The different way of understanding and way of thinking between gender on the problem the linear equations of two variables

M T Bakar1*, D Suryadi2 and D Darhim2
1Program Studi Pendidikan Matematika, Universitas Khairun Jl. Bandara Babullah Akehuda Ternate Indonesia
2Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia Jl. Dr. Setiabudi No. 229, Bandung Indonesia

*Corresponding author’s email: tmarwia@gmail.com

Abstract. This article discusses about way of understanding and way of thinking college students between gender in solve math problems. A way of understanding is a cognitive product of a mental act perform by someone, and way of thinking is a cognitive characteristic of a mental act. The mental act is obtained by observing their statements and actions. Open interviews conducted by researchers after watching the learning video and check the college student works. This research uses phenomenographic approach. Rusdi and Ira are prospective elementary teachers who have equal ability of between gender in solving four problems related to the linear equation of two variables. The results of this study revealed that female college students tend to relate previous concepts to problems on the subject, solution representation more than one way, and way of thinking appear are to use empirical solutions and tend to follow frequently used patterns. While the male college students answered practically, that is using the clue of the question to obtain a solution, and way of thinking appear are to as non referential symbolic. This implies that way of understanding college students depends on heavily on the characteristics of their way of thinking.

1. Introduction
Mathematics instructors, including lecturers, still see mathematics as subject matter and not as conceptual tools this is not something that is wrong because mastering the subject matter, such as, definitions, theorems, proofs, problems and solutions, is necessary in the teaching and learning mathematics. But this is not enough. Lecturers must also focus on conceptual tools such as problems solving is needed to construct mathematical objects. Both of these perspectives are not the same. Mathematics as a subject matter, is to view mathematics as a product that has its own meaning and is understood by everyone [1] have objects that have meaning, only in the system the object is defined [2]; mathematics as a priori knowledge [3] is knowledge based only on reason and not based on observation. While mathematics as a conceptual tool is looking at mathematics as a tool used to produce or construct mathematical objects

The assertion related to differences in perspective can be seen in the mathematical definition proposed by Harel [4] that mathematics consists of two complementary subsets: “The first subset is a collection, or structure, of structures consisting of particular axioms, definitions, theorems, proofs, problems, and solutions. This subset consists of all institutionalized ways of understanding in
mathematics that have been institutionalized ways of understanding in mathematics throughout the history, it is denoted by WoU. The second subset consists of all ways of thinking, which are characteristic of the mental acts whose products comprise the first set, it is denoted by WoT”. This definition clearly distinguishes between ways of understanding and ways of thinking in mathematics as something different, but has a reciprocal relationship called duality, which is the theoretical framework and principles of DNR-based instructions, which then in short DNR; duality, necessity, and repeated reasoning.

WoU and WoT can be studied by observed on what someone’s mental action and his statement. The intended mental action is to interpret, conclude, prove, explain, generalize, analyze, and communicate [5]. A statement and action of a person can signify a cognitive product from the mental action carried out by that person. This cognitive product is called a way of understanding related to mental actions. Repeated observations from ways of understanding can reveal certain cognitive characteristics of these actions. This cognitive characteristic is called the way of thinking. This article adapts and uses Harel's opinions about Wou and WoT (2001 [6], 2005 [7], 2007 (a) [8], 2007 (b) [9], 2008 (a) [10], 2008 (b) [11], 2008 (c) [12], 2010 (a) [13], 2010 (b) [14].

Many researches have been done on how to understand mathematics as a concept, for example, [15] examines students' learning obstacle in solving linear equations found that students experience epistemological obstacle, ontogenic obstacle and didactical obstacle. Epistemological obstacle because students have not been able to translate problems into mathematical models; miscalculated and unable to provide an explanation on the answers obtained as an epistemological obstacle, ontogenic obstacle, where students cannot connect the knowledge acquired previously with the problem at hand. Didactical obstacle occurs because the teacher is unable to create learning that is able to arouse students' intellectual need. Wahyudin 1999 in [5] revealed the ability of teachers and prospective teachers and students towards mathematical abilities to find at least five weaknesses that are possessed by students, namely: (1) lack of knowledge of the prerequisites possessed, (2) lack of understanding deep on the basic concepts of mathematics, (3) lack of focus in listening and inaccurate in recognizing a given problem, (4) less able to identify the given problem, and (5) weak ability of logical reasoning.

Furthermore, our reading of several studies that is related to wou and wot, we did not find many scholars have studied. But Harel Guerson has clearly discussed the two in several papers relating to DNR. But specifically, no one has tried to analyze students' wou and wot when given a mathematical problem based on gender, so this is what is considered to be a novelty of this research.

2. Methods

This study uses a phenomenographic approach [16] to reveal the differences of WoU and WoT of students in solving the problem of linear equations. This approach describes qualitatively of how mathematical problems are understood and experienced by students [17]. Three questions related to linear equations that students have worked on have been published in the previous article (see Bakar 2018), with a different focus of analysis. The research subjects were students of prospective primary school teachers who were in the initial semester at one of the State Universities in Ternate City, North Maluku - Indonesia. Both selected students have equal mathematical abilities, namely in the medium category, with different gender. The results of the students' work were analyzed to determine their mathematical abilities in the dimensions of wou and wot, then conducted an in-depth interview to find out what students understood regarding the material being tested. These three problems are shown in Table 1.
Table 1. Problem description.

| No | Problems |
|----|----------|
| 1  | Mr. Amir bought five recreational park tickets for two adults and three children for Rp105,000.00. Meanwhile Pak Iksan bought three tickets for adults and five tickets for children at Rp165,000.00. State this situation in the most appropriate form you think (among SPLDV settlement methods) to determine the price of each recreational park ticket. |
| 2  | A convenience store sells two types of rice. Rice type I and rice type II. The price of rice type I is Rp 10,000, - and the price of rice type II is Rp. 13,000, - The sales on that day were Rp. 2,935,000, - and the amount of rice sold were as much as 250 liters. |
|    | a. What kind of rice is the bestseller? Explain your reasons. |
|    | b. Suppose that rice sold for 370 and sales were of Rp 4,450,000. How much is each type of rice sold? |
|    | c. What conclusions are derived from these two problems? |
| 3  | The circumference of an isosceles triangle is 20cm. If the length of both feet added by 3 cm and the length of the base two times the original length then the circumference to be 34 cm. Represent your answers using more than two ways to get the lengths of all three sides of the isosceles triangle. |

3. Results and Discussion

This section presents the results analysis of two students work in terms of WOU and WOT who has different gender. The work of Rusdi (male) in number 1 is presented in Figure 1.

Figures 1. Rusdi work (male) in no. 1.

Based on Figure 1. It shows that Rusdi was able to change the statement on the problem into the form of a mathematical equation. Rusdi worked these two equations into 2d + 3a = 105,000 as a form of the equation of Pak Amir's problem and dd + 5a = 165,000 as a form of equation for Pak Iksan, indicating that Rusdi has a concept of equality. Overall, it identified that Rusdi has more knowledge about equality, understanding operations in algebra, solving problems differently, following the flow of thinking that tends to explore, for example Rusdi uses a strategy by using the other equations obtained to solve problems in problem no. 1. It can be concluded that the equation concept and algebraic operations (WoU) are interpreted differently in a way which is done by the teacher / lecturer. The dominant way of thinking used by Rusdi is flexible in interpret the symbols, rarely connecting with other concepts, and subject to the process of algebraic operations. When it was interviewed related to his work, Rusli stated
that "I tried to understand the problem and tried to solve based on how I understood it, I think it made me not forget, compared to memorizing formulas or following procedures taught by the lecturer but it was too long or I don't understand".

The work of Ira (female) in number 1 is presented in Figure 2. Based on Figure 2. It can be seen that Ira was able to interpret the two equations into a mathematical model. It shows that Ira also has skills related to equality, but the way of solving the problem, Ira adapted the way the teacher / lecturer that was often used in solving the problems, inflexible interpreting symbol, and the alternative solution is based on deduction rules.

![Figures 2. Ira work (female) in No.1](image1)

The way of Rudi works on number 2 is as described on figure number 3. In Figure 3. It shows that Rusdi solved the problem by not using the same method, he was able to produce a mathematical model of the problem in that case, and the result of his work was identified that Rusdi was very familiar with the method that he used. When he was interviewed that Rusdi stated "he wanted to answer in another way, because the results obtained were the same".

![Figures 3. Rusdi work (male) in no. 2](image2)
The way of Ira works on number 2 is described on figure 4 as follow:

![Figures 4](image)

It can be seen In Figure 4 that Ira was able to interpret and solve problems by adopted the method which is used by the teacher/lecturer, in which it was still rigid in interpreting mathematical symbols, without a quantitative reference and the method of completion was based on deduction rules. When he was interview about the way of how she solves the problem, Ira stated that "I prefer to use the elimination method and be more confident with the results obtained.

The way of Rusdi and Ira works on number 3 is described on figure 5 and 6 as follow:

![Figures 5](image)

![Figures 6](image)

It can be seen in figure 5 that the way of Rusdi’s is very flexible to mathematical symbol interpretation, able to connecting questions with the applicable formula, and to interpret the formulas in different way from Ira does, even though using the same way of solving the problem. In terms of ways of thinking male students are more flexible in interpret symbols, they use different ways of solve the problem, they tend to use the rules of induction and their way of thinking used symbolic non-referential
Figure 6 shows that Ira was able to show completion in more than one way, able to relate the problem to the previous concept, although there was still a slight error in algebra operations, she is still using the same pattern of resolution as before. Therefore, it can be concluded that in terms of ways of thinking female students are less flexible in interpret symbols, they always use the same method, they tend to use a valid formula, and the way of their thinking, they used non-referential symbolic.

4. Conclusions

The findings of this study illustrate that the way of thinking used by male students in general is flexible in using mathematical symbols, they tend to use different ways with lecturers / teachers use; they use clue on questions to solve problems, and they rarely use formulas to solve problems, they do not use examples or visual perceptions in solving problems, and they tend to follow deduction rules. Whereas the way of thinking used by female students in this study is less flexible in using symbols, the way to solve the problem, they used the way of the lecturer’s used. Therefore, the solution is based on examples, or visual perception to solve the problem. In general, this research is in line with the findings of Harel & Showder (1998) [18] that the settlement schemes commonly used among students are authoritative schemes and empirical schemes whose resolution depends on examples or visual perception.

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