Moment of inertia of superconductors

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We find that the bulk moment of inertia per unit volume of a metal becoming superconducting increases by the amount \( m_e/(\pi r_e) \), with \( m_e \) the bare electron mass and \( r_e = e^2/m_e c^2 \) the classical electron radius. This is because superfluid electrons acquire an intrinsic moment of inertia \( m_e(2\lambda_L)^2 \), with \( \lambda_L \) the London penetration depth. As a consequence, we predict that when a rotating long cylinder becomes superconducting its angular velocity does not change, contrary to the prediction of conventional BCS-London theory that it will rotate faster. We explain the dynamics of magnetic field generation when a rotating normal metal becomes superconducting.

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I. INTRODUCTION

A superconducting body rotating with angular velocity \( \vec{\omega} \) develops a uniform magnetic field throughout its interior, given by

\[
\vec{B} = -\frac{2m_e c}{e} \vec{\omega} \equiv B(\omega)\hat{\omega}
\]

(1)

with \( e \) \((<0)\) the electron charge, and \( m_e \) the bare electron mass. Numerically, \( B = 1.137 \times 10^{-7} \) \( \omega \), with \( B \) in Gauss and \( \omega \) in rad/s. The magnetic field is parallel to the angular velocity. The phenomenon was predicted in 1933, just before the Meissner effect was discovered, by Becker, Heller and Sauter [1] for a perfect conductor set into rotation. Subsequently London predicted that the same final state should result when a rotating normal metal is cooled into the superconducting state [2]. The resulting magnetic moment will depend on the shape of the body and is called the “London moment”. In this paper we propose that this effect reveals fundamental physics of superconductors not predicted by conventional BCS theory [3, 4]. For a preliminary treatment where we argued that BCS theory is inconsistent with the London moment we refer the reader to our earlier work [5]. Other non-conventional explanations of the London moment have also been proposed [6, 7].

Eq. (1) has been verified experimentally for a variety of superconductors [8-15]. The same result is obtained whether the sample is first cooled and then set into rotation, or cooled while rotating, as predicted by London [2]. Rotation speeds used in experiment are typically of order 20 to 100 revolutions/second. Equation (1) follows from London’s equation [4]

\[
\vec{\nabla} \times \vec{v}_s = -\frac{e}{m_e c} \vec{B}.
\]

(2)

with \( \vec{v}_s \) the superfluid velocity. In the interior of the body, the superfluid rotates together with the body, hence the superfluid velocity at distance \( r \) from the rotation axis is given by

\[
\vec{v}_s = \vec{\omega} \times \vec{r}
\]

(3)

and using \( \vec{\nabla} \times (\vec{\omega} \times \vec{r}) = 2\vec{\omega} \) in Eq. (2), Eq. (1) results.

Consider the simplest geometry, a long cylinder of radius \( R \), where no demagnetization effects exist. A section of such a cylinder is shown in Figure 1. For a supercurrent flowing within the London penetration depth \( (\lambda_L) \) of the surface, the superfluid velocity that excludes an applied magnetic field \( \vec{B} \) is

\[
\vec{v}_s = -\frac{e\lambda_L}{m_e c} \vec{B}.
\]

(4)

Therefore, to generate the interior London field Eq. (1) in a rotating superconductor in the absence of applied external field, the superfluid velocity in a rim of thickness \( \lambda_L \) at the surface has to be

\[
\vec{v}_s = \vec{\omega} \times \vec{r}(1 - \frac{2\lambda_L}{R})
\]

(5)
so the superfluid velocity in the rim lags the rotation of the body by the small amount
\[ \Delta v_s = -2\lambda_L \omega. \] (6)
The full behavior of superfluid velocity versus radius is given by Laue [16] for a cylindrical geometry and by Becker et al [1] and London [2] for a spherical geometry. The London penetration depth is given by [4]
\[ \frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2} \] (7)
with \( n_s \) the superfluid density.

For a perfect conductor that is set into rotation, Eq. (1) follows from Maxwell’s equations, as discussed by Becker et al [1], assuming the superfluid electrons are completely detached from the lattice. As the body is set into rotation the moving ions generate an electric current and hence a time-dependent magnetic field that generates a Faraday electric field that pushes the superfluid electrons to follow the motion of the ions, albeit with a small lag near the surface that gives rise to the surface current that generates the magnetic field Eq. (1). The derivation is reviewed in Appendix A.

In this paper we explain the behavior of rotating superconductors using the theory of hole superconductivity [17], and we argue that the conventional theory of superconductivity is inconsistent with the physics of rotating superconductors.

II. THE CONVENTIONAL VIEW

We consider a normal metallic cylinder rotating around its axis with angular velocity \( \omega \) that is cooled into the superconducting state. We assume the cylinder is floating in a gravitation-free environment and there is no friction. Therefore, total angular momentum is conserved.

In the normal state, electrons and ions rotate together at the same speed. When the cylinder becomes superconducting, electrons near the surface have to spontaneously slow down to the velocity Eq. (5) upon entering the superconducting state. The conventional theory of superconductivity does not explain the dynamics of this process, which London himself considered to be “quite absurd” from the perfect conductor viewpoint [18] and did not provide an explanation for. Furthermore, the body has to spontaneously speed up to compensate for the electronic angular momentum change. The conventional theory of superconductivity does not explain what is the physical process that causes the body to increase its rotation velocity.

The total mechanical angular momentum of electrons plus ions is
\[ \tilde{L} = \tilde{L}_e + \tilde{L}_i. \] (8)
The electronic angular momentum in the rotating normal state is
\[ L_e = I_e \omega \] (9a)

\[ I_e = \frac{\pi}{2} n_s m_e h R^4 \] (9b)

where \( h \) is the height of the cylinder. \( I_e \) is the moment of inertia of the superfluid electrons. As the system goes superconducting, electrons in the rim slow down according to Eq. (5), and the electronic angular momentum decreases by
\[ \Delta L_e = (2\pi R \lambda_L n_s) \times (m_e \Delta v_s R) \] (10)
with \( \Delta v_s \) given by Eq. (6). In Eq. (10), the first factor is the number of superfluid electrons in the surface rim, and the second factor is the change in angular momentum of one electron. Using Eq. (7), Eq. (10) can be rewritten as
\[ \Delta L_e = -(\frac{m_e c^2}{e})^2 R^2 h \omega. \] (11)

The kinetic energy of the superfluid electrons in the normal state due to the body rotation is
\[ K_e = \frac{1}{2} I_e \omega^2 = \frac{L_e^2}{2I_e} \] (12)
and when the system goes superconducting and the electrons in the rim slow down it decreases by the amount
\[ \Delta K_e = \frac{L_e \Delta L_e}{I_e} = \Delta I_e \omega \] (13)
or, using Eqs. (11) and (1)
\[ \Delta K_e = -\frac{B^2}{4\pi} (\pi R^2 h). \] (14)

For the ions, the kinetic energy of rotation is
\[ K_i = \frac{I_i^2}{2I_i} \] (15)
with \( I_i \) the moment of inertia for the ions. From Eq. (8) and conservation of total angular momentum we deduce
\[ \Delta I_i = -\Delta L_e, \] (16)
therefore, angular momentum has to be transferred from the electrons to the body. This gives rise to a change in the kinetic energy of the ions,
\[ \Delta K_i = \frac{I_i}{I_i} \Delta I_i = \Delta L_i \omega = -\Delta K_e \] (17)
hence
\[ \Delta K_e + \Delta K_i = 0. \] (18)

Therefore, conservation of angular momentum requires that the change in ionic and electronic kinetic energies of rotation exactly compensate each other. The body’s rotation speed slightly increases to compensate for the slowing down of the rim electrons, and its kinetic energy of rotation slightly increases. The conventional theory of
superconductivity does not explain how the kinetic energy saved by the lagging electrons is transferred to the body to make it speed up. The frequency of rotation when the rotating normal metal becomes superconducting will be larger than $\omega$ by the amount

$$\Delta \omega = \frac{\Delta L_i}{I_i} = \frac{m_e n s \lambda_n^2}{m_p A n R^2 \omega}$$

(19)

where $m_p$ is the nucleon mass, $A$ the atomic weight and $n$ the ionic number density.

For a type I superconductor it is expected [19] that when the body is rotating at frequency $\omega$ the transition to the superconducting state will occur when the system is cooled below $T_c$ to the temperature $T$ for which

$$H_c(T) = -\frac{2m_e c}{e} \omega = B.$$  

(20)

with $H_c(T)$ the thermodynamic critical field. Now the magnetic field energy associated with the magnetic field Eq. (1) for a cylinder with zero demagnetizing factor is given by

$$E_B = \frac{B^2}{8\pi}(\pi R^2 h) = \frac{1}{2}|\Delta K_s|.$$  

(21)

The condensation energy for a superconducting cylinder at rest is given by

$$F_{\text{cond}} = \frac{H_c(T)^2}{8\pi} n_s (\pi R^2 h).$$

(22)

In the Meissner effect, the condensation energy provides the energy required to expel the magnetic field. Here, the condensation energy Eq. (22) provides the energy to generate the magnetic field Eq. (21). Therefore, conservation of energy requires that the condensation energy Eq. (22) is the same for rotating and non-rotating superconductors.

However, we argue that for a superconductor rotating at high speeds it is inconsistent to assume that the binding energy of Cooper pairs, that determines the condensation energy, would be independent of rotation speed. According to the conventional theory this should be true for arbitrarily high frequencies, even for frequencies giving rise to a magnetic field $B$ larger than the thermodynamic critical field at zero temperature. If we consider two electrons in a Cooper pair separated by a typical distance $\xi \sim \lambda_L$, the energy associated with rotation at frequency $\omega$ is

$$\epsilon_{\text{rot}} = m_e \lambda_n^2 \omega^2 \sim B^2 / (8\pi n_s)$$

(23)

i.e. the magnetic field energy per electron. We argue that it is inconceivable that the binding energy of Cooper pairs would not be lowered by the rotation energy $\epsilon_{\text{rot}}$ for any magnitude of $B$.

However, if the condensation energy is decreased at finite rotation frequency, there is not enough energy to account for the creation of the magnetic field Eq. (1) when the system becomes superconducting, because the energy saved in the slowing down of the rim electrons was entirely used up in speeding up the body to satisfy angular momentum conservation. Therefore, we argue that the conventional theory cannot account for the physics of rotating superconductors without violating either conservation of energy or conservation of angular momentum.

The implausibility of the conventional picture also follows from the following argument. From the Meissner effect we learned that the final state of a superconductor in a magnetic field $B$ is unique, independent of history. Whether we apply a magnetic field $B$ to a normal metal and then cool it to the superconducting state, or we apply the same magnetic field $B$ to a metal already superconducting, the final state of the system is exactly the same. Similarly here, the rotation frequency $\omega$ plays the role that $B$ plays in the Meissner effect. Whether we apply the rotation $\omega$ to the normal metal and then cool it to the superconducting state, or instead apply the same rotation $\omega$ to a metal already superconducting, the final state of the system should be exactly the same, with the same $\omega$ [18]. This implies that the rotation speed of the body should not change when the rotating metal enters the superconducting state, contrary to the prediction of the conventional theory that the rotation frequency should change by the amount given by Eq. (19).

Finally, as reviewed in Appendix A, for the case of a superconductor at rest that is set into rotation, the derivation of Eq. (1) requires that there is no direct interaction between the superfluid electrons and the lattice. If that is the case, how can there be a transfer of angular momentum from electrons to the body as a rotating normal metal becomes superconducting?

III. ALTERNATIVE VIEW

Instead, we propose that when the rotating cylinder becomes superconducting its angular velocity does not change and hence that the ionic angular momentum doesn’t change. This then implies that the total electronic angular momentum does not change either.

The angular momentum of electrons in the rim decreases by the amount Eq. (10). Hence the angular momentum of electrons in the bulk has to increase by the same amount:

$$\Delta L_e + \Delta L_e^{\text{bulk}} = 0$$

(24a)

$$\Delta L_e^{\text{bulk}} = m_e (2\lambda_L^2) n_s (\pi R^2 h \omega)$$

(24b)

Since the number of electrons in the bulk is

$$N_e = n_s \pi R^2 h$$

(25)

this implies that each electron in the bulk acquires an additional ‘intrinsic’ angular momentum

$$\ell = m_e (2\lambda_L^2) \omega.$$  

(26)
Eq. (26) says that when the rotating metal becomes superconducting there is an additional contribution to its electronic angular momentum that comes from the electron mass being spread out in a ring of radius $2\lambda_L$. Equivalently, that the electron’s orbit expands from a microscopic radius to radius $2\lambda_L$. Such physics was predicted by the theory of hole superconductivity \cite{20,21} to explain the dynamics of the Meissner effect: as an electron expands its orbit from point-like to radius $2\lambda_L$ in the presence of a magnetic field, azimuthal current of the electron mass being spread out in a ring of radius 2.

In Eq. (26) we find a direct confirmation of this essential part of the theory. It implies that when electrons enter the superfluid state they acquire an intrinsic moment of inertia

$$i_{el} = \frac{m_e}{8\pi}(2\lambda_L)^2.$$  

so the total electronic moment of inertia increases by

$$\Delta I_e = \frac{m_e}{8\pi}(2\lambda_L)^2 n_z (\pi R^2 h)$$

and the electronic angular momentum increase in the bulk Eq. (24b) is

$$\Delta L_e^\text{bulk} = \Delta I_e \omega$$

Using Eq. (7), we can write the increase in moment of inertia as

$$\Delta I_e = \frac{V}{\pi r_e}$$

where $V = \pi R^2 h$ is the volume of the body and $r_e = e^2/m_e c^2$ is the classical electron radius. Because Eq. (30) no longer depends on the geometry of the body we believe it is very likely that it is a general result for a body of arbitrary shape and for any rotation axis.

Note the interesting fact that Eq. (30) is also independent of the superfluid density and the London penetration depth. Thus it expresses a qualitative difference between the normal and superconducting states of matter. Presumably the volume factor in Eq. (30) corresponds to the volume of the sample that is in the superconducting state.

How does the energetics work in this scenario? We have from Eqs. (13) and (14) that the decrease in electronic kinetic energy because of the rim slowing down is

$$\Delta K_e = \Delta I_e \omega = -\Delta I_e \omega^2 = -\frac{B^2}{4\pi}(\pi R^2 h)$$

The ions do not acquire extra kinetic energy since the frequency of rotation $\omega$ doesn’t change. The bulk electronic kinetic energy is given by Eq. (12), so when the moment of inertia increases it increases by

$$\Delta K_e^\text{bulk} = \frac{1}{2} \Delta I_e \omega^2 = \frac{B^2}{8\pi}(\pi R^2 h)$$

which is half of the decrease in rim kinetic energy Eq. (31). The other half goes into paying the cost in magnetic energy Eq. (1). This then implies that the condensation energy of the rotating superconductor at temperature $T$ is, instead of Eq. (22)

$$E_{\text{cond}}(T, \omega) = \left(\frac{H^2(T)}{8\pi} - \frac{B^2(\omega)}{8\pi}(\pi R^2 h)\right)$$

with $B(\omega)$ given by Eq.(1). The transition occurs at the frequency or temperature where the condensation energy vanishes.

### IV. KINETICS OF THE TRANSITION

Let us consider the process by which the magnetic field attains the value Eq. (1). Consider a point at distance $r$ from the origin. From Faraday’s law

$$\oint E \cdot dl = -\frac{1}{c} \frac{\partial \phi}{\partial t}$$

and assuming cylindrical symmetry we have

$$E_F(r, t) = -\frac{1}{2\pi r e} \frac{\partial \phi(r, t)}{\partial t}$$

where $E_F(r, t)$ is the induced Faraday electric field and $\phi(r, t)$ the magnetic flux through $\pi r^2 < r$, at time $t$. Integrating over time,

$$\int_0^\infty dt E_F(r, t) = \frac{B}{2\pi r} = \frac{m_e}{e} (\pi r_L^2)$$

If the location $r$ is superconducting, a superfluid electron at $r$ obeys the equation of motion

$$\frac{dv_s}{dt} = \frac{e}{m_e} E(r, t)$$

where $v_s$ and $E$ are in the azimuthal direction. Assuming the point $r$ was superconducting during the entire process we have upon integrating Eq. (37) and using Eq. (36)

$$\Delta v_s = \int_0^\infty \frac{dv_s}{dt} = \omega r.$$  

Therefore, this equation describes the process shown in Fig. 2 (a), where the body is initially at rest in the superconducting state and the electron initial speed is $v_s(R, t = 0) = 0$, and attains final speed $\omega r$ when the body acquires angular velocity $\omega$. Eq. (38) does not apply when $r$ is within $\lambda_L$ of the surface because the magnetic field does not acquire its full bulk value Eq. (1) in that region.

This reasoning also shows that if we are considering the process where the rotating normal metal is cooled into the superconducting state, an electron at radius $r$ has to be in the normal state during the entire time where the magnetic flux $\phi(r, t)$ changes. This is because initially
electrons at radius \( r \) in the normal state rotate together with the body with azimuthal speed \( \omega r \). If at any time while \( \phi \) is changing Eq. (37) was valid, the electron at \( r \) would attain a final velocity different from \( \omega r \), however we know that in the final state electrons in the interior rotate together with the body with azimuthal speed \( \omega r \). We conclude that when a rotating metal is cooled into the superconducting state the superconducting region necessarily expands from the inside out, as shown schematically in Fig. 2 (b). The Faraday field acts on electrons at point \( r \) in the normal state while the magnetic field is growing in the region \( r' < r \), during this time it generates Joule heat but does not change the electron’s azimuthal speed.

V. DYNAMICS OF THE TRANSITION

Next we analyze the dynamics of the transition when the rotating normal metal becomes superconducting, along the same lines that we analyzed the dynamics of the Meissner effect in Refs. [23, 24]. There are many similarities but some important subtle differences. Figures 3 and 4 show schematically both processes, where \( r_0 \) denotes the radius of the expanding phase boundary. The magnetic field pointing out of the paper is denoted by black circles, with their diameter illustrating the intensity.

The driving force for the generation of the surface current in both cases is expansion of the electronic orbits from a microscopic radius to radius \( 2\lambda_L \) when the elec-

FIG. 2: Growth of the magnetic field in rotating superconductors (from left to right). (a) A superconductor at rest starts rotating until it reaches frequency \( \omega \). (b) A normal metal (N) enters the superconducting state (S) upon cooling, while rotating at frequency \( \omega \). Dots indicate magnetic field pointing out of the paper, their density indicates the strength of the field. At a given point \( r \) in the interior, the change in magnetic flux through the region \( r' < r \) between the initial and final states is given by \( \Delta \phi = \pi r^2 B \) and determines the time integral of the Faraday electric field at point \( r \), Eq. (36).

FIG. 3: Rotating normal metal becoming superconducting. The phase boundary at radius \( r_0 \) expands outward. Black dots indicate magnetic field pointing out of the paper, grey crosses indicated demagnetizing field pointing into the paper (see Sect. VI). Electrons at the phase boundary expand their orbits to radius \( 2\lambda_L \), and a backflow of normal electrons takes place to compensate for the radial charge imbalance.

FIG. 4: Meissner effect. See text and ref. [23] for details.
trons enter the superconducting state. In the Meissner case, this expansion in the presence of magnetic field $B$ produces the Meissner counterclockwise current through the action of the magnetic Lorentz force. Similarly, in the rotating case the Coriolis force acts

$$\vec{F}_C = 2m_e \vec{v} \times \vec{\omega} = -\frac{e}{c} \vec{v} \times \vec{B} (\vec{\omega})$$  \quad (39)$$

imparting clockwise speed (relative to the body) to the outgoing electron with radial velocity $\vec{v}$.

In the Meissner effect, the body acquires momentum in the opposite direction through the backflow process illustrated in Fig. 4: backflowing electrons with negative effective mass are subject to a force from the ions in the counterclockwise direction ($F_{\text{latt}}$), causing a clockwise reaction force to be exerted on the body $F_{\text{on-latt}}$ that makes the body turn. That force is partially compensated by the counterclockwise force exerted on the positive ions by the Faraday field. The quantitative analysis is given in ref. [23].

The momentum transferred by the backflowing electrons to the body in the Meissner effect can be thought of as momentum that was stored in the electromagnetic field when electrons expanded their orbits and acquired azimuthal momentum through the Lorentz force [23]. Therein lies the essential difference with the case of the rotating superconductor: there, when the orbit expands and acquires azimuthal momentum through the Coriolis force (Fig. 3), no momentum is stored in the electromagnetic field because there is no magnetic field in the lightly shaded region in Fig. 3 for a long cylinder. Instead, the same Coriolis force that deflections the electron expanding its orbit in the clockwise direction transfers momentum to the ions in counterclockwise direction. That momentum is cancelled by the clockwise momentum transferred by the Faraday electric field $E_F$ to the ions in that region.

The balance of forces for the backflowing electrons for the rotating superconductor is shown in Fig. 3. Coriolis and electric forces act counterclockwise, exactly balanced by the clockwise force exerted on the electrons by the lattice of ions ($F_{\text{latt}}$) so that backflowing electrons flow radially in, just like for the Meissner effect. However unlike for the Meissner case, here the backflowing electrons do not transfer net momentum to the body because the reaction to the force exerted by the ions on the electrons, $F_{\text{on-latt}}$, is cancelled by the sum of electric force $F_E$ and the reaction to the Coriolis force on the electrons $F_C$ (Fig. 3).

In summary, for the reasons explained above, unlike for the Meissner effect there is no net transfer of angular momentum to the body for the rotating superconductor as the superconducting region expands. The fact that backflowing electrons have negative effective mass of course still plays an essential role: if they had positive effective mass, backflowing electrons would be deflected counterclockwise and transfer their momentum to the body by collisions. This would of course not alter the angular momentum balance compared to the scenario described above, but would cause dissipation and entropy production rendering the transition irreversible in contradiction with theory and experiment [23, 25].

VI. NON-ZERO DEMAGNETIZING FACTOR

For samples other than long cylinders a demagnetizing magnetic field will exist, and the above considered simplest situation needs to be modified. Consider for example the case of a rotating sphere [1, 2]. The lagging velocity of electrons in the rim is not constant as given by Eq. (6) but rather depends on the location of electrons relative to the equator [2]:

$$\Delta v_s = 3\lambda_L (\sin \theta) \omega$$  \quad (40)$$

where $\theta$ is the angle between the position vector and the axis of rotation. So at the equator, the lagging speed of electrons is larger than for the cylinder by a factor 3/2. How can this be understood within our scenario, and what are its consequences?

For the case of the Meissner effect (Fig. 4), it is immediately clear why the electrons in the Meissner current at the equator acquire the higher speed: the magnetic field that imparts the azimuthal speed through the Lorentz force acting on the expanding orbits is larger than the applied field precisely by the factor 3/2, due to demagnetization. Recall that the critical magnetic field for a sphere is $(2/3)H_c$ rather than $H_c$ [4].

Similarly we can understand the larger lagging speed for the spherical rotating superconductor. In Fig. 4, in the lightly shaded region where the electron orbit expands and is deflected clockwise by the Coriolis force, there is now a magnetic field pointing into the paper because of demagnetization (indicated by crosses in Fig. 3). The magnetic Lorentz force on the expanding orbit provides additional azimuthal momentum in the clockwise direction in addition to the one imparted by the Coriolis force. It also stores some momentum in the electromagnetic field. The angular momentum density in the electromagnetic field is

$$\vec{L}_{\text{em}} = \frac{1}{4\pi c} \vec{r} \times (\vec{E} \times \vec{B}).$$  \quad (41)$$

The electric field $\vec{E}$ created by the outflow of negative charge resulting from orbit enlargement points radially outward, the demagnetizing field $\vec{B}$ points into the paper, hence $\vec{L}_{\text{em}}$ points parallel to $\vec{\omega}$ (out of the paper) and will increase the angular velocity when transferred to the body by the backflowing electrons. Thus, unlike the case of the long cylinder where there is no demagnetizing field, here (and for any sample with non-zero demagnetizing factor) there will be a small change (increase) in the velocity of rotation of the body when the rotating normal metal becomes superconducting.
VI. EXPERIMENTAL CONSEQUENCES

Our theory predicts that upon cooling a rotating long cylinder into the superconducting state, its rotation frequency should not change. Instead, the conventional theory predicts the body’s rotation frequency should increase by the amount given by Eq. (19). For Al, with $A = 27$, density $\rho = 2.7 g/cm^3$, $\lambda_L(T = 0) = 500 A$, this is

$$\Delta \omega = 3.05 \times 10^{-5} (\lambda_L / R)^2 \omega$$

Unfortunately, even for very thin cylinders with radius approaching $\lambda_L$, this small change would be extremely difficult to detect by direct measurement of the frequency or by measuring the resulting very small change in the magnetic field, $\Delta B = 1.37 \times 10^{-4} \Delta \omega$.

What about directly measuring the change in moment of inertia between normal and superconducting states? Using a sensitive torsional oscillator, experiments attempting to measure ‘non-classical rotational inertia’ in solid $^4He$ were performed to study possible ‘supersolid’ behavior [26].

In this paper we have proposed that when a rotating normal metal becomes superconducting the rotation speed of the body does not change, contrary to what the conventional theory predicts, when the electrons near the surface slow down relative to the body motion and create the interior magnetic field Eq. (1). This unexpected effect occurs because electrons increase their contribution to the bulk moment of inertia of the body when they enter the superconducting state according to the theory of hole superconductivity, thus increasing the bulk electronic angular momentum and thereby compensating for the decrease in the surface electrons angular momentum. Each superfluid electron acquires an intrinsic angular momentum $\hbar$ that adds to the total angular momentum of the body without increasing its angular velocity.

Our scenario certainly resolves the question of how to explain a speed-up of the rotational velocity of a normal metallic cylinder when it becomes superconducting, for which the conventional theory has provided no mechanism: there is no speedup. We also explain the mechanism that causes the rim electrons to slow down when the rotating normal metal becomes superconducting. In a nutshell, the Coriolis force acting on expanding orbits. The conventional theory provides no explanation, it simply postulates that it happens.

In fact, we found that except in the case of zero demagnetizing factor our theory also predicts a speedup of rotation when the normal metal becomes superconducting, albeit by a smaller amount than predicted by the

\[ \frac{\Delta I}{I} = -\frac{\Delta \omega}{\omega} \]
conventional theory. We explained how the demagnetizing field leads to a larger decrease in the speed of surface electrons and to transfer of angular momentum to the body, using the same concepts that we recently used to explain the dynamics of the Meissner effect, in particular the fact that normal state charge carriers have to be hole-like [23].

Within the theory of hole superconductivity, electrons expand their orbits from microscopic radius \( k_F^{-1} \) \((k_F=\) Fermi momentum\) to radius \( 2\lambda_L \) when they pair up and become superconducting [20]. This explains why the diamagnetic susceptibility grows from the normal metal Landau susceptibility to \(-1/(4\pi)\) [27], since the Larmor diamagnetic susceptibility is proportional to the square of the radius of the orbit. In addition this provides a pictorial understanding of the development of macroscopic phase coherence due to overlapping orbits [27]. The fact that the moment of inertia of the body increases by \( m r_0^2 \) per electron when the orbit of the electron increases from essentially zero radius to radius \( r_0 = 2\lambda_L \) follows from simple geometry illustrated in Fig. 5. Fig. 5 shows a point-like particle labeled 1 at radius \( r \) from the center, and another point-like particle labeled 2 that moves in an orbit of radius \( r_0 \) centered at distance \( r \) from the center. Simple geometry shows that the moment of inertia of particle 1 relative to an axis going through the center of the large circle is \( mr^2 \) and that of particle 2 is \( mr^2 + r_0^2 \) (\( m=\) mass of the particles) assuming particle 2 is uniformly distributed along its orbit of radius \( r_0 \).

The reason why the increase in the bulk moment of inertia of the body Eq. (30) is independent of the direction of the rotation axis is easy to understand. When the body rotates, the expanded orbits will orient themselves so that they lie on planes perpendicular to the rotation axis to minimize their energy, as classical particles would.

It was pointed out by Bethe long ago [28] that the moment of inertia of the electronic charge distribution in a solid determines its mean inner electric potential. Shortly thereafter, Rosenfeld [29] pointed out that the mean inner potential is proportional to the diamagnetic susceptibility. In other work we have pointed out [30] that the same physics discussed here, increase in electronic moment of inertia when a normal metal becomes superconducting, should lead to an increase in both its diamagnetic susceptibility (as observed) and in its mean inner potential, which can be measured by electron holography [31] (not yet observed).

Within the theory of hole superconductivity, superconductors have a macroscopically inhomogeneous charge distribution in their ground state, with more negative charge near the surface [32]. One may wonder whether this will give an additional effect at sufficiently low temperatures where the resulting electric field is not screened by normal quasiparticles (that would also cancel the mass imbalance). This was suggested in Ref. [5]. In fact, it will not. The expelled electrons will rotate together with the body and the speed of body rotation will not change. The extra angular momentum of the expelled electrons will be exactly compensated by angular momentum stored in the electromagnetic field and the angular momentum balance discussed in this paper will not change.

We have pointed out that there is an inconsistency in the conventional theory, that assumes that there is no direct interaction between electrons and the body when a superconductor at rest is set into rotation (Appendix A), yet requires a transfer of momentum between electrons and the body when a rotating normal metal becomes superconducting. How is that inconsistency resolved in our theory? In our theory, there can be a net momentum transfer between electrons and the body when there is radial charge flow. Such charge flow occurs in the process of the normal metal becoming superconducting, but not when a superconductor is set into rotation. We can explain both situations where there is no momentum transfer and where there is, provided in the latter case there is also radial charge flow. Within the conventional theory, there never is radial charge flow.

We point out that the physics discussed here is closely related to physics of superfluid \(^4\)He. \(^4\)He has maximum density at the superfluid transition temperature \([33]\). Below the superfluid transition the system expands when cooled further. As a consequence its bulk moment of inertia will increase as it enters the superfluid state, as found here for superconductors. We have argued elsewhere that this commonality between superconductors and superfluid \(^4\)He derives from the fact that both superconductivity and superfluidity are kinetic energy driven, originating in expansion of the wavefunction driven by quantum pressure \([34, 35]\). The physics of the Meissner effect discussed in refs. [22, 23] involving flow of superfluid and counterflow of normal fluid is also closely related to physics found in \(^4\)He that gives rise to the fountain effect [36]. We conjecture that the concepts discussed here may be relevant to the understanding of rotation experiments in superfluid \(^4\)He [37–39].

In summary we find that superconducting matter has a new distinct property. In addition to its electrical conductivity becoming infinite and its magnetic susceptibil-

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**FIG. 5:** Geometric illustration of increase in moment of inertia caused by orbit enlargement (see text).
ity becoming that of a perfect diamagnet, a metal becoming superconducting will increase its bulk moment of inertia per unit volume by the amount \( (m_e c/e)^2 / \pi = 1.03 \times 10^{-15} \text{g/cm}^2 \). This extra moment of inertia arises from the development of an intrinsic moment of inertia for each superfluid electron. Superfluid electrons behave as an extended rim of mass \( m_e \), radius \( 2\lambda_L \), intrinsic moment of inertia \( m_e (2\lambda_L)^2 \), and intrinsic orbital angular momentum \( \hbar / 2 \) [21] rather than as point particles. This leads to a dynamical understanding of the Meissner effect and the London moment, and follows from the quantization of orbital angular momentum in the presence of Dirac’s spin orbit interaction predicted by the theory [21]. Unfortunately as we saw in Sect. VII it appears very difficult in practice to test the different predictions of our theory vis-a-vis the conventional theory for rotating superconductors experimentally.

Appendix A: Derivation of Eq. (1) from Maxwell’s equations

When a superconducting body with cylindrical symmetry is set into rotation, the superfluid electrons obey the equation of motion

\[
m_e \frac{dv_s}{dt} = eE + F_{\text{latt}} \tag{A1}
\]

where the first term is the force on the electrons from the induced azimuthal electric field, the second term is a direct force that may be exerted by the ions on the superfluid electrons, and \( m_e \) is the bare electron mass. The electric field \( E \) is determined by Faraday’s law Eqs. (34) and (35), which yield at radial position \( r \)

\[
E(r, t) = -\frac{1}{2c} \frac{dB(r, t)}{dt}. \tag{A2}
\]

In the interior, superfluid electrons rotate together with the body, i.e. with azimuthal velocity \( v_s = \omega r \). Using this and combining Eqs. (A1) and (A2) then yields

\[
m_e r \frac{d\omega}{dt} = -\frac{e}{2c} \frac{dB(r, t)}{dt} + F_{\text{latt}}. \tag{A3}
\]

Under the assumption that \( F_{\text{latt}} = 0 \), Eq. (A3) integrates to

\[
m_e \omega = -\frac{e}{2c} B \tag{A4}
\]

i.e. Eq. (1), assuming the initial conditions are \( \omega = B = 0 \).

In other words, Eq. (1) is obtained for a superconductor set into rotation under the assumption that there is no direct force \( F_{\text{latt}} \) acting between the ions and electrons, i.e. that the electrons are perfectly free from interactions with the ions. This is precisely what was assumed in the original work by Becker et al [1]: that “die mittlere Geschwindigkeit der Elektronen nur unter der Wirkung eines elektrischen Feldes ändern” (“the mean velocity of the electrons only changes under the effect of an electric field”).

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while it is rotating. The perfect conductor theory would, of course, furnish no reason for the electrons near the surface of the metal to lag suddenly behind when the rotating sphere goes into the superconducting state. It would simply lead to a state of zero magnetic moment in which all charges move in phase - the same below as above the transition point.

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