Mesoscopic mechanism of exchange interaction in magnetic multilayers

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Abstract

We discuss a mesoscopic mechanism of exchange interaction in ferromagnet-normal metal-ferromagnet multilayers. We show that in the case when the metal’s thickness is larger than the electron mean free path, the relative orientation of magnetizations in the ferromagnets is perpendicular. The exchange energy between ferromagnets decays with the metal thickness as a power law.

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Both the experiment and the theory of ferromagnet-normal metal-ferromagnet multilayers have attracted a lot of attention [1−6]. An example of such a structure consisting of two ferromagnetic films separated by a nonferromagnetic metallic film is shown in Fig.1. In the case when the metal thickness $L$ is much smaller than the electron scattering mean free path $l$ the sign of the exchange interaction energy between the ferromagnet’s magnetizations oscillates as a functions of $L$ with a period of order of the Fermi wave length. As a result the magnetic structure of the system oscillates between ferromagnetic and antiferromagnetic orientations of the ferromagnet’s magnetizations [1−6]. The explanation of this phenomenon is based on the fact that the interlayer exchange energy is due to Ruderman-Kittel interaction between electron spins in different ferromagnets.

In the case of low temperatures and at $|r − r'| \gg l$ the exchange energy between two localized spins $< J(r, r') >$ averaged over the scattering potential configurations decays exponentially with $|r − r'|$ [7]. Here $r$ and $r'$ are coordinates of spins and brakets <> stand for averaging over realizations of the scattering potential in the metal and the ferromagnets. Recent experiments on ferromagnet-metal-ferromagnet multilayers [8] imply, however, that the exchange energy between the ferromagnets does not decay exponentially at $L \gg l$ and that the equilibrium relative orientation of the ferromagnet’s magnetizations is perpendicular independently of $L$. Phenomenologically, this situation can be described by an effective energy per unit area

$$E = -J^0(m_1^0 \cdot m_2^0) + B[(m_1^0 \cdot m_2^0)^2 - 1]$$

in the case $2B \gg |J^0|$. Here $m_1^0$ and $m_2^0$ are, averaged over the film’s volumes, unit vectors parallel to magnetizations of the ferromagnetic films. Indices 1,2 indicate the first and the second ferromagnetic film respectively, $J^0$ and $B(\theta)$ are bilinear and biquadratic coupling coefficients. In general, $B(\theta)$ is a smooth function of the angle $\theta$ between $m_1^0$ and $m_2^0$.

In this paper we discuss a theory of this phenomenon. It has been shown in [9−11] that the exponential decays of the average $< J(r, r') >$ is connected to the fact that it has a random sign at large $L$. The modulus of the exchange interaction decay with $L$ as a power
law.

We can introduce a local exchange energy \( J(\rho) \) between the ferromagnets as an average of \( J(r, r') \) over a ferromagnet’s surface area of order of \( L^2 \). Here \( \rho \) is coordinate along the films. We assume that \( J(\rho) \) is small enough and spatial dependence of the magnetizations on the scale of order of \( L \) can be neglected. According to Slonczewski \cite{12,13}, biquadratic term proportional to \( B \) in Eq.1 can originate from the existence of spatial fluctuations of the sign of exchange interaction \( J(\rho) = \langle J \rangle + \delta J(\rho) \) along the layers. The fluctuations of \( J(\rho) \) cause fluctuations of directions of magnetizations. Energy associated with spatial fluctuations in the magnetization’s directions can be represented as

\[
E(J(\rho), m_i(\rho)) = - \int d^2 \rho J(\rho)(m_1(\rho) \cdot m_2(\rho)) + \alpha d \int d^2 \rho \left[ \frac{\partial m_1(\rho)}{\partial \rho} \cdot \frac{\partial m_1(\rho)}{\partial \rho} + \frac{\partial m_2(\rho)}{\partial \rho} \cdot \frac{\partial m_2(\rho)}{\partial \rho} \right]
\]

(2)

where the first term corresponds to the interfilms exchange energy, the second term is associated with the gradients of magnetizations inside the films, \( d \) is the ferromagnetic film’s thickness and \( \alpha \) is a coefficient characterizing the exchange energy value in the ferromagnets.

In the case when \( \delta J(\rho) \gg \langle J \rangle \) and \( J(\rho) \) has a random sign, the energy \( E(J, m_i(\rho)) \) has a minimum at a sample specific realization \( m_i(\rho) = m^0_i + \delta m_i(\rho; [\delta J]) \) with \( m^0_1 \perp m^0_2 \) \cite{12,14} and

\[
B \equiv \frac{B_0}{\alpha d} G(\theta)
\]

(3)

\[
G = \int d^2 \rho \langle \delta J(\rho) \delta J(0) \rangle
\]

(4)

Here \( B_0 \) is a number of order unity \cite{14}.

Let us consider the case when \( J(\rho) \) has random sign due to mesoscopic fluctuations of Ruderman-Kittel oscillations inside the metal \cite{9–11}. We assume that the ferromagnetic film’s thickness \( d > L_s = \sqrt{D/\omega_s} \) and \( L_T = \sqrt{D_T} > d, L \). The latter inequality allows us to neglect temperature dependence of \( B \). Here \( D \) is the diffusion constant, which is assumed to be the same in the ferromagnetic and nonferromagnetic parts of the sample, and \( \omega_s \) is the exchange spin splitting energy in the ferromagnets. We will show that in the case \( L_s < L \),
$$G = \gamma(\theta)\left(\frac{E_c}{L}\right)^2$$

(5)

while in the case $L_s > L$, 

$$G = \gamma_1(\theta)\left(\frac{\omega_s}{L_s}\right)^2.$$

(6)

Here $\gamma$ and $\gamma_1$ are smooth functions of $\theta$ of order unity and $E_c = \frac{\nu_0}{\tau}$ is the Thouless energy.

Qualitatively, Eqs. 5, 6 can be understood as follows: In the case $|r - r'| \gg l$ the random oscillations of $J(r, r')$ exhibit a long range sign correlations $[15]$. In the case $L_s \ll L$ these long range correlations should be cut off at a length of the order of $L$. As a result, the fluctuations of the exchange energy averaged over the area of order of $L^2$ is of order $E_c$; and they are $\delta$-correlated at a distances larger than $L$. This leads to Eq. 5. We think that the estimate presented above can be relevant for the experiment $[8]$. In the opposite limit $L_s \gg L$ the cut off length is $L_s$. The fluctuations of the exchange energy averaged over the area of order $L_s^2$ is of order $\omega_s$. This leads to Eq. 6, which is independent of $L$.

To derive the results presented above we describe the exchange energy splitting in ferromagnet with the help of an effective Hamiltonian

$$H = H_0 + h(r; \theta)\sigma$$

(7)

Here $H_0$ is the Hamiltonian of free electron gas in a random potential $U(r)$, $h(r, \theta) \equiv \omega_s m(r, \theta)$ is the effective magnetic filed which is acting only on electron spins, $m(r, \theta)$ denotes a unit vector parollel to the magnetic moment in ferromagnets in the case when the angle between $m_1^0$ and $m_1^0$ is $\theta$, and $\sigma = \{\sigma_x, \sigma_y, \sigma_z\}$ is the Pauli matrixes vector.

We assume the following correlation properties of random potential: $< U(r) >= 0$ and $< U(r)U(r') >= \frac{1}{2\pi\nu_0\tau}\delta(r - r')$. Here $\nu_0$ is density of states at fermi level, $\tau$ is mean free scattering time of electrons.

To get the correlation function $<\delta J(0)\delta J(\rho)>$ we consider sample specific fluctuations of thermodynamic potential $\Omega(\theta)$ of the electrons as a function of $\theta$,

$$\Omega(\theta) = \langle\Omega(\theta)\rangle + \delta\Omega(\theta).$$

(8)
Using the identity \( \frac{d\Delta\Omega(\theta)}{d\theta} = \int d^2\rho \delta J(\rho) \) we get
\[
G(\theta) = \langle (\frac{d\delta\Omega}{d\theta})^2 \rangle
\] (9)

In the case of noninteracting electrons we can express thermodynamic potential as \( \Omega = \int_0^\mu d\mu N(\mu) \), where \( N(\mu) \) is the number of electrons at given chemical potential \( \mu \). Then, the correlation function of fluctuations of thermodynamic potential has the form
\[
\langle \delta\Omega(\theta_1)\delta\Omega(\theta_2) \rangle = \int_0^\mu d\mu_1 d\mu_2 \langle \delta N(\mu_1, \theta_1)\delta N(\mu_2, \theta_2) \rangle
\] (10)

To calculate it we use the usual diagram technique for averaging over configurations of disordered potential [16]. Diagrams for correlation function of number of electrons are shown in Fig.2. As a result we have
\[
\langle \delta N(\mu_1, \theta_1)\delta N(\mu_2, \theta_2) \rangle = \frac{2}{\pi} \sum_{\omega_n > 0} \omega_n \text{Re} \int_V d^3r d^3r' D_{\alpha\beta}^{\gamma\nu}(r, r'; \omega_n) D_{\beta\alpha}^{\gamma\nu}(r', r; \omega_n) =
\]
\[
= \frac{2}{\pi} \sum_{\omega_n > 0} \text{Re} \int \frac{d^2q}{(2\pi)^2} \sum_m (E_m(\theta_1, \theta_2) + Dq^2 + \omega_n + i(\mu_1 - \mu_2))^2
\] (11)

where \( \omega_n = 2\pi nT \) is the Matsubara frequency, \( n = 1, 2, \ldots \) and \( \alpha, \beta, \gamma, \mu \) are spin indices. Diffusion propagators \( D_{\alpha\beta}^{\gamma\nu}(r, r'; \omega_n) \) obey the equation
\[
\left[ -D\Delta + \omega_n + i(\mu_1 - \mu_2) \right]\delta_{\alpha\xi}\delta_{\gamma\mu} + i(h(r; \theta_1)\sigma_{\gamma\mu}\delta_{\alpha\xi} - h(r; \theta_2)\sigma_{\alpha\xi}\delta_{\gamma\mu}) \right] D_{\xi\beta}^{\mu\nu}(r, r'; \omega_n) =
\]
\[
= \delta(r - r') \delta_{\gamma,\nu} \delta_{\alpha,\beta}
\] (12)

The second equality in Eq.11 is the representation in terms of eigenvalues of Eq.12. In the case of geometry of the system, shown in Fig.1, the eigenvalues are equal to \( Dq^2 + E_m(\theta_1, \theta_2) \).

Here the spectrum \( E_m(\theta_1, \theta_2) \) is determined by equation
\[
[-D\delta_{\alpha\xi}\delta_{\gamma\mu}\frac{d^2}{dz^2} + i(h(r; \theta_1)\sigma_{\gamma\mu}\delta_{\alpha\xi} - h(r; \theta_2)\sigma_{\alpha\xi}\delta_{\gamma\mu})]\Psi_m(z; \mu, \xi) = E_m\Psi_m(z; \gamma, \alpha)
\] (13)

To calculate Eq.11 we use following equalities
\[
\int \frac{d^2q}{(2\pi)^2} \sum_m (E_m + Dq^2 + \omega_n + i(\mu_1 - \mu_2))^2 =
\]
\[
= -\frac{d}{d\omega_n} \int \frac{d^2q}{(2\pi)^2} 4\pi i \int dp \frac{dp}{L^2} \frac{d}{dp} (\frac{dp}{L^2} + Dq^2 + \omega_n + i(\mu_1 - \mu_2))
\]
\[
= \frac{1}{8\pi D} \int \frac{d}{d\omega_n} \ln(det(ip_0)det(-ip_0))
\] (14)
Here \( \text{det}(p) = \prod_n (p - E_n) \) is the spectral determinant of Eq.13, and \( p_0 = \sqrt{\frac{L^2}{D}} (\omega_n + i(\mu_1 - \mu_2)) \). In expression Eq.14 the integration contour \( C \) runs around zeros of \( \text{det}(p) \). Let us note that although Eqs.10,11 are formally divergent, their contribution to \( \langle (\frac{d\Omega}{dp})^2 \rangle \) is finite.

Let us consider the case \( L_s \ll L, d \) when results do not depend on \( \omega_s \). To define boundary conditions for Eq.13 it is convenient to introduce operators

\[
S_\pm = \frac{1}{2} \left[ 1 \pm \left( m(z; \theta_1) \sigma_1 \right) \left( m(z; \theta_2) \sigma_2 \right) \right] \tag{15}
\]

Then the boundary conditions for \( \Psi_m(z; \gamma, \alpha) \) are: \( \frac{d}{dz} S_+ \Psi_m = 0 \) at \( z = \pm (d + L/2) \) and \( S_- \Psi_m = 0 \) at \( L/2 < |z| < d + L/2 \). As a result, a solution of the eigenvalue problem Eq.13 gives the following spectral determinant

\[
\text{det}(ip) = \left( \text{sinh} p \text{ sinh}(1 + 2d/L)p + \left( 1 - \cos(\theta_1 - \theta_2) \right) \frac{1 + \cosh(2dp/L)}{4} \right) \times
\left[ 1 + \cosh(2 + 2d/L)p - \left( 1 + \cos(\theta_1 + \theta_2) \right) \frac{1 + \cosh(2dp/L)}{2} \right] \tag{16}
\]

Integrating over the chemical potentials, suming over Matsubara frequencies and considering case \( L_T \equiv \sqrt{D/T} \gg L; d \) we get

\[
G = \frac{2S}{(4\pi L)^2} (D/L^2)^2 \int_0^\infty dp \sinh p \sinh(1 + 2d/L)p \left[ 1 - \frac{1}{2} \frac{d}{d\theta} \left( \frac{1 + \cos(\theta)}{4\Phi(p)} \right) \right] \tag{17}
\]

where

\[
\Phi(p) = \frac{1 + \cosh 2dp/L}{\text{sinh} p \sinh(1 + 2d/L)p} \tag{18}
\]

The case when \( L_s \gg L \) can be studied in the same way giving Eq.6.

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FIG. 1. A schematic picture of the ferromagnet (F)-normal metal (N)- ferromagnet system
FIG. 2. Diagrams for calculation of the correlation function \( \langle \delta N(\mu_1, \theta_1) \delta N(\mu_2, \theta_2) \rangle \). Solid lines correspond to electron Green functions and dashed lines correspond to the correlation function of the scattering potential \( \langle U(r)U(r') \rangle \).