CURVE SELECTION LEMMA IN INFINITE DIMENSIONAL ALGEBRAIC GEOMETRY AND ARC SPACES

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ABSTRACT. We prove a Curve Selection Lemma valid in infinite-dimensional algebraic geometry and apply it, together with a structure theorem of Grinberg-Kazhdan and Drinfeld, to obtain a Curve Selection Lemma in arc spaces.

1. Introduction

Curve Selection Lemma is valid in many situations in finite algebraic and analytic geometry, and it is a very useful tool in many geometric situations. To give an idea of its content we state one of its versions. Let $X$ be a scheme of finite type over an algebraically closed field $K$. If a $K$-valued point $a$ is in the Zariski closure $\overline{A}$ of a constructible subset $A$, then there is a non-constant morphism

$$\alpha : \text{Spec}(K[[s]]) \to \overline{A}$$

sending the closed point to $a$ and the generic one to a point in $A$. If $K$ is equal to $\mathbb{C}$ the parametrization can be found convergent, or even given by algebraic power series.

One of the reasons why it is important to generalize Curve Selection Lemma to infinite dimensional algebraic geometry is its applications to arc spaces of algebraic varieties, and, more concretely to the study of Nash problem. See for example [5],[6],[9],[10],[11],[14],[15]. In these cases and in particular in the proof of Nash Problem for surfaces in [6], it suffices the version of the Curve Selection Lemma in [15] and further developments in [5]. Here we do a stronger version of it.

Unfortunately in infinite dimensional algebraic geometry a plain formulation of Curve Selection Lemma as stated above is not true (see Example 4). In this paper we prove a version of Curve Selection Lemma under the assumption that the set $\overline{A}$ is of finite codimension in an affine space (possibly of infinite dimension). This is the content of Theorem 2 and Corollary 3 of the second section.

In the last section we show how to apply Theorem 2 in order to obtain a Curve Selection Lemma in arc spaces. The difficulty is that arc spaces are not finite codimensional in the affine space of infinite dimensions. The idea is to use a theorem of Grinberg, Kazhdan and Drinfeld, which tells that the formal neighbourhood of the arc space at a $K$-arc not contained in the singular set indeed looks like the formal neighbourhood of a finite codimensional algebraic set of an affine space. The

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versions of the Curve Selection Lemma in arc spaces that we obtain are Theorem 7 and Corollary 13.

In [15], Corollary 4.8, A. Reguera proves a Curve Selection Lemma in arc spaces of the following form. Let $X$ be an algebraic variety and let $N$ and $N'$ two irreducible subsets of the arc space $L(X)$ of $X$. Suppose that $N$ is generically stable (see Definition 10) and that we have a proper inclusion $N \subset N'$. Let $K$ be the residue field of $N$. Then there is a finite field extension $K \subset L$ and a morphism

$$\alpha : \text{Spec}(L[[s]]) \to L(X)$$

whose special point is sent to the generic point of $N$ and such that the image of the generic point of $\text{Spec}(L[[s]])$ falls in $N' \setminus N$.

Notice that the output of the previous result is a parametrization defined over the field $L$ which is of infinite transcendence degree over the base field. In many applications it is necessary to obtain a Curve Selection Lemma whose outcome is defined over the base field. In [5] and in [11], assuming that $K$ is uncountable and using a specialization procedure, it is deduced from A. Reguera’s result the existence of parametrizations

$$\alpha : \text{Spec}(K[[s]]) \to L(X)$$

whose special point is sent to a sufficiently general point of $N$ and such that the image of the generic point of $\text{Spec}(K[[s]])$ falls in $N' \setminus N$. However these specialization procedures do not allow to choose the point $a \in N$. Applying the results of this paper we prove that, given any $a \in N$ there exists a morphism

$$\alpha : \text{Spec}(K[[s]]) \to L(X)$$

whose special point is sent to the point $a$ and such that the image of the generic point of $\text{Spec}(K[[s]])$ falls in $N' \setminus N$. Furthermore we do not need to assume uncountability of the base field.

The last improvement is that we assume finite codimensionality for the big set $N'$ instead of the small one $N$, which in our result may be taken to be even a point.

We thank the referee for detecting a gap in a previous version of proof of Proposition 8 and for pointing out Example 9.

2. A curve selection lemma in infinite dimensional algebraic geometry

Let $K$ be an algebraically closed field and $I$ any set of indices. Let $\{x_i\}_{i \in I}$ be a set of variables indexed by $I$. Denote by $K[\{x_i\}_{i \in I}]$ the ring of polynomials, and by $K[[\{x_i\}_{i \in I}]]$ the ring of formal power series with finitely many summands at each degree. We define the affine space $\mathbb{A}^I$ to be the scheme

$$\mathbb{A}^I := \text{Spec}(K[\{x_i\}_{i \in I}])$$

and we denote it simply by $\mathbb{A}^I$. Any $K$-valued point of $a \in \mathbb{A}^I$ corresponds to a maximal ideal of the form $m_a := (\{x_i - a_i\}_{i \in I})$, with coordinates $a_i \in K$ for any $i$. The origin $O$ is the point with coordinates $a_i = 0$ for any $i \in I$. The ring $K[[\{x_i\}_{i \in I}]]$ is the $m_0$-adic completion of $K[\{x_i\}_{i \in I}]$. Denote by $\mathbb{A}^I_O$ the scheme

$$\mathbb{A}^I_O := \text{Spec}(K[[\{x_i\}_{i \in I}]]).$$

We have an scheme morphism

$$\mathbb{A}^I_O \to \mathbb{A}^I.$$
Let \( N \subset \mathbb{A}^I \) be a Zariski closed subset. Denote by \( \mathcal{I}(N) \) its defining ideal.

**Definition 1.** Consider an inclusion of Zariski closed subsets \( N \subset N' \subset \mathbb{A}^I \), being \( N \) irreducible. The codimension of \( N \) in \( N' \) is defined to be the height of \( \mathcal{I}(N)/\mathcal{I}(N') \) in \( \mathbb{K}[[\{x_i\}_{i \in I}]]/\mathcal{I}(N') \). We denote it by \( \text{codim}(N', N) \).

**Theorem 2.** Let \( N \) be an irreducible finite codimensional subset of \( \mathbb{A}^I \). Let \( Z \subset N \) be a Zariski closed subset properly contained in \( N \). Then there exists a morphism
\[
\alpha : \text{Spec}(\mathbb{K}[[s]]) \rightarrow N
\]
mapping the closed point to the origin \( O \) and the generic point to \( N \setminus Z \).

**Proof.** If \( N \) is proper then \( \mathcal{I}(N) \) has a non-zero element \( f \). Let \( k \) be the order of \( f \) (the lowest degree of a monomial in \( f \)). Choose an element \( i_0 \in I \). After linear change of variables (involving in fact only finitely many variables) we may assume that \( f \) contains the monomial \( x_{i_0}^k \). Weierstrass Division Theorem (ch. VII, §3, n°8, Propositions 5 and 7) implies that the ring \( \mathbb{K}[[\{x_i\}_{i \in I}]]/(f) \) is a free \( \mathbb{K}[[\{x_i\}_{i \in I \setminus \{i_0\}}]] \)-module generated by \( \{1, x_{i_0}, \ldots, x_{i_0}^{k-1}\} \). Consequently the \( (x_{i_0}) \)-projection
\[
\pi_{i_0} : \text{Spec}(\mathbb{K}[[\{x_i\}_{i \in I}]]/(f)) \rightarrow \text{Spec}(\mathbb{K}[[\{x_i\}_{i \in I \setminus \{i_0\}}]])
\]
is finite and flat.

Since \( \pi_{i_0} \) is finite and \( N \) is irreducible, by Theorem 9.3 of [12] the images \( N_1 := \pi_{i_0}(N) \) and \( Z_1 := \pi_{i_0}(Z) \) are Zariski closed subsets of \( \text{Spec}(\mathbb{K}[[\{x_i\}_{i \in I \setminus \{i_0\}}]])\) and \( Z_1 \) is a proper subset of \( N_1 \).

On the other hand, since \( \pi_0 \) is flat, by Theorem 9.5 of [12], we have the inequality
\[
\text{codim}(\text{Spec}(\mathbb{K}[[\{x_i\}_{i \in I \setminus \{i_0\}}]]), N_1) \leq \text{codim}(\text{Spec}(\mathbb{K}[[\{x_i\}_{i \in I}]]/(f)), N).
\]

Since any prime ideal containing \( f \) has height at least 1 in \( \mathbb{K}[[\{x_i\}_{i \in I}]] \) we conclude that
\[
\text{codim}(\mathbb{A}^I \setminus \{i_0\}, N_1) \leq \text{codim}(\mathbb{A}^I, N) - 1. \tag{1}
\]

We iterate the procedure of changing coordinates and projecting until we find a set of indices \( J := \{i_0, \ldots, i_r\} \) such that the restriction
\[
\pi_J : N \rightarrow \text{Spec}(\mathbb{K}[[\{x_i\}_{i \in I \setminus J}]])
\]
to \( N \) of the \( (x_0, \ldots, x_r) \)-projection is finite and surjective, and moreover \( Z_r := \pi_J(Z) \) is a proper Zariski closed subset of \( \text{Spec}(\mathbb{K}[[\{x_i\}_{i \in I \setminus J}]])) \). The inequality (1) ensure that the process finishes after finitely many steps.

Choose any morphism
\[
\beta : \text{Spec}(\mathbb{K}[[t]]) \rightarrow \text{Spec}(\mathbb{K}[[\{x_i\}_{i \in I \setminus J}]])
\]
such that the image of the closed point is the origin and the image of the generic point does not belong to \( Z_r \). This may be done, for example in the following way. Consider an element \( g \) of \( I(Z_r) \) and let \( k_0 \) be its order. Let \( x_{i_1}, \ldots, x_{i_m} \) be the variables that appear on the homogeneous form \( g_{k_0} \) of degree \( k_0 \) of \( g \). Consider a vector \( v \) in \( \mathbb{K}^m \) such that \( g_{k_0}(tv) \) is not identically 0. Then, the line \( L \) in the direction of \( v \) in \( \mathbb{A}^I \) is not contained in \( Z \) and we can parametrize it.

Consider \( L \times \mathbb{A}^J \). This is a finite dimensional affine subspace of \( \mathbb{A}^I \) that cuts \( N \) in positive dimension. Then, there exists a formal parametrization
\[
\alpha : \text{Spec}(\mathbb{K}[[s]]) \rightarrow (L \times \mathbb{A}^J) \cap N.
\]

\[\square\]
Corollary 3. Let $Z \subset N$ be a proper inclusion of algebraic subsets in $\mathbb{A}^I$. Let $a$ be a $\mathbb{K}$-valued point of $N$. If at least one of the minimal primes of the completion of the ring $\mathbb{K}[\{x_i\}_{i \in I}]/I(N)$ for the $\mathfrak{m}_a$-adic filtration does not contain the ideal that $I(Z)$ generates in the completion, then there exists a morphism

$$\alpha : \text{Spec}(\mathbb{K}[[s]]) \to N$$

mapping the closed point to $a$ and the generic point outside $Z$.

The following example shows that the finite codimension condition for $N$ is essential.

Example 4. Consider $I := \mathbb{N}$ and $N := V(\{x_1 - x_k^k\}_{k \in \mathbb{N}})$. Let $Z$ be equal to the origin. There is no morphism

$$\alpha : \text{Spec}(\mathbb{K}[[s]]) \to N$$

such that $\alpha(0)$ is equal to the origin and such that the image of the generic point is not the origin; otherwise the formal power series $x_1(\alpha(s))$, which is not identically zero and is divisible by $s$, would have $k$-th roots for any positive $k$.

3. An application to arc spaces

Let $X$ be an algebraic variety defined over an algebraically closed field $\mathbb{K}$. We denote by $L(X)$ its space of arcs. All over the section $\gamma$ denotes an arc not contained in the singular locus,

$$\gamma : \text{Spec}(\mathbb{K}[[t]]) \to X.$$

Let $O_{L(X),\gamma}$ be the local ring of $L(X)$ at $\gamma$ and $\widehat{O_{L(X),\gamma}}$ be its completion. We denote by $\varphi_\gamma$ the natural map

$$\varphi_\gamma : \text{Spec}(\widehat{O_{L(X),\gamma}}) \to L(X).$$

We will use the previous results to prove a Curve Selection Lemma in arc spaces with the help of the following theorem, which was proved by Grinberg and Kahzdan [7] in characteristic 0 and by Drinfeld [3] in arbitrary characteristic. Another proof was provided by C. Bruschek and H. Hauser in [2].

Theorem 5 (Grinberg-Kahzdan, Drinfeld). There exists a scheme of finite type $Z$ over $\mathbb{K}$, a $\mathbb{K}$-point $z \in Z$ and an isomorphism of formal schemes

$$\widehat{L(X)}_{\gamma} \cong \widehat{Z} \times \widehat{\mathbb{A}^N}_{\mathbb{O}}$$

where $\widehat{L(X)}_{\gamma}$, $\widehat{Z}$ and $\widehat{\mathbb{A}^N}_{\mathbb{O}}$ denote the formal neighbourhoods of $\gamma$, $z$ and the origin in $L(X)$, $Z$ and $\mathbb{A}^N$ respectively.

Since the topological spaces underlying the formal schemes appearing in the isomorphism [2] consist of one point their structure sheaves are just rings. The isomorphism [2] just expresses that the rings $\widehat{O_{L(X),\gamma}}$ and $\widehat{O_{Z,z}} \times \mathbb{K}[\{x_i\}_{i \in \mathbb{N}}]$ are isomorphic.

Since $Z$ is of finite type, the structure sheaf of $\widehat{Z}$ is the quotient of a formal power series ring in finitely many variables by an ideal, which is finitely generated by noetherianity. Hence we have the following:
Corollary 6. We have an isomorphism
\[ \mathcal{O}_{\hat{\text{L}(X)}}, \gamma \cong \mathbb{K}[[\{x_k\}_{k \in \mathbb{N}}]]/J \]

where \( J \) is finitely generated by series in finitely many variables.

The Curve Selection Lemma for arc spaces that we obtain is the following.

Theorem 7. Let \( Z \subset N \subset \mathcal{L}(X) \) be Zariski closed subsets of the arc space of \( X \) and let \( \gamma \) be in \( N \). If codim(\( \varphi_\gamma^{-1}(N), \text{Spec}(\mathcal{O}_{\hat{\text{L}(X)}, \gamma}) \)) is finite and \( \varphi_\gamma^{-1}(Z) \) is properly included in \( \varphi_\gamma^{-1}(N) \) then there exists a morphism
\[ \alpha : \text{Spec}(\mathbb{K}[s]) \to N \]
mapping the special point to \( \gamma \) and the generic point outside \( Z \).

Proof. By [3] we get that if the ideal \( I(\varphi_\gamma^{-1}(N)) \) defining \( \varphi_\gamma^{-1}(N) \) in \( \text{Spec}(\mathcal{O}_{\hat{\text{L}(X)}, \gamma}) \) is of finite height in \( \mathcal{O}_{\hat{\text{L}(X)}, \gamma} \) then the ideal defining \( \varphi_\gamma^{-1}(N) \) as a subscheme of \( \text{Spec}(\mathbb{K}[\{x_k\}_{k \in \mathbb{N}}]) \) is also of finite height.

Then, a direct application of Theorem 2 produces a morphism
\[ \alpha' : \text{Spec}(\mathbb{K}[s]) \to \text{Spec}(\mathcal{O}_{\hat{\text{L}(X)}, \gamma}) \]
such that the image of the special point is \( \gamma \) and the image of the generic point belongs to \( \varphi_\gamma^{-1}(N) \setminus \varphi_\gamma^{-1}(Z) \). Composing with \( \varphi_\gamma \) we obtain \( \alpha \). \( \square \)

For application of the previous theorem to certain sets of the arc spaces we need a generalization of the Principal Ideal Theorem to power series rings in infinitely many variables:

Proposition 8. Let \( \mathcal{I} \) be a proper ideal of \( \mathbb{K}[[\{x_1\}_{k \in \mathbb{N}}]] \) generated by \( r \) elements. The height of any minimal prime ideal containing \( \mathcal{I} \) is at most \( r \).

Proof. We start proving the proposition in the special case where there is a unique minimal prime ideal \( \mathfrak{p}_1 \) containing \( \mathcal{I} \). Consider a chain of distinct prime ideals
\[ \mathfrak{p}_1 \supset \mathfrak{p}_2 \supset \ldots \supset \mathfrak{p}_s = 0. \]

We claim that, after a linear change of coordinates, the restriction to \( V(\mathcal{I}) \) of the projection
\[ \sigma : \text{Spec}(\mathbb{K}[[\{x_1\}_{k \in \mathbb{N}}]]) \to \text{Spec}(\mathbb{K}[[\{x_i\}_{i \geq s}]]) \]
is finite.

The claim implies the proposition in the considered special case. Indeed, let \( \tilde{f}_i \in \mathbb{K}[[x_1, \ldots, x_{s-1}]] \) be the result of evaluating at 0 in \( f_i \) the variables \( \{x_i\}_{i \geq s} \). The fibre of the restriction of \( \sigma \) to \( V(\mathcal{I}) \) is
\[ \text{Spec}[[x_1, \ldots, x_{s-1}]]/(\tilde{f}_1, \ldots, \tilde{f}_r). \]

Since the restriction is finite we conclude that \( r \geq s - 1 \). Therefore the height of \( \mathcal{I} \) is at most \( r \).

Let us show the claim by induction. Let \( g \neq 0 \) be an element in \( \mathfrak{p}_{s-1} \). It is easy to find a linear change of coordinates such that all \( g \) has a monomial that is a power of the single variable \( x_1 \). Using Weierstrass Preparation Theorem ([1] chap. VII, §3, n°8, Proposition 6), we may assume that \( g \) is a Weierstrass polynomial in the variable \( x_1 \). The restriction
\[ \pi_1 : V(g) \to \text{Spec}(\mathbb{K}[[\{x_i\}_{i \geq 2}]]). \]
is finite and flat as in the proof of Theorem 2.

Define \( q_i := p_i \cap \mathbb{K}[[\{x_i\}_{i \geq 2}]] \). We claim that \( q_i \) is the only minimal prime containing \( I \cap \mathbb{K}[[\{x_i\}_{i \geq 2}]] \). Indeed, the defining ideal of the set \( \pi(V(I)) \) is the intersection of the defining ideal of \( V(I) \) with \( \mathbb{K}[[\{x_i\}_{i \in I \setminus \{i_0\}}]] \), and the defining ideal of \( V(I) \) is the intersection of the minimal primes containing it.

By Theorem 9.3 of [12] all the inclusions in the chain

\[
q_1 \supset q_2 \supset \ldots \supset q_{s-1}
\]

are strict. Then, by induction, after a possible linear change of coordinates, there is a finite projection from \( V(q_1) \) to \( \text{Spec}(\mathbb{K}[[\{x_i\}_{i \geq 1}]] \). Composing with \( \pi_1 \) we obtain the desired projection \( \sigma \). The claim is proved.

For the general case, suppose that \( I \) is generated by \( r \) elements \( f_1, \ldots, f_r \). Fix a minimal prime ideal \( p_1 \) containing \( I \). Let \( I_1 \) be the intersection of the minimal prime ideals containing \( I \) which are different from \( p_1 \). Choose an element \( g \) in \( I_1 \setminus p_1 \). Consider the localization \( \mathbb{K}[[\{x_i\}_{i \in I}]]_g \) at \( g \). The ideal \( I' \) generated by \( I \) in the localization \( \mathbb{K}[[\{x_i\}_{i \in I}]]_g \) has a unique minimal prime ideal, namely the ideal generated by \( p_1 \). We denote it by \( p_1' \). Moreover the height of \( p_1 \) coincides with the height of \( p_1' \). Observe that the ring \( \mathbb{K}[[\{x_i\}_{i \in I}]]_g \) is isomorphic to the ring

\[
\mathbb{K}[[\{z\} \cup \{x_i\}_{i \in I}]/(gz - 1).
\]

The ideal \( I'' \) in \( \mathbb{K}[[\{z\} \cup \{x_i\}_{i \in I}]] \) generated by \( f_1, \ldots, f_r, gz - 1 \) has also a unique minimal prime ideal, namely the ideal generated by the preimage of \( p_1' \) and \( gz - 1 \). We denote it by \( p_1'' \). Hence we can apply the special case of the proposition to the ideal \( I'' \) and deduce that the height of \( I'' \) in

\[
\mathbb{K}[[\{z\} \cup \{x_i\}_{i \in I}]]
\]

is at most \( r + 1 \).

Now, if there is a chain of prime ideals \( p_1 \supset \ldots \supset p_s \) in \( \mathbb{K}[[\{x_i\}_{i \in I}]] \), localizing we obtain a chain of prime ideals \( p_1' \supset \ldots \supset p_s' \) in \( \mathbb{K}[[\{x_i\}_{i \in I}]]_g \). Adding the zero ideal at the end of the chain formed by the preimage of these ideals in

\[
\mathbb{K}[[\{z\} \cup \{x_i\}_{i \in I}]]
\]

we obtain a chain of prime ideals \( p_1'' \supset \ldots \supset p_s'' \supset (0) \) of length \( s + 1 \). Therefore \( s + 1 \leq r + 1 \) and we deduce the general case of the proposition. \( \Box \)

The following example shows that for general non-Noetherian rings, the Principal Ideal Theorem, and hence the previous Proposition, is not true in general:

**Example 9.** Consider the valuation ring in \( \mathbb{K}(x, y) \) given by the rank two valuation

\[
\nu(f) := (\text{ord}_x(f), \text{ord}_y(x^{-\text{ord}_x(f)}f)|_{x = 0}).
\]

The maximal ideal is generated by \( y \) and has height 2 (note that \( (0) \subset (x) \subset (y) \)).

Let’s recall the following definition of [15].

**Definition 10.** A Zariski closed subset \( N \) in the arc space \( \mathcal{L}(X) \) is generically stable if there exists an affine open subset of the arc space in which \( N \) is defined as a set by finitely many equations.

**Remark 11.** Most of the Zariski closed subsets considered in the literature are generically stable. Examples are Zariski closed cylindrical sets, contact loci with
ideals and maximal divisorial sets (see [1] and [8]). Some other very important examples in connection with Nash problem are the following: let

\[ \pi : Y \to X \]

be a proper birational morphism and \( E \) an irreducible component of the exceptional set of \( \pi \). Denote by \( N_E \) the Zariski closure in \( \mathcal{L}(X) \) of the set of arcs \( \gamma \) having a lifting \( \tilde{\gamma} \) to \( Y \) such that \( \tilde{\gamma}(0) \) belongs to \( E \) (these sets were introduced in [13]).

**Lemma 12.** Let \( N \) be an irreducible generically stable subset of the arc space \( \mathcal{L}(X) \) of a variety \( X \). Consider \( \gamma \) in \( N \). Then \( \text{codim}(\text{Spec}(\widehat{\mathcal{O}_{\mathcal{L}(X)}}(\gamma)), \varphi^{-1}(N)) \) is finite.

**Proof.** Let \( U \) be a Zariski open subset in \( \mathcal{L}(X) \) containing \( \gamma \) and \( U' \) a Zariski open subset in which \( N \) is defined by the vanishing of finitely many regular functions. Then \( N \cap U \cap U' \) is defined by the restriction of these equations. The functions defining \( N \cap U' \) are rational functions defined in \( U \). The ideal generated by the numerators of these rational functions defines a Zariski closed subset having \( N \cap U \) as one of its irreducible components. A direct application of Proposition 8 gives what we need.

Then a direct application of Theorem 7 gives the following:

**Corollary 13.** Let \( N \) be an irreducible generically stable subset of the arc space \( \mathcal{L}(X) \) of a variety \( X \) and let \( \gamma \) be in \( N \). Let \( Z \) be another Zariski closed subset of \( \mathcal{L}(X) \) not containing \( N \). Then there exists a morphism

\[ \alpha : \text{Spec}(\mathbb{K}[[s]]) \to N \]

mapping the special point to \( \gamma \) and the generic point outside \( Z \).

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