Ultra-high energy head-on collisions without horizons or naked singularities: general approach

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Recently, alternatives to the Bañados, Silk and West (BSW) effect were proposed which are (i) due to the existence of naked singularities instead of the horizon, (ii) require neither horizon nor naked singularity. We reveal the main features of such alternatives in a model-independent way. The metric should be close to that of the extremal black hole but the horizon should not form. Then, one can gain unbound the energy $E_{\text{c.m.}}$ in the centre of mass frame due to head-on collision of particles near the would-be horizon. The energy measured at infinity can also be unbound. If instead of particles self-gravitating shells collide, the underlying reason leading to unbound $E_{\text{c.m.}}$ is the same.

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I. INTRODUCTION

In recent years, interest to high-energy collisions of particles in strong gravitational field increased significantly after observation made by Bañados, Silk and West (hereafter, BSW). They noticed that if two particles collide in the vicinity of the Kerr black hole, the energy in the centre of mass (CM) frame $E_{\text{c.m.}}$ may grow unbound \[1\]. Later on, it was shown that the BSW effect is due to the general properties of the black hole horizon \[2\], so in this sense, the effect has an universal character. Meanwhile, there are some difficulties in astrophysical realization and observation of the BSW effect. The first one consists in that one of particles should have fine-tuned relation between the energy and angular momentum

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(so-called critical particle). The second one is due to the fact that enormous $E_{c.m.}$ lead to relatively modest energies measured at infinity $[3] - [5]$.

Meanwhile, alternative mechanisms of getting ultra-high energies in the CM frame also exist. One of them comes back to the works $[6], [7]$ where unbound $E_{c.m.}$ were also obtained. The crucial difference between $[6], [7]$ and $[1]$ consists in the type of trajectories of colliding particles. In the BSW effect, both particles approach the horizon. In $[6], [7]$, they move in opposite directions, so one of particles has to move away from the horizon that is rather difficult to realize for a black (not white) hole. However, there is no need for fine-tuning parameters in this case $[8]$.

Another mechanism does not require the presence of the horizon at all. The following scenario in the background of naked Kerr $[9]$ and Reissner-Nordström (RN) $[10]$ metrics were considered. The first particle reflects from an infinite potential barrier and collides with the second one. It turned out that $E_{c.m.}$ can be made as large as one likes provided the parameters of a metric are close to the threshold of forming an extremal black hole, so the charge $Q = M(1 + \varepsilon)$ or $J = M^2(1 + \varepsilon)$ where $M$ is the mass, $Q$ is the charge, $J$ is the angular momentum, $\varepsilon \ll 1$. Quite recently, it was shown that unbound $E_{c.m.}$ are still possible even if both horizons and naked singularities are absent $[11]$. Such a scenario was further considered in detail for a particular example of colliding spherical dust shells $[12]$.

The aim of the present work is to extend the approach of $[11]$ from the spherically symmetric case to axially symmetric rotating configurations. We draw attention that scenario with neither horizons nor naked singularities has a general character. It does not require the knowledge of the whole dynamics. We consider generic configurations with matter and reveal the main features of the effect in a model-independent way. The key ingredients are (i) the metric on the threshold of forming the extremal horizon, (ii) head-on collision that is similar to $[6], [7]$ but without horizons. Point (i) is a feature typical of quasiblack holes (QBH) $[13]$ for which the ultra-high energy collisions are also possible $[14]$. However, the mechanism of getting large $E_{c.m.}$ in both cases is essentially different (see below). Thus instead of considering particular metrics, trajectories or models $[9], [10], [12], [15]$ we reveal and discuss underlying factors that ensure the existence of the effect.

All features considered in the present paper imply that although there is no horizon as such, there exists a time-like surface ”close” to it. There, the value of the lapse function becomes small although not exactly equal to zero. Meanwhile, there is also another type of
the high energy process which is due to the ergosphere, not the horizon [16], [17]. We do not discuss it here.

II. GENERAL FORMALISM

Let us consider the metric

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2,$$

where the coefficients do not depend on $t$ and $\phi$. We use units in which fundamental constants $G = c = \hbar = 1$.

Equations of motion for a particle with the mass $m$ in the background (1) read

$$m \ddot{t} = \frac{X}{N^2},$$

$$m \ddot{\phi} = \frac{L}{g_\phi} + \frac{\omega X}{N^2},$$

where dot denotes derivative with respect to the proper time. Here,

$$X = E - \omega L,$$

$$E = -mu_0, \quad L = mu_\phi.$$

From the normalization condition it follows that

$$m \dot{r} = \pm \sqrt{\frac{A}{N}} Z,$$

$$Z = \sqrt{X^2 - N^2 \left( \frac{L^2}{g_\phi} + g_\theta (p_\theta)^2 + m^2 \right)},$$

where $p_\theta = m \dot{\theta}$.

For geodesic motion, $E$ and $L$ are conserved and have the meaning of the energy and angular momentum, respectively. However, the equations (2) - (6) are valid even if $E$ and $L$ are not conserved.

The key quantity of interest is the energy in the CM frame $E_{c.m}$. If two particles collide in some point, it can be defined in this point by analogy with the standard relation for one particle. For two particles with masses $m_1$ and $m_2$ and four-velocities $u_1^\mu$ and $u_2^\mu$, the energy $E_{c.m.}$ at the collision event is the norm of their total four-momentum,

$$E_{c.m.}^2 = -(p_1^\mu + p_2^\mu)(p_1^\mu + p_2^\mu) = m_1^2 + m_2^2 + 2m_1m_2\gamma$$
where
\[ \gamma = -u_1 u_2^\mu \] (8)
is the relative Lorentz factor.

Then, by direction substitution into (8), one can find that
\[ m_1 m_2 \gamma = \frac{X_1 X_2 + \delta Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi} - g_\phi P_1 P_2. \] (9)
where \( \delta = -1 \) for particles moving in the same radial direction before collision and \( \delta = +1 \) otherwise. From now on, we consider scenarios for which \( \delta = +1 \) in (9) (head-on collisions).

**III. METRIC ON THRESHOLD OF FORMING THE EXTREMAL HORIZON**

In what follows we assume that (i) \( N^2 > 0 \) everywhere, (ii) for some value \( r = r_0 \), it can be made as small as one likes, (iii) collision occurs in the point \( r = r_0 \). With these assumptions, a natural representation is
\[ N^2(r, \theta) = B(r, \theta)(r - r_+)(r - r_-) \] (10)
with \( B(r_0, \theta) > 0 \) separated from zero. It follows from (i) that both roots are complex and mutually conjugate. It is convenient to introduce the new parameter \( \varepsilon \) and write \( r_{\pm} = r_0 \pm ir_0 \varepsilon \), so
\[ N^2 = B(r, \theta)[(r - r_0)^2 + r_0^2 \varepsilon^2] \] (11)

Then, requirement (ii) leads to \( \varepsilon \ll 1 \). Then, near the point of collision \( r - r_0 = r_0 O(\varepsilon) \), \( N^2 = O(\varepsilon^2) \). Correspondingly,
\[ \gamma = O(\varepsilon^{-2}) \] (12)
and can be made as large as one likes. In doing so, the metric is perfectly regular in the vicinity of \( r_0 \). It is seen from above consideration that under continuous change of the parameter \( \varepsilon \), the system passes through the state of the extremal black hole when \( \varepsilon = 0 \). For \( \varepsilon \ll 1 \), eq. (10) gives us now
\[ E_{\text{c.m.}}^2 \approx \frac{4X_1(r_0, \theta)X_2(r_0, \theta)}{B(r_0, \theta) r_0^2 \varepsilon^2}. \] (13)

If one tries to repeat the procedure for the nonextremal would-be horizons, one is led to take real distinct roots in (10). However, this is inconsistent with assumption (i) since \( N^2 \)
changes the sign when \( r \) passes through \( r_- \) and \( r_+ \). Therefore, the effect under consideration is impossible. It is worth reminding that, by contrast, the BSW effect for nonextremal horizons is possible \([18], [2]\).

### A. Example: the Reissner-Nordström metric

To illustrate the general situation, one can compare it to the previous results for the RN metric \([10]\). In this case,

\[
N^2 = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \tag{14}
\]

Let for simplicity two particles have the same mass \( m \) and collision occurs in the point where \( N^2 \) reaches the minimum value. Then, according to eq. 16 of \([10]\),

\[
E_{\text{c.m.}}^2 = \frac{4m^2}{1 - \frac{M^2}{Q^2}}. \tag{15}
\]

Thus, unbound \( E_{\text{c.m.}}^2 \) correspond to \( Q \to M \), so the metric looks "almost" like the extremal RN black hole. There is a naked singularity inside at \( r = 0 \) due to the last term in (14). This terms is also responsible for repulsion and reflection of the first particle. However, in a general case one can imagine some distribution of matter which smooths the singularity.

### IV. MECHANISM OF COLLISION

Let particle 1 pass over \( r = r_0 \) and bounce back at some \( r_1 < r_0 \). Then, it collides with particle 2 that moves from the outside region. If the point of collision is adjusted to be at \( r_0 \) or in its immediate vicinity, we obtain \( E_{\text{c.m.}} \sim \varepsilon^{-2} \) in accordance with \([7], [12]\). To make particle 1 to reflect at \( r = r_1 \), some potential barrier should exist at \( r < r_0 \). In some cases it is infinite like in the RN case \([10]\). Then, the effect under discussion reveals itself for any Killing energies. However, even if the potential barrier is of some finite height, the effect of unbound \( E_{\text{c.m.}} \) persists, although with the restriction on the permitted range of energies \( E \). As \( N \to 0 \) near \( r_0 \) but \( N = O(1) \) inside, it is clear that such a barrier does exist.

To some extent, the situation resembles that for quasiblack holes (QBH) in that the horizon is "almost" formed but does not form. However, there are crucial differences. We do not require \( N \) to be small everywhere inside in contrast to the QBH case \([13]\). And, now
particles move in the opposite direction before collision whereas it was assumed in [14] that they move in the same direction before collision, the BSW effect being due to the difference in the energy scales outside and inside the quasihorizon.

V. TIME BEFORE COLLISION

It is instructive to evaluate the time needed for collision and compare it with the corresponding time in the case of the BSW process. As is known, if we want the BSW effect to occur, we should choose one of particle to be 'critical' (with fine-tuned parameters). And, for such a particle the proper time required to reach the horizon diverges [19], [18], [2]. As a result, this mechanism prevents the actual release of infinite energy, as it should be in any physically meaningful process: the energy $E_{c.m.}$ remains finite in any act of collision although it can be made as large as one likes.

Now, one can expect that in the situation under discussion the proper time remains finite since particles are assumed to be usual, without special fine-tuning. Let us consider, for simplicity, motion in the equatorial plane $\theta = \frac{\pi}{2}$. It follows from (5), (6) that, in the absence of the turning point, motion between $r_i$ and $r_f < r_i$ takes the proper time

$$\tau = m \int_{r_f}^{r_i} \frac{drN}{\sqrt{AZ}}. \quad (16)$$

As the particle is taken to be usual, $X > 0$ everywhere, $Z > 0$ is separated from zero. Assuming additionally that $\sqrt{A} \sim N$ (like it happens, say, for the Kerr metric), we see that the integrand in (16) is finite, so $\tau$ is also finite. When a particle reflects from the turning point and returns to $r_0$, the corresponding time is also finite because of the same reasonings. Thus in a general model-independent way and without calculations we can conclude that the proper time before collision is finite.

For the coordinate time $t$, it follows from (2) that

$$t = \int_{r_f}^{r_i} \frac{drX}{ZN\sqrt{A}}. \quad (17)$$

This time is also finite. But, in contrast to $\tau$, the time of travel between $r_i$ and $r_f = r_0$ grows unbound when $\varepsilon \to 0$ in (11). Taking $A = N^2b^2$ where $b$ is some model-dependent nonzero coefficient, we obtain for time of motion between $r_i > r_0$ and $r_0$

$$t_0 \approx \frac{\pi}{2b(r_0)\varepsilon B(r_0, \frac{\pi}{2})}. \quad (18)$$
If one takes into account the time for back motion to $r_0$, $t$ acquires an additional factor 2. Comparison to (13) gives us that

$$E_{c.m.} \sim t_0.$$  \hfill (19)

The contents of the present section agrees with that of Sec.II D of [10] where the particular case of the Reissner-Nordström metric was considered.

VI. CASE OF MOTION ALONG THE AXIS OF SYMMETRY

There is the case deserving special attention. Let the coordinate $z$ have the meaning of the polar angle. Let us consider motion along the polar axis, so $\theta = 0$ or $\theta = \pi$. The regularity of the metric near the axis $\theta = 0$ requires $g_\phi \sim \theta^2$. Then, the finiteness of the term $L^2/g_\phi$ in (6) entails $L = 0$, so in (11) $X = E$. It follows from (6) that

$$Z^2 = E^2 - N^2m^2.$$  \hfill (20)

Let $N(0,0) = N_0$ and $N(\infty,0) = N_\infty$. Then, we can choose any value of $E$ such that $E < N_0$ since this guarantees the presence of the turning point. If $N_\infty < N_0$, the particle with an intermediate energy $N_\infty < E < N_0$ can fall from infinity. Otherwise, it oscillates between turning points. Assuming that the presentation (11) is valid with $\varepsilon \ll 1$, we obtain the unbound energy $E_{c.m.}$ according to (12). This generalizes observation made in [15] for the Kerr metric. The same formula (20) applies to the case $\theta = \frac{\pi}{2} = const$, $L = 0$.

VII. AFTER COLLISION

Let us consider the scenario described above. One sends particle 1 towards the centre and, later, particle 2. Particle 1 enters the inner region, reflects from the potential barrier and collides with particle 2 near $r = r_0$. As a result, particles 3 and 4 are created. We assume that in the act of collision both the energy and angular momentum are conserved:

$$E_1 + E_2 = E_3 + E_4,$$  \hfill (21)

$$L_1 + L_2 = L_3 + L_4,$$  \hfill (22)

whence

$$X_1 + X_2 = X_3 + X_4.$$  \hfill (23)
We also assume the forward in time condition

\[ X_i > 0, \ 1 \leq i \leq 4, \quad (24) \]

which follows from \((2)\) and \(\dot{t} > 0\). In contrast to the black hole case, where \(N = 0\) on the horizon, now \(N > 0\) everywhere, so the case \(X_i = 0\) is excluded. Also, we assume that \(X_i = O(1)\) do not become small near \(r_0\).

The conservation of the radial momentum gives us, according to \((5)\), that

\[ Z_1 - Z_2 = Z_4 - Z_3. \quad (25) \]

For small \(N\),

\[ Z_i \approx X_i - \frac{N^2}{2X_i} \left( \frac{L_i^2}{g_\phi} + m_i^2 \right). \quad (26) \]

The main terms give us

\[ X_1 - X_2 \approx X_4 - X_3. \quad (27) \]

It follows from \((23), (27)\) that

\[ X_1 \approx X_4, \ X_2 \approx X_3. \quad (28) \]

The main corrections give us from \((25), (26)\) that

\[ \frac{1}{X_2} \left( \frac{L_3^2}{g_\phi} + \frac{L_3^2}{g_\phi} + m_2^2 + m_3^2 \right) \approx \frac{1}{X_1} \left( \frac{L_3^2}{g_\phi} + \frac{L_3^2}{g_\phi} + m_1^2 + m_3^2 \right), \quad (29) \]

where \(g_\phi\) is taken in the point of collision \(r = r_0\).

For fixed \(E_{1,2}\) and \(L_{1,2}\) (hence, \(X_1\) and \(X_2\)), we are interested in the solutions for which \(E_3\) grows with \((24)\) satisfied. According to \((28)\), \(X_3 = X_2\) is also fixed, hence this implies that \(L_3 = \frac{E_3 - X_3}{\omega}\) is large. Let us assume that \(\omega > 0\) everywhere. Our goal can be achieved if, say, \(E_4 \to -\infty, L_4 \to -\infty, E_3 \to \infty, L_3 \to \infty\). This implies that orbits with large negative energy do exist. In principle, this is possible even in the absence of the horizon. Then, insofar as all masses \(m_i \ll M\) where \(M\) is the mass corresponding to the metric \((1)\), there are no bounds on the ratio \(\frac{E_3}{E_1 + E_2}\) on this scale. In this sense, this is the standard situation for the Penrose process.
VIII. EXAMPLE: REGULAR STAR-LIKE CONFIGURATIONS VERSUS VACUUM-LIKE BLACK HOLES

In this section, we give an example of physically relevant objects to which the scenario of collision under discussion can apply. (Another example based on the Bardeen spacetime [20] was given in Sec. IV of [11]). In a sense, the RN or Kerr naked singularity can be obtained by deformation of the metric of the corresponding extremal black hole. In a similar way, the required starlike configuration can be obtained by deformation of a regular extremal black hole. As such an example, we can choose the regular black hole with the de Sitter core proposed in [21]. Let us consider the spherically symmetric metric

$$ds^2 = -f dt^2 + \frac{d\rho^2}{f} + r^2 (u) d\omega^2. \quad (30)$$

In [21], it is assumed that the matter satisfies the vacuum-like equation of state $p_r = -\rho$ ($p_r$ is the radial pressure, $\rho$ is the energy density). Then, $r = u$. However, this is not necessary. We can consider (30) with more general equations of state (which become vacuum-like near the origin to ensure regularity). We only require (i) the de Sitter core for small $r$ [21], (ii) asymptotic flatness, $f \to 1$ when $u \to \infty$. As $f = 1$ both at infinity and near the origin $r = 0$, it must have a minimum in between. For simplicity, we assume that there is only one such a minimum. If $f$ has two zeros, we have a nonextremal black hole, if it has one double zero in the point of minimum, a black hole becomes extremal, if $f > 0$ there is no black hole at all (starlike configuration). The main purpose of [21] was to obtain a regular black hole. By contrary, now we are interested in a starlike configuration which is close to the extremal black hole in the sense described above (see eq. [11]). If

$$f = f_0 + a(u - u_0)^2 \quad (31)$$

near $u = u_0$, and $f_0 \to 0$, previous general consideration applies. Then, in the background (30) with aforementioned properties, one can obtain unbound $E_{c.m.}$ for test particles without the horizon or naked singularity. In the absence of the ergoregion, extraction of energy does not occur. However, due to large $E_{c.m.}$, creation of superheavy particles is possible.
IX. SHELLS

Instead of test particles, let us consider collision of shells which move in opposite directions. We can divide the act of collision to the set of individual collisions of small constituents. For example, if shells are spherical, natural division consists in collision between particles with the same value of angle variables. Then, for each pair of colliding elements, we can apply again eq. (7), (9). The values of $X_i$ and $Z_i$ can be obtained (for given initial conditions) from equations of motion. These equations differ from (2) - (6) due to the effect of self-gravitation. Say, for collision of charged shells, one obtains (see eq. 74 of [10]) that

$$E_{c.m.}^2 = 2m^2 + \frac{2m^2}{f} \left( |\dot{R}_1 \dot{R}_2| + \sqrt{\dot{R}_1^2 + f \dot{R}_2^2 + f} \right),$$  \hspace{0.5cm} (32)

$f \equiv N^2$ is taken in the region between shells in the coincidence limit. The law that governs the dependence $R_i(\tau)$ is different for test particles and constituents of self-gravitation shells and can be described in terms of different effective potentials. However, the structure of the expression (9) is universal. Therefore, insofar as $f$ is small, $E_{c.m.}^2$ is large. And, previous explanation based on presentation (11) is still valid. (It is worth stressing that it is important that shells move in the opposite directions before collision. For motion in the same direction, the effect of inbound $E_{c.m.}$ is absent [24].)

Thus inasmuch as we are interested in the effect of gaining unbound $E_{c.m.}$ only, there is no need to analyze the whole history of the shell (which, however, is of interest by itself). What is important is smallness of $N^2$ and the possibility that an inner shell bounces back from some surface. This can be achieved either due to the naked singularity or the effective potential barrier of a finite height. Actually, some restrictions (not related to our subject) on $E_{c.m.}$ come from requirement that description of shells is macroscopic, so they should contain a large number of constituents (see Sec. III C of [10]).

X. DISCUSSION AND CONCLUSION

In previous scenarios, the following difficulties were present: (i) the necessity to ensure fine-tuning, (ii) severe bounds on the energy of products of collisions measured at infinity, (iii) if collisions are arranged due to naked singularities, the problem with the cosmic censorship arises, (iv) if collisions with large $E_{c.m.}$ occur with no horizons or naked singularities, this requires effects of self-gravity, so for test particles such collisions could not be
realized. Meanwhile, now we see that all these difficulties can be avoided for regular star-like configurations, so the effect exists even for test particles. In particular, if there exists an ergoregion (that is, in principle, is possible even without the horizon - see some example in [22]), the collisional Penrose process should become much more efficient than for the BSW effect. We described an unified picture of the scenario that ensures the unbound $E_{c.m.}$ in head-on collisions with the metrics close to forming the horizon but when the horizon does not form.

Apart from this, the advantage of collisions under discussion consists in that we can safely neglect the role of gravitational radiation [23]. Such radiation bounds the BSW effect since it "spoils" special (critical) trajectories with fine-tuning of parameters required for this effect. However, now fine-tuning is not required at all, so small perturbation due to an additional force do not change the whole picture qualitatively.

It is instructive to classify main types of the effect under discussion - see Table 1.

| Relevant references | Relative direction | Horizon | Naked singularity | Fine-tuning | Self-gravity |
|---------------------|--------------------|---------|--------------------|-------------|--------------|
| [6], [7]            | −                  | +       | −                  | −           | −            |
| [1]                 | +                  | +       | −                  | +           | −            |
| [14]                | +                  | −       | −                  | −           | −            |
| [9], [10], [15]     | −                  | −       | +                  | −           | −            |
| [12]                | −                  | −       | −                  | −           | +            |
| present paper       | −                  | −       | −                  | −           | −            |

Table 1. Different types of high energy processes with horizons or would-be horizons.

For shortness, in this table, "self-gravity” means ”necessity of self-gravity to have unbound $E_{c.m.}$” (collision of shells), etc.

Thus the present kind of high energy collision in a strong gravitational field looks promising since it relaxes or weakens strong restrictions typical of the BSW effect required for observation (at least in principle). In doing so, only the vicinity of the would-be horizon is important in accordance with the spirit of black hole physics (even in the absence of a black hole!). In this sense, in all effects described in Table 1, it is necessary that a system posses either the true horizon or time-like surface which in a sense is close it.

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