Neutron EDM in Four Generation Standard Model

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A fourth generation of quarks, if it exists, may provide sufficient CP violation for the baryon asymmetry of the Universe. We estimate the neutron electric dipole moment in the presence of a fourth generation, and find it would be dominated by the strange quark chromoelectric dipole moment, assuming it does not get wiped out by a Peccei-Quinn symmetry. Both the three electroweak loop and the two-loop electroweak/one-loop gluonic contributions are considered. With $m_s$, $m_N$ at 500 GeV or so that can be covered at the LHC, and with a Jarlskog CPV factor that is consistent with hints of New Physics in $b \rightarrow s$ transitions, the neutron EDM is found around $10^{-21}$ cm, still far below the $10^{-28}$ cm reach of the new experiments being planned or under construction.

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I. INTRODUCTION AND MOTIVATION

The Kobayashi–Maskawa (KM) mechanism\textsuperscript{1} for CP violation (CPV) has been verified by the Belle and BaBar experiments\textsuperscript{2}. Constituting the flavor and CPV part of the Standard Model (SM), it falls short of the observed Baryon Asymmetry of the Universe (BAU) by many orders of magnitude. However, it was pointed out that, by extending to four quark generations\textsuperscript{3}, SM4, the KM picture may have enough\textsuperscript{4} CPV phase for BAU. The strength of phase transition, needed to satisfy the third Sakharov condition, i.e. departure from equilibrium, remains an issue. But interest has renewed\textsuperscript{5} in the direct search of fourth generation quarks at hadron colliders, where the LHC should finally be able to discover, or rule out once and for all\textsuperscript{6}, fourth generation quarks.

The long quest for neutron electric dipole moment (nEDM) has been motivated by BAU, as the latter implies the existence of new CPV sources beyond SM. Given the large jump in CPV, it is of interest to ask what nEDM value one might expect for SM4. The KM mechanism cleverly shields itself from nEDM. At the one weak loop level, the CKM factors come always conjugate to each other so the CPV phase cancels. It was then shown\textsuperscript{7} that the sum over all diagrams other so the CPV phase cancels. It was shown\textsuperscript{7} by Shabalina, at two loop in SM, that the sum over all diagrams for single quark electric dipole moments (qEDM) vanish. It was then shown that, bringing in a further gluon loop (two-loop electroweak/one-loop gluonic) breaks the identical cancelation, leading to $d_n \sim 10^{-34} \text{ cm}\textsuperscript{3}$\textsuperscript{8}. Considerations of long distance (LD) effects allow a value of $d_n$ that is two orders of magnitude higher\textsuperscript{10}.

The current limit for nEDM, $2.9 \times 10^{-26} \text{ cm}$ at 90\% C.L.\textsuperscript{11}, is from the RAL-Sussex-ILL experiment which operated at Grenoble. Compared with dropping an order of magnitude per decade\textsuperscript{12} since the 1950s, the pace has slowed. The chief limiting factor is the number of neutrons. There is, however, a renewed effort, by the CryoEDM collaboration at Grenoble, the nEDM collaboration at PSI, and the nEDM collaboration at the Spallation Neutron Source (SNS) at Oak Ridge Na-
ated in the QCD sum rule framework. In terms of quark EDMs and chromo-EDMs (CEDM), one has [21]

\[
d_n = (0.4 \pm 0.2) \left[ \chi m_+ (4c_d - e_u) \left( \tilde{\theta} - \frac{1}{2} m_0 \tilde{d}_s \right) + \frac{1}{2} \chi m_0^2 \left( \tilde{d}_d - \tilde{d}_u \right) \frac{4e_d m_d + e_u m_u}{m_u + m_d} + \frac{1}{8} \left( 4d_d \alpha_d^+ - \tilde{d}_d \alpha_d^0 + (4d_d - d_u) \right) \right],
\]

where \(1/m_+ = 1/m_u + 1/m_d + 1/m_s \approx 1/m_u + 1/m_d\), \(\tilde{\theta} = \sum \theta_q + \theta_G\) is the combined quark and gluonic \(\theta\) term, \(\alpha_d^\pm = e_q (2\kappa \pm \xi)\), and \(\chi, m_d^2, \kappa\) and \(\xi\) are condensate susceptibilities. The large factor of 3 uncertainty inherent in the overall \(0.4 \pm 0.2\) coefficient reflects the large hadronic uncertainty, as determined in the sum rule approach. Thus, our estimates that follow are aimed at the order of magnitude.

The interesting subtlety is that, when a Peccei-Quinn symmetry [22] is invoked to remove the \(\tilde{\theta}\) term (setting it to zero), it induces additional CPV terms [23] to the axion potential. While remarkable, as we shall see, the sCEDM is of the greatest interest in SM4.

Furthermore, three decades of axion search has so far ignored the \(\bar{\chi} q\) condensation.

Analyzing the flavor structure of a typical three loop diagram shows why the strange CEDM is highlighted, despite a smaller coefficient in Eq. [2]. A typical three loop diagram involves two nonoverlapping \(W\) boson loops, with one \(Z\) or gluon loop. Following the quark line, the \(f\) quark (C)EDM has the following form [17, 26]

\[
i \sum_{j,k,l} \text{Im} (V^*_{ij} V_{jk} V_{kl} V_{lf}) f_{jkl} = \frac{i}{2} \sum_{j,k,l} \text{Im} (V^*_{ij} V_{jk} V_{kl} V_{lf}) f_{jkl}. \tag{3}
\]

where \(f, j, k, l\) stand for both flavor indices and the corresponding Green function. The antisymmetry in Eq. [4] is at the root of Shabalin’s argument.

Since we shall consider rather heavy \(t'\) and \(b'\) quarks in the loop, typical loop momenta would be at these large values. Therefore, one can take \(c = u \equiv u, d = s = b \equiv d\) as all effectively massless in loop propagators. One then easily sees that the (C)EDM of the \(u\) quark vanishes. That is, performing the sum over \(j\) and \(l\) in Eq. [5] for \(f = u\), using the unitarity relation \(V^*_{ud} V_{kd} + V^*_{us} V_{ks} + V^*_{ub} V_{kb} = \delta_{uk} - V_{ub} V_{kb} '\) and the “degeneracy” in mass for the \(d, s, b\) propagators, one gets

\[
i \sum_{j,k,l} \text{Im} (V^*_{uj} V_{jk} V^*_{kl} V_{ul}) u_{jkl} = \frac{1}{2} \sum_{k} \text{Im} (V^*_{uk} V_{kb} V^*_{kb} V_{uk}) u (dk b' - b' k d) u = 0, \tag{4}
\]

as phases pairwise cancel. Effectively there are only two generations in the loop.

The case for \(f = d, s\) is therefore more interesting. By similar token, one has

\[
i \sum_{j,k,l} \text{Im} (V^*_{uj} V_{jk} V^*_{kl} V_{ul}) f_{jkl} = i \text{Im} (V^*_{uf} V_{fb} V^*_{fb} V_{uf}) \left[ t (d - b') t' t' (d - b') t + \right. \text{U} (d - b') t + \text{U} (d - b') - t (d - b') \text{U} f]. \tag{5}
\]

In Eq. [5], we have spelled out the sum over \(k\), after utilizing CKM unitarity as before. The sum of Green function factors contain the “degeneracy” of \(c = u \equiv u\) and \(d = s = b \equiv d\) in the loop. But one should treat the CKM coefficient with care, where use has been made of the rather good approximate relation [17]

\[
\text{Im} (V^*_{uf} V_{fb} V^*_{fb} V_{uf}) \cong -\text{Im} (V^*_{tf} V_{tb} V^*_{tb} V_{tf}) \equiv J_f, \tag{6}
\]

which is a consequence of the smallness of \(V_{ub}\) (assuming that other CKM elements that enter are not much smaller). As noted in Ref. [22], for \(f = s, \), this relation effectively means that the CKM “triangle” (degenerated from a quadrangle) governing \(b' \rightarrow s\) transitions have essentially the same area as the \(b \rightarrow s\) triangle. We will turn to numerical analysis in the next section, but we can already see that the CKM factor for \(f = s\) is much larger than for \(f = d\), which is the reason why we highlight the strange quark CEDM.

The \(s\) quark CEDM arising from the two-\(W\) loop plus one gluon loop diagram was estimated [17] using the external field method, to double log accuracy and in the large \(N_C\) limit, with the result of

\[
\tilde{a}_s^{(g)} = -\mathcal{J}_s m_s \frac{G_F}{\sqrt{2}} \frac{\alpha_s \alpha_W}{(4\pi)^4} \frac{5N_c}{6} \frac{m_s^2}{M_W^2} \frac{1}{4} \log^2 \left( \frac{m_s^2}{M_W^2} \right). \tag{7}
\]

Replacing the gluon by a \(Z\) boson loop, one might think it should be subdominant. But since there are rather large quark masses in the loop, it implies that rather large loop momenta might be relevant. The derivative coupling nature of the longitudinal \(Z\) boson, or equivalently, that the Goldstone boson couples to the heavy quark masses hence is nondecoupled, effectively voids the above intuition. By an ingenious argument of
limiting to large loop momenta and involving longitudinal vector bosons, the authors of Ref. [18] were able to reduce the three-loop calculation effectively to calculating three one-loop integrals, and the core of it is an effective $i \to fZ$ transition involving the heavy fourth generation quark mass. This is the familiar $Z$ penguin [22, 23], and indeed it has been found [3] that $b' \to bZ$ and $b' \to b\gamma$ transitions are not too different in strength. The upshot of the estimate (with the brutality of setting all logarithms to order 1) of Ref. [13], done in 1996, is

$$d_s^{(Z)} = -\mathcal{J}_s m_t \frac{G_F}{\sqrt{2}} \frac{\alpha_s^2}{(4\pi)^2} \frac{m_t^2 m_s^2}{4M^4_W} \log \left( \frac{m_t^2}{m_s^2} \right). \quad (8)$$

Comparing Eqs. (7) and (8), aside from the double versus single logarithm, one can see from $\alpha_W/M^2_W = \sqrt{2} G_F/\pi = 1/\pi v^2$ that one is comparing $5 N c_\alpha /6$ with $\lambda^2_3/4\pi$. The gluonic effect is enhanced by the color factor, but the Yukawa coupling grows with $m_t^2$. Compared literally, they are actually comparable. On the other hand, in arriving at Eq. (8), one has set all logarithms to 1. In this spirit, both the double log (including the 1/2!) in Eq. (7) and the single log in Eq. (8) should be treated as order one. Then, the gluonic effect would be subdominant to the $Z$ effect, for $t'$ and $b'$ masses of order 500 GeV (or higher), a nominal value used by Ref. [13], and which we shall use in the next section.

Given the roughness of these calculations, and the great difficulty in calculating genuine three electroweak loop diagrams, we shall take the estimate of Eq. (8) for our subsequent numerics.

III. NUMERICAL ESTIMATE

We shall use $m_{t'} \approx m_{b'} \approx 500$ GeV as our nominal fourth generation quark mass. Above this value, one would pass through the unitarity bound [30], and the Yukawa coupling starts to turn nonperturbative. For the other quark masses, we take $m_u = 2.5$ MeV, $m_d = 5$ MeV, $m_s = 100$ MeV, $m_c = 1.3$ GeV, $m_b = 4.2$ GeV, and $m_t = 165.5$ GeV, where heavy quark masses are in SM scheme. The light quark mass values were in fact implicit in Eq. (2). For the CKM products $\mathcal{J}_s$ and $\mathcal{J}_d$, we take the nominal fit [31] to flavor data performed for $m_t \approx 500$ GeV, where $V_{tb} \approx 0.1$, $V_{ts} \approx -0.06 e^{-175}$, and $V_{td} \approx -0.003 e^{-188}$, we get

$$\mathcal{J}_s \approx \text{Im}(V_{ts}^* V_{tb}^* V_{tb} V_{ts}) \approx 2.4 \times 10^{-4}, \quad (9)$$
$$\mathcal{J}_d \approx \text{Im}(V_{td}^* V_{tb}^* V_{tb} V_{td}) \approx 1.7 \times 10^{-7}. \quad (10)$$

Note that $\mathcal{J}_s$ could be measured [32] in the next two years at the LHC, but $\mathcal{J}_d$ would be harder to disentangle.

Although the range of uncertainty is large, it is clear that $|\mathcal{J}_s| \gg |\mathcal{J}_d|$, which correlates with the fact that $b \to d$ transitions, including in $B_d$ mixing, show little sign of deviation from SM expectations, while for $b \to s$ transitions, and especially in $B_s$ mixing, we have several indications for sizable deviations from three generation SM. Note that the study of Ref. [31] predated the summer Tevatron update on sin $2\Phi_{B_s}$, the CPV phase of the $B_s$ to $B_s$ mixing amplitude, and predicted a value lower than the $-0.5$ to $-0.7$ value given [33] in 2007 for $m_{t'} \approx 300$ GeV. In any case, for our purpose, we see from Eqs. (9) and (10) that, despite the smaller coefficient in Eq. (2), the sCEDM is the most important quark contribution to $d_n$, because of the CKM factor.

Putting in numbers, we find from Eq. (8) that

$$d_s^{(4)} \approx -4 \times 10^{-16} \text{GeV}^{-1} \approx -0.8 \times 10^{-29} \text{ cm}, \quad (11)$$

where the $W-W-g$ 3-loop effect of Eq. (7) is treated as subdominant. Treating the sCEDM as the leading effect in Eq. (2), then

$$d_n^{(4)} = (2.2 \pm 1.1) \times 10^{-31} \text{ e cm}, \quad (12)$$

where the superscript indicates that this is the estimated effect from the fourth generation.

IV. DISCUSSION AND CONCLUSION

Though the estimate of Eq. (12) is considerably larger than the SM result with three quark generations, be it the quark level [9], or hadronically enhanced [10], it is very far from the sensitivities of next generation experiments [12–15], nominally at $10^{-28}$ e cm. What is worse, or perhaps intriguing, is that the sCEDM effect of Eq. (11) is at the mercy of the Peccei-Quinn symmetry. If a PQ symmetry is operative in Nature, then the sCEDM effect is precisely canceled [22, 24]. In this case, one has a reduced formula. Rather than Eq. (2), one gets [21]

$$d_n^{\text{PQ}} = (0.4 \pm 0.2) \left[ 1.6 e (2d_d + d_u) + (4d_d - d_u) \right] \text{ cm}, \quad (13)$$

i.e. only dependent on the naive constituents of the neutron. Note that the axion potential has also modified the $d$ and $u$ quark CEDMs, as it brings in analogous terms to the one that canceled away the sCEDM effect.

From Eq. (11), $d_u$ and $\bar{d}_u$ vanish. Thus, we need only consider $d_d$ and $\bar{d}_d$ for SM4. Assuming again that the analogue of Eq. (8) dominates over the gluonic counterpart, we use the formulas of Ref. [21] for $d_u$ and Eq. (8) to obtain $d_d$ and $\bar{d}_d$ by simply shifting the CKM index, i.e. shifting from $\mathcal{J}_s$ of Eq. (9) to $\mathcal{J}_d$ of Eq. (11), and replacing $m_s$ by $m_d$. We find

$$d_d^{(4)} \approx -3 \times 10^{-34} \text{ cm}, \quad d_d^{(4)} \approx -4 \times 10^{-34} \text{ cm}, \quad (14)$$

and, from Eq. (13),

$$d_n^{(4)\text{PQ}} = -(1 \pm 0.5) \times 10^{-33} \text{ e cm}, \quad (15)$$

where $d_d$ contributes roughly twice as $\bar{d}_d$. These should be taken as very rough estimates. Note that, unlike Eq. (9), it would take some time to refine Eq. (10).

We see that $d_d^{(4)}$ is stronger than in SM [3], while $d_n^{(4)\text{PQ}}$, as estimated through sum rules, is at $10^{-33}$ e cm.
level. Other LD effects might bring about another order (or maybe two) of magnitude enhancement. But it should be clear that, with PQ symmetry operative, even with four quark generations, nEDM is way below the sensitivity of the next generation of experiments.

But what about LD effects to the sCEDM-driven result of Eq. (12), without PQ symmetry? Operators beyond dimension 5, such as dimension 6 operators involving two quarks in the neutron, are beyond the scope of our investigation. Even with WWZ loops, Eq. (7), of similar order of magnitude and constructive with the WWZ loop result of Eq. (8) (hence Eq. (11)), $d_n^{(4)}$ without PQ symmetry still seems a couple of orders below $10^{-28} \text{ cm}$. One could also estimate from $s$ to $d$ transition elements induced by gluon or $Z$, enhanced by pion-nucleon coupling at long distance. It is possible that $K_L \rightarrow \pi^0 \bar{\nu} \nu$ gets enhanced by a factor of 100 in SM4, which means a factor of 10 at amplitude level. Given the two orders of magnitude LD enhancement in SM4, it could be brought up by another order of magnitude in SM4, hence to $10^{-33} \text{ cm}$. This is rather similar to the result in Eq. (12). We remark that, one way to probe the sEDM and sCEDM effects, even if PQ symmetry is operative, would be to measure hyperon EDM, which is very interesting in its own right. A rough estimate, by extrapolating from Eq. (13), gives the order at $10^{-29} - 10^{-27} \text{ cm}$. But if $10^{-28} - 10^{-27} \text{ cm}$ is still the challenge of the next decade for neutron EDM experiments, the measurement of the EDM of the less abundant, shorter-lived hyperons would be still farther away.

The CMS experiment has recently stated that “data exclude SM4 Higgs boson with mass between 120 and 600 GeV at 95% C.L.”, while the $t'$ quark is not seen below 450 GeV [33]. One may be entering the strong coupling, beyond the unitarity bound regime of $m_{t'} > 500 \text{ GeV}$, with an associated composite, massive (hence broad) “Higgs” boson. In this regime, the short distance results of Eqs. (7) and (8) start to fail even as rough estimates, as the loop functions themselves turn nonperturbative.

In conclusion, with four quark generations and with Peccei-Quinn symmetry operative, the neutron EDM is slightly enhanced above the SM value, but no more than an order of magnitude, hence much below the sensitivities of the next generation of experiments. If PQ symmetry is absent, then a large enhancement is possible through the $s$ quark CEDM, which is correlated with possible effects in $b \rightarrow s$ transitions that are of current interest. However, it is still unlikely that the neutron EDM could reach the $10^{-28} \text{ cm}$ level sensitivity that may be probed during the next decade.

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