Synthetic Stellar Photometry – I. General considerations and new transformations for broad-band systems.

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ABSTRACT
After a pedagogical introduction to the main concepts of synthetic photometry, colours and bolometric corrections in the Johnson-Cousins, 2MASS, and HST-ACS/WFC3 photometric systems are generated from MARCS synthetic fluxes for various [Fe/H] and [α/Fe] combinations, and virtually any value of $E(B-V) \lesssim 0.7$. The successes and failures of model fluxes in reproducing the observed magnitudes are highlighted. Overall, extant synthetic fluxes predict quite realistic broad-band colours and bolometric corrections, especially at optical and longer wavelengths: further improvements of the predictions for the blue and ultraviolet spectral regions await the use of hydrodynamic models where the microturbulent velocity is not treated as a free parameter. We show how the morphology of the colour-magnitude diagram (CMD) changes for different values of [Fe/H] and [α/Fe]; in particular, how suitable colour combinations can easily discriminate between red giant branch and lower main sequence populations with different [α/Fe], due to the concomitant loops and swings in the CMD. We also provide computer programs to produce tables of synthetic bolometric corrections as well as routines to interpolate in them. These colour–metallicity relations may be used to convert isochrones for different chemical compositions to various bandpasses assuming observed reddening values, thus bypassing the standard assumption of a constant colour excess for stars of different spectral type. We also show how such an assumption can lead to significant systematic errors. The MARCS transformations presented in this study promise to provide important constraints on our understanding of the multiple stellar populations found in globular clusters (e.g., the colours of lower main sequence stars are predicted to depend strongly on [α/Fe]) and of those located towards the Galactic bulge.

Key words: techniques: photometric — stars: atmospheres — stars: fundamental parameters — Hertzsprung-Russell and colour-magnitude diagrams — globular clusters: general

1 INTRODUCTION
Photometric systems and filters are designed to be sensitive temperature, gravity, or metal abundance indicators and thereby to complement spectroscopic determinations of the fundamental properties of stars. Moreover, when studying more complex systems such as star clusters and galaxies, the integrated magnitudes and colours of stars can be used to infer the ages, metallicities, and other properties of the underlying stellar populations. To accomplish this, filter systems are tailored to select regions in stellar spectra where the variations of the atmospheric parameters leave their characteristic traces with enough prominence to be detected in photometric data. Beginning with the influential papers by Johnson (1966) and Strömgren (1966), which describe the basis of broad- and intermediate-band photometry, a large number of systems exists nowadays, and more are being developed with the advent of extensive photometric surveys (see e.g., Bessell 2005, for a review).

Broad-band colours are usually tightly correlated with the stellar effective temperature ($T_{\text{eff}}$), although metallicity ([Fe/H]) and (to a lesser extent) surface gravity (log $g$) also play a role, especially towards the near ultraviolet (UV) and the Balmer discontinuity (e.g., Eggen et al. 1962; Ridgway et al. 1980; Bell & Gustafsson 1989; Alonso et al. 1996; Ivezić et al. 2008; Casagrande et al. 2010). On the other hand, intermediate- and/or narrow-band filters centred on specific spectral features can have a much higher sensitivity to stellar parameters other than $T_{\text{eff}}$ (e.g., Strömgren 1966; Wing 1967; McClure & van den Bergh 1968; Golay 1972; Mould & Siegel 1982; Worthey et al. 1994). While broad-band photometry can be easily used to map and study extended stellar populations and/or large fractions of the sky, also at faint magnitudes (e.g., Stetson et al. 1998; Bedin et al. 2004; Ivezić et al. 2007; Saito...
et al. 2012), intermediate- and narrow-band photometry is more limited in this respect, although still very informative (e.g., Mould & Bessell 1982; Bonnell & Bell 1982; Yong et al. 2008; Arnadóttir et al. 2010; Casagrande et al. 2014a).

In principle, determining stellar parameters from photometric data is a basic task, yet empirical calibrations connecting them are generally limited to certain spectral types and/or involve substantial observational work. In recent years, we have made a considerable effort to derive empirical relations that link photometric indices to the effective temperatures and metallicities of dwarf and subgiant stars (Casagrande et al. 2010, 2011). In particular, our studies of solar twins have enabled us to accurately set the zero-points of both the [Fe/H] and $T_{\text{eff}}$ scales for the first time (Meléndez et al. 2010; Ramírez et al. 2012; Casagrande et al. 2012) (Similar work is currently underway for giants, see also Casagrande et al. 2014b). The zero-point of photometrically derived $T_{\text{eff}}$ is also intimately connected to the absolute calibration of photometric systems; i.e., it is crucial to convert magnitudes into fluxes and vice versa, be they synthetic or observed.

Parallel to this empirical approach, in the spirit of VandenBerg & Clem (2003) and Clem et al. (2004), we have also tested the performance of synthetic colours against field and cluster stars of different evolutionary stages (VandenBerg et al. 2010; Brassée et al. 2010). Such comparisons, which have been carried out using the Victoria-Regina models (VandenBerg et al. 2012) and an initial set of colour transformations based on one-dimensional MARCS synthetic stellar fluxes (Gustafsson et al. 2008), have validated the accuracy of the adopted library in predicting broad-band colours for a wide range of $T_{\text{eff}}$ and [Fe/H] relevant to dwarf stars and subgiants. Some discrepancies between the predicted and observed colours of giants were found, but they could be telling us that the model temperatures are not realistic because of deficiencies in, e.g., the treatment of convection or the atmospheric boundary condition. It is also possible that these differences stem from abundance anomalies in some of the globular clusters that were used in these studies.

In this paper, we continue our efforts to provide reliable MARCS synthetic colours for different [Fe/H] and [$\alpha$/Fe] combinations in the most commonly used optical and infrared (IR) photometric systems (Figure 1). In particular, we take full advantage of the accuracy we have established concerning the absolute calibration of photometric systems to compute synthetic colours with well defined zero-points. Also, for the first time, we generate synthetic colours accounting for different values of the reddening, $E(B - V)$, thus bypassing the usual assumption that the colour excess is independent of spectral type. Our purpose here is thus twofold: together with assessing the performance of the latest MARCS grids of synthetic fluxes in different bands, we provide several computer programs (in Fortran) to produce, and interpolate in, tables of bolometric corrections (BCs, see Appendix A). The resultant colour–$T_{\text{eff}}$–metallicity relations are given for the most commonly used broad-band systems.

2 SYNTHETIC PHOTOMETRY

The basic idea of synthetic photometry is to reproduce observed colours (be they stars, planets, or galaxies) based simply on input (usually theoretical) spectra $f_{\lambda}$, and a characterization of the instrumental system response function $T_{\lambda}$ under which the photometric observations are performed (i.e., the total throughput of the optics, detector, filter, . . . over a spectral range $x_a$ to $x_b$). If observations are ground-based, then the atmospheric transmission must also be included in the throughput: this is often done at a nominal airmass of 1.3. Thus, in its most general and simplest form, a synthetic magnitude can be written as proportional (via a logarithmic factor, see Section 2.1) to $\int_{x_a}^{x_b} f_{T_{\lambda}} \, dx$, where the integration over $x$ is carried out either in wavelength or frequency space. This relation also highlights the fact that a magnitude can be simply thought of as an heterochromatic measurement, where the information encoded in the spectrum is weighted by $T_{\lambda}$. Depending on the spectral coverage and the transmissivity of $T_{\lambda}$, the result will depend more or less strongly upon (some of) the physical parameters of the underlying spectrum. By employing different filter combinations, it is then possible to highlight certain spectral features. If we are interested in stellar colours, these could be, for instance, the slope of the continuum, the depression owing to metal lines, the Balmer discontinuity, etc., which in turn tell us about the basic stellar parameters.

The advantage of using synthetic libraries is that they cover uniformly and homogeneously a range of stellar parameters — a feature which is appealing for a number of reasons including, in particular, the transposition of theoretical stellar models from the $T_{\text{eff}}$-luminosity plane onto the observational colour-magnitude diagram (CMD). Several papers deal with this topic, and excellent discussions on synthetic photometry can be found in e.g., Bessell et al. (1998), Girardi et al. (2002) and Bessell & Murphy (2012).

2.1 From fluxes to magnitudes

Even though the physical quantities that we work with are fluxes, in astronomy it is customary to deal with magnitudes, a concept that is thought to have originated with Hipparchus. In his system, the brightest stars in the sky, first magnitude, were considered to be about twice as bright as those of second magnitude, and so forth until the faintest stars visible with the naked eye, which are about one hundred times fainter, were classified as sixth magnitude. This concept was formalized by Pogson (1856), who conveniently proposed that a first magnitude star be 100$^{-5} \approx 2.5$ times as bright as a second magnitude star. This underpins the definition of magnitudes as proportional to $-2.5 \log$ of the flux, i.e.,

$$m_1 - m_2 = -2.5 \log \int_{x_a}^{x_b} f_{\lambda_1} T_{\lambda_1} \, dx \int_{x_a}^{x_b} f_{\lambda_2} T_{\lambda_2} \, dx$$

(1)

where in the case of Pogson $T_{\lambda_1}$ was the transmission of the eye, and using $m_2$ and $f_{\lambda_2}$ as standard star (or the Hipparchus “first magnitude stars”), the heterochromatic definition of a photometric system is readily obtained (see e.g., Eq. 8). The fact that a non-linear transformation relates magnitudes to fluxes, is a potential source of bias, which we deal with in Appendix B. Here it suffices to mention that this bias arises when using poor quality data, and assuming that Gaussian random errors in fluxes are mapped into Gaussian random errors in magnitudes, and vice versa.

2.2 Photon counting versus Energy integration

Before dealing with specific magnitude systems one further concept must be introduced. The system response function $T_{\lambda}$ represents the total throughput in reaching the observer, which is affected by everything lying between a photon as it arrives at the...
top of the Earth’s atmosphere and its final detection (e.g., Golay 1974). A proper characterization of $T_f$ is therefore non-trivial, although nowadays it is accurately known for most photometric systems. Among the different stages and components which define the total throughput, the most important ones are arguably the filter itself and the response function of the detector. The latter must be taken into account in the response function of the overall system because, in all instances, the flux is evaluated after entering the detector. Because of this, one must distinguish between photon-counting (e.g., CCD) and energy-integration (e.g., photomultiplier) detectors (e.g., Bessell 2000; Maíz Apellániz 2006; Bessell & Murphy 2012).

For a CCD system, the rate of arrival of photons (number per unit time per detector area) from a source having a flux $f_i$ is $\int \frac{d\lambda}{\lambda} T_f d\lambda$, while the energy measured in the same window of time over the same detector area is simply $\int f_i T_\lambda d\lambda$. Since a CCD counts photons (or actually converts photons into electrons with a given efficiency), it follows that $\int \frac{d\lambda}{\lambda} T_\lambda d\lambda = T_\lambda \int f_i T_\lambda d\lambda$; i.e., the energy formalism can also be used for a CCD, if $T_f$ is replaced by $\lambda T_\lambda$ (or equivalently, $T_f$ by $\lambda$ if working in frequency space). A common source of confusion when using published response functions for different photometric systems is whether or not $T_f$ has already been multiplied by $\lambda$. Throughout this paper, the formalism that we introduce for heterochromatic measurements always assumes that $T_f$ is provided in energy integration form and that all synthetic colours are generated for photon-counting systems; i.e., we explicitly introduce the $\lambda T_\lambda$ (or $\int f_i T_\lambda d\lambda$) term in the equations of synthetic photometry. (Note that the $hc$ constant disappears because of the normalization over $T_\lambda$). Since energy integration response functions are the correct ones to use in the photon-counting formalism, they are sometimes also called photonic response functions (e.g., Bessell & Murphy 2012). Among the existing photometric packages, the most commonly used one is Synphot (Laidler et al. 2008), which comply with the photon-counting formalism (and photonic response functions) described here. Needless to say, in a number of cases where the published values of $T_f$ already include the $\lambda$ dependence (see Table 1), the photon-counting synthetic quantities have been calculated using the energy-integration formalism.

2.3 ST mag system

ST monochromatic magnitudes (i.e. per unit wavelength) are defined as

$$m_{ST} = -2.5 \log f_i + ZP_\lambda$$

(2)

where, by construction, a constant flux density per unit wavelength $f_\lambda^0 = 3.631 \times 10^{-22}$ erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$ is defined to have $m_{ST} = 0.0$, thus implying $ZP_\lambda = -21.10$. Since in reality all measurements are heterochromatic, it follows that ST magnitudes are in fact defined over the wavelength range $\lambda_1$ to $\lambda_2$ which characterizes a given bandpass $\xi$ that has a system response function $T_\xi$.

$$m_{ST} = -2.5 \log \int_{\lambda_1}^{\lambda_2} f_i T_\xi d\lambda - 21.10 = -2.5 \log \int_{\lambda_1}^{\lambda_2} f_i T_\xi d\lambda / \int_{\lambda_1}^{\lambda_2} T_\xi d\lambda$$

(3)

2 http://www.stsci.edu/institute/software_hardware/stsdas/synphot and its python version http://stsdas.stsci.edu/pySynphot

To express the above $\lambda$-formalism in terms of a constant flux density per unit frequency, the only change that needs to be made to Eq. (3) is to replace $f_i^0$ by $f_i^0 / \nu$.

2.4 AB mag system

Conceptually identical to the ST magnitudes, the AB system is defined as

$$m_{AB} = -2.5 \log f_i + ZP_\nu$$

(4)

where, in this case, $m_{AB} = 0.0$ corresponds to a constant flux density per unit frequency $f_i^0 = 3.631 \times 10^{-20}$ erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$, thus implying $ZP_\nu = -48.60$. Integrating over frequencies (heterochromatic measurement)

$$m_{AB} = -2.5 \log \int f_i^0 T_\nu d\nu - 48.60 = -2.5 \log \int f_i^0 T_\nu d\nu / \int T_\nu d\nu$$

(5)

which is identical to, e.g., the photon-counting definition given by Fukugita et al. (1996). Recasting Eq. (5) in terms of wavelength results in

$$m_{AB} = -2.5 \log \frac{\int f_i^0 T_\lambda d\lambda}{\int T_\lambda d\lambda} - 2.5 \log \frac{f_i^0}{c} - 2.5 \log \frac{\int f_i^0 T_\lambda d\lambda}{\int T_\lambda d\lambda} =$$

$$-2.5 \log \frac{\int f_i^0 T_\lambda d\lambda}{\int T_\lambda d\lambda} - 2.5 \log \frac{f_i^0}{c} - 2.5 \log \frac{\int f_i^0 T_\lambda d\lambda}{\int T_\lambda d\lambda}$$

(6)

where the dimensionless quantity $-2.5 \log \frac{f_i^0}{c} = 18.6921$. With $\lambda_{Pivot,\xi} = \left(\frac{\int f_i^0 T_\xi d\lambda}{\int T_\xi d\lambda}\right)^{\frac{1}{2}}$, we obtain

$$m_{AB} = m_{ST} - 5 \log \lambda_{Pivot,\xi} + 18.6921.$$  

(7)

From Eq. (7) it follows that AB and ST magnitudes are identical at a wavelength $\lambda = 5475$ Å. As Oke & Gunn (1983) chose to use the absolute flux of $\alpha$ Lyr (Vega) at 5480 Å to define the wavelength in AB and ST magnitudes, the flux values $f_i^0$ and $f_i^0$ are chosen so that, for convenience, $\alpha$ Lyr has very similar magnitudes in all systems, ST, AB, and VEGA. Note that a revision of the actual flux scale of $\alpha$ Lyr (or any network of spectrophotometric standards) affects only the way that photometric observations are standardized to those systems, not their definitions. (That is, when standardizing observations, zero-point shifts can be applied in order to exactly match the original definitions).

2.5 VEGA mag system

This system, which is the most well known one, uses $\alpha$ Lyr (Vega) as the primary calibrating star, as in the case of the most famous and still widely used Johnson-Cousins system (and many others). Historically, the zero-points of the Johnson system were defined in terms of unreddened main-sequence (MS) stars of class A0... with
an accuracy sufficient to permit the placement of the zero-point to about 0.01 mag\(^{\prime}\) (Johnson & Morgan 1953). While observationally this zero-point is often defined by a network of standard stars, formally it can be anchored to just one object, the usual choice being \(\alpha\) Lyr. The definition of the VEGA system is given only for heterochromatic measurements

\[
m_{\text{VEGA}} = -2.5 \log \frac{\int_{\lambda_{\text{f}}}^{\lambda_{\text{h}}} \lambda f_\lambda T_\lambda \text{d}\lambda}{\int_{\lambda_{\text{f}}}^{\lambda_{\text{h}}} \lambda T_\lambda \text{d}\lambda} + ZP_\zeta
\]

where \(ZP_\zeta\) is derived for each bandpass \(\zeta\) using a star of known absolute flux \(f_\lambda = f_\star\) and observed magnitude \(m_{\text{VEGA}} = m_{\star,\zeta}\) (e.g., Bessell et al. 1998; Girardi et al. 2002; Casagrande et al. 2006, where our adopted magnitudes for Vega are reported in the third column of Table 1). Thus

\[
ZP_\zeta = m_{\star,\zeta} + 2.5 \log \frac{\int_{\lambda_{\text{f}}}^{\lambda_{\text{h}}} \lambda f_\star T_\lambda \text{d}\lambda}{\int_{\lambda_{\text{f}}}^{\lambda_{\text{h}}} \lambda T_\lambda \text{d}\lambda} = m_{\star,\zeta} + 2.5 \log f_\star(\zeta)
\]

where \(f_\star(\zeta)\) is the absolute calibration of the photometric system in the \(\zeta\) band. \(ZP_\zeta\) is thus of crucial importance for the synthetic photometry that we want to generate. To achieve the correct standardization, it is possible to modify either the adopted absolute calibration or the standard magnitude observed in the \(\zeta\) band (or both). When generating synthetic magnitudes, an evaluation of both quantities must be provided (see Table 1).

As already noted, \(\alpha\) Lyr is commonly used as the zero-point standard. Should its theoretical spectrum \(F_\lambda\) (see next Section) be modelled with sufficient precision, and its angular diameter \(\theta\) accurately measured, its absolute flux would be straightforward to obtain. Practice, its pole-on and rapidly rotating nature, as well as its suspected variability (although longward of 12\(\mu\)m, Aumann et al. 1984, and thus of no impact on the colours investigated here) complicate things (also see the discussion in Casagrande et al. 2006). In practice, the best approach resorts to the use of a composite absolutely calibrated spectrum for different wavelength regions. Arguably, the best absolute flux \(f_\star\) for \(\alpha\) Lyr is currently available from the CALSPEC library\(^4\), which intermingles Hubble Space Telescope (HST) absolute spectrophotometry in the range 1675 – 5350 \(\AA\), with an especially tailored model flux longward of this wavelength. The overall accuracy of this absolute flux is ~ 1\% (Bohlin 2007), which translates to about 0.01 mag in \(ZP_\zeta\). If not otherwise specified, this is the absolute flux that we have used when generating synthetic magnitudes in the VEGA system.

2.6 Reality Check

It follows from the previous sections that the formalism to define a photometric system is sound, independently of whether the ST, AB, or VEGA system is used. However, its actual realization at the telescope is far from being simple (see also Bessell 2005). Factoring in all possible sources of uncertainty in terms of the quantities we are discussing here means that the absolute flux, as well as the magnitudes, of the standard star might not be accurate (nor precise) enough (as is probably the case for \(\alpha\) Lyr), and/or the actual system transmission curves might not be in exact agreement with those tabulated. In fact, more often than not, (small) zero-point corrections are still needed to replicate the definition of a given photometric system as closely as possible (and more generally, linear terms rather than zero-points might even be more appropriate, even though they are rarely, if ever, considered). That is, for certain filters, a correction term \(\epsilon_\zeta\) might be added to the right-hand sides of Eq. (3), (5) and (8). Table 1 reports when such corrections are necessary. It is worth mentioning that these corrections depend on the definitions adopted for each photometric system; consequently, different authors might use opposite signs. The values listed here comply with the definitions adopted in the present paper.

2.7 Bolometric corrections

Libraries of synthetic stellar spectra provide the flux on a surface element \(F_\lambda\) (usually in erg s\(^{-1}\) cm\(^{-2}\) \(\AA^{-1}\)) for a given set of stellar parameters (see Section 3) from which it is straightforward to compute the theoretical absolute flux

\[
f_\star = 10^{-0.4A_\lambda} \left(\frac{R^2}{d}\right) F_\lambda,
\]

for a star of radius \(R\), at a distance \(d\), and (if present) suffering from interstellar extinction of magnitude \(A_\lambda\) at wavelength \(\lambda\). Such the-

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\(^4\) Regularly updated absolute spectrophotometry can be found at http://www.stsci.edu/hst/observatory/cdbs/calspec.html. If not otherwise specified, we have used the STIS005 spectrum in this investigation.
Table 1. Characteristic parameters defining the photometric systems studied here. The absolute calibration $f_{\epsilon,\lambda}$ (obtained using a spectrum of Vega as defined in Section 2.5) is given in erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$.

| Filter | $T_\epsilon$ | VEGA system | AB system | ST system |
|--------|--------------|--------------|-----------|-----------|
|        | $m_{\epsilon,\lambda}$ | $f_{\epsilon,\lambda}$ | $\epsilon_\xi$ | $\xi_\epsilon$ | $\xi_\epsilon$ |
| UX     | 1' +0.020    | 4.0275E-9    | 0         | –         | –         |
| BX     | 1' +0.020    | 6.2712E-9    | 0         | –         | –         |
| B      | 1' +0.020    | 6.3170E-9    | 0         | –         | –         |
| V      | 1' +0.030    | 3.6186E-9    | 0         | –         | –         |
| RC     | 1' +0.039    | 2.1652E-9    | 0         | –         | –         |
| IC     | 1' +0.035    | 1.1327E-9    | 0         | –         | –         |
| U      | 2 – – – –    | –0.761       | –         | –         | –         |
| B      | 2 – – – –    | +0.130       | –         | –         | –         |
| V      | 2 – – – –    | +0.020       | –         | –         | –         |
| RC     | 2 – – – –    | –0.163       | –         | –         | –         |
| IC     | 2 – – – –    | –0.404       | –         | –         | –         |
| J      | 3' −0.001    | 3.129E-10    | −0.017    | –         | –         |
| H      | 3' +0.019    | 1.133E-10    | +0.016    | –         | –         |
| Ks     | 3' −0.017    | 4.283e-11    | +0.003    | –         | –         |
| u      | 4 – – – –    | +0.037       | –         | –         | –         |
| g      | 4 – – – –    | −0.010       | –         | –         | –         |
| r      | 4 – – – –    | +0.003       | –         | –         | –         |
| i      | 4 – – – –    | −0.006       | –         | –         | –         |
| z      | 4 – – – –    | −0.016       | –         | –         | –         |
| F435W  | 5 0         | 6.4530E-9    | 0         | 0         | 0         |
| F475W  | 5 0         | 5.3128E-9    | 0         | 0         | 0         |
| F555W  | 5 0         | 3.8191E-9    | 0         | 0         | 0         |
| F606W  | 5 0         | 2.8705E-9    | 0         | 0         | 0         |
| F814W  | 5 0         | 1.1353E-9    | 0         | 0         | 0         |
| F218W  | 6 0         | 4.6170E-9    | 0         | 0         | 0         |
| F225W  | 6 0         | 4.1887E-9    | 0         | 0         | 0         |
| F275W  | 6 0         | 3.7273E-9    | 0         | 0         | 0         |
| F336W  | 6 0         | 3.0228E-9    | 0         | 0         | 0         |
| F350LP | 6 0         | 2.7464E-9    | 0         | 0         | 0         |
| F390M  | 6 0         | 6.5403E-9    | 0         | 0         | 0         |
| F390W  | 6 0         | 5.7932E-9    | 0         | 0         | 0         |
| F438W  | 6 0         | 6.6896E-9    | 0         | 0         | 0         |
| F475W  | 6 0         | 5.2240E-9    | 0         | 0         | 0         |
| F547M  | 6 0         | 3.6694E-9    | 0         | 0         | 0         |
| F555M  | 6 0         | 3.9511E-9    | 0         | 0         | 0         |
| F606W  | 6 0         | 2.9094E-9    | 0         | 0         | 0         |
| F625W  | 6 0         | 2.4383E-9    | 0         | 0         | 0         |
| F775W  | 6 0         | 1.3104E-9    | 0         | 0         | 0         |
| F814W  | 6 0         | 1.1486E-9    | 0         | 0         | 0         |
| F850LP | 6 0         | 8.0146E-10   | 0         | 0         | 0         |
| F909M  | 6 0         | 6.6878E-10   | 0         | 0         | 0         |
| F110W  | 6 0         | 4.0648E-10   | 0         | 0         | 0         |
| F125W  | 6 0         | 3.0448E-10   | 0         | 0         | 0         |
| F140W  | 6 0         | 2.0837E-10   | 0         | 0         | 0         |
| F160W  | 6 0         | 1.4553E-10   | 0         | 0         | 0         |

An asterisk in the second column indicates that $T_\epsilon$ given in the reference is already multiplied by $\lambda$ and renormalized. –1: Bessell (1990). –2: Bessell & Murphy (2012); notice that the formalism introduced in that paper for the $UBV(RI)c$ system is in terms of $AB$ magnitudes, although the system is actually VEGA based; $\epsilon_\xi$ values are obtained from their tables 3 and 5 (but with the opposite sign, following our definition of $\epsilon_\xi$). –3: Cohen et al. (2003); $\epsilon_\xi$ values from Casagrande et al. (2010). –4: SDSS website; $\epsilon_\xi$ values are obtained comparing the observed magnitudes of a number of HST CALSPEC standards with the magnitudes generated in the $AB$ system using the adopted $T_\epsilon$ with the HST absolute spectrophotometry. –5: http://www.stsci.edu/hst/acs/analysis/zeropoints. –6: http://www.int.stsci.edu/~WFC3/UVIS/SystemThroughput.

Theoretical determinations of $f_\xi$ can then be inserted into the formalism outlined in the previous sections in order to compute synthetic magnitudes. Often only synthetic colours (i.e., the difference in the magnitudes measured through two filters) are computed, since doing so cancels out the dependence on the dilution factor $\frac{\delta}{\epsilon}$. Similarly, if one wishes to compute the BC in a given band $BC_\xi$

$$BC_\xi = m_{\text{bol}} - m_\xi = M_{\text{bol}} - M_\xi$$

(11)

where the lower and upper cases refer to apparent and absolute magnitudes, respectively. For simplicity, when dealing with synthetic fluxes, we use the same dilution factor $R_\delta/d_\delta$ for all models, where $d_\delta = 10$pc, i.e. for the solar spectrum we also compute its
absolute magnitudes. For any models it follows that
\[
M_\lambda = -2.5 \log \left( \frac{R_\odot}{d_{10}} \right)^2 K_\lambda + zp_\lambda
\]  
(12)
where the exact form of \( zp_\lambda \) (which now includes \( \epsilon_\lambda \), if necessary) and \( K_\lambda \) (i.e., the integration of \( f_\lambda \) over the system response function) depends whether synthetic magnitudes are computed according to Eq. (3), (5) or (8). Also, from the definition of the absolute bolometric magnitude
\[
M_{bol} = -2.5 \log \frac{R^2 T^4_{\text{eff}}}{R^2_{\odot} T^{4}_{\text{eff},\odot}} + M_{bol,0},
\]  
(13)
because of the above choice on the dilution factor, the BC then becomes
\[
BC_\lambda = -2.5 \log \left( \frac{T_{\text{eff}}}{T^{\text{eff},\odot}} \right)^4 + M_{bol,0} + 2.5 \log \left( \frac{R_\odot}{d_{10}} \right)^2 K_\lambda - zp_\lambda,  
\]  
(14)
In this paper we have adopted \( M_{bol,0} = 4.75 \), but it should be appreciated that this value is simply a definition, not a measurement. In fact, BCs were originally defined in the visual only, in such a way as to be negative for all stars (Kuiper 1938). From the spectral energy distribution, we expect BCs in the visual to be largest for very hot and for very cool stars. The minimum would then occur around spectral type F, which then would set the zero-point of the bolometric magnitudes. For the Sun, this implied a value \( BC_{V\odot} \) between 0.11 (Aller 1963) and 0.07 (Morton & Adams 1968). However, with the publication of a larger grid of model atmospheres, e.g., the smallest BC in the Kurucz’s grid (1979) implied \( BC_{V\odot} = 0.194 \). These difficulties can instead be avoided by adopting a fixed zero-point, which is currently the preferred choice. Therefore, any value of \( M_{bol,0} \) is equally legitimate, on the condition that once chosen, all BCs are rescaled accordingly (e.g., Torres 2010). As a matter of fact, whenever bolometric magnitudes are published, be they apparent or absolute, the adopted \( M_{bol,0} \) should always be specified.\(^5\) We also note that using the solar values reported in Table 2, our choice implies \(-0.07 \leq BC_{V\odot} \leq -0.06.\)

2.8 Caveats with extinction

Before dealing with extinction, it is useful to clarify the wavelength with which heterochromatic measurements (magnitudes in this case) are associated. Any heterochromatic measurement can always be reduced to a monochromatic flux \( \tilde{f} \) (also called effective flux) via
\[
\tilde{f} = \int_{\lambda_1}^{\lambda_2} A \tilde{f}_\lambda T_\lambda d\lambda,
\]  
(15)
Assuming that the function \( f_\lambda \) is continuous and that \( T_\lambda \) does not change sign in the interval \( (\lambda_1, \lambda_2) \), the generalization of the mean value theorem states that there is at least one value of \( \lambda \) in the above interval such that
\[
f_\lambda \int_{\lambda_1}^{\lambda_2} A T_\lambda d\lambda = \int_{\lambda_1}^{\lambda_2} A \tilde{f}_\lambda T_\lambda d\lambda.
\]  
(16)
\(^5\) It follows from Eq. (13) that adopting \( M_{bol,0} = 4.75 \) with the GONG collaboration solar luminosity value \( 3.846 \times 10^{26} \) W, a star with \( M_{bol} = 0 \) has a radiative luminosity of \( 3.055 \times 10^{26} \) W, which is the flux zero-point of the BC scale according to the definitions adopted by IAU Commissions 25 and 36 at the IAU 1997 General Assembly.

Rearranging this, we obtain
\[
\tilde{f} = f_\lambda = \frac{\int_{\lambda_1}^{\lambda_2} A f_\lambda T_\lambda d\lambda}{\int_{\lambda_1}^{\lambda_2} A T_\lambda d\lambda}
\]  
(17)
where \( \tilde{\lambda} \) is the isophotal wavelength. The latter is thus the wavelength which must be attached to the monochromatic quantity \( \tilde{f} \) that is obtained from a heterochromatic measurement. Stellar spectra do not necessarily satisfy the requirement of the mean value theorem for integration, as they exhibit discontinuities. Although the mean value of the intrinsic flux is well defined, the determination of the isophotal wavelength becomes problematic because spectra contain absorption lines, and hence the definition of \( \tilde{\lambda} \) can yield multiple solutions (e.g., Tokunaga & Vacca 2005; Rieke et al. 2008). It is possible to avoid these complications by introducing the effective wavelength (here in photon-counting formalism)
\[
\lambda_{eff} = \frac{\int_{\lambda_1}^{\lambda_2} \lambda A f_\lambda T_\lambda d\lambda}{\int_{\lambda_1}^{\lambda_2} A f_\lambda T_\lambda d\lambda}
\]  
(18)
i.e., the mean wavelength of the passband weighted by the energy distribution of the source over that band, which is a good approximation for \( \tilde{\lambda} \) (Golay 1974). The effective wavelength thus shifts with \( f_\lambda \), depending on the source under investigation since it depends on both the extinction and the intrinsic stellar spectrum (Eq. 10). If present, extinction is usually parameterized as follows: \( A_f = R_f E(B-V) a(A^{-1}) + b(A^{-1}) R_f \), where \( E(B-V) \) is the colour excess (or reddening) and \( R_f \equiv \frac{A_f}{E(B-V)} \) is the ratio of total to selective extinction in the optical, assumed to be constant \( = 3.1 \) for most line of sights (e.g., Cardelli et al. 1989). The above parameterization is known as the extinction law. Note that the extinction law is derived primarily using early O- and B-type stars, i.e. it is expressed in terms of an \( A_f = R_f E(B-V) \) relation that is valid for these spectral types. Also, despite being given in a monochromatic form (i.e. with a \( \lambda \) dependence), it is actually derived using photometric (i.e. heterochromatic) quantities, which are valid at the effective wavelengths appropriate for the stars plus filters used to derive the extinction law. While the implications of these facts are discussed in the literature (e.g., Fitzpatrick 1999; McCall 2004; Maiz Apellániz 2013), their importance is often overlooked. We call the \( R_f \) and \( E(B-V) \) values which enter the extinction law “nominal”. The amount of attenuation in a given bandpass \( \zeta \) (i.e. the magnitude difference, where the subscript 0 indicates an unreddened source) is then
\[
m_\zeta - m_{\zeta,0} = -2.5 \log \left( \frac{1}{\int_{\lambda_1}^{\lambda_2} \lambda 10^{-0.4 A_f T_\lambda d\lambda}} \right) = A_f
\]  
(19)
and a given nominal \( E(B-V) \) and \( R_f \) will produce a different effective flux associated with a different effective wavelength, depending on the intrinsic spectral energy distribution \( F_\lambda \) of the source under investigation. In other words, the colour excess between two bands \( E(\zeta - \eta) = A_\zeta - A_\eta \) will vary with spectral class as well as with \( E(B-V) \) and \( R_f \). For example, assuming \( R_f = 3.1 \), the ratio \( E(a-\zeta)/E(B-V) \) in a metal-poor subgiant star ([Fe/H] = -3.0, log \( g = 3.5, T_{\text{eff}} = 6500 \) K) will vary from 3.333 to 3.319 assuming a nominal \( E(B-V) = 0.01 \) or 0.70, respectively. The same ratio will be 3.283 or 3.275 in a solar metallicity giant (log \( g = 2.0, T_{\text{eff}} = 4000 \) K), and 3.375 or 3.362 in a solar metallicity M dwarf (log \( g = 5.0, T_{\text{eff}} = 2500 \) K). If \( R_f = 2.5 \), which seems to be representative of selected directions towards the bulge (Nataf et al. 2013), the aforementioned numbers would change to 3.188
and 3.164 (metal-poor subgiant), 3.123 and 3.108 (solar metallicity giant), and 3.211 and 3.187 (solar metallicity M dwarf).

Based on the above considerations, it is also important to stress that the nominal $R_V$ and $E(B-V)$ values entering the extinction law — as derived from early type stars — change when moving towards later spectral types as a result of the change in effective wavelength. As an example, in the case of a late type star, the effective wavelengths of the $B$ and $V$ filters shift to longer wavelengths, where the extinction decreases, lowering both $A_B$ and $A_V$ for a given amount of dust. As a result, $R_V = \frac{A_V}{E(B-V)} = \frac{A_V}{E(V)}$ increases (even if the amount and physical properties of the dust are the same!) because the decrease in the denominator of this ratio overwhelms the decrease of the numerator. Thus, it is important to take this effect into account to distinguish whether a variation in $R_V$ stems from shifts in the effective wavelengths or from changes in the nature of the dust (see e.g., the excellent discussion in McCall 2004). For reference, a nominal $R_V = 3.1$ and $E(B-V) = 0.60$ in the aforementioned extinction law return $R_V = 3.4$ and $E(B-V) = 0.55$ for a moderately metal-poor turn-off star. (We will elaborate on our discussion of this practical example in Section 5 and Appendix A.)

These subtleties are usually neglected when, e.g., isochrones are fitted to the CMDs of halo globular clusters that have low values of the reddening, but they might not be negligible if the object under consideration is in a heavily obscured region of the Galactic disc or bulge. Furthermore, when using an $E(B-V)$ value from the literature, it should be kept in mind how this value was obtained; i.e., whether it is based on stars (and if so, which spectral types, e.g., Schlafly & Finkbeiner 2011), or even background galaxies (e.g., Schlegel et al. 1998). Indeed, because of the above issues, it should not come as a surprise to find that small adjustments to literature values of $E(B-V)$ are needed to obtain a satisfactory fit an isochrone to a given CMD (though this is just one of many possible causes of discrepancies between predicted and observed colours). In addition, as we already pointed out, the use of a constant colour excess across the entire CMD is not optimal in the presence of high extinction. We discuss in Section 5 how the set of reddened colours provided in this investigation enables one to address these subtleties.

### 2.9 Johnson-Cousins $UBV(RI)_C$ and 2MASS $JHK_S$

Synthetic colours in the Johnson-Cousins and 2MASS filter sets have been computed in the VEGA system only, and they have already been used in the investigations by VandenBerg et al. (2010) and Brasseur et al. (2010), where further details can be found. Briefly, the absolute calibration of the Johnson-Cousins system is obtained using the updated CALSPEC absolute spectrophotometry of $\alpha$ Lyr (Bohlin 2007) and the filter transmission curves of Bessell (1990b); the results are forced to match the observed magnitudes of Vega (Bessell 1990a). Because of problems standardizing the $U$ magnitudes, Bessell (1990b) provides two transmission curves for the $B$ system: one dubbed “$BX$” to be used in conjunction with “$UX$” for the computation of the $U-B$ colour index, and the other labeled “$B$” for use with any of the redder bandpasses $V(RI)_C$ to calculate,

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**Figure 2.** *Left panels:* difference between the $ubvri9$ and $ubvri12$ magnitudes. Each point is a model flux of different $T_{eff}$, log $g$ and [Fe/H], according to the values available from the MARCS library ($\alpha$-standard composition). *Right panels:* same difference for selected colour indices.
e.g., $B - V$ or $V - I_C$ colours. We have chosen to use the name $uvbri120$ to identify these magnitudes.

An additional set of synthetic Johnson-Cousins colours has also been computed following the prescriptions of Bessell & Murphy (2012), i.e., using improved filter transmission curves for the $U$ and $B$ bands (thereby removing the need to distinguish between, e.g., “BX” and “B”) and newly derived zero-points. The novel approach of Bessell & Murphy (2012) is that the Johnson-Cousins system is formulated in terms of $AB$ magnitudes, despite being on the VEGA system. We have named these magnitudes $uvbri12$. It is very comforting to find that the differences between the two sets of Johnson-Cousins magnitudes are relatively minor and essentially constant, never exceeding 0.02 mag (0.05 mag) above (below) 4000 K. The same is also true when colour indices are compared instead of magnitudes; with regard to $BVR_C$, in particular, the constant offsets found in magnitudes conspire to give nearly the same colours above 4000 K (see Figure 2).

For the 2MASS $JHK_s$ system, the $\alpha$ Lyr magnitudes, absolute calibration, and system response functions of Cohen et al. (2003) are used, including the small zero-point corrections found in Casagrande et al. (2010). The difference with respect to the CALSPEC absolute flux in the near-infrared is minor (see discussion in Casagrande et al. 2010), and our choice is motivated by the fact that the 2MASS absolute calibration adopted here has been thoroughly tested (Rieke et al. 2008; Casagrande et al. 2012). Note that both Bessell (1990b) and Cohen et al. (2003) system response functions are photon-counting, i.e., they are already multiplied by $\lambda$ and renormalized. Thus, $T_\lambda$ (not $\lambda T_\lambda$) must be used in the equations of synthetic photometry (Section 2). With these choices, the absolute calibration in these filters is essentially identical to that underlying the $T_{\text{cal}}$ scale of Casagrande et al. (2010) and verified via analyses of solar twins\(^7\).

### 2.10 Sloan Digital Sky Survey $ugriz$

SDSS photometry is defined to be on the $AB$ system, although small post-facto zero-point corrections are necessary (Eisenstein et al. 2006; Holberg & Bergeron 2006). We have generated $AB$ magnitudes using the SDSS filter transmission curves\(^8\) and determined the necessary zero-point corrections $\epsilon_z$ using stars with absolute flux measurements from the CALSPEC library and having SDSS $ugriz$ measurements (Casagrande et al., in prep.).

Two remarks must be made: first, the exact definition of SDSS magnitudes is on the asinh scale, where the classical Pogson logarithmic scale (Section 2.1) is replaced by the inverse hyperbolic sine function (“asinh magnitudes”, see Lupton et al. 1999). Conveniently, this definition is virtually identical to the standard Pogson magnitudes at high signal-to-noise, but it behaves better at magnitudes fainter than the detection repeatability limit (i.e. much fainter than 20$^{\text{th}}$ mag for SDSS). Brighter than this threshold the difference between the asinh and Pogson magnitudes is negligible and rapidly goes to zero. This makes the Pogson formulation outlined so far fully appropriate for generating synthetic magnitudes, besides being more convenient given the dependence of the asinh magnitudes\(^9\) on a softening parameter (survey dependent) and the difficulty of defining quantities such as bolometric corrections and absolute magnitudes in the asinh formulation (see e.g., Girardi et al. 2004).

The second remark concerns the potential confusion between the primed $ugr'iz'$ and unprimed $ugriz$ magnitudes. The basis of the photometric calibration of the SDSS is defined by the 158 primary standards measured by the USNO 40-inches telescope (Smith et al. 2002), which however has filters with different effective wavelengths than those of the SDSS 2.5m survey telescope\(^10\). The latter defines the unprimed $ugriz$ system for which photometry has been published from SDSS DR1 onward, and for which we provide synthetic colours in this work. Primed magnitudes should thus be transformed into the unprimed system before comparing and/or using them with the synthetic photometry published here (see e.g., Tucker et al. 2006).

### 2.11 HST

The distinctive feature of Hubble Space Telescope (HST) photometry is the use of the $AB$ and $ST$ systems, in addition to the VEGA system. We have generated BCs and colours for a number of filters commonly used in two instruments: the Advanced Camera for Surveys (ACS) is a third generation HST instrument which has provided some of the deepest photometric optical images ever obtained (Sarajedini et al. 2007), while the Wide Field Camera 3 (WFC3) is a fourth generation instrument (replacing the Wide Field Planetary Camera 2, WFPC2) that covers the electromagnetic spectrum from UV to the IR. For both cameras, a large number of filters is available, covering long-pass (LP), broad (W), intermediate (M) and narrow (N) bands. Some of them closely match such commonly used filters as those in the Johnson-Cousins, SDSS, Strömgren, and Washington systems (e.g., Bessell 2005). To provide continuity with previous observations, several of the WFC3 filters are very similar to those used in the WFPC2 and ACS instruments (e.g., Rajan et al. 2011). We have produced transformations in the $AB$, $ST$ and VEGA systems for the following filters: $F435W, F475W, F555W, F606W$ and $F814W$ (ACS camera); $F218W, F225W, F275W, F336W, F350LP, F390M, F390W, F438W, F475W, F547M, F555W, F606W, F625W, F775W, F814W, F850LP$ (WFC3 optical) and $F098M, F110W, F125W, F140W, F160W$ (WFC3 infrared).

Updated values for the system response functions as well as regular improvements to the absolute flux of $\alpha$ Lyr are provided by the HST team. We list in Table 1 our adopted values. The absolute calibration of $\alpha$ Lyr that we derived for the ACS system differs slightly from the values currently listed in the website\(^11\). These differences in absolute flux correspond to zero-point shifts of $\pm 0.01$ mag in $F435W$ and $< 0.005$ mag in the other ACS filters considered here. At any given time, users can readily compute the difference in magnitude implied by the $f_\lambda$ that we adopted, and they may use any updated/preferred value, shifting if necessary our computed colours by this amount.

We conclude this section with a cautionary note: particular

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\(^7\) The only difference is the $BV(RI)_{\text{cal}}$ absolute calibration in Casagrande et al. (2010), which is based on earlier HST spectrophotometry of $\alpha$ Lyr (as reported in Table A2 of Casagrande et al. 2006), producing $T_{\text{cal}}$ values that differ by only a few Kelvin. However, this difference essentially cancels out when the solar twins are used in the calibrating process; i.e., the absolute calibration derived from Bohlin (2007) agrees with that based on the solar twins.

\(^8\) http://www.sdss3.org/instruments/camera.php#Filters

\(^9\) https://www.sdss3.org/dr10/algorithms/magnitudes.php#asinh

\(^10\) see http://www.sdss.org/dr7/algorithms/fluxcal.html for an extensive explanation

\(^11\) http://www.stsci.edu/hst/acs/analysis/zeropoints/#vega
Figure 3. Difference in synthetic magnitudes and $BC_{\xi}$ as function of model $T_{\text{eff}}$ when using a microturbulence of 1 km s$^{-1}$ (blue) or 5 km s$^{-1}$ (red) with respect to standard value of $\xi = 2$ km s$^{-1}$. The filter used is indicated in each panel. In all instances models with $\alpha$-standard composition have been used. A fixed value of $\log g = 4.0$ and 2.5 has been used when comparing between 1 and 2 km s$^{-1}$ and 5 and 2 km s$^{-1}$, respectively.

care should be taken when working with stars that have cool effective temperatures and/or high value of reddening (the exact values of these quantities depend on the filter being used). Some of the UV filters are known to have red leaks towards longer wavelengths. Although these leaks are usually confined to much less than 1%, they may not be negligible in some cases (see table 6.5 in Rajan et al. 2011), which could cause the synthetic colours to be anomalous and the effective wavelengths to be considerably redder than the values expected from the nominal filter bandpasses.

3 MARCS SYNTHETIC LIBRARY

Magnitudes and colours have been computed using the MARCS grid of synthetic stellar spectra$^{12}$, which is available for $2500 \leq T_{\text{eff}}(K) \leq 8000$ (in steps of 100 K below 4000 K, and 250 K above this limit), $-0.5 \leq \log g \leq 5.5$ (in steps of 0.5 dex) and different chemical compositions (Gustafsson et al. 2008). In all instances, a microturbulence $\xi = 2$ km s$^{-1}$ has been adopted: we discuss later the effect of changing this parameter.

The reference solar abundance mixture is that of Grevesse et al. (2007). All radiative transfer is treated assuming local thermodynamic equilibrium. The bulk of the synthetic colours has been computed using MARCS model fluxes assuming the so-
called “standard chemical composition” (α-standard); i.e., a metals mixture that reflects the values of [α/Fe] typically observed in solar neighbourhood stars. To be specific, [α/Fe] = 0.4 for [Fe/H] ≤ −1.0, [α/Fe] = 0.0 for [Fe/H] ≥ 0.0, and [α/Fe] decreasing linearly from 0.4 to 0.0 for −1.0 ≤ [Fe/H] ≤ 0.0. While this α-enrichment is broadly representative of field stars in the Galactic disc and halo (e.g., Edvardsson et al. 1993; Nissen et al. 1994; Fuhrmann 2008), different abundance ratios can be more appropriate for other stellar populations, where the trend of [α/Fe] versus metallicity depends primarily on the initial mass function and the star-formation rate (e.g., Tinsley 1979; McWilliam 1997; Tolstoy et al. 2009). Models with different [α/Fe] are available from the MARCS website and we have generated additional synthetic colours for the following cases: i) α-enhanced (i.e. [α/Fe] = 0.4 above [Fe/H] = −1.0), ii) α-poor (i.e [α/Fe] = 0.0 below [Fe/H] = 0.0) and iii) α-negative (i.e [α/Fe] = −0.4 from super-solar to metal-poor). The MARCS library covers the range −5.0 ≤ [Fe/H] ≤ 1.0, although for certain combinations of stellar parameters the grid contains relatively few models. As discussed in Appendix A, computer programs have been provided to generate BCs (and hence colour indices) for nearly the entire MARCS grid as it presently exists.

When computing synthetic magnitudes, we assume one solar radius at a 10 pc distance for all models, i.e. \( f_\lambda = 10^{-0.4(A_\lambda)} \left( \frac{R}{R_\odot} \right)^2 F_\lambda \). While this choice is arbitrary (and actually irrelevant), it has the practical advantage that BCs can be computed very easily from synthetic magnitudes (see Eq. 14). To determine the effect of varying the \( E(B-V) \) from 0.0 to 0.72 on the BCs, we reddened each theoretical flux using the extinction law of O’Donnell (1994) in the range 3030 – 9090 Å and the one by Cardelli et al. (1989) everywhere else. In all instances, we adopted \( R_V = 3.1 \); an extension of our computations to other values of \( R_V \) is currently in progress.

Figure 4. Same as Figure 3, but as function of [Fe/H].
3.1 The role of microturbulence

In one-dimensional model atmospheres, the microturbulence $\xi$ is a free parameter which is introduced to broaden spectral lines when synthesizing a spectrum. It conserves the width of weak lines, and it can reduce the saturation of strong lines, thus increasing their total absorption. Its physical interpretation is that of a non-thermal small-scale motion (compared to unit optical depth, or the mean free path of a photon), stemming from the convective velocity field in a stellar atmosphere. Insofar as our investigation is concerned, the effect of microturbulence is to partly redistribute the flux in spectral regions that are crowded with lines, and thus its effects are expected to be more evident towards the blue and the ultraviolet, and in filters with smaller wavelength coverages.

There is no unique, nor clear-cut, relation between the microturbulence and stellar parameters, although the available evidence points towards $\xi$ increasing at higher luminosities. For reference, a typical value for dwarfs and subgiants is around $1 - 1.5 \text{ km s}^{-1}$, going up to $2 - 2.5 \text{ km s}^{-1}$ for stars on the red-giant branch (RGB), and possibly as high as $5 \text{ km s}^{-1}$ in supergiants (e.g., Nissen 1981; Gray et al. 2001; Allende Prieto et al. 2004; Roederer & Sneden 2011).

A constant value of $\xi = 2 \text{ km s}^{-1}$ is usually assumed in large grids of synthetic stellar spectra (e.g., Castelli & Kurucz 2004; Brott & Hauschildt 2005) and the same is true for the MARCS library as well. Fortunately, subsets of MARCS models that assume $\xi = 1 \text{ km s}^{-1}$ and $5 \text{ km s}^{-1}$ are also available, which enables us to investigate how a change in the microturbulence affects synthetic magnitudes.

Figures 3–4 compare the magnitudes which are obtained using $\xi = 1$ or $5 \text{ km s}^{-1}$ instead of $\xi = 2 \text{ km s}^{-1}$. Notice that it follows from Eq. (11) that these figures also illustrate the differences of $BC_{\xi}$ as a function of $T_{\text{eff}}$ and $[\text{Fe/H}]$. We opted to use the WFC3 optical filters for this demonstration, as they are representative of most of the colours computed in this work, and they provide a rich diversity in terms of wavelength coverage and filter widths. For the blue and ultraviolet filters, the effects of microturbulence are significant at all $T_{\text{eff}}$ values and they increase with increasing $[\text{Fe/H}]$: in certain cases, the differences are as much as 0.1 mag if $\xi$ is reduced from 2 to 1 km s$^{-1}$, or up to 0.5 mag if the microturbulence is increased to $5 \text{ km s}^{-1}$. While such a high value of $\xi$ might not be representative of the values commonly encountered in stars, it provides a very instructive example of how a fudge factor such as the microturbulence can impact synthetic colours. The only filter essentially insensitive to the effects of microturbulence is to partly redistribute the flux in spectral regions that can be generated when taking into account zero-point and absolute calibration uncertainties.

The impact of microturbulence in cool stars must therefore be kept in mind when comparing models with observations at wavelengths roughly bluer than $V$, especially for objects which are known to have microturbulent velocities that depart considerably

\begin{table}[h]
\centering
\caption{Solar absolute magnitudes, obtained using the MARCS synthetic solar spectrum ($\xi = 1 \text{ km s}^{-1}$, $T_{\text{eff}} = 5777 \text{ K}$, log $g = 4.44$ and [Fe/H] = 0.0) and by interpolating in the $\xi = 2 \text{ km s}^{-1}$ grid on the assumption of the same physical parameters. The filters are the same as in Table 1.}
\begin{tabular}{ccc}
\hline
M$_{\odot}$ & Solar & grid \\
\hline
UX & 5.569 & 5.615 \\
BX & 5.427 & 5.438 \\
B & 5.444 & 5.455 \\
V & 4.823 & 4.818 \\
RC & 4.469 & 4.459 \\
IC & 4.129 & 4.120 \\
U & 5.584 & 5.631 \\
B & 5.435 & 5.446 \\
V & 4.814 & 4.808 \\
RC & 4.453 & 4.444 \\
IC & 4.126 & 4.118 \\
J & 3.650 & 3.643 \\
H & 3.362 & 3.359 \\
K$_{S}$ & 3.277 & 3.274 \\
$\mu$ & 6.430 & 6.479 \\
g & 5.075 & 5.079 \\
r & 4.649 & 4.640 \\
i & 4.535 & 4.526 \\
z & 4.508 & 4.500 \\
F435W_{\text{ACS}} & 5.442 & 5.454 \\
F475W_{\text{ACS}} & 5.169 & 5.172 \\
F555W_{\text{ACS}} & 4.832 & 4.827 \\
F606W_{\text{ACS}} & 4.631 & 4.623 \\
F814W_{\text{ACS}} & 4.109 & 4.100 \\
F218W & 9.220 & 9.263 \\
F225W & 8.446 & 8.517 \\
F275W & 6.986 & 7.063 \\
F336W & 5.465 & 5.523 \\
F350LP & 4.751 & 4.748 \\
F390M & 5.987 & 6.018 \\
F390W & 5.611 & 5.638 \\
F438W & 5.447 & 5.460 \\
F475W & 5.146 & 5.149 \\
F547M & 4.800 & 4.794 \\
F555W & 4.868 & 4.864 \\
F606W & 4.639 & 4.652 \\
F625W & 4.493 & 4.484 \\
F775W & 4.156 & 4.147 \\
F814W & 4.115 & 4.106 \\
F850LP & 4.001 & 3.993 \\
F098M & 3.956 & 3.949 \\
F110W & 3.787 & 3.780 \\
F125W & 3.662 & 3.656 \\
F140W & 3.520 & 3.515 \\
F160W & 3.393 & 3.390 \\
\hline
\end{tabular}
\end{table}

HST magnitudes are all in the VEGA system.

from the standard assumption of 2 km s$^{-1}$. This dependence on $\xi$ clearly introduces an additional degree of freedom which can be avoided only by hydrodynamic simulations that treat the velocity field in a consistent manner (e.g., Steffen et al. 2013). At the present time, the impact of three-dimensional model atmospheres on synthetic colours is still largely unexplored (Kučinskas et al. 2005, 2009; Casagrande 2009), but hopefully the recent advent of hydrodynamic grids of model atmosphere (Beeck et al. 2013; Magic et al. 2013; Tremblay et al. 2013; Tanner et al. 2013) will soon make such studies possible.

\footnote{It is for this reason that the colours which are obtained by interpolating in our transformation tables for the solar values of log $g$, $T_{\text{eff}}$ and metallicity are somewhat different from those derived from the MARCS synthetic spectrum for the Sun, which assumes $\xi = 1 \text{ km s}^{-1}$.}
4 Comparisons with Observations

The testing of synthetic fluxes is normally done by comparing the observed and modelled spectral energy distributions for a range of wavelengths and spectral types (e.g., Edvardsson 2008; Bohlin 2010; Bessell 2011) or by comparing synthetic and observed colours in different bands and for different values of $T_{\text{eff}}$, log $g$, and [Fe/H]. Here we follow the latter approach. First, we compare synthetic colours in different filters with observations of field stars that have well determined physical parameters and magnitudes, as well as with empirical relations that link colours to stellar parameters. Then, we turn our attention to star clusters, which enable us to test the consistency of the fits of isochrones to the observed photometry on several different colour-magnitude planes.

Readers should bear in mind the caveats of generating synthetic photometry that have been discussed in Section 2, as well as the difficulty of standardizing photometric observations (ground-based, in particular) to better than about 0.01 mag (e.g., Stetson 2005). Therefore, when comparing synthetic and observed colours, agreement to within 0.01 to 0.02 mag should usually be regarded as excellent.

4.1 The Sun and other field stars

Arguably, the Sun is the star with the best-known parameters, and it is the most important benchmark in stellar astrophysics. However, for obvious reasons, it cannot be observed with the same instruments and under the same conditions as more distant stars, thus making it virtually impossible to tie its colours to other photometric observations (e.g., Stebbins & Kron 1957). Recently, the use of solar analogs and twins has overcome this limitation, and reliable colours are now available in the Johnson-Cousins and 2MASS systems (Ramírez et al. 2012; Casagrande et al. 2012). Figure 5 compares these measurements with the colour indices obtained using the MARCS solar synthetic flux (open circles). Differences are usually well within 0.02 mag, which is approximately the conjugated uncertainty of the observed and synthetic colours, and hence the accuracy at which our colours are testable. Also shown in Figure 5 is a similar comparison, but of colours which are obtained by interpolating in the $\xi = 2$ km s$^{-1}$ grid at the solar $T_{\text{eff}}$, log $g$, and [Fe/H]. The differences with respect to those based on the solar synthetic spectrum are negligible longward of the V band, whereas at wavelengths bluer than the B band they are more substantial (in accordance with the effects of microturbulence discussed in Section 3.1).

Figure 6 shows the comparison between the observed and synthetic $ugriz$ colours for a sample of about 13500 stars in the SEGUE DR8 catalogue (Yanny et al. 2009), flagged to have good photometric quality in all bands, and to cover a broad range of stellar parameters as determined from the SSPP pipeline (Lee et al. 2011, and references therein). MARCS synthetic colours are clearly quite successful in reproducing the observed trends on the different colour planes as function of both [Fe/H] and log $g$, especially given that the SSPP values have their own measurement uncertainties. Also, the match is decreasing sensitivity of $ugriz$ broad-band colours to [Fe/H] in the direction of reduced metallicities: note the reduced separations of the loci between $-1 > [\text{Fe/H}] > -3$, and even more so between $-3 > [\text{Fe/H}] > -5$.

For all of the stars in Figure 6, we also determine $T_{\text{eff}}$ and $f_{\text{bol}}$ using the Infrared Flux Method (IRFM) described in Casagrande et al. (2010). Briefly, multi-band optical and $ugriz$ and infrared $JHK_s$ photometry is used to recover the bolometric flux of each star, from which its effective temperature can then be readily obtained. The method is to a large extent empirical: it depends very mildly on the input [Fe/H] and log $g$ of each star (adopted from SSPP in our case) and it uses so many photometric bands that it relies very little on synthetic flux libraries — a claim that has been extensively tested in the literature (e.g., Blackwell et al. 1980; Alonso et al. 1995; Casagrande et al. 2006). In Figure 7, we compare, for each star, the empirical BCs derived in the $ugriz$ and $JHK_s$ systems when using $f_{\text{bol}}$ from the IRFM, or as obtained instead by interpolating in our grids of MARCS synthetic photometry at the [Fe/H], log (from SSPP) and $T_{\text{eff}}$ (from the IRFM) of each star.

This comparison allows us to easily quantify the performance of the MARCS synthetic fluxes in predicting broad-band colours on a star-by-star basis. In fact, it follows from Eq. (11) that a difference of $\Delta V$ mag in a bolometric correction corresponds to a fractional uncertainty of $10^{-0.4\Delta V}$ in the derived bolometric flux. The BCs calculated from MARCS models usually agree to within 0.02±0.07 mag with the values determined empirically from the IRFM for large ranges of $T_{\text{eff}}$, [Fe/H], and log $g$. Only in the $V$ band is the uncertainty larger, and it shows systematic deviations towards the lowest surface gravities. The above numbers translate into a typical uncertainty of about $2 \pm 7$ per cent in the bolometric fluxes that are obtained from the MARCS tables. It should also be kept in mind that $f_{\text{bol}}$ values from the IRFM typically have errors of a few percent. Overall, this implies that the grid of MARCS synthetic broad-band colours reported here can be used to reconstruct the bolometric flux of a star of known $T_{\text{eff}}$, log $g$, and [Fe/H] to within an accuracy of a few per cent, and with a precision comparable to what can be achieved with empirical approaches. Also, the comparisons presented in Fig. 7 for a wide range of physical parameters in the $ugriz$/$JHK_s$ system is representative of the wavelength domain explored in this work, and of the parameter space covered by the MARCS library as a whole.

4.2 Colour–$T_{\text{eff}}$–metallicity relations

Empirical colour–$T_{\text{eff}}$–metallicity relations offer another way to test how well the interplay among these quantities is reproduced by synthetic flux models — before we “attach” them to stellar isochrones
Synthetic photometry for stellar models

Figure 6. Comparison between observed and synthetic \textit{ugriz} colours (\( \alpha \)-standard, continuous lines) for a sample of about 13500 stars in SEGUE DR8. Stars are dereddened using \( E(B-V) \) values provided by SEGUE, sorted into panels according to surface gravity, and colour coded by metallicity (log \( g \) and [Fe/H] from the SSPP pipeline). MARCS (\( \alpha \)-standard) colours are shown for fixed [Fe/H] from \(-5.0\) to \(+0.5\) (as indicated) and log \( g = 2 \) (left), 3.5 (centre) and 4.5 (right-panel), for \( T_{\text{eff}} \geq 4000 \) K.

The subject of Section 4.3.) Using the routines described in Appendix A, we generated BCs in the \( BVI_{C}JK_{S} \) system (from which colour indices in any combination of the above filters can be derived) for two metallicities, [Fe/H] = 0.0 and \(-2.5\), and \( T_{\text{eff}} \) values from 4500 K to 6500 K, in steps of 100 K, at two fixed values of log \( g = 4.0 \) and 4.5. These choices broadly encompass the properties of most of the subgiants and dwarf stars in the Galaxy.

For a fixed log \( g \) we can then produce synthetic colour–\( T_{\text{eff}} \)–metallicity relations to be compared with the polynomials given by Casagrande et al. (2010): such comparisons are presented in Figure 8. The empirical relations have a “built-in” surface gravity dependence as they are valid for MS and subgiant stars. As a consequence, we chose to generate synthetic colours for values of \( T_{\text{eff}}, \) [Fe/H], and log \( g \) that are representative of the majority of the stars on which the empirical polynomials are based to do a better sampling of the parameter space which contains the observed stars. Figure 8 shows the effect of using two different gravities mainly to demonstrate that the exact choice of log \( g \) is not the dominant factor in this comparison.

It is also worth noticing that the absolute calibration, zero-points, and filter transmission curves used in Casagrande et al. (2010) to derive the empirical relations closely match those used to produce synthetic MARCS BCs in this study (see Section 2.9). The comparisons performed here are thus largely free from any systematic biases that the above choices could introduce, and thus allows us to directly assess the degree to which the synthetic fluxes are able to match empirical relations. Overall, synthetic colours reproduce the main trends that are seen in empirical relations, as well as the crossing-over of the fiducial sequences for different values of [Fe/H] in certain colour indices. The agreement is particularly good around 5500 K in all of the colour indices which are examined in Figure 8. The largest offsets are found for the \( B-V \) colour index at lower temperatures and higher metallicities — which is not too surprising, considering the increased crowding of molecular lines in metal-rich dwarfs. Similarly, the predicted \( J-K_{S} \) colours that we have derived from hot metal-rich (\( T_{\text{eff}} \gtrsim 6000 \) K) and cool metal-poor (\( T_{\text{eff}} \lesssim 5200 \) K) models increasingly depart from the empirical relations. In this case, however, it should be kept in mind that the scatter around the empirical \( J-K_{S} \) relation is quite large, partly stemming from its limited colour baseline, and that it seems to underestimate \( T_{\text{eff}} \) above 6000 K (Pinsonneault et al. 2012). Nevertheless, all things considered, the comparisons given in Figure 8 validate the MARCS effective temperature scale (for dwarfs and subgiants, in particular) when using indices such as \( V-I_{C} \) or \( V-K_{S} \), which have been shown to agree with the empirical Casagrande et al. (2010) scale, typically to much better than 100 K.

4.3 Open and Globular Clusters

Comparisons of computed isochrones with the observed colour-magnitude diagrams (CMDs) of star clusters provide valuable tests of the consistency of synthetic colour–\( T_{\text{eff}} \) relations because the same interpretation of the data should be obtained on all CMDs. This is not to say that the models must provide perfect reproductions of the observed morphologies, but rather that whatever discrepancies exist between theory and observations in one CMD

...
should be consistent with those that are apparent on other colour-magnitude planes. When that is not the case, this information can be used to pinpoint where inaccuracies occur in synthetic fluxes. To illustrate this, we have opted to consider the extensively studied globular cluster M 5, which has [Fe/H] = −1.33 according to Carretta et al. (2009), the old, super-metal-rich open cluster NGC 6791 ([Fe/H] ≈ +0.3; see Brogaard et al. 2012), and the near solar-metallicity open cluster M 67. (It should be appreciated that some discrepancies between predicted and observed CMDs in an absolute sense will generally be present due to the uncertainties associated with, e.g., the treatment of convection and the atmospheric boundary conditions in stellar models, as well as those connected with the photometric data and such basic cluster properties as the distance, reddening, and chemical abundances.)

In what follows, observations of these three systems are compared with the latest Victoria-Regina isochrones (VandenBerg et al. 2014) on several different CMDs. Rather than correcting the photometry for the effects of reddening and distance to obtain the intrinsic colours and absolute magnitudes, the model luminosities and temperatures have been converted to the observed magnitudes and colours using the reddening-adjusted transformations presented in this investigation. To accomplish this, values of $(m - M)_0$ (true
distance moduli) that agree well with published estimates have been assumed. In all instances, well-supported determinations of the metallicity have been adopted, along with $E(B-V)$ values from Schlafly & Finkbeiner (2011, hereafter SF11), except in the case of NGC 6791. For this cluster, a slightly higher reddening, by $\approx 0.02$ mag, appears to result in improved consistency over most of the colour planes that have been considered (see also discussion in Section 5, though it may instead be the assumed [Fe/H] value that is not quite right). Such issues are not of particular importance for the present analysis, however, as our primary goal is to study how the differences between the isochrones and the cluster CMDs vary with the particular colour index that has been plotted.

### 4.3.1 M 5 (NGC 5904)

Thanks to the efforts of P. B. Stetson, very high-quality, homogeneous Johnson-Cousins $BVRI_0$ photometry is available for relatively large samples of stars in many open and globular clusters (Stetson 2005). For a few of them, including M 5, $U$-band data may also be obtained from his “Photometric Standard Fields” archive at the Canadian Astronomy Data Centre. Indeed, his observations yield exceedingly tight, well-defined CMDs for M 5 from the tip of the RGB to $\sim 2$ mag below the turn-off (TO). A further reason for choosing to use this system to examine the colour transformations for metal-poor stars is that it is nearly unreddened [$E(B-V) = 0.032$ according to SF11].

Figure 9 plots the measured $V$ magnitudes of the stars in the observed fields of M 5 as a function of their $U - B$, $B - V$, and $B - R_c$ colours. The solid curve in each panel is an 11.75 Gyr (VandenBerg et al. 2013) isochrone for [Fe/H] = $-1.33$, [$\alpha$/Fe] = 0.4, and a helium abundance $Y = 0.25$. To place it on the observed planes, the so-called ubvri12 transformations for $E(B-V) = 0.032$ were used, together with $(m - M)_0 = 14.35$. Dashed loci represent the same isochrone, except that the ubvri1990 transformations have been employed: differences between them and the solid curves are apparent only in the left-hand panel. Historically, synthetic $U - B$ colours have not been very reliable, but those based on the latest MARCS model atmospheres appear to be quite good (if the metallicity and distance that we have assumed for M 5 are accurate). Indeed, if the predicted $U - B$ colours were adjusted to the blue by only $0.02$–$0.03$ mag, the isochrone would provide reasonably consistent fits to the observations on all three of the CMDs that are shown in Fig. 9. That is, independently of the colour that is considered, the isochrone would match the TO photometry satisfactorily and be somewhat too red along the RGB — except at $V \lesssim 15$ on the $(U - B, V)$-diagram, which suggests that the $U - B$ colours become less trustworthy at low gravities and/or cool temperatures. This is also in agreement to what was shown in Figure 7 for the $u$ filter.

The left-hand panel of Figure 10 shows that the same consistency is found on the $(V - I_c, V)$-plane. Interestingly, this plot indicates a slight preference for the ubvri12 transformations to $V - I_c$ over those that we have labelled ubvri90, though this result will depend on the model $T_{\text{eff}}$ scale, which is subject to many uncertainties. On the other hand, one would not expect to find such good agreement between the predicted and observed RGB morphologies unless the temperatures along the isochrones are quite close to those implied by the MARCS colour-$T_{\text{eff}}$ relations. As illustrated in the right-hand panel, very similar results are obtained when the same isochrone is compared with the HST ACS observations of

![Figure 8. Comparison between the empirical colour–$T_{\text{eff}}$–metallicity relations of Casagrande et al. 2010 ([Fe/H] = 0.0 red, [Fe/H] = $-2.5$ blue), and those obtained from the MARCS ($\alpha$-standard) fluxes at the same metallicity ([Fe/H] = 0.0 black, [Fe/H] = $-2.5$ grey) for two different gravities, log $g = 4.0$ (dotted line) and 4.5 (continuous line).](image-url)
M5 that were obtained by Sarajedini et al. (2007). However, in this case, the isochrone does not match the observations quite as well as in the left-hand panel: to achieve a fully consistent fit, the models would need to be adjusted to the blue by 0.01–0.015 mag. It is not clear whether this zero-point correction should be made to the colour transformations or to the photometric data (from either source).

The availability of Sloan (ugriz) observations of M5 by An et al. (2008) gives us the opportunity to extend our comparisons to this photometric system. Four of the many possible CMDs that could be constructed from their observations are shown in Figure 11, along with the same isochrone that has been plotted in the two previous figures, on the assumption of the same reddening and true distance modulus, etc. The fits to the data on the $g-i$ and $g-z$ colour planes closely resemble the fits to the $BV(RI)c$ observations discussed above, which implies satisfactory consistency between them. However, as in the case of the ACS photometry of M5, the isochrone lies ~0.02 to the red of the cluster MS and TO on the $(g-r, r)$-diagram, while it reproduces the observed $u-g$ colours (in the mean) for both the MS and and RGB nearly perfectly. To try to identify whether the $g$ or $r$ magnitudes are primarily responsible for the offset in the $g-r$ colours, we produced a plot on the $(r-i, r)$-plane (not shown). It revealed that the isochrone is too blue relative to the observed TO by about 0.02 mag, from which we conclude that the difficulty with the $g-r$ colours is mainly due to a small problem with the $r$ magnitudes. Whether this should be attributed to our transformations or to the observations, or both, is not known.

To improve our understanding of star clusters and of the photometric systems that are used, it is clearly advantageous to obtain observations through as many filters as possible. Encouragingly, most of the plots that we have produced for M5 contain basically the same fit of the selected isochrone to the observations. In other words, the models generally provide a consistent interpretation of the data, regardless of the particular CMD that is considered. However, we did find a few “exceptions to the rule”, which is to be expected given the difficulty of establishing the zero-points of synthetic or observed photometry to within ~0.01–0.02 mag and the uncertainties associated with current model atmospheres and synthetic spectra. In fact, the small colour offsets that we found on some colour-magnitude planes may also be a reflection of the assumed metal abundances; i.e., perhaps improved overall consistency would have been obtained had our analysis used stellar models that assumed a different mixture of the heavy elements. Although it is beyond the scope of the present investigation to do so, this possibility should be fully explored.

4.3.2 NGC 6791

Brogaard et al. (2011, 2012) have carried out the most exhaus-
tive studies of NGC 6791 to date, finding an age of 8–8.5 Gyr if \( Y \approx 0.30 \) and [Fe/H] = 0.30–0.35, when fits of stellar models to the mass-radius diagrams of the binaries known as V18 and V20 are taken into account. Their results are based predominately on isochrones that assume the solar mix of the heavy elements as reported by Grevesse & Sauval (1998), suitably scaled to the [Fe/H] values of interest. For the present work, which uses the latest Victoria-Regina isochrones (VandenBerg et al. 2014), the updated solar abundances by Asplund et al. (2009) provide the reference metals mixture. According to VandenBerg et al., their models require \( Y \approx 0.28 \) to satisfy the same binary constraints if NGC 6791 has [Fe/H] = 0.30 — and we have therefore adopted these estimates of the helium and metallicity.

Unfortunately, the reddening of NGC 6791 has been particularly hard to pin down. The value of \( E(B-V) = 0.155 \) from the original Schlegel et al. (1998) dust maps has been revised downwards to 0.133 mag by SF11. Moreover, as discussed by Brogaard et al. (2012), who concluded that the current best estimate of the reddening is \( 0.14+0.02 \), it appears to be difficult to obtain consistent fits to the \((B-V, V)-\) and \((V-I_c, V)\)-diagrams unless a small systematic shift is applied to one of the predicted colours. Part of the difficulty, as noted by them, is that the value of \( E(V-I_c)/E(B-V) \) appears to be uncertain (e.g., Hendricks et al. 2012, their Tables 11–13). As it turns out, it is possible to obtain nearly indistinguishable fits of isochrones to observations of NGC 6791 on many different CMDs (using our predictions for the effects of reddening on the synthetic magnitudes and colours) if the cluster has \( E(B-V) = 0.16 \).

This is shown in Figures 12 and 13 where, in turn, an 8.5 Gyr isochrone for the aforementioned chemical abundances has been overlaid onto a number of CMDs derived from \( BVJcJ \) and \( ugriz \) photometry of NGC 6791. All fits assume \((m-M)_0 = 13.05\). The \( BVJc \) photometry that we have used is contained in the NGC6791.KFB and NGC6791.COR files that can be downloaded from the “Homogeneous Photometry” link at the same CADC website address (see footnote 14) that was used to obtain the M 5 data. Although \( U\)-band observations are not available, we have been able to extend our comparisons to the near-infrared by adding a panel that plots the Johnson \( V \), 2MASS \( J \) photometry from Brasser et al. (2010). For whatever reason, the isochrone lies along the red edge of the distribution of MS stars on the \((V-J, V)\)-diagram, while it coincides closely with the blue edge in the other panels, as well as in at least three of the four CMDs shown in Fig. 13. [The isochrone appears to match the mean MS fiducial on the \((u-g, r)\)-plane, but the TO is close to the limit of the \( u \) photometry.] As in the case of M 5, the Sloan photometry of NGC 6791 that we have used has been taken from An et al. (2008).

Even in an absolute sense, the isochrone clearly reproduces the observed CMDs of NGC 6791 rather well from 3–4 mag below the TO to approximately the luminosity of the horizontal branch clump. At higher luminosities, the predicted colours tend to be too blue, indicating a problem with either the MARCS transformations or the model temperatures or both. (For quite a thorough discussion of this issue, reference may be made to VandenBerg et al. 2014.) Although Figs. 12 and 13 appear to provide compelling support for the derived (or adopted) properties of NGC 6791, we have not explored whether equally good fits of the isochrones to the observed CMDs (and the cluster binaries) can be obtained for different combinations of [Fe/H], the mix of heavy elements, \( Y \), \( E(B-V) \), \((m-M)_0 \), and age. This would be well worth investigating. Note that the un-
usual morphology of the upper RGB portion of the isochrone in the \((B-V, V)-(u-g, r)-(g-r, r)\)-diagrams is consistent with the predicted behaviour of the synthetic magnitudes in the respective filters, and are not a consequence of, say, extrapolations in the colour transformation tables. On the other hand, that morphology will be a sensitive function of the model \(T_{\text{eff}}\) scale; i.e., if the predicted RGB were appreciably cooler or hotter, the blueward excursion of upper RGB in the left-hand panel of Fig. 13 and the loop in the adjacent panel would be accentuated or minimized.

4.3.3 M 67 (NGC 2682)

Our emphasis thusfar has been on upper MS and more evolved stars because several recent papers have already shown that the slope of the MS down to \(M_V \sim 7–8\) in open and globular clusters and as defined by local subdwarfs, over a wide range in [Fe/H], can be reproduced quite well by stellar models that employ the MARCS colour transformations (see VandenBerg et al. 2014, 2013, 2010; Brasseur et al. 2010). Hence, for our final comparisons of isochrones with cluster observations, we decided to focus on the lower-MS extension of M 67, given that \(ugriz\) (An et al. 2008) and \(BVI_cJK_S\) photometry is available for low-mass stars belonging to this system. [The 2MASS observations (Skrutskie et al. 2006) were kindly provided to us by A. Dotter (see Sarajedini et al. 2009), while the Johnson-Cousins photometry is from the same CADC website mentioned previously (see footnote 14).]

In Figures 14 and 15, eight of the possible CMDs that can be produced from these data have been superimposed by the MS segment of the same 4.3 Gyr, [Fe/H] = 0.0 (e.g., Randich et al. 2006, Onehag et al. 2011), \(Y = 0.255\) isochrone that was compared with observations for the upper MS and RGB stars of M 67 by VandenBerg et al. (2014). In all cases, \(E(B-V) = 0.030\) (SF11) and \((m-M)_0 = 9.60\) has been assumed. Except for the \(J-K_S\) and \(u-g\) colour planes, the isochrone matches the MS stars that have \(V\) or \(r \lesssim 17\) very well. As expected, the observed \(B-V\) colours of cool, faint stars are more problematic than their \(V-K_S\) colours. Indeed, the close agreement of the isochrone with the observations on the \((V-K_S, V)\)-diagram suggests that the temperatures of the models are quite good (for additional discussion of this issue see VandenBerg et al. 2014). If this is correct, then the deviations which are apparent at faint magnitudes in the other CMDs must be telling us something about the MARCS transformations, or the photometric data, or perhaps that the assumed metallicity or metals mixture is wrong. As discussed in the next section, colours for metal-rich, lower MS stars are especially dependent on [Fe/H] and \([\alpha/\text{Fe}]\) (and presumably on any metal with a high abundance). (We note, in closing this section, that fits of completely independent isochrones to the Sloan data for M 67 were obtained by An et al. 2009.)

5 MORPHOLOGY OF THE CMD IN VARIOUS COLOUR PLANES AND AS FUNCTION OF REDDENING

With our set of stellar isochrones and colours, it is of some interest to study how variations in chemical composition — \([\alpha/\text{Fe}]\), to be specific — affect the morphology of the CMD when using different filter combinations. For this purpose, we consider two different cases. In Figure 16, we examine the effects of changing the \(\alpha\)-element abundances on the location and shape of an [Fe/H] = −0.8, 12 Gyr isochrone, which is roughly representative of moderately metal-rich globular clusters in the Milky Way. The upper left-hand panel shows three isochrones in the theoretical \(T_{\text{eff}}-M_{\text{bol}}\) diagram, all for the same iron content, but different values of \([\alpha/\text{Fe}]\). Because of this difference, the total mass-fraction abundance of the
metallicity (Z) is somewhat different in the three isochrones, which explains why they are slightly offset from each other. However, apart from this, there are no morphological changes.

The other panels of Figure 16 show the same three isochrones, but as transformed to various CMDs that involve the magnitudes predicted for various optical and infrared filters. In nearly all cases, the only part of the CMD that is substantially affected by variations in the abundances of the α-elements is the lower-MS (at $T_{\text{eff}} \lesssim 4000$ K). A detailed explanation of the cause(s) of this sensitivity is beyond the scope of this paper, but it stems essentially from the onset of various molecules (e.g., H$_2$O and TiO, which form more easily with increasing [α/Fe]) as important sources of blanketing. Interestingly, recent F110W and F160W photometry of two globular clusters (namely, NGC 2808 and M4, which both have [Fe/H] $\approx -1.2$, e.g., Carretta et al. 2009, making them at least 0.4 dex more metal-poor than the isochrones considered here) has revealed that they have split lower main sequences (Milone et al. 2012, 2014). Indeed, the observed morphologies are qualitatively quite similar to the results shown in the bottom right-hand panel, which suggests that star-to-star variations in the abundances of one or more of the α-elements is responsible for this phenomenon. It may be the case, for instance, that a split main sequence is one of the consequences of the presence of O-rich and O-poor populations within a given globular cluster. Further work is clearly needed to check out this speculation, but the main point of this example is that chemical abundance anomalies can be expected to leave detectable photometric signatures.

Figure 17 is identical to the previous one (in particular, the same variations of [α/Fe] are considered), except that somewhat younger isochrones (10 Gyr) for [Fe/H] = 0.0 have been plotted. The split in the lower-MS is even more dramatic, and certain filter combinations reveal interesting features for the upper CMD (above the TO) as well. As the abundances of the α-elements increase, the RGB swings to blue and red colours and even makes loops in the CMD in some cases. Again, a detailed discussion of the causes of such effects is beyond our purposes, but a visual inspection of synthetic RGB optical spectra provides a clue as to what is going on. At low α-element abundances ([α/Fe] = −0.4), the overall shapes of spectra essentially scale with decreasing $T_{\text{eff}}$ (implying that, e.g., the $g − r$ index gets monotonically redder). However, in the α-enhanced models, there is a dramatic dampening of the flux with decreasing $T_{\text{eff}}$, likely driven by the strengthening TiO bandheads. We suspect that this prevents some colour indices from getting monotonically redder with decreasing $T_{\text{eff}}$.

While we defer a more detailed explanation of these effects to a subsequent investigation, optical spectrophotometry of cool metal-rich giants (e.g., the Next Generation Spectral Library – NGSL– Heap & Lindler 2010 and the Medium Resolution INT Library of Empirical Spectra – MILES– Sánchez-Blázquez et al. 2006) can be used to assess the reliability of synthetic spectra in this regime. Nevertheless, the apparent high sensitivity of many colours to the temperatures and chemical abundances of metal-rich stars (both of giants and dwarfs, depending on the filters used) has considerable potential in advancing our understanding of, in particular, the Galactic bulge given that it may be possible to distinguish between α-rich and -poor populations.

Our final topic of discussion concerns the effects of extinction on the CMD. In most investigations, the extinction coefficient in a given band $R_{\lambda}$ is assumed to be constant for stars of different spectral types. That is, once a value of $E(B-V)$ is known, the colour excess in any colour other than $B-V$ is determined from $E(\zeta - \eta) = A_{\zeta} - A_{\eta} = (R_{\zeta} - R_{\eta})E(B-V)$, from which the unreddened colour index is

$$\langle \zeta - \eta \rangle = \langle \zeta - \eta \rangle - E(\zeta - \eta).$$

Similarly, for magnitudes

$$m_{\zeta,\eta} = m_\zeta - A_\zeta = m_\zeta - R_{\zeta}E(B-V),$$

and the distance modulus in a given band $\zeta$ (affected by extinction $A_\zeta$) is related to the true distance modulus via $(m - M)_\zeta = (m - M)_\eta - A_\zeta$. In Figure 18, we show in red a metal-poor isochrone on different colour-magnitude planes that has been reddened at each point with an input $R_V = 3.1$ extinction law, assuming a nominal $E(B-V) = 0.6$. As discussed in Section 2.8, our nominal extinction coefficient and colour excess are valid for early type stars. In panel a) we show in green the same isochrone, but in this case the reddening has been calculated assuming the same colour excess (0.6) and $R_V = 3.1$ in the relations (20) - (21) above. Likewise, in panel b) we show the effect of using the constant SDSS extinction coefficients provided by An et al. (2009, in blue) and McCall (2004, in cyan). The extinction coefficients of An et al. (2009) closely match the absolute position of our reddened isochrone, simply because those coefficients were derived using a synthetic spectrum for log $g = 4.5$, [Fe/H] = 0.0 and $T_{\text{eff}} = 5750$ K, which is similar to those that are appropriate for our reference isochrone. In contrast, the extinction coefficients given by McCall (2004) were derived using the α Lyr reference spectrum, which is responsible for the relatively large offset between the cyan and red isochrones. These two panels thus illustrate the risk involved in trying to fit isochrones to photometric data on the assumption of a literature value of $E(B-V)$ when a slightly different value may actually be more correct for the stars under investigation.

In the lower panels c) and d), we re-register all of the isochrones to match our reference isochrone at the TO point, applying whatever colour shifts $\delta$ are required (as noted). Alternatively, it would be possible to obtain the same result using in the relations (20) - (21) the extinction and reddening coefficient appropriate for a moderately metal-poor turn-off star (i.e., $E(B-V) = 0.55$ and $R_V = 3.40$ for the case of panel c) and $E(B-V) = 0.55$, $E(g-r) = 0.62$ and $R_g = 2.95$ for the case of panel d), see Section 2.8 and Appendix A). Nevertheless, it is clear from these panels that, despite this renormalization, the morphologies of the CMDs for the different cases are not identical along the lower-MS or the RGB. While these subtleties are usually negligible in the presence of low reddening, this is not the case for high values of the extinction. We emphasize that our reddened colours account for the effects of extinction in a fully self-consistent way. We also reiterate our comment that, when using literature values of $E(B-V)$, attention should be paid to how they were derived in order to minimize the inconsistencies highlighted above. More often than not, small adjustments to literature values of $E(B-V)$ might be applied. However, this should not be regarded as an excuse to use reddening as a free parameter, since an error in the reddening is only one of many possible reasons why an isochrone may not reproduce the observed TO colour of a cluster CMD even when well-supported estimates of the distance, metallicity, and $E(B-V)$ are adopted.

6 CONCLUSIONS

The large grids of MARCS model atmospheres, now available for different chemical compositions, together with the fluxes calculated at ~ $10^4$ wavelength points, represent an especially valuable resource for the study of stars and stellar populations (Gustafsson
To facilitate their application to photometric data, we have provided the means to generate, and to interpolate in, tables of synthetic BCs (and thus colours) applicable to several broad-band systems, for a wide range of physical and chemical abundance parameters (see Appendix A for detailed information on the parameter space covered). The photometric systems for which such transformations are provided include Johnson-Cousins-2MASS (UBVR, I), Sloan (ugriz), and HST-ACS/WFC3 (26 different filters, each in the ST, AB, and VEGA mag systems). The differences between the latter have been fully described, and the most updated photometric zero-points and absolute calibrations used. A novel feature of our transformations is that they can be corrected for the effects of reddening (for any $E(B-V)$ value between 0.0 and 0.72, inclusive) in a fully consistent way; i.e., the variation in the colour excess with the spectral type of a star is correctly taken into account. So far, our results assume the standard $R_V = 3.1$ for the extinction law, although the inclusion of other values (as e.g. more appropriate towards certain Galactic sightlines) is currently underway. The impact of varying the assumed microturbulent velocity on synthetic magnitudes in UV, optical, and IR bandpasses has also been examined in some detail: all of our results assume $\xi = 2$ km/s. While this value is generally appropriate for stars across most of the CMD, its variation can seriously impact colours for $T_{\text{eff}} \lesssim 4000$ K, and increasing metallicities, in particular towards the UV.

Encouragingly, we have found that the MARCS colour–$T_{\text{eff}}$–metallicity relations and BCs in most of the broad-band filters agree with those derived empirically above $\approx 4500$ K using the IRFM (Casagrande et al. 2010) to within a couple/few percent. This is easily within the uncertainties associated with the zero-points of the photometric systems and of the temperature scale. Moreover, the predicted metallicity dependence of such relations appears to
reproduce observed trends quite well according to our analyses of a large sample of solar neighbourhood stars that have $ugriz$ photometry. In general, the fits of isochrones to globular and open cluster CMDs (specifically, those of M 5, NGC 6791, and M 67) yielded consistent interpretations of the data on different colour planes, though small (0.01-0.02 mag) colour offsets were sometimes found in some CMDs relative to others for the same cluster. Even the synthetic $U$ magnitudes did not appear to be seriously discrepant, at least for metal-poor TO stars. Larger deficiencies might affect synthetic colours of late M-stars, although this regime has only been partly explored in this work (and certainly not at the coolest $T_{\text{eff}}$ of the MARCS grid).

Our brief consideration of Victoria-Regina isochrones on many different CMDs revealed the strong sensitivity of the colours of lower-MS stars to [$\alpha$/Fe], which undoubtedly has relevance for the bifurcated CMDs of M 4 and NGC 2808. Not surprisingly, the colours of cool stars, are very dependent on the chemical composition. As such, changing [$\alpha$/Fe] can modify quite dramatically the morphology of the CMD at its coolest extremities, both along the lower-MS and upper-RGB.

We defer to a future investigation a more detailed exploration of the effects of chemical abundances on colours, in particular accounting for the anti-correlations observed in many globular clusters (e.g., Sbordone et al. 2011). In this respect, it will also be interesting to extend our grid of synthetic BCs to intermediate and narrow-band systems, where synthetic models seem to perform less satisfactorily in comparison with observations (e.g., Önehag et al. 2009; Meléndez et al. 2010). The Strömgren system will also be of particular interest, because of its capability to help distinguish between multiple populations in globular clusters (e.g., Grundahl et al. 1998; Yong et al. 2008; Carretta et al. 2011).

Figure 17. Same as previous Figure, but using instead a 10 Gyr isochrone having $Y = 0.25$ and [Fe/H] = 0.0.
Figure 18. Effect of reddening $E(B-V) = 0.6$ on a 12 Gyr, $Y = 0.25$, [Fe/H] = $-0.8$, [$\alpha$/Fe] = +0.4 isochrone when using our published colours (in red), or assuming constant extinction coefficients (green, blue and cyan). See text for discussion.

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APPENDIX A: INTERPOLATION ROUTINES

The transformations described in this paper and the codes (in FORTRAN) that are needed to tabulate, and interpolate in the BCs for the filters and reddenings of interest are contained in the two files BCtables.tar.gz and BCcodes.tar.gz, along with some details concerning the execution of their various programs. A quick step-by-step set of instructions (MIX/Linux-oriented) is contained in the file INSTRUCTIONS.txt. The BCtables.tar.gz file contains the following folders, each of which contains many sub-directories:

1. alpha_s04  2) alpha_p00  3) alpha_s04  4) alpha_std

The BCcodes.tar.gz file contains the following Fortran computer programs:

1) getphotos.txt for code to retrieve BC data for several fixed (B-V) values
2) gettable.for code to generate BC transformation tables for any (B-V)
3) gettables.for code to retrieve original PARSEC data (see below)
4) unbicubics.for verification program (see below)
5) brevitt.for subroutines to interpolate in BC transformation tables
6) colstar.for program to illustrate the use of brevitt for
7) brestars.for code to generate BCs for stars of known Teff,l,int,[Fe/H]

In addition, BCcodes.tar.gz also contains the following files:
8) select_data ... to select photometric systems and MARCS models to use
9) input_sample ... an example of input file for brestars.for
10) bpaint ... an executable code to compile, link and run brestars for

Figure A1. Excerpt of the README file provided with our package.

5.5, 2500 ≤ Teff ≤ 8000 K, and −0.4 ≤ [α/Fe] ≤ +0.4, the coverage of parameter space is not complete. For instance, there are no model atmospheres at low gravities and high temperatures (which would be of limited usefulness, anyway), and relatively few computations at the lowest temperatures, gravities, and metallicities, or at log g = 5.5. Moreover, the number of Teff values for which model atmospheres were generated varies with log g, [Fe/H], and [α/Fe]. To deal with this nonuniformity, which obviously presents some challenges for the setting up of interpolation tables, cubic splines were used whenever possible to determine the transformations, which consist of BCs for each filter bandpass that we have considered, when model atmospheres were lacking at gravities and temperatures in the middle of their respective ranges. Linear or 3-point extrapolations were also made, as needed, to “fill in” missing BC entries located near the upper and lower limits of the dependent variables.

In order to minimize the number of extrapolations, while maximizing the size of the transformation tables, the decision was made to restrict the latter to the ranges in gravity and temperature shown in Figure A2 (the region enclosed by the dashed lines). Victoria-Regina isochrones (VandenBerg et al. 2014) for [Fe/H] = −2.4 and +0.4, which include masses as low as 0.12M⊙ and extend to the RGB tip, have been included in this plot simply to illustrate where such models are located in this diagram. In this figure, the grid values of log g and Teff are indicated by the intersection of the dotted grey lines, and it is for these values that BCs are provided. (Tables of exactly the same dimensions have been created for all of the grid values of [Fe/H] and [α/Fe].) Thus, no data are provided for [Fe/H] and [α/Fe].) Thus, no data are provided for Teff < 2600 K, while BCs for log g = 5.5 are given only for 5000 < Teff ≤ 8000 K, which is a consequence of the fact that MARCS model atmospheres were not computed for higher temperatures. As a result, they cannot be used for, e.g., hot MS or blue horizontal-branch stars — but hopefully an extension of the tables to higher temperatures will be made at some future date.

Once BCtables.tar.gz has been retrieved from CDS, it would be prudent to open it in a separate directory as it contains a fairly extensive set of sub-directories (each containing the BCs for a specific photometric system). BCcodes.tar.gz, which con-
Synthetic photometry for stellar models

2: = ialf (= [alpha/Fe] variation; select from choices listed below)
4: = nfil (= number of filter bandpasses to be considered; maximum = 5)
7: = photometric system and filter (select from menu below)
7: = photometric system and filter (select from menu below)
1: = photometric system and filter (select from menu below)

i alf: “standard variation” of [alpha/Fe] with [Fe/H]
1: = [alpha/Fe] = +0.4 (for [Fe/H] from −2.5 to +0.5, inclusive)
2: = [alpha/Fe] = +0.4 (for [Fe/H] from −2.5 to +0.5, inclusive)
3: = [alpha/Fe] = +0.4 (for [Fe/H] from −2.5 to +0.5, inclusive)
4: = [alpha/Fe] = −0.4 (for [Fe/H] from −2.5 to +0.5, inclusive)

Photometric system
Fil lers
1 = main 3 = $H$
2 = $V$
3 = $I$
5 = $V$
6 = $B$
7 = $R$
8 = $I$
9 = $V$

x-axis, as in the case of a ($B - V, V$)-diagram; in which case, i alf would be set to a value of 2. However, nfil = 5 would be the natural choice for observations obtained in, e.g., the ugriz system. The next 4–7 lines specify the photometric system(s) and filters of choice. In the example given in Fig. A3, the 1990 calibration (named u_bvri90, see Section 2.9) of the $B$, $V$, and $I_c$ filters, along with the 2MASS $K_s$ filter, have been selected. (The $I_u$ line, which specifies the integers 1 and 3, will not be read in this case because nfil = 4.) The photometric systems and filters can be identified in any order.

Once the parameters in selectbc.data have been set, two programs must be executed in order to obtain the desired BC transformation tables. The first one that needs to be compiled and executed is getinputbc.for. Because reddening-corrected BC values have been derived for $E(B - V) = 0.00$ to 0.72, in steps of 0.02, the user can request transformations for any reddening within this range. However, rather than storing such a large dataset, only the tables for $E(B - V) = 0.00, 0.12, 0.24, \ldots 0.72$ have been retained. We found that spline interpolations in tables for these 7 reddenings were able to reproduce the reddening-adjusted transformations in the full set of tables to within 0.001 mag, independently of the selected filter. Consequently, the purpose of getinputbc.for is to retrieve from the appropriate subdirectories the BCs for $0.0 \leq E(B - V) \leq 0.72$, in steps of 0.12 mag. These data are placed in the seven files that have been given the names inputbc_r00.data, inputbc_r12.data, etc. (These files are rewritten each time getinputbc is executed.)

Figure A2. BCs are given for $T_{\text{eff}}$ and log $g$ values within the region enclosed by the dashed lines. (Tables of the same dimensions are provided for each of the grid values of [Fe/H] and [$\alpha$/Fe], which vary from −4.0 to +1.0 and −0.4 to +0.4 (for a subset of the [Fe/H] values), respectively (see the text). Isochrones (VandenBerg et al. 2014) have been plotted to show that the transformations provide sufficient temperature and gravity coverage for stellar models over a wide range in mass ($\gtrsim 0.12 M_\odot$) and evolutionary state.

tains the software that manipulates the transformation data, should be opened in the same “home” directory because the computer programs assume a particular directory structure for the data files and they will not execute properly unless that structure is in place. Their execution is controlled by the included selectbc.data file, a sample of which is shown in Figure A3. It essentially provides a menu (the information given below the dashed line) that enables the user to select up to 5 filters for which a table of BCs will be created for subsequent interpolations (so as to, e.g., transform stellar models from the theoretical plane to various CMDs of interest). The main advantage of tabulating BCs, instead of colours, is that the transformations provide sufficient temperature and gravity coverage for stellar models over a wide range in mass ($\gtrsim 0.12 M_\odot$) and evolutionary state.
The next step is to compile and execute getbctable.for, which prompts the user for a value of \( E(B - V) \) and then interpolates in the inputbc_* .data files to produce the requested BC transformations for that choice of the reddening. Since the ranges in [Fe/H] are different for the cases that are specified by the parameter ialf (see above), the output file generated by getbctable is given the name bc_std.data if ialf = 1, bc_p04.data if ialf = 2, bc_p00.data if ialf = 3, or bc_m04.data if ialf = 4. Perhaps the main reason for doing this is to facilitate interpolations in the resultant transformations for arbitrary values of [a/Fe] within the range \(-0.4 \leq [a/Fe] \leq +0.4\). (To perform such interpolations, it is clearly necessary to cycle through the above procedure in order to create the respective bc_* .data files corresponding to ialf = 2, 3, and 4.) Note that getinputbcs and getbctable are completely self-contained; i.e., all file assignments are done internally. In summary, to obtain reddening-adjusted BC transformations for up to 5 different filters, one simply sets the parameters in selectbc.data to the desired values, and then executes getinputbcs and getbctable.

The output data files so obtained have been organized in a straightforward way. Figure A4 reproduces the beginning of the bc_p04.data file that would be created using the example of selectbc .data given in the previous figure. The first (title) line is included primarily to reinforce the fact that the quantities in the tables are BCs (not colours). The second line specifies the number of \( T_{\text{eff}} \)'s, gravities, [Fe/H] values, and BCs, as well as the reddening value that was entered when prompted for it by getbctable. The next four lines list the grid values of \( T_{\text{eff}} \) and log \( g \), while the following (7th) line specifies how many BC values, beginning with the first entry at 2600 K, are included in the table at each log \( g \) value. For example, the minimum gravity in this table is log \( g = 0.0 \), and according to the first entry on the seventh line, BC transformations for that choice of the reddening. Since the ranges in [Fe/H] and [a/Fe] are completely self-contained; i.e., all file assignments are done internally. In summary, to obtain reddening-adjusted BC transformations for up to 5 different filters, one simply sets the parameters in selectbc.data to the desired values, and then executes getinputbcs and getbctable.

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The rest of the sample bc_p04.data file lists, in the form of mini-tables, BCs in the order of increasing [Fe/H], and within each table, they are organized in rows for the grid values of log \( g \), with a maximum of 10 numbers per row. Hence the 19 BC values at log \( g = 0.0 \), in the order of increasing \( T_{\text{eff}} \), are contained in the two rows immediately after the header line that gives the values of [Fe/H] and [a/Fe] which are applicable to the BC data that follow. All of the transformations for a given filter, which is also identified in the mini-table header lines are grouped together. Thus, all of the transformations for the \( uvbri90.B \) filter are listed before all of those for the \( V \) filter (which are likewise organized in the direction of increasing [Fe/H]), etc.

In addition to subroutines (such as getbcs_p04.for) that perform interpolations in the BC tables, BColors .tar .gz contains programs that can be used to verify that everything is working correctly. To be more specific, gettestbcs.for retrieves from the appropriate sub-directories, the original MARCS transformations that were used in the construction of the interpolation tables (for one of six possible values of \( E(B - V) \); namely, 0.0, 0.08, 0.14, 0.28, 0.44, or 0.56). These data are stored in files with names like sph90 .data and pp140 .data, where the prefixes sph and pp1 indicate that the data are based on “spherical” or “plane-parallel” model atmospheres, respectively, and the two digits indicate the log \( g \) value (e.g., 00 implies 0.0 whereas 40 implies 4.0; log \( g = -0.5 \) is coded as m05). These values of \( E(B - V) \) were chosen so as to be offset by different amounts from the values (0.0, 0.12, 0.24, 0.36, ...) used in the spline fits.

The second of three steps in the verification process is to run getbctable for the BC table for each of the six reddening values. In the final step, which requires the execution of chkbctable .for, the [Fe/H], \( T_{\text{eff}} \), and log \( g \) values listed in the sph90 .data and pp140 .data files are used to obtain BC values, by interpolating in the output file from getbctable; these are then compared with the values in the original MARCS tables (i.e., sph90 .data, etc.). In our experience, the differences between the interpolated and tabulated BCs are always \(< 0.001 \text{mag}\); they tend to be the largest for blue and UV bandpasses. Differences will be exactly 0.0 only when the BCs for \( E(B - V) = 0.0 \) are selected by gettestbcs given that exactly the same data are employed in the construction of the splines. A short program, with results, is also included in the package to illustrate how to perform interpolations in the tables produced by getbctable. As already pointed out, a detailed explanation of these auxiliary codes is contained in a README file that is also provided.

Finally, it is worth mentioning that the extinction coefficients (and reddening values) that apply to reddened stars of different spectral types may be easily calculated using our interpolation routines. Consider, for instance, an old, moderately metal-poor ([Fe/H] = -0.8, [a/Fe] = +0.4), turnoff star \( T_{\text{eff}} = 5950 \text{K} \), log \( g = 4.2 \) that is affected by a nominal reddening \( E(B - V) = 0.6 \). On the assumption of the canonical extinction law, \( R_V = 3.1 \), the following BCs are obtained in a number of selected bands: \( B^\odot = -3.03, B_V^\odot = -1.97, B_C^\odot = -2.54, B_{C^\odot} = -1.58 \). If that star were unreddened, the values of the same BCs would be: \( B^\odot = -0.61, B_V^\odot = -0.10, B_C^\odot = -0.30, \) and \( B_{C^\odot} = 0.04 \). It thus follows that, at the turnoff, \( E(B - V) = \)\( (-1.97 + 3.03)\) = \(-0.10 \) and \( E(B - V) = \)\( (-1.58 + 2.54)\) = \(0.04 + 0.30\) = 0.62, with ex-
tion coefficients \( R_B = \frac{1.03-0.61}{0.55} = 4.40, \) \( R_V = \frac{1.97-0.10}{0.55} = 3.40, \) \( R_R = \frac{2.54-0.30}{0.55} = 4.07, \) and \( R_i = \frac{1.63-0.14}{0.55} = 2.95. \) Note, in particular, that the difference \( R_B - R_V \) is exactly 1 because the extinction coefficients were calculated assuming the value of \( E(B-V) \) that applies to the star in question.

This difference would be \(< 1 \) if the \( E(B-V) \) value appropriate to early-type stars is adopted — which is the usual convention for reddening determinations that are reported in the literature. Returning to the example considered in the previous paragraph, \( R_B = \frac{1.03-0.61}{0.55} = 4.03 \) and \( R_V = \frac{1.97-0.10}{0.55} = 3.12 \), with the result that \( R_B - R_V = 0.91 \) and \( E(B-V) = 0.91(0.6) = 0.55. \) Reinforcing the point already made in Section 5 (also see Bessell et al. 1998), these calculations show that the \( E(B-V) \) value produced by a given amount of dust will be significantly less for a 5950 K (late F-type) turnoff star than for early O- and B-type stars. Similarly, colour excess ratios will scale according to spectral type since, for filters \( \zeta \) and \( \eta \), the difference \( R_\eta - R_\zeta \) is equal to \( E(\zeta - \eta)/E(B-V) \), where the denominator of this term is the nominal reddening. Clearly, to correct colours (Eq. 20) and magnitudes (Eq. 21) for the effects of interstellar extinction, one should use either 0.55 and (3.4)(0.55) or (0.60)(0.91) and (3.12)(0.60), respectively, in the case of the present example.

In comparisons of isochrones with observed CMDs, it is traditional to apply constant offsets to the observed magnitudes and/or colours (depending on whether an apparent or true distance modulus is assumed) to account for the effects of foreground gas and dust. This is much simpler than correcting the observations (or, more likely, the isochrones) on a point-by-point basis prior to fitting the models to the data, and indeed, there is no reason to abandon this practice if, e.g., the main goal of the work is to derive the turnoff age(s). (Recall the discussion in Section 5 concerning Fig. 18, which showed that, even when the reddening is high, the two approaches lead to no more than minor differences along the RGB and the lower MS.) Since published \( R \) values are generally based on hot stars (e.g., Vega, in the case of McCall 2004), we have calculated the extinction coefficients relevant to turnoff stars (log \( g = 4.1 \), \( 5250 \leq T_\text{eff} \leq 7000 \) K in \( 250 \) K increments), and \(-2.0 \leq [\text{Fe/H}] \leq +0.25 \) (at the specific values adopted in the MARCS library). (This involves nothing more than evaluating the extinction coefficients for the standard extinction law, \( R_V = 3.1 \).) and the rms deviations, which are typically one-third as large, are reduced by about a factor of 5 if the least-squares fit to \( R \) is adopted instead of the mean value. Only in the case of the F218W, F225W, F275W, and F350LP filters it was not possible to obtain a good fit to the actual values of \( R_\zeta \); consequently, they have been omitted from this table. (The mean values were likewise very poor approximations, undoubtedly due to the strong \( T_\text{eff} \) dependence of the BCs in these four filters.) In any case, Table A1 provides two ways of determining approximate (but still quite accurate) values of \( R_\zeta \) that can be used to evaluate the colour excess ratio for any of the filters that are listed. Importantly, such ratios involve the nominal reddening given that all of our \( R \) results are based on the standard extinction law, \( R_V = 3.1 \).

APPENDIX B: BIAS IN A NON-LINEAR TRANSFORMATION

Every time a non-linear transformation is applied to a variable that is subject to random errors (in this context e.g., an observed quantity with a Gaussian Probability Distribution Function; PDF) a skewed PDF arises as a consequence of the transformation. In this case the expectation value differs from the mode and the difference between the two introduces a bias. This traces directly back to a better known example, such as the bias between parallaxes and distances (e.g. Lutz & Kelker 1973; Casagrande et al. 2007).

Let us consider the case of transforming a magnitude \( m \) into a heterochromatic flux \( F \), which (from Eq. 1) involves the following non-linear relation

\[
\mathcal{F} = \alpha 10^{-0.4 m}
\]

where \( \alpha \) is a constant that incorporates all of the details concerning the absolute calibration and zero-point of the photometric system under consideration.

Assuming a Gaussian distribution for the errors around \( m \) (as obtained e.g., by averaging different independent measurements)

\[
\mathcal{F}(m) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(m-m_0)^2}{2\sigma^2}},
\]

the expectation value for the flux \( \mathcal{F} \) is

\[
E[\mathcal{F}] = \int_{-\infty}^{+\infty} \mathcal{F}(\tilde{m}) f(\tilde{m}) d\tilde{m}
\]

\[
= \alpha \sqrt{2\pi\sigma} \int_{-\infty}^{+\infty} 10^{-0.4m} e^{\frac{u^2}{2\sigma^2}} du
\]

\[
= \alpha 10^{-0.4 m_0} e^{0.08 \sigma^2 \ln^2(10)}
\]

where \( m_0 = \bar{m} - m \). The amplitude of the bias can be immediately estimated by comparing Eq. (B3) and Eq. (B1), or in fractional terms

\[
\frac{E[\mathcal{F}] - \mathcal{F}}{\mathcal{F}} = e^{0.08 \sigma^2 \ln^2(10)} - 1.
\]

Fortunately, this difference is negligible in most investigations, when uncertainties are of the order of few hundredths of a magnitude, although it quickly rises from 1 percent at \( \sigma \approx 0.2 \) mag to 10 percent at \( \sigma \approx 0.7 \) mag.

Notice that the opposite bias is also present. In fact, in the above example we have assumed that the errors arise as a consequence of averaging a number of independent measurements. In reality, when measuring magnitudes, several steps are involved, which essentially consist of transforming number counts into fluxes.

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15 Users are warned not to extrapolate these relations outside of the range of their applicability. Such caution is advisable even though the dependence of the extinction coefficients on \( T_\text{eff} \) is usually less dramatic above 10,000 K — as we have found using ATLAS9 models.
Table A1. Values of $R_L$ relevant to turnoff stars with $T_{\text{eff}} \leq 7000$ K. Notice that for a nominal $E(B-V)$, the excess in any given $\zeta-\eta$ colour is $E(\zeta-\eta) = (R_L - R_0)E(B-V)$, and the attenuation for a magnitude $m_L$ is $R_L E(B-V)$. See also discussion in the text.

| Filter | $<R_L>$ | $R_L = a_0 + T_4 (a_1 + a_2 T_4) + a_3$ [Fe/H] | Filter | $<R_L>$ | $R_L = a_0 + T_4 (a_1 + a_2 T_4) + a_3$ [Fe/H] |
|--------|--------|---------------------------------|--------|--------|---------------------------------|
|        | $a_0$  | $a_1$  | $a_2$  | $a_3$  | $a_0$  | $a_1$  | $a_2$  | $a_3$  |
| 2MASS  |        |        |        |        |        |        |        |        |
| $J$    | 0.899  | 0.9095 | -0.0526| 0.0583 | —      | 0.567  | 0.5611 | 0.0176 | -0.0115 | —      |
| $K_S$  | 0.366  | 0.3667 | -0.0026| 0.0021 | —      |        |        |        |        |        |
| ACS    |        |        |        |        |        |        |        |        |        |        |
| F435W  | 4.106  | 3.4929 | 1.7330 | -1.2071| -0.0135| F475W  | 3.720  | 3.1735 | 1.4366 | -0.8569 | -0.0078 |
| F555W  | 3.217  | 3.0197 | 0.4900 | -0.2738| -0.0018| F606W  | 2.876  | 2.5200 | 0.8308 | -0.4074 | -0.0018 |
| F814W  | 1.884  | 1.7763 | 0.2522 | -0.1227| —      |        |        |        |        |        |
| WFC3b  |        |        |        |        |        |        |        |        |        |        |
| F336W  | 5.148  | 4.9002 | 0.6329 | -0.3806| -0.0062| F390M  | 4.503  | 4.3391 | 0.5133 | -0.4007 | -0.0015 |
| F390W  | 4.432  | 3.6722 | 2.3798 | -1.8697| -0.0165| F438W  | 4.135  | 3.8431 | 0.8230 | -0.5695 | -0.0055 |
| F475W  | 3.698  | 3.1588 | 1.4014 | -0.8529| -0.0071| F547M  | 3.140  | 3.0439 | 0.2472 | -0.1483 | -0.0015 |
| F555W  | 3.274  | 2.8862 | 0.9399 | -0.5000| -0.0023| F606W  | 2.894  | 2.5345 | 0.8514 | -0.4306 | -0.0020 |
| F625W  | 2.678  | 2.5954 | 0.1656 | -0.0481| —      | F775W  | 2.049  | 2.0174 | 0.0527 | -0.0003 | —      |
| F814W  | 1.897  | 1.8027 | 0.2058 | -0.0818| —      | F850LP | 1.457  | 1.3997 | 0.1673 | -0.1192 | —      |
| F098M  | 1.290  | 1.2709 | 0.0432 | -0.0193| —      | F110W  | 1.047  | 0.9272 | 0.2848 | -0.1428 | —      |
| F125W  | 0.891  | 0.8675 | 0.0394 | -0.0027| —      | F140W  | 0.753  | 0.7180 | 0.0754 | -0.0290 | —      |
| F160W  | 0.634  | 0.6196 | 0.0301 | -0.0100| —      |        |        |        |        |        |
| SDSS   |        |        |        |        |        |        |        |        |        |        |
| $u$    | 4.862  | 4.5725 | 0.9248 | -0.7397| -0.0051| $g$    | 3.762  | 3.2679 | 1.2843 | -0.7840| -0.0076 |
| $r$    | 2.704  | 2.6623 | 0.0691 | -0.0017| —      | $i$    | 2.120  | 2.0927 | 0.0464 | -0.0019| —      |
| $z$    | 1.528  | 1.4713 | 0.1576 | -0.1052| —      |        |        |        |        |        |
| ubvri12|        |        |        |        |        |        |        |        |        |        |
| $U$    | 4.801  | 4.3796 | 1.3861 | -1.1400| -0.0065| $B$    | 4.046  | 3.3504 | 1.8900 | -1.2437| -0.0143 |
| $V$    | 3.122  | 2.9364 | 0.4604 | -0.2568| -0.0022| $R_C$  | 2.552  | 2.4248 | 0.2672 | -0.0932| —      |
| $I_C$  | 1.902  | 1.8662 | 0.0657 | -0.0098| —      |        |        |        |        |        |
| ubvri90|        |        |        |        |        |        |        |        |        |        |
| $U_X$  | 4.814  | 4.3241 | 1.6005 | -1.3063| -0.0073| $B_X$  | 4.032  | 3.2999 | 2.0123 | -1.3425| -0.0140 |
| $B$    | 4.049  | 3.3155 | 2.0119 | -1.3400| -0.0145| $V$    | 3.129  | 2.9256 | 0.5205 | -0.3078| -0.0022 |
| $R_C$  | 2.558  | 2.4203 | 0.3009 | -0.1220| —      | $I_C$  | 1.885  | 1.8459 | 0.0741 | -0.0151| —      |

\[ E[m|F] = \int_{-\infty}^{\infty} m(\tilde{F}) f(\tilde{F}) d\tilde{F} \]

\[ = -\frac{2.5}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \log\left(\frac{\tilde{F}}{\alpha}\right) e^{-\frac{\tilde{F}^2}{2\sigma^2}} d\tilde{F} \]

\[ = 2.5 \log{\alpha} - \frac{2.5}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \log(\tilde{F}) e^{-\frac{\tilde{F}^2}{2\sigma^2}} du \]  

(B5)

where $u = \tilde{F} - F$, and expanding $\log(\tilde{F})$ around the maximum of $-u^2$, we obtain

\[ E[m|F] = 2.5 \log{\alpha} - \frac{2.5}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \log(F) \]

(for CCD detectors, and fluxes into magnitudes, which then are corrected for atmospheric extinction (in case of ground-based observations) and standardized using photometric equations (e.g., Harris et al. 1981). Assuming that the uncertainty in the photometric equations is small, which is usually the case if a sufficient number of standards has been observed spanning different colours and airmasses, with decreasing signal–to–noise the main uncertainty of a magnitude stems from its flux measurement. We also assume that the uncertainty in flux measurements is Gaussian, which is appropriate since a Poisson distribution is well approximated by a Gaussian after relatively few counts (e.g., Taylor 1982).

For objects with high signal–to–noise, the flux uncertainty has a narrow Gaussian peak, which transforms to negligible uncertainty in magnitude. However, as the signal–to–noise decreases, the uncertainty in the flux broadens (and other sources of random errors can also become important e.g., Howell 1989). As before, if we know the flux $F$, we can compute the expectation value for a magnitude using the full PDF

\[ E[m|F] = \int_{-\infty}^{\infty} m(\tilde{F}) f(\tilde{F}) d\tilde{F} \]
\[
+5 \log(e) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left( \frac{u}{F} \right)^n e^{-\frac{u^2}{2\sigma^2}} \text{du}
\]

\[
= -2.5 \log \left( \frac{F}{\alpha} \right) - 2.5 \frac{5 \log(e)}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \left[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left( \frac{u}{F} \right)^n \right] e^{-\frac{u^2}{2\sigma^2}} \text{du}
\]

(B6)

whose solution is not trivial, especially when large errors will formally return negative fluxes. Nevertheless, for errors up to \(\approx 30\%\) in flux (i.e. the limit where no negative flux is found within \(3\sigma\)), the above expression can be conveniently approximated using the first term of the expansion, thus leading to an estimate of the bias

\[
E[m|F] + 2.5 \log \left( \frac{F}{\alpha} \right) \approx 0.6 \left( \frac{\sigma F}{F} \right)^2 .
\]

(B7)

Notice that the two biases discussed here, \(E[F|m]\) and \(E[m|F]\) do not cancel each other, but are representative of two approaches typically used for estimating errors. In the first case, the bias in the flux arises if the error in magnitude is obtained by averaging different measurements. In the second, the bias follows from the fact that symmetric errors in the fluxes are asymmetric once transformed into magnitudes. In either case, when publishing photometry, the procedure used to determine the errors should always be described, in particular for the object with the lowest precision (although formally the bias is always present). If this is not done, users should refrain from (or be very cautious in) analysing statistically large samples of poor quality data, in the hope that average quantities (with formally small uncertainties) will still be useful: severe biases could undermine the mean results so obtained.