Study of torque matching of revolving vane compressor and expander

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Abstract. An investigation was carried out to find the most optimum configuration, particularly the torque matching characteristics, of an integrated Revolving Vane compressor-expander. To carry out the study, a mathematical model of the integrated compressor-expander was developed. An open cycle air refrigeration system was adopted. The controlled parameter was the angle shift between the compressor and the expander. The observed parameters were the peak torque requirement and the bearing load. The results show that when properly matched, the peak torque can be reduced by more than 65\% while the bearing loads can be reduced by up to 25\%, depending on operating conditions. Unfortunately, the optimum angle shifts for peak torque do not always coincide with those for bearing load. When the pressure and inertial components of the torques are comparable or when the inertial component is dominant, the optimum angle shifts for peak torque and bearing load are around 180\° and 330\°, respectively. When the pressure component is dominant, the optimum angle shift for peak torque is equal to the angle difference between the pressure peak torques of the compressor and the expander while for bearing load is around 150\°.

1. Introduction
Revolving vane (RV) mechanism was introduced in 2006 [1] as a novel rotary mechanism that is simple and has low frictional losses. In its basic form, the mechanism has only 3 main components, i.e. a cylinder, a rotor and a vane. Unlike typical rotary mechanisms where the cylinders are stationary, the cylinder of a RV machine rotates together with the rotor. This results in reduced relative velocities between the rubbing surfaces and hence, the frictional losses are reduced too. The mechanism has been employed for compressor [2-6] and expander [7-10] applications. Moreover, various alternative designs have been introduced and studied [11-12]. The main components and the working principle as an expander of one of the variants, called “RV-I” elsewhere [11], are shown in figure 1.

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Figure 1. Schematic and working principles of a revolving vane expander [7].

As shown in the figure, the rotor and the cylinder rotate together eccentrically at their respective axes. A line contact is always maintained. Together with the vane, this line contact separates the space between the cylinder and the rotor into two working chambers. As the components rotate, one chamber decreases in volume while the other increases. In practice, the first is the discharge chamber and the latter is the suction chamber. It is important to note that to get a superior performance, the driving shaft must be connected to the component where the vane is attached to [11]. So, the driving shaft of the RV machine in figure 1 is at the rotor.

Expanders have been proposed in the recent years to increase the energy efficiency of refrigeration systems by recovering the usually wasted expansion energy in the expansion device. In conventional systems, the increase in Coefficient of Performance (COP) is more than 10% [13] while it is more than 50% in transcritical CO\textsubscript{2} refrigeration systems [14]. Expanders are also economically feasible and attractive once the technology is mature enough and mass production begins [15]. One of the remaining technological challenges left is in the integration of expander and compressor. This issue encompasses many aspects including design, manufacturing, performance, etc.
In this study, the torque matching aspect of an integrated RV expander and RV compressor for performance optimisation is studied. The RV design variant adopted was that shown in figure 1. It was assumed that the compressor and the expander shared a common shaft at their rotors. The controlled design parameter was the angle difference between the expander and the compressor. The performance was quantified by the peak torque requirement and the frictional loss. A low peak torque requirement is desirable as it allows for the use of a smaller motor. It also indirectly reduces the mechanical losses. Among all the frictional forces, the integration of RV compressor and RV expander affects particularly those at the shaft bearings due to the coupling of the loads. Therefore, the frictional loss analysis was focused on the bearing loads. A small average bearing load is desirable to minimise the bearing frictional loss.

2. Methodology
To carry out the study, a mathematical model of the integrated RV compressor-expander was developed. It combined the RV compressor and RV expander models developed previously [2-10], covering the geometrical, kinematics, dynamics and thermodynamics processes of the machine. The integrated compressor-expander machine was assumed to operate in an open cycle air refrigeration system where the compressor and the expander were linked with a shared shaft as illustrated in figure 2. In such a system, ambient air is first compressed by the compressor to increase its temperature and pressure. The discharged air is then channelled to a heat exchanger to reduce its temperature. This air is then expanded by the expander to reduce its temperature and pressure and is then used.

![Figure 2. Schematic of an open cycle refrigeration system.](image)

Assumptions were adopted to simplify the model. They include:
- Negligible pressure drop in the heat exchanger, so the compressor’s outlet pressure was equal to the expander’s inlet pressure
- Constant and equal operational speeds of compressor and expander
- Compressor and expander rotate in the same direction
- Negligible vane thickness, so the vane was always in contact with the driven component
- Negligible centrifugal forces
- Frictional losses were neglected in the analysis of peak torque
- Adiabatic compression and expansion processes
- Ambient air conditions were 30°C and 1 atm
- Compressor inlet was at ambient conditions
- Expander inlet temperature was 10°C hotter than ambient
- Expander outlet was at ambient pressure
- Negligible internal leakages in the compressor and the expander
Negligible dead volume

Ideal valves and ports

Size of expander was calculated according to compressor capacity and operating conditions, expander’s cylinder radius is constant while rotor radius is adjustable.

Mathematical models of RV machines have been proposed and verified before [6,10]. The most important equations after the adoption of the assumptions above are reviewed briefly here. Detailed descriptions of the equations and terms are available in the relevant references.

The chamber volumes are shown in equations (1) and (2). The corresponding parameters are computed according to equations (3-5).

\[ V_{sv} = \frac{r_c^2}{2} \left( r_r^2 - e_r^2 \sin \varphi_r - \varphi_r r_r^2 \right) \]  
(1)

\[ V_{dcv} = l_{CV} \pi \left( r_c^2 - r_r^2 \right) - V_{sv} \]  
(2)

\[ l_{se} = -e \cos \varphi_r - r_e + \sqrt{r_r^2 - e_r^2 \sin^2 \varphi_r} \]  
(3)

\[ \cos \varphi_e = \frac{e^2 + r_r^2 - (r_e + l_{se})^2}{2r_e e} \]  
(4)

where \( e \) is eccentricity (m), \( l_{CV} \) is working chamber length (m), \( l_{se} \) is exposed vane length (m), \( r_c \) and \( r_r \) are radii of cylinder and rotor, respectively (m), \( V_{dcv} \) and \( V_{scv} \) are volumes of discharge and suction chambers, respectively (m³), \( \varphi_c \) and \( \varphi_r \) are angles of cylinder and rotor, respectively (rad).

The torque required by compressor generated by expander is calculated according to equation (6). The corresponding parameters are computed with equations (7-9). Equation (6) gives out a positive value for torque requirement of the compressor and a negative value for torque generated by the expander. Equation (6) also shows that the torque of a RV machine comprises three components, i.e. pressure, inertial and losses components. The first two are more dominant than the last. The nett torque of the integrated RV compressor-expander machine is the summation of the RV compressor’s torque requirement and the RV expander’s torque generated.

\[ T = (p_{dcv} - p_{sv}) l_{CV} \left( r_r + \frac{l_{se}}{2} \right) + \frac{r_r + l_{se}}{r_r \cos \varphi_r} I_c \]  
(6)

\[ \cos \varphi_r = \frac{r_r^2 + (r_r + l_{se})^2 - e^2}{2r_e (r_r + l_{se})} \]  
(7)

\[ \alpha_r = \omega_r^2 \frac{d l_{se}}{dt} \left( \frac{4(r_r + l_{se})(r_r^2 - e^2)}{(r_r^2 + (r_r + l_{se})^2 - e^2)^2} \right) \]  
(8)

\[ \frac{d l_{se}}{dt} = \omega_r e \sin \varphi_r \left( 1 - \frac{e \cos \varphi_r}{\sqrt{e^2 - e^2 \sin^2 \varphi_r}} \right) \]  
(9)

where \( I_c \) is the inertia of cylinder (kg·m²), \( p_{scv} \) and \( p_{dcv} \) are pressures of suction and discharge chambers, respectively (Pa), \( T \) is torque (N·m), \( t \) is time (s), \( \alpha_r \) is angular acceleration of cylinder (rad/s²), \( \omega_r \) is angular velocity of rotor (rad/s).

The shaft bearing load's components in the x and y directions are according to equations (10-11).

\[ F_x = \left( \frac{I_c \alpha_r}{r_r \cos \varphi_r} - (p_{sv} - p_{dcv}) l_{CV} (r_r + l_{se}) \right) \cos \varphi_r + (p_{sv} - p_{dcv}) l_{CV} r_r \]  
(10)

\[ F_y = \left( -\frac{I_c \alpha_r}{r_r \cos \varphi_r} + (p_{sv} - p_{dcv}) (r_r + l_{se}) l_{CV} \right) \sin \varphi_r \]  
(11)

where \( F_x \) and \( F_y \) are shaft bearing load components in the x and y axes, respectively (N)
A computer spreadsheet was developed to solve all the equations. The independent variable in the model was the rotor angle of the compressor, $\phi_{r,\text{compressor}}$. It is important to note that some of the equations need inputs from other equations. Therefore, they had to be solved in order. The models were applied for the RV compressor and the RV expander. An “angle shift” was introduced between the compressor and the expander as expressed in equation (12). This shifted the torque profiles of the compressor and the expander, allowing for control of the matching of the corresponding torques. The shift will modify the amplitude and shape of the nett torque of the integrated machine. The average value of the nett torque was affected too, mainly through the change in the bearing loads. However, the average values of the pressure and inertia components of the torque were unaffected by this shift.

$$\phi_{r,\text{expander}} = \phi_{r,\text{compressor}} + \Delta\phi$$

(12)

Major dimensions of the model and operating parameters are tabulated in Table 1. The dimensions were based on the RV expander prototype built and tested before [10]. The inertia values of the components were obtained with the SolidWorks design software.

### Table 1. Major dimensions and operating parameters

| Item                  | Value                  |
|-----------------------|------------------------|
| **Compressor**        |                        |
| Rotor radius          | 29 mm                  |
| Cylinder radius       | 35 mm                  |
| Length                | 25 mm                  |
| Capacity              | 30.2 cm$^3$            |
| Cylinder inertia      | 0.00281 kg·m$^2$       |
| Rotor inertia         | 0.00021 kg·m$^2$       |
| Suction pressure      | 1 atm                  |
| Discharge pressure    | 8 - 80 bar (abs.)      |
| **Expander**          |                        |
| Cylinder radius       | 35 mm                  |
| Length                | 25 mm                  |
| Cylinder inertia      | 0.00281 kg·m$^2$       |
| Discharge pressure    | 1 atm                  |
| **Operating Parameters** |                      |
| Working fluid         | Air                    |
| Ambient temperature   | 30°C                   |
| Ambient pressure      | 1 atm                  |
| Polytropic constant   | 1.4                    |
| Operating speed       | 1000 - 3000 rev/min    |

### 3. Results and discussion

Results of the simulation are presented and discussed in this section. It is useful to start the discussion by observing the major components of a RV machine’s torque, which are the pressure and the inertia components. Some of these are plotted in figure 3. Ratio between the pressure and the inertial components changed with different operating pressure and speed conditions. In the figure, it can be seen that when compressor discharge pressure was 8 bar and speed was 1000 rev/min, the pressure and
inertial components were comparable. However, the total peak torque was practically following that of the inertial component. When pressure was 80 bar and speed was 1000 rev/min, the pressure component was dominant and hence, the total peak torque was dictated by the pressure component. When pressure was 8 bar and speed was 3000 rev/min, the inertial components was dominant. Consequently, the total peak torque followed that of the inertial component. It is also useful to observe that the profiles of the inertial components of RV compressor and RV expander torques were similar in shape. They were sinusoidal, positive in the first half of the cycle and negative in the second half. However, the trends of the pressure components were different. The peak of the pressure component of the RV compressor was in the second half of the cycle while that of the RV expander was in the first half.

**Figure 3.** Components of torques of RV compressor and RV expander at various compressor discharge pressures and operating speeds.

The values of nett peak torque of the integrated RV compressor-expander machine at various angle shifts, compressor discharge pressures and operating speeds are plotted in figure 4. In general, it is found that it was not preferable to have an angle shift of 0°. When properly matched, the nett peak torque can be more than 65% smaller as compared to that with an angle shift of 0°, such as that in the case of 8 bar compressor discharge pressure and 3000 rev/min operating speed. On the other hand, when not properly matched, the nett peak torque can be up to 35% higher than when angle shift is 0°.
This change of nett peak torque was because of the modification in the nett torque profile as the angle shift changes as illustrated in figure 5. For example, when compressor discharge pressure was 8 bar, operating speed was 1000 rev/min and angle shift was 0°, the nett peak torque was more than 15% higher than that of the compressor alone. This was because when the compressor torque peaked, the expander was also positive. Hence, the nett torque is even higher than the compressor’s peak torque. However, when the angle shift was 180°, the nett peak torque was more than 50% lower than the compressor peak torque because when the compressor torque peaked, the expander torque was negative. Hence, the nett torque was a smaller than the compressor’s original peak torque.

![Figure 4. Variations of nett peak torque of the integrated RV compressor-expander with different angle shifts, operating speeds and compressor discharge pressure conditions.](image)

The optimum angle shift depended on the operating speed and compressor discharge pressure. When the inertial and pressure components of torque were comparable, such as that in the case of 8 bar compressor discharge pressure and 1000 rev/min operating speed, or when the inertial component was dominant, such as when compressor discharge pressure was 8 bar and speed was 3000 rev/min, the optimum angle shift was in the vicinity of 180°. In such cases, the nett peak torque followed that of the inertial component as shown in figure 3 above. Moreover, the inertial components of RV compressor’s and RV expander’s torques were of similar profiles. Therefore, a shift of around 180° causes the two torque components to cancel each other as shown in figure 5. When the pressure component of the torque was dominant, such as when the compressor discharge pressure was 80 bar and operating speed was 1000 rev/min, the optimum angle shift was equal to the angle difference between the pressure peak torques of the compressor and the expander. These usually occur at the start of discharge in the compressor and at the end of suction in the expander, respectively.
The variations of average bearing load with angle shift, operating speed and compressor discharge pressure are shown in figure 6. When properly matched, the average bearing load can be up to 25% smaller than that with an angle shift of 0° (i.e. 8 bar compressor discharge pressure and 1000 rev/min operating speed). On the other hand, when not properly matched, the average bearing load can be up to 50% higher than when the angle shift is 0° (i.e. at 8 bar compressor discharge pressure and 2000 rev/min speed). In general, when the inertial component dominated (i.e. when speed is high or compressor discharge pressure is low), the optimum angle shift was in the vicinity of 330°. On the other hand, when the pressure component was dominant, the optimum angle shift was around 150°.

A note is necessary here to discuss the anomaly of the 8 bar, 3000 rev/min data in Figure 6. In general, bearing load is usually a weak function of the operational speed. This is true only when the pressure component of the load is significantly stronger than the inertial (speed) component. However, at 8 bar and 3000 rpm, the pressure component is smaller than the inertial component as mentioned in the previous sections. Hence, the load is dictated mostly by speed in such condition.

Comparing figures 4 and 6 shows, unfortunately, that the optimum angle shifts for bearing load reduction do not always coincide with those for peak torque reduction. Therefore, the designer will eventually have to decide which angle shift to use.

**Figure 5.** Torques of compressor, expander and integrated machine at various compressor discharge pressures, operating speeds and angle shifts.
4. Conclusions

Torque matching of an integrated revolving vane (RV) compressor-expander has been studied. The controlled design parameter was the angle shift between the expander and the compressor. The performance was quantified by the peak torque requirement and the shaft bearing load. To carry out the study, a mathematical model of the integrated RV compressor-expander was developed. The integrated compressor-expander machine was assumed to operate in an open cycle air refrigeration system. The low-side pressure was 1 atm, the high-side pressure was between 8-80 bar and the operating speed was between 1000-3000 rev/min. The compressor capacity was 30.2 cm$^3$.

From the study, the followings were observed:

- In general, it was not preferable to have no angle shift between the compressor and the expander as it may result in a high nett peak torque and/or a high average bearing load.
- When properly matched, the nett peak torque can be more than 65% smaller as compared to that with an angle shift of 0°. On the other hand, when not properly matched, the nett peak torque can be up to 35% higher than when angle shift is 0°.
- The optimum angle shift for nett peak torque depended on the operating speed and compressor discharge pressure. When the pressure component of the torque was dominant, i.e. when the pressure ratio was large, the optimum angle shift was equal to the angle difference between the pressure peak torques of the compressor and the expander. These usually occur at the start of discharge in the compressor and at the end of suction in the expander. In other cases, optimum angle shift was in the vicinity of 180°.
- When properly matched, the average bearing load can be up to 25% smaller than that with an angle shift of 0°. On the other hand, when not properly matched, the average bearing load can be up to 50% higher than when the angle shift is 0°.
- In general, when the inertial component was dominant (i.e. when speed is high or compressor discharge pressure is low), the optimum angle shift was in the vicinity of 330°. On the other hand, when the pressure component was dominant, the optimum angle shift was around 150°.
- The optimum angle shifts for bearing load reduction do not always coincide with those for peak torque reduction.

The study provides simple guidelines to help in the design process of an integrated RV compressor-expander. The method developed is simple and flexible enough to be used for further studies. However, more detailed studies and experimental validation will be conducted for better understanding of the phenomena.
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