Massless Three Dimensional Quantum Electrodynamics and Thirring Model Constrained by Large Flavor Number.

A.R. Fazio

Departamento de Física, Universidad Nacional de Colombia
Ciudad Universitaria, Bogotá, D.C. Colombia
arfazio@unal.edu.co

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Abstract

We explicitly prove that in three dimensional massless quantum electrodynamics at finite temperature, zero density and large number of flavors the number of infrared degrees of freedom is never larger than the corresponding number of ultraviolet. Such a result, strongly dependent on the asymptotic freedom of the theory, is reversed in three dimensional Thirring model due to the positive derivative of its running coupling constant.
1 Introduction

A crucial question to understand a physical phenomenon is related to the number of degrees of freedom of the physical system under investigation. That task is not always easy because, depending on the scale of observation of the system, degrees of freedom appear different. Some remain coupled, some have to decouple, some are inaccessible to our theoretical and experimental probes. It is nice to have an universally valid constraint on quantum field theories involving the number of the degrees of freedom of the theory.

Some years ago, an inequality between the number of the degrees of freedom of asymptotically free field theories in infrared and ultraviolet regime was proposed in [1] to constrain these theories. The count of these degrees of freedom in ultraviolet ($f_{UV}$) and in infrared ($f_{IR}$) happens through the thermal pressure of the system, following the definition provided later in the paper in (14) and (15). This constraint for asymptotically free field theories appears in form of the inequality

$$f_{IR} \leq f_{UV},$$

(1)
to be reversed if the theory is not asymptotically free. This inequality has not been yet proved for a general asymptotically free field theory, but it is consistent with the known results and it has been used to derive new constraints for several strongly coupled vector-like gauge theories.

In this paper we prove the validity of this inequality for three-dimensional quantum electrodynamics (QED3) with a large number of fermions. This theory is super-renormalizable, UV-complete and rapidly damped at momentum scales beyond its own mass scale. The asymptotic freedom allows for the calculation of $f_{UV}$ like in a gas of non-interacting fermions and photons. The existence of an infrared fixed point relies on the calculations of the quantum corrections of $f_{IR}$, based only on perturbation theory.

An analogous calculation is eventually performed in three-dimensional Thirring model, weakly coupled at energy scales much less than its own typical mass scale and in general strongly coupled at high energy. For large number of fermions the existence of an ultraviolet fixed point allows for the use of perturbation theory to compute quantum corrections of $f_{UV}$ and to reverse the inequality (1).

Let us now suppose to extend the inequality (1) to finite $N$, where a strong coupling dynamics might allow for a spontaneous symmetry breaking of some global symmetry with corresponding dynamical mass generation. The count of degrees of freedom involving now the Goldstone bosons, appearing in the spectrum, might determine a critical number of fermion to generate this mass. Intensive studies have been devoted to this critical value of $N$ in QED3 on the continuum and on the lattice [2]. It seems to be little agreement about such a critical value, due essentially to our rudimentary understanding of most strongly coupled quantum field theories. The constraint (1) could help in this important task.

The paper is organized as follows. In section 2 we analyze the case of quantum electrodynamics in 2+1 dimensions. We prove that with a large number $N$ of fermions this theory is weakly coupled at all momentum scales. In the subsection 2.2 we compute the thermal pressure in the large $N$ limit and we show that our gauge invariant result provides a negative correction to the count of infrared degrees of freedom. Section 3 is devoted to Thirring model in 2+1 dimensions. We compute the running of its dimensionful coupling constant and we find an opposite behaviour to asymptotic freedom. The bosonized equivalent version of this model allows for an easy calculation of the quantum corrections of the pressure and for a
subsequent non-positive correction to $f_{UV}$. Section 4 is for conclusions and perspectives.

2 Three dimensional quantum electrodynamics

2.1 Large $N$ weak coupling at all momentum scale

Let’s consider $N$ massless fermions interacting with photons in the 2 + 1 dimensional Minkowsky space by the model

$$S_{\text{QED}} = \int d^3x \left[ \sum_{j=1}^{N} \bar{\psi}_j (i\hat{\partial} - e\hat{A}) \psi_j - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \right] (2)$$

$$\hat{\partial} = \gamma^\mu (i\partial_\mu - eA_\mu)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The squared of the coupling constant $e$ has dimension of a mass and the interaction is super-renormalizable. We will be working in the large $N$ limit by keeping fixed the mass $\frac{e^2 N}{8}$. It is remarkable that $\psi_j$ is a set of $N$ 4-component fermion fields. This sort of redoubling of the fermions number avoids any Chern-Simon terms in the Lagrangian.

As a first step let’s prove that for large $N$ the theory remains weakly coupled at all momentum scales. In a general covariant gauge the leading quantum correction to the gauge boson propagator of momentum $p$ is

$$N e^2 \left[ \frac{g_{\mu\rho}}{p^2} - (1 - \xi) \frac{p_\mu p_\rho}{p^4} \right] \int d^3 q (2\pi)^3 Tr \left( \frac{1}{q^2} \frac{1}{q - p} \right) \left[ \frac{g_{\sigma\nu}}{p^2} - (1 - \xi) \frac{p_\sigma p_\nu}{p^4} \right] (3)$$

After cutting the external legs and making some $\gamma$ gymnastic one easily obtains

$$4 Ne^2 \int d^3 q (2\pi)^3 \frac{1}{q^2(q - p)^2} (2q^\rho q^\sigma - g^{\rho\sigma} q^2 - q^\rho p^\sigma - p^\rho q^\sigma + g^{\rho\sigma} q \cdot p). (4)$$

The use of Feynman’s parameters implies

$$\frac{1}{q^2(q - p)^2} = \int_0^1 dx \frac{1}{(-q^2 + 2xq \cdot p - xp^2)^2} (5)$$

$$\int \frac{d^3 q}{(2\pi)^3} \frac{g_{\rho\sigma}}{q^2(q - p)^2} = \frac{1}{64\sqrt{p^2}} (3p_\rho p_\sigma - g_{\rho\sigma} p^2) (6)$$

$$\int \frac{d^3 q}{(2\pi)^3} \frac{g_{\rho}}{q^2(q - p)^2} = \frac{p_\rho}{16\sqrt{p^2}}. (7)$$

Collecting all the previous results the photon propagator till to one loop is given, in a general $\xi$-covariant gauge, by

$$- \frac{i}{p^2} \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \left( 1 - \frac{Ne^2}{8\sqrt{-p^2}} \right) - \frac{i}{p^2} \frac{p_\mu p_\nu}{p^4}, (8)$$

which is an exact result to leading order in the $1/N$ expansion, showing that only the transverse gauge independent part is renormalized by interactions.
It is worth to remark that due to the dimensionality of the space-time no infrared divergences are encountered. In spite of superficial ultraviolet divergences, they too are absent. In fact, in (4) the only possible divergent term is
\[ \int \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2}. \] (9)
That is zero, as it is easy to see by using the Feynman parameter \( x \) in
\[ \int \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2} \frac{(p+q)^2}{(p+q)^2} \] (10)
which reduces to
\[ i \frac{\pi^{3/2}}{(2\pi)^3} \Gamma \left( \frac{1}{2} \right) \sqrt{\frac{4}{\pi}} \int_0^1 (x^2 - x)^{-1/2} (4x^2 - 3x) = 0. \] (11)
This correction amounts to extract from the gauge boson propagator \( \delta \) a dimensionless coupling running constant \[ \bar{\alpha}(p^2) = \frac{e^2 \sqrt{-p^2}}{-p^2 + \frac{N_c^2}{8} \sqrt{-p^2}} \] (12)
which in the Euclidean three-dimensional space-time appears as
\[ \bar{\alpha}(p) = \frac{e^2 p}{p^2 + \frac{N_c^2}{8} p}. \] (13)
This expression exhibits asymptotic freedom at large momentum and conformal symmetry with an infrared fixed point of strength \( \frac{8}{N} \) as \( p \) tends to zero. For large \( N \) the coupling is always weak and we assume no dynamical fermion mass generation.

### 2.2 Appelquist, Cohen, Schmaltz’s conjectured inequality

In \( [1] \) and \( [6] \) a constraint on this theory in the large flavor number was conjectured in form of an inequality stating that the number of infrared degrees of freedom \( f_{IR} \), defined using the thermal pressure \( P(T) \) at the absolute temperature \( T = \frac{1}{\beta} \) of the above interacting fermions and photon gas, is never larger than the number of the corresponding ultraviolet degrees of freedom. To be more specific
\[ f_{IR} = \lim_{T \to 0} \frac{P(T)f(d)}{T^d} \] (14)
and
\[ f_{UV} = \lim_{T \to +\infty} \frac{P(T)f(d)}{T^d}, \] (15)
where \( P(T) \) is related to the grand canonical partition function \( Z \) as
\[ P(T) = \frac{T \log Z}{V} \] (16)
and \( f(d) \) is a function of the number of Euclidean space-time dimensions \( d \), defined such that the contribution from a free bosonic field is equal to 1, amounting therefore to:
\[ f(d) = 2^{d-1} \pi^{\frac{d-1}{2}} \Gamma \left( \frac{d+1}{2} \right) \times \frac{1}{\Gamma(d)\zeta(d)}. \] (17)
It is a straightforward exercise to compute the above numbers of degrees of freedom of a gas of non-interacting fermions and non-interacting photons \[7\]. In 2 + 1 dimensions one obtains that the 4\(N\) fermion components are Boltzman-weighted by a \(3/4\) factor.

Due to the above proved asymptotic freedom the result for \(f_{UV}\) is

\[
f_{UV} = \frac{3}{4}(4N) + 1
\]

where we have taken into account that in three space-time dimensions only one degree of freedom per gauge boson field is propagating.

The leading contribution to \(f_{IR}\) is given by \[18], but due to the weak coupling \[13\] it can be corrected using perturbation theory. The lowest order correction to \(f_{IR}\) due to interactions comes from the usual two loop exchanging term contribution to the pressure \(P(T)\) \[16\], which in the formalism of the imaginary time \[8\] is

\[
-\frac{1}{2} NT\bar{\alpha}(T) \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} (2\pi)^2 \delta^2(p - q - \vec{k}) \sum_{n_p,n_q,n_k} \beta \delta_{n_p,n_q+n_k} T^3 \frac{T r(\gamma^\mu \bar{p} \gamma^\mu \bar{q})}{p^2 q^2 k^2}.
\]

(19)

In the analytic expression of this contribution we took into account the \(1/N\) resummation using \(T \bar{\alpha}(T)\) as perturbative parameter and the free gauge boson propagator in \[19\].

By simply dimensional analysis the contribution \[19\] is proportional to \(N \bar{\alpha}(T) T^3\) and, since

\[
\lim_{T \to 0} \bar{\alpha}(T) = \frac{8}{N},
\]

it is expected to be \(O(1)\) in this large \(N\) perturbative expansion. It is easy to check that higher corrections are suppressed like \(1/N\) powers.

The calculation of the above diagram is performed in Feynman gauge, eventually we give an argument in favour of the gauge invariance of our result. The simplicity of the calculations in Feynman gauge, already exploited in four dimensions \[8\], will be recovered in our case, where the task will be simplified due to the absence of nasty subdivergences.

Firstly let us make the sum on the Matsubara frequencies

\[
\sum_{n_p,n_q,n_k} \beta \delta_{n_p,n_q+n_k} T^3 \frac{T r(\gamma^\mu \bar{p} \gamma^\mu \bar{q})}{p^2 q^2 k^2}.
\]

(20)

Due to the periodic and antiperiodic boundary conditions for, respectively, gauge bosons and fermions one can perform an analytic continuation by writing

\[
\beta \delta_{n_p,n_q+n_k} = \int_0^\beta d\theta \exp[\theta(p^0 - q^0 - k^0)]
\]

and multiplying \[20\] by the quantity

\[
-\exp[\beta(k^0 + q^0)]
\]

without making any change. The above Matsubara sum becomes

\[
4 T \Sigma_{n_p} \frac{1}{p^2} T \Sigma_{n_q} \frac{1}{q^2} T \Sigma_{n_k} \frac{1}{k^2} \frac{p \cdot q}{p^0 - q^0 - k^0} \{\exp(\beta p^0) - \exp[\beta(k^0 + q^0)]\}.
\]

(21)
The contour integral result for the $n_p$ summation is

$$T \Sigma_{n_p} I(p^0, q^0, k^0) = \frac{I(E_{p'}, q^0, k^0)}{2E_p} N_F(p) + \frac{I(-E_{p'}, q^0, k^0)}{2E_p} (N_F(p) - 1) \quad (22)$$

where $I(p^0, q^0, k^0)$ is an arbitrary analytical function and

$$E_p = |\vec{p}| \quad N_F(p) = \frac{1}{1 + \exp(\beta E_p)};$$

the $n_k$ summation amounts to

$$T \Sigma_{n_k} I(p^0, q^0, k^0) \frac{k^2}{k^2} = -\frac{I(p^0, q^0, E_k)}{2E_k} N_B(k) - \frac{I(p^0, q^0, -E_k)}{2E_k} (N_B(k) + 1) \quad (23)$$

where

$$E_k = |\vec{k}| \quad N_B(k) = \frac{1}{\exp(\beta E_k) - 1}.$$

Taking in particular

$$I(p^0, q^0, k^0) = \frac{p \cdot q}{p^0 - q^0 - k^0} \{ \exp(\beta p^0) - \exp[\beta (k^0 + q^0)] \} \quad (24)$$

our Matsubara frequencies sum (21) can be written as

$$-\frac{1}{2E_k E_p E_q} \left\{ N_F(p) N_F(q) \left[ I(E_p, E_q, -E_k)(N_B(k) + 1) + I(E_p, E_q, E_k)N_B(k) \right] + N_F(q)(N_F(p) - 1)[I(-E_p, E_q, -E_k)(N_B(k) + 1) + I(-E_p, E_q, E_k)N_B(k)] + (N_F(q) - 1)N_F(p)[I(E_p, -E_q, -E_k)(N_B(k) + 1) + I(E_p, -E_q, E_k)N_B(k)] + (N_F(p) - 1)(N_F(q) - 1)[I(-E_p, -E_q, E_k)(N_B(k) + 1) + I(-E_p, -E_q, E_k)N_B(k)] \right\}. \quad (25)$$

After some algebra this result can be simplified to

$$-\frac{1}{E_k E_p E_q} \left\{ \frac{E_p E_q - \vec{p} \cdot \vec{q}}{E_p - E_q + E_k} (N_F(q) + N_B(k)N_F(q) - N_F(p)N_F(q) - N_B(k)N_F(p)) + \frac{E_p E_q - \vec{p} \cdot \vec{q}}{E_p - E_q - E_k} (N_F(p)N_F(q) + N_B(k)N_F(q) - N_F(p)N_B(k) - N_F(p)) + \frac{E_p E_q + \vec{p} \cdot \vec{q}}{E_p + E_q - E_k} (N_B(k) - N_B(k)N_F(q) - N_F(p)N_F(q) - N_B(k)N_F(p)) + \frac{E_p E_q + \vec{p} \cdot \vec{q}}{E_p + E_q + E_k} (1 + N_B(k) - N_F(q) - N_F(p) - N_B(k)N_F(q) + N_F(p)N_F(q) - N_B(k)N_F(p)) \right\}. \quad (26)$$

Since the set of values of the physical thermal pressure has the algebraic structure of a torsor, the temperature independent vacuum term can be neglected. The linear terms in the occupation number provide a vanishing contribution to the pressure by symmetric integration and using the result (13), which implies

$$\int \frac{d^2 p}{(2\pi)^2} \frac{1}{2E_q} = 0. \quad (27)$$
The final expression for our exchange term (19) is
\[
-3NT\tilde{\alpha}(T) \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} (2\pi)^2 \delta^2(\vec{p} - \vec{q} - \vec{k}) \frac{N_F(p) N_F(q)}{E_p E_q} \tag{28}
\]
and since
\[
\int \frac{d^2q}{(2\pi)^2} \frac{N_F(p)}{E_p} = \frac{1}{2\pi} \int_0^{+\infty} dx \frac{1}{e^{\beta x} + 1} = \frac{\log 2}{2\pi \beta} \tag{29}
\]
the one-loop corrected \( f_{IR} \) is
\[
f_{IR} = 3N + 1 - \frac{12 \log^2 2}{\pi \zeta(3)} \tag{30}
\]
confirming the conjectured inequality
\[
f_{IR} \leq f_{UV} \tag{31}
\]
in Feynman gauge.

Since this result is based on the calculation of a physical observable as the thermal pressure, one can be convinced about its gauge independence, also if it is not so easy to provide an explicit proof about that. Let’s say that in non-covariant gauges as the Coulomb and temporal gauge the pressure was computed for QCD in 3 + 1 dimensions in [9] obtaining the same result as in Feynman gauge, therefore an analogous invariance is expected for QED3.

In a general \( \xi \) covariant gauge one should add to the simple expression (19) the term
\[
\frac{(1 - \xi)}{2} NT\tilde{\alpha}(T) \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} (2\pi)^2 \delta^2(\vec{p} - \vec{q} - \vec{k}) \times \\
\sum_{n_p,n_q,n_k} \beta \delta_{n_p,n_q+n_k} T^3 \frac{1}{p^2 q^2 k^4} Tr(\hat{k} \hat{p} \hat{k} \hat{q}). \tag{32}
\]
The calculation of (32) proceeds in the same way as in Feynman gauge, although the contour integral method involves now a double pole residue, providing the Matsubara frequency sum on \( n_k \)
\[
\frac{1}{\beta} \sum_{n_k} \frac{1}{(k^2)^2} I(p^0, q^0, k^0) = - \frac{N_B(k)}{4E_k^2} \frac{\partial I}{\partial k^0}(p^0, q^0, E_k) + \frac{N_B(k) + 1}{4E_k^2} \frac{\partial I}{\partial k^0}(p^0, q^0, -E_k) \\
+ \beta \frac{(N_B(k) + N_B(k)^2)}{4E_k^2} I(p^0, q^0, E_k) + \beta \frac{(N_B(k) + N_B(k)^2)}{4E_k^2} I(p^0, q^0, -E_k) \\
+ \frac{N_B(k)}{4E_k^3} I(p^0, q^0, E_k) + \frac{N_B(k)}{4E_k^3} I(p^0, q^0, -E_k). \tag{33}
\]

However the term (32) doesn’t contribute to the count of infrared degrees of freedom of the three dimensional quantum electrodynamics, as it can be seen by the following simple argument [10]. Let’ write (32) as
\[
\frac{(1 - \xi)}{2} NT\tilde{\alpha}(T) \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} (2\pi)^2 \delta^2(\vec{p} - \vec{q} - \vec{k}) \times \\
T^3 \sum_{n_p,n_q,n_k} \beta \delta_{n_p,n_q+n_k} \frac{1}{k^4} Tr\left(\frac{\hat{k}}{\hat{p}} \frac{\hat{1}}{\hat{q}} \frac{\hat{1}}{\hat{q}}\right). \tag{34}
\]
Integration on $\vec{p}$ and summation on $n_p$ reduce this integral to

$$\frac{(1 - \xi)}{2} NT \tilde{\alpha}(T) \int \frac{d^2 q}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} T^2 \sum_{n_q, n_k} \frac{1}{k^4} Tr \left( \frac{1}{k + q} \frac{1}{\hat{k} \hat{k}} \right),$$

but

$$\frac{1}{k + q} \frac{1}{\hat{k} \hat{k}} = \frac{1}{\hat{q} \hat{k}} - \frac{1}{k + q} \frac{1}{\hat{k} \hat{k}}$$

and due to the convergence of (32) which guarantees the shifting of the origin in $q$-momentum space, the integral (32) vanishes.

3 Thirring Model

The 2 + 1 dimensional action for Thirring model with a large number $N$ of fermions is:

$$S = \int d^3 x \left[ \sum_{j=1}^{N} \bar{\psi}_j \hat{\partial} \psi_j - \frac{g^2}{2} \sum_{j=1}^{N} (\bar{\psi}_j \gamma_\mu \psi_j)^2 \right]$$

The coupling constant $g^2$ has dimension of a inverse of a mass, and the theory is nonrenormalizable. However it has been proved [11] that in 2+1 dimensions no ultraviolet divergences appear in a scheme preserving the current conservation, if an expansion in the dimensionless parameter $1/N$ is performed. Supported by this result we claim that in the three dimensional Euclidean space-time it is possible to extract from the 4-point Green function of this model a dimensionless running coupling constant

$$\tilde{g}^2(p) = \frac{g^2 p}{1 + \frac{N}{8} g^2 p}.$$ 

It shows that the model becomes non-interacting in the infrared ($g^2 p \ll 1$) and it has a weak ultraviolet fixed point at $\frac{8}{N}$, in strength exactly equal to the infrared one of three dimensional quantum electrodynamics. This result is in perfect agreement with that one obtained in [11], working in the bosonized version of the model, whose Lagrangian in the three dimensional Euclidean space is

$$\mathcal{L} = \sum_{j=1}^{N} \bar{\psi}_j \hat{\partial} \psi_j + ig \sum_{j=1}^{N} \bar{\psi}_j \hat{A} \psi_j + \frac{1}{2} A_\mu A_\mu$$

the field $A_\mu$ is an auxiliary vector; it may be integrated over to recover the original model (37). To leading order in $1/N$ the auxiliary propagator receives a contribution from vacuum polarization, so we write

$$D_{\mu\nu}(p) = \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{1}{1 + \frac{N}{8} g^2 p} + \frac{p_\mu p_\nu}{p^2}.$$ 

Extending the Lagrangian (39) at finite temperature, it is possible to extract the physical degrees of freedom following the definitions (14) and (15). At the leading order one finds:

$$f_{IR} = f_{UV} = 3N,$$
then \( f_{UV} \) can be corrected in perturbation theory. The calculation of \( f_{UV} \) next-to-leading correction proceeds in the same way as in QED. Up to a positive proportionality factor that correction is

\[
\frac{N g^2(T)}{2 T^4} \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 q}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} (2\pi)^2 \delta^2(\vec{p} - \vec{q} - \vec{k}) \sum_{n_p,n_q,n_k} \beta \delta_{n_p,n_q+n_k} T^3 Tr(\gamma^\mu \vec{p} \gamma^\nu \vec{q}) \frac{T^4}{p^2 q^2}.
\]

The summation on the Matsubara frequencies is now highly simplified, the result is

\[
T^3 \sum_{n_p,n_q,n_k} \beta \delta_{n_p,n_q+n_k} \frac{Tr(\gamma^\mu \vec{p} \gamma^\nu \vec{q})}{p^2 q^2} = \\
\frac{1}{\beta E_p E_q} \left[ (N_F(p) - N_F(q)) \frac{E_p E_q - \vec{p} \cdot \vec{q}}{E_p - E_q} + (N_F(p) + N_F(q)) \frac{E_p E_q + \vec{p} \cdot \vec{q}}{E_p + E_q} \right].
\]

Substituting (43) into (42) we obtain

\[
\frac{2\pi N g^2(T)}{\beta T^4} \int_0^{+\infty} dE_p dE_q \frac{E_p N_F(q) - E_q N_F(p)}{E_p^2 - E_q^2}.
\]

But (44) is convergent and

\[
\forall E_p, E_q \quad \frac{E_p N_F(q) - E_q N_F(p)}{E_p^2 - E_q^2} < 0,
\]

moreover

\[
\lim_{T \to +\infty} N g^2(T) = 8
\]

we therefore conclude that the \( f_{UV} \) next-to-leading correction is non-positive and

\[
f_{UV} \leq f_{IR}.
\]

4 Conclusions and perspectives

The existence of a conjectured inequality between infrared and ultraviolet degrees of freedom has been explicitly verified in QED3 and in 3D-Thirring model with large \( N \) number of fermions. The calculations have been done in the formalism of imaginary time with large \( N \) limit resummation and gauge invariant results have been obtained.

In a subsequent paper we plan to verify this inequality when the global symmetry \( U(N) \otimes U(N) \) of QED3 is spontaneously broken and a non-linear realization of that symmetry is working at low energy through \( 2N^2 \) Goldstone bosons and massive fermions. In that case the expected critical number of fermions for dynamical mass generation is \( \frac{3}{2} \). As it was pointed out in [6] a natural question about the application of \( f_{IR} \leq f_{UV} \) in the context of spontaneous symmetry breaking of QED3 has to do with the Mermin-Wagner-Coleman theorem [12], stating that spontaneous symmetry breaking of a continuous symmetry cannot happen in \( 2 + 1 \) dimensions, as for example it has been verified in the renormalizable theory \( \lambda \phi^6 \) [13]. It is in fact expected [14] that at small temperature Goldstone bosons acquire masses, but it would not affect the count of infrared degrees of freedom [14]. An explicit computation in this sense is in progress.
Following the recent proposal of [15] we are investigating about the implications of this inequality, in quantum chromo-dynamics too. Since the analytic results about the condensate quark-antiquark in orbifold QCD [16] it would be interesting to see if a critical number of fermions, compatible with our inequality, could be extracted from the analytic value of this condensate.

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