Roll angle estimation of guided gun-launched projectile using MEMS angular rate gyro and magnetometer

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Abstract. The previous method to estimate roll angle of guided gun-launched projectile can be completely invalid in case of GPS is disabled in battlefield. To solve that problem, this paper proposes roll angle estimation used MEMS angular rate gyro and magnetometer. Comparing to the previous method with GPS, the addition of magnetometer outputs increases the measurement accuracy, but increases the dimension and nonlinearity of the measurement model. Therefore the Extended Kalman Filter is introduced. Through trajectory simulation, this study indicates that the absolute roll estimation error is less than 7 degrees after 3 seconds in whole flight trajectory, which is completely met the guided system requirement.

1. Introduction

Facing the harsh requirement of lower cost and higher precision in modern warfare, Conventional shells urgently need to improve the accuracy of the second dimension, in which high-precision rolling angle measurement is critical to the accuracy of the whole guiding system.

In previous methods, Global Positioning System (GPS) and angular rate are combined in roll angle estimation\cite{1}. Artillery shell’s roll angle is related to pitch rate, yaw rate and gyroscope’s output in Body Frame coordinate, pitch rate and yaw rate can calculated from velocity information of GPS. However GPS may be disabled under the certain circumstance in the battlefield, there has to be an immune method to solve the roll angle measurement anyway.

In case GPS signal is not available, a magnetometer can be used as an important measurement\cite{2}. It is assumed that variation of earth magnetism can be ignored when the artillery projectile’s flight distance is shorter than 100km, in another words, earth magnetism is constant in the study. Three-axis magnetometer is used to measure the geomagnetic field signal in the projectile coordinate system, considering the nonlinearity of the measurement model, combining with Extended Kalman Filter (EKF) to measure.

2. Previous Estimation Using Satellite Position System and Angular Rate Gyro\cite{3-8}

This section introduces the previous roll estimation method, which uses the satellite position information and angular rate gyro outputs.
Figure 1 shows a Body Frame (b) and Navigation Frame (n) for a gun-launched projectile. The x axis of nb is defined by a forward direction of the projectile, the y axis is defined by an up direction perpendicular to the x axis and the z axis is defined by a right direction perpendicular to the x-y plane, while x_ny_nz_n is defined as Navigation Frame. A pitch rate (\( \dot{\theta} \)) and a yaw rate (\( \dot{\psi} \)) defined in Body Fixed Plane and a roll defined in Body Frame. The attitude of gyroscope is aligned with the Body Frame.

In figure 1, it is assumed that guided projectile rotates with respect to x axis by roll angle (\( \gamma \)), so the y axis and z axis outputs of gyroscope will measure projection of the pitch rate and the yaw rate with respect to roll angle’s sinusoidal variation when the shell is flying. According to this knowledge, following equation is given.

\[
\begin{align*}
\omega_x &= \dot{\theta} \cos \gamma + \dot{\psi} \sin \gamma \\
\omega_z &= -\dot{\theta} \sin \gamma + \dot{\psi} \cos \gamma
\end{align*}
\]  

The pitch rate (\( \dot{\theta} \)) and yaw rate (\( \dot{\psi} \)) mentioned above are obtained from the GPS velocity as shown in equation (2) and equation (3).

\[
\begin{align*}
\dot{\theta} &= \tan^{-1}\left(\frac{-V_y}{\sqrt{V_x^2 + V_z^2}}\right) \\
\dot{\psi} &= \tan^{-1}\left(\frac{V_x}{V_z}\right)
\end{align*}
\]  

Then the pitch rate (\( \dot{\theta} \)) and yaw rate (\( \dot{\psi} \)) are held by equation (4) and equation (5) as follow.

\[
\begin{align*}
\dot{\theta} &= \frac{\Delta \theta}{t} \\
\dot{\psi} &= \frac{\Delta \psi}{t}
\end{align*}
\]  

Combining equation (4) and equation (5), the analytic formula of gyroscope’s output in Body Frame and pitch/yaw rate in Body Fixed Plane are established, and roll angle calculation is available.

3. **Roll Angle Estimation Using Gyro and Magnetometer without GPS**

In this section, a method of using gyro and magnetometer without GPS’s data is introduced, then an Extended Kalman Filter is also brought in because of the high non-linear measurement model.

3.1. **Coordinate Definition**
Normally, earth magnetism consists of two parts: declination and inclination. Magnetic declination is defined as an angle between the magnetic north and the geographic north. Magnetic inclination is defined as an angle between a magnetic vector and a tangential plane of the earth. It is also assumed that both declination and inclination are constant because trajectory of gun-launched shell is mostly nearer than 100km.

An attitude of a magnetometer is exactly aligned with a Body Frame \((\mathbf{b}_{zyx})\). Each axis of magnetometer output is same to earth magnetism in the Body Frame. Equation (6) presents the coordinate transformation and the matrix \(C^b_n\) is coordinate transform matrix from a Navigation Frame to a Body Frame.

\[
\begin{bmatrix}
 x_b \\
 y_b \\
 z_b \\
\end{bmatrix} = C^b_n \begin{bmatrix}
 x_n \\
 y_n \\
 z_n \\
\end{bmatrix} = C^b_n \begin{bmatrix}
 x_0 \\
 y_0 \\
 z_0 \\
\end{bmatrix}
\]

(6)

Where 
\[
C^b_n = \begin{bmatrix}
 \cos \varphi & 0 & -\sin \varphi \\
 0 & 1 & 0 \\
\sin \varphi & 0 & \cos \varphi \\
\end{bmatrix},
\]

\[
C^2_y = \begin{bmatrix}
 \cos \theta & \sin \theta & 0 \\
 -\sin \theta & \cos \theta & 0 \\
 0 & 0 & 1 \\
\end{bmatrix},
\]

\[
C^2_z = \begin{bmatrix}
 1 & 0 & 0 \\
 0 & \cos \gamma & \sin \gamma \\
 0 & -\sin \gamma & \cos \gamma \\
\end{bmatrix}.
\]

When the earth magnetic frame \((\mathbf{em}_{zyx})\) is introduced, the y and z axes are determined by the coordinate transformation from the Navigation Frame to the Earth Magnetic Frame. In this coordinate transformation, a yaw is declination (\(\beta\)), a pitch is inclination (\(\alpha\)) and a roll is zero. The normalized magnetometer output in Earth Magnetic Frame is \([0 0 1]^T\), which is a theoretical value. Equation (7) represents coordinate transformation from the Navigation Frame to the Earth Magnetic Frame.

\[
\begin{bmatrix}
 1 \\
 0 \\
 0 \\
\end{bmatrix} = \begin{bmatrix}
 \cos \beta & \sin \beta & 0 \\
 -\sin \beta & \cos \beta & 0 \\
 0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
 \cos(-\alpha) & 0 & -\sin(-\alpha) \\
 0 & 1 & 0 \\
\sin(-\alpha) & 0 & \cos(-\alpha) \\
\end{bmatrix} \begin{bmatrix}
 x_0 \\
 y_0 \\
 z_0 \\
\end{bmatrix} = C^{em}_n \begin{bmatrix}
 x_0 \\
 y_0 \\
 z_0 \\
\end{bmatrix}
\]

(7)

Thus the earth magnetometer’s output in Body Frame is as equation (8) as follow.

\[
\begin{bmatrix}
 x^b_0 \\
 y^b_0 \\
 z^b_0 \\
\end{bmatrix} = C^{em}_n C^b_n \begin{bmatrix}
 x_{em} \\
 y_{em} \\
 z_{em} \\
\end{bmatrix} = C^{em}_n (C^b_n)^{-1} \begin{bmatrix}
 1 \\
 0 \\
 0 \\
\end{bmatrix}
\]

(8)

3.2. System Model and Measurement Model for Magnetic Measurement Using EKF

A state vector, a measurement vector and a system model are shown below.

\[
x_k = \begin{bmatrix}
 \varphi \\
 \gamma_k \\
 \omega_x \\
 \omega_y \\
 \omega_z \\
 \psi \\
\end{bmatrix}
\]

(9)

\[
x_k = \begin{bmatrix}
 \sin \gamma(k) \\
 \cos \gamma(k) \\
 \psi \\
 \dot{\varphi} \\
 x^b_0 \\
 y^b_0 \\
 z^b_0 \\
\end{bmatrix}
\]

(10)

\[
x_{k+1} = H_{6x6} \times x_k + q_k
\]

(11)

Where \(\gamma(k)\) is roll angle and \(q_k\) is system noise. Then measurement matrix for EKF is given following.
\[ H_1 = \begin{bmatrix} k \cos(p_k + \gamma_0) & \cos(p_r + \gamma_0) & \theta & -\psi & \frac{\theta}{\theta^2 + \psi^2} & -\frac{\psi}{\theta^2 + \psi^2} \\ -k \sin(p_k + \gamma_0) & -\sin(p_k + \gamma_0) & 0 & 0 & \frac{\theta}{\theta^2 + \psi^2} & \frac{\psi}{\theta^2 + \psi^2} \\ 0 & 0 & \cos(p_k + \gamma_0) / \cos \alpha & -\sin(p_k + \gamma_0) / \cos \alpha & \sin(p_k + \gamma_0) & \cos(p_k + \gamma_0) \end{bmatrix} \]

\[ H_2 = \frac{\partial[x_r \ y_r \ z_r]}{\partial\psi} \quad H_3 = \frac{\partial[x_r \ y_r \ z_r]}{\partial\theta} \quad H_4 = \frac{\partial[x_r \ y_r \ z_r]}{\partial\psi} \quad H_5 = \frac{\partial[x_r \ y_r \ z_r]}{\partial\theta} \]

\[ H_k = \begin{bmatrix} H_2 & H_3 & H_4 & 0_{4 \times 2} \end{bmatrix} \]

(12)

4. Simulation Results

In this section, an algorithm based on angular rate gyro and magnetometer using EKF without GPS velocity is simulated. Table 1 shows simulation conditions.

| Characteristics     | Value            |
|---------------------|------------------|
| Roll rate           | 10~60 rad/s      |
| Initial roll        | 0~360 deg (random) |
| Initial pitch       | 70 deg           |
| Initial yaw         | 0 deg            |
| Algorithm step      | 1 ms             |
| Gyro bias           | 200 deg/hr       |
| Rate noise density  | 0.3 deg/sec/hr^{0.5} |
| Non-orthogonal angle| 3 deg            |
| Magnetometer noise  | 2×10^{-3}        |

Figure 2 shows estimation error is nearly zero using theoretical analytic formula without measurement noise that indicates algorithm’s validity. In figure 3, however, estimation error is drastically increase because of measurement noise, then an EKF is needed to bring into algorithm instead of analysis formula.
Figure 4 shows real roll angle and calculated roll angle using EKF, figure 5 indicates estimation error using EKF.

![Figure 4. Roll Angle with EKF and Roll Angle Nominal Value.](image1)

![Figure 5. Estimation Error Using EKF.](image2)

In figure 5, the absolute roll estimation error is less than 7 degrees after 3 seconds in whole flight trajectory, which is fully met the guided system requirement. The reason of larger estimation error in the beginning and the final trajectory is inaccurate measurement noise this simulation chooses, estimation precision can be improved by enhancing the gyro and magnetometer measurement model.

5. Conclusion

This paper discusses the previous roll estimation method, then introduces a new way to estimate the roll angle using gyro and magnetometer without GPS and an Extended Kalman Filter algorithm is also brought in to analyze. The simulation results show that the absolute roll estimation error is less than 7 degrees after 3 seconds in whole flight trajectory, which is completely met the guided system requirement.

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