CPT-violating leptogenesis induced by gravitational defects

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Abstract We explore leptogenesis induced by the propagation of neutrinos in gravitational backgrounds that may occur in string theory. The first background is due to linear dilatons and the associated Kalb–Ramond field (axion) in four non-compact space–time dimensions of the string, and can be described within the framework of local effective Lagrangians. The axion is linear in the time coordinate of the Einstein frame and gives rise to a constant torsion which couples to the fermion spin through a gravitational covariant derivative. This leads to different energy–momentum dispersion relations for fermions and antifermions. As a result leptogenesis and baryogenesis can arise in various scenarios. The next two backgrounds go beyond the local effective Lagrangian framework. One is a stochastic (Lorentz violating) Finsler metric which again leads to different dispersion relations between fermions and antifermions. The third background of primary interest is the one due to populations of stochastically fluctuating point-like space–time defects that can be encountered in string/brane theory (D0-branes). Only neutral matter interacts non-trivially with these intrinsic defects, as a consequence of charge conservation. Hence, such a background singles out neutrinos among the Standard Model excitations as the ones interacting predominantly with the defects. The back-reaction of the defects on the space–time due to their interaction with neutral matter results in stochastic Finsler-like metrics (similar to our second background). On average, the stochastic fluctuations of the D0-brane defects preserve Lorentz symmetry, but their variance is nonzero. Interestingly, the particle–antiparticle asymmetry comes out naturally to favour matter over antimatter in this third background, once the effects of the kinematics of the scattering of the D-branes with matter is incorporated.

1 Introduction

One of the most important issues of fundamental physics, relates to an understanding of the magnitude of the observed baryon asymmetry $n_B - n_{\overline{B}}$ (where $B$ denotes baryon, $\overline{B}$ denotes antibaryon, $n_B$ is the number density of baryons and $n_{\overline{B}}$ the number density of antibaryons in the universe). The numbers of protons and neutrons far exceeds the number of antiprotons and antineutrons. The universe is overwhelmingly made up of matter rather than antimatter. According to the standard Big Bang theory, matter and antimatter have been created in equal amounts in the early universe. However, the observed charge-parity (CP) violation in particle physics \cite{1}, prompted A. Sakharov \cite{2} to conjecture that for baryon asymmetry in the universe (BAU) we need:

1. Baryon number violation to allow for states with $\Delta B \neq 0$ starting from states with $\Delta B = 0$ where $\Delta B$ is the change in baryon number.
2. If C or CP conjugate processes to a scattering process were allowed with the same amplitude then baryon asymmetry would disappear. Hence C and CP need to be broken.
3. Chemical equilibrium does not permit asymmetries. Hence Sakharov required that chemical equilibrium does not hold during an epoch in the early universe.

Hence nonequilibrium physics in the early universe together with baryon number (B), charge (C) and charge-parity (CP) violating interactions/decays of antiparticles in the early universe, may result in the observed BAU. In fact there are two types of nonequilibrium process in the early universe which can produce this asymmetry: the first type concerns processes generating asymmetries between leptons and antileptons (leptogenesis), while the second produces asymmetries between baryons and antibaryons (baryogenesis). The near complete observed asymmetry today, is estimated in the Big-Bang theory \cite{3} to im-
In this article we will consider leptogenesis due to CPTV from a non-Riemannian point of view, inspired by a stringy model of gravitational backgrounds and defects interacting with neutral fermions. The theory of strings motivates the axion background and non-Riemannian framework that we will consider. The non-Riemannian structure that we investigate is Finsler geometry [16] where momentum as well as position are explicitly involved in its metric. The defects that we will consider are point-like solitonic structures in some string theories; they are known as D0-branes (or D-particles) [17]. Our generic model involves three-space-dimensional brane universes, obtained from compactification of higher-dimensional branes, which are embedded in a bulk space punctured by D-particles (see e.g. [18]). One of these brane universes constitutes our observable world, which moves in the bulk. As a consequence of this motion, D-particles cross the brane. Open strings for electrically neutral particles on the brane can attach an end to the D-particle; subsequently this string can detach [19, 20]. Scattering off a population of D-particles (D-foam) affects the kinematics of the stringy matter and leaves an imprint on the background geometry [21] on the brane world. This geometry is similar to Finsler metrics but with stochastic parameters [22–24]. We first investigate the consequences for gravitational leptogenesis of this metric structure in the general setting of Finsler geometry (without reference explicitly to D-foam). For the underlying stringy model however, since it is microscopic, we can consider in addition the kinematics of D particle scattering. This kinematical aspect leads naturally to the asymmetry between the particle and antiparticle abundances having the right sign. The kinematical argument, which involves recoil kinetic energy, does not fit into an effective local field theory approach and represents a new approach that is relevant to leptogenesis.

The structure of the article will be as follows: in Sect. 2 we shall briefly review some relevant existing models for fermionic asymmetry, which entail CPTV in the early universe as alternatives to scenarios involving sterile neutrinos and/or supersymmetry. The discussion will also help to differentiate between our viewpoint and those prevailing in the literature. The models for leptogenesis that we will discuss are of gravitational type, namely CPTV-induced differences in the dispersion relations between particles and antiparticles propagating in these backgrounds. In Sect. 3, we propose a new model for gravitational leptogenesis which follows broadly an earlier framework [25, 26], but differs crucially in that the full gravitational multiplet [27] that arises in string theory is used. The leptogenesis in this model is due to CPTV dispersion relations between fermions/antifermions, induced by the (constant) torsion associated with the antisymmetric Kalb–Ramond tensor field in the gravitational multiplet. The torsion is space–time independent for a particular solution of the conformal invariance equations of the
string associated with a linear dilaton. In the proposals discussed in Sect. 2 the space–time independence of the torsion was not guaranteed. In Sect. 4, we consider another scenario for gravitational leptogenesis that involves a non-Riemannian Finsler metric, with stochastically fluctuating parameters, a variant of which appears in our string/brane model in Sect. 5. In Sect. 5 we present our model for parameters, a variant of which appears in our string/brane Riemannian Finsler metric, with stochastically fluctuating scenario for gravitational leptogenesis that involves a non-

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\[ n - \bar{n} = g_{\text{d.o.f.}} \int \frac{d^3 p}{(2\pi)^3} \left[ f(E, \mu) - f(\bar{E}, \bar{\mu}) \right], \]

where \( g_{\text{d.o.f.}} \) denotes the number of degrees of freedom of the particle species under study. An example of spontaneous Lorentz violation [35] is provided by a vector field \( A_{\mu} \) with a nonzero time-like expectation value which couples to a global current \( J^\mu \) such as baryon number through an interaction Lagrangian density

\[ \mathcal{L} = \lambda A_\mu J^\mu. \]

This leads to \( m \neq \bar{m} \) and \( \mu \neq \bar{\mu} \). Alternatively, following [36] we can make the assumption that the dominant contributions to baryon asymmetry come from quark–antiquark mass differences, and that their masses “run” with the temperature i.e. \( m \sim gT \) (with \( g \) the QCD coupling constant).

One can provide estimates for the induced baryon asymmetry on noting that the maximum quark–antiquark mass difference is bounded by the current experimental bound on the proton–antiproton mass difference, \( \delta m_p = |m_p - \bar{m}_p| \), known to be less than \( 2 \times 10^{-9} \) GeV. Taking \( n_\gamma \sim 0.24T^3 \) (the photon equilibrium density at temperature \( T \)) we have [36]:

\[ \beta_T = \frac{n_B}{n_\gamma} = \frac{8.4 \times 10^{-3} m_u \delta m_u + 15 m_d \delta m_d}{T^2}, \]

\[ \delta m_q = |m_q - \bar{m}_q|. \]

\( \beta_T \) is too small compared to the observed one. To reproduce the observed \( \beta_{T=0} \sim 6 \times 10^{-10} \) one would need \( \delta m_q(T = 100 \text{ GeV}) \sim 10^{-5} \) \( \text{GeV} \gg \delta m_p \), which is somewhat unnatural.

However, active (light) neutrino–antineutrino mass differences alone may reproduce BAU; some phenomenological models in this direction have been discussed in [37], considering, for instance, particle–antiparticle mass differences for active neutrinos compatible with current oscillation data. This leads to the result

\[ n_B = n_\nu - n_\bar{\nu} \simeq \frac{\mu_\nu T^2}{6}, \]

\[ \frac{n_B}{n_\nu - n_\bar{\nu}} \simeq \frac{\mu_\nu T^2}{6}, \]
yielding \( n_B/s \sim \frac{n_B}{B} \sim 10^{-11} \) at \( T \sim 100 \text{ GeV} \), in agreement with the observed BAU. (Here \( s, n_\nu, \) and \( \mu_\nu \) are the entropy density, neutrino density and chemical potential, respectively.)

2.2 CPTV decoherence models

Particle–antiparticle mass differences, however, may not be the only way to obtain CPTV. As discussed in [38, 39], quantum gravity fluctuations in the structure of space–time, may be strong in the early universe; the fluctuations may act as an environment inducing decoherence for the neutrinos and antineutrinos. However, the couplings between the particles and the environment are different for the neutrino and antineutrino sectors. Once there is decoherence for an observer with a low energy (as compared to the Planck scale \( M_p \sim 10^{19} \text{ GeV} \)), the effective CPT symmetry generator may be ill-defined as a quantum mechanical operator, according to a theorem by R. Wald [40], leading to an intrinsic violation of CPT symmetry. This type of violation may characterise models of quantum gravity with stochastic space–time fluctuations due, for instance, to gravitational space–time defects, in certain brane models [18, 20, 23]. In such a case, a slight mismatch in the strength of the stochastic space–time fluctuations between particle and antiparticle sectors, can lead to different decoherence parameters to describe the interaction of the gravitational environment with matter.

In [38, 39], simple models of Lindblad decoherence [41], conjectured to characterise CPTV due to decoherence induced by quantum gravity [42, 43], have been considered for neutrinos [44]. Decoherence is described in the framework of a master equation of Lindblad type for a density matrix \( \rho_\mu \):

\[
\frac{\partial \rho_\mu}{\partial t} = [H, \rho_\mu] + L_{\mu\nu} \rho_\nu, \quad \mu, \nu = 0, \ldots, N^2 - 1
\]

for an \( N \)-flavour neutrino system, where we use the Einstein summation convention for indices, \( t \) is the time, \( H \) is the Hamiltonian, and \( L_{\mu\nu} \) is the Lindblad decoherence matrix, which cannot be written as a commutator of an operator with \( \rho \). The Lindblad term describes an effective interaction of the low-energy neutrino system with Hamiltonian \( H \) with an “environment”, which in the models of [38, 39] is assumed to be provided by space–time fluctuations induced by quantum gravity. It was assumed on phenomenological grounds that non-trivial decoherence parameters were \emph{only} present in the antiparticle sector: this is consistent with the lack of any experimental evidence to date [45–47] for vacuum decoherence in the particle sector. The antineutrino decoherence parameters (with dimension of energy) had different dependences on energy. The model of [38, 39] assumes a diagonal \( L_{\mu\nu} \). A diagonal Lindblad decoherence matrix for three-generation neutrinos requires eight coefficients \( \gamma_i \). Some of the eight coefficients were assumed for simplicity in [38, 39] to be proportional to the antineutrino energy

\[
\gamma_i = \frac{T}{M_p} E, \quad i = 1, 2, 4, 5
\]

while the remaining (subdominant) ones were inversely proportional to it

\[
\gamma_j = \frac{10^{-24} (\text{GeV})^2}{E}, \quad j = 3, 6, 7, 8.
\]

The model was proposed without any microscopic justification; its choice was phenomenological and motivated by fitting the LSND “anomalous data” in the antineutrino sector [48] with the rest of the neutrino data. This fit required \( T/M_p \sim 10^{-18} \), i.e. in the temperature range of electroweak symmetry breaking. One can derive [38, 39] an active (light) \( v-\bar{\nu} \) asymmetry of order

\[
A = \frac{|n_\nu - n_{\bar{\nu}}|}{n_\nu + n_{\bar{\nu}}} = \frac{\gamma_1}{\sqrt{\Delta m^2}} = \frac{T}{M_p} \cdot \frac{E}{\sqrt{\Delta m^2}}, \quad (8)
\]

where \( \Delta m^2 \) denotes the (atmospheric) neutrino mass squared difference, which plays the role of a characteristic low mass scale in the problem. This lepton number violation is communicated to the baryon sector by means of baryon number \( (B) \) plus lepton number \( (L) \) conserving sphaleron processes. (In this case we need an antilepton excess in order to produce a baryon excess.) These processes lead to an estimate [38] for the current value of \( B \) to be

\[
B \approx \frac{n_{\bar{\nu}} - n_\nu}{s} \sim \frac{A}{g^* n_\gamma}
\]

with \( n_\gamma \) the photon number density, \( g^* \) the effective number of degrees of freedom (at the temperature where the asymmetry developed, i.e. the electroweak symmetry breaking temperature in the model of [38]), \( g^* \) depends on the matter content of the model (with typically \( g^* \) lying between \( 10^2 \) and \( 10^3 \)). For such parameter values \( A \sim 10^{-6} \) and so the observed BAU may be reproduced in this case without the need for extra sources of CP violation such as sterile neutrinos. Such models, however, do not provide a microscopic understanding of CPTV. In particular there is missing an understanding of the preferential role of the neutrino compared to other particles of the Standard Model in the CPT violating decoherence process. Within some microscopic models of space–time foam, involving populations of point-like brane defects (D particles) puncturing brane worlds [18, 20, 23] with three (or higher)-spatial dimension, we shall discuss in Sect. 5 that such a preferred role may be justified. Moreover D particles also imply that the framework of Riemannian geometry will need to be generalised to Finsler geometries. Finsler geometries (with a stochastic background) will be investigated in order to evaluate the possibilities of CPTV occurring.
2.3 CPTV-induced by curvature effects in background geometry

Although the role of gravity was alluded to in the last subsection, associated features of space–time were not discussed. The explicit role of gravity is usually considered within a local effective action framework such as that of Eq. (4). A coupling to scalar curvature $\mathcal{R}$ through a CP violating interaction Lagrangian $L$

$$L = \frac{1}{M_\ast^2} \int d^4x \sqrt{-g} (\partial_\mu \mathcal{R}) J^\mu, \quad (10)$$

where $M_\ast$ is a cut-off in an effective field theory of gravity and $J^\mu$ could be the current associated with $B - L$. There is an implicit choice of sign in front of the interaction (10), which has been fixed so as to ensure matter dominance. In an expanding universe the interaction violates CPT. Indeed it has been shown that [29]

$$\frac{n_{B \rightarrow L}}{s} = \frac{\mathcal{R}}{M_\ast^2 T_d}. \quad (11)$$

$T_d$ being the freeze-out temperature for $B$ violating interactions. This CPTV leads to baryon number asymmetry. Note that to leading order in $M_\ast^{-2}$ we have $\mathcal{R} = 8\pi G (1 - 3w) \rho$ where $\rho$ is the energy density of matter and the equation of state is $p = w\rho$ where $p$ is pressure. For radiation $w = 1/3$ and so in the radiation dominated era of the Friedmann–Robertson–Walker cosmology $\mathcal{R} = 0$. However, $w$ is precisely 1/3 when $T_d^\mu = 0$. In general $T_d^\mu \propto \beta(g) F^{\mu\nu} F_{\mu\nu}$ where $\beta(g)$ is the beta function of the running gauge group $g$ in a $SU(N_c)$ gauge theory with $N_c$ colours. This allows $w \neq 1/3$. Further issues in this approach can be found in [25, 29, 30, 32].

Another approach involves an axial vector current [26, 31, 33, 34] instead of $J^\mu$. The scenario is based on the well-known fact that fermions in curved space–times exhibit a coupling of their spin to the curvature of the background space–time. The Dirac Lagrangian density of a fermion can be re-written as

$$L = \sqrt{-g} \bar{\psi} \left( i \gamma^\alpha \partial_\alpha - m + \gamma^5 B_\mu \right) \psi, \quad (12)$$

$$B^\mu = e^{abcd} e_{b\lambda} \left( \partial_\lambda e^\mu_c + \Gamma_{\mu\lambda}^{\nu} e^{\nu}_c e^\mu_d \right),$$

in a standard notation, where $e_{b\lambda}$ are the vierbeins, $\Gamma_{\mu\lambda}^{\nu}$ is the Christoffel connection and Latin (Greek) letters denote tangent space (curved space–time) indices. The space–time curvature background has, therefore, the effect of inducing an “axial” background field $B_\mu$ which can be non-trivial in certain anisotropic space–time geometries, such as Bianchi-type cosmologies [26, 31, 33, 34]. For an application to particle–antiparticle asymmetry it is necessary for this axial field $B_\mu$ to be a constant in some local frame. The existence of such a frame has not been demonstrated for anisotropic space–time geometries. If it can be arranged that $B_\mu \neq 0$ for $a = 0$, then for constant $B_0$ CPT is broken: the dispersion relation of neutrinos in such backgrounds differs from that of antineutrinos. Explicitly we have

$$E = \sqrt{(p - B)^2 + m^2 + B_0}, \quad (13)$$

$$\bar{E} = \sqrt{(p + B)^2 + m^2 - B_0}. \quad (14)$$

The relevant neutrino asymmetry emerges on following steps similar to those in Eqs. (2) and (3)). As a consequence the following neutrino–antineutrino density difference is found in Bianchi II Cosmologies [26, 31, 33, 34]:

$$\Delta n_\nu \equiv n_\nu - n_\bar{\nu} \sim g^* T^3 \left( \frac{B_0}{T} \right)$$

with $g^*$ the number of degrees of freedom for the (relativistic) neutrino. An excess of particles over antiparticles is predicted only when $B_0 > 0$, which had to be assumed in the analysis of [26, 31, 33, 34]; we should note, however, that the sign of $B_0$ and its constancy have not been justified in this phenomenological approach.1

At temperatures $T < T_d$, with $T_d$ the decoupling temperature of the $L$ violating processes, the ratio of the net lepton number $\Delta L$ (neutrino asymmetry) to the entropy density (which scales as $T^3$) remains constant,

$$\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d}. \quad (15)$$

For $T_d \sim 10^{15}$ GeV and $B_0 \sim 10^5$ GeV, a lepton asymmetry (leptogenesis) of order $\Delta L \sim 10^{-10}$ is implied, in agreement with observations. The latter can then be communicated to the baryon sector to produce the observed BAU (baryogenesis) by a $B - L$ conserving symmetry in the context of either GUT [26], or sphaleron processes in the standard model. (Note that in the scenario of decoherence-induced CPTV [38, 39], mentioned previously, $B + L$ was assumed to be conserved in the corresponding sphaleron processes; in such a case one needs an antilepton excess to produce baryogenesis.)

In closing this section, let us recapitulate some unsatisfactory issues with the above proposals. The simple assumption of mass differences between particles and antiparticles dominated by the quark/antiquark mass differences cannot

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1The above considerations concern the dispersion relations for any fermion, not only neutrinos. However, when one considers matter excitations from the vacuum, as relevant for leptogenesis, we need chiral fermions to get non-trivial CPTV asymmetries in populations of particle and antiparticles, because $\langle \psi^\dagger \gamma^5 \psi \rangle = -\langle \psi^\dagger \gamma^5 \psi_L \rangle + \langle \psi^\dagger \gamma^5 \psi_R \rangle$. 
reproduce the observed BAU, because the latter are naturally bound by the current proton/antiproton mass difference. Moreover, phenomenological models involving neutrino/antineutrino mass differences, although capable of reproducing the observed BAU, nevertheless are ad hoc and lack microscopic justification. In addition, the models involving spin/curvature coupling require constancy of the axial vector $B^\mu$, which is assumed without justification. The same holds for the sign of the asymmetry between matter and antimatter in these models. The models we will propose below, address these problems in a more microscopic way and offer partial resolution.

3 CPTV-induced
in (string-inspired) background geometry with torsion

We will discuss the case of a constant $B^0$ “axial” field that appears due to the interaction of the fermion spin with a string-theory background geometry with torsion. This is a novel observation (as far as we are aware). In the presence of torsion the Christoffel symbol contains a part that is antisymmetric in its lower indices: $\Gamma^\lambda_{\mu\nu} \neq \Gamma^\lambda_{\nu\mu}$. Hence the last term of the right-hand side of Eq. (12) is not zero. Since the torsion term is of gravitational origin it couples universally to all fermion species. The role of the coupling to neutrinos will be clarified below.

The massless gravitational multiplet in string theory contains the dilaton ($\phi$, graviton (spin 2, symmetric tensor), $g_{\mu\nu}$, and the spin 1 Kalb–Ramond antisymmetric tensor $B_{\mu\nu}$. The field $B$ appears in the string effective action only through its totally antisymmetric field strength, $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$, where [...] denotes antisymmetrisation of the indices within the brackets. From the calculation of string amplitudes [49] it can be deduced that $H_{\mu\nu\rho}$ plays the role of torsion in a generalised connection. In the Einstein frame, the four-dimensional bosonic part of the effective action of the string to $O(\alpha')$ is

$$S = \frac{M_s^2 V^c}{16\pi} \int d^4 x \sqrt{-g} \left( R(g) - 2\phi \partial\bar{\partial} \phi \right)$$

$$- \frac{1}{12} \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} H_{\sigma} + \ldots$$

(16)

where $M_s = 1/\sqrt{\alpha'}$ is the string mass scale, $V^c$ denotes the (dimensionless) compactification volume, and the compact radii are expressed in units of $\sqrt{\alpha'}$. It can be shown that the term in $S$ that is the square of the field strength and the Einstein scalar curvature term $R(g)$, can be combined to form a generalised curvature $\mathcal{R}(g, T)$ term where the generalised Christoffel symbol (connection) $\mathcal{T}$ is

$$\mathcal{T}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + \epsilon^{\lambda\mu\nu} \phi \equiv \Gamma^\lambda_{\mu\nu} + T^\lambda_{\mu\nu}.$$

(17)

$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$ is the torsion-free Einstein-metric connection, and $T^\lambda_{\mu\nu} = -T^\lambda_{\nu\mu}$ is the torsion. This decomposition of $T^\lambda_{\mu\nu}$ persists in higher orders of $\alpha'$ after appropriate field redefinitions that leave the scattering amplitudes invariant. We will assume its validity to all orders in stringy $\sigma$-model perturbation theory. In such an effective theory the relevant Lagrangian terms for fermions (to lowest order in $\alpha'$) will have the form given in Eq. (12).

The four-dimensional Lagrangian $\mathcal{L}_f$ for a spin-1/2 fermion $\psi$ in our background with torsion is (up to constants of proportionality):

$$\mathcal{L}_f \sim \sqrt{-g} (\bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi),$$

$$D_a = \partial_a - i \frac{\epsilon}{4} \bar{\omega}_{bc} \sigma^{bc} \cdot \epsilon^{a} \left[ \gamma^a, \gamma^b \right].$$

(18)

where Latin indices refer to tangent-space, and $\omega$, the generalised spin connection (with torsion), is

$$\omega_{bc} = \epsilon_{bc} \left( \partial_a e^a + T^\lambda_{\gamma\mu} \epsilon^\gamma^a \epsilon^\mu_a \right).$$

(19)

The torsion $T_{\mu\nu\rho}$ associated with the generalised connection $\mathcal{T}^\sigma_{\mu\nu}$ (cf. Eq. (17)) provides a background for the fermions through the term $\epsilon_{bc} \mathcal{T}^\lambda_{\gamma\mu} \epsilon^\gamma^a \epsilon^\mu_a$, which appears in the covariant derivative term in the Lagrangian. Even in the case of constant vierbeins there are non-trivial interactions between the fermions and the torsion part of the spin-connection.

In Ref. [52] exact solutions to the conformal invariance conditions (to all orders in $\alpha'$) of the low energy effective action of strings have been presented. In four “large” (uncompactified) dimensions of the string, the antisymmetric tensor field strength can be written uniquely as

$$H_{\mu\nu\rho} = e^{2\phi} \epsilon_{\mu\nu\rho} \partial^\sigma b(x)$$

(20)

with $\epsilon_{0123} = \sqrt{g}$ and $\epsilon_{\mu\nu\rho\sigma} = |g|^{-1} \epsilon_{\mu\nu\rho\sigma}$, with $g$ the metric determinant. The field $b(x)$ is a “pseudoscalar” axion-like field. The dilaton $\phi$ and axion $b$ fields are fields that appear as Goldstone bosons of spontaneously broken scale symmetries of the string vacua, and so are exactly massless classically. In the effective string action such fields appear only through their derivatives. The exact solution of [52] in the string frame requires that both dilaton and axion fields are linear in the target time $X^0$, $\Phi(X^0) \sim X^0$, $b(X^0) \sim X^0$. In this dilaton and axion background, the minima of all fields in the effective action which couple to the dilaton and axion are shifted by a space–time independent amount.

In the “physical” Einstein frame, relevant for cosmological observations, the temporal component of the metric is normalised to $g_{00} = +1$ by an appropriate change of the

We note that fermions coupled to Kalb–Ramond torsion tensors $H_{\mu\nu\rho}$ have been considered previously [50, 51] but from a different perspective than ours.
time coordinate. In this setting, the solution of [52] leads to a Friedmann–Robertson–Walker (FRW) metric, with scale factor \( a(t) \sim t \), where \( t \) is the FRW cosmic time. Moreover, the dilaton field \( \Phi \) behaves as \( -\ln t + \Phi_0 \), with \( \Phi_0 \) a constant, and the axion field \( b(x) \) is linear in \( t \). There is an underlying world-sheet conformal field theory with central charge \( c = 4 - 12Q^2 - \frac{\Phi_0^2}{M_s^2} + c_f \) where \( Q^2 > 0 \) is the central-charge deficit and \( c_f \) is the central charge associated with the world-sheet conformal field theory of the compact "internal" dimensions of the string model [52]. The condition of cancellation of the world-sheet ghosts that appear because of the fixing of reparametrisation invariance of world-sheet co-ordinates requires that \( c = 26 \). The solution for the axion field is

\[
b(x) = \sqrt{2e^{-\Phi_0}} \sqrt{Q^2} \frac{M_s}{\sqrt{n}} t, \tag{21}
\]

where \( M_s \) is the string mass scale and \( n \) is a positive integer, associated with the level of the Kac–Moody algebra of the underlying world-sheet conformal field theory. For nonzero \( Q^2 \) there is an additional dark energy term in (16) of the form \( \int d^4x \sqrt{-g} e^{2\Phi} (-4Q^2) / a' \). The linear axion field (21) remains a non-trivial solution even in the static space–time limit with a constant dilaton field [52]. In such a case space time is an Einstein universe with positive cosmological constant and constant positive curvature proportional to \( 6/(n + 2) \).

For the solutions of [52], the covariant torsion tensor \( e^{-2\Phi} H_{\mu
u\rho} \) is constant. (This follows from (17) and (20) since the exponential dilaton factors cancel out in the relevant expressions.) Only the spatial components of the torsion are nonzero in this case,

\[
T_{ijk} \sim \epsilon_{ijk} \dot{b} = \epsilon_{ijk} \sqrt{2Q^2} e^{-\Phi_0} \frac{M_s}{\sqrt{n}}, \tag{22}
\]

where the overdot denotes derivative with respect to \( t \). From (17), (19), (20) and (12), we also observe that only the temporal component \( B^0 \) of the \( B^i \) vector is nonzero. Note that the torsion-free gravitational part of the connection (for the FRW or flat case) yields a vanishing contribution to \( B^0 \). From (12), (19) and (22) then we obtain a constant \( B^0 \) of order

\[
B^0 \sim \sqrt{2Q^2} e^{-\Phi_0} \frac{M_s}{\sqrt{n}} \text{ GeV} > 0. \tag{23}
\]

We follow the conventions of string theory for the sign of \( B^0 \). From phenomenological considerations \( M_s \) and \( g_s^2 / 4\pi \) are taken to be larger than \( \mathrm{O}(10^3) \) GeV and about 1/20, respectively. A stringent constraint on the Kac–Moody level is not imposed from the requirement of CPTV-induced leptogenesis. We observe from (23) that the central charge deficit \( Q^2 \) of the underlying conformal field theory determines the order of particle–antiparticle asymmetries in this model.

The particle–antiparticle asymmetry occurs already in thermal equilibrium, due to the background-induced [26] difference in the dispersion relations between particles and antiparticles. Since the coupling of fermions to torsion is universal, the axion background can also couple to quarks and charged leptons. For CPT-violating leptogenesis at the GUT scale [26] \( B^0 \sim 10^5 \text{ GeV} \). Hence a constant torsion-induced \( B^0 \) could lead directly to baryogenesis at the quark decoupling temperature \( T_q \sim 100 \text{ MeV} \), provided \( B^0 \) is much lower than \( B^0 \sim 10^5 \text{ GeV} \). In such a scenario, the observed BAU could be realised directly through the quark–H-torsion interactions; unfortunately, the corresponding neutrino asymmetry would be too high, given that the neutrino decoupling temperatures are at 1 MeV scale. Note that standard model \( B–L \) conserving sphaleron processes freeze-out at temperatures of order 100 GeV but such interactions cannot produce sufficient baryon asymmetry in the universe without extra sources for CP violation [4].

A phenomenologically viable alternative scenario is one in which \( B^0 \) is not constant with time. In such a scenario the universe undergoes a phase transition at GUT scales \( (\sim 10^{15} \text{ GeV}) \). Above the GUT temperature heavy right-handed Majorana neutrinos decouple or \( L \) violating (but \( B–L \) conserving) processes occur. At the phase transition the value of the \( H \)-torsion background changes from a large to a smaller value. The possibility of such phase transitions is based on conformal field theories with different central charges which characterise different epochs of the early universe [52]. Transitions between such conformal (fixed) points in moduli space correspond to nonconformal (Liouville) time evolution [53] and represent phase transitions in the early universe. An enlightening example of such a scenario is provided by the quantum Hall system in condensed matter where, as the temperature varies, there are transitions between plateaux with fixed values of the conductivity. The plateaux correspond to underlying conformal field theories of a “stringy” nature [54, 55], with different values of the central charge. Transitions between the plateaux are not described by conformal field theories. In the approach of [53] the cosmic time evolution in the universe is associated with a world-sheet renormalisation group flow in a string theory sigma model, whose description lies beyond the scope of this article. Here leptogenesis occurs at GUT scales (just after inflation). Following Ref. [26], the leptogenesis can be communicated to the baryon sector by means of \( B–L \) conserving sphaleron processes. A more detailed study of phenomenologically realistic models obtained from strings in such CPTV backgrounds will be discussed elsewhere. The population difference between fermions and antifermions is due to the interaction with the torsion of a CPTV background and occurs already in thermal equilibrium. Provided
the background torsion is constant in time over the appropriate epoch of the evolution of the universe (until the freeze out of the pertinent processes characterising the epoch), such differences cannot be eroded during the expansion of the universe. In contrast to the case of conventional leptogenesis [5, 56–59], no enhanced CP violation in the lepton sector is essential for leptogenesis in this approach. (This observation does not completely remove the need for right-handed sterile neutrinos. The latter may be essential, for instance, for explaining the smallness of the active neutrino masses, through see-saw mechanisms, or may play the role of dark matter [5, 59].)

4 CPTV in stochastic Finsler geometries

Although all the models displaying CPTV, which we have considered so far, are based on local effective field theories, there is no compelling reason for a restriction to such a framework. In fact a microscopic model involving space–time defects based on string-brane theory suggests the use of a non-Riemannian metric background similar to that which occurs in Finsler geometry [16, 60]. (This model will be discussed in a subsequent section.) Independently there has been much interest in Finsler geometry [61–70] for characterising the early universe [71–75] and for descriptions of modified dispersion relations for particle probes [60, 72, 76, 77]. Finsler geometry has a metric which, in addition to space–time coordinates, depends also on “velocities”. Lorentz symmetry is broken through some fixed vectors in the metric. We explore the consequences of making such vectors having components which are stochastic with possibly zero mean. This is a feature that arises in the defect model of D-foam [18, 20, 76] that we will consider in the next section. The defects stochastically fluctuate, due to both statistical and quantum stringy effects in large populations of such D-particles which exist in the early universe. The result of the interaction of neutrinos with these defects, leads to stochastically fluctuating Finsler-like metrics. However, in this section we will consider the consequences of this stochasticity for CPTV and matter–antimatter asymmetry in a general context. This underlying model provides our main motivation to study this class of space–times in this section and to contrast our findings on the induced CPTV for such cases with the corresponding ones for the D-foam model. For clarity, we commence our discussion with a brief reminder of the definition and properties of Finsler geometries, and then we proceed to discuss CPTV issues in a particular, but representative, Finsler-like geometry, which, however, is stochastically fluctuating.

A Finsler geometry on a manifold \( M \) is defined in terms of a Finsler norm \( F(x, y) \), a real function of two arguments \( x \) and \( y \), where \( x \in M \) and \( y \in T_x M \) (the tangent space at \( x \)) [16, 60]. The norm \( F(x, y) \) satisfies

1. \( F(x, y) \neq 0 \) if \( y \neq 0 \)
2. \( F(x, \lambda y) = |\lambda| F(x, y) \) for \( \lambda \in \mathbb{R} \).

The Finsler geometry is defined in terms of a metric \( g_{\mu \nu}(x, y) \) which is given in terms of the Finsler norm

\[
\frac{1}{2} \frac{\partial^2 F^2(x, y)}{\partial y^\mu \partial y^\nu}.
\]

The inverse of \( g_{\mu \nu}(x, y) \) is represented by \( g^{\mu \nu}(x, y) \). It is necessary to define a similar structure in phase space (i.e. for the co-tangent space). The dual \( \omega^\mu(y) \) to \( y^\mu \) is defined by [60]

\[
\omega^\mu(y) = g_{\mu \nu}(x, y)y^\nu.
\]

Furthermore this relation can be inverted so that corresponding to \( \omega \) there is a dual \( y(\omega) \). The Finsler norm \( G(x, \omega) \) on cotangent space is defined by

\[
G(x, \omega) = F(x, y(\omega)).
\]

The Finsler metric \( h^{\mu \nu}(x, \omega) \) in phase space is defined analogously to the usual Finsler metric in (24)

\[
\frac{1}{2} \frac{\partial^2 G^2(x, \omega)}{\partial \omega^\mu \partial \omega^\nu}.
\]

It is interesting to note that Cartan’s torsion tensor \( C_{\mu \nu \lambda} = \frac{1}{2} \frac{\partial g_{\mu \lambda}}{\partial y^\nu} \) vanishes when \( g_{\mu \nu} \) is Riemannian. Much of the mathematical literature has dealt with Finsler extensions of Riemannian geometry when the metric signature has been Euclidean. However, we need to consider pseudo-Riemannian structures. Although this can be formally done, the Finsler norm leads to a metric which can lead to singularities in the metric for off-shell test particles.

We focus on on-shell neutrinos (which are now known to have small masses). Again, this is motivated by our desire to discuss leptogenesis in such geometries. Moreover, for reasons that will become clear in Sect. 5, it is neutrinos that play a preferential rôle in interacting non-trivially with the D-particle foam background, which induces stochastically fluctuating Finsler-like space times. As our main motivation is to compare the generic Finsler-like case with the D-foam model, as far as CPTV is concerned, we restrict our attention here on the effects of stochastically fluctuating Finsler geometries on dispersion relations of neutrinos and antineutrinos. We shall consider a particular type of Finsler metric on a manifold \( M \) which is known as the Randers metric [16].

The Randers norm \( F_R \) for the Randers metric

\[
F_R(x, y) = \alpha(x, y) + \beta(x, y),
\]
where
\[ \alpha(x, y) = \sqrt{r_{\mu\nu}(x)y^\mu y^\nu} \]
and
\[ \beta(x, y) = b_\mu(x)y^\mu. \]

The conditions for a Finsler norm are satisfied. It was noted in [80] that the geodesics of this metric coincided with the minimum time trajectories of a particle moving on a Riemannian manifold in the presence of a time independent drift given by a vector field. This is similar to Fermat’s principle for propagation in refractive media. Similarities of D-particle foam to a refracting medium [76, 81, 82], will be mentioned briefly in the next section. If we were to assume that the result on minimum time trajectories was true for a pseudo-Riemannian situation and the drift was given by collisions due to D-particle scattering, then at a heuristic level a stochastic drift could be a reasonable generic phenomenological model of the back-reaction of low dimensional re-coiling branes on matter. We shall write
\[ b_\mu(x) = \phi(x)l_\mu, \]
where \( l_\mu \) is a constant vector. In our model \( \phi(x) \) will be a Gaussian stochastic variable. On average the metric will be like a Riemannian metric if the mean of \( \langle \phi \rangle \) vanishes. From (24) we deduce that
\[
g_{\mu\nu}(x, y) = r_{\mu\nu}(x) + \phi^2(x)l_\mu l_\nu + \frac{r_{\alpha\beta}(x)r_{\nu\sigma}(x)y^\alpha y^\sigma}{\alpha(x, y)^3}\phi(x)l_\epsilon y^\epsilon + \frac{1}{\alpha(x, y)}(r_{\nu\rho}(x)y^\rho \phi(x)l_\mu + r_{\nu\phi}(x)y^\phi \phi(x)l_\mu). \]

We shall consider now a situation with \( r_{\mu\nu}(x) = 0 \) where \( \eta_{\mu\nu} \) is the diagonal Minkowski matrix with entries \((1, -1, -1, -1)\). (The summation convention of repeated indices will be always understood unless explicitly stated otherwise.) Within the framework of a Robertson–Walker metric we shall ignore effects on the time-scale of the inverse expansion rate. Let us introduce some notation:
\[
\tilde{\alpha} = \sqrt{\eta_{\mu\nu}y^\mu y^\nu}, \quad \tilde{\beta} = l_\mu y^\mu.
\]

and
\[ C_{\mu} = y^\nu \eta_{\nu\mu}. \]

We can then rewrite \( g_{\mu\nu}(x, y) \) as
\[
g_{\mu\nu} = \eta_{\mu\nu} + \phi^2l_\mu l_\nu + \left( \frac{\eta_{\mu\nu}}{\alpha} - \frac{C_{\mu}C_{\nu}}{\alpha^3} \right)\phi \tilde{\beta} + (C_{\mu}l_\nu + C_{\nu}l_\mu)\phi \tilde{\alpha}^{-1}.
\]

However, in order to construct energy-momentum dispersion relations we will need to calculate the dual metric \( h^{\mu\nu} \) (cf. (27)) which involves \( y^\mu(\omega) \). We shall consider \( \phi \) to be small and work to lowest order in the evaluation of \( G(x, \omega) \). First we write \( y^\mu = y^\mu_0 + \phi y^\mu_1 \). From (25) we have
\[
\omega_\mu = g_{\mu\nu}(x, y(0) + \phi y_1(0))\left( y^\nu_0 + \phi y^\nu_1 \right).
\]

We can solve for \( y_0(0) \) and \( y_1(0) \). We find for \( y_0(0) \)
\[
y_0^0(0) = -\omega_0, \quad y_0^1(0) = \omega_1, \quad y_0^2(0) = \omega_2, \quad y_0^3(0) = \omega_3.
\]

We find for \( y_1^\mu(1) \)
\[
y_1^0(1) = -\omega_0, \quad y_1^1(0) = \omega_1, \quad y_1^2(0) = \omega_2, \quad y_1^3(0) = \omega_3.
\]

Hence
\[
G(x, \omega) = \sigma - \phi \frac{\omega_0 l_1 + \omega_1 l_2 + \omega_2 l_3 + \omega_3 l_0}{\omega_0^2 + \omega_1^2 + \omega_2^2 + \omega_3^2}.
\]

We deduce that
\[ h^{\mu\nu} = \eta^{\mu\nu} + \phi H^{\mu\nu}, \]
where
\[
H^{0j} = \frac{1}{\sigma} \left[ 2l_j \omega_j \sigma^2 - \sigma^2 \omega_0^2 + 2\omega_0 \omega_j \sigma \right],
\]
and for \( i < j \) for \( i = 1, 2, 3 \)
\[
H^{ij} = \frac{\omega_i l_j (\sigma^2 + 2\omega_0^2)}{\sigma^3} + \omega_j \left( l_i (\sigma^2 + 2\omega_0^2) + \omega_0 (\sigma^2 + 2\omega_0^2) \right). \]

persion relations of the spin-curvature type discussed in Sect. 2.3. In fact such VSR-related models have been proposed in the past as candidates for the generation of \( L \) conserving neutrino masses [79], and hence our \( L \) violating considerations in this work do not apply.
\[
+ \left( \frac{\sigma^2 - 6\omega_1^2}{\sigma^2} \omega_1 (l_1^2 \omega_2 - l_1 \omega_3) \right)
+ \left( \frac{2 \omega_2}{\sigma^2} (l_1 \sigma^2 - \omega_1 l_1 \omega_3) \right).
\]

We have assumed a homogeneous \( \phi \) with \( \phi \) being \( x \) independent. The mass shell condition \( h^{\mu \nu} \omega_\mu \omega_\nu = m^2 \) leads to the following equation for \( \omega_0 \):

\[
\omega_0^2 - \frac{2 \phi}{m} \left[ l_0 \left( \omega_0^2 + \omega_0 \omega^2 \right) + l_1 \left( \omega_0^2 + \omega^2 \right) \right] + l_2 \left( \omega_2^2 + \omega^2 \right) = m^2
\]

where \( \omega_0^2 = \omega_1^2 + \omega_2^2 + \omega_3^2 \).

In the model it is possible to choose \( l_\mu \). Not all choices will lead to asymmetric population distributions between particles and antiparticles. The space-like choice \( l_0 = 0 \) gives a degenerate spectrum for particle and antiparticle and hence no CPTV in dispersion relations:

\[
\omega_0 = \left[ \omega^2 + \left( 1 + \frac{2 \phi}{m} \left( l_1 \omega_1 + l_2 \omega_2 + l_3 \omega_3 \right) \right) + m^2 \right]^{1/2}
\]

\[
\times \left[ 1 - \frac{2 \phi}{m} \left( l_1 \omega_1 + l_2 \omega_2 + l_3 \omega_3 \right) \right]^{-1/2}.
\]

Therefore this case cannot be used for leptogenesis in our framework.

More generally the dispersion relation is

\[
\omega_0 = \pm \frac{\phi}{m} l_0 (2 \omega^2 + m^2) + \mathcal{R}(\phi, \omega, m)
\]

where

\[
\mathcal{R}(\phi, \omega, m) = \left( \omega^2 + m^2 + \frac{2 \phi}{m} (2 \omega^2 + m^2) \right) \left( l_1 \omega_1 + l_2 \omega_2 + l_3 \omega_3 \right) \right)^{1/2}.
\]

The + sign is for the particle and the - sign is for the antiparticle. For the “time-like” case \( l_0 = l_1 = l_2 = l_3 = 0 \) the dispersion relation reduces to

\[
\omega_0 = \sqrt{\omega^2 + m^2} \pm \frac{\phi}{m} \left( 2 \omega^2 + m^2 \right).
\]

The sign of \( l_0 \) can be reabsorbed in \( \phi \).

For the “null” case \( l_0 = l_1 = 1 \) and \( l_2 = l_3 = 0 \) the dispersion relation reduces to

\[
\omega_0 = \sqrt{\omega^2 + m^2} \left( 1 + \frac{\phi}{m} \left( 2 - \frac{m^2}{\omega^2 + m^2} \right) \right) \omega_1
\]

\[
\pm \frac{\phi}{m} \left( 2 \omega^2 + m^2 \right).
\]

The parameters \( \omega \) play the r\( o\)le of momenta \( p \) in our case of neutrinos of mass \( m = m_\nu \) propagating in these spacetimes, and the Finsler metric may be seen as sort of back reaction on the space–time of such a propagation (to better appreciate this, the reader is invited to the discussion in the next Sect. 5, where a particular model of D(efect)-foam is considered as a medium for neutrino propagation in the early universe, leading to Finsler-like metric distortions as a consequence of medium–particle interactions).

Corresponding to such models involving D-foam, the parameter \( \phi \) is modelled as a stochastic Gaussian process with a mean \( a \) and standard deviation \( \sigma \). The interesting features in earlier analyses [23, 24] were retained when \( a = 0 \). The fermion number distribution \( n \) from equilibrium statistical mechanics is given by

\[
n = \text{g.d.o.f.} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\exp(\beta(\omega_0 - \mu)) + 1}.
\]

where we have ignored degeneracy factors. In terms of spherical polarss \( d^3 p = d\xi \sin \theta d\theta dp \) where \( p = |p| \), \( \theta \) lies in \([0, \pi]\) and \( \xi \) lies in \([0, 2\pi]\). In the regime of validity of our analysis (i.e. relativistic neutrinos (\( \mu \approx 0 \) that decouple at high temperatures \( T \gg m_\nu \)) and for small stochastic fluctuations of the background geometry) we will consider \( \beta = 1/T \) (in units of the Boltzmann constant \( k_B = 1 \)). We expand the relevant expressions in a power series. We denote \( n \) averaged over the distribution of \( \phi \) as \( \langle n \rangle \). It is given by

\[
\langle n \rangle \equiv \text{g.d.o.f.} \int_{-\infty}^{\infty} d\phi \frac{\exp(-\frac{(\phi-a)^2}{\sigma^2})}{\sqrt{\pi \sigma}} n.
\]

First of all, it immediately follows from (37) that for the “time-like” case, when \( a = 0 \), there is no particle–antiparticle asymmetry. This is to be expected, given that \( a \neq 0 \) corresponds in a sense to an averaged Lorentz violation in this stochastic geometry, and hence one of the basic assumptions for CPT Invariance of the effective theory of neutrinos in this “medium” is relaxed.

For the “time-like” case, when \( a \neq 0 \) and \( \beta \) small, we obtain to leading order in \( T/m \gg 1 \):

\[
\langle n \rangle \sim -\frac{2}{\pi^2} a \text{g.d.o.f.} T^3 \left( \frac{T}{m} \right) \int_0^\infty dx \frac{x^4 e^x}{(1 + e^x)^2}
\]

\[
= -a \text{g.d.o.f.} T^3 \frac{7\pi^2}{15} \left( \frac{T}{m} \right)
\]

We require \( a < 0 \) in order to have a particle–antiparticle asymmetry where the particle distribution dominates the antiparticle distribution. This yields the following lepton (neutrino) asymmetry, assumed to freeze at the neutrino decoupling temperature \( T_d \):

\[
\Delta L(T \sim T_d) = \frac{\Delta n_\nu}{s} \sim -10 a \frac{T_d}{m_\nu}.
\]
where, as usual, $s$ denotes the entropy density, which for relativistic species is assumed to be $s \sim 8 \text{d.o.f.} \frac{2 \pi^2}{45} T^3$.

To obtain the phenomenologically correct value of $\Delta L(T \sim T_d) \sim 10^{-10}$, which is then communicated to the baryon sector via $B+L$ violating sphaleron processes, or $B-L$ conserving GUT (assumed appropriately embedded in such space–time geometries), one needs to take into account that, according to current data, the masses of the active neutrinos that are assumed to participate in (42) must be smaller than $m_\nu < 0.2 \text{ eV}$. In GUT-scale $L$-violation models $T_d \sim 10^{15} \text{ GeV}$, and so, in this stochastic Finsler space time, we need the magnitude of the violation of Lorentz symmetry on average, $a \sim 10^{-36}$, to be extremely small in order to reproduce the observed BAU. The fixing of the sign of $a$ is considered as fine tuning, and is a feature that is common to many current models of gravitational leptogenesis/baryogenesis [25, 26, 28–34].

Let us next calculate the asymmetry for the null case (38). For $a = 0$ there is no asymmetry as we can see from considering (40) where we will exchange the order of the integrations over $p$ and $\phi$. The $\phi$ averaged expression for the particle distribution leads to

\[
\frac{1}{\sigma \sqrt{\pi}} \int_{-\infty}^{\infty} d\phi \exp \left( -\frac{(\phi - a)^2}{\sigma^2} \right) \int_0^\infty dx x^2 \int_0^\pi d\theta \sin \theta \frac{1}{1 + e^x} (1 + A(x, \theta)), \tag{43}
\]

where

\[
A(x, \theta) = \left\{ -\frac{e^x}{1 + e^x} \left[ \frac{2 \phi^2}{\epsilon} x^2 (1 + \cos \theta) + \frac{2}{\epsilon^2} x^4 (1 + \cos \theta)^2 \right] + \frac{e^{2x}}{(1 + e^x)^2} \frac{4 \phi^2}{\epsilon^2} x^2 x^4 (1 + \cos \theta)^2 \right\}
\]

we have expressed $p$ in terms of spherical polars, $x = \frac{\sqrt{1 + \sin \theta}}{\sqrt{1 + \epsilon^2 \cos \theta}}$, and the $\theta = 0$ axis has been taken for convenience to coincide with the 1-axis; The corresponding expression for the antiparticle is

\[
\frac{1}{\sigma \sqrt{\pi}} \int_{-\infty}^{\infty} d\phi \exp \left( -\frac{(\phi - a)^2}{\sigma^2} \right) \int_0^\infty dx x^2 \int_0^\pi d\theta \sin \theta \frac{1}{1 + e^x} (1 + B(x, \theta)), \tag{44}
\]

where

\[
B(x, \theta) = \left\{ -\frac{e^x}{1 + e^x} \left[ -\frac{2 \phi^2}{\epsilon} x^2 (1 + \cos \theta) + \frac{2}{\epsilon^2} x^4 (1 + \cos \theta)^2 \right] + \frac{e^{2x}}{(1 + e^x)^2} \frac{4 \phi^2}{\epsilon^2} x^2 x^4 (1 + \cos \theta)^2 \right\}
\]

Although $A(x, \theta) \neq B(x, \theta)$, for $a = 0$, on integrating over $\theta$ (as part of the integration over $p$) we have

\[
\left\langle \int_0^\pi d\theta \sin \theta A(x, \theta) \right\rangle = \left\langle \int_0^\pi d\theta \sin \theta B(x, \theta) \right\rangle.
\]

Hence when $a = 0$ there is no particle–antiparticle asymmetry. Hence we need Lorentz violation in the mean to obtain leptogenesis. This contrasts with our earlier work on correlations in neutral meson pairs created in meson factories where a signature of CPT violation [23] was present even for $a = 0$.

When $a \neq 0$ the counterpart of (43) is

\[
\frac{2}{\sigma \sqrt{\pi}} \int_{-\infty}^{\infty} d\phi \exp \left( -\frac{(\phi - a)^2}{\sigma^2} \right) \times \left( \frac{1}{1 + e^x} \left\{ \frac{2}{\epsilon^2} x^2 \frac{x^4 e^x}{1 + e^x} \right\} \right) \tag{45}
\]

and the counterpart of (44) is

\[
\frac{2}{\sigma \sqrt{\pi}} \int_{-\infty}^{\infty} d\phi \exp \left( -\frac{(\phi - a)^2}{\sigma^2} \right) \times \left( \frac{1}{1 + e^x} \left\{ \frac{2}{\epsilon^2} x^2 \frac{x^4 e^x}{1 + e^x} \right\} \right). \tag{46}
\]

Hence the analysis reduces to that for the time-like case. Consequently we have again (41) and require $a < 0$ to obtain an asymmetry with particle distribution exceeding that for the antiparticle.

This need to tune the sign of the asymmetry in the approaches described so far leads us to pursue a different route to relations of the form (13) in the remainder of this paper. The route is not based on an approach involving effective local field theory. The framework we will follow fits naturally into a picture which has been advocated in the past to understand gravitational decoherence [22, 23] and dark matter abundance [83], and may be considered to be a more microscopic approach than the earlier proposals that we reviewed. For D-foam the asymmetry is controlled by a parameter whose sign does not need adjustment. Furthermore, the constancy of the parameter, which has a stochastic origin, seems to be a more reasonable assumption than the same requirement for curvature. The D-particle foam model...
also has an interesting feature in that a non-Riemannian metric is induced on the motion of matter on the brane world. Our mechanism of CPTV in D-foam did not directly rely on this aspect. However, since this non-Riemannian structure is somewhat similar to the structure of Finsler metrics (see e.g. [60] and references therein) we have explored the implications for CPTV within the more general framework of stochastic Finsler metrics. We have found that it is indeed possible to have CPTV but in general it is necessary to choose the appropriate sign of the mean of a stochastic parameter.

5 Stringy-defect(D-)foam-induced CPTV and leptogenesis

In this section we shall consider a population of D0-branes (or, if there are no intrinsic D0-branes, lower dimensional compactified D-branes which are effectively point-like from the point of view of a brane world observer\(^4\)) interacting with neutral fermions such as the neutrino and antineutrino. This interaction leads to different dispersion relations for neutrinos and antineutrinos which in turn leads to an excess of the population of neutrinos over antineutrinos. The freeze-out of neutrinos at the decoupling temperature of neutrinos leads to leptogenesis given by the standard cosmological considerations. The latter results, through standard B and L violating sphaleron processes or B–L conserving interactions in GUT describing matter excitations on the brane, to the observed baryon asymmetry in the universe, with complete dominance of matter over antimatter, in a rather natural way, as we shall discuss below. Moreover, as we shall explain below, in this model of (effect)-foam, the prevalence of matter over antimatter, i.e. the positive sign of the asymmetry \(\Delta n > 0\), follows naturally, as a consequence of loss of energy of neutrinos during their interactions with the space–time defects, due to recoil of the latter. Thus, the sign of the induced asymmetry need not be fixed by hand, unlike the cases of gravitational leptogenesis discussed in previous sections. For instructive purposes, we first discuss the properties of the foam model, in the next subsection, before moving onto issues of CPTV and leptogenesis.

5.1 The D-foam model of the universe and neutrinos

D-foam models [18, 20, 23] are stringy models of space–time foamy geometries, which involve brane universes, propagating in higher-dimensional bulk geometries. The bulk contains point-like D-brane defects (“D-particles” or D0-branes) whose population density is constrained by the amount of CPTV that we observe. In many string theories (such as bosonic and type IIA string theories) they are stable zero-dimensional defects. However, for our purposes we will consider them to be present in string theories of phenomenological interest [81] since, even when elementary D-particles cannot exist consistently, as is the case of type IIB string models, there can be effective D-particles formed by the compactification of higher dimensional D-branes [82] (e.g. three-branes wrapped around three-cycles, with relatively small radii). In general the construction of a model involves a number of parallel brane worlds with three large spatial dimensions, the required number being determined by target space SUSY. (Phenomenologically realistic models may require stacks of intersecting branes arranged in particular ways [27].) These brane worlds move in a bulk space–time containing a gas of point-like bulk branes, called D-particles, which are stringy space–time solitonic defects [84] (cf. Fig. 1). One of these branes is the observable universe. On this brane the D-particles will appear as space–time defects. Typically open strings interact with D-particles and satisfy Dirichlet boundary conditions when attached to them. Closed and open strings may be “cut” by D-particles, a process that involves capture of the incident open string and creation of stretched strings between the (recoiling) D-particle and the brane world (string “splitting”), and subsequent re-emission of the open string. It has also been speculated that nucleation of localised compactified defects [28] from such a D brane world (at the very high temperatures in the early universe) can be considered as a generation of compactified effective (metastable [85] but very long lived) D-particles.

The preferential role of neutrinos in interactions with D-foam, and hence for CPTV, is discussed below; since electric charge is conserved, the representation of SM particles as open strings, with ends attached to the brane world, prevents capture and splitting of open strings carrying electric fluxes by the D-particles. (We should recall that in string theory the electric charge is at the end point of an open string.) D-particles are electrically neutral and thus electric charge would not have been conserved if such processes had taken place. This is also consistent with the effective D-particles which may have formed as a result of nucleation [28]. Hence, the D-particle foam is transparent to charged excitations of the SM, leaving neutral particles, in particular neutrinos, susceptible to the foam effects. The different behaviour of neutrinos from charged leptons implies a background-induced breaking of the SU(2) gauge symmetry of the standard model. In type IIB string theories the SU(2) symmetry is not broken since the compactified D-particles [82] can interact with the full SU(2) lepton doublet. However, the lepton interactions are suppressed compared to

\(^4\)It should be remarked that for the effective compactified D-“particles” the interactions with the charged matter excitations are suppressed relative to the neutral ones [82]. Hence, even in this case, it is the electrically neutral excitations which interact primarily with the D-foam.
those of photons [82]. The interactions of D-foam with sterile right-handed Majorana neutrinos remain unsuppressed, since the sterile neutrinos do not have any standard model charges. Such heavy states can participate in lepton-number violating processes which freeze out at GUT scales.

As discussed in detail in [24] the density of D-particles on the brane world is permitted to be relatively large, even at late eras of the universe, given the fact that bulk D-particles exert forces on the brane universe with mixed sign contributions to the brane vacuum energy, depending on the distance of the bulk D-particles from the brane [19, 86]. Such forces are due to stretched strings between the defect and the brane. These energy contributions depend only on the transverse components of the relative velocities of the defect with respect to the brane worlds. In fact, depending on the distance of the bulk D-particle from the brane world, the sign of the contributions on the D-brane vacuum energy from the moving defect in the bulk, with velocity \( v \) perpendicular to the brane world, may be negative or positive. In particular, the interaction of a single D-particle that lies far away from the D3 brane (D8-compactified world) and, moves adiabatically with a small velocity \( v_\perp \) in a direction transverse to the brane, results in the following potential\footnote{For brevity, in what follows we ignore potential contributions induced by compactification of the D8 brane worlds to D3 branes, stating only the expressions for the induced potential on the uncompactified brane world as a result of a stretched string between the latter and the D-particle—the compactification does not affect our arguments on the negative energy contributions to the brane vacuum energy.} [20]:

\[
V_{D0-D8}^{\text{long}} = \frac{r (v_\perp^\text{long})^2}{8\pi \alpha'}, \quad r \gg \sqrt{\alpha'}. \tag{47}
\]

On the other hand, a D-particle close to the D3-brane (compactified D8), at a distance \( r' \ll \sqrt{\alpha'} \), moving adiabatically in the perpendicular direction with a velocity \( v_\perp^{\text{short}} \) will induce the following potential to it:

\[
V_{D0-D8}^{\text{short}} = -\frac{\pi \alpha' (v_\perp^{\text{short}})^2}{12r'^3}. \tag{48}
\]

This difference in sign, then, implies that one can arrange for the densities of far away and nearby bulk D-particles, which are not in general homogeneous, to be such that the total contribution to the brane world’s vacuum energy is always subcritical, so that issues such as overclosure of the universe by a significant population of D-particle defects can be avoided.

For our purposes in this work we may therefore consider that statistically significant populations of D-particles existed in the early eras of the brane universe. As the time elapses, the brane universe, which propagates in the higher-dimensional bulk (cf. Fig. 1), enters regions characterised by D-particle depletion, in such a way that the late eras cosmology of the universe is not affected. Nevertheless, as we shall discuss below, the early D-particle populations may still have important effects in generating neutrino–antineutrino populations differences (asymmetries), which are then communicated to the baryon sector via the standard sphaleron processes [87] or \( B-L \) conserving GUT symmetries in unified particle physics models.

To this end, we need to consider the effective dispersion relation of a (anti)neutrino field in a brane space–time punctured with statistically significant populations of D-particles. The latter is a dynamical population, consisting of defects crossing the brane all the time, thereby appearing to a brane observer as flashing “on” and “off” space–time “foamy”
structures. The (anti)neutrino excitations are represented as matter open strings with their ends attached on the brane. The number density of (anti) neutrinos on the brane world is limited by the requirement that they do not overclose the universe. If neutrinos are assumed to have a chemical potential \( \mu \), then standard cosmological neutrino models predict that the number densities of a single flavour of relativistic neutrinos plus antineutrinos in thermal equilibrium at temperature \( T_v \) is estimated by \[ n_\nu \sim \frac{3}{2\pi^2} \left( \frac{2\ln 2}{3T_v^2 \xi_\nu} + \frac{\mu_v^4}{72T_v^4 \xi_\nu} + O\left( \frac{\mu_v^6}{T_v^6} \right) \right) \] (49)

upon making the standard assumption that \( \mu_v \ll T_v \) for all neutrino flavours. The quantity \( \xi_\nu \equiv \frac{\mu_v}{T_v} \) is called the degeneracy parameter and is invariant under cosmic expansion. If we assume that the electron-neutrino chemical potential is the only one with significant presence in the early universe, then Big-Bang-Nucleosynthesis (BBN) constraints imply \( -0.04 < \xi_\nu < 0.07 \). Thus, the order of magnitude of the neutrino plus antineutrino number density is agrees with naive standard estimate \( n_\nu \sim \frac{1}{T^3} n_\gamma \), where \( n_\gamma \) is the photon density. Thus, today, where the temperature of the universe is of order \( T_0 = 2.728 \text{ K} \) (Cosmic Microwave Background), corresponding to an energy of \( k_B T_0 \sim 2.35 \times 10^{-13} \text{ GeV} \) (with \( k_B \) Boltzmann constant), the density of neutrinos is found to be of order \( n_\nu^{(0)} \sim 112 \text{ cm}^{-3} \) and scales roughly with the cubic power of the temperature: \( n_\nu \sim n_\nu^{(0)} \left( \frac{T}{T_0} \right)^3 \). So, for the decoupling temperatures of neutrinos, \( k_B T_d \sim 10^{15} \text{ GeV} \), where we are interested in this work, in order to compute the frozen CPT Violating neutrino–antineutrino population differences, one obtains a number density of neutrino plus antineutrino populations of order \[ n_\nu(T = T_d \sim 10^{15} \text{ GeV}) \sim 10^{85} \text{ cm}^{-3}. \] (50)

On the other hand, as already mentioned, there are no similar restrictions on the population of the D-particle defects on the brane, in view of the negative contributions on the potential energy of the brane universe by bulk D-particle populations [24]. Thus, at the early universe, at the above neutrino-decoupling temperatures, we may even assume D-particle densities of one defect per string volume on the three brane world, without overclosing the universe. The assumption that the string length can take on values in the phenomenologically acceptable (post LHC era) range \( 10^{-27} - 10^{-32} \text{ cm} \), corresponding to string mass scales from 10 TeV to 10^{18} \text{ GeV}, yields then a D-particle number density in the range

\[ n_D(T = T_d \sim 10^{15} \text{ GeV}) \sim 10^{54} - 10^{66} \text{ cm}^{-3}, \] (51)

respectively. Thus we observe that in order to be able to treat the D-particle populations as providing a more-or-less uniform “medium” over which neutrinos propagate, with non-trivial effective dispersion relations, we need to have at the decoupling temperature much higher densities of D-particles than those of neutrinos plus antineutrinos. Comparing (50) with (51), we observe that if one assumes one D-particle per three-dimensional string volume on the brane, then this latter requirement excludes the low values of the string mass scale, implying an allowed range

\[ 10^{-5} M_P \leq M_s \leq 10^{-1} M_P, \] (52)

with \( M_P \sim 10^{19} \text{ GeV} \) the four-dimensional Planck mass. One of course could have much more dense D-particle gases in the early universe, which would allow for lower string scales.

5.2 Kinematics of D-particle scattering and CPTV induced leptogenesis

We will now estimate the modification of the dispersion relations of neutrinos in such a “media” of D particles in the early universe. The interaction of a string with a D particle implies that at least one of the ends of the string is attached to the D particle defect. Furthermore, the simultaneous creation of virtual strings stretched between the defect and the brane, describes the recoil of the D-particle. During the interaction time, the D-particle undergoes motion characterised by non-trivial velocities, \( u_j = \frac{2}{3} \Delta p_i = 2/3 r_i p_i \) along the brane longitudinal dimensions, where \( r_i \) denotes the proportion of the incident neutrino momentum that corresponds to the momentum transfer \( \Delta p_i \) during the scattering, and \( v_\perp \) in directions transverse to the brane world [22].

As discussed in [23, 76, 86] the non-trivial capture and splitting of the open string during its interaction with the D particle, and the recoil of the latter, result in a local effective metric distortion of the form:

\[ ds^2 = g_{\mu \nu} dx^\mu dx^\nu \equiv (u_{\mu \nu}^a + h_{\mu \nu}) dx^\mu dx^\nu, \]

\[ h_{0i} = \langle u_i^a | \sigma_a \rangle, \] (53)

where \( u_{i \parallel} \) is the recoil velocity of the D-particle on the D-brane world, with \( i = 1, 2, 3 \) a spatial space–time index, \( \sigma_a \) are the \( 2 \times 2 \) Pauli flavour matrices with \( a = 1, 2, 3 \) (assuming two-flavour oscillations for simplicity). On average over a population of stochastically fluctuating D-particles including flavour changes, one may have the conditions (58), the second of which in the case of flavour oscillations can be generalised to

\[ \langle u_{i \parallel}^a u_{j \parallel} \rangle = \sigma_a^2 \delta_{ij} \delta_{ab}. \] (54)

(We still assume that \( \langle u_{i \parallel}^a \rangle = 0 \).) As a result of (54), on average, the flavour change during the interactions of neutrinos with the D-foam can be ignored. In such a case, any
flavour structure in the metric (53) is ignored. This result is the motivation for the consideration of a more general structure: Finsler geometry with stochastic parameters examined previously. However, the effects of D-foam go beyond those encoded in the induced Finsler like metric. The fine tuning that is required in stochastic Finsler metrics to get the correct sign for the particle–antiparticle asymmetry is a feature, although commonplace in other approaches, that is not entirely satisfactory. However, because we have a microscopic model we can consider the kinematics of D-particle scattering. On considering string theory scattering amplitudes we find that the four momentum is conserved in the scattering of D-particles and strings. D particles in the bulk exert forces on the vacuum energy of the brane world of mixed sign, depending on their relative distance. Thus, during the scattering process of a neutrino field with a D particle, the vacuum energy of the brane fluctuates by an amount $\Delta V$ which depending on the process can be of either sign. From energy-momentum conservation, at each individual scattering event between a neutrino field and a recoiling D-particle, one could thus write

$$P_{\text{before}} + P_{\text{after}} + M_s u_\parallel = 0,$$

$$E_{\text{before}} = E_{\text{after}} + \frac{1}{2} \frac{M_s}{g_s} u_\parallel^2 + \Delta V,$$  

(57)

$^6$Ignoring the flavour structure, the metric (53) can be written as

$$ds^2 = dt^2 + 2u_i dx^i dt - \delta_{ij} dx^i dx^j.$$  

(55)

This metric was determined from world-sheet conformal field theory considerations [76] and represents a dragging of the frame by the Galilean (slowly moving) D particle, which moves on a flat space–time background. However, the string excitations represent relativistic particles, and as such they move according to the rules of special relativity. Any four vectors attached to the strings, such as a four velocity, will evolve by a series of infinitesimal Lorentz boosts induced by the change of the D particle velocity relative to the particle. In this sense, one may perform a time coordinate change in the metric (55) to write in the form, up to terms $u^3$ for small recoil velocities $|u| \ll 1$,

$$ds^2 = dt^2 + 2u_i dx^i dt - \delta_{ij} (dx^i - u^j dx^j) (dx^l - u^l dx^l) + O(u^3).$$  

(56)

The metric (56) is nothing but the so-called Gullstrand–Painlevé metric [89], representing the geometry in the exterior of a Schwarzschild black hole, where the falling space into the black hole is represented as a Galilean river on a flat space–time in which relativistic fishes swim. The river represents the frame of the recoiling D particle, while the fishes are the relativistic matter strings. Here $t_0$ is the time of a free-floating observer who is at rest at infinity (compared to the centre of the black hole). In the case of a black hole the relative velocities $u^i$ are coordinate dependent, of course, unlike our approximation in the D-foam case, although one may easily consider more general cases, where the recoil velocities of the D-particles in the foam are non-uniform, in which case the analogy with the Gullstrand–Painlevé river would become stronger.

where $\langle p, E \rangle_{\text{before/after}}$ denote the incident (outgoing) neutrino momenta, energies respectively and we used the fact that the recoiling heavy D particle of mass $M_s/g_s$ (with $M_s$ the string scale and $g_s < 1$ the string coupling, assumed weak, so that string perturbation theory applies) has a non-relativistic kinetic energy $\frac{1}{2} M_s u_\parallel^2$. We have also assumed that the fraction of the neutrino momentum transfer in the direction perpendicular to the brane world is negligible. The importance of the term $\Delta V$ not having a fixed sign in each individual scattering process is associated with the possibility of D-particle induced neutrino flavour oscillations [23].

Indeed, upon averaging $\langle \langle \cdot \cdot \cdot \rangle \rangle$ over a statistically significant number of events, due to multiple scatterings in a D-foam background, we may use the following stochastic hypotheses [23]:

$$\langle \langle u_i \rangle \rangle = 0, \quad \langle \langle u_i u_j \rangle \rangle = \frac{1}{2} \delta_{ij},$$  

(58)

implying that Lorentz invariance holds only as an average symmetry over large populations of D particles in the foam. At a microscopic level, (58) translates to momentum conservation on average in (57), since $\langle \langle u_i \rangle \rangle = 0$. At an individual scattering process, if one represents the energy of the incident neutrino on-shell as $\sqrt{p^2 + m_1^2}$, where $p$ is the magnitude of the conserved spatial momentum of the neutrino, and the outgoing one as $\sqrt{p^2 + m_2^2}$, we observe that the energy-conservation equation (57) implies in general $m_1 \neq m_2$. Which one is larger depends on the signature of the term $\frac{1}{2} \frac{M_s}{g_s} u_\parallel^2 + \Delta V$, which as mentioned is not of fixed sign, thereby allowing for neutrino oscillations to take place. The situation is somewhat analogous to the standard Mössbauer effect [90], where the emitted or absorbed photon from a nucleus of an atom bound in a solid may sometimes be free of nuclear recoil, in contrast to the case of gases, thereby attributing the phenomena of nuclear resonances to such recoil-free fraction of nuclear events. In our case the role of the “nuclei bound in a lattice” is played by the D-particle lattice. In addition to the D-particle recoil energy during scattering with stringy matter, which would lead to energy losses for the neutrinos, there are vacuum energy fluctuations, as a consequence of the motion of bulk particles in the foam, thus the neutrino experiences losses and gains from the vacuum, which results in the induced flavour oscillations. The analogue of resonances in this case would correspond to the loss-and-gain-free fraction of events, in which the neutrino does not oscillate.

However, the effects of the D-foam go beyond the above-mentioned kinematical ones. On assuming isotropic momentum transfer, $r_i = r$ for all $i = 1, 2, 3$. The dispersion relation of a neutrino of mass $m$ propagating on such a deformed isotropic space–time, then, reads

$$p^\mu p^\nu g_{\mu\nu} = p^\mu p^\nu (\eta_{\mu\nu} + h_{\mu\nu}) = -m^2$$

$$\Rightarrow E^2 - 2E p \cdot u_\parallel - p^2 - m^2 = 0,$$  

(59)
This on-shell condition implies that
\[ E = u \cdot p \pm \sqrt{(u \cdot p)^2 + p^2 + m^2}. \] (60)

We take the average \( \langle \cdots \rangle \) over D-particle populations with the stochastic processes (54), (58). Hence we arrive at the following expression for an average neutrino energy in the D-foam background:
\[ \langle E \rangle = \langle p \cdot u \rangle \pm \sqrt{p^2 + m^2 + (p \cdot u)^2} \]
\[ \simeq \pm \sqrt{p^2 + m^2 \left( 1 + \frac{1}{2} \sigma^2 \right), \quad p \gg m,} \] (61)

for the active light neutrino species. The last relation in Eq. (61) expresses the corrections due to the space–time distortion of the stochastic foam to the free neutrino propagation. It is this expression for the neutrino energies that should be used in the averaged energy-momentum conservation equation (57) that characterises a scattering event between a neutrino and a D-particle. On further making the assumption for the brane vacuum energy that \( \langle \langle \Delta V \rangle \rangle = 0 \), the total combined effect on the energy-momentum dispersion relations, from both capture/splitting and metric distortion, can then be represented as
\[ \langle E_2 \rangle = \pm \sqrt{p^2 + m^2 \left( 1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} M_s \sigma^2}, \] (62)

Since antiparticles of spin 1/2 fermions can be viewed as “holes” with negative energies, we obtain from (57) and (61) the following dispersion relations between particles and antiparticles in this geometry (for Majorana neutrinos, the roles of particles/antiparticles are replaced by left/right handed fermions):
\[ \langle E_\nu \rangle = \sqrt{p^2 + m^2 \left( 1 + \frac{1}{2} \sigma^2 \right) - M_s \sigma^2}, \]
\[ \langle E_\bar{\nu} \rangle = \sqrt{p^2 + m^2 \left( 1 + \frac{1}{2} \sigma^2 \right) + M_s \sigma^2}, \] (63)

where \( \bar{E} > 0 \) represents the positive energy of a physical antiparticle. In our analysis above we have made the symmetric assumption that the recoil-velocities fluctuation strengths are the same between particle and antiparticle sectors. (Scenarios for which this symmetry were not assumed have also been considered in an early work [23].) There can thus be local CPTV in the sense that the effective dispersion relation between neutrinos and antineutrinos are different. This is a consequence of the local violation of Lorentz symmetry (LV), as a result of the non-trivial recoil velocities of the D-particle, leading to the LV space–time distortions (53).

The discussion of CPTV in such foamy universes now follows the line of argument adopted by others: the difference in the dispersion relations between particles and antiparticles will imply differences in the relevant populations of neutrinos (\( n \)) and antineutrinos (\( \bar{n} \)), (cf. the dispersion (63)). This difference between neutrino and antineutrino phase-space distribution functions in D-foam backgrounds generates a matter–antimatter lepton asymmetry in the relevant densities
\[ \langle n - \bar{n} \rangle = g_{\text{d.o.f.}} \frac{d^3p}{(2\pi)^3} \left\{ f(E) - f(E) \right\}, \] (64)

where \( g_{\text{d.o.f.}} \) denotes the number of degrees of freedom of relativistic neutrinos, and \( \langle \langle \cdots \rangle \rangle \) denotes an average over suitable populations of stochastically fluctuating D-particles (58).

Let us first make the plausible assumption that \( \sigma^2 \) is constant i.e. independent of space. It is a parameter which can only be positive. Furthermore we will assume that \( \sigma^2 \) is independent of the (anti)neutrino energy. This is for estimation purposes only. We shall come back to a more detailed analysis later. We should note that the form of the dispersion relations (63) is analogous to the case of CPTV axisymmetric geometries in the early universe, discussed previously (cf. (13)) with the role of the axial curvature scalar potential \( B_0 \) being played here by the quantity \( \frac{M_s}{g_s} \sigma^2 \). In fact, it is easily seen that to leading order in \( \sigma^2 \) the \( (1 + \frac{1}{2} \sigma^2) \) prefactors of the square roots in (63) play no role, and hence the leading in \( \sigma^2 \) contribution to the leptonic asymmetry comes from the constant \( \frac{M_s}{g_s} \sigma^2 \) terms in the dispersion relations. The induced lepton asymmetry can therefore be calculated following similar steps as those leading to (15), upon the replacement of \( B_0 \) by \( \frac{M_s}{g_s} \sigma^2 \), the difference being that here the value of \( B_0 \) (i.e. of the D-foam recoil fluctuations \( \sigma^2 \)) is to be fixed phenomenologically.

The result for the D-foam-induced lepton asymmetry can be estimated from (64), using (63). Ignoring neutrino mass terms and \( (1 + \frac{\sigma^2}{2}) \) square-root prefactors in (63), setting the (anti-)neutrino chemical potential to zero (which is a sufficient approximation for relativistic light neutrino matter) and performing a change of variables \( |p|/T \to \tilde{u} \) we obtain from (64) the result
\[ \Delta n_\nu = \frac{g_{\text{d.o.f.}}}{2\pi^2} T^3 \int_0^\infty d\tilde{u} \tilde{u}^2 \left[ \frac{1}{1 + e^{-\frac{M_s\sigma^2}{g_s T}}} - \frac{1}{1 + e^{\frac{M_s\sigma^2}{g_s T}}} \right] \]
\[ = \frac{g_{\text{d.o.f.}}}{\pi^2} T^3 \left( \text{Li}_3(-e^{-\frac{M_s\sigma^2}{g_s T}}) - \text{Li}_3(-e^{\frac{M_s\sigma^2}{g_s T}}) \right) \]
\[ \approx \frac{g_{\text{d.o.f.}}}{\pi^2} T^3 \left( \frac{M_s \sigma^2}{g_s T} \right) > 0, \] (65)

to leading order in \( \sigma^2 \), where in the last step we took into account the formal definition as a series of the Polylogarithm function \( \text{Li}_3(z) = \sum_{k=1}^\infty \frac{z^k}{k^3} \) which is valid for \( |z| < 1 \), while the cases \( |z| \geq 1 \) are defined by analytic continuation. We thus observe that the CPTV term \( -\frac{1}{2} \frac{M_s}{g_s} \sigma^2 \) in the dispersion relation (63) for the neutrino, which corresponds to the
energy ‘loss’ due to the D-particle recoil kinetic energies, comes with the right sign (‘loss’) so as to guarantee an excess of particles over antiparticles. Unlike the model of [26, 31, 33, 34], then, where the sign of the $B_0$ parameter had to be assumed, in our D-foam case there is no such freedom, and the positive $\Delta n_\nu$ is derived from first principles. This is a good feature of our model.

As in standard scenarios of leptogenesis, the lepton asymmetry (65) decreases with decreasing temperature up to a freeze-out point, which occurs at temperatures $T_d$ at which the lepton-number-violating processes decouple. In Sect. 3 we argued that in a string theory background the phenomenologically realistic scenario seems to require GUT scale lepton-number violating processes, which freeze out the lepton–antilepton number at temperatures $T_d \sim 10^{15}$ GeV. For our D-particle brane universe we assume that at the GUT scale there is a sufficiently dense D-particle gas in the bulk and on the brane, so that phenomenologically realistic leptogenesis takes place. The latter is then communicated via $B–L$ conserving GUT processes to the baryon sector. After the GUT-scale freeze out the bulk population of D-particles can be depleted: the brane world representing our universe passes through an area with a diminished density of D-particles can be depleted: the brane world represents a good feature of our model.

In Sect. 3 we argued that in a string theory background metric geometries in the early universe [26, 31, 33, 34] is derived from first principles. This is a good feature of our model.

The above estimate ignores the dependence of the stochastic variable $\sigma^2$ on the neutrino energy. We may parameterise (see Refs. [22, 23]) the momentum transfer in D-particle collisions by the stochastic parameter $r$, which is in turn assumed stochastic:

$$u_i = \frac{g_s}{M_s} \Delta p_i \rightarrow g_s r_i \frac{p_i}{M_s}, \quad \text{no sum over } i,$$

$$\langle r_i \rangle = 0, \quad \langle r_i r_j \rangle = \Delta^2 \delta_{ij}. \quad (69)$$

In this case, the dispersion relations (63) are modified by the replacement of

$$\sigma^2 \rightarrow \frac{g_s^2}{M_s^2} \Delta^2 p^2, \quad (70)$$

with the momentum dependent quantity:

$$\langle E_\nu \rangle = \sqrt{p^2 + m_\nu^2} \left( 1 + \frac{g_s^2}{2M_s^2} \Delta^2 p^2 \right) - \frac{g_s}{2M_s} \Delta^2 p^2,$$

$$\langle E_\bar{\nu} \rangle = \sqrt{p^2 + m_\nu^2} \left( 1 + \frac{g_s^2}{2M_s^2} \Delta^2 p^2 \right) + \frac{g_s}{2M_s} \Delta^2 p^2. \quad (71)$$

We shall evaluate the integral (64) for the case (58) with $\Delta^2 \ll 1$ sufficiently small so that a truncation to order $\Delta^2$ will be sufficient. $\Delta^2$ will be assumed to be the same between particle and antiparticle sectors.

In D-foam models the upper limit of the momentum integration $p_{\text{max}}$:

$$p_{\text{max}} \equiv |p|_{\text{max}} = \frac{M_s}{g_s \sqrt[4]{\Delta^2}}, \quad (72)$$

corresponds to a D-particle recoil velocity of magnitude the speed of light in vacuo where $r$ is the stochastic variable satisfying (69). The resulting integrals in (64) are

$$\Delta n_\nu = \frac{g_{\text{d.o.f.}}}{2\pi^2} T^3 \int_0^{M_s \tau_{gs} \sqrt{\Delta^2}} d\tilde{u} \left[ \frac{1}{1 + e^{\tilde{u} - \tilde{u}_e \Delta^2 T / 2m_p}} - \frac{1}{1 + e^{\tilde{u} + \tilde{u}_e \Delta^2 T / 2m_p}} \right]. \quad (73)$$

The upper limit of the $\tilde{u}$ integration cannot be taken as $\infty$ since $\Delta n_\nu$ should be evaluated at the decoupling temperature $T_d \sim 10^{15}$ GeV where it freezes out. An advantage of this CPTV scenario over the ones associated with axisymmetric geometries in the early universe [26, 31, 33, 34] is that the CPTV parameter, $\Delta^2$, is a phenomenological parameter rather than a function. This parameter depends on the density of D particles in the early universe and cannot be significantly constrained by the cosmology of the early universe. We shall now analyse (73); first we set

$$y \equiv \frac{M_s}{T g_s \sqrt[4]{\Delta^2}} \left( \equiv \frac{\alpha}{\sqrt{\Delta^2}} \right).$$

We note that

$$\Delta n_\nu = \frac{g_{\text{d.o.f.}}}{2\pi^2} T^3 \left[ \int_0^y \frac{du}{1 + e^{u - u_e \Delta / 2y}} - \int_0^y \frac{du}{1 + e^{u + u_e \Delta / 2y}} \right].$$
We write \( I_1 = \int_0^1 \frac{d\alpha}{1 + \exp(\alpha x)} \) and \( I_2 = \int_0^1 \frac{d\alpha}{1 + \exp(\alpha x^2)} \).

We will evaluate \( \Delta n_\nu \) in two limits: (i) \( \alpha \to \infty, \quad \Delta \sim O(1) \)
and (ii) \( \Delta \to 0, \quad \frac{\Delta}{\alpha} \to 0 \). For case (i) let \( x = \frac{\Delta}{\alpha} - \frac{\alpha}{2} \) and so
\[
I_1 = \int_0^{\frac{\pi}{2}} \frac{d\alpha}{1 - v} \frac{1}{1 + \exp(\alpha x)}.
\]
where \( v = \frac{1 - \sqrt{1 - 2x\Delta^2}}{\Delta} \).

This integral can be evaluated (using Watson’s lemma) to give
\[
I_1 \sim \Delta \left( \frac{1}{\alpha} + \frac{\Delta^2}{\alpha^2} + \frac{3\Delta^4}{\alpha^3} + \cdots \right).
\]
A similar analysis for \( I_2 \) gives
\[
I_2 \sim \Delta \left( \frac{1}{\alpha} - \frac{\Delta^2}{\alpha^2} + \frac{3\Delta^4}{\alpha^3} + \cdots \right).
\]

Hence \( \Delta n_\nu \) in limit (i) is given by
\[
\Delta n_\nu \sim \frac{\text{d.o.f.} \pi^2}{2} T^3 \frac{\Delta^2}{\alpha} + \cdots = \frac{\text{d.o.f.} \pi^2}{2} T^3 \frac{\Delta^2}{s} \frac{g_s T}{M_s}.
\]

For case (ii) we have
\[
I_1 \sim \int_0^1 \frac{e^{-\frac{\pi}{2} v}}{\pi} e^{-\frac{\pi}{2} v^2} \quad \text{and} \quad I_2 \sim \int_0^1 \frac{e^{-\frac{\pi}{2} v^2}}{\pi} e^{-\frac{\pi}{2} v^2}.
\]

However, the analysis again leads to the result (78). The lepton asymmetry resulting from (78) freezes out at temperature \( T_d \) is
\[
\Delta L(T < T_d) = \Delta n_\nu \frac{\Delta^2}{s} \frac{g_s T_d}{M_s}.
\]

We note that this result is compatible formally with (66) if one takes into account (70) and associates the momentum magnitude \( p \) of the relativistic neutrino (i.e. its energy) with the temperature \( T \).

For \( \Delta^2 \sim 10^{-6} \) a Planck-size D-particle mass \( M_s/g_s \sim 10^{19} \text{ GeV} \) is required so that the D-foam provides the physically observed lepton and, thus, baryon asymmetry. For the unnaturally small \( \Delta^2 < 10^{-21} \) one arrives at \( M_s/g_s \sim 10 \text{ TeV} \). Unfortunately, for \( \Delta^2 \sim O(1) \) transPlanckian D-particle masses are required. We should stress that the above conclusions were based on the assumption that the freeze-out temperature was the temperature at decoupling of neutrinos in standard big-bang cosmology.

Our approach to leptogenesis is distinguished from others in that a local effective field theoretical description is not adopted. Because of D-particle recoil when scattering off matter strings the background of D-particles can be modelled as a stochastic medium [22–24]. The underlying string theoretic description provides the rigorous description of the scattering of D-particles. The D-particles backreact (as seen from infra-red divergences in perturbation theory) and change the metric which influences the space in which matter is moving. Furthermore as discussed at length in [24], and mentioned briefly above, the D-particle foam model does not lead to overclosing the universe. Hence despite having statistically significant populations of D-particles in the early universe, which provide the CPTV background on which neutrinos propagate, the assumption of a subcritical energy density for the universe can still hold.

6 Conclusions and outlook

In this work we have considered some models leading to CPTV gravitational leptogenesis, based on string-inspired constructions. One such model involves a special dilaton background and an antisymmetric Kalb–Ramond field, which leads to a space–time independent torsion. Coupling of this torsion to fermions results in corrections to the standard dispersion relations which differentiate between particles and antiparticles, leading to leptogenesis. The primary model that we discuss here involves a brane universe propagating in a higher-dimensional bulk space–time, which contains populations of (“point-like”) D0-brane defects (D particles). The topologically non-trivial interaction of the D-particles with the neutrinos, in the sense of capture and subsequent re-emergence of the latter by the defects, is allowed by electric charge conservation; in contrast the medium of defects is “transparent” for charged excitations of the standard model (such as charged leptons and quarks). The propagation of neutrinos in this medium of space–time defects results in CPTV dispersion relations, which are different for neutrinos and antineutrinos. It is essential to notice that this difference is obtained by considering the kinematics of the defect/neutrino scattering on the assumption that the antiparticle has negative energies following the “hole theory” of Dirac. These considerations apply only to fermions, as
opposed to bosons, because the “hole theory” is based on the fermion exclusion principle. This model has two aspects. One is the induced metric in space–time, which depends on momenta as well as space–time coordinates, and the other is the particle-like kinematics involved in the D0-brane scattering of matter. The first aspect is similar to stochastic Finslerian metrics which we have considered in a general way without necessarily tying it to the brane model. The second, kinematical aspect is important in leading naturally to matter dominance over antimatter, in contrast to earlier proposals of gravitational leptogenesis, where this sign has to be adjusted by hand.

The cosmology and particle-physics phenomenology of such scenarios, although currently in their infancy, are worth pursuing in our opinion. In particular, for the defect model, at early epochs of the universe, despite the fact that significant populations of massive D-particles are assumed present, we have argued here and in our previous works [24, 83] that one can avoid overclosure of the brane universe: the defects are allowed to propagate in the bulk and thus they exert forces on the brane world which are such that there are mixed sign contributions to the brane vacuum energy, depending on the distance of the bulk defect populations from the brane. The population of D-particles in the bulk is also constrained by the amount of Lorentz violation at late eras of the universe, which leave their imprint on the Cosmic Microwave Background [91, 92] and in vacuum-refractive-index tests of arrival times of cosmic photons [15, 93–95].

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