Some comments on the note

"Some comments on the paper
"Filter design for the detection of compact sources based on the Neyman-Pearson detector"

by M. López-Caniego et. al (2005, MNRAS 359, 993)"

by R. Vio and P. Andreani
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Abstract. In this note we stress the necessity of a careful check of the arguments used by Vio & Andreani (2005) (VA hereinafter) to criticise the superior performance of the biparametric scale adaptive filter (BSAF) with respect to the classic matched filter (MF) in the detection of sources on a random Gaussian background. In particular, we point out that a defective reading and understanding of previous works in the literature (Rice 1954; Barreiro et al. 2003; López-Caniego et al. 2005) leads the authors of VA to the derivation of an incorrect formula and to some misleading conclusions.

Key words. Methods: data analysis – Methods: statistical

1. Introduction

In recent times, some controversy has arisen about the design of “optimal” filters for the detection of sources embedded in a noisy background. The controversy seems to be focused on the following question: do we have already an optimal tool for detecting such sources or is it worth trying to find better methods for the task?

In López-Caniego et al. (2005) and some previous works (Barreiro et al. 2003; López-Caniego et al. 2004, 2005), we have explored the detection problem in the context of astronomical data mining. The motivation of our work has been the need to detect extragalactic objects, often referred to as “point sources” due to their small angular size, in microwave Astronomy. Since the number of these objects increases very quickly as their flux decreases, even a small improvement in our ability to notice faint extragalactic objects can lead to a significant rise in the number of detections. Hence the importance of working out new and more powerful detection procedures.

López-Caniego et al. (2005) have proposed a detection procedure based on a common practice in Astronomy, that consists in identifying possible sources through the presence of “peaks” in the data. Commonly, the data is previously filtered in order to improve the detectability of the sources. Then, some decision rule is applied on the peaks in order to determine whether they correspond to sources or not.

A typical decision rule is based on the idea of amplitude thresholding, that is, the hypothesis that a source is present in any considered point is accepted if the amplitude of the observed data at that point is higher than a certain value. A decision rule based only on the amplitude at the point where the decision has to be made is missing information on the local structure of the source and the background where it is embedded. Thus, in order to increase the power of the decision rule, in
Following this idea, in López-Caniego et al. (2005) we considered as decision rule the Neyman-Pearson detector, which gives the highest number of detections for a given number of false alarms; since the Neyman-Pearson detector was to be applied to local maxima (peaks), first we derived the expressions for the number of detections for peaks in an interval \((x + dx)\) with amplitude \((\nu + dv)\) and curvature \((\kappa + dk)\) both in presence and in absence of a source. These number densities depend on the properties of the background, and these properties can be modified, up to a certain extent, through filtering. Then, we explored the performance of a linear filter, the Biparametric Scale-Adaptive Filter (BSAF), that depends on a small number of parameters that can be chosen so that the performance of the Neyman-Pearson detector is optimised. We found that the BSAF outperforms other filters such as the Mexican Hat Wavelet and the standard matched filter (MF) in some interesting cases.

Very recently, Vio & Andreani (2005) have criticised the work presented in López-Caniego et al. (2005). We feel compelled to warn the community against some of their comments, that result to be either fruit of a bad interpretation of our work or just plainly wrong.

2. Some comments on the comments by VA

In the introduction of their note, VA reproduce some very well-known results about the matched filter in the context of the Neyman-Pearson theorem, which can be found in any basic signal detection textbook (for example Wainstein & Zubakov 1962). Though this is unquestionably correct, only the amplitude of the signal is considered and, besides, it is still necessary to provide a criterion to localise and define a single source among the set of pixels above the threshold. We have followed a different approach that incorporates the identification of any single source through the presence of a local maximum and information about the curvature. Hence, our work is not in contradiction with the scheme reproduced by VA, because we are following a different, more complete, approach.

In addition, VA make three comments about our approach. In the first comment, VA point out that one of our equations (equation (8) in López-Caniego et al. 2005) is not correct. However, this statement is not true and the alternative equation they propose is actually wrong. Regarding the second comment of VA, they criticise the fact that we work on a filtered version of the original signal. We would like to stress that the common procedure in astronomy (and other fields) for object detection is to filter the original image in order to enhance the sources and then detect and identify those sources. Thus, an important issue is not only to find the optimal filter, but also which is the criterion to identify the sources. In our approach, we a priori identify the maxima of the filtered image as source candidates. Then, we apply a Neyman-Pearson detector to decide whether the maximum is due by dividing the previous equation by the total number density \(n_b\).

To obtain the probability density in the presence of a point source of amplitude \(\nu_s\) and curvature \(\kappa_s\), VA simply substitute \(\nu \to \nu - \nu_s\) and \(\kappa \to \kappa - \kappa_s\) in \(p_b(\nu, \kappa)\):

\[
p(\nu, \kappa | \nu_s) = \frac{1}{\sqrt{2\pi}} \frac{\kappa - \kappa_s}{\sqrt{1 - \rho^2}} \exp \left[ -\frac{(\nu - \nu_s)^2 + (\kappa - \kappa_s)^2 - 2\rho(\nu - \nu_s)(\kappa - \kappa_s)}{2(1 - \rho^2)} \right],
\]

and indicate that \(\nu \in (-\infty, +\infty)\) and \(\kappa \in (\kappa_s, +\infty)\). However, the derivation of this equation cannot be done in such a simple way. First of all, one needs to construct the joint probability density of the field, its first and its second derivative (where terms of the form \(\nu - \nu_s\) and \(\kappa - \kappa_s\) appear). From this joint probability, one follows the procedure explained in Rice (1954), Bardeen et al. (1986) and Bond & Efstathiou (1987) obtaining the number density of maxima in the intervals \((x + dx), (\nu + dv)\) and \((\kappa + dk)\):

\[
n(\nu, \kappa | \nu_s) = \frac{n_b}{\sqrt{2\pi}} \frac{\kappa}{\sqrt{1 - \rho^2}} \exp \left[ -\frac{(\nu - \nu_s)^2 + (\kappa - \kappa_s)^2 - 2\rho(\nu - \nu_s)(\kappa - \kappa_s)}{2(1 - \rho^2)} \right],
\]

where \(\nu \in (-\infty, +\infty)\) and \(\kappa \in (0, +\infty)\). Note that the factor \(\kappa\) that multiplies the exponential comes in from imposing the condition of having a maximum and it refers to the total curvature given by the background plus source (not only to the background as stated by VA). We would like to remark that equation (3) gives the number density of maxima coming from the combination of the background plus source. This does not mean, at all, that the maximum of the source has to coincide with a maximum of the noise process as stated by VA. In addition, VA claims that \(\kappa \in (\kappa_s, +\infty)\), since they wrongly assume that the maximum of the global field has to coincide with a maximum of the background. However this is not true and therefore there is no reason to restrict \(\kappa\) to such interval. In fact \(\kappa\) can take values from \((0, +\infty)\). Note that this is another indication of the fact that equation (2) proposed by VA is wrong, since this probability can take negative values when considering the correct interval for \(\kappa\).
or not to the presence of a point source. Note that since we are considering the identification through the idea of maxima, it is natural to define the Neyman-Pearson detector in terms of number density of maxima. Taking into account these ideas, we explore different filters and find that the BSAF outperforms the other filters (including the MF) in some cases. Furthermore, VÁ suggest that we are trying to find an approximate solution to the decision problem based on the likelihood ratio

\[ L(x, \nu, \kappa) = \frac{p(x, \nu, \kappa | H_1)}{p(x, \nu, \kappa | H_0)} > \gamma \]  

(4)

It is not clear to us what VÁ mean with this notation. If \( x \) is the observed 1-dimensional signal (as VÁ defined in their introduction), \( \nu \) is redundant. Also, if \( \kappa \) refers to the whole image, it should be a vector, \( \kappa \). We understand that what VÁ mean is to construct a likelihood ratio using the amplitude and curvature of all the pixels in the image (in fact, if using all the pixels, one should also introduce the information on the first derivative). However, this procedure does not make sense in our approach, since we are considering only the maxima of the image. Note that in the approach suggested by VÁ, one would also need, a posteriori, a criterion to identify which points of the image (from those above the threshold \( \gamma \)) correspond to each source. In addition, VÁ claim that our conclusions are drawn only on the basis of numerical experiments. However, most of the work of López-Caniego et al. (2005) is devoted to present the theoretical framework of our method. Simulations are then performed in order to test the theoretical results. Therefore, the criticisms of VÁ in their second comment are not well founded.

The third comment of VÁ refers to the procedure followed in the numerical experiments. They criticise that we consider only those sources whose peak is not moved to another pixel. We would like to remark that the aim of our work was to present a novel theoretical framework for object detection and to test it with numerical simulations. Therefore, we try to reproduce exactly the theoretical scheme with our simulations and focus only on what happens in one pixel of the image, the pixel in which the source is located. In fact, this point was already discussed in Barreiro et al. (2003), finding that the different filters there considered lead to similar number of detections in the neighbouring pixels of the source and, thus, it did not affect the conclusions. In any case, in the more realistic case when all the pixels of the image are considered, the conclusion that the BSAF detects more sources than the other filters in the correct localisation remains true.

Finally, VÁ comment in their conclusions that the performance of our filter is based on strong a priori assumptions such as the Gaussianity of the background and the symmetry of the source profile. We would like to remark that many real fields do follow a Gaussian distribution and therefore this is a very common and realistic assumption. In fact, in their introduction, VÁ also assume the Gaussianity of the background to show that the statistic given by the Neyman-Pearson detector (when only information about the amplitude is used) leads to the MF. Regarding the symmetry of the source profile, the filters can be generalised without any difficulty to non-symmetrical profiles.

3. Conclusions

In a note recently appeared in astro-ph, VÁ have made some comments about our work (López-Caniego et al. 2005). In this note, we have carefully checked their arguments. The main comments made by VÁ are three. Let us summarise:

In their first comment, VÁ have questioned an allegedly unproven formula in our work, which is in fact rigorously derived from previous works in the literature (Rice 1954; Bardeen et al. 1986; Bond & Efstathiou 1987). Instead, VÁ have proposed an incorrect formula.

In their second comment, VÁ criticise the lack of generality of the approach proposed in López-Caniego et al. (2005). In particular, VÁ criticise the idea of filtering the data and applying the Neyman-Pearson detector to the local maxima. Instead, they suggest that a generalisation of the derivation of the Neyman-Pearson detector, including not only amplitudes but also the second derivatives of the field, should be done on a purely theoretical basis. Nevertheless, they are not able to provide such a theoretical derivation, and the likelihood ratio they propose is not general either, since it does not include the first derivative of the field, that outside the maxima is not zero. Our approach, however, is consistent and it leads to an improvement in the number of detections.

In their third comment, VÁ criticise a set of numerical experiments designed to test our theoretical arguments precisely for being designed to test our theoretical arguments. They suggest instead to make numerical experiments in order to test what the theory does not say. We have derived the number densities of maxima in two cases: when a source is located at the position of the maxima (not “nearby the maxima”) and when there is no source. The way to test the hypothesis expressed by these formulae is exactly the one explained in López-Caniego et al. (2005).

Besides the three main comments mentioned above, VÁ made a few others. One of them is that VÁ claim that our conclusions are drawn only on the basis of numerical experiments, which is plainly false. In López-Caniego et al. (2005), we give a theoretical foundation for our method, we make predictions based on the theory and then we check those predictions with numerical simulations. The agreement is excellent.

Other main objection is that our proposed method seems rather complicated. Though it is true that simplicity is an aesthetically admirable quality, we feel that a little complexity should not scare scientists in their work. As mentioned in the introduction of this note, it is worth to work hard to improve the capability of detection of our statistical methods, even if the improvement is a few percent, because it may lead to a significant rise in the number of extragalactic objects detected.
Finally, VA blame our method for doing stringent a priori assumptions, namely two: symmetry of the source profile and Gaussianity of the background. It is false that our method requires symmetry of the source: it was assumed only for simplicity but the filters can be generalised to non-symmetric profiles just as the standard matched filter can. Regarding Gaussianity, VA make in their introduction exactly the same assumption as we do.

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