We present an application of the recent CS-ARDL methodology in the context of a country’s trade balance–exchange rate relationship. The trade balance is expected to deteriorate first before improving in response to currency depreciation and vice versa, widely known as the J-curve effect satisfying the Marshall–Lerner condition in the long run. Combining bilateral and aggregate analysis in one setting by constructing specific panel data with one reference country, we find that aggregate analysis is sensitive to our allowance for heterogeneity. Estimates using the aggregate time series data show evidence favoring the J-curve relation, whereas the aggregate analysis resulting from the panel time series data shows that currency appreciation improves trade balance in Bangladesh in the long run, which goes against the Marshall–Lerner condition. With the reference of the existing commodity-level literature, we argue that this atypical scenario lines with the realities of a ‘small’ economy like Bangladesh, where her exporters attempt to maintain their market share with some government support. The study provides essential policy suggestions by identifying the significant
contributors to Bangladesh’s trade balance–exchange rate relationship: China, Japan, and Singapore.

**Keywords** Exchange rate · Trade balance · Cross-sectionally augmented nonlinear ARDL · Panel time series · Common correlated effects · Aggregation bias

**JEL Classification** C13 · C32 · C33 · F14 · F31 · F32

1 **Introduction**

In this research, we attempt to estimate the cross-sectionally dependent aggregate relationship between a country’s trade balance and real exchange rate, where we can also extract the partner-specific heterogeneous relationships for policy implication. This attempt is new in the literature investigating trade balance–exchange rate relationship, which is predominantly based on the analysis of either bilateral relationships or single-country aggregate (over all trading partners) analysis without considering the common factors. We use Bangladesh as our country of interest and construct a panel data with her 25 major trading partners to disentangle partner relationships from the aggregate analysis. We replace the domestic and foreign GDP variables with foreign GDP relative to domestic GDP to preserve enough variation. We construct the aggregate time series data from our panel data following Beyer et al. (2000) flexible weights aggregation method. Our structure facilitates a comparative analysis of single-country aggregate time series, bilateral time series, and panel time series estimates using the same data and therefore contributes to the analysis of aggregation bias in this regard. We also include both linear (symmetric) and nonlinear (asymmetric) effects of exchange rate movement following Shin et al. (2014), Bahmani-Oskooee et al. (2019), Bahmani-Oskooee and Kanitpong (2017): whether the effect of currency appreciation on the trade balance is different to that of currency depreciation.

The relationship between the real exchange rate and the country’s trade balance is still a topic of empirical interest, with the current focus on finding evidence favoring J-curve due to the econometric methodology advancements. The literature, pioneered by Magee (1973) to date, investigates whether currency depreciation improves the trade balance of a country, noting different short run and long run effects due to adjustment lags. Based on the Marshall–Lerner condition, this theory is briefly referred to as the J-curve effect; currency depreciation causes a short run deterioration of a country’s trade balance before improving it in the long run. Broadly, there are three lines of research: estimating the aggregate effect (see, e.g., Bahmani-Oskooee and Ratha 2004, Bahmani-Oskooee and Hegerty 2010 for surveys), estimating the bilateral effect (e.g., Bahmani-Oskooee and Fariditavana 2015, Bahmani-Oskooee and Kanitpong 2017),

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1 Here, we refer to aggregation over trading partner countries. This is different from aggregation over time (e.g., daily to monthly or monthly to yearly data) and aggregation over industry or sectors. All our data is in monthly form and remains aggregate over industries and sectors.

2 See details in ‘Appendix A.’
using time series analysis\textsuperscript{3}, and estimation using panel time series data (e.g., Yazgan and Ozturk 2019). While estimating the bilateral effect, some recent research focuses on industry-level trade data, with (e.g., Lucarelli et al. 2018) or without (e.g., Bahmani-Oskooee et al. 2016) incorporating asymmetric effects analysis.

The studies so far, however, cannot provide any conclusive results on the topic. For example, in China’s case, Bahmani-Oskooee and Wang (2006) based on linear bilateral time series did not find any reliable evidence of J-curve. Wang et al. (2012) using panel data with no correction for cross-section dependence found some inverted J-curve evidence. Later, Bahmani-Oskooee and Fariditavana (2015) found an absence of J-curve with linear aggregate time series analysis, but presence allowing for nonlinear (asymmetric) adjustment. Recently, Bahmani-Oskooee et al. (2018) found evidence of the J-curve with 5 out of 21 trading partners of China using nonlinear bilateral analysis. In the case of Bangladesh, based on aggregate time series (vector error correction model) analysis using monthly data, Younus and Chowdhury (2014) found no effect of real effective exchange rate on trade balance both in the short and long run. Bahmani-Oskooee et al. (2019), on the other hand, using a bilateral nonlinear ARDL model with quarterly data, found short run asymmetric effects in favor of J-curve with most trading partners but only a few long run asymmetric effects. In the case of panel data for Bangladesh, the applications are limited to the basic techniques such as fixed effects and random effects estimations (Khan and Hossain 2010) without considering the cross-section dependence. Notably, none of those mentioned above studies considered the exchange rate regime shifts, neither for China nor Bangladesh.

The third line of research emerged very recently using panel time series data, Yazgan and Ozturk (2019) analyzed the J-curve effect of 33 countries applying the pooled Common Correlated Effects estimator (CCE estimator, due to Pesaran 2006) admitting the possible cross-section correlation. Our analysis falls in this third line of research. We contribute to this ongoing debate by presenting a rigorous comparative analysis: aggregate time series, bilateral time series, and panel time series analysis of a single country experience. While the aggregate time series analysis suffers from possible aggregation bias, the bilateral time series ignores the country-specific common factors and has limited policy suggestions. A panel time series is tempting under this argument, but that needs to consider the cross-section dependence. While Yazgan and Ozturk (2019) admitted this and constructed the panel including mostly developed countries\textsuperscript{4}, we argue that the experience of the developing countries can vary considerably depending on how they manage their exchange rate determination, their trade agreements, and custom policies and practices\textsuperscript{5}. In the country-panel they considered, no allowance has been made to recognize country-specific exchange rate regime shifts (e.g., from a fixed exchange rate system to free-floating exchange rate system). The analysis, therefore, is limited to merely testing the J-curve hypothesis for the world as a whole without any reasonable country-specific policy implications.

\textsuperscript{3} Two popular time series methods used in this research are based on Johansen and Juselius (1990) cointegration methodology and Pesaran et al. (2001) autoregressive distributed lag (ARDL) methodology.

\textsuperscript{4} A few developing countries were included, e.g., China, India.

\textsuperscript{5} For example, India followed a fixed exchange rate system before 1993 (Patnaik 2007), the Chinese system is still a managed float after relaxing the fixed exchange rate regime in 2005 (Das 2019).
To be more specific, our research is focused on one country. We construct a unique panel data with Bangladesh’s trade with her 25 major trading partners, using Bangladesh as the reference country. We then construct the aggregate over all trading partners’ data with flexible weights of partner-specific trade share and present a comparative analysis of the aggregate and disaggregate estimates of the trade balance–exchange rate relationship. To avoid confusion regarding exchange rate regime shifts, we consider monthly data from 2003, from when the country has been under a flexible exchange rate regime. We found significant cross-section dependence, and the CCE-type estimators provide surprising results for Bangladesh: trade balance improves with currency appreciation in the long run. This unusual finding seems to support the theory presented in Arize et al. (2017): if exporters aim to maintain market share, they often increase the volume of exports with government support in response to a currency appreciation. The opposite does not happen with currency depreciation, which strengthens the importance of keeping the market share for exporters of the small country where they can have the opportunity to offset any margin reduction they apply during currency appreciation.

After this brief background, the paper is organized as below. Section 2 presents a brief literature review, Sect. 3 elaborates the model, econometric methods used and the data, Sect. 4 places a comparative discussion of the results from different estimation methods, Sect. 5 presents a discussion of the findings and Sect. 6 concludes the paper.

2 Literature review

Investigating aggregation bias in trade balance–exchange rate relation can be dated back to work by Rose and Yellen (1989), who dealt with aggregate and bilateral data for the US and strictly rejected the J-curve hypothesis. Bahmani-Oskooee and Brooks (1999) revisited the same problem as Rose and Yellen (1989) with advancements in methodology and found some evidence for the J-curve. Bahmani-Oskooee and Goswami (2003) highlighted the aggregation problem again, using quarterly data for Japan, presenting both separate bilateral and aggregated analysis. With the increasingly available data and econometric methodologies, a vast amount of literature emerged where bilateral analysis focuses on a specific country relationship with her major trading partners, covering both developed and developing countries as the country of interest (see, e.g., Bahmani-Oskooee and Hegerty 2010 for a survey). The studies differ in terms of their conclusion about the existence of the J-curve effect and its extent, and no consensus has been reached using the bilateral analysis, still leaving room for the debate of ‘aggregation bias’ to continue (e.g., Gürtler 2019).

In the search for evidence for and against the J-curve phenomenon, the focus sharply changed, recognizing the differences in currency appreciation effects than that from currency depreciation. Following the seminal work by Bahmani-Oskooee and Fariditavana (2015), bilateral analysis performed in the previous decade has been revisited for many countries allowing for asymmetric effects largely using the methodology proposed by Shin et al. (2014) (e.g., Bahmani-Oskooee and Arize 2019, Arize et al. 2017, Hunter 2019, Bahmani-Oskooee et al. 2018, Lee 2018, to name a few). The findings mostly confirm more evidence supporting the J-curve compared to the ‘aggre-
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The analysis using Bangladesh data follows no exception as above, from the use of aggregate time series analysis (e.g., Khatoon and Rahman 2009, Younus and Chowdhury 2014, Hassan et al. 2017), to asymmetric bilateral analysis (e.g., Bahmani-Oskooee et al. 2019) and asymmetric bilateral commodity-level analysis (Bahmani-Oskooee and Rahman 2017). The findings are mixed in support of the J-curve effect but indicate that more evidence in favor is found from asymmetric analysis compared to linear analysis.

To our knowledge, except for Yazgan and Ozturk (2019), there is no literature analyzing the exchange rate–trade balance relationship in a panel format, combining both aggregate and disaggregate analysis, and allowing for the presence of common factors. Yazgan and Ozturk (2019)’s work has a world perspective with 33 countries using the common correlated effects (CCE) estimator (Pesaran 2006). Pesaran (2006) introduced the CCE estimator in a static panel. Using this method, Yazgan and Ozturk (2019) found evidence for the J-curve effect as a global phenomenon without considering the dynamic structure of the relationship, the effects of structural breaks, and asymmetric adjustment possibilities. While it is interesting to find the global perspective allowing for country-specific heterogeneity, combining the experiences of small and large economies, developing and developed economies are often misleading. For individual country context, policy recommendations need to be based on a country-specific analysis of complex bilateral relations with various trading partners (Gürtler 2019). We aim to bridge this gap in the literature using Bangladesh as a case.

3 Variables and data sources

We start with the base model with four variables: trade balance, real exchange rate, domestic and foreign GDP, following the literature (e.g., Yazgan and Ozturk 2019, Lucarelli et al. 2018, Rose and Yellen 1989). We measure trade balance (TB) as the ratio of exports to imports of a country to facilitate taking logs by avoiding negative numbers. The log of the trade balance is denoted as LTB. This includes the trade balance of goods only. This definition aligns with our definition of the real exchange rate (RER) as Bangladeshi Taka per foreign currency, where we define depreciation of Bangladeshi Taka as an increase in RER. RER is calculated following the method in Khatoon and Rahman (2009), as the monthly trade-weighted real exchange rate of each partner countries:
\[ RER_{it} = E_{it}^{BD} \left[ CPI_{it}^f / E_{it}^f \right] / CPI_{it}^d, \]

where \( RER_{it} \) = real exchange rate of Taka against partner \( i \)’s currency, \( E_{it}^{BD} \) = exchange rate of Taka against US dollar, \( CPI_{it}^f \) = over time Consumer Price Index (CPI) of the \( i \)th partner country, \( E_{it}^f \) = over time exchange rate of the \( i \)th partner country’s currency against US dollar, and \( CPI_{it}^d \) = over time CPI of Bangladesh. The log of RER is used, denoted as LRER.

In Bangladesh, the GDP is only measured annually. In order to use data with a higher frequency to capture the dynamics of the exchange rate, we used the monthly industrial production index as a proxy to both domestic and foreign GDP.\(^6\) It is to note that rather than modeling a conventional country panel, our goal is to use Bangladesh’s case of the trade balance–exchange rate relationship, where we need the partners’ relations with Bangladesh. To allow for a disaggregated analysis within the same framework, we constructed a panel data of major trading partners of Bangladesh observed over time. The trade balance, RER, and foreign GDP data were readily available in the panel format, whereas the domestic (Bangladesh) GDP data was the same for all 25 partners. To overcome this limitation, we transformed the basic model and included relative GDP (RGDP): foreign GDP to domestic GDP. The inclusion of the variable

\[ RGDP_{it} = \log \left( \frac{GDP_{it}^F}{GDP_{it}^D} \right), \]

compared to the log of domestic GDP and the log of the foreign GDP restricts the model: the absolute values of the domestic and foreign income elasticity of trade balance are equal. This restriction turned out to be an empirical regularity for Bangladesh (see, e.g., Iqbal et al. 2019), and the results in the aggregate time series analysis using the standard model are statistically equivalent to the one used in here.\(^7\) The relative GDP can increase by either increasing foreign GDP, decreasing domestic GDP, or both and vice versa. Therefore, the interpretation of the coefficient of RGDP is in line with the theory; a positive sign is expected given that the increase in foreign GDP (or a decrease in domestic GDP) will favor exports and thereby increase trade balance. This transformed model allows us to present the Bangladesh context in her trading-partner-based disaggregate form to construct a panel time series.

All the aggregate variables are formed by trade-weighted totals of the disaggregated variables, thereby providing a scope to analyze aggregation bias, if any.

We highlight the importance of gravity model factors in explaining the trade dynamics of a developing country like Bangladesh, situated geographically to the east (e.g., Khan and Hossain 2010). We include the log of import weighted distance (LMWD) as a proxy to transportation cost in Bangladesh’s trade balance equation. The distances

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\(^6\) The rationale behind taking industrial production (IP) as a proxy of GDP for both domestic and foreign is that (1) The index of industrial production is an index of physical output while real GDP is a measure of the value of output. So, real GDP is largely determined by industrial production. (2) There is a high positive correlation between real GDP and IP; see, e.g., Bahmani-Oskooee and Kutan 2009). Besides, Bangladesh’s GDP is measured only annually, and industrial production is measured monthly.

\(^7\) Detailed results are available upon request.
are approximated by Bangladesh’s capital city Dhaka to the partner country capital city in kilometers.\(^8\)

To analyze possible asymmetric effects of exchange rate depreciation and appreciation on trade balance, we further construct two variables, POS and NEG, respectively, as the positive and negative partial sum decomposition of the real exchange rate (LRER) variable following Bahmani-Oskooee and Fariditavana (2015):

\[
\begin{align*}
POS_{it} & = \sum_{j=1}^{t} \Delta LRER_{ij}^+ = \sum_{j=1}^{t} \max(\Delta LRER_{ij}, 0), \\
NEG_{it} & = \sum_{j=1}^{t} \Delta LRER_{ij}^- = \sum_{j=1}^{t} \min(\Delta LRER_{ij}, 0).
\end{align*}
\]

where given the real exchange rate is measured as Taka per foreign currency, \(POS_{it}\) measures depreciation of Bangladeshi Taka and \(NEG_{it}\) measures Taka appreciation.\(^9\)

We report their Kernel density functions compared to the normal distribution in Fig. 4, where it shows that \(NEG_{it}\) has a slightly positively skewed distribution compared to \(POS_{it}\). We chose 25 major trading partners of Bangladesh of which the top ten partners are: Hong Kong, China, France, Germany, India, Japan, Korea, UK, USA, and Singapore. These 25 countries are selected based on average trade volume for consideration from 2003 to 2017. These countries cover, on average, 68% of the total trade of Bangladesh. Figure 1 presents the comparative time series of the overall trade balance of Bangladesh and trade balance with her 25 major trading partners. The figure shows some spikes in all partner trade balance data not used here for the analysis. The 25 partners trade balance series (the solid line) has more stability than the entire series, reflecting no significant possibility of structural changes in the data under consideration.\(^10\) Figure 2 shows the partner-specific LTB data where it seems that the trade balance with some partner countries is more volatile than the rest of the partners. We think this is more likely due to the type of commodity trade with these countries.\(^11\) The disaggregated LRER data in Fig. 3 does not have any notable features, reflecting moderate fluctuations and overtime slight appreciation patterns.

We used data from the International Financial Statistics (IFS), The Direction of Trade Statistics (DOTS), both published by International Monetary Fund (IMF), and the country-specific national websites to fill in a few missing observations (e.g., Singapore IP index from the government of Singapore website\(^12\)). All the variables are

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8 https://www.distancecalculator.net/.
9 To our understanding, these are not equivalent to simple dummy variables to capture currency appreciation and depreciation effects, as the real exchange rate variable is nonstationary, \(I(1)\) and a binary transformation will not preserve the dynamics of the variables.
10 The CUSUM test results reported in Sect. 5 show no major parameter instability in the aggregate time series model in both linear and nonlinear cases.
11 We need to do some volatility spillover analysis to comment on this, which we think is out of scope of the present paper.
12 https://www.singstat.gov.sg/find-data/search-by-theme/industry/manufacturing/latest-data.
Fig. 1  Trade balance of Bangladesh, total compared to that with her top 25 trading partners

Fig. 2  Log of trade balance (exports to imports ratio) of Bangladesh with her top 25 trading partners

converted to index with January 2015 as the base. There is no seasonality found in the variables.\textsuperscript{13}

4 The model and methods

As discussed in Sect. 2, the popular method used in the literature for trade balance–exchange rate relationship analysis is cointegration, using either aggregate time series,\textsuperscript{13} Detailed results are available upon request.
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Fig. 3 Log of real exchange rate of Bangladeshi Taka with her trading partner currency

bilateral time series, or panel time series data. The most common cointegration method used is the autoregressive distributed lags (ARDL) models, both in linear (Pesaran et al. 2001) and in the asymmetric form (Shin et al. 2014), for its flexibility in dealing with a mixture of stationary and nonstationary variables. We aim to use the ARDL model too, as Table 4 shows that the variables under consideration are a mixture of stationary and nonstationary variables, I(0) and I(1), and varies depending on the unit root tests.

In the long panel literature, the cross-section dependence, often defined as the interdependence across partners in this context (Yazgan and Ozturk 2019), gained popularity following the Common Correlated Effects (CCE) estimator proposed by Pesaran (2006). We expect that Bangladesh’s trade with her partners will have some common factors together with some partner-specific heterogeneity. While the CCE in Pesaran (2006) is proposed for static model, we found the ideal model to analyze the relationship between Bangladesh’s trade balance and exchange rate as the common correlated effects estimators for dynamic panel data allowing for cross-section dependence, if any (CS-ARDL hereafter) (Chudik and Pesaran 2015, Chudik et al. 2016, Ditzen 2018).14 The method allows us to estimate the aggregated and disaggregated effects and assuming symmetric and asymmetric effects of currency appreciation and

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14 Note that the Cross-sectionally augmented Distributed Lags (CS-DL) model (Chudik et al. 2016) is not consistent if there is feedback effect, that is, reverse causality, and (or) if there are a mixture of stationary and nonstationary variables. Given that Bangladesh is under a floating exchange rate regime, we cannot ignore the possibility of reverse causality with some controversy of managed float. The central bank interventions in the floating exchange rate regime imply that the trade balance changes may affect the real exchange rates, too.
depreciation on Bangladesh’s trade balance. We discuss the method in detail and justify the suitability of the CS-ARDL estimator in this context.

4.1 CS-ARDL methodology

Here, we present the CS-ARDL methodology using general notations following Chudik and Pesaran (2015). Suppose that the dependent variable is $y$ and we have one regressor $x$. The heterogeneous coefficients dynamic panel data model is given as:

\[
\begin{align*}
y_{it} &= \alpha_i + \lambda_i y_{i,t-1} + \beta_{0i} x_{it} + \beta_{1i} x_{i,t-1} + u_{it} \\
u_{it} &= \gamma_i f_t + e_{it}
\end{align*}
\]  

(1)

where $\alpha_i$ are individual fixed effects, $f_t$ is a vector of unobserved factors common for each cross-sectional unit, $\gamma_i$ is the heterogeneous factor loading, and $e_{it}$ is the iid error term. If the unobserved common factors, $f_t$, are correlated with the regressors, ignoring this can lead to biased estimates. The CCE estimator proposed by Pesaran (2006) is based on approximating $f_t$ by cross-section averages of the regressors ($\bar{y}_t$, $\bar{x}_t$). The estimators is consistent under the assumption of strict exogeneity of the regressors, which is violated in the above model (1) due to the dynamic structure with the presence of the lagged dependent variable. Chudik and Pesaran (2015) show that the common correlated effects estimator yields to consistent and unbiased estimates in a panel with weakly exogenous regressors by adding $T^{1/3}$ lags of the cross-sectional averages\(^\text{15}\):

\[
\begin{align*}
y_{it} &= \alpha_i + \lambda_i y_{i,t-1} + \beta_{0i} x_{it} + \beta_{1i} x_{i,t-1} + \sum_{l=0}^{T/3} \delta_{il} \bar{Z}_{t-l} + e_{it} \\
\end{align*}
\] 

(2)

where $\bar{Z}_t = [\bar{y}_t, \bar{x}_t]$. Chudik et al. (2016) show that the dynamic CCE estimator derived in Chudik and Pesaran (2015) is valid for the estimation of long run coefficients as well. Following them, the error correction form of Eq. (2) can be presented as:

\[
\begin{align*}
\Delta y_{it} &= \phi_i (y_{i,t-1} - \theta_{1i} x_{i,t-1}) + \alpha_i + \beta_{0i} \Delta x_{it} + \sum_{l=0}^{T/3} \delta_{il} \bar{Z}_{t-l} + e_{it} \\
\end{align*}
\]  

(3)

with $\phi_i = (1 - \lambda_i)$, the error correction speed of adjustment parameter with an expected negative sign, $(y_{i,t-1} - \theta_{1i} x_{i,t-1})$ is the error correction term, $\Delta$ refers to the first difference of the respective variables, and $\theta_{1i} = (\beta_{0i} + \beta_{1i})/\phi_i$ and $\beta_{0i}$ are the long run and short run coefficients, respectively. In line with Chudik and Pesaran (2015)\(^\text{16}\) and Ditzen (2018), we assume that the heterogeneous coefficients follow a random coefficient model which provides us both the aggregate and disaggregated results from one regression. The model can be generalized including additional lags.

\(^{15}\) Theorem 1 in Chudik and Pesaran (2015).

\(^{16}\) See Assumption 4 in Chudik and Pesaran (2015).
of the dependent variable and the regressors, as well as observed common effects and deterministic intercepts and time trends (Chudik et al. 2017). Chudik and Pesaran (2015) specify the ARDL formulation using same lag orders for the dependent variable and the regressors to avoid potential problems of persistence. Here, we follow their structure.

4.2 Trade balance model

In our initiative to identify both aggregate and disaggregate effects of real exchange rate on the trade balance of a specific country, Bangladesh, under consideration, we begin to build up our model keeping the questions in mind: is there any unobserved common factors? If so, what are they? Are these common factors causing cross-sectional dependence? We believe our analysis of one country’s effect implies some common factors, e.g., several macroeconomic factors including the trade and financial integration state of Bangladesh affecting all her partners, and some partner-specific heterogeneous effects due to the separate bilateral trade agreements in place. We test the existence of weak cross-section dependence following Pesaran (2015), in the variables (Table 3) as well as, by estimating separate bilateral linear and asymmetric (nonlinear) ARDL models. As shown in Table 3, we reject the null hypothesis of weak cross-section dependence when the test is applied to the variables series. We use the specification in Eq. (1) for the asymmetric ARDL models using bilateral time series, ignoring the $i$ subscripts and assuming $\gamma'_i f_t$, a constant.

Following Chudik and Pesaran (2015) and Chudik et al. (2013), and the error correction representation in Eq. (3), our trade-balance exchange-rate model in the linear form is represented below:

$$\Delta LT_{Bi} = \phi_i (LT_{Bi}, -\theta_1 \Delta LRE_{R,t-1} - \theta_2 \Delta RGDP_{i,t-1} - \theta_3 \Delta LMWD_{i,t-1}) + \alpha_i$$

$$+ \sum_{k=0}^{K} \beta_{1ki} \Delta LRE_{R,t-k} + \sum_{k=0}^{K} \beta_{2ki} \Delta RGDP_{i,t-k} + \sum_{k=0}^{K} \beta_{3ki} \Delta LMWD_{i,t-k}$$

$$+ \sum_{l=0}^{\rho_T} \delta_{il} \Delta \bar{Z}_{it-l} + e_{it}$$

(4)

where the variables are as defined in Sect. 3, $\bar{Z}_t = [LT_{Bi}, LRE_{R,t}, RGDP_{i,t}, LMWD_{i,t}]$, are the cross-section averages of the dependent variable and the other (strongly or weakly) exogenous variables. The optimum number of lags $\rho_T = \sqrt{T} = \sqrt{170} = 5.54 \approx 6$ of the cross-section averages are included to achieve consistency (Chudik and Pesaran 2015).

In the above notations, $i = 1, 2, \ldots, 25$ refers to the 25 major trading partners of Bangladesh, $t$ refers to the period between June 2003 to November 2017 under the flexible exchange rate regime. By construction, $\phi_i$ is the error correction speed of the adjustment parameter, and $\Delta$ refers to the first difference of the respective variables. $k = 0, 1, \ldots, K$ are the lag orders to capture short run dynamics, set to be the same 17 Not presented here, available from the authors upon request.
for all variables (as CS-ARDL is consistent irrespective of lag order selection, Chudik et al. 2016). The lag orders are determined to apply a general-to-specific methodology to ensure that the resulting error term does not have strong cross-sectional dependence (Pesaran 2015, Ditzen 2018).

Among others, Bahmani-Oskooee and Fariditavana (2015) extended the linear autoregressive distributed lag (ARDL) model to one of the nonlinear ARDL models following Shin et al. (2014) by splitting the exchange rate variable into appreciation-only and depreciation-only components. We extend the CS-ARDL model of Eq. 4 to incorporate this as below:

\[
\Delta LT B_{it} = \phi_i (LT B_{it-1} - \theta_1 POS_{i,t-1} - \theta_2 NEG_{i,t-1} - \theta_3 RGDP_{i,t-1} - \theta_4 LMWD_{i,t-1}) + \alpha_i \\
+ K \sum_{k=0}^{K} \beta_{1ki} \Delta POS_{it-k} + K \sum_{k=0}^{K} \beta_{2ki} \Delta NEG_{it-k} + K \sum_{k=0}^{K} \beta_{3ki} \Delta RGDP_{it-k} \\
+ K \sum_{k=0}^{K} \beta_{4ki} \Delta LMWD_{it-k} + \rho \delta_i \tilde{Z}_{t-l} + e_{it}
\]  

(5)

where $\tilde{Z}_t = \{LTB_t, POS_t, NEG_t, RGDP_t, LMWD_t\}$.

We use the cross-sectionally augmented panel unit root (CIPS) test (Pesaran 2007) of the CS-ARDL residuals to test for cointegration.

Table 6 reports the long run estimates and the residual cross-section dependence test p values for different lag orders $K = [1, 2, \ldots, 12]$ \(^{18}\), for both CS-ARDL and asymmetric CS-NLARDL models. Except for the lag order 8 to 12 in CS-NLARDL models, all residuals are rejected to have strong cross-sectional dependence, using the weak cross-section dependence test proposed by Pesaran (2015). The test uses the null hypothesis that the series is weakly cross-sectionally dependent against the alternative that the cross-section dependence is strong.

The coefficients are not very sensitive to the choice of the lag order. For example, the coefficients of $POS$ are not statistically significant in any models considered, while the coefficients of $LRER$ are significant at only 10% level. $NEG$ is significant in all lags, except in lag 6 in the asymmetric model, is positive, and ranges between 0.585 and 1.617. Import weighted distance $LMWD$ is negative and statistically significant in all cases considered. The error correction speed adjustment coefficient is negative, less than one, and statistically significant in all settings. The relative GDP in the linear model is significant, while the asymmetric model becomes significant in lag 5. Based on the results, we choose $K = 5$ in our model. We keep the lag order the same for the linear and the asymmetric models to facilitate comparison. The $p$ values in Table 6 reflect that with $K = 5$, we cannot reject the null of weak dependence at even 10% level of significance for both linear and the asymmetric models.

\(^{18}\) It is conventional to use 12 lags for monthly data.
5 Results

We present two types of analysis here. We focus on the panel time series analysis, which yields both aggregate and partner country-specific estimates. For comparison, we offer time series analysis, both aggregate and bilateral, too. Aggregate time series analysis is based on the variables’ aggregate measures: a trade-weighted sum of the trade balance\(^{19}\), relative GDP, real exchange rate index, and the import weighted distance of the top 25 trading partners of Bangladesh.

We start with the bilateral time series analysis of the asymmetric ARDL model following Bahmani-Oskooee et al. (2019) and Bahmani-Oskooee and Kanitpong (2017), presented in Table 5. Based on the Pesaran et al. (2001) bounds test, each of the analysis with 25 trading partners shows a cointegrated relationship between trade balance and real exchange rate. In the long run, depreciation of Bangladeshi Taka against the Indian rupee, US dollar, Brazilian real, Norwegian krone, Russian ruble, and Turkish lira improves Bangladesh’s trade balance significantly. Appreciation of Taka against the Indian rupee and US dollar improves trade balance, whereas that against the UK pound hurts the trade balance. These results are not the same, but partly agree with Bahmani-Oskooee et al. (2019).\(^{20}\) Based on the Wald test of parameter equality, our results suggest that the asymmetric effect, in the long run, exists for some partners including China, Germany, India, Korea, and the UK. However, none of these results are reliable, as in all variables considered, there is strong evidence of cross-sectional dependence.

Columns 3 and 4 of Table 1 present aggregate results from Panel time series estimators: CS-ARDL and CS-NLARDL, which corrects for cross-section dependence. As discussed in Sect. 4.2, we use five lags in the short run model and six lags of the cross-section averages to control for the unobserved common factors. The linear model with Taka’s real exchange rate against Bangladesh’s 25 trading partners shows no statistically significant effect of real exchange rate depreciation on the trade balance in the long run. However, there is a significant negative effect in the short run with a 3-month lag and an error correction adjustment factor of 0.637, showing it takes less than two months to absorb the negative influence of depreciation in Bangladesh. No significant long run effect is not a surprise, as the central bank still keeps managing Bangladesh’s competitiveness by regular interventions (Bank 2019). The aggregate time series estimate of the linear model (column 1) shows a similar effect in magnitude in the long run which is not statistically significant as well. We include the impulse response function for the response of LTB to a one standard deviation impulse in LRER following the linear model in Fig. 6 simply for illustrative purposes.\(^{21}\) The figure shows some deterioration following currency depreciation and some improvement in the first couple of months, without any significant long run improvement in trade balance.

To investigate the asymmetric effects of currency appreciation and depreciation, the cross-sectionally augmented asymmetric ARDL estimates are presented in col-

\(^{19}\) See ‘Appendix A’ for the aggregation method used.

\(^{20}\) It is to note that there are some differences in the set of partner countries that Bahmani-Oskooee et al. (2019) considered to our set. They did not consider the regime shift from fixed to flexible and the import-weighted distance variable, which we found highly statistically significant in all partner cases except China.

\(^{21}\) The statistical foundations of impulse response functions from an ARDL or CS-ARDL model have not been established in the literature, to our knowledge.
Table 1  Aggregate ARDL estimation results

| Variables | Time series | Panel time series |
|-----------|-------------|------------------|
|           | (1)         | (2)              | (3)              | (4)              |
| Linear\(^a\) | NL\(^b\) | CS-ARDL\(^c\) | CS-NLARDL\(^d\) |

Panel A: Long Run\(^e\)

|          | \(lrer_t\) | \(pos_t\) | \(neg_t\) | \(rgdp_t\) | \(lmwd_t\) |
|----------|------------|-----------|-----------|------------|------------|
|          | 0.649      | 0.983*    | −0.030    | −0.321     | −0.275     |
|          | (0.572)    | (0.512)   | (0.838)   | (0.369)    | (0.674)    |

Panel B: Diagnostics

|          | \(ECM_{t-1}\) | Cointegration\(^f\) | Observations | R-squared | Number of groups | Wald: \(p\) value LR\(^g\) | Wald: \(p\) value SR\(^h\) | CD: \(p\) value\(^i\) |
|----------|----------------|---------------------|---------------|-----------|------------------|--------------------------|--------------------------|---------------------|
|          | −0.224***     | Yes                 | 170           | 0.371     | 25               | 0.106                    | –                        | 0.227               |

Standard errors in parentheses. *** \(p < 0.01\), ** \(p < 0.05\), * \(p < 0.1\)

\(^a\) Linear ARDL estimate based on the aggregate time series data. The country-specific variables are combined together with their trade-weighted shares to generate the aggregate time series variables

\(^b\) Asymmetric ARDL estimate on aggregate time series data

\(^c\) Cross-sectionally augmented ARDL estimator (Chudik et al. 2016, Ditzen 2018). The cross-section averages each has \(\sqrt{T} \approx 6\) lags. The short run lags (5, same for each variable) are chosen based on the general-to-specific methodology to ensure that there is no strong cross-sectional dependence (Pesaran 2015)

\(^d\) Asymmetric Cross-sectionally augmented ARDL estimator (Chudik et al. 2016, Bahmani-Oskooee and Kanitpong 2017). The lags are the same as the linear model (column 3), selected in the same way

\(^e\) The table reports the long run estimates. The full table including short run coefficients is available from the authors upon request

\(^f\) Cointegration is tested using Pesaran et al. (2001) for time series models and using Pesaran (2007) for the panel time series models

\(^g\) \(P\) value of the Wald test statistic of symmetry of long run coefficients. In the asymmetric ARDL analysis, we test the coefficients of \(POS\) and \(NEG\) in the long run, with the null hypothesis that they are the same

\(^h\) \(P\) value of the Wald test statistic of symmetry of cumulative short run coefficients. In the asymmetric ARDL analysis, we test the coefficients of Eq. (5): \(\sum_{k=0}^{7} \beta_{1ki} = \sum_{k=0}^{7} \beta_{2ki}\) in the short run: testing short run cumulative symmetry

\(^i\) \(P\) value of the weak cross-section dependence test (Pesaran 2015, Ditzen 2018), the null hypothesis is the errors are weakly cross-sectionally dependent, with the alternative that they are strongly cross-sectionally dependent
umn (4) of Table 1. Similar asymmetric effects estimation obtained from aggregate time series data is given in column (2) for comparison. Interestingly, there is clear disagreement between the results of the aggregate time series and the panel time series analyses in terms of the signs and statistical significance of the coefficients of currency appreciation ($NEG$) and depreciation ($POS$). While the asymmetric time series model ignoring cross-section dependence supports J-curve for Bangladesh in the aggregate level, the CS-NLARDL model yields a positive long run effect of currency appreciation ($NEG$) on the Bangladesh trade balance. This finding based on the CS-NLARDL estimate contradicts the theory that currency appreciation increases export competitiveness here rather than decreasing it for Bangladesh. The findings indicate that without considering the cross-section dependence, the asymmetric aggregate time series analysis overestimates the effects of currency depreciation (statistically significant under time series, but not under panel time series estimates) underestimates the impact of currency appreciation in the present context. Our estimates are robust to different lag orders in CS-ARDL and CS-NLARDL models (Table 6).

Following Bahmani-Oskooee et al. (2019), we conducted Wald tests to test for the presence of asymmetric effects using the estimated coefficients of $POS$ and $NEG$ in both columns 2 and 4 of Table 1. The time series coefficients do not reject the Wald test null hypothesis of equality ($p$ value presented in Panel B). For the panel time series CS-NLARDL estimates, there is not enough precision in the $POS$ coefficient estimate, but still we reject the null supporting evidence for an asymmetric effect. Comparing the estimates presented in columns 2, 3, and 4, it is evident that there is an asymmetric effect of currency appreciation and depreciation in the case of Bangladesh, with currency appreciation having a positive impact and depreciation having no significant impact on the trade balance in the long run. We also report the $p$ value of the Wald test for cumulative short run estimates of the CS-NLARDL model, which suggests that the cumulative effects of currency appreciation and that of currency depreciation in the short run are not statistically different.

In our analysis of the other coefficients, we focus on column 4, the CS-NLARDL estimates, as the time series estimates in columns 1 and 2 are more likely to be biased, and there seem to be asymmetric effects. The coefficient of foreign to domestic GDP is positive as expected and is statistically significant in both the short and long run. The negative and significant impact of import weighted distance is as expected; distance plays an important role in Bangladesh’s trade relations, which is also evident from the composition of her major trading partner countries; six out of 25 are located in the same continent, Asia.

Panel B of Table 1 presents the error correction speed of the adjustment coefficient. The speed of adjustment in presence of any shock is $-0.699$ in our chosen model, which implies that it takes approximately a month and a half to reach the equilibrium. The coefficient is statistically significant, following Pesaran et al. (2001), which implies that the system represents cointegration.\footnote{Also, the F test for the joint hypothesis that the coefficients of the level variables are simultaneously zero is rejected significantly.} In line with Yazgan and Ozturk (2019), we use the simple cross-sectional dependence augmented DF test (CADF)
which confirms that the residual from CS-NLARDL is stationary, to support cointegration.23

We now focus on the disaggregate, country-specific analysis. Table 2 presents the long run country-specific estimates of the CS-NLARDL model, obtained together with the aggregate effects. Among the 25 partner countries considered, only depreciation against Malaysia and Mexico and appreciation against China, Japan, and Singapore currency seems to have a significant long run effect on the trade balance. The effects are positive for currency appreciation and negative for depreciation, as opposed to the theory. In the short run, depreciation against the Singapore currency has a high positive significant impact on the Bangladesh trade balance, which gets mostly opposed by the adverse effects it causes against Korea, China, Brazil, and Russia. A robust positive effect of currency appreciation in the short run is seen to be in place against India, Brazil, and Malaysia in two months’ time lag but then fades away with similar negative effects against Singapore and Korea within four months. The primary role players in the trade balance–exchange rate relationship of Bangladesh are, therefore, China, Japan, and Singapore, with some influence from Malaysia, Mexico, Brazil, and Korea.

Our findings above compared with the conclusions from bilateral time series analysis in Table 5, is quite different, where Korea did not have any significant long run effect, and Singapore did not have any significant short run effect of currency depreciation or appreciation on the trade balance. The major role players in affecting Bangladesh’s trade balance via exchange rate movements using bilateral time series are India, UK, USA, Canada, Russia, Sweden, and Turkey, with significant coefficient estimates for both currency appreciation and depreciation. Our findings using monthly data and the period under flexible exchange rate regimes are not in line with those of Bahmani-Oskooee et al. (2019), using quarterly data and a period covering a mixture of Bangladesh’s exchange rate regimes. Though Bahmani-Oskooee et al. (2019) found some evidence of the conventional J-curve-type relationship in the case of Bangladesh, their findings probably suffer from biases caused by common observed and unobserved factors and ignoring the impact of transportation cost in international trade.

The coefficient of foreign to domestic GDP in the disaggregated form comes out to be significantly negative for Singapore, positive for Brazil, Malaysia, and Russia, and not significant for the other trading partners under consideration. The import weighted distance is significantly negative in most countries. These findings worth further discussion in line with the country-specific trade composition of Bangladesh. The error correction terms are all individually statistically insignificant, reflecting the aggregate nature of the analysis of the heterogeneous effects, where, as a whole, the stability of the system is confirmed in Table 1, Panel B.

The comparative linear CS-ARDL model estimates are given in Appendix Table 7, for completeness of the analysis. The dominance of India, Japan, and Singapore in both the long run and the short run is evident in the linear model. An increase in $LRER$, referring to currency depreciation, has a positive effect against India, Japan, Turkey, and Singapore in the long run. Combining the short run and long run effects based on the estimates obtained from the linear CS-ARDL model, we see that the J-curve

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23 To confirm panel cointegration in presence of cross-section dependence, we used the panel cointegration test by Westerlund (2007) (Persyn and Westerlund 2008) separately, available from the authors upon request.
| Variables | (1)  | (2)  | (3)  | (4)  | (5)  | (6)  | (7)  | (8)  | (9)  | (10) | (11) | (12) | (13) |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| Hong Kong | 2.227| −1.155| 2.467| −1.855| −3.330| 1.183| 1.294| −2.334| 0.664| 1.646| −6.482| −2.988| −0.369|
| China     | 2.723| (2.636)| (2.574)| (3.061)| (2.898)| (1.155)| (1.212)| (2.070)| (2.365)| (2.444)| (4.465)| (2.523)| (4.458)|
| France    | 2.749| 3.616*| −0.969| −1.829| 0.988| 2.333***| −1.007| 0.496| 1.870| 4.988**| −0.442| 0.929| 1.090|
| Germany   | 1.936| −1.099| 1.716| 1.267| −0.666| 1.025| −1.594| 0.003| −0.580| −2.056***| 1.821| 6.759***| −2.428|
| India     | (3.323)| (1.074)| (2.616)| (3.290)| (0.904)| (1.533)| (2.114)| (3.636)| (3.121)| (0.791)| (3.188)| (1.659)| (3.852)|
| Japan     | −1.714***| −0.672| −0.933*| −0.953| −0.721| −1.645***| −0.798**| −1.404***| −0.998*| −0.235| −1.068***| −1.370***| −1.210*|
| Korea     | (0.627)| (0.968)| (0.487)| (0.660)| (0.997)| (0.552)| (0.325)| (0.482)| (0.575)| (0.223)| (0.314)| (0.250)| (0.621)|
| UK        | −0.457| −0.718| −0.699| −0.576| −0.604| −0.829| −0.858| −1.012| −0.668| −1.035| −0.644| −0.471| −0.548|
| USA       | (12.818)| (8.166)| (7.560)| (7.763)| (8.596)| (4.020)| (5.725)| (5.074)| (5.803)| (8.995)| (33.996)| (12.930)| (12.375)|
| Singapore | r GDP         | −0.574| 0.143| 0.629| −3.767*| −2.946*| 2.992| 0.973| −2.373| −3.761| 3.122| 0.596|
| Belgum    | (3.376)| (2.434)| (2.651)| (3.923)| (1.973)| (1.717)| (4.397)| (2.344)| (2.974)| (3.713)| (2.946)| (0.625)|
| Brazil    | −0.142| 0.276| −0.276| −0.567| −1.281| 0.699| 3.273| −0.488| −0.806| 0.135| 0.065| 0.495|
| Canada    | (1.879)| (1.673)| (2.161)| (2.457)| (1.100)| (1.991)| (2.735)| (1.549)| (2.165)| (1.844)| (2.070)| (0.774)|
| Egypt     | −1.065| 1.028| 0.309| 0.907| 4.312***| 0.512| 1.164| 8.547**| 1.076| 0.443| −2.340| 1.356|
| Indonesia | (1.878)| (1.651)| (0.703)| (2.253)| (1.388)| (4.533)| (1.493)| (4.041)| (1.320)| (1.437)| (1.946)| (1.565)|
|        | Denmark | Finland | Ireland | Italy | Malaysia | Mexico | Norway | Russia | Spain | Sweden | Turkey | Ukraine |
|--------|---------|---------|---------|-------|----------|--------|--------|--------|-------|-------|--------|---------|
| $lm w_{t-1}$ | $-0.799^{**}$ | $1.058^{***}$ | $0.869^{***}$ | $-0.792$ | $1.044^{***}$ | $1.403^{***}$ | $-0.953^{***}$ | $-0.973^{***}$ | $-0.955^{**}$ | $1.072^{***}$ | $-0.857^{***}$ | $-0.970^{***}$ |
|        | (0.351) | (0.168) | (0.228) | (0.273) | (0.131) | (0.276) | (0.205) | (0.254) | (0.409) | (0.334) | (0.299) | (0.120) |
| $ECM_{t-1}$ | $-0.690$ | $0.820$ | $0.672$ | $-0.508$ | $1.191$ | $-0.531$ | $-0.598$ | $-0.530$ | $-0.696$ | $-0.687$ | $-0.520$ | $-0.901$ |
|        | (2.764) | (1.631) | (0.822) | (3.889) | (10.019) | (6.449) | (19.914) | (34.115) | (6.947) | (13.982) | (8.723) | (1.942) |

Standard errors in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Cross-sectionally augmented asymmetric ARDL estimator (Chudik et al. 2016, Ditzen 2018); disaggregated results obtained together with the results in column 4 of Table 1. The cross-section averages each has $\sqrt{T} \approx 6$ lags. The short run lags, 5, are the same for each variable. The table reports the long run estimates only; short run estimates are available from the authors upon request.
phenomenon is in place against India and to some extent against Japan, Turkey, and Singapore.

6 Discussion

Some of the findings presented in Sect. 5 are worth further discussion. First of all, we found that currency appreciation improves Bangladesh’s trade balance in the long run, which goes against the Marshall–Lerner condition. Specifically, currency appreciation is expected to reduce a country’s relative export competitiveness against her trading partner, increase imports, thereby deteriorating her trade balance in the long run. In our setting using the CS-NLARDL approach, we found the opposite, a one percent increase in \( \text{NEG} \) (Taka appreciation) improves trade balance by 0.65 percent in the long run. One possibility can be that Bangladesh’s export and import elasticities are low and mostly based on long term bilateral agreements. In general, being a small developing country with low bargaining power, Bangladesh’s trade policies, market development, technology transfers, capital accumulation, and direct investment do not exhibit much changes over time. In 2015\(^{24}\), the highest share of the country’s import goes to the intermediate goods (46% of total imports), where 95% of the exports are consumer goods. All of the top 5 export commodities in the six-digit level are ready-made garments, primarily exported to the USA, UK, and Germany, based on the long term trading agreements between the supermarkets and the factory owners. The ready-made garments exports contribute to a large part of the country’s imports of raw materials, mainly from China, India, and the USA. In effect, if Bangladesh’s export relies on her imports to a large extent, it is more likely to be inelastic, and we are more likely to observe slow responses in trade balance following currency fluctuations. This is supported by the theory presented in Arize et al. (2017), where exporters tend to stay if the currency strengthens and often increases the volume of exports with government support and (or) by lowering their export prices and markups to maintain market share. There is further evidence that supports our findings of Taka appreciation, improving Bangladesh’s trade balance. When analyzed commodity-specific asymmetric effects for Bangladesh–US relations, Bahmani-Oskooee and Rahman (2017) found evidence of Taka appreciation against the US dollar improving Bangladesh trade balance for the clothing industry, and the improvement is more than that from Taka depreciation. This finding implies clothing exports from Bangladesh increases following Taka appreciation more than that following Taka depreciation.\(^{25}\) On the other hand, they also found that Taka appreciation hurts Bangladesh’s trade balance for cotton and fabrics, implying increased import of the raw materials to support her exports.

Looking further into the composition of Bangladesh’s exports and imports, the top import commodity (6 digits) is fuel, where half of the fuel is imported from Singapore. Narayan (2013) discusses that oil price predicts exchange rate returns for Bangladesh. According to them, a rise in international oil price will cause Taka to appreciate. However, Bangladesh provides subsidies to oil prices to bring them closer to the world market price, which is more likely to prevent imports from rising (or

\(^{24}\text{The most recent data available from the World Integrated Trade Solution, the World Bank}\)

\(^{25}\text{Note that Bahmani-Oskooee and Rahman (2017) used opposite definition of } RER, POS, \text{ and } NEG \text{ than ours, and reported the data by US.}\)
rather fall) following an appreciation and vice versa. This may help explain the highly significantly positive short run coefficients of Taka depreciation against the Singapore dollar. However, we cannot come to a precise conclusion without further research on this possibility and is therefore left for future research.

Another finding regarding the negative effect of foreign to domestic GDP in case of China and Singapore worth discussing in this connection. We expect the relative GDP to have a positive sign, as the increase in foreign GDP over domestic GDP is expected to improve trade balance by increasing exports and reducing imports. However, Bangladesh’s GDP growth depends on raw material imports from China and fuel imports from Singapore. If the domestic GDP growth slows down compared to foreign GDP growth (thereby increasing $RGDP$), Bangladesh would need to increase imports from China and Singapore to give the economy the necessary supply of raw materials and energy. Again, a combined analysis and a disaggregated commodity-level investigation are needed to confirm this, which is out of the scope of this research.

Looking into these surprising findings from the methodological perspective, we agree that we have a small cross-section sample ($N = 25$) and the CS-ARDL methodology relies on $N, T \sim \infty$ asymptotics with $T^{1/3}$ lags of the cross-section averages (Chudik and Pesaran 2015). However, the simulation results in Chudik et al. (2016) show that the bias is sensitive to $T$ and to a lesser extend to $N$. For the mean group estimator in a setting without strong cross-section dependence, Chudik and Pesaran (2019) report significant size distortions for sample sizes where the ratio $N/T$ is not sufficiently small and the size remains close to the nominal level where $T$ is not too small as compared to $N$. Juodis et al. (2021) also shows the asymptotic normality of the pooled common correlated estimator (CCE) based on $N/T \rightarrow \kappa \in (0, \infty)$ asymptotics, which is mostly satisfied in our analysis ($N/T = 0.15$). Even if $T$ is large, the CS-ARDL (or the CCEMG) estimator could be sensitive to a smaller $N$; our experiment with $N = 10$ for the given $T$ results in a difference in magnitude of the estimated coefficients, though largely preserves our conclusion.

7 Conclusion

We found some significant aggregation bias using time series analysis compared to the panel time series analysis controlling for cross-section dependence in the trade balance–exchange rate relationship for Bangladesh. The effect is indeed asymmetric, with currency appreciation improving trade balance in the long run. This finding is contrary to the Marshall–Lerner condition and thereby the J-curve effect. Our analysis aligns with the literature that considering asymmetric effect increases the evidence of trade balance–exchange rate relationship (at least) for Bangladesh, which is not present under linear analysis. However, the disaggregate separate bilateral time series analysis failed to identify significant role players in this connection. We found that Singapore, Japan, and China substantially influence Bangladesh’s exchange rate–trade balance relationship. From the policy perspective, it is crucial to recognize that the country’s exports and GDP are heavily dependent on expensive imports from a few partners, which worth diversifying. Understanding the fact that currency appreciation helps the trade balance of Bangladesh may help release the pressure of managed float in Bangladesh’s context.
Acknowledgements We would like to express our gratitude to the Bureau of Economic Research, University of Dhaka, Bangladesh, for funding the study. We are grateful to the anonymous referees and the associate editor for their valuable comments on an earlier version of the paper.

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Appendix A Beyer et al. (2000) Flexible weight aggregation method

To construct our time series aggregate variables from the disaggregate panel data, we followed the Beyer et al. (2000) aggregation method using flexible weights of partner-specific trade share. We aggregate the growth rates of the variable, say, \( x \) with variable weights using the formula:

\[
\Delta x_t = \sum_{i=1}^{25} \Delta x_{it} w_{it}, \quad t = 1, 2, \ldots 170, 
\]

where the weight \( w_{it} \) is over time trade share of Bangladesh with the \( i \)th partner country, with \( \sum_{i=1}^{25} w_{it} = 1 \). Taking the initial value as the simple average of the start time period \( (t = 1) \), the level of the aggregate is recovered as

\[
x_t = \Delta x_t + x_{t-1}, \quad t = 2, 3, \ldots 170.
\]

Appendix B Tables

See Tables 3, 4, 5, 6, and 7.

Table 3 Pesaran (2015) weak cross-section dependence test of the variables

| Variable | ltb | lrer | rgdp | lmwd | pos | neg |
|----------|-----|------|------|------|-----|-----|
| Panel A: Levels | | | | | | |
| CD test | -2.69 | 116.48 | 195.22 | 209.91 | 227.77 | 227.06 |
| P value | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Cross-sectional Exponent (\( \alpha \)) | -0.037 | 0.93 | 1.00 | 0.91 | | |
| Panel B: First differences | | | | | | |
| CD test | 23.70 | 129.37 | 138.41 | 6.96 | 150.87 | 145.58 |
| P value | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Cross-sectional Exponent (\( \alpha \)) | 0.92 | 0.99 | 0.99 | 0.88 | 0.99 | |

Under the null hypothesis of weak cross-section dependence \( CD \sim N(0, 1) \) asymptotically. \( 0.5 \leq \alpha < 1 \) implies strong cross-sectional dependence.
| Variables | ltb | lrer | pos | neg | rgdp | lmwd | Panel time series |
|-----------|-----|------|-----|-----|------|------|------------------|
| CADF      | I(0)| I(0) | I(1)| I(1)| I(1) | I(0) |
| ADF-Fisher| I(0)| I(1) | I(1)| I(1)| I(1) | I(0) |
| PP-Fisher | I(0)| I(1) | I(1)| I(1)| I(0) | I(0) |

| Variables | ltb | lrer | pos | neg | rgdp | lmwd | Time series (Aggregate) |
|-----------|-----|------|-----|-----|------|------|------------------------|
| ADF       | I(0)*| I(1) | I(1)| I(1)| I(1) | I(1) |
| PP        | I(0) | I(1) | I(1)| I(1)| I(0) | I(0) |

| Variables | ltb | lrer | pos | neg | rgdp | lmwd | Bilateral time series |
|-----------|-----|------|-----|-----|------|------|-----------------------|
| Hong Kong | ADF | I(1) | I(1) | I(1)| I(1) | I(1) | ADF                  |
|           | PP  | I(0) | I(1) | I(1)| I(1) | I(1) | PP                   |
| China     | ADF | I(1) | I(1) | I(1)| I(1) | I(1) | ADF                  |
|           | PP  | I(0) | I(1) | I(1)| I(0) | I(1) | PP                   |
| France    | ADF | I(0) | I(1) | I(1)| I(0) | I(0) | ADF                  |
|           | PP  | I(0) | I(1) | I(1)| I(0) | I(0) | PP                   |
| Germany   | ADF | I(1) | I(1) | I(1)| I(1) | I(1) | ADF                  |
|           | PP  | I(0) | I(1) | I(1)| I(0) | I(0) | PP                   |
| India     | ADF | I(0) | I(1) | I(1)| I(0) | I(0) | ADF                  |
|           | PP  | I(0) | I(1) | I(1)| I(0) | I(0) | PP                   |

| Denmark   | ADF | I(0) | I(1) | I(1)| I(1) | I(1) | I(1) | I(1) | I(0) |
| Finland   | PP  | I(0) | I(1) | I(1)| I(1) | I(1) | I(1) | I(1) | I(0) |
| Ireland   | ADF | I(1) | I(1) | I(1)| I(1) | I(1) | I(1) | I(1) | I(1) |
|           | PP  | I(0) | I(1) | I(1)| I(1) | I(0) | I(0) | I(0) | I(0) |
| Italy     | ADF | I(1) | I(1) | I(1)| I(1) | I(1) | I(1) | I(1) | I(1) |
|           | PP  | I(0) | I(1) | I(1)| I(1) | I(0) | I(0) | I(0) | I(0) |

| Malaysia  | ADF | I(0) | I(1) | I(1)| I(1) | I(0) | I(1) | I(1) | I(1) |
|           | PP  | I(0) | I(1) | I(1)| I(0) | I(0) | I(1) | I(1) | I(1) |
Table 4 continued

| Variables | ltb | lrer | pos | neg | rgdp | lmwd | ADF | ltb | lrer | pos | neg | rgdp | lmwd |
|-----------|-----|------|-----|-----|------|------|-----|-----|------|-----|-----|------|------|
| ADF       | I(1)| I(1) | I(1) | I(1) | I(1) | I(1) | ADF | I(1)| I(1) | I(1) | I(1) | I(1) | I(1) |
| PP        | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) | PP  | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) |
| Japan     |     |      |     |     |      |      |     |     |      |     |     |      |      |
| ADF       | I(1)| I(1) | I(1) | I(1) | I(0) | I(0) | ADF | I(1)| I(1) | I(1) | I(1) | I(1) | I(0) |
| PP        | I(0)| I(1) | I(1) | I(1) | I(0) | I(0) | PP  | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) |
| Korea     |     |      |     |     |      |      |     |     |      |     |     |      |      |
| ADF       | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) | ADF | I(1)| I(1) | I(1) | I(1) | I(1) | I(0) |
| PP        | I(0)| I(1) | I(1) | I(1) | I(0) | I(0) | PP  | I(0)| I(1) | I(1) | I(1) | I(1) | I(1) |
| UK        |     |      |     |     |      |      |     |     |      |     |     |      |      |
| ADF       | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) | ADF | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) |
| PP        | I(0)| I(1) | I(1) | I(1) | I(0) | I(0) | PP  | I(0)| I(1) | I(1) | I(1) | I(1) | I(1) |
| USA       |     |      |     |     |      |      |     |     |      |     |     |      |      |
| ADF       | I(1)| I(1) | I(1) | I(1) | I(1) | I(1) | ADF | I(0)| I(1) | I(1) | I(1) | I(1) | I(1) |
| PP        | I(0)| I(1) | I(1) | I(1) | I(0) | I(0) | PP  | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) |
| Singapore |     |      |     |     |      |      |     |     |      |     |     |      |      |
| ADF       | I(1)| I(1) | I(1) | I(1) | I(1) | I(1) | ADF | I(0)| I(1) | I(1) | I(1) | I(1) | I(1) |
| PP        | I(0)| I(1) | I(1) | I(1) | I(0) | I(0) | PP  | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) |
| Belgium   |     |      |     |     |      |      |     |     |      |     |     |      |      |
| ADF       | I(1)| I(1) | I(1) | I(1) | I(1) | I(1) | ADF | I(1)| I(1) | I(1) | I(1) | I(1) | I(0) |
| PP        | I(0)| I(1) | I(1) | I(1) | I(0) | I(0) | PP  | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) |
| Brazil    |     |      |     |     |      |      |     |     |      |     |     |      |      |
| ADF       | I(1)| I(1) | I(1) | I(1) | I(1) | I(1) | ADF | I(1)| I(1) | I(1) | I(1) | I(1) | I(0) |
| PP        | I(0)| I(1) | I(1) | I(1) | I(0) | I(0) | PP  | I(0)| I(1) | I(1) | I(1) | I(1) | I(0) |
Table 4 continued

| Variables | ltb | lrer | pos | neg | rgdp | lmwd | ltb | lrer | pos | neg | rgdp | lmwd |
|-----------|-----|------|-----|-----|------|------|-----|------|-----|-----|------|------|
| ADF       | I(1) | I(1) | I(1) | I(1) | I(1) | I(1) | ADF | I(1) | I(1) | I(1) | I(1) | I(1) |
| PP        | I(0) | I(1) | I(1) | I(1) | I(1) | I(0) | PP  | I(0) | I(1) | I(1) | I(1) | I(0) |
| Canada    |     |      |      |      |      |      |     |      |      |      |      |      |
| ADF       | I(1) | I(1) | I(1) | I(1) | I(1) | I(1) |     |      |      |      |      |      |
| PP        | I(0) | I(1) | I(1) | I(1) | I(0) | I(0) |     |      |      |      |      |      |

All tests with level variables include 12 lags and trend (except for the ones with asterisk), while tests with first differenced variables don’t have trends. Most of the decisions are based on 5% critical values, which occasionally has been relaxed to 10% CV.

a) Test for unit roots in heterogenous panels with cross-section dependence (Pesaran 2007)

b) Fisher-type ADF unit root tests for panel data (Choi 2001)

c) Fisher-type Phillips–Perron unit root tests for panel data (Choi 2001)

d) Phillips–Perron unit root test
### Table 5: Bilateral time series: asymmetric ARDL estimation results

| Variables | Hong Kong | China | France | Germany | India | Japan | Korea | UK | USA | Singapore | Belgium | Brazil | Canada |
|-----------|-----------|-------|--------|---------|-------|-------|-------|----|-----|-----------|---------|--------|--------|
| $pos_t$   | 3.436     | −0.257| −0.0678| −0.250  | 2.382***| −0.731| 0.907 | −1.097***| 0.812**| 0.986     | −0.458   | 1.624** | −0.558**|
|           | (2.695)   | (0.812)| (0.164)| (0.183) | (0.450)| (0.779)| (0.606)| (0.271)   | (0.383) | (0.666)   | (0.292) | (0.712) | (0.251) |
| $neg_t$   | 2.159     | −1.929***| −0.162| 0.193   | 1.886***| 0.964 | −0.0372| 0.236 | 1.397***| 1.346*   | 0.00590 | −0.289 | 0.776***|
|           | (1.877)   | (0.840)| (0.162)| (0.186) | (0.469)| (0.734)| (0.502)| (0.185)   | (0.397) | (0.782)   | (0.265) | (1.404) | (0.166) |
| $rgdp_t$  | 1.059     | 1.221**| 0.244  | −0.724***| −0.419| −3.441***| 1.035**| −1.454***| −0.336| −1.650** | −0.349 | 1.882   | −1.689***|
|           | (0.929)   | (0.512)| (0.223)| (0.238) | (0.352)| (1.280)| (0.476)| (0.295)   | (0.316) | (0.689)   | (0.251) | (2.345) | (0.356) |
| $lmwd_t$  | −1.828***| −0.749| −0.882**| −1.147***| −1.844***| −1.602***| −1.435***| −1.088***| −1.086***| −1.030***| −1.050***| −0.820**| −1.028***|
|           | (0.563)   | (0.612)| (0.0467)| (0.0887)| (0.404)| (0.591) | (0.358)| (0.0836) | (0.0916)| (0.235) | (0.0686) | (0.388) | (0.0542) |
| $ECM_{t-1}$ | −0.190***| −0.384***| −0.918***| −0.747***| −0.573***| −0.228***| −0.725***| −0.803***| −1.061***| −0.565***| −0.281***| −0.748***|
|           | (0.0671) | (0.0791)| (0.0486)| (0.0575)| (0.0681)| (0.0658)| (0.0750)| (0.0840) | (0.0636) | (0.0756) | (0.0693) | (0.0744) | (0.0495) |
| $T$       | 170       | 170   | 170    | 170     | 170    | 170    | 170    | 170       | 170    | 170       | 170    | 170     | 170    |
| $R^2$     | 0.445     | 0.360 | 0.722  | 0.538   | 0.391  | 0.502  | 0.398  | 0.664     | 0.606  | 0.551     | 0.535  | 0.566   | 0.731  |
| $p$ value  | Yes       | Yes   | Yes    | Yes     | Yes    | Yes    | Yes    | Yes       | Yes    | Yes       | Yes    | Yes     | Yes    |

| Denmark   | 0.45      | 0.00   | 0.58   | 0.00    | 0.00   | 0.09   | 0.00   | 0.00      | 0.00   | 0.14      | 0.29   | 0.00   | 0.15   | 0.00   |
| Finland   | 0.739     | 0.247  | -0.259 | 0.918   | 0.717  | 2.700**| 2.607***| −0.611*** | −0.516**| 1.903***  | 0.294  |
| Ireland   | 0.461     | 0.468  | 0.251  | (0.681) | (0.470) | (1.042) | (0.454) | (0.221)   | (0.215) | (0.313)   | (0.314) |
| Italy     | 0.468     | 0.251  | (0.681)| 0.470   | (1.042) | (0.454) | (0.221) | (0.215)   | (0.313) | (0.314)   |
| Malaysia  | 0.407     | 0.269  | 0.190  | 0.396   | 0.497  | 0.759  | 0.303  | 0.199     | 0.214  | 0.281     | 0.270  |
| Mexico    | 1.424***  | 0.246  | −0.105 | −1.351***| 2.316***| −0.941 | 0.784**| −0.303    | −0.496**| 1.209***  | 1.033***|
| Norway    | (0.228)   | (0.506)| (0.269)| (0.190) | (0.396)| (0.497)| (0.759) | (0.303)   | (0.199) | (0.214)   | (0.281) | (0.270) |
| Russia    | −1.190*** | −0.705 | −0.203 | −0.197  | 1.422  | −3.785***| 3.236**| 1.232     | −0.579**| 0.449     | −0.237 | −1.708***|
| Spain     | (0.333)   | (0.510)| (0.366)| (0.130) | (0.880)| (0.859)| (1.625) | (0.774)   | (0.250) | (0.503)   | (0.599) | (0.563) |
| Country       | Denmark | Finland | Ireland | Italy | Malaysia | Mexico | Norway | Russia | Spain | Sweden | Turkey | Ukraine |
|--------------|---------|---------|---------|-------|----------|--------|--------|--------|-------|--------|--------|---------|
| $l m w d_{t}$| $-0.855***$ | $-1.037***$ | $-0.886***$ | $-0.873***$ | $-0.824***$ | $-1.065***$ | $-0.860***$ | $-0.957***$ | $-0.798***$ | $-0.993***$ | $-0.749***$ | $-1.127***$ |
|              | $(0.0642)$ | $(0.0682)$ | $(0.0597)$ | $(0.134)$ | $(0.120)$ | $(0.0533)$ | $(0.137)$ | $(0.135)$ | $(0.0753)$ | $(0.0442)$ | $(0.122)$ | $(0.0445)$ |
| $E C M_{t-1}$| $-0.754***$ | $-0.695***$ | $-0.487***$ | $-0.550***$ | $-0.848***$ | $-0.572***$ | $-0.263***$ | $-0.379***$ | $-0.599***$ | $-0.677***$ | $-0.382***$ | $-0.988***$ |
|              | $(0.0755)$ | $(0.0834)$ | $(0.0673)$ | $(0.107)$ | $(0.0865)$ | $(0.0641)$ | $(0.0823)$ | $(0.0759)$ | $(0.0700)$ | $(0.0724)$ | $(0.0733)$ | $(0.0429)$ |
| $T$          | 170     | 170     | 170     | 170   | 170      | 170    | 170    | 170    | 170   | 170    | 170    | 170     |
| $R^2$        | 0.798   | 0.788   | 0.848   | 0.445 | 0.552    | 0.840  | 0.919  | 0.674  | 0.767 | 0.788  | 0.594  | 0.810   |
| Coint$^b$    | Yes     | Yes     | Yes     | Yes   | Yes      | Yes    | Yes    | Yes    | Yes   | Yes    | Yes    | Yes     |
| $p$ value$^c$| 0.08    | 0.05    | 0.99    | 0.26  | 0.00     | 0.00   | 0.01   | 0.00   | 0.17  | 0.86   | 0.00   | 0.07    |

Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

$^a$ Asymmetric ARDL estimate on bilateral time series data. The optimum lag length is chosen based on AIC. Estimates for the long run coefficients are reported here; the full set of results available from the authors upon request.

$^b$ Based on the Pesaran-Shin-Smith bound test for cointegration (Pesaran et al. 2001).

$^c$ $p$ value of the Wald test statistic of symmetry. In the asymmetric ARDL analysis, we test the coefficients of $POS$ and $NEG$ in the long run, with the null hypothesis that they are the same.
### Table 6  Long run estimates from CS-ARDL models based on different lag orders

| Variables       | Lag (1)          | Lag (2)          | Lag (3)          | Lag (4)          | Lag (5)          | Lag (6)          | Lag (7)          | Lag (8)          | Lag (9)          | Lag (10)         | Lag (11)         | Lag (12)         |
|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| **Panel A: CS-ARDL** |                  |                  |                  |                  |                  |                  |                  |                  |                  |                  |                  |                  |
| $ECM_{t-1}$     | $-0.611^{***}$   | $-0.618^{***}$   | $-0.623^{***}$   | $-0.631^{***}$   | $-0.637^{***}$   | $-0.640^{***}$   | $-0.649^{***}$   | $-0.668^{***}$   | $-0.680^{***}$   | $-0.685^{***}$   | $-0.698^{***}$   | $-0.703^{***}$   |                  |
|                 | $(0.0390)$       | $(0.0393)$       | $(0.0401)$       | $(0.0397)$       | $(0.0396)$       | $(0.0397)$       | $(0.0397)$       | $(0.0394)$       | $(0.0393)$       | $(0.0383)$       | $(0.0389)$       | $(0.0369)$       |                  |
| $lret_{t-1}$    | $0.467^*$        | $0.458^*$        | $0.473^*$        | $0.445$          | $0.519$          | $0.533^*$        | $0.523$          | $0.687^*$        | $0.634^*$        | $0.633^*$        | $0.666^*$        | $0.851^{**}$     |                  |
|                 | $(0.245)$        | $(0.252)$        | $(0.276)$        | $(0.282)$        | $(0.319)$        | $(0.316)$        | $(0.323)$        | $(0.354)$        | $(0.376)$        | $(0.345)$        | $(0.363)$        | $(0.383)$        |                  |
| $rgdp_{t-1}$    | $0.881^*$        | $1.133^{**}$     | $1.143^{**}$     | $1.311^{**}$     | $1.574^{**}$     | $1.563^{**}$     | $1.392^{**}$     | $1.106^*$        | $1.025^*$        | $1.151^{**}$     | $1.311^{**}$     | $1.310^{**}$     |                  |
|                 | $(0.456)$        | $(0.495)$        | $(0.457)$        | $(0.513)$        | $(0.654)$        | $(0.630)$        | $(0.651)$        | $(0.573)$        | $(0.594)$        | $(0.577)$        | $(0.547)$        | $(0.535)$        |                  |
| $lmwd_{t-1}$    | $-1.025^{***}$   | $-1.015^{***}$   | $-1.013^{***}$   | $-1.030^{***}$   | $-1.027^{***}$   | $-1.062^{***}$   | $-1.106^{***}$   | $-1.076^{***}$   | $-1.089^{***}$   | $-1.141^{***}$   | $-1.188^{***}$   | $-1.222^{***}$   |                  |
|                 | $(0.0401)$       | $(0.0394)$       | $(0.0474)$       | $(0.0488)$       | $(0.0543)$       | $(0.0530)$       | $(0.0647)$       | $(0.0725)$       | $(0.0679)$       | $(0.0694)$       | $(0.0773)$       | $(0.0827)$       |                  |
| $R^2$-squared   | $0.516$          | $0.287$          | $0.277$          | $0.268$          | $0.256$          | $0.249$          | $0.242$          | $0.230$          | $0.220$          | $0.214$          | $0.205$          | $0.200$          |                  |
| CD: $p$ value^c | $0.516$          | $0.287$          | $0.344$          | $0.426$          | $0.227$          | $0.168$          | $0.224$          | $0.281$          | $0.208$          | $0.133$          | $0.256$          | $0.180$          |                  |
| **Panel B: CS-NLARDL** |                  |                  |                  |                  |                  |                  |                  |                  |                  |                  |                  |                  |                  |
| $ECM_{t-1}$     | $-0.675^{***}$   | $-0.681^{***}$   | $-0.687^{***}$   | $-0.691^{***}$   | $-0.699^{***}$   | $-0.703^{***}$   | $-0.720^{***}$   | $-0.742^{***}$   | $-0.755^{***}$   | $-0.766^{***}$   | $-0.780^{***}$   | $-0.796^{***}$   |                  |
|                 | $(0.0368)$       | $(0.0372)$       | $(0.0386)$       | $(0.0381)$       | $(0.0376)$       | $(0.0380)$       | $(0.0392)$       | $(0.0396)$       | $(0.0395)$       | $(0.0378)$       | $(0.0379)$       | $(0.0367)$       |                  |
| $post_{t-1}$    | $-0.272$         | $-0.394$         | $-0.642$         | $-0.790$         | $-0.526$         | $-0.584$         | $-0.743$         | $-0.606$         | $-0.385$         | $-0.416$         | $-0.578$         | $-0.295$         |                  |
|                 | $(0.520)$        | $(0.531)$        | $(0.548)$        | $(0.549)$        | $(0.495)$        | $(0.514)$        | $(0.495)$        | $(0.544)$        | $(0.601)$        | $(0.625)$        | $(0.687)$        | $(0.702)$        |                  |
| $negt_{t-1}$    | $0.748^{**}$     | $0.792^{***}$    | $0.891^{***}$    | $0.817^{**}$     | $0.648^{*}$      | $0.585$          | $0.945^{**}$     | $1.238^{***}$    | $1.062^{**}$     | $1.008^{**}$     | $1.237^{***}$    | $1.617^{***}$    |                  |
| Variables | Lag (1) | Lag (2) | Lag (3) | Lag (4) | Lag (5) | Lag (6) | Lag (7) | Lag (8) | Lag (9) | Lag (10) | Lag (11) | Lag (12) |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| \( \text{rgdp}_{t-1} \) | 0.397   | 0.541   | 0.461   | 0.654   | 0.894*  | 0.913   | 0.793   | 0.675   | 0.886   | 1.027   | 0.981   | 1.048   |
| \( \text{lmw}_{dt-1} \) | 1.069*** | 1.048*** | 1.032*** | 1.042*** | 1.019*** | 1.069*** | 1.106*** | 1.100*** | 1.065*** | 1.047*** | 1.096*** | 1.057*** |
| \text{R}-squared | 0.281   | 0.266   | 0.252   | 0.243   | 0.226   | 0.219   | 0.207   | 0.192   | 0.176   | 0.166   | 0.153   | 0.142   |
| CD: \( p \) value\(^c\) | 0.704   | 0.512   | 0.285   | 0.389   | 0.246   | 0.177   | 0.132   | 0.042   | 0.017   | 0.064   | 0.038   | 0.028   |
| Observations | 4,150   | 4,125   | 4,100   | 4,075   | 4,050   | 4,025   | 4,000   | 3,975   | 3,950   | 3,925   | 3,900   | 3,875   |
| Number of groups | 25      | 25      | 25      | 25      | 25      | 25      | 25      | 25      | 25      | 25      | 25      | 25      |

Standard errors in parentheses. ***\( p < 0.01, ** p < 0.05, * p < 0.1 \)

\(^a\) Cross-sectionally augmented ARDL estimator (Chudik et al. 2016, Ditzen 2018). The cross-section averages each has \( \sqrt[3]{T} \approx 6 \) lags. The columns present the long run estimates considering various short run lags from 1 to 12, using monthly data

\(^b\) Asymmetric Cross-sectionally augmented ARDL estimator (Chudik et al. 2016, Bahmani-Oskooee and Kanitpong 2017). The lags are the same as in the linear model

\(^c\) \( p \) value of the weak cross-section dependence test (Pesaran 2015, Ditzen 2018), the null hypothesis is the errors are weakly cross-sectionally dependent, with the alternative that they are strongly cross-sectionally dependent.
Table 7  Panel Time Series: Linear ARDL Estimation Results by Country\textsuperscript{a}

| Variables | (1)  | (2)  | (3)  | (4)  | (5)  | (6)  | (7)  | (8)  | (9)  | (10) | (11) | (12) | (13) |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| lrer\textsubscript{t\rightarrow 1} | 1.019 | 1.943 | 0.465 | −1.411 | 3.470* | 2.036*** | 0.794 | 0.356 | 1.677 | 3.196*** | −0.025 | 0.535 | 0.008 |
|           | (1.896) | (1.635) | (1.979) | (2.209) | (1.898) | (0.671) | (0.490) | (0.493) | (1.716) | (0.890) | (2.338) | (0.740) | (1.665) |
| rgdp\textsubscript{t\rightarrow 1} | 1.709 | 0.381 | 1.528 | −0.217 | 1.365 | 2.603*** | 3.931 | −0.438 | −1.909*** | −2.437 | 7.275*** | −0.958 |
|           | (2.658) | (0.676) | (2.362) | (3.095) | (0.909) | (1.378) | (0.954) | (2.598) | (1.806) | (0.543) | (2.412) | (1.461) | (2.494) |
| lmwd\textsubscript{t\rightarrow 1} | −1.614*** | −0.534 | −0.966* | −1.103 | −0.688 | −1.569*** | −0.794*** | −1.176*** | −1.030* | −0.589*** | −0.990*** | −1.424*** | −1.185*** |
|           | (0.636) | (0.781) | (0.524) | (0.768) | (0.965) | (0.517) | (0.296) | (0.454) | (0.571) | (0.177) | (0.312) | (0.207) | (0.524) |
| ECM\textsubscript{t\rightarrow 1} | −0.421 | −0.606 | −0.572 | −0.475 | −0.512 | −0.778 | −0.863 | −0.830 | −0.644 | −1.040 | −0.525 | −0.459 | −0.562 |
|           | (4.241) | (3.153) | (3.643) | (6.832) | (6.542) | (3.191) | (2.727) | (10.445) | (3.432) | (3.077) | (5.836) | (10.677) | (2.736) |
|          | (14) | (15) | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) | (24) | (25) |
| Denmark | 0.427 | 1.306 | −0.342 | 0.273 | −3.583*** | −0.296 | −1.762 | 2.026 | −1.379 | −0.135 | 2.508* | −0.129 |
|           | (1.538) | (1.371) | (1.458) | (2.397) | (1.072) | (1.264) | (2.810) | (1.131) | (1.969) | (1.411) | (1.497) | (0.556) |
| rgdp\textsubscript{t\rightarrow 1} | 0.343 | 0.454 | 0.460 | 1.629 | 0.860 | 4.204 | −0.038 | 13.614*** | 1.869 | 0.019 | −1.679 | 2.872*** |
|           | (1.162) | (1.407) | (0.667) | (2.233) | (1.113) | (4.382) | (1.455) | (3.789) | (1.327) | (1.351) | (2.291) | (1.030) |
| lmwd\textsubscript{t\rightarrow 1} | −0.953*** | −1.187*** | −0.889*** | −0.972 | −0.876*** | −1.395*** | −1.110*** | −0.831*** | −0.844** | −0.885*** | −0.986*** | −1.073*** |
|           | (0.312) | (0.169) | (0.217) | (0.682) | (0.905) | (0.311) | (0.238) | (0.234) | (0.362) | (0.307) | (0.358) | (0.095) |
| ECM\textsubscript{t\rightarrow 1} | −0.735 | −0.787 | −0.676 | −0.448 | −1.137 | −0.433 | −0.480 | −0.478 | −0.563 | −0.631*** | −0.404 | −0.873 |
|           | (0.854) | (1.914) | (0.629) | (4.011) | (3.493) | (18.669) | (4.930) | (52.393) | (4.262) | (0.209) | (4.695) | (2.952) |

Standard errors in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1

\textsuperscript{a} Cross-sectionally augmented ARDL estimator (Chudik et al. 2016, Ditzen 2018): disaggregated results obtained together with the results in column 3 of Table 1. The cross-section averages each has \( \sqrt{T} \approx 6 \) lags. The short run lags, 5, are the same for each variable. The table reports the long run estimates only; short run estimates are available from the authors upon request.
Appendix C Figures

See Figs. 4, 5, and 6.

Fig. 4 Kernel density estimate of POS and NEG

Fig. 5 CUSUM of squares test of parameter stability in the aggregate linear (left) and asymmetric (right) ARDL model

Fig. 6 Illustrative impulse response function from CS-ARDL model against 25 trading partners
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