The interaction of surface acoustic waves with an array of quantum wires

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(February 21, 1999)

We describe the interaction of surface acoustic waves with electrons in an array of quantum wires, patterned out of a two dimensional electron gas. Two specific geometries are considered, in which the surface acoustic wave travels parallel, or perpendicular to the wires. Although we assume the electron wave functions in different wires do not overlap, the screening of the phonon potential by the electrons in the wire array is a collective phenomenon. It is shown that the surface acoustic wave absorption cannot be described via the ac conductivity in the usual manner. We derive an integral equation for the dielectric function of the electrons in the quantum wire system, and solve it in the narrow wire approximation. Using the dielectric function we find the absorption and the change in velocity of the surface acoustic waves.

I. INTRODUCTION

Experiments which include interactions of surface acoustic waves (SAW’s) with electrons in a two dimensional electron gas (2DEG) are of great interest [3–5], have attracted great attention, since they strongly support the composite fermion approach to the compressible state at \( \nu = 1 \). Unlike conductivity measurements in which a current is driven through the 2DEG and the voltage is measured, where only the conductivity at zero wave vector can be probed, SAW measurements allow one to probe the finite wave vector conductivity. Experiments that have been carried out in the fractional quantum Hall regime near a filling factor \( \nu = 1/2 \) [3,3], have attracted great attention, since they strongly support the composite fermion approach to the compressible state at \( \nu = 1/2 \).

Experimental work done in modulated 2DEG [3] triggered theoretical works on SAW’s in such systems. In Ref. [3] Weiss oscillations in SAW propagation were considered, while composite fermions in modulated structures were treated in [3].

This work was motivated by an experiment [10], in which the transmission of a SAW by an array of parallel quantum wires (QWR’s) was measured, as a function of a strong magnetic field (that tunes the electron subbands in the QWR’s relative to the Fermi energy). The measurements were performed in two geometries with the SAW traveling parallel or perpendicular to the QWR’s. In the latter case a structure that corresponds to the crossing of the Fermi level by the bottom of the subbands was observed, while in the former case no such structure was seen.

In this work we will calculate the absorption and the change in velocity of a SAW that propagates either parallel or perpendicular to a QWR array. In section II we describe the model that we consider. We then show why the “usual” - “classical” approach cannot be used to describe the SAW attenuation by the electrons in the QWR array. When the Boltzmann equation is used in order to calculate the phonon relaxation rate due to electron transitions, one finds that the SAW absorption is negligible, since the phase space available for these transitions in one dimension is highly restricted by energy and momentum conservation. The approach in which the SAW absorption is given in terms of the dc conductivity also fails since in this system the dc conductivity is zero. When the SAW travels parallel to the wires the dc conductivity is zero due to localization, while in the perpendicular case it is zero since the electron wave functions in different wires do not overlap.

In section III we will define what the dielectric matrix is, and explain how it is related to the SAW absorption and change in velocity. We then find an equation for the inverse dielectric matrix in terms of the polarization (that is also defined in this section). In section IV we describe the random potential that we consider, and the wave functions that are related to it. We quote the result for a specific average over the wave functions, that we shall encounter in the calculation of the polarization, and discuss the conditions for its validity. The calculations of the polarization and the inverse dielectric matrix are presented in section V. The results will be discussed in section VI.

II. FORMULATION OF THE PROBLEM

The geometry of our system is as follows: There are \( N \) wires of length \( L_y \), and width \( w \), in a periodic array of period \( d \) (\( L_x = N d \)). The coordinate along the wires is \( y \), and the coordinate perpendicular to the wires is \( x \). The electrons in a single wire are described by a wave function \( \psi_{\nu \alpha}(x, y) \), that corresponds to the eigenenergy \( E_{\nu \alpha} \). We assume that the electron wave functions in different wires do not overlap. SAW’s of wave vector \( \mathbf{q} = (q_x, q_y) \), and frequency \( \omega_q = v_S q \) (where \( q = |\mathbf{q}| \)), interact with the electrons in this array. Since the effect that is seen in the experiment is clearly connected with the crossing of the Fermi level by the electron energy subbands, we
consider the magnetic field only as a mechanism for moving the position of the bottom of the subbands, and we do not consider the manner in which it affects the wave functions of the electrons.

When the electrons are free i.e., they do not interact and they are not under the influence of a random potential, the electron wave functions and energies are separable, and can be written as \( \psi_{n,k}(x,y) = \phi_n(x) e^{iky} / \sqrt{L_y} \), and \( E_{n,k} = E_n + \epsilon_k \). Here \( E_n \) are the energy subbands created by the constriction in the \( x \) direction, and \( \phi_n(x) \) are the corresponding wave functions. The kinetic energy is given by \( \epsilon_k = k^2 / 2m^* \) where \( m^* \) is the effective mass.

Let us first show that the absorption of the SAW cannot be described using a simple Boltzmann formalism. If one were to use the Boltzmann equation and the “free” electron states in order to calculate the phonon decay rate 1/\( \tau \), one would obtain

\[
\frac{1}{\tau} = \frac{1}{d} \sum_n |M_n(q)|^2 \int dk \delta(\epsilon_k - \epsilon_{k-q} - \omega_q) \\
\times \left[ f(E_{n,k-q}) - f(E_{n,k}) \right]. \tag{1}
\]

The phonon energy \( \omega_q \) is much smaller than the subband difference \( \Delta E_n \), therefore no intersubband transitions occur due to the electron-phonon interaction. The matrix element of the electron-phonon interaction is given by \( M_n(q) = M^o \int f_0 dx |\phi_n(x)|^2 e^{iqx} \), where \( M^o = 4\pi \beta e(h/4\pi e)^{1/2} \). The constant \( \beta \) describes the piezoelectric coupling, \( \rho \) is the mass density of the material, and \( a \) and \( C \) are numerical factors that depend on the SAW: on the direction of its propagation, its velocity, and the elastic constants of the material through which it propagates. The constant \( a \) is of the order of 1 and the constant \( C \) is of the order of 0.1. The function \( f(E_{n,k}) \) is the electron distribution function, and at equilibrium it is given by the Fermi distribution function, \( f_{Fr}(E_{n,k}) \).

The delta function that appears in expression \( (1) \) for 1/\( \tau \) stands for energy and momentum conservation. When the phonon travels parallel to the wires i.e., when \( \mathbf{q} \parallel \hat{y} \), then one obtains from Eq. \( (1) \)

\[
\frac{1}{\tau} = \frac{m^*}{\alpha d} \sum_n |M_n(0)|^2 \\
\times \left[ \int_{Fr}(E_n + \epsilon_{m^*v_0} - q/2) - \int_{Fr}(E_n + \epsilon_{m^*v_0} + q/2) \right]. \tag{2}
\]

If the temperature \( T \gg \omega_q \), SAW absorption occurs when \( \epsilon_{Fr} \equiv \epsilon_F - E_n \) (the Fermi energy measured from the bottom of the \( n^{th} \) subband) is within \( T \) of \( m^*v_0^2 \). If \( T \ll \omega_q \), SAW absorption occurs only when \( \epsilon_{Fr} \) is within \( \omega_q \) of \( m^*v_0^2 \). For both temperature regions the peaks are expected to be of square shape. For \( T = 1.3 \) K, and \( \omega_q = 300 \) neV, the temperature and phonon frequency at which the experiment \( [10] \) was carried out, one would expect peaks of the width of \( \Delta B = 0.06 \) T, while in practice no peaks were seen.

If \( \mathbf{q} \parallel \hat{x} \), Eq. \( (1) \) leads to,

\[
\frac{1}{\tau} = \frac{1}{d} \sum_n |M_n(q)|^2 \delta(\omega_q) \\
\times \int dk \left[ f_{Fr}(E_{n,k-q}) - f_{Fr}(E_{n,k}) \right] = 0, \tag{3}
\]

regardless of the temperature. Thus, in this case there is no absorption at all, contrary to the absorption peaks that were observed in the experiment, that were of the width of \( \Delta B \approx 1 \) T, and were not of a square shape.

From the two cases described above it is clear that if one wants to explain a finite SAW absorption in both the parallel and the perpendicular cases, as is seen for example in \([10]\), a nonzero temperature does not suffice, and one has to take into account either inelastic, or elastic scattering. These will introduce a level broadening that will relax the energy and the momentum conservation requirements. We shall consider only elastic scattering, specifically that due to a random potential. In addition we assume zero temperature for simplicity.

In the usual approach the SAW attenuation is given in terms of the dc conductivity by

\[
\Gamma = -\frac{\alpha^2}{2} \frac{\sigma_{dc}/\sigma_M}{1 + (\sigma_{dc}/\sigma_M)^2}, \tag{4}
\]

where \( \alpha \) is the piezoelectric coupling constant, \( \sigma_M = \kappa_0 \omega_q^2 / 2\pi \) is the Maxwell conductivity, and \( \kappa_0 \) is the dielectric constant of the material. We cannot use \( (4) \) since in our case \( \sigma_{dc} = 0 \). For \( \mathbf{q} \parallel \) QWR’s the dc conductivity is zero since, as is well known, the states in a one dimensional system in the presence of disorder are localized (see for example the review \([12]\) and references therein). In the case of \( \mathbf{q} \perp \) QWR’s the dc conductivity is zero since the functions in different wires do not overlap.

Although the electron wave functions in different wires do not overlap, so that the absorption is additive, the electrons in different wires interact via the Coulomb potential, giving rise to a strong collective screening effect.

### III. THE DIELECTRIC FUNCTION

A function that will describe both the absorption of the SAW by the electrons in the QWR array, and the strong screening, is the dielectric function. When a SAW of frequency \( \omega \) travels through the sample, the potential exerted by it \( \varphi_\omega \), is screened by the electrons. The screened potential \( \varphi_\omega^{sc} \) is related to \( \varphi_\omega \) through the inverse dielectric function \( \epsilon_\omega^{-1} \)

\[
\varphi_\omega^{sc}(\mathbf{r}) = \int d\mathbf{r}' \epsilon_\omega^{-1}(\mathbf{r}, \mathbf{r}') \varphi_\omega(\mathbf{r}'), \tag{5}
\]

therefore the absorption of the SAW by the electrons in the wire array is related to the inverse dielectric function.

If the 2DEG is periodically patterned in the \( x \) direction with a period \( d \), and homogeneous in the \( y \) direction,
then the nonzero Fourier components \( \epsilon_{s'}^{(1)}(q', q'') \) have
\[ q'_x = q_x + 2\pi s'/d \equiv q_s', \quad q''_x = q_x + 2\pi s''/d \equiv q_s'', \quad (s', s'' \text{ are integers}), \quad \text{and} \quad q'_y = q''_y \equiv q_y. \] These components can be written in the form of a matrix \((\epsilon^{-1})_{s',s''}(q, \omega) \), where \( q = (q_x, q_y) \) (as we will explain below). We shall denote \((\epsilon^{-1})_{s',s''} \) by \( \epsilon_{s',s''}^{-1} \) from now on in order to simplify the notations. In terms of these components the attenuation per unit length, and the change of the velocity of the SAW, are given by

Then we Fourier transform Eq. (8) and obtain

\[
\Pi(x_1, x_2; y_1 - y_2; \omega_q) = \frac{i}{2} \int \frac{d\epsilon}{2\pi} \left\{ G^s(x_1, x_2; y_1 - y_2; \epsilon) G^a(x_2, x_1; y_2 - y_1; \epsilon - \omega_q) + G^a(x_1, x_2; y_1 - y_2; \epsilon) G^s(x_2, x_1; y_2 - y_1; \epsilon - \omega_q) \right\},
\]

where \( G^s, G^a, \) and \( G^r \) are the statistical, advanced, and retarded one particle electron Green functions (for definitions of these functions see (14)). The electrons in the QWR array are at equilibrium, therefore expression (9) is simplified by the substitution of \( G^s(r, r'; \epsilon) = 2i(1 - 2f(\epsilon)) \text{Im} G^r(r, r'; \epsilon) \), where \( f \) is the Fermi distribution. The angular brackets \((\cdot\cdot\cdot)_U\) represent averaging over all impurity configurations.

The dielectric function and the polarization are considered as quantities that are averaged over all impurity configurations (correlations between the two are neglected), therefore they are translationally invariant in the \( y \) direction. Since we assume the electron wave functions in different wires do not overlap, the two variables of \( \Pi \) in the \( x \) direction are restricted to the same wire. This is not the case with \( \epsilon^{-1} \), where two electrons in different wires can interact via the Coulomb interaction.

We now wish to Fourier transform the equation above. Due to the homogeneity in the \( y \) coordinate, the equation will be diagonal in \( q_y \). Using the following definitions

\[
\epsilon^{-1}(q_x, q'_x; q_y; \omega_q) = \frac{1}{L_x L_y} \int_{0}^{L_x} dx \int_{0}^{L_y} dy \int_{0}^{L_y} dy' \int_{0}^{L_y} dy'' 
\times e^{-i q_x x + i q'_x x'} - i q_y y - i q_y y' \epsilon^{-1}(x, x'; y; \omega_q),
\]

\[
V(r) = \frac{1}{L_x L_y} \sum_{q} e^{i q \cdot r} V(q),
\]

where \( V(q) = 2\pi e^2/(\kappa_0 q) \), and

\[
\Gamma = -\frac{q^2}{2} \text{Im} \epsilon_{0,0}^{-1}(q, \omega_q),
\]

and

\[
\Delta v = \frac{\alpha^2}{2} \text{Re} \left[ \epsilon_{0,0}(q, \omega_q) - 1 \right].
\]

Our goal is therefore to find \( \epsilon_{0,0}^{-1}(q, \omega_q) \).

The dielectric function is given by the following equation (for example see (13))

\[
\Pi(x_1, x_2; y_1 - y_2; \omega_q) = \frac{1}{L_x L_y} \sum_{q_x \neq q'_x, q_y} e^{-i q_x x - i q'_x x'} + i q_y y \Pi_N(q_x, q'_x; q_y; \omega_q),
\]

we Fourier transform Eq. (8) and obtain

\[
\epsilon^{-1}(q_x, q'_x; q_y; \omega_q) = \delta_{q_x, q'_x}
\]

\[
+ V(q_x, q_y) \frac{1}{L_x} \sum_{q''_y} \Pi_N(q_x, q'_x; q_y; \omega_q) \epsilon^{-1}(q''_x, q'_x; q_y; \omega_q).
\]

Due to the periodicity of the array, \( \epsilon^{-1}(x_1, x_2; y; \omega_q) \) and \( \Pi(x_1, x_2; y; \omega_q) \) remain unaltered when their two \( x \) variables, \( x_1 \) and \( x_2 \), are replaced by \( x_1 + n d \) and \( x_2 + n d \) (\( n \) is any integer). Therefore the \( x \) components of the two \( q \) variables of \( \epsilon^{-1}(q_1x, q_2x; q_y; \omega_q) \) and \( \Pi_N(q_1x, q_2x; q_y; \omega_q) \) will differ by \( 2\pi s/d \), where \( s \) is an integer. This is due to the fact that in an unmodulated structure the screened field will have the same wave vector as the unscreened field, while in a modulated structure the screened field will have all the Umklapp wave vectors that are related to the inverse lattice of the modulation, as well. Thus if we define \( q_s = q_x + 2\pi s/d \), where \( q_x \) is the \( x \) component of the phonon wave vector, then

\[
\epsilon^{-1}(q_1x, q_2x; q_y; \omega_q) = \epsilon^{-1}(q_s, q_s; q_y; \omega_q) \equiv \epsilon_{s,s}^{-1}(q, \omega_q).
\]

Rewriting Eq. (13) in terms of the new \( s \) variables we obtain
\[
\epsilon_{s,s'}^{-1}(q, \omega_q) = \delta_{s,s'} + V_s(q) \frac{1}{L_x} \sum_{s''} \Pi_{s,s''}^N(q, \omega_q) \epsilon_{s'',s'}^{-1}(q, \omega_q),
\]

where \(V_s(q) = 2\pi e^2/[\kappa_o(q_x^2 + q_y^2)^{1/2}]\). Note that in our case \(q_y\) that appears in the equation is the \(y\) component of the phonon momentum, while \(q_x\), the basis for all \(q_x\), is the \(x\) component of the phonon momentum. Thus \(q\) is the phonon momentum.

Reversing the relation given by (12), and taking into account the fact that the \(x\) coordinates of \(\Pi\) are restricted to the same wire we find that

\[
\Pi_{s,s''}^N(q, \omega_q) = \sum_{n=0}^N \Pi_{s,s''}^n(q, \omega_q),
\]

where

\[
\Pi_{s,s''}^m(q, \omega_q) = \int_{m d}^{md+w} dx \int_0^{L_y} dy \int_0^{L_y} d\omega \epsilon_{s,s'}^{-1}(q, \omega_q) \exp\left[i q, x + i q', x' - i q_y y\right] \Pi(x, x'; y, \omega_q),
\]

IV. THE RANDOM POTENTIAL AND THE WAVE FUNCTIONS

We will assume that the random potential responsible for the states \(\psi_{n\alpha}\) can be written as a sum of the random potential \(U_0(x, y)\) for the unpatterned 2DEG (with amplitude \(\langle U_0^2 \rangle\) and correlation length \(\Lambda_0\)), and an effective potential \(V_n(y)\) which describes the scattering by the QWR’s boundaries. When the width of the QWR \(w \ll \Lambda_0\), the total potential within the QWR can be written as \(U(x, y) = U_n(y) + x W(y)\), where \(U_n(y) = V_n(y) + U_0(0, y)\), and \(W(y) = \partial U_0(0, y)/\partial x\). We shall assume that \(U_0(0, y)\) is a Gaussian correlated random variable with \(\langle U_0(0, y) U_0(0, y') \rangle = \langle U_0^2 \rangle \exp\left[(y - y')^2/2\Lambda_0^2\right]\), therefore \(W(y)\) and \(U_0(0, y)\) are uncorrelated. In addition, \(V_n(y)\) and \(W(y)\) are uncorrelated (since they stem from different unrelated physical processes). We thus conclude that \(W(y)\) and \(U_n(y)\) are uncorrelated.

When the term containing \(W\) can be neglected, the wave functions and the energies are separable i.e.,

\[
\psi_{n\alpha}(x, y) = \phi_n(x) \chi_{n\alpha}(y), \quad E_{n\alpha} = E_n + \epsilon_{n\alpha},
\]

where \(\phi_n\) and \(E_n\) are defined by the confining potential, while \(\chi_{n\alpha}\) and \(\epsilon_{n\alpha}\) are defined by the potential \(U_n\). Thus only \(\chi_{n\alpha}\) and \(\epsilon_{n\alpha}\) are random. This approximation will suffice in the case of \(q \parallel\) QWR’s, but for \(q \perp\) QWR’s a higher order approximation that includes the linear term in \(xW\) is needed.

is the polarization of the \(m\)th wire. Since all the wires are identical, and there is no overlap between the electron wave functions, the polarization of all wires must be the same and the suffix \(m\) can be dropped

\[
\Pi_{s,s''}^N(q, \omega_q) = N \Pi_{s,s''}^N(q, \omega_q).
\]

This leads to the final form of the equation for the inverse dielectric matrix

\[
\epsilon_{s,s'}^{-1}(q, \omega_q) = \delta_{s,s'} + \frac{1}{d} V_s(q) \sum_{s''} \Pi_{s,s''}(q, \omega_q) \epsilon_{s'',s'}^{-1}(q, \omega_q).
\]

In order to solve Eq. (19) for \(\epsilon_{s,s'}^{-1}\) we must find the polarization \(\Pi_{s,s''}\). It can be written in terms of the electron wave functions and energies in the following form

\[
\psi_{n\alpha}(x, y) = \phi_n(x) \chi_{n\alpha}(y) + \sum_{n'\alpha'} \frac{x_{n'n'\alpha'} W_{n'n'\alpha'; n\alpha}}{E_{n'n'} - E_{n\alpha}} \phi_{n'}(x) \chi_{n'\alpha'}(y),
\]

where

\[
x_{n'n'\alpha'} = \int_0^w dx \phi_{n'\alpha'}^\ast(x) x \phi_n(x),
\]

\[
W_{n'n'\alpha'; n\alpha} = \int_0^{L_y} dy \chi_{n'n'}^\ast(y) W(y) \chi_{n\alpha}(y).
\]

Under the assumption of symmetric quantum wires, the diagonal matrix elements of \(x\) are \(x_{n,n} = 0\).

We assume the wires to be narrow \(w \ll d\), and approximate the following matrix elements by

\[
\int_0^w dx \phi_n^\ast(x) e^{iq_x x} \phi_n(x) = \begin{cases} \delta_{n,n'} + iq_x x_{n'n'} & \text{if } s \leq d/2\pi w; \\ 0 & \text{otherwise.} \end{cases}
\]

The justification for this approximation is that for \(s \ll d/2\pi w\) we can expand the exponent, while for \(s \gg d/2\pi w\) the matrix elements are exponentially small.

In our calculations we shall encounter the following average over electron wave functions and energies.
\[ \left\langle \sum_{\alpha,\alpha'} \delta(\epsilon_{n\alpha} - \epsilon) \delta(\epsilon_{n\alpha'} - \epsilon') \chi_{n\alpha}(y) \chi_{n\alpha'}(y') \chi_{n\alpha}(y) \chi_{n\alpha'}(y') \right\rangle_{U} \].

(25)

This average has been calculated in [13], where the averages of electron wave functions in one dimensional structures, over ensembles of random impurities, are calculated. The following result was obtained in [13]

\[ F_{n}^{(1)}(\xi) = \frac{\nu^2(\epsilon_{Fn})}{\epsilon_{Fn}} \langle \sum_{\alpha,\alpha'} \delta(\epsilon_{n\alpha} - \epsilon) \delta(\epsilon_{n\alpha'} - \epsilon') \chi_{n\alpha}(y) \chi_{n\alpha'}(y) \rangle_{U} \]

\[ = \begin{cases} \frac{2\nu^2(\epsilon_{Fn})}{\pi v_{F}} = \frac{2}{3}(\pi v_{F})^{-2} & \text{if } k_{Fn}^{-1} \ll \xi \ll l_{n} \\ \frac{\pi^{1/2}\nu^2(\epsilon_{Fn})}{\epsilon_{Fn}} (l_{n}/\xi)^{3/2} \exp(\xi/4l_{n}) & \text{if } l_{n} \ll \xi \ll a_{n} l_{n} , \\ -\nu^2(\epsilon_{Fn}) (4\pi a_{n})^{-1/2} \exp\left(-\left[\frac{(\xi-a_{n}l_{n})^{2}}{4a_{n}l_{n}^2}\right]\right) & \text{if } \xi \simeq a_{n} l_{n} \end{cases} \]

(26)

where \( \xi = |y-y'| \), \( \epsilon_{Fn} = m^{*} v_{F}^{2}/2 = k_{Fn}^{2}/2m^{*} \), \( l_{n} \) is the electron localization length in the \( n^{th} \) subband, \( a_{n} = 2 \ln(8/\omega_{Tn}) \), \( \tau_{n} = l_{n}/v_{Fn} \), and \( \omega = \epsilon - \epsilon' \). The dependence of \( F_{n}^{(1)} \) on \( \omega \) is weak, so that we can neglect it. The result that is quoted above is valid for weak scattering \( k_{Fn} l_{n} \gg 1 \), and for a short range random potential \( \Lambda_{n} \ll l_{n} \), where \( \Lambda_{n} \) is the correlation length of \( U_{n} \).

We are interested in small \( \epsilon_{Fn} \), since the transmission peaks were observed when \( \epsilon_{Fn} \) approaches zero [14]. However, when approaching the threshold, we are limited by the conditions quoted above under which [20] is valid. We shall now estimate how close we can approach the threshold.

The localization length \( l_{n} \) is related to the one dimensional transport scattering time via \( l_{n} = v_{Fn} \tau_{n}^{1D} \), where \( \tau_{n}^{1D} \) is given by

\[ \frac{1}{\tau_{n}^{1D}} = \left(\frac{\pi}{8}\right)^{1/2} k_{Fn} l_{n} \frac{U_{n}^{2}}{\epsilon_{Fn}} e^{-\left(k_{Fn} l_{n}\right)^{2}/2}. \]

(27)

Here we considered \( U_{n}(y) \) to be Gaussian correlated with an amplitude of \( U_{n}^{2} \). Since we have no information on the effective potential due to the roughness of the QWR boundaries, we shall carry our estimates with \( U_{0} \), the random potential for the unpatterned 2DEG, only. The correlation length of \( U_{0} \) is given by the spacer width, which in the particular experiment that we are interested in is 1050 Å. The potential amplitude \( U_{0}^{2} \) is related to the two dimensional transport time via

\[ \frac{1}{\tau_{n}^{2D}} = \left(\frac{\pi}{8}\right)^{1/2} k_{F}^{2D} \Lambda_{0} \frac{U_{0}^{2}}{\epsilon_{F}}, \]

(28)

where \( k_{F}^{2D} \) is the inverse Fermi wave length for the unpatterned 2DEG, that is related to the 2DEG density via \( n^{2D} = (k_{F}^{2D})^{2}/2\pi \), and to the two dimensional Fermi energy via \( \epsilon_{F}^{2D} = (k_{F}^{2D})^{2}/2m^{*} \). For \( 1/\tau_{n}^{2D} = 3 \times 10^{-2} \) meV and \( n^{2D} = 2.1 \times 10^{11} \) cm\(^{-2} \) we find that \( U_{0}^{2} = 3.9 \) meV\(^{2} \).

Returning to the conditions for the validity of [20], \( l_{n} \gg \Lambda_{n} \), and \( k_{Fn} l_{n} \gg 1 \), and replacing \( U_{n} \) and \( \Lambda_{n} \), by \( U_{0} \) and \( \Lambda_{0} \), we find that we can approach the threshold up to \( \epsilon_{F}^{th} = 1 \) meV. This corresponds to a change of the magnetic field of \( \Delta B = 0.6 \) T.

**V. Calculation of \( \Pi \) and \( \epsilon^{-1} \)**

Let us begin by dealing with the parallel case - that in which the phonons propagate along the direction parallel to that of the QWR’s (\( q \parallel \hat{y} \)). In this case it suffices to use the zeroth order wave functions in the matrix elements that appear in the expression for \( \Pi \) [20]. Moreover, since the contribution of terms with \( n \neq n' \) is much smaller than that of the terms with \( n = n' \), due to the large energy denominator \( E_{n} - E_{n'} \), we keep only the diagonal terms. Thus we have

\[ \Pi_{s,s'}(q,\omega)\big|_{q=a\hat{y}} = \frac{1}{L_{y}} \sum_{\nu} \int_{0}^{L_{y}} \int_{0}^{L_{y}} dy \, dy' \, e^{-iq(y-y')} \left\langle \sum_{\alpha,\alpha'} \chi_{n\alpha}(y) \chi_{n\alpha'}(y') \chi_{n\alpha'}(y) \chi_{n\alpha}(y') \right\rangle_{U} \frac{f(E_{n\alpha} - \epsilon_{Fn}) - f(E_{n\alpha} + \epsilon)}{\omega - (\epsilon_{Fn} - \epsilon_{n\alpha}) + i\delta}. \]

(29)

Using the results presented in [13], we find

\[ \Pi_{s,s'}(q,\omega)\big|_{q=a\hat{y}} = \frac{1}{L_{y}} \sum_{\nu} \int_{0}^{L_{y}} \int_{0}^{L_{y}} dy \, dy' \, e^{-iq(y-y')} \int_{0}^{\infty} dc \, dc' \frac{\epsilon_{Fn}}{\sqrt{\epsilon \epsilon'}} \frac{f(E_{n} + \epsilon') - f(E_{n} + \epsilon)}{\omega - (\epsilon - \epsilon') + i\delta} F_{n}^{(1)}(y - y') \]

\[ = \sum_{n} F_{n}^{(1)}(q) \int_{0}^{\infty} dc \, dc' \frac{\epsilon_{Fn}}{\sqrt{\epsilon \epsilon'}} \frac{f(E_{n} + \epsilon') - f(E_{n} + \epsilon)}{\omega - (\epsilon - \epsilon') + i\delta}. \]

(30)
The function $\tilde{F}_n^{(1)}(q)$ is the Fourier transform of $F_n^{(1)}(y)$, and for $q l_n \ll 1$ it can be approximated by $\tilde{F}_n^{(1)}(q) \big|_{q l_n \ll 1} \approx (q l_n)^2 l_n / v_{F_n}^2$.

In order to conclude the calculation of $\Pi_{s,s'}(q, \omega)$, we still have to evaluate the integral over energies that appears in expression (30)

$$
\int \int \frac{d\epsilon d\epsilon'}{\sqrt{\epsilon \epsilon'}} \frac{1}{\omega_q - (\epsilon - \epsilon') + i\delta} = - \left( a + \frac{i \pi \omega_q}{\epsilon_{F_n}} \right),
$$

where $a$ is a numerical factor of the order of one. We have used the following expression

$$
\int \frac{d\epsilon}{\sqrt{\epsilon(\epsilon - \omega_q)}} [f(E_n + \epsilon - \omega_q) - f(E_n + \epsilon)] \approx \frac{\omega_q}{\epsilon_{F_n}},
$$

since due to the distribution functions $\epsilon$ and $\epsilon - \omega_q$ must both be close to $\epsilon_{F_n}$, and we consider $\omega_q \ll \epsilon_{F_n}$.

We finally obtain the following expression for $\Pi_{s,s'}$ in the parallel case

$$
\Pi_{s,s'}(q, \omega) = - \sum_n \epsilon_{F_n} \tilde{F}_n^{(1)}(q) \left( a + \frac{i \omega_q}{\epsilon_{F_n}} \right)
$$

$$
= \Pi_0(q, \omega_q),
$$

where

$$
\langle n'\alpha'| e^{iq_{s'}x} n\alpha \rangle = S_{n'\alpha':n\alpha}(\delta_{n',n} + iq_{s'}x_{n'n}) + iq_{s'} \sum_{m,\beta} \left[ \frac{x_{mn} W_{m\beta,n\alpha}}{E_{n\alpha} - E_{m\beta}} S_{n'\alpha':m\beta} x_{n'm} + \frac{x_{n'm} W_{n'\alpha':m\beta}}{E_{n'\alpha'} - E_{m\beta}} S_{m\beta:n\alpha} x_{mn} \right],
$$

where the summation is only over occupied subbands.

In the parallel case $\Pi_{s,s'}(q, \omega)$ has no dependence on $s$ and $s'$. Using this characteristic of $\Pi$ we can now solve Eq. (19) for the inverse dielectric function. We try a solution of the form $\delta_{s,s'} + V_s(q) g(q_{s'}) / d$, substitute it into Eq. (19) and find the function $g(q_{s'})$. For the specific case of $s = 0$ and $s' = 0$ we obtain

$$
\epsilon^{-1}_{0,0}(q, \omega_q) = 1 + \frac{V(q) \Pi(q, \omega_q) / d}{1 - V(q)^2 \Pi(q, \omega_q) / d},
$$

where $V(q) = V(q)[1 + qd / \pi \ln(dq / 2\pi \omega)]$, and $\gamma \approx 1.8$ is the Euler constant.

We now turn our attention to the perpendicular case. If we were to keep only the zeroth order wave functions in the matrix elements that appear in expression (20), we would obtain that the imaginary part of $\Pi_{s,s'}(q, \omega)$ is zero, and thus no absorption occurs. We must therefore include higher order corrections to the wave functions. In fact, we must evaluate $\Pi_{s,s'}$ up to the second order in $W$ since the first order terms give zero contribution because they are proportional to $(W' / \gamma) = 0$. We shall, however, consider the wave functions only up to the first order in $W$, and on we shall explain why the second order correction to the wave function is negligible in its contribution to $\Pi_{s,s'}$.

The matrix element that appears in expression (20) for $\Pi_{s,s'}(q, \omega_q) \big|_{q=q_{s'}}$, up to the first order in $W$ is

over powers of $W$ from that over the wave functions and the eigenenergies. Thus, if there is a term that is proportional to $W$, after averaging it will be proportional to $\langle W(y) \rangle_W = 0$.

Expression (20) includes the energy denominator $\omega_q - (E_0 - E_{n'\alpha'}) + i\delta$. When $n \neq n'$ the energy difference $E_{n\alpha} - E_{n'\alpha'}$ is large, and $\omega_q + i\delta$ can be neglected compared to it (we have in mind the phonon frequencies for $\omega_q$ and these are much smaller than the subband energy difference). Thus these terms will contribute only to the real part of $\Pi_{s,s'}$, and only terms with $n = n'$ will contribute to the imaginary part. There are additional energy denominators in the expression for $\Pi_{s,s'}$ that come from the matrix elements. We shall keep only the largest terms, those with the least number of large energy denominators. Had we kept the second order correction to the wave function, it’s contribution to $\Pi_{s,s'}$ would have been neglected at this point, since it includes too many large energy denominators. We are left with the following expressions for the real and imaginary parts of $\Pi_{s,s'}$.
Thus we can finally write
\[ \text{Re} \Pi_{s,s'}(\mathbf{q}, \omega_q) |_{\mathbf{q}=\mathbf{q}_k} = \frac{q_s q_{s'}}{L_y} \sum_{n,n'} |x_{nm}|^2 \left\langle \sum_{\alpha,\alpha'} S_{n',\alpha':n\alpha} \left( \frac{2 f(E_{n'\alpha'}) - f(E_{n\alpha})}{E_{n\alpha} - E_{n'\alpha'}} - f(E_{n\alpha}) \right) \right\rangle_U. \] (36)

and
\[ \text{Im} \Pi_{s,s'}(\mathbf{q}, \omega_q) |_{\mathbf{q}=\mathbf{q}_k} = -\pi \frac{q_s q_{s'}}{L_y} \times \sum_{n} \left| \sum_{\alpha,\alpha'} \sum_{m\beta} x_{nm} \right|^2 \left( S_{n',m\beta} W_{m\beta;n\alpha} E_{n\alpha} - E_{m\beta} + S_{m\beta;n\alpha} W_{n',m\beta} E_{n\alpha} - E_{m\beta} \right)^2 \left\langle \frac{2 f(E_{n'\alpha'}) - f(E_{n\alpha})}{E_{n\alpha} - E_{n'\alpha'}} \right\rangle_U. \] (37)

In expression (36) \( n \neq n' \) due to the factor of \( |x_{nm}|^2 \), therefore it can be simplified by estimating the energy denominator to be \( E_{n'\alpha'} - E_{n\alpha} \approx E_{n'} - E_n \). We assume that the electron wave functions in different subbands are uncorrelated. Using the following definitions
\[ N_n = \frac{1}{L_y} \left\langle \sum_{\alpha} f(E_{n\alpha}) \right\rangle_U, \] (38)
the electron density in subband \( n \), and
\[ p_n = \sum_{n'} \frac{|x_{nm}|^2}{E_n - E_{n'}}, \] (39)

the polarization related to subband \( n \), we can rewrite (36) as
\[ \text{Re} \Pi_{s,s'}(\mathbf{q}, \omega_q) |_{\mathbf{q}=\mathbf{q}_k} = -2 q_s q_{s'} \sum_n N_n p_n. \] (40)

In expression (37) the energy denominators \( E_{n\alpha} - E_{m\beta} \) and \( E_{n'\alpha'} - E_{n\alpha} \) can be approximated by \( E_n - E_m \), since \( n \neq m \) due to the factor \( |x_{nm}|^2 \). Furthermore, by separating the averaging over \( W \) from that over the wave functions, and using the definitions of \( F_{n}(1) \) (26), and \( p_n \) (38), this term can be written as
\[ \text{Im} \Pi_{s,s'}(\mathbf{q}, \omega_q) |_{\mathbf{q}=\mathbf{q}_k} = -4 \pi \frac{q_s q_{s'}}{L_y} \omega_q \sum_n p_n^2 \int_0^{L_y} dy dy' (W(y)W(y'))_U F_{n}(1)(y - y'). \] (41)

The summation here is only over occupied subbands.

The correlation \( (W(y)W(y'))_U \) is a function of \( y - y' \) only, and the length scale in which it changes is \( \Lambda_n \), while the length scale in which \( F_{n}(1) \) changes is \( L_n \). Since we consider the case in which \( \Lambda_n \ll L_n \) we may write
\[ \int_0^{L_y} dy dy' (W(y)W(y'))_U F_{n}(1)(y - y') = L_y F_{n}(1)(0) \int_{-\infty}^{\infty} dy (W(y)W(0))_U = L_y F_{n}(1)(0)(W^2)_{q=0}. \] (42)

Thus we can finally write
\[ \Pi_{s,s'}(\mathbf{q}, \omega_q) |_{\mathbf{q}=\mathbf{q}_k} = -2 q_s q_{s'} \sum_n \left( N_n p_n + \frac{i}{3 \pi} m^* n_p^2 \omega_q W(2)^2 q=0 \right) = \frac{q_s q_{s'}}{q^2} \Pi \perp(q, \omega_q). \] (43)

In the perpendicular case \( \Pi_{s,s'}(\mathbf{q}, \omega_q) \propto q_s q_{s'} \). Using this characteristic of \( \Pi \) we substitute a solution of the form \( \delta_{s,s'} + q_s V_s(\mathbf{q}) g(q_{s'}) \) for \( \epsilon_{s,s'}^{-1} \) into Eq. (19), and find \( g(q_{s'}) \). For the specific case of \( s = 0 \) and \( s' = 0 \) we find
\[ \epsilon_{0,0}^{-1}(\mathbf{q}, \omega_q) |_{\mathbf{q}=\mathbf{q}_k} = 1 + \frac{V(q) \Pi \perp(q, \omega_q)}{1 - V \perp(q)\Pi \perp(q, \omega_q)/d}, \] (44)
where \( V \perp(q) = V(q)(1 + d/2\pi q u^2) \).

The SAW absorption, and its relative change of velocity, can now be found by substituting expressions (34) and (44) for \( \epsilon_{0,0}^{-1} \) in the parallel and perpendicular cases, into expressions (6) and (5).

VI. DISCUSSION

Let us begin by discussing expressions (34) and (14) for \( \epsilon_{0,0}^{-1} \). From these expressions one can see that we could not have replaced \( \sigma_{dc} \) by \( \sigma_{ac} \) in the “usual” expression for the attenuation \( \Gamma \) (7). The numerators in these expressions describe the unscreened interaction, while the denominators describe the screening. This can be seen by expanding the expressions for \( \epsilon_{0,0}^{-1} \) in the electron charge.
The combinations of V II that enter the screening and the interaction are different, therefore the attenuation cannot be described by $\Sigma_{dc}$, replaced by $\Sigma_{ac}$.

One can see from expressions (23) and (13) that in both cases $\text{Im} \Pi \ll \text{Re} \Pi$ due to the factor of $\omega / \epsilon_{Fn}$, thus the screening is dominated by $\text{Re} \Pi$. The ratio $\Pi/d$ that appears in the expressions for $\epsilon_{1,0}$ is the effective two dimensional polarization of the patterned 2DEG.

In the parallel case, as $\epsilon_{Fn}$ increases, that is, as we move away from the threshold, the localization $l_n$ increases, and so does the screening. This can be understood in the following manner: When the electron energy increases, the random potential seems weaker, and therefore the localization length grows. When the localization length increases, the electrons can move more freely, and the thus the screening is enhanced.

In the perpendicular case the mechanism through which the screening increases as $\epsilon_{Fn}$ increases is different. As the bottom of the subband is lowered, the electron number in the QWR increases, and therefore the screening increases. One can see that as the wire width $w$ decreases towards zero $\epsilon_{0,0}^{-1}$ approaches one, so there is no screening. This is due to the fact that $x_{nn'}$ that appears in $p_n$ (13), which in turn appears in the expression for $\Pi^{2 \perp}$ (13), is proportional to $w$.

We find that in both the parallel and the perpendicular geometries $\text{Im} \epsilon_{0,0}^{-1} \propto 1/\epsilon_{Fn}$ (in the parallel case it is due to $F_n^{(1)}(q) \propto 1/v_{Fn}^2$), and therefore the absorption should show a structure that is related to the crossing of the Fermi energy by the bottom of the subbands in both geometries, unlike what was seen in the experiment (10).

We can now make some numerical estimates of the magnitude of the phonon absorption per unit length $\Gamma$, and the phonon change in velocity $\Delta v/v$, for both the parallel and the perpendicular cases. We consider an array of wires of period $d = 10^4 \, \text{Å}$ and of width $w = 5000 \, \text{Å}$ manufactured in GaAs, as in Ref. (10). We take the localization length $l_n \simeq 1 \, \mu\text{m}$, in accordance with experimental work done in quantum wires (13). We find that in both cases $\Gamma \simeq 10^{-4} \, \text{cm}^{-1}$ and $\Delta v/v \simeq -(10^{-2} - 10^{-4})$.

The fact that our results do not fully agree with the experimental findings raises the question whether the magnetic field should not be considered more accurately. We considered it only as a mechanism for moving the bottom of the energy subbands, and yet the magnetic field should affect both the electron wave functions, and the localization length (that should increase when a magnetic field is turned on).

**ACKNOWLEDGMENTS**

We would like to thank D. Khmelnitskii for helpful discussions. One of us (Y. L.) would like to thank G. R. Nash and S. J. Bending for the discussions of the experimental aspects, and the British Council in Tel Aviv for its support.

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