Non-axisymmetric flow excited by fluid oscillations in a rotating cylinder with sloping ends

Stanislav Subbotin

Perm State Humanitarian Pedagogical University, Laboratory of vibrational hydromechanics, Sibirskaya St. 24, 614990, Perm, Russia.

E-mail: subbotin_sv@pspu.ru

Abstract. Fluid flow in non-uniform rotating (librating) cylinder with sloping ends is experimentally investigated. Periodical changes in the rotation rate lead to the appearance of inertial oscillations of the fluid. Due to the azimuthal inhomogeneity of the geometry of the ends, inertial oscillations are non-axisymmetric. The most intense pulsating motion of the fluid is observed at the frequency corresponding to the mode \{1, 1, 1\}, which is a global single-vortex flow. As a result of the nonlinear response, a non-axisymmetric steady flow arises along the cavity side wall.

1. Introduction

In rotating fluids, the Coriolis force supports an internal oscillating motion known as inertial waves. In enclosed cavities, these waves can experience spatial resonance in the form of large-scale vortex structures, which are the eigenmodes in the cavity of a given geometry and are called inertial modes. In the experiments, inertial modes can be excited in different ways: librations (periodic changes in the rotation rate) [1–3], precession of the rotation axis [4, 5], tidal deformations of the cavity boundary [6, 7], and differential rotation of the inner core [8–10].

Fluid flow in a librating cylinder with straight ends was studied in [3, 11]. Due to the azimuthal symmetry of the cavity geometry, only axisymmetric inertial oscillations where generated. In the work [3] it was found that oscillations lead to the appearance of steady circulation in the dynamic Stokes layer on the side wall of the cavity. The flow structure has the form of a system of averaged axisymmetric toroidal vortices the number of which depends on the axial wave number of the inertial mode. Violation of the geometric symmetry of the cavity leads to the emergence of new interesting flow regimes. Thus, due to the presence of one sloping wall, the inertial waves can be focused into the wave attractor, where the energy is concentrated [12]. At the same time, a lot of interest is non-axisymmetric modes and the related non-linear response in the form of steady flow. To generate non-axisymmetric oscillations in the present study the slope of both end walls is used. In this case, the pulsational velocity field has a spatial azimuthal inhomogeneity with \(k = 1\). The main attention is paid to the study of the instantaneous flow structure in the
axial section of the cavity depending on the libration phase, as well as the structure and intensity of steady flow.

2. Experimental setup and technique

The cavity is a cylinder of circular cross-section, the ends of which are inclined at an angle of $\alpha = 8.5^\circ$ (Fig. 1). The distance between the ends is $L = 90.0$ mm, and the diameter of the cavity is $D = 52.0$ mm. To describe the fluid motion a cylindrical coordinate system $(r, \phi, z)$ with the origin in the center of the cavity is used. The cavity is filled with a water-glycerin solution of kinematic viscosity $\nu = 34$ cSt and rotates around the axis of symmetry in the laboratory reference frame according to the law:

$$\mathbf{\omega}(t) = \Omega + \Delta \varphi \Omega_{\text{lib}} \sin(\Omega_{\text{lib}} t),$$

where $\Omega$ is the mean rotation rate of the cavity, $\Omega_{\text{lib}}$ is the angular libration radian frequency, and $\Delta \varphi \Omega_{\text{lib}}$ is the amplitude of modulation of rotation rate. As a dimensionless characteristic of the libration frequency, the parameter $\sigma = \Omega_{\text{lib}} / \Omega$ is used. The value of the mean rotation rate in all experiments is $\Omega = 31.4$ s$^{-1}$, while the frequency of librations varies in the range $\Omega_{\text{lib}} = 11.3$ – 41.4 s$^{-1}$.

The cavity rotation is set by a stepper motor FL86STH118-6004A controlled by a driver SMD-8.0 and powered by a DC power supply Mastech HY5005E. The rotation of the motor shaft is adjusted by a Zet 210 Sigma USB generator. To minimize the optical distortions from the sidewall, the cylindrical cavity is placed in a hollow Plexiglas parallelepiped filled with a working fluid.

The experimental technique is as follows. The cavity is brought into uniform rotation around the axis of symmetry with the speed $\Omega$. After the transition to the steady-state rotation of the fluid, librations with the frequency $\Omega_{\text{lib}}$ and the amplitude $\varepsilon \varphi = \Delta \varphi \sigma$ are set. The velocity field in the axial section of the cavity is investigated by the PIV method. For this purpose, particles of the Resin Amberlite Visualizer with a size of $d \approx 60 \mu$m and a density of $\rho \approx 1.04$ g/cm$^3$ are added to the working fluid. The liquid is illuminated along the axis of rotation by a continuous laser Z-Laser Z 500 Q, and the position of the light-scattering particles is recorded on a video camera Optronis CamRecord CL600x2.

Figure 1. Scheme of the experimental setup

To calculate the averaged fluid velocity $\bar{\mathbf{u}}$, pairs of frames with a time interval multiple of the period of the mean rotation rate are selected. Note that the averaged velocity field is calculated over at least ten libration periods. Cross-correlation analysis of the sequence of images by the PIV method is carried out in the PIVlab program [13].
3. Inertial fluid oscillations and steady flow

The variations in the rotation rate of the cavity lead to the appearance of the oscillating motion of the fluid. Perturbations are generated at the junction of the side and end walls of the cavity, and propagate in the fluid bulk in the form of inertial waves [14]. The characteristic surface along which waves propagate has the form of a cone formed by free oscillating shear layers. Since the cavity ends are not axisymmetric, the cone is displaced from the axis of rotation. Thus, the inertial waves propagating from each cavity end are azimuthally inhomogeneous with the azimuthal wavenumber $k = 1$.

Depending on the libration frequency, the vorticity created by inertial waves after their multiple reflections from the cavity walls can add up. The observations show that the strongest response to librations occurs at a frequency of $\sigma = 0.52$. A series of instantaneous velocity fields obtained in the axial section passing through the largest and smallest diagonals of the cylinder is shown in fig. 2. During the half-period of librations, a single-vortex flow is formed and destroyed. This vortex is oriented predominantly along the larger diagonal of the cavity. During the next half-period of librations, the vorticity of the flow changes to the opposite. It can be seen that the most intense oscillating motion occurs in the phases of librations $\Omega_{\text{lib}}t = \pi/4$ and $\Omega_{\text{lib}}t = 5\pi/4$.

Figure 2. Snapshots of the velocity fields in the axial section of the cavity at $\sigma = 0.52$ and $\epsilon = 0.08$ in different phases of librations: $\Omega_{\text{lib}}t = \pi/4, \pi/2, 3\pi/4, \pi$ (left row, top down); $\Omega_{\text{lib}}t = 5\pi/4, 3\pi/2, 7\pi/4, 2\pi$ (right row, top down). The color shows the $z$-component of the fluid velocity.
The velocity fields in a rectangular section of the cavity are shown in fig. 3. In this section, the fluid oscillations are symmetrical with respect to the ends. The maximums of the pulsation flow occur in phases \( \Omega_{\text{lib}} t = 3\pi/4 \) and \( \Omega_{\text{lib}} t = 7\pi/4 \), which differs from the phase of the pulsation flow maximum in another section of the cavity (fig. 2) by \( \pi/2 \). At the same time, the vorticity pulsations \( \text{rot}_v u \) during the libration period are similar (where \( u \) is the \( z \)-component of the fluid velocity). The difference between the times of occurrence of the maximum pulsational velocity in different axial sections indicates that during the libration period the vortex structure turns in the azimuthal direction. With a change in the libration frequency, the system leaves the resonance region, resulting in the intensity of the oscillatory motion decreases.

The appearance of intense fluid oscillations is associated with the excitation of inertial modes [15]. Since we consider non-axisymmetric oscillations of the fluid for the spatial characteristic of the inertial mode the wave numbers \( \{n, m, k\} \) are used. The flow regime described at \( \sigma = 0.52 \) corresponds to the inertial mode \( \{1, 1, 1\} \). This value is in satisfactory agreement with the natural frequency of the inviscid inertial mode, \( \sigma^\prime_{111} = 0.48 \).

In previous works [3, 11], inertial modes in a librating cylinder with straight ends were considered. Due to the symmetry of the perturbations propagated from the cavity ends, only axisymmetric modes with an even axial wave number \( n \) were excited. Here we found that the violation of the azimuthal symmetry of the cavity geometry qualitatively changes the pulsational velocity field. Due to the small slope of the ends, the fluid oscillations are non-axisymmetric manner, resulting in inertial modes with an odd wavenumber \( n \). The observations show that with an increase in the libration frequency, the number of vortices in the cavity axial section monotonically increases.

![Figure 3. Snapshots of the velocity fields in the axial section of the cavity at \( \sigma = 0.52 \) and \( \varepsilon = 0.08 \) in different phases of librations: \( \Omega_{\text{lib}} = \pi/4, \pi/2, 3\pi/4, \pi \) (left row, top down); \( \Omega_{\text{lib}} = 5\pi/4, 3\pi/2, 7\pi/4, 2\pi \) (right row, top down). The color shows the \( z \)-component of the fluid velocity.](image-url)
Fluid oscillations excited by the inertial mode lead to the appearance of steady flow in the viscous Stokes boundary layer near the cavity side wall. The steady flow structure is manifested in the experiment after averaging the velocity field over the period of librations. Fig. 4 shows the averaged velocity fields obtained in two mutually perpendicular planes for the libration frequency $\sigma = 0.52$. The flow has the form of jets that beat along the entire side wall. Note, that the direction of fluid motion in the jets changes to the opposite through the azimuth angle $\approx \pi$. The transverse size of the jets is determined by several thicknesses of the Stokes boundary layer $\delta = \sqrt{2\nu/\Omega_{\lib}}$. It is seen that at a distance $r/R \approx 0.4$, the opposite flow with a significantly lower intensity arises. Note, that in previous experiments with axisymmetric ends [3] the steady flow has the form of a system of toroidal vortices. The size of vortex structures along the rotation axis was consistent with the spatial period of the pulsational flow. In this case, an oscillating vortex cell excites fluid oscillations along the entire length of the Stokes layer, resulting in the steady jets extend from one end to the other.

As it was shown in the work [3], the most important parameter that determines the transverse size of steady flow is the dimensionless frequency of fluid oscillations $\omega = \Omega_{\lib} D^2 / 4\nu$, which characterizes the relative thickness of the dynamic boundary layer $\delta$. Moreover, this parameter is responsible for the transformation of the flow structure. Thus, in the high-frequency region ($\omega > 2 \cdot 10^3$) the number of steady vortices is determined by the expression $n + 2$. The additional two vortices are generated in the cavity corners and are associated exclusively with rotational oscillations of the cavity rather than inertial modes. In the case under consideration, the value of the frequency is $\omega = 420$, which corresponds to the region of moderate frequencies. In this region of parameters, a strong interaction of flows caused by different mechanisms is observed [3]. It can be expected that with an increase in $\omega$, the transverse size of the jets will decreases, while the steady corner flow caused by the rotational oscillations will manifest itself.

![Figure 4. Velocity field averaged over the libration period in the axial section of the cavity at the parameters corresponding to Figures 2-3](image-url)
4. Conclusions
The structure of oscillatory and steady flow excited by librations of a rapidly rotating cylinder with sloping ends is experimentally investigated. The non-axisymmetric flow occurs due to the azimuthally inhomogeneous geometry of the cylinder ends. It is found that modes with different azimuthal wave numbers appear in the cavity. Thus, librations with a frequency $\sigma = 0.52$ excite the mode $\{1, 1, 1\}$, which is a single-vortex flow that turns in the azimuthal direction at an angle $\pi/2$ during the libration period. The steady flow has the form of jets that beat along the side wall of the cavity. PIV-studies of the velocity field in two different axial sections show that the direction of fluid motion in the jets changes to the opposite through the azimuth angle $\sim \pi$.

Acknowledgments
The research was supported by a grant of the President of the Russian Federation (Project No. MK-1779.2021.1.1) and the Ministry of Education and Science of the Perm region (grant No. C-26/1191).

References
[1] Aldridge K D and Toomre A 1969 J. Fluid Mech. 37(2) 307–323
[2] Boisson J et al. 2012 Phys. Fluids 24 076602
[3] Subbotin S 2020 Phys. Rev. Fluids 5(1) 014804
[4] Malkus W 1968 Science 160(3825) 259–264
[5] Lin Y et al. 2014 Phys. Fluids 26 046604
[6] Morize C et al. 2010 Phys. Rev. Lett. 104 214501
[7] Favier B et al. 2014 Mon. Not. R. Astron. Soc. 439 845–860
[8] Kelley D H et al. 2010 Phys. Rev. E 81(2) 026311
[9] Rieutord M et al. 2012 Phys. Rev. E 86(2) 026304
[10] Hoff M et al. 2016 Phys. Rev. Fluids 1 043701
[11] Sauret A et al. 2012 Phys. Fluids 24 026603
[12] Manders A M M and Maas L R M 2003 J. Fluid Mech. 493 59–88
[13] Thielicke W and Stamhuis E J 2014 J. Open Res. Softw. 2(1) e30
[14] Messio L et al 2008 Exp. Fluids 44 519–528
[15] Greenspan H P 1968 The Theory of Rotating Fluids (University Press, Cambridge)