Self-Supervised Super-Resolution for Multi-Exposure Push-Frame Satellites

Ngoc Long Nguyen¹, Jérémy Anger¹,², Axel Davy¹, Pablo Arias¹, Gabriele Facciolo¹
¹ Université Paris-Saclay, CNRS, ENS Paris-Saclay, Centre Borelli, France
² Kayrros SAS
https://centre borelli.github.io/HDR-DSP-SR/

Abstract

Modern Earth observation satellites capture multi-exposure bursts of push-frame images that can be super-resolved via computational means. In this work, we propose a super-resolution method for such multi-exposure sequences, a problem that has received very little attention in the literature. The proposed method can handle the signal-dependent noise in the inputs, process sequences of any length, and be robust to inaccuracies in the exposure times. Furthermore, it can be trained end-to-end with self-supervision, without requiring ground truth high resolution frames, which makes it especially suited to handle real data. Central to our method are three key contributions: i) a base-detail decomposition for handling errors in the exposure times, ii) a noise-level-aware feature encoding for improved fusion of frames with varying signal-to-noise ratio and iii) a permutation invariant fusion strategy by temporal pooling operators. We evaluate the proposed method on synthetic and real data and show that it outperforms by a significant margin existing single-exposure approaches that we adapted to the multi-exposure case.

1. Introduction

High resolution (HR) satellite imagery is a key element in a broad range of tasks, including human activity monitoring and disaster relief. Super-resolution by computational methods has recently been adopted [7, 41] by the remote sensing industry (Planet SkySat, Satellogic Aleph-1). By leveraging high frame rate low-resolution (LR) acquisitions, low-cost constellations can be effective competitors to more traditional high-cost satellites. In order to capture the full dynamic range of the scene, some satellites use exposure bracketing, resulting in sequences with varying exposures. While several works have addressed multi-image super-resolution (MISR) of single-exposure sequences, almost no previous work considers the multi-exposure case.

MISR techniques exploit the aliasing in several LR acquisitions to reconstruct a HR image. The maximum attainable resolution is capped by the spectral decay of the blur kernel resulting from the sensor’s pixel integration and the camera optics, which imposes a frequency cutoff beyond which there is no usable high frequency information.

Aggregating many frames is also interesting as it allows significant noise reduction. If the LR frames are acquired with bracketed exposures, it is possible to integrate them in a super-resolved high dynamic range (HDR) image. Long exposures have higher signal-to-noise ratio (SNR) which helps reduce the noise in dark regions, whereas short exposures provide information in bright regions which can cause saturation with longer exposure times.

In this work, our goal is to perform joint super-resolution and denoising from a time series of bracketed satellite images. We focus on push-frame satellite sensors such as the SkySat constellation from Planet. We increase the resolution by a factor of two, which is the frequency cutoff of the combined optical and sensor’s imaging system. The SkySat satellites [41] contain a full-frame sensor capable of capturing bursts of overlapping frames: a given point on the ground is seen in several consecutive images. However, our technique is general and can be applied to other satellites, or beyond satellite imagery to consumer cameras capable of multi-exposure burst or video acquisition.

Several methods have addressed either MISR or HDR imaging from multiple exposures, but their combination has received little attention. Existing works consider an ideal setup in which frames can be aligned with an affinity [7, 53] or a homography [55], and the number of acquisitions is large enough to render the problem an overdetermined system of equations. Such motion models are good approximations for satellite bursts, but ignore parallax [8], which can be noticeable for mountains and tall buildings.

In the case of satellite imaging, push-frame cameras capable of capturing multi-exposure bursts are relatively recent, which explains why all previous works on MISR focus on the single-exposure case [7, 17, 40, 43], except for SkySat’s proprietary method [41] producing the L1B product, whose details are not public. Deep learning methods currently outperform traditional model-based approaches [47]. In general, learning-based methods require large realistic datasets with ground truth to be trained, as
methods trained on synthetic data [4] fail to generalize to real images [14]. One of such datasets is the PROBA-V dataset [37], acquired with a satellite equipped with two cameras of different resolutions. This dataset has fostered the publication of several deep learning approaches to satellite MISR [9,17,39]. However, the PROBA-V dataset is not appropriate for MISR of LR image bursts acquired at a high frame rate, as the PROBA-V sequences are multi-date and present significant content and illumination changes.

A promising direction is to use self-supervised learning techniques, which have been applied to video restoration tasks such as denoising and demosaicing [18–20, 51, 58], and recently to MISR [43]. These techniques benefit from the temporal redundancy in videos. Instead of using ground truth labels, one of the degraded frames in the input sequence is withheld from the network and used as label.

Our work builds upon Deep Shift-and-Add (DSA) [43], a self-supervised deep learning method for MISR of single-exposure bursts of satellite images. The model is trained without supervision by exploiting the frame redundancy.

**Contributions.** In this work, we propose **High Dynamic Range Deep Shift-and-Pool**, HDR-DSP a self-supervised method for joint super-resolution and denoising of bracketed satellite imagery. The method is able to handle time-series with a variable number of frames and is robust to errors in the exposure times, as the ones provided in the metadata are often inaccurate. This makes our method directly applicable to real image data (see Figure 1). This is, to the best of our knowledge, the first multi-exposure MISR method for satellite imaging, and beyond satellite imagery, it is the first approach based on deep-learning.

Our contributions are the following:

**Feature Shift-and-Pool.** We propose a shift-and-pool module that merges features (computed by an encoder network on each input LR frame) into a HR feature map by temporal pooling using permutation invariant statistics: average, maximum, and standard deviation. This gives a rich fused representation which yields a substantial improvement over the average [43], in both single and multiple exposure cases.

**Robustness to inaccurate exposure times via base-detail decomposition.** We propose normalizing the input frames and decomposing them into base and detail. The errors caused by the inaccurate exposure times affect mainly the base, whereas the detail containing the aliasing required for super-resolution can be safely processed by the network. Note that vignetting and stray light can also cause exposure issues that affect single and multi-exposure MISR alike.

**Noise-level-aware detail encodings.** The noise present in the LR images is signal-dependent, its variance being an affine function of the intensity. To deal with such noise, we provide the un-normalized LR images to the encoder in addition to the normalized detail components. This gives the encoder information about the noise level of each pixel, necessary for an optimal fusion.

**Self-supervised loss with grid shifting.** Using random shifts of the high-resolution grid, we make the self-supervised loss of [43] translation equivariant, leading to improved results.

We validate our contributions with an ablation study on a synthetic dataset (§5.2), designed to model the main characteristics of real bracketed SkySat sequences. Since there are no previous works on multi-exposure MISR, we compare against state-of-the-art single-exposure MISR methods which we adapt and retrain to multi-exposure inputs (§5.3).

We also introduce a dataset of 2500 multi-exposure real SkySat bursts (§5.4). The dataset only consists of noisy LR images, but we can nevertheless train our network on it, since it is self-supervised. Both on synthetic and real data, the proposed HDR-DSP method attains the best results by a significant margin even though it is trained without high resolution ground truth data. The dataset is available for download on the project website.
2. Related work

Most works on video and burst super-resolution focus on the single-exposure case [7, 12, 17, 34, 39, 43, 49, 52]. The problem of super-resolution from multi-exposure sequences has received much less attention. In [53] it is modeled as an overdetermined system and solved via a non regularized least-squares approach. An affine motion model and exact knowledge of the exposure times are assumed. The authors in [55] address the case in which the images have motion blur due to the camera shake. They also consider a static scene and do not consider noise. A related method for HDR imaging uses dual exposure sensors, which interface two exposures in even and odd columns of the image [15, 26]. This can be seen as horizontally super-resolving the video.

Other works perform a related task: joint super-resolution and reverse tone-mapping [30–32]. The difference with our problem is that the input video is a single-exposure LR video, and the goal is to artificially increase its dynamic range to adapt it to HDR screens.

Methods for HDR imaging from multiple exposures need to deal with the noise. Granados et al. [23] address the case of signal-dependent noise and propose a fixed point iteration of the MLE estimator which is close to the Cramer-Rao bound [3]. In these works, the denoising comes only from the temporal fusion. In [1, 2], this is incorporated into spatio-temporal patch-based denoisers.

Our work can also be related to burst and video joint denoising and demosaicing [19, 25, 56], as demosaicing can be regarded as a super-resolution problem.

3. Observation model

We denote by \( \mathcal{J}_i \) a dynamic infinite-resolution ideal scene. The camera on the satellite captures a sequence of \( m \) low-resolution images \( I_i^{LR} \) with different exposures. For the \( i \)-th acquisition, the dynamic scene \( \mathcal{J}_i \) is integrated during an exposure time \( e_i \) centered at \( t_i \). Even if satellites travel at a very high speed relative to the ground, precise electro-optical image stabilization systems (with piezo-electric actuators [29, 33] or steering mirrors [46]) assure that the observed scene \( \mathcal{J}_i \) is mostly constant during the exposure time (\( \sim 2 \text{ms} \)), which allows us to approximate the temporal integration with a product in our observation model

\[
\tilde{I}_i^{LR} = e_i \Pi_1 (\mathcal{J}_i \ast k) + n_i = e_i I_i^{LR} + n_i.
\]

(1)

Here \( k \) is the Point Spread Function (PSF) modeling jointly optical blur and pixel integration, \( \Pi_1 \) is the bi-dimensional sampling operator due to the sensor array, \( I_i^{LR} \) is the clean low-resolution image corresponding to an exposure of 1 unit of time and \( n_i \) denotes the noise. Throughout the text, calligraphic fonts \( \mathcal{I} \) denote noise-free images and regular fonts \( I \) noisy ones. A bar \( \bar{I} \) indicates that the image is multiplied by its exposure time (i.e. as it is acquired by the sensor), while its absence denotes images normalized to an exposure time of 1. We consider the \( r \)-th image \( I_r^{LR} \) in the time series as the reference, and without loss of generality we assume its exposure time to be one, \( e_r = 1 \).

We model the noise as spatially independent, additive Gaussian noise with zero mean and signal-dependent variance

\[
\sigma^2(\tilde{I}^{LR}_i(x)) = ae_i \sigma^2(I^{LR}_i(x)) + b,
\]

(2)

is an approximation of the Poisson shot noise plus Gaussian readout noise [21, 45], with parameters \( a \) and \( b \).

Because of the spectral decay imposed by the pixel integration and optical blur \( (k) \), the images \( \mathcal{J}_i \ast k \) are band limited with a cutoff at about twice the sampling rate of the LR images for SkySat. Our goal is to increase the resolution by a factor 2 by estimating \( \hat{I}_r^{HR} \), a non-aliased sampling of \( \mathcal{J}_i \ast k \) from several LR observations \( \{I_i^{LR}\}_m^{k} \) with varying exposures \( \{e_i\}_m^{k} \). A sharp super-resolved image can then be recovered by partially deconvolving \( k \).

In order for the method to be applicable in practice, it needs to handle time series with a variable number of frames \( m \), and to be robust to inaccuracies in the exposure times \( e_i \), as the exposure times in the image metadata are only a coarse approximation of the real ones.

4. Proposed method

Our method builds upon the DSA method for MISR introduced in [43], which can be regarded as a trainable generalization of the traditional shift-and-add (S&A) algorithms [6, 22, 24, 28, 38]. A feature S&A is used to fuse feature representations produced from the LR images by an encoder network. A motion estimation network computes the optical flows between each input LR frame and the reference frame. The output of the feature S&A is a high-resolution aggregated feature map, which is then decoded by another network to produce the output image.

The DSA method could be extended to multi-exposure sequences by applying it to the normalized images \( \tilde{I}_i^{LR} = \tilde{I}_i^{LR}/e_i \). This approach however is sub-optimal because it neglects the fact that the normalization alters the noise variance model, and fails if the reported exposure times are inaccurate, which is the case in practice.

To better exploit multiple exposures, we propose two modifications: (1) A base-detail decomposition, which provides robustness to errors in the exposure times; (2) An encoding of the images that is made dependent on the noise variance, which allows the encoder to weight different contributions according to their signal-to-noise ratio. In addition, we also propose a new feature pooling fusion intended to capture a richer picture of the encoded features, leading to a substantial improvement in reconstruction quality, both for single and multiple exposure cases. The resulting net-
work can be trained end-to-end with self-supervision, i.e. without requiring ground truth.

4.1. Architecture

Figure 2 shows a diagram of our proposed architecture, which takes as input a sequence of multi-exposure LR images \( \{I_i^{LR}\}_{i=1}^m \) along with the corresponding exposure times \( e_i \) and produces one super-resolved image \( \hat{I}_e^{HR} \). The input LR images are first normalized to unit exposure time. The normalized LR images \( \{I_i^{LR}\}_{i=1}^m \) are then decomposed into base \( \{B_i^{LR}\} \) and detail \( \{D_i^{LR}\} \) components. The bases contain the low frequencies. We align and average them to reduce the low frequency noise and upsample the result using bilinear zooming to produce the HR base component. The LR detail images are fed to a shared convolutional Encoder network that outputs a feature representation of each LR image. The features are then merged into a HR feature map by our shift-and-pool block (FSP), which aligns the LR features into the HR grid of the reference frame, and applies different pooling operations. The pooled features are then concatenated and fed to a Decoder CNN module that produces the HR detail image. The final HR image is obtained by adding the HR base and detail \( \hat{I}_e^{HR} = B_e^{HR} + D_e^{HR} \).

The trainable modules of the proposed architecture (shown in red in Figure 2) include the Motion Estimator, the Encoder, and the Decoder.

Base-Detail decomposition. As mentioned above, normalizing a sequence of the frames \( I_i^{LR} \) by their reported exposures \( e_i \) does not result in stable intensity levels across the sequence. This can be due to small errors in \( e_i \). However, uncorrected vignetting or stray light also contribute the same effect, even in single-exposure imagery.

The nature of the super-resolution task makes it very sensitive to these exposure fluctuations. The shift-and-add operation would merge the LR features into an incoherent high-resolution feature map, making the task of the decoder more difficult, resulting in loss of details or high-frequency artifacts (see Figure 3). Refining the initial \( e_i \) could limit this problem. But this entails its own challenges, especially if one also considers vignetting and stray light sources.

Instead, in this paper we propose a more robust and simple alternative, which is based on a base-detail decomposition [44] of the normalized LR images defined as follows

\[
B_i^{LR} = I_i^{LR} + G, \quad D_i^{LR} = I_i^{LR} - B_i^{LR},
\]

for \( i = 1, \ldots, m \). Here \( G \) is a Gaussian kernel of standard deviation 1. We then process independently the details \( \{D_i^{LR}\} \) and the bases \( \{B_i^{LR}\} \) to produce the corresponding high resolution estimates \( \hat{D}_i^{HR} \) and \( \hat{B}_i^{HR} \). This decomposition is linear and does not affect the super-resolution since the alias is preserved in the detail components \( \{D_i^{LR}\} \).

As the detail images span a smaller intensity range than the complete image \( I_i^{LR} \), an error \( \delta \) in the exposure time results in a small deviation in the detail and a large one in the base: \( \delta B_i^{LR} + \delta D_i^{LR} = \delta I_i^{LR} \). The small error in the detail can be handled by a super-resolution method.

On the other hand, the base images do not need to be super-resolved, but still need to be denoised. In this work we propose a simple processing that aligns and averages the bases and upsamples the result. To fully exploit the high signal-to-noise ratio of longer exposures, the average is weighted by the exposure times \( e_i \)

\[
B_i^{HR} = \text{Zoom} \left( \sum_i e_i \text{Warp}(B_i^{LR}) \right). \tag{4}
\]

This weighting is an approximation of the ML estimator of Granados et al. [23] (details in the supplementary material).

Base and detail decompositions have been used in super-resolution networks [27, 31] to focus the network capacity on the details. In our case, the decomposition also provides robustness to errors in the radiometric normalization.

Motion Estimator. We follow the works of [43, 49] to build a network (with the same hourglass architecture) that esti-
Our HDR-DSP (with BD) noise variances SPMC module [52]. Each LR frame is upscaled by introducing zeros between samples and motion compensated following the flows \( F_{i \rightarrow r} \). This is differentiable with respect to the intensities and the optical flows. Each splatted pixel is assigned a bilinear weight depending on the fractional part of its position in the HR grid. See [43, 52] for details.

This results in a set of aligned sparse HR feature maps

\[
J_i^{HR} = \text{SPMC}(J_i^{LR}, \{ F_i \rightarrow r \}) \in \mathbb{R}^{sH \times sW \times N},
\]

and the corresponding bilinear splatting weights \( W_i^{HR} = \text{SPMC}(1, \{ F_i \rightarrow r \}) \). The upsampling factor \( s \) is set to 2.

As in [43], we use a weighted average pooling in the temporal direction (8). In addition, we propose computing the standard deviation and the max (9):

\[
J_A^{HR} = (\sum_i J_i^{HR}) / (\sum_i W_i^{HR})^{-1},
J_M^{HR} = \max_i J_i^{HR},
J_S^{HR} = \text{std}_i J_i^{HR}.
\]

Note that this block does not have any trainable parameters, a trainable layer may attain a similar performance at a much higher computational cost (see the supplementary material).

These feature pooling operations render the architecture invariant to permutations of the input frames [5]. The key idea is that through end-to-end training, the encoder network will learn to output features for which the pooling is meaningful. Therefore, it is essential that the pooling operation is capable of passing all the necessary information to the decoder. Indeed, average pooling captures a consensus of the features, which amounts to a temporal denoising. But in aliased image sequences, it is common to come across features that are only visible in a single frame. Thus, the idea of the max-pooling operation is to preserve these unique features that would otherwise be lost in the average. The standard deviation pooling completes the picture by measuring the point-wise variability of the features.

The pooled features are independent of the number of processed frames. But this information is important as the decoder may interpret features resulting from aggregating many images differently than those resulting from just a few. For this reason, the aggregation weights \( W^{HR} = \sum_i W_i^{HR} \) are also concatenated with the pooled features. As we will see in §5.2, incorporating \( W^{HR} \) improves the network ability to handle a variable number of input frames.

**Decoder.** The Decoder network reconstructs the HR detail image \( \hat{D}_r^{HR} \) from the pooled features

\[
\hat{D}_r^{HR} = \text{Decoder}(J_A^{HR}, J_M^{HR}, J_S^{HR}, W^{HR}, \Theta_{D}) \in \mathbb{R}^{sH \times sW},
\]

where \( \Theta_D \) denotes the set of parameters of the decoder. The architecture is detailed in the supplementary material.

### 4.2. Self-supervised learning

To train the HDR-DSP detail fusion network, we adapt the fully self-supervised framework of [43], which requires no ground truth HR images. During training, the LR frames

\[
F_i \rightarrow r = \text{MotionEst}(I_i^{LR}, J_i^{LR}, \Theta_M) \in [-R, R]^{H \times W \times 2},
\]

where \( \Theta_M \) denotes the network parameters. A small Gaussian filter (\( \sigma = 1 \)) is applied to the input images to reduce the alias [43, 54]. The network is trained with a maximum motion range of \([ -R, R ]^2 \) (with \( R = 5 \) pixels). The training was adapted to better handle the noise difference due to the multi-exposure setting (see §4.2).

**Noise-level-aware detail encodings.** The Encoder module generates relevant features \( J_i^{LR} \) for each normalized LR detail image \( D_i^{LR} \) in the sequence

\[
J_i^{LR} = \text{Encoder}(D_i^{LR}, I_i^{LR}, \Theta_E) \in \mathbb{R}^{H \times W \times N},
\]

where \( \Theta_E \) is the set of parameters of the encoder and \( N = 64 \) is the number of produced features. The network architecture is detailed in the supplementary material.

The un-normalized low resolution frames \( \tilde{I}_i^{LR} \) are also fed to the encoder. This is motivated by the fact that the maximum likelihood fusion of noisy acquisitions into a (HR) image is a weighted average, where the weights are the inverse of the noise variances \([3, 23]\). In the proposed architecture, the normalized details \( D_i^{LR} \) are fused to produce a high resolution detail \( \hat{D}_i^{HR} \). The noisy un-normalized images are unbiased estimators of an affine function of the noise variances \( \sigma^2(\tilde{I}_i^{LR}) / a - b / a \), thus they provide to the encoder the information required to compute the optimal fusion weights. The resulting features \( J_i^{LR} \) are then aggregated via a set of pooling operations, without any particular handling related to different source exposures.

**Feature Pooling.** We propose the Feature Shift-and-Pool block (FSP) which maps the LR features into their positions on the reference HR grid and pools them. First the features are “splatting” bilinearly onto the HR grid by the SPMC module [52]. Each LR frame is upscaled by introducing zeros between samples and motion compensated following the flows \( F_i \rightarrow r \). This is differentiable with respect to
are randomly selected and for every sequence, one frame is set apart as the reference \( I^r_{LR} \). Then, all the other LR images in each sequence are registered against the reference using the MotionEst network yielding the flows \( F_{i \rightarrow r} \). The reference frame serves as the target for the self-supervised training similarly to noise-to-noise [20, 35]. The procedure relies on the minimization of a reconstruction loss in the LR domain plus a motion estimation loss to ensure accurate alignment of the frames. The losses and the proposed adaptations are detailed in the following paragraphs.

Self-supervised SR loss. The self-supervised loss forces the network to produce an HR detail \( D^{HR}_{r} \) such that when subsampled, it coincides (modulo the noise) with the withheld target detail \( D^{LR}_{r} \)

\[
\ell_{self}(D^{HR}_{r}, D^{LR}_{r}) = \| \Pi_2(\hat{D}^{HR}_{r} \ast k) - D^{LR}_{r} \|_1, \tag{11}
\]

where \( \hat{D}^{HR}_{r} = \text{Net}(\{D^{LR}_{i}\}_{i \neq r}, \{\hat{I}^{LR}_{i}\}_{i=1}^{m}) \) is the SR output, and \( \Pi_2 \) is the subsampling operator that takes one pixel over two in each direction. As in [43] we include the convolution kernel \( k \) in the loss. This forces the network to produce a deconvolved HR image that once convolved with \( k \) and subsampled matches the optical blur present in \( D^{LR}_{r} \).

During training, the LR reference is only used in the motion estimator to compute the optical flows, but it is not fused into the HR result to avoid unwanted trivial solutions [11, 18, 43]. At inference time we use the reference as this leads to improved results [43].

Grid shifting. The self-supervised loss (11) downsamples the super-resolved detail to compare it with the reference LR detail. But since the downsampling is fixed, only the sampled positions intervene in the loss, which breaks the translation equivariance of the method. To avoid this issue, during training we augment the data by adding to the estimated optical flows a random shift of 0.5\( \epsilon \) in each dimension (\( \epsilon \in \{0, 1\} \)). As a result, the super-resolved image is shifted by \( \epsilon \), which is easily compensated before computing the loss. This yields an improvement in PSNR of 0.2dB.

Motion estimation loss. The motion estimator is trained with unsupervised learning as in [57]. The loss consists of a warping term and a regularization term. We observed that the optical flow is very sensitive to the intensity fluctuations between frames (as in our normalized LR frames \( I^r_{LR} \)), which result in imprecise alignments. To prevent this issue we compute the warping loss on the details rather than on the images, which is common in traditional optical flow [36, 49]. The loss is computed for each flow \( F_{i \rightarrow r} \) estimated by the MotionEst module

\[
\ell_{mse}(\{F_{i \rightarrow r}\}_{i=1}^{m}) = \lambda_1 TV(F_{i \rightarrow r}) + \sum_i \| \text{Detail}(I^r_{i} - \text{Pullback}(I^r_{LR}, F_{i \rightarrow r})) \|_1, \tag{12}
\]

where \( \text{Pullback} \) computes a bicubic warping of \( I^r_{i} \) according to a flow. \( \text{Detail} \) applies a high-pass filter, \( TV \) is the finite difference discretization of the classic Total Variation regularizer [48], and \( \lambda_1 = 0.003 \) is a hyperparameter controlling the regularization strength.

Training. The self-supervised training of HDR-DSP is done in two stages. We first pretrain the motion estimator on the simulated data to ensure that it produces accurate flows. Then, we train the entire system end-to-end with the pretrained MotionEst using the self-supervised loss (\( \lambda_2 = 3 \))

\[
\text{loss} = \ell_{self} + \lambda_2 \ell_{mse}. \tag{13}
\]

Other training details are in the supplementary material.

5. Experiments

For our experiments, we use real multi-exposure push-frame images (L1A) acquired by SkySat satellites [41]. For the quantitative evaluations we also simulated a multi-exposure and a single-exposure datasets from L1B products (super-resolved products by Planet with a factor of 1.25).

5.1. Simulated multi-exposure dataset

The two simulated datasets were generated from 1371 crops of L1B products (1096 train, 200 test, 75 val). First, we generate the noise-free LR images normalized to an exposure time of 1. Random subpixel translations of \( \{\Delta_i\}_{i=1}^{m} \) are applied to the ground truth followed by \( \times 2 \) subsampling

\[
\mathcal{I}^{LR}_{r} = \Pi_2(\mathcal{I}^{HR}_{r}),
\]

\[
\mathcal{I}^{LR}_{i} = \Pi_2(\text{Shift}_{\Delta_i}(\mathcal{I}^{HR}_{r})), \quad i \neq r \tag{14}
\]

where \( \Pi_2 \) is the subsampling operator. The exposure times are simulated as \( e_i = \alpha \epsilon_i \), where \( \epsilon_i \in \{-5, \ldots, 5\} \) and \( \alpha = \text{uniform}(1.2, 1.4) \). The noises \( n_i = \sqrt{ae_i} \mathcal{I}^{LR}_{i} + b N(0, 1) \) are then added to all the un-normalized frames to produce the noisy multi-exposure sequence \( \mathcal{I}^{LR}_{i} = e_i \mathcal{I}^{LR}_{i} + n_i \). The constants \( a = 0.119, b = 12.050 \) were estimated from real SkySat images with the Ponomarenko noise curve estimation method [16, 45]. The single-exposure dataset is generated in the same manner but with all \( e_i = 1 \). To simulate the exposure inaccuracies, during training and testing the \( e_i \) values are contaminated with noise within a range of 5%.

We use a PSNR score in our evaluation. The SkySat L1A images have a dynamic range of 12 bits, but we observed that the peak signal is at about 3400 DN. Therefore, our PSNR is normalized with a peak of 3400. We denote PSNR ME (resp. PSNR SE) as the average PSNRs computed on all the multi-exposure (resp. single-exposure) test sequences.

5.2. Ablation study

We study in Table 1 the importance of the base-detail decomposition. We consider simulated multi-exposure (ME)
Table 1. Handling of multi-exposure sequences with base-detail decomposition (BD) and using the un-normalized LR frames $I_{LR}^{HR}$ as an additional encoder input.

| Methods (all HDR-DSP ) | full | w/o BD | w/o BD (trained SE) | w/o LR |
|------------------------|------|--------|---------------------|-------|
| PSNR(db) ME            | 54.70| 53.76  | 52.91               | 53.94 |
| PSNR(db) SE            | 54.72| 54.16  | 54.54               | 54.16 |

Table 2. Feature pooling choice. Using average (A), maximum (M), and standard deviation (S) pooling improves the results.

| Features         | AMS (HDR-DSP ) | AS | AM | A |
|------------------|----------------|----|----|---|
| PSNR(db) ME      | 54.70          | 54.46| 54.44| 54.17 |
| PSNR(db) SE      | 54.72          | 54.47| 54.48| 54.20 |

Table 3. Handling variable number of frames (PSNR ME (dB)).

| Methods (all HDR-DSP ) | full | w/o $W^{HR}$ | HDR-DSP 4 | HDR-DSP 14 |
|------------------------|------|---------------|-----------|------------|
| 4 frames               | 52.61| 52.53         | 52.55     | 51.31      |
| 14 frames              | 55.85| 55.29         | 54.26     | 55.53      |
| variable n frames      | 54.70| 54.45         | 53.85     | 54.07      |

and single-exposure (SE) sequences presenting small exposure errors that match the ones observed in real sequences. If we train HDR-DSP without the proposed base-detail (w/o BD), the performance drops noticeably, which is also visible on real sequences (Figure 3). Even when training specifically for a single-exposure setting, as with DSA [43], the performance with base-detail is superior. In addition, we can see that removing the un-normalized LR frame from the encoder inputs (w/o LR) leads to a large performance drop for both single- and multi-exposure.

The experiment shown in Table 2 studies the impact of using multiple feature pooling strategies: average, maximum, and standard deviation. It shows that using the three greatly improves the results: about 0.5dB with respect to just using average. We observed that not including the average among the pooling strategies yields much worse results.

The aggregation weight feature $W^{HR}$ was added to improve the handling by the decoder of sequences with variable number of input frames. The results in Table 3 confirm the importance of providing these weights. We also compare with networks trained for a fixed number of frames (HDR-DSP 4 and 14) and observe that in this case the performance drops even when testing for those specific configurations. We conclude that the weights become useless if the training does not consider a variable number of frames.

Lastly, removing the grid shifting (§4.2) from the training also reduces the PSNR ME: from 54.70 to 54.49dB.

5.3. Comparison with the state-of-the-art

We compare our self-supervised network on the simulated dataset against state-of-the-art MISR methods for satellite images: DSA [43], HighRes-net (HR-net) [17], RAMS [50], and ACT [7]. A weighted Shift-and-add [38] with bicubic splatting adapted to multi-exposure sequences (ME S&A) serves as the baseline. HR-net and RAMS are two supervised networks designed to perform super-resolution of multi-temporal PROBA-V satellite images. In the context of push-frame satellites, we use the reference-aware version [42] of HR-net and RAMS rather than the original approaches, as they achieve higher quality results. DSA and ACT are two state-of-the-art super-resolution methods for SkySat imagery. ACT also serves as a proxy for comparison with other interpolation-based methods from the literature [56].

We adapt these methods to multi-exposure sequences. The deep learning approaches are fed with the normalized input images, whereas for ACT method we apply the same base-detail decomposition described in §4.1 and use ACT to restore the details (denoted BD-ACT). The registration step of ME S&A, BD-ACT, and RAMS are done with the inverse compositional algorithm [10, 13], which is robust to noise and brightness changes. The motion estimator of DSA is also trained with the loss on the details (§4.2).

Table 4 shows a quantitative comparison of the methods over the test set in the case of adding exposure time errors of 5% (as during training) and 20%. These errors are estimated from SkySat data (exposures ranging from 0.5 to 4.5 ms); see the supplementary material for details. Note that even with exact exposure times (row 0%), vignetting or stray light effects still justify the use of the proposed base-detail decomposition. Our self-supervised network ranks first in all cases with a significant gain of more than 1dB over all others (see Figure 4). Interestingly, the performance of most methods degrades quickly for large inaccuracy in exposure times. Only the methods using the base-detail decomposition (BD-ACT and ours) are robust to these inaccuracies. Note that HDR-DSP has never seen errors of 20% during training.

5.4. Results on real data

The proposed self-supervised training allows to train HDR-DSP on real multi-exposure sequences taken from SkySat satellites. From the L1A product of Planet SkySat, we extracted 2500 sequences (128 × 128 pixels) pre-registered up-to an integer translation. Out of 2500 sequences, 300 are used for testing. Each sequence contains
Figure 4. Super-resolution from a synthetic multi-exposure sequence (5% exp. error) of 15 aliased LR images. Methods are trained on a synthetic dataset and receive as inputs the normalized ME images except BD-ACT and HDR-DSP, which use the base-detail decomposition.

Figure 5. Super-resolution from a real multi-exposure sequence of 9 SkySat images. The first line corresponds to 4 normalized LR images in that sequence with different exposure times. The second line shows the reconstructions by Planet (L1B), DSA, BD-ACT and our method HDR-DSP.

from 4 to 15 frames. In about 75% of the sequences the exposure time varies within each sequence and we used the exposure time information provided in the metadata.

Figure 5 compares HDR-DSP against Planet L1B, DSA, and BD-ACT. The top row shows four normalized frames of the sequence, where we can notice the dependence of the noise level on the exposure time. The method used in the Planet L1B product is unknown. It super-resolves by a factor of 1.25 but contains noticeable artifacts and lacks fine details. The result from DSA exhibits a high-frequency pattern due to the imprecise exposure times. BD-ACT is able to cope with the exposure changes thanks to the base-detail decomposition, but the result is still very noisy. In contrast, HDR-DSP shows a clean and detailed reconstruction.

Figure 1 also shows a multi-exposure LR sequence along with the results from ME S&A, Planet L1B, ACT, DSA and HDR-DSP. Comparing HDR-DSP with DSA, we see that the former provides a cleaner result thanks to the base-detail decomposition and the proposed improvements over the DSA architecture and training procedure, which is also observed in the synthetic experiments.

6. Conclusion and limitations

The proposed HDR-DSP method is able to reconstruct high-quality results from multi-exposure bursts, providing fine details, low-noise, and high dynamic range. The proposed base-detail processing allows robustness to errors in the exposure time that are common in practice. In addition, a significant performance improvement is obtained by making the image encoding dependent on the noise variance, and using a new feature pooling designed to capture richer representations. Thanks to its fully self-supervised training, the method requires no ground truth and can thus be applied on real data. We show its effectiveness by training a model that super-resolves multi-exposure SkySat L1A acquisitions, leading to a substantial resolution gain with respect to the state-of-the-art.

Limitations. The context of remote sensing allows one to make additional assumptions that do not hold in more general settings: 1. The considered noise levels are away from the challenging photon-limited regime; 2. Motion and occlusions are much easier to handle. In particular, the latter point should be improved to apply this method to video or burst super-resolution. Besides, the proposed method does not handle saturation. This will be studied in future work.

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