Electromagnetic scattering of two-dimensional rough cylinder based on statistical integral equation

Mengnan Zhu, Anqi Wang, Zhixiang Huang

Key Laboratory of Intelligent Computing & Signal Processing, Ministry of Education, Anhui University, Hefei, China.
nzxhuang@ahu.edu.cn  *Corresponding author’s e-mail: 2111483088@qq.com

Abstract. The study of electromagnetic scattering from Gaussian rough surface is of great significance in radar reconnaissance, target tracking and ocean remote sensing. The moment method (MOM) is a commonly used method with high accuracy. However, in the past research, Monte Carlo method is used to simulate the Gaussian rough surface. In this method, a large number of random simulation operations are carried out, and then a statistical convergence result is obtained by counting the results of all operations. In this paper, the stochastic integral equation method SIEM(Stochastic Integral Equation Method) is proposed to calculate the electromagnetic scattering of Gaussian rough surface. SIEM avoids the difficulty of high difficulty precise modeling and repeated calculation many times, and only single modeling and calculation are carried out for the target problem. In this paper, the electromagnetic scattering of two-dimensional cylinder with Gaussian rough surface is solved, and the effectiveness of SIEM is proved by comparing with mom.

1. Introduction
With the development of modern science and technology, whether military or civilian, what we use the frequency of electromagnetic waves is getting higher and higher. The unstoppable research aspect is the study of the electromagnetic characteristics of random rough surfaces. The traditional method is to use the Monte Carlo simulation method (Monte Carlo Method) for modeling. This method has tens of thousands of simulation calculations. The obtained results are counted, and the calculation amount is very large, in order to solve the problem of stochastic rough surface, a method is urgently needed to solve the problem quickly. This paper presents a new method of statistical integral equation method (Stochastic Integral Equation Method, SIEM)[9]-[13], it can quickly simulate the electromagnetic scattering of Gaussian rough surfaces.

This article does not require samples of Gaussian rough surfaces. By comparing the numerical results of SIEM and MoM [14]-[15]to evaluate the effectiveness and efficiency of SIEM. In the following, this method will be studied with the scattering of a two-dimensional rough cylinder.

2. Theoretical derivation
First explain two basic formulas:

2.1. Gaussian Hermit formula:

\[ \int_{-\infty}^{\infty} e^{-x^2} f(x) dx \approx \sum_{i=1}^{n} A_i f(x_i) \]  

(1)
Generally, five-point integration can be used to achieve relatively high accuracy. In the above formula, \( n \) is the number of integration points, \( A_i \) is the weight coefficient, and \( x_i \) is the integration point. The five-point Gaussian Hermit integration points and weight coefficients are listed below:

| Points | \( n \) | Integration points \( x_i \) | Weight coefficients \( A_i \) |
|--------|--------|-------------------------------|-------------------------------|
| 5      | 0      | \( \pm 0.95857246 \)           | 0.94530572                   |
|        |        | \( \pm 2.02018287 \)           | 0.39361932                   |
|        |        |                               | 0.01995324                   |

2.2. Gauss-Legendre formula:

\[
\int_a^b f(x)dx = \sum_{i=1}^{n} W_i f(x_i)
\]

(2)

\[
\int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^{1} f\left(\frac{b-a}{2} t + \frac{b+a}{2}\right)dt
\]

(3)

Generally, three-point integration can be used to achieve accuracy. In the above formula, \( I \) is the number of integration points, \( W_i \) is the weight coefficient, and \( x_i \) is the integration point. The integration points and weight coefficients of the three-point Gauss-Legendre are listed below:

| Points | \( i \) | Integration points \( x_i \) | Weight coefficients \( W_i \) |
|--------|--------|-------------------------------|-------------------------------|
| 3      | ±0.7745967 | 0.0000000                     | 0.5555556                     |
|        |         |                               | 0.8888889                     |

2.3. Theoretical derivation

Because TE wave has only Z component of magnetic field in[1]and[3], when it irradiates infinite rough cylinder, the induced current \( J \) on conductor cylinder has only transverse component, so MFRE can be obtained as follows: From the field source relationship:

\[
-H_z'(r) = J_z(r) - \frac{1}{4j} \int \frac{\partial H_0^{(2)}(k|r-r'|)}{\partial n'} J_z(r')dl'
\]

(4)

Using SIEM theory to average the above formula, we can get:

\[
<-H_z'(r) >=< J_z(r) - \frac{1}{4j} \int \frac{\partial H_0^{(2)}(k|r-r'|)}{\partial n'} J_z(r')dl' >
\]

(5)

In addition, the scattering field in [2] can be expressed as:

\[
\overline{H^t} = \hat{u}_z \cdot \nabla \times \int_G JGdl'
\]

(6)

Where \( G \) is a two-dimensional Green function,

\[
G(r,r') = \frac{1}{4j} H_0^{(2)}(k|r-r'|)
\]

(7)

The cylinder contour is divided into \( N \) segments, each segment is \( \Delta C_n \), and the following pulse basis functions are used as the basis functions in[5]:

\[
\rho_n(t) = \begin{cases} 
1 & t \in \Delta C_n \\
0 & \text{other} 
\end{cases}
\]

(8)
The following Point matching method in [8] is used to solve this electromagnetic scattering. So the above formula can be written as

$$\begin{align*}
Z[J] &= \begin{bmatrix} H_z \end{bmatrix} \\
(9)
\end{align*}$$

Where the impedance matrix is:

$$
Z_{mn} = \frac{jk}{4} \Delta C_n (\mathbf{n} \cdot \mathbf{R}) H_1^{(2)} (k |r_m - r_n|)
$$

$$
= \frac{jk}{4} \int_{\Delta C_n} (\mathbf{n} \cdot \mathbf{R}) \int_{\infty}^{\infty} P_2(h_1, h_2, r_1, r_2) H_1^{(2)} (kR)
$$

$$
= \frac{jk}{4} \Delta C_n (\mathbf{n} \cdot \mathbf{R}) \int_{\infty}^{\infty} P_2(h_1, h_2, r_1, r_2) H_1^{(2)} (kR)
$$

(10)

The voltage matrix is:

$$
\overline{H} = \langle H_z^j \rangle = \int_{\infty}^{\infty} P(h) H_z^j dh
$$

(11)

Where \( \mathbf{n} \) is the external normal direction of the cylindrical boundary, \( J_s \) is the current of each section, and \( \mathbf{R} = (r - r') / |r - r'| \) is the unit vector from the source point to the field point. \( P_2(h_1, h_2, r_1, r_2) \) in the above formula is the joint probability density, \( P(h) \) is the probability density, the expressions of the two formulas are as follows:

$$
P_2(h_1, h_2, r_1, r_2) = \frac{\exp(-h_1^2 - 2C(r_1, r_2)h_2 + h_2^2)}{2\pi^2(1-C(r_1, r_2)^2)}
$$

$$
P(h) = \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{h^2}{2\sigma^2})
$$

(12)

(13)

When \( m = n \), the singularity of the impedance matrix will appear, that is, the surface element and the source surface element coincide, and the joint probability density at this time will degenerate into the probability density of the surface element, which is \( P_2(h_1, h_2, r_1, r_2) = P(h) \), also due to the integral amount \( h \) it is independent of cell \( m \) and can exchange order, and according to the calculation formula, the impedance matrix in [7] can be written as:

$$
Z_{mn} = 1 / 2
$$

(14)

The RCS in [4][and6] is calculated by

$$
\sigma(\phi) = \frac{k}{4} \int_{c} J(x', y') \mathbf{n} \cdot \mathbf{R} e^{j(k(x\cos\phi + y\sin\phi) - l)} dl
$$

(15)

Let the integral be approximately the sum of all \( \Delta C_n \)'s, and there are \( J_n = \alpha_n \), \( x = x_n \), \( y = y_n \), Results for each \( \Delta C_n \) in the integration.

$$
\sigma(\phi, \phi') = \frac{k}{4} \left| [H_n^m] [Z_{mn}] [H_m^m] \right|^2
$$

(16)

In the formula:

$$
[H_n^m] = [\Delta C_n e^{jk(x_n\cos\phi + y_n\sin\phi)}]
$$

$$
[H_m^m] = [\Delta C_n \mathbf{n} \cdot \mathbf{R} e^{jk(x\cos\phi + y\sin\phi)}]
$$

(17)

(18)
3. Experimental results
For a radius of 2, the Gaussian root mean square is 0.01, and the correlation length is 0.1. The incident wave is a 300M TM wave, incident along the x direction, and the incident angle $\phi=0^\circ$, the RCS results are as follows:

![Figure 1. TE wave incident infinitely long conductive cylinder current distribution diagram (MC200)](image1)

![Figure 2. TE wave incident infinitely long conductive column current distribution result diagram (SIEM method)](image2)
4. Summary
It can be compared with SIEM in time method. By comparison, we can see that the method of using the statistical integral equation is basically consistent with the experimental results of MOM, but there are still errors. The traditional method of modeling using Monte Carlo requires thousands or more times, but the statistical integral equation does not need. The more prominent advantage is that the time used by the SIEM method is greatly reduced, and the efficiency is greatly improved. As the frequency of electromagnetic problems increases, the model becomes more and more complex. For random problems with rough surfaces, research will become more and more important. The traditional methods for solving random rough problems face huge difficulties. Especially for complex problems, the traditional Monte Carlo method will be difficult to use and calculate, so the focus of this article is that applying the statistical integral equation method will be meaningful for studying stochastic rough problems.

Acknowledgment
This work was supported by the National Natural Science Foundation of China (Grant No. 61971002, 61722101), by the Natural Science Foundation of Anhui Province (Grant No. 1608085QF141, 1608085MF135), and by the Provincial Program of Natural Science of Anhui Higher Education (Grant No. KJ2019A0028). The authors would like to thank the reviewers for their constructive suggestions.

References
[1] M.A.Karamn and A.K.Fung,"Electromagnetic wave scattering from some vegetation
samples,"IEEE Trans. Geosci. Remote Sensing, vol.26, pp. 799-808,1988.

[2] Ishimaru A. Wave propagation and scattering in random media[M]. New York: Academic press,1978.

[3] L. Leung, J. Kong, K. Ding, and C. Ao, Scattering of electromagnetic waves, Vol. 2 Numerical simulations, Wiley, New York, 2001.

[4] Millet F, Warnick K. Validity study of rough surface scattering models[C]//Antennas andPropagation Society International Symposium, 2003. IEEE. IEEE, 2003, 1: 565-568.

[5] R. F. Harrington. Field Computation by Moment Methods [M]. New York: Macmillan, 1968.

[6] L.E.Allan and G.M.McCormick "Measurements of the backscatter matrix of dielectric bodies "IEEE Trans. Antennas Propag., vol. 28, no. 2, pp. 166-169, 1980.

[7] Jennings A, Mc Keown JJ. Matrix computation[M]. John Wiley & Sons Inc, 1992.

[8] Beckmann P, Spizzichino A. The scattering of electromagnetic waves from rough surfaces[J].Norwood, MA, Artech House, Inc., 1987, 511 p., 1987.

[9] Zhu Z, White J. Fast Sies: a fast stochastic integral equation solver for modeling the rough surface effect[C] Proceedings of the 2005 IEEE/ACM International conference on Computer-aided design.IEEE Computer Society,2005: 675-682.

[10] Zhu Z. Efficient integral equation based algorithms for parasitic extraction of interconnects with smooth or rough surface[D]. Massachusetts Institute of Technology, 2004.

[11] Knott E F,Sheaffer J F,Tuley M T. Radar cross section: Its prediction, measurement and reduction [M].London: Ar-tech House Inc,1985.

[12] Chen Q, Wong N. A stochastic integral equation method for resistance extraction of conductors with random rough surfaces[C] Intelligent Signal Processing and Communications, 2006.ISPACS'06. International Symposium on. IEEE, 2006: 411-414.

[13] L. Leung, J. Kong, K. Ding, and C. Ao, Scattering of electromagnetic waves, Vol. 2, Numerical simulations, Wiley, New York, 2001.

[14] R. F. Harrington. Field Computation by Moment Methods [M]. New York: Macmillan, 1968.

[15] Beale, B. and Cramer, H.T., “Analyzing the effects of bond wires on signal integrity”, Electronics Engineer,1995, pp.54-58.