8Dim calculations of the third barrier in \(^{232}\text{Th}\) and a conflict between theory and experiment on uranium nuclei.

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We find the height of the third fission barrier \(B_{III}\) and energy of the third minimum \(E_{III}\) in \(^{232}\text{Th}\) using the macroscopic - microscopic model, very well tested in this region of nuclei. For the first time it is done on an 8-dimensional deformation hypercube. The dipole distortion is included among the shape variables to assure that no important shapes are missed. The saddle point is found on a lattice containing more than 50 million points by the immersion water flow (IWF) method. The shallow third minimum, \(B_{III} - E_{III} \approx 0.36\) MeV, agrees with experimental data of Blons et al. This is in a sharp contrast with the status of the IIrd minima in \(^{232-238}\text{U}\): their experimental depth of \(\geq 3\) MeV contradicts all realistic theoretical predictions. We emphasize the importance of repeating the experiment on \(^{232}\text{Th}\), by a technique similar to that used in the uranium nuclei, for settling the puzzle of the third minima in actinides.

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Search for a hyperdeformation (HD), i.e., extremely elongated shapes of atomic nuclei (spheroid with a major to minor axis ratio of 3:1), is one of the most difficult challenges of modern nuclear structure studies. There is still no convincing evidence for discrete gamma transitions of HD rotational bands. The only experimental evidence for hyperdeformation, except in light nuclei, has been reported in the region of light actinides, in particular in uranium isotopes \(^{232}\text{U}\), \(^{234}\text{U}\) and \(^{236}\text{U}\), see [1] and references therein. In these experiments, the fission probability was studied as a function of the excitation energy via different reactions. Strong resonances were observed and a fine structure of some interpreted as a signature of multiple hyperdeformed bands. Measurements of the angular distribution of the fragments arising from the induced fission support this interpretation. Moreover, by measuring the angular distribution of the fission fragments one can obtain some information about the spin and \(K\) - quantum number. Due to the predicted static mass-asymmetry of the third minima, their intrinsic parity would be broken, so the low-energy excitations should show a pattern of alternating parity (even nuclei) or parity doublet (odd nuclei) bands, different from that in the second minima. Since the third minima are predicted axially symmetric, the excitations would have an approximate (up to a Coriolis mixing, much weaker than in the first or second minimum) \(K\) quantum number.

The current experimental status of the third barrier in \(^{232}\text{Th}\) is different from that in uranium isotopes. No recent data exist, the only available are those from the prior 30 years [2,3]. The experiment conducted by Blons et al pointed to a shallow third minimum (no deeper than 0.5 MeV). Only a little more than 1 MeV deep third minimum in a neighboring nucleus \(^{232}\text{Pa}\) was recently reported by the Munich-Debrecen group [6]. The technique used to resolve observed resonances was the same as in the study of uranium nuclei by the same group. Deep third minima in uranium (3 ÷ 4 MeV) disagree with all calculations [7,13] except those of S. ´Cwiok et al, within the Woods-Saxon model [14,15]. Self-consistent calculations based on the Skyrme SkM* interaction do not give a hyperdeformed minimum in \(^{232}\text{Th}\) at all. With the help of a temperature effect, a slight minimum appears, but does not exceed 500 keV. Moreover, with an increasing excitation, the depth of this minimum decreases again. Situation is very similar in \(^{232}\text{U}\) and \(^{234}\text{U}\) nuclei [8]. Self-consistent calculations with the Gogny D1S interaction do not give third minima in those nuclei [10,12] as well. Moreover, for \(^{236}\text{U}\), a recent result of Möller et al [13] within the macro-micro approach also does not support a deep minimum reported in [17]: one can read from the map (Fig. 8. in ref. [13]) that the minimum is not deeper than 300 keV, ten times smaller than the experimental value.

In fact, hyperdeformed minima predicted in [14,16] were quite abundant: they not only appeared in nuclei where nobody saw them in experiment, but were double, with different octupole deformations \(\beta_3 \approx 0.3\) and 0.6. Using the same model, we have just showed in [19] that both types of the IIIrd minima in uranium nuclei disappear completely after proper nuclear shapes are considered. The more mass-asymmetric minima were unphysical from the beginning. Their quadrupole moment is large, \(Q_{20} \approx 170\) b, which situates them behind the barrier, closer to the scission point. The existence of minima with a smaller mass asymmetry (and quadrupole moments) requires a more detailed study. Finding the barrier in many-dimensional space requires hypercube

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calculations. This is the case of $^{232}$Th, where the less mass-asymmetric minimum is the deeper one.

Previously, we found in [19] a tentative upper limit of 330 keV for the IIId saddle in $^{232}$Th and 0 in even uranium isotopes $^{232-236}$U by 6Dim hypercube calculations and by probing various trial 8Dim fission paths. The main aim of the present study is to ascertain the prediction for $^{232}$Th, showing that a third minimum is there. To accomplish this, an 8-dimensional grid calculation has been done. All deformations were treated on the same footing, without introducing any subdivision into relevant and irrelevant subspaces. Let us emphasize that till now only 5-dimensional macroscopic-microscopic calculations of fission saddles were available in the literature [13, 20, 21].

There is one more important motivation for such studies. To distinguish between resonances in the second and third minimum, the excitation energy should be higher than the first barrier but simultaneously lower than the second one. We show that our multidimensional calculations predict such a possibility in $^{232}$Th, which is a natural candidate for the future experimental study.

The energy is calculated within the microscopic-macroscopic method with a deformed Woods-Saxon potential. Nuclear shapes are defined in terms of the nuclear surface [22]. Since we found that nonaxiality in the region of the third barrier is less important than a proper treatment of the axially symmetric shapes, we parameterize the surface as

$$R(\theta) = c(\{\beta\}) R_0 (1 + \sum_{\lambda=1} \beta_\lambda Y_\lambda(\theta)), \quad (1)$$

where $c(\{\beta\})$ is the volume fixing factor. The macroscopic energy is calculated using the Yukawa plus exponential model [23] with parameters specified in [24]. All other parameters of the model are the same as in a number of our previous studies, e.g. in [25-27]. A hyperdeformed minimum was searched by exploring energy surfaces in a region of deformations beyond the second minimum. We generate the grid in axially symmetric deformations:

$$\beta_{10} = -0.35 \ (0.05) \ 0.00; \ \beta_{20} = 0.55 \ (0.05) \ 1.50, \quad (2)$$
$$\beta_{30} = 0.00 \ (0.05) \ 0.35; \ \beta_{40} = -0.10 \ (0.05) \ 0.35,$$
$$\beta_{50} = -0.20 \ (0.05) \ 0.20; \ \beta_{60} = -0.15 \ (0.05) \ 0.15,$$
$$\beta_{70} = -0.20 \ (0.05) \ 0.20; \ \beta_{80} = -0.15 \ (0.05) \ 0.15.$$

Numbers in the parentheses specify the step with which the calculation is done for a given variable. Thus, we finally have the values of energy at a total of 50803200 grid points. On a such giant 8-dimensional grid the IWF saddle point searching method is used.

Let us emphasize that our model, without the $\lambda = 1$ term in (1), very well reproduces first [23] and second barriers [26] and second minima [27] in actinide nuclei.

One could hope that an extrapolation to more elongated shapes would be still valid within such a parametrization. What then causes troubles beyond the second peak? They result from the truncation of the expansion (3) that restricts possible shapes for large deformations. Two shapes close to the IIId saddle are shown in Fig. 1, one from the 8D calculation (nonzero $\beta_{10}-\beta_{80}$, in red) and the other, from the 7D calculation $\beta_{20} = \beta_{80}$ (in blue). Even if they seem not much different, the difference in barrier is 4 MeV. It appears that the dipole deformation $\beta_{10}$, usually associated only with a shift of the center of mass, effectively makes up for truncated multipoles in very deformed, mass-asymmetric configurations. We emphasize that our program keeps automatically the center of mass at the coordinate origin, thus $\beta_{10}$ serves only probing somewhat different shapes.

A modification of the energy landscape by including $\beta_{10}$ is shown in Fig. 2. Since dipole is the first spherical harmonic (Eq.1), one can suspect its pronounced effect. Indeed, that effect is significant. Not only the height of the third saddle is clearly reduced, but the whole landscape changed. The result indicates a large effect of new shapes, unattainable previously. Figures show also examples of two fission paths. In the version of calculation without the dipole (blue trajectory), after passing through the second saddle, the nucleus falls into a deep third minimum. To be split, it needs to tunnel through a more than 4 MeV barrier. In the case with included $\beta_{10}$ (red trajectory), after passing the second barrier nucleus can easily split. One can notice that $\beta_{10}$ deformation has almost no effect on the position and height of the second saddle. Precise lowering of the third barrier by $\beta_{10}$ may be read from Fig. 3 as 4.1 MeV. Beside the significant reduction, one can also see a change in the barrier shape. This rather dramatic change of the fission path in the
multidimensional deformation space has a direct effect on fission half-life.

A detailed information about the structure of the fission barrier in $^{232}$Th is collected in Table I. The recent theoretical results and experimental data for $^{232}$U are also given. It is seen that the model calculation well reproduces essential features of the fission barrier up to superdeformed shapes in both nuclei. The inner fission barrier $B_{II}^{th}$ has been found according to the prescription presented in [25].

Here, a comment is needed. Usually the calculated first saddles are substantially higher than experimental values but the opposite situation takes place in the light Thorium isotopes what is called in the literature the "Thorium anomaly". In our case the nonaxiality reduces the first saddle for light Thorium only non-significantly, to a nearly proper height (see column 3 of Table I). Thus, the mentioned anomaly does not occur in our case (we use the modern data of [18]).

In order to determine the second peak $B_{II}^{th}$, a mass asymmetry is included, details can be found in [25]. One can see that our calculated second minimum and second barrier reasonably agree with experiment. As already mentioned, the experimental situation (when it comes to the depth of hyperdeformed minima) in two isobars: $^{232}$Th and $^{232}$U is completely different. While in $^{232}$U, the IIIrd minimum is quite deep, in $^{232}$Th it is very shallow. Our results support the existence of a shallow third minimum in $^{232}$Th as in old experiments of Blons and thus agree with the majority of modern theoretical models.

A designed experiment for $^{232}$Th, by a technique described in ref [1, 29], would be crucial for solving the mystery of the third minima in actinides. Particularly promising are experiments employing highly monochromatic $\gamma$-ray beams for photofission studies. With a high quality of photon and spectral intensity, exceeding the performance of existing facilities by several orders of magnitude seems to be possible in the near future [30]. If the result of Blons et al is confirmed, we will have to understand why between $^{232}$Th and $^{232}$U, two beta decays away, the energy landscape changes so dramatically. On the other hand, if in the future experiment, a depth of the IIIrd minimum is obtained similar as that in $^{232}$U, we will have a total contradiction between theory and experiment. Assuming that the existing resonances cannot be interpreted otherwise, all current meaningful theoretical models would have to be reconstructed anew to give deep hyperdeformed minima.

One can expect that in hyperdeformed nuclei, some high particle states at normal deformation become occupied with increasing deformation. Since there are the orbits that are occupied in superheavy nuclei at moderate deformations, the whole question may have some impact on the understanding of superheavy nuclei.

In summary:

(i) We have presented for the first time an 8D hypercube calculation for $^{232}$Th. To find the third barrier on a giant grid, the IWF method has been applied. After a
FIG. 3: Energy along a sequence of smoothly elongating shapes, beginning at the superdeformed minimum. Appropriate shapes along trajectories are shown.

(proper inclusion of the dipole deformation we found the depth of the third minimum of about 0.36 MeV. This would rather exclude spectroscopic studies in the IIIrd well.

(ii) Including a dipole distortion lowers the third saddle by more than 4 MeV. It seems likely that, with the shape parametrization [1], the dipole deformation is important everywhere, where large elongation and necking is combined with a sizable mass asymmetry. For example, it may be the case of the Jacobi shape transition at high spins in medium-heavy nuclei.

(iii) New experimental study dedicated to hyperdeformation in $^{232}$Th seems essential for the understanding of the third minima in actinide nuclei.

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