Hidden Negative Energies in Strongly Accelerated Universes

Ignacy Sawicki
Institut für Theoretische Physik, Ruprecht-Karls-Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany

Alexander Vikman
CERN, Theory Division, CH-1211 Geneva 23, Switzerland, and Department of Physics, Stanford University, Stanford, CA 94305, USA

(Dated: 2nd May 2014)

We point out that theories of cosmological acceleration which have equation of state, \( w \), such that \( 1 + w \) is small but positive may still secretly violate the null energy condition. This violation implies the existence of observers for whom the background has infinitely negative energy densities, despite the fact that the perturbations are free of ghosts and gradient instabilities.

I. INTRODUCTION

One frequently posits that dark energy (DE) is described by a perfect-fluid energy-momentum tensor (“EMT”),

\[
T_{\mu\nu} = (\varepsilon_u + P) u_\mu u_\nu - g_{\mu\nu} P ,
\]

where \( u^\mu \) is the velocity of an observer while \( \varepsilon_u \) is the energy density and \( P \) is the pressure measured by the observer and we use the \((+ - - -)\) convention for the metric. The equation-of-state parameter is then defined as \( w \equiv P/\varepsilon_u \).

In cosmological setups, one then considers homogeneous and isotropic Friedmann universes. In such a case, even for more general, imperfect-fluid DE models, the configuration of the background EMT takes the form of Eq. (1) with \( u^\mu \) taking on the meaning of the cosmological observer. The Universe accelerates for \( w < -1/3 \) and does so more strongly the more negative is \( w \). The cosmological constant, with \( w = -1 \) exactly, is compatible with all the observations. However, the data show, see e.g. [1–3], a statistically insignificant preference for DE with a slightly more negative equation of state for at least a part of the history of the universe. Moreover, given realistic future surveys, even if \( w \) is constant, the measurement error will remain at 2%, see e.g. [4]. The possibility of strong acceleration, \( w < -1 \), sometimes called a phantom [5], is striking, because it implies a that the EMT violates the null energy condition (“NEC”), which states that for all null vectors \( n^\mu \) and \( T_{\mu\nu} : T_{\mu\nu} n^\mu n^\nu \geq 0 \) and for the EMT given by (1) is equivalent to \( \varepsilon_u + P \geq 0 \). This is the weakest of all classical energy conditions and its violation may have dramatic implications for the future of the universe [6, 7].

The question of whether it is at all possible to violate the NEC without any perturbative instabilities has only been answered recently. Perfect fluids and canonical scalar fields which violate the NEC always suffer from either ghosts or gradient instabilities or both, see e.g. [8]. These are short-timescale instabilities that are most rapid at energies near the cut-off and therefore immediately disqualify the background solution, see however [9, 10]. In Refs [11–14] noncanonical scalar-field models were constructed that can serve as a medium with \( w < -1 \) and where perturbations have a positive kinetic term and real sound speed. These scalar fields belong to theories which contain higher-order derivatives in the action but retain second-order dynamical equations [15–17] extending the so-called galileons [18–21]. A feature shared by all these models is that, contrary to scalar fields without higher derivatives in the action, these models do not have EMTs of perfect-fluid form (1) on generic configurations.

In this paper, we point out two peculiarities of DE configurations which violate the NEC. First we provide a general proof that a violation of the NEC by any matter on some arbitrary (not necessarily homogeneous or isotropic) configuration implies that the corresponding energy density as a function in phase space at a given point is necessarily unbounded from below. This is true even if there are no ghosts or gradient instabilities around this configuration. Thus the existence of a single NEC-violating configuration always implies that there are other configurations of this matter on which the Hamiltonian is arbitrarily negative, if, as is usually the case, the Hamiltonian is equal to the energy density. As a consequence, observers boosted with speed higher than \((-w)^{-1/2}\) with respect to the cosmological background see negative energy densities of the NEC-violating DE, the more negative the faster they move, without any limit. Moreover, these observers see the flows of DE energy density as spacelike. In particular, given the current knowledge of \( w \), this may apply to e.g. massive particles in cosmic rays.

Further we point out that even measuring \( w > -1 \) does not preclude the possibility of a violation of the NEC by means of the purely spatial parts of the EMT: energy flows and anisotropic stresses which can be relevant in perturbations around strongly accelerated back-
grounds with positive but small $1 + w$. As we have mentioned above, the only known systems violating the NEC without immediate pathologies in perturbations necessarily possess such imperfect EMs. Thus again there would be observers measuring infinitely negative energy densities of DE.

All of this applies equally well to NEC-violating models of the early universe, e.g. [12, 14, 22, 23].

Whether the unbounded negative energy densities we discuss in this paper definitely imply a problem, we are unsure. However, their existence is definitely puzzling and somewhat uncomfortable.

II. $w < -1$ IMPLIES UNBOUNDED NEGATIVE ENERGIES

The violation of the NEC for matter with the EMT $T_{\mu\nu}$ is defined as the existence of at least one future-directed ray of null vectors $n^\mu$ such that $T_{\mu\nu}n^\mu n^\nu < 0$. In a frame $w^\mu$, the observed energy density $\varepsilon_u = T_{\mu\nu}w^\mu w^\nu$. Below we show that in that case, there always exists a frame $V^\mu$ such that $V^\mu = T_{\mu\nu}V^\nu V^\nu < \varepsilon_u$. In particular, for any positive $\varepsilon_u$, there are frames where $\varepsilon_V < 0$. Let us prove this statement.

Let us chose the null vector from the NEC-violating ray such that $n^\mu n_\mu = 1$. We parameterise the new frame as

$$V^\mu = \alpha u^\mu + \eta n^\mu.$$  

This vector is timelike provided

$$\alpha^2 + 2\alpha\eta = 1,$$  

and the new frame is future-directed when $V^\mu u_\mu > 0$. The frame $V^\mu$ moves with respect to the frame $u^\mu$ with the speed

$$v = \frac{\eta}{\alpha + \eta} = \frac{1 - \alpha^2}{1 + \alpha^2}.$$  

Substituting $\eta$ from (3) we obtain

$$\varepsilon_V (\alpha^2) = \alpha^2 \varepsilon_u + (1 - \alpha^2) T_{\mu\nu}h^\mu n^\nu +$$  

$$+ \frac{1}{4\alpha^2} (1 - \alpha^2)^2 T_{\mu\nu}n^\mu n^\nu,$$

so that

$$\frac{\partial^2 \varepsilon_V}{\partial \alpha^2 \partial \alpha^2} = \frac{1}{2\alpha^0} T_{\mu\nu}n^\mu n^\nu.$$  

Thus the energy density as a function of $\alpha^2$ can have a minimum only if the NEC holds. Indeed, in frames very close to the NEC-violating vector $n^\mu$, i.e. for small $\alpha$, the energy density $\varepsilon_V (\alpha^2)$ behaves as

$$\varepsilon_V (\alpha^2) \simeq \frac{1}{4\alpha^2} T_{\mu\nu}n^\mu n^\nu < 0,$$

and can be made arbitrarily negative.

We can now reinterpret the effect of this passive Lorentz transformation, as an active one: instead of observing the same configuration from different reference frames we can change the configuration itself. By virtue of Lorentz symmetry, these boosted configurations are solutions as well and therefore belong to the phase space. Thus if a boosted observer can observe arbitrarily negative energy density, this implies that the phase space contains configurations with arbitrarily negative energy, i.e. that the Hamiltonian is unbounded from below. There are two types of systems for which this conclusion may not be applicable: (i) systems without fundamental Lorentz invariance, (ii) systems where the Hamiltonian does not coincide with the energy density given by the EMT.

Let us now assume that the cosmological background of DE has a positive energy density, $\varepsilon_u$, in (1) and violates the NEC only slightly: namely $w < -1$ but so close to $-1$ that it may remain unobservable for future probes. Further the perturbations are not ghosty and have positive sound speed squared, $c_s^2 > 0$, i.e. this DE seemingly does not suffer from any perturbative instabilities. Let us now boost away from the cosmological frame with speed $v$ in any direction. Then Eq. (5) implies that the energy density of DE in the new frame, $\varepsilon_V$, becomes negative when

$$v > v_{\text{crit}} = \frac{1}{\sqrt{-w}}.$$  

Inside structures the total EMT is dominated by dark matter (“DM”) and therefore the total energy density measured by all observers will still be positive. However, for a sufficiently smooth DE there are large voids where DE dominates the total EMT over DM.

Now let us recall that the energy of the protons at the LHC is 4 TeV so that $1 - v_p \simeq 10^{-7}$, whereas cosmic rays have been observed up to energies of order $10^{19}$ eV [24], giving $1 - \nu_{\text{ray}} \simeq 10^{-20}$, assuming that cosmic rays consist of protons.

This implies that in order that the energy density of the cosmological background of DE be positive in the rest frame of observed particles,

$$1 + w \gtrsim 10^{-20}.$$  

This is clearly a strong requirement far beyond the precision of any future experiments. Even stronger conditions can be obtained from ultra-high energy neutrinos e.g. by the IceCube detector [25] where neutrino up to 400 TeV were measured which gives roughly $\nu_{\mu} \simeq 1 - 10^{-28}$.

In addition, the density of the fluid’s 4-momentum as measured by an observer moving with $V^\mu$ is $p_\mu = T_{\mu\nu}V^\nu$. This vector becomes spacelike when $v > 1/|w|$, thus at even smaller observer speeds than those corresponding to negative $\varepsilon_V$. Under normal circumstances, the spatial part of $p_\mu$ points against the direction of the boost
potentially providing a force to resist further acceleration. When the NEC is violated, the opposite is true, seemingly aiding the motion.

At this point we should ask whether this peculiar observation necessarily signifies a problem. This should be a question of the structure of interactions. If there are no direct interactions between usual particles and the DE, except through gravity, then it is unlikely that any process could occur sufficiently quickly to have an effect. However, one should keep in mind that, if it is possible to extract this negative energy density, the particle could be boosted even further and see an even more negative energy density of the background. Also this could lead to violations of the second law of thermodynamics and run-away instabilities.

The situation is more interesting for small fluctuations of DE itself – “phonons”. They definitely interact with the background. If their maximal speed of propagation – i.e. the sound speed $c_s$ – lies in the interval $v_{\text{crit}} < c_s < 1$ then these fluctuations in their own rest frame interact with the negative energy density of their own background. Since we have required that the perturbations be healthy as it is in e.g. [13], the phonons’ energy is positive and they behave like normal particles.

If the presence of this negative energy density is a problem it would result in the pumping of energy into phonons making the background less and less homogeneous.

NEC-violating theories can also have interesting applications in models of the early universe. In particular, G-inflation [14] can create a blue spectrum of gravitational waves, while models [12, 22, 23] provide insight on the initial state of the universe. It is interesting to note that in the particular case studied in detail in [14] the sound speed is $c_s \lesssim 0.18$, while $w + 1$ can be made arbitrary small therefore avoiding these issues.

In the case of Galilean Genesis [12], at the beginning of the universe, $\tau \to -\infty$, one has $c_s \simeq 1 - \frac{3}{\pi^2} \tau^2$ and $w \simeq -1 - 4\tau^2$ corresponding to $v_{\text{crit}} \simeq \frac{1}{4} \tau^{-1} < c_s$ so that phonons in their rest frame interact with a background of a very negative energy density.

Here one should keep in mind that to bounce the universe one has to require that the energy density of the NEC-violating medium in the cosmological reference frame at least vanishes or is even negative to compensate for the presence of the normal matter, see [23]. However, this is only necessary around the bounce / turnaround point. For vanishing energy density of the medium the above argument would imply that $v_{\text{crit}} = 0$ at that moment.

To finish this section on a more hopeful note we would like to remark that if our observation can signify a stability problem for the above models, it could also be considered as a feature allowing for a defragmentation of the inflaton background, i.e. reheating to end inflation.

III. VIOLATING THE NEC WITH $w > -1$

All models known so far which violate the NEC and have healthy perturbations possess an EMT which is not of the perfect-fluid form (1) on realistic cosmological configurations which are locally slightly inhomogeneous and anisotropic. The simplest such example is imperfect DE from a scalar field $\phi$ with Kinetic Gravity Braiding [13] which for all timelike $\partial_\mu\phi \propto u_\mu$ has EMT

$$T_{\mu\nu} = (\varepsilon_u + P) u_\mu u_\nu - g_{\mu\nu} P + u_\mu q_\nu + u_\nu q_\mu \ ,$$

(10)

corresponding to an imperfect fluid [26]. Usually the purely spatial $q_\mu$, orthogonal to the frame $q^\mu u_\mu = 0$, is attributed to heat flow and entropy production. However, in case of [13, 26] it is just the flow of charges, resulting from the fact that $w^\mu$ does not correspond to the Eckart frame. The four-velocity $w^\mu$ is uniquely chosen by requiring there be no anisotropic stress.

On large scales, the spatial vector $q_\mu$ averages to zero. However, locally it contributes to flows of DE perturbations. On average, $w^\mu$ coincides with the cosmological frame. Note that more general scalar-field models violating the NEC would necessarily possess anisotropic stress, see e.g. [27]. Motivated by the above example, below we consider DE systems with EMT given by (10) without making any reference to the underlying theory.

Unsurprisingly the restrictions put on four-scalars $P$, $\varepsilon_u$ and the four-vector $q^\mu$ by the various energy conditions differ from those for the perfect fluid. In particular $T_{\mu\nu}n^\mu n^\nu$ is minimal on the ray with $n^\mu = w^\mu + q^\mu/q$ where $q = \sqrt{-q_\mu q^\mu} \geq 0$. Hence the NEC requires that

$$\varepsilon_u + P \geq 2q \ ,$$

(11)

which is a stronger restriction than the one for a perfect fluid. Since the expressions $T_{\mu\nu}n^\mu n^\nu$ (with total $T_{\mu\nu}$) and $\varepsilon_u + P - 2q$ (for total i.e. locally defined quantities) are four-scalars, they are invariant under coordinate transformations: their negativity in any one set of coordinates implies that the NEC is violated at this space-time point for all observers.

Since the universe is on average isotropic, the magnitude of the vector $q_\mu$ will generically be of the order of the perturbations. Moreover, in linear perturbation theory this vector $q_\mu$ is invariant under gauge and coordinate transformations because it does not have any background value, see e.g. [28, Eq. (7.14)].

In general, a configuration of the perturbed fluid will feature non-zero perturbations $\delta \varepsilon$, $\delta P$ and $q_\mu$ on top of the background values, and where we have defined them in a comoving gauge with $\delta u^\mu = 0$. In this case, the NEC is just

$$\tau + \mathcal{P} + \delta \varepsilon + \delta P \geq 2q \ ,$$

(12)

where overlines denote background quantities. Dependence on metric potentials is absent in this expression since it is purely a local measurement of a four-scalar quantity.
On a background which just satisfies the NEC, with \( w \) close to \(-1\), these perturbations can be large enough to violate the condition locally. For example, given an appropriate pressure perturbation, the local equation of state \( w_{\text{loc}} = (P + \delta P)/(\varepsilon + \delta \varepsilon) \) can fluctuate sufficiently to do so. Violations of this type would be in principle thinkable even in a matter with an EMT of perfect-fluid form Eq. (1), although we stress every such model known so far is unacceptably unstable, see e.g. [8], and therefore unphysical.

Furthermore, only in an imperfect fluid, there exist additional configurations in which \( w_{\text{loc}} > -1 \), and yet the NEC is violated,

\[
0 < 1 + w_{\text{loc}} < \frac{2q}{\varepsilon}.
\] (13)

A naive observer performing measurements of the cosmological background, or even of the pressure and energy density of the perturbations, would from \( w > -1 \) conclude that the NEC is satisfied and the universe is free from the negative energies discussed in the previous section. However, when the perturbations are taken into account, it turns out that the NEC is secretly violated. On the other hand, the knowledge of \( w \) and requirement of the NEC puts an upper bound on \( q \) in this situation.

We have expressed everything up to now in comoving gauge. One can appropriately derive the condition in other gauges and \( T_{\mu \nu} n^\mu n^\nu \) will not necessarily have the same value. This is not worrying, but purely a result of the fact that gauge transformations change the space-time point assigned to particular coordinates. By changing a gauge, we are investigating the violation of the NEC in a neighboring space-time point. Working in any one fixed gauge is enough to ascertain whether the NEC is violated at some point in the universe.

Interestingly the requirements from other energy conditions are also stronger in the presence of the energy flow \( q \), so that by observing only \( w \) one can easily be misled. Below we provide the analytical form of the conditions, which we have plotted for convenience in Figure 1; for discussion and derivation see [29]. To compare these conditions with the one for a canonical scalar field or a perfect fluid see e.g. [30].

The Weak Energy Condition (WEC), \( T_{\mu \nu} V^\mu V^\nu \geq 0 \) for all time-like future-directed vectors \( V^\mu \), is stronger than NEC and is also important in various fundamental theorems in GR. Below we formulate restrictions on \( q \), \( \varepsilon_u \) and \( P \) necessary for the WEC to hold. Provided the NEC holds there now exists a minimal energy density that can be seen by any observer,

\[
\varepsilon_{\text{min}} = \frac{\varepsilon_u - P}{2} + \sqrt{\left(\frac{\varepsilon_u + P}{2}\right)^2 - q^2}. \quad (14)
\]

The WEC is equivalent to the condition \( \varepsilon_{\text{min}} > 0 \) on top of the NEC (11). These inequalities can be also written in the form of the following two options

\[
\begin{align*}
\text{either} & \quad \varepsilon_u > q, \quad \text{and} \quad \varepsilon_u + P \geq 2q, \\
\text{or} & \quad q \geq \varepsilon_u > 0, \quad \text{and} \quad \varepsilon_u P \geq q^2.
\end{align*}
\]

The observer measuring \( \varepsilon_{\text{min}} \) is boosted against the energy flow \( q^\mu \) with the speed

\[
v_{\text{min}} = \frac{\varepsilon_u + P}{2q} - \sqrt{\left(\frac{\varepsilon_u + P}{2q}\right)^2 - 1}. \quad (15)
\]

The frame of this observer, \( V_{\mu} \propto u_{\mu} + v_{\text{min}} q_{\mu}/q \), is the Landau-Lifshitz frame. If NEC is violated there is no Landau-Lifshitz frame comoving with the energy (for a discussion of the choice of cosmological frames see e.g. [31]). The Dominant Energy Condition (DEC): assumes that the WEC holds and that the density of the fluid’s 4-momentum \( p_{\mu} = T_{\mu \nu} V^\nu \) is non-spatial: \( p_{\mu} P^\mu \geq 0 \) and future directed for all timelike and null future-directed vectors \( V^\mu \). At the end these conditions can be written as the WEC plus on top of that \( \varepsilon_u > q \) and either \( \varepsilon_u - P > q \)
\[ q \geq \varepsilon_u - P \geq -q, \quad \text{and} \quad P (\varepsilon_u - P) \geq \frac{1}{2} q^2. \]

In particular, from these conditions it follows that the DEC always implies that

\[ \varepsilon_u \geq \sqrt{2} q, \quad (16) \]

see Figure 1.

\section*{ACKNOWLEDGMENTS}

It is a pleasure to thank Asimina Arvanitaki, Fedor Bezrukov, Savas Dimopoulos, Valerio Marra, Slava Mukhanov, Subodh Patil, Leonardo Senatore, Glenn Starkman and Wessel Valkenburg for very useful and valuable discussions. IS is supported by the DFG through TRR33 “The Dark Universe”. The work of AV is supported by ERC grant BSMOXFORD no. 228169. IS would like to thank the CERN Theory Group for hospitality during the preparation of this paper.

[1] WMAP Collaboration, E. Komatsu et al., “Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation,” \textit{Astrophys. J. Suppl.} \textbf{192} (2011) 18, \texttt{arXiv:1001.4538 [astro-ph.CO]}.  

[2] Supernova Cosmology Project Collaboration, N. Suzuki et al., “The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints Above \( z > 1 \) and Building an Early-Type-Hosted Supernova Sample,” \textit{Astrophys. J.} \textbf{746} (2012) 85, \texttt{arXiv:1105.3470 [astro-ph.CO]}.  

[3] WMAP Collaboration, G. Hinshaw, D. Larson, E. Komatsu, D. Spergel, C. Bennett, \textit{et al.}, “Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results,” \texttt{arXiv:1212.5226 [astro-ph.CO]}.  

[4] Euclid Collaboration, L. Amendola \textit{et al.}, “Cosmology and fundamental physics with the Euclid satellite,” \texttt{arXiv:1206.1225 [astro-ph.CO]}.  

[5] R. Caldwell, “A Phantom menace?,” \textit{Phys. Lett.} \textbf{B545} (2002) 23–29, \texttt{arXiv:astro-ph/9908168 [astro-ph]}.  

[6] A. A. Starobinsky, “Future and origin of our universe: Modern view,” \textit{Grav. Cosmol.} \textbf{6} (2000) 157–163, \texttt{arXiv:astro-ph/9912054 [astro-ph]}.  

[7] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, “Phantom energy and cosmic doomsday,” \textit{Phys. Rev. Lett.} \textbf{91} (2003) 071301, \texttt{arXiv:astro-ph/0305206 [astro-ph]}.  

[8] S. Dubovsky, T. Gregoire, A. Nicolis, and R. Rattazzi, “Null energy condition and superluminal propagation,” \textit{JHEP} \textbf{0603} (2006) 025, \texttt{arXiv:hep-th/0512260 [hep-th]}.  

[9] R. Emparan and J. Garriga, “Non-perturbative materialization of ghosts,” \textit{JHEP} \textbf{0603} (2006) 028, \texttt{arXiv:hep-th/0512274 [hep-th]}.  

[10] J. Garriga and A. Vilenkin, “Living with ghosts in Lorentz invariant theories,” \texttt{arXiv:1202.1239 [hep-th]}.  

[11] A. Nicolis, R. Rattazzi, and E. Trincherini, “Energy’s and amplitudes’ positivity,” \textit{JHEP} \textbf{05} (2010) 095, \texttt{arXiv:0912.4258 [hep-th]}.  

[12] P. Creminelli, A. Nicolis, and E. Trincherini, “Galilean Genesis: An Alternative to Inflation,” \textit{JCAP} \textbf{1011} (2010) 021, \texttt{arXiv:1007.0027 [hep-th]}.  

[13] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, “Imperfect Dark Energy from Kinetic Gravity Braiding,” \textit{JCAP} \textbf{1010} (2010) 026, \texttt{arXiv:1008.0048 [hep-th]}.  

[14] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, “G-inflation: Inflation driven by the Galileon field,” \textit{Phys. Rev. Lett.} \textbf{105} (2010) 231302, \texttt{arXiv:1008.0603 [hep-th]}.  

[15] G. W. Horndeski, “Second-order scalar-tensor field equations in a four-dimensional space,” \textit{International Journal of Theoretical Physics} \textbf{10} (1974) 363–384.  

[16] C. Deffayet, X. Gao, D. Steer, and G. Zahariade, “From k-essence to generalised Galileons,” \textit{Phys. Rev.} \textbf{D84} (2011) 064039, \texttt{arXiv:1103.3260 [hep-th]}.  

[17] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, “Generalized G-inflation: Inflation with the most general second-order field equations,” \textit{Prog. Theor. Phys.} \textbf{126} (2011) 511–529, \texttt{arXiv:1105.5723 [hep-th]}.  

[18] A. Nicolis, R. Rattazzi, and E. Trincherini, “The galileon as a local modification of gravity,” \textit{Phys. Rev.} \textbf{D79} (2009) 064036, \texttt{arXiv:0811.2197 [hep-th]}.  

[19] C. Deffayet, G. Esposito-Farese, and A. Vikman, “Covariant Galileon,” \textit{Phys. Rev.} \textbf{D79} (2009) 084003, \texttt{arXiv:0901.1314 [hep-th]}.  

[20] C. Deffayet, S. Deser, and G. Esposito-Farese, “Generalized Galileons: All scalar models whose curved background extensions maintain second-order field equations and stress-tensors,” \textit{Phys. Rev.} \textbf{D80} (2009) 064015, \texttt{arXiv:0906.1967 [gr-qc]}.  

[21] C. de Rham and A. J. Tolley, “DBI and the Galileon reunited,” \textit{JCAP} \textbf{1005} (2010) 015, \texttt{arXiv:1003.5917 [hep-th]}.  

[22] T. Qiu, J. Evslin, Y.-F. Cai, M. Li, and X. Zhang, “Bouncing Galileon Cosmologies,” \textit{JCAP} \textbf{1110} (2011) 036, \texttt{arXiv:1108.0593 [hep-th]}.  

[23] D. A. Easson, I. Sawicki, and A. Vikman, “G-Bounce,” \textit{JCAP} \textbf{1111} (2011) 021, \texttt{arXiv:1109.1047 [hep-th]}.  

[24] \textbf{Pierre Auger} Collaboration, M. Settino, “Measurement of the Cosmic Ray Energy Spectrum Using Hybrid Events of the Pierre Auger Observatory,” \textit{Eur. Phys. J. Plus} \textbf{127} (2012) 87, \texttt{arXiv:1208.6574 [astro-ph.HE]}.  

[25] IceCube Collaboration, R. Abbasi \textit{et al.}, “Measurement of the atmospheric neutrino energy spectrum from 100 GeV to 400 TeV with IceCube,” \textit{Phys. Rev.} \textbf{D83} (2011) 012001, \texttt{arXiv:1010.3980 [astro-ph.HE]}.  

[26] O. Pujolas, I. Sawicki, and A. Vikman, “The Imperfect Fluid behind Kinetic Gravity Braiding,” \textit{JHEP} \textbf{1111} (2011) 156, \texttt{arXiv:1103.5360 [hep-th]}. 

or

\[ q \geq \varepsilon_u - P \geq -q \quad \text{and} \quad P (\varepsilon_u - P) \geq \frac{1}{2} q^2. \]
[27] A. Barreira, B. Li, C. M. Baugh, and S. Pascoli, “Linear perturbations in Galileon gravity models,” Phys.Rev. D86 (2012) 124016, arXiv:1208.0600 [astro-ph.CO].
[28] V. Mukhanov, Physical foundations of cosmology. Cambridge University Press, 2005.
[29] C. A. Kolassis, N. O. Santos, and D. Tsoubelis, “Energy conditions for an imperfect fluid,” Classical and Quantum Gravity 5 (1988) 1329.
[30] S. M. Carroll, M. Hoffman, and M. Trodden, “Can the dark energy equation - of - state parameter w be less than -1?,” Phys.Rev. D68 (2003) 023509, arXiv:astro-ph/0301273 [astro-ph].
[31] I. Sawicki, I. D. Saltas, L. Amendola, and M. Kunz, “Consistent perturbations in an imperfect fluid,” JCAP 1301 (2013) 004, arXiv:1208.4855 [astro-ph.CO].