Roles of Diquarks
in the Nucleon for the Deep Inelastic Scattering
and the Non-leptonic Weak Transitions

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Abstract: We study roles of quark-quark correlations in the baryons for the deep inelastic structure function and the $I=1/2$ rule of the non-leptonic weak hyperon decay. The quark-quark correlation is incorporated phenomenologically as diquarks within the Nambu and Jona-Lasinio model. The strong diquark correlations in the spin-0 channel enhance the $I=1/2$ weak matrix elements, and account for the $I=1/2$ rule. The ratio of the nucleon structure functions $F_2^n(x) = F_2^p(x)$ also comes to agree with experiment due to the diquark correlations.

1 Introduction

Quark-quark correlations inside the nucleon

Recently models of the baryons have attracted considerable interest. These models, e.g., MIT bag model and Skyrme ion, possess some essential aspects of the low energy QCD, and give successful descriptions for static baryon properties such as mass and magnetic moments. However, these models do not take into account quark-quark correlations inside the baryon. In the QCD sum rule calculation, asymmetric moment distributions in the proton are found, where the most part of the proton moment is carried by the u-quark with its spin directed along the proton spin and the remaining small part is carried by the u-d quarks with combined spin-0 [1]. Instanton liquid model of the QCD vacuum also shows that the quark correlations in the spin-0 and spin-1 channels are completely different, due to the large attractive correlation in the spin-0 channel [2]. These results indicate that the quark-quark correlation in the spin-0 channel is quite strong, and is of great importance to discuss the baryon structure. This strong correlation in the spin-0 channel may be understood as a close connection between the quark pair in the spin-0 channel ($q_i \bar{q}_j$) and the pion ($\pi^0$), through a particle-antiparticle symmetry, i.e., Pauli-Gursey symmetry [3,4]. (Here, $q_i$ are the SU(3) color antitriplet operators, and $\bar{q}_i$ the charge conjugation of the quark $q$.) Due to the spontaneous breakdown of chiral...
symmetry, the Goldstone pion is a collective state of the QCD vacuum. Hence, we expect that the strong correlation exists in the corresponding spin-0 \(qq\) channel.

In this study, we introduce "D quirks", i.e. correlated two quark states inside the baryons, to incorporate the quark-quark correlation in the phenomenological way [5]. Assuming the SU(6) spin–avor symmetry, the baryon wave functions are written as products of diquarks and quarks. For example, the proton wave function is given by

\[
\hat{p}^+ = \frac{1}{18} [S(ud)u^+ + 2A(uu)^+ d^+ + P_{2A}(uu)^0 d^+] + A(ud)^0 u^+];
\]

where \(S(ud)\) denotes the scalar (spin-0, \(0^+\)), isospin-0 diquark, and \(A(\mathbf{i}j)\) the axial-vector (spin-1, \(1^+\)) diquark with the \(\mathbf{a}v\) or content \(\mathbf{i}\) and \(\mathbf{j}\) and the helicity \(\sigma\). These quantum numbers are determined by the antisymmetrization of the diquark wave function. In order to incorporate the non-perturbative quark-quark correlations, namely, structures of diquarks, we use the Nambu and Jona-Lasinio (NJL) model [6]. This model demonstrates the spontaneous chiral symmetry breaking, and reproduces the SU(3) \(f\) meson properties very well with the parameters fixed by the pion and kaon properties [3]. The constituent quark mass, the diquark mass, and the diquark-quark coupling constant are obtained by solving the gap equation and the Bethe-Salpeter equation simultaneously.

2. \(I=1=2\) non-leptonic hyperon decay and diquark correlation

The puzzle of the non-leptonic weak decay and the \(I=1=2\) rule has a long history. In the standard factorization approximation, theoretical calculations show serious disagreements with experiments (Table 1). Although the renormalization group method for gluon exchange contributions indicates an enhancement of \(I=1=2\) amplitude and a suppression of \(I=3=2\) amplitude, this effect is too small.

Recently, Stech and his collaborators pointed out that the scalar diquarks play a crucial role for the parity conserving part (\(P\)-wave) of \(I=1=2\) transitions [7]. We consider a possible intermediate state contribution, which is the so called pole diagram as shown in Fig.1, where an initial hyperon \(B_1\) changes to an intermediate state baryon \(B_m\) by the weak interaction, and then \(B_m\) emits a pion to produce a final state baryon \(B_f\). The scalar diquark appears in the calculation of these weak transition matrix elements, since the effective weak Hamiltonian is rewritten by the diquark annihilation and creation operators by virtue of the Fierz transformation [7].

\[
H = \frac{G_F \sin \theta}{2} \cos \phi \delta \left[ \mu \begin{pmatrix} 1 & \bar{s} \\ s & 1 \end{pmatrix} \right] \delta \left[ \mu \begin{pmatrix} 1 & \bar{s} \\ s & 1 \end{pmatrix} \right] + c_2 \left[ \mu \begin{pmatrix} 1 & \bar{s} \\ s & 1 \end{pmatrix} \right] \delta \left[ \mu \begin{pmatrix} 1 & \bar{s} \\ s & 1 \end{pmatrix} \right] + \cdots
\]

\[
= \frac{G_F \sin \theta}{2} \cos \phi \left( c_1 c_2 \frac{2}{3} \delta \left[ \mu \begin{pmatrix} 1 & \bar{s} \\ s & 1 \end{pmatrix} \right] \delta \left[ \mu \begin{pmatrix} 1 & \bar{s} \\ s & 1 \end{pmatrix} \right] + \cdots \right)
\]

Hence, this weak process occurs as a transition between spin-0 diquarks, \((us)^{0+}\) to \((ud)^{0+}\) with the remaining quark unchanged. Note that \((ud)^{0+}\) scalar diquark necessarily has
I = 0, and \((us)^{0+}\) has I = 1/2 due to the Pauli principle. Therefore, the strong scalar diquark correlations enhance only the I = 1/2 weak matrix elements, and thus explain the I = 1/2 rule. Using the NJL model to calculate the diquark weak decay matrix elements, we obtain numerical results in Table 1 [8]. The results of the factorization including the penguin contributions are listed in the second column, and the diquark contributions in the third column. In the fourth column, the sum of the diquark and the factorization is listed, which is compared with experimental data indicated in the fifth column. The diquark process enhances largely the transition amplitudes, and the result is in a good agreement with experiment.

Table 1 P-wave amplitude \(10^{-7}\)

|       | Fact. | Diquark | Total | Exp |
|-------|-------|---------|-------|-----|
| \(^{+}\) | 0.00  | 37.0    | 37.0  | 41.8 |
| \(^{+}\) | 2.05  | 24.5    | 26.5  | 26.7 |
| \(0\)   | 3.92  | 2.47    | 1.45  | 1.44 |
| \(0\)   | 9.60  | 12.0    | 21.6  | 22.4 |
| \(k\)   | 2.40  | 18.8    | 16.4  | 17.5 |

We note that estimations of the pole diagram within other effective models of the baryon yield much smaller values than experimental data, due to the absence of the quark correlation in these models. For instance, the MIT bag model gives 25.3 for \(^{+}\) amplitude, which disagrees with data.

3 Deep Inelastic Nucleon Structure Function and Diquarks

On the other hand, the nucleon structure functions provide us with the information on the spin-avor structure in the nucleon. The ratio of nucleon structure functions \(F_{2}^{n}(x) = F_{2}^{p}(x)\) shows a clear deviation from the naive quark-parton model value; 2/3. We evaluate the nucleon structure function within the diquark-quark model, and discuss how the avor structure of the structure function depends on the diquark correlation.

In this model, both quark and diquark scattering processes contribute to the structure functions [9]. The quark scattering process (Fig. 2a), in which a quark is struck out by the virtual photon with the residual diquark being a spectator, is evaluated by using the standard method in the impulse approximation [10]. For the diquark part (Fig. 2b), we obtain the distribution function as the convolution of quark distributions in diquarks with the diquark distributions in the nucleon. This diagram represents the leading-twist contributions of diquarks, where diquarks break up completely after absorbing the virtual photon. Concerning the quark distributions in diquarks, we use the same procedure as done in the meson case in terms of the NJL model [11].
As a result, the proton and neutron structure functions are written as

\[
F_2^p(x) = \frac{2}{9} q_S^p(x) + \frac{1}{9} q_V^p(x) + \frac{5}{18} Q_S^p(x) + \frac{7}{18} Q_V^p(x),
\]
\[
F_2^n(x) = \frac{1}{18} q_S^n(x) + \frac{1}{6} q_V^n(x) + \frac{5}{18} Q_S^n(x) + \frac{1}{6} Q_V^n(x);
\]

where \(q_S^p\) and \(q_V^p\) are the quark distributions with the residual diquarks being the scalar and the axial-vector diquarks, respectively (Fig. 2a). \(Q_S^p\) and \(Q_V^p\) are the quark distributions obtained by the scalar and the axial-vector diquark scattering processes, respectively (Fig. 2b). The flavor dependence of the structure functions arises from the difference of the distribution contents in this model. The correlations in the spin-0 and spin-1 diquarks affect the quark distributions, mainly through their masses. We note that this approach naturally reproduces the asymmetry of momentum distributions in the nucleon obtained by the QCD sum rule [1]. To see this point, we calculate the momentum fraction carried by each quark. In the scalar channel, we find \(<xq_S^p>_1 <xQ_S^p>_2 \approx 2:1\), which is consistent with the result of ref. [1], while we get \(<xq_V^p>_1 <xQ_V^p>_2 \approx 1:1\) in the axial-vector channel. These results reflect the strength of the correlation in each channel. In the axial-vector channel, the quark correlation is too weak that the asymmetry of the momentum distribution is negligible.

Our calculated quark distribution gives a boundary condition for the QCD perturbation at the low energy model scale \(\sqrt{\frac{2}{1 GeV^2}}\), at which the effective quark model is supposed to work [12]. The distribution functions are evolved to the experimental high momentum scale with the help of the Altarelli-Parisi equation.

The resulting ratio \(F_2^n(x)=F_2^p(x)\) is shown in Fig. 2. The ratio \(F_2^n(x)=F_2^p(x)\) depends on the strength of the quark-diquark correlation, and is the same value as the parton model prediction without the spin-0 diquark correlations (dashed curve). If the correlation in the scalar channel is so strong as to reproduce the non-leptonic transitions, it is in a reasonable agreement with the data (solid curve), except for the small x region, where the sea quark effects dominate.

4 Summary

We have studied the diquark correlation inside the baryon, and its contribution to the non-leptonic hyperon weak decay and the deep inelastic scattering. The diquark correlation, suggested by the QCD sum rule and the instanton liquid model, is the strongest in the spin-0 and isospin-0 channel. We comment on the result of the lattice QCD calculation, in which the diquark-like clustering is not observed [13]. At the present, such lattice simulations are performed within the quenched approximation. However, it is essential to include the light dynamical quark for the discussion of the hadron structure. Thus, the quenched approximation may be inadequate to study such diquark clusters.

We have used the simple SU(6) diquark-quark model to incorporate the quark correlations within the NJL model. Such a diquark correlation enhances the \(I = 1\)
weak transition amplitudes, and reproduces the experimental data very well. The diquark correlations also produce the asymmetric momentum distribution inside the nucleon, as obtained in the QCD sum rule calculation. Due to this asymmetric distribution, the ratio of the nucleon structure functions becomes smaller than the naive parton model value at the large Bjorken $x$, and is consistent with experiment. Our results indicate the importance of the quark-diquark correlation in the nucleon.

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Fig. 1: Pole diagram for the non-leptonic hyperon decay

Fig. 2: The forward scattering amplitude of the nucleon. The thin solid line represents the quark, and the shaded line the diquark. The nucleon and the virtual photon are depicted by the solid and the wavy lines.

Fig. 3: The ratio of the neutron-proton structure functions at $Q^2 = 15$ GeV$^2$. The experimental data are taken from the EMC, BCDMS, and NMC experiments. The solid curve is obtained by taking into account the strong scalar diquark correlation. The dashed curve is the result where the channel difference of the diquark mass is neglected [9].
Fig. 3

Ratio of the nucleon structure functions