p-wave holographic superconductors with Weyl corrections

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Abstract – We study the (3+1)-dimensional p-wave holographic superconductors with Weyl corrections both numerically and analytically. We describe numerically the behavior of the critical temperature $T_c$ with respect to charge density $\rho$ in a limited range of Weyl coupling parameter $\gamma$ and we find in general that the condensation becomes harder with the increase of the parameter $\gamma$. In the strong-coupling limit of Yang-Mills theory, we show that the minimum value of $T_c$ obtained from the analytical approach is in good agreement with the numerical results, and finally show how we got remarkably a similar result for the critical exponent $\frac{1}{2}$ of the chemical potential $\mu$ and the order parameter $\langle J_1^x \rangle$ with the numerical curves of the superconductors.

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Introduction. – The strongly coupled field theories and superconductor phase transitions have been studied by the powerful theoretical methods of Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence linking a d-dimensional strongly coupled conformal field theory on the boundary to a (d+1)-dimensional weakly coupled dual gravitational description in the bulk [1–3]. Coupling of a complex scalar field with an Einstein-Maxwell theory is necessary to explain the simplest model for holographic superconductors. The key point of holographic superconductors is that, in the gravity theory, a black hole coupled with matter fields will have symmetry-breaking solutions. More specifically, below a critical temperature, the gauge symmetry breaks and a black hole is constructed by the unstable developing scalar hair near the horizon. According to the AdS/CFT correspondence, the complex scalar field is dual to a charged operator at the boundary, therefore a superconductor phase transition will occur in both the $U(1)$ symmetry breaking in the gravity and the global $U(1)$ symmetry breaking in the dual boundary theory [4].

Holographic superconductors have been properly considered in two different models with different matter sectors. The first one is an Abelian-Higgs model, which is the gravity dual of an s-wave superconductor with a scalar order parameter. This model has been studied by many authors see [2,3,5–13]. The other type is an Einstein-Yang-Mills (EYM), $SU(2)$ gauge theory in which the condensate carries angular momentum [14–21].

Gubser [22] suggested that solutions spontaneously breaking the Abelian gauge symmetry via a charged complex scalar condensate near the horizon of the black hole can be obtained by coupling the Abelian-Higgs model to gravity with a negative cosmological constant. Hartnoll et al. [3] investigated further this Abelian-Higgs model of superconductivity, and according to AdS/CFT correspondence constructed s-wave holographic superconductors solutions with a scalar order parameter displaying the phase transition process at the critical temperature $T_c$ below which the charge condensates forms. The EYM model of holographic superconductors was constructed later by Gubser [23], where spontaneous symmetry-breaking solutions through a condensate of non-Abelian gauge fields is presented, and also p-wave and $(p+ip)$-wave backgrounds have been studied by Gubser and Pufo [14], where the CFT has a global $SU(2)$ symmetry and hence three conserved currents $J_{\mu}^a (a = 1, 2, 3$ label the generators of $SU(2))$. All of these works are based on the numerical analysis of the equation of motion (EOM) by a suitable shooting method in the
asymptotic limit. The first effort on analytic methods in this topic was Herzog’s work [24], where the critical exponent and the expectation values of the dual operators were attained.

Analytical studies of superconductors have been established according to major methods: the small parameter perturbation theory as in [24], and the variational method [25,26]. In the present paper, we study the Weyl-corrected $p$-wave holographic superconductor composed of a non-Abelian SU(2) gauge field (the matter sector) and a black-hole background (the gravity sector) by using the variational method giving only the critical temperature $T_{c}^{Min}$. As has been mentioned in [27], for an Abelian gauge field and large range of the Weyl coupling value, $\frac{1}{16} < \gamma < \frac{1}{24}$, a universal relation for the critical temperature, $T_{c} \approx \sqrt{\frac{\pi}{\gamma}}$, has been found. In this paper we have explored the same validity of the critical-temperature relation in a non-Abelian gauge field with Weyl correction numerically and analytically.

The organization of this paper is as follows. In the second section we reconstruct the Weyl-corrected superconductor’s solution of the EYM theory, which is dual to a $p$-wave superconductor. In the third section we present numerical results for condensation and the critical temperature of the holographic superconductor. Then we analytically investigate the behavior of the critical temperature $T_{c}$ and the dual of the chemical potential $\mu$ with respect to the Weyl coupling parameter $\gamma$, the dual of charge density $\rho$, and the order parameter $\langle J_{z}^{4} \rangle$ in the next section. The conclusion and some discussion are given in the fifth section.

Weyl-corrected $p$-wave superconductors. – In this section we study the holographic phase transition phenomena for the probe $SU(2)$ Yang-Mills (YM) field $A_{\mu}^{a}$ in a five-dimensional spacetime. The bulk action of the Weyl gravity with an $SU(2)$ Yang-Mills field in a five-dimensional spacetime is

$$ S = \int d^{5}x \sqrt{-g} \left\{ \frac{1}{16\pi G_{5}} (R + 12) - \frac{1}{4g^{2}} F_{\mu\nu}^{a} F^{\mu\nu}_{a} + \gamma C_{\mu\nu\rho\sigma} F_{\mu\nu}^{a} F_{\rho\sigma}^{a} \right\} f. \tag{1} $$

Here $G_{5}$ is the gravitational constant, $g$ is the Yang-Mills coupling constant, and the negative cosmological constant is satisfied by the factor 12 in the first parenthesis. The field strength component is given as below, where $A_{\mu}^{a}$’s are the non-commutative corresponding to a non-Abelian gauge field:

$$ F_{\mu\nu}^{a} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + \varepsilon^{abc} A_{\mu}^{b} A_{\nu}^{c}, \quad \{a = 1, 2, 3, \mu = 0, 1, 2, 3\}. \tag{2} $$

Weyl’s coupling $\gamma$ is limited such that its value is in the interval $-\frac{1}{16} < \gamma < \frac{1}{24}$, more precise details have been discussed in [28]. In the probe limit, we neglect the back reactions and in this case, the gravity sector is effectively decoupled from the matter field’s sector. In this probe limit, the metric is given by an AdS-Schwarzschild black hole:

$$ ds^{2} = r^{2}(- f dr^{2} + dx^{2} + dx_{i} dx_{i}) + \frac{dr^{2}}{r^{2} f}, \tag{3} $$

where

$$ f = 1 - \frac{T_{c}}{r^{4}}, \tag{4} $$

and the black-hole horizon locates at $r = r_{+}$. The Hawking temperature of the black hole is determined by the Schwarzschild radius as $T = \frac{\pi}{2r_{+}}$, which is the temperature of the conformal field theory on the boundary of the AdS spacetime. Applying the Euler-Lagrange equation, we can derive the generalized Yang-Mills equation as [27]

$$ \nabla_{\mu} (F^{a}_{\mu\nu} - 4\gamma C^{a\rho\sigma\tau} F^{a}_{\rho\sigma\tau}) = -\epsilon_{abc} A_{\mu}^{b} F^{c\mu\nu} + 4\gamma C^{a\rho\sigma\tau} \epsilon_{abc} A_{\mu}^{b} F^{c}_{\rho\sigma\tau}, \tag{5} $$

where $C^{a\rho\sigma\tau}$ is the Weyl tensor and has the following nonzero components in AdS5:

$$ C_{0}^{0} = f(r) r^{4} \delta_{ij}, \quad C_{0}^{r0} = - \frac{3r^{4}}{r^{2}}, \quad C_{r}^{i} = - \frac{r^{4}}{r^{2}} \delta_{ik} \delta_{ij}. \tag{6} $$

Usually the realization of a holographic $p$-wave superconductor is prepared with the following anzatz for Yang-Mills gauge field [29]:

$$ A = \varphi(r) \sigma^{3} dt + \psi(r) \sigma^{1} dx. \tag{7} $$

Here $\sigma^{i}$ are the three $SU(2)$ generators. The condensation of $\psi(r)$ will break the $SU(2)$ symmetry and lead to the superconductor phase transition. As we were taught from Gauss-Bonnet superconductors [30], the gauge function $\psi(r)$ is dual to the $J_{z}^{4}$ operator on the boundary; by choosing the $x$-axis as a special direction, the condensation of $\psi(r)$ breaks the rotational symmetry and leads to a phase transition, which can be interpreted as a $p$-wave superconductor phase transition on the boundary. The resulting Yang-Mills equations for metric (3) are given by

$$ \begin{align*}
(1 - \frac{24 \gamma r^{4}}{r^{4}}) \varphi'' + (\frac{3}{\gamma} + \frac{24 \gamma r^{4}}{r^{4}}) \varphi' - (1 + \frac{8 \gamma r^{4}}{r^{4}}) \psi^{2} \varphi f &= 0, \tag{8} \\
(1 - \frac{8 \gamma r^{4}}{r^{4}}) \psi'' + (\frac{3}{\gamma} + \frac{8 \gamma r^{4}}{r^{4}}) \left( \frac{1}{r} f^{'} + \frac{8 \gamma r^{4}}{r^{4}} \right) \psi' + (1 + \frac{8 \gamma r^{4}}{r^{4}}) \frac{\varphi^{2}}{r^{2} f^{2}} &= 0, \tag{9} \end{align*} $$

where the prime represents derivative with respect to $r$. It is more convenient to work in terms of the dimensionless
parameter $z = \frac{r_c}{r}$, in which at the horizon $z = 1$, and the boundary at the infinity locates at $z = 0$. Then the equations of motion (EOMs) (8) and (9) can be re-expressed as

$$\begin{align*}
(1 - 24\gamma z^4) \varphi'' - \frac{1}{z} (1 + 72\gamma z^4) \varphi' &+ \left(1 + 8\gamma z^4 \right) \psi^2 \varphi = 0, \\
(1 - 8\gamma z^4) \psi'' + \left[ - \frac{1}{z} + \frac{f'}{f} - 8\gamma z^4 \left( \frac{3}{z} + \frac{f'}{f} \right) \right] \psi' &+ \left(1 + 8\gamma z^4 \right) \frac{\varphi^2 \psi}{f^2} = 0,
\end{align*}$$

(10)

where the prime now denotes derivative with respect to $z$. The boundary conditions at infinity, i.e. $z \to 0$, are

$$\begin{align*}
\varphi &\approx \mu - \rho z^2, \\
\psi &\approx \psi^{(0)} + \psi^{(2)} z^2,
\end{align*}$$

(12)

(13)

$\mu$ and $\rho$ are dual to the chemical potential and charge density of the CFT boundary, $\psi^{(0)}$ and $\psi^{(2)}$ are dual to the source and expectation value of the boundary operator $J^1_z$, respectively. Further to have a normalizable solution, we always set the source $\psi^{(0)}$ to zero.

**Numerical treatment.** – According to construction of the (3+1)-dimensional Weyl-corrected $p$-wave superconductor in the previous section, in this section we will present numerical results for the condensation and critical temperature due to the shooting method. From EOMs (10) and (11) and the asymptotic behavior of $\psi$ and $\varphi$ at infinity (12) and (13), we can obtain the regularity condition as

$$\varphi(1) = \varphi'(0) = \psi'(0) = \psi'(1) = 0$$

(14)

combining boundary condition (12) and (13) with regularity condition (14), we can solve EOMs (10) and (11) numerically by using a shooting method, and plot fig. 1 to demonstrate the condensation as a function of temperature for the operator $(J^1_z)$. The curve in fig. 1 is qualitatively similar to that obtained in the holographic superconductors $[27,31]$, where the condensation of $(J^1_z)$ goes to a constant at zero temperature. As we can see from fig. 1, it is easy to find that the critical temperatures of Weyl-corrected superconductors is increasing as the parameter $\gamma$ varies in the range of $-0.06$ to $0.04$, therefore we conclude that when $\gamma < 0$ the critical temperature is smaller and the formation of the scalar hair is harder and vice versa when $\gamma > 0$.

We have also presented the critical temperature $T_c$ with different values of the parameter $\gamma$ in table 1. According to the results in table 1, we can conclude that by minimizing the coupling parameter $\gamma$, the critical temperature decreases smoothly.

**Analytical treatment.** – In this section we compute the critical temperature and critical exponent via an analytical method, which has been proposed recently $[25]$. In this method by defining appropriate equation matching with field’s boundary conditions, the field’s EOM will be transformed into the Sturm-Liouville form. Therefore according to the general variational method to solve the Sturm-Liouville problem $([32] \text{ or appendix of } [25])$, the eigenvalue $\lambda^2$ minimizing the Sturm-Liouville equation can be found. Using this minimum value of $\lambda$, one can obtain the minimum critical temperature $T_c^{Min}$. In this section we calculate this $T_c^{Min}$ and discuss the critical exponent.

**Critical temperature $T_c^{Min}$.** Considering the nonlinear system (10), (11). If there is a second-order continuous phase transition at the critical temperature, the solution of the EOMs at the $T_c$ should be

$$\psi(z) = 0, \quad \varphi(z) = \lambda h_c(1 - z^2).$$

(15)

Here $\lambda = \frac{\lambda \gamma}{\hbar}$, $h_c$ is the radius of the horizon corresponding to $T = T_c$. At a temperature slightly below $T_c$, the EOM for $\psi$ becomes

$$
\begin{align*}
z^2 &\frac{d}{dz} \left( \frac{(1 - z^4)}{2 g^2 z^3} \frac{d \psi}{dz} \right) + \lambda^2 \left[ \frac{h_c^2}{2 g^2} + 4\gamma z^5 \right] \frac{1 - z^4}{1 + z^4} \psi = 0.
\end{align*}

(16)

It is appropriate to define

$$\psi(z) = \frac{\langle J^1_z \rangle}{\hbar} z^2 F(z).$$

(17)

Matching the boundary condition at the boundary $z = 0$, we normalize the function as $F(0) = 1, F'(0) = 0$. The equation for $F(z)$ is

$$
\begin{align*}
\frac{d}{dz} \left( k(z) \frac{dF(z)}{dz} \right) - p(z) F(z) + \lambda^2 q(z) F(z) &= 0,
\end{align*}

(18)
$$\lambda_{\alpha}^2 = \frac{2g^2}{h_c^2}\left(-1.6h_c^2\alpha^2\gamma + 8.5h_c^2\alpha\gamma - 5.33333h_c^2\gamma + \frac{0.533333a^2}{\sigma^2} - \frac{a}{\sigma^2} + \frac{0.666667}{\sigma^2}\right),$$  \tag{23}

$$T_{c_{\text{Min}}}^{\text{Min}(\pm)} = 0.256926 \sqrt{-0.128539 \pm 0.3125 g^2} \sqrt{\frac{1.90164\gamma^2 p^2 (1.00743\gamma^2 P + 0.134769) + 0.169187}{\gamma^2 g^2}}. \tag{24}$$

\begin{table}
\begin{center}
\begin{tabular}{c|c|c|c|c|c|c}
\hline
$\gamma$ & 0.06 & -0.04 & -0.02 & 0 & 0.02 & 0.04 \\
\hline
$T_{c}$ & 0.1701$\rho^{1/3}$ & 0.1774$\rho^{1/3}$ & 0.1869$\rho^{1/3}$ & 0.2005$\rho^{1/3}$ & 0.2239$\rho^{1/3}$ & 0.3185$\rho^{1/3}$ \\
\hline
\end{tabular}
\end{center}
\caption{The critical temperature $T_{c}$ for different values of Weyl coupling parameter $\gamma$ (numerical results).}
\end{table}

where

$$k(z) = \frac{z^3(1 - z^4)}{2g^2} + 4\gamma h_c^2 z^2, \tag{19}$$

$$p(z) = -2z^5 - \frac{2}{g^2} + 16\gamma h_c^3, \tag{20}$$

$$q(z) = h_c^2(\frac{h_c}{2g^2} + 4\gamma z^5)(1 - z^4). \tag{21}$$

The eigenvalue $\lambda$ minimizes expression (18) and is obtained from the following functional:

$$\lambda^2 = \frac{\int_0^1 (k(z)F''(z)^2 + p(z)F(z)^2)dz}{\int_0^1 q(z)F(z)^2dz}. \tag{22}$$

To estimate it, we use the trial function $F(z) = 1 - \alpha z^2$. We then obtain

$$\text{see eq. (23) above}$$

which attains a minimum at $\alpha = 0.304936$, and from the $\lambda = \frac{\rho}{\gamma}$ and $T_{c} = \frac{\rho}{\gamma}$ the minimum value of the critical temperature can be written as

$$\text{see eq. (24) above}$$

We know that in strong-coupling regime of the YM theory, the quantities can be expanded in series of $\frac{1}{\rho}$ [33]. Since for some values of $g$, we may be have be $T_{c_{\text{Min}}}^{\text{Min}(-)} < 0$, therefore only the $T_{c_{\text{Min}}}^{\text{Min}(+)}$ is acceptable and can be read in strong limit of order $\frac{1}{\rho^2}$ as

$$T_{c_{\text{Min}}}^{\text{Min}(+)} \approx 0.1943040830 \rho^{\frac{1}{2}} + \frac{1.047604(-0.128539\gamma + 0.028931219\gamma\rho)}{\gamma^2 \rho^2 g^2}. \tag{25}$$

In probe limit by neglecting the back reaction, the large values of the YM coupling is accessible. Comparing eq. (25) with table 1 we observe that the analytic value of the leading order $T_{c_{\text{Min}}}^{\text{Min}} \approx 0.1943040830 \rho^{\frac{1}{2}}$ obeys the well-known role $T_{c} \propto \rho^{\frac{1}{2}}$ and it is the lower bound for tabulated values of $T_{c}$, given in table 1. According to the numerical results of table 1, the minimum value of $T_{c}$ reads as

$$T_{c_{\text{Min}}}^{\text{Min}(+)} \approx 0.1701 \rho^{\frac{1}{2}}, \tag{26}$$

which shows that the analytic values in relation (25) and the numerical estimate in eq. (26) are in good agreement with each other. It seems that there is a deep relation between the strong limit of YM part of the action and the analytical results of the $p$-wave superconductors.

Relation of $\langle J_{\parallel}^1 \rangle - (\mu - \mu_c)$. If we want to know the behavior of the order parameter at $T_{c}$, we need to solve the equation for the scalar potential close to $T_{c}$, therefore by substituting (17) in (10) we have

$$\frac{d}{dz} \left( -\frac{1}{2zg^2} + 12\gamma z^3 \right) \frac{d\varphi}{dz} + \left( \frac{z}{h_c} \right)^2 \left( \frac{z}{2(1 - z^4)g^2} + \frac{4\gamma z^5}{1 - z^4} \right) \left( \frac{\langle J_{\parallel}^1 \rangle}{h_c} \frac{\langle J_{\parallel}^1 \rangle}{F(z)^2}\varphi = 0. \tag{27}$$

Since the order parameter $\langle J_{\parallel}^1 \rangle$ is small, we can expand $\varphi$ in this small parameter as below:

$$\varphi = \mu_c + \langle J_{\parallel}^1 \rangle \chi(z) + \cdots. \tag{28}$$

The boundary condition at the tip imposes the correction function $\chi(z)$ to be $\chi(1) = 0$. The EOM for $\chi(z)$ can be obtained as below:

$$\frac{d}{dz} \left( -\frac{1}{2zg^2} + 12\gamma z^3 \right) \frac{d\chi(z)}{dz} + \left( \frac{z}{h_c} \right)^2 \left( \frac{z}{2(1 - z^4)g^2} + \frac{4\gamma z^5}{1 - z^4} \right) \frac{J_{\parallel}^1}{F(z)^2} = 0. \tag{29}$$

Making integration of both sides of (29), the EOM for $\chi(z)$ can be reduced to

$$\left( \frac{1}{2zg^2} + 12\gamma z^3 \right) \frac{d\chi(z)}{dz} = -\mu_c \frac{\langle J_{\parallel}^1 \rangle}{h_c^2} \times \int z^2 \left( \frac{z}{2(1 - z^4)g^2} + \frac{4\gamma z^5}{1 - z^4} \right) F(z)^2 dz. \tag{30}$$
From the regularity condition, we must take $\chi'(0) = 0$ and therefore by taking $F(z) = (1 - z^2)(1 - \alpha z^2)$. Near $z = 0$, $f(z)$ can be expanded as

$$f(z) \sim e^{-\mu z^2} \approx e^{-\mu z^2} + O(z) \ldots .$$

(31)

Now from (31), comparing the coefficients of the $z^0$ term on both sides of the above formula, we can have

$$\mu = -\mu_c \approx \frac{\left(\frac{\langle J_1^z \rangle^2}{\mu_c} \right)}{4 \pi G N_c} \frac{(3003.22 + 3/2 \cdot g^3)}{(8 \cdot g^2 + 1)}$$

$$\times \text{Li}_2 \left( \frac{12 \cdot \sqrt{g} - 2.44949}{12 \cdot \sqrt{g}} \right) + (9047.79 + 6350.31) \gamma^2 g^4,$$

(32)

where $\alpha = 0.304936$ is a parameter minimizing eq. (23), and $\text{Li}_2(z)$ gives the polylogarithm function.

This critical exponent $\gamma$ for the condensation value and $(\mu - \mu_c)$ qualitatively match the numerical curves for superconductors with Weyl corrections [27].

Conclusions. – In the Weyl-corrected $p$-wave holographic superconductors at the probe limit, we have investigated the numerical and analytical solutions for the behavior of the critical temperature $T_c$ with respect to the dual of charge density $\rho$ at different values of the Weyl coupling parameter $\gamma$ in the range $\frac{1}{N} < \gamma < \frac{1}{2}$. We have found that the critical temperature will be higher as we amplify the Weyl coupling parameter, therefore the condensation gets harder when $\gamma < 0$, and vice versa when $\gamma > 0$. As a final point, obtaining the critical exponent $\frac{1}{2}$ for the chemical potential $\mu$ and order parameter $\langle J_1^z \rangle$ shows a good agreement with the numerical curves of Weyl-corrected superconductors. Furthermore, we have shown that in the strong limit of YM theory, at order $O(1/g^2)$, the analytical and numerical values of the minimum value of the critical temperature are in good agreement.

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REFERENCES

[1] Maldacena J. M., Adv. Theor. Math. Phys., 2 (1998) 231; Int. J. Theor. Phys., 38 (1999) 1113.
[2] Gubser S. S., Phys. Rev. D, 78 (2008) 065034.
[3] Hartnoll S. A., Herzog C. P. and Horowitz G. T., Phys. Rev. Lett., 101 (2008) 016401.
[4] Weinberg S., Prog. Theor. Phys. Suppl., 86 (1986) 43.
[5] Hartnoll S. A., Herzog C. P. and Horowitz G. T., JHEP, 12 (2008) 015; Horowitz G. T. and Roberts M. M., Phys. Rev. D, 78 (2008) 126008.
[6] Herzog C. P., Kovtun P. K. and Son D. T., Phys. Rev. D, 79 (2009) 066002.
[7] Albash T. and Johnson C. V., arXiv:0906.0519.
[8] Albash T. and Johnson C. V., Phys. Rev. D, 80 (2009) 126009.
[9] Horowitz G. T. and Roberts M. M., JHEP, 11 (2009) 015.
[10] Gubser S. S. and Nellore A., Phys. Rev. D, 80 (2009) 105007.
[11] Keranen V., Keski-Vakkuri E., Nowling S. and Yogendran K. P., Phys. Rev. D, 80 (2009) 121901.
[12] Keranen V., Keski-Vakkuri E., Nowling S. and Yogendran K. P., Phys. Rev. D, 81 (2010) 126011.
[13] Chen J. W., Kao Y. J. and Wen W. Y., Phys. Rev. D, 82 (2010) 066007.
[14] Gubser S. S. and Pufu S. S., JHEP, 11 (2008) 033; Herzog C. P. and Pufu S. S., JHEP, 04 (2009) 126; Ammon M. et al., Phys. Lett. B, 686 (2010) 192.
[15] Roberts M. M. and Hartnoll S. A., JHEP, 08 (2008) 035.
[16] Ammon M., Erdmenger J., Kaminski M. and Kerner P., Phys. Lett. B, 680 (2009) 516.
[17] Ammon M., Erdmenger J., Kaminski M. and Kerner P., JHEP, 10 (2009) 067.
[18] Ammon Martin, Erdmenger Johanna, Grass Viviane, Kerner Patrick and O’Bannon Andy, Phys. Lett. B, 686 (2010) 192.
[19] Sonner J., Phys. Rev. D, 80 (2009) 084031.
[20] Herzog C. P., J. Phys. A, 42 (2009) 343001.
[21] Basu Pallab, He Jianyang, Mukherjee Anindya and Sheih Hsien-Heng, Phys. Lett. B, 689 (2010) 45.
[22] Gubser S. S., Phys. Rev. D, 78 (2008) 065034.
[23] Gubser S. S., Phys. Rev. Lett., 101 (2008) 191601.
[24] Herzog C. P., Phys. Rev. D, 81 (2010) 126009.
[25] Zeng Hua-Bi, Gao Xin, Jiang Yu and Hong-Shi Zong, JHEP, 05 (2011) 002.
[26] Memeni D. and Setare M. R., Mod. Phys. Lett. A, 26 (2011) 2889.
[27] Wu Jian-Pin, Cao Yue, Kuang Xiao-Mei and Li Wei-Jia, Phys. Lett. B, 697 (2011) 153; Ma Da-Zhu, Cao Yue and Wu Jian-Pin, Phys. Lett. B, 704 (2011) 604.
[28] Ritz A. and Ward J., Phys. Rev. D, 79 (2009) 066003.
[29] Manvelyan R., Radu E. and Tchrakian D. H., Phys. Lett. B, 677 (2009) 79.
[30] Gregory R., Kanno S. and Soda J., JHEP, 10 (2009) 010 (arXiv:0907.3203 [hep-th]); Brihaye Y. and Hartmann B., Phys. Rev. D, 81 (2010) 126008; Cai Rong-Gen, Nie Zhang-Yu and Zhang Hui-Qing, Phys. Rev. D, 82 (2010) 066007; Pan Qiyuan and Wang Bin, Phys. Lett. B, 693 (2010) 159; Setare M. R. and Memeni D., EPL, 96 (2011) 6006.
[31] Horowitz G. T. and Roberts M. M., Phys. Rev. D, 78 (2008) 126008.
[32] Hartwan P., Ordinary Differential Equations, 2nd edition (SIAM, Philadelphia) 2002; Shao Huimin, Mathematical Physics Method (Science Press, Beijing) 2004.
[33] Gorski Andrzej, J. Phys. A: Math. Gen., 16 (1983) 849.