Search for $C = +$ charmonium and $XYZ$ states in $e^+e^- \rightarrow \gamma + H$ at BESIII

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Abstract: Within the framework of nonrelativistic quantum chromodynamics, we study the production of $C = +$ charmonium states $H$ in $e^+e^- \rightarrow \gamma + H$ at BESIII with $H = \eta_c(nS)$ $(n=1, 2, 3, \text{and} 4)$, $\chi_{cJ}(nP)$ $(n=1, 2, \text{and} 3)$, and $^1D_2(nD)$ $(n=1 \text{ and} 2)$. The radiative and relativistic corrections are calculated to next-to-leading order for $S$ and $P$ wave states. We then argue that the search for $C = +$ XYZ states such as $X(3872)$, $X(3940)$, $X(4160)$, and $X(4350)$ in $e^+e^- \rightarrow \gamma + H$ at BESIII may help clarify the nature of these states. BESIII can search XYZ states through two body process $e^+e^- \rightarrow \gamma H$, where $H$ decay to $J/\psi\pi^+\pi^-$, $J/\psi\phi$, or $D\bar{D}$. This result may be useful in identifying the nature of $C = +$ XYZ states. For completeness, the production of $C = +$ charmonium in $e^+e^- \rightarrow \gamma + H$ at B factories is also discussed.
1 Introduction

During the last 10 years, many heavy quarkonium or heavy quarkonium-like $XYZ$ states had been discovered (more details can be found in Ref.[1] and related papers). The $X(3872)$ state is the first and the most famous state among them. It was discovered by the Belle collaboration[2], and confirmed by the CDF [3], D0[4], BaBar[5], LHCb[6], and CMS[7] collaborations. One of the most conspicuous properties of $X(3872)$ is its mass, which is close to the $D^0\bar{D}^{*0}$ threshold within 1 MeV; hence, $X(3872)$ is suggested to be a $D^0\bar{D}^{*0}$ molecule [8–11]. The contribution of the charged component $D^+D^{*-}$ is also considered in Ref.[12, 13]. The molecule model may be puzzled to explain the production cross-sections of $X(3872)$ in hadron colliders (which may be large in some phenomenological models[14]) [15]. The quantum numbers of $X(3872)$ have been determined to be $J^{PC}=1^{++}$ by LHCb collaboration [16]. The $J^{PC}$ of $X(3872)$ is the same as $\chi_{c1}(nP)$. On the contrary, the mass 3.872 GeV seems too low for a $\chi_{c1}(2P)$ state. The coupled-channel and screening effects may draw its mass down to 3.87 GeV [17]. However, next-to-leading order (NLO) prediction of $X(3872)$ production in hadron colliders within nonrelativistic quantum chromodynamics (NRQCD) disfavors the interpretation of $X(3872)$ as pure $\chi_{c1}(2P)$ [18]. The possibility that $X(3872)$ might be a mixture state with the $\chi_{c1}(2P)$ and the $D^0\bar{D}^{*0}$ components was proposed in Ref.[19]. The prompt $X(3872)$ hadroproduction is studied at NLO in $\alpha_s$[20] and the result is consistent with the CMS [7] and the CDF data[3]. This idea is also favored the data of some other measurements and predictions [15, 17, 21, 22].

Besides $X(3872)$, other $C = +$ $XYZ$ states are listed in Table 1. These states are particularly interesting and the interpretations for their nature are still inconclusive[23]. $X(3915)$
However, charmonium or charmonium-like states, recoiling against the easily reconstructed $1^-$ $e^+e^-$, have not provided a clear understanding of charmonium-like states. The double charmonium production in the two-photon process is also proposed\[6\]. The relativistic correction of $e^+e^- \to \gamma^* \to \gamma + \eta_c$ is also included in Ref.[31]. Indirect measurement of quarkonium in the two-photon process is also proposed\[32\].

Recently, BesIII reports the cross-sections of $e^+e^- \to \gamma X(3872)$\[46, 47\]

\[
\begin{align*}
\sigma[e^+e^- \to \gamma X(3872)] \times \text{Br}[J/\psi\pi\pi] & < 0.13 \text{pb at 90\% CL.} & \sqrt{s} = 4.009 \text{GeV} \\
\sigma[e^+e^- \to \gamma X(3872)] \times \text{Br}[J/\psi\pi\pi] & = 0.32 \pm 0.15 \pm 0.02 \text{pb} & \sqrt{s} = 4.230 \text{GeV} \\
\sigma[e^+e^- \to \gamma X(3872)] \times \text{Br}[J/\psi\pi\pi] & = 0.35 \pm 0.12 \pm 0.02 \text{pb} & \sqrt{s} = 4.260 \text{GeV} \\
\sigma[e^+e^- \to \gamma X(3872)] \times \text{Br}[J/\psi\pi\pi] & < 0.39 \text{pb at 90\% CL.} & \sqrt{s} = 4.360 \text{GeV} \ (1.1)
\end{align*}
\]

Where $\text{Br}[J/\psi\pi\pi]$ means $\text{Br}[X(3872) \to J/\psi\pi\pi]$. And the studies of $\psi(4160) \to X(3872)\gamma$
Many NLO relativistic and radiative corrections for heavy quarkonium production are considered within nonrelativistic QCD (NRQCD)[50]. By introducing the color octet mechanism, one can obtain the infrared-safe calculations for the decay rates of P wave [51–53] and D wave[54–56] quarkonium states. The color octet contributions of the diphoton decay of P wave quarkonium states are calculated in Ref.[57]. O(αs v2) corrections to the decays of h_c, h_b and η_b are studied in Ref.[58, 59]. The NLO QCD corrections[60–70], relativistic corrections[71–78], and O(αs v2) corrections [79, 80] largely compensate for the discrepancies between theoretical values and experimental measurements at B factories. The contributions of higher-order QCD corrections for charmonium production [18, 20, 81–88] and polarization [89–92] in hadron colliders are also significant. The relativistic corrections to J/ψ hadroproduction are significant[93–95].

We calculate the production of C = + charmonium at e+e− annihilation at BESIII to test the nature of C = + XYZ states. Our paper is organized as follows. The calculation framework is given in Sec. 2. The numerical results of the cross-sections of C = + charmonium are discussed in Sec. 3. A discussion of X(3872) and other C = + XYZ states is given in Sec. 4. The summary is given in Sec. 5.

2 The frame of the calculation

In the NRQCD factorization framework, we can express the amplitude in the rest frame of H as[28, 30, 31]

\[
\mathcal{A}(e^-(k_1)e^+(k_2) \rightarrow H_{cc}^{(2S+1)L_J})(2p_1) + \gamma) = \sum_{L_s S_z} \sum_{s_1 s_2} \sum_{j k} \int d^3\bar{q}\Phi_{cc}(\bar{q}) \langle s_1; s_2 | SS_z \rangle \langle 3j; 3k | 1 \rangle \times \mathcal{A} \left[ e^-(k_1)e^+(k_2) \rightarrow c_{j_1}^s(p_1 + q) + \bar{c}_{j_2}^s(p_1 - q) + \gamma(k) \right], \tag{2.1}
\]

where \( \langle 3j; 3k | 1 \rangle = \delta_{jk}/\sqrt{N_c} \), \( \langle s_1; s_2 | SS_z \rangle \) is the color Clebsch-Gordan coefficient for cc pairs projecting out appropriate bound states, and \( \langle s_1; s_2 | SS_z \rangle \) is the spin Clebsch-Gordan coefficient. \( \mathcal{A} \left[ e^-(k_1)e^+(k_2) \rightarrow c_{j_1}^s(p_1 + q) + \bar{c}_{j_2}^s(p_1 - q) + \gamma(k) \right] \) is the quark level scattering amplitude. In the rest frame of \( H \), \( q = (0, \bar{q}) \), and \( p_1 = (\sqrt{m_c^2 + \bar{q}^2}, 0, 0, 0) \). \( \Phi_{cc}^{H}(\bar{q}) \) is the cc component wave function of hadron \( H \) in momentum space. For \( v^2 = \bar{q}^2/m_c^2 \ll 1[50] \), we can expand Eq.(2.1) with \( v^2 \):

\[
\mathcal{A}(q) = \mathcal{A}(0) + \frac{\partial \mathcal{A}(\bar{q})}{\partial \bar{q}^0} \bigg|_{q=0} \bar{q}^0 + \frac{\partial^2 \mathcal{A}(\bar{q})}{\partial \bar{q}^0 \partial \bar{q}^3} \bigg|_{q=0} \frac{\bar{q}^0 \bar{q}^3}{2} + \frac{\partial^3 \mathcal{A}(\bar{q})}{\partial \bar{q}^0 \partial \bar{q}^3 \partial \bar{q}^6} \bigg|_{q=0} \frac{\bar{q}^7}{3!} + \ldots \tag{2.2}
\]

[48] and \( \psi(4260) \rightarrow X(3872)\gamma \) [49] are proposed to probe the molecular content of the \( X(3872) \).
Here $A(q) = A \left[ e^{-i(k_1)}e^{i(k_2)} \rightarrow c'_{J}(p_1 + q) + c''_{J}(p_1 - q) + \gamma(k) \right]$. We consider the Fourier transform between the momentum space and position space as: [50, 94],

$$
\int d^3\bar{q} \Phi_{cc}(\bar{q}) \propto \sqrt{Z^H_{cc}}R_{cc}(0)
$$

$$
\int d^3\bar{q} \bar{q}^0 \Phi_{cc}(\bar{q}) \propto \sqrt{Z^H_{cc}^l}R'^{l}_{cc}(0)
$$

$$
\int d^3\bar{q} \bar{q}^0 \bar{q}^2 \Phi_{cc}(\bar{q}) \propto \sqrt{Z^H_{cc}^m}R''_{cc}(0)
$$

$$
\int d^3\bar{q} \bar{q}^0 \bar{q}^4 \Phi_{cc}(\bar{q}) \propto \sqrt{Z^H_{cc}^n}R''''_{cc}(0).
$$

(2.3)

Here $Z^H_{cc}$ is the possibility of $c\bar{c}$ component in hadron $H$. $R_{cc}(0)$ is the radial Schrodinger wave function at the origin. $R_{cc}(0)$ is the derivative of the radial Schrodinger wave function at the origin

$$
R_{cc}(0) = \frac{d^1R_{cc}(r)}{dr} \bigg|_{r=0}
$$

(2.4)

$R_{cc}(0)$ corresponds to the $O(v^0)$ S wave matrix element, $R_{cc}'(0)$ corresponds to the $O(v^0)$ P wave matrix element, $R_{cc}''(0)$ corresponds to the $O(v^0)$ S wave matrix element or $O(v^0)$ D wave matrix element, and $R_{cc}''''(0)$ corresponds to the $O(v^0)$ P wave matrix element.

$R_{cc}(0)$ is also written as long-distance matrix elements (LDMEs) as discussed in Ref.[94]. For example,

$$
\langle 0|O^{\chi_{c1}}(3P^1_1)|0 \rangle = \frac{27}{2\pi} |R'_{1P}(0)|^2,
$$

(2.5)

We calculated the relativistic corrections for the S wave and P wave states and obtain two LDMEs for $\eta_c$, four LDMEs for $\chi_{cJ}$, and one LDMEs for $^1D_2$ states. To simplify the discussion of the numerical result, we assumed that

$$
< 0|O^{\chi_{cJ}}(3P^1_J)|0 > = (2J + 1) < 0|O^{\chi_{cJ}}(3P^1_J)|0 > .
$$

(2.6)

$$
v^2 = \frac{\langle 0|P^H (2s+1L^H_J)|0 \rangle}{m^2\langle 0|O^H (2s+1L^H_J)|0 \rangle}.
$$

(2.7)

Then there is only one LDME for $S$ wave, $P$ wave, and $D$ wave respectively. More details can be found in Ref.[94].

The relativistic correction $K$ factor is

$$
K_{v^2}[\eta_c] = -\frac{5v^2}{6} - \frac{rv^2}{1-r}
$$

$$
K_{v^2}[\chi_{c0}] = -\frac{(5r^2 - 28r + 13) v^2}{10(3r^2 - 4r + 1)} - \frac{rv^2}{1-r}
$$

$$
K_{v^2}[\chi_{c1}] = -\frac{(21r^2 + 30r - 11) v^2}{10(r^2 - 1)} - \frac{rv^2}{1-r}
$$

$$
K_{v^2}[\chi_{c2}] = -\frac{(90r^3 + 113r^2 + 4r - 7) v^2}{10(r - 1)(6r^2 + 3r + 1)} - \frac{rv^2}{1-r}.
$$

(2.8)
where $r = 4m_c^2/s$. $-\frac{r^2}{1 - r}$ is the relativistic correction of the phase space. If we select $r \to 0$, the $K_{\alpha\pi}$ factor is consistent with the $K$ factor at large $p_T$ in Ref.[94].

Our leading order (LO) cross-sections of $e^+e^- \to \gamma^* \to \gamma + H$ is consistent with Ref.[28, 30, 31]. The QCD corrections of $e^+e^- \to \gamma^* \to \gamma + H$ is consistent with Ref.[30, 31]. And the relativistic corrections of $e^+e^- \to \gamma^* \to \gamma + \eta_c$ is consistent with Ref.[31, 77, 78].

We can estimate the cross-sections for pure $\chi_{cJ}(nP)$ ($n=1$, 2, and 3), and $1D_2(nD)$ ($n=1$ and 2) are several hundreds of MeV and several MeV, respectively. If $Z_H^2 \sim Z_D^2$, we can consider the $c\bar{c}$ contributions only.

In the numerical calculation, we consider the charm quark mass as half of the hadron mass consistent with the physics phase space. With a large charm quark mass, the wave functions at the origin are identified as the Cornell potential result in Ref.[96]. The selected parameters are as follows:

$$m_c = \frac{m_H}{2}, \quad \alpha_s = 0.23, \quad \alpha = 1/133,$$

$$v^2 = 0.23, \quad R_{1S} = 1.454\text{GeV}^3, \quad R_{2S} = 0.927\text{GeV}^3,$$

$$R_{3S} = 0.791\text{GeV}^3, \quad R_{1P} = 0.131\text{GeV}^5, \quad R_{2P} = 0.186\text{GeV}^5,$$

$$R_{1D}'' = 0.031\text{GeV}^7. \quad (2.10)$$

The wave functions at origin for higher states are estimated as

$$R_{4S} = 2 \times R_{3S} - R_{2S} = 0.655\text{GeV}^3,$$

$$R_{5P} = (R_{1P} + R_{2P})/2 = 0.159\text{GeV}^5,$$

$$R_{2D}'' = R_{1D}'' = 0.031\text{GeV}^7. \quad (2.11)$$

In the numerical result, "$\sigma_{LO}$" is the LO cross-section, "$\sigma_{\alpha\pi}$" is the cross-section including the LO and the relativistic correction, "$\sigma_{\alpha\pi}$" is the cross-section including the LO and the radiative correction, and "$\sigma_{\alpha\pi\alpha\pi}$" is the cross-section including the LO, the relativistic correction, and the radiative correction. In addition, "LO" is the LO cross-section, "RC" is the relativistic correction, "QCD" is the radiative correction, and "Total" is the cross-section including the LO, the relativistic correction, and the radiative correction.

For the LO, the cross-section is $O(\alpha_s^0 v^0)$. As $\alpha_s = 0.23 \pm 0.03$ and $v^2 = 0.23 \pm 0.03$ are reasonable estimates, we can estimate that the uncertainty of the numerical result from $\alpha_s$ and $v^2$ is < 10%.

### 3 Pure $C = +$ charmonium states

We can estimate the cross-sections for pure $C = +$ charmonium states $H$ in $e^+e^- \to \gamma + H$ at BESIII with $H = \eta_c(nS)$ ($n=1$, 2, 3, and 4), $\chi_{cJ}(nP)$ ($n=1$, 2, and 3), and $1D_2(nD)$ ($n=1$ and 2).
and 2). The mass of the lower states can be found in Ref.[24], and the mass of the higher states is selected from Ref.[17].

\[ \sigma[e^+e^→\eta_c\gamma](\text{fb}) \]

![Graph 1](image1)

**Figure 1.** The cross-sections of \( e^+e^− \to \eta_c + \gamma \) as a function of \( \sqrt{s} \) in fb. The cross-section \( \sigma_{LO} \), \( \sigma_{v^2} \), \( \sigma_{\alpha_s} \), and \( \sigma_{\alpha_s,v^2} \) are defined near the end of Section 2.

\[ \sigma[e^+e^−\to\gamma\eta_{c2}(nD)](\text{fb}) \]

![Graph 2](image2)

**Figure 2.** The cross-sections of \( e^+e^− \to \eta_{c2}(1D,2D) + \gamma \) as a function of \( \sqrt{s} \) in fb.

The cross-section of \( e^+e^− \to \eta_c + \gamma \) as a function of \( \sqrt{s} \) is shown in Fig.1. The cross-sections of \( e^+e^− \to \eta_{c2}(1D,2D) + \gamma \) as a function of \( \sqrt{s} \) are shown in Fig.2. The numerical results for \( nS \) with \( n = 1,2,3,4 \) and \( nD \) with \( n = 1,2 \) are listed in Table 2. We determined
Table 2. The cross-sections of $e^+e^- \rightarrow H + \gamma$ for $\eta_c(nS)$ with $n = 1, 2, 3, 4$ and $\eta_c(2nD)$ for $n = 1, 2$ charmonium states in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2. The mass of $\eta_c(3S)$, $\eta_c(4S)$, $\eta_c(1D)$, and $\eta_c(2D)$ are selected from Ref.[17]. The other mass can be found in Ref.[24].

$$\sqrt{s} (\text{GeV})$$ | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 10.6 | 11.2 |
|----------------|------|------|------|------|------|------|------|
| $\eta_c(2981)$ | LO   | 2781 | 2494 | 2192 | 1906 | 1652 | 117  | 95  |
|                 | RC   | -1332| -1033| -814 | -650 | -526 | -25  | -20 |
|                 | QCD  | -909 | -807 | -700 | -598 | -508 | -22  | -16 |
|                 | Total | 540  | 653  | 678  | 658  | 617  | 70   | 58  |
| $\eta_c(2S)(3639)$ | LO | 563  | 684  | 706  | 679  | 629  | 58   | 48  |
|                 | RC   | -730 | -563 | -442 | -352 | -284 | -13  | -10 |
|                 | QCD  | -177 | -221 | -231 | -222 | -205 | -13  | -10 |
|                 | Total | -344 | -100 | 33   | 105  | 141  | 32   | 27  |
| $\eta_c(3S)(3994)$ | LO | 233  | 337  | 374  | 377  | 44   | 36   |
|                 | RC   | -450 | -352 | -279 | -225 | -10  | -8   |
|                 | QCD  | -72  | -107 | -121 | -123 | -10  | -8   |
|                 | Total | -228 | -122 | -27  | 29   | 24   | 20   |
| $\eta_c(4S)(4250)$ | LO | 133  | 198  | 225  | 34   | 28   |
|                 | RC   | -279 | -221 | -178 | -8   | -6   |
|                 | QCD  | -41  | -63  | -73  | -8   | -7   |
|                 | Total | -186 | -86  | -26  | 17   | 15   |
| $\eta_c(1D)(3796)$ | LO | 4.0  | 6.4  | 7.3  | 7.3  | 7.0  | 0.71 | 0.58 |
| $\eta_c(2D)(4099)$ | LO | 1.5  | 2.9  | 3.5  | 3.7  | 0.47 | 0.38 |

that the radiative and relativistic corrections are negative and large for $\eta_c(nS)$, respectively. The LO cross-sections for $\eta_c(2D)$ is very small at BESIII; hence, the high order corrections are ignored.

The cross-sections of $e^+e^- \rightarrow \chi_{cJ} + \gamma$ as a function of $\sqrt{s}$ are shown in Fig.3, Fig.4, and Fig.5 for $J = 0, 1, 2$, respectively. The numerical results for $\chi_{cJ}(nP)$ with $n = 1, 2, 3$ are listed in Table 3, Table 4, and Table 5 for $J = 0, 1, 2$, respectively. We determined that the QCD corrections are large but negative and the relativistic corrections are large and positive. Hence, many $P$ wave states can be searched at BESIII.

The NRQCD requires that the energy of photon at the center of the mass frame of $e^+e^-$

$$E_{\gamma} = \frac{s - M_H^2}{2\sqrt{s}} \sim \sqrt{s} - M_H + O\left[(1 - M_H/\sqrt{s})^2\right]$$

be larger than $\Lambda_{QCD} \sim 300$ MeV $\sim m_c v^2$. Although this process is a QED process, the prediction is not reliable and only a reference value if this requirement is not satisfied. If we replace photon with gluon, the soft photon contributions correspond to the long-distance color octet contributions[31, 50].
Figure 3. The cross-sections of $e^+e^- \rightarrow \chi_c 0 + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section "$\sigma_{LO}$", "$\sigma_{v^2}$", "$\sigma_{\alpha s}$", and "$\sigma_{\alpha s,v^2}$" are defined near the end of Section 2.

Table 3. The cross-sections of $e^+e^- \rightarrow \chi_c 0(nP)+\gamma$ with $n = 1, 2, 3$ in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2. The $\chi_c 0(2P)$ is considered as $X(3915)(X(3945)/Y(3940))$ [1, 33]. The mass of $\chi_c 0(3P)$ are selected from Ref.[17]. The other mass can be found in Ref.[24].

| $\sqrt{s}$ (GeV) | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 10.6 | 11.2 |
|------------------|------|------|------|------|------|------|------|
| $\chi_c 0(3415)$ | LO   | 877  | 328  | 132  | 21   | 1.81 | 1.6  |
|                  | RC   | 825  | 268  | 107  | 48   | 22   | -0.77 | -0.63 |
|                  | QCD  | -528 | -228 | -107 | -52  | -26  | -0.38 | -0.29 |
|                  | Total| 1173 | 368  | 131  | 49   | 17   | 1.42  | 1.22  |
| $\chi_c 0(2P)(3918)$ | LO | 1991 | 665  | 271  | 119  | 1.30 | 1.18 |
|                  | RC | 3102 | 680  | 230  | 96   | -0.64 | -0.54 |
|                  | QCD | -1013 | -384 | -177 | -89  | 0.39 | 0.30 |
|                  | Total | 4080 | 962  | 324  | 127  | 1.04 | 0.94 |
| $\chi_c 0(3P)(4131)$ | LO | 1073 | 384  | 164  | 0.82 | 0.75 |
|                  | RC | 1600 | 391  | 140  | -0.44 | -0.38 |
|                  | QCD | -551 | -223 | -107 | 0.29 | 0.23 |
|                  | Total | 2121 | 554  | 198  | 0.67 | 0.61 |

4 $C = + XYZ$ states

$X(4160)$ and $Y(4274)$ are found in the B decay $B \rightarrow K + H \rightarrow K + \phi J/\psi$ by CDF collaboration[44]. No signal of $X(4160)$ or $Y(4274)$ is reported by B factories. Hence, the cross-sections for $X(4160)$ or $Y(4274)$ at BESIII may be too small. The cross-sections of $e^+e^- \rightarrow \gamma H$ for $X(3872)$, $X(3940)$, $X(4160)$, and $X(4350)$ are discussed here. The $1^{-+}$
Figure 4. The cross-sections of $e^+e^- \rightarrow \chi_{c1} + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-sections $\sigma_{LO}$, $\sigma_{v^2}$, $\sigma_{\alpha_s}$, and $\sigma_{\alpha_s,v^2}$ are defined near the end of Section 2.

Table 4. The cross-sections of $e^+e^- \rightarrow \chi_{c1}(nP) + \gamma$ with $n = 1, 2, 3$ in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2. The mass of $\chi_{c1}(2P,3P)$ are selected from Ref.[17]. And the mass of $\chi_{c1}(1P)$ can be found in Ref.[24].

| $\sqrt{s}$(GeV) | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 10.6 | 11.2 |
|-----------------|------|------|------|------|------|------|------|
| $\chi_{c1}(3511)$ LO | 7186 | 3874 | 2392 | 1597 | 1124 | 23.5 | 18.5 |
| RC | 4448 | 1296 | 459  | 168  | 52  | -4.8 | -3.8 |
| QCD | -3327 | -1791 | -1091 | -715 | -492 | -6.5 | -4.9 |
| Total | 8307 | 3379 | 1760 | 1051 | 685 | 12.3 | 9.7 |
| $\chi_{c1}(2P)(3901)$ LO | 8854 | 4244 | 2495 | 1624 | 25.7 | 20.0 |
| RC | 9585 | 2297 | 789  | 312  | -4.9 | -3.9 |
| QCD | -4041 | -1967 | -1152 | -741 | -7.7 | -5.70 |
| Total | 14397 | 4573 | 2131 | 1195 | 13.2 | 10.3 |
| $\chi_{c1}(3P)(4178)$ LO | 1073 | 384  | 164  | 0.82 | 0.75 |
| RC | 1600 | 391  | 140  | -0.44 | -0.38 |
| QCD | -551 | -223 | -107 | 0.29 | 0.23 |
| Total | 2121 | 554  | 198  | 0.67 | 0.61 |

resonance contributions are ignored here.

4.1 $X(3872)$

In the light of the mixture state of the $\chi_{c1}(2P)$ and $D^0\bar{D}^{*0}$ molecule, the cross-sections of $X(3872)$ at hadron collides can be expressed as[20]:

$$d\sigma[X(3872) \rightarrow J/\psi\pi^+\pi^-] = d\sigma[\chi_{c1}(2P)] \times k,$$  \hspace{1cm} (4.1)
\[ \sigma[e^+e^- \to \chi_c2\gamma](\text{fb}) \]

![Graph](image)

**Figure 5.** The cross-sections of \( e^+e^- \to \chi_c2 + \gamma \) as a function of \( \sqrt{s} \) in fb. The cross-section \( \sigma_{\text{LO}} \), \( \sigma_{v^2} \), \( \sigma_{\alpha_s} \), and \( \sigma_{\alpha_s,v^2} \) are defined near the end of Section 2.

**Table 5.** The cross-sections of \( e^+e^- \to \chi_c(nP) + \gamma \) with \( n = 1, 2, 3 \) in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2. \( \chi_c2(2P) \) is considered as \( Z(3930) \), [1, 33]. The mass of \( \chi_c2(3P) \) are selected from Ref.[17]. And the mass of \( \chi_c2(1P) \) can be found in Ref.[24].

| \( \sqrt{s} \) (GeV) | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 10.6 | 11.2 |
|----------------------|------|------|------|------|------|------|------|
| \( \chi_c2(3556) \) |      |      |      |      |      |      |      |
| LO       | 10149 | 4724 | 2590 | 1562 | 1004 | 9.66 | 7.37 |
| RC       | 8587  | 2385 | 880  | 376  | 173  | -1.16| -0.93 |
| QCD      | -5056 | -2455| -1384| -851 | -557 | -6.27| -4.82 |
| Total    | 13679 | 4655 | 2087 | 1086 | 621  | 2.22 | 1.63 |
| \( \chi_c2(2P)(3927) \) |      |      |      |      |      |      |      |
| LO       | 13419 | 5581 | 2931 | 1927 | 11.29| 8.53 |
| RC       | 17835 | 3965 | 1355 | 565  | -1.22| -0.99 |
| QCD      | -6423 | -2822| -1533| -926 | -7.25| -5.52 |
| Total    | 24862 | 6723 | 2754 | 1368 | 2.82 | 2.03 |
| \( \chi_c2(3P)(3408) \) |      |      |      |      |      |      |      |
| LO       | 8938  | 3607 | 1886 | 8.55 | 6.40 |
| RC       | 14212 | 2949 | 995  | -0.83| -0.68 |
| QCD      | -4210 | -1803| -977 | -5.43| -4.10 |
| Total    | 18941 | 4753 | 1904 | 2.28 | 1.62 |

where \( k = Z_{c\bar{c}}^{X(3875)} \times Br[X(3872) \to J/\psi\pi^+\pi^-] \). \( Br[X(3872) \to J/\psi\pi^+\pi^-] \) is the branching fraction for \( X(3872) \) decay to \( J/\psi\pi^+\pi^- \). \( Z_{c\bar{c}}^{X(3875)} \) is the possibility of the \( \chi_c1(2P) \) component in \( X(3872) \). And \( k = 0.018 \pm 0.04 \) [19, 20].

To clarify the nature of \( X(3872) \), we also give the numerical calculation of \( e^+e^- \to \)
**Figure 6.** The cross-sections of $e^+e^- \rightarrow \chi_{c2} + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section $\sigma_{LO}$, $\sigma_{v2}$, $\sigma_{\alpha s}$, and $\sigma_{\alpha s,v2}$ are defined near the end of Section 2. The uncertainty band of $\sigma_{\alpha s,v2}$ is from the uncertainty of $k = 0.018 \pm 0.04$.

**Table 6.** The cross-sections of $e^+e^- \rightarrow X(3872) + \gamma \rightarrow J/\psi\pi\pi + \gamma$ in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2.

| $\sqrt{s}$ (GeV) | 4.15 | 4.20 | 4.25 | 4.30 | 4.35 | 4.40 | 4.45 | 4.50 |
|------------------|------|------|------|------|------|------|------|------|
| LO               | 221±49 | 180±40 | 150±33 | 127±28 | 110±24 | 84±19 | 66±15 |
| RC               | 310±69 | 208±46 | 146±32 | 106±24 | 80±18 | 47±10 | 30±7 |
| QCD              | -100±22 | -82±18 | -69±15 | -59±13 | -51±11 | -39±9 | -31±7 |
| Total            | 431±96 | 306±68 | 227±51 | 175±39 | 138±31 | 92±20 | 65±14 |

| $\sqrt{s}$ (GeV) | NRQCD prediction for continue BESIII [46, 47] |
|------------------|-----------------------------------------------|
| 4.009            | <130 at 90% CL. |
| 4.160            | 401 ± 89 |
| 4.230            | 255 ± 57 |
| 4.260            | 215 ± 48 |
| 4.360            | 133 ± 29 |
| 4.415            | 105 ± 23 |
| 4.660            | 47 ± 10 |

$\gamma X(3872) \rightarrow J/\psi\pi^+\pi^-\gamma$ in this picture

$$\sigma[e^+e^- \rightarrow \gamma X(3872)] \times \text{Br}[X \rightarrow J/\psi\pi\pi] = \sigma[e^+e^- \rightarrow \gamma \chi_{c1}(2P)(3872)] \times (0.018 \pm 0.004)$$

(4.2)
The cross-sections as a function of $\sqrt{s}$ is shown in Fig. 6. Many $1^{--}$ states with $M_H < 5$ GeV are also observed. We can predict the cross-sections from continuous contributions at this point, and the result is listed in Table 6. We ignore the $1^{--}$ resonances contributions here. We emphasize that if we select $\sqrt{s} = 4.009$ GeV, the energy of photon $E_\gamma = 134$ MeV and smaller than $\Lambda_{QCD} \sim m_c v^2 \sim 300$ MeV. Hence, NRQCD cannot accurately predict the cross-sections with a soft photon with $\sqrt{s} = 4.009$ GeV[50]. If $\sqrt{s} = 4.160$ GeV, the energy of photon is $E_\gamma = 270$ MeV. Although this process is a QED process, the prediction is not reliable and only a reference value[31]. We determined that the NRQCD prediction of the continuous contributions can be compared with the BESIII data of the cross-sections of $e^+e^- \rightarrow \gamma X(3872)$ [46, 47] in Eq.(1.1).

When we only considered the continuum production, the resonance contributions can be estimated as that:

$$
\sigma_{Res}[s] = \frac{12\pi[H[Res \rightarrow e^+e^-]|\Gamma[Res \rightarrow \gamma X]|^2}{(s - M^2)^2 + (\Gamma_{tot}[Res])^2}. \tag{4.3}
$$

We take into account only one resonance here and ignore continuum and other resonances here. If we ignore the interference between one resonance and continuum and other resonances, the gamma energy dependence of the $\Gamma[Res \rightarrow \gamma X]$, and $D\bar{D}$ contributions of decay of $Res \rightarrow \gamma X$, we can estimate the resonance contributions. With $X(3872)$ considered as $2P$ states, the largest decay widths are $\psi(4040)$ and $\psi(4160)$, which are considered as the mixing of $\psi(3S)$ and $\psi(2D)$ [97, 98]. The $\Gamma[Res \rightarrow \gamma X]$ for other states will be less than 1 keV [98], and $\Gamma_{tot} \sim 100$ MeV, $\Gamma[Res \rightarrow e^+e^-] \sim 1$ keV. Hence, we ignore the contributions from other resonances. With the parameters for $\psi(4040)$ and $\psi(4160)$[24, 98]:

$$
\Gamma[\psi(4040) \rightarrow e^+e^-] = 0.87 \text{ keV} \quad \Gamma[\psi(4040) \rightarrow \gamma X] = 40 \text{ keV} \quad \Gamma_{tot}[\psi(4040)] = 80 \text{ MeV}
$$

$$
\Gamma[\psi(4160) \rightarrow e^+e^-] = 0.83 \text{ keV} \quad \Gamma[\psi(4160) \rightarrow \gamma X] = 140 \text{ keV} \quad \Gamma_{tot}[\psi(4160)] = 103 \text{ MeV}
$$

Hence, we can determine the contributions of these parameters to $X(3872) \gamma \rightarrow J/\psi \pi^+\pi^-\gamma$

$$
(\sigma_{\psi(4040)}[4.23] + \sigma_{\psi(4160)}[4.23]) \times k = (62 \pm 14) \text{ fb}
$$

$$
(\sigma_{\psi(4040)}[4.26] + \sigma_{\psi(4160)}[4.26]) \times k = (37 \pm 8) \text{ fb} \tag{4.4}
$$

If we considered the interference, the result would be more complex. On the other hand, we have calculated the quark-level intermediate states, which do not clearly deal with the hadron-level intermediate states.

4.2 X(3940) and X(4160)

X(3940) and X(4160) are observed in $e^+e^- \rightarrow J/\psi (D\bar{D})$ at B factories [43]. $\eta_c$ and $\chi_{c0}$ are recoiled with $J/\psi$, but $\chi_{c1}$ and $\chi_{c2}$ are missed[43]. The theoretical predictions are consistent with the experimental data[61, 69, 99, 100]. So there should be large $\eta_c(nS)$ and $\chi_{c0}(nP)$ component in X(3940) and X(4160), respectively. The mass of $\eta_c(3S)$ and $\chi_{c0}(3P)$ are predicted as 3994 MeV and 4130 MeV respectively[17]. Compared with Table 2 and Table
we can found that the cross-sections of $\eta_c(3S)$ is small even negative at $\sqrt{s} < 5$ GeV. But $\chi_{c0}(3P)$ is large. The cross-sections as a function of $\sqrt{s}$ is shown in Fig 7. Here $Z_{c\bar{c}}^X \leq 1$ is the possibility of $\eta_c(3S)$ and $\chi_{c0}(3P)$ component in $X(3940)$ and $X(4160)$ respectively. The BESIII collaboration can search $X(3940)$ and $X(4160)$ in the process $e^+e^- \rightarrow \gamma + X(DD)$. The result may be useful in identifying the nature of $X(3940)$ and $X(4160)$.

![Graph](image)

**Figure 7.** The cross-sections of $e^+e^- \rightarrow X(3940)(X(4160)) + \gamma$ as a function of $\sqrt{s}$ in fb.

4.3 $X(4350)$

$X(4350)$ are found in $\gamma\gamma \rightarrow H \rightarrow \phi J/\psi$ at B factories \cite{45}. And $J^{PC}$ is $0^{++}$ or $2^{++}$. So there should be large $\chi_{c0}(nP)$ or $\chi_{c2}(nP)$ component in $X(4350)$. In Ref.\cite{17}, The mass of $\chi_{c2}(3P)$ is 4208 MeV. Ignore more detail of the mass, we considered it as $\chi_{c0}(nS)$ or $\chi_{c2}(nP)$, the wave function at origin are estimated as

$$R' = R'_{3P} = (R'_{1P} + R'_{2P})/2 = 0.159\text{GeV}^5,$$

(4.5)

The cross-sections of $e^+e^- \rightarrow X(4350) + \gamma$ as a function of $\sqrt{s}$ is show in Fig.8. Here $Z_{c\bar{c}}^X$ is the possibility of $\chi_{c0}(nP)$ or $\chi_{c2}(nP)$ component in $X(4350)$. The cross-section for $\chi_{c2}(nP)$ is larger than $\chi_{c0}(nP)$ by a factor of 6. The result may be useful in identifying the nature of $X(4350)$.

5 Summary and discussion

While BESIII and Belle have collected a large amount of data, some final states may be searched by the experimentalists. We can estimate the possible event number at BESIII and Belle. The possible event number is

$$N = \sigma[e^+e^- \rightarrow \gamma + c\bar{c}[n]] \times Z_{c\bar{c}}^H \times Br \times L \times \epsilon,$$

(5.1)
Figure 8. The cross-sections of $e^+e^- \rightarrow X(4350) + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section $\sigma_{LO}$, $\sigma_{\alpha_s}$, $\sigma_{\alpha_s,x}$, and $\sigma_{\alpha_s,x,x}$ are defined near the end of Section 2. And $Z_{c\bar{c}}^X$ is the possibility of $\chi_{c0}(nP)$ or $\chi_{c2}(nP)$ component in $X(4350)$.

where $\epsilon$ is the efficiency of detectors selected as 20%, $Br$ is the branch ratio of $H$ to the decay mode, and $\mathcal{L}$ is the luminosity. The result is listed in Table 7.

Table 7. The possible event number of $C = +$ charmonium and XYZ states through $e^+e^- \rightarrow \gamma + H$ at BESIII and Belle. The efficiency of detectors are selected as 20%. The integrated luminosity is $1.0 fb^{-1}@4.23$ GeV, $1.0 fb^{-1}@4.26$ GeV, $0.5 fb^{-1}@4.66$ GeV, and $1 ab^{-1}@10.6$ GeV. The decay mode of $nKm\pi$ corresponds to $D\bar{D}$ decay, and the branch ratio is estimated as 1%.

| $H$                      | Decay    | $Br$ | $Z_{c\bar{c}}^H$ | 4.23 | 4.26 | 4.66 | 10.6 |
|------------------------|----------|------|-----------------|------|------|------|------|
| $\eta_c$               | $K\bar{K}\pi$ | 7.2% | 1 | 9 | 9 | 5 | 1012 |
| $\chi_{c0}$            | $2\pi^+2\pi^-$ | 2.2% | 1 | 2 | 2 | 6 | |
| $\chi_{c1}$            | $\gamma l^-l^- (\gamma J/\psi)$ | 4.1% | 1 | 29 | 27 | 5 | 101 |
| $\chi_{c2}$            | $\gamma l^-l^- (\gamma J/\psi)$ | 2.3% | 1 | 23 | 20 | 3 | 10 |
| $\eta_{c2}(1D)$        | $\gamma \gamma K\bar{K}\pi$ | 1.5% | 1 | | | 2 | |
| $\eta_{c}(2S)$         | $K\bar{K}\pi$ | 1.9% | 1 | | | | 123 |
| $X(3872)(\chi_{c1}(2P))$ | $\pi^+\pi^-l^+l^- (\pi^+\pi^- J/\psi)$ | 0.6% | 0.36 | 6 | 5 | 1 | 6 |
| $X(3915)(\chi_{c0}(2P))$ | $\pi^+\pi^-\rho^0l^- (\omega J/\psi)$ | 1% | 1 | 9 | 8 | | 2 |
| $Z(3930)(\chi_{c2}(2P))$ | $nKm\pi(D\bar{D})$ | 1% | 1 | 57 | 46 | 4 | 6 |
| $X(3940)(\eta_c(3S))$  | $nKm\pi(D\bar{D})$ | 1% | 1 | | | | 48 |

As a summary, we study the production of $C = +$ charmonium states $H$ in $e^+e^- \rightarrow \gamma + H$ at BESIII with $H = \eta_c(nS)$ (n=1, 2, 3, and 4), $\chi_{cJ}(nP)$ (n=1, 2, and 3), and $1D_2(nD)$ (n=1 and 2) within the framework of NRQCD. The radiative and relativistic corrections are
calculated to next-to-leading order for $S$ and $P$ wave states. We then argue that the search for $C = + XYZ$ states such as $X(3872)$, $X(3940)$, $X(4160)$, and $X(4350)$ in $e^+e^- \rightarrow \gamma + H$ at BESIII may help clarify the nature of these states. BESIII can search $XYZ$ states through two body process $e^+e^- \rightarrow \gamma H$, where $H$ decay to $J/\psi \pi^+\pi^-$, $J/\psi \phi$, or $D \bar{D}$. This result may be useful in identifying the nature of $C = + XYZ$ states. For completeness, the production of $C = +$ charmonium in $e^+e^- \rightarrow \gamma + H$ at B factories is also discussed.

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