Backlash Analysis of Compound Planetary Gear Set in Rotary Actuators

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Abstract. This paper proposes a model for backlash analysis of compound planetary gear set in rotary actuators. Three main types of errors in the compound planetary gear set are analysed, that is, manufacturing errors, assembly errors and deformation errors. Due to the randomness of the errors, the backlash model of the gear set was established based on probability theory to improve the backlash estimation for the practical applications. Finally, the backlash of a compound planetary gear set was calculated and the influences of main errors to the backlash were also investigated.

1. Introduction
Aviation actuator is an important transmission mechanism of automatic flight control system. It receives the control instructions of the control system to drive the deflection of plane rudder, and then completes the control of the flight attitude and trajectory. Its performance affects the flight quality of the aircraft, such as maneuverability, stability, safety and reliability. The structure size of the actuator is limited by the aircraft space, and its weight should be as lighter as possible. Meanwhile, sufficient precision and load requirements should be ensured. Therefore, the design of actuator needs to meet these requirements of the special working environment of the aircraft [1-2].

One of the main factors influencing the performance of the actuator is the gear backlash, which is defined as the lag of the output shaft at the rotation angle when the input shaft changes the direction of rotation in the working state [3]. So in the design stage of the actuator, it is necessary to estimate the numerical range of the backlash in order to evaluate whether the design scheme is reliable.

In the previous studies, the extremum method [4-5] is often used to calculate the gear backlash. However, all the influencing factors are not simultaneously at the maximum. So the system synthetic error obtained from the extremum method exceeds the actual value, thus unreasonably improving the manufacturing precision and resulting in the uneconomic consequences [6].

In the design stage, the tolerances of each part are given, and the errors of these parts are usually distributed in certain statistical rules. Therefore, it is more reasonable to use probability method to establish the statistical synthesis formula of transmission chain backlash [7-8].

2. Design
According to the working conditions of the aircraft, basic structure and transmission technical requirements of the actuator are transmission ratio over 100, output torque over 200 Nm, output angular velocity about 30r/min, structure diameter within 150mm, and backlash angle within 0.5°.

In the design process of the actuator, the new structure adopted is compound planetary gear set and the diagram of compound planetary gear set is shown in Figure 1.
Figure 1. Diagram of compound planetary gear set.

Three planetary gears on the same shaft form a planetary gear set and there are four meshes on each planet gear set. The power system controls the sun gear S1 which engages with gear P1 to drive the planetary gear set. The planetary gear P1 drives the ring gear R1 as output. The fixed ring gear R2 and R2’ adopt symmetrical design in order to reduce the wear of the tooth surface caused by bending moment of the large output torque. In order to realize power distribution, the whole mechanism uses three planetary gear sets. The gear ratio \(i_{S1R1}\) can be expressed as:

\[
\frac{Z_{R1}(Z_{P1}Z_{R2} + Z_{S1}Z_{P2})}{Z_{S1}(Z_{R1}Z_{P2} - Z_{R2}Z_{P1})}
\]

In this scheme, the gear module is 1mm and the pressure Angle is 20 degrees. The gear geometry parameter and precision are shown in Table 1. According to the equation (1), the ratio is 178. And the transmission efficiency is 0.631.

Table 1. Gear geometry of the compound planetary gear set.

| Gear | S1 | P1 | R1 | P2 | R2 |
|------|----|----|----|----|----|
| Tooth number  | 18 | 45 | 108 | 42 | 105 |
| Tooth Width | 15 | 15 | 15 | 12 | 15 |
| Precision grade (GB2363-90) | 7e | 7e | 7e | 7e | 7e |

3. Modelling

3.1. Error sources

The main factors affecting the backlash mainly cover three categories including manufacturing error caused by the machine tool error, assembly error caused by errors of gearbox assembly and deformation error caused by the load. Due to these errors, there is a backlash between the gear pair in the process of meshing, and the circumferential backlash is defined as \(j_i\).

3.2. Manufacturing error

The influence of manufacturing error on the backlash is reflected in two aspects, tooth thickness deviation and geometric eccentricity.

3.2.1. Tooth thickness deviation. Tooth thickness deviation is an important error in gear manufacturing. The tooth thickness deviation can be estimated according to the deviation \(\Delta E_{\text{mm}}\) of the common normal line length. \(\Delta E_{\text{mm}}\) is a negative value, so the circumferential backlash can be expressed as:

\[
j_{\text{mm}} = \Delta E_{\text{mm}} \cos \alpha_n
\]
3.2.2. Geometric eccentricity. Due to the positioning error of the machine tool worktable, there is an eccentricity between the base axis of the gear blank and the rotary axis of the worktable. In the probability method, the influence of the eccentricity is analyzed by the total composite radial deviation $\Delta F''_i$ of the gear. $\Delta F''_i$ represents the difference between the maximum and minimum center distance, so the value of the eccentricity is approximately equivalent to $\frac{1}{2}\Delta F''_i$. The eccentricity $e_g$ follows Rayleigh distribution and the numerical characteristics are:

$$E_{eg} = \pi \frac{1}{2} \Delta F''_i \approx 0.62 \Delta F''_i$$

The circumferential backlash can be expressed as:

$$j_{eg} = 2e_g \tan \alpha_n \approx 1.24 \Delta F''_i \tan \alpha_n$$

3.3. Assembly error

3.3.1. Parallelism error of bearing mounting holes. The error of gearbox during the assembly results in the parallelism error of gear shaft axis. In this paper, a rectangular coordinate system of gear is established, with the shaft axis as X axis, the center line direction of meshing gear pair as Z axis, and the direction perpendicular to X axis and Z axis as Y axis. The deflection $\Delta f_{xy}$ of gear X axis in XY plane is shown in Figure 2. When the deflection occurs, $\Delta f_{xy}$ increases the actual tooth thickness in the meshing plane, and the change of effective tooth thickness is:

$$\Delta S = S(\cos \lambda_{xy} - 1) + b \sin \lambda_{xy} \approx b \lambda_{xy} = b \frac{\Delta f_{xy}}{b} = \Delta f_{xy}$$

In this equation: $S$ -The original tooth thickness; $b$-the tooth width; $\lambda_{xy}$ -A tiny angular displacement.

![Figure 2. The deflection of gear X axis in XY plane.](image)

The circumferential backlash can be expressed as:

$$j_{xy} = -\Delta S \approx -\Delta f_{xy}$$

The deflection $\Delta f_{xz}$ of gear X axis in XZ plane is shown in Figure 3.

![Figure 3. The deflection of gear X axis in XZ plane.](image)
The radial displacement at any point of the involute is:
\[
j_{sc} = \frac{b}{2} \sin \lambda_{sc} - r(1 - \cos \lambda_{sc}) = \frac{b}{2} \Delta f_{sc} - r(1 - \cos \lambda_{sc}) = \frac{\Delta f_{sc}}{2} - r(1 - \cos \lambda_{sc}) \approx \frac{1}{2} \Delta f_{sc}
\] (7)

In this equation: \(b\) - the tooth width; \(\lambda_{sc}\) - A tiny angular displacement.

The circumferential backlash can be expressed as:
\[
j_{sc} = -2j_{sc} \tan \alpha_n \approx -\Delta f_{sc} \tan \alpha_n
\] (8)

### 3.3.2. Error of bearing

The clearance errors introduced by bearing include radial clearance \(\Delta r\) of bearing, assembly gap \(\Delta C_{bi}\) of bearing inner ring and shaft, assembly gap \(\Delta C_{bo}\) of bearing outer ring and mounting hole. The bearing clearance causes gear movement in the radial direction increasing the center distance of gear pair. The circumferential backlash can be expressed as:
\[
j_{sc} = \Delta r \tan \alpha_n; j_{sc} = \Delta C_{bi} \tan \alpha_n; j_{sc} = \Delta C_{bo} \tan \alpha_n
\] (9)

### 3.4. Deformation error

#### 3.4.1. Torsional deformation of the shaft

When the rotating shaft is under the load of output torque \(T\), the torsion deformation of the driving shaft is:
\[
\varphi_n = \frac{180 \times 60}{\pi} \frac{T}{G I_n}
\] (10)

#### 3.4.2. Bending deformation of the shaft

In the process of gear transmission, under the action of normal force \(F_n\) on the tooth surface, the shaft will bend and deform. The bending deformation \(f_t\) can be divided into tangential component \(f_t\) and radial component \(f_r\). According to the equation (6) and (8), the following relationship is derived:
\[
j_{pn} = 2f_t + 2f_r \tan \alpha_n = 2f_t \cos \alpha_n + 2f_r \sin \alpha_n \tan \alpha_n = \frac{2f_t}{\cos \alpha_n}
\] (11)

### 3.5. The calculation model of backlash

The backlash angle caused by \(j_i\) is:
\[
B_p = \frac{j_i}{2mp_2} \times \frac{180}{\pi} \times 60 = 6.876 \times \frac{j_i}{m_Z}
\] (12)

So the backlash angle of gear A caused by the errors is:
\[
B_A = 6.876 \times \frac{1}{m_Z} (2j_{iZ_{1A}} + j_{iP_{2A}} + j_{iP_{1A}} + j_{iA} + j_{iC_{3A}} + j_{iC_{4A}} + j_{iP_{1A}} + j_{iP_{2A}} + 2\varphi_{nA})
\] (13)

The backlash angle of gear A caused by gear pair A and B is:
\[
B_{AB} = B_A + \frac{Z_B}{Z_A} B_B
\] (14)

For the compound planetary gear set in this paper, the backlash converted to the output gear R1 is:
\[
B_{output} = \frac{1}{i_{SR1}} B_{S1P1} + B_{R1P1} + \frac{Z_{R2} Z_{P1}}{Z_{P2} Z_{R1}} B_{R2P2}
\] (15)

### 4. Results

#### 4.1. The backlash angle of the compound planetary gear set
On the basis of accuracy standards, Table 2 shows the main tolerances of the error. According to the equation (13), (14) and (15), the backlash angle of each gear pair and output gear is shown in Table 3.

**Table 2. The main tolerances of the error.**

| Gear               | S1     | P1     | R1     | P2(P2') | R2(R2') |
|--------------------|--------|--------|--------|---------|---------|
| Manufacturing error| $\Delta E_{sw}$ / $\mu m$ | -20    | -20    | -27.5   | -20     |
|                    | $\Delta F^*$ / $\mu m$    | 25     | 32     | 39      | 32      |
| Parallelism error  | $\Delta f_{sc}$ / $\mu m$ | 5      |        |         |         |
|                    | $\Delta f_{sy}$ / $\mu m$ | 3      |        |         |         |
| Bearing error      | $\Delta \eta$ / $\mu m$   | 10.5   |        |         |         |
|                    | $\Delta C_{b1}$ / $\mu m$ | 8      |        |         |         |
|                    | $\Delta C_{bc}$ / $\mu m$ | 19     |        |         |         |
| Deformation        | $\phi_{c}$ / arc min      | 0      | 3.05   | 0       | 1.53    |
|                    | $f_{c}$ / $\mu m$          | 0      | 0.08   | 0       | 0.41    |

**Table 3. The backlash angle of the compound planetary gear set.**

| The backlash between gear pair | Backlash angle/arc min |
|-------------------------------|------------------------|
| Gear pair S1 and P1           | $B_{S1P1}$             |
| Gear pair P1 and R1           | $B_{P1R1}$             |
| Gear pair P2 and R2           | $B_{P2R2}$             |
| Sum(Convert to R1)            | $B_{Output}$           |

4.2. The error analysis

Figure 4 shows the influence of tooth thickness deviation, composite radial error and parallelism deflection of each gear on the backlash. According to Figure 4, the errors of gear P2 and R2 have the greatest influence and the gear P1 and R1 play an important role on the backlash. Because of the large ratio, the backlash angle caused by the errors of gear S1 is much smaller than the errors of other gears.

From Figure 4, it can be seen from the results that manufacturing errors are fundamental and the main source of the backlash. The errors of assembly have little impact on the backlash. Because the value of parallelism deflection is usually less than other errors and the actual backlash angle by the bearing will decrease with three sets of evenly distributed gears.

Deformation error is a dynamic error, varying according to the load. In order to meet the strength requirements, the deformation error should be kept within a reasonable range.
Figure 4. The effect of the main errors on backlash.

5. Conclusions
In this paper, the actuator uses a new structure of compound planetary gear set and realizes the transmission of the actuator with large transmission ratio, high torque, small size and high precision. For the compound planetary gear set, this paper systematically analyzed the main factors of the gear set including manufacturing error, assembly error and deformation error, and the system backlash analysis model is established by probability method.

According to the model, the backlash angle is 24.8 arcmin which meets the accuracy requirements of actuator. The influences of the errors on the backlash can be obtained from the model. The main factors affecting backlash is the manufacturing error which mainly caused by the tooth thickness deviation. It should be noted that the influence of each gear error on output backlash angle can also be achieved by the model. This model can be a reference to the error allocation.

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