Spatial curvature sensitivity to local $H_0$ from the Cepheid distance ladder

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Abstract

Over the last few years, low- and high-redshift observations set off a tension in the measurement of the present-day expansion rate, $H_0$. Adding to the riddle, observational data from the Planck mission point to a $3.4\sigma$ evidence for a closed universe, further challenging the $\Lambda$CDM concordance model of cosmology. Recently, a direct-observational test has been proposed to discriminate effects of the spatial curvature in the cosmological model. The test is based on the fundamental distance–flux–redshift relation of the luminosity distance modulus, $\Delta \mu$. We reexamine the outcomes of this test and show that achieving the required $\Delta \mu$ sensitivity to discriminate among cosmological models is materially far more challenging than previously thought. Armed with supernova type Ia (SN Ia) data, calibrated using Cepheid measured distances, we apply the test to archetypal spatially non-flat models that ameliorate the $H_0$ tension and show that the $3\sigma$ contour of $\Delta \mu$ predicted by these models overlaps the $68\%$ CL SN Ia residuals with respect to $\Lambda$CDM. This implies that the spatial curvature remains insensitive to local $H_0$ measurements from the Cepheid distance ladder.

Keywords: cosmological parameters – distance scale – dark energy – dark matter.

1. Introduction

Almost a century after the expansion of the Universe was established (Hubble 1929), the Hubble constant, which measures its rate ($H_0 \equiv 100 h \text{ km/s/Mpc}$), continues to encounter challenging shortcomings. According to the latest observations, the measured expansion rate (Riess et al. 2020a) is about $9\%$ faster than predicted by observations of the cosmic microwave background (CMB) on the basis of the spatially flat $\Lambda$ cold dark matter (CDM) cosmological model (Aghanim et al. 2020a). The statistical significance of this discrepancy is about $4.4\sigma$, which gives rise to the so-called $H_0$ tension (Di Valentino et al. 2021a; Shah et al. 2021).

A plethora of models extending $\Lambda$CDM have been proposed to ameliorate the $H_0$ tension; see e.g. (Di Valentino et al. 2021b) for a recent review. These models either reduce the size of the sound horizon at recombination, modifying the expansion rate in the early-universe, or else shift the matter–dark energy equality to earlier times than it otherwise would in $\Lambda$CDM with new physics in the post-recombination universe. Then, to keep the locations of the peaks in the CMB angular power spectrum fixed, $H_0$ increases, diminishing the tension. A particular class of models of interest herein are those in which the background geometry is not spatially flat. These models are motivated by observational data from the Planck mission (Aghanim et al. 2020a), which point to a $3.4\sigma$ evidence for a closed universe (Di Valentino et al. 2019; Handley 2019). Recently, a direct-observational test has been proposed to discriminate effects of the spatial curvature in the cosmological model (Shirokov et al. 2020). The test pivots on variations of the luminosity distance modulus, $\Delta \mu$. In this work we reexamine the outcomes of this test and show that achieving the required $\Delta \mu$ sensitivity to discriminate among cosmological models is more complex than previously thought.

2. Relative luminosity distance as a discriminator of space-curvature

The classical distance-ladder approach to measure $H_0$ combines Cepheid period-luminosity relations with absolute-distance measurements to local anchors so as to calibrate distances to supernova type Ia (SN Ia) host galaxies in the Hubble flow. For each (homogeneous and isotropic) cosmological model, a key parameter of local $H_0$ measurements is the predicted distance modulus, which is given by

$$\mu(z; \theta) = 5 \log_{10} \left( \frac{D_L}{10 \text{ Mpc}} \right) + 25,$$

(1)

where $\theta$ are the cosmological parameters,

$$D_L(z) = \frac{c}{H_0} \frac{1 + z}{\sqrt{\Omega_k}} \sinh \left( \sqrt{\Omega_k} \int_0^z \frac{dz'}{h(z')} \right),$$

(2)

is the luminosity distance, $\Omega_k$ is the present-day value of the curvature density normalized to the critical density, $h(z) = H(z)/H_0$ is the normalized Hubble parameter, and
where \( \sin(x) = \sin(x) \), \( x \), \( \sinh(x) \) for closed (\( \Omega_k < 0 \)), flat (\( \Omega_k = 0 \)), and open (\( \Omega_k > 0 \)) universes (Diwan et al. 2020). For low-redshift probes, the contribution of the radiation density parameter \( \Omega_r \) to \( h(z) \) can be safely neglected, and thus

\[
h(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_D(1+z)^{3(1+w)} + \Omega_k(1+z)^2},
\]

where \( \Omega_m \) is the present day value of the nonrelativistic matter density and \( \Omega_D \) the corresponding dark energy density parameter, with \( \Omega_D + \Omega_m + \Omega_k + \Omega_r = 1 \). The scaling of \( \Omega_D \) is characterized by the “equation-of-state” parameter \( w \equiv p_D/\rho_D \), the ratio of the spatially homogeneous dark energy pressure to its energy density.

In the spirit of (Shirokov et al. 2020), we define the relative luminosity distance, \( D_{\text{rel}} \), as the ratio of the luminosity distance in a given cosmological model to the luminosity distance in the fixed spatially flat ΛCDM model,

\[
D_{\text{rel}}(z) = \frac{D_L(z)}{D_{\Lambda\text{CDM}}(z)} = F(z; \Omega_k, \Omega_m, w),
\]

which is independent of \( H_0 \). To discriminate models featuring \( \Omega_k \neq 0 \) from \( \Omega_k = 0 \), we adopt the relative luminosity distance modulus, given by the difference between a given model of luminosity distance modulus and the spatially flat ΛCDM luminosity distance modulus,

\[
\Delta m(z; \Omega_k, \Omega_m, w) = 5 \log_{10} [F(z; \Omega_k, \Omega_m, w)].
\]

We specify the range of cosmological parameters to scan over using results of Markov chain Monte Carlo analyses, which confront the growth of perturbations and of CMB fluctuations with experimental data. These analyses, which have been carried out elsewhere (Di Valentino et al. 2021c Anchordoqui et al. 2021), use the publicly available Boltzmann solver CAMB (Lewis et al. 2000) in combination with the sampler CosmoMC (Lewis et al. 2002 Lewis 2013), and the following data sets: (i) the CMB temperature and polarization angular power spectra \( p_{TT+EE}+lowl+lowE \) from the Planck 2018 legacy release (Aghanim et al. 2020a); (ii) measurements of baryon acoustic oscillations (BAO) from different galaxy surveys (6dFGS (Beutler et al. 2011), SDSS-MGS (Ross et al. 2015), and BOSS DR12 (Alam et al. 2017)); (iii) the 1048 SN type Ia data points of the Pantheon sample distributed in the redshift interval \( 0.01 < z < 2.3 \) (Scolnic et al. 2018); (iv) a gaussian prior on the Hubble constant in agreement with measurements obtained by the SH0ES Collaboration (Riess et al. 2019, 2021).

We consider two representative cosmological models in which particular combinations of data samples provide evidence for \( \Omega_k \neq 0 \), with a statistical significance larger than 3σ and relax the \( H_0 \) tension:

- A ΛCDM extension with three extra free parameters, which are \( \Omega_k \neq 0 \), \( w \neq -1 \), and \( N_{\text{eff}} \); the latter characterizes the number of “equivalent” light neutrino species prior to recombination (Anchordoqui et al. 2021). In the ΛCDM model \( N_{\text{eff}} = 3.046 \) for three families of massless (Standard Model) neutrinos (Mangano et al. 2005). The 9-parameter space is given by

\[
P_1 \equiv \left\{ \Omega_b h^2, \Omega_{\text{CDM}} h^2, 100 \theta_{\text{MC}}, \tau, n_s, \ln[10^9 A_s],
\right.
\]

\[
w, \Omega_k, N_{\text{eff}} \right\},
\]

where \( \Omega_b h^2 \) is the density of baryons, \( \Omega_{\text{CDM}} h^2 \) is the density of CDM, \( \theta_{\text{MC}} \) is the ratio of sound horizon to the angular diameter distance, \( \tau \) denotes the reionization optical depth, \( n_s \) is the scalar spectral index, and \( A_s \) is the amplitude of the primordial scalar power spectrum.

- A ΛCDM extension with four extra free parameters, which are \( \Omega_k \neq 0 \), \( w \neq -1 \), the sum of neutrino masses \( \sum_i m_{\nu_i} \), and the running of the spectral index of inflationary perturbations \( \alpha_s \) (Di Valentino et al. 2021c). In the ΛCDM model a minimal \( \sum_i m_{\nu_i} = 0.06 \text{ eV} \) is assumed (Aghanim et al. 2020a). A 3σ indication for a negative running of \( \alpha_s \) has been observed in the combined analysis of Planck, BAO, and Lyman-α forest data (Palanque-Delabrouille et al. 2020), while a positive value at more than 2.4σ has been measured by the ACT Collaboration (Aiola et al. 2020); see also (Forconi et al. 2021). The 10-parameter space is given by

\[
P_2 \equiv \left\{ \Omega_b h^2, \Omega_{\text{CDM}} h^2, 100 \theta_{\text{MC}}, \tau, n_s, \ln[10^9 A_s],
\right.
\]

\[
w, \Omega_k, \alpha_s, \sum_i m_{\nu_i} \right\}.
\]

To discriminate the models from ΛCDM, we require the
predicted free parameters in the likelihood fit to be $3\sigma$ away from the $\Lambda$CDM result for any particular combination of the data samples yielding a solution of the $H_0$ tension. In Fig. 1 we show the $\Omega_k$ one-dimensional posterior distributions of $P_1$. A larger than $3\sigma$ evidence of $\Omega_k \neq 0$ is only achieved for the combination of CMB data with the SH0ES gaussian prior on the Hubble constant (dubbed R20). The best-fit values of relevant cosmological parameters to our analysis are: $\Omega_k = -0.020^{+0.0065}_{-0.0075}$, $w = -1.90^{+0.41}_{-0.25}$, and $\Omega_m = 0.264^{+0.010}_{-0.012}$ (Anchordoqui et al. 2021). For $P_2$, both CMB data alone and the combination of CMB data with the SH0ES prior give a larger than $3\sigma$ effect. However, when analyzing the CMB data alone, the best-fit gives a relative small evidence for the curvature density, $\Omega_k = -0.074^{+0.055}_{-0.06}$, which exacerbates the Hubble tension: $H_0 = 53^{+6}_{-16}$ km/s/Mpc (Di Valentino et al. 2021c). Therefore, we only consider for our investigation the results of the likelihood analysis to CMB data with the SH0ES prior. The best-fit values of the relevant parameters are as follows: $\Omega_k = -0.0192^{+0.0036}_{-0.0039}$, $w = -2.11^{+0.35}_{-0.77}$, $\Omega_m = 0.264^{+0.010}_{-0.013}$ (Di Valentino et al. 2021c).

In Fig. 2 we show the predicted relative distance modulus by $P_1$ and $P_2$ when varying the relevant parameters between $1\sigma$, $2\sigma$, and $3\sigma$. We can see that for $P_1$, the predicted lower boundary of the $3\sigma$ region (corresponding to $\Omega_m = 0.294$) is consistent with the $\Lambda$CDM prediction, whereas for $P_2$, gives $\Delta \mu \lesssim 10^{-2}$. Altogether, this challenges the feasibility of using the relative luminosity distance modulus to probe the spatial curvature.

For $P_2$, the predicted $\Delta \mu$ lower boundary of the $3\sigma$ region has a maximum around $z \sim 1$, which is currently probed by the Pantheon data sample. As an illustration, in Fig. 3 we show the residuals for the SN Ia data plotted relative to the best fit $\Lambda$CDM model (Dhawan et al. 2020).

Now, a direct comparison of Figs. 2 and 3 demonstrates that the $68\%$ CL residuals of SN Ia data relative to the best fit $\Lambda$CDM overlap the predicted $\Delta \mu$ contours obtained from the $3\sigma$ variation of relevant model parameters. This implies that for the models analyzed herein, the spatial curvature remains insensitive to local $H_0$ measurements from the Cepheid distance ladder.

3. Conclusions

We have reexamined the idea of using the relative luminosity distance modulus $\Delta \mu$ as a discriminator of space-curvature (Shirokov et al. 2020). We have shown that achieving the required $\Delta \mu$ sensitivity to discriminate among cosmological models will be vastly more complicated than once thought. An improvement in the sensitivity to $\Delta \mu$ of future probes must be accompanied by a reduction of the systematic uncertainties driving the determination of cosmological parameters.
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Data availability

We have used data from standard cosmological probes which are freely available.

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