A Review of pQCD Calculations of Electromagnetic Form Factors of Hadrons

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Abstract

We review the current status of perturbative QCD calculation of hadronic electromagnetic form factors.

1 INTRODUCTION

The applicability of perturbative QCD to exclusive processes at large momenta is an interesting research problem. The Brodsky-LePage\textsuperscript{[1]} pQCD based factorization has been only partially successful. In this case the process is factorized into a perturbatively calculable hard scattering piece and the soft distribution amplitude. The pion electromagnetic form factor\textsuperscript{[1, 2, 3]} at momentum transfer $q^2 = -Q^2$, for example, can be written as

$$F_\pi(Q^2) = \int dx_1 dx_2 \phi(x_2, Q) H(x_1, x_2, Q) \phi(x_1, Q)$$ (1)

where $\phi(x, Q)$ are the distribution amplitudes which can be expressed in terms of the pion wave function $\psi(x, \vec{k}_T)$ as

$$\phi(x, Q) = \int^Q d^2 k_T \psi(x, \vec{k}_T).$$ (2)

Here $x$ is the longitudinal momentum fraction and $\vec{k}_T$ the transverse momentum carried by the quark. The factorization is possible provided the external photon momentum $Q^2$ is much larger than the intrinsic quark transverse momentum $k^2_T$, in which case the $k_T$ dependence of the hard scattering $H$ can be neglected.
The formalism predicts that at large momenta the cross section for exclusive processes \( d\sigma/dt \), where \( t \) is the momentum transfer squared, scales like \( 1/t^{n-2} \) up to logs, where \( n \) is the total number of elementary partons participating in the process. The underlying reason for the power law is scale invariance of the fundamental theory. The extra logarithmic dependence is given by QCD scaling violations. The dominant contribution to this scattering arises from the valence quark, since every additional parton leads to an additional suppression factor of \( 1/t \). Physically the scattering probes the short distance part of the hadron wave function. Dominance by the short distance wave functions leads to several predictions such as helicity conservation, color transparency \([4, 5]\) etc.

The successes and failures of this scheme are well known. The predicted momentum dependence of exclusive processes, in particular the hadronic electromagnetic form factors, have generally been found to be in good agreement with data. However more detailed dynamical predictions such as helicity conservation in hadron-hadron collisions fail to agree. Calculation of electromagnetic form factors using this factorization scheme has been criticised by several authors. The basic problem is that the momentum scales of the exchanged gluons tend to become rather small, and the applicability of pQCD becomes doubtful. The normalization of form factors is largely unknown; use of asymptotic distribution amplitudes tends to give small normalizations compared to data. Form factor magnitudes can be enhanced by use of model distribution amplitudes which peak closer to the endpoints, namely \( x \to 0, 1 \), which then exacerbates the problem of small internal momentum transfers.

\section{The Sudakov Form Factor}

In order to investigate this problem in more detail, Botts and Sterman \([6]\) and Li and Sterman \([7]\) developed an alternate factorization which does not neglect the \( k_T \) dependence of the hard scattering. This formalism also includes use of a Sudakov form factor. For the case of pion form factor \([8]\) the starting point is,

\[ F_{\pi}(Q^2) = \int dx_1 dx_2 d\vec{k}_{T1} d\vec{k}_{T2} \psi^*(x_2, \vec{k}_{T2}, P_2) H(x_1, x_2, Q^2, \vec{k}_{T1}, \vec{k}_{T2}) \psi(x_1, \vec{k}_{T1}, P_1), \]

where it is again assumed that the process is factorizable into hard scattering and soft hadronic wave functions \( \psi(x, \vec{k}_T, P) \). The calculation is simplified
by dropping the $k_T$ dependence in the quark propagators in hard scattering kernel $H$, in which case only the combination $k_{T1} + k_{T2}$ appears in the calculation. The authors \cite{7} work in configuration space where this can be written as

$$F_\pi(Q^2) = \int dx_1 dx_2 \frac{d^2 \vec{b}}{(2\pi)^2} \mathcal{P}(x_2, b, P_2, \mu) \tilde{H}(x_1, x_2, Q^2, \vec{b}, \mu) \mathcal{P}(x_1, b, P_1, \mu),$$

where $\mathcal{P}(x, b, P, \mu)$ and $\tilde{H}(x_1, x_2, Q^2, \vec{b}, \mu)$ are the Fourier transforms of the wave function and hard scattering respectively; $\vec{b}$ is conjugate to $k_{T1} + k_{T2}$, $\mu$ is the renormalization scale and $P_1, P_2$ are the initial and final momenta of the pion.

Sudakov form factors are obtained by summing the leading and next to leading logarithms using renormalization group (RG) techniques. The wave function at small $b$ with a transverse momentum $k_T$ cutoff equal to $\omega = 1/b$ can be approximated by the distribution amplitude $\phi(x, 1/b)$. Large $k_T$ corrections can be evaluated perturbatively, which result in the Sudakov form factor. The final result is given by:

$$\mathcal{P}(x, b, P, \mu) = \exp \left[ -s(x, \omega, Q) - s(1 - x, \omega, Q) - 2 \int_\omega^\mu \frac{d\mu}{\mu} \gamma_q(\alpha_s(\bar{\mu})) \right] \times \phi(x, 1/b) + O(\alpha_s(\omega)).$$

where $\gamma_q(\alpha_s)$ is the quark anomalous dimension. The explicit formula for the function $s(x, \omega, \mu)$ is given in Li and Sterman \cite{7}. Here $\omega = 1/b$ plays the role of the factorization scale, above which QCD corrections give the perturbative evolution of the wave function $P$, and below which QCD corrections are absorbed into the nonperturbative distribution amplitude $\phi$. For the case of the pion, $1/b$ is the natural choice for this scale. However as discussed below, for the proton the relevant scale is not obvious and several possibilities exist in the literature.

The final formula for the form factor, after incorporating the renormalization group evolution of the hard scattering from the renormalization scale $\mu$ to $t$, $t = \max(\sqrt{x_1 x_2 Q}, 1/b)$, is given by \cite{6},

$$F_\pi(Q^2) = 16\pi C_F \int_0^1 dx_1 dx_2 \phi(x_1)\phi(x_2) \int_0^\infty bdb \alpha_s(t) K_0(\sqrt{x_1 x_2 Q}b)$$

$$\times \exp[-S(x_1, x_2, \omega, Q)],$$

(6)
Figure 1: The Sudakov form factor $\exp(-S)$ with $Q^2 = 4$ GeV$^2$. For this calculation the QCD scale parameter $\Lambda$ was taken to be 0.1 GeV.

where

$$S(x_1, x_2, b, Q) = \sum_{i=1}^{2} [s(x_i, b, Q) + s(1 - x_i, b, Q)] - 4 \int_{\omega}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})).$$

The function $e^{-S}$ is plotted in fig. 1. It cuts off large $b$ regions of the integral and hence the calculation is infrared finite, without needing any arbitrary infrared cutoff such as a gluon mass. At small $b$ the function has been set equal to one.

The resulting form factor using asymptotic as well as CZ distribution amplitudes is shown in fig. 2. A remarkable fact is that the correct asymptotic $Q^2$ behavior is seen beyond the scale of about $Q = 1$ GeV, irrespective of the choice of wave function. In contrast to the Brodsky-LePage factorization, the $k_T$ dependence of the hard scattering is not neglected, and hence this $Q^2$ dependence does not follow trivially. It is rather a detailed dynamical prediction of the theory and depends on the relative size of intrinsic $k_T^2$ and $x_1 x_2 Q^2$. The prediction is robust, since it is independent of the details of the distribution amplitude used. This simple yet important point justifies the basic idea of Brodsky-LePage factorization, namely that $k_T$ can be treated as negligible in the hard scattering.

We note that the normalization of the theoretical result falls below the experimental data for both choices of distribution amplitude. However, the
Figure 2: The pion form factor $F_\pi(Q^2)$ using the asymptotic (dotted line) and the CZ (solid line) distribution amplitudes. The experimental data, taken from Ref. 9 with errorbars are also shown.

large difference between theory and experiment at high momenta should be interpreted with caution, since as emphasized by Sterman and Stoler [10], there may be large systematic errors in the experimental extraction of the form factor which are not shown in the figure. Further theoretical issues in this extraction have been raised in Ref. [11].

In any event, the theoretical normalization of the form factor is comparatively murky, because it depends on the details of the distribution amplitude. Furthermore, the leading order pQCD amplitudes that are practical to calculate may not give a very reliable estimate of the normalization. One can investigate this further by considering the transverse separation cutoff ($b_c$) dependence of the form factor. This can give an idea about the integration regions important for the calculation. We show the $b_c$ dependence in fig. 3 as originally discussed in Ref. [7]. Based on this plot Li and Sterman argue that roughly 50% of the contribution can regarded as perturbative, since it is obtained from the region where $\alpha_s/\pi < 0.7$. The observation also implies that higher order contributions in $\alpha_s$ are not negligible, and the leading order predictions for the normalization of the form factor cannot be regarded as accurate. The next to leading order calculation [12] of the pion form factor also leads to the same conclusion.

We are left with the following interesting situation: Although the ba-
Figure 3: Dependence of $Q^2 F_\pi(Q^2)$ on the transverse distance cutoff $b_c$ with the asymptotic (dotted line), and CZ (solid line) distribution amplitudes for $Q^2 = 4$ GeV$^2$. The QCD scale parameter $\Lambda$ was chosen to be 0.1 GeV for this calculation.

sic Brodsky-LePage factorization is correct, one may need to go to higher orders in $\alpha_s$ in order to obtain an accurate prediction for the form factor normalizations. However the predicted $Q^2$ dependence appears to be quite robust, and independent of the theoretical uncertainties such as the choice of distribution amplitude.

3 THE PROTON

The improved factorization has also been applied to the proton Dirac form factor $F_1^p(Q^2)$\textsuperscript{[13]}. The calculation is considerably more complicated compared to the pion. Here also it is necessary to use distribution amplitudes which peak close to the end points in order to obtain the experimental normalization of the form factor. In contrast to pion, there is no natural choice for the infrared cutoff $\omega$ in the Sudakov exponent, due to the presence of three quarks and resulting three distances.

The Sudakov resummation of large logarithms in $\mathcal{P}$ leads to

$$\mathcal{P}(x_i, b_i, P, \mu) = \exp \left[ -\sum_{l=1}^{3} s(x_l, w, Q) - 3 \int_{w}^{\mu} \frac{d\mu}{\mu} \gamma_q(\alpha_s(\mu)) \right] \times \phi(x_i, w),$$

(7)
where the quark anomalous dimension $\gamma_q(\alpha_s) = -\alpha_s/\pi$ in axial gauge governs the RG evolution of $P$. The function $\phi$ is the standard proton distribution amplitude. The exponent $s$ is given in Ref. [13].

In equation 7 we use the same infrared parameter $\omega$ for all the three $s(x_l, \omega, Q)$ for $l = 1, 2, 3$ as well as in the integrals over the anomalous dimension. Earlier Li [13] chose to use different infrared cutoffs $b_l$ for each exponent $s$ and for each integral involving $\gamma_q$. As pointed out in [14] this choice does not does not suppress the soft divergences from $b_l \rightarrow 1/\Lambda$ completely, where $\Lambda$ is the QCD scale parameter. For example, the divergences from $b_1 \rightarrow 1/\Lambda$, which appear in $\phi(x, w)$ as $w \rightarrow \Lambda$, survive as $x_1 \rightarrow 0$, since $s(x_1, b_1, Q)$ vanishes and $s(x_2, b_2, Q)$ and $s(x_3, b_3, Q)$ remain finite. On the other hand, $w$ should play the role of the factorization scale, above which QCD corrections give the perturbative evolution of the wave function $P$ in Eq. (7), and below which QCD corrections are absorbed into the initial condition $\phi$. It is then not reasonable to choose the cutoffs $b_l$ for the Sudakov resummation different from $w$.

A modified choice of the cutoffs, $w = 1/b_{\text{max}}$, $b_{\text{max}} = \max(b_l), l = 1, 2, 3$, was proposed in Ref. [14]. This choice was found to suppress the soft enhancements, and the form factor was found to saturate as $b_c \rightarrow 1/\Lambda$. The authors also included a model non-perturbative soft wave function in the calculation. Unfortunately, it turned out that the normalization of the resulting $Q^4F_1$ was found to be less than half of that of the data for all the distribution amplitudes explored [14]. Bolz et al [14] then concluded that pQCD is unable to fit the experimental form factor.

Kundu et al [15] reexamined the situation. The group argued that the form factor normalization is sensitive to the precise choice of the infrared cutoff $w$. They used $w = c/b_{\text{max}}, b_{\text{max}} = \max(b_l), l = 1, 2, 3$, as the infrared cutoff in the Sudakov exponent, instead of the one used by Bolz et al [14]. The introduction of parameter $c$ is natural from the viewpoint of the resummation, since the scale $cw$, with $c$ of order unity, is as appropriate as $w$ [8]. Kundu et al [15] find that the calculation is in good agreement with data using the King-Sachrajda (KS) [16] distribution amplitude and setting $c = 1.14$.

The choice $c = 1.14$ can also be motivated physically by considering the proton as a quark-diquark type configuration. The diquark constituents are the two quarks closest to each other in the transverse plane. Let $d_{\text{typ}}$ be the distance between the center of mass of the diquark and the remaining third quark. Then the infrared cutoff scale $\omega$ can be taken to be $1/d_{\text{typ}}$. We choose $c$ such that for a large number of randomly chosen triangles, we get
for the average $\langle d_{\text{typ}}/b_{\text{max}} \rangle = 1/c$. Defining $c$ in such a way, gives $c \approx 1.14$.

The results of the calculation using KS [16] and CZ [8] distribution amplitudes and $c = 1$ and 1.14 are shown in fig. 4. It is found that with $c = 1.14$ the KS distribution amplitude gives good agreement with data. The $b_c$ dependence of the form factor is shown in fig. 5, which shows saturation at about $b_c = 0.8/\Lambda$. The result after including a model nonperturbative soft wave function are displayed in fig. 6. Again we find that choosing $c$ of order unity gives pQCD calculations in agreement with data. For all choices of the distribution amplitude and parameter $c$, independent of whether the model soft wave function is included or not, the $Q^2$ dependence of the form factor is in good agreement with data. An analogous situation was found for pion form factor.

The natural agreement of $Q^2$ dependence of the pQCD calculations is in contrast to data fits obtained using soft model [18, 19]. In such models the $Q^2$ dependence depends on the details of the model wave function. Soft model predictions at high momentum have a tendency to fall more strongly than experimental data. As also noted for the pion, the approximate power law behaviour in $Q^2$ is not directly implied by the factorization and is a detailed dynamical prediction of the calculation. This could have
Figure 5: Dependence of $Q^4 F^p_1$ on the cutoff $b_c$ with the KS distribution amplitude for $Q^2 = 12 \text{ GeV}^2$ (dotted line), $Q^2 = 16 \text{ GeV}^2$ (dashed line), $Q^2 = 25 \text{ GeV}^2$ (dense-dot line), and $Q^2 = 36 \text{ GeV}^2$ (solid line). The QCD scale parameter $\Lambda$ was taken to be 0.2 GeV for this calculation.

been achieved only if the intrinsic $k_T$ were negligible in the hard scattering kernel. Thus the observation of power-law dependence in the data lends considerable support to the basic factorization of Brodsky-LePage. Nevertheless, the relatively small magnitude of internal momentum scales and the sensitive dependence of final result on the infrared scale $w$ indicates that calculation of normalizations using leading order diagrams is not reliable. While higher order contributions are probably non-negligible, there is every reason to believe that the power-law dependence of the calculations is robust.

We have reviewed the current status of pQCD calculations of hadronic electromagnetic form factors. We argue that the normalization of the form factor cannot be predicted reliably by a leading order calculation in $\alpha_s$. Detailed calculations including the soft $k_T$ dependence, however, support the basic factorization scheme. One finds the correct asymptotic $Q^2$ evolution of the form factors for $Q$ as small as 2-3 GeV, independent of the choice of distribution amplitude and other theoretical uncertainties. Hence we conclude that agreement of quark counting scaling predictions is not accidental but is well supported by detailed dynamical calculations.

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Figure 6: $Q^2$ dependence of $Q^4F_1^p$ including a soft model wave function with $< k_T^2 > = 0.271$ GeV$^2$. The four upper curves at $Q^2 = 35$ GeV$^2$ use the KS distribution amplitude with the infrared parameter $c = 1.4$ (solid curve), $c = 1.2$ (dashed curve) and $c = 1$ (dashed-dotted curve). The dotted curve shows the result without including model soft wave function with $c = 1$. The two lower curves at $Q^2 = 35$ GeV$^2$ use CZ distribution amplitude with $c = 1$. The lowest curve includes the soft model wave function, whereas the upper curve does not. The experimental data with error bars [17] are also shown.

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