Are We Really Measuring the $\rho$-Value?

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Abstract

The “normal” analysis of $\bar{p}p$ and $pp$ elastic scattering uses a ‘spinless’ Coulomb amplitude, i.e., a Rutherford amplitude $(2\sqrt{\pi}\alpha/t)$ multiplied by a Coulomb form factor $G^2(t)$, an ansatz that pretends that the nucleon does not have any magnetic scattering. In this note, we investigate the role of the anomalous magnetic moment of the nucleon, $\kappa \approx 1.79$. Given the method of analysis currently used by most published experiments, we conclude that the current experimentally inferred values of $\rho$ for $\bar{p}p$ should be systematically lowered by $\approx 0.005–0.0100$ and, correspondingly, the $\rho$ values for $pp$ should be systematically raised by the same amount. We discuss the theoretical uncertainties and a method of experimentally minimizing them.

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I Introduction

We will only calculate electromagnetic amplitudes accurate to order $\alpha$, i.e., one-photon exchange diagrams. Further, we will consider only high energy scattering ($E_{\text{lab}} \gg m$, where $m$ is the nucleon mass) in the region of small $|t|$, where $t$ is the squared 4-momentum transfer. We will measure $m$ and $E_{\text{lab}}$ in GeV and $t$ in $(\text{GeV})^2$, and will use $\hbar = c = 1$.

I.a ‘Spinless’ Coulomb Scattering

If we consider ‘spinless’ proton-antiproton Coulomb scattering, the relevant Feynman diagram is shown in Fig. 2. The electromagnetic differential cross section is readily shown to be

$$
\frac{d\sigma}{dt} = 4\pi G^4(t) \frac{\alpha^2}{\beta_{\text{lab}}^2 t^2} \times \left(1 - \frac{|t|}{4mE_{\text{lab}}}\right)^2,
$$

where the upper (lower) sign is for like (unlike) charges, $t$ is the (negative) 4-momentum transfer squared, and $m$ is the nucleon mass.

For small angle scattering, the term $\left(1 - \frac{|t|}{4mE_{\text{lab}}}\right)^2 \approx 1 - \frac{|t|}{2mE_{\text{lab}}}$ and

$$
\frac{d\sigma}{dt} \approx 4\pi G^4(t) \frac{\alpha^2}{\beta_{\text{lab}}^2 t^2} \times \left(1 - \frac{|t|}{2mE_{\text{lab}}}\right),
$$

At high energies, the correction term $\frac{|t|}{2mE_{\text{lab}}}$ becomes negligible and $\beta_{\text{lab}} \to 1$, so eq. (2) goes over into the well-known Rutherford scattering formula,

$$
\frac{d\sigma}{dt} = \pi \left| \frac{\mp 2\alpha G^2(t)}{|t|} \right|^2,
$$

where the electromagnetic charge form factor $G(t)$ is commonly parameterized by the dipole form

$$
G(t) = \frac{1}{\left(1 - \frac{t}{4\Lambda^2}\right)^2},
$$
where $\Lambda^2 = 0.71$, if $t$ is measured in $(\text{GeV})^2$. We note that this is the Coulomb amplitude that is normally used in the analysis of $\bar{p}p$ and $pp$ elastic scattering, i.e., the ‘spinless’ analysis[1].

I.b $\bar{p}p$ Scattering, Including Magnetic Scattering

The relevant Feynman diagram is shown in Fig. 2, where magnetic scattering is explicitly taken into account via the anomalous magnetic moment $\kappa$ ($\approx 1.79$). The fundamental electromagnetic interaction is

$$eV^\mu = e \left( F_1 \gamma^\mu + i \frac{\kappa}{2m} F_2 \sigma^{\nu\mu} q_\nu \right), \quad q = p_f - p_i \tag{5}$$

which has two form factors $F_1(q^2)$ and $F_2(q^2)$ that are normalized to 1 at $q^2 = 0$. The anomalous magnetic moment of the nucleons is $\kappa$, and $m$ is the nucleon mass. Because of the rapid form factor dependence on $t$, the annihilation diagram for $\bar{p}p$ scattering (or the exchange diagram for $pp$ scattering) is negligible in the small $|t|$ region of interest and has been ignored. The interaction of eq. (5) is most simply treated by using Gordon decomposition and can be rewritten as

$$eV^\mu = e \left[ (F_1 + \kappa F_2) \gamma^\mu - \kappa F_2 \frac{p_f + p_i}{2m} \right]. \tag{6}$$

Thus, using eq. (6), the matrix element for the scattering is

$$M = e\bar{u}(p_f) \left[ -\kappa F_2 \left( \frac{p_f + p_i}{2m} \right) \gamma^\mu + (F_1 + \kappa F_2) \gamma^\mu \right] u(p_i) \times \frac{1}{t} \times$$

$$+ e\bar{u}(p'_f) \left[ -\kappa F_2 \left( \frac{p'_f + p'_i}{2m'} \right) + (F_1 + \kappa F_2) \gamma^\mu \right] u(p'_i), \tag{7}$$

where the upper (lower) sign is for $\bar{p}p$ ($pp$) scattering. A straightforward, albeit laborious calculation, gives a differential scattering cross section

$$\frac{d\sigma}{dt} = 4\pi \frac{\alpha^2}{\beta_{\text{lab}}^2 t^2} \times$$

$$\left\{ (F_1 + \kappa F_2)^4 \left[ 1 + \frac{t}{2} \left( \frac{1}{mE_{\text{lab}}} + \frac{1}{E_{\text{lab}}^2} \right) + \frac{t^2}{8m^2 E_{\text{lab}}^2} \right] \right\}$$

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We now introduce the electric and magnetic form factors, \( G_E(t) \) and \( G_M(t) \), defined as

\[
G_E(t) \equiv F_1(t) + \frac{\kappa t}{4m^2} F_2(t) \quad \text{and} \quad G_M(t) \equiv F_1(t) + \kappa F_2(t),
\]

and rewrite the differential cross section of eq. (8) as

\[
\frac{d\sigma}{dt} \approx 4\pi \frac{\alpha^2}{\beta_{lab}^2} t^2 \times \left\{ \left( \frac{G_E^2(t) - \frac{t}{4m^2} G_M^2(t)}{1 - \frac{t}{4m^2}} \right)^2 \left( 1 + \frac{t}{2mE_{lab}} \right) + \frac{G_M^2(t) - \frac{t}{4m^2} G_E^2(t)}{1 - \frac{t}{4m^2}} \frac{t}{2E_{lab}^2} \right. \\
\left. + \left[ G_M^4(t) + \frac{1}{2} \left( \frac{G_E^2(t) - G_M^2(t)}{1 - \frac{t}{4m^2}} \right)^2 \right] \frac{t^2}{8m^2E_{lab}^2} \right\}
\] (10)

Note in eq. (11) that there is no term in \( G_E G_M \), but only squares of these form factors. We can parameterize these new form factors with

\[
G_E(t) = G(t) = \frac{1}{(1 - \frac{t}{\Lambda^2})^2} \quad \text{where} \quad \Lambda^2 = 0.71,
\]

\[
G_M(t) = (1 + \kappa) G(t) = \frac{1 + \kappa}{(1 - \frac{t}{\Lambda^2})^2},
\]

with \( t \) in GeV/c\(^2\), and where \( G(t) \) is the dipole form factor already defined in eq. (4), i.e., the form factor that is traditionally used in experimental analyses

We now expand eq. (11) for very small \( |t| \), and find that

\[
\frac{d\sigma}{dt} \approx 4\pi \frac{\alpha^2}{\beta_{lab}^2} t^2 G^4(t) \left\{ 1 - \kappa(\kappa + 2) \frac{t}{2m^2} + \frac{t}{2mE_{lab}} + (\kappa + 1)^2 \frac{t}{2E_{lab}^2} \right\},
\]

(12)

where the new term in \( t \), compared to eq. (2), is \(-\frac{\kappa(\kappa+1)}{2m^2} t + \kappa(\kappa+1) \frac{t}{2E_{lab}^2} \approx 1 + 3.86|t| - \frac{339}{E_{lab}} |t|\), and is due to the anomalous magnetic moment of the proton (antiproton). To get an estimate of its effect, we note that \( G^4(t) \approx 1 - 11.26|t| \), in our units where \( t \) is in GeV/c\(^2\). We note that the new term is not negligible in comparison to the squared form factor, reducing the form factor effect by about 35% if the energy \( E_{lab} \) is large compared to \( m \). In this limit, we find that independent of the energy \( E_{lab}^2 \),

\[
\frac{d\sigma}{dt} \approx 4\pi \frac{\alpha^2}{t^2} G^4(t) \{ 1 + 3.86|t| \},
\]

(13)

and is to be compared with the ‘spinless’ Rutherford formula of eq. (3).
II Effects on Experimental Analysis of Elastic Scattering

UA4/2 has recently made a precision measurement of $p-p$ scattering at $\sqrt{s} = 541$ GeV, at the SppS at CERN, in order to extract the $\rho$ value for elastic scattering. We now reanalyze this experiment, taking into account the magnetic scattering. They constrain the total cross section by an independent measurement of $(1+\rho^2)\sigma_{\text{tot}} = 63.3 \pm 1.5$ mb. For their published $\rho$-value of $0.135 \pm 0.015$, this implies that they fix the total cross section at $\sigma_{\text{tot}} = 62.17 \pm 1.5$ mb. The main purpose of the UA4/2 experiment was the measurement of the $\rho$ value, defined by $\rho = \frac{\Re f_n(t=0)}{3m f_n(t=0)}$, where $f_n(t=0)$ is the forward nuclear scattering amplitude.

II.a Spinless Analysis Neglecting Magnetic Scattering

The experimenters parameterized the nuclear slope amplitude as $f_n(|t|) = \frac{\sigma_{\text{tot}}(\rho \pm i)}{4\sqrt{\sigma}} e^{-b|t|/2}$, and measured the nuclear slope parameter as $b = 15.5 \pm 0.2$ GeV$^{-2}$. They fit $\bar{p}p$ elastic scattering data at $\sqrt{s} = 541$ GeV over the $t$-interval $0.00075 \leq |t| \leq 0.12$ GeV$^2$. They used for the Coulomb amplitude the ‘spinless’ Rutherford amplitude, modified by a Coulomb phase factor $i\alpha \phi(t)$, i.e., $f_c(t) = \frac{2\sqrt{\pi}\alpha}{|t|} G^2(t) e^{i\alpha \phi(t)}$, where the phase is given by

$$\phi(t) = \mp \left\{ \gamma + \ln \left( \frac{|t|}{2} \right) + \ln \left( 1 + \frac{8}{b\Lambda^2} \right) + \left( \frac{4|t|}{\Lambda^2} \right) \ln \left( \frac{4|t|}{\Lambda^2} \right) + \frac{2|t|}{\Lambda^2} \right\},$$

where $\gamma = 0.577\ldots$ is Euler’s constant, $b$ is the slope parameter, $\Lambda^2 [= 0.71$ GeV$^2]$ appears in the dipole fit to the proton’s electromagnetic form factor, $G(t)$. The upper sign is for $pp$ and the lower sign for $\bar{p}p$. Using these parameterizations, the differential elastic scattering cross section is

$$\frac{d\sigma}{d|t|} = |f_c + f_n|^2 = \frac{4\pi \alpha}{t^2} G^4(t) + \frac{\alpha \sigma_{\text{tot}}}{|t|} (\rho + \alpha \phi(t)) G^2(t) e^{-b|t|/2} + \frac{\sigma_{\text{tot}}^2 (1 + \rho^2)}{16\pi} e^{-b|t|}.$$  

We now introduce the parameter $t_0$, defined as the absolute value of $t$ where the nuclear and Coulomb amplitudes have the same magnitude, i.e., $t_0 = \frac{8\pi \alpha}{\sigma_{\text{tot}}} = \frac{1}{14.00\sigma_{\text{tot}}}$, when $\sigma_{\text{tot}}$ is in mb, and $t_0$ is in GeV$^2$. For $\sigma_{\text{tot}} = 62.17$ mb, we find that $t_0 = 0.00115$ GeV$^2$. We can now rewrite the differential cross section as

$$\frac{d\sigma}{d|t|} = \frac{\sigma_{\text{tot}}^2}{16\pi} \left\{ G^4(t) t_0^2 + 2 t_0 \left( \rho + \alpha \phi(t) \right) G^2(t) e^{-b|t|/2} + \left( 1 + \rho^2 \right) e^{-b|t|} \right\},$$

for $t_{\text{min}} \leq t \leq t_{\text{max}}$, which was the form used by the UA4/2 group to analyze their experimental data. They extracted the value $\rho = 0.135 \pm 0.015$, with a statistical error of 0.007. We emphasize that their analysis, using eq. (16), neglected the effects of the anomalous magnetic moments of the nucleons.

II.b ‘Spinless’ Analysis, Taking into Account the Magnetic Scattering

For our small $|t|$ analysis, we approximate $G^4(t)$ as $G^4(t) \approx 1 - 2a|t|$, where $a \equiv \frac{4}{\Lambda^2} = 5.6338$. We further write $G^2(t) e^{-b|t|/2} \approx 1 - (a + \frac{b}{2})|t|$. However, if we take into account
the anomalous magnetic moments, we see from eq. \(13\) that we could have written \(f_c(t) \approx 2\sqrt{\frac{\pi}{|t|}} e^{\alpha \phi(t)}\), an energy-independent result. Literally, we have used the ‘spinless’ ansatz that the Coulomb amplitude is given by the square root of the Coulomb cross section. Expanding the above equation in \(|t|\), incorporating the new factor of \(1+1.93|t|\), we can rewrite in a concise form the ‘correct’ Coulomb amplitude as

\[
f_c(t) \approx 2\sqrt{\frac{\pi}{|t|}} G_{\text{eff}}(t) e^{i\alpha \phi(t)},
\]

where

\[
G_{\text{eff}}^2(t) \approx 1 - a' |t| = 1 - (a - 1.93)|t| = 1 - 3.674|t|.
\]

Thus, \(a' = 3.674\), and we can now write a modified form for the cross section which mimics eq. \(16\) as

\[
\frac{d\sigma'}{dt} = \frac{\sigma_{\text{tot}}^2}{16\pi} \left\{ G_{\text{eff}}^4(t) \frac{t_0^2}{t^2} + 2 \frac{t_0}{|t|}(\rho' + \alpha \phi(t))G_{\text{eff}}^2(t)e^{-b|t|/2} + (1 + \rho'^2)e^{-b|t|} \right\},
\]

for \(t_{\min} \leq |t| \leq t_{\max}\), by using \(G_{\text{eff}}(t)\) in place of \(G(t)\), i.e., by replacing \(a\) by \(a'\) and \(\rho\) by \(\rho'\).

In order to extract the new value of \(\rho'\) from eq. \(19\) without having to refit directly the experimental data, we will require that the integral of eq. \(19\) be equal to the integral of eq. \(16\), from \(t_{\min}\) to \(t_{\max}\). This insures that we fit the measured events with both formulae, allowing us to solve for \(\rho'\), given the UA4/2 published value of \(\rho = 0.135\). Thus, we require that

\[
\int_{t_{\min}}^{t_{\max}} \frac{d\sigma'}{dt} = \int_{t_{\min}}^{t_{\max}} \frac{d\sigma}{dt},
\]

To obtain an approximate analytical solution, we can set equal the contributions of the terms \((1 + \rho'^2)e^{-b|t|}\) and \((1 + \rho'^2)e^{-b|t|}\), and expand in \(|t|\) to first order. We find the approximate solution for \(\rho'\), the corrected value of \(\rho\), valid when we set the contribution of \(\alpha \phi(t) = 0\),

\[
\rho' = \frac{(a' - a)t_0 + \rho \left(1 - (a + b/2)\frac{(t_{\max} - t_{\min})}{\ln(t_{\max}/t_{\min})}\right)}{1 - (a' + b/2)\frac{(t_{\max} - t_{\min})}{\ln(t_{\max}/t_{\min})}}.
\]

\[
\approx \rho + (a' - a) \left(t_0 + \rho \frac{t_{\max} - t_{\min}}{\ln(t_{\max}/t_{\min})}\right) \quad \text{if} \quad t_0 \ll \rho \frac{t_{\max} - t_{\min}}{\ln(t_{\max}/t_{\min})}
\]

We will later see numerically that neglecting the \(\alpha \phi(t)\) contribution is a reasonable approximation, since the average value of \(\alpha \phi(t) = -0.00036 \approx 0\) over the \(t\)-range in question. We find, using the UA4/2 extracted values described above, that \(\Delta \rho = \rho' - \rho = \rho' - 0.135 = -0.0114\). An exact numerical solution of eq. \(20\), where we expand to first order in \(|t|\) yields the final answer \(\Delta \rho_{\text{UA4}} = -0.0105\). This shift in \(\rho\) is comparable to the quoted total error of \(\pm 0.015\) and is larger than the statistical error of \(\pm 0.007\). The solution in completely insensitive to the slope parameter \(b\), as well as to \(\alpha \phi(t)\). It mainly depends on the value of \(\rho\) that’s measured, and critically on the \(t\) interval that was measured. We note that these conclusions are in contrast with the analysis of this problem made by N. Buttimore\(6\), which did not take into account the experimental \(t\) interval.
Since all high energy $\rho$ values come from experiments that measure about the same range of $t$ and similar $b$ values, it is likely that all the $\rho$ values for $\bar{p}p$ scattering need to be lowered by $\sim 0.005-0.010$ and that all $pp$ values need to be raised by about the same amount. For example, E710 measured $\rho_{\bar{p}p} = 0.134 \pm 0.069$, $\sigma_{\text{tot}} = 72.2 \pm 2.7$ mb and $b = 16.72$ GeV$^{-2}$, with $t_{\text{min}} = 0.00075$ GeV$^2$ and $t_{\text{max}} = 0.077$ GeV$^2$, at $\sqrt{s} = 1.8$ TeV. In this case, the exact numerical solution of eq. (20) gives $\Delta \rho_{E710} = -0.004$. Thus, any global fit to the $\rho$ values of $\bar{p}p$ and $pp$ scattering should take these systematic shifts in the $\rho$ values (at all energies) into account.

II.c A Nuclear Model Taking Account of Magnetic Scattering

In the preceding Section, we emulated the ‘spinless’ analysis by ignoring the structure of the nuclear scattering, and not distinguishing between spin flip and non-spin flip of the nuclear scattering, which we will now remedy. In this Section, we will introduce a toy nuclear model, which mimics the known electromagnetic scattering, which we will now introduce. In this Section, we will introduce a toy nuclear model, which mimics the known electromagnetic scattering, and will also introduce “electric” and “magnetic” nuclear form factors $G_{EN}$ and $G_{MN}$, analogous to eq. (5), with it. We thus write the total matrix element for coherent nuclear and Coulomb scattering, taking into account the Bethe phase as expressed by Cahn[4], and adds coherently with it. We thus write the total matrix element for coherent nuclear and Coulomb scattering, taking into account the Bethe phase as expressed by Cahn[4], as

$$M'_t = \pm \frac{4\pi \alpha \exp i\delta(t)}{|t|} \bar{u}(p_f) \left[ -\kappa F_2 \left( \frac{p_f + p_i}{2m} \right)_\mu + (F_1 + \kappa F_2 )\gamma_\mu \right] u(p_i) \times$$

$$\bar{u}(p'_f) \left[ -\kappa F_2 \left( \frac{p'_f + p'_i}{2m'} \right)_\mu + (F_1 + \kappa F_2 )\gamma_\mu \right] u(p'_i) +$$

$$\frac{4\pi g}{m^2} \bar{u}(p_f) \left[ -\kappa N H_2 \left( \frac{p_f + p_i}{2m} \right)_\mu + (H_1 + \kappa N H_2 )\gamma_\mu \right] u(p_i) \times$$

$$\bar{u}(p'_f) \left[ -\kappa N H_2 \left( \frac{p'_f + p'_i}{2m'} \right)_\mu + (H_1 + \kappa N H_2 )\gamma_\mu \right] u(p'_i),$$

where now the upper sign is for $\bar{p}p$ and the lower sign is for $pp$ scattering, since we introduced $|t|$ into eq. (22). We have substituted $e^2 = 4\pi \alpha$ and defined the strong coupling analog of $\alpha$ as $h^2 = 4\pi g$, where $h$ is the (complex) nuclear ‘charge’, $\kappa_N$ is the nuclear ‘anomalous magnetic moment’ and $H_1(t)$ and $H_2(t)$ are the nuclear form factors. We have replaced the electromagnetic propagator $|t|$ by the nuclear propagator $m^2$. Later, we will use the optical theorem to fix the real and imaginary portions of $h^2$ and will also introduce “electric” and “magnetic” nuclear form factors $G_{EN}$ and $G_{MN}$, analogous to eq. (4), with

$$G_{EN}(t) \equiv H_1(t) + \frac{\kappa_N}{4m^2} H_2(t) \quad \text{and} \quad G_{MN}(t) \equiv H_1(t) + \kappa_N H_2(t),$$

and

$$G_{EN}(t) = G_N(t) = e^{bt/4}$$

$$G_{MN}(t) = (1 + \kappa_N)G_N(t) = (1 + \kappa_N)e^{bt/4}.$$

The squaring of $M'_t$ in eq. (22) will give rise to three terms,

$$|M_t|^2 = |M_c|^2 + 2|M_c| (\Re M_N + \alpha \phi) + |M_N|^2,$$
since $e^{i\alpha \phi(t)} \approx 1 + i\alpha \phi(t)$. The first term of eq. (25) corresponds to pure Coulomb scattering, the last term to pure nuclear scattering and the term $2|M_c|(\Re e M_N + \alpha \phi)$ to the coherent interference cross section between nuclear and Coulomb amplitudes. We note that the Dirac structure of all three terms in eq. (25) is the same, which greatly simplifies the evaluation. We have already calculated the term $|M_c|^2$ and the substitution of $\alpha \rightarrow g$, $F_1 \rightarrow H_1, F_2 \rightarrow H_2$ and $1/b_{1m} \rightarrow 1/m$ in the Coulomb term gives us the nuclear term $|M_N|^2$. Thus we find by inspection of eq. (12) that the nuclear differential cross section is given by

$$
\frac{d\sigma_N}{dt} = 4\pi \frac{g^2}{\beta_{lab}^2 m^4} \times
$$

$$
\left\{ \frac{G_{EN}^2(t) - \frac{t}{4m^2} G_{M_N}^2(t)}{1 - \frac{t}{4m^2}} \right\}^2 (1 + \frac{t}{2mE_{lab}})
+ G_{M_N}^2(t) \frac{G_{EN}^2(t) - \frac{t}{4m^2} G_{M_N}^2(t)}{1 - \frac{t}{4m^2}} \frac{t}{2E_{lab}}
+ \left[ G_{M_N}^4(t) + \frac{1}{2} \left( \frac{G_{EN}^2(t) - G_{M_N}^2(t)}{1 - \frac{t}{4m^2}} \right)^2 \right] \frac{t^2}{8m^2E_{lab}^2} \right\}.
$$

We now expand eq. (26) for very small $|t|$, using eq. (24) and find that

$$
\frac{d\sigma_N}{dt} \approx 4\pi \frac{g^2}{\beta_{lab}^2 m^4} e^{\beta t} \left\{ 1 - \kappa_N (\kappa_N + 2) \frac{t}{2m^2} + \frac{t}{2mE_{lab}} + (\kappa_N + 1)^2 \frac{t}{2E_{lab}^2} \right\}.
$$

Using the optical theorem, we now rewrite

$$
\left( \frac{d\sigma_N}{dt} \right)_{t=0} = \frac{\sigma_{tot}^2 (1 + \beta^2)}{16\pi} = \frac{4\pi}{\beta_{lab}^2 m^4} g^2,
$$

since $G_N(0) \equiv 1$. Inspection of eq. (28) yields the equivalent statement that the (complex) value of the nuclear coupling is $\frac{g}{\beta_{lab} m^4} = (\rho + i) \frac{2\kappa_N}{8\pi}$. Using eq. (28), we can rewrite the nuclear differential scattering cross section of eq. (27) as

$$
\frac{d\sigma_N}{dt} = \frac{\sigma_{tot}^2 (1 + \beta^2)}{16\pi} e^{\beta t} \left\{ 1 - \kappa_N (\kappa_N + 2) \frac{t}{2m^2} + \frac{t}{2mE_{lab}} + (\kappa_N + 1)^2 \frac{t}{2E_{lab}^2} \right\}.
$$

The linear term in the brackets of eq. (27) or eq. (29) is clearly the spin flip term induced by the nuclear “anomalous magnetic moment” $\kappa_N$ and goes to zero in the forward direction, as must be true for spin flip amplitudes. At ultra-high energies, eq. (27) goes over to

$$
\frac{d\sigma_N}{dt} = \frac{\sigma_{tot}^2 (1 + \beta^2)}{16\pi} e^{\beta t} \left( 1 - \frac{\kappa_N (\kappa_N + 2)}{2m^2} \right)
$$

Again, with a lengthy, but straightforward calculation, we find that the interference cross section $\frac{d\sigma_{CN}}{dt}$ is given by

$$
\frac{d\sigma_{CN}}{dt} = \pm \frac{\alpha}{\beta_{lab} |t|} \sigma_{tot} (\rho + \alpha \phi(t)) \times
$$

$$
\left\{ \left[ \frac{G_E(t) - \frac{t}{4m^2} G_M(t)}{1 - \frac{t}{4m^2}} \right] \left( \frac{G_{EN}(t) - \frac{t}{4m^2} G_{M_N}(t)}{1 - \frac{t}{4m^2}} \right) \right\}.
$$
Finally, in the high energy limit, eq. (32) simplifies to

\[
- \left( \frac{G_E(t) - G_M(t)}{1 - \frac{t}{4m^2}} \right) \left( \frac{G_{E_N}(t) - G_{MN}(t)}{1 - \frac{t}{4m^2}} \right) \frac{t}{4m^2} \left( 1 + \frac{t}{2mE_{lab}} \right) \\
+ G_M(t)G_{MN}(t) \frac{G_E(t)G_{E_N}(t) - \frac{t}{4m^2}G(t)G_{MN}(t)}{1 - \frac{t}{4m^2}} \frac{t}{2E_{lab}^2} \\
+ \left[ (G_M(t)G_{MN}(t))^2 + \frac{1}{2} \left( \frac{G_E(t)G_{E_N}(t) - G_M(t)G_{MN}(t)}{1 - \frac{t}{4m^2}} \right)^2 \right] \frac{t^2}{8m^2E_{lab}^2} \right). \tag{30}
\]

Taking the limit of eq. (30) for small \( t \), we find that

\[
\frac{d\sigma_{CN}}{dt} = \pm \frac{\alpha G^2(t)}{\beta_{lab}^2 |t|} \sigma_{tot} e^{b/t/2} \left[ \rho + \alpha \phi(t) \right] \times \\
\left[ 1 - \left( \frac{\kappa + \kappa_N + \kappa\kappa_N}{2m^2} + \frac{1}{2mE_{lab}} - \frac{(\kappa + 1)(\kappa_N + 1)}{2E_{lab}^2} \right) |t| \right] \tag{31}
\]

Introducing again the parameter \( t_0 = \frac{8\alpha}{\sigma_{tot}} \), we write, in the small \( t \) limit, the elastic differential scattering cross section \( \frac{d\sigma}{dt} \) for coherent Coulomb and nuclear interactions as

\[
\frac{d\sigma}{dt} = \frac{\sigma_{tot}^2}{16\pi} \left\{ \frac{t_0^2}{t^2} G^4(t) \left[ 1 + \left( \frac{\kappa(\kappa + 2)}{2m^2} - \frac{1}{2mE_{lab}} + \frac{1}{2E_{lab}^2} \right) |t| \right] \\
\mp 2 \frac{t_0}{|t|} \frac{G^2(t)}{\beta_{lab}} e^{-b|t|/2} \left[ \rho + \alpha \phi(t) \right] \times \\
\left[ 1 + \left( \frac{\kappa + \kappa_N + \kappa\kappa_N}{2m^2} - \frac{1}{2mE_{lab}} + \frac{(\kappa + 1)(\kappa_N + 1)}{2E_{lab}^2} \right) |t| \right] \\
+(1 + \rho^2) e^{-b|t|} \left[ 1 + \left( \frac{\kappa_N(\kappa_N + 2)}{2m^2} - \frac{1}{2mE_{lab}} + \frac{1}{2E_{lab}^2} \right) |t| \right] \right\} \tag{32}
\]

Finally, in the high energy limit, eq. (33) simplifies to

\[
\frac{d\sigma}{dt} = \frac{\sigma_{tot}^2}{16\pi} \left\{ \frac{t_0^2}{t^2} G^4(t) \left[ 1 + \frac{\kappa(\kappa + 2)}{2m^2} |t| \right] \\
\mp 2G^2(t) \frac{t_0}{|t|} \left( \rho + \alpha \phi(t) \right) e^{-b|t|/2} \left[ 1 + \frac{\kappa + \kappa_N + \kappa\kappa_N}{2m^2} |t| \right] \\
+(1 + \rho^2) e^{-b|t|} \left[ 1 + \frac{\kappa_N(\kappa_N + 2)}{2m^2} |t| \right] \right\} \tag{33}
\]

In order to use eq. (33), we must expand the nuclear term for small \( t \)

\[
\left( \frac{d\sigma}{dt} \right)_N = \frac{\sigma_{tot}^2}{16\pi} (1 + \rho^2) e^{-b'|t|} \left[ 1 + \frac{\kappa_N(\kappa_N + 2)}{2m^2} |t| \right] \\
\approx \frac{\sigma_{tot}^2}{16\pi} (1 + \rho^2) \left[ 1 - \left( b - \frac{\kappa_N(\kappa_N + 2)}{2m^2} \right) |t| \right] \\
\approx \frac{\sigma_{tot}^2}{16\pi} (1 + \rho^2) e^{-b'|t|}, \quad \text{where} \quad b' = b - \frac{\kappa_N(\kappa_N + 2)}{2m^2}. \tag{34}
\]
Effectively, it is $b'$ that is measured, not $b$. Thus we rewrite eq. (33) as

$$\frac{d\sigma}{dt} \approx \frac{\sigma_{\text{tot}}^2}{16\pi} \left\{ \frac{t^2}{t^2} G^4(t) \left[ 1 + \frac{\kappa(\kappa + 2)}{2m^2}\frac{1}{t} \right] \right. \\
+ 2G^2(t) - 1 + \frac{2\kappa + 2\kappa\kappa_N - \kappa^2}{4m^2} \frac{1}{t} \right\} e^{-\kappa b'|t|/2} \left( 1 + \frac{1}{t} \right) \left[ 1 + \frac{\kappa(\kappa + 2)}{2m^2}\frac{1}{t} \right]$$

(35)

We really have no deep knowledge of a value that is appropriate for $\kappa_N$, let alone its sign. Since the known polarization at Fermilab energies [7, 8] is small, of the order of several percent, a magnitude compatible with it being produced by Coulomb interactions, this suggests that $|\kappa_N| \lesssim \kappa$. It is tempting, however, to set $\kappa_N = \kappa$, since the conventional interpretation of the anomalous magnetic moment $\kappa$ is that it arises from the strong interactions. If we equate $\kappa$ and $\kappa_N$, we find the same interference term of eq. (33) where we “ignore” spin, i.e., $2G^2(t) b|t| e^{-\kappa b'|t|/2} (1 + 1.93|t|)$. Thus, the numerical evaluation made earlier in Section II.A is valid—namely, for UA4/2, the $\rho$ value changes by $\approx -0.01$.

On the other hand, even if we set $\kappa_N = 0$, we still would have an interference term proportional to $(1 + 1.02|t|)$.

We see from eq. (21) that for $\kappa_N = 0$ that the shift in $\rho$, which is proportional to $a' - a$, is now reduced by $1.02/1.93 = 0.53$. Thus, the $\rho$ value for $\kappa_N = 0$ is shifted by $\approx -0.006$. We conclude that for reasonable values of the parameter $\kappa_N$, there is a significant shift in the $\rho$ value of UA4/2 due to the anomalous magnetic moment of the nucleon. The theoretical uncertainty in the $\rho$-value can be reduced if the experimenters break their data up into two distinct regions—Region 1 being the interference region from $t_{\text{min}}$ to $\approx 10 \times t_0$ and Region 2 from $10 \times t_0$ to $t_{\text{max}}$. In the case of UA4/2, this would change the uncertainty in $\Delta \rho_{\text{UA4}}$ from $\approx 0.006$ to $\approx 0.001$, if we found $\rho$ from Region 1 and $\sigma_{\text{tot}}$ and $b$ from Region 2, even assuming that $\kappa_N = 0$.

The other theoretical uncertainty is in the value of $\alpha \Phi(t)$. The original derivation [4] assumed that we had ‘spinless’ scattering, that the nuclear amplitude was $e^{b't/2}$, where $b'$ was the measured slope, and that the Coulomb amplitude was $2G^2(t)$. We see that we really should be using $\frac{4}{3} G_{\text{eff}}^2(t)$, or, using $\Lambda^2 \approx 1.0$ rather than $\Lambda^2 = 0.71$, in eq. (14). Fortunately, this is a very small change, and increases the $\bar{p}p \rho$-value by $\approx 0.001$, thus contributing negligibly to $\Delta \rho_{\text{UA4}}$.

In conclusion, it seems sensible for experimenters to redo their data analysis using $\kappa_N = \kappa$, i.e., using $G_{\text{eff}}(t)$ and $\Lambda^2$ rather than $G(t)$ and $\Lambda^2$, in the two distinct $t$-regions described above. This procedure allows the experimenter to control the theoretical uncertainties, i.e., our lack of knowledge of the nuclear amplitudes.

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