Abstract

Objects in the std::map class of C++ associate a value to each of a set of keys. Accessing values or keys of such an object is problematic in the R programming language because the value-key pairs are not stored in a well-defined order. This document motivates and discusses the concept of “disordered vector” as implemented by the disordR package which facilitates the handling of map objects. Values and keys of a map are stored in an implementation-specific way so certain extraction and replacement operations should be forbidden. For example, if values are real, then the “first” value is implementation specific... but the maximum value has a well-defined result. The disordR package makes forbidden operations impossible while allowing transparent R idiom for permitted operations. An illustrative R session is given in which the package is used abstractly, without reference to any particular application, and then shows how it can be used to manipulate multivariate polynomials. The disordR package is a dependency of clifford, freealg, hyper2, mvp, spray, stokes, and weyl. The disordR package is available on CRAN at https://CRAN.R-project.org/package=disordR.

Keywords Disordered vectors

1 Introduction

In C++ [2], the std::map class [3] is an object that associates a value to each of a set of keys. Accessing values or keys of a map object is problematic because the value-key pairs are not stored in a well-defined order. The situation is applicable to any package which uses the map objects. Consider, for example, the mvp package [1] which deals with multinomials using std::maps. An mvp object is a map from terms to coefficients, and a map has no intrinsic ordering: the maps

x -> 1, xy -> 3, xy^3 -> 4 and xy^3 -> 4, xy -> 3, x -> 1

are the same map and correspond to the same object (symbolically, x + 3xy + 4xy^3 = 4xy^3 + 3xy + x). Thus the coefficients of the multinomial might be c(1,3,4) or c(4,3,1), or indeed any ordering. Internally, the elements are stored in some order but the order used is implementation-specific. Quite often, I am interested in the coefficients per se, without consideration of their meaning in the context of a multivariate polynomial. I might ask:

• “How many coefficients are there?”
• “What is the largest coefficient?”
• “Are any coefficients exactly equal to one?”
• “How many coefficients are greater than 2?”

These are reasonable and mathematically meaningful questions. Compare a meaningless question: “what is the first coefficient?”. This is meaningless because of the order ambiguity discussed above: the answer is at best implementation-specific, but fundamentally it is a question that one should not be allowed to ask.

To deal with the coefficients in isolation in R \cite{6}, one might be tempted to use a multiset. However, this approach does not allow one to link the coefficients with the terms. Suppose I coerce the coefficients to a multiset object (as per the \texttt{sets} package \cite{4,5}, for example): then it is impossible to extract the terms with coefficient greater than 2 (which would be the polynomial $3xy + 4xy^3$) because the link between the coefficients and the terms is not included in the multiset object. Sensible questions involving this aspect of \texttt{mvp} objects might be:

- Give me all terms with coefficients greater than 2
- Give me all terms with positive coefficients
- Give me all terms with integer coefficients

and these questions cannot be answered if the the coefficients are stored as a multiset (compare inadmissible questions such as “give me the first three terms”). Further note that replacement methods are mathematically meaningful, for example:

- Set any term with a negative coefficient to zero
- Add 100 to any coefficient less than 30

Again these operations are reasonable but precluded by multiset formalism (compare inadmissible replacements: “replace the first two terms with zero”, or “double the last term” would be inadmissible).

\textit{What we need is a system that forbids stupid questions and stupid operations, while transparently permitting sensible questions and operations}

The \texttt{disord} class of the \texttt{disordR} package is specifically designed for this situation. This class of object has a slot for the coefficients in the form of a numeric R vector, but also another slot which uses hash codes to prevent users from misusing the ordering of the numeric vector.

For example, a multinomial $x + 2y + 3z$ might have coefficients $c(1,2,3)$ or $c(3,1,2)$. Package idiom to extract the coefficients of a multivariate polynomial \texttt{a} is \texttt{coeffs(a)}; but this cannot return a standard numeric vector. If stored as a numeric vector, the user might ask “what is the first element?” and this question should not be asked \cite{6} and certainly not answered!, because the elements are stored in an implementation-specific order. The \texttt{disordR} package uses \texttt{disord} objects which are designed to return an error if such inadmissible questions are asked. But \texttt{disord} objects can answer admissible questions and perform admissible operations.

Suppose we have two multivariate polynomials, \texttt{a} as defined as above with $a = x + 2y + 3z$ and $b = x + 3y + 4z$. Even though the sum $a + b$ is well-defined algebraically, idiom such as \texttt{coeffs(a) + coeffs(b)} is not defined because there is no guarantee that the coefficients of the two multivariate polynomials are stored in the same order. We might have $c(1,2,3)+c(1,3,4)=c(2,5,7)$ or $c(1,2,3)+c(1,4,3)=c(2,6,6)$, with neither being more “correct” than the other. In the package, this ambiguity is rendered void: \texttt{coeffs(a) + coeffs(b)} will return an error. Note carefully that \texttt{coeffs(a+b)} is perfectly well defined, although the result is subject to the same ambiguity as \texttt{coeffs(a)}.

In the same way, \texttt{coeffs(a) + 1:3} is not defined and will return an error. Further, idiom such as \texttt{coeffs(a) <- 1:3} and \texttt{coeffs(a) <- coeffs(b)} are not defined and will also return an error. However, note that

\begin{verbatim}
coeffs(a) + coeffs(a)
coeffs(a) + coeffs(a)^2
coeffs(a) <- coeffs(a)^2
coeffs(a) <- coeffs(a)^2 + 7
\end{verbatim}

are perfectly well defined, with package idiom behaving as expected. In the assignments, one does not need to know the order of the left hand side, so long as the order is the same on both sides. The idiomatic English equivalent would be: “the coefficient of each term of \texttt{a} becomes its square”; note that this operation
The `disordR` package is insensitive to the order of coefficients. The whole shebang is intended to make idiom such as `coeffs(a) <- coeffs(a)%%2` possible, so we can manipulate polynomials over finite rings, here \( \mathbb{Z}/2\mathbb{Z} \).

The replacement methods are defined so that an expression like `coeffs(a)[coeffs(a) < 5] <- 0` works as expected; the English idiom would be “replace any coefficient less than 5 with 0”. To fix ideas, consider a fixed small `mvp` object:

```r
R> library("mvp")
R> a <- as.mvp("5 a c^3 + a^2 d^2 f^2 + 4 a^3 b e^3 + 3 b c f + 2 b^2 e^3")
R> a
```

```
  mvp object algebraically equal to
  5 a c^3 + a^2 d^2 f^2 + 4 a^3 b e^3 + 3 b c f + 2 b^2 e^3
```

Extraction presents issues; consider `coeffs(a) < 3`. This object has Boolean elements but has the same ordering ambiguity as `coeffs(a)`. One might expect that we could use this to extract elements of `coeffs(a)`: specifically, those elements less than 5. We may use replace methods for coefficients if this makes sense. Idiom such as

```r
R> coeffs(a)[coeffs(a)<5] <- 4 + coeffs(a)[coeffs(a)<5]
R> coeffs(a) <- pmax(coeffs(a),3)
```

is algebraically meaningful and allowed in the package. Idiomatically: “Add 4 to any element less than 5”; “coefficients become the parallel maximum of themselves and 3” respectively. Further note that `coeffs(a) <- rev(coeffs(a))` is disallowed (although `coeffs(a) <- rev(rev(coeffs(a)))` is meaningful and admissible).

So the output of `coeffs(x)` is defined only up to an unknown rearrangement. The same considerations apply to the output of `vars()`, which returns a list of character vectors in an undefined order, and the output of `powers()`, which returns a numeric list whose elements are in an undefined order. However, even though the order of these three objects is undefined individually, their ordering is jointly consistent in the sense that the first element of `coeffs(x)` corresponds to the first element of `vars(x)` and the first element of `powers(x)`. The identity of this element is not defined—but whatever it is, the first element of all three accessor methods refers to it.

Note also that a single term (something like `4a^3*b*c^6`) has the same issue: the variables are not stored in a well-defined order. This does not matter because the algebraic value of the term does not depend on the order in which the variables appear and this term would be equivalent to `4bc^6*a^3`.

### 2 An R session with the `disordR` package

We will use the `disordR` package to show how the idiom works.

```r
R> library("disordR")
R> set.seed(0)
R> a <- rdis()
R> a
```

```
A disord object with hash 5b7279f3c05d00cf1e8f999a755151e0451c56ec and elements
[1] 9 4 7 1 2 6 3 8 5
(in some order)
```

Object `a` is a `disord` object but it behaves similarly to a regular numeric vector in many ways:

```r
R> a^2
```

```
A disord object with hash 5b7279f3c05d00cf1e8f999a755151e0451c56ec and elements
[1] 81 16 49 1 4 36 9 64 25
(in some order)
```

```r
R> a+1/a
```
A disord object with hash 5b7279f3c05d00cf1e8f999a755151e0451c56ec and elements
[1] 9.111111 4.250000 7.142857 2.000000 2.500000 6.166667 3.333333
[8] 8.125000 5.200000
(in some order)

Above, note how the result has the same hash code as \( a \). Other operations that make sense are \texttt{max()} and \texttt{sort()}:

\begin{verbatim}
R> max(a)
[1] 9
R> sort(a)
[1] 1 2 3 4 5 6 7 8 9
\end{verbatim}

Above, see how the result is a standard numeric vector. However, inadmissible operations give an error. For example, we cannot extract the “first” element of \( a \):

\begin{verbatim}
R> try(a[1])
Error in .local(x, i, j = j, ..., drop) :
  if using a regular index to extract, must extract each element once and once only (or none of them)
nor can we replace it:

R> try(a[1] <- 1000)
Error in .local(x, i, j = j, ..., value) :
  if using a regular index to replace, must specify each element once and once only
\end{verbatim}

However, the package is designed so that standard R operations generally work as expected for permissible operations:

\begin{verbatim}
R> x <- a + 1/a
R> x
A disord object with hash 5b7279f3c05d00cf1e8f999a755151e0451c56ec and elements
[1] 9.111111 4.250000 7.142857 2.000000 2.500000 6.166667 3.333333
[8] 8.125000 5.200000
(in some order)

R> y <- a*2-9
R> y
A disord object with hash 5b7279f3c05d00cf1e8f999a755151e0451c56ec and elements
[1] 9 -1 5 -7 -5 3 -3 7 1
(in some order)

R> x+y
A disord object with hash 5b7279f3c05d00cf1e8f999a755151e0451c56ec and elements
[1] 18.111111 3.250000 12.142857 -5.000000 -2.500000 9.166667
[7] 0.333333 15.125000 6.200000
(in some order)
\end{verbatim}

Above, observe that objects \( a \), \( x \) and \( y \) have the same hash code: they are “compatible”, in disordR idiom. However, if we try to combine object \( a \) with another object with different hash, we get errors:

\begin{verbatim}
R> (b <- rdis())
\end{verbatim}
A disord object with hash 488e1c6f4e2c062379d47b5511730a9785661318 and elements
[1] 2 3 8 1 5 6 9 7 4
(in some order)

R> try(a+b)

a + b
Error in check_matching_hash(e1, e2, match.call()) :
hash codes 5b7279f3c05d00cf1e8f999a755151e0451c56ec... and 488e1c6f4e2c... do not match

The error is given because objects a and b are stored in an implementation-specific order (we say that a and b are incompatible). In the package, many extract and replace methods are implemented whenever this is admissible:

R> a[a<0.5] <- 0 # round down
R> a

A disord object with hash 5b7279f3c05d00cf1e8f999a755151e0451c56ec and elements
[1] 9 4 7 1 2 6 3 8 5
(in some order)

R> b[b>0.6] <- b[b>0.6] + 3 # add 3 to every element greater than 0.6
R> b

A disord object with hash 488e1c6f4e2c062379d47b5511730a9785661318 and elements
[1] 5 6 11 4 8 9 12 10 7
(in some order)

Usual semantics follow, provided one is careful to maintain the hash code:

R> d <- disord(1:10)
R> d

A disord object with hash 65e11d78de79b7f584068ad856749e3748cb837c and elements
[1] 1 2 3 4 5 6 7 8 9 10
(in some order)

R> e <- 10 + 3*d - d^2
R> e

A disord object with hash 65e11d78de79b7f584068ad856749e3748cb837c and elements
[1] 12 12 10 6 0 -8 -18 -30 -44 -60
(in some order)

R> e<4

A disord object with hash 65e11d78de79b7f584068ad856749e3748cb837c and elements
 [1] FALSE FALSE FALSE FALSE TRUE TRUE TRUE TRUE TRUE TRUE
(in some order)

R> d[e<4] <- e[e<4]
R> d

A disord object with hash 65e11d78de79b7f584068ad856749e3748cb837c and elements
[1] 1 2 3 4 0 -8 -18 -30 -44 -60
(in some order)

Above, the replacement command works because d and e, and e<4 [which is a Boolean disord object] all have the same hash code.
3 An R session with the mvp package

The mvp package implements multivariate polynomials using the STL map class. Following commands only work as intended here with mvp >= 1.0-12. Below we see how disordR idiom allows mathematically meaningful operation while suppressing inadmissible ones:

R> set.seed(0)
R> a <- rmvp()
R> b <- rmvp()
R> a

mvp object algebraically equal to
3 a b^9 e^4 f + 7 a^2 b^4 d^6 e f^4 + 4 a^4 b^6 c^5 d^11 f^4 +
6 a^6 b^3 c^14 f^2 + 5 a^11 e^6 f^6 + b^8 e^7 f^12 + 2 b^10 d^10 f^4

R> b

mvp object algebraically equal to
5 a c^2 e^8 f^7 + 4 a^2 b^5 c^6 e^3 + 7 a^2 b^7 c^4 d e^2 +
a^4 d^6 e^5 f + 6 a^6 d^6 f^6 + 3 b^7 c^7 e^5 + 2 b^10 c^3 f^7

Observe that standard multivariate polynomial algebra works:

R> a + 2*b

mvp object algebraically equal to
3 a b^9 e^4 f + 10 a c^2 e^8 f^7 + 7 a^2 b^4 d^6 e f^4 +
8 a^2 b^5 c^6 e^3 + 14 a^2 b^7 c^4 d e^2 + 4 a^4 b^6 c^5 d^11 f^4 +
2 a^4 d^6 e^5 f + 6 a^6 d^6 f^6 + 5 a^11 e^6 f^6 + 6 b^7 c^7 e^5 +
2 b^10 d^10 f^4

R> (a+b)*(a-b) == a^2-b^2 # should be TRUE (expression is quite long)
[1] TRUE

We can extract the coefficients of these polynomials using the coeffs() function:

R> coeffs(a)

A disord object with hash 76b070e3d27bf2e3a548b56a02678d79881de0ce and elements
[1] 3 7 4 6 5 1 2
[in some order]

R> coeffs(b)

A disord object with hash 40b9beff42bebe889cb596f78d096c90ef279834 and elements
[1] 5 4 7 1 6 3 2
[in some order]

observe that the coefficients are returned as a disord object. We may manipulate the coefficients of a polynomial in many ways. We may do the following things:

R> coeffs(a)[coeffs(a) < 4] <- 0 # set any coefficient of a that is <4 to zero
R> a

mvp object algebraically equal to
7 a^2 b^4 d^6 e f^4 + 4 a^4 b^6 c^5 d^11 f^4 + 6 a^6 b^3 c^14 f^2 +
5 a^11 e^6 f^6

R> coeffs(b) <- coeffs(b)%%2 # consider coefficients of b modulo 2
R> b
The mvp object algebraically equal to
\[ a \cdot c^2 \cdot e^8 \cdot f^7 + a^2 \cdot b^7 \cdot c^4 \cdot d \cdot e^2 + a^4 \cdot d^6 \cdot e^5 \cdot f + b^7 \cdot c^7 \cdot e^5 \]

However, many operations which have reasonable idiom are in fact meaningless and are implicitly prohibited.

For example:
```
R> x <- rmvp()  # set up new mvp objects x and y
R> y <- rmvp()
```

Then the following should all produce errors:
```
coeffs(x) + coeffs(y)  # order implementation specific
coeffs(x) <- coeffs(y)  # ditto
coeffs(x) <- 1:2  # replacement value not length 1
coeffs(x)[coeffs(x) < 3] <- coeffs(x)[coeffs(y) < 3]
```

4 Functions vars() and powers() return disord objects

The disord() function takes a list argument, and this is useful for working with mvp objects:
```
R> (a <- as.mvp("x^2 + 4 - 3*x*y*z"))
```

mvp object algebraically equal to
\[ 4 - 3 x y z + x^2 \]

```
R> vars(a)
```

A disord object with hash 9395842c6e674f0be871e04b3a8964a1c9b9bd5 and elements
[[1]]
character(0)
[[2]]
[1] "x" "y" "z"
[[3]]
[1] "x"

(in some order)
```
R> powers(a)
```

A disord object with hash 9395842c6e674f0be871e04b3a8964a1c9b9bd5 and elements
[[1]]
integer(0)
[[2]]
[1] 1 1 1
[[3]]
[1] 2

(in some order)
```
R> coeffs(a)
```

A disord object with hash 9395842c6e674f0be871e04b3a8964a1c9b9bd5 and elements
[1] 4 -3 1

(in some order)

Note that the hash of all three objects is identical, generated from the polynomial itself (not just the relevant element of the three-element list that is an mvp object). This allows us to do some rather interesting things:
The \texttt{disordR} package

\begin{verbatim}
R> double <- function(x){2*x}
R> (a <- rmvp())
mvp object algebraically equal to
a^2 c^10 d^2 f^2 + 7 a^3 d^5 e^14 + 6 a^5 c^7 d^4 e^2 + 4 a^8 c d^5 e^6 +
2 b^2 c^4 d^10 e^5 f + 5 b^2 c^6 d^2 e^7 f^6 + 3 c^6 d^4 e^2 f^6

R> pa <- powers(a)
R> va <- vars(a)
R> ca <- coeffs(a)
R> pa[ca<4] <- sapply(pa,double)[ca<4]
R> mvp(va,pa,ca)
mvp object algebraically equal to
7 a^3 d^5 e^14 + a^4 c^20 d^4 f^4 + 6 a^5 c^7 d^4 e^2 +
4 a^8 c d^5 e^6 + 5 b^2 c^6 d^2 e^7 f^6 + 2 b^4 c^8 d^20 e^10 f^2 +
3 c^12 d^8 e^4 f^12

Above, \(a\) was a multivariate polynomial and we doubled the powers of all variables in terms with coefficients
less than 4. Or even:

R> a <- as.mvp("3 + 5*a*b - 7*a*b*x^2 + 2*a*b^2*c*d*x*y -6*x*y + 8*a*b*c*d*x")
R> a
mvp object algebraically equal to
3 + 5 a b + 8 a b c d x - 7 a b x^2 + 2 a b^2 c d x y - 6 x y

R> pa <- powers(a)
R> va <- vars(a)
R> ca <- coeffs(a)
R> va[sapply(pa,length) > 4] <- sapply(va,toupper)[sapply(pa,length) > 4]
R> mvp(va,pa,ca)
mvp object algebraically equal to
3 + 8 A B C D X + 2 A B^2 C D X Y + 5 a b - 7 a b x^2 - 6 x y

Above, we took multivariate polynomial \(a\) and replaced the variable names in every term with more than
four variables with their uppercase equivalents.

5 Conclusions

In the context of \texttt{stl} map class, the \texttt{disordR} package allows permissible idiom transparently and without
being noticed; but traps inadmissible constructions with an error.

References

[1] Robin K. S. Hankin. \textit{mvp: Fast Symbolic Multivariate Polynomials}, 2022. R package version 1.0-12.
[2] ISO Central Secretary. Programming languages, C++. ISO/IEC 144882. Technical report, American
National Standard Institute, 1998.
[3] N. M. Josuttis. \textit{The C++ Standard Library: A Tutorial and Reference}. Addison-Wesley, 1999.
[4] David Meyer and Kurt Hornik. Generalized and customizable sets in R. \textit{Journal of Statistical Software},
31(2):1–27, 2009.
[5] David Meyer and Kurt Hornik. \textit{sets: Sets, Generalized Sets, Customizable Sets and Intervals}, 2022. R
package version 1.0-21.
[6] R Core Team. \textit{R: A Language and Environment for Statistical Computing}. R Foundation for Statistical
Computing, Vienna, Austria, 2022.