Warm Gauge-Flation

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Abstract

Non-abelian gauge field inflation is studied in the context of warm inflation scenario. We introduce this scenario as a mechanism that gives an end for gauge-flation model. Slow-roll parameters and perturbation parameters are presented for this model. We find the general conditions which are required for this model to be realizable in slow-roll approximation. We also develop our model in the context of intermediate and logamediate scenarios which are exact solutions of inflationary field equation in the Einstein theory. General expressions of slow-roll parameters, tensor-scalar ratio and scalar spectral index are presented in terms of inflaton field for these two cases. Our model is compatible with recent observational data from Planck satellite.

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I. INTRODUCTION

Inflation model presents better description of the early phase of the universe. Problems of the Big Bang model, namely the numerical density of monopoles, flatness, homogeneity and the horizon problem, may find explanations in the framework of the inflationary universe models \[1\]. Inflation also predicts a mechanism to generate the anisotropy for cosmological microwave background (CMB) and the inhomogeneity for structure formation \[2\]. Cosmic microwave background (CMB) and large scale structure (LSS) observational data denote the isotropic space-time at the background level \[2, 3\]. Therefore, many models of inflation were proposed versus either single or multi scalar field theories which are isotropic and compatible with the observations of CMB and LSS. These scalar field models have (non)standard kinetic and potential terms which are coupled to gravity. In slow-roll limit, when the kinetic energy of scalar fields is relatively small compared to the potential energy, the inflation period has been derived. After this period, in reheating period, the scalar field oscillates around the minimum of the potential while losing its energy to massless particles \[4\]. Recently, a new model of inflation was proposed \[5\] and studied \[6\] which is generally driven by gauge field theory. This model is widely used in particle physics models but is against the isotropic symmetry of the universe at the background level. This problem could be solved by using three gauge fields which can rotate among each other by non-Abelian gauge transformation SU(2). The rotational symmetry is the global part of SU(2) and isotropy is retained in gauge-flation model \[5\].

In the warm inflationary models, radiation production process occurs during inflationary period while reheating is avoided \[7\]. Thermal fluctuations may be obtained during warm inflation. These fluctuations could play a dominant role to produce initial fluctuations which are necessary for Large-Scale Structure (LSS) formation. Thus, the density fluctuation arises from thermal fluctuation rather than quantum fluctuation \[8\]. Warm inflationary period ends when the universe stops inflating. After this period, the universe enters radiation phase smoothly \[7\]. Finally, remaining inflatons or dominant radiation fields create the matter components of the universe. Some extensions of this model can be found in Ref. \[9\]. The main problem of the inflation theory is how to attach the universe to the end of the inflation period. One of the solutions of this problem is to study the inflation in the context of warm inflation \[7\]. In this model, radiation is produced during inflation period where its
energy density is kept nearly constant. This is phenomenologically fulfilled by introducing the dissipation term $\Gamma$. The study of warm inflation model as a mechanism that gives an end for gauge-flation motivates us to consider the warm gauge-flation model.

In the warm inflation, there has to be continuously a particle production. For this to be possible, the microscopic processes that produce these particles must occur at a timescale much faster than Hubble expansion. Thus, the decay rates $\Gamma_i$ (not to be confused with the dissipative coefficient) must be larger than $H$. The produced particles need also to be thermalized. Thus, the scattering processes amongst these produced particles must occur at a rate larger than $H$. These adiabatic conditions were outlined since the publication of the early warm inflation papers, such as Ref. [10]. More recently, there has been a considerable explicit calculation from Quantum Field Theory (QFT) that explicitly computes all the relevant decay and scattering rates in the warm inflation models [11, 12].

In one of the sections of the present work, we would like to consider the warm gauge-flation model in the context of "intermediate inflation". This scenario is one of the exact solutions of inflationary field equation in the Einstein theory with scale factor $a(t) = a_0 \exp(At^f)$ ($A > 0, 0 < f < 1$). The study of this model is motivated by string/M theory [17]. By adding the higher order curvature correction, which is proportional to Gauss-Bonnent (GB) term, to Einstein-Hilbert action, one can obtain a free-ghost action [18]. Gauss-Bonnent interaction is the leading order in the expansion of inverse string tension, "$\alpha$" to low-energy string effective action [18]. This theory may be applied for black hole solutions [19], acceleration of the late time universe [20] and initial singularity problems [21]. The GB interaction in 4d with dynamical dilatonic scalar coupling leads to an intermediate form of the scale factor [17]. Expansion of the universe in the intermediate inflation scenario is slower than standard de sitter inflation with scale factor $a = a_0 \exp(H_0 t)$ ($a_0, H_0 > 0$), but faster than power-low inflation (with the scale factor $a = t^p$ ($p > 1$)). Harrison-Zeldovich [22] spectrum of the density perturbation i.e. $n_s = 1$ for the intermediate inflation models driven by scalar field is presented for exact values of parameter $f$ [23].

On the other hand, we will also study our model in the context of "logamediate inflation" with the scale factor $a(t) = a_0 \exp(a[\ln t]^\lambda)$ ($\lambda > 1, A > 0$) [24]. This model is converted to power-law inflation for $\lambda = 1$ cases. This scenario is applied in a number of scalar-tensor theories [25]. The study of the logamediate scenario is motivated by imposing weak general conditions on the cosmological models which have indefinite expansions [24].
effective potential of the logamediate model has been considered in dark energy models \[26\]. These forms of potentials are also used in supergravity, Kaluza-Klein theories and superstring models \[25, 27\]. For logamediate models, the power spectrum could be either red or blue tilted \[28\]. In Ref.\[24\], we can find eight possible asymptotic scale factor solutions for cosmological dynamics. Three of these solutions are non-inflationary scale factor, another three solutions gives power-low, de sitter and intermediate scale factors. Finally, two cases of these solutions have asymptotic expansions with logamediate scale factor.

II. WARM INFLATION DRIVEN BY NON-ABELIAN GAUGE FIELDS

Gauge-flation model in a flat-space Friedmann-Robertson-Walker (FRW) background is described by an effective Lagrangian \[5\]

\[\mathcal{L} = \sqrt{-g} \left( -\frac{R}{2} - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{k^2}{384} (\epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu}^a F_{\lambda\sigma}^a)^2 \right) \quad (1)\]

where \(8\pi G = M_p^{-2} = 1\), \(\epsilon^{\mu\nu\rho\sigma}\) is an antisymmetric tensor and

\[F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g \epsilon_{bc}^a A_\nu^b A_\mu^c \quad (2)\]

where \(\epsilon_{bc}^a\) is also antisymmetric tensor. It was shown that \(F^4\) term in Eq.(1) can be derived by integrating out an axion field in Chromo-Natural inflation \[6\]. To obtain isotropy symmetry of space-time, we can introduce the effective inflaton field as \[5\] 

\[A_\mu^a = \begin{cases} \phi(t) \delta_i^a & \mu = i, \mu = 0 \\ \psi(t) \delta_i^a & \mu = i, \mu = 0 \end{cases} \quad (3)\]

Using the above ansatz, we could find a reduced effective Lagrangian from Eq.(1)

\[\mathcal{L}_{red} = \frac{3}{2} \left( \frac{\dot{\phi}^2}{a^2} - \frac{g^2 \phi^4}{a^4} + \kappa \frac{g^2 \dot{\phi}^2 \phi^4}{a^6} \right) \quad (4)\]

Pressure and energy density have the following forms

\[P_\phi = \frac{1}{3} \rho_{YM} - \rho_{F^4} \quad \rho_\phi = \rho_{YM} + \rho_{F^4} \quad (5)\]

where

\[\rho_{YM} = \frac{3}{2} \left( \frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} \right) \quad \rho_{F^4} = \frac{3}{2} \kappa \frac{g^2 \dot{\phi}^2 \phi^4}{a^6} \quad (6)\]
$F^4$ term has been chosen because the contribution of this term to pressure and energy density leads to $P = -\rho$ which gives the inflationary dynamic. The gauge field without $F^4$ term has the equation of state of radiation ($P = \frac{4}{3}\rho$) and could not be source of inflation. The dynamics of phenomenological warm gauge-flation in spatially flat FRW model is described by these equations

$$H^2 = \frac{1}{3}(\rho_\phi + \rho_\gamma)$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -\Gamma\frac{\dot{\phi}^2}{a^2}$$

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\frac{\dot{\phi}^2}{a^2}$$

where $\rho_\gamma$ is energy density of the radiation, $H$ is Hubble parameter and $\Gamma$ is the dissipative coefficient. From Eqs.(5) and (6), the equation of motion is reduced to

$$(1 + \kappa g^2\frac{\dot{\phi}^4}{a^4})\ddot{\phi} + (1 + r - 3\kappa g^2\frac{\dot{\phi}^4}{a^4})H\frac{\dot{\phi}}{a} + (1 + \kappa \frac{\dot{\phi}^2}{a^2})\frac{2g^2\dot{\phi}^3}{a^3} = 0$$

where $r = \frac{\Gamma}{3H}$. In the above equations dots "." mean derivative with respect to cosmic time.

During inflation epoch, the energy density $\rho_\phi$ is of the order of potential energy density $\rho_F$ ($\rho_\phi \sim V$) and dominates over the radiation energy $\rho_\phi > \rho_\gamma$. Using slow-roll approximation when $\dot{\phi} \ll (3H + \Gamma)\dot{\phi}$ and when inflation radiation production is quasi-stable ($\dot{\rho}_\gamma \ll 4H\rho_\gamma$, $\dot{\rho}_\gamma \ll \Gamma\frac{\dot{\phi}^2}{a^2}$), we can see the dynamic equations (7) and (8) are reduced to

$$H^2 = \frac{1}{72\kappa g^2 \psi^2}$$

$$\dot{H} + \frac{r}{2}H^2\psi^2 + g^2\psi^4 = 0, \quad \psi^2 \simeq -\frac{6}{\Gamma}H$$

$$\frac{\dot{\phi}}{a} = -\frac{6\kappa g^2\psi^2}{\Gamma}$$

$$\rho_\gamma = \frac{\Gamma\dot{\phi}^2}{4H a^2} \simeq \frac{\Gamma^2}{4H^2\psi^2} = \frac{\Gamma^2}{24(2\kappa g^2)^{\frac{1}{2}}} = \sigma T_r^4$$

where $\sigma$ is Stefan-Boltzmann constant and $T_r$, is the temperature of thermal bath. Slow-roll parameters of the warm gauge-flation are

$$\epsilon = -\frac{1}{H} \frac{d}{dt} \ln H = -\frac{\Gamma'}{\Gamma} - 2\frac{\psi'}{\psi} = \frac{1}{2}r\psi^2$$

$$\eta = \frac{1}{2}\epsilon - \frac{\dot{\epsilon}}{H\epsilon} = \frac{1}{2}r\psi^2 - \frac{r'}{r} - 2\frac{\psi'}{\psi}$$

$$\delta = -\frac{\dot{\psi}}{H\psi} = -\frac{\psi'}{\psi}$$
where primes (′) denote a derivative with respect to the number of e-folds \( N \) \((dN = H dt)\). The scalar field at the beginning of inflation is given by \( \psi_1^2 = \frac{2}{r} \) when \( \epsilon = 1 \). From Eqs. (9) and (10), we could find a relation between \( \rho_\phi \) and \( \rho_\gamma \)

\[
\rho_\gamma = \frac{\epsilon}{2} \rho_\phi
\]  

(11)

Condition of inflation epoch \( (\ddot{a} < 1) \) may be obtained by inequality \( \epsilon < 1 \) \([5]\).

\[
\rho_\phi > 2 \rho_\gamma
\]  

(12)

Warm inflation epoch ends when \( \rho_\phi = 2 \rho_\gamma \). The number of e-folds has the following form

\[
N = \int H dt = \frac{1}{\sqrt{72 \kappa g^2}} \int_0^{\phi_f} \frac{\Gamma a}{\phi} dt
\]  

(13)

where \( f \) denotes the end of inflation and \( \ast \) denotes the epoch when the cosmological scale exits the horizon.

Now we study the perturbation of our model at the smallest level in spatially flat FRW background. We will consider the perturbation theory in isotropic universe using variation of inflaton \( \psi \). In warm inflation scenario, the variation of field is presented by thermal fluctuation. In non-warm scenarios quantum fluctuation predicts \([5, 6]\)

\[
\langle \delta \psi \rangle_{\text{quantum}} = \frac{H^2}{2\pi}
\]  

(14)

but in warm inflation model the thermal fluctuation provides \([7, 8]\)

\[
\langle \delta \psi \rangle_{\text{thermal}} = \left( \frac{\Gamma HT^2}{(4\pi)^3} \right)^{\frac{1}{4}}
\]  

(15)

Scalar and tensor perturbation emerging during inflation epoch will be studied in warm gauge-flation model. These perturbations may leave an imprint in the CMB anisotropy and on the LSS \([13, 14]\). Power spectrum and a spectral index, are characteristics of each fluctuation: \( \Delta_2^2(k) \) and \( n_T \) for the tensor perturbation, \( \Delta_2^2_R(k) \) and \( n_R \) for the scalar perturbation. In warm and cool inflation models, scalar power spectrum is given by \([7]\)

\[
\Delta_2^2_R(k) = \left( \frac{H}{\phi} \langle \delta \phi \rangle \right)^2
\]  

(16)

where \( k \) is co-moving wavenumber. In our model, the power-spectrum of the scalar perturbation is presented from Eqs. (9) and (15)

\[
\Delta_2^2_R(k) = \frac{1}{\left( \frac{\Gamma T}{(4\pi)^{\frac{1}{2}}(72 \kappa g^2)^{\frac{1}{2}}} \right)^{\frac{1}{4}} \frac{\psi^5}{2}}
\]  

(17)
The largest value of the density perturbation is produced when $\psi = \psi_i$ \[15\]. The scalar spectral index of our model is presented by

$$n_s - 1 = -\frac{d\ln \Delta_R^2(k)}{d\ln k} = \frac{3}{2} \epsilon - \eta$$  \hspace{1cm} (18)$$

The Planck measurement constraints this parameter as \[2\]:

$$n_s = 0.96 \pm 0.0073$$  \hspace{1cm} (19)$$

In the warm inflation scenario, the thermal fluctuations are considered instead of the quantum fluctuations that generate the scalar perturbations. Therefore, the density fluctuation of the scalar perturbation is modified while the tensor perturbation shows the same spectrum as in the usual non-warm inflation \[16\]

$$\Delta^2_T = \frac{2H^2}{\pi^2} = \frac{1}{36\kappa g^2\pi^2} \Gamma^2$$  \hspace{1cm} (20)$$

The spectral index $n_T$ may be found as

$$n_T = -2\epsilon$$  \hspace{1cm} (21)$$

From observational data, $\Delta^2_T$ could not be constrained directly, but the tensor-scalar ratio could be constrained

$$R = \frac{\Delta^2_T}{\Delta^2_R} = \frac{2^{\frac{1}{2}}(4\pi)^{\frac{3}{2}} \Gamma \psi^{\frac{1}{2}}}{(36\kappa g^2)^{\frac{1}{2}} T}$$  \hspace{1cm} (22)$$

The Planck measurement constraints this parameter as \[2, 3\]:

$$R \leq 0.11$$  \hspace{1cm} (23)$$

### III. INTERMEDIATE INFLATION

Intermediate inflation will be studied in this section, where the scale factor of this model is given by

$$a = a_0 \exp(At^f) \quad 0 < f < 1$$  \hspace{1cm} (24)$$

where $A$ is a positive constant. Using the above equation, we obtain the number of e-fold as

$$N = \int_{t_1}^{t} H dt = A(t^f - t_1^f)$$  \hspace{1cm} (25)$$
where $t_1$ is the beginning time of the inflation. From Eqs. (9) and (24), we could find the Hubble parameter and scalar field as

$$H(\psi) = fA(\frac{\psi^2}{\beta})^{f-1}$$

$$\psi^{-2} = \beta t$$

where $\beta = \frac{\Gamma}{6(1-f)}$ and $\Gamma = const.$ Important slow-roll parameters $\epsilon$ and $\eta$ are given by

$$\epsilon = \frac{1-f}{fA} \left(\frac{\psi^2}{\beta}\right)^{-f}$$

$$\eta = \frac{2-f}{fA} \left(\frac{\psi^2}{\beta}\right)^{-f}$$

respectively. The number of e-fold between two fields $\psi_1$ and $\psi$ is presented, using Eq. (25), by

$$N = A((\frac{\psi^2}{\beta})^f - (\frac{\psi_1^2}{\beta})^f) = A((\frac{\psi^2}{\beta})^f - \frac{1-f}{fA})$$

At the beginning of the inflation epoch where $\epsilon = 1$, we find the scalar field in terms of constant parameters of the model

$$\psi_1^{-2} = \beta \left(\frac{1-f}{fA}\right)^{\frac{1}{2}}$$

FIG. 1: In this graph, we plot the spectral index $n_s$ in terms of the number of e-folds $N$ for intermediate scenario ($A = 1, f = \frac{1}{2}, \Gamma \propto \frac{1}{fA}$).
By using the above equations, we may find the scalar field $\psi(t)$ in terms of the number of e-folds

$$\psi^{-2} = \beta \left( \frac{N}{A} + \frac{1 - f}{fA} \right)^{\frac{1}{3}}$$  \hspace{1cm} (30)

Perturbation parameters versus the scalar field $\psi$, and constant parameters of intermediate scenario are presented for warm gauge-flation model. In the slow-roll limit, the power-spectrum of scalar perturbation could be found, using Eqs. (15), (16) and (30), as

$$\Delta^2_R = T \left( \frac{\Gamma fA}{(4\pi)^3} \right)^{\frac{1}{2}} \psi^{-2} \left( \frac{\psi^{-2}}{\beta} \right)^{\frac{f+1}{3}}$$  \hspace{1cm} (31)

Another important perturbation parameter is spectral index $n_s$ which is given by

$$n_s - 1 = -\frac{d \ln \Delta^2_R}{dN} = -f + 1 - \frac{N}{2fA} + \frac{1 - f}{fA}^{-1}$$  \hspace{1cm} (32)

In Fig. (1), the spectral index $n_s$ versus the number of e-folds is plotted (where $f = \frac{1}{2}$). Our model is compatible with observational data [2], ($N \simeq 70$ case leads to $n_s \simeq 0.96$). Tensor
power spectrum and its spectral index are given by

\[
\Delta^2_T = \frac{2 f^2 A^2}{\pi^2} \frac{(\psi^2)^{2(f-1)}}{\beta} = \left( \frac{N}{A} + \frac{1 - f}{fA} \right)^{2f/2} 
\] (33)

\[
n_T = \frac{2f - 2}{fA} \left( \frac{\psi^2}{\beta} \right)^{-f-nT}
\]

Tensor-scalar ratio has the following form

\[
R = \frac{2}{TT^{3/2}} \frac{(fA)^{3/2}}{\pi^{1/2}} \frac{\beta^2}{\psi^2} (\psi^2 - 2\beta)^{3f/3} (f - 1)^{3f/3}
\] (34)

\[
= \frac{2}{TT^{3/2}} \frac{(fA)^{3/2}}{\pi^{1/2}} \beta^2 \left( \frac{N}{A} + \frac{1 - f}{fA} \right)^{3f/3} 
\] (35)

Tensor-scalar ratio \( R \) in terms of the number of e-folds is plotted in Fig.(2). Standard case \( N \geq 70 \), leads to \( 0.1 < 0.11 \), which agrees with observational data [2, 3].

**IV. LOGAMEDIATE INFLATION**

In this section, we study warm gauge field logamediate inflation where the scale factor has the following form

\[
a(t) = a_0 \exp(A[\ln t]^{\lambda}) \quad \lambda > 1, \quad (36)
\]

\( A \) is a constant parameter. By using the above equation, we may find the number of e-folds as

\[
N = \int_{t_1}^{t} H dt = A[(\ln t)^{\lambda} - (\ln t_1)^{\lambda}] 
\] (37)

where \( t_1 \) denotes the beginning time of inflation epoch. Using Eqs.(36) and (37), we may find the inflaton \( \psi \)

\[
\psi = \Xi(t) 
\]

where \( \Xi(t) = \left( \frac{r^{\lambda A[\ln t]^{\lambda-1}}}{t} \right)^{\frac{1}{2}} (1 + \sqrt{1 + \frac{16 \lambda^2 A^2}{r^{2(\ln t)^{2\lambda-1}}}}) \). The Hubble parameter in terms of scalar field \( \psi \) is presented as:

\[
H = \lambda A \frac{(\ln \Xi^{-1}(\psi))^{\lambda-1}}{\Xi^{-1}(\psi)} 
\] (39)

The slow-roll parameters of the model in this case are given by
FIG. 3: Spectral index $n_s$ in terms of the number of e-folds $N$: (a) for $\lambda = 10$ and (b) for $\lambda = 50$ (where $\Gamma \propto \frac{1}{T^2}$).

\[
\epsilon = \frac{[\ln \Xi^{-1}(\psi)]^{1-\lambda}}{\lambda A} \quad (40)
\]
\[
\eta = \frac{2[\ln \Xi^{-1}(\psi)]^{1-\lambda}}{\lambda A}
\]

The number of e-folds between two fields $\psi_1$ and $\psi(t)$ may be determined (From Eqs. (37) and (38)).

\[
N = A((\ln \Xi^{-1}(\psi))^\lambda - (\ln \Xi^{-1}(\psi_1))^\lambda) = A((\ln \Xi^{-1}(\psi))^\lambda - (\lambda A)^{\frac{1}{1-\lambda}}) \quad (41)
\]

where $\psi_1$ is the inflaton at the beginning of the inflation epoch where $\epsilon = 1$. Inflaton field in the inflation period may be obtained in terms of the number of e-folds (From the above equation).

\[
\psi = \Xi[\exp([\frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}}]^\frac{1}{\lambda})]
\]

Power spectrum and tensor-scalar ratio in this case are given by

\[
\Delta^2_R = T^2 \frac{\Gamma \lambda A}{(4\pi)^3} \frac{[\ln \Xi^{-1}(\psi)]^{\frac{\lambda-1}{\lambda}}}{\psi^2 \Xi^{-1}(\psi)} \quad (43)
\]
\[
= T^2 \frac{\Gamma \lambda A}{(4\pi)^3} [\frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}}]^\frac{1}{\lambda} \times \exp(-[\frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}}]^\frac{1}{\lambda}) \Xi^2[\exp([\frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}}]^\frac{1}{\lambda})]
\]
\[
R = \frac{2(\lambda A)^{\frac{3}{2}}}{\pi^2 T^2} \frac{(4\pi)^3}{\Gamma} \frac{[\ln \Xi^{-1}]^{\frac{3\lambda-3}{2}}}{(\Xi^{-1})^\frac{1}{2}} \psi^2
\]
Spectral indices for our model have the following forms

\[ n_s - 1 = \frac{3}{2} \epsilon - \eta = -\frac{1}{2} \frac{[\ln \Xi^{-1}(\psi)]^{1-\lambda}}{\lambda A} \tag{44} \]

\[ = -\frac{1}{2\lambda A^2} \left( \frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}} \right)^{1-\lambda} \]

\[ n_T = -2 \frac{(\ln \Xi^{-1})^{1-\lambda}}{\lambda A} \]

In Fig.(3), the spectral index \( n_s \) in terms of the number of e-folds is plotted (for \( \lambda = 10, \lambda = 50, \) cases). We can see the small values of number of e-folds are assured for large values of \( \lambda \) parameter. We could find the tensor-scalar ratio in terms of the number of e-folds and spectral index \( n_s \)

\[ R = \frac{2(\lambda A)^{\frac{3}{2}}}{\pi^2 T^2} \left( \frac{(4\pi)^2}{\Gamma} \right)^{\frac{3}{2}} \Xi^2 \left[ \exp \left( \frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}} \right)^{\frac{1}{1-\lambda}} \right] \]

\[ \times \left( \frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}} \right)^{\frac{1}{1-\lambda}} \exp \left[-\frac{3}{2} \frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}} \right] \tag{45} \]

\[ \times \left( \frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}} \right)^{\frac{1}{1-\lambda}} \exp \left[-\frac{3}{2} \frac{N}{A} + (\lambda A)^{\frac{1}{1-\lambda}} \right] \tag{46} \]

In Fig.(4), the tensor-scalar ratio versus the number of e-folds is plotted for \( \lambda = 10, \lambda = 50. \)

**FIG. 4:** Tensor-scalar ratio in term of number of e-folds: (a) for \( \lambda = 10 \) and (b) for \( \lambda = 50 \) (where \( \Gamma \sim \frac{1}{T^2} \)).

We find the model to be compatible with observational data \([2, 3]\) (60 < \( N < 100 \), leads to \( R < 0.11 \)).

**V. CONCLUSION**

In this work we have studied the gauge-field inflation model in the context of warm inflation. The main problem of inflation theory is how to attach the universe to the end of
the inflation period. The study of phenomenological warm inflation scenario as a mechanism that gives an end for inflation models motivates us to consider the gauge-flation model using warm inflation theory. We have found the general conditions which are required for our model to be realizable in slow-roll limit. The model is compatible with Planck observational data. We have developed our model in the intermediate and logamediate scenarios. In the intermediate scenario, the numerical study for $f = \frac{1}{2}$ case leads to best compatibility with observational data ($N > 70$, leads to $R < 0.11$, and $N \simeq 70$, leads to $n_s \simeq 0.96$ ). In the logamediate scenario, we have studied $\lambda = 10$, and $\lambda = 50$, cases. In $\lambda = 50$ case, where $60 < N < 100$ we have found Planck result, $R < 0.11$.

[1] A. Guth, Phys. Rev. D 23, 347, (1981); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220, (1982).
[2] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].
[3] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO].
[4] B. A. Bassett, S. Tsujikawa and D.Wands, Rev. Mod. Phys. 78, 537 (2006), arXiv:astro-ph/0507632.
[5] A. Maleknejad, M. M. Sheikh-Jabbari, Phys. Rev. D 84, 043515 (2011); A. Maleknejad, M. M. Sheikh-Jabbari, arXiv:1102.1513 [hep-ph].
[6] M. M. Sheikh-Jabbari, arXiv:1203.2265[hep-th]]; P. Adshead, M. Wyman, Phys. Rev. Lett. 108, 261302 (2012).
[7] A. Berera, Phys. Rev. Lett. 75 (1995) 3218 [astro-ph/9509049]; Phys. Rev. D 55 (1997) 3346 [hep-ph/9612239].
[8] L. M. H. Hall, I. G. Moss and A. Berera, Phys.Rev.D 69, 083525 (2004); I.G. Moss, Phys.Lett.B 154, 120 (1985); A. Berera, Nucl.Phys B 585, 666 (2000).
[9] Y. -F. Cai, J. B. Dent and D. A. Easson, Phys. Rev. D 83, 101301 (2011) arXiv:1011.4074 [hep-th]]; R. Cerezo and J. G. Rosa, JHEP 1301, 024 (2013) arXiv:1210.7975 [hep-ph]]; S. Bartrum, A. Berera and J. G. Rosa, Phys. Rev. D 86, 123525 (2012) arXiv:1208.4276 [hep-ph]]; M. Bastero-Gil, A. Berera, R. O. Ramos and J. G. Rosa, Phys. Lett. B 712, 425 (2012) arXiv:1110.3971 [hep-ph]]; M. Bastero-Gil, A. Berera and J. G. Rosa, Phys. Rev. D 84, 103503 (2011) arXiv:1103.5623 [hep-th].
[10] A. Berera, M. Gleiser and R.O. Ramos, Phys. Rev. D 58 (1998) 123508 [hep-ph/9803394]; Phys. Rev. Lett. 83 (1999) 264 [hep-ph/9809583].

[11] M. Bastero-Gil, A. Berera and R. O. Ramos, JCAP 1109, 033 (2011) [arXiv:1008.1929 [hep-ph]].

[12] M. Bastero-Gil, A. Berera, R. O. Ramos and J. G. Rosa, JCAP 1301, 016 (2013) [arXiv:1207.0445 [hep-ph]].

[13] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49, 1110 (1982).

[14] V. F. Mukhanov and G. V. Chibisov, Pisma Zh. Eksp. Teor. Fiz. 33, 549 (1981) [JETP Lett. 33, 532 (1981)]; S. W. Hawking, Phys. Lett. B 115, 295 (1982); A. A. Starobinsky, Phys. Lett. B 117, 175 (1982); J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. D 28, 679 (1983).

[15] K. Freese, J. A. Frieman, A. V. Olinto, Phys. Rev. Lett. 65, 3233 (1990).

[16] I. G. Moss and C. Xiong, On the consistency of warm inflation, JCAP 0811, 023 (2008) [astro-ph/0808.0261].

[17] A.K. Sanyal, Phys. Lett. B 645 (2007) 1 [astro-ph/0608104].

[18] T. Koivisto and D.F. Mota, Phys. Lett. B 644 (2007) 104 [astro-ph/0606078]; Phys. Rev. D 75 (2007) 023518 [hep-th/0609155].

[19] S. Mignemi and N. Stewart, Phys. Rev. D 47 (1993) 5259 [hep-th/9212146].

[20] S. Nojiri, S.D. Odintsov and M. Sasaki, Phys. Rev. D 71 (2005) 123509 [hep-th/0504052]; G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov and S. Zerbini, Phys. Rev. D 73 (2006) 084007 [hep-th/0601008].

[21] I. Antoniadis, J. Rizos and K. Tamvakis, Nucl. Phys. B 415 (1994) 497 [hep-th/9305025].

[22] J.D. Barrow and A.R. Liddle, Phys. Rev. D 47 (1993) 5219 [astro-ph/9303011]; A. Vallinotto, E.J. Copeland, E.W. Kolb, A.R. Liddle and D.A. Steer, Phys. Rev. D 69 (2004) 103519 [astro-ph/0311005]; A.A. Starobinsky, JETP Lett. 82 (2005) 169 [Pisma Zh. Eksp. Teor. Fiz. 82 (2005) 187] [astro-ph/0507193].

[23] M. R. Setare and V. Kamali, JCAP 1208, 034 (2012) [arXiv:1210.0742 [hep-th]].

[24] J. D. Barrow, Class. Quantum Grav. 13, 2965 (1996).

[25] J. D. Barrow, Phys. Rev. D 51, 2729 (1995).

[26] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003).

[27] P. G. Ferreira and M. Joyce, Phys. Rev. D 58, 023503 (1998).
[28] J. D. Barrow and N. J. Nunes, Phys. Rev. D 76, 043501 (2007); J. Yokoyama and K. Maeda, Phys. Lett. B 207, 31 (1988); A. K. Sanyal, Phys. Lett. B 645, 1 (2007).