Isometries of half supersymmetric time-like solutions in five dimensions

J B Gutowski\(^1\) and W A Sabra\(^2\)

\(^1\) DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, UK  
\(^2\) Centre for Advanced Mathematical Sciences and Physics Department, American University of Beirut, PO Box 11-0236, Riad El-Solh 1107-2020, Beirut, Lebanon

E-mail: J.B.Gutowski@damtp.cam.ac.uk and ws00@aub.edu.lb

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Abstract
Spinorial geometry techniques have recently been used to classify all half supersymmetric solutions in gauged five-dimensional supergravity with vector multiplets. In this paper we consider solutions for which at least one of the Killing spinors generates a time-like Killing vector. We obtain coordinate transformations which considerably simplify the solutions, and in a number of cases, we obtain explicitly some additional Killing vectors which were hidden in the original analysis.

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1. Introduction

The classification of supersymmetric supergravity solutions is of interest in a number of contexts, such as the conjectured AdS/CFT correspondence, the microscopic analysis of black hole entropy and a deeper understanding of string theory dualities. In particular, in five dimensions, considerable progress has been made in the classification and analysis of solutions preserving various fractions of supersymmetry of \( N = 2 \) gauged supergravity \([1–11]\). In these theories, the only solution which preserves all of the supersymmetries is AdS\(_5\) with vanishing gauge field strengths and constant scalars. The Killing spinor equations when expressed in terms of Dirac spinors are linear over \( \mathbb{C} \), implying that supersymmetric solutions preserve 2, 4, 6 or 8 of the supersymmetries. However, in the ungauged theory it was found that supersymmetric solutions can only preserve 4 or 8 of supersymmetries \([12, 13]\). The first supersymmetric solutions which were constructed \([1]\) have naked singularities or naked closed time-like curves. Domain walls and magnetic strings were also constructed in \([2]\). Later, a more systematic approach, motivated by the results of Tod \([14]\), was employed to construct and classify supersymmetric solutions. Quarter supersymmetric solutions of minimal gauged
$N = 2$, $D = 5$ supergravity were classified in [3]. The first asymptotically $\text{AdS}_5$ solutions with no closed time loops or naked singularities were constructed for the minimal supergravity theory in [4]. Generalizations to solutions with null Killing spinors as well as to solutions with vector multiplets were later given in [5, 6].

Half supersymmetric solutions possess two Dirac Killing spinors thus enabling the construction of two Killing vectors as bilinears in the Killing spinors. These vectors could be either time-like or null and thus there exist three classes of solutions, depending on the nature of the Killing spinors and vectors considered. All of these solutions have been fully classified in [7, 8], using the spinorial geometry method. In this approach to the analysis of the Killing spinor equations, the Killing spinors are expressed in terms of differential forms [15–17], which are further simplified by making appropriately chosen gauge transformations. These techniques were originally developed to analyse more complicated supergravity theories in 10 and 11 dimensions, see for example [18–21]; it is also straightforward to use them to analyse lower dimensional theories as well. The remaining class of supersymmetric solutions, preserving $3/4$ of the supersymmetry, has also been considered recently in [9] where it was shown, again using spinorial geometry techniques, that these solutions are merely cosets of $\text{AdS}_5$.

It must be noted that supersymmetric solutions are not automatically solutions of the equations of motion. For the $1/4$ supersymmetric time-like solutions, one must solve the gauge equations and the Bianchi identities in addition to the Killing spinor equations. In the null case, however, one must additionally solve one of the components of the Einstein equations of motion. For the $1/2$ supersymmetric solutions, considered in our present work, it was found that supersymmetry together with Bianchi identities is sufficient to imply that all the Einstein, gauge and scalar equations of motion are satisfied.

Our present work is concerned with half supersymmetric solutions of gauged $N = 2$, $D = 5$ supergravity. We examine the time-like solutions of [7], which split into six different classes of solutions. Three of these classes are already presented in the most explicit possible form, and their metrics cannot be simplified in any meaningful fashion using further coordinate transformations. The geometries of these solutions correspond to certain types of fibration over squashed $S^3$, $\mathbb{H}^3$, a three-dimensional Nil-manifold, $\mathbb{H}^2 \times \mathbb{H}^2$, $\mathbb{H}^2 \times \mathbb{R}^2$ or $\mathbb{H}^2 \times S^2$. Moreover, as it turns out to be particularly straightforward to investigate black hole near-horizon geometries associated with these solutions, we shall concentrate on the simplification of the remaining three classes of solutions, which appear to have a rather more complicated structure.

The paper is organized as follows. In section 2 we present the basics of the theories we are studying and their half supersymmetric solutions which are presented in six classes. In section 3, we simplify the three remaining types of solution which are not fibrations of the type described above, and we obtain some hidden symmetries of these solutions.

2. Half supersymmetric time-like solutions

In this section, we present a summary of all half supersymmetric solutions of gauged $N = 2$, $D = 5$ supergravity, for which at least one of the Killing spinors generates a time-like Killing vector. Such solutions were completely classified in [7] and are specified in terms of a spacetime metric, together with a number of scalars $\lambda^I$ and 2-form gauge field strengths $F^2$. In the classification of these solutions, the Killing spinors were expressed as differential forms on $\Lambda^\ast (\mathbb{R}^2) \otimes \mathbb{C}$. One starts with the most generic form for a symplectic Majorana spinor; then, by applying appropriately chosen gauge transformations, the spinor can be written in a number of particularly simple canonical forms. The conditions for quarter supersymmetric
solutions with a time-like Killing vector (generated from the Killing spinor) are then derived. The conditions are then substituted into the Killing spinor equations, acting on the second spinor, and further constraints on the Kähler base are then determined. Using the integrability conditions of the Killing spinor equations, it was demonstrated that for a given background preserving at least half of the supersymmetry, where at least one of the Killing spinors generates a time-like Killing vector, all of the Einstein, gauge and scalar field equations of motion hold automatically provided that the Bianchi identity is satisfied.

In listing all possible solutions, it will be particularly useful to introduce coordinates \( t, x \) and decompose the mostly negative signature metric as

\[
d s^2 = f^4 (dt + \Omega)^2 - f^{-2} d\Omega^2, \tag{1}\]

where \( \frac{\Omega}{\psi} \) is a Killing vector which is a symmetry of the whole solution, \( d\Omega^2 \) is the metric on a 4-manifold \( B \) which is constrained by supersymmetry to be Kähler; \( f \) is a \( t \)-independent function and \( \Omega \) is a \( t \)-independent 1-form on \( B \). The bosonic action of the theory is

\[
S = \frac{1}{16\pi G} \int \left( (-R + 2\chi^2 V) \ast 1 + Q_{IJ}(dX^I \wedge \ast dX^J - F^I \wedge \ast F^J) - \frac{C_{IJK}}{6} F^I \wedge F^J \wedge A^K \right) \tag{2}
\]

where \( I, J, K \) take values \( 1, \ldots, n \). The scalar fields \( X^I \) are subject to the constraint

\[
X_I X^I = 1, \tag{3}
\]

where

\[
X_I = \frac{1}{6} C_{IJK} X^J X^K. \tag{4}
\]

The coupling \( Q_{IJ} \) and the scalar potential depend on the scalars via

\[
Q_{IJ} = \frac{9}{2} X_I X_J - \frac{1}{6} C_{IJK} X^K, \quad \chi = 9 V_I V_J (X^I X^J - \frac{1}{2} Q_{IJ}), \tag{5, 6}
\]

where \( V_I \) are constants.

Bosonic backgrounds are said to be supersymmetric if there exists a spinor \( \epsilon^a \) for which the supersymmetry variations of the gravitino and dilatino vanish in the given background. For the gravitino this requires

\[
\left[ \nabla_\mu + \frac{1}{8} X_I (\gamma_\mu F^I \rho_\sigma \gamma^{\rho \sigma} - 6 F^I \rho_\sigma \gamma^{\rho \sigma}) \right] \epsilon^a - \frac{\chi}{2} V_I (X^I \gamma_\mu - 3 A^I_\mu) \epsilon^{ab} \epsilon^b = 0, \tag{7}
\]

and for the dilatino it requires

\[
\left[ \frac{1}{4} (Q_{IJ} \gamma^{\mu \nu} F^J_{\mu \nu} + 3 \gamma^a \nabla_\mu X_I) \right] \epsilon^a - \frac{3 \chi}{2} V_I \epsilon^{ab} \epsilon^b \right] \partial_\mu X^I = 0. \tag{8}
\]

The Einstein equations derived from (2) are given by

\[
R_{\mu \nu} + Q_{IJ} \left( F^I_{\mu \lambda} F^J_{\nu \lambda} - \nabla_\mu X^I \nabla_\nu X^J - \frac{1}{6} g_{\rho \mu} F^{I}_{\rho \nu} F^{J}_{\mu \nu} \right) - \frac{2}{3} g_{\rho \mu} \chi^2 V = 0. \tag{9}
\]

The Maxwell equations are

\[
d(Q_{IJ} \wedge F^J) = -\frac{1}{6} C_{IJK} F^J \wedge F^K. \tag{10}
\]
The scalar equations are
\[
\left[-d(\star dX_I) + \left(X_M X^P C_{NP} L - \frac{1}{6} C_{MNP}\right) (F^M \wedge \star F^N)
- dX_M \wedge \star dX_N - \frac{3}{2} \chi^2 V_M Q^M L Q^{NP} L C_{NP} \text{ vol} \right] \partial_r X^I = 0.
\] (11)

We refer the reader to [7] for the details of the analysis of the Killing spinor equations. The solutions can be divided into six classes.

1. The Kähler base metric has coordinates $\tau, \eta, u, v$ and is given by
\[
ds^2_B = H^2 \left( d\tau + \eta \left( \frac{\partial Y}{\partial u} + H^2 \sin Y \right) du + \eta \frac{\partial Y}{\partial v} dv \right)^2 + H^2 \left( d\eta - \eta \cot Y \frac{\partial Y}{\partial u} du - \eta \left( \cot Y \frac{\partial Y}{\partial v} + \frac{1}{v} \right) dv \right)^2 + \frac{dv^2}{H^2} + H^2 v^2 \sin^2 Y du^2
\] (12)
\[
\Omega = -\frac{1}{2cv} \left( H^2 + \frac{c^2 v^2}{f^6} \right) d\tau + \eta \left( \frac{1}{2} \theta \sin Y - H^2 \frac{\partial Y}{\partial v} \right) dv
- \eta \left( \theta H^2 \cot Y + \frac{c}{f^5} H^2 v - \frac{1}{cv} H^4 \right)
+ \frac{H^2}{cv} \left( \frac{\partial Y}{\partial u} + H^2 \sin Y \right) du
\] (13)
and the scalars $X^I$ and function $f$ are constrained by
\[
\frac{\partial}{\partial u} \left( \frac{X_I}{f^2} \right) = \frac{\chi H^2 V_I}{c} \sin^2 Y,
\] (14)
\[
\frac{\partial}{\partial v} \left( \frac{X_I}{f^2} \right) = \frac{1}{v} \left( \frac{\chi V_I (\cos Y - 1)}{c} - \frac{X_I}{f^2} \right).
\] (15)

For this solution, the scalars $X^I$ and $f$ depend only on $u, v; c, \theta$ are constants with $c \neq 0$, and $H, Y$ are functions of $u, v$ such that $\sin Y \neq 0$, which are constrained by
\[
\frac{\partial H^2}{\partial u} = H^2 v \sin^2 Y \left( 3 \frac{\chi cv V_I X^I}{f^4} - \theta \right),
\] (16)
\[
\frac{\partial H^2}{\partial v} = -\frac{cv}{f^4} \left( 3 \frac{\chi V_I X^I}{f^2} + \frac{c^2 v^2}{f^6} \right) + \cos Y \left( 3 \frac{\chi V_I X^I}{f^4} - \theta \right),
\]
and
\[
\frac{\partial Y}{\partial u} = \sin Y \left( -H^2 + 3 \frac{\chi cv V_I X^I}{f^4} + \frac{c^2 v^2}{f^6} \right) + \frac{v}{2} \sin 2Y \left( 3 \frac{\chi cv V_I X^I}{f^4} - \theta \right),
\]
\[
\frac{\partial Y}{\partial v} = -\frac{1}{H^2} \sin Y \left( 3 \frac{\chi cv V_I X^I}{f^4} - \theta \right).
\] (17)

If $\theta \neq 0$, then one can integrate the constraints (14) to obtain
\[
X_I = f^2 \left( q I + \frac{\chi}{c} \left( \frac{c^2 v}{f^6 \theta} - \frac{H^2}{\theta v} - 1 \right) V_I \right).
\] (18)
The gauge field strengths are given by
\[ F^I = d \left[ f^2 X^I (dt + \Omega) + \frac{cv}{f^4} (dt + \eta H^2 \sin Y du) \right. \]
\[ \left. - \frac{3\chi \eta}{f^2} \sin Y \left( X^I X^J - \frac{1}{2} Q^{IJ} \right) V_I (-H^2 v(1 + \cos Y) du + dv) \right]. \]  

(20)

(2) For the second class of solutions, one can choose a coordinate \( v \) on \( B \) together with three \( v \)-independent 1-forms \( \sigma^i (i = 1, 2, 3) \) on \( B \) orthogonal to \( \frac{\partial}{\partial v} \). One also has constants \( c, \theta \) \((c \neq 0)\) and the solution takes one of the three types according to whether \( c\theta \) is negative, zero or positive. If \( \theta \neq 0 \), then
\[ ds_B^2 = \frac{1}{\theta v + c^2 v^2 f^{-6}} dv^2 + \frac{v}{|\theta|} ((\sigma^2)^2 + (\sigma^3)^2) + \frac{v}{\theta^2} (\theta + c^2 v f^{-6})(\sigma^1)^2, \]
and if \( \theta = 0 \), then
\[ ds_B^2 = \frac{1}{c^2 v^2 f^{-6}} dv^2 + 4c^8 f^{-6} v^2 (\sigma^1)^2 + 2c^3 v((\sigma^2)^2 + (\sigma^3)^2). \]

(21)

The 1-forms \( \sigma^i \) satisfy
\[ d\sigma^i = -\frac{1}{2} \epsilon_{ijk} \sigma^j \wedge \sigma^k : \text{if } c\theta > 0, \]
\[ d\sigma^1 = \sigma^2 \wedge \sigma^3, \quad d\sigma^2 = \sigma^1 \wedge \sigma^3, \quad d\sigma^3 = -\sigma^1 \wedge \sigma^2 : \text{if } c\theta < 0, \]
\[ d\sigma^1 = \sigma^2 \wedge \sigma^3, \quad d\sigma^2 = d\sigma^3 = 0 : \text{if } c\theta = 0. \]

If \( \theta \neq 0 \), then
\[ \Omega = -\frac{cv}{|\theta|} f^{-6} \sigma^1, \]
whereas if \( \theta = 0 \), then
\[ \Omega = 2c^3 v f^{-6} \sigma^1. \]

(23)

(24)

In all cases, the scalars \( f \) and \( X^I \) are constrained by
\[ X_I = \frac{f^2}{c} \left( -2 \chi V_I + \frac{\rho_I}{\sqrt{2}v} \right) \]
for constants \( \rho_I \) and
\[ F^I = d(f^2 X^I dt). \]

(25)

(26)

(3) For the third class of solutions one can again choose a coordinate \( v \) on \( B \) together with three \( v \)-independent 1-forms \( \sigma^i (i = 1, 2, 3) \) on \( B \), orthogonal to \( \frac{\partial}{\partial v} \). For these solutions, the scalars \( X^0 \) are constant, and it is convenient to define
\[ \Lambda = c\theta + 9\sqrt{2} \chi^2 (X^I X^I - \frac{1}{4} Q^{IJ}) V_I V_J \]
for constants \( c, \theta \) \((c \neq 0)\). The scalar \( f \) is given by
\[ f^2 = \sqrt{2}cv. \]

(27)

\[ \Lambda^2 \]

(28)

The solution takes one of the three types. If \( \Lambda \neq 0 \), then
\[ ds_B^2 = \frac{1}{\frac{1}{2\sqrt{2}cv} - \theta v + \frac{3}{c} V_I X^I} dv^2 + \frac{cv}{|\Lambda|} ((\sigma^2)^2 + (\sigma^3)^2) \]
\[ + \frac{c^2}{\Lambda^2} \left( \frac{1}{2\sqrt{2}cv} - \theta v + \frac{3}{c} V_I X^I \right) (\sigma^1)^2, \]

(29)
and if $\Lambda = 0$, 
\[
\begin{align*}
ds_B^2 &= \frac{1}{(\frac{1}{2\sqrt{2}}v - \theta v + \frac{1}{2}v J)} \, dv^2 + \sqrt{2}cv((\sigma^2)^2 + (\sigma^3)^2) \\
&\quad + 2c^2\left(\frac{1}{2\sqrt{2}}v - \theta v + \frac{3}{c}v J\right)(\sigma^1)^2. 
\end{align*}
\tag{30}
\]

The 1-forms $\sigma^i$ satisfy 
\[
\begin{align*}
d\sigma^i &= -\frac{1}{2}\epsilon_{ijk}\sigma^j \wedge \sigma^k, & \text{if } \Lambda > 0, \\
d\sigma^1 &= \sigma^2 \wedge \sigma^3, & \text{if } \Lambda < 0, \\
d\sigma^3 &= 0 & \text{if } \Lambda = 0.
\end{align*}
\]

If $\Lambda \neq 0$, then 
\[
\begin{align*}
\Omega &= \frac{1}{\Lambda cv^2} \left(\frac{1}{2\sqrt{2}}v + \frac{3}{2}v J\right)\sigma^1, \\
F^I &= d\left(\sqrt{2}cvX^I \, dt + \frac{3}{\sqrt{2}}v \left(Q^{IJ} - X^I X^J\right) v J\sigma^1\right), 
\end{align*}
\tag{31}
\]

whereas if $\Lambda = 0$, then 
\[
\begin{align*}
\Omega &= \frac{3\sqrt{3}}{cv^2} \left(\frac{1}{2\sqrt{2}}v + \frac{3}{2}v J\right)\sigma^1, \\
F^I &= d\left(\sqrt{2}cvX^I \, dt + \frac{3}{\sqrt{2}}v \left(Q^{IJ} - X^I X^J\right) v J\sigma^1\right). 
\end{align*}
\tag{32}
\]

(4) For the fourth class of solution, the scalars $X^I$ are constant ($V_I X^I \neq 0$), and 
\[
\begin{align*}
f &= 1. 
\end{align*}
\tag{33}
\]

The Kähler base-metric is the product of two 2-manifolds 
\[
\begin{align*}
ds_B^2 &= ds^2(M_1) + ds^2(M_2) 
\end{align*}
\tag{34}
\]

where $M_1$ is $\mathbb{H}^2$ with Ricci scalar $R = -18\chi^2(V_I X^I)^2$, and $M_2$ is $\mathbb{H}^2, \mathbb{R}^2$ or $S^2$ with Ricci scalar $R = 18\chi^2(Q^{IJ} - X^I X^J) V_I V_J$. In addition, we have 
\[
\begin{align*}
d\Omega &= 3\chi V_I X^I \, dvol(M_1), \quad F^I = 3\chi(X^I X^J - Q^{IJ}) V_I \, dvol(M_2), 
\end{align*}
\tag{35}
\]

where $dvol(M_1), dvol(M_2)$ are the volume forms of $M_1, M_2$.

(5) For the fifth class of solution, one takes the coordinates $\psi, \phi, x^1, x^2$ on the base space $B$, whose metric is given by 
\[
\begin{align*}
ds_B^2 &= e^{\frac{\chi^2}{2}v} \left[(d\phi + \beta)^2 + d\psi^2 + T^2((dx^1)^2 + (dx^2)^2)\right], 
\end{align*}
\tag{36}
\]

where $\beta = \beta_i(x^1, x^2) \, dx^i, \psi$ is a non-zero constant and $T = T(x^1, x^2)$ is a scalar. $\beta$ is constrained by the relation 
\[
\begin{align*}
T^2 &= \frac{1}{\sqrt{2}v^2} \left(\frac{\partial \beta_2}{\partial x^1} - \frac{\partial \beta_1}{\partial x^2}\right). 
\end{align*}
\tag{37}
\]
The scalars $X'$ depend only on $x^1$, $x^2$, and
\[ f = e^{2\phi/\psi} u \] (38)
for a function $u$ which depends only on $x^1$, $x^2$. There also exist two purely imaginary functions $G$, $H$ which depend only on $x^1$, $x^2$ and satisfy the constraints
\[ \frac{\partial}{\partial x^2} (T H) = - \frac{\partial}{\partial x^1} (T G), \] (39)
and
\[ \frac{\partial}{\partial x^1} \left( \frac{H}{T} \right) = \frac{\partial}{\partial x^2} \left( \frac{G}{T} \right), \quad \frac{\partial}{\partial x^1} \left( \frac{G}{T} \right) = - \frac{\partial}{\partial x^2} \left( \frac{H}{T} \right). \] (40)
The scalars $u$ and $X'$ are constrained by
\[ X_1 = u^2 q_1 + \chi u^2 \left( -\frac{i}{\sqrt{2}} \frac{T}{T^2 \partial x^2} + \frac{H}{T^2 \partial x^1} \right) V_J X_J + 3 \chi \frac{3}{T^2 u^2} V_J X_J, \] (41)
for constant $q_1$, and $T$ satisfies the equation
\[ \Box \log T + 2 \sqrt{2} T^2 = 18 \chi^2 \left( X' X' - \frac{1}{2} Q^{ij} \right) V_J V_J T^2 \mu^2, \] (42)
where $\Box = \left( \frac{\partial}{\partial x^1} \right)^2 + \left( \frac{\partial}{\partial x^2} \right)^2$ is the Laplacian on $\mathbb{R}^2$. Finally, $\Omega$ is given by
\[ \Omega = - e^{-\sqrt{2} \phi \psi} \left[ i T e^\left(-2 \psi dx^1 + \frac{H}{T} dx^2 \right) \right. \]
\[ + \left( -\frac{i}{\sqrt{2}} \frac{T}{T^2 \partial x^2} + \frac{H}{T^2 \partial x^1} \right) \left( d\phi + \beta \right) \]
\[ \left. + \frac{3 \chi}{\sqrt{2}} u^2 V_J X_J \right], \] (43)
and the gauge field strengths are
\[ F^I = d(f^2 X^I (dt + \Omega)) + 6 \chi \left( X' X' - \frac{1}{2} Q^{ij} \right) V_J V_J T^2 \mu^2 dx^1 \wedge dx^2. \] (44)

(6) For the sixth class of solutions, it is again convenient to introduce the coordinates $\phi$, $\psi$, $x^1$, $x^2$ on the Kähler base. The base space metric is then
\[ ds_B^2 = d\phi^2 + d\psi^2 + T^2 (dx^1)^2 + (dx^2)^2, \] (45)
where $T = T(x^1, x^2)$. The scalars $f$ and $X'$ depend only on $x^1$, $x^2$. Again, there also exist two purely imaginary functions $G$, $H$ which depend only on $x^1$, $x^2$ and satisfy the constraints
\[ \frac{\partial}{\partial x^2} (T H) = - \frac{\partial}{\partial x^1} (T G), \] (46)
and
\[ \frac{\partial}{\partial x^1} \left( \frac{H}{T} \right) = \frac{\partial}{\partial x^2} \left( \frac{G}{T} \right), \quad \frac{\partial}{\partial x^1} \left( \frac{G}{T} \right) = - \frac{\partial}{\partial x^2} \left( \frac{H}{T} \right). \] (47)
The scalars $f$ and $X'$ are constrained via
\[ \frac{\partial}{\partial x^1} \left( \frac{X_1}{f^2} \right) = \sqrt{2} i \chi T H V_J, \quad \frac{\partial}{\partial x^2} \left( \frac{X_2}{f^2} \right) = \sqrt{2} i \chi T G V_J, \] (48)
and $T$ satisfies
\[ \Box \log T = 18 \frac{X^2}{f^2} \left( X' X' - \frac{1}{2} Q^{ij} \right) V_J V_J T^2. \] (49)
Finally, \( \Omega \) is constrained by
\[
\begin{align*}
\mathrm{d}\Omega &= -\frac{i}{\sqrt{2}} \left( T \frac{\partial G}{\partial x^1} + \mathcal{H} \frac{\partial T}{\partial x^1} \right) \left( \mathrm{d}x^1 \wedge \mathrm{d}x^2 - \frac{1}{T^2} \mathrm{d}\phi \wedge \mathrm{d}\psi \right) \\
&\quad - \frac{3\chi}{T^2} V_j X^j \left( T^2 \mathrm{d}x^1 \wedge \mathrm{d}x^2 + \mathrm{d}\phi \wedge \mathrm{d}\psi \right),
\end{align*}
\]
and the gauge field strengths are then given by
\[
F^I = \mathrm{d} \left( f^2 X^I (\mathrm{d}t + \Omega) \right) + 6\chi \frac{T^2}{f} V_j \left( X^I X^J - \frac{1}{2} Q^{IJ} \right) \mathrm{d}x^1 \wedge \mathrm{d}x^2.
\] (51)

3. Simplification of the solutions

The solutions of types (2), (3), (4) are given in the most explicit possible form. Hence, we shall concentrate on the solutions of types (1), (5), (6).

3.1. Simplification of type (1) solutions

We find it convenient to simplify the expression for the metric and gauge field strengths of these solutions by changing coordinates from \( (t, \tau, \eta, u, v) \) to \( (t', \phi, w, u, v) \). There are two cases, corresponding to \( \theta \neq 0 \) and \( \theta = 0 \).

3.1.1. Solutions with \( \theta \neq 0 \)

If \( \theta \neq 0 \), it is convenient to make the coordinate transformation
\[
\begin{align*}
t &= t' - \frac{w}{2c} \left( H^2 - c^2 v^4 f^{-6} \right) \cos Y - \frac{w v \theta}{2c}, \\
\eta &= v \sin Y w, \\
\tau &= \phi + \frac{w}{\theta} \left( H^2 - c^2 v^4 f^{-6} \right) + v w \cos Y.
\end{align*}
\] (52)

In these new coordinates, the solution is specified by
\[
\begin{align*}
\mathrm{d}s_B^2 &= H^2 (\mathrm{d}\phi + (v \cos Y + \theta^{-1} (H^2 - c^2 v^4 f^{-6})) \mathrm{d}w)^2 + H^{-2} \mathrm{d}v^2 + H^2 v^2 \sin^2 Y (\mathrm{d}u^2 + \mathrm{d}w^2) \\
\mathrm{d}t + \Omega &= \mathrm{d}t' - \frac{1}{2cv} (H^2 + c^2 v^4 f^{-6}) \mathrm{d}\phi \\
&\quad - \left( \frac{1}{2c\theta v} (H^4 - c^4 v^8 f^{-12}) + \frac{1}{c} \left( H^2 \cos Y + \frac{\theta v}{2} \right) \right) \mathrm{d}w \\
F^I &= \mathrm{d} \left( f^2 X^I \left( \mathrm{d}t' - \frac{1}{2cv} (H^2 - c^2 v^4 f^{-6}) \mathrm{d}\phi \right) \right. \\
&\quad \left. - \left( \frac{1}{2c\theta v} (H^2 - c^2 v^4 f^{-6})^2 + \frac{H^2}{c} \cos Y + \frac{\theta v}{2c} + c v^2 f^{-6} \right) \mathrm{d}w \right) \\
X_I &= f^2 \left( \frac{q_I}{v} + \frac{\chi}{c} \left( \frac{c^2 v^2}{f \theta v} - \frac{H^2}{\theta v} - 1 \right) V_j \right)
\] (53)
for constants \( q_I \), where \( Y, H \) are the functions of \( u, v \) (\( \sin Y \neq 0 \)) satisfying the constraints (16) and (17). Note that \( \frac{\partial}{\partial t}, \frac{\partial}{\partial \phi} \) and \( \frac{\partial}{\partial \omega} \) are commuting Killing vectors which are also symmetries of the full solution.
3.1.2. Solutions with $\theta = 0$. In the special case when $\theta = 0$, note that
\[
\frac{d}{\chi} \left( cvf^{-2}X_I + vV_I \right) = V_I (\cos Y \, dv + H^2 v \sin^2 Y \, du); \tag{54}
\]
as not all of the $V_I$ vanish, fix some $\tilde{I}$ with $V_I \neq 0$, so that
\[
\frac{d}{\chi} \left( cvf^{-2}X_I + v \right) = \cos Y \, dv + H^2 v \sin^2 Y \, du. \tag{55}
\]
It is also convenient to define
\[
X = v f^{-2} \frac{X_I}{V_I}. \tag{56}
\]
The coordinate transformation is then given by
\[
t = t' - \frac{H}{2c} (H^2 - c^2 v^2 f^{-6}) \cos Y, \tag{57}
\]
\[
\eta = v \sin Y \, w, \tag{57}
\]
\[
\tau = \phi - w \left( v + \frac{c}{X} X \right) + v w \cos Y.
\]
In these new coordinates, the solution is specified by
\[
\frac{d}{\chi} s_B^2 = H^2 \left( d\phi + \left( v (\cos Y - 1) - \frac{c}{X} X \right) dw \right)^2 + H^{-2} v^2 + H^2 v^2 \sin^2 Y (dw^2 + du^2),
\]
\[
\frac{d}{\chi} \Omega = \frac{d}{\chi} t' - \frac{1}{2cv} (H^2 + c^2 v^2 f^{-6}) d\phi + \left( \frac{1}{2cv} (H^2 + c^2 v^2 f^{-6}) \left( v + \frac{c}{X} X \right) - \frac{H^2}{c} \cos Y \right) dw, \tag{58}
\]
\[
F^I = \frac{d}{\chi} \left( f^2 X^I \left( d\phi - \frac{1}{2cv} (H^2 - c^2 v^2 f^{-6}) d\phi + \left( \frac{1}{2cv} (H^2 - c^2 v^2 f^{-6}) \left( v + \frac{c}{X} X \right) - \frac{H^2}{c} \cos Y - cv f^{-6} \right) dw \right) \right),
\]
v$ f$^{-2}X_I = XV_I + q_I$, (58)

for constants $c, q_I$ ($q_I = 0$), where $Y, H$ are the functions of $u, v$ ($\sin Y \neq 0$) satisfying constraints (16) and (17) with $\theta = 0$.

Again, note that $\frac{d}{\eta}, \frac{d}{\tau}$ and $\frac{d}{\xi}$ are commuting Killing vectors which are also symmetries of the full solution.

3.2. Simplification of type (5) solutions

To simplify the solutions further, define
\[
Q = -\frac{i}{\sqrt{2}} \theta \left( T^{-1} \frac{\partial G}{\partial x^2} + H T^{-2} \frac{\partial T}{\partial x^1} \right) + 3 X u^{-2} V_I X^I \tag{59}
\]
so that
\[
\Omega = -\frac{e^{-\sqrt{2} \theta} \psi}{\sqrt{2} \theta^2} [i T e^\theta (-G dx^1 + H dx^2) + Q (d\phi + \beta)] \tag{60}
\]
and noting that
\[
dQ = \sqrt{2} i \theta^2 (T H dx^1 + T \phi dx^2) \tag{61}
\]
one finds
\[ d \left[ Q^2 - \varphi^6 \left( \frac{2u^{-6}}{Q^2} + G^2 + \mathcal{H}^2 \right) \right] = 0 \] (62)
and hence
\[ Q^2 = \xi + \varphi^6 \left( \frac{2u^{-6}}{Q^2} + G^2 + \mathcal{H}^2 \right) \] (63)
for constant \( \xi \). Also note that
\[ u^{-2} X_I = \frac{\chi}{\varphi^4} \Omega V_I + q_I. \] (64)

There are then a number of subclasses of solutions, according to whether \( \mathcal{H}^2 + G^2 \neq 0 \) or \( \mathcal{H} = G = 0 \).

3.2.1. Solutions with \( \mathcal{H}^2 + G^2 \neq 0 \). Suppose we consider a neighbourhood in which \( \mathcal{H}^2 + G^2 \neq 0 \). Note that as \( T^{-1}(\mathcal{H} + iG) \) is a holomorphic function of \( x^1 + ix^2 \), it follows that \( \frac{\partial}{\partial x^1} (\mathcal{H} + iG) \) is also a holomorphic function of \( x^1 + ix^2 \), and so
\[ \frac{\partial}{\partial x^1} \left( \frac{T \mathcal{H}}{\mathcal{H}^2 + G^2} \right) = -\frac{\partial}{\partial x^2} \left( \frac{T G}{\mathcal{H}^2 + G^2} \right) \] (65)
or equivalently
\[ d \left( \frac{T G}{\mathcal{H}^2 + G^2} \right) dx^1 - \frac{T \mathcal{H}}{\mathcal{H}^2 + G^2} dx^2 = 0, \]
\[ d \left( \frac{T \mathcal{H}}{\mathcal{H}^2 + G^2} \right) dx^1 + \frac{T G}{\mathcal{H}^2 + G^2} dx^2 = 0; \] (66)
hence, one obtains (locally) real functions \( z = z(x^1, x^2) \), \( y = y(x^1, x^2) \) such that
\[ \frac{T G}{\mathcal{H}^2 + G^2} dx^1 - \frac{T \mathcal{H}}{\mathcal{H}^2 + G^2} dx^2 = i dz, \]
\[ \frac{T \mathcal{H}}{\mathcal{H}^2 + G^2} dx^1 + \frac{T G}{\mathcal{H}^2 + G^2} dx^2 = i dy, \] (67)
with
\[ \frac{\partial}{\partial z} = \frac{i}{T} \left( G \frac{\partial}{\partial x^1} - \mathcal{H} \frac{\partial}{\partial x^2} \right), \]
\[ \frac{\partial}{\partial y} = \frac{i}{T} \left( \mathcal{H} \frac{\partial}{\partial x^1} + G \frac{\partial}{\partial x^2} \right). \] (68)

Next, note that one can solve for the 1-form \( \beta \) to find (up to a total derivative which can be neglected)
\[ \beta = -\frac{i}{\varphi^3} \frac{Q T}{\mathcal{H}^2 + G^2} (-G dx^1 + \mathcal{H} dx^2) = -\frac{Q}{\varphi^3} dz \] (69)
and the expression for \( \Omega \) can be further simplified to
\[ \Omega = -\frac{1}{\sqrt{2} \varphi^2} e^{-\sqrt{2} \varphi^2} \left( Q d\phi - \frac{1}{\varphi^3} (\xi + 2 \varphi^2 u^{-6}) dz \right). \] (70)
It is then straightforward to see that \( \frac{\partial}{\partial z} \) is an additional Killing vector, which is also a symmetry of the full solution: \( Q, X^I, u \) are the functions only of \( y \), and the metric on the Kähler base space simplifies to
\[
\mathbf{ds}^2 = e^{\frac{1}{2}\sqrt{2}\varrho^2 \psi} \left( \left( \frac{\partial}{\partial \phi} - \frac{Q}{\varrho^3} \frac{\partial}{\partial z} \right)^2 + d\psi^2 - (H^2 + G^2)(dz^2 + dy^2) \right). \tag{71}
\]
Finally, it is most useful to change coordinates from \( (t, \psi, \phi, z, y) \) to \( (t, \psi, \phi, z, Q) \), where
\[
dQ = -\sqrt{2}\varrho^5 (H^2 + G^2) \frac{\partial}{\partial y}. \tag{72}
\]
In these new coordinates, the solution is given by
\[
\mathbf{ds}^2 = e^{\frac{1}{2}\sqrt{2}\varrho^2 \psi} \left[ \left( \frac{\partial}{\partial \phi} - \frac{Q}{\varrho^3} \frac{\partial}{\partial z} \right)^2 + d\psi^2 + \frac{1}{2(2u - 6 \varrho - 2 - \varrho)(Q - \xi)} dQ^2 \right], \tag{73}
\]
\[
u^{-2} X_I = \frac{\chi \varrho^2}{Q} V_I + q_I,
\]
\[
\Omega = -\frac{1}{\sqrt{2}\varrho^2} e^{-\sqrt{2}\varrho^2 \psi} \left( Q \frac{\partial}{\partial \phi} - \frac{1}{\varrho^3} (\xi + 2\varrho^2 u - 6) \frac{\partial}{\partial z} \right),
\]
\[
F^I = 3\sqrt{2} \chi \varrho^{-2} V_I \left( X^I X^J - \frac{1}{2} Q^{IJ} \right) dx^I \wedge dQ + d(\varrho^2 e^{\sqrt{2}\varrho^2 \psi} X^I (dt + \Omega)),
\]
\[
f = e^{\frac{\varrho^2}{2} \psi} u\tag{74}
\]
for constants \( \varrho \neq 0, q_I, \xi \). Note that \( \frac{\partial}{\partial \phi}, \frac{\partial}{\partial x^I}, \frac{\partial}{\partial z} \) are commuting Killing vectors which are symmetries of the full solution.

### 3.2.2. Solutions with \( \mathcal{H} = G = 0 \)

If \( \mathcal{H} = G = 0 \), then the scalars \( X^I \) are constant, as is \( u \). Without loss of generality, set \( u = 1 \). With these constraints, the function \( T \) which is introduced \([7]\) must satisfy
\[
T^{-2} \Box \log T = \Lambda \tag{75}
\]
where
\[
\Lambda = -2\varrho^4 + 18 \chi^2 (X^I X^J - \frac{1}{2} Q^{IJ}) V_I V_J \tag{76}
\]
is constant. Let \( M \) be a 2-manifold equipped with metric
\[
\mathbf{ds}^2(M) = T^2 ((dx^I)^2 + (dz)^2). \tag{77}
\]
Then (75) implies that \( M \) has Ricci scalar \( ^{(M)} R = -2\Lambda \), so is isometric to \( S^2, \mathbb{R}^2 \) or \( \mathbb{H}^2 \) according to whether \( \Lambda < 0, \Lambda = 0 \) or \( \Lambda > 0 \) respectively. Note also that the 1-form \( \beta \) must satisfy
\[
d\beta = \sqrt{2}\varrho^2 \text{dvol } (M). \tag{78}
\]
Hence, to summarize, these solutions have constant \( X^I \), and
\[
\mathbf{ds}^2 = e^{\sqrt{2}\varrho^2 \psi} ((\frac{\partial}{\partial \phi} + \beta)^2 + d\psi^2 + \mathbf{ds}^2(M)), \quad d\beta = \sqrt{2}\varrho^2 \text{dvol } (M), \tag{79}
\]
\[
^{(M)} R = 4\varrho^4 - 36 \chi^2 \left( X^I X^J - \frac{1}{2} Q^{IJ} \right) V_I V_J, \tag{80}
\]
\[ \Omega = -3 \chi V_I X^I e^{-\sqrt{2} \phi} \psi \] (81)

\[ F^I = d(e^{-\sqrt{2} \phi} X^I (dt + \Omega)) + 6 \chi \left( X^I X^I - \frac{1}{2} Q^2 \right) V_J d \text{vol} (M), \] (82)

\[ f = e^{\frac{2}{\sqrt{2}} \phi}, \] (83)

where \( \phi \) is a non-zero constant.

Note that \( \frac{\partial}{\partial \phi} \) and \( \frac{\partial}{\partial \rho} \) are commuting Killing vectors which are symmetries of the full solution. A Killing vector of \( M \) can also be obtained which commutes with both \( \frac{\partial}{\partial \phi} \) and \( \frac{\partial}{\partial \rho} \) and is also a symmetry of the full solution. So again, there are three commuting Killing vectors which are symmetries of the full solution.

3.3. Simplification of type (6) solutions

Again, for these solutions, there are two sub-classes, according to \( G^2 + H^2 \neq 0 \), or \( G = H = 0 \).

3.3.1. Solutions with \( H^2 + G^2 \neq 0 \). In order to simplify the solutions of type (6), note that the functions \( G, H, T \) satisfy the same constraints (46) and (47) as the type (5) solutions, and hence we again introduce coordinates \( z, y \) such that

\[ \frac{T G}{H^2 + G^2} dx^1 - \frac{T H}{H^2 + G^2} dx^2 = i dz, \] (84)

\[ \frac{T H}{H^2 + G^2} dx^1 + \frac{T G}{H^2 + G^2} dx^2 = i dy, \]

with

\[ \frac{\partial}{\partial z} = \frac{i}{T} \left( G \frac{\partial}{\partial x^1} - H \frac{\partial}{\partial x^2} \right), \quad \frac{\partial}{\partial y} = \frac{i}{T} \left( H \frac{\partial}{\partial x^1} + G \frac{\partial}{\partial x^2} \right). \] (85)

It is also convenient to define the scalar \( Q \) by

\[ Q^2 = -(G^2 + H^2). \] (86)

Then the constraints (46) and (47) imply that \( Q, f \) and \( X^I \) are the functions only of \( y \), with

\[ \frac{d}{dy} (f^{-2} X_I) = \sqrt{2} \chi Q^2 V_I. \] (87)

Next, note that

\[ d\Omega = \left( \frac{1}{\sqrt{2}} \frac{d \log Q}{dy} - 3 \chi f^{-2} V_I X^I \right) d\phi \wedge d\psi \]

\[ + \left( -\frac{1}{\sqrt{2}} \frac{d \log Q}{dy} - 3 \chi f^{-2} V_I X^I \right) T^2 dx^1 \wedge dx^2. \] (88)

The integrability condition of this constraint is given by

\[ \frac{1}{\sqrt{2}} \frac{d \log Q}{dy} - 3 \chi f^{-2} V_I X^I = \xi \] (89)

for constant \( \xi \).

It is then straightforward to show that

\[ d\Omega = \xi d\phi \wedge d\psi + d \left( \frac{1}{\sqrt{2}} \left( \frac{1}{2} Q^2 + f^{-2} \right) dz \right) \] (90)
and also
\[ F^I = df^2 X^I (dr + \Omega) - \sqrt{2} f^{-4} X^I dz. \] (91)

Finally, consider the constraints (87) and (89). As not all \( V_I \) vanish, choose \( \tilde{I} \) such that \( V_{\tilde{I}} \neq 0 \); then (87) implies
\[ Q^2 = \frac{1}{\sqrt{2} \chi} \frac{d}{dy} \left( f^{-2} X_{\tilde{I}} \right). \] (92)
It is then convenient to define
\[ X = f^{-2} X_{\tilde{I}} / V_{\tilde{I}}. \] (93)

Combining all of the above constraints, one finds that the solution is specified by
\[ f^{-2} X_I = X V_I + q_I, \] where \( X \) satisfies \( \frac{1}{4 \chi} \frac{dX}{dy} - \frac{1}{2} f^{-6} = \frac{\xi}{\sqrt{2} \chi} X. \)
\[ ds^2_B = d\phi^2 + d\psi^2 + 2\sqrt{2} \left( \frac{1}{2} f^{-6} + \frac{\xi}{\sqrt{2} \chi} X \right) (dy^2 + dz^2), \] (dy^2 + dz^2)
\[ d\Omega = \frac{\xi}{\chi} d\phi \wedge d\psi + d \left( \sqrt{2} f^{-6} + \frac{\xi}{\sqrt{2} \chi} X \right) dz, \] (94)
\[ F^I = df^2 X^I (dr + \Omega) - \sqrt{2} f^{-4} X^I dz. \] (95)

for constants \( q_I, (q_{\tilde{I}} = 0), \xi. \) Note that \( \frac{\partial}{\partial t}, \frac{\partial}{\partial \phi}, \frac{\partial}{\partial z} \) are commuting Killing vectors which are symmetries of the full solution.

### 3.3.2. Solutions with \( \mathcal{H} = \mathcal{G} = 0. \) For these solutions, the scalars \( X^I \) and \( f \) are constant; without loss of generality set \( f = 1. \) (96)

Then
\[ ds^2_B = d\phi^2 + d\psi^2 + ds^2(M), \] (97)
where \( M \) is a 2-manifold which is either \( S^2, R^2 \) or \( H^2 \) according to whether the Ricci scalar
\[ (M) R = -36 \chi^2 (X^I X^J - \frac{1}{2} Q^{IJ}) V_I V_J \] (98)
is positive, zero or negative. In addition, one has
\[ d\Omega = -3 \chi V_I X^I (dvol(M)) + d\phi \wedge d\psi, \] (99)
\[ F^I = -3 \chi X^I X^J V_J d\phi \wedge d\psi + 3 \chi (X^I X^J - Q^{IJ}) V_J dvol(M). \] (100)

It is clear that this solution also admits three Killing vectors which are symmetries of the full solution.

### 4. Summary

In summary, we have revisited the classification of 1/2 supersymmetric solutions of the theory of \( N = 2, D = 5 \) supergravity which have at least one time-like Killing spinor. Three of the six classes of these solutions were given in their most explicit form in [7]. Our purpose was to recast the remaining three solutions in a form which enabled us to extract some hidden isometries of the metric solutions. The presence of these additional symmetries is to be
expected, as a consequence of the enhancement of supersymmetry; however, they were not explicit in the original construction of the solutions. We found coordinate transformations that simplified these three classes of solutions and allowed the explicit construction of their Killing vectors. It is of interest to investigate whether there are any regular asymptotically AdS\(_5\) black ring solutions. Supersymmetric rings exist in the ungauged theory [23–26] and it is known that supersymmetry is fully restored at the ring horizon. Furthermore, supersymmetric regular asymptotically AdS\(_5\) black holes undergo supersymmetry enhancement from 1/4 to 1/2 supersymmetry in their near horizon limits. So, if AdS\(_5\) black rings exist in the gauged theory, one might also expect that supersymmetry is enhanced from 1/4 to 1/2 at the horizon.

Recent work [22] has shown that there are no black rings which have horizons with a \([U(1)]^2\) symmetry. In our work, we have shown that the 1/2 supersymmetric solutions in the ‘time-like’ class possess, in addition to the time-like Killing vector generated by the time-like Killing spinor, two further commuting Killing vectors. However, these two additional Killing vectors are not necessarily space-like. Hence, in order to determine if there exists a 1/2 supersymmetric solution corresponding to the near-horizon geometry of a black ring, it will be necessary to construct a more detailed analysis of half-supersymmetric near horizon geometries. The simplifications to the half-supersymmetric solutions presented in this work will be useful in constructing such a classification.

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References

[1] Behrndt K, Chamseddine A H and Sabra W A 1998 BPS black holes in \(N = 2\) five-dimensional AdS supergravity Phys. Lett. B 442 97 (arXiv:hep-th/9807187)

Klemm D and Sabra W A 2001 Charged rotating black holes in 5d Einstein–Maxwell-(A)dS gravity Phys. Lett. B 503 147 (arXiv:hep-th/0010200)

Klemm D and Sabra W A 2001 General (anti-)de Sitter black holes in five dimensions J. High Energy Phys. JHEP02(2001)031 (arXiv:hep-th/0011016)

[2] Cacciatori S L, Klemm D and Sabra W A 2003 Supersymmetric domain walls and strings in D = 5 gauged supergravity coupled to vector multiplets J. High Energy Phys. JHEP03(2003)023 (arXiv:hep-th/0302218)

Klemm D and Sabra W A 2000 Supersymmetry of black strings in D = 5 gauged supergravities Phys. Rev. D 62 024003 (arXiv:hep-th/0001131)

[3] Gauntlett J P and Gutowski J B 2003 All supersymmetric solutions of minimal gauged supergravity in five dimensions Phys. Rev. D 68 105009 (arXiv:hep-th/0304064)

[4] Gutowski J B and Reall H S 2004 Supersymmetric AdS\(_5\) black holes J. High Energy Phys. JHEP02(2004)006 (arXiv:hep-th/0401042)

[5] Gutowski J B and Reall H S 2004 General supersymmetric AdS\(_5\) black holes J. High Energy Phys. JHEP04(2004)048 (arXiv:hep-th/0401129)

[6] Gutowski J B and Sabra W A 2005 General supersymmetric solutions of five-dimensional supergravity J. High Energy Phys. JHEP10(2005)039 (arXiv:hep-th/0505185)

[7] Gutowski J B and Sabra W A 2007 Half-supersymmetric solutions in five-dimensional supergravity J. High Energy Phys. JHEP12(2007)025 (arXiv:0706.3147 [hep-th])

[8] Grover J, Gutowski J B and Sabra W A 2008 Null half-supersymmetric solutions in five-dimensional supergravity J. High Energy Phys. JHEP10(2008)103 (arXiv:0802.0231 [hep-th])

[9] Grover J, Gutowski J B and Sabra W A 2007 Vanishing preons in the fifth dimension Class. Quantum Grav. 24 417 (arXiv:hep-th/0608187)

Figueras-O’Farrell J, Gutowski J B and Sabra W A 2007 The return of the four- and five-dimensional preons Class. Quantum Grav. 24 4429 (arXiv:0705.2778 [hep-th])
[10] Kunduri H K and Lucietti J 2007 Near-horizon geometries of supersymmetric AdS(5) black holes J. High Energy Phys. JHEP12(2007)015 (arXiv:0708.3695 [hep-th])

[11] Kunduri H K, Lucietti J and Reall H S 2006 Supersymmetric multi-charge AdS5 black holes J. High Energy Phys. JHEP04(2006)036 (arXiv:hep-th/0601156)

[12] Gauntlett J P, Gutowski J B, Hull C M, Pakis S and Reall H S 2003 All supersymmetric solutions of minimal supergravity in five dimensions Class. Quantum Grav. 20 4587 (arXiv:hep-th/0209114)

[13] Gutowski J B 2004 Uniqueness of five-dimensional supersymmetric black holes J. High Energy Phys. JHEP08(2004)049 (arXiv:hep-th/0404079)

[14] Tod K P 1983 All metrics admitting supercovariantly constant spinors Phys. Lett. B 121 241

[15] Lawson H B and Michelsohn M 1989 Spin Geometry (Princeton, NJ: Princeton University Press)

[16] Wang M Y 1989 Parallel spinors and parallel forms Ann. Glob. Anal. Geom. 7 59

[17] Harvey F R 1990 Spinors and Calibrations (London: Academic)

[18] Gillard J, Gran U and Papadopoulos G 2005 The spinorial geometry of supersymmetric backgrounds Class. Quantum Grav. 22 1033 (arXiv:hep-th/0410155)

[19] Gran U, Gutowski J B and Papadopoulos G 2005 The spinorial geometry of supersymmetric IIB backgrounds Class. Quantum Grav. 22 2453 (arXiv:hep-th/0501177)

[20] Gran U, Gutowski J B, Papadopoulos G and Roest D 2006 Maximally supersymmetric G-backgrounds of IIB supergravity Nucl. Phys. B 753 118 (arXiv:hep-th/0604079)

[21] Gran U, Gutowski J B, Papadopoulos G and Roest D 2007 $N = 3$ is not IIB J. High Energy Phys. JHEP02(2007)044 (arXiv:hep-th/0606049)

[22] Kunduri H K, Lucietti J and Reall H S 2007 Do supersymmetric anti-de Sitter black rings exist? J. High Energy Phys. JHEP02(2007)026 (arXiv:hep-th/0611351)

[23] Elvang H, Emparan R, Mateos D and Reall H S 2004 A supersymmetric black ring Phys. Rev. Lett. 93 211302 (arXiv:hep-th/0407065)

[24] Elvang H, Emparan R, Mateos D and Reall H S 2005 Supersymmetric black rings and three-charge supertubes Phys. Rev. D 71 024033 (arXiv:hep-th/0408120)

[25] Gauntlett J P and Gutowski J B 2005 Concentric black rings Phys. Rev. D 71 025013 (arXiv:hep-th/0408010)

[26] Gauntlett J P and Gutowski J B 2005 General concentric black rings Phys. Rev. D 71 045002 (arXiv:hep-th/0408122)