Lepton masses and mixing and lepton number violating processes in the minimal 3-3-1 model

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Abstract. We consider the minimal 3-3-1 model with sterile neutrinos transforming as singlet under the \( SU(3)_L \otimes U(1)_X \) symmetry and we find numerical solutions for these matrices by imposing that besides the observed lepton masses, they also give the observed mixing matrix in the lepton sector, the so called Pontecorvo-Maki-Nakawaga-Sakata (PMNS) matrix. After that we study its phenomenological consequences in various processes involving flavor number violating interactions. In the allowed leptonic decays \( l_i \rightarrow l_j l_k \bar{l}_k \) which occurs at the tree level with \( l_i = \mu, \tau \), \( l_j,k = e, \mu \), we find in particular that the channel \( \mu \rightarrow ee \bar{e} \) imposes a lower mass limit on the vector doubly charged bilepton of 4.58 TeV and that the scalar contributions are negligible.

1. Introduction
At present it is very well established that neutrinos are massive particles and that there is mixing in the leptonic charged current. It means that is necessary to show that the mechanism for generating neutrino, charged lepton masses and a realistic PMNS matrix is also compatible with the data of violating lepton flavor numbers processes like \( l_i \rightarrow l_j l_k \bar{l}_k \), where \( l_i = \mu, \tau \), \( l_j,k = e, \mu \), \( l_i \rightarrow l_j \gamma \) and \( h^0 \rightarrow l_i \bar{l}_j \) [1].

Here we will study these processes in the context of the minimal 3-3-1 model (m331 for short) in which the three lepton families transform as triplets \( \Psi = (\nu_l, l, l^c) \sim (3, 0) \) under the \( SU(3)_L \otimes U(1)_X \) symmetry [2, 3, 4, 5]. Moreover, since we will work in the context where the VEVs of the triplets \( \eta \) and \( \rho \) saturate the electroweak scale, \( (246 \text{ GeV})^2 \), the sextet is not enough to give the correct mass to the charged leptons. Hence, the sextet is used just to induce the dimension five effective operator. And finally we add three sterile neutrinos.

The 3-3-1 models have doubly charged vector and scalar bileptons, generically denoted by \( X^{--} \), with interactions that violate the individual lepton flavor number by single or two units, for instance \( X^{--} \rightarrow \mu^- e^- \) and \( X^{--} \rightarrow e^- e^- \), respectively [6]. Hence, in this models the decays \( \tau \rightarrow \mu \mu \bar{e}, e e \bar{e} \) (the three leptons may be all different) and \( \mu \rightarrow e e \bar{e}, e \gamma, ... \) are allowed and can be used to constraints the masses of the bileptons once the unitary matrices, here denoted by \( V_{L,R}^{l} \) and \( V_{L}^{\nu} \), needed to diagonalize the lepton matrices, are fixed. In fact, the processes are a prediction of this sort of model in a given context.

The processes with lepton flavor number violation \( \Delta L_i = \pm 1 \) in this model were considered many years ago by Liu and Ng. However, at that time almost nothing was known about the lepton mixing and neutrino masses. Here we will take into account this new information and
also consider the effects of the neutral Higgs bosons which in this model have non-diagonal interactions in the flavor space. We stress that even if neutrinos were massless (at tree level) the lepton flavor number is not conserved in this model. Here we will not consider neither CP violation.

After adjusting the masses and the matrices $V_{LR}$ and $U_{LR}$, respectively, we are left with the following free parameters: $\Lambda$, which is related with the mass scale of the scalar sextet and the matrices relating the mass and symmetry eigenstates in the scalar sectors. In particular in the processes analyzed in the paper those relevant matrices are $A$ appearing in the doubly charged sector, $U^h$ in the CP even sector, and $U^A$ in the CP odd sector. Next, we were able to identify the SM Higgs $h^0$, which from the experimental data for $h^0 \to \tau\tau$ allowed us to determine $U_{h1}^h = 0.096$, while the parameter $U_{h1}^h$ is not important. Hence, the tree level flavor number violating Higgs decays are $\text{Br}(h^0 \to e\mu\tau)^{m_{331}} \approx 10^{-14}$ and $\text{Br}(h^0 \to e\tau)^{m_{331}} \approx 10^{-13}$, being highly suppressed respect to the reported data of $\text{Br}(h^0 \to \mu\tau)^{\text{Exp}} \approx 10^{-5}$. We stress that this decay also could be generated via loop interactions of the SM Higgs with new possible virtual scalars, that kind of process might be responsible of the surprising experimental data.

In the flavor number violating processes $l_i \to j_l l_k l_k$, the channel $\mu \to e\bar{e}$ imposed the bound $m_{U^+} > 4584$ GeV respecting the experimental upper limit of $\text{Br}(\mu \to e\bar{e})^\text{Exp} < 10^{-12}$. For the tau decays we estimate for all of the reactions $\text{Br}(\tau \to j_l l_k l_k)^{m_{331}} \approx 10^{-15}$ using $m_{U^+} = 4590$ GeV, which have resulted 7 orders of magnitude suppressed respect to the experimental upper limits.

Regarding to the one-loop processes $l_i \to l_j \gamma$, the $\text{Br}(\mu \to e\gamma)^{m_{331}} \approx 10^{-33} - 10^{-30}$, this is up to 18 orders of magnitude larger than the SM estimation but 17 orders of magnitude below the experimental upper limit; and similar behaviour for the tau decays being $\text{Br}(\tau \to e\gamma)^{m_{331}} \approx 10^{-29} - 10^{-24}$ and $\text{Br}(\tau \to \mu\gamma)^{m_{331}} \approx 10^{-31} - 10^{-25}$, and in contrast to the channels $l_i \to j_l l_k l_k$ where the $U^{\pm \pm}$ vector bielectron was responsible for the signals, in these one-loop processes the vector bielectron provided the more suppressed contribution of the considered new particles interacting with leptons.

In order to verify how these predictions depend on the numerical values for the entries of those matrices we have considered different solutions. With the first one we obtain the lower limit $m_{U^+} > 51.8$ TeV; the second solution also adjusts the lepton masses and the PMNS and predicts a $m_{U^+} > 16.49$ TeV; the third solution predicts $m_{U^+} > 3.34$ TeV, but unfortunately we were not able to fit the PMNS matrix. Finally, our principal solution, the one reported throughout the text, adjusts the masses and the mixing matrix and predicts a $m_{U^+} > 4.584$ TeV, we have chosen this solution because it allow us to explore the phenomenological consequences of the model with a bielectron gauge boson of just few TeV. We include the various solutions in order to leave clear that the lower limit obtained by phenomenological analysis strongly depend on the matrices obtained previously by fitting the fermion masses and mixing matrices. However, we consider this procedure is better or more realistic that the one when the entries of the matrices are still arbitrary and it is not known if they give also the correct masses and mixing matrices [7].

Moreover, in models with FCNC the unitary matrices that are needed to diagonalize the mass matrix in each charged sector survive in different places of the Lagrangian when it is written in terms of the mass eigenstates fields. Besides the usual combinations $V_{CKM} = V_{dL}^\dagger V_L^d$ and $V_{PMNS} = V_{L}^\dagger V_{\mu}$, some combinations involving the left- with right-handed matrices also survive. For instance, in the present model, in the lepton sector we have interactions with vector fields that involve $V_U = V_R^\dagger V_L^d$ in (15), and with scalars as in (18) with $K_L = V_R^\dagger G^* V_L^d$ where $G^*$ is a symmetric matrix. Similar situation occurs in the quark sector. For this reason, it is much better to fit first all of these unitary matrices by getting the known masses and mixing matrices in the interactions with $W^\pm_\mu$ and only then study the phenomenological consequences constraining the masses of the extra particles in the model. After doing this, the only free parameters are the
masses of the new particles and the unitary matrices in the scalar sectors which we have denoted by $O, U^{h,A}$.

The decays $\mu \rightarrow eee$ and $\mu \rightarrow e\gamma$ can also be considered in the 3-3-1 model with right-handed neutrinos (331RN by short) of the sort proposed, i.e., when the leptons are in triplets $\psi_L = (\nu_l, l, \nu_c)^T \sim (\mathbf{3}, -1/3)$. In the latter model only three triplets as $\eta, \rho, \rho'$ are needed to break the gauge symmetry and give correct masses to all fermions in the model. However, it was shown that in this model the processes above are suppressed as in the standard model unless a sextet is added giving also a natural small masses for neutrinos. We note that in that model there is no doubly charged vector boson and the lepton flavor violating processes are mediated only by the doubly charged Higgs scalar in the sextet.

For more details see Ref.[8] and all references cited there.

![Figure 1](image)

**Figure 1.** The channel $\mu \rightarrow eee$ impose a strong limit on the vector doubly charged bilepton mass, as shown below. Each solution for the masses of leptons and the PMNS have very distinct phenomenological predictions for the mass of the $m_{U^{++}}$.

**References**

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