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Static Modeling of Braided Pneumatic Muscle Actuator: An Amended Force Model

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Abstract.
The present study reports an amended static force model for a pneumatic muscle actuator (PMA) used in different aerodynamic and fluid power system applications. The PMA is a fluid actuator, made of a polymeric bladder enclosed in a braided mesh sleeve. A physics-based static model is developed to predict the deformation response of the actuator for different applied pressure. The significant losses, like braid-to-braid friction, non-cylindrical ends, and bladder hyperelasticity effect, have been considered to enhance the model’s practical feasibility. However, a combined effect of all these losses in the PMA was ignored in the literature. The findings of the derived model agree well with existing experimental results.

INTRODUCTION

In general, the pneumatic muscle actuator (PMA), also known as McKibben actuator, is an axially-contracting actuator initially patented by Gaylord [1]. The significant advantages of PMA over conventional actuators are high power to weight ratio, compliance, high operating range, low cost, and high technology readiness level. These advantages have broadened its application from the medical field to the aerospace industry [2, 3, 4]. PMA can be applied at the trailing edges of an aerofoil to delay the flow separation [3] as shown in 1 (a). The primary components of PMA include a cylindrical rubber (elastomer) bladder, a braided mesh sheath, and rigid metal end-caps. Consequently, as the internal pressure of the actuator increases, the bladder starts expanding radially. This radial expansion is restricted via a braided sleeve, which generates a contractile motion in the longitudinal direction. In essence, PMA converts fluidic power into mechanical forces.

In literature, the output of a PMA is measured in terms of blocked force and free contraction [4, 5, 6]. The blocked force is defined as the maximum amount of force exerted by PMA at its end for an applied pressure keeping the actuator’s length fixed. Similarly, the free contraction determines the full contraction length of the PMA for a given pressure until the force reduces to null [6]. Moreover, the experimental observations suggest that the hysteresis losses are more prominent in static than the actuator’s dynamic characterization [5]. The friction between the braids, rigid end fixtures, and material nonlinearity majorly contribute to the losses in PMA. In Chou and Hannaford [5], the loss terms were added to the force model directly as constant. Next, Davis and Caldwell [7] modified the frictional loss in PMA. They [7] described the braid-to-braid friction in terms of braid geometrical parameters and observed that both the output force and the contraction ratio increase by 16 % and 7 %, respectively, by halving the number of strands used to create the PMA. However, the material model was linear, as in the earlier works [5, 8]. Tsagarakis and Caldwell [8] included the loss due to the non-cylindrical ends. They [8] calculated the surface area of the non-cylindrical ends and found an improvement compared to [5]. Later, Doumit et al. [9] considered the losses in PMA due to rigid end fixtures. However, those models [8, 9] also used a linear material model to describe the final output force of PMA. Later, Wang et al. [10] modeled the material nonlinearity but did not consider other losses.

In this work, we combined all those loss factors and developed a physics-based static force model that can predict the actuator’s behavior accurately. The amended static force model connects the output actuation force with the contraction ratio for an applied pressure. The model includes a combined effect of the actuation losses due to braid-to-braid friction, non-cylindrical end effect, and elastic energy storage by the bladder. In contrast, other researchers [7, 9, 10] either considered linear material or braid friction. Our model predicts the combined effect of all those loss factors.

The primary aim of this study is to model the static behavior of PMA, including the losses during its operation. The proposed model includes the collective significant loss factors during actuation, namely, the braid-to-braid friction, end-effects consideration, and bladder hyperelasticity effect. The developed physics-based model presents the output...
force with the corresponding applied pressure. Further, the effect of end fixtures can be predicted from the present model.

**MATHEMATICAL MODELING**

In the current section, an amended force model for a pneumatic muscle actuator (PMA) is developed analytically. The model is used to relate the actuation of PMA with an applied input pressure considering the losses in the actuator.

The PMA consists of an elastic thick-walled circular cylindrical bladder with the nylon fibers-based braided mesh and two end-cap fittings, as shown in Figure 1. The following assumptions are made to develop the model:

1. The bladder material is considered incompressible.
2. The threads of the braided sleeve are considered inextensible.
3. Only braid-to-braid frictional losses are considered. However, the friction between sleeve and bladder is ignored due to non-existent of the relative motion between them.
4. The deformation is considered axisymmetric as the braided sleeve is sufficiently dense.

![Figure 1: Schematic representation of a pneumatic muscle actuator (PMA) (a) used to control flow in an aerofoil, (b) the undeformed configuration, (c) geometrical relations between the actuator dimensions and braid angle in the undeformed configuration, (d) the deformed configuration.](image)

**Elastic deformation**

In undeformed configuration $\Omega_0$, the coordinates of any point at the inner cylindrical bladder of PMA may be chosen as $(R, \Theta, Z)$. With an application of pressure, the point gets a new position at $(r, \theta, z)$ in deformed configuration $\Omega$. The occupied volume of the inner bladder in undeformed configuration $\Omega_0$ is defined as

$$A \leq R \leq B, \quad 0 \leq \Theta \leq 2\pi, \quad 0 \leq Z \leq L,$$

wherein $A$, $B$ and $L$ denote inner, outer radii and length of the cylindrical bladder, respectively. After inflation, the occupied volume of the inner bladder is defined as

$$a \leq r \leq b, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq z \leq l,$$
wherein $a$, $b$ and $l$ denote inner, outer radii and length of the bladder, respectively. For an axisymmetric deformation, the deformation map is expressed as

$$r = r(R), \quad \theta = \Theta, \quad z = \lambda Z,$$

wherein $\lambda = \frac{l}{L}$ represents the axial-stretch of the bladder. An inflation pressure $P$ is applied at inner radius of the cylindrical bladder. The deformation gradient tensor $F$ for the above deformation mapping (3) may be obtained as

$$F = \frac{dr}{dR} e_{rr} + \frac{r}{R} e_{\theta \theta} + \lambda e_{zz},$$

wherein $e_{rr}, e_{\theta \theta}, e_{zz}$ are unit directional vector along $r, \theta, z$ respectively. The corresponding principal stretches is defined as

$$\lambda_r = \frac{dr}{dR}, \quad \lambda_\theta = \frac{r}{R}, \quad \lambda_z = \lambda.$$

The invariants of the left Cauchy-Green deformation tensor $B = FF^T$ for an incompressible isotropic elastic deformation of PMA is given by [11]

$$I_1 = \text{tr}B = \left( \frac{dr}{dR} \right)^2 + \left( \frac{r}{R} \right)^2 + \lambda^2,$n

$$I_2 = \frac{1}{2} \left[ (\text{tr}B)^2 - \text{tr}B^2 \right] = \left( \frac{dR}{dr} \right)^2 + \left( \frac{R}{r} \right)^2 + \frac{1}{\lambda^2},$$

$$I_3 = \det B = \left( \frac{dr}{dR} \right)^2 \left( \frac{r}{R} \right)^2 \lambda^2 = 1.$$

On integrating $\frac{dr}{dR} \frac{r}{R} \lambda = 1$, we have

$$R^2 = \lambda \left( r^2 - c \right),$$

wherein $c$ is an integration constant that can be found from the boundary conditions.

**Deformed cylinder radii**

The initial braid angle ($\alpha_{\text{min}}$) is related to the geometrical parameters of the cylinders as shown in Figure 1(c). The length of the inextensible braid ($w$) remains constant during inflation due to relatively high stiffness. The relation is given by

$$w = \sqrt{L^2 + (2\pi Bn)^2} = \sqrt{l^2 + (2\pi bn)^2},$$

wherein $n$ denotes the number of turns of a single thread about the PMA axis. This parameter connects the braid angle at the undeformed configuration ($\alpha_{\text{min}}$) as

$$n = \frac{L \tan \alpha_{\text{min}}}{2\pi B}.$$

By substituting $\lambda = \frac{l}{L}$ in the above relation (8), we may obtain the deformed outer radius $b$ given as

$$b = \sqrt{\frac{L^2(1 - \lambda^2) + 4\pi^2 n^2 B^2}{4\pi^2 n^2}}.$$

On using equation (7) and $c = b^2 - \frac{B^2}{\lambda^2} = a^2 - \frac{A^2}{\lambda^2}$, we get the deformed inner radius $a$ given by

$$a = \sqrt{b^2 - \frac{B^2 - A^2}{\lambda}}.$$
Equilibrium equations and stress solutions

Following the theory of hyperelasticity [11, 12], the Cauchy stress tensor $S$ for an incompressible isotropic hyperelastic material in terms of strain energy-density function $W(I_1, I_2)$ is defined as

$$ S = -pI + 2\frac{\partial W}{\partial I_1} B + 2\frac{\partial W}{\partial I_2} (I_1 B - B^2), \tag{12} $$

wherein, $p$ is the Lagrange multiplier due to incompressibility constraint. Herein, we consider the Mooney-Rivlin type of strain energy-density function in terms of the invariants (6) given by

$$ W = C_1(I_1 - 3) + C_2(I_2 - 3), \tag{13} $$

wherein, $C_1$ and $C_2$ are the material constant parameters. Now, the associated stress components in the principal directions may be obtained from the above relations (6), (12) and (13) as

$$ S_r = -p + 2C_1 \frac{R^2}{\lambda^2 r^2} + 2C_2 \frac{R^2}{\lambda^2 r^2} \left( \frac{r^2}{R^2} + \lambda^2 \right), $$

$$ S_\theta = -p + 2C_1 \frac{r^2}{R^2} + 2C_2 \frac{r^2}{R^2} \left( \frac{R^2}{\lambda^2 r^2} + \lambda^2 \right), \tag{14} $$

$$ S_z = -p + 2C_1 \lambda^2 + 2C_2 \lambda^2 \left( \frac{R^2}{\lambda^2 r^2} + \frac{r^2}{R^2} \right). $$

The unknown material parameters $C_1$ and $C_2$ introduced in (14) may be obtained from the experiments [10]. The equilibrium equation in radial direction in absence of body forces is expressed as

$$ \frac{dS_r}{dr} - \frac{S_\theta - S_r}{r} = 0. \tag{15} $$

The above equilibrium equation (15) is solved under the following boundary condition, given by

$$ S_r(r = a) = -P. \tag{16} $$

By applying the above boundary condition (16) in the equilibrium equation (15) with (7) and (14), we have

$$ S_r = -P + C_1 \left[ \frac{1}{\lambda} \ln \left( \frac{r^2-c}{a^2-c} \right) - \frac{1}{2\lambda} \left( r - a + c \left( \frac{1}{a^2} - \frac{1}{r^2} \right) \right) \right] + C_2 \left[ \lambda \ln \left( \frac{r^2-c}{a^2-c} \right) - \lambda^2 \ln \left( \frac{r}{a} \right) \right]. \tag{17} $$

Similarly, from the relations (7) and (14), we also have

$$ S_\theta = -P + \left( \frac{C_1}{\lambda} + C_2 \lambda \right) \left[ \ln \left( \frac{r^2-c}{a^2-c} \right) - 2 \ln \frac{r}{a} + c \left( \frac{1}{a^2} - \frac{1}{r^2} \right) \right] - \left( \frac{C_1}{\lambda} + C_2 \lambda \right) \frac{2c}{r^2} \left( \frac{2r^2-c}{r^2-c} \right), $$

$$ S_z = -P + \left( \frac{C_1}{\lambda} + C_2 \lambda \right) \left[ \ln \left( \frac{r^2-c}{a^2-c} \right) - 2 \ln \frac{r}{a} + c \left( \frac{1}{a^2} - \frac{1}{r^2} \right) \right] - 2C_1 \left[ \frac{1}{\lambda^2} - \frac{\lambda^2 r^2}{r^2-c} \right] + 2C_2 \left[ \frac{\lambda^2 - r^2}{\lambda r^2} \right]. \tag{18} $$

Force solutions

The output force $F_{out}$ of the actuator is determined by equating forces in circumferential and axial directions. Herein, we first obtain the corresponding circumferential and axial cylindrical force $F_\theta$ and $F_z$, respectively. The resultant force may be expressed as

$$ F_\theta = l \int_a^b S_\theta dr, \quad F_z = \int_0^{2\pi} \int_a^b S_z r dr d\theta. \tag{19} $$
The friction force model

To estimate the braid-to-braid friction force term, we adopt the parallel analogy proposed by Davis and Caldwell [7] as shown in Figure 2. In line with that, the friction force in PMA with the coefficient of friction $\mu$ is given by

$$ f = \mu Q_c P, $$

where $Q_c$ is the total contact area between the braids, and $P$ is the applied pressure. The total contact area between the braids can be expressed in terms of geometrical parameters of the strands as shown in Figure 2(a). Herein, $\alpha, G, d_i$ represents the angle between two overlapping braids in the deformed configuration, distance between two consecutive nodes, and inter-braid distance, respectively. The single crossover of two overlapping braids can be assumed as rhombus as shown in Figure 2(b). The geometrical parameters $X, x, Y$ as shown in Figure 2(b) is given by

$$ X = \frac{w_b}{2}, \quad Y(\alpha) = \frac{w_b}{2\cos \alpha}, \quad x(\alpha) = \frac{Y}{\tan \alpha}, $$

where $w_b$ is the width of the single strand. Thus, the contact area of a single crossover braid ($Q_{one}$) is expressed in terms of above parameters (21) as

$$ Q_{one} = 2xY = \frac{w_b^2}{\sin 2\alpha}. $$

To evaluate the total contact area ($Q_c$), we need to estimate the all the contact points between the overlapping strands ($N_c$). We here assume that the total number of contact points between the overlapping strands remains the same before and after actuation, i.e, using surface area relation $N_c Q_{one}(\alpha = \alpha_{\text{min}}) = 2\pi BL$ similar to [7]. The total number of contact points of the overlapping braids $N_c$ is estimated using equations (8), (9), and (22) as

$$ N_c = \frac{2w^2 \sin^2 \alpha_{\text{min}} \cos^2 \alpha_{\text{min}}}{n w_b^2}, $$

where $w$ is the non-extensible strand length. Thus, the total contact area ($Q_c$) is obtained as

$$ Q_c(\alpha) = Q_{one}(\alpha) N_c = \frac{w^2 \sin^2 \alpha_{\text{min}} \cos^2 \alpha_{\text{min}}}{n \sin \alpha \cos \alpha}. $$

![FIGURE 2: Geometrical description of (a) a part of unrolled braid sleeve, (b) one trapezoid of mesh and one single braid crossover point.](image)
An amended force model

To obtain the final output force $F_{\text{out}}$ in PMA with an applied pressure considering all the significant losses (braid friction, end-fixture effects, and bladder hyperelasticity) throughout the actuation of PMA, we equate all the forces in circumferential and axial directions as shown in Figure 3. For the equilibrium in circumferential direction in the cylindrical portion we have

$$F_\theta + nT \sin \alpha - R_t - P_{al} = 0,$$

wherein $T$ denotes the tension in the braids during actuation in the cylindrical portion. And,

$$R_t = \left[ \int_{0}^{\pi} \frac{T}{2\pi a} \sin \alpha \cos \theta \, d\theta \right] \pi a$$

is the tension component at section R-R due to braiding. To include the end cap effects specifically, the net tension

$T$ in the braids is transferred to the end caps [9] through a spatial angle $\beta$ as shown in Figure 3. In addition, the area reduction through a tapered zone contributes the reduction in the forces as well as the braid tension. The reduced tension is $T' < T$ as well as the reduced forces are $F_z' = \int_{r_i}^{r_f} S_z r dr d\theta$ (wherein $r_i = a - \frac{2}{2} \tan \beta$ and $r_f = b - \frac{2}{2} \tan \beta$), and $F_z'' = \int_{r_i}^{r_f} A_z S_z r dr d\theta$ at the cross-sections $X_2$ and $X_3$, respectively shown in Figure 3. For the equilibrium in axial-direction at the cross-section $X_2$, we have

$$T' \cos \alpha = P \pi a^2 + T \cos \alpha - F_z + F_z' - P A_{X_2},$$

wherein $A_{X_2}$ is calculated at $z = \frac{l_2}{2}$ for the simplification, and $l_i$ is the length of the tapered section as shown in Figure 1 (c). However, one may calculate $A_{X_2}$ at any cross-section in the tapered zone. Further, the output force $F_{\text{out}}$ including

![Figure 3: Free body diagram of PMA.](image-url)
the friction force $f$ through net force balance at section $X_3$ is expressed as

$$F_{out} = F''_z - P\pi A^2 - T' \cos \alpha - f \quad \text{(while contraction)},$$

$$F_{out} = F''_z - P\pi A^2 - T' \cos \alpha + f \quad \text{(while expansion}).$$

(27)

The above amended static force model (27) of PMA with the consideration of significant loss factors (braid-to-braid friction, end-effects consideration and hyperelasticity of the bladder) collectively, is obtained followed by the continuum mechanics-based approach.

RESULT AND DISCUSSION

In the current section, the response of PMA is analyzed utilizing the derived model (27), corresponding to various input parameters. The amended force model is validated with the existing experimental data [10]. Furthermore, the model can be used to improve the output of the PMA.

Experimental validation

To validate the proposed model (27), we compare the same with Wang et al. [10] experimental data at two different 40 psi and 60 psi pressure levels, as shown in Figure 8. However, the model can be validated at any other input pressure. We here consider a PMA of the nominal length 128.5 mm, the outer diameter 15.9 mm, and the bladder thickness 1.59 mm, similar to Wang et al. [10] experimental data. The experimental output force and axial contraction were measured using an MTS model 312.12 (50 kip load frame with a 20 kip hydraulic actuator) in [10]. The comparison of our model with Wang et al. [10] for output force versus contraction ratio data is shown in Figure 8. The material constants $C_1 = 172$ kPa, $C_2 = 127$ kPa, $\mu = 0.04$, and $\beta = 10^9$ are used at two different pressure level 40 psi and 60 psi. The findings of the proposed force model (27) are in good agreement with Wang et al. [10] experimental data.

![Figure 4: Comparison of the static force model with the Wang et al. [10] experimental data (a) at 40 psi pressure, and (b) at 60 psi pressure.](image)

Key observations

The actuation force versus contraction ratio plot is drawn using the amended force model (27) for different applied pressure levels 40 psi, 60 psi, 80 psi, and 100 psi, as shown in Figure 4. We observe an increase in both the actuation
force and the contraction ratio, increasing the applied pressure. The model predicts a similar behavior to existing experimental results [6, 7, 10]. Herein, we have considered the actuator dimensions similar to [10], and spatial angle $\beta = 10^0$. Figure 5 describes the response of the actuator with and without consideration of loss factors at an applied pressure of 60 psi, where $\mu = 0.04$ and $\beta = 10^0$. We observe that the significant losses (braid-to-braid friction, end-effects consideration, and hyperelasticity of the bladder) in PMA reduce the output force and the contraction ratio. However, the model (27) depicts the reduction in the actuator’s output may not be significant for non-cylindrical ends than braid-to-braid friction. We also plot the force versus applied pressure results based on the proposed force model (27). Moreover, a similar linear trend is followed in line with the existing literature [6, 9] as shown in Figure 6.

Figure 7 illustrates the effect of spatial angle $\beta$ on the actuation force and contraction ratio of the actuator. As

![Figure 5: Effect of the variation in input pressure on amended force model (27) for PMA containing all the significant losses.](image1)

![Figure 6: Comparison of the static force model with and without consideration of the significant losses at same pressure.](image2)
expected, both the output force and contraction ratio decreases with an increase in the spatial angle. The change in spatial angle is connected with the end fixture size [9]. Thus, it can be recommended to use small end fixtures to improve the actuator force and contraction.

CONCLUSIONS

In the present paper, a continuum mechanics-based static force model for a pneumatic muscle actuator (PMA) is developed considering all the significant loss factors like braid-to-braid friction, non-cylindrical ends, and nonlinear bladder. The model is used to analyze the output force and contraction of the actuator for different applied pressures. The findings of the proposed amended static force model are in good agreement with existing Wang et al. [10] experimental results. As a result, we find that considering the braid-to-braid friction is more significant than non-
cylindrical ends. Moreover, the loss due to end fixtures can be reduced by reducing its size. In line with this work, the possible future work might include braid-to-bladder friction in the present model.

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