Effective operator contributions to the oblique parameters

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Abstract

We present a model and process independent study of the contributions from non-Standard Model physics to the oblique parameters $S$, $T$ and $U$. We show that within an effective lagrangian parameterization the expressions for the oblique parameters in terms of observables are consistent, while those in terms of the vector-boson vacuum polarization tensors are ambiguous. We obtain the constraints on the scale of new physics derived from current data on $S$, $T$ and $U$ and note that deviations in $U$ from its Standard Model value would favor a scenario where the underlying physics does not decouple.

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The oblique parameters $S$, $T$ and $U$ \cite{1} are known to be sensitive probes of non-Standard Model physics. Because of this they are often used in deriving bounds on the scale (and other properties) of new physics from the existing and expected experimental bounds \cite{2}. These parameters are often defined in terms of the vector-boson vacuum polarization tensors \cite{1}, but a practical definition requires them to be expressed in terms of direct observables \cite{3}. Thus, whenever the contributions from new interactions to the oblique parameters are calculated, the modifications to all observable quantities involved should be included.

When dealing with specific models the calculation of the oblique parameters is a straightforward exercise. In contrast, when considering the same calculation using a model-independent (effective Lagrangian) parameterization of the new-physics effects, some subtleties arise. The reason is that the effective Lagrangian parameterization is not unique in the sense that one can change the effective Lagrangian without affecting the $S$ matrix \cite{4}, yet these modifications do alter the vector boson vacuum polarization and, with this, the corresponding definition of the oblique parameters. Because of this the definition of $S$, $T$ and $U$ in terms of the the vector-boson vacuum polarization tensors is ambiguous.

We will show that this problem can be avoided by defining $S$, $T$ and $U$ in terms of observables (which unfortunately is seldom the case \cite{2,4}). We will obtain the complete expressions for the contributions from non-Standard Model physics to the oblique parameters within an effective Lagrangian parameterization. From these expressions unambiguous limits on the scale of new physics can be derived. Finally we will also argue that accurate measurements of the $U$ parameter will provide information on whether the heavy physics decouples.

Within the Standard Model the oblique parameters vanish at tree-level, but they are non-zero at one loop \cite{3}: new physics will also, in general, generate non-vanishing contributions \cite{4}. To lowest order we expect $S = \delta_{\text{rad}} S + \Delta S$ (with similar expressions for $T$ and $U$), where $\delta_{\text{rad}} S$ denotes the radiative Standard Model contributions and $\Delta S$ the contributions generated by the heavy physics. The quantities $\delta_{\text{rad}}(S,T,U)$ are well known and have been studied extensively \cite{4}. In this paper we concentrate on $\Delta(S,T,U)$ keeping in mind that these quantities denote the deviations from the Standard Model predictions with radiative
corrections included; in calculating $\Delta S$, $\Delta T$, $\Delta U$ we will ignore all Standard Model loop effects.

The oblique parameters can be expressed \cite{3} in terms of the fine-structure constant $\alpha$ (measured at the $Z$ mass), the vector boson masses $M_Z$ and $M_W$, the Fermi constant $G_F$, and the width $\Gamma(Z \to \ell^+\ell^-)$ and forward-backward asymmetry, $A_{FB}(Z \to \ell^+\ell^-)$, for the decay of the $Z$ into charged leptons. From $A_{FB}$ we obtain $g_V/g_A$ (the vector-coupling to axial-coupling ratio of the $Z$ to the charged leptons); with this result and using the other observables the oblique parameters are obtained from

\[
\Gamma(Z \to \ell\ell) = \frac{G_F M_Z^3}{24\sqrt{2} \pi} \left(1 + \frac{g_V^2}{g_A^2}\right)(1 + \alpha T),
\]

\[
\frac{M_W^2}{M_Z^2(1 - s_0^2)} = 1 + \frac{1 - s_0^2}{1 - 2s_0^2} \alpha T - \frac{\alpha S}{2(1 - 2s_0^2)} + \frac{\alpha U}{4s_0^2},
\]

where

\[
s_0^2(1 - s_0^2) = \frac{\pi \alpha}{\sqrt{2} G_F M_Z^2}.
\]

To obtain the heavy physics contributions to the oblique parameters it is then necessary to determine the contributions to the observables used in the above definitions. In this paper we will use an effective Lagrangian parameterization of the heavy physics \cite{6,8} which has the advantage of being model and process independent. The detailed form of the effective Lagrangian depends crucially on the low energy spectrum, we will include three families of fermions as well as the usual Standard Model gauge bosons. For the scalars we will consider two possibilities: in the first, which we label the linear case \cite{9}, we assume a single light scalar doublet; in the second, which we call the chiral case \cite{10}, we assume that there are no light physical scalars. In both cases we denote the scale of new physics by $\Lambda$.

For the linear case the part of the effective Lagrangian which contributes to the oblique parameters takes the form \cite{9,11}

\[
L_{\text{eff}} = \frac{1}{\Lambda^2} \sum_i b_i O_i + O\left(\frac{1}{\Lambda^3}\right)
\]
where

\[ O_{\phi W} = \frac{1}{2} \left( \phi^\dagger \phi \right) W_{\mu \nu}^I W^{I \mu \nu} \quad O_{\phi B} = \frac{1}{2} \left( \phi^\dagger \phi \right) B_{\mu \nu} B^{\mu \nu} \quad O_{W B} = \left( \phi^\dagger \tau^I \phi \right) W_{\mu \nu}^I B^{\mu \nu} \]

\[ O_{\phi (1)} = \left( \phi^\dagger \phi \right) \left[ (D_{\mu} \phi)^\dagger D^\mu \phi \right] \quad O_{\phi (3)} = \left( \phi^\dagger D^\mu \phi \right) \left[ (D_{\mu} \phi)^\dagger \phi \right] \quad O_{\phi (1)}^{(1)} = i \left( \phi^\dagger D^\mu \phi \right) \left( \bar{\ell} \gamma_{\mu} \ell \right) \]

\[ O_{\phi (3)}^{(3)} = \frac{i}{2} \left( \bar{\ell} \tau^I \gamma_{\mu} \ell \right) \left( \bar{\ell} \tau^J \gamma_{\mu} \ell \right) \]

(4)

The coefficients \( b_i \) parameterize all heavy physics contributions to the oblique parameters. The choice of operators is, however, not unique [4]; we will discuss this issue below.

When there are no-light scalars (chiral case) the effective Lagrangian can be obtained from (3) by replacing

\[ \phi \rightarrow \phi_{\text{chir}} = \Sigma \begin{pmatrix} 0 \\ v \end{pmatrix} ; \quad \Sigma^\dagger \cdot \Sigma = 1 \tag{5} \]

where \( v \simeq 246 \text{GeV} \). In this case the effects of the operators \( O_{\phi W}, O_{\phi B} \) and \( O_{\phi (1)}^{(1)} \) can be absorbed by an appropriate renormalization of the Standard Model parameters. Therefore, in the chiral case we set \( b_{\phi W} = b_{\phi B} = b_{\phi (1)}^{(1)} = 0 \). A consistent expansion of the chiral effective Lagrangian [3] requires that we include all operators if \( f \) fermion fields and \( d \) derivatives are such that \( d + f/2 \leq 4 \), hence we must also include the operator [12]

\[ O_{W W} = \frac{1}{2} \left( \phi_{\text{chir}}^\dagger \tau^I \phi_{\text{chir}} \right) \left( \phi_{\text{chir}}^\dagger \tau^J \phi_{\text{chir}} \right) W_{\mu \nu}^I W^{I \mu \nu} \tag{6} \]

---

1 We use the following conventions [11]: \( I, J, K \) denote \( SU(2) \) indices, the Pauli matrices are denoted by \( \tau^I \); \( W_{\mu}^I \) and \( B_{\mu} \) denote the \( SU(2) \) and \( U(1) \) gauge fields, and \( W_{\mu \nu}^I \) and \( B_{\mu \nu} \) the corresponding curvatures; the gauge coupling constants are denoted by \( g \) and \( g' \) respectively. Left-handed quark and lepton doublets are denoted by \( q \) and \( \ell \) respectively; right-handed up and down-type quarks correspond to \( u \) and \( d \), while the right-handed charged lepton corresponds to \( e \); all fermion fields have implicit family indices. The scalar doublet is denoted by \( \phi \) and the covariant derivative by \( D_{\mu} \). The scalar vacuum expectation value is denoted by \( v \) defined so that \( v \simeq 246 \text{GeV} \).

2 This is a generalization of the derivative expansion when fermions are present.
(which does not appear in (4) since in the linear case it corresponds to a dimension 8 operator which will generate subdominant contributions to the oblique parameters).

Using (3, 4) (together with (5, 6) in the chiral case) we obtain the heavy physics contributions to the $Z$ couplings and mass, the $W$ mass, $\alpha$ and $G_F$. We first provide the expressions in terms of the $SU(2)$ and $U(1)$ gauge coupling constants, $g$ and $g'$ respectively, and the vacuum expectation value $v$; and then express these in terms of direct observables.

We consider first the case where there is a single light scalar doublet (the linear case). The $Z$ axial and vector couplings to the charged leptons equal, respectively

$$g_A = g_R - g_L, \quad g_V = g_R + g_L,$$

where

$$g_L = -\frac{1}{2} + x - \frac{1}{2} \left[(1-x)(1+2x)b_{\phi W} - (x + 2y^2)b_{\phi B} - 2y(1-2x)b_{W B} + b^{(1)}_{\phi e} + b^{(3)}_{\phi e}\right] \epsilon + O(\epsilon^2);$$

$$g_R = x - \frac{1}{2} \left[b_{\phi e} + 2y^2b_{\phi W} - 2x(2-x)b_{\phi B} - 4y(1-x)b_{W B}\right] \epsilon + O(\epsilon^2);$$

and

$$\epsilon = \frac{1}{2} \frac{v^2}{\Lambda^2}, \quad x = \frac{g'^2}{g^2 + g'^2}, \quad y = \frac{g'}{g^2 + g'^2}.$$

For the remaining observables we find

$$M_W = \frac{g v}{2} \left[1 + \left(b_{\phi W} + \frac{1}{2}b^{(1)}_{\phi}\right) \epsilon \right] + O(\epsilon^2);$$

$$M_Z = \frac{g v}{2\sqrt{1-x}} \left[1 + \left[xb_{\phi B} + (1-x)b_{\phi W} + 2yb_{W B} + b^{(1)}_{\phi} + \frac{1}{2}b^{(3)}_{\phi}\right] \epsilon \right] + O(\epsilon^2);$$

$$G_F = \frac{1}{\sqrt{2}} \frac{v^2}{v^2} \left[1 + \left(2b^{(3)}_{\ell \ell} + 4b^{(3)}_{\phi \ell} - b^{(1)}_{\phi} - 4b_{\phi W}\right) \epsilon \right] + O(\epsilon^2);$$

$$\alpha = \frac{g^2 x}{4\pi} \left[1 + 2 \left[xb_{\phi W} + (1-x)b_{\phi B} - 2yb_{W B}\right] \epsilon \right] + O(\epsilon^2);$$

which can be used to express $x, g, \epsilon, \alpha, g'$, etc. in terms of observables. These expressions do not contain the Standard Model radiative corrections since, as discussed above, we expect the corresponding contributions to the oblique parameters to be additive and we are interested only in the contributions generated by the heavy physics. Substituting (7-10) into (1) we get
\[ \Delta_{\text{lin}} T = -\frac{4\pi}{g^2 x} \left( 2b_{\ell\ell}^{(3)} + 2b_{\phi\ell}^{(3)} - 2b_{\phi\ell}^{(1)} + b_{\phi}^{(3)} + 2b_{\phi e} - 4b_{\phi W} \right) \epsilon + O(\epsilon^2), \]
\[ \Delta_{\text{lin}} S = \frac{8\pi}{g^2 x} \left[ -b_{\phi e} + 2x \left( b_{\phi\ell}^{(1)} + b_{\phi\ell}^{(3)} \right) + 4yb_{WB} \right] \epsilon + O(\epsilon^2), \]
\[ \Delta_{\text{lin}} U = \frac{16\pi}{g^2} \left( 2b_{\ell\ell}^{(3)} + 2b_{\phi\ell}^{(3)} - 2b_{\phi\ell}^{(1)} + b_{\phi e} + 2b_{\phi W} \right) \epsilon + O(\epsilon^2). \] 

(11)

In the chiral case, using (5) and (6) we obtain
\[ \Delta_{\text{chir}} T = -\frac{4\pi}{g^2 x} \left( 2b_{\ell\ell}^{(3)} + 2b_{\phi\ell}^{(3)} - 2b_{\phi\ell}^{(1)} + b_{\phi}^{(3)} + 2b_{\phi e} - 4b_{\phi W} \right) \epsilon + O(\epsilon^2), \]
\[ \Delta_{\text{chir}} S = \frac{8\pi}{g^2 x} \left[ -b_{\phi e} + 2x \left( b_{\phi\ell}^{(1)} + b_{\phi\ell}^{(3)} \right) + 4yb_{WB} \right] \epsilon + O(\epsilon^2), \]
\[ \Delta_{\text{chir}} U = \frac{16\pi}{g^2} \left( 2b_{\ell\ell}^{(3)} + 2b_{\phi\ell}^{(3)} - 2b_{\phi\ell}^{(1)} + b_{\phi e} - 2b_{WW} \right) \epsilon + O(\epsilon^2). \] 

(12)

where \( b_{WW} \) is the coefficient of \( O_{WW} \) in (3) (which has dimension 4).

The above expressions can be re-written in terms of observables using the tree-level relations
\[ g^2 = 4\sqrt{2} G_F M_W^2, \quad x = 1 - \left( M_W/M_Z \right)^2, \quad \epsilon = \frac{1}{\sqrt{8} G_F \Lambda^2}. \] 

(13)

These expressions were obtained using the effective Lagrangian (3, 4), together with (5, 6) in the chiral case. But it is well known [4] that there is no unique choice of operators in an effective Lagrangian parameterization. Given two operators \( O_1 \) and \( O_2 \) such that \( O_1 - O_2 \) vanishes when the classical equations of motion are imposed, then the term \( b_1 O_1 + b_2 O_2 \) in the effective Lagrangian generates modifications to the S matrix which depend only on \( b_1 + b_2 \) [4], but not on \( b_1 \) and \( b_2 \) independently.

Our expressions for \( \Delta S, \Delta T \) and \( \Delta U \) satisfy this property. As an example consider the operator
\[ O_{DW} = (D_\mu W_{\nu\rho})^I (D^\mu W^{\nu\rho})^I, \] 

(14)

which, up to terms which vanish when the classical equations of motion are imposed, satisfies
\[ O_{DW} = 2gO_W + \frac{g^2}{2} \left[ 6O_{\phi}^{(1)} + 2m^2 \left( \phi^I \phi \right)^2 - 6\lambda O_{\phi} + 4O_{\phi\ell}^{(3)} + 4O_{\phi q}^{(3)} + 2O_{\ell\ell}^{(3)} + 2O_{\ell q}^{(3)} + 2O_{qq}^{(3)} \right] \] 

(15)
where \( m \) denotes the scalar mass, \( \lambda \) the scalar self-coupling and where the operators not defined in (11) are
\[
\mathcal{O}_\phi = \frac{1}{3} \left( \phi^\dagger \phi \right)^3 \quad \mathcal{O}^{(3)}_{\phi q} = \frac{1}{2} \left( \phi^\dagger \tau^I D^\mu \phi \right) \left( \bar{q} \gamma^\mu q \right) \\
\mathcal{O}^{(3)}_{\ell q} = \frac{i}{2} \left( \bar{\ell} \gamma^\mu (\bar{q} \gamma^\mu q) \right) \quad \mathcal{O}^{(3)}_{qq} = \frac{1}{2} \left( \bar{q} \gamma^\mu q \right) \left( \bar{q} \gamma^\mu q \right) \\
\mathcal{O}_W = \varepsilon_{IJK} W_\mu^{I\nu} W_\nu^{J\lambda} W_\lambda^{K\mu}.
\]
It then follows that the replacement
\[
\mathcal{L}_{\text{eff}} \rightarrow \mathcal{L}_{\text{eff}} + b_{DW} \mathcal{O}_{DW}/\Lambda^2
\]
is equivalent to \( b_i \rightarrow b_i + \delta b_i \) where
\[
\frac{1}{3} \delta b_{(1)}^\phi = -\frac{1}{3} \delta b_\phi = \frac{1}{2} \delta b_{(3)}^{\ell q} = \frac{1}{2} \delta b_{(3)}^{\ell \ell} = \delta b_{(3)}^{\ell q} = \delta b_{(3)}^{qq} = \frac{1}{\lambda \epsilon} \delta \lambda = b_{DW} g^2
\]
which, in fact, leave \( \Delta(S, T, U) \) invariant.

This result can also be obtained without using the equations of motion on \( \mathcal{O}_{DW} \). Adding a term \( b_{DW} \mathcal{O}_{DW}/\Lambda^2 \) to the effective Lagrangian generates a quadratic term in the vector bosons, \( b_{DW} W_\mu^I \partial^2 W^I \). When the quadratic part of the vector-boson Lagrangian is re-diagonalized the \( W \) and \( Z \) masses and the vacuum expectation value \( v \) are modified, \( \delta M_W^2/M_W^2 = \delta M_Z^2/M_Z^2 = \delta v/v = g^2 b_{DW} \epsilon \). The Fermi constant is unaffected, \( \delta G_F = 0 \), and the coupling of the \( Z \) to the left-handed fermionic current \( J_L \) becomes \( -(1 + g^2 b_{DW} \epsilon) \sqrt{g^2 + g'^2} J_L \cdot Z \). It is a tedious exercise (for which we used Ref. [11] after correcting a few typographical errors) to show that these modifications correspond to (18). This illustrates the fact that [11] are consistent definitions of the heavy physics to the oblique parameters.

In contrast, the naive definition of the oblique parameters in terms of the vacuum polarization tensors, are not invariant under the replacement (17). Indeed, using an \( SU(2) \times U(1) \) basis,
\[
S_{\text{vac.pol.}} = -\frac{8\pi}{M_Z^2} \left[ \Pi_{3Y} (M_Z^2) - \Pi_{3Y}(0) \right] \\
T_{\text{vac.pol.}} = \frac{16\pi}{\sin^2(2\theta_W) M_Z^2} \left[ \Pi_{11}(0) - \Pi_{33}(0) \right] \\
U_{\text{vac.pol.}} = \frac{16\pi}{M_W^2} \left[ \Pi_{11} (M_W^2) - \Pi_{11}(0) \right] - \frac{16\pi}{M_Z^2} \left[ \Pi_{33} (M_Z^2) - \Pi_{33}(0) \right]
\]

(19)
from which, using (3), we obtain

\[ \Delta_{\text{lin}} T_{\text{vac.pol.}} = -\frac{4\pi}{g^2 x} \alpha^{(3)}_\phi \epsilon + O(\epsilon^2), \quad \Delta_{\text{lin}} S_{\text{vac.pol.}} = \frac{32\pi y}{g^2 x} b_{W} \epsilon + O(\epsilon^2), \quad \Delta_{\text{lin}} U_{\text{vac.pol.}} = O(\epsilon^2), \]

(20)

but in this case (17) does not leave \( U_{\text{vac.pol.}} \) invariant,

\[ \Delta_{\text{lin}} U_{\text{vac.pol.}} \rightarrow 16\pi g^2 b_{DW} \epsilon + O(\epsilon^2) \]

(21)

which illustrates the importance of using the definitions (1) for the oblique parameters.

It must be noted that the operator \( O_{DW} \) generates a \( p^4 \) contribution to the vacuum polarizations \( \Pi(p) \), and within the linear approximation (14) in \( p^2 \), this operator will not affect the oblique parameters. This does not mean that the effective lagrangian contributions to \( S_{\text{vac.pol.}}, T_{\text{vac.pol.}}, \) and \( U_{\text{vac.pol.}} \) within the linear approximation are unambiguous. Consider, for example, the operator \( (\phi^\dagger D^\mu \phi) \partial^\nu B_{\nu\mu} \) which contributes to \( \Delta_{\text{lin}} S_{\text{vac.pol.}} \); using the equations of motion this operator is equivalent to \( (i g'/2) \left( 2 O^{(3)}_\phi + O^{(1)}_\phi \right) \) — plus a string of operators involving fermions which do not contribute to (19) — and \( O^{(3)}_\phi \) contributes to \( \Delta_{\text{lin}} T_{\text{vac.pol.}} \) only. In contrast, the definitions (1) present no such ambiguity and should be used whenever an effective Lagrangian computation is performed.

Using (11) or (12) and currently available data we can derive limits on the scale of new physics \( \Lambda \). The operators \( O^{(1,3)}_{\phi\ell} \) and \( O_{\phi e} \) modify the \( Z \) coupling to the fermions and the corresponding coefficients can be bounded using data from LEP1 (13),

\[ \epsilon |b^{(1,2)}_{\phi\ell}| < 0.0016, \quad \epsilon |b_{\phi e}| < 0.0014; \]

(22)

the operator \( O^{(3)}_{\ell\ell} \) contributes to \( e^+e^- \to \mu^+\mu^- \) and it coefficient can be correspondingly bounded (14),

\[ -0.105 < \epsilon b^{(3)}_{\ell\ell} < 0.056. \]

(23)

\(^3\)There are many other operators that contribute to this reaction, we assume there are no significant cancelations
Finally the limits on the oblique parameters are

\[-0.0414 < \frac{g^2 x}{8\pi} S < 0.0060, \quad -0.0875 < \frac{g^2 x}{4\pi} T < 0.0951, \quad -0.0072 < \frac{g^2}{16\pi} U < 0.0020.\]  

(24)

Note however that the operators \( O_{\ell\ell}^{(3)} \), \( O_{\phi\ell}^{(3)} \) and \( O_{\phi W} \) also contribute to \( G_F \) and this can be used to impose better bounds on the corresponding coefficients (again assuming no cancellations). Using \( G_F, \alpha \) and \( M_Z \) as input parameters the uncertainty in the predictions of \( M_W \) requires \( \epsilon |b_{\ell\ell}^{(3)}|, \epsilon |b_{\phi\ell}^{(3)}|, \epsilon |b_{\phi W}| \lesssim 5 \times 10^{-4} \). In the linear case we then have, to a good approximation,

\[
\Delta_{\text{lin}} T \approx -\frac{4\pi}{g^2 x} (-2b_{\phi\ell}^{(1)} + b_{\phi}^{(3)} + 2b_{\phi e}) \epsilon + O(\epsilon^2),
\]

\[
\Delta_{\text{lin}} S \approx \frac{8\pi}{g^2 x} (-b_{\phi e} + 2xb_{\phi\ell}^{(1)} + 4yb_{W B}) \epsilon + O(\epsilon^2),
\]

\[
\Delta_{\text{lin}} U \approx \frac{16\pi}{g^2} (-2b_{\phi\ell}^{(1)} + b_{\phi e}) \epsilon + O(\epsilon^2).
\]

(25)

In the chiral case,

\[
\Delta_{\text{chir}} T = -\frac{4\pi}{g^2 x} (-2b_{\phi\ell}^{(1)} + b_{\phi}^{(3)} + 2b_{\phi e}) \epsilon + O(\epsilon^2),
\]

\[
\Delta_{\text{chir}} S = \frac{8\pi}{g^2 x} (-b_{\phi e} + 2xb_{\phi\ell}^{(1)} + 4yb_{W B}) \epsilon + O(\epsilon^2),
\]

\[
\Delta_{\text{chir}} U = \frac{16\pi}{g^2} (-2b_{\phi\ell}^{(1)} + b_{\phi e} - 2b_{W W}) \epsilon + O(\epsilon^2).
\]

(26)

Using these expressions and the above experimental constraints we find the following bounds,

\[
\epsilon |b_{\phi}^{(3)}| \lesssim 0.1, \quad \epsilon |b_{W B}| \lesssim 0.02, \quad \epsilon |b_{W W}| \lesssim 0.006,
\]

(27)

(where the last is relevant only for the chiral case).

In the linear case the natural size [15] for the coefficients are \( |b_{\phi}^{(3)}| \lesssim 1 \) and \( |b_{W B}| \lesssim gg'/(4\pi)^2 \). The limit on \( b_{\phi}^{(3)} \) implies \( \Lambda \gtrsim 550 \text{GeV} \) while, from \( b_{W B} \), \( \Lambda \gtrsim 50 \text{GeV} \). This disparity is due to the fact that \( O_{\phi}^{(3)} \) can be generated at tree level by the heavy physics, while \( O_{WW} \) is necessarily loop generated [15]. The 550GeV limit refers to the mass of a heavy scalar or vector boson whose interactions violate the custodial symmetry [16].
For the chiral case the natural sizes \([14]\) are \(|b^{(3)}_\phi| \lesssim 1, |b_{WB}| \lesssim g g',\) and \(|b_{WW}| \lesssim g^2;\) moreover we also have \(\Lambda \sim 4\pi v \sim 3\text{TeV}\) so that \(\epsilon \lesssim 1/(4\pi)^2.\) The above limits are not sufficiently precise to provide useful information in this case; for example, the limit on \(b_{WW}\) implies \(\Lambda \gtrsim 1.5\text{TeV}.\)

Finally we note a peculiarity of the parameter \(U:\) the heavy physics contributions generated by \(O^{(1)}_{\phi\ell}\) and \(O_{\phi e}\) were measured at LEP1 and are known to be small; this means that in the linear case current data implies \(\Delta U \sim 0 (U \sim \epsilon^2 \lesssim 0.01 \text{ for } \Lambda > 550\text{GeV}).\) In contrast there are no severe bounds on the contributions generated by \(O_{WW};\) in the chiral case we therefore have \(|\Delta_{\text{chir}} U| \sim 32\pi|b_{WW}|\epsilon \lesssim 2/\pi.\) Should a future measurement produce a deviation of order 0.1 in the measurement of \(U,\) this observation would not only indicate the presence of new physics, but would strongly disfavor the existence of light Higgs-like scalars. Note, however, that a bound \(\Delta U \lesssim 0.1\) does \textit{not} imply the presence of light scalars since this could also occur within the chiral case for a sufficiently large \(\Lambda.\)

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