Shallow crustal mechanics from volumetric strain data: Insights from Soufrière Hills Volcano, Montserrat

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Abstract
Volumetric strain data from the 29 July 2008 Vulcanian explosion of the Soufrière Hills Volcano provide an excellent opportunity to probe the mechanical properties of the volcanic edifice and the shallow crust beneath Montserrat. We use Finite Element Analysis to constrain the geometry of the shallow plumbing system as well as edifice and crustal mechanical properties for mechanically plausible pressure drops associated with dilatational volumetric strains recorded during the explosive activity. Our results from both forward and inverse models indicate a conduit radius of ~40 m and a length of ~1500 m, and hence much larger conduit dimensions than previously suggested. In order to fit the syneruptive volumetric strain data for a conduit pressure drop of < 10 MPa, the conduit needs to be surrounded by a halo of mechanically compliant rocks for which the best fit models indicate a width of ~600 m and a Young’s modulus of ~1 GPa. Young’s modulus for the main edifice and the shallow crust up to ~1000 m below sea level are found to be ~10 GPa. Our best fit inverse model predicts a syneruptive radial conduit contraction by 0.24 m with a corresponding volume loss of ~0.1 Mm³, implying only partial emptying of the conduit upon the explosion. This study demonstrates the critical role of edifice and shallow crustal mechanics for strain partitioning on Montserrat. Our findings may have implications for the assessment of conduit processes at other dome-building volcanoes.

1. Introduction
The Soufrière Hills Volcano (SHV; Figure 1) is an andesitic volcano located on the Island of Montserrat in the West Indies. The Island of Montserrat is a compound volcanic edifice peaking at the SHV whose height currently stands approximately 1 km above sea level [Wadge et al., 2014]. Volcanic activity at the SHV since 1995 has so far involved five phases of dome formation interspersed by periods of eruptive pause [Odbert et al., 2014].

A wide range of activity has been documented including periods of dome collapses, pyroclastic flows, and Vulcanian and sub-Plinian explosions [Sparks and Young, 2002]. Key insights on preeruptive and syneruptive processes of Vulcanian explosions as well as on the size and geometry of SHV’s shallow plumbing system have been gained through the analysis of multiparameter geophysical data [Thomas and Neuberg, 2012; Hautmann et al., 2009; Chardot et al., 2010; Hautmann et al., 2010; Linde et al., 2010; Mattioli et al., 2010; Voight et al., 2010; Gottsmann et al., 2011; Odbert et al., 2014].

The emerging geodetic model of SHV’s plumbing system is composed of vertically stacked magma chambers which connect to a dyke conduit complex [Elsworth et al., 2008; Hautmann et al., 2009; Linde et al., 2010; Mattioli et al., 2010; Gottsmann and Odbert, 2014]. The concerted view in the published literature regarding the shallowest part of the system indicates that a 1.2 to 2 km long cylindrical conduit connects to the dyke [Costa et al., 2007b; Hautmann et al., 2009; Linde et al., 2010; Voight et al., 2010]. A typical conduit radius of around 15 m has been inferred from the size of extruded andesitic spines [Voight et al., 1999; Sparks et al., 2000; Melnik and Sparks, 2002] though spines of 25 m radius have also been reported [Wadge et al., 2014]. Volumetric strain data have provided important insights on plumbing system dynamics beneath SHV during explosive activity [Chardot et al., 2010; Linde et al., 2010; Voight et al., 2010; Hautmann et al., 2013, 2014].

Volumetric strains induced by pressure transients in the conduit are thought to be masked when lower sections of the plumbing system are involved in explosions; this has been shown by strain modeling of the various system components of the Soufrière Hills Volcano [Hautmann et al., 2014; Odbert et al., 2014]. Consequently, the conduit dimensions consistently used in the literature have not been tested by strain modeling [Chardot et al., 2010; Linde et al., 2010; Voight et al., 2010; Gottsmann et al., 2011]. Similarly, the
effects of topography or mechanical heterogeneity on the strain signature of explosions have so far not been assessed in detail [Chardot et al., 2010; Linde et al., 2010; Voight et al., 2010; Gottsmann et al., 2011].

Topography has been shown to have a measurable effect on strain models [Cayol and Cornet, 1998], yet it has commonly been neglected in the analysis of near-surface processes at SHV. Many volcano geodetic models applied to explain surface and volumetric strains at SHV ignore mechanical heterogeneity and assume elastic homogeneous isotropic mechanics. This is an oversimplification for two main reasons. First, typical rock strengths vary considerably by type and degree of fracturing [Barnett, 2008; Gudmundsson, 2011]. Fracturing in conduit walls is common, and some evidence suggests that this is the case for the SHV [Widiwijayanti et al., 2005]. Second, layering of mechanically stiff and soft rocks (as is common in stratovolcanoes) has been shown to affect the amplitude of vertical and horizontal surface displacements [Geyer and Gottsmann, 2010; Gudmundsson, 2012] leading, for example, to strain amplification with important implications for deduced source pressure changes [Manconi et al., 2007; Geyer and Gottsmann, 2010]. Vertically extended pressure sources in particular can have their strain signal significantly amplified in the presence of mechanical anisotropies [Folch and Gottsmann, 2006]. Mechanical heterogeneities of the shallow and upper crust beneath Montserrat have been found to significantly influence strain partitioning from the pressurization of middle and upper crustal reservoirs beneath SHV [Hautmann et al., 2010; Gottsmann and Odbert, 2014]. Although it is known from recent seismic and gravimetric data that the crustal architecture of Montserrat is very complex, with structures such as horizontal layering [Byerly et al., 2010], stiff andesitic volcanic cores in the island, and weakened edifice flanks [Paulatto et al., 2010; Shalev et al., 2010; Hautmann et al., 2013], geodetic models to explain volumetric strain signals have yet to include such complexities.

In this paper we focus our mechanical analysis on the 29 July 2008 Vulcanian explosion at SHV. This eruption has been interpreted as having been triggered and fed exclusively by dynamics in the shallowest part of the plumbing system, i.e., the conduit [Chardot et al., 2010; Hautmann et al., 2014]. Extensive strain modeling of the entire SHV plumbing system has isolated and identified the individual signatures of almost every component; conduit signals typically demonstrate a low-amplitude strain change on the order of tens of nS with the same polarity at all sites [Chardot et al., 2010; Hautmann et al., 2014]. This renders it an ideal candidate for a study on the effects of topography and heterogeneity.
Using the syneruptive volumetric strain signature of this explosion, we investigate (1) the effects of topography and mechanical heterogeneity on volumetric strain partitioning in the shallow crust of Montserrat and (2) how volumetric strain data can be used to infer source parameters for shallow conduit mechanics. The aim is to utilize the strain records to inform on the mechanical response of the conduit walls upon decompression, the mechanical properties of the SHV edifice, the amount of conduit closure, preruptive conduit excess pressure, and the wavelength and amplitude of resultant surface deformation.

2. Background on the 29 July 2008 Eruption

The eruption on 29 July 2008 caused plumes up to 12 km high and generated pyroclastic flows by column collapse [Chardot et al., 2010; Voight et al., 2010]. The eruption produced a total estimated volume of between 0.2 to 1.4 Mm³ according to field data [Komorowski et al., 2010; Stewart et al., 2009]. Assuming a conduit length of 2000 m and a cylindrical conduit of 15 m radius, the eruption evacuated either a part of the conduit or the entire conduit. The preeruptive and syneruptive phase of the 29 July explosion was documented in detail by a multiparameter data set including seismic, barometric, dilatometric, infrasonic, and gravimetric observations [Gottsmann et al., 2011]. The eruption was preceded by intense seismicity at 03:32 UTC 29 July 2008, and the first explosion occurred at 03:38 UTC [Chardot et al., 2010], accompanied by a sharp increase in seismicity and acoustic emissions [Gottsmann et al., 2011].

The strain signal showed a 6 min long precursory phase, followed by a near-linear increase during the main eruption [Chardot et al., 2010]. This syneruptive volumetric strain signal (Figure 2) caused the CALIPSO Sacks-Everton borehole strainmeters Air Studios (AIRS) (4.6 km radial distance to dome) and TRNT (6 km radial distance to dome) to expand recording maximum strain amplitudes of ∼31 nS and ∼12 nS at AIRS and TRNT, respectively [Chardot et al., 2010]. Strainmeters Geralds (GERD) and Olveston (OVLS) were not in operation at the time. The signal flattened and then a slow recovery of strain occurred over the next few hours [Chardot et al., 2010; Gottsmann et al., 2011].
For this eruption, both strainmeters recorded positive similar amplitude strains (the dike, for example, typically causes strains on the order of 100 nS at AIRS and TRNT, positive at the former, negative at the latter); therefore, the eruption has been attributed to the explosive depressurization of the conduit only [Chardot et al., 2010; Gottsmann et al., 2011; Hautmann et al., 2014].

3. Methods
3.1. Numerical Modeling and Volumetric Strain Data
We use a finite element solver (COMSOL Multiphysics 5.0) to simulate the decompression of a cylindrical conduit as a proxy of the proposed shallow plumbing system at SHV. Because of the quasi-instantaneous response of the strainmeters to the eruption initiation as recorded by seismic and barometric data shown by Gottsmann et al. [2011], we invoke a linear relationship between crustal stress $\sigma$ and resulting strain $\varepsilon$ and attribute elastic mechanical conditions to the modeling domain via Hooke's law, whereby $\sigma = E \times \varepsilon$ and $E$ is the Young's modulus.

We first explore a series of forward models which have two aims: (1) to test for the effects of topography and heterogeneity and refine the model parameter space and (2) to therefore allow us to define a suitable parameter range of conduit dimensions, pressure drops, and domain material properties. We then invert for these properties using the volumetric strain signals from the eruption.

Both suites of models are based in a 2-D axisymmetric domain as shown in Figure 3 with dimensions of 30 km in the $r$ and $z$ direction. The top of the domain was set to $z = 0$ m. Figure 3 also shows details of the boundary conditions invoked on the modeling domain according to Hickey and Gottsmann [2014]. The model space size and mesh resolution were tested for conversion and found to be adequate. A series of simple numerical
models were benchmarked against analytical models (see supporting information) for code verification. For simplicity, a cylindrical conduit geometry was used such that uniform boundary conditions, e.g., pressure change, could be applied along the conduit wall. Parameters are defined in Table 1, and domain properties for the forward and inverse model suites are defined in Table 2. Volumetric strains at AIRS and TRNT were evaluated by imposing negative pressures on the cylindrical cavity to mimic conduit depressurization and by varying model geometries and material properties. Best fit solutions were found by matching the observed strain signals within their 1σ error (AIRS 31 ± 1 nS, TRNT 12 ± 3 nS). We evaluated the quality of fit to the data with a χ² test.

$$\chi^2 = \frac{(\text{predicted} - \text{observed})^2}{\text{observed}}$$

Best fit models hence have χ² values < 0.032 for AIRS and < 0.75 for TRNT, respectively.

Using the best fit model parameters to match data from AIRS and TRNT, we also forward modeled volumetric strains at 9.4 km distance from the vent to predict the signal at strainmeter GERD. Although data from GERD are not available for the eruption, forward modeling enables the assessment of strain ratios between the three strainmeters and their comparison against signals from other conduit-dominated Vulcanian explosions such as in July 2003 and January 2009 [Chardot et al., 2010; Voight et al., 2010]. In addition, we also model ground displacements for the best fit model parameters at all strainmeter sites (see Figure 1; TRNT, OLVS, GERD, and AIRS).

### 3.2. Crustal Heterogeneity

We adopt a heterogeneous crustal rheological model for those models incorporating heterogeneity. The mechanical properties (density, ρᵣ, and Young’s modulus, E) of crustal rocks for SHV are parameterized using P wave velocity (vₚ) data by Sevilla et al. [2010] for greater depth (> 7 km) and Paulatto et al. [2010] for shallow depths (< 7 km). We use the relationship presented by Brocher [2005] for the ρᵣ (kg/m³) versus vₚ (km/s):

$$\rho_r = 1661.2v_p^2 - 472.1v_p^4 + 67.1v_p^6 - 4.3v_p^8 + 0.106v_p^{10}$$

### Table 1. Model Parameters

| Parameter                  | Symbol | EVR      | Unit |
|----------------------------|--------|----------|------|
| Conduit radius             | rₑ     | 15–50 m  | m    |
| Conduit length             | l      | 1000–2000 m | m    |
| Halo radius                | rₙ     | 30–500 m | m    |
| Young’s modulus            | E      | m.d. see Table 2 | GPa  |
| Poisson’s ratio            | ν      | 0.25 (0.4 for domain 3) | -    |
| Pressure drop              | ΔP     | m.d.     | MPa  |
| Crustal density            | ρᵣ     | m.d.     | kg/m³|

*EVＲ = explored value range; m.d. = model dependent.

### Table 2. Model Parameterization

| Model ID | Topo. | Het. | Domain 1 E (GPa) | Domain 2 E (GPa) | Domain 3 ν | E (GPa) |
|----------|-------|------|------------------|------------------|-------------|---------|
| SBM      | none  | none | 12.5             | NA               | NA          | NA      |
| TBM      | yes   | none | 12.5             | NA               | NA          | NA      |
| HBM      | none  | yes  | equation (4)     | NA               | NA          | NA      |
| CBM      | yes   | yes  | equation (4)     | NA               | NA          | NA      |
| SCAM     | yes   | yes  | equation (4)     | 1–10             | NA          | 15–50   |
| DCAM     | yes   | yes  | equation (4)     | 1–10             | NA          | 15–50   |
| SHAM     | yes   | yes  | equation (4)     | 1–10             | 0.4         | 0.5–1   | 15–50   | 30–500 |
| DHAM     | yes   | yes  | equation (4)     | 1–10             | 0.4         | 0.5–1   | 15–50   | 30–500 |
| ISHAM    | yes   | yes  | equation (4)     | 1–10             | 0.4         | 0.1–5   | 15–50   | 75–500 |
| IDHAM    | yes   | yes  | equation (4)     | 1–10             | 0.4         | 0.1–5   | 15–50   | 75–500 |

*The details of each model setup are shown in terms of invoked topography (Topo.), mechanical heterogeneity (Het.), and geometry (conduit radius rₑ and halo radius rₙ). NA, not applicable.
Young’s modulus versus depth profile for DHAM intersecting all three domains. Domain 1 is as per equations (2)–(4), domain 2 has $E$ values of 1 GPa (green), 5 GPa (red), or 10 GPa (black), and domain 3 was assigned an $E$ value of either 1 GPa or 0.5 GPa (0.5 GPa not shown).

and

$$E = \frac{\nu^2 \rho (1 + \nu)(1 - 2\nu)}{1 - \nu}.$$  \hspace{1cm} (3)

Poisson’s ratio $\nu$ is set at 0.25, consistent with values deduced for Montserrat where the Moho has been imaged at a depth of around 30 km [Sevilla et al., 2010], except for domain 3 where it is set at 0.4 mimicking a very compliant material [Gercek, 2007].

The derived $E$ values were then fitted using a third-order polynomial to obtain a continuous function of $E$ versus $z$ values which then informed the mechanical properties of model domain 1:

$$E = 0.09245 \times 10^{10} - 1.1361 \times 10^7 z - 608.74z^2 - 0.016872z^3,$$  \hspace{1cm} (4)

where $E$ is in pascal and $z$ is in meters.

3.3. Forward Models

We explored a series of forward models whose complexities were incrementally increased (see Table 2). The Basic Model set examined the effect of topography (Topographic Basic Model (TBM)), heterogeneity (Heterogenous Basic Model (HBM)), and a combination of both (Combined Basic Model (CBM)), compared to the standard model widely used in the literature (Standard Basic Model (SBM)). SBM is a single-domain HoE (Homogenous Elastic) model with a flat surface, TBM is a single-domain HoE model with topography, HBM is a single-domain heterogenous elastic (HeE) model, and CBM is a single-domain HeE with topography. The Advanced Model set incorporated more domains with differing material properties. The Shallow Crustal Advanced Model (SCAM) and Deep Crustal Advanced Model (DCAM) are two-domain HeE models including topography. The Shallow Halo Advanced Model (SHAM) and Deep Halo Advanced Model (DHAM) are three-domain HeE models including topography (see Table 2).

The simplified topography invoked in models TBM, CBM, SCAM, DCAM, SHAM, and DHAM extends to $z = 0$ m (taken as the summit elevation of the SHV edifice in 2008) with a gentle slope toward sea level. Rotation during computation results in an even semiconical shape. Although we expect erroneous strain

| Model  | SBM | TBM | HBM | CBM |
|--------|-----|-----|-----|-----|
| $l = 1000$ m | 142 | 295 | 95 | 290 |
| $l = 1500$ m | 107 | 150 | 70 | 155 |
| $l = 2000$ m | 98 | 112 | 60 | 118 |

Strain Amplitude at TRNT (nS)

| $l = 1000$ m | 15.6 (1.11) | **13.4** (0.17) | 11.6 (0.01) | **11.7** (0.01) |
| $l = 1500$ m | 16.1 (1.39) | **13.9** (0.32) | 12.1 (0.00) | **11.8** (0.00) |
| $l = 2000$ m | 18.4 (3.41) | 15.1 (0.8) | **12.8** (0.05) | **12.7** (0.04) |

*Model fits to AIRS have $\chi^2$ values < 0.032. Pressure drops reported in MPa and modeled TRNT strain amplitudes in nS with their respective $\chi^2$ value. The $\chi^2$ values for TRNT must be < 0.75 to fit the volumetric strain data (shown in bold). See Table 2 for model parameterizations.*
Table 4. DCAM Results

| Model     | | E (Domain 2) | TRNT (nS) |
|-----------|-----------------|-------------|
| DCAM 2000 | 50              | 10          | 12.1 (0.0009) |
| DCAM 1500 | 5               | 5           | 10.3 (0.3)    |
| DCAM 2000 | 50              | 5           | 10.8 (0.02)   |
| DCAM 1000 | 25              | 1           | 8.5 (1.0)     |
| DCAM 1500 | 25              | 1           | 8.6 (1.0)     |
| DCAM 2000 | 25              | 1           | 8.7 (0.9)     |
| DCAM 2000 | 50              | 1           | 8.7 (0.9)     |

*Best fit results for which acceptable pressure drops (1 to 10 MPa) match observed volumetric strains at AIRS within $\chi^2$ values of <0.032. Predicted volumetric strains at TRNT are shown with the associated $\chi^2$ values. The $\chi^2$ values for TRNT must be <0.75 to fit the volumetric strain data (shown in bold).

results at the intersection between the top of the conduit at $z = -10$ to $-30$ m and the top of the edifice due to meshing errors and unstable solutions, solutions at a radial distance $r < 4$ km from the conduit remain unaffected by such effects.

The Advanced Model set is more complex than the Basic Model set, whereby the modeling domain was divided into several subdomains (Figure 3 and Table 2) in order to examine the joint effects of different edifice and crustal mechanical properties. Domain 1 represents the crust with mechanical heterogeneity as described in equations (2)–(4). Domain 2 represents a fractured edifice and shallow crust

invoked by a low Young’s modulus between 1 and 10 GPa (to mimic a mechanically compliant and soft material) and extending to $z = -1100$ m (SCAM) or $z = -2000$ m (DCAM). The $E$ value range chosen for domain 2 is based on data for Merapi’s upper edifice ($E \sim 1$ GPa [Beauducel et al., 2000]), for fractured lavas ($E \sim 1–10$ GPa [Gudmundsson, 2011]), as well as from laboratory measurements of andesitic tuff from SHV ($E = 2.25$ GPa [Voight et al., 1999]).

Domain 3 represents a cylindrical halo of mechanically compliant and soft material around the conduit with $E$ of either 0.5 or 1 GPa, and with a radius $2r_c$, $5r_c$, and $10r_c$ (SHAM and DHAM). The parameter space presented in Table 2 was explored within a Mode-I tensile failure criterion with conduit pressure drops in the range of 1 to 10 MPa, a range consistent with shallow crustal tensile rock strengths [Gudmundsson, 2012]. See Figure 4 for details of Young’s modulus change with depth.

3.4. Inverse Models

We used the SHAM and DHAM model setups as starting points (ISHAM and IDHAM) to invert for geometrical and material properties within set ranges informed by our forward models (Tables 1 and 2). First, we ran a Monte Carlo simulation inverting for $l$, $r_c$, $r_h$, and pressure drop (keeping the material properties constant), and selected those results which were within our $1\sigma$ error for the volumetric strain signal. Second, we ran a new inversion for material properties of the halo and the edifice, while maintaining a constant geometry provided by the best fits solution to the prior geometric inverse model. These material properties were permitted to vary from 0.1 to 5 GPa for the halo and from 1 to 20 GPa for the edifice, respectively. We iterated between geometric property inversions and material property inversions given the best fit solution of each previous model until a solution converged or the iterations remained with the standard deviation of the results.

Table 5. SHAM and DHAM Results for Halo Radius $r_h = 2r_c$.

| $E$ | $E$ %cbf | $l$ | $l$ %cbf | $r_c$ | $r_c$ %cbf | $E$ | $E$ %cbf | $l$ | $l$ %cbf | $r_c$ | $r_c$ %cbf |
|-----|----------|-----|----------|-------|------------|-----|----------|-----|----------|-------|------------|
| 10  | 33.33    | 2000| 50       | 50    | 100        | 10  | 37.5     | 2000| 37.5     | 50    | 75         |
| 5   | 33.33    | 1500| 50       | 25    | 0          | 5   | 62.5     | 1500| 37.5     | 25    | 25         |
| 1   | 33.33    | 1000| 0        | 15    | 0          | 1   | 0        | 1000| 25       | 15    | 0          |

*Best fit results for $\chi^2 < 0.032$ at AIRS and $\chi^2 < 0.75$ at TRNT for predicted volumetric strains. The percentage contribution of tested values (domain 2 $E$, conduit length $l$, and conduit radius $r_c$) to the total number of best fit models (%cbf) is shown. Only results which fit the $\chi^2$ criteria are listed. Results are for a halo with $E = 0.5$ GPa; results for a 1 GPa halo are very similar and available in the supporting information.
Table 6. SHAM and DHAM Results for Halo Radius $r_h = 5r_c$.

|       | SHAM ($r_h = 5r_c$) | DHAM ($r_h = 5r_c$) |
|-------|---------------------|---------------------|
| $E$   | $E$ %cbf            | $l$ %cbf            | $r_c$ %cbf |
| 10    | 43                  | 2000                | 43         |
| 5     | 28.5                | 1500                | 43         |
| 1     | 28.5                | 1000                | 14         |
|       | No. of best fit models | 7                   |            |

$^a$Best fit results for $\chi^2 < 0.032$ at AIRS and $\chi^2 < 0.75$ at TRNT for predicted volumetric strains. The percentage contribution of tested values (domain $2E$, conduit length $l$, and conduit radius $r_c$) to the total number of best fit models (%cbf) is shown. Only results which fit the $\chi^2$ criteria are listed. Results are for a halo with $E = 0.5$ GPa; results for a 1 GPa halo are very similar and available in the supporting information.

3.5. Conduit Volume Loss

We used the best fit model parameters from ISHAM and IDHAM to constrain the radial contraction of the conduit during the eruption. By imposing a prescribed radial displacement on the conduit walls instead of negative pressure, we could fit the volumetric strain signals due to a conduit wall contraction and could hence calculate the conduit volume loss.

4. Results

4.1. Forward Models

The Basic Model set, which assumed $r_c = 15$ m and varying conduit length, did not produce any geologically realistic solutions which fit the volumetric strain data within the 1σ error. SBM, TBM, and CBM require pressure drops on the order of 100 MPa to reproduce the strain signal. TBM requires an additional 40 MPa pressure drop in comparison to SBM, similarly CBM requires an additional 50 MPa pressure drop (for $l = 1500$ m). The plausible pressure range ($-1$ to $-10$ MPa) was not met for any of the simulations (see Table 3). As can be seen in Table 3, the magnitude of the pressure change is controlled by $l$, whereby the longer the conduit length, the lower the required pressure drop. The SBM did not attain the required $\chi^2$ quality of fit for TRNT of $< 0.75$; however, the TBM did match for $l < 2000$ m. The HBM and CBM fitted the data for all explored scenarios (Table 3).

The Advanced Model set yields satisfactory solutions from DCAM (Table 4), SHAM, and DHAM (Tables 5–7). Model predictions for SCAM do not provide any acceptable fit to the data. Predictions from SHAM and DHAM provide a high number of best fit solutions. The overall best fit scenario is obtained by DHAM for a conduit length $l = 1500–2000$ m, conduit radius $r_c = 50$ m, embedded within a mechanically compliant halo with a radius of between $5r_c$ and $10r_c$. The best fit mechanical properties for the halo are a Young’s modulus of 0.5 GPa and a Young’s modulus of 5–10 GPa for the shallow crust and edifice. Forward modeling of vertical

Table 7. SHAM and DHAM Results for Halo Radius $r_h = 10r_c$.

|       | SHAM ($r_h = 10r_c$) | DHAM ($r_h = 10r_c$) |
|-------|---------------------|---------------------|
| $E$   | $E$ %cbf            | $l$ %cbf            | $r_c$ %cbf |
| 10    | 44.44               | 2000                | 56         |
| 5     | 33.33               | 1500                | 33         |
| 1     | 22.22               | 1000                | 11         |
|       | No. of best fit models | 9                   |            |

$^a$Best fit results for $\chi^2 < 0.032$ at AIRS and $\chi^2 < 0.75$ at TRNT for predicted volumetric strains. The percentage contribution of tested values (domain $2E$, conduit length $l$, and conduit radius $r_c$) to the total number of best fit models (%cbf) is shown. Only results which fit the $\chi^2$ criteria are listed. Results are for a halo with $E = 0.5$ GPa; results for a 1 GPa halo are very similar and available in the supporting information.
Table 8. Best Fit Parameters for Inverse Model Suites ISHAM and IDHAM and Associated 1σ Uncertainties

| Parameter                  | ISHAM  | IDHAM  | Forward Model |
|----------------------------|--------|--------|---------------|
| \( r_c \) (m)             | 38 ± 7 | 38 ± 6 | 25–50         |
| Conduit length (m)        | 1550 ± 243 | 1470 ± 275 | 1500–2000    |
| \( r_h \) (m)             | 283 ± 105 | 263 ± 111 | 125–500      |
| Pressure drop (MPa)       | 7 ± 2  | 6 ± 2  | 3–10          |
| Domain 2 \( E \) (GPa)    | 13 ± 5 | 8 ± 3  | 5–10          |
| Domain 3 \( E \) (GPa)    | 1 ± 0.2 | 1 ± 0.7 | 0.5           |
| AIRS (nS)                 | 31 ± 0.5 | 31 ± 0.5 | 31 ± 1       |
| TRNT (nS)                 | 12 ± 0.3 | 11 ± 0.3 | 12 ± 3       |

The solutions from the Monte Carlo simulations for ISHAM and IDHAM converged on similar geometric properties with \( r_c \sim 40 \) m, \( l \sim 1500 \) m, and \( r_h \sim 300 \) m. The results on material properties do not converge as well as those obtained by forward modeling but are reasonably close with \( E \) values for the halo of \( \sim 1 \) GPa and \( \sim 10 \) GPa for the shallow crust and edifice. Based on our inverse model results, the best fit conduit dimensions suggest a conduit volume of \( \sim 6.5 \) Mm\(^3\). Furthermore, best fit modeling of conduit wall displacements (\( \sim 0.24 \) m) predicts a volume change within the conduit of \( \sim 0.1 \) Mm\(^3\) during the eruption.

4.3. Summary of Best Fit Models

Figure 5 shows our best fit volumetric strain predictions from model DHAM compared to predictions from models SBM, TBM, HBM, CBM, and DCAM. Each model has the same parameter inputs as derived from the best fit inverse model IDHAM. The latter models predict lower strain amplitudes than observed and require either higher pressure drops and/or lower values of Young’s modulus for domain 1 or 2 to match the observations. Additionally, those models do not appear to partition the volumetric strain according to observations at both AIRS and TRNT. From our set of models, the implementation of the mechanically weak halo in model IDHAM allows us to match the observed strain values within their uncertainties while maintaining geologically and mechanically plausible parameters.

5. Discussion

5.1. Conduit Dimensions and Syneruptive Contraction

The SHV’s uppermost plumbing system is thought to be composed of a 1.2 km to 2 km long cylindrical conduit [Costa et al., 2007b; Hautmann et al., 2009; Linde et al., 2010; Voight et al., 2010] with \( r_c \sim 15 \) m matching dimensions of extruded andesitic spines [Voight et al., 1999; Sparks et al., 2000; Melnik and Sparks, 2002]. However, our best fit models require conduit dimensions that do not fully match those quoted above. The results indicate that the conduit radius might be closer to \( \sim 40 \) m, similar to data available from other dome-forming volcanoes. Mount St. Helens, for example, is postulated to have a conduit of 50 m radius [Scandone and Malone, 1985], although the longevity of this conduit is debated [Scandone et al., 2007]. During the 2006 spine extruding event the conduit exit was thought to be 50–100 m in radius [Pollister et al., 2008]. In our forward models larger conduit widths permit shorter and horizontal displacement at the strainmeter installations AIRS, TRNT, OLV, and GERD for the best fit models produces deformation on the order of micrometers to hundreds of micrometers, respectively. Forward modeling of the strain signal at GERD produces a strain ratio for AIRS:GERD of \( \sim 16:1 \).
Figure 6. Proposed conceptual model of the conduit and shallow crustal mechanics at SHV based on best fit results presented in Table 8. The conduit narrows from an average radius of \( \sim 40 \) m to about 15 m (based on observed spine dimensions [Voight et al., 1999; Sparks et al., 2000; Melnik and Sparks, 2002]) and is embedded within a halo of mechanically compliant material (dotted grey color). The edifice and the shallow crust are also composed of weakened material (light grey). At depth the rock is mechanically stronger (dark grey). The dyke has not been taken into account in our modeling and is shown here for illustration purposes. The transition zone is drawn after the data presented in Thomas and Neuberg [2012] and Costa et al. [2007a].

5.2. Edifice and Shallow Crustal Mechanics

Published geodetic models for this part of the system commonly neglect topography and mechanical heterogeneities [Chardot et al., 2010; Voight et al., 2010; Gottsmann et al., 2011]. We find these assumptions to be unjustified.

First, we demonstrate that topography and simple mechanical layering affect strain partitioning in the shallow crust. In fact, invoking these complexities required an additional pressure drop of \( \sim 150 \) MPa for a conduit of length 1000 m and 20 MPa for a conduit of 2000 m length, compared to a HoE model without topography. We could not achieve solutions yielding mechanically plausible pressure drop values for the Basic Model set, indicating a lack of critical details.

Second, the implementation of subdomains in the Advanced Model set with distinct mechanical properties significantly improves the quality of fit to the observed volumetric strains for plausible conduit pressure drops (\( \sim 1–10 \) MPa). We propose that the way volumetric strain partitions in the shallow crust beneath Montserrat is most sensitive to the mechanical properties of the edifice and the immediate shallow crust. We find best fit pressure drop solutions for a Young's modulus of \( \sim 10 \) GPa for the volcanic edifice and the immediate shallow crust (ISHAM and IDHAM). We find a greater number of solutions for DHAM than SHAM in our forward models; however, our best fit inverse models cannot discriminate between them. We do find,
however, that attributing a Young's modulus of less than 5 GPa to the first 2000 m of the crust does not yield acceptable solutions, particularly to the data recorded at TRNT.

The deduced mechanical properties of the shallow crust do not match the properties suggested by Chardot et al. [2010] and Voight et al. [2010] who invoke very low Young's moduli (~3 GPa) for the entire crustal domain. Such a low $E$ values allows these authors to explain volumetric strains recorded during the 29 July 2008 eruption or similar Vulcanian explosions that are thought to only involve the shallow conduit such as on 13 July 2003. As we have shown above, invoking such low Young's modulus value for the entire model domain appears to be an oversimplification and unjustified.

5.3. Mechanically Compliant Halo

Our models show the importance of a halo of compliant rocks around the conduit to explain the strain signature during the eruption. The mechanical properties deduced by the modeling indicate a best fit Young's modulus of ~1 GPa, with a radius of ~300 m for the halo. This may indicate a zone of mechanically damaged rock around the conduit which profoundly affects strain partitioning. It is conceivable that inelastic effects (including stress corrosion and cyclic fatigue) around the conduit at SHV play a much more crucial role than perhaps hitherto explored. While time-dependent stress corrosion is most sensitive to the mean stress level, cyclic fatigue is most sensitive to the amplitude of the stress cycles [Costin and Holcomb, 1981]. Explosions can cause blasting fractures and loss of cohesion on joints, thus reducing the overall rock strength and country rock stability [Barnett, 2008]. Finally, the conduit wall can also experience erosion due to particles above the fragmentation level and fluid shear stresses [Macedonio et al., 1994].

5.4. Volumetric Strain Ratios

Strain data recorded at GERD for eruptions similar to the 29 July 2008 eruption, such as on 13 July 2003 and 3 January 2009, indicate a ratio of 4:1 between strain amplitudes at AIRS and GERD [Chardot et al., 2010; Voight et al., 2010]. Although data from GERD are not available for the 29 July 2008 eruption, forward modeling of volumetric strain at GERD indicates a ratio of 16:1 between AIRS and GERD. This difference may indicate that the 2-D axisymmetric models employed in this study do not fully capture the three-dimensional mechanical complexities such as those discovered by recent seismic and gravimetric studies [Sevilla et al., 2010; Hautmann et al., 2013]. It will be necessary to test the influence of such structures within the older volcanic edifices on Montserrat as well as large-scale tectonic lineaments on strain partitioning, particularly for eruptions involving the deeper plumbing system.

6. Conclusions

This study shows that topography and mechanical heterogeneity fundamentally influence strain partitioning from pressure transients in the near-surface plumbing system of SHV. The results indicate much larger conduit dimensions than have been suggested previously including a halo of damaged and highly compliant rocks around the conduit with a width of ~600 m. The immediate shallow crust (to about 1 km below sea level) of SHV appears to be mechanically compliant with $E$ ~10 GPa and increasing with depth.

Our preferred conceptual model of domain mechanics and conduit dimensions is shown in Figure 6. We find that the exceptional volumetric strain data available for eruptions from SHV provide outstanding opportunities to probe the mechanical properties of the very shallow crust beneath Montserrat and is potentially superior to seismic studies. Invoking mechanical complexities is a necessity to constrain geologically and mechanically plausible conduit pressure drops to fit the volumetric strain signals. Our findings should have implications for the quantification of conduit dynamics and mechanics at other dome-forming volcanoes. In addition, the modeling introduced here may be a useful tool to elucidate shallow crustal mechanics at similar volcanoes.

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