The search of the best mode of the reserve power supply consumption during the nuclear reactor’s emergency shutdown procedures in case of force majeure circumstances

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Abstract. This article deals with the problem of the control mode choice for a power supply system in case of force majeure circumstances. It is not known precisely, when a force majeure incident occurs, but the threatened period is given, when the incident is expected. It is supposed, that force majeure circumstances force nuclear reactor shutdown at the moment of threat coming. In this article the power supply system is considered, which consists of a nuclear reactor and a reserve power supply, for example, a hydroelectric pumped storage power station. The reserve power supply has limited capacity and it doesn’t undergo the threatened incident. The problem of the search of the best reserve supply time-distribution in case of force majeure circumstances is stated. The search is performed according to minimization of power loss and damage to the infrastructure. The software has been developed, which performs automatic numerical search of the approximate optimal control modes for the reserve power supply.

1. Introduction
Nuclear reactor today is one of the most widespread types of energy sources. Nuclear energy is continuously spreading all over the world, approving to be convenient for a variety of purposes.

In present, the growth of nuclear power industry allows nuclear power plants to be built in areas of unstable climatic conditions or with an increased risk of seismic activity, as well as in isolated regions, which are difficult of access and don’t have a developed power supply system infrastructure, so a nuclear reactor may be the only source of energy there [1].

Such power plants may undergo force majeure circumstances, which cause abrupt shutdown of nuclear reactors and their consequent idleness. The consumers of energy produced by the reactor may suffer damages induced by an unexpected shutdown and outage of the chief source of power [1, 2].

In this article a power supply system is considered, which consists of a nuclear reactor and reserve power supply unit. The reserve power supply unit is meant to have huge capacity and energy emission rates, comparable to the same values for a nuclear reactor. For example, such conditions may be satisfied by hydroelectric pumped storage power stations [3, 4, 5]. It is supposed, that the reserve power supply capacity is limited and does not undergo the incident. In the system analysed, the reserve supply can only be released in case of force majeure circumstances.

If such power supply system is isolated, then there is no other way of obtaining electricity or thermal energy for the infrastructure, but from the nuclear reactor. Any lack of energy production can’t be compensated. In regions, which are difficult of access, the resumption of power production
after unplanned reactor shutdown may occur not earlier than the reactor is restored or an external
source of power is connected; both scenarios may take very long time.

Thus, in isolated systems any interruption in power production may cause considerable damage on
the infrastructure. This is why methods of nuclear reactor control should be considered, which may
help to bypass an unexpected shutdown or at least minimize the damage, caused by the reactor outage.

The time when force majeure situation occurs can’t be predicted precisely. A menacing period is
considered, during which the incident is expected, given the function of probability distribution.
Force-majeure circumstances considered force nuclear reactor shutdown at the moment of the incident
coming. After the reactor is shut down, there is a period of time, when the consequences of the
accident are eliminated, and the reactor can’t be started up. After that, the reactor power may be
maximized again.

In this article the problem of the search for the best way of the reserve power supply time-
distribution is considered, with aim of the minimization of damage, caused by the nuclear reactor
power losses due to emergency control being applied and unexpected shutdown being carried out.

2. Mathematical model of the nuclear reactor
The nuclear reactor and the reserve power supply unit are considered separately, with two different
control sets. The control process for both elements of the system starts at the time, when the menacing
period has been declared.

The control process for the nuclear reactor has to comply with the limitation of xenon-135
concentration during the reactor operation [6, 7]. To simulate the situation, one-group point model of a
nuclear reactor [8] was used in the research. The process of xenon poisoning is described by the
system of differential equations [9]. The reactor is supposed to be in steady state at the initial time.

\[
\begin{align*}
\frac{di(t)}{dt} &= \sigma_X \phi(t) - \lambda_I i(t) \\
\frac{dx(t)}{dt} &= \lambda_I i(t) - \lambda_X x(t) - \sigma_X \phi(t) x(t)
\end{align*}
\]

(1)

where \( t \) – time, h,
\( i(t) \), \( x(t) \) – iodine-135 and xenon-135 concentrations, normalized by the xenon-135 concentration for
the “infinite” neutron flux, relative unit,
\( i(t) = \frac{\sigma_X}{\gamma_X} I(t) \) – iodine-135 concentration, relative unit,
\( x(t) = \frac{\sigma_X}{\gamma_X} X(t) \) – xenon-135 concentration, relative unit,
\( \gamma_X \) – xenon-135 yield, relative unit,
\( \Sigma_f \) – macroscopic fission cross-section, \( \text{sm}^{-1} \),
\( \lambda_I, \lambda_X \) – decay constants for iodine-135 and xenon-135, \( \text{s}^{-1} \),
\( \sigma_X \) – microscopic xenon-135 absorption cross-section, \( \text{sm}^2 \),
\( \phi \) – neutron flux density, \( \text{sm}^2 \text{s}^{-1} \).

Then, the system, which describes the change of state of the reactor after shutdown, appears to be (1)
with zero neutron flux (\( \phi = 0 \)):

\[
\begin{align*}
\frac{di(t)}{dt} &= -\lambda_I i(t) \\
\frac{dx(t)}{dt} &= \lambda_I i(t) - \lambda_X x(t)
\end{align*}
\]

(2)

The control of the reactor before the menace comes usually causes power production loss due to
local power decreases or high iodine concentration, which leads to outage after shutdown. During time
intervals, when the reactor power is less then maximum, the reserve power supply comes into use to
compensate the lack of the reactor energy production. It is difficult to search all variety of reactor and
reserve supply controls simultaneously, so it was decided to fix the set of possible reactor controls.

In this research, the nuclear reactor control has been fixed with respect to the search of optimal
reserve supply consumption mode. Also, the control to be applied does not depend on the moment
when menace comes – the power of the reactor at this moment is just set to zero. The control fixed has been defined by the following.

In case of force majeure circumstances, when the time of menace coming is unknown, it is consistent, that the reactor should be shut down as fast as possible. But instant shutdown at the time when menacing period was declared may cause strong poisoning of the reactor with xenon-135. So, some control should be chosen, which shuts down the reactor and satisfies the limitation on xenon-135 concentration.

The control, which complies with the above, is the solution of optimal performance problem. Such control does the optimum time transition of the state of a nuclear reactor to the xenon-safe state, in which the reactor shutdown won’t cause xenon limitation violation. On the other hand, in case when shutdown occurs earlier, than the reactor has reached xenon-safe state, xenon poisoning duration is equal or less than in case of instant shutdown. Also, such optimal performance control does not have to end with the reactor shutdown: if menace has not come yet, the reactor may produce energy according to the mode, which leaves the concentrations of iodine-135 and xenon-135 in safe bounds. The control is formed according to principles given in [10]. The form of the optimal performance control is presented in fig. 1:

Besides the optimal performance control has been chosen, two more reactor control alternatives have been considered to compare them to each other. These two controls are: instant stop at the moment of menacing period start and last-minute stop, which instantly stops the reactor at the time of menace coming.

3. Mathematical model of the reserve power supply
As the reserve power supply capacity is limited, the distribution of the supply in time should be carried out by some criterion, defining the effect of power compensation in time.

Let \( Q_R \) be the energy capacity of the reserve power supply. The capacity is sequenced in time by fractions, which sizes are defined by the chosen criterion. So, the aim of the search is to find such fractions of which fit the limitation conditions. For example, the timeline of the optimal performance control may be split into 4 time intervals. Then, the conditions of the search are:

\[
\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4: 0 \leq \epsilon_i \leq 1, 0 \leq \sum_{i=1}^{4} \epsilon_i \leq 1
\]

According to the \( \epsilon_i \) fractions, the control function \( u_R(t) \) for the reserve power supply is formed. The control function is proportional to the power of the supply. It is formed on principle of smoothing the total power graph within a time interval to the uniform state.

The lack of total power at the moment \( t \) is proportional to the following: \( \Delta u(t) = u_{\text{max}} - u(t) - u_R(t) \).

The cost functional is defined by some parameter, which depends on the state of the system and the control function. For example, the power generation loss before the reactor shutdown, as well as the reactivity margin or the concentration of xenon-135 in the reactor core at the time of the shutdown, etc. may be used as such parameters.
Penalty function was introduced, which defines the value of damage, caused by the lack of power supply system energy production due to decreased power of the reactor. The penalty is defined by the difference between designed reactor capacity and current total power, produced by the power supply system.

So, in this article, the optimization criterion is represented by the functional to be minimized. The functional describes total energy production loss:

\[ \Delta Q(u_R) = \int_{t_0}^{t_1} C(t) \Delta u(t) \, dt \]  

(4)

where \( C(t) \) – penalty function, for example: \( C(t) = e^{\Delta u(t)} \).

The form of penalty function reflects the situation, when the lack of power production causes more damage, than

The force-majeure incident during the emergency period may occur at any moment of time \( \tau \), which is random. The probability is defined by the probability distribution \( f(\tau) \). Given \( f(\tau) \) and boundaries of the emergency period \([t_0; T]\), mathematical expectation of total power production loss is calculated as follows:

\[ M[\Delta Q] = \int_{t_0}^{T} f(\tau) \int_{t_0}^{\tau} C(t) \Delta u(t) \, dt \, d\tau = \int_{t_0}^{T} f(\tau) \Delta Q(\tau, u_R) \, d\tau \]  

(5)

In this article \( f(\tau) \) given is the uniform probability distribution. In this case, practically, the average value of losses is calculated:

\[ \overline{\Delta Q} = \frac{1}{T-t_0} \int_{t_0}^{T} \Delta Q(\tau, u_R) \, d\tau \]  

(6)

The software has been developed to perform automatic numerical search of reserve power supply optimal time-distribution approximations.

4. Simulation and results

The algorithm implemented accepts the steady state parameters of the reactor, the xenon-135 concentration limitation value, the emergency period duration and the capacity of the emergency power supply as input parameters, and calculates numerically the control modes for the nuclear reactor and reserve power supply unit, average power supply system losses and relative loss values.

The system power loss for three different controls applied has been calculated. These calculations have been performed for different values of the limitation on xenon-135 concentration (see fig. 2).

Figure 2. The dependence of power production losses for different types of reactor control from the value of xenon-135 limitation

In the figure, it can be noticed, that the best result has been derived for the optimal speed (optimal performance) reactor control. It can be explained: instant shutdown of the reactor leads to long-time xenon poisoning, which means practically full-power loss for the whole time until real menace comes;
as for the last-minute shutdown, the control is more profitable due to maximum power production for
the whole time until menace comes, but after the reactor is stopped, xenon poisoning may end even
later, than in previous case.

The minimal losses almost for each value of xenon limitation are shown in graph for the optimal
performance control. This means, that power production losses reduction with the reserve power
supply should be performed for this type of nuclear reactor control.

The time-distribution of the reserve supply is retrieved in the following form:

![Figure 3. The distribution of reserve power supply in time by fractions, depending on the moment of menace coming](image)

Actually, optimal modes strongly depend on the specific system chosen, because, for example,
practical power supply capacity may vary during day and night workload changes and penalty
functions are defined by the infrastructure features.

The algorithms implemented and results derived may be used for planning control modes for power
supply systems during menacing period.

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