Sensitivity and uncertainty analyses of retention performance of green roofs considering aleatory and epistemic uncertainties

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Abstract. A methodological framework is presented to integrate epistemic uncertainties into an aleatory-based probabilistic green roof model. In doing so, the sensitivity and uncertainty of the mean and standard deviation of retention ratio are assessed. Regression-based global sensitivity analysis showed that the first-order effect of model parameters plays the dominant role over the higher-order effect in contributing the overall uncertainty of statistical moments of retention ratio from the probabilistic green roof model used. Inclusion of epistemic uncertainties allows comprehensive evaluation of uncertainty associated with the performance of a green roof system, which further facilitates risk-based design of green roof systems.

1. Introduction

The use of green roofs (GRs) is a well-accepted practice in source control of runoff quantity and quality in sustainable urban stormwater management [1]. Comparing with general impervious rooftops, GRs not only can retain precipitation rendering runoff volume reduction, but also attenuate runoff peak discharged [2].

Hydrologic models of green roof (GR) performance vary from simple lumped-parameter models to complex physical-based distributed parameter models. For retention evaluation, a simple lumped hydrologic model considering event-based water balance would generally be sufficient. Notice that, retention performance of a GR system is affected by various uncertainties which can be broadly categorized into aleatory and epistemic types. The former is due to inherent randomness of rainfall characteristics (e.g., rainfall depth, duration, and inter-event dry period). The latter, epistemic uncertainties, arise from knowledge deficiency about rainfall-runoff transformation process as well as lack of complete characterization of soil-plant-climatic properties. Thus, the performance of a GR system is subject to uncertainty. It would be desirable to quantify the uncertainty features of the GR performance so that a more comprehensive assessment or design can be made. Elaborations of the importance of incorporating both aleatory and epistemic uncertainties for reliability evaluation of engineering systems can be found elsewhere [3,4]. Examples considering aleatory and epistemic uncertainties can be found in seismic modelling [5] and hydrosystem design capacity determination [6].

Reviewing the literature of probabilistic models for GR evaluation, great majority of the work
considers aleatory uncertainty due to inherent randomness of rainfall characteristics [7-9]. This study presents a methodological framework to further incorporate epistemic uncertainties in an aleatory-based probabilistic GR model for assessing the statistical properties of GR retention performance. The approach provides a simple and direct assessment of the uncertainty features of the system performance involved aleatory and epistemic uncertainties without intensive simulation. In doing so, not only the overall effect of uncertain factors in GR systems can be accounted for, but also the relative importance of uncertain factors in terms of their contributions to the overall uncertainty of model outputs can be assessed and identified.

2. Probabilistic evaluation of GR performance

To evaluate the hydrologic performance of a GR system, a simple lumped-parameter hydrologic model is adopted herein [9]. The water balance for each rainfall and dry period cycle can be expressed as:

\[ R_c = S_i + S_c + (\theta_f - \theta_i)h \] (1)

in which \( R_c \) = retention capacity; \( S_i \) = interception loss; \( S_c \) = storage capacity of storage layer; \( \theta_f \) = field capacity; \( \theta_i \) = initial soil moisture content; and \( h \) = growing medium depth. Assuming that the soil moisture in the growing medium is maintained above the wilting point of the plants, \( \theta_w \), the maximum retention capacity for the GR system can be obtained as:

\[ R_{c,max} = S_i + S_c + (\theta_f - \theta_w)h \] (2)

By incorporating rainfall event depth, inter-event dry period, and evapotranspiration into equations (1) and (2), the expression for GR runoff can be derived as [16]:

\[ v_{rg} = \begin{cases} 0 & , \quad \left[ v \leq R_{c,max}, b > \frac{W_i}{E_a} \right] \text{ or } \left[ v > R_{c,max} - W_i + E_a b, b \leq \frac{W_i}{E_a} \right] \\ v + W_i - R_{c,max} - E_a b, & \left[ v > R_{c,max} - W_i + E_a b, b > \frac{W_i}{E_a} \right] \\ v - R_{c,max} & , \quad \left[ v > R_{c,max}, b > \frac{W_i}{E_a} \right] \end{cases} \] (3)

in which \( v \) = rainfall depth; \( b \) = inter-event dry period; \( v_{rg} \) = GR runoff volume; \( E_a \) = evapotranspiration rate; and \( W_i \) = evapotranspirable water content in soil medium at the beginning of the dry period.

2.1. Considering aleatory uncertainty

In this study the indicator for hydrologic performance of a GR system is the rainfall retention ratio (\( R_r \)) defined as:

\[ R_r = \frac{V - V_{rg}}{V} = 1 - \frac{V_{rg}}{V} \] (4)

Under the conditions that rainfall and inter-event dry period are independent random variables and each has an exponential distribution:

Rainfall volume, \( V \): \( f_V(v) = \zeta e^{-\zeta v}, \quad v \geq 0 \) (5)

Inter-event dry time, \( B \): \( f_B(b) = \psi e^{-\psi b}, \quad b \geq 0 \) (6)

where \( \zeta \) and \( \psi \) are, respectively, the distribution parameters for the rainfall depth, \( V \), and inter-event dry period, \( B \), Tung and You [10] recently extended the analysis of Zhang and Guo [9] to derive the CDF and PDF of retention ratio, \( R_r \), as:
\[ F_{R_r}(\eta') = P \ e^{-\left(\frac{\psi W_i + \xi R_{c,\text{max}}}{\eta} \right)} + \left( \frac{\psi}{\psi + \xi E_a \eta} \right) \left( e^{\frac{\xi(R_{c,\text{max}} - W_i)}{\eta} - \frac{\xi E_a}{\eta} - e^{-\left(\frac{\xi R_{c,\text{max}}}{\eta} - \frac{\psi W_i}{\eta} \right)} \right) \]  

\[ f_{R_r}(\eta') = \begin{cases} 
\frac{\zeta e^{\frac{-\xi R_{c,\text{max}}}{\eta}}}{\eta^2} \left( e^{-\frac{\psi W_i}{\eta}} \right)^2 & \frac{R_{c,\text{max}} - \psi E_a - \zeta E_a \eta}{\psi + \zeta E_a \eta} \left( \psi + \zeta E_a \eta \right)^{2} + 1, \quad 0 \leq \eta' < 1 \\
Pr(V_{rg} = 0) & \eta' = 1 
\end{cases} \]  

in which \( \eta' \) is dummy variable for the \( R_r \). According to the PDF of \( R_r \), its statistical moments of any order \( s \) can be analytically derived by:

\[ E(R_r^s) = \int_0^1 (\eta')^s f_{R_r}(\eta') \ d\eta', \ s = 1, 2, 3, ... \]  

Mean retention ratio, \( E(R_r) \), can be obtained from equation (9) with \( s=1 \) and the variance, \( Var(R_r) \), can be calculated as:

\[ Var(R_r) = E(R_r^2) - E^2(R_r) \]  

Note that, by considering only aleatory uncertainty in rainfall characteristics, a unique functional relation can be established between the distributional properties of \( R_r \) and growing medium depth. However, the situation no longer is valid when one considers various epistemic uncertainties in the parameter values in defining the functional relation. In other words, parameter values describing the characteristics of climate, soils, plants \( (\theta_f, \theta_w, E_a, S_t) \), and available soil moisture content for evapotranspiration \( (W_i) \) in each rainfall-dry period cycle are not exactly known. Hence, the statistical properties for \( R_r \) presented above are in fact subject to some degrees of uncertainty.

2.2. Incorporating epistemic uncertainty

In GR system performance evaluation, epistemic uncertainties are attributed to the lack of perfect knowledge about rainfall-runoff transformation process as well as the inability to accurately characterize the soil, plant and climatic properties involved in the GR system. As mentioned above, the five parameters in the GR model, i.e., equations (1)-(3), that are subjected to epistemic uncertainties are \( E_a, \theta_f, \theta_w, S_t \) and \( c \).

Field capacity, \( \theta_f \), is the soil properties indicating soil’s capability to retain water against gravitational pull [11]. It can also be considered as a parameter representing the combined effect of soil and plant properties. Wilting point, \( \theta_w \), is the soil water content that is held so tightly by the soil matrix that roots cannot extract from it. At this stage, a plant starts to wilt. It is a parameter determined by the interactions of soil, plant and climatic properties. Due to non-homogeneity of soil properties in the growing medium and variation of plants features, it is not unusual to find that \( \theta_f \) and \( \theta_w \) vary over a range of values. Raghuvanshi and Mailapalli [12] provided a typical range of \( \theta_f \) and \( \theta_w \) for different soil types. Interception of a plant, \( S_t \), is the quantity of rainwater storing in vegetation canopy. Zou et al [13] pointed out that plant interception, \( S_t \), is more effected by canopy structure than plant species. Miralles et al [14] in their event-based interception model considered that \( S_t \) ranged with \( 1.2 \pm 0.4 \text{ mm} \) at global scale. Evapotranspiration is the process of water transport through the growing medium.
surface and vegetation into the atmosphere. Therefore, evapotranspiration rate, $E_a$, is a parameter that encompasses the integral effect of soil, plant and climatic components in a GR system. Badgley et al. [15] conducted an analysis to determine the uncertainty in global $E_a$ estimates. The constant, $c = W_1/R_{c,max}$, represents the ratio of evapotranspirable water in the soil at the beginning of a dry period and the maximum retention capacity of a GR system. Since the constant, $c$, is a conceptual model parameter, there is no direct measurement available to quantify its uncertainty. Therefore, its uncertainty is subjectively imposed by the authors in the numerical study.

3. Sensitivity and uncertainty analyses considering epistemic uncertainty

3.1. Sensitivity analysis (SA)

In SA of model involving uncertainty inputs/parameters, the main focus is to assess the relative importance of model parameters with regard to their contributions to the overall uncertainty of model outputs. In this study, the SA of the statistical properties the retention ratio from the probabilistic GR model is performed. To facilitate the SA, Latin hypercube sampling (LHS) approach is used to synthesize model input-output database for establishing a simple, yet creditable input-output relationship for the underlying model, $g(X_1, X_2, ..., X_K)$, by regression analysis:

$$ Y = g(X_1, X_2, ..., X_K) = a_0 + \sum_{k=1}^{K} a_k X_k + \sum_{i=1}^{K-1} b_k X_k^2 + \sum_{j=k+1}^{K} c_{ij} X_k X_j + \varepsilon $$

where $Y =$ model output; $X_k =$ the $k$-th model parameter; $K =$ total number of model parameters; $a$’s, $b$’s, $c$’s = regression coefficients; and $\varepsilon =$ residual. In the above regression model, the first two terms describe the linear component of the model; the terms in the bracket are the $2^{nd}$-order relationship representing the non-linear components of the model approximated by the quadratic and interaction terms; the residual term represents the inadequacy of the hypothesized regression model in mimicking the underlying model, $g(X_1, X_2, ..., X_K)$. To remove the scale effect of model parameters in equation (11), their standardized form are used so that all model parameters are placed on an equal level in the SA.

Suppose that the regression model, equation (11), or its standardized form can adequately represent the model input-output relationship with a high correlation of determination. Then, the percentage of variance contribution of the linear (first-order) and the non-linear (second-order) terms can be obtained, respectively, by:

$$ \%VarC(I) = \frac{\text{SSE}(I)}{\text{SSR}}; \%VarC(II) = \frac{\text{SSE}(II)}{\text{SSR}} $$

where $\%VarC(I), \%VarC(II) =$ percentage of variance contributions to total model output variation by the first (linear) and second (non-linear) terms, respectively; $\text{SSE}(I), \text{SSE}(II) =$ sum of squared errors associated with the first and second terms, respectively; and $\text{SSR} =$ regression sum of squared errors associated with the regression model. Assuming that the linear terms dominate the total output variability, the relative importance of each individual model parameter, $X_k$, can be similarly assessed in terms of its variance contribution to model output variation as:

$$ \%VarC_k = \frac{\text{SSE}_k}{\text{SSE}(I)} $$

in which $\text{SSE}_k =$ sum of squared errors associated with the model parameter $X_k$.

3.2. Uncertainty analysis (UA)

In the study, LHS approach, in conjunction with antithetic variates (AV) technique, are utilized to estimate the uncertainty features of the statistical properties of GR retention ratio. The AV technique [16] achieves variance-reduction by generating random variates that would induce a negative correlation for the quantity of interest between separate simulation runs. By the AV technique, the
concerned model output, $Y$, can be estimated as:

$$\hat{Y} = \frac{1}{2} \left[ \hat{Y}_1(u) + \hat{Y}_2(1 - u) \right]$$

(14)

in which $\hat{Y}$ = estimator of the quantity $Y$; and $\hat{Y}_1(u), \hat{Y}_2(1 - u)$ = unbiased estimators of the quantity $Y$ based on $u \sim U[0,1]$ and $1 - u \sim U[0,1]$, respectively. The adopted estimator, $\hat{Y}$, will have smaller variance than the individual estimators, $\hat{Y}_1, \hat{Y}_2$, for estimating the quantity of interest $Y$.

4. Application

4.1. Uncertainty features of model parameters

Aleatory uncertainty associated with rainfall characteristics, based on the rainfall data at Detroit Airport, Michigan [9], are treated to be statistical independent having an exponential distribution. The mean values of rainfall depth and inter-event dry period are listed in table 1(a) which can be used to determine the distribution parameters in equations (5) and (6). As for epistemic uncertainties, the values of soil, plant and climatic properties of the GR system used in Zhang and Guo [9] were taken to be the mean values of the system parameters (see table 1(a)). Five model parameters were treated as independent random variables following a uniform distribution with range defined in table 1(b) according to literature review of model parameters described in Section 2.2.

Table 1. Statistical properties of uncertain factors in the example extensive GR. (a) Model inputs subject to aleatory uncertainty [9] and (b) Model parameters subject to epistemic uncertainty.

| (a)                                              |
|-------------------------------------------------|
| Rainfall event amount, $V$                       |
| Dry period between rainfall events, $B$         |
| $E(V) = 14.35$mm (Exponential)                  |
| $E(B) = 97.95$hr (Exponential)                  |

| (b)                                              |
|-------------------------------------------------|
| ET rate, $E_a$ (mm/hr), $\theta_f$              |
| Field capacity, $\theta_f$                      |
| Wilting point, $\theta_w$                       |
| Interception, $S_i$ (mm)                        |
| Multiplier constant, $c$                        |
| $0.11 \pm 25\%$ (Uniform)                      |
| $0.232 \pm 15\%$ (Uniform)                     |
| $0.116 \pm 20\%$ (Uniform)                     |
| $2 \pm 30\%$ (Uniform)                         |
| $0.5 \sim 0.9$ (Uniform)                        |

4.2. Sensitivity analysis

The task of SA is to assess the relative importance of different model parameters to the outputs. In the context of probabilistic GR model, the concerned system outputs herein are $Y = [E(R_r | X), SD(R_r | X)]$ with $X$ being a vector of model parameters, $X = (\theta_f, \theta_w, S_i, E_a, c)$. Based on the distributional properties of the model parameters, LHS approach was applied to generate random samples of the model parameters to form the input database which, in turn, were used to compute the corresponding values of model outputs $Y$. Databases of model parameters $X$ and outputs $Y$ were used in equation (11) for evaluating the relative importance of the model parameters.

According to equation (12), the variance contributions to the two outputs of the probabilistic GR model by the linear and non-linear terms of the model parameters are presented in table 2. As can be seen that the first-order (linear) terms dominate the variance contribution to $E(R_r)$ and $SD(R_r)$ as their values of percentage variance contribution are close to 100%. This is to say, the use of linear terms alone is sufficiently accurate enough to represent the responses of the mean and standard deviation of $R_r$. 
Table 2. Percentage contributions of linear and nonlinear terms of model parameters.

| Output | Order | Growing medium depth (mm) |
|--------|-------|---------------------------|
|        |       | 50 | 75 | 100 | 125 | 150 |
| $E(R_c)$ | 1-st  | 97.8% | 97.6% | 97.4% | 97.2% | 97.0% |
|        | 2-nd  | 2.2% | 2.4% | 2.6% | 2.8% | 3.0% |
| $SD(R_c)$ | 1-st  | 96.3% | 97.7% | 98.5% | 99.0% | 99.2% |
|        | 2-nd  | 3.7% | 2.3% | 1.5% | 1.0% | 0.8% |

As for the variance contribution to the overall variability of model output by individual model parameter, equation (13) was applied and the results are shown in figure 1 for the two concerned model outputs. For $E(R_c)$, figure 1(a) shows that the $\theta_f$ and $c$ are the two most influential parameters over the range of growing medium depth for the mean GR retention. However, as the growing medium depth gets thicker the importance of the former diminishes whereas the latter grows more influential and becomes more dominant over other model parameters. $\theta_w$ has the same trend as $\theta_f$, but with about a third of variance contribution as compare with that of $\theta_f$. The relative importance of $E_a$ increases with the growing medium depth and its relative importance ranks the third among the five model parameters considered. Among the model parameters, $S_1$ has the least contribution to the uncertainty of $E(R_c)$. For $SD(R_c)$, the first-order sensitivity of model parameters shown in figure 1(b) reveals that $c$ is the most influential parameter over other parameters. The $E_a$ ranks as the distant second.

Figure 1. Percentage contributions to overall variation in $E(R_c)$ and $SD(R_c)$ by the model parameters.

4.3. Quantification of GR model outputs uncertainties

UA is to determine the uncertainty features of the system responses as influenced by the uncertainties associated with the stochastic basic parameters in the system. Specifically, UA herein is concerned with the estimation of statistical moments of $E(R_c)$ and $SD(R_c)$. Through LHS, in conjunction with AV, the mean, standard deviation, and skew coefficient of $E(R_c)$ are shown in figure 2(a). It reveals that, under the influence of epistemic uncertainties, the expected value of mean retention ratio, $E[E(R_c)]$, increases steadily with the growing medium depth (solid red line). Without considering epistemic uncertainty, the value of mean retention ratio, $E(R_c)$, is slightly higher (dash red line). Interestingly, the standard deviation of $E(R_c)$ stays invariant with the growing medium depth whereas the corresponding skew coefficient becomes more negative with increase in growing medium depth.

Figure 2(b) shows that the mean of $SD(R_c)$, $E[SD(R_c)]$, decreases slowly with the growing
medium depth considering epistemic uncertainties (solid red line). The figure also shows that \( SD(R_r) \) without considering epistemic uncertainty practically coincides with \( E[SD(R_r)] \). Like figure 2(a), the standard deviation of \( SD(R_r) \) remains rather constant with the growing medium depth. However, the distribution of the standard deviation of \( R_r \) becomes less negatively skewed with increase in growing medium depth.

![Figure 2](image)

**Figure 2.** Variation of statistical moments of retention ratio with respect to growing medium depth.

5. Conclusions

Like any engineering systems, GR systems, once installed, are placed in a natural environment subject to elements (e.g., rain, temperature, etc.) whose occurrences are inherently random. Due to aleatory nature of rainfall characteristics, the rainfall retention performance of a GR system over its service life is not certain. Probabilistic GR models have been developed to explicitly consider inherent randomness of rainfall characteristics for evaluating the statistical properties of GR retention ratio, \( R_r \). It should be noted that, in addition to rainfalls, there exist non-rain system parameters describing soil, vegetation and climatic properties that play important roles affecting the rainfall-runoff transformation process in GR systems. These system parameters often are subject to uncertainty of epistemic nature due to knowledge deficiency. The presence of epistemic uncertainties in model parameters implies that results of aleatory-based probabilistic GR models are not certain. In this paper, a methodological framework is presented to incorporate uncertainties of epistemic nature in GR model parameters to analyze the sensitivity and uncertainty of statistical properties of retention ratio from the probabilistic GR model.

Regression-based global SA showed that the first-order effect of model parameters plays the dominant role over the higher-order effect in contributing the overall uncertainty of \( E(R_r) \) and \( SD(R_r) \) from the probabilistic GR model used. Among the five model parameters, \( \theta_f \) and \( c \) are the two most influential parameters in their contributions to the overall uncertainty of \( E(R_r) \). The former plays the dominant role under the shallower growing medium depth whereas the latter gets more influential under the thicker growing medium depth. \( \theta_m \) has the same trend as \( \theta_f \), but with about a third of variance contribution as compare with that of \( \theta_f \). The relative importance of \( E_a \) ranks the third among the five model parameters considered. Moreover, \( S_1 \) has the weakest contribution to the uncertainty of \( E(R_r) \). For \( SD(R_r) \), \( c \) is the most influential parameter and \( E_a \) ranks as the distant second.

The study provides a systemic framework for analyzing the sensitivity and uncertainty of an aleatory-based probabilistic GR model by incorporating involved epistemic uncertainties. The results from the proposed analysis framework promote a more comprehensive design and analysis of a GR system.

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