Comparative analysis of Rectangular Plate by Finite element method and Finite Difference Method

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Abstract: Analysis of rectangular plates is common when designing the foundation of civil, traffic, and irrigation works. The current research presents the results of the analysis of rectangular plates using the finite difference method and Finite Element Method. The results of the research verify the accuracy of the FEM and are in agreement with findings in the literature. The plate is analyzed considering it to be completely solid. The ordinary finite difference method is used to solve the governing differential equation of the plate deflection. The proposed method can be easily programmed to readily apply on a plate problem. The work covers the determination of displacement components at different points of the plate and checking the result by software (STAAD.Pro) analysis. Keywords: rectangular plate, FEM, Finite Difference Method

I. INTRODUCTION

These methods adopted are computationally utilized. However, the exact accuracy of approximate analysis methods is the matter of high concern. Thus, the efforts should be taken to check out the deviation range in results of such different methods, so that the design should be stable, accurate and economical. The finite element method is a numerical technique for solving problems which are described by partial differential equations or can be formulated as functional minimization. A domain of interest is represented as an assembly of finite elements. Approximating functions in finite elements are determined in terms of nodal values of a physical field which is sought and continuous physical problem is transformed into discretized finite element problem with unknown nodal values. For a linear problem a system of linear algebraic equations should be solved. The finite difference method has emerged as a powerful tool of structural analysis. In mathematical approach the continuous formulation is reduced to discrete formulation by simply replacing derivatives with finite difference approximation. In short differential equation is converted into difference equation. Thus the idea behind this method is to replace the governing differential equation and the equations defining boundary conditions by corresponding finite difference quantities as some selected points.

II. LITERATURE DETAILS

To provide a detailed review of the literature related to FEM and FDM would be difficult to address in this chapter. A brief review of previous studies on FEM and FDM is presented in this section. This literature review focuses on recent contribution related to FEM and FDM from IJIRSET, IJNME, IJERD etc. The study uses the approximate technique of finite element method for analyzing a rectangular plate based on the classical plate theory. In this paper free vibration analysis has been carried out for a rectangular plate. By varying the thickness of the plate, it has been concluded that the frequency parameter is constant. The result showed current methodology solutions closely converged to the exact solutions. Increase in thickness of the plate does not affect the frequency parameter. The research work entitled in this paper deals with the finite element formulation applied to a plate made with isotropic and laminated plates subjected to uniform pressure with various end conditions. This study applied to the results for circular, elliptical and square holes. Finally, from the results obtained, it can be predicted that the value of stresses deflections can be further enhanced by experimenting with various cut-outs, bluntness, loading angle, various materials particularly with composites they can be varied by using proper ply sequence, number of layers and ply angles. Finite difference method as one of the existing numerical methods is relatively strong method for numerical solution of the plate equations with different loading and support solutions conditions. As observed, increasing repeat steps in finite difference method does not result in increased attention to the problem and may also act in the opposite way. The study has investigated validity of the Finite Difference method to simulate thin sectorial steel plate under the effect of uniform and concentrated loads with different types of boundary conditions. The differential equation schemes of curved plate were solved by means of the Finite Difference method. Also bending moments Mr, M\phi and Mr\phi schemes were derived.
The effects of curvature, aspect ratio, load type and boundary conditions of the plate were studied. The behavior of thin sectorial circular plate was investigated in this paper. A Finite Difference approach was presented to analyze the plate. The validity of the adopted approach was compared with Finite Element approach using software package (ABAQUS). The investigation was carried out to simulate thin sectorial plate under the effect of uniform and concentrated loads with different types of boundary conditions, aspect ratios and mesh size.

III. METHODS

Methods of analysis:- There are mainly two methods of analysis a. Classical method- Which gives us realistic solutions, but quite cumbersome and some time it becomes almost impossible for complex structures to analyze for given loading and boundary conditions. b. Approximate methods- It includes Finite Difference Method (FDM), Finite Element Method (FEM), and Finite Strip Method (FSM) etc. All these methods give approximate solutions near to exact one.

IV. FINITE ELEMENT METHOD (FEM)

Finite element analysis (FEA), also called the finite element method (FEM), is a method for numerical solution of field problems. Mathematically, a field problem is described by differential equations or by an integral expression. Either description may be used to formulate finite elements. Finite element (FE) formulations, in ready to use form, are contained in general purpose FEA programs. Individual finite elements can be visualized as small pieces of a structure. The word “finite” distinguishes these pieces from infinitesimal elements used in calculus. The actual variation in the region spanned by an element is almost certainly more complicated, so FEA provides an approximate solution. Elements are connected at points called nodes.

V. FINITE DIFFERENCE METHOD (FDM)

The finite difference method has emerged as a powerful tool of structural analysis. The method is economical and competitive with the well known Finite Element Method. The finite difference method is probably the most transparent and the most general. Especially plate bending analysis is the classical field of the FDM. Today despite the existence of numerous finite element- based software packages, the FDM still be regarded as a numerical method that has merit due to its straight forward approach and a minimum requirement on hardware. In applying the FDM, the derivatives in the governing differential equations are replaced by difference quantities at some selected points of the plates. These points are located at the joints of a square, rectangular, triangular or other reference network, called a finite difference mesh.

VI. OBSERVATIONS

A. Case I : Mesh Density (4 x 4)
Nodal Displacement
Nodal location
Z axis = 1m
Interval length 'h' along X axis = 1m
Total span on X axis = 4m
| Nodal location (h) | FEM displacement | FDM displacement | (%) Difference |
|-------------------|------------------|------------------|----------------|
| 0                 | 0                | 0                | 0%             |
| 1                 | 0.628            | 0.578            | 8.2%           |
| 2                 | 1.149            | 1.053            | 8.7%           |
| 3                 | 0.628            | 0.578            | 8.2%           |
| 4                 | 0                | 0                | 0%             |

Case I : Nodal Displacement

Max Absolute N/mm²:
- X: 1.21
- Y: 1.21
- Z: 1.22
- M: 1.23
- N: 1.24
- O: 1.25
- P: 1.26
- Q: 1.27
- R: 1.28
- S: 1.29
- T: 1.30
- U: 1.31
- V: 1.32
- W: 1.33

Nodal Displacement (mm):
B. Case II: Mesh Density (8 x 8)
Nodal Displacement
Nodal location Z axis = 1m
Interval length 'h' along X axis = 1m
Total span on X axis = 4m

| Nodal location (h) | FEM displacement | FDM displacement | (% Difference) |
|---------------------|------------------|------------------|----------------|
| 0                   | 0                | 0                | 0%             |
| 1                   | 0.238            | 0.221            | 7.4%           |
| 2                   | 0.705            | 0.654            | 7.5%           |
| 3                   | 1.104            | 1.030            | 7.0%           |
| 4                   | 1.257            | 1.170            | 7.6%           |
| 5                   | 1.104            | 1.030            | 7.0%           |
| 6                   | 0.705            | 0.654            | 7.5%           |
| 7                   | 0.238            | 0.221            | 7.4%           |
| 8                   | 0                | 0                | 0%             |

VII. REMARK
From the results obtained for considered plate element using Finite Element Method and Finite Difference Method, it can be concluded that however, when the comparison is made for the results obtained for above considered cases solved in this report by both the methods that is Finite Element Method and Finite Difference Method, it can be seen that the percentage difference in FEM and FDM is about in the range of 7 to 10%. The result of FEM are more towards accuracy as it is considering the nodal location at mesh intervals, where as in FDM the results are influenced by the nodal location.

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