Student’s Mathematical Representation in Solving Geometry Problems Based on Cognitive Style

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Abstract. This research aims to describe the student’s mathematical representation of junior high school who had field independent cognitive style and field dependent in solving geometry problem. Subjects were selected based on the test results of the Group Embedded Figures Test (GEFT), and had equivalent mathematical abilities. The research subjects consisted of two female students who had field independent and field dependent cognitive style. This research uses a qualitative method. The results showed that the subject used mathematical representation in expressing his idea to solve geometry problems by using Polya's problem solving. The mathematical representation of field independent in solving geometry problem is: understanding information and what is asked in verbal, devising a plan in a visual of geometric images, then presenting verbal plans, carrying out the plan in symbolic by doing calculations, and looking back in symbolic by doing recalculation. Where as field dependent mathematical representations to solving geometry problems is: understanding information and asking in visual of geometrical images and verbal, devising a plan in a visual in the form of subsequent images presenting verbal plans, carrying out the plan in symbolic by performing calculations, and looking back in symbolic in the form of recalculation.

1. Introduction
Representation is one of the five standard processes in learning mathematics [1]. Five processes should students know and be able to use as they progress through school: problem solving, reasoning and proof, communication, connections, and representations [1]. We use representational forms to communicate ideas and as tools for reasoning [2]. Problem Solving an important role in mathematics learning, there is a general acceptance of the idea that the primary goal of mathematics instruction should be to have competent students problem solvers [3]. There is a close relationship representation to solve problems such as statements empirical studies suggest that mathematics problem solving competency depend on one's ability to think in terms of different representational system during problem solving process [4]. The linkage occurs when students begin to instruct the right representation for problems in obtaining the right solution.

The branch of mathematics consists of algebra, geometry, numbers, statistics and opportunities [5]. One of the material that has stronger problems solving at the junior level is a matter of geometry. Geometry provides a rich source of visualization for understanding arithmetic, algebraic, and statistical concepts [6]. Geometry deals with the nature and relationships of points, lines, shapes, and
spaces [7]. Geometry has a very important role to learn mathematics, as a link between events in everyday life and mathematical concepts.

Representations which each student is different from the other students, because each student has their own characteristics, such as that complexities of representation as cognitive and social processes and of how it is inextricably linked with the knowledge people have the situation being presented [8]. Various perceptions possessed by someone in exploring possible information for exploration, one of which is cognitive style [9]. That is because the representation as cognitive activities associated with the knowledge of the students, then the style of the students' cognitive affect representations to problem solving. In this study the characteristics of students are seen from their cognitive style. Typical ways of perceiving and remembering information, and typical ways of solving problems and making decisions [10]. Solving a problem is an attempt to determine a way out of a difficulty to get the right solution of the problem [11]. Solving problems in this study using the stages revealed by Polya consisting of understanding the problem, devising a plan, carry out the plan, and looking back [12].

2. Theoretical Framework

Representation plays an important role in efforts to develop and optimize students' mathematical abilities. NCTM includes a representation as the standard process of the fifth after problem solving, reasoning, communication, and connection [1]. According to the context or the use of the representation: a Cartesian graph, for example, can represent a function or the solution on the set of an algebraic equation [13].

Pape & Tchoshanov states that there are four ideas used in understanding the concept of representation. First, representations may be broadly as mental states. These internal representations are mental images of, for example, a set of five objects. Second, representations may more narrowly be thought of as mental representations of a former mental state. Here, the numerals, 5, or the number, five, are examples. Finally, the last two formulations include a structurally equivalent 'pre-sentation' through pictures, symbols or signs, and something 'in place of' something else [14].

The main purpose of mathematical representation as a standard process in school mathematics is to create and use representations to organize, record and communicate mathematical ideas. Translating, choosing, and applying mathematical representations to solve problems. Representation is a basic ability that will help in increasing students' understanding and mathematical abilities.

This study uses a mathematical representation form proposed by Goldin & Shteingold, namely: Symbolic (notations, mathematical models, doing calculations and manipulation of mathematical models), visual (number line, graph, diagram, table or picture), and verbal.

Referring to the explanation of the experts' definitions above, it can be concluded that the mathematical representation in this study is an expression displayed by students as mathematical ideas or concepts that describe, represent, or symbolize something in mathematics in visual, verbal, or symbolic form.

3. Method

3.1. Subject

The subjects of this study were two female students of grade VIII SMP Al-Falah Surabaya, consisting of a student who had field independent cognitive style and field dependent. Students are selected based on the results of the GEFT [16]. Scoring on GEFT is Subject scoring less (0-11) than the national mean for the GEFT (11.4) were categorized as field dependent. Subjects scoring the national mean (11.4) or greater (12-18) were categorized as field independent [17]. The next step students are given a math ability test to choose subjects with equal ability. The subject is said to have an equivalent mathematical ability, if the difference in test results is less than 5 with a maximum score of 100. In order to obtain two female students as subjects with details of field independent students’ (FIS) with a
score of 18 on the GEFT and a score of 77 on the math ability test, field dependent students’ (FDS) with a score of 5 on GEFT and a score of 76 on the math ability test.

Table 1. The Problem Solving Task of Mathematical Representation

| For example, ABCD is a isosceles trapezoid. AD and BC are extended so that it intersects at point E. Length $\overline{AB} = 30$cm, $\overline{CD} = 18$ cm and trapezoid height 8cm. If F and G respectively midpoints of $\overline{AD}$ and $\overline{BC}$. Length $\overline{FG} = 24$cm, find the area of the triangle EFG! |

3.2. Procedure
The design of this study is research with a qualitative approach. This study aims to find out how the student’s mathematical representation in solving geometry problems. The description of the phenomenon described in this study is a mathematical representation of junior high school students in solving geometry problems in terms of cognitive style. Moleong explains that qualitative research is a research procedure that produces descriptive data in the form of written or spoken words from people and observable behavior [18].

Data collection procedure begins with giving a GEFT test to students, then students are given a math ability test and then the research subject is determined. Subjects were given the task of solving problems to obtain mathematical representation. After that, researchers conducted interviews based on the results of student work to explore mathematical representations. The data validation process is carried out using triangulation. Data analysis refers to aspects of mathematical representation through the steps of categorizing data, reducing, discussing, and making conclusions about mathematical representation.

4. Result

4.1. Mathematical Representation of Field Independent Student’s (FIS)
- Understanding the Problem

![Figure 1. Mathematical Representation of FIS in Understanding the Problem](image_url)

In fig.1 above, FIS present information that is known in the form of sentences in which there are several symbols. FIS write information on the sides of trapezoid with symbols of line segments such as $\overline{AD}$, $\overline{AB}$, $\overline{CD}$ but not consistently on the side $\overline{BC}$ are not using symbols segments like the others. FIS write that the sides of the trapezoid $\overline{AD}$ and $\overline{BC}$ intersect at points E, F and G midpoint of $\overline{AD}$ and $\overline{BC}$.
• Devising a Plan

![Figure 2. Mathematical Representation of FIS in the Devising a Plan](image)

In fig.2 above, FIS present ideas that help determine the relationship between what is known and asked by using mathematical representations in the form of visual trapezoid images that are complete according to the information obtained at the stage of understanding the problem. There is also a symbol on the trapezoid \(\n\) to represent the same length at the foot of the trapezoid, but there is no symbol that marks the parallel side of the trapezoid.

• Carrying Out the Plan

![Figure 3. Mathematical Representation of FIS in Carrying Out the Plan](image)

In fig.3 above, FIS calculating based on the mathematical model she has made, namely "\(L \text{ small triangle} = L \text{ large triangle} - L \text{ small trapezoid}\)". The purpose of the model is that the area of the small triangle (EFG) is equal to the area of the large triangle (EAB) minus the small trapezoid (FABG). In the calculation process student use the symbol \(t\) to express the height of a triangle, \(t + 4\) for height which is asked the extent. In the final results of calculations FIS write \(cm^2\) as a unit area of the EFG triangle.
• Looking Back

![Mathematical Representation of FIS in the Looking Back](image)

Figure 4. Mathematical Representation of FIS in the Looking Back

In Fig.4 above, FIS re-examine the problem solving, expressing whether or not the problem solving steps in the symbolic form, which is to recalculate the results of the problem solving on the frosted sheet in which there is the $L_{EFG}$ symbol as the area of the EFG triangle being asked, the symbol $a$ as the base of the EFG triangle and $t + 4$ as the height of the EFG triangle.

4.2. Mathematical Representation of Field Dependent Student’s (FDS)

• Understanding the Problem

![Mathematical Representation of FDS in Understanding the Problem](image)

Figure 5. Mathematical Representation of FDS in Understanding the Problem

In fig.5 above, FDS present information that is known using a visual representation of drawing a trapezoid with information that is known in accordance with what is stated in the problem. Student thought that the 4cm in size also in terms of $\overline{AF}$, $\overline{FD}$, $\overline{BG}$, and $\overline{GC}$. FSD wrote were asked to determine the area of the EFG triangle. The triangle is obtained from the extension side $\overline{AD}$ and $\overline{BC}$ the intersecting in E.

• Devising a Plan

![Mathematical Representation of FDS in the Devising a Plan](image)

Figure 6. Mathematical Representation of FDS in the Devising a Plan
In fig.6 above, FDS present devising a plans in verbal form in the form of sentences which are sequential steps, FDS write down first determine the height of triangle is asked, then substitute the high value into the triangle area of the formula, in the plan students explain that the symbol $a$ is the base of a triangle is $FG$ 24cm.

- **Carrying Out the Plan**

![Image of a mathematical model]

**Figure 7. Mathematical Representation of Field Dependent Student’s in Carrying Out the Plan**

In fig. 7 above, FDS make a mathematical model "$L\Delta EFG = L\Delta EAB - L ABGF$" in the model there is a symbol $\Delta$ to represent a triangle, but not in the symbol for the trapezoid, FDS only writes $L ABFG$, and calculating. In the calculation student also use the symbol $t$ to express the height of the triangle and the trapezoid in the model. After obtaining the value of $t$, FDS draw an EFG triangle with arrows indicating that the height is $t + 4$. Student write that the area of the EFG triangle is $192cm^2$. FDS write in complete unit area, which is $cm^2$.

- **Looking Back**

![Image of a mathematical model]

**Figure 8. Mathematical Representation of FDS in the Looking Back**

In fig.8 above, FDS re-examine the problem solving, revealing whether or not the problem solving step is to do a recalculation of the results of solving the problem in which there is a symbol not as a high value.

5. **Discussion**

In the stage of understanding the problem, mathematical representation of FIS express information that is known in the problem in verbal. FIS write elements that are known to the problem in accordance with those listed in the problem, in the sentence written by FIS consistently used the symbol of the line segment to express the sides of the trapezoid. Furthermore, FIS revealed what was asked in the verbal
form, FIS wrote questions asking to determine the area of the EFG triangle. Mathematical representation of FDS presents information that is known in the form of visual images in accordance with information that is understood in the problem, but in the picture there are no parallel symbols and in the FDS pictures’ misplaced a size of side. Furthermore, FDS express information that is asked for problems in the form of verbal. FDS present that in the problem asked to determine the area of the EFG triangle.

At the stage of devising a plan, the mathematical representation of FIS presents ideas that help determine the relationship between what is known and asked in the visual form in the form of a trapezoid image that matches the information that has been obtained at the stage of understanding the problem. Furthermore, FIS presented a devising a plan in the form of verbal words, the FIS said that the area of the triangle in question could be obtained from the process of subtracting a large triangle with a small trapezoid in accordance with the drawing he had made beforehand. Mathematical representation of FDS presents ideas that help determine the relationship between the known and asked in verbal form in the form of written sentences that are steps to solve the problem given, in the sentence there are symbols \( a \) as the base and \( t \) axle height on the triangle asked.

At the stage of carrying out the plan, the mathematical representation of FIS makes a mathematical model in accordance with the information obtained at the stage of understanding the problem and a plan, then FIS make calculations, and consistent in using the symbol \( t \) as the height of the triangle and trapezoid. Mathematical representation of FDS make mathematical models in accordance with information on problems in symbolic form. Furthermore, FDS express interpretations of the results of the solutions obtained in the form of visual images in the form of triangles that are asked and make conclusions on the results of calculations in the form of written sentences. At the stage of looking back, mathematical representation of FIS re-examining problem solving in symbolic form, FIS re-calculate the opaque paper in accordance with the mathematical model they have made. Mathematical representation of FDS re-examining in symbolic forms.

There are some differences in representation between field independent and field dependent students which are in line with the opinion of Witkin which states that differences in cognitive styles affect students in solving problems, namely influencing students’ strategies in receiving, thinking problem solving, and learning [16]. Complexities of representation as a cognitive and social process and of how it is inextricably linked with knowledge people have the situation being presented [8]. Representation as cognitive activities associated with the knowledge of the students, then the style of the students' cognitive Affect representations to problem solving.

6. Conclusion
There are several differences in the mathematical representation of students based on the cognitive style they have. Mathematical representation of FIS in solving geometry problems is understanding information and what is asked in verbal form in writing and verbally, while mathematical representation of FDS in expressing their ideas to solve geometry problems by understanding information and being asked in visual form in the form of geometry drawings and verbal and verbal forms. FIS devising a plan in a visual form in the form of a geometry drawing then present the plan in verbal form, while FDS devising a a plan in visual form in the form of a picture then present a plan in verbal form verbally. FIS carrying out plans in symbolic form by doing calculations in accordance with the mathematical model, while FDS carrying out plans in symbolic form by doing calculations in accordance with the mathematical model. FIS re-examine in symbolic form by recalculating, while FDS re-examine in symbolic form in the form of recalculation and writing conclusions.

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