Probing mirror anomaly and classes of Dirac semimetals with circular dichroism

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We theoretically investigate the optical activity of three dimensional Dirac semimetals (DSMs) using circular dichroism (CD). We show that DSMs in the presence of a magnetic field in any one of the mirror-symmetric planes of the materials exhibit a notable dichroic behavior. In particular, for different orientations of the light field with respect to the mirror-symmetric plane, the CD in type-II DSMs can detect the presence of mirror anomaly by showing sharply distinct patterns at the mirror-symmetric angle. Interestingly, CD can also distinguish type-II DSMs having only one Dirac point at a time-reversal invariant momentum from type-I DSMs with a pair of Dirac points on the rotation axis of the crystals.

I. INTRODUCTION

Nearing a decade of their theoretical and experimental advances, the three dimensional (3D) Dirac semimetals (DSMs) continue to evolve as promising materials for studying several unusual phenomena in condensed matter physics such as large magnetoresistance, giant diamagnetism, oscillating quantum spin Hall effect, etc. Additionally, they can host topological insulators and Weyl metals in the presence and/or absence of certain symmetries such as time-reversal (TR) symmetry, inversion symmetry (IS), and crystalline uniaxial rotational symmetry. Based on these symmetries, DSMs can further be classified as type-I and type-II. In type-I DSMs, Dirac points generally appear in pairs on the rotation axis and they are protected by crystalline symmetry. In contrast, type-II DSMs contain a single Dirac point at the time-reversal-invariant momentum (TRIM) on the rotation axis. Na$_8$Bi$_{20–22}$ and Cd$_3$As$_2$$_5$,$13$,$18$,$23$,$24$ compounds are found to show type-I behavior, while TBI$(S_{1–x}Se_x)_2$$_5$,$21$, (Bi$_{1–x}$In$_x)_2$Se$_3$$_5$,$16$, and ZrTe$_5$$_7$,$28$ are confirmed to be type-II Dirac semimetals.

Although these two types of DSMs differ in underlying symmetries, apparently there are no as such observable properties that are significantly different in these two class of DSMs. Since both of them possess chiral anomaly, it is expected to observe negative longitudinal magnetoresistance or planar Hall effect in both of these DSMs in the presence of electric and magnetic fields. In addition to the chiral anomaly, DSMs have been shown to possess mirror anomaly which is manifested in a step-like anomalous Hall conductivity (AHC) as a function of polar angle of the applied magnetic field perpendicular to the mirror-symmetric plane, resembling the AHC due to the parity anomaly in 2D systems with massive Dirac fermions. However, the step-like behavior is found to be broadened in the presence of an additional perturbation that breaks mirror symmetry. Accordingly, the Hall measurement can be thought of as a key observable to infer the presence or absence of mirror anomaly in type-II DSMs. However, such measurements are always limited by impurities or it may have contribution from orbital effect, which in turn may lead to the deviation from the step-like behavior. Thus the anomalous Hall measurement may not be an ideal probe to discriminate between type-II DSMs with or without mirror symmetry. In addition, very recently, it has been shown that this step-like feature is not generic to all DSMs. In type-I Dirac semimetals, the step-like AHC is smoothen out with the polar angle across the mirror-symmetric plane even in the presence of mirror symmetry. Therefore the types of DSMs discussed herein may also not be distinguished experimentally with the help of Hall measurement. Hence, it is desirable to find easily accessible diagnostic tools to discriminate DSMs with or without mirror symmetry as well as their types. This may eventually help us understanding the optical responses of DSMs with different underlying symmetries and their potential applications in opto-electronic devices.

In view of that, we theoretically study the optical activity of these two types of DSMs via circular dichorism (CD) which deals with differential absorption of left- and right-circularly polarized light of materials under consideration. It has already been shown that the chiral anomaly of Weyl semimetals with pairs of Weyl nodes with opposite chiralities can be probed by the CD. In contrast, the CD provides vanishing result for pristine DSMs since the Weyl nodes with opposite chiralities coincide in energy and momentum space. The reason for the null results can further be understood from the connection between the CD and Berry curvature. The momentum resolved circular dichorism is found to be proportional to the component of the Berry curvature along the direction of incident light. Thus for gapless DSMs, the Berry curvature is zero everywhere in momentum space except at the gapless point. This leads to the null CD. However, DSMs under symmetry breaking may give rise to finite Berry curvature, hence nonzero CD. To verify this, we apply magnetic field along any one of the mirror-symmetric planes of the DSMs. With this, we find that the optical responses of both types of DSMs depend on the orientation of the light field. In particular, the CD due to the light field along $x$ direction differs from the CD with the light field along $y$ direction for both types of DSMs. Moreover, we find that the presence or absence of mirror symmetry in type-II DSMs is well manifested in the CD, specially at the mirror-symmetric angle. We furthermore show that there is a sharp contrast between the CD in type-I and type-II DSMs. When the light field is incident along the...
x direction, at the mirror-symmetric angle, the CD diverges at
the zone center in type-II DSMs, whereas it is found to be
zero in type-I DSMs. Additionally, the CD is obtained to be
an odd function of each component of momentum for type-
II, but it differs for type-I. Indeed, these signatures are key
to identify different types of DSMs with or without mirror sym-
metry, which in turn may give us indirect hint on if a DSM
can possess mirror anomaly.

The rest of the paper is organized as follows. In Sec. II
we provide a general framework for the CD, relating it to the
Berry curvature. In Sec. III, we discuss the Berry curvature
and corresponding CD in type-II DSMs for different orien-
tations of the light field. We compare results between the
cases with and without mirror symmetry. This is followed by
Sec. III where we only focus on the type-I DSMs with mirror symmetry and compare the results for two different directions
of the applied light field. We also compare the results with
the type-II DSMs. We conclude in Sec. V with a discussion on
possible applications.

II. GENERAL THEORY: OPTICAL TRANSITION

We begin by reviewing light-induced optical activity in a
two-band model, particularly focusing on the Berry-curvature
dependent interband transition probabilities. The light field
couples to the electron of a material in two ways: a) orbital
coupling where momentum \( k \) is replaced by \( k - eA \) due to
minimal coupling and b) Zeeman coupling via \( \nabla \times A \), where
\( A \) is the vector potential due to the light field. Thus the total
Hamiltonian of light-electron interaction reads

\[
\mathcal{H}_{te} = -e \mathbf{v} \cdot \mathbf{A} + g_e \mu_B (\nabla \times \mathbf{A}) \cdot \sigma,
\]

where \( e \) is the charge of electron, \( m_e \) is the mass of the
electron, \( g_e \) is the Lande-g factor, \( \mu_B \) is the Bohr magneton and
\( \mathbf{v} \) is the vector potential due to the light field. The total
Hamiltonian \( \mathcal{H}_{te} \) is

\[
\mathcal{H}_{te}^{orb \pm} = -e A_0 (v_k^x \pm i v_k^y).
\]

Accordingly, for a two-band model, the optical transition matrix is defined as

\[
\mathcal{P}_\pm(k) = \mathcal{P}_x \pm i \mathcal{P}_y = (c|H_{te}^{orb \pm}|v),
\]

where \(|c\rangle\) and \(|v\rangle\) are conduction and valence bands, respectively. Then the \( k \)-resolved circular dichroism is given by

\[
\eta(k) = \frac{|\mathcal{P}_+(k)|^2 - |\mathcal{P}_-(k)|^2}{|\mathcal{P}_+(k)|^2 + |\mathcal{P}_-(k)|^2}.
\]

The numerator of \( \eta \) can further be expressed in terms of
the valence band Berry curvature (\( \Omega \)) of a two-band model as follows:

\[
|\mathcal{P}_+(k)|^2 - |\mathcal{P}_-(k)|^2 = 2i(\mathcal{P}_x^* \mathcal{P}_y - \mathcal{P}_y^* \mathcal{P}_x)
\]

\[
= 2\frac{e^2 A_0^2}{\hbar^2} (\langle v | \partial \mathcal{H} / \partial k_x | c \rangle \langle c | \partial \mathcal{H} / \partial k_y | v \rangle - \langle c | \partial \mathcal{H} / \partial k_y | v \rangle \langle v | \partial \mathcal{H} / \partial k_x | c \rangle)
\]

\[
= \frac{2e^2 A_0^2 (\epsilon_e - \epsilon_v)^2 \Omega \cdot \hat{n}}{\hbar^2},
\]

For an arbitrary direction \( \hat{n} \) of the incident light, we can
generalize \( \eta(k) \) by rotating the coordinate frame accordingly to obtain

\[
\eta(k) = \frac{e^2 A_0^2 (\epsilon_e - \epsilon_v)^2 \Omega \cdot \hat{n}}{\hbar^2},
\]

where \( \mathcal{P} = |\mathcal{P}_+(k^0)|^2 + |\mathcal{P}_-(k^0)|^2 \) and \( k^0 = R k \) with \( R \)
being the usual rotation matrix.

We next aim to investigate \( \eta \) in type-I and type-II Dirac
semimetals in the presence of symmetry breaking, considering
lowest conductive and highest valence bands. The simple
form of the low-energy Hamiltonian of type-II DSMs allows
us to find analytical expressions for the CD for some specific
directions, while type-I is needed to be solved numerically due
to complexities in the structure of the Hamiltonian. We there-
fore first discuss the CD for type-II followed by the same for

III. CIRCULAR DICHORISM IN TYPE-II DIRAC SEMIMETAL

The linearized low energy model Hamiltonian for a type-II
DSM is given by

\[
\mathcal{H}_0(k) = v (k_x \tau_x \sigma_y + k_y \tau_y \sigma_x + k_z \tau_z),
\]

where \( v \) is the quasiparticle velocity, \( \sigma \)'s and \( \tau \)'s are the Pauli
matrices for spin and orbital, respectively. Note that, we work
here with \( \hbar = 1 \). This Hamiltonian in Eq. (6) has only one
Dirac node at the time-reversal invariant momentum (TRIM)
\( \Gamma \). Eq. (6) commutes with the Mirror symmetry operator
\( M = i \sigma_x \hat{K} \) at \( k_x = 0 \) plane:

\[
[H_0(k_x = 0), M] = 0.
\]

Thus the Hamiltonian possesses mirror symmetry in \( yz \) plane.
The same is true for \( xz \) plane as well. In addition, the Hamil-
tonian in Eq. (6) is invariant under time reversal \( (T = i \sigma_y \hat{K}) \),
where \( \hat{K} \) is complex conjugation operator and inversion \( (I =
\sigma_x) \) symmetry. Consequently, we obtain vanishing Berry cur-
vature which in turn may give rise to null CD (cf. Eq. 5).
To have non-zero Berry curvature, we consider an external magnetic field in one of the mirror-symmetric planes (say, $xz$). This gives Zeeman term as

$$\mathcal{H}_{2mn}^{\text{II}}(\theta) = b \cos(\theta)\sigma_z + b \sin(\theta)\sigma_x,$$

where $b$ is the strength of the field and $\theta$ is the angle between $z$-axis and the direction of the magnetic field. Eq. (8) preserves mirror symmetry but breaks tetrahedral symmetry which splits the single Dirac node into two Weyl nodes with opposite chiralities on the $z$ axis at $k_z = \pm b/v$ for all values of $\theta$ except at $\theta = \pi/2$. At this value, the valence and conduction bands touch each other along a nodal line in the $yz$ plane, satisfying $k_y^2 + k_z^2 = b^2/v^2$. This nodal line is found to be protected by the mirror symmetry with respect to the reflections along the $yz$ plane. Note that $\theta = \pi/2$ is a critical angle, namely mirror-symmetric angle where the chiralities of the Weyl nodes on the $z$ axis interchange sign. It turns out that the emergent mirror symmetry at this critical angle is manifested in the CD as will be discussed shortly.

Since the momentum resolved CD is proportional to the Berry curvature, it is instructive to find an analytical expression of it to have an intuitive understanding on the CD. A block diagonal form of the the mirror-symmetric Hamiltonian $\mathcal{H}^{\text{II}} + \mathcal{H}_{2mn}^{\text{II}}$ may help finding such an expression of the Berry curvature. It turns out that Eq. (6) can be block-diagonalized by rotating the $z$-axis along $\theta$ followed by similarity transformations $\sigma_x \rightarrow \tau_z\sigma_x$ and $\tau_x \rightarrow \tau_x\sigma_z$. With this, we obtain

$$\mathcal{H}_{2\times2} = v k_y \cos \theta \sigma_x - v k_x \sigma_y + p_\pm \sigma_z,$$

where, $p_\pm = b \pm v \sqrt{k_y^2 \sin^2 \theta + k_z^2}$. The $+$ and $-$ signs correspond to two blocks of the block-diagonalized Hamiltonian. We focus on the block, associated with the $-$ sign, that contains the lowest conduction and highest valence bands with energies

$$\epsilon_{\pm,k} = \pm \epsilon_k = \pm \sqrt{v^2k_z^2 + v^2k_y^2 \cos^2 \theta + p^2_\pm}.$$  

The corresponding valence band Berry curvature is found to be

$$\Omega(k) =$$

$$\left\{ \begin{array}{c}
\frac{k_x k_z v^3 \cos(\theta)}{2\epsilon_k \sqrt{k_y^2 \sin^2(\theta) + k_z^2} + k_z} - \frac{k_y k_z v^3 \cos(\theta)}{2k_z \sqrt{k_y^2 \sin^2(\theta) + k_z^2}} \\
\frac{v^2 \cos(\theta)}{2\epsilon_k} \left( b - \frac{k_y^2 v}{\sqrt{k_y^2 \sin^2(\theta) + k_z^2}} \right) \end{array} \right\}$$

(11)

Note that all three components of the Berry curvature vanish at the mirror-symmetric angle $\theta = \pi/2$. This fact is expected to be reflected in the CD.

To compare the CD with the case without mirror symmetry, we add a cubic term $\mathcal{H}'(\lambda) = \frac{2}{3}(k_x^2 - k_z^2)k_y\tau_z\sigma_z$ to $\mathcal{H}^{\text{II}} + \mathcal{H}_{2mn}^{\text{II}}$. Although the positions of the Weyl points along $k_z$ remain unchanged, instead of $\theta = \pi/2$, they now interchange their chiralities at a critical angle $\theta_c = \cot^{-1}(\delta)$, where $\delta = \lambda b^2/2v^3$. In addition to these two Weyl points, four additional Weyl points at $(b_y, k_z) = (\pm b \cos(\theta)/\delta, \pm b \sqrt{1 - \cos^2(\theta)/\delta})$ emerge in the $yz$-plane for $\theta > \theta_c$. These extra Weyl points move slowly towards the $y$-axis and they annihilate with each other at $\theta = \pi/2$. The original two Weyl points retain with their interchanged chiralities.

The Hamiltonian $\mathcal{H}^{\text{II}} + \mathcal{H}_{2mn}^{\text{II}} + \mathcal{H}'(\lambda)$ can be written in a block-diagonalized form using similarity transformations mentioned earlier:

$$\mathcal{H}_{2\times2}^{\lambda} = [vk_y \cos \theta - k_y(k_z^2 - k_x^2)/2]\sigma_x - v k_x \sigma_y + q_\pm \sigma_z,$$

(12)

where $q_\pm = b \pm v \sqrt{v^2k_z^2 + [v \sin \theta + \cos \theta(k_z^2 - k_x^2)/2]k_y^2}$. Diagonalizing Eq. (12), we obtain the energies for lowest conduction and highest valence bands as

$$\epsilon_{\pm,k,\lambda} = \pm \sqrt{v^2k_z^2 + [v \cos \theta - \sin \theta(k_z^2 - k_x^2)/2]k_y^2 + q_\pm^2}.$$

Subsequently, one can easily compute all the three components of the Berry curvature which turns out to be very lengthy to present herein. However, for the sake of further discussions on the CD, we only show two components for two specific limits:

$$\Omega_x(k_y = 0) = -\frac{k_xk_z v^3 \cos \theta + \frac{1}{2}(k_z^2 - k_x^2) \sin \theta}{2|k_y|\epsilon_{k,\lambda}}$$

(13)

$$\Omega_y(k_x = 0) = -\frac{k_y k_z v}{4\epsilon_{k,\lambda} (b + q_-)} [v (2v^2 + k_y^2k_z^2/\lambda^2) \cos \theta + (2bq_+ - 2b^2 + (2k_y^2 + k_z^2)v^2)\lambda \sin \theta]$$

(14)

Note that both of these cases exhibit finite Berry curvature at $\theta = \pi/2$ in contrary to the mirror-symmetric case. Hence, these features are expected to be reflected in the CD.

We next turn to analyze circular dichroism for two distinct cases: (i) incoming light in the plane of the applied magnetic field (along $x$-axis) and (ii) incoming light perpendicular to the plane of the applied magnetic field (along $y$-axis).

### A. Light parallel to the plane of magnetic field

Having discussed the Berry curvature for type-II DSMs with and without mirror symmetry, we now investigate $\eta(k)$. For light field along $x$-direction, only $x$-component of $\Omega(k)$ contributes to $\eta(k)$ as evident from Eq. (5). Since $\Omega_x(k)$ is trivially zero for both the $k_y = 0$ and $k_z = 0$ planes, we concentrate on the $k_y = 0$ plane. Together with Eq. (5) and Eq. (11), we obtain

$$\eta^x(k_x, k_z) = \frac{2 \epsilon_k k_x k_z v \cos \theta}{|k_z| (\epsilon_k (1 + \cos^2 \theta) - p_z^2)}.$$

(15)
The upper row of the Fig. (1) shows contour plot of \( \eta^z \) in the \( xz \) plane for various \( \theta \) in the case of a mirror-symmetric type-II DSM. For \( \theta \ll \pi/2 \), \( \eta^z \rightarrow \pm 1 \) except at or in the vicinity of \((k_x, k_z) \equiv 0\) (cf. Eq. (10)). \( \eta^z \) starts to deviate as \( \theta \) approaches \( \pi/2 \). At \( \theta = \pi/2 \), \( \eta^z \) diverges at \((k_x, k_z) \equiv (0, 0)\) although Berry curvature itself vanishes everywhere in the \((k_x-k_z)\) plane except at the two Weyl points in this plane. This is due to the fact that the denominator in Eq. (15) goes as \( k_x^2 \) at \( \theta = \pi/2 \), hence \( \eta^z \sim 1/k_x \). Notice that \( \eta^z \) reverses its pattern as \( \theta \) crosses \( \pi/2 \), corroborating the switching of topological charge of Weyl points across the mirror-symmetric angle.

To substantiate the behavior of \( \eta^z \) in Fig. (1), we show the variation of \( \eta^z \) with \( \theta \) in Fig (2) at two different limits \((k_x, k_z) \rightarrow 0 \) and \((k_x, k_z) \rightarrow 1 \). Clearly, in the vicinity of \((k_x, k_z) \rightarrow 0 \), \( \eta^z \) is found to be negligibly small for all \( \theta \) except at \( \theta = \pi/2 \) where it diverges. On the other hand, \( \eta^z \sim 1 \) for large \((k_x, k_z)\) for almost all \( \theta \) except at \( \theta = \pi/2 \) where it vanishes. The reason for this behavior can be easily traced back to the \( \theta \) dependent numerator i. e., Berry curvature and the denominator (\( P_t \)) of \( \eta^z \) (cf. Eq. 5). For \((k_x, k_z) \rightarrow 0 \), \( \Omega \) goes as \( \cos(\theta) \), whereas \( P_t \) goes as \( \cos^2 \theta \), hence \( \eta^z(x) \sim \frac{1}{\cos \theta} \).

In the limit of \((k_x, k_z) \rightarrow 1 \), \( \Omega \) goes as \( \cos(\theta) \) as before but \( P_t \) goes as \( 1 + \cos^2 \theta \), which in turn leads to \( \eta \sim \frac{\cos(\theta)}{1+\cos(\theta)} \).

The lower row of the Fig. 1 illustrates the behavior of \( \eta^z \) when mirror symmetry is broken. For smaller \( \theta \), the pattern of \( \eta^z \) is obtained to be similar to that of upper row of the Fig. 1. This can be understood easily from the Berry curvature as shown in Eq. 13. The symmetry breaking term involving \( \lambda \) contains \( \sin(\theta) \), hence it does not modify \( \Omega_{zz} \) significantly for \( \theta \ll \pi/2 \). As \( \theta \) increases, it starts contributing to \( \Omega_{zz} \), consequently we find different pattern in \( \eta^z \). We note that \( \eta^z \) diverges in the limit \((k_x, k_z) \rightarrow 0 \) as before, but it is finite for non-zero \((k_x, k_z)\) at the mirror-symmetric angle. This is in contrast to the case which preserves mirror symmetry. Indeed, this fact can be used to identify whether a type-II DSM can host the mirror anomaly.

**B. Light perpendicular to the plane of magnetic field**

For incident light along \( y \)-direction and for mirror-symmetric DSMs, we find \( \eta^y \) in \( k_x = 0 \) plane as

\[
\eta^y(k_y, k_z) = \frac{2\epsilon_k k_z v \cos(\theta)}{\sqrt{k_y^2 \sin^2(\theta) + k_z^2 \left( \epsilon_k^2 + \frac{k_y^2 v^2 (\epsilon_k^2 - p_-^2)}{(b - p_-)^2} \right)}}
\]

(16)
The circular dichroism $\eta^y$ as a function of $\theta$ for both the cases with and without mirror symmetry is depicted in Fig. 3. For any finite $\theta \ll \pi/2$, $\eta^y$ with mirror symmetry is found to be nearly zero in the $k_y$-$k_z$ plane, satisfying $k_y^2 + k_z^2 \ll b^2$ and on the other hand it is maximum above the line $k_y^2 + k_z^2 \geq b^2$ as evident from upper row of the Fig. 3. For the same range of $\theta$, the pattern differs from $\eta^x$. This is also apparent from their corresponding equations (16) and (15). Notably, the crucial difference between $\eta^y$ and $\eta^x$ is found at the mirror-symmetric angle $\theta = \pi/2$. At $\theta = \pi/2$, $\eta^y$ is zero everywhere in the $(k_y, k_z)$ plane except at the nodal line, reflecting the vanishing Berry curvature at $\theta = \pi/2$. This in contrast to the $\eta^x$ which diverges due to the denominator although Berry curvature individually vanishes at this specific angle.

For the case without mirror symmetry, $\eta^y$ shows similar pattern for $\theta \ll \pi/2$. However, as $\theta$ increases a complete contrasting feature is observed. Indeed this is due to the presence of six Weyl points for $\theta > \theta_c$. For illustration we have indicated the location of Weyl points. At $\theta = \pi/2$, we find finite $\eta$ for some specific values of momentum in contrary to the case with mirror symmetry where $\eta^y$ is zero for any finite momentum except those on the nodal line. Thus, the effect of broken mirror symmetry is well manifested in $\eta^y$, specifically at $\theta = \pi/2$ (cf. Fig. 3). To justify this distinction, we also plot $\eta^y$ as a function of $\theta$ in Fig. (4) for $k_y, k_z \neq 0$. Note that this result also differs from $\eta^x$ without mirror symmetry, where it diverges at the zone center.

To close this section, we comment that in all the cases, the patterns of $\eta$ turn out to be some reflection symmetric about all the 3-axes $k_x$, $k_y$, and $k_z$ with polarities inverted. It is similar to a reflection in a plane followed by a rotation about an axis perpendicular to that plane, also known as rotoreflection. In other words, $\eta$ turns out to be an odd function of each component of $k$.

IV. CIRCULAR DICHRISM IN TYPE-I DIRAC SEMIMETAL

We now investigate the CD in type-I DSM to see if there is any distinction between these two types of DSMs. Since type-
I DSMs do not process any mirror anomaly\textsuperscript{30} in the anomalous Hall conductance as mentioned before, we only concentrate on the case with mirror symmetry. The low energy effective Hamiltonian for this type of DSMs near $\Gamma$ point is given by\textsuperscript{34-36},

\[ \mathcal{H}^I(k) = M(k)\tau_z\sigma_0 + A(k)\tau_x\sigma_z + B(k)\tau_y\sigma_0 + C(k)\tau_x\sigma_x + D(k)\tau_x\sigma_y, \]  \hspace{1cm} (17)

where, $M(k) = M_0 - M_1(k_x^2 + k_y^2) - M_z k_z^2$ is the momentum dependent mass parameter, $A(k) = A_1 k_x$, $B(k) = -A_1 k_y$, $C(k) = (\beta + \gamma)k_z(k_y^2 - k_x^2)$ and $D(k) = -2(\beta - \gamma)k_x k_y k_z$. The band parameters $M_0, M_1, M_z, A_1, \beta$ and $\gamma$ are material dependents and can be obtained from first-principles calculations. The degenerate Dirac points occur at $(0,0,\pm\sqrt{\frac{M_0}{M_z}})$. Although the Hamiltonian in Eq. 17 has several mirror-symmetric planes such as $(001), (100)$ and $(110)$\textsuperscript{34}, we only focus on the $k_y = 0$ plane for simplicity. As before, we consider a magnetic field in the $k_y = 0$ plane as

\[ \mathcal{H}^I_{\text{spin}}(\theta) = b \cos(\theta)\tau_0\sigma_z + b \sin(\theta)\frac{(\tau_0 + \tau_z)}{2}\sigma_z. \]  \hspace{1cm} (18)

Eq. (18) breaks time-reversal symmetry, which results in two pairs of Weyl points at $(0,0,\pm\sqrt{\frac{M_0 B_z}{M_z}})$, where $B_z$ is the $z$-component of the applied field.

For finite $\theta$, the Hamiltonian $\mathcal{H}^I(k) + \mathcal{H}^I_{\text{spin}}(\theta)$ cannot be expressed in block diagonalized form by the simple similarity transformations used for type-II DSMs. Thus we resort to the Eq. (4) to calculate the CD. In Fig. 5, we compare $\eta$ along two different directions, $x$ and $y$, of the incident light. As before, the pattern in $\eta^x$ and $\eta^y$ differs from each other. However, this distinction becomes sharp at $\theta = \pi/2$ where $\eta^y$ is found to be zero everywhere except at the Weyl points on the $k_y$-$k_z$ plane. This is in contrast to the $\eta^x$ which is finite except along two semi-circular lines in the $k_x$-$k_z$ plane.

FIG. 5. Contour plot of circular dichirism, $\eta(k)$ in $xz$ plane and $yz$ plane for type-I DSMs with mirror symmetry considering the incident light along the $x$ direction ($\eta^x$ upper row) and $y$ direction ($\eta^y$ lower row) respectively. The applied magnetic field is rotated in the $xz$ plane. $\eta^x$ and $\eta^y$ differ from each other as clearly shown in figure. This distinction becomes sharp at mirror-symmetric angle $\theta = \pi/2$ where $\eta^y$ is found to be zero everywhere in the ($k_x$-$k_z$) plane in contrast to the $\eta^x$ which is finite except along two semi-circular lines in the $k_x$-$k_z$ plane. The Weyl nodes are marked with black dots. Here we use $A_1/b = 10$ in units of $\mathcal{A}$.

V. CONCLUSION

In conclusion, the DSMs can be classified into type-I and type-II based on their implicit symmetries such as TRS, IS and crystalline uniaxial rotational symmetry. Although type-I and type-II DSMs differ in underlying symmetries, there are no as such observable properties that are significantly different in these two classes of DSMs. Moreover, it has been argued that unlike type-I DSMs, the type-II DSMs in the presence of mirror symmetry can possess a novel anomaly, namely the mirror anomaly in addition to the celebrated chiral anomaly. Although it has been proposed that the anomalous Hall conductivity is one of the observable consequences to detect the presence of mirror anomaly, there is no concrete experimental evidence yet. Specifically, an easy reliable probe to distinguish between type-I and type-II DSMs based on their sym-


metries as well as the presence/absence of mirror anomaly in DSMs is still lacking.

To answer this, we study momentum-resolved circular dichroism $\eta(k)$ in these two types of DSMs in the presence of a rotating magnetic field applied in any one of the mirror-symmetric planes. Interestingly, we find that $\eta(k)$ shows significantly different patterns in both types of DSMs for the light field parallel or perpendicular to the plane of the applied magnetic field. For a specific direction of the incident light and at the mirror-symmetric angle, the CD pattern in type-II DSMs with mirror symmetry is found to show a distinct feature than the case without mirror symmetry. Therefore, the presence or absence of mirror symmetry in type-II DSMs is well manifested in the CD, specially at the mirror-symmetric angle which may help finding type-II DSMs that host step-like Hall conductance as a function of magnetic field, in particular, the mirror anomaly. In addition, we find that for the light field incident along the $x$-direction, the CD at the mirror-symmetric angle diverges at the zone center in type-II DSMs, whereas it vanishes in type-I. Moreover, in type-II DSMs, the CD is obtained to be an odd function of individual components of momentum, whereas it differs in type-I. Therefore, these signatures which are experimentally verifiable can be used to distinguish the different class of DSMs. Since type-II DSMs with mirror symmetry can have the transistor-like action, our study may contribute to potential technological applications by detecting mirror anomaly along with the application in opto-electronic devices.

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