Effects of hygrothermal conditions on free vibration behaviour of laminated composite structures

Trupti R Mahapatra¹* and Subrata K Panda²

¹KIIT University-Bhubaneswar, Odisha, India, 751024
²National Institute of Technology-Rourkela, Odisha, India, 769008

*E-mail: trmahapatrafme@kiit.ac.in

Abstract. Free vibration behaviour of laminated composite structure under combined hygrothermal load is investigated in this article. The responses are obtained numerically through a finite element model using hygrothermal dependent composite material properties. The convergence behaviour of the present numerical model has been established and validated by comparing the responses with those available open literatures. The effects of various hygrothermal conditions and geometrical parameters (curvature ratio, side-to-thickness ratio and aspect ratio) are examined in details.

1. Introduction

Laminated structures made of composites are exposed to combined hygrothermal loading both during manufacturing as well as in service. It is well known that, the mechanical properties (Young’s modulus and rigidity modulus) of laminated composites are significantly influenced due to hygrothermal and high strain rate loading. The corrugated fiber properties due to uneven environment and fiber volume fractions may alter the mechanical responses substantially. The structural components made up of laminated composites found in aerospace, marine, automotive, agriculture, sports, and biomedical industries are exposed to vibration which experiences fatigue/cyclic loading during their service life. In recent years researchers quest to analyse (analytical, numerical and/or experimental) the complex loading in conjunction with degraded material properties of laminated structures accurately for finished products. It is noted that, free vibration behavior of laminated shell panels under hygrothermal loading have been analysed by using various displacement models say, classical laminated shell theory (CST), the first order shear deformation theory (FSDT), and the higher order shear deformation theory (HSDT). In this regard some of the important contributions are discussed briefly in the following line to address the objective and necessity of the present study. Sai Ram and Sinha [1] investigated the effects of moisture and temperature on the free vibration responses of laminated composite plates using finite element method (FEM). A quadratic isoparametric finite element (FE) formulation is presented by Parhi et al. [2] for the hygrothermal free vibration and transient analysis of multiple delaminated plates and shells. Their formulation is based on the FSDT. Patel et al. [3] analyzed the static and dynamic behavior laminated composite plates using FEM. Their model is based on the modified HSDT and includes hygrothermal dependent material properties. Huang and Zheng [4] analysed nonlinear free and forced vibration of moderately thick laminated plates using HSDT and von-Karman type nonlinear kinematics under transient loading. Shen et al. [5] reported dynamic behaviour of laminated composite plate resting on elastic foundation under
hygrothermal load through a micromechanical based HSDT model. Naidu and Sinha [6] investigated the nonlinear vibration response of laminated composite shells in hygrothermal environment using nonlinear FEM steps. They have developed the model based on the FSDT and the Green-Lagrange type nonlinearity. Analytical solution of the hygrothermal stresses in thick multilayered composite plates by taking the hygrothermal dependent material properties was presented by Benkhedda et al [7]. Lo et al. [8] presented the effect temperature dependent material properties and hygrothermal loading on multilayer composites using global–local HSDT mid-plane kinematics in conjunction with four-node quadrilateral plate element. Nonlinear free vibration and transient response of laminated composite spherical and cylindrical shells with imperfections under hygrothermal environment is investigated by Nanda and Pradyumna [9] using FSDT and von-Karman nonlinearity. The severity due to combined variation in temperature and moisture concentrations on the nonlinear vibration behaviour of laminated composite structure is reported by Ashraf [10]. The effects of temperature and moisture content on natural frequencies of woven fiber glass/epoxy delaminated composite plate is analysed through combined numerical (FEM) and experimental steps by Panda et al. [11]. They modelled the composite plate based on the FSDT kinematics by considering the degraded hygrothermal properties. Panda and Mahapatra [12] studied the nonlinear thermal free vibration behaviour of laminated composite spherical shell panel using the HSDT and Green-Lagrange nonlinearity.

We note that limited studies related to linear/nonlinear vibration behaviour of laminated composite flat/curved panels under hygrothermal environment have been reported in the open literature. Based on the authors’ knowledge, studies related to free vibration behaviour of laminated composite spherical shell panel under hygrothermal loading by using HSDT mid-plane kinematics are also scarce. The present work aims to analyse the free vibration behaviour of laminated composite shear deformable spherical shell panels under hygrothermal loading using a mathematical model based on the HSDT. The temperature and/or moisture dependent material properties are considered in order to obtain a realistic response. The sets of governing equations are obtained using Hamilton’s principle and discretised using FEM steps. The effect of various geometrical parameters on the free vibration behaviour of laminated composite spherical panels under combined hygrothermal loading is investigated through subsequent numerical illustrations.

2. Numerical Methodology

Figure 1 shows the geometry and lay-up of a laminated spherical shell panel of length a, width b and thickness h with composed of ‘N’ number of equally thick anisotropic layers. R1 and R2 are the radii of curvature of the shell panel at the mid surface (Z=0). In order to derive the hygro-thermo elastic mathematical model of laminated composite spherical shell panel, a HSDT based displacement field is adopted which is assumed to be in following form:

\[
\begin{align*}
\overline{u}(x, y, z, t) &= u(x, y, t) + z\phi_1(x, y, t) + z^2\psi_1(x, y, t) + z^3\theta_1(x, y, t) \\
\overline{v}(x, y, z, t) &= v(x, y, t) + z\phi_2(x, y, t) + z^2\psi_2(x, y, t) + z^3\theta_2(x, y, t) \\
\overline{w}(x, y, t) &= w(x, y, t)
\end{align*}
\]

where, \(\overline{u}\), \(\overline{v}\) and \(\overline{w}\) are the displacements at any point of the shell panel along the (X, Y, Z) coordinates corresponding to any point of time t. \((u, v, w)\) are the mid plane displacements and \(\phi_1\) and \(\phi_2\) are the shear rotations about the y and x-axes, respectively. \(\psi_1, \psi_2, \theta_1, \theta_2\) represent the higher order terms of the Taylor series expansion defined in the mid-plane and account for the parabolic variation of shear stress.
2.1 Strain Displacement Relation
The strain-displacement relation for a shell panel can be defined as:
\[
\{e\}^T = \left[\varepsilon_{xx} \; \varepsilon_{yy} \; \gamma_{xy} \; \gamma_{xz} \; \gamma_{yz}\right]^T
\]
\[
= \left(\frac{\partial \varepsilon}{\partial x} + \frac{\partial \gamma_{xy}}{\partial z}, \frac{\partial \varepsilon}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}, \frac{\partial \varepsilon}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}, \frac{\partial \varepsilon}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}, \frac{\partial \varepsilon}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)^T
\]  
(2)
Using Equations (1) and (2), the strain vector \(\{e\}\) may be expressed in terms of unknowns (for the structural deformation) as
\[
\{e\}^T = [H]\{\varepsilon\}^T
\]  
(3)
where, \(\{\varepsilon\} = \left[\varepsilon_0 \; \varepsilon_0 \; \varepsilon_0 \; \gamma_0 \; \gamma_0 \; \gamma_0 \right]^T\), the terms in \(\{\varepsilon\}\) having superscripts ‘0’, ‘1’, ‘2’ and ‘3’ are bending, curvature and higher order terms and \([H]\) is the function of thickness coordinate.

2.2 Hygro-thermo elastic equations
The hygro-thermo elastic constitutive matrix equation of generalized stress tensor considering plane stress condition for any general \(k\)th isotropic composite lamina and any fibre orientation angle \(\theta\), can be expressed as
\[
\{\sigma\}^k = \left[Q_{ij}\right]^k \{\varepsilon\} = \{\varepsilon\}^k - \alpha_{ij} \Delta T - \beta_{ij} \Delta C
\]  
(4)
where, \(\{\sigma\}^k = \{\sigma_1 \; \sigma_2 \; \sigma_6 \; \sigma_3 \; \sigma_4\}^T\) is the stress vector for the \(k\)th layer, \(\{\varepsilon\}^k = \{\varepsilon_1 \; \varepsilon_2 \; \varepsilon_6 \; \varepsilon_5 \; \varepsilon_4\}^T\) the strain vector for the \(k\)th layer, \(\left[Q_{ij}\right]^k\) the transferred reduced elastic constant for the \(k\)th layer, \(\{\alpha_{ij}\}^k = \{\alpha_1 \; \alpha_2 \; 2\alpha_{12}\}^T\) the thermal expansion/contraction coefficient vector, \(\{\beta_{ij}\}^k = \{\beta_1 \; \beta_2 \; 2\beta_{12}\}^T\) the moisture expansion/contraction coefficient vector, \(\Delta T\) the uniform change in temperature and \(\Delta C\) the uniform change in moisture concentration.
2.3 Work done, kinetic energy and strain energy
The in-plane hygro thermal forces are obtained using the steps in [14] and given by
\[
\{N\}_k = \int \left[ \bar{Q} \cdot \{ \alpha \}' \{1, z, z' \} \Delta T + \{ \beta \}' \{1, z, z' \} \Delta C \right] dz
\]
where, \( \{ N \} \), \( \{ M \} \) and \( \{ P \} \), are the resultant vectors of compressive in-plane hygrothermal forces, moments and the higher order terms due to combined temperature and moisture variation.

The work done \((W)\) due to in-plane hygrothermal load is given by
\[
W = \frac{1}{2} \int \{ \varepsilon \}_g^T \left[ D_\varepsilon \right] \{ \varepsilon \}_g \ dA
\]
(6)
The kinetic energy of a vibrating laminated plate is expressed in following step
\[
\varepsilon = \frac{1}{2} \int \sum_{k=1}^{NL} \int \Delta L \left[ \varrho \right] \{ f \}^T \{ f \} \ dA = \frac{1}{2} \int \left[ \delta \right]^T \left[ m \right] \{ \delta \} \ dA
\]
(7)
where, \( \{ \varrho \} \), \( \{ \delta \} \) and \( \{ \delta \} \) are the density, global displacement vector and first order derivative of the global displacement vector with respect to time.

\([m]\) is the inertia matrix written as
\[
[m] = \sum_{k=1}^{NL} \int \Delta L \varrho \left[ f \right]^T \{ f \} \ dA
\]
and \( \{ \delta \} = \{ \bar{u} \ \bar{v} \ \bar{w} \}^T = \{ f \} \{ \delta \} \)
where, \( \{ f \} \) is the function of thickness coordinate.

The total strain energy of the laminated shell panel is expressed as
\[
U = \frac{1}{2} \int \{ \varepsilon \}^T \{ \sigma \} \ dV
\]
(8)
Substituting the values of stress and strain components from Equations (3) and (4) the strain energy expression can be re-written as
\[
U = \frac{1}{2} \int \int \left[ \bar{Q} \right]^T \left[ D \right] \{ \varepsilon \}_i \ dx dy
\]
(9)
where, \([D] = \sum_{k=1}^{NL} \int \left[ H \right] \left[ \bar{Q} \right]^T \left[ H \right] dZ \) and ‘NL’ is number of layer.

3. Finite element implementation
The FEM has proved to be a powerful and widely appreciable method for the vibration analysis of laminated composite structures whose analytical solutions are difficult to obtain. A nine-noded isoparametric element having nine degrees of freedom per node is employed in the present model.

The displacement vector \( \{ \delta \} \) can be expressed after implementing the FEM as
\[
\{ \delta \} = \sum_{i=1}^{NN} N_i \delta_i
\]
(10)
where, \( N_i \) is the nodal interpolation function for \( i^{\text{th}} \) node and \( NN \) is the numbers of node per element.

Employing FEM the strain vector in Eq. (3), can be rewritten in terms of global displacement vector
\[
\{ \varepsilon \} = [B] \{ \delta \}
\]
(11)
where, \([B]\) is the product of operator matrix and interpolation functions of structural displacement vector.
4. System governing Equation

The system governing equation of free vibrated laminated orthotropic spherical shell panel under hygrothermal loading can be obtained using Hamilton’s principle as follows.

\[ \delta \int_0^t L \, dt = 0 \]  

(12)

where, \( L = [T-(U+W)] \)

Now employing the Equation (10) into Equation (6) and (9), the elemental form is obtained as in [12] and subsequently Equation (12) can be expressed as in [15] as

\[ ([K] - \omega^2 [M])\delta = 0 \]

(13)

where, \( \delta^T = [u \, v \, w \, \phi_1 \, \phi_2 \, \psi_1 \, \psi_2 \, \theta_1 \, \theta_2] \) is the displacement vector, \([M]\) and \([K]\) are the global mass and stiffness matrices, respectively.

Now, the numerical solutions are obtained using a direct iterative method.

5. Results and Discussion

A homemade finite element computer code is developed in MATLAB 7.10.0 environment based on the present mathematical formulation for the laminated composite spherical shell panel. The graphite/epoxy composite lamina properties are considered for computation purpose throughout the analysis. The composite properties are also taken as temperature and moisture dependent ([3]) and presented in Table 1. \( \nu_{12}=0.3, \quad G_{12} = G_{13}, \quad G_{23} =0.5G_{12}, \quad \alpha_1=-0.3\times10^{-6}/K, \quad \alpha_2= 28.1\times10^{-6}/K, \quad \beta_1=0, \quad \beta_2=0.44). \)

| Table 1 (a) Elastic moduli of graphite/epoxy lamina at different temperatures |
|-----------------------------|---|---|---|---|---|---|
| Elastic Moduli (Gpa)        | 300| 325| 350| 375| 400| 425|
| \( E_{11} \)                | 130| 130| 130| 130| 130| 130|
| \( E_{22} \)                | 9.5| 8.5| 8.0| 7.5| 7.0| 6.75|
| \( G_{12} \)                | 6.0| 6.0| 5.5| 5.0| 4.75| 4.5|

| Table 1(b) Elastic moduli of graphite/epoxy lamina at different moisture concentration |
|-----------------------------|---|---|---|---|---|
| Elastic Moduli (Gpa)        | 0 | 0.25| 0.5| 0.75| 1 |
| \( E_{11} \)                | 130| 130| 130| 130| 130|
| \( E_{22} \)                | 9.5| 9.25| 9| 8.75| 8.5|
| \( G_{12} \)                | 6.0| 6.0| 6.0| 6.0| 6.0|

The following sets of support conditions are considered in the present analysis.

(a) All edges simply supported condition (SSSS):

\[ v = w = \phi_2 = \psi_2 = \theta_2 = 0 \text{ at } x = 0, \text{ a and } y = 0, \text{ b } \]

(b) All edges clamped condition (CCCC):

\[ u = v = w = \phi_1 = \phi_2 = \psi_1 = \psi_2 = \theta_1 = \theta_2 = 0 \text{ for both } x = 0, \text{ a and } y = 0, \text{ b. } \]
The fundamental frequency in nondimensional form is obtained using the equation
\[ \bar{\omega} = \frac{\omega \alpha^2}{h \sqrt{\rho / E_z}}. \]

5.1. Convergence and Comparison Study

In order to establish the convergence behaviour the free vibration under unlike environmental conditions, responses computed using the present model is plotted with respect to mesh divisions in figure 1. The geometrical, material properties, lamination scheme and support conditions are taken same as [6] and [9]. The results show very good convergence rate with respect to mesh refinement. Based on the convergence study, it is concluded that a (5×5) mesh is adequate to obtain desired responses throughout the analysis. For validation study, simply supported square (a=b=0.5 m), laminated composite spherical shells (R1=R2, h=5mm, a/h=100) are considered. The material properties are taken same as [2]. The responses obtained using the present model along with the reference values are presented in Table 2. It is clearly observed that the present results are in good agreement with the reference values. The difference in results is because the fact that the reference uses FSDT based mathematical model where as the present model is developed based on HSDT mid plane kinematics, which is more realistic in nature.

![Figure 1](image_url)

**Figure 1.** Convergence study of simply supported, square, laminated composite spherical panel (R1=R2, R/a=10, a/h=10, [0/0/30/-30]2).
Table 2. Comparison of fundamental frequency (Hz) of simply supported laminated composite spherical shells

| Source | Lamination | R/a=5 | R/a=10 |
|--------|------------|-------|--------|
|        |            | C=0.25 | C=0.75 | C=1.0  |
| Present|[0/90]2     | 217.100 | 196.7728 | 185.6448 |
|        | [45/-45]2  | 385.3415 | 370.9074 | 358.5730 |
| [2]    | [0/90]2    | 201.9100 | 201.7200 | 201.6400 |
|        | [45/-45]2  | 346.3500 | 344.1800 | 343.1800 |
| Difference (%) |          | 6.9970 | -2.5140 | -8.6160 |
|        |            | 10.1190 | 7.2050 | 4.2920 |

5.2. Parametric Study

In order to address the effect of various geometrical parameter like curvature ratio (R/a = 5, 10, 50, 100), thickness ratio (a/h = 5, 10, 50, 100) and aspect ratio (a/b = 1, 1.5, 2, 2.5) on the nondimensional fundamental frequency of laminated composite spherical shell panel under hygrothermal environment is computed and discussed in detail in the following examples.

5.2.1. Example 1

In this example, a simply supported square (a/b=1) eight layer anti-symmetric ([0°/90°]4) cross-ply laminated composite spherical panel (a/h=20) is used for the computational purpose under different hygrothermal load. The responses are presented in figure 2. It is observed that the nondimensional fundamental frequency parameter increases as temperature and moisture concentrations increases for each case. It can also be seen that frequency parameter decreases as the curvature ratio increases.

5.2.2. Example 2

Figure 3 shows the effect of thickness ratio (a/h) and hygrothermal conditions on the nondimensional linear frequency responses of angle-ply ([±45]2), square (a/b=1) laminated composite spherical (R/a=50) panel under CFCF support condition. It is observed that the nondimensional fundamental frequency responses are higher for thin shell panels in comparison to the thick panels.

5.2.3. Example 3

Square hybrid ([±30 /0/75]) laminated spherical panels (R/a=50, a/h=100), under CSCS support condition and different environmental conditions are considered. The computed results are presented in figure 4. It can be seen that the nondimensional fundamental frequency parameter increases with hygrothermal load and aspect ratios.
Figure 2. Variation of nondimensional fundamental frequency parameters with curvature ratio under hygrothermal loading

Figure 3. Variation of nondimensional fundamental frequency parameters with thickness ratio under hygrothermal loading
Figure 4. Variation of nondimensional fundamental frequency parameters with aspect ratio under hygrothermal loading

6. Conclusion
Effect of hygrothermal load and geometrical parameter on the free vibration behaviour of laminated composite spherical shell panels have been investigated in the present study. A mathematical model based on the HSDT mid-plane kinematics is developed and discretised using FEM. The convergence and validation of the present model has been established. The significance of the present HSDT based model for the analysis of laminated structures under combined hygrothermal loading condition is observed. The subsequent numerical illustrations indicate that considering the effect of hygrothermal conditions is inevitable for laminated composite structures, particularly when they are exposed to a severe environmental condition. Irrespective of the hygrothermal load and support conditions, the nondimensional fundamental frequency responses decrease with curvature ratios whereas they increase with increase in thickness ratios and aspect ratios under all sorts of hygrothermal conditions.

Acknowledgement
Authors are thankful to AICTE (All India Council for Technical Education) for partial support through the sanction 8023/RID/RPS/56/11/12 Dated: 26/03/2012.

Reference
[1] Sai Ram K S and Sinha P K 1992 Journal of Sound and Vibration 158(1) 133–148
[2] Parhi P K, Bhattacharyya S K and Sinha P K 2001 Journal of Sound and vibration 248(2) 195-214
[3] Patel, B P, Ganapathi M and Makhecha D P 2002 Composite Structures 56 25–34
[4] Huang X L and Zheng J J 2003 Engineering Structures 25 1107–1119
[5] Shen H S, Zheng J J and Huang X L 2004 Journal of Reinforced Plastics and Composites 23 (1095) DOI: 10.1177/0731684404037038
[6] Naidu N V S and Sinha P K 2007 Composite Structure 77 475-483
[7] Benkhedda A, Tounsi A and Adda Bedia E 2008 Composite Structures 82 623–35
[8] Lo S H, Zhen W, Cheung Y K and Wanji C 2010 Composite Structures 92 633–646
[9] Nanda N and Pradyumna S 2011 Journal of Composite Material 45(20) 2103-2112
[10] Ashraf M Z 2012 Composite Structures 94 3685–3696
[11] Panda H S, Sahu S K and Parhi P K 2013 Composite Structures 96 502–513
[12] Panda S K and Mahapatra T R 2014 Meccanica 49 191–213
[13] J N Reddy (2004), Mechanics of laminated Composite Plates and Shells, (Second Edition), CRC Press
[14] Cook R D, Malkus D S and Plesha M E 2000 Concepts and applications of finite element analysis Third Edition John Willy and Sons (Asia) Pvt. Ltd., Singapore
[15] Sundaramoorthy R, David W and Murray 1973 Incremental finite element Matrices (ASCE J. Structural Division 99 (ST12) 2423-2438