Searching for the Scalar Glueball

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Abstract. Existence of gluonic resonances is among the early expectations of QCD. Today, QCD calculations predict the lightest glueball to be a scalar state with mass within a range of about 900-1700 MeV but there is no consensus about its experimental evidence. In a re-analysis of the phase shifts for $\pi\pi$ scattering up to 1800 MeV where such states should show up we find the broad resonance $f_0(600)/\sigma$ contributing to the full mass range and the narrow $f_0(980)$ and $f_0(1500)$ but no evidence for $f_0(1370)$. Phenomenological arguments for the broad state to be a glueball are recalled. It is argued that the large radiative width of $f_0(600)/\sigma$ reported recently is not in contradiction to this hypothesis but is mainly due to $\pi\pi$-rescattering. The small “direct” radiative component is consistent with QCD sum rule predictions for the light glueball.

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EXPECTATIONS FOR THE LIGHTEST GLUEBALL IN QCD

Quantitative results on glueballs are available today from 1. Lattice QCD: The existence of glueballs within the purely gluonic theory is established and the lightest state is found in the scalar sector with mass around 1.7 GeV. In full QCD both glue and $q\bar{q}$ states couple to the flavour singlet $0^{++}$ states and first “unquenched” results for the lightest gluonic state point towards a lower mass of around 1 GeV [1] (recent review [2]). Further studies concerning the dependence on lattice spacing and the quark mass appear important.

2. QCD sum rules: Results on the scalar glueball and various decays are obtained in [3]. The lightest gluonic state is found in the mass range (950-1100) MeV with a decay width of $\sim$1000 MeV into $\pi\pi$ and the width into $\gamma\gamma$ of (0.2-0.6) keV. Other analyses find similar or slightly higher masses (around 1250 MeV) for the lightest glueball [4].

THE SCALAR MESON SPECTRUM

In the search for glueballs one attempts to group the experimentally observed scalar mesons into flavour multiplets (either $q\bar{q}$ or tetraquarks) and to identify supernumerous states. The existence of such states could be a hint for glueballs either pure or mixed with $q\bar{q}$ isoscalars. A signature of gluonic states is its abundance in so-called “gluon rich” processes and their suppression in $\gamma\gamma$ reactions.

In a popular scheme the light scalars $\sigma(600)$, $\kappa(800)$, $f_0(980)$, $a_0(980)$ are put together into one multiplet, either as $q\bar{q}$ or 4$q$ nonet (see, for example Refs. [5], [6]). Then, a $q\bar{q}$ multiplet can be formed with the heavier $a_0(1450)$, $K_0^*(1430)$; with nearby
masses three isoscalars can be found at 1370, 1500 and 1710 MeV and this suggests their interpretation as mixtures of the two $q\bar{q}$ nonet members and one glueball (for an early reference, see [7]).

A potential problem in this scheme for the glueball is the very existence of $f_0(1370)$, otherwise there is no supernumerous state in this mass range. Some problems with this state will be discussed below, see also the review [8]. The low mass multiplet depends on the existence of $\kappa$ which we consider as not beyond any doubt: the observed related phase motion in $K\pi$ scattering [9] is less pronounced as the corresponding one of “$\sigma$” in $\pi\pi$ scattering, see Fig. 1, discussed below.

In the scheme we prefer [10] the lightest $q\bar{q}$ nonet contains $f_0(980)$, $f_0(1500)$ together with $a_0(1450)$, $K'_0(1430)$. The supernumerous state $\sigma/f_0(600)$, called previously $f_0(400−1200)$, corresponds to a very broad object which extends from low energy up to about 2 GeV and is interpreted as largely gluonic. No separate $f_0(1370)$ is introduced, nor $\kappa(800)$. Our classification is consistent with various findings on production and decay processes including $D, D_s, B$ and $J/\psi$ decays [10, 11, 12].

Related schemes are the Bonn model [13] with a similar mixing scheme for the isoscalars and the K-matrix model [14] which finds a similar classification (but with $f_0(1370)$ included) and a broad glueball, but centered at the higher masses around 1500 MeV.

**PHASE SHIFT RESULTS ON $\pi\pi \to \pi\pi$ UP TO 1800 MEV**

Our focus here is on the significance of $f_0(1370)$ and the appearance of $\sigma/f_0(600)$ which was $f_0(400−1200)$ before and is sometimes treated as “background” where we describe results from an ongoing analysis (see also [15]).

Information on $\pi\pi$ scattering can be obtained from production experiments like $\pi p \to \pi\pi n$ by isolating the contribution of the one-pion-exchange process. In an unpolarised target experiment these amplitudes can be extracted by using dynamical assumptions, such as “spin and phase coherence”, which have been tested by experiments with polarised target. At the level of the process $\pi\pi \to \pi\pi$ in different charge states one measures the distribution in scattering angle, $z = \cos \theta^*$, or their moments $\langle Y^L_M \rangle$, in a sequence of mass intervals. The $\pi\pi$ partial wave amplitudes $S, P, D, F, \ldots$ can be obtained in each bin from the measured moments up to the overall phase and a discrete ambiguity (characterised by the “Barrelet Zeros”). The overall phase can be fixed by fitting a Breit Wigner amplitude for the leading resonances $\rho, f_2(1270)$ and $\rho_3(1690)$ to the respective experimental moments.

Energy-independent phase shift analyses of this type for $\pi^+\pi^-$ scattering have been performed by the CERN-Munich group: an analysis guided by a global resonance fit (CM-I [16]) and an analysis to reveal all ambiguities by CM-II [18] and by Estabrooks and Martin [17]; 4 different solutions have been found above 1 GeV in mass. Up to 1400 MeV a unique solution has been established [19] using results from polarised target and unitarity. Two solutions remain above 1400 MeV, classified according to Barrelet zeros in [18] as (−−−) and (−+−) corresponding to sols. A,C in [17].

The remaining ambiguity has been resolved recently in [15] by comparison with the isoscalar S wave $S_0$ reconstructed from the $\pi^+\pi^- \to \pi^0\pi^0$ data (GAMS collaboration
FIGURE 1. Argand diagram for the corrected partial wave $S_0$ (CM-I/II) in comparison with the resonance fit in Eq. (1); right panel: broad component $f_0(600)/\sigma$ from the fit.

and results on $I = 2$ scattering. The $S_0$ wave obtained shows a qualitatively similar behaviour to the $S_0$ solution above. In particular, both solutions show an $f_0(1500)$ resonance circle in the complex plane (Argand diagram) above a slowly moving circular background amplitude. The $S_0(-++-)$ amplitude is shown in Fig. 1.

The resulting amplitude $S_0(-++-)$ is shown in Fig. 1 using the CM-II data after correction for the more recent $I = 2$ amplitudes. The curves refer to a fit of the data (CM-II for $M_{\pi\pi} > 1$ GeV, CM-I for $M_{\pi\pi} < 1$ GeV) to a unitary S-matrix in the space of 3 reaction channels ($\pi\pi$, $K\bar{K}$, $4\pi$) as product of individual S-matrices for resonances $S_R = 1 + 2iT_R$

\begin{align}
S &= S_{f_0(980)}S_{f_0(1500)}S_{\text{broad}} \quad (1) \\
T_R &= \rho_1^2 \mathcal{I} (g_1g_2) \rho_2 \frac{1}{[M_0^2 - M_{\pi\pi}^2 - i(\rho_1 g_1^2 + \rho_2 g_2^2 + \rho_3 g_3^2)]^{-1}} \quad (2)
\end{align}

where $\rho_i = 2k_i/\sqrt{s}$. This simplified amplitude for the unitary S matrix is a generalisation of the so-called Dalitz-Tuan form: it is well suited to locally describe the superposition of a smooth background with a narrow resonance. The fit including 3 resonances gives a reasonable description of the measured inelasticities $\eta$ and phase shifts $\delta$. For $f_0(1500)$ the fit parameters $M_0 = 1510$ MeV, $\Gamma_{\text{tot}} = 88$ MeV, $B(f_0 \rightarrow \pi\pi) = 38\%$ are obtained, comparable to the PDG results. This is remarkable as the latter values come from the $\pi\pi$ fraction of all observed decay rates but our value from elastic scattering.

The broad object is also described by a Breit-Wigner form (2) with parameters [15]

\[ M_{\text{BW}} \sim 1100 \text{ MeV}, \quad \Gamma_{\text{BW}} \sim 1450 \text{ MeV}. \quad (3) \]

The elastic width is about 85% whereas the GAMS data suggest rather a smaller value around 70%. This should be considered as systematic uncertainty. More details will be given elsewhere. This amplitude is shown in Fig. 1 right panel. It completes about 3/4 of the full resonance circle. The parameter $M_{\text{BW}}$ refers to the mass where the amplitude is purely imaginary. It may be different from the pole mass which is referred to as resonance mass. The determination of this mass requires an analytic continuation of the propagator into the deep complex region and in many analyses it appears to be considerably smaller.
The data in Fig. 1 suggest the existence of a broad state in $\pi\pi$ scattering, centered around 1000 MeV along the physical region and what is called $f_0(600)$ or $\sigma$ refers to the same state. A large difference between both masses is revealed in a simple analytical model (one channel, one pole) [22] in extension of the model [23] where the propagator real part is calculated from a dispersion relation. The model is fit to the $\pi\pi$ data at lower energies (< 700 MeV), then the “on-shell” mass $M^{os}$ where the amplitude is purely imaginary (phase shift at $90^\circ$), the pole mass $M^{pole}$ and the corresponding widths are obtained as

$$M^{os} = 920 \text{ MeV}, \Gamma^{os} = 1020 \text{ MeV}; \quad M^{pole} = 422 \text{ MeV}, \Gamma^{pole} = 580 \text{ MeV}. \quad (4)$$

The on shell results from this low energy fit resemble the Breit Wigner parameters in [3]. The very different pole and on-shell masses are related by the analytic extrapolation and refer to the same state. On the other hand, fits with a pole mass near 1000 MeV are possible as well as shown at this conference [24].

We note that the data presented in Fig. 1 do not give any indication of the existence of $f_0(1370)$ which is expected to show up as a second circle in the Argand diagram besides $f_0(1500)$ with respective signals in $\eta_0^0$ and $\delta_0^0$. In fact, none of the energy-independent bin by bin phase shift analyses of the CM or CKM data [16, 17, 18, 19] nor of the GAMS data [21, 15] gave such an indication. From our analysis we exclude an additional state with branching ratio $B(f_0(1370) \rightarrow \pi\pi) > 0.1$ near 1370 MeV (this would correspond to a circle of diameter 0.1). It should be noted that the “experimental” bin-by-bin $\eta_0^0$ and $\delta_0^0$ values are obtained from the original moments in very good fits ($\chi^2$/data point $\sim 0.1$).

The existence of $f_0(1370)$ is still controversially discussed and very different views and numbers are recommended. At this conference two other analyses [25, 26] show this. In the analysis [25] CM-I moments have been fitted directly by a model amplitude with resonances in all relevant partial waves ($\chi^2$/data point $\gtrsim 2$). The resulting amplitude $S_0$ includes $f_0(1370)$ which appears with extra circle of diameter 0.25 in the Argand diagram near the mass of 1300 MeV. Such an effect is hard to reconcile with any of the energy independent phase shift analyses. Another presentation [26] based on phase shift results does not produce any extra circle in the 1300 MeV region. Above 1400 MeV a set of phase shifts from CKM is used, different from ours, which does not reveal any extra circle at all, neither for $f_0(1500)$ nor for $f_0(1370)$ in the mass range below 1800 MeV. The $f_0(1370)$ phenomenon is clearly of a different nature as in Ref. [25]. The ambiguity in the phase shifts at the higher energies in both analyses [15] and [26] needs further study, in particular, the consistency with the GAMS data [21].

GLUEBALL INTERPRETATION OF THE BROAD OBJECT $f_0(600)$

Arguments in favour of glueball have been put forward in [10, 11, 12].

1. This state is produced in almost all “gluon rich” processes, including central production $pp \rightarrow p(\pi\pi)p, p\bar{p} \rightarrow 3\pi, J/\psi \rightarrow \gamma\pi\pi(?)$, $\gamma K\bar{K}, \gamma 4\pi$, $\psi' \rightarrow \psi\pi\pi, \Upsilon', \Upsilon' \rightarrow \Upsilon\pi\pi$ and finally $B \rightarrow \pi\pi, B \rightarrow K\bar{K}K$ related to $b \rightarrow s g$. The high mass tail above 1 GeV is
seen as “background” in $J/\psi \rightarrow \gamma K\bar{K}$ and in $B$ decay channels where it leads to striking interference phenomena with $f_0(1500)$ [12]. Only the channel $J/\psi \rightarrow \gamma \pi\pi$ is problematic.

2. Within our classification scheme [10] without $f_0(1370)$ the state $f_0(600)$ is supernumerous.

3. The mass and large width is in agreement with the QCD sum rule results (see below) and the mass also with the first results from unquenched lattice QCD.

4. Suppression in $\gamma\gamma$ production.

Recently, the radiative width $\Gamma(f_0(600) \rightarrow \gamma\gamma) = (4.1 \pm 0.3)$ keV has been determined by Pennington [27] from the process $\gamma\gamma \rightarrow \pi\pi$. As this number is larger than expected for glueballs, he concluded this state “unlikely to be gluonic”. A resolution of this conflict has been suggested recently [22] as follows.

The physical processes in $\gamma\gamma \rightarrow \pi\pi$ at low energies are different from the ones at high energies. At low energies, the photons couple to the charged pions and the Born term with one pion exchange dominates in $\gamma\gamma \rightarrow \pi^+\pi^-$, in addition there is a contribution from $\pi^+\pi^-$ rescattering. Explicit models with $\pi\pi$ scattering as input and with $\sigma/f_0(600)$ pole, can explain the low energy processes [23], also calculations in $\chi PT$ with non-resonant $\pi\pi$ scattering at low energies. In this case of the rescattering contribution, a resonance decaying into $\pi\pi$ would also decay into $\gamma\gamma$ irrespective of the constituent nature of the state.

At high energies, the photons do resolve the constituents of the produced resonances: for example, the radiative widths of tensor mesons $f_2, f_2', a_2$ in the region 1200-1500 MeV follow the expectations from a $q\bar{q}$ state; the rescattering contribution for $f_2 \rightarrow \gamma\gamma$ is limited to be lower than 10-20% [23].

The model by Mennessier [23] satisfies the constraints from unitarity (Watson theorem and generalisations) and analyticity. The dispersion relations create a polynomial ambiguity and this allows introducing an arbitrary “direct” coupling of resonances into two photons besides the coupling of photons to charged pions and the coupling of hadrons among themselves within a field theoretic approach. The case of tensor mesons (and others) suggest attributing the direct terms to parton annihilation processes.

In our application to the $\sigma/f_0(600)$ resonance we restrict the analysis to the $\pi\pi$ channel only in a mass range $m_{\pi\pi} < 700$ MeV. Once the parameters of $\pi\pi \rightarrow \pi\pi$ scattering are determined the $\gamma\gamma \rightarrow \pi\pi$ processes in both charge states are calculable as superposition of Born term, rescattering and direct contribution with the direct coupling as only free parameter. As a result we obtain [22]

$$\Gamma^\text{dir}_{\sigma \rightarrow \gamma\gamma} \simeq (0.13 \pm 0.05) \text{ keV} , \quad \Gamma^\text{resc}_{\sigma \rightarrow \gamma\gamma} \simeq (2.7 \pm 0.4) \text{ keV} ;$$

this corresponds to the total radiative width of $\Gamma^\text{tot}_{\sigma \rightarrow \gamma\gamma} \simeq (3.9 \pm 0.6)$ keV which is compatible with the range $1.2 \sim 4.1$ keV obtained in other analyses. The direct radiative width in Eq. (5) is then to be compared with the predictions for different intrinsic structures of this resonance.

In comparing with the predictions from the QCD sum rules it is more appropriate to consider on shell “physical” quantities determined from the physical region along the real axis. As the resonance mass is above the mass region fitted these numbers should be considered as crude approximation. The predictions for the lightest gluonium state are
quoted in [22] as
\[ M \simeq (950 \sim 1100) \text{ MeV}, \quad \Gamma \simeq 1050 \text{ MeV}, \quad \Gamma_{\gamma\gamma} \simeq (0.2 \sim 0.6) \text{ keV}. \]  

There is a remarkable agreement with the results from our analysis for the on shell quantities in (4) and with the direct decay width \( \Gamma_{\gamma\gamma}^{\text{dir}} = (1.0 \pm 0.4) \text{ keV} \) (see also Eq. (5)). On the other hand, a large \( q\bar{q} \) or \( 4q \) component is found disfavoured.

In conclusion, the broad state \( f_0(600) / \sigma \) observed in \( \pi\pi \) scattering with maximal amplitude around 1 GeV and in other channels is a good glueball candidate. The large width into two photons is not in contradiction with this view if the large contribution from \( \pi\pi \) rescattering is taken into account. The observed parameters are in remarkable agreement with QCD sum rule expectations.

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