SU(3) and Nonet Breaking Effects in $K_L \to \gamma\gamma$

Induced by $s \to d + 2\text{gluon}$ due to Anomaly

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Abstract

In this paper we study the effects of $s \to d + 2\text{gluon}$ on $K_L \to \gamma\gamma$ in the Standard Model. We find that this interaction can induce new sizeable SU(3) and U(3) nonet breaking effects in $K_L - \eta, \eta'$ transitions and therefore in $K_L \to \gamma\gamma$ due to large matrix elements of $\langle \eta(\eta') | \alpha_s C_{\mu\nu}^a \tilde{G}^\mu\nu_a | 0 \rangle$ from QCD anomaly. These new effects play an important role in explaining the observed value. We also study the effects of this interaction on the contribution to $\Delta m_{K_L - K_S}$.

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It is well known that contributions from intermediate hadronic state effect play an important role in many low energy processes. Some of the notable examples are $K_L \to \gamma \gamma$ [1,2] and $\Delta m_K = m_{K_L} - m_{K_S}$ [3–7]. For $K_L \to \gamma \gamma$, the direct contribution due to quark level $s \to d \gamma \gamma$ alone accounts for only a small portion of the amplitude measured experimentally [2,7]. For $\Delta m_K$, the direct contribution due to $\Delta S = 2$ four quark operator is again only a fraction of the experimental value depending on the value of the bag factor $B_K$ [3,4]. A simple method to estimate the contributions from intermediate hadronic states is the pole dominance approximation in which one assumes that a few low lying resonances saturate the contribution. The commonly identified resonances in the above two cases are $\pi^0$, $\eta$ and $\eta'$. Combining with U(3) flavor symmetry, the $K_L \to \gamma \gamma$ amplitude can be estimated [2,6]. If U(3) nonet is a good symmetry, the calculations are straightforward. However, not only nonet but also SU(3) are known to be broken, there are large uncertainties in these calculations. One should also study that if there are some new contributions in the Standard Model (SM) which have not been examined so far. In this paper we show that indeed there is a new contribution to to $K_L \to \gamma \gamma$ and $\Delta m_K$. This new contribution comes from $s \to d + 2$gluon induced $K - \eta(\eta')$ transition, and the intermediate $\eta(\eta')$ subsequently decay into $\gamma \gamma$ or change to another neutral kaon through the usual $\Delta s = 1$ interaction. We find that, because of the large QCD anomaly hadronic matrix element $\langle \eta(\eta') | \alpha_s G_{\mu\nu}^{a} \tilde{G}_{\mu\nu}^{a} | 0 \rangle$ ($\tilde{G}_{\mu\nu}^{a} = \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^{a}$), the new contributions are sizeable and also induce new sizeable SU(3) and U(3) breaking effects.

The decay amplitude $A_{dir}$ of the direct contribution to $K_L \to \gamma \gamma$ from quark level interaction $s \to d \gamma \gamma$ in the SM has been studied before [1,2,7]. Here we improve the calculations by including QCD corrections which also serve to set up our notations. In the SM, $s \to d \gamma \gamma$ can be generated at one loop level by exchanging a $W$ boson and quarks with two photons emitted from particles in the loop and particles in the external legs. The QCD corrected effective Hamiltonian for $s \to d \gamma \gamma$ is given by

$$H_{\text{eff}}(s \to d \gamma \gamma) = M_{\text{IR}}^{\gamma} + M_{R}^{\gamma},$$

(1)
where $M_{IR}^{\gamma\gamma}$ is the irreducible contribution with the two photons emitted from particles in the loop. $M_{R}^{\gamma\gamma}$ is the reducible contribution with at least one photon emitted from an external $s$ or $d$ quark.

The irreducible contribution $M_{IR}^{\gamma\gamma}$ is given by [2,7,8]

$$M_{IR}^{\gamma\gamma} = -i\frac{16\sqrt{2}\alpha_{em}G_F}{9\pi}N\alpha_2\epsilon^{*\mu}(k_2)\frac{1}{2k_1 \cdot k_2} \sum_{i=u,c,t} V_{id}V_{is}F(x, x_i)\bar{d}\gamma^\rho LR_{\mu\nu\rho\sigma}\epsilon^{*\mu}(k_1), \quad (2)$$

Here $\epsilon^\mu(k)$ is the photon polarization vector with momentum $k$, $L(R) = [1 - (+)\gamma_5]/2$, $N = 3$ is the number of colors, $a_2 = c_1 + c_2/N$, $x = 2k_1 \cdot k_2/m_W^2$, $x_i = m_i^2/m_W^2$, and $R_{\mu\nu\rho} = k_{1\mu}\epsilon_{\mu\nu\rho\lambda}k_{2\lambda}^0 - k_{2\mu}\epsilon_{\nu\rho\lambda}k_{1\lambda}^0 + k_1 \cdot k_2\epsilon_{\mu\nu\rho\sigma}(k_2 - k_1)^\sigma$. The function $F(x, x_i)$ is given by

$$F(x, x_i) = \frac{x_i}{x} \int_0^1 \frac{\ln[1 - y(1-y)x/x_i]}{y} dy. \quad (3)$$

The reducible contribution $M_{R}^{\gamma\gamma}$ is given by [7,8]

$$M_{R}^{\gamma\gamma} = \frac{\sqrt{2}\alpha_{em}}{6\pi}\sum_{i=u,c,t} V_{id}V_{is}c_i x_{12}d[\frac{1}{p_d \cdot k_1} - \frac{1}{p_s \cdot k_2}]\sigma_{\mu\beta}\sigma_{\nu\alpha} k_1^\beta k_2^\alpha + 2i(\frac{p_{d\mu}}{p_d \cdot k_1} - \frac{p_{s\mu}}{p_s \cdot k_1})\sigma_{\nu\beta} k_2^\beta + (k_1 \rightarrow k_2, k_2 \rightarrow k_1; \mu \rightarrow \nu, \nu \rightarrow \mu)(m_dL + m_sR)\epsilon^{*\mu}(k_1)\epsilon^{*\nu}(k_2). \quad (4)$$

In the above $c_i$ are the Wilson coefficients defined in the following $\Delta S = -1$ effective Hamiltonian [9]

$$H_{eff}(\Delta S = -1) = \frac{4G_F}{\sqrt{2}}[V_{qd}^*V_{qs}(c_1O_1 + c_2O_2) - \sum_k \sum_{i=u,c,t} V_{id}^*V_{is}(c_i^kO_k)], \quad (5)$$

where the summation over $k$ is on all possible operators, four quark operators, quark-photon and quark-gluon operators, which are defined in Ref. [9]. The operators directly relevant to our calculations to the leading order are

$$O_1 = \bar{q}\gamma_\mu Lq\bar{d}\gamma^\mu Ls, \quad O_2 = \bar{d}\gamma_\mu Lq\bar{q}\gamma^\mu Ls,$$

$$O_{7\gamma} = \frac{e}{16\pi^2}\bar{d}\sigma_{\mu\nu}F^{\mu\nu}(m_dL + m_sR)s, \quad O_{SG} = \frac{g_s}{16\pi^2}\bar{d}\sigma_{\mu\nu}T^aG_a^{\mu\nu}(m_dL + m_sR)s,$$

where $G_a^{\mu\nu}$ and $F^{\mu\nu}$ are the gluon and photon field strengths. Here we have also written down the operator $O_{SG}$ which is needed for the study of $s \rightarrow dgg$. 

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To obtain the amplitude $A_{dir}$ for $K_L \to \gamma\gamma$ from the effective Hamiltonian $H_{\text{eff}}(s \to d\gamma\gamma)$, one needs to bind the $d$ and $s$ quarks to form a kaon which involves long distance non-perturbative QCD effects. This effect cannot be calculated at present and is usually parameterized by a decay constant $f_K$ as, $\langle 0|\bar{d}\gamma^\mu\gamma_5s|\bar{K}^0\rangle = -if_KP_\mu^k$ with $f_K$ determined from data. We have,

$$A_{dir}(\bar{K}^0 \to \gamma\gamma) = \langle \gamma\gamma|H_{\text{eff}}(O_{7\gamma})|\bar{K}^0\rangle$$

$$= \frac{2\sqrt{2}\alpha_{em}G_F}{9\pi}f_K[i(Na_2V_{ud}V_{us} + 3\xi c^\gamma_{7\gamma}V_{td}^*V_{ts})F_{\mu\nu}\tilde{F}^{\mu\nu} + 3\xi c^\gamma_{7\gamma}V_{td}^*V_{ts}F_{\mu\nu}F^{\mu\nu}],$$

where $H_{\text{eff}}(O_{7\gamma})$ indicates the term proportional to $O_{7\gamma}$ in the effective Hamiltonian of eq. (5). $\tilde{F}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$.

In obtaining the above result, we have used the fact that $F(x, x_c,t) \approx -1/2$ (large $x_c,t/x$), and $F(x, x_u) \approx 0$ (small $x_u/x$). We also neglected small contributions from $c^u_{7\gamma}$ which are proportional to $x_u,c$ [10], but have kept $c^\gamma_{7\gamma}$ which is $-0.3$ in the SM.

The parameter $\xi$ is an average value of the quantity, $\kappa = -(m_K^2/16)(1/p_d\cdot k_1 - 1/p_s\cdot k_2 + 1/p_d\cdot k_2 - 1/p_s\cdot k_2)$. If one assumes that the $d$ and $s$ quarks share equally the kaon momentum, then $\xi = 1$ [2]. We have also estimated $\xi$ by calculating the quantity $<0|\kappa\bar{d}(1 + \gamma_5)s|\bar{K}^0>$ using perturbative QCD method and appropriate distribution amplitude of quarks in the kaon [11]. This approach also obtains a value of order one for $\xi$. One should be aware that the applicability of pQCD may not be a good one here. However, we find that contribution related to $\xi$ is not important as long as $\xi$ is of order one. That is, the precise value of $\xi$ is not important here and we will use $\xi$ to be one in our later discussions.

To estimate the irreducible contribution, one needs to know the quantity $a_2 = c_1 + c_2/N$. Without QCD corrections, $c_1 = 0$ and $c_2 = 1$. This gives a $a_2 = 1/3$. With QCD corrections the value for $a_2$ will be altered. The leading and next-leading order corrections to $c_i$ have been calculated [9]. The values of $c_i$ depend on the renormalization scale $\mu$. Since one does not know precisely where is the matching scale $\mu$, this causes uncertainty in $a_2$. For example at the leading order, $a_2 = -0.27$ at $\mu \approx 1$ GeV, while at $\mu = 1.3$ GeV, $a_2 = -0.17$ with $\Lambda_{\overline{MS}} = 325$ MeV. At the next leading order the dependences on $\mu$ for each of the $c_i$
and $c_2$ are reduced, but leaves $a_2$ still sensitive to $\mu$. For example, in the NDR scheme, for $\Lambda_{\overline{MS}} = 325$ MeV, $a_2$ is -0.08 and -0.1 at $\mu = 1.0$ GeV and $\mu = 1.3$ GeV, respectively. Allowing the QCD parameter $\Lambda_{\overline{MS}}$ to vary within the allowed range $215 \sim 435$ MeV, $a_2$ can vary in the range $-0.1 \sim -0.35$ depending whether NDR or HV scheme is used [9]. That is, the value of $a_2$ is not well determined even from the next leading order perturbative calculations. When all effects, perturbative and non-perturbative, are correctly treated, the final physical observables will not depend on the renormalization scale $\mu$. Unfortunately, such a calculation is not possible at present. The parameter $a_2$ behaves similarly to the one in hadronic $B$ and $D$ decays. In both $D$ and $B$ decays, the parameter $a_2$ determined from data ($|a_2| \sim (0.2 \sim 0.5)$) is very different from factorization value by inserting $c_{1,2}$ at relevant scale in the expression for $a_2$ [12]. One would expect similar thing happens in kaon decays although the details may be different. To take into account uncertainties in theoretical calculations of $a_2$, we will treat it as a free parameter and allow it to vary in the range of $-0.5 \sim 0.5$. One can also turn the argument around to obtain information about $a_2$ from $K_L \to \gamma\gamma$ data.

For $\xi$ of order one, and $a_2$ in the range of $-0.5 \sim 0.5$, we find that the dominant direct $\bar{K}^0 \to \gamma\gamma$ amplitude is from the irreducible contribution. We have

$$A_{\text{dir}}(K_L \to \gamma\gamma) = i\tilde{A}_{\text{dir}} \frac{1}{2} F_{\mu\nu} \tilde{F}^{\mu\nu},$$

$$\tilde{A}_{\text{dir}} = \frac{8\alpha_{\text{em}} G_F}{9\pi} f_K N a_2 \text{Re}(V_{ud}^* V_{us}).$$

(8)

Using $V_{ud} = 0.9735$ and $V_{us} = 0.2196$ and $f_K = 1.27 f_\pi$ [13], we obtain,

$$\tilde{A}_{\text{dir}} = 2.54 \times 10^{-12} a_2 \text{MeV}^{-1}.$$  

(9)

For $|a_2| = 0.5$, it is only about 35% of the experimental value of $3.5 \times 10^{-12}$ MeV$^{-1}$ [13]. Without QCD corrections $a_2 = 1/3$, $\tilde{A}_{\text{dir}}$ is about 24% of the total amplitude. There must be some other contributions to this process. These effects may come from contributions with intermediate hadronic states or even contributions from new physics beyond the SM. If one has a good understanding of all SM contributions, one can have a detailed study of
new physics beyond the SM. It is probably too early to say that new physics is needed here due to large uncertainties in possible hadronic intermediate contributions. Therefore we will work within the SM and see how contribution from hadronic intermediate states can affect the results.

Several analyses have been carried out using pole model with $\pi^0$, $\eta$ and $\eta'$ poles to calculate the hadronic intermediate contribution. In this model, the amplitude $A_{\text{had}}$ from exchange of intermediate hadronic states is given by [6]

$$\tilde{A}_{\text{had}} = \tilde{A}(\pi^0 \to \gamma\gamma) \frac{\langle \pi^0 | H_W | K_L \rangle}{m_K^2 - m_\pi^2} \times \left[ 1 + \frac{m_\pi^2}{m_K^2} \tilde{A}(\eta \to \gamma\gamma) \left( \frac{1 + \delta}{\sqrt{3}} \cos \theta + \frac{2\sqrt{2}}{\sqrt{3}} \rho \sin \theta \right) + \frac{m_\eta^2}{m_K^2} \tilde{A}(\eta' \to \gamma\gamma) \left( \frac{1 + \delta}{\sqrt{3}} \sin \theta - \frac{2\sqrt{2}}{\sqrt{3}} \rho \cos \theta \right) \right], \quad (10)$$

where $\theta$ is the $\eta - \eta'$ mixing angle, $\delta$ is the SU(3) breaking parameter [6]. The parameter $\rho$ parameterizes U(3) nonet breaking effect and is defined as

$$\rho = -\sqrt{\frac{3}{8}} \frac{\langle \eta_1 | H_W | K^0 \rangle}{\langle \pi^0 | H_W | K^0 \rangle}. \quad (11)$$

In the nonet limit $\rho = 1$. Chiral Lagrangian analysis gives $\langle \pi^0 | H_W | K_L \rangle = 1.4 \times 10^{-7} m_K^2$ [6]. Using experimental values for $\pi^0, \eta, \eta' \to \gamma\gamma$, $\tilde{A}_{\text{had}}$ can be estimated.

The above contributions can be viewed as obtained by $A_{\text{had}} = \sum_i \langle \gamma|\gamma_i \rangle \langle i|\tilde{H}_{\text{eff}}(\Delta S = -1)|K_L \rangle$ with $i = \pi^0, \eta, \eta'$ in the pole model approximation. Here $\tilde{H}_{\text{eff}}(\Delta S = -1)$ is the full $\Delta S = -1$ effective Lagrangian with $O_7\gamma$ term removed since it has been counted as the contribution to $A_{\text{dir}}$. Therefore $A_{\text{dir}}$ and $A_{\text{had}}$ are contributions from different sources. In previous calculations the contributions for $A_{\text{had}}$ from $O_{8\gamma}$ were not considered [2,6]. We now study in detail the effect of this interaction on $K_L \to \gamma\gamma$.

At the quark-gluon level, $O_{8\gamma}$ induces $s \to d + gg$. To obtain $A_{\text{had}}$, one needs to estimate the contribution from $s \to d + gg$ to $K^0 - \eta, \eta'$ through $gg \to \eta, \eta'$. The effective Hamiltonian $M_{\text{I}_R,R}^{gg}$ for $s \to dgg$, with color singlet $\bar{d}s$ bi-spinor product, can be obtained by some simple replacements from $M_{\text{I}_R,R}^{\gamma\gamma}$. To obtain $M_{\text{I}_R,R}^{gg}$ one first replaces the photon polarization vectors.
\(e^\mu(k_1)e^\nu(k_2)\) by the gluon polarization vector \(e^\mu_a(k_1)e^\nu_a(k_2)\) with the color index \(a\) summed over. Then one replaces \(\alpha_{em}\) by \(\alpha_s(9/4)/(2N)\) and \(\alpha_{em}c^a_{12}\) by \(\alpha_s c^a_{8G}/(2N)\) for \(M^{gg}_I\) and \(M^{gg}_R\), respectively [7]. The factor \(1/(2N)\) comes from picking up the color singlet part.

Similar to the procedure in obtaining the amplitude \(A_{dir}\) for \(K_L \rightarrow \gamma \gamma\), one can obtain the amplitude for \(K_L \rightarrow gg\). We find that with \(\xi\) of order one, \(a_2\) in the range of \(-0.5 \sim 0.5\) and \(c^j_{8G} \approx -0.15\) as given in the SM, the irreducible contribution, again, dominates the amplitude. We have

\[
A(K_L \rightarrow gg) = \frac{1}{2N} \frac{2\alpha_s G_F}{\pi} f_K N a_2 Re(V_{ud}^* V_{us}) i \frac{1}{2} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a. \tag{12}
\]

The above interaction can induce large \(K_L - \eta, \eta'\) transitions and therefore contribution to \(K_L \rightarrow \gamma \gamma\), because QCD can induce large matrix elements for \(\langle \eta(\eta')|\alpha_s G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a|0\rangle\).

QCD anomaly implies that the divergence of the singlet current, \(a^1_\mu = \bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d + \bar{s}\gamma_\mu \gamma_5 s\), is not zero in the limit of zero quark masses, and is given by

\[
\langle \eta(\eta')|\partial^\mu a^1_\mu|0\rangle = \langle \eta(\eta')|2i(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d + m_s \bar{s}\gamma_5 s)|0\rangle

- \langle \eta(\eta')|3\alpha_s G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a|0\rangle. \tag{13}
\]

While for the octet current, \(a^8_\mu = \bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d - 2\bar{s}\gamma_\mu \gamma_5 s\), one obtains [14]

\[
\langle \eta(\eta')|\partial^\mu a^8_\mu|0\rangle = \langle \eta(\eta')|2i(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d - 2m_s \bar{s}\gamma_5 s)|0\rangle. \tag{14}
\]

Since \(m_u,d\) are much smaller than \(m_s\), one can neglect terms proportional to \(m_u,d\). One then obtains

\[
\langle \eta'(p)|\frac{3\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a|0\rangle = \sqrt{\frac{3}{2}}(\sqrt{2}f_1 \cos \theta + f_8 \sin \theta)p^2,

\langle \eta(p)|\frac{3\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a|0\rangle = \sqrt{\frac{3}{2}}(-\sqrt{2}f_1 \sin \theta + f_8 \cos \theta)p^2, \tag{15}\]

where \(f_{1,8}\) are the singlet and octet pseudo-scalar decay constants.

If there is no \(\eta - \eta'\) mixing and all quark masses are equal, the \(gg\) state being a flavor singlet can only have transition to \(\eta_1\). Because the \(\eta - \eta'\) mixing and the different quark masses, both U(3) nonet and SU(3) symmetries are broken. The \(K_L \rightarrow \eta, \eta'\) transitions
induced by \( s \to dgg \) will induce nonet and SU(3) breaking in the total amplitude \( \tilde{A}^{total} \).

Normalizing the signs of each contributions to theoretical calculations, we finally obtain

\[
\tilde{A}^{total} = \tilde{A}_{dir} + \tilde{A}(\pi^0 \to \gamma\gamma) \frac{\langle \pi^0 | H_W | K_L \rangle}{m_K^2 - m_{\pi}^2} \times \left[ 1 + \frac{m_N^2 - m_{\pi}^2}{m_K^2 - m_{\pi}^2} \tilde{A}(\eta \to \gamma\gamma) \left( \frac{1 + \delta + \delta^{gg}}{\sqrt{3}} \cos \theta + \frac{2\sqrt{2}}{\sqrt{3}} (\rho + r^{gg}) \sin \theta \right) \right.
\]

\[
\left. + \frac{m_{\eta}^2 - m_{\pi}^2}{m_K^2 - m_{\pi}^2} \tilde{A}(\eta' \to \gamma\gamma) \left( \frac{1 + \delta + \delta^{gg}}{\sqrt{3}} \sin \theta - \frac{2\sqrt{2}}{\sqrt{3}} (\rho + r^{gg}) \cos \theta \right) \right],
\]

where \( \delta^{gg} \) and \( r^{gg} \) are the SU(3) and nonet breaking induced by the \( s \to dgg \) interaction.

They are given by

\[
\delta^{gg} = -\sqrt{2} f_{K} f_{8} m_{K}^2 \frac{G_F Re(V_{ud}^* V_{us})}{\langle \pi^0 | H_W | K_L \rangle} a_2,
\]

\[
r^{gg} = -\frac{f_1}{2 f_8} \delta^{gg}.
\]

We find

\[
\delta^{gg} = 0.96 \frac{f_8}{f_K} a_2, \quad r^{gg} = -0.48 \frac{f_1}{f_K} a_2.
\]

We see that the corrections can be sizeable and can not be neglected.

We now provide some details for numerical calculations. There are several parameters involved in \( \tilde{A}_{had} \), the mixing angle \( \theta \), the decay constants \( f_{1,8} \), the SU(3) and U(3) nonet breaking parameters \( \delta \) and \( \rho \), and the parameter \( a_2 \). Chiral perturbation calculations and fitting data not involving \( K_L \to \gamma\gamma \) have obtained \( \theta \approx -20^\circ, \delta \approx 0.17, f_8 \approx 1.28 f_\pi \) and \( f_1 \approx 1.10 f_\pi \) [15]. We will use these values for these parameters in the calculation of \( K_L \to \gamma\gamma \). There is not a reliable estimate for the parameter \( \rho \). Since we are interested to see how the new \( s \to dgg \) interaction induces U(3) nonet breaking effect, we will take \( \rho = 1 \) and attribute nonet breaking solely to \( r^{gg} \). As have been discussed \( s \to dgg \) also induce SU(3) breaking effect. This effect was not included in other fittings. We therefore should include this new SU(3) breaking effect also.

Without the \( s \to dgg \) effect, we find that the amplitude \( \tilde{A}^{total} \) is equal to \( 5.5(1 + 0.46 a_2) \times 10^{-12} \) MeV\(^{-1} \) which is considerably larger than the experimental value \( 3.5 \times 10^{-12} \) MeV\(^{-1} \) [13] for \( |a_2| < 0.5 \). With the new effect, we find
\[ \tilde{A}_{\text{total}}^{\text{total}} = 5.5(1 + 2.14a_2) \times 10^{-12}\text{MeV}^{-1}. \]  

(19)

To reproduce the central experimental value, \( a_2 \) is required to be \(-0.17 \) which is a reasonable value to have.

The detailed numerical results depend on several parameters. Even with other parameters fixed, one can introduce also a phase to \( a_2 \). To fit the \( K_L \rightarrow \gamma\gamma \) data, the values for the magnitude and phase of \( a_2 \) can vary. We, however, would like to emphasize that the new effect discussed can play an important role in \( K_L \rightarrow \gamma\gamma \) independent of the details.

The new contributions for \( K_L \rightarrow \eta(\eta') \) transitions also induce new hadronic intermediate state effect to the \( K_L \) and \( K_S \) mass difference parameter \( \text{Re}(M_{12}) \) in the pole dominance approximation. We find [6]

\[
2m_K \text{Re}(M_{12}) = \frac{|\langle \pi^0|H_W|K^0\rangle|^2}{m_K^2 - m_\pi^2} \times \left[ 1 + \frac{m_K^2 - m_\eta^2}{m_K^2 - m_{\eta'}^2} \left( \frac{1 + \delta + \delta^{gg}}{\sqrt{3}} \cos\theta + \frac{2\sqrt{2}}{\sqrt{3}} (\rho + r^{gg}) \sin\theta \right)^2 \right.
\]

\[
+ \left. \frac{m_K^2 - m_\eta^2}{m_K^2 - m_{\eta'}^2} \left( \frac{1 + \delta + \delta^{gg}}{\sqrt{3}} \sin\theta - \frac{2\sqrt{2}}{\sqrt{3}} (\rho + r^{gg}) \cos\theta \right)^2 \right].
\]  

(20)

Without the new effects, the above would lead to \( \Delta m_K = -0.5 \times 10^{-12} \) MeV which is a non-negligible portion of the experimental value of \( 3.5 \times 10^{-12} \) MeV. With the new effects and \( a_2 = -0.17 \) as determined from \( K_L \rightarrow \gamma\gamma \), the contribution to \( \Delta m_K \) is \(-0.9 \times 10^{-12} \) MeV, and again it can not be neglected. The new effect in \( K_L \rightarrow \pi^0, \eta, \eta' \) transitions can have sizeable contribution to \( \Delta m_K \).

The \( s \rightarrow dgg \) process can also induce \( K_L \)-glueball mixing, which would also affect \( K_L \rightarrow \gamma\gamma \) and \( \Delta m_{S-L} \), as pointed out in Ref. [7] where a light glueball mass 1.4 GeV was used. Recent lattice calculations indicate that the pseudo-scalar glueball mass is about 2.3 GeV [16]. With such a large mass the glueball-\( \eta(\eta') \) mixing contribution should be small and therefore the effects are smaller than effects discussed earlier.

In conclusion we have evaluated additional contributions to \( K_L \rightarrow \eta(\eta') \) transitions from \( s \rightarrow dgg \) in the Standard Model. These transitions induce new sizeable SU(3) and U(3) breaking effects and have significant effects on contributions to \( K_L \rightarrow \gamma\gamma \) and \( \Delta m_K \).
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