A quantitative theory of spectral lags for $\gamma$-ray bursts (GRBs) is given. The underlying hypothesis is that GRB subpulses are photons that are scattered into our line of sight by local concentrations of baryons that are accelerated by radiation pressure. For primary spectra that are power laws with exponential cutoffs, the width of the pulse and its fast rise, slow decay asymmetry is found to increase with decreasing photon energy, and the width near the exponential cutoff scales approximately as $E_{\text{max}}^\eta$, where $\eta \sim 0.4$, as observed. The spectral lag time is naturally inversely proportional to luminosity, all else being equal, also as observed.

**Subject heading:** gamma rays: bursts

1. INTRODUCTION

The fast rise and slow decay of subpulses in $\gamma$-ray bursts (GRBs) is a common feature. There could be many ways to explain it (e.g., impulsive energy infusion followed by slower cooling or light echoing). It is therefore desirable to discriminate among the different models with quantitative tests and predictions whenever possible.

In a previous paper (Eichler & Manis 2007, hereafter EM07), it was suggested that fast rise, slow decay subpulses constitute a qualitative manifestation of baryons being accelerated by radiation pressure. More generally, the basic idea can apply to any flow in which a light, fast fluid imparts energy to a clumpy, denser component of the flow by overtaking the clumps from the rear, but for convenience in this discussion we refer to the fast, light component as photons that scatter off the clumps. It was proposed that the fast rise of a subpulse is the stage where a cloud of baryons scatters photons into a progressively narrowing beaming cone of width $1/\Gamma$, where $\Gamma$ is the bulk Lorentz factor of the accelerating cloud. This narrowing of the $1/\Gamma$ cone causes brightening as long as $1/\theta > v_c$, where $\theta$ is the viewing angle offset between the observer’s line of sight and the velocity vector of the scattering cloud. Once the scattering cloud accelerates to a Lorentz factor exceeding $\sqrt{\eta}c$, the unscattered radiation is beamed and directed slightly away from the observer’s line of sight, so that the scattering of photons into the line of sight creates a “flash-in-the-pan” type of brightening. This assumption is nontrivial but has been suggested as an explanation for the Amati (2002) relation in earlier papers (Eichler & Levinson 2004, 2006; Levinson & Eichler 2005). In this series of papers, it was suggested that a significant fraction of all GRBs are actually brighter and harder in spectrum than they appear to be, and that they appear dimmer and softer because we, the observers, are viewing them from a slightly offset angle relative to the direction of the fireball. The interpretation of the subpulses given here and in EM07 is thus in accord with this picture.

2. PULSE PROFILES AT DIFFERENT PHOTON ENERGIES

The equations describing matter that is being accelerated by highly collimated radiation pressure were presented in EM07. Here we apply the solutions described in EM07 to calculate the profile of a subpulse as a function of photon energy. We assume that the differential primary photon spectrum $N_p(E)$ has the form $E_p^\delta \exp(-\beta E_p)$, where $E_p$ is the photon energy in the frame of the central engine. This form is consistent with a Comptonized thermal component. It does not, however, exclude a power-law photon spectrum produced further downstream by internal shocks. After scattering off a baryon clump that moves with velocity $\beta c$, the photon energy as seen by an observer at angle $\theta$ is

$$E = E_p/|\Gamma^2(1 + \beta)(1 - \beta \cos \theta)| = E_p(1 - \beta)(1 - \beta \cos \theta).$$

(1)

Together with the solution for the accelerating trajectory $\beta(t)$ given in EM07, the source/observer frame invariance of the number of photons $N(E)dE\text{d}t\Omega$ scattered within energy interval $dE$, time interval $dt$, and solid angle $d\Omega$, equation (1) determines the light curve $N(E,t)$ as a function of observed photon energy $E$ and observer time $t$.

In Figure 1 the subpulse light curves are plotted for three different frequencies. It is clear that the pulse width is larger and the rise-fall asymmetry is more pronounced at lower fre-
quencies, as reported by Fenimore et al. (1995 and references therein). In Figure 2 the width is plotted as a function of photon energy. At high energies, which correspond to the BATSE measurements used by these authors, the width is seen to scale approximately as the photon energy to the power \(-0.4\), as reported by Fenimore et al., above 10^2 keV. Similar calculations with varying values for the low-energy power-law index \(a\) of the primary spectrum show that this dependence is weakly dependent on \(a\) and on viewing angle. For a viewing offset angle of 10^\(\circ\), the width depends on \(E^{-\nu}\), with \(0.4 \leq \eta \leq 0.5\) when \(-0.75 \leq \alpha \leq 0\) with the sensitivity \(d\eta/d\alpha \sim 0.08\) at \(\alpha = -0.7\). For a viewing offset of 15^\(\circ\), the value of \(\eta\) is increased by about 0.033 so that a given range of \(\eta\) is occupied by a somewhat lower (i.e., more negative) range of \(\alpha\) than for smaller viewing offsets. For an extended beam, some contribution from larger offsets is inevitable, but a synthesis of light curves from extended beams is deferred for future work. It can be seen from Figure 2 that the value of \(\eta\) increases with \(\xi\), and the range of \(\xi\) that corresponds to BATSE sensitivity depends on cosmological redshift: larger \(z\) implies larger intrinsic values of \(\xi\). Hence steeper \(\xi\) dependence of the pulse width, over a given range of observed photon energies. Finally, the primary source spectrum, which we argue is not a direct observable, is somewhat uncertain. Altogether, the range of \(0.4 \leq \eta \leq 0.5\) is consistent with the ranges \(-1 \leq \alpha \leq 0\), \(1 \leq \xi \leq 3\), \(0.5 \leq \xi \leq 2\), and \(0.10 \leq \theta \leq 0.25\). It is predicted that the dependence of width on \(E\) weakens (i.e., \(\eta\) decreases) at lower photon energies, and this should be testable with detectors that are more sensitive at lower energies, such as the Fermi Gamma-Ray Burst Monitor.

As the acceleration time is inversely proportional to the radiation flux on the scatterer, it is clear, all other things being equal, that the rise time of the pulse and spectral lag are inversely proportional to source luminosity, as observed (e.g., Gehrels et al. 2006). Of course, scatter in other variables, such as the distance from the source of illumination, the optical depth of the scatterer, etc., creates scatter in the constant of proportionality.

If the scattering is isotropic (or backwardly biased due to high optical depth) in the scattering frame, it follows from equation (1) that the scattered radiation, averaged over an angle, is a factor of 2 (or more) softer than the primary emission. As explained in EM07, the other half of the energy goes into the acceleration of the scatterer. On the other hand, at most viewing angles \(\theta\), the scattered radiation is harder than scattered radiation after the scatterer has reached the terminal Lorentz factor \(\Gamma_t\) if \(\Gamma_t \geq 1/\theta\). As the scattered radiation during the acceleration phase of the scatterer is likely to be the most time-dependent, it may be possible to separate out this component from the other two. The extent to which the scattering affects the spectrum depends, of course, on the fraction of primary radiation that is scattered. Equating the scattered photon energy with the baryon afterglow energy and applying the results of Eichler & Jontof-Hutter (2005), which estimated the afterglow efficiency, we may tentatively estimate that about 30% of the primary emission is scattered, with about half of that 30% going into baryons and the other half ending up in a scattered subpulse component. Clearly, there is variation in the scattered fraction, as well as uncertainty in theoretical inferences of the baryon energy from afterglow calorimetry, so this estimate should be considered rough and preliminary.

The time-integrated spectrum at a given viewing angle can be different from the average, because the scattered radiation is not isotropic, but rather beamed in an ever-narrowing cone as the scatterer accelerates. Consider a primary emission spectrum that is a delta function \(\delta(E - E_0)\). At a given \(\mu \equiv \cos \theta\) and a given observed photon energy \(E\), a monochromatic primary spectrum \(\delta(E - E_0)\) is, after scattering, monochromatic at a photon energy \(E[\beta(t)]\) given by equation (1), so the contribution to the emitted power at energy \(E\) comes only at

\[
\beta(t) = \frac{1 - E/E_0}{1 - \mu E/E_0}.
\]

The time-integrated energy \(d^2F(E,\theta)/dEd\Omega\) of the scattered radiation at observed photon energy \(E\) and viewing angle \(\theta\) is

\[
d^2F/d\Omega dE = d \left[ (dP/d\Omega) \, d\Omega \right] /dE = (dP/d\Omega)(d\beta/dt)^{-1} \times (-dE/d\beta)^{-1},
\]

where \(P(E,\theta)\) is the power of the scattered radiation as observed at photon energy \(E\) in the frame of the pri-
A high, time-varying optical depth would be more complicated. In indefinite. If the scatterer reaches a terminal velocity, that this result assumes that the scatterer’s acceleration proceeds than the one the observer is on, so that the observer sees t in Rybicki & Lightman 1979); and evaluating in units of from equation (1), it follows that
\[ dE/d\Omega \propto \left[ (E/E_0)(2 - (E/E_0)(1 + \mu)) \right]^{1/2}/E_0(1 - \mu)^{3/2}. \]

In the limit that \( \mu \) is sufficiently below unity that \( (1 - \mu E_0) \) does not depend significantly on \( E \), \( d^2F/d\Omega dE \) is proportional to \( E^{1/2} \), i.e., \( \alpha = -0.5 \). That this is softer than the average over all viewing angles, for which \( \alpha = 0 \), can be understood as the result of most of the emission at large \( t \) going into a narrower cone than the one the observer is on, so that the observer sees only the soft fringes of this dominant component. Also note that this result assumes that the scatterer’s acceleration proceeds indefinitely. If the scatterer reaches a terminal velocity \( \beta_c \), then the observer will not see any of the primary radiation originally at \( E_0 \) scattered to an energy below \( E = E_0(1 - \beta^2)/(1 - \beta^2 \cos \theta) \).

While the scattered radiation is not the only observable component, the hypothesis that it comprises a significant fraction of the total fluence of many GRBs is broadly consistent with the tendency of the low-energy spectral index to not exceed zero. The question of how much radiation is scattered before and after the scatterers have reached terminal velocity remains somewhat open at the quantitative level, but the Fermi Gamma-Ray Burst Monitor data may provide the opportunity to address these questions quantitatively, as well as qualitatively.

3. DISCUSSION AND CONCLUSIONS

We have presented evidence that subpulses in \( \gamma \)-ray bursts (GRBs) are photons that are scattered into our line of sight by scatterers with lower Lorentz factors than the frame in which the prescattered photons had zero net momentum. Because scattering can never increase the intensity of a beam of photons, the hypothesis presupposes that the observed intensity is lower in the observer’s direction than in the direction of the beam and that the scattering by the slower baryons broadens the beam enough that it engulfs the observer’s line of sight. It is this widening of the beam that allows the observer to see enhanced flux. This fits the picture already put forth to explain the Amati (2002) relation.

As most GRBs with known redshifts have spectral peaks and energies that are below those of the hardest, brightest GRBs, it would follow, according to our interpretation, that a large fraction of, and perhaps most, GRBs are observed from an offset viewing angle. The question then arises as to why there are so many offset observers relative to those that are within the \( 1/T_{\text{sh}} \) beaming cone of the primary radiation. In an earlier paper (Eichler & Levinson 2004), it was proposed that the complex shape of the primary beam—e.g., an annular shape—would allow a number of viewers just off (i.e., within several times \( 1/T_{\text{sh}} \)) the periphery of the primary beam comparable to the number of viewers within this periphery. It was shown that a thick annulus, in which the separation between the inner and outer radii is comparable to half the outer radius, gives a distribution of offsets that is consistent with the observed distribution of spectral peaks. There may be several reasons for GRB jets to have an annular or otherwise complex morphology; it could be that baryons are entrained in the flow from (or are fed neutrons by) the confining walls (Levinson & Eichler 2003) and that much of the liberated energy is due to dissipation associated with this entrainment. Or it could be that dissipation from shocks associated with wall impact could preferentially liberate energy from near the confining walls (M. C. Begelman 2004, private communication).

Here we suggest another simple mechanism that would give the scattered \( \gamma \)-radiation some measure of annular bias, depending on the fraction scattered: consider the region of flow where the \( \gamma \)-rays make their last scattering off baryons within the flow. If, as seems more likely than not, the baryons are clumpy, it is likely that the clumps are optically thick, while the interclump medium is optically thin. In this case the photons are most likely to make their last scattering off the surface of a clump. If the clumps are moving more slowly than the primary fireball, then the photons are most likely to overtake them from the rear, and, because the clumps are optically thick, the photon is likely to emerge from its last scattering from the rear end of the clump. It is then obscured from a viewer who lies along the velocity vector of the clump, just as sunlight scattering off the Moon is obscured to a viewer on the dark side of the Moon. The viewers best positioned to see the back-side illumination are those who see photons that are emitted backward in the frame of the cloud, i.e., those that are offset by more than \( 1/T \) from the velocity vector of the clump. This, we suggest, could be a reason so many GRBs are observed from an offset angle of more than \( 1/T \). Some subpulses have such fast rises that they can be interpreted as the \( 1/T \) shadow of an optically thick cloud narrowing from above to below the offset angle of the observer, \( \theta \), as the cloud accelerates (EM07). The very sudden rise is then attributed to the observer emerging from the shadow of the clump.

To summarize, we suggest that spectral lags from long bursts are a result of high \( \Gamma \) radiation (where \( \Gamma \) is the Lorentz factor of the frame in which the radiation is isotropic) impacting slower baryons from behind and scattering off them while accelerating them. As in EM07, the quantitative agreement with the data is excellent. The inverse correlation between lag and luminosity (e.g., Gehrels et al. 2006) follows from that between acceleration time and luminosity. The assumptions needed to make the general scheme work are minimal. In any case, the minimal conclusion to be drawn from the quantitative success of this model in explaining spectral lags of long GRBs is that baryons are still undergoing considerable acceleration by the time the fireball as a whole is becoming optically thin. Were energy systematically removed from baryons beyond the photosphere (e.g., because they collided with other scatterers in their shadow), one would expect negative spectral lags.

1 Here \( P \) has units of power rather than power per unit energy, as the primary spectrum is taken to be monochromatic. The minus sign in front of \( dE/d\beta \) is to make it a positive quantity.

2 This expression is for an optically thin source, in the approximation that the primary radiation is radially combed and under the assumption that the scattered radiation \( dP/d\Omega \) has front-back symmetry in the frame of the scatterer. A high, time-varying optical depth would be more complicated.

3 The result could even be obtained from the internal shock model of GRBs if the photon energy of radiation from the rear end of the accelerating clump were to scale linearly with the Lorentz factor of the high-\( \Gamma \) fluid in the frame of the clump. However, in the simplest internal shock model, where the average electron energy, magnetic field, and blueshift all scale with \( \Gamma \), the spectral peak varies as a high power of \( \Gamma \).
It is predicted that the time-integrated spectra of the subpulses should be slightly softer than the primary emission and harder than the emission that is scattered at the terminal Lorentz factor. The Fermi Gamma-Ray Burst Monitor offers the potential opportunity for unraveling these three components.

For short hard bursts (SHBs), the subpulses are about a factor of 10–30 shorter than for long ones, and the spectral lags are much smaller. This difference can perhaps be attributed to the difference in timescale over which the baryons are undergoing acceleration. For example, SHBs are likely to be observable somewhat closer to the central engine, being unobscured by the envelope of a massive host star, and at a wider angle (Eichler et al. 2008). If this is indeed the case, then we may be able to see baryonic clumps at an earlier stage of their acceleration, when the acceleration time is considerably shorter. There are many unknowns in this hypothesis—e.g., the optical depth of the baryons, their point of injection, and their covering factor—on which the qualitative nature of the subpulses may depend, and future work will focus on the question of whether reasonable ranges for these parameters can explain the wide diversity of GRB light curves and spectra.

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REFERENCES

Amati, L., et al. 2002, A&A, 390, 81
Eichler, D., Guetta, D., & Manis, H. 2008, ApJ, submitted (arXiv: 0810.3013)
Eichler, D., & Jontof-Hutter, D. 2005, ApJ, 635, 1182
Eichler, D., & Levinson, A. 2004, ApJ, 614, L13
———. 2006, ApJ, 649, L5
Eichler, D., & Manis, H. 2007, ApJ, 669, L65 (EM07)
Fenimore, E. E., in ’t Zand, J. J. M., Norris, J. P., Bonnell, J. T., & Nemiroff, R. J. 1995, ApJ, 448, L101
Gehrels, N., et al. 2006, Nature, 444, 1044
Levinson, A., & Eichler, D. 2003, ApJ, 594, L19
———. 2005, ApJ, 629, L13
Rybicki, G. B., & Lightman, A. P. 1979, Radiative Processes in Astrophysics (New York: Wiley)
Ryde, F. 2004, ApJ, 611, L41