The Extent of the Spectral Bias in BATSE: The True Distribution of the $\nu F_{\nu}$ Peak Energy

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Abstract. The distributions of spectral characteristics and their correlations with fluence, peak flux, or duration, are essential in understanding the nature of GRBs. However, the selection effects involved in detecting GRBs can distort these distributions. Here, we discuss how to deal with selection effects involving the peak energy $E_p$ of the GRB $\nu F_{\nu}$ spectrum, which suffers from both an upper and lower threshold. We describe a new method to account for this double-sided truncation, and show that the true distribution of $E_p$ is significantly different from the observed distribution.

INTRODUCTION

The spectral properties of GRBs provide the most direct information about the physical processes associated with the event. In particular, the distribution of the spectral parameters and the correlation between these and other GRB characteristics can shed significant light on the radiation mechanisms and energy production of the burst.

Several studies have obtained the spectral parameters for the brightest GRBs (1) and investigated correlations between these parameters and peak flux (2), duration (3) and spatial distributions (4).

However, as pointed out by Piran and Narayan (5), caution is required in obtaining these distributions and correlations. Several selection effects come into play in detecting GRBs, which limit the information we can obtain from the data. These selection criteria often truncate the data, which - if not accounted for properly - can result in incorrect distributions and correlations.

In some cases, such as with $\log(N) - \log(S)$ distributions, the data is truncated from only one side; methods dealing with this kind of truncation were discussed by Petrosian (6) and Efron and Petrosian (7) and applied to GRBs by Lee and Petrosian (8).
However, in other cases the data truncation can be more complicated - some parameters can have both a lower and upper limit. A good example of this is the peak energy $E_p$ of the $\nu F_\nu$ spectrum; this is the focus of our paper.

**SPECTRAL PARAMETERS**

It is well known that the $\nu F_\nu$ spectrum of a GRB peaks at an energy $E_p$ with power law indices of $\alpha$ and $\beta$ below and above this energy, respectively. Band et al. have presented a useful, smooth form of such a spectrum:

$$F(E) = \begin{cases} 
\frac{A}{E_p} \left(\frac{E}{E_p}\right)^\alpha \exp\left[\left(\beta - \alpha\right)\left(\frac{E}{E_p} - 1\right)\right], & E < E_p \\
\frac{A}{E_p} \left(\frac{E}{E_p}\right)^\beta, & E > E_p 
\end{cases}$$

(1)

$F(E)$ can be the photon or energy fluence or flux. In this paper, we will deal with the energy fluence in which case $EF(E) \propto \nu F(\nu)$, and $A$ is in units of ergs/cm$^2$.

Our aim is to demonstrate how to obtain bias free distributions for $E_p$.

**DATA**

Determining accurate values for the spectral parameters of a large number of GRBs is difficult. We use the ratio of the fluences in the four channel BATSE LAD detectors to determine $\alpha$, $E_p$, and $A$ via the downhill simplex method. The index $\beta$ was derived from an assumed gaussian distribution; the form of this distribution was inconsequential to the calculations as a whole.

Given the values of $\alpha$, $\beta$, $A$, and the fluence thresholds for each burst, we may ask (in the spirit of the $V/V_{max}$ test): What $E_p$ brings the fluence below the threshold value? [See Petrosian and Lee (9) for how the threshold is obtained from the $C_{max}$ and $C_{min}$ values in the catalog.] It can be shown, as a result of the BATSE trigger condition, that there is both an upper limit $E_{p\max}$ and lower limit $E_{p\min}$ (another way of stating this is that BATSE is most sensitive to bursts with $E_p$ in the 50-300 keV trigger range). As pointed out by Piran and Narayan (5), this introduces some bias in the distribution of $E_p$. In order to quantify the severity of this bias, we have determined the values of $E_{p\min}$ and $E_{p\max}$ for 433 bursts in the 3B catalog (See Figure 1); from this, we obtain the corrected distribution of $E_p$ and compare it with the raw distribution.

**METHOD**

Our aim is to correct the observed distribution given that the observations can detect only bursts with $E_p$ limited to the interval $[E_{p\min}, E_{p\max}]$. Recently, in collaboration with B. Efron, we have developed a method to deal with this kind of data truncation (this method is a generalization of the one sided monotonic truncation case employed in our earlier studies (6), (7)).
Consider data points \((x_i, y_i)\) (or in our case \((E_p, F)\)), where \(x_i\) has lower and upper limits \(l_i\) and \(u_i\), respectively; \(x_i \in T_i = [l_i, u_i]\). Let \(f(x)\) be the true distribution of \(x\), which would be observed if there were no truncations. However, because \(x_i\) is limited to \(T_i\), we observe the conditional distribution \(f(x_i|T_i) = f(x_i)/F(T_i)\) where \(F(T_i) = \sum f(x_j) : x_j \in T_i\) is the probability that \(x\) exists in \(T_i\). We define

- \(f_i = f(x_i)\),
- \(F_i = F(T_i)\), and
- \(J_{i,j} = \begin{cases} 1, & x_j \in T_i \\ 0, & x_j \notin T_i \end{cases}\).

The goal is to estimate \(f(x)\) from \(l_i, u_i\), and \(x_i\) assuming all \(N\) cases are independently distributed. The procedure for this amounts to solving the following three equations iteratively:

1. \(F_i = J_{i,j}f_j\) \hspace{1cm} (definition),
2. \(1/f_i = J_{j,i}F_j + constant\) \hspace{1cm} (maximum likelihood assertion),
3. \(\sum_{i=1}^N f_i = 1\) \hspace{1cm} (normalization).

Convergence is reached when \(constant\) goes to zero.

This method can be used to determine univariate cumulative and differential distributions, as well as correlations between relevant variables (e.g. fluence and \(E_p\) or peak flux and \(E_p\)).
FIGURE 2. Figure (a) shows the corrected and uncorrected cumulative distributions (calculated from above and below) of $E_p$, where the total number of gamma ray bursts is normalized to 1. Figure (b) displays the corrected and uncorrected differential distributions of $E_p$, normalized to the low end of the distribution to emphasize the relative difference between the number of bursts with $E_p$ above and below 1MeV.

RESULTS

In Figure 2, we present the uncorrected and corrected cumulative and differential distributions of $E_p$. Note the large number of $E_p$'s above 1MeV in the corrected distribution, not present in the uncorrected distribution.

We test the hypothesis that the two distributions are from the same parent distribution, using the Kolmogorov-Smirnov (K-S) test (for the cumulative distributions) and $\chi^2$ test (for the differential distributions). The $\chi^2$ test is performed dividing the uncorrected data into two and three bins of equal numbers of bursts. The probabilities that the above hypothesis is true are shown below for each test.

| Test                              | Statistic | Probabilities |
|----------------------------------|-----------|---------------|
| K-S                              | 0.26      | $5 \times 10^{-13}$ |
| $\chi^2$ (2 bins, 216 bursts/bin) | 42.7      | $6 \times 10^{-11}$ |
| $\chi^2$ (3 bins, 144 bursts/bin) | 58.5      | $2 \times 10^{-13}$ |

As shown elsewhere in these proceedings (Harris et al (11)), the corrected distribution agrees well with the distribution of $E_p$ obtained from untriggered SMM data.

As a by product of this work, we are able to use the spectral parameters we obtained for each burst to correct the observed fluence (50-300keV) to the total fluence. Figure 3 shows the $\log(N) - \log(S)$ distributions for total fluence (essentially the distribution of $A$ in equation (1)) of our entire sample, as well as the distribution for bursts with $E_p$ above 1MeV and $E_p$ below 400keV.

In the future, with more accurate values of the spectral parameters, we can not only refine the corrected distribution of $E_p$, but also correct the observed distribu-
FIGURE 3. Cumulative Distributions of the total fluence for 433 GRBs, as well as the distributions for GRBs with $E_p < 400\text{keV}$ (172 GRBs) and $E_p > 1\text{MeV}$ (129 GRBs). Deviation from the dashed line indicates deviation from homogeneous, isotropic, static, and Euclidean geometry (HISE).

... of $\alpha$ and $\beta$; we can then determine correlations between these parameters and fluence or flux, accounting for the double sided truncation.

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