Numerical solution of direct and inverse problems of non-isothermal gas filtration in zone-inhomogeneous reservoir

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Abstract. In this paper we propose formulation and method for solving the inverse problem of interpretation of the results of research thermal gas dynamic vertical gas wells. We investigate the influence of the form-input information on the solve of inverse problem. In this work we also demonstrate that based on pressure and temperature results of measurements at the well bottom after its start-up the bottomhole zone parameters can be estimated.

1. Introduction
The source of information on thermal processes in oil and gas reservoir formations is a well. We have widely used thermohydrodynamic methods for studying wells and seams based on deep measurement technology. The special value of these studies is due to the fact that, according to the measurement data, the filtration and thermophysical parameters of the exploited reservoirs [1, 6–8].

In this paper, a mathematical model is constructed for describing thermodynamic processes in the vicinity of a vertical well in a zone-inhomogeneous gas reservoir. The influence of the bottomhole zone parameters on changes in temperature and pressure at the bottom of the well after its start has been analyzed. A numerical approach to the mathematical processing of measurement results obtained during the operation of vertical gas wells is proposed, based on the application of the theory of inverse problems of mathematical physics. It is shown that the parameters of the formation and the parameters of the bottomhole zone can be estimated from the results of pressure and temperature changes at the bottom of the well after its start. The knowledge of the bottomhole zone parameters (permeability, radius) allows choosing the treatment technology for this zone to intensify the flow of gas to the well and to assess the effectiveness of the measures carried out.

2. Statement and numerical solution of the direct problem
The process of non-isothermal filtration of a real gas in a zone-inhomogeneous formation during its selection with a flow rate $Q$ through a single well located in the center of the deposit is described by a system of differential equations:

$$
\frac{m}{\rho} \left( \frac{\partial p}{\partial t} \right) = \frac{1}{r} \left( \frac{\partial}{\partial r} \left( \frac{k(r)}{\mu Tz} \frac{\partial p}{\partial r} \right) \right), \quad r \in (r_w, R_{res}), \quad t > 0, \quad (1)
$$

$$
C_{res} \frac{\partial T}{\partial t} = \frac{1}{r} \left( r \lambda_{res} \frac{\partial T}{\partial r} \right) + \frac{Q}{RTz} \left( m\eta \frac{\partial p}{\partial t} + \frac{k}{\mu \rho r} \left[ \frac{\partial T}{\partial r} - \frac{\partial p}{\partial r} \right] \right), \quad r \in (r_w, R_{res}), \quad t > 0, \quad (2)
$$

with initial
\[ p(r,0) = p_{res}, \quad T(r,0) = T_{res}, \quad r \in [r_u, R_{res}] \]  \tag{3} \\
and boundary conditions

\[ 2\pi H \left( \frac{k(r) p_{Tst}}{\mu} \right) \left[ \frac{\partial p}{\partial r} \right]_{r=r_u} = Q + C_{well} \frac{\partial}{\partial (T_z)} \left( \frac{p}{T_z} \right), \quad t > 0, \]  \tag{4} \\

\[ \left[ C_{res} \frac{\partial T}{\partial t} - C_p \frac{p}{RT_z} \left( m \eta \frac{\partial p}{\partial \tau} + k(r) \frac{\partial}{\partial r} \left[ \frac{\partial T}{\partial r} - \frac{\partial p}{\partial r} \right] \right) \right]_{r=r_u} = 0, \quad t > 0, \]  \tag{5} \\

\[ p(R_{res}, t) = p_{res}, \quad T(R_{res}, t) = T_{res}, \quad t > 0, \]  \tag{6} \\

\[ k(r) = \begin{cases} k_{\text{skin}}, & r \leq r_{\text{skin}}, \\ k_{res}, & r > r_{\text{skin}}, \end{cases} \]

where \( p = p(r,t), \) \( T = T(r,t) \) - pressure and temperature of gas in the reservoir, \( H \) - thickness of gas reservoir, \( p_{res}, p_{Tst} \) - reservoir and standard pressure, \( T_{res}, T_{st} \) - reservoir and standard temperature, \( r_u \) - well radius, \( R_{res} \) - gas reservoir boundary radius, \( \mu \) - gas viscosity, \( \eta \) - adiabatic expansion coefficient, \( \varepsilon \) - Joule-Thomson coefficient, \( C_{res} = mC_p \rho/RT_z + (1-m)C_{rock} \rho_{rock} \) - bulk heat capacity of a reservoir, \( C_p, C_{rock} \) - specific heat of gas and medium, \( \rho_{rock} \) - density of medium, \( m \) - porosity of the gas reservoir, \( z \) - supercompressibility coefficient of gas, \( R \) - gas constant, \( \lambda_{res} \) - thermal conductivity of a gas reservoir, \( C_{well} \) - wellbore volume effect ratio [3, 4]. The gas supercompressibility coefficient \( z \) is calculated by the formula of Gurevich-Latunov [2], the adiabatic expansion coefficients \( \eta \) and the Joule-Thomson coefficient \( \varepsilon \) are calculated from the formulas given in [2].

For the numerical solution of the nonlinear system of differential equations (1) - (6), the finite difference method is applied. When the boundary value problem is approximated, the construction of a finite-difference grid is carried out by means of a transition to a new spatial variable \( u = \ln r \).

In the region \( \Omega = \{u_{min} < u < u_{max} = \ln R_{res}, 0 < t \leq t_{exp} \} \) grid nodes are introduced

\[ \sigma_1 = \{u_{min}, u_{min} < u < u_{max} = u_{res}, u_1 = u_t (i-1)h, h = (u_{max} - u_{min})/(N-1), i = 0, N \} \]

\[ \sigma_2 = \{j \cdot 0 = t_0 < t_2 < \cdots < t_N = t_{exp}, j = t_N/\tau, \tau = 1/\bar{M} \} \]

and relied on \( p_{ij} = p(u_i, t_j), \quad T_{ij} = T(u_i, t_j) \).

Difference derivatives are introduced as follows:

\[ A_i[\zeta] = \frac{\xi_i - \xi_{i-1}}{h}, \quad A_i[\xi] = \frac{\xi_i - 2\xi_{i-1} + \xi_i}{h^2}, \]

\[ A_i[\tau] = \frac{1}{h} (\theta_i A_i[\zeta] - \theta_{i-1} A_i[\zeta]), \quad A_i[\xi] = \frac{\xi_j - \xi_{j-1}}{\tau}. \]

Then the discrete analogue of the boundary value problem (1) - (6) can be written in the form:

\[ m \Delta_i \left[ \frac{p}{T_z} \right]_{ij} = \frac{1}{\mu \varepsilon_{2n}} A_i \frac{k_i}{\mu h_j} \left[ p_{ij} \right], \quad i = 1, N - 1, \quad j = 1, M \]  \tag{7} \\

\[ \left[ C_{res} A_i \frac{\xi_{2n} \Delta_i}{2} - C_p \frac{R h_j}{\mu \varepsilon_{2n}} A_i \left[ p_{ij} A_i \right] \right] \left[ T \right]_{ij} = \frac{C_p}{R} h_j \left[ m \eta_i A_i + \frac{k_i \xi_{2n} \Delta_i}{\mu \varepsilon_{2n}} \right] \left[ p_{ij} \right], \quad i = 2, N - 1, \quad j = 1, M \]  \tag{8} \\

\[ p_{i0} = p_{res}, \quad T_{i0} = T_{res}, \quad i = 1, N \]  \tag{9} \\

\[ 2\pi H h_j \frac{k_i}{\mu} A_i \left[ p_{ij} \right] = Q + C_{well} \left[ \frac{p}{T_z} \right]_{ij}, \quad j = 1, M \]  \tag{10} \\

\[ \left[ C_{res} A_i \frac{\xi_{2n} \Delta_i}{2} - C_p \frac{R h_j}{\mu \varepsilon_{2n}} A_i \left[ p_{ij} A_i \right] \right] \left[ T \right]_{ij} - \frac{C_p}{R} h_j \left[ m \eta_i A_i + \frac{k_i \xi_{2n} \Delta_i}{\mu \varepsilon_{2n}} \right] \left[ p_{ij} \right] = 0, \quad j = 1, M \]  \tag{11} \\

\[ p_{Nj} = p_{res}, \quad T_{Nj} = T_{res}, \quad j = 1, M \]  \tag{12}
The obtained system of nonlinear algebraic equations (7) - (12) is solved by a sweep method with iterations, while the initial approximation on each time layer takes values from the previous layer.

3. Analysis of temperature and pressure variation curves

An analysis of the numerical solution of system (7) - (12) is carried out. Condition (11) allows us to solve the equation with respect to temperature (8) without using the splitting method for physical processes. Figure 1 shows a comparison of the numerical solution of equation (8) with the analytical solution [5]. The change in the temperature at the bottom of the well at its start with a constant production rate, calculated under conditions (fig. 1, curve 2), \( \lambda_{res} = 0, C_{well} = 0, \eta = 0, \varepsilon = \text{const}, C_{res} = \text{const} \) (fig. 1, curve 1), is in good agreement with the analytical. The deviation of the final section of the derivative temperature is characterized by the influence of the formation boundary (fig. 1, curve 1). It should be noted here that the change in pressure at the bottom of the well, calculated under the same conditions, practically coincides with the analytical solution [3].

Next, the effect of the bottomhole formation zone on the pressure and temperature changes at the bottom of the well after its start is investigated. The states of the bottomhole formation zone are characterized by the skin effect \( S \). In fig. 2 - 3 are the curves of pressure and temperature changes at the bottom of the well, depending on the skin effect value. Contamination of the bottomhole zone \( (S > 0) \) leads to an increase in the filtration resistance in the vicinity of the well, respectively, to an increase in depression (fig. 2, curve 3) and a decrease in temperature (fig. 3, curve 3). The initial sections of the pressure derivative curves differ depending on the value of the skin effect (fig. 2), but they can not diagnose the bottomhole zone state because the effect of the wellbore volume is affected. If the coefficient \( C_{well} \) is not taken into account, the initial sections of the pressure derivative curves deviate either downward \( (S > 0) \) or up \( (S < 0) \), depending on the value of the skin effect. The state of the bottomhole zone affects the character of the change in the final section of the curves of the temperature derivatives (fig. 3), regardless of the presence of the influence of the volume of the wellbore. This makes it possible to diagnose the state of the bottomhole zone from the final section of the temperature derivative curve (fig. 3, curves - 2, 3).
4. Statement and numerical solution of the inverse problem

To assess the parameters of the formation and the bottomhole zone, the data on the change in bottomhole pressure and temperature are used, recorded by deep instruments after the well was put into operation. The inverse problem is to determine the permeability $k_{\text{res}}$, the porosity $m$ of the gas reservoir, the permeability $k_{\text{skin}}$ and the radius $r_{\text{skin}}$ of the bottomhole zone when the process of non-isothermal filtration of a real gas to a vertical well in a zone-inhomogeneous formation is described by a system of equations (1) - (6). The following initial information is considered known:
\[ p(r_{out}, t) = \phi(t), \quad T(r_{out}, t) = \psi(t), \quad \text{(13)} \]

where, \( \phi(t), \psi(t) \) - observed values of pressure and temperature at the bottom of the well.

The solution of the inverse problem (1) - (6) and (13) reduces to minimizing one of the following functionals, depending on the type of initial information:

\[ F_1(a) = \int_0^T [\phi(t) - p(r_{out}, t)]^2 \, dt, \quad \text{(14)} \]
\[ F_2(a) = \int_0^T [\psi(t) - T(r_{out}, t)]^2 \, dt, \quad \text{(15)} \]
\[ F_3(a) = \int_0^T \left[ \xi[\phi(t) - p(r_{out}, t)]^2 + [\psi(t) - T(r_{out}, t)]^2 \right] \, dt, \quad \text{(16)} \]

where \( a = (k_{true}, m, k_{skin}, r_s) \), \( 0 < a_i \leq a < b_i \) (\( a_i, b_i = \text{const} \)), \( \xi \) - весовой коэффициент. An iteration sequence for minimizing the functionals is constructed on the basis of the Levenberg-Marquardt method. The convergence and stability of the iterative process with different input information have been investigated using standard examples.

For the exact values of the initial information, the minimization processes for the functionals (14), (15), and (16) converge in 15-20 iterations. To study the stability in the model curves, changes in bottomhole pressure and temperature randomly introduced errors \( \phi_\delta(t) = \phi(t) + \omega \delta_1, \quad \psi_\delta(t) = \psi(t) + \omega \delta_2 \), where \( \delta_1 = 0.1 \text{ MPa}, \quad \delta_2 = 0.1 \text{ K}, \quad \omega \) - a random variable distributed according to a different law on the interval [-1, 1]. The results of the calculations are shown in fig. 5 - 6. From the obtained results it follows that small changes in the initial information correspond to small changes in the solution of the inverse problem, i.e. The proposed method is stable with respect to the errors of the initial information.

In fig. 5 one of the characteristic calculations of the convergence of the iterative process of minimizing the functional (16) with the perturbed initial data (fig. 6, curves - 1), where \( a_i \) - true parameters, \( a_i^{abs} \) - calculated parameters. The calculated pressure and temperature variation curves are shown in fig. 6 (curves - 2). The iterative process converges over 25 iterations.

The results of the calculations showed that the estimates of the formation and bottomhole zone parameters obtained from minimizing the functional (16) have the smallest error. When using only the data on the pressure change (functional \( F_1 \)) as the initial information, the formation permeability and
skin effect are determined with sufficient accuracy, and if only the temperature change (functional $F_2$) is used, the permeability of the near-wellbore zone and the porosity. Thus, to evaluate the formation and bottomhole zone parameters, it is preferable to use simultaneous measurements of bottomhole pressure and temperature as the initial information.

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