Current Uncertainties in the Use of Cepheids as Distance Indicators

Michael Feast

Astronomy Department, University of Cape Town, Rondebosch, 7701, South Africa. mwf@artemisia.ast.uct.ac.za

Abstract. The methods of calibrating the luminosities of galactic Cepheids and determining Cepheid reddenings are considered in some detail. Together with work on NGC4258 this suggests that the calibration presented is valid to about 0.1mag (s.e.) at least for Cepheids with near solar abundances. Metallicity effects are considered, partly through the use of non-Cepheid moduli of the LMC. To reduce the uncertainty substantially below ~ 0.1mag will require extensive work on metallicity effects. Non-linearities in period-luminosity and period-colour relations will also need to be considered as will the need to distinguish unambiguously between fundamental and overtone pulsators.

1 Introduction

The assigned title of this paper might suggest that Cepheids are poor or untrustworthy distance indicators. In fact they are currently the best fundamental distance indicators that we have. For (classical) Cepheids within our own Galaxy, the zero-point of the period-luminosity relation in the V-band (PL(V)) is known to ~ 0.1mag. However, if we wish to confirm this level of accuracy and improve on it, we need to consider a number of possible constraints and complexities.

I begin by considering the calibration of the zero-point of the PL(V) relation within our own Galaxy and later discuss possible complications with this relation, particularly those related to the chemical abundance of Cepheids. In this connection the indirect calibration of Cepheid luminosities through independent estimates of distances to other galaxies, particularly the LMC, will be considered.

In our own Galaxy there are basically four methods that can be used to calibrate a PL relation; trigonometrical parallaxes of Cepheids; statistical parallaxes of Cepheids; Cepheids in clusters or binary systems, and; Pulsation parallaxes (Baade-Wesselink type analyses).

2 Basic Relationships

In any determination of absolute magnitudes, the interstellar extinction to the objects used must be taken into account. Reddenings of individual Cepheids can be obtained from multicolour photometry (e.g. BVI photometry [1][2]). Relative reddenings of good individual accuracy can be obtained in this way for Cepheids of a given metallicity, despite the method having been questioned on
non-quantitative grounds [120]. This is clear from a discussion of LMC and SMC data [2]. The spread (standard deviation) in the derived individual reddenings, $E(B - V)$, after allowance for small photometric uncertainties is only 0.03mag (LMC) and 0.02mag (SMC). These are therefore upper limits to the intrinsic scatter in the method. In addition the derived reddenings show no dependence on period [2]. Laney and Stobie [4] quote results showing good agreement between individual BVI reddenings of galactic Cepheids and those determined in other ways, although full details have not yet been published. The zero-point of the BVI reddening system in our Galaxy is determined from Cepheids in open clusters of known reddening. However as noted below a knowledge of this zero-point is not necessary in some important distance-scale applications.

Using BVI-based reddenings, or reddenings consistent with these, period-colour relations (PC) can be constructed (e.g. [3][4]). For the present discussion the following PC and PL relations have been adopted.

$$< B >_o - < V >_o = 0.416 \log P + 0.314,$$  \hspace{1cm} (1)

$$< M_V > = -2.811 \log P + \rho$$  \hspace{1cm} (2)

The PC relation is for galactic Cepheids [4]. The slope of the PL relation is that for the LMC [5]. These are the basic relations used by Feast and Catchpole [6] in their work on the Cepheid calibration using trigonometrical parallaxes. Later we shall require a PC relation in $(V - I)$ and adopt [7][8] for galactic Cepheids,

$$< V >_o - < I >_o = 0.297 \log P + 0.427.$$  \hspace{1cm} (3)

Which is based on the same BVI reddening system as equation (1).

There is evidence in the literature of some misunderstanding regarding the use of a PC relation. Both the PC and PL relations are approximations to a period-luminosity-colour (PLC) relation and both have significant scatter. In view of the scatter, using a PC relation does not produce the best possible estimates of the reddenings of individual Cepheids. These can best be obtained from multicolour photometry. However because of the relation between the PC and the PL relations, through the PLC relation, deviations of PC-based reddenings from true reddenings compensate for deviations of luminosities from the mean PL relation. Thus the use of the two relations (equations (1) and (2)) together effectively reduces the scatter in the PL relation. This reduction in scatter, i.e. in width of the PL relation, is by a factor $(R/\beta - 1)$, where $R$ is the ratio of total to selective absorption ($A_V/E(B - V)$) and $\beta$ is the colour coefficient in the PLC relation in $V$ and $(B - V)$. Since for the Cepheids, $R$ is $\sim 3.3$ and $\beta \sim 2.5$, the PL width is, in this way, reduced by a factor of more than 3. This is important, not only in reducing the scatter of estimates of the PL zero-point

\footnote{But note that the angle between the intrinsic and reddening lines becomes less favourable with decreasing intrinsic colour (i.e. decreasing period). So the precision is a function of period.}
from individual stars, and hence reducing the uncertainty in the mean value, but also in reducing the effects of bias which are discussed below.

The reddening derived from a PC relation is a combination of the true reddening with a measure of the deviation of the Cepheid from the mean PC and PL relations. This being the case, negative PC reddenings can be expected and must be used, as must, of course, negative reddenings which are simply due to statistical scatter. These negative reddenings are sometimes dismissed in the literature as being “unphysical”, but this is due to a misunderstanding of the PC/PL method.

The zero-point of the PC relation is not of importance in distance determination provided it is used consistently for both calibrating and programme Cepheids (but note the exceptions discussed in sections 5 and 6).

3 Trigonometrical Parallaxes

It is sometimes claimed or implied that since the trigonometrical parallaxes which are currently available for many Cepheids have large percentage errors, they cannot be used to derive a trustworthy PL zero-point. This is not the case provided there is a significant sample of stars and that good estimates have been made of the standard errors of the individual parallaxes. The massive and homogeneous astrometric survey carried out by the Hipparcos mission [9] produced data which appears to satisfy these requirements. Nevertheless, the method of combining the data has to be carefully chosen.

Consider a group of objects all of the same absolute and apparent magnitude and so at the same (true) distance. The uncertainties in the absolute magnitudes derived from their measured parallaxes ($\pi$) are proportional to $\sigma_\pi/\pi$, where $\sigma_\pi$ is the standard error of $\pi$. Suppose $\sigma_\pi$ is the same for all the objects. Whilst the (weighted) mean parallax of the sample will be unbiased, the absolute magnitudes from underestimated parallaxes would have larger computed standard errors than those from overestimated parallaxes and a weighted mean absolute magnitude will be biased. This type of argument can be generalized as was done by Lutz & Kelker [10] and others (e.g. [11][12] see also [13]). The correction to the derived mean absolute magnitude depends on the space distribution of the objects concerned as well as on $\sigma_\pi/\pi$. In the case of the Hipparcos parallaxes for Cepheids the necessary corrections would be large for most of the stars. Since corrections of this type, unless very small, have considerable uncertainties, they are best avoided. This can be done by working in parallax space as will now be discussed.

If objects are selected by apparent brightness (which will be so in the cases of interest here) there will be a selection bias if there is a spread in absolute magnitude about the mean, or about a relation such as the Cepheid PL relation. Bias of this kind was discussed quantitatively by Eddington [14], Malmquist [15] and others. The treatment in this section is from [16].

Consider first a group of objects with a mean absolute magnitude per unit volume of $M_o$ and an intrinsic dispersion of $\sigma_{M_o}$. It is assumed there has been
no selection of the sample to be analysed according to $\pi$ or $\sigma_\pi/\pi$. The method of reduced parallaxes scales the measured parallaxes to the values they would have at the same apparent magnitude. This can be written:

$$10^{0.2M} = \sum 0.01\pi 10^{0.2m_o p} / \sum p$$

(4)

where the parallaxes are in milliarcsec, $m_o$ is the absorption free absolute magnitude and $p$ is the weight given by:

$$(0.01\sigma_T 10^{0.2m_o})^2 = 1/p$$

(5)

and

$$\sigma_T^2 = \sigma_\pi^2 + b^2 \pi_M^2 (\sigma_{m_o}^2 + \sigma_{M_o}^2)$$

(6)

In equation (6), $\sigma_T$ is derived from the uncertainty in the parallax ($\sigma_\pi$), the intrinsic scatter in the absolute magnitude ($\sigma_{M_o}$) and the uncertainty in the reddening corrected apparent magnitude ($\sigma_{m_o}$); also, $b = 0.2 \log_{10} 10 = 0.4605$.

$\pi_M$ is the photometric parallax derived using the PL relation [12]. Put $x = (m_o - M_o)$. Due to observational errors in $m_o$ and intrinsic scatter in $M_o$, $x$ will differ from the true distance modulus by $\epsilon$ (say). It is then evident that equation (4) yields an estimate of:

$$10^{0.2M} = 10^{0.2(M_o+x)} = e^{bM_o} e^{b\epsilon}$$

(7)

Consider objects all of the same $m_o$ (and $x$). Then [15][17];

$$e^{b\epsilon} = e^{0.5b^2\sigma_T^2} v(x - b\sigma_T^2) / v(x),$$

(8)

where,

$$\sigma_T^2 = \sigma_{m_o}^2 + \sigma_{M_o}^2$$

(9)

and $v(x)$ is the frequency distribution of $x$ which would have been observed if a complete survey had been made. It is important to note that this is the case, whether or not the objects under consideration actually form a complete survey. That is, the fraction of objects of a given apparent magnitude, $m_o$, actually observed may be a function of $m_o$ but this does not affect the quantity $e^{b\epsilon}$.

Evidently at a given $m_o$ an unbiased estimate of $10^{0.2M_o}$ is obtained by combining equations (4), (7) and (8). In general equation (8) is a function of $x$ (that is $m_o$). Furthermore if we apply the method to large volumes of space (as is likely to be possible with GAIA parallaxes), the function $v(x)$ may not be the same in all heliocentric directions. If however we assume a constant underlying density distribution, the r.h.s of equation (8) is independent of $x$ and becomes $10^{-2.5b^2\sigma_T^2}$ (see e.g. [17] equation (9)). In this approximation, the best unbiased estimate of $M_o$ is obtained by combining equation (4) above, with;

$$M_o = 5 \log(10^{0.2M}) + 1.151\sigma_T^2 - 0.23\sigma_T^2$$

(10)
where $\sigma_1$ is the standard error of the derived value of $M_o$, and the final term in equation (10) accounts for the conversion between natural and logarithmic quantities.

The above discussion refers to a set of objects assumed to have a mean absolute magnitude $M_o$ with a gaussian scatter. If this is not the case but instead the relative absolute magnitudes of the objects are a function of some measured quantity (e.g. in the case of Cepheids, the period) then two cases need to be considered. If the measuring errors of this auxiliary quantity introduce errors in $M_o$ which are small compared to $\sigma_{M_o}$, the formulation just given can, with obvious modification, be used to find the absolute magnitude zero-point. If this is not the case a different formulation is required [16][17]. In the case of the Cepheids the periods are usually known with good accuracy and their uncertainty has a negligible effect on the predicted relative values of the absolute magnitudes, so the formulation just given can be used.

Feast and Catchpole [6] analysed the Hipparcos parallaxes of Cepheids by the method of reduced parallaxes. Similar results have been obtained by [18][19]. Because they, [6], used the PC/PL approach discussed above to reduce the effective width of the PL relation, the bias term given in equation (10) is very small (0.010mag). This would change their derived PL zero-point ($\rho$) from $-1.43$ to $-1.42$. If, in an analysis of the Cepheid data, the reddenings were derived in some other way then it would be necessary to take into account the full (true) width of the PL(V) relation. There is some evidence (e.g [3]) that Cepheids are distributed rather uniformly through a strip in the PL plane. If the half-width of this strip in magnitudes is $\Delta$, then for a constant space density distribution, equation (8) becomes;

$$e^{\overline{b\epsilon}} = \frac{3\sinh(2b\Delta)}{2\sinh(3b\Delta)}$$  

(11)

At longer periods in the LMC, $2\Delta$ is approximately 0.7mag [3]. If this width applies to the calibrating Cepheids, the bias correction terms amount to $\sim 0.05$mag. This much larger bias shows the value of the PC/PL approach. Note that this bias remains the same however accurate or numerous the individual parallaxes are. It is perhaps worth noting that the need for a bias correction of this type is not necessarily avoided by working in magnitude space and applying a Lutz-Kelker type correction.

Whilst there are a large number ($\sim 220$) of Cepheids with Hipparcos parallaxes, most of the weight in the reduced parallax analysis is in a relatively small number of stars. The final result adopted [6] depends on the 26 stars of highest weight. This should now be corrected by the bias term in equation (10) and results in the value shown in table 1. If the overtone pulsator, $\alpha$ UMi, (see below) which carries about half the final weight in this solution is omitted, a negligibly different result is obtained, though of course with an increased standard error.

An important recent development has been the publication of a rather precise parallax of $\delta$ Cephei itself from HST observations [20]. The main uncertainty in this result probably comes from the need to convert from relative to absolute parallax. Since this star was presumably chosen for measurement and analysis
because of its bright apparent magnitude and not (retrospectively) because of its parallax, a determination of its absolute magnitude does not require a correction for Lutz-Kelker bias [16]. It will however be subject to magnitude selection bias. A zero-point for the PL relation from this one star is best derived using the PL/PC method and equation (10). One then obtains the result shown in table 1. The “26” star solution [6] can now be improved by replacing the Hipparcos parallax of δ Cep by a weighted mean of this value with that from the HST result. This leads to the value also shown in the table 1. Incorporating the result of Benedict et al. leads to a distinct lowering of the uncertainty in the zero-point. The standard errors quoted are those derived directly from the analyses. These, of course, have their own uncertainties and Monte Carlo simulations by Pont [21] suggest that in the case of the “26” star solution of [6], a more realistic estimate of the standard error is 0.12 rather than 0.10 [8]. In view of this one might feel that the uncertainty in the final value of table 1 (0.08) should be somewhat increased though it would seem unlikely to be greater than 0.10.

### Table 1. Cepheid Trigonometrical Parallax Zero-Points (Bias Corrected)

| Method                  | $\rho$      |
|-------------------------|-------------|
| 25 high weight          | $-1.43 \pm 0.13$ |
| $\alpha$ UMi fundamental | $-2.05 \pm 0.14$ |
| $\alpha$ UMi 1st overtone | $-1.41 \pm 0.14$ |
| $\alpha$ UMi 2nd overtone | $-0.97 \pm 0.14$ |
| 26 high weight          | $-1.42 \pm 0.10$ |
| δ Cep (HST)             | $-1.32 \pm 0.10$ |
| 26 high weight revised  | $-1.36 \pm 0.08$ |

Not all Cepheids pulsate in the fundamental mode and overtone pulsators are most frequent amongst stars with short (fundamental) periods. Double mode pulsators [22] provide the period ratio of the fundamental ($P_0$) to the first overtone ($P_1$) for galactic Cepheids, e.g.

$$P_1/P_0 = 0.720 - 0.027 \log P_0,$$

and this can be used to derive the fundamental periods of known overtone pulsators. These overtone Cepheids may be identified using the Fourier components of their light curves (e.g. [23]). They can also be identified in an (observed) period - radius diagram using Baade-Wesselink type radii. Early Baade-Wesselink work did not generally give radii of individual stars of sufficient accuracy to do this. However more recent work (e.g. using infrared photometry [24][25]) seems to be sufficiently consistent for this purpose. It would therefore be important to obtain radii of high accuracy for all the parallax stars of high weight. Only
a few of them seem to have the necessary data (e.g. $\beta$ Dor, l Car, Y Oph and U Sgr are confirmed as fundamental pulsators in this way, and SZ Tau as an overtone [24][25][26]). However the speculation [26] that the misidentification of overtone Cepheids for fundamental pulsators amongst the high weight parallax stars could have led to a significant overestimation of Cepheid luminosities seems rather unlikely to be correct.

Polaris ($\alpha$ UMi) is treated in the analysis as a first overtone pulsator on the basis of its derived absolute magnitude. If it were either a fundamental or second overtone pulsator it would yield a PL zero-point discrepant with the other high weight stars [6]. Evans et al. [27] have discussed other evidence that Polaris pulsates in the first overtone, including the diameter of the star derived using the interferometric angular diameter [28]. It is known from the LMC [29] that overtone Cepheids obey the normal PLC relation (at their fundamental periods). However they are in the mean brighter than the standard PL relation and intrinsically bluer than a standard PC relation. Because of the PL/PC method of analysis this means that the zero-point derived from overtone Cepheids will be slightly too faint. In the present sample the effect of this is expected to be negligible.

It has to be stressed that $\alpha$ UMi gives a PL zero-point in accord with that of the other high weight stars. The apparent discrepancy discussed by Di Benedetto [30] arises entirely because of the PL zero-point he adopts ($-1.27 \pm 0.17$). However, this value is derived from a non-optimal selection of Cepheids (i.e. it does not contain all the high weight Cepheids). Note, however, that due to its larger error it is not significantly different from the values in table 1.

In later sections there will be a discussion of possible chemical abundance effects on Cepheid luminosities. This is an area of some uncertainty. The parallax Cepheids are all in the general solar neighbourhood where the variation of chemical abundance amongst young stars such as Cepheids is expected to be small. However, abundance determinations for all the high weight Cepheids would be desirable.

4 Statistical Parallaxes

The method of statistical parallaxes combines proper motions and radial velocities to obtain a PL or PLC zero-point. In common with the method of reduced parallaxes discussed above, this method assumes that the relative distances of the stars are known and only a scale value is to be derived. In order to carry out an analysis of this type we require a kinematic model. Both the proper motions [31] and the radial velocities [32] show clearly and independently, the dominant effect of differential galactic rotation on Cepheid motions in the Galaxy. Thus the required model must be based on differential galactic rotation. This is even more apparent when one considers that to a first approximation the amplitude of the differential rotation effect in the proper motions is independent of distance whereas for the radial velocities it is proportional to distance. Adjusting the analyses for equality of the Oort constant ($A$) in proper motions and radial
velocities provides the best statistical parallax result for Cepheids. This is particularly the case since in this method the weight is spread over a large volume of the Galaxy and so avoids problems due to local deviations from an idealized model which almost certainly occur. In this way zero-points were found [31][33] for a PLC and for a PL relation. The zero-point of the latter, corrected for a possible magnitude bias of $\sim 0.01\text{mag}$ (as discussed in section 3) is given in table 2.

In view of some discussions in the literature it is important to stress that in deriving a PL zero-point from statistical parallaxes, there is a great advantage, as has just been mentioned, in treating the proper motions and the radial velocities separately [31][32].

One can also attempt a solution using the solar motion obtained from a combined discussion of solar motion and differential galactic rotation using proper motions and radial velocities. In this way the solar motion has a value which is averaged out over the whole large region of the Galaxy covered by the proper motion (Hipparcos) and radial velocity Cepheids and is not confined to a small region round the Sun where local deviations from the idealized model may lead to false results. The resulting scale [34] is only $0.04 \pm 0.26\text{mag}$ larger than that just given. However the standard error of this result is too large for the method to have any significant weight.

The above discussion refers to the use of the systematic motions of the Cepheids. In principle one can obtain a Cepheid scale from a comparison of the dispersions about an adopted solution in radial velocities and proper motions. However the velocity dispersion of Cepheids is small. Thus any such solution will be sensitive to the treatment of observational scatter in radial velocities and proper motions. It will probably also be sensitive to the effects of group motions. For these reasons no attempt along these lines has been made here. A further discussion of statistical parallax solutions is given in [8].

The Cepheids used in the statistical parallax solutions cover a significant fraction of the galactic plane. Most of the stars lie in the range, $(R_o - 3)\text{kpc}$ to $(R_o + 4)\text{kpc}$, where $R_o$ is the distance of the Sun from the galactic centre. If there is a galacto-centric gradient in chemical abundances of Cepheids over this range it might affect the PL and PC relations, particularly the latter. Evidently the work now in progress on chemical abundances of Cepheids (e.g. [35] etc.) should eventually allow us to take these effects into account. However since the sample of Cepheids used in the statistical parallax work is (roughly) centred on the Sun it may well be that any effect in the final mean result will be small.

5 Cepheids in Open Clusters

The (re)discovery of Cepheids in open clusters by Irwin [36] was a major step in the Cepheid calibration problem. Whilst the use of this method is of considerable importance, there are a number of special problems associated with it. These are: (1) Uncertainty of cluster membership; (2) Effects of reddening and photometric
uncertainties; (3) Effects of metallicity; (4) Absolute calibration of the cluster
distance scale.

(1). There are 30 open clusters or associations in our Galaxy which have
been listed as containing Cepheids [8]. Since that list was drawn up SZ Tau
has been shown [37] from proper motions to be a non-member of the cluster to
which it was formerly assigned. In addition TW Nor is not used here because its
cluster membership appears doubtful [109]. Definite confirmation of membership
of several others would be very valuable. It seems desirable that membership
should be based on position in the cluster, radial velocity and proper motion.
In the past a decision on membership has sometimes been made on the basis of
whether or not the derived Cepheid luminosity fitted with preconceived ideas.
This seems dangerously like an application of Merrill’s [38] principle which states
that when the discordant observations are rejected the remainder are found to
agree very well.

(2). The relative distances of the various clusters are obtained by a main-
sequence fitting procedure. Because of the steepness of the main sequence this
fitting procedure is very vulnerable to errors in the photometry or in the adopted
reddenings. For instance an error in \((B - V)\), of \(\Delta(B - V)\), leads to an error
in the derived distance modulus of \(\sim 6\Delta(B - V)\) if the fitting is done in the
\(V,(B-V)\) plane. For some clusters, distance moduli with standard errors as small
as 0.02mag have been claimed. However, even the adopted \((B - V)\) colours of
photometric standard stars can vary by 0.02mag or more between standard star
observers [39]. Thus estimates of the uncertainties of cluster moduli of \(\sim 0.15\)mag
as in Walker and Laney [40] seem more realistic, and the errors could be greater
in some cases. If the cluster fitting is done in the \(V,(V-I)\) plane an error of
\(\Delta(V - I)\), produces an error of \(\sim 5\Delta(V - I)\) in the derived modulus.

In the case of the analysis of trigonometrical parallaxes and statistical paral-
laxes of Cepheids it was pointed out that the zero-point of the reddening system
was not important so long as it was used consistently for both the calibrating and
programme stars. This is not the case when calibrating Cepheids using clusters.
Thus a change in the reddening zero-point by \(\Delta E\) changes the distance modulus
derived from \(V,(B-V)\) by \(\sim 6\Delta E\) and only \(\sim 3\Delta E\) of this is recovered when
dereddening the Cepheid itself.

(3). The position of the main sequence is sensitive to metallicity effects. A
change in [Fe/H] of 0.1 dex leads to a change in absolute magnitude at a given
\((B - V)\), of \(\sim 0.1\) mag, e.g. [46]. It is generally assumed that all the clusters
containing Cepheids are of solar metallicity or at least of solar metallicity in the
mean. The latter at least seems likely but it has not been proved and further
work on the metallicities of the clusters and their Cepheids would be desirable.

(4). In the past the absolute calibration of the cluster distance scale was
based on an adopted distance modulus for the Pleiades. The value which has
generally been used, 5.57mag, is the value derived by van Leeuwen [41] by fitting
nearby field main-sequence stars with known parallaxes to the Pleiades main
sequence though this figure has been revised by others from time to time [42][43].
It came as something of a surprise when the Hipparcos parallaxes of Pleiades
stars themselves gave a smaller distance modulus, $5.37 \pm 0.07 \text{mag}$ [44][45]. One reason for this surprise was that the van Leeuwen distance fitted rather well with theoretical results for solar-metallicity main-sequences [46]. It has been suggested that the Hipparcos distance can be reconciled with main-sequence theory if the Pleiades are metal poor ($[\text{Fe/H}] \approx -0.15$)[47]. There appears to be some evidence in Geneva-system photometry for such a suggestion [48] ($[\text{Fe/H}] = -0.12 \pm 0.03$) but neither the Stromgren photometry [49] ($[\text{Fe/H}] = +0.02 \pm 0.03$) nor spectroscopic abundances [50] ($[\text{Fe/H}] = -0.03 \pm 0.02$) show evidence for significant metal pooress. These abundances are derived assuming that the Hyades cluster members have $[\text{Fe/H}] = +0.13$. Alternatively the Hipparcos mean parallax of the Pleiades may have a greater uncertainty than given by its formal error or there is some problem with observations (see [8]) or theory. No final agreement on this point seems to have yet been reached. However a rereduction of the Hipparcos data for the Pleiades stars suggests a possible way out of this problem [121].

In view of this uncertainty it seems best at the present to base the cluster scale on the Hyades for which there is an excellent Hipparcos-based parallax. The problem with this is that it is generally agreed that the Hyades stars are slightly metal-rich, so a correction for this has to be made, if we make the common assumption that the clusters with Cepheids are of solar metallicity in the mean. The Hipparcos based distance modulus for the Hyades is $(m-M)_o = 3.33 \pm 0.01$, [51] and the metallicity adopted by e.g. Pinsonneault et al. [46] is $[\text{Fe/H}] = 0.13$. The theoretical metallicity correction adopted by these latter authors then shows that the Hyades main sequence in $V$, $(B-V)$ corresponds to that expected for a solar metallicity cluster at $(m-M)_o = 3.17 \text{mag}$, or $3.12 \text{mag}$ if the metallicity corrections of Robichon et al. [45] are used. I adopt a mean value $3.14 \text{mag}$. Since most work on clusters containing Cepheids is referred to Turner’s Pleiades main sequence [52], we need to see how this is affected by the Hyades result. The Pleiades - Hyades magnitude difference in a $V,(B-V)$ diagram , corrected for reddening but not metallicity is $2.52 \text{mag}$ [53]. Thus the Turner main sequence is that expected for a solar metallicity cluster at,

$$(m-M)_o = 3.14 + 2.52 = 5.66.$$  

Adopting this value and assuming the clusters containing Cepheids which are listed in [8] are in the mean of solar metallicity we obtain a PL zero-point of, 

$$\rho_1 = -1.45 \pm 0.05 \text{ (internal) \ mag}.$$  

Here SZ Tau and TW Nor have been omitted for the reasons given above. If the Hyades metallicity suggested by Taylor [54] ($[\text{Fe/H}] = +0.11$) were adopted we would obtain a brighter PL zero-point ($-1.47$). The error of this result is internal. The true error may well be larger, partly due to uncertainties in the metallicity correction. The standard deviation of the result is $0.26 \text{mag}$. Some of this is due to the width of the PL strip ($\sigma_{PL} = 0.21$ [5]) which comes in with full force here (unlike the case discussed in section 3). Subtracting this quadratically gives $0.15 \text{mag}$ as the standard deviation of the cluster moduli. This agrees with a recent comparison of Baade-Wesselink and cluster moduli by Turner and Burke [122] (their table 3) from which one finds a standard deviation
of 0.14 mag, presumably mainly due to the scatter in the cluster moduli since the Baade-Wesselink results are believed to have very high internal accuracy.

Whilst the adopted zero-point from clusters avoids the use of the Pleiades modulus, the cluster method cannot be considered entirely trustworthy until the problem of the Pleiades distance is fully understood.

Of the same nature as the cluster method is the use of physical companions to Cepheids whose luminosity can be independently estimated. This method has been used, notably by Evans and collaborators [55][56]. At the present time the accuracy obtained is not as good as that from other methods (see [8]).

6 Pulsation Parallaxes

An estimate of the luminosities of Cepheids can be made using pulsation parallaxes (Baade-Wesselink method). This method is dealt with extensively elsewhere in this volume. The procedure normally used gives results of high internal accuracy especially when implemented using infrared photometry [24][25]. The method is currently being strengthened by interferometric measurements of the angular diameters of Cepheids and their variation with phase [57][58][59][60]. It remains difficult to estimate in a realistic way the true uncertainty in the results from pulsation parallaxes which depend on possible systematic errors in the derived radii and in the surface brightness estimates. For the present discussion I have adopted a PL(V) zero-point derived from the results of Lane [61] (see [8]).

7 Summary of the Galactic Calibration

The results discussed above are summarized in Table 2. The cluster and pulsation parallax methods both have small internal errors but their real (external) uncertainty is difficult to quantify. The results of Monte Carlo simulations by Pont [21] suggest that in the case of the “26” high weight parallax-solution Cepheids [6] the error might have been slightly underestimated. That may still be the case here but it is unlikely to be significantly greater than \( \sim 0.1 \) mag. Both the trigonometrical and statistical parallax methods seem rather robust. In the present paper an unweighted mean has been adopted as best galactic zero-point and this is shown in Table 2.

8 A Cepheid Zero-point from NGC4258

A distance to the galaxy NGC4258 has been derived from the motions of H2O masers in the central region combined with a model [62]. In this way a distance modulus of 29.29 \( \pm \) 0.09 was obtained. Newman et al. [63] have obtained V,I data for Cepheids in this galaxy using the HST. Reducing their data with the PC relation in \( (V - I) \) given above and a PL(V) relation of slope \(-2.81\) leads to a PL zero-point of \(-1.17 \pm 0.13\). Here the error in the maser distance has been combined with the internal uncertainty in the Cepheid result. In obtaining this
value of the zero-point a small correction for metallicity has been applied. HII region measurements [64] suggest that [O/H] is −0.05. A correction of 0.20 mag [O/H]$^{-1}$ was adopted (see sections 9 and 10). Unless the adopted metallicity of the galaxy is grossly in error or the metallicity effect much greater than assumed, the metallicity correction is very small.

Including this zero-point (−1.17 ± 0.13) with the galactic values (Table 2) yields an unweighted mean of −1.35 ± 0.05, which is possibly the best current estimate of the zero-point for metal-normal Cepheids. However it has been suggested recently [111] that model uncertainties lead to possible uncertainties in the mass of the central black hole in NGC4258 of at least 25 percent and it remains to explore what effect this has on the deduced distance.

### 9 Metallicity Effects

A remaining source of uncertainty in deriving distances of Cepheids is the effect of metallicity variations on the PLC, PL and PC relations, and on multi-band intrinsic colours. In any distance derivation, the interstellar reddening and absorption must be derived as well as a prediction of the absolute magnitude of the star. The effect of metallicity change on PL relations will vary with wavelength. The effect on the derived interstellar absorption will depend on the method used for its derivation. For instance there is good evidence from a comparison of the LMC and SMC that Cepheids become bluer in $(B − V)$ at a given period with decreasing metallicity [43]. Thus a standard PC relation in $(B − V)$ will give too small a reddening for a metal-poor Cepheid. However if the reddening of a metal-poor Cepheid is derived from a standard two-colour, $BVI$, plot, the reddening will be too high (see e.g. [2]). If we knew precisely the dependence on
metallicity of all the quantities involved and had sufficient data we could solve in any given case for the reddening and metallicity of a Cepheid and also for its luminosity and distance. An indication of how this this could be done in practise using $BVI$ photometry was given in [65].

So far as the use of a PL relation to derive luminosities together with either multi-colour data or a PC relation to derive redenning is concerned, the metallicity problem may be broken down into three part.

1. A possible change in bolometric luminosity at a given period.
2. A possible change in colour at a given temperature.
3. A possible change of temperature at a given period.

Laney and Stobie [66] showed that at a given period the metal-poor Cepheids in the Magellanic Clouds were slightly hotter than those in our Galaxy. Laney[67] then showed that the radii of Magellanic Cepheids as determined from Baade-Wesselink type analyses fitted the galactic period-radius relation. These observations seem to show that at a given period the bolometric luminosity of a Cepheid increases with decreasing metallicity. However the effect is small and within the uncertainties of the observations. Further work along these lines would be valuable. An effect on the bolometric luminosity obviously affects the results at all wavelengths in the same way.

The effects of items 2 and 3 above, on reddening and luminosity depend on the wavelengths and methods used. Here we consider the effects when using $V, I$ photometry as in the HST work on extragalactic Cepheids. Other cases have also been considered, e.g. [8].

The HST work essential uses a PC relation in $(V-I)$ and a PL(V) relation to determine reddening and distance. There is some confusion in the literature regarding metallicity effects using $V$ and $I$. This arises because the effects of metallicity on equations (2) and (3) are such that the changes affect the derived distance modulus in opposite directions. It is thus important to consider these two equations together. A direct test of this was made by Kennicutt et al. [68]. They observed Cepheids in the galaxy M101 at different distances from the centre of the galaxy where abundances had been estimated for HII regions. The abundance is above solar in the inner field and below solar in the outer field. Their results lead to a metallicity effect on a distance modulus derived using equations (2) and (3) of $0.24 \pm 0.16$ mag $[O/H]^{-1}$. This is in the sense that without the correction the distance of a metal-poor Cepheid would be overestimated. This result suggests there is a small metallicity effect in the $V, I$ method. However the uncertainty in the result is large. It should also be borne in mind that although there seems little doubt that there is a strong metallicity gradient in M101, the absolute values of the metallicities in the fields studied by Kennicutt et al. [68] remain somewhat uncertain (see their figure 2 and the accompanying discussion).

Laney [67][69] (see Feast [8]) discussed Baade-Wesselink radii and colours of Cepheids in the Galaxy, the LMC and the SMC and these lead [8] to an effect in the moduli of $\sim 0.09 \pm \sim 0.04$ (int) mag $[O/H]^{-1}$ in the same sense as the Kennicutt et al. correction. Much of the weight of the Laney result depends on the SMC.
10 Non-Cepheid Distances to the LMC

The LMC is of great importance for the Cepheid problem. It provides the slope of the PL(V) relation currently in use. Also the LMC showed clearly that when Cepheids are dereddened using three colour photometry, the PL relation has a significant width which is reduced to within the observational errors when a PLC relation is used [70]. The zero-point of the LMC PL relation can be established if the LMC distance can be independently determined. However it is known that the LMC Cepheids are metal-deficient compared with those in the solar neighbourhood, e.g. [71]. Thus a comparison of the Cepheid luminosities in our Galaxy and in the LMC can be an important test of metallicity effects on Cepheids. This is obviously a major source of concern in the use of Cepheids as general distance indicators. It is particularly important to note that a non-Cepheid distance to the LMC does not give a PL zero-point for normal metallicity Cepheids independent of some knowledge of the metallicity effect.

The distance to the LMC is discussed elsewhere in this volume but it is necessary to give here the basis for the present discussion on the luminosities of LMC Cepheids. In the following subsections some non-Cepheid methods of determining the LMC distance will be considered. Whilst many of these methods appear promising it should be remembered that none of them have yet been subjected to the intense scrutiny that has been applied to the Cepheids themselves. In each case, some of the issues that need resolving before that particular distance indicator can be fully relied on, are mentioned. Only methods which are, or have been claimed to be, largely independent of theory are considered. For instance the magnitude of the Red-Giant-Branch tip seems to be a good indicator of relative distances but requires an absolute calibration either from stellar evolution models or through some other indicator (such as Cepheids).

10.1 The RR Lyrae Variables

RR Lyrae variables have long been regarded as valuable distance indicators. However the dependence of their absolute magnitudes on metallicity has been a matter for debate. Furthermore if globular clusters are taken as a guide [72] there is a significant spread in $M_V$ at a given metallicity. Other papers in this volume discuss the RR Lyraes in detail and a full discussion is not given here. The most important recent development has been the publication by Benedict et al. [73] of a trigonometrical parallax of RR Lyrae itself. This can be used together with equation (4) above, to obtain an estimate of the mean absolute magnitude of RR Lyrae stars of this metallicity ($\text{Fe}/H = -1.39$). Account needs to be taken of the fact that at this metallicity globular cluster results suggest that RR Lyraes fill a strip of width $\sim 0.4\text{mag}$. One obtains [16], $+0.64 \pm 0.11$ correcting for the resulting bias using equation (11) above. Then, adopting the relation:

$$M_V = 0.18[\text{Fe}/H] + \gamma$$

(13)

from Carretta et al. [74] and a mean reddening-corrected apparent magnitude of $V_0 = 19.11$ at a metallicity of $[\text{Fe}/H] = -1.5$ for the LMC field RR Lyraes,
one obtains an LMC modulus of $18.49 \pm 0.11$. Note however that this standard error should probably be increased, possibly to $\sim 0.16$ due to the uncertainty in the bias correction as applied to the one calibrating star. Other RR Lyrae-based estimates of the LMC modulus are listed in [75] where a mean of $18.54$ was adopted. The uncertainty in this latter value is probably somewhat over $0.10$ mag. Whilst the determination of an accurate trigonometric parallax for RR Lyrae is a great step forward, the accuracy of the LMC modulus derived from it must be limited if the spread in absolute magnitudes at a given metallicity is as great as that adopted above. It may however be possible to use this parallax result together with an infrared period-luminosity relation [76] to obtain a more precise result if this latter relation has a small scatter.

10.2 The Mira Variables

Multi-epoch infrared photometry of Mira variables in the LMC shows that both carbon-rich (C-type) and oxygen-rich (O-type) variables have a well-defined PL relation in the $K$ band ($2.2\mu m$) [77]. For the O-Miras the relation can be written,

$$M_K = -3.47 \log P + \gamma.$$  

(14)

The scatter about this relation is only $0.13$ mag. Miras in the SMC, in globular clusters, as well as those with spectroscopic parallaxes from companions, all fit a PL($K$) relation with the same slope [78][79][80]. The zero-point may be calibrated using Hipparcos parallaxes of Miras. This yields, $\gamma = +0.86 \pm 0.14$ mag [81] when small bias effects (see equation (10), above) [16] are taken into account. A zero-point can also be obtained from Miras in globular clusters. This method gives, $\gamma = 0.93 \pm 0.14$ mag [80]. To this may now be added a result from the parallaxes of OH-maser spots in Miras obtained using VLBI [113]. The four Miras with distances from this method, together with infrared photometry [114] yield, $\gamma = +1.04$ mag. The internal standard error of this result is small ($0.13$ mag) but the uncertainties in the individual determinations suggest that this is an underestimate and that the standard error of the mean is probably about $0.23$ mag. In view of this, the last method is given half weight in combining the three estimates. One then obtains $\gamma = +0.92$ and an LMC modulus of $18.56$. If full weight had been given to the third method the zero-point would only have been increased by $0.02$ mag. The standard error of the adopted result is less than $0.10$ mag.

In globular clusters the periods of Mira variables are a function of metallicity e.g. [82]. It is not clear whether, at a given period, the metallicity of Miras differs from system to system. However there is some evidence that the infrared colours of O-Miras at a given period are systematically different in the LMC from those of Miras in the galactic bulge. This is likely to be due to weaker $H_2O$ bands in the LMC stars [82][83]. This could be due either to a deficiency of oxygen (an $\alpha$ element) or to a higher C/O ratio resulting from an overabundance of carbon. The effect of this on the PL($K$) relation is not known empirically. However theoretical work [110] suggests that, if anything, a general metal deficiency, if not taken into account, will lead to an underestimation of the distance modulus.
It is perhaps worth pointing out that whilst Miras seem reliable distance indicators in the case of the Magellanic Clouds, caution is require if only a few such variables are identified in a system. In the LMC, whilst the bolometric PL relation found at short periods is continued out to periods of \( \sim 1000 \) days by dust-enshrouded Miras [84][85], there are a number of stars with periods over 420 days which lie above this relation [77] and some similar objects at shorter period. Whitelock [84][85] has pointed out that, of these, those studied by Smith et al. [86] show evidence for surface lithium and can be interpreted as hot bottom burning stars which would not be expected to obey the PL relation. Note that the Mira-like variable in IC1613 which is clearly too bright for the PL relation [87] has an unusually early spectral type for its period (641 days) and is a likely candidate for a hot bottom burning object [85].

10.3 Eclipsing Binaries

Deriving distances from eclipsing binaries has much in common with the determination of pulsation parallaxes by a Baade-Wesselink type analysis. In both cases a stellar radius is combined with an estimate of the surface brightness to obtain a luminosity. The method has been applied to three LMC eclipsing variables [88][89][90] and rediscussion of some of these results have been published [91][92]. In view of the spread in distance moduli derived, Fitzpatrick et al. [90] suggest only that they lead to an LMC modulus of \( \sim 18.40 \). Uncertainties and assumptions in the method used have been discussed [91][75].

10.4 SN 1987A

Panagia [93] deduced a distance to the LMC centroid from the ring round SN1987A \( (18.58 \pm 0.05) \). The distance depends, amongst other things, on the assumed ellipticity of the ring. A spectral-fitting expanding-atmosphere model gives a similar result though with considerable uncertainty \( (18.5 \pm 0.2) \) [94].

10.5 The Red Giant Clump

The use of the red giant clump as a distance indicator has been much discussed in recent years. As applied to the LMC this has led to conflicting results. Girardi and Salaris [95] investigated theoretically the dependence of the clump absolute magnitude on age and metallicity. Coupling their results with a population synthesis model of the LMC they obtained an LMC modulus of 18.55. More recently Alves et al. [96] have applied the method in the K-band and find 18.49 \( \pm 0.04 \). However the need to assume theoretical age and metallicity corrections and to adopt an LMC model, reduces the usefulness of the clump as a distance indicator [95].
10.6 Open Clusters

It has long been realized that main-sequence fitting to young clusters in the LMC provides a method to estimate its distance. Since this is the same procedure as that used to derive distances to Cepheids in open clusters in our Galaxy (section 5, above), the qualifications discussed there apply also in the case of LMC clusters. A particularly interesting result is that derived from extensive work on the LMC cluster NGC1866 by Walker et al. [97]. These authors deduce a reddening-corrected distance modulus of $18.35 \pm 0.05$ for the cluster and point out that if it lies in the plane of the LMC the mean modulus of this galaxy is 18.33.

The NGC1866 modulus is derived differentially with respect to the Hyades, adopting the Hipparcos distance for this cluster and applying a correction for the metallicity effect. The comparison between the two main sequences is made in the $V, (V-I)$ plane and a value of $A_V/E_{V-I}$ of 2.08 was adopted. Walker et al. obtain reddening corrected moduli of 18.37 and 18.33 for assumed values of $[\text{Fe}/\text{H}]$ of $-0.30$ and $-0.50$. Since these metallicities span the range of metallicities measured in various ways for the cluster, they adopt an NGC1866 modulus of 18.35. It is interesting to note (as can be deduced from the diagrams in their paper) that the distance modulus of NGC1866 corrected for reddening but not for the metallicity difference between it and the Hyades ($[\text{Fe}/\text{H}] = +0.13$) is $\sim 18.9$.

A comparison of this with the results for the two different assumptions as to the metallicity of NGC1866 shows that the applied metallicity corrections are highly nonlinear. These results may be compared with those which are obtained using the (linear) metallicity corrections of Pinsonneault et al. [46]. These are moduli of $\sim 18.5$ and $\sim 18.3$ for $[\text{Fe}/\text{H}]$ of $-0.30$ and $-0.50$. Clearly the metallicity model is crucial.

One might be concerned that the relative abundances of the heavy elements might be different in the Magellanic Clouds from the Sun. Hill et al. [98] found no significant enhancement or depletion in the ratio of $\alpha$-elements to iron for stars in NGC1866 ($[[\alpha/\text{Fe}]] = 0.1 \pm 0.1$). However the situation is not entirely clear since depletion of the $\alpha$-element oxygen seems rather general in the LMC [98][99][112].

Finally it is worth noting that the uncertainty assigned by Walker et al. to their favoured modulus for NGC1866 (0.05mag) should probably be regarded as an internal error only. The external error is likely to be larger due to the effects of magnitude transformations and other causes. For instance they find from a comparison of their HST data with overlapping ground based data that there is a mean difference in colour of $\Delta(V-I) = -0.07 \pm 0.06$ (s.d) in the sense, ground based minus HST data. They do not apply this as a correction to their HST data in view of the fact that it is much reduced if outliers are omitted. However had they applied this correction their distance modulus would have been $\sim 0.35$mag greater for the same adopted reddening. This follows since a comparison with the ground based data [100] shows that the HST results are the relevant data set for the main sequence fitting. This of course does not necessarily prove that the
cluster distance modulus is $18.35 + 0.35 = 18.70$. However it does indicate the uncertainties encountered in this type of work.

10.7 LMC Summary

Mean distance moduli from various types of objects are listed in table 3. These can be combined in a variety of ways. This generally leads to mean moduli near 18.5. In view of the various points discussed above one should probably consider this to have a realistic standard error of about 0.1 despite internal accuracies better than this being claimed for some determinations. In the present connection we are not concerned with the LMC modulus per se. We require a distance which can be compared with that derived from Cepheids so as to estimate the effects of metallicity on the Cepheid scale. In doing this a remaining uncertainty is whether or not the reddenings adopted in deriving the moduli listed in table 3 are consistent with those used for the Cepheids.

11 Tests of Metallicity Effects

Table 4 shows the adopted non-Cepheid LMC modulus. Also shown is the Cepheid distance modulus of the LMC based on equations (2) and (3) and with the zero-point of the PL(V) relation from Table 2 ($-1.35$) without any metallicity correction, with the corrections used by the HST Key project group [107], $0.20\text{mag} [\text{O/H}]^{-1}$, and that derived from the work of Laney.

It is clear that the various estimates are in better agreement than we might reasonably have expected.

Some caution has to be used with these results. For instance there is evidence (see section 10.6) that young objects in the LMC may be deficient in oxygen (an $\alpha$ element) relative to iron and this could affect the luminosities of some calibrators. There remains also the problem of the depth of the LMC. It is generally assumed to be small, at least for young objects. But more evidence bearing on this is required, especially for the old objects such as RR Lyrae stars which are used as LMC distance indicators. It is worth recalling that the SMC Cepheids show evidence of a considerable depth of this galaxy in the line-of-sight [3] and this tends to preclude the use of the SMC for stringent tests of the Cepheid scale and its metallicity dependence.

A similar comparison of Cepheid and non-Cepheid moduli to that described above for the LMC can be made for other galaxies. Dolphin et al. [101] and Udalski et al. [102] have published such discussions for IC1613 in which the Cepheids are believed to have a low metallicity ($[\text{Fe/H}] \sim -1.0$). These discussions suggest a relatively small metallicity effect using $V, I$ for Cepheids of metallicity lower than that of the LMC. Dolphin et al. [101] found $-0.07 \pm 0.16\text{mag} [\text{O/H}]^{-1}$ and Udalski et al. [101] suggest there is no significant metallicity effect. In the case of IC1613 there is some uncertainty, due amongst other things to the lack of good estimates of the metallicities of the Cepheids and other objects used to derive distances.
### Table 3. Non-Cepheid LMC Distance Moduli

| Object      | Method       | Modulus  | Mean Modulus |
|-------------|--------------|----------|--------------|
| RR Lyraes   | Trig. Par.   | 18.49 ± 0.11 |              |
|             | Hor. Branch  | 18.50 ± 0.12 |              |
|             | Glob. Cl.    | 18.64 ± 0.12 |              |
|             | δ Sct        | 18.62 ± 0.10 |              |
|             | Stat. Par.   | 18.32 ± 0.13 |              |
|             |              | unweighted mean | 18.51      |
| Miras       |              | 18.56 ± < 0.10 | 18.56        |
| Eclipsers   |              | ~ 10.40 | 18.40        |
| Red Clump   | V-band       | 18.55 |              |
|             | K-band       | 18.49 ± (0.04) | 18.52       |
|             |              | unweighted mean | 18.52      |
| SN1987A     |              | 18.58 ± 0.05 | 18.58        |
| NGC1866     |              | 18.33 ± (0.05) | 18.33        |
| All         | Unweighted Mean | 18.48 ± (0.04) |              |

### Table 4. LMC Moduli: Cepheid - Non-Cepheid Comparison

| Object      | Method                      | Modulus  |
|-------------|-----------------------------|----------|
| Non-Cepheid |                             | 18.48 ± (0.04) |
| Cepheid     | V,I Uncorrected             | 18.60    |
|             | V,I Laney Correction        | 18.56    |
|             | V,I HST Adopted Correction  | 18.52 (Range 18.57 - 18.47) |
The above discussion suggests that in \( V, I \) there is a small but probably significant metallicity effect, at least for Cepheids more metal rich than the LMC. It is evident that further progress requires amongst other things improved abundances for Cepheids in both our Galaxy and nearby galaxies.

In view of the uncertainty in the metallicity correction it is advisable to avoid the need for using it if possible. This is the case for the galaxies in the HST key project \([107]\). The mean metallicity of these galaxies, weighted by their contribution to the finally adopted value of \( H_0 \) is close to solar \([O/H] = –0.08\). It has been hypothesised by some workers that the metallicity effect at \( V, I \) could be as high as 0.6mag \([O/H]^{-1}\). Whilst it seems unlikely that it could be as high as this, even such a large value will only have a small effect on a value of the HST key project \( H_0 \) if this is based on the galactic (or NGC4258) calibration.

12 Cepheids – General Problems

There is evidence of a metallicity gradient for Cepheids in our own Galaxy e.g. \([35]\) and this will need to be taken into account in future analyses of Cepheid data (see e.g. sections 3 and 4 above). Abundance determinations are also important since they can help distinguish first- and later-crossing Cepheids. Most Cepheids are expected to be second-crossing stars and abundance analyses (see \([103]\) and papers referenced there) which show that they have undergone first dredge-up, are consistent with this. The Cepheid SV Vul does not seem to have undergone first dredge-up \([104]\) and is therefore likely to be a first-crossing Cepheid. Evidently chemical analyses together with accurate parallaxes will be a powerful way of investigating the multiple crossings and their effect on the use of Cepheids as distance indicators.

In most discussions of Cepheid luminosities it is assumed that the slope of the PL(\( V \)) relation can be taken from the LMC. In using this in our own and other galaxies, this assumes that the slope is independent of metallicity. The empirical evidence on this point is not strong. Caldwell and Laney \([5]\) found a slope of \(-2.63 ± 0.08\) for the SMC Cepheids. The value (adopted in the present paper) for the LMC is \(-2.81 ± 0.06\) \([5]\). Udalski et al. \([105]\) found \(-2.76 ± (0.03)\) for the LMC. The standard error is placed in brackets since much of the weight of this determination is in short-period Cepheids. Gieren et al. \([106]\) obtained \(-3.04 ± 0.14\) for Cepheids in the general solar vicinity using pulsation parallax results. There is a slight hint of a trend SMC, LMC, Galaxy i.e. a metallicity effect. The evidence for such a trend is evidently marginal and requires confirmation. The slope is of importance since, for instance, the weighted mean log-period of the Cepheids used in the Hipparcos parallax solution is smaller than the weighted mean log-period of the extragalactic Cepheids used to determine \( H_0 \), e.g. \([107]\). If the Gieren et al. slope had been used this would have resulted in an approximately seven percent increase in the parallax distance scale as applied to the HST key-project galaxies. Adopting the OGLE slope \([105]\) would have had only a small effect.
In the present discussion it has been assumed that the reddenings of Cepheids are derived from a PC relation. The relations in $B, V$ and $V, I$ (equations (1) and (3) above) are both based on the system of BVI reddenings and should be compatible. However there still remains work to make certain that these form a completely self-consistent set. Some estimates of the PC slope in $(V - I)$ are shown in Table 5. The value for the LMC derived from Udalski et al. [105] is based primarily on short period Cepheids and is uncomfortably different from the galactic value. It suggests the possibility of a change in slope at about 10 days [108]. This is currently a cause for concern. The LMC slope of Caldwell and Coulson [3] which is weighted to longer periods that of Udalski et al. differs from the latter and the LMC and SMC slopes also seem to differ. However none of these differences are vastly larger than their standard errors. The question of changes of slope with metallicity and at a period of about 10 days thus remains to be finally settled.

### Table 5. Slope of the PC($V - I$) Relation

| Method                        | Slope     |
|-------------------------------|-----------|
| Galaxy (Caldwell, Coulson) [7]| 0.297 ± 0.014 |
| LMC (Udalski et al.) [105]    | 0.202 ± 0.037 |
| Difference                    | 0.095 ± 0.040 |
| LMC (Caldwell, Coulson) [3]   | 0.318 ± 0.054 |
| SMC (Caldwell, Coulson) [3]   | 0.227 ± 0.038 |
| Difference                    | 0.091 ± 0.066 |

I have chosen to adopt here the galactic PC slope in $(V - I)$ for two reasons. Firstly, as just mentioned, the determination of Udalski et al. [105] is heavily weighted by short-period Cepheids. Such stars have little weight in the zero-point point calibrations discussed earlier or in the current applications of Cepheids in galactic astronomy and in the determination of $H_0$. Secondly, it happens that the mean metallicity of the galaxies studied in the HST key-project is close to solar when weighted according to their contributions to the final value of $H_0$. It is thus of some interest to compare the present calibration for metal-normal Cepheids with that adopted by the HST key-project group [107]. Some years ago [115] there was a difference of about 8 percent (0.17mag) between the Cepheid scale derived from Hipparcos parallaxes [6] and that adopted by the Key-project group. This difference has been essentially eliminated by two factors. Firstly, the Cepheid
zero-point for metal-normal Cepheids adopted from the various estimates in Table 2 is 0.08 mag fainter than that derived from Hipparcos parallaxes alone by Feast and Catchpole [6]. Secondly, though the Key-project group still adopt an LMC modulus of 18.50, they apply a metallicity correction which implies that their scale for metal-normal Cepheids has been increased by 0.08 mag. The true value of \( H_0 \), however, still remains somewhat contentious e.g. [108].

13 Very Short Period Cepheids

The discussions above have left out of consideration the very short period Cepheids (periods less than \( \sim 2 \) days). For instance the analysis of Udalski et al. [105], although heavily weighted to shorter period stars, omits Cepheids with periods less than 2.5 days. However, very short period Cepheids are known to be numerous in the SMC and other young metal-poor systems and although they are relatively faint, they are potentially important as distance indicators for systems such as the metal-poor dwarf galaxies [116].

In the SMC there is a steepening of the PL relations for periods shorter than 2 days, as was originally pointed out by Bauer et al. [117]. The OGLE data for these very short period Cepheids in the SMC [118] has been used by Dolphin et al. [116] to derive slopes of \(-3.10\) and \(-3.31\) for the PL(V) and PL(I) relations of fundamental mode pulsators and \(-3.30\) and \(-3.41\) for the first overtone pulsators (which constitute about half the OGLE sample).

Dolphin et al. [116] have also discussed the relative distances of the SMC and the dwarf galaxies, IC1613, Leo A and Sex A. The Cepheids in these systems are expected to be metal poor since the values of \( 12 + \log(O/H) \) from HII regions are: SMC, 8.0; IC1613, 7.9; Leo A, 7.3; Sex A, 7.6 [119]. From a comparison of the relative distances derived from the very short period Cepheids in these systems with relative distances from other indicators (RR Lyrae variables, RGB tip, red clump) Dolphin et al. conclude that there is no significant effect of metallicity on the luminosities of these very short period metal-poor Cepheids.

14 Conclusions

The present discussion suggests that the luminosities of metal-normal Cepheids are now known within a standard error of 0.1 mag or possibly less. However it has to be recognized that deviations from the true value considerably in excess of one standard error are entirely possible statistically. It is therefore very desirable to further strengthen the empirical determinations. Some remaining issues that need to be resolved are as follows.

1. Do the slopes of PL (and PC) relations at different wavelengths vary with metallicity?
2. Are there non-linearities in the PL and PC relations? Particularly is there a significant slope difference between short and long period (\( > \sim 10 \) days) Cepheids that would seriously affect the calibration and use of PL and PC relations?
3. Are reddening effects being correctly and consistently treated in the calibration and use of Cepheids?
4. Is the reddening law being used really applicable to all Cepheids in our own and other galaxies?
5. Can better empirical estimates be obtained of the effects of metallicity variations on Cepheid luminosities and colours?
6. Do the relative abundances of heavy elements (e.g. the ratio of $\alpha$ elements, such as oxygen, to iron) affect Cepheid PL and PC relations?
7. Are a significant number of calibrating or programme Cepheids undiscovered overtone pulsators?
8. Can we distinguish (probably by spectroscopic analysis) first-crossing Cepheids from others? Are the luminosities of such stars significantly different from others of the same period? If so, are these stars sufficiently numerous to create a problem for the use of Cepheids as distance indicators?

References

1. J. F. Dean, P. R. Warren, A. W. J. Cousins: MNRAS, 183, 569 (1978)
2. J. A. R. Caldwell, I. M. Coulson: MNRAS, 212, 879 (1985) (erratum 214, 639)
3. J. A. R. Caldwell, I. M. Coulson: MNRAS 218, 223 (1986)
4. C. D. Laney, R. S. Stobie: MNRAS, 266, 441 (1994)
5. J. A. R. Caldwell, C. D. Laney: In: The Magellanic Clouds, ed. by R. Haynes, D. Milne (Kluwer, Dordrecht 1991) p.249
6. M. W. Feast, R. M. Catchpole: MNRAS, 286, L1 (1997)
7. J. A. R. Caldwell, I. M. Coulson: AJ, 93, 1090 (1987)
8. M. W. Feast: PASP, 111, 775 (1999)
9. M. A. C. Perryman et al.: The Hipparcos and Tycho Catalogues, ESA SP-1200 (1999)
10. T. E. Lutz, D.H. Kelker: PASP, 85, 573 (1973)
11. R. B. Hanson: MNRAS, 186, 875 (1979)
12. C. Koen, C. D. Laney: MNRAS, 301, 582 (1998)
13. A. Sandage, A. Saha: AJ, 123, 2047 (2002)
14. A. S. Eddington: MNRAS, 73, 359 (1913)
15. K. G. Malmquist: Lund Medd. Ser. II, no 22 (1920)
16. M. W. Feast: MNRAS, 337, 1035 (2002)
17. M. W. Feast: Vistas in Astronomy, 13, 207 (1972)
18. M. A. T. Groenewegen, R. D. Oudmaijer: A&A, 356, 849 (2000)
19. P. Lanoix, G. Paturel, R. Garnier: MNRAS, 308, 969 (1999)
20. G. F. Benedict et al.: astro-ph/0206213 (2002)
21. F. Pont: In, ASP Conf. Ser. 167, Harmonizing the Cosmic Distance Scale in the Post-Hipparcos Era, ed. by D Egret, A. Heck (San Francisco) p.113 (1999)
22. C. Alcock et al.: AJ, 109, 1653 (1995)
23. E. Poretti: A&A, 285, 524 (1994)
24. C. D. Laney, R. S. Stobie: MNRAS, 274, 337 (1995)
25. W. P. Gieren, P. Fouqué, M. Gómez: ApJ, 496, 17 (1998)
26. M. E. Sachkov: Astron. Lett. 28, 589 (2002)
27. N. R. Evans, D. D. Sasselov, C. I. Short: ApJ, 567, 1121 (2002)
28. T. E. Nordgen et al.: AJ, 118, 3032 (1999)
29. J. P. Beaulieu et al.: A&A, 303, 137 (1995)
30. G. P. Di Benedetto: AJ, 124, 1213 (2002)
31. M. W. Feast, P. A. Whitelock: MNRAS, 291, 693 (1997)
32. F. Pont, M. Mayor, G. Burki: A&A, 285, 415 (1994)
33. M. W. Feast, F. Pont, P. A. Whitelock: MNRAS, 298, L43 (1998)
34. M. W. Feast: MNRAS, 313, 596 (2000)
35. S. M. Andrievsky et al.: astro-ph/0208056 (2002)
36. J. B. Irwin: Mon. Not. Ast. Soc Sth. Afr., 14, 38 (1955)
37. H. Baumgardt, C. Dettbarn, R. Wielen: A&A Sup., 146, 251 (2000)
38. P. W. Merrill: The Spectra of Long-Period Variable Stars (Chicago University Press) p. 63 (1940)
39. J. W. Menzies et al.: MNRAS, 248, 642 (1991)
40. A. R. Walker, C. D. Laney: MNRAS, 224, 61, (1987)
41. F. van Leeuwen: Ph.D. Thesis. Leiden University (1983)
42. A. R. Sandage, C. Cacciari: ApJ, 350, 645 (1990)
43. M. W. Feast: In, Observational Tests of Cosmological Inflation, ed T. Shanks et al. (Kluwer, Dordrecht) p. 147 (1991)
44. F. van Leeuwen: A&A, 341, L71 (1999) and in, ASP. Conf. Ser., 198, Stellar Clusters and Associations, ed. by R. Pallavicini et al. (San Francisco, ASP), p.85 (2000)
45. N. Robichon et al.: In, ASP Conf. Ser., 198, Stellar Clusters and Associations, ed. by R. Pallavicini et al. (San Francisco, ASP) p141 (2000)
46. M. H. Pinsonneault et al.: ApJ, 504, 170 (1998)
47. V. Castellani et al.: MNRAS, 334, 193 (2002)
48. M. Grenon: In, ASP Conf. Ser., 223, Proc. 11th Cambridge Workshop on Cool Stars, ed. by R. J. Garcia Lopez et al. (San Francisco, ASP) p. 359 (1999)
49. D. Stello, P. E. Nissen: A&A, 374, 105 (2001)
50. E. Friel, A. M. Boesgaard: ApJ, 387, 480 (1990)
51. M. A. C. Perryman et al.: A&A, 331, 81 (1998)
52. D. G. Turner: PASP, 91, 642 (1979)
53. J-W. Pel: In, Cepheids: Theory and Observations, ed. by B. Madore (Cambridge, CUP) p. 1 (1985)
54. B. J. Taylor: A&A, 362, 563 (2000)
55. N. Evans: ApJ, 372, 597 (1991)
56. N. Evans: ApJ, 389, 657 (1992)
57. P. Kervella et al.: A&A, 367, 876 (2001)
58. T. E. Nordgren et al.: ApJ, 543, 972 (2000)
59. J. T. Armstrong et al.: AJ, 121, 476 (2001)
60. B. F. Lane et al.: ApJ, 573, 330 (2002)
61. C. D. Laney: In, ASP Conf. Ser., 135, A Half-Century of Stellar Pulsation Interpretations, ed. by P. A. Bradley, J. A. Guzik (San Francisco, ASP), p180 (1998)
62. J. R. Herrnstein et al.: Nature, 400, 539 (1999)
63. J. A. Newman et al.: ApJ, 553, 562 (2001)
64. D. Zaritsky, R. C. Kennicutt, J. P. Huchra: ApJ, 420, 87 (1994)
65. M. W. Feast: In, ASP Conf. Ser. 4, The Extragalactic Distance Scale, ed. by S. van den Bergh (San Francisco, ASP), p. 9 (1988)
66. C. D. Laney, R. S. Stobie: MNRAS, 222, 449 (1986)
67. C. D. Laney: In IAU Symp. 192, The Stellar Content of Local Group Galaxies, ed. by P. A. Whitelock, R. Cannon (San Francisco,ASP) p. 459 (1999)
68. R. C. Kennicutt et al.: ApJ, 498, 181 (1998)
69. C. D. Laney: In, ASP Conf. Ser. 203, The Impact of Large-Scale Surveys on Pulsating Star Research, ed. by L. Szabados, D. W. Kurtz (San Francisco, ASP), p. 199 (2000)
70. W. L. Martin, P. R. Warren, M. W. Feast: MNRAS, 188, 139 (1979)
71. R. E. Luck et al.: AJ, 115, 605 (1998)
72. A. R. Sandage: ApJ, 350, 603 (1990)
73. G. F. Benedict et al.: AJ, 123, 473 (2002)
74. E. Carretta et al.: ApJ, 533, 215 (2000)
75. M. W. Feast: In IAU Symp. 201, New Cosmological Data and the Values of the Fundamental Parameters in press
76. A. J. Longmore et al.: MNRAS, 247, 684 (1990)
77. M. W. Feast et al.: MNRAS 241, 375 (1989)
78. P. R. Wood: In, ASP Conf. Ser. 83, Astrophysical Applications of Stellar Pulsations, ed. by R. S. Stobie, P. A. Whitelock (San Francisco, ASP) p. 127 (1995)
79. P. W. Whitelock et al.: MNRAS, 267, 711 (1994)
80. M. W. Feast, P. A. Whitelock, J. W. Menzies: MNRAS, 329, L7 (2002)
81. P. A. Whitelock, M. W. Feast: MNRAS, 319, 759 (2000)
82. M. W. Feast, P. A. Whitelock: In, The Evolution of the Milky Way, ed. by F. Matteucci, F. Giovannelli (Kluwer, Dordrecht) p. 229 (2000)
83. M. W. Feast: MNRAS, 278, 11 (1996)
84. P. A. Whitelock, M. W. Feast: Mem. Soc. Ast. It. 71, 601, (2000)
85. P. A. Whitelock: In, Mass-losing Pulsating Stars and their Circumstellar Matter, ed. by Y. Nakada, M. Honma (Kluwer, Dordrecht) in press
86. V. V. Smith et al.: ApJ, 441, 735 (1995)
87. R. Kurtev et al.: A&A, 378, 449 (2001)
88. E. F. Guinan et al.: ApJ, 509, L21 (1998)
89. E. L Fitzpatrick et al: ApJ, 564, 260 (2002)
90. I. Ribas et al.: ApJ, 574, 771 (2002)
91. C. A. Nelson et al.: AJ, 119, 1205 (2000)
92. M. A. T. Groenewegen, M. Salaris: A&A, 366, 752 (2001)
93. N. Panagia: Mem. Soc. Ast. It. 69, 225 (1998)
94. R. C. Mitchell et al.: ApJ, 574, 293 (2002)
95. L. Girardi, M. Salaris: MNRAS, 323, 109 (2001)
96. D. R. Alves et al.: ApJ, 573, L51 (2002)
97. A. R. Walker et al.: ApJ, 560, L139 (2001)
98. V. Hill et al.: A&A, 364, L19 (2000)
99. G. Gilmore, R. F. G. Wyse: ApJ, 367, L55 (1991)
100. A. R. Walker: AJ, 110, 638 (1995)
101. A. E. Dolphin et al.: ApJ, 550, 554 (2001)
102. A. Udalski et al.: Act. Ast., 51, 221 (2001)
103. I. S. Usenko, V. V. Kovyukh, V. G. Klochkova: A&A, 377, 156 (2001)
104. R. E. Luck, V. V. Kovyukh, S. M. Andrievsky: A&A, 373, 589 (2001)
105. A. Udalski et al.: Act. Ast., 49, 201 (1999)
106. W. P. Gieren et al.: ApJ, 496, 17 (1998)
107. W. L. Freedman et al.: ApJ, 553, 47 (2001)
108. G. A. Tammann et al.: In, ASP Conf. Ser. —, A New Era in Cosmology, ed. by T Shanks, N. Metcalfe (San Francisco, ASP) in press, Basel preprint 129
109. A. M. Orsatti, E. I. Vega, H. G. Marraco: A&A, 380, 130 (2001)
110. P. R. Wood: In, From Miras to Planetary Nebulae, ed. by M. O, Mennessier, A. Omont (Editions Frontières, Gif-sur-Yvette), p. 67 (1990)
111. J-M. Huré: In, ASP Conf. Ser. —, Active Galactic Nuclei, ed. by S. Collin, F. Combes, I. Shlosman (San Francisco, ASP) in press, astro-ph 0210421

112. V.V. Smith et al.: astro-ph 0208417

113. W. Vlemmings: Circumstellar Maser Properties through VLBI Observations. Ph. D. Thesis, Leiden University, Leiden (2002)

114. P. A. Whitelock, F. Marang, M. W. Feast: MNRAS, 319, 728 (2000)

115. M. W. Feast: MNRAS, 293, L27 (1998)

116. A. E. Dolphin et al.: astro-ph 0211486

117. F. Bauer et al.: A&A, 348, 175 (1999)

118. A. Udalski et al.: Act. Ast. 49, 437 (1999)

119. S. van den Bergh: The Galaxies of the Local Group (Cambridge University Press, 2000)

120. B. F. Madore, W. L. Freedman: PASP, 103, 933 (1991)

121. V. V. Makarov: AJ, 125, 3299 (2002)

122. D. G. Turner, J. F. Burke: AJ, 124, 2931, (2002)