Devices for the transformation of motion in the structure of oscillatory system: dynamic damping of oscillations

A I Orlenko¹, N K Kuznetsov², Q T Vuong³

¹ Krasnoyarsk Institute of Railway Transport of Irkutsk State Transport University, 89, Lado Ketckhoveli St., Krasnoyarsk, 660028, Russia
² Irkutsk National Research Technical University, 83, Lermontov St., Irkutsk, 664074, Russia
³ Irkutsk State Transport University, 15, Chernyshevskiy St., Irkutsk, 664074, Russia

E-mail: trucvq1990@gmail.com

Abstract. Dynamic properties of technical object with two degrees of freedom in the regimes of dynamic damping of oscillations are considered. Technologies of constructing transfer functions and determining conditions for dynamic damping of oscillations are offered. The purpose of the work is to develop a method for constructing mathematical models for estimating the dynamic properties, determining conditions for realizing the dynamic effects of “nullifying” motions on individual coordinates under the simultaneous action of several harmonic perturbations. Methods of structural mathematical modeling are used. The scientific novelty of the work is to evaluate the possibilities of applying devices for the transformation of motion (DTM) in the structures of mechanical oscillatory systems. The technology of transition from transfer functions of systems to the equations for estimating frequencies of dynamic damping of oscillations is developed. Method of constructing frequency diagrams for finding the required frequency of dynamic damping of oscillations is developed. It is shown that the system has the potential to implement different regimes of dynamic damping of oscillation.

1. Introduction

Many technical objects of modern production, including vehicles, operate under conditions of increased dynamic loads. Calculation schemes based on mechanical oscillatory systems with lumped parameters are widely used as calculated schemes for machines, equipment and apparatus. The effectiveness of preliminary assessments of the dynamic capabilities of the created technical means at the stages of preliminary, pre-project research and development can be significantly improved by developing analytical approaches, building mathematical models and using experience in solving problems of system dynamics in various applied directions [1 ÷ 4].

Mechanical oscillatory systems can be interpreted by the structural schemes of dynamically equivalent automatic control systems [5, 6].

The proposed article develops a method for constructing mathematical models of technical objects subject to the action of several external disturbances, on the basis of which problems of controlling the dynamic state of systems could be solved, in particular, with the possibilities of creating regimes for simultaneous dynamic damping of oscillations for two coordinates.
2. The features of technical object

Many technical objects, in particular, traction electric motors of vehicles (locomotives) are considered as systems with two degrees of freedom, consisting of a solid body that performs plane oscillatory motion. External impacts in such problems are determined by periodic motions of the supporting surface and are assumed to be known. A principal diagram of technical object of this kind in the form of a locomotive traction motor (Figure 1, a) with a calculated diagram in the form of a mechanical oscillatory system (Figure 1, b) with two degrees of freedom is shown in Figures 1 a, b. The motion of the system is considered in the coordinate system $y_1, y_2$, connected with the fixed basis. In the system are used elastic elements and additional connections in the form of DTM based on non-self-locking screw mechanisms having nut-flywheels [7-9].

![Figure 1](image1.png)

**Figure 1.** Principal and calculated diagrams of a traction electric motor: a – principal diagram of motor with a support axle suspension; b – calculated diagram of a traction motor with devices for the transformation of motion.

In Figure 1 a number of notations is adopted, in particular: $M, J$ – mass and moment of inertia of a solid body relative to the center of gravity; $L_1, L_2$ – reduced masses of DTM; $l_1, l_2$ – determine the position of the center of gravity; $z_1(t), z_2(t)$ – the kinematic perturbations.

The structural mathematical model (Figure 2) of the system can be obtained on the basis of using the known technologies, outlined in [4-6].

![Figure 2](image2.png)

**Figure 2.** The structural mathematical model of the initial system of Figure 1 with kinematic perturbation.

In Figure 2 it is assumed that $p = j\omega$ is a complex variable; the “−” icon above the variable means its Laplace image. The system has two input effects: $z_1(t), z_2(t)$ – the kinematic perturbations. Dynamic state is evaluated by variables $\bar{y}_1, \bar{y}_2$.

3. Mathematical model and its possibilities

Using the structural diagram, we write down the expressions for the transfer functions by $\bar{y}_1$ and $\bar{y}_2$ under the combined action of two harmonic in-phase perturbations:
\[ W_1(p) = \frac{\bar{y}_1}{\tau_1} = \frac{(L_1 p^2 + k_1)[(Mb^2 + Jc^2 + L_2)p^2 + k_2]}{A(p)} \cdot (L_1 p^2 + k_2)[(Jc^2 - Mab)p^2] + (A(p)) \]  
\[ W_2(p) = \frac{\bar{y}_2}{\tau_2} = \frac{(L_2 p^2 + k_2)[(Ma^2 + Jc^2 + L_2)p^2 + k_1]}{A(p)} \cdot (L_2 p^2 + k_1)[(Jc^2 - Mab)p^2] + (A(p)) \]  

where \( a = \frac{l_2}{l_1 + l_2}, \) \( b = \frac{l_1}{l_1 + l_2}, \) \( c = \frac{1}{l_1 + l_2}, \) and 
\[ A(p) = [(Ma^2 + Jc^2 + L_4)p^2 + k_1] - [(Mb^2 + Jc^2 + L_2)p^2 + k_2] - [(Jc^2 - Mab)p^2] \]  

is the characteristic frequency equation.

In the case, when \( \bar{\tau}_1 = \bar{\tau}_2 = \bar{\tau} \), the possibility of a functional interrelation of the values of the reduced masses DTM (\( L_1, L_2 \)) is considered in the form
\[ L_1 = \alpha \cdot L_2. \]  

Such a dependence of the parameters can be provided by means of a special control system for the interactions of the elements. For example, to change the coefficient \( \alpha \) in the DTM based on non-self-locking screw mechanisms (Figure 1, b), a controlled friction moment on the rim can be introduced – flywheel nuts, etc. [10 ÷ 12].

The introduction of the coupling coefficient \( \alpha \) changes the transfer functions of the system (1), (2) and its characteristic frequency equation:
\[ W_1'(p) = \frac{\bar{y}_1}{\tau_1} = \frac{(\alpha L_2 p^2 + k_1)[(Mb^2 + Jc^2 + L_2)p^2 + k_2]}{A_1(p)} \]  
\[ W_2'(p) = \frac{\bar{y}_2}{\tau_2} = \frac{(L_2 p^2 + k_2)[(Ma^2 + Jc^2 + \alpha L_2)p^2 + k_1]}{A_1(p)} \cdot (L_2 p^2 + k_1)[(Jc^2 - Mab)p^2] + (A_1(p)) \]  

where
\[ A_1(p) = [(Ma^2 + Jc^2 + \alpha L_2)p^2 + k_1] - [(Mb^2 + Jc^2 + L_2)p^2 + k_2] - [(Jc^2 - Mab)p^2]. \]

The partial frequencies of the system in this case are given by expressions
\[ n_1^2 = \frac{k_1}{Ma^2 + Jc^2 + \alpha L_2}, \]  
\[ n_2^2 = \frac{k_2}{Mb^2 + Jc^2 + L_2}. \]

The frequencies of the dynamic damping of oscillations in the considered case, in contrast to the usual approaches, will no longer coincide with the values of the partial frequencies in the usual formulation scheme, but will be determined from the condition of “nullifying” the numerators of the fractional-rational expressions for the corresponding transfer functions.
\[ p^4[\alpha L_2(Mb^2 + Jc^2 + L_2) + L_2(Jc^2 - Mab)] + p^2[k_1(Mb^2 + Jc^2 + L_2)] + k_2(\alpha L_2 + Jc^2 - Mab)] + k_1k_2 = 0, \]  
\[ p^4[L_2(Ma^2 + Jc^2 + \alpha L_2) + \alpha L_2(Jc^2 - Mab)] + p^2[k_2(Ma^2 + Jc^2 + \alpha L_2)] + k_1(L_2 + Jc^2 - Mab)] + k_1k_2 = 0. \]

Thus, the evaluation of the possibilities for the manifestation of regimes of dynamic damping of oscillations will be related to the presence of the corresponding roots of equations (10), (11).

4. Construction of frequency diagram
The frequency diagram is formed as a set of dependency graphs \( n_1^2(\alpha), n_2^2(\alpha), \alpha_{1,\text{din}}^2(\alpha), \alpha_{2,\text{din}}^2(\alpha), \alpha_{1,\text{sov}}^2(\alpha), \alpha_{2,\text{sov}}^2(\alpha) \), as shown in Figure 3. The dependency graphs \( \alpha_{1,\text{din}}^2(\alpha), \alpha_{2,\text{din}}^2(\alpha) \) show the relationships between the dynamic damping of oscillations.

For numerical simulation, the following model parameters: \( M = 7000 \text{ kg}; J = 2000 \text{ kg.m}^2; \)
\( a = 0.57; \ b = 0.43; \ c = 0.71; \ k_1 = 1000 \text{ kN/m}; \ k_2 = 2000 \text{ kN/m}; \ l_1 = 0.6 \text{ m}; \ l_2 = 0.8 \text{ m}; \ L_2 = 200 \text{ kg}; \ L_1 = \alpha \cdot L_2. \)

**Figure 3.** Frequency diagram for determining regimes of dynamic damping of oscillations under condition \( L_1 \neq 0, \ L_2 \neq 0 \) and different values of coefficient of connectivity \( \alpha \).

We note the presence of characteristic points on the diagram. In particular, p. (1) determines the value of \( \alpha \) and the frequency of the dynamic damping of oscillations simultaneously to the coordinates \( \bar{y}_1, \ \bar{y}_2 \). In pp. (2) and (2') a specific regime of dynamic damping of oscillations is realized, when for one value of \( \alpha \) there will be two regimes of dynamic damping of oscillations.

**Figure 4.** Amplitude-frequency characteristics of system with \( \alpha = 0.5 \) for p.(1) in graph, given in Figure 3.

In pp. (I), (II) graph \( \frac{\bar{y}_1}{\bar{z}} (\omega) \) twice crosses the axis of abscissas and determines the frequencies of dynamic damping of the oscillations for coordinate \( \bar{y}_1 \). Graph \( \frac{\bar{y}_2}{\bar{z}} (\omega) \) also twice crosses the abscissa.
axis: in p. (I'), and also in p. (II). We note that in p. (II) there occurs a simultaneous intersection of the graphs \( \frac{\overline{y}_1}{z}(\omega) \) and \( \frac{\overline{y}_2}{z}(\omega) \). On a larger scale, the details of the intersection are shown in Figure 5.

**Figure 5.** Intersection of amplitude-frequency characteristics \( \frac{\overline{y}_1}{z}(\omega) \), \( \frac{\overline{y}_2}{z}(\omega) \) under simultaneous dynamic damping of oscillations for two coordinates \((\overline{y}_1 \text{ and } \overline{y}_2)\) with \( \alpha = 0.5 \).

5. **Discussion of results and conclusion**

The transfer function of the system is the basis for evaluating the dynamic properties of systems, both in general and in individual coordinates. In the presence of DTM in the system, the transfer function of the system is a fourth-order fractional-rational expression relative to the complex variable \( p \). The numerators and denominators of the transfer function are biquadratic equations, which allow to estimating the possible diversity of the forms of amplitude-frequency characteristics (AFC).

In the general case, each of the coordinates \( \overline{y}_1 \) and \( \overline{y}_2 \) individually can realize two regimes of dynamic damping of oscillations. For certain values of the coupling coefficient \( \alpha \), simultaneous dynamic damping of oscillations at a certain frequency is possible simultaneously for two coordinates. The variation \( \alpha \) provides an opportunities to obtain a greater variety of AFC. A characteristic property for the system is the presence of limiting values of AFC with increasing frequency of the external impact, which follows from the structure of the expression for the transfer function.

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