Supercritical charged objects and $e^+e^-$ pair creation

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We investigate the stability and $e^+e^-$ pair creation of supercritical charged superheavy nuclei, udQM nuggets, strangelets, and strangeon nuggets based on Thomas-Fermi approximation. The model parameters are fixed by reproducing their masses and charge properties reported in earlier publications. It is found that udQM nuggets, strangelets, and strangeon nuggets may be more stable than $^{56}$Fe at $A \gtrsim 315, 5 \times 10^4$, and $1.2 \times 10^8$, respectively. For those stable against neutron emission, the most massive superheavy element has a baryon number $\sim 965$, while udQM nuggets, strangelets, and strangeon nuggets need to have baryon numbers larger than $39$, $433$, and $2.7 \times 10^6$. The $e^+e^-$ pair creation will inevitably start for superheavy nuclei with charge numbers $Z \gtrsim 177, 2 \times 163, 2 \times 192$, and $2 \times 212$. A universal relation $Q/R_e = (m_e - \mu_e)/\alpha$ is obtained at a given electron chemical potential $\mu_e$, where $Q$ is the total charge and $R_e$ the radius of electron cloud. This predicts the maximum charge number by taking $\mu_e = -m_e$. For supercritical charged objects with $\mu_e < -m_e$, the decay rate for $e^+e^-$ pair production is estimated based on the JWKB approximation. It is found that most positrons are emitted at $t \lesssim 10^{-15}$ s, while a long lasting positron emission is observed for large objects with $R \gtrsim 1000$ fm. The emission and annihilation of positrons from supercritical charged objects may be partially responsible for the short $\gamma$-ray burst during the merger of binary compact stars, the $511$ keV continuum emission, as well as the narrow faint emission lines in X-ray spectra from galaxies and galaxy clusters.

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I. INTRODUCTION

The possible existence of objects heavier than the currently known nuclei has been a long-standing and intriguing question. As early as in 1960s, it was suggested that there exist unusually stable superheavy nuclei due to quantum shell effects, i.e., the superheavy island [1–3]. Based on cold and hot fusion reactions, superheavy elements with charge number $Z$ up to $118$ have been synthesized [4–7]. The quest to obtain heavier elements is still ongoing, which is focused both on their properties [8–13] and synthesis mechanism [14–23]. Meanwhile, there exist other possibilities. For example, it was argued that strange quark matter (SQM) comprised of approximately equal numbers of $u$, $d$, and $s$ quarks may be more stable than nuclear matter (NM) [24–26]. This indicates the possible existence of stable SQM objects such as stranelets [27–30], nuclearites [31, 32], meteorlike compact ultradense objects [33], and strange stars [34–36]. Nevertheless, if we consider the dynamical chiral symmetry breaking [37, 38], the stability window of SQM vanishes. An interesting proposition was raised recently suggesting that quark matter comprised of only $u$ and $d$ quarks (udQM) may be more stable [39]. It was shown that the energy per baryon of udQM nuggets become smaller than $930$ MeV at $A \gtrsim 300$ [39], while the properties of nonstrange quark stars are still consistent with current pulsar observations [40, 41]. Inspired by various astrophysical observations [42], instead of deconfined quark matter, it was proposed that a solid state comprised of strangeons (quark-clusters with three-light-flavor symmetry) can be the true ground state [43, 44], then small strangeon nuggets could also be stable and persist in the universe [45].

To synthesize these heavy objects with terrestrial experiments is very difficult. The fusion evaporation-residue cross sections in producing superheavy elements with $Z > 118$ are extremely small and synthesizing them requires great efforts [21–23]. The possible production of strangelets via heavy-ion collisions was proposed in the 1980s [46, 47], while up till now no any evidence

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of their existence is obtained \[48, 49\]. Meanwhile, the \(ud\)QMs and strangeon nuggets have not been observed in any of heavy-ion collision experiments either. The situation may be very different in astrophysical environments. Being one of the most dense celestial objects in the universe, pulsars provide natural laboratories for strongly interacting matter (termed simply strong matter) at highest densities. Due to a first-order liquid-gas phase transition at subsaturation densities, nuclear matter could form pasta phase in the inner crust region of a neutron star \[50\], where giant nuclei with \(Z\) up to \(10^3\) are expected \[51, 52\]. Meanwhile, if any of the arguments on SQM, \(ud\)QM, or strangeon matter (SM) is true, pulsars may in fact be strange stars \[34–36, 53–58\], nonstrange quark stars \[40, 41\], or strangeon stars \[42–44\].

The matter inside compact stars can be released during the merger of a binary system by both tidal disruption and squeezing as the stars come into contact \[59, 60\]. With a simple estimation on the balance between the tidal force and surface tension \(\sigma\), the mass of heaviest objects ejected into space is \(M_{\text{max}} \approx 3R_c^3\sigma/GM_c\), where \(R_c\) is the distance to the centre and \(M_c\) the total mass of the binary system. Nevertheless, in such a violent environment, the ejecta is heated and further collisions between those objects are expected, then most of the heavy objects are expected to decay. For example, in the binary neutron star merger event GW170817 \[61\], the ejecta quickly evolves into a standard neutron-rich environment for \(r\)-process nucleosynthesis and produces the transient counterpart AT2017gfo \[60, 62\], which is recently confirmed by the identification of the neutron-decay element strontium \[63\]. For the merger of strange stars, strangelets and strangeon nuggets are then obtained in \(\text{Sec. III}  \) based on the method adopted in our previous publications \[72–75\], and the \(e^+e^-\) pair creations for supercritical charged objects are investigated in \text{Sec. IV}. Our conclusion is given in \text{Sec. V}.

### II. PROPERTIES OF STRONG MATTER

The properties of various types of strong matter can be well approximated by expanding the energy per baryon to the second order, i.e.,

\[
\frac{E_{\text{DM}}}{n_b} = \varepsilon_0 + \frac{K_0}{18} \left( \frac{n_b}{n_0} - 1 \right)^2 + 4\varepsilon_s (f_Z - f_{Z0})^2.
\]

Here \(E_{\text{DM}}\) is the energy density, \(n_b\) the baryon number density, and \(f_Z\) the charge fraction with the charge density \(f_{Z0}\). The parameter \(\varepsilon_0\) is the minimum energy per baryon at saturation density \(n_0\) and charge fraction \(f_{Z0}\), while \(K_0\) is the incompressibility parameter and \(\varepsilon_s\) the symmetry energy. The exact values for those parameters are fixed according to the properties of strong matter obtained based on various studies. In this work, we adopt four representative parameter sets for \(NM, ud\)QM, SQM, and SM, which are summarized in Table I.

The baryon chemical potential \(\mu_b = \frac{\partial E_{\text{DM}}}{\partial n_b}\) and charge chemical potential \(\mu_Q = \frac{1}{n_b} \frac{\partial E_{\text{DM}}}{\partial f_Z}\) of strong matter are obtained with

\[
\mu_b = \varepsilon_0 + \frac{K_0}{18} \left( \frac{3n_b^2}{n_0^3} - 4\frac{n_b}{n_0} + 1 \right) + 4\varepsilon_s (f_{Z0} - f_Z)^2.
\]

Then the pressure is fixed according to the basic thermodynamic relations, i.e.,

\[
P_{\text{DM}} = \mu_b n_b + \mu_Q f_Z n_b - E_{\text{DM}} = \frac{K_0 n_b^2}{9n_0^2} (n_b - n_0).
\]

In nuclear matter, the minimum energy per baryon is obtained at \(f_Z = f_{Z0} = 0.5\) and the saturation density \(n_0 \approx 0.15–0.16 \text{ fm}^{-3}\), where \(\varepsilon_0 = n_N - B\) with the binding energy \(B \approx 16 \text{ MeV}\), the incompressibility \(K_0 = 240 \pm 20 \text{ MeV}\) \[76\], and the symmetry energy \(\varepsilon_s = 31.7 \pm 3.2 \text{ MeV}\) \[77, 78\] are constrained with terrestrial experiments and nuclear theories. In this work, we take their central values with \(n_0 = 0.16 \text{ fm}^{-3}\), \(\varepsilon_0 = 922 \text{ MeV}\), \(K_0 = 240 \text{ MeV}\), and \(\varepsilon_s = 31.7 \text{ MeV}\).

The properties of \(ud\)QM obtained with linear sigma model in Ref. 39 can be well reproduced if we take \(n_0 = 0.22 \text{ fm}^{-3}\), \(\varepsilon_0 = 887 \text{ MeV}\), \(K_0 = 2500 \text{ MeV}\), and \(\varepsilon_s = 17.35 \text{ MeV}\) with \(f_{Z0} = 0.5\). Note that the symmetry energy \(\varepsilon_s\) adopted here is small and contains only the kinetic term. In fact, extensive investigations on the values
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1. Introduction

The study of finite-sized objects in strong matter has gained significant attention in the past few years, with a particular focus on strange matter (SM) and quark matter (QM). These objects, which include strange stars, strangelets, and quark stars, are of great interest in astrophysics and particle physics due to their unique properties and potential for testing fundamental aspects of QCD.

2. Theoretical Framework

To investigate the properties of finite-sized objects, we adopt a unified description that involves the strong matter and the β-stability condition should be fulfilled, i.e.,

\[ \mu_e = -\mu_Q. \]  

3. Finite-Sized Objects

To reach the energy minimum, electrons interact with strong matter and the β-stability condition should be fulfilled, i.e.,

\[ \mu_e = -\mu_Q. \]  

4. Numerical Results

The obtained properties of strangeon matter are then obtained with Table I: The adopted parameter sets in Eq. (1) for nuclear matter (NM) [77, 78], quark matter (udQM) [39], strange quark matter (SQM) [75], and strangeon matter (SM) [85].

| Parameter | NM | udQM | SQM | SM |
|-----------|----|------|-----|----|
| \( n_0 \) | 0.16 | 0.22 | 0.296 | 0.27 |
| \( f_{Z0} \) | 0.5 | 0.5 | 0.1 | 0.0063 |
| \( \varv_0 \) | 922 | 887 | 924.9 | 927.6 |
| \( K_0 \) | 240 | 2500 | 2266 | 4268 |
| \( \varepsilon_s \) | 31.7 | 18.2 | 18.2 | 250 |
| \( \sigma \) | 1.34 | 19.35 | 15 | 100 |

The electron energy density is obtained with

\[ E_e = \int_0^{v_e} \frac{p^2}{\pi} \sqrt{p^2 + m_e^2} dp = \frac{m_e^2}{8\pi^2} (x_e(2x_e + 1) + \sqrt{x_e^2 + 1} - \arcsinh(x_e)). \]  

Here \( x_e \equiv v_e/m_e \) with \( v_e \) being the Fermi momentum of electrons and \( m_e = 0.511 \text{ MeV} \) the electron mass. The number density, chemical potential, and pressure of electron gas are given by

\[ \nu_e = \frac{v_e^2}{3\pi^2}, \]  

\[ \mu_e = \sqrt{\nu_e^2 + m_e^2}, \]  

\[ P_e = \mu_e n_e - E_e. \]

5. Conclusion

The energy density of strangeon matter is then obtained with

\[ E_{SM} = 2U_0 (6.2r_0^2n^5 - 8.4r_0^5n^3) + M_q n, \]  

where \( n = n_b/A_q \) is the number density of strangeons. In this work we take the potential depth \( U_0 = 50 \text{ MeV} \), the range of interaction \( r_0 = 2.63 \text{ fm} \), the baryon number of a strangeon \( A_q = 6 \), and the mass of a strangeon \( M_q = 975A_q \text{ MeV} \). The obtained properties of strangeon stars well reproduce the current constraints on pulsar-like compact objects [86]. The energy density obtained with Eq. (9) around the saturation density can be approximated with Eq. (1) if we take \( n_0 = 0.27 \text{ fm}^{-3} \), \( \varepsilon_0 = 927.6 \text{ MeV} \), \( K_0 = 4268 \text{ MeV} \). Meanwhile, since stable strangeon matter is slightly positively charged due to the larger current mass of s-quarks, we take \( f_{Z0} = 0.0063 \) and \( \varepsilon_s = 250 \text{ MeV} \).

Since the strong matter considered here are positively charged with \( f_{Z0} > 0 \), the contribution of electrons should be considered due to the attractive Coulomb interaction. The electron energy density is obtained with

\[ E_e = \int_0^{v_e} \frac{p^2}{\pi} \sqrt{p^2 + m_e^2} dp = \frac{m_e^2}{8\pi^2} (x_e(2x_e + 1) + \sqrt{x_e^2 + 1} - \arcsinh(x_e)). \]  

Here \( x_e \equiv v_e/m_e \) with \( v_e \) being the Fermi momentum of electrons and \( m_e = 0.511 \text{ MeV} \) the electron mass. The number density, chemical potential, and pressure of electron gas are given by

\[ \nu_e = \frac{v_e^2}{3\pi^2}, \]  

\[ \mu_e = \sqrt{\nu_e^2 + m_e^2}, \]  

\[ P_e = \mu_e n_e - E_e. \]
density profiles are then obtained based on the properties predicted in Sec. II. At a given surface tension value \( \sigma \), the radius of the core \( R \) is fixed according to the dynamic stability of the hadron/quark-vacuum interface, i.e.,

\[
P_{DM}(R) = \frac{2\sigma}{R}.
\]

The mass \( M \), total baryon number \( A \), net charge number \( Z \), and total charge number \( Q \) of the object are determined by

\[
M = \int_0^\infty \left[ 4\pi r^2 E(r) + \frac{r^2}{2a} \left( \frac{d\varphi}{dr} \right)^2 \right] dr + 4\pi R^2\sigma,
\]

\[
A = \int_0^R 4\pi r^2 n_b(r)dr,
\]

\[
Z = \int_0^R 4\pi r^2 f_Z(r)n_b(r)dr,
\]

\[
Q = \int_0^\infty 4\pi r^2 n_{ch}(r)dr.
\]

Note that the local energy density is obtained with \( E = E_{DM} + E_e \) and charge density \( n_{ch} = f_Z n_b - n_e \) at \( r \leq R \), while the region at \( r > R \) is occupied by electrons with \( E = E_e \) and \( n_{ch} = -n_e \). The energy densities for strong matter \( E_{DM} \) and electrons \( E_e \) are obtained with Eqs. (1) and (10), while the electron density is determined by Eq. (11).

Based on the parameter sets indicated in Table I, we can study finite-sized objects comprised of NM, udQM, SQM, and SM, i.e., finite nuclei, udQM nuggets, strangelets, and strangeon nuggets. The energy correction from the surface can be well approximated with a surface tension \( \sigma \). For finite nuclei, to reproduce the masses of known atomic nuclei \([87–89]\), we take \( \sigma = 3.4 \text{ MeV/fm}^2 \). The surface tension value for udQM nuggets is indicated in Ref. [39] with \( \sigma = 19.35 \text{ MeV/fm}^2 \). For strangelets, it was shown that the curvature term is important for small strangelets \([90]\). However, small strangelets are unstable according to our previous calculation \([75]\), we thus neglect the the curvature term and take \( \sigma = 15 \text{ MeV/fm}^2 \), which well reproduce the strangelets’ masses at \( A \geq 200 \).

The surface tension value \( \sigma \) for strangeon nuggets is undetermined and should be fixed based on the interaction between strangeons \([91]\).

In this work, however, we take a reasonable surface tension value \( \sigma = 100 \text{ MeV/fm}^2 \) since strangeon matter is in a solid-state. The adopted surface tension values are summarized in Table I.

At given \( \mu_b \) and \( \mu_e \), Eq. (16) is solved numerically and the density profiles are obtained according to Eq. (15).

The properties of a finite-sized object is then fixed based on Eqs. (17-21). It is found that varying \( \mu_e \) has little impact on the obtained masses of finite-sized objects. To investigate the properties of supercritical charged objects, we thus adopt \( \mu_e = -m_e \) in our calculation.

In Fig. 1 we present the energy per baryon of finite-sized objects fulfilling the \( \beta \)-stability condition. The experimental values for finite nuclei obtained from the 2016 Atomic Mass Evaluation \([87–89]\) are well reproduced in our framework. A minimum value corresponding to \(^{56}\text{Fe}\) is identified with \( M/A = 930 \text{ MeV} \), which is mainly due to the small surface tension of nuclear matter. For other exotic objects such as udQM nuggets, strangelets, and strangeon nuggets, the obtained energy per baryon is decreasing with \( A \) due to the dominant surface energy correction. As indicated in Table II, a critical baryon number \( A_{\text{crit}} \) can then be fixed for those objects, where at \( A > A_{\text{crit}} \) they become more stable than \(^{56}\text{Fe}\), i.e., \( M/A < 930 \text{ MeV} \). Note that the critical baryon number may vary with surface tension. In fact, if a small enough \( \sigma \) is adopted, it was shown there also exists a local energy minimum for strangelets, where strangelets of a certain size are more stable than others \([54, 92, 93]\).

Similar situations may occur for other exotic objects. A crossing between the curves of finite nuclei and udQM nuggets is found at \( A \approx 266 \). In such cases, with the heaviest element \(^{294}\text{Og}\) synthesized by far \([94]\), producing udQM nuggets may be imminent via heavy ion collisions or the decay of superheavy elements if udQM is the true ground state for strong matter.
The stability of those objects against particle emission can be observed through their chemical potentials. In Fig. 2 we present the baryon chemical potential $\mu_b$ as functions of the baryon number $A$. The neutron separation energy is then obtained with $S_n = m_n - \mu_b$, which becomes negative once $\mu_b > m_n$ and spontaneous neutron emission is thus inevitable. The corresponding baryon number ranges for objects that are stable against neutron emission ($S_n > 0$) are listed in Table II. For the emission of charged particles such as protons and $\alpha$ particles, the existence of a Coulomb barrier effectively reduces the rate of emission, which is insignificant comparing with neutron emissions at $S_n > 0$. For superheavy elements with $A < 965$, however, the emission of charged particles as well as spontaneous fission should play important roles on their stability, which is expected to be sensitive to the shell effects and pairing. A more detailed investigation on these aspects is thus necessary, e.g., those in Refs. [8–13, 95, 96].

With a given electron potential $\bar{\mu}_e$, the structures of the core and electron cloud are obtained by solving Eq. (16). The net charge number $Z$ of the core is determined by subtracting the contributions of electrons, while the total charge number $Q$ includes contributions of all charged particles. As was done in our previous studies [72–75], taking $\bar{\mu}_e = m_e$ corresponds to the global charge neutrality condition with $Q = 0$, while here we have adopted $\bar{\mu}_e = -m_e$. In such cases, $Q$ represents the maximum charge number without causing $e^+e^-$ pair creation. The obtained net and maximum charge numbers are presented in Fig. 3. The predicted proton numbers for nuclei coincide with the experimental $\beta$-stability line as indicated with solid squares. For udQM nuggets, the obtained charge numbers are slightly smaller than finite nuclei, which is mainly due to the small symmetry energy adopted here. By taking $f_{20} = 0.1$ and 0.0063 instead of 0.5, the obtained charge numbers for strangelets and strangeon nuggets are much smaller than the two-flavor cases. The net charge-to-mass ratios vary smoothly from $f_{20}$ at $A < 100$ to small values at $A \gtrsim 10^3$, which are presented in Table III. Meanwhile, as was discussed in our previous works [72–75], a constant surface charge density $Q(R)/R^2$ (as indicated in Table III) is obtained at $A \gtrsim 10^5$ if we also consider the contribution of electrons in the core.

Since the single particle levels for electrons are degenerate in spin, a critical charge number $Z_{\text{crit}}$ is obtained at $Z - Q = 2$ according to Fig. 3. The corresponding upper limits of baryon and charge numbers for objects that are stable against $e^+e^-$ pair creation with $Z - Q \leq 2$ are presented in Table II. For objects with larger $Z$, with the critical electric field built around the core, electrons will inevitably appear due to $e^+e^-$ pair creation, which effectively reduces the charge number from $Z$ to $Q$ ($Q < Z$). The corresponding decay rates can be estimated by Eq. (24). Note that the critical charge number for superheavy elements was a long-standing problem and

![FIG. 2: Baryon chemical potential of finite-sized objects as functions of the baryon number A. The mass of a free neutron $m_n$ is indicated with the horizontal line.](image)

![FIG. 3: The net (Z) and maximum (Q) charge numbers of finite-sized objects as functions of the baryon number A, obtained by taking $\bar{\mu}_e = -m_e$.](image)

| TABLE III: The charge properties of maximum charged objects obtained at $\bar{\mu}_e = -m_e$, i.e., the net charge-to-mass ratios $Z/A$, the surface charge density of the core $Q(R)/R^2$ (R in fm), and the ratio of maximum charge number to baryon number $Q/A^{1/3}$. |
|-----------------|-----------------|-----------------|
| $Z/A$ | $Q(R)/R^2$ | $Q/A^{1/3}$ |
| $A \lesssim 100$ | $A \gtrsim 10^3$ | $A \gtrsim 10^5$ |
| finite nuclei | 0.5 | 0.0047 | 1.4 | 0.81 |
| udQM nuggets | 0.5 | 0.0064 | 0.56 | 0.73 |
| strangelets | 0.1 | $4.6 \times 10^{-5}$ | 0.028 | 0.66 |
| strangeon nuggets | 0.0063 | $3.2 \times 10^{-5}$ | 0.020 | 0.68 |
many efforts were made in the past decades. For example, the critical charge number $Z_{\text{crit}} = 137$ is obtained for a pointlike nucleus [97, 98]. For more realistic cases, adopting different radii for finite-sized nuclei predicts various critical charge numbers with $Z_{\text{crit}} = 171-178$ [99-101], while our prediction in Fig. 3 with $Z_{\text{crit}} = 177$ lies within this range. Finally, the maximum charge numbers $Q$ for different types of objects are converging at $A \gtrsim 10^5$ or $R \gtrsim 1000$ fm, where the variations on the core structures become insignificant.

In Fig. 4 we present the radii of the core $R$ and electron cloud $R_e$ obtained at $\tilde{\mu}_e = -m_e$. The boundary of the electron cloud $R_e$ is fixed at vanishing $n_e$, i.e., $\mu_e(R_e) = m_e$. In fact, since there is no electron persists at $r > R_e$, the Coulomb potential is simply $\varphi(r) = \alpha Q/r$. According to Eq. (15), at given $\tilde{\mu}_e$ one obtains the following relation

$$\frac{Q}{R_e} = \frac{m_e - \tilde{\mu}_e}{\alpha}.$$  \hfill (22)

The maximum charge an object can carry without causing $e^+e^-$ pair production can then be obtained by taking $\tilde{\mu}_e = -m_e$, which gives $Q = 0.71R_e$ ($R_e$ in fm) [102]. The radii of electron cloud $R_e$ are thus linked with the maximum charge number $Q$, which is indeed the case according to our numerical calculation. The relation also predicts the trend on the maximum charge numbers with $Q = 0.71R$ (or $Q = 0.71r_0A^{1/3}$ with $r_0^3 = 3/4\pi n_0$) as we increase $A$, which should be valid at $R \gtrsim 10^5$ fm or $A \gtrsim 10^{15}$ with $R$ and $R_e$ being nearly the same. For finite nuclei, as indicated in Fig. 3, adopting $n_0 = 0.16$ fm$^{-3}$ gives $Q = 0.81A^{1/3}$. For other exotic objects, as indicated in Table III, $Q$ is smaller due to larger values for $n_0$.

\section*{IV. $e^+e^-$ PAIR PRODUCTION}

For $e^+e^-$ pair production in the electric field of a positively charged object, an example of the tunneling process is illustrated in Fig. 5. Electrons located in the Dirac sea propagate into the Fermi sea (from $r_-$ to $r_+$), leaving behind a hole at $r_-$, i.e., positrons. The electron chemical potential of the system is $\tilde{\mu}_e \leq -m_e$, with the total charge number $Q$. A potential for electrons is then obtained with $V(r) = -\varphi(r) = -\alpha Q/r$ for $r \gtrsim R_e$. Note that the screening effects of electrons are included in the total charge number, where the charge number without electrons $Z$ is larger than $Q$. The tunneling process is only possible for electrons with energy $\tilde{\mu}_e \leq \varepsilon \leq -m_e$, where the levels at $\varepsilon \leq \tilde{\mu}_e$ are already occupied. According to the Thomas-Fermi approximation, a boundary for electrons is obtained at $r = R_e$ with $\tilde{\mu}_e = V(R_e) + m_e$, beyond which electrons do not exist. The relation between $Q, R_e$, and $\tilde{\mu}_e$ is indicated in Eq. (22), while the maximum charge an object can carry without causing $e^+e^-$ pair production was obtained by taking $\tilde{\mu}_e = -m_e$ [102].

The decay rate of the vacuum for $e^+e^-$ pair production in an arbitrary constant electric field $E$ is given by [66]

$$\Gamma \equiv \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left( -\frac{n\pi E_c}{E} \right),$$ \hfill (23)

where the critical electric field is $E_c = m_e^2/e \sqrt{4\pi \alpha}$.

For a supercritical charged object, the decay rate can then be estimated based on the JWKB approximation [99, 103], i.e.,

$$\Gamma = \frac{1}{\pi} \int_{V(0)+m_e}^{\infty} \sum_{l=0}^{l_{\text{max}}} (2l+1)f(\varepsilon)P_{\text{JWKB}}(\varepsilon, l)d\varepsilon,$$ \hfill (24)

with the electron transmission probability at given en-
FIG. 6: The number of $e^+e^-$ pairs created by objects with $Z = 300$, $1000$, $10^4$, and $10^5$ as functions of time.

energy $\varepsilon$ and angular momentum $l$

$$P_{\text{JWKB}} = \exp \left[ 2 \int_{r_-}^{r_+} \sqrt{\frac{l(l+1)}{r^2} + m_e^2 - \left( \varepsilon + \frac{\alpha Q}{r} \right)^2} \, dr \right].$$

(25)

Here $f(\varepsilon)$ predicts the empty states of electrons. If the $e^+e^-$ pair creation rate is much smaller than the rate of electron thermalization, we can adopt the Fermi-Dirac distribution of electrons and have

$$f(\varepsilon) = 1 - \left[ 1 + \exp \left( \frac{\varepsilon - \bar{\mu}_e}{T} \right) \right]^{-1},$$

(26)

where a lower limit $\bar{\mu}_e$ in the integral of Eq. (24) is obtained for zero temperature cases ($T = 0$) due to the requirement of Pauli exclusive principle, and the maximum angular momentum is given by $l_{\text{max}} = \text{Int} \left( \sqrt{\alpha^2 Q^2 + 1/4} - 1/2 \right)$. The two real turning points $r_\pm$ are obtained by solving

$$\varepsilon + \frac{\alpha Q}{r_{\pm}} = \pm \sqrt{\frac{l(l+1)}{r_{\pm}^2} + m_e^2},$$

(27)

which gives

$$r_{\pm} = -\frac{\alpha Q \varepsilon \pm \sqrt{\alpha^2 Q^2 m_e^2 + l(l+1)(\varepsilon^2 - m_e^2)}}{\varepsilon^2 - m_e^2}.$$  

(28)

Note that the turning points may become smaller than the electron-vacuum boundary ($r_\pm < R_e$) at $l > 0$. The tunneling process for $\varepsilon > \bar{\mu}_e$ is still possible without violating the Pauli exclusive principle. However, the Coulomb potential $V(r) = -\alpha Q/r$ is not valid at $r < R_e$, since the charge number enclosed within the sphere of radius $r$ becomes larger than $Q$ [104]. In such cases, $r_+$ may become slightly larger and the transmission probability $P_{\text{JWKB}}(\varepsilon, l)$ at $l > 0$ increases. In this work, for simplicity, we neglect the variation of the Coulomb potential at
$r < R_c$. The integral in Eq. (25) can then be obtained with
\[ P_{JWKB} = \exp \left[ 2\pi \sqrt{\alpha^2 Q^2 - l(l + 1)} + \frac{2\pi \alpha Q \varepsilon}{\sqrt{\varepsilon^2 - m_e^2}} \right]. \]

By taking $l$ as continuum values, the summation in Eq. (24) can be obtained via integration and gives
\[ \Gamma = \frac{1}{4\pi^2} \int_{\mu_e}^{-m_e} \exp \left( \frac{2\pi \alpha Q \varepsilon}{\sqrt{\varepsilon^2 - m_e^2}} \right) d\varepsilon. \]

Assuming a constant Coulomb potential inside a core of radius $R$ and net charge number $Z$, the electron distributions at given $\bar{\mu}_e$ can be obtained based on Eqs. (15) and (16). Note that for the ultra-relativistic cases of radius $R_b$ at given $\bar{\varepsilon}$, objects are fixed according to the results indicated in Figs. 3 and 4, where $\bar{\mu}_e = -m_e$ was adopted. The $e^+e^-$ pair production rate is predicted by Eq. (30), then the time evolution of $e^+e^-$ pair creation can be obtained.

In Fig. 6 we present our results for supercritical charged nuclei, udQM nuggets, strangelets, and strangeon nuggets with $Z = 300$, 1000, 10$^4$, and 10$^5$. The $e^+e^-$ pair creation is most effective at $t \lesssim 10^{-15}$ s, where most electrons are created and fill in the Fermi sea. At the same time, positrons will leave the system due to Coulomb repulsion. During the merger of binary compact stars, the positron emission due to the release of supercritical charged objects may thus be partially responsible for the short $\gamma$-ray burst [70, 71]. For a fixed net charge number $Z$, more $e^+e^-$ pairs are produced by objects with smaller $R$, where $R$ increases in the order of finite nuclei, udQM nuggets, strangelets, and strangeon nuggets. For the superheavy nuclei 918, 300, to create one $e^+e^-$ pair requires at least few $10^{-22}$ s, while longer duration is expected for smaller $Z$ [106]. Note that at small charge numbers such as $Z = 300$, the pair creation quickly stops at $t \gtrsim 10^{-15}$ s since the Coulomb field is easily screened by electrons. This is not the case for larger objects, where the positron emission tends to last much longer since they possess larger charge numbers.

In fact, a long-lasting positron emission could explain the 511 keV emission from positron annihilation in the Galaxy [67, 68].

For larger objects, as an example, we consider the cases with $R = 1000$ fm, which correspond to the net charge numbers $Z = 3.1 \times 10^7$, $6.4 \times 10^6$, 91698, and 60487 for finite nuclei, udQM nuggets, strangelets, and strangeon nuggets, respectively. The evolutions of the charge numbers $Q$ are presented in Fig. 7, which quickly drop from $Z$ to $14000$ within $10^{-20}$ s. The $e^+e^-$ pair creation still effectively reduces the charge number $Q$ throughout time, i.e., a continued source of positron emission. Comparing with the charge numbers $\bar{Q} = Z - N_{\gamma}$ indicated in Fig. 6, the values obtained here for objects with same radii are close to each other and evolve similarly with time, which is what we have observed in Fig. 3 for objects with $A \gtrsim 10^8$ or $R \gtrsim 1000$ fm. Meanwhile, similar to static cases, the charge number $Q$ increases with $Z$. As was discussed in Fig. 4, a universal relation $Q/R = (m_e - \bar{\mu}_e)/\alpha$ can be obtained based on Eq. (22) for very large objects with $R \gtrsim 10^8$ fm or $A \gtrsim 10^{15}$. By substituting this relation into Eq. (30), the decay rate for objects with $R \gtrsim 10^5$ fm can be determined.

With most $e^+e^-$ pairs created at $t \lesssim 10^{-15}$ s, the maximum number of positrons emitted by supercritical charged objects at $T = 0$ can be obtained with $N_{\gamma} \approx Z - Q$ based on the charge numbers indicated in Fig. 3. In Fig. 8 a rough estimation on the energy release of positron annihilation during the merger of binary
compact mass of ejected objects with baryon number $A$ is $M_{ej} = 0.001 M_{\odot}$. The obtained isotropic energy release for the ejected superheavy nuclei and udQM nuggets are comparable with the estimated value $(3.1 \pm 0.7) \times 10^{46}$ erg of GRB 170817A [70, 71], while smaller values for strangelets and strangeon nuggets are obtained. Nevertheless, we should mention there may be other important energy sources, e.g., the thermonuclear reactions, the thermal radiation such as the outflowing $\nu\bar{\nu}$ [107] and/or $e^+e^-$ [108] fluxes, the decay of strangelets [64] and strangeon nuggets [65], etc. In such cases, the energy release in $\gamma$-rays during the merger of binary strange stars or strangeon nuggets can be attributed to those processes instead of positron emissions from strangelets or strangeon nuggets.

A substantial amount of positrons and supercritical charged objects may finally escape the binary system, which later create the 511 keV continuum emission observed in the Galaxy via positronium decay [67, 68]. In fact, it was shown that the observed positron annihilation mainly comes from the bulge with a large bulge-to-disk ratio around 1.4 [68], which seems to correlate with the distribution of binary systems in the Milky Way. Such kinds of correlations have recently been adopted as tracers of binary neutron star mergers [109]. Meanwhile, before the emission of positrons, the $e^+e^-$ pairs produced around the surfaces of supercritical charged objects would oscillate with alternating electric field for a short time, and emit electromagnetic radiations with a characteristic frequency around 4 keV [69]. We suspect these radiations are actually responsible for the narrow faint emission lines around 3.5, 8.7, 9.4 and 10.1 keV observed in the Milky Way center, nearby galaxies and galaxy clusters [110, 111].

V. CONCLUSION

We study the properties of finite-sized objects that are heavier than the currently known nuclei, i.e., superheavy nuclei, udQM nuggets, strangelets, and strangeon nuggets. The structures of those objects are obtained based on the UDS model [72–75], where the Thomas-Fermi approximation is adopted. The local properties of nuclear matter, ud quark matter, strange quark matter, and strangeon matter are determined by expanding the energy per baryon to the second order, while a surface tension is introduced for the hadron/quark-vacuum interface. The parameters are fixed by reproducing the masses and charge properties of $\beta$-stable nuclei [87–89], udQM nuggets [39], large strangelets [75], and strangeon matter [85].

Comparing with the most stable nucleus $^{56}\text{Fe}$, udQM nuggets, strangelets, and strangeon nuggets are more stable at $A > A_{\text{crit}}$ with $A_{\text{crit}} \approx 315, 5 \times 10^4$, and $1.2 \times 10^8$, respectively. The masses of finite nuclei and udQM nuggets become similar at $A \approx 266$, which increases the possibility in synthesizing udQM nuggets via heavy ion collisions. The stability of those objects is investigated by examining their chemical potentials, where we have obtained a maximum baryon number for superheavy elements with $A_{\text{max}} \approx 965$, and minimum baryon numbers $A_{\text{min}} \approx 39, 433$, and $2.7 \times 10^5$ for udQM nuggets, strangelets, and strangeon nuggets that are stable against neutron emission. The charge properties of those objects are obtained, where the net charge fraction $(Z/A)$ vary smoothly from 0.5, 0.5, 0.1, and 0.0063 ($A \lesssim 100$) to 0.047, 0.0064, $4.6 \times 10^{-5}$, and $3.2 \times 10^{-5}$ ($A \gtrsim 10^9$) for finite nuclei, udQM nuggets, strangelets, and strangeon nuggets, respectively. For objects with large enough net charge numbers $Z \geq Z_{\text{crit}}$, $e^+e^-$ pair creation inevitably starts, where $Z_{\text{crit}} = 163, 177, 192$, and 212 for udQM nuggets $(^{609,163}i)$, finite nuclei $(^{480,177}i)$, strangelets $(^{162,85}i)$, and strangeon nuggets $(^{90,796,212}i)$, respectively. The maximum charge numbers that are stable against $e^+e^-$ pair creation are investigated, which increase with $Z$ and are converging at $A \gtrsim 1000$ fm or $A \gtrsim 10^8$ for different types of objects. A universal relation $Q/R_e = (m_e - \bar{m}_e)/\alpha$ is obtained at given $\bar{m}_e$, where $Q$ the charge and $R_e$ the radius of electron cloud. The maximum charge can be obtained by taking $\bar{m}_e = -m_e$. At $R \gtrsim 10^5$ fm or $A \gtrsim 10^{15}$, $R \approx R_e$ and the universal charge radius relation is obtained with $Q = 0.71R$, which is consistent with those predicted in Ref. [102].

For supercritical charged objects, the decay rate for $e^+e^-$ pair production is estimated based on the JWKB approximation [99, 103]. It is found that most positrons are emitted at $t \lesssim 10^{-15}$ s, which should be partially responsible for the short $\gamma$-ray burst due to the release of supercritical charged objects during the merger of binary compact stars [70, 71]. For the superheavy nucleus $^{510,300}$, to create one $e^+e^-$ pair requires at least $10^{-22}$ s, while longer duration is expected for smaller Z. The $e^+e^-$ pair creation for small objects ($Z = 300$) quickly stops at $t \gtrsim 10^{-15}$ s due to the screening effects of electrons. For larger objects, positron emission last much longer, which may be responsible for the 511 keV emission from positron annihilation in the Galaxy [67, 68] as well as the narrow faint emission lines in X-ray spectra observed in the Milky Way center, nearby galaxies and galaxy clusters [110, 111].

Finally, it is worth mentioning that the temperature of newly created supercritical charged objects may reach up to $\sim 50$ MeV during the merger of a binary system [59]. In such cases, the rate of $e^+e^-$ pair creation becomes much larger since the electronic states with $\varepsilon < \bar{m}_e$ may not be completely occupied as predicted in Eq. (26). The thermal ionization should also be considered, where bound electrons are excited to the continuum of free electron states so that the charge $Q$ of those objects is increased. In fact, the emission of positrons due to $e^+e^-$ pair creation combined with the evaporation of thermalized electrons was shown to create an outflowing plasma of $\sim 10^{51}$ ergs/s on strange stars’ surfaces with $T \approx 10^{11}$ K [108]. Meanwhile, the environment of these objects created dur-
ing the merger of a binary system may be filled with $e^+e^-$ plasma, which could reduce $Q$ by capturing the surrounding electrons. In such cases, to determine the final state of those charged objects, more detailed studies on the evolution of $Q$ with $e^+e^-$ pair creation, thermal ionization, and electron capturing combined with the time evolution of their surrounding environment are necessary, which is intended in our future works.

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