Measurements of $B \to J/\psi$ at forward rapidity in $p+p$ collisions at $\sqrt{s}=510$ GeV

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We report the first measurement of the fraction of $J/\psi$ mesons coming from $B$-meson decay ($F_{B \to J/\psi}$) in $p+p$ collisions at $\sqrt{s} = 510$ GeV. The measurement is performed using the forward silicon vertex detector and central vertex detector at PHENIX, which provide precise tracking and distance-of-closest-approach determinations, enabling the statistical separation of $J/\psi$ due to $B$-meson decays from prompt $J/\psi$. The measured value of $F_{B \to J/\psi}$ is $8.1\% \pm 2.3\%$ (stat) $\pm 1.9\%$ (syst) for $J/\psi$ with transverse momenta $0 < p_T < 5$ GeV/$c$ and rapidity $1.2 < |y| < 2.2$. The measured fraction $F_{B \to J/\psi}$ at PHENIX is compared to values measured by other experiments at higher center of mass energies and to fixed-order-next-to-leading-logarithm and color-evaporation-model predictions. The $b\bar{b}$ cross section per unit rapidity ($d\sigma/dy(pp\to b\bar{b}))$ extracted from the obtained $F_{B \to J/\psi}$ and the PHENIX inclusive $J/\psi$ cross section measured at 200 GeV scaled with color-evaporation-model calculations, at the mean $B$ hadron rapidity $y = \pm 1.7$ in 510 GeV $p+p$ collisions, is $3.63^{+1.92}_{-1.70}\mu b$. It is consistent with the fixed-order-next-to-leading-logarithm calculations.

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I. INTRODUCTION

The measurement of bottom ($B$) mesons in $p+p$ and $p+\bar{p}$ collisions is of interest to constrain the total bottom cross section as well as test our understanding of bottom quark production mechanisms and hadronization. There are extensive direct measurements of various $B$ mesons, as well as measurements of $B \to J/\psi$ contributions over a broad range in $J/\psi$ transverse momentum and rapidity from the Tevatron in $p+\bar{p}$ at $\sqrt{s} = 1.8, 1.96$ TeV [1–3] and the Large Hadron Collider (LHC) in $p+p$ at $\sqrt{s} = 7–13$ TeV [4–8]. In contrast, measurements from UA1 in $p+\bar{p}$ at $\sqrt{s} = 630$ GeV [9] are statistically limited and only for $p_T(J/\psi) > 5$ GeV/$c$. Adding new measurements at lower energies and covering different kinematic regions is valuable for testing perturbative quantum chromodynamics (pQCD) calculations and constraining production mechanisms.

The Relativistic Heavy Ion Collider (RHIC) provides $p+p$ collisions at $\sqrt{s} = 200, 500$ and 510 GeV, which extends the kinematic reach for bottom measurements. At these smaller energies, bottom production is dominated by gluon-gluon fusion, while higher energy bottom production contains a larger fraction of flavor excitation and gluon splitting processes [10]. The STAR experiment measured $B \to J/\psi$ at midrapidity for $J/\psi$ $p_T > 5$ GeV/$c$ in $p+p$ at $\sqrt{s} = 200$ GeV [11]. Our measurement at forward rapidity and $p_T$ within 0–5 GeV/$c$ in $\sqrt{s} = 510$ GeV $p+p$ collisions at PHENIX can provide the validation of parton distribution functions (PDF) in a different gluon fractional momentum range ($FVTX$) [17] are used. The BBC detector, which comprises 128 quartz Čerenkov counters with a pseudorapidity coverage of 3

II. EXPERIMENTAL SETUP

The data set used in this analysis is from the 2012 run of $p+p$ at $\sqrt{s} = 510$ GeV and the detector configuration of PHENIX for that running period is shown in Fig. 1. For this measurement, the beam-beam counters (BBC) [13], the muon arm spectrometers [14], the central silicon vertex detector (VTX) [15, 16] and the forward silicon vertex detector (FVTX) [17] are used. The BBC detector, which comprises 128 quartz Čerenkov counters with a pseudorapidity coverage of 3.0 $< |\eta| < 3.9$, determines when a collision event has taken place. The BBC provides the minimum-bias (MB) trigger, by requiring a coincidence between at least one hit in both the positive and negative acceptance of the BBC. The PHENIX muon detectors are divided into the North ($1.2 < y < 2.4$) and the South ($-2.2 < y < -1.2$) arms. Each muon arm spectrometer has full azimuthal coverage and is composed of hadron absorbers, a muon tracker (MuTr) which resides in a radial field magnet, and a muon identifier (MuID). The MuTr comprises three cathode strip wire chamber stations inside a magnet which provides a radial magnetic field with an integrated bending power of around 0.8 T·m. The MuTr measures track momentum $p$ with a resolution of $\delta p/p \approx 0.05$ at $p < 10$ GeV/$c$. The hadron absorber comprises 19 cm of copper, 60 cm of iron and 36.2 cm of stainless steel along the beam axis. The absorbers are situated in front of the MuTr to provide hadron (mostly pion and kaon) rejection. The MuTr has a position resolution at each station of around 100 µm, which, together with a precisely determined vertex, results in a mass resolution of around 95 MeV for dimuon pairs within the $J/\psi$ mass region and $0 < p_T(J/\psi) < 5$ GeV/$c$. The downstream MuID comprises five sandwiched planes of Iarocci proportional tubes and steel. The MuTr+MuID
FIG. 1. The PHENIX detector setup for the 510 GeV $p+p$ data taking in 2012.

The VTX (installed in 2011) comprises two inner pixel layers and two outer strip layers distributed from 2.5 cm to 14.0 cm along the radial direction, covering $\Delta \varphi \approx 5.0$ radians in azimuth and $|z(\text{VTX})| < 10$ cm along the $z$ axis (beam direction). The radii of the inner silicon pixel detectors are 2.5 and 5.0 cm, and the radii of the outer silicon strip detectors are on average 10.0 and 14.0 cm. Each pixel of the inner VTX layers covers a 50 $\mu$m $\times$ 450 $\mu$m active area [15, 16]. The FVTX, installed in front of the hadron absorbers in 2012, comprises 4 silicon disks perpendicular to the beam axis and placed at approximately $z = \pm 20.1$ cm, $\pm 26.1$ cm, $\pm 32.2$ cm and $\pm 38.2$ cm. The rapidity coverage of the FVTX overlaps the muon arm coverage. Each FVTX disk comprises 48 individual silicon sensors (wedges) and each wedge contains two columns of strips that each span an azimuthal segmentation of $3.75^\circ$. The column comprises mini-strips with 75 $\mu$m width in the radial direction. The strip length in the azimuthal ($\varphi$) direction varies from 3.4 mm at the inner radius to 11.5 mm at the outer radius for the largest stations [17]. Tracks passing through the forward muon arms are unlikely to pass through the VTX outer strip layers due to the angular acceptance of the strips. In addition, the two inner pixel layers can help improve the DCA resolutions as they are closer to the vertex and have finer pixel sizes compared to the outer strip layers. Therefore, for track reconstruction with the combined FVTX+VTX detectors, only the two inner pixel layers in the VTX are used.

The FVTX enhances the existing muon arm tracking performance in several ways. The FVTX helps reject hadrons that undergo multiple scattering or decay inside the hadron absorber by requiring a good joint fit of FVTX and MuTr tracks. It also provides a better opening angle determination than the MuTr alone can provide, which results in an improved mass resolution for dimuon pairs. Finally, the additional precision tracking added in front of the hadron absorber by the FVTX makes the measurement of displaced tracks possible when combined with a determination of the primary vertex position.

Due to limited resolutions in the $z$ and azimuthal $\varphi$ components of the FVTX detector, the separation of prompt and decay muons is realized with the FVTX using the DCA measurement instead of measuring the displaced vertex of decayed muons. Because the FVTX has better resolution in the radial direction than in the azimuthal direction, the radial DCA (DCA$_R$) is the primary variable used in this analysis. The primary vertex is reconstructed using all FVTX and VTX tracks which pass the track quality cut $\chi^2/\text{NDF} < 4$, where NDF is the number of degrees of freedom. Figure 2 illustrates the projection of a muon from a $B$ meson to $J/\psi$ decay in the transverse vertex plane and
FIG. 2. (a) 3D and (b) projection of a muon from a $B$ meson to $J/\psi$ to dimuon decay to the transverse vertex plane ($x$-$y$) and definition of DCA$_R$.

how to calculate the DCA$_R$. A track reconstructed in the FVTX is extrapolated to the transverse plane ($x$-$y$) at the $z$ location of the primary collision vertex. DCA is defined as the vector $\vec{L}_{DCA}$ formed between this intersection point and the $x$-$y$ collision vertex point in the same transverse plane of the collision vertex. The DCA$_R$ is the component of the DCA which is measured in the same radial direction as the FVTX strips,

$$DCA_R = \vec{L}_{DCA} \cdot \hat{R} = \vec{L}_{DCA} \cdot \frac{\vec{R}}{|\vec{R}|}.$$  (1)

Prompt particles from the primary collision vertex have a symmetric DCA$_R$ distribution centered at zero, with the width determined by the intrinsic detector and vertex resolutions, while the shape is asymmetric for decay particles from a displaced decay vertex. As illustrated in Fig. 2(b), the definition of DCA$_R$ results in an asymmetric distribution for muons from $B \to J/\psi$ decay due to the projection onto the transverse $x$-$y$ plane of the primary vertex. This is confirmed by the full simulation shown in Section III C.

III. ANALYSIS PROCEDURE

This analysis starts with the identification of good $J/\psi$ candidates by selecting dimuon pairs found by the MuTr that are matched to MuID tracks. Separately, track finding is performed in the FVTX/VTX system where reconstructed tracks are required to contain at least one FVTX hit and a total of at least three FVTX+VTX hits. Then, for each reconstructed MuTr track, the FVTX/VTX tracks are searched for potential matches.

The collision point is determined from VTX and FVTX tracks. First, regions where there is a concentration of track crossings are determined. The center of gravity of each of these regions defines a collision point. For each region, the center of gravity is used to initiate a minimization of the vector sum of the DCAs of the tracks. During the minimization, tracks with large displacements are removed to improve the fidelity of the final vertex reconstruction. The vertex determination in each event is strongly affected by the small VTX and FVTX track multiplicities in $p+p$ collisions. Events containing $b\bar{b}$ decay products can also skew the vertex determination. Therefore, in this analysis we take advantage of the beam stability in $x$ and $y$ during the fill (5–12 hours) and use the measured average $x$ and $y$ position of all events in the fill to determine our primary $x$ and $y$ vertex. The spread of the primary $x$ and $y$ vertex position based on the beam spot size is around 80 $\mu$m in RMS. The $z$ position is still determined on an event-by-event basis for events that have a VTX+FVTX track multiplicity $\geq 2$. Events with smaller multiplicity are thrown out. For events with more than one reconstructed vertex, the vertex with the best reconstruction quality is selected as the primary vertex. For the reconstructed events, we obtain an average $z$ resolution of approximately 180 $\mu$m in 510 GeV $p+p$ collisions. After matching to the FVTX tracks, the DCA$_R$ is determined using the MuTr+MuID+FVTX combined track fit and the primary vertex location.

The next step in the analysis is to characterize the DCA$_R$ of muons from prompt $J/\psi$ decay and $J/\psi$ from $B$-meson decay through simulation. The final analysis step uses a fit function for the muon DCA$_R$ spectra that includes the
prompt $J/\psi$, $J/\psi$ from $B$-meson decay, and background components to extract the fraction of $J/\psi$ from $B$-meson decay in the data, using a log-likelihood fit. Details of the analysis procedure are explained step by step in the following sections.

### A. Data Quality Assurance

The precise primary $z$ vertex reconstruction is limited by the VTX acceptance and therefore only events within a $z$ vertex ($z_{\text{VTX}}$) window of (-10, 10) cm are selected for this analysis. Events with poorly determined primary $z$ vertices are removed by requiring less than 400 $\mu$m calculated uncertainty on the $z$-vertex. Runs without an accurately determined average $x$, $y$ position of the beam center are rejected. The number of events with MB and dimuon triggers surviving after these vertex selections is $3.5 \times 10^6$, which is equivalent to a total integrated luminosity of 0.47 pb$^{-1}$. The event rejection fraction is around 67%.

During the 2012 $p+p$ run, there were some areas of the FVTX detector which were not yet operational due to various electronics issues. When the FVTX-MuTr matching algorithm tries to find an FVTX track in a dead area, there is a tendency for it to match to a track in a live region neighboring the dead one instead, pulling the matching distributions away from the central value of 0. Because of this tendency to pull tracks away from a symmetric distribution, fiducial corrections described above are applied, the DCA$_p$ is then extracted for these tracks and checked for any indications of residual misalignments. The mean of these distributions is found to be flat along the $z$ direction (within the measurement precision) and the overall offsets of the distributions are within 30 $\mu$m in both arms. These offset values are much smaller than the detector position resolution. Variations of the DCA$_R$ mean and spread which could occur if there were beam instability, detector, trigger or acceptance×efficiency changes, are checked by examining the DCA$_R$ distributions as a function of run and BBC instantaneous rate. The mean values of the DCA$_R$ distributions across all runs are found to be within one standard deviation (of the intrinsic DCA$_R$ distribution width) after quality assurance checks.

### TABLE I. Quality cuts for $J/\psi$ candidates in $p+p$ collisions.

| Variable (Meaning) | $1.2 < |p| < 2.2$ |
|--------------------|-------------------|
| $|z_{\text{VTX}}| (\text{collision vertex measured by the FVTX/VTX})$ | $< 10$ cm |
| $z_{\text{VTX}}$ uncertainty! (collision vertex uncertainty measured by the FVTX/VTX) | $< 400$ $\mu$m |
| $p \cdot DG0$ (Track momentum times the spatial difference between the MuTr and MuID track at the first MuID layer) | $< 80$ GeV/$c \cdot$ cm |
| $p \cdot DDG0$ (Track momentum times the slope difference between the MuTr and MuID track at the first MuID layer) | $< 40$ GeV/$c \cdot$ $^\circ$ |
| $\chi^2_{\text{MuTr}}$ ($\chi^2$/NDF of the MuTr track) | $< 10$ |
| $\chi^2_{\text{MuID}}$ ($\chi^2$/NDF of the MuID track) | $< 3$ |
| Track $\chi^2_{\text{FVTX-MuTr}}$ ($\chi^2$/NDF of the FVTX-MuTr matching $\mu$ track) | $< 5$ |
| Radial residual between FVTX and MuTr projection at FVTX station 4 | $< 3\sigma$ |
| Azimuthal residual between FVTX and MuTr projection at FVTX station 4 | $< 3\sigma$ |
| Last gap (Last MuID plane that the $\mu$ track penetrated) | $= 4$ |
| nidhits (Number of hits in the MuID, out of the maximum 10) | $> 6$ |
| ntrhits (Number of hits in the MuTr, out of the maximum 16) | $> 11$ |
| nfvtxhits (Number of hits in the FVTX+VTX, out of the maximum 6) | $> 2$ |
| $|p_z| (\text{GeV}/c)$ (Momentum of the $\mu$ along the beam axis) | $> 3$ |
| dimuon pair vertex $\chi^2$/NDF | $< 3$ |
B. \( J/\psi \) Reconstruction

Tracks formed in the MuTr are required to contain at least 12 (out of 16) hits in the various cathode strip planes. We start with a loose quality cut \( \chi^2/NDF < 10 \) on the MuTr tracks to make sure all potentially good tracks are included in the analysis. The MuTr tracks which reach the last gap of the MuID and have longitudinal momentum \( > 3 \text{ GeV}/c \) are treated as muon track candidates. Muon candidates in this analysis need to have good associations between the MuTr track and the MuID road in both position and angle. The momentum-dependent position and angle differences between the MuTr track and the MuID road are required to be within three standard deviations as calculated using the Kalman Filter track fitting and error propagation method. In addition, the associated MuID road should contain at least 6 (out of 10) hits in different MuID planes. Because the MuID road is not included in the fully reconstructed tracks, we apply a tighter quality cut which is \( \chi^2/NDF < 3 \).

Good matching between the FVTX tracks and the MuTr+MuID tracks is also required. This requirement helps remove mis-reconstructed and bad quality tracks as well as some hadronic background. The matched FVTX tracks should contain at least 3 (out of 6) FVTX+VTX hits. The differences in azimuthal angle, polar angle and radial distance between matched FVTX and MuTr+MuID combined tracks are required to be within three standard deviations as determined by the Kalman Filter fits and error propagation. Fits on the combined FVTX+MuTr tracks should satisfy \( \chi^2/NDF < 5 \). Dimuon pairs are created from muons passing all the quality cuts. A slightly different selection which requires at least one muon of the dimuon pair passing through the quality cuts is tested. No bias is found as consistent results are achieved between the two selections. The fit of the vertex point plus the two muon tracks with opposite charges must satisfy \( \chi^2/NDF < 3 \) to ensure the two muon tracks are not separated by more than 1 mm. The complete set of quality cuts is listed in Table I.

Raw yields of the invariant mass of dimuon pairs after applying the quality cuts are shown in Figs. 3(a) and (b). A smaller number of events is measured in the forward than the backward rapidity due to larger MuTr dead areas and lower MuID efficiency in the forward rapidity region during this data taking period. These spectra contain a combination of \( J/\psi \) events, combinatorial background (random combinations of reconstructed tracks within an event) and heavy flavor background. The heavy flavor background determination will be discussed in Section III E 3.

We start with a loose quality cut \( \Delta R \), defined in Eq. (2), is applied to the mass distribution of dimuon pairs and muon DCAR distribution in mixed events:

\[
\text{Norm}_{\text{mix}} = \sqrt{\frac{N_{\text{same}}^{++} \cdot N_{\text{same}}^{--}}{N_{\text{mix}}^{++} \cdot N_{\text{mix}}^{--}}} \tag{2}
\]

where \( N_{\text{same}}^{++}, N_{\text{same}}^{--} \) are the like-sign yields in same events and \( N_{\text{mix}}^{++}, N_{\text{mix}}^{--} \) are the like-sign yields in mixed events, for dimuon mass \( \geq 2 \text{ GeV}/c^2 \). As shown in Fig. 3, the invariant mass distributions determined by these methods are consistent with each other within statistical uncertainties. The mixed event method is then used to determine the combinatorial background for the final analysis in order to reduce statistical fluctuations. After the combinatorial background subtraction, clear \( J/\psi \) peaks are found in both muon arms, as shown in Figs. 3(c) and (d). A mass window cut \( 2.7 < m < 3.5 \text{ GeV}/c^2 \) is applied to the dimuon pair invariant mass distribution to select \( J/\psi \) candidates. The signal (combinatorial background subtracted yields) to the combinatorial background ratio in the \( J/\psi \) mass window is 18.6 in the \( 1.2 < y < 2.2 \) region and 19.9 in the \( -2.2 < y < -1.2 \) region.

C. Simulation Setup

The full simulation framework, which comprises \textsc{pythia8}[18]+\textsc{geant4}[19]+reconstruction, is set up to characterize the DCAR distributions of muons from prompt \( J/\psi \) and \( J/\psi \) from \( B \)-meson decay. Dead areas in the detector are determined from data on a run-by-run basis and the same vertex and tracking reconstruction algorithms as in data analysis are used. The width of the simulated primary vertex distributions along the \( x \) and \( y \) axes is 80 \( \mu \)m as determined from Vernier Scan measurements [20]. The vertex distribution along the \( z \) axis used in the simulation has been determined from the real data. To get an accurately reproduced \( z \) vertex resolution in simulation, which is dependent on the multiplicity in the event, additional simulated MB events (with \( z \) vertex matched to the hard QCD events) are embedded into the prompt \( J/\psi \) events, or events with a \( J/\psi \) from \( B \)-meson decay. To ensure that the accessed kinematic region of the probed parton distribution function (PDF) in the MB events is the same in prompt \( J/\psi \) events or in \( B \)-meson \( \rightarrow J/\psi \) events, the renormalization scale \( Q_{\text{renorm}} \) defined in \textsc{pythia}, which determines the PDF shape, is kept at the same value between the MB event and the triggered event.
FIG. 3. The invariant mass of dimuons in the (a,c) $1.2 < y < 2.2$ and (b,d) $-2.2 < y < -1.2$ regions. Raw yields (black solid), the combinatorial background using mixed events (red open rectangular) and like-sign dimuon pairs (green open triangle) are shown in panels (a) and (b). The combinatorial background subtracted yields are shown in panels (c) and (d). The magenta dashed lines represent the mass cut used to select $J/\psi$ candidates.

To verify that the simulations accurately represent the real data, we have compared the simulated and measured muon DCA$_R$ distributions from inclusive $J/\psi$ events. The inclusive $J/\psi$ events in simulation are obtained by combining 90% prompt $J/\psi$ events and 10% $J/\psi$ from $B$-meson decay. This fraction of $B$-meson decays to $J/\psi$ is selected based on the average result from global data measured in the same inclusive $J/\psi$ $p_T$ region [3–7]. A single Gaussian function is fit to the centroid of the DCA$_R$ distributions in data and simulation to derive the resolutions of the prompt component of the DCA$_R$. The momentum dependence of this DCA$_R$ resolution extracted from the core region ($|\text{DCA}_R| < 500\mu$m) is compared between data and simulation. As shown in Fig. 4, good agreement between data and simulation is achieved in both of the measured rapidity regions.
D. Signal Determination

The shapes of the DCA\textsubscript{R} distributions of muons from prompt $J/\psi$ and those from $B$-meson $\rightarrow J/\psi$ are characterized using the full simulation. Figure 5 shows the resulting normalized distribution of DCA\textsubscript{R} for muons from prompt $J/\psi$ events (blue open circle) and from $B$-meson $\rightarrow J/\psi$ events (green circle). As explained at the end of Section II, the shape of the muon DCA\textsubscript{R} distribution in prompt $J/\psi$ events is symmetric, which is consistent with expectations for prompt particle decays. The $\Lambda_b$, $B^\pm$, $B^0$, $B^0_s$ hadrons have a finite lifetime of 1.4–1.6 ps on average, resulting in a displaced vertex at forward rapidity of approximately 0.8 mm from the primary collision vertex for the $J/\psi$ from $B$-meson decay. Due to the displacement between the decay vertex and the primary collision vertex, the negative side of the muon DCA\textsubscript{R} distribution shows a clear deviation from symmetry for $B$-meson $\rightarrow J/\psi$ events. The respectively
symmetric and asymmetric DCA_{R} distributions allow the separation of prompt J/\psi from B-meson \rightarrow J/\psi.

Several functions were tested to describe the line shapes of the muon DCA_{R} in both prompt J/\psi and J/\psi from B-meson decay in simulations. The final fit functions which will be described below are selected based on the best fits to the simulation spectra with the maximum log-likelihood method and the convolution of the intrinsic DCA_{R} resolution with a function which represents B meson decay kinematics is used. Variations of the fit functions and the simulation setup were then used to account for systematic uncertainties in the fit function. A convolution fit is used to describe the shape of the muon DCA_{R} from prompt J/\psi decay, with the definition shown in Eq. (3).
\[
f_{\text{prompt } J/\psi}(\text{DCAR}) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(\text{DCAR} - \mu)^2}{2\sigma^2}\right] \otimes \frac{\sigma_1^2 \text{DCAR}^2}{(\text{DCAR}^2 - \mu_1^2)^2 + \text{DCAR}^4(\sigma_1^2 / \mu_1^2)},
\]

where \(\mu, \sigma, \mu_1\) and \(\sigma_1\) are determined from the fit to the prompt \(J/\psi\) simulation spectra. Parameter \(\sigma\) and \(\sigma_1\) determine the width of the muon DCAR shape in prompt \(J/\psi\) events, which comes from the detector and vertex resolutions. Values of these parameters defined in Eq. (3) are fixed in the next step: the fit to the measured DCAR distributions. For \(B\)-meson decay to \(J/\psi\) events, the convolution fit function defined in Eq. (4) is used:

\[
f_{B\rightarrow J/\psi}(\text{DCAR}) = f_{\text{prompt } J/\psi}(\text{DCAR}) \otimes f_B(\text{DCAR}),
\]

where the function \(f_{\text{prompt } J/\psi}(\text{DCAR})\) is defined in Eq. (3). The parameters of \(f_{\text{prompt } J/\psi}(\text{DCAR})\) are already determined, as explained above, in the fit of muon DCAR in the prompt \(J/\psi\) simulation. Function \(f_B(\text{DCAR})\), which stands for the decay kinematics of \(B\)-meson, is defined as:

\[
f_B(\text{DCAR}) = \left\{ \begin{array}{ll}
\exp\left[-\frac{(\text{DCAR} - \mu_2)^2}{2\sigma_2^2}\right], & \text{DCAR} - \mu_2 > -\alpha \\
(n/|\alpha|)^n \exp\left(-\frac{|\alpha|^2}{2}\right) & (n/|\alpha| - |\alpha| - \frac{\text{DCAR} - \mu_2}{\sigma_2})^{-n}, \text{DCAR} - \mu_2 \leq -\alpha
\end{array} \right.
\]

where \(\mu_2, \sigma_2, n\) and \(\alpha\) are parameters determined from the fit to the \(B \rightarrow J/\psi \rightarrow \mu^+\mu^-\) simulation. The average value of the muon DCAR from \(B \rightarrow J/\psi\) decay is determined by \(\mu_2\). Parameters \(\sigma_2, n\) and \(\alpha\) determine the asymmetric shape of this DCAR distribution. The determined values of these parameters defined in this section and used in Eq. (4) and Eq. (5) are then fixed in the fit to the measured DCAR distributions.

Fits of the simulated muon DCAR distributions for prompt \(J/\psi\) (blue open circle) and \(B \rightarrow J/\psi\) (green circle) are shown in Fig. 5. The DCAR spectra can be modeled by the two functions defined in Eq. (3) and Eq. (4).

### E. Background Determination

For this analysis, backgrounds come from three different sources: combinatorial, MuTr-FVTX track mis-matching and heavy flavor decay continuum which represents unlike-sign dimuon pairs from \(b\bar{b} \rightarrow BB \rightarrow \mu^+\mu^- + X\) and \(c\bar{c} \rightarrow D\bar{D} \rightarrow \mu^+\mu^- + X\) events. The combinatorial background and the background from mis-matching between FVTX and MuTr tracks are determined by data-driven methods. The fraction of the contribution from the heavy flavor continuum background is determined by fitting the dimuon pair invariant mass spectra in real data, and the DCAR shape is determined from simulation. Details of the background determinations will be discussed in sections III E 1 through III E 3.

#### 1. Combinatorial Background Determination

The combinatorial background, which comes from combining randomly associated tracks in an event, is evaluated using unlike-sign dimuons formed by muon tracks from two different events (referred to as the mixed event procedure). The events to be mixed are required to have \(z\) vertices with no more than 1.5 cm difference from each other. The muon DCAR distribution of the combinatorial background from normalized mixed events (the normalization factor is defined in Eq. (2)) is shown as magenta open triangles in Fig. 6.

#### 2. FVTX-MuTr mis-matching determination

The last FVTX plane and the first MuTr station are 150 cm apart and have approximately 1 m of absorber material in between. MuTr tracks with momentum above 3 GeV/c projected to the fourth station of the FVTX therefore cover a circle with a radius of up to 2 cm for muons, due to the multiple scattering in the absorber. As a result, some
FIG. 6. The raw yields of data (black closed circles) and estimated background DCA $R_{d}$ distributions within the $J/\psi$ mass window (2.7–3.5 GeV/c$^{2}$) are shown for (a) rapidity $1.2<y<2.2$ and (b) $-2.2<y<-1.2$. The combinatorial background defined as $f_{\text{combinatorial}}$ in Eq. (8) (magenta open triangle), the heavy flavor continuum ($c\bar{c}+b\bar{b}$) background defined as $f_{c\bar{c}+b\bar{b}}$ in Eq. (8) (green solid triangle) and the detector mis-matching background defined as $f_{\text{mismatch}}$ in Eq. (8) (blue open circle) are determined using techniques described in the text.
FIG. 7. Fit of dimuon mass spectra to determine the heavy flavor continuum background for (a) rapidity $1.2 < y < 2.2$ and (b) $-2.2 < y < -1.2$. The fit function (black solid curve) includes the $J/\psi$ and $\psi'$ yields which already include the FVTX-MuTr mismatching background, the combinatorial background (red dashed curve) and the heavy-flavor-continuum background (blue dash-dotted curve). The total background (yellow solid-dotted curve) shows the combinatorial and the heavy-flavor-continuum background.
fraction of the MuTr projections will find more than one FVTX track or a single but incorrect FVTX track inside its projected circle, and have a certain probability of selecting an incorrect FVTX match. We refer to these incorrect matches as “mis-matching background”.

To estimate the amount of mis-matching, we attempt to match MuTr tracks from one event to FVTX tracks from a separate event (referred to as swapped events). To be as realistic as possible, the swapped events need to belong to the same z-vertex category, meaning the difference of the z vertex between the swapped event and the true event should be less than 1mm. The selection of 1mm $z$-vertex difference does not introduce any bias to the DCA$_R$ distribution. In addition to this, we also count the mis-matching tracks from swapped events only when the matching track in the swapped event has a better $\chi^2$ than the matching track in the real event, so that we do not overestimate the mismatches in real events. The mis-matching background in the analyzed events is dominated by $J/\psi$ MuTr tracks which do not have a corresponding FVTX track in the real event and accidentally match to a random background track. The fraction of candidate FVTX tracks in swapped events which are found to be wrongly associated with a MuTr track from a good $J/\psi$ dimuon pair, and that pass the quality cuts shown in Table I, is 3% (2%) in the $1.2 < y < 2.2$ ($-2.2 < y < -1.2$) rapidity region.

### 3. Heavy Flavor Background Determination

After subtracting the combinatorial background from the dimuon invariant mass distribution within the 2–6 GeV/$c^2$ region (shown in Fig. 3), there are remaining backgrounds in the sideband regions outside the $J/\psi$ mass window. This remaining background is dominated by the heavy flavor continuum and indicates that this continuum is not negligible in the $J/\psi$ mass region. To determine the fraction of the heavy flavor background, a fit function which includes yields from $J/\psi$, $\psi'$, the combinatorial background and heavy flavor continuum background is applied to the invariant mass distribution of dimuon pairs. In the dimuon pair mass region > 4 GeV/$c^2$, the heavy flavor continuum background also contains Drell-Yan. Because the fraction of Drell-Yan events within the $J/\psi$ mass region (2.7–3.5 GeV/$c^2$) is negligible, the fit in this mass region does not include a Drell-Yan component.

Figure 7 shows the fit of the dimuon mass distribution to determine the heavy flavor continuum background. The total background (yellow) determined by the fit to the invariant mass spectrum, which comprises the combinatorial (red) and the heavy flavor background (blue), follows the mass distribution outside the $J/\psi$ mass window well. The fraction of the heavy flavor background within the $J/\psi$ mass window is found to be $7.1\% \pm 1.1\%$ ($5.5\% \pm 0.8\%$) in the $1.2 < y < 2.2$ ($-2.2 < y < -1.2$) regions.

The relative $b\bar{b}$ and $c\bar{c}$ dimuon contributions within the $J/\psi$ mass window are not well known, and extrapolation from previous midrapidity dimuon invariant mass yields in 200 GeV $p+p$ collisions would introduce a large systematic uncertainty. We therefore first fit the unlike-sign dimuon invariant mass spectrum near the $J/\psi$ region including the PYTHIA8-simulated shape of $b\bar{b}$ and $c\bar{c}$ components and an unconstrained normalization scale to estimate the contribution. The fit suggests there is a 33% $b\bar{b}$ fraction in the heavy flavor continuum within the $J/\psi$ mass region. However, we do note there is systematic uncertainty in the PYTHIA8 shape. Because of this uncertainty, for this analysis the fraction of the $b\bar{b}$ contribution to the heavy flavor yields within the $J/\psi$ mass window is set to be 50%, and varied from 0 to 100% to take into account all possibilities in the systematic uncertainty.

### F. Fitting Procedure

The DCA$_R$ distributions are selected from dimuon pairs within the mass window 2.7–3.5 GeV/$c^2$. A fit function is developed to simultaneously extract the prompt $J/\psi$ and B-meson $\rightarrow J/\psi$ yields from the real data DCA$_R$ distributions with the maximum log-likelihood method. This fit function comprises five components: 1) muons from prompt $J/\psi$, 2) muons from $B$-meson $\rightarrow J/\psi$, 3) combinatorial background determined by mixed events, 4) mismatching between FVTX and MuTr determined by swapped events, and 5) heavy flavor ($c\bar{c}$ + $b\bar{b}$) continuum background. The fit function which is used to determine the shape of muon DCA$_R$ distributions from prompt $J/\psi$ ($B$-meson $\rightarrow J/\psi$) events is $f_{\text{prompt}} J/\psi(DCA_R) \times (f_B \rightarrow J/\psi(DCA_R))$ as discussed in Section III D. Parameters defined in both Eq. (3) and Eq. (4) are fixed according to the fit to the simulated spectra and the detector resolution smearing is fine-tuned in the data fit. The functions which represent the three background contributions are $f_{\text{combinatorial}}(DCA_R)$, $f_{\text{mismatch}}(DCA_R)$ and $f_{c\bar{c}+b\bar{b}}(DCA_R)$ as discussed in Section III E. Histograms of muon DCA$_R$ from different background contributions after normalization are used to represent each component in Eq. (8). Fluctuations of the fit methods, signal and background determinations are studied in the systematic uncertainty evaluations. These functions used to describe the data spectrum, are summarized in Eq. (6),
where \( \text{Yield}_{\text{incl}} \rightarrow J/\psi \) is the total yield of inclusive \( J/\psi \) which comprises both prompt \( J/\psi \) and \( B \)-meson decayed \( J/\psi \). Normalization and shapes of most of the components are fixed in previous steps. In the final stage of the fit, the fraction of muons from \( B \)-meson \( \rightarrow J/\psi \) (i.e. \( F_{B \rightarrow J/\psi} \)), is the main free parameter in the total fit function (defined in Eq. (6)), together with the \( J/\psi \) yield and a last tuning of the resolution that is described below. As the DCA\(_R\) resolution in data can be affected by additional factors which may not be well captured by the simulation (such as event-by-event variations in the vertex resolution, additional smearing from multiple scattering in the nonuniform detector materials, part of the detector randomly dropping out within a run and beam-beam collision geometry fluctuations), an additional free parameter, \( \sigma t \), is introduced in the convolution fit functions for prompt \( J/\psi \) (defined in Eq. (3)) and \( B \)-meson \( \rightarrow J/\psi \) (defined in Eq. (4)). It accounts for detector resolution smearing and also captures any uncertainty of the beam spot size. The fit is then performed with the parameter \( \sigma t^* \) instead of \( \sigma t \), where \( \sigma t^* = \sigma t + \sigma t \). The resolution smearing parameter \( \sigma t \) determined from the fit to the data is within 20 \( \mu m \) with approximately 20 \( \mu m \) statistical uncertainty for the 1.2 < \( |y| \) < 2.2 region. The size of the smearing is much smaller than the the average \( x-y \) beam profile value (around 80 \( \mu m \)) and the DCA\(_R\) resolution (around 230 \( \mu m \)). The value of the resolution smearing \( \sigma t \) varies from 5 to 70 \( \mu m \) when different beam profile values in the \( x-y \) plane are used in the simulation (from 80 to 180 \( \mu m \)). Variation of the smearing parameter \( \sigma t \) will be included in the systematic uncertainty evaluation. Applying the fit procedure to the DCA\(_R\) distributions, assuming 50% of the heavy flavor continuum contribution comes from \( b \bar{b} \) (see discussions in IIIE3), allows the raw fraction of \( J/\psi \) mesons from \( B \) decays in inclusive \( J/\psi \) yields to be extracted. The corresponding raw ratios \( B \rightarrow J/\psi \) are 7.3\%±3.7\% (stat) for (1.2 < \( y < 2.2 \)) and 8.1\%±2.8\% (stat) for (-2.2 < \( y < -1.2 \)). The spectra and fit results are shown in Fig. 8. The fit parameter values are summarized in Table II.

| Fit parameter | -2.2 < \( y < -1.2 \) | 1.2 < \( y < 2.2 \) |
|---------------|-----------------|-----------------|
| \( \mu \)     | -15 ± 5 \( \mu m \) | 6 ± 5 \( \mu m \) |
| \( \sigma \)   | 209 ± 8 \( \mu m \) | 210 ± 6 \( \mu m \) |
| \( \mu_1 \)    | 0 \( \mu m \)    | 0 \( \mu m \) |
| \( \sigma_1 \) | 60 ± 11 \( \mu m \) | 50 ± 9\( \mu m \) |
| \( \sigma_t \) | 7 ± 14 \( \mu m \) | 10 ± 18 \( \mu m \) |
| \( \mu_2 \)    | -135 ± 15 \( \mu m \) | -123 ± 18 \( \mu m \) |
| \( \sigma_2 \) | 169 ± 10 \( \mu m \) | 150 ± 16 \( \mu m \) |
| \( \alpha \)   | 0.74 ± 0.06 | 0.60 ± 0.08 |
| \( n \)        | 3.50 ± 0.51 | 4.26 ± 0.75 |

G. Acceptance × Efficiency Correction

In \( p+p \) collisions, the DCA\(_R\) resolution is dominated by the VTX/FVTX vertex resolution. Higher event multiplicity can lead to a better vertex resolution and a higher probability that a vertex can be reconstructed for a given event. The \( B \rightarrow J/\psi \) events have higher average VTX/FVTX multiplicity in comparison with prompt \( J/\psi \) events. Conversely, due to their different \( p_T \) distributions, \( B \rightarrow J/\psi \) events have a somewhat lower probability of having both muons accepted into the muon arm than prompt \( J/\psi \) events. These differences in VTX/FVTX event multiplicities and kinematics result in somewhat different values of the acceptance × efficiency for the two sets of events.
The raw ratio $F_{\text{raw}}^{B \to J/\psi}$ as discussed in section III F must be corrected for the relative acceptance\times efficiency difference between prompt $J/\psi$ and $B \to J/\psi$ events, using the PYTHIA8+GEANT4+reconstruction simulation described previously in section III C, $\frac{A_{\varepsilon, \text{prompt}}^{J/\psi \to \mu\mu}}{A_{\varepsilon, B \to J/\psi \to \mu\mu}}$, where $A_{\varepsilon, \text{prompt}}^{J/\psi \to \mu\mu}$ ($A_{\varepsilon, B \to J/\psi \to \mu\mu}$) is the acceptance\times efficiency for prompt $J/\psi$ ($B \to J/\psi$) events.

The acceptance\times efficiency for prompt $J/\psi$ events is $0.455\% \pm 0.007\%$ ($0.506\% \pm 0.008\%$) and for $B \to J/\psi$ events is $0.446\% \pm 0.007\%$ ($0.473\% \pm 0.007\%$) in the $-2.2 < y < -1.2$ ($1.2 < y < 2.2$) rapidity region. The extracted relative ratio of $B \to J/\psi$ acceptance\times efficiency to prompt $J/\psi$ acceptance\times efficiency is $0.980 \pm 0.022$ ($0.935 \pm 0.020$) in the $-2.2 < y < -1.2$ ($1.2 < y < 2.2$) rapidity region. The $B \to J/\psi$ fraction $F_{B \to J/\psi}$ which is defined as $\frac{N_{B \to J/\psi}}{N_{\text{prompt} \ J/\psi} + N_{B \to J/\psi}}$ ($N_{\text{prompt} \ J/\psi}$ is the yield for prompt $J/\psi$, $N_{B \to J/\psi}$ is the yield for $B \to J/\psi$) can be derived according to Eq. (9).

$$F_{B \to J/\psi} = \frac{1}{1 + \left( \frac{1}{F_{B \to J/\psi}} - 1 \right) \cdot \frac{\varepsilon_{B \to J/\psi \to \mu\mu}}{\varepsilon_{\text{prompt} \ J/\psi \to \mu\mu}}}$$ (9)

H. Systematic Uncertainty

The systematic uncertainty for $F_{B \to J/\psi}$ is evaluated by taking into account any factors which can affect the DCA_R mean, the DCA_R resolution, or the overall normalization of the signals. The following items are considered in the systematic uncertainty evaluation, along with a description of the methods performed to extract the uncertainties. For each item we compare the nominal $B \to J/\psi$ fraction extracted from our analysis to that obtained with alternate methods to extract the systematic uncertainty:

a: $p_T$ uncertainties: the $B$-meson $\rightarrow J/\psi \ p_T$ distributions were re-weighted in $B \rightarrow J/\psi$ simulations according to the prompt $J/\psi \ p_T$ distribution. The inclusive $J/\psi \ p_T$ spectrum was also varied with different fractions of prompt $J/\psi$ and $B$-meson $\rightarrow J/\psi$.

b: Background determination uncertainties: smooth fit functions were used to characterize the combinatorial, mis-matching and heavy flavor backgrounds instead of histograms and their effect on the fit result was evaluated.

c: Background determination uncertainties: deviation of fit results from the average value with different fractions of $bb$ contribution in the heavy flavor background. The $bb$ fraction of the heavy flavor background is varied from 0, 50% to 100%. Even though the assumption of 0 or 100% $bb$ heavy flavor continuum background is unrealistic, to be conservative, the maximum variation between the average value of the fitted $B$ to $J/\psi$ fraction and the fit result assuming 0 or 100% $bb$ fraction of heavy flavor background is quoted as the systematic uncertainty.

d: Background determination uncertainties: the combinatorial background normalization $\text{Norm}_{\text{mix}}$ defined in Eq. (2) was calculated within different dimuon mass ranges and compared to the nominal values.

e: Fitting method uncertainties: multiple tests of the DCA_R fit function with varied DCA_R means and resolutions were applied to pseudo data, including different fractions of prompt $J/\psi$ and $J/\psi$ from $B$-meson decay with muon DCA_R shape determined in simulation and realistic backgrounds. The stability of the extracted ratios was checked and deviation from the average value is accounted for in the systematic uncertainty.

f: Signal determination uncertainties: different functions were used to represent the muon DCA_R distributions in both prompt $J/\psi$ and $J/\psi$ from the $B$-meson decay events in simulation. A triple Gaussian function was used for prompt $J/\psi$ events and a Crystal-Ball plus single Gaussian function was used for $J/\psi$ from the $B$-meson decay events. The stability of the extracted ratios was checked.

g: $J/\psi$ selection uncertainties: good $J/\psi$ candidates were selected in different dimuon pair mass windows (shifted by 0.15 GeV/c^2) and the extracted ratio results were compared to the nominal ratios.

h: Alignment determination uncertainties: different misalignment residuals were applied to the DCA_R mean to determine their effect on the fit.

i: Event quality cut uncertainties: different vertex resolution cuts were used and their effect on the fit evaluated.

j: Dependence of simulation on different $x$-$y$ vertex smearing: the vertex smearing was varied from the reconstructed value in real data (around 200 $\mu$m) to the average beam profile value (around 80 $\mu$m) and the effect on the fit evaluated.
TABLE III. Systematic uncertainty summary for the fraction of $J/\psi$ from $B$-meson decay in the $1.2 < y < 2.2$ and $-2.2 < y < -1.2$ rapidity regions. Values are in absolute scale. See the specific meaning of each item in Section III H.

| Source | $1.2 < y < 2.2$ | $-2.2 < y < -1.2$ | Specific meaning |
|--------|----------------|-------------------|------------------|
| a      | $< 0.1\%$     | $< 0.1\%$        | $p_T$ uncertainties. |
| b      | 0.1\%         | 0.2\%            | Backgrounds shape variations with fit functions. |
| c      | 1.4\%         | 1.1\%            | $b \bar{b}$ fraction variations in the heavy flavor background. |
| d      | $< 0.1\%$     | $< 0.1\%$        | Combinatorial background normalization variation. |
| e      | 0.5\%         | 0.5\%            | Fit method variations. |
| f      | 0.3\%         | 0.3\%            | Signal determination variations. |
| g      | 0.4\%         | 0.5\%            | $J/\psi$ selection variation. |
| h      | 0.3\%         | 0.5\%            | Alignment correction variations. |
| i      | 0.4\%         | 0.6\%            | Event quality cut variations. |
| j      | 1.0\%         | 1.0\%            | Vertex smearing in the $x - y$ plane. |
| k      | 0.1\%         | 0.2\%            | Variations of the acceptance $\times$ efficiency. |

**Total syst uncertainty** 1.9\% 1.9\%

FIG. 8. $B \to J/\psi$ fraction fit to muon DCA$_R$ in the (a) $1.2 < y < 2.2$ and (b) $-2.2 < y < -1.2$ regions. The ([red] solid curve) stands for the total fit, which includes the prompt $J/\psi$ (solid blue), the $B$-meson $\to J/\psi$ ([green] filled region), the combinatorial background ([magenta] dashed curve), the $c \bar{c} + b \bar{b}$ background ([brown] long-dashed curve) and the detector mismatching background ([purple] short-dashed curve).

**k**: Variation of the acceptance $\times$ efficiency: the renormalization scale factors were varied in simulation to get different $p_T$ distributions for prompt $J/\psi$ and $B$ meson decays, then the acceptance $\times$ efficiency correction factors were re-calculated and their effect on the fit was evaluated.

Table III gives the values and specific meanings for each evaluated contribution to the systematic uncertainty on the extracted fraction for $J/\psi$ from $B$-meson decay. As indicated, the total systematic uncertainty is 1.9\% in absolute scale for each muon arm in the $1.2 < |y| < 2.2$ rapidity coverage.

IV. RESULTS AND DISCUSSIONS

After applying the acceptance $\times$ efficiency factors shown in Table IV, the corrected $B \to J/\psi$ fraction in the rapidity interval ($1.2 < y < 2.2$) is $7.8\% \pm 3.9\%$ (stat) and the fraction in the rapidity interval ($-2.2 < y < -1.2$) is $8.3\% \pm 2.9\%$.
TABLE IV. Relative ratio of acceptance efficiency between prompt $J/\psi$ and $B \rightarrow J/\psi$ events, uncorrected $B \rightarrow J/\psi$ fraction ($F_{B \rightarrow J/\psi}^{\text{raw}}$) and corrected $B \rightarrow J/\psi$ fraction ($F_{B \rightarrow J/\psi}$). Uncertainties are statistical only.

| $-2.2 < y < -1.2$ | $1.2 < y < 2.2$ |
|-------------------|------------------|
| $\frac{A_{\text{B \rightarrow J/\psi} \rightarrow p p}}{A_{\text{prompt \ J/\psi} \rightarrow p p}}$ | $0.980 \pm 0.022$ |
| $F_{B \rightarrow J/\psi}^{\text{raw}}$ | $8.1\% \pm 2.8\%$ |
| $F_{B \rightarrow J/\psi}$ | $8.3\% \pm 2.9\%$ |

TABLE V. Fraction of $B$-meson decays in $J/\psi$ samples obtained in $p+p$ collisions at $\sqrt{s} = 510$ GeV.

| $|y| < 2.2$ |
|------------------|
| $F_{B \rightarrow J/\psi}$ |
| $-2.2 < y < -1.2$ | $8.3\% \pm 2.9\%(\text{stat}) \pm 1.9\%(\text{syst})$ |
| $1.2 < |y| < 2.2$ | $7.8\% \pm 3.9\%(\text{stat}) \pm 1.9\%(\text{syst})$ |
| $1.2 < |y| < 2.2$ | $8.1\% \pm 2.3\%(\text{stat}) \pm 1.9\%(\text{syst})$ |

The final results are summarized in Table V. Because the $p+p$ system is a symmetric, the results from the two arms are combined into a statistical average, giving a fraction of $J/\psi$ from $B$-meson decays in the $1.2 < |y| < 2.2$ region of $8.1\% \pm 2.3\%(\text{stat}) \pm 1.9\%(\text{syst})$. This result is integrated in the interval $0 < p_T(J/\psi) < 5$ GeV/c.

Comparisons to global measurements within the same inclusive $J/\psi$ $p_T$ region from CDF [3], ALICE [4], CMS [5] and LHCb [6, 7] experiments are shown in Fig. 9(a). The result from PHENIX is also compared with the $p_T$-dependent fraction from other experiments using the average $p_T = 2.2$ GeV/c of our inclusive $J/\psi$ sample as shown in Fig. 9(b). The LHCb experiment has measurements over a wide rapidity range, $2.0 < y < 4.5$; only results from $2.0 < y < 2.5$ and $3.0 < y < 3.5$ are shown in Fig. 9. The $2.0 < y < 2.5$ rapidity range is close to the kinematic range accessed by other measurements. The $F_{B \rightarrow J/\psi}$ result from this measurement is consistent with those from the higher energy collisions within uncertainties, although it does not exclude the possibility of a decrease of the $F_{B \rightarrow J/\psi}$ toward lower collision energy.

Figure 10 presents the comparison between the 510 GeV $p+p$ PHENIX result and the fixed-order next-to-leading-log
FIG. 10. In p+p collisions at $\sqrt{s} = 510$ GeV and $1.2 < |y| < 2.2$ rapidity region, comparison of PHENIX $B \to J/\psi$ fraction ($F_{B\to J/\psi}$) measured in integrated $J/\psi$ $p_T$ range of $p_T < 5$ GeV/c with $J/\psi$ $p_T$ dependent (shown in solid red) and $p_T$ integrated within 0–5 GeV/c region (shown in dashed blue) $B \to J/\psi$ fraction predicted by the FONLL+CEM model in 500 GeV p+p collisions. The uncertainty of the PHENIX measurement is statistical and systematic combined.

plus color-evaporation-model (FONLL+CEM) [12, 21, 22] predictions for the $B \to J/\psi$ fraction ($F_{B\to J/\psi}$) in 500 GeV p+p collisions. The CEM $J/\psi$ calculation uses the results of fitting the scale parameters to the energy dependence of the open charm total cross section for the charm quark mass $m_c = 1.27 \pm 0.09$ GeV/$c^2$. The factorization and renormalization scales, relative to the mass of the charm quark in the total cross section were found to be $\mu_F/m = 2.1^{+2.55}_{-0.85}$ and $\mu_R/m = 1.6^{+0.11}_{-0.12}$ [22]. The same central values were used to fix the $J/\psi$ normalization parameter in the CEM to the total cross section at $x_F > 0$ and $y > 0$ as a function of energy. The $J/\psi$ distributions were calculated with the same mass and scale parameters but to include the $p_T$ dependence instead of $\mu_F,R/m$, $\mu_F,R/m_T$ was used, where $m_T = \sqrt{(p_T^2 + p_T^2) + m_c^2}$. The shape of the $p_T$ distribution at low $p_T$ is determined by a $k_T$ kick of 1.29 GeV/c at $\sqrt{s} = 500$ GeV. The energy difference between 500 GeV and 510 GeV is small, so the difference in the $B \to J/\psi$ fraction is negligible. The measured fraction at PHENIX is consistent with the FONLL+CEM model prediction within uncertainties. The CMS nonprompt and prompt $J/\psi$ cross section measurements at 7 TeV...
FIG. 11. The average $b\bar{b}$ cross section per unit rapidity ($d\sigma/dy(pp\to b\bar{b}+X)$) is determined by the $B\to J/\psi$ fraction ($F_{B\to J/\psi}$) discussed in the paper and the inclusive $J/\psi$ cross section in 510 GeV $p+p$ collisions extrapolated from PHENIX 200 GeV $p+p$ measurements and the energy scaling factor provided by the CEM [22]. The extrapolated $d\sigma/dy(pp\to b\bar{b})$ (shown as open red circles) at $B$ hadron mean rapidity $y = \pm 1.7$ in 510 GeV $p+p$ collisions is compared with the rapidity dependent $B$ cross section (shown as blue solid line) calculated in FONLL. The PHENIX result is also comparable with the value of UA1 630 GeV $p+\bar{p}$ $d\sigma/dy(p\bar{p}\to b\bar{b})$ extracted from $p_T > 8$ GeV/$c$ to $p_T > 0$ range [23, 24] and unscaled with energy. The uncertainty of the extrapolated value at PHENIX (UA1) combines the statistical and systematic uncertainty from experiment with the CEM uncertainty. The uncertainty of the FONLL calculations contains both $b$ quark mass and scaling uncertainties.

$p+p$ collisions [5] have been compared to the FONLL+CEM calculations as well. The old CEM model underestimated the prompt $J/\psi$ cross section within $1.6 < |y| < 2.4$ and $J/\psi$ $p_T < 5$ GeV/$c$ region measured by the CMS experiment in 7 TeV $p+p$ collisions, while the nonprompt $J/\psi$ cross section measured in the same kinematic region and experiment is consistent with the FONLL calculations. Calculations with the CEM parameters from [22] give a better agreement between the FONLL+CEM prediction and the $B\to J/\psi$ fraction measured by CMS [5]. The FONLL calculations can reasonably describe the nonprompt $J/\psi$ cross section results at LHCb for $p_T > 0$ [6, 7].

The $B\to J/\psi$ fraction $F_{B\to J/\psi}$ is also related to the inclusive $J/\psi$ cross section per unit rapidity $d\sigma/dy(pp\to J/\psi)$
and the $b\bar{b}$ cross section per unit rapidity $\frac{d\sigma}{dy(pp\to b\bar{b})}$,

$$
F_{B\to J/\psi} = \frac{2 \times \frac{d\sigma}{dy(pp\to b\bar{b})} \times \text{Br}(B\to J/\psi + X)}{\frac{d\sigma}{dy(pp\to J/\psi)}},
$$

(10)

where $\text{Br}(B\to J/\psi + X)$ is the branching ratio of $B$ hadron decays to $J/\psi$ and the $b$ ($\bar{b}$) quark to $B$-hadron fragmentation is assumed to be 1. The factor of two in Eq. (10) accounts for the fact that both $B\to J/\psi$ and $\bar{B}\to J/\psi$ contribute to the $B\to J/\psi$ fraction $F_{B\to J/\psi}$. Eq.(10) can be rewritten as:

$$
\frac{d\sigma}{dy(pp\to b\bar{b})} = \frac{1}{2} \frac{\frac{d\sigma}{dy(pp\to J/\psi)} \times F_{B\to J/\psi}}{\text{Br}(B\to J/\psi + X)}. 
$$

(11)

Therefore, $\frac{d\sigma}{dy(pp\to b\bar{b})}$ can be derived from Eq. (11). To do this, we use $\frac{d\sigma}{dy(pp\to J/\psi)} = 1.00 \pm 0.11$ nb (0.97\pm0.11 nb) at mean rapidity $y = 1.7$ (−1.7) in 510 GeV $p+p$ collisions, and $\text{Br}(B\to J/\psi + X) = 1.094\pm0.032\%$ [25]. Here, $\frac{d\sigma}{dy(pp\to J/\psi, 510 \text{ GeV})}$ is extrapolated as $\frac{d\sigma}{dy(pp\to J/\psi, 200 \text{ GeV})} \times R(510/200)$, where the scaling factor $R(510/200)$ is 2.08$^{+0.72}_{-0.55}$ according to the CEM [22], and $\frac{d\sigma}{dy(pp\to J/\psi, 200 \text{ GeV})} = 0.48\pm0.05$ nb (0.47\pm0.05 nb) at mean rapidity $y = 1.7$ (−1.7) [26].

The extracted $\frac{d\sigma}{dy(pp\to b\bar{b})}$ is $3.57^{+3.38}_{-2.22}$ $(3.68^{+2.08}_{-1.88})$ mb at $B$ hadron mean rapidity = $1.7$ (−1.7) in 510 GeV $p+p$ collisions. The weighted average of the two measurements is $\frac{d\sigma}{dy(pp\to b\bar{b})} = 3.63^{+1.92}_{-1.70}$ mb at $B$-hadron rapidity=±1.7. As shown in Fig. 11, these values are comparable with the FONLL-calculated rapidity-dependent $B$ cross section within large uncertainties [27–29]. The PHENIX extracted values are also comparable to the AU1 $\sqrt{s} = 630$ GeV $p+p$ average $b\bar{b}$ cross section per unit rapidity ($\frac{d\sigma}{dy(pp\to b\bar{b})}$, 630 GeV) = $4.3^{+2.51}_{-2.10}$ mb within $|y| < 1.5$ [23, 24] which is extrapolated from $p_T > 8$ GeV/c to the $p_T > 0$ range. The FONLL calculation assumes $m_b = 4.75 \pm 0.25$ GeV/c$^2$ while the renormalization and factorization scales are varied by a factor of two around the central value, $\mu_{R,F} = \sqrt{p_T^2 + m_b^2}$ [12, 29].

V. SUMMARY

We have presented a new measurement of the nonprompt over inclusive $J/\psi$ production ratio $F_{B\to J/\psi}$ in $p+p$ collisions at $\sqrt{s} = 510$ GeV, integrated over the $J/\psi$ kinematical domain, $p_T < 5$ GeV/c and rapidity 1.2 < $|y|$ < 2.2. The result is $F_{B\to J/\psi} = 8.1\% \pm 2.3\%$ (stat) $\pm 1.9\%$ (syst). This measurement extends the previously measured $F_{B\to J/\psi}$ values at CDF and LHC to lower energy, and is comparable to measurements at higher energies; it is also within 1.0 standard deviation of the FONLL+CEM calculation which has a negligibale dependence on $\sqrt{s}$, $p_T$ and $y$. The extrapolated $\frac{d\sigma}{dy(pp\to b\bar{b})}$ is $3.63^{+1.92}_{-1.70}$ mb at $B$ hadron mean rapidity, ±1.7, in 510 GeV $p+p$ collisions, which is comparable with the FONLL calculations in 500 GeV $p+p$ collisions.

The weak dependence on the center of mass energy in Fig. 9(a) for the $F_{B\to J/\psi}$ fraction could indicate that the variation of the bottom yield with energy is compensated by a similar variation of the prompt $J/\psi$ yield. It is also noteworthy that only a factor of two decrease of the $b$ over the $c$ yield is expected going from LHC energies to $\sqrt{s} = 510$ GeV, as calculated with FONLL [27, 28]. However, modeling the hadronization of the bound $c\bar{c}$ at low $p_T$ is still a challenge to QCD calculations. The present results provide complementary information to the surprisingly weak evolution of $F_{B\to J/\psi}$ in 0.51 < $\sqrt{s}$ < 13 TeV domain, for central or near central rapidity and low $p_T$ production.

The analysis procedure developed in this study will be applied to other data sets recorded by PHENIX at different center of mass energies. A similar method can also be applied to the study of $B$- and $D$-meson semileptonic decays to muons, which will help to understand the production mechanism of charm and bottom, and provide a complementary measurement to the one presented in this paper.

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