Surface properties of neutron star within coherent density fluctuation model using the relativistic mean-field density

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8 December 2021

ABSTRACT
A detailed study of the structural properties of neutron star (NS) is performed within the coherent density fluctuation model using the recently developed G3 and widely used NL3 and IU-FSU parameter sets in the relativistic mean-field formalism. The masses, moment of inertia, and density profiles of the NS at various mass limits are studied. The incompressibility $K_{\text{star}}$, symmetry energy $S_{\text{sym}}$, slope parameter $L_{\text{sym}}$, and curvature coefficient $K_{\text{sym}}$ of the NS at various masses are analyzed. The surface properties ($K_{\text{star}}, S_{\text{sym}}, L_{\text{sym}}$ and $K_{\text{sym}}$) are found to be model dependent, NL3 is the stiffest equation of state gives the higher magnitude of surface quantities as compared to the G3 and IU-FSU forces.

Key words: equation of state– stars: neutron– surface properties

OVERVIEW

Background: In our earlier work, we studied the surface properties of neutron stars (NS), assuming it is a huge finite nucleus containing protons, neutrons, electrons, and muons. For the first time, we reported these properties of a neutron star for a few representative masses. In this paper, we want to give a detailed study of these quantities to draw definite conclusions.

Method: To carry forward the above idea, the energy density functional of the momentum space of neutron star matter converted to the coordinate degree of freedom in a local density approximation. This functional was again used to derive the neutron star surface properties in the star matter limit. From these star matter properties, the corresponding finite mass NS properties are obtained within the coherent density functional model using the weight function obtained from the density profile of the neutron star.

Results: We get systematic results for the surface properties of neutron star, such as incompressibility $K_{\text{star}}$, symmetry energy $S_{\text{sym}}$, slope parameter $L_{\text{sym}}$, and curvature coefficient $K_{\text{sym}}$. The volume and surface components of the total symmetry energy are decomposed with the help of the $\kappa$ factor obtained from the volume to surface components in the liquid drop limit of Danielewicz.

1 INTRODUCTION

Among the known objects in the universe, the NS is the densest. It is known that the neutron star cores are $10^3$ times denser than the density at “neutron drip” line (Engvik et al. 1994). To understand the properties of NS, a thorough knowledge of both nuclear physics and astrophysics is demanded. In a layman concept, the neutron star can be treated as a giant asymmetric nucleus comprised mostly degenerated neutrons gas with a small fraction of protons and electrons to maintain a charge-neutral body along with some exotic particles, such as muon and baryon octet (Sharma et al. 2007). The neutron star is bounded by the attractive gravitational force balanced by the short-range strong nuclear interaction generated by the baryons and mesons. In addition to these two opposite forces, the electromagnetic interactions are also vital for the stability of NS. In contrast, the normal nucleus is governed by strong nuclear and relatively weaker electromagnetic interactions. As a consequence, inside a nucleus, the density is flat, and inside a neutron star, the density increases. Because of this difference in the densities distribution, it is not straightforward that the behavior of similarly defined properties are the same. One can not generalise the normal nuclear properties to the neutron star and needs a separate analysis to understand the NS properties.

In Kumar et al. (2021a), the conventional Brückner energy density functional (B-EDF) (Brueckner et al. 1968b, 1969) is replaced by the effective field theory motivated relativistic mean-field (E-RMF) density functional in a local density approximation (LDA) to calculate the nuclear surface properties in the framework of the coherent density fluctuation model (CDFM). They demonstrated that the microscopic E-RMF energy functional is able to incorporate the structural effects of the nucleus and reproduce the peak in the symmetry energy of the Pb isotopic chain at neutron number N = 126, which is generally failed by the B-EDF functional (Quddus et al. 2020a; Gaidarov et al. 2011a). Recently, this formalism is extended successfully to study the surface properties of neutron star (Kumar et al. 2021a) and esti-
mated for the first time the \( K^{\star} \), \( g^{\star} \), \( L^{\star} \), and \( K^{\star}_{\mathrm{sym}} \). These properties of NS are quite informative for experimental observations and theoretical modeling. For example, assuming the NS as a huge nucleus, with the mass number \( A \sim 10^{57} \), it must possess most of the properties of normal finite nucleus (Kumar et al. 2021a). It should have multi-pole moments and all possible collective oscillations.

The parameters obtained by expanding the symmetry energy near the saturation density (slope and curvature parameter) control the cooling rate of the neutron star, and the core-crust transition density and transition pressure (Alam et al. 2016; Schneider et al. 2019). These parameters are also used to constraint the nuclear equation of state (EoS), which is a key ingredient for the study of NS properties as well as the properties of supernovae explosion, binary neutron star merger, and the physics of gravitational wave (Gil et al. 2021). A prominent bridge between the finite nuclei and the nuclear/neutron matter (interstellar bodies) is the comprehensive knowledge of the nuclear EoS. The EoS is the key component for determining the properties of the neutron star. Also, it controls the dynamics of core-collapse supernovae remnants, and the cooling of NS (Schneider et al. 2019; Bombaci & Logoteta 2018). With the help of observational gravitational wave data (GW170817) (Abbott et al. 2017), Einstein Observatory (HEAO-2) (Boguta 1981) and X-ray radio telescopes (Greif et al. 2020), a large number of constraints are implemented to get a proper EoS at high density regime.

From last few decades, the non-relativistic (Skyrme (Skyrme & Schonland 1961), Gogny forces (Dechargé & Gogny 1980)), and relativistic (Walecka 1974; Gambhir et al. 1990) theoretical approaches have been used as a consistent formalism to construct the EoS and calculate the properties of strongly-interacting dense matter systems. The relativistic class of models are an alternative approach for low-energy Quantum Chromodynamics with all the built-in non-perturbative properties (Adam et al. 2020, 2015). In the present paper, we will use the latest form of E-RMF Lagrangian to evaluate the EoS with the recently developed G3 parameter set, and the results will be compared with the widely used NL3 and IU-FSU forces.

The paper is organised as follows: In Sub-Sec. 2.1, the relativistic mean-field formalism is briefly described. The coherent density fluctuation model is given in Sub-Sec. 2.2 and NS properties are calculated in Sub-Sec. 2.3. The results and discussions are given in Sec. 3. In this section, the mass, radius, the moment of inertia, density profile, and the weight function of neutron star obtained using the E-RMF equation of states are explained in Sub-Sec. 3.1 and 3.2. The parameters such as incompressibility, symmetry energy, slope parameter, and curvature co-efficient of neutron star are illustrated in Sub-Sec. 3.3. The summary and concluding remarks are drawn in Section 4.

2 THEORY

2.1 Effective field theory relativistic mean field model

As it is mentioned, we used NL3 (Lalazissis et al. 1997), G3 (Kumar et al. 2017b) and IU-FSU (Carbone & Schwenk 2019) parameter sets of the E-RMF Lagrangian. The NL3 set is the stiffest, and the newly reported G3 is the softest EoS. The numerical values of nuclear matter properties at saturation are listed in Table 1. The empirical/experimental data are also given for comparison. The nuclear matter incompressibility, which controls the stiffness/softness of the EoS, are 271.38, 243.96, and 231.31 MeV for NL3, G3, and IU-FSU, respectively. And the symmetry energies are 37.43, 31.84, and 32.71 MeV for these three sets. These values are within the range given by various experimental observations, and theoretical predictions (Kumar et al. 2021a).

It is motivated by the work of Brückner et al. (Brueckner et al. 1968a,c), using the LDA, the momentum space energy functional is converted to the coordinate space through a generator coordinate x. The detailed procedure can be found in Refs. (Kumar et al. 2021b). It is worth mentioning that the B-EDF (Brueckner et al. 1968a,c) fails to reproduce the Coester band (Coester et al. 1972). Consequently, the peak that appears in the symmetry energy at the magic number for larger nuclei (like Pb isotopes) does not find in the appropriate position (Pattnaik et al. 2021b,a). The coordinate space E-RMF energy density functional within LDA for neutron star matter is defined as (Kumar et al. 2021a):

\[
E = C_k n^{2/3} + C_c n^{4/9} + \sum_{i=3}^{14} (b_i + a_i \alpha^2) m_i^{1/3},
\]

where \( C_k = 0.3(\hbar^2/2M)(3n^2)^{2/3}[(1+\alpha)^{5/3}+(1-\alpha)^{5/3}] \) (Brueckner et al. 1968c) is the coefficient of the kinetic energy for protons and neutrons and \( C_c = b_c(1-\alpha)^{5/3} \) is the kinetic energy coefficient term for electrons and muons, with \( b_c \) as a variable obtained from the conversion of the E-RMF energy density from momentum space to coordinate space in the local density approximation. The last term is the potential interaction of the nucleons and the coefficients \( b_i \) and \( a_i \) obtained from the fitting for different E-RMF models. It is shown in Ref. (Kumar et al. 2021b) that the accuracy of the fitting increases with the number of coefficients \( b_i \) and \( a_i \) in the series of the potential term of Eq. (1).

The mean deviation \( \delta = \sum_i^{N} \left( (E/A)_{i,Fitted} - (E/A)_{i, RMF} \right) / N \), = 18\%, 6\% and 0.5\% for \( i = 8, 10 \) and 12 respectively. Here \( N \) is the total number of points and \( i \) = 3 to 10 in Eq. (1). The coefficients of the expansion series Eq. (1) for \( e_i, b_i \) and \( a_i \) are tabulated in Table 2.

2.2 Coherent density fluctuation model

The CDFM is a well established formalism to calculate the properties of finite nuclei (Antonov et al. 1980, 1994; Gaidarov et al. 2020) from the infinite nuclear/NS matter. The CDFM is used a generator coordinate x to evaluate the one-body density matrix \( n(r, r') \) of a finite nucleus/NS as the superposition of infinite number of one-body density matrices \( n_x(r, r') \), called “Fluctons” (Gaidarov et al. 2011b; Kaur et al. 2020). The density of a Flucton is written as (Antonov et al. 1980, 1994; Gaidarov et al. 2020):

\[
n_x(r) = n_0(x) \Theta(x-|r|),
\]
Table 2. The fitted coefficients $a_i$, $b_i$ and $c_i$ of the Eq. (1) for NL3, G3 and IU-FSU forces. The values are scaled by $10^{-8}$ factor i.e. each should be multiplied by a factor of $10^8$ to get the exact magnitude of the coefficient.

|        | NL3   | G3    | IU-FSU |
|--------|-------|-------|--------|
| $a_1$  | 0.00017 | 0.00011 | 0.00011 |
| $b_1$  | -0.00054 | -0.000085 | -0.000088 |
| $c_1$  | 0.00898 | 0.00048 | 0.00043 |
| $a_2$  | -0.08078 | -0.00158 | -0.00091 |
| $b_2$  | 0.04609 | 0.00346 | 0.00336 |
| $c_2$  | -1.774 | -0.00547 | 0.00241 |
| $a_3$  | 4.742 | 0.00648 | -0.00592 |
| $b_3$  | -8.896 | -0.00579 | 0.00698 |
| $c_3$  | 11.65 | 0.00379 | -0.00498 |
| $a_4$  | -10.43 | -0.01714 | 0.00227 |
| $b_4$  | 6.0713 | 0.00523 | -0.00606 |
| $c_4$  | -2.069 | -0.000092 | 0.00105 |
| $a_5$  | 0.3132 | 0.000007 | -0.000007 |
| $b_5$  | -0.00088 | -0.00198 | -0.00199 |
| $c_5$  | 0.02289 | 0.002919 | 0.002475 |
| $a_6$  | -0.02359 | -0.01797 | -0.013555 |
| $b_6$  | 1.639 | 0.06395 | 0.04266 |
| $c_6$  | -6.864 | -0.1472 | -0.08664 |
| $a_7$  | 19.60 | 0.2303 | 0.1198 |
| $b_7$  | -39.03 | -0.2502 | -0.1156 |
| $c_7$  | 54.27 | 0.1891 | 0.07793 |
| $a_8$  | -51.73 | -0.09762 | -0.03605 |
| $b_8$  | 32.25 | 0.03281 | 0.01092 |
| $c_8$  | -11.84 | -0.00647347 | -0.00194919 |
| $a_9$  | 1.945 | 0.000569 | 0.000155 |

The saturation density of the Flucton $n_0(x) = 3A/4\pi x^3$, $A$ is the total number of protons and neutrons in the neutron star matter. In the coherent density fluctuation model, the density of the spherical finite neutron star matter of radius ‘$r$’ is (Antonov et al. 2018, 2016a),

$$n(r) = \int_0^\infty dx |F(x)|^2 n_0(x) \Theta(x - |r|),$$

where $|F(x)|^2$ is the weight function in the generator coordinate ‘$x$’ with the local density $n(r)$ written as (Antonov et al. 2018):

$$|F(x)|^2 = \frac{1}{n_0(x)} \frac{dn(r)}{dr} \bigg|_{r=x}.$$  

The $K^{NSM}$, $S^{NSM}$, $L^{NSM}$ and $K^{NSM}_y$ are expressed with the weight function as (Kumar et al. 2021a; Gaidarov et al. 2011b, 2012a; Antonov et al. 2016a):

$$K^{NSM} = \int_0^\infty dx |F(x)|^2 K^{NSM}(n(x)),$$  

$$S^{NSM} = \int_0^\infty dx |F(x)|^2 S^{NSM}(n(x)),$$  

$$L^{NSM}_y = \int_0^\infty dx |F(x)|^2 L^{NSM}_y(n(x)),$$  

$$K^{NSM}_y = \int_0^\infty dx |F(x)|^2 K^{NSM}_y(n(x)),$$

where $K^{NSM}$, $S^{NSM}$, $L^{NSM}_y$ and $K^{NSM}_y$ are the incompressibility, symmetry energy, slope parameter and curvature of the neutron star matter (NSM).

The converted energy density functional of the neutron star matter from momentum space to the coordinate space ‘$x$’ in a local density approximation is Eq. (1). The expressions for $K^{NSM}$, $S^{NSM}$, $L^{NSM}_y$ and $K^{NSM}_y$ are obtained from this Eq. (1) with the definitions (Fetter & Walecka 1971; Chen et al. 2009; Chen & Piekarewicz 2014), i.e., the NM parameters $K^{NM}$, $S^{NM}$, $L^{NM}_y$ and $K^{NM}_y$ are obtained from the following standard relations (Gaidarov et al. 2011a; Antonov et al. 2016b; Gaidarov et al. 2012b; Kumar et al. 2021a):

$$K^{NM} = 9\rho_0^2 \frac{\partial^2}{\partial \rho^2} \left( \frac{E}{\rho} \right) \bigg|_{\rho = \rho_0},$$

$$S^{NM} = \frac{1}{2} \frac{\partial^2}{\partial \rho^2} \left( E(\rho) \right) \bigg|_{\rho = \rho_0},$$

$$L^{NM}_y = \frac{3 \rho_0}{\rho_0} \frac{\partial S(\rho)}{\partial \rho} \bigg|_{\rho = \rho_0} = \frac{3P}{\rho_0},$$

$$K^{NM}_y = 9\rho_0^2 \frac{\partial^2 S(\rho)}{\partial \rho^2} \bigg|_{\rho = \rho_0},$$

which are given as follow using Eq. (1):

$$K^{NSM} = -150.12 n_0^{2/3}(x) - 2.22 b_e n_0^{4/9}(x) + \sum_{i=4}^{14} i(i-3) b_i n_0^{i/3}(x),$$

$$S^{NSM} = 41.7 n_0^{2/3}(x) - 0.12 b_e n_0^{4/9}(x) + \sum_{i=4}^{14} a_i n_0^{i/3}(x),$$

$$L^{NSM}_y = 83.4 n_0^{2/3}(x) - 0.16 b_e n_0^{4/9}(x) + \sum_{i=3}^{14} i a_i n_0^{i/3}(x),$$

$$K^{NSM}_y = -83.4 n_0^{2/3}(x) + 0.266 b_e n_0^{4/9}(x) + \sum_{i=3}^{14} i(i-3) a_i n_0^{i/3}(x),$$

The symmetry energy for finite nuclei, i.e., a neutron star with nucleon number $A$, can be further expressed as the components of volume $S_V$ and surface $S_S$ contributions using the Danielewicz’s liquid drop prescription, which is written as (Pattnaik et al. 2021b, 0):

$$S = \frac{S_V}{1 + \frac{S_S}{S_V} A^{-1/3}} = \frac{S_V}{1 + A^{-1/3}/k},$$

$$\text{MNRA}$$
where the ratio \( \kappa \equiv \frac{\rho V}{3r^2} \) is defined as (Patnaik et al. 2021b, 0):

\[
\kappa = \frac{3}{R \rho_0} \int_0^\infty dx |F(x)| x \rho_0(x) \left( \frac{\rho_0}{\rho(x)} \right)^\gamma - 1.
\]  

The value of \( \gamma = 0.3 \) is used in Eq. (18) following Ref. (Antonov et al. 2018). An alternative method has been reported by Gaidarov et al. to obtain the volume, and surface symmetry energy components (Gaidarov et al. 2021).

2.3 Neutron star properties

The neutron star equation of state is calculated using the E-RMF model with the assumption that the neutron star is in \( \beta \)-equilibrium and charge neutrality conditions (Glendenning 1997). The EoS of the NS is completely model dependent and also depends on the types of extra particles such as hyperons (Schaffner & Mishustin 1996; Bhowmick et al. 2014; Fortin et al. 2017; Kumar et al. 2017a; Bhuyan et al. 2017; Biswal 2019; Biswal et al. 2019), kaons (Pal et al. 2000; Gupta & Arumugam 2012), dark matter (Das et al. 2019, 2020a, 2021c,a,b) etc. are present in this system. In this model, we limit to the nucleons and leptons only, which satisfy the \( \beta \)-equilibrium and charge neutrality condition.

To calculate the NS macroscopic/structural properties such as \( M \) and \( R \), one has to solve the following Tolman-Oppenheimer-Volkoff (TOV) equations (Tolman 1939; Oppenheimer & Volkoff 1939)

\[
\frac{dP}{dr} = -\frac{[E + P][m + 4\pi r^3 P]}{r^2(1 - 2m/r)},
\]

\[
\frac{dm}{dr} = 4\pi r^2 E.
\]

The coupled equations are solved by using boundary conditions as: \( r = 0 \), \( P = P_c \) and \( r = R, P = 0 \) at fixed central density. The maximum mass and radius of the NS are calculated, where the pressure vanishes, which defines the surface of the star.

For slowly and uniformly rotating NS, the metric is given by (Stergioulas 2003)

\[
ds^2 = -e^{2\nu} dt^2 + e^{2\phi} (d\phi - \omega dt)^2 + e^{2\sigma} (r^2 d\theta^2 + d\phi^2).
\]

The moment of inertia \( I \) of the NS is calculated with the slow rotation approximation and is given by (Stergioulas 2003; Jha et al. 2008; Sharma & Jha 2009; Friedman & Stergioulas 2013; Paschalidis & Stergioulas 2017; Quddus et al. 2020b; Koliogiannis & Moustakidis 2020):

\[
I \approx \frac{8\pi}{3} \int_0^R dr (E + P) e^{-\phi(r)} \left[ 1 - \frac{2m(r)}{r} \right]^{-1} \frac{\omega}{\Omega} r^4.
\]

where \( \omega \) is the dragging angular velocity for a uniformly rotating star. The \( \omega \) satisfying the boundary conditions are

\[
\omega(r = R) = 1 - \frac{2I}{R^3}, \quad \left. \frac{d\omega}{dr} \right|_{r=0} = 0.
\]

3 RESULTS AND DISCUSSIONS

In this section, we present the macroscopic properties of the NS, such as, mass (\( M \)), radius (\( R \)), and \( I \) for NL3, G3, and IU-FSU parameter sets. After getting a broad knowledge of the bulk properties of NS, we extend our calculations to the surface properties of NS with these above three forces. For this, we estimate the \( S_{\text{star}}, K_{\text{star}}, L_{\text{sym}} \) and \( K_{\text{sym}} \) of neutron star with respect to their masses form 0.8 \( M_\odot \) to \( M_{\text{max}} \) for NL3, G3 and IU-FSU sets. To calculate these properties, the NS densities are extracted by using the EoSs and feeding the EoSs into the TOV equations. For the stability of the neutron star, the \( \beta \)-equilibrium condition and charge neutrality have also been taken care of in the EoSs. Considering the obtained density as the local density of the star, with the help of CDFM, we construct the weight function \( |F(x)|^2 \), which is folded with the derived infinite neutron star matter quantities Eqns. [9, 10, 11, 12] \( S^\text{NSM}, K^\text{NSM}, L^\text{NSM} \) and \( K^\text{NSM} \) to produce the results for NS using Eqns. (5, 6, 7, 8). Further, the results and discussions are made in the following sub-sections.

3.1 Relation between mass-radius and the momentum of inertia of neutron star

The mass-radius profiles of the NS are calculated with three different cases of masses with \( M = 1.0M_\odot, 1.4M_\odot \) and maximum mass \( M = M_{\text{max}} \) with fixing the central densities corresponding to these masses, which are depicted in Fig. 1 with NL3, G3, and IU-FSU parameter sets. Among them, NL3 being the stiffest EoS compared to both G3 and IU-FSU, predicts a larger maximum mass compared to both G3 and IU-FSU. Since we fix the central densities corresponding to these three masses \( 1.0M_\odot, 1.40M_\odot \) and \( M_{\text{max}} \), the radii are also found to be different for each of the parameter sets. This can be seen clearly from Fig. 1. The NL3 set predicts a larger mass and radius as compared to G3 and IU-FSU forces. Similarly, we calculate the moment of inertia \( I \) of the NS for these three sets, which is shown in Fig. 2. The value of \( I \) increases with the mass of the NS due to the linear relationship between them. The NL3 predicts higher \( I \) as...
compared to G3 and IU-FSU. This is because of the stiffer EoS of NL3 than G3 and IU-FSU sets.

### 3.2 Neutron star density and its weight function

The densities and their corresponding weight functions $|F(x)|^2$ versus radius of the NS with masses $1.0 \, M_\odot$, $1.4 \, M_\odot$ and $M_{\text{max}}$ are depicted in Fig. 3. The densities are in the upper panel, and their weight functions are in the lower panel. The results are presented for NL3, G3, and IU-FSU parameter sets. The chosen masses cover the lower, canonical, and maximum mass of the neutron star. The maximum mass for NL3, G3, IU-FSU are $2.77 \, M_\odot$, $1.99 \, M_\odot$, $1.94 \, M_\odot$ respectively. The $M_{\text{max}}$ for NL3 is distinctly larger than the other two sets. This behavior is reflected in the $I - M$ and $M - R$ profiles curves and is clearly seen in the densities and weight functions. Unlike the normal nucleus, which is bound by strong interaction, the NS is balanced by the attractive gravitational and the repulsive force due to the degenerated neutrons gas. Thus, the NS’s central density is quite different from the density distribution of nucleons in the normal nucleus. In addition, the density is influenced by the presence of electrons and muons. The density obtained by the NL3 set has the minimum central density followed by IU-FSU and G3 models. However, the $M_{\text{max}}$ of NL3 is more as compared to the maximum masses of G3 and IU-FSU. The corresponding weight functions for NL3, G3, and IU-FSU sets are given just below their densities. The shape of the $|F(x)|^2$ is like an exponential rise, and it is maximum at the surface of the neutron star. The values of $K_{\text{star}}$, $S_{\text{star}}$, $L_{\text{star}}$ and $K_{\text{star}}^{\text{sym}}$ are determined by folding the weight function with the corresponding neutron star matter $K_{\text{NSM}}$, $S_{\text{NSM}}$, $L_{\text{NSM}}$ and $K_{\text{NSM}}^{\text{sym}}$ (see Eqs. 5, 6, 7, 8). Thus, the maximum contribution comes from the neutron star’s surface and is termed as a surface phenomenon. Precisely, the values of $|F(x)|^2$ gather momentum at $\sim 6$ km, and it is maximum at the surface ($\sim 10 - 12$ km).

### 3.3 Surface properties of the neutron star

In this sub-section, we analyse the surface properties of our results obtained from the CDFM calculations for neutron star as a function of mass. Here, the incompressibility $K_{\text{star}}$, symmetry energy $S_{\text{star}}$, slope parameter $L_{\text{star}}$ and curvature co-efficient $K_{\text{star}}^{\text{sym}}$ of the NS are discussed. The results are depicted in Figs. (4,5,6,7) in the following sub-sections.

#### 3.3.1 Neutron star incompressibility

The incompressibility $K$ of an object is a prominent quantity to know its nature. It defines how much the object can be compressed or expanded. So, it directly connects with the collective motion of the system. As a matter of fact, $K$ is a significant quantity in the case of the properties of the neutron star. The EoS governs by incompressibility and plays a crucial role in determining the mass and radius of the NS. Because the EoS is used as an input while solving the TOV equations. It is shown in Ref. (Kumar et al. 2021a) that the $K_{\text{star}}$ is much less than the nuclear matter incompressibility at saturation $K_\infty$. For example, $K_\infty = 271.38$ MeV for NL3 set as compared to the $K_{\text{star}} = 44.956$ MeV for the maximum mass of the neutron star. This can also be related to the asymmetric nature of the medium $\alpha$. The $K_\infty$ is obtained at the asymmetric limit $\alpha = 0$ and $K_{\text{star}}^{\text{sym}}$ is evaluated at $\alpha \sim 1$, i.e., the incompressibility of a system decreases with asymmetric nature (Kumar et al. 2021a). One can see the trend of $K_{\text{star}}$ as a function of mass $M$ of the NS in Fig. 4 for the three considered sets. The three forces predict almost similar incompressibilities up to mass $M = 1.8M_\odot$. More explicitly, the G3 and IU-FSU give almost the same results, while NL3 predicts a comparatively larger value of $K_{\text{star}}$ as can be seen from the inset of the figure. Beyond mass $M = 1.8M_\odot$, the incompressibility increases suddenly, as shown in Fig. 4. The similarity of $K_{\text{star}}$ between G3 and IU-FSU forces could be related with the $K_\infty$ which are 243.96 and 231.31 MeV for G3 and IU-FSU, respectively. On the other hand, the incompressibility of NL3 at saturation is quite high as compared to G3 and IU-FSU. The decrease of incompressibility with the density of a system is reported in one of our earlier works (Kumar et al. 2020). The numerical range
of $K_\infty$ can not be fixed unrestricted. For larger incompressibility, we get more a stiffer equation of state, and the requirement of the compatibility of the EoS with causality is needed to be respected, keeping the adiabatic sound speed less than the speed of light (Das et al. 2020b). Although the NL3 gives the highest NS mass, it has the lowest central density because of the result of causality restriction (Olson 2000). After realizing that the EoS can be made softer/stiffer by reducing/increasing the $K$, which in turn reduces/enhances the maximum mass of the neutron star considerably. To see the variation of incompressibility with the mass of the neutron star, we present the results of $K_{\text{star}}$ with mass in Fig. 4 with the three forces for masses $M = 0.8M_\odot - M_{\text{max}}$. This shows that the value of $K_{\text{star}}$ increases marginally up to $M = 1.8M_\odot$ and suddenly increases beyond, indicating the mass dependence of the incompressibility.

3.3.2 Symmetry energy and its higher-order derivatives

The symmetry energy of a neutron star in its maximum mass is predicted to be higher in comparison to the value of symmetric nuclear matter at saturation. This observation is noticed in all the three considered E-RMF models. The symmetry energy at saturation $J_0$ for NM with NL3, G3, and IU-FSU sets are 37.43, 31.84, and 32.71 MeV, respectively. These results are 146.002, 66.813, and 60.758 MeV for the NS at the limit of maximum mass. These values of $S_{\text{star}}$ are very small as compared to the value at the maximum mass of the star. The results of $S_{\text{star}}$ are depicted in Fig. 5 as a function of neutron star mass for all the three-parameter sets. The symmetry energy for G3 and IU-FSU are found to be almost similar to each other, while the values with the NL3 set are a bit higher. This can be seen clearly from the inset of the figure. For the maximum mass of the NS, the values of $S_{\text{star}}$ are quite large for all the models. This behavior shows the structural dependence of the symmetry energy. The symmetry energy is obtained from the derivative of the energy density with respect to asymmetry $\alpha$, which shows a significant variation in the $S_{\text{star}}$ as compared to $K_{\text{star}}$. The higher derivatives of the symmetry energy, i.e., the slope $L_{\text{sym}}^{\text{star}}$ and curvature parameter $K_{\text{sym}}^{\text{star}}$, are quite useful quantities. These are shown in Figs. 6 and 7 as a function of neutron star mass. The magnitude of $L_{\text{sym}}^{\text{star}}$ and $K_{\text{sym}}^{\text{star}}$ increases with the mass of the star. The $L_{\text{sym}}^{\text{star}}$ values for all the models are found to be positive, contrary to the values of $K_{\text{sym}}^{\text{star}}$, which are negative in nature, as shown in the figures. The negative sign of $K_{\text{sym}}^{\text{star}}$ is correlated by the 1-$\sigma$ constraint, and 90% confident limits on its saturation value of normal nuclear matter as reported by Zimmerman et al. from the experimental data of PSR J0030+0451 and GW170817 event (Zimmerman et al. 2020; Riley et al. 2019; Abbott et al. 2017). It is noted that these bounds are not well matched to explain the $K_{\text{sym}}^{\text{star}}$ of NS; however, it indicates the possibility of the negative value of the curvature parameter and predicts the range with ~ 90% confidence limit of the observational data. Sometimes
Figure 8. (color online) The ratio of the volume to surface components of the symmetry energy $\kappa$ as a function of NS mass for NL3 (red), G3 (green) and IU-FSU (magenta) parameter sets. The $\kappa$ values are evaluated from Eq. (18).

it is beneficial to analyze the terrestrial data related to the exotic nuclei, and heavy-ion collisions (Chen et al. 2007; Centelles et al. 2009) by separating the contribution of isovector incompressibility or curvature parameter part.

Finally, the total symmetry energy of the neutron star can be divided into its volume $S_V$, and surface $S_S$, components, which are derived from Eq. (17) through $\kappa$ (the ratio of the volume to surface symmetry energy) and $\kappa$ is obtained from Eq. (18). The value of $\kappa$ as a function of the mass of the neutron star is shown in Fig. 8. In our calculation, $\gamma = 0.3$ is used following Antonov et al. (2018). The $\kappa$ values are consistently larger for the NL3 set followed by IU-FSU and least for the G3 force. These trends are in accordance with the forces used in the calculations. It goes on the increase with the mass of the NS to some value, as shown in the figure. Beyond that, the $\kappa$ value decreases considerably. When the mass number of the system approaches a large value, i.e., in the limit of $A \rightarrow \infty$, the system deals mostly with the volume. For example, for $1.4 \ M_\odot$ of G3 set, the total symmetry energy is 8.904 MeV, while the volume part contributes as a whole and the surface component contributes nearly 9% only of total $S^{star}$. In such a case, the major contribution of symmetry energy comes from the volume part of the NS as referring Eq. (17). To take care of such a system properly, the alternative method of Gaidarov et al. (2021) may be useful. The $\kappa$ is defined as $\kappa = S_V / S_S$, and even though the total symmetry energy is contributed by the volume component, i.e., $S \rightarrow S_V$, it has a surface component due to the finite value of $\kappa$.

4 SUMMARY AND CONCLUSIONS

The structural properties of neutron stars, mass, radius, and the moment of inertia are studied within the well-known E-RMF formalism. Three established forces (NL3, G3, and IU-FSU) are used in the calculations. Although NL3 is one of the oldest sets, it gives an understanding of the properties of finite nuclei. Also, this set is used extensively, so it may give a piece of known information, and a comparison of other sets with the results of NL3 may be more familiar. The main motivation of the present work is the surface properties of the neutron star in terms of incompressibility, symmetry energy, and its higher coefficients, such as the slope and curvature coefficients.

To our knowledge, there are no prior theoretical results or empirical/experimental data are available to support our calculations of NS symmetry energy and incompressibility and, of course, the slope and curvature parameters. In recent work, we suggested the formalism for the computation of these quantities (Kumar et al. 2021a) for NS. In the present paper, we extended the model to study the properties systematically. Thus the results reported here are the first of such kind. We know that the neutron star is a highly iso-spin asymmetric system, and we expected that the NS surface properties could be very different than the normal nuclear matter. The larger $L^{sym}_S$ for NL3 force is in agreement with the stiff EoS (Bednarek et al. 2020) and moderate slope parameter predict for the softer G3 and IU-FSU sets. A proper understanding of the range of $L^{sym}_S$, $L^{sym}_V$ and $K^{sym}_S$ generally help to fix the radius of a NS. A precise correlation is established via Danielewicz’s liquid drop prescription with the factor $\kappa$, i.e., the ratio of the volume to the surface component of the symmetry energy. This correlation can be extended to $L^{sym}_V$ in the liquid drop mass formula with the help of surface $S^{sym}_S$, and volume $S^{sym}_V$ symmetry energy (Lattimer 2015). From the analysis of the surface properties, we noticed that almost all these parameters, along with $\kappa$ for all the three sets, have coincided with each other in the vicinity of mass range $M \sim 1.8M_\odot$ indicating a possible correlation with the mass of the NS. It is shown in (Lattimer 2015; Lattimer & Lim 2013) that the static dipole and quadrupole polarizability and the neutron-skin thickness are strongly related between symmetry energy and slope parameter. Thus, we expect that the NS radius can be synchronized with its surface properties. The magnitude of $K^{star}_S$, $S^{star}_V$, $L^{sym}_S$ and $K^{sym}_S$ increases with mass of the neutron star. That means, for smaller NS, we find smaller values of all the surface properties estimated in this paper. The second derivative of symmetry energy is the $K^{sym}_S$, which is obviously more sensitive to the mass and also most ambiguous.

The present systematic calculations may provide a better theoretical bound on $K^{sym}_S$ and a better pathway to constrain the experimental setup for the isoscalar giant resonances for the properties of astrophysical objects. In spite of the absence of the direct experimental data for these surface properties of neutron stars (incompressibility, symmetry energy, slope, and curvature parameters), the calculated results using the E-RMF densities and CDFM approach seems reasonable. The present theoretical calculations can be validated using different relativistic and non-relativistic energy density functionals and suitable force parameters. The present method of accessibility to NS considering the neutron star as a finite nucleus system favors a bridge between the two unequal size objects. The theoretical approach adopted here presents a new way for the nuclear and astrophysicist to reveal the wealth of information on the exotic nuclei and dense astronomical objects.

ACKNOWLEDGEMENT

Mr. J. A. Pattnaik thanks the Institute of Physics, Bhubaneswar, for facilities. SERB partly reinforces this work, Department of Science and Technology, Govt. of India, Project No. CRG/2019/002691.

DATA AVAILABILITY

This manuscript has no associated data or the data will not be deposited. Data sharing not applicable to this article as no data sets were generated or analysed during the current study.
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