Properties of asymmetric nuclear matter in different approaches

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Properties of asymmetric nuclear matter are derived from various many-body approaches. This includes phenomenological ones like the Skyrme Hartree-Fock and relativistic mean field approaches, which are adjusted to fit properties of nuclei, as well as more microscopic attempts like the Brueckner-Hartree-Fock approximation, a self-consistent Greens function method and the so-called V_{lowk} approach, which are based on realistic nucleon-nucleon interactions which reproduce the nucleon-nucleon phase shifts. These microscopic approaches are supplemented by a density-dependent contact interaction to achieve the empirical saturation property of symmetric nuclear matter. The predictions of all these approaches are discussed for nuclear matter at high densities in \( \beta \)-equilibrium. Special attention is paid to behavior of the isovector component of the effective mass in neutron-rich matter.

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I. INTRODUCTION

The equation of state for nuclear matter under exotic conditions is one of the main topics in modern nuclear physics. This interest in the properties of nuclear matter at high densities and large proton-neutron asymmetries is partly motivated by the fact that this information is required in theoretical models of compact objects in astrophysics like neutron stars or the simulation of supernovae. However, the study of nuclear systems with large isospin asymmetries are also the subject the forthcoming radioactive-ion-beam facilities such as FAIR at GSI or SPIRAL2 at GANIL.

Different ways have been developed to obtain predictions for the properties of nuclear systems under exotic conditions. One way is to start from phenomenological models which successfully describe the properties of stable nuclei. A very popular approach along this line is the use of an effective density dependent Skyrme-type interaction. Modern Skyrme parameterizations have been developed, which were constrained in their fitting procedures to obtain results for neutron-rich nuclear matter compatible to those of microscopic calculations. Here we mention the Skyrme-Lyon (SLy) forces, which have been used in studies of the neutron-star crust.

Also the relativistic mean-field approximation has very successfully been used to describe the properties of stable nuclei. Attempts have been made to derive a set of density-dependent meson-nucleon coupling constants of density dependent relativistic mean field (DDRMF) calculations from microscopic Dirac Brueckner Hartree Fock calculations with a readjustment in order to reproduce the bulk properties of stable nuclei and the saturation point of symmetric nuclear matter.

The parameters of such Skyrme Hartree Fock and DDRMF calculations have been fitted to the data of stable nuclei and the predictive power of these simple phenomenological approaches could be rather limited. This would mean that the predictions for nuclear systems with exotic values for density and proton-neutron asymmetries may not be very reliable.

Therefore we will also consider some so-called microscopic approaches, which start from models of the nucleon-nucleon (NN) interaction, which are adjusted to describe the experimental phase shifts of NN scattering at energies below the pion threshold. The traditional models of such realistic NN interactions like e.g. the charge-dependent Bonn potential CDBONN or the Argonne potential V18 contain rather strong short range components, which make it inevitable to employ non-perturbative approximation schemes for the solution of the many-body problem for the nuclear hamiltonian based on such interactions. Such non-perturbative approximations include the Brueckner hole-line expansion with the Brueckner Hartree Fock (BHF) approximation, the self-consistent evaluation of Green’s function using the T-matrix approximation (SCGF) and also variational approaches using correlated basis functions.

If such microscopic calculations would reproduce the properties of nuclear systems under normal conditions, one could argue that a scheme which reproduce the data of two nucleons in the vacuum (NN phase shifts) as well as nuclear data at normal densities should also provide reliable results for nuclear systems at densities beyond the saturation density of nuclear matter. Unfortunately, however, such microscopic calculations fail to reproduce the saturation point of symmetric nuclear matter or the bulk properties of finite nuclei with good precision. For example in nuclear matter such calculations yield results for the saturation point, which are located on the so-called Coester band, i.e. they either yield too little binding energy or a saturation density well above the empirical value of \( \rho_0 = 0.16 \text{ fm}^{-3} \).
In recent years large progress has been made developing tools for essentially exact calculations for nuclear system with mass number up to about $A = 126$. These calculations demonstrate that using realistic models for the NN interaction in a non-relativistic hamiltonian precise results for nuclear systems are obtained if and only if these realistic two-body interactions are supplemented by a three-body force.

Therefore we will consider in the present study the results of BHF and SCGF calculations, adding a simple density-dependent contact interaction which is adjusted to describe the saturation point of symmetric nuclear matter.

During the last years a different calculation scheme has evolved, which is also based on the NN scattering data but tries to decouple the low- and high-momentum components in the nuclear hamiltonian using renormalization group methods\textsuperscript{[27, 28, 29, 30]}. The interaction resulting for the low-momentum regime of nuclear structure, which we will refer to as $V_{\text{lowk}}$ is rather soft, which implies that non-perturbative tools of many-body theory can be applied\textsuperscript{[31]}. Also it is very attractive, that $V_{\text{lowk}}$ turns out to be essentially model-independent, if the cut-off for the low-momentum regime is chosen appropriately.

However, evaluating the properties of nuclear matter by using $V_{\text{lowk}}$ in a Hartree-Fock or BHF calculation, one does not obtain a saturation point\textsuperscript{[28, 32]}, the binding energy per nucleon increases with increasing density and nuclear systems tend to collapse to high densities. This will be compensated by adding a density-dependent contact interaction, which is adjusted in the same fashion as for the BHF and SCGF cases discussed above.

Therefore we will apply two different schemes: two phenomenological methods (Skyrme Hartree Fock and relativistic mean field DDRMF) employing parameterizations, which tend to reproduce some features of microscopic calculations and three microscopic approaches (BHF, SCGF and Hartree-Fock with $V_{\text{lowk}}$), which are supplemented by a simple contact interaction to reproduce the empirical saturation properties of symmetric nuclear matter. For all these methods we study bulk features of nuclear matter at large densities and proton neutron asymmetries. A comparison of the resulting values for nuclear compressibility, symmetry energy, proton fraction in $\beta$ equilibrium and effective masses, which characterize the density of states around the Fermi energy, should provide information about the uncertainties in the extrapolation of nuclear properties in exotic regions of density and asymmetry. Also we expect some hints about the reliability and problems of the individual approaches.

After this introduction the section two shall briefly outline the different approximation schemes which we are going to consider. The results are discussed in section three and the main conclusions are summarized in the final section.

\section*{II. DIFFERENT MANY-BODY APPROACHES}

\subsection*{A. Skyrme Hartree-Fock}

A very popular many-body approach in nuclear physics is the Skyrme-Hartree-Fock approach which can be found e.g. in \textsuperscript{[1, 2, 3, 4]}. The Skyrme interaction leads to an energy functional

$$E = \int \mathcal{H}(r) \, dr,$$

where $\mathcal{H}$ is the Hamiltonian density in the Hartree-Fock approximation. In case of infinite asymmetric nuclear matter the Hamiltonian density writes \textsuperscript{[3, 13]}

$$\mathcal{H} = \mathcal{H}_K + \mathcal{H}_{\text{eff}} + \mathcal{H}_0 + \mathcal{H}_3$$

where $\mathcal{H}_K$ is the kinetic energy term, $\mathcal{H}_{\text{eff}}$ an effective mass term, $\mathcal{H}_0$ a zero range term and $\mathcal{H}_3$ a density dependent term. These terms are given by

$$\begin{align*}
\mathcal{H}_K &= \frac{\hbar^2}{2m} \tau, \\
\mathcal{H}_{\text{eff}} &= \frac{1}{8} \left[ t_1(2 + x_1) + t_2(2 + x_2) \right] \tau \rho \\
&\quad + \frac{1}{8} \left[ t_2(2x_2 + 1) - t_1(2x_1 + 1) \right] \left[ \tau \rho_p + \tau \rho_n \right], \\
\mathcal{H}_0 &= \frac{1}{4} t_0 \left[ 2 + x_0 \right] \rho^2 - (2x_0 + 1)(\rho_p^2 + \rho_n^2), \\
\mathcal{H}_3 &= \frac{1}{24} t_3 \rho^\alpha \left[ (2 + x_3) \rho^2 - (2x_3 + 1)(\rho_p^2 + \rho_n^2) \right],
\end{align*}$$

where the coefficients $t_i$, $x_i$, $W_0$, and $\alpha$ are the parameters of a generalized Skyrme force. In the present study we apply the commonly used parameterization SLy4.
The densities $\rho$ and $\tau$ are defined in terms of the corresponding densities for protons and neutrons $\rho = \rho_p + \rho_n$ and $\tau = \tau_p + \tau_n$. If we identify the isospin label ($i = p, n$), the matter densities for protons and neutrons are given by

$$\rho_i = 2 \int_{k_F}^{k_F} \frac{d^3k}{(2\pi)^3} = \frac{1}{3\pi^2}k_F^3, \quad (4)$$

where $k_F$ denotes the corresponding Fermi momentum and the spin degeneracy gives a factor of 2. We obtain the kinetic energy density by

$$\tau_i = 2 \int_{k_F}^{k_F} \frac{d^3k}{(2\pi)^3} k^2 = \frac{1}{5\pi^2}(3\pi^2 \rho_i)^{5/3}. \quad (5)$$

The Hamiltonian density leads to the energy per nucleon $E/A = \mathcal{H}/\rho$ and the single particle energy may be written as a function of momentum $k$

$$\varepsilon_i(k) = \frac{k^2}{2m^*_i} + V_i \quad (6)$$

with an effective mass $m^*_i$ and a density dependent Potential $V_i$.

**B. Brueckner-Hartree-Fock**

Starting from realistic NN interactions, we have to use more advanced many-body approximations like the Brueckner-Hartree-Fock (BHF) approach, which have the capability to account for the effects of correlations, which are due to the strong tensor and short-range components of such realistic NN interactions. In the BHF approximation this is achieved by evaluating the so-called $G$-matrix, which corresponds to the in-medium scattering matrix. The self-energy of a nucleon with isospin $i$, momentum $k$ and energy $\omega$ in asymmetric nuclear matter is defined in the BHF approximation by $[13, 20]$

$$\Sigma_{ij}^{BHF} = \sum_j \int d^3q \langle k|G(\Omega)|k\rangle_{ij} n_j^0(q). \quad (7)$$

In this equation $n_j^0(q)$ refers to the occupation probability of a free Fermi gas of protons ($j = p$) and neutrons ($j = n$) like in the mean-field or Hartree-Fock approach. This means that for asymmetric matter with a total density $\rho = \rho_p + \rho_n$ this probability is defined by

$$n_j^0(q) = \begin{cases} 1 \text{ for } |q| \leq k_{Fj}, \\ 0 \text{ for } |q| > k_{Fj}, \end{cases} \quad (8)$$

with Fermi momenta for protons ($k_{Fp}$) and neutrons ($k_{Fn}$).

The antisymmetrized $G$ matrix elements in eq. (7) are obtained from a given NN interaction by solving the Bethe-Goldstone equation

$$\langle k|G(\Omega)|k\rangle_{ij} = \langle k|V|k\rangle_{ij} + \int d^3p_1 d^3p_2 \langle k|V|p_1p_2\rangle_{ij} \times \frac{Q(p_{1i}, p_{2j})}{\Omega - (\varepsilon_{p_{1i}} + \varepsilon_{p_{2j}}) + i\eta} \times \langle p_1p_2|G(\Omega)|k\rangle_{ij}. \quad (9)$$

The single-particle energies $\varepsilon_{pi}$ of the intermediate states should be the corresponding BHF single-particle energies which are defined in terms of the real part of the BHF self-energy of eq. (7) by

$$\varepsilon_{ki} = \frac{k^2}{2m^*_i} + \text{Re} \left[ \Sigma^{BHF}_i(k, \omega = \varepsilon_{ki}) \right], \quad (10)$$

with a starting energy parameter $\Omega = \omega + \varepsilon_{qj}$ in the Bethe-Goldstone equation (9).
The Pauli operator \( Q(p_1, p_2) \) restricts the intermediate states to particle states with momenta \( p_1, p_2 \), which are above the corresponding Fermi momentum. However, the single-particle spectrum is often parameterized in the form of an effective mass

\[
\varepsilon_{ki} \approx \frac{k^2}{2m_i^*} + U_i,
\]

so that a so-called angle-averaged propagator can be defined, which reduces the Bethe-Goldstone equation to an integral equation in one dimension. The exact Pauli operator has been treated in [33].

C. Relativistic models

An interesting extension of the BHF approach is the Dirac-Brueckner-Hartree-Fock (DBHF) approach which accounts for relativistic effects as well as correlation effects as described in terms of the G matrix. In the DBHF approximation one evaluates the self-energy \( \Sigma_i(k) \) in a way very similar to the BHF approximation keeping track of the Lorentz structure. Attempts have been made to fit a density dependent Relativistic Mean Field model (DDRMF) to the results of such DBHF calculations [10, 34]. Here we employ such a DDRMF model which has been obtained by fitting density dependent coupling constants for the meson-nucleon vertices to reproduce the self-energy of the DBHF calculations of van Dalen et al. [35], which was based on the Bonn A potential.

Both approaches, the DDRMF as well as the DBHF, start from a Lagrangian, which includes baryons, mesons and the interaction

\[
\mathcal{L} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{\text{int}}.
\]

However, they differ in the included mesons and the coupling operators [10, 35, 36, 37]. In the DDRMF model the \( \sigma, \delta, \omega, \) and \( \rho \) mesons are included in scalar and vector couplings, respectively. The variation of the Lagrangian leads to a Dirac equation for the nucleons

\[
[\gamma \cdot k_i^* + m_{Di}^*] u(k, s, i) = \gamma_0 E_i^* u(k, s, i),
\]

where \( u(k, s, i) \) denotes the plane wave Dirac spinor with momentum \( k \), spin \( s \) and isospin \( i \)

\[
u(k, s, i) = \sqrt{k_i^* + m_i^* + \Sigma_0, i},
\]

The starred quantities contain the different components of the nucleon self-energy: the scalar, time-like vector and space-like contributions

\[
m_i^* = M + \Sigma_{S,i}(k, k_F),
\]

\[
E_i^* = E_i(k) - \Sigma_{0,i}(k, k_F),
\]

where \( \hat{k}_i \) is the unit vector along the momentum \( k_i \) of the nucleon. The general form of the nucleon self-energy in infinite matter is obtained by evaluation of the meson exchange in spin saturated nuclear matter

\[
\Sigma_i(k, k_F) = \Sigma_{S,i}(k, k_F) + \gamma_0 \Sigma_{0,i}(k, k_F) + \gamma \cdot \hat{k}_i \Sigma_V,i(k, k_F).
\]

In the fitting process special attention has to be payed to the rearrangement contribution to the time-like vector self-energy [38], the relativistic operator structure [39], and the proper renormalization due to the space-like vector contribution to the self-energy [10].

The single particle energy in the DDRMF model is obtained from the Dirac equation [13]

\[
E_i(k) = \sqrt{k_i^* + m_i^* + \Sigma_{0,i}},
\]

and the energy-momentum tensor leads to the energy density in asymmetric nuclear matter

\[
\mathcal{E} = \langle T_{00} \rangle = \frac{1}{\pi^2} \sum_{i=p,n} \int_0^{k_F,i} k^2 dk \sqrt{k_i^* + m_{Di}^*} + \frac{1}{2} \sum_{i=p,n} (\Sigma_{S,i} \rho_i^* + \Sigma_{0,i} \rho_i),
\]
with the scalar densities $\rho_i^s$ and the baryon densities $\rho_i$

$$\rho_i^s = \frac{1}{\pi^2} \int_{0}^{k_{F,i}} k^2 \, dk \, \frac{m^*_{Di} \, \rho_{Di}}{E_i(k)}$$

$$\rho_i = \frac{1}{\pi^2} \int_{0}^{k_{F,i}} k^2 \, dk.$$

(18)

D. Self-consistent Green’s function

One of the drawbacks of the BHF approximation is the fact that it does not provide results for the equation of state, which are consistent from the point of view of thermodynamics. As an example we mention that BHF results do not fulfill e.g. the Hugenholtz van Hove theorem. This is due to the fact that the BHF approximation does not consider the propagation of particle and hole states on equal footing. An extension of the BHF approximation, which obeys this symmetry is the self-consistent Green’s function (SCGF) method using the so-called $T$-matrix approximation. During the last years techniques have been developed, which allow to evaluate the solution of the SCGF equations for microscopic NN interactions [15, 16, 17, 18, 19]. Those calculation demonstrate that for the case of realistic NN interactions, the contribution of particle-particle ladders dominates the contribution of corresponding hole-hole propagation terms. This justifies the use of the BHF approximation and a procedure, which goes beyond BHF and accounts for hole-hole terms in a perturbative way [40, 41]. This leads to a modification of the self-energy in the BHF approximation by adding a hole-hole term of the form

$$\Delta \Sigma_2^{hh}(k, \omega) = \sum_j \int_{k_{F,j}}^{\infty} d^3 p \int_{0}^{k_{F,i}} d^3 h_1 \int_{0}^{k_{F,j}} d^3 h_2 \times \langle kp|G(\Omega)|h_1 h_2\rangle^2_{ij} \frac{(\omega + \epsilon_{pj} - \epsilon_{h_i} - \epsilon_{h_j} - i\eta)}{\omega - k^2/2m - \text{Re} \Sigma_i(k, \omega)^2 + |\text{Im} \Sigma_i(k, \omega)|^2},$$

(19)

The quasi-particle energy for the extended self-energy can be defined as

$$\varepsilon_{qp}^{kj} = \frac{k^2}{2m} + \text{Re} [\Sigma_i^{BHF}(k, \omega = \varepsilon_{qp}^{kj}) + \Delta \Sigma_2^{2hh}(k, \omega = \varepsilon_{qp}^{kj})],$$

(20)

Accordingly, the Fermi energy is obtained evaluating this definition at the Fermi momentum $k = k_{F,i}$ for protons and neutrons, respectively,

$$\varepsilon_{F_i} = \varepsilon_{qp}^{k_{F,i}}.$$  

(21)

The spectral functions for hole and particle strength, $S_i^h(k, \omega)$ and $S_i^p(k, \omega)$, are obtained from the real and imaginary part of the self-energy $\Sigma = \Sigma_i^{BHF} + \Delta \Sigma_i^{2hh}$

$$S_i^{h(p)}(k, \omega) = \pm \frac{1}{\pi} \frac{\text{Im} \Sigma_i(k, \omega)}{\omega - k^2/2m - \text{Re} \Sigma_i(k, \omega)^2 + |\text{Im} \Sigma_i(k, \omega)|^2},$$

(22)

where the plus and minus sign on the left-hand side of this equation refers to the case of hole ($h, \omega < \varepsilon_{F_i}$) and particle states ($p, \omega > \varepsilon_{F_i}$), respectively. The hole strength represents the probability that a nucleon with isospin $i$, momentum $k$, and energy $\omega$ can be removed from the ground state of the nuclear system with the removal energy $\omega$, whereas the particle strength denotes the probability that such a nucleon can be added to the ground state of the system with $A$ nucleons resulting in a state of the $A+1$ particle system which has an energy of $\omega$ relative to the ground state of the $A$ particle system. Hence the occupation probability is obtained by integrating the hole part of the spectral function

$$n_i(k) = \int_{-\infty}^{\varepsilon_{F_i}} d\omega \, S_i^h(k, \omega).$$

(23)

Note that this yields values for the occupation probability, which ranges between values of 0 and 1 for all momenta $k$, leading to a partial depletion of the hole-states in the Fermi gas model ($k < k_{F}$) and partial occupations for states
with momenta $h > k_F$. A similar integral yields the mean energy for the distribution of the hole and particle strength, respectively

$$
\langle \varepsilon_{h\tau}(k) \rangle = \frac{\int_{-\infty}^{\xi_{F\tau}} d\omega \omega S^h_0(k, \omega)}{n_i(k)}, \quad (24)
$$

$$
\langle \varepsilon_{p\tau}(k) \rangle = \frac{\int_{\xi_{F\tau}}^{\infty} d\omega \omega S^p_0(k, \omega)}{1 - n_i(k)}. \quad (25)
$$

Our self-consistent Green’s function calculation is defined by identifying the single particle energy in the Bethe-Goldstone equation as well as in the $2h1p$ correction term in eq. (19) by

$$
\varepsilon_{k\tau} = \begin{cases} 
\langle \varepsilon_{h\tau}(k) \rangle & \text{for } k < k_{F\tau}, \\
\langle \varepsilon_{p\tau}(k) \rangle & \text{for } k > k_{F\tau}.
\end{cases} \quad (26)
$$

This definition leads to a single particle Greens function, which is defined for each momentum $k$ by just one pole at $\omega = \varepsilon_{k\tau}$. Hence in the calculation of the self-energy the mean value of the spectral distribution is considered. However, the modified occupation of nucleons obtained by the spectral functions are not included in the calculation of the self-energies. The total energy per nucleon is expressed by

$$
\frac{E}{A} = \sum_i \int d^3k \int_{-\infty}^{\xi_{F\tau}} d\omega \omega S^h_0(k, \omega)(k^2/2m + \omega)/2 \sum_i \int d^3k n_i(k). \quad (27)
$$

E. Renormalization of the NN interaction

It is very reasonable to assume that the long-range or low-momentum part of the NN interaction is fairly well described in terms of meson-exchange, while different (quark) degrees of freedom are getting important to describe the short-range or high-momentum components of the NN interaction. Therefore it is quite attractive to disentangle these low-momentum and high-momentum components from each other. This means that one tries to define a model space, which accounts for the low-momentum degrees of freedom and renormalizes the effective Hamiltonian for this low-momentum regime to account for the effects of the high-momentum parts, which are integrated out.

This concept of a model space and effective operators appropriately renormalized for this model space has a long history in approaches to the nuclear many-body physics. As an example we mention the effort to evaluate effective operators to be used in Hamiltonian diagonalization calculations of finite nuclei. For a review on this topic see e.g. [42].

During the last years this concept has received a lot of attention and led to the definition of the so-called $V_{\text{lowk}}$ interaction. One way to determine this interaction is to follow the unitary-model-operator approach (UMOA) [46]. We follow the usual notation and define a projection operator $P$, which projects onto the model space of two-nucleon wave functions with relative momenta $k$ smaller than a chosen cut-off $\Lambda$. The operator projecting on the complement of this subspace is identified by $Q$ and these operators satisfy the usual relations like $P + Q = 1$, $P^2 = P$, $Q^2 = Q$, and $PQ = 0 = QP$. It is the aim of the Unitary Model Operator Approach (UMOA) to define a unitary transformation $U$ in such a way, that the transformed Hamiltonian does not couple the $P$ and $Q$ space, i.e.

$$
QU^{-1}HUP = 0. \quad (28)
$$

The technique to determine this unitary transformation is nicely been described by Fujii et al. [47] (see also [31]). It leads to an effective Hamiltonian $H_{\text{eff}} = h_0 + V_{\text{eff}}$, which contains the term of the kinetic energy $h_0$ and an effective interaction $V_{\text{eff}}$ given by

$$
V_{\text{eff}} = V_{\text{lowk}} = U^{-1}(h_0 + V)U - h_0. \quad (29)
$$

Diagonalising this effective Hamiltonian in the low-momentum model-space, one obtains eigenvalues which are identical to the diagonalisation of the original Hamiltonian $h_0 + V$ in the complete space. This means solving the Lipmann Schwinger equation for NN scattering using this $V_{\text{lowk}}$ with a cut-off $\Lambda$ yields the same phase shifts as obtained for the realistic interaction $V$ without a cutoff. One finds that the resulting $V_{\text{lowk}}$ is essentially independent of the underlying realistic interaction $V$, if is fitted to the experimental phase shifts and if the cut-off $\Lambda$ is chosen around $\Lambda = 2$ fm$^{-1}$, which means that the model space includes scattering up to the pion threshold. In that sense $V_{\text{lowk}}$ is unique and, as it reproduces the NN scattering phase shifts it can also be regarded as a realistic interaction like e.g. the CDBONN or Argonne V18 interactions.
FIG. 1: (Color online) Comparison of binding energy per nucleon of symmetric nuclear matter as obtained from Skyrme SLy4, DDRMF, BHF, SCGF, and Vlowk. Results of approaches based on realistic NN interactions are also compared with an additional contact interaction of the form displayed in eq. (30).

Since, however, the high-momentum or short-range components have been integrated out by means of the unitary transformation of eq. (28), the $V_{\text{lowk}}$ does not induce any short-range correlations into the nuclear wave function. This leads to the nice feature that mean-field calculations using $V_{\text{lowk}}$ lead to reasonable results and corrections of many-body theories beyond mean field are weak [31]. On the other hand, however, it is this lack of short-range correlation effects, which are modified in the medium, which prevents the emergence of a saturation point in calculations of symmetric nuclear matter [28, 32].

In order to achieve saturation in nuclear matter one has to add three-body interaction terms or a density-dependent two-nucleon interaction. This is not very astonishing as it is known that a renormalization of a two-body operator leads to many-body terms [48, 49]. Therefore it is quite natural to supplement the effective interaction $V_{\text{lowk}}$ by a simple contact interaction, which we have chosen following the notation of the Skyrme interaction to be of the form

$$\Delta H = \frac{1}{2}t_0 \rho^2 + \frac{1}{12}t_3 \rho^{2+\alpha},$$

where $\rho$ is the matter density, $t_0$, $t_3$ and $\alpha$ are parameters. For a fixed value of $\alpha$ (typically $\alpha = 0.5$) we have fitted $t_0$ and $t_3$ in such a way that a Hartree-Fock calculation using $V_{\text{lowk}}$ plus the contact term of eq. (30) yields the empirical saturation point for symmetric nuclear matter.

The same parameterization of a contact term has been used to evaluate corrections to the self-energy of BHF and SCGF in such a way that also these calculations reproduce the saturation of symmetric nuclear matter. Note that this contact term is an isoscalar term and does not influence the symmetry energy, proton fractions in $\beta$-equilibrium, and effective masses.

III. RESULTS AND DISCUSSION

All results of calculations, which refer to realistic NN interactions, have been obtained using the CDBONN [11] interaction. This includes all BHF and SCGF calculations. Also the evaluation of $V_{\text{lowk}}$ has been based on the proton-neutron part of CDBONN. Using a cut-off parameter $\Lambda = 2$ fm$^{-1}$, these results do not significantly depend on the underlying interaction. The Skyrme Hartree-Fock calculations have been done using the parameterization SLy4 and for the relativistic mean-field calculation the parameterization for DDRMF in [10] has been used.

First let us turn to the binding energy of symmetric nuclear matter, which are displayed in Fig. 1. Compared to other realistic NN interactions the CDBONN potential, which we have chosen here is a rather soft NN interaction with a weak tensor force. This is indicated by the results for the saturation point of symmetric nuclear matter as
| $\rho_0$ [fm$^{-3}$] | SLy4 | DDRMF | BHF | BHF(+ct) | SCGF | SCGF(+ct) | $V_{lowk}$ + ct |
|-----------------|-------|-------|-----|---------|------|----------|----------------|
| 0.160           | 0.178 | 0.374 | 0.161| 0.212   | 0.160| 0.160    |
| $E/A(\rho_0)$ [MeV] | -15.97 | -16.25 | -23.97 | -16.01 | -11.47 | -16.06 | -16.0 |
| $K$ [MeV]       | 230   | 337   | 286  | 214     | 203  | 270      | 258            |
| $a_S(\rho_0)$ [MeV] | 32.0 | 32.1  | 51.4 | 31.9    | 34.0 | 28.3     | 21.7           |

TABLE I: Properties of symmetric nuclear matter are compared for Skyrme SLy4, DDRMF, BHF, SCGF, and $V_{lowk}$. The results, which are listed in the columns labeled with +ct are obtained employing the additional contact interaction of eq.30 with parameters as listed in table II. The quantities listed include the saturation density $\rho_0$, the binding energy at saturation $E/A$, the compressibility modulus $K$ and the symmetry energy at saturation density $a_S(\rho_0)$.

| $t_0$ [MeV fm$^{-3}$] | BHF | SCGF | $V_{lowk}$ |
|-----------------------|-----|------|------------|
| -153                  | -311| -438.1|
| $t_3$ [MeV fm$^{-3}$] | 2720| 3670 | 6248 |

TABLE II: Parameters $t_0$ and $t_3$ defining the contact interaction of eq.30 as obtained for the fit to the saturation point $\rho = 0.16f m^{-3}$ and $E/A = -16.0$ MeV at $\alpha = 0.5$ for various realistic approaches.

Some of these results for the EoS for BHF, SCGF and $V_{lowk}$ are rather sensible to the choice of the exponent $\alpha$ in the $t_3$ term of the contact interaction. A larger value of $\alpha$ would lead to a softer EoS. We have picked the value $\alpha = 0.5$ to obtain results for the EoS, which are similar to those resulting from the empirical approaches. Note, that the choice of $\alpha$ does not have any effect on the results referring to proton neutron asymmetries, which is the main focus of this study.

Table II also displays results for the symmetry energy

$$a_S(\rho) = \frac{\partial(E/A)}{\partial I^2} \bigg|_\rho,$$  

$$I = \frac{N - Z}{A} = 1 - 2Y_p,$$  

(32)
evaluated for each approach at the corresponding saturation density $\rho_0$. The two phenomenological approaches SLy4 and DDRMF yield results which are in the range of the experimental value of $32 \pm 1$ MeV. Also the BHF and SCGF approach lead to results which are rather close to the empirical value, if the contact term has been added. The BHF and SCGF calculations without the contact term lead to non-realistic values for $a_S(\rho_0)$ since these values are calculated at the corresponding saturation densities, which are larger than the empirical saturation density.

The symmetry energy calculated in the SCGF approach is slightly smaller than the one obtained from the BHF approximation. This is valid for all densities under consideration (see Fig. 2). This difference can easily be explained: As we already mentioned above, the contribution of the hole-hole terms is repulsive, which leads to larger energies for SCGF as compared to BHF for all densities in symmetric nuclear matter (Fig. 1) as well as in pure neutron matter (see Fig. 3). Since, however, the contribution of ladder diagrams is larger in the proton-neutron interaction (due to the strong tensor terms in the $^3S_1 - ^3D_1$ partial wave) than in the neutron-neutron interaction, this repulsive effect is stronger in symmetric nuclear matter than in neutron enriched matter. Therefore the symmetry energy calculated in SCGF is slightly smaller if the hole-hole terms are included in SCGF\[50\].

The symmetry energy at saturation density obtained from $V_{\text{lowk}}$ plus contact term is only about two third of the experimental value (see table I) and the value is significantly below the other two microscopic approaches also at higher densities. This can be understood from the following considerations: The Hartree-Fock calculations using $V_{\text{lowk}}$ do not account for the attractive contributions due to the NN ladder terms involving NN states with relative momenta below the cut-off $\Lambda$. This missing attraction is compensated by the fit of the contact interaction to the empirical saturation point of symmetric matter. While the contact interaction is chosen to be identical for proton-neutron and neutron-neutron interaction, the ladder terms are more attractive for the isospin $T = 0$ partial waves (see above), i.e. the proton-neutron interaction. This leads to a significant underestimate for the symmetry energy at all densities. Note, however, that this failure of $V_{\text{lowk}}$ should disappear if $V_{\text{lowk}}$ would be employed in an appropriate many-body calculation beyond the mean field approximation.

The symmetry energy rises as a function of density for all approaches considered. Note, however, that the two phenomenological approaches Skyrme Hartree-Fock using SLy4 and DDRMF provide rather different predictions at high densities although the symmetry energy at normal density is identical. The relativistic approach predicts symmetry energies for high densities, which are well above all those derived from the microscopic calculations, while the Skyrme interaction yields a symmetry energy which is even below the $V_{\text{lowk}}$ estimate at densities above four times saturation density.

Rather similar features also observed, when we inspect the properties of nuclear matter in $\beta$-equilibrium, neutralizing the charge of the protons by electrons, displayed in Fig. 4. The upper panel of this figure displays the proton abundance $Y_p = Z/A$, which are to some extent related to the symmetry energy: large symmetry energy should correspond to large proton abundances. So the largest proton abundances are predicted within the DDRMF approach. Already at a density around $0.4$ fm$^{-3}$ $Y_p$ exceeds the about 10%, which implies that the direct URCA process could be enabled, which should be reflected in a fast cooling of a neutron star.
FIG. 3: (Color online) Energy per nucleon of pure neutron matter as a function of density as obtained from Skyrme SLy4, DDRMF, BHF, SCGF, and $V_{\text{lowk}}$ approaches.

FIG. 4: (Color online) Results for a system of infinite matter consisting of protons, neutrons and electrons in $\beta$-equilibrium. The upper panel show the proton abundances and the lower panel displays the energy per nucleon as a function of density using the various approximation schemes discussed in the text.
The other extreme case is the prediction derived from SLy4. In this approach the proton abundance does not exceed a value of 6%.

The $V_{\text{lowk}}$ and SCGF approaches lead to similar proton abundances at large densities. This demonstrates that the evaluation of the proton abundance in $\beta$-equilibrium cannot directly be deduced from the symmetry energy, since the former observable is derived from proton and neutron energies at large asymmetries ($Z << N$), whereas the symmetry energy is calculated from the second derivative at $N = Z$ (see eq. (32)). The BHF approach shows slightly lower values for $Y_p$ at high density, but the results are still in the same range as SCGF and $V_{\text{lowk}}$.

At low densities the Skyrme HF approach yields large proton fractions as compared to the results of the other calculations. Large proton fractions at low densities tend to enhance density inhomogeneities and thus favor the existence of a large variety of pasta structures. Therefore the Skyrme HF (Sly4) and the DDRMF approach, which have been explored in detail in [8, 10], should favor the formation of pasta structures as compared to the microscopic approaches.

Comparing the energies of matter in $\beta$-equilibrium derived from the various approaches as a function of density (Fig. 4, lower panel) we find the same trends as in the case of pure neutron matter displayed in Fig. 5. The absolute values are lower in the case $\beta$-equilibrium (about 75% of the energies for neutron matter). Furthermore the relative differences between the various approaches are smaller. While in the case of neutron matter the energy differences between the various predictions are as large as 50% of the mean value, the corresponding number for matter in $\beta$-equilibrium is only around 25%. The approximation schemes leading to large energies for neutron matter also show large symmetry energies, which result into relatively large proton abundances and smaller energies for matter in $\beta$-equilibrium.

The equation of state of nuclear matter in $\beta$-equilibrium is the main input to predict mass and radii of neutron stars. A stiffer equation of state supports a larger maximum mass and a lower central density. In addition a thicker crust is found for the stiffer equation of state.[51]

Another important information for the evaluation of dynamical features of matter in neutron stars is the density of states, which can be characterized by an effective mass. The term effective mass is used in various connections in many-body physics. This includes the effective Dirac mass of the relativistic mean field approach $m_D^*$ (see eq. (15)), as well as effective masses, which express the non-locality of the self-energy in space and time, which corresponds to a momentum and energy dependence. The density of states, however, is related to the single-particle spectrum close to the Fermi energy, which in the case of nuclear matter can be parameterized in terms of an effective mass by the expression

$$\epsilon(k) = \frac{k^2}{2m^*} + U .$$

Such effective masses for protons and neutrons determined for nuclear matter in $\beta$-equilibrium are displayed in Fig. 5 as a function of density considering non-relativistic approximation schemes.
It is a general feature of all approaches considered that the effective masses for protons as well as neutrons decrease with increasing density. However, there is a striking difference between the phenomenological Skyrme approximation and the BHF and $V_{\text{lowk}}$ approach, which are based on realistic NN interactions: The effective mass for protons is smaller than the corresponding one for neutrons in neutron rich matter for the calculations using realistic interactions, while it is opposite applying the Skyrme parameterization. In fact, if we define the effective masses for protons $m^*_p$ and neutrons $m^*_n$ in terms of isoscalar $m^*_S$ and isovector masses $m^*_V$, by

$$\frac{1}{m^*_n} = \frac{1}{m^*_S} + I \left( \frac{1}{m^*_S} - \frac{1}{m^*_V} \right)$$

$$\frac{1}{m^*_p} = \frac{1}{m^*_S} - I \left( \frac{1}{m^*_S} - \frac{1}{m^*_V} \right)$$

with $I = \frac{N - Z}{A}$, (34)

it turns out most of the Skyrme parameterizations yield an effective isovector mass $m^*_V$, which is even larger than the bare nucleon mass $M$, which implies that it is larger than the effective isoscalar mass $m^*_S$. This means that the effective mass for neutrons is smaller than the corresponding one for the protons in neutron rich matter ($I > 0$). These Skyrme parameterizations leading to a large effective isovector mass are usually favored as they correspond within the mean-field approach to an enhancement factor $\kappa$ of the Thomas-Reiche-Kuhn sum-rule \[4, 52\]. Attempts have been made to distinguish between effective masses, which describe the energies around the Fermi energy, and those characterizing the bulk spectrum by introducing a term, which leads to a surface peaking of the effective mass term in finite nuclei \[53\].

Non-relativistic descriptions of nuclear matter, which are based on realistic interactions yield an effective isovector mass $m^*_V$ which is smaller than the corresponding effective isoscalar mass, which leads to a larger effective mass for neutrons than for protons in neutron-rich matter (see Fig. 5). In order to analyze this finding we inspect the dependence of the nucleon self-energy in the BHF approximation $\Sigma^{BHF}$, defined in eq. (7), as a function of energy $\omega$ and momentum $k$ of the nucleon considered. Following the discussion of Mahaux and Sartor \[52\] one can define the effective $k$-mass

$$m_k(k) = \left[ 1 + \frac{M}{k} \frac{\partial \Sigma(k, \omega)}{\partial k} \right]^{-1}$$

and the effective $E$-mass

$$m_E(\omega) = \left[ 1 - \frac{\partial \Sigma(k, \omega)}{\partial \omega} \right].$$

The effective mass can then be calculated from the effective $k$-mass and the effective $E$-mass by

$$m^*(k) = \frac{m^*_k(k)}{M} m^*_E(\omega = \varepsilon(k)).$$

(35)

(36)

Results for the effective $k$-mass and $E - mass$ as obtained from BHF calculations for asymmetric nuclear matter at a density $\rho = 0.17 \text{ fm}^{-3}$ and a proton abundance $Y_p$ of 25% ($I = 0.5$) are displayed in Fig. 6. We notice that the effective $k$-mass for the protons is significantly below the corresponding value for the neutrons at all momenta. Since the $k$-masses tend to increase as a function of the nucleon momentum $k$, the difference in the Fermi momenta for protons and neutrons enhance the difference $m^*_{k,n}(k_{F,n}) - m^*_{k,p}(k_{F,p})$.

The effective $k$-mass describes the non-locality of the BHF self-energy. This non-locality and thereby also these features of the effective $k$-mass are rather independent on the realistic interaction used. Furthermore it turns out that the values for the $k$-mass are essentially identical if one derives them from the nucleon BHF self-energy using the $G$-matrix or from the bare interaction $V$ or from $V_{\text{lowk}}$ \[41\]. This non-locality of the self-energy is dominated by Fock-exchange contribution originating from $\pi$-exchange. In neutron-rich matter this contribution leads to a stronger depletion for the proton mass than for the neutron mass \[20, 55\].

The effective $E$-mass, representing the non-locality of the self-energy in time, yield values larger than $M$ for momenta around $k_{F}$. Also in this case the deviation of $m^*$ from $M$ is larger for protons than for neutrons. The effective $E$-mass originates from the energy-dependence of the $G$-matrix and is due to the admixture of 2-particle 1-hole configurations to the single-particle states. One may also say that the effective $E$-mass is due to correlations beyond mean field. It accounts to the coupling of vibrational modes.

Anyway, the enhancement of the effective mass $m^*$, which is due to the effective $E$-mass in eq.(37) is not strong enough to compensate the effects of the $k$-mass. Therefore the final effective mass is below the bare mass $M$ and the effective mass for neutrons remains larger than the corresponding one for protons.
FIG. 6: (Color online) Effective $k$-mass $m^*_k(k)$ (solid lines) and effective $E$-mass $m^*_E(k)$ (dashed lines) for neutrons and protons (lines with symbol) as obtained from the BHF calculations for asymmetric nuclear matter at the density $\rho = 0.17$ fm$^{-3}$ and a proton abundance of 25%. The Fermi momenta for protons and neutrons are indicated by vertical dotted lines.

The effects of the $E$-mass are weaker for $V_{\text{low}}$ than for the $G$-matrix. This is obvious since $V_{\text{low}}$ only accounts for ladder diagrams included with particle-particle states above the cut-off, whereas the $G$-matrix includes all particle-particle states above the Fermi-momenta. This explains the lower effective masses obtained for $V_{\text{low}}$ than for the BHF approximation (see Fig. 5). Results on effective masses obtained from SCGF are rather similar to the BHF results, therefore we do not discuss them here explicitly.

We want to add that the results for the effective $E$-mass can be rather different in finite nuclei than in nuclear matter. As it has already been mentioned above, this $E$-mass is due to the admixture of vibrational modes, which are quite different in finite nuclei as compared to nuclear matter. This may explain the differences in the predictions for the effective isovector mass resulting from Skyrme interactions, which are based on fits of properties for finite nuclei, and those originating from realistic interactions.

Finally we are going to address the results for effective masses as they originate from relativistic mean-field approximations. Here we have to distinguish between the values for the Dirac mass and the effective mass parameterizing the single-particle spectrum according to eq. (33). The Dirac mass $m^*_D$ is defined in terms of the scalar part of the self-energy (see eq. (15)). In the case of the relativistic mean-field approximation the self-energies do not depend on energy or momentum and the scalar self-energies for protons and neutrons are solely due to the direct contributions of the scalar mesons $\sigma$ and $\delta$. The Dirac masses obtained from DDRMF in nuclear matter at $\beta$-equilibrium are displayed in Fig. 7. We see that the Dirac masses decrease with density but show larger values for the protons than for the neutrons.

In order to compare these results for the Dirac mass with the corresponding non-relativistic effective mass, we have to compare the expressions for the single-particle energies eq. (17) and eq. (33) and adjust the parameters in such a way that the expressions yield identical results and slopes as a function of $k^2$ at the Fermi momentum. This leads to

$$m^*_i = \sqrt{k^2_{Fi} + (m^*_D)_{Fi}},$$

(38)

where the label $i$ refers to the case of proton and neutron. Results for these non-relativistic masses are displayed in Fig. 7 by solid lines. We find that the enhancement of the Dirac mass by the corresponding Fermi momenta in eq. (38) is significant for the neutrons in particular. This leads to the effect that at high densities the non-relativistic effective mass for neutrons gets larger than the corresponding mass for protons, a behavior which is opposite to the one observed for the Dirac masses.

It is worth noting that this feature, the difference of the Dirac masses $m^*_{Dn} - m^*_{Dp}$, is negative while the difference of the corresponding non-relativistic masses is positive has been observed before within the framework of Dirac Brueckner...
FIG. 7: (Color online) Effective masses originating from the DDRMF calculations of nuclear matter in β-equilibrium. The Dirac masses for protons and neutrons are represented by the dashed lines, while solid lines are used to identify the effective masses according to eq. (33). The results for protons are shown in terms of lines with symbols.

Hartree Fock (DBHF) even at small densities\[^{35, 39, 56, 57}\]. The parameterization of the DDRMF approach has been made to reproduce the bulk properties of the DBHF of \[^{35}\]. However, this adjustment cannot account for details like the non-locality of the self-energies in DBHF. Therefore it does not reproduce such details as the effective isovector mass with good accuracy.

IV. CONCLUSION

Various approaches to the nuclear many-body problem have been investigated to explore their predictions for nuclear matter at high density and large proton-neutron asymmetries. Two of these approaches, the Skyrme Hartree-Fock and the Density Dependent Relativistic Mean Field approach are predominantly of phenomenological origin. Their parameters have been adjusted to reproduce data of finite nuclei. However, the parameters have been selected in such a way that also bulk properties of asymmetric nuclear matter derived from microscopic calculations are reproduced. The other three approaches are based on realistic NN interactions, which fit the NN scattering phase shifts. In these approximation schemes (Brueckner Hartree Fock BHF, Self-consistent Greens Function SCGF and Hartree Fock using a renormalized interaction \(V_{\text{lowk}}\)) a isoscalar contact interaction has been added to reproduce the empirical saturation point of symmetric nuclear matter.

These various approximation schemes lead to rather similar predictions for the energy per nucleon of symmetric and asymmetric nuclear matter at high densities. In detail one finds that the relativistic DDRMF leads to a rather stiff Equation of State (EoS) for symmetric matter while the BHF approach leads to a relatively soft EoS, a feature which is compensated within the microscopic framework by the repulsive features of the hole-hole ladders included in SCGF. The phenomenological approximation schemes DDRMF (Skyrme Hartree Fock) over (under) estimate the symmetry energy at high densities as compared to the microscopic approaches. The lack of long range (low energy) correlation effects in \(V_{\text{lowk}}\) leads to a symmetry energy which is too small already at normal density. These features are also reflected in the study of nuclear matter in the β-equilibrium and lead to moderate differences in the predictions for proton abundances and EoS.

More significant differences are observed when we inspect details like the effective masses, in particular the isovector effective mass. In neutron-rich matter the microscopic approaches predict a positive difference between neutron and proton effective masses. This feature can be related to the non-locality of the self-energy induced by one-pion exchange term and is expressed in terms of an effective \(k\)-mass. This feature may partly be compensated by the effects of vibrational excitation modes on the nucleon mean fields. The effects of such low-energy excitations might lead to different results in nuclear matter and finite nuclei. This could be study e.g. in many-body calculations employing \(V_{\text{lowk}}\), which account for the effects of vibrational modes explicitly.

We also discuss the differences between effective Dirac masses and corresponding non-relativistic masses in neutron-
rich matter. While the difference between the neutron and proton Dirac masses is negative, the differences of the corresponding non-relativistic masses tend to get positive. Further studies of the non-localities in space and time of various components in the Dirac self-energy would be useful to explore the connections to the non-relativistic microscopic approaches more in detail.

V. ACKNOWLEDGEMENTS

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