Tunable exciton Aharonov-Bohm effect in a quantum ring

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Abstract. We studied the optical Aharonov-Bohm effect for an exciton in a semiconductor quantum ring. A perpendicular electric field applied to a quantum ring with large height, is able to tune the exciton ground state energy such that it exhibits a weak observable Aharonov-Bohm oscillations. This Aharonov-Bohm effect is tunable in strength and period.

1. Introduction

The electron and hole wave functions acquire an extra phase when moving in the presence of a perpendicular magnetic flux. The phase difference between the electron and the hole wave function can be observed through photoluminescence (PL) experiments, which exhibits an optical Aharonov-Bohm (AB) effect. Experimentally, this effect has been reported in PL measurements of radially polarized neutral excitons in a type-II quantum dot structure. [1, 2]. But the optical AB effects will be strongly suppressed [3] when both the electron and the hole are spatially confined within the same geometry, i.e. in a quantum ring (as in a type-I quantum dot). Theoretically, this optical AB effect in a quantum ring can be enhanced when the exciton is radially polarized, either by the application of an external electric field [6, 7] or due to a radial asymmetry in the effective confinement for electrons and holes [8, 9].

Studies [3, 8, 9] on one dimensional rings predict that for a quantum ring whose radial size is comparable to the exciton Bohr radius (in the weakly bound regime), the ground state energy can display a nonvanishing AB oscillation. However, numerical calculations on two dimension narrow rings [4, 5] show that there is no AB oscillation observable for the exciton ground state energy, except for some low-lying energy levels. [5]. In this paper, we calculate the exciton energy in three dimension volcano-like quantum rings by using the configurational interaction method for small quantum rings where the exciton is in the weakly bound regime. We found that for such quantum rings with large height and in the presence of a strong perpendicular electric field, the exciton ground state energy shows a weak but nonvanishing Aharonov-Bohm oscillation as function of the external magnetic field.

2. Model

The geometry of the quantum ring is schematically depicted in Fig. 1: a volcano-like GaAs ring surrounded by a Al₀.₃Ga₀.₇As barrier. We consider a ring with inner (outer) radius $R₁ = 8$
of the wave function, we calculate the total exciton energy by using the configurational interaction (CI) method. The total Hamiltonian of the exciton is of the form \( \Psi(\rho, z, \theta) = \psi_{n_e(h)}(\rho, z)e^{-\frac{|qE_\rho|}{|\rho|}} \). After averaging out the angular part of the wave function, we calculate \( \psi_{n_e(h)}(\rho, z) \) by solving the resulting 2D Schrödinger equation using the finite element method:

\[
\left( -\frac{\hbar^2}{2m_e(h)} \left( \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} - \frac{l_e^2(h)}{\rho^2} - \frac{q^2B^2\rho^2}{4m_e(h)} \right) - \frac{qBl_e(h)\hbar}{2m_e(h)} + V(\vec{r}_e(h)) + qE_\rho(\rho, z) \right) \psi_{n_e(h)}(\rho, z) = E_{\rho, z} \psi_{n_e(h)}(\rho, z),
\]

where \( q \) is \(-e\) for electron and \( e\) for hole.
Figure 2. Single particle energy levels of (a) the electron and (b) the hole for $n_e = 1$ (solid line), $n_e = 2$ (dash line) and different values of $l_e$. The top (bottom) subplot is for $E = 0$ ($E = 200\, kV$)

Figure 2 shows our numerical results for the magnetic field dependence of the electron (hole) energy for different values of the quantum number $n_e$ ($n_h$) and $l_e$ ($l_h$) ($l_e = - l_h$), in the presence of a perpendicular electric field directed from the bottom to the top of the ring structure. Notice that the energy difference between levels with the same angular quantum number $l$ and different $n$ are much larger than those between the same $n$ but different $l$. Both the electron and the hole ground state energy exhibit angular momentum transitions from small absolute values of angular moment $l$ to large ones. But as the inner radius of the ring is small, the period of the oscillation is large and thus we can only observe few transitions for a magnetic field up to $B = 30\, T$. In the presence of a strong perpendicular electric field, the energy difference between the electron (hole) levels with different quantum number $n_e$ ($n_h$) are smaller (larger). Notice that there is another angular momentum transition of the hole ground state energy in the $B = 30\, T$ range. The reason is that the electric field pushes the hole upwards in the quantum ring where the effective radius is larger, resulting in Aharonov-Bohm oscillations with a smaller period.

As a result of the cylindrical symmetry, the total angular moment is a good quantum number. The exciton wave function is expanded as $\Psi_L (\vec{r}_e, \vec{r}_h) = \sum_k C_k \Phi_k (\vec{r}_e, \vec{r}_h)$ for fixed total angular moment $L$. Here, $\Phi_k (\vec{r}_e, \vec{r}_h) = \psi_{n_e} (\rho_e, z_e) e^{-il_e\varphi_e} \psi_{n_h} (\rho_h, z_h) e^{-il_h\varphi_h}$, with $l_e + l_h = L$, and $k$ stands for the collection of indices ($n_e, n_h, l_e, l_h$). With these wave functions, we construct the matrix of the total Hamiltonian and after diagonalizing the obtained matrix, we find the eigenvalues and eigenvectors. As the energies of the eigenstates with large $l_e$ ($l_h$) and large $n_e$ ($n_h$) are much larger as compared to the one with small quantum numbers, only several tens of low lying levels have to be included, in order to obtain sufficient accuracy.

Our results for the exciton ground state energy is shown in Fig. 3, the blue (green) line is for $E = 0$ ($E = 200\, kV$), we should specify here that the ground state is always the exciton state with total angular moment quantum number $L = 0$ for $B < 30\, T$. From Fig. 3 no oscillatory behavior is seen for $E = 0$ as a function of magnetic field $B$, which is similar for a two dimension quantum ring as shown in Ref. 4. The magnetic field dependence of the exciton energy for $E = 0$ is almost a parabola, but from the second derivative of the energy with respect to the magnetic field a weak oscillation appears (see inset of Fig. 3). But when we apply a strong perpendicular
electric field, the exciton ground state energy shows an observable Aharonov-Bohm oscillation. The reason is that the perpendicular electric field pushes the hole to the top area of the ring with a larger value of $z$ where the exciton will be more extended in the $\rho$-direction, while it pushes the electron to the bottom of the ring. This polarizes the exciton and weakens the Coulomb interaction between the electron and the hole. From the inset of Fig. 3, we find that although the applied electric field makes the Aharonov-Bohm effect stronger and more easy to observe, the period of oscillation increases. The first oscillation ends at $B = 18T$ ($B = 20.5T$) for $E = 0$ ($E = 200$ kV). We should mention here that in our case the electric field is in the perpendicular direction, which is different from Refs. 6 and 7 where an electric field in the lateral plane was applied.

4. Conclusion
In this paper we studied the ground state energy of a neutral exciton in a semiconductor quantum ring within the configurational interaction method. No observable Aharonov-Bohm effect was found for the neutral exciton ground state energy. We showed how a perpendicular electric field applied to a pyramidal shaped quantum ring is able to induce the Aharonov-Bohm effect.

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