AN ORIENTIFOLD OF THE SOLITONIC FIVEBRANE

Stefan Förste, Debasish Ghoshal and Sudhakar Panda

Sektion Physik, Universität München
Theresienstraße 37, 80333 München, Germany

Abstract

We study an orientifold of the solitonic fivebrane of type II string theory. The consideration is restricted to a space-time domain which can be described by an exact conformal field theory. There are no IR divergent contributions to tadpole diagrams and thus no consistency conditions arise. However, extrapolating the results to spatial infinity leads to consistency conditions implying that there are four (physical) D-6-branes sitting at each of the two orientifold fixed planes.

1Permanent Address: Mehta Research Institute of Mathematics & Mathematical Physics, Chhatnag Road, Jhusi, Allahabad 211506, India
2E-mail: Stefan.Foerste@physik.uni-muenchen.de
Debashis.Ghoshal@physik.uni-muenchen.de
panda@mri.ernet.in
1 Introduction

One of the first solitonic solutions to be found in superstring theory is a fivebrane that carries the magnetic charge of the rank two antisymmetric gauge potential \([1]\). This is dual to the elementary string itself which is electrically charged. The fivebrane configuration is a solution of the equations of motion derived from the low energy effective action. Although it is constructed as a classical solution, the fivebrane breaks half of the spacetime supersymmetries and hence its energy and charge saturate the BPS bound. This makes the fivebrane stable. Since the gauge potential arises in the Neveu-Schwarz sector, this solution is called an NS fivebrane.

In the last couple of years there has been a fair amount of progress in our understanding of the non-perturbative aspect of superstrings (see Ref.\([2]\) for some recent reviews). One finds extended solitonic objects, \(p\)-branes, of all dimensions that couple to the variety of Ramond-Ramond (RR) gauge potentials in type II strings\([3]\). These \(p\)-branes are topological defects arising from open strings that have Dirichlet boundary condition in \((9 - p)\) directions and hence are tied to a \(p\) dimensional subspace at all times\([4]\)\([5]\). The end-points of the open strings thus sweep the worldvolume of the D(irichlet)-\(p\)-branes. Various properties of D-branes can therefore be computed using perturbation theory of open strings with appropriate modification of boundary conditions\([6]\).

Another kind of topological defect arises when one generalizes space-time orbifold by combining it with the world-sheet parity of type II strings\([7]\)\([5]\). The orientifold hyperplane obtained this way is not a dynamical object (at least in perturbation theory of open strings) but nevertheless carries charge of an appropriate RR field. Near the orientifold, unoriented string diagrams contribute. Indeed type I string itself can be obtained by quotienting the ten dimensional type IIB theory by the world-sheet parity. More general examples of this type provide new string vacua in lower dimensions\([8]\)\([9]\).

In this paper we would like to study an orientifold of type II string in the background of an NS fivebrane. The fivebrane modifies the flat geometry of the space transverse to it. The orientifold construction therefore needs to be carried out in a curved geometry. What makes this tractable is that there exists an exact description of the NS fivebrane as a superconformal field theory in some region of spacetime. We analyze the spectrum and compute the one-loop tadpole and look for possible divergence as a signal of inconsistency. Such divergences appear due to massless fields. In a consistent theory the total divergence from all diagrams must cancel to have conservation of charge. This requires to have D-branes with specific properties. Surprisingly we will not encounter any real divergence. This would imply that an orientifold of the NS fivebrane is always consistent. However subtleties arise
due to the curved background and nonconstant dilaton. We will discuss this issue in more
detail later.

One motivation to look at this problem came from the ‘derivation’ of the Seiberg-duality
in $N = 1$ supersymmetric gauge theories using simple manipulations with branes. The
duality for orthogonal and symplectic gauge groups require one to have NS fivebranes
and orientifolds simultaneously. In any case, the problem is interesting in its own
right as one applies the orientifold construction to a non-trivial conformal field theory. As
far as we know the orientifold technique has been applied in detail to flat spacetime only.

2 Geometry and Conformal Field Theory of the NS Fivebrane

The worldvolume of the fivebrane spans six space-time dimensions and is transverse to the re-
remaining four. Let the fivebrane lie along the coordinates $\{x^0, x^1, \ldots, x^5\}$, and $\{x^6, x^7, x^8, x^9\}$
be the transverse space. Let us recall the explicit solution for the metric, dilaton and the
antisymmetric tensor field:

$$
\begin{align*}
\eta_{\mu \nu} dx^\mu dx^\nu + e^{-2\varphi} \left( dr^2 + r^2 d\Omega_3^2 \right) \\
e^{-2\varphi} = e^{-2\varphi_0} \left( 1 + \frac{k}{r^2} \right) \\
H = -2kd\Omega_3
\end{align*}
$$

The geometry of the transverse space is modified by the presence of the brane. It is still
asymptotically flat, but as we approach the ‘position’ of the brane, distances stretch out
resembling a semi-wormhole throat. This part of space is cylindrical with a three-sphere $S^3$
as its base (see figure). The string coupling gets stronger along the length of the cylinder
as the dilaton field is linear in this direction, and finally diverges at the core of the brane.

In the region of the wormhole throat, the fivebrane is exactly described by a supercon-
formal field theory. The SCFT relevant for the transverse coordinates is an $N = 4$ supers-
symmetric $SU(2)$ WZW model tensored with a free scalar field with a background charge
(a Feigin-Fuchs model). The level of the WZW model is the charge of the NS fivebrane (upto a
factor of $\alpha'$), which is also the size of the $S^3$. The total central charge is exactly six (due to
supersymmetry), and this determines the background charge of the Feigin-Fuchs field. The
fermions are essentially free. For more details, see Ref.

The SCFT in the longitudinal directions is a theory of free bosons and fermions with
c = 9. The bosonic part of the $SU(2)_k$ WZW action is, (see Ref. for an excellent text on
Figure 1: The geometry of the NS-5-brane orientifold.

CFT in general and WZW models in particular,

\[ S_{WZW} = \frac{k}{8\pi} \int_{\mathcal{B}} \text{Tr}(\partial g^{-1} \bar{\partial} g) - \frac{ik}{12\pi} \int_{\mathcal{B}} \text{Tr}(g^{-1} dg)^{3} \]  \hspace{1cm} (2)

where \( \mathcal{B} \) is a three manifold with the string worldsheet as its boundary. As is clear from the solution \( \text{(1)} \) the Wess-Zumino term in action \( \text{(2)} \) describes the coupling of the fivebrane to the NS antisymmetric tensor.

There are two conserved currents

\[ J(z) = (\partial g)g^{-1} \text{ and } \bar{J}(\bar{z}) = g^{-1}\bar{\partial}g \]  \hspace{1cm} (3)

corresponding to the Kac-Moody gauge symmetries

\[ g(z, \bar{z}) \rightarrow \Lambda(z)g(z, \bar{z})\Lambda^{-1}(\bar{z}) \]  \hspace{1cm} (4)

The central charge is given in terms of the level \( c_{WZW} = 3k/(k + 2) \). The Feigin-Fuchs part is a free boson with a background charge \( Q \) induced by the linear dilaton of the fivebrane. The central charge shifts to \( c_{FF} = 1 + 3Q^{2} \) with \( Q = \sqrt{2/(k + 2)} \) (the \( \alpha' \sim 1/k \) corrections have been taken into account here). The total bosonic contributions from the WZW and FF part add up to four, equal to that of four flat coordinates.
This model has a discrete symmetry, whose action is the worldsheet parity transformation $\Omega$ combined with a ‘reflection’ $R$ taking the group element to its inverse:

$$\Omega R : \begin{align*}
g(z, z) &\to g^{-1}(\bar{z}, z) \\
X(z, \bar{z}) &\to X(\bar{z}, z)
\end{align*} \tag{5}$$

where $X(z, \bar{z})$ stands for the other bosons. The first term in the WZW action (2) is invariant separately under the action of $\Omega$ and $R$ whereas the Wess-Zumino term is invariant only under the combined action. So (part of) the NS antisymmetric tensor survives the $\Omega R$ projection. Notice that under this symmetry the Kac-Moody currents are interchanged

$$J(z) \leftrightarrow -\bar{J}(\bar{z}) \tag{6}$$

which is consistent with Eq.(4). The orientifold of the fivebrane we want to study is obtained by gauging the symmetry $\Omega R$ in Eq.(5).

The transformation (5) leaves two fixed points on the $SU(2)$ group manifold $S^3$. Hence the resulting orientifold plane is extended along all directions parallel to the fivebrane and one direction transverse to it, namely the Feigin-Fuchs coordinate. We therefore obtain an orientifold 6-plane. This carries charge of the RR 7-form gauge potential, which is part of the spectrum of type IIA string. This charge can be neutralized by adding D-6-branes parallel to the orientifold plane. Taking some of the directions $(x_1, \cdots, x_5)$ to be compact and performing T-duality on them, one can go back and forth from IIA to IIB theory.

As can be seen from (5) the two fixed points are in the centre of $SU(2) g = \pm 1$. There is another symmetry in the WZW theory $[15][16]$, namely $\Omega R' : g(z, \bar{z}) \to -g^{-1}(\bar{z}, z)$. This results in a fixed $S^2$ surface at the equator of $S^3$. This would lead to orientifold 8-surfaces and D-8-branes wound on the $S^2$, but we will not discuss this case any farther.

### 3 Massless Spectrum of the Fivebrane Orientifold

Before going into the details of the spectrum, let us point out one subtlety. As described, for example in $[17]$, the presence of the linear dilaton shifts the mass square by a term proportional to the square of the background charge. This can be understood by dividing the vertex operator by the string coupling ‘constant’. Down the throat of the wormhole the string coupling increases, and particles which are massless at asymptotic regions acquire an effective mass. The notion of the massless spectrum to be discussed presently is understood with this caveat.

\(^{3}\text{This is unlike the type I string obtained by an } \Omega \text{ projection of type IIB theory where the NS antisymmetric tensor gets eliminated.}\)
The fivebrane breaks the ten dimensional little group $SO(8)$ to the six dimensional little group $SO(4) \simeq SU(2) \times SU(2) \times SU(2)$ of the WZW part of the transverse space. We will label the states by their six dimensional Lorentz quantum numbers. The massless spectrum of type IIA theory is constructed out of an $8_v$ from the left and right NS-sector, an $8_s$ from the left R-sector, and an $8_c$ from the right R-sector. These $SO(8)$ representations decompose as follows

\[
8_v \rightarrow (2, 2) + 4(1, 1) \\
8_s \rightarrow 2(2, 1) + 2(1, 2) \\
8_c \rightarrow 2(2, 1) + 2(1, 2).
\]

Taking $\Omega R$ invariant combinations from the left and right sectors, we arrive at the bosonic spectrum

\[
\text{NS-NS : } (3, 3) + 4(2, 2) + 11(1, 1) \\
\text{R-R : } (3, 1) + (1, 3) + 4(2, 2) + 6(1, 1).
\]

A part of the NS antisymmetric tensor field survives as vectors and scalars. Together with the fermions these states combine into the gravity multiplet and four vector multiplets of $N = 4$ non-chiral supergravity in six dimensions.

As stated in Ref.\[8\] $\Omega R$ does not give rise to twisted sector states but may require the inclusion of open strings in order to obtain a consistent theory. These open strings can have NN, DN, ND or DD boundary conditions. The oscillator parts in the mode expansion of a free boson and fermion are\[6\]

\[
\partial X(\sigma, 0) = \sum_m \alpha_m \left( e^{im\sigma} \pm e^{-im\sigma} \right) \\
\psi(\sigma, 0) = \sum_r \psi_r e^{ir\sigma} \\
\tilde{\psi}(\sigma, 0) = \pm \sum_r \psi_r e^{-ir\sigma}
\]

where the $\pm$ refer to NN or DD boundary conditions for integer moded $\alpha$, and to ND or DN for half-integrally moded ones. Under worldsheet parity $\Omega : \sigma \rightarrow \pi - \sigma$, the oscillators transform as

\[
\alpha_m \rightarrow \pm e^{i\pi m} \alpha_m \quad \text{and} \quad \psi_r \rightarrow \pm e^{i\pi r} \psi_r.
\]

In the WZW sector the role of the $U(1)$ current is played by the Kac-Moody currents Eq.(4). The $R$ action gives an additional sign in the bosons and fermions of the WZW sector. Here we should point out that our treatment for the WZW sector of open strings is purely algebraic.
While it would be nice to have further insight the algebraic viewpoint suffices for our present purpose.

In the open string spectrum, one gets a vector

\[ \psi^\mu \left| 0, \alpha \beta \right\rangle \lambda_{\beta \alpha}, \quad \lambda = \mp \gamma_{\Omega R} \gamma_{\Omega R}^{-1} \]  

(14)

where the \( \mp \) refers to DD or NN boundary conditions respectively. Additionally one gets scalars from the WZW and FF fermions

\[ \psi_{WZW} \left| 0, \alpha \beta \right\rangle \lambda_{\beta \alpha}, \quad \lambda = \pm \gamma_{\Omega R} \gamma_{\Omega R}^{-1} \]  

(15)

\[ \psi_{FF} \left| 0, \alpha \beta \right\rangle \lambda_{\beta \alpha}, \quad \lambda = \mp \gamma_{\Omega R} \gamma_{\Omega R}^{-1} \]  

(16)

The restriction on the Chan-Paton matrices arise for open strings with end points stuck at the orientifold fixed plane. Otherwise \( \Omega R \) maps them to their images and puts no further condition. To be able to form a supermultiplet one needs to have the same projections for the four scalars and the vector. For consistency therefore we choose DD conditions in the WZW sector and NN for the rest. This results in D-6-branes parallel to the orientifold.

4 One Loop Calculation

Now we come to the computation of the one loop amplitudes. The general framework is discussed at length in Ref.[8] which we will follow closely. We will only point out special features of our model and the reader is referred to [8] for additional details. Let us begin by recalling the expression for the one loop amplitude

\[ \int \frac{dt}{2t} \left( \text{Tr}_C \left( (-)^F P e^{-2\pi t (L_0 + \tilde{L}_0)} \right) + \text{Tr}_O \left( (-)^F P e^{-2\pi t L_0} \right) \right) \]  

(17)

where \( P = \frac{1+\Omega R}{2} \frac{1+(-)^F}{2} \) is the orientifold and GSO projections and \( F \) is the spacetime fermion number. The traces \( \text{Tr}_O \) and \( \text{Tr}_C \) refer to worldsheets with or without boundaries respectively. Diagrams with an \( \Omega \) insertion correspond to nonorientable surfaces, the Möbius strip and the Klein bottle; and those without are the cylinder and the torus respectively. Since modular invariance of the torus removes the region of \( t \) integration that would lead to IR divergence, we need to consider the other three diagrams only.

The ingredients needed for the computation are the characters of the free bosonic and fermionic fields and the bosonic WZW model. The nontrivial part is the character of the \( SU(2)_k \) WZW model [14]

\[ \chi_j(q) = \frac{q^{(2j+1)^2/4(k+2)}}{\eta^3(q)} \sum_{n=-\infty}^{\infty} (2j + 1 + 2n(k+2)) q^{n(2j+1+(k+2)n)} \]  

(18)
where \((2j + 1)\) is the dimension of the spin \(j\) representation of \(SU(2)\). In the WZW model \(j\) takes values from 0 to \(k/2\). When taking the trace in Eq. (17), one has to sum over the representations. This is the analog of the sum over momenta and windings in the compact coordinates in Ref. [8].

The \(\Omega R\) projection in the Klein bottle implies that we only keep excitations that are identical in the left and the right sectors. On the \(SU(2)\) quantum numbers this puts the restriction \(\tilde{\jmath} = j, \tilde{m} = -m\) relating the ground states of the left and the right sector\(^4\). This results in the following expression

\[
(1 - 1) \frac{V_6 L}{8} \int_0^\infty \frac{dt}{t^4 \sqrt{t}} f_3^8(e^{-2\pi t}) \sum_{j=0}^{k/2} \chi_j(e^{4\pi t})
\]

where \(V_6\) and \(L\) are the regularized volume of the noncompact space and the Feigin-Fuchs direction. The functions \(f_i(q)\) are

\[
f_1(q) = q^{1/12} \prod_{n=1}^{\infty} (1 - q^{2n}), \quad f_2(q) = \sqrt{2} q^{1/12} \prod_{n=1}^{\infty} (1 + q^{2n}),
\]

\[
f_3(q) = q^{-1/24} \prod_{n=1}^{\infty} (1 + q^{2n-1}), \quad f_4(q) = q^{-1/24} \prod_{n=1}^{\infty} (1 - q^{2n-1}).
\]

(The function \(\eta(q)\) in (18) is the same as \(f_1(\sqrt{q})\).) These functions have the modular property

\[
f_1(e^{-\pi t}) = \sqrt{t} f_1(e^{-\pi t}), \quad f_2(e^{-\pi t}) = f_4(e^{-\pi t}), \quad f_3(e^{-\pi t}) = f_3(e^{-\pi t}).
\]

The corresponding expression for the Möbius strip is

\[
- (1 - 1) \frac{V_6 L}{64\sqrt{2}} \sum_{I=1,2} \text{Tr}(\gamma_{I,\Omega R}^{-1} \gamma_{I,\Omega R}^T) \int_0^\infty \frac{dt}{t^4 \sqrt{t}} f_3^8(e^{-\pi t + \frac{i\pi}{2}}) \sum_{j=0}^{k/2} e^{-i\pi(2j+1)^2 / 4(k+2)} \chi_j(e^{2\pi t + i\pi})
\]

(Notice that in the open string sector the zero modes \(J_0\) have \(\Omega R\) eigenvalue +1 due to DD boundary conditions.) The shift in the argument of the characters comes from the fact that the \(\Omega R\) insertion is equivalent to an \(e^{i\pi L_0}\) insertion. Finally the cylinder diagram contributes

\[
(1 - 1) \frac{V_6 L}{64\sqrt{2}} \sum_{I=1,2} \langle \text{Tr} \gamma_{I,1} \rangle^2 \int_0^\infty \frac{dt}{t^4 \sqrt{t}} f_3^4(e^{-\pi t}) \sum_{j=0}^{k/2} \chi_j(e^{-2\pi t})
\]

We only consider D-6-branes that are stuck to the orientifold planes.

\(^4\)For the other symmetry \(\Omega R'\), the analogous condition is \(\tilde{j} = j, \tilde{m} = m\).
To extract the IR behaviour (the \( t \to 0 \) limit), the following identities pertaining to the characters of the WZW model are useful

\[
e^{-\frac{\pi (2j+1)^2 t}{(k+2)}} \sum_{n=-\infty}^{\infty} \left( 2j + 1 + 2(k + 2)n \right) e^{-4\pi tn(2j+1+(k+2)n)}
= \frac{1}{\sqrt{4t^3(k+2)}} \sum_{n=1}^{\infty} ne^{-\frac{\pi n^2}{4t(k+2)}} \sin \left( \frac{\pi (2j + 1)n}{(k + 2)} \right)
\]

(25)

\[
e^{-\frac{\pi (2j+1)^2 t}{2(k+2)}} \sum_{n=-\infty}^{\infty} \left( 2j + 1 + 2(k + 2)n \right) e^{-2\pi (t+\frac{1}{2})n(2j+1+(k+2)n)}
= \frac{1}{\sqrt{8t^3(k+2)}} \sum_{n=1}^{\infty} ne^{-\frac{\pi n^2}{8t(k+2)}} \sin \left( \frac{\pi (2j + 1)n}{2(k + 2)} \right) \left( 1 - e^{i\pi(k+2j+n)} \right)
\]

(26)

The above formulas are easily obtained by Poisson resummation.

The length of the Klein bottle, Möbius strip and cylinder are related to the loop parameter \( t \) by \( t = \frac{1}{4T}, \frac{1}{8T}, \frac{1}{2T} \) respectively\cite{8}. Taking this into account and using Eqs.(22), (25) and (26), we obtain the leading IR contribution

\[
\int_{l \to \infty} dl \left( 64 - 8 \sum_{I=1,2} \text{Tr}(\gamma_I^{-1} \Omega_R \gamma_I^T) + \frac{1}{2} \sum_{I=1,2} (\text{Tr} \gamma_I) \right) e^{-\pi l/3}.
\]

(27)

In the above we have taken the value \( k = 1 \) corresponding to the singly charged NS fivebrane. The expression (27) is clearly convergent as \( l \to \infty \), but the argument of the exponential is proportional to the shift in mass due to the linear dilaton. In the asymptotic region the dilaton approaches a constant value and there are massless particles in the spectrum. The one loop amplitude is then truly divergent. We assume that the results of the calculation carried out in the region of the wormhole throat can be extrapolated to the asymptotically flat region.

If the branes are distributed equally at the two fixed points, the ‘divergence’ can be cancelled for four physical D-6-branes with Chan-Paton factors transforming in \( SO(8) \) at each orientifold plane. If D-6-branes away from the fixed planes are allowed, then they contribute additional divergence that cannot be cancelled. This fits nicely with the fact that the \( \Omega R \) projection does not put any constraint on such branes since it maps them to their images. The final result is that we get four (physical) D-6-branes stuck at each of the two orientifold 6-planes leading to an \( SO(8) \times SO(8) \) gauge symmetry.

5 Summary

In this paper we have studied an orientifold of the conformal field theory of the NS fivebrane. Unlike the orientifolds of flat spacetime, we do not encounter any real IR divergence in
the one loop tadpole diagrams. However in the curved background of the fivebrane the asymptotically massless particles acquire an effective mass induced by the linear dilaton field. Extrapolating the one loop results of the CFT to spatial infinity, consistency conditions arise. This requires that four D-6-branes sit at each of the two orientifold 6-planes resulting in an $SO(8) \times SO(8)$ gauge symmetry in the effective six dimensional theory. Perhaps we should emphasize that this is the first attempt at carrying out the orientifold construction explicitly in a curved background. It would be desirable to understand some of the subtleties that arise in dealing with nontrivial background better. Especially the role of the space dependent dilaton field deserves further study.

Acknowledgement: We are grateful to Stefan Theisen for extensive discussion. The work of S.F. is supported by GIF, the German Israeli Foundation for Scientific Research. The research of D.G. is supported by the Alexander von Humboldt Foundation. S.P. is grateful to the Physics Department of Universität München, and particularly Stefan Theisen for hospitality during the course of this work. The work presented here is supported in part by TMR program ERBFMX-CT96-0045.

References

[1] C. Callan, J. Harvey and A. Strominger, *Worldsheet approach to heterotic instantons and solitons*, Nucl. Phys. **B359** (1991) 611;
C. Callan, J. Harvey and A. Strominger, *Supersymmetric string solitons*, [hep-th/9112030](http://arxiv.org/abs/hep-th/9112030).

[2] J. Schwarz, *Lectures on superstrings and M theory dualities*, [hep-th/9607201](http://arxiv.org/abs/hep-th/9607201);
A. Sen, *Unification of string dualities*, [hep-th/9609176](http://arxiv.org/abs/hep-th/9609176);
P. Townsend, *Four lectures on M-theory*, [hep-th/9612121](http://arxiv.org/abs/hep-th/9612121);
S. Förste and J. Louis, *Duality in string theory*, [hep-th/9612192](http://arxiv.org/abs/hep-th/9612192).

[3] J. Polchinski, *Dirichlet branes and Ramond-Ramond charges*, Phys. Rev. Lett. **75** (1995) 4724, [hep-th/9510017](http://arxiv.org/abs/hep-th/9510017).

[4] P. Hořava, *Background duality of open string models*, Phys. Lett. **B231** (1989) 251.

[5] J. Dai, R. Leigh and J. Polchinski, *New connections between string theories*, Mod. Phys. Lett. **A4** (1989) 2073.
[6] J. Polchinski, S. Chaudhury and C. Johnson, Notes on D-branes, hep-th/9602052.
C. Bachas, (Half) a lecture on D-branes, hep-th/9701019.
J. Polchinski, TASI lectures on D-branes, hep-th/9611050.

[7] A. Sagnotti, Open strings and their symmetry groups, in Proceedings of the 1987 Cargèse Summer Institute, Pergamon Press (1988);
J. Govaerts, Quantum consistency of open string theories, Phys. Lett. B220 (1989) 77;
P. Hořava, Strings on worldsheet orbifolds, Nucl. Phys. B327 (1989) 461.

[8] E. Gimon and J. Polchinski, Consistency conditions for orientifolds and D-manifolds, Phys. Rev. D54 (1996) 1667, hep-th/9601038.

[9] A. Dabholkar and J. Park, An orientifold of type IIB theory on K3, Nucl. Phys. B472 (1996) 207, hep-th/9602030.
E. Gimon and C. Johnson, K3 orientifolds, Nucl. Phys. B477 (1996) 715, hep-th/9604129.
M. Berkooz and R. Leigh, A D = 4 N = 1 orbifold of type I strings, Nucl. Phys. B483 (1997) 187, hep-th/9605049.

[10] K. Intriligator and N. Seiberg, Lectures on supersymmetric gauge theories and electric-magnetic duality, Nucl. Phys. Proc. Suppl. 45BC (1996) 1, hep-th/9509066.

[11] S. Elitzur, A. Giveon and D. Kutasov, Branes and N = 1 duality in string theory, hep-th/9702014.

[12] N. Evans, C. Johnson and A. Shapere, Orientifolds, branes, and duality of 4D gauge theories, hep-th/9703210.

[13] S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and A. Schwimmer, Brane dynamics and N = 1 supersymmetric gauge theory, hep-th/9704104.

[14] P. Di Francesco, P. Mathieu and D. Sénéchal, Conformal field theory, Springer-Verlag (1997).

[15] P. Hořava, Two dimensional stringy black holes with one asymptotically flat domain, Phys. Lett. B289 (1991) 293, hep-th/9203031.

[16] C. Johnson, On the orientifolding of type II NS-fivebranes, hep-th/9705148.

[17] P. Di Francesco and D. Kutasov, Worldsheets and spacetime physics in two-dimensional (super)string theory, Nucl. Phys. B375 (1992) 119, hep-th/9109005.