Residual vibration control of a single-link flexible manipulator with variable stiffness and damping magnetorheological joint

Xiaomin Dong¹*, Kaiyuan Shi¹ and Wenfeng Li¹

¹ State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqing, 400044, People’s Republic of China

*Corresponding author’s e-mail: xmdong@cqu.edu.cn

Abstract. Residual vibrations of large flexible manipulator can be easily excited due to large inertia and low damping, which greatly reduces the efficiency of the tasks. This paper describes an approach for the use of smart materials, specifically, magnetorheological fluid, in vibration control of a single-link flexible (SLF) manipulator. Considering that the systems with variable stiffness and damping have demonstrated excellent performance, a compact variable stiffness and damping magnetorheological joint (VSD-MRJ) was designed. A mathematical model of the SLF manipulator with a VSD-MRJ was established. Two independent fuzzy controllers were designed and applied to the stiffness and damping control of the joint. Finally, a test platform was built and the experimental results verified that the residual vibration suppression performance of the manipulator with the VSD-MRJ is better than the traditional rigid manipulator.

1. Introduction

Manipulators play an important role in improving the safety and efficiency of completing construction for large complex structure, specifically, space structures. The Canadarm and the European Robotic Arm are examples of construction and maintenance operation manipulator. Traditional joints of the manipulator are usually rigid which has the advantages of high transmission accuracy. Unfortunately, the ability of the vibration suppression is weak. In addition, due to the slender structural characteristics of large manipulators, links are flexible. In these cases, the vibration can be easily excited and last longer, which greatly reduces the working efficiency and increases the economic cost [1-2].

To make the manipulator efficiently to stabilize, in recent years, many scholars design and develop more advanced structures and more automated control algorithms. At present, two common methods can be followed: passive and active. A passive method is simple which can be based on trajectory planning or increasing system damping by using some damping material [3]. Both S Yavuz [4] and Meng [5] verified an appropriate trajectory planning can effectively reduce residual vibration. An active method through control of actuators such as piezoelectric actuators has advantages of good performance and strong robustness [6]. In literatures [7-10], full-state feedback neural network control, PD control, optimum control, and nonlinear decoupling feedback control are implemented to improve the stability of the flexible manipulator system. However, the shortcomings of active control systems and passive control systems are obvious. On the one hand, the energy consumption of active control is large, which is undesirable in some situations, such as the space manipulator; on the other hand, the passive control does not require additional energy, but it has poor adaptive performance.
Compared with the above two control systems, a semi-active control system needs only a small amount of energy to change parameters such as stiffness and damping. A key technology is represented by magnetorheological (MR) fluid, a kind of smart material. Benefits from characteristics of rapid response and controllability [11], the MR dampers have been widely used in vibration control of vehicles, helicopter, and civil structure [12-15], etc. Unfortunately, there are few studies on MR fluid applied to manipulators. Huynh et al. verified that a space manipulator system with MR damper can effectively avoid the stability decreases caused by the collision and the angular momentum of the target [16]. Recently, a new research hotspot is the simultaneous variability of structural stiffness and damping. In literature [17-19], scholars design variable stiffness and damping (VSD) configurations and propose that a VSD system can make it better to reduce vibration than a variable damping system.

Based on the review above, we aim to suppress the vibration of the single-link flexible (SLF) manipulator effectively by control of the variable stiffness and damping magnetorheological joint (VSD-MRJ). The paper is organized as follows: in section 2, the dynamic model of a SLF manipulator with a VSD-MRJ is established; in section 3, a fuzzy controller is carried out; And then, an experimental platform is built and the residual vibration suppression performance of the SLF manipulator with a VSD-MRJ is tested and analyzed in section 4. Finally, the conclusions are drawn in section 5.

2. Dynamic modeling of SLF manipulator with VSD-MRJ

As shown in figure 1, a flexible link carrying a payload is modeled as a uniform cantilever beam with a tip mass that can rotate about the Z-axis. A VSD-MRJ is connected in series to change the transmission stiffness and damping of the entire system. It consists of two parts, one is the rigid rotation displacement (the position of \( p \) moves to the position of \( p' \)) and the other is the small displacement caused by the deformation of the link (the position of \( p' \) moves to the position of \( p'' \)).

![Figure 1. SLF manipulator with VSD-MRJ](image)

![Figure 2. 3D cross-sectional view of the joint](image)

In our prior study [20], a compact VSD-MRJ was designed and manufactured. It consists of two springs and two MR dampers, as shown in figure 2. The dynamic model of the joint and its approximately equivalent model are given in Figure 3. The equivalent stiffness and damping coefficient can be obtained [21].

![Figure 3. Dynamic model of the VSD-MRJ: the original model (left) and its equivalent model (right)](image)
\[
c_s = c_1 + \left[ (1 + \eta)^2/c_s + w^2c_s/k_{s1}^2 \right]^{-1} \\
k_s = k_{s1} \left[ 1 - \left[ 1 + \eta + (c_2w/\kappa_{s1})^2(1 + \eta) \right]^{-1} \right]
\]

where \( \eta = k_{s2}/k_{s1} = 0.3 \), \( w \) is the excitation angular frequency. From equations above, it can be considered that the equivalent stiffness of the joint is determined by damper 2, and the equivalent damping coefficient of the joint is similar to damper 1.

For system modeling, the flexible link is considered to be a Euler-Bernoulli beam whose axial deformation is neglected; The dynamic model of the SLF manipulator requires doubling of the generalized coordinates in a Lagrange approach, i.e. Both the link and joint deformation coordinates \( q_i \) and \( \theta_i \) (angular displacements of the motor and output shaft \( \theta_1 \) and \( \theta_2 \)) as the generalized coordinates, respectively.

The coordinate system \( OXY \) is the fixed inertial frame. The local coordinate system \( OX_1Y_1 \) is fixed to the link and rotates around the point \( O \). The location vector of the point \( P \) in the \( OXY \) frame is given as \( R_p = A r_p \), where \( A \) is the transformation matrix, \( r_p = [X \ W P]^T \) is the location vector in the \( OX_1Y_1 \) frame, \( x \) is the distance from point \( P \) to the point \( O \) in the \( OX_1 \) direction; \( W_P \) is the transverse displacement of the point \( P \). The deflection \( w_p \) can be described using the Assume Modal Method \([22]\) by two-order mode equation \( w_p = a_1(t)\phi_1(x) + a_2(t)\phi_2(x) \), where \( \phi_1(x) \) and \( a_i(t) \) represent the mode shape and the modal coordinates, respectively.

Denote \( L, I, E, \rho \) and \( S \) be the length, the area moment inertia of the cross-section, Young’s modulus, the density and cross-sectional area of flexible link, respectively. Since the moment of inertia of the joint is much smaller than moments of the link and the tip load, it is not considered in this modeling. The kinetic energy of the SLF manipulator can thus be expressed as:

\[
T = 0.5\rho S \int_0^L \left( \frac{\partial R_p}{\partial t} \right)^T \left( \frac{\partial R_p}{\partial t} \right) \, dx + 0.5m_m \left( \frac{\partial R_L}{\partial t} \right)^T \left( \frac{\partial R_L}{\partial t} \right) + 0.5m_m \left( \frac{\partial^2 w_p}{\partial x^2} + \frac{\partial^2 w_p}{\partial x \partial t} \right)^2 \frac{dx}{x=L} \]

where \( m_m \) and \( J_m \) are mass and moment of inertia of payload, respectively. \( R_L \) which is the location vector of the terminal of the link in the \( OXY \) frame. Let \( k \) be the equivalent stiffness coefficient of the VSD-MRJ, the potential energy of the whole system can be respectively expressed as:

\[
V = 0.5 \int_0^L E I \left( \frac{\partial^2 w_p}{\partial x^2} \right)^2 \, dx + 0.5k(\theta_1 - \theta_2)^2
\]

The dynamic equation of the SLF manipulator can be obtained as:

\[
\ddot{M}\ddot{\Theta} + K\dot{\Theta} + C\Theta = \Phi + \partial \Phi/\partial \Theta - \ddot{M}\ddot{\Theta}
\]

where \( M \in \mathbb{R}^{4x4} \) represents the inertia mass matrix, \( Q = [\theta_1 \ \theta_2 \ a_1 \ a_2]^T \in \mathbb{R}^4 \) is the deflection vector, \( K \in \mathbb{R}^{4x4} \) represents the global stiffness matrix, \( C\Theta \in \mathbb{R}^4 \) is the damping vector, \( F \in \mathbb{R}^4 \) is the input torque vector, \( \partial \Phi/\partial \Theta \) and \( \ddot{M}\ddot{\Theta} \) represents the centripetal force and Coriolis force respectively. For the dynamic model of SLF manipulator with a VSD-MRJ, the matrix \( K \) and vector \( C\Theta \) both are variable and controllable.

### 3. Controller formulation

Considering that the flexible manipulator system is a strong nonlinear system, to reduce the vibration magnitude of the tip, a controller for the vibration suppression of the manipulator system was constructed by using fuzzy control herein.

Due to the independence of the joint damping and stiffness control, two different fuzzy controllers are designed respectively. It is difficult to obtain an accurate control model of the VSD-MRJ. Besides, the more complex the control model will cause the longer the adjustment time of the whole system, which is unfavorable in practical applications. For this reason, control current \( (I_c \text{ and } I_s) \) of the joint as the output variables of fuzzy controllers. The control flow chart of the SLF manipulator system is designed as shown in figure 4. For the damped fuzzy controller, the relative rotational speed \( \nu_r = \frac{\partial(\theta_1 - \theta_2)}{\partial t} \) and relative rotational acceleration \( \alpha_r = \frac{\partial^2(\theta_1 - \theta_2)}{\partial t^2} \) between the input and
output shafts of the joint are used as the input variables of the damping fuzzy controller, while the control current of the joint damping is the output. For the stiffness fuzzy controller of the joint, the system receives the relative rotation angle \( x_r = (\theta_1 - \theta_2) \) and relative rotation speed \( v_r \) as the input variables. NB, NS, ZE, PS and PB are used as 5 membership function of \( v_r \), \( a_r \) and \( x_r \), while stiffness and damping control current both are implemented with 4 triangle membership function: O (zero), S (small), M (medium), and B (big). All inputs of the controller are scaled between -1 and 1, and the range of the output can be set as [0, 3].

![Diagram of the SLF manipulator](image)

**Figure 4.** Control flow chart of the SLF manipulator

**Figure 5.** Fuzzy surface

After defining the membership, the control rules should be defined. The final rules are given in Tables 1 and 2. Control surfaces are obtained from the rule tables above, as illustrated in figure 5.

| \( x_r \) | NB | NS | ZE | PS | PB |
|---|---|---|---|---|---|
| NB | B | B | M | O | O |
| NS | B | M | S | O | O |
| ZE | M | O | O | O | S |
| PS | O | O | S | B | B |
| PB | O | O | M | B | B |

| \( v_r \) | NB | NS | ZE | PS | PB |
|---|---|---|---|---|---|
| NB | B | M | S | O | O |
| NS | M | S | O | O | O |
| ZE | M | S | O | S | M |
| PS | O | O | S | S | M |
| PB | O | O | S | M | B |

**Table 1.** Fuzzy control rules of \( I_k \)

| \( a_r \) | NB | NS | ZE | PS | PB |
|---|---|---|---|---|---|
| NB | B | M | S | O | O |
| NS | M | S | O | O | O |
| ZE | M | S | O | S | M |
| PS | O | O | S | S | M |
| PB | O | O | S | M | B |

**Table 2.** Fuzzy control rules of \( I_C \)

4. **Test and discussion**

4.1. **Experimental setup**

A test platform is conducted to verify the residual vibration suppression performance of the SLF manipulator with the VSD-MRJ. The experimental setup is shown in figure 6. The driving torque generated by a DC motor, an aluminum flexible single beam which rotates only in the horizontal plane with a rectangular cross-section is connected to the output shaft of the joint. For the control system, two encoders are used to measure the relative rotation angle and angular velocity between input and output shafts of the joint. Those measured signals are fed back to the Micro-control box (dSPACE), and then two voltage signals are output. The voltage signals are converted into two desired current signals by two developed power amplifiers that are used for the stiffness and damping adjustments independently. In
addition, the deformation vibration of the beam is measured by the strain gauge and the reduced ratio of reducer is 4.

![Diagram of the ground experiment system]

**Figure 6.** Ground experiment system: system photo (left) and scheme (right)

**4.2 Result and discussion**

The objective of the experiment is to make the flexible manipulator system rotate to a desired angle of 21.75°. A trapezoidal trajectory is applied and the rotation speed of the DC motor is 40 RPM. The sampling frequency of the whole experimental system is 2000 Hz. The experimental results include the response of the input-output shafts and the tip deformation of the flexible link.

According to the designed fuzzy controller in Section 3, the fuzzy controller is adjusted. The input gain of $x_r$, $v_r$, and $a_r$ in the fuzzy controllers are 0.2, 0.1, and 0.2, respectively. The output gain of the stiffness controller and the damping controller is 0.8 and 0.9, respectively. Figure 7 and 8 illustrate the responses of the SLF manipulator system under ‘VD control’, ‘VS control’, ‘VSD-control’ and ‘Passive-Rigid’ cases. ‘VD control’ and ‘VS control’ indicates that currents $I_c$ and $I_k$ are co-controlled only by fuzzy controller respectively; ‘VSD-MR’ indicates that the currents $I_k$ and $I_c$ both are controlled by the fuzzy controller; ‘Passive-Rigid’ indicates that the stiffness adjustment current $I_k$ is 1.2A and the damping adjustment current $I_c$ is 0A.

![Graph of absolute deviation angle responses]

**Figure 7.** Absolute deviation angle responses

![Graph of absolute tip deformation]

**Figure 8.** Absolute tip deformation
As can be seen in results, the VSD fuzzy controller has a good performance in the suppression of the residual vibration. Figure 7 shows the time histories of absolute deviation angle responses under 4 cases. The maximum value of ‘Passive-Rigid’ cases is the smallest and the deviation angle can eventually be stabilized at 0 degrees; However, when it comes to the ability of residual vibration suppression, ‘VS control’, ‘VD control’, and ‘VSD control’ all greatly reduce the system stability time with a decrease of 24.9%, 26.9%, and 27.2% respectively. Table 3 shows comparisons of vibration-attenuation performance and the parentheses in the table are the rates of change compared to ‘Passive-Rigid’. According to comparison results, the ‘VSD control’ has the best performance. Comparing to ‘Passive-Rigid’, the maximum absolute value of the tip deformation under ‘VSD control’ is decreased by a factor of 11.2%.

| Setting time (sec) | Maximum absolute value | Deviation angle (Deg.) | Tip deformation (Deg.) |
|-------------------|------------------------|------------------------|------------------------|
| Passive-Rigid     | 1.622                  | 4.000                  | 11.97                  |
| VD control        | 1.187 (-26.9%)         | 4.374 (+9.35%)         | 11.20 (-7.5%)          |
| VS control        | 1.219 (-24.9%)         | 4.966 (+24.15%)        | 11.23 (-6.2%)          |
| VSD control       | 1.181 (-27.2%)         | 4.185 (+4.625%)        | 10.63 (-11.2%)         |

According to the comparison between ‘VD control’ and ‘VS control’, the ‘VD control’ have fewer values in maximum deviation angle and maximum tip deformation than ‘VS control’. It is due to joint assembly and processing problems, and the adjustable range of the joint stiffness is much smaller than the adjustable range of the damping. From figure 7 it is worth noting that there are still small amplitude vibrations after 4.4 seconds, which are caused by the residual vibration of the flexible link. There is less oscillation in the curve of the ‘VSD control’, and the ‘VSD control’ has an ability to suppress the small-amplitude vibrations efficiently.

5. Conclusion
In this study, a rotary VSD-MRF is used to control the residual vibration of the SLF manipulator. On the basis of the Lagrange approach and Assume Modal Method, the dynamic model of a SLF manipulator with the VSD-MRJ was established. Considering the strong nonlinear system, a fuzzy controller was built. An experimental test platform comprising the SLF manipulator, a VSD-MRJ, and a fuzzy controller was built to evaluate its performance under Trapezoidal trajectory driving. The analysis and test results verified that the VSD-MRJ have the best residual vibration suppression performance than the traditional passive, VS control and VD control system. Experimental results indicate that the setting time value and tip deformation value are decreased by a factor of 27.2% and 11.2% than these of the Passive-Rigid.

Acknowledgments
This work was financially supported by graduate scientific research and innovation foundation of Chongqing, China (Grant No. CYB17023, No. CYB19009 and No. CYB20008). These supports are gratefully acknowledged.

References
[1] Lee N N, Burdick J W, Backes P, Pellegrino S, Hogstrom K, Fuller C and Wu Y H 2016 Architecture for in-space robotic assembly of a modular space telescope. Journal of Astronomical Telescopes, Instruments, and Systems, 2(4), 041207.
[2] N Seth and Rattan K S 1991 Vibration reduction in computer controlled machines. IEEE International Conference on Systems Engineering. IEEE.
[3] Yang D, Xiong Q, Wang Y and Zhai D 2009 Passive control for uncertain singular bilinear systems. In 2009 Chinese Control and Decision Conference. IEEE. pp 3874-3877
[4] Yavuz Ş, Malgaca L and Karagülle H 2016 Vibration control of a single-link flexible composite
manipulator. *Composite Structures*. 140 684-691.

[5] Meng D, She Y, Xu W, Lu W and Liang B 2018 Dynamic modeling and vibration characteristics analysis of flexible-link and flexible-joint space manipulator. *Multibody System Dynamics*, 43(4), 321-347.

[6] Aggogeri F, Borboni A, Merlo A, Pellegrini N and Ricatto R 2016 Real-time performance of mechatronic PZT module using active vibration feedback control. *Sensors*, 16(10), 1577.

[7] Gao H, He W, Zhou C and Sun C 2018 Neural network control of a two-link flexible robotic manipulator using assumed mode method. *IEEE Transactions on Industrial Informatics*, 15(2), 755-765.

[8] Mohamed Z, Martins J M, Tokhi M O, Da Costa J S and Botto M A 2005 Vibration control of a very flexible manipulator system. *Control Engineering Practice*, 13(3), 267-277.

[9] Biswas S K and Klafter R D 1988 Dynamic modeling and optimal control of flexible robotic manipulators. In *Proceedings. 1988 IEEE International Conference on Robotics and Automation* (pp. 15-20). IEEE.

[10] Tourassis V D and Neuman C P 1985 Robust nonlinear feedback control for robotic manipulators. In *IEEE Proceedings D (Control Theory and Applications)* (Vol. 132, No. 4, pp. 134-143). IET Digital Library.

[11] Sun S S, Ning D H, Yang J, Du H, Zhang S W and Li W H 2016 A seat suspension with a rotary magnetorheological damper for heavy duty vehicles. *Smart Materials and Structures*, 25(10), 105032.

[12] Ahmadian M, Song X and Sandu C 2005 Designing an adaptive semiactive magneto-rheological seat suspension for heavy truck applications. In *Smart structures and materials 2005: damping and isolation* (Vol. 5760, pp. 247-255). International Society for Optics and Photonics.

[13] Hiemenz G J, Hu W and Wereley N M 2008 Semi-active magnetorheological helicopter crew seat suspension for vibration isolation. *Journal of Aircraft*, 45(3), 945-953.

[14] Daniel C, Hemalatha G, Magdalene A, Tensing D and Manoharan S S 2017 Magnetorheological Damper for Performance Enhancement Against Seismic Forces. In *International Congress and Exhibition" Sustainable Civil Infrastructures: Innovative Infrastructure Geotechnology"* (pp. 104-117). Springer, Cham.

[15] Dong X M 2015 Design and characterization of axial flux permanent magnet energy harvester for vehicle magnetorheological damper. *Smart Materials and Structures* 25(1):1–20.

[16] Nguyen Huynh T C and Sharf I 2010 Capture of Spinning Target With Space Manipulator Using Magneto Rheological Damper. In *AIAA Guidance, Navigation, and Control Conference* (p. 7753).

[17] Dong X M 2013 Semi-active control of magneto-rheological variable stiffness and damping seat suspension with human-body model. *International journal of vehicle design*, 63(2-3), 119-136.

[18] Deng H, Deng J, Yue R, Han G, Zhang J, Ma M and Zhong X 2019 Design and verification of a seat suspension with variable stiffness and damping. *Smart Materials and Structures*, 28(6), 065015.

[19] Sun S S, Deng H, Du H, Li W H, Yang J and Liu G *et al.* 2015 A compact variable stiffness and damping shock absorber for vehicle suspension. *Mechatronics IEEE/ASME Transactions on*, 20(5), 1-9.

[20] Dong X M, Liu W Q, An G P, Zhou Y Q, Yu J Q and Lin Q 2018 A novel rotary magnetorheological flexible joint with variable stiffness and damping. *Smart Materials and Structures*, 27(10), 105045.

[21] Dong X M, Yu M, et al. (2010) ‘Absorbing control of Magneto rheological variable stiffness and damping system under impact load’, *Transaction of the Chinese Society for Agricultural Machinery*, Vol. 41, No. 3, pp.20–26.

[22] Ding W and Shen Y 2017 Analysis of transient deformation response for flexible robotic manipulator using assumed mode method. In *2017 2nd Asia-Pacific Conference on Intelligent Robot Systems (ACIRS)* (pp. 331-335). IEEE.