Method for measuring the position of the normal reaction on the vehicle wheel

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Abstract. The paper deals with the study of the reaction positions of the support surface on the vehicle wheels. The problems is that the longitudinal drifts of the normal reaction are not fully described. There is no initial data on the rolling radii in the presence of a moment on the wheel. The existing problem does not allow considering components of the longitudinal drifts of normal reactions. The authors noted that the accuracy of predicting the stability and controllability of the vehicle deteriorates. The way out of this is to search other ways to determine the values of these drifts. In conclusion, the authors admitted dividing the general drift of the normal reaction into three components. The paper proposes a measurement method to determine the existence and numerical value of the third component. This method includes indirectly measuring the normal reaction drift that was implemented on an experimental setup.

1. Introduction
The reaction positions of the support surface on the vehicle wheels significantly affect stability and controllability [1-4]. Today, the longitudinal drifts of the normal reaction are not fully described [5]. The use for these purposes of the existing calculated and experimental dependences [5] is difficult due to the lack of initial data on the rolling radii in the presence of a moment on the wheel. This leads to the lack of the possibility of taking into account all the components of the longitudinal drifts of normal reactions. As a result, the accuracy of predicting the stability and controllability of the vehicle deteriorates. In this regard, it is important and relevant to search for other ways to determine the values of these drifts, which provide the availability of initial data for calculations. It is proposed to divide the general drift of the normal reaction into three components. Defining two of them is not a mathematical problem. To determine the existence and numerical value of the third component, a measurement method is proposed. It is a method of indirectly measuring the normal reaction drift. The method was implemented on an experimental setup. The measurement error of this value for small and large car tires does not exceed 12%.

\[ b = a_0 + c + e \]  

where \( b \) – longitudinal drift of the normal reaction (the distance from the action line of the normal reaction of the support surface to the central transverse plane of the wheel);
\( a_0 \) – rolling resistance arm (a longitudinal drift component of the normal reaction, which causes the appearance of a rolling resistance moment and characterizes the hysteresis losses due to wheel
rolling) [5,6-8];

- longitudinal drift component of the normal reaction, which characterizes the longitudinal elastic displacement of the wheel axis relative to the geometric center of the contact patch [5];

- longitudinal drift component of the normal reaction, which characterizes the elastic angular deformations of the tire.

Depending on the driving mode of the vehicle, these components have different directions as shown in Figure 1, and they were calculated as shown in Table 1.

**Table 1. Formulas for calculating the total longitudinal drift \(b\) of the normal reaction of the support surface from the geometric center of the wheel contact patch**

| Driving mode of the vehicle | Formula | Driving mode of the vehicle | Formula |
|----------------------------|---------|----------------------------|---------|
| Driven                     | \(|b| = |a| + |c|\) | Neutral                    | \(|b| = |a| + |c| + |e|\) |
| Driving                    | \(|b| = |a| - |c| + |e|\) | Braking                     | \(|b| = |e| - |c| - |a|\) |

**Figure 1.** Drifts of the normal reaction of the support surface in different driving modes of the vehicle: a – driven; b – driving; c – neutral; d – braking
Calculating components $a_0$ and $c$ is not a mathematical problem:

$$a_0 = f_k \cdot r_k$$
$$c = P_x / C_\alpha$$

where $f_k$ – rolling resistance coefficient;
$r_k$ – wheel rolling radius in the driven mode;
$P_x$ – pushing force of the wheel;
$C_\alpha$ – longitudinal stiffness coefficient of a tire.

2. The aim of the research
The aim of the research is to develop a method for measuring the drift component $e$ of the normal reaction to the wheel, which characterizes the elastic angular deformations of the tire.

Before developing a method for measuring the drift component $e$ of the normal reaction to a wheel, it is necessary to develop a measuring scheme and a method for calculating this value. In the future, the measuring scheme and the measurement method should not contradict the essence of the issue.

3. The measuring scheme and the measurement method for calculating the drift component $e$ of the normal reaction to the vehicle wheel
To calculate the component $e$, the following dependence was obtained. Figure 2 shows the calculation scheme for determining the drift component $e$ of the normal reaction of the support surface caused by elastic angular deformations of an elastic wheel.

![Figure 2](image-url)

**Figure 2.** The calculation scheme for determining the drift component $e$ of the normal reaction of the support surface
Let us consider a section of the tire along the contact patch length $l_c$. The vertical effect of the braking moment $M_b$ on the contact patch is replaced by a pair of forces $\Delta P_e$ or a moment $M_b'$ reduced to the contact patch. To keep the contact patch horizontal, the normal reaction $R_e$ is shifted by an amount to counter the moment $M_b'$:

$$M_b' = R_e \cdot e. \quad (4)$$

We consider an arbitrary pair of points $A$ on the disc periphery with longitudinal coordinates $|X_i|$ from the wheel axis. Each of the two tangential forces $F = M_b/2r$ has two components:

$$F_x = F \cdot \cos \beta; \quad (5)$$

$$F_y = F \cdot \sin \beta; \quad (6)$$

$$\cos \beta = \frac{X_i}{r} \quad (7)$$

where $r$ – tire landing radius;

$$F_z = \frac{X_i}{2r^2} \cdot M_b. \quad (8)$$

With $X_i = l_c/2$ it creates an additional load $\Delta P_e$ on the contact patch, the maximum value of which is:

$$\Delta P_{e,\text{max}} = \frac{l_c}{4r^2} \cdot M_b. \quad (9)$$

Therefore:

$$M_b' = 2\Delta P_{e,\text{max}} \cdot \frac{l_c}{2} = \frac{1}{4} \left( \frac{l_c}{r} \right)^2 \cdot M_b. \quad (10)$$

Accordingly:

$$R_e \cdot e_{\text{max}} = \frac{1}{4} \left( \frac{l_c}{r} \right)^2 \cdot M_b; \quad (11)$$

$$e_{\text{max}} = \frac{1}{4} \left( \frac{l_c}{r} \right)^2 \cdot \frac{M_b}{R_e}. \quad (12)$$

The maximum value of the braking moment:

$$M_{b,\text{max}} = P_e \cdot \varphi_s \cdot r_k \quad (13)$$

where $\varphi_s$ – friction coefficient [13-16]; $r_k$ – wheel rolling radius in free mode [4];

$$e_{\text{max}} = \frac{1}{4} \left( \frac{l_c}{r} \right)^2 \cdot r_k \cdot \varphi_s. \quad (14)$$

The parameters of the contact patch have been studied by many authors [9-12]. The contact patch length can be calculated from the experimental dependence [2]:

$$l_c = 2 \cdot k \cdot \sqrt{\Delta z} \cdot (2R_0 - \Delta z) \quad (15)$$

where $\Delta z$ – normal deflection (radial deformation) of the tire; $k$ – experimental coefficient of reduction of the contact patch length, calculated by the Hedekel formula ($k = 0.6$ for low-profile tires and 0.7 – for all others). This formula is valid for the passport pressure in the tire or it differs from the standard value by no more than 20%:

$$\Delta z = P_e / C_{le} \quad (16)$$

where $C_{le}$ – coefficient of normal (radial) tire stiffness.
Let’s accept \( k_i^2 = 0.5 \). In practical calculations, it is permissible to use the value of the free radius \( R_0 \) as a value \( r_k \).

After algebraic transformations:

\[
e = \left( \frac{R_0}{r} \right)^2 \cdot \Delta z \cdot \varphi_x - \left( \frac{\Delta z}{r} \right)^2 \frac{R_0}{2} \cdot \varphi_x .
\]  

Due to the smallness of the second term in comparison with the first, it can be ignored. In this way:

\[
e = \left( \frac{R_0}{r} \right)^2 \cdot \Delta z \cdot \varphi_x ;
\]  

or

\[
e = \left( \frac{R_0}{R_0 - H_t} \right)^2 \cdot \Delta z \cdot \varphi_x \]

where \( R_0 \) – free wheel radius; \( r \) – tire landing radius; \( H_t \) – tire profile height; \( \Delta z \) – normal deflection (radial deformation) of the tire; \( \varphi_x \) – friction coefficient.

From the obtained expressions it follows that the longitudinal drift component \( e \) of the normal reaction of the supporting surface is determined by the elastic and friction tire properties. It is proportional to the radial deformation and tire free radius and is inversely proportional to the landing radius. If there is no moment on the wheel \( \varphi_x = 0 \) and, accordingly, the drift \( e = 0 \).

With the longitudinal redistribution of normal loads in the process of braking the car, the radial tire deformations \( \Delta z \) of the front wheels will increase, and those of the rear wheels will decrease. Thus, when the vehicle is braking, the drift values \( e \) on the front wheels will be greater than those on the rear ones.

For low-profile tires, in which the free radius is closer to the landing radius than that for normal profile tires, the values of the drift component \( e \) of the normal reaction caused by elastic angular deformations of the tire are very small.

The additional drift component \( e \) of the normal reaction caused by elastic angular deformations of an elastic wheel, in contrast to the rolling resistance arm \( a_0 \), is not associated with power losses. But it simply characterizes the «skew» of the normal stress diagram and is associated with tire deformations, which practically do not affect its shape. The reaction \( R_e \) on the arm \( e \) creates a moment that compensates for the downforce moment \( M_{b+} \) in the contact patch from the braking moment \( M_{b-} \).

4. Method for measuring the drift component \( e \) of the normal reaction to the wheel that characterizes the elastic angular deformations of the tire

The aim of the experimental research was to determine the fundamental presence of a part \( e \) of the longitudinal drift of the normal reaction of a solid support surface to an elastic wheel and its quantitative value. The measurement scheme is shown in Figure 3.
Figure 3. The measurement scheme: 1 – hub; 2 – rim; 3 – wheel; 4 – frame; 5 – hinges; 6 – wheel axle; 7 – tire; 8 – support platform; 9 – hinges; 10 – platform axis; 11 – force sensor; 12 – beam; 13 – a load that creates a normal reaction; 14, 15, 19 – lifting devices; 16 – a load that creates a moment; 17 – flexible rods; 18 – blocks

A special feature of the setup is that its design makes it possible to exclude the measurement of the other two components $a_0, c$ of drift: $a_0$ – wheel did not rotate; $c$ – wheel axis did not move in the longitudinal direction; only the normal load and moment in the main plane of the wheel were applied. A photograph of the setup is shown in Figure 4.

Figure 4. A photograph of the setup
Metrological support is shown in Table 2.

**Table 2. Metrological characteristics of the used measuring instruments**

| Measuring instrument name | Accuracy class | Measurement limit | Measured parameter             |
|---------------------------|----------------|-------------------|--------------------------------|
| Dynamometer               | 2              | 0.1-2 kN          | Tensile forces                 |
| Electronic dynamometer    | 2              | 0.5-5 kN          | Compressive forces             |
| Strain gauge sensor       | 3              | 10 kN             | Force and weight measurement   |

The results of the experiment to determine the drift dependence of the normal reaction on the braking moment $e = f(M_b)$ are shown in Figure 5.

![Figure 5](image)

**Figure 5.** Measurement results $e = f(M_b)$ with the following tire pressure: 1 – $p = 1.5$ atm; 2 – $p = 1.6$ atm; 3 – $p = 1.7$ atm

The experimental research has shown the presence of a drift component $e$ of the normal reaction to the wheel, which characterizes the elastic angular deformations of the tire. In terms of the quantitative value, it can be several times higher than other two components $a_{0, c}$.

5. **Conclusion**

A method for measuring the drift component of the normal reaction to the wheel, which characterizes the elastic angular deformations of the tire, has been developed. It is a method of indirectly measuring a quantity. The method was implemented on an experimental setup. Experimental research has shown the presence of a drift component $e$ of the normal reaction to the wheel, which characterizes the elastic angular deformations of the tire. In terms of the quantitative value, it can be several times higher than other two components $a_{0, c}$. The measurement error of this value for small and large car tires does not exceed 12%.

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