A phenomenological theory of zero-energy Andreev resonant states

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(Dated: November 9, 2018)

A conceptual consideration is given to a zero-energy state (ZES) at the surface of unconventional superconductors. The reflection coefficients in normal-metal / superconductor (NS) junctions are calculated based on a phenomenological description of the reflection processes of a quasiparticle. The phenomenological theory reveals the importance of the sign change in the pair potential for the formation of the ZES. The ZES is observed as the zero-bias conductance peak (ZBCP) in the differential conductance of NS junctions. The split of the ZBCP due to broken time-reversal symmetry states is naturally understood in the present theory. We also discuss effects of external magnetic fields on the ZBCP.

PACS numbers: 74.50.+r, 74.25.Fy, 74.70.Tx

I. INTRODUCTION

Transport phenomena in unconventional superconductors have attracted considerable interest in recent years because high-$T_c$ superconductors may have the $d$ wave pairing symmetry. The unconventional pairing symmetry causes the anisotropy in transport properties such as the electric conductance and the thermal conductivity. In normal-metal / high-$T_c$ superconductor junctions, for instance, the shape of the differential conductance reflects the density of states when the $a$ axis of high-$T_c$ materials is perpendicular to the junction interface. When $a$ axis deviates from the interface normal, on the other hand, the conductance shows a large peak at the zero bias-voltage. Such anisotropy in the conductance is now explained by the formation of a zero-energy state (ZES) at the interface of junctions. Since the ZES appears just on the Fermi energy, it drastically affects transport properties through the interface of unconventional superconductor junctions. The low-temperature anomaly of the Josephson current between the two unconventional superconductors is explained in terms of the resonant tunneling of Cooper pairs via the ZES. So far a considerable number of studies have been made on the ZES itself and related phenomena of transport properties in both spin-singlet and spin-triplet unconventional superconductor junctions.

The conductance in normal-metal / superconductor (NS) junctions is calculated from the normal and the Andreev reflection coefficients, which are obtained by solving the Bogoliubov-de Gennes (BdG) equation under appropriate boundary conditions at the junction interface. Consequently we easily find the zero-bias conductance peak (ZBCP) in NS junctions of high-$T_c$ superconductors. Although the algebra itself is straightforward, it is not easy to understand the physics behind the calculation. In a previous paper we briefly discussed reasons for the appearance of the ZBCP by a phenomenological argument. The phenomenological analysis has several advantages. For instance, it shows the importance of the unconventional pairing symmetry for the formation of the ZES without directly solving the BdG equation. Moreover we easily understand that the ZES is a result of the interference effect of a quasiparticle. The applicability of the analysis in the previous paper, however, is very limited because of its simplicity.

In this paper, we reconstruct the phenomenological theory of the Andreev reflection to meet the mathematical accuracy. We calculate the reflection coefficients of an electronlike quasiparticle incident from a normal metal into a NS interface. Near the junction interface, a quasiparticle suffers two kinds of reflection: (i) the normal reflection by the barrier potential at the NS interface and (ii) the Andreev reflection by the pair potential in the superconductor. In the present theory, we consider the two reflections separately to calculate the transport coefficients. As a consequence, the Andreev reflection coefficient is decomposed into a series expansion with respect to the normal reflection probability of NS junctions. The expression of the Andreev reflection probability enables us to understand the importance of the unconventional pairing symmetry for the formation of the ZES. In unconventional superconductors, the pair potential in the electron branch ($\Delta_+$) differs from that in the hole branch ($\Delta_-$). The Andreev reflection probability at the zero-energy is expressed as the summation of the alternating series when $\Delta_+$ and $\Delta_-$ have the same sign with each other. In this case, the zero-bias conductance becomes a small value proportional to $|t_N|^2$, where $|t_N|^2$ is the normal transmission probability of junctions.
hand when $\Delta_+ \Delta_- < 0$, all the expansion series have the same sign and the conductance has a large peak at the zero-bias. The phenomenological theory can be applied to superconductors with a broken time-reversal symmetry state (BTRSS) and NS junctions under external magnetic fields.

This paper is organized as follows. In Sec. II, the Andreev and the normal reflection coefficients are derived from a phenomenological description of a quasiparticle’s motion near the NS interface. In Sec. III, we discuss the conductance peaks in NS junctions. A relation between the broken time-reversal symmetry states and the peak position in the conductance is discussed in Sec. IV. We apply the phenomenological theory to NS junctions under magnetic fields in Sec. V. In Sec. VI, we summarize this paper.

II. QUASIPARTICLE’S MOTION NEAR NS INTERFACES

Let us consider two-dimensional NS junctions as shown in Fig. 1 where a normal-metal ($x < 0$) and a superconductor ($x > 0$) are separated by a potential barrier $V(r) = V_0 \delta(x)$. We assume the periodic boundary condition in the $y$ direction and the width of the junction is $W$. The NS junctions are described by the Bogoliubov-de Gennes equation:

$$
\int dr' \left( \begin{array}{c} \delta(r-r')h_0(r') \\
\Delta(r-r') e^{i\varphi_s} \\
\Delta^* (r-r') e^{-i\varphi_s} \\
-\delta(r-r') h_0(r') \end{array} \right) \\
\times \left( \begin{array}{c} u(r') \\
v(r') \end{array} \right) = E \left( \begin{array}{c} u(r) \\
v(r) \end{array} \right),
$$

$$
h_0(r) = -\frac{\hbar^2 \nabla^2}{2m} + V_0 \delta(x) - \mu_F,
$$

$$
\Delta(R_c, r_r) = \left\{ \begin{array}{ll}
\frac{1}{\kappa_{\text{vol}}} \sum_k \Delta(k)e^{ik \cdot r_r} & : X_c > 0 \\
0 & : X_c < 0,
\end{array} \right.
$$

where $\varphi_s$ is a macroscopic phase of the superconductor, $R_c = (X_c, Y_c) = (r + r')/2$ and $r_r = r - r'$. Here we assume spin-singlet superconductors for simplicity. The argument in the following can be extended to spin-triplet superconductors as shown in Appendix. When an electronic quasiparticle is incident from the normal metal as shown in Fig. 1 the wave function in the normal metal is given by:

$$
\Psi_N(r) = \left[ \begin{array}{c}
\left( \begin{array}{c}
0 \\
1 \\
0 \\
1
\end{array} \right) e^{ik_x x} + \left( \begin{array}{c}
0 \\
1 \\
1 \\
0
\end{array} \right) e^{-ik_x x}
\end{array} \right] e^{ik_y y} \sqrt{W},
$$

where $k_x$ and $k_y$ are the wave numbers on the Fermi surface and they satisfy $k_x^2 + k_y^2 = k_F^2$ with $k_F$ being the Fermi wave number. Throughout this paper we assume that $E \sim \Delta_0 \ll \mu_F$, where $\Delta_0$ is the amplitude of the pair potential and $E$ is the energy of a quasiparticle measured from the Fermi energy, $\mu_F = \hbar^2 k_F^2 / (2m)$. In Eq. (4), $r_{ee}$ and $r_{he}$ are the normal and the Andreev reflection coefficients, respectively.

When a quasiparticle is incident from the normal metal in the electron branch, directions of the outgoing waves are indicated by arrows as shown in Fig. 1. The trajectories of a quasiparticle in the electron branch and those in the hole branch are denoted by solid and broken lines, respectively. In (b), the pair potentials of $s, d_{x^2-y^2}$ and $d_{xy}$ wave symmetries are schematically illustrated.

![FIG. 1: A normal-metal / superconductor junction is illustrated in (a). The trajectories of a quasiparticle in the electron branch and those in the hole branch are denoted by solid and broken lines, respectively. In (b), the pair potentials of $s, d_{x^2-y^2}$ and $d_{xy}$ wave symmetries are schematically illustrated.]

continued...
potentials,
\begin{align}
\Psi_S(r) &= \left[ e^{-i\phi^+}e^{-i\varphi^+} \psi_+ \right] e^{i(k^x x + k^y y)} \\
&+ \left( e^{i\varphi^+}e^{i\varphi^-} \psi_+ \right) e^{-i(k^x x + k^y y)} \sqrt{|W|},
\end{align}

\begin{align}
u_\pm(v_\pm) &= \pm \frac{\Delta^2}{|\Delta|},
\end{align}

where \( t^{ee} (t^{he}) \) is the transmission coefficient to the electron (hole) branch in superconductors. The wave numbers of a quasiparticle are approximately given by \( k^{(h)} \approx k_0 + (-)i(2\xi_0) \) for \( E \approx 0 \), where \( \xi_0 = h v_F/(\pi \Delta_0) \) is the coherence length and \( v_F = h k_F/m \) is the Fermi velocity. Thus a quasiparticle penetrates into the superconductor within a range of \( \xi_0 \). In Eqs. (6)–(9), a phase \( e^{i\varphi^+} \) represents the sign (internal phase) of the pair potential and appears in the wave function in addition to a macroscopic phase of the superconductor. The transmission and the reflection coefficients are obtained from the boundary conditions of these wave functions. Near the junction interface, an incident quasiparticle suffers two kinds of reflection: (i) the normal reflection by the barrier potential at the NS interface and (ii) the Andreev reflection by the pair potential in the superconductor. In this paper, we consider separately contributions of the two reflection processes to the reflection coefficients.

We first consider NS junctions with no barrier potential at the interface,
\begin{equation}
z_0 \equiv \frac{V_0}{h v_F} = 0,
\end{equation}
where \( z_0 \) represents the strength of the potential barrier. The Andreev reflection coefficients become
\begin{align}
r^{he}_0 &= -i \nu_+ e^{-i\phi^+} e^{-i\varphi^+},

r^{he}_0 &= -i \nu_- e^{i\phi^+} e^{i\varphi^+},

\nu_\pm &= i \frac{E - \Omega_\pm}{|\Delta_\pm|},
\end{align}
where \( r^{he}_0 \) is the Andreev reflection coefficients from the electron branch to the hole branch in the absence of the potential barrier. We also give the Andreev reflection coefficient from the hole branch to the electron branch \( (r^{he}_0) \). In the case of \( E^2 - \Delta^2 < 0 \), \( \nu_\pm \) can be described as
\begin{align}
\nu_\pm &= \frac{\sqrt{\Delta^2 - E^2}}{|\Delta|} + i \frac{E}{|\Delta|},

\equiv \cos \theta_\pm + i \sin \theta_\pm = e^{i\theta_\pm}.
\end{align}
Thus the Andreev reflection coefficients include only the phase information in the limit of \( z_0 = 0 \).

We next consider the reflection by the potential barrier in a phenomenological way. In the presence of the potential barrier, the Andreev reflection processes are shown in Fig. 2. In the electron branch, the normal transmission and the normal reflection coefficients of the barrier are calculated to be \( t_N = k_0/(k_0 + iz_0) \) and \( r_N = -iz_0/(k_0 + iz_0) \), respectively with \( k_0 = k_F/\sqrt{2} \). Those in the hole branch are \( t^*_N \) and \( r^*_N \). The Andreev reflection coefficient in the 1st order process is given by,
\begin{equation}
\nu_\pm^{(1)} = t_N \cdot \nu_\pm r^{he}_0 \cdot t_N.
\end{equation}

At first an electron-like quasiparticle starting from \( r_i \) transmits into the superconductor through \( r_0 \), \((t_N)\). In Fig. 2 the vectors in real space are surrounded by squares to avoid confusion. While traveling the superconductor within the range of \( \xi_0 \), the quasiparticle is reflected into the hole branch by the pair potential at \( r_1 \) (\( r^{he}_0 \)). Then the quasiparticle goes back to the normal metal in the hole branch through \( r_0 \) (\( t^*_N \)). The 2nd order Andreev reflection process in Fig. 2 (b) can be estimated in the same way,
\begin{equation}
r^{he}_1 = t^*_N \cdot \nu_\pm r^{he}_0 \cdot t_N.
\end{equation}

\begin{align}
A_S &= r^{he}_1 \cdot r^{*h}_N \cdot r^{*h}_N, \\
&= - |r_N|^2 \nu_\pm e^{i(\phi^+ - \phi^+)}. 
\end{align}
After the first Andreev reflection into the hole branch, the quasiparticle suffers the normal reflection ($r_{0h}^{*}$). Next the holelike quasiparticle experiences the 2nd Andreev reflection to the electron branch at $r_{0}^{e}$ ($r_{0}^{eh}$). Then the electronlike quasiparticle suffers the normal reflection ($r_{N}$) followed by the 3rd Andreev reflection into the hole branch ($r_{0}^{he}$). Finally the holelike quasiparticle goes back to the normal metal through $r_{0}'$ ($t_{N}'$). We only show the expression of the Andreev reflection coefficient in the 3rd order process,

$$r_{he}^{3} = t_{N}'^{*} A_{S}^{2} r_{0}'^{he} t_{N}. \tag{20}$$

The corresponding trajectory is shown in Fig. 2 (c). The total Andreev reflection coefficient is obtained by the summation of these reflection processes up to the infinite order,

$$r_{he} = |t_{N}|^{2} \cdot r_{he}^{0} \cdot \sum_{n=1}^{\infty} A_{S}^{2n-1}. \tag{21}$$

In the similar way, the normal reflection coefficient results in

$$r_{ee} = r_{N}^{*} t_{N}^{2} \cdot r_{he}^{0} r_{0}^{eh} \sum_{n=1}^{\infty} A_{S}^{2n-1}. \tag{22}$$

Although the reflection coefficients in Eqs. (21) and (22) are obtained based on the phenomenological description of a quasiparticle’s motion, they are mathematically identical to the exact expressions calculated from the boundary conditions of the wave functions in the presence of the potential barrier.

**III. CONDUCTANCE**

The differential conductance is calculated from the normal and the Andreev reflection coefficients,

$$G_{NS} = \frac{2e^{2}}{h} \sum_{k_{y}} \left[ 1 - |r_{ee}|^{2} + |r_{he}|^{2} \right] \bigg|_{E=\epsilon_{bias}} \left( t_{N} \right), \tag{23}$$

where $V_{bias}$ is the bias voltage applied to NS junctions. We focus on the limit of $E \rightarrow 0$ for a while, where the Andreev reflection probability dominates the zero-bias conductance because the conductance can be described by

$$G_{NS} = \frac{4e^{2}}{h} \sum_{k_{y}} |r_{he}|^{2} \bigg|_{E=\epsilon_{bias}}. \tag{24}$$

A quasiparticle after the Andreev reflection traces back the original trajectory of a quasiparticle before the Andreev reflection. This is called the retro property of a quasiparticle. When we estimate the reflection coefficients in Eqs. (21) and (22), we only consider the phase factor of the Andreev reflection. A quasiparticle, however, may suffer additional phase shift while moving around the NS interface. Actually, an electron acquires a phase $e^{i\phi_{-}}(r_{1}-r_{0})$ while traveling from $r_{0}$ to $r_{1}$ as shown in Fig. 2 (a). In addition to this, a phase factor $e^{i\phi_{+}}(r_{0}'-r_{1})$ is multiplied while traveling from $r_{1}$ to $r_{0}'$ in the hole branch. These two phase factors exactly cancel each other out when the retro property holds because $r_{0} = r_{0}'$. Thus $r_{he}$ indicates the same position for all $n$. In particular for $E=0$, a relation $r_{n} = r_{n}'$ for all $n$ holds, which means the retro property of a quasiparticle in the normal metal. In the limit of $E \rightarrow 0$, we find in Eq. (13) that $\nu_{+} \rightarrow 1$ irrespective of symmetries of the pair potential. The Andreev reflection probability becomes

$$|r_{he}|^{2} = |t_{N}|^{4} \sum_{n=0}^{\infty} |r_{n}|^{2n} \left[ -e^{i(\phi_{-} - \phi_{+})} \right]^{n} = 0. \tag{25}$$

Firstly, we consider superconductors where the pair potentials in the two branches ($\Delta_{+}$ and $\Delta_{-}$) have the same sign, (i.e., $e^{i(\phi_{-} - \phi_{+})} = 1$). For examples, the pair potentials below satisfy the condition irrespective of the wave numbers of a quasiparticle,

$$\Delta_{s}(k) = \Delta_{0} (s \text{ wave}), \tag{26}$$

$$\Delta_{d_{x^{2}-y^{2}}}(k) = \Delta_{0}(k_{x}^{2} - k_{y}^{2}) (d_{x^{2}-y^{2}} \text{ wave}). \tag{27}$$

where $k_{x} = k_{x}/k_{F}$ and $k_{y} = k_{y}/k_{F}$ are the normalized wave number on the Fermi surface in the $x$ and $y$ directions, respectively. The schematic figures of the pair potentials are shown in Fig. 11 (b). Equation (20) represents the pair potential of $s$ wave superconductors. The pair potential in Eq. (24) is realized in a junction where the $a$ axis of a high-$T_{c}$ superconductor is perpendicular to the interface normal. When $e^{i(\phi_{-} - \phi_{+})} = 1$ is satisfied, Eq. (24) becomes the summation of the alternating series. The Andreev reflection probability results in

$$|r_{he}|^{2} = \frac{2|t_{N}|^{4}}{2-|t_{N}|^{2}}. \tag{28}$$

In low transparent junctions, (i.e., $z_{0}^{2} \gg 1$), the Andreev reflection probability becomes a small value $|t_{N}|^{4}/2 \propto 1/z_{0}^{4}$. Therefore the zero-bias conductance in Eq. (24) is proportional to $1/z_{0}^{4}$. Secondly we consider that the signs of the two pair potential are opposite to each other. The pair potential

$$\Delta_{d_{xy}}(k) = 2\Delta_{0} \bar{k}_{x} \bar{k}_{y}, \tag{29}$$

satisfies $e^{i(\phi_{-} - \phi_{+})} = 1$ for all wave numbers and is realized in a junction where the $a$ axis of a high-$T_{c}$ superconductor is oriented by 45 degrees from the interface normal. All the expansion series in Eq. (24) have the same sign and the Andreev reflection probability becomes

$$|r_{he}|^{2} = 1. \tag{30}$$

Thus the zero-bias conductance in Eq. (24) takes its maximum value. The sign of the pair potentials characterizes the interference effect of a quasiparticle near the NS
interface. For $e^{i(\phi_+ - \phi_-)} = 1$, the alternating series in Eq. (20) reflect the destructive interference among the partial waves of a quasiparticle in the expansion series. Hence the conductance becomes small at the zero-bias. On the other hand for $e^{i(\phi_+ - \phi_-)} = -1$, the expansion series with the same sign imply that the partial waves in the expansion series interfere constructively, which leads to the large zero-bias conductance. The constructive interference at the interface causes a resonant state which is now referred to as the ZES. The Andreev reflection probability is unity independent of the normal transmission probability of junctions as shown in Eq. (30). This can be interpreted as a result of the resonant transmission of a quasiparticle through the ZES. A microscopic calculation shows that the ZES has a large local density of states around $x = \xi_0$ at the zero-energy. Similar arguments have been done in normal-metal/insulator/normalmetal/insulator/superconductor junctions and at the surface of high-$T_c$ superconductors.

In Eq. (30), we can explain a large conductance at the zero-bias. In what follows, we will show that the conductance has a peak structure around the zero-bias. When $E \neq 0$ but still $E \lesssim \Delta_0$, the degree of resonance is suppressed because $\nu \pm$ is no longer unity as shown in Eq. (15). In the superconductor, the argument of the phase cancellation in the round-trip between $r_0$ and $r_1$ in Fig. 2(a) is still valid as far as $E^2 - |\Delta_\pm|^2 > 0$ being satisfied. In the electron branch on the way to $r_1$, the $x$ component of the wavenumber is given by

$$k_x \simeq k_x + i \frac{k_F}{k_x} \sqrt{|\Delta_+|^2 - E^2}.$$

The real part determines the direction of the quasiparticle’s motion. The inverse of the imaginary part characterizes the dumping of the wave function and is roughly estimated to be $\xi_0$. It is also shown that $k_x$ is the real part of the wave number in the hole branch on the way back to $r_0$. The Andreev reflection probability for finite $E$ is given by

$$|p^{he}|^2 = \frac{|t_N|^4}{|t_N|^4 + 4|t_N|^2 [1 + \text{Re} \phi \pm e^{i(\phi_+ - \phi_-)}]}.$$

To make clear a relation between the peak positions of the conductance and the relative sign of the pair potentials, we consider the pair potential

$$\Delta_{\text{sign}}(k) = \Delta_0 \text{sign}(k_x k_y),$$

instead of Eq. (20). Here the anisotropy of pair potential is taken into account only through the phase $e^{i\phi_\pm}$ and the $k$ dependence of the pair potential is neglected. The pair potential in Eq. (33) is illustrated in Fig. 1(b). We will check the validity of Eq. (33) later. The Andreev reflection probability for $\Delta_{\text{sign}}$ becomes

$$|p^{he}|^2 = \frac{|t_N|^4}{|t_N|^4 + 4|t_N|^2 \sin^2 \theta} = \frac{E_0^2}{E^2 + E_0^2},$$

$$E_0 = \frac{\Delta_0 |t_N|^2}{2|t_N|^2}.$$

where we use a relation $\theta = \theta_+ = \theta_-$. In Eq. (15), The Andreev reflection probability has a peak structure at $E = 0$ and the width of the peak is characterized by $E_0$ which is $\Delta_0/2\xi_0$ in the limit of $\xi_0^2 \gg 1$. On the other hand in $s$ wave junctions (i.e., $e^{i(\phi_+ - \phi_-)} = 1$), we find

$$|p^{he}|^2 = \frac{|t_N|^4}{|t_N|^4 + 4|t_N|^2 \cos^2 \theta} = \frac{E_0^2}{(\Delta_0^2 - E^2) + E_0^2}.$$

The Andreev reflection probability has a peak at $E = \Delta_0$ reflecting a peak of the bulk density of states in $s$ wave superconductors. In Fig. 3 we plot the conductance, where $\xi_0 = 3$ and $N_c = W k_F / \pi$ is the number of propagating channels on the Fermi surface. The results for $s$ wave junction are indicated by the broken line. The conductance for $d_{x^2-y^2}$ symmetry is amplified by 5 times in the dotted line.

![Fig. 3: The conductance is plotted as a function of $E$, where $\xi_0 = 3$. The anisotropy of the pair potential in $d_{xy}$ symmetry is taken into account only through the phase factor $e^{i(\phi_+ - \phi_-)}$ and $k$ dependence of the pair potential is neglected in the dash-dotted line. The $d_{xy}$ symmetry is fully taken into account in the solid line. The conductance for $d_{x^2-y^2}$ symmetry is amplified by 5 times in the dotted line.](image-url)
The spatial dependence of the pair potential may affect the width of the ZBCP through $E_0$ in Eq. (35).

We note that there is no remarkable differences between the mathematical origin of the peaks at $E = 0$ for $e^{i(\phi_+ - \phi_-)} = -1$ and that at $E = \Delta_0$ for $e^{i(\phi_+ - \phi_-)} = 1$. Actually it is easy to confirm at $E = \Delta_0$ that the Andreev reflection probability in s wave junctions becomes

$$|r^{hc}|^2 = |t_N|^4 \left| \sum_{n=0}^{\infty} r_{N}^{2n} \left[ e^{i(\phi_+ - \phi_-)} \right]^{2n} \right|^2. \tag{37}$$

All the expansion series have the same sign for $e^{i(\phi_+ - \phi_-)} = 1$.

In above arguments, we have assumed that the junctions have non zero transmission probabilities. In the end of this section, we briefly mention that the ZES becomes a real bound state in the limit of $z_0 \to \infty$. A quasiparticle motion is spatially limited at the surface of the semi-finite superconductor because of the perfect normal reflection by the surface and the Andreev reflection by the pair potential. The ZES becomes a bound state because there is no quasiparticle excitations which extend into the bulk superconductors at $E = 0$. In the density of states, such ZES is found as the $\delta$-function peak. For finite transmission probability of junctions, the finite propagation into normal metals gives a finite life time of the ZES which is given by $\hbar/\Delta_0$. On the other hand for $e^{i(\phi_+ - \phi_-)} = 1$, the resonant state at $E = \Delta_0$ does not become a bound state because there are excitations extend into the bulk superconductors at $E = \Delta_0$. In superconductor/insulator/superconductor (SIS) junctions, the ZES is also a bound state irrespective of the transmission probability of junctions. The description of the Andreev bound states in SIS junctions was given, for example, in Ref. 27.

IV. PAIRING WITHOUT TIME-REVERSAL SYMMETRY

In recent experiments, a possibility of the broken time reversal symmetry state (BTRSS) at the surface of high-$T_c$ superconductors has been discussed. These experiments found the split of the ZBCP in the zero magnetic field. It is pointed out that such surface states may have $s + id_{xy}$ or $d_{xy} + id_{xy}^2$ pairing symmetry. Theoretical studies showed the split of the surface density of states when $s + id_{xy}$ wave pairing symmetry is assumed at the surface of the $d_{xy}$ wave superconductor. Within the present phenomenological theory, it is also possible to discuss the split of the conductance peak by the BTRSS in terms of the shift of the resonance energy. We assume the pair potential as

$$\Delta_{s + id_{xy}}(k) = \alpha \Delta_0 + i \beta \Delta_{d_{xy}}(k), \quad (s + id_{xy} \text{ wave}), \tag{38}$$

with $\alpha^2 + \beta^2 = 1$. We find

$$\Delta_{\pm} = |\Delta| = \sqrt{\alpha^2 \Delta_0^2 + \beta^2 \Delta_{d_{xy}}^2(k)}, \tag{39}$$

$$e^{i(\phi_+ - \phi_-)} = e^{i\phi_-} = \left[ \frac{\alpha \Delta_0 - i \beta \Delta_{d_{xy}}}{|\Delta|} \right]. \tag{40}$$

In Fig. 4 we show the conductance in the $s + id_{xy}$ symmetry for several $\alpha$. For $\alpha = 0$, the results are identical to the conductance of $d_{xy}$ wave junctions in Fig. 3. The ZBCP splits into two peaks for $\alpha \neq 0$. The splitting width increases almost linearly with increasing $\alpha$. In the limit of $\alpha = 1$, the results coincide with the conductance of $s$ wave junctions in Fig. 3. The peak position can be explained by the expression of the Andreev reflection probability

$$|r^{hc}|^2 = \frac{|t_N|^4}{|t_N|^4 + 4|r_N|^2 \cos^2(\theta + \phi_-)}, \tag{41}$$

$$\cos(\theta + \phi_-) = \frac{\sqrt{|\Delta|^2 - E^2}}{|\Delta|^2} \alpha \Delta_0 + \frac{E}{|\Delta|^2} \beta \Delta_{d_{xy}}, \tag{42}$$

$$\approx 4 \frac{\Delta_0 - E^2}{\Delta_0} \alpha + \frac{E}{\Delta_0} \beta \sgn(k_x k_y). \tag{43}$$

In the last equation, we replace $\Delta_{d_{xy}}$ by $\Delta_{d_{s_{xy}}}$. The conductance peak (the resonance energy) is expected at $E = 0$ as shown in Eq. (41). The resonance energies for $\alpha = 0$ and $\alpha = 1$ are $E = \Delta_0$ and $E = 0$, respectively. These resonance energies are independent of the wave numbers. Consequently the peak heights for $\alpha = 0$ and $\alpha = 1$ become unity. The peak heights for finite $\alpha$, however, are always less than unity as shown in Fig. 4 because the resonance energy depends on wave numbers as shown in Eq. 42.

![FIG. 4: The conductance is plotted as a function of $E$ for $s + id_{xy}$ symmetry, where $z_0 = 3$.](image)
resonance condition of \( \cos(\theta + \phi_-) = 0 \) in Eq. (13). Since
the peak position is determined by \( \alpha \), relative amplitudes
of \( s \) and \( d_{xy} \) components can be estimated from
the peak splitting width observed in experiment. In the
phenomenological theory, effects of the BTRSS on the
conductance can be understood in terms of the shift of the
resonance energy.

In theoretical studies, it is shown that the \( s + id_{xy} \)
wave BTRSS splits zero-energy peak of the local density
of states \( \Delta_0 = 2 \mu_F/(k_F \hbar c B/m_c) \) and the ZBCP.\footnote{Experimental
results are, however, still controversial. Some ex-
}periments reported the split of the ZBCP at the zero
magnetic field\footnote{55,56,57,58,59} other did not observe the
splitting.\footnote{7,10,12,13,16,17,18} Thus opinions are still divided
among scientists on the BTRSS in high-
\( T \) superconductors. If the BTRSS does not exist, we have to find
another reasons for the peak splitting observed in
experiments. In recent papers, we have showed that the
interfacial randomness causes the split of the ZBCP in
the zero magnetic field in both numerically\footnote{82,83,84,85,86,87} using
the recursive Green function method\footnote{79,80} and analytically\footnote{74}
using the single-site approximation.\footnote{81} Our conclusion,
however, contradicts to those of a number of theories
\footnote{82,83,84,85,86,87} based on the quasiclassical Green function
method.\footnote{88,89,90,91,92} The drastic suppression of the ZBCP
by the interfacial randomness is the common conclusion
of all the theories. The theories of the quasiclassical
Green function method, however, concluded that the ran-
dom potentials do not split the ZBCP.

V. EFFECTS OF MAGNETIC FIELD

The TRS is also broken by applying external magnetic
fields onto NS junctions. The resonance at \( E = 0 \) is
suppressed because a quasiparticle acquires a Aharonov-
Bohm like phase from magnetic fields.\footnote{93,94,95} Actually it is
pointed out that the ZBCP in NS junctions splits into
two peaks under the magnetic field.\footnote{54,55,68,69,70} The re-

tection process in Fig. 5(a) corresponds to \( A_s \) in Eq. (15).

We consider uniform magnetic fields perpendicular to
the \( xy \) plane (i.e., \( B \hat{z} \)) and assume the Landau gauge
\( \mathbf{A}_{\text{ext}} = B x \hat{y} \). Effects of magnetic fields are taken into
account through the phase of the wave function by using
the gauge transformation. While traveling from \( r_0 \) to \( r_1 \),
an electronlike quasiparticle acquires a phase \( e^{i \phi_m} \) with
\( \phi_m = \int_{r_0}^{r_1} \mathbf{A}_{\text{ext}}(l) \cdot dl \).

Since the magnetic field is sufficiently weak, the integra-
tion path can be replaced by a straight line between \( r_0 \) and \( r_1 \) which is denoted by \( C_1 \) in Fig. 5(a). This approx-
imation is justified when the radius of the cyclotron
motion of a quasiparticle, \( 2 \mu_F / (k_F \hbar c B / m_c) \), is much
larger than \( \xi_0 \). The condition is equivalent to the relation
\( \pi \Delta_0 \gg \hbar c B / m_c \). In high-\( T \) materials, \( \Delta_0 \sim 30 - 40 \)
meV, whereas \( \hbar c B / m_c \) is \( 10^{-1} \) meV for \( B = 1 \) Tesla,
where we use the bare mass of an electron. The phase
shift on the way from \( r_1 \) to \( r_0 \) (\( C_2 \)) in the hole branch is
equal to \( e^{i \phi_m} \). This is because the direction of a quasiparticle’s motion and the sign of the charge on \( C_1 \) are opposite to those on \( C_2 \) at the same time. In the same
way, we can show that the phase shifts on \( C_3 \) and \( C_4 \) in
Fig. 5(a) are also \( e^{i \phi_m} \). Under the gauge transformation,
the pair potential should be changed to
\[ \Delta(r, r') \exp \left[ \frac{ie}{\hbar c} \left( \int_r^{r'} dl \cdot \mathbf{A}_{\text{ext}}(l) \right) \right]. \]

At \( r_1 \), a phase factor
\[ \exp \left[ -i2 e \int_{r_1}^{r_0} dl \cdot \mathbf{A}_{\text{ext}}(l) \right] \]
is multiplied to the Andreev reflection coefficients, where \( r \) and \( r' \) in Eq. (15) are set to be \( r_1 \). A phase factor
\[ \exp \left[ i2 e \int_{r_1}^{r_2} dl \cdot \mathbf{A}_{\text{ext}}(l) \right] \]
is also multiplied to the Andreev reflection coefficients
at \( r_2 \). The total phase shift by the magnetic field along
\( C_1 \sim C_4 \) in Fig. 5(a) \( (e^{i2 \phi_B}) \) is then given by
\[ \phi_B = 2 \phi_m + \frac{e}{\hbar c} \int_{r_1}^{r_2} dl \cdot \mathbf{A}_{\text{ext}}(l), \]
\[ = - \frac{eB}{\hbar c}(y_1 - y_0)(x_1 - x_0), \]
\[ = - \frac{B k_y}{B_0 k_x} \frac{y_1 - y_0}{x_1 - x_0}. \]

where \( \phi_m = 2 \pi \hbar c / e \). On the way to Eq. (49), we use a relation
\( (x_1 - x_0)/(y_1 - y_0) = k_x / k_y \) and \( x_1 - x_0 \sim \xi_0 \).
We note that \( 2 \phi_B \) is the gauge invariant magnetic flux
passing through the gray area in Fig. 5(b), where \( r_3 =
(2x_1 + x_0, y_0) \). Thus \( 2 \phi_B \) remains unchanged in another
gauges such as \( \mathbf{A}_{\text{ext}} = -B y \hat{x} \) and \( \hat{y} \) with \( \lambda_0 \gg \xi_0 \), where \( \lambda_0 \) is the

\[ \frac{2 \phi_0}{2 \pi \xi_0}. \]
and (b), respectively. In high-
T_B material, B_0 is estimated to be 160 Tesla.

penetration depth. In high-T_c materials, \( \xi_0 \approx 2 \text{nm and } \lambda_0 \approx 200 \text{nm.} \)

Effects of magnetic field can be taken into account in the present theory by

\[
A_s \rightarrow A_s e^{2i\phi_B}, \tag{51}
\]

where \( A_s \) is defined in Eq. 19. We show the conductance in \( d_{xy} \) wave junctions calculated from Eqs. 21-24 and 25 in Fig. 6 where \( z_0 = 3 \) and 10 in (a) and (b), respectively. In high-\( T_c \) superconductors, \( B_0 \) is about 160 Tesla. The ZBCP decreases with increasing \( B \) in both Figs. 6(a) and (b). The degree of suppression due to magnetic fields depends on the transmission probability of the junction. More drastic suppression can be seen in lower transparent junctions. In Fig. 6(b), the ZBCP almost disappears for \( B = 0.05B_0 \). The ZBCP, however, remains one peak and does not split into two peaks even in the strong magnetic fields. The results in Fig. 6 are qualitatively well described by the analytical expression

of the Andreev reflection probability for \( E \ll \Delta_0 \),

\[
|\tau^h|^2 = \frac{|t_N|^4}{|t_N|^4 + 4|r_N|^2 \sin^2(\theta + \phi_B)}, \tag{52}
\]

\[
|\tau^h|^2 \approx \frac{|t_N|^4|\Delta|^2}{|t_N|^4|\Delta|^2 + 4|r_N|^2(E + |\Delta|\phi_B)^2}. \tag{53}
\]

We linearize the magnetic fields in \( \sin(\theta + \phi_B) \) in Eq. 53. Equation 53 implies that the resonance energy may be shifted from \( E = 0 \) by magnetic fields. In contrast to the splitting of the ZBCP by the BTRSS in Sec. IV, we do not find the peak splitting under magnetic fields in Fig. 6.

In the BTRSS, the shift of the resonance energy is caused by the supercurrent component which has the resonance energy at \( E = \Delta_0 \). On the other hand, any resonant states are not associated with magnetic fields. Thus the magnetic fields only suppress the resonance of the ZES as shown in Fig. 5.

In a previous paper, however, the split of the ZBCP in magnetic fields was reported within the quasiclassical approximation (QCA). The results in Eq. 53 are similar to that in the argument of the Dopplar shift in the QCA. The supercurrents flows along the interface shift the energy of a quasiparticle as

\[
E \rightarrow E + v_F \cdot p_s, \tag{54}
\]

\[
p_s = -\frac{eA}{c} \frac{eB\lambda_0}{c} e^{-\frac{x}{\lambda_0}} y. \tag{55}
\]

where \( p_s \) is the condensate momentum at the interface. In Eq. 55, \( d \) wave character of the supercurrent is not considered. The corresponding approximation in the present theory is replacing \( E + |\Delta|\phi_B \) by \( E + \Delta_0\phi_B \) in Eq. 53 and we find

\[
|\tau^h_{QCA}|^2 = \frac{|t_N|^4|\Delta|^2}{|t_N|^4|\Delta|^2 + 4|r_N|^2(E + \Delta_0\phi_B)^2}. \tag{56}
\]

In Fig. 6(a), we show the conductance calculated from Eq. 52 for \( z_0 = 10 \). In contrast to Fig. 6(b), we find split of the ZBCP when magnetic fields are larger than the threshold magnetic field, \( B_c \). The threshold depends on \( z_0 \) as shown in Fig. 6(b), where \( B_c \) is plotted as a function of \( 1/z_0^2 \) which is proportional to the normal transmission probability of junctions. The threshold increases with increasing the transmission probability of junctions. This has been pointed out in the conductance calculated on the lattice model by using the QCA. In the lattice model, it was also shown that \( B_c \) decreases with the increase of the doping rate. The Fermi energy is a decreasing function of the doping rate. Therefore the transmission probability of junctions decreases with increasing the doping rate.

Although Eqs. 55 and 56 are similar to each other, the response of the ZBCP to magnetic fields are qualitatively different. To make clear if a magnetic field splits the ZBCP or not, we need some numerical simulations, where effects of magnetic field are taken into account accurately. In experiments, some papers show the split of
the ZBCP in magnetic fields. On the other hand, several papers report no splitting of the ZBCP. A microscopic scattering theory indicates that the sensitivity of the ZBCP to magnetic fields depends on the degree of potential disorder near the NS interface.

Finally we briefly discuss an important difference of the conductance in the present theory and that in the QCA. The phenomenological theory reaches the conductance of the ZES, which is almost the same as the conductance expression given by the vector potential which is not an observable value. Thus the QCA does not satisfy the gauge invariance. In the present theory, on the other hand, we consider uniform magnetic field and the normalization for the penetrating magnetic fields in the QCA is originally about 16 Tesla with $\lambda_0 \sim 100\xi_0$. In Eq. (55), $p_\parallel$ in the QCA is originally given by the vector potential which is not an observable value. Thus the QCA does not satisfy the gauge invariance. In the present theory, on the other hand, we consider uniform magnetic field and the normalization of magnetic fields ($B_0$) is about 160 Tesla. This value remains unchanged even if we consider penetrating magnetic field as $B e^{-x/\lambda_0} \xi$ with $\lambda_0 \gg \xi_0$. For example in Fig. (b), we find that $B_c$ is about 0.16 Tesla at $z_0 = 3$. Therefore $B_c$ is estimated to be 16 Tesla in the present theory. The same results are interpreted as $B_c=0.16$ Tesla if we use $B_0^{QCA}$ in the QCA. The threshold magnetic field in the QCA is estimated to be much smaller than that in the present theory. This disagreement may be important because the maximum value of magnetic fields in experiments is about 10 Tesla.

VI. CONCLUSION

We have presented a phenomenological theory of the Andreev reflection to make clear reasons for the appearance of the zero-bias conductance peak (ZBCP) in normal-metal / unconventional superconductor junctions. The phenomenological theory reveals that the zero-energy state (ZES) is a consequence of the constructive interference effect of a quasiparticle. The expression of the Andreev reflection probability enables us to understand an importance of the unconventional pairing symmetry for the formation of the ZES. The phenomenological theory is applied to superconductors with a broken time-reversal symmetry state (BTRSS) and junctions under magnetic fields. The split of the ZBCP in $s+id_{xy}$ wave superconductors is understood in terms of the shift of the resonance energy by the $s$ wave component. The Aharonov-Bohm like phase received from magnetic fields suppresses the degree of resonance of the ZES, which explains the suppression of the ZBCP in magnetic fields.

APPENDIX A: ANDREEV REFLECTION BY SPIN-TRIPLET SUPERCONDUCTORS

In the text, we consider two-dimensional spin-singlet superconductors and $\delta$-function type potential barrier for simplicity. Here we generalize the phenomenological theory to spin-triplet superconductors in three-dimension. The pair potential in superconductors is given by

$$\hat{\Delta}(k) = \begin{cases} \frac{i}{2}d(k) \cdot \vec{\sigma} \hat{\sigma}_2 : \text{triplet}, \\ \frac{i}{2}d(k) \hat{\sigma}_2 : \text{singlet}, \end{cases}$$

where $\hat{\sigma}_j$ for $j = 1, 2$ and 3 are Pauli matrices representing the spin degree of freedom. We assume that the current is in the $x$ direction and consider a potential barrier

$$V(r) = V_0 [\Theta(x) - \Theta(x - L)],$$

where $L$ is the thickness of the insulating layer. The Andreev reflection coefficients in the absence of the insulator are calculated analytically

$$\tilde{\tau}_0^{he} = -e^{-i\varphi} \hat{\Delta}_{(+)}^\dagger \hat{R}_{(+)},$$

$$\tilde{\tau}_0^{eh} = -e^{i\varphi} \hat{R}_{(-)} \hat{\Delta}_{(-)},$$

$$\hat{\Delta}_{(\pm)} = id_{\pm} \cdot \vec{\sigma} \hat{\sigma}_2,$$

$$\hat{R}_{(\pm)} = \frac{1}{2|q_{\pm}|} \sum_{l=1}^2 \left[ \frac{K_{l,\pm}}{Q_{l,\pm}} \hat{R}_{l,\pm} \right],$$
\[ \Delta_{l,\pm} = \sqrt{|d_{l\pm}|^2 - (1)^{|d_{l\pm}|}}, \quad \text{(A7)} \]
\[ K_{l,\pm} = U E^2 - \Delta_{l,\pm}^2 - E, \quad \text{(A8)} \]
\[ \hat{P}_{l,\pm} = |q_{l\pm}| \hat{\sigma}_0 - (1)^{|q_{l\pm}|} \hat{\sigma}, \quad \text{(A9)} \]
\[ q_{l\pm} = i d_{l\pm} \times d^*_{l\pm}, \quad \text{(A10)} \]
\[ d_{l\pm} = d(\pm k_x, k_y, k_z), \quad \text{(A11)} \]

where \( \varphi_s \) is a macroscopic phase of superconductor, \( l(=1 \text{ or } 2) \) indicates the two spin branches of Cooper pairs and \( \hat{\sigma}_0 \) is the 2 x 2 unit matrix. The normal transmission and the normal reflection coefficients of the insulator are calculated as

\[ \hat{t}_N = \frac{-2i k_p \bar{p}_e e^{-ik_p L}}{z_1^*} \hat{\sigma}_0, \quad \text{(A12)} \]
\[ \hat{r}_N = \frac{z_0}{z_1} \hat{\sigma}_0, \quad \text{(A13)} \]
\[ z_0 = \frac{V_0}{\mu_F} \sinh(p_x L), \quad \text{(A14)} \]
\[ z_1 = (\bar{p}_e^2 - k^2) \sinh(p_x L) + 2i k_x \bar{p}_e \cosh(p_x L), \quad \text{(A15)} \]

where \( p_x = \sqrt{(V_0/\mu_F)(k_x/k_F)^2} \) is the wave number at the insulator and \( \bar{p}_e = p_x/k_F \).

The argument in Sec. II leads to the exact expression of the Andreev and the normal reflection coefficients which are given by

\[ \hat{r}^{ee} = -z_0 z_1 \left[ \frac{\hat{\sigma}_0 - \bar{W}^*}{|1|} \right] \left| z_1^2 \hat{\sigma}_0 - z_0^2 \bar{W}^* \right|^{-1}, \quad \text{(A16)} \]
\[ \hat{r}^{he} = -e^{-i\varphi_x} 4 \bar{p}_e^2 \hat{\sigma}_0 \hat{R}_{(\pm)} \left[ z_1^2 \hat{\sigma}_0 - z_0^2 \bar{W}^* \right]^{-1}, \quad \text{(A17)} \]
\[ \bar{W} = \hat{R}_{(-)} \hat{\Delta}_{(-)}^\dagger \hat{R}_{(+)}. \quad \text{(A18)} \]

The results of unitary states including the spin-singlet states can be obtained when we use following relations

\[ \hat{R}_{(\pm)} = \frac{\sqrt{E^2 - \Delta_{l\pm}^2 - E}}{|D_{l\pm}|^2} \hat{\sigma}_0, \quad \text{(A19)} \]
\[ |D_{l\pm}| = \begin{cases} |d_{l\pm}| & : \text{singlet} \\ |d_{l\pm}| & : \text{triplet}, \end{cases} \quad \text{(A20)} \]

in Eqs. (A10)-(A18). The differential conductance is given by

\[ G_{NS} = \frac{e^2}{h} \sum_{k_y, k_z} \text{Tr} \left[ \hat{\sigma}_0 - \hat{r}^{ee} (\hat{r}^{ee})^\dagger + \hat{r}^{he} (\hat{r}^{he})^\dagger \right], \quad \text{(A21)} \]

A relation \( d_- = -d_+ \) represents the condition for the perfect formation of the ZES. Actually when \( d_+ = d = \nu d_- \) with \( \nu = \pm 1 \), the Andreev reflection probability becomes

\[ \text{Tr} [\hat{r}^{he} (\hat{r}^{he})^\dagger] = \frac{2}{\nu} \left( \frac{4 k_p^2 p_e^2 \Delta_{l^{\pm}} K_l}{4 k_p^2 p_e^2 \Delta_{l^{\pm}}^* + z_0^2 (\Delta_{l^{\pm}} - \nu K_l^2)} \right)^2, \quad \text{(A22)} \]

where \( K_l = K_{l^+} = K_{l^-} \) and \( \Delta_l = \Delta_{l^+} = \Delta_{l^-} \).

In the limit of \( E \rightarrow 0 \) and \( z_0 \gg 1 \), we find

\[ \text{Tr} [\hat{r}^{he} (\hat{r}^{he})^\dagger] = \begin{cases} 2 \left( \frac{4 k_p^2 p_e^2}{z_0^2} \right)^2 & : \nu = 1 \\ 2 & : \nu = -1, \end{cases} \quad \text{(A23)} \]

where spin degree of freedom give rise to a factor 2. Thus the zero-bias conductance is independent of the transmission probability of junctions when \( d_- = -d_+ \) is satisfied.

In spin-singlet superconductors, we show that the internal phase of a Cooper pair is responsible for the ZES. In spin-triplet superconductors, the internal spin degree of freedom of a Cooper pair has another possibilities for the formation of some resonant states in subgap energies.

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