Non-Local effective SU(2) Polyakov loop model from inverse Monte-Carlo methods

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Motivation

Theory
- Inverse Monte-Carlo Method
- Geometric Ward-Identities and Geometric DSEs
- Polyakov Models: Linear and Logarithmic

Numerical Results
- Local Polyakov Models: Linear vs. Logarithmic
- Non-local Logarithmic Polyakov Model
- Non-local Linear Polyakov Model
- Long-Distance Behaviour of Non-Local Couplings

Conclusion and Outlook
Motivation

- QCD difficult to solve. Sign problem for finite $\mu$
- There are many ideas to deal with this, e.g. effective models
- Sign problem in Polyakov loop models are expected to be less severe
- Problem: How do we get the effective action from a known action?
Wilson action $S[U]$ integrate out $U$ effective action $S_{\text{eff}}[\chi]$

$C(U)$ $\chi = \chi(U)$ $C(\chi)$

MC IMC

configurations

eff. configurations

Effective SU(2) Polyakov loop model from inverse Monte-Carlo methods
Inverse Monte-Carlo Method II

- Take an effective action $S_{\text{eff}}(\lambda)$ with yet to find coupling constants $\lambda$

- Remember how DSEs are derived

$$\left\langle \frac{\delta S}{\delta \varphi}(\lambda) \right\rangle_{\text{eff}} = 0$$

- Demand that the effective theory approximates the full theory well, i.e.

$$\left\langle \frac{\delta S}{\delta \varphi}(\lambda) \right\rangle_{\text{full}} = 0$$

- Solve this equation numerically for $\lambda$
Geometric Ward-Identities and Geometric DSEs

Geometric Ward-Identities I

- Left invariance of the Haar measure yields the (mathematical) identity

\[
\int d\mu(g) (L_a f)(g) = 0, \quad f \in L^2(G).
\]

- For class functions \( F, \tilde{F} \) we obtain

\[
\int d\mu_{\text{red}} \left[ \mathbf{L} \cdot (F \mathbf{L} \tilde{F}) \right] = \int d\mu_{\text{red}} (F \mathbf{L}^2 \tilde{F} + \mathbf{L} F \cdot \mathbf{L} \tilde{F}) = 0
\]
Geometric Ward-Identities and Geometric DSEs

Geometric Ward-Identities II

- Use character expansion for class functions

\[ F(g) = F(\chi_1(g), ..., \chi_r(g)), \quad r = \text{rank}(G) \]

\[ L_a F(\chi) = \sum_q \frac{\partial F(\chi)}{\partial \chi_q(g)} L_a \chi_q(g) \]

- Set \( \tilde{F} = \chi_p \), with \( p \in \{1, ..., r\} \)
- Use

\[ \chi_\mu \chi_\nu = \sum_\lambda C^\lambda_{\mu\nu} \chi_\lambda, \quad \sum_a L^2_a \chi_p(g) = -c_p \chi_p(g) \]

Geometric Ward-Identity

\[
0 = \int_G d\mu_{\text{red}} \left\{ \frac{1}{2} \sum_q \left[ (c_p + c_q) \chi_p \chi_q - \sum_\lambda C^\lambda_{\mu\nu} c_\lambda \chi_\lambda \right] \frac{\partial F(\chi)}{\partial \chi_q(g)} - c_p \chi_p(g) F \right\} =: K_q
\]
Geometric Ward-Identities and Geometric DSEs

**Geometric DSEs**

- Insert \( \exp(-S_{\text{eff}}) \) and take sum over all lattice points

\[
V^{-1} \sum_{i \in L} \left\langle \frac{1}{2} \sum_{q} K_{q,i} \frac{\partial F_i}{\partial \chi_{q,i}} \exp(+S_{\text{eff}}) - c_p \chi_{p,i} F_i \exp(+S_{\text{eff}}) \right\rangle_{\text{eff}} = 0
\]

- Take this DSE for IMC-method to calculate \( \vec{\lambda} \) via

\[
V^{-1} \sum_{i \in L} \left\langle \frac{1}{2} \sum_{q} K_{q,i} \frac{\partial \vec{F}_i}{\partial \chi_{q,i}} \exp(+S_{\text{eff}}(\vec{\lambda})) - c_p \chi_{p,i} \vec{F}_i \exp(+S_{\text{eff}}(\vec{\lambda})) \right\rangle_{\text{full}} = 0
\]

(Need \( \dim(\vec{F}) = \dim(\vec{\lambda}) \) different class-functions \( F_i \) to solve the equations for \( \vec{\lambda} \))

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Non-Local effective SU(2) Polyakov loop model from inverse Monte-Carlo method
Integrating out all spatial links and applying the strong coupling expansion yields

### The linear Polyakov model

\[ S = \sum_p \sum_r \sum_{\langle i,j \rangle=r} \lambda_{p,r} \chi_{p,i} \chi_{p,j} \]

Expanding the action term and applying a resummation of higher order terms yields

### The logarithmic Polyakov model

\[ S = - \sum_p \sum_r \sum_{\langle i,j \rangle=r} \log \left( 1 + g_{p,r} \chi_{p,i} \chi_{p,j} \right) \]

[J. Langelage, S. Lottini, O. Philipsen 2010]
Polyakov Models: Linear and Logarithmic

Geometric DSEs for the Logarithmic Polyakov Model

- Neglecting terms with $r \geq r_{\text{max}}$, and representations with $p \geq p_{\text{max}}$ yields

$$e^{-S} = \prod_{p=1}^{p_{\text{max}}} \prod_{r=1}^{r_{\text{max}}} \prod_{<i,j>=r} \exp \left( -\lambda_{p,r} \chi_{p,i} \chi_{p,j} \right),$$

- Insert into geometric DSE and set $\vec{F}_i = \vec{f}_i \exp(-S_{\text{eff}})$, with $\vec{f}_i = \{f_{p,r,i}\}$

$$f_{p,r,i} = \frac{1}{g_{p,r}} \frac{\partial (e^{-S})_{p,r,i}}{\partial \chi_{1,i}}$$

- Now match the effective model to the full theory
NUMERIC RESULTS
Lin. model improves if we add representations
Log. resummation seems to work quite well. No higher representations needed.

For small $\beta$ the log. resummation is expected to improve results.

Still far from the full theory for large $\beta$. $\rightarrow$ Try non-local models.
Adding larger distances for the interaction improves the result.

We “overshoot” when we include too large distances.
Motivation  
Theory  
Numerical Results  
Conclusion and Outlook

Non-local Logarithmic Polyakov Model

Non-local Logarithmic Polyakov Model

log. model with 1 rep.

log. model 2 rep.

- Larger Lattice seems to fix overshooting.
- But: Higher representations change the result. We overshoot again.
- Logarithmic resummation not doing well for large $\beta$ (many non-local terms)
Non-local Linear Polyakov Models

- Linear model does not overshoot, even on the smaller lattice.
Motivation
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Non-local Linear Polyakov Models

Non-local Linear Polyakov Models

lin. model with 2 rep.

We get close to the full theory with 2 representations.
Adding more rep. seems not to spoil the result. Still close to full theory.

Approaches the full theory very slowly near $\beta_c$ (large correlation length)
Non-local Linear Polyakov Models

- Same result for larger lattice. No overshooting. Close to full theory with 2 rep.
Higher representations seem not to spoil the result.

Adding non-local terms for large $\beta$ works much better than for log. model.

Approaches the full theory very slowly around $\beta_c$. 
Look at long-distance behaviours of couplings to make model predictable and compare to analytical models.
Compare to Greensite’s and Langfeld’s analytical model

\[ S = c_0 \sum_x P_x \left( - \frac{1}{2} c_1 \sum_x P_x^2 - 2 c_2 \sum_{x,y} P_x Q(x-y) P_y \right), \]

\[ Q(x-y) = \begin{cases} (\sqrt{-\nabla^2})_{xy} & |x-y| \leq r_{\text{max}} \\ 0 & |x-y| > r_{\text{max}} \end{cases} \]

\[ \beta = 2.22 \]

- Shape agrees quite well.
- Fitted coupling \( c_2 \approx 0.227(53) \) does not agree with prediction of \( 0.491(1) \)
- Maybe if we insert additional terms (linear P-term)?
Fitting linear part and extracting "correlation length" yields peak around $\beta_c \approx 2.29$

- Dependence seems not to scale with the volume
- Might suggest model becomes local again in the continuum
Conclusion

- IMC-method works well to fix theories
- Logarithmic resummation does not work well for $\beta \gtrsim \beta_c$
- Non-local linear Polyakov model seems to work well for $\beta > \beta_c$
- Difficult around $\beta_c$. Need more non-local terms.
- Model might become local in the continuum limit

Outlook

- Improvements around $\beta_c$
- Maybe add additional terms in our ansatz (linear polyakov term)
- Check larger lattices and continuum limit
- Add fermions
- Other gauge groups ($SU(3)$, $G_2$)
THANK YOU FOR YOUR ATTENTION