A New Look at Those Old Black Holes: Existence of Universal Horizons

Kai Lin a,b, O. Goldoni c,d, M.F. da Silva d, and Anzhong Wang a,d*

a Institute for Advanced Physics & Mathematics, Zhejiang University of Technology, Hangzhou 310032, China
b Instituto de Física, Universidade de São Paulo, CP 66318, 05315-970, São Paulo, Brazil
c Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, Maracanã, CEP 20550013, Rio de Janeiro, RJ, Brazil
d GCAP-CASPER, Physics Department, Baylor University, Waco, TX 76798-7316, USA

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In this paper, we study the existence of universal horizons in static spacetimes, and find that the khronon field can be solved explicitly when its velocity becomes infinitely large, in which the universal horizons coincide with the sound horizon of the khronon. Choosing the timelike coordinate aligned with the khronon, the static metric takes a simple form, from which it can be seen clearly that the metric now is free of singularity at the Killing horizons, but becomes singular at the universal horizons. Applying such developed formulas to three well-known black hole solutions, the Schwarzschild, Schwarzschild anti-de Sitter, and Reissner-Nordström, we find that in all these solutions universal horizons exist and are always inside the Killing horizons. The peeling-off behavior of the khronon depends on the coordinates adopted. In particular, in the Schwarzschild coordinates the khronon is peeling off at both Killing and universal horizons, while in the Eddington-Finkelstein and Painlevé-Gullstrand coordinates, the peeling-off behavior is found only when across the universal horizons. We also calculate the surface gravity at each of the universal horizons, and find that the surface gravity at the universal horizon is always greater than that at the Killing horizon.

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I. INTRODUCTION

The studies of black holes have been one of the main objects both theoretically and observationally over the last half of century [1, 2], and so far there are many solid observational evidences for their existence in our universe. Theoretically, such investigations have been playing a crucial role in the understanding of the nature of gravity in general, and quantum gravity in particular. They started with the discovery of the laws of black hole mechanics [3] and Hawking radiation [4], and led to the profound recognition of the thermodynamic interpretation of the four laws [5] and the reconstruction of general relativity (GR) as the thermodynamic limit of a more fundamental theory of gravity [6]. More recently, they are essential in understanding the AdS/CFT correspondence [7, 8] and firewalls [9].

Lately, such studies have gained further momenta in the framework of gravitational theories with broken Lorentz invariance (LI) [10–13]. In particular, Blas and Sibiryakov showed that an absolute horizon exists with respect to any signal with any large velocity, including the one with infinitely large one (instantaneous propagation) [10]. Such a horizon is dubbed as the universal horizon, which is always located inside a Killing horizon. A critical point is the existence of a globally well-defined hypersurface-orthogonal and timelike vector field $u_\mu$,

$$u_{[\mu} D_{\nu]} u_{\beta]} = 0.$$  \hspace{1cm} (1.1)

Then, it implies that there exists a scalar field $\phi$ [14], so that

$$u_\mu = \frac{\phi,_{\mu}}{\sqrt{X}},$$  \hspace{1cm} (1.2)

where $\phi,_{\mu} \equiv \partial \phi / \partial x^\mu, X \equiv -g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi$. Clearly, $u_\mu$ is invariant under the gauge transformations,

$$\tilde{\phi} = F(\phi),$$  \hspace{1cm} (1.3)

where $F(\phi)$ is a monotonically increasing and otherwise arbitrary function of $\phi$. Such a scalar field was often referred to as the khronon [15], and is equivalent to the Einstein-aether ($\mathcal{E}$-) theory [16], when the aether $u_\mu$ is hypersurface-orthogonal, as showed explicitly in [17] (See also [18]).

Note that in the studies of the existence of the universal horizons carried out so far [10–13], the khronon field is always part of the underlined theory of gravity. To generalize such definitions to any theories that violate LI, recently the khronon $\phi$ was promoted to a probe field, and assumed that it plays the same role as a Killing vector field in a given space-time, so its existence does not affect the background, but defines the properties of it [19]. By this way, such a field is no longer part of the gravitational field and it may or may not exist in a given space-time. Applied such a generalized definition of the universal horizons to static charged solutions of the healthy extensions [15] of the Hořava-Lifshitz (HL)
gravity \[20\], it was showed explicitly that universal horizons exist in some of these solutions \[19\]. Such horizons exist not only in the IR limit of the HL gravity, as has been considered so far \[10, 11\] but also in the full HL gravity, that is, when high-order operators are taken into account.

In this paper, we shall apply such a definition of universal horizons to the well-known black holes, Schwarzschild, Schwarzschild anti-de Sitter, and Reissner-Nordström, as they are often also solutions of gravitational theories with broken LI \[21\]. We shall show that in all these solutions universal horizons always exist inside the Killing horizons. We also investigate the peeling-off behavior of the khronon in three different systems of coordinates, the Schwarzschild, Eddington-Finkelstein, and Painleve-Gullstrand. Specifically, the paper is organized as follows: In Sec. II, we give a brief review on the definition of universal horizons in terms of khronon, while in Sec. III, we apply it to static spacetimes. In Sec. III, we consider the problem in the three well-known systems of coordinates, the Schwarzschild, Eddington-Finkelstein, and Painleve-Gullstrand, and show explicitly how to make coordinate transformations to the khronon coordinates, so the metric takes the form,

\[
ds^2 = -\frac{(F\alpha^2 + 1)^2}{4\alpha^2} d\varphi^2 + \frac{F^2 - 1}{4\alpha^2} d\psi^2 + r^2 d\Omega^2,
\]

from which we can see that the metric is free of coordinate singularity at the Killing horizons \(F(r) = 0\), but becomes singular at the universal horizons \(F\alpha^2 + 1 = 0\), where \(\alpha = \alpha(r)\). In Section IV, we show that the khronon equation can be solved explicitly when the speed of the khronon becomes infinitely large. Then, we apply such formulas to the Schwarzschild, Schwarzschild anti-de Sitter, and Reissner-Nordström solutions, and show explicitly the existence of universal horizons in each of these solutions. The paper is ended in Sec. V, in which we present our main conclusions. An appendix is also included, in which we calculate the speed of the khronon mode in the Minkowski background.

II. UNIVERSAL HORIZONS AND BLACK HOLES

The khronon is described by the action \[16\],

\[
S_\phi = \int d^{D+1}x \sqrt{|g|} \left[ c_1 (D_{\mu} u_\nu)^2 + c_2 (D_\nu u_\mu)^2 + c_3 (D_{\mu} u_\nu) (D_\nu u_\mu) - c_4 a_{\mu} a_{\nu} \right],
\]

where \(c_i\)'s are arbitrary constants, and \(a_\mu = u_\nu D_\nu a_\mu\). The operator \(D_\mu\) denotes the covariant derivative with respect to the background metric \(g_{\mu\nu}\). Note that the above action is the most general one in the sense that the resulting differential equations in terms of \(u_\mu\) are second-order \[16\]. However, with the hypersurface-orthogonal condition \[1.1\], it can be shown that only three of the four coupling constants \(c_i\) are independent. In fact, now we have the identity \[10\],

\[
\Delta L_\phi = -a^\mu a_\mu (D_\alpha u_\beta)(D^\alpha u^\beta) + (D_\alpha u_\beta)(D^\beta u^\alpha) = 0.
\]

Then, we can always add the term,

\[
\Delta S_\phi = -c_0 \int \sqrt{|g|} d^{D+1}x \Delta L_\phi,
\]

into \(S_\phi\), where \(c_0\) is an arbitrary constant. This is effectively to shift the coupling constants \(c_i\) to \(c'_i\), where

\[
c'_1 = c_1 + c_0, \quad c'_2 = c_2, \quad c'_3 = c_3 - c_0, \quad c'_4 = c_4 - c_0.
\]

Thus, by properly choosing \(c_0\), one can always set one of \(c_{1,3,4}\) to zero. However, in the following we shall leave this possibility open.

The variation of \(S_\phi\) with respect to \(\phi\) yields the khronon equation,

\[
D_\mu A^\mu = 0,
\]

where \[18\] 1,\[
A^\mu = \frac{(\delta^\mu_\nu + u^\mu u_\nu)}{\sqrt{X}} E^\nu,
\]

\[
E^\nu = D_\nu J^\nu + c_4 a_\nu u^\gamma, \quad J^\alpha = (c_1 g^{\alpha \beta} g_{\mu \nu} + c_2 g^{\alpha \beta} g_{\nu \gamma} + c_3 g^{\alpha \beta} g_{\gamma \mu} - c_4 u^\mu u^\nu g_{\mu \nu}) D_\beta u^\gamma.
\]

Eq. \(2.5\) is a second-order differential equation for \(u_\mu\), and to uniquely determine it, two boundary conditions are needed. These two conditions in stationary and asymptotically flat spacetimes can be chosen as follows \[10\] 2: (i) \(u_\mu\) is aligned asymptotically with the time translation Killing vector \(\zeta^\mu\),

\[
\zeta^\mu \propto \zeta^\mu.
\]

(ii) The khronon has a regular future sound horizon, which is a null surface of the effective metric \[22\],

\[
g^{(\phi)}_{\mu \nu} = g_{\mu \nu} - (c^2_\phi - 1) u_\mu u_\nu,
\]

where \(g^{(\phi)}\) denotes the speed of the khronon given by [cf. Appendix A],

\[
c^2_\phi = \frac{c_{123}}{c_{14}},
\]

where \(c_{123} = c_1 + c_2 + c_3, \ c_{14} = c_1 + c_4\). It is interesting to note that such a speed does not depend on the redefinition of the new parameters \(c'_i\), as it is expected.

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1 Notice the difference between the signatures of the metric chosen in this paper and the ones in \[13\].

2 These conditions can be easily generalized to asymptotically anti-de Sitter spacetimes.
A Killing horizon is defined as the existence of a hypersurface on which the time translation Killing vector \( \zeta^\mu \) becomes null,

\[
\zeta^\lambda \zeta_\lambda = 0. \tag{2.10}
\]

On the other hand, a universal horizon is defined as the existence of a hypersurface on which \( \zeta^\mu \) becomes orthogonal to \( u_\mu \),

\[
u_\lambda \zeta^\lambda = 0. \tag{2.11}
\]

Since \( u_\mu \) is timelike globally, Eq. (2.11) is possible only when \( \zeta_\mu \) becomes spacelike. This can happen only inside the apparent horizon. Then, we can define region inside the universal horizon as black hole, since any signal cannot escape to infinity, once it is trapped inside it, no matter how large its velocity is.

The corresponding surface gravity is defined as [13].

\[
\kappa \equiv \frac{1}{2} \mu^\alpha D_\alpha (u_\lambda \zeta^\lambda). \tag{2.12}
\]

III. STATIC SPACETIMES

From the last section, it can be seen that the Killing and universal horizons, as well as the surfer gravity, are all defined in covariant form, so they are gauge-invariant. However, the singular behaviors of the khronon \( \psi \) will depend on the choices of coordinates. In this section, we shall consider three different and often-used systems of coordinates.

A. Eddington-Finkelstein Coordinates

In terms of the Eddington-Finkelstein coordinates \((v, r)\), static spacetimes are described by the metric,

\[
ds^2 = -F(r) dv^2 + 2f(r) dvdr + r^2 d\Omega_k^2, \tag{3.1}\]

where \( k = 0, \pm 1, \) and

\[
d\Omega_k^2 = \begin{cases} 
d\theta^2 + \sin^2 \theta d\varphi^2, & k = 1, \\
d\theta^2 + d\varphi^2, & k = 0, \\
d\theta^2 + \sinh^2 \theta d\varphi^2, & k = -1.
\end{cases} \tag{3.2}\]

In these coordinates, the time-translation Killing vector \( \zeta^\mu \) is given by

\[
\zeta^\mu = \delta^\mu_v, \tag{3.3}
\]

and the location of the Killing horizons are the roots of the equation,

\[
F(r)|_{r=r_{EH}} = 0, \tag{3.4}
\]

on which \( \zeta^\mu \) becomes null, \( \zeta^\lambda \zeta_\lambda |_{r=r_{EH}} = 0 \). The four-velocity of the khronon is parametrized as [12] \(^3\),

\[
u^\mu = -\alpha \delta^\mu_t - \beta \delta^\mu_r, \
u_\mu = \frac{F^2 + 1}{2\alpha} \delta^\mu_r - \alpha f \delta^\mu_v, \tag{3.5}\]

where

\[
\beta \equiv \frac{F^2 - 1}{2\alpha f}. \tag{3.6}\]

Then, the location of the universal horizon is at \( \zeta^\lambda u_\lambda = (F^2 + 1)/(2\alpha) = 0 \), or

\[
F^2 + 1 = 0, \tag{3.7}
\]

which is possible only inside the Killing horizon, because only in that region \( F(r) \) can be negative.

It is interesting to note that \( g_{\mu\nu} \) and \( g^{\mu\nu} \) in this set of coordinates are not singular at both Killing and universal horizons, as one can see from the expressions,

\[
g_{vv} = -F(r), \quad g_{vr} = f(r), \quad g_{rr} = 0, 

g^{vv} = 0, \quad g^{vr} = \frac{1}{f(r)}, \quad g^{rr} = \frac{F(r)}{f^2(r)}. \tag{3.8}\]

On the other hand, introducing the spacelike unit vector \( s_\mu \),

\[
s_\mu = \alpha \delta^\mu_t + \frac{F^2 + 1}{2\alpha f} \delta^\mu_r, 

s_\mu = -f \left( \beta \delta^\mu_t - \alpha \delta^\mu_r \right), \tag{3.9}\]

which is orthogonal to \( u_\mu \), i.e., \( s_\lambda u^\lambda = 0 \), we find that it defines a family of timelike hypersurfaces, \( \psi = \) Constant, where

\[
\psi \equiv -v - \int \frac{s_r}{s_v} dr 

= -v + \int \frac{2\alpha^2 f}{F^2 - 1} dr. \tag{3.10}\]

Similarly, the kronon field \( \phi \) is given by

\[
\phi \equiv v + \int \frac{u_r}{u_v} dr 

= v - \int \frac{2\alpha^2 f}{F^2 - 1} dr. \tag{3.11}\]

From Eqs. (3.10) and (3.11) we can see that in general both \( \phi \) and \( \psi \) are smoothly crossing the Killing horizons. But this is no longer the case when across the universal

\(^3\) Note the sign difference of \( u_\mu \) used here and the one used in [12]. In the current case, one can see that \( \phi \) is asymptotically given by \( t \) in asymptotically flat spacetimes.
horizons, as \( \phi \) becomes singular there. It is interesting to note that, in contrast to the khronon \( \phi \), the spacelike coordinate \( \psi \) is well-defined at the universal horizon.

In terms of \( d\phi \) and \( d\psi \), we find that

\[
dv = \frac{F \alpha^2 + 1}{2} d\phi + \frac{F \alpha^2 - 1}{2} d\psi,
\]

\[
dr = \frac{F^2 \alpha^4 - 1}{4\alpha^2 f} (d\phi + d\psi).
\]

Inserting the above expressions into Eq.(3.1), we obtain

\[
ds^2 = -\frac{(F \alpha^2 + 1)^2}{4\alpha^2} d\phi^2 + \frac{(F \alpha^2 - 1)^2}{4\alpha^2} d\psi^2 + r^2 d\Omega^2_k,
\]

from which we can see that the metric is free of coordinate singularity at the Killing horizons, but becomes singular at the universal horizons. It is interesting to note that the metric component \( g^{\phi \psi} \) behaves as

\[
g^{\phi \psi} \simeq (r - r_{UH})^{-2n},
\]

as \( r \to r_{UH} \), where \( n \geq 1 \). Thus, the nature of the coordinate singularities of the metric at the universal horizons is more like that of the Killing horizon in the extreme charged black hole, rather than that of a normal Killing horizon [23]. This may indicate that the universal horizons are not stable [10].

B. Schwarzschild Coordinates

Setting

\[
dv = dt + \frac{f(r)}{F(r)} dr,
\]

the metric (3.1) becomes

\[
ds^2 = -F(r) dt^2 + \frac{f^2(r)}{F(r)} dr^2 + r^2 d\Omega^2_k,
\]

while \( u^\mu, s^\mu \) and \( \zeta^\mu \) take the forms,

\[
\begin{align*}
u^\mu &= -\frac{F \alpha^2 + 1}{2\alpha F} \delta^\mu_t - \beta \delta^\mu_r, \\
u_\mu &= \frac{F \alpha^2 + 1}{2\alpha} \delta^\mu_t - \beta \delta^\mu_r, \\
s^\mu &= \frac{\beta f}{F} \delta^\mu_t + \frac{F \alpha^2 + 1}{2\alpha f} \delta^\mu_r, \\
s_\mu &= -\beta f \delta^\mu_t + \frac{f(F \alpha^2 + 1)}{2\alpha F} \delta^\mu_r, \\
\zeta^\mu &= \delta^\mu_r.
\end{align*}
\]

In terms of \( t \) and \( r, \phi \) and \( \psi \) are given by,

\[
\phi = t + \int \frac{u_r}{u_\phi} dr,
\]

\[
\psi = -t - \int \frac{s_r}{s_\phi} dr
\]

from which we can see that both \( \phi \) and \( \psi \) now are singular at the Killing horizons \( F(r) = 0 \), while only \( \phi \) is singular at the universal horizons \( F \alpha^2 + 1 = 0 \). The main reason is that the metric now becomes singular at the Killing horizon, as it can be seen from Eq.(3.16), while in the Eddington-Finkelstein coordinates the metric is regular at both Killing and universal horizons. Note that the locations of the Killing and universal horizons remain the same, as they are all defined in gauge-invariant forms, as mentioned above. From Eqs.(3.18) we find that

\[
\begin{align*}
dt &= \frac{F + \beta f^2}{F} d\phi + \frac{\beta^2 f^2}{F} d\psi, \\
dr &= \frac{\beta(F \alpha^2 + 1)}{2\alpha} (d\phi + d\psi).
\end{align*}
\]

Inserting it into Eq.(3.16), we find that the metric takes the same form as that given by Eq.(3.13) in terms of \( d\phi \) and \( d\psi \).

C. Painleve-Gullstrand Coordinates

Setting [23],

\[
\tau = t + \int \frac{f \sqrt{1 - F}}{F} dr,
\]

the metric (3.16) becomes

\[
ds^2 - dr^2 + f^2 \left( dr + \frac{\sqrt{1 - F}}{f} d\tau \right)^2 + r^2 d\Omega^2_k,
\]

from which we find that,

\[
\begin{align*}
g_{\tau \tau} &= -F, \\
g_{\tau \rho} &= f \sqrt{1 - F}, \\
g_{\rho \rho} &= f^2, \\
g^{\tau \tau} &= -1, \\
g^{\tau \rho} &= \frac{\sqrt{1 - F}}{f}, \\
g^{\rho \rho} &= \frac{F}{f^2}.
\end{align*}
\]

Therefore, at the both Killing and universal horizons, the metric is also non-singular. But, to have the metric real, we must assume that \( F(r) \leq 1 \). In terms of \( \tau \) and \( r \), we find that

\[
\begin{align*}
u^\mu &= -\frac{4\alpha^2 + (F \alpha^2 - 1)^2}{2\alpha F} \delta^\mu_t - \beta \delta^\mu_r, \\
u_\mu &= \frac{4\alpha^2 + (F \alpha^2 - 1)^2}{2\alpha} \delta^\mu_t - \beta \delta^\mu_r, \\
s^\mu &= \frac{4\alpha^2 - (F \alpha^2 + 1)^2}{2\alpha F} \delta^\mu_t + \frac{e\alpha^2 + 1}{2\alpha f} \delta^\mu_r, \\
s_\mu &= \frac{4\alpha^2 - (F \alpha^2 + 1)^2}{2\alpha} \delta^\mu_t - \frac{e\alpha^2 + 1}{2\alpha f} \delta^\mu_r, \\
\zeta^\mu &= \delta^\mu_r.
\end{align*}
\]
where
\[ \Delta_+ \equiv (F\alpha^2 + 1) + \sqrt{1-F} (1-F\alpha^2), \]
\[ \Delta_- \equiv \sqrt{1-F} (F\alpha^2 + 1) + (1-F\alpha^2). \]  
(3.24)

Then, we have
\[ \phi = \tau + \int \frac{u_r}{ur} dr \]
\[ = \tau + \int \frac{f[4\alpha^2 - (F\alpha^2 + 1)^2]}{(F\alpha^2 + 1) \Delta_-} dr, \]
\[ \psi = -\tau - \int \frac{u_r}{ur} dr \]
\[ = -\tau - \int \frac{f[4\alpha^2 + (F\alpha^2 - 1)^2]}{(1-F\alpha^2) \Delta_+} dr. \]  
(3.25)

Therefore, similar to that in the Eddington-Finkelstein coordinates, now only \( \phi \) peels off at the universal horizons, while both \( \phi \) and \( \psi \) are smoothly crossing the Killing horizon.

In terms of \( d\phi \) and \( d\psi \), the metric \( [3.21] \) also reduces to that given by Eq.\( (3.13) \).

**IV. EXISTENCE OF UNIVERSAL HORIZONS IN WELL-KNOWN BLACK HOLE SPACETIMES**

In most of the well-known black hole solutions, we have
\[ f(r) = 1. \]  
(4.1)

Thus, in this section we consider static space-times with this condition. Then, from the definition \( [2.6] \) of \( \mathcal{A}^\mu \) we find that
\[ \mathcal{A}^t = \frac{c_{12} U(F + U^2)}{r^2 F} \left( r^2 U'' + 2r U' - 2U \right) \]
\[ - \frac{c_{14} U}{rF(F + U^2)} \left( -4rUF'U'' - rF'' \right. \]
\[ + 2F \left( rF'' + 2rU'' + 2rU^2 + 4U^2 \right) \]
\[ \left. + 2U' \left( rF'' + 2rU' + 4U \right) \right) \]
\[ = \frac{F\sqrt{F + U^2}}{U} \mathcal{A}^t, \quad \mathcal{A}^\theta = \mathcal{A}^\phi = 0, \]  
(4.2)

where \( U \equiv u^r = Fu_r \), and
\[ u_r^2 = F(r) + U^2. \]  
(4.3)

When space-time is asymptotically flat, the khronon equation \( [2.3] \) is equivalent to \( [19] \).
\[ \mathcal{A}^\mu = 0. \]  
(4.4)

Then, from Eq.\( (4.2) \) we find that
\[ c_{12} \left( F + U^2 \right)^2 \left( r^2 U'' + 2r U' - 2U \right) \]
\[ - \frac{c_{14} r U}{4} \left[ -4rUF'U'' - rF'' \right. \]
\[ + 2F \left( rF'' + 2rU'' + 2rU^2 + 4U^2 \right) \]
\[ \left. + 2U' \left( rF'' + 2rU' + 4U \right) \right) \]
\[ = 0. \]  
(4.5)

This is a nonlinear equation for \( U \), and is found difficult to solve in the general case. However, when \( c_{14} = 0 \), it reduces to
\[ r^2 U'' + 2r U' - 2U = 0, \]  
(4.6)

which has the general solution, \( U = rA^\mu - (r_o/r)^2 \), where \( r_A \) and \( r_o \) are two integration constants. However, the asymptotical condition Eq.\( (2.7) \) requires \( r_A = 0 \), so finally we have
\[ U = \frac{-r^2}{r^2}. \]  
(4.7)

Then, from Eq.\( (4.3) \) we obtain \( u_t = \sqrt{G(r)} \), where
\[ G(r) \equiv U^2 + F(r). \]  
(4.8)

Hence, we get
\[ u^\mu = -\frac{\sqrt{G(r)}}{F(r)} \delta^\mu_0 - U \delta^\mu_t, \]
\[ u_\mu = \frac{G(t)}{F(t)} \delta_\mu - U \delta_\mu_t. \]  
(4.9)

Now several remarks are in order. First, in order for the khronon field \( \phi \) to be well-defined, we must assume that
\[ G(r) \geq 0, \]  
(4.10)

in the whole space-time, including the internal region of a Killing horizon, in which we have \( F(r) < 0 \). Second, for the choice \( c_{14} = 0 \), the khronon has an infinitely large speed \( c_\phi = \infty \), as can be seen from Eq.\( (2.9) \). Then, by definition the universal horizon coincides with the sound horizon of the spin-0 khronon mode. So, the regularity of the khronon on the sound horizon now becomes the regularity on the universal horizon. On the other hand, from Eq.\( (4.9) \) we find that
\[ u_\mu U^\mu = \sqrt{G(r)}. \]  
(4.11)

Then, from the regular condition \( (4.10) \) we can see that the universal horizon located at \( r = r_{UH} \) must be also a minimum of \( G(r) \), where \( r_{UH} \) is a real root of \( G(r_{UH}) = 0 \). Therefore, at the universal horizons we must have \( [12] \),
\[ G(r) \vert_{r=UH} = 0 = G'(r) \vert_{r=UH}, \]  
(4.12)

as illustrated in Fig. \( \[1 \] \). Clearly, in general \( G(r) \) can have several such minimums, and we shall define the one with maximal radius as the universal horizon.

On the other hand, setting
\[ \phi = t + \varphi(r), \]  
(4.13)

and from Eq.\( (4.2) \) we find that
\[ \varphi(r) = -\int \frac{U(r)}{F\sqrt{G(r)}} dr. \]  
(4.14)
Assuming

$$G(r) = (r - r_{UH})^{2n}\mathcal{G}(r, r_{UH}), \ (n \geq 1),$$

(4.15)

where $\mathcal{G}(r_{UH}, r_{UH}) \neq 0$, we find that $\varphi$ in the neighborhood $r = r_{UH}$ is singular, and to the leading order is given by

$$\varphi(r) = \begin{cases} -\alpha_0 \ln (r - r_{UH}) + \hat{\varphi}_1(r, r_{UH}), & n = 1, \\ \frac{-\alpha_0}{(n-1)(r-r_{UH})^{n-1}} + \hat{\varphi}_n(r, r_{UH}), & n > 1, \end{cases}$$

(4.16)

where

$$\frac{U}{F\sqrt{\mathcal{G}}} = \sum_{m=0}^{\infty} \alpha_m (r - r_{UH})^m,$$

$$\hat{\varphi}_n(r, r_{UH}) = -\sum_{m=1}^{\infty} \frac{\alpha_m (r - r_{UH})^{m-n+1}}{m - n + 1}.$$  

(4.17)

It is interesting to note that $\varphi$ is also singular at the Killing horizon $r = r_{KH}$, as now we have $F(r_{KH}) = 0$. Therefore, peeling-like behavior happens at both universal and Killing horizons in the Schwarzschild coordinates. However, all these singularities are coordinate ones, and can be removed by the field redefinition. In addition, as shown in the last section, in the Eddington-Finkelstein and Painleve-Gullstrand coordinates, the peering-off behavior only happens at the universal horizons.

From Eq.(4.13), on the other hand, we find that

$$d\phi = dt - \frac{Udr}{F\sqrt{\mathcal{G}}}.$$  

(4.18)

Inserting the above into Eq.(3.16) we obtain

$$ds^2 = -Fd\phi^2 - \frac{2Ud\phi dr}{\sqrt{\mathcal{G}}} + \frac{dr^2}{G} + r^2 d\Omega_k^2,$$

(4.19)

which is singular at both the Killing and the universal horizons.

The corresponding surface gravity, on the other hand, is given by,

$$\kappa_{UH} = \frac{1}{2} u^a D_a \left( u^\lambda \xi^\lambda \right) = \frac{r_o^2}{4r^4} \frac{G'}{\sqrt{\mathcal{G}}} \bigg\vert_{r=r_{UH}},$$

(4.20)

which is different from that normally defined in GR. Inserting Eq.(4.15) into the above, we find that

$$\kappa_{UH} = \frac{nr_o^2\sqrt{\mathcal{G}}}{2r_{UH}^2} (r - r_{UH})^{n-1} \bigg\vert_{r=r_{UH}}$$

$$= \begin{cases} \frac{r_o^2\sqrt{\mathcal{G}}}{2r_{UH}^2}, & n = 1, \\ 0, & n > 1. \end{cases}$$

(4.21)

Now, let us apply the above formal to some specific solutions.

### A. Schwarzschild Solution

The existence of the universal horizon in the Schwarzschild background was already studied numerically for various $c_\phi$ in [10]. When $c_\phi = \infty$, their results are the same as ours to be presented below. Here we shall provide more detailed studies, including the slices of $\phi = \text{Constant}$, and the ones of $\psi = \text{Constant}$ in all the three systems of coordinates. The Schwarzschild solution is given by

$$F(r) = 1 - \frac{r_o}{r}, \ k = 1.$$  

(4.22)

Then, we find that

$$G(r) = 1 - \frac{r_o}{r} + \frac{r_o^4}{r^4} = \begin{cases} \infty, & r = 0, \\ 1, & r = \infty, \end{cases}$$

$$G'(r) = \frac{r_o}{r^4} \left( r^3 - r_o^3 \right) U_{UH},$$

(4.23)

where $r_{UH} \equiv \left( \frac{4r_o^4}{r_s} \right)^{1/3}$, or inversely, $r_o = \left( \frac{4r_s}{U_{UH}} \right)^{1/3}$. Fig.2 shows the curve of $G(r)$ vs $r$. Then, from Eq.(4.12) we find that

$$r_o = \frac{3^{3/4}}{4} r_s, \ r_{UH} = \frac{3}{4} r_s.$$  

(4.24)

Note that $r_{UH}$ given above is the same as that found in [10] for $c_\phi = \infty$. Hence,

$$G(r) = \left( \frac{r - r_{UH}}{r_o} \right)^2 \left( \frac{r^2 + r_o^2}{2} + \frac{3r_o^2}{16} \right).$$

(4.25)

Then, in terms of $\phi$ and $\psi$ the Schwarzschild solution takes the form,

$$ds^2 = -\left( \frac{r - r_{UH}}{r_o} \right)^2 \left( \frac{r^2 + r_o^2}{2} + \frac{3r_o^2}{16} \right) d\phi^2$$

$$+ \left( \frac{r_o}{r} \right)^4 d\psi^2 + r^2 (\phi, \psi) d\Omega_k^2,$$

(4.26)
Killing and universal horizons.
from which we can see that
In Fig. 3 we draw the curves
Therefore, the surfaces
\( \phi = 0 \) is a constant, and \( \mathcal{U}_H \equiv \text{sign}(r - r_{UH}) \). Requiring that \( \phi(r)|_{r \to 0} \to 0 \), we find that,
\[
\varphi_0 = -r_s \frac{r_s \mathcal{U}_H}{8\sqrt{3}} \left[ 8\ln(16) - 3\sqrt{6}\ln \left( 2 + \sqrt{6} \right) \right].
\]
Therefore, the surfaces \( \phi = \phi_0 \) in the \((t, r)\)-plane have the properties,
\[
t(r) = \phi_0 - \varphi(r) = \begin{cases} 
+\infty, & r = r_{UH}^- \\
-\infty, & r = r_{UH}^+ \\
+\infty, & r = r_s 
\end{cases}
\]
In Fig. 3 we draw the curves \( t(r) \) vs \( r \) for different \( \phi_0 \)'s, from which we can see that \( t(r) \) becomes singular at both Killing and universal horizons.
On the other hand, from Eq. (4.18) we obtain,
\[
\psi(r) = -t + \int \frac{\sqrt{G}}{F} dr
\]
which now is free of coordinate singularity at the Killing horizon \( r = r_s \). Inserting \( G(r) \) given above into Eq. (4.14) we obtain,
\[
\begin{align*}
\phi(r) &= -\int \frac{U(r)}{F \sqrt{G(r)}} dr = \varphi_0 \\
&+ \int \frac{3\sqrt{3}r^2}{16(r - r_s)\sqrt{(r - \frac{3}{4}r_s)^2(r^2 + r^2 r + \frac{3r^2}{16})}} dr \\
&= \varphi_0 + \frac{r_s \mathcal{U}_H}{8\sqrt{3}} \times \left\{ 9\sqrt{2} \ln \left[ \frac{16r + 6r_s + 3\sqrt{2}16r^2 + 8r_s r + 3r_s^2}{4(r - r_{UH})} \right] \\
&+ 8\sqrt{3}\ln \left( \frac{20r + 7r_s + 3\sqrt{5}16r^2 + 8r_s r + 3r_s^2}{r - r_s} \right) \right\}.
\end{align*}
\]
where \( \varphi_0 \) is a constant, and \( \mathcal{U}_H \equiv \text{sign}(r - r_{UH}) \). Requiring that \( \phi(r)|_{r \to 0} \to 0 \), we find that,
\[
\varphi_0 = -r_s \left[ 8\ln(16) - 3\sqrt{6}\ln \left( 2 + \sqrt{6} \right) \right].
\]
(4.28)
Then, we find that on the hypersurface \( \psi = \psi_0 \), the coordinate \( t \) behaves like,
\[
t(r) = \begin{cases} 
\text{finite}, & r = 0, \\
-\psi_0, & r = r_{UH}, \\
+\infty, & r = r_s, \\
-\infty, & r \to \infty.
\end{cases}
\]
(4.33)
In Fig. 3 we draw the curves \( t(r) \) vs \( r \) for the surfaces \( \psi = \psi_0 \) in the \((t, r)\)-plane for different values of \( \psi_0 \)'s.
The surface gravities on the universal and killing horizons are given by,
\[
\kappa_{UH} = \left( \frac{2}{3} \right)^{3/2} \frac{1}{r_s},
\]
\[
\kappa_{EH} = \frac{37}{256r_s},
\]
\[
\kappa_{GR} = \frac{1}{2} \frac{F'(r)|_{r=r_s}}{r_{s}},
\]
(4.34)
FIG. 4: The surfaces of $\psi(t, r) = \psi_0$ in the $(t, r)$-plane with different $\psi_0$’s for the Schwarzschild solution.

FIG. 6: The surfaces of $\phi(v, r) = \phi_0$ in the $(v, r)$-plane for the Schwarzschild solution.

FIG. 5: The surface gravities on the killing and universal horizons for the Schwarzschild solution given by Eq. (4.34), which are plotted in Fig. 5 vs $r_s$, where $\kappa_{EH}^{GR}$ denotes the surface gravity at the Killing horizons normally defined in GR. In the current case, $\kappa_{UH}$ is always greater than $\kappa_{EH}$.

As we already emphasized, $\phi$ is peeling off not only at the universal horizon but also at the Killing horizon, while $\psi$ is peeling off only at the Killing horizon. This is because of the choice of the coordinates $(t, r)$. In fact, in the $(v, r)$- and $(\tau, r)$-planes, the hypersurfaces of $\phi = \text{Constant}$ are given, respectively, in Figs. 6 and 7, while the hypersurfaces of $\psi = \text{Constant}$ are given, respectively, in Figs. 8 and 9, from which it can be seen that the peeling-off behavior shows only at the universal horizons.

B. Schwarzschild Anti-de Sitter Solution

The Schwarzschild anti-de Sitter solution is given by,

$$F(r) = 1 - \frac{r_s}{r} + \frac{r^2}{\ell^2}, \quad k = 1,$$

(4.35)
where \( \ell \equiv \sqrt{3/|\Lambda|} \), \( r_s \equiv 2m = (1 + \frac{2M}{r_s}) r_{EH} \), where \( r_{EH} \) denotes the Killing horizon of the Schwarzschild anti-de Sitter black hole.\(^4\) Then, from Eq.\((4.12)\) we find that
\[
r_0^2 = \frac{1}{18C_r^{4/3}} \left[ 2^{1/3} C_r^{4/3} \ell^4 + 64 \times 2^{1/3} \ell^8 \right. \\
-27 \times 2^{1/3} C_r (r_{EH} \ell^4 + r_{EH}^3 \ell^2) \\
+108 \times 2^{1/3} C_r^{4/3} (r_{EH}^6 + r_{EH}^3 \ell^4) \\
+2^{1/3} (81 \ell^4 r_{EH}^2 - 32 \ell^6) \\
+162 r_{EH}^2 \ell^4 + 81 r_{EH}^6 \right],
\]
\[
C_r = 27r_{EH} (\ell^2 + \ell^2) \\
+ \sqrt{1286^3 + 729 (r_{EH}^2 + r_{EH}^3)^{1/2}},
\]
\[
r_{EH} = \frac{2^{1/3} C_r^{4/3} - 28/3 \ell^2}{6C_r^{1/3}}.
\]
Thus, in terms of \( r_{EH} \) and \( r_{EH} \), we find that
\[
G(r) = 1 - \frac{r_s}{r} + \frac{r^2}{\ell^2} + \frac{r^4}{\ell^4} \\
= \left( \frac{r - r_{UH}^2}{\ell^2} \right)^2 \left[ r_0^2 - 2 r_{UH} r^3 \right. \\
+ \ell^2 + 3 r_{UH}^2 \left. \right] - 4 r_{UH}^2 - r_{EH} \ell^2
\]

\(^4\) It should be noted that in this case we also impose the condition \([4.4]\), so that the khronon equation \((1.2)\) is satisfied identically for the particular solution of \( U \) given by Eq.\((4.7)\), although the space-time now is no longer asymptotically flat. For such a particular solution, the boundary conditions for \( u_\mu \) are also satisfied.

In Fig.\(10\) we show the curves of \( G(r) \) and \( F(r) \) defined in Eqs.\((4.35)\) and \((4.36)\) for the Schwarzschild Anti-de Sitter solution.

In Fig.\(11\) we draw the curves \( t(r) \) vs \( r \) on the hypersurface \( \phi = \phi_0 \) for different \( \phi_0 \)'s. On the other hand, the integrand of \( \psi(t,r) \) becomes singular only at the Killing horizon, and behaves as \((r-r_{EH})^{-1}\). In Fig.\(12\) we draw the curves \( t(r) \) vs \( r \) for the surfaces \( \psi = \psi_0 \) in the \((t,r)\)-plane for different values of \( \psi_0 \)'s.

Finally, the surface gravities on the universal and killing horizons are given by
\[
\kappa_{UH} = \frac{\sqrt{3} r_0^2}{2 \ell r_{UH}^{7/2}} \sqrt{5 r_{UH}^3 - r_{EH}^3 - \ell^2 (r_{EH} - 2 r_{UH})},
\]
\[
\kappa_{EH} = \frac{r_0^2 (3 \ell^2 + 3 r_{EH}^2 + 4 r_{EH} r_{UH} + 5 r_{UH}^2)^{1/2}}{4 \ell r_{EH}^5 |r_{EH} - r_{UH}| r_{UH}} \\
\times \left[ 3 r_{EH}^6 - 9 r_{EH}^2 r_{UH}^3 + 8 r_{EH}^3 r_{UH} \right] \\
+ 18 r_{EH} r_{UH}^5 - 20 r_{UH}^6 + \ell^2 \left( r_{EH}^4 - 9 r_{EH}^2 r_{UH}^2 + 12 r_{UH}^4 \right),
\]
\[
\kappa_{GR}^{EH} = \frac{1}{2} F'(r) \bigg|_{r=r_{EH}} = \frac{1}{2} \left( \frac{1}{r_{EH}} + 3 r_{EH} \ell^2 \right),
\]
which are shown in Fig.\(13\). It is interesting to note \( \kappa_{UH} \) is always greater than \( \kappa_{EH} \). However, \( \kappa_{UH} \) is larger than

![FIG. 9: The surfaces of \( \psi(\tau, r) = \psi_0 \) in the \((\tau, r)\)-plane with different \( \psi_0 \)'s for the Schwarzschild solution.](image)

![FIG. 10: The functions \( F(r) \) and \( G(r) \) defined in Eqs.\((4.35)\) and \((4.36)\) for the Schwarzschild Anti-de Sitter solution.](image)

![FIG. 11: The functions \( F(r) \) and \( G(r) \) defined in Eqs.\((4.35)\) and \((4.36)\) for the Schwarzschild Anti-de Sitter solution.](image)

![FIG. 12: The functions \( F(r) \) and \( G(r) \) defined in Eqs.\((4.35)\) and \((4.36)\) for the Schwarzschild Anti-de Sitter solution.](image)

![FIG. 13: The functions \( F(r) \) and \( G(r) \) defined in Eqs.\((4.35)\) and \((4.36)\) for the Schwarzschild Anti-de Sitter solution.](image)
The Reissner-Nordström (RN) solution is given by,

\[
F(r) = 1 - \frac{r_s}{r} + \frac{Q^2}{r^2}, \quad k = 1, \tag{4.40}
\]

\(\kappa_{EH}^{GR}\) only when \(r_{EH}\) is small. There exists a critical value \(r_c\) at which \(\kappa_{UH} = \kappa_{EH}^{GR}\). When \(r_{EH} > r_c\), we have \(\kappa_{UH} < \kappa_{EH}^{GR}\). It should be also noted that in Fig. 13 we plot the curves only for \(\ell = 1\). However, for other values of \(\ell\), similar properties are found, as it can be seen from Figs. 14 and 15.

On the other hand, in the \((v, r)\)-plane, the hypersurfaces of \(\phi = \) Constant are given in Figs. 16 while the hypersurfaces of \(\psi = \) Constant are given in Figs. 17.

Again, peeling-off behavior happens only at the universal horizon.

Note that the Schwarzschild Anti-de Sitter solution in the Painleve-Gullstrand coordinates \((\tau, r)\) is not well-defined for \(r \gg \ell\), as now \(N^r = \sqrt{1 - F(r)}\) becomes imaginary when \(r\) is sufficiently large.

C. Reissner-Nordström Solution

The Reissner-Nordström (RN) solution is given by,
where $r_s \equiv 2m = r_{EH} + r_{IH}$, $Q^2 = r_{EH}r_{IH}$, where $r_{EH}$ and $r_{IH}$ denote the event and inner horizons of the RN black hole, respectively. Setting $r_{IH} = b r_{EH}$, where $0 < b \leq 1$, from Eq. (4.12) we find that

$$r_o^2 = \frac{r_{EH}^2}{16\sqrt{2}} [27 - 36b + 2b^2 - 36b^3 + 27b^4] + (9 - 5b - 5b^2 + 9b^3) C_b \, \frac{r_{EH}}{8},$$

$$r_{UH} = (3 + 3b + C_b) \frac{r_{EH}}{8},$$

$$C_b = \sqrt{9 - 14b + 9b^2}. \quad (4.41)$$

Thus, in terms of $r_{UH}$, $r_{IH}$ and $r_{EH}$, we obtain

$$G(r) = 1 - \frac{r_s}{r} + \frac{Q^2}{r^2} + \frac{r_o^4}{r^3},$$

$$= \frac{(r - r_{UH})^2}{r^4} (r^2 + A_1 r + A_0), \quad (4.42)$$

where

$$A_1 = 2r_{UH} - (1 + b)r_{EH},$$

$$A_0 = b r_{EH} - 2br_{UH} - 2r_{UH}r_{EH} + 3r_{UH}^2. \quad (4.43)$$

In Fig. 18 we show the curves of $G(r)$ and $F(r)$ vs $r$ in the non-extreme case ($0 < b < 1$) and the extreme case ($b = 1$), respectively.

In the extreme case $b = 1$, the inner, event and universal horizons all coincide. This is because the position of universal horizon is always between the inner and event horizons, in which the killing vector is space-like. Then, from Eq. (4.41) we find that $r_{UH}^2 = 0$, so that $U = 0$ and $G(r) = F(r)$. Hence, from Eq. (4.18) we obtain $d\phi = dt$, that is, now the khrnon is aligned along the $t$-axis. Redefining $\psi$ as $d\psi = \frac{dr}{\sqrt{F}}$, the metric takes the form,

$$ds^2 = -\frac{(r - r_{UH})^2}{r^2} d\phi^2 + d\psi^2 + r^2 d\Omega_1^2; \quad (b = 1). \quad (4.44)$$

In the non-extreme case $0 < b < 1$, we obtain

$$\phi = t - \int \frac{U(r)}{F \sqrt{G(r)}} dr = t - \varphi(r) + \varphi(0), \quad (4.45)$$

where

$$\varphi(r) = c_{UH} r_{UH}^2 \left[ A(r_{EH}) A_C(r_{IH}) A_C(r_{UH}) \right] \left( (r_{EH} - r_{EH}) (r_{EH} - r_{UH}) (r_{IH} - r_{UH}) \right)^{-1}$$

$$\times \left[ r_{EH}^2 A(r_{IH}) A_C(r_{UH}) (r_{IH} - r_{UH}) \right]$$

$$\ln \left| \frac{2A_0 + A_1 r + A_1 r_{EH} + 2r_{EH}}{r - r_{EH}} \right|$$

$$+ \frac{2A_C(r_{EH}) A_C(r)}{r - r_{EH}} + A_C(r_{EH}) A_C(r_{UH}) r_{IH}^2$$

$$\times (r_{UH} - r_{IH}) \ln \left| \frac{2A_0 + A_1 r + A_1 r_{IH} + 2r_{IH}}{r - r_{IH}} \right|$$

$$+ \frac{2A_C(r_{IH}) A_C(r)}{r - r_{IH}} + A_C(r_{EH}) A_C(r_{UH}) r_{IH}^2.$$
In Fig. 20 we draw the curves where the killing horizons are given by
\[ \psi \left( r \right) = \frac{A_c(r)}{r_{EH}} \left[ 27 - 36b + 2b^2 - 36b^3 + 27b^4 \right] \]
\[ + C_6 \left( 9 - 5b - 5b^2 + 9b^3 \right) \right]^{-1} (\Psi(r) - \Psi(0)). \]
\[ \Psi(r) = \frac{16}{3} A_0 - 2A_1^2 + \frac{4}{3} A_1 r + 4A_1 (r_{EH} + r_{I H}) \]
\[ - r_{U H} + \frac{8}{3} (2r^2 + 3r (r_{EH} + r_{I H} - r_{U H})) \]
\[ + 6 (r_{E H}^2 + r_{E H}r_{I H} + r_{I H}^2 - r_{E H}r_{U H} - r_{I H}r_{U H})] \]
\[ - A_c(r)^{-1} \left\{ 4A_0 A_1 + 2 \left( A_1^2 - 4A_0 \right) (r_{E H} + r_{I H}) \]
\[ - r_{U H} - A_1^2 - 8A_1 \left[ r_{E H}^2 + \left( r_{I H} + r_{E H} \right) (r_{I H} - r_{U H}) \]
\[ - r_{E H}^2 + 16 \left[ r_{E H}^2 + \left( r_{I H}^2 + r_{I H}r_{E H} + r_{E H}^2 \right) (r_{I H} - r_{U H}) \]
\[ + A_c(r) \ln \left[ \frac{A_c(r)}{r_{EH}} \right] \frac{A_c(r)}{r_{EH} - r_{I H}} \]
\[ + 16 \left[ A_c(r) \ln \left[ \frac{A_c(r)}{r_{EH} - r_{I H}} \right] \frac{A_c(r)}{r_{EH} - r_{I H}} \right] \]
\[ A_c(r) = 2A_0 + A_1 r + A_1 r_H + 2rr_H \]
\[ + 2A_1 (r_H - r_{EH}). \]  (4.48)

Thus, on the hypersurface $\psi = \psi_0$ we have
\[ t(r) = \tilde{\psi}(r) - \tilde{\psi}(0) = \left\{ \begin{array}{ll}
-\infty, & r = r_{I H}; \\
+\infty, & r = r_{E H}.
\end{array} \right. \]  (4.50)

In Fig 20 we draw the curves $t(r)$ vs $r$ for the surfaces $\psi = \psi_0$ in the $(t, r)$-plane for different values of $\psi_0$.

Finally, the surface gravities on the universal and killing horizons are given by
\[ \kappa_{U H} = \frac{32\sqrt{2}}{(3 + 3b + C_6)^3 r_{EH}} \left\{ \left[ 27 - 36 + 2b^2 - 36b^3 \right] \right. \]
\[ + 27b^4 + C_6 \left( 9 - 5b - 5b^2 + 9b^3 \right) \right\} \]  (4.51)

The curves of $\kappa_{U H}$, $\kappa_{E H}$ and $\kappa_{E H}^G$ vs $r_{E H}$ are given in Fig 21. It is interesting to note, similar to the case of the Schwarzschild anti-de Sitter solution given in Fig 13, $\kappa_{U H}$ is always greater than $\kappa_{E H}^G$. However, $\kappa_{U H}$ is larger than $\kappa_{E H}^G$ only when $r_{E H}$ is small. There exists a critical value $r_c$ at which $\kappa_{U H} = \kappa_{E H}^G$. When $r_{E H} > r_c$, we have $\kappa_{U H} < \kappa_{E H}^G$.

In the $(t, r)$-plane, the hypersurfaces of $\phi = \text{Constant}$ are given in Figs. 22 while the hypersurfaces of $\psi = \text{Constant}$ are given in Figs. 23 which are peeling off only at the universal horizon.

Similar to the Schwarzschild Anti-de Sitter solution, the RN solution is also not well-defined in the Painleve-Gullstrand coordinates $(t, r)$, as now $N^r = \sqrt{1 - F(r)}$ will become imaginary when $r$ is sufficiently small.
FIG. 21: The surface gravities on the killing and universal horizons for the Reissner-Nordström solution with $r_{EH} = 1$.

FIG. 22: The surfaces of $\phi(v, r) = \phi_0$ in the $(v, r)$-plane for the Reissner-Nordström solution.

FIG. 23: The surfaces of $\psi(v, r) = \psi_0$ in the $(v, r)$-plane for the Reissner-Nordström solution.

V. CONCLUSIONS

In this paper, we have studied the existence of universal horizons in static spacetimes, and found that the khronon field can be solved explicitly when its velocity becomes infinitely large, at which the universal horizons coincide with the sound horizon of the khronon. Choosing the timelike coordinate aligned with the khronon, the static metric takes the simple form (3.13), which shows clearly that the metric now is free of coordinate singularity at the Killing horizons, but becomes singular at the universal horizons. These singularities are coordinate ones, and can be removed by properly coordinate transformations.

Applying such definition to the three well-known black hole solutions, Schwarzschild, Schwarzschild anti-de Sitter, and Reissner-Nordström, which are often also solutions of gravitational theories with broken LI [21], we have shown that in all these solutions universal horizons always exist inside the Killing horizons. The peeling-off behavior of the khronon depends on the coordinates adopted. In particular, in the Schwarzschild coordinates $\phi$ is peeling off at both Killing and universal horizons, while in the Eddington-Finkelstein and Painlevé-Gullstrand coordinates, the peeling-off behavior is found only when across the universal horizons.

We have also calculated the surface gravity at each of the universal horizons, using the definition given in [13], which yields the standard relation $T = \kappa_{UH} / 2\pi$ between the Hawking temperature $T$ [12] and the surface gravity $\kappa_{UH}$. In all these cases we have found that the surface gravity at the universal horizon is always greater than that at the Killing horizon. We have also compared them with the surface gravity often defined in general relativity.

Appendix A: The Khronon Mode

In the Minkowski background, we have $\phi = t$. Considering the perturbations of the khronon in this background,

$$\phi = t + \chi(t, x^i),$$

(A.1)

where $\chi$ denotes the perturbations, we find that to the second-order, the khronon action takes the form,

$$S^{(2)}_{\phi} = \int dt dDx \left[ c_{123} (\nabla^2 \chi)^2 - c_{14} (\nabla_i \chi)^2 \right],$$

(A.2)

where $\dot{\chi} = \partial_t \chi$. Then, $\chi$ satisfies the equation,

$$\nabla^2 (\ddot{\chi} - c_{2\phi}^2 \nabla^2 \chi) = 0,$$

(A.3)

where $c_{\phi}$ is defined by Eq.(2.9). The above equation shows that there are two different modes, one is propagating with a speed $c_{\phi}$, and the other is propagating with an infinitely large speed (instantaneous propagation) [10]. It should be also noted the difference between the speed of the Khronon and the speed of the spin-0 mode of the aether [23],

$$c_{\phi, JM}^2 = \frac{c_{123} (2 - c_{14})}{c_{14} (1 - c_{13}) (2 + c_{13} + 3c_2)}.$$

When $|c_1| \ll 1$, it reduces to the one given by Eq.(2.9).
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