Probing polarized strangeness in the proton:
the use of heavy quarks and the renormalization group

STEVEN D. BASS

High Energy Physics Group,
Institute for Experimental Physics and Institute for Theoretical Physics,
Universität Innsbruck,
Technikerstrasse 25, A 6020 Innsbruck, Austria

Once heavy-quark corrections are take into account $\nu p$ elastic scattering provides a complementary probe of polarized strangeness in the nucleon to measurements from inclusive and semi-inclusive polarized deep inelastic scattering. We review the different types of experiment and a recent NLO calculation of heavy-quark contributions to the weak neutral-current axial-charge (measureable in $\nu p$ elastic scattering) performed using Witten’s heavy-quark renormalization group method.

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1. Introduction

Understanding the internal spin structure of the proton is one of the most challenging problems facing subatomic physics: How is the spin of the proton built up out from the intrinsic spin and orbital angular momentum of its quark and gluonic constituents? A key issue is the contribution of polarized strangeness in building up the spin of the proton. Fully inclusive and semi-inclusive measurements of polarized deep inelastic scattering together with elastic $\nu p$ scattering provide complementary information about the transverse momentum and Bjorken $x$ distributions of strange quark polarization in the proton. In this paper we review the different experiments and outline the vital role of charm (and other heavy quarks) in helping to pin down the size and interpretation of polarized strangeness in the proton.

Our present knowledge about the spin structure of the nucleon comes from polarized deep inelastic scattering. Following pioneering experiments at SLAC [1], recent experiments in fully inclusive polarized deep inelastic
scattering have extended measurements of the nucleon’s $g_1$ spin dependent structure function to lower values of Bjorken $x$ where the nucleon’s sea becomes important [2]. From the first moment of $g_1$, these experiments have revealed a small value for the flavour-singlet axial-charge:

$$g_A^{(0)}_{\text{inv}} = 0.2 - 0.35$$

This result is particularly interesting [3] because $g_A^{(0)}$ is interpreted in the parton model as the fraction of the proton’s spin which is carried by the intrinsic spin of its quark and antiquark constituents. The value (1) is about half the prediction of relativistic constituent quark models ($\sim 60\%$). It corresponds to a negative strange-quark polarization

$$\Delta s = -0.10 \pm 0.04.$$ (2)

(polarized in the opposite direction to the spin of the proton).

The small value of $g_A^{(0)}$ measured in polarized deep inelastic scattering has inspired vast experimental and theoretical activity to understand the spin structure of the proton. New experiments are underway or being planned to map out the proton’s spin-flavor structure and to measure the amount of spin carried by polarized gluons in the polarized proton. These include semi-inclusive polarized deep inelastic scattering [4], polarized proton-proton collisions at the Relativistic Heavy Ion Collider (RHIC) [5], and polarized ep collider studies [6]. It is essential to ensure that the theory and experimental acceptance are correctly matched when extracting new information from present and future experiments [7]. For example, the spin-flavor structure of the sea extracted from semi-inclusive measurements of polarized deep inelastic scattering may depend strongly on the angular acceptance of the detector.

A clean measurement of parity violating $\nu p$ elastic scattering [8,9] would provide an exciting new opportunity to probe strangeness polarization in the proton. This experiment measures the weak axial charge $g_A^{(Z)}$ through elastic $Z^0$ exchange. Because of anomaly cancellation in the Standard Model the weak neutral current couples to the combination $u - d + c - s + t - b$, viz.

$$J_{\mu}^Z = \frac{1}{2} \left\{ \sum_{q=u,c,t} - \sum_{q=d,s,b} \right\} \bar{q} \gamma_\mu \gamma_5 q$$

It measures the combination:

$$2g_A^{(Z)} = (\Delta u - \Delta d - \Delta s) + (\Delta c - \Delta b + \Delta t)$$

where $\Delta q$ refers to the expectation value

$$\langle p, s \bar{q} \gamma_\mu \gamma_5 q | p, s \rangle = 2m_p s_\mu \Delta q$$
for a proton of spin $s_\mu$ and mass $m_p$. The contribution $\Delta u - \Delta d$ in Eq. 4 is just the axial charge measured in neutron beta-decays ($g_A^{(3)} = 1.267 \pm 0.004$). Hence, once heavy-quark corrections [10, 11, 12, 13] have been taken into account, $g_A^{(Z)}$ is related (modulo the issue of $\delta$–function terms at $x = 0$ [14]) to the strange-quark axial-charge (polarized strangeness), defined scale invariantly, which is extracted from polarized deep inelastic scattering. A quality $\nu p$ elastic measurement would be independent of assumptions about the $x \sim 0$ behaviour of the proton’s $g_1$ spin structure function. A definitive measurement of $\nu p$ elastic scattering may be possible using the miniBooNE set-up at FNAL [9].

We next review the different types of experiment: inclusive and semi-inclusive polarized deep inelastic scattering, and elastic $\nu p$ scattering respectively (Sections 2-4). The renormalization scale invariant axial-charge $\Delta q_{|\text{inv}}$ is defined in Section 2. Section 4 summarizes the recent NLO calculation [13] of the heavy-quark contributions to $g_A^{(Z)}$, which was performed using the rigor of Witten’s heavy-quark renormalization group [15, 16].

2. Polarized deep inelastic scattering

The value of $g_A^{(0)}_{|\text{inv}}$ extracted from polarized deep inelastic scattering is obtained as follows. The first moment of the structure function $g_1$ to the scale-invariant axial charges of the target nucleon by:

\begin{equation}
\int_0^1 dx \, g_1^p(x, Q^2) = \left( \frac{1}{12} g_A^{(3)} + \frac{1}{36} g_A^{(8)} \right) \left\{ 1 + \sum_{\ell \geq 1} c_{\text{NS} \ell} \alpha_s^\ell(Q) \right\} \\
+ \frac{1}{9} g_A^{(0)}_{|\text{inv}} \left\{ 1 + \sum_{\ell \geq 1} c_{\text{S} \ell} \alpha_s^\ell(Q) \right\} + \mathcal{O}\left( \frac{1}{Q^2} \right). \tag{5}
\end{equation}

Here $g_A^{(3)}$, $g_A^{(8)}$ and $g_A^{(0)}_{|\text{inv}}$ are the isoriplet, SU(3) octet and scale-invariant flavour-singlet axial charges respectively. The flavour non-singlet $c_{\text{NS} \ell}$ and singlet $c_{\text{S} \ell}$ Wilson coefficients are calculable in $\ell$-loop perturbative QCD. The main source of experimental error on deep inelastic measurements of $g_A^{(0)}_{|\text{inv}}$ comes from the extrapolation of the measured $g_1$ spin structure function to lower values of Bjorken $x$.

Note that the first moment of $g_1$ is constrained by low energy weak interactions. For proton states $|p, s\rangle$ with momentum $p_\mu$ and spin $s_\mu$:

\begin{align}
2ms_\mu \, g_A^{(3)} &= \langle p, s | \left( \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d \right) | p, s \rangle \\
2ms_\mu \, g_A^{(8)} &= \langle p, s | \left( \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d - 2\bar{s} \gamma_\mu \gamma_5 s \right) | p, s \rangle \tag{6}
\end{align}
Here $g_A^{(3)} = 1.267 \pm 0.004$ is the isotriplet axial charge measured in neutron beta-decay; $g_A^{(8)} = 0.58 \pm 0.03$ is the octet charge measured independently in hyperon beta decay. The scale-invariant flavour-singlet axial charge $g_A|_{inv}^{(0)}$ is defined as follows. Let $\alpha_f = g_f^2/4\pi$ and $\beta_f(\alpha_f)$ be the gluon coupling and beta function for $\overline{\text{MS}}$ renormalized quantum chromodynamics (QCD) with $f$ flavours and $N_c = 3$ colours, and let $\gamma_f(\alpha_f)$ be the gamma function for the singlet current

\begin{equation}
(\bar{u}\gamma_\mu\gamma_5u + \bar{d}\gamma_\mu\gamma_5d + \ldots)_f = \sum_{k=1}^{f} (\bar{q}_k\gamma_\mu\gamma_5q_k)_f
\end{equation}

which is induced by the QCD axial anomaly

\begin{equation}
\partial^\mu \sum_{k=1}^{f} (\bar{q}_k\gamma_\mu\gamma_5q_k)_f = 2f\partial^\mu K_\mu + \sum_{i=1}^{f} 2im_i\bar{q}_i\gamma_5q_i
\end{equation}

where $K_\mu$ is the gluonic Chern-Simons current. A scale-invariant current $(S_{\mu5})_f$ is obtained when (7) is multiplied by

\begin{equation}
E_f(\alpha_f) = \exp \int_0^{\alpha_f} dx \gamma_f(x) / \beta_f(x)
\end{equation}

The invariant singlet charge is given by

\begin{equation}
g_A|_{inv}^{(0)} = E_3(\alpha_3)(\Delta u + \Delta d + \Delta s)_3 = (\Delta u + \Delta d + \Delta s)|_{inv}
\end{equation}

Flavour-dependent, scale-invariant axial charges $\Delta q|_{inv}$ such as

\begin{equation}
\Delta s|_{inv} = \frac{1}{3} \left( g_A|_{inv}^{(0)} - g_A^{(8)} \right)
\end{equation}

can then be obtained from linear combinations of (10) and

\begin{align}
g_A^{(3)} &= \Delta u - \Delta d = (\Delta u - \Delta d)|_{inv} \\
g_A^{(8)} &= \Delta u + \Delta d - 2\Delta s = (\Delta u + \Delta d - 2\Delta s)|_{inv}
\end{align}

Modulo heavy-quark corrections, $g_A^{(3)}$ and $g_A^{(8)}$ together with $g_A^{(Z)}$ would provide a weak interaction determination of $\Delta s|_{inv}$, complementary to the DIS measurement.
3. Semi-inclusive polarized deep inelastic scattering

Semi-inclusive measurements of fast pions and kaons in the current fragmentation region with final state particle identification can be used to reconstruct the individual up, down and strange quark contributions to the proton’s spin \[18,19\]. In contrast to inclusive polarized deep inelastic scattering where the $g_1$ structure function is deduced by detecting only the scattered lepton, the detected particles in the semi-inclusive experiments are high-energy (greater than 20% of the energy of the incident photon) charged pions and kaons in coincidence with the scattered lepton. For large energy fraction $z = E_h/E_\gamma \to 1$ the most probable occurrence is that the detected $\pi^\pm$ and $K^\pm$ contain the struck quark or antiquark in their valence Fock state. They therefore act as a tag of the flavour of the struck quark.

New semi-inclusive data reported by the HERMES experiment \[20,4\] (following earlier work by SMC \[21\]) suggest that the light-flavoured (up and down) sea measured in these semi-inclusive experiments contributes close to zero to the proton’s spin. Furthermore, recent HERMES data \[4\] also tends to favour slightly positively polarized strangeness in the kinematical range probed by the experiment.

An important issue for semi-inclusive measurements is the angular coverage of the detector \[7\]. The non-valence spin-flavor structure of the proton extracted from semi-inclusive measurements of polarized deep inelastic scattering may depend strongly on the transverse momentum (and angular) acceptance of the detected final-state hadrons which are used to determine the individual polarized sea distributions. The present semi-inclusive experiments detect final-state hadrons produced only at small angles from the incident lepton beam (about 150 mrad angular coverage) whereas the perturbative QCD “polarized gluon interpretation” \[22\] of the inclusive measurement \[2\] involves physics at the maximum transverse momentum \[23,24\] and large angles.

New semi-inclusive measurements with increased luminosity and a 4π detector, as proposed for the next generation Electron Ion Collider facility in the United States, would be extremely useful to map out the transverse momentum distribution of the total polarized strangeness \[2\] measured in inclusive deep inelastic scattering.

To understand this physics, consider the polarized photon-gluon fusion contribution to the polarized sea \[23,24\]. In leading twist, the first moment of the $g_1$ spin structure function for polarized photon-gluon fusion ($\gamma^*g \to q\bar{q}$) receives a positive contribution proportional to the mass squared of the struck quark or antiquark which originates from low values of quark transverse momentum, $k_t$, with respect to the photon-gluon direction. It also receives a negative contribution from $k_t^2 \sim Q^2$, where $Q^2$ is the virtuality...
Fig. 1. $\int_0^1 dx \ g_1^{(\gamma^*g)}|_{\text{soft}}$ for polarized strangeness production with $k_t^2 < \lambda^2$ in units of $\frac{\alpha_s}{2\pi}$. Here $Q^2 = 2.5\text{GeV}^2$ (dotted line) and $10\text{GeV}^2$ (solid line).

Fig. 2. $\int_0^1 dx \ g_1^{(\gamma^*g)}|_{\text{soft}}$ for light-flavor ($u$ or $d$) production with $k_t^2 < \lambda^2$ in units of $\frac{\alpha_s}{2\pi}$. Here $Q^2 = 2.5\text{GeV}^2$ (dotted line) and $10\text{GeV}^2$ (solid line).

of the hard photon. Thus, the spin-flavor structure of the sea extracted from semi-inclusive measurements depends strongly on the $k_t$ distribution of the detected hadrons. The positive mass-dependent contribution from low $k_t^2$ can safely be neglected for light-quark flavour (up and down) production.
It is very important for strangeness (and charm [25, 26]) production. Let \( P^2 \) and denote virtuality of the target gluon and \( m \) denote the mass of the struck quark. The fully inclusive first moment \( \int_0^1 dx g_1^{(\gamma^* g)} \) is equal to \(-\frac{\alpha_s}{2\pi}\) in the limit \( m^2 \ll P^2 \) and vanishes in the limit \( m^2 \gg P^2 \). The vanishing of \( \int_0^1 dx g_1^{(\gamma^* g)} \) in the limit \( m^2 \ll P^2 \) to leading order in \( \alpha_s(Q^2) \) follows from an application [27] of the fundamental Drell-Hearn-Gerasimov sum-rule [28].

The practical consequence [7] of the strange quark mass on polarized photon-gluon fusion is shown in Figs. 1-2. Here we let \( g_1^{(\gamma^* g)} \mid_{\text{soft}}(\lambda) \) denote the contribution to \( g_1^{(\gamma^* g)} \) for photon-gluon fusion where the hard photon scatters on the struck quark or antiquark carrying transverse momentum \( k^2_t < \lambda^2 \). Figs. 1 and 2 show the first moment of \( g_1^{(\gamma^* g)} \mid_{\text{soft}} \) for the strange and light (up and down) flavour production respectively as a function of the transverse momentum cut-off \( \lambda^2 \). Here we set \( Q^2 = 2.5\text{GeV}^2 \) (corresponding to the HERMES experiment) and 10\text{GeV}^2 (SMC). Following [23], we take \( P^2 \sim \Lambda^2_{\text{QCD}} \) and set \( P^2 = 0.1\text{GeV}^2 \). Observe the small value for the light-quark sea polarization at low transverse momentum and the positive value for the integrated strange sea polarization at low \( k^2_t \): \( k_t < 1.5\text{GeV} \) at the HERMES \( Q^2 = 2.5\text{GeV}^2 \). When we relax the cut-off, increasing the acceptance of the experiment, the measured strange sea polarization changes sign and becomes negative (the result implied by fully inclusive deep inelastic measurements). Note that for \( \gamma^* g \) fusion the cut-off \( k^2_t < \lambda^2 \) is equivalent to a cut-off on the angular acceptance \( \sin^2 \theta < 4\lambda^2/(s - 4m^2) \) where \( \theta \) is defined relative to the photon-gluon direction and \( s \) is the centre of mass energy for the photon-gluon collision. Leading-twist negative sea polarization at \( k^2_t \sim Q^2 \) corresponds, in part, to final state hadrons produced at large angles. For HERMES the average transverse momentum of the detected final-state fast hadrons is less than about 0.5 GeV whereas for SMC the \( k_t \) of the detected fast pions was less than about 1 GeV.

Semi-inclusive measurements of charm production in polarized lepton-proton scattering will be made by the COMPASS experiment at CERN and the SLAC experiment E-161. The aim of these experiments is to shed new light on the role of polarized glue in the nucleon, which will, in turn, help to resolve the origin of the negative polarized strange sea [2] extracted from inclusive polarized deep inelastic scattering.

4. \( \nu p \) elastic scattering

The \( \nu p \) elastic process [8] measures the neutral current axial-charge \( g_A^{(2)} \), where

\[
2g_A^{(2)} = (\Delta u - \Delta d - \Delta s) + (\Delta c - \Delta b + \Delta t)
\]  

(13)
Bass, Crewther, Steffens and Thomas have recently combined Witten’s renormalization group with the matching conditions of Bernreuther and Wetzel to calculate at next-to-leading order the complete heavy-quark contribution to $g_A^{(2)}$. One finds that, when first $t$, then $b$, and finally $c$ are decoupled from (4), the full NLO result is

$$2g_A^{(2)} = (\Delta u - \Delta d - \Delta s)_{\text{inv}} + \mathcal{P}(\Delta u + \Delta d + \Delta s)_{\text{inv}} + O(m_{t,b,c}^{-1}) \quad (14)$$

where $\mathcal{P}$ is a polynomial in the running couplings $\bar{\alpha}_h$,

$$\mathcal{P} = \frac{6}{23\pi}(\bar{\alpha}_b - \bar{\alpha}_t)\left\{1 + \frac{125663}{82800\pi}\bar{\alpha}_b + \frac{6167}{3312\pi}\bar{\alpha}_t - \frac{22}{75\pi}\bar{\alpha}_c\right\}$$

$$- \frac{6}{27\pi}\bar{\alpha}_c - \frac{181}{648\pi^2}\bar{\alpha}_c^2 + O(\bar{\alpha}_t^3, \bar{\alpha}_c^3) \quad (15)$$

Here $(\Delta q)_{\text{inv}}$ denotes the scale-invariant version of $\Delta q$ and $\bar{\alpha}_h$ denotes Witten renormalization group invariant running couplings. These Witten couplings are defined for the full theory (including the heavy-quark $h$) as follows. Let $m_h$ be the MS$_F$ renormalized mass and $\alpha_F$ denote the $F$-flavour running coupling. Then the Witten coupling

$$\bar{\alpha}_h = \bar{\alpha}_h(\alpha_F, \ln(m_h/\bar{\mu})) \quad (16)$$

is defined via

$$\ln(m_h/\bar{\mu}) = \int_{\alpha_F}^{\bar{\alpha}_h} dx \left(1 - \delta_F(x)/\beta_F(x)\right) \quad (17)$$

where $\delta_F$ denotes the mass anomalous dimension and $\beta_F$ is the $F$-flavour $\beta$ function. It satisfies the constraints

$$\bar{\alpha}_h(\alpha_F,0) = \alpha_F \quad , \quad \bar{\alpha}_h(\alpha_F, \infty) = 0 \quad (18)$$

the latter being a consequence of the asymptotic freedom of the $F$ flavour theory ($F \lesssim 16$). The Witten coupling is renormalization group invariant:

$$\mathcal{D}_F\bar{\alpha}_h = 0 \quad (19)$$

Taking $\bar{\alpha}_t = 0.1$, $\bar{\alpha}_b = 0.2$ and $\bar{\alpha}_c = 0.35$ in (15), we find a small heavy-quark correction factor $\mathcal{P} = -0.02$, with LO terms dominant.

These results are manifestly renormalization group invariant. They will permit a theoretically clean determination of the strange-quark axial-charge, $\Delta s_{\text{inv}}$, from the neutrino-proton elastic process. The results extend to NLO and make more precise the well known work of Collins, Wilczek and
Zee \cite{10} and Kaplan and Manohar \cite{11}, where heavy-quark effective theory was used to estimate $g_A^{(Z)}$ in leading order (LO) for sequential decoupling of $t, b$ and $t, b, c$ respectively.

There is interest \cite{9} to perform the experimental measurement at FNAL using the mini-BooNE set-up with very low duty factor neutrino beam to control background. The estimated error on the strange quark polarization one could extract from this experiment is 0.03, competitive with the error from the present polarized deep inelastic measurements.

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