Charge Neutrality Effects on 2-flavor Color Superconductivity

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The effect of electric and color charge neutrality on 2-flavor color superconductivity has been investigated. It has been found that the effect of the charge neutrality on a 2-flavor quark system is very different from that on a 3-flavor system. The BCS diquark pair in 2-flavor color superconducting phase has been largely reduced by the large Fermi momentum of electron, the diquark condensate first increases with the increase of quark’s chemical potential $\mu$, then decreases rapidly and finally disappears at about $\mu = 535\text{MeV}$, at which the thermodynamic potential equals to that in the neutral normal 2-flavor quark matter.
I. INTRODUCTION

Because of the existence of an attractive interaction in the anti-triplet quark-quark channel in QCD, the cold and dense quark matter has been believed to favor the formation of diquark condensate and being in the superconducting phase [1]-[3].

The QCD phase structure at high baryon density is determined by how many kinds of quarks can participate in the pairing. If we have an ideal system with two massless quarks $u$ and $d$, and the three colored $u$ quarks and three colored $d$ quarks have the same Fermi surface, the system will be in 2 flavor color superconducting phase (2SC); and if we have an ideal system with three massless quarks $u$, $d$ and $s$, and all the nine quarks have the same Fermi surface, then all the quarks will participate in pairing and form a color-flavor-locking phase (CFL) [4].

In the real world, the strange quark mass $m_s$ should be considered, which will reduce the strange quark’s Fermi surface. If $m_s$ is large enough, there will be no pairing between $us$ and $ds$, and a 2SC+s phase will be favored [5].

Also, in reality, the system should be electric and color neutral, the existence of eletrons in the system will shift the Fermi surface of the two pairing quarks, and the system can be in BCS phase or crystalline phase [6] or normal quark matter, depending on how large the difference in the chemical potentials of the two pairing quarks is.

In a recent paper [8], it has been argued that the 2SC+s phase would not appear if the electric and color charge neutrality condition is added, which is also agreed with the results in [9] by using SU(3) NJL model and taking into account the self-consistent effective quark mass $m_f(\mu)$, where $f$ refers to $u, d, s$. It is found in [9] that the charge neutrality has a large effect on the $s$ quark mass, i.e., $m_s(\mu)$ begins to decrease at a smaller $\mu \simeq 400 MeV$ than the case if no charge neutrality is considered like in [10], [11], where $m_s(\mu)$ begins to decrease at about $\mu \simeq 550 MeV$.

In this paper, complementary to [9], we investigate the effect of charge neutrality on 2-flavor color superconductivity assuming that there is no strange quark involved in the
chemical potential regime $\mu < 550\text{MeV}$.

This paper is organized as following: in Sec. II, we extend our method in [12] to derive the thermodynamic potential when charge neutrality is considered; the gap equations and charge neutrality conditions will be derived in Sec. III, and in Sec. IV, we will give our numerical results; the conclusions and discussions will be given at the end.

II. THERMODYNAMIC POTENTIAL

A. The Lagrangian

Assuming that the strange quark does not appear in the system, we use the SU(2) Nambu-Jona-Lasinio model, and only consider scalar, pseudoscalar mesons and scalar diquark. The Lagrangian density has the form as

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m_0)q + G_S[(\bar{q}q)^2 + (\bar{q}\gamma_5\bar{q}q)^2] + G_D[(i\bar{q}C\varepsilon\gamma_5q)(i\bar{q}\varepsilon\gamma_5q^C)],$$

where $q^C = C\bar{q}^T$, $\bar{q}^C = q^TC$ are charge-conjugate spinors, $C = i\gamma^2\gamma^0$ is the charge conjugation matrix (the superscript $T$ denotes the transposition operation), the quark field $q \equiv q_{i\alpha}$ with $i = 1, 2$ and $\alpha = 1, 2, 3$ is a flavor doublet and color triplet, as well as a four-component Dirac spinor, $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ are Pauli matrices in the flavor space, and $(\varepsilon)^{ik} \equiv \varepsilon^{ik}$, $(\varepsilon^{b})^{\alpha\beta} \equiv \epsilon^{\alpha\beta b}$ are antisymmetric tensors in the flavor and color spaces respectively. $G_S$ and $G_D$ are independent effective coupling constants in the scalar quark-antiquark and scalar diquark channel.

After bosonization, one can obtain the linearized version of the model (1)

$$\tilde{\mathcal{L}} = \bar{q}(i\gamma^\mu \partial_\mu - m)q - \frac{1}{2}\Delta^{ab}(i\bar{q}C\varepsilon\gamma_5q)(i\bar{q}\varepsilon\gamma_5q^C) - \frac{\sigma^2}{4G_S} - \frac{\Delta^{ab}\Delta^b}{4G_D},$$

where we have assumed that there will be no pion condensate and introduced the constituent quark mass.
\[ m = m_0 + \sigma. \] (3)

From general considerations, there should be eight scalar diquark condensates \[13\] \[14\]. In the case of the NJL type model, the diquark condensates related to momentum vanish, and there is only one \(0^+\) diquark gap with Dirac structure \(\Gamma = \gamma_5\) for massless quark, and another \(0^+\) diquark condensate with Dirac structure \(\Gamma = \gamma_0\gamma_5\) at nonzero quark mass. In this paper, we assume that the contribution of the diquark condensate with \(\Gamma = \gamma_0\gamma_5\) is small, and only consider the diquark condensate with \(\Gamma = \gamma_5\). The diquark condensate with Dirac structure \(\gamma_5\gamma_0\) has been recently discussed in \[15\].

We can choose the diquark condensates in the third color direction, i.e., only the first two colors participate in the condensate, while the third one does not.

The model is non-renormalizable, and a momentum cut-off \(\Lambda\) should be introduced. The parameters \(G_S\) and \(\Lambda\) in the chiral limit \(m_0 = 0\) are fixed as

\[ G_S = 5.0163\text{GeV}^{-2}, \quad \Lambda = 0.6533\text{GeV}. \] (4)

The corresponding effective mass \(m = 0.314\text{GeV}\), and we will choose \(G_D = 3/4G_S\) in our numerical calculations.

**B. Partition function and thermodynamic potential**

The partition function of the grand canonical ensemble can be evaluated by using standard method \[16\] \[17\],

\[ Z = N' \int \left[ dq \right] \left[ d\bar{q} \right] \exp \{ \int_0^\beta d\tau \int d^3x (\hat{\mathcal{L}} + \mu \bar{q}_0 q) \}, \] (5)

where \(\beta = 1/T\) is the inverse of temperature \(T\), and \(\mu\) is the chemical potential. When electric and color charge neutrality is considered, the chemical potential \(\mu\) is a diagonal \(6 \times 6\) matrix in flavor and color space, and can be expressed as

\[ \mu = \text{diag}(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6), \] (6)
where $\mu_1, \mu_2, \mu_3$ are for the three colored $u$ quarks, and $\mu_4, \mu_5, \mu_6$ are for the three colored $d$ quarks.

Like in [8] [9], the chemical potential for each color and flavor quark is specified by its electric and color charges $\mu_i = \mu - Q_e \mu_e + T_3 \mu_{3c} + T_8 \mu_{8c}$, where $Q_e$, $T_3$ and $T_8$ are generators of $U(1)_Q$, $U(1)_3$ and $U(1)_8$. Because the diquark condenses in the third color direction, and the first two colored quarks are degenerate, we can assume $\mu_{3c} = 0$. For the same flavor, the difference of chemical potentials between the first two colored quarks and the third colored quark is induced by $\mu_{8c}$, and for the same color, the difference of chemical potentials between $u$ and $d$ is induced by $\mu_e$.

The explicit expressions for each color and flavor quark’s chemical potential are:

$$
\begin{align*}
\mu_1 &= \mu_2 = \mu - \frac{2}{3} \mu_e + \frac{1}{3} \mu_{8c}, \\
\mu_4 &= \mu_5 = \mu + \frac{1}{3} \mu_e + \frac{1}{3} \mu_{8c}, \\
\mu_3 &= \mu - \frac{2}{3} \mu_e - \frac{2}{3} \mu_{8c}, \\
\mu_6 &= \mu + \frac{1}{3} \mu_e - \frac{2}{3} \mu_{8c}.
\end{align*}
$$

(7)

For the convenience of calculations, we define the mean chemical potential $\bar{\mu}$ for the pairing quarks $q_1, q_5,$ and $q_2, q_4$

$$
\bar{\mu} = \frac{\mu_1 + \mu_5}{2} = \frac{\mu_2 + \mu_4}{2} = \mu - \frac{1}{6} \mu_e + \frac{1}{3} \mu_{8c},
$$

(8)

and the difference of the chemical potential $\delta \mu$

$$
\delta \mu = \frac{\mu_5 - \mu_1}{2} = \frac{\mu_4 - \mu_2}{2} = \mu_e/2.
$$

(9)

Because the third colored $u$ and $d$, i.e., the 3rd and 6th quarks do not participate in the diquark condensate, the partition function can be written as a product of three parts,

$$
\mathcal{Z} = \mathcal{Z}_{\text{const}} \mathcal{Z}_{36} \mathcal{Z}_{15,24}.
$$

(10)

The constant part is
\[
Z_{\text{const}} = N' \exp\{- \int_0^\beta d\tau \int d^3 \vec{x} \left[ \frac{\sigma^2}{4G_S} + \frac{\Delta^* \Delta}{4G_D} \right].
\]

\(Z_{36}\) part is for the unpairing quarks \(q_3(u_3)\) and \(q_6(d_3)\), and \(Z_{15,24}\) part is for the quarks participating in pairing, \(q_1(u_1)\) paired with \(q_5(d_2)\), and \(q_2(u_2)\) paired with \(q_4(d_1)\). In the following two subsections, we will derive the contributions of \(Z_{36}\) and \(Z_{15,24}\).

1. Calculation of \(Z_{36}\)

Introducing the 8-spinors for \(q_3\) and \(q_6\),
\[
\bar{\Psi}_{36} = (\bar{q}_3, \bar{q}_6; \bar{q}_3^C, \bar{q}_6^C),
\]
we can express \(Z_{36}\) as
\[
Z_{36} = \int [d\Psi_{36}] \exp \left\{ \frac{1}{2} \sum_{n,\vec{p}} \bar{\Psi}_{36} \left[ G_{\vec{0}}^{-1} \right]_{36} \Psi_{36} \right\} = \text{Det}^{1/2}(\beta[G_{\vec{0}}^{-1}]_{36}),
\]
where the determinantal operation Det is to be carried out over the Dirac, color, flavor and the momentum-frequency space, and \([G_{\vec{0}}^{-1}]_{36}\) has the form of
\[
[G_{\vec{0}}^{-1}]_{36} = \begin{pmatrix}
[G_{\vec{0}}^{-1}]_3 & 0 & 0 & 0 \\
0 & [G_{\vec{0}}^{-1}]_6 & 0 & 0 \\
0 & 0 & [G_{\vec{0}}^{-1}]_3 & 0 \\
0 & 0 & 0 & [G_{\vec{0}}^{-1}]_6
\end{pmatrix},
\]
with
\[
[G_{\vec{0}}^{-1}]_i^{-1} = \not{p} + \not{p}_i - m.
\]
Here we have used \(\not{p} = p_\mu \gamma^\mu\) and \(\not{p}_i = \mu_i \gamma_0\).

For the two quarks not participating in the diquark condensate, from Eq. (13), we can have
\[
\ln Z_{36} = \frac{1}{2} \ln \det(\beta [G_0^{-1}])_{36} \\
= \frac{1}{2} \ln [\det(\beta [G_0^+])_{3}] \det(\beta [G_0^-])_{3} \det(\beta [G_0^+]_{16}^{-1}) \det(\beta [G_0^-]_{16}^{-1}).
\] (16)

We first work out
\[
[\det(\beta [G_0^+])_{3}] \det(\beta [G_0^-])_{3} = \beta^4 [p_0^2 - E_3^+ - E_3^+],
\]
\[
[\det(\beta [G_0^+]_{16}) \det(\beta [G_0^-]_{16})] = \beta^4 [p_0^2 - E_6^+ - E_6^+],
\] (17)

with \( E_3^\pm = E \pm \mu_3 \) and \( E_6^\pm = E \pm \mu_6 \) where \( E = \sqrt{p^2 + m^2} \). Considering the determinant in the flavor, color, spin spaces and momentum-frequency space, we get the expression
\[
\ln Z_{36} = \sum_n \sum_{\vec{p}} \{ \ln(\beta^2 [p_0^2 - E_3^+]) + \ln(\beta^2 [p_0^2 - E_3^-]) \\
+ \ln(\beta^2 [p_0^2 - E_6^+]) + \ln(\beta^2 [p_0^2 - E_6^-]) \}. 
\] (18)

2. Calculation of \( Z_{15,24} \)

The calculation of \( Z_{15,24} \) here is much more complicated than that when the two pairing quarks have the same Fermi surface \[12\].

Also, we introduce the Nambu-Gokov formalism for \( q_1, q_2, q_4 \) and \( q_5 \), i.e.,
\[
\bar{\Psi} = (\bar{q}_1, \bar{q}_2, \bar{q}_4, \bar{q}_5; \bar{q}_1^C, \bar{q}_2^C, \bar{q}_4^C, \bar{q}_5^C).
\] (19)

The \( Z_{15,24} \) can have the simple form as
\[
Z_{15,24} = \int [d\bar{\Psi}] \exp \left\{ \frac{1}{2} \sum_{n,\vec{p}} \bar{\Psi} G_{15,24}^{-1} \Psi \right\} \\
= \det^{1/2} (\beta [G^{-1}]_{15,24}),
\] (20)

where
\[
G_{15,24}^{-1} = \begin{pmatrix}
[G_0^+]_{15,24}^{-1} & \Delta^- \\
\Delta^+ & [G_0^-]_{15,24}^{-1}
\end{pmatrix},
\] (21)

with
\[ [G^+_0]^{-1}_{15,24} = \begin{pmatrix}
[G^+_0]^{-1}_{1} & 0 & 0 & 0 \\
0 & [G^+_0]^{-1}_{2} & 0 & 0 \\
0 & 0 & [G^+_0]^{-1}_{4} & 0 \\
0 & 0 & 0 & [G^+_0]^{-1}_{5}
\end{pmatrix}, \tag{22}\]

and the matrix form for \( \Delta^\pm \) is

\[ \Delta^- = -i\Delta\gamma_5 \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 \\
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}, \quad \Delta^+ = \gamma^0(\Delta^-)^t\gamma^0. \tag{23}\]

From Eq. \( \text{(20)} \), we have

\[ \ln Z_{15,24} = \frac{1}{2} \ln \text{Det}(\beta G^{-1}_{15,24}). \tag{24}\]

For a \( 2 \times 2 \) matrix with elements \( A, B, C \) and \( D \), we have the identity

\[ \text{Det} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \text{Det}(-CB + CAC^{-1}D) = \text{Det}(-BC + BDB^{-1}A). \tag{25}\]

Replacing \( A, B, C \) and \( D \) with the corresponding elements of \( G^{-1}_{15,24} \), we have

\[ \text{Det}(G^{-1}_{15,24}) = \text{Det}D_1 = \text{Det}(-\Delta^+\Delta^- + \Delta^+[G^+_0]^{-1}_{15,24}[\Delta^-]^{-1}[G^-_0]^{-1}_{15,24}) \]

\[ = \text{Det}D_2 = \text{Det}(-\Delta^-\Delta^+ + \Delta^-[G^-_0]^{-1}_{15,24}[\Delta^+]^{-1}[G^+_0]^{-1}_{15,24}). \tag{26}\]

Using the massive energy projectors \( \Lambda^\pm \) in [12] for each flavor and color quark, we can work out the diagonal matrix \( D_1 \) and \( D_2 \) as

\[ (D_1)_{11} = (D_1)_{22} = [(p_0 + \delta \mu)^2 - [\tilde{E}_\Delta^-]^2]\Lambda_- + [(p_0 + \delta \mu)^2 - [\tilde{E}_\Delta^+]^2]\Lambda_+ \]

\[ (D_1)_{33} = (D_1)_{44} = [(p_0 - \delta \mu)^2 - [\tilde{E}_\Delta^-]^2]\Lambda_- + [(p_0 - \delta \mu)^2 - [\tilde{E}_\Delta^+]^2]\Lambda_+ \]

\[ (D_2)_{11} = (D_2)_{22} = [(p_0 - \delta \mu)^2 - [\tilde{E}_\Delta^-]^2]\Lambda_- + [(p_0 - \delta \mu)^2 - [\tilde{E}_\Delta^+]^2]\Lambda_+ \]

\[ (D_2)_{33} = (D_2)_{44} = [(p_0 + \delta \mu)^2 - [\tilde{E}_\Delta^+]^2]\Lambda_- + [(p_0 + \delta \mu)^2 - [\tilde{E}_\Delta^-]^2]\Lambda_+, \tag{27}\]
where $\bar{E}_\Delta^\pm = \sqrt{(E \pm \bar{\mu})^2 + \Delta^2}$.

With the above equations, Eq. (24) can be expressed as

$$\ln Z_{15,24} = 2 \sum_n \sum_p \left\{ \ln[\beta^2(p_0^2 - (\bar{E}_\Delta^- + \delta\mu))^2] + \ln[\beta^2(p_0^2 - (\bar{E}_\Delta^+ - \delta\mu))^2] \\
+ \ln[\beta^2(p_0^2 - (\bar{E}_\Delta^+ + \delta\mu))^2] + \ln[\beta^2(p_0^2 - (\bar{E}_\Delta^- - \delta\mu))^2] \right\}. \quad (28)$$

C. The thermodynamic potential

Using the helpful relation

$$\ln Z_f = \sum_n \ln[\beta^2(p_0^2 - E_n^2)] = \beta[E_p + 2T\ln(1 + e^{-\beta E_p})], \quad (29)$$

we can evaluate the thermodynamic potential of the quark system

$$\Omega_q = \frac{m^2}{4G_S} + \frac{\Delta^2}{4G_D} - 2 \int \frac{d^3p}{(2\pi)^3} \left[ 2E + T\ln(1 + e^{-\beta E_3^+}) \right. \\
+ T\ln(1 + e^{-\beta E_3^-}) + T\ln(1 + e^{-\beta E_6^+}) + T\ln(1 + e^{-\beta E_6^-}) \\
+ 2\bar{E}_\Delta^+ + 2\bar{E}_\Delta^- + 2T\ln(1 + e^{-\beta \bar{E}_\Delta^+_+}) + 2T\ln(1 + e^{-\beta \bar{E}_\Delta^+_-}) \\
+ 2T\ln(1 + e^{-\beta \bar{E}_\Delta^-_+}) + 2T\ln(1 + e^{-\beta \bar{E}_\Delta^-_-}) \left. \right] \quad (30)$$

with $\bar{E}_\Delta^\pm = \bar{E}_\Delta \pm \delta\mu$.

For the total thermodynamic potential, we should include the contribution from the electron gas, $\Omega_e$. Assuming the electron’s mass is zero, we have

$$\Omega_e = -\frac{\mu_e^4}{12\pi^2}. \quad (31)$$

The total thermodynamic potential of the system is

$$\Omega = \Omega_q + \Omega_e. \quad (32)$$
III. GAP EQUATIONS AND CHARGE NEUTRALITY CONDITION

From the thermodynamic potential Eq.(32), we can derive the gap equations of the order parameters \( m \) and \( \Delta \) for the chiral and color superconducting phase transitions.

A. Gap equation for quark mass

The gap equation for quark mass can be derived by using

\[
\frac{\partial \Omega}{\partial m} = 0. \tag{33}
\]

The explicit expression for the above equation is

\[
m[1 - 4G_S \int \frac{d^3p}{(2\pi)^3} \frac{1}{E^\Delta} \frac{1}{E}(1 - \tilde{f}(E_{\Delta^+}) - \tilde{f}(E_{\Delta^-})) + 2 \frac{E^+}{E^\Delta} (1 - \tilde{f}(E_{\Delta^+}) - \tilde{f}(E_{\Delta^-})) + (2 - \tilde{f}(E_{3^+}) - \tilde{f}(E_{3^-}) - \tilde{f}(E_{6^+}) - \tilde{f}(E_{6^-}))] = 0, \tag{34}
\]

with the Fermi distribution function \( \tilde{f}(x) = 1/(\exp\{\beta x\} + 1) \). \( m = 0 \) corresponds to the chiral symmetric phase, \( m \neq 0 \) corresponds to the chiral symmetry breaking phase.

B. Gap equation for diquark condensate

Using

\[
\frac{\partial \Omega}{\partial \Delta} = 0, \tag{35}
\]

we can derive the gap equation for diquark condensate

\[
\Delta[1 - 4G_D \int \frac{d^3p}{(2\pi)^3} \frac{1}{E^\Delta} (1 - \tilde{f}(E_{\Delta^+}) - \tilde{f}(E_{\Delta^-})) + 2 \frac{1}{E^\Delta} (1 - \tilde{f}(E_{\Delta^+}) - \tilde{f}(E_{\Delta^-}))] = 0. \tag{36}
\]
C. Color charge neutrality

The color charge neutrality condition is to choose $\mu_{8c}$ such that the system has zero net charge $T_8$, i.e.,

$$ T_8 = \frac{\partial \Omega}{\partial \mu_{8c}} = 0. \quad (37) $$

Evaluating the above equation, we have the color charge neutrality condition as

$$ \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{\vec{E}^-}{E^-_\Delta} (1 - \tilde{f}(\vec{E}^-_{\Delta+}) - \tilde{f}(\vec{E}^-_{\Delta-})) 
+ \frac{\vec{E}^+}{E^+_{\Delta}} (1 - \tilde{f}(\vec{E}^+_{\Delta+}) - \tilde{f}(\vec{E}^+_{\Delta-})) 
+ (\tilde{f}(\vec{E}^+_{3+}) - \tilde{f}(\vec{E}^+_{3-})) 
+ (\tilde{f}(\vec{E}^+_{6+}) - \tilde{f}(\vec{E}^+_{6-})) \right] = 0. \quad (38) $$

D. Electric charge neutrality

Similarly, the electric charge neutrality condition is to choose $\mu_e$ such that the system has zero net electric charge $Q_e$, i.e.,

$$ Q_e = \frac{\partial \Omega}{\partial \mu_e} = 0. \quad (39) $$

From the above equation, we obtain

$$ \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{\vec{E}^-}{E^-_\Delta} (1 - \tilde{f}(\vec{E}^-_{\Delta+}) - \tilde{f}(\vec{E}^-_{\Delta-})) 
- 2 \frac{\vec{E}^+}{E^+_{\Delta}} (1 - \tilde{f}(\vec{E}^+_{\Delta+}) - \tilde{f}(\vec{E}^+_{\Delta-})) 
+ 6 (\tilde{f}(\vec{E}^-_{\Delta+}) + \tilde{f}(\vec{E}^+_{\Delta+}) - \tilde{f}(\vec{E}^-_{\Delta-}) - \tilde{f}(\vec{E}^+_{\Delta-})) 
+ 4 (\tilde{f}(\vec{E}^+_{3+}) - \tilde{f}(\vec{E}^+_{3-})) - 2(\tilde{f}(\vec{E}^+_{6+}) - \tilde{f}(\vec{E}^+_{6-})) \right] + \frac{\mu_e^3}{\pi^2} = 0. \quad (40) $$

IV. NUMERICAL RESULTS

In the numerical results, we investigate the following four different cases: 1), no color superconducting phase, no charge neutrality condition; 2), no color superconducting phase,
electric charge neutrality condition added, the electron’s chemical potential in this phase will be characterized by $\mu_e^0$; 3), color superconducting phase exists, no charge neutrality condition, the diquark condensate in this phase will be characterized by $\Delta_0$; 4), color superconducting phase exists, electric and color charge neutrality condition added, the electron’s chemical potential and diquark condensate in this phase will be characterized by $\mu_e$ and $\Delta$ respectively.

Fig. 1 is for the chiral phase transition in the case of no electric neutrality considered, which is familiar to all of us, the quark posses a large constituent mass in the low baryon density regime due to chiral symmetry breaking, and restores chiral symmetry at high baryon density.

Fig. 2 is the chiral phase diagram when the electric charge neutrality is considered, the solid squares and the solid circles are for quark mass $m$ and electron’s chemical potential $\mu_e^0$ respectively. It is found that, in the chiral symmetric phase, there is a large Fermi surface for electrons, and the electron’s chemical potential $\mu_e^0$ increases linearly with the increase of quark’s chemical potential $\mu$.

Fig. 3 is the phase diagram for color superconductivity without charge neutrality constraints, the quark mass $m$ (squares) and diquark condensate $\Delta_0$ (circles) are plotted as functions of quark’s chemical potential $\mu$. It is found that in the color superconducting phase, the magnitude of the diquark condensate $\Delta_0$ is about $100 MeV$ by using the parameter $G_D = 3/4G_S$. $\Delta_0$ first increases with increasing $\mu$, and this tendency stops at about $\mu = 530 MeV$, which is related to the momentum cut-off $\Lambda$.

Fig. 4 is the phase diagram for color superconductivity when electric and color charge neutrality condition is considered, the quark mass $m$ (squares), diquark condensate $\Delta$ (triangles) and the electron’s chemical potential $\mu_e$ (circles) are plotted as functions of quark’s chemical potential $\mu$. It is found that when chiral symmetry restores, the color superconducting phase appears. In the color superconducting phase, it is found that both the diquark condensate $\Delta$ and the electron’s chemical potential first increase with increasing $\mu$, and reaches their maximum at about $\mu = 475 MeV$, then decrease rapidly with increasing
\( \mu \), and the diquark condensate disappears at about \( \mu = 535 \text{MeV} \).

In Fig. 5, we show the chemical potential \( \mu_{sc} \) as a function of \( \mu \), which ensures the system having zero net color charge \( T_8 \). It is found that \( \mu_{sc} \) can be negative and positive, but its value is very small, only about several \( \text{MeV} \). It means that for the same flavor quark \( u \) or \( d \), the difference of Fermi momenta between the third colored quark and the first two colored quarks can be neglected.

In order to explicitly see what happens to a color superconducting phase when charge neutrality condition is added, we compare the electron’s chemical potential in the neutral normal quark matter \( \mu_e^0 \) (light circles) and in the neutral superconducting phase \( \mu_e \) (solid circles) in Fig. 6, and compare the diquark condensate \( \Delta_0 \) (light squares) and \( \Delta \) (solid circles) in the electric charged and neutral superconducting phases in Fig. 7.

From Fig. 6, we find that the electron has a larger Fermi surface in the neutral superconducting phase than that in the neutral normal quark matter. This is because in the superconducting phase, the isospin is mainly carried by unpaired quarks only, and \( \mu_e \) has to be larger in order to get the same isospin density. \( \mu_e \) is equal to \( \mu_e^0 \) at about \( \mu = 535 \text{MeV} \), at which the diquark condensate disappears. We separate \( \mu_e \) into two parts as \( \mu_e = \mu_e^0 + \delta \mu_e \), where \( \delta \mu_e \) reflects the effect induced by diquark condensate, comparing \( \delta \mu_e \) (light squares) and \( \Delta \) (solid squares) in Fig. 6, we can see that \( \delta \mu_e \) increases when \( \Delta \) increases, and decreases when \( \Delta \) decreases, and when \( \Delta = 0 \), \( \delta \mu_e = 0 \), too.

From Fig. 7, we can see that the diquark condensate \( \Delta \) in the neutral superconducting phase has a much smaller value than \( \Delta_0 \) in the charged superconducting phase, it means that the difference of the Fermi surfaces of the two pairing quarks reduces the magnitude of the diquark condensate \( \Delta \).

From Fig. 6 and Fig. 7, we can see a distinguished characteristic in the neutral superconducting phase, i.e., both the electron’s chemical potential \( \mu_e \) and the magnitude of the diquark condensate \( \Delta \) first increase with increasing \( \mu \), then at about \( \mu = 475 \text{MeV} \), decrease with increasing \( \mu \). Now we try to understand this phenomena.

The magnitude of the diquark condensate is not only affected by \( \mu \), but also by \( \delta \mu = \mu_e/2 \),
which describes the difference of Fermi surface between the two paired quarks and reduces the diquark condensate as seen in Fig. 7. The diquark condensate first increases with increasing $\mu$, which is the result of the increasing density of states. At about $\mu = 475\text{MeV}$, where $\mu_d = \mu + 1/3\mu_e$ is about $530\text{MeV}$, the diquark condensate starts to be affected by the momentum cut-off $\Lambda$. Like that in the charged color superconducting phase, the diquark condensate stops increasing with $\mu$, so does $\mu_e$. However, when $\mu > 475\text{MeV}$, $\mu_e^0$ keeps increasing with $\mu$, which reduces the diquark condensate largely. This is why we see $\Delta$ decreases with increasing $\mu$. Finally at a certain chemical potential $\mu = 535\text{MeV}$, where $\mu_d$ is about $580\text{MeV}$, the diquark condensate disappears. Because $\delta\mu_e$ decreases with the decrease of $\Delta$, the total $\mu_e$ decreases in the chemical potential regime $\mu > 475\text{MeV}$.

At last, in Fig. 8, we show the thermodynamic potentials in the four different cases as functions of chemical potential $\mu$, the light squares and circles are for the cases with and without charge neutrality and without diquark condensate, the dark circles and squares are for the cases with and without charge neutrality and with diquark condensate. It can be seen that in the chiral symmetric phase, the charged superconducting phase has the lowest thermodynamic potential, and the charged normal quark matter has the second lowest $\Omega$. The neutral superconducting phase has a little bit lower $\Omega$ than that of the neutral quark matter in the chemical potential region $\mu < 535\text{MeV}$, and at $\mu = 535\text{MeV}$, the two $\Omega$s coincide with each other. Therefore, in the chiral symmetric phase, the stable state of the neutral system is the color superconducting phase for $\mu < 535\text{MeV}$. At $\mu = 535\text{MeV}$, the stable phase is the normal neutral quark matter.

V. CONCLUSIONS

We investigated the effect of charge neutrality on a two flavor quark system. It has been found that the BCS diquark pair has been largely reduced by the large Fermi momentum of electron, the diquark condensate first increases with the increase of quark’s chemical potential $\mu$, then decreases rapidly and disappears at about $\mu = 535\text{MeV}$, at which the
thermodynamic potential equals to that in the neutral normal quark matter. In the chemical potential region \(330 MeV < \mu < 535 MeV\), the stable neutral system is in the color superconducting phase if no strange quark involved.

As we mentioned in the introduction, we did not consider the strange quark in the system, which should be considered and has been investigated self-consistently in [9]. Comparing our results with their results about the diquark gap \(\Delta\), the electron’s chemical potential \(\mu_e\) and the chemical potential \(\mu_{8c}\) in 2SC, it can be found that the main difference lies in the chemical potential region \(\mu > 400 MeV\). In [9], the diquark condensate in the neutral superconducting phase is largely reduced only in a small chemical potential region \(370 MeV < \mu < 400 MeV\). \(\mu_e\) first increases with \(\mu\) then begins to decrease at about \(\mu = 400 MeV\). As for \(\mu_{8c}\), the tendency also becomes different from our results at about \(\mu = 400 MeV\). The reason lies in that, in [9], the strange quark involves in the system when \(\mu > 400 MeV\). This shows that the effect of the charge neutrality condition on three- and two- flavor quark system is quite different.

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[1] D.Bailin and A.Love, Phys. Rep. **107**, 325(1984).

[2] M.Alford, K. Rajagopal and F. Wilczek, Phys.Lett.**B 422**, 247(1998).

[3] R.Rapp, T.Schäfer,E.V.Shuryak and M.Velkovsky, Phys.Rev.Lett.**81**,53(1998).

[4] K.Rajagopal and F.Wilczek, [hep-ph/0011333](https://arxiv.org/abs/hep-ph/0011333).

[5] M.Alford, K. Rajagopal, F.Wilczek, Nucl.Phys.**B537** (1999) 443.

[6] M.Alford, J. Berges, K. Rajagopal, Nucl.Phys.**B538**(1999) 219.

[7] M. Alford, J. A. Bowers, K. Rajagopal, Phys.Rev.**D63**(2001) 074016; J. A. Bowers, J. Kundu, K. Rajagopal, E. Shuster, Phys.Rev.**D64**(2001) 014024; A. K. Leibovich, K. Rajagopal, E. Shuster, Phys.Rev.**D64**(2001), 094005; J. Kundu, K. Rajagopal, Phys.Rev.**D65**(2002), 094022; J. A. Bowers, K. Rajagopal, [hep-ph/0204079](https://arxiv.org/abs/hep-ph/0204079).

[8] M. Alford, K. Rajagopal, [hep-ph/0204001](https://arxiv.org/abs/hep-ph/0204001).

[9] A.W. Steiner, S. Reddy, M. Prakash, [hep-ph/0205201](https://arxiv.org/abs/hep-ph/0205201).

[10] F. Gastineau, R. Nebauer, J. Aichelin, Phys. Rev. **C65**(2002) 045204.

[11] M. Buballa, M. Oertel, Nucl.Phys. **A703** (2002) 770.

[12] M. Huang, P. Zhuang and W.Q. Chao, Phys. Rev. **D65** (2002) 076012.J

[13] R.D.Pisarski, D.H.Rischke, Phys.Rev.**D 60**(1999) 094013.

[14] R.D.Pisarski, D.H.Rischke, Phys.Rev.Lett.**83**(1999) 37.

[15] T. Fugleberg, [hep-ph/0206033](https://arxiv.org/abs/hep-ph/0206033).

[16] J.Kapusta, ”Finite-temperature Field Theory”, Cambridge University Press, 1989.

[17] M. L. Bellac, ”Thermal Field Theory”, Cambridge University Press, 1996.
FIG. 1. The quark mass $m$ as a function of chemical potential $\mu$ in the case of no color superconducting phase and no charge neutrality condition.
FIG. 2. The quark mass $m$ (squares) and the electron’s chemical potential $\mu_e^0$ (circles) as functions of chemical potential $\mu$ in the case of neutral normal quark matter.
FIG. 3. The quark mass $m$ (squares) and the diquark condensate $\Delta_0$ (circles) as functions of chemical potential $\mu$ in the case of no charge neutrality on color superconducting phase.
FIG. 4. The quark mass $m$ (squares), the diquark condensate $\Delta$ (triangles) and the electron's chemical potential $\mu_e$ (circles) as functions of chemical potential $\mu$ in the case of considering charge neutrality on color superconducting phase.
FIG. 5. The $\mu_{8e}$ as a function of chemical potential $\mu$ in the case of considering charge neutrality on color superconducting phase.
FIG. 6. The electron’s chemical potential in the neutral normal quark matter $\mu_e^0$ (light circles) and in the neutral superconducting phase $\mu_e$ (solid circles) as functions of chemical potential $\mu$, and $\delta \mu_e = \mu_e - \mu_e^0$ (light squares) comparing with $\Delta$ (solid squares) as functions of $\mu$. 
FIG. 7. The diquark condensate in the electric charged $\Delta_0$ (light squares) and in the neutral $\Delta$ (solid circles) superconducting phase as a function of chemical potential $\mu$. 
FIG. 8. The thermodynamic potentials as functions of chemical potential $\mu$, the light squares and circles are for the cases with and without charge neutrality and without diquark condensate, the dark circles and squares are for the cases with and without charge neutrality and with diquark condensate.