How the weak and strong links affect the evolution of prisoner’s dilemma game

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Abstract

The complex interactions between individuals are intertwined with time in multilayer networks. In this paper, we propose a prisoner’s dilemma game on a two-layered network, including the weak-link layer and the strong-link layer. Based on the mean-field theory and numerical simulations, we show that if the players update their strategies primarily depending on the information received in the weak-link layer, i.e. the weak relations have the dominate influences on the individuals’ strategies choice, the cooperation becomes the most abundant strategy. More comparative analysis of the different structures on the strong-link layer indicates that a higher connectivity heterogeneity of the strong-link layer could be conducive to promote the individuals’ cooperation behavior. In the situation while the relation between the players’ connections in the strong-link layer and their activity rates in the weak-link layer is negative correlated, i.e. ones’ connections increase with the decrease of their activity rates, the propensity for cooperation can be greatly enhanced.

1. Introduction

Recent experimental evidences provide that social preferences and moral preferences are the important force to promote cooperation [1−3]. However, the influence of the interactions networks formed by individuals’ activities cannot be ignored. How to quantitatively analyze human behavior is a very important topic in network science, which has been the focus of sociology, psychology and economics in the past years [4−7]. Exploring the collaboration of selfish individuals in complex network systems remains a huge challenge yet [8, 9], however, the evolutionary game theory has provided a powerful mathematical framework to shed insight [10−13].

Since social interactions are traditionally described by networks, evolutionary games on networks and structured populations have received ample attention in the recent past [14−16]. In general, the study of game evolution in complex networks mainly focuses on the influence of network topology, the choice mechanism of individuals’ strategies and external environment disturbance [17−19]. The evolutionary version of the prisoner’s dilemma (PD) game was first proposed in small-world networks in [20], after that the game model based on the mutual interactions (such as the stag hunt game, the boxed pigs’ game and the snowdrift game) or the group interactions (such as the public goods game) have received extensive attentions [21−25]. The factors such as weighted edges, individuals’ visibility and limited resources have been taken into account where the evolutionary game evolves in a single-layer network [26−29]. For instance, Matjaz et al explored how the coevolution of link weight affects cooperation in the spatial PD game [30]. Kleineberg et al studied the metric clusters in evolutionary games on scale-free networks [31]. Wu et al claimed that to enhance cooperation, individuals could give high-degree neighbors more help [32].

Different types of relations may interrelate with each other, and the understanding of multilayer networks offers a new promising approach of such relations. Therefore, the focus on network connectivity pattern has moved from single-layered networks to interdependent, multiplex or multilayer networks [33]. On the other
hand, most of the previous researches of evolutionary game were concentrated on non-time-varying networks. Whereas the individuals’ interactive networks are not only intertwined, but also evolving with time [34–39]. In fact, for the time-varying networks, such as social networks, the structure is usually twofold. For one thing, some links are temporally dynamical during network evolution, which can be treated as weak connections between nodes. For another, some links are static and invariant, we treat these as strong connections. For example, we could view the individuals who comment on an event in some online social platform as a link, and when all individuals comment on different news events, different relations are established between the pair of individuals. As the relations may change over time, so we call such varying relations as the weak links. However, an individual’s offline relations generally do not change or will not change in a period, such as the kinship or friendship, which we call these constant relations as the strong links [40, 41].

Weak links and strong links play a very important role in the social structure, and are the effective bridge to transmit information between different social clusters. As a result, the different links of the players would heavily affect the change of their strategies. However, the influence of weak links and strong links on human behavior has remained unclear. In order to uncover how the weak and strong links affect the evolution of players’ strategies, we explore the PD game in a two-layered network. We use the activity-driven model to describe the network structure of the weak-link layer [42]. In addition, we analyze and simulate the PD game, mainly focusing on three types of networks of the strong-link layer, which are the lattice network, the ER random network and the BA scare-free network, respectively. If the weak relations have the dominate influences on the individuals’ strategy choice, the cooperation becomes the most abundant strategy. We find that the propensity for cooperation can be greatly enhanced, while the heterogeneity of the strong-link structure is higher or the players’ strong connections and their activity rates is in negative correlation.

2. The PD game in a two-layered network

The traditional setup of the PD game assumes N players occupying vertices of an interaction network. The available strategies are cooperation (C) and defection (D). Mutual cooperation yields the reward R, mutual defection leads to punishment P, and the mixed choice gives the cooperator the sucker’s payoff S and the defector the temptation gain T. For the sake of simplicity, we focus on the weak evolutionary game, where $S = P = 0, R = 1, T > 1$.

The proposed two-layered network includes the weak-link layer and the strong-link layer. The strong-link layer does not evolve with time. Player $i$ in the strong-link layer has a counterpart in the weak-link layer (see figure 1). The weak-link layer is constructed by the activity-driven model [37, 42]. Player $i$ is characterized by an activity rate $a \in [\epsilon, 1]$, where $\epsilon$ is set to be a very small positive number to avoid divergence when $a \to 0$. The activity rates are the ability to create contacts or interactions with other individuals per unit time, and are assigned according to a given probability distribution $F(a)$. An activated player generates $m$ links that connect with other players randomly. The average degree of node $i$ characterized by activity rate $a$ is $k = am + \langle a \rangle m$, where $\langle a \rangle$ is the first moment of the activity distribution. By integrating in finite game time $t$, the degree distribution in the weak-link layer follows $\frac{1}{tm} F(k/tm)$ [37].

The players in the weak-link layer could spread their corresponding strategies to others according to the weak relations. Meanwhile, the players in the strong-link layer may collect and adopt their counterparts’ strategies. Remarkably, the players in the weak-link layer simply duplicate or diffuse its counterpart strategy. Therefore, we consider the change of individuals’ strategies in the strong-link layer.

We determine a parameter $\beta \in [0, 1]$ to reflect the personal selection preference. At each game round, a player in the strong-link layer either follows one of his neighbors’ strategies or adopts the strategy based on his counterpart gathered. Specifically, for player $i$ in the strong-link layer, one has the following two ways to change one’s strategy.

(1) According to the strong links, player $i$ will select one of the neighbors with probability $\beta$ to imitate the strategy. Particularly, player $i$ randomly picks up neighbor $j$ to compare their profits. Player $i$ adopts the strategy $s_j$ from player $j$ with the probability determined by the Fermi function

$$P(s_i \to s_j) = \frac{1}{1 + \exp\left(-\frac{f_j - f_i}{\kappa}\right)},$$

where $s_i$ denotes the strategy of player $i$, if player $i$ chooses to be a cooperator, then $s_i = 1$, otherwise, $s_i = -1$. $f_i$ and $f_j$ account for the accumulated profits of individuals $i$ and $j$, respectively. $\kappa$ denotes the amplitude of noise or its inverse the so-called intensity of selection. In this paper, we fix the value of $\kappa$ as $\kappa = 0.1$ [30].
An individual with strategy $s_i$ interacts with an $s_j$ neighbor can acquire payoff $f_{ij}$:

$$f_{ij} = \frac{1}{4}(1 + s_j)(1 + s_j)R + \frac{1}{4}(1 + s_j)(1 - s_j)S + \frac{1}{4}(1 - s_j)(1 + s_j)T + \frac{1}{4}(1 - s_j)(1 - s_j)P.$$  (2)

Therefore, player $i$ will get the accumulated profits $f_i$ as follows:

$$f_i = \sum_{j \in \partial i} f_{ij},$$  (3)

where $\partial i$ is the set of neighbors of player $i$.

(2) Based on the gathered strategy from the weak-link layer, player $i$ in the strong-link layer will adopt the strategy based on his counterpart's strategy collected with probability $1 - \beta$. Assume that in the weak-link layer, the two connected individuals can transmit their strategies to each other. For individual $i$, we define the probability that individual $i$ holds the cooperative cognition in the weak-link layer is

$$P(s_i = C) = \frac{m_C}{m_C + m_D},$$  (4)

where $m_C$ is the received cooperative strategy packets (the total number of cooperative neighbors) and $m_D$ is the received defective strategy packets (the total number of defective neighbors). Notice that if player $i$ in the strong-link layer decides to follow his counterpart's strategy, this player will take the cooperative strategy with probability $P(s_i = C)$.

### 3. The analysis of the PD game in a two-layered network

#### 3.1. The strong-link layer is a homogeneous network

When the strong-link layer is a homogeneous network with the average degree $\langle k \rangle$, the evolution processes of the fraction of cooperators $\rho_i^C$ in the strong-link layer is written as

$$\rho_i^C(t + 1) = \rho_i^C(t) + \beta[\rho_i^{D-C} - \rho_i^{C-D}] + (1 - \beta)\pi^{s,w},$$  (5)

where $\rho_i^C(t)$ is the fraction of cooperators at the game round $t$, $\rho_i^{D-C}$ is the transition probability from cooperative to defective; $\rho_i^{C-D}$ is the transition probability from defective to cooperative. $\pi^{s,w}$ denotes the coupling between the strong-link layer and weak-link layer.
The transition probabilities of $\rho_s^{D-C}$ and $\rho_s^{C-D}$ corresponding to the game dynamics are expressed as

$$\rho_s^{D-C} = (1 - \rho_s^c) \rho_s^c P(D \rightarrow C) = (1 - \rho_s^c) \rho_s^c \frac{1}{1 + \exp\left(-\frac{E_D - E_C}{k}\right)}$$

and

$$\rho_s^{C-D} = \rho_s^c (1 - \rho_s^c) P(C \rightarrow D) = \rho_s^c (1 - \rho_s^c) \frac{1}{1 + \exp\left(-\frac{E_C - E_D}{k}\right)}.$$  

(6)

The average profits of the cooperators and defectors are $f_C = \langle k \rangle \rho_s^c$ and $f_D = \langle k \rangle \rho_s^c T$, respectively. Accordingly, we further get

$$\rho_s^{D-C} - \rho_s^{C-D} = \rho_s^c (1 - \rho_s^c) \tan h\left(\frac{(1 - T) \langle k \rangle \rho_s^c}{2k}\right).$$

(7)

Through the weak links, a player with activity rate $a$ would receive the expected information packets from cooperators are $C_w = am \int_s \rho_s^{C,a} da' + P(a) \int_s ma' \rho_s^{C,a} da'$, where $am \int_s \rho_s^{C,a} da'$ represents the influence that the players are active and connected with other cooperators, $P(a) \int_s ma' \rho_s^{C,a} da'$ indicates the effect that the players are linked by the active cooperators. Thus, we write the expectation probability that the individuals with activity rate $a$ hold the cooperative cognition in the weak-link layer as follows:

$$P_a(s = C) = \frac{am \int_s \rho_s^{C,a} da' + P(a) \int_a ma' \rho_s^{C,a} da'}{am + \langle a \rangle m} = \frac{a \rho_s^c + P(a) \int_a ma' \rho_s^{C,a} da'}{a + \langle a \rangle}.$$  

(9)

Summing along all the classes, we obtain the density of individuals $\rho_s^c$ who hold the cooperative cognition in the weak-link layer:

$$\rho_s^c = \int_a P_a(s = C) = \int_a \frac{a \rho_s^c + P(a) \int_a ma' \rho_s^{C,a} da'}{a + \langle a \rangle} da.$$  

(10)

We write the coupling term $\pi_s^{v,w}$ as follows:

$$\pi_s^{v,w} = (1 - \rho_s^c) \rho_s^c - \rho_s^c (1 - \rho_s^c).$$

(11)

Therefore, the evolution of cooperators in the strong-link layer is as follows:

$$\frac{d\rho_s^c}{dt} = \beta (1 - \rho_s^c) \rho_s^c \tan h\left(\frac{(1 - T) \langle k \rangle \rho_s^c}{2k}\right) + (1 - \beta) [(1 - \rho_s^c) \rho_s^c - \rho_s^c (1 - \rho_s^c)].$$

(12)

In general, it is very difficult to theoretically analyze equation (12). Next, we further analyze the PD game in some special cases.

1. In the case of $\beta = 0$, an individual updates his strategy only according to his strategy cognition. Equation (12) reduces to the following form

$$\frac{d\rho_s^c}{dt} = (1 - \rho_s^c) \rho_s^c - \rho_s^c (1 - \rho_s^c).$$

(13)

According to equation (4), we can get the probability that an individual in the weak-link layer receives the cooperative information is $\rho_s^c(t)$ at game round $t$. In addition, an individual in the strong-link layer updates his strategy with replicating his/her strategy cognition in the weak-link layer. Hence, the initial proportion of the cooperators in the strong-link layer determines the final size of cooperators. Once the initial cooperative density in the strong-link layer $\rho_s^c(0) > 0.5$, and $\rho_s^c > 1 - \rho_s^c$, the individuals in the system tends to be cooperative. Conversely, if the initial cooperative density in the strong-link layer $\rho_s^c(0) < 0.5$, we could get $\rho_s^c < 1 - \rho_s^c$, the individuals in the system will tend to be defective.

2. In the case of $\beta = 1$, an individual will decide his strategy only based on his strong relations. Equation (12) can be written as follows
\[
\frac{d\rho_s^C}{dt} = (1 - \rho_s^C) \rho_s^C \tan h \left( \frac{(1 - T)(k)}{2\kappa} \right).
\]

Assuming that the strong-link layer is a star network: (i) if the center node adopts the cooperative strategy, the expected gain of this central node \( i \) is \( f_{i_{\text{Hub}}} = N \rho_s^C \). For any non-central node \( j \), once he/she adopts a cooperative strategy, the expected profit is \( f_j^C = R \), otherwise, \( f_j^D = T \). Thus, when \( f_{i_{\text{Hub}}} > f_j^D \), that is \( \rho_s^C > T/N \), the propensity for cooperation could be greatly enhanced. (ii) However, if the center node takes the defective strategy, the expected gain of the central node is \( f_{i_{\text{Hub}}} = N T \rho_s^C \). For any non-central node \( k \), if he/she adopts a cooperative strategy, the expected profit is \( f_k^C = S \). However, once he/she adopts the defective strategy, the expected profit is \( f_k^D = P \). In general, \( f_{i_{\text{Hub}}} > f_j^C \) and \( f_{i_{\text{Hub}}} > f_j^D \), as a result the cooperation becomes the most abundant strategy.

3.2. The strong-link layer is a heterogeneous network

When the strong-link structure is a heterogeneous network with the degree distribution \( P(k) \), where the average degree is \( \langle k \rangle \), we define \( \rho_{s,k}^C \) as the fraction of cooperators with the given degree \( k \) in the strong-link layer. Then, the evolution of the fraction of cooperators \( \rho_{s,k}^C \), reads

\[
\rho_{s,k}^C(t + 1) = \rho_{s,k}^C(t) + \beta [\rho_{s,k}^{D-C} - \rho_{s,k}^{C-D}] + (1 - \beta) \pi^{s,w}.
\]

The second term and the third term in the right-hand side of equation (15) stand for the change of the cooperators’ density affected by the strong links and weak links, respectively. Note that weak links are memoryless, that is, the weak links formed in each game round are independent. Thus, the item of \( \pi^{s,w} \) in equations (15) and (5) are the same.

The transition probabilities of \( \rho_{s,k}^{D-C} \) and \( \rho_{s,k}^{C-D} \) corresponding to the game dynamics are expressed by

\[
\rho_{s,k}^{D-C} = \rho_{s,k}^D \sum_{k'} \frac{k' P(k') \rho_{s,k'}^C}{\langle k \rangle} P(D \rightarrow C)
\]

\[= \rho_{s,k}^D \sum_{k'} \frac{k' P(k') \rho_{s,k'}^C}{\langle k \rangle} \frac{1}{1 + \exp \left( -\frac{f_j^C - f_{i_{\text{Hub}}}^C}{\kappa} \right)}\]

(16)

and

\[
\rho_{s,k}^{C-D} = \rho_{s,k}^C \sum_{k'} \frac{k' P(k') \rho_{s,k'}^D}{\langle k \rangle} P(C \rightarrow D)
\]

\[= \rho_{s,k}^C \sum_{k'} \frac{k' P(k') \rho_{s,k'}^D}{\langle k \rangle} \frac{1}{1 + \exp \left( -\frac{f_j^D - f_{i_{\text{Hub}}}^D}{\kappa} \right)}\]

(17)

where \( f_j^k = k \rho_s^C, f_j^T = k \rho_s^C T, f_j^C = \langle k \rangle \rho_s^C, f_j^D = \langle k \rangle \rho_s^C T, \rho_s^C + \rho_s^D = P(k) \) and \( \sum_k P(k) = 1 \).

According to equation (15), we get that the change of individual strategy is independent of the strong links when \( \beta = 0 \). That is, an individual alters his strategy on the basis of his strategy cognition in the weak-link layer. Consequently, no matter how the strong-link structure is, we can get the same conclusion as formula (13).

When \( \beta = 1 \), an individual will decide one’s strategy only based on his/her neighbor’s strategy in the strong-link layer. The change of the fraction of cooperators \( \rho_{s,k}^C \) is described by

\[
\frac{d\rho_{s,k}^C}{dt} = \rho_{s,k}^{D-C} - \rho_{s,k}^{C-D}.
\]

(18)

Let \( \frac{d\rho_{s,k}^C}{dt} = 0 \), combining equations (15)–(18), we obtain

\[
\frac{d\rho_{s,k}^C}{dt} = \rho_{s,k}^D \sum_{k'} \frac{k' P(k') \rho_{s,k'}^C}{\langle k \rangle} \frac{1}{1 + \exp \left( -\frac{f_j^C - f_{i_{\text{Hub}}}^C}{\kappa} \right)}
\]

\[- \rho_{s,k}^C \sum_{k'} \frac{k' P(k') \rho_{s,k'}^D}{\langle k \rangle} \frac{1}{1 + \exp \left( -\frac{f_j^D - f_{i_{\text{Hub}}}^D}{\kappa} \right)} = 0.
\]

(19)
strong-link structure is:

We focus on the following three cases to perform the numerical simulations on the proposed PD game.

4. Numerical simulations

It is easy to see that equation (19) has the trivial solutions \( r^C_{s,k} = 0 \) and \( r^D_{s,k} = 0 \). Let \( \phi^C_{s} = \sum_{k'} k' P(k') \rho^C_{s,k} / \langle k \rangle \),

\[
\phi^D_{s} = \sum_{k'} k' P(k') \rho^D_{s,k} / \langle k \rangle \quad \text{and} \quad \Gamma = \frac{1 + \exp(-g)}{1 + \exp(-b)}
\]

then the general solution of formula (19) can be obtained as follows:

\[
\rho^C_{s,k} = \frac{\phi^C_{s} P(k)}{\phi^C_{s} + \Gamma \phi^D_{s}}
\]

Therefore, when \( \frac{d\rho^C_{s,k}}{dt} = 0 \), that is \( \rho^C_{s,k} = 0 \), \( r^C_{s,k} = P(k) \) or \( r^C_{s,k} = \frac{\phi^C_{s} P(k)}{\phi^C_{s} + \Gamma \phi^D_{s}} \), the game system is in equilibrium.

The condition \( \rho^C_{s,k} = \frac{\phi^C_{s} P(k)}{\phi^C_{s} + \Gamma \phi^D_{s}} \) supports the coexistence of cooperators and defectors.

4. Numerical simulations

We focus on the following three cases to perform the numerical simulations on the proposed PD game. The strong-link structure is: (a) a lattice network with the periodic boundary, where the total players are \( N = L^2 = 1024 \) and the average degree \( \langle k \rangle = 4 \), or (b) an ER random network with \( N = 1000 \) players and the average degree \( \langle k \rangle = 6 \), or (c) a BA scale-free network with \( N = 1000 \), the degree obeys the distribution \( P(k) \sim k^{-\gamma_1} \) with \( \gamma_1 = 2.1 \) and the average degree \( \langle k \rangle = 6 \). The players’ activity rates obey the distribution \( F(a) \sim a^{-\gamma_2} \), \( \gamma_2 = 2.1 \), \( \epsilon = 0.1 \). An activated player generates \( m = 3 \) links that are connected to other randomly selected weak relations (we find that the evolution trend of the fraction of cooperators almost keeps the same with different value of \( m \)). Each player \( i \) in the strong-link layer is initially designated either as a cooperator \( (s_i = 1) \) or defector \( (s_i = -1) \) with equal probability. Based on the Monte Carlo method, we simulate the PD game in two-layered networks. Since the results differ for each Monte Carlo trial, we present the results averaged over 100 independent runs. For simplicity, we use \( \rho_C(t) \) to denote the fraction of cooperators at time \( t \) in the strong-link layer.

Figures 2(a1)–(c1) illustrate the fraction of cooperators \( \rho_C \) for different \( \beta \), and figures 2(a2)–(c2) depict the final fraction of cooperators \( \rho_C(\infty) \) where the strong-link structure is the ER random networks or the BA
networks. We conclude that the cooperators could not be extinct only when the value of $\beta$ is large enough. However, when the strong-link structure is the lattice networks, even though the value of $\beta$ is large enough, the final fraction of cooperators $\rho_C(\infty) = 0$. The results indicate the higher the value of $\beta$, the larger the stationary fraction of cooperators $\rho_C(\infty)$. When $\beta$ is sufficiently large, it means that the players prefer to update their strategies rely on their strong relations. Whereas the weak links have less influence on individuals’ behavior choice. Consequently, if the players actively acquire or passively receive the information indiscriminately, and blindly follow his counterpart’s strategy information, the individuals’ cooperation behavior could be restrained. On the one hand, the defector strategy can bring more advantages, on the other hand, weak links promote information sharing. As a result, if the players update their strategies primarily depending on the information they received in the weak-link layer, the cooperation becomes the most abundant strategy. Comparing with information sharing. As a result, if the players update their strategies primarily depending on the information they received in the weak-link layer, the cooperation becomes the most abundant strategy. Comparing with information sharing. As a result, if the players update their strategies primarily depending on the information they received in the weak-link layer, the cooperation becomes the most abundant strategy.

In figures 3(a)–(c), we find that the increase of $\beta$ leads to the emergence of the cooperators in the case of a small $T$. However, with the increase of the temptation gain $T$, the advantage for cooperators disappears quickly, and they earn a smaller payoff than the defectors. A larger $T$ results in the decrease of the final fraction of cooperators $\rho_C(\infty)$. Once the temptation gain $T$ is larger than a certain threshold, no matter how large the parameter $\beta$ is, the cooperation cannot persist and all individuals will choose to defect in the end.

The individuals who take the cooperative strategy in one game round are called as the Cooperation-Preference. If the whole population has played $M$ rounds of game, the frequency of player $i$ to adopt the cooperative strategy is noted as $C$-Preference($i$) = $\frac{M}{T} \sum_{t=1}^{M} \delta(s_t(i), C)$, where $\delta(s_t(i), C)$ is the Dirac function, when $s_t(i) = C$, $\delta(s_t(i), C) = 1$, otherwise, $\delta(s_t(i), C) = 0$. Moreover, the Pearson correlation coefficient between the players’ activity rate $a$ and the C-Preference(CP) is $R = \frac{\sum_{i=1}^{N}(CP(i) - \langle CP \rangle)(a(i) - \langle a \rangle)}{\sqrt{\sum_{i=1}^{N}(CP(i) - \langle CP \rangle)^2} \sqrt{\sum_{i=1}^{N}(a(i) - \langle a \rangle)^2}}$, where $\langle CP \rangle$ and $\langle a \rangle$ is average value of C-Preference and activity rate, respectively. From figure 4, we find the correlation between activity rate $a$ and C-Preference is not significant, and the value of C-Preference almost concentrate around a certain numerical value. That is, when we fix $\beta$ and $T$, the players’ activity rates in the weak-link layer have less effect on their strategies’ selection in the strong-link layer. By comparing figures 4(a2), (b2) and (c2), we may observe that the difference of the strong-link networks may lead a great change of the frequency distribution of C-Preference dramatically.

At some given rounds of game, the individuals’ strategy states in the strong-link layer are shown in figure 5. We observe that even when the temptation gain $T$ is extreme small, the fraction of cooperators $\rho_C$ gradually reduces as well. Although none cooperators exist in the middle bottom of figure 5(b), the cooperators appear in the same place of figure 5(c). In addition, even if some individuals are surrounded by the defectors, those isolated individuals may remain cooperation due to the weak links. From figures 5(e) and (f), one can observe that the cooperative cluster may appear, however, it is vulnerable to the invasion of defectors and become difficult to expand.

In the last, we further analyze how the correlation between the individuals’ strong connections and their activity rate influences the evolutionary game. We consider the following two cases: the activity rate in the weak-link layer is in negative correlation, or positive correlation with the individuals’ strong connections. Through the comparative analysis in figures 6(a) and (b), we find that when the relation between the players’ connections in

Figure 3. Color map encoding the fraction of cooperators $\rho_C(\infty)$ on the $T - \beta$ parameter plane. From left to right, we set the strong-link networks are the lattice networks, the ER random networks, and the BA networks. The increase of the temptation gain $T$ results in the decrease of the final cooperative density $\rho_C(\infty)$.
strong-link layer and their activity rates in weak-link layer is in the negative correlation, the individuals’ cooperation behavior could be promoted markedly. Conversely, if the individuals’ strong connections are large enough, and the individuals are very active in the weak-link layer, then it is not conducive to improve the individuals’ cooperative behavior.

5. Conclusions

In this paper, we have explored the PD game on two-layered networks with the weak-link layer and the strong-link layer. If the weak relations have the dominate influences on the choice of the individuals’ strategies, the cooperation becomes the most abundant strategy. A higher heterogeneity of the strong-link layer could be conducive to promote the individuals’ cooperation behavior. In addition, the situation that while the relation

Figure 4. The relation between the activity rate and the Cooperation-Preference, as the strong-link layer are the lattice networks, the ER random networks and the BA networks. The first row presents the relation between the C-Preference and the players’ activity rates. The second row shows the frequency distribution diagram of C-Preference. Other parameters are set as: $T = 1.1, M = 1000, \beta = 0.95$. (a1)–(a2) the strong-link layer is the lattice networks; (b1)–(b2) the strong link layer is the ER random networks; (c1)–(c2) the strong-link layer is the BA networks.

Figure 5. The typical snapshot distribution of cooperators is illustrated for the game round $t = 1, t = 10, t = 50$ in the strong-link layer, as the strong-link layer is the lattice networks. The yellow part represents the cooperative individuals, the blue part corresponds to the defective individuals. Other parameters are set to be: $\beta = 0.98$. (a)–(c) $T = 1.01$; (d)–(f) $T = 1.05$. 

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between the players’ strong connections and their activity rates is in the negative correlation, leading to the dominance of cooperation. Nevertheless, more analytical investigations deserve more efforts on the interdependence of weak-link and strong-link with the eyes of empirical data and experimental supports.

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Figure 6. The final fraction of cooperators $\rho_c(\infty)$ for different combinations of the temptation gain $T$ and $\beta$, as the strong-link layer is the BA scale-free networks. The first column indicates the activity rate $\alpha$ in the weak-link layer is in the negative correlation with the players’ strong connections $k$, and $\rho_{\alpha,k} = -0.1966$. The last column indicates the activity rate $\alpha$ in the weak-link layer is in the positive correlation with the players’ strong connections $k$ in the strong-link layer, and $\rho_{\alpha,k} = 0.8436$. 
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