Cosmology with moving bimetric fluids

Carlos García-García, Antonio L. Maroto and Prado Martín-Moruno

Departamento de Física Teórica I, Universidad Complutense de Madrid, E-28040 Madrid, Spain

E-mail: cargar08@ucm.es, maroto@ucm.es, pradomm@ucm.es

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Abstract. We study cosmological implications of bigravity and massive gravity solutions with non-simultaneously diagonal metrics by considering the generalized Gordon and Kerr-Schild ansätze. The scenario that we obtain is equivalent to that of General Relativity with additional non-comoving perfect fluids. We show that the most general ghost-free bimetric theory generates three kinds of effective fluids whose equations of state are fixed by a function of the ansatz. Different choices of such function allow to reproduce the behaviour of different dark fluids. In particular, the Gordon ansatz is suitable for the description of various kinds of slowly-moving fluids, whereas the Kerr-Schild one is shown to describe a null dark energy component. The motion of those dark fluids with respect to the CMB is shown to generate, in turn, a relative motion of baryonic matter with respect to radiation which contributes to the CMB anisotropies. CMB dipole observations are able to set stringent limits on the dark sector described by the effective bimetric fluid.

Keywords: modified gravity, CMBR theory, dark energy theory

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1 Introduction

Despite the recent advances in cosmology, there are fundamental questions about the universe that still remain unsolved. One of such questions is its composition. Recent measurements from type Ia supernovae confirmed by cosmic microwave background (CMB) and large scale structure observations have shown that our Universe is expanding in an accelerated way [1–3]. In order to explain the observed behavior, cosmologists postulated the existence of a cosmological constant or dark energy fluid that can only be detected by its gravitational effects. Assuming the validity of General Relativity (GR), it should comprise around the 68% of the total energy density of the universe. In addition, around the 27% of the total content is taken to be cold dark matter, some kind of fluid with low pressure that seems to weakly interact with ordinary matter so that, to date, its existence can only be inferred from its gravitational effects. This leaves us with only a 5% of known origin that would be composed of baryonic matter and a negligible part (0.01%) of radiation. In addition, yet another dark fluid could exist: dark radiation. However, it would only be a 10%, at most, of the radiation energy density [3]. On the other hand, the CMB dipole, quadrupole and octupole exhibit an unexpected allignment [4, 5], which might suggest the possible existence of a preferred direction in the Universe. Furthermore, the possible existence of a dark flow [3, 6], that is, a coherent motion of matter with respect to the CMB on cosmological scales, would also support this idea.
One could think that this complicated picture may be just a signal of the breakdown of GR on very large scales. Following this line of thought we will explore the cosmological implications of well-known theories as massive gravity and bigravity. Massive gravity dates from 1939, when Fierz and Pauli tried to give mass to the carrier particle of the gravitational interaction, i.e. the graviton [7]. In the first place, it was found that the massless limit of massive gravity could not recover GR, where the graviton only has 2 propagating degrees of freedom in contrast to a massive graviton which necessarily propagates 5. This fact lies under the appearance of the vDVZ (van Dam-Veltman-Zakharov) discontinuity [8, 9]. However, Vainshtein showed some years later that the extra degree of freedom responsible for that discontinuity could be screened by its own interaction, which dominates over the linear terms in the massless case [10]. In the second place, non-linear massive gravity was thought to be always affected by the Boulware-Deser instability [11], that is a state with negative kinetic energy leading to an unbounded Hamiltonian. Nonetheless, it has been recently discovered that there are particular theories [12] in which this ghost can be avoided [13] (see also references [14, 15] and references therein). On the other hand, the existing astrophysical and cosmological observations can constraint the value of the graviton mass to be \( m < 7.2 \times 10^{-23} \) eV. In any case, a graviton with \( m \ll 10^{-33} \) eV could not be distinguished from a massless graviton [16, 17].

Non-linear massive gravity is formulated using a non-dynamical reference metric in addition to the dynamical one describing our spacetime [18]. This bimetric nature of massive gravity connects it directly with bigravity [19], in which both metrics are dynamical, although it must be kept in mind that they are conceptually and phenomenologically different theories [20]. However, the consideration of the same term of interaction between the metrics that avoids the Boulware-Deser ghost in massive gravity [12] has led to the formulation of a stable bigravity theory [21]. The existence of a second gravitational sector in bigravity allows the presence of two kinds of matter fields, each one minimally coupled to its respective dynamical metric. Furthermore, the consequences of more general kinds of couplings, which we will not consider in the present work since they re-introduce the instability, have been also investigated [22, 23].

Cosmological solutions of both types of ghost-free theories have been obtained recently in the literature. In the case of stable massive gravity theories formulated using different reference metrics [24, 25], it has been shown that either there is no homogeneous and isotropic accelerating solutions [26, 27] or they are affected by the Higuchi instability [28]. On the other hand, although self-accelerating cosmological solutions have been found in stable bigravity [29, 30], early-time fast growing modes [31–33] have to be carefully avoided [34, 35]. These works considered solutions where both metrics can be written in a diagonal way in the same coordinate patch (see reference [36] and references therein). In the present paper we will go beyond that assumption studying the cosmological consequences of massive gravity and bigravity solutions where both metrics are not necessarily diagonal in the same coordinate patch. In order to simplify the treatment of these bimetric theories, we will focus our attention on solutions with particular causal relations between both metrics, described by the generalized Gordon ansatz and the Kerr-Schild ansatz [37]. As we will show, the non-diagonality of both metrics in the same coordinate patch can be re-interpreted from the point of view of standard GR as an effective perfect fluid which is non-comoving with respect to the CMB. The relative motion of the effective bimetric fluid induces, in turn, a relative motion of baryons with respect to the CMB after recombination thus contributing to the CMB dipole. This effect opens the possibility to observationally constrain the possible values of the free parameters of these moving bimetric theories.
This work is organized as follows. In section 2, ghost-free massive gravity and bigravity will be briefly reviewed. In section 3, we will impose the generalized Gordon ansatz and study the resulting effective fluids in section 3.1. Its cosmological implications will be studied in section 4, considering the slow-moving regime compatible with a FLRW solution. In section 4.2, the monocomponent effective fluid and its effect on the CMB dipole will be investigated, while in section 4.3 the multicomponent case will be considered. In section 5, the Kerr-Schild anstaz will be used to consider the fast-moving regime. In section 5.1, the resulting null-fluid and its natural subcomponents will be studied. The cosmological case with a Bianchi I metric will be investigated in section 6. Finally, in section 7 the conclusions will be summarized.

2 Massive gravity and bigravity

The action of massive gravity and that of bigravity are written in terms of two metric tensors $g_{\mu\nu}$ and $f_{\mu\nu}$, where $g_{\mu\nu}$ is the standard metric to which the observed physical fields are coupled, whereas $f_{\mu\nu}$ is the new reference metric introduced with the theory in massive gravity and an additional dynamical field in bigravity. Thus the action can be written in general as

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R(g) + 2\Lambda - 2m^2L_{\text{int}}(g,f) \right] + S_{(m)} - \frac{\kappa}{16\pi G} \int d^4x \sqrt{-f} \left[ \bar{R}(f) + 2\bar{\Lambda} \right] + \epsilon \bar{S}_{(m)},$$

(2.1)

where $R$ is the curvature scalar of $g_{\mu\nu}$, overlined variables are defined using $f_{\mu\nu}$, $m$ is the graviton mass, $G$ the gravitational constant, and $\kappa$ and $\epsilon$ are constants which vanish in the case of massive gravity (since $f_{\mu\nu}$ is non-dynamical). We will work in natural units $\hbar = c = 1$. We consider that the matter sectors described through $S_{(m)}$ and $\bar{S}_{(m)}$ are minimally coupled to $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively. These theories are ghost-free [13, 21] if the interaction term is given in terms of the matrix $\gamma = \sqrt{g^{-1}f}$, that is

$$\gamma^\mu_\nu \gamma^\nu_\rho = g^{\mu\rho}f_{\nu\nu},$$

(2.2)

as [12]

$$L_{\text{int}} = \beta_1 e_1(\gamma) + \beta_2 e_2(\gamma) + \beta_3 e_3(\gamma),$$

(2.3)

where the elementary symmetric polinomials are

$$e_1(\gamma) = \text{tr}(\gamma),$$

(2.4)

$$e_2(\gamma) = \frac{1}{2} \left( \text{tr}(\gamma)^2 - \text{tr}(\gamma^2) \right),$$

(2.5)

$$e_3(\gamma) = \frac{1}{6} \left( \text{tr}(\gamma)^3 - 3\text{tr}(\gamma)\text{tr}(\gamma^2) + 2\text{tr}(\gamma^3) \right),$$

(2.6)

with $\beta_n$ being arbitrary constants. It is important to note that there are two additional terms proportional to $\beta_0$ and $\beta_4$ respectively, that have been absorbed into the two cosmological constants $\Lambda$ and $\bar{\Lambda}$ [20, 38] as they appear in the interaction Lagrangian through $\beta_0 e_0(\gamma) = \beta_0$ and $\beta_4 e_4(\gamma) = \beta_4 \det(\gamma)$. 

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Varying the action (2.1) with respect to the dynamical metrics (only one for massive gravity), one obtains the Einstein equations for both sectors [20]

\[ G_{\mu \nu} - \Lambda \delta_{\mu \nu} = m^2 T_{\mu \nu} + 8\pi G T^{(m)}_{\mu \nu}, \]  

(2.7) and

\[ \kappa \left( G_{\mu \nu} - \Lambda \delta_{\mu \nu} \right) = m^2 T_{\mu \nu} + \epsilon 8\pi G T^{(m)}_{\mu \nu}, \]

(2.8)

where \( T^{(m)}_{\mu \nu} \) and \( \overline{T}^{(m)}_{\mu \nu} \) are the stress-energy tensors of the material components of each sector, while \( T_{\mu \nu} \) and \( \overline{T}_{\mu \nu} \) are the effective stress-energy tensors that encapsulate the interaction between both metrics. These effective tensors are [39]

\[ T_{\mu \nu} = \tau_{\mu \nu} - \delta_{\mu \nu} L_{\text{int}}, \]

(2.9) and

\[ \overline{T}_{\mu \nu} = -\sqrt{-g} \sqrt{-f} \tau_{\mu \nu}, \]

(2.10)

with

\[ \tau_{\mu \nu} = \gamma_{\mu} \frac{\partial L_{\text{int}}}{\partial \gamma_{\nu \rho}}. \]

(2.11)

It should be noted that indices in equations (2.7) and (2.8) are raised and lowered with \( g \) and \( f \), respectively. In addition, it is important to remark that in massive gravity, equation (2.8) is absent. As the Bianchi constrain is satisfied, one has

\[ \nabla_{\mu} T_{\mu \nu} = 0 \quad \text{and} \quad \nabla_{\mu} \overline{T}_{\mu \nu} = 0. \]

(2.12)

It can be seen that both constraints are equivalent [39]. Thus, the conservation of the effective fluid in the \( g \)-sector implies the conservation of the other effective fluid in the \( f \)-sector, and vice versa. Finally, it is important to remark that both \( T_{\mu \nu} \) and \( \overline{T}_{\mu \nu} \) are linear in \( \beta_i \). Then, we can redefine them so that \( m^2 T_{\mu \nu} \rightarrow 8\pi G T_{\mu \nu} \) and similarly for the overlined tensor. This way, \( \beta_i \rightarrow \beta_i' = m^2 / (8\pi G) \beta_i \). We will drop the prime and use these \( \beta_i' \) in the subsequent sections.

3 Perfect bimetric fluid

In this section we will restrict our attention to a particular set of solutions of massive gravity and bigravity. We will consider a particular relation between the causal structures of both metrics that leads to an effective stress-energy tensor with the form of a perfect fluid. This relation is given by the generalized Gordon ansatz [37]:

\[ f_{\mu \nu} = \Omega^2 (g_{\mu \nu} + \xi V_{\mu} V_{\nu}), \]

(3.1)

where \( \Omega \) and \( \xi \) are arbitrary functions, and \( V_{\mu} \) is a timelike vector with \( g_{\mu \nu} V^\mu V^\nu = -1 \). One can also work with the function \( \xi \neq 0 \), defined by \( \xi = 1 - \zeta^2 \), that better parametrizes the fact that we need \( \xi < 1 \) to have both metrics with Lorentzian signature, implying also that \( V_{\mu} \) is timelike with respect to \( f_{\mu \nu} \). The generalized Gordon ansatz can be interpreted as a conformal transformation combined with a stretch along the timelike direction parallel to \( V_{\mu} \). This ansatz relates the position of the light cones of both metrics, that is the light cones of \( f_{\mu \nu} \) lie strictly outside (inside) the light cones of \( g_{\mu \nu} \), if \( \xi < 0 \) (\( \xi > 0 \)), having the metrics
the same null vectors if $\xi = 0$. It must be emphasized that we are only restricting attention to a particular kind of solutions. In the case of bigravity we are assuming that the material content in the $f$-sector is such that it generates a metric $f_{\mu\nu}$ of the form given by equation (3.1) through equations (2.8). Then, one should obtain $g_{\mu\nu}$ (completely fixing $f_{\mu\nu}$) by considering equations (2.7). For massive gravity our approach consists in considering only theories with a reference metric of the form expressed in the ansatz (3.1). This implies that not all stable massive gravity theories that we could construct will have this kind of solutions, as $g_{\mu\nu}$ has to be determined through equation (2.7) taking into account this $f_{\mu\nu}$ and the material content. Indeed, if one completely fixes the functions appearing in the ansatz (3.1), then only a given kind of material content will be compatible with these solutions. This issue will be further clarified in section 4.

The Gordon ansatz implies that the square root matrix takes the form

$$\gamma^{\mu}_{\nu} = \Omega\{(\delta^{\mu}_{\nu} + V^{\mu}V_{\nu}) - \zeta V^{\mu}V_{\nu}\},$$

(3.2)

where the principal square root of the matrix, corresponding to a positive value of $\zeta$, was taken in reference [37]. Therefore, the stress-energy tensor of the effective fluid takes the perfect fluid form [37]

$$T^{\mu}_{\nu} = (\rho_{BF} + p_{BF})V^{\mu}V_{\nu} + p_{BF}\delta^{\mu}_{\nu},$$

(3.3)

where BF stands for bimetric fluid and with

$$\rho_{BF} = \Omega(3\beta_{1} + 3\beta_{2}\Omega + \beta_{3}\Omega^{2}),$$

(3.4)

$$p_{BF} = -\Omega[(\beta_{1} + \beta_{2}\Omega) + (\beta_{1} + 2\beta_{2}\Omega + \beta_{3}\Omega^{2})\zeta] .$$

(3.5)

Furthermore, one can obtain that the effective stress-energy tensor for the $f$-sector is [37]

$$\mathcal{T}^{\mu}_{\nu} = (\mathcal{p}_{BF} + \mathcal{p}_{BF})\mathcal{V}^{\mu}\mathcal{V}_{\nu} + \mathcal{p}_{BF}\delta^{\mu}_{\nu},$$

(3.6)

with

$$\mathcal{p}_{BF} = -\frac{1}{\Omega^{3}}(\beta_{1} + 3\beta_{2}\Omega + 3\beta_{3}\Omega^{2}),$$

(3.7)

$$\mathcal{p}_{BF} = -\frac{1}{\Omega^{3}\zeta}\left[\beta_{1} + 3\beta_{2}\Omega(2 + \zeta) + \beta_{3}\Omega^{2}(1 + 2\zeta)\right],$$

(3.8)

and $\mathcal{V}_{\mu} = \Omega \zeta V_{\mu}$ with $f_{\mu\nu}\mathcal{V}^{\mu}\mathcal{V}^{\nu} = -1$. Note that the pre-factors in equations (3.7) and (3.8) are due to the term $\sqrt{-f} = \Omega^{4}\zeta\sqrt{-g}$ in equation (2.10). In the present paper we will not restrict our study to the principal square root, $\zeta > 0$, due to the phenomenological interest of solutions with $\zeta < 0$. It should be pointed out, however, that when considering the branch $\zeta < 0$ one should consistently take all the negative signed square roots in the $f$-sector, in particular those appearing in the $f$-volume element $d^{4}x\sqrt{-f}$ and in the vector $\mathcal{V}_{\mu}$, which would be pointing to the opposite direction to $V^{\mu}$. Consequently, solutions of the negative branch may propagate a spin-2 ghost. Therefore, the stability and consistency of solutions with $\zeta < 0$ should be carefully considered.

### 3.1 Interacting effective perfect fluids

As we have summarized, in massive gravity and bigravity the effects of the interaction of both metrics can be encapsulated in an effective stress-energy tensor, which takes the form of a perfect fluid when one restrict attention to solutions satisfying the generalized Gordon
ansatz. We can go further and split the effective fluid in other three different perfect fluids that will share a common velocity and evolve in a way that conserve energy and momentum as a whole, but not necessarily separately. Taking into account equations (3.4), (3.5), (3.7), and (3.8), such a natural decomposition yields

\[
\begin{align*}
\rho_1 &= 3\beta_1 \Omega, \\
\rho_2 &= 3\beta_2 \Omega^2, \\
\rho_3 &= 3\beta_3 \Omega^3, \\
\omega_1 &= -\frac{1}{3} (2 + \zeta), \\
\omega_2 &= -\frac{1}{3} (1 + 2\zeta), \\
\omega_3 &= -\zeta; \\
\end{align*}
\]

and

\[
\begin{align*}
\bar{\rho}_1 &= \beta_1 \Omega^{-3}, \\
\bar{\rho}_2 &= 3\beta_2 \Omega^{-2}, \\
\bar{\rho}_3 &= 3\beta_3 \Omega^{-1}, \\
\bar{\omega}_1 &= -\zeta^{-1}, \\
\bar{\omega}_2 &= -\frac{2 + \zeta}{3\zeta}, \\
\bar{\omega}_3 &= -\frac{1 + 2\zeta}{3\zeta}. \\
\end{align*}
\]

The fluids of both sectors have the same effective equation of state parameter when \( \omega_n = \bar{\omega}_n = \{-1, 1/3\} \). It must be noted that the first case corresponds to that of two conformally related metrics, which is known to be equivalent to having an extra contribution to the cosmological constant for both sectors. Moreover, both effective densities can be expressed in general by

\[
\rho_n = \rho_{n0} (\Omega/\Omega_0)^n \quad \text{and} \quad \bar{\rho}_n = \bar{\rho}_{n0} (\Omega/\Omega_0)^{n-4},
\]

with \( n = 1, 2, 3 \); and \( \bar{\omega}_n \) can be written in terms of \( \omega_n \) as

\[
\begin{align*}
\bar{\omega}_1 &= \frac{1}{3\omega_1 + 2}, \\
\bar{\omega}_2 &= \frac{1 - \omega_2}{3\omega_2 + 1}, \\
\bar{\omega}_3 &= \frac{1 - 2\omega_3}{3\omega_3}. \\
\end{align*}
\]

In order to better understand the nature of these effective fluids, \( \omega(\zeta) \) and \( \bar{\omega}(\zeta) \) have been plotted in figure 1, where we have restricted to values \( \omega_n \lesssim 1 \). As one could expect from the arguments at the beginning of this section, the \( \bar{\omega} \) plot reveals that the case \( \zeta = 0 \) is not well defined. It must be noted that for \( \zeta = 1 \), both metrics are conformally related and \( \omega_n = \bar{\omega}_n = -1 \) for \( n = 1, 2, 3 \). Finally, it should be emphasized that two more effective components are generated by these bimetric theories when \( \beta_0 \neq 0 \) and \( \beta_3 \neq 0 \), which, as mentioned before, naturally describe a cosmological constant in each gravitational sector.

\section{4 Slow-moving bimetric fluid: FLRW solutions}

In this section we will study cosmological solutions based on the generalized Gordon ansatz introduced above. Thus, we will restrict ourselves to the case in which the free functions appearing in the ansatz are just function of time, i.e. \( \Omega = \Omega(\eta), \xi = \xi(\eta) \) and \( V^\mu = a^{-1}(1, \vec{v}_{\text{BF}}(\eta)) \), where we have assumed \( |\vec{v}_{\text{BF}}| \ll 1 \) in order to avoid the generation of large anisotropies. Then, the stress-energy tensor for each fluid will be of the form

\[
T^\mu_{(\alpha)\nu} = (\rho_{(\alpha)} + p_{(\alpha)}) V^\mu_{(\alpha)} V_{(\alpha)\nu} + p_{(\alpha)} \delta^\mu_{\nu},
\]

with \( \rho_{(\alpha)} \), the density function, \( p_{(\alpha)} \) the pressure, and \( V_{(\alpha)} = a^{-1}(1, \vec{v}_{(\alpha)}(\eta)) \) the velocity for the fluid \( \alpha = R, \text{DR}, \text{DM}, \text{B}, \text{BF}, \cdots \) (radiation, dark radiation, dark matter, baryonic matter,
Figure 1. Equation of state parameter of the effective fluid’s components of the \(g\)-sector (left) and the \(f\)-sector (right) as a function of \(\zeta\). As it should be expected, the case \(\zeta = 0\) is not well defined.

bimetric fluid, \ldots\) which we also assume to be small, i.e. \(|\vec{v}_{(\alpha)}| \ll 1\). We will use \(\alpha\) for all fluids in our universe, including the bimetric one, while we will use the subscript \(n = 1, 2, 3\) for the bimetric fluid subcomponents. Therefore, a metric compatible with this scenario in the \(g\)-sector is

\[
ds_g^2 = a^2(\eta)\left(-d\eta^2 - 2S_i d\eta dx^i + (\delta_{ij} + 2F_{ij})dx^i dx^j\right),
\]

where \(S_i\) and \(F_{ij}\) are two homogeneous time-dependent functions.

To first order in velocities, the total stress-energy tensor is

\[
T^0_0 = -\sum_\alpha \rho_{(\alpha)},
\]

\[
T^i_0 = -\sum_\alpha \left(\rho_{(\alpha)} + p_{(\alpha)}\right)v^i_{(\alpha)},
\]

\[
T^0_i = \sum_\alpha \left(\rho_{(\alpha)} + p_{(\alpha)}\right)\left(v^i_{(\alpha)} - S^i\right),
\]

\[
T^i_j = \sum_\alpha p_{(\alpha)} \delta^i_j,
\]

where the sum goes through the different kind of fluids. Considering the modified Einstein equations (2.7), we see from the \((i\ j)\) component that \(F_{ij}\) vanishes to first order in velocities and from the \((0\ i)\) components, we get

\[
\vec{S} = \frac{\sum_\alpha (\rho_{(\alpha)} + p_{(\alpha)})\vec{v}_{(\alpha)}}{\sum_\alpha (\rho_{(\alpha)} + p_{(\alpha)})},
\]

revealing that \(\vec{S}\) is the cosmic center of mass velocity [40]. We can choose to work in the center of mass frame with \(\vec{S} = 0\), leaving the metric as

\[
ds_g^2 = a(\eta)^2\left[-d\eta^2 + \delta_{ij}dx^i dx^j\right].
\]

Considering equation (3.1), the metric in the \(f\)-sector is

\[
ds_f^2 = \Omega^2 a^2\left[(-1 + \xi)d\eta^2 - 2\xi\vec{v}_B dx^i dx^j + \delta_{ij}dx^i dx^j\right].
\]

It can be checked that it is indeed Lorentzian for all \(\xi < 1\). It is easy to note that when \(V^\mu\) is the comoving timelike vector, both metrics will be FLRW and expressed in a diagonal way in
the same coordinate patch, as it was seen in reference [37]. The fact that the ansatz entails a stretching along the timelike direction parallel to \( V^\mu \), which is not necessarily parallel to the comoving timelike vector, is the ultimate responsible for having a non diagonal \( f_{\mu \nu} \) and a moving bimetric dark fluid with respect to the comoving coordinates. Finally, let us note that the stress energy tensor for the material components of the \( f \)-sector, described by \( T^{(m)\mu \nu} \), is also of the perfect fluid form. It can be seen from the modified Einstein equations (2.8) that if there were no material components in the \( f \)-sector, \( G_{\mu \nu} \propto T_{\mu \nu} \), where \( T_{\mu \nu} \) is known from equation (3.6) to be a perfect fluid, as well. Then, \( G_{\mu \nu} \) must be compatible with perfect fluids solutions. As a consequence, if both \( T_{\mu \nu} \) and \( G_{\mu \nu} \) are to preserve this character, the additional component \( T^{(m)\mu \nu} \) must be a perfect fluid as well.

We can now find the conservation equations in the \( g \)-sector up to first order in \( \nu_\alpha \), which are the same as those obtained in general relativity with moving fluids [40]. These are:

- **Energy conservation**
  \[
  \rho'(\alpha) + (\rho(\alpha) + p(\alpha))3H = 0, \quad (4.10)
  \]

- **Momentum conservation**
  \[
  \partial_\eta [a^4(\rho(\alpha) + p(\alpha))i(\alpha)] = 0. \quad (4.11)
  \]

We have used \( ' = \partial_\eta \) and \( H \equiv a'/a \). We see that each fluid will preserve energy and momentum separately and that \((\rho(\alpha) + p(\alpha))i(\alpha) \propto a^{-4}\). This implies, for an equation of state parameter which can depend on time, that

\[
  i(\alpha) = \frac{1 + \omega_0}{1 + \omega(\alpha)} \frac{\rho_0}{\rho(\alpha)} a^{-4}, \quad (4.12)
\]

where the subscript 0 is for their present values and \( \alpha = R, DR, BF, \cdots \). The conservation equations have the well-known solutions for a constant equation of state parameter

\[
  \rho(\alpha) = \rho_0 a^{-3(1+\omega_0)}, \quad (4.13)
  \]

\[
  i(\alpha) = \frac{1 + \omega_0}{1 + \omega(\alpha)} \frac{\rho_0}{\rho(\alpha)} a^{-4}. \quad (4.14)
\]

Now, let us focus our attention on the effective bimetric fluid. In this case, the energy conservation equation can be rewritten in terms of \( \Omega \) using equations (3.4) and (3.5). This leads to

\[
  \frac{\Omega'}{\Omega} \frac{1}{1 - \zeta} + H = 0, \quad (4.15)
\]

when \( \zeta \neq 1 \). As before, for the particular case of a constant function \( \zeta \) this equation can be easily solved and leads to

\[
  \Omega = \Omega_0 a^{\zeta^{-1}}. \quad (4.16)
\]

If we split the effective fluid in its three subcomponents, the energy conservation equation becomes

\[
  \sum_n [\rho'_n + (\rho_n + p_n)3H] = 0, \quad n = 1, 2, 3. \quad (4.17)
\]

As each \( \rho_n \propto \Omega^n \), each summand must vanish individually, implying

\[
  \rho'_n + (\rho_n + p_n)3H = 0, \quad n = 1, 2, 3. \quad (4.18)
\]
That is, each component of the effective fluid conserves energy individually. Nevertheless, as they all move with the same velocity, no such separation can be done in the momentum conservation equation, which remains

$$\partial_t \left[ a^4 \sum_n (\rho_n + p_n) \vec{v}_{BF} \right] = 0 \quad n = 1, 2, 3. \quad (4.19)$$

Therefore, this effective fluid originated by the interaction between both metrics can be interpreted as having three subcomponents, which are perfect fluids as well, but characterized by the fact that while their energy is individually conserved, their momenta are not. These three fluids share the same velocity (the effective fluid velocity) which is given by equation (4.12).

Finally, it must be noted that we have started by restricting attention to solutions of the form given by equations (4.8) and (4.9), keeping free the functions $a(t)$, $\Omega(t)$, $\zeta(t)$, and $v(t)$. We want to emphasize that we have not altered the theory considering an additional requirement, but we have focused our attention to a particular kind of solutions. Considering the Bianchi inspired constraint, that is the conservation of the energy and momentum of the effective fluid, we are left only with two free functions, for example $\zeta(t)$ and $a(t)$. In bigravity these functions will be determined by the material content of both gravitational sectors through the equations of motion, which has to satisfy the conservation equations (4.10) and (4.11), so by $\rho_m(t)$ and $\bar{\rho}_m(t)$. On the other hand, in massive gravity the functions $\zeta(t)$ and $a(t)$ are given by the reference metric $f_{\mu\nu}$. These solutions only exist for some theories of massive gravity, those with a reference metric given by expression (4.9) with $\Omega(t)$ and $v(t)$ fixed in terms of $\zeta(t)$ and $a(t)$ through equations equations (4.15) and (4.19). Fixing $\zeta(t)$ and $a(t)$ at this point means fixing the particular massive gravity theory under investigation, being the Gordon solutions compatible only with a given matter content present in our gravitational sector that can be calculated through the equations of the dynamics. At this point we prefer to take a family of theories and not to fix the theory under investigation completely, keeping the freedom to consider different functions $\rho_m(t)$ and $\zeta(t)$ (and, therefore, $a(t)$) of phenomenological interest.

### 4.1 CMB dipole

Let us now study the effect of slow moving fluids over the CMB. As shown before, in the center of mass frame, the ordinary fluids acquire non-vanishing velocities in order to cancel the momentum density of the bimetric fluid. This fact implies in particular that because of the different scaling of the baryon and radiation velocities given by equation (4.14), after decoupling, baryon velocity will scale as $|\vec{v}_B| \propto a^{-1} \ (\omega_B = 0)$ whereas radiation velocity $|\vec{v}_R| = \text{const.} \ (\omega_R = 1/3)$. In other words, a non-vanishing $|\vec{v}_{BF}|$ induces a relative motion of baryonic matter and radiation after decoupling. This motion in turn contributes by Doppler effect to the CMB dipole anisotropy.

The photon energy measured by an observer with four velocity $U^\mu = a^{-1}(-1, \vec{u})$ is given by [41]

$$\varepsilon = U_\mu P^\mu, \quad (4.20)$$

where

$$P^\mu = E \frac{dx^\mu}{d\lambda}, \quad (4.21)$$

is the photon four-momentum where $x^\mu(\lambda)$ is the photon geodesics, $\lambda$ the corresponding affine parameter and $E$ parametrizes the energy. Using the geodesics equation for the FLRW
metric (4.8), it is easy to show that the geodesics can be written as \( x^\mu = n^\mu \eta \) with \( n^\mu = (1, \vec{n}) \) and \( \vec{n}^2 = 1 \) with \( d\lambda = a^2 d\eta \). Thus, the photon momentum results

\[
P^\mu = \frac{E}{a^2} n^\mu, \tag{4.22}
\]

and

\[
\varepsilon \simeq E(1 + \vec{n} \vec{u}), \tag{4.23}
\]
to first order in velocities. The CMB temperature anisotropy is then given by

\[
\frac{\delta T}{T} = \frac{a_0 \varepsilon_0 - a_{\text{dec}} \varepsilon_{\text{dec}}}{a_{\text{dec}} \varepsilon_{\text{dec}}} \simeq \vec{n}\vec{v}_{\text{dec}}, \tag{4.24}
\]

where the subscripts 0 and dec refer to the present time and decoupling, respectively.

As mentioned before in our model baryonic matter is moving with respect to the cosmic center of mass, as it was considered in reference [40] in a general relativistic framework, and, therefore, it will contribute to the observer and emitter velocities. Taking into account that \( \vec{v}_B = \vec{v}_{B0} a^{-1} \), the present value of \( |\vec{v}_B| \) must be much smaller than the one at decoupling and we can neglect it. Therefore, we have

\[
\frac{\delta T}{T} \simeq \vec{n}\vec{v}_{\text{dec}}, \tag{4.25}
\]

which is just a dipole contribution. The experimental amplitude for the CMB dipole is [42]

\[
\left| \frac{\delta T}{T} \right|_{\text{dipole}} = 1.23 \times 10^{-3}, \tag{4.26}
\]

so that we can set a limit on the baryonic velocity at decoupling \( |\vec{v}_{B,\text{dec}}| < 1.23 \times 10^{-3} \), which implies that today \( |\vec{v}_{B0}| < 1.1 \times 10^{-6} \). In addition, since prior to decoupling baryonic matter and radiation should have shared the same velocity, we have \( \vec{v}_R \simeq \vec{v}_{B,\text{dec}} \) and, therefore, \( |\vec{v}_R| < 1.23 \times 10^{-3} \).

### 4.2 Monocomponent bimetric fluid

In this section we will study the case when the effective fluid is composed only of one component, i.e. only one non-vanishing \( \beta_n \). Then, this fluid will be identified with one of the dark fluids existing in our universe. In fact, as there is only a component, it is equivalent to work with \( \zeta \) and \( \Omega \) or \( \omega \) and \( \rho \), since they are related by

\[
1 - \zeta = 3(\omega_n + 1)/n, \tag{4.27}
\]

\[
\rho_n = \rho_{n0}(\Omega/\Omega_0)^n, \tag{4.28}
\]

where the subscript 0 stands for the present value of the function, and where the other constants present in equations (3.9) have been absorbed into \( \rho_{n0} \). For the sake of simplicity, we will assume that all the fluids move along the x-axis so that \( \vec{v}(\alpha) = (v(\alpha),0,0) \). We can express the spatial velocity of the effective fluid, \( \vec{v}_{\text{BF}} = v_n \), in terms of \( \zeta \) and \( \Omega \), using the fact that \( (\rho_n + p_n)v_n \propto a^{-4} \), as shown in equation (4.11). This is

\[
v_n = v_{n0} a^{-4} \left( \frac{1 - \zeta_0}{1 - \zeta(a)} \right)^n \left( \frac{\Omega_0}{\Omega} \right)^n. \tag{4.29}
\]
It must be emphasized that the particular form of the functions $\zeta(\eta)$ and $\Omega(\eta)$ will be fixed by the theory itself in case of massive gravity since $f_{\mu\nu}$ in equation (4.9) is given a priori. In contrast, for bigravity, these functions can be obtained from the modified Einstein equations of the $f$-sector, equation (2.8), once the material content coupled to that sector is fixed.

In the following, we will consider some phenomenological interesting cases assuming that the reference metric or the material content of the $f$-sector is such that it gives compatible parameters with our calculations in massive gravity and bigravity, respectively. Hence, considering that the function $\zeta$ is constant, which implies $\omega_n = \text{const}$, we have

$$\rho_n = \rho_{n0} a^{-3(1+\omega_n)} \quad \text{or} \quad \Omega = \Omega_0 a^{\zeta-1}, \quad (4.30)$$

$$v_n = v_{n0} a^{3\omega_n-1} \quad \text{or} \quad v_n = v_{n0} a^{n(1-\zeta)-4}. \quad (4.31)$$

Note that a cosmological constant fluid can already be obtained in bigravity or massive gravity from the contributions of $\beta_0$ and $\beta_4$ to $g$-sector and $f$-sector, respectively. Therefore, we will not consider the case ($\zeta = 1$) in which the bimetric fluid behaves as cosmological constant.

### 4.2.1 Bimetric dark radiation

Let us consider that the monocomponent bimetric fluid corresponds to dark radiation, while the other cosmic fluids (radiation, dark and baryonic matter and cosmological constant) will be described by $T^{(m)\mu}_\nu$. Taking into account equation (4.27), it can be noted that in order to describe a bimetric fluid with $\omega_n = 1/3$, $\zeta$ has to take negative values: $\zeta = -3$ if we want to have dark radiation originated by component $n = 1$, $\zeta = -1$ for $n = 2$, and $\zeta = -1/3$ for $n = 3$. Moreover, from equation (4.30) one has $\Omega = \Omega_0 a^{-4/n}$. We will work with $\rho$ and $\omega$ instead of $\Omega$ and $\zeta$ for the following calculations, since they are the physical observables.

Using the gauge choice $\vec{S} = 0$, it is possible to find a relation between the different fluids velocities. From equation (4.7) this is

$$\frac{4}{3} \Omega_{\text{DR}} v_{\text{DR}} + \Omega_B v_{\text{B}} + \Omega_{\text{DM}} v_{\text{DM0}} + \frac{4}{3} \Omega_R v_R = 0, \quad (4.32)$$

where the $\Omega$ are density parameters and DR, R, B and DM stand for dark radiation, radiation, baryonic matter and dark matter, respectively. It is important to remark that an exact cosmological constant fluid does not contribute to the momentum density and, therefore, it does not enter in momentum conservation.

Imposing the additional conditions $|v_R| < 1.23 \times 10^{-3}$ and $|v_{\text{B0}}| < 1.1 \times 10^{-6}$, we obtain a relation between dark matter and dark radiation velocities. In figure 2, the allowed values of $v_{\text{DR}}$ and $v_{\text{DM0}}$ have been plotted. Dark matter velocity is seen to be able to vary only in a range of order $10^{-8}$ for a given value of $rv_{\text{DR}}$, with $r$ that fraction of dark radiation with respect to total radiation density ($\Omega_{\text{DR}} = r \Omega_R$, $r < 0.1$ [3]).

Within the range of solutions compatible with CMB dipole observations shown in figure 2, we will concentrate in the case in which dark matter is at rest with respect to the cosmic center of mass. In the standard thermal dark matter scenario, dark matter decoupled from radiation and baryonic matter in the early universe, and its velocity would have been falling as $a^{-1}$ since then, making it negligible at present. Then, if we take $v_{\text{DM0}} \simeq 0$ together with $|v_{\text{R}}| < 1.23 \times 10^{-3}$ and $|v_{\text{B0}}| < 1.1 \times 10^{-6}$, the momentum conservation equation (4.32) yields

$$v_{\text{DR}} = -r^{-1} \left[ \frac{3}{4} \Omega_B v_{\text{B0}} + v_{\text{R}} \right], \quad (4.33)$$
Figure 2. Compatible values of dark matter and dark radiation velocity with the CMB dipole. The horizontal line is the present velocity of baryonic matter. Remember $r < 0.1$, as $\Omega_{DR} = r \Omega_{R}$.

Figure 3. Velocities (left) and densities (right) for a universe filled with dark and usual radiation, dark and baryonic matter and a cosmological constant. Their present values have been taken to be compatible with the CMB dipole. Note that $v_{DR}$ has been obtained supposing dark matter is at rest with respect to the cosmic center of mass. In addition, it has been taken that $\Omega_{\text{dark}} = 0.1 \Omega_{R}$. It should be noted that for early times the linear approximation breaks and our treatment is not valid.

Thus, we see that in this scenario, a dark radiation bimetric fluid with a non-negligible velocity is still compatible with observations.

In figure 3, the evolution of the fluids velocities (except for cosmological constant, that is irrelevant) and densities has been plotted for the case that saturates all inequalities. It should be noted that, as we are working linearly in velocities, they will not be valid for arbitrary early times; whereas in the future our approximation will become even better.

and

$$|v_{DR}| < 1.8 \times 10^{-2}.$$ (4.34)
4.2.2 Bimetric dark matter

Let us now identify the bimetric fluid with dark matter. As $\omega_n = 0$, from equation (4.27) we get $\zeta = -2$ if only $n = 1$ fluid is present, $\zeta = -1/2$ for $n = 2$, and $\zeta = 0$ for $n = 3$. The only possible purely matter fluid would be due to component 1 or 2, since third component is forbidden by the condition $\zeta \neq 0$ to preserve both metrics Lorentzian. We can use, nevertheless, the positive square root branch of $\zeta$ with fluid 3 if we take $\zeta$ arbitrarily near to zero, instead of using the exact matter solution. From (4.30) we get in this case $\Omega = \Omega_0 a^{-3/n}$.

We assume that the universe is filled with radiation, dark and baryonic matter and cosmological constant. From the previous section we know that $v_R$ and $v_B$ are already fixed by dipole with values $|v_R| < 1.23 \times 10^{-3}$ and $|v_B| < 1.1 \times 10^{-6}$. Therefore, we only need to calculate the velocity of dark matter. Using momentum conservation and $\vec{S} = \vec{0}$, we have

$$\frac{4}{3} \Omega_R v_R + \Omega_{DM} v_{DM0} + \Omega_B v_B = 0,$$

yielding,

$$v_{DM0} = -\frac{4}{3} \frac{\Omega_R}{\Omega_{DM}} v_R - \frac{\Omega_B}{\Omega_{DM}} v_B,$$

and

$$|v_{DM0}| < 6.9 \times 10^{-7}.$$

4.2.3 Interpolating bimetric dark fluid

We consider now the case in which dark matter and dark energy are indeed the same fluid, which we identify with the bimetric fluid. We parametrize the equation of state parameter of this interpolating bimetric dark fluid as

$$\omega_D(a) = \frac{\rho_{DM} + p_{DM}}{\rho_{DM} + p_\Lambda} = -1 + \Omega_{DM} a^{-3/\Omega_\Lambda},$$

where the subscript D stands for “dark”. This equation of state parameter has been plotted in figure 4. It must be noted that the only bimetric fluid component able to evolve in such a way is the third one, since the others cannot avoid the singular behavior at $\zeta = 0$, then $\beta_3 \neq 0$.

Therefore, from equation (3.9), $\zeta = -\omega_D$ and, from equation (4.15), $\Omega = \Omega_0 \left(\frac{\Omega_\Lambda + \Omega_{DM} a^{-3/\Omega_\Lambda}}{\Omega_\Lambda + \Omega_{DM}}\right)^{1/n}$.

In the first place, we solve the conservation equations for such a fluid, equations (4.10) and (4.11), to obtain

$$\rho_D = \rho_\Lambda + \rho_{DM0} a^{-3},$$

$$v_D = v_{D0} a^{-1}.$$

It should be noted that the velocity of this interpolating fluid will always behave as that of matter even though at late times the fluid behaves as a cosmological constant. This is so because the cosmological constant does not contribute to the momentum density as mentioned before. In the second place, we consider again the condition $\vec{S} = \vec{0}$ to fix the present values of velocities. This reads

$$(1 + \omega_D) \Omega_D v_{D0} + \frac{4}{3} \Omega_R v_R + \Omega_B v_B = 0.$$

So, the present velocity for the dark sector is

$$v_{D0} = -\frac{4}{3} \frac{\Omega_R v_R + \Omega_B v_B}{(1 + \omega_D) \Omega_D}.$$
yielding

\[ |v_{D0}| < 6.4 \times 10^{-7}. \]  

(4.43)

The energy densities and the absolute values of the velocities have been plotted in figure 5, considering their maximum possible values.

Finally, it should be noted that, since \( \rho_{n0} \propto \beta_n \Omega_0 \) we cannot fix, for a monocomponent fluid at least, both \( \Omega_0 \) and \( \beta_n \) from observations.

### 4.3 Multicomponent bimetric fluid

As we have seen in section 3.1, the effective fluid can be split in three subcomponents. In the previous section we have studied the case when only one of those subcomponents was present. Now, we will describe qualitatively two cases when there is more than one component.

It is easy to see from the definition of \( \rho_{BF} \) and \( \rho_n \), equations (3.4) and (3.9), that when \( \Omega \) depends on time and we have several subcomponents, the bimetric fluid energy density will exhibit the behavior of the dominant subcomponent at every time. This change in the evolution will be reflected also in the velocity through equation (4.12). Before studying some interesting cases, we want to emphasize again that when we refer to fixing the theory.
functions $\Omega$ and $\zeta$, we actually mean that we choose a theory of massive gravity with a reference metric $f_{\mu \nu}$ given by equation (4.9) with those functions, or that we consider that the material content of the $f$-sector is such that it leads to a $f_{\mu \nu}$ metric corresponding to those functions in bigravity.

Now, let us consider an example in which we have a matter behaviour at late time with an early phase in which the bimetric fluid is self-accelerated. For that purpose we need a bimetric fluid with only components 2 and 3 and with $\zeta = -1/2$. Then, from the equation (4.27), we have $\omega_2 = 0$ and $\omega_3 = 1/2$. Taking into account equation (4.19), the velocity is given by

$$v_{BF} = v_{BF0}(1 + r)(a + r/\sqrt{a})^{-1}, \quad (4.44)$$

with $r = \Omega_0\beta_3/\beta_2$. Note that we cannot constrain simultaneously both constants, $v_{BF0}$ and $r$ with the CMB dipole. For this reason we have plotted only its functional behavior for different values of $r$ in figure 6. It can be seen that the fluid accelerates at short times. The maximum velocity is found at $a = (r/2)^{2/3}$ with $v \propto r^{-2/3}$; i.e. the lower the $r$ the higher the peak. The density associated to this effective fluid is

$$\frac{\rho_{BF}}{\rho_{BF0}} = a^{-3} + \frac{r}{1 + r} a^{-9/2}, \quad (4.45)$$

the matter contribution dominates at late times whereas at early time a decrease faster than radiation is exhibited. The dependence on $r$ is highly suppressed on time (see figure 6).

Other interesting situation is that when $\zeta = -1$ and, therefore, $\omega_1 = -1/3$ and $\omega_2 = 1/3$; therefore, the second component of the effective fluid behaves as radiation. Now, the velocity of the effective fluid is given by

$$v_{BF} = v_{BF0}(1 + r)(a^2 + r)^{-1}, \quad (4.46)$$

and its density

$$\frac{\rho_{BF}}{\rho_{BF0}} = a^{-4} + \frac{r}{1 + r} a^{-2}, \quad (4.47)$$

with $r = \Omega_0\beta_2/\beta_1$. These functions have been plotted in figure 7. This time we see that the velocity is constant up to some time when it starts to decrease. Its energy density is again
almost unaffected by the value of \( r \). As in the previous case, the bimetric fluid can be split into two contributions, one that behaves as radiation, which dominates at early times, and another driving the cosmological evolution at late times.

More general situations with all three subcomponents could be explored. Nevertheless, the qualitative behaviour will be similar to those shown in this section.

5 Null bimetric fluid

In this section, we will restrict our attention to solutions of massive gravity and bigravity with metrics that are related through a different kind of causal relation. We consider the Kerr-Schild ansatz which assumes that both metrics have one null vector in common. This ansatz can be expressed as [37]

\[
f_{\mu \nu} = \Omega^2 (g_{\mu \nu} + \xi \, l_{\mu} l_{\nu}) ,
\]

(5.1)

where \( l_{\mu} \) is the common null vector and \( \xi \) is a function that can take any value. It must be noted that for \( \xi = 0 \) we recover a conformal transformation, whose effect for the bimetric theory is that of a cosmological constant. When considering the Kerr-Schild ansatz (5.1) it can be seen that [37]

\[
\gamma_{\mu \nu} = \Omega (\delta_{\mu \nu} + \xi \, l^\mu l_\nu) .
\]

(5.2)

The stress-energy tensor of the effective fluid obtained in reference [37] can be expressed as

\[
T_{\mu \nu} = (\rho_N + p_N) l^\mu l_\nu + p_N \delta_{\mu \nu} ,
\]

(5.3)

where \( N \) stands for bimetric null fluid and the density and pressure are given by

\[
\rho_N = \frac{\Omega}{2} \left[ \beta_1 (6 - \xi) + 2 \beta_2 \Omega (3 - \xi) + \beta_3 \Omega^2 (2 - \xi) \right] ,
\]

(5.4)

\[
p_N = -\Omega (3 \beta_1 + 3 \Omega \beta_2 + \Omega^2 \beta_3 ) ,
\]

(5.5)

and whose sum takes the form

\[
\rho_N + p_N = -\frac{\Omega}{2} \xi (\beta_1 + 2 \Omega \beta_2 + \Omega^2 \beta_3 ) ,
\]

(5.6)
which will be useful in the next section to obtain $\xi$. On the other hand, for the other sector one has

$$
\mathcal{T}^\mu_{\nu} = (\rho_N + p_N)l^\mu l^\nu + p_N \delta^\mu_{\nu},
$$

(5.7)

where

$$
\rho_N = \frac{1}{2\Omega^3} \left[ \beta_1 (2 + \xi) + 2\beta_2 \Omega (3 + \xi) + \beta_3 \Omega^2 (2 + \xi) \right],
$$

(5.8)

$$
p_N = -\frac{1}{\Omega^3} (\beta_1 + 3\beta_2 \Omega + \beta_3 \Omega^2).
$$

(5.9)

Thus, we see that unlike the generalized Gordon ansatz which was related to perfect fluids with time-like velocities, the Kerr-Schild ansatz introduces an effective perfect fluid with null velocity. It should be noted that whereas in reference [37] $\rho_N + p_N$ has been interpreted as the flux of the null fluid, here we have chosen this formulation to emphasize that, in contrast to the previous section, we are now considering the high velocity limit.

### 5.1 Interacting effective null fluids

As in the Gordon case, we can split the bimetric fluid in three subcomponents. For our gravitational sector we have

$$
\begin{align*}
\rho_1 &= \frac{1}{2} \beta_1 \Omega (6 - \xi), \\
p_1 &= -3\beta_1 \Omega, \\
\omega_1 &= -\frac{6}{6 - \xi};
\end{align*}
$$

$$
\begin{align*}
\rho_2 &= \beta_2 \Omega^2 (3 - \xi), \\
p_2 &= -3\beta_2 \Omega^2, \\
\omega_2 &= -\frac{3}{3 - \xi};
\end{align*}
$$

$$
\begin{align*}
\rho_3 &= \frac{1}{2} \beta_3 \Omega^3 (2 - \xi), \\
p_3 &= -3\beta_3 \Omega^3, \\
\omega_3 &= -\frac{2}{2 - \xi};
\end{align*}
$$

(5.10)

and for the $f$-sector

$$
\begin{align*}
\bar{\rho}_1 &= \frac{1}{2} \beta_1 \Omega^{-3} (2 + \xi), \\
\bar{p}_1 &= -\beta_1 \Omega^{-3}, \\
\varpi_1 &= -\frac{2}{2 + \xi};
\end{align*}
$$

$$
\begin{align*}
\bar{\rho}_2 &= \beta_2 \Omega^{-2} (3 + \xi), \\
\bar{p}_2 &= -3\beta_2 \Omega^{-2}, \\
\varpi_2 &= -\frac{3}{3 + \xi};
\end{align*}
$$

$$
\begin{align*}
\bar{\rho}_3 &= \frac{1}{2} \beta_3 \Omega^{-1} (2 + \xi), \\
\bar{p}_3 &= -\beta_3 \Omega^{-1}, \\
\varpi_3 &= -\frac{2}{2 + \xi}.
\end{align*}
$$

(5.11)

Both groups of equations of state parameters have been plotted in figure 8. It is obvious that the $\omega$ will diverge for some value of $\xi$. Nevertheless, this is not a physical problem since the divergences do not appear in $\rho$ nor $p$. In fact, they are exactly canceled when multiplied by $\rho$ to obtain $p$. One can note that $\varpi_1 = \varpi_3$, although their densities and pressures have very different behaviors due to the different dependence in $\Omega$.

On the other hand, as in section 3, it should be pointed out that the energy densities follow the relations $\rho_n \propto (\Omega/\Omega_0)^n$ and $\bar{\rho}_n \propto (\Omega/\Omega_0)^{n-4}$. In this case, however, the only value of $\xi$ that gives the same equation of state parameter in both sectors is $\xi = 0$, which correspond to the conformal transformation. We can relate the equation of state parameter of each subcomponent in the different sectors through

$$
\begin{align*}
\varpi_1 &= -\frac{\omega_1}{4\omega_1 + 3}, \\
\varpi_2 &= -\frac{\omega_2}{2\omega_2 + 1}, \\
\varpi_3 &= -\frac{\omega_3}{2\omega_3 + 1}.
\end{align*}
$$

(5.12)
Figure 8. Equation of state parameter, seen from the $g$-sector (left) and the $f$-sector (right), for each fluid as a function of $\zeta$. The vertical lines show where the divergences take place. These divergences cause no problem since they are canceled when multiplied by $\rho$ to obtain $p$. Equivalently, they are harmless since they do not appear in $\rho$ nor $p$.

6 Fast-moving bimetric fluid: Bianchi I solutions

In section 4 we considered a FLRW spacetime, which was compatible with slow moving fluids. In the present case, however, we have a bimetric null fluid, which in principle requires an exact treatment of the problem. Thus, we consider a Bianchi I metric that is compatible with this solution \[ ds^2_g = -a^2_\parallel d\eta^2 + a^2_\parallel dx^2 + a^2_\perp dy^2 + a^2_\perp dz^2, \] which continues being homogeneous. Here, we have assumed that the fluids motion takes place along the $x$-axis, with $a_\parallel$ and $a_\perp$ the scale factors parallel and perpendicular to the direction of motion, respectively. We have also chosen the gauge $\vec{S} = \vec{0}$. We consider the null vector being $l^\mu = a^{-1}_\parallel (1, 1, 0, 0)$ without loss of generality, as any global factor can be absorbed into $\xi$. As we focus our attention on solutions satisfying the Kerr-Schild ansatz (5.1), the metric of the other gravitational $f$-sector is

\[ ds^2_f = \Omega^2 \left[ -a^2_\parallel (1 - \xi) d\eta^2 - 2a^2_\parallel^2 d\eta dx + a^2_\parallel^2 (1 + \xi) dx^2 + a^2_\perp dy^2 + a^2_\perp dz^2 \right]. \] In this case, the non-vanishing components of the total stress-energy tensor are

\[ T^0_0 = - \sum_\alpha (\rho_\alpha + p_\alpha) \gamma_\alpha^2 - \rho_N, \]  
\[ T^1_0 = - \sum_\alpha (\rho_\alpha + p_\alpha) \gamma_\alpha^2 v_\alpha - (\rho_N + p_N), \]  
\[ T^0_1 = \sum_\alpha (\rho_\alpha + p_\alpha) \gamma_\alpha^2 v_\alpha + (\rho_N + p_N), \]  
\[ T^1_1 = \sum_\alpha (\rho_\alpha + p_\alpha) \gamma_\alpha^2 v^2_\alpha + p_\alpha + (\rho_N + 2p_N), \]  
\[ T^i_i = \sum_\alpha p_\alpha + p_N, \quad i = 2, 3, \] where the sum goes only through the radiation and dark and baryonic matter fluids, since we will be identifying the null bimetric fluid with dark energy. The $\alpha$-fluid velocity is given by

\[ v_\alpha = \frac{a_\parallel}{a} \frac{\frac{\partial a_\parallel}{\partial \eta}}{\frac{\partial a_\parallel}{\partial x}} + \frac{a_\perp}{a} \frac{\frac{\partial a_\perp}{\partial \eta}}{\frac{\partial a_\perp}{\partial x}}. \]
\[ V_{(\alpha)} = a_{(\alpha)}^{-1} \gamma_{\alpha}(1, v_{(\alpha)}, 0, 0) \] with \( \gamma_{\alpha} = (1 - v_{(\alpha)}^2)^{-1} \) from the normalization \( g_{\mu\nu} V_{(\alpha)}^\mu V_{(\alpha)}^\nu = -1 \). In this case, we consider all orders in velocities since we are considering the exact solution. In addition, as before, the stress-energy tensor for the material components of the \( f \)-sector, \( \mathcal{T}^{(m)}_{\mu\nu} \), is also of the perfect fluid form.

As the fluids are non-interacting, we can calculate their conservation equations separately. The exact conservation equations for the null fluid, as found in reference [43], are

\[ p_N' = 0, \quad (\rho_N + p_N)' + 2(\mathcal{H}_\parallel + \mathcal{H}_\perp)(\rho_N + p_N) = 0, \]

where we have defined \( \mathcal{H}_\parallel = a_\parallel'/a_\parallel \) and \( \mathcal{H}_\perp = a_\perp'/a_\perp \), with \( ' \equiv \partial_\eta \), and whose solutions are

\[ p_N = p_{N0}, \quad \rho_N = (\rho_{N0} + p_{N0})(a_\parallel a_\perp)^{-2} - p_{N0}, \]

and where we have taken, for the sake of simplicity, \( a_\parallel = a_\parallel = 1 \). Therefore, the equation of state parameter for the null fluid is

\[ \omega_N = \frac{p_{N0}}{(\rho_{N0} + p_{N0})(a_\parallel a_\perp)^{-2} - p_{N0}}. \]

It can be noted that the null fluid can be split into two contributions: one that behaves as radiation and other behaving as a cosmological constant. In contrast, although at late times its equation of state parameter is in agreement with its energy density behavior (\( \omega_N \rightarrow -1 \) and \( \rho_N \rightarrow -p_{N0} = \text{const} \)), this is not the case at early times. In fact, while its density behaves as a radiation fluid, \( \rho_N \sim (a_\parallel a_\perp)^{-2} \), its equation of state parameter approximates to that of matter (\( \omega_N \rightarrow 0 \)). This result comes from the fact that this time the bimetric fluid is moving at the highest speed.

From equation (5.5) and (6.10), one can obtain

\[ \Omega(3\beta_1 + 3\Omega\beta_2 + \beta_3\Omega^2) = -p_{N0}, \]

where it has been taken into account that null fluid pressure is constant in time by equation (6.10). This equation implies that \( \Omega \) is constant in time and fixed once \( p_{N0} \) is. On the other hand, from the other equations (5.6) and (6.11), one gets

\[ \xi = \frac{2(\rho_{N0} + p_{N0})}{\Omega(\beta_1 + 2\beta_2\Omega + \beta_3\Omega^2)}a_\parallel^{-2}a_\perp^{-2}, \]

or equivalently, \( \xi = \xi_0 a_\parallel^{-2}a_\perp^{-2} \) with \( \xi_0 \) fixed once \( \rho_{N0} \) and \( p_{N0} \) are. This result highly constrains the possible scenarios. In fact, in massive gravity the reference metric \( f_{\mu\nu} \) must be given by equation (6.2) with a constant \( \Omega \), satisfying (6.13), and \( \xi \) evolving as expressed in equation (6.14). In bigravity, the material component of the \( f \)-sector must be of the form that, through the modified Einstein equations (2.8), fixes \( \Omega \) and \( \xi \) of such form. Therefore, in this case there is no remaining freedom and the system constants \( \beta_n \) will not qualitatively change the dynamics.

Let us consider the limits imposed by the CMB dipole in this case. Thus, imposing our gauge choice \( \mathcal{S} = 0 \) [43]

\[ \sum_\alpha (\rho_{(\alpha)} + p_{(\alpha)}) \gamma_{\alpha}^2 v_{(\alpha)} = \sum_\alpha (\rho_{(\alpha)} + p_{(\alpha)}) \gamma_{\alpha}^2 v_{(\alpha)} + (\rho_N + p_N) = 0. \]
Figure 9. Allowed values of present dark matter velocity related to the present value of the equation of state parameter of the null bimetric fluid. We have supposed $\Omega_N = \Omega_{DE}$. The dashed line is the value of $\omega_{DE}$ and the dotted lines show its one-sigma region. These data have been taken from Planck 2015 [3]: $\omega_{DE} = -1.006 \pm 0.045.$

Nevertheless, as we showed in section 4.1, the observed value of the CMB dipole anisotropy imposes very strong constrains over the possible values of baryonic matter and radiation velocities, being $|v_{B0}| < 1.1 \times 10^{-6}$ and $|v_R| < 1.23 \times 10^{-3}$, respectively. In figure 9, $\omega_{N0}$ and $v_{DM0}$ are depicted under the assumption $\Omega_N = \Omega_{DE}$, where the subscript DE denotes dark energy with constant equation of state parameter $\omega_{DE}$. In addition, the value of $\omega_{DE}$ has also been plotted together with the corresponding one-sigma region as measured by Planck satellite. Notice that the dipole constrains the parameters to lie in a narrow strip along the line shown in figure 9, (this is shown in more detail in the small panel within the figure), thus the allowed range for dark matter velocity, given a $\omega_{N0}$, is of order $10^{-8}$. In addition, there are two important conclusions that can be obtained from (6.15). On the one hand, if dark matter moves in the same direction as the other fluids and $\omega_{N0} < -1$, the bimetric null fluid energy density will be negative while $p_N > \rho_{N0}(1 + \omega_{N0}) a^{-4}$. The reason is that, since its velocity is completely fixed, density is the only variable that can change sign to preserve momentum conservation. On the other hand, as we can see in the figure, within the one-sigma region of $\omega_{DE}$ the dark matter velocity could reach non-negligible velocities $-0.13 < v_{DM0} < 0.10$, which could be interesting within the bulk flows problem.

Nevertheless, let us study in further detail the case where the cosmic fluids move slowly and dark energy is given by the null bimetric fluid. Considering the Bianchi I metric, the conservation equations to first order in velocity are as follows:

- **Energy conservation**
  \[
  \rho'_{(\alpha)} + (\rho_{(\alpha)} + p_{(\alpha)})(\mathcal{H}_\parallel + 2\mathcal{H}_\perp) = 0. \tag{6.16}
  \]

- **Momentum conservation**
  \[
  \partial_\eta[a_\parallel^2 a_\perp^2 (\rho_{(\alpha)} + p_{(\alpha)}) v_{(\alpha)}] = 0. \tag{6.17}
  \]

Therefore, for low velocities we get
\[
  v_{(\alpha)} = v_{\alpha 0} a_\parallel^2 a_\perp^{-2} \frac{1 + \omega_{\alpha 0} \rho_{\alpha 0}}{1 + \omega_{\alpha}(a) \rho_{(\alpha)}}, \tag{6.18}
\]
Figure 10. Energy densities in a universe with radiation, dark and baryonic matter and a dark energy null-fluid. As the null-fluid has been subdominant until now, we can neglect the anisotropy for past times. The present values have been taken to be compatible with CMB dipole, and we have not plotted future times when the anisotropy will presumably increase. In the right figure the case of radiation and barionic matter moving in the same direction as dark energy ($v_R, v_B > 0$) has been depicted; in this case the energy energy of the null fluid changes sign at $a = 0.023$. In the right figure, which corresponds to $v_R, v_B < 0$, the null fluid’s energy density is always positive.

and, if the equation of state parameter is constant ($\omega_\alpha = \text{const}$), the solutions are

$$\rho(\alpha) = \rho_\alpha 0 a^{-2(1+\omega_\alpha)} a^{-1(1+\omega_\alpha)}$$

$$v(\alpha) = v_\alpha 0 a^{2\omega_\alpha}$$

These equation reduce to those found for a FLRW metric in section 4 (equations (4.13) for density and equation (4.14) for velocity) for past times, since the null-fluid must have been subdominant until now. Also, as a consequence, the anisotropy in the past must have been much lower than nowadays. Then, if we neglected the present anisotropy, it is safer to neglect it at earlier times. Only in the future the anisotropy may be important enough to be taken into account.

We consider now the interesting case with dark matter velocity negligible at present as a consequence of having decoupled from the radiation-baryonic fluid at early times. In this case we have, $v_{DM0} \approx 0$ and $|v_R| < 1.23 \times 10^{-3}$, $|v_{B0}| < 1.1 \times 10^{-6}$, from the CMB dipole observations. Then, the gauge choice $\vec{S} = \vec{0}$ yields

$$1 + \omega_{N0} = -\frac{1}{\Omega_N} \left(\frac{4}{3} \Omega_R v_R + \Omega_B v_{B0}\right),$$

which, choosing the present null fluid energy density to be that of dark energy in the Standard Model ($\Lambda$CDM), $\Omega_N = \Omega_{DE}$, one has

$$|1 + \omega_{N0}| < 3.0 \times 10^{-7},$$

which is compatible with 2015 Planck data [3]: $\omega_{DE} = -1.006 \pm 0.045$.

In figure 10 the fluid energy densities have been plotted. One can see that, depending on the relative direction of the motion of the bimetric fluid and the radiation and baryonic matter, one can have positive or negative energy densities for the bimetric fluid, since being null, the bimetric fluid velocity is fixed. In fact, when the other two fluids move in the same sense as the null fluid, its energy density has to be negative for times earlier than $a = 0.023$.
in order to cancel the momentum of the other two fluids. One can compare the density behavior with the equation of state parameter plotted in figure 11. In both cases, the figures for parallel moving fluids show a peak as a result of the change of sign of the density. It must also be noted that the null dark fluid has been subdominant until present, when it starts to dominate. Finally, fluids velocities have been plotted in figure 12.

### 7 Conclusions

In this work we have studied the cosmological consequences of massive gravity and bigravity solutions whose metrics are not necessarily diagonal in the same patch of coordinates, as assumed in previous literature. Focusing our attention on solutions in which both causal structures have a particular relation, we have explored situations in which the interaction between both metrics is described by an effective bimetric fluid with an apparent velocity with respect to the comoving coordinates.
In the first place, we have considered solutions with both metrics related through the
generalized Gordon ansatz, which implies that the light cones of one of the metrics are strictly
inside those of the other metric. As the conserved stress-energy tensor for the effective bi-
metric fluid of these solutions is of the perfect fluid form, with a four-velocity given by the
timelike vector of the ansatz, and there are three free parameters of the theory apart from
those leading to cosmological constant contributions, we have been able to decompose the
bimetric fluid in three perfect fluid components with the same velocity. Although these com-
ponents are not separately conserved in general, their effective equation of state parameters
depend only on one function of the ansatz, allowing to determine the behavior of the effective
fluids that could coexist in the same bimetric solution.

Considering these bimetric solutions in a cosmological scenario, we have been able to
describe a universe that satisfies the cosmological principle in the limit of a slow-moving
perfect bimetric fluid, that is the case in which the timelike vector of the Gordon ansatz does
not depart significantly from the comoving vector. As we have chosen to work in the cosmic
center of mass frame, the material content of our gravitational sector has to be in motion. In
this scenario, the energies of the three bimetric components are individually conserved but
their momenta are not. Restricting attention to theories in which the effective bimetric fluid
only has one of these components, we have explored in detail the cases where the bimetric
fluid can be identified with dark radiation, dark matter and a fluid behaving as dark matter
at early times and as a cosmological constant at late times. Each case corresponds to consider
a particular theory of massive gravity, with fixed reference metric, or a hypothetical material
content hidden in the gravitational sector that we are not inhabiting compatible with that
solution in bigravity. Moreover, we have been able to constrain the possible values of the
velocities of the fluids taking into account the data available about the density parameters
and the magnitude of the CMB dipole. On the other hand, we have also considered an
effective fluid with two components, due to two non-vanishing parameters in the interaction
Lagrangian, obtaining a new purely bimetric phenomenon consisting of a transition from an
accelerating to a decelerating fluid velocity.

In the second place, we have studied the effects of a null bimetric fluid, which can be
obtained by restricting attention to solutions that satisfy the Kerr-Shild ansatz. The null
vector describing the motion of the effective fluid in this case is just the only common null
vector of both metrics. Therefore, this is the opposite limit to the slowly moving bimetric
fluid that we had previously studied. In this case, one necessarily has to consider an exact
solution to describe our Universe, which can be a Bianchi I spacetime. Although the bimetric
fluid can also be decomposed in three components, such decomposition is not so useful as
in the previous case since the free functions appearing in the ansatz (and, therefore, in the
fluid) are completely fixed once the conservation equation for the null fluid is considered in
this scenario. Moreover, such fixing also implies that the scenario is only compatible with a
group of massive gravity theories, with reference metrics set accordingly, or that a particular
material content has to be present in the other gravitational sector if the cosmological solution
is due to a bigravity theory. As in the slow-moving fluids case, we have used the available
data to constrain the fluids velocities. In this framework it is possible, at least in principle,
to obtain scenarios beyond the slowly moving dark matter case. On the other hand, we have
also constrained the null fluid equation of state parameter identifying this fluid with dark
energy and taking into account that the rest of cosmic fluids moves slowly.
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