Determination of multistoried buildings stiffness properties in the inverse dynamics solution

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Abstract. The article is devoted to the development of methods and evaluation of the accuracy of determining the horizontal stiffness of multi-storey buildings basing on the results of instrumental measurements of the self-induced vibrations parameters carried out with the help of high-precision instruments. The proposed studies are extremely relevant in estimating the unsoundness degree of the buildings that have passed a certain period of operation or buildings that have been subjected to severe exposure. The use of dynamic methods of load-bearing structure condition analysis has undeniable advantages, since it excludes the need for a detailed examination, often associated with the necessity to open the enclosing structures and evict the residents of the building. The advantages of using this approach are particularly evident in the survey of a large array of residential buildings that are in heterogeneous operating conditions and require rapid results. The practical application of the methods was tested in the survey of the high-rise apartment buildings of 1-335 series in the city of Irkutsk. However, the need to use dynamic models of small dimension in the evaluation of stiffness parameters makes it inevitable and urgent to solve the problems of assessing the accuracy of such approximations.

1. Introduction
One of the simplest versions of the inverse dynamics problem is to determine the stiffness of a multi-storey building by the actual values of natural oscillations measured using high-precision instruments. Among the various problems of inverse dynamics [1-3] the determination of the stiffness properties of the building is certainly stable in computational terms and does not belong to the category of ill-posed problems [3-7].

However, to evaluate the stiffness parameters based on the results of determining the base frequency, one has to use some approximation of the design scheme, which introduces errors in assessing the building's inherent parameters [4, 8].

It should be noted that the determination of the limpness value characterizes the quantitative assessment of the accumulation of defects of the building exposed to a certain period of operation [4], which affects the functional properties of the bearing structures [8].

Such estimation methods basing on the dynamic tests have obvious advantages, as they exclude the necessity of the enclosing structures demolishing for the internal bearing building designs inspection [9].

It should be noted that the mandatory periodic determination of the natural vibration frequencies of the fundamental tone of the building for seismic regions are identified by the conditions of state standards [9].
The solution of the inverse dynamics problem is not the only reason conditioning the need to assess the accuracy of the fundamental dynamic parameters determination; it is necessary to give examples of the necessity for such an assessment when choosing a discrete design model and determining the level of discretization, which, in most cases, is too high [7, 10, 11].

The overwhelming predominance of overestimation of the dynamics problems dimension in computational practice does not exclude the possibility of forming the dynamic models of "small" dimension, that do not reflect the necessary dynamic properties of the calculated object with a sufficient degree of approximation [12].

Among other problems of estimating the error of determining the intrinsic response parameters of multi-storey buildings, the problem of specifying the error when using simple design models with lumped inertia is relevant [6, 12]. Some common methods for determining solutions to such problems are mainly based on the use of similarity transformations [13, 14-16].

The use of such estimates does not take into account the specifics of the construction elements of the calculated structure, and the solution of such a problem is, in our opinion, extremely important.

2. Theoretical Basis and Method
Let us consider the error values obtained by approximating various construction elements with single-mass dynamic models.

Using a dynamic deflection method [19-20], the design scheme of such a structural element can be represented as a vertical member with a distributed mass of intensity \( \rho \), length \( l \) and shear stiffness \( GF \), where \( G \) is the shear modulus, \( F \) is the cross-sectional area (Fig. 1).

![Figure 1. Design scheme of the element with shear deformations, with distributed mass \( \rho \).](image)

The equation of self-induced vibrations of such an element is a partial differential equation and has the form:

\[
\frac{\rho \partial^2 v}{\partial t^2} + \frac{\partial^2 v}{\partial x^2} GF = 0,
\]

Where \( v \) - the horizontal displacement of the element points during the self-induced vibrations, \( t \) – the time parameter [21-24]. Let us assume that the self-induced vibrations of such a system are carried out with a certain frequency \( \omega \).

Therefore, using the method of separating variables [15, 25-29], it can be written:

\[
v(t, x) = y(x) \cdot \sin(\omega t),
\]

When substituting (2) into (1), there is an ordinary differential equation:

\[-\rho \cdot \omega^2 \cdot y(x) \cdot \sin(\omega t) + GF''(x) \cdot \sin(\omega t) = 0\]

After the simplification of which, the following equation can be obtained
The characteristic equation for (3) is:
\[ \mu^2 \frac{\rho \omega^2}{GF} = 0 \]

The roots of this equation are:
\[ \mu_1 = \omega \sqrt{\frac{\rho}{GF}} \quad \mu_2 = -\omega \sqrt{\frac{\rho}{GF}} \]

Or \( \mu_1 = \omega \mu \); \( \mu_2 = -\omega \mu \),

where \( \mu = \omega \sqrt{\frac{\rho}{GF}} \).

The general solution of equation (3) is a linear combination of two linearly independent partial solutions forming a basic vector-function:
\[ h(x) = [\sin(\omega \mu x), \cos(\omega \mu x)]. \]

In this case, the solution is given by the following:
\[ y(x) = C_1 \sin(\omega \mu x) + C_2 \cos(\omega \mu x), \]

where \( C_1, C_2 \) - the coefficients of the linear combination determined by the boundary conditions of the equation (3) [16]. If \( x = 0 \), there is an equality \( C_2 \cos(\omega \mu x) = 0 \), from which \( C_2 = 0 \).

Thus, taking into account the locking of joints at the point \( x=0 \), the solution is
\[ y(x) = C_1 \sin(\omega \mu x). \]

Since the function of the oscillatory form during the self-induced vibrations is determined within the accuracy of a factor, let us take \( C_1 \) equal to 1, then
\[ y(x) = \sin(\omega \mu x). \]

When the point \( B \) moves to a certain amount \( \Delta \), the equality is:
\[ \sin(\omega \mu l) = \Delta. \]

In particular, for \( \Delta = 1 \), the equality of the form is obtained:
\[ C_1 \sin(\omega \mu l) = \sin \left( \omega \sqrt{\frac{\rho}{GF}} \cdot l \right) = 1. \]

If the point \( B \), moved by the value \( \Delta = 1 \), imposes a lateral brace (Fig. 2), then there is a reaction \( H_B \), equal to:
\[ E I \cdot \frac{\partial y(x)}{\partial x} \bigg|_{x=l} = \cos(\omega \mu l) \cdot E I \cdot \omega \mu. \]

Figure 2. Calculation scheme for determining the reaction \( H_B \) in the linear connection of the upper node of the element AB.

3. Under the influence of a single force in the connection line the amplitude of harmonic displacement \( y(l) \) is determined by the expression:
In case of the magnitude of the resonance, the amplitude is achieved at the denominator of expression (4) equal to zero. Herewith, 
\[ \cos(\omega l) = 0. \]

The lowest frequency of self-induced vibrations is determined in this case from the equality:
\[ \gamma_{1,\mu l} = \frac{\pi}{2l}. \]

Thus, the frequency \( \gamma_1 \) of the lower tone oscillations of such a system is determined by the expression:
\[ \gamma_1 = \frac{\pi}{2l} \cdot \frac{GF}{\rho}. \]

Let us consider the approximation of the lowest frequency of the self-induced vibrations of wall structures with the masses localized in the levels of flooring.

Let us discuss the approximation of the simplest dynamic system represented by a two-storey building. The natural-vibration frequencies of such a system can be determined quite simply. There is also the opportunity to simply evaluate the approximation of the natural vibration frequency of the basic tone in a one-dimensional system.

The design scheme of a two-storey structure with the masses localized in the level of flooring is shown in Fig. 3.

![Dynamic model of a two-storey building with wall bearing structures](image)

**Figure 3.** Dynamic model of a two-storey building with wall bearing structures – 1, 2 - link numbers.

To construct a system of dynamic equilibrium equations, the method of displacements is used, for which linear relations in the horizontal direction are imposed on the displacements of the discrete mass points (Fig. 3).

Localizing the masses of the building at the points, let us denote the values of the masses as \( m_1 \) and \( m_2 \).

The matrix \( R \) of single reactions of such a system has the form:
\[
R = \begin{bmatrix}
\frac{2GF}{h} & \frac{GF}{h} \\
\frac{GF}{h} & \frac{GF}{h}
\end{bmatrix}
\]

The matrix \( D \) of dynamic reactions of the model has the form:
\[
D = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]
\[ D = \begin{bmatrix} \frac{2GF}{h \cdot m_1} & -\frac{2GF}{h \cdot m_3} \\ -\frac{2GF}{h \cdot m_2} & \frac{2GF}{h \cdot m_3} \end{bmatrix} \]  

(5)

If \( m_1 = m_2 = m \), the natural-vibration frequencies can be determined from the assumption

\[ |D - \lambda \cdot E| = 0, \]

where \( \lambda \) – eigenvalues, \( E \) – a unit matrix.

In expanded form, the equation (6) can be written as follows:

\[ \left( \frac{2GF}{h \cdot m} - \lambda \right) \left( \frac{GF}{h \cdot m} - \lambda \right) - \left( \frac{GF}{h \cdot m} \right)^2 = 0 \]  

(7)

The roots of the equation (7) are defined as:

\[ \lambda_{12} = \frac{3GF}{h \cdot m} \mp \sqrt{9 \left( \frac{GF}{h \cdot m} \right)^2 - \left( \frac{GF}{h \cdot m} \right)^2} \]

Thus, the minimum eigenvalue is:

\[ \lambda_1 = \frac{GF}{h \cdot m} \left( 3 - 2\sqrt{2} \right), \]

The minimum natural-vibration frequency \( \gamma_1 \) is defined as:

\[ \gamma_1 = \sqrt{\lambda_1} = \frac{GF}{h \cdot m} \cdot \sqrt{3 - 2\sqrt{2}} \]  

(8)

Let us consider the error in determining the minimum natural-vibration frequency when replacing the two-mass model by one-dimensional model with a mass lumping of 2m at one point of it (Fig. 4).

**Figure 4.** Simplifying of a two-dimensional dynamic model with a one-dimensional console model.

The frequency \( \omega_1 \) of the natural oscillations of such a one-dimensional system has the form:

\[ \omega_1 = \sqrt{\frac{GF}{3 \cdot h \cdot m}}. \]

As is seen from above, \( \omega_1 \) is much larger than \( \gamma_1 \).

This circumstance is explained by the fact that the actual center of mass in the oscillations of the system in the first oscillatory form must take into account the unevenness of the mass movements of the system. At the same time, the most distant mass from the bearing point has greater displacements than
the mass located below.

Let us form the center of concentration taking into consideration the first oscillatory form. The components of the vector $A_i$ of self-induced vibrations are determined from the solution of a system of equations:

$$(D - \lambda_i \cdot E)A = 0,$$  \hspace{1cm} (9)$$

where $A = (a_1, a_2)^T$ - the eigenvector of a two-dimensional dynamical system corresponding to the eigenvalue $\gamma_1$.

As $\text{Det}(D - \lambda_i \cdot E) = 0$, in the system of equations there are two linearly dependent equations.

To define $A$, let us assume that $a_1 = 1$, then:

$$1 \cdot \left( \frac{2GF}{h \cdot m} - \frac{GF}{h \cdot m} \cdot (3 - 2\sqrt{2}) \right) - \frac{GF}{h} a_2 = 0.$$  

From here it follows that $a_2 = 2\sqrt{2} - 1$.

Thus, the vector $A$ is written as:

$$A_2 = (1, 2\sqrt{2} - 1)^T.$$  

4. Conclusion

The advantages of using this approach are particularly evident in the survey of a large array of residential buildings that are in heterogeneous operating conditions and require rapid results. The practical application of the methods was tested in the survey of the high-rise apartment buildings of 1-335 series in the city of Irkutsk. However, the need to use dynamic models of small dimension in the evaluation of stiffness parameters makes it inevitable and urgent to solve the problems of assessing the accuracy of such approximations.

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