Prediction of Hydrodynamic Forces Acting on Wigley Hull based on SQCM

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Abstract. Prediction of hydrodynamic forces acting on a ship hull is important to understand the manoeuvring performance of a ship at the initial design stage. To achieve the hydrodynamic forces, CFD calculations that directly solve the Navier-Stokes equation are often used recently, but they require much computational costs. Panel methods can also accurately represent the shape of a body and predict hydrodynamic forces acting on the body. SQCM (Source and Quasi Continuous Vortex Lattice Method) is one of panel methods which was developed at Kyushu University, and it has been confirmed that has a good accuracy to predict hydrodynamic force produced by a propeller. In this study, the SQCM is introduced to predict hydrodynamic forces acting on a Wigley hull in manoeuvring motion. In order to represent flow field around the Wigley hull appropriately, two kinds of free vortex models to be applied to predict hydrodynamic forces acting on the Wigley hull using the SQCM are examined. Predicted results are compared with model test results to verify the effectiveness of the free vortex models. The purpose of this study is to investigate the applicability of the SQCM to the prediction of hydrodynamic forces acting on a Wigley hull.

1. Introduction
Prediction of the hydrodynamic forces acting on a ship hull is important to understand the manoeuvring performance of a ship at the initial design stage. It is well known that there are three kinds of method to obtain various hydrodynamic coefficients to express the hydrodynamic forces acting on a hull: Conducting captive model tests, using approximate formulae with the parameters of principal dimensions of a ship based on past measurement data, and theoretical calculation methods. Captive models test can provide accurate hydrodynamic forces, but it requires enormous amount of time and cost. Approximate formulae are simple calculation method which can be used easily, but the estimation accuracy of the method becomes poor if the hull shape of a target ship is different from mother data used to develop the approximate formulae.

As the theoretical calculation methods, there are CFD calculation that directly solves the Navier-Stokes equation numerically, and a panel method that can relatively accurately represent the shape of a body. The former is being put to practical prediction of hull resistance, but in the field of ship manoeuvrability, standard calculation methods based on CFD have not been established.
Hess and Smith [1] introduced a method to calculate incompressible potential flow about arbitrary, non-lifting, and three-dimensional bodies. In this method, the body surface is divided into quadrilateral planes and sources are distributed on the planes. The source distribution is solved to make the normal component of velocity of the fluid becomes zero on the surface. By using quadrilateral surface, integral equation for the source distribution is replaced by a set of linear algebraic equations. The calculation method has two advantages over other methods, which is the equations that must be solved are two-dimensional over the body surface rather than the three-dimensional over the entire flow field and by working straightly on the body surface it can avoid the difficulty that other methods may encounter when the body surface converge the coordinate net in an arbitrary form.

On the other hand, QCM method has been developed by Lan [2] to solve the thin wing problem with considering the wing edge and Cauchy singularities, but is still keeping the flexibility and simplicity of the conventional Vortex Lattice Method. In order to fulfilling the conditions of wing boundary, chord wise vortex integral is lowered to finite sum through an altered trapezoidal rule and the theory of Chebychev while the distribution of span wise is assumed to be step wise. The result of his research stated that the current estimation for two-dimensional and three-dimensional wings are more accurate than the conventional Vortex Lattice Method for an airfoil with a flap deflection. The results of planar lifting surface are proportionate to those by some continuous loading methods, however this method requires much less computing time in some comparison.

A calculation method for three-dimensional unsteady wing problem has been developed by Nakatake et al. [3]. This simple surface panel method (SQCM, Source and Quasi Continuous Vortex Lattice Method) can easily satisfy the Kutta condition in steady and unsteady condition problems. By using Hess and Smith [1] method on the wing surface, the vortex strength and the source strength are determined simultaneously and the Kutta condition is satisfied automatically as same as QCM (Quasi-Continuous vortex lattice Method) developed by Lan [2]. By combining the source panel and unsteady QCM, SQCM can be extended to the unsteady problem for three-dimensional wing. It has been confirmed that this calculation method can accurately predict hydrodynamic force produced by a propeller.

In this study, Wigley Hull is considered as a thick wing. By applying SQCM to the Wigley hull, hydrodynamic forces acting on the hull are predicted. The influence of free vortex models on predicted results are examined. Predicted hydrodynamic forces are compared with model test results.

2. Calculation Method of SQCM

2.1. Basic theory

This chapter describes a method for estimating hydrodynamic forces acting on a wing-shaped body using the SQCM. Schematic view of the SQCM for two-dimensional body is shown in Figure 1 for simplicity. In the SQCM method, the shape of the body is represented by discrete panels based on the Hess and Smith Method [1] and sources are distributed for each panel. Furthermore, according to Lan [2], circulation around the body is expressed by vortices arranged along the central longitudinal plane of the body. By locating calculation points on the surface panels and the vortex plane as shown by red cross marks in Figure 1 and calculating the strength of the sources and the vortices that satisfy boundary conditions at the calculation points, hydrodynamic forces acting on a body in uniform flow can be estimated. As the strength of vortices become zero at the trailing edge of the wing-shaped body, the Kutta condition is automatically satisfied like the QCM. Since there is no need to perform repetitive calculation, the SQCM is an efficient calculation method with respect to calculation time.
Consequently, total flow velocity around a body in uniform flow is expressed as follows,

\[ \vec{v} = \vec{v}_0 + \vec{v}_s + \vec{v}_v, \]  

(1)

where,

- \( \vec{v} \) : total flow velocity vector,
- \( \vec{v}_0 \) : flow velocity vector by uniform flow around the body,
- \( \vec{v}_s \) : induced velocity vector by source panels,
- \( \vec{v}_v \) : induced velocity vector by vortices.

2.2. Calculation method of induced velocity by sources

2.2.1. Hess and Smith method. Assuming double body model of a body in uniform flow, the origin of the basic coordinate system \( o-xyz \) is defined as shown in Figure 2. The surface of the body is expressed by discrete points which are placed on its surface. These are identified as a group of four to form a quadrilateral surface element which is treated as a source panel. In order to form the quadrilateral surface element from a four given points, two “diagonal” vectors, each of which is simply the vector between the two of the four points are formed. Here, \( \vec{n} \) is a cross product of the two diagonal vectors, taken as the normal vector to the quadrilateral surface element. The four points which are projected parallel to the normal vector into the plane of the element to obtain the points at the corners of the quadrilateral. The element plane as defined here is equidistant from the four points used to form the element.

The basic formula of Hess and Smith method gives induced velocity by a quadrilateral source panels with a unit source density at a point in the space. Let the surface of the body have an equation of the form,

\[ F(x, y, z) = 0, \]  

(2)

where, \( x \), \( y \), and \( z \) are the coordinates in the basic coordinate system. The undisturbed flow coming to the body is taken as uniform flow of unit magnitude. This flow is described by the following equation,

\[ V_w = (V_{wx}^2 + V_{wy}^2 + V_{wz}^2)^{1/2} = 1. \]  

(3)

The fluid velocity at an arbitrary point may be denoted as the negative gradient of a velocity potential function \( \Phi \), which must satisfy three conditions; it must satisfy Laplace’s equation, it should have a zero normal derivative on the surface body, and it should reach the proper uniform flow potential at infinity, as follows,

\[ \Delta \Phi = 0 \quad \text{in} \quad R’, \]  

(4)

\[ \frac{\partial \Phi}{\partial n} \bigg|_S \equiv \vec{n} \cdot \text{grad} \Phi \bigg|_{\vec{n}=0} = 0, \]  

(5)

\[ \Phi \rightarrow -(xV_{wx} + yV_{wy} + zV_{wz}) \quad \text{for} \quad (x^2 + y^2 + z^2)^{1/2} \rightarrow \infty. \]  

(6)
Here, \( \Delta \) stands for the Laplacian operator and \( \vec{n} \) is the unit normal vector at an arbitrary point of the body surface. For convenience, \( \Phi \) can be written as:
\[
\Phi = \varphi_\infty + \varphi,
\]
where \( \varphi_\infty \) is the uniform flow potential and \( \varphi \) is the disturbance potential due to the body.

\( \varphi \) now will be denoted as the potential of source density distribution over the surface \( S \) of the body. The potential at arbitrary point \( P(x,y,z) \) in the basic coordinate system due to a unit point source located at a point \( q \) on the body surface is \( 1/r(P,q) \), where, \( r(P,q) \) is the distance between the points \( P \) and \( q \). Accordingly, the potential \( \varphi \) at \( P \) due to a source density distribution \( \sigma(q) \) on the surface of the body is given by,
\[
\varphi(x,y,z) = \iiint_S \frac{\sigma(q)}{r(P,q)} \, dS.
\]

2.2.2. Induction Velocity by Source of the Quadrilateral Plane. The panel fixed coordinate system \( O - \xi\eta\zeta \) is defined as shown in Figure 3. Note that the \( \zeta \) axis is set in the direction of the outward normal vector \( \vec{n} \) on the quadrilateral surface. In addition, one point is selected where the fluid velocity normal to the quadrilateral surface is required to vanish and where tangential velocity and pressure are eventually evaluated. The origin \( O \) of the panel fixed coordinate system is designated as a null point which is a point that doesn’t receive the induced velocity by the panel itself.

Let \( (\xi_k, \eta_0, 0) \) \( (k = 1, 2, 3, 4) \) be represented as the coordinates of the points at the corners of the quadrilateral in the panel fixed coordinate system as shown in Figure 4. Further, the point at the corners of the quadrilateral are set to clockwise direction as seen from the angle of the unit normal vector to the plane of the quadrilateral. The velocity potential \( \varphi \) at the point \( P(x,y,z) \) by the quadrilateral is given by the following equation,
\[
\varphi = \iiint_A \frac{dA}{r^3} = \frac{1}{r^3} \iiint_A \frac{d\xi d\eta}{[(x-\xi)^2 + (y-\eta)^2 + z^2]^{3/2}},
\]
where, \( r \) is the distance between the point \( P(x,y,z) \) and the point \( (\xi, \eta, 0) \) on the quadrilateral surface, and the range of integration is the area \( A \) of the quadrilateral surface.

Therefore, if the velocity vector induced at the point \( P \) in the panel fixed coordinate system is \( \vec{V}_{sl}(V_{slx}, V_{sly}, V_{slz}) \), then the components of the velocity are given by,
\[
V_{slx} = -\frac{\partial \varphi}{\partial x} = -\iiint_A \frac{(x-\xi)d\xi d\eta}{r^3},
\]
\[
V_{sly} = -\frac{\partial \varphi}{\partial y} = -\iiint_A \frac{(y-\eta)d\xi d\eta}{r^3},
\]
\[
V_{slz} = -\frac{\partial \varphi}{\partial z} = -\iiint_A \frac{zd\xi d\eta}{r^3}.
\]
The potential function for the inside and the outside of the quadrilateral source panel shown in Figure 4 is composed as the sum of the potentials of two semi-infinite source strips, each of whose boundaries consists of the side of the quadrilateral and two semi-infinite lines parallel to the one of the coordinate axes. The region corresponding to the inside of the quadrilateral has a value of source density $\sigma = +1/2$, and for the outside $\sigma = -1/2$. From Figures 4 and 5, the source densities on the strips cancel outside the quadrilateral and add inside to give a unit value. Thus, the potential and velocity of the quadrilateral are given as the sum of the potentials and velocities of the four sets of semi-infinite strips. Now, assuming that the induced velocity component on the $y$-axis direction at the point $P$ due to the pair of semi-infinite strips corresponding to the points $\left( \xi_k, \eta_k \right)$ and $\left( \xi_{k+1}, \eta_{k+1} \right)$ is $V_{sl, j, k+1} (k = 1, \cdots; \xi_j = \xi_k, \eta_j = \eta_k)$, it is given by the following equation,

$$V_{sl, j, k+1} = \frac{1}{2} \int_{\xi_k}^{\xi_{k+1}} d\xi \int_{\eta_j}^{\eta_{k+1}} \left[ (y - \eta) d\eta \right] \frac{(y - \eta)d\eta}{[(x - \xi_j)^2 + (y - \eta_j)^2 + z^2]^{1/2}}, \quad (11)$$

where, $\eta_{k+1}$ indicates the $\eta$ coordinate of a point on the side of the quadrilateral.

Furthermore, the following equations are obtained by executing the integration shown in the equation (11).

$$V_{sl, j, k+1} = \frac{\eta_{k+1} - \eta_k}{d_{k, k+1}} \ln \left( \frac{r_k + r_{k+1} - d_{k, k+1}}{r_k + r_{k+1} + d_{k, k+1}} \right) - \tan^{-1} \frac{m_{k, k+1} e_k - h_k}{z r_k} - \tan^{-1} \frac{m_{k, k+1} e_{k+1} - h_{k+1}}{z r_{k+1}}, \quad (12)$$

where,

$$r_k = [(x - \xi_j)^2 + (y - \eta_j)^2 + z^2]^{1/2}, \quad r_{k+1} = [(x - \xi_{k+1})^2 + (y - \eta_{k+1})^2 + z^2]^{1/2},$$

$$d_{k, k+1} = [(\xi_{k+1} - \xi_k)^2 + (\eta_{k+1} - \eta_k)^2]^{1/2},$$

$$e_k = z^2 + (x - \xi_k)^2, \quad e_{k+1} = z^2 + (x - \xi_{k+1})^2,$$

$$h_k = (y - \eta_j) - (x - \xi_k), \quad h_{k+1} = (y - \eta_{k+1}) - (x - \xi_{k+1}),$$

$$m_{k, k+1} = \frac{\eta_{k+1} - \eta_k}{\xi_{k+1} - \xi_k}.$$

Finally, induced velocity vector $\vec{V}_{sl}$ are given by the sum of the induced velocity by the four pairs of semi-infinite strips as follows,

$$\vec{V}_{sl} = \sum_{k=1}^{4} \left( \vec{t}_x V_{sl, j, k+1} + \vec{t}_y V_{sl, j, k+1} + \vec{k} V_{sl, j, k+1} \right), \quad (14)$$

where, $\vec{t}_x$, $\vec{t}_y$, and $\vec{k}$ are unit vectors in the panel coordinate system.

Because hydrodynamic forces acting on the body is handled in the basic coordinate system, after calculating the induced velocity from the sources expressed in the panel fixed coordinate system, it is
necessary to convert it back to the basic coordinate system. The induced velocity vector $\vec{V}_s$ at arbitrary point by a source of strength $\sigma$ in the basic coordinate system is expressed by the following equation using the induced velocity vector $\vec{V}_s$,

$$\vec{V}_s = \sigma M^{-1}\vec{V}_s,$$

(15)

where, $M$ is the conversion matrix to perform the conversion from the panel fixed coordinate system $O-\xi\eta\zeta$ to the basic coordinate system $o-xyz$.

2.2.3. Induced Velocity by All Quadrilateral Surface. Total induced velocity vector by all sources distributed on quadrilateral panels is expressed as the sum of the induced velocity vectors due to each quadrilateral panel.

Let $M$ be the number of quadrilateral panels used to approximate the body surface and $i$ be the number of arbitrary quadrilateral panel, then the induced velocity vector $\vec{V}_s$ by the entire source panels is given by the following equation,

$$\vec{V}_s = \sum_{i=1}^{M} \sigma_i M_i^{-1}\vec{V}_s,$$

(16)

By using normal vector $\vec{n}$ at an arbitrary calculation point, the boundary condition at the calculation point is expressed by the following equation,

$$\vec{V}_s \cdot \vec{n} = \sum_{i=1}^{M} \sigma_i C_{si},$$

(17)

where,

$$C_{si} = M_i^{-1}\vec{V}_s \cdot \vec{n}.$$  

(18)

$C_{si}$ is an influence coefficient representing the velocity induced by the $i$-th source panel having unit source strength.

2.3. Calculation method of induced velocity by horseshoe vortex

2.3.1. Horseshoe Vortices. Assuming a body in uniform flow is represented by its central longitudinal plane, the basic coordinate system shown in Figure 6 is defined as same as the coordinate system shown in Figure 2. Here, the positive directions of $x$, $y$, and $z$ axes are backward, port side, and upward, respectively. Then discrete horseshoe vortices having shedding angle $\Theta$ with respect to the...
Arbitrary calculation point

2.3.3. Induced Velocity by Horseshoe Vortex

Let assume that a horseshoe vortex $A'ABB'$ shown in Figure 6 is the one of the horseshoe vortices distributed on the central longitudinal plane and it has unit vortex strength, velocity vector $\vec{v}$ at an arbitrary point $P(x, y, z)$ induced by the horseshoe vortex $A'ABB'$ is given by the following equation,

$$\vec{v} = \left( K^{(b)}(x, y, z) + K^{(f_1)}(x, y, z) + K^{(f_2)}(x, y, z) \right) \vec{e},$$

where,

$$K^{(b)}(x, y, z) = \frac{1}{4\pi \left| PH_b \right|} \left( \frac{PB \cdot AB}{PB \parallel AB} - \frac{PA \cdot AB}{PA \parallel AB} \right),$$

$$K^{(f_1)}(x, y, z) = \frac{1}{4\pi \left| PH_{f_1} \right|} \left( \frac{PA \cdot AB}{PA \parallel AB} + 1 \right),$$

$$K^{(f_2)}(x, y, z) = \frac{1}{4\pi \left| PH_{f_2} \right|} \left( 1 - \frac{PB \cdot AB}{PB \parallel AB} \right).$$

$H_b$, $H_{f_1}$, and $H_{f_2}$ are the feet of perpendicular drawn from the point $P$ to the line segments of the bound vortex $AB$, the free vortex $AA'$, and the free vortex $BB'$, $K^{(b)}$, $K^{(f_1)}$, and $K^{(f_2)}$ are influence functions of the vortices, respectively. The vector $\vec{e}$ is a unit vector representing the direction of the induced velocity.

2.3.4. Induced Velocity by Horseshoe Vortices Distributed on the Central Longitudinal Plane of the Body

Let the strength of each horseshoe vortex be $\gamma_{\mu\nu}$, where $\mu = 1, \ldots, N_\mu$, $\nu = 1, \ldots, N_\nu$, and the induced velocity vector by a single horseshoe vortex having unit strength be $\vec{\nu}_{\mu\nu}$, the induced velocity vectors $\vec{V}$ by all the horseshoe vortices on the central longitudinal plane of the body are given by the following equation,

$$\vec{V} = \frac{\pi}{2N_\nu} \sum_{\mu=1}^{N_\mu} L \sum_{\nu=1}^{N_\nu} \gamma_{\mu\nu} \vec{\nu}_{\mu\nu} \sin \left( \frac{2\nu - 1}{2N_\nu} \pi \right).$$

Further, assuming that the number of horseshoe vortices is $N_\nu$ and normal vector at an arbitrary calculation point $\hat{n}$, equation (22) can be expressed as the following equation.
\[
\vec{V}_v \cdot \vec{n} = \sum_{j=1}^{N} \gamma_j C_{ij},
\]
where,
\[
C_{ij} = \frac{\pi L}{2N_v} \sin \left( \frac{2v-1}{2N_v} \pi \right) \vec{V}_{ij} \cdot \vec{n}, \quad j = (\mu - 1) \times N_v + \nu, \quad (\mu = 1, \cdots, N_v; \ \nu = 1, \cdots, N_v).
\]

\(j\) represents the sequential number of the horseshoe vortices, and \(C_{ij}\) is an influence coefficient representing the induced velocity by the \(j\)-th horseshoe vortex having unit strength. \(\nu\) and \(\mu\) is the number of divisions in the \(x\)-axis and \(z\)-axis directions, respectively.

2.4. Boundary conditions
In SQCM, the source strength \(\sigma_i (i = 1, \cdots, M)\) and the vortex strength \(\gamma_j (j = 1, \cdots, N)\) are resolved by satisfying the boundary condition assuming that the flow does not pass through the body surface where the source panel is placed and the central longitudinal plane of the body where the vortex is arranged,
\[
\vec{V}_k \cdot \vec{n}_k = 0 \quad (k = 1, 2, \cdots, M + N),
\]
where, \(\vec{V}_k\) and \(\vec{n}_k\) are the flow velocity and the normal vector at the \(k\)-th calculation point. From equation (25), the following equation can be obtained,
\[
\sum_{i=1}^{M} \sigma_i C_{ki} + \sum_{j=1}^{N} \gamma_j C_{kj} + H_k = 0 \quad (k = 1, 2, \cdots, M + N),
\]
where,
\[
H_k = \vec{V}_0 \cdot \vec{n}_k \quad (k = 1, 2, \cdots, M + N),
\]
\(\vec{V}_0\) is flow velocity vector by uniform flow around the body. When equation (26) displayed in a matrix, it can be expressed as follows,
\[
\begin{pmatrix}
C_{b1} & \cdots & C_{bM} & C_{b1} & \cdots & C_{bM} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
C_{s1} & \cdots & C_{sM} & C_{s1} & \cdots & C_{sM} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
C_{sM1} & \cdots & C_{sM1} & C_{sM1} & \cdots & C_{sM1} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
C_{sM} & \cdots & C_{sM} & C_{sM} & \cdots & C_{sM}
\end{pmatrix}
\begin{pmatrix}
\sigma_1 \\
\vdots \\
\sigma_M \\
\sigma_1 \\
\vdots \\
\gamma_N
\end{pmatrix}
+ \begin{pmatrix}
H_1 \\
\vdots \\
H_M \\
H_{M+1} \\
\vdots \\
H_{M+M}
\end{pmatrix}
= 0.
\]

By solving equation (28), the source strength \(\sigma\) of each hull panel and the vortex strength \(\gamma\) of each horseshoe vortex can be determined.

2.5. Calculation Method of Hydrodynamic Force Coefficients
When the source strength and the vortex strength are obtained, the flow velocity vector \(\vec{V}\) of the entire flow field can be determined. By using this entire flow velocity vector, the pressure coefficient \(C_p\) on the body surface is given by the following equation according to Bernoulli’s theorem,
\[
C_p = 1 - \frac{\vec{V}^2}{|\vec{V}_0|^2}.
\]
Further, the lateral force $F_y$ and the yawing moment $M_z$ acting on the hull can be obtained by the following equations using the pressure coefficient $C_p$.

$$
egin{align*}
F_y &= -\frac{\rho}{2} \int_S C_p n_i dS = -\frac{\rho}{2} \sum_{i=1}^{M} C_p n_i A_i,
M_z &= -\frac{\rho}{2} \int_S C_p (n_y y - n_x x) dS = -\frac{\rho}{2} \sum_{i=1}^{M} C_p n_i A_i (n_y y - n_x x),
\end{align*}
$$

(30)

where,
- $\rho$ : fluid density,
- $i$ : source panel number,
- $n_x, n_y$ : $x$ and $y$ components of the $i$-th panel normal vector,
- $x, y$ : $x$ and $y$ coordinate of the $i$-th panel,
- $A_i$ : area of the $i$-th panel.

### 3. Application of SQCM for Wigley hull

#### 3.1. Target Ship.

The SQCM is applied to the Wigley hull. The Wigley hull is a mathematical hull form that is symmetrical with respect to all directions expressed by the following formula,

$$
y(x, z) = \frac{B}{2} \left[ 1 - \left( \frac{x}{l} \right)^{2} \right] \left[ 1 - \left( \frac{z}{d} \right)^{2} \right],
$$

(31)

where, $B$ is the ship breadth, $l$ is the half ship length ($= L/2$), and $d$ is the draft. Table 1 shows the principal particulars of the Wigley hull used in calculations.

Table 1. Principal particulars of Wigley hull.

|       |         |
|-------|---------|
| $L$ (m) | 2.500   |
| $B$ (m) | 0.250   |
| $d$ (m) | 0.156   |
| $C_p$  | 0.444   |

Figure 7 shows the body plan and side views of the Wigley hull. As shown in the body plan, the Wigley hull does not have a flat part at the bottom of the hull but draws an arc from the keel part in the width direction of the hull. Further, as shown in the side view, the bow and the stern are vertical, and the distance between the bow and the stern is equal regardless of the height from the bottom. Therefore, the shape is different from the actual ship hull form.

**Figure 7.** Body plan and side view of Wigley hull
3.2. Calculation Conditions

The numbers of divisions for the arrangement of the vortices were selected as \( \nu = 120 \) in longitudinal direction and \( \mu = 20 \) in vertical direction. As for the source panels arranged on the port and starboard sides of the hull, the number of divisions in longitudinal direction is 240 (= 120 \times 2). In addition, the ship breadth at the stern was set to be 1\% wider than the exact breadth to avoid the divergence of induced velocity due to the computational singularity of the stern edge. Figure 8 shows the hull shape of the Wigley hull under these calculation conditions.

The ship speed \( V_0 \) is set at 0.5 (m/s) and it is used as uniform flow velocity and vortex shedding angle \( \Theta \) is assumed as the half of drift angle \( \beta \) [4]. Finally, the following formulae are used to non-dimensionalize the hydrodynamic forces acting on the Wigley hull,

\[
C_y = \frac{F_y}{\frac{1}{2} \rho S V_0^2}, \quad C_m = \frac{M_z}{\frac{1}{2} \rho L^2 d V_0^2}, \quad (32)
\]

where, \( S \) is the wetted surface area of the Wigley hull.

3.3. Vortex Models

In this study, two models of free vortex shed from the Wigley hull are investigated. The first model (later referred as model 1) is shown in Figure 9. In this model, all free vortices of the horseshoe vortices distributed along the central longitudinal plane are assumed to be shed from the stern of the Wigley hull. On the other hand, in the second model (later referred as model 2) shown in Figure 10, free vortices of horseshoe vortices located at the bottom of the Wigley hull are assumed to be shed from the bottom. It is considered that this model is close to the real phenomenon around the hull. Free vortices of other horseshoe vortices are shed from the stern as same as the model 1.

3.4. Calculation Results

3.4.1. Lateral Force and Yawing Moment Coefficient. Figures 11 and 12 show the calculation results of the lateral force coefficient \( C_y \) and the yawing moment coefficient \( C_m \) of the Wigley hull for the

Figure 8. Wigley hull shape used for calculation

Figure 9. Model 1 outflow method for Wigley hull form

Figure 10. Model 2 outflow method for Wigley hull form
model 1 and the model 2 as the function of drift angle $\beta$. It can be observed that calculated lateral force coefficient $C_y$ of the model 1 shown with red solid line in Figure 11 is too small comparing with the measured results. Same tendency is also found in the results of yawing moment coefficient $C_m$ presented in Figure 12. On the other hand, improvement of prediction accuracy for the lateral force coefficient $C_y$ is observed for the model 2 as shown with blue solid line in Figure 11. However, yawing moment coefficient $C_m$ shown with blue solid line in Figure 12 is still too small comparing with measured results, though the value of $C_m$ for large drift angle is bigger than that for the model 1.

3.4.2. Pressure Coefficients. Pressure distribution around the Wigley hull form at drift angle $\beta = 20^\circ$ are presented in Figures 13 to 16. These figures show the values of pressure coefficient $C_p$ with the positive pressure being red colour, the negative pressure being blue colour, and the pressure 0 being white colour. For simplicity, mirror part of the double body model is omitted in the figures.

For the pressure coefficients on the port side for both vortex models shown in Figures 13 and 15, no significant differences are seen in any of the models except for the pressure distribution at the bottom of the hull. Lower pressure distribution is observed for the model 1 as shown in Figure 13.

On the other hand, significant difference can be observed between the pressure coefficients on the starboard side for the vortex models 1 and 2 as shown in Figures 14 and 16. Pressure distribution on the starboard side for the model 1 is almost same as that on the port side. It can be considered that the small lateral force coefficient $C_y$ for the model 1 is caused by these small difference of pressure distribution between the starboard and the port sides. In contrast, difference in pressure distributions between the starboard and the port side is clear for the model 2. It means that free vortices which are assumed to be shed from the bottom have remarkable effect on hydrodynamic forces acting on the hull. Therefore, in order to improve the prediction accuracy of hydrodynamic forces acting on the hull, it is necessary to investigate vortex model including the influence of shedding angle based on the model 2.

4. Conclusions
Prediction of hydrodynamic forces acting on a Wigley hull in manoeuvring motion using SQCM has been investigated from the viewpoints of calculation accuracy and physical phenomena. Two kinds of
free vortex model were applied, and calculated results were compared with model test results.

As for the lateral force coefficient $C_y$, it was confirmed that free vortices shed from the bottom of the hull contribute to the improvement of prediction accuracy, but there still be significant difference between calculated yawing moment coefficient and measured results. In order to improve the prediction accuracy of hydrodynamic forces acting on the hull, it is necessary to investigate vortex model including the influence of shedding angle based on the model 2.

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