Ring Formation from an Oscillating Black Hole

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24 November 2009

ABSTRACT

Massive black hole (BH) mergers can result in the merger remnant receiving a “kick,” of order 200 km s$^{-1}$ or more, which will cause the remnant to oscillate about the galaxy centre. Here we analyze the case where the BH oscillates through the galaxy centre perpendicular or parallel to the plane of the galaxy for a model galaxy consisting of an exponential disk, a Plummer model bulge, and an isothermal dark matter halo. For the perpendicular motion we find that there is a strong resonant forcing of the disk radial motion near but somewhat less than the “resonant radii” $r_R$ where the BH oscillation frequency is equal one-half, one-fourth, (1/6, etc.) of the radial epicyclic frequency in the plane of the disk. Near the resonant radii there can be a strong enhancement of the radial flow and disk density which can lead to shock formation. In turn the shock may trigger the formation of a ring of stars near $r_R$. As an example, for a BH mass of $10^8 M_\odot$ and a kick velocity of 150 km s$^{-1}$, we find that the resonant radii lie between 0.2 and 1 kpc. For BH motion parallel to the plane of the galaxy we find that the BH leaves behind it a supersonic wake where star formation may be triggered. The shape of the wake is calculated as well as the slow-down time of the BH. The differential rotation of the disk stretches the wake into ring-like segments.

Key words: galaxies: kinematics and dynamics — galaxies: nuclei — galaxies: structure

1 INTRODUCTION

Recent breakthroughs in numerical General Relativity have led to predictions of large recoil velocities of merged binary supermassive black-holes (BH) as the binary radiates away linear momentum as gravitational waves during the final stages of merger (González et al. 2007; Campanelli et al. 2007; Lousto, Campanelli, & Zlochower 2009). Typical kick velocities are of order $\sim 200$ km s$^{-1}$ (Bogdanović, Reynolds, & Coleman 2007). One possible result of these mergers is to cause the resulting remnant black hole (BH) to oscillate through the stellar disk on timescales on the order of Gyr (Kornreich & Lovelace 2008, hereafter KL08; Blecha & Loeb 2008; Fujita 2009). If the merged BH receives an impulse from the merger, some of that motion will be transmitted to the galaxy by dynamical friction. It is clearly of interest to know whether the motion of the BH through the disk generates observable changes in morphology and dynamics in the galaxy. Binary black-holes have already been observed as a double nucleus in a quasar (e.g., Decarli et al. 2009). Further, Comerford et al. (2009) show that BH binaries resulting from recent mergers are observable in the spectra of as many as 40% of Seyfert 2 galaxies. The possible observable effects of free and “wandering” BH merger remnants on disk galaxies, however, have not yet been fully analyzed. de la Fuente Marcos & de la Fuente Marcos (2008) discuss the formation of stars in the wakes of runaway black holes ejected from the host galaxy. Here we consider more slowly moving BHs which remain bound to the host galaxy.

This work first analyzes the case where the ejected BH oscillates with angular frequency $\Omega_{bh,z}$ through the galaxy center in a direction normal to the disk. We discuss the case where the BH is ejected parallel to the plane of the galaxy. Sufficiently large gas accretion by the binary BH system is predicted to drive the orbital and BH spins into alignment normal to the plane of the galaxy (Campanelli et al. 2007). However, the amount is uncertain due to the uncertainty in the ratio of the two viscosity coefficients for the $(r, z)$ and $(r, \phi)$ motion in the disk (e.g., Natarajan & Armitage 1999). This alignment favors BH ejection in the plane of the galaxy (Campanelli et al. 2007). The merging process preceding the ejection is assumed to be sufficiently slow that the host galaxy has returned to an axisymmetric equilibrium state. The BH oscillations are observed in simulations to persist for many periods (KL08; Blecha & Loeb 2008; Fujita 2009). Under these...
conditions we find that there is a strong resonant forcing of the disk radial motion near radii $r_R$ where $2n\Omega_{bh,z} = \Omega_r(r_R)$, with $n = 1, 2, \ldots$, where $\Omega_r$ is the radial epicyclic frequency. Near $r_R$ there can be a strong enhancement of the density in a ring of the disk and shock formation which may trigger the formation of a ring of stars. We note that models for the formation of observed “ring galaxies” assume the (single) passage of one galaxy nearly through the center of another (Thyes & Spiegel 1976, 1977; Lynds & Toomre 1976). The gravitational interaction of the two galaxies causes the formation of a pronounced ring(s) of enhanced density (in one or both of the galaxies) where star formation is expected to be enhanced.

For the case of BH ejection in the plane of the galaxy we find that the BH leaves behind it a supersonic wake where star formation may be triggered. The shape of this wake is calculated and the slow-down time of the BH is estimated. The differential rotation of the disk stretches the wake into ring-like segments.

Section 2.1 describes the gravitational potential of the equilibrium disk galaxy at the time of the BH merger. This includes an exponential disk of stars and gas, a Plummer model the bulge, and an isothermal dark matter halo. Section 2.2 derives the dynamical equations of the system as driven by the radial force due to a BH oscillating perpendicular to the disk. The frequencies and amplitudes of the BH oscillations are taken from the $N$–body simulations of KL08, Section 2.3 discusses the forced oscillations of the gas disk, and §2.4 the possible parametric instability of the disk. Section 3 discusses the response of a galaxy disk to a BH ejected in the plane of the galaxy. Section 4 gives the conclusions of this work.

2 THEORY

2.1 Equilibrium

The equilibrium galaxy is assumed to be axisymmetric and to consist of a thin disk of stars and gas and a spheroidal distributions consisting of a bulge component and a halo of dark matter. The model is the same as that used in the simulations of KH08. The model is similar to that of Fujita (2008, 2009). We use an inertial cylindrical $(r, \phi, z)$ and Cartesian $(x, y, z)$ coordinate systems with the disk and halo equatorial planes in the $z = 0$ plane. The total gravitational potential is written as

$$\Phi(r, z) = \Phi_d + \Phi_b + \Phi_h,$$  (1)

where $\Phi_d$ is the potential due to the disk, $\Phi_b$ is due to the bulge, and $\Phi_h$ is that for the halo. The galaxy may have a central massive black hole of mass $M_{bh}$ in which case a term $\Phi_{bh} = -G M_{bh}/\sqrt{r^2 + z^2}$ is added to the right-hand side of (1). The particle orbits in the equilibrium disk are approximately circular with angular rotation rate $\Omega(r)$, where

$$\Omega^2(r) = \left. \frac{1}{r} \frac{\partial \Phi}{\partial r} \right|_{z=0} = \Omega^2_d + \Omega^2_b + \Omega^2_h.$$  (2)

The equilibrium disk velocity is $v = r \Omega(r) \dot{\phi}$. A central black hole is accounted for by adding the term $\Omega^2_{bh} = GM_{bh}/r^3$ to the right-hand side of (2).

The surface mass density of the (optical) disk is taken to be $\Sigma_d = \Sigma_{d0} \exp(-r/r_d)$ with $\Sigma_{d0}$ and $r_d$ constants and

$$M_d = 2\pi r_d^2 \Sigma_{d0}$$

the total disk mass. The potential due to this disk matter is

$$\Phi_d(r, 0) = -\frac{G M_d}{r_d} R [I_0(R)K_1(R) - I_1(R)K_0(R)],$$  (3)

and the corresponding angular velocity is

$$\Omega^2_d = \frac{GM_d}{2 r_d^3} [I_0(R)K_0(R) - I_1(R)K_1(R)],$$  (4)

where $R \equiv r/(2r_d)$ and the $I$’s and $K$’s are the usual modified Bessel functions (Freeman 1970; Binney & Tremaine 1987, p.77). We consider the mass of the disk stars and gas is $M_d = 2.8 \times 10^{10} M_\odot$ and $r_d = 3.5$ kpc following KL08. For these values, $v_d \equiv \sqrt{GM_d/r_d} = 186$ km/s.

The potential due to the bulge component is taken as a Plummer (1915) model

$$\Phi_b = -\frac{G M_{bh}}{(r^2 + r^2 + z^2)^{1/2}},$$  (5)

where $M_b$ is the mass of the bulge and $v_b$ is its characteristic radius (Binney & Tremaine 1987, p.42). This component is similar to the spherical component considered by Fujita (2008, 2009). We have

$$\Omega^2_b = \frac{GM_b}{(r^2 + r^2)^{1/2}}.$$  (6)

We take $M_b = 10^{10} M_\odot$ and $r_b = 1$ kpc so that $v_b \equiv \sqrt{GM_b/r_b} = 208$ km/s again following KL08.

The dark matter halo is assumed to have the isothermal distribution with the potential

$$\Phi_h = \frac{1}{2} v_{h0}^2 \ln(r_c^2 + r^2 + z^2),$$  (7)

where $r_c$ is the core radius of the halo, and $v_{h0}$ is the circular velocity at distances larger than $r_c$. This potential is assumed to apply out to a large distance say 50 kpc. Equation (7) implies

$$\Omega^2_h = \frac{v_{h0}^2}{r_c^2 + r^2}.$$  (8)

Representative values are $v_{h0} = 250$ km/s and $r_c = 2$ kpc. The total dynamical mass of the galaxy model is $M_{tot} \approx 6 \times 10^{11}$. We do not attempt to model the dark matter distribution at small radii where ΛCDM simulations predict a cusp with
\[ \rho_{\text{dm}} \propto r^{-1} \] (Navarro, Frenk, & White 1996). Observed rotation curves of spiral galaxies are however well fit by profiles \( \rho_{\text{dm}} \propto (r + r_B)^{-1} \) with \( r_B > 5 \) kpc (Salucci 2001; Salucci & Burkert 2000; Burkert 1995).

Figure 1 shows an illustrative rotation curve \( \Omega(r)/2\pi \) not including the contribution of the BH. The contribution of the BH for \( M_{\text{bh}} = 10^8 M_\odot \) is not significant for \( r > 0.2 \) kpc.

### 2.2 Oscillating Black Hole

We first consider the case where the BH of mass \( M_{\text{bh}} \) is ejected vertically and oscillates about the midplane of the galaxy,

\[ z_{\text{bh}} = z_m \left[ \sin(\Omega_{\text{bh}} t) + c_3 \sin(3\Omega_{\text{bh}} t) + \ldots \right], \tag{9} \]

where \( \Omega_{\text{bh}} = 2\pi f_{\text{bh}} \) is the angular frequency of the vertical oscillation, and \( c_3, c_5, \ldots \) account for the fact that the motion is not in general simple harmonic, but the effective potential is an even function of \( z \). However, for \( z_m \lesssim 1 \) kpc, the magnitudes the \( c \)'s is small compared with unity (KL08), and we neglect them. Figure 2 shows the dependence of \( f_{\text{bh}} \) on the initial kick velocity of the BH from KL08. We assume that the damping time-scale of the oscillations due to dynamical friction is significantly longer than the oscillation period as found in equation (11) and obtained numerically.

\[ \frac{du_r}{dt} = -\frac{\nabla P}{\Sigma} + u_\phi \frac{\partial \Phi}{\partial r} + a_r(r, t), \]

\[ \frac{d(r u_\phi)}{dt} = 0, \tag{12} \]

where \( \Sigma = \Sigma_g \) and different 'sound' speeds \( (c_4 \) and \( c_5) \) following the approach of Lin & Shu (1970). In our galaxy \( \Sigma_g \ll \Sigma \) and \( c_g \ll c_s \). The Toomre (1964) stability factor for axisymmetric perturbations of the gas \( Q_g \propto c_g/\Sigma_g \) is significantly less than that for the stars \( Q_s \propto c_s/\Sigma_s \). A number of studies (e.g., Jog & Solomon 1984; Wang & Silk 1994; Rafikov 2001) point out that the disk stability is determined mainly by the small-\( Q \) (less stable) component of the disk. For this reason we analyze the response of the gaseous component of the disk and let \( \Sigma = \Sigma_g \). We have

\[ \frac{\partial \Sigma}{\partial r} + \frac{1}{r} \frac{\partial (r u_\phi \Sigma)}{\partial r} = 0. \tag{13} \]

There is also an equation for the gravitational potential.

We linearize equations (12) and (13) by letting \( u_r = 0 + \delta u_r, u_\phi = r \Omega + \delta u_\phi, P = P_0 + \delta P, \Sigma = \Sigma_0 + \delta \Sigma, \) and \( \Phi = \Phi_0 + \delta \Phi \). The disk equilibrium has \( 0 = -\Sigma_0^{-1} \partial P_0/\partial r + \)
The equation for the gravitational potential for \( r \) given by equation (10) and included in equation (15), we recover the dispersion relation for axisymmetric perturbations of the disk, \( \omega^2 = \Omega_r^2 + (k \cdot c_g)^2 - 2\pi G \Sigma_0 T |k| \) (Toomre 1964).

With \( \alpha_r = 0 \), given by equation (10) and included in equation (15), there are forced oscillations of the disk due to the \( \partial a_r / \partial t \) term at radii where \( \Omega_r \approx 2\Omega_{bh} \) (for the \( g_1 \) term in equation 11), \( \Omega_r \approx 4\Omega_{bh} \) (for the \( g_2 \) term), etc. The radii where these resonances occur are shown in Figure 5.

There can be a parametric instability of the disk due to the \( \partial a_r / \partial t \) term at radii where \( \Omega_r \approx 2\Omega_{bh} \) (for the \( g_1 \) term), \( \Omega_r \approx 4\Omega_{bh} \) (for the \( g_2 \) term), etc. We first discuss the case of forced oscillations.

### 2.3 Forced Oscillations of the Disk

In the case of forced oscillations in equation (15) we have
\[
\frac{\partial a_r}{\partial t} = 2n\Omega_{bh}^2 r g_1\Omega_{bh} \sin(2n\Omega_{bh} t)
\]
for \( n = 1, 2, \ldots \) in equation (11). At the resonant radius \( r_R \) in the disk, \( \Omega_r(r_R) = 2n\Omega_{bh} \). Note that \( \Omega_r \) varies slowly across the disk so that we can write \( \Omega_r^2 = \Omega_{bh}^2[1 - 2\eta (r - r_R)/r_R] \), where \( \Omega_R = \Omega_r(r_R) \) and \( \eta \equiv (r / \Omega_r)(d\Omega_r / dr) \) which is found to be negative for the considered conditions. We let \( \delta u_r(r, t) = \delta u_c(r) \sin(2n\Omega_{bh} t) \) so that equation (15) becomes
\[
\frac{d^2 \delta u_r}{dt^2} - ia \frac{d \delta u_r}{dx} + bx \delta u_r = K(x),
\]
where
\[
x \equiv r - r_R \quad \text{and} \quad \eta \equiv \frac{2\pi G \Sigma_0 T}{c_g \Omega_0}, \quad b \equiv 2 \left( \frac{\Omega_R}{\Omega_0} \right)^2 \eta \left( \frac{c_g}{u_0} \right),
\]
\[
\Omega_0 \equiv \Omega(r_R), \quad u_0 \equiv r_R \Omega_0, \quad K = -gn \left( \frac{\Omega_{bh} \Omega_R}{\Omega_0^2} \right) \left( \frac{u_0}{c_g} \right).
\]

Note that for the considered thin disks \( c_g / u_0 \ll 1 \) so that the \( x \)-coordinate is an expanded version of \( r - r_R \). For the assumed conditions we find \( T = 1 \). We can write \( a = 2(\Omega_R / \Omega_0)/|Q| \), where \( Q = \Omega_r c_g / (\pi G \Sigma_0) \) is Toomre’s (1964) factor with \( Q > 1 \) disks being stable to axisymmetric perturbations.

We can simplify equation (16) by letting \( \delta u_r / c_g = U(x) \) and choosing \( F = \exp(iax/2) \). Then,
\[
\frac{d^2 U}{dx^2} + \left( \frac{a^2}{4} + bx \right) U = \exp(-iax/2)K(x).
\]

A WKBJ solution of the homogenenous part of the equation with \( U \propto \exp[i (x^2/2a - bx)] \) gives \( a = (a^2/4 + bx)^{-1/2} \). Thus there is wave-like propagation for \( x > -a^2/4b \) which begins inside the resonant radius where \( x = 0 \). The region \( x < -a^2/4b \) is forbidden and we assume \( U(x \to -\infty) \to 0 \).

The exact inhomogeneous solution to equation (16) is
\[
\frac{\delta u_r}{c_g} = \frac{\pi e^{ax/2}}{\beta} \frac{\beta}{\pi G \Sigma_0 T} \int_{x_R}^x dy \text{Bi}(\alpha - \beta y) K(x) e^{iax/2},
\]
\[
- \text{Bi}(\alpha - \beta x) \int_{x_R}^x dy \text{Ai}(\alpha - \beta y) K(x) e^{iax/2},
\]
where \( \alpha \equiv a^2/(4b^2/3), \beta \equiv b^1/3, \) and \( \text{Ai} \) and \( \text{Bi} \) are the usual Airy functions. The value of \( x_R \) is chosen to be well inside the forbidden region; that is, \( x_R \approx -a^2/4b \).

Figure 6 shows an illustrative solution of equation (16) for \( \delta u_r / c_g \) and the corresponding fractional surface density variations \( \delta \Sigma / \Sigma \). For this figure we consider a BH with \( M_{bh} = 10^6 M_\odot \), a resonant radius \( r_R = 0.5 \) kpc where \( \Omega_r(r_R) = 4\Omega_{bh} \) (that is, \( n = 2 \)). This corresponds approximately to the results of KL08 with \( v_{\text{init}} \approx 150 \) km/s and \( z_m \approx 1.3 \) kpc. From Figure 4 we find \( g_2 \approx 0.16 \). For the galaxy model of §2.1 we find \( f_{\Omega_{bh}} = \Omega_{bh} / 2\pi = 16.9 \) Gyr^{-1} (or a period of 59.2 Myr), \( 2\pi / \Omega_{bh} \approx 14.8 \) Myr, \( \Omega_{bh} / \Omega_0 \approx 1.89 \), and \( \eta \approx -0.311 \). We assume the galaxy disk of §2.1 has a gas mass-fraction of 0.22 and that \( c_g / u_0 = 0.05 \) so that the Toomre factor of the gas disk, \( Q = 2.1 \) which corresponds to the disk being stable to axisymmetric perturbations. For the §2.1 model, \( u_0 = r_R \Omega_0 / (2 \pi) \approx 110 \) km s^{-1} so that \( c_g = 5.5 \) km s^{-1}. We assume \( x_L = -10 \). For these values we find \( a = 1.8, b = 0.111, \alpha = 3.52, \beta = 0.480, \) and \( K \approx -0.438[1 + (c_g / u_0)^{-2}]^{-3} \).

The full-width of the surface-density peak in Figure 6 at half-maximum is \( \Delta x \approx 2 \). In terms of the actual radius this translates to \( \Delta r \approx r_R(c_g / u_0)^{2/3} K \), where...
Figure 6. Illustrative solution of equation (16) for $\Re(\delta u_r)/c_g$ (solid curve) and the corresponding $\delta \Sigma/\Sigma_0$ (dashed curve) for conditions described in the text. For this case $x = 20(r - r_R)/r_R$. Note that $\delta u_r(x, t) = \Re[\delta u_r(x)]\sin(\Omega_{rt})$ and $\delta \Sigma(x, t) = \delta \Sigma(x)\cos(\Omega_{rt})$ with $\delta \Sigma(x)/\Sigma_0 = (\Omega_{0}/\Omega_{r})d[\Re(\delta u_r(x)/c_g)]/dx$. The location of the peak of $\delta u_r$ at $x \approx -5.4$ coincides approximately with the location of the maximum of the Airy function $Ai(-\alpha - \beta x)$.

$K \approx (2\Omega_0/\Omega_{r})^{2/3}|\eta|^{-1/3} \approx 1.53$. We find $\Delta r \approx 0.1$ kpc. This length is larger than the “Toomre-length” $k_T^{-1} \approx r_R(c_g/u_0)/(\Omega_0/\Omega_{r})Q$, where $k_T$ is the “least stable wavenumber” (where $\omega(k_T)$ is a minimum) for axisymmetric perturbations. For the conditions of Figure 6, $k_T^{-1} \approx 0.028$ kpc.

The magnitude of the disk response, $\delta u_r/c_g$ and $\delta \Sigma/\Sigma_0$, to the oscillating BH is directly proportional to the BH mass $M_{bh}$ and inversely proportional to the $Q$-factor of the gas disk. The response decreases strongly as the resonant radius $r_R$ increases, roughly as $r_R^{-1}$.

A sufficiently strong disk response can lead to shock formation. The time-scale for a shock to form is of the order of $t_{sh} = |\delta u_r/d\Omega|^{-1}$. If $t_{sh}$ is less than half the period of oscillation of $\delta u_r$, then a shock has time to form. Using the continuity equation, the condition for a shock to form is $|\delta \Sigma/\Sigma_0 > \pi^{-1}$. This condition is satisfied for a range of $r$ for the conditions of Figure 6. The compression of gas in the shock wave may in turn lead the formation of a ring of stars.

The amplitude of oscillation of the BH, $z_m$, decays on a time-scale longer than its oscillation period (KL08) due to dynamical friction of the BH with the different components of the galaxy. Part of the decrease in the BH kinetic energy goes into the energy of the disk perturbation. The decrease of $z_m$ leads to an increase in the BH oscillation frequency $\Omega_{bhr}$ (KL08). Thus the radius of a given resonance $r_R$ decreases with time. For the case of Figure 6, we estimate that shock formation occurs only for $r_R \leq 1$ kpc, but continues down to $r_R \sim 0.2$ kpc.

### 2.4 Parametric Instability of the Disk

Here we discuss briefly the influence of the term $(\Omega/r)\delta u_r d\Omega/\partial r$ in equation (15) which can give rise to a parametric instability in the disk. This term can be written as $G_n\Omega_n^2\delta u_r \cos(2n\Omega_{rt})$, for $n = 1, 2, ...$, where $G_n = -(\Omega/r)\Omega_n^2\delta r^2\Omega_{bhr}g_r/\partial r$ is a dimensionless factor. The dominant parametric instability occurs when $n\Omega_{bhr} \approx \Omega_r$. Omitting for simplicity the $r$-derivative terms and the $\partial \Omega_r/\partial t$ term in equation (15) gives

$$\frac{\partial^2\delta u_r}{\partial t^2} + \frac{\Omega_r^2\delta u_r \left\{ 1 - G_n \cos[2(\Omega_{r} + \delta \Omega_r)t] \right\}}{\delta u_r = 0,}$$

where in this case the “resonant radius” $r_R$ is such that $n\Omega_{bhr} = \Omega_r(r_R)$, and $\delta \Omega_r = \eta \Omega_r \delta r/r$ is a measure of the radial distance from $r_R$. The solution of this equation is oscillatory with angular frequency $\Omega_{r} + \delta \Omega_r$ with an amplitude exponentially growing with growth rate

$$\omega_i \approx 0.04 \Omega_r \left[ G_n - \left( \frac{4\Omega_r}{\Omega_{r}} \right)^{2} \right]^{1/2}$$

for $\delta \Omega_r \leq \Omega_{r} |G_n|/4$ (Landau & Lifshitz 1960). The maximum growth rate is $\max(\omega_i) = \Omega_{r}|G_n|/4$. For the galaxy of §2.1 and a BH mass $M_{bh} = 10^{6.5} M_\odot$, we find that $|G_n|$ decreases from roughly 0.15 at $r_R = 0.2$ kpc to $10^{-3}$ at $r_R = 1.5$ kpc assuming $\eta = 0.16$. We conclude that the parametric instability is unimportant compared with the forced motion (§2.3) for $r_R > 0.2$ kpc.

### 3 BH Ejection Parallel to the Plane of the Galaxy

For the case where the BH is eject parallel to the plane of the galaxy we can assume that the motion is in the $x-$direction, $x_{bh} = x_m \sin(\Omega_{bhr}t)$, where $x_m$ is the amplitude of the motion and $\Omega_{bhr}$ the angular frequency as determined by KL08. The motion of the BH through the gas disk of the galaxy is highly supersonic. Thus the BH will leave behind it a narrow conical shock wave or wake. This shock wave may trigger star formation particularly in regions of high density such as dense molecular clouds. When the BH is at $x_{bh}(t)$ it leaves a “shocksection” at the radius $|x_{bh}(t')|$ at a later time $t \geq t'$ this shock-section rotates about the galaxy’s centre by an amount $\Delta \phi = \Omega(x_{bh}(t) - t') \geq 0$. Following the rotation curve of the galaxy, Figure 7 shows the geometry of the wake for the case where the BH has gone through one period of radial oscillation, $2\pi/\Omega_{bhr}$. For the case shown this period is $59$ Myr. The differential rotation of the disk stretches the wake into ring-like segments of radii approximately equal to the maximum excursion of the BH, $x_m$, where the BH most slowly.

The relative velocity of the BH and galactic gas disk is $\Delta v = \phi u_\phi - \delta x_{bh}/\partial t$, where $u_\phi$ is the rotation velocity of the disk. As mentioned the flow is highly supersonic with $|\Delta v| \gg c_g$, where $c_g$ is the sound speed in the gas. In this limit there is Bondi, Hoyle, & Lyttleton accretion to the moving BH (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944) where gas approaching the BH with an impact parameter less than $b_a = 2GM_{bh}/|\Delta v|^2$ accretes to the BH if $b_a \leq h$ with $h$ the half-thickness of the disk. In this case the cross-section for accretion is $\pi b_a^2$. If $b_a > h$ the cross-section is approximately $4b_a h$. We assume $b_a < |x_{bh}|$ during most of the BH’s orbit. The relevant case is $b_a > h$ and this gives the accretion rate $\dot{M}_{bh} = 4b_a h \rho_g |\Delta v|$ where $\rho_g$ is the gas density. The accretion time-scale is then

$$T_a = \frac{M_{bh}}{\dot{M}_{bh}} = \frac{|\Delta v|}{8Gh \rho_g},$$
The differential rotation of the disk stretches the wake into ring-like segments of radii approximately equal to the maximum excursion of the BH, \( x_m \), where the BH most slowly.

Many other processes may be involved in the formation and evolution of nuclear rings in galaxies (e.g., van de Ven & Chang 2009).

ACKNOWLEDGEMENTS

We thank Drs. L.E. Kidder, M.S. Tiscareno, M.M. Hedman, and Profs. D. Lai and R. Giovanelli for helpful discussions. This work has made use of the computational facilities of the National Astronomy and Ionosphere Center, which is operated by Cornell University under a cooperative agreement with the National Science Foundation. RVEL was supported in part by NASA grant NNX08AH25G and by NSF grants AST-0607135 and AST-0807129.

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4 CONCLUSIONS

Here, we first analyzed the linear axisymmetric perturbations of a gas disk driven by a black hole oscillating vertically along the axis of symmetry. We find that there is a strong resonant forcing of the disk radial motion near “resonant radii” \( r_R \) where the BH oscillation frequency is equal one-half, one-fourth, \((1/6, \text{ etc.)}\) of the radial epicyclic frequency in the plane of the disk. Near the resonant radii there can be a strong enhancement of the radial flow velocity and disk density which can lead to shock formation. The shock formation occurs during one period of oscillation of the BH which is assumed longer than the BH damping time due to dynamical friction. This shock may trigger the formation of a ring of stars near \( r_R \). As an example, for a BH mass of \( 10^8 M_\odot \) and a kick velocity of \( 150 \text{ km s}^{-1} \), we find that the resonant radii lie between 0.2 and 1 kpc. The magnitude of the disk response is proportional to the BH mass, inversely proportional to the Toomre \( Q \) of the gas disk, and decreases rapidly as \( r_R \) increases. The resonant radii increase as the initial BH kick velocity increases (KL08).

For BH motion parallel to the plane of the galaxy we find that the BH leaves behind it a supersonic wake which over time gets contorted into a complicated shape by the galaxy’s differential rotation. The shape of the wake is calculated for an illustrative case as well as the slow-down time of the BH.

For these reference values \( h \approx 87 \text{ pc} \) and \( \dot{M}_{bh} \approx 0.43 M_{\odot} \text{ yr}^{-1} \). The drag force on the BH is simply \( F_{bh} = \frac{4 \rho_b h |\Delta v| \Delta v}{\rho_g} \). This acts to both slow down the BH radial motion and impart to it azimuthal motion in the rotation direction of the galaxy. The time-scales for the changes in the motion are all of order \( T_b \).

\[ \approx 230 \text{Myr} \left( \frac{50 \text{pc}}{h} \right) \left( \frac{10 \text{Hcm}^{-3}}{\rho_g} \right) \left( \frac{|\Delta v|}{100 \text{km/s}} \right). \] (23)

Figure 7. Wake of the BH ejected in the plane of the galaxy for the case where the BH has gone through one period of radial oscillation, \( 2\pi/\Omega_{bh} \). For this plot \( x_m = 1 \text{ kpc} \) and \( f_{bh} = \Omega_{bh}/2\pi = 17 \text{ Gyr}^{-1} \) corresponding to a period of 59 Myr. The points marked by \( a, b, \ldots \) correspond to the \( t' = 0, t' = \pi/(2\Omega_{bh}), \text{ etc.} \)
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