DVCC Based (2 + α) Order Low Pass Bessel Filter Using Optimization Techniques

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Abstract
This paper proposes the design and analysis of (2 + α) order low pass Bessel filter using different optimization techniques. The coefficients of the proposed filter are obtained by minimizing the error between transfer functions of (2 + α) order low pass filter and third-order Bessel approximation using simulated annealing (SA), interior search algorithm (ISA), and nonlinear least square (NLS) optimization techniques. The best optimization technique based on the error in gain, cut-off frequency, roll-off, passband, stopband, and phase is chosen for designing the proposed filter. The stability analysis of the proposed filter has also been done in W-plane. The simulated responses of the best optimized proposed filter are attained using the FOMCON toolbox of MATLAB and SPICE. The circuit realization of 2.5 order low pass Bessel filter is done using two DVCCs (differential voltage current conveyors), one generalized impedance converter (GIC) based inductor, and one fractional capacitor. The proposed filter is implemented for the cut-off frequency of 10 kHz using a wideband fractional capacitor. Monte Carlo and noise analyses are also performed for the proposed filter. The MATLAB and SPICE results are shown in good agreement.

Keywords Bessel filter · Optimization · DVCC · GIC · Monte Carlo

1 Introduction

Recently, fractional order systems have shown great attraction to researchers in the field of science and engineering. These fields contain bioengineering, control systems, signal processing, nanotechnologies, biology, electrical engineering, medicine, finances, etc. The concepts of fractional calculus can be used to model various fractional order systems since it provides various novel features along with design flexibilities. The continuous progress of fractional order systems and circuits requires the study of their mathematical explanation as well as their physical implementation [1, 2]. Various signal processing blocks such as fractional order

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oscillators, filters, differentiators, integrators multivibrators, etc. have been explored in the fractional order domain. Many definitions have been proposed for fractional order derivatives [3] such as

Caputo definition is as follows

\[ D_0^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-u)^{m-\alpha-1} f^{(m)}(u) du \]  

(1)

Riemann–Liouville is as follows

\[ \text{RL} a D_\alpha^a f(t) = \frac{d^m}{dt^m} \left[ \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} f(\tau) d\tau \right] \]

\[ m - 1 < \alpha \leq m, m \in \mathbb{N}. \]  

(2)

Grunwald-Letnikov is as follows

\[ GL a D_\alpha^a f(t) = \lim_{h \to 0} \sum_{k=0}^{\left\lfloor \frac{t}{h} \right\rfloor} (-1)^k \binom{a}{k} f(t-kh) h^a, \quad \alpha \in \mathbb{R}, t-a = nh \]  

(3)

where \( \Gamma(.) \) is the gamma function, \( m \) is an integer and \( \alpha \) is fractional order.

Initially, fractional order filters have been designed for first and second order systems [4, 5]. Further, the active and passive realization of fractional Butterworth filters has been done by Ali et al. [6]. Nowadays, the performance of fractional order filters is being improved by using optimization techniques [7–9]. Freeborn et al. realized fractional order Butterworth, Chebyshev, and Inverse Chebyshev filters using optimization techniques [10–14]. In addition to these, the comparison of different optimization techniques for designing fractional filters (Butterworth, Chebyshev and Bessel) has also been done [15–18]. Thus, fractional order Butterworth, Chebyshev, Inverse Chebyshev, and Bessel filters have been designed using optimization techniques in the literature.

However, there is a need to design a higher order fractional filter. Here, higher order Bessel filter is designed using optimization techniques as it is not attempted previously. In the proposed work, \( (2 + \alpha) \) order low pass Bessel filter is approximated using SA, ISA, and NLS optimization techniques. The best technique out of these three is chosen and then the proposed filter is realized using DVCC based circuit. DVCC is an advanced and most effective block for realizing analog circuits. It has the benefits of the differential difference amplifier and second generation current conveyor (CC-II).

This paper is organized as follows: Sect. 2 describes the optimization techniques used in the proposed filter. Section 3 highlights the use of SA, ISA and NLS optimization methods to obtain the filter coefficients. Section 4 presents the stability analysis in W-plane. Section 5 deals with the performance parameters obtained using various techniques. Section 6 focuses on the comparison of proposed filters with existing counterparts. Section 7 emphasizes the analog realization of the proposed filter. Section 8 discusses results and finally, the main facts are summarized in Sect. 9.
2 Optimization Techniques

In the proposed work, \((2 + \alpha)\) order low pass Bessel filter coefficients are optimized using SA, ISA, and NLS. These optimization techniques have been shown using flow charts in Figs. 1, 2 and 3.
Over the last few years, the SA technique has been attained great attention to obtain good solutions for challenging optimization problems. Various attempts have been taken to apply this approach to multiple problems in areas of VLSI design, code generation, and pattern recognition with significant achievement. SA is a meta-heuristic algorithm that can escape from local optima. This technique is popular due to ease of execution, use of hill-climbing moves, and convergence properties. Discrete and partly continuous optimization problems can be usually dealt with by SA. This algorithm has an analogy with the process in that a crystalline solid is heated and then cooled very slowly till it reaches the most regular possible crystal lattice arrangement (physical annealing of solids), and thus is free of crystal defects. It forms an algorithmic link between the search for global minima and thermodynamic behavior to solve discrete optimization issues [19–22].
Start

Form a matrix or vector of input \( x_{\text{data}} \).

Define Lower bound and upper bound in the form of vectors or matrices of same size as \( x \). Coefficients of \( x \) are to be found out using Iscurvefit.

Define \( y_{\text{data}} \) observed output matrices or vectors.

For \( F(x,x_{\text{data}}) \) in the vector form \([x,x_{\text{data}(1)}] \ldots [x,x_{\text{data}(k)}]\)\), it is vector-valued or matrix-valued function of similar size as \( y_{\text{data}} \).

Lscurvefit solve the problem using \( \min_x |F(x,x_{\text{data}})-y_{\text{data}}|^2 = \min_x \sum (F(x,x_{\text{data}})-y_{\text{data}})^2 \).

Analyze and compare the outcomes of levenberg-marquardt algorithm and default trust-region-reflective algorithm.

Are the outcomes same?

No

Better outcomes out of two can be taken into consideration.

Yes

Any of the algorithm outcomes can be chosen.

End

Fig. 3 Flow chart of NLS technique
The ISA algorithm is used for solving wide areas of optimization problems. It is a metaheuristic algorithm which has fast convergence speed and large search space. This novel algorithm can solve complex optimization tasks efficiently. It has some benefits over conventional optimization techniques such as only one tuning parameter, simple and solves the problems of local and premature convergence. It can also find the global minimum much more efficiently [23, 24].

The NLS problems have arisen when the parameterized function is fitted to a set of measured data points to minimize the sum of the squares of the errors between the function and data points. The Levenberg–Marquardt and trust-region approaches have been used to solve NLS problems. The Levenberg–Marquardt curve-fitting method is essentially an arrangement of two minimization methods such as the gradient descent method and the Gauss–Newton method. If the parameters are far from their optimal value, the Levenberg–Marquardt method acts as a gradient-descent method. It is the sharpest descent approach that updates parameter values in the “downhill” direction. For problems with simple objective functions the gradient descent approach converges well. Gradient descent approaches are sometimes the only practical method when the number of parameters is thousands. When the parameters are close to their optimal value, the Levenberg–Marquardt method behaves as the Gauss–Newton method. It converges much faster than gradient-descent methods for moderately-sized problems. This method reduces the sum of the squared errors to assume that the least-squares function is locally quadratic. The Levenberg–Marquardt method adaptively fluctuates the parameter updates between the Gauss–Newton and the gradient descent. The important and common class of minimization problems having upper and lower bound for some of the variables. These types of problems have been solved by many algorithms, few are restricted to the quadratic objective function and few are more general. Unconstrained minimization problems have been solved using Trust region methods that form a respected class of algorithms. Its strong convergence properties, naturalness, reliability, and efficiency make it more attractive. While the Levenberg–Marquardt algorithm can suffer from slow convergence. On the other hand, the algorithm may easily become lost in parameter space when the least-squares function is very flat [25, 26].

3 Filter Coefficient Selection

To approximate the passband behavior of the proposed filter, the transfer function of $(2 + \alpha)$ order low pass filter and the 3rd order Bessel transfer function with cut off frequency 1 rad/sec are compared in the frequency range from $\omega$ equals $10^{-5}$ rad/sec to 1.5 rad/sec for reducing the error function.

The transfer function of $(2 + \alpha)$ order low pass filter is given as follows

$$T_{LP}^{2+\alpha}(s) = \frac{a_0}{a_1s^{2+\alpha} + a_2s^{1+\alpha} + a_3s + a_4s^\alpha + 1}$$  \hspace{1cm} (4)

The 3rd order Bessel transfer function with cut off frequency 1 rad/sec is given by

$$B_3(s) = \frac{0.2506}{s^3 + 3.4175s^2 + 4.8664s + 2.7718}$$  \hspace{1cm} (5)

Equation 6 is used to minimize the error between Eqs. 4 and 5 with SA, ISA, and NLS optimization techniques.
where vector of filter coefficients denoted as $x$, magnitude response of Eq. 1 is $T(x, \omega)$ and $B_3(\omega_i)$ is the third-order Bessel approximation with frequency $\omega_i$, and the total number of data points are $k$. SA, ISA, and NLS optimized filter coefficients $(a_0, a_1, a_2, a_3, a_4)$ are found out for $\alpha$ value ranging from 0.1 to 0.9 and summarized in Table 1.

**Table 1** Filter coefficients of $(2 + \alpha)$ order Bessel filter using SA, ISA, and NLS

| $\alpha$ | SA          | ISA          | NLS          |
|---------|-------------|--------------|--------------|
| $0.2$   | $0.2000$    | $0.2000$     | $0.2000$     |
| $0.5$   | $0.9957$    | $0.9997$     | $1.0000$     |
| $0.8$   | $0.9972$    | $0.9981$     | $0.9999$     |
| $1.0$   | $1.0000$    | $0.9997$     | $0.9999$     |

\[
\min_x \left\| |T(x, \omega)| - |B_3(\omega)| \right\|^2 = \min_x \sum_{i=1}^{k} \left( |T(x, \omega_i)| - |B_3(\omega_i)| \right)^2
\]

\[
s.t. x > 0.1
\]

where vector of filter coefficients denoted as $x$, magnitude response of Eq. 1 is $T(x, \omega_i)$ and $B_3(\omega_i)$ is the third-order Bessel approximation with frequency $\omega_i$, and the total number of data points are $k$. SA, ISA, and NLS optimized filter coefficients $(a_0, a_1, a_2, a_3, a_4)$ are found out for $\alpha$ value ranging from 0.1 to 0.9 and summarized in Table 1.

**4 Stability Analysis**

Stability analysis is an important aspect to confirm the possibility of analog realization of the proposed filter. To explore the stability of the proposed $(2 + \alpha)$ order Bessel filter, conversion of the s-plane transfer function into the W-plane transfer function is required [27–32]. This conversion is done in the following manner.

(i) Convert $s = W^m$ and $\alpha = k/m$

(ii) Choose $k$ and $m$ for the required value of $\alpha$.

(iii) Converted W-plane transfer function is solved for all poles.

(iv) Evaluate the absolute pole angles $|\theta|$, if all are greater than $\pi/2$ m then the system is stable otherwise not.

The above steps have been used to find the root angles for different values of $\alpha$ using SA, NLS, and ISA optimized filter coefficients. For example, the stability analysis of the 2.2 order NLS optimized filter can be done using the denominator of Eq. 4. It can be written as follows

\[
D(s) = a_1s^{2.2} + a_2s^{1.2} + a_3s + a_4s^{0.2} + 1
\]  

Equation 7 can be rewritten for $W = s^{1/10}$ with $m = 10$ and modified as

\[
D(W) = a_1W^{22} + a_2W^{12} + a_3W + a_4W^2 + 1
\]  

Equation 8 can be written as follows

\[
D(W) = 1.6842W^{22} + 0.6050W^{12} + 1.2695W + 0.0351W^2 + 1
\]
The minimum root angle ($\theta_{w_{\text{min}}}$) for the above equation is 12.2 degrees (mentioned in Table 2). Table 2 reported the minimum pole angle $|\theta_{w_{\text{min}}}|$ of $(2 + \alpha)$ order low pass Bessel filter using SA, ISA, and NLS techniques for $\alpha$ equals to 0.2, 0.5 and 0.8. It can be seen that all pole angles ($|\theta_{w_{\text{min}}}|$) are greater than 9 degrees (minimum value of $\pi/2$ for $m=10$). Hence, it confirms that all the techniques used for optimizing the proposed filter are physically realizable.

**5 Performance Parameters**

SA, ISA, and NLS optimized filter coefficients are compared for errors such as gain, cut-off frequency, roll-off, passband, stopband, and phase. The best optimization technique out of these three in terms of the above mentioned parameters is chosen for analog realization of the proposed filter.

The following parameters are compared to check the performance of the optimization techniques [33]:

(i) Gain error: It is the error between maximum gain of the ideal Bessel filter and the maximum gain of fractional order low pass Bessel filter.

(ii) Cut-off frequency error: It is the error obtained when the cut-off frequency of the proposed filter is compared with the ideal Bessel filter (1 rad/sec).

(iii) Roll-off error: The roll-off error can be obtained by comparing the roll-off rate of the proposed filter with the ideal Bessel filter.

(iv) Passband error (PE): It is the error measured in the passband when compared to the ideal Bessel response. PE can be calculated as follows:

$$PE = 20 \times \log_{10} \left\{ \sum_{i=1}^{K} \frac{\left| T_{LP}^{2+\alpha} (\omega_i) \right| - \left| B_3 (\omega_i) \right|}{K} \right\} dB$$  \hspace{1cm} (10)

where $K = 500$ and $0.01 \leq \omega_i \leq 1$.

(v) Stopband error (SE): It is measured in the stopband (1 rad/sec to 10 rad/sec) while compared with ideal Bessel response. It is calculated as follows

$$SE = 20 \times \log_{10} \left\{ \sum_{i=1}^{K} \frac{\left| T_{LP}^{2+\alpha} (\omega_i) \right| - \left| B_3 (\omega_i) \right|}{K} \right\} dB$$  \hspace{1cm} (11)
where $K = 500$ and $1 \leq \omega_i \leq 10$.

(vi) Phase error: It is observed in the phase response while compared with ideal Bessel response. It is measured as follows

$$\text{Phase Error} = \frac{\sum_{i=1}^{K} \left| \tan^{-1} T_{LP}^{2+\alpha}(\omega_i) \right| - \left| \tan^{-1} B_3(\omega_i) \right|}{K}$$

where $K = 500$ and $0.01 \leq \omega_i \leq 10$.

Table 3 shows the comparison of parameters for different optimization techniques (SA, ISA, and NLS). It has been observed that NLS gives the minimum error of all parameters (gain error, cut-off frequency error, roll-off error, PE, SE, and phase error) as compared to SA, and ISA for $\alpha$ equal to 0.2, 0.5, and 0.8. Gain and roll-off of ideal third-order Bessel filter are $-20.9$ dB and $-54.6$ dB/decade.

The frequency and phase response of SA, ISA, and NLS optimized $(2 + \alpha)$ order Bessel filters for the orders 2.2, 2.5, and 2.8 have been plotted in Fig. 4a–c. Further, the frequency and phase responses have also been plotted for $B_3(s)$ to show the deviation of fractional order filters. These responses show that the roll-off increases as the order are increasing from 2.2 to 2.8.

| $\alpha$ Parameters | SA | ISA | NLS |
|---------------------|----|-----|-----|
| 0.2 Gain Error (dB)  | 6.1| 5.6 | 0.1 |
| Cut-off frequency error (rad/sec) | 0.9364 | 0.9922 | 0.051 |
| Roll-off error (dB/dec) | 11.4 | 12.3 | 9.5 |
| PE (dB) | 56.1936 | 66.1824 | 92.3605 |
| SE(dB) | 87.6072 | 93.7984 | 92.5471 |
| Phase error(radians) | 2.5 | 2.77 | 1.12 |
| 0.5 Gain error (dB) | 6.9 | 6.9 | 0 |
| Cut-off frequency error (rad/sec) | 0.741 | 0.964 | 0.11 |
| Roll-off error (dB/dec) | 2.1 | 4.3 | 1.7 |
| PE(dB) | 47.9189 | 60.2175 | 108.7552 |
| SE(dB) | 80.8105 | 90.8799 | 112.7983 |
| Phase error(radians) | 0.515 | 0.62 | 0.381 |
| 0.8 Gain error(dB) | 6.9 | 6.9 | 0 |
| Cut-off frequency error (rad/sec) | 0.43 | 0.883 | 0.01 |
| Roll-off error(dB/dec)) | 7.2 | 4.5 | 0.4 |
| PE(dB) | 42.5550 | 57.2557 | 127.8340 |
| SE(dB) | 67.3757 | 90.7401 | 137.4023 |
| Phase error(radians) | 0.445 | 0.587 | 0.122 |
Fig. 4 Frequency and phase response of $(2 + \alpha)$ order Bessel filter using a SA b ISA c NLS
6 Comparison of Proposed Filter with Existing Filters

The comparison of the proposed filter with existing fractional filters in literature is needed to show the merits of the \((2 + \alpha)\) order Bessel filter. The comparative analysis of the proposed Bessel filter with existing filters has been done in Table 4 for roll-off error, cut-off frequency error, PE, SE, phase error, degree of freedom and applied optimization technique. It can be seen that the phase error of the NLS optimized proposed Bessel filter is less or comparable to existing filters.

7 Analog Realization of the Proposed Filter

It has been discussed earlier that the NLS gives the least gain error, cut-off frequency error, roll-off error, PE, SE, and phase error as compared to SA, and ISA techniques for the proposed filter. So, there is a requirement to verify the results obtained from the NLS optimization technique. Here, DVCC is chosen to design NLS optimized \((2 + \alpha)\) order low pass Bessel filter. DVCC is defined using the following matrix:

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
V_X \\
I_Z
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
V_Y \\
V_Z
\end{bmatrix}
\tag{13}
\]

Figure 5 shows the circuit diagram of the NLS optimized \((2 + \alpha)\) order low pass Bessel filter using 2 DVCCs, 1 GIC based inductor, and 1 fractional capacitor. The internal structure of DVCC is given in Fig. 6 using 3 AD844 [38–40].

GIC based inductor is used in the proposed circuit for \(L = 1\) mH. In Fig. 7, the desired value of the inductor \((L = 1\) mH) is achieved by choosing \(R = 1\) KΩ, \(R_5 = 100\) Ω, and \(C_4 = 0.01\) µF. Equivalent input impedance to ground of above ckt. (Fig. 7) is given by

\[
Z = \frac{Z_1Z_2Z_5}{Z_2Z_4}
\tag{14}
\]

Using \(Z_1 = Z_2 = Z_3 = R, Z_4 = 1/sC_4, Z_5 = R_5\), Eq. 14 becomes

\[
Z = sC_4RR_5
\tag{15}
\]

\[
L = C_4RR_5
\tag{16}
\]

DVCC based 2.5 order proposed filter is used \(C_1\) as a fractional capacitor, this capacitor is used for a wide frequency range. It is made up of 10 resistances and nine capacitances. Figure 8 shows the structure of a wideband fractional capacitor and Table 5 gives the values of resistances and capacitances used in \(C_1\) [41].

To get the overall transfer function of DVCC based proposed filter (Fig. 5), the following steps are required:

\[
\frac{V_1}{V_{in}} = \frac{1}{LC_1s^{1+\alpha} + R_1C_1s^\alpha + 1}
\tag{17}
\]
Table 4  Comparison of NLS optimized proposed filter with existing counterparts

| Parameters                  | Order of filter | [17] | [18] | [34] | [9]  | [35] | [36] | [37] | NLS optimized Proposed filter |
|-----------------------------|-----------------|------|------|------|------|------|------|------|-------------------------------|
| Cut-off frequency error (rad/sec) | 1.1             | 1.300| 0.82 | 0.878| 0.003| 0.999| 0.798| 0.034| 0.051                        |
|                             | 1.2             | –    | 1.088| 0.876| –    | 0.921| 0.799| 0.048|                             |
|                             | 1.3             | 1.070| 0.85 | 0.878| –    | 0.901| 0.819| 0.078|                             |
|                             | 1.4             | 0.880| 0.84 | 0.884| –    | 0.891| 0.759| 0.081|                             |
|                             | 1.5             | –    | 0.731| 0.885| 0.002| 0.601| 0.788| 0.071| 0.11                         |
|                             | 1.8             | –    | 0.892| –    | –    | –    | –    | –    | 0.01                         |
|                             | 1.9             | –    | 0.545| 0.913| 0.001| 0.209| 0.912| 0.071|                             |
| Roll-off error (dB/decade)  | 1.1             | 1.600| 1.60 | 2.450| 4.893| 4.060| 1.800| 1.059|                             |
|                             | 1.2             | –    | 2.540| 2.540| –    | 4.070| 2.860| 1.930| 9.5                          |
|                             | 1.3             | 1.600| 1.3  | 2.790| –    | 3.800| 2.670| 2.727|                             |
|                             | 1.4             | 2.100| 1.5  | 2.860| –    | 4.000| 2.100| 3.375|                             |
|                             | 1.5             | –    | 3.210| 3.210| 4.741| 4.480| 2.900| 3.785| 1.7                          |
|                             | 1.8             | –    | 2.8  | –    | –    | –    | –    | –    | 0.4                          |
|                             | 1.9             | –    | 5.130| 5.130| 14.655|4.900| 4.300| 0.481|                             |
| PE(dB)                      | 1.1             | –67.7175| –74.9487| –35.59| –37.8| –37.8| –66.1824|                             |
|                             | 1.2             | –93.011| –8.783| –40.96| –11.38| –40.1| –60.2175|                             |
|                             | 1.3             | –88.2934| –80.9750| –33.7| –24.97| –8.288| –30.6| –57.2557|                             |
|                             | 1.4             | –78.6026| –69.7839| –33.7| –25.4| –25.4| –57.2557|                             |
|                             | 1.5             | –77.522| –8.35 | –33.58| –24.97| –8.288| –30.6| –60.2175|                             |
|                             | 1.8             | –87.0294| –8.828| –31.74| –11.55| –9.399| –25.2| –57.2557|                             |
|                             | 1.9             | –87.0294| –8.828| –31.74| –11.55| –9.399| –25.2| –57.2557|                             |
| Parameters | Order of filter | [17] | [18] | [34] | [9] | [35] | [36] | [37] | NLS optimized | Proposed filter |
|------------|----------------|------|------|------|-----|------|------|------|----------------|-----------------|
| SE(dB)     |                |      |      |      |     |      |      |      |                |                 |
| 1.1        |                | −68.1378 | −68.2453 |      |     |      |      |      | −44.9           |                 |
| 1.2        | −              | −74.764 | −28.73 | −6.872 | −34.55 | −29.37 | −38 | −93.7984 |
| 1.3        | −83.3390      | −85.3804 | −    | −    | −    | −    | −74.764 | −28.73 | −6.872 | −34.55 | −29.37 | −38 | −93.7984 |
| 1.4        | −91.7671      | −92.8286 | −    | −    | −    | −    | −    | −    | −    | −    | −    | −    | −33.6 |
| 1.5        | −              | −95.768 | −31.79 | −19.47 | −37.72 | −31.00 | −33.4 | −90.8799 |
| 1.8        | −              | −99.042 | −    | −    | −    | −    | −    | −    | −    | −    | −    | −    | −41.3 | −90.7401 |
| 1.9        | −108.523      | −38.62 | −30.81 | −36.63 | −39.81 | −45.4 |
| Phase error (radians) | |    |      |      |     |      |      |      |                |                 |
| 1.1        | 0.982         | 0.970 | 1.180 | 1.159 | −    | 1.180 | 1.197 |
| 1.2        | −              | 1.040 | 1.040 | −    | 1.2013 | 1.040 | 1.062 | 1.12 |
| 1.3        | 0.668         | 0.64 | 0.909 | −    | −    | 0.091 | 0.929 |
| 1.4        | 0.525         | 0.502 | 0.774 | −    | −    | 0.720 | 0.797 |
| 1.5        | −              | 0.643 | 0.643 | 1.161 | 0.7229 | 0.635 | 0.664 | 0.381 |
| 1.8        | −              | 0.107 | −    | −    | 0.0892 | −    | −    | 0.122 |
| 1.9        | −              | 0.144 | 0.144 | 1.173 | −    | 0.142 | 0.143 |
| Degree of freedom | |    |      |      |     |      |      |      |                |                 |
| 2          | 1              | 1     | 1     | 1     | 1     | 1     | 1 |
| Applied optimization technique | Yes | Yes | No | Yes | No | No | No | Yes |
After dividing the numerator and denominator of Eq. 19 by $(R_2 + R_3)$, then compare this equation with Eq. 4. The outcome of comparison gives the values of $R_1 = 5163.9 \, \Omega$, $R_2 = 168,200 \, \Omega$, $R_3 = 19070 \, \Omega$ with $C_1 = 2.5 \, nF s^{-\alpha-1}$, $C_2 = 0.02 \, \mu F$, and $L = 1 \, mH$. These values are used to get the magnitude response of DVCC based NLS optimized $(2 + \alpha)$ order low pass Bessel filter, magnitude is scaled by 10,000 and frequency shifted to 10 kHz.
Fig. 7 The internal structure of GIC based inductor

Fig. 8 Wideband Fractional capacitor \((C_1)\) for \(\alpha = 0.5\)

| Resistors | Values (Ω) | Capacitors | Values (F) |
|-----------|------------|------------|------------|
| R_2       | 537.6      | –          | –          |
| R_3       | 394.1      | C_3        | 803.2 p    |
| R_4       | 974        | C_4        | 1506 p     |
| R_5       | 2153       | C_5        | 3164 p     |
| R_6       | 4665       | C_6        | 6779 p     |
| R_7       | 10.07 K    | C_7        | 14.58 n    |
| R_8       | 21.75 K    | C_8        | 31.33 n    |
| R_9       | 47.4 K     | C_9        | 66.7 n     |
| R_{10}    | 108.2 K    | C_{10}     | 135.6 n    |
| R_{11}    | 341.4 K    | C_{11}     | 84 n       |
8 Result and Discussion

8.1 SPICE Simulated Magnitude Response

The SPICE simulated magnitude response of the proposed 2.5 order NLS optimized Bessel filter is shown in Fig. 9. The MATLAB and SPICE simulated results of 2.5 order Bessel filters have been compared. The absolute error in MATLAB and SPICE simulated results of gain and cut-off frequency is 3.5 dB and 0.37 rad/sec, respectively. It specifies that the results of MATLAB and SPICE are close to each other as desired for realization at the circuit level using approximated fractional order capacitor.

In addition to it, the Monte Carlo analysis of 2.5 order DVCC based NLS optimized Bessel filter for all the resistances and capacitances used in the circuit (Fig. 5) within 5% tolerance has been done for n = 100 runs. The resultant plots are shown in Fig. 10a, b. The maximum variation in gain, cut-off frequency and roll-off rate for 2.5 order proposed filter are (−20.18 dB to −20.22 dB), (15.80 kHz to 16.33 kHz) and (−39.65 dB/decade to −41.27 dB/decade) respectively. Thus, it shows the reasonable variation in the above mentioned parameters.

8.2 Noise Analysis

Noise analysis is an important aspect to see the impact of noise on the proposed circuit. There are different kinds of noise in any electronic circuit such as shot noise, flicker noise, and thermal noise. The collective effect of all such noises on the proposed circuit (Fig. 5) is determined in the SPICE environment. The behavior of input and output noise voltage of 2.5 order NLS optimized proposed filter is shown in Fig. 11. As can be seen from this figure (Fig. 11) that both the input and output noise are low in the entire passband.
9 Conclusion

This work presents the designing of \((2 + \alpha)\) order low pass Bessel filter using SA, ISA, and NLS techniques. These techniques are used to optimize the filter coefficients. Further, the best optimization technique based on gain error, cut-off frequency error, roll-off error, PE, SE, and phase error has been chosen to design the proposed filter using DVCCs. The NLS optimized \((2 + \alpha)\) order low pass Bessel filter gives good similarity with SPICE simulated
DVCC based circuit. Therefore, MATLAB and SPICE results show a good similarity between results. This work can be further extended for other approximations of the filter.

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**Data Availability** Enquiries about data availability should be directed to the authors.

**Code Availability** Not applicable.

**Declarations**

**Conflict of interest** The authors declare that they have no conflict of interest.

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