Information-theoretically Secure Key Agreement 
over Partially Corrupted Channels

Reihaneh Safavi-Naini and Pengwei Wang
Department of Computer Science, University of Calgary, Canada
reisaf@ymail.com and pengwwan@ucalgary.ca

Abstract. Key agreement is a fundamental cryptographic primitive. It has been proved that key agreement protocols with security against computationally unbounded adversaries cannot exist in a setting where Alice and Bob do not have dependent variables and communication between them is fully public, or fully controlled by the adversary. In this paper we consider this problem when the adversary can “partially” control the channel. We motivate these adversaries by considering adversarial corruptions at the physical layer of communication, give a definition of adversaries that can “partially” eavesdrop and “partially” corrupt the communication. We formalize security and reliability of key agreement protocols, derive bounds on the rate of key agreement, and give constructions that achieve the bound. Our results show that it is possible to have secret key agreement as long as some of the communicated symbols remain private and unchanged by the adversary. We relate our results to the previous known results, and discuss future work.

Key words: Wiretap Channel, Active Adversary, Key Agreement, Secure Message Transmission, Physical Layer Security, Information Theoretic Security.

1 Introduction
One of the fundamental problems in cryptography is establishing a shared secret key between two parties. The problem has been studied in different settings. Important distinctions among settings are based on, (i) the adversary’s computational power, (ii) if communicants have access to a public authenticated channel, or if the communication is over tamperable channel, and, (iii) if there are initial shared information, possibly in the form of dependent random variables.

We consider key agreement with security against computationally unlimited adversaries. Information theoretic key agreement was first considered by Maurer [25], with the motivation of providing positive results for scenarios that secure communication, with security against a computationally unlimited adversary, had been proved to be impossible. The two main approaches to securely sending a message over a channel that is eavesdropped by an unlimited adversary are due to Shannon [34], who formalized the notion of perfect secrecy when Alice and Bob are connected by a public reliable channel, and second, Wyner [43], who
introduced wiretap model in which communicants use the noise in the channel to provide prefect secrecy for the communication without requiring a shared secret key. Security guarantee in both these models although strong, is only achievable under very restrictive assumptions. Perfect secrecy in Shannon’s model requires communicants to share a secret key of the length at least the size of the message. Secure communication in wiretap setting is only possible if Eve’s view of the codeword is “noisier” than the Bob’s view of the codeword, which does not hold for settings that the eavesdropper is closer to Alice than Bob, and has a better communication channel to Alice. Thus positive result in both above approaches are under conditions that are of limited practical applications.

Maurer considered the more basic problem of secret key agreement (secure communication is possible, if one can have a shared key) where Alice and Bob want to share a secret key while the communication channel between them is eavesdropped by a computationally unlimited adversary. Maurer considered a minimum setting where Alice, Bob and Eve hold dependent variables $X, Y$ and $Z$, with a joint distribution $P_{XYZ}$, and Alice and Bob can interact over a public discussion (PD) channel: an authenticated channel that is fully visible by all system participants. The setting is “minimum” in the sense that it was proved [26] that without any initial joint distribution, secure key agreement is impossible. The joint partially leaked distribution $P_{XYZ}$ is the only resource of Alice and Bob, and so a basic question is when a secure shared key can be derived by interacting over the PD. The joint distribution can be generated by receiving transmission of a public random beacon (e.g broadcast by a satellite) that broadcasts samples of a random variable, and this is received by all parties over their individual channels. The distribution can also be simply generated by Alice sending a random string $X^n$ of length $n$ to Bob, over a discrete memoryless wiretapped channel, resulting in $Y^n$ and $Z^n$, at Bob and Eve respectively.

Maurer showed that interaction over PD in the above setting is indeed powerful, and could result in a shared random key and hence secure communication, when Shannon and Wyner models give negative results.

Maurer [23] later considered the case that no PD is available, and communications are over a tamperable channel. The adversary in this setting can completely control the communication and modify or drop the sent messages. Intuitively when the adversary is computationally unbounded and fully controls the communication channel, one cannot expect any secure key to be established. Maurer proved that without initial correlated variables, this is indeed the case even if extra restrictions (e.g. secure channel in one direction) are placed on the adversary. He also proved existential results for the case that $P_{XYZ}$ exists.

The works [27,31,20,11] all consider this powerful model of adversary, corresponding to Dovel-Yao model of adversary that is applicable in networks [15]. To enable establishment of shared key however, authors assume strongly correlated variables, in particular identical [31] or “close” secrets [20,11], that are partially leaked. The role of interaction in these settings is “reconciliation” that results
in a shared secret (in the case of close secrets), and privacy amplification that extracts the randomness in the shared variable.

In this paper we consider a setting where no dependent variables is held by Alice and Bob. Maurer showed impossibility of key agreement when there is no shared correlated variables, and the adversary can either completely eavesdrop the communication channel, or fully control the channel. We ask if positive results could be obtained when the adversary “partially” sees and “partially” controls the channel.

Adversaries with partial access to the channel can be naturally defined at the lowest layer of OSI (Open Systems Interconnection) protocol stack, known as the physical layer. This is the same OSI layer that is used in the Wyner wiretap model where partial view of the adversary (due to noise in Eve’s channel) is used to provide secure communication. Here we consider adversaries with partial view of the message and partial tampering ability.

Information units at the physical layer of communication correspond to elements of a q-ary alphabet \([14,17,18,19,34]\). The goal of encoding at this layer traditionally is to provide reliability against channel noise. We consider the case that these units (channel symbols) can be individually accessed, changed, or blocked by a jamming adversary. In practice, an adversary with a transceiver, depending on their location and transceiver capability, can intercept some of the transmitted symbols, and/or add adversarial noise to corrupt them. An adversary with full control of all communicated symbols corresponds to the network layer adversary in [23] and the follow up works (same as Dolev-Yao model). By moving to physical layer, we are able to consider adversaries with different levels of control over the communicated messages, and study the key agreement problem against a more refined classes of adversaries, that capture corruption at physical layers of communication.

### 1.1 Our work

We initiate cryptographic study of key agreement problem in presence of physical layer adversaries, and show positive results when key agreement in presence of network layer full eavesdropping and/or corrupting adversaries, is impossible.

We consider key agreement problem between Alice and Bob who are connected by a channel that is partially controlled by Eve, and the partially controlled communication is their only resource. Alice and Bob send messages back and forth, over the channel. We define the partial control of Eve by their ability to select, (i) a subset \(S_r\) of the transmitted components for eavesdropping, and (ii) a subset \(S_w\) of transmitted components, to corrupt by adding (jamming) a noise vector. The two sets are chosen adaptively in each round and may have overlap. We impose the restriction that in each round, \(|S_r| \leq \rho_r n\) and \(|S_w| \leq \rho_w n\), where \(n\) is the length of transcript in that round, and \(\rho_r\) and \(\rho_w\) are fixed constants in the range \([0, 1]\), specifying the adversaries capabilities. A network layer adversary corresponds to \(\rho_r = \rho_w = 1\), and a perfectly secure authenticated channel corresponds to \(\rho_r = \rho_w = 0\).
Parameters $\rho_r$ and $\rho_w$ model wireless adversaries’ limitation of receiving antenna and receiver, and jamming capability, respectively. As experimentally shown in [30], because of the constraints of real systems and channels, making a deterministic change to an individual transmitted symbol is “hard”, if possible at all. Assuming additive noise captures the uncertain effect of corruption on the transmitted symbol. We do not consider an adversary with unlimited jamming power; such adversaries can always completely disrupt the communication. Our adversary has a fixed eavesdropping and corruption budget in each round. A stronger adversary is when the adversary has a total budget for the whole protocol, and can plan how to spend it in different round with the restriction that, $\sum |S_r^i| \leq \rho_r n$ and $\sum |S_w^i| \leq \rho_w n$. We start with the less demanding case that the budget of each round is fixed. This is also the more realistic case as the partial view of the adversary in most cases is due to limitations of the adversary’s hardware and processing capabilities.

We also consider a third parameter, $\rho = \frac{|S_r \cup S_w|}{n}$ which is the fraction of transmitted symbols that are either leaked or corrupted.

**Models and definition.** We define a $(\rho_r, \rho_w)$-channel, and use the definition of (interactive) key agreement protocol in [25,23], replacing the PD with $(\rho_r, \rho_w)$-channels. The protocol proceeds in rounds, in each Alice or Bob sends a message. At the end of the protocol, Alice and Bob output keys $K_A$ and $K_B$, respectively. The security properties of a secure key agreement protocol are as follows.

- **Strong reliability:** The probability that Alice and Bob do not derive the same key satisfy, $P(K_A \neq K_B) \leq \delta$;
- **Security:** The generated key is private, given Eve’s view of the communication;
- **Randomness.** The generated key is statistically close to uniform distribution.

Defining reliability in information theoretic key agreement, when adversary tampers with the communication, is subtle. Maurer [23] and the follow up work [20], consider the case that at the end of the protocol, Alice and Bob either output a key, or $\bot$. That is they either output a key, or declare the protocol unsuccessful. The protocol is successful when at least one of the communicants output $\bot$ (and so guaranteeing that a shared leaked key will not be established) or, a secret shared key is established. Using this definition, a protocol may have negligible failure probability but at that same time in most cases no shared secret key be established. This is a natural definition considering a network layer adversary that completely controls the communication.

Our definition of strong reliability is the same as in [26,24] where communication is over PD and the adversary is passive. In such setting (initial $P_{XYZ}$ and communication over PD) Alice and Bob may output different keys because of the working of the protocol and properties of the functions that are used for the derivation of the key, and not because of corruption by the channel adversary.

A surprising result of our work is that by slightly reducing the control of the adversary on the channel (\(\rho_r\) and \(\rho_w\) can be close to 1), one can expect strong reliability in presence of corrupted communication.
Definition of secrecy and key randomness is by requiring that the distribution of secret key given Eve’s observation $K|Z$, is statistically close to a uniformly distributed variable $U$ which is of the same length as $K$.

**Rate and impossibility results.** Following [23], we define the secret key rate of a protocol as the rate that Alice and Bob agree on a shared key, while Eve’s total information remains bounded by $\epsilon$. The rate is given by $R = \frac{\log |K|}{n \log |\Sigma|}$, where $\Sigma$ is the channel alphabet (symbols sent over the channel).

We prove,

$$R \leq 1 - \rho.$$

The bound effectively shows that the fraction of symbols that are either eavesdropped or tampered with in each round, cannot contribute to the secret key. This is intuitively expected and the proof shows that interaction cannot overcome this restriction.

In this definition of rate, all communicated symbols contribute to the communication cost. In the definition of rate in [23], however, communication over public channel is free and the rate only considers the number of shared triplets $(X,Y,Z)$ used by the protocol. When $P_{X^nY^nZ^n}$ is the result of Alice sending $X^n$ to Bob over a wiretap channel, only the cost of this communication (over physical layer) is considered in the calculation of rate. The communication over the PD is for reconciliation and extraction, and are considered free.

The bound shows impossibility of secure key agreement when $\rho > 1$.

The above bound is derived for key agreement protocols assuming strong reliability. The bound also holds for AWTP-PD channel under the same condition.

In Section 3 we derive the upper bound

$$R \leq 1 - \rho_r$$

on the secret key rate of key agreement protocols under weak reliability condition. The same bound holds for key agreement over AWTP-PD channels also.

Maurer’s bound $R \leq \max(I(X^n,Y^n),I(X^n,Y^n|Z^n))$ [23], can be written as $R \leq 1 - \rho$ because $\max(I(X^n,Y^n),I(X^n,Y^n|Z^n)) = 1 - \rho$. Note that the rate in Maurer’s bound does not consider communication over PD, while in our setting the rate includes all the communication.

Using weak reliability, the bound on the secret key rate is $R_{SK} \leq 1 - \rho_r$. Note that in our setting the shared dependent variables $X, Y, Z$ are influenced by Eve through the choices of $S_r$ and $S_w$ and the added noise. Although Maurer’s general setting [26] allows for the distribution $P(X,Y,Z)$ to be adversarially influenced, but in the case of multiple instances of triplets $(X,Y,Z)$ received through noisy channels from a single randomness source, they are not influenced by the adversary.

**Constructions.** We give two constructions of secure key agreement protocols, with strong and weak reliability.

The first construction is an efficient three round protocol that achieves the rate $1 - \rho$ when $\rho_r + 2\rho_w < 1 - \rho$ when strong reliability is considered. The second protocol is an efficient one round protocol that achieves the rate $1 - \rho_r$, but only for weak reliability. Both protocols have constant size alphabet.
1.2 Motivation and applications

Considering adversaries at physical layer of communication gives a realistic model of adversaries in wireless communication systems, evidenced by growing research in physical layer security [15,28,30] in recent years. This research however has been primarily in networking and engineering communities with emphasis on tools and techniques, such as modulation techniques, multiple input, multiple output antennas [38,29,4] and signal design, to achieve security goals that is “informally” stated.

Our adversary model is motivated by physical layer adversaries in wireless communication, where entities interact with its neighbour over a channel that can be “partially controlled” by the adversary. However our formulation of the problem can also be used to model networks that are partially controlled by an adversary. The network between Alice and Bob can be modelled by node disjoint paths between them. The adversary selects two subsets of paths (possibly overlapping), some for eavesdropping and some for tampering. The goal is to for Alice and Bob to share a secret key. A similar model of network is considered in SMT [12,16,15,21] problem. In its full generality and when $S_r \neq S_w$, the tampering is algebraic and by adding a noise vector on the set $S_w$. However for $S_r = S_w$ (and more generally any component that is both read and written to), the tampering will be arbitrary as Eve can determine the noise component $z_i = x^2_i - x^1_i$, where $x^2_i$ and $x^1_i$ are the new and old (read) value of the component.

Using physical layer properties of the system for secure communication has the interesting intuition that massive surveillance would translate to the requirement of everywhere physical presence which would significantly raise the bar for successful surveillance. This paper effectively shows that under the reasonable assumption that adversary cannot fully access the physical layer communication system (channel or network), Alice and Bob can generate shared randomness, with provable security against a computationally unlimited adversary, and are able to securely communicate. This provides an interesting new research direction for using physical layer security as a source of individualization and diversification in security systems with the goal of improved security against massive surveillance. A well-known approach to increasing security against massive blanket attacks in computer systems is using diversification and individualization of software and hardware systems. Diversification has been successfully used by computer virus writers to avoid detection by making each copy distinct [37]. It has also been used by security system designers, for example by using multiple operating systems and protection software, to protect against mass infections. In cryptography, massive surveillance through techniques such as algorithm substitution and backdoors, has motivated new research [44,2]. An important source of diversity in secure communication is physical layer properties of the communication systems. To thwart systems that use physical layer properties of the system as a resource, the adversary must exert higher level of control over the physically environment which would be significantly more intrusive, visible and demanding on the adversary. Our work is an step in this direction of exploring this resource for security against massive surveillance.
1.3 Relation to previous work

Maurer [25] considered a setup where Alice, Bob and Eve have correlated variables $X, Y,$ and $Z$, distributed as $P_{XYZ}$. Alice and Bob want to share a shared secret key by exchanging messages over an authenticated insecure channel that is fully readable by the adversary. He derived an upper bound on the entropy of the key that can be obtained from such a protocol,

$$H(K) \leq \min\{I(X;Y), I(X:Y|Z)\} + H(K, K') + I(K;C^tZ)$$ (1)

To obtain more concrete results, he considered a scenario where a discrete memoryless channel generates sequences $X^n = (X_1 \cdots X_n)$, $Y^n = (Y_1 \cdots Y_n)$, and $Z^n = (Z_1 \cdots Z_n)$. In such a setting, rate of secret key agreement is introduced [25]. This is the maximum rate at which Alice and Bob can agree on a secret key, while the rate at which Eve obtains information is arbitrarily small. This definition was later [23] strengthened by requiring that the total leakage of the key approaches zero, when the communication length $n$ over the channel approaches infinity. They extend a lower bound on the rate of secret key agreement in [23] to the key rate with strong secrecy. The bound states,

$$R \geq I(X;Y) - \min\{I(Z;X), I(Z;Y)\}$$ (2)

In this model communication over public discussion channel is assumed free.

**NOT Authenticated channels.** In [23] Maurer removed the assumption that the channel connecting Alice and Bob is authenticated. He allowed the adversary to be able to control this channel and completely control the messages that are sent over it. The goal of the protocol is to establish a shared key that is perfectly unknown to the adversary. This means that this goal could be achieved while some tampered communication remains undetected. Maurer considered two cases: (1). Alice, Bob and Eve do not share the initial information. (2). Alice, Bob and Eve share the initial information $X, Y, Z$ with distribution $P_{XYZ}$.

Theorem 1 [23] shows that in case (1) no secret key can be established. This intuitively says that if there is no initial common randomness and communications are completely tamperable, then no secret shared key can be expected. The theorem further shows impossibility of key agreement even when all messages are authenticated (but public), or communication is authentic in one direction, and secret in the opposite direction. So without initial shared randomness, even if no tampering and only one direction visible by the adversary, one cannot expect a shared secret key. Thus initial setup is necessary for any information theoretic secret key agreement protocol.

Maurer defines $X$-simulatable ($Y$-simulatable) and shows impossibility of key agreement if $P_{XYZ}$ is $X$-simulatable ($Y$-simulatable). When the triplet $X^n$, $Y^n$, and $Z^n$ is generated by many applications of the same experiment, the secret key rate $S^* (P_{XYZ})$ can be defined. A surprising result of the paper is that $S^* (P_{XYZ}) = 0$, or $S^* (P_{XYZ}) = S (P_{XYZ})$, and this distinction is based on or $P_{XYZ}$ is $X$-simulatable (or $Y$-simulatable ) or not.
That is if $P_{XYZ}$ is not $X$-simulatable ($Y$-simulatable), using non-authenticated communication gives the same rate as using authenticated communication.

$(\rho_r, \rho_w)$-Correlation. To compare our results with the above models, we consider a distribution $P_{X^nY^nZ^n}$ that is generated by Alice sending $X^n = (X_1 \cdots X_n)$ over a $(\rho_r, \rho_w)$-channel to Bob. Here $Y^n$ and $Z^n$ denote the observations of Bob and Eve, under the restrictions imposed by the channel. We refer to such correlated variables, as $(\rho_r, \rho_w)$-triplet of length $n$. Maurer notes [23] “In general the distribution $P_{XYZ}$ may be under Eve partial control, and may only partly be known to Alice and Bob.” $(\rho_r, \rho_w)$-correlation generates $P_{XYZ}$ under the influence of Eve.

We consider this initial correlation in, (i) Maurer’s setting of [25] where communication is over a PD, and (ii) Maurer setting of [23] where the channel is fully controlled by the adversary. In $(\rho_r, \rho_w)$-correlation in setting of [25], $P_{X^nY^nZ^n}$ is a $(\rho_r, \rho_w)$-correlation of length $n$, with interaction over PD and using strong reliability definition. Maurer upper bound for a key agreement protocol with security $I(K; Z) \leq \epsilon$ and reliability bounds $\delta$ is,

$$H(S) \leq \min\{I(X; Y), I(X : Y | Z)\} + h(\epsilon) + \epsilon(|S| - 1).$$

The key rate of a protocol in Maurer’s setting however only considers the number $n$ of instances of the triplets $(X, Y, Z)$ shared by parties, and assumes free communication over public channel. That is the rate in fact is the expected amount of entropy that can be derived from each instance.

The secret key in our setting however is the result of interaction over a partially observed channel, and takes the total number of channel uses into account.

The two bounds give the same result. A partially adversarially controlled channel can be used to generate secret keys at the same rate as key agreement protocols in a setting of initially shared dependent vector variables, and having access to a PD channel. This means that the requirement of the existence of PD can be replaced with channels that are partially controlled by the adversary, without incurring rate loss.

$(\rho_r, \rho_w)$-correlation in setting of [23]. Again $P_{X^nY^nZ^n}$ is $(\rho_r, \rho_w)$-correlation, with interaction over a network layer adversary, and using weak definition of reliability. The question that one can ask is, if the distribution $P_{X^nY^nZ^n}$ is $X$-simulatable ($Y$-simulatable),

**Adversary model.** The adversarial model for physical layer adversaries was first proposed in [40]. The goal of the protocol however was to provide reliable communication. Secure communication using the same adversary model was studied in [41,39]. We use the same adversary model and refer to it as $(\rho_r, \rho_w)$-Adversarial Wiretap Channel ($(\rho_r, \rho_w)$-AWTP). A secure message transmission protocol can be used to send a random key and establish a shared key. The secrecy and reliability definition in [41,39] ensure that the received random string satisfies security properties of definition 3 and so can be used as a key. It was proved that the secrecy rate $R_M$ of a message transmission protocol (the highest number of bits per channel use, where transmission has perfect secrecy and reliability approaches perfect reliability with increased message length) is bounded
by \( R_M \leq 1 - \rho_r - \rho_w \). This means that no secure message transmission is possible if \( \rho_r = \rho_w = 0.5 \), that is \( S_r \) and \( S_w \) are chosen each to be half of the codeword, even if \( S_r = S_w \) and half of the components of each round are left untouched. The results of this paper shows that secure communication is possible if \( S_r = S_w \) and each set is as large as \( (1 - \nu)n \), where \( \nu \) is a negligible constant.

The adversary corrupts the codeword by adding a noise vector, with non-zero element over \( S_w \), to the codeword. If the adversary chooses \( S_r = S_w \), then they know the component that is corrupted (because it is in \( S_r \)) and so can design the noise to change the component to any desired value. That is arbitrarily change the component. Our work shows possibility of secure communication if \( (1 - \nu)n \) components are arbitrarily corrupted.

**Other works.** Key agreement is a fundamental problem with a very large body of research. Directly related work on secure key agreement can be grouped into those that assume a shared partly leaked string \([9,10,20,11]\) at the start, and those that do not assume a shared string, but assume a close or highly correlated variable \([25,23,31,24,7]\). In each group, communication can be over PD, or a tamperable channel.

Renner et al. consider the general setting of key agreement protocol between Alice and Bob with variables, \( X \) and \( Y \), that are similar but not identical, while Eve’s information about \( X \) and \( Y \) is incomplete, with communication over completely insecure channel. They find bound and propose a protocol. Kanukurthi et al. \([20]\) propose an efficient key agreement protocol in the same setting.

This problem has also been considered under robust fuzzy extractors \([10]\) and \([9]\) for also the case that parties may have a long-term small secret key.

Extracting secret and shared randomness (key) when parties have weak, partially leaked secret has been studied in \([3]\). Maurer et al. consider privacy amplification against passive and active adversaries with incomplete information about a shared string between two parties.

Dodis et al. \([11]\) give a two round protocol that optimally extracts the randomness in a shared string of length \( n \), by interacting over a channel that is controlled by the adversary.

The work in \([7]\) gives characterization of distributions \( P_{XYZ} \) in different communication setting such as when there is no communication, there is a one-way communication and there is a helper.

## 2 Definitions of Key Agreement Protocol

### 2.1 Channel Models

Let \( n \) be the length of codeword, \( [n] = \{1, \cdots, n\} \). We denote set \( S_r = \{i_1, \cdots, i_{\rho r}n\} \subseteq [n] \) and \( S_w = \{j_1, \cdots, j_{\rho w}n\} \subseteq [n] \) be the two subsets of the \( n \) coordinates. Let \( \text{SUPP}(x) \) of vector \( x \in \Sigma^n \) be the set of positions in which the component \( x_i \) is non-zero.
Definition 1. A \((\rho_r,\rho_w)\)-Adversarial Wiretap Channel is an adversarially corrupted communication channel between Alice and Bob such that it is (partially) controlled by an adversary Eve, with two capabilities: Reading and Writing. For a codeword of length \(n\), Eve can do the following:

1. Reading (eavesdropping): Adversary selects a subset \(S^r \subseteq [n]\) of size \(|S^r| \leq \rho_r n\) and reads the components of the sent codeword \(c\) on \(S^r\).
2. Writing (modifying): Adversary chooses a subset \(S^w \subseteq [n]\) of size \(|S^w| \leq \rho_w n\) for writing, and adds to \(c\) an error vector \(e\) with \(\text{SUPP}(e) = S^w\).

For each channel, the subset \(S = S^r \cup S^w\) with size \(|S| \leq \rho n\) is the set of codeword components that are either read or write to, by the adversary. We assume the adversary is adaptive and selects components of the codeword for reading and writing one by one, using its current view of the communication. In each communication round, the two subsets \(S^r\) and \(S^w\), chosen by Eve, may be different but will satisfy the bounds \(|S^r| \leq \rho_r n, |S^w| \leq \rho_w n\) and \(|S| \leq \rho n\).

A key agreement protocol is an interactive protocol that uses the \((\rho_r,\rho_w)\)-AWTP channel in two directions, from Alice to Bob, or from Bob to Alice.

To compare our results with previous ones, we also consider public discussion channels.

Definition 2. A (Public Discussion Channel) is an authenticated channel between Alice and Bob, that can be read by everyone including Eve. The adversary’s reading capability is \(S^r = [n]\), while the writing capability is \(S^w = 0\).

2.2 Key Agreement Protocols

We study key agreement protocols over \((\rho_r,\rho_w)\)-AWTP channels. We consider the case that the channel is the only resource, and interaction over this channel generates the key.

Interactive Key Agreement Protocols over \((\rho_r,\rho_w)\)-AWTP Channels.

There are a pair of forward and backward \((\rho_r,\rho_w)\)-AWTP channels, from Alice to Bob and Bob to Alice, respectively (Figure 1). To establish a secret key, Alice and Bob follow the an \(\ell\)-round key agreement protocol, sending coded messages over the two channels. The protocol is defined by a sequence of randomized function pairs \((\Pi^r_A, \Pi^r_B)\) for \(r = 1, \cdots, \ell\), and a pair of deterministic key derivation functions \((\Phi_A, \Phi_B)\). Each protocol function outputs a vector over alphabet symbols \(\Sigma\). In the \(i^{th}\) round, Alice transmits the protocol message \(c_i\) to Bob, or Bob transmits the protocol message \(d_i\) to Alice. Eve reads and writes to the channel. In the \(i^{th}\) round, Eve reads on the set \(S^r_i\) and adds error on the set \(S^w_i\), and the sizes of \(S^r_i\) and \(S^w_i\) are bounded as \(|S^r_i| \leq \rho_r n_i, |S^w_i| \leq \rho_w n_i\), respectively. At the end of the \(i^{th}\) round, Bob (or Alice) receives a corrupted word \(x_i\) (or \(y_i\)).
Let $r_A$ and $r_B$ denote the randomness of Alice and Bob, and $v_A^i$ and $v_B^i$ denote the views of Alice and Bob, respectively. The view of Alice $v_A^i$ consists of all messages received and sent by her at the end of round $i - 1$,

$$c_i = \Pi_A(r_A, v_A^i) \quad \text{and} \quad d_i = \Pi_B(r_B, v_B^i).$$  \hfill (3)

At the end of the $\ell$th round, Alice and Bob generate the keys $k_A$ and $k_B$, using their sent and received messages that form their views of the protocol,

$$\Phi_A(v_A^\ell) = k_A \quad \text{and} \quad \Phi_B(v_B^\ell) = k_B.$$  \hfill (4)

The key derivation algorithm is deterministic. Since there is no initial dependent variables held by Alice and Bob, the key will only depend on the communication transcripts.

We also consider a second case that the key derivation function outputs, either a key or detects an error and outputs $\perp$. That is,

$$\Phi_A(v_A^\ell) = k_A \quad \text{and} \quad \Phi_B(v_B^\ell) = k_B.$$  \hfill (5)

**Security and Efficiency.** The protocol has $\ell$ rounds. In the $i$th round of communication, the length of the protocol message is $n_i$ protocol alphabet, where $i = 1, \ldots, \ell$. The total length of communication is $n = \sum_{i=1}^{\ell} n_i$.

The following gives correctness, security and reliability definitions of the key agreement protocol.

**Definition 3.** $(\epsilon, \delta)$-Secure Key Agreement ($(\epsilon, \delta)$-SKA) Protocol: An $(\epsilon, \delta)$-key agreement protocol satisfies the following properties:

1. Correctness: If Eve is passive, that is $S_e = \emptyset$, then $P(K_A = K_B) = 1$.
2. Secrecy: Let $U$ be a uniformly distributed variable over $K$. For any adversary view $Z$, the statistical distance between the distribution of key and $U$ is bounded by $\epsilon$. That is,

$$\text{SD}(P_{K|Z,U}) \leq \epsilon$$  \hfill (6)
3. **Strong Reliability**: If the protocol key derivation function follows (4), the probability that Alice or Bob output different keys is bounded by $\delta$. That is, Alice and Bob output a common key with probability at least $1 - \delta$, 

$$P(K = K_A = K_B) \geq 1 - \delta$$

(7)

4. **Weak Reliability**: If the protocol key derivation function follows (5), Eve wins if $K_A \neq \perp, K_B \neq \perp, K_A \neq K_B$. That is, the probability that Alice and Bob either output error, or output the correct key, is at least $1 - \delta$, 

$$P(K_A = \perp, \text{ or } K_B = \perp, \text{ or } K = K_A = K_B) \geq 1 - \delta$$

(8)

The key agreement protocol is **perfectly secure** if $\epsilon = 0$, and **perfectly reliable** if $\delta = 0$.

The transmission efficiency of a key agreement protocol is measured by the secret rate $R_{SK}$ which is the rate at which a secret key is agreed between Alice and Bob, assuming the adversary uses their best possible adversarial strategies. For a protocol with total transcript length $n$ the secret key rate is given by, 

$$\frac{\log |K|}{n \log |\Sigma|}.$$ 

The rate of key agreement protocol is the maximum rate of key that Alice and Bob can generate by communicating over AWTP channel. A key agreement is parametered with the total length $n$ of protocol.

**Definition 4.** The rate $R_{SK}$ of key agreement protocol is achievable for protocol $SK = \{\Pi_r^{\ell}, \Pi_r^{\ell}\}$ for $r = 1, \cdots, \ell$, if for any $\xi > 0$, there exist $n_0$ such that for any $n \geq n_0$, there is

$$\frac{\log |K|}{n \log |\Sigma|} \geq R_{SK} - \xi$$

and,

$$\delta < \xi$$

The $\epsilon$-secret key capacity $C_{SK}$ of key agreement protocol is the largest achievable rate of all key agreement protocol over the channel with $\epsilon$-secrecy. The perfect secret key capacity $C_{SK}^p$ is the largest achievable secret key rate of all key agreement protocols over the channel.

The computational efficiency of a key agreement protocol refers to the computational complexity of the protocol algorithms run by Alice and Bob. The key agreement protocol is efficient if both parties computations are polynomial time. Otherwise, the key agreement protocol is inefficient.

**Key Agreement Protocol over AWTP-PD Channels**

To better compare and contrast our results with the known results in key agreement, and in particular Maurer’s setting in [25, 26], we will consider a setting
where Alice and Bob have access to a one-way $(\rho_r, \rho_w)$-AWTP channel, and a two-way PD channel. They will use the $(\rho_r, \rho_w)$-AWTP channel to establish dependent and partially leaked variables, and then use the PD channel to extract the entropy captured in the established dependent variables.

There is a one-way AWTP channel from Alice to Bob, and two-way PD channels between Alice and Bob (Figure 2). To establish a secret key, Alice and Bob follow an $\ell$ round key agreement protocol. In the first round of the key agreement protocol, Alice sends a sequence of variables $X^n$ to Bob over the $(\rho_r, \rho_w)$-AWTP channel. In the following rounds, Alice and Bob communicate over the PD channel.

![Fig. 2: Key Agreement Protocol over AWTP-PD Channel](image)

Description of the protocol messages and key derivation functions, and the definitions of correctness, security and reliability are the same as the interactive case above. The only difference is that the $(\rho_r, \rho_w)$-AWTP channel is used only in the first round.

In the first round of the protocol, Eve reads on the set $S_r$, and adds error on the set $S_w$ of the $(\rho_r, \rho_w)$-AWTP channel and we have $|S_r| \leq \rho_r n_1$, $|S_w| \leq \rho_w n_1$. At the end of the first round, Bob receives a corrupted word $x_1$, and Eve has the selected partial view given by $z_1$. In the following rounds, communication is over PD and is fully accessible to Eve. This means that in the $i^{th}$ round of the protocol with $i \geq 2$, Eve read and write sets are $S_r = [n_i]$ and $S_w = \emptyset$, with $|S_r| \leq n_i$ and $|S_w| = 0$. That is at the end of $i^{th}$ round, $i \geq 2$, Bob or Alice correctly receives the sent codeword.

3 Rate Bounds

We derive upper bounds on the secret key rate of key agreement protocols over $(\rho_r, \rho_w)$-AWTP channel.

3.1 Interactive key agreement

This is the main setting considered in this paper. The only resource available to the adversary is the channel that is partially controlled by the adversary. We consider strong reliability.
Theorem 1. The upper bound on the secret key rate of an interactive key agreement protocol over a \((\rho_r, \rho_w)\)-AWTP channel, is \(R_{SK} \leq 1 - \rho\). The bound is for strong reliability.

Proof. We assume the protocol has \(\ell\) rounds. The length of the protocol message in the \(i\)th invocation of the AWTP channel is \(n_i\), and \([n] = \bigcup_{i=1}^{\ell} [n_i]\). For the \(i\)th communication round, let \(c_i\) and \(d_i\) be the codewords sent by Alice and Bob, respectively; \(c_{i,j}\) and \(d_{i,j}\) denote the \(j\)th components of codeword \(c_i\) and \(d_i\), respectively; \(c^i\) and \(d^i\) denote concatenations of all codewords sent in all invocations up to, and including, the \(i\)th round transmission. We use capital letters to refer to the random variables associated with, \(c_i, d_i, c_{i,j}, d_{i,j}, c^i\) and \(d^i\), as \(C_i, D_i, C_{i,j}, D_{i,j}, C^i\) and \(D^i\), respectively. Let \(C^{\ell,r}, C^{\ell,w}, D^{\ell,r}, D^{\ell,w}\) be the random variables of the protocol messages on the adversarial reading sets and writing sets. Let \(C^{\ell,a}\) and \(D^{\ell,a}\) be the random variables on adversarial read only sets, \(C^{\ell,b}\) and \(D^{\ell,b}\) be the random variables on the adversarial read and write sets, \(C^{\ell,c}\) and \(D^{\ell,c}\) be the random variables on adversarial write only sets, \(C^{\ell,d}\) and \(D^{\ell,d}\) be the random variables corresponding to the sets without adversarial corruptions, respectively. We use \(X_i\) and \(Y_i\) to denote the corrupted word received by Bob and Alice when \(C_i\) and \(D_i\) are sent (by Bob and Alice, respectively), and define similarly \(X_{i,j}, X^i\) and \(Y_{i,j}, Y^i\) corresponding to \(C_{i,j}, C^i\) and \(D_{i,j}, D^i\), respectively.

Step 1: We first define an adversary \(Adv_1\) that works as follow:

1. Selects the reading sets and writing sets in all \(\ell\) rounds before the start of the protocol.
2. During the protocol, in round \(i\), chooses a random error vector \(e\), and adds it to the set \(S_{w}^i\) of that round.
3. During round \(i\), the adversary reads the components of \(S_{r}^i\).

Note that this adversary does not use the information seen during the protocol to improve their chance of making the protocol to fail. We give two lemmas that follow from secrecy and reliability.

Lemma 1. For an \((\epsilon, \delta)\)-key agreement protocol, the following holds for adversary \(Adv_1\):

\[
I(K; C^{\ell,r} D^{\ell,r}) \leq 4\epsilon n \log(\frac{|\Sigma|}{\epsilon})
\]

and,

\[
\log |K| - H(K) \leq 4\epsilon n \log(\frac{|\Sigma|}{\epsilon})
\]

Proof is in Appendix A.

Lemma 2. For any \((\epsilon, \delta)\)-key agreement protocol, the following holds:

\[
H(K | C^{\ell,a} C^{\ell,d} D^{\ell,a} D^{\ell,d}) \leq 2H(\delta) + 2\delta \log |K|
\]
Proof is in Appendix B.

**Step 2:** We prove the upper bound:

\[
\frac{\log |K|}{2n \log |\Sigma|} \leq 1 - \rho + 2\epsilon(1 + \log|\Sigma| \frac{1}{\epsilon})
\]

We have,

\[
H(K) = I(K; C^{\ell,a}D^{\ell,a}|C^{\ell,b}D^{\ell,b}) + H(K|C^{\ell,a}C^{\ell,b}D^{\ell,a}D^{\ell,b}).
\]  

(12)

The first item \(I(K; C^{\ell,a}D^{\ell,a}|C^{\ell,b}D^{\ell,b})\) is upper bounded using Lemma 1 (Eq. (9)). For second item, \(H(K|C^{\ell,a}C^{\ell,b}D^{\ell,a}D^{\ell,b})\), we have,

\[
H(K|C^{\ell,a}C^{\ell,b}D^{\ell,a}D^{\ell,b}) = H(K|C^{\ell,b}D^{\ell,b}|C^{\ell,a}D^{\ell,a}) - H(C^{\ell,b}D^{\ell,b}|C^{\ell,a}D^{\ell,a})
\]

\[
= H(K|C^{\ell,a}D^{\ell,a}|C^{\ell,d}D^{\ell,d}) + H(C^{\ell,d}D^{\ell,d}|C^{\ell,a}D^{\ell,a}) - H(C^{\ell,d}D^{\ell,d}|KC^{\ell,a}D^{\ell,a})
\]

\[
+ H(C^{\ell,b}D^{\ell,b}|KC^{\ell,a}D^{\ell,a}) - H(C^{\ell,b}D^{\ell,b}|C^{\ell,a}D^{\ell,a})
\]

\[
\leq 2H(\delta) + 2\delta \log |K| + H(C^{\ell,d}D^{\ell,d})
\]

\[
\leq 2H(\delta) + 2\delta \log |K| + (1 - \rho)2n \log |\Sigma|.
\]

(13)

So the bound on \(H(K)\) is,

\[
H(K) \leq 4\epsilon n \log\left(\frac{|\Sigma|}{\epsilon}\right) + 2H(\delta) + 2\delta \log |K| + (1 - \rho)2n \log |\Sigma|. 
\]

(14)

From (Eq. (10) and (14)), and letting \(\delta \to 0\) as \(n \to \infty\), we have,

\[
R_{SK} = \frac{\log |K|}{2n \log |\Sigma|} \leq 1 - \rho + 4\epsilon(1 + \log|\Sigma| \frac{1}{\epsilon}).
\]

(15)

\[\Box\]

**Weak reliability.** A natural question is if the upper bound will be affected if strong reliability is replaced by weak reliability, where the protocol success also includes cases that Alice and/or Bob output \(\bot\). We prove the following theorem using an approach similar to above. The proof is in full version of paper.

**Theorem 2.** The upper bound on rate of key agreement protocol with weak reliability over AWTP channel is bounded by \(R_{SK} \leq 1 - \rho_r\).

The bound suggests that the corrupted components of protocol messages (corresponding to \(S^{w\ell}_r\)) can be detected and so the secret key rate is limited by the leakage of the components in \(S^{r\ell}_r\).
3.2 Rate of Key Agreement Protocol over AWTP-PD channel

In this setting the \((\rho_r, \rho_w)\)-AWTP channel is used to establish the initial dependent variables, and the remaining construction is over a two-way PD. The following theorem gives the upper bound on the secret key rate of key agreement protocol in this setting. Proof strategy is similar to above and is given in the full version of the paper.

**Theorem 3.** The upper bound on the secret key rate of key agreement protocol over \((\rho_r, \rho_w)\)-AWTP-PD channel, with strong reliability is, \(R_{SK} \leq 1 - \rho_r\). For weak reliability the upper bound is \(R_{SK} \leq 1 - \rho_r\).

**Remark 1.** The rate in \((\rho_r, \rho_w)\)-AWTP-PD does take into account the communication over the PD. This is different from Maurer’s definition \cite{Maurer1993} where PD is free. The bound however is the same as the interactive case where all communications is over \((\rho_r, \rho_w)\)-AWTP. This is surprising and shows that the secret key rate could could stay the same if the channel is partially corrupted.

4 Constructions

We first introduce the building blocks that are use in our construction, and then describe the constructions of protocols that achieve the upper bounds in Section 3.1 and 3.2.

4.1 Cryptographic primitives

**Universal Hash Family**

An \((N, n, m)\)-hash family is a set \(\mathcal{F}\) of \(N\) functions, \(f : \mathcal{X} \rightarrow \mathcal{T}\), \(f \in \mathcal{F}\), where \(|\mathcal{X}| = n\) and \(|\mathcal{T}| = m\). Without loss of generality, we assume \(n \geq m\).

**Definition 5.** \cite{Maurer1993} Suppose the range \(\mathcal{T}\) of an \((N, n, m)\)-hash family \(\mathcal{F}\) is an additive Abelian group. \(\mathcal{F}\) is called \(\epsilon\)-universal, if for any two elements \(x_1, x_2 \in \mathcal{X}\), \(x_1 \neq x_2\), and for any element \(t \in \mathcal{T}\), there are at most \(\epsilon N\) functions \(f \in \mathcal{F}\) such that \(f(x_1) = f(x_2) = t\), were the operation is from the group.

Let \(q\) be a prime and \(u \leq q - 1\). Let the message be \(x = \{x_1, \cdots, x_u\}\). For \(\alpha \in \mathbb{F}_q\), define the universal hash function \(\text{hash}_\alpha\) by the rule,

\[
t = \text{hash}_\alpha(x) = x_1\alpha + x_2\alpha^2 + \cdots + x_u\alpha^u \mod q
\]

Then \(\{\text{hash}_\alpha : \alpha \in \mathbb{F}_q\}\) is a \(\frac{u}{q}\)-universal \((q, q^u, q)\)-hash family. This is a known construction of \(\frac{u}{q}\)-universal hash family \cite{Maurer1993}.
Message Authentication Code

A message authentication code (MAC) is a cryptographic primitive that allows a sender who shares a secret key with the receiver to construct authenticated messages to be sent over a channel that is tampered by an adversary, and the receiver to be able to verify the integrity of the received message.

**Definition 6.** A message authentication code consists of two algorithms (MAC, Ver) that are used for authentication and verification, respectively. For a message $m$ an authentication tag, or simply a tag, is computed,

$$t = \text{MAC}(m, r),$$

and a tagged message $(m, t)$ is constructed. The verifier accepts a tagged pair $(m, t)$ if $\text{Ver}((m, t), r) = 1$. Security of a one-time MAC is defined as,

$$\Pr[(m', t'), \text{Ver}((m', t'), r) = 1 | (m, t), t = \text{MAC}(m, r)] \leq \delta$$

We use a MAC construction that uses polynomials over $\mathbb{F}_q$. Let $m$ be a vector of length $\ell$, and $r = (\alpha, \beta)$, $t$ is over $\mathbb{F}_q$. Define the MAC generation function $\text{MAC} : \mathbb{F}_q^\ell \times \mathbb{F}_q^2 \to \mathbb{F}_q$, where $t = \text{MAC}(m, (\alpha, \beta))$ as,

$$t = \text{MAC}(m, (\alpha, \beta)) = \sum_{i=0}^{\ell-1} x_i \alpha^i + \beta \mod q.$$

**Lemma 3.** For the MAC construction above, the success probability of the adversary in forging a tagged message $(m', t')$ that pass MAC verification is no more than $\frac{\ell}{q}$.

The proof is a direct extension of the proof in [22].

Algebraic Manipulation Detection Code

Algebraic manipulation detection code (AMD code) can be used to encode a source into a value stored on $\Sigma(G)$ so that any tampering by an adversary will be detected, except with a small error probability $\delta$.

**Definition 7 (AMD Code [8] ).** Let $G$ be an additive group. An $(X, G, \delta)$-Algebraic Manipulation Detection code $(X, G, \delta)$-AMD code) consists of two algorithms (AMDec and AMDec) that are used for encoding and decoding, respectively. Encoding is a probabilistic mapping $\text{AMDec} : X \to G$ that maps an element of $X$ to an element of the group $G$. Decoding is a deterministic mapping $\text{AMDec} : G \to X \cup \{\perp\}$ and for any $x \in X$ satisfies $\text{AMDec}(\text{AMDec}(x)) = x$. The security of AMD codes requires,

$$\Pr[\text{AMDec}(\text{AMDec}(x) + \Delta) \notin \{x, \perp\}] \leq \delta, \quad (17)$$

for all $x \in X, \Delta \in G$. 
An AMD code is systematic if the encoding has the form $\text{AMDenc} : \mathcal{X} \rightarrow \mathcal{X} \times \mathcal{G}_1 \times \mathcal{G}_2$, mapping $x$ to $(x, r, t = f(x, r))$ for some function $f$, where $r \overset{\$}{\leftarrow} \mathcal{G}_1$. The decoding function outputs $\text{AMDdec}(x, r, t) = x$ if and only if $t = f(x, r)$, and $\bot$ otherwise.

We use a systematic AMD code, given in [8], over an extension field. Let $d$ be an integer such that $d + 2$ is not divisible by $q$. Define the encoding $\text{AMDenc} : \mathbb{F}_q^d \rightarrow \mathbb{F}_q^d \times \mathbb{F}_q \times \mathbb{F}_q$ as $\text{AMDenc}(x) = (x, r, f(x, r))$, where:

$$f(x, r) = \left( r^{d+2} + \sum_{i=1}^{d} x_i r^i \right) \mod q.$$  \hfill (18)

**Lemma 4.** For the AMD code above, the success chance of an adversary in tampering with a stored codeword $(x, r, t)$ and constructing a new codeword $(x', r', t') = (x' = x + \Delta x, r' = r + \Delta r, t' = t + \Delta t)$, that satisfies $t' = f(x', r')$, is no more than $\frac{d+1}{q}$.

### Randomness Extractor

A randomness extractor is a function, which is applied to a weakly random entropy source (i.e., a non-uniform random variable), to obtain a uniformly distributed source.

**Definition 8.** [10] A (seeded) $(n, m, r, \delta)$-strong extractor is a function $\text{Ext} : \mathbb{F}_q^n \times \mathbb{F}_d \rightarrow \mathbb{F}_q^m$ such that for any source $X$ with $H_\infty(X) \geq r$, we have

$$\text{SD}((\text{Ext}(X, \text{Seed}), \text{Seed}), (U, \text{Seed})) \leq \delta$$

with the Seed uniformly distributed over $\mathbb{F}_q^n$.

A function $\text{Ext} : \mathbb{F}_q^n \rightarrow \mathbb{F}_q^m$ is a (seedless) $(n, m, r, \delta)$-extractor if for any source $X$ with $H_\infty(X) \geq r$, the distribution $\text{Ext}(X)$ satisfies $\text{SD}(\text{Ext}(X), U) \leq \delta$.

A seedless extractor can be constructed from Reed-Solomon (RS) codes [6]. The construction works only for a restricted class of sources, known as symbol-fixing sources.

**Definition 9.** An $(n, m)$ symbol-fixing source is a tuple of independent random variables $X = (X_1, \cdots, X_n)$, defined over a set $\Omega$, such that $m$ of the variables take values uniformly and independently from $\Omega$, and the remaining variables have fixed values.

We show a construction of a seedless $(n, m, m \log q, 0)$-extractor from RS-codes. Let $q \geq n+m$. Consider an $(n, m)$ symbol-fixing source $X = (X_1, \cdots, X_n) \in \mathbb{F}_q^n$ with $H_\infty(X) \geq m \log q$. The extraction has two steps:

1. Construct a polynomial $f(x) \in \mathbb{F}_q[X]$ of degree $\leq n-1$, such that $f(i) = x_i$ for $i = 0, \cdots, n-1$.
2. Evaluate the polynomial at $i = \{n, \cdots, n+m-1\}$. That is, $\text{Ext}(x) = (f(n), f(n+1), \cdots, f(n+m-1))$
Limited-View Adversary Code

Limited-view adversary codes provide reliable communication over an \((\rho_r, \rho_w)\)-AWTP. \[32,42\].

**Definition 10.** An \((n, k, \delta)\)-Limited-View Adversary Code (or \((n, k, \delta)\)-LV adversary code) for a \((\rho_r, \rho_w)\)-AWTP channel, is a code of dimension \(k\) and length \(n\). Encoding and decoding algorithms are \((\text{LVACenc}, \text{LVACdec})\). The probability that the receiver output a message \(m' \neq m\), is bounded by \(\delta\). That is for any \(m \in \mathcal{M}\), and adversary’s observation \(z\) we have,

\[
P(\text{LVACdec}(\text{LVACenc}(m) + \text{Adv}(z)) \neq m) \leq \delta.
\]

LV adversary codes provide reliable communication over AWTP channels. Previous constructions achieves capacity \(1 - \rho_w\) \[12,33\], but with the condition that \(\rho_r + \rho_w < 1\). In Appendix \[C\] we give a simple construction of LV adversary code with low rate of communication, but get rid of the restriction \(\rho_r + \rho_w < 1\) (Appendix \[C\]). Security of this construction is given by the following theorem.

**Theorem 4.** The LV adversary code has rate \(R_{LV} = 1 + \rho - \rho_r - 2\rho_w\) over \((\rho_r, \rho_w, \rho)\)-AWTP channel. The probability of error for a length \(n\) code is \(\delta \leq \frac{un_1}{q}\). The computation is polynomial in code length \(n\).

4.2 Interactive Key Agreement Protocol with Strong Reliability

We introduce a three round interactive protocol with strong reliability, over AWTP channel. The idea behind the protocol is as follows. In the first transmission round, Alice sends a sequence of randomly selected components to Bob. The adversary reads over a set \(S_r\), and adds errors on a \(S_w\). Bob receives a vector that is partially corrupted and partially leaked to the adversary. In the second round, Bob generates a key for each component, and uses a MAC algorithm to construct a tag for each component, using its ow attached key. The keys and tag pairs are sent to Alice using an LV-code and so are received correctly by Alice who will use the received key and tag pair to check the correctness of the \(i^{th}\) component. In the key derivation step, Alice and Bob use a randomness extractor to generate a shared key from their shared randomness which is partially leaked to the adversary.

The construction of secure key agreement protocol uses universal hash function, seedless random extractor, and LV adversary code. Let \(n_1, n_2, n_3\) be the length of the protocol messages sent by Alice and Bob in each round. The total communication length is \(n = n_1 + n_2 + n_3\). Let the protocol be over \(\mathbb{F}_q^u\). We use:

1. the \(\frac{n}{q}\)-universal \((q, q^{u-1}, q)\)-hash family;
2. the seedless \((un_1, \ell, \ell \log q, 0)\)-extractor;
3. the \((n, k, \delta)\)-LV adversary code with \(k = \frac{2n_1}{u}\), \(n = \frac{k}{R_w}\), \(\delta \leq \frac{un_1}{q}\), over alphabet \(\Sigma = \mathbb{F}_q^u\).
Theorem 5. The key agreement protocol has rate $R_{SK} = 1 - \rho - \xi$ when $\rho + 2\rho w < 1 + \rho$, over AWTP channel. The alphabet size is $|\Sigma| = O(q^{\xi})$. The key agreement protocol is perfectly secure and the decoding error is bounded by $\delta \leq \xi$. The number of round is three. The computation complexity is $O((n \log q)^2)$.

The secure key agreement protocol is given in Figure 4.2.

**Fig 4.2** Secure Key Agreement Protocol over AWTP channel

1. R1: Alice $\xrightarrow{\text{AWTP}}$ Bob. For each $i \in n_1$, Alice chooses a vector $r_i$ that is uniformly distributed over $\mathbb{F}_q^{n-1}$, and $\beta_i$ over $\mathbb{F}_q$. Alice sends over the forward AWTP channel to Bob, the codeword $c_1 = (c_{1,1}, \cdots, c_{1,n_1})$ where $c_{1,i} = (r_i, \beta_i)$.
2. R2: Bob $\xrightarrow{\text{AWTP}}$ Alice. Bob receives $x_1 = (x_{1,1}, \cdots, x_{1,n_1})$ with $x_{1,i} = (r'_i, \beta'_i)$ and generates a vector of random values $(\alpha_1, \cdots, \alpha_{n_1})$ over $\mathbb{F}_q$. Bob generates $t = (t_1, \cdots, t_{n_1})$ over $\mathbb{F}_q$ such that,
   \[ t_i = \text{MAC}(r'_i, \alpha_i) + \beta'_i \mod q, \quad \text{for } i = 1, \cdots, n_1. \tag{19} \]
   Bob encodes $(\alpha_1, \cdots, \alpha_{n_1}, t_1, \cdots, t_{n_1})$ into LV adversary code $d_2$ over $\mathbb{F}_q^n$. Bob sends the codeword $d_2$ over backward AWTP channel to Alice.
3. R3: Alice $\xrightarrow{\text{AWTP}}$ Bob. Alice receives $y_2$, and decode into $(\alpha_1, \cdots, \alpha_{n_1}, t_1, \cdots, t_{n_1})$ using LV adversary code decoding algorithm. For each $i = 1 \cdots n_1$, Alice checks if,
   \[ t_i = \text{MAC}(r_i, \alpha_i) + \beta_i \mod q. \tag{20} \]
   Alice generates a binary vector $(v_1, \cdots, v_{n_1})$ where $v_i = 1$ if $(r_i, \beta_i)$ pass the authentication test, and $v_i = 0$ if not. Alice encodes $(v_1, \cdots, v_{n_1})$ into an LV adversary code $c_3$, and sends it over the forward AWTP channel to Bob.
4. Key Derivation. Alice and Bob use a key derivation algorithm to generate secret key.
   - Alice generates a vector $(s_1, \cdots, s_{n_1})$ with $s_i = r_i$ if $v_i = 1$, $s_i = 0$ if $v_i = 0$ for $i = 1 \cdots n_1$. Alice generates a key $k_A$ using randomness extractor,
     \[ k_A = \text{Ext}(s_1, \cdots, s_{n_1}) \tag{21} \]
   - Bob receives $x_3$ from Alice, and decodes it into $(v_1, \cdots, v_{n_1})$ using LV adversary code decoding algorithm. Bob generates a vector $(s'_1, \cdots, s'_{n_1})$ with $s'_i = r'_i$ if $v_i = 1$, $s'_i = 0$ if $v_i = 0$ for $i = 1 \cdots n_1$. Finally Bob uses the extractor to generate a security key,
     \[ k_B = \text{Ext}(s'_1, \cdots, s'_{n_1}) \tag{22} \]
Secrecy and Reliability

Let the secret key \( k = k_A \). We will show that the probability that Alice and Bob output different keys is bound by \( \delta \); that is \( P(K = K_A \neq K_B) \leq \delta \).

Moreover and the distribution of the secret key given the adversary’s observation, is uniform, that is \( SD(P_K | Z, U) = SD(P_{K_A} | Z, U) = 0 \).

Lemma 5. The probability that Alice and Bob do not output the same key is bounded by \( \delta \leq \frac{mn}{q} \) if \( \rho_r + 2\rho_w \leq 1 + \rho \).

Proof. First we consider the case \( \rho_r + 2\rho_w \geq 1 + \rho \). If \( \rho_r + 2\rho_w \geq 1 + \rho \), we can not use the LV adversary code to transmit messages \((\alpha_1, \ldots, \alpha_{n_1}, t_1, \ldots, t_{n_1})\) and \((v_1, \ldots, v_{n_1})\) reliably to the other party, respectively. This is because the rate of LV adversary code \( R_{LV} = 1 + \rho_w - \rho_r - 2\rho_w \). If \( \rho_r + 2\rho_w \geq 1 + \rho \), the rate of LV adversary code \( R_{LV} \leq 0 \). Alice and Bob can not receive \((\alpha_1, \ldots, \alpha_{n_1}, t_1, \ldots, t_{n_1})\) and \((v_1, \ldots, v_{n_1})\) except with negligible error. So the Alice and Bob can not generate secret key using key agreement protocol with negligible error probability.

Then we consider the case \( \rho_r + 2\rho_w \leq 1 + \rho \). The secret key generated by Alice not equal to Bob happens in two cases:

1. Alice and Bob decode the correct message \((\alpha_1, \ldots, \alpha_{n_1}, t_1, \ldots, t_{n_1})\) and \((v_1, \ldots, v_{n_1})\) from \( y_2 \) and \( x_3 \), respectively, using LV adversary code decoding algorithm, except with probability at most \( \delta_1 \leq \frac{u(n_2 + n_3)}{q} \).

This is because \((\alpha_1, \ldots, \alpha_{n_1}, t_1, \ldots, t_{n_1})\) and \((v_1, \ldots, v_{n_1})\) are encoded by LV adversary code. Since \( \rho_r + 2\rho_w \leq 1 + \rho \), the message \((\alpha_1, \ldots, \alpha_{n_1}, t_1, \ldots, t_{n_1})\) and \((v_1, \ldots, v_{n_1})\) can be encoded by LV adversary code with rate \( R_{LV} > 0 \). From Theorem [4] the probability that receiver (Alice or Bob) does not output the correct messages \((\alpha_1, \ldots, \alpha_{n_1}, t_1, \ldots, t_{n_1})\) and \((v_1, \ldots, v_{n_1})\) are bounded by \( \frac{un_2}{q} \) and \( \frac{un_3}{q} \), respectively. So both parties outputs correct message from \( y_2 \) and \( x_3 \), except with probability at most \( \frac{un(n_2 + n_3)}{q} \).

2. Given Alice and Bob share the same \((\alpha_1, \ldots, \alpha_{n_1}, t_1, \ldots, t_{n_1})\) and \((v_1, \ldots, v_{n_1})\), the two parties will generate common randomness \((s_1, \ldots, s_{n_1}) = (s'_1, \ldots, s'_{n_1})\), except with probability at most \( \delta_2 \leq \frac{um_n}{q} \).

This is from,

\[
P((s_1, \ldots, s_{n_1}) \neq (s'_1, \ldots, s'_{n_1})) = \sum_{i=1}^{n_1} P(s_i \neq s'_i) = \sum_{i=1}^{n_1} P(s_i \neq s'_i, v_i = 1) \leq \sum_{i=1}^{n_1} P(r_i \neq r'_i \text{ and } MAC(r_i, \alpha_i) - MAC(r'_i, \alpha_i) = \beta'_i - \beta_i) \leq \frac{un_1}{q}.
\]
Since the secret key $k_A$ and $k_B$ are extracted from randomness $(s_1, \cdots, s_{n_1})$ and $(s'_1, \cdots, s'_{n_1})$, the probability that Alice and Bob generate same secret key such that $k_A = k_B$, is bounded by,

$$1 - \delta = P(K = K_A = K_B)$$

$$= P\left( \text{LVACdec}(y_2) = (\alpha_1, \cdots, \alpha_{n_1}, t_1, \cdots, t_{n_1}) \text{ and } \text{LVACdec}(x_3) = (v_1, \cdots, v_{n_1}) \right)$$

$$= P\left( \text{LVACdec}(y_2) = (\alpha_1, \cdots, \alpha_{n_1}, t_1, \cdots, t_{n_1}) \text{ and } \text{LVACdec}(x_3) = (v_1, \cdots, v_{n_1}) \right)$$

$$\geq (1 - \delta_1)(1 - \delta_2) \geq \left(1 - \frac{u(n_2 + n_3)}{q}\right) \left(1 - \frac{u n_1}{q}\right)$$

$$\geq 1 - \frac{u(n_2 + n_3)}{q} - \frac{un_1}{q} = 1 - \frac{un}{q} \quad \text{(24)}$$

So it implies the reliability of key agreement protocol is bounded by $\delta \leq \frac{u}{q}$.

**Lemma 6.** The key agreement protocol has perfectly secrecy if $\ell \leq (u - 1)(1 - \rho)n_1$

**Proof.** To show the prefect security of key agreement protocol, we assume the adversary reads on the last $\rho n_1$ fraction of codeword. The general adversary attacking can be proved similarly.

First, the vector $(r_1, \cdots, r_{(1-\rho)n_1})$ is perfectly secure for any adversary’s observation $z$. Since adversary reads $\rho r$ fraction of codeword in first round, and read the message encoded by LV adversary code in the second and third, the adversary’s observation is no more than the following set of components,

$$Z = \left\{ r_{(1-\rho)n_1+1} \cdots r_{n_1}, \beta_{(1-\rho)n_1+1} \cdots \beta_{n_1}, \alpha_1, \cdots, \alpha_{n_1}, t_1, \cdots, t_{n_1}, v_1, \cdots, v_{n_1} \right\} \quad \text{(25)}$$

For the set of components $\left\{ r_1, \cdots, r_{(1-\rho)n_1} \right\}$, it is perfectly secure for any adversary’s observation. It implies the vector $(r_1, \cdots, r_{(1-\rho)n_1})$ has min-entropy at least $\ell \log q$.

Second, since vector $(s_1, \cdots, s_{n_1})$ is generated from $(r_1, \cdots, r_{n_1})$, which has min-entropy at least $\ell \log q$, it implies $(s_1, \cdots, s_{n_1})$ also has min-entropy at least $\ell \log q$.

Finally, since the $k_A$ is generated from $(s_1, \cdots, s_{n_1})$ using randomness extractor, and $(s_1, \cdots, s_{n_1})$ is $(n_1, \ell)$ symbol-fixing source, it implies the secret key $\text{SD}(K_A|Z) = \text{SD}(U)$. □
Rate of Key Agreement Protocol

Lemma 7. The rate of key agreement protocol approaches $R_{SK} = 1 - \rho$.

Proof. For a small $\xi \geq 0$, let the parameter be chosen as $u \geq \frac{1}{\xi} + \frac{2}{R_{LV}}$, $q \geq 2un^2$, $\ell = (u-1)(1-\rho)n_1$, $n_0 \geq u$, and $\Sigma = F_q^u$. Let $R_{SK} = 1 - \rho$. For any $n \geq n_0$, the rate of secure key agreement protocol family is given by,

$$\frac{\log |K|}{n \log |\Sigma|} = \frac{\ell \log q}{un \log q} = \frac{(u-1)(1-\rho)n_1 \log q}{(n_1 + n_2 + n_3)u \log q} = \frac{(u-1)(1-\rho)n_1 \log q}{(n_1 + \frac{2n_1}{alv} + \frac{2n_1}{alv})u \log q} = \frac{u-1}{u + \frac{1}{R_{LV}}}(1-\rho) \geq 1 - \rho - \xi = R_{SK} - \xi$$

The decoding error probability is bounded by,

$$\delta \leq \frac{un}{q} \leq \frac{1}{2n} \leq \xi$$

From Definition[4] the rate of secure key agreement is $R_{SK} = 1 - \rho$.

4.3 An SKA Protocol with Strong Reliability over AWTP-PD Channel

We introduce the key agreement protocol with strong reliability over AWTP-PD channel. Both AWTP channel and public discussion is over alphabet $\Sigma = F_q^u$, where $q$ is a prime, and $u$ is an integer. The key agreement protocol has three rounds, uses AWTP channel once and the public discussion channel twice.

The construction of key agreement protocol is similar to the key agreement protocol over AWTP channel (Fig 4.2). Since the communication is over PD channel, after the first round of the protocol, Bob can directly transmit $(\alpha_1, \cdots, \alpha_{n_1}, t_1, \cdots, t_{n_1})$ to Alice in the second round, and Alice can also directly transmit $(v_1, \cdots, v_{n_1})$ to Bob in the third round, without using LV adversary code. The difference between key agreement protocol over AWTP-PD channel and key agreement protocol over interactive AWTP channel is that messages are directly transmitted in the second and third round of the protocol. This means that the condition $\rho_r + 2\rho_w < 1 + \rho$ that was imposed by the LV-code will not be required.

Theorem 6. The key agreement protocol has rate $R_{SK} = 1 - \rho - \xi$ over transmission alphabet for the AWTP channel is of size $|\Sigma| = O(q^{\frac{1}{2}})$. The computation complexity is $O((n \log q)^2)$. 

4.4 An SKA Protocol with weak reliability

We consider weak reliability. The key agreement protocol is one round.

The construction uses AMD codes and randomness extractors. The proof is in the full version of the paper.

**Theorem 7.** The key agreement protocol in [4.4] has one round over AWTP channel, and achieves rate $R_{SK} = 1 - \rho$. The alphabet size is $|\Sigma| = O(q^{1/4})$. The protocol has polynomial time computation.

**Fig. 4.4** Secure Key Agreement Protocol with Weak Reliability.

Alice does the following:
1. Chooses a vector $s = (s_1, \cdots, s_n)$, that is uniformly distributed over $\mathbb{F}_q^{u-2}$.
2. Chooses a vector $(r_1, \cdots, r_n)$ that is uniformly distributed over $\mathbb{F}_q$, and generates $(t_1, \cdots, t_n)$ using AMD code (Eq 18),
   \[ t_i = f(s, r_i) \mod q \]  \hspace{1cm} (28)
3. Sends the codeword $c = (c_1, \cdots, c_n)$ over $\mathbb{F}_q^n$ with $c_i = (s_i, r_i, t_i)$, to Bob over AWTP channel.
4. Alice generate $k_A$ using randomness extractor.
   \[ k_A = \text{Ext}(s_1, \cdots, s_n) \]  \hspace{1cm} (29)

Bob does the following:
1. Receives the word $x = (x_1, \cdots, x_n)$ with $x_i = (s'_i, r'_i, t'_i)$ and checks if $x$ is tampered by Eve by checking:
   \[ t'_i = f(s'_i, r'_i) \]  \hspace{1cm} (30)
2. Output $\perp$ if $x$ is tampered by Eve. Otherwise, Bob generates $k_B$ using randomness extractor.
   \[ k_B = \text{Ext}(s'_1, \cdots, s'_n) \]  \hspace{1cm} (31)

The above protocol shows that under weak reliability, very efficient key agreement protocols can be constructed.
5 Concluding remarks

We motivated and defined a new setting for key agreement protocols where the adversary partially controls the communication channel, and interaction over this channel is the only resource of the adversary. Previous works had considered the cases that the channel was fully authenticated, or fully corrupted. In such a setting channel by itself cannot be the only resource for establishing a secret shared key: in the former case no secrecy for the can be provided, and in the latter no guarantee on communication. All protocols in these settings assume prior dependent variables as communicants’ resource for establishing a shared key. In our setting, the limited control of the adversary makes the channel a resource for extracting a shared key. We formalized the model, derived the secret key rate bounds, and gave constructions that achieve the bounds.

There are numerous open questions that follow from this work. First and foremost, construction of protocols for small alphabets. Our constructions although have constant size alphabet, but the alphabet size depends on how close the rate of the protocol is to the upper bound. The alphabet size determines granularity of the physical layer adversaries. In network setting, each component of a protocol message (codeword) will be sent over a path and so larger size alphabets could be acceptable. In wireless communication however, the alphabet size must be reduced.

Secondly, we defined leakage and corruption as constant ratios of the transmitted word. One can consider other measures of leakage and corruption to limit the adversary’s power.

Thirdly, we motivated the use of physical layer properties of communication systems for providing security against massive surveillance systems. We showed partially controlled physical environments can be used to establish shared secret keys between two participants. Designing other cryptographic primitives that use partial access of the adversary to the physical resources of a system is an interesting direction for future work.

Finally, the three round protocol in Section 4.2 has the requirement $\rho_r + 2\rho_w \leq 1 + \rho$ among parameters. Achieving the bound without this requirement, and finding the minimum number of rounds for protocols with similar property (achieve the upper bound), are open problems.

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A Proof of Lemma 1

Proof. 1). First, we show \( I(K;C^{\ell,r}D^{\ell,r}) \leq 4en \log(\frac{|\Sigma|}{\epsilon}) \).

The proof uses Pinsker’s Lemma:

Lemma 8. Let \( P, Q \) be probability distributions. Let \( \text{SD}(P,Q) \leq \epsilon \). Then

\[
H(P) - H(Q) \leq 2\epsilon \cdot \log\left(\frac{|P \cup Q|}{\epsilon}\right)
\]

From the definition of secrecy of key agreement (Eq. 6), we have,

\[
\text{SD}(P_K|Z,U) \leq \epsilon
\]

From Pinsker’s lemma and adversarial reading sets \( Z = (C^{\ell,r}D^{\ell,r}) \), it implies,

\[
H(U) - H(K|C^{\ell,r}D^{\ell,r}) \leq 2\epsilon \cdot \log\left(\frac{|K|}{\epsilon}\right) \quad (32)
\]

Since \( U \) is the uniform distribution over \( \mathcal{K} \), it implies, \( H(K) \leq \log |\mathcal{K}| = H(U) \). Since Alice and Bob’s randomness \( r_A \) and \( r_B \) are not correlated, it implies \( |\mathcal{K}| \leq |\Sigma|^{2n} \). So there is,

\[
I(K;C^{\ell,r}D^{\ell,r}) \leq H(K) - H(K|C^{\ell,r}D^{\ell,r})
\]

\[
\leq H(U) - H(K|C^{\ell,r}D^{\ell,r})
\]

\[
\leq 2\epsilon \cdot \log\left(\frac{|K|}{\epsilon}\right)
\]

\[
\leq 4en \log\left(\frac{|\Sigma|}{\epsilon}\right) \quad (33)
\]
2). Second, we show \( \log |\mathcal{K}| - H(K) \leq 2\epsilon \cdot \log \left( \frac{|\mathcal{K}|}{\epsilon} \right) \).

From \( H(K|C^{\ell,r}D^{\ell,r}) \leq H(K) \leq \log |\mathcal{K}|, H(U) = \log |\mathcal{K}| \), and (Eq. 32 and 33), it implies,

\[
\log |\mathcal{K}| - H(K) \leq 4\epsilon n \log \left( \frac{|\Sigma|}{\epsilon} \right) \tag{34}
\]

\[\square\]

**B Proof of Lemma 2**

*Proof.* From the definition of reliability (Eq. (9)), the probability that Bob outputs the wrong key is bounded as follows,

\[
\Pr(K_A \neq K) \leq \delta \quad \text{and} \quad \Pr(K_B \neq K) \leq \delta \tag{35}
\]

From Fano’s lemma and (Eq. 35), it implies,

\[
H(K|C^{\ell,Y}) = H(K|K_A) \leq H(\delta) + \delta \log |\mathcal{K}| \tag{36}
\]

and,

\[
H(K|D^{\ell,X}) \leq H(K|K_B) \leq H(\delta) + \delta \log |\mathcal{K}| \tag{37}
\]

Since \( C^{\ell,a} = X^{\ell,a}, C^{\ell,d} = X^{\ell,d}, D^{\ell,a} = Y^{\ell,a}, D^{\ell,d} = Y^{\ell,d} \), it implies,

\[
H(K|C^{\ell,a}C^{\ell,c}C^{\ell,d}D^{\ell,a}Y^{\ell,b}Y^{\ell,c}D^{\ell,d}) \leq H(\delta) + \delta \log |\mathcal{K}| \tag{38}
\]

and,

\[
H(K|D^{\ell,a}D^{\ell,b}D^{\ell,c}D^{\ell,d}C^{\ell,a}X^{\ell,b}X^{\ell,c}C^{\ell,d}) \leq H(\delta) + \delta \log |\mathcal{K}| \tag{39}
\]

From,

\[
H(K|D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d})
\]

\[
= H(K|D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}) + H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d})
\]

\[
= H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}K) + H(K|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}) \tag{40}
\]

it implies,

\[
H(K|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d})
\]

\[
\leq H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}) - H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}K)
\]

\[+ H(\delta) + \delta \log |\mathcal{K}| \tag{41}
\]
From (Eq. 45 and 46), it implies, 

\[
H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c})
\]

\[
= H(K|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c})
\]

\[
+ H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c})
\]

\[
- H(K|D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c})
\]

\[
\leq H(K|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c})
\]

\[
+ H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c})
\]

\[
(42)
\]

It implies, 

\[
H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c})
\]

\[
\leq H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c}) + H(\delta) + \delta \log |K|
\]

\[
(43)
\]

From (Eq. 41 and 44), it implies, 

\[
H(K|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d})
\]

\[
\leq H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d})
\]

\[
- H(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}C^{\ell,b}Y^{\ell,b}Y^{\ell,c}) + 2H(\delta) + 2\delta \log |K|
\]

\[
= l(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}Y^{\ell,b}Y^{\ell,c}) + 2H(\delta) + 2\delta \log |K|
\]

\[
(45)
\]

From adv1, since \((X^{\ell,b}, X^{\ell,c}) = (C^{\ell,b}, C^{\ell,c}) + (E^{\ell,b}, E^{\ell,c})\) and \((Y^{\ell,b}, Y^{\ell,c}) = (D^{\ell,b}, D^{\ell,c}) + (E^{\ell,b}, E^{\ell,c})\), it implies, 

\[
l(D^{\ell,b}D^{\ell,c}X^{\ell,b}X^{\ell,c}, C^{\ell,b}C^{\ell,c}Y^{\ell,b}Y^{\ell,c}|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}) = 0
\]

\[
(46)
\]

From (Eq. 45 and 46) it implies, 

\[
H(K|D^{\ell,a}D^{\ell,d}C^{\ell,a}C^{\ell,d}) \leq 2H(\delta) + 2\delta \log |K|
\]

\[
\square
\]

C LV Adversary Code

The construction of LV adversary code use the algebraic manipulation detection code (AMD code).
LV adversary code Construction

The construction of LV adversary code achieves reliable communication over LV adversary channel (or AWTP channel), even with the condition of $\rho_r + \rho_w \geq 1$. The idea of LV adversary code construction is that sender first encode the message using Reed-Solomon code. Then for each components of Reed-Solomon code, sender uses AMD code to authentication each component. When the receiver receives the LV adversary code, for each components the receiver uses AMD code verification algorithm to check whether the components has been tampered by adversary or not. For the components of LV adversary code that adversary only write, there is high chance to detect the error. For the rest of components of codeword, the receiver use the Reed-Solomon code decoding algorithm to output the correct message.

Let the message of LV adversary code has length $\ell$. The length of LV adversary code is $n$. The LV adversary code is over $\mathbb{F}_u$. To construct LV adversary code, we use $(\ell, n)$-Reed-Solomon code over $\mathbb{F}_u^{-2}$, and $(\mathbb{F}_q^{-u-2}, \mathbb{F}_q^{-u}, d+1)$-AMD code. The encoding algorithm is in Figure 7 and decoding algorithm is in Figure 7.

Fig 7. LV adversary code Encoding algorithm

1. Step1: For message $m$, the sender encodes the message into Reed-Solomon code $c_{RS}$.
2. Step2: For each component $c_i$ for $i = 1, \cdots, n$, the sender encodes $c_i$ into AMD code $(c_i, r_i, t_i)$.

The LV adversary can read on set $S_r$ and add error on set $S_w$. The receiver receives corrupted word $y$.

Fig 7. LV adversary code Decoding algorithm

1. Step1: For each component $y_i = (c'_i, r'_i, t'_i)$ for $i = 1, \cdots, n$, the receiver uses AMD code verification algorithm to check if the AMD code is valid, that is $t'_i \overset{\Delta}{=} f(c'_i, r'_i)$.
2. Step2: The receiver discard the error components. For the rest components passing the AMD code verification algorithm, the receiver uses Reed-Solomon code decoding algorithm to output the message $m$.

We show the rate of the LV adversary code.

**Theorem 8.** The LV adversary code achieves rate $R = 1 - \rho_r - 2\rho_w + \rho$ over $(\rho_r, \rho_w, \rho)$-AWTP channel, except with error probability $\delta \leq \frac{wu}{q}$, in Poly(n).
Proof. We denote the components \(|S_r \cap S_w| = \rho_0 n\). Since \(|S_r \cap S_w| + |S_r \cup S_w| = |S_r| + |S_w|\), it implies, \(\rho_0 + \rho = \rho_r + \rho_w\).

The components of codeword can be divided into four categories: not corrupted, read only, read and write, write only.

For the write only components \((c_i, r_i, t_i)\), since the adversary does not know the AMD code \((c_i, r_i, t_i)\), the probability that adversary tampered AMD code \((c'_i, r'_i, t'_i)\) passes verification is bounded by \(u_q\). Since there are at most \(n\) write only components, the probability of any writing only components pass AMD code verification algorithm is bounded by \(u_q n\).

For the rest components including not corrupted, read only, read and write components, the length of codeword is \(n' = n - (\rho_w - \rho_0) n\). Since the length of error is \(\rho_0 n\), the receiver can uniquely output the correct message if the length of message \(\ell \leq n' - 2\rho_0 n\). So the rate of codeword is bounded as follow,

\[
R \leq \frac{\ell}{n} = \frac{n' - 2\rho_0 n}{n} = 1 - (\rho_w - \rho_0) - 2\rho_0 \\
= 1 - \rho_w - \rho_0 = 1 + \rho - \rho_r - 2\rho_w 
\]

(D) Relation Between Upper Bound of Key Agreement

We show the relation between the upper bound of key agreement [23] in which Alice and Bob generate key using public discussion from the shared triple variable with distribution \(P_{XYZ}\), and key agreement over AWTP-PD channel.

Lemma 9. The rate of key agreement is

\[
R_{SK} \leq \frac{1}{n \log |\Sigma|} \min(I(X^n; Y^n), I(X^n; Y^n | Z^n))
\]

The relation between mutual information entropy and reading and writing parameters \((\rho_r, \rho_w)\) of adversary wiretap channel are \(I(X^n; Y^n | Z^n) = (1 - \rho)n \log |\Sigma|\) and \(I(X^n; Y^n) \leq (1 - \rho_w)n \log |\Sigma|\). So the rate of key agreement protocol is bounded by \(R_{SK} \leq 1 - \rho\).

Proof. We assume that Eve reads on the \(S_r\) components of \((X_1, \cdots, X_n)\) with \(|S_r| = \rho_r n\), and add random error on the \(S_w\) components of codeword with \(|S_w| = \rho_w n\). The components that adversary either read or write is \(S = S_r \cup S_w\) with \(|S| = \rho n\).

First, from Theorem 4 [26], the rate of key agreement protocol \(R_{SK}\) is bounded by,

\[
R_{SK} \leq \frac{1}{n \log |\Sigma|} \min(I(X^n; Y^n), I(X^n; Y^n | Z^n))
\]

Second, we show that \(I(X^n; Y^n) \leq (1 - \rho_w)n \log |\Sigma|\).

We have,

\[
H(Y^n | X^n) = H(E^n | X^n) = H(E^n) = \rho_w n \log |\Sigma|
\]
So,
\[
I(X^n; Y^n) = H(Y^n) - H(Y^n|X^n) \leq n \log |\Sigma| - \rho_w n \log |\Sigma|
= (n - \rho_w n) \log |\Sigma|
\]

Third, we show that \( I(X^n; Y^n|Z^n) \leq (1 - \rho)n \log |\Sigma| \). Let \( Z^n \) be the random variable that adversary does not read. Since \( Z^n \) is equal to \( X^n \) on set \( S_r \), and zero on \( [n]/S_r \), it implies \( Z^n \) is equal to \( X^n \) on set \( [n]/S_r \), and zero on \( S_r \). So there is \( X = Z + \overline{Z} \). We have,

\[
I(X^n; Y^n|Z^n) = H(X^n|Z^n) - H(X^n|Y^n, Z^n)
= H(X^n, Z^n) - H(Z^n) - (H(X^n, Y^n, Z^n) - H(Y^n, Z^n))
= H(Y^n|Z^n) - H(Y^n|X^n)
= H(X^n + E^n|Z^n) - \rho_w n \log |\Sigma|
= H(Z^n + E^n + Z^n) - \rho_w n \log |\Sigma|
\leq H(Z^n + E^n) - \rho_w n \log |\Sigma|
\leq (n - (\rho - \rho_w)n) \log |\Sigma| - \rho_w n \log |\Sigma|
= (1 - \rho)n \log |\Sigma|
\]

\( \square \)