A Realistic Technicolor Model
from 150 TeV down

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Abstract

A realistic technicolor model is presented with the dynamics below 150 TeV treated explicitly. Electroweak symmetry is broken by the condensates of a ‘minimal’ doublet of technifermions. The new feature of the model is that the the third generation quarks are unified with the technifermions into multiplets of a walking gauge force down to a scale of 10 TeV. The remaining quarks and leptons are not involved in this unification however. The walking dynamics enhances the higher dimension interactions which give the ordinary fermions their masses and mixing, while leaving flavor-changing neutral currents suppressed. Because the third generation quarks actually feel the walking force their masses can be much larger than those of the other quarks and the leptons. The only non-standard particles with masses below several TeV are the single doublet of technifermions, so electroweak radiative corrections are estimable and within experimental limits.

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1 Introduction

Models of dynamical electroweak (EW) symmetry breaking must satisfy several stringent experimental and theoretical constraints in order to be realistic. First, they must include interactions to communicate EW symmetry breaking to the ordinary fermions, strong enough to give the observed masses (in particular the top quark mass) and mixings, while suppressing interactions which lead to unacceptably strong flavor-changing neutral currents (FCNC’s) \[1\]. It is preferable that this be done without fine-tuning the couplings of the theory. In this regard walking technicolor (WTC) theories are a large step in the right direction \[2\] since these theories contain a dynamical enhancement of the interactions responsible for fermion masses but not for those interactions leading to FCNC’s. Also, improving precision electroweak experiments have become sensitive to the virtual effects of TeV-scale particles, thereby further restricting models of the undiscovered Higgs sector \[3\].

In this paper I will outline a technicolor model, made explicit below a cutoff $\Lambda = 150$ TeV, which is consistent with the present experimental constraints. In this model EW symmetry is broken by the condensates of a techni-doublet, $(T, B)$, just as in minimal technicolor (MTC) \[4\]. Between $\Lambda$ and a scale $\mu_U = 9$ TeV the technicolor (TC) group is unified with the color group of the top and bottom quarks. The resulting unified gauge force has walking gauge dynamics, due to the presence of several heavy EW-singlet fermions transforming under the same gauge symmetry. From the point of view of the third generation quarks, this is a ‘walking extended technicolor’ theory as contemplated by Georgi \[5\]. The fact that the third generation quarks feel the walking force leads to extra enhancement of their low-energy interactions and masses beyond that which occurs in the standard WTC scenario, where all the standard fermions are singlets of the walking gauge group.

The extra walking enhancement is not required for the remaining quark and lepton masses. In fact the first two generation quarks must not feel the walking force because otherwise FCNC interactions would also be enhanced, as noted by Georgi \[5\]. Therefore, I take the quarks of the first two generations to transform under a separate color group from the top and bottom at high energies. Thus at high energies the third generation quarks are more closely associated with
the technifermions than with the standard fermions. At accessible energies this
is not the case. The transition is made at $\mu_U = 9$ TeV where a new ‘ultra-
color’ (UC) sector, becomes strongly coupled and condensates of UC-fermions
form. The quantum number assignments of the UC-fermions are such that the
condensation breaks the various high-energy gauge groups down to QCD and
TC. In order for this all to go through I must make the dynamical assumption
that gauge symmetries can be broken though their couplings are not perturbatively
weak but are nevertheless below the critical coupling necessary to trigger chiral
symmetry breaking.

Due to the walking dynamics from $\Lambda$ down to $\mu_U$, large anomalous di-
mensions in the renormalization group (RG) flow enhance the necessary higher-
dimension operators. However, additional enhancement turns out to be neces-
sary in order to obtain realistic masses and mixings for the quarks and leptons.
This is provided by enhancing the low-energy effects of a four-technifermion op-
erator beyond its natural strength. This requires tuning of the corresponding
high-energy coupling \[6\]. However, the tuning is only at the ten percent level.

The walking effects are estimated using a large-N approximation, modified
for a walking gauge theory. In this approximation one can calculate the
anomalous dimensions of the important higher-dimension operators given the
anomalous mass dimension of fermions feeling the walking force. (More details
on this type of RG analysis of walking gauge theories in the modified large-N
expansion will appear in Ref. \[7\].)

Most of the non-standard particles are very massive and decouple from
EW physics at accessible energies. The exceptions are the technifermions which
therefore dominate the EW radiative corrections from the non-standard physics.
These effects are both easily estimable and within experimental limits.

The cutoff in this effective theory, $\Lambda$, has been taken as large as the scale of
the strongest four-fermion operator appearing in it. If the cutoff were larger, the
physics underlying the higher dimension operators would have to be described
explicitly. However, the effective field theory language allows us to postpone
the details of this $\Lambda$-scale physics while focussing on the problem of dynamical
EW symmetry breaking. The physics above $\Lambda$ may be supersymmetric, preonic,
extended technicolor or something else. Recently, Einhorn and Nash \[8\] have also
discussed technicolor theories compatible with the large top quark mass from a similar viewpoint (though they stayed within the purely extended technicolor scenario).

The paper is organized as follows. In Section 2, the particle content of the effective theory below $\Lambda$ is detailed. In Section 3, I describe the approximation in which the dynamics between $\Lambda$ and $\mu_U$ is analysed. In Section 4, the effective lagrangian is written down including the higher-dimension operators which communicate EW symmetry breakdown to the ordinary fermions, giving rise to fermion masses and mixings. In Section 5, I define a quantitative measure of the tuning necessary to enhance the low-energy effects of the four-technifermion coupling. Its value within the present model is given in Section 7. In Section 6, I describe the dynamics of the UC sector which breaks the high energy gauge symmetries down to those of MTC. In Section 7, the masses and mixings of the ordinary particles are estimated. In Section 8, the strengths of FCNC’s due to the non-standard physics are estimated. In Section 9, EW radiative corrections due to the technifermions are estimated. Section 10 provides the conclusions of this paper.

2 The model

At the scale $\Lambda \sim 150$ TeV the effective theory has a gauge symmetry

$$SU(2)_{EW} \times U(1)_Y \times SU(3) \times SU(5) \times SU(2)' \times SU(M)_{UC} \times SU(M)_{UC}'. \tag{1}$$

$SU(2)_{EW}$ is the weak gauge group. $SU(3)$ is a weak color group carried by the quarks of the first two generations. $SU(5)$ is the walking unified group for the three colors of the third generation quarks and two technicolors. $SU(M)$ and $SU(M)'$ are UC gauge groups. $SU(2)'$ is carried only by certain UC-fermions in order to ensure the correct gauge symmetry breaking pattern at $\mu_U$. At $\mu_U$ the weak $U(1)_Y$ group combines with an $SU(5)$ generator to give ordinary hypercharge, $U(1)_Y$, QCD emerges, $SU(3)_{QCD} < SU(3) \times SU(5)$, and so does TC, $SU(2)_{TC} < SU(5) \times SU(2)'$. (Readers concerned about the vacuum alignment problem associated with TC theories with two technicolors [4] are assured that the matter will be satisfactorily resolved in Section 7.)
The model contains fermions belonging to four basic sectors. Each sector is separately anomaly-free.

(i) Minimal Technicolor (MTC)

This sector includes all the ordinary particles of the standard model (minus the Higgs, of course) and the technifermions which eventually break EW symmetry. These particles only transform under the first four of the above gauge groups, as

\[
\begin{align*}
(u, d)_L, \quad (c, s)_L & \quad (2, 3, 1)_{1/3} \\
(u, d)_R, \quad (c, s)_R & \quad (1, 3, 1)_{(4/3, -2/3)} \\
L = (\nu_e, e)_L, \quad (\nu_\mu, \mu)_L, \quad (\nu_\tau, \tau)_L & \quad (2, 1, 1)_1 \\
l_R = e_R, \quad \mu_R, \quad \tau_R & \quad (1, 1, 1)_{-2} \\
\psi_L = (T, B)_L & = \left( \begin{pmatrix} t \\ T \\ b \\ B \end{pmatrix} \right)_L \quad (2, 1, 5)_{1/5} \\
(T, B)_R & = \left( \begin{pmatrix} t \\ T \\ b \\ B \end{pmatrix} \right)_R \quad (1, 1, 5)_{(6/5, -4/5)}.
\end{align*}
\]

As can be seen, the technifermions \(T, B\) are unified with the top and bottom quarks at high energies, the three quark colors being the first three components of each 5 and the two technicolors being the last two components of each 5. The first two generations of quarks and all the leptons carry \(Y'\) assignments given by their standard model hypercharge assignments, while \(T\) and \(B\) have \(Y'\) assignments designed to cancel anomalies and to make \(t, T, b, B\) transform in standard fashion under the hypercharge subgroup of the above gauge groups. The \(Y'\) assignments have been written on the bottom-right as conventionally done. The quantum number assignments for the third generation quarks and the technifermions are identical to those of the quarks and certain exotic fermions in the color-\(SU(5)\) model of Foot and Hernandez [10]. The assignment of different color groups to the third generation quarks and the quarks of the first two generations is similar to the construction of the ‘Topcolor’ model of Chris Hill [11].

(ii) Exotic fermions

These are heavy Dirac fermions which will mix with \(t\) and \(b\) to a small extent and will allow interactions to emerge which will give rise to KM mixing.
between the third generation quarks and the first two generation quarks. They transform under $SU(3) \times U(1)_Y$, only, as

\[(t', b')_L \quad 3_{(4/3, -2/3)}\]
\[(t', b')_R \quad 3_{(4/3, -2/3)}\].

(iii) Ultra-color (UC)

The purpose of this sector is to break $SU(3) \times SU(2') \times SU(5) \times U(1)_Y$, down to $SU(3)_{QCD} \times SU(2)_{TC} \times U(1)_Y$. It is achieved by the QCD-like condensation of the UC fermions due to the strong $SU(M)$ UC gauge force. These fermions transform under the full gauge symmetry as

\[Q_L = \begin{pmatrix} q \\ Q \end{pmatrix}_L \quad (1, 1, 5, 1, 1, M, 1)_{6/5}\]
\[q_R \quad (1, 3, 1, 1, M, 1)_{4/3}\]
\[Q_R \quad (1, 1, 1, 2', M, 1)_{1},\]
\[p_L \quad (1, 3, 1, 1, 1, M')_{4/3}\]
\[P_L \quad (1, 1, 1, 2', 1, M')_{1},\]
\[P_R = \begin{pmatrix} p \\ P \end{pmatrix}_R \quad (1, 1, 5, 1, 1, M')_{6/5}\] (4)

The $p$’s and $P$’s have been introduced purely to cancel anomalies. They will be referred to as the UC’ sector, to distinguish them from the $q$’s and $Q$’s. We will take $M = M' = 3$ to be specific but will continue to refer to $SU(M)$ and $SU(M')$ in order to distinguish them from the $SU(3)$ group already in the theory. The basic structure of this sector of the theory is similar to that employed in Ref. [12].

(iv) Extra $SU(5)$ fermions.

These I denote by $\chi$ and they are Dirac fermions, assumed to transform purely under $SU(5)$, in such a way as to make it a walking gauge theory between $\Lambda$ and $\mu_U$. This is their sole purpose in the model.
3 The method of analysis

I will now specify the approximation in which the RG flow between $\Lambda$ and $\mu_U$ is treated, concentrating on providing a working set of rules to capture the effects of walking. Further details and a more careful discussion of the assumptions made will appear in Ref. [7]. Readers familiar with gap equation analyses of WTC will find no surprises here.

The task at hand is to estimate the anomalous dimensions of the various four-fermion operators which can appear in our effective lagrangian so that their infrared enhancement can be calculated. The walking $SU(5)$ dynamics will be analysed in the leading order of a $1/N$ expansion capable of capturing the walking dynamics, where $N = 5$ here. Such an expansion was suggested by Appelquist and Wijewardhana [2] and first made use of in Ref. [13]. The only modification of standard large-$N$ QCD diagrammatics which occurs in the walking theory is that $\chi$ loops are not suppressed because there are many flavors of them or because they are in higher representations of $SU(5)$. This is of course crucial in making the $SU(5)$ coupling walk and not run. I will assume that a suitable choice of $\chi$ representations exist so that the $SU(5)$ coupling walks over the hierarchy from $\Lambda$ down to $\mu_U$, and that the anomalous mass dimension for fermions in the 5 representation due to the walking force is

$$\gamma \sim 0.8.$$ \hfill (5)

The normalization of $\gamma$ is such that $m(\mu)/m(\mu') = (\mu'/\mu)^\gamma$ for a running fermion mass parameter $m$. Gap equation analysis [2] shows that chiral symmetry breaking occurs when the gauge coupling is so strong that $\gamma > 1$, so we are assuming a gauge coupling which is walking close to, but below the critical value for chiral symmetry breaking.

Between $\Lambda$ and $\mu_U$ I will keep only the effects of the walking force on the RG flow of higher dimension operators, dropping the effects of the (weaker) running gauge forces. The only exception is the UC force acting in the UC sector, which is presumed to be getting stronger in the infrared in QCD-like fashion so that it can break the high-energy gauge groups at $\mu_U$. Also the $SU(5)$ coupling will be treated as approximately standing. It is clear that four-fermion operators which contain only fields which are $SU(5)$ singlets have zero anomalous
dimension in this approximation. This leaves those operators with two or four fields transforming under $SU(5)$. It is convenient to think of these operators as being due to the exchanges of heavy $SU(5)$-singlet bosons. These auxiliary fields have mass terms but no kinetic terms. They act as a convenient book-keeping device. Depending on the four-fermion operator the auxiliary boson is chosen to be a scalar or a vector, whichever is the $SU(5)$-singlet. Thus the four-fermion operators can be rewritten as dimension-four couplings of fermions to auxiliary scalar and vector fields.

The reason for doing this is that the diagrams which dominate for large N are those in which the auxiliary fields do not appear in loops. This implies that the anomalous dimensions of the four-fermion operators are determined purely by the running of the fermion couplings to the auxiliary fields, and the running of the auxiliary field masses. Auxiliary scalars, $S$, have Yukawa couplings, $\bar{f} S f$, while auxiliary vectors, $V_\mu$, couple to fermionic currents as $V_\mu J_\mu$. The running due to the walking gauge dynamics is now simple. If the $f$’s are $SU(5)$-singlets neither type of coupling runs. If they are both 5’s then Yukawa couplings run with the mass anomalous dimension $\gamma$. However, because $J_\mu$ is a conserved flavor current from the point of view of the $SU(5)$ dynamics, the vector coupling still does not run.

The masses for the auxiliary fields are naturally of order $\Lambda$ at that scale, corresponding to four-fermion operators with strengths of order $1/\Lambda^2$. At lower scales, interactions can renormalize these masses, but these are $O(1)$ effects unless large cancellations occur.

The reader may wonder whether the auxiliary fields develop kinetic terms and become truly propagating at lower energies. This actually does occur but the particles are still very heavy and it is inessential to keep these kinetic terms for the purposes of this paper. The properties of these kinetic terms are discussed in Ref. \[14\] and will be further discussed in the present large-N RG language in Ref. \[7\].

The rules outlined in this section will become clearer in the next section where they are used to determine the effective lagrangian below the scale $\Lambda$. 

7
4 The effective lagrangian

The effective lagrangian at a scale \( \mu, \mu_U < \mu < \Lambda \), can be written as

\[
\mathcal{L}(\mu) = \mathcal{L}_{MTC} + \mathcal{L}_{3-\text{mix}} + \mathcal{L}_{\mu-\text{mass}} + \mathcal{L}_{\text{misc}}. \tag{6}
\]

The various parts have been named according to the role they play in our story. I will not write the kinetic and mass terms of the fermions and the gauge fields explicitly because there is no mystery concerning them. The only particles which can have mass terms are the \( \chi \)'s and the \( t' \) and \( b' \). I will take the \( \chi \) masses to be such that they decouple from the theory once their job of maintaining walking is done, namely at \( \mu_U \). The \( t' \) and \( b' \) are assigned masses of order 20 TeV. Also I will omit all four-fermion operators which remain completely unenhanced by the rules of the last section. The effects due to these interactions to which we are most sensitive are FCNC’s, which are examined separately in Section 8 and found to be acceptably weak. All other four-fermion interactions which can occur will be explicitly shown. I now explain the various parts of the lagrangian one at a time.

- \( \mathcal{L}_{MTC} \):

Enhanced four-fermion interactions involving only the particles of the MTC sector can be written in terms of the exchange of a single auxiliary scalar, \( \phi \), which transforms under the weak group, \( SU(2)_{EW} \times U(1)_{Y'} \) as a \( 2_1 \).

\[
\mathcal{L}_{MTC} = (U \overline{D})_L y_D(\Lambda) \phi D_R + (U \overline{D})_L y_U(\Lambda) \overline{\phi} U_R + \overline{L} y_l(\Lambda) \phi l_R \\
+ (\Lambda/\mu) \overline{\psi}_L y_b(\Lambda) \phi B_R + (\Lambda/\mu) \overline{\psi}_L y_t(\Lambda) \overline{\phi} T_R + h.c. \\
- x(\mu)\Lambda^2 \phi^\dagger \phi, \tag{7}
\]

Here, \( U \) and \( D \) represent \((u,c)\) and \((d,s)\), so that \( y_D \) and \( y_U \) are really \( 2 \times 2 \) matrices in flavor space and \( y_l \) is a three component vector (one for each charged lepton). The enhancement factors follow the rules of the last section.

At \( \mu = \Lambda \), integrating out \( \phi \) yields a set of four-fermion interactions with strengths of order \( y^2/\Lambda^2 \) if \( x(\Lambda) = 1 \). \( x(\mu)\Lambda^2 \) is the running mass-squared of \( \phi \). As mentioned in the last section, \( x(\mu) \) is naturally of order one but can be tuned to be smaller. Its role is discussed in the next section. The \( \phi \) kinetic terms induced below \( \Lambda \) play no part in this paper and are therefore neglected.
The interactions in $L_{MTC}$ are responsible for communicating the TC-induced EW breakdown to the quarks and leptons in order to give them their masses. In the language of the auxiliary scalar this proceeds through $\phi$ first getting a non-zero vacuum expectation value.

• $L_{3-\text{mix}}$:

Although $y_{D,U}$ can provide mixing between the quarks of the first two generations through their off-diagonal elements, Yukawa couplings which can produce mixing involving the third generation quarks are forbidden by gauge invariance ($t, b$ are in $SU(5)$ multiplets while the other quarks are not). This is where the exotic fermions, $t', b'$, come in. The important part of the effective lagrangian involving them is

$$L_{3-\text{mix}} = (\bar{U} \bar{D})_L y_b(\Lambda) \phi b'_R + (\bar{U} \bar{D})_L y_t(\Lambda) \tilde{\phi} t'_R + \frac{g_b(\Lambda)(\Lambda/\mu)^\gamma}{\Lambda^2} (\bar{q}_{LqR})(\bar{Q}_L B_R) + \frac{g_t(\Lambda)(\Lambda/\mu)^\gamma}{\Lambda^2} (\bar{t}_L q_R)(\bar{Q}_L T_R) + \text{h.c.}$$

(8)

I have written some of the above operators without auxiliary fields, but the enhancement rules are obvious. $y_b$ and $y_t$ each have two components, one for each of the first two generations. Integrating out the heavy $t', b'$ fermions below their masses will lead to operators which will permit mixing between the third and first two generation quarks.

• $L_{\text{P-mass}}$:

This part of the lagrangian is designed to give sizeable masses for the lightest UC’ states so that they decouple from the low-energy theory. I have not bothered to include the $O(1)$ $\Lambda$-scale coefficients of each operator. With this in mind,

$$L_{\text{P-mass}} \sim \frac{(\Lambda/\mu)^\gamma}{\Lambda^2} (\bar{P}_R Q_L)(\bar{q}_{RqL}) + \frac{(\Lambda/\mu)^\gamma}{\Lambda^2} (\bar{P}_R Q_L)(\bar{Q}_R P_L).$$

(9)

The $SU(M)'$ UC’ force is taken to be considerably weaker than the UC force. When the UC condensate forms the above operators induce TeV-scale ‘current mass’ terms for the UC’ fermions.

• $L_{\text{misc}}$: 

9
Finally, the lagrangian contains various miscellaneous operators, most compactly written in the form

$$\mathcal{L}_{\text{misc}} \sim \frac{(A/\mu)^{2\gamma}J^L_\mu J^R_\mu}{\Lambda^2} (\mathcal{T} t' \chi \chi)$$

$$+ \frac{(A/\mu)^{\gamma}}{\Lambda^2} (\mathcal{B} b' \chi \chi),$$

where $J^L_\mu$ denotes any of the $SU(5)$-adjoint currents $\bar{\psi}_L \gamma_\mu \psi_L$, $\bar{Q}_L \gamma_\mu Q_L$, $\bar{\chi}_L \gamma_\mu \chi_L$, and $J^R_\mu$ denotes any of the $SU(5)$-adjoint currents $\bar{T}_R \gamma_\mu T_R$, $\bar{B}_R \gamma_\mu B_R$, $\bar{P}_R \gamma_\mu P_R$, $\bar{\chi}_R \gamma_\mu \chi_R$. $\mathcal{L}_{\text{misc}}$ plays little part in our model, although in Section 8 FCNC’s induced by some of these operators will be estimated.

5 Enhanced fermion masses by tuning four-fermion interactions

Four-fermion couplings at high energies can be tuned in such a way as to enhance their effects at lower energies beyond what is expected from dimensional analysis. This phenomenon occurs even in the presence of walking gauge dynamics and has been exploited in the past to obtain enhanced fermion masses beyond the pure walking effect [6, 15].

In the present model, it will be necessary to tune the strength of the operator, $(\bar{\psi}_L T_R)(\bar{T_R} \psi_L)$, in order to give extra enhancement to fermion masses. It is convenient to continue using the auxiliary scalar language in discussing this issue. Recall that $x(\mu)\Lambda^2$ is the mass-squared of the auxiliary scalar at the scale $\mu$. The dominant effect renormalizing this parameter is due to $\mathcal{T}$ loops in the auxiliary scalar’s self-energy graphs. ($\mathcal{T}$ loops dominate because they have the largest Yukawa couplings to $\phi$.)

The result is,

$$x(\mu)\Lambda^2 \sim \Lambda^2 - \frac{5(1 - (\mu/\Lambda)^2(1-\gamma))}{(1 - \gamma)8\pi^2} y_t(\Lambda)^2 \Lambda^2.$$  \(11\)

The 5 is just the number of colors running around a loop. This estimate has been made in the RG approach [7]. The basic result is however simple to understand. Putting $\gamma = 0$ recovers the one-loop result, which neglects the effects of walking
dynamics. The first term is just the ‘bare’ scalar mass-squared \( x(\Lambda) = 1 \). The dressing of the loop by \( SU(5) \)-gluons is an \( O(1) \) effect.

Now, \( y_t(\Lambda) \) can be tuned in order to make \( x \) small and positive. Because the four-fermion interactions in \( \mathcal{L}_{MTC} \) are represented by \( \phi \) exchange this amounts to enhancing the strength of these interactions at low energies, which in turn will help to enhance the masses of ordinary fermions. Using the above equation as a rough guide, \( x \) is tuned to be small for \( y_t^2 \sim 5 \).

I will quantify the amount of tuning necessary by \( x/x_{\text{generic}} \). The generic choice would appear to be \( x_{\text{generic}} \sim 1/2 \), so \( 2x \) is the measure of tuning needed. In the end roughly a ten percent tuning will be required, \( x \sim 0.05 \).

6 The effective theory below the UC scale

From \( \Lambda \) down to \( \mu_U \), walking dynamics leads to the effective lagrangian described in Section 4. I assume that at this scale the running \( SU(M) \) gauge coupling, taken to be much stronger than the \( SU(M)' \) coupling, rapidly gets strong and breaks the chiral symmetries of the UC-fermions in QCD-like fashion. I will take the associated Goldstone boson decay constant, \( f_U \), to be \( \sim 7 \) TeV. The resulting condensates are estimated from our experience with QCD as

\[
\langle \bar{q}q \rangle \sim \langle \bar{Q}Q \rangle \sim -20f_U^3.
\]

I will suppose that the UC' force is much weaker, \( f'_U \sim 1.5 \) TeV.

The formation of the UC-condensate breaks the full gauge symmetry of the model, Eq. (1), down to

\[
SU(2)_{EW} \times U(1)_Y \times SU(3)_{QCD} \times SU(2)_{TC}.
\]

The MTC particles now transform as

\[
\begin{align*}
(u, d)_L, \quad (c, s)_L, \quad (t, b)_L & \quad (2, 3, 1)_{1/3} \\
(u, d)_R, \quad (c, s)_R, \quad (t, b)_R & \quad (1, 3, 1)_{(4/3, -2/3)} \\
L = (\nu_e, e)_L, \quad (\nu_\mu, \mu)_L, \quad (\nu_\tau, \tau)_L & \quad (2, 1, 1)_{-1} \\
l_R = e_R, \quad \mu_R, \quad \tau_R & \quad (1, 1, 1)_{-2}
\end{align*}
\]
Hypercharge has emerged as a linear combination of the $Y'$ generator and a diagonal $SU(5)$ generator, so that for former members of 5’s,

$$ Y = Y' + 1/15 \text{ diagonal}(2, 2, 2, -3, -3), $$

and for $SU(5)$-singlets, $Y = Y'$. Thus the low-energy quantum numbers are indeed just those of standard MTC. The UC' fermions now transform as

$$ p_{L,R} \rightarrow (1, 3, 1)_{4/3} $$

$$ P_{L,R} \rightarrow (1, 1, 2)_1. $$

The breaking of gauge symmetries makes 24 gauge bosons massive, which means that all 24 UC Goldstone bosons are eaten. One of these heavy gauge bosons is a $Z'$ but it is so heavy (O(10 TeV) in mass) that its mixing with the ordinary $Z$ can be neglected. The masses of the gauge bosons are estimated by working to leading order in their gauge couplings, which give mass-squares $\sim \pi \alpha_5 f_U^2$.

We will therefore match effective lagrangians at $\mu_U^2 \sim \pi \alpha_5 f_U^2$. Gap equation analyses suggest $\alpha_5 \sim 1/2 \ [2]$. Thus,

$$ \mu_U = 9 \text{ TeV}. $$

Below this scale the effective lagrangian has the gauge symmetry Eq. (13), whereas above it the theory has the full gauge symmetry of Eq. (1). At this level of approximation the matching of couplings reads

$$ \frac{1}{\alpha_{QCD}(\mu_U)} \sim \frac{1}{\alpha_3(\mu_U)} + \frac{1}{\alpha_5(\mu_U)} $$

$$ \frac{1}{\alpha_{TC}(\mu_U)} \sim \frac{1}{\alpha_2'(\mu_U)} + \frac{1}{\alpha_5(\mu_U)}. $$

At the matching scale QCD is weak. This is arranged by taking the $SU(3)$ force to be weak.

Because the $P$’s feel TC, the masses of the light UC' states must be known before determining the TC coupling at $\mu_U$. In the low-energy phase, the $p$’s and
$P$'s are permitted mass terms, and the interactions in $\mathcal{L}_{P\text{-mass}}$ provide them upon UC condensation,

$$\mathcal{L}_{P\text{-mass}} \sim \frac{(\Lambda/\mu)\gamma}{\Lambda^2} 5f_U^3 \bar{p}p + \frac{(\Lambda/\mu)\gamma}{\Lambda^2} 5f_U^3 \bar{P}P.$$  \hspace{1cm} (19)

One must decide what choice to make for $\mu$, the scale down to which the walking enhancement occurs. This is not just $\mu_U$ now because the $Q$'s have constituent masses $\sim 30$ TeV, scaling up from QCD. This means that they effectively decouple at this scale, so $\mu \sim 30$ TeV is a better estimate. The induced $\mathcal{P}$ masses are then of order 1 TeV. Because $f_U' = 1.5$ TeV, this means that even the lightest UC' states, the UC' Goldstone bosons, have masses $\sim \mu_U$. Thus UC' states do not affect the running of TC below $\mu_U$. Below this scale, the only fermions which feel TC are $T$ and $B$.

I estimate that

$$\alpha_{TC}(\mu) \sim 1/3.$$  \hspace{1cm} (20)

This follows by taking the TC Goldstone decay constant to be the weak scale, $f_T = 250$ GeV, and scaling up QCD data with the appropriate modification due to the difference in the number of colors. From this estimate it follows that $\alpha_{2'}(\mu_U) \sim 1$. This is a rather strong coupling but below the critical coupling necessary for the $SU(2')$ force to break the chiral symmetries of the fermions which feel it, which is $\alpha_{2'}^{\text{critical}} \sim 3/2$ by gap equation analysis [2].

Since the $t'$ and $b'$ have masses $\sim 20$ TeV, these particles should be integrated out at scales below their masses. Integrating them out of the effective lagrangian and then proceeding down to $\mu_U$ yields

$$\mathcal{L}_{3\text{-mix}}(\mu_U) = \frac{g_{b'}(\Lambda)y_{b'}(\Lambda)(\Lambda/\mu)^\gamma}{\Lambda^2 m_{b'}} (\bar{U} D)_{L\phi} q_R \bar{Q}_{L} B_R$$

$$+ \frac{g_{t'}(\Lambda)y_{t'}(\Lambda)(\Lambda/\mu)^\gamma}{\Lambda^2 m_{t'}} (\bar{U} D)_{L\phi} q_R \bar{Q}_{L} T_R + h.c. + \ldots$$  \hspace{1cm} (21)

The ellipsis refers to other operators obtained upon integrating out the $t'$ and $b'$ but which are irrelevant at low energies. Once again the appropriate choice for the scale below which walking enhancement is cut off is $\mu \sim 30$ TeV. Inserting the UC condensate, for scales below $\mu_U$,

$$\mathcal{L}_{3\text{-mix}} \sim 0.02g_{b'}(\Lambda)y_{b'}(\Lambda)(\bar{U} D)_{L\phi} b_R + 0.02g_{t'}(\Lambda)y_{t'}(\Lambda)(\bar{U} D)_{L\phi} t_R.$$  \hspace{1cm} (22)
These Yukawa couplings mix the first two generation quarks with the third as required.

Only the MTC particles remain in the theory below $\mu_U$ with a set of four-fermion interactions mediated by $\phi$ exchange which I will now show lead to masses and mixings for the quarks and leptons.

7 Masses and couplings of the MTC particles

The technifermions condense, break EW symmetry and are confined in standard fashion. With more than two technicolors this statement would require no explanation, but there is a subtlety when there are only two technicolors since the doublet representation of $SU(2)_{TC}$ is pseudo-real. This implies that the $T$ and $B$ give rise to an $SU(4)$ chiral flavor symmetry for the pure TC theory instead of the familiar $SU(2) \times SU(2)$. TC breaks the $SU(4)$ symmetry down to $Sp(4)$ instead of $SU(2)_V$. Five Goldstone bosons result instead of three. Suppose that the only forces present in the theory were EW and TC. Then one can show that in the standard model vacuum, where $U(1)_{EM}$ is the preserved symmetry, three of the Goldstone bosons would be eaten but the remaining two Goldstone bosons would have negative mass-squares due to EW forces $\sim -(100 \text{ GeV})^2$. This shows that the standard model vacuum is not the right one. If this were the case it would be a phenomenological disaster. Fortunately there are relatively strong four-technifermion forces in the present model which stabilize the standard model vacuum. For the moment the reader should accept that the correct EW breaking pattern arises and I will show that this choice of vacuum is stable later.

The dominant mass contributions to MTC particles come from their four-fermion couplings to the technifermions. In the language where the auxiliary scalar field $\phi$ is present, these contributions are written in terms of $\langle \phi \rangle$, which develops as the result of TC condensation. This quantity is found by extremizing the dominant terms in the effective potential,

$$V_{eff}(\phi) \sim -(\Lambda/\mu_U)^{\gamma}y_b(\Lambda)\langle \bar{\psi}_L B_R \rangle \phi - (\Lambda/\mu_U)^{\gamma}y_t(\Lambda)\langle \bar{\psi}_L T_R \rangle \tilde{\phi} + \text{h.c.} + x\Lambda^2 \phi^\dagger \phi$$

Note that the walking enhancement ceases when $SU(5)$ is broken at $\mu_U$. Scaling
up from QCD, and accounting for the difference in the number of colors,

\[ \langle TT \rangle \sim \langle BB \rangle \sim -25 f_T^3. \]  

(24)

Using the fact that \( y_b \ll y_t \),

\[ \langle \phi \rangle \sim \frac{y_t(\Lambda)}{x} 100 \text{ MeV}. \]  

(25)

Substituting \( \langle \phi \rangle \) into \( \mathcal{L}_{MTC} \) gives a set of mass terms for the fermions. Notice that fermion masses will be enhanced because of the enhancement of four-fermion operators as measured by \( 1/x \).

There is an extra contribution to the masses of the third generation quarks due to the operators arising from the exchange of massive \( SU(5) \) gauge bosons which link \( t \) with \( T \) and \( b \) with \( B \). Such operators are not described by \( \phi \) exchange. Estimating the operators by one-\( SU(5) \)-gluon exchange, this ‘extended technicolor’ contribution follows in standard fashion. It gives a common mass to the \( t \) and \( b \) quarks,

\[ m_{ETC} \sim 2\langle TT \rangle / f^2_U \sim 15 \text{ GeV}. \]  

(26)

Before attempting to pick couplings in order to obtain a fully realistic spectrum, let us see what the generic prediction is. That is, take \( y_f(\Lambda) \sim 1 \) for all fermions, \( f \), so that \( x \sim 1/2 \). Then \( \langle \phi \rangle \sim 400 \text{ MeV} \). Referring to the expression for \( \mathcal{L}_{MTC} \) in Section 4, and taking into account the extra ETC mass contribution for the third generation quarks, one obtains

\[ m_{t,b} \sim 20 \text{ GeV} \]

\[ m_{\text{other}} \sim 200 \text{ MeV}. \]  

(27)

Using Eq. (21), the mixing between the third generation quarks and the other quarks is given by a mixing mass

\[ m_{3-\text{mix}} \sim 10 \text{ MeV}. \]  

(28)

This is in fact a decent caricature of the observed particle spectrum. There is a large hierarchy between the masses of the third generation quarks and the
other quarks and leptons, along with small mixing angles involving the third generation.

Now I will choose couplings in order to get a realistic spectrum. Third generation quark masses are given by

\[ m_{t,b} = (\Lambda/\mu_U)^2 y_{t,b}(\Lambda) \langle \phi \rangle + m_{ETC}. \]  

(29)

A large top mass requires that \( x \) be tuned to be small in order to enhance \( \langle \phi \rangle \) sufficiently. In order to get a mass of 100 GeV, I estimate that, \( x \sim 0.05 \), \( y_t(\Lambda)^2 \sim 5 \). This is the necessary ten percent tuning discussed in Section 5. With this choice,

\[ \langle \phi \rangle \sim 4.5 \text{ GeV}. \]  

(30)

Thus \( y_b(\Lambda) \sim -1/5 \), in order to get the observed bottom quark mass. The remaining quark and lepton masses and mixing masses are of the form

\[ m_f = y_f(\Lambda) \langle \phi \rangle, \]  

(31)

so they can all be realistically obtained with \( y_f \leq 1/2 \).

We can also associate ‘current’ masses for the technifermions \( T, B \) due to their Yukawa couplings to \( \phi \), in \( \mathcal{L}_{MTC} \). These couplings are identical to those for \( t \) and \( b \), by \( SU(5) \)-invariance. Therefore we find that

\[ m_T^{\text{current}} \sim 100 \text{ GeV} \]
\[ m_B^{\text{current}} \sim 10 \text{ GeV}. \]  

(32)

This will provide the major effect in estimating the non-standard contribution to the \( T \) parameter of Peskin and Takeuchi [3]. These current masses also represent the crucial vacuum stabilizing effect of the four-technifermion interactions. In particular one can use chiral perturbation theory to estimate the contribution they make to the mass-squares of the two pseudo-Goldstone bosons (PGB’s) mentioned at the beginning of this section. The result is \( \sim + (500 \text{ GeV})^2 \). This completely overwhelms the EW forces which are trying to destabilize the standard model vacuum as represented by their negative mass-square contributions to the PGB’s, so disaster has been averted.
From Eq. (21), the typical size of the mixing masses involving the third
generation is

\[ m_{3-mix} \sim 0.02g_{\nu',\nu}(\Lambda)y_{\nu',\nu}(\Lambda)\langle \phi \rangle \]

\[ \sim g_{\nu',\nu}(\Lambda)y_{\nu',\nu}(\Lambda)100 \text{ MeV}. \]  

(33)

For \( g \)'s of order one this gives mixing of the correct size.

8 Flavor-changing neutral currents

To begin, consider FCNC’s due to four-fermion operators that can occur in the
effective lagrangian at \( \Lambda \). Because of the different quantum numbers carried by
the third generation quarks, these \( \Lambda \)-scale flavor-changing operators can only
involve the first two generations. At this scale, quarks from the first two genera-
tions, \( f \), must have four-fermion couplings at most as strong as \( y_f/\Lambda^2 \), as can
be seen by integrating \( \phi \) out. Following the analysis of Dimopoulos and Ellis
\[ \overline{d}d \overline{d} \] this suggests that flavor-changing four-fermion operators, such as \( \overline{d}d \overline{d} \) will
naturally be present with (Cabibbo suppressed) strengths of order \( y_f/(20\Lambda^2) \).
These operators are not enhanced by walking dynamics. Recall that \( \langle \phi \rangle \sim 4.5 \)
GeV, so \( y_s \sim 0.05 \). Thus \( \overline{s}d \overline{s}d \) is suppressed in the effective lagrangian by
\( 1/(3000 \text{ TeV})^2 \). This is phenomenologically acceptable. Other flavor-changing
operators are less constrained by experiment.

If we turn off the interactions which give mixing involving the third gen-
eration (\( \mathcal{L}_{3-mix} \rightarrow 0 \)) the non-standard physics below \( \Lambda \) does not give rise to
new FCNC’s. FCNC’s develop because the third generation quarks couple dif-
finitely to the non-standard sectors than the first two generations (\( \mathcal{L}_{misc} \) and
\( SU(5) \)-gauge-boson exchange) and when we rotate from the gauge basis to the
mass basis for the quarks these couplings which were flavor diagonal in the
gauge basis become slightly off-diagonal, so heavy non-standard states can med-
iate FCNC’s. Therefore the new FCNC’s involve the third generation quarks,
and are suppressed by the small third generation mixing angles and the masses
of the non-standard states, which are several TeV. This suggests for example
that \( \overline{b}d \overline{b}s \) has a strength at most \( \sim V_{bc}^2/(10\text{ TeV})^2 \), \( \overline{b}d \overline{b}d \) has a strength at most
\( \sim V_{bs}^2/(10\text{ TeV})^2 \) and \( \overline{b}d \overline{d} \mu \mu \) has a strength at most \( \sim V_{bc}^2/(10\text{ TeV})^2 \). These rough
upper bounds are below the experimental limits on \( \Delta B \neq 0 \) FCNC’s \[ \square \].

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9 Electroweak radiative corrections

The dominant non-standard contributions to the $S$ and $T$ parameters of Peskin and Takeuchi \[3\] come from the weak scale TC sector, which we have seen is just MTC. Peskin and Takeuchi \[17\] have given the estimates for MTC with two technicolors (where the $T - B$ mass splitting is given by $\sim m_t$, as in the present model),

$$S \sim 0.22$$
$$T \sim 0.3.$$ \hspace{1cm} (34)

The values above lie within the 90 percent confidence level ellipse in the experimental $S - T$ plane \[17\].

One may wonder about the effects of the two PGB’s on $S$, not taken into account in the formula above. The answer is that virtual pairs of these PGB’s do not contribute to the EW gauge boson mixing parameter $S$ because PGB-pair couplings to EW currents are proportional to their custodial $SU(2)_{\text{cust}}$ quantum numbers, but the PGB’s in this model are singlets under $SU(2)_{\text{cust}}$.

10 Conclusion

I have constructed a realistic technicolor model of EW symmetry breaking, effective below 150 TeV. It uses a combination of walking dynamics and strong four-fermion interactions to enhance operators responsible for giving the ordinary fermions their masses, while leaving operators responsible for FCNC’s unenhanced. For the leptons and quarks of the first two generations this is very similar to the schemes already in the literature. However the third generation quarks are treated differently in this model because they feel the walking force. This gives extra enhancement to their interactions and to the final top quark mass. At an ultra-color scale of several TeV, the theory becomes identical to minimal technicolor, with the desired set of four-fermion interactions emerging from the high energy walking dynamics. Mixing with the third generation quarks is naturally small because they feel a different high energy color force from the first two generations. Mixing then necessarily proceeds through very high dimension operators, produced in the present model by integrating
out some heavy fermions. The minimal low-energy content of the theory results in electroweak radiative corrections which are estimable, moderate and phenomenologically acceptable.

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