Implications of multiwavelength spectrum on cosmic-ray acceleration in blazar TXS 0506+056

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\textbf{ABSTRACT}

\textbf{Context.} The MAGIC collaboration has recently analyzed data from a long-term multiwavelength campaign of the γ-ray blazar TXS 0506+056. In December 2018 it was flaring in the very high-energy (VHE; \(E > 100\text{ GeV}\)) γ-ray band, but no simultaneous neutrino event was detected.

\textbf{Aims.} We modeled the observed spectral energy distribution (SED) using a one-zone leptohadronic emission.

\textbf{Methods.} We estimated the neutrino flux through the restriction from the observed X-ray flux on the secondary radiation due to the hadronic cascade, initiated by protons with energy \(E_p \lesssim 0.1\text{ EeV}\). We assumed that ultra-high-energy cosmic rays (UHECRs; \(E \gtrsim 0.1\text{ EeV}\), with the same slope and normalization as the low-energy spectrum, are accelerated in the jet but escape efficiently. We propagate the UHE protons in a random turbulent extragalactic magnetic field (EGMF).

\textbf{Results.} The leptonic emission from the jet dominates the GeV range, whereas the cascade emission from CR interactions in the jet contributes substantially to the X-ray and VHE range. The line-of-sight cosmogenic γ-rays from UHECRs produce a hardening in the VHE spectrum. Our model prediction for neutrinos from the jet is consistent with the 7.5-year flux limit by IceCube and shows no variability during the MAGIC campaign. Therefore, we infer that the correlation between GeV-TeV γ-rays and neutrino flare is minimal. The luminosity in CRs limits the cosmogenic γ-ray flux, which in turn bounds the RMS value of the EGMF to \(\lesssim 10^{-2}\) mG. The cosmogenic neutrino flux is lower than the IceCube-Gen2 detection potential for 10 yr of observation.

\textbf{Conclusions.} Very high-energy γ-ray variability should arise from increased activity inside the jet; thus, detecting steady flux at multi-TeV energies may indicate UHECR acceleration. Upcoming γ-ray imaging telescopes, such as the CTA, will be able to constrain the cosmogenic γ-ray component in the SED of TXS 0506+056.

\textbf{Key words.} astroparticle physics – galaxies: active – gamma rays: general – neutrinos

1. Introduction

Blazars are a subclass of radio-loud active galactic nuclei (AGNs) with a highly relativistic jet collimated toward the observer’s line of sight. They are considered to be prominent candidates for the origin of IceCube-detected diffuse astrophysical neutrino flux beyond \(\sim 10\text{ TeV} (\text{IceCube Collaboration 2013; Eichler 1979; Sikora et al. 1987; Petropoulou et al. 2015; Murase et al. 2018; Yuan et al. 2020})\) and may also contribute in the PeV-EeV energy range (Kalashev et al. 2013; Kochcki et al. 2021; Das et al. 2021). For the first time in September 2017, a high-energy muon-neutrino event IC-170922A (\(E_\nu \sim 0.3\text{ PeV}\)) was associated with the γ-ray flaring blazar TXS 0506+056 at \(3\sigma\) significance (IceCube Collaboration 2018a). Subsequently, other events having positional coincidence with Fermi-LAT detected blazars are also observed with lower statistical significance (Garrappa et al. 2019; Franckowiak et al. 2020). Although there are several studies on this object, it is crucial to revisit the spectral properties in order to predict the multi-messenger signals from similar sources.

The explanation of the neutrino event requires synchrotron (SYN) and synchrotron self-Compton (SSC) photons as the target for \(p\gamma\) interactions (Gao et al. 2019; Cerruti et al. 2019; Sahu et al. 2020). Whereas, other models require an external photon field, resulting in external inverse-Compton (IC) emission (Reimer et al. 2019; Keivani et al. 2018; Rodrigues et al. 2019; Petropoulou et al. 2020). In the hadronuclear interpretation via \(pp\) interaction, the shock-accelerated protons may interact with gas clouds in the vicinity of the supermassive black hole (Liu et al. 2019) or cold protons in the jet (Banik & Bhadra 2019). Some studies invoke neutrino production from the interaction of relativistic neutron beams in the jet, originating in \(p\gamma\) interactions with external photons (see, e.g., Zhang et al. 2020). In Fraija et al. (2020), \(p\gamma\) interactions occur with seed photons produced by the annihilation of electron-positron pairs from the accretion disk.

The cosmic-ray-induced cascade from Bethe-Heitler (BH) pair production contributes near the X-ray energies in the \(p\gamma\) scenario, thus also limiting the hadronic component in GeV-TeV γ-rays. As a result, this constrains the astrophysical neutrinos, in many cases, to a flux level lower than that predicted by IceCube observations. Thus, an additional photon field of energy \(\epsilon \approx m_e c^2/20E_\nu \approx 440\text{ eV}\) (i.e., in the UV to soft X-ray energy band) is required. However, an “orphan” neutrino flare...
from this source from September 2014 to March 2015 is revealed from the analysis of archival data, at 3.5σ statistical significance (IceCube Collaboration 2018b) with 13 ± 5 signal events above the atmospheric background. This flare was not accompanied by increased activity in γ-rays, indicating that different astrophysical processes may dominate for neutrino and γ-ray flares. Often, a two-zone model is employed, considering a high opacity for GeV γ-rays in the neutrino production region (Sahakyan 2018; Xue et al. 2019, 2021).

The MAGIC collaboration recently modeled the spectrum of TXS 0506+056, observed during a multiwavelength campaign lasting 16 months from November 2017 to February 2019, covering the radio band, optical/UV, high-energy, and very high-energy (VHE; $E > 100$ GeV) γ-rays (Acilleri et al. 2022). A γ-ray flaring activity was observed by MAGIC during December 2018. Fermi-LAT observed several short flares on timescales of days to weeks, unlike the long-term flare of 2017. At lower energies, no significant variability was observed. The observed flare was not associated with any neutrino event. Their model infers the neutrino luminosity to be lower than the detection threshold of currently operating instruments.

We analyzed the multiwavelength spectral energy distribution (SED) using a one-zone leptohadronic model. The low-energy peak results from the SYN radiation of relativistic electrons. The high-energy peak is produced by SSC and IC scattering of external photons, also called external Compton (EC) emission. These external photons may originate from the broad-line region (BLR). Although broad-line emission in the optical low-energy peak results from the SYN radiation of relativistic distribution (SED) using a one-zone leptohadronic model. The

in our study essentially a one-zone, model, with all the jet parameters constrained by the multiwavelength SED. These jet parameters are used to calculate the luminosity in cosmic rays required inside the jet for SED and neutrino modeling. However, the parameter space is degenerate and hence adjusted to maximize the neutrino production inside the jet. Using the same normalization of the proton spectrum required for this luminosity, we also calculated the luminosity in escaping UHECRs. Hence, it is essentially the same proton spectrum, with the same normalization. However, we assume at energies $\geq 0.1$ EeV that they escape the source because the observed SED does not allow for UHECR interactions inside the source and the simultaneous explanation of quiescent state neutrino flux.

We present the methods of our one-zone modeling in Sect. 2. In Sect. 3.1 we present the results for leptohadronic emissions inside the jet. In Sect. 3.2 we show the contribution of line-of-sight resolved cosmogenic γ-ray contribution to the SED and the subsequent constraints on the EGMF strength. We discuss our results and draw our conclusions in Sect. 4.

2. Radiative modeling

We consider the emission region in the jet to be a spherical blob of radius $R'$, consisting of a relativistic plasma of electrons and protons moving through a uniform magnetic field $B$. The bulk Lorentz factor of the jet is $\Gamma$ and the doppler factor is given by $\delta_D = (\Gamma(1 - \beta \cos \theta))^{-1}$, where $\beta c$ is the velocity of the emission and $\theta$ is the viewing angle. For $\theta < 1/\Gamma$, $\Gamma \approx \delta_D$. We inject electrons in the blob with a spectrum

$$Q'(\gamma'_e) = A_e(\gamma'_e/\gamma_0)^{-\alpha_0}\log^{\delta}(\gamma'_e/\gamma_0)$$

for $\gamma'_e, min < \gamma'_e < \gamma'_e, max$ (1)

to fit the observed broadband SED. The normalization of the spectrum $A_e$ depends on the luminosity of injected electrons, and $\gamma_0m.ec^2$ is a reference energy fixed at 500 MeV. A quasi-steady state is reached when the injection is balanced by radiative cooling and/or escape. Empirically, the steady-state electron density distribution is given as $N'_e(\gamma'_e) = \xi e^{b_{\gamma}}$, where $t'_e = \min\{t'_e, cool\}$.

We consider the escape timescale $t'_e = \frac{2R'}{c}$. The radiative cooling timescale is given as

$$t'_e, cool = \frac{3mc}{4(u'_B + \kappa_{KN}u'_p)\sigma_{T\gamma}\gamma'_e}$$

(2)

where $u'_B = B^2/8\pi$ is the energy density in magnetic field, $u'_p$ is the energy density of soft photons, $\sigma_{T\gamma}$ is the Thomson scattering cross section, and $\kappa_{KN}$ accounts for the suppression of IC emission due to the Klein-Nishina effect. We use the open-source code GAMERA to solve the transport equation for obtaining the injection spectrum at time $t'$ (Hahn 2016)

$$\frac{\partial N'_e}{\partial t} = \xi e^{b_{\gamma}} - \frac{\partial}{\partial \gamma_e}(bN'_e) - \frac{N'_e}{t'_e, esc},$$

(3)

where $b = b_{\gamma}$ is the energy loss rate of electrons.

The steady-state electron spectrum yields the SYN and SSC emission. In addition, we consider an external photon field, which is Compton upscattered by the same electrons. It is considered to be a blackbody with temperature $T'$ and energy density $u'_{ext} = (4/3)kT'\rho_{ext}$ in the jet frame, where the energy density in the AGN frame is $u_{ext} = u_{ext}R_{disk}/4\pi R_{disk}^2c$ and $\rho_{ext}$ is the fraction of the disk luminosity. Here $R_{ext}$ is the radius of the region.
containing the external photons. The emission blob is assumed to be at this distance along the axis of the jet. These photons can enter the relativistic jet and become doppler boosted in the comoving frame.

The steady-state proton injection spectrum is given by a power law \( N_p(\gamma_p) = A_p \gamma_p^{-\alpha_p} \). The main energy loss processes of the protons are pion production \((p\gamma \rightarrow p + \pi^0 + n + \pi^0)\) and BH process \((p\gamma \rightarrow p + e^+ - e^-)\). The seed photons are the leptonic emission and external photons. The charged pions decay to produce neutrinos. The timescale of these interactions can be expressed as

\[
\frac{1}{\tau_p} = \frac{c}{2\gamma_p^2} \int_{E_{0\gamma}}^{\infty} \frac{d\epsilon'}{\epsilon'} \int_{\epsilon_0}^{\epsilon_1} \frac{\epsilon^2_{\text{e}}}{\epsilon^2_{\text{p}}} d\epsilon \sigma(\epsilon) K(\epsilon) f_{\text{e}}(\epsilon, f_{\text{p}}),
\]

where \(\sigma(\epsilon)\) and \(K(\epsilon)\) are respectively the cross section and inelasticity of photopion production or BH pair production as a function of photon energy \(\epsilon\) in the proton rest frame, and \(n(\epsilon')\) is the target proton number density (Stecker 1968; Chodorowski et al. 1992; Mücke et al. 2000). The interaction timescale of protons inside the jet is many orders of magnitude greater than the dynamical timescale, below tens of PeV energies. Hence, in order to increase the efficiency of \(p\gamma\) interactions required for appreciable neutrino production, it is compelling to ignore their escape if the Eddington luminosity budget is maintained. Otherwise, the production of the same neutrino flux will require higher kinetic power in protons if the escape rate is comparable to the \(p\gamma\) interaction rate. Hence, an escape timescale greater than the \(p\gamma\) interaction timescale is assumed. The normalization \(A_p\) of the proton spectrum is calculated from the luminosity requirement arising from the in-jet hadronic contribution to the SED.

The spectrum of \(\gamma\)-rays from the decay of neutral pions and the spectrum of \(e^+ - e^-\) due to the BH process are calculated using the parameterization by Kelner & Aharonian (2008). When calculating the pion decay gamma rays and electron spectra, the input proton spectrum is weighted by the rate of the corresponding process; for example, in the case of electron spectrum from the BH process, we inject \(N_p(\gamma_p^{\prime}) \times R_{\text{BH}}/R_{\text{tot}}\), where \(R_{\text{BH}}\) is the BH interaction rate (calculated using Eq. (4)). The quantity \(R_{\text{tot}}\) is the total interaction rate considering photopion production and BH pair production. The high-energy \(\gamma\)-rays are absorbed by \(\gamma\gamma \rightarrow e^+ e^-\) pair production with the leptonic radiation and also with the external blackbody radiation, leading to the attenuation of TeV \(\gamma\)-rays. The escaping \(\gamma\)-ray flux is given as

\[
Q_{\gamma,\text{esc}}(\gamma_p^{\prime}) = Q_{\gamma,\text{BH}}(\gamma_p^{\prime}) \left[ 1 - \exp\left(\frac{-\tau_{\gamma\gamma}}{\tau_p}\right) \right].
\]

We calculate \(\tau_{\gamma\gamma}\) using the formalism given by Gould & Schréder (1967) to calculate the absorption probability per unit path length for an isotropic photon field

\[
\tau_{\gamma\gamma}(\epsilon') = \frac{1}{2} \int \int n(\epsilon') \sigma_{\gamma\gamma}(\epsilon', \epsilon'_b, \theta) (1 - \cos \theta) \sin \theta d\theta d\epsilon'_b,
\]

where \(\sigma_{\gamma\gamma}\) is the full pair-production cross section and the \(n(\epsilon')\) is the combined density of soft photons and external radiation.

The high-energy electrons and positrons produced in \(\gamma\gamma\) pair production \(Q_{e,\gamma\gamma}\), charged pion decay \(Q_{e,\gamma}^{\prime}\), and BH process \(Q_{e,BH}\) can initiate cascade radiation from the jet. We solve the steady-state spectrum of secondary electrons \(N_{\text{esc}}(\gamma'_e)\) in the jet frame using the analytical approach of Boettcher et al. (2013), including \(Q_{e,BH}'\) in the source term and the escape term to be the same as primary electrons. In a synchrotron-dominated cascade, emission from secondary electrons is given by

\[
Q_e(\epsilon_e') = A_0 \epsilon_e'^{-3/2} \int_1^{\infty} \frac{d\gamma_e'}{\gamma_e'} N_{e,\text{esc}}(\gamma_e'}) \epsilon_e' [\epsilon_e'/\epsilon_e^2]^{1/2} \epsilon_e'^{-5/2} e^{-\epsilon_e'/\gamma_e^2}.
\]

with \(A_0 = c \sigma_T B^2 / [16 \pi m_e Z^2 (4/3)^{3/2}]\) being a normalization constant, where \(b = B'/B_{\text{crit}}\) and \(B_{\text{crit}} = 4.4 \times 10^{13} \text{ G}\).

Our results in Sect. 3.1 suggest that the proton spectrum inside the source is required to be cut off beyond a specific energy to explain the multiwavelength SED. The resulting neutrino flux is thus limited by this value of \(E_{\nu,\text{max}}\). We model the protons to escape beyond this energy if accelerated inside their source. An energy-independent escape timescale of the order \(\sim R/c\) is sufficient for the escape to dominate over photohadronic interactions inside the jet. However, considering a diffusion faster than \(\sim E^1\) leads to negligible interaction efficiency beyond tens of PeV energies in the comoving jet frame. This is reasonable when a quasi-ballistic propagation is assumed instead of diffusive propagation inside the jet emission region.

The resulting muon neutrino flux from \(p\gamma\) interactions is calculated as

\[
E^2_{\nu} J_{\nu} = \frac{1}{3} \frac{V^2_{\nu}}{4 \pi d_L^2} E^2 \nu_{\gamma} Q_{\nu,\gamma\gamma},
\]

where the factor 1/3 corresponds to neutrino oscillation and \(Q_{\nu,\gamma\gamma}\) is the total electron and muon neutrino flux from charged pion decay in the comoving frame.

3. Results

3.1. Leptohadronic emission inside the jet

During the multiwavelength campaign, the source was monitored using the Swift-XRT, Swift-UVOT, and NuSTAR, maximizing the simultaneity of observation with the MAGIC telescope (Acciari et al. 2022). From November 2017 to February 2019, a total of \(-79\) h of good-quality data were collected by MAGIC. During most of this period (\(-74\) h), the source was found to be in a low state with average photon flux \(F(>90\text{ GeV})=(2.7 \pm 2.1) \times 10^{-11} \text{ erg cm}^{-2} \text{s}^{-1}\). Fermi-LAT data was also used in the spectral analysis. For the flaring state of December 2018, only simultaneous data is obtained by Fermi-LAT and MAGIC in the GeV and VHE \(\gamma\)-rays, and by ASAS-SN in the optical. The integral photon flux observed by MAGIC rose by an order of magnitude compared to the low state. The most significant variability was observed in the GeV band, while the X-ray variability was found to be at a lower level. The radio, optical, and UV bands showed moderate variability (Acciari et al. 2022).

The \(\gamma\)-ray variability timescale of TXS 0506+056 observed in October 2017 was shown to be \(t_{\text{var}} \leq 10^5 \text{ s}\) (Keivani et al. 2018). The \(\gamma\)-ray flare of December 2018 was found to be very similar. The size of the emission region inferred from the variability is \(R' \leq \delta_{\text{pc}} c t_{\text{var}} (1 + z) = 6.75 \times 10^{15} (\delta_{\text{pc}}/30) t_{\text{var}}/10^5 \text{ s}\) cm. We assume the radius of the emission region to be \(10^{16} \text{ cm}\) and \(\Gamma \approx \delta_{\text{pc}}\) during the high and the low states. The value of the magnetic field was fine-tuned by fitting the optical and gamma-ray data. It is also assumed to be the same in the two states, \(B' = 0.28 \text{ G}\). The muons and pions produced in hadronic interactions do not suffer significant energy losses before decaying, for this value of \(B'\).

The luminosity distance of TXS 0506+056 is \(d_L \approx 1837 \text{ Mpc}\), with a redshift of \(z = 0.3365\). The total kinetic power of the jet in the AGN frame is calculated as \(L_{\text{kin}} = L_\gamma + L_\nu + L_B = \ldots\)
\( \pi R^2 \Gamma^2 c (u'_e + u'_p + u'_\gamma) \), where \( u'_e, u'_p, \) and \( u'_\gamma \) are the energy densities of electrons, protons, and magnetic field, respectively. The maximum electron energy changes from \( \gamma_{e,\text{max}} \approx 2 \times 10^5 \) in the low state to \( \gamma_{e,\text{max}} = 5 \times 10^4 \) in the high state to account for the spectral variability. We vary the maximum proton energy \( (E_{p,\text{max}}) \), in the comoving jet frame over a wide range to find the best-fit value of 6.3 PeV, fixed for both the low and high states. The cascade emission from the steady-state secondary electron spectrum \( N_{e,\text{sec}} \) is shown by the red lines in Fig. 1. The low-energy peak of the cascade emission originates from the secondary emission of \( e^+ \) pair produced in BH process, which is severely constrained by the X-ray data. As a result, the pion decay cascade at higher energies is also limited, and the contribution to the high-energy peak is not significant.

We obtain \( T^* = 2 \times 10^5 \) K and \( u_{e,\text{sec}} = 0.01 \) erg cm\(^{-3} \) for the external photon field from fitting the SED, which is the most important target of \( p\gamma \) interaction for neutrino production and for IC scattering, crucial for explaining the VHE spectrum. It is also vital for \( \gamma\gamma \) absorption in the jet, beyond a few hundred GeV. For a typical disk luminosity \( L_{\text{disk}} \approx 10^{46} \) erg s\(^{-1} \) and the scattered disk emission to be a fraction \( \eta_{\text{disk}} \approx 0.01 \) of the disk photon energy density, \( R_{\text{ext}} \) is found to be a few times \( 10^{18} \) cm.

The VHE flare of December 2018 does not have simultaneous observations at lower energies, and thus the constraints on the theoretical model are moderate. Nevertheless, to reduce the uncertainties, only the electron primary distribution and its corresponding luminosity is changed with respect to the low state. The parameter values used in the modeling are given in Table 1.

We fit the low-state spectrum first, and optimize the parameters \( \delta_D, B', \) and spectral indices, using the leptonic emission alone. The SYN spectrum peaks at the optical band and a log-parabola injection spectrum of electrons explains the data well. The hadronic component is then added and the power and maximum proton energy are varied to fit the X-ray data with a BH cascade. The VHE photon flux upper limits constrain the contribution from the pion-decay cascade. We also consider the absorption of VHE \( \gamma\) rays in the extragalactic background light (EBL) using the Gilmore et al. model (Gilmore et al. 2012). It can be seen from the left panel of Fig. 1 that the neutrino flux is comparable to the 7.5-year averaged flux prediction from this source by IceCube. We find the neutrino flux to be roughly unchanged in modeling the low- and high-state SEDs obtained by the MAGIC multiwavelength campaign. This corresponds to a flux that produces on average one neutrino detection (as for IC-170922A) over a period of 7.5 years and explains the non-observation of neutrino events during December 2018 flare in the MAGIC waveband. A further increase in the neutrino flux leads to the violation of the X-ray data.

### 3.2. Cosmogenic \( \gamma \) -rays from UHECRs

The maximum proton energy for photopion interactions in the AGN frame is \( E_{p,\text{max}} = \Gamma E'_{p,\text{max}} \approx 0.17 \) EeV from modeling multiwavelength SEDs of TXS 0506+056. The proton spectrum has to be cut off at \( \mathcal{O} \sim 0.1 \) EeV inside the jet to

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**Table 1. Model parameters for the multiwavelength SED, indicating the electron and proton luminosities in the AGN rest frame.**

| Parameters | Low state | High state |
|-----------|-----------|------------|
| \( \delta_D \) | 28 \( \approx \) | 28 \( \approx \) |
| \( B' \) [G] | 0.28 \( \approx \) | 0.28 \( \approx \) |
| \( R' \) [cm] | \( 10^{16} \) \( \approx \) | \( 10^{16} \) \( \approx \) |
| \( u'_{e,\text{sec}} \) [erg/cm\(^3\)] | 0.01 \( \approx \) | 0.01 \( \approx \) |
| \( T' \) [K] | \( 2 \times 10^5 \approx \) | \( 2 \times 10^5 \approx \) |
| \( \alpha (e/p \) spectral index) | 2.0 \( \approx \) | 2.0 \( \approx \) |
| \( \beta (\log \text{parabola index}) \) | 0.3 \( \approx \) | 0.3 \( \approx \) |
| \( E_0 \) [MeV] | 500 \( \approx \) | 500 \( \approx \) |
| \( E_{e,\text{min}} \) [GeV] | 0.20 \( \approx \) | 0.25 \( \approx \) |
| \( E_{e,\text{max}} \) [GeV] | 10 \( \approx \) | 25 \( \approx \) |
| \( L_e \) [erg s\(^{-1}\)] | \( 5.8 \times 10^{44} \approx \) | \( 7.6 \times 10^{44} \approx \) |
| \( E_{p,\text{min}} \) [GeV] | 10 \( \approx \) | 10 \( \approx \) |
| \( E_{p,\text{max}} \) [PeV] | 6.3 \( \approx \) | 6.3 \( \approx \) |
| \( L_p \) [erg s\(^{-1}\)] | \( 1.6 \times 10^{41} \approx \) | \( 1.6 \times 10^{41} \approx \) |
satisfy the constraints from the X-ray data and to simultaneously produce neutrinos with a flux in the PeV range inferred from the detection of one event in 7.5 yr of IceCube operation. According to the Hillas condition, the maximum acceleration energy \( E_{\text{acc}}^{\text{max}} \approx 2\zeta_c E_{\gamma} B R \), where the bulk Lorentz factor \( \Gamma \) takes into account the frame transformation from comoving jet frame to AGN frame. The gyration radius of \( 10^{20} \text{eV} \) protons from this simplistic expression can be calculated as \( r_L \approx 2.13 \times 10^{18} \text{cm} \), which is comparable to the blob radius in our modeling. Thus, it is possible to produce protons of energy higher than \( 0.17 \text{EeV} \) in the same blob for the magnetic field and the length scale considered. The \( \rho \gamma \) opacity in the jet is higher than \( 10^5 \) for protons with energy \( >6 \times 10^{17} \text{eV} \) (see, e.g., Das et al. 2022). The jet is opaque to photons beyond \( 30 \text{TeV} \), the angular resolution of IceCube -ray detector (IceCube Collaboration 2021). The black and red lines show the 5 times higher than the currently operating IceCube detector (IceCube Collaboration 2021). The survival fraction \( \xi_B \) as a function of RMS field strength is shown in Fig. 2.

The value of \( B_{\text{ms}} \) can be constrained from the required luminosity in cosmogenic \( \gamma \)-rays by the following expression, under isotropic approximation

\[
L_{\text{UHE} \gamma} \equiv \frac{L}{4\pi d_L^2} \approx \frac{1}{\xi_B f_{\gamma,p}} \int_{E_{\gamma,\text{min}}}^{\bar{E}_{\gamma,\text{rms}}} \frac{d\nu}{d\epsilon_{\gamma}} d\epsilon_{\gamma},
\]

where \( d\nu/d\epsilon_{\gamma} d\epsilon_{\gamma} \) is the differential flux of cosmogenic \( \gamma \)-rays constrained by the SED. We calculate the electromagnetic cascade using the external code DINT integrated with CRPropa-3 (Heiter et al. 2018). The factor \( f_{\gamma,p} \) takes into account the fraction of injected UHECR power that goes into cosmogenic \( \gamma \)-rays and is fairly constant with the variation of \( B_{\text{ms}} \). For TXS 0506+056 we find \( f_{\gamma,p} \approx 0.156 \). The integrated flux of cosmogenic photons, along the line of sight of the observer, allowed by the observed SED is \( \sim 1 \times 10^{-15} \text{erg cm}^{-2} \text{s}^{-1} \), as seen in Fig. 1. The luminosity of protons interacting inside the jet is found to be \( L_{\rho} = 1.6 \times 10^{48} \text{erg s}^{-1} \) (Table 1). Using the same normalization for the escaping proton spectrum beyond \( 0.1 \text{EeV} \), the luminosity in UHECR photons (i.e., in the range 0.1–100 \text{EeV} \) is \( L_{B_{\text{ms}}} \approx 8 \times 10^{47} \text{erg s}^{-1} \). According to Eq. (9), for the allowed flux of cosmogenic \( \gamma \)-rays, this implies \( \xi_B \leq 0.05 \). This indicates an EGMF with an RMS value higher than a few times \( 10^{-2} \text{nG} \), as shown in Fig. 2. There may be no cosmogenic component along the line of sight for magnetic field strength much greater than this, otherwise, the luminosity budget is violated. It should be noted that this result is an order-of-magnitude estimate. The precise value is sensitive to the angular resolution to detect high-energy \( \gamma \)-rays, the coherence length of EGMF, the angular spread of jet emission, and the numerical precision of cascade calculation.

We calculate the flux of cosmogenic neutrinos produced simultaneously during the propagation of UHECRs and use the same normalization as obtained for the allowed flux of cosmogenic \( \gamma \)-ray spectrum. The cosmogenic neutrino spectrum peaks at \( 2.8 \times 10^{18} \text{eV} \) with peak flux \( \sim 4.5 \times 10^{-13} \text{cm}^{-2} \text{s}^{-1} \), shown in Fig. 3. The IceCube-Gen2 detector will be capable of detecting neutrinos from TeV to EeV energies, with sensitivity five times higher than the currently operating IceCube experiment. We also show the 5\( \sigma \) sensitivity for detection of muon neutrino flux from TXS 0506+056, using the IceCube-Gen2 detector (IceCube Collaboration 2021). The black and red lines correspond to 100 days and 10 years of observation and indicate the sensitivity for neutrino flares and the time-averaged neutrino

Fig. 2. Survival fraction of UHECRs along the line of sight, i.e., within 0.1° of the initial direction of propagation, as a function of the RMS strength of EGMF.
Fig. 3. All-flavor cosmogenic neutrino flux from TXS 0506+056 due to UHECR propagation along the line of sight, i.e., using the same normalization as the cosmogenic γ-ray spectrum in Fig. 1. The black and red curves correspond to 100 days and 10 years of observations of muon neutrino fluxes by IceCube-Gen2 and indicate the sensitivity for neutrino flares and the time-averaged neutrino emission, respectively.

4. Summary and conclusions

The flux variability observed by MAGIC in December 2018 was very similar to that seen in 2017. No neutrino event was detected in 2018, however, in contrast to the 2017 flare. In our one-zone modeling of the multiwavelength data from November 2017 to February 2019, we see that increased γ-ray activity does not yield increased neutrino flux, and the latter is comparable to the 7.5 yr IceCube upper limit. In our study the neutrino event rate is $N_{\nu^+}\Delta t = 3 \times (\Delta T/7.5 \text{ yr})$. Hence, along the lines of the one-zone leptohadronic model adopted here, our results lead to similar conclusions using yet another epoch of the blazar and thus add to the literature.

The lack of simultaneous X-ray data in the high state is a drawback to the SED modeling, although the data points shown are for the nearest observation on 2018 December 8. The parameters are varied minimally from the low state to account for this. Interestingly, our modeling does not predict any significant flare of the optical flux, where the SYN spectrum peaks, but the UV and soft X-ray fluxes are expected to change moderately. We note that a higher X-ray flux can allow for an increased neutrino flux; however, explaining the ΔT = 0.5 yr neutrino flux remains difficult, due to excess X-ray production. Many plausible alternatives exist in the literature, such as neutrino production near the supermassive black hole of the AGN, in the accretion disk, or the corona (Stecker 2013; IceCube Collaboration 2022), or multiple emission zones with increased γ opacity in the neutrino production zone (Xue et al. 2021). Nevertheless, production of one neutrino event in 0.5 yr is achieved in Cerruti et al. (2019) and Fraija et al. (2020), among others, but the neutrino flux peak is shifted compared to the mean energy of the observed event.

The origin of external photons is a question of fundamental importance in the modeling of TXS-like blazars. In their modeling, MAGIC collaboration used an external field originating from the spine layer or the jet sheath (Ansoldi et al. 2018; Acciari et al. 2022). In our analysis, we consider it to originate from the BLR. It provides a substantial target for neutrino production by $p\gamma$ processes and also inverse-Compton scattering by electrons. For this to be true, the radius of the emission region must be smaller than the radius of the BLR. In our analysis, $R_{\text{ext}}$ is a few times $10^{16}$ cm, which is large compared to usual estimates for the BLR, $R_{\text{BLR}} = 10^{17} E_{50}$ cm (see Eq. (4) in Ghisellini & Tavecchio 2008). One possibility is that the blob lies at the edge or even outside the BLR leading to a decrease in the effective BLR photon density (Tavecchio & Ghisellini 2008). The typical energy of the photons in the AGN frame is $\epsilon_{\text{ext}} \approx 3k_B T / \Gamma \approx 17 \text{ eV}$. This is comparable to that obtained in other studies (Keivani et al. 2018) and can also be considered as scattered emission from the disk. The contribution from the disk photon itself is negligible. We consider a log-parabola spectrum for the injection of electrons, to improve the fit to the observed SED. Other assumptions are often made in the literature, such as a broken power-law spectrum Xue et al. (2019).

In our modeling, for photodisintegration interaction rate to dominate over escape, we need an escape timescale greater than $10^6 (R / c) / t_{\text{esc}} \approx 1 \text{ PeV}$ inside the emission region, and even higher at lower energies, assuming an energy-independent escape. In an energy-dependent parameterization, the escape timescale can be expressed as $t_{\text{esc}} > 10^6 (R / c) (E / 10^{17} \text{ TeV})^{-1}$. For the proton energy range interacting inside the jet. In the one-zone model, the efficient escape of UHECRs requires a rigidity-dependent diffusion rate, for example $D(E) \propto E^\alpha$ at higher energies (Globus et al. 2008; Harari et al. 2014; Muzio et al. 2022). As an alternative to the step function for the escape, as we had assumed, one can also assume a separate emission zone for the acceleration of UHE protons with lower photodisintegration opacity. We do not present the analytical estimates of such an
astrophysical scenario in this paper but assume a single proton population. In our analysis, the cosmogenic γ-ray spectrum remains fixed for both the low and high states. A change in the primary proton distribution will not affect the cosmogenic flux significantly because the spectrum is driven greatly by parameters guiding the extragalactic propagation. Since UHECRs are delayed in the EGMF, any observed variability in the VHE regime occurs, most likely due to increased activity inside the jet. The required luminosity in UHE protons can also be translated into a resulting flux of neutrinos at EeV energies. The cosmogenic neutrino flux predicted here, from constraints on γ-ray flux, is found to be lower than that in our earlier study (Das et al. 2022). Thus, detection of cosmogenic neutrinos from TXS 0506+056 seems unlikely with the next generation upgrade of IceCube, leaving ground-based γ-ray detectors such as CTA to test UHECR signature in the SED of blazars.

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