A General Lower Bound for Radiative B Decay

David Atwood\textsuperscript{a}, C.P. Burgess\textsuperscript{b} and A. Soni\textsuperscript{a}

\textsuperscript{a} Department of Physics, Brookhaven National Laboratory
Upton New York 11973 USA.

\textsuperscript{b} Physics Department, McGill University
3600 University St., Montréal, Québec, CANADA, H3A 2T8.

Abstract

In a wide class of models – including the standard model – flavour-changing neutral currents are suppressed by an approximate chiral symmetry of the low-energy lagrangian. This symmetry allows the derivation of general relations among low-energy flavour-changing processes. We derive one such: the relative sizes of the decay rates $b \to s\gamma$, $b \to d\gamma$ and $s \to d\gamma$. Together with a unitarity-related lower bound on the rate for the strangeness-changing hyperon decay $\Omega^- \to \Xi^-\gamma$ we obtain a reasonably model-independent lower bound for the inclusive $b \to s\gamma$ rate.

1. Introduction

Flavour-changing neutral-current processes have long been known to be extremely rare. They are much weaker than would be naively expected to be generated through loops involving the known flavour-changing charged-current interactions. This small size
is understood within the standard model as being due to the GIM mechanism: a partial cancellation among loop-induced effects.

This fairly delicate cancellation provides a major clue as to the nature of the new physics that is expected to replace the standard model at higher energies. A minimal requirement for any candidate model for such new physics is that it not ruin this cancellation and so generate unacceptably large flavour-changing neutral currents.

In the absence of any direct knowledge of the physics at higher scales, and barring the possibility of an unnatural fine tuning, the absence of such flavour-changing effects is most efficiently summarized in terms of an approximate symmetry of the low-energy effective lagrangian that is obtained once the unknown heavy particles are integrated out of the underlying theory. Such a symmetry formulation has arisen [1] within the context of technicolor models, for example.

The main purpose of this note is to point out that such an approximate symmetry strongly constrains the generation dependence of all of the interactions in such an effective theory. As such it leads to general relations amongst the potential flavour-changing effects that can be probed purely at low energies. Most importantly, since these relations follow purely from the symmetries of the low-energy effective theory their validity transcends that of any particular model for new physics and may be considered a general consequence of the given symmetry-breaking pattern.

We illustrate this point here by using such an approximate symmetry to derive the relative strength of the flavour-changing processes $s \to d\gamma$, $b \to s\gamma$ and $b \to d\gamma$. This relation holds in a very broad class of models, including the standard model. Using a lower bound for the $s \to d\gamma$ rate which follows from unitarity we are then able to derive a lower bound for the inclusive branching ratio $b \to s\gamma$ which turns out to be 8.5 times smaller than the standard model prediction. For $100 \text{ GeV} < m_t < 200 \text{ GeV}$ the lower bound is $B(b \to s\gamma) > (3 - 5) \cdot 10^{-5}$. The corresponding inclusive $b \to d\gamma$ bound is twenty-five times smaller: $B(b \to d\gamma) > (1 - 2) \cdot 10^{-6}$. Should these decays not be measured at this level then this would serve to rule out not just the standard model but any model within which flavour-changing neutral currents are suppressed via the same type of approximate symmetry.

The weakest link in the arguments that establish this lower bound is the extraction of the quark-level bound for $s \to d\gamma$ from the lower limit on the rate for radiative $\Omega^- \to \pi^+\pi^-\gamma$ decay. This relies on the expected dominance in this decay of the single-quark transition. An experimental check on this reasoning is given by polarization-sensitive measurements such as the decay distribution of outgoing photons relative to the initial $\Omega^-$ polarization. The single-quark picture of this decay unambiguously predicts an asymmetry parameter $\alpha =$

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1. A similar check for $b$ decays involves detecting the correlation between the direction of the final hadrons relative to the photon polarization in decays such as $B \rightarrow K^*\gamma \rightarrow K\pi\gamma$. [2]

We now turn to the guts of our argument. We start by briefly summarizing the formulation of the GIM mechanism in terms of an approximate symmetry. This is next used to fix the relative size of the quark-level amplitudes for $b \rightarrow s\gamma$, $b \rightarrow d\gamma$ and $s \rightarrow d\gamma$ leading to the lower bound on the total branching ratio for radiative $b$ decay. We conclude with a short discussion of two possible polarization-dependent observables which might be used to check the validity of the single-quark mechanism for mediating these flavour-changing transitions once they have been reliably observed.

2. The Approximate Symmetry

The standard model enjoys an approximate $SU_L(3) \times SU_{uR}(3) \times SU_{dR}(3)$ symmetry under which the quark fields transform in the following representation

\[
\begin{pmatrix}
  u_i \\
  d_i 
\end{pmatrix}_L \sim (3, 1, 1), \quad
(u_a)_R \sim (1, 3, 1), \quad
(d_n)_R \sim (1, 1, 3).
\] (1)

If unbroken, this symmetry would completely forbid flavour-changing neutral currents. In the standard model this symmetry is broken only by the Yukawa couplings which generate fermion masses. As a result the generation dependence of any flavour-changing neutral-current amplitude is completely determined by its dependence on these Yukawa couplings. Once expressed in a basis of mass eigenstates for which the Yukawa couplings are diagonal this ensures the GIM-type suppression of all flavour changing amplitudes by fermion masses and Kobayashi-Maskawa mixing angles.

Once phrased in this manner it is clear how to ensure the same properties for a general low-energy effective lagrangian which need not include scalars and Yukawa couplings. The effective lagrangian must first enjoy the same approximate $SU_L(3) \times SU_{uR}(3) \times SU_{dR}(3)$ symmetry with the fermion transformation law of eq. (1). The content of the GIM mechanism may then be summarized by the requirement that the unknown underlying physics only involve two types of (small) order parameters which break this symmetry by ‘transforming’ in the following way

\[
\lambda^u \sim (\overline{3}, 3, 1), \quad
\lambda^d \sim (\overline{3}, 1, 3).
\] (2)

Notice that these transformation rules are precisely those that are satisfied by the Yukawa couplings in the standard model.
This transformation property ensures an acceptable quark mass matrix:

\[ \mathcal{L}_m = -\frac{M}{2} \left[ \lambda_{ui} \bar{u}_a \gamma_L u_i + \lambda_{d_i} \bar{d}_a \gamma_L d_i \right] + \text{h.c.}, \quad (3) \]

in which \( M \) is a large new-physics scale which might reasonably be several hundreds of GeV. Comparison with the known masses would then imply that \( \lambda^u \) and \( \lambda^d \) have the same flavour-structure and size as have the standard-model Yukawa couplings.

Consider now the implications that may be inferred from such an approximate symmetry for radiative flavour-changing transitions. Lorentz invariance requires that the matrix element for the process \( d_i \to d_n \gamma \) must take the general form

\[ \mathcal{M}(d_i \to d_n \gamma) = \frac{e}{M} \xi_{ni} \bar{d}_n \gamma^{\mu \nu} \gamma_L d_i \, q_\nu \epsilon_\mu(q), \quad (4) \]

where \( q \) and \( \epsilon_\mu(q) \) are the photon’s four-momentum and polarization.

The main point is now this: for small \( \lambda^u \) and \( \lambda^d \) the generation structure of the coefficients \( \xi_{im} \) is determined up to an overall normalization. For example, since the operator in eq. (4) transforms under the chiral symmetry as \((3, 1, 3)\) we must have

\[ \xi_{ni} = \lambda^d_{nj} \left[ A(\lambda^{u^\dagger} \lambda^u, \lambda^{d^\dagger} \lambda^d) \right]_{ji} + \left[ B(\lambda^d \lambda^{d^\dagger}) \right]_{nm} \lambda^d_{mi}. \quad (5) \]

Should these functions be analytic near \( \lambda^u = \lambda^d = 0 \), \( A \) and \( B \) could be replaced by their Taylor expansions about this point. In the standard model, however, these functions turn out to be logarithmically singular for small \( \lambda^u \) and \( \lambda^d \) [3] once QCD corrections are included. This leads instead to the following asymptotic expression

\[ A \simeq a_1 \ln (\lambda^{u^\dagger} \lambda^u) + a_2 \ln (\lambda^{d^\dagger} \lambda^d) + a_3 + a_4 \lambda^{u^\dagger} \lambda^u + a_5 \lambda^{d^\dagger} \lambda^d + \cdots, \quad (6) \]
\[ \text{and} \quad B \simeq b_1 \ln (\lambda^d \lambda^{d^\dagger}) + b_2 + b_3 \lambda^d \lambda^{d^\dagger} + \cdots. \quad (7) \]

The GIM mechanism now reveals itself once these interactions are expressed in a basis of fermion mass eigenstates: \( u_{Li} = (U_u)_{ij} u'_{Lj}, \ u_{Ra} = (U_u)^{*}_{ab} u'_{Rb}, \ d_{Li} = (U_d)_{ij} d'_{Lj} \) and \( d_{Rm} = (U_d)^{*}_{mn} d'_{Rn} \). The unitary matrices \( U_u \) and \( U_d \) here are chosen to diagonalize the appropriate mass matrices: \( m^u = U_u^T \lambda^u U_u \ M = \text{diag}(m_u, m_c, m_t) \) and \( m^d = U_d^T \lambda^d U_d \ M = \text{diag}(m_d, m_s, m_b) \). In this basis all terms in eq. (5) that involve only \( \lambda^d \) become automatically diagonal. As a result the only off-diagonal coefficient which can remain in eq. (4) is

\[ \xi_{ni} = \sum_{x=u,c,t} V_{xn}^{*} V_{xi} \frac{m^d_{m}}{M} \left\{ a_1 \ln \left[ \frac{(m_x^u)^2}{M^2} \right] + a_4 \frac{(m_x^u)^2}{M^2} + \cdots \right\}. \quad (8) \]
The matrix $V = U_{d}^{\dagger}U_{d}$ that appears in this expression is the usual Kobayashi-Maskawa matrix whose elements govern the charged-current weak interactions.

In a world for which the QCD scale is much smaller than the current quark masses the logarithmic dependence on $\lambda^{u}$ and $\lambda^{d}$ reflects an infrared mass singularity as the up-quark masses tend to zero. In the real world, for which $\Lambda_{QCD} > m_{u}$, such a dependence on $m_{u}$ requires some comment. In this case, although the $m_{c}$ and $m_{t}$-dependence do not change, the simple perturbative expression for the $m_{u}$-dependence gets complicated by perturbatively incalculable QCD effects. The logarithmic dependence on $m_{c}$ and $m_{t}$, together with the chiral $SU_{L}(3) \times SU_{uR}(3) \times SU_{dR}(3)$ symmetry nevertheless still imply logarithmic dependence of $A$ and $B$ on $\lambda^{u}$. This must then combine with strong QCD contributions to the nonlogarithmic terms — such as $a_{3}$ — to convert $\ln(m_{u}^{2}/M^{2})$ into a logarithm involving an appropriate hadronic scale with the same chiral behaviour: $\ln(m_{\pi}^{4}/M^{4})$.

As advertised, to the extent that powers of $\lambda^{u}$ may be neglected, only the single complex number $a_{1}$ in eq. (8) is left undetermined by the approximate chiral symmetry. In this limit this is the only quantity which can distinguish various underlying theories from one another.

Notice also that the explicit dependence of the coefficients of this equation on the quark masses and on the Kobayashi-Maskawa matrix precisely mirrors those that are obtained when this vertex is generated by loops within the standard model. In the standard-model case $a_{1}$ turns out to be real. More generally a nontrivial phase for $a$ is strongly bounded by limits on the neutron electric dipole moment. This is because such a phase violates $CP$ and so generates a quark electric dipole moment

$$D_{f} = \text{Im } a \frac{m_{f}}{M^{2}} \sum_{x=u,c,t} |V_{xf}^{\ast}|^{2} \ln \left[ \frac{(m_{x}^{u})^{2}}{M^{2}} \right], \quad (9)$$

which must be less than $\sim 10^{-26} \text{ e-cm}$. [4].

3. A Lower Bound for $b \to s\gamma$

As is clear from the previous section, the approximate $SU_{L}(3) \times SU_{uR}(3) \times SU_{dR}(3)$ symmetry relates the strength of the effective $s \to d$, $b \to d$ and $b \to s$ operators in the following way

$$\frac{\Gamma(b \to s\gamma)}{\Gamma(s \to d\gamma)} = m_{s}^{2} \left[ V_{cs}^{\ast} V_{cb} \ln \left( \frac{m_{s}^{2}}{m_{c}^{2}} \right) + V_{ts}^{\ast} V_{tb} \ln \left( \frac{m_{s}^{2}}{m_{t}^{2}} \right) \right]^{2} \left[ V_{cd}^{\ast} V_{cs} \ln \left( \frac{m_{c}^{2}}{m_{u}^{2}} \right) + V_{td}^{\ast} V_{ts} \ln \left( \frac{m_{t}^{2}}{m_{u}^{2}} \right) \right].$$
\[
\Gamma(b \to d\gamma) = \frac{V_{cd} V_{cb} \ln \left( \frac{m^2}{m_u} \right) + V_{td}^* V_{tb} \ln \left( \frac{m^2}{m_u} \right)}{V_{cs} V_{cb} \ln \left( \frac{m^2}{m_u} \right) + V_{ts}^* V_{tb} \ln \left( \frac{m^2}{m_u} \right)} \approx (10 - 20) \quad \text{for} \quad 100 \text{ GeV} < m_t < 200 \text{ GeV};
\]

\[
\Gamma(b \to s\gamma) = \left[ V_{cs} V_{cb} \ln \left( \frac{m^2}{m_u} \right) + V_{ts}^* V_{tb} \ln \left( \frac{m^2}{m_u} \right) \right]^2 \approx 0.04 \quad \text{for} \quad 100 \text{ GeV} < m_t < 200 \text{ GeV}.
\]

We use current-quark masses in the numerical estimates and take \( m_s \simeq 200 \text{ MeV} \).

We next wish to use this relation together with experimental information concerning the process \( s \to d \) to derive a lower bound for the inclusive \( b \to s\gamma \) and \( b \to d\gamma \) rates. Our starting point is the recognition that the radiative \( s \to d \) transition can be observed in radiative hyperon decays. Although detailed study of these decays over the past ten years [5] has indicated that they are generally not dominated by the single-quark decay \( s \to d\gamma \), there are exceptions to this rule. In particular, the single-quark contribution appears to dominate the decay \( \Omega^- \to \Xi^-\gamma \) – dominating both penguin and long-distance effects [6]. An evaluation of the matrix elements in ref. [6] gives the result

\[
B_{\text{sm}}(\Omega^- \to \Xi^-\gamma) \simeq 6.8 \cdot 10^{-5} \quad \text{for} \quad \xi_{ds} = 0.8 \frac{G m_s}{16\pi^2}.
\]

\( G \) here represents the usual Fermi constant.

Unfortunately this decay has not yet been observed, the present 90\% c.l. upper bound being \( B_{\text{exp}}(\Omega^- \to \Xi^-\gamma) < 2.2 \cdot 10^{-3} \) [7]. Experiment does, however, furnish a lower bound to this branching ratio [8] as may be seen by cutting the long-distance contributions such as that shown in Fig. 1. Both halves of the cut diagram may be determined from experiment together with \( SU_f(3) \) flavour symmetry. The lower bound obtained in this way in ref. [8] is \( B(\Omega^- \to \Xi^-\gamma) > 8 \cdot 10^{-6} \). When taken together with the dominance of the single-quark decay in this mode, this gives the following lower bound

\[
\frac{B(b \to s\gamma)}{B_{\text{sm}}(b \to s\gamma)} = \frac{B(s \to d\gamma)}{B_{\text{sm}}(s \to d\gamma)} = \frac{B(\Omega^- \to \Xi^-\gamma)}{B_{\text{sm}}(\Omega^- \to \Xi^-\gamma)} > 0.12.
\]

Together with the standard-model prediction [9]: \( B_{\text{sm}}(b \to s\gamma) = (2 - 4) \cdot 10^{-4} \) for \( 50 \text{ GeV} < m_t < 200 \text{ GeV} \) this leads to the lower bounds quoted above:

\[
B(b \to s\gamma) > (3 - 5) \cdot 10^{-5}, \quad \text{and} \quad B(b \to d\gamma) > (1 - 2) \cdot 10^{-6}.
\]

This establishes the main result of this letter. We now turn to a brief discussion of how the measurement of polarization-sensitive observables in these decays can be used to check the importance of the single-quark mechanism in both hyperon and \( B \) decays.
4. Polarized Observables

Until flavour-changing radiative $b$ decays are directly observed it is necessary to look to strange-quark processes in order to determine the normalization of the interaction of eq. (4). As outlined in the previous section we have used for these purposes the rate for the hyperon decay $\Omega^- \to \Xi^-\gamma$. The key part of the argument is the expectation that it is the single-quark process that is responsible for this particular decay. In ref. [6] this conclusion is reached by comparing the single-quark rate with that arising from long-distance contributions as well as penguin-type $s \to dg$ gluon transitions.

Much of the uncertainty in these arguments may be removed once the $\Omega^- \to \Xi^-\gamma$ decay is directly observed. There are two reasons for this. The first reason is that once the rate for this decay is experimentally known it can be compared to the total inclusive rates or upper limits for $b \to s\gamma$ and $b \to d\gamma$. More importantly, once this hyperon decay is observed it becomes possible to measure the decay distribution of the $\Xi^-$ relative to the direction of $\Omega^-$ polarization in its rest frame.

As a function of the mass ratio, $x \equiv m_\Xi/m_\Omega$, and the cosine, $z = \cos \theta$, of the angle in the $\Omega^-$ rest frame between the $\Xi^-$ momentum and the direction of $\Omega^-$ polarization the decay distribution is given by:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dz} \bigg|_{h=3/2} = \frac{3}{4(x^2 + 3)} \left[(1 + x^2) + (3 - x^2)z^2 - \alpha z\left((3 + x^2) - (1 - x^2)z^2\right)\right], \quad (15)$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{dz} \bigg|_{h=1/2} = \frac{1}{4(x^2 + 3)} \left[(x^2 + 9) + (3x^2 - 9)z^2 + \alpha z\left((5x^2 - 9) + 9(1 - x^2)z^2\right)\right]. \quad (16)$$

The shape parameter $\alpha$ in this equation is predicted to satisfy $\alpha = -1$ if the decay is dominantly mediated by the single-quark interaction of eq. (4). A measurement of this value for $\alpha$ would provide convincing support for the single-quark mechanism for the decay.

A similar check would also be useful when diagnosing flavour-changing radiative $B$-meson decays. This is particularly so for the $b \to d$ transition for which the single-quark interpretation is less clean than it is for $b \to s$. Unfortunately such observables are harder to come by for $B$ mesons. One possibility arises should the polarization of the outgoing photon prove to be measureable [2] In this case the parity-preserving and parity-violating contributions to the $b \to s$ or $b \to d$ transition operator of eq. (4) may be disentangled from one another by measuring the angular distribution of decay products relative to the direction of polarization of the photon.

For example, in the decay $B \to K^*\gamma \to K\pi\gamma$ (or $B \to \rho\gamma \to \pi\pi\gamma$) choose, in the $B$ rest frame, the directions of the momentum and polarization of the outgoing photon to
define the $z$- and $x$-axes. Denote in this frame the polar angles defining the direction of the momentum difference $p_\pi - p_K$ (or $p_{\pi_1} - p_{\pi_2}$) by $(\theta, \phi)$. Then the decay distribution as a function of $\phi$ is predicted to behave as

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\phi} = \frac{1}{2\pi} \left[ 1 + \sqrt{1 - \alpha^2 \cos 2\phi} \right].$$

(17)

As before $\alpha = -1$ if the single-quark transition dominates the decay.

5. Conclusions

In summary, we have shown how the natural expression of the observed suppression of flavour-changing neutral currents in terms of an approximate chiral symmetry of the effective lagrangian leads to definite relations amongst low-energy flavour-changing processes. We show in particular that within a wide class of models – including, of course, the standard model – the relative strength of the quark-level radiative transitions $s \to d\gamma$, $b \to s\gamma$ and $b \to d\gamma$ is completely fixed in terms of particle masses and Kobayashi-Maskawa mixing angles.

Although this same prediction is well known within the standard-model context, our derivation makes explicit the much wider validity of the result. This would be of most interest should these predictions fail, in which case a broad framework for suppressing flavour-changing neutral currents would be ruled out.

We finally combine the relation among these amplitudes with two facts concerning the hyperon radiative decay $\Omega^- \to \Xi^-\gamma$ in order to establish a model-independent lower bound for the inclusive rates $B(b \to s\gamma)$ and $B(b \to d\gamma)$. The two facts needed to obtain these results are (i) the long-distance unitarity lower bound for $B(\Omega^- \to \Xi^-\gamma)$ derived in ref. [8], and (ii) the dominance for this process of the single-quark decay found in ref. [6].

The lower bounds obtained in this way are $B(b \to s\gamma) > (3 - 5) \cdot 10^{-5}$ and $B(b \to d\gamma) > (1 - 2) \cdot 10^{-6}$. The ranges quoted indicate the dependence of the result on the top mass which we take to lay between 100 and 200 GeV. Of these bounds the first is of most immediate interest given that branching ratios of the order of $10^{-4}$ are presently being probed at CLEO.

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**Figure Caption**

A long-distance contribution to the decay $\Omega^- \rightarrow \Xi^- \gamma$ from whose imaginary part the lower bound is derived.
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