Building a case for a Planck-scale-deformed boost action: 
the Planck-scale particle-localization limit\(^1\)

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ABSTRACT

“Doubly-special relativity” (DSR), the idea of a Planck-scale Minkowski limit that is still a relativistic theory, but with both the Planck scale and the speed-of-light scale as nontrivial relativistic invariants, was proposed (gr-qc/0012051) as a physics intuition for several scenarios which may arise in the study of the quantum-gravity problem, but most DSR studies focused exclusively on the search of formalisms for the description of a specific example of such a Minkowski limit. A novel contribution to the DSR physics intuition came from a recent paper by Smolin (hep-th/0501091) suggesting that the emergence of the Planck scale as a second nontrivial relativistic invariant might be inevitable in quantum gravity, relying only on some rather robust expectations concerning the semiclassical approximation of quantum gravity. I here attempt to strengthen Smolin’s argument by observing that an analysis of some independently-proposed Planck-scale particle-localization limits, such as the “Generalized Uncertainty Principle” often attributed to string theory in the literature, also suggests that the emergence of a DSR Minkowski limit might be inevitable. I discuss a possible link between this observation and recent results on logarithmic corrections to the entropy-area black-hole formula, and I observe that both the analysis here reported and Smolin’s analysis appear to suggest that the examples of DSR Minkowski limits for which a formalism has been sought in the literature might not be sufficiently general. I also stress that, as we now contemplate the hypothesis of a DSR Minkowski limit, there is an additional challenge for those in the quantum-gravity community attributing to the Planck length the role of “fundamental length scale”.

Beyond Special Relativity

One might say that special relativity went into retirement at a very young age, since the advent of general relativity, in 1916, took away from special relativity the status of “fundamental” ingredient of the laws of Nature. But actually special relativity, now 100 years old, continues to work hard for physicists. In quite a few contexts involving gravitational phenomena one gets away describing the spacetime metric \(g_{\mu\nu}\) in terms of a Minkowski background metric, \(\eta_{\mu\nu}\), and a “gravity field” Lorentz tensor \(h_{\mu\nu}\), related to \(g\) and \(\eta\) by the relation \(h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}\). And special relativity still reigns supreme in the vast class of phenomena studied in particle physics, where one can safely

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assume that the processes unfold in a Minkowski background spacetime, with metric \( \eta_{\mu\nu} \), and that “gravitational interactions” among particles are negligible. Special relativity still holds an essentially fundamental status within particle physics. This longevity is due mostly to the fact that special relativity, initially introduced to address some issues of concern in classical mechanics (notably the form of Maxwell equations), turned out to deal rather well with the new structures of quantum mechanics. Special relativity and quantum mechanics coexist rather peacefully in the framework provided by relativistic quantum field theory. But several authors have argued that the transition from (special-) relativistic quantum field theory to (the still unknown) “quantum gravity” might force special relativity to relinquish even its present privileged status within particle physics, as soon as we acquire sensitivity to Planck-scale corrections to particle-physics processes.

The description of Planck-scale corrections to particle-physics processes will be a key aspect of the Minkowski limit of quantum gravity. In our current conceptual framework special relativity emerges in the Minkowski limit, where one deals with situations that allow the adoption of a Minkowski metric throughout, and one might wonder whether the Minkowski limit of quantum gravity could still be governed by special relativity. The issue will be of particular interest if quantum gravity admits a limit in which one can assume throughout a (expectation value of the) metric of Minkowski type, but some Planck-scale features of the fundamental description of spacetime (such as spacetime discreteness and/or spacetime noncommutativity) are still not completely negligible. I will denominate “nontrivial Minkowski limit” this type of Minkowski limit in which essentially the role of the Planck scale in the description of gravitational interactions (expressing the gravitational constant \( G \) in terms of the Planck scale) can be ignored, but the possible role of the Planck scale in spacetime structure/kinematics is still significant. It is of course not obvious that the correct quantum gravity should admit such a nontrivial Minkowski limit. With the little we presently know about the quantum-gravity problem we must be open to the possibility that the Minkowski limit would actually be trivial, i.e. that whenever the role of the Planck scale in the description of gravitational interactions can be neglected one should also neglect the role of the Planck scale in spacetime structure/kinematics. But the hypothesis of a nontrivial Minkowski limit is worth exploring: it would be “extremely kind” of quantum gravity to admit such a limit, since it might open a wide range of opportunities for accessible experimental verification (see, e.g., Refs. [1, 2, 3, 4, 5, 6, 7]).

For various approaches to the quantum-gravity problem evidence as emerged in support of this possibility of a nontrivial Minkowski limit. While there is no fully-developed proposed solution of the quantum-gravity problem based on a fundamentally noncommutative spacetime picture, it has been observed that the hypothesis that in general the correct fundamental description of spacetime should involve noncommutativity can imply that in particular the Minkowski limit is described in terms of noncommuting spacetime coordinates, and this is found [8, 9, 10, 11, 12, 13, 14] to naturally lead to a nontrivial Minkowski limit with departures from classical Poincaré symmetry. In the literature on the loop-quantum-gravity approach one finds a large number (although all of preliminary nature) of arguments [2, 15, 16, 17, 18] supporting the possibility of a nontrivial Minkowski limit, primarily characterized by a Planck-scale-modified energy-momentum (dispersion) relation. For the string-theory approach, while there are no studies arguing that the availability of a nontrivial Minkowski limit is necessary, there is a large literature (see, e.g., Refs. [11, 12] and references therein) on a nontrivial Minkowski limit with broken Lorentz symmetry.

The fate of Poincaré symmetry in such nontrivial Minkowski limits is of course a key issue both phenomenologically and from a conceptual perspective. In the large number of studies produced between 1997 and 2000 on the possibility of a nontrivial Minkowski limit it was always assumed that Poincaré (and in particular Lorentz) symmetry would be broken: the Galilei Relativity Principle
would not hold with Planck-scale accuracy. On the basis of an analogy with the century-old process which led from Galilei/Newton Relativity, through the analysis of Maxwell’s electrodynamics (first viewed as a manifestation of an ether violating the relativity principle, but ultimately understood as a manifestation of a needed transition in the formulation of the relativistic theory), to Einstein’s Special Relativity, I argued in Ref. [19] that the Minkowski limit of quantum gravity might be characterized by a “doubly special relativity” (DSR), a relativistic theory with two, rather than one, nontrivial relativistic invariants (the Planck scale in addition to the speed-of-light scale), but still fully compatible with the Galilei Relativity Principle.

At present this DSR proposal is still confined in the limbo of a physics scenario for which good mathematics (or at least a fully satisfactory use of mathematics) is being sought. The prototypical example of a quantum-gravity theory that would need the DSR idea is a quantum-gravity theory in which in the Minkowski limit one finds that the Planck (length) scale sets an observer-independent minimum allowed value of wavelength. But of course there is a plethora of other requirements which would could be advocated in order to find a DSR Minkowski limit. The abundance of possible physical principles that one might consider from a DSR perspective is counter-acted by an apparently scarce ensemble of mathematical formalisms that could be used to describe such a DSR Minkowski limit. For example attempts based on the κ-Minkowski noncommutative spacetime, and an associated Hopf-algebra structure [8, 9, 13, 20], provide a description which, while exposing some structures that are of potential relevance for the DSR idea, at present is still unsatisfactory as far as interpretation of the formalism in the one-particle sector, and both mathematically unsettled and interpretationally unclear for multiparticle systems [21]. It is legitimate to hope that an improved scheme based on this Hopf-algebra mathematics might turn out to circumvent these limitations, but some key difficulties must be overcome. Some arguments suggesting a possible relevance of the DSR proposal for the Minkowski limit of loop quantum gravity have been proposed [22, 18], but in order to render these arguments more precise it would of course be first necessary to establish what procedure actually leads to the Minkowski limit in loop quantum gravity.

A gravity rainbow?

One of the many attempts to introduce mathematical structures suitable for the description of a DSR Minkowski limit is based on the introduction of an energy-dependent metric. This technique was named “gravity rainbow” by Magueijo and Smolin, who first proposed it as a tool for DSR research [22]. It is probably fair to say that also this proposal still lacks a fully developed interpretation and implementation. It is easy to see however how the language of an energy-dependent metric could be used for an effective description of modified energy-momentum relations in the Minkowski limit. For example, if the modified dispersion relation is of the type

\[ C = p^2 + f(E, E_p)E^2, \]

where \( C \) is the deformed “mass Casimir” and \( E_p \) denotes the Planck energy scale, one could introduce an energy-dependent metric \( g_{\mu\nu}(E, E_p) \) such that \( g_{00}(E, E_p) = f(E, E_p)\eta_{00} \) and \( g_{ij} = \eta_{ij} \) and describe the same dispersion relation as

\[ C = p^\mu g_{\mu\nu}(E, E_p)p^\nu. \]

The possibility of an observer-independent Planck-scale modification of the energy-momentum dispersion relation is one of the most studied possibilities as a “physics ingredient” for a DSR Minkowski limit, and therefore the gravity rainbow formulation might indeed be relevant for DSR research. However, it might be harder to formalize simply as an energy-dependent metric other aspects of a DSR
Minkowski limit (other than the dispersion relation). The dispersion relation is by construction a link between the energy of a given particle and its momentum, so it is clear in that context to what energy one should calibrate the metric, but in other contexts, especially when several energy scales are involved, a simple-minded implementation of an energy dependence of the metric might lead to ambiguities. I shall get back to this issue at a later point of the essay.

**Motivation for DSR from a Planck-scale particle-localization limit**

The key point I intend to make in this essay concerns a perspective on some much-discussed Planck-scale particle-localization limits that also suggests, in the same sense of the argument presented by Smolin in Ref. [23], that the availability of a DSR Minkowski limit might indeed be inevitable.

I intend to focus on the relation that describes the minimum uncertainty \( \delta x \) in the position of a particle of given (expectation value of) energy \( E \). In relativistic quantum field theory it is well established that [24, 25]

\[
\delta x \geq \frac{1}{E},
\]

and this relation is a key aspect of the interplay between (and ultimately responsible for the compatibility of) special relativity and quantum mechanics. If one wanted to localize a particle \( \mathcal{P} \) of mass \( m \), in a frame where it is at rest, with accuracy better than \( 1/m \), the procedure should necessarily (because of the Heisenberg uncertainty principle) involve an exchange of energy greater than \( m \) between the probe used in the procedure and \( \mathcal{P} \). But the availability of energy greater than \( m \) would be sufficient (according to special relativity) to create additional copies of \( \mathcal{P} \), thereby rendering the procedure results not meaningful [24, 25] as a localization of \( \mathcal{P} \). In the rest frame of the particle one therefore finds the following localization limit

\[
\delta x_{\text{rest}} \geq \frac{1}{m},
\]

where \( \delta x_{\text{rest}} \) denotes the uncertainty in any of the three spatial coordinates \( \delta x_{1,\text{rest}}, \delta x_{2,\text{rest}}, \delta x_{3,\text{rest}} \). Since the \( \delta x_i \) are lengths (the size of the “uncertainty interval” on the \( x_i \) axis), and are therefore subject to FitzGerald-Lorentz (boost) contraction, if indeed in its rest frame the particle is confined to a volume of size \( 1/m \), then an observer moving (with respect to the particle’s rest frame) with speed \( V \) along the “\( x_1 \)” direction would attribute to the particle a \( \delta x_1 = \delta x_{1,\text{rest}} \sqrt{1 - V^2} = \sqrt{1 - V^2} / m = 1/E \), while \( \delta x_2 = \delta x_3 = 1/m \). So clearly (3) must hold in any frame.

Whereas in nonrelativistic quantum mechanics one characterizes completely the limitations on particle localization through the Heisenberg relation

\[
\delta x \geq \frac{1}{\delta p},
\]

and there is still no absolute limitation on the accuracy of localization of a particle of any energy \( E \) (if one accepts a correspondingly large uncertainty in the momentum of the particle), in relativistic quantum mechanics (quantum field theory) one must take into account (as stressed above) the possibility of particle production, which introduces a further (and absolute) limitation, codified by (3), on the accuracy reachable in localization of a particle of energy \( E \).

The introduction of special-relativistic effects within the conceptual framework of quantum mechanics leads to an absolute limit on the localization of a particle of energy \( E \), but still allows the abstraction of a particle that can be sharply localized, in the \( E \to \infty \) limit. If one then also introduces
general-relativistic effects, according to an intuition which finds support in a large literature (see, e.g., Refs. [26, 27, 28, 29, 30]), the abstraction of sharp localization of a particle should be completely removed. Essentially the classical concept of a spacetime point would lose operative meaning, since no particle can ever provide “physical identity” to that spacetime point.

The expectation that (3) should fail at the Planck scale is based on the fact that the Planck length appears to set an absolute limit on the localization of a particle. According to (3) one could measure the position of a ultrahigh-energy particle with corresponding ultrahigh precision, and for particles of sufficiently high energy one could achieve even localization better than the Planck length. Several arguments [26, 27, 28, 29, 30] suggest that sub-Planckian localization accuracy should be impossible. The simplest such argument observes that in order to localize a particle with accuracy \( \delta x \), such that \( \delta x < L_p \) (where \( L_p \) denotes the Planck length), there should be a stage in the measurement procedure in which the probe and the particle whose position is being measured exchange energy greater than the Planck scale localized in a region of size smaller than the Planck length. This should lead to the formation of a black hole, which then prevents the localization procedure from completing successfully.

The intuition that sub-Planckian localization cannot be possible is shared by a large majority of the quantum-gravity community, but there has been no previous discussion of possible implications for equation (3), and the issue of consistency between the familiar form of Lorentz boosts and the Planck-length localization limit was never addressed. Most attempts of introducing formal structures that would reflect the Planckian localization limit were confined (sometimes implicitly) to the context of Planck-scale modifications of nonrelativistic quantum mechanics. In particular several papers (see, e.g., Ref. [31]) have used the Planck-length limit as motivation for the study of some Hilbert-space pictures that would be consistent with the existence of a minimum length.

I propose that any fruitful characterization of Planck-scale localization limits should include an analysis of the corresponding modifications of the relation (3). This is rather clear if one thinks of relativistic quantum field theory as the starting point for the search of quantum gravity, and it also reflects the fact that (3) is the ultimate uncertainty principle for localization in our current classical-spacetime theories, and therefore, as we seek, with quantum gravity, a new description of spacetime structure, modifications of (3) will provide the most intuitive characterization of possible nonclassical features in spacetime geometry. I will consider modifications of (3) such that the Planck-length localization limit is enforced, and it will become quickly clear that such modifications inevitably require departures from at least some special-relativistic laws. Let us consider first the possibility that boost still act in the familiar Lorentz way on energy, so that the relation between the energy \( E \) in our chosen frame and the rest energy \( m \) is still given by \( E = m/\sqrt{1-V^2} \), and energy is still unbounded from above. In this case the only way to ensure \( \delta x \geq L_p \) is to replace (3) with a formula of the type \( \delta x \geq f(E; L_p) \) where the function \( f \) has minimum value \( L_p \). But then of course boost-contraction of \( \delta x \) (the relation between \( \delta x \) and \( \delta x_{\text{rest}} \)) should be accordingly modified. For an explicit illustrative example of this possibility let us consider a simple “see-saw formula”

\[
\delta x \geq \frac{1}{E} + \frac{L_p^2}{4} E ,
\]

which is inspired by the “Generalized Uncertainty Principle” \( \delta x \geq \frac{1}{m_p} + \alpha \delta p \) that is often attributed to string theory in the literature [27, 31]. The added term \( L_p^2 E/4 \) ensures that \( \delta x \) can never be smaller than \( L_p \). If \( E \) still transforms under boosts in the special-relativistic way, then the covariance of (6) would require a Planck-scale-deformed boost contraction of lengths such that

\[
\delta x = \frac{\delta x_{\text{rest}} + \sqrt{\delta x_{\text{rest}}^2 - L_p^2}}{2} \sqrt{1-V^2} + \frac{L_p^2}{2 \left( \delta x_{\text{rest}} + \sqrt{\delta x_{\text{rest}}^2 - L_p^2} \right) \sqrt{1-V^2}}
\]
If instead we allow a Planck-scale-deformed action of boosts on energy, then actually equation (3) could even preserve its original form, as a relation between $\delta x$ and $E$ (but necessarily changing form as a relation between $\delta x$ and the rest mass of the particle $m$). To give once again an illustrative example, let me consider the following Planck-scale-modified boost relation between the energy in the chosen frame and the rest mass

$$E = \frac{1}{L_p} \tanh \left( \frac{L_p m}{\sqrt{1 - V^2}} \right),$$

(8)

which, as a result of familiar properties of the hyperbolic tangent, introduces a maximum energy $1/L_p$ for fundamental particles (but as long as $E \ll 1/L_p$ is in excellent agreement with $E = m/\sqrt{1 - V^2}$).

If this deformed boost relation between $E$ and $m$ was assumed, equation (3) would then automatically be consistent with the requirement $\delta x \geq L_p$.

Of course, as long as the only hint we have is the $\delta x \geq L_p$ relation, we cannot establish whether (3) should be modified explicitly, as in the illustrative example (7), or only implicitly, because of a change in the relation between $E$ and $m$, as in the illustrative example (8). Alternative candidate solutions of the quantum-gravity problem might actually lead to different modifications of relation (3). But we have seen that in any case a Planck-scale deformation of the action of boosts on at least some observables is required. If indeed the Minkowski limit of quantum gravity is affected by a Planck-scale localization limit, and if this emerges without spoiling the Galilei Relativity Principle, one should then conclude that this Minkowski limit is a DSR Minkowski limit.

**Encouragement from black-hole entropy results?**

Since the perspective I am adopting is the one of building a case for a DSR Minkowski limit on the basis of the argument reported by Smolin in Ref. [23] and on the basis of my description of a Planck-scale particle-localization limit, it is useful to stress that a Planck-scale particle-localization limit can also be independently motivated (as implicitly argued in Refs. [34, 35]) on the basis of the expectation [32, 33] of log corrections to the area-entropy relation for black holes. I can quickly reach this conclusion by revisiting the Bekenstein argument [36] for the area-entropy relation, using (6) in place of $\delta x \geq 1/E$.

As in the original Bekenstein argument [36], I take as starting point the general-relativity result [37] which establishes that the area of a black hole changes according to $\Delta A \geq 8\pi L_p^2 E s$ when a classical particle of energy $E$ and size $s$ is absorbed. Whereas Bekenstein describes the size of the particle in terms of the uncertainty in its position according to $s \sim \delta x \sim 1/E$, I shall assume $s \sim \delta x \sim 1/E + L_p^2 E/4$, thereby obtaining

$$\Delta A \geq \alpha L_p^2 - \beta \frac{L_p^4}{(\delta x)^2}. \quad (9)$$

where $\alpha$ and $\beta$ are numerical coefficients whose precise value is irrelevant for my point. I then use [34, 35] the fact that in falling in the black hole the particle acquires [38, 39, 40] position uncertainty $\delta x \sim R_S$, where $R_S$ is the Schwarzschild radius (and of course $A = 4\pi R_S^2$) to obtain

$$\Delta A \geq \alpha L_p^2 - \beta \frac{4\pi L_p^4}{A}. \quad (10)$$
From (10) one derives an area-entropy relation assuming (again following Bekenstein) that the entropy of the black hole depends only on its area and that the minimum increase of entropy should be, independently of the value of the area, $\ln 2$:

$$\frac{dS}{dA} \simeq \frac{\min(\Delta S)}{\min(\Delta A)} \simeq \frac{\ln 2}{\alpha L_p^2 - \beta \frac{4\pi L_p^4}{A}},$$

(11)

which gives (up to an irrelevant constant contribution to entropy)

$$S \sim \alpha' \frac{A}{L_p^2} + \beta' \ln \frac{A}{L_p^2}.$$  

(12)

Again for notational convenience I introduced numerical coefficients $\alpha'$ and $\beta'$. These coefficients are anyway not reliably computed using the Bekenstein argument. Of course, the original version of the Bekenstein argument, using only $\delta x \geq 1/E$, only includes the leading-order term, i.e. a linear relation between entropy and area. The famous result $\alpha' = 1/4$ for the coefficient of this linear term was first obtained by Hawking [41]. Improving the Bekenstein argument by using the Planck-scale particle-localization limit $\delta x \geq 1/E + L_p^2 E/4$ one finds also a log correction to the entropy-area relation, and this is indeed encouraging since both in String Theory and in Loop Quantum Gravity it has been argued that such log corrections should be present. Looking at this from a perspective bottom-to-top one could say that the arguments providing support for log corrections to the entropy-area relation also provide indirect support for a Planck-scale particle-localization limit of the type (6).

**A new avenue for DSR research**

The Doubly-special relativity idea was proposed [19] as an intuition for an open physics problem (rather than for a certain mathematical formalism): the Minkowski limit of quantum gravity should be described by a relativistic theory (a theory compatible with the Galilei Relativity Principle) with both the Planck scale and the speed-of-light scale as nontrivial relativistic invariant (scales which enter in the formulas that connect different observers). However, most of the attempts trying to introduce formalisms suitable for the description of a DSR Minkowski limit focused on the possibility that the modification of the laws of transformation between inertial observers would be such that space rotations and boosts should be described in terms of nonlinear realizations of the classical Lorentz symmetry group. The relevant work was so focused on this possibility that many readers from outside the “DSR community” have identified the concept of a “DSR Minkowski limit” and the mathematics of nonlinear realizations of the classical Lorentz symmetry group. Moreover, it is assumed that the classical Lorentz symmetry group should be realized nonlinearly in exactly the same way throughout the formalism (or at least on all aspects of the energy-momentum-space).

However, if one looks at the type of structures that quantum gravity might impose on its Minkowski limit one easily finds encouragement to look into other possible mathematical formalisms. This is in particular true of the Magueijo-Smolin “gravity rainbow” proposal for a DSR Minkowski limit and also for the argument I gave above based on a Planck-scale localization limit.

Let us first consider the possibility of a gravity-rainbow-type DSR Minkowski limit. As stressed above, the abstraction of an energy-dependent metric provides a rather intuitive description of Planck-scale modifications of the energy-momentum dispersion relation, but attempts to apply the generic
concept of an energy-dependent metric to other physically meaningful quantities, especially when several energy scales are involved, might easily run into ambiguities. I intend to argue that a scheme which may not run into ambiguities even when naively implemented is obtained if one replaces the energy-dependent metric with a corresponding statement of nonlinear relation between covariant four-momentum and contravariant four-momentum. After all (in an appropriate sense) the Minkowski limit does not really require us to make explicit reference to a metric. The ordinary $\eta_{\mu\nu}$ is only used to lower and raise indices, and in particular it is used to relate (linearly) the covariant four-momentum and the contravariant four-momentum. The energy dependence of the metric in the Minkowski limit could be a simple way to express a requirement of nonlinear relation between the covariant four-momentum and the contravariant four-momentum. One of the two (say, the covariant four-momentum) could still transform according to ordinary special relativity, but then the relativistic properties of the other would codify departures from the special-relativistic predictions. This leads one to consider a previously unexplored possibility for the construction of DSR Minkowski limits. Whereas usually in DSR research one assumes that the same nonlinear realization of the Lorentz symmetry group should be applied to all energy-momentum-space quantities, one should perhaps also contemplate the possibility that, say, the covariant four-momentum still transforms linearly under Lorentz transformations, while the contravariant four-momentum might indeed transform nonlinearly.

One can look from an analogous perspective at the point concerning a Planck-scale particle-localization limit which I made in this essay. One of the scenarios I considered assumed that the energy $E$ still transforms linearly under Lorentz boosts, but the position uncertainty $\delta x$ would transform nonlinearly (and of course then the relation between $E$ and $\delta x$ should be nonlinear).

In general one could notice that in the classical Minkowski limit various quantities, such as the covariant four-momentum, the contravariant four-momentum, and the frequency/wavenumber fourvector, all transform in the same linear way under Poincaré transformations, but in a Planck-scale-accurate description of the Minkowski limit some differences may arise, and, for example, the transformation rules of some of these quantities might still be linear, while some other of these quantities might transform nonlinearly.

**On the criteria for a DSR Minkowski limit**

Part of the thesis presented in this essay is that the search of formalisms suitable for the description of a DSR Minkowski limit might have been too narrow. There is a variety of physical postulates that one could consider for the Minkowski limit of a quantum-gravity theory, and it appears likely that each of this possibilities might require different mathematics for its description. The tendency by some authors to identify the “physics project” of a DSR Minkowski limit with some specific formalism has also led to some inconsistency in the terminology. Additional confusion is generated by studies...
in which the authors quickly conclude that they are proposing a DSR Minkowski limit whenever “the Planck length takes the role of an absolute scale”, without verifying that the “absolute scale” is such to require departures from some standard special-relativistic laws.

In light of this possibility of confusion it is perhaps useful to contemplate explicitly some possible roles for the Planck scale that would indeed require a DSR Minkowski limit, and some that would not. And let me start by observing that if in the Minkowski limit of a given quantum-gravity theory one had the Planck length setting an observer-independent minimum allowed value of wavelength, then of course one would be dealing with a DSR Minkowski limit. Under special-relativistic boosts wavelengths contract, and therefore in order to enforce an observer-independent minimum-wavelength law one should necessarily introduce departures from special relativity, and the observer independence of the postulated new law should allow to accommodate the departures from special relativity in such a way that the Galilei Relativity Principle would still hold. A role for the Planck length as observer-independent minimum wavelength would require a modification of special relativity just like one needs to modify Galilei Relativity in order to accommodate a maximum-speed law (speeds transform linearly under Galilei boosts).

Similarly a DSR Minkowski limit would necessarily arise in a quantum-gravity theory which in the Minkowski limit predicts the existence of some absolutely fundamental particles whose energy is constrained by an observer-independent bound \( E \leq 1/L_p \). But of course if the Minkowski limit introduces instead a bound on the mass (rest energy) of the particles then instead there is no a priori reason for expecting DSR structures. Mass is an invariant of Poincaré transformations, so an observer-independent bound on mass does not necessarily affect Poincaré symmetry. A useful example of the situation in which an absolute scale does not affect symmetries is provided by the Planck constant \( \hbar \) in the quantum mechanics of angular momentum. Angular momentum transforms under space rotations, but \( \hbar \) is most fundamentally a scale affecting the square modulus of angular momentum, \( L_x^2 + L_y^2 + L_z^2 \), which is an invariant under space rotations, and in fact the scale \( \hbar \) can be introduced without affecting space-rotation symmetry.

For what concerns the fate of Poincaré symmetry in the Minkowski limit of quantum gravity it is therefore crucial to establish whether the Planck scale is introduced “a la c” (the speed bound introduced through \( c \) required a deformation of Galilei boosts) or is introduced “a la \( \hbar \)” (the properties of \( L_x^2 + L_y^2 + L_z^2 \) introduced through \( \hbar \) do not affect in any way space-rotation symmetry). An early attempt to introduce a length scale (possibly the Planck length) in spacetime structure in such a way that it would not require any modification of Poincaré symmetry is the one of Snyder [44], who indeed postulated some spacetime noncommutativity and then went to great length to show that the system is still Poincaré invariant. Some confusion may arise from the fact that in some recent papers (see, e.g., Ref. [45]) there has been some discussion of a “Snyder-type modification of special relativity”\(^3\), as this terminology misses the point that Snyder was trying to prove\(^4\) just the opposite: an absolute length scale can be introduced without modifying Poincaré symmetry.

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\(^3\)Besides the possibly confusing terminology for “Snyder deformations” another potential element of confusion originating from the terminology adopted in Ref. [45] (and references therein) concerns the fact that the authors rename doubly special relativity as “deformed special relativity”, but the name “deformed special relativity” had already been used in the literature to describe research with completely different physics motivation and formal approach.

\(^4\)While Snyder should be credited for the idea of introducing the Planck scale in spacetime structure in such a way not to affect Poincaré symmetry, it is actually still unclear whether Snyder succeeded. Some of the tools more recently developed to analyze noncommutative geometries were not available to Snyder. Even now we only have a reliable description of rotation and boost transformations in the Snyder spacetime, which are indeed undeformed, whereas it is still unclear how to properly describe translations, which are often affected by severe ambiguities in noncommutative geometry (see, e.g., Ref. [13]).
Going back to a list of concepts which require or do not require the concept of a DSR Minkowski limit, let me now focus on the minimum-uncertainty intuition that is common in the quantum-gravity literature, and is relevant for my observation concerning the Planck-scale particle-localization limit. If the Minkowski limit of quantum gravity predicts an observer-independent bound on the measurability of lengths ($\delta L \geq L_p$) then necessarily this would have to be a DSR Minkowski limit. Instead there is no need to introduce departures from Poincaré symmetry if there is a bound on the measurability of proper lengths (the length of an object in its rest frame is of course a Poincaré-invariant quantity).

And in closing, since there is literature on the possibility of a maximum acceleration [47, 48], let me also stress that acceleration is a Poincaré invariant, and therefore a quantum-gravity theory predicting an observer-independent upper bound on acceleration will not necessarily lead to a DSR Minkowski limit.

While this is only a very limited list of examples it should suffice as a warning that it is not sufficient to argue that “the Planck length takes the role of an absolute scale” in order to provide support for a DSR Minkowski limit. One must go through the (sometimes tedious, but extremely valuable) exercise of deriving the explicit form of the laws of transformation between observers, verifying that indeed the transformation laws are Planck-scale modified.

**Outlook and some challenges to the community**

The argument presented by Smolin in Ref. [23] and the argument presented here combine to provide the first pieces of a “physics case” favouring the hypothesis of a DSR Minkowski limit, as proposed in Ref. [19]. It is of course important for the overall research programme to look for other possible hints in the DSR direction, but also to look for possible loopholes in these two arguments.

Whether or not these two arguments fully establish that a DSR Minkowski limit is inevitable for quantum gravity, they clearly suggest that the search of formalisms suitable for the description of a DSR Minkowski limit has been too narrow. Some authors have identified the hypothesis of a DSR Minkowski limit with one or another specific formalism. In turn this has led other authors to conclude that their own preferred framework is not suitable for DSR research, without an explicit verification of the role of the Planck scale in relativistic transformations, but simply observing some differences from the most popular formalisms so far adopted in the DSR literature. An example of this type of situations is provided by research on “stable symmetry algebras” (see, e.g., Ref. [49]), which, understandably, has been viewed (see, e.g., Ref. [50]) as an alternative to the type of mathematics most popular in the DSR literature. Still, a fully physical characterization of the laws of transformation of observables between different observers in frameworks based on stable symmetry algebras is still missing, and it may well be that actually some stable symmetry algebras turn out to be useful in the description of a DSR Minkowski limit.

In developing the thesis presented in this essay I also implicitly raised a challenge for those researchers in the quantum-gravity community who are seeking a “fundamental role for the Planck scale” without paying attention to the differences between the various types of fundamental scales that are possible in physics. For example, several papers adopt the hypothesis that the Planck length should set the minimum allowed value for wavelengths, but before the proposal in Ref. [19] of the idea of a DSR Minkowski limit this minimum-wavelength studies never explored the implications for special relativity. Similarly, there is a large literature on a vague hypothesis that the Planck length should set the absolute limit on the measurability of lengths, but the relevant studies often do not even provide an explicit statement concerning whether this absolute limit applies to the measurement of proper lengths or to the measurement of the length in any frame. And when a limit on the measurability of
lengths in any frame is assumed, the authors often still (even now that there is some literature on DSR Minkowski limits) do not comment on the implications for special relativity. This point is particularly embarrassing in the case of some of the studies based on a “Generalized Uncertainty Principle” attributed to string theory: nobody appears to notice that any attempt of enforcing in full generality an uncertainty principle of the type $\delta x \geq \frac{1}{\delta p} + \alpha \delta p$ will of course require departures from special relativity. The opposite attitude is equally dangerous for what concerns the amount of confusion produced in the literature: some authors, once they have established that in a chosen framework the Planck length has the role of “fundamental scale”, quickly jump to the conclusion that they are dealing with a DSR Minkowski limit, whereas in order to draw such a conclusion one should first make sure that the Planck scale affects nontrivially the laws of transformation between inertial observers.

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