THE \( \gamma \)-RAY SPECTRUM OF GEMINGA AND THE INVERSE COMPTON MODEL OF PULSAR HIGH-ENERGY EMISSION

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ABSTRACT

We reanalyze the Fermi spectra of the Geminga and Vela pulsars. We find that the spectrum of Geminga above the break is well approximated by a simple power law without the exponential cutoff, making Geminga’s spectrum similar to that of Crab. Vela’s broadband \( \gamma \)-ray spectrum is equally well fit with both the exponential cutoff and the double power-law shapes. In the broadband double power-law fits, for a typical Fermi spectrum of a bright \( \gamma \)-ray pulsar, most of the errors accumulate due to the arbitrary parameterization of the spectral roll-off. In addition, a power law with an exponential cutoff gives an acceptable fit for the underlying double power-law spectrum for a very broad range of parameters, making such fitting procedures insensitive to the underlying Fermi photon spectrum. Our results have important implications for the mechanism of pulsar high-energy emission. A number of observed properties of \( \gamma \)-ray pulsars—i.e., the broken power-law spectra without exponential cutoffs and stretching in the case of Crab beyond the maximal curvature limit, spectral breaks close to or exceeding the maximal breaks due to curvature emission, patterns of the relative intensities of the leading and trailing pulses in the Crab repeated in the X-ray and \( \gamma \)-ray regions, presence of profile peaks at lower energies aligned with \( \gamma \)-ray peaks—all point to the inverse Compton origin of the high-energy emission from majority of pulsars.

Key words: pulsars: general – pulsars: individual (Geminga) – radiation mechanisms: non-thermal

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1. VERITAS DETECTION OF CRAB PULSAR: A CASE FOR INVERSE COMPTON SCATTERING ORIGIN OF \( \gamma \)-RAY EMISSION

The recent launch of the Fermi Gamma-ray Space Telescope and subsequent detection of a large number of pulsars (Abdo et al. 2010c) revolutionized our picture of the non-thermal emission from pulsars in the \( \gamma \)-ray band from 100 MeV up to about 10 GeV. At even higher energies, in the very high energy (VHE) band, above 10 GeV, the detection of the Crab pulsar at 25 GeV by the Magic Collaboration (Aliu & MAGIC Collaboration 2008) and recently at 120 GeV by the VERITAS Collaboration (VERITAS Collaboration et al.2011) clearly demonstrated the inverse Compton origin of the high-energy emission from majority of pulsars.

Lyutikov et al. (2012) have argued that in the case of Crab the inverse Compton (IC) scattering may be the dominant emission mechanism of the VHE emission. This was based on the detection of the Crab pulsar by the VERITAS Collaboration above 150 GeV (VERITAS Collaboration et al. 2011) with non-exponential cutoff above the spectral break. The non-exponential cutoff was later confirmed by the MAGIC Collaboration (Aleksić et al. 2011, which also preferred the IC model over the curvature emission).

The curvature emission in pulsars is limited to energies below

\[ \epsilon_{\text{br}} = (3\pi )^{7/4} \frac{\eta}{(\epsilon e)^{3/4}} \left( \frac{R_{\text{NS}}}{R_{\text{NS}}} \right)^{3/4} \sqrt{\frac{\eta}{\xi}} \left( \frac{R_{\text{NS}}}{R_{\text{NS}}} \right)^{9/4} \frac{E_{\text{br}}^{9/4}}{p^{7/4}}, \]  

where \( R_{\text{L}} \) is the light cylinder radius, \( P \) is the pulsar period of rotation, \( \xi \) is a dimensionless scaling parameter \( \xi = R_{\text{L}}/R_{\text{NS}}, \)

\( R_{\text{NS}} \) is the radius of curvature of magnetic field lines, \( B = B_{\text{NS}}(R_{\text{NS}}/R)^3, \)

where \( B_{\text{NS}} \) is the magnetic field on the surface of the neutrons star and \( R_{\text{NS}} \) is the star’s surface, and \( \eta = E / B \leq 1 \)

is the relative strength of the accelerating electric field (Lyutikov et al. 2012).

If the \( \gamma \)-ray photons are due solely to the curvature emission of a radiation reaction-limited population of leptons, the spectrum above the break must show an exponential cutoff. The detection of the Crab pulsar by VERITAS Collaboration (VERITAS Collaboration et al. 2011) clearly demonstrated the non-exponential cutoff above the spectral break; see Figure 1. Lyutikov et al. (2012) have argued that this is inconsistent with the curvature emission.

2. SPECTRAL BREAKS IN \( \gamma \)-RAY PULSARS

The curvature and the IC model of the high-energy emission offer different interpretations of the spectral breaks. In the case of the curvature emission, the spectral break corresponds to the maximal energy of the radiation reaction-limited acceleration. Above the break, the spectrum is expected to have an exponential cutoff. In the IC model, the spectral break corresponds to the break in the particle distribution function, which is likely to be of the power-law type both below and above the break. Thus, observations of the non-exponential cutoff above the break favor the IC model.

The Crab pulsar is a bright emitter in all spectral bands. Thus, there is a lot of low-frequency target photons available for Compton scattering. What about other pulsars? In the present paper, we argue that there are evidence for a universal dominance of the IC scattering mechanism in \( \gamma \)-ray pulsars.

First, Lyutikov et al. (2012) compared the observed spectral breaks of Fermi pulsars from the first Fermi catalog (Abdo et al. 2010c) with the predicted breaks due to curvature emission, Equation (1), by calculating the ratio of the observed \( E_{\text{br}} \) and predicted spectral break \( \epsilon_{\text{br}} \); see Figure 2. For a significant number of pulsars the ratio is close to one and for one pulsar,
PSR J1836+5925, the ratio is even larger than one. In order to explain the spectral break for these pulsars as a result of curvature radiation, an accelerating electric fields should be close to or even larger than the magnetic fields.

What is more, the example of Crab demonstrates that the spectral break may not be related to the maximal curvature photons. The presence of non-exponential breaks in pulsars others than Crab would be a clear indication that the break is not related to maximal energy of curvature emission by radiation-limited acceleration of leptons. Due to low photon counts, only the brightest $\gamma$-ray pulsars—Crab, Vela, and Geminga—allow precise enough measurements of fluxes to distinguish between different spectral shapes. In addition, typically, only phased average properties have enough photon counts to distinguish between the models. (This is expected to change with the forthcoming Fermi data.)

3. SPECTRA OF GEMINGA AND VELA PULSARS

3.1. Geminga: A Non-exponential Cutoff

Geminga is one of the brightest $\gamma$-ray pulsars. Its spectrum is, conventionally, fit with a power law plus exponential cutoff (Abdo et al. 2010a). We have performed an independent fit to the Geminga spectrum; see Table 2 and Figures 3–5. For the fits, the processed data (energy flux) provided by the Fermi Collaboration were used (Table 1).

First, we performed $\chi^2$ fits of the whole spectrum using a particular type of double power-law prescription (with four independent parameters, row (a) in Table 2); the power law plus exponential cutoff (with three independent parameters,
Figure 4. Errors for the broadband fit to Geminga data using double power-law function. The lower panel shows the residuals (on a linear scale) for the double power-law fit, while the upper panel shows relative errors that are used to calculate $\chi^2$. The errors are not random indicating the inadequacy of the arbitrarily chosen spectral roll-off. Also, most of the $\chi^2$ is accumulated near the break energy due to the arbitrary parameterization of the spectral roll-off. Note that the highest energy data points actually have the smallest error bars.

(A color version of this figure is available in the online journal.)

Figure 5. Fits to the high-energy tail of the Geminga spectrum: power law (solid line, $\chi^2 = 0.1$) and exponential cutoff (dashed line, $\chi^2 = 2$).

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row (b) in Table 2), and a softened exponential cutoff (with four independent parameters, row (c) in Table 2); see Table 2 and Figure 3. Weighted and unweighted reduced $\chi^2$ fits were not statistically distinguishable. (Overall normalization is the additional fit parameter to the ones listed in Table 2.)

Inspection of Table 2 and Figures 3 and 4 shows, first, that since the reduced $\chi^2$ is close to unity all three models are statistically acceptable. (In the Appendix, we also demonstrate that for a typical Fermi spectrum of a bright $\gamma$-ray pulsar a fit with an exponential cutoff can actually give a satisfactory approximation to the underlying power-law spectrum for a very wide regime of fit parameters, e.g., over two orders of magnitude in break energy.)

Second, the fit that looks the best down to low flux levels, fit (a), actually has the largest reduced $\chi^2$ among the models. The errors for this fit are accumulated at intermediate energies due to an arbitrarily chosen parameterization of the spectral roll-off. Since we are mostly interested in the high-energy scaling of the spectrum, we can take only the highest energy points. Having chosen the last six points we derive the fit for a function $e^{\alpha}$: $\alpha = 3.04$, reduced $\chi^2 = 0.1$ (with 4 degrees of freedom), see also Figure 5.

Thus, the high-energy part of the Geminga spectrum is exceptionally well fit with a power-law function, giving reduced $\chi^2 = 0.1$. This exceptional small value of $\chi^2$ is a bit surprising: despite the lower overall count the highest energy data points have the smallest error bars, Table 1, due to the very low background counts above $\sim 2$ GeVs. A small value of $\chi^2$ for the high-energy fit may be due to the fact that the fit has only 4 degrees of freedom—in this case the random chance for all the points being very close to the model is not small. This may also be an indication that the number of energy channels was chosen not in an optimal way, e.g., too broad energy bins. (If only five highest energy points are used, the fit values are $\alpha = 2.8$ and $\chi^2 = 0.03$ for 3 degrees of freedom.)
Table 1
Phase-averaged Data for the Geminga Pulsar

| $E$ (GeV) | $E^2$ Flux (erg cm$^{-2}$ s$^{-1}$) | $dE^2$ Flux (erg cm$^{-2}$ s$^{-1}$) |
|----------|----------------------------------|-------------------------------|
| 0.114    | 4.02E-10                         | 2.3E-11                       |
| 0.153    | 5.29E-10                         | 4.0E-11                       |
| 0.204    | 5.90E-10                         | 2.2E-11                       |
| 0.273    | 6.82E-10                         | 6.5E-11                       |
| 0.365    | 8.04E-10                         | 6.1E-11                       |
| 0.487    | 9.47E-10                         | 7.2E-11                       |
| 0.643    | 1.10E-09                         | 1.0E-10                       |
| 0.860    | 1.22E-09                         | 9.2E-11                       |
| 1.149    | 1.32E-09                         | 1.3E-10                       |
| 1.518    | 1.36E-09                         | 1.3E-10                       |
| 2.053    | 1.36E-09                         | 1.0E-10                       |
| 2.711    | 1.25E-09                         | 9.4E-11                       |
| 3.580    | 9.65E-10                         | 1.1E-10                       |
| 4.785    | 7.34E-10                         | 1.2E-10                       |
| 6.319    | 4.74E-10                         | 5.5E-11                       |
| 8.446    | 2.69E-10                         | 3.1E-11                       |
| 11.153   | 1.12E-10                         | 1.5E-11                       |
| 14.728   | 4.94E-11                         | 1.1E-11                       |
| 19.214   | 2.66E-11                         | 9.0E-12                       |
| 25.681   | 1.01E-11                         | 6.7E-12                       |
| 33.505   | 5.35E-12                         | 5.3E-12                       |

Table 2
Fits to the Broadband Spectrum of Geminga

| Model | Fit Function | $\alpha$ | $\beta$ | $\epsilon_{\text{br}}$ | Reduced, $b$ | dof | Unweighted $\chi^2$ |
|-------|--------------|-----------|---------|------------------------|--------------|-----|---------------------|
| $a$   | $(\frac{E}{\epsilon_{\text{br}}})^\alpha + (\frac{E}{\epsilon_{\text{br}}})^{-\beta}$ | 2.38 | 0.45 | 3.32 | 1.26 | ... | 17 |
| $b$   | $\epsilon_{\text{br}}^{-\beta} e^{-\frac{E}{\epsilon_{\text{br}}}}$ | ... | 0.70 | 2.35 | 1.13 | ... | 18 |
| $c$   | $\epsilon_{\text{br}}^{-\beta} e^{-\frac{E}{\epsilon_{\text{br}}}}$ | ... | 0.75 | 1.98 | 0.83 | 0.91 | 17 |

Note. The values for fit (c) can be compared with the best fit done by the Fermi team, $\beta = 1.12$, $\epsilon_{\text{br}} = 1.58$, and $\text{dof} = 0.81$, (Abdo et al. 2010a).

3.1.1. Geminga to be Seen at Very High $\gamma$-ray Energies?

The spectral index above the break $dN/dE \propto E^{-p}$ with $p \approx 5$ for Geminga is steeper than that of the Crab pulsar ($p = 3.8$ above the break; Lyutikov et al. 2012). Geminga is nearly two and a half times brighter than Crab at the peak of its spectral distribution of $\sim 1$ GeV, but its has lower break energy and most importantly the steeper spectrum. The expected flux at $\sim 120$ GeV, the low-energy threshold for the VERITAS Cherenkov telescope, is then few $\sim 10^{-5}$ MeV cm$^{-2}$ s$^{-1}$. This is approximately 10 times lower than the flux from the Crab pulsar. Geminga, still, has a chance to be detected, since unlike Crab it has no strong background from the pulsar wind nebula. (It is not clear whether the excess TeV flux from the general direction of Geminga (Abdo et al. 2008) is related to the pulsar (see, though, Salvati & Sacco 2008)).

3.1.2. Geminga: Phase-resolved Fit

Phase variation in the cutoff energy can affect the overall spectral fit, but not at the highest energies. For example, Abdo et al. (2010a) cite the cutoff energy variations between approximately 1 and 3 GeV, while our high-energy fit starts at 8.5 GeV, nearly three times higher; see Table 2 and Figure 5. In addition, due to a low photon count in the phase-resolved spectral energy distributions (SEDs), they naturally miss the highest energy, lowest counts tails.

To test the spectral shape in phase-resolved spectra we have performed spectral fitting at one particular phase, $0.614 < \phi < 0.623$, corresponding, approximately to the second peak (Abdo et al. 2010a). Since phase-resolved data were not available, we assumed the error on the flux to be equal to $2 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$; see Table 3 and Figure 6.

Thus, both the double power law and the exponential cutoff provide a statistically viable description of data for a particular phase-resolved spectrum of Geminga.

3.2. Vela: Consistent with the Non-exponential Tail

Vela, the brightest $\gamma$-ray pulsar, offers tantalizing, but non-conclusive evidence of the non-exponential break; see Figure 7. We performed an unweighted $\chi^2$ fit with phase-averaged data provided by the Fermi Collaboration (Table 4). We used various double power-law prescriptions (with four independent parameters), the power law plus exponential cutoff (with three independent parameters), and a softened exponential cutoff (with four independent parameters); see Table 5. The spectrum of Vela is equally well fit with a softened exponential cutoff and a double power-law spectrum.

In the case of Vela, a particular choice of the parameterization of the double power-law spectrum illustrates that an arbitrary choice of the roll-off between the two spectral power-law components has an important effect on the fit: while the more conventional double power-law parameterization in line (a), Table 5, gives an unacceptable fit and double power-law spectrum parameterization in line (b) in Table 5 gives an acceptable fit.

4. OPTICAL–X-RAY–$\gamma$-RAY CORRELATION

Within the framework of the synchrotron-self-Compton model, the power emitted by IC is related to the power of
the seed photons. Photons of different energies that are emitted by the same particles should in principle produce similar pulse profiles. In our model, one expects, therefore, the pulse profiles in X-rays and in $\gamma$-rays to be similar because the secondary plasma emits synchrotron radiation in X-rays and IC scatters UV photons into the VHE band. And indeed, the ratio of the amplitudes of the two pulses in the pulse profile of the Crab pulsar changes consistently in the X-rays/soft $\gamma$-ray band and in the high-energy $\gamma$-ray band; see Figure 8. In X-rays, the main pulse dominates over the interpulse. The ratio changes toward higher energies and reverses in the soft $\gamma$-ray band at about 1 MeV. Mirroring the evolution at lower energy, the main pulse dominates at 100 MeV, while at 120 GeV the interpulse clearly again dominates over the main pulse (Abdo et al. 2010c; VERITAS Collaboration et al. 2011).

Vela also offers tantalizing evidence for the correlation between the optical/X-ray and $\gamma$-ray bands (Figure 9): All three $\gamma$-ray peaks (P1, P2, and P3 in the top panel) have lower frequency analogs: P1 in the X-rays, P2 in optical, and P3 in both bands. This is expected in the IC model, though relative...
The Crab pulsar is somewhat exceptional among the brightest X-ray–γ-ray pulsars since it has no indication of the thermal component. The X-ray emission of Vela and Geminga are strongly dominated by the thermal components coming from the surface (Halpern & Ruderman 1993; Pavlov et al. 2001; Manzali et al. 2007). In Vela, the non-thermal component in X-rays is barely detected at some orbital phases, roughly aligned with the γ-ray peaks, especially with P2; see Figure 9 of Manzali et al. (2007). The relative weakness of the non-thermal X-ray components does not allow a measurement of the evolution of the P1/P2 ratio with energy, as in Crab. Overall, both Vela and Geminga spectra show a complicated phase- and energy-dependent mix of thermal and non-thermal components (e.g., Kargaltsev et al. 2005; Romani et al. 2005). In both cases the optical emission, the main target for IC scattering, is highly non-thermal.

5. DISCUSSION

In this paper we demonstrate that, first, broadband pulsar high-energy spectra are generally consistent with the double power-law shapes. This result is based on three brightest γ-ray pulsars. In Geminga, the high-energy part of the SED is well fit with a power law, while the spectrum of Vela is inconclusive. Importantly, in Geminga most errors in the broadband double power-law fit are accumulated due to an arbitrary parameterization of the spectral roll-off between the two asymptotic power laws. Thus, the standard χ² fit underestimates the goodness of fit in this particular case. In addition, the low background at high energies, above 2 GeV, make Fermi data especially sensitive (smallest error bars) to the shape of the high-energy spectrum (though at low flux level systematic errors may start to become important).

If pulsar spectra generally show non-exponential cutoffs, a fact well established in the Crab pulsar by VERITAS (VERITAS Collaboration et al. 2011) and in Geminga in the present paper, this has important implications for models of pulsar γ-ray emission. This would imply the importance of the IC scattering for the production of γ-ray photons (Lyutikov et al. 2012).

Lyutikov et al. (2012) argued that in Crab and in a number of other pulsars the IC scattering is the main emission mechanism of the VHE emission. The authors tried to give the most general arguments in favor of IC, mostly independent of the numerous possible particular details of a full radiative model. Overall, Lyutikov et al. (2012) adopted the Cheng et al. (1986) paradigm of pulsar high-energy emission, but with IC scattering playing a more important role that previously assumed. Recall that
Cheng et al. (1986) argued that IC scattering is not dominant in Crab, but may be important for Vela. VERITAS and MAGIC results on Crab indicate that IC is the dominant mechanism in Crab.

In the model of Lyutikov et al. (2012), the IC scattering occurring in the Klein–Nishina (KN) regime by the secondary particles results in a picture that, overall, is consistent with observations without any fine-tuning. The key features of our model are (1) a population of primaries that is accelerated in a modest electric field, which is a fraction  of the magnetic field strength near the light cylinder with a typical value of . The suppression of the scattering cross-section in the KN regime favors the lower energy optical–UV photons. In this view, the example of non-exponential cutoff in Geminga, and the implied importance of the IC scattering, are quite surprising given the low-observed optical–X-ray luminosity. This poses challenges for radiative modeling. It is likely to be related to the fact that IC scattering is highly dependent on the details of both the soft photon distribution and the particle’s distribution (e.g., their anisotropy).

Finally, let us recapitulate the main arguments in favor of IC scattering being the dominant pulsar radiation mechanism at GeV energies.

1. The spectrum of Crab pulsar extends beyond the upper limit for curvature emission.
2. The beak energy in many pulsars approaches and even exceeds the upper limit for curvature emission.
3. In the three brightest pulsars, Crab, Geminga, and possibly Vela, the spectrum above the break is not exponentially suppressed, implying that the break is not due to curvature remission of leptons in the radiation reaction-limited regime.
4. In Crab, the energy dependence of the relative intensity of emission peaks at γ-ray energies mirrors the lower energy dependence, as expected in the IC model.

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Figure 10. Numerical fit to the function $f = 2/(x + 1/x)$ using a softened exponential function, $\propto e^{\alpha x} e^{-(\epsilon/\epsilon_{br})^b}$. Two curves correspond to extreme values of the parameters ($\epsilon_{br} = 0.01, \alpha = 1.87, b = 0.35$ and $\epsilon_{br} = 2, \alpha = 1.68, b = 0.98$), but both give acceptable fits (reduced $\chi^2 = 0.33$ and 1.01 correspondingly).

(A color version of this figure is available in the online journal.)

APPENDIX

ON THE EXPONENTIAL FIT TO FERMI-LIKE DATA WITH POWER-LAW DISTRIBUTION

Often, Fermi pulsar data are fit with a softened exponential function, $\propto e^{\alpha x} e^{-(\epsilon/\epsilon_{br})^b}$, which is supposed to imitate variations in the exponential cutoff energy. In this appendix, we address a question: if the data come from double power-law distribution, how well will they be represented by the softened exponential fit? As an example, we fit a function $f = 2/(x + 1/x)$ sampled at equally spaced logarithmic intervals from 0.1 to 40 with a typical error of 0.01 and random scatter 0.01. This choice of the parameters simulates a typical bright pulsar spectrum sampled in unites of GeV and maximum flux of the order of $\sim 100$ of the statistical error.

We find that the fit function provides an acceptable fit (reduced $\chi^2 \leq 1$) for an exceptionally wide range of the break energies $0.01 \leq \epsilon_{br} \leq 2$, see Figure 10. For less bright pulsars, where due to low photon statistics the spectrum can be measured up to a lower maximal energy, the reduced $\chi^2$ remains smaller than unity even for larger values of the fitted $\epsilon_{br}$.

Thus, if a typical spectrum of a bright Fermi pulsar is a broken power law, the spectral fit with a softened exponential function, $\propto e^{\alpha x} e^{-(\epsilon/\epsilon_{br})^b}$, gives acceptable fits for a very wide range of parameters and, thus, does not provide a sufficiently sensitive probe of the underlying spectrum.

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