We study a class of models for tri-bimaximal neutrino mixing in $SO(10)$ grand unified SUSY framework. Neutrino masses arise from both type-I and type-II seesaw mechanisms. We use dimension five operators in order to not spoil tri-bimaximal mixing by means of type-I contribution in the neutrino sector. We show that it is possible to fit all fermion masses and mixings including also the recent T2K result as deviation from the tri-bimaximal.

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I. INTRODUCTION

Neutrino mixing leads to large atmospheric angle (maximal), large solar angle (trimaximal) and small reactor angle. In particular recently T2K collaboration has given indication of non zero reactor angle $\sin^2 \theta_{13} = 0.013^{+0.022}_{-0.012}$ [2], $\sin^2 \theta_{13} = 0.025^{+0.025}_{-0.020}$ [3]. This interesting result seems in contradiction with tri-bimaximal (TBM) mixing ansatz [4] that predicts zero reactor angle. However TBM solar and atmospheric mixing angles can be used as first approximation. Deviation from zero reactor angle can arises in grand unified theory (GUT) like $SU(5)$ and $SO(10)$ from the charged sector, see for instance [5]. While neutrino mass matrix is diagonalized from TBM unitary matrix, charged leptons are not diagonal giving deviation to the TBM. In this paper we consider such a possibility in the framework of a supersymmetric (SUSY) $SO(10)$ model. In this scenario charged leptons and CKM mixings are strongly related, we therefore consider the TBM as a good starting point to be corrected in general by small (CKM-like) deviations. In Ref.[6] has been shown that in a renormalizable $SO(10)$ model this is not possible in case of type-I seesaw. Such a difficulty arises from the fact that up quark and Dirac neutrino Yukawa couplings are strongly related in renormalizable $SO(10)$ models. Some interesting attempts to obtain TBM with a flavour symmetry are developed in Ref.[7] and Ref.[8], assuming type-II seesaw to be dominant $^1$. In Ref.[12], in the context of SUSY renormalizable $SO(10)$ with type-II seesaw dominance, a fit of all the fermion masses and mixing has been done (see also [13]). The superpotential considered is of the form

$$w = h 16 16 10 + f 16 16 \overline{126} + h' 16 16 120,$$

where $h$ is a symmetric matrix, $h'$ is antisymmetric and $f$ has the TBM structure, namely

$$f = \begin{pmatrix} f_2 & f_1 \\ f_1 & f_2 + f_0 \\ f_2 - f_0 & f_2 + f_0 \end{pmatrix},$$

where $m_{\nu 1} = f_2 - f_1$, $m_{\nu 2} = f_2 + 2f_1$ and $m_{\nu 3} = f_2 - f_1 + 2f_0$. It is well know that a mass matrix with the above structure is diagonalized by TBM mixing matrix, see for instance [14]. No assumptions have been made taking $f$ to be TBM because we can always go to this basis by rotating the 16 of fermions [12]. The matrices $h, h'$ are assumed to be hermitian that can correspond to an underlying parity $^1$.

---

$^1$ For an incomplete list of papers with TBM in GUT see [9], for $SO(10)$ models with discrete flavour symmetry and no TBM mixing see [10] and for a collection of general $SO(10)$ models see at the References in Ref. [11].
Another possibility to reproduce TBM mixing in the framework of \( SO(10) \) GUT models is to use non-renormalizable operators containing a scalar field transforming as a 45\(_H\) of \( SO(10) \). This field allows to distinguish up quarks from neutrinos permitting TBM mixing also in the case where neutrino masses arise from type-I seesaw mechanism. In particular in Ref.\(^{[12]}\) for this purpose the dimension five operator 16 \( 16 \) 45\(_H\) has been used. This operator yields a contribution to the up-quark mass matrix and not to the Dirac neutrino one allowing to distinguish the up-quark from Dirac neutrino sectors. In this way it is possible to obtain both Dirac and Majorana neutrino masses TBM and hierarchical structure in charged fermions sector.

A full fit of quark and lepton masses and mixing in models with TBM mixing from type-I seesaw in \( SO(10) \) is still missing. In this paper we consider such a problem. We link the idea of distinguish up-quark and Dirac neutrino 

\[ \text{by means of } 16 \ 16 \ 120 \ \text{H} \]

still missing. In this paper we will translate the superpotential \( (2) \) of Ref.\(^{[12]}\) in the language of dimension five operators.

In section III we give some examples of models and the corresponding fits, in section IV we discuss the possibility to obtain a renormalizable model, then in section V we give our conclusions.

## II. DIMENSION FIVE EFFECTIVE OPERATORS

In this section we report the main ingredients that will be useful to construct our model in the next section. It contains some of the result of table VIII of Ref.\(^{[6]}\) that we report in appendix A for the useful of the reader.

As discussed in the introduction, one possibility to reproduce TBM mixing in \( SO(10) \) in the case of type-I seesaw is by means of the dimension five operator 16 \( 16 \) 45\(_H\) operator with the result of Ref.\(^{[12]}\) where has been shown that from the superpotential \( (2) \) it is possible to fit all the data having TBM mixing in the neutrino sector. In this paper we will translate the superpotential \( (2) \) in the language of dimension five operators.

In the next section we will review some of the \( SO(10) \) dimension five operator that will be useful to construct an \( SO(10) \) model giving TBM mixing with type-I as well as type-II seesaw following the indication given in the superpotential \( (2) \) of Ref.\(^{[12]}\). In section III we give some examples of models and the corresponding fits, in section IV we discuss the possibility to obtain a renormalizable model, then in section V we give our conclusions.

The dimension five operators that will be used are:

- **16 \( 16 \) \( \overline{16} \) \( H \)**

  This operator can be obtained, for example, integrating out a \( SO(10) \) singlet \( 1_\chi \) or a 45-plet \( 45_\chi \) of heavy messenger fermions:

  \[
  f (16 \overline{16})_1 (16 \overline{16})_1, \quad f (16 \overline{16})_{15} (16 \overline{16})_{45}.
  \]

- **16 \( 16 \) \( 45 \)\(_H\)**

  This operator can be obtained by integrating out a couple \( 16_\chi - \overline{16}_\chi \)

  \[
  h (16 \overline{16} \overline{16}) (16 45 \overline{16} \overline{16}).
  \]

It can yield a contribution to the up-quark mass matrix (and to the down-quark and charged lepton mass matrices) and not to the Dirac neutrino one, allowing to distinguish the up-quark from Dirac neutrino sectors. This can be described naively in the \( SU(5) \) language as follows. The up-type Higgs doublet in a 45\(_H\) of \( SU(5) \) (contained into the 120\(_H\)) couples antisymmetrically to the two matter multiplets 10 that give the up-quark mass \( M_u \) while it does not contribute to the Dirac neutrinos, when contracted as in eq. \( (5) \). This is a well know feature...
of SO(10) where the operator $16 \times 16 \times 120_H$ contributes to $M_u \propto \langle 45_{SU(5)} \rangle$ and $M_\nu \propto \langle 5_{SU(5)} \rangle$ with antisymmetric Yukawa. But with the dimension five operator the resulting mass matrix is not antisymmetric and so can give the hierarchical structure of the quark sector. In fact with the insertion of a $24_H$ scalar multiplet (contained in the $45_H$ of SO(10)) that takes vev in the hypercharge direction, results that $M_u$ is not antisymmetric (but a generic matrix) since the Clebsch-Gordan coefficients for the isospin doublet $Q$ and the isosinglet $u^c$ are different with a relative factor $(-4)$ that arise from the hypercharge.

We can describe the property of the $16 \times 120_H$ $45_H$ operator in more detail as follow. The $45_H$ can take vev in its singlet $1_{SU(5)}$ component called $X$-direction\(^2\) or along the adjoint $24_{SU(5)}$ component, that is the hypercharge $Y$-direction (see for instance $[18]$). We indicate their vevs as

$$b_1 = \langle 1_{SU(5)} \rangle, \quad b_{24} = \langle 24_{SU(5)} \rangle. \quad (6)$$

Equivalently, the $45_H$ can take vev along the isospin direction or the $B - L$ direction and their corresponding vev are denoted as $b_3$ and $b_{15}$ respectively and are given by

$$b_1 = \frac{1}{3}(b_3 + 3b_{15}), \quad b_{24} = \frac{1}{3}(-b_3 + 2b_{15}). \quad (7)$$

The $SU(5)$ components of the $120_H$ of SO(10) that contain $SU(2)$ doublet (giving rise to the Dirac masses terms for the fermions) are the $45_{SU(5)}$, $\bar{10}5_{SU(5)}$, $5_{SU(5)}$ and $5_{SU(5)}$. We denote their vevs as

$$a_5 = \langle 5_{SU(5)} \rangle, \quad a_5 = \langle \bar{5}_{SU(5)} \rangle, \quad a_{45} = \langle 45_{SU(5)} \rangle, \quad a_{45} = \langle \bar{10}5_{SU(5)} \rangle. \quad (8)$$

\(^2\)From the table in appendix A we have for instance that

$$M_u = h a_{45}(b_1 - 4b_{24}) - h^T a_{45}(b_1 + b_{24}), \quad (9)$$

$$M_\nu = 5h a_5 b_1 - h^T a_5 (-3b_1 - 3b_{24}), \quad (10)$$

$$M_d = h(a_5^* + a_5)(-3b_1 + 2b_{24}) - h^T(a_5^* + a_5)(b_1 + b_{24}), \quad (11)$$

$$M_e^T = h(a_5^* - 3a_5)(-3b_1 - 3b_{24}) - h^T(a_5^* - 3a_5)(b_1 + 6b_{24}). \quad (12)$$

Then if $a_5 = 0$ this operator contributes to $Y_u$, $Y_d$, $Y_e$ and not to $Y_\nu$. So $h$ can be in a hierarchical form, with $(3, 3)$ element dominant, as required by charged fermion phenomenology, without changing the TBM result. Because of the $45_H$, the mass matrix that results from such an operator is not antisymmetric but general. Note that with a $45_H$ in $B - L$ direction the resulting mass matrix is symmetric, in fact using eq. (7) we have

$$M_u = h a_{45}(b_1 - b_{15}) - h^T a_{45}(b_{15}), \quad (13)$$

$$M_\nu = h a_5 (b_3 + 3b_{15}) + h^T a_5 (3b_{15}), \quad (14)$$

$$M_d = h(a_5^* + a_5)(-b_3 - b_{15}) - h^T(a_5^* + a_5)b_{15}, \quad (15)$$

$$M_e^T = h(a_5^* - 3a_5)(-3b_{15}) - h^T(a_5^* - 3a_5)(3b_{15} - b_3). \quad (16)$$

and setting $b_1 = 0$ then all the mass matrices are symmetric.

- $16 \times 16 \times 10_H 45_H$

This operator can be obtained by contracting with a couple $16\chi - \bar{16}\chi$:

$$h'(16_{10H})_{16}(16_{45H})_{\bar{16}}. \quad (17)$$

We denote the vev of the $SU(5)$ components of the $10_H$, containing all the possible $SU(2)$ doublets, as

$$a_5 = \langle 5_{SU(5)} \rangle, \quad a_5 = \langle \bar{5}_{SU(5)} \rangle. \quad (18)$$

If $a_5 = 0$ this operator contributes only in $Y_e$ and $Y_d$ as usual in $SU(5)$. In fact from Appendix A we have

$$M_u = h' a_5 (b_1 - 4b_{24}) + h'^T a_5 (b_1 + b_{24}), \quad (19)$$

$$M_\nu = 5h' a_5 b_1 + h'^T a_5 (-3b_1 - 3b_{24}), \quad (20)$$

$$M_d = h' a_5 (-3b_1 + 2b_{24}) + h'^T a_5 (b_1 + b_{24}), \quad (21)$$

$$M_e^T = h' a_5 (-3b_1 - 3b_{24}) + h'^T a_5 (b_1 + 6b_{24}). \quad (22)$$

\(^2\) This is the extra $U(1)$ contained in $SO(10) \supset SU(5) \times U(1)$.\)
Again, because of the $45_H$, the resulting mass matrix is not symmetric but a generic matrix and it can contribute to the down-quark and lepton masses. Note that with a $45_H$ in B-L direction the resulting mass matrix is antisymmetric in fact

\[ M_u = h'a_5(b_1 - b_{15}) + h'Ta_5b_{15}, \]
\[ M_\nu = 5h'a_5(b_3 + 3b_{15}) - h'Ta_53b_{15}, \]
\[ M_d = h'a_5(-3b_3 - b_{15}) + h'Ta_5b_{15}, \]
\[ M_e^T = h'a_5(-3b_{15}) + h'Ta_5(-b_3 + 3b_{15}). \]

and putting $b_3 = 0$ the mass matrices are clearly antisymmetric.

- Adding an $SO(10)$ scalar singlet $1_H$ we can consider also the dimension five operators

\[ 16 \bar{16} \bar{16} 1_H, \quad 16 \bar{16} 10_H 1_H, \quad 16 \bar{16} 120_H 1_H, \]

that behave in the same way as the renormalizable ones (see appendix A) introduced in eq. (2). Also these operators can be obtained integrating out a $16\chi_−16\chi$ of heavy messenger fermions

\[ (16\bar{16}H)_{16}(161_H)_{\bar{16}}, \quad (1610_H)_{16}(161_H)_{\bar{16}}, \quad (16120_H)_{16}(161_H)_{\bar{16}}. \]

We see that the key ingredient to obtain type-I seesaw and TBM mixing is that the up-type $SU(2)$ Higgs doublets in the $5_{10}$ and $5_{120}$ do not have vevs and so that they are not in linear combination of the light Higgs doublet. This can be a potentially problem since $\bar{5}_{10}$ takes a vev and it is mixed with the other light Higgs doublets. However the study of the complete scalar potential is beyond the scope of this paper and will be studied elsewhere.

### III. MODELS FOR TBM AND FIT OF FERMION MASSES AND CKM

In Ref. [12] has been studied a model for TBM mixing with dominant type-II seesaw mechanism given in eq. (2). In this section we present some possible modifications of the model given in eq. (2). In the models we will present below, TBM arises from both type-I and type-II seesaw mechanisms differently from Ref. [7, 12] where dominant type-II seesaw mechanism has been assumed for neutrino masses. We remark that the main problem with type-I seesaw is that the tree-level operator $16\bar{16} 10$ gives equal contribution to the up-quark and Dirac neutrino mass matrix. But in order to fit quark masses and mixings with TBM neutrino mixing, the structure of the two mass matrices must be very different, namely the up quark mass matrix must be hierarchical while the Dirac neutrino mass matrix must be of TBM-type as in eq. (3) or the identity. So we need to disentangle the two sectors, leaving Dirac and Majorana neutrino masses of TBM-type and Dirac charged fermions masses hierarchical and almost diagonal. From the previous section it is clear that one possibility is to replace the operator $16\bar{16} 10$ of eq. (2) with the operator $16\bar{16} 45 120$.

In the following we will assume an underlying parity, like in [15], making all the mass matrices hermitian and so reducing the number of free parameters. Another way to reduce the sometimes high number of parameters is to assume that the 45 get vev in the B-L direction. In this case the fermion mass matrices are symmetric or antisymmetric and not arbitrary.

Examples of models with TBM neutrino mixing are listed below. The details of the fit are given in appendix B and C. We fit all charged fermion masses, the two neutrino mass square differences, leptons and quarks mixings, and the CKM phase for a total of 18 observables. For the operators $16\bar{16} 120_H 45_H$ and $16\bar{16} 10_H 45_H$ we always take zero vev for the component $5_{SU(5)}$ of $120_H$ and $10_H$, as described in the previous section ($a_5 = 0$).

- **case A:** $w = f 16\bar{16} 16\bar{16} H 16 + h 16\bar{16} 45_H 120_H + h' 16\bar{16} 45_H 10_H$

  where the $45_H$ takes vev in a general direction, that is $b_1$ and $b_{24}$ (see eq. (10)), are both different from zero. The
mass matrices are:

\[ M_u = h + f, \]
\[ M_d = r_1[h(2\frac{b_1}{b_{24}} - 3) + h^T(2\frac{b_1}{b_{24}} + 2) + h'], \]
\[ M_e = r_1[c_e[h(2\frac{b_1}{b_{24}} + 7) + h^T(2\frac{b_1}{b_{24}} + 2)] + h'(-5\frac{b_{24}}{4b_1 - b_{24}}) + h'^T(4\frac{b_1 + b_{24}}{4b_1 - b_{24}})], \]
\[ M_{\nu, D} = \frac{1}{2}f, \]
\[ M_R = r_Rf, \]
\[ M_L = r_Lf. \]

where \( h \) and \( h' \) are generic matrices, \( r_i, c_e \) and \( b_i \) are combinations of vevs (see appendix A).

Results:

\[ \chi^2 = 0.0050, \] (35)

with 26 parameters.

We note that with the 45\(_H\) taking vev in B-L direction the number of parameters is considerably reduced but a good fit can not be performed.

• case B: \( w = f 16 16 \overline{126}_H 1_H + h 16 16 45_H 120_H + h' 16 16 45_H 10_H \)

where the 45\(_H\) takes vev in B-L direction. The mass matrices are:

\[ M_u = h^S + r_2 f, \]
\[ M_d = r_1(h^S + h^A + f), \]
\[ M_e = r_1(c_e h^S - 3h^A - 3f), \]
\[ M_{\nu, D} = -3r_2 f, \]
\[ M_R = r_R f, \]
\[ M_L = r_L f. \]

Results:

\[ \chi^2 = 5.6, \] (42)

with 16 parameters (2 d.o.f).

• case C: \( w = f 16 16 \overline{126}_H 1_H + h 16 16 45_H 120_H + h' 16 16 120'_{H} 1_H \)

where the 45\(_H\) takes vev in B-L direction. The mass matrices are:

\[ M_u = h^S + r_3 h^A + r_2 f, \]
\[ M_d = r_1(h^S + h^A + f), \]
\[ M_e = r_1(c_e h^S + c_e h^A - 3f), \]
\[ M_{\nu, D} = -3r_2 f, \]
\[ M_R = r_R f, \]
\[ M_L = r_L f. \]

Results:

\[ \chi^2 = 0.0015, \] (49)

with 18 parameters.
The last case reproduces basically the same structure of the renormalizable case (eq. (2)) with type-II seesaw dominance studied for example in ref. [12], with just one more parameter $c^c_S$. We note that the analysis performed in [12] is based on a previous set of data (before the T2K and MINOS recent results). For this reason we show also an updated fit for that case, that can be used for comparison:

$$\chi^2 = 0.14, \quad d_{FT} = 461863$$

with 17 parameters (1 d.o.f), where $d_{FT}$ is a parameter introduced in [12]. We note that the goodness of the fit is substantially unchanged compared with the old analysis, showing that in this class of models it is possible to obtain the desired (very small before T2K or more sizeable now) corrections to zero $\theta_{13}$ from the charged lepton sector, taking into account an appreciable amount of finetuning. In fact the neutrino mass matrix is of TBM-type and it is diagonalized by TBM mixing matrix. The charged fermion mass matrices have hierarchical structure. Assuming all the parameters to be real, the charged lepton mass matrix is diagonalized by a rotation matrix $O_l$ characterized by three angles $\theta_{13}^l, \theta_{12}^l$ and $\theta_{23}^l$. The lepton mixing matrix $V_{lep}$ is given by the product $V_{lep} = O_l^T \cdot V_{TBM}$ so we have

\begin{align*}
(V_{lep})_{13} & = \frac{1}{\sqrt{2}}(s_{13} - c_{13}s_{12}), \\
(V_{lep})_{12} & = \frac{1}{\sqrt{3}}(c_{12}c_{13} + s_{12}c_{13} + s_{13}), \\
(V_{lep})_{23} & = \frac{1}{\sqrt{2}}(-c_{12}c_{23} + s_{23}(c_{13} + s_{12}s_{13}))
\end{align*}

where $s_{ij} = \sin \theta_{ij}^l$ and $c_{ij} = \cos \theta_{ij}^l$. We can have a large value for the reactor angle in agreement with the result of the T2K collaboration, and at the same time $(V_{lep})_{12} \approx 1/\sqrt{3}$ and $(V_{lep})_{23} \approx 1/\sqrt{2}$ fine-tuning the mixing angles $\theta_{ij}^l$.

We observe that from type-I and type-II seesaw mechanisms we have for all the cases presented above

$$m_\nu = M_L - M_\nu \frac{1}{M_R} M_D^T f = r_\nu f,$$

where we have used the fact that $f = f^T$. Note that only a combination of the $r_L, r_R$ parameters enters in the neutrino sector. So counting the number of free parameters, $r_L$ and $r_R$ are equivalent to one free parameter instead of two.

IV. RENORMALIZABLE THEORY

The dimension five operators assumed in the previous section can be obtained from a renormalizable theory integrating out heavy messengers fields. In general the operator $16 \phi_a \phi_b$ can be obtained from

$$w = 16 \phi_a \phi_b + \chi \overline{\chi} \chi + M_\chi 16 \chi \overline{\chi},$$

where $\chi - \overline{\chi}$ is a couple of sets of fermion messengers$^3$ and it gives rise to the operator $16 \phi_a \phi_b$ at a scale $E \ll M_\chi$.

Moreover it is easy to take a symmetry forbidding the direct tree level operators $16 \phi_a$, for $\phi_a = 10_H, 120_H, 126_H$. For example we can take a $Z_2$ symmetry acting as

$$\phi_{a,b} \rightarrow (\phi_{a,b}, -\phi_{a,b}), \quad \chi, \overline{\chi} \rightarrow -\chi, -\overline{\chi}.$$

Below we report explicit examples of renormalizable models from which the effective dimension five superpotentials assumed in the previous section can be obtained:

- case $A$
  The matter and scalar fields content of a possible renormalizable model that can give the effective superpotential of the case $A$ is given by:

$^3$ If only one messenger field is assumed the effective Yukawa mass matrix is rank one.
then the renormalizable superpotential is

\[
w = 16 \chi_1 \overline{10}_H + 16 \chi_5 \overline{16}_H + 16 \chi_7 120_H + 16 \overline{10}_H \chi_5 45_H + 16 \chi_6 10_H + M_1 \chi_1 \chi_5 + M_4 45_5 \chi_5. \quad (57)
\]

- case B
  The matter and scalar field content of the model is

\[
\begin{array}{c|cccc|cccc}
\hline
& 16 & 10_H & 16_H & 45_H & 120_H & 16_H & 45_H & 120_H \\
\hline
Z_2 & + & - & - & - & - & - & - & - \\
Z'_2 & + & + & - & - & + & + & - & - \\
\hline
\end{array}
\]

and the superpotential is given by

\[
w = 16 \chi_1 \overline{10}_H + 16 \overline{16}_H \chi_1 10_H + 16 \chi_2 120_H + 16 \overline{10}_H \chi_2 45_H + M_1 \chi_1 \overline{16}_H + M_2 \chi_2 \overline{16}_H. \quad (58)
\]

- case C
  The matter and scalar field content of the model is

\[
\begin{array}{c|cccc|cccc}
\hline
& 16 & 1_H & 10_H & 45_H & 120_H & 120_H & 10_H & 45_H \\
\hline
Z_2 & + & - & - & - & - & - & - & - \\
Z'_2 & + & + & - & - & + & + & - & - \\
\hline
\end{array}
\]

and the superpotential is given by

\[
w = 16 \chi_1 \overline{10}_H + 16 \chi_1 \overline{16}_H + 16 \chi_2 \overline{10}_H + 16 \chi_2 \overline{45}_H + M_1 \chi_1 \overline{16}_H + M_2 \chi_2 \overline{16}_H. \quad (59)
\]

V. CONCLUSIONS

Neutrino mixing data are in well agreement with maximal atmospheric angle, tri-maximal solar angle and may be with a non-zero and quite large (namely of order of the Cabibbo angle) reactor angle. TBM mixing gives zero reactor angle however it can be a reasonable starting point. In fact in GUT framework large deviation of the 1 - 3 angle can arise from the charged sector. However a simple picture for TBM in SO(10) is still missing. In order to approach the problem recently has been studied models where light-neutrino mass matrix arises only from type-II seesaw mechanism. In this paper we studied the possibility that both type-I and type-II seesaw mechanisms yield TBM neutrino mixing in a SO(10) model. We have assumed that the superpotential contains only dimension five non-renormalizable operators. We studied three different possible scenarios for TBM neutrino mixing. In each case proposed we make the fits of all the fermion masses and mixing angle. One case corresponds to the model studied already in Ref. [12] for type-II seesaw dominance, while the other two are new.

We found in both cases a good fit of all the data including the recent T2K result. In particular for the first model we found an excellent fit ($\chi^2 = 0.005$) but with a high number (26) of parameters. We therefore can conclude that this case can be considered as a good starting point for a flavour theory that can reduce the number of the free parameters of the theory (for example introducing a flavour symmetry). Moreover in this case we did not need to introduce extra SO(10) singlets. For the second case we obtained $\chi^2 = 5.6$ with only 16 free parameters and 2 degree of freedom, making this case the most predictive. For third case we found a very good fit $\chi^2 = 0.002$ with 18 free parameters. This case also can be considered as a good starting point for a complete flavour theory. Even if we needed to introduce one SO(10) singlet we consider the last two cases as the most promising for the moment.

For the three cases proposed, we give possible renormalizable realizations where we have introduced messenger fields and extra Abelian symmetries.

We remark that in this paper we focused on the flavour sector and we do not make a full analysis of the model. In particular we leave to a future analysis the study of the Higgs potential and related issues such as the breaking pattern of SO(10) to the SM, problems related to the doublet-triplet splitting (proton-decay) and the achieving of exact coupling unification considering the possible breaking steps and the related threshold corrections.
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Appendix A

Here we report for convenience of the reader the table VIII of ref. In general an $SO(10)$ dimension five operator can be written as $16 \times 16 \times \phi_a \phi_b$ where $\phi_{a,b}$ are scalar fields $\phi_{a,b} = 1_H, 16_H, 16_{\overline{H}}, 45_H, \ldots$ and so on. For simplicity we assume that $SO(10)$ is broken through $SU(5)$ and we describe the contribution of the dimension five operators to the fermion mass matrices in the $SU(5)$ language. When one of the components of $\phi_a$ and $\phi_b$ take vev $a_i$ and $b_i$ respectively (where $i$ is the $SU(5)$ index of the component), one generates contributions to the quark and lepton masses.

| case | $SO(10)$ operator | mass matrices |
|------|-------------------|---------------|
| IV   | $(16,16_H)_{10}(16,16_H)_{10}$ | $M_d = K a_1 b_1 + K^T a_1 b_1$  
$M_d^T = K a_1 b_1 + K^T a_1 b_1$ |
| V    | $(16,16_{\overline{H}})_{1}(16,16_{\overline{H}})_{1}$ | $M_u = K a_5 b_1 + K^T a_1 b_5$  
$M_L = K a_s a_5 b_5$  
$M_R = K a_s a_1 b_1$ |
| VI   | $(16,16_{\overline{H}})_{45}(16,16_{\overline{H}})_{45}$ | $M_u = 8K a_5 a_1 b_1$  
$M_L = 3(K a_5 b_1 + K^T a_1 b_5)$  
$M_R = -5K a_s a_5 b_5$  
$M_R = -5K a_s a_1 b_1$ |
| VII  | $(16,16_{10})_{16}(16,45_{\overline{H}})_{16}$ | $M_u = K a_5 b_1 - 4b_{24}$  
$M_L = 5K a_5 b_1 + K^T a_5 (-3b_1 - 3b_{24})$  
$M_d = K a_5 b_1 + K^T a_5 (b_1 + b_{24})$  
$M_d^T = K a_5 (-3b_1 - 3b_{24}) + K^T a_5 (b_1 + b_{24})$  
|
| VIII | $(16,120_{H})_{16}(16,45_{\overline{H}})_{16}$ | $M_u = K a_5 (b_1 - 4b_{24}) - K^T a_5 (b_1 + b_{24})$  
$M_d = 5K a_5 b_1 - K^T a_5 (-3b_1 - 3b_{24})$  
$M_d = 5K a_5 b_1 + K^T a_5 (b_1 + b_{24})$  
$M_d^T = K a_5 (-3b_1 + 3b_{24}) - K^T a_5 (b_1 + 6b_{24})$  
|
| IX   | $(16,16_H)_{120}(16,16_{\overline{H}})_{120}$ | $M_d = K (a_5 b_1 + 2a_1 b_5) + K^T (a_1 b_1 + 2a_5 b_1)$  
$M_d^T = K (a_5 b_1 + 2a_1 b_5) + K^T (a_1 b_5 + 2a_5 b_1)$  
|

TABLE I: The contributions to the mass matrices from $SO(10)$-invariant dim-5 operators, from table VIII of ref. $K$ is an arbitrary matrix.

Below we report the contributions to the mass matrices from $SO(10)$ invariant renormalizable Yukawa couplings. Different VEVs of the same $SO(10)$ Higgs multiplet carry a subscript indicating the $SU(5)$ component they belong to.

| case | $SO(10)$ operator | mass matrices |
|------|-------------------|---------------|
| I    | $16_M 16_{10}H$  | $M_u = M_v = Y_{10}v_5$  
$M_d = M_{\nu} = Y_{10}\nu_{\overline{\nu}}$  
|
| II   | $16_M 16_{120}H$ | $M_u = Y_{120}v_{45}$  
$M_v = Y_{120}\nu_{5}$  
$M_d = Y_{120}(\nu_{\overline{\nu}} + v_{\overline{\nu}})$  
$M_d^T = Y_{120}(\nu_{\overline{\nu}} - 3v_{\overline{\nu}})$  
|
| III  | $16_M 16_{126}H$ | $M_u = Y_{126}v_{5}$  
$M_v = -3Y_{126}\nu_{5}$  
$M_d = Y_{126}v_{\overline{\nu}}$  
$M_d^T = Y_{126}v_{\overline{\nu}}$  
$M_L = Y_{126}v_{15}$  
$M_R = Y_{126}v_{1}$  
|
Appendix B

In this section we show the fitting procedure used in our analysis. For charged fermions and CKM mixings the fit are performed on the set of data evolved at the GUT scale showed in Tab. II. The threshold effects are not considered, because they are model dependent and we try to make a general analysis valid for the various models. In these theories there are no constrains on the value of $\tan\beta$, so we use the high scale evolved data in the case of $\tan\beta = 10$.

| Observables | Input data |
|-------------|------------|
| $m_u[\text{MeV}]$ | $0.55 \pm 0.25$ |
| $m_c[\text{MeV}]$ | $210 \pm 21$ |
| $m_t[\text{GeV}]$ | $82.4^{+30.7}_{-14.8}$ |
| $m_d[\text{MeV}]$ | $1.24 \pm 0.41$ |
| $m_s[\text{MeV}]$ | $21.7 \pm 5.2$ |
| $m_b[\text{GeV}]$ | $1.06^{+0.14}_{-0.09}$ |
| $m_e[\text{MeV}]$ | $0.3585 \pm 0.0003$ |
| $m_\mu[\text{MeV}]$ | $75.672 \pm 0.058$ |
| $m_\tau[\text{GeV}]$ | $1.3922 \pm 0.0013$ |
| $V_{us}$ | $0.2243 \pm 0.0016$ |
| $V_{cb}$ | $0.0351 \pm 0.0013$ |
| $V_{ub}$ | $0.0032 \pm 0.0005$ |
| $J \times 10^{-5}$ | $2.2 \pm 0.6$ |

TABLE II: GUT scale data for charged fermions for $tg\beta = 10$ (ref. [20], [21])

For neutrino masses and PMNS mixings we use the results in Tab. III. These values are obtained with a global fit considering also the recent results from T2K and MINOS. In the models we considered we never obtain degenerate neutrino mass spectrum, so the effects of the evolution from the low energy scale to the GUT scale can be considered negligible to a good approximation for these observables.

| Observable | Input data |
|------------|------------|
| $\Delta m_{21}^2 \times 10^{-5}[\text{eV}^2]$ | $7.59^{+0.20}_{-0.18}$ |
| $\Delta m_{31}^2 \times 10^{-3}[\text{eV}^2]$ | $2.50^{+0.09}_{-0.16}$ |
| $\sin^2\theta_{13}$ | $0.013^{+0.005}_{-0.005}$ |
| $\sin^2\theta_{12}$ | $0.312^{+0.017}_{-0.015}$ |
| $\sin^2\theta_{23}$ | $0.52^{+0.06}_{-0.07}$ |

TABLE III: Neutrino masses and mixing in normal hierarchy (ref. [2])
Appendix C

Here we give some other details on the results of the numerical analysis. In particular for the three cases we analysed we give the best fit parameters and the values for the observables that we obtain.

- **case A:**

\[
\begin{align*}
h & = \begin{pmatrix}
h_{11} & h_{12} e^{i\delta_{h_{12}}} & h_{13} e^{i\delta_{h_{13}}} \\
h_{12} e^{-i\delta_{h_{12}}} & h_{22} & h_{23} e^{i\delta_{h_{23}}} \\
h_{13} e^{-i\delta_{h_{13}}} & h_{23} e^{-i\delta_{h_{23}}} & h_{33}
\end{pmatrix} \\
\delta h_{12} & = -1.39 \\
h_{13} & = 13.1 \\
\delta h_{13} & = 0.232 \\
h_{22} & = 5.10 \\
\delta h_{22} & = 0.9 {\text{s}} \\
h_{23} & = 15.0 \\
\delta h_{23} & = 1.81 \\
h_{33} & = 79.1 \\
h_{11} & = -6.42 \\
\delta h_{11} & = 0.901 \\
h_{12} & = -6.22 \\
\delta h_{12} & = -0.652 \\
h_{13} & = 2.62 \\
\delta h_{13} & = -1.06 \\
h_{22} & = 4.55 \\
\delta h_{22} & = 0.901 \\
h_{23} & = -31.4 \\
\delta h_{23} & = -1.06 \\
h_{33} & = 27.3 \\
\delta h_{33} & = -1.06 \\
f_0 & = -3.23 \\
f_1 & = -0.155 \\
f_2 & = 0.987 \\
\delta f_0 & = -0.652 \\
\delta f_1 & = -1.06 \\
\delta f_2 & = -1.06 \\
\delta f_{12} & = 0.901 \\
\delta f_{13} & = 0.901 \\
\delta f_{23} & = 0.901 \\
\delta f_{33} & = 0.901 \\
\end{align*}
\]

| Observables | Parameter | \(m_{u}[\text{MeV}]\) | \(m_{c}[\text{MeV}]\) | \(m_{t}[\text{GeV}]\) | \(m_{d}[\text{MeV}]\) | \(m_{s}[\text{MeV}]\) | \(m_{b}[\text{GeV}]\) | \(m_{c}[\text{MeV}]\) | \(m_{t}[\text{GeV}]\) | \(V_{us}\) | \(V_{cb}\) | \(V_{ub}\) | \(J \times 10^{-3}\) | \(\Delta m_{21}^{2} \times 10^{-3} \text{[eV}^{2}\text{]}\) | \(\Delta m_{32}^{2} \times 10^{-3} \text{[eV}^{2}\text{]}\) | \(\sin^{2}\theta_{13}\) | \(\sin^{2}\theta_{12}\) | \(\sin^{2}\theta_{23}\) | \(\chi^{2}\) | \(v_{u}/v_{u} \times 10^{-9}\) |
|-------------|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.550       | 210       | 1.24            | 21.7            | 1.06            | 0.3585          | 75.67           | 1.922           | 0.224           | 0.0351          | 0.00329        | 2.20            | 7.59            | 2.50            | 0.0132          | 0.312           | 0.516           | 0.00500         | 0.00947         |
$$h^S = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{12} & h_{22} & h_{23} \\ h_{13} & h_{23} & h_{33} \end{pmatrix}$$  \quad (63)

$$h^A = i \begin{pmatrix} 0 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & 0 & \sigma_{23} \\ \sigma_{13} & -\sigma_{23} & 0 \end{pmatrix}$$  \quad (64)

$$f = \begin{pmatrix} f_2 & f_1 & f_1 \\ f_1 & f_2 + f_0 & f_1 - f_0 \\ f_1 & f_1 - f_0 & f_2 + f_0 \end{pmatrix}$$  \quad (65)

| Observable | Best fit value | Parameter | Best fit value |
|------------|----------------|-----------|----------------|
| $m_u$ [MeV] | 0.465          | $h_{11}v_u$ [GeV] | 0.785          |
| $m_c$ [MeV] | 210            | $h_{12}v_u$ [GeV] | 0.346          |
| $m_t$ [GeV] | 81.5           | $h_{13}v_u$ [GeV] | 7.83           |
| $m_d$ [MeV] | 2.95           | $h_{22}v_u$ [GeV] | 3.09           |
| $m_s$ [MeV] | 23.2           | $h_{23}v_u$ [GeV] | 4.35           |
| $m_b$ [GeV] | 1.09           | $h_{33}v_u$ [GeV] | 82.2           |
| $m_s$ [MeV] | 0.3585         | $\sigma_{12}v_u$ [GeV] | -0.298        |
| $m_t$ [GeV] | 75.67          | $\sigma_{13}v_u$ [GeV] | 0.229          |
| $m_b$ [GeV] | 1.292          | $\sigma_{23}v_u$ [GeV] | 2.29          |
| $V_{us}$    | 0.224          | $f_{1}v_u$ [GeV] | -0.935         |
| $V_{cb}$    | 0.0352         | $f_{2}v_u$ [GeV] | 0.0316         |
| $V_{ub}$    | 0.00321        | $r_1/\tan \beta$ | 0.0132         |
| $J \times 10^{-5}$ | 2.15 | $v_e$ | 1.15         |
| $\Delta m^2_{21} \times 10^{-5}$ [eV$^2$] | 7.59 | $r_2$ | 2.50          |
| $\Delta m^2_{32} \times 10^{-3}$ [eV$^2$] | 2.50 | $v_e/v_u \times 10^{-9}$ | 0.0249 |
| $\sin^2 \theta_{13}$ | 0.0118        | $\sin \theta_{13}$ | 0.315         |
| $\sin^2 \theta_{12}$ | 0.516        | $\sin^2 \theta_{23}$ | 0.516         |
| $\chi^2$   | 5.64           |           |                |

TABLE V: Fit result for the case B (16 parameters) described in Sect. 3
• case C:

\[
    h^S = \begin{pmatrix}
        h_{11} & h_{12} & h_{13} \\
        h_{12} & h_{22} & h_{23} \\
        h_{13} & h_{23} & h_{33}
    \end{pmatrix}
\]

(66)

\[
    h^A = i \begin{pmatrix}
        0 & \sigma_{12} & \sigma_{13} \\
        \sigma_{12} & 0 & \sigma_{23} \\
        \sigma_{13} & -\sigma_{23} & 0
    \end{pmatrix}
\]

(67)

\[
    f = \begin{pmatrix}
        f_2 & f_1 & f_1 \\
        f_1 & f_2 + f_0 & f_1 - f_0 \\
        f_1 & f_1 - f_0 & f_2 + f_0
    \end{pmatrix}
\]

(68)

| Observable   | Best fit value | Parameter   | Best fit value |
|--------------|---------------|-------------|---------------|
| \(m_u\) [MeV] | 0.550         | \(h_{11} v_u\) [GeV] | 0.584         |
| \(m_c\) [MeV] | 210           | \(h_{12} v_u\) [GeV] | -0.548        |
| \(m_t\) [GeV] | 82.2          | \(h_{13} v_u\) [GeV] | -5.49         |
| \(m_d\) [MeV] | 1.24          | \(h_{22} v_u\) [GeV] | 3.55          |
| \(m_s\) [MeV] | 21.6          | \(h_{23} v_u\) [GeV] | 3.99          |
| \(m_\tau\) [GeV] | 1.06         | \(h_{33} v_u\) [GeV] | 81.8          |
| \(\sigma_{12}\) [GeV] | 0.3585       | \(\sigma_{12} v_u\) [GeV] | -0.317       |
| \(\sigma_{13}\) [GeV] | 75.67        | \(\sigma_{13} v_u\) [GeV] | 2.79          |
| \(\sigma_{23}\) [GeV] | 1.292        | \(\sigma_{23} v_u\) [GeV] | -7.09        |
| \(V_{us}\) | 0.224         | \(f_{0u} v_u\) [GeV] | -0.999        |
| \(V_{cb}\) | 0.0351        | \(f_{1u} v_u\) [GeV] | -0.207        |
| \(V_{ub}\) | 0.00320       | \(f_{2u} v_u\) [GeV] | 0.0290        |
| \(J \times 10^{-5}\) | 2.20         | \(r_1 / \tan \beta\) | 0.0129       |
| \(\Delta m^2_{23} \times 10^{-5}[eV^2]\) | 7.59         | \(r_2\) | 1.85          |
| \(\Delta m^2_{32} \times 10^{-3}[eV^2]\) | 2.50         | \(r_3\) | 1.37          |
| \(\sin^2 \theta_{13}\) | 0.0131       | \(c^2\) | 1.48          |
| \(\sin^2 \theta_{12}\) | 0.312         | \(c^4\) | 1.17          |
| \(\sin^2 \theta_{23}\) | 0.520         | \(v_{u} / v_{s} \times 10^{-9}\) | 0.0287       |
| \(\chi^2\) | 0.00149       | |

TABLE VI: Fit result for the case C (18 parameters) described in Sect. 3

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