Propagation characteristics of THz radiation in hollow rectangle metal waveguide

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Abstract. The secular equation of medium coating were obtained by considering the particularity of THz, which had some reference value in designing of THz waveguides. The attenuation of medium coating metal hallow waveguides were theoretically calculated, the conclusion is that the power loss is less than the metal waveguides. Also we obtained the mode characters and attenuation in metal waveguides, and we noted that there exists an absorption peak. And we studied the affection of shape and size on propagation and got some useful conclusion. The split rectangular waveguide (SRW) is suitable for THz transmission which is confirmed by experiment. Our secular equation can be considered to be a theoretically discussion on it.

1. Introduction
The generation of THz signals at high power levels and the efficient transmission of THz signals using wave guiding structures are two major challenges researchers are currently facing while trying to satisfy the strong demand for compact, Practical THz waveguides with losses below the level of 1 dB/m only recently low-cost and robust THz systems have been realized[1-5]. Certain key applications anticipated at microwave or optical frequencies (e.g. integrated circuit devices or fiber-based lasers, respectively) seem to be impossible at THz frequencies due to the lack of sufficiently efficient waveguides. Transmission properties of dielectric single-mode flexible rod and tube waveguides made of plastic have been investigated recently at millimeter-wave frequencies between 33 GHz and 50 GHz [6]. Here we investigate loss, dispersion and mode confinement properties of the proposed structures in the THz range and compare the results with state-of-the-art THz waveguides.

2. Transmission characteristics of THz pulse in dielectric coated metal pipe waveguide
The structure of the waveguide is shown in figure 1. Suppose the wave transmits along z axis, the refractive index of the inner and outer is \( n_1 \) and \( n_2 \) respectively, and \( n_1 > n_2 \).

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Figure 1. Structure of rectangle dielectric coated metal-pipe waveguides.

2.1 Mode analysis of THz transmit in the waveguide

Assume that the refractive index of the waveguide film have symmetry distribution and the differential coefficient equation concerning to mode field only have one variable x, partial differential coefficient equation can be expressed as [7]:

\[
\frac{d^2 E}{dx^2} + (k^2 n^2 - \beta^2) E = 0
\]

(1)

\[
\frac{d^2 H}{dx^2} + (k^2 n^2 - \beta^2) H = 0
\]

(2)

Take TE mode for example, under boundary condition, \( E_y \) is limited in the core film, when film x goes to unlimited, \( E_y \) tend to 0. We have expression of TE even mode as:

\[
E_y = \begin{cases} 
  b_1 \cos(\sqrt{k^2 n_1^2 - \beta^2} x) & (|x| < a) \\
  b_2 \exp(-\sqrt{\beta^2 - k^2 n_2^2} |x|) & (|x| > a)
\end{cases}
\]

(3)

\[
H_z = \begin{cases} 
  b_1 \frac{k^2 n_1^2 - \beta^2}{\omega \mu_0} \sin(\sqrt{k^2 n_1^2 - \beta^2} x) & (|x| < a) \\
  b_2 \frac{\beta^2 - k^2 n_2^2}{\omega \mu_0} \exp(-\sqrt{\beta^2 - k^2 n_2^2} |x|) & (|x| > a)
\end{cases}
\]

(4)

Where, \( b_1 \) and \( b_2 \) are integral constants.

2.2 Attenuation characteristics of THz transmit in the waveguide

The attenuation of the dielectric film can be expressed as [8, 9]:

\[
2a(\theta) = \frac{1 - R(\theta)}{2b \cot(\theta)}
\]

(5)

Where \( \theta \) the incident angle, \( b \) is is the distance between layers, \( R(\theta) \) is the refractive index of different polarized directions,

\[
R(\theta) = \frac{R_s(\theta) + R_p(\theta)}{2}
\]

(6)

The power of transmitting distance of z is:

\[
P(z) = \int_0^{\theta_{\text{max}}} p_0(\theta) \exp[-2a(\theta) z \sin\theta d\theta = \int_0^{\theta_{\text{max}}} p_0(\theta) \exp\left[-\frac{1 - R(\theta)}{2T \cot(\theta)} z \right] \sin\theta d\theta
\]

(7)

Take the roughness into consideration and the refractive index of power loss is:
\[ R'(\theta) = R(\theta) \exp\left[-\frac{4\pi n_1 \sigma \sin \theta}{\lambda}\right] \]  

(8)

For mental waveguide coated with dielectric film, the refractive index for inner layer can be expressed as [9]

\[ r_{1p} = \frac{n_p \cos \psi_1 - \cos \psi_2}{n_p \cos \psi_1 + \cos \psi_2} \]  

(9)

\[ r_{1s} = \frac{\cos \psi_1 - n_p \cos \psi_2}{\cos \psi_1 + n_p \cos \psi_2} \]  

(10)

For outer layer, the refractive index can be expressed by two parameters:

\[ r_2 = \rho_2 \exp(j\phi_2) \]  

(11)

The expression of two parameters is as follows:

\[ \rho_{2p}^2 = \frac{\left((n^2 - \kappa^2) \cos \psi_2 - n_p u \right)^2 + (2n\kappa \cos \psi_2 - n_p u)^2}{\left((n^2 - \kappa^2) \cos \psi_2 + n_p u \right)^2 + (2n\kappa \cos \psi_2 + n_p u)^2} \]  

(12)

\[ \tan \varphi_{2p} = \frac{-2nku - (n^2 - \kappa^2)u}{(n^2 + \kappa^2) \cos^2 \psi_2 - n_p^2(u^2 + u^2)} \]  

(13)

\[ \rho_{2s}^2 = \frac{(n_p \cos \psi_2 - u)^2 + u^2}{(n_p \cos \psi_2 + u)^2 + u^2} \]  

(14)

\[ \tan \varphi_{2s} = \frac{-2nu_p \cos \psi_2}{u^2 + u^2 - n_p^2 \cos^2 \psi_2} \]  

(15)

Where \( n \) and \( \kappa \) are the image and real part of the refractive index:

\[ u^2 = \frac{1}{2} \left[ n^2 - \kappa^2 - n_p^2 \sin^2 \psi_2 + \left((n^2 - \kappa^2 - n_p^2 \sin^2 \psi_2)^2 + 4n^2 \kappa^2 \right)^{\frac{1}{2}} \right] \]  

(16)

\[ v^2 = \frac{1}{2} \left[ -(n^2 - \kappa^2 - n_p^2 \sin^2 \psi_2) + \left((n^2 - \kappa^2 - n_p^2 \sin^2 \psi_2)^2 + 4n^2 \kappa^2 \right)^{\frac{1}{2}} \right] \]  

(17)

Thus, we get the refractive index of dielectric film

\[ r = \frac{r_1 + r_2 \exp(-j2\Gamma)}{1 + r_1 r_2 \exp(-j2\Gamma)} = \frac{r_1 + \rho_2 \exp\left[j(\phi_2 - 2\Gamma)\right]}{1 + r_1 \rho_2 \exp\left[j(\phi_2 - 2\Gamma)\right]} \]  

(18)

where \( \Gamma = k \Phi n_p d \cos \psi_2 \). Power refractive index \( R(\theta) \) can be expressed as:

\[ |r|^2 = \frac{r_1^2 + \rho_2^2 + 2r_1 \rho_2 \cos(\phi_2 - 2\Gamma)}{1 + r_1^2 \rho_2^2 + 2r_1 \rho_2 \cos(\phi_2 - 2\Gamma)} \]  

(19)

3. Transmitting character of THz of THz pulse in metal-coated pipe waveguide

3.1 The cut-off frequency of THz radiation in the waveguide

The cut-off frequency and wavelength of TE (or TM) modes are:

\[ f_c = \frac{k_c}{2\pi \sqrt{\mu \varepsilon}} = \nu \sqrt{(m/a)^2 + (n/b)^2} / 2, \lambda_c = \nu / f_c = 2 \sqrt{(m/a)^2 + (n/b)^2} \]  

3
3.2 Mode analysis of THz radiation in the waveguide

Using right angle coordinate system, Helmholtz equation can be expressed as:

\[
\begin{align*}
\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} &= -k_c^2 H_z, \\
\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} &= -k_c^2 E_x
\end{align*}
\]  

(20)

The boundary condition of inner wall is:

\[
E_y \bigg|_{x=0} = 0, E_x \bigg|_{y=0} = 0 ; H_z = H_{mn} \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta_{uv} z}
\]  

(21)

By solving the equation, we get all the field values (take TE mode for example):

\[
\begin{align*}
E_x &= \frac{j\omega \mu m \pi}{k_c^2 b} H_{mn} \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta_{uv} z} \\
E_y &= -\frac{j\omega \mu m \pi}{k_c^2 a} H_{mn} \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta_{uv} z} \\
E_z &= 0 \\
H_x &= \frac{j\beta m \pi}{k_c^2 a} H_{mn} \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta_{uv} z} \\
H_y &= \frac{j\beta n \pi}{k_c^2 b} H_{mn} \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta_{uv} z}
\end{align*}
\]  

(22)

3.3 Attenuation characteristics

Commonly, loss contains the loss of waveguide wall and the filled. Thus, transmission constant is \(\gamma = \alpha + j \beta\). When wave transmits in waveguide, its electromagnetic field decrease by index rule. Suppose wave transmits from \(z=0\) to \(z\), the relation of power is

\[
P(z) = P_0 \exp(-2\alpha z) \quad \text{Obviously, the power loss in transmitting unit length is} \quad P_l = P_0[1 - \exp(-2\alpha)] = 2\alpha P_0.
\]

Therefore, the attenuation coefficient of the waveguide should be:

\[
\alpha = \frac{P_l}{2P_0} = \alpha_c + \alpha_d
\]  

(23)

where \(\alpha_c\) is attenuation constant of conductivity, \(\alpha_d\) is attenuation constant of filled medium. \(\alpha_c\) is caused by the current on the wall. The power loss of conductivity of unit length is

\[
P_l = 0.5R_s \int l [J^2] dl = 0.5R_s \int l [H_i^2] dl
\]  

(24)

where \(R_s = (\sigma \delta)^{-1}\) is the surface resistance of waveguide wall in unit width, and \(s\) is the conductance rate, \(\delta = 2/(\omega \mu \sigma)^{-1/2}\) is the skin depth. Integral trace \(l\) is the closed line along the cross section of inner wall. \(H_i\) is the tangential component of magnetic field at the inner wall of waveguide.

Using the equations above, we can easily derive the expressions for attenuation of TE and TM mode:

\[
a_{\text{TE}} \approx \frac{2R_s}{\eta b} \sqrt{1 - \left(\lambda/\lambda_s\right)^2} \left\{ \frac{b}{a} \left(\frac{b}{a} \left(\frac{b}{a} \frac{m^2 + n^2}{m^2 + n^2} + \left(\frac{b}{a} \lambda_s / \lambda_c\right)^2 + \left(1 + \frac{b}{a} \lambda_s / \lambda_c\right)^2 \right) \right) (Np/m) \right\}
\]  

(25)
\[ a_{\text{TM}_{\text{of}}} = \frac{2R_c}{\eta b \sqrt{1 - (\lambda / \lambda_c)^2}} \frac{(b/a)^3 m^2 + n^2}{(b/a)^2 m^2 + n^2} (Np/m) \]  

(26)

where \( \varepsilon_{0a} = \begin{cases} 1, & n = 0 \\ 2, & n \neq 0 \end{cases} \). When loss of dielectric exists, its dielectric constant takes complex number as \( \varepsilon' = \varepsilon(1 + j \sigma / (\omega \varepsilon)) \). Then the longitudinal transmission constant of waveguide changes to:

\[ \gamma = \sqrt{k_e^2 - k^2} = j \omega \sqrt{\mu \varepsilon} \sqrt{1 - j \frac{\sigma}{\omega \varepsilon} - (\lambda / \lambda_c)^2} \approx j \omega \sqrt{\mu \varepsilon} \sqrt{1 - (\lambda / \lambda_c)^2} \left( 1 - j \frac{\sigma / \omega \varepsilon}{2[1 - (\lambda / \lambda_c)^2]} \right) \]  

(27)

\[ = a_d + f \beta \]

where \( \alpha_d = \frac{\sigma \eta}{2 \sqrt{1 - (\lambda / \lambda_c)^2}} = \frac{\pi \tan \delta}{\lambda \sqrt{1 - (\lambda / \lambda_c)^2}} (Np/m) \) is the attenuation of medium, \( \tan \delta = \sigma / (\omega \varepsilon) \) is tangent of dielectric loss angle.

4. Numerical simulations

Taking aluminum for example, we calculated the attenuation constant using Matlab programs, as shown in figure 3. We can see from figure 3 that, at higher value of \( b/a \), higher mode has bigger loss.

Figure 4 shows that attenuation changes almost exponentially with wavelength, the attenuations of TE_{1X} modes are far below those of TE_{2X} modes.

Figure 2. Dependence of cut-off frequency on \( b/a \).

Figure 3. Attenuation changes with \( b/a \) of TE modes @ 0.1THz, \( b=0.01 \text{m} \), aluminium.
The results show that TM01 mode has a bigger loss than that of other modes. The results can be used in designing THz devices, and TM01 mode should be avoided.
Figure 7 and figure 8 show that the waveguide has a bigger loss when b/a is close to 3 or 7 for different modes. That is to say, when designing waveguides we should avoid the two situations. Figure 8 shows that silver is better than other metals in transmitting THz wave. While brass is not suitable for coating waveguides.

5. Conclusions
The results show that for TM$_{20}$ mode, it is a good choice when b/a is less than 3 for transmitting THz waves. While for TM$_{11}$ mode and TM$_{10}$ mode, it is good when b/a is less than 6. THz pulse transmits in metal waveguide suffering more loss of current on the surface of metal wall. Dielectric film can lower this loss by reducing its power loss on the metal. The loss of rectangle waveguide is lower than that of circular hollow waveguide as discussed in reference [10]. The attenuation is bellow 0.2dB/m when b/a is bellow 1. The TM$_{01}$ mode is at a high loss than other modes, so the TM mode should be avoided in application.

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