Laser pulse-length effects in trident pair production

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Abstract

Laser pulses facilitate multiphoton contributions to the trident pair production $e^{-}_L \rightarrow e^{-}_L + e^{-}_L + e^{+}_L$, where the label $L$ indicates a laser field dressed electron ($e^{-}$) or positron ($e^{+}$). We isolate the impact of the pulse envelope in the trident $S$ matrix element, formulated within the Furry picture, in leading order of a series expansion in the classical nonlinearity parameter $a_0$. Generally, the Fourier transform of the envelope carries the information on the pulse length, which becomes an easily tractable function in the case of a $\cos^2$ pulse envelope. The transition to a monochromatic laser wave can be handled in a transparent manner, as also the onset of bandwidth effects for short pulses can be factorized out and studied separately.

Keywords: trident process, pair production, weak-field expansion, strong-field QED

(Some figures may appear in colour only in the online journal)

1. Introduction

High-intensity laser beams in the optical regime are customarily generated by the chirped pulse amplification (see [1]). Intensities up to $10^{22}$ W cm$^{-2}$ are achievable nowadays in several laboratories [2], yielding a classical nonlinearity parameter of $a_0 = \mathcal{O}(10^{-50})$ in the focal spot. Ongoing projects [4–6] of 10 PW class lasers envisage even larger values of $a_0$. Due to higher frequencies in XFEL beams, $\omega = \mathcal{O}(10 \text{ keV})$, the parameter $a_0$ stays significantly below unity, despite similar intensities of $\mathcal{O}(10^{22} \text{ W cm}^{-2})$ when tight focusing is attained [7]. Given such a variety of laser facilities, the experimental exploration of nonlinear QED effects became feasible and is currently further promoted. Elementary processes are under consideration with the goal of testing QED in the strong-field regime. Most notable is the nonlinear Compton process $e^{-}_L \rightarrow e^{-}_L + \gamma$, also w.r.t. the subsequent use of the high energy photons ($\gamma$), up to prospects of industrial applications. While in the pioneering theoretical studies [8, 9] the higher harmonics, related to multi-photon effects, i.e. the simultaneous interaction of the electron with a multitude of photons, in monochromatic laser beams have been considered, the study of laser pulses revealed a multitude of novel structures in the $\gamma$ spectrum [3, 10–18].

The nonlinear Breit–Wheeler process, $\gamma \rightarrow e^{-}_L + e^{+}_L$ [3, 13, 19–29], as cross channel of the nonlinear Compton process, is in contrast a threshold process—sometimes termed a genuine quantum process—since the probe photon $\gamma$ energy in combination with the laser must supply sufficient energy to produce a $e^{-}e^{+}$ pair. When considering the seminal SLAC experiment E-144 [30, 31] as a two-step process (first step: generation of a high-energy photon $\gamma$ by Compton backscattering [32], second step: Breit–Wheeler process $\gamma + L \rightarrow e^{-}e^{+}$), also the nonlinear Breit–Wheeler process has been identified with the simultaneous interaction of up to five photons in the elementary subprocess.

Strictly speaking, the mentioned two-step process is only a part of trident pair production $e^{-}_L \rightarrow e^{-}_L + e^{-}_L + e^{+}_L$, as stressed in [33, 34]. Since the trident process is the starting point of seeded QED avalanches, expected to set in at high-intensities,
it is currently a subject of thorough analyses [35–37], also for benchmarking PIC codes [38].

Given the high repetition rate of the European XFEL [39] a potentially interesting option is to combine it with a synchronized electron beam of about 50 MeV (to operate slightly above the threshold) in order to facilitate a high-statistics search for the dark photon. Such a dark photon (also dubbed U boson or hidden photon) is a candidate for Dark Matter beyond the standard model of particle physics; it is a possible extension which enjoys intense theoretical [40–42] and experimental [43–46] considerations. A corresponding theoretical analysis of the trident process can be found in [47]. In fact, the trident process—in a perturbative QED (pQED) language—includes sub-diagrams of the type $\gamma^* \to e^+ e^-$, i.e. an intermediate (virtual) photon which decays into a $e^+ e^-$ pair. Via kinetic mixing, that virtual photon may ‘temporarily’ couple to a dark photon $A'$, e.g. $\gamma^* \to A' \to \gamma^*$, thus signalizing its presence as a peak of the invariant mass distribution of $e^+ e^-$. The peak would be at the mass of the dark photon and its width is related to the kinetic mixing strength.

We briefly mention the trident option of the LUXE project [48, 49] at DESY/Hamburg, which however is primarily dedicated to explore the ‘boiling of the vacuum’ by means of the nonlinear Breit–Wheeler process in the Ritus corner, i.e. a kinematical region with a nonperturbative field strength dependence and coupling constant $|\epsilon|$ dependence analog to the Schwinger pair creation probability.

While most of the above quoted papers focus on nonlinear effects in strong laser pulses, that is the impact of multiphoton contributions, we aim here at the study of apparent multiphoton effects due to bandwidth effects in weak and moderately strong laser pulses with $a_0 < 1$. The analysis of the Breit–Wheeler pair production in [22, 50] revealed that in such a regime interesting features appear for short and ultra-short pulses. For instance, despite of $a_0 < 1$ a significant subthreshold pair production is enabled. Roughly speaking, in short pulses the frequency spectrum contains high Fourier components, thus enabling the subthreshold pair creation. In that respect, we are going to study the relevance of the pulse duration for the trident process. In contrast to the elementary one-vertex processes, the trident process as a two-vertex process obeys a higher complexity, similar to the two-photon Compton scattering [51–53].

Our paper is organized as follows. In section 2, the matrix element is evaluated with emphasis on a certain regularization required to uncover the perturbative limit. The weak-field expansion is presented in section 3, where the Fourier transform of the background field amplitude is highlighted as a central quantity. The case of a constant envelope is elaborated in dome detail in section 4, where also numerical examples are exhibited. The conclusion can be found in section 5.

2. Matrix element in the Furry picture

The leading-order tree level Feynman diagram of the trident process is exhibited in figure 1(a). The corresponding $S$ matrix reads

$$S_{\text{fi}} = e^2 \int d^3x \int d^3y \{\bar{\psi}(x; p_1)\gamma^\mu\psi(x; p)D_{\mu\nu}(x - y) \times \bar{\psi}(y; p_2)\gamma^\nu\gamma^\rho\gamma^\lambda(y; p_3)\} - (p_1 \leftrightarrow p_2),$$

(1)

where $\psi(x; p)$ stands for the Volkov solution of Dirac’s equation with a classical external electromagnetic (laser) field $A_\mu(\phi) = a_0 \frac{m}{c} \epsilon_\mu f(\phi)$, $\bar{\psi}$ its adjoint, and $D_{\mu\nu}$ is the photon propagator. The $p_1 \leftrightarrow p_2$ term ensures the antisymmetrization of two identical fermions (mass $m$, charge $|\epsilon|$) in the final state. The laser field $A_\mu$, and its polarisation four-vector $\epsilon_\mu$ and phase $\phi = k \cdot x$ is specialized further on below. The momenta $p_{1,2,3}$ and $k$ are four-vectors as well, and $\gamma^\mu$ stands for Dirac’s gamma matrices. Transforming the photon propagator into momentum space, $D_{\mu\nu}(x, y) = \int d^4k/(2\pi)^4 e^{-i(k \cdot y)}D_{\mu\nu}(k')$ and employing the Feynman gauge, $D_{\mu\nu}(k') = \frac{\gamma_\mu}{k^2}$, and a suitable splitting of phase factors of the Volkov solution, e.g. $\psi(x; p) = \left(1 + e^{\frac{k \cdot x}{2p_0}}\right)A_p e^{-i(k \cdot x) - ie^2 x^2}$, one can cast the above matrix element in the form

$$S_{\text{fi}} = (2\pi)^2 e^2 \int d^3r \int d^3s\{(\pi(p_1)\Gamma^\mu(r; C)u(p)) \times \frac{g_{\mu\nu}}{k^{2} + i\epsilon} - (\pi(p_2)\Gamma^\nu(s; \phi)\nu(p_3)) \delta^{(4)}(p_1 + p_2 + p_3 - (r + s)k) - (p_1 \leftrightarrow p_2)\}$$

(2)

upon Fourier transform of the vertex function $(1 + \bar{\Pi}_\mu(k \cdot x))\gamma^\mu$

$$(1 + \bar{\Omega}_\mu(k \cdot x))\epsilon^{-i\bar{\Sigma}_\mu(k \cdot x) - \bar{\Delta}_\mu(k \cdot x)} = \int \frac{dk'}{2\pi} \Gamma^\mu(r, q_1, q_2)e^{-i\hat{k} \cdot x}.$$  

A key for that is $\bar{\Omega}_\mu(\phi = k \cdot x) = \frac{\epsilon^{\hat{k} \cdot p}}{2k_0}$ as well as

$$\hat{S}(\phi = k \cdot x; p) = \frac{1}{2k_0} \int_0^{k_0} d\phi'(2e^2 p \cdot A(\phi') - e^2 \hat{\Lambda}(\phi'))$$

as the nonlinear Volkov phase part. The quantities $u(p)$ and $\nu(p)$ are free-field Dirac bispinors, with spin indices suppressed for brevity. In intermediate steps, one meets the local energy-momentum balance $p_1 + p + k - sk = 0$ and $p_2 + p_3 - k' - rk = 0$, which combine to the overall conservation in $\delta^{(4)}$. The corresponding representation of the matrix element in momentum space is exhibited in figure 1(b)-left, where we exposed the interaction with $s$ and $r$ laser photons marked by the crosses. In figure 1(b)-right, we suppress these explicit representations of the laser background field. Note that the momentum space diagrammatics differs from the notation in [54, 55]. We introduce the short-hand notations $C$ and $BW$ to mean momentum dependences on $(p_1, -p_2)$ and $(p_2, p_3)$, respectively. For the currents $\Delta^\nu(r, C) = \bar{\Gamma}(p_1)\Gamma^\nu(u(p))$ and $\Delta^\nu(r, BW) = \bar{\Gamma}(p_2)\Gamma^\nu(p_3)$ in (2), we note the decomposition, emerging from inserting the Volkov solution, $\Gamma^\nu(r, *) = \Gamma^\nu_0(r, *) + \Gamma^\nu_1(r, *) + \Gamma^\nu_2(r, *)$ with $\bar{\epsilon}^* C$ or $\bar{\epsilon}^* BW$ and, by using a generic momentum pair $(q_1, q_2)$ for them,

$$\Gamma^\nu_0(r, q_1, q_2) = \int_{-\infty}^{\infty} d\phi e^{i\hat{\phi}q_1}e^{-i\hat{\Sigma}_\mu(k \cdot x) - \hat{\Delta}_\mu(k \cdot x)}\gamma^\mu,$$

(3a)

$$\Gamma^\nu_1(r, q_1, q_2) = \int_{-\infty}^{\infty} d\phi e^{i\hat{\phi}q_1}e^{-i\hat{\Sigma}_\mu(k \cdot x) - \hat{\Delta}_\mu(k \cdot x)} \times [\hat{\Pi}_\mu(\phi)\gamma^\mu + \gamma^\mu \hat{\Omega}_\mu(\phi)].$$

(3b)
Figure 1. Diagrams for the trident process. (a) Lowest order Feynman diagram in position space Furry picture with \( \psi(x, p) \) for the Volkov state, \( * = -ie \int d^4x \, \gamma^\mu \) for the vertex at \( x \), \( \sim \sim = D_{\mu\nu}(x - y) \) for the bare photon propagator. (b) The translation into momentum space to arrive at the right diagram with \( \Theta_0 = -ie \int d^4x \, \gamma^\mu(s, p, p_1) \) with local four-momentum balance, e.g. \( p + sk = k' + p_1 \) and \( \sim \sim = D_{\mu\nu}(k') = \frac{i\kappa_{\mu\nu}}{k'^2 + i\varepsilon} \), for the bare photon propagator in Feynman gauge. (c) Using the regularized vertex \( \hat{O}_\pi = -ie \int \frac{d^4x}{(2\pi)^4} \Gamma^\mu(s, p, p_1) \) from \( \Gamma^\mu_0 = \bar{\psi} \gamma^\mu \psi \) in \( \Gamma^\mu_0 = \Gamma^\mu_0 + \Gamma^\mu_1 + \Gamma^\mu_2 \), i.e. \( \Gamma^\mu_{a\mu} = \Gamma^\mu_0 + \Gamma^\mu_1 + \Gamma^\mu_2 \) and the one photon vertex \( \delta = \hat{G}_\pi \, \frac{d}{ds} \gamma^\mu \psi \). We note \( \Gamma^\mu_0 \propto e_{a\mu}, \Gamma^\mu_0 \propto (e_{a\mu})^2 \), while \( \Gamma^\mu_{a\mu} \) has terms \( e_{a\mu} \delta \gamma^\mu \).
\[ \hat{B}_{1,2} = B_{1,2} \text{ we arrive at} \]
\[ g_{\mu\nu}\Delta^\mu(r; C)\Delta^\nu(s; BW) = \pi^2G^2\delta(r)\delta(s)M_0 + \pi^2\delta(r)M_1(s) + \pi^2\delta(s)M_2(r) + M_2(r, s), \]
(10)

where
\[ M_0 = g_{\mu\nu}J^\mu(C)J^\nu(BW), \]
(11a)
\[ M_{11}(s) = g_{\mu\nu}J^\mu_0(C) \left( \sum_{l=0}^{2} B_l(BW)J^\nu_l(BW) \right), \]
(11b)
\[ M_{12}(r) = g_{\mu\nu}J^\mu_0(BW) \left( \sum_{l=0}^{2} B_l(C)J^\nu_l(C) \right), \]
(11c)
\[ M_2(r, s) = g_{\mu\nu} \left( \sum_{l=0}^{2} B_l(s; BW)J^\nu_l(BW) \right) \times \left( \sum_{l=0}^{2} B_l(s; C)J^\nu_l(C) \right), \]
(11d)

These four expressions correspond to the momentum space diagrams exhibited in figure 1(c).

3. Weak-field expansion

With the argumentation given in the introduction we now attempt an expansion in powers of \( a_0 \). We note \( J^\mu \propto a_0^\mu \); again, \( l \) is a label (power) on the lhs (rhs). The leading-order terms of the phase integrals (6) thus become
\[ \hat{B}_0(s) = -\frac{a_0}{s} \int_{-\infty}^{\infty} \text{d}\phi f(\phi)e^{is\phi} + \mathcal{O}(a_0^2), \]
(12a)
\[ \hat{B}_l(s) = \int_{-\infty}^{\infty} \text{d}\phi f^{(l)}(\phi)e^{is\phi} \times \left[ 1 + ia_0\alpha_l \int_{0}^{\infty} \text{d}\phi F(\phi') \right] + \mathcal{O}(a_0^2), \]
(12b)
\[ \mathcal{G} = 2 + ia_0\alpha_l \lim_{\eta \to -\infty} \left( \int_{0}^{\eta} f(\phi)d\phi + \int_{-\infty}^{0} f(\phi)d\phi \right) + \mathcal{O}(a_0^2). \]
(12c)

Denoting the Fourier transform of \( f(\phi) \) by
\[ F(s) = \int_{-\infty}^{\infty} \text{d}\phi f(\phi)e^{is\phi}, \]
(13)
we recognize that
\[ M_0 \propto a_0^0, \]
(14a)
\[ M_{11}(s) = a_0J^0_0(C) \left( \frac{\alpha_1(BW)}{s} F(s) \right) J_{00}(BW) + J_{10}(BW)F(s) + \mathcal{O}(a_0^2), \]
(14b)
\[ M_{12}(r) = a_0J^0_0(BW) \left( \frac{\alpha_1(C)}{r} F(r) \right) J_{00}(C) + J_{10}(C)F(r) + \mathcal{O}(a_0^2), \]
(14c)
\[ M_2(r, s) \propto \mathcal{O}(a_0^2). \]
(14d)

The two delta distributions in the \( M_0 \) term in (10) enforce for the overall momentum conservation in (2) a factor
\[ \delta^{(4)}(p_1 + p_2 + p_3 - p) \] implying a zero contribution of the \( M_0 \) term (14a). In the spirit of the \( a_0 \) series expansion we neglect (14d) at all. (This term would give rise to on/off-shell contributions which require some care.) The remaining leading order terms in (14b) and (14c) generate the contributions
\[ S_s = 4G\pi^2 \int_{-\infty}^{\infty} ds \frac{M_1(s)}{(p - p_1)^2 + i\epsilon}, \]
(15a)
\[ S_s = 4G\pi^2 \int_{-\infty}^{\infty} dr \frac{M_2(r)}{(p_2 + p_3)^2 + i\epsilon}, \]
(15b)

plus the corresponding exchange terms upon \( p_1 \leftrightarrow p_2 \). Introducing light-front coordinates\(^4\) and the light-front delta distribution \( \delta^{(4)}(p) = \delta(p) \delta^{(3)}(p^+) \) as well as choosing \( k^0 \) as the non-zero component of the laser four-momentum \( k^\mu \) yields
\[ S_s = \frac{2G\pi^2 e^2}{k^+} \delta^{(4)}(p_1 + p_2 + p_3 - p) \frac{M_1(s)}{(p - p_1)^2 + i\epsilon}, \]
(16a)
\[ S_s = \frac{2G\pi^2 e^2}{k^+} \delta^{(4)}(p_1 + p_2 + p_3 - p) \frac{M_2(r)}{(p_2 + p_3)^2 + i\epsilon}, \]
(16b)

where \( M_{11} \) refers to (11b) and \( M_{12} \) to (11c) and one has to use in both cases
\[ r = s = \frac{p^+ - p_1^+ - p_2^+ - p_3^+}{k^+} = 0, \]
(17)
and the principal value can be dropped due to the last inequality. For given entry channel parameters, equation (17) implies the dependence \( s(E_{2,3}, \cos\theta_{2,3}, \varphi_{2,3}) \) with spherical coordinates \( E_s, \cos\theta_s, \varphi_s \) for the particles 1, 2 and 3 (see figure 1(a)) in the exit channel. Fixing \( E_s, \cos\theta_s, \varphi_s \) yields the contour plot \( s(E_2, \varphi_2) \). An example is exhibited in figure 3. The locus of the apparent one- (two-) photon contribution with \( s = 1 \) (2) is highlighted by fat curves.

The final result is the leading-order matrix element
\[ S_{gg} = \frac{2G\pi^2 a_0}{k^+} \left( \frac{M(s; BW)}{(p - p_1)^2 + i\epsilon} + \frac{M(s; C)}{(p_2 + p_3)^2 + i\epsilon} \right) \times F(s) \delta^{(4)}(p_1 + p_2 + p_3 - p), \]
(18)
with
\[ M(s; BW) = g_{\mu\nu}J^\mu_0(C) \left( \frac{\alpha_1(BW)}{s} J^\nu_0(BW) + J^\nu_0(BW) \right), \]
(19a)
\[ M(s; C) = g_{\mu\nu}J^\mu_0(BW) \left( \frac{\alpha_1(C)}{s} J^\nu_0(C) + J^\nu_0(C) \right). \]
(19b)

\(^4\) We use the definition \( q^\pm = \frac{1}{2}(q^1 \pm q^0) \) and \( q^0 = (q^0, q^1, q^2, q^3) \).
where the tildes indicate here that the factor $a_0$ is scaled out. These structures are suggestive: $M(BW)$ may be read as the coupling of the free Compton current $J^\mu_C$ to the modified Breit–Wheeler current (in parenthesis of (19a)) and a free Breit–Wheeler current $J^\mu_B(W)$ to a modified Compton current (in parenthesis of (19b)), both depicted in the middle panels of figure 1(c). The interaction with the external field is encoded in the modified currents. For practical purposes we replace the modified currents by the proper Volckov currents $\Delta^\alpha(C)$ and $\Delta^\alpha(BW)$ when evaluating numerically the matrix elements for $a_0 < 1$. The differential probability is

$$
\frac{d\omega}{d\Omega} = \left(\frac{2G_F^2\pi^2}{k^2}\right) |M|^2 (2\pi)^4 \delta^4(p_1 + p_2 + p_3 - p) |F(s)|^2 V_{ff} d\Omega,
$$

with three-body phase space element

$$
d\Omega_3 = \frac{6\Theta(p_1)\Theta(p_2)\Theta(p_3) dp_1^+ dp_2^+ dp_3^+ dp_2^+ dp_3^+ dp_3^+}{(2\pi)^9} \frac{d^3p_1}{2p_1} \frac{d^3p_2}{2p_2} \frac{d^3p_3}{2p_3}.
$$

The Heaviside step-function $\Theta$ and the differential cross section is

$$
\sigma = \frac{d\omega}{NV_{ff}}
$$

with the light-front volume $V_{ff}$ and the normalization factor $N = \frac{a_0^2 m^2}{2 \pi} \int_{-\infty}^{\infty} g^2(\phi) d\phi$ (see [16]). The matrix element $M$ is given by (18) but without the pre-factor:

$$
M = M(s; BW) \frac{(p - p_1)^2 + i\epsilon}{(p_1 - p_2 + p_3)^2 + i\epsilon} + M(s; C) \frac{(p_1 - p_2 + p_3)^2 + i\epsilon}{(p_2 + p_3)^2 + i\epsilon}.
$$

We emphasize that the weak-field limit (18) in (20) corresponds to the standard perturbative tree level QED diagrams depicted in the figure 2, supposed

$$
|F(s)|^2 \rightarrow \delta(s - 1) + \delta(s + 1).
$$

$s = 1$ selects then the admissible kinematics, and the phase space in (20) becomes five-dimensional. The decomposition (8) is essential for catching the proper weak-field perturbative limit. This is obvious when considering Möller or Bhabha scattering in an ambient background field as the cross channels of the trident process: for $A \rightarrow 0$ the standard pQED must be recovered.

4. The case of a $\cos^2$ envelope

We now specialize the linearly polarized external e.m. field $A^\mu$ with $e^\mu = (0, 1, 0, 0)$ to a $\cos^2$ envelope times the oscillating part yielding

$$
f(\phi) = \cos^2\left(\frac{\pi \phi}{2\Delta\phi}\right) \sin^2(\frac{\phi}{\Delta\phi}) \cos(\phi + \phi_{cep}),
$$

where $\sin^2(\frac{\phi}{\Delta\phi})$ is a box profile of width $2\Delta\phi$ centered at $\phi = 0$. We leave a discussion of the carrier envelope phase $\phi_{cep}$ for separate work, i.e. put $\phi_{cep} = 0$. Then (13) can be integrated analytically with the result

$$
\frac{1}{2\Delta\phi} \int_{-\Delta\phi}^{\Delta\phi} F(s, \Delta\phi) = \frac{\pi^2}{2} \int_{s}^{s + 1} \left[ \frac{\sin((s + 1))}{(s - 1)} + \frac{\sin((s - 1))}{(s + 1)} \right] d\phi.
$$

where $\sin(x) = \frac{\sin(x)}{x}$ for $x \neq 0$ and $\sin(0) = 1$ is the cardinal sine function. With a proper normalization, (20) is proportional to $\frac{|F(s)|^2}{\Delta\phi}$. $F(s)$ is a real function due to the even symmetry of the special field (25), but in general it acquires also an imaginary part. We therefore keep the notation $|F(s)|^2$. Given the properties of the sinc functions entering (26) we find

$$
\lim_{\Delta\phi \rightarrow \infty} \frac{|F(s)|^2}{\Delta\phi} = \frac{1}{4}(\delta(s + 1) + \delta(s - 1)),
$$

thus making (24) explicit. Since $\frac{|F(s)|^2}{\Delta\phi} > 0$ for $s \neq 1$, in particular for $s > 1$, at finite values of $\Delta\phi$, we see that this signals bandwidth effects, despite $a_0 \ll 1$. Such effects have been observed in [50] for the Breit–Wheeler pair production below the threshold and the Compton process as well [22, 23]. Since $s$ is a continuous variable one must not identify it with a ‘photon number’; instead, $s$ could be interpreted as fraction of energy or momentum in units of $\omega = [k]$ participating in creating a final state different from the initial state (see [9] for discussions of that issue). Even more, $s > 1$ does not mean proper multiphoton effects due to our restriction on leading order in $a_0$, rather one could speak an ‘apparent multiphoton effects’ caused by finite bandwidth of the pulse. The dependence of $\frac{|F(s)|^2}{\Delta\phi}$ on $s$ for various values of $\Delta\phi$ is displayed in figure 4. The function is symmetric, $|F(s)|^2 = |F(-s)|^2$, with main maxima at $s = \pm 1$ and the
envelopes are
\[
\frac{\text{env}}{\Delta \phi} = \frac{1}{\Delta \phi} \int_{-\infty}^{\infty} d\phi \left| 2\Theta(\phi)f(\phi)e^{i\omega_0\phi} \right|^2.
\] (28)

We use the absolute value of the analytic signal of \(F(s)\), i.e. the Fourier transform of the amplitude function (25) constrained to the positive half-line [57]. The first side maxima are located between \(1 \pm 2\frac{\Delta \phi}{\Delta \phi}\) and \(1 \pm 3\frac{\Delta \phi}{\Delta \phi}\) and their heights are \(7 \times 10^{-4}\) of the respective main maximum, meaning that their contribution is not negligible, in particular for a kinematical situation where \(s > 1\) (see figure 3).

From the definition of \(s\) in equation (17), we note (i) the dependence \(s(E_2, E_3, \theta_{2,3}, \varphi_{2,3})\) and (ii) the U shape of \(s(E_2)\) when keeping constant the other variables of the momenta in polar coordinates. Denoting the minimum of \(s(E_2)\) by \(s_{\text{min}}\), then for kinematical situations, where \(s_{\text{min}} > 1\) the side peaks of \(\frac{|F(s)|^2}{\Delta \phi}\) with spacing of about \(\frac{\Delta \phi}{\Delta \phi}\) become relevant. In fact, \(\frac{|F(s)|^2}{\Delta \phi}\) carries much of the energy dependence of the differential cross section. As an example, we exhibit in figure 5 (bottom) the differential cross section \(d^6\sigma/(dE_2\, dcos\theta_2\, d\varphi_2\, dE_3\, dcos\theta_3\, d\varphi_3)\) (multiplied by \(m^4\) to make it dimensionless) as a function of \(E_2\) and compare it with \(\frac{|F(s(E_2), \Delta \phi)|^2}{\Delta \phi}\) scaled by a factor \(5.2 \times 10^{-7}\). Note the near-perfect match. The selected kinematics implies \(s_{\text{min}} \approx 1\), i.e. in the limit \(\Delta \phi \to \infty\), this setting would be kinematically forbidden, but bandwidth effects for finite values of \(\Delta \phi\) enable the selected kinematics. In fact, increasing \(\Delta \phi\) causes (i) a rapid dropping of the differential cross section and (ii) make the oscillatory pattern more dense. At the heart of the behavior is essentially the quantity \(\frac{|F(s(E_2), \Delta \phi)|^2}{\Delta \phi}\) of figure 4; the additional \(E_2\) dependence is fairly smooth.

The relevance of the function \(\frac{|F(s)|^2}{\Delta \phi}\) continues of course for kinematical situations which are not forbidden in the limit \(\Delta \phi \to \infty\). Examples are exhibited in figure 6. For the selected kinematic situation, \(s_{\text{min}} = 0.976\), meaning that with increasing values of \(\Delta \phi\) the differential cross section does not drop but gets concentrated at two values of \(E_2\) (at given \(E_3\)) which are allowed for \(s = 1\). In the limit \(\Delta \phi \to \infty\), two delta peaks arise when \(E_3\) is appropriately fixed. They are the result of cutting the strength distribution over the \(E_2 - E_3\) plane at \(E_3 = \text{const.}\), i.e. there is a sharp ring given by the solution of \(E_2(E_3)\). The tilt of the U shaped function \(s(E_2)\) at fixed other parameters to the right makes the region \(s \approx 1\) larger at large but finite values of \(\Delta \phi\). Correspondingly, the rhs peak structure is wider, as seen e.g. in the right bottom panel.

To compare with the standard pQED result, based on the evaluation of the Feynman diagrams depicted in figure 2, one has to perform the \(\varphi_2\) integration. In fact, making \(\Delta \phi\) larger and larger, the differential cross section \(d^6\sigma/(dE_2\, dcos\theta_2\, dE_3\, dcos\theta_3\, d\varphi_3)\) from equation (22) approaches the pQED result which is for a monochromatic photon beam, see figure 7. This figure illustrates how the pQED is approached for \(\Delta \phi \to \infty\). It also demonstrates that for \(\Delta \phi < 100\) a significantly larger phase space beyond the perturbatively accessible (one-photon) region (indicated by \(\rightarrow\)) is occupied due to bandwidth effects in laser pulses.

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Figure 3. Contour plot \(s(E_2, \varphi_2)\) for \(E_1 = 1.76\) m, cos \(\theta_{2,3} = 0.965\) and \(\varphi_3 = 0\). The initial electron is at rest in this frame, and laser frequency amounts to \(\omega = k_0 = 5.12\) m, i.e. the center of momentum energy is \(\sqrt{(k + p)^2} = 3.353m\).

Figure 4. The quantity \(\frac{|F(s)|^2}{\Delta \phi}\) as a function of \(s\) for several values of \(\Delta \phi\) (blue: \(\Delta \phi = 25\), green: \(\Delta \phi = 50\), red: \(\Delta \phi = 250\), yellow: \(\Delta \phi = 500\)). Dashed curves exhibit the envelopes according to equation (26). The inset zooms into the region \(s \approx 1\). Note the symmetry property \(|F(s)|^2 = |F(-s)|^2\).
5. Summary

The length of laser pulses has a decisive impact on the pair production in the trident process. The rich phase space patterns, already found and analyzed in some detail for nonlinear one-vertex processes à la Compton and Breit–Wheeler, show up also in the two-vertex trident process. Even for weak laser fields, a region becomes accessible which would be kinematically forbidden in a strict perturbative, leading-order tree level QED approach. The key is the frequency distribution in a pulse which

Figure 5. The differential cross section \( \frac{d\sigma}{dE_2 \, d\Omega_2 \, dE_3 \, d\Omega_3} \) from equations (22) with (19a) and (19b) for an intensity parameter \( a_0 = 10^{-4} \) with \( d\Omega \equiv d\cos \theta 2,3 \, d\varphi 2,3 \), for \( \cos \theta 2,3 = 0.95, \, \varphi 2 = \pi/2, \, \varphi 3 = 0 \) over the \( E_2 - E_3 \) plane (top) and as a function of \( E_3 \) for \( E_3 = 1.76 \) in this frame. The pulse length parameters are \( \Delta \phi = 25, \ldots, 500 \) as indicated. In the bottom panels, the function \( F(s, \Delta \phi) \) is exhibited by dashed black curves, scaled up by a common factor of \( 5.2 \times 10^{-7} \).

Figure 6. As figure 5 but for \( \cos \theta 2,3 = 0.965 \) and the same scaling factor of \( \frac{F(s, \Delta \phi)}{\Delta \phi} \). Again a near perfect agreement of the solid and dashed curves is achieved.
The differential cross section $d^4\sigma/(d\omega d\Omega_1 d\Omega_2 d\Omega_3)$ from (22) with (19a) and (19b) as a function of $E_2/m$ for $E_1 = 2.0\mu m$, $\cos \theta_{1,3} = 0.965$ and $\phi_3 = 0$ for various values of $\Delta \phi$ (solid curves, $\Delta \phi = 25$: blue, 50: orange, 250: green, 500: red) at $a_0 = 10^{-4}$. The initial electron is at rest and the laser frequency amounts to $\omega = k^0 = 5.12\mu m$ in this frame. The pQED result is depicted by the black dashed curve (we checked our pQED-software package by comparing with [58-60] and get confidence of our numerical evaluation and normalization of (22) by the smooth approach towards the pQED result for large values of $\Delta \phi$).

Figure 7. The differential cross section $d^4\sigma/(d\omega d\Omega_1 d\Omega_2 d\Omega_3)$ from (22) with (19a) and (19b) as a function of $E_2/m$ for $E_1 = 2.0\mu m$, $\cos \theta_{1,3} = 0.965$ and $\phi_3 = 0$ for various values of $\Delta \phi$ (solid curves, $\Delta \phi = 25$: blue, 50: orange, 250: green, 500: red) at $a_0 = 10^{-4}$. The initial electron is at rest and the laser frequency amounts to $\omega = k^0 = 5.12\mu m$ in this frame. The pQED result is depicted by the black dashed curve (we checked our pQED-software package by comparing with [58-60] and get confidence of our numerical evaluation and normalization of (22) by the smooth approach towards the pQED result for large values of $\Delta \phi$).

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References

[1] Mourou G A, Tajima T and Bulanov S V 2006 Optics in the relativistic regime Rev. Mod. Phys. 78 309–71
[2] Yanovsky V et al 2008 Ultra-high intensity—300 TW laser at 0.1 Hz repetition rate Opt. Express 16 2109–14
[3] Di Piazza A, Müller C, Hatsagortsyan K Z and Keitel C H 2012 Extremely high-intensity laser interactions with fundamental quantum systems Rev. Mod. Phys. 84 1177
[4] Mourou G A et al (ed) 2011 ELI White Book (Berlin: THOSS Media GmbH)
[5] Papadopoulos D N et al 2016 The Apollon 10 PW laser: experimental and theoretical investigation of the temporal characteristics High Power Laser Sci. 4 09
[6] Yu L et al 2018 High-contrast front end based on cascaded XPWG and femtosecond OPA for 10 PW level Ti:sapphire laser Opt. Express 26 2628–33
[7] Ringwald A 2001 Pair production from vacuum at the focus of an x-ray free electron laser Phys. Lett. B 510 107–16
[8] Nikishov A I and Ritus V I 1964 Quantum processes in the field of a plane electromagnetic wave and in a constant field Sov. Phys. JETP 19 529–41
[9] Nikishov A I and Ritus V I 1964 Zh. Eksp. Teor. Fiz. 46 776
[10] Ritus V I 1985 Quantum effects of the interaction of elementary particles with an intense electromagnetic field J. Sov. Laser Res. 6 497–617
[11] Mackenroth F and Di Piazza A 2011 Nonlinear Compton scattering in ultra-short laser pulses Phys. Rev. A 83 032106
[12] Krajewska K, Twardy M and Kamiński J Z 2014 Supercontinuum and ultrashort-pulse generation from nonlinear Thomson and Compton scattering Phys. Rev. A 89 032125
[13] Seipt D, Kharin V, Rykovonov S, Surzhykov A and Fritzsche S 2016 Analytical results for nonlinear Compton scattering in short intense laser pulses J. Plasma Phys. 82 655820203
[14] Titov A I, Kämpfer B, Hosaka A, Nousch T and Seipt D 2016 Determination of the carrier envelope phase for short, circularly polarized laser pulses Phys. Rev. D 93 045010
[15] Seipt D and Kämpfer B 2014 Laser assisted Compton scattering of x-ray photons Phys. Rev. A 89 023433
[16] Seipt D and Kämpfer B 2013 Asymmetries of azimuthal photon distributions in nonlinear Compton scattering in ultra-short intense laser pulses Phys. Rev. A 88 012127
[17] Seipt D and Kämpfer B 2011 Nonlinear compton scattering of ultrashort and ultraintense laser pulses Phys. Rev. A 83 022101
[18] Heinzl T, Seipt D and Kämpfer B 2010 Beam-shape effects in nonlinear compton and thomson scattering Phys. Rev. A 81 022125
[19] Narozhnyi N B and Nikishov A I 1974 Pair production by a periodic electric field Sov. Phys. JETP 38 427
[20] Narozhnyi N B and Nikishov A I 1974 Zh. Eksp. Teor. Fiz. 65 862
[19] Reiss H R 1962 Absorption of light by light J. Math. Phys. 3 59–67
[20] Reiss H R 1971 Production of electron pairs from a zero-mass state Phys. Rev. Lett. 26 1072–5
[21] Titov A I, Kämper B, Hosaka A and Takabe H 2016 Quantum processes in short and intensive electromagnetic fields Phys. Part. Nucl. 47 456–87
[22] Titov A I, Kämper B, Takabe H and Hosaka A 2013 Breit–Wheeler process in very short electromagnetic pulses Phys. Rev. A 87 042106
[23] Titov A I, Takabe H, Kämper B and Hosaka A 2012 Enhanced subthreshold electron–positron pair production in short laser pulses Phys. Rev. Lett. 108 240406
[24] Heinzl T, Ilderton A and Marklund M 2010 Finite size effects in stimulated laser pair production Phys. Lett. B 692 250–6
[25] Krajewska K and Kamiński J Z 2012 Breit–Wheeler process in intense short laser pulses Phys. Rev. A 86 052104
[26] Kamiński J Z, Twardy M and Krajewska K 2018 Diffraction at a time grating in electron–positron pair creation from the vacuum Phys. Rev. D 98 056009
[27] Jansen M J A, Kamiński J Z, Krajewska K and Müller C 2016 Strong-field Breit–Wheeler pair production in short laser pulses: relevance of spin effects Phys. Rev. D 94 013010
[28] Jansen M J A and Müller C 2017 Strong-field Breit–Wheeler pair production in two consecutive laser pulses with variable time delay Phys. Lett. B 766 71–6
[29] Jansen M J A and Müller C 2016 Strong-field Breit–Wheeler pair production in short laser pulses: identifying multiphoton interference and carrier-envelope-phase effects Phys. Rev. D 93 053011
[30] Bamber C et al 1999 Studies of nonlinear QED in collisions of 46.6 GeV electrons with intense laser pulses Phys. Rev. D 60 092004
[31] Burke D L et al 1997 Positron production in multiphoton light-by-light scattering Phys. Rev. Lett. 79 1626–9
[32] Bula C et al 1996 Observation of nonlinear effects in compton scattering Phys. Rev. Lett. 76 3116–9
[33] Hu H, Müller C and Keitel C H 2010 Complete QED theory of multiphoton trident pair production in strong laser fields Phys. Rev. Lett. 105 080401
[34] Ilderton A 2011 Trident pair production in strong laser pulses Phys. Rev. Lett. 106 020404
[35] Dinu V and Torgrensson G 2018 Trident pair production in plane waves: coherence, exchange, and spacetime inhomogeneity Phys. Rev. D 97 036021
[36] King B and Fedotov A M 2018 Effect of interference on the trident process in a constant crossed field Phys. Rev. D 98 016005
[37] Mackenroth F and Di Piazza A 2018 Nonlinear trident pair production in an arbitrary plane wave: a focus on the properties of the transition amplitude Phys. Rev. D 98 116002
[38] Blackburn T G, Ilderton A, Murphy C D and Marklund M 2017 Scaling laws for positron production in laser-electron-beam collisions Phys. Rev. A 96 022128
[39] Altarelli M (ed) 2007 The Technical Design Report of the European XFEL (Hamburg: DESY XFEL Project Group)
[40] Rizzo T G 2018 Kinetic mixing and portal matter phenomenology Phys. Rev. D 99 115024
[41] Bauer M, Foldenauer P and Jaeckel J 2018 Hunting all the hidden photons J. High Energy Phys. JHEP07(2018)004
[42] Denig A 2016 Review of dark photon searches EPJ Web Conf. 130 01005
[43] Beranek T, Merkel H and Vanderhaeghen M 2013 Theoretical framework to analyze searches for hidden light gauge bosons in electron scattering fixed target experiments Phys. Rev. D 88 015032
[44] Curciarello F 2016 Review on dark photon EPJ Web Conf. 118 01008
[45] Adrian P H et al 2018 Search for a dark photon in electroproduced $e^+e^-$ pairs with the Heavy Photon Search experiment at JLab Phys. Rev. D 98 091101
[46] Raggi M 2018 Status of the PADME experiment and review of dark photon searches EPJ Web Conf. 179 01020
[47] Gakh G I, Konchatnij M I and Merenkov N P 2018 Photoproduction of triplets on free electrons and the search for the dark photon J. Exp. Theor. Phys. 127 279–98
[48] Wing M 2017 LUXE: Introduction and Experiment Technical Report European XFEL
[49] Hartin A, Ringwald A and Tapia N 2019 Measuring the boiling point of the vacuum of quantum electrodynamics Phys. Rev. D 99 036008
[50] Nousch T, Seipt D, Kämper B and Titov A I 2012 Pair production in short laser pulses near threshold Phys. Lett. B 715 246–50
[51] Lötstedt E and Jentschura U D 2009 Nonperturbative treatment of double compton backscattering in intense laser fields Phys. Rev. Lett. 103 110404
[52] Seipt D and Kämper B 2012 Two-photon compton process in pulsed intense laser fields Phys. Rev. D 85 101701
[53] Mackenroth F and Di Piazza A 2013 Nonlinear double compton scattering in the ultrarelativistic quantum regime Phys. Rev. Lett. 110 070402
[54] Mitter H 1975 Quantum electrodynamics in laser fields Acta Phys. Austriaca Suppl. 14 397–498
[55] Meuren S, Keitel C H and Di Piazza A 2013 Polarization operator for plane-wave background fields Phys. Rev. D 88 013007
[56] Boca M and Florescu V 2009 Nonlinear Compton scattering with a laser pulse Phys. Rev. A 80 053403
[57] Cohen L 1995 Time-frequency Analysis: Theory and Applications vol 778 (Upper Saddle River, NJ: Prentice Hall)
[58] Haug E 1975 Bremsstrahlung and pair production in the field of free electrons Z. Naturforsch. A 30 1099–113
[59] Jarp S and Mork K J 1973 Differential cross sections for pair production by photons on electrons Phys. Rev. D 8 159–68
[60] Mork K J 1967 Pair production by photons on electrons Phys. Rev. 160 1065–71