Numerical Simulation on the Cavity-Induced Boundary Layer Transition

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Abstract. The boundary layer transition can be triggered in either natural or forced way. In this paper, a Mach 3 cavity-induced boundary layer transition is simulated using the implicit large eddy simulation, in order to study the transition caused by the surface gap on the vehicles in the real condition. One of the characteristics of this paper is that no disturbance is imposed in the inflow condition such that the boundary layer transition is triggered by the self-sustained fluctuation in the upstream cavity. Results show that the transition appears in the natural scenario and the stage of the linear instability seems to be followed by the process of the secondary instability. Linear stability analyses are also utilized to show the development of the unstable modes.

1. Introduction

The boundary layer transition draws considerable attention both in theoretical studies and industry applications due to its significance, in particular, in the prediction and control of force and heat transfer of the vehicles. In the real condition, the boundary layer transition can be triggered either in the natural scenario or in the forced situation. For the latter way, rough elements or surface gaps may induce artificial transition. The flow inside the surface gap can be simplified to the classical cavity flow, which has received extensive attention in both aircraft design and basic hydrodynamic research. However, the supersonic boundary layer transition caused by the disturbance produced by the cavity flow has hardly been studied deeply and systematically.

To clarify the mechanism of the cavity-induced boundary layer transition, the characteristics of the cavity flow should be understood. For the infinite-width cavity flow, when the length-depth-ratio $L/D$ is relatively small, the flow pattern is open cavity flow where the shear layer forms across the cavity and the Kelvin-Helmholtz instability would grow in the shear layer. When the ratio $L/D$ is high, the closed cavity flow appears and the shear layer enters the cavity and reattaches on the bottom of the cavity, rather than crossing the cavity. This paper mainly focuses on the open cavity flow, where the self-sustained disturbance can exist in the cavity flow in either the shear layer mode or the low-frequency wake mode. The cavity flow in the shear layer mode produces the self-sustained fluctuation through the “feedback mechanism”. That is, the disturbance develops in the shear layer through the Kelvin-Helmholtz instability and reflects at the trailing edge of the cavity, and then feeds back the upstream disturbance. One representative work on the theory of the shear layer mode is the Rossiter’s [1] semi-empirical formula, which gives the resonance frequency of the cavity flow. The disturbance produced by the cavity flow can be the source of the downstream boundary layer transition.

With respect to the boundary layer transition caused by the cavity whose length scale is significantly larger than the boundary layer thickness, relevant studies are relatively in lack and mostly focus on the finite-width cavity. Chang et.al [2] conducted high-fidelity numerical simulation on the...
Mach 6 cavity-induced boundary layer transition, which happened on the NASA’s Project Orion crew entry vehicle. In their cases, the cavity takes the shape of a cylinder. Results show that when the Reynolds number based on the depth of the cavity is 800, only weak instability is found in the downstream boundary layer. With a higher Reynolds number, strong wake instability is observed in the downstream boundary layer. Nevertheless, the instability is caused by the finite width of the cavity, i.e. its three-dimensionality. For some other occasions, for example, in the joint part on the surface of the vehicle, the width of the cavity is much larger than its length as well as the boundary layer thickness, and thus the cavity can be viewed as an infinite-width two-dimensional one. We wonder if the self-sustained disturbance in the cavity flow can cause the boundary layer transition, either in the natural or the bypass scenario.

In this paper, an infinite-width cavity with the length-depth-ratio of about 2 is inserted in the Mach 3 plate laminar boundary layer flow, so the forced boundary layer transition downstream of the cavity can be expected. One of the interesting respects of this work is that no external disturbance is imposed at the inlet of the computational domain, and thus the boundary layer transition can only be triggered by the self-sustained oscillation produced inside and just above the cavity. The main numerical methods are the implicit large eddy simulation (LES) and the linear stability analyses, which are capable of high-fidelity numerical simulation and hydrodynamic stability studies.

2. Numerical Methods and Computational Setups

The LES in this paper is conducted using the code ACANS, based on the finite difference method for compressible flows. The governing equations are compressible Favré-filtered Navier-Stokes equations which are listed as follow:

\[
\begin{align*}
\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial (\mathbf{u} \cdot \mathbf{u})}{\partial x} &= 0 \\
\frac{\partial (\mathbf{u} \rho)}{\partial t} + \frac{\partial (\mathbf{u} \mathbf{u} \rho)}{\partial x} &= \frac{\partial \sigma}{\partial x} + \frac{\partial T}{\partial x} \\
\frac{\partial (\mathbf{u} E)}{\partial t} + \frac{\partial (\mathbf{u} \mathbf{u} E)}{\partial x} &= \frac{\partial (u \sigma_{ij} - \mathbf{q}_i)}{\partial x}
\end{align*}
\]

where the superscript “~” refers to the Favré filtering operation for a quantity \( f \), which is given by

\[
\tilde{f} = \frac{f}{\rho}
\]

For LES, the superscript “-” denotes the spatial-filtering operation. The filtered/averaged viscous stress tensor, specific total energy and heat flux are given by

\[
\begin{align*}
\sigma_{ij} &= \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right) \\
E &= \frac{1}{\gamma - 1} \frac{\rho}{\rho} + \frac{1}{2} u_i u_i \\
\mathbf{q}_i &= -\mu \left( \frac{\partial T}{\partial x_i} \right)
\end{align*}
\]

The subgrid stress tensor denotes

\[
\tau_{ij} = -\rho(u_i u_j - \tilde{u}_i \tilde{u}_j)
\]

which is modeled by the numerical dissipation of the scheme implicitly, and no additional subgrid stress model is added.

As for the discretization scheme, the fifth-order WENO scheme is chosen for the inviscid flux, and the second-order central difference scheme is selected for the viscous flux. The time marching method is the implicit dual-time stepping scheme with a second order accuracy, and the time step is set as \( 1 \times 10^{-7} \)s that ensures the resolution of the time scales of unstable modes during the transition. Statistical stage for the unsteady flow field starts when the time-averaged flow field converges.
The boundary condition is set as follows. The inflow condition is given by the laminar boundary layer profile that is taken from the ending of a 1.0 m length plate boundary layer, where the free Mach number is 3 and unit Reynolds number is $7.7 \times 10^7$. Figure 1 shows the comparison of the computed profile by ACANS and the compressible self-similar solution. The velocity and temperature is nondimensionalized by the corresponding free flow parameters while the $y$ coordinate by the length scale $l = (\nu x / U_\infty)^{1/2}$. Very good agreement is reached and the inflow profile given by ACANS is verified. Note that no external disturbance is added in the inflow condition that allows the boundary layer transition to be triggered by the fluctuation in the cavity. For the upper and outflow condition, the extrapolation method is used. For the wall condition, the adiabatic no-slip condition is chosen while for the spanwise boundary the periodic condition is selected.

The computational mesh is multi-block structured, and the total grid size is about 50 million. The size of the computational domain is around 0.4m×0.08m×0.03m, while the streamwise and spanwise grid spacings are 0.3mm and 0.2mm that is at least one order of magnitude less than the corresponding wave lengths of the first mode in order to describe the transition process correctly. The minimum grid spacing in the wall-normal direction is $2.5 \times 10^{-6}$ m. It should be clarified that the grid size is not able to reach the requirement of DNS for the fully-turbulent region, though is large enough for the transition process. With respect to the cavity, a 21.6mm×10mm×30mm cavity is located at $x=24$mm where $x=0$ denotes the inflow boundary. The grid size inside the cavity is 72×150×150 which is able to resolve small-scale vortexes.

![Figure 1. Comparison of the inflow boundary layer profile with the self-similar solution](image)

The other numerical method applied in this paper is the linear stability analyses. The linear stability theory (LST) solves the linear stability equations and gives the stability characteristics of the local boundary layer. The small disturbance is given by

$$\phi(x, y, z, t) = \phi(y)e^{j(\alpha x + \beta z - \omega t)}, \phi = u, v, w, T, p$$

where the streamwise wavenumber $\alpha$, the spanwise wavenumber $\beta$ and the circular frequency $\omega$ is nondimensionalized by the freestream velocity $U_\infty$ and the length scale $l = (\nu x / U_\infty)^{1/2}$. For the spatially developed boundary layer, LST solves the complex dispersion relation

$$\alpha = \alpha(\alpha, \beta)$$

where $\alpha_i$ denotes the streamwise wavenumber and $-\alpha_i$ denotes the growthrate. The numerical details can be found in Malik’s [3] work. Our code integrates the global and local methods, which gives the whole spectrum of the eigenvalues and the accurate solution of a single eigenvalue, respectively.

3. Validation of the Code

Here we present the validation of the ACANS code and LST code. The case of the development of the Mack second mode in the plate boundary layer in Zhao et al.’s [4] paper is chosen as the test case. The Mach number is 6 while the unit Reynolds number is $1.05 \times 10^7$. The Mack second mode disturbance is added using the suction-blowing method on the wall near the leading edge. Then two-dimensional direct
numerical simulation (DNS) is conducted to simulate the growth of the disturbance. All the simulation setups can be found in Zhao et al.’s [4] paper. Figure 2 shows the comparison of the fluctuated pressure distribution on the wall between ACANS and the reference [4]. It can be found that ACANS predicts the development of the disturbance as accurately as the code in Zhao et al.’s [4] paper. To compare the DNS results with those given by LST, the growthrate can be calculated from the fluctuated pressure through the formula:

\[
\alpha_i = -\frac{1}{p'_{ij}} \frac{d|p'|}{dx}
\]  

(7)

Thus the comparison of the growthrate computed by DNS and LST is given in Figure 3. The three kind of LST results denote the results given by [4], results given by our LST code based on the ACANS laminar flow field and the compressible self-similar solution, respectively. Figure 3 illustrates that the growthrate given by ACANS agrees well with that by LST in the linear unstable stage at 0.4m<x<0.7m. For x<0.4m, the second mode is not amplified significantly and not dominant, and thus some other disturbances affect the growthrate.

In conclusion, the DNS and LST results in [4] validate the ACANS and LST codes used in this paper.

**Figure 2.** Comparison of the $p_{rms}$ on the wall between results by ACANS and the reference [4]

**Figure 3.** Comparison of the growthrates between DNS and LST

4. Results and Discussions

Figure 4 shows the instantaneous temperature and velocity contours both in the $x$-$z$ slice and $x$-$y$ slice. It illustrates that the boundary layer transition begins downstream the cavity without any inflow disturbances. In the cavity flow, the flow pattern is the open cavity flow and the shear layer as well as the self-sustained oscillation is observed. From the instantaneous results in the $x$-$y$ slice, a finite-
amplitude disturbance wave appears in the boundary layer when the x coordinate is over around 0.13m, and in a short distance the spanwise streaks form. When the x coordinate is over around 0.15m, the three dimensionality is strong and the boundary layer changes to be turbulent. Therefore, it is numerically proved that the boundary layer transition happens at certain conditions just induced by the disturbance inside the upstream cavity.

Figure 5 provides the results of the mean skin friction coefficient $C_f$ and the sound pressure level (SPL) on the wall, which represent the critical time-averaged and fluctuated information. It can be seen from the result of $C_f$ that $C_f$ is negative on the bottom of the cavity due to the recirculation flow and rises rapidly when $x>0.15m$. The transitional region ranges from about 0.15m to 0.26m. As for the sound pressure level, it reaches more than 160dB in the cavity owing to the cavity-flow resonance. In the laminar boundary layer, the SPL goes down until the transition point. The SPL rises in the transitional region and approaches 180dB in the fully-turbulent region.

In order to study the boundary layer transition quantitatively on the level of the stability, Figure 6 shows the growthrate results given by LST and ACANS. The basic flow field for LST is obtained from the time-averaged results by ACANS. Furthermore, results from three different statistical intervals are shown, and it is concluded that the statistical converge is reached. It should be noted that the LST result denotes the growthrate of the most unstable local first mode and shows the limitation of the growthrate in the linear theory rather than the real growthrate in this case.

Figure 4. The instantaneous temperature and velocity contours in the x-z slice and x-y slice

Figure 5. The distribution of the time-averaged skin friction coefficient (left) and the sound pressure level on the wall (right)
Figure 6. Comparison of the growth rate given by the numerical simulation and LST

It is revealed from Figure 6 that the $\alpha_i$ oscillates in the cavity and then keeps positive in the early stage of the reattached plate boundary layer which means the boundary layer is stable at the beginning. At $0.08m < x < 0.1m$, the growth rate given by ACANS approaches the LST solution, which indicates that linear instability appears in this region. However, the boundary layer grows stable sequently and then unstable again. At around $0.125m < x < 0.18m$, the growth rate exceeds the limitation of the growth rate predicted by LST. The most probably true explanation for this region is that secondary instability happens. Another interesting point is that at $x=0.15m$ the transition occurs according to Figure 5 and the growth rate reaches the maxima. Breakdown of the boundary layer can be imagined near this position.

Figure 7. The power spectrum density at different streamwise locations

Figure 8. The streamwise wavenumber (left) and the growth rate (right) of the 30kHz mode
Figure 7 displays the power spectrum density at six streamwise locations. It is found that at the beginning of the reattached boundary layer where \(x=60\text{mm}\), only the two-dimensional mode with the frequency equal to 30kHz is remarkable, and this is also one of the main frequency of the cavity flow. Then a mode with a higher frequency of 337 kHz and a small phase angle rises, and seems to dominate the transition location.

Using the Fourier transformation method, we can get the wave number and the growthrate of the 30 kHz component as shown in Figure 8. In the left figure, it is illustrated that the result by the numerical simulation agrees well with LST on the wave number. This result demonstrates that a counterpart can be found in the linear stability theory for the 30kHz component. Meanwhile, in the right figure, the growthrate of the 30kHz component shows that this mode is generally stable until \(x\) is over 0.15m, which means the instability wave is not dominant by the 30kHz component. Naturally, we doubt that the dominant mode is the 337Hz component which rises in the energy with the increase of the \(x\) coordinate as shown in Figure 7. However, this high frequency component is not found in LST and should be explained by the nonlinear stability theory, which will be presented in the following work.

5. Conclusion
A Mach 3 cavity-induced boundary layer transition is simulated using the implicit large eddy simulation, in order to study the transition caused by the surface discontinuities on the vehicles. The simulation is conducted with no disturbance imposed in the inflow condition such that the boundary layer transition is triggered by the self-sustained fluctuation in the upstream cavity. Meanwhile, the linear stability analyses are applied in this paper. First, the LES and LST codes are validated. Then, results show that the transition appears in the natural scenario and the stage of the linear instability seems to be followed by the process of the secondary instability. Thus the boundary layer transition triggered by the self-sustained fluctuation in the cavity is confirmed. Using the Fourier transformation and LST, the 30kHz and 337kHz components are revealed and the latter one rises rapidly in the downstream boundary layer. The 30kHz component is found and verified in LST while the 337kHz component is still unclear. The research on the specific mechanism is still in process.

6. Acknowledgements
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7. References
[1] Rossiter J E 1964 Wind-Tunnel Experiments on the Flow over Rectangular Cavities at Subsonic and Transonic Speeds Aeronautical Research Council R&M 3438 (U.K.: RAE Farnborough).
[2] Chang C L, Choudhari M M and Li F 2011 Effects of Cavities and Protuberances on Transition over Hypersonic Vehicles AIAA paper 2011-3245.
[3] Malik M R 1990 Numerical methods for hypersonic boundary layer stability J. Comput. Phys. 86 376-413.
[4] Zhao R, Wen C, Tian X, et al. 2018 Numerical simulation of local wall heating and cooling effect on the stability of a hypersonic boundary layer Int. J. Heat Mass Transf. 121 986 – 998.