Pion–deuteron scattering length in Chiral Perturbation Theory up to order $\chi^{3/2}$

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Abstract

A complete calculation of the corrections to pion-deuteron scattering length up to order $\chi^{3/2}$ with $\chi = m_\pi / M_N$ is performed. The calculation includes the dispersive contributions and corrections due to the explicit treatment of the $\Delta$ resonance. $s$-wave pion-nucleon scattering parameters are extracted from a combined analysis of modern experimental data.

1 Introduction

The pion-nucleon ($\pi N$) scattering lengths are fundamental quantities of low-energy hadron physics since they test the QCD symmetries and the pattern of chiral symmetry breaking. As stressed by Weinberg long time ago, chiral symmetry suppresses the isoscalar $\pi N$ scattering length $a^+$ substantially compared to its isovector counterpart $a^-$. Thus, a precise determination of $a^+$ demands in general high accuracy experiments.

Here pion-deuteron ($\pi d$) scattering near threshold plays an exceptional role for $\text{Re}(a_{\pi d}) = 2a^+ + (\text{few–body corrections})$. The first term $\sim a^+$ is simply generated from the impulse approximation (scattering off the proton and off the neutron) and is independent of the deuteron structure. Thus, if one is able to calculate the few–body corrections in a controlled way, $\pi d$ scattering is a prime reaction to extract $a^+$ (most effectively in combination with pionic hydrogen measurements).

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A method how to calculate processes on few nucleon systems with external probes was proposed by Weinberg in one of his classical papers [1]. As a first step the perturbative transition operators need to be calculated using the rules of ChPT. Then those transition operators must be convoluted with the appropriate $NN$ wave functions. This scheme was already applied to a large number of reactions like $\pi d \rightarrow \pi d$ [2], $\gamma d \rightarrow \pi^0 d$ [3, 4], $\pi^3\text{He} \rightarrow \pi^3\text{He}$ [5], $\pi^-d \rightarrow \gamma nn$ [6], and $\gamma d \rightarrow \pi^+nn$ [7], where only the most recent references are given. The standard expansion parameter $\chi = m_\pi / M_N$, where $m_\pi (M_N)$ is the pion (nucleon) mass, was used in most of these references.

It was also Weinberg who calculated the leading order few-body corrections to the $\pi d$ system, the most important of which – the diagram when pion rescatters on two nucleons with the Weinberg-Tomozawa (WT) vertices – is almost as large as the experimental value for the $\pi d$ scattering length. The diagrams at leading order were calculated in the fixed center kinematics, i.e. with static nucleons. An accounting for the nucleon recoils leads to potentially sizable corrections of order of $\chi^{1/2}$ in the standard Weinberg counting. The non-analyticity of this correction is related to the few–body singularities that are employed in some pion–few-nucleon diagrams as demonstrated in Refs. [7, 8]. It was the main result of Ref. [8] that the importance of the resulting effect of all recoil terms is directly connected to the Pauli principle for the nucleons in the intermediate state. In particular, if the s-wave $NN$-state is not allowed by quantum numbers, which is fulfilled in the $\pi d$ process, the net effect of the recoil correction is to be small due to a cancellation of individually large terms. At next-to-leading (NLO) order there are basically the same diagrams as at LO but with subleading vertices. The calculation performed in Ref. [2] showed that the sum of diagrams contributing at NLO vanishes. Furthermore, the solution for $\{a^+, a^-\}$ was found in Ref. [2] from a common intersection of three bands corresponding to the shift and width of pionic hydrogen atom [9] and to the shift of pionic deuterium [10]. Due to a cancellation of terms at orders $\chi^{1/2}$ and $\chi$ the results for $\{a^+, a^-\}$ extracted in ChPT [2] turned out to be quite similar to the phenomenological calculations [11, 12]. However, the recent measurement of the width of $\pi^-p$ atom with much better accuracy [13] seriously changes the picture. The problem is that an intersection region of the three bands and thus a unique solution for $\{a^+, a^-\}$ does not exist anymore. This finding means that something important is missing in our understanding of the $\pi d$ system. This could be isospin symmetry breaking (ISB) effects that were recently found in Ref. [14] to give a huge effect to the $\pi d$ scattering although with large uncertainty. Another possibility is that higher order effects to the transition operators could be important. In this presentation we discuss both possibilities. The influence of ISB effects on the extraction of the s-wave $\pi N$ scattering lengths
is considered in sec. 2. In sec. 3 and 4 we discuss corrections to the $\pi d$ scattering length emerging at order $N^{3/2}\text{LO (}\chi^{3/2})$. At this order two new classes of diagrams start to contribute. One of them, the so-called dispersive correction due to the process $\pi d \to NN \to \pi d$ is the subject of sec. 3. In sec. 4 we discuss the effect of the $\Delta$ isobar as explicit degree of freedom. The main results are summarized in sec. 5.

2 ISB effects and s-wave $\pi N$ scattering lengths

Since the leading one-body contribution ($\sim a^+$) to the $\pi d$ scattering length is chirally suppressed the role of ISB effects in this process becomes significant. For the $\pi d$ system so far only leading ISB corrections were evaluated [14]. They were found to give a very large effect of order of 40% to the $\pi d$ scattering length. In this section we would like to investigate the influence of this correction on the combined analysis of experimental data and thus on the s-wave $\pi N$ scattering lengths. To account for the ISB correction at leading order we should replace $2a^+ + a^- + a^-$ in the expression for $a_{\pi d}$, which agrees to the former only, if isospin were an exact symmetry. The expressions for the $\pi N$ amplitudes with inclusion of ISB effects were derived in Ref. [14]:

$$a_{\pi^- p} = a^+ + a^- + \frac{1}{4\pi(1+\chi)} \left( \frac{4(m_{\pi}^2 - m_{\pi^0}^2)}{F_\pi^2} c_1 - \frac{e^2}{2} (4f_1 + f_2) \right),$$

$$a_{\pi^- n} = a^+ - a^- + \frac{1}{4\pi(1+\chi)} \left( \frac{4(m_{\pi}^2 - m_{\pi^0}^2)}{F_\pi^2} c_1 - \frac{e^2}{2} (4f_1 - f_2) \right).$$

(1)

Here $m_{\pi}(m_{\pi^0})$ is the charged (neutral) pion mass, $F_\pi = 92.4$ MeV and $c_1$ and $f_1, f_2$ correspond to the strong and electromagnetic LECs. Whereas $f_2$ and $c_1$ are known more or less well ($f_2 = -(0.97 \pm 0.38)$ GeV$^{-1}$ [15] and $c_1 = -0.9^{+0.2}_{-0.5}$ GeV$^{-1}$ [16]) the value for $f_1$ ($|f_1| \leq 1.4$ GeV$^{-1}$) is very uncertain – a naive dimensional analysis was used in Ref. [14] to fix the latter. At the same time it is $f_1$ and $c_1$ that give the largest contribution to the ISB correction for $\pi d$ scattering thus introducing a large uncertainty in the extraction of $\{a^+ , a^-\}$ from the data. At this stage we would like to note that the parameters $a^+, c_1$ and $f_1$ enter the expressions for $a_{\pi^- p}$ and $a_{\pi^- n}$ in the same linear combination (see Eq. (1)). Note that the expression for the charge exchange amplitude $\pi^- p \to \pi^0 n$ does not depend on the LECs $c_1$ and $f_1$ at all. Therefore let us introduce the quantity $\tilde{a}^+$ which is defined as

$$\tilde{a}^+ = a^+ + \frac{1}{4\pi(1+\chi)} \left( \frac{4(m_{\pi}^2 - m_{\pi^0}^2)}{F_\pi^2} c_1 - 2e^2 f_1 \right).$$

(2)
The leading isospin breaking terms that are also the main source of the uncertainty are contained now in $\tilde{a}^+$. Using this the expressions for $a_{\pi^-p}$, $a_{\pi^-n}$ and $\text{Re} a_{\pi d}$ take the form

$$a_{\pi^-p} = \tilde{a}^+ + a^- - \frac{1}{4\pi(1 + \chi)} \frac{e^2}{2} f_2,$$

$$a_{\pi^-n} = \tilde{a}^+ - a^- + \frac{1}{4\pi(1 + \chi)} \frac{e^2}{2} f_2,$$

$$\text{Re} a_{\pi d} = 2\tilde{a}^+ + \langle\text{few–body corrections } (a^-)\rangle,$$  \hspace{1cm} (3)

Thus, we get a system of three equations for $a_{\pi^-p}$, $\text{Re} a_{\pi d}$ and $a_{\pi^-d \rightarrow \pi^0n}$ (the explicit expression for the latter is given in Ref. [14]) to determine $\tilde{a}^+$ and $a^-$. Once this determination is performed and provided that new and less uncertain information about the LECs $c_1$ and $f_1$ is available from elsewhere, one will be able to extract $a^+$ directly without doing the analysis of pionic data once again. Let us discuss Eq. (3) in more detail. Note that the equations for the hydrogen and deuterium shifts ($a_{\pi^-p}$ and $\text{Re} a_{\pi d}$ respectively) written in terms of $\tilde{a}^+$ and $a^-$ and including ISB effects at leading order are very close to those obtained in the isospin symmetric case in Ref. [2] in terms of $a^+$ and $a^-$. The difference basically consists in the term proportional to $f_2$ for the pionic hydrogen shift that gives a relatively small effect. Thus the main modification due to the inclusion of ISB effects at leading order consists in the replacement of $a^+$ by $\tilde{a}^+$. At the same time we know that for the isospin symmetric case there is no unique solution for $\{a^+, a^-\}$ if the new data for the hydrogen width are utilized. Thus we conclude that the system of equations for $\{\tilde{a}^+, a^-\}$ does not have a unique solution either at least as long as the few body corrections of Ref. [2] are used. In the following sections we present the recent progress in this sector.

### 3 Dispersive corrections

Experimental measurement of the $\pi d$ scattering length shows that its imaginary part is relatively large, about 1/4 of its real part [10]. The imaginary part can be expressed in terms of the $\pi d$ total cross section through the optical theorem. One gets

$$4\pi \text{Im}(a_{\pi d}) = \lim_{q \rightarrow 0} q \sigma(\pi d \rightarrow NN) + \sigma(\pi d \rightarrow \gamma NN)), \hspace{1cm} (4)$$

\[\text{The idea of using some linear combinations of observables to reduce the uncertainty was suggested in Refs [14,17]. In particular the combination } 2a_{\pi^-p} - a_{\pi^-d}\text{ that depends solely on } a^-\text{ was considered in Ref. [17].}\]
where \( q \) denotes the relative momentum of the initial \( \pi d \) pair. The ratio 
\[ R = \lim_{q \to 0} (\sigma(\pi d \to NN)/\sigma(\pi d \to \gamma NN)) \]
was measured to be \( 2.83 \pm 0.04 \) [18]. At low energies diagrams that lead to a sizable imaginary part of some 
amplitude are expected to contribute also significantly to its real part. Those 
contributions are called dispersive corrections. As a first estimate Br"uckner 
speculated that the real and imaginary part of these contributions should be of the same order of magnitude [19]. This expectation was confirmed 
within Faddeev calculations in Refs. [20]. Here we present the first consistent 
ChPT calculation of the dispersive corrections that emerge from the hadronic 
\( \pi d \to NN \to \pi d \) and photonic \( \pi d \to \gamma NN \to \pi d \) processes [21]. We define 
dispersive corrections as contributions from diagrams with an intermediate 
state that contains only nucleons, photons and at most real pions. Therefore, 
all the diagrams shown in Fig. 1 are included in our work. All these diagrams 
contribute at order \( \chi^{3/2} \) as compared to the leading double scattering dia-
gram (see Ref. [21] for details). The hatched blocks in the diagrams of Fig. 1 
refer to the relevant transition operators for the reaction \( NN \to NN \pi \) that 
consist of the direct and rescattering mechanisms. Note that the latter is to 
be calculated with the on–shell \( \pi N \to \pi N \) vertices (\( 2m_\pi \)) as was derived in 
Ref. [22]. Using the CCF potential [23] for the \( NN \) distortions we found for 
the dispersive correction from the purely hadronic transition

\[ \delta a_{\pi d}^{\text{disp}} = (-6.5 + 1.3 + 2.4 - 0.2) \times 10^{-3} \ m_\pi^{-1} = -3.0 \times 10^{-3} \ m_\pi^{-1}, \]

where the numbers in the first bracket are the individual results for the 
diagrams shown in Fig. 1 in order. Note that the diagrams with intermediate \( NN \) interactions and the crossed ones (diagram (c) and (d)), neither 
of them were included in most of the previous calculations, give significant 
contributions. When repeating the calculation with the four different phe-
nomenological \( NN \) potentials CD Bonn [24], Paris [25], AV18 [26] we find

\[ \delta a_{\pi d}^{\text{disp}} = (-2.9 \pm 1.4) \times 10^{-3} \ m_\pi^{-1}, \]

where the first number is the mean value for the various potentials and the 
second number reflects the theoretical uncertainty of this calculation esti-
mated conservatively — see Ref. [21] for details. Note that the same calculation gave very nice agreement for the corresponding imaginary part [21].

In Ref. [21] also the electromagnetic contribution to the dispersive correction was calculated. It turned out that the contribution to the real part was tiny — $-0.1 \times 10^{-3} m_\pi^{-1}$ — while the sizable experimental value for the imaginary part was described well.

### 4 Role of the Delta resonance

From phenomenological studies it is well known that the delta isobar $\Delta(1232)$ plays a very special role in low energy nuclear dynamics [27] as a consequence of the relatively large $\pi N \Delta$ coupling and the quite small delta–nucleon mass difference $\Delta = M_\Delta - M_N \simeq 2m_\pi$, where $M_\Delta$ denotes the mass of the delta.

In the present section we investigate the role of the $\Delta$ isobar in the reaction $\pi d \rightarrow \pi d$ at threshold in EFT. In the delta-less theory the effect of the $\Delta$ resonance is hidden in the LEC $c_2$ which is the leading term in the so-called boost correction to the $\pi d$ scattering length [2]. This correction is known to be quite sizable ($\sim 10\text{–}20\%$ of $a_{\pi d}$) although very model dependent. The pertinent one–body operator scales with the square of the nucleon momentum and therefore the corresponding expectation value is proportional to the nucleon kinetic energy inside the deuteron — this quantity is strongly model-dependent [28]. However, the value of $c_2$ is reduced by a large factor once the delta contribution is taken out [29, 30] so that the residual boost correction becomes negligible [31].

The reason why the explicit inclusion of the delta in pionic reactions on the two–nucleon system is beneficial is that the dynamical treatment of the $\Delta$ allows to improve the convergence of the transition operators. Let us, for example, focus on the one–body terms with the delta (see, e.g., second diagram in Fig. 2). Then the corresponding $\pi N \rightarrow \pi N$ transition potential is proportional to $p^2/(m_\pi - \Delta - \vec{p}^2/M_N)$. For static deltas, the nucleon-delta propagator reduces to $1/(m_\pi - \Delta)$. Thus, in the latter case the transition operator behaves like $\vec{p}^2$, whereas in the former it approaches a constant for momenta larger than $|\vec{p}_\Delta| \sim \sqrt{(\Delta - m_\pi)M_N} \sim 2.7m_\pi$ with the effect that the static amplitude is much more sensitive to the short range part of the deuteron wave function and must be balanced by appropriate counter terms.

The value of $p_\Delta$ is numerically very close to $p_{\text{thr}} = \sqrt{M_Nm_\pi} \sim 2.6m_\pi$ — the minimum initial momentum for the reaction $NN \rightarrow NN\pi$ (for a recent review of this class of reactions see Ref. [32]). This automatically puts the delta contributions in the same order as the dispersive corrections [31]. In Fig. 2 we show diagrams with the $\Delta$ isobar that contribute at order $\chi^{3/2}$. Diagrams with crossed external pions are not shown explicitly but are taken
into account in the calculation. The resulting correction is \[31\]

\[ \delta a_{\pi d}^\Delta = (2.38 \pm 0.40) \times 10^{-3} m_\pi^{-1}, \] (7)

where the central value is the arithmetic average of the results for the seven different potentials and the uncertainty reflects the variations in the results. Note that we used phenomenological \(NN\) models without \([24–26]\) and with \([23]\) explicit delta degree of freedom, as well as three variants of \(NN\) wave functions derived within EFT \([33]\). These numbers were obtained with the \(\pi N\Delta\) coupling constant \(h_A = 2.77\). In contrast to earlier treatments of the boost correction, the results we found with the explicit treatment of the \(\Delta\) depend only very weakly on the \(NN\) model used. In Ref. \([31]\) also a detailed comparison to previous phenomenological works is given.

\section{5 Results and Conclusions}

We performed a complete calculation of the isospin-conserving corrections to the pion-deuteron scattering length up to order \(\chi^{3/2}\). The calculation includes the dispersive contributions and corrections due to the dynamical treatment of the \(\Delta\) resonance. Although these corrections are quite significant individually the net effect of the diagrams that contribute at order \(\chi^{3/2}\) is very small:

\[ \delta a_{\pi d}^\Delta + \delta a_{\pi d}^{\text{disp}} = (-0.6 \pm 1.5) \times 10^{-3} m_\pi^{-1}. \] (8)

However, an important consequence of our investigations is that once the \(\Delta\) is treated dynamically, as it is done here, the so-called boost corrections contribute insignificantly to the \(\pi d\) scattering length.

Also we analyzed the role of ISB effects at leading order in the combined analysis of pionic data. It was observed that the LEC \(f_1\), that is known very poorly, appears in the expressions for \(a_{\pi^-p}\) and \(a_{\pi^-n}\) in the same linear combination with \(a^+\) and the LEC \(c_1\). We called it \(\tilde{a}^+\). Thus, the inclusion of ISB effects at leading order consists basically in the replacement of \(a^+\)
by $\tilde{a}^+$ in the combined analysis of pionic data. This drastically reduces the uncertainty of the analysis that originates mainly from our ignorance regarding $f_1$. The solution for the s-wave $\pi N$ parameters $\{\tilde{a}^+, a^-\}$ is shown in Fig. 3. The black band stems from the analysis of the pionic hydrogen shift [13]. The blue vertical band corresponds to the new preliminary data for the pionic hydrogen width [34]. The red solid and dashed bands correspond to the pion deuteron scattering length calculated with and without corrections at order $\chi^{3/2}$. Also the boost correction was not included in the full calculation corresponding to the solid red band as a consequence of the explicit treatment of the $\Delta$ resonance. It is basically the latter effect that improves the situation resulting in some intersection region for all three bands. However, it still remains to be seen if the corrections at NLO of the isospin violation do not distort this picture. Corrections at this order for the $\pi^-p$ system were evaluated in Refs. [15, 35] and turned out to be quite sizable, especially those that come from the pion mass difference. In order to push also the calculation for the $\pi d$ system to a similar level of accuracy in isospin violation, the $\pi^-n$ scattering amplitude as well as some virtual photon exchanges in the $\pi^-d$ system are still to be calculated.

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