Anomalous dimensions of twist-2 conformal operators in supersymmetric Wess-Zumino model.

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Abstract

In the present paper we are studying scale properties of twist-2 conformal operators in supersymmetric Wess-Zumino model. In particular, we are interested in a construction of multiplicatively renormalized conformal operators. We show, that in order to find multiplicatively renormalized operators in this model, it is sufficient to find multiplicatively renormalized operators only for one member of operator supermultiplet. We found a closed analytical solution for fermionic conformal operator, which together with supersymmetry transformations could be used to find remained multiplicatively renormalized bosonic operators. Moreover, we found, that the knowledge of fermionic diagonal and non-diagonal anomalous dimensions matrices allows us completely reconstruct the forward anomalous dimensions matrix in singlet case.

1 Introduction

Parton distributions in QCD satisfy the Balitsky-Fadin-Kuraev-Lipatov (BFKL) \cite{1} and Dokshitzer-Gribov-Lipatov-Altarelli-Parizi (DGLAP) \cite{2} equations. When considered in supersymmetric limit these equations reveal some remarkable properties. For example, in the maximally supersymmetric $N = 4$ Yang-Mills theory there is a deep relation between BFKL and DGLAP evolution equations \cite{3}. In particular, the anomalous dimensions of Wilson twist-2 operators in $N = 4$ SYM could be found from the eigenvalues of the kernel of BFKL equation. Moreover, the anomalous dimensions matrix of the twist-2 operators in $N = 4$ Supersymmetric Yang-Mills theory has a very simple form with its elements being expressed in terms of one universal anomalous dimension with shifted arguments \cite{3, 4}. In leading order, universal anomalous dimension was found to be proportional to the $\Psi$-function, what may be considered as an argument in favor of integrability of this model \cite{5}. Generally, the conformal invariance of the theory in leading order allows us to construct multiplicatively renormalized quasi-partonic operators \cite{6} up to this order in perturbation theory. However, in next-to-leading order in perturbation theory multiplicative renormalization of conformal operators is violated due to necessity in regularization of arising ultraviolet divergences.

The Efremov-Radyushkin-Brodsky-Lepage (ER-BL) equation \cite{7} may be viewed as some kind of the generalization of the DGLAP equation for the case of non-forward distribution functions. In this case in and out hadronic states in matrix elements of conformal operators are different, what allows
us to study scale properties of hadron wave functions. In the latter case the matrix elements of corresponding conformal operators are considered between vacuum and hadronic state. It is known, that the eigenfunctions of ER-BL evolution equation are directly related to multiplicatively renormalized conformal operators. Moreover, knowing ER-BL evolution kernels one may find both diagonal and non-diagonal parts of anomalous dimensions matrix for corresponding conformal operators.

Recently, there was a great interest in properties of before mentioned operators in the context of famous AdS/CFT correspondence [8]. Namely, there are some calculations of the anomalous dimensions of such operators in the limit of large $j$ (Lorentz spin) from both sides of the AdS/CFT correspondence [9, 10, 11]. There are, also some predictions for anomalous dimensions of other types of operators from string theory [12], partially confirmed by field theory calculations. It should be noted, that up to this moment only diagonal part of anomalous dimension matrix have been studied in the context of AdS/CFT correspondence and it would be interesting to compare the results for its non-diagonal part with the appropriate result from string theory. So, we believe, that non-diagonal parts of anomalous dimensions matrices in supersymmetric theories may reveal some interesting properties and thus are worth to study.

In this paper we start our study with a simple supersymmetric model - supersymmetric Wess-Zumino model [13] which contains only matter superfields. This model allow us to understand general properties of the ER-BL equations and their solutions in the supersymmetric theories. As the next step we suppose to perform a similar analysis for field theories with more supersymmetries. We will be mainly interested in the construction of multiplicatively renormalized conformal operators for non-forward kinematics and subsequent understanding of their string counterparts. This will be a subject of one of our next papers.

To be precise, in the present work we calculate evolution kernels for non-singlet and singlet unpolarized conformal operators together with similar quantities in polarized case. We study the transformation properties of twist-2 conformal operators in Wess-Zumino model under supersymmetry transformations. At this point, it is natural to introduce fermionic operators (with respect to quantum numbers), as components of corresponding supersymmetric chiral multiplets. It is shown, that the latter allow us reduce the problem of finding the solution of evolution equations in singlet case (here we have operator mixing adding extra complexity) to the similar problem for fermionic operators. The corresponding problem for fermionic operators turns out to be much simpler and we will be able to obtain a closed analytical solution for multiplicatively renormalized fermionic operators, which together with supersymmetry transformation could used to obtain a solution of evolution equations in the singlet case. Moreover, we found, that the knowledge of fermionic diagonal and non-diagonal anomalous dimensions matrices allows us completely reconstruct the forward anomalous dimensions matrix in singlet case.

Our present consideration is based on two approaches developed for studying non-forward partonic distribution functions. The first one follows the articles of Mikhailov and Radyushkin [14, 15, 16], where an effective method for calculation of the evolution kernels of ER-BL equation was proposed [14, 15]. The properties of evolution kernels were later studied in [16]. The absence of the gauge fields in our problem considerably simplifies the whole calculation, so here, we are following Ref. [14], where the scalar $\phi^3_0$ theory in the space-time dimension six was considered. The second approach is based on papers of Müller [17, 18], where the renormalized conformal Ward identities were extensively exploited to find non-diagonal parts of anomalous dimension.

The present paper is organized as follows. In section 2, after briefly recalling some necessary background about distribution amplitudes, conformal operators and supersymmetric Wess-Zumino model, we give our results for ER-BL evolution kernels in unpolarized and polarized cases together with some comments on the details of this calculation. In section 3 we introduce an approach for the
evaluation of non-diagonal parts of anomalous dimensions matrices for conformal operators, we are interested in, based on the analysis of conformal Ward identities for Green functions with operator insertions. The obtained non-diagonal anomalous dimensions matrices were checked to coincide with those coming from ER-BL evolution kernels evaluated before. In section 4, we combine our conformal operators into one supersymmetric chiral multiplet and study the constrains on anomalous dimensions of conformal operators following from supersymmetric Ward identity. Next, in section 5 we construct multiplicatively renormalized fermionic operator up to next-to-leading order in perturbation theory and explain how corresponding solutions could be found for other operators. Finally, section 6 contains our conclusion.

2 ER-BL evolution kernels in the supersymmetric Wess-Zumino model

2.1 Preliminary

The field-theoretical background for the study of the distribution amplitudes is provided by their relation to the matrix elements of non-local operators sandwiched between hadron and vacuum states

\[ \phi(x) = \frac{1}{2\pi} \int dz_- e^{i z_- p_+} <0|\varphi(0)\varphi(z_-)|h(p_+)> , \]  

where \( p_+ \) is hadron momentum, \( x \)-momentum fraction carried by field \( \varphi \) and \( z_- \) stands for light-cone coordinate.

ER-BL equation describes the scale dependence of the distribution amplitudes (DA) or more general non-forward parton distribution in contrast to scale dependence of forward distribution functions governed by DGLAP equation. The ER-BL equation is generally written as

\[ \mu^2 \frac{d}{d\mu^2} \phi(x, \mu) = \int_0^1 dy V(x, y|\alpha(\mu))\phi(y, \mu) , \]  

where evolution kernel

\[ V(x, y|\alpha) = \frac{\alpha}{4\pi} V^{(0)}(x, y) + \left( \frac{\alpha}{4\pi} \right)^2 V^{(1)}(x, y) + ... \]  

is given as a series in the coupling constant and is calculated in perturbation theory. In leading order the solution of evolution equation (2) could be constructed introducing moments of DA with respect to Gegenbauer polynomials in the case of bosonic (on quantum numbers) DA or Jacobi polynomials in the case of fermionic DA. For example, for non-singlet bosonic DA we have:

\[ \int_0^1 dx C_k^{3/2}(x - \bar{x}) \phi(x, Q^2) = \langle 0|\mathcal{O}_{k,k}^{\varphi}(\mu^2)|P\rangle_{\mu^2=Q^2}^{red} , \]  

where \( \bar{x} = 1 - x \) and \( \langle 0|\mathcal{O}_{k,k}^{\varphi}(\mu^2)|P\rangle^{red} = \langle 0|\mathcal{O}_{k,k}^{\varphi}(\mu^2)|P\rangle / P^{k+1}_+ \) denotes reduced expectation value of non-singlet bosonic conformal twist-2 operator:

\[ \mathcal{O}_{j,l} = \frac{1}{2} \bar{\psi}_+(i \partial_+) j^0 \mathcal{C}_j^{3/2} \left( \frac{\mathcal{D}_+}{\partial_+} \right) \psi_+ \]  

where \( \mathcal{D} = \bar{\partial} - \bar{\partial}_t, \partial = \bar{\partial} + \bar{\partial}_t \). In leading order this and other conformal operators, we will consider later, are multiplicatively renormalized, so that the solution of evolution equation can be easily
found. The evolution equation for DA (2) leads to renormalization group equations for corresponding conformal operators:

\[ \mu \frac{d}{d\mu} \mathcal{O}^\psi_{j,l}(\mu^2) = \sum_{k=0}^{l} \gamma_{j,k}(\alpha(\mu^2)) \mathcal{O}^\psi_{k,l}(\mu^2), \]  

(6)

where the anomalous dimensions matrix

\[ \gamma_{j,k} = \left( \frac{\alpha}{4\pi} \right) \gamma_{k}^{(0)} \delta_{j,k} + \left( \frac{\alpha}{4\pi} \right)^2 \gamma_{j,k}^{(1)} + \ldots \]  

(7)

is diagonal in leading order and could be extracted from the expression for evolution kernel:

\[ \int_{0}^{1} dx C_{j}^{3/2}(x - \bar{x}) V(x, y; \alpha) = \sum_{k=0}^{j} \gamma_{j,k}(\alpha) C_{k}^{3/2}(y - \bar{y}). \]  

(8)

So, we see that the problem of determining the evolution kernels for DA and the problem of calculation of anomalous dimensions matrix for corresponding conformal operators are closely related. In what follows, we are using two different approaches to determine DA evolution kernels and anomalous dimensions matrices of our conformal operators. Equations, similar to (8) will be used only to check, that both methods give us the same results.

2.2 Wess-Zumino model

The field content of supersymmetric Wess-Zumino model [13] consists of the (anti)chiral supermultiplets containing off mass shell a complex scalar field \( \varphi \), one Majorana fermion field \( \psi \) (“quark” field) and a complex auxiliary scalar field \( F \):

\[
\Phi(y, \theta) = \varphi(y) + \sqrt{2} \theta \psi(y) + \theta \theta \mathcal{F}(y) \\
\Phi(x, \theta, \bar{\theta}) = \varphi(x) + \sqrt{2} \theta \psi(x) + \theta \theta \mathcal{F}(x) \\
- i \theta \sigma^\mu \bar{\theta} \partial_\mu \varphi(x) - \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \partial^\mu \varphi(x) - \frac{i}{\sqrt{2}} \theta \theta \bar{\theta} \bar{\theta} \bar{\sigma}^\mu \partial_\mu \psi(x)
\]

(10)

The lagrangian for this model is conveniently written using superfield formalism and is given by

\[
\mathcal{L} = \int d^2 \theta d^2 \bar{\theta} \Phi_i \Phi^i - \frac{1}{3!} \int d^2 \theta Y_{ijk} \Phi^i \Phi^j \Phi^k - \frac{1}{3!} \int d^2 \bar{\theta} \bar{Y}^{ijk} \bar{\Phi}_i \bar{\Phi}_j \bar{\Phi}_k,
\]

(11)

where \( Y_{ijk} \) is Yukawa coupling, symmetric with respect to all indices. The first term in the lagrangian stands for kinetic terms of all fields present in this model, second and third terms denote superpotential (and its hermitian conjugate), which remains non-renormalized to all orders in perturbative theory. Due to non-renormalization of the superpotential the \( \beta \)-function for Yukawa coupling completely determined by the wave function renormalization of fields. To simplify notation it is convenient to put \( Y^{ijk} = g \). The first coefficient of Yukawa \( \beta \)-function \( \beta(g^2) = b_0 \frac{g^4}{(4\pi)^2} + \ldots \) in Wess-Zumino model is \( b_0 = -3 \) and the leading order anomalous dimension of the chiral superfield is \( \gamma_\Phi = \frac{g^2}{(4\pi)^2} \).

In what follows, we will also need this lagrangian written in component notation. Rewriting complex scalar field \( \varphi = (A + iB)/\sqrt{2} \) in terms of real scalar \( A \) and pseudoscalar \( B \) fields, complex
auxiliary scalar field $\mathcal{F} = (F + iG)/\sqrt{2}$ in terms of real scalar $F$ and pseudoscalar $G$ auxiliary fields the Wess-Zumino lagrangian takes the form

$$
\mathcal{L} = \frac{1}{2} \partial^\mu A \partial_\mu A + \frac{1}{2} \partial^\mu B \partial_\mu B + \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} \left( F^2 + G^2 \right) \\
+ g \left[ F(A^2 + B^2) + 2GAB - i\bar{\psi}(A + \gamma_5 B)\psi \right].
$$

(12)

The supersymmetric transformations for the component fields could be summarized as follows

$$
\delta^Q A = \bar{\epsilon} \psi, \quad \delta^Q B = \bar{\epsilon} \gamma_5 \psi, \\
\delta^Q \psi = i\gamma^\mu \partial_\mu (A + \gamma_5 B) \epsilon + i(F + \gamma_5 G) \epsilon, \\
\delta^Q F = \bar{\epsilon} \gamma^\mu \partial_\mu \psi, \quad \delta^Q G = \bar{\epsilon} \gamma^\mu \gamma_5 \partial_\mu \psi.
$$

(13)

After eliminating auxiliary fields $F$ and $G$ with the use of their equations of motion the lagrangian in Eq. (12) becomes

$$
\mathcal{L} = \frac{1}{2} \partial^\mu A \partial_\mu A + \frac{1}{2} \partial^\mu B \partial_\mu B + \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi \\
- \frac{1}{2} g^2 (A^2 + B^2)^2 - i\bar{\psi}(A + \gamma_5 B)\psi.
$$

(14)

Without auxiliary fields supersymmetric transformations (13) do not form a closed algebra. However, introducing light-cone coordinates and defining light-cone fields one can find closed superalgebra for “plus” - components of the fields. In light-cone coordinates general four-vector $X^\mu$ is represented as $X^\pm = (1/\sqrt{2})(X^0 \pm X^3)$, and $X^\mu Y^\nu = X_+ Y_- + X_- Y_+ - X^i Y^i$, where $i=1,2$. For spinors the appropriate definition is $\lambda_+ = \frac{1}{2}(\gamma_+ \gamma_5 \lambda)$, so that $\lambda = \lambda_+ + \lambda_-$. Then choosing $\epsilon = \epsilon$ the supersymmetric transformations take the following form (15)

$$
\delta^Q A = \bar{\epsilon}_- \psi_+, \\
\delta^Q B = \bar{\epsilon}_- \gamma_5 \psi_+, \\
\delta^Q \psi_+ = i\partial_+(A - \gamma_5 B)\gamma_- \epsilon_-.
$$

(15)

Now, let us introduce the local conformal Wilson twist-2 operators appearing in this model for unpolarized and polarized cases

$$
\mathcal{O}_{\psi}^{\phi} = \frac{1}{2} \bar{\psi}_+ (i\partial_+) \gamma_+ C^{3/2}_j \left( \frac{D_+}{\partial_+} \right) \psi_+, \\
\mathcal{O}_{\psi}^{\phi} = \frac{1}{2} \bar{\psi}_+ (i\partial_+) \gamma_+ \gamma_5 C^{3/2}_j \left( \frac{D_+}{\partial_+} \right) \psi_+, \\
\mathcal{O}_{\phi}^{\psi} = A(i\partial_+)^{1+1} C_j^{1/2} \left( \frac{D_+}{\partial_+} \right) A + B(i\partial_+)^{1+1} C_j^{1/2} \left( \frac{D_+}{\partial_+} \right) B, \\
\mathcal{O}_{\phi}^{\psi} = A(i\partial_+)^{1+1} C_{j+1}^{1/2} \left( \frac{D_+}{\partial_+} \right) B + B(i\partial_+)^{1+1} C_{j+1}^{1/2} \left( \frac{D_+}{\partial_+} \right) A,
$$

(16)

(17)

(18)

(19)

where $D = \bar{\not}{\partial} - \frac{\not}{\partial} $, $\partial = \bar{\not}{\partial} + \frac{\not}{\partial}$ and $C^{\nu}_n(z)$ - Gegenbauer polynomials

$$
C^{\nu}_n(z) = \frac{(-1)^n 2^n}{n!} \frac{\Gamma(n + \nu)}{\Gamma(n)} \frac{\Gamma(n + 2\nu)}{\Gamma(2n + 2\nu)} (1 - z^2)^{-\nu - 1/2} \frac{\partial^n}{\partial z^n} [ (1 - z^2)^{\nu - 1/2} ].
$$

(20)
Moreover in this model one can introduce also the following fermionic (by quantum numbers) operators

\[ O^{\text{fer}}_{j,l} = \bar{\psi} + (i \partial_l + 1) P_{j+1}^{(1,0)} \left( \frac{D_+}{\partial_+} \right) (A + \gamma_5 B), \]  

\[ \tilde{O}^{\text{fer}}_{j,l} = \bar{\psi} + (i \partial_l + 1) \gamma_5 P_{j+1}^{(1,0)} \left( \frac{D_+}{\partial_+} \right) (A + \gamma_5 B), \]  

where \( P_n^{(\alpha, \beta)}(z) \) - Jacobi polynomials

\[ P_n^{(\alpha, \beta)}(z) = \frac{(-1)^n}{n! 2^n} (1 - z)^{-\alpha} (1 + z)^{-\beta} \frac{d^n}{dz^n} \left[ (1 - z)^{\alpha} (1 + z)^{\beta} (1 - z^2)^n \right]. \]  

As was already mentioned in introduction, we will be interested in the renormalization properties of these operators. It should be noted, that in the singlet case there is mixing between bosonic operators formed by fermion and scalar fields. Also, in the non-forward kinematics, in contrast to forward case, the operators (16) and (18) will mix under renormalization not only with each other, but also with the total derivatives of these operators.

The scale properties of twist-2 conformal operators, we are interested in, could be deduced from scale properties of corresponding distribution amplitudes. So, in what follows, we will first determine the evolution kernels for DA in the cases of interest.

### 2.3 Unpolarized case

#### 2.3.1 Non-singlet case

In non-singlet case we need to calculate the evolution kernel of DA, which Gegenbauer moments are given by matrix elements of operator (16) sandwiched between quark states. In leading order there is only one diagram shown in Fig. 1a and in next-to-leading order there are three diagrams shown in Fig. 2a, 2b and 2c. For the diagram evaluation we used the method proposed by Mikhailov and Radyushkin [14, 15]. The calculation details together with diagram by diagram answers may be found in Appendices A and B. The final result for the non-singlet non-forward evolution kernel is

\[ \alpha = \frac{q^2}{4\pi} \]

\[ V_{NS}(x, y) = \frac{\alpha}{4\pi} V^{(0)}_{NS}(x, y) + \left( \frac{\alpha}{4\pi} \right)^2 V^{(1)}_{NS}(x, y) \]  

\[ V^{(0)}_{NS}(x, y) = \theta(x < y) \left\{ \frac{2x}{y} - \delta(x - y) \right\} + \left( x \leftrightarrow \bar{x}, y \leftrightarrow \bar{y} \right) \]  

\[ V^{(1)}_{NS}(x, y) = \theta(x < y) \left\{ -4 \frac{x}{y} - \frac{\bar{x}}{\bar{y}} \ln(\bar{x}) - \frac{\bar{x}}{\bar{y}} \ln(x) + \frac{\bar{x}}{\bar{y}} \ln \left( 1 - \frac{x}{y} \right) - \frac{\bar{x}}{\bar{y}} \ln \left( 1 - \frac{\bar{x}}{\bar{y}} \right) \right\} + \left( x \leftrightarrow \bar{x}, y \leftrightarrow \bar{y} \right) \]  

In the forward case the evolution kernels are given by

\[ P^{(0)}_{NS}(z) = 2\bar{z} - \delta(1 - z) \]  

\[ P^{(1)}_{NS}(z) = -4\bar{z} - 2\bar{z} \ln(\bar{z}) + 2\bar{z} \ln(z) + (1 + z) \ln^2(z) + \delta(1 - z) \]

\[ \text{One need to include also graphs with external self-energies which give } \delta \text{-function contribution in the coordinate space} \]
First three terms in the second line in Eq. \((26)\) represent non-symmetric piece of the kernel \(y\bar{y}V_{NS}(x, y)\) with respect to the exchange of variables \((x \leftrightarrow y)\)

\[
y\bar{y}V_{NS}(x, y) = x\bar{x}V_{NS}(y, x) \quad (29)
\]

and, as was shown in Ref. [14, 16], lead to non-diagonal part of anomalous dimensions matrix in the basis of Gegenbauer polynomials (see below).

### 2.3.2 Singlet case

In singlet case there is mixing between operators \([16]\) and \([18]\) and the evolution kernel takes now the matrix form

\[
V(x, y) = \begin{pmatrix}
V_{\psi\psi}(x, y) & V_{\psi\varphi}(x, y) \\
V_{\varphi\psi}(x, y) & V_{\varphi\varphi}(x, y)
\end{pmatrix}
\]

with the elements of this matrix in leading order (diagrams Fig. [1]a - [1]d) given by

\[
V_{\psi\psi}^{(0)}(x, y) = \theta(x < y) \left\{ 2\frac{x}{y} - \delta(x - y) \right\} + \left( x \leftrightarrow \bar{x} \atop y \leftrightarrow \bar{y} \right)
\]

\[
V_{\psi\varphi}^{(0)}(x, y) = \theta(x < y) 2x - \left( x \leftrightarrow \bar{x} \atop y \leftrightarrow \bar{y} \right)
\]

\[
V_{\varphi\psi}^{(0)}(x, y) = \theta(x < y) \left\{ -\frac{2}{y} \right\} - \left( x \leftrightarrow \bar{x} \atop y \leftrightarrow \bar{y} \right)
\]

\[
V_{\varphi\varphi}^{(0)}(x, y) = \theta(x < y) \left\{ -2 - \delta(x - y) \right\} + \left( x \leftrightarrow \bar{x} \atop y \leftrightarrow \bar{y} \right)
\]

and next-to-leading orders (diagrams Figs. [2] - [5])

\[
V_{\psi\psi}^{(1)}(x, y) = \theta(x < y) \left\{ -6\frac{x}{y} + \frac{x}{y} \ln\left(\frac{x}{y}\right) - \frac{\bar{x}}{\bar{y}} \ln(\bar{x}) - \frac{x}{y} \ln(x) + \frac{1}{y} \left( 1 - \frac{x}{y} \right) \ln\left( 1 - \frac{x}{y} \right) \\
+ \frac{\bar{x}}{\bar{y}} \ln^2(\bar{x}) + 2\frac{x}{y} \ln(x) \ln(y) - \frac{x}{y} \ln^2(y) + \delta(x - y) \right\} + \left( x \leftrightarrow \bar{x} \atop y \leftrightarrow \bar{y} \right)
\]

\[
V_{\psi\varphi}^{(1)}(x, y) = \theta(x < y) \left\{ -6x - x \ln\left(\frac{x}{y}\right) + 3\bar{x} \ln(\bar{x}) - 3x \ln(x) - \ln\left( 1 - \frac{x}{y} \right) \\
+ \bar{x} \ln^2(\bar{x}) - 2x \ln(x) \ln(y) + x \ln^2(y) \right\} - \left( x \leftrightarrow \bar{x} \atop y \leftrightarrow \bar{y} \right)
\]

\[
V_{\varphi\psi}^{(1)}(x, y) = \theta(x < y) \left\{ \frac{6}{y} + \frac{1}{y} \ln\left(\frac{x}{y}\right) - \frac{1}{y} \ln(\bar{x}) + \frac{1}{y} \ln(x) + \frac{1}{y} \ln\left( 1 - \frac{x}{y} \right) \\
+ \frac{1}{y} \ln^2(\bar{x}) - \frac{2}{y} \ln(x) \ln(y) + \frac{1}{y} \ln^2(y) \right\} - \left( x \leftrightarrow \bar{x} \atop y \leftrightarrow \bar{y} \right)
\]

\[
V_{\varphi\varphi}^{(1)}(x, y) = \theta(x < y) \left\{ 2 + \ln\left(\frac{x}{y}\right) + \ln^2(\bar{x}) + 2 \ln(x) \ln(y) - \ln^2(y) + \delta(x - y) \right\} \\
+ \left( x \leftrightarrow \bar{x} \atop y \leftrightarrow \bar{y} \right)
\]
The corresponding evolution kernels in the case of forward scattering have the following form

\[
P^{(0)}(x, y) = \begin{pmatrix}
  2z - \delta(1 - z) & 2 \\
  2z & -\delta(1 - z)
\end{pmatrix}
\] (36)

\[
P^{(1)}_{\psi\psi}(z) = -7\bar{z} - 2\bar{z} \ln(\bar{z}) + \bar{z} \ln(\bar{z}) + (1 + z) \ln^2(z) + \delta(1 - z)
\] (37)

\[
P^{(1)}_{\varphi\psi}(z) = -7 + z - 2 \ln(\bar{z}) + (1 - 2z) \ln(z) + \ln^2(z)
\] (38)

\[
P^{(1)}_{\varphi\varphi}(z) = \bar{z}(1 - 2 \ln(z)) + \delta(1 - z)
\] (39)

\[
P^{(1)}_{\varphi\varphi}(z) = \bar{z}(1 - 2 \ln(z)) + \delta(1 - z)
\] (40)

### 2.3.3 Fermionic operator

The calculation of evolution kernel for fermionic DA is similar to the calculation of evolution kernels for bosonic DA. However, in this case there is one subtlety worth to mention. It is necessary to keep track of what are the meanings of \(x\) and \(y\) in the expression for evolution kernels.

Now, for example if we agreed that \(x\) and \(y\) are the fractions of momentum for scalar field in fermionic operator, then we have the following answers for evolution kernels in leading and next-to-leading orders (see diagrams in Fig. 1.e and Fig. 6)

\[
V^{(0)}_{\text{fer}}(x, y) = \theta(x < \bar{y}) \left\{ -\frac{2}{\bar{y}} \right\} - \delta(x - y)
\] (41)

\[
V^{(1)}_{\text{fer}}(x, y) = \theta(x < \bar{y}) \left\{ 2 \frac{\ln(x) \ln(y)}{\bar{y}} - \frac{\ln^2(y)}{\bar{y}} \right\} + \theta(x > y) \left\{ \frac{\ln^2(x)}{\bar{y}} \right\}
\]

\[
+ \frac{1}{\bar{y}} \ln(x) - \theta(x > \bar{y}) \left\{ \frac{1}{\bar{y}} \ln \left( 1 - \frac{x}{\bar{y}} \right) \right\}
\]

\[
+ \theta(x < \bar{y}) \left\{ \frac{6}{\bar{y}} + \frac{1}{\bar{y}} \ln \left( 1 - \frac{x}{\bar{y}} \right) + \frac{1}{\bar{y}} \ln \left( \frac{x}{\bar{y}} \right) \right\} + \delta(x - y)
\] (42)

Evolution kernel in the case of forward scattering has the following form

\[
P^{(0)}_{\text{fer}}(z) = 2 - \delta(1 - z)
\] (43)

\[
P^{(1)}_{\text{fer}}(z) = -6 - \ln(z) - 2 \ln(\bar{z}) + \ln^2(z) + \delta(1 - z)
\] (44)

### 2.4 Polarized case

In polarized case the results for each contributing diagram differ from those in unpolarized case only by overall sign.
2.4.1 Non-singlet case

In non-singlet case the diagrams Fig. 1.a and Fig. 2.b change the overall sign and thus the final result for the non-singlet evolution kernel in polarized case has the following form:

\[
\tilde{V}_{NS}(x, y) = \frac{\alpha}{4\pi} \tilde{V}_{NS}^{(0)}(x, y) + \left( \frac{\alpha}{4\pi} \right)^2 \tilde{V}_{NS}^{(1)}(x, y) \tag{45}
\]

\[
\tilde{V}_{NS}^{(0)}(x, y) = \theta(x < y) \left\{-2 \frac{x}{y} - \delta(x - y) \right\} + \left( x \leftrightarrow \bar{x} \right. \quad \left( y \leftrightarrow \bar{y} \right) \tag{46}
\]

\[
\tilde{V}_{NS}^{(1)}(x, y) = \theta(x < y) \left\{ 4 \frac{x}{y} + \frac{\bar{x}}{y} \ln(\bar{x}) + \frac{x}{y} \ln(x) - \frac{1}{y} \left( 1 - \frac{x}{y} \right) \ln \left( 1 - \frac{x}{y} \right) \right. \\
+ \left. \frac{\bar{x}}{y} \ln^2(\bar{x}) + 2 \frac{x}{y} \ln(x) \ln(y) - \frac{x}{y} \ln^2(y) + \delta(x - y) \right\} + \left( x \leftrightarrow \bar{x} \right. \quad \left( y \leftrightarrow \bar{y} \right) \tag{47}
\]

In the forward case the evolution kernels are given by

\[
\tilde{P}_{NS}^{(0)}(z) = -2\bar{z} - \delta(1 - z) \tag{48}
\]

\[
\tilde{P}_{NS}^{(1)}(z) = 4\bar{z} + 2\bar{z} \ln(\bar{z}) + 2\bar{z} \ln(z) + (1 + z) \ln^2(z) + \delta(1 - z) \tag{49}
\]

2.4.2 Singlet case

Changing the overall sign of diagrams in the cases where it is necessary and taking into account mixing between operators (17) and (19) the evolution kernel in the singlet case has the following form

\[
\tilde{V}(x, y) = \begin{pmatrix}
\tilde{V}_{\psi\psi}(x, y) & \tilde{V}_{\psi\phi}(x, y) \\
\tilde{V}_{\phi\psi}(x, y) & \tilde{V}_{\phi\phi}(x, y)
\end{pmatrix} \tag{50}
\]

with the elements of this matrix in leading order given by

\[
\tilde{V}_{\psi\psi}^{(0)}(x, y) = \theta(x < y) \left\{-2 \frac{x}{y} - \delta(x - y) \right\} + \left( x \leftrightarrow \bar{x} \right. \quad \left( y \leftrightarrow \bar{y} \right) \tag{51}
\]

\[
\tilde{V}_{\psi\phi}^{(0)}(x, y) = \theta(x < y) \left\{-x \right\} - \left( x \leftrightarrow \bar{x} \right. \quad \left( y \leftrightarrow \bar{y} \right) \tag{51}
\]

\[
\tilde{V}_{\phi\psi}^{(0)}(x, y) = \theta(x < y) \left\{ \frac{1}{y} \right\} - \left( x \leftrightarrow \bar{x} \right. \quad \left( y \leftrightarrow \bar{y} \right) \tag{51}
\]

\[
\tilde{V}_{\phi\phi}^{(0)}(x, y) = \theta(x < y) \left\{ 2 - \delta(x - y) \right\} + \left( x \leftrightarrow \bar{x} \right. \quad \left( y \leftrightarrow \bar{y} \right) \tag{51}
\]
and in next-to-leading order

\[
\hat{\mathcal{V}}^{(1)}_{\psi\psi}(x, y) = \theta(x < y) \left\{ 6 \frac{x}{y} + \frac{x}{y} \ln \left( \frac{x}{y} \right) + \frac{\bar{x}}{y} \ln(\bar{x}) + \frac{\bar{x}}{y} \ln(x) - \frac{1}{y} \left( 1 - \frac{x}{y} \right) \ln \left( 1 - \frac{x}{y} \right) \right. \\
+ \left. \frac{\bar{x}}{y} \ln^2(\bar{x}) + 2 \frac{x}{y} \ln(x) \ln(y) - \frac{x}{y} \ln^2(y) + \delta(x - y) \right\} + \left( \frac{x}{y} \leftrightarrow \bar{x} \bar{y} \right) \tag{52}
\]

\[
\hat{\mathcal{V}}^{(1)}_{\psi\phi}(x, y) = \theta(x < y) \left\{ 6x + x \ln \left( \frac{x}{y} \right) + \bar{x} \ln(\bar{x}) - x \ln(x) + \ln \left( 1 - \frac{x}{y} \right) \right. \\
+ \left. \bar{x} \ln^2(\bar{x}) - 2x \ln(x) \ln(y) + x \ln^2(y) \right\} - \left( \frac{x}{y} \leftrightarrow \bar{x} \bar{y} \right) \tag{53}
\]

\[
\hat{\mathcal{V}}^{(1)}_{\phi\psi}(x, y) = \theta(x < y) \left\{ -6 \frac{y}{x} + \frac{1}{y} \ln \left( \frac{y}{x} \right) + \frac{1}{y} \ln(\bar{x}) - \frac{1}{y} \ln(x) - \frac{1}{y} \ln \left( 1 - \frac{x}{y} \right) \right. \\
+ \left. \frac{1}{y} \ln^2(\bar{x}) - 2 \frac{y}{x} \ln(x) \ln(y) + \frac{1}{y} \ln^2(y) \right\} - \left( \frac{x}{y} \leftrightarrow \bar{x} \bar{y} \right) \tag{54}
\]

\[
\hat{\mathcal{V}}_{\phi\phi}(x, y) = \theta(x < y) \left\{ -2 - \ln \left( \frac{x}{y} \right) + \ln^2(\bar{x}) + 2 \ln(x) \ln(y) - \ln^2(y) + \delta(x - y) \right\} \\
+ \left( \frac{x}{y} \leftrightarrow \bar{x} \bar{y} \right) \tag{55}
\]

The corresponding kernels in the case of forward scattering are found to be

\[
\hat{\mathbf{P}}^{(0)}(x, y) = \begin{pmatrix} -2\bar{z} - \delta(1 - z) & -2 \\
-2z & -\delta(1 - z) \end{pmatrix} \tag{56}
\]

\[
\hat{\mathbf{P}}^{(1)}_{\psi\psi}(z) = 7\bar{z} + 2\bar{z} \ln(\bar{z}) + \bar{z} \ln(z) + (1 + z) \ln^2(z) + \delta(1 - z) \tag{57}
\]

\[
\hat{\mathbf{P}}^{(1)}_{\psi\phi}(z) = 7 - z + 2 \ln(\bar{z}) + (3 - 2z) \ln(z) + \ln^2(z) \tag{58}
\]

\[
\hat{\mathbf{P}}^{(1)}_{\phi\psi}(z) = -1 + 7z + 2z \ln(\bar{z}) + (3z - 2) \ln(z) - z \ln^2(z) \tag{59}
\]

\[
\hat{\mathbf{P}}^{(1)}_{\phi\phi}(z) = -\bar{z}(1 + 2 \ln(z)) + \delta(1 - z) \tag{60}
\]

2.4.3 Fermionic operator

As it can be easily seen from Eqs. 21 and 22 there is a simple relation between fermionic operators in polarized and unpolarized cases

\[
\tilde{\tilde{\mathcal{O}}}^\text{fer}_{\delta,\delta} = -\gamma_5 \mathcal{O}^\text{fer}_{\delta,\delta} \tag{61}
\]

Thus, the evolution kernel for fermionic DA in polarized case differs only by overall sign from unpolarized one.

### 3 Conformal Ward Identity approach

As was shown in Refs. 17, 18 non-diagonal part of anomalous dimension matrix may be obtained by studying renormalized Conformal Ward Identities (CWI) for the Green functions of elementary fields with the insertions of corresponding conformal operators. To deduce the necessary identities,
one starts with the generating functional for renormalized disconnected Green functions of conformal operators. Exploiting its invariance with respect to transformations of collinear conformal group we get
\[ \langle [\delta G \mathcal{O}_{jl}] \chi \rangle = -\langle ([\delta G \mathcal{O}_{jl}] \chi) - \langle [\delta G \mathcal{O}_{jl}] \mathcal{S} \rangle \chi \rangle , \] (62)
where \( < A > \) denotes averaging over the vacuum of the time ordered product \( T A \exp(i\mathcal{S}) \) and \( \chi = \prod_i \phi_i \) stands for the product of elementary fields at different space-time points. The left hand side of the above equality is the differential operator acting on renormalized Green function \( \langle [\mathcal{O}_{jl}] \chi \rangle \), which thus should be finite. Hence, the right hand side of the equality is also finite. Using this fact we may determine the scale and special conformal anomalous matrices for conformal operators under consideration.

Now, the crucial point is to note, that scale and special conformal anomalies are in fact related. The relation between matrices of scale and special conformal anomalies could be obtained by considering the action of the commutator of special conformal transformation with dilation \( [\delta^D, \delta^K] = \delta^K \) on Green function with conformal operator insertion. As a result we obtain the following relation
\[ \hat{a}(l) + \hat{\gamma}_c(\alpha, l) + \beta(\alpha) \hat{b}(l), \hat{\gamma}(\alpha) \] = 0 , \] (63)
where \( \hat{\gamma}_c(\alpha, l) \) is matrix of special conformal anomalies, \( \hat{a}(l) \), \( \hat{b}(l) \) is some matrix coefficients and \( \hat{\gamma}(\alpha) \) are the anomalous dimensions matrix, we want to find. Due to the fact, that matrices \( \hat{a}(l) \) and \( \hat{b}(l) \) do not depend on coupling constant (in contrast with all other quantities, appearing in Eq. (63)), it is possible to solve this equation recursively at a given order of perturbation theory and find the relation between non-diagonal elements of anomalous dimension matrix \( \gamma_{ND}^{\alpha} \) in \( n \)-order and matrix of special conformal anomalies \( \hat{\gamma}_c(\alpha, l) \) in \( (n - 1) \)-order of perturbation theory. For example, at LO we have \( \gamma_{ND}^{\alpha} = \delta_{jk} \gamma_k^D \). It should be noted, that in the absence of gauge field the final expression is considerably simplified and one only needs to know the matrices \( a(l) \) \( b(l) \). More details about this method in the case of fermionic operator could be found in Appendix C. Below we present the results, obtained within this framework, for non-diagonal parts of anomalous dimensions matrix of conformal operators, we are interested in.

3.1 Unpolarized case

3.1.1 Non-singlet case

Consider first the case of non-singlet bosonic operator. Solving Eq. (63) in the next-to-leading order, it is easy to find the following general expression for non-diagonal anomalous dimensions matrix \[ \psi \gamma_{ND}^{\alpha} = (\psi \gamma_{\alpha}^{(0)} - \psi \gamma_{\alpha}^{(0)}) d_{j,k} \left( b_0 + \gamma_\Phi - \psi \gamma_{\alpha}^{(0)} \right) , \] (64)
where
\[ \psi \gamma_{\alpha}^{(0)} = \frac{2}{n + 1} - \frac{2}{n + 2} - 1 \]
is the corresponding leading order anomalous dimension of operator \( \mathcal{O}_\alpha \) in forward case and
\[ d_{j,k} = \begin{cases} \frac{2k + 3}{(j + k + 3)(j - k)} & \text{if } j - k > 0 \text{ and even} \\ 0 & \text{otherwise} \end{cases} \] (65)
Taking into account the next-to-leading order anomalous dimension in forward case

\[
\psi^{(1)}_{\gamma,n} = \frac{2}{(n+1)^3} - \frac{2}{(n+1)^2} - \frac{4}{n+1} + \frac{2}{(n+2)^3} + \frac{2}{(n+2)^2} + \frac{4}{n+2}
\]

it not hard to verify that the relation holds true.

### 3.1.2 Singlet case

In the singlet case the expression for non-diagonal anomalous dimensions matrix should be modified to take into account operator mixing. In the case of Wess-Zumino model the corresponding expressions are [20]

\[
\begin{align*}
\psi^\text{ND}_{\gamma,j,k} &= (\psi_{\gamma,j}^{(0)} - \psi_{\gamma,k}^{(0)}) d_{j,k} (b_0 - \psi_{\gamma,k}^{(0)}) - (\psi_{\gamma,j}^{(0)} - \psi_{\gamma,k}^{(0)}) d_{j,k} \psi_{\gamma,k}^{(0)} \\
\phi^\text{ND}_{\gamma,j,k} &= (\phi_{\gamma,j}^{(0)} - \phi_{\gamma,k}^{(0)}) d_{j,k} (b_0 - \phi_{\gamma,k}^{(0)}) - (\phi_{\gamma,j}^{(0)} - \phi_{\gamma,k}^{(0)}) d_{j,k} \phi_{\gamma,k}^{(0)} \\
\phi^\text{ND}_{\phi,j,k} &= (\phi_{\phi,j}^{(0)} - \phi_{\phi,k}^{(0)}) d_{j,k} (b_0 - \phi_{\phi,k}^{(0)}) - (\phi_{\phi,j}^{(0)} - \phi_{\phi,k}^{(0)}) d_{j,k} \phi_{\phi,k}^{(0)} \\
\end{align*}
\]

where \(\phi_{\omega,j}^{(0)}\) are leading order anomalous dimensions of singlet operators [16] and [18]:

\[
\Gamma_{WZ} = \begin{bmatrix}
\psi_{\gamma,n} & \psi_{\phi,n} \\
\phi_{\gamma,n} & \phi_{\phi,n}
\end{bmatrix} = \begin{bmatrix}
\frac{2}{n+1} - \frac{2}{n+2} - 1 & \frac{2}{n+1} - \frac{2}{n+2} - 1 \\
\frac{2}{n+1} - \frac{2}{n+2} - 1 & \frac{2}{n+1} - \frac{2}{n+2} - 1
\end{bmatrix}
\]

Note, that in our normalization of Gegenbauer polynomials, the anomalous dimensions of conform operators [16] and [18] in singlet case differ from corresponding anomalous dimensions of the usual (non-conformal) Wilson twist-2 operators appearing in the description of DIS by shift in argument \(n \to n+1\) and normalization factors. Here and in what follows we will show explicitly additional normalization factors \(\frac{2}{n+1}\) or \(\frac{n+1}{2}\). For example, our results for \(\psi_{\phi,n}^{(0)}\) and \(\phi_{\phi,n}^{(0)}\) in Eq. (71) contain explicit multiplication by these factors.

The next-to-leading order diagonal elements of anomalous dimensions matrix in this case are given by

\[
\begin{align*}
\psi^{(1)}_{\gamma,n} &= \frac{2}{(n+1)^3} - \frac{1}{(n+1)^2} - \frac{7}{n+1} + \frac{2}{(n+2)^3} - \frac{1}{(n+2)^2} + \frac{7}{n+2} + \frac{2}{S_1(n+1)} - \frac{2}{S_1(n+2)} + 1 \\
\psi^{(1)}_{\phi,n} &= \left( \frac{2}{(n+1)^3} - \frac{1}{(n+1)^2} - \frac{7}{n+1} + \frac{2}{(n+2)^3} + \frac{1}{n+2} + \frac{2}{S_1(n+1)} \right) \frac{n+1}{2} \\
\phi^{(1)}_{\gamma,n} &= \left( \frac{2}{(n+1)^2} + \frac{1}{n+1} - \frac{2}{(n+2)^3} + \frac{1}{n+2} - \frac{7}{n+2} + \frac{2}{S_1(n+1)} \right) \frac{2}{n+1} \\
\phi^{(1)}_{\phi,n} &= \frac{2}{(n+1)^2} + \frac{1}{n+1} - \frac{2}{(n+2)^3} - \frac{1}{n+2} + 1
\end{align*}
\]
In singlet case the relation similar to Eq. (8) has the following matrix form
\[
\int_0^1 dx \left( C_n^{3/2}(x - \bar{x}) C_{n+1}^{1/2}(x - \bar{x}) \right) \begin{pmatrix}
V_{\psi\psi}(x, y) & V_{\psi\varphi}(x, y) \\
V_{\varphi\psi}(x, y) & V_{\varphi\varphi}(x, y)
\end{pmatrix} = 
\sum_{k=0}^n \begin{pmatrix}
\phi_{\psi\gamma_{n,k}} & \phi_{\varphi\gamma_{n,k}} \\
\phi_{\psi\gamma_{n,k}} & \phi_{\varphi\gamma_{n,k}}
\end{pmatrix} \begin{pmatrix}
C_n^{3/2}(y - \bar{y}) \\
C_{n+1}^{1/2}(y - \bar{y})
\end{pmatrix}
\]
(76)

It could be easily verified substituting explicit expressions for NLO evolution kernels Eqs. (32-35) and NLO anomalous dimensions Eqs. (72-75).

### 3.1.3 Fermionic operator

The expression for non-diagonal anomalous dimensions matrix of operators with fermionic quantum numbers (21) could be derived similar to the case of bosonic operators (for more details see Appendix C). It is given by
\[
\text{fer}_{\gamma_{j,k}} = \left( \text{fer}_{\gamma_{j}^{(0)}} - \text{fer}_{\gamma_{k}^{(0)}} \right) d_{\text{fer}}^{j,k} \left( \beta_0 - \text{fer}_{\gamma_{k}^{(0)}} \right) ,
\]
(77)
where
\[
\text{fer}_{\gamma_{n}^{(0)}} = (-1)^{n+1} \frac{2}{n+1} - 1
\]
(78)
is the corresponding leading order anomalous dimension of fermionic operator (21) in forward case. The matrix \(d_{\text{fer}}^{j,k}\) is defined through the derivative of corresponding Jacobi polynomial over its indices:
\[
\frac{d}{d\nu} P_{j}^{(1+\nu,\nu)}(z) \bigg|_{\nu=0} = 2 \sum_{k=0}^{j} d_{\text{fer}}^{j,k} P_{k}^{(1,0)}(z)
\]
(79)
The explicit expression for \(d_{\text{fer}}^{j,k} (j > k)\) has the following form
\[
d_{\text{fer}}^{j,k} = (-1)^{j-k} \frac{1}{j-k} \frac{(j+1) + (-1)^{j-k}(k+1)}{j+k+2} \frac{k+1}{j+1}
\]
(80)

Taking into account the next-to-leading order anomalous dimension of this operator in forward case
\[
\text{fer}_{\gamma_{n}^{(1)}} = \frac{2}{(n+1)^3} + 1 + (-1)^{n+1} \left( \frac{1}{(n+1)^2} - \frac{6}{n+1} + \frac{S_1(n+1)}{n+1} \right)
\]
(81)
it easy to verify, that the relation similar to Eq. (8)
\[
\int_0^1 dx P_{j}^{(1,0)}(x - \bar{x}) V_{\text{fer}}(x, y|\alpha) = \sum_{k=0}^{j} \text{fer}_{\gamma_{jk}(\alpha)} P_{k}^{(1,0)}(y - \bar{y})
\]
(82)
is valid.

### 3.2 Polarized case

In the polarized case non-diagonal anomalous dimensions matrices have the same expression as in unpolarized case (see. Eqs. (64), (67)-(70) and (77)). The only change, which should be done, is the replacement of unpolarized forward anomalous dimensions \(\gamma_{j}\) with polarized ones \(\tilde{\gamma}_{j}\).
3.2.1 Non-singlet case

Taking into account the change of sign in diagrams in Fig. 1a and Fig. 2b we find that polarized leading order anomalous dimension is given by

\[ \psi \tilde{\gamma}_n^{(0)} = -\frac{2}{n+1} + \frac{2}{n+2} - 1, \quad (83) \]

whereas in next-to-leading order we have

\[
\psi \tilde{\gamma}_n^{(1)} = -\frac{2}{(n+1)^3} - \frac{2}{(n+1)^2} + \frac{4}{n+1} + \frac{2}{(n+2)^3} + \frac{2}{(n+2)^2} - \frac{4}{n+2} \\
- \frac{2 S_1(n+1)}{n+1} + \frac{2 S_1(n+2)}{n+2} + 1 \\,(84)\]

It easy to see, that the relation (8) holds true.

3.2.2 Singlet case

In singlet case the leading order anomalous dimensions are

\[
\tilde{\Gamma}_{WZ} = \begin{pmatrix}
\psi \tilde{\gamma}_n & \psi \phi \tilde{\gamma}_n \\
\phi \tilde{\gamma}_n & \phi \phi \tilde{\gamma}_n
\end{pmatrix} = \begin{pmatrix}
-\frac{2}{n+1} + \frac{2}{n+2} - 1 & -\frac{2}{n+1} + \frac{2}{n+2} \\
-\frac{2}{n+2} n+1 & -1
\end{pmatrix} \quad (85)
\]

In the next-to leading order the diagonal elements of the anomalous dimensions matrix have the following expressions

\[
\psi \tilde{\gamma}_n^{(1)} = -\frac{2}{(n+1)^3} - \frac{3}{(n+1)^2} + \frac{7}{n+1} + \frac{2}{(n+2)^3} + \frac{3}{(n+2)^2} - \frac{7}{n+2} \\
- \frac{2 S_1(n+1)}{n+1} + \frac{2 S_1(n+2)}{n+2} + 1 \\,(86)\]

\[
\psi \phi \tilde{\gamma}_n^{(1)} = \left( \frac{2}{(n+1)^3} - \frac{3}{(n+1)^2} + \frac{7}{n+1} + \frac{2}{(n+2)^3} - \frac{1}{n+2} - \frac{2 S_1(n+1)}{n+1} \right) \frac{n+1}{2} \\,(87)\]

\[
\phi \tilde{\gamma}_n^{(1)} = \left( \frac{2}{(n+1)^2} - \frac{1}{n+1} - \frac{2}{(n+2)^2} + \frac{7}{n+2} - \frac{2 S_1(n+1)}{n+2} \right) \frac{n+1}{2} + 1 \\,(88)\]

\[
\phi \phi \tilde{\gamma}_n^{(1)} = \frac{2}{(n+1)^2} - \frac{1}{n+1} - \frac{2}{(n+2)^2} + \frac{1}{n+2} + 1 \\,(89)\]

4 Supersymmetric Ward Identity

Now, let us recall, that all operators considered above form the representation of $N = 1$ supersymmetry algebra. Similar to CWI we may consider the supersymmetric Ward identity [21] (its derivation goes along the same lines as the derivation of CWI). As we will see in a moment, there is a host of remarkable consequences for anomalous dimensions of our operators following from supersymmetric Ward identity. To begin with, let us introduce the following combinations of conformal operators for
unpolarized (Eqs. (16), (18) and (21)) and polarized (Eqs. (17), (19) and (22)) operators.

\[
S^1_{j,l} = \frac{2}{j+1} \mathcal{O}^\psi_{j,l} + \mathcal{O}^\phi_{j,l} \quad P^1_{j,l} = \frac{2}{j+1} \tilde{\mathcal{O}}^\psi_{j,l} + \tilde{\mathcal{O}}^\phi_{j,l} \\
S^2_{j,l} = \frac{j+1}{j+2} \frac{2}{j+1} \mathcal{O}^\psi_{j,l} + \mathcal{O}^\phi_{j,l} \quad P^2_{j,l} = \frac{j+1}{j+2} \frac{2}{j+1} \tilde{\mathcal{O}}^\psi_{j,l} + \tilde{\mathcal{O}}^\phi_{j,l} \quad U_{j,l} = 2 \tilde{\mathcal{O}}^\text{fer}_{j,l},
\]

where the coefficients in front of operators define the matrix, diagonalizing the matrices of forward anomalous dimensions (71) and (85).

Under restricted light-cone supersymmetry transformations the combinations of conformal operators introduced above transform as follows

\[
\delta^Q S^1_{j,l} = (1 - (-1)^j) i \delta \mathcal{V}^1_{j-1,l} \quad (93)
\]
\[
\delta^Q S^2_{j,l} = (1 - (-1)^j) i \delta \mathcal{V}^2_{j,l} \quad (94)
\]
\[
\delta^Q P^1_{j,l} = (1 + (-1)^j) i \delta \mathcal{V}^1_{j-1,l} \quad (95)
\]
\[
\delta^Q P^2_{j,l} = (1 + (-1)^j) i \delta \mathcal{V}^2_{j,l} \quad (96)
\]
\[
\delta^Q \mathcal{V}^i_{j,l} = i \partial_+ (S^1_{j+1,l} + \gamma_5 P^2_{j,l}) \gamma^- \epsilon + i \partial_+ (S^2_{j+1,l} + \gamma_5 P^1_{j+1,l}) \gamma^- \epsilon. \quad (97)
\]

The transformation low for the operators \(U_{j,k}\) following from the observation \(U_{j,k} = -\gamma_5 \mathcal{V}_{j,k}\) and the Eq. (96). Now, we see that it is natural to combine bosonic and fermionic conformal operators into \(N = 1\) chiral supermultiplet.

\[
\Phi = (S^1 + iP^2) + \sqrt{2} \theta \mathcal{X} + \theta \theta (S^2 + iP^1), \quad (98)
\]

where \(\mathcal{V}_{j-1,l}\) is the Majorana fermion build from the Weyl spinor \(\mathcal{X}\).

It is easy to find, that the renormalized supersymmetric Ward identity in the regularization scheme, preserving supersymmetry, has the following form (21) (\(S_{j,l}\) denotes vector of operators \(S^1_{j,l}\) and \(S^2_{j,l}\))

\[
\langle [S_{jl}] \delta^Q \mathcal{X} \rangle = -\langle [S_{jl}] \mathcal{X} \rangle - \langle [i[S_{jl}] \delta^Q S] \mathcal{X} \rangle \quad \text{and} \quad \langle \delta^Q [S_{jl}] \mathcal{X} \rangle = \text{finite}, \quad (99)
\]

where we used the fact, that renormalized action in supersymmetric regularization is invariant with respect to supersymmetry transformations \(\langle [i[S_{jl}] \delta^Q S] \mathcal{X} \rangle = 0\). The operators (93) and (94) mix under renormalization and thus we define renormalized operators as (square brackets correspond to renormalized quantities)

\[
\begin{bmatrix}
S^1 \\
S^2
\end{bmatrix}_{jl} = \sum_{k=0}^{j} \begin{pmatrix}
\frac{11}{21} Z_S & \frac{12}{22} Z_S \\
\frac{21}{21} Z_S & \frac{22}{22} Z_S
\end{pmatrix}_{jk} \begin{pmatrix}
Z^{-1}_\phi & 0 \\
0 & Z^{-1}_\phi
\end{pmatrix}_{kl} \begin{bmatrix}
S^1 \\
S^2
\end{bmatrix}_{kl}, \quad (100)
\]

and as a consequence the renormalization group equation for these operators is given by

\[
\frac{d}{d \ln \mu} \begin{bmatrix}
S^1 \\
S^2
\end{bmatrix}_{jl} = \sum_{k=0}^{j} \begin{pmatrix}
\frac{11}{21} \gamma^S & \frac{12}{22} \gamma^S \\
\frac{21}{21} \gamma^S & \frac{22}{22} \gamma^S
\end{pmatrix}_{jk} \begin{bmatrix}
S^1 \\
S^2
\end{bmatrix}_{kl}. \quad (101)
\]

Now, from supersymmetric Ward identity (99) we get \((\sigma_k = \frac{1}{2} (1 - (-1)^k)\) and \(Z_{jk} = 0\) for \(k > j)\)

\[
\sum_{k=0}^{j} \sum_{k'=0}^{k} \begin{pmatrix}
\frac{11}{21} Z_S & \frac{12}{22} Z_S \\
\frac{21}{21} Z_S & \frac{22}{22} Z_S
\end{pmatrix}_{jk} \sigma_k \left\{ Z^{-1}_V \right\}_{k-1,k'} \left\{ Z^{-1}_V \right\}_{kk'} \left[ V_{kl} \right] = \text{finite} \quad (102)
\]
1/\epsilon \text{ poles in (102) cancel, provided }

\sum_{k=0}^{j} \left\{ 11Z_{k}^{[1]} \right\}_{jk} \sigma_{k} [\mathcal{V}_{k-1,i}] + \sum_{k=0}^{j} \left\{ 12Z_{k}^{[1]} \right\}_{jk} \sigma_{k} [\mathcal{V}_{kl}] = \sigma_{j} \sum_{k=0}^{j} \left\{ Z_{k}^{[1]} \right\}_{j-1,k} [\mathcal{V}_{kl}], \quad (103)

\sum_{k=0}^{j} \left\{ 21Z_{k}^{[1]} \right\}_{j,k} \sigma_{k} [\mathcal{V}_{k-1,i}] + \sum_{k=0}^{j} \left\{ 22Z_{k}^{[1]} \right\}_{jk} \sigma_{k} [\mathcal{V}_{kl}] = \sigma_{j} \sum_{k=0}^{j} \left\{ Z_{k}^{[1]} \right\}_{j,k} [\mathcal{V}_{kl}], \quad (104)

Noting, that \( U_{j,k} = -\gamma_{j}V_{j,k} \), it is easy to derive analogous relations for polarized operators \( P_{j,k} \). Taking into account linear independence of fermionic operators [\mathcal{V}_{kl}] we finally get the following relations [21]

\begin{align*}
11S_{2n+1,2m+1} &= 22P_{2n,2m} = \text{fer} \gamma_{2n,2m}, \quad m \leq n, \\
12S_{2n+1,2m+1} &= 21P_{2n,2m+2} = \text{fer} \gamma_{2n,2m+1}, \quad m \leq n - 1, \\
21S_{2n+1,2m+1} &= 12P_{2n+2,2m} = \text{fer} \gamma_{2n+1,2m}, \quad m \leq n, \\
22S_{2n+1,2m+1} &= 11P_{2n+2,2m+2} = \text{fer} \gamma_{2n+1,2m+1}, \quad m \leq n,
\end{align*}

and

\begin{align*}
12S_{2n+1,2n+1} &= 0, \quad 12P_{2n,2n} = 0. \quad (109)
\end{align*}

These relations allow us to find anomalous dimensions of bosonic operators from known anomalous dimensions of fermionic operators and vice versa. In particular, substituting explicit expression for non-diagonal anomalous dimension of bosonic operators, one can determine matrix \( d_{j,k} \) for fermionic operators Eq. (100). The found relations between anomalous dimensions of conformal operators is special case of existing superconformal relations for anomalous dimensions of conformal operators in supersymmetric theories.

To obtain from the results above the corresponding relations for anomalous dimensions of conformal operators considered earlier, one will need the following transition formulae among anomalous dimensions of conformal operators (90) and (91) and anomalous dimensions of operators (16) and (18)

\[
\left( \begin{array}{c}
\frac{1}{k+1}^{11\gamma} \\
\frac{1}{k+2}^{12\gamma} \\
\frac{1}{k+1}^{21\gamma} \\
\frac{1}{k+2}^{22\gamma}
\end{array} \right)_{jk} = \frac{1}{2k+3} \left( \begin{array}{cccc}
k+2 & k+2 & 2 & 1 \\
\frac{k+2}{j+1} & \frac{k+2}{j+1} & \frac{2}{j+1} & -1 \\
\frac{k+2}{k+1} & \frac{k+2}{k+1} & \frac{2}{k+1} & -1 \\
\frac{k+2}{k+1} & \frac{k+2}{k+1} & \frac{2}{j+2} & 1
\end{array} \right) \left( \begin{array}{c}
\psi\psi_{\gamma} \\
\phi\phi_{\gamma} \\
\psi\phi_{\gamma} \\
\phi\psi_{\gamma}
\end{array} \right)_{jk}
\] 

(110)

Now, let us explore some particular limits of equations (105)-(109). From Eq. (109) it follows, that (here and below \( \gamma_{jj} \equiv \gamma_{j}, \gamma \equiv \gamma^{V}, \tilde{\gamma} \equiv \gamma^{A} \) and indexes \( S (P) \) correspond to unpolarized (polarized) cases):

\[
\psi\psi_{\gamma}^{j} + \frac{j+1}{2} \phi\phi_{\gamma}^{i} = \frac{2}{j+1} \psi\phi_{\gamma}^{j} + \phi\psi_{\gamma}^{i}, \quad i = S, P,
\]

(111)

where \( j \) is odd for unpolarized case and even for polarized case. This equation is analogous (up to redefinition of anomalous dimensions) to the well-known Dokshitzer relation (last paper in Ref. [2]),
which was found first empirically in LO in the case of supersymmetric QCD (the anomalous dimensions in supersymmetric QCD could be obtained from those in ordinary QCD through substitutions $C_A = N_c, C_F = N_c$ and $N_f = \frac{1}{2}N_c$).

From (105) and (108), we also have (for odd $j$)

$$\psi\gamma_j^S + \frac{j + 1}{2} \phi\gamma_j^S = \psi\gamma_j^P - \frac{2}{j} \phi\gamma_j^P = \text{fer}_\gamma_{j-1},$$  \hspace{0.5cm} (112)

$$\psi\gamma_{j+1}^P + \frac{j + 2}{2} \phi\gamma_{j+1}^P = \psi\gamma_j^S - \frac{2}{j + 1} \phi\gamma_j^S = \text{fer}_\gamma_j.$$  \hspace{0.5cm} (113)

Moreover, from equations (106) and (107) we get the following relations between diagonal and non-diagonal elements of anomalous dimensions matrices of conformal operators

$$\frac{2}{j + 2} \phi\gamma_j^i - \frac{j + 1}{2} \phi\gamma_j^i = \frac{j + 1}{2j + 1} \Delta_{j+1,j-1} = \text{fer}_\gamma_{j,j-1},$$ \hspace{0.5cm} (114)

where $j$ is odd for $i = S$ and $k = P$, $j$ is even for $i = P$ and $k = S$ and

$$\Delta_{j+1,j-1} \equiv \frac{j}{j + 2} \phi\gamma_{j+1,j-1} + \frac{j + 2}{2} \phi\gamma_{j+1,j-1} - \frac{2}{j + 1} \phi\gamma_{j+1,j-1} = \phi\gamma_{j+1,j-1}.$$ \hspace{0.5cm} (115)

In the leading order $\Delta_{j+1,j-1}$ is equal to zero, but starting from next-to-leading order it acquires nonzero contribution. Here, we would like to note, that all these relations are valid in all orders of perturbation theory provided anomalous dimensions were evaluated in supersymmetric scheme. In Wess-Zumino model such scheme could be identified with the usual $\overline{\text{MS}}$-scheme.

Next, the forward anomalous dimensions matrix in singlet case (71) is diagonalized with the help of matrix $D$

$$D = \begin{pmatrix} \frac{2}{j + 1} & 1 \\ \frac{2}{j + 2} & 1 \\ -1 & 0 \\ 0 & -\frac{2}{j + 1} \end{pmatrix}$$ \hspace{0.5cm} (116)

so, that in leading order we have

$$[D\Gamma_{\text{wz}}D^{-1}]^{(0)} = \begin{pmatrix} \frac{2}{j + 1} & -1 & 0 \\ 0 & -\frac{2}{j + 1} \end{pmatrix}$$ \hspace{0.5cm} (117)

In next-to-leading order of perturbation theory the elements of forward anomalous dimensions matrix of bosonic operators are given by (72)-(75). Using the same diagonalizing matrix $D$ (116), as in leading order, we get

$$[D\Gamma_{\text{wz}}D^{-1}]^{(1)} = \begin{pmatrix} \frac{2}{(j + 1)^3} + \frac{1}{(j + 1)^2} - \frac{6}{j + 1} + \frac{2 S_1(j + 1)}{j + 1} & 0 \\ -\frac{4}{j + 1} - \frac{4}{j + 2} & \frac{2}{(j + 2)^3} - \frac{1}{(j + 2)^2} + \frac{6}{j + 2} - \frac{2 S_1(j + 2)}{j + 2} \end{pmatrix}$$ \hspace{0.5cm} (118)
So, in leading order the following relations between elements of anomalous dimensions matrix are valid
\[
\psi\psi\gamma_j + \frac{j+1}{2}\phi\psi\gamma_j = \frac{2}{j+1}\psi\phi\gamma_j + \phi\phi\gamma_j \quad (119)
\]
\[
\psi\psi\gamma_j - \frac{j+1}{2}\phi\psi\gamma_j = -\frac{2}{j+1}\psi\phi\gamma_j + \phi\phi\gamma_j \quad (120)
\]
However, in next-to-leading order the relation (120) does not hold, while the relation (119) is valid to all orders of perturbation theory. Note, also, that from the relation (119) it follows that
\[
\psi\psi\gamma_j - \frac{2}{j+1}\psi\phi\gamma_j = -\frac{j+1}{2}\phi\psi\gamma_j + \phi\phi\gamma_j \quad (121)
\]
which is also valid to all orders in perturbation theory and in leading order coincides with the relation (120). Moreover, modifying the diagonalizing matrix (116) it is possible to show, that the forward anomalous dimensions matrix will remain diagonal also in next-to-leading order, and, what is more important, in all orders of perturbation theory. Such matrix \( \hat{D} \) is given by \( \text{widetext}{[22]} \)²:
\[
\hat{D} = \begin{pmatrix}
\frac{2}{j+1} & 1 \\
-j+1 & \frac{j+1}{2}\phi\psi\gamma_j \frac{2}{\psi\phi\gamma_j} & 1
\end{pmatrix}
\quad (122)
\]
In polarized case the forward anomalous dimensions matrix (85) may be diagonalized with the help of the same matrix \( D \) (116), as in unpolarized case and it is possible to find, that
\[
[\hat{D}\bar{\Gamma}_{wz}\hat{D}^{-1}]^{(0)} = \begin{pmatrix}
-\frac{2}{j+1} & 0 \\
0 & \frac{2}{j+2} - 1.
\end{pmatrix}
\quad (123)
\]
In next-to-leading order the elements of forward anomalous dimensions matrix in polarized case are given by Eqs. (86)-(89). Using the same diagonalization matrix, as in leading order, we get
\[
[\hat{D}\bar{\Gamma}_{wz}\hat{D}^{-1}]^{(1)} = \begin{pmatrix}
\frac{4}{j+1} - \frac{4}{j+2} + \frac{8}{(j+2)^2} & \frac{2}{(j+2)^3} - \frac{1}{(j+2)^2} + \frac{6}{j+2} - \frac{2S_1(j+2)}{j+2} & 0 \\
\frac{2}{(j+1)^3} + \frac{6}{j+1} + 2S_1(j+1) & -\frac{6}{j+1} - 1 & 0
\end{pmatrix}
\quad (124)
\]
At LO we have three independent relations among elements of forward anomalous dimensions matrix: Eq. (111) together with Eq. (112), Eq. (113) and Eq. (114), where at LO we have \( \Delta^i_{j+1,j-1} = 0 \). Therefore, we may conclude, that in forward case in leading order of perturbation theory, it is sufficient to know only one diagonal element of anomalous dimensions matrix \( T \) or anomalous dimension of fermionic operator \( \text{fer}_{\gamma_j} \) to reconstruct the whole matrix.

At NLO the righthand side of Eq. (114) is nonzero and as a consequence matrices (118) and (124) become triangular. So, to reconstruct the forward anomalous dimensions matrix one needs to know additionally any other element of forward anomalous dimensions matrix or non-diagonal anomalous dimension of fermionic operator \( \text{fer}_{\gamma_{j,j-1}} \).

²We thank Anatoly Kotikov for fruitful discussions of this issue.
5 Solution of NLO evolution equation

Our primary goal is the construction of multiplicatively renormalized conformal operators up to next-to-leading order in perturbation theory. The non-singlet case is simple and the corresponding solution could be easily found in full analogy with $\phi^3$-theory [14]. So, here we will be interested only in singlet case with bosonic operator mixing. In previous section we shown, that the twist-2 conformal operators form a chiral supermultiplet of $N = 1$ supersymmetry algebra and derived their transformation rules under supersymmetry. It turns out, that in the singlet case, it is sufficient to find only the expression for multiplicatively renormalized fermionic operator. The multiplicatively renormalized bosonic operators up to next-to-leading order in perturbation theory could be obtained then via supersymmetry transformations from the expression for multiplicatively renormalized fermionic operator.

As we already mentioned before there is a close relation between DA and conformal operators, we are studying. To see it explicitly, let us consider the case of non-singlet bosonic operator. The eigenvalues of the anomalous dimensions matrix $\hat{\gamma}$ of this operator are $\gamma_j = \gamma_{jj}$. Therefore, the renormalization group analysis provides for multiplicatively renormalized operators $\tilde{O}_{jl}$:

$$\tilde{O}_{jl}(\mu^2) = \exp\left(\int_{\mu_0^2}^{\mu^2} \frac{dt}{t} \gamma_j(g(t))\right) \tilde{O}_{jl}(\mu_0^2).$$

Moreover, the operators $O_{jl}$ could be completely expressed through $\tilde{O}_{jl}$:

$$O_{jl}(\mu^2) = \sum_{k=0}^{\infty} B_{jk}(g(\mu^2)) \tilde{O}_{kl}(\mu^2).$$

Then, the evolution of distribution amplitude is given by:

$$\phi(x, Q^2) = \sum_{n=0}^{\infty} \phi_n(x, \alpha(Q^2)) \exp\left(\int_{\mu_0^2}^{Q^2} \frac{dt}{t} \gamma_n(g(t))\right) \langle 0 | O_{nn}(Q_0^2) | P \rangle_{\text{red}},$$

where $\phi_n(x, \alpha(Q^2)) = \sum_{k=n}^{\infty} x \bar{x}^k C_k^{3/2}(x - \bar{x}) B_{kn}(\alpha(Q^2)),$

where $N_k$ is some normalization factor (see below). So, in this section we will first determine the eigenfunctions of the fermionic evolution kernel. Then, it is straightforward to write down the multiplicatively renormalized fermionic operators. Our treatment here is close to the work of Mikhailov and Radyushkin on $\phi^3$-theory in 6-dimensional space-time [14, 16].

In leading order of perturbation theory the evolution equation (2) for the fermionic operator (21) could be easily solved employing separation of variables.

$$\phi(x, Q^2) = \sum_{n=0}^{\infty} a_n \left(\ln \frac{\mu^2}{\Lambda^2}\right)^{-\left(\frac{\alpha_{\text{fer}}(0)/b_0}{\alpha_{\text{fer}}(0)/b_0}\right)} P_{n}^{(1,0)}(x - \bar{x}),$$

where $P_n^{(1,0)}(x - \bar{x}) = \psi_n(x)$ is the solution of the eigenvalue equation

$$\int_0^1 v_0(x, y) \psi_n(y) dy = \lambda_n \psi_n(x),$$

where $\alpha_{\text{fer}}(0)$ is the LO anomalous dimension of the fermionic operator [21] and $\psi_n(x)$ is the solution of the eigenvalue equation

$$\int_0^1 v_0(x, y) \psi_n(y) dy = \lambda_n \psi_n(x),$$

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where \( \lambda_n = (-1)^n / (n+1) \) and \( v_0 \) is defined as

\[
V^{(0)}_{\text{fer}}(x, y) = v_0(x, y) + \delta(x - y)
\] (131)

The easiest way to find \( \psi_n \)'s is to note, that the combination \( \tilde{y}v_0(x, y) \equiv \tilde{v}(x, y) \) is symmetric under the change \( x \leftrightarrow y \): \( \tilde{v}(x, y) = \tilde{v}(y, x) \). Another property of \( v_0(x, y) \) is that its convolution with the polynomial of degree \( N \) is the polynomial of degree \( M \) with \( M \leq N \). Thus, the functions \( \psi_n(x)/\bar{x} \) should be the polynomials orthogonal to each other on \( [0, 1] \) with measure \( \bar{x} \).

Now, let us proceed with NLO evolution kernel. It is convenient to rewrite the resulting expression Eq. (42) for NLO evolution kernel of the fermionic operators in the following form

\[
V^{(1)}_{\text{fer}} = \theta(x > \bar{y}) \left\{ \frac{1}{\bar{y}} \ln(x) - \frac{1}{\bar{y}} \ln\left(1 - \frac{x}{\bar{y}}\right) \right\} + \theta(x < \bar{y}) \left\{ \frac{1}{\bar{y}} \ln(\bar{y} - x) + \frac{6}{\bar{y}} \right\} + \theta(x < y) \left\{ -\frac{\ln^2(y)}{y} + 2 \ln(x) \ln(y) \right\} + \theta(x > y) \left\{ \frac{\ln^2(x)}{y} \right\}
\] (132)

It is easy to verify, that the combination \( \tilde{y}V^{(1)}_{\text{fer}} \) is symmetric with respect to transformation \( x \leftrightarrow y \) if one takes into account only the first line of Eq. (132). The second and third lines of the expression above constitute non-symmetric piece and thus lead to the non-diagonal terms in the Jacobi polynomial basis (excluding \( \delta \)-function, of course). To find a solution of NLO evolution equation we will need the expression of NLO evolution kernel in the basis of LO eigenfunctions. To simplify further notation it is convenient to introduce following definitions:

\[
\Psi^{(1,0)}_{\nu,n}(x) = \frac{w(1 + \nu, \nu|x - \bar{x})}{\mathcal{N}_n(1 + \nu, \nu)} (-1)^n P^{(1+\nu, \nu)}_n(x - \bar{x})
\] (133)

\[
\Psi^{(1,0)}_{n}(x) = \left. \Psi^{(1,0)}_{\nu,n}(x) \right|_{\nu=0},
\] (134)

where \( \mathcal{N}_n(\alpha, \beta) = 2^{\alpha+\beta+1} \frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{\Gamma(n+\alpha+\beta+1)\Gamma(n+\alpha+\beta+1)} \) is the normalization factor and \( w(\alpha, \beta, x) = (1 - x)\alpha (1 + x)^\beta \) is weight function. These are just the solutions of the eigenvalue equation (130) and its generalization:

\[
\int_0^1 dy V^{(0)}_{\nu}(x, y) \Psi^{(1,0)}_{\nu,n}(y) = -\frac{(-1)^{n+1}}{n+1+\nu} \Psi^{(1,0)}_{\nu,n}(y),
\] (135)

where

\[
V^{(0)}_{\nu}(x, y) = \theta(x < \bar{y}) \frac{1}{\bar{y}} \left( \frac{x}{\bar{y}} \right)^\nu
\] (136)

To write down the evolution kernel in the basis of LO eigenfunctions, it is convenient to express the NLO kernel as the convolution of LO evolution kernels and their derivatives:

\[
v_0(x, y) = \theta(x < \bar{y}) \frac{1}{\bar{y}}
\] (137)

\[
\dot{v}_0(x, y) = \left. \left( \frac{d}{d\nu} V^{(0)}_{\nu}(x, y) \right) \right|_{\nu=0} = \theta(x < \bar{y}) \frac{1}{\bar{y}} \ln \left( \frac{x}{\bar{y}} \right)
\] (138)

Then it is an easy exercise to derive, that:

\[
V^{(1)}(x, y) = -2\dot{v}_0(x, z) \otimes v_0(z, y) + 6v_0(x, y) + 2\dot{v}_0(x, y) + u(x, y),
\] (139)
where $\otimes$ denotes the convolution and

$$ u(x, y) = \theta(x > y) \left\{ \frac{1}{y} \ln(x) - \frac{1}{y} \ln\left(1 - \frac{x}{y}\right) \right\} + \theta(x < y) \left\{ \frac{1}{y} \ln(y - x) \right\} $$ (140)

It is not hard to show that $u(x, y)$ is diagonal in the basis of LO eigenfunctions:

$$ \int_0^1 dy \, u(x, y) \Psi_n^{(1,0)}(y) = (-1)^{n+1} \left\{ -\frac{1}{(n+1)^2} + \frac{2S_1(n+1)}{n+1} \right\} \Psi_n^{(1,0)}(x). $$ (141)

Now, what is left is to specify the action of dot kernels on LO eigenfunctions. As a starting point we take the following eigenvalue equation

$$ V^{(0)}_{\nu}(x, y) \otimes \Psi_{\nu,n}^{(1,0)}(y) = -\frac{(-1)^{n+1}}{n+1+\nu} \Psi_{\nu,n}^{(1,0)}(x) $$ (142)

Differentiating this equation with respect to $\nu$ and putting it afterwards to zero we get

$$ \hat{v}_0(x, y) \otimes \Psi_n^{(1,0)}(y) + v_0(x, y) \otimes \frac{d}{d\nu} \left( \Psi_{\nu,n}^{(1,0)}(y) \right) \bigg|_{\nu=0} = \frac{(-1)^{n+1}}{(n+1)^2} \Psi_n^{(1,0)}(x) - \frac{(-1)^{n+1}}{n+1} \frac{d}{d\nu} \left( \Psi_{\nu,n}^{(1,0)}(x) \right) \bigg|_{\nu=0} $$ (143)

So, to find an action of dot kernels on LO eigenfunctions we need to know an expression for the $\nu$ derivative of $\Psi_{\nu,n}^{(1,0)}$. The latter is given by

$$ \frac{d}{d\nu} \left( \Psi_{\nu,n}^{(1,0)}(x) \right) \bigg|_{\nu=0} = 2 \sum_{k>n} \hat{d}_{kn} \Psi_k^{(1,0)}(x) + \text{diagonal part}, $$ (144)

where

$$ \hat{d}_{jk} = \frac{1}{j - k} \frac{(j + 1) + (-1)^j k (k + 1)}{j + k + 2} \frac{k + 1}{j + 1} $$ (145)

We failed to find diagonal part in the equation above, however it is quite easy to derive diagonal part directly for $\hat{v}_0(x, y) \otimes \Psi_n^{(1,0)}(y)$. In this way, we get from Eq. (143)

$$ \hat{v}_0(x, y) \otimes \Psi_n^{(1,0)}(y) = \frac{(-1)^{n+1}}{(n+1)^2} \Psi_n^{(1,0)}(x) - \sum_{k>n} d_{kn} (\gamma_k^{(0)} - \gamma_n^{(0)}) \Psi_k^{(1,0)}(x) $$ (146)

We can summarized the actions of building blocks of NLO evolution kernel on LO eigenfunctions as follows

$$ v_0(x, y) \otimes \Psi_n^{(1,0)}(y) = -\frac{(-1)^{n+1}}{n+1} \Psi_n^{(1,0)}(x) $$ (147)

$$ \hat{v}_0(x, y) \otimes \Psi_n^{(1,0)}(y) = \frac{(-1)^{n+1}}{(n+1)^2} \Psi_n^{(1,0)}(x) - \sum_{k>n} d_{kn} (\gamma_k^{(0)} - \gamma_n^{(0)}) \Psi_k^{(1,0)}(x) $$ (148)

$$ u(x, y) \otimes \Psi_n^{(1,0)}(y) = (-1)^{n+1} \left\{ -\frac{1}{(n+1)^2} + \frac{2S_1(n+1)}{n+1} \right\} \Psi_n^{(1,0)}(x) $$ (149)
With all these formula at hand it is easy to derive that
\[-2\dot{v}_0(x, z) \otimes v_0(z, y) \otimes \Psi_n^{(1,0)}(y) = \frac{2}{(n+1)^3} \psi_n^{(1,0)}(x) + \sum_{k>n} d_{kn} \left( \gamma_k^{(0)} - \gamma_n^{(0)} \right) (-1 - \gamma_n^{(0)}) \psi_k^{(1,0)}(x) \]  
(150)

Note, that substituting the expression for NLO evolution kernel in the basis of LO eigenfunction into Eq. (82) we can easily determine analytical expression for non-diagonal part of anomalous dimensions matrix of fermionic operator.

Now we are ready to write down the solution of ER-BL evolution equation for fermionic operator \( b_0 = 0 \) in next-to-leading order. Note, first, that if the next-to-leading order kernel \( V^{(1)}(x, y) \) is diagonal in the Jacobi polynomials basis, then the solution of evolution equation (2) would be
\[ \phi_n^{(\text{diag})} (x, \mu^2) = a_n (\mu^2_0) \exp \left( - \int_{\mu_0^2}^{\mu^2} \frac{\text{fer}_{\gamma_n}(g(t))}{t} \right) x P_n^{(1,0)}(x - \bar{x}), \]  
(151)

where \( \text{fer}_{\gamma_n}(g) = \left( \frac{\alpha}{4\pi} \right) \text{fer}_{\gamma_n}^{(0)} + \left( \frac{\alpha}{4\pi} \right)^2 \text{fer}_{\gamma_n}^{(1)} \) \( \text{fer}_{\gamma_n}^{(0)} \) and \( \text{fer}_{\gamma_n}^{(1)} \) given by Eqs. (78) and (81) correspondingly. In the case when \( V^{(1)}(x, y) \) contains the non-diagonal terms, the solution of the evolution equation will differ from \( \phi_n^{(\text{diag})} \) by \( \mathcal{O}(\alpha) \) terms. Therefore, we will look for the solution in the following form
\[ \phi_n = \left( 1 + \frac{\alpha}{4\pi} W \right) \otimes \phi_n^{(\text{diag})}, \]  
(152)

Consider first a more simple case \( b_0 = 0 \). Substituting (152) into the evolution equation and using the explicit form of the total kernel (139) one can find, that in this case \( W = W(x, y) \) should satisfy the following equation
\[ [v_0, W]_+ + \dot{v}_0 \otimes (\gamma_\psi - v_0) = 0. \]  
(153)

This equation could be easily solved noticing, that it is similar to eigenvalue equation for the \( V_{\nu}^{(0)} \) kernel written up to \( \mathcal{O}(\nu) \) terms
\[ [v_0, \omega]_+ + \dot{v}_0 = 0, \]  
(154)

where \( \omega \) stands for the generator of \( \nu \)-shifts:
\[ \Psi_{\nu,n}^{(1,0)} \equiv \Psi_n^{(1,0)} + \nu \frac{d}{d\nu} \Psi_n^{(1,0)} \bigg|_{\nu=0} + \mathcal{O}(\nu^2) = (1 + \nu \omega) \otimes \Psi_n^{(1,0)} + \mathcal{O}(\nu^2) \]  
(155)

As a result, we have the following expression for \( W \):
\[ W = -\omega \otimes (v_0 - \gamma_\psi) \]  
(156)

and as a consequence we have
\[ \phi_n = \left( 1 + \frac{\alpha}{4\pi} W \right) \otimes \Psi_n^{(1,0)} = \left( 1 - \frac{\alpha}{4\pi} (\lambda_n - \gamma_\psi) \right) \otimes \Psi_n^{(1,0)} \]  
(157)

Then, the multiplicatively renormalized operators in this \( (b_0 = 0) \) case could be written through the coefficients \( \tilde{d}_{kn} \) of the expansion (144):
\[ O_n = \mathcal{O}_n + \frac{\alpha}{4\pi} \sum_{n>k} \tilde{d}_{kn} \gamma_k^{(0)} \mathcal{O}_n \]  
(158)
In the $b_0 \neq 0$ case the equation for the rotation kernel $W$ has even more complicated structure than Eq. (153)
\[ b_0 W + [v_0, W]_+ + \dot{v}_0 \otimes (b_0 + \gamma_\psi - v_0) = 0. \] (159)

The effective method to solve this equation is to use matrix representation for the kernels in the Jacobi polynomial basis. It then follows, that $W \otimes \Psi_n^{(1,0)}$ has the following form:
\[ W \otimes \Psi_n^{(1,0)} = (b_0 + \gamma_\Phi - \lambda_n) (\lambda_n - v_0 - b_0)^{-1} \otimes \dot{v}_0 \otimes \Psi_n^{(1,0)} \] (160)

Now using the expression for the dot kernel in the Jacobi polynomial basis derived previously, we get
\[ W \otimes \Psi_n^{(1,0)} = \left( b_0 + \text{fer}_n^{(0)} \right) \left( \bar{P}_n \Psi_n^{(1,0)} + b_0 \sum_{k>n} \frac{\hat{d}_{kn}}{\text{fer}_k^{(0)} - \text{fer}_n^{(0)} - b_0} \Psi_k^{(1,0)} \right), \] (161)

where $\bar{P}_n$ is the projection operator $\bar{P}_n \Psi_n^{(1,0)} = (1 - \delta_{nk}) \Psi_n^{(1,0)}$ subtracting the diagonal part. The multiplicatively renormalized operators in this case are given by
\[ O_n = O_n + \frac{\alpha}{4\pi} (b_0 + \text{fer}_n^{(0)}) \sum_{n>k} \hat{d}_{kn} \frac{\text{fer}_k^{(0)} - \text{fer}_n^{(0)}}{\text{fer}_k^{(0)} - \text{fer}_n^{(0)} - b_0} O_k. \] (162)

6 Conclusion

It follows from our analysis, that to find multiplicatively renormalized twist-2 conformal operators in Wess-Zumino model it is sufficient to find multiplicatively renormalized operators only for one member of operator supermultiplet. It is convenient in this case to find a solution for fermionic operator. For the latter the problem of finding multiplicatively renormalized operators is much simpler compared to singlet bosonic operators, where we have operator mixing. All other multiplicatively renormalized operators could be then obtained from fermionic operator with the use of supersymmetry transformations. Moreover, we found, that the knowledge of fermionic diagonal and non-diagonal anomalous dimensions matrices allows us completely reconstruct the forward anomalous dimensions matrix in singlet case. Also, the analysis was performed in a simplified supersymmetric Wess-Zumino model, we think that most of our findings could be straightforwardly applied to field theories with more supersymmetries.

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Here we give the diagram by diagram results. The whole calculation was automated with the help of Feynman diagram analyzer DIANA [23], calling QGRAF [24] for the Feynman diagram generation, and computer algebra system FORM [25].

The results for each diagram include contributions of this diagram to forward anomalous dimension \( \gamma_m \), to forward evolution kernel \( P(z) \) and to non-forward evolution kernel \( V(x, y) \). Here and below, the contributions of diagrams marked by star ‘\(^\ast\)’ change the overall sign in polarized case. The tadpole diagrams (similar to \([d]\)) give zero contribution in forward case and are proportional to constant in non-forward case. So, in what follows we do not write explicit expressions for contributions of these diagrams. Also, graphs containing external self-energies, giving \( \delta \)-function contribution to evolution kernels, were taken into account in calculation, but not shown explicitly here.
Figure 1: One-loop diagrams

|   | \( \frac{2}{m+1} - \frac{2}{m+2} \) |
|---|----------------------------------|
| 1.a * | \( 2z \) |
| 1.a * | \( \theta(x < y) \frac{2}{y} + (x \leftrightarrow \bar{x}, \ y \leftrightarrow \bar{y}) \) |
| 1.b * | \( \frac{2}{m+1} \) |
| 1.b * | \( 2 \) |
| 1.b * | \( \theta(x < y) 2x - (x \leftrightarrow \bar{x}, \ y \leftrightarrow \bar{y}) \) |
| 1.c * | \( \frac{2}{m+2} \) |
| 1.c * | \( 2z \) |
| 1.c * | \( \theta(x < y) \frac{2}{y} - (x \leftrightarrow \bar{x}, \ y \leftrightarrow \bar{y}) \) |
| 1.d | 0 |
| 1.d | 0 |
| 1.d | \( \theta(x < y) + (x \leftrightarrow \bar{x}, \ y \leftrightarrow \bar{y}) \) |
| 1.e * | \( (-1)^{m+1} \frac{2}{m+1} \) |
| 1.e * | \( 2z \) |
| 1.e * | \( \theta(x < \bar{y}) \frac{2}{y} \) |
Figure 2: Two-loop diagrams for quark operator Eq. (16) sandwiched between quark states

|   |   |   |
|---|---|---|
| 2.a | $-\frac{2}{(m+1)^3} + \frac{4}{(m+1)^2} - \frac{6}{m+1} - \frac{2}{(m+2)^3} + \frac{6}{m+2}$ | $-6\bar{z} - 4\ln(z) - (1 + z)\ln^2(z)$ |
|   | $\theta(x < y) \left\{ \frac{4x}{y} + 2\frac{\bar{x}}{y} \ln(\bar{x}) + 2\frac{x}{y} \ln(y) + \frac{x}{y} \ln^2(\bar{x}) + 2\frac{x}{y} \ln(x) \ln(y) - \frac{x}{y} \ln^2(y) \right\} + \left( \begin{array}{c} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{y} \end{array} \right)$ |   |
| 2.b* | $-\frac{2}{(m+1)^2} + \frac{6}{m+1} - \frac{2}{(m+2)^2} - \frac{6}{m+2}$ | $6\bar{z} + 2(1 + z)\ln(z)$ |
|   | $\theta(x < y) \left\{ -4\frac{x}{y} - 2\frac{\bar{x}}{y} \ln(\bar{x}) - 2\frac{x}{y} \ln(y) \right\} + \left( \begin{array}{c} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{y} \end{array} \right)$ |   |
| 2.c | $\frac{4}{m+1} - \frac{4}{m+2} - 2\frac{S_1(m+1)}{m+1} + 2\frac{S_1(m+2)}{m+2}$ | $2\bar{z} (2 + \ln(z))$ |
|   | $\theta(x < y) \left\{ -4\frac{x}{y} - \frac{\bar{x}}{y} \ln(\bar{x}) - \frac{x}{y} \ln(x) + \frac{1}{y} \left( 1 - \frac{x}{y} \right) \ln \left( 1 - \frac{x}{y} \right) \right\} + \left( \begin{array}{c} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{y} \end{array} \right)$ |   |
| 2.d* | $-\frac{1}{(m+1)^2} + \frac{3}{m+1} + \frac{1}{(m+2)^2} - \frac{3}{m+2}$ | $\bar{z} (3 + \ln(z))$ |
|   | $\theta(x < y) \frac{x}{y} \left\{ -2 - \ln \left( \frac{x}{y} \right) \right\} + \left( \begin{array}{c} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{y} \end{array} \right)$ |   |
Figure 3: Two-loop diagrams for quark operator Eq. (16) sandwiched between scalar states

|   | 3.a | 3.b* | 3.c* |
|---|-----|------|------|
|   | \[-\frac{2}{(m+1)^3} + \frac{2}{(m+1)^2} - \frac{2}{(m+2)^2} \right) \frac{m+1}{2} \] | \[\frac{4}{m+1} - \frac{2S_1(m+1)}{m+1} \right) \frac{m+1}{2} \] | \[-\frac{1}{(m+1)^2} + \frac{3}{m+1} - \frac{1}{m+2} \right) \frac{m+1}{2} \] |
|   | \[(-2 \ln(z) - \ln(z)) \ln(z) \] | \[2 (2 + \ln(\bar{z})) \] | \[3 - z + \ln(z) \] |
|   | \[\theta(x < y) \left\{ 2 \ln(\bar{x}) + \bar{x} \ln^2(\bar{x}) - 2x \ln(x) \ln(y) + x \ln^2(y) \right\} \] | \[\bar{x} \left\{ 2 + \ln(\bar{x}) \right\} - \theta(y > x) \left\{ 2 + \ln \left( 1 - \frac{x}{y} \right) \right\} \] | \[\theta(y > x) x \left\{ -2 - \ln \left( \frac{x}{y} \right) \right\} \] |
|   | \[\frac{m+1}{2} \] | \[\frac{m+1}{2} \] | \[\frac{m+1}{2} \] |

\( S_1 = \frac{2}{m+1} \)
Figure 4: Two-loop diagrams for scalar operator Eq. (18) sandwiched between quark states

|   | Equation                                                                 |
|---|-------------------------------------------------------------------------|
| 4.a | $\left(-\frac{2}{(m+1)^2} + \frac{2}{(m+2)^3} + \frac{2}{(m+2)^2}\right) \frac{2}{m+1}$ |
|    | $z \ln^2(z) + 2\bar{z} \ln(z)$                                          |
|    | $\theta(x < y) \left\{ \frac{1}{y} \ln^2(\bar{x}) - \frac{2}{\bar{y}} \ln(x) \ln(y) + \frac{1}{\bar{y}} \ln^2(y) \right\} - \left( x \leftrightarrow \bar{x} \right)$ |
| 4.b*| $\left( \frac{4}{m+2} - \frac{2S_1(m+2)}{m+2} \right) \frac{2}{m+1}$     |
|    | $2z \left( 2 + \ln(\bar{z}) \right)$                                    |
|    | $-\frac{1}{\bar{y}} \left\{ 2 + \ln(\bar{x}) \right\} + \theta(y > x) \frac{1}{yy} \left\{ 2 + \ln \left( 1 - \frac{x}{y} \right) \right\} - \left( x \leftrightarrow \bar{x} \right)$ |
| 4.c*| $\left( -\frac{1}{m+1} + \frac{1}{(m+2)^2} - \frac{3}{m+2} \right) \frac{2}{m+2}$ |
|    | $-1 + 3z + z \ln(z)$                                                   |
|    | $\theta(y > x) \frac{1}{y} \left\{ 2 + \ln \left( \frac{x}{y} \right) \right\} - \left( x \leftrightarrow \bar{x} \right)$ |
Figure 5: Two-loop diagrams for scalar operator Eq. (18) sandwiched between scalar states

|   | \(-\frac{2}{m + 2} + \frac{2}{(m + 2)^2} + \frac{2}{m + 1} - \frac{2}{(m + 1)^2}\) |
|---|---|
| 5.a | \(2\tilde{z}(1 + \ln(z))\) |
|     | \(\theta(x < y) \left\{ -4 - 2 \ln\left(\frac{x}{y}\right) + \ln^2(\bar{x}) + 2 \ln(x) \ln(y) - \ln^2(y) \right\} + \left( \begin{array}{c} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{y} \end{array} \right)\) |

|   | \(\frac{3}{m + 2} - \frac{3}{m + 1}\) |
|---|---|
| 5.b | \(-3\tilde{z}\) |
|     | \(\theta(x < y) \left\{ 6 + 3 \ln\left(\frac{x}{y}\right) \right\} + \left( \begin{array}{c} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{y} \end{array} \right)\) |

Figure 6: Two-loop diagrams for operator Eq. (21) with fermion quantum numbers
|   |   |
|---|---|
| 6.a | $\frac{2}{(m+1)^3} - \frac{2}{(m+1)^2}$ |
|   | $(2 + \ln(z)) \ln(z)$ |
|   | $\theta(x < y) \left\{ 2 \frac{\ln(x) \ln(y)}{\bar{y}} + 2 \frac{\ln(y)}{\bar{y}} - \frac{\ln^2(y)}{\bar{y}} \right\} + \theta(y < x) \left\{ 2 \frac{\ln(x)}{\bar{y}} + \frac{\ln^2(x)}{\bar{y}} \right\}$ |
| 6.b | $(-1)^{m+1} \left( -\frac{2}{m+1} + \frac{S_1(m+1)}{m+1} \right)$ |
|   | $-2 - \ln(\bar{z})$ |
|   | $\theta(x < \bar{y}) \frac{2}{\bar{y}} - \theta(x > \bar{y}) \frac{1}{\bar{y}} \ln \left( 1 - \frac{\bar{x}}{\bar{y}} \right) + \frac{1}{\bar{y}} \ln(x)$ |
| 6.c | $(-1)^{m+1} \left( \frac{1}{(m+1)^2} - \frac{2}{m+1} \right)$ |
|   | $-2 - \ln(\bar{z})$ |
|   | $\theta(x < \bar{y}) \left\{ \frac{2}{\bar{y}} + \frac{1}{\bar{y}} \ln \left( \frac{x}{\bar{y}} \right) \right\}$ |
| 6.d | $(-1)^{m+1} \left( -\frac{2}{m+1} + \frac{S_1(m+1)}{m+1} \right)$ |
|   | $-2 - \ln(\bar{z})$ |
|   | $\theta(x < \bar{y}) \left\{ \frac{2}{\bar{y}} + \frac{1}{\bar{y}} \ln \left( 1 - \frac{x}{\bar{y}} \right) \right\}$ |
| 6.e | $\frac{2}{(m+1)^2}$ |
|   | $-2 \ln(\bar{z})$ |
|   | $-\theta(x < y) \frac{2}{y} \ln(y) - \theta(y < x) \frac{2}{\bar{y}} \ln(x)$ |
Appendix B

Here we present details on the evaluation of momentum integrals present in the calculation of evolution kernels of ER-BL equations. Our consideration follows the original work of Mikhailov and Radyushkin [15].

The main complication of the integrals involved compared to usual propagator-type integrals (for which a lot of powerful technics were developed over the past years) is the presence of $\delta$-function related to effective vertex. To derive all the formula we used $\alpha$-presentation for momentum integrals as was originally proposed in [15]. For propagators and $\delta$-function insertion we have:

$$\frac{1}{(k^2 + i\varepsilon)^\nu} = \left(-i\right)^\nu \frac{1}{\Gamma(\nu)} \int_0^\infty d\alpha \alpha^{\nu-1} \exp \left\{ i\alpha (k^2 + i\varepsilon) \right\}$$

(163)

$$\delta \left( x - \frac{k \cdot n}{P \cdot n} \right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\alpha \exp \left\{ i\alpha \left( x - \frac{k \cdot n}{P \cdot n} \right) \right\}$$

(164)

In what follows we used the same integration measure convention as used by [15], that is we use $\overline{MS}$ prescription. Technically, it could be accomplished by multiplying each integration $d^d k$ by additional factor $M(\varepsilon) = (4\pi)^{-d} \Gamma(1 - \varepsilon)$. At one-loop level all integrals obtained after projecting out Dirac structure of the conformal operator under consideration and expanding scalar products of momenta in the numerator over the denominator factors could be reduced to the following two most general integrals

$$\int \frac{d^d k}{(2\pi)^d \left\{ [(k - aP)^2 - m_1^2]^{n_1} [(k - bP)^2 - m_2^2]^{n_2} \right\}} \frac{\delta \left( 1 - \frac{n_k}{n_P} \right)}{(2\pi)^d \Gamma(n_1) \Gamma(n_2)} \int_1 \frac{d\alpha \alpha^{n_1-1} (1 - \alpha)^{n_2-1} \delta \left( x - a\alpha - b(1 - \alpha) \right) A^{d-n_1-n_2}}{(n_1 + n_2 - \frac{d}{2})}$$

(165)

and

$$\int \frac{d^d k}{(2\pi)^d \left\{ [(k - aP)^2 - m_1^2]^{n_1} [(k - bP)^2 - m_2^2]^{n_2} (k - cP)^2 - m_3^2 \right\}} \frac{\delta \left( 1 - \frac{n_k}{n_P} \right)}{(2\pi)^d \Gamma(n_1) \Gamma(n_2) \Gamma(n_3)} \int_1 \frac{d\alpha \alpha^{n_1-1} (1 - \alpha)^{n_2-1} \beta_3^{n_3-1} \delta \left( x - a\beta_2 - b\beta_3 \right) B^{d-n_1-n_2-n_3}}{(n_1 + n_2 + n_3 - \frac{d}{2})} \times \int_0 \left\{ d\beta_2 d\beta_3 \right\} (1 - \beta_2 - \beta_3)^{n_1-1} \beta_2^{n_2-1} \beta_3^{n_3-1} \delta \left( x - a\beta_2 - b\beta_3 \right)$$

(166)

where the following notation is introduced: $d = 4 - 2\varepsilon$, $\left\{ d\beta_2 d\beta_3 \right\} = \theta(1 - \beta_2 - \beta_3) d\beta_2 d\beta_3$ and

$$A = -\alpha m_1^2 - (1 - \alpha) m_2^2$$

(167)

$$B = -\beta_2 m_2^2 - \beta_3 m_3^2 - (1 - \beta_2 - \beta_3) m_1^2$$

(168)

The masses $m_1^2, m_2^2, m_3^2$ provide infrared regularization in the cases where it is necessary.

On the other hand these expressions for one-loop integrals could be used as building blocks to obtain expressions for necessary two-loop integrals. Here we would like to note that evolution kernels,

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3It should be noted that expressions for $J(z)$ and $S(z)$ functions (to be introduced below) have misprints in [15]. We thank Sergei Mikhailov for fruitful discussions on this subject and performed crosschecks.
in which we are interested, are being obtained as a result of application $KR'$-operation to diagrams describing renormalization of effective vertices corresponding to multiparticle distribution functions. So, in what follows we give answers for all necessary two-loop integrals before and after application of $KR'$-operation. Note, that it would be incorrect to use just $KR'$-subtracted scalar integrals as extra $\varepsilon$ terms could follow from Dirac algebra for $\gamma$-matrices and procedure used for subtracting subdivergences provided by $R'$-operation should take them into account.

To perform the calculation described in this paper the following general formulas for two-loop scalar integrals are sufficient

$$V(b|c,d|1) = -(4\pi)^4 \int \frac{1}{(l-cP)^2(l-dP)^2} \int \frac{\delta(x - \frac{k_n}{P \cdot n})}{(k-l)^2(k-bP)^2} d^d \vec{k} \frac{d^d \vec{k}}{2^d}$$ (169)

Before $KR'$-operation we have

$$V(b|c,d|1) = \frac{1}{2\varepsilon y_2} \left\{ \theta(-x_1)\theta(x_1 > y_1) \bar{J}\left(\frac{x_1}{y_1}\right) - \theta(-x_1)\theta(x_1 > y_1 + y_2) \bar{J}\left(\frac{x_1}{y_1 + y_2}\right) + \theta(x_1)\theta(x_1 < y_1 + y_2) \bar{J}\left(\frac{x_1}{y_1 + y_2}\right) - \theta(x_1)\theta(x_1 < y_1) \bar{J}\left(\frac{x_1}{y_1}\right) \right\}$$ (170)

Here we have introduced the notation

$$d^d \vec{k} = M(\varepsilon) d^d k, \quad x_1 = x - b, \quad y_1 = d - b, \quad y_2 = c - d > 0$$

$$\bar{J}(z) = -\frac{1}{\varepsilon} \ln z + \frac{1}{2} \ln^2 z, \quad \bar{S}(z) = \bar{J}(z) - \frac{1}{\varepsilon} \ln z$$

$$J(z) = \frac{1}{\varepsilon} \ln z + \frac{1}{2} \ln^2 z, \quad S(z) = \frac{1}{\varepsilon} \ln z$$

The result for $V(b|c,d|1)$ after $KR'$-operation could be obtained from the result without $KR'$-operation performed with the substitution $\bar{J}(z) \rightarrow J(z)$.

$$V\left(b|c,d\left|\frac{l \cdot n}{P \cdot n}\right\right) = -(4\pi)^4 \int \frac{l \cdot n/P \cdot n}{(l-cP)^2(l-dP)^2} \int \frac{\delta(x - \frac{k_n}{P \cdot n})}{(k-l)^2(k-bP)^2} d^d \vec{k} \frac{d^d \vec{k}}{2^d}$$ (171)

Before $KR'$-operation we have

$$V\left(b|c,d\left|\frac{l \cdot n}{P \cdot n}\right\right) = \frac{1}{2\varepsilon y_2} \left\{ \theta(x_1 > y_1)\theta(-x_1) \left[ x_1 \bar{S}\left(\frac{x_1}{y_1}\right) + b\bar{J}\left(\frac{x_1}{y_1}\right) \right] - \theta(x_1 > y_1 + y_2)\theta(-x_1) \left[ x_1 \bar{S}\left(\frac{x_1}{y_1 + y_2}\right) + b\bar{J}\left(\frac{x_1}{y_1 + y_2}\right) \right] + \theta(x_1 < y_1 + y_2)\theta(x_1) \left[ x_1 \bar{S}\left(\frac{x_1}{y_1 + y_2}\right) + b\bar{J}\left(\frac{x_1}{y_1 + y_2}\right) \right] - \theta(x_1 < y_1)\theta(x_1) \left[ x_1 \bar{S}\left(\frac{x_1}{y_1}\right) + b\bar{J}\left(\frac{x_1}{y_1}\right) \right] \right\}$$ (172)
The result for $V(b|c,d|\frac{ln}{P^\mu})$ after $KR'$-operation could be obtained from the result without $KR'$-operation performed with the substitutions: $\tilde{J}(z) \rightarrow J(z)$ and $S(z) \rightarrow \tilde{S}(z)$

$$W \left( a, b \big| c \bigg| \left\{ \frac{1}{(l \cdot n/P \cdot n)} \right\} \right) = -(4\pi)^4 \int \frac{\delta(x - \frac{k \cdot n}{P \cdot n})}{(k - aP)^2(k - bP)^2} \times \int \frac{1}{(l - k)^2(l - cP)^2} \left\{ \frac{2}{(l \cdot n/P \cdot n)} \right\} \frac{d^dkd\bar{d}}{(2\pi)^{2d}}. \quad (173)$$

Before $KR'$-operation we have

$$W \left( a, b \big| c \bigg| \left\{ \frac{1}{(l \cdot n/P \cdot n)} \right\} \right)$$

$$a \neq b = \frac{1}{2\varepsilon} \left\{ \frac{1}{2}(x_1 + y + 1) + b \right\} \frac{1}{y_1 + y_2} \left\{ \theta(x_1)\theta(y_1 + y_2 > x_1) \left[ -\frac{1}{\varepsilon} + 2 \right] + \theta(x_1)\theta(y_1 > x_1) \ln \left( \frac{x_1}{y_1} \right) - \theta(x_1)\theta(y_1 > x_1) \theta(x_1 > y + y_2) \ln \left( \frac{y_1 + y_2 - x_1}{y_1 + y_2 - y_1} \right) \right\}$$

$$a - b \neq c = \frac{1}{2\varepsilon} \left\{ \frac{1}{2}(x_1 + y_1 + b) \right\} \left( \theta(-x_1)\theta(x_1 > y_1) + \theta(x_1)\theta(y_1 > x_1) \right) \left( \frac{1}{y_1} + \frac{1}{x_1} \right), \quad (174)$$

where $x_1 = x - b$, $y_1 = c - b$, $y_2 = a - c$, $y_1 + y_2 = a - b \geq 0$, $x > 0$. After $KR'$-operation was performed the integral above takes the following form

$$W \left( a, b \big| c \bigg| \left\{ \frac{1}{(l \cdot n/P \cdot n)} \right\} \right)$$

$$a \neq b = \frac{1}{2\varepsilon} \left\{ \frac{1}{2}(x_1 + y + 1) + b \right\} \frac{1}{y_1 + y_2} \left\{ \theta(x_1)\theta(y_1 + y_2 > x_1) \left[ -\frac{1}{\varepsilon} + 2 \right] + \theta(x_1)\theta(y_1 > x_1) \ln \left( \frac{x_1}{y_1} \right) - \theta(x_1)\theta(y_1 > x_1) \theta(x_1 > y + y_2) \ln \left( \frac{y_1 + y_2 - x_1}{y_1 + y_2 - y_1} \right) \right\}$$

$$a - b \neq c = \frac{1}{2\varepsilon} \left\{ \frac{1}{2}(x_1 + y_1 + b) \right\} \left( \theta(-x_1)\theta(x_1 > y_1) + \theta(x_1)\theta(y_1 > x_1) \right) \left( \frac{1}{y_1} + \frac{1}{x_1} \right). \quad (175)$$
Appendix C

Here, we following Refs. [17, 18], present the details for the derivation of non-diagonal part of anomalous dimensions matrix for fermionic operator, based on the analysis of conformal Ward identity for Green functions with fermionic operator insertion. In what follows, we will consider somewhat more general fermionic operator, then one considered in the main body of the paper:

\[ O_{\text{fer}}^{j,l} = \bar{\psi}(i\partial_{+})^{l}P_{j}^{(\alpha,\beta)} \left( \frac{D_{+}}{\partial_{+}} \right) \phi. \]  \hspace{1cm} (176)

Even so, this operator will be highest vector in corresponding conformal representation only for \( \alpha = 1 \) and \( \beta = 0 \), some of the formulas given here will certainly have applications in other supersymmetric models with more rich field content. To start with, let us remained reader some basic facts about conformal symmetry group.

Conformal group is defined as the most general group leaving invariant light-cone in \( n \)-dimensional Minkowski space. The algebra of this group contains, in addition to generators of Poincare group: \( P_{\mu} \) and \( M_{\mu\nu} \), the generator of dilations \( D \) and \( n \) generators \( K_{\mu} \) of special conformal transformations. The latter generators satisfy the following commutation relations [26]:

\[
[D, K_{\mu}] = iK_{\mu}, \quad [K_{\mu}, P_{\nu}] = -2i(g_{\mu\nu}D + M_{\mu\nu}), \quad [D, P_{\mu}] = -iP_{\mu}, \quad [K_{\rho}, M_{\mu\nu}] = i(g_{\rho\mu}K_{\nu} - g_{\rho\nu}K_{\mu}), \quad [K_{\mu}, K_{\nu}] = 0, \quad [D, M_{\mu\nu}] = 0. \]  \hspace{1cm} (177)

Here we will work only with the subalgebra of conformal algebra, the so called collinear conformal algebra, isomorphic to \( SU(1,1) \cong SO(2,1) \). The representations of collinear conformal algebra could be obtained from those of full conformal algebra via a projection to the light cone. The necessary projection itself could be build with the help of two light-cone vectors, \( n \) and \( n^* \) with the properties: \( n^2 = n^*2 = 0 \) and \( n \cdot n^* = 1 \). The generators of collinear conformal group obtained have the following expressions:

\[
K_- = n^*_\mu K^\mu, \quad D, \quad P_+ = n_\mu P^\mu, \quad M_{-+} = n^*_\mu M^{\mu\nu}n_\nu. \]  \hspace{1cm} (178)

In what follows, we will also need the transformation laws of fermion and scalar fields under special conformal transformations:

\[
K_- \phi = i(2x_-(d_\phi + x \cdot \partial) - x^2 \partial_-) \phi, \quad K_- \psi = i(2x_-(d_\psi + x \cdot \partial) + 2\Sigma_{-\nu}x^\nu - x^2 \partial_-) \psi \]  \hspace{1cm} (179)

where \( d_\phi \) and \( d_\psi \) are scale dimensions for the scalar and fermion fields correspondingly. \( \Sigma_{-\nu} \) is the spin operator:

\[
\Sigma_{\mu\nu} \psi = \frac{1}{4} [\gamma_\mu, \gamma_\nu] \psi. \]  \hspace{1cm} (180)

The fermion conformal operators, we are interested in, form an infinite dimensional representation of collinear group. The members of each series are labeled by their spin \( l \). The generators \( D \) and \( M_{-+} \) are diagonal with respect to representations, whereas \( P_+ \) acts as a raising operator and \( K_- \) as a lowering operator. The action of \( K_- \) on \( O_{j,l}^{\text{fer}} \) is

\[
K_- O_{j,l}^{\text{fer}} = \int d^4x \left\{ (K_- \phi(x)) \frac{\delta}{\delta \phi(x)} + (K_- \bar{\psi}(x)) \frac{\delta}{\delta \bar{\psi}(x)} \right\} O_{j,l}^{\text{fer}}. \]  \hspace{1cm} (181)
Now, it is an easy exercise to find that
\[ K_j O_{j,l}^{\text{fer}} = a(j, l, \alpha, \beta) O_{j,l-1}^{\text{fer}}, \] (182)
where
\[ a(j, l, \alpha, \beta) = 2(j - l)(j + l + \alpha + \beta + 1). \] (183)

To derive the conformal Ward identity itself we start with generating functional for renormalized disconnected Green functions with operator insertions
\[ Z_{j,l}(\bar{\eta}, \eta, J) = \frac{1}{N} \int D\Phi \exp \left\{ i[S] + i \int d^d x [\bar{\eta}(x)\psi(x) + \bar{\psi}(x)\eta(x) + J(x)\phi(x)] \right\}, \] (184)
where \([O_{j,l}^{\text{fer}}]\) and \([S]\) denote corresponding renormalized quantities. Using the invariance of the generating functional with respect to special conformal transformations and performing differentiation over field sources, one gets the following conformal Ward identity
\[ \langle [O_{j,l}^{\text{fer}}](\delta K \chi) \rangle = -\langle [O_{j,l}^{\text{fer}}](\delta K [S]) \chi \rangle - \langle [O_{j,l}^{\text{fer}}](\delta K [S]) \chi \rangle, \] (185)
where \(\langle A \rangle\) stands for vacuum averaging of the time ordered product \(TA exp(i[S])\) and \(\chi = \Pi_i \phi_i\) denotes product of elementary fields taken at different space-time points. Note, that the left-hand side of Eq. (185) is finite, so the right-hand side also should be finite. The first term at right-hand side is a variation of the renormalized fermion operator under special conformal transformations. It is given by
\[ \delta^K [O_{j,l}^{\text{fer}}] = i \sum_{k=0}^{j} \left\{ \hat{Z} \hat{A}(l, \varepsilon) \hat{Z}^{-1} \right\}_{j,k} [O_{k,l-1}^{\text{fer}}], \] (186)
where we introduced the following notation \(^4\)
\[ [O_{j,l}^{\text{fer}}] = \sum_{k=0}^{j} Z_{j,k}(\varepsilon, g) O_{k,l}^{\text{fer}}, \quad k \leq l, \] (187)
and\(^5\)
\[ \hat{A}(l, \varepsilon) = \hat{a}(l) + 2(\gamma \Phi + \varepsilon)\hat{b}(l), \] (188)
\[ \hat{a}(l) = \{ a(j, l, \alpha, \beta) \delta_{jk} \}, \] (189)
\[ \hat{b}(l) = 2 \int_{-1}^{1} dx w(x|\alpha, \beta) P^{(\alpha, \beta)}_{k}(x) \hat{\mathcal{L}} P^{(\alpha, \beta)}_{j}(x), \] (190)
with
\[ \hat{\mathcal{L}} = l - x \frac{d}{dx}, \quad w(x|\alpha, \beta) = (1 - x)^{\alpha}(1 + x)^{\beta}, \]
\[ \hat{\mathcal{N}}_{k}(\alpha, \beta) = 2^{\alpha+\beta+1} \frac{\Gamma(k + \alpha + 1)\Gamma(k + \beta + 1)}{(2k + \alpha + \beta + 1)\Gamma(k + 1)\Gamma(k + \alpha + \beta + 1)}. \] (191)

\(^4\)The renormalization of conformal operators does not depend on \(l\) due to Lorentz invariance and the matrix \(Z_{jk}\) is triangular.
\(^5\)Note, that here we used the fact that anomalous dimensions of scalar and fermion fields are equal, as they belong to the same supermultiplet.
The evaluation of integral in the definition of matrix $\hat{b}(l)$ gives

$$b_{jk}(l) = 2(l - k)\delta_{jk} - \frac{\theta(j > k)}{(2k + \alpha + \beta + 1)\Gamma(k + \alpha + \beta + 1)} \left\{ (-1)^{j-k} \frac{\Gamma(j + \alpha + 1)}{\Gamma(k + \alpha + 1)} + \frac{\Gamma(j + \beta + 1)}{\Gamma(k + \beta + 1)} \right\}.$$  

The second term on the right-hand side of Eq. (185) is the product of two renormalized operators: $[O^\text{fer}_{j,l}]$ and the variation of renormalized action. For the latter we have the following expression

$$i\delta^K[S] = \frac{\beta(\varepsilon, g)}{g}[\Delta_-] \quad \text{with} \quad \Delta_- = g \frac{\partial}{\partial g} \int d^d x 2 x_\mu \mathcal{L}(x), \quad \beta(\varepsilon, g) = -\varepsilon g + \beta(g). \quad (192)$$

As it is known, the product of two renormalized operators may still contain ultraviolet divergences, so an additional subtraction of divergences is necessary

$$[O^\text{fer}_{j,l} \Delta_-] = [O^\text{fer}_{j,l}][\Delta_-] - i \sum_{k=0}^{j} Z_{jk}(l)[O^\text{fer}_{k,l-1}]. \quad (193)$$

Substituting Eqs. (180), (192) and (193) into conformal Ward identity and taking limit $\varepsilon \to 0$, one gets

$$\langle [O^\text{fer}_{j,l}] \delta^K \chi \rangle = i \sum_{k=0}^{j} \{ \hat{a}(l) + \hat{\gamma}(g, l) \}_{jk} \langle [O^\text{fer}_{k,l}] \chi \rangle - \frac{\beta}{g} \langle [O^\text{fer}_{j,l} \Delta_-] \rangle. \quad (194)$$

Here the definition for the matrix of special conformal anomalies was introduced

$$\hat{\gamma}(g, l) = \lim_{\varepsilon \to 0} \left( \hat{Z} A \hat{Z}^{-1} - \hat{a} - \frac{\beta}{g} \hat{Z}^* \right). \quad (195)$$

It is easy to derive, that in Wess-Zumino model we are considering here $\hat{Z}^{[1]}(l)$ (the coefficient of simple pole in $\varepsilon$) is given by

$$\hat{Z}^{[1]}(l) = g^2 \hat{\gamma}(1) \hat{b}(l) + \mathcal{O}(g^2) \quad (196)$$

and as a consequence we have $\hat{\gamma}(g, l) = \hat{b}(l)(\hat{\gamma}(1) + 2 \gamma^{(1)}_{\phi} \hat{1})$ with $\hat{1} = \{ \delta_{jk} \}$.

Now, the crucial point is to note, that the scale and special conformal anomalies are in fact related. The relation between matrices of scale and special conformal anomalies could be obtained by considering the action of the commutator of special conformal transformation with dilation $[\delta^K, \delta^K] = \delta^K$ on Green function with fermion operator insertion. Taking into account that $\delta^K$ acts on $\langle [O^\text{fer}_{j,l}] \chi \rangle$ as $\mu \frac{\partial}{\partial \mu} + N_\psi \gamma_\psi + N_\phi \gamma_\phi$ ($N_\psi$ and $N_\phi$ are numbers of $\psi$ and $\phi$ fields in $\chi$) one gets the following identity

$$\left[ \hat{a}(l) + \hat{\gamma}(g, l) + \frac{2 \beta(g)}{g} \hat{b}(l), \hat{\gamma}(g) \right] = 0. \quad (197)$$

Since $\hat{a}(l)$ does not depend on coupling constant, solving this equation recursively we see that $\gamma_{j,k}^{(n)}$ in $(n)$-loop order is determined by the $(n - 1)$-loop approximation of $\hat{\gamma}$, $\beta$ and $\hat{\gamma} = \hat{\gamma} + \hat{\gamma}^{ND}$. At two first order we have

$$\gamma_{j,k}^{(0)} = \delta_{jk} \gamma_k^{(D)}, \quad (198)$$

$$[\gamma_{j,k}^{ND}]^{(1)} = \frac{\gamma_{j}^{(0)} - \gamma_{k}^{(0)}}{2(j - k)(j + k + 2)} (\gamma_{j,k}^{(0)} - \beta_0 b_{jk}). \quad (199)$$

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Note, that at this point the finiteness of the right-hand side of CWI is used
