$B$ Decays in the Upsilon Expansion

Zoltan Ligeti

Theory Group, Fermilab, P.O. Box 500, Batavia, IL 60510, USA
E-mail: ligeti@fnal.gov

Abstract

Theoretical predictions for $B$ decay rates are rewritten in terms of the Upsilon meson mass instead of the $b$ quark mass, using a modified perturbation expansion. The theoretical consistency is shown both at low and high orders. This method improves the behavior of the perturbation series for inclusive and exclusive decay rates, and the largest theoretical error in the predictions coming from the uncertainty in the quark mass is eliminated. Applications to the determination of CKM matrix elements, moments of inclusive decay distributions, and the $B \to X_s \gamma$ photon spectrum are discussed.

1. Introduction

Testing the Cabibbo–Kobayashi–Maskawa (CKM) description of quark mixing and CP violation is a large part of the high energy experimental program in the near future. The goal is to overconstrain the unitarity triangle by directly measuring its sides and (some) angles in several decay modes. If the value of $\sin 2\beta$, the CP asymmetry in $B \to \psi K_S$, is near the CDF central value [1], then searching for new physics will require precise measurements of the magnitudes and phases of the CKM matrix elements. Inclusive $B$ decay rates can give information on $|V_{cb}|$, $|V_{ub}|$, $|V_{ts}|$, and $|V_{td}|$.

The theoretical reliability of inclusive measurements can be competitive with the exclusive ones. For example, for the determination of $|V_{cb}|$ model dependence enters at the same order of $\Lambda_{QCD}^2/m_{c,b}^2$ corrections from both the inclusive semileptonic $B \to X_c e\bar{\nu}$ width and the $B \to D^* e\bar{\nu}$ rate near zero recoil. It is then important to test the theoretical ingredients of these analyses via other measurements.

The main uncertainty in theoretical predictions for inclusive $B$ decay rates arise from the poorly known quark masses which define the phase space, and the bad behavior of the series of perturbative corrections when it is written in terms of the pole mass. Only the product of these quantities, the decay widths, are well-defined physical quantities; while perturbative multi-loop calculations are most conveniently done in terms of the pole mass. Of course, one would like to eliminate any quark mass from the predictions in favor of physical observables. Here we present a new method of eliminating $m_b$ in terms of the $\Upsilon(1S)$ meson mass [2].

2. Upsilon Expansion

Let us consider, for example, the inclusive $B \to X_u e\bar{\nu}$ decay rate. At the scale $\mu = m_b$,

$$
\Gamma(B \to X_u e\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} m_b^5 \left[ 1 - \frac{2.41}{\pi} \frac{\alpha_s}{\epsilon} - 3.22 \frac{\alpha_s^2}{\pi^2} \beta_0 \epsilon^2 - \ldots - \frac{9\lambda_2 - \lambda_1}{2m_b^6} + \ldots \right].
$$

(1)

The variable $\epsilon \equiv 1$ denotes the order in the modified expansion, and $\beta_0 = 11 - 2n_f/3$. In comparison, the expansion of the $\Upsilon(1S)$ mass in terms of $m_b$ has a different structure,

$$
m_{\Upsilon} = 2m_b \left\{ 1 - \frac{(\alpha_s/\pi)}{8} \left[ \epsilon + \frac{\alpha_s}{\pi} \left( \ell + \frac{11}{6} \right) \beta_0 \epsilon^2 + \ldots \right] \right\},
$$

(2)

where $\ell = \ln[\mu/(m_b\alpha_s C_F)]$ and $C_F = 4/3$. In this expansion we assigned to each term one less power of $\epsilon$ than the power of $\alpha_s$. It is also convenient to choose the same renormalization scale $\mu$. The prescription of counting $\alpha_s^n$ in $B$ decay rates as order $\epsilon^n$, and $\alpha_s^n$ in $m_{\Upsilon}$ as order $\epsilon^{n-1}$ is the upsilon expansion. It combines different orders in the $\alpha_s$ perturbation series in Eqs. (1) and (2), but as it is sketched below, this is the consistent way of combining these expressions.

At large orders in perturbation theory, the coefficient of $\alpha_s^n$ in Eq. (1) has a part which grows as $Cn!\beta_0^{n-1}$. For large $n$, this divergence is cancelled by the $\alpha_s^{n+1}$ term in Eq. (2), whose coefficient behaves as $(1/\alpha_s)(C/5)n!\beta_0^{n-1}$ [3–5]. The crucial $1/\alpha_s$ factor arises because the coefficient of $\alpha_s^{n+1}$ in Eq. (2) contains a series of the form $\epsilon^{n+1}$.
\( (\ell^{n-1} + \ell^{-2} + \ldots + 1) \) which exponentiates for large \( n \) to give \( \exp(\ell) = \mu/(m_b \alpha_s \bar{C}_F) \), and corrects the mismatch of the power of \( \alpha_s \) between the two series.

The infrared sensitivity of Feynman diagrams can be studied by introducing a fictitious infrared cutoff \( \lambda \). The infrared sensitive terms are non-analytic in \( \lambda^2 \), such as \( (\lambda^2)^{n/2} \) or \( \lambda^{2n} \ln \lambda^2 \), and arise from the low momentum part of Feynman diagrams. Linear infrared sensitivity (terms of order \( \sqrt{\lambda^2} \)) are a signal of \( \Lambda_{QCD}^2 \) effects, quadratic sensitivity (terms of order \( \lambda^2 \ln \lambda^2 \)) are a signal of \( \Lambda_{QCD}^2 \) effects, etc. From Refs. [3][4] it follows that the linear infrared sensitivity cancels in the upsilon expansion to order \( \epsilon^2 \) (probably to all orders as well, but the demonstration of this appears highly non-trivial).

Thus, the upsilon expansion is theoretically consistent both at large orders for the terms containing the highest possible power of \( \beta_0 \), and to order \( \epsilon^2 \) including non-Abelian contributions.

The most important uncertainty in this approach is the size of nonperturbative contributions to \( m_\tau \) other than those which can be absorbed into \( m_b \). If the mass of heavy quarkonia can be computed in an operator product expansion then this correction is of order \( \Lambda_{QCD}^2 / (\alpha_s m_b)^3 \) by dimensional analysis. Quantitative estimates, however, vary in a large range, and it is preferable to constrain such effects from data. We use 100 MeV to indicate the corresponding uncertainty. Finally, if the nonperturbative contribution to \( m_\tau \), \( \Delta m_\tau \), were known, it could be included by replacing \( m_\tau \) by \( m_\tau - \Delta m_\tau \) on the left hand side of Eq. (2).

There are three surprising facts that are either accidental or indicate that the nonperturbative contributions may be small: 1) applications in terms of the \( Y(2S) \) mass give consistent results [3, 4]; 2) the \( D \rightarrow X e\bar{\nu} \) rate in terms of the \( \psi \) mass works (un)reasonably well [3, 4]; 3) sum rule calculations for \( e^+ e^- \rightarrow bb \) find that the 1S \( b \) quark mass (defined as half of the perturbative part of \( m_{T(1S)} \)) is only 20 MeV different from \( m_{T(1S)/2} \).

3. Application

Substituting Eq. (2) into Eq. (4) and collecting terms of a given order in \( \epsilon \) gives

\[
\Gamma(\bar{B} \rightarrow X_e e\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192 \pi^3} \left( \frac{m_T}{2} \right)^5 \eta_{QED} \times \left[ 1 - 0.115 \epsilon - 0.031 \epsilon^2 - \ldots \right].
\]

The complete order \( \alpha_s^2 \) result calculated recently [5] is included. Keeping only the part proportional to \( \beta_0 \), the coefficient of \( \epsilon^2 \) would be \(-0.035 \). The perturbation series, \( 1 - 0.115 \epsilon - 0.031 \epsilon^2 \), is better behaved than the series in terms of the \( b \) quark pole mass, \( 1 - 0.17 \epsilon - 0.10 \epsilon^2 \), or the series expressed in terms of the \( \overline{\text{MS}} \) mass, \( 1 + 0.30 \epsilon + 0.13 \epsilon^2 \). The uncertainty in the decay rate using Eq. (3) is much smaller than that in Eq. (6), both because the perturbation series is better behaved, and because \( m_\tau \) is better known (and better defined) than \( m_b \). The relation between \( |V_{ub}| \) and the total semileptonic \( B \rightarrow X_e e\bar{\nu} \) decay rate is

\[
|V_{ub}| = (3.04 \pm 0.06 \pm 0.08) \times 10^{-3}
\]

\[
\left( \frac{B(\bar{B} \rightarrow X_e e\bar{\nu})}{1.6 \text{ps}} \right)^{1/2},
\]

The first error is obtained by assigning an uncertainty in Eq. (3) equal to the value of the \( \epsilon^2 \) term and the second is from assuming a 100 MeV uncertainty in Eq. (4). The scale dependence of \( |V_{ub}| \) due to varying \( \mu \) in the range \( m_b/2 < \mu < 2m_b \) is less than 1%. The uncertainty in \( \lambda_1 \) makes a negligible contribution to the total error. Although \( B(\bar{B} \rightarrow X_e e\bar{\nu}) \) cannot be measured without significant experimental cuts, for example, on the hadronic invariant mass, this method will reduce the uncertainties in such analyses as well.

The \( \bar{B} \rightarrow X_e e\bar{\nu} \) decay depends on both \( m_b \) and \( m_c \). It is convenient to express the decay rate in terms of \( m_\tau \) and \( \lambda_1 \) instead of \( m_b \) and \( m_c \), using Eq. (3) and

\[
m_b - m_c = m_B - m_D + \left( \frac{\lambda_1}{2m_B} - \frac{\lambda_1}{2m_D} \right) + \ldots,
\]

where \( m_B = (3m_{B^*} + m_B)/4 = 5.313 \text{ GeV} \) and \( m_D = (3m_{D^*} + m_D)/4 = 1.973 \text{ GeV} \). We then find

\[
\Gamma(\bar{B} \rightarrow X_e e\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192 \pi^3} \left( \frac{m_T}{2} \right)^5 \eta_{QED} \times \left[ 1 - 0.096 \epsilon - 0.029_{\text{BLM}} \epsilon^2 \right],
\]

where the phase space factor has also been expanded in \( \epsilon \), and the BLM [9] subscript indicates that only the corrections proportional to \( \beta_0 \) have been kept. For comparison, the perturbation series in this relation written in terms of the pole mass is \( 1 - 0.12 \epsilon - 0.07_{\text{BLM}} \epsilon^2 \). Including the terms proportional to \( \lambda_{1,2} \), Eq. (6) implies

\[
|V_{ub}| = (41.6 \pm 0.8 \pm 0.7 \pm 0.5) \times 10^{-3}
\]

\[
\times \eta_{QED} \left( \frac{B(\bar{B} \rightarrow X_e e\bar{\nu})}{1.6 \text{ps}} \right)^{1/2},
\]

where \( \eta_{QED} \sim 1.007 \) is the electromagnetic radiative correction. The uncertainties come from...
assuming an error in Eq. (3) equal to the $\epsilon^2$ term, a 0.25 GeV$^2$ error in $\lambda_1$, and a 100 MeV error in Eq. (2), respectively. The second uncertainty can be reduced by determining $\lambda_1$ (see below).

The theoretical uncertainty hardest to quantify in the predictions for inclusive $B$ decays is the size of quark-hadron duality violation. This was neglected in Eqs. (4) and (7). It is believed to be exponentially suppressed in the $m_b \to \infty$ limit, but its size is poorly known for the physical $b$ quark mass. Studying the shapes of inclusive decay distributions is the best way to constrain this experimentally, since duality violation would probably show up in a comparison of different spectra. The shape of the hadronic mass-squared predictions for inclusive rates.

One must be careful in comparing different spectra. The shape of the lepton energy distribution in $e^+e^-$ annihilation [11–14] and hadron invariant mass spectra [1, 2, 4, 5] in semileptonic $B \to X_c e\bar{\nu}$ decays, and the photon spectrum in $B \to X_s \gamma$ [17–21] can also be used to determine the heavy quark effective theory (HQET) parameters $\Lambda$ and $\lambda_1$. The extent to which these determinations agree with one another will indicate at what level to trust predictions for inclusive rates.

Last year CLEO measured the first two moments of the hadronic invariant mass $m_H^2$ spectra in semileptonic $B \to X_c e\bar{\nu}$ decay, and the photon spectrum in $B \to X_s \gamma$ [17–21] can also be used to determine the heavy quark effective theory (HQET) parameters $\Lambda$ and $\lambda_1$. The extent to which these determinations agree with one another will indicate at what level to trust predictions for inclusive rates.

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Expansions in terms of $m_\Upsilon$ and $\alpha_s$ for inclusive decay rates using the conventional expansion and the upsilon expansion. The second order terms are the BLM parts only.

| Decay | Expansions in terms of $m_\Upsilon$ and $\alpha_s$ |
|-------|------------------------------------------------|
| $B \to X_c e^+ e^-$ | $1 - 0.12 \epsilon - 0.07 \epsilon^2$ |
| $B \to X_b \tau^+ \tau^-$ | $1 - 0.17 \epsilon - 0.13 \epsilon^2$ |
| $B \to X_u \tau^+ \tau^-$ | $1 - 0.10 \epsilon - 0.06 \epsilon^2$ |
| $B \to X_c (u+d)$ | $1 - 0.16 \epsilon$ |
| $B \to X_b (u+d)$ | $1 - 0.05 \epsilon - 0.04 \epsilon^2$ |
| $B \to X_u (u+d)$ | $1 + 0.20 \epsilon + 0.15 \epsilon^2$ |
| $B \to X_c (u+d)$ | $1 - 0.10 \epsilon$ |
| $B \to X_u (u+d)$ | $1 + 0.09 \epsilon$ |

Table 1. Comparison of the perturbation series for inclusive decay rates using the conventional expansion and the upsilon expansion. The second order terms are the BLM parts only.

4. Conclusions

- The biggest uncertainty is the nonperturbative contribution to $m_\Upsilon$ unrelated to the quark mass. It will be possible to estimate/constrain this from data in the future.

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