Gravidynamics, spinodynamics and electrodynamics within the framework of gravitational quantum field theory

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By noticing the fact that the charged leptons and quarks in the standard model are chirality-based Dirac spinors since their weak interaction violates maximally parity symmetry though they behave as Dirac fermions in electromagnetic interaction, we show that such a chirality-based Dirac spinor possesses not only electric charge gauge symmetry U(1) but also inhomogeneous spin gauge symmetry WS(1,3)=SP(1,3)⊗W_{1,3}, which reveals the nature of gravity and spacetime. The gravitational force and spin gauge force are governed by the gauge symmetries W_{1,3} and SP(1,3), respectively, and a biframe spacetime with globally flat Minkowski spacetime as base spacetime and locally flat gravigauge spacetime as a fiber is described by the gravigauge field through emergent non-commutative geometry. The gauge-geometry duality and renormalizability in gravitational quantum field theory (GQFT) are carefully discussed. A detailed analysis and systematic investigation on gravidynamics and spinodynamics as well as electrodynamics are carried out within the framework of GQFT. A full discussion on the generalized Dirac equation and Maxwell equation as well as Einstein equation and spin gauge equation is made in biframe spacetime. New effects of gravidynamics as extension of general relativity are particularly analyzed. All dynamic equations of basic fields are demonstrated to preserve the spin gauge covariance and general coordinate covariance due to the spin gauge symmetry and emergent general linear group symmetry GL(1,3,R), so they hold naturally in any spinning reference frame and motional reference frame.

inhomogeneous spin gauge symmetry, locally flat gravigauge spacetime, gravitational relativistic quantum mechanics with generalized Dirac equation, gravidynamics with generalized Einstein equation, generalized Maxwell equation in any motional and spinning reference frame

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1 Introduction

The theory of electromagnetism was built by Maxwell to unify the electricity and magnetism, which is regarded as the first classical unified field theory. The constancy of the speed of light in Maxwell’s theory inspired Einstein to build up the special theory of relativity (SR) [1] which unifies space and time into a four-dimensional spacetime. Such a spacetime is referred to as four-dimensional Minkowski spacetime which is characterized by the Lorentz symmetry SO(1,3). Later on, Einstein [2, 3] extended the SR to the theory of general relativity (GR) based on the principle of general coordinate covariance under arbitrary transformations of coordinates, which is characterized by the general linear group symmetry GL(1,3,R). In GR, the gravity is described by the dynamics of Riemannian geometry of curved spacetime instead of globally flat four-dimensional Minkowski spacetime in SR. The GR indicates that space and time cannot be well defined in such a way that the differences of the spatial coordinates or time coordinates can directly be measured by the standard ways in SR. On the other hand, the relativistic quantum mechanics built by Dirac has successfully combined the quantum mechanics and special relativity, which led to the establishment of quantum field theory (QFT) in globally flat four-dimensional Minkowski spacetime. The framework of QFT was initiated to extend the classical electromagnetism into quantum electrodynamics (QED) [4-11], which is governed by the Abelian gauge symmetry U(1). Later on, Yang and Mills [12] extended such an Abelian gauge symmetry to a non-Abelian gauge symmetry SU(2) in describing the isotopic spin gauge invariance.

The success of the standard model (SM) shows that the basic forces of electroweak and strong interactions are governed by the gauge symmetries U(1)×SU(2)×SU(3) [13-18]. Namely, the three basic forces in SM can well be described by the dynamics of gauge fields within the framework of quantum field theory formulated in globally flat Minkowski spacetime, which is in contrast to the gravity described by Einstein’s GR built based on the dynamics of Riemannian geometry of curved spacetime. It is such a contradiction that causes an obstacle to make a unified description on the gravity and other three basic forces. Recently, the quantum field theory of gravity was constructed based on the spin and scaling gauge symmetries when treating the gravitational interaction on the same footing as the electroweak and strong interactions, such a gravitational gauge field theory provides a framework of gravitational quantum field theory (GQFT) [19, 20]. The concept of biframe spacetime was shown to play an essential role in GQFT, where a bicovariant vector field defined in biframe spacetime is introduced as a basic gravitational field instead of a metric field in GR to describe the gravitational interaction. In GQFT, the action was proposed to possess a joint symmetry, namely, the Poincaré group symmetry PO(1,3) in Minkowski spacetime of coordinates and a proposed internal Poincaré-type gauge symmetry PG(1,3) in noncoordinate spacetime spanned by the bicovariant vector field. Such a bicovariant vector field was postulated to be a gauge-type field in the coset PG(1,3)/SP(1,3) of internal Poincaré-type gauge symmetry group PG(1,3) with spin gauge group SP(1,3) as a subgroup.

It is clear that the key point is to realize explicitly the group structure of internal Poincaré-type gauge symmetry PG(1,3), which comes to the main purpose in our present paper. Our starting point in this paper is to examine carefully the chiral structure of Dirac fermion. This is because the electron and charged leptons and quarks as basic constituents of matter in SM are actually Weyl fermions although they behave as Dirac fermions in the electromagnetic interaction. The reason is simple by noticing the well-known fact that the weak interactions of leptons and quarks in SM violate maximally the law of parity conservation [21-24]. To make a systematical analysis and detailed investigation, our paper is organized as follows: after a brief introduction in sect. 1, we will show in sect. 2 how an inhomogeneous spin symmetry WS(1,3) in Hilbert space is obtained to provide an internal Poincaré-type symmetry. Specifically, by viewing the Dirac fermion in four-dimensional Hilbert space as the superposition of left-handed and right-handed Weyl fermions and representing it into a chirality-based Dirac spinor in a chiral spinor representation of eight-dimensional Hilbert space, we are able to demonstrate that such a chirality-based Dirac spinor gets an enlarged inhomogeneous spin symmetry group WS(1,3)=SP(1,3)×W\text{1,3}, which is a semi-direct product group with SP(1,3) being the spin group and W\text{1,3} referring to as W\text{r}-spin group. Such a W\text{r}-spin group W\text{1,3} is shown to be a translation-like chiral-type spin group in eight-dimensional Hilbert space. Meanwhile, an internal Abelian symmetry group U(1) remains as electric charge symmetry group. A free motion chirality-based Dirac spinor is verified to possess a maximal associated symmetry when combining the inhomogeneous spin symmetry WS(1,3) in eight-dimensional Hilbert space with the Poincaré group symme-
try PO(1,3) in four-dimensional Minkowski spacetime of coordinates. In sect. 3, the inhomogeneous spin symmetry WS(1,3) and electric charge symmetry U(1) are gauged to be local symmetries based on the gauge invariance principle. It is found that the \( W_\mu \)-spin invariant-gauge field brings on the genesis of gravigauge field, which reveals the nature of gravity and brings on the gravitational origin of spin gauge symmetry. The group structure of internal Poincaré-type gauge symmetry PG(1,3) is explicitly realized to be as the inhomogeneous spin gauge symmetry, i.e., PG(1,3)\( \equiv \)WS(1,3)=SP(1,3)\( \times \)W\(^{1,3}\). We further present in sect. 4 a general demonstration that the non-coordinate spacetime spanned by the gravigauge field basis generates a locally flat gravigauge spacetime characterized by emergent non-commutative geometry. A fiber bundle structure is analyzed in light of biframe spacetime with globally flat Minkowski spacetime as base spacetime and locally flat gravigauge spacetime as a fiber. In sect. 5, we will show the scaling property of basic fields and make a general definition on gauge fields and field strengths as vector and tensor fields in locally flat gravigauge spacetime. We are going to formulate in sect. 6 the various formalisms of gauge and scaling invariant actions for the chirality-based Dirac spinor based on the action principle of path integral formulation, which enables us to prove the gauge-gravity and gravity-geometry duality relation within the framework of gravitational quantum field theory. A generalized Dirac equation is derived in sect. 7 to describe the gravitational relativistic quantum theory. Meanwhile, a gravigauge Dirac equation is obtained in locally flat gravigauge spacetime. In sect. 8, we will present a detailed investigation on the gravodynamics by deriving the gauge-type gravitational equations of gravigauge field in locally flat gravigauge spacetime and biframe spacetime. A geometric gravodynamics is discussed as the extension of general relativity and also analyzed in locally flat gravigauge spacetime. We provide a particular investigation on the new effects of gravidynamics beyond the Einstein theory of general relativity. The spinodynamics is studied in sect. 9 by deriving the equation of motion for the spin gauge field. In sect. 10, we provide a systematic analysis on the electrodynamics in the presence of gravitational interaction. Various generalized Maxwell equations are derived to characterize the so-called gravigeometry-medium electrodynamics in the presence of gravigravitational field and the gravigauge-mediated electrodynamics obtained in locally flat gravigauge spacetime. Meanwhile, the electromagnetic-like gravimetric-gauge field is introduced in generalized Maxwell equations to describe the gravimetric-gauge-mediated electrodynamics formulated in dynamic Riemannian spacetime. We will further show in sect. 11 that the dynamics of all basic fields obeys the general coordinate covariance in curved Riemannian spacetime and the spin gauge covariance in locally flat gravigauge spacetime. It is demonstrated that the gravigeometry-medium electrodynamics holds in any motional reference frame and the gravigauge-mediated electrodynamics is applicable in any spinning reference frame. In particular, the gravigeometry-medium Maxwell equations in special background medium are explicitly demonstrated to maintain the general coordinate covariance in motional reference frame, and the gravigauge Maxwell equations in special background medium are examined to keep the spin gauge covariance. Our conclusions and discussions are presented in the final section.

2 Inhomogeneous spin symmetry WS(1,3) and maximal associated symmetry for chirality-based Dirac spinor

The basic constituents of nature are known to be the leptons and quarks as indicated in SM. The observed matter consists of the electrons and nucleons which are made of the up and down quarks, they all appear as Dirac fermions in the electromagnetic interaction. Their free motion is described by the following relativistic quantum Dirac equation [25]:

\[
\left( \mathbf{E} - c \mathbf{a} \cdot \mathbf{p} - \beta mc^2 \right) \psi = 0,
\]

or

\[
\left( \mathbf{E} + c \mathbf{a} \cdot \mathbf{p} + \beta mc^2 \right) \psi = 0,
\]

with \( \mathbf{a} = (a_1, a_2, a_3) \) and \( \beta \) the \( 4 \times 4 \) Hermitian matrices satisfying the conditions \( \alpha \beta = -\beta \alpha, \alpha_i \alpha_j = -\delta_{ij} (i \neq j) \) and \( \alpha_i^2 = \beta^2 = 1 \). \( \psi \) is the Dirac fermion with a complex four-component entity \( \psi^T = (\psi_1, \psi_2, \psi_3, \psi_4) \) in the spinor representation of four-dimensional Hilbert space. A general covariant formulation of the Dirac equation can be rewritten as follows:

\[
(\gamma^\mu \partial_\mu - m) \psi = 0,
\]

with \( \mu = 0, 1, 2, 3 \), \( \gamma^0 = \beta \) and \( \gamma^i = \beta a^i \), where \( \partial_\mu = \partial / \partial x^\mu \) is the 4-dimensional coordinate derivative and \( \gamma_\mu \) the \( 4 \times 4 \) matrices which satisfy the anticommutation relations,

\[
\{ \gamma_\mu, \gamma_\nu \} = \eta_{\mu \nu}, \quad \eta_{\mu \nu} = \text{diag}(1, -1, -1, -1).
\]

The Hermitian action in obtaining the above Dirac equation can be constructed as follows:

\[
S_D = \int d^4x \frac{1}{2} \left( \bar{\psi}(x) \gamma^\mu \partial_\mu \psi(x) + H.c. \right) - m \bar{\psi}(x) \psi(x), \quad (1)
\]

with \( \bar{\psi}(x) = \psi^\dagger(x) \gamma^0 \) and \( m \) being the mass of Dirac fermion.

In the SM of electroweak and strong interactions, the charged leptons and quarks are actually Weyl fermions due
to their weak interactions which bring on the maximal parity violation. Such a Dirac fermion in the electromagnetic interaction should be regarded as the superposition of left-handed and right-handed Weyl fermions. By representing the Dirac fermion in a chiral spinor representation of eight-dimensional Hilbert space, we can express the action of free motion Dirac fermion in a chiral spinor representation of eight-dimensional Hilbert space, we can express the action of free motion Dirac fermion in a chiral spinor representation of eight-dimensional Hilbert space, where we provide the chiral project operators via the chiral Γ-matrices in coordinate spacetime and noncoordinate space.

$$S = \int d^4x \frac{1}{2} \left\{ \bar{\Psi}_-(x) \Gamma_- \delta_\mu^\nu \partial_\mu \Psi_-(x) - \bar{\Psi}_-(x)(m_3 \Gamma_- + m_5 \Gamma_5) \Psi_-(x) + H.c. \right\},$$

where we have introduced the following definitions:

$$\Psi_+(x) = \Gamma_+ \Psi(x) \equiv \begin{pmatrix} \psi_L(x) \\ \psi_R(x) \end{pmatrix},$$

$$\psi_{L,R}(x) = \gamma_\pm \psi(x),$$

$$\Gamma_\pm = \frac{1}{2} (1 \mp \gamma_5), \quad \gamma_\pm = \frac{1}{2} (1 \mp \gamma_5),$$

$$\Gamma^\alpha = \sigma_\alpha \otimes \gamma^\alpha, \quad \gamma^\alpha = \delta^\alpha_\mu \gamma^\mu,$n

$$\Gamma^5 = i \sigma_1 \otimes \gamma_5, \quad \Gamma^6 = i \sigma_2 \otimes \gamma_5,$n

$$\Gamma^a = 2(\Sigma^a + \Sigma^a_3), \quad \Sigma^a_3 = \frac{1}{2} \Gamma^a \gamma_5,$n

$$\gamma_5 = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \sigma_3 \otimes \sigma_0,$n

$$\gamma_7 = -\sigma_0 \Gamma^1 \Gamma^2 \Gamma^{\alpha} \Gamma^6 = \sigma_3 \otimes \gamma_5,$n

with \( a = (a, 6, 7) \), where \( \gamma^\alpha \) and \( \gamma_5 \) are the usual \( \gamma \) matrices and \( \delta^\alpha_\mu \) is the Kronecker symbol. Note that \( \gamma_5 \) and \( \gamma_7 \) provide the chiral project operators via the chiral \( \gamma \)-matrices \( \gamma_5 \) and \( \gamma_7 \) defined in four-dimensional and eight-dimensional Hilbert spaces, respectively.

It is noticed that \( \Psi_-(x) \) defines a chirality-based Dirac spinor in the chiral spinor representation of eight-dimensional Hilbert space with \( \psi_{L,R}(x) \) denoting the ordinary left-handed and right-handed Weyl fermions in four-dimensional Hilbert space. Note that we have introduced the Greek alphabet \( (\mu, \nu = 0, 1, 2, 3) \) and Latin alphabet \( (a, b = 0, 1, 2, 3) \) to distinguish four-vector indices in coordinate spacetime and noncoordinate spacetime, respectively. Both the Greek and Latin indices are raised and lowered by the constant metric matrices, i.e., \( \eta_{\mu\nu} \) and \( \eta^{ab} \) with \( \eta_{\mu\nu}(\eta_{\mu
u})=\text{diag}(1, -1, -1, -1) \) and \( \eta^{ab}(\eta^{ab})=\text{diag}(1, -1, -1, -1) \). All the scalar product of vectors and tensors is obtained via the contraction with the constant metric matrices \( \eta_{\mu\nu} \) and \( \eta^{ab} \), i.e., \( V^\mu V_\mu = \eta_{\mu\nu} V^\mu V^\nu \) and \( A^a A_b = \eta^{ab} A^a A_b \). The system of units is chosen such that \( c = \hbar = 1 \).

The mass term of Dirac fermion is produced via the following relations:

$$\Psi_-(x)(m_3 \Gamma^5 \Gamma_- + m_5 \Gamma^5 \Gamma_-) \Psi_-(x) = m \bar{\Psi}_-(x) \Gamma^5 \Gamma_- e^{i2\alpha_1 \gamma_5} \Psi_-(x) \equiv m \bar{\Psi}_-(x) \Gamma^6 \Gamma_- e^{i2\alpha_2 \gamma_5} \Psi_-(x) \rightarrow m \bar{\Psi}_-(x) \Gamma^6 \Gamma_- \Psi_-(x) = m \bar{\psi}_\gamma \psi,$$

where we have made a replacement in obtaining the last form:

$$\Psi_-(x) \rightarrow e^{-i\alpha_1 \gamma_5} \Psi_-(x) \equiv e^{i\alpha_2 \gamma_5} \Psi_-(x),$$

with \( \gamma_7 = \sigma_3 \otimes \sigma_0 \otimes \sigma_0 \), tan \( 2\alpha_1 = m_5 / m_6 \), \( m = \sqrt{m_5^2 + m_6^2} \).

Note that such a replacement for the chirality-based Dirac spinor preserves the kinetic term of the action in eq. (2) due to its invariance under such a transformation.

The \( \Gamma \)-matrices \( \Gamma^a \) with \( \hat{a} = (a, 5, 6) \) and \( \hat{a} \gamma_7 \) defined in eight-dimensional Hilbert space satisfy the anticommuting relations of Clifford algebra:

$$\{ \hat{a}, \Gamma^a \} = \eta^{ab}, \quad \{ \Gamma^a, \hat{a} \gamma_7 \} = 0,$$

which indicates that the mass of Dirac fermion may have a geometric origin via the extra dimension of spacetime as discussed in ref. [26].

It can be verified that the action given in eq. (2) possesses the following associated symmetry:

$$G_S = P_{1}^{13} \ltimes SO(1, 3) \otimes SP(1, 3) \times W_{13} \times U(1)$$

$$= PO(1, 3) \otimes WS(1, 3) \times U(1),$$

where the symbol “\( \otimes \)” is adopted to notate the associated symmetry in which the transformation of spin symmetry group \( SP(1, 3) \) in Hilbert space must be coincidental to that of the isomorphic Lorentz symmetry group \( SO(1, 3) \) in Minkowski spacetime.

\( PO(1, 3) \) denotes the inhomogeneous Lorentz symmetry group or Poincaré symmetry group in four-dimensional Minkowski spacetime.

\( PO(1, 3) = P_{1}^{13} \ltimes SO(1, 3), \)

where the symbol “\( \ltimes \)” is used to indicate that \( PO(1, 3) \) is a semi-direct product group with \( P_{1}^{13} \) representing the translational symmetry group of coordinates in Minkowski spacetime.

\( WS(1, 3) \) is an enlarged spin symmetry group, which is also a semi-direct product group expressed as follows:

$$WS(1, 3) \equiv SP(1, 3) \times W_{13}.$$

(9)

It can be checked that \( SP(1, 3) \) group generators \( \Sigma^{ab} \) and \( W_{13} \) group generators \( \Sigma^a \) have the following explicit forms:

\( \Sigma^{ab} = \frac{1}{4} \{ \Gamma^a, \Gamma^b \}, \quad \Sigma^a = \frac{1}{2} \Gamma^a \Gamma_-, \quad \Gamma_- = \frac{1}{2} (1 - \gamma_7). \)
which satisfy the group algebra:
\[
[\Sigma^{ab}, \Sigma^{cd}] = i(\Sigma^{ad} \eta^{bc} - \Sigma^{bd} \eta^{ac} - \Sigma^{ac} \eta^{bd} + \Sigma^{bc} \eta^{ad}),
\]
\[
[\Sigma^{ab}, \Sigma^c] = i(\Sigma^{a} \eta^{bc} - \Sigma^{b} \eta^{ac}),
\]
\[
[\Sigma^a, \Sigma^c] = 0,
\]
where \(W^{1,3}\) appears as a translation-like Abelian-type symmetry group in Hilbert space.

Let us now discuss explicitly the translation-like Abelian-type symmetry group \(W^{1,3}\). The chirality-based Dirac spinor \(\Psi_-(x)\) transforms under \(W^{1,3}\) group operation as follows:
\[
\Psi'_-(x) \rightarrow \Psi'_-(x) = S(\sigma)\Psi_-(x),
\]
\[
S(\sigma) = e^{i\sigma_i \Sigma^i/2} = 1 + i\sigma_\mu \Sigma^\mu/2,
\]
which can be rewritten into the following forms:
\[
\Psi'_-(x) \equiv \Psi^{(w)}_-(x) + \Psi^{(v)}_-(x),
\]
\[
\Psi^{(w)}_-(x) \equiv \Psi_-(x), \quad \Psi^{(v)}_-(x) \equiv \frac{1}{2} \sigma_\mu \Sigma^\mu \Psi_-(x),
\]
where \(\Psi^{(w)}_-(x)\) and \(\Psi^{(v)}_-(x)\) are associated with the initial and shifted new parts, respectively, in order to distinguish what appears as a translation-like Abelian-type symmetry just for the free motion Dirac fermion expressed in the chiral spinor representation of eight-dimensional Hilbert space.

\[\hat{\gamma}_\tau \Psi^{(w)}_-(x) = -\Psi^{(w)}_-(x), \quad \hat{\gamma}_\tau \Psi^{(v)}_-(x) = \sigma_\mu \Sigma^\mu \Psi^{(v)}_-(x).\]

For that, \(\Psi^{(w)}_-(x)\) and \(\Psi^{(v)}_-(x)\) are referred to as westward and eastward chirality-based Dirac spinors indicated by the upper indices “w” and “v”, respectively, in order to distinguish with the left-handed and right-handed Weyl fermions \(\psi_L(x)\) and \(\psi_R(x)\) defined in four-dimensional Hilbert space.

As such an Abelian-type spin symmetry group \(W^{1,3}\) is associated with the sign flip in chirality of westward and eastward spinor fields for the initial and shifted new parts, we may refer to the translation-like chiral-type symmetry \(W^{1,3}\) as \(W^{1,3}\)-spin symmetry for short. So we may refer to the enlarged spin symmetry group \(WS(1,3)\) as an inhomogeneous spin symmetry group acting on the chirality-based Dirac spinor \(\Psi_-(x)\), which is a semi-direct product group as indicated by the symbol “\(\times\)”. It is intriguing to notice that such an inhomogeneous spin symmetry group is in correspondence to the inhomogeneous Lorentz symmetry group (or Poincaré symmetry group) \(PO(1,3)\) as an inhomogeneous spin symmetry group acting on the chirality-based Dirac spinor \(\Psi_-(x)\) in Hilbert space and the latter operates on the coordinates in Minkowski spacetime.

\[\hat{\gamma}_\mu \rightarrow i\hat{\gamma}_\mu \equiv i\hat{\gamma}_\mu + \hat{A}_\mu(x) + A_\mu(x),\]

which describes the electric charge symmetry.

It is clear that the symmetry shown in eq. (8) presents the maximal associated symmetry for a free motion Dirac fermion in Hilbert space.

### 3 Inhomogeneous spin gauge symmetry with the genesis of gravigauge field and the gravitational origin of spin gauge symmetry

It is the essential point to distinguish the group symmetries in Hilbert space and Minkowski spacetime when proposing the gauge invariance principle to study the fundamental interaction. The gauge invariance principle states that the laws of nature should be independent of the choice of local field configurations, from which the inhomogeneous spin symmetry \(WS(1,3)\) and electric charge symmetry \(U(1)\) of chirality-based Dirac spinor \(\Psi_-(x)\) are postulated to be local gauge symmetries. Meanwhile, the inhomogeneous Lorentz group symmetry (Poincaré group symmetry) \(PO(1,3)\) of coordinates is considered to remain a global symmetry. In such a consideration, the inhomogeneous spin gauge symmetry \(WS(1,3)\) in Hilbert space of spinor field does become distinguishable from the inhomogeneous Lorentz group symmetry \(PO(1,3)\) in Minkowski spacetime of coordinates. It becomes natural that the inhomogeneous spin gauge symmetry group \(WS(1,3)\) provides the so-called internal Poincaré-type gauge symmetry group \(PG(1,3)\) proposed in ref. [19], i.e., \(PG(1,3) = WS(1,3) \times W^{1,3}\).

#### 3.1 Inhomogeneous spin gauge symmetry with the genesis of gravigauge field

Based on the gauge invariance principle, the gauge fields \(\hat{A}_\mu(x)\) and \(A_\mu(x)\) in correspondence to the inhomogeneous spin gauge symmetry \(WS(1,3)\) and electromagnetic gauge symmetry \(U(1)\) are introduced to ensure the theory be gauge invariant. To realize such a gauge invariant theory, it is practically carried out by replacing the usual derivative operator of coordinates in Minkowski spacetime into the following covariant derivative operator:

\[i\partial_\mu \rightarrow i\hat{D}_\mu \equiv i\partial_\mu + \hat{A}_\mu(x) + A_\mu(x),\]

with

\[\hat{A}_\mu(x) \equiv A_\mu(x) + \hat{A}_\mu(x), \quad A_\mu(x) \equiv A_\mu^{ab}(x) \frac{1}{2} \Sigma_{ab}, \quad \hat{A}_\mu(x) \equiv A_\mu^a(x) \frac{1}{2} \Sigma_{-a},\]
where \( \hat{A}_\mu(x) \) is referred to as inhomogeneous spin gauge field with \( \hat{A}^{ab}_\mu(x) \) representing the spin gauge field relevant to the spin gauge symmetry \( \text{Sp}(1,3) \) and \( \mathcal{R}^{ab}_\mu(x) \) denoting the \( \mathcal{W}_c \)-spin gauge field relating to the translation-like \( \mathcal{W}_c \)-spin Abelian gauge symmetry \( \mathcal{W}^{1,3} \). \( A_\mu(x) \) is the electromagnetic gauge field corresponding to the \( U(1) \) gauge symmetry.

From the general covariant derivative \( \hat{D}_\mu \), the field strength of gauge field \( \hat{A}_\mu(x) \) is obtained as follows:

\[
\hat{F}_{\mu\nu} = \hat{D}_\mu \hat{A}_\nu - \hat{D}_\nu \hat{A}_\mu + i[\hat{A}_\mu, \hat{A}_\nu],
\]

\[
\hat{F}_{\mu
u} = \mathcal{F}_{\mu\nu} + \hat{F}_{\mu\nu} + F_{\mu
u},
\]

or

\[
A^{ab}_\mu(x) = \Lambda^a_\mu(x) \Lambda^b_\mu(x) - \Lambda^b_\mu(x) \Lambda^a_\mu(x),
\]

\[
\mathcal{R}^{ab}_\mu(x) = \Lambda^a_\mu(x) \mathcal{R}^{ab}_\mu(x), \quad \Lambda^a_\mu(x) \in \text{Sp}(1,3),
\]

under the gauge transformation of \( \text{Sp}(1,3) \) gauge symmetry, and

\[
\mathcal{R}^{ab}_\mu(x) \rightarrow \mathcal{R}^{ab}_\mu(x) = \mathcal{R}^{ab}_\mu(x) + \mathcal{D}_\mu \sigma^a(x),
\]

\[
\mathcal{D}_\mu \sigma^a(x) = \partial_\mu \sigma^a(x) + \mathcal{R}^{ab}_\mu(x) \sigma^b(x),
\]

under the \( \mathcal{W}_c \)-spin gauge transformation of \( \mathcal{W}^{1,3} \) gauge symmetry.

It is noticed that the presence of gauge symmetry in the action involves actually redundant degrees of freedom. To eliminate the redundant degrees of freedom arising from the gauge symmetry, it needs to make a gauge prescription by imposing a gauge fixing condition. In order to extract the redundant degrees of freedom, we are initiated to decompose the \( \mathcal{W}_c \)-spin gauge field \( \mathcal{R}^{ab}_\mu(x) \) into the following two parts:

\[
\hat{A}_\mu(x) \equiv \mathcal{R}^{ab}_\mu(x) \frac{1}{2} \Sigma_{ab} = (\Omega^{ab}_\mu(x) + A^{ab}_\mu(x)) \frac{1}{2} \Sigma_{ab},
\]

where \( \Omega^{ab}_\mu(x) \) is supposed to get an inhomogeneous gauge transformation and \( A^{ab}_\mu(x) \) becomes gauge invariant under the \( \mathcal{W}_c \)-spin gauge transformation. Meanwhile, they should transform as covariant vector fields under the gauge transformation of spin gauge symmetry \( \text{Sp}(1,3) \). The explicit transformations are presented as follows:

\[
\Omega^{ab}_\mu(x) \rightarrow \Omega^{ab}_\mu(x) = \Omega^{ab}_\mu(x) + \mathcal{D}_\mu \sigma^a(x),
\]

\[
A^{ab}_\mu(x) \rightarrow A^{ab}_\mu(x) = A^{ab}_\mu(x),
\]

under the gauge transformation of \( \mathcal{W}_c \)-spin gauge symmetry \( \mathcal{W}^{1,3} \), and

\[
\Omega^{ab}_\mu(x) \rightarrow \Omega^{ab}_\mu(x) = \Lambda^a_\mu(x) \Omega^{ab}_\mu(x),
\]

\[
A^{ab}_\mu(x) \rightarrow A^{ab}_\mu(x) = \Lambda^a_\mu(x) A^{ab}_\mu(x),
\]

under the gauge transformation of spin gauge symmetry \( \text{Sp}(1,3) \).

We may refer \( \Omega^{ab}_\mu(x) \) to be as a \( \mathcal{W}_c \)-spin non-homogeneous gauge field and \( A^{ab}_\mu(x) \) as a \( \mathcal{W}_c \)-spin invariant-gauge field. From the property of \( \mathcal{W}_c \)-spin gauge transformation, it is natural to express \( \Omega^{ab}_\mu(x) \) into the following form:

\[
\Omega^{ab}_\mu(x) \equiv \mathcal{D}_\mu \sigma^a(x) = \partial_\mu \sigma^a(x) + \mathcal{R}^{ab}_\mu(x) \sigma^b(x),
\]

where we have introduced a vector field \( \sigma^a(x) \) in Hilbert space. Correspondingly, the field strength of \( \mathcal{W}_c \)-spin gauge field can also be decomposed into two parts:

\[
\hat{F}_{\mu\nu}(x) = \mathcal{F}^{ab}_\mu(x) + \mathcal{F}^{ab}_\mu(x),
\]

\[
\mathcal{F}^{ab}_\mu(x) = \mathcal{D}_\mu \sigma^a(x) + \mathcal{D}_\mu A^{ab}_\mu(x) = \mathcal{R}^{ab}_\mu(x) \sigma^b(x),
\]

\[
\mathcal{D}_\mu \sigma^a(x) = \partial_\mu \sigma^a(x) + \mathcal{R}^{ab}_\mu(x) \sigma^b(x),
\]

under the \( \mathcal{W}_c \)-spin gauge transformation, the vector field \( \sigma^a(x) \) transforms as follows:

\[
\sigma^a(x) \rightarrow \tilde{\sigma}^a(x) = \sigma^a(x) + \mathcal{R}^{ab}_\mu(x) \sigma^b(x),
\]

which behaves as a translation of vector field \( \sigma^a(x) \) in non-coordinate field spacetime. As such a transformation reflects the symmetry property of translation-like \( \mathcal{W}_c \)-spin Abelian-type gauge group \( \mathcal{W}^{1,3} \), we may mention \( \tilde{\sigma}^a(x) \) as \( \mathcal{W}_c \)-spin vector field. It is seen that under the \( \mathcal{W}_c \)-spin gauge transformation, \( \mathcal{D}_\mu \) becomes \( \mathcal{W}_c \)-spin invariant-gauge field strength and the field strength \( \mathcal{R}^{ab}_\mu(x) \) transforms as follows:

\[
\mathcal{R}^{ab}_\mu(x) \rightarrow \tilde{\mathcal{R}}^{ab}_\mu(x) = \mathcal{R}^{ab}_\mu(x) + \mathcal{F}^{ab}_\mu(x) \sigma^b(x),
\]

(28)

It indicates that to remove the degrees of freedom, we are able to take an appropriate \( \mathcal{W}_c \)-spin gauge transformation to make a gauge fixing condition, so that the following relation holds, i.e.,

\[
\tilde{\sigma}^a(x) = \sigma^a(x) + \mathcal{R}^{ab}_\mu(x) \sigma^b(x), \quad i.e., \quad \sigma^a(x) = -\tilde{\sigma}^a(x),
\]

(29)

\[
\tilde{\mathcal{R}}^{ab}_\mu(x) = 0, \quad \tilde{\mathcal{R}}^{ab}_\mu(x) = 0.
\]
Therefore, by choosing the gauge fixing condition, we can always take the gauge field and field strength to be as follows:

$$
\mathcal{A}_\mu(x) \equiv \mathcal{A}_\mu^{(a)}(x) \equiv \frac{1}{2} \Sigma_{\mu} = \mathcal{A}_\mu(x) \equiv \frac{1}{2} \Sigma_{\mu},
$$

(30)

$$
\mathcal{F}_\mu(x) \equiv \mathcal{F}_\mu^{(a)}(x) \equiv \mathcal{F}_\mu(x) \equiv \frac{1}{2} \Sigma_{\mu}.
$$

3.2 Gravitational origin of spin gauge symmetry

In analogous, to clarify the redundant degrees of freedom from the spin gauge symmetry, we are motivated to decompose the spin gauge field $\mathcal{A}_\mu(x)$ into the following two parts:

$$
\mathcal{A}_\mu(x) = \Omega_\mu(x) + \mathcal{A}_\mu(x) = \left( \Omega_\mu^{ab}(x) + \mathcal{A}_\mu^{ab}(x) \right) \frac{1}{2} \Sigma_{ab},
$$

(31)

where $\Omega_\mu(x)$ obeys inhomogeneous gauge transformation of the spin gauge symmetry $\text{SP}(1,3)$ and $\mathcal{A}_\mu(x)$ transforms homogeneously under the spin gauge transformation, i.e.,

$$
\Omega_\mu(x) \rightarrow \Omega'_\mu(x) = S(\Lambda) \partial_\mu S^{-1}(\Lambda) + S(\Lambda) \Omega_\mu S^{-1}(\Lambda),
$$

$$
\mathcal{A}_\mu(x) \rightarrow \mathcal{A}'_\mu(x) = S(\Lambda) \mathcal{A}_\mu S^{-1}(\Lambda), \quad S(\Lambda) \in \text{SP}(1,3).
$$

(32)

Unlike the usual internal gauge field for which the inhomogeneous gauge transformation part is thought to be characterized by a pure gauge field with a vanishing field strength, the gauge field $\Omega_\mu^{ab}$ under the inhomogeneous gauge transformation is considered to be relevant to the $\mathcal{W}_c$-spin invariant-gauge field $\mathcal{A}_\mu^{ab}$ in order to maintain the independent degrees of freedom arising from the inhomogeneous spin gauge symmetry $\text{WS}(1,3)$. Namely, the total numbers of independent degrees of freedom for the gauge field should be fixed to be the same as the initial ones required from the inhomogeneous spin gauge symmetry $\text{WS}(1,3)$ which brings about both the spin gauge field $\mathcal{A}_\mu^{ab}(x)$ and the $\mathcal{W}_c$-spin gauge field $\mathcal{A}_\mu^{ab}(x)$.

Indeed, it can be verified that $\Omega_\mu^{ab}$ is completely determined through the $\mathcal{W}_c$-spin invariant-gauge field $\mathcal{A}_\mu^{ab}$ with the following explicit form:

$$
\Omega_\mu^{ab} = \frac{1}{2} \left( \mathcal{F}^{\mu}_{\nu \rho} - \mathcal{A}^{\mu \nu} \mathcal{F}^{\rho}_{\nu \rho} - \mathcal{A}^{\mu \rho} \mathcal{F}^{\nu}_{\nu \rho} - \mathcal{A}^{\nu \rho} \mathcal{F}^{\mu}_{\rho \nu} \right),
$$

(33)

$$
\mathcal{F}^{\mu}_{\nu \rho}(x) = \partial_\nu \mathcal{A}^{\mu \rho}_{\nu}(x) - \partial_\rho \mathcal{A}^{\mu \nu}_{\nu}(x),
$$

where we have introduced the dual $\mathcal{W}_c$-spin invariant-gauge field $\mathcal{A}^{\mu \rho}_{\nu}(x)$ defined as follows:

$$
\mathcal{A}^{\mu \rho}_{\nu}(x) = \mathcal{A}^{\nu \rho}_{\nu}(x) = \mathcal{A}^{\nu \rho}_{\rho}(x) = \mathcal{A}^{\nu \rho}_{\mu}(x),
$$

(34)

$$
\mathcal{A}^{\nu \rho}_{\mu}(x) = \mathcal{A}^{\nu \rho}_{\rho}(x) = \mathcal{A}^{\nu \rho}_{\mu}(x) = \mathcal{A}^{\nu \rho}_{\rho}(x) = \mathcal{A}^{\nu \rho}_{\mu}(x),
$$

(35)

which indicates that when regarding $\mathcal{A}^{\nu \rho}_{\mu}(x)$ as a matrix field, $\mathcal{A}^{\nu \rho}_{\mu}(x)$ is viewed as the inverse matrix field of $\mathcal{A}^{\nu \rho}_{\mu}(x)$. The existence of $\mathcal{A}^{\nu \rho}_{\mu}(x)$ requires a non-zero determinant of matrix field $\mathcal{A}^{\nu \rho}_{\mu}(x)$,

$$
\det A^{\mu \rho}_{\nu}(x) \neq 0, \quad \det \mathcal{A}^{\nu \rho}_{\mu}(x) \neq 0.
$$

Note that the field strength $\mathcal{F}^{\mu}_{\nu \rho}(x)$ is $\mathcal{W}_c$-spin gauge invariant but no longer spin gauge covariant, which distinguishes, as indicated by a different letter style, from the spin gauge covariant field strength $\mathcal{F}^{\mu}_{\nu \rho}(x)$ defined in eq. (26).

It can be checked that the gauge transformation of $\mathcal{A}^{\rho}_{\mu}(x)$ in the vector representation of spin gauge symmetry $\text{SP}(1,3)$ does bring $\Omega_\mu^{ab}$ to a proper gauge transformation in the adjoint representation of spin gauge symmetry $\text{SP}(1,3)$, i.e.,

$$
\mathcal{A}^{\rho}_{\mu}(x) = \Lambda^{\rho}_{\mu}(x) \mathcal{A}^{\ast}_{\mu}(x), \quad \mathcal{A}^{\ast}_{\mu}(x) = \mathcal{A}^{\mu}_{\ast}(x) \mathcal{A}^{\mu}_{\ast}, \quad \Lambda^{\ast}_{\mu}(x) \in \text{SP}(1,3),
$$

$$
\Omega_\mu^{ab}(x) = \Lambda^{a}_{\mu}(x) \Lambda^{b}_{\mu}(x) \Omega^{d}_{\mu}(x) + \frac{1}{2} \left( \Lambda^{a}_{\mu}(x) \partial_\mu A^{bc}_{\nu} - A^{a}_{\mu}(x) \partial_\mu \Lambda^{bc}_{\nu} \right) - \Lambda^{b}_{\mu}(x) \partial_\mu \Lambda^{ac}_{\nu}(x).
$$

(36)

In general, there is no way to eliminate the spin gauge field part $\Omega_\mu^{ab}(x)$ by making a spin gauge transformation, which differs completely from the usual internal gauge field. This can be verified from the spin gauge field strength which can also be decomposed into two parts:

$$
\mathcal{F}^{\mu \rho}_{\nu \rho}(x) \equiv \mathcal{R}^{\mu \rho}_{\nu \rho}(x) + \mathcal{G}^{\mu \rho}_{\nu \rho}(x),
$$

(37)

with the following explicit forms:

$$
\mathcal{R}^{\mu \rho}_{\nu \rho}(x) = \partial_\mu \Omega^{\rho}_{\nu \rho} - \partial_\nu \Omega^{\rho}_{\mu \rho} + \Omega^{\mu}_{\nu \rho} \Omega^{\rho}_{\mu \rho} - \Omega^{\rho}_{\mu \rho} \Omega^{\rho}_{\nu \rho},
$$

$$
\mathcal{G}^{\mu \rho}_{\nu \rho}(x) = \partial^\mu \mathcal{A}^{\rho}_{\nu \rho} - \partial^\nu \mathcal{A}^{\rho}_{\mu \rho} + \mathcal{A}^{\mu \rho}_{\nu \rho} \mathcal{A}^{\rho}_{\mu \rho} - \mathcal{A}^{\mu \rho}_{\nu \rho} \mathcal{A}^{\rho}_{\mu \rho},
$$

(38)

where $\mathcal{R}^{\mu \rho}_{\nu \rho}$ is purely the field strength of $\Omega_\mu^{ab}(x)$. Since $\Omega_\mu^{ab}(x)$ is uniquely determined by the $\mathcal{W}_c$-spin invariant-gauge field $\mathcal{A}^{\rho}_{\mu}(x)$, there are no additional independent degrees of freedom involved in the decomposition of spin gauge field $\mathcal{A}_\mu^{ab}(x)$.

It can be verified that the field strength $\mathcal{R}^{\mu \rho}_{\nu \rho}(x)$ is related to Riemann-type curvature tensor as follows:

$$
\mathcal{R}^{\mu \rho}_{\nu \rho}(x) = \mathcal{A}^{\mu \rho}_{\nu}(x) \mathcal{A}^{\nu \rho}_{\mu}(x) \mathcal{R}^{\mu \rho}_{\nu \rho}(x),
$$

(39)

with the Riemann-type curvature tensor $\mathcal{R}^{\mu \rho}_{\nu \rho}(x)$ given by

$$
\mathcal{R}^{\mu \rho}_{\nu \rho}(x) = \partial_\mu \mathcal{G}^{\rho \nu \rho}_{\mu \rho} - \partial_\nu \mathcal{G}^{\rho \mu \rho}_{\nu \rho} + \mathcal{G}^{\rho \mu \rho}_{\nu \rho} \mathcal{G}^{\rho \nu \rho}_{\mu \rho} - \mathcal{G}^{\rho \nu \rho}_{\nu \rho} \mathcal{G}^{\rho \mu \rho}_{\mu \rho},
$$

(40)

where $\mathcal{G}^{\mu \rho}_{\nu \rho}(x)$ is defined as follows:

$$
\mathcal{G}^{\mu \rho}_{\nu \rho}(x) = \mathcal{A}^{\mu \rho}_{\nu}(x) \partial_\rho \mathcal{A}^{\nu \rho}_{\mu}(x) + \mathcal{A}^{\mu \rho}_{\nu}(x) \Omega^{\rho}_{\mu \rho} + \frac{1}{2} \mathcal{G}^{\mu \rho}_{\nu \rho}(x),
$$

(41)

In the second equality we have introduced the gauge invariant dual tensors,

$$
\mathcal{H}^{\mu \rho}_{\nu \rho}(x) = \mathcal{A}^{\mu \rho}_{\nu}(x) \mathcal{A}^{\nu \rho}_{\mu}(x) \mathcal{H}^{\mu \rho}_{\nu \rho}(x),
$$

(42)
Geometrically, we will show later on that the tensor $H_{\mu}(x)$ (\(\hat{H}^{\mu}(x)\)) composed of $A_{\mu}^{ab}(x)$ ($\hat{A}_{\mu}^{ab}(x)$) represents a metric-like tensor which characterizes the gravitational interaction in Einstein’s general theory of relativity. It is clear that the $W_{\mu}$-spin invariant-gauge field $A_{\mu}^{ab}(x)$ should be the basic gauge field which brings the gravitational interaction as fundamental gauge interaction. For convenience of mention, $A_{\mu}^{ab}(x)$ is referred to as gravimetric field. Meanwhile, $\Omega_{\mu}^{ab}(x)$ is referred to as gravigauge field. Correspondingly, the gravimetric field is regarded as gravicoordinate displacement and derivative as follows: $\delta x_{\mu} \equiv \hat{A}_{\mu}^{ab}(x)dx^{a}$, $\hat{\delta}_{\mu} \equiv \hat{A}_{\mu}^{ab}(x)\partial_{a}$, which enables us to define the dimensionful gravicoordinate displacement and derivative as follows: $\hat{\delta}_{\mu} \equiv \hat{A}_{\mu}^{ab}(x)\partial_{a}$, (43)

Similarly, we should introduce a coordinate-like field displacement $\delta x_{\mu}$ in correspondence to the ordinary coordinate displacement $dx^{\mu}$. The $W_{\mu}$-spin invariant-gauge field $A_{\mu}^{a}(x)$ as gravigauge field is regarded as a bi-covariant vector field under the transformations of spin gauge symmetry $\text{SP}(1,3)$ in Hilbert space and global Lorentz symmetry $\text{SO}(1,3)$ in coordinate spacetime. It is natural to propose the displacement correspondence that the coordinate-like field displacement $\delta x_{\mu}$ is directly associated to the ordinary coordinate displacement $dx^{\mu}$ via the bi-covariant vector field $A_{\mu}^{a}(x)$. To be explicit, such a displacement correspondence is presented by the following relations:

$$\delta x^{\mu} \equiv A_{\mu}^{a}(x)dx^{a}, \quad \hat{\delta}_{\mu} \equiv \hat{A}_{\mu}^{ab}(x)\partial_{a},$$

where we may refer to the displacement $\delta x_{\mu}$ as dimensionless gravicoordinate displacement and the derivative $\hat{\delta}_{\mu}$ as dimensionless gravicoordinate derivative.

It is useful to introduce the corresponding dimensionful gravicoordinate displacement and derivative. For that, let us express the gravigauge field $A_{\mu}^{a}(x)$ and its dual field $\hat{A}_{\mu}^{ab}(x)$ into the following forms:

$$A_{\mu}^{a}(x) \equiv \phi(x)x^{a}_{\mu}, \quad \hat{A}_{\mu}^{ab}(x) \equiv \phi^{-1}(x)x^{b}_{\mu},$$

where $\phi(x)$ is considered as a scalar field with unit canonical dimension, which is referred to as graviscalar field. $x^{a}_{\mu}(x)$ provides a dimensionless gravigauge field with $\hat{x}^{a}_{\mu}(x)$ as a dimensionless dual gravigauge field, they satisfy the following conditions:

$$x^{a\mu}(x) \equiv x^{a}_{\mu}(x)x^{b}_{\nu}(x)\eta_{ab}, \quad \hat{x}^{a\mu}(x) \equiv \hat{x}^{a}_{\mu}(x)\hat{x}^{b}_{\nu}(x)\eta^{ab},$$

which enables us to define the dimensionful gravicoordinate displacement and derivative as follows:

$$\delta x^{a\mu} \equiv x^{a\mu}(x)dx^{\mu} \equiv \phi^{-1}(x)\delta x^{a},$$

$$\delta_{a} \equiv \delta/\delta x^{a} = \hat{x}^{a}_{\mu}(x)\partial_{\mu} = \phi(x)\hat{\delta}_{a},$$

where the dimensionless gravicoordinate displacement $\delta x_{\mu}$ and derivative $\hat{\delta}_{\mu}$ with respect to gravigauge field $x^{a}$ are related to the corresponding dimensionful gravicoordinate displacement $\delta x_{\mu}$ and derivative $\hat{\delta}_{\mu}$ via the graviscalar field $\phi(x)$.

Mathematically, it is known that the ordinary derivative vector operator $\partial_{\mu} \equiv \partial/\partial x^{\mu}$ at point $x$ of Minkowski spacetime $M_{4}$ defines a tangent basis $[\partial_{\mu}] \equiv [\partial/\partial x^{\mu}]$ for tangent spacetime $T_{4}$ over $M_{4}$. A displacement vector $dx^{\mu}$ at point $x$
of \( M_4 \) forms a dual tangent basis \( \{dx^\mu\} \) for dual tangent spacetime \( T^*_4 \) over \( M_4 \). These dual bases satisfy the dual condition:

\[
\langle dx^\mu, \delta_x \rangle = \frac{\delta x^\mu}{\delta x^\mu} = \eta^\mu_{\nu}.
\]  

(48)

In analogous, the gravicoordinate displacement \( \delta x^a \) and gravicoordinate derivative \( \delta_a \) form a pair of dual field bases \( \{\delta x^a\} \) and \( \{\delta_a\} \). Such dual field bases meet to the following dual condition:

\[
\langle \delta x^a, \delta_b \rangle = A^a_b(x)\tilde{A}^b_\nu(x)\langle dx^\nu, \delta_x \rangle = A^a_b(x)\tilde{A}^b_\nu(x)\eta^\mu_{\nu} = \eta^a_b,
\]  

(49)

which span a pair of dual locally flat spacetimes over globally flat Minkowski spacetime \( M_4 \).

For convenience, the dual field bases \( \{\delta_a\} \) determined by the gravigauge field \( A^a_\mu(x) \) as bi-covariant vector field are referred to as a pair of dual gravigauge field bases. Meanwhile, the dual locally flat field spacetimes spanned by the dual gravigauge field bases are referred to as locally flat dual gravigauge spacetimes, where the gravigauge spacetime spanned from gravigauge field basis \( \{\delta_a\} \) is mentioned as tangent-like gravigauge spacetime \( G_4 \), and the dual gravigauge spacetime formed from the dual gravigauge field basis \( \{\delta^a\} \) is referred to as dual tangent-like gravigauge spacetime \( G^*_4 \). Therefore, the gravigauge field \( A^a_\mu(x) \) as bi-covariant vector field is thought to be sided on the dual tangent Minkowski spacetime \( T^*_4 \) and valued on the dual gravigauge spacetime \( G^*_4 \). Thus \( A^a_\mu(x) \) transforms as a bi-covariant vector field under the transformations of both spin gauge symmetry \( \text{SP}(1,3) \) and global Lorentz group symmetry \( \text{SO}(1,3) \).

It is noticed that the gravigauge field basis \( \{\delta_a\} \) is no longer commutative, its non-commutation relation is explicitly given by

\[
\left[ \delta_a, \delta_b \right] = \Omega^c_{[cd]} \delta_c,
\]

\[
\Omega^c_{[cd]} = - \tilde{A}^c_\mu(x)\tilde{A}^\mu_\nu(x)F_{\mu\nu}(x) = \Omega^a_c(x) - \Omega^a_d(x),
\]  

(50)

\[
F_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x),
\]

which brings on a non-Abelian Lie algebra of gravicoordinate derivative operator \( \delta_a \), where the structure factor \( \Omega^c_{[cd]} \) is no longer a constant and determined by the gravigauge field strength \( F^a_{\mu\nu} \). Such a non-Abelian Lie algebra leads to the emergence of non-commutative geometry in locally flat gravigauge spacetime \( G_4 \). To explore the property of non-commutative geometry in locally flat gravigauge spacetime \( G_4 \), it is necessary to study the dynamics of spin gravigauge field \( \Omega^a_{\mu\nu} \).

In general, the gravigauge spacetime \( G_4 \) and tangent Minkowski spacetime \( T_4 \) bring on a biframe spacetime \( T_4 \times G_4 \) which is associated with a dual biframe spacetime \( T^*_4 \times G^*_4 \) over coordinate spacetime \( M_4 \). Mathematically, the globally and locally flat vector spacetimes allow for a canonical identification of vectors in tangent Minkowski spacetime \( T_4 \) at points with vectors in Minkowski spacetime itself \( M_4 \), and also for a canonical identification of vectors at points with its dual vectors at the same points. In physics, either tangent or dual tangent Minkowski spacetime over globally flat Minkowski spacetime is viewed as a free-motion spacetime \( M_4 \). The locally flat gravigauge spacetime characterized by the gravigauge field is regarded as an emergent spacetime \( G_4 \).

The canonical identification for the vector spacetimes simplifies the structure of biframe spacetime to be

\[
B_4 = M_4 \times G_4,
\]

\[
M_4 \equiv T_4 \equiv T^*_4 \equiv M_4, \quad G_4 \equiv G_4 \equiv G^*_4.
\]  

(51)

Mathematically, such a biframe spacetime structure brings about a gravigauge fiber bundle \( E_4 \), where the globally flat free-motion Minkowski spacetime is considered as a base spacetime \( M_4 \) and the locally flat emergent gravigauge spacetime is viewed as a fiber \( G_4 \). For a trivial case, we have

\[
E_4 \sim B_4 = M_4 \times G_4.
\]  

(52)

In general, the gravigauge fiber bundle \( E_4 \) is related to the product biframe spacetime \( M_4 \times G_4 \) through a continuous surjective map \( \Pi_x \) which projects the bundle \( E_4 \) to the base spacetime \( M_4 \). Thus the gravigauge fiber bundle \( E_4 \) with the surjective map \( \Pi_x \) is in general expressed as follows:

\[
\Pi_x : E_4 \rightarrow M_4.
\]  

(53)

Geometrically, the gravigauge fiber bundle structure of biframe spacetime is represented by \( (E_4, M_4, \Pi_x, G_4) \).

5 Scaling property of basic fields and the gauge fields and field strengths in gravigauge spacetime

It is useful to analyze the scaling property of basic fields in order to construct a scaling invariant action. Meanwhile, the introduction of biframe spacetime enables us to redefine the gauge fields and field strengths in locally flat gravigauge spacetime, which is crucial to build a gauge invariant and coordinate independent action.

5.1 Scaling property of basic fields

From the covariant derivative operator presented in eq. (16), it is clear that the gauge field has the same scaling property
as coordinate derivative. Under the scaling transformation of coordinates, the gauge fields transform as follows:

$$\chi^\mu \rightarrow \chi^\nu = \lambda^{-1} \chi^\mu, \quad \partial_\mu \rightarrow \partial'_\mu = \lambda \partial_\mu,$$

$$\left( A_\mu(x), \bar{A}_\mu(x), A'_\mu(x) \right) \rightarrow \left( A'_\mu(x'), \bar{A}'_\mu(x'), A''_\mu(x') \right)$$

(54) \hspace{1cm} \lambda \left( A_\mu(x), \bar{A}_\mu(x), A'_\mu(x) \right),

with $\lambda$ the constant scaling factor, where the power of $\lambda$ defines the canonical dimension of coordinates and gauge fields.

The dimensionless gravivector $\chi^\nu(x)$ in gravigauge spacetime should be a scaling invariant vector field indicated from the definitions of gravicoordinatn displacement $\delta x^a$ and derivative $\delta_\nu$ operators given in (44). A graviscalar field $\phi(x)$ is rescaled from the gravigauge field in eq. (45) to provide the dimensional gravicoordinatn displacement $\delta x^a$ and derivative $\delta_\nu$ operators in eq. (47). The presence of graviscalar field enables us to analyze the local and global scaling properties of basic fields:

$$\phi(x) \rightarrow \phi'(x) = \xi(x)\phi(x),$$

$$\chi^\nu(x) \rightarrow \chi'^\nu(x) = \xi^{-1}(x)\chi^\nu(x),$$

$$\bar{A}_\mu(x) \rightarrow \bar{A}'_\mu(x) = \xi(x)\bar{A}_\mu(x),$$

$$A_\mu(x) \rightarrow A'_\mu(x) = \xi(x)A_\mu(x) - \Lambda^{-1}\chi'^\nu(x)\chi^\mu \chi'^\nu,$$

$$\hat{A}_\mu(x) \rightarrow \hat{A}'_\mu(x) = \Lambda^{-1}\chi'^\nu(x)\chi^\mu \chi'^\nu,$$

(55) \hspace{1cm} \chi^\nu(x) \rightarrow \chi'^\nu(x) = \xi(x)\chi^\nu(x),$$

under the scaling gauge transformation, and

$$\chi^\nu \rightarrow \chi'^\nu = \lambda^{-1} \chi^\nu, \quad \phi(x) \rightarrow \phi'(x') = \phi(x),$$

$$\chi^\nu(x) \rightarrow \chi'^\nu(x') = \lambda^{-1}\chi^\nu(x),$$

$$\bar{A}_\mu(x) \rightarrow \bar{A}'_\mu(x') = \lambda\bar{A}_\mu(x),$$

$$A_\mu(x) \rightarrow A'_\mu(x') = \lambda A_\mu(x),$$

$$\hat{A}_\mu(x) \rightarrow \hat{A}'_\mu(x') = \lambda \hat{A}_\mu(x) - \Lambda^{-2}\chi'^\nu(x')\chi^\mu \chi'^\nu,$$

(56) \hspace{1cm} \under the global scaling transformation.

Let us assign the powers of $\lambda$ and $\xi(x)$ under the global and local scaling transformations of fields to be as the global and local scaling charges $C_\mu$ and $\hat{C}_\nu$ of fields, respectively. The canonical dimension $C_d$ of field is defined as the sum of the global and local scaling charges, i.e.,

$$C_d = C_\mu + \hat{C}_\nu.$$

(57) \hspace{1cm} It is obvious that the gauge fields $A_\mu^{ab}(x)$ and $A_{\mu a}(x)$ as well as $A^a_\mu(x)$ all have global and local scaling charges $C_\mu = 1$ and $\hat{C}_\nu = 0$ with the canonical dimension $C_d = 1$, while the gravivector field $\chi^\nu(x)(\hat{\chi}^\mu(x))$ has global and local scaling charges $C_\nu = 1$ ($\hat{C}_\nu = -1$) and $\hat{C}_\nu = 1$ ($\hat{C}_\nu = 1$) with the canonical dimension $C_d = 0$. A field with zero canonical dimension $C_d = 0$ is referred to as dimensionless field and a field with zero local scaling charge $\hat{C}_\nu = 0$ is called as scaling gauge invariant field. In general, a field with both zero global and local scaling charges $C_\nu = 0$ and $\hat{C}_\nu = 0$ is referred to as scaling gauge invariant dimensionless field.

In general, taking the dimensionless gravigauge field $\chi^\mu(x)$ and graviscalar field $\phi(x)$, we can equivalently decompose, instead of using the explicit scaling gauge invariant ones $\Omega_\mu^{ab}(x)$ and $A_\mu^{ab}(x)$, the spin gauge field into the following two alternative parts:

$$\vec{A}_\mu^{ab}(x) \equiv \Omega_\mu^{ab}(x) + A_\mu^{ab}(x) \equiv \Omega_\mu^{ab}(x) + A_\mu^{ab}(x),$$

(58) \hspace{1cm} \text{where the alternative spin gravigauge field } \Omega_\mu^{ab}(x) \text{ and spin covariant gauge field } A_\mu^{ab}(x) \text{ are defined as follows:}

$$\Omega_\mu^{ab}(x) = \frac{1}{2} \left( \chi^\mu \chi^\nu F_\mu^{ab} - \chi^\nu \chi^\mu F_\nu^{ab} - \chi^\mu \chi^\nu F_\nu^{ab} \right),$$

$$F_\mu^{ab}(x) = \partial_\mu \chi^\nu(x) - \partial_\nu \chi^\mu(x),$$

(59) \hspace{1cm} \text{with the relations:}

$$\Omega_\nu^{ab}(x) \equiv \Omega_\mu^{ab}(x) - S_\nu^{ab}(x),$$

$$A_\mu^{ab}(x) \equiv A_\mu^{ab}(x) - S_\mu^{ab}(x),$$

(60) \hspace{1cm} \text{and the scaling invariant gravigauge field strength,}

$$F_\mu^{ab}(x) = d_\mu \chi^\nu(x) - d_\nu \chi^\mu(x) \equiv F_\mu^{ab} + S_\mu^{ab},$$

$$S_\mu^{ab} = S_\mu^{cb} \chi^b(x) - S_\mu^{cb} \chi^b(x),$$

(61) \hspace{1cm} \text{which indicates that } \Omega_\mu^{ab}(x) \text{ and } A_\mu^{ab}(x) \text{ become no longer scaling gauge invariant. Under the scaling gauge transformation, they transform explicitly as follows:}

$$\Omega_\mu^{ab}(x) \rightarrow \Omega_\mu^{ab}(x'),$$

$$\Omega_\mu^{ab}(x) \rightarrow \Omega_\mu^{ab}(x) + \left( \chi^\mu \chi^\nu F_\mu^{ab} - \chi^\mu \chi^\nu F_\mu^{ab} \right) \partial_\nu \ln (\xi(x)),$$

$$A_\mu^{ab}(x) \rightarrow A_\mu^{ab}(x'),$$

(62) \hspace{1cm} \text{and the relations:}

$$\vec{R}_\mu^{ab}(x) \equiv \vec{R}_\mu^{ab}(x) + \vec{S}_\mu^{ab}(x),$$

$$\vec{F}_\mu^{ab}(x) = \vec{F}_\mu^{ab}(x) - \vec{S}_\mu^{ab}(x),$$

(63) \hspace{1cm} \text{with the definitions:}

$$R_\mu^{ab}(x) = \partial_\mu \Omega_\mu^{ab} - \partial_\mu \Omega_\mu^{ab} + \Omega_\mu^{ab} \Omega_\mu^{ab} - \Omega_\mu^{ab} \Omega_\mu^{ab},$$

$$F_\mu^{ab}(x) = \partial_\nu \hat{A}_\mu^{ab} - \partial_\mu \hat{A}_\mu^{ab} + \partial_\nu \hat{A}_\mu^{ab} - \partial_\mu \hat{A}_\mu^{ab},$$

$$D_\mu \hat{A}_\mu^{ab}(x) = \partial_\mu \hat{A}_\mu^{ab} + \Omega_\mu^{ab} \hat{A}_\mu^{ab} + \Omega_\mu^{ab} \hat{A}_\mu^{ab},$$

(64) \hspace{1cm} \text{and the relations:}

$$\vec{R}_\mu^{ab}(x) \equiv \vec{R}_\mu^{ab}(x) + \vec{S}_\mu^{ab}(x),$$

(65) \hspace{1cm} \text{with the definitions:}

$$R_\mu^{ab}(x) = \partial_\mu \Omega_\mu^{ab} - \partial_\mu \Omega_\mu^{ab} + \Omega_\mu^{ab} \Omega_\mu^{ab} - \Omega_\mu^{ab} \Omega_\mu^{ab},$$

$$F_\mu^{ab}(x) = \vec{F}_\mu^{ab}(x) - \vec{S}_\mu^{ab}(x),$$

$$S_\mu^{ab}(x) = D_\nu S_\mu^{ab} - D_\mu S_\mu^{ab} + S_\mu^{cb} S_\mu^{ab} - S_\mu^{cb} S_\mu^{ab}. $$
5.2 Gauge fields and field strengths in locally flat gravigauge spacetime

The gravitational origin of spin gauge symmetry indicates that the scaling gauge invariant gravigauge field $A^\mu_a (\hat{A}^\mu_a)$ as bi-covariant vector field in biframe spacetime can be regarded as Goldstone-like boson, which enables us to project the spin gauge field $\mathcal{A}^{\mu a}_c (x)$ and electromagnetic gauge field $A^\mu_c (x)$ as vector fields in free-motion Minkowski spacetime $M_4$ into the vector fields in locally flat gravigauge spacetime $G_4$:

\[ \mathcal{A}^{ab}_c \equiv \hat{A}^{\mu}_c (x) \mathcal{A}^{\mu a}_c (x) \equiv \phi^{-1} \mathcal{A}^{ab}_c, \]
\[ A_c \equiv \hat{A}^{\mu}_c (x) A^\mu_c (x) \equiv \phi^{-1} A_c, \]  
\[ \mathcal{R}^{ab}_c \equiv \hat{R}^{\mu}_c (x) \mathcal{R}^{\mu a}_c (x), \quad A_c \equiv \hat{R}^{\mu}_c (x) A^\mu_c (x), \]  

where $\mathcal{R}^{ab}_c$ and $A_c$ appear as scaling gauge invariant dimensionless fields, and $\mathcal{R}^{ab}_c$ and $A_c$ become local scaling charged gauge fields. The scaling gauge invariant dimensionless covariant derivative operator in locally flat gravigauge spacetime is defined as:

\[ D_c \equiv \delta_c - iA_c \equiv \delta_c - i\mathcal{A}^{ab}_c \frac{1}{2} \Sigma_{ab} \equiv \phi^{-1} D_c, \]
\[ D_c \equiv \delta_c - iA_c \equiv \delta_c - i\mathcal{A}^{ab}_c \frac{1}{2} \Sigma_{ab}, \]  

for the spin gauge field, and

\[ D_c \equiv \delta_c - iA_c \equiv \phi^{-1} D_c, \quad D_c \equiv \delta_c - iA_c, \]  

for the electromagnetic gauge field, where $\delta_c \equiv \hat{R}^{\mu}_c \partial_{\mu}$ is the scaling gauge invariant dimensionless gravigaucoodinate derivative operator given in eq. (44) and $\delta_c \equiv \hat{R}^{\mu}_c \partial_{\mu}$ is the local scaling charged one. The commutator of the covariant derivative operator is given by

\[ [D_c, D_d] = \Omega^{\mu}_{[cd]} D_\mu - iF^{cd}, \]
\[ [D_c, D_d] = \Omega^{\mu}_{[cd]} D_\mu - iF^{cd}, \]
\[ [D_c, D_d] = \Omega^{\mu}_{[cd]} D_\mu - iF^{cd}, \]
\[ [D_c, D_d] = \Omega^{\mu}_{[cd]} D_\mu - iF^{cd}, \]  

which define the scaling gauge invariant dimensionless gravigaucoodinate field strengths in locally flat gravigauge spacetime:

\[ \Omega^{\mu}_{[cd]} = -\hat{A}^{\mu}_c (x) \hat{A}^{\mu}_d (x) F^{\mu}_{;cd}(x) \equiv -F^{\mu}_{;cd}, \]
\[ F^{cd} = \delta_c A_d - \delta_d A_c + [A_c, A_d] - \Omega^{\mu}_{[cd]} A_\mu \]
\[ F^{cd} = \delta_c A_d - \delta_d A_c - \Omega^{\mu}_{[cd]} A_\mu, \]  

and the corresponding local scaling charged gauge field strengths:

\[ \Omega^{\mu}_{[cd]} = -F^{\mu}_{;cd} \equiv -\hat{A}^{\mu}_c (x) \hat{A}^{\mu}_d (x) F^{\mu}_{;cd}(x), \]
\[ F^{cd} = \delta_c A_d - \delta_d A_c + [A_c, A_d] - \Omega^{\mu}_{[cd]} A_\mu \]
\[ F^{cd} = \delta_c A_d - \delta_d A_c - \Omega^{\mu}_{[cd]} A_\mu, \]  

The spin gauge and scaling gauge invariant gravigauge field strength is defined as:

\[ F^{\mu}_{;cd} = \delta_c A_d - \delta_d A_c - \Omega^{\mu}_{[cd]} A_\mu \equiv \phi^{-1} F^{\mu}_{;cd}, \]
\[ F^{\mu}_{;cd} = \delta_c A_d - \delta_d A_c - \Omega^{\mu}_{[cd]} A_\mu \equiv \phi^{-1} F^{\mu}_{;cd}, \]  

with $F^{\mu}_{;cd}(x)$ being the spin gauge and scaling gauge invariant gravigauge field strength defined in eq. (69).

The field strengths $F^{\mu}_{;cd}(F^{\mu}_{;cd})$ possess the local scaling charge $\hat{C}_s = 2$, while $F^{\mu}_{;cd}$ and $\Omega^{\mu}_{[cd]}(F^{\mu}_{;cd})$ have the local scaling charge $\hat{C}_s = 1$ as indicated from their relations to the scaling gauge invariant dimensionless gravigauge field strengths, where $\Omega^{\mu}_{[cd]} \equiv F^{\mu}_{;cd}$ brings on the gravigaucoodinate field strength arising from the non-commutation relation of the gravigaucoodinate derivative operator. $F^{ab}_{;cd}(F^{ab}_{;cd})$ defines the field strength of spin gauge field and $F_{cd}(F_{cd})$ represents the field strength of electromagnetic gauge field.

The gravitational origin of spin gauge symmetry brings the scaling gauge invariant dimensionless spin gauge field $\mathcal{A}^{ab}_c$ into the two parts:

\[ \mathcal{A}^{ab}_c = \Omega^{ab}_c + A^{ab}_c \equiv \phi^{-1} \mathcal{A}^{ab}_c \equiv \phi^{-1} (\Omega^{ab}_c + A^{ab}_c), \]  

where $\Omega^{ab}_c$ and $A^{ab}_c$ correspond to the scaling gauge invariant dimensionless spin gravigaucoodinate field and spin covariant-gauge field in locally flat gravigaucoodinate spacetime. $\Omega^{ab}_c$ characterizes the gravitational origin of gauge symmetry determined by the dual gravigaucoodinate fields $A^{\mu}_a$ and $\hat{A}^{\mu}_a$ with the following explicit form:

\[ \Omega^{ab}_c = \frac{1}{2} \left[ \hat{A}^{\mu}_a \hat{A}^{\mu}_b - \hat{A}^{\mu}_b \hat{A}^{\mu}_a - \hat{A}^{\mu}_c \left( \delta^{\mu}_a \delta^{\mu}_b - \delta^{\mu}_b \delta^{\mu}_a \right) \right] \]
\[ \Omega^{ab}_c = \frac{1}{2} \left[ \hat{A}^{\mu}_a \hat{A}^{\mu}_b - \hat{A}^{\mu}_b \hat{A}^{\mu}_a - \hat{A}^{\mu}_c \left( \delta^{\mu}_a \delta^{\mu}_b - \delta^{\mu}_b \delta^{\mu}_a \right) \right] \equiv \eta^{ab}_{;cd} \Omega^{ab}_{;cd}. \]  

The scaling gauge invariant dimensionless field strength of spin gauge field is correspondingly expressed into the following two parts:

\[ F^{\mu}_{;cd} = R^{\mu}_{;cd} + F^{\mu}_{;cd} \equiv \phi^{-1} \left( R^{\mu}_{;cd} + F^{\mu}_{;cd} \right), \]  

where $R^{\mu}_{;cd}$ and $F^{\mu}_{;cd}$ are the local scaling charged field strengths.
with the explicit forms:

\[ \mathcal{F}^{ab}_{cd} = \hat{\nabla}_c \mathcal{A}^{ab} - \hat{\nabla}_d \mathcal{A}^{ab} + (\mathcal{A}^{cb}_c, \mathcal{A}^{db}_d) \]
\[ \mathbf{R}^{ab}_{cd} = \hat{\nabla}_c \mathbf{A}^{ab} - \hat{\nabla}_d \mathbf{A}^{ab} + \mathbf{A}^{bc}_c, \mathbf{A}^{db}_d \]
\[ \mathbf{F}^{ab}_{cd} = \hat{\nabla}_c \mathbf{A}^{ab} - \hat{\nabla}_d \mathbf{A}^{ab} + \mathbf{A}^{bc}_c, \mathbf{A}^{db}_d \]

where \( \mathbf{R}^{ab}_{cd} \) and \( \mathbf{F}^{ab}_{cd} \) correspond to the scaling gauge invariant dimensionless spin gravigauge field strength and spin covariant-gauge field strength. \( \hat{\nabla}_c \) is regarded as the scaling gauge invariant dimensionless covariant derivative of gravicoordinate derivative \( \delta_c \) under the presence of spin gravigauge field in locally flat gravigauge spacetime. It can be checked that \( \mathbf{R}^{ab}_{cd} \) and \( \mathbf{F}^{ab}_{cd} \) with corresponding derivatives \( \hat{\nabla}_c \) and \( \delta_c \) get the similar definitions in light of the scaling charged quantities.

6 Gauge and scaling invariant action and gauge-gravity-geometry correspondence in the framework of gravitational quantum field theory

With the above discussions and analyses, we are able to construct the gauge and scaling invariant action in locally flat gravigauge spacetime and biframe spacetime. Meanwhile, we are going to present a general demonstration on the gauge-gravity and gravity-geometry correspondences within the framework of gravitational quantum field theory. We shall begin with discussing the scaling gauge fixing and the Einstein basis.

6.1 Scaling gauge fixing and Einstein basis

The gravigauge field expressed in eq. (45) indicates a hidden scaling gauge invariance, which allows us to take an essential scaling gauge transformation \( \xi(x) \) to set a special gauge fixing condition, so that the graviscalar field \( \varphi(x) \) can be transformed into a constant scale \( M_e \):

\[ \varphi(x) \to \varphi(x) \xi(x) \equiv M_e. \]  \( \tag{77} \)

The dimensionless gravigauge field \( \chi^a(x) \) gets a corresponding scaling gauge transformation in order to preserve the scaling gauge invariance of gravigauge field \( A^a(x) \), i.e.,

\[ \chi^a(x) \to \chi^a(x) \xi^1(x) \equiv \xi^1_e \chi^a(x). \]  \( \tag{78} \)

In such a special scaling gauge transformation, the scaling gauge invariant gravigauge field and gravimetric field can be written into the following forms:

\[ A^a(x) \equiv \varphi(x) A^a(x) \to A^a(x) \equiv M_e \chi^a(x), \]
\[ H^\mu(x) \equiv \varphi^2(x) H^\mu(x) \to H^\mu(x) \equiv M_e^2 \chi^\mu(x), \]  \( \tag{79} \)

which fixes the scaling gauge symmetry into the so-called Einstein basis, where the constant mass \( M_e \) plays a role as a fundamental mass scale. \( \chi^\mu(x) \) and \( \chi^\mu(x) \) are mentioned to be the dimensionless gravigauge field and gravimetric field in Einstein basis.

6.2 Gauge and scaling invariant action in locally flat gravigauge spacetime

We are now in the position to construct the gauge and scaling invariant action for the chirality-based Dirac spinor field appearing in the SM. It is believed that the SM is an effective theory as it involves 18 unknown parameters, so the action that we are going to build should also be an effective action. For our present purpose, we just provide an action with keeping the lowest order terms by following along the gauge and scaling invariance principle. The explicit form for such an action in locally flat gravigauge spacetime is found to have the following simple form:

\[ S_D = \int \left[ \delta^4 \chi \right] \Psi \]
\[ \equiv \int \left[ \delta^4 \chi \right] \left( \left( \bar{\Psi}, \mathcal{D} \Psi \right) - \frac{m}{M_e} \bar{\Psi}, \mathcal{D} \Psi \right) + H.c. \]
\[ - \frac{1}{4} \delta^{2 \eta^c} \bar{\Psi}^d \eta^d \mathcal{D}_a \mathcal{D}_c \bar{\Psi} - \frac{1}{4} \delta^{2 \eta^c} \bar{\Psi}^d \eta^d \mathcal{F}^{ab}_{cd} \mathcal{F}^{cd}_{ab} \]
\[ + \frac{1}{4} \delta^{2 \eta^c} \bar{\Psi}^d \mathcal{M}_G \mathcal{F}^{ab}_{cd} \mathcal{F}^{cd}_{ab} - \frac{1}{4} \delta^{2 \eta^c} \bar{\Psi}^d \mathcal{F}^{ab}_{cd} \mathcal{F}^{cd}_{ab} \]  \( \tag{80} \)

with the total covariant derivative,

\[ i \mathcal{D}_a \equiv i \delta_a + \mathcal{A}_a + \mathcal{A}_a \equiv i \delta_a + \mathcal{A}_a \delta_a + \mathcal{A}_a \]
\[ \equiv \frac{1}{2} \mathcal{D}_a + \mathcal{A}_a. \]  \( \tag{81} \)

It is noted that the gravigauge field \( \mathcal{A}_a(x) \) does not couple to the spinor field via the usual covariant derivative due to the chirality property of the group generators \( \Sigma_e \Sigma_e = 0 \), it couples alternatively to the spinor field through the gravicoordinate derivative \( \Sigma_e \delta_a \equiv \Sigma_e \mathcal{A}_a \delta_a \equiv \mathcal{H}^\mu \mathcal{A}_a \Sigma_e \delta_a \) in locally flat gravigauge spacetime. In fact, the kinematics of all basic fields acts as the source of gravitational interaction along with the gravicoordinate derivative characterized by the dual gravigauge field \( \mathcal{A}_a^a(x) \), which actually brings on the comprehension why all motional fields get universal gravitational interactions.
It is noticed that all basic fields in the action appear as scaling gauge invariant dimensionless fields. Namely, all basic fields and operators in the above action have zero global and local scaling charges $C_i = 0$ and $\bar{C}_i = 0$. The mass terms are also rescaled into dimensionless quantities in light of the fundamental mass scale, i.e., $m_{M_i}$ and $m_{C_i}/M_c$.

The tensor $\eta_{aad'}^{dc'd'}$ is given by the following general structure:

$$\eta_{aad'}^{dc'd'} \equiv \alpha_G \eta_c^{\prime \prime} \eta_d^{\prime \prime} \eta_{aa'} - \frac{1}{2} \alpha_W (\eta_c^{\prime \prime} \eta_d^{\prime \prime} \eta_a^{\prime \prime} + \eta_d^{\prime \prime} \eta_{aa'}^{\prime \prime})$$

(82)

with $\alpha_G$ and $\alpha_W$ being two coupling constants, and the tensor structure $\eta_{aad'}^{dc'd'}$ is defined as follows:

$$\eta_{aad'}^{dc'd'} \equiv \eta_c^{\prime \prime} \eta_d^{\prime \prime} \eta_{aa'} + \eta_d^{\prime \prime} (\eta_c^{\prime \prime} \eta_a^{\prime \prime} - 2\eta_c^{\prime \prime} \eta_d^{\prime})
\eta_d^{\prime \prime} (\eta_{aa'}^{\prime \prime} - 2\eta_{aa'}^{\prime})$$

(83)

which preserves the spin gauge invariance for gravitational interaction of gravigauge field. It can be verified explicitly from the following relation and identity:

$$\frac{1}{4} \bar{\eta}^{cd'd'} F_{cd'} F_{e'd'} = \eta_c^{\prime} \eta_d^{\prime} \bar{\eta}^{ab} R_{ab}^{cd'} + 2 \delta_{\mu} \bar{\eta}^{cd'} (\varphi^2 \eta_{ab} F_{cd'})$$

$$\equiv \frac{1}{4} \bar{\eta}^{cd'd'} \chi^2 F_{cd'} F_{e'd'}$$

$$= \frac{1}{4} \bar{\eta}^{cd'd'} \chi^2 (F_{cd'} + S_{cd'}) (F_{e'd'} + S_{e'd'})$$

$$= \eta_c^{\prime} \eta_d^{\prime} \varphi^2 (R_{ab}^{cd'} + S_{ab}^{cd'} + 2 \delta_{\mu} (\varphi^2 \eta_{ab} F_{cd'}),$$

(84)

where the term $\delta_{\mu} (\bar{\eta}^{cd'} \varphi^2 F_{cd'})$ (or $\delta_{\mu} (\varphi^2 \eta_{ab} F_{cd'})$) on the right-hand side of equality appears as a total derivative in the action.

From the above identity and relation, the above spin gauge invariant action via a hidden scaling gauge formalism (eq. (80)) can be rewritten into the following spin gauge and scaling gauge invariant action:

$$S_D \equiv \int [\delta^4 \zeta^{(3)}] L$$

$$= \int [\delta^4 \zeta^{(3)}] \left\{ (\varphi^{\prime \prime} \Sigma^{\prime \prime} \partial_2 D_{ab} \varphi - \frac{m}{M_k} \varphi \Sigma^{\prime \prime} \varphi + H.c.)
- \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'}
+ \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'}
+ \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'} \right\}$$

$$= \int [\delta^4 \zeta^{(3)}] \left\{ (\varphi^{\prime \prime} \Sigma^{\prime \prime} \partial_2 D_{ab} \varphi - \frac{m}{M_k} \varphi \Sigma^{\prime \prime} \varphi + H.c.)
- \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'}
+ \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'} \right\},$$

(85)

where we have introduced the following definition:

$$\Psi_- \rightarrow \Psi_- \equiv \phi^{3/2} \Psi_-,$$

(86)

and made the replacement in the second formalism of the action:

$$A_i^{bc} \rightarrow g_{C_i} A_i^{bc}, \quad A_\mu \rightarrow g_{C_i} A_\mu,$$

$$i D_\mu \rightarrow i D_\mu + i \partial_\mu,$$

(87)

$$\equiv i \partial_\mu + g_{C_i} R_{ab}^{bc} \Sigma^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'} + g_{C_i} \frac{2}{2} \Sigma^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'}.$$}

Such a redefined chirality-based Dirac spinor $\Psi_-$ has a local scaling charge $\bar{C}_s = 3/2$ and canonical dimension $C_d = 3/2$. Note that the covariant derivative $D_\mu$ and the integral measure $[d^4 \zeta^{(3)}]$ have local scaling charges $\bar{C}_s = 1$ and $\bar{C}_i = -4$, respectively.

When making special scaling gauge transformation and taking the scaling gauge fixing condition to be Einstein basis, i.e., $\phi \rightarrow M_k$ and $\chi^{\prime \prime} \rightarrow \chi^{\prime \prime}$ as shown in eqs. (77)-(79), we are able to express the gauge invariant action in eq. (85) into the following simple form:

$$S_D \equiv \int [\delta^4 \zeta^{(3)}] L$$

$$= \int [\delta^4 \zeta^{(3)}] \left\{ (\varphi^{\prime \prime} \Sigma^{\prime \prime} \partial_2 D_{ab} \varphi - \frac{m}{M_k} \varphi \Sigma^{\prime \prime} \varphi + H.c.)
- \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'}
+ \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'} \right\},$$

(88)

6.3 Gauge and scaling invariant action with gauge-gravity-geometry correspondence in gravitational quantum field theory

As the gravigauge field $A_i^{\prime \prime} (\hat{\Delta}_i^{\prime \prime})$ or $\chi^{\prime \prime} (\hat{\chi}^{\prime \prime})$ plays a role as Goldstone-like boson and appears as a bi-covariant vector field in biframe spacetime, it enables us to project the above actions built in locally flat gravigauge spacetime into the ones represented in biframe spacetime within the framework of gravitational quantum field theory.

In light of the scaling gauge invariant fields, we can rewrite the spin gauge invariant action into the following form:

$$S_D \equiv \int [d^4 x] \bar{\Psi}(x) \Psi$$

$$= \int [d^4 x] \bar{\Psi}(x) \left\{ \left( \frac{m}{M_k} \bar{\Psi} \Sigma^{\prime \prime} \Psi + H.c. \right)
- \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'}
+ \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'} \right\} + \frac{1}{4} \bar{\eta}^{cd'd'} \frac{m_{C_i}^2}{M_k^2} \eta_{aa'} \eta_d^{\prime \prime} \eta_c^{\prime \prime} \varphi^2 F_{cd'} F_{e'd'}$$

(89)
where we have used the definitions:
\[ A^a_{\mu} \rightarrow g_G R^a_{\mu}, \quad A^a_{\mu} \rightarrow g_E A^a_{\mu}, \]
\[ i\partial_{\nu} + \mathcal{A}_a + A^a_{\mu} \equiv i\partial_{\nu} + g_G A^a_{\mu} \frac{\gamma}{2} \Sigma_{bc} + g_E A^a_{\mu}, \] (90)
\[ \bar{\chi}^{a\mu} \partial_{\mu} \chi^{a\nu} + \frac{1}{4} \bar{\chi}^{a\rho \nu} \partial_{\rho} \chi^{a\sigma} F_{\mu \nu}^{a} = \chi R + 2\partial_\mu (\chi^{a\mu} \chi^{a\nu} F_{\mu \nu}^{a}), \] (94)
\[ \equiv \bar{\chi}^{a\mu} R_{a\nu} \equiv \bar{\chi}^{a\mu} \chi^{a\nu} R_{a\nu}. \] (95)

Such relation and identities reveal the gauge-gravity-geometry correspondence.

Geometrically, \( R_{a\nu} \) is the so-called Riemann curvature tensor, \( R_{a\nu} \) and \( R \) correspond to the Ricci curvature tensor and Ricci curvature scalar. \( R_{a\nu} \) is explicitly given by
\[ R_{a\nu}(x) = \partial_\mu R_{a\nu}^{\mu} - \partial_\nu R_{a\mu}^{\mu} + \Gamma_{a\mu}^{\rho} R_{\mu \nu}^{\rho} - \Gamma_{a\nu}^{\rho} R_{\mu \mu}^{\rho}, \] (96)
with \( \Gamma_{a\nu}^{\rho}(x) \) defined as follows:
\[ \Gamma_{a\nu}^{\rho}(x) = \frac{1}{2} \bar{\chi}^{a\rho} \partial_\mu \chi^a_{\nu} + \frac{1}{2} \bar{\chi}^{a\rho} \sigma_{\mu \nu}^{a}, \] (97)
which is the so-called affine connection or Christoffel symbol in geometry.

When applying for the gauge-gravity-geometry correspondence and adopting the vector-like spinor representation of Dirac fermion in four-dimensional Hilbert space, the action in eq. (93) can be further simplified into the following form:
\[ S_D \equiv \int [d^4 x] \mathcal{L}(x) (e) \equiv \int [d^4 x] \mathcal{L}(x) \left\{ \left( \frac{1}{2} \bar{\chi}^{a\mu} \psi^{b} \mathcal{A}_{a} \mathcal{D}_{\mu} \psi + H.c. \right) - m \bar{\psi} \psi \right. \]
\[ - \frac{\lambda}{4} \bar{\chi}^{a\mu} \chi^{a\nu} F_{\mu \nu}^{a} F_{\rho \sigma}^{a} - \frac{\lambda}{4} \bar{\chi}^{a\mu} \chi^{a\nu} F_{\mu \nu}^{a} F_{\rho \sigma}^{a} \]
\[ + \frac{1}{4} g_G^{2} \bar{\chi}^{a\mu \nu} F_{\mu \nu}^{a} F_{\rho \sigma}^{a} + \frac{1}{4} g_G^{2} \bar{\chi}^{a\mu \nu} F_{\mu \nu}^{a} F_{\rho \sigma}^{a} \]
\[ = 2 \partial_\mu (\chi^{a\mu} \chi^{a\nu} F_{\mu \nu}^{a}), \] (98)
with \( G_N \) the gravitational constant,
\[ \frac{1}{16 \pi G_N} = \frac{M_{G}^{2}}{M_{G}^{2}} = \frac{M_{P}^{2}}{16 \pi}. \] (99)

So far, we have provided various formalisms for the gauge and scaling invariant action of Dirac spinor field within the framework of gravitational quantum field theory following along the action principle of path integral formulation, which indicate the gauge-gravity and gravity-geometry correspondences and lead to the gauge-geometry duality relation.

### 6.4 Hidden gauge formalism of the action with flowing unitary gauge and the gauge-geometry duality with emergent general linear group symmetry GL(1,3,R)

The above formalisms of the action are built based on the scaling and gauge invariance principle through the inhomogeneous spin gauge symmetry in locally flat gravigauge spacetime. The coordinate spacetime remains to maintain the
Poincaré group symmetry in globally flat Minkowski spacetime. The gauge fixing \( W_e \)-spin gauge field as a bicovariant vector field spanned in biframe spacetime is shown to play an essential role as a projection operator. In general, such a projection operator should allow us to define a spacetime gauge field from the spin gauge field and provide an equivalent action in hidden gauge formalism, so that we are able to investigate more profound correlation between the gravitational interaction and Riemann geometry of spacetime.

Let us demonstrate explicitly how the general linear group symmetry \( GL(1,3,\mathbb{R}) \) emerges automatically as a consequence of gauge invariance principle, which corroborates the gauge-geometry duality in curved Riemannian spacetime. The \( W_e \)-spin gauge field with imposing the gauge fixing condition turns out to be characterized by the gravigauge field \( A_\mu^a \), which is associated in general with the spin gauge field \( \mathcal{A}_\mu^a \) through the following covariant relation of spin gauge symmetry:

\[
\partial_\mu A_\nu^a + \mathcal{A}_{\mu b}^a A^b_\nu - \mathcal{A}_{\nu a}^b A^b_\mu = 0.
\]

Such a relation enables us to define a hidden spin gauge invariant field \( \overline{\mathcal{A}}_{\mu \nu}^a \) from the spin gauge field as follows:

\[
\overline{\mathcal{A}}_{\mu \nu}^a \equiv \hat{A}_a^b \partial_\mu A^b_\nu + \hat{A}_a^b \mathcal{A}^{b \sigma}_{\mu \nu} = \hat{A}_a^b \partial_\mu \mathcal{A}^b_\nu, \tag{101}
\]

which brings on a gauge field represented in coordinate spacetime. For short, we may refer to such a gauge field \( \overline{\mathcal{A}}_{\mu \nu}^a \) as a spacetime gauge field.

As shown in eq. (31), the spin gauge symmetry gets a gravitational origin by decomposing the spin gauge field \( \mathcal{A}_{\mu \nu}^a \) into the sum of spin gravigauge field \( \Omega_{\mu \nu}^a \) and covariant-gauge field \( A_{\mu \nu}^a \). In light of such a decomposition, the spacetime gauge field \( \overline{\mathcal{A}}_{\mu \nu}^a \) can be devided into the corresponding two parts:

\[
\overline{\mathcal{A}}_{\mu \nu}^a \equiv \Gamma_{\mu \nu}^a + A_{\mu \nu}^a, \tag{102}
\]

with \( \Gamma_{\mu \nu}^a \) satisfying the following gauge covariant relation:

\[
\Gamma_{\mu \nu}^a = \partial_\mu A_\nu^a + \Omega_{\mu b}^a A^b_\nu - \Gamma_{\mu \nu}^a A_\rho^a = 0, \tag{103}
\]

and \( A_{\mu \nu}^a \) being defined as follows:

\[
A_{\mu \nu}^a \equiv \hat{A}_a^b \partial_\mu A^b_\nu. \tag{104}
\]

From the relation eq. (103) and the explicit form of \( \Omega_{\mu \nu}^a \) presented in eq. (33), we arrive at the following form for \( \Gamma_{\mu \nu}^a \):

\[
\Gamma_{\mu \nu}^a = \hat{A}_a^b \partial_\mu A^b_\nu + \hat{A}_a^b \Omega_{\mu b}^a A^b_\nu
= \frac{1}{2} \hat{F}_{\mu \nu} (\partial_\mu H_{\nu b} + \partial_\nu H_{\mu b} - \partial_b H_{\mu \nu}) = \Gamma_{\mu \nu}^a,
\]

which comes to the identity given in eq. (41).

The gravigauge field \( A_\mu^a \) may be regarded as Goldstone-like boson field which transforms as bicovariant vector field under both spin gauge group and Lorentz group transformations. In light of the spacetime gauge field, the spin gauge symmetry \( SP(1,3) \) is transmuted into a hidden gauge symmetry. The symmetric tensor fields \( H_{\mu \nu} \) and \( \hat{F}_{\mu \nu} \) as the product of two gravigauge fields by contracting the vector indices in locally flat gravigauge spacetime are considered as composite fields with both hidden spin gauge symmetry and scaling gauge symmetry, which are referred to as Goldstone-like dual gravimetric fields geometrically.

The symmetric gauge field \( \Gamma_{\mu \nu}^a = \Gamma_{\nu \mu}^a \) is completely determined by the dual Goldstone-like gravimetric fields \( H_{\mu \nu} \) and \( \hat{F}_{\mu \nu} \), we may refer to \( \Theta_{\mu \nu}^a \) as scaling gauge invariant spacetime gravimetric-gauge field. For convenience of mention, the gauge field \( A_{\mu \nu}^a \) is called as spacetime covariant-gauge field. In terms of the spacetime gauge field, we obtain the corresponding field strength:

\[
\Theta_{\mu \nu}^a = \nabla_\mu \mathcal{A}^a_{\nu \rho} - \nabla_\nu \mathcal{A}^a_{\mu \rho} + \mathcal{A}^{a \rho \sigma}_{\mu \sigma} \mathcal{A}^\rho_{\nu \lambda},
\]

\[
\nabla_\mu \mathcal{A}^a_{\nu \rho} = \partial_\mu \mathcal{A}^a_{\nu \rho} - \Gamma_\mu^b \mathcal{A}^b_{\nu \rho} - \Gamma_\nu^b \mathcal{A}^b_{\mu \rho} - \Gamma_{\nu \rho}^b \mathcal{A}^b_{\mu}.
\]

which is rewritten into the following two parts in correspondence to the decomposition of spacetime gauge field shown in eq. (102):

\[
\Theta_{\mu \nu}^a = \mathcal{R}^a_{\mu \nu} + \mathcal{F}^a_{\mu \nu},
\]

\[
\mathcal{R}^a_{\mu \nu} = \partial_\mu \mathcal{A}^a_{\nu \rho} - \partial_\nu \mathcal{A}^a_{\mu \rho} + \mathcal{A}^{a \rho \sigma}_{\mu \sigma} \mathcal{A}_\rho^\lambda - \mathcal{A}^{a \lambda \rho}_{\mu \rho} \mathcal{A}^\lambda_{\nu},
\]

\[
\mathcal{F}^a_{\mu \nu} = \nabla_\mu \mathcal{A}^a_{\nu \rho} - \nabla_\nu \mathcal{A}^a_{\mu \rho} + \mathcal{A}^{a \rho \sigma}_{\mu \sigma} \mathcal{A}^\rho_{\nu \lambda} - \mathcal{A}^{a \lambda \rho}_{\mu \rho} \mathcal{A}^\lambda_{\nu}.
\]

Such defined tensor fields \( \mathcal{R}^a_{\mu \nu} \) and \( \mathcal{F}^a_{\mu \nu} \) in the hidden gauge formalism are referred to as scaling gauge invariant spacetime gravimetric-gauge field strength and spacetime covariant-gauge field strength, respectively.

In terms of the dimensionless gravigauge field \( \chi_\mu^a (\hat{\chi}_a^\mu) \), we arrive at the following gauge covariant relation:

\[
\partial_\mu \chi_\nu^a + \mathcal{A}^{a \rho \sigma}_{\mu \sigma} \chi_\nu^b - \Theta_{\rho \nu}^a \chi_\rho^b = 0.
\]

Again decomposing the spin gauge field into two parts:

\[
\mathcal{A}_{\mu \sigma}^a \equiv \mathcal{A}_{\mu b}^a + \mathcal{A}^{a \rho \sigma}_{\mu \sigma} \mathcal{A}^b_{\nu},
\]

with the spin gravigauge field \( \Omega_{\mu \nu}^a \) being the so-called spin connection, which is completely determined by the dimensionless gravigauge field \( \chi_\mu^a (\hat{\chi}_a^\mu) \) as presented in eqs. (58) and (59). In this case, the spacetime gauge field \( A_{\mu \nu}^a \) can also be expressed into the following form:

\[
\mathcal{A}_{\mu \nu}^a \equiv \Gamma_{\mu \nu}^a + A_{\mu \nu}^a, \tag{108}
\]

where the spacetime gravimetric-gauge field \( \Gamma_{\mu \nu}^a \) satisfies the well-known identity:

\[
\partial_\mu \chi_\nu^a + \Omega_{\mu b}^a \chi_\nu^b - \Theta_{\rho \nu}^a \chi_\rho^b = 0, \tag{109}
\]
with the explicit form:

\[ \Gamma_{\mu\nu}^\rho(x) = \frac{1}{2} \hat{\chi}^\rho (\partial_\mu \chi_{\nu\lambda} + \partial_\nu \chi_{\mu\lambda} - \partial_\lambda \chi_{\mu\nu}) = \Gamma_{\mu\nu}^\rho, \]

which is called as affine connection or Christoffel symbol represented in eq. (97). The spacetime covariant-gauge field \( A^\rho_{\mu\nu} \) is defined as follows:

\[ A^\rho_{\mu\nu} \equiv \hat{\chi}^\rho A^\mu_{\nu} \chi^\nu. \]  

(110)

It is noticed that by directly taking the symmetric feature of \( \Gamma_{\mu\nu}^\rho \), i.e., \( \Gamma_{\mu\nu}^\rho = \Gamma_{\nu\mu}^\rho \), one can obtain from the relation in eq. (109) the explicit form of spin gravigauge field \( \Omega_{\mu\rho}^a \) in eq. (59) by simply solving the following equation:

\[ \partial_\mu \chi_{\nu}^a - \partial_\nu \chi_{\mu}^a + \Omega_{\mu\rho}^b \chi_{\nu}^b - \Omega_{\nu\rho}^b \chi_{\mu}^b = 0. \]  

(111)

The corresponding spacetime gauge field strength can be written into the following form:

\[ F_{\mu\nu}^\rho = R_{\mu\nu}^\rho + F_{\mu\nu}^\rho, \]

\[ R_{\mu\nu}^\rho = \partial_\mu \chi_{\nu}^a + \Omega_{\mu\rho}^b \chi_{\nu}^b - \Omega_{\nu\rho}^b \chi_{\mu}^b = 0, \]  

(112)

\[ F_{\mu\nu}^\rho = \nabla_\mu A^\rho_{\nu} - \nabla_\nu A^\rho_{\mu} + \hat{\chi}^\rho \hat{\chi}^\mu \chi^\nu - \hat{\chi}^\rho \hat{\chi}^\nu \chi^\mu, \]

where \( R_{\mu\nu}^\rho \) is called as Riemann curvature tensor presented in eq. (96). In general, we can verify the following relations:

\[ F_{\mu\nu}^{\rho\sigma} = \partial_\mu \chi_{\nu\sigma} - \partial_\nu \chi_{\mu\sigma} + \Omega_{\mu\rho}^a \chi_{\nu\sigma} - \Omega_{\nu\rho}^a \chi_{\mu\sigma} = 0, \]

(113)

by using the following general identities of covariant derivative:

\[ \nabla_\mu \chi_{\nu}^a = \partial_\mu \chi_{\nu}^a + \Omega_{\mu\rho}^b \chi_{\nu}^b - \Gamma^b_{\rho\alpha} \chi_{\mu}^\alpha = 0, \]

\[ \nabla_\mu \hat{\chi}^a = \partial_\mu \hat{\chi}^a + \Omega_{\mu\rho}^b \hat{\chi}^b + \Gamma^b_{\rho\alpha} \hat{\chi}^\alpha = 0, \]

\[ \nabla_\mu \chi_{\rho\sigma} = \partial_\mu \chi_{\rho\sigma} + \Gamma^\rho_{\mu\lambda} \chi_{\sigma}^\lambda + \Gamma^\sigma_{\mu\lambda} \chi_{\rho}^\lambda = 0, \]

(114)

By taking the above identities and relations, we can reformulate the actions in eqs. (80) and (89) into the following alternative form in hidden gauge formalism:

\[ S_D \equiv \int [d^4x] \Psi (x) \varphi \]

\[ \equiv \int [d^4x] \Psi (x) \left( \left( \bar{\Phi} \Sigma^\mu \delta \phi_{\mu} - \frac{m}{\sqrt{\Lambda}} \bar{\Psi} \Sigma_{\mu} \Psi_{\mu} + H.c. \right) + \frac{1}{4 \sqrt{\Lambda}} F^\rho_{\mu\nu} F_{\mu\nu} + \right. \]

\[ + \frac{1}{4} \sqrt{\Lambda} \Psi \hat{\Psi} \hat{F}_{\mu\nu} F_{\mu\nu} + \frac{1}{8 \sqrt{\Lambda} \sqrt{\Lambda}} \bar{\Psi} \Psi \bar{\Phi} \Phi \right), \]  

(115)

where we have used the following definitions in the hidden gauge formalism:

\[ F_{\mu\nu}^\rho = \hat{A}_\mu \Gamma_{\nu}^\rho - \hat{A}_\nu \Gamma_{\mu}^\rho, \]

\[ \bar{F}_{\mu\nu}^\rho = \hat{A}_\mu \Gamma_{\nu}^\rho - \hat{A}_\nu \Gamma_{\mu}^\rho, \]

\[ \hat{F}^\rho_{\mu\nu} = \hat{A}_\mu \hat{A}_\nu \hat{F}_{\rho}^\gamma \Gamma^{\gamma}_{\mu\nu} - \hat{A}_\nu \hat{A}_\mu \hat{F}_{\rho}^\gamma \Gamma^{\gamma}_{\mu\nu}, \]

\[ \bar{F}^\rho_{\mu\nu} = \hat{A}_\mu \hat{A}_\nu \hat{F}_{\rho}^\gamma \Gamma^{\gamma}_{\mu\nu} - \hat{A}_\nu \hat{A}_\mu \hat{F}_{\rho}^\gamma \Gamma^{\gamma}_{\mu\nu} = \hat{A}_\rho \Omega_{\mu\nu}^a \hat{A}_a b \hat{A}_b \gamma, \]  

(116)

\[ \Omega_{\mu\rho}^a = \frac{1}{3} \hat{A}_\mu \hat{A}_\nu \Omega_{\rho\sigma}^a \hat{A}_a b \hat{A}_b \gamma, \]

\[ \Omega_{\mu\rho}^a = \frac{1}{3} \hat{A}_\mu \hat{A}_\nu \Omega_{\rho\sigma}^a \hat{A}_a b \hat{A}_b \gamma, \]

\[ \Sigma_{\mu}^\gamma = \frac{1}{2} \hat{\chi}^\mu \chi_{\mu}^\gamma, \]

\[ \hat{\Sigma}_{\mu}^\gamma = \frac{1}{3} \hat{\chi}^\mu \chi_{\mu}^\gamma, \]

\[ \hat{\chi}_{\mu}^\gamma \equiv \hat{\chi}_{\mu}^\gamma, \]

\[ \chi_{\mu}^\gamma \equiv \chi_{\mu}^\gamma, \]

\[ \bar{\chi}_{\mu}^\gamma \equiv \chi_{\mu}^\gamma. \]

It is noted that the spacetime gravigauge field \( \Omega_{\mu\rho}^a \) actually reflects the gravitational origin of gauge symmetry. The \( \gamma \)-matrices \( \Gamma^\gamma_{\mu\nu} \) are regarded as gravigauge-dressed \( \gamma \)-matrices as they are no longer constant matrices. The matrices \( \Sigma^\rho_{\mu\nu} \) and \( \bar{\Sigma}^\rho_{\mu\nu} \) provide the gravigauge-dressed inhomogeneous spin group generators.

By choosing the scaling gauge fixing condition to be in Einstein basis, the above action can be expressed as follows:

\[ S_D \equiv \int [d^4x] \Psi (x) \varphi \]

\[ \equiv \int [d^4x] \Psi (x) \left( \left( \bar{\Psi} \Sigma^\mu \delta \phi_{\mu} - \frac{m}{\sqrt{\Lambda}} \bar{\Psi} \Sigma_{\mu} \Psi_{\mu} + H.c. \right) + \frac{1}{4 \sqrt{\Lambda}} F^\rho_{\mu\nu} F_{\mu\nu} + \right. \]

\[ - \frac{1}{4} \bar{\Psi} \hat{\Psi} \hat{F}_{\mu\nu} F_{\mu\nu} - \frac{1}{4} \bar{\Psi} \hat{\Psi} \hat{F}_{\mu\nu} F_{\mu\nu} + \frac{1}{8 \sqrt{\Lambda}} \bar{\Psi} \Psi \bar{\Phi} \Phi \right), \]  

(118)

with the definitions:

\[ iD_{\mu} \equiv i\partial_{\mu} + (\Omega_{\mu\rho}^a + \hat{A}_a b \gamma^a) \frac{1}{\sqrt{\Lambda}} \hat{\chi}_{\mu}^\gamma, \]

\[ A_{\mu\rho}^a = \frac{1}{3} (\hat{A}_{\mu\rho} + A_{\mu\rho} + A_{\rho\mu}), \]

\[ \Omega_{\mu\rho}^a = \frac{1}{3} (\hat{A}_{\mu\rho} + A_{\mu\rho} + A_{\rho\mu}), \]

(119)
It becomes manifest that the actions presented in eqs. (115) and (118) bring on an emergent general linear group symmetry in coordinate spacetime:

$$G_S = \text{GL}(1,3,\mathbb{R}),$$

(120)

which lays the foundation for Einstein theory of GR and governs the gravitational interaction in curved Riemannian spacetime. It indicates that such a group symmetry actually appears as a hidden local symmetry in the action built based on the scaling and gauge invariance principle. In general, the action in eq. (115) reformulated in the hidden gauge formalism possesses the following maximal joint symmetry:

$$G_S = \text{GL}(1,3,\mathbb{R}) \rtimes \text{WS}(1,3),$$

(121)

which extends naturally the global Poincaré group symmetry PO(1,3) in globally flat Minkowski spacetime to be a local group symmetry GL(1,3,\mathbb{R}) in curved Riemannian spacetime with emergence of Riemann geometry.

It is noted that the inhomogeneous spin gauge symmetry WS(1,3) still holds for the chirality-based Dirac spinor field in the action eq. (115) though it appears as a hidden spin gauge symmetry. This can be seen from the coupling of totally antisymmetric spacetime gravigauge field $\Omega^{[\mu\nu\rho]}$ which is characterized by the gravigauge field $\chi^a_{\mu} (\hat{x}_a^\mu)$ rather than gravimetric field $\chi_{\mu\nu} (\bar{x}^{\mu\nu})$. It is interesting to notice that the spacetime gravimetric-gauge field $\Gamma^{\rho}_{pp}$ actually decouples from the Dirac spinor field due to its symmetric property $\Gamma^{\rho}_{pp} = \Gamma^{\rho}_{pp}$ and the hermiticity requirement of the action. Nevertheless, in the purely bosonic interactions of the action, the spacetime gauge interaction is described by the antisymmetric spacetime gauge field $\mathbf{A}^{\rho}_{\mu\nu} = -\mathbf{A}^{\rho}_{\nu\mu}$ together with the symmetric gravimetric field $\chi_{\mu\nu}$, which characterizes the spacetime gravimetric-gauge field $\Gamma^{\rho}_{\mu
u} = \Gamma^{\rho}_{\nu\mu}$ as affine connection/Christoffel symbols.

It is observed that the symmetric Goldstone-like gravimetric field $\chi_{\mu\nu}$ concerns 10 degrees of freedom, while the basic gravitationnal field characterized by the Goldstone-like gravigauge field $\chi_{\mu}^a (\hat{x}^\mu_a)$ contains 16 degrees of freedom, which involves additional six degrees of freedom. Such extra degrees of freedom reflect exactly the equivalence classes of spin gauge symmetry SP(1,3), which corresponds to the six group parameters of SP(1,3) gauge transformation. Such an observation motivates us to eliminate the redundant degrees of freedom caused from the spin gauge symmetry by simply making a gauge prescription of spin gauge symmetry SP(1,3).

Unlike the usual internal gauge symmetry, the spin gauge symmetry gets a gravitational origin characterized by the gravigauge field $\chi_{\mu\nu}(x)$, a simple gauge prescription should be realized by taking an appropriate spin gauge transformation. Let us choose such a spin gauge transformation $\hat{\Lambda}_a^b(x)$ that transmutes the Goldstone-like gravigauge field into the following symmetric one:

$$\chi_{\mu\nu}(x) \rightarrow \hat{\chi}_{\mu\nu}(x) = \chi_{\mu\nu}(x) \hat{\Lambda}_a^b(x) = \hat{\chi}_{ab}(x),$$

(122)

which presents a natural gauge fixing condition of spin gauge symmetry SP(1,3). We may refer to such a gauge prescription as flowing unitary gauge, which holds locally for a coordinate system at point to point in spacetime. In such a flowing unitary gauge, the Goldstone-like symmetric gravigauge field $\hat{\chi}_{\mu\nu}(x)$ gets exactly the same degrees of freedom as the gravimetric field $\chi_{\mu\nu}(x)$ via the hidden gauge symmetry, i.e.,

$$\chi_{\mu\nu}(x) = \chi_{\mu\nu}(x) \eta^{\mu\nu} \chi_{\mu\nu}(x) \equiv \hat{\chi}_{\mu\nu}(x) \eta^{\mu\nu} \hat{\chi}_{\mu\nu}(x) \equiv (\hat{\chi}_{\mu\nu}(x))^2.$$

(123)

When fixing gauge to be in the flowing unitary gauge, the independent degrees of freedom for the spacetime gauge interactions are represented by the symmetric Goldstone-like gravigauge field $\hat{\chi}_{\mu\nu}(x) = \hat{\chi}_{ab}(x)$ (or symmetric Goldstone-like gravimetric field $\chi_{\mu\nu}(x)$) and the spacetime gauge field $\mathbf{A}^{\rho}_{\mu\nu}(x)$. Meanwhile, the total independent degrees of freedom in the theory should remain unchanged since the extra degrees of freedom appearing in the Goldstone-like gravigauge field $\chi_{\mu\nu}(x)$ are actually absorbed into the antisymmetric spacetime gauge field $\mathbf{A}^{\rho}_{\mu\nu}(x)$ which behaves as a massive-like gauge field as shown in eq. (118). It can be checked that in such a flowing unitary gauge, the action can still possess an associated global symmetry:

$$G_S = \text{SO}(1,3) \rtimes \text{SP}(1,3),$$

(124)

which transforms the gauge fixing symmetric gravigauge field $\hat{\chi}_{\mu\nu}(x)$ to remain symmetric, i.e., $\hat{\chi}_{\mu\nu}(x) = \hat{\chi}_{\nu\mu}(x')$.

We now come to the conclusion that the laws of nature should be independent of the choice of coordinate systems, which arises from the intriguing observation that when the fundamental symmetry of basic fields in Hilbert space is gauged to be local symmetry by following along the scaling and gauge invariance principle, the fundamental symmetry of coordinates in Minkowski spacetime automatically obeys the general covariance principle of coordinate system with the emergence of general linear group symmetry GL(1,3,\mathbb{R}) and the genesis of Riemann geometry in curved Riemannian spacetime, which indicates that the scaling and gauge invariance principle plays an essential role not only to the fundamental symmetry of basic fields in Hilbert space but also to the fundamental symmetry in coordinate spacetime. In fact, such a feature reflects intrinsically a gauge-geometry duality in GQFT.
6.5 Finiteness and renormalizability of gravitational quantum field theory

Before studying the gravidynamics and spinodynamics as well as electrodynamics, it is useful to present a general analysis and discussion on the finiteness and renormalizability of GQFT though the investigation on the quantum contributions to the dynamics of basic fields is beyond the scope of present paper. Actually, the finiteness and renormalizability have been long-standing issues involved in all QFTs established based on the special relativity and quantum mechanics. In the GQFT, there gets the universal coupling with the situation. They say: ‘Quantum electrodynamics renormalizability within the framework of QFT. Nevertheless, in the perturbative treatment of QFT, it still meets the ultraviolet (UV) divergence problem due to infinite Feynman integrals involved in the closed loops of virtual particles, which may destroy the basic symmetries of the original theory. Mathematically, the divergence is caused from the integral region where all virtual particles will get infinite large energy momentum. Physically, it arises from the very short wavelength/high frequency fluctuations of basic fields in the path integral. To deal with the divergences, the so-called renormalization scheme is developed to absorb all divergences into basic fields and coupling constants in the theory through their redefinitions, which is referred to as the renormalizability of the theory.

The theoretical consistency of the SM mainly relies on its renormalizability within the framework of QFT. Nevertheless, such a renormalization scheme does not satisfy to all physicists, like Dirac and Feynman, which may be seen from their criticisms [27-29].

Dirac has commented that ‘Most physicists are very satisfied with the situation. They say: ‘Quantum electrodynamics (QED) is a good theory and we do not have to worry about it anymore.’ I must say that I am very dissatisfied with the situation, because this so-called ‘good theory’ does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small, not neglecting it just because it is infinitely great and you do not want it.’

Feynman has also remarked that ‘The shell game that we play . . . is technically called ‘renormalization’. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics (QED) is mathematically self-consistent. It’s surprising that the theory still hasn’t been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate.’

Therefore, QFT becomes well-defined only when it can be regularized properly to avoid the infinities through appropriate regularization method. Namely, such a renormalization scheme should eliminate divergences and get only finite quantities. On the other hand, the development of renormalization group technique has inspired us to figure out a well-defined QFT with the presence of physically meaningful energy scale. Nevertheless, the usual regularization schemes always bear some limitations for realizing a satisfied description on QFT. To achieve an infinity-free and symmetry-preserving regularization scheme, we have developed a novel regularization method which is the so-called loop regularization (LORE) method [30, 31]. Such a LORE method has turned out to be a consistent regularization scheme which enables us to introduce intrinsically the physically meaningful energy scale and avoid infinities without spoiling basic symmetries of the original theory. For a detailed description on the Lore method, it is referred to the review article [32].

The key concept in LORE method is the so-called irreducible loop integrals (ILIs) which are evaluated from the Feynman integrals by using Feynman parametrization and ultraviolet divergence-preserving (UVDP) parametrization [30,31]. For illustration on the ILIs, let us represent the one loop calculation of Feynman diagrams, where all Feynman integrals of the one-particle irreducible graphs can be expressed into the following sets of loop integrals by adopting the Feynman parametrization method:

\[
I_{2n} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^{2+n}},
\]

\[
I_{2n}^{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{(k^2 - M^2)^{3+n}},
\]

\[
I_{2n}^{\mu\nu\rho\sigma} = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu k_\rho k_\sigma}{(k^2 - M^2)^{4+n}},
\]

with \(\alpha = -1, 0, 1, \cdots\) where the subscript \((-2n)\) labels the power counting dimension of energy momentum in the loop integrals. For the cases with \(\alpha = -1\) and \(\alpha = 0\), we arrive at the scalar and tensor type ILIs corresponding to the quadratic
divergent integrals ($I_2, I_{2\mu
u} \cdots$) and logarithmic divergent integrals ($I_0, I_{0\mu
u} \cdots$). $M^2$ is the mass factor obtained as a function of Feynman parameters and external momenta $p_i$, $M^2 = M^2(\mu_1^2, \mu_2^2, \cdots)$. For the high loop overlapping Feynman integrals, the ILIs are evaluated to get rid of, in the denominator, the overlapping momentum factors $(k_i - k_j + p_ij)^2$ ($i \neq j$) which appear in the original overlapping Feynman integrals of loop momenta $k_i$ ($i = 1, 2, \cdots$), and eliminate the scalar momentum factors $k^2$ in the numerator.

The essential feature demanded for a consistent regularization scheme is that it must preserve the symmetry principles of the original theory, such as gauge invariance, Lorentz invariance and translational invariance, and meanwhile avoid the infinities of loop integrals with maintaining the initial divergent behavior and structure of original theory, such as quadratic and logarithmic divergent behaviors and structures. It can generally be proved that to preserve the gauge invariance principle in QFT, the regularized tensor-type and scalar-type ILIs should satisfy the following necessary and sufficient conditions:

\[ I_{2\mu
u}^R = \frac{1}{4} g_{\mu\nu} I_0^R, \]
\[ I_{0\mu
u}^R = \frac{1}{4} (g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma}) I_0^R, \]
\[ I_{4\mu
u}^R = \frac{1}{2} (g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma}) I_0^R, \]

which is referred to as the gauge invariance consistency conditions [30-32].

To achieve such consistency conditions, a simple regularization prescription in LORE method has consistently been realized as follows: firstly rotating the momentum to the Euclidean space via a Wick rotation and then replacing the loop integrating variable $k^2$ and loop integrating measure $\int d^4k$ of the ILIs through the corresponding regularized ones $[k^2]_l$ and $\int [d^4k]_l$, i.e.,

\[ k^2 \to [k^2]_l \equiv k^2 + M_l^2, \]
\[ \int d^4k \mathcal{F}(k^2) \to \int [d^4k]_l \mathcal{F}(k^2 + M_l^2) \]
\[ = \lim_{N,M_l^2 \to 0} \sum_{i=0}^{N} c_i^N \mathcal{F}(k^2 + M_l^2), \]

where $M_l^2$ ($l = 0, 1, \cdots$) are regarded as regulator masses and $\mathcal{F}(k^2)$ is considered to be any integration function. The notation $\lim_{N,M_l^2 \to 0}$ denotes the limiting case $\lim_{N \to \infty, M_l^2 \to \infty}$ ($i = 1, 2, \cdots, N$).

The key point in LORE is to choose the appropriate coefficients $c_i^N$, so that any loop divergence with the power counting dimension larger than or equal to the space-time dimension will vanish. From that, we come to the general conditions for the coefficients $c_i^N$ as follows:

\[ \int d^4k \lim_{N,M_l^2} \sum_{i=0}^{N} c_i^N (k^2 + M_l^2)^n = 0, \quad (n = 0, 1, \cdots), \]

which brings on the following relations for the regulator masses:

\[ \sum_{i=0}^{N} c_i^N (M_l^2)^n = 0, \quad (n = 0, 1, \cdots), \]

where we shall choose $M_l^2 = \mu_l^2 + lM_0^2$, $l = 0, 1, 2, \cdots$.

To completely fix the coefficients $c_i^N$ and make them independent of the regulator masses, it is simple to take the following string-mode regulators:

\[ M_l^2 = \mu_l^2 + lM_0^2, \quad l = 0, 1, 2, \cdots, \]

which enables us to determine uniquely the coefficients $c_i^N$ from the general relations of regulator masses given in eq. (130) and obtain the following explicit solution:

\[ c_i^N = (-1)^i \frac{N!}{(N-i)!i!}, \]

which is known as the sign-associated combinations. The factor $(-1)^i c_i^N$ may be regarded as the number of combinations of $N$ regulators taken $i$ at a time.

To be more explicit and concrete, when applying such a regularization prescription in the LORE method with the simple solution of the regulators to the quadratically and logarithmically divergent ILIs, the divergent integrations become well-defined and can safely be performed to obtain the regularized divergent ILIs $I_{2\mu
u}^R$ and $I_{0\mu
u}^R$, as well as $I_0^R$ and $I_0^R[30-32]$. We just represent them as follows:

\[ I_{2\mu
u}^R = \frac{-i}{16\pi^2} \left[ M_c^2 - \mu_M^2 - \mu_M^2 \left( \ln \frac{M_c^2}{\mu_M^2} - \gamma_E + \varepsilon \left( \frac{\mu_M^2}{M_c^2} \right) \right) \right]^{1/2}, \]
\[ I_0^R = \frac{i}{16\pi^2} \left[ \ln \frac{M_c^2}{\mu_M^2} - \gamma_E + \varepsilon \left( \frac{\mu_M^2}{M_c^2} \right) \right], \]
\[ I_{2\mu
u}^R = \frac{1}{2} g_{\mu\nu} I_2^R; \quad I_{0\mu
u}^R = \frac{1}{4} g_{\mu\nu} I_0^R, \]
with the definitions,
\[
\mu^2_s = \mu^2 + M^2,
\]
\[
M^2_s = \lim_{N,M \to \infty} \left( \frac{M^2}{\ln N} \right),
\]
(134)
where the primary regulator mass \(M_R\) is taken to be infinity so as to recover the original integrals and the regulator number \(N\) is set to be infinity so as to make the regularized theory independent of the regularization prescription. \(\gamma_E\) is the Euler constant \((\gamma_E = 0.577215 \ldots)\) and \(\varepsilon(x)\) is a special function with \(x = \mu_s^2/M^2\) and \(\varepsilon(x) \equiv \partial_x \varepsilon(x)\). It is noted that \(M_c\) and \(\mu_s\) are regarded as intrinsic mass scales. In general, the mass scale \(M_c\) can always be made to be finite as long as the ratio \(M_c^2/\ln N\) is kept as a finite quantity when the primary regulator mass \(M_R\) and regulator number \(N\) are approaching to infinity.

It is seen that the LORE method truly brings on an infinity-free and symmetry-preserving regularization method with the presence of two intrinsic mass scales and the satisfaction of symmetry invariance consistency conditions. Specifically, \(M_c\) provides an ultraviolet (UV) “cutoff,” and \(\mu_s\) sets an infrared (IR) “cutoff” for massless case \(M^2 = 0\). It is clear that the LORE method does maintain the original divergent structure of integrals when taking \(M_c \to \infty\) and \(\mu_s \to 0\). In comparison to the dimensional regularization (DR) scheme, although the DR scheme can also result in the same symmetry invariance consistency conditions to preserve gauge invariance, while its regularized quadratic divergent ILIs are actually suppressed to be a logarithmic divergence multiplying by the mass scale \(M^2\) or vanishes \((I_{2s}^2 = 0)\) for the massless case \(M^2 = 0\). This is because the dimensional regularization scheme modifies the original theory by making an analytical extension for the space-time dimensions of the original theory, which makes the DR scheme to be limited in directly applying to theories which require to keep well-defined spacetime dimensions and maintain quadratic divergent structure, such as chiral type theories and supersymmetric theories as well as theories with dynamically spontaneous symmetry breaking.

Furthermore, the LORE method becomes simple at one loop level and gets very applicable at high loop level for understanding the overlapping divergence structure of Feynman diagrams. Nevertheless, LORE method is not motivated for working out a much simpler regularization scheme but for figuring out an infinity-free and symmetry-preserving regularization method without modifying the original theory. It has been demonstrated in refs. [33, 34] that the derivation of ILIs from Feynman integrals by adopting UVDP parametrization brings about the so-called Bjorken-Drell’s circuit analogy between Feynman diagrams and electric circuits [35], which enables us to pick up one-to-one correspondence between the divergences of UVDP parameters and the subdiagrams of Feynman diagrams.

Therefore, the LORE method does overcome the shortages and limitations appearing in some widely-used regularization schemes. In fact, the LORE method has been applied to make numerous interesting applications and carry out various consistent calculations [36-45], such as: how it preserves non-Abelian gauge symmetry [36] and supersymmetry [37], how it results in consistent analyses on the chiral anomaly [38] and radiatively induced Lorentz and CPT-violating Chern-Simons term in the extended QED [39] as well as QED trace anomaly [40], how it enables us to derive the gap equation for the dynamically generated spontaneous chiral symmetry breaking at low energy QCD and obtain reliably the dynamic quark masses and mass spectra of light scalar and pseudoscalar mesons in the chiral effective field theory [41], and meanwhile understand the chiral symmetry restoration in chiral thermodynamic model [42]. Especially, the LORE method has been utilized to perform the consistent computations on the quantum gravitational contribution to gauge theories and obtain asymptotic free power-law running [43-45], and also to clarify the important issues raised in ref. [46] for the Higgs decay process \(H \to gg\) [47]. It is intriguing to observe that the LORE method has also been adopted to achieve a dynamical spontaneous symmetry breaking mechanism for the electroweak symmetry in the SM and arrive at a sliding electroweak symmetry breaking scale to be around \(\mu_{EW} \sim 760\) GeV [48]. In general, the LORE method was proved to be applicable to theories in high dimensional spacetime [49, 50].

With the above analyses, we should turn to make discussions on the finiteness and renormalizability of QFT. The crucial point in LORE is the appearance of two intrinsic energy scales \(M_c\) and \(\mu_s\) which play the role as ultraviolet (UV) cut-off and infrared (IR) cut-off of loop integrals to avoid infinities in QFT without spoiling symmetries of the original theory. These two energy scales should get physically meaningful as the characteristic energy scale (CES) \(M_c\) and sliding energy scale (SES) \(\mu_s\), indicated from the so-called folk’s theorem addressed by Weinberg [51, 52]. Such a folk’s theorem states that: any quantum theory that at sufficiently low energy and large distances looks Lorentz invariant and satisfies the cluster decomposition principle will also at sufficiently low energy look like a quantum field theory. It implies that there must exist a characteristic energy scale \(M_c\) which makes the statement of sufficiently low energy to be reliable. It also means that such a CES \(M_c\) can be either a fundamental mass scale for a basic theory or a dynamically generated
energy scale for an effective theory. In the GQFT, the CES $M_c$ is considered to be a fundamental mass scale, which distinguishes to the QFT in Minkowski spacetime. The reason is that the presence of gravigauge field in GQFT will bring about the scaling gauge invariance of the action, which always allows us to set up a fundamental mass scale $M_e$ by imposing a suitable scaling gauge fixing condition. Such a fundamental mass scale $M_e$ is naturally identified to the CES $M_c$ for characterizing the ultraviolet behavior of the theory, i.e.,

$$M_e \approx M_c,$$

which makes any theory in the framework of GQFT to be infinity-free. Namely, there exists in principle no divergences in GQFT and any quantum contribution from high dimensional interactions in GQFT should be finite and physically meaningful.

In addition to the above consideration, let us follow along the key concept of biframe spacetime in GQFT to understand the finiteness and renormalizability of GQFT. In general, the biframe spacetime forms the fiber bundle structure with the base spacetime taken to be a globally flat Minkowski spacetime and the fiber bringing on a locally flat gravigauge spacetime. As the base spacetime acts as an inertial reference frame for describing the motion of basic fields, the energy momentum of basic fields in such a base spacetime can approach to be infinite large and small as it is considered to be a continue and differential spacetime to ensure the laws of energy momentum conservation, which becomes the main reason why the path integral in such a globally flat Minkowski spacetime gets divergences as the virtual particles can in principle have very short wavelength/high frequency fluctuations or infinite large energy momentum $k_\mu \to \infty$. In contrast, the fiber as a locally flat gravigauge spacetime emerges as a non-commutative geometry characterized by the field strength of gravigauge field and reflects the gravitational interaction, which functions as an interaction frame of basic fields. Therefore, the actual energy momentum of all basic fields in such a locally flat gravigauge spacetime is always associated to the dual gravigauge field and presented explicitly as follows:

$$\mathcal{K}_a \equiv \hat{\chi}_a^\mu(q)k_\mu \sim \int [dq]e^{-i\chi^\mu_\alpha(q)\hat{\chi}_a(q)},$$

$$\hat{\chi}_a \equiv \hat{\chi}_a^\mu(q)k_\mu,$$

which provides a physically meaningful energy momentum in GQFT. For convenience, we may refer to $\hat{\chi}_a$, (or $\mathcal{K}_a$) as gravigauging energy momentum. Unlike the globally flat Minkowski spacetime, the locally flat gravigauge spacetime as a matter field spacetime with noncommutative geometry is in principle no longer to appear as an infinite continue spacetime, so the gravigauging energy momentum $\hat{\chi}_a$ (or $\mathcal{K}_a$) should not approach to be an infinite large/small quantity even when the ordinary energy momentum $k_\mu$ in globally flat Minkowski spacetime goes to be infinite, i.e.,

$$\hat{\chi}_a \neq \infty, \quad k_\mu \to \infty,$$

which may be comprehended as follows: once at a short distance with infinite large energy momentum $k_\mu \to \infty$ in the base spacetime of coordinates, the gravigauge field $\chi_\mu^\alpha$ will become infinite large $\chi_\mu^\alpha \to \infty$ and its dual/inverse gravigauge field $\hat{\chi}_a^\mu$ gets infinitely small $\hat{\chi}_a^\mu \to 0$, whereas the gravigauging energy momentum $\hat{\chi}_a$ (or $\mathcal{K}_a$) as a combination quantity $\hat{\chi}_a \equiv \hat{\chi}_a^\mu(q)k_\mu$ (or $\mathcal{K}_a \equiv \mathcal{K}_a^\mu(x)k_\mu$) remains to keep a physically meaningful finite energy momentum. Therefore, it appears in principle no infinite gravigauging energy momentum in GQFT. On the other hand, there should exist a potential horizon $L_H$ for the locally flat gravigauge spacetime as a matter field spacetime, which can provide a natural infrared cutoff scale, i.e.,

$$\mu_s \to 1/L_H,$$

which allows us to remove the infrared divergence in GQFT.

From the above analyses and discussions, we are in the position to conclude that any divergence should in principle disappear in GQFT, which provides a reasonable answer to the criticisms raised by Dirac and Feynman about the treatment of infinities and divergences via the usual renormalization schemes in QFT. Actually, the globally flat Minkowski spacetime as an inertial frame of free motion fields is observ-able, only the locally flat gravigauge spacetime as a gravitational frame of dynamic interaction fields becomes observable, which constitutes our observed universe. Moreover, by applying the concept of renormalization group developed in refs. [53, 54], we should be able to study physical phenomena at any interesting energy scale by integrating out physical effects at a higher energy scale, and define a finite renormalized theory at any interesting renormalization scale. In fact, the existence of both the CES $M_c \approx M_e$ and SES $\mu_s \sim 1/L_H$ in GQFT allows us to choose the renormalization scale at any scale of interest between them. In this sense, GQFT becomes more natural to provide a profound theoretical framework for describing all basic forces.

7 Gravitational relativistic quantum theory on Dirac spinor field

The above various formalisms of the action enable us to derive the equations of motion for all basic fields based on the
least action principle. These equations can be applied to investigate the dynamics of basic fields in the gravitational relativistic quantum theory. Let us first study the equation of motion for the chirality-based Dirac spinor $\Psi_-(x)$ in the presence of gravitational interaction and spin gauge interaction as well as electromagnetic interaction, which generalizes the usual relativistic quantum mechanics to gravitational relativistic quantum theory.

### 7.1 Generalized Dirac equation in gravitational relativistic quantum theory

Following along the least action principle, we are able to arrive at the following two types of the equations of motion for the chirality-based Dirac spinor field:

\[
\chi^{\mu
u\Sigma} a_i (\mathcal{D}_\nu - V_\nu) \Psi_-(x) - m_\Sigma \chi^{\nu\Sigma}(x) = 0, \tag{139}
\]

\[
\chi^{\mu
u\Sigma} a_i (\mathcal{D}_\nu - V_\nu) \Psi_-(x) - m_{\Sigma} \phi(x) \chi^{\nu\Sigma}(x) = 0,
\]

which correspond to the scaling gauge invariant dimensionless fields and local scaling charged fields, where we have introduced the following vector-like field:

\[
\begin{align*}
V_\mu &\equiv \frac{1}{2} g A_\mu D_{\chi}(\hat{\chi} A_\mu) = - \frac{1}{2} A_{\mu a} \hat{\chi}_a \chi^{\mu\nu} A^{\mu \nu}, \tag{140} \\
V_\mu &\equiv \frac{1}{2} \chi^{\nu\Sigma}(x) - m_{\Sigma} \chi^{\nu\Sigma}(x) = 0,
\end{align*}
\]

which is given by the graviscaling field and spin covariant gauge field. Such a vector-like field preserves the scaling gauge invariance for the equation of motion, which can be verified explicitly from the second equation in eq. (139) when all fields are represented as local scaling charged fields. For convenience, we may refer to such a vector-like field as a graviscaling induced gauge field, which is distinguished from the ordinary gauge field of Lie group due to the associated imaginary factor in the covariant derivative.

For simplicity of discussions, let us consider the equation of motion of chirality-based Dirac spinor by taking the scaling gauge fixing condition to be in Einstein basis, i.e.,

\[
\chi^{\mu
u\Sigma} a_i (\mathcal{D}_\nu - V_\nu) \Psi_-(x) - m_{\Sigma} \chi^{\nu\Sigma}(x) = 0. \tag{141}
\]

In terms of the vector-like spinor representation of Dirac spinor field in four-dimensional Hilbert space, the above equation can be simplified into the following form:

\[
\gamma^\nu \hat{\chi}_a^\nu i (\mathcal{D}_\nu - V_\nu) \psi - m_\nu \psi = 0, \tag{142}
\]

with the covariant derivative defined as follows:

\[
i (\mathcal{D}_\nu - V_\nu) \equiv i \partial_\mu + \frac{1}{2} g g_i \chi^{\nu\Sigma}(x) A^{\mu \nu} + (\Omega_{\mu}^{bc} + g G^a_{\mu b}) \frac{1}{2} \Sigma_{bc} + g E A_\mu. \tag{143}
\]

Let us now investigate the quadratic form of the equation of motion for the Dirac spinor in the presence of gravitational interaction and spin gauge interaction as well as electromagnetic interaction. Its explicit form is found to be

\[
\chi^{\mu \nu \Sigma} (\nabla_\mu - V_\mu) (\mathcal{D}_\nu - V_\nu) \psi + m^2 \psi = \Sigma_{\nu \Sigma}^{\chi} \hat{\chi}_a^\nu \chi^{\nu \Sigma} (\mathcal{D}_\nu - V_\nu) - i V_\nu \psi, \tag{144}
\]

with the following definitions:

\[
\begin{align*}
V_\mu (\mathcal{D}_\nu - V_\nu) &\equiv \frac{1}{2} (\mathcal{D}_\mu - V_\mu) (\mathcal{D}_\nu - V_\nu) - \Gamma_{\mu \nu} (\mathcal{D}_\nu - V_\nu), \\
\Gamma_{\mu \nu} &\equiv \chi^{\nu \Sigma}(x) - m_{\Sigma} \chi^{\nu \Sigma}(x), \\
\chi^{\nu \Sigma}(x) &\equiv \frac{1}{2} A_{\mu a} \hat{\chi}_a \chi^{\mu \nu}, \\
\psi &\equiv \frac{1}{2} \chi^{\nu\Sigma}(x) - m_{\Sigma} \chi^{\nu\Sigma}(x) = 0,
\end{align*}
\]

where $F_{\mu \nu}$ and $F_{\nu \rho}$ are spin gauge field strength and electromagnetic field strength, respectively. $\Gamma_{\mu \nu}$ denotes the affine connection and $\psi_{\mu}$ represents the graviscaling induced gauge field strength.

It is intriguing to notice that although the gauge invariant action is Hermitian, while the equation of motion of Dirac spinor field gets an emergent dissipative term characterized by the presence of gravitational interaction and spin gauge interaction via the graviscaling field $\chi^{\nu \Sigma}$ and spin covariant gauge field $A_{\mu \nu}$.

To display explicitly the effects of gravitational interaction and spin gauge interaction on the dynamics of Dirac fermion, we represent the above equation of motion into the following form:

\[
\begin{align*}
\chi^{\mu \nu \Sigma} (\nabla_\mu - V_\mu) (\mathcal{D}_\nu - V_\nu) \psi + \left( \partial_\mu \chi^{\nu \Sigma} + 2 \chi^{\mu \nu \Sigma} \partial_\mu \ln \chi + g G A^{a \nu \Sigma} \hat{\chi}_a \chi^{\nu \Sigma} (\mathcal{D}_\nu - V_\nu) \right) \psi + \left( m^2 + \frac{1}{16} R + \frac{1}{16} g G F_{\mu \nu \Sigma} \hat{\chi}_a \chi^{\nu \Sigma} - \frac{1}{8} g G a b c d F_{\mu \nu \Sigma} \hat{\chi}_a \chi^{\nu \Sigma} \right) \psi \\
= \Sigma_{\nu \Sigma}^{\chi} \hat{\chi}_a \chi^{\nu \Sigma} (\mathcal{D}_\nu - V_\nu) + \Gamma_{\mu \nu} (\mathcal{D}_\nu - V_\nu), \\
\end{align*}
\]

where we have used the algebra relation of $\gamma$-matrices:

\[
\Sigma_{\nu \Sigma}^{\chi} = \frac{1}{2} i \left( \Sigma_{\nu \Sigma}^{\chi} a b c d e f - \Sigma_{a b c d e f}^{\chi} \right) + \frac{1}{16} \left( R_{\mu \nu \Sigma}^{\chi} - R_{\nu \mu \Sigma}^{\chi} \right) + \frac{1}{4} \Sigma_{\mu \nu \Sigma}^{\chi}, \tag{145}
\]

and the properties of Riemann and Ricci curvature tensors:

\[
R_{\nu \mu \rho \sigma} + R_{\nu \rho \mu \sigma} + R_{\nu \rho \sigma \mu} = 0, \quad R_{\nu \rho \mu} = R_{\nu \rho \mu}. \tag{146}
\]
7.2 Gravigauge Dirac equation in locally flat gravigauge spacetime

From the gauge and scaling invariant action presented in eqs. (80)-(85) and (88), it is noticed that the Dirac fermion actually couples to the spin gauge field $\mathcal{A}_{ab}^{\rho \sigma}$ defined in locally flat gravigauge spacetime. Therefore, it is particularly interesting to derive the equation of motion of Dirac spinor field in locally flat gravigauge spacetime via a hidden coordinate formalism. Taking the scaling gauge fixing condition to be in Einstein basis, we can rewrite the equation of motion for the Dirac spinor into the following simple form:

$$y'\iota(D_c - V_c)\psi - m\psi = 0,$$  \hspace{1cm} (149)

which appears formally to be analogous to the Dirac equation in globally flat Minkowski spacetime. Whereas they are essentially distinguished due to the presence of spin gauge field $\mathcal{A}_{ab}^{\rho \sigma}$ and the emergent non-commutation relation of covariant derivative operator in locally flat gravigauge spacetime shown in eq. (69), which can be seen from the following definitions and relations:

$$iD_c \equiv i\delta_c + g_G A_{ab}^{\rho \sigma} \frac{1}{2} \Sigma_{ab} + g_E E_a,$$

$$[\delta_c, \delta_d] = \Omega_c^{\rho \sigma}[\delta_d, \delta_c = \chi_c^{\rho \sigma} \delta_d],$$

$$V_c \equiv \frac{1}{2} g_G A_{bc}^{\rho \sigma} \equiv \frac{1}{2} g_G \chi_c^{\rho \sigma} A_{bc}^{\rho \sigma}.$$  \hspace{1cm} (150)

To be more explicit, let us check the covariant quadratic form for the equation of motion in locally flat gravigauge spacetime:

$$([\nabla_c - V_c]) (D^c - V^c) \psi + m^2 \psi$$

$$= \Sigma_{ab}^{\rho \sigma} \bigg[ F_{cd}^{\rho \sigma} \frac{1}{2} \Sigma_{ab} + g_E F_{cd} - g_G F_{cd} i(D_a - V_a) - iV_c \bigg] \psi,$$  \hspace{1cm} (151)

with the definitions:

$$\nabla_c (D_d - V_d) \equiv \mathcal{D}_c (D_d - V_d) - \mathcal{A}_{cd}^{\rho \sigma} (D_a - V_a),$$

$$F_{cd}^{\rho \sigma} \equiv \mathcal{A}_{cd}^{\rho \sigma} - \Omega_c^{\rho \sigma}[\delta_d, \delta_c = \chi_c^{\rho \sigma} A_{cd}^{\rho \sigma},$$

$$V_c \equiv \delta_c V_d - \delta_d V_c - \delta_{cd} V_a.$$  \hspace{1cm} (152)

where all the gauge fields and field strengths are defined in locally flat gravigauge spacetime as shown in eqs. (66) and (71). It can be further demonstrated that eq. (151) can be rewritten into the following explicit form in locally flat gravigauge spacetime:

$$\left[ (\nabla_c - V_c) (D^c - V^c) + \left( \partial \chi^{\rho \sigma} + \delta_c \ln \chi + \mathcal{A}_{cd}^{\rho \sigma} (D^d - V^d) \right) \right] \psi$$

$$+ \left[ m^2 + \frac{1}{16} R + \frac{1}{16} g_G F_{cd}^{\rho \sigma} \frac{1}{2} \Sigma_{ab} + g_E F_{cd} i(D_a - V_a) \right] \psi$$

$$= \Sigma_{ab}^{\rho \sigma} \left[ g_G \Sigma_{ab} - g_E A_{cb}^{\rho \sigma} i(D_a - V_a) \right] \psi.$$  \hspace{1cm} (153)

It is interesting to notice that the Ricci curvature scalar $R \equiv R_{ab} f^{\rho \sigma} \equiv R_{ab} f^{\rho \sigma}$ and the spin covariant gauge field strength scalar $F_{ab}^{\rho \sigma} \chi^{\rho \sigma} \equiv F_{ab}^{\rho \sigma} \chi^{\rho \sigma}$ emerge as mass-like terms in the quadratic form of equation of motion for the Dirac fermion, which indicates that a non-zero Ricci curvature scalar $R \neq 0$ appears to generate an effective mass for the Dirac fermion when it propagates in locally flat gravigauge spacetime. Meanwhile, the totally antisymmetric tensor $\epsilon^{cdab} F_{cdab}$ arising from the spin covariant gauge field strength behaves as a pseudoscalar.

In conclusion, the equation of motion of Dirac spinor field in the presence of gravitational interaction and spin gauge interaction characterized by the spin gravigauge field (or gravigauge field) and spin gauge field generalizes the relativistic quantum mechanics described by Dirac equation in globally flat Minkowski spacetime to obtain a gravitational relativistic quantum theory in locally flat gravigauge spacetime with the emergence of non-commutative geometry.

8 Gravidynamics with gauge-type and geometric gravitational equations as extension to Einstein theory of general relativity

To study the dynamics of gauge fields, we are going to take the gravigauge field $\mathcal{A}_{ab}^{\rho \sigma}(x) \equiv \mathcal{A}(x)$ and spin gauge field $\mathcal{A}_{ab}^{\rho \sigma}(x)$ as independent degrees of freedom based on the inhomogeneous spin gauge symmetry $\text{WS}(1,3)$. In particular, we will discuss the potential new effects of gravidynamics beyond the Einstein theory of general relativity.

8.1 Gravidynamics with gauge-type gravitational equations of gravigauge field

To describe the gravidynamics, let us derive the equation of motion for the $\mathcal{D}_c$-spin invariant-gauge field as gravigauge field. The gauge covariant and scaling invariant equations of motion in correspondence to the scaling gauge invariant gravigauge field $\mathcal{A}_{ab}^{\rho \sigma}$ and scaling charged gravigauge field $\chi_{ab}^{\rho \sigma}$ are found to be

$$\partial_a \mathcal{F}_{ab}^{\rho \sigma} = \mathcal{J}_{ab}^{\rho \sigma}, \quad \partial_a \mathcal{F}_{ab}^{\rho \sigma} = \mathcal{J}_{ab}^{\rho \sigma},$$  \hspace{1cm} (154)

where the field strengths $\mathcal{F}_{ab}^{\rho \sigma}$ and $\mathcal{F}_{ab}^{\rho \sigma}$ are defined as follows:

$$\mathcal{F}_{ab}^{\rho \sigma} \equiv M_a \chi^{\rho \sigma} \mathcal{A}_{ab}^{\rho \sigma}, \quad F_{\mu \nu}^{\rho \sigma} \equiv \partial_{\mu} A_{ab}^{\rho \sigma} - \partial_{\nu} A_{ab}^{\rho \sigma},$$

$$\mathcal{D}_a \mathcal{F}_{ab}^{\rho \sigma} = \chi M_a \phi^{\rho \sigma} \mathcal{A}_{ab}^{\rho \sigma}, \quad \mathcal{D}_{\mu} \mathcal{F}_{ab}^{\rho \sigma} = d_{\mu} A_{ab}^{\rho \sigma}, \quad \mathcal{D}_{\mu} \mathcal{F}_{ab}^{\rho \sigma} = d_{\mu} A_{ab}^{\rho \sigma}, \quad S_{\mu} \equiv \partial_{\mu} \ln \phi,$$  \hspace{1cm} (155)
and the currents $\mathcal{J}_a^\mu$ and $\mathcal{F}_a^\mu$ are given explicitly by

$$\mathcal{J}_a^\mu = 16\pi G_N \mathcal{J}_a^\mu + \bar{\mathcal{J}}_a^\mu, \quad \mathcal{J}_a^\mu = \mathcal{J}_a^\mu + \bar{\mathcal{J}}_a^\mu,$$

$$\mathcal{F}_a^\mu = \mathcal{A}_a^\mu \mathcal{F}_{\rho\nu} \mathcal{F}^{\rho\nu} - \frac{1}{4} \mathcal{A}_a^\mu \mathcal{F}_{\rho\nu} \mathcal{F}^{\rho\nu},$$

$$\mathcal{J}_a^\mu = M_k \left( \left( \mathcal{A}_a^\mu \mathcal{A}_c^\mu - \mathcal{A}_a^\nu \mathcal{A}_c^\nu \right) \left( \Psi_\Sigma \Delta \mathcal{D}_\rho \Psi_\rho + H.c. \right) + \mathcal{A}_a^\mu \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \right),$$

$$\mathcal{A}_a^\mu = M_k \left( g_s^2 \left( \mathcal{H}^{\mu\nu} \mathcal{A}_a^\mu - \frac{1}{4} \mathcal{H}^{\mu\nu} \mathcal{A}_a^\mu \right) \mathcal{H}^{\rho\nu} \mathcal{T}_{\rho\mu\nu} \mathcal{T}_{\rho\mu\nu} + \frac{m_G^2}{M_k} \left( \mathcal{H}^\rho_{\mu\nu} \mathcal{A}_a^\rho - \frac{1}{4} \mathcal{H}^\rho_{\mu\nu} \mathcal{A}_a^\rho \right) \mathcal{T}_{\rho\mu\nu} \mathcal{T}_{\rho\mu\nu} \right) - \frac{m_G^2}{M_k} \mathcal{D}_\rho \left( \phi \mathcal{H}^\rho_{\mu\nu} \mathcal{E}_{\rho\mu\nu} \right),$$

and

$$\mathcal{J}_a^\mu = 16\pi G_N \mathcal{J}_a^\mu + \bar{\mathcal{J}}_a^\mu, \quad \mathcal{J}_a^\mu = \mathcal{J}_a^\mu + \bar{\mathcal{J}}_a^\mu.$$

Taking the scaling gauge fixing condition to be in Einstein basis, we arrive at the following gauge-type gravitational equation:

$$\partial_\rho F_{\rho\mu} = J_a^\mu,$$  \hspace{1cm} (159)

where the field strength $\mathcal{F}_{\rho\nu}^\mu$ and current $J_a^\mu$ are defined as follows:

$$F_{\rho\nu}^\mu = \mathcal{A}_a^\mu \mathcal{F}_{\rho\nu} \mathcal{F}^{\rho\nu} - \frac{1}{4} \mathcal{A}_a^\mu \mathcal{F}_{\rho\nu} \mathcal{F}^{\rho\nu},$$

$$J_a^\mu = 16\pi G_N \mathcal{J}_a^\mu + \bar{\mathcal{J}}_a^\mu, \quad \mathcal{J}_a^\mu = \mathcal{J}_a^\mu + \bar{\mathcal{J}}_a^\mu.$$

and

$$J_a^\mu = 16\pi G_N \mathcal{J}_a^\mu + \bar{\mathcal{J}}_a^\mu.$$

The following equation of motion:

$$F_{\rho\nu}^\mu = \mathcal{A}_a^\mu \mathcal{F}_{\rho\nu} \mathcal{F}^{\rho\nu} - \frac{1}{4} \mathcal{A}_a^\mu \mathcal{F}_{\rho\nu} \mathcal{F}^{\rho\nu},$$

$$J_a^\mu = \frac{m_G}{M_k} \mathcal{D}_\rho \left( \phi \mathcal{H}^\rho_{\mu\nu} \mathcal{E}_{\rho\mu\nu} \right),$$

with the conserved current,

$$\partial_\rho J_a^\mu = 0.$$  \hspace{1cm} (162)

Such equations of motion for gravigauge field are obtained to describe the gravidynamics and referred to as gauge-type gravitational equations.

### 8.2 Gauge-type gravidynamics in locally flat gravigauge spacetime

From the gauge and scaling invariant action built in hidden coordinate formalisms in eqs. (80)-(85) and (88), it is seen that all interactions emerge in locally flat gravigauge spacetime spanned by the gravigauge bases. It should be appropriate to derive the gauge-type gravitational equation in locally flat gravigauge spacetime. When taking the scaling gauge fixing condition to be in Einstein basis, we arrive at the following equation of motion:

$$\bar{D}_\rho F_{\rho\mu} = J_b^\mu,$$  \hspace{1cm} (163)

with the definitions:

$$F_{\rho\nu}^\mu = \mathcal{A}_a^\mu \mathcal{F}_{\rho\nu} \mathcal{F}^{\rho\nu}, \quad F_{\rho\nu}^\mu = \mathcal{A}_a^\mu \mathcal{F}_{\rho\nu} \mathcal{F}^{\rho\nu},$$

$$\bar{D}_\rho F_{\rho\mu} = \partial_\rho F_{\rho\mu} + \Omega_{\rho\mu} F_{\rho\nu} + \Omega_{\rho\nu} F_{\rho\mu}.$$

(164)
the gravigauge field $\chi_\mu^a(\tilde{\kappa}_d^a)$, we arrive at the following two geometric gravitational equations:

$$R_{\mu\nu} - \frac{1}{2} \chi_{\mu
u} R + 8\pi G N \bar{T}_{\mu\nu} = -8\pi G N \bar{T}_{\mu\nu},$$

and

$$\bar{\psi}_\mu \bar{F}^\rho_{\mu\nu} + \bar{T}^\rho_{\mu\nu} = -m_G^2 T^\rho_{\mu\nu}.$$  

The above two equations correspond to the symmetric tensor and antisymmetric tensor in the base spacetime of coordinates.

The geometric gravitational equation presented in eq. (171) brings on the extension to Einstein theory of general relativity in the presence of spin gauge field, which is characterized by the symmetric tensor $\bar{T}_{\mu\nu}$. The explicit forms of the tensors are given as follows:

$$T_{\mu\nu} + \bar{T}_{\mu\nu} = T_{\mu\nu} + \bar{T}_{\mu\nu} = \frac{1}{2} \left( \chi_{\mu\nu} F^a_{\mu\nu} + \chi_{\nu\mu} F^a_{\nu\mu} \right).$$

$$T_{\mu\nu} = \frac{1}{4} \left( \chi_{\mu\nu} \psi_\gamma \nabla_\gamma \psi_\alpha + \chi_{\nu\mu} \psi_\gamma \nabla_\gamma \psi_\alpha + H.c. \right).$$

$$\bar{T}_{\mu\nu} = -\left( \eta_\mu^a \eta_\nu^a - \frac{1}{4} \chi_{\mu\nu} \bar{\psi}_\partial \bar{\psi}_\partial \right) F^\rho_{\mu\nu} F^\rho_{\partial\partial},$$

with the definitions:

$$\bar{\psi}_\rho \bar{F}^\rho_{\mu\nu} = \partial_\mu \bar{F}^\rho_{\nu\rho} - \Gamma^\rho_{\mu\nu} \bar{F}^\lambda_{\rho\lambda} - \frac{1}{2} (T^\rho_{\mu\nu} F^\lambda_{\rho\lambda} + \bar{T}^\rho_{\mu\nu} \bar{F}^\lambda_{\rho\lambda}),$$

$$\bar{F}^\rho_{\mu\nu} = \frac{1}{2} \left( \bar{F}^\rho_{\mu\nu} + \bar{F}^\rho_{\nu\mu} \right).$$

where $T_{\mu\nu}$ is the usual four energy momentum tensor arising from the Dirac fermion and electromagnetic gauge field in the presence of spin gauge field and gravigauge field, and $\bar{T}_{\mu\nu}$ is purely attributed to the inhomogeneous spin gauge field governed by the inhomogeneous spin gauge symmetry.

The geometric gravitational equation given in eq. (172) is obtained from the antisymmetric tensors, which indicates that the gauge invariant field strength $\bar{F}^\rho_{\mu\nu}$ is governed by the
relevant antisymmetric tensors $T_{[\mu\nu]}$ and $\bar{T}_{[\mu\nu]}$. Their explicit forms are defined as follows:

\[
\begin{align*}
\tilde{\nabla}_\mu \tilde{F}_{[\mu\nu]}^
u &= \partial_\mu \tilde{F}_{[\mu\nu]} - \Gamma_\mu^\alpha \tilde{F}_{[\lambda\nu]}^\alpha - \frac{1}{2} (\Gamma_\mu^\alpha \tilde{F}_{[\mu\nu]}^\alpha - \Gamma_\nu^\alpha \tilde{F}_{[\mu\nu]}^\alpha \\
&\quad + g_G \bar{\chi}_{\mu\nu} \tilde{F}_{[\mu\nu]}^\alpha - g_G \bar{\chi}_{\mu\nu} \tilde{F}_{[\mu\nu]}^\alpha), \\
\tilde{F}_{[\mu\nu]}^
u &= \frac{1}{2} (\tilde{F}_{[\mu\nu]}^\nu - \tilde{F}_{[\nu\mu]}^\nu) \\
&= \frac{1}{2} (\partial_\nu \chi_{\mu\nu} - \partial_\mu \chi_{\nu\mu}) \bar{X}_{\mu\nu} \tilde{F}_{[\mu\nu]}^{\alpha\beta}, \\
\bar{T}_{[\mu\nu]} &= \frac{1}{2} (\partial_\nu \chi_{\mu\nu} - \partial_\mu \chi_{\nu\mu}) \bar{X}_{\mu\nu} \tilde{F}_{[\mu\nu]}^{\alpha\beta}, \\
T_{[\mu\nu]} &= \frac{1}{4} (\partial_\nu \chi_{\mu\nu} - \partial_\mu \chi_{\nu\mu}) \bar{X}_{\mu\nu} \tilde{F}_{[\mu\nu]}^{\alpha\beta} \tilde{F}_{[\mu\nu]}^{\beta\alpha} + \bar{J}_{\mu\nu},
\end{align*}
\]

(175)

which reflect the basic feature of gravigauge field $\chi_{\mu\nu}^a$ as the fundamental gravitational field instead of the symmetric gravimetric field $\chi_{\mu\nu}$.

### 8.4 Geometric gravidynamics in locally flat gravigauge spacetime

Let us now project all tensors and currents into the ones in locally flat gravigauge spacetime by adopting the gravigauge field $\chi_{\mu\nu}^a$ ($\tilde{\chi}_{\mu\nu}^a$) as Goldstone-like boson. The following gravitational equations are obtained:

\[
R_{cb} = \frac{1}{2} \eta_{cb} R + 8\pi G_N \bar{T}_{cb} = -8\pi G_N T_{cb},
\]

(176)

and

\[
\bar{D}_d \tilde{F}_{[cb]} + \tilde{T}_{[cb]} = -m_G^2 T_{[cb]},
\]

(177)

where the tensors are given by the field strength of spin gravigauge field $\Omega_{\alpha\beta}^a$:

\[
R_{cb} = \eta_{cb} R_{\alpha\beta}, \quad R = \eta_{cb} R_{\alpha\beta},
\]

\[
R_{\alpha\beta \delta} = \delta_{\alpha \delta} \Omega^{\epsilon \nu}_{\beta} + \Omega_{\alpha \epsilon} \Omega_{\beta}^{\nu \epsilon} - \Omega^{\nu \epsilon}_{\beta} \Omega_{\alpha \epsilon} - \Omega_{\epsilon \alpha \beta} \Omega_{\epsilon \nu},
\]

(178)

The symmetric tensors $T_{cb}$ and $\bar{T}_{cb}$ are provided by the following explicit forms:

\[
T_{cb} = \frac{1}{4} \left[ \bar{\psi} \gamma_a i D_b \bar{\psi} + \bar{\psi} \gamma_a D_c \bar{\psi} + H.c. \right] \\
- \frac{1}{2} \eta_{cb} \frac{1}{2} \left( \bar{\psi} \gamma_a i D_b \bar{\psi} + H.c. \right) - m \bar{\psi} \bar{\psi} \\
- \left( \eta_{cb} \eta_{cd} \right) \frac{1}{4} \left( \bar{\psi} \gamma_a \bar{\psi} \gamma_b \bar{\psi} \gamma_c \bar{\psi} \gamma_d \bar{\psi} \gamma_e \bar{\psi} \gamma_f \bar{\psi} \right) \tilde{F}_{[\alpha \beta]}^{\epsilon \nu} \tilde{F}_{[\alpha \beta]}^{\epsilon \nu} \\
- \frac{1}{4} m_G^2 \bar{\psi} \gamma_a \bar{\psi} \gamma_b \bar{\psi} \gamma_c \bar{\psi} \gamma_d \bar{\psi} \gamma_e \bar{\psi} \gamma_f \bar{\psi} \gamma_g \bar{\psi} \gamma_h \bar{\psi} \gamma_i \bar{\psi} \gamma_j \bar{\psi} \gamma_k \bar{\psi} \gamma_l \bar{\psi}
\]

(179)

with the definitions:

\[
\begin{align*}
\bar{D}_d \tilde{F}_{[cb]} &= \partial_d \tilde{F}_{[cb]} + \Omega^{\epsilon \nu}_{\alpha \beta} \tilde{F}_{[\alpha \beta]}^{\epsilon \nu} - \frac{1}{2} \left( \bar{\psi} \gamma_a \bar{\psi} \gamma_b \bar{\psi} \gamma_c \bar{\psi} \gamma_d \bar{\psi} \right) \tilde{F}_{[\epsilon \nu]}^{\alpha \beta} \\
&\quad + \Omega^{\epsilon \nu}_{\alpha \beta} \tilde{F}_{[\alpha \beta]}^{\epsilon \nu} + g_G \bar{\chi}_{\epsilon \nu} \tilde{F}_{[\epsilon \nu]}^{\alpha \beta} + g_G \bar{\chi}_{\epsilon \nu} \tilde{F}_{[\epsilon \nu]}^{\alpha \beta}, \\
\tilde{F}_{[cb]} &= \eta_{cb} \eta_{cd} \eta_{cd} \eta_{cd} \tilde{F}_{[cd]}^{\epsilon \nu} \tilde{F}_{[cd]}^{\epsilon \nu} \\
\tilde{F}_{[cb]} &= \frac{1}{2} \left( \tilde{F}_{[cb]} + \tilde{F}_{[bc]} \right),
\end{align*}
\]

(180)

\[
\begin{align*}
\tilde{F}_{[cb]} &= \frac{1}{2} \left( \tilde{F}_{[cb]} + \tilde{F}_{[bc]} \right) = \frac{1}{2} \left( \eta_{cb} \eta_{cd} \eta_{cd} \eta_{cd} \tilde{F}_{[cd]}^{\epsilon \nu} \tilde{F}_{[cd]}^{\epsilon \nu} \\
&\quad - \frac{1}{4} \left( \eta_{cb} \eta_{cd} \eta_{cd} \eta_{cd} \tilde{F}_{[cd]}^{\epsilon \nu} \tilde{F}_{[cd]}^{\epsilon \nu} \right) \right),
\end{align*}
\]

(181)

and

\[
\begin{align*}
\bar{T}_{[cb]} &= \frac{1}{2} \left( \eta_{cb} \eta_{cd} \eta_{cd} \eta_{cd} \tilde{F}_{[cd]}^{\epsilon \nu} \tilde{F}_{[cd]}^{\epsilon \nu} \\
&\quad - \frac{1}{4} \left( \eta_{cb} \eta_{cd} \eta_{cd} \eta_{cd} \tilde{F}_{[cd]}^{\epsilon \nu} \tilde{F}_{[cd]}^{\epsilon \nu} \right) \right) \right),
\end{align*}
\]

(182)

The gravitational equations in eqs. (176) and (177) provide the equivalent formalisms to characterize the gravidynamics in locally flat gravigauge spacetime spanned by the gravigauge bases.

### 8.5 New effects of gravidynamics beyond Einstein theory of general relativity

The gravidynamics described by the geometric gravitational equation presented in eq. (171) or eq. (176) brings about the extension to Einstein theory of general relativity, which is explicitly shown from the additional energy momentum tensor $\tilde{T}_\mu^\nu$ in coordinate spacetime or $\bar{T}_\mu^\nu$ in gravigauge spacetime. Such an additional energy momentum tensor arises solely from the inhomogeneous spin gauge field which is governed by the inhomogeneous spin gauge symmetry.

To explore potential new effects caused from such a gravidynamics beyond the Einstein theory of general relativity, let us turn to investigate the property of spin gauge field. It is interesting to notice from the action shown in eq. (88) that the term given by the spin gauge invariant gravigauge field strength $F_{\epsilon \mu \nu}^{\alpha \beta}$, can be rewritten into the following form:

\[
\begin{align*}
\frac{1}{4} \eta_{\alpha \beta} \bar{\psi} \gamma_{\epsilon \mu \nu} \bar{\psi} \gamma_{\alpha \beta} \tilde{F}_{\epsilon \mu \nu}^{\gamma \delta} \\
= \frac{3}{4} \left( \alpha_G + \frac{3}{2} \bar{a}_W \right) m_G^2 \bar{G} (\bar{\Omega}_{\epsilon \mu \nu} - \Omega_{\epsilon \mu \nu}/G) (\bar{\Omega}_{\epsilon \mu \nu} - \Omega_{\epsilon \mu \nu}/G) \\
+ \frac{1}{4} \left( \alpha_G - \frac{3}{2} \bar{a}_W \right) m_G^2 \bar{G} (\bar{\Omega}_{\epsilon \mu \nu} - \Omega_{\epsilon \mu \nu}/G) (\bar{\Omega}_{\epsilon \mu \nu} - \Omega_{\epsilon \mu \nu}/G)
\end{align*}
\]

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\[ \equiv \frac{1}{2} m_{\gamma}^2 (\mathcal{A}_{(c \bar{a})} - \Omega_{(c \bar{a})}/g_G)(\mathcal{A}_{c \bar{a}} - \Omega_{c \bar{a}})/g_G) + \frac{1}{2} m_{\gamma}^2 (\mathcal{A}_{(c \bar{a})} - \Omega_{(c \bar{a})}/g_G)(\mathcal{A}_{c \bar{a}} - \Omega_{c \bar{a}})/g_G), \quad (183) \]

with

\[ m_{\gamma}^2 \equiv 2(\alpha_G + \alpha_w)m_G^2/s_G, \]

\[ m_{\gamma}^2 \equiv \frac{1}{2}(\alpha_G - \frac{2}{3} \alpha_w)m_G^2/s_G, \quad (184) \]

where we have introduced the following definitions:

\[ \mathcal{A}_{c \bar{a}} \equiv \mathcal{A}_{(c \bar{a})} + \mathcal{A}_{(c \bar{a})}, \quad \Omega_{c \bar{a}} \equiv \Omega_{(c \bar{a})} + \Omega_{(c \bar{a})}, \quad (185) \]

with

\[ \mathcal{A}_{(c \bar{a})} \equiv \frac{1}{3} (\mathcal{A}_{c \bar{a}} + \mathcal{A}_{abc} + \mathcal{A}_{bac}), \]

\[ \mathcal{A}_{c \bar{a}} \equiv \frac{1}{3} (2\mathcal{A}_{c \bar{a}} - \mathcal{A}_{abc} - \mathcal{A}_{bac}). \quad (186) \]

A similar definition holds for \( \Omega_{c \bar{a}} \) and \( \Omega_{c \bar{a}} \), where \( \mathcal{A}_{c \bar{a}} \) defines a totally antisymmetric spin gauge field which couples directly to spinor field, and \( \mathcal{A}_{c \bar{a}} \) represents a mixing symmetric spin gauge field.

It indicates that the action of spin gauge invariant gravigauge field strength \( F_{a \bar{c}} \) brings a gauge invariant mass term for the spin gauge field. In general, the totally antisymmetric spin gauge field \( \mathcal{A}_{c \bar{a}} \) as a torsion like field and the mixing symmetric spin gauge field \( \mathcal{A}_{c \bar{a}} \) have different masses. Only when the coupling constants satisfy a special relation, they arrive at the same mass, i.e.,

\[ \alpha_w = -\frac{2}{3} \alpha_G, \]

\[ m_{\gamma}^2 = m_{\gamma}^2 = \frac{2}{3} \alpha_G m_G^2/s_G, \quad (187) \]

which leads to the mass \( m_{\gamma} \) for whole spin gauge field \( \mathcal{A}_{\gamma} \).

As the spin gauge field couples to all spinor fields, i.e., leptons and quarks in the standard model, so that it can in general decay into leptons and quarks when its mass becomes much larger than the masses of those particles. On the other hand, if the spin gauge field is not heavy enough, it should be produced from the colliders of the electron and hadron. As there is no any signal indicated from the experiments on such a spin gauge field, the large hadron collider LHC as the highest energy collider at present provides the most stringent constraint on the mass of spin gauge field up to several TeV for the spin gauge coupling constant being compatible to the electroweak coupling constant. In fact, various tests on the general relativity at short distance arrive at the same order of magnitude though its consistency has well been established in the macroscopic world.

Let us first consider the gravidynamics at the distance \( l_G > l_A = 1/m_A \). In such a case, the spin gauge field goes away due to its decay into other light particles, so the gravidynamics is characterized by the following simplified equation:

\[ R_{\mu \nu} - \frac{1}{2} \gamma G R_{\mu \nu} = -8\pi G N T_{\mu \nu}, \quad (188) \]

with the definitions:

\[ \gamma G \equiv 8\pi G N m_G^2, \]

\[ \tilde{R}_{\mu \nu} = \tilde{\nabla}_a F_{a \mu \nu} + \frac{1}{2} \left( \chi_{\mu a} \eta_{\nu} + \chi_{a \nu} \eta_{\mu} \right) \chi_{a \mu \nu}, \]

\[ -\frac{1}{4} \chi_{\theta a} \chi_{a \mu \nu} \tilde{F}_{\mu \nu}, \]

\[ \tilde{F}_{\mu \nu} \equiv \tilde{\nabla}_a \chi_{a \mu \nu}, \]

\[ \tilde{F}_{\mu \nu} - \frac{\partial}{\partial x_{\nu}} F_{\mu a} - \frac{\partial}{\partial x_{\mu}} F_{\nu a} - \frac{1}{2} \left( F_{\mu a} F_{\nu a} + F_{\nu a} F_{\mu a} \right), \]

\[ T_{\mu \nu} = \frac{1}{4} \left[ \chi_{\mu a} \tilde{\partial}_a \chi_{a \nu} \tilde{D}_a \psi + \chi_{\nu a} \tilde{\partial}_a \chi_{a \mu} \tilde{D}_a \psi + H.c. \right] - \chi_{\mu a} \frac{1}{2} \left( \tilde{\chi}_{\nu a} \tilde{\chi}_{a \nu} \tilde{D}_a \psi + H.c. \right) - m_{\tilde{\chi}_{\nu a}} \psi \]

\[ - \left( \eta_{\mu a} \eta_{\nu a} - \frac{1}{4} \chi_{\nu a} \chi_{a \mu} \right) \chi_{a \mu \nu} \tilde{F}_{\mu \nu}, \]

\[ i\tilde{D}_a = i\tilde{\partial}_a + g e A_{a}, \]

where \( T_{\mu \nu} \) is the usual energy momentum tensor arising from the Dirac fermion and electromagnetic gauge field.

It is seen from the above equation that the new effect beyond Einstein theory of general relativity will arise from the tensor \( R_{\mu \nu} \) which is characterized by the gravigauge field \( \chi_{\mu a} \).

As the spin gauge symmetry always allows us to make the gravigauge field to be symmetric \( \chi_{\mu a} = \chi_{a \mu} \) by taking the physical unitary gauge, so that its degrees of freedom are the same as those of gravimetric field \( \chi_{\mu a} \) with the relation \( \chi_{\mu a} = \chi_{\mu a} \). As the tensor \( \tilde{R}_{\mu \nu} \) involves a quadratic derivative over the gravigauge field, it brings on a similar effect as that of Ricci tensor \( R_{\mu \nu} \). To be more explicit, let us write down its gauge-type gravitational equation based on eqs. (159)-(161) as follows:

\[ \partial_a F_{\mu \nu} = \tilde{J}_{a \mu} + \gamma_G J_{a \mu} + 16\pi G N J_{a \mu}, \quad (191) \]

with

\[ F_{\mu \nu} \equiv \chi_{\alpha \mu \nu} F_{\alpha \mu \nu} \]

\[ J_{a \mu} = \tilde{\partial}_a F_{\mu \nu} F_{\nu \alpha} - \frac{1}{4} \chi_{a \mu} F_{\mu \alpha} F_{\alpha \mu}, \]

\[ J_{a \mu} = \chi \left( \tilde{\partial}_a F_{\mu \nu} F_{\nu \alpha} - \frac{1}{4} \chi_{a \mu} F_{\mu \alpha} F_{\alpha \mu} \right), \]

\[ J_{a \mu} = \chi \left( \tilde{\partial}_a F_{\mu \nu} F_{\nu \alpha} - \frac{1}{4} \chi_{a \mu} F_{\mu \alpha} F_{\alpha \mu} \right), \]

\[ J_{a \mu} = \chi \left( \tilde{\partial}_a F_{\mu \nu} F_{\nu \alpha} - \frac{1}{4} \chi_{a \mu} F_{\mu \alpha} F_{\alpha \mu} \right), \]
where \( \mathbf{J}_a \) reflects the new effect of gravidynamics beyond Einstein theory of general relativity. It is seen that the usual gravitational current \( \mathbf{J}_a \) and the extra current \( \tilde{\mathbf{J}}_a \) are all described by the gravigauge field strength \( F_{\mu\nu}^a \).

It is manifest that the magnitude of new effect caused from the gravidynamics based on the inhomogeneous spin gauge symmetry at a distance larger than 1/m_G is characterized by the mass scale \( m_G \) via the mass ratio \( \gamma_G \):

\[
\gamma_G = 8\pi G N m_G^2 = \frac{8\pi m_G^2}{M_P^2} \equiv \frac{m_G^2}{M_P^2},
\]

with \( M_P \) the Planck mass. Such a ratio indicates that when the mass of spin gauge field is much smaller than the Planck mass, i.e., \( m_G^2 \ll M_P^2 \) and \( \gamma_G \ll 1 \), the resulting new effect only brings a tiny modification to the general relativity. It is expected that the future experiments could provide a reliable constraint on the mass of spin gauge field.

Let us now discuss an alternative case that the considering energy scale \( \mu_G \) is higher than the mass scale of spin gauge field, i.e., \( \mu_G > m_A \). For such a short distance case, it is useful to take into account the gravitational origin of spin gauge symmetry, which brings the spin gauge field decomposed into two parts as shown in eq. (58), i.e., \( \mathcal{A}_{\mu\nu} = \Omega_{\mu\nu} + \mathcal{A}_{\mu\nu}^a \). By applying for such a decomposition of spin gauge field with the replacement \( \mathcal{A}_{\mu\nu}^a \to g_G \mathcal{A}_{\mu\nu}^a \), we can rewrite the gravidynamical equation given in eq. (171) into the following form:

\[
R_{\mu\nu} - \frac{1}{2} \chi_{\mu\nu} R + 8\pi G N \tilde{G}_{\mu\nu} = -8\pi G N (T_{\mu\nu} + \tilde{T}_{\mu\nu}),
\]

with the definitions:

\[
\tilde{G}_{\mu\nu} = \left( \eta_{\mu} \eta_{\nu} - \frac{1}{4} \chi_{\mu\nu} \chi^{\mu\nu} \right) \left( R^{\mu\sigma\nu}_{\rho \sigma\tau} R_{\rho \sigma\tau} - \frac{1}{4} \chi_{\mu\nu} R^{\rho \sigma\nu}_{\rho \sigma\tau} R_{\rho \sigma\tau} \right),
\]

\[
\tilde{T}_{\mu\nu} = - \left( \eta_{\mu} \eta_{\nu} - \frac{1}{4} \chi_{\mu\nu} \chi^{\mu\nu} \right) \chi^{\rho \sigma} \left( \left( 2 g_G^{-1} F_{\rho \sigma} F_{\rho \sigma \omega} + F_{\rho \sigma \omega} F_{\rho \sigma \omega} \right) + m_G^2 g_G \tilde{\mathbf{A}}_{(\rho \sigma)} \right)
\]

\[
\times \left( \frac{1}{2} \chi_{\rho \sigma} \eta_{\mu} \eta_{\nu} + \chi_{\rho \sigma} \eta_{\mu} \eta_{\nu} \right),
\]

\[
\tilde{\mathbf{A}}_{(\rho \sigma)} = \frac{1}{4} \left( \eta_{\rho} \eta_{\sigma} \eta_{\mu} \eta_{\nu} \right),
\]

\[
\tilde{\mathbf{A}}_{\mu\nu} = \partial_{\mu} \tilde{\mathbf{A}}_{\nu} - \gamma_{\mu} \tilde{\mathbf{A}}_{\nu} - \gamma_{\nu} \tilde{\mathbf{A}}_{\mu} - \Gamma_{\mu\nu}^{\rho} \tilde{\mathbf{A}}_{\rho} - \Gamma_{\nu\mu}^{\rho} \tilde{\mathbf{A}}_{\rho} - \frac{1}{8} g_G \left( \mathbf{A}_{\rho\sigma} \tilde{\mathbf{A}}_{\rho\sigma} + \mathbf{A}_{\rho\sigma} \tilde{\mathbf{A}}_{\rho\sigma} \right),
\]

9 Spinodynamics via equations of motion of spin gauge field in biframe spacetime

Following along the least action principle and taking the spin gauge field and gravigauge field as independent degrees of freedom, we are able to derive the following equations of motion to describe the spinodynamics:

\[
\mathcal{D}_\nu \tilde{\mathbf{F}}^{\mu\nu}_{\alpha\beta} = \frac{m_G^2}{M_P^2} \chi_{\beta} A_{\alpha\mu} = \tilde{\mathbf{J}}_{\alpha\mu},
\]

\[
\mathcal{D}_\nu \tilde{\mathbf{F}}^{\mu\nu}_{\alpha\beta} - \frac{m_G^2}{M_P^2} \chi_{\beta} A_{\alpha\mu} = \tilde{\mathbf{J}}_{\alpha\mu},
\]

where the second equation is resulted by making the scaling gauge fixing condition to be in Einstein basis. The field
strength and gauge covariant derivative are defined as follows:

\[
\hat{J}^{\mu \nu}_{ab} = \Theta^{[\mu}_{a} \chi^{\nu]b},
\]

\[
D_{\mu} \hat{J}^{\mu \nu}_{ab} = \partial_{\mu} \hat{J}^{\mu \nu}_{ab} - \hat{\omega}_{ab} \hat{J}^{\mu \nu}_{cb}, \tag{201}
\]

\[
\hat{F}^{\mu \nu}_{ab} = \Theta^{[\mu}_{a} \hat{F}^{\nu]b},
\]

\[
\hat{D}^{\mu \nu}_{ab} = \partial_{\mu} \hat{F}^{\nu}_{ab} - \hat{\omega}_{ab} \hat{D}^{\mu \nu}_{cb}, \tag{202}
\]

\[
\sigma^{\mu \nu}_{a} = \chi^{[\mu}_{a} \chi^{\nu]b} F^{b}, \tag{203}
\]

where \( \hat{\omega}_{ab} \) and \( \chi^{[\mu}_{a} \chi^{\nu]b} \) are defined in eqs. (90) and (92), respectively. The field strengths \( F^{\mu \nu}_{ab} \), \( F^{a} \) and \( F^{\mu \nu} \) are defined correspondingly in eqs. (18), (26) and (72). The corresponding currents are given by the following explicit forms:

\[
\hat{J}^{\mu \nu}_{ab} = \Theta^{[\mu}_{a} \chi^{\nu]b} \sigma_{ab} \chi^{\nu}, \tag{204}
\]

The above equations of motion characterize the spinodynamics of spin gauge field.

By applying the relations shown in eqs. (183)-(187), we are able to rewrite the above equation of motion into the following forms:

\[
D_{\mu} \hat{F}^{\mu \nu}_{[ab]} + m_{F}^{2} \hat{\omega}_{[ab]} = m_{F}^{2} \eta^{-1} \Omega_{[ab]} - J_{[ab]}, \tag{205}
\]

\[
D_{\mu} \hat{F}^{\mu \nu}_{[ab]} + m_{F}^{2} \hat{\omega}_{[ab]} = m_{F}^{2} \eta^{-1} \Omega_{[ab]}, \tag{206}
\]

with

\[
\hat{F}^{\mu \nu}_{[ab]} = \eta^{\mu \nu}_{[ab]} \hat{F}^{d}_{ab} = \eta^{\mu \nu}_{[ab]} \hat{F}^{d}_{ab} + \hat{\omega}_{ab} \hat{F}^{d}_{cb}, \tag{207}
\]

for the field strength and gauge covariant derivative, and

\[
J_{[ab]}^{\mu} = g_{G} \hat{F}^{d}_{[ab]} \hat{\psi} \sigma_{ab} \psi, \tag{208}
\]

for the corresponding current.

10 Electrodynamics in gravitational quantum field theory and generalized Maxwell equations in biframe spacetime

Based on the maximal joint symmetry for the action of chirality-based Dirac spinor, namely the inhomogeneous spin gauge symmetry and inhomogeneous Lorentz symmetry (or Poincaré symmetry), we have provided a detailed analysis and investigation on the gravitational relativistic quantum theory of Dirac spinor as well as the gravidynamics and spinodynamics, which brings about the theoretical framework of gravitational quantum field theory. In this section, we are going to investigate systematically the electrodynamics in the presence of gravitational interaction and to derive the generalized Maxwell equations in Minkowski spacetime of coordinates and also in locally flat gravigauge spacetime spanned by the gravigauge basis.

10.1 Gravigeometry-medium electrodynamics and generalized Maxwell equations with gravigeometry-medium electromagnetic field

Following along the least action principle, we obtain the following equations of motion for the electromagnetic gauge field:

\[
\partial_{\mu} \hat{F}^{\mu \nu} = -J^{\mu}, \tag{209}
\]

\[
\partial_{\mu} \hat{F}^{\mu \nu} = -J^{\mu}, \tag{210}
\]
with the field strengths and currents defined by

\[ \mathbf{F}^{\mu \nu} = \partial (\mathbf{A}^{\mu \nu}) \mathbf{F}_{\mu \nu}, \]
\[ \mathbf{F}^{\mu \nu} = \chi \partial (\mathbf{A}^{\mu \nu}) \mathbf{F}_{\mu \nu}, \]
\[ \mathbf{F}^{\mu \nu} \equiv \frac{1}{2} \left( \mathbf{F}^{\mu \nu} - \mathbf{F}^{\nu \mu} \right), \]
\[ \chi \equiv \frac{1}{2} \left( \chi^{\mu \nu} - \chi^{\nu \mu} \right). \] (211)

and

\[ \mathbf{J}^{\mu} = \mathbf{A}^{\mu \nu} \mathbf{J}_{\nu}, \quad \mathbf{J}^{\nu} = -e^{\alpha \beta} \mathbf{A}_{\nu} \mathbf{J}_{\nu}, \]
\[ \mathbf{J}^{\mu} = \chi \mathbf{A}^{\mu \nu} \mathbf{J}_{\nu} - e^{\alpha \beta} \mathbf{A}_{\nu} \mathbf{J}_{\nu}, \] (212)

The second equation in eq. (171) holds in the Einstein basis for the gauge fixing condition of scaling gauge symmetry, where \( \mathbf{F}^{\mu \nu} \) and \( \mathbf{J}^{\mu} \) define the electromagnetic gauge field strength and the corresponding current density in the presence of gravimetric field, and \( \mathbf{J}^{\mu} \) is regarded as a free current density in the absence of gravitational interaction.

Meanwhile, the gauge field strength \( \mathbf{F}_{\mu \nu} \) satisfies the Bianchi identity,

\[ \partial \mu \sigma \mathbf{F}_{\mu \sigma} = 0. \] (213)

The above equations of motion in the Einstein basis can be rewritten into the following four sets of equations:

\[ \nabla \cdot \mathbf{E} = \mathbf{j}, \]
\[ \nabla \times \mathbf{B} = \mathbf{j} \quad \text{with the coefficient functions:} \]
\[ \mathbf{E} \equiv \mathbf{E} \mathbf{E} + \mathbf{e} \mathbf{E} + \mathbf{E} \mathbf{e}, \]
\[ \mathbf{B} \equiv \mathbf{B} \mathbf{B} + \mathbf{e} \mathbf{B} + \mathbf{B} \mathbf{e}, \]
\[ \nabla \cdot \mathbf{E} = \nabla \times \mathbf{B} = 0, \]

where we have introduced the definitions:

\[ \mathbf{E} \equiv \left( \mathbf{E}^{01}, \mathbf{E}^{02}, \mathbf{E}^{03} \right), \]
\[ \mathbf{B} \equiv \left( \mathbf{B}^{01}, \mathbf{B}^{02}, \mathbf{B}^{03} \right), \]
\[ \mathbf{E} \equiv \left( \mathbf{E}^{1}, \mathbf{E}^{2}, \mathbf{E}^{3} \right), \]
\[ \mathbf{B} \equiv \left( \mathbf{B}^{1}, \mathbf{B}^{2}, \mathbf{B}^{3} \right), \] (214)

In the ordinary Maxwell equations yielded in globally flat Minkowski spacetime, the corresponding field strength with \( \mathbf{E} \equiv \left( \mathbf{E}^{0} = -F^{0} \right) \) \((i, j, k = 1, 2, 3)\) defines a free-motion electromagnetic field in the absence of gravitational interaction. In the gravigauge-medium Maxwell equations shown in eq. (214) due to the presence of gravimetric field, the field strength defined via \( \mathbf{E} \equiv \left( \mathbf{E}^{0} = -F^{0} \right) \) and \( \mathbf{B} \equiv \left( \mathbf{B}^{0} = -\frac{1}{2} \mathbf{F}^{0} \right) \) are regarded as gravigauge-medium electromagnetic field. The relations between the gravigauge-medium electromagnetic field and the free motion electromagnetic field can directly be written down from the definitions in eq. (18) as follows:

\[ \mathbf{E}^{0} = \mathbf{E}^{0} + \mathbf{A}^{0} \mathbf{E}^{0}, \]
\[ \mathbf{B}^{0} = \mathbf{B}^{0} - \mathbf{A}^{0} \mathbf{B}^{0}, \] (215)

with the coefficient functions:

\[ \mathbf{E}^{0} \equiv \chi \mathbf{E}^{0} \mathbf{E}^{0}, \]
\[ \mathbf{B}^{0} \equiv \mathbf{B}^{0} \mathbf{B}^{0}, \]

Note that all the coefficients are functions of coordinate spacetime and they are in general the vectors and tensors in spatial dimensions.

It is useful to rewrite the above relations into the following forms in the vector representation:

\[ \mathbf{E} \equiv \mathbf{e} \mathbf{E} \mathbf{E} + \mathbf{e} \mathbf{E} \mathbf{E} + \mathbf{E} \mathbf{e}, \]
\[ \mathbf{B} \equiv \mathbf{B} \mathbf{B} + \mathbf{e} \mathbf{B} + \mathbf{B} \mathbf{e}, \]

with the vector bi-vector coefficient functions:

\[ \mathbf{E} \equiv \left( \mathbf{E}^{0} \mathbf{E}^{0} \right), \]
\[ \mathbf{B} \equiv \left( \mathbf{B}^{0} \mathbf{B}^{0} \right), \]

These coefficient functions are in principle governed by the gravidyamics.

10.2 Gravigauge-mediated electrodynamics and generalized Maxwell equations in locally flat gravigauge spacetime

In the absence of gravitational interaction, the electromagnetic gauge field \( \mathbf{A}_{\mu} \) is regarded as a vector field in globally flat Minkowski spacetime. The inhomogeneous spin gauge symmetry with the introduction of gravigauge field \( \chi^{\mu}_{\nu} \) brings \( \mathbf{A}_{\mu} \) to couple with Dirac fermion via a vector gauge
field $A_a \equiv \hat{\chi}_a^\mu A_{a\mu}$ defined in locally flat gravigauge spacetime. The least action principle leads the equation of motion of gauge field $A_a$ to have the following form in locally flat gravigauge spacetime:

$$D_b F^{ab} = -J^a,$$  \hspace{1cm} (221)  

with the field strength and covariant derivative defined as follows:

$$D_b F^{ab} \equiv \delta_b F^{ab} + \Omega_b^{cb} F^{cb} + \Omega_b^{ac} F^{ac},$$

$$F^{ab} \equiv \eta^{ac} \eta^{bd} F_{cd} \equiv \hat{\chi}^{ac}_{\mu} \chi^{bd}_{\nu} F_{\mu\nu},$$

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]} - \partial_{\nu} A_{\mu} - \Omega_{\mu\nu}^{b} A_{b},$$

and the current density is simply given by

$$J^a = -g_{\mu\nu} \bar{\psi} \gamma^a \psi,$$  \hspace{1cm} (223)  

which is considered as the current density for freely moving Dirac fermion in locally flat gravigauge spacetime.

The field strength $F_{ab}$ satisfies the following identity:

$$\epsilon^{abcd} D_c F_{ab} = 0,$$  \hspace{1cm} (224)  

which results from the Bianchi identity of Riemann curvature tensor $\epsilon^{abcd} R_{cab}^{\alpha} = 0$ in locally flat gravigauge spacetime. It can directly be verified as follows:

$$\epsilon^{abcd} D_c F_{ab} = 2 \epsilon^{abcd} D_c (D_a A_b) = -\epsilon^{abcd} R_{cab}^{\alpha} A_{\alpha} = 0.$$  \hspace{1cm} (225)  

It can be proved from eq. (221) that the current density $J^a$ is conserved in a gauge covariant form, i.e.,

$$D_a J^a = -D_a F_{a\mu} = 0,$$

where the last equality is due to the symmetric feature of Ricci curvature tensor $R_{a\mu} = R_{\mu a}$ and the antisymmetric property of field strength $F^{cb} = -F^{bc}$. It is straightforward to write down the quadratic form for the equation of motion of gauge field $A_a$ shown in eq. (221) as follows:

$$D_a D^a A_a + R_{a\mu} A^\mu = D_a (D_a A^\mu) = 0.$$  \hspace{1cm} (227)  

When taking the following covariant gauge fixing condition:

$$D_a A^\mu = \delta_a A^\mu + \Omega_{\alpha a}^c A^\alpha = 0,$$  \hspace{1cm} (228)  

and utilizing the equation of motion of gradvodynamics presented in eq. (176), we arrive at the following equation of motion for the gauge field $A_a$ in locally flat gravigauge spacetime:

$$D_a D^a A_a + \frac{1}{2} R A_a - 8 \pi G_N (T_{ac} + \bar{T}_{ac}) A^c = J_{a},$$  \hspace{1cm} (229)  

which indicates that a non-zero Ricci curvature scalar $R \neq 0$ brings on an effective mass to the electromagnetic gauge field $A_a$ when it propagates in locally flat gravigauge spacetime.

It is useful to rewrite the equations of motion in eqs. (221) and (224) into the following forms:

$$\delta_b F^{ba} - \tilde{A}_b F^{ba} - \frac{1}{2} F_{bc} F^{bc} = J^a,$$  \hspace{1cm} (230)  

$$\epsilon^{abcd} (\delta_b F_{cd} + F_{ce} F_{db}) = 0,$$  \hspace{1cm} (231)  

with the definitions:

$$F_{bc} \equiv -\Omega_{[bc]}^{d} = \Omega_{bc}^{d} - \Omega_{cb}^{d} \equiv \hat{\chi}^{\mu}_{\beta} \hat{\chi}^{\nu}_{\alpha} F_{\mu\nu}^{a} \equiv \hat{F}^{\mu}_{\beta\alpha} \hat{\chi}^{\mu}_{\alpha} \hat{\chi}^{\nu}_{\beta},$$

where $F_{\mu\nu}$ is the gravigauge field strength in locally flat gravigauge spacetime. The equations of motion in locally flat gravigauge spacetime with $a, b, (0, \alpha)$ ($\alpha = 1, 2, 3$) can be expressed as follows:

$$\delta_a E^a - \tilde{A}_a E^a - E_a \bar{E}^a = J^a - \tilde{B}_a \bar{B}^a,$$

$$\epsilon^{\alpha\beta\gamma}(\delta_b \bar{E}_\gamma - \tilde{A}_b \bar{B}_\gamma) + \tilde{B}_\alpha E^\alpha = J^a - \tilde{B}_a \bar{B}^a,$$

$$\delta_a B^a + 2 \tilde{A}_a \bar{B}^a + 2 \bar{E}_a E^a = \tilde{B}_a \bar{E}^a,$$

$$\delta_0 \bar{B}^a + 2 \tilde{A}_0 \bar{B}^a = 2 E_0 E^a,$$

where the field strengths defined to be

$$\bar{E}^a = -F^{0a}, \quad \bar{B}^a = -\frac{1}{2} \epsilon_{\beta\gamma} F^{\beta\gamma},$$

$$\tilde{E}_a = -F^{0a}, \quad \tilde{B}_a = -\frac{1}{2} \epsilon_{\alpha\beta\gamma} F_{\alpha\beta\gamma},$$

$$\tilde{E}^{\beta}_a = -F_{\beta 0}, \quad \tilde{B}^{\beta}_a = -\frac{1}{2} \epsilon^{\alpha\beta\gamma} F_{\alpha\beta\gamma},$$  \hspace{1cm} (233)  

The field strengths $\bar{E}^a$ and $\bar{B}^a$ are defined as electromagnetic field in locally flat gravigauge spacetime, which may be called as gravigauge-dressed electric field and gravigaude-dressed magnetic field, respectively, in order to distinguish with the free-motion electromagnetic field $E$ and $B$ defined in Minkowski spacetime. $\hat{E}_a$ and $\hat{B}_a$ as well as $\tilde{E}^{\beta}_a$ and $\tilde{B}^{\beta}_a$ are defined as electromagnetic-like gravigauge fields to characterize the gravitational interactions. For convenience, $\hat{E}_a$ and $\hat{B}_a$ are referred to as electric-like gravigauge field and magnetic-like gravigauge field, respectively, while $\tilde{E}^{\beta}_a$ and $\tilde{B}^{\beta}_a$ are mentioned as electric-like gravigauge tensor field and magnetic-like gravigauge tensor field, respectively. $\hat{A}_0$ and $\hat{A}_a$ are related to the electromagnetic-like gravigauge field as follows:

$$\hat{A}_0 = \eta^{\beta}_{\alpha} \hat{E}^{\alpha}_{\beta}, \quad \hat{A}_a = \hat{E}_a - \epsilon_{\alpha\beta\gamma} \hat{B}^{\beta}_{\gamma}.$$  \hspace{1cm} (234)
The equations of motion given in eqs. (221) and (224) or eqs. (230)-(255) characterize the electrodynamics in locally flat gravigauge spacetime, which may be referred to as gravigauge-mediated electrodynamics.

In light of vector representation, the equations of motion for the gravigauge-dressed electromagnetic fields $\mathcal{E}$ and $\mathcal{B}$ as shown in eq. (232) can be rewritten into the following forms in locally flat gravigauge spacetime:

\[
\begin{align*}
\hat{\mathcal{V}} \cdot \mathcal{E} - \hat{\mathcal{A}} \cdot \mathcal{E} - \mathcal{E} &= \mathcal{J}^0 + \mathcal{B} \cdot \mathbf{B}, \\
\hat{\mathcal{V}} \times \mathcal{B} - \hat{\mathcal{A}} \times \mathcal{B} + \mathbb{B} \cdot \mathbf{B} &= \mathcal{J} + \partial_0 \mathcal{E} - \hat{\mathcal{A}}_0 \mathcal{E} + \mathbb{B} \cdot \mathbf{E}, \\
\hat{\mathcal{V}} \times \mathcal{E} + \mathcal{E} \times \mathcal{E} - \mathbb{E} \cdot \mathbf{E} &= -\partial_0 \mathcal{B} - 2\hat{\mathcal{A}}_0 \mathcal{B} + \mathbb{B} \cdot \mathbf{B}, \\
\hat{\mathcal{V}} \cdot \mathcal{B} + 2\mathcal{A} \cdot \mathcal{B} + 2\mathcal{E} \cdot \mathbf{B} &= \mathbf{B} \cdot \mathbf{E},
\end{align*}
\]

where we have introduced the following vector-like notations:

\[
\hat{\mathcal{E}} \equiv (\hat{E}^1, \hat{E}^2, \hat{E}^3), \quad \hat{\mathcal{B}} \equiv (\hat{B}^1, \hat{B}^2, \hat{B}^3), \\
\mathcal{E} \equiv (E^1, E^2, E^3), \quad \mathcal{B} \equiv (B^1, B^2, B^3),
\]

\[
\mathcal{E}^a \equiv (E_1^a, E_2^a, E_3^a), \quad \hat{\mathcal{B}}^a \equiv (\hat{B}_1^a, \hat{B}_2^a, \hat{B}_3^a), \\
\hat{\mathcal{A}} \equiv (\hat{A}_1, \hat{A}_2, \hat{A}_3),
\]

and

\[
\begin{align*}
\mathbb{E} &\equiv \mathcal{E}^a, \\
\mathbb{B} &\equiv \hat{\mathcal{B}}^a
\end{align*}
\]

\[
\begin{align*}
\mathbb{E} &= \{(\hat{E}^1_1, \hat{E}^2_1, \hat{E}^3_1), (\hat{E}^1_2, \hat{E}^2_2, \hat{E}^3_2), (\hat{E}^1_3, \hat{E}^2_3, \hat{E}^3_3)\}, \\
\mathbb{B} &= \{(\hat{B}^1_1, \hat{B}^2_1, \hat{B}^3_1), (\hat{B}^1_2, \hat{B}^2_2, \hat{B}^3_2), (\hat{B}^1_3, \hat{B}^2_3, \hat{B}^3_3)\},
\end{align*}
\]

\[
\begin{align*}
\hat{\mathcal{V}} &= (\partial_1, \partial_2, \partial_3) = (\hat{x}^0_0 \partial_0 + \hat{x}^1_0 \partial_1 + \hat{x}^2_0 \partial_2 + \hat{x}^3_0 \partial_3), \\
\partial_0 &= \hat{x}^0_0 \partial_0 = \hat{x}^1_0 \partial_1 + \hat{x}^2_0 \partial_2 + \hat{x}^3_0 \partial_3,
\end{align*}
\]

with $\mathbb{E}$ and $\mathbb{B}$ denoting electromagnetic-like gravigauge bivector fields, where $\hat{\mathcal{V}}$ and $\partial_0$ represent the gravi-coordinate derivatives, which are related to the ordinary coordinate derivatives as follows:

\[
\begin{align*}
\hat{\mathcal{V}} &= (\partial_1, \partial_2, \partial_3) = (\hat{x}^0_0 \partial_0 + \hat{x}^1_0 \partial_1) \\
\partial_0 &= \hat{x}^0_0 \partial_0 + \hat{x}^1_0 \partial_1 + \hat{x}^2_0 \partial_2 + \hat{x}^3_0 \partial_3,
\end{align*}
\]

with the definitions:

\[
\begin{align*}
\hat{k}(x) &= \{\hat{x}^0_0, \hat{x}^1_0, \hat{x}^2_0, \hat{x}^3_0\}, \\
\hat{k}_a(x) &= \{\hat{x}^0_0, \hat{x}^1_0, \hat{x}^2_0, \hat{x}^3_0\}, \\
\hat{k}(x) &= \{\hat{x}^0_1, \hat{x}^1_1, \hat{x}^2_1, \hat{x}^3_1\}
\end{align*}
\]

\[
\begin{align*}
\hat{k}(x) &= \{\hat{x}^0_1, \hat{x}^1_1, \hat{x}^2_1, \hat{x}^3_1\}, \\
\hat{k}_a(x) &= \{\hat{x}^0_0, \hat{x}^1_0, \hat{x}^2_0, \hat{x}^3_0\}, \\
\hat{k}(x) &= \{\hat{x}^0_1, \hat{x}^1_1, \hat{x}^2_1, \hat{x}^3_1\}
\end{align*}
\]

In terms of the above definitions and notations for the gravi-coordinate derivatives, the gravigauge Maxwell equations in eq. (235) can be expressed as follows:

\[
\begin{align*}
\hat{k} \cdot \frac{\partial \mathcal{E}}{\partial \hat{x}^0} + (\hat{k} \cdot \nabla) \cdot \mathcal{E} - \hat{\mathcal{A}} \cdot \mathcal{E} - \mathcal{E} &= \mathcal{J}^0 + \mathcal{B} \cdot \mathbf{B}, \\
\hat{k} \times \frac{\partial \mathcal{B}}{\partial \hat{x}^0} + (\hat{k} \cdot \nabla) \times \mathcal{B} - \hat{\mathcal{A}} \times \mathcal{B} + \mathbb{B} \cdot \mathbf{E},
\end{align*}
\]

\[
\begin{align*}
\mathcal{J} + \hat{k} \cdot \partial \mathcal{E} + (\hat{k} \cdot \nabla) \mathcal{E} - \hat{\mathcal{A}}_0 \mathcal{E} + \mathbb{B} \cdot \mathbf{E}, \\
\hat{k} \times \frac{\partial \mathcal{E}}{\partial \hat{x}^0} + (\hat{k} \cdot \nabla) \mathcal{E} - \hat{\mathcal{A}}_0 \mathcal{E} + \mathbb{B} \cdot \mathbf{E},
\end{align*}
\]

(240)

\[
\begin{align*}
\hat{k} \cdot \frac{\partial \mathcal{B}}{\partial \hat{x}^0} + (\hat{k} \cdot \nabla) \mathcal{B} - \hat{\mathcal{A}}_0 \mathcal{B} + \mathbb{B} \cdot \mathbf{E}, \\
\hat{k} \cdot \frac{\partial \mathcal{B}}{\partial \hat{x}^0} + (\hat{k} \cdot \nabla) \mathcal{B} - \hat{\mathcal{A}}_0 \mathcal{B} + \mathbb{B} \cdot \mathbf{E}
\end{align*}
\]

It is noticed that the equations in eqs. (235) and (240) (or eq. (232)) bring on the generalized Maxwell equations in locally flat gravigauge spacetime. To distinguish with the gravigeometry-medium Maxwell equations presented in eqs. (241) and (242)-(249) in coordinate spacetime, they may be referred to as gravigauge-mediated Maxwell equations which are derived to describe the gravigauge-mediated electrodynamics in locally flat gravigauge spacetime.

The gravigauge-mediated Maxwell equations show explicitly how the gravigauge-dressed electromagnetic fields $\mathcal{E}$ and $\mathcal{B}$ interact directly with the electromagnetic-like gravigauge fields $\mathbb{E}$ and $\mathbb{B}$ as well as electromagnetic-like gravigauge vector fields $\mathbb{E}$ and $\mathbb{B}$ in locally flat gravigauge spacetime, which reveals how the gravitational effect emerges to bring the gravigauge-mediated electrodynamics in locally flat gravigauge spacetime.

### 10.3 Gravimetric-gauge-mediated electrodynamics and generalized Maxwell equations in dynamic Riemannian spacetime

The equations of motion for the gauge field $A_\mu$ can also be expressed as the following form in dynamic Riemannian spacetime:

\[
\nabla^\nu F_{\mu \nu} = -J_\mu,
\]

(241)

\[
\epsilon^{\mu \nu \rho \sigma} \partial_\rho F_{\nu \sigma} = 0,
\]

with the covariant derivative defined as follows:

\[
\nabla^\nu F_{\mu \nu} \equiv \hat{x}^\nu_\psi \epsilon_\nu_\mu_F_{\nu \psi} = \hat{x}^\nu_\psi \partial_\psi F_{\nu \nu} - \Gamma_\nu_\mu F_{\nu \psi} - \Gamma_\nu_\nu F_{\mu \psi}.
\]

(242)

The second equation in eq. (241) is the Bianchi identity which can also be rewritten in terms of the covariant derivative:

\[
\epsilon^{\mu \nu \rho \sigma} \partial_\rho F_{\nu \sigma} \equiv \epsilon^{\mu \nu \rho \sigma} \nabla_\rho F_{\nu \sigma} = 0,
\]

(243)
where we have used the symmetric property of the affine connection (Christoffel symbol) $\Gamma^i_{\mu\nu} = \Gamma^i_{\nu\mu}$.

The current density in eq. (241) is defined by

$$J_\mu \equiv \chi_\mu^a J_a = -g_{Ea} \chi_\mu^a \tilde{\psi} \gamma_\mu \psi,$$

(244)

which is conserved via the covariant derivative:

$$\nabla_\mu J_\mu = -R^\nu_{\mu\rho} \nabla_\nu F_{\rho\mu} + \frac{1}{2} \left( R^\nu_{\mu\rho} F_{\rho\mu} + R^\nu_{\mu\rho} F_{\rho\mu} \right) = R^\nu_{\mu\rho} F_{\rho\mu} = 0,$$

(245)

where the last equality is due to the symmetric feature of Ricci curvature tensor $R^\nu_{\mu\rho} = R^\nu_{\rho\mu}$ and the antisymmetric property of electromagnetic field strength $F_{\rho\mu} = -F_{\mu\rho}$.

In can be checked that the equation of motion of gauge field $A_\mu$ gets the following form:

$$\nabla^\nu \nabla_\nu A_\mu + R_{\mu\rho\sigma} A^\rho - \nabla_\mu (\nabla^\nu A_\nu) = J_\mu.$$

(246)

By imposing the covariant gauge fixing condition $\nabla_\nu A_\nu = 0$, we come to the following equation of motion for the gauge field $A_\mu$:

$$\nabla^\nu \nabla_\nu A_\mu + R_{\mu\rho\sigma} A^\rho = J_\mu.$$

(247)

When applying for the geometric gravitational equation shown in eq. (171), we have

$$\left( \nabla^\nu \nabla_\nu + \frac{1}{2} R \right) A_\mu - 8\pi G_N \left( T^\nu_{\mu\rho} + T^\nu_{\rho\mu} \right) A^\nu = J_\mu,$$

(248)

which indicates that a non-zero Ricci curvature scalar $R \neq 0$ leads to an effective mass-like term for the electromagnetic gauge field $A_\mu$ when it is propagating in a curved Riemannian spacetime.

To reflect the effect of geometric gravidynamics on the electrodynamics, it is useful to express the equations of motion in eq. (241) into the following form:

$$(\tilde{\partial}^\nu + A^\nu) F_{\rho\sigma} + \frac{1}{2} \chi_{\rho\tau} \chi^\tau_{\sigma} F^\nu_{\rho\sigma} \eta_{\rho\sigma} F_{\nu\nu} = J_\mu,$$

(249)

where we have introduced the definitions:

$$F^\nu_{\rho\sigma} \equiv \partial_\rho \chi^\nu_{\sigma} - \partial_\sigma \chi^\nu_{\rho}, \quad \chi^\nu_{\rho} \equiv \chi^\nu_{\rho} \eta^\mu_{\rho},$$

$$\chi^\nu_{\rho} \eta_{\rho\nu} \equiv \tilde{\partial}^\nu \ln \chi + \partial_\nu \chi^\nu_{\rho},$$

$$\tilde{\partial}^\nu \equiv \chi^\nu_{\rho} \partial_\rho.$$

(250)

Such a defined $F^\nu_{\rho\sigma}$ is considered to be a gauge field strength when viewing the gravimetric field $\chi^\nu_{\rho}$ to be a gauge-type field (which is actually a combined field of gaugivauge field $\chi^\nu_{\rho} \equiv \chi^\nu_{\rho} \chi^\mu_{\lambda}$). For convenience of mention, $\chi^\nu_{\rho}$ is referred to as gravimetric-gauge field and $F^\nu_{\rho\sigma}$ is called as electromagnetical-like gravimetric-gauge field strength.

The equations of electromagnetic field in dynamic Riemannian spacetime are obtained as follows:

$$(\tilde{\partial}^\nu + \hat{A}^\nu) E_\nu + \chi^\rho \chi^\sigma F^\nu_{\rho\sigma} E_\nu + \frac{1}{2} \chi^\rho \chi^\sigma F^\nu_{\rho\sigma} \epsilon_{\rho\sigma} E_\nu = J_\nu,$$

(251)

$$(\tilde{\partial}^\nu + \hat{A}^\nu) J_\nu + \frac{1}{2} \chi^\rho \chi^\sigma F^\nu_{\rho\sigma} \epsilon_{\rho\sigma} J_\nu = J_\nu,$$

(252)

which can simply be expressed as follows:

$$(\tilde{\partial}^\nu + \hat{A}^\nu) E_\nu + \hat{E}^\nu = J_\nu,$$

where we have introduced the definitions:

$$\hat{E}^\nu \equiv \epsilon_{\nu\rho}^\nu \hat{J}_\rho + \epsilon_{\nu\rho}^\nu \hat{B}_\rho, \quad \hat{B}_\nu \equiv \mu_{\nu\rho} \hat{B}_\rho - \nu_{\nu\rho} \hat{E}_\rho, \quad \hat{E}^\nu \equiv \epsilon_{\nu\rho} \hat{E}_\rho + \nu_{\nu\rho} \hat{B}_\rho.$$

(253)

and the covector functions and field strengths defined as follows:

$$\epsilon_{\nu\rho}^\nu \chi_{\nu\rho} \equiv \chi_{\nu\rho} \chi_{\nu\rho},$$

$$\mu_{\nu\rho} \chi_{\nu\rho} \equiv \chi_{\nu\rho} \chi_{\nu\rho},$$

$$\nu_{\nu\rho} \chi_{\nu\rho} \equiv \chi_{\nu\rho} \chi_{\nu\rho}.$$

(254)

It is noticed that $\hat{E}_\nu$ and $\hat{B}_\nu$ as well as $\hat{E}_\nu^i$ and $\hat{B}_\nu^i$ are introduced as electromagnetical-like gravimetric-gauge fields to characterize the gravitational interactions in dynamic Riemannian spacetime. In analogous to electromagnetic field, $\hat{E}_\nu$ and $\hat{B}_\nu$ are referred to as electric-like gravimetric-gauge field and magnetic-like gravimetric-gauge field, respectively, while $\hat{E}_\nu^i$ and $\hat{B}_\nu^i$ are called as electric-like gravimetric-gauge tensor field and magnetic-like gravimetric-gauge tensor field, respectively.

In terms of vector representation, the equations presented in eqs. (252) and (241) can be rewritten into the following forms:

$$\hat{\nabla} \cdot \hat{E} + \hat{\nabla} \cdot \hat{A} + \hat{E} \cdot \hat{E} + \hat{B} \cdot \hat{B} = J_0,$$

$$\hat{\nabla} \times \hat{B} + \hat{\nabla} \times \hat{A} + \hat{B} \cdot \hat{B} = J_0 + (\tilde{\partial}^\nu + \hat{A}^\nu) \hat{E},$$

$$\nabla \cdot \hat{E} = 0,$$

(256)

which bring on the generalized Maxwell equations in dynamic Riemannian spacetime, where we have used the vector notations $\hat{E} \equiv \{ \hat{E}_\nu \}, \hat{B} \equiv \{ \hat{B}_\nu \}, \hat{A} \equiv \{ \hat{A}_\nu \}, \hat{\nabla} \equiv \{ \hat{\delta}_\nu \}$ and bi-vector notations $\hat{E} \equiv \{ \hat{E}_i^\nu \}, \hat{B} \equiv \{ \hat{B}_i^\nu \}$. 

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To distinguish with the gravitau-mediated Maxwell equations in locally flat gauge spacetime and gravigometry-medium Maxwell equations in coordinate spacetime, we may refer to the above generalized Maxwell equations as gravimetric-gauge-mediated Maxwell equations, which are provided to describe the gravimetric-gauge-mediated electrodynamics in dynamic Riemannian spacetime.

11 General covariance of dynamic equations and electrodynamics in any motional and spinning reference frames

It is shown that the Hermitian action constructed in locally flat gauge spacetime based on the gauge invariance principle becomes independent of the choice of coordinates. Namely, such an action holds in any coordinate system and gets invariant under the general coordinate transformation. In other word, such a gauge invariant action possesses automatically a hidden general linear group symmetry GL(1,3,R) which is known to lay the foundation of Einstein’s general theory of relativity. Such a consequence indicates that the gauge invariance principle brings naturally on the emergence of general linear group symmetry GL(1,3,R) as a local symmetry in coordinate spacetime.

11.1 General covariance of dynamic equations

To be concrete, the general coordinate transformation is defined as an arbitrary reparametrization of coordinate systems, i.e.,

$$x^\mu \rightarrow x'^\mu \equiv x'^\mu(x),$$  \hspace{1cm} (257)

which provides a local transformation in curved Riemannian spacetime and leads the displacement and derivative of coordinate systems to satisfy the following transformation laws:

$$dx^\mu \rightarrow dx'^\mu \equiv T^\mu_\nu dx^\nu, \quad T^\mu_\nu \equiv \frac{\partial x'^\mu}{\partial x^\nu},$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \rightarrow \partial'_\mu \equiv \frac{\partial}{\partial x'^\mu} = T^\nu_\mu \partial_\nu, \quad T^\nu_\mu = \frac{\partial x^\nu}{\partial x'^\mu},$$ \hspace{1cm} (258)

which preserves the inner product to be invariant under the general coordinate transformation,

$$\langle dx^\mu, dx'^\nu \rangle \equiv T^\mu_\sigma T^{\nu}_\rho \langle dx^\mu, dx^{\nu} \rangle = T^\mu_\sigma T^{\nu}_\rho \eta^{\sigma}_\rho = \eta^{\nu}_\mu \equiv \langle dx^\nu, dx^\mu \rangle. \hspace{1cm} (259)$$

From the above transformation law, $\partial_\mu$ is usually regarded as covariant vector and $dx^\mu$ as contravariant vector in coordinate spacetime. In general, any covariant vector field $V_\mu(x)$ with a lower index and contravariant vector field $\hat{V}^\nu(x)$ with an upper index transform as follows:

$$V'_\mu(x') = T^\nu_\mu V_\nu(x), \quad \hat{V}^\nu(x') = T^\nu_\nu \hat{V}^\nu(x),$$ \hspace{1cm} (260)

which keeps the scalar product between the covariant vector field and contravariant vector field to be invariant:

$$V'_\mu(x')\hat{V}^\nu(x') = T^\nu_\nu T^\nu_\rho V_\rho(x)\hat{V}^\nu(x) = \eta^{\nu}_\rho V_\rho(x)\hat{V}^\nu(x) \equiv \eta^{\nu}_\mu V_\mu(x)\hat{V}^\nu(x).$$ \hspace{1cm} (261)

The gravigauge field $A_\mu^a(x)(\chi^a_\mu(x))$ and its dual field $\hat{A}_a^\mu(x)(\hat{\chi}_a^\mu(x))$ are considered to be bi-contravariant vector fields in biframe spacetime, while $A_\mu^a(x)(\chi^a_\mu(x))$ is regarded as a covariant vector field in coordinate spacetime and a contravariant vector field in locally flat gravigauge spacetime, while $\hat{A}_a^\mu(x)(\hat{\chi}_a^\mu(x))$ is viewed as a contravariant vector field in coordinate spacetime and a covariant vector field in locally flat gravigauge spacetime. Namely, they transform as follows:

$$A_\mu^a(x') = T^\nu_\mu A_\nu^a(x), \quad \chi^a_\mu(x') = T^\nu_\nu \chi^a_\nu(x),$$

$$\hat{A}_a^\mu(x') = T^\nu_\nu \hat{A}_a^\nu(x), \quad \hat{\chi}_a^\mu(x') = T^\nu_\nu \hat{\chi}_a^\nu(x),$$ \hspace{1cm} (262)

under the general coordinate transformation of general linear group symmetry GL(1,3,R), and

$$A_\mu^a(x) = \Lambda_\mu^a(x)A_\mu^a(x), \quad \chi_\mu^a(x) = \Lambda_\mu^a(x)\chi_\mu^a(x),$$

$$\hat{A}_a^\mu(x) = \Lambda_\mu^a(x)\hat{A}_a^\mu(x), \quad \hat{\chi}_a^\mu(x) = \Lambda_\mu^a(x)\hat{\chi}_a^\mu(x),$$ \hspace{1cm} (263)

under the gauge transformation of spin gauge group symmetry Spin(1,3). So the gravicoordinate displacement $\delta x^\mu (\delta \chi_\mu^a)$ and derivative $\partial_{\mu} (\partial_{\mu})$ defined in locally flat gauge spacetime become invariant under the general coordinate transformation. Explicitly, we have

$$\delta x^\mu \equiv A_\mu^a(x)dx^\mu = A_a^a(x)dx^\mu,$$

$$\delta \chi_\mu^a(x)dx^\mu = \chi_\mu^a(x)dx^\mu,$$

$$\delta_{\mu} \equiv \chi_\mu^a(x)\partial_\mu = \chi_\mu^a(x)\partial_\mu,$$ \hspace{1cm} (264)

which indicates that the gravicoordinate displacement $\delta x^\mu (\delta \chi_\mu^a)$ and derivative $\partial_{\mu} (\partial_{\mu})$ introduced in eqs. (44) and (47) are the scalar product between the covariant vector and contravariant vector in curved Riemannian spacetime of coordinates. Therefore, the gravicoordinate displacement $\delta x^\mu (\delta \chi_\mu^a)$ and derivative $\partial_{\mu} (\partial_{\mu})$ naturally possess the general linear group symmetry GL(1,3,R). It can be checked that the inner product of $\delta x^\mu (\delta \chi_\mu^a)$ and derivative $\partial_{\mu} (\partial_{\mu})$ becomes invariant under the spin gauge transformation,

$$\langle \delta_{\mu}^a(x), \delta_{\mu}^b(x) \rangle = \Lambda_\mu^a \Lambda_\mu^b \langle \delta_{\mu}^a(x), \delta_{\mu}^b(x) \rangle = \Lambda_\mu^a \Lambda_\mu^b \eta_{ab} \equiv \eta_{ab} = \langle \delta_{\mu}^a(x), \delta_{\mu}^b(x) \rangle. \hspace{1cm} (265)$$

On the other hand, the gravimetric field $H_\rho^\mu(x)(\chi^a_\rho(x))$ and dual gravimetric field $\hat{H}^\rho_\mu(x)(\hat{\chi}^a_\rho(x))$ defined in eqs. (42)
and (46) as the scalar products of bi-covariant vector field $A^\mu_a(x)(\chi^a(x))$ and $\hat{A}^{\nu}_b(x)(\hat{\chi}^b(x))$ in locally flat gravigauge spacetime correspond to the covariant tensor and contravariant tensor fields in curved Riemannian spacetime of coordinates. They obey the following transformation laws:

$$H^\mu(x) = T^\mu_{\nu}H^\nu(x),$$

$$\chi^\mu(x) = T^\mu_{\nu}\chi^\nu(x),$$

$$\tilde{H}^{\mu\nu}(x) = T^\mu_{\rho}T^\nu_{\sigma}\tilde{H}^{\rho\sigma}(x),$$

$$\hat{\chi}^{\mu\nu}(x) = T^\mu_{\rho}\hat{A}^\nu_{\sigma}(x),$$

(266)

with $\Gamma^\mu_{\nu\sigma}(x)$ the Christoffel symbols with appearance of inhomogeneous term under general linear group GL(1,3,R), which is analogous to the gauge-type transformation of spin gauge group symmetry GL(1,3,R) in locally flat gravigauge spacetime. Such a property of transformations keeps the covariant derivative on the covariant vector field $V^\mu(x)$ and contravector vector field $\hat{V}^\mu(x)$ to be general covariance under the general coordinate transformations, i.e.,

$$\nabla^\mu V^\nu(x) = T^\mu_{\rho}T^\nu_{\sigma}\nabla^\rho V^\sigma(x),$$

$$\nabla^\nu V^\mu(x) = \partial_\nu V^\mu(x) - T^\mu_{\rho}\nabla^\rho V^\nu(x),$$

$$\nabla_\mu \hat{V}^\nu(x) = \partial_\mu \hat{V}^\nu(x) + \Gamma^\nu_{\mu\sigma}(x)\hat{V}^\sigma(x).$$

(268)

With the above transformation laws, it can be verified explicitly that all actions constructed in the previous sections are invariant under the general coordinate transformation via an arbitrary reparametrization of coordinates shown in eqs. (257)-(259), which displays the emergent general linear group symmetry GL(1,3,R) of the actions. As a consequence, we come to the statement that the gauge invariance principle naturally leads the action formulated in locally flat gravigauge spacetime to possess a maximal joint symmetry:

$$G^3 = \text{GL}(1,3,\mathbb{R}) \rtimes \text{WS}(1,3),$$

which implies that the Poincaré group symmetry PO(1,3) in globally flat Minkowski spacetime will automatically be extended to the local general linear group symmetry GL(1,3,R), which brings on the appearance of Riemann geometry in curved Riemannian spacetime.

Such a joint symmetry of the action indicates that the dynamic equations of motion for the basic fields derived based on the least action principle all become covariance in biframe spacetime. Namely, the gravodynamics and spinodynamics as well as electrodynamics discussed in the previous sections should obey the general coordinate covariance in curved Riemannian spacetime and spin gauge covariance in locally flat gravigauge spacetime.

11.2 Gravigeometry-medium electrodynamics in any motional reference frame

Let us examine explicitly the general coordinate covariance and spin gauge covariance for the electrodynamics. The general coordinate covariance indicates that the dynamic equations of electromagnetic gauge field presented in eqs. (171)-(212) hold in any reference frame in the Einstein basis for the scaling gauge symmetry, i.e.,

$$\partial_\mu \tilde{F}^{\mu\nu}(x) = -\tilde{J}^{\nu}(x),$$

(269)

with

$$\tilde{F}^{\mu\nu}(x) = \tilde{\chi}^{\mu}(x)\chi^{\nu}(x)F^{\mu\nu}(x),$$

$$\tilde{F}^{\mu\nu}(x) = \tilde{\chi}^{\mu}(x)\tilde{\chi}^{\nu}(x)F^{\mu\nu}(x),$$

$$\tilde{J}^{\mu}(x) = \tilde{\chi}^{\mu}(x)\tilde{\chi}^{\rho}(x)\tilde{J}^{\rho}(x),$$

$$\tilde{J}^{\mu}(x) = -g_{\mu\nu}\tilde{\psi}(x)^\nu\psi(x),$$

(270)

where the relevant fields and vectors in arbitrary new reference frame obey the following transformation properties:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \rightarrow \partial'_{\mu} \equiv \frac{\partial}{\partial x'^\mu} = T^\mu_{\nu}\partial_\nu,$$

$$A_\mu(x) \rightarrow A'^\mu_\nu(x) = T^\mu_{\nu}A_\nu(x),$$

$$F^{\mu\nu}(x) \rightarrow F'^{\mu\nu}(x) = T^\mu_{\rho}T^\nu_{\sigma}F^{\rho\sigma}(x),$$

$$\tilde{\chi}^{\mu}(x) \rightarrow \tilde{\chi}'^{\mu}(x) = T^\nu_{\rho}\tilde{\chi}^{\rho}(x),$$

$$\tilde{\chi}^{\mu}(x) \equiv \eta^{\mu\nu}\tilde{\chi}^{\nu}(x)\tilde{\chi}^{\rho}(x) \rightarrow \tilde{\chi}'^{\mu}(x) = T^\nu_{\rho}\tilde{\chi}'^{\rho}(x).$$

(271)

It is noticed that the above dynamic equations possess a hidden gauge invariance.

To demonstrate the general coordinate covariance, it is useful to rewrite the equation of motion in terms of the covariant derivative. Explicitly, we have the following identity:

$$\partial_\nu \tilde{F}^{\nu\mu}(x) = \tilde{\chi}(x)\nabla_\mu \tilde{F}^{\nu\mu}(x)$$

$$\equiv \tilde{\chi}(x)[\partial_\nu \tilde{F}^{\nu\mu}(x) + \Gamma^\mu_{\nu\sigma}\tilde{F}^{\nu\sigma}(x) + \Gamma^\nu_{\nu\rho}\tilde{F}^{\rho\mu}(x)],$$

(272)
where we have used the following definition and property:

$$\tilde{F}^{\mu\nu}(x) \equiv \tilde{\chi}^{\mu\nu}(x)\tilde{x}_{\nu}^{\rho}(x)F_{\mu\rho}(x) \equiv \tilde{\chi}^{\mu\nu}(x)F_{\mu\rho}(x)$$

$$= -\tilde{F}^{\nu\mu}(x),$$

$$\Gamma^\nu_{\mu\rho} \equiv \Gamma^\mu_{\rho\nu}, \quad \Gamma^\nu_{\mu\rho} \equiv \partial_\nu \ln \chi(x),$$

$$\Gamma^\nu_{\mu\rho} \tilde{F}^{\rho\nu}(x) \equiv \frac{1}{2}(\Gamma^\mu_{\nu\rho} - \Gamma^\rho_{\nu\mu})\tilde{F}^{\nu\rho}(x) = 0,$$

$$\chi(x)\Gamma^\nu_{\mu\rho} \tilde{F}^{\rho\nu}(x) \equiv (\partial_\mu \chi(x))\tilde{F}^{\rho\nu}(x).$$

(273)

So the equations of motion in terms of the explicit covariant form can be expressed as follows:

$$\nabla_\nu \tilde{F}^{\mu\nu}(x) = -\tilde{J}^{\mu}(x),$$

(274)

with the current density,

$$\tilde{J}^{\mu}(x) \equiv \tilde{\chi}^{\mu\nu}(x)\tilde{x}_{\nu}^{\nu}(x)\tilde{\psi}(x)\gamma^\mu \psi(x).$$

(275)

Therefore, the gravigometry-medium Maxwell equations hold in any motional reference frame and can be expressed into the following forms:

$$\nabla' \cdot \tilde{E}' = \tilde{J}^0,$$

$$\nabla' \times \tilde{B}' = \tilde{J}' - \frac{\partial \tilde{E}'}{\partial \tilde{x}^0},$$

$$\nabla' \cdot \tilde{B}' = 0,$$

$$\nabla' \times \tilde{E}' = -\frac{\partial \tilde{B}'}{\partial \tilde{x}^0},$$

(276)

with

$$\tilde{E}'(x') \equiv \tilde{E}' \cdot \tilde{E}' + \tilde{\alpha}' \cdot \tilde{B}',$$

or

$$\tilde{E}'(x') \equiv \tilde{E}' \cdot \tilde{E}' + \tilde{\alpha}' \cdot \tilde{B}',$$

$$\tilde{B}'(x') \equiv \tilde{\mu}' \cdot \tilde{B}' - \tilde{\alpha}' \cdot \tilde{B}',$$

(277)

and

$$\tilde{\alpha}'(x') \equiv [\tilde{\alpha}'(x')],$$

or

$$\tilde{\alpha}'(x') \equiv [\tilde{\alpha}'(x')],$$

$$\tilde{\alpha}'(x') \equiv [\tilde{\alpha}'(x')],$$

(278)

where the coefficient functions are in general governed by the gravidynamics in the new reference frame.

11.3 Gravigauge-mediated electrodynamics in any spinning reference frame

We now turn to discuss the electrodynamics in locally flat gravigauge spacetime. The dynamic equation presented in eq. (221) is spin gauge covariance, which holds in any spinning reference frame in locally flat gravigauge spacetime, i.e.,

$$D'_\beta F'^{\alpha\beta}(x) = -\tilde{J}'^\alpha(x),$$

(279)

with

$$D'_\beta F'^{\alpha\beta}(x) \equiv \partial'_\beta F'^{\alpha\beta}(x) + \Omega'^{\alpha\beta}_{\mu\nu}(x)F'^{\mu\nu}(x)F'^{\alpha\beta}(x) - \Omega'^{\beta\alpha}_{\nu\rho}(x)F'^{\nu\rho}(x)F'^{\alpha\beta}(x),$$

$$F'^{\alpha\beta}(x) \equiv \eta'^{\alpha\beta} F'^{\alpha\beta}(x),$$

$$F'^{\alpha\beta}(x) \equiv \eta'^{\alpha\beta} F'^{\alpha\beta}(x),$$

$$F'^{\alpha\beta}(x) \equiv \Omega'^{\alpha\beta}_{\nu\rho}(x)F'^{\mu\nu}(x),$$

(280)

$$\psi(x) \rightarrow \psi'(x) = S(\Lambda)\psi(x), \quad S(\Lambda) = e^{i\pi_{\alpha\beta}(x)\gamma^{\alpha\beta}},$$

$$\tilde{\alpha}_c(x) \rightarrow \tilde{\alpha}'_c(x) \equiv \Lambda_{\alpha\beta}^c(x)(\Lambda^a_{\mu\nu}(x)\Lambda_{\beta\gamma}^b(x)\Omega^{\alpha\beta}_{\nu\rho}(x) + \frac{1}{2} \Lambda_{d\alpha}(x)\Lambda_{\alpha\beta}^d(x) - \Lambda_0^d(x)\partial_\gamma \Lambda_{\alpha\beta}^d(x)],$$

$$\tilde{\alpha}'_c(x) \rightarrow \tilde{\alpha}'_c(x),$$

$$\tilde{\alpha}'_c(x) \rightarrow \tilde{\alpha}'_c(x),$$

(281)

So the gravigauge-mediated Maxwell equations in eq. (235) are valid in any spinning reference frame, which can be expressed as follows:

$$\nabla' \cdot \tilde{E}' - \tilde{\alpha}' \cdot \tilde{B}' - \tilde{\psi}' \cdot \tilde{E}' = \tilde{J}^0 + \tilde{B}' \cdot \tilde{B}',$$

$$\nabla' \times \tilde{B}' - \tilde{\alpha}' \cdot \tilde{B}' + \tilde{\psi}' \cdot \tilde{B}' = \tilde{J}^0 + \tilde{\psi}' \cdot \tilde{E}',$$

$$\nabla' \times \tilde{E}' + \tilde{B}' \cdot \tilde{E}' = \tilde{J}^0 + \tilde{\psi}' \cdot \tilde{B}' - 2\tilde{\psi}' \cdot \tilde{B}' + 2\tilde{\psi}' \cdot \tilde{E}',$$

(282)

$$\nabla' \cdot \tilde{B}' + 2\tilde{\alpha}' \cdot \tilde{B}' + 2\tilde{\psi}' \cdot \tilde{B}' = \tilde{B}' \cdot \tilde{B}',$$

with

$$\tilde{E}' \equiv (\tilde{E}'_A - \tilde{\psi}'_E) - \tilde{E}'_E,$$

$$\tilde{B}' \equiv (\tilde{B}'_A - \tilde{\psi}'_B) - \tilde{B}'_B,$$

$$\tilde{E}' \equiv (\tilde{E}'_A - \tilde{\psi}'_E) - \tilde{E}'_E,$$

$$\tilde{B}' \equiv (\tilde{B}'_A - \tilde{\psi}'_B) - \tilde{B}'_B,$$

(283)

$$\tilde{\psi}'_A \equiv \tilde{\psi}'_A,$$

$$\tilde{\psi}'_B \equiv \tilde{\psi}'_B,$$

$$\tilde{\psi}'_E \equiv \tilde{\psi}'_E,$$

$$\tilde{\psi}'_B \equiv \tilde{\psi}'_B.$$
11.4 Gravigeometry-medium Maxwell equations in special background medium and general coordinate covariance of equations in motional reference frame

In general, either the gravigeometry-medium electric field \( \mathbf{E} \) or gravigeometry-medium magnetic field \( \mathbf{B} \) depends on both free-motion electric field \( \mathbf{E} \) and magnetic field \( \mathbf{B} \) defined in globally flat Minkowski spacetime. Only when the symmetric gravigermic field \( \hat{\chi}(\mathbf{x}) = \chi(\mathbf{x}) \) goes to be vanishing and brings the coefficient function \( \hat{\alpha}^i_j(x) \) to be zero, so that the gravigeometry-medium electric field \( \mathbf{E} \) or gravigeometry-medium magnetic field \( \mathbf{B} \) becomes to be proportional only to the electric field \( \mathbf{E} \) or magnetic field \( \mathbf{B} \). Explicitly, such a case can be expressed into the following relations:

\[
\hat{\chi}^0(\mathbf{x}) = \chi^0(\mathbf{x}) \rightarrow 0, \quad \hat{\alpha}^i_j(x) \rightarrow 0, \\
\mathbf{E} = \hat{\mathbf{E}}(\mathbf{x}) = \hat{\mathbf{E}}^i_j(\mathbf{x}) \mathbf{E}^j, \quad \mathbf{B} = \hat{\mathbf{B}}(\mathbf{x}) = \hat{\mathbf{B}}^i_j(\mathbf{x}) \mathbf{B}^j.
\]  

As a simple case, let us examine a special background medium in which the gravigermic field is supposed to have the following simple geometric distributions:

\[
\hat{\chi}^0 = \alpha_0^2 \eta^0, \quad \hat{\chi}^{ij} = \alpha_0^2 \eta^{ij}, \quad \hat{\chi}^0_i = 0,
\]  

with \( \alpha_0 \) and \( \alpha_s \) being constants. Such a special background medium is homogeneous and isotropic in three spatial dimensions. The global scaling invariance of the action enables us to make the following scale transformations for the gravigermic field and coordinates:

\[
\hat{\chi}^\mu_a = \alpha_s \chi^\mu_a, \quad \hat{\chi}^{\nu\sigma} = \alpha_s^2 \chi^{\nu\sigma}, \\
\chi^{\mu}_{\nu} = \alpha_s^{-1} \chi^{\mu}_{\nu}, \quad \chi^{\nu\sigma} = \alpha_s^{-2} \chi^{\nu\sigma}, \\
\chi^\mu = \alpha_s^{-1} \chi^\mu,
\]  

which simplifies the gravigermic distribution into the following form:

\[
\hat{\chi}^0 = c_s^{-2} \eta^0, \quad \hat{\chi}^{ij} = \eta^{ij}, \quad \hat{\chi}^0_i = 0,
\]  

with

\[
c_s \equiv \frac{\alpha_s}{\alpha_0},
\]  

which indicates that the basic feature of such a special background medium is solely charactrized by the constant ratio \( c_s \).

So the corresponding gravigeometry-medium electric field \( \mathbf{E} \) and gravigeometry-medium magnetic field \( \mathbf{B} \) get the following simple relations:

\[
\mathbf{E} = c_s \mathbf{E} \equiv \mathbf{D}, \\
\mathbf{B} = \frac{\mathbf{B}}{\mu_s} \equiv \mathbf{H},
\]  

with the constant coefficients:

\[
\varepsilon_s \equiv c_s^{-1}, \quad \mu_s \equiv c_s^{-1}, \quad \frac{1}{\sqrt{\varepsilon_s \mu_s}} = c_s.
\]  

The current density in such a special background medium has a simple form:

\[
\mathbf{j}^0 = J^0, \quad \hat{\mathbf{j}}^0 = -g_0 \hat{\psi}(\mathbf{x}) \gamma^0 \psi(\mathbf{x}), \\
\hat{\mathbf{j}}^i = c_s J^i, \quad \hat{j}^i = -g_0 \hat{\psi}(\mathbf{x}) \gamma^{i} \psi(\mathbf{x}),
\]  

with \( J^0 \) and \( J^i \) being the free charge density and current density, respectively. From the above analyses and the general gravigeometry-medium Maxwell equations, we arrive at the following Maxwell equations in such a special background medium:

\[
\nabla \cdot \mathbf{D} = J^0, \\
\nabla \times \mathbf{H} = \frac{1}{\sqrt{\varepsilon_s \mu_s}} \mathbf{J} + \frac{\partial \mathbf{D}}{\partial \tilde{x}^0}, \\
\nabla \cdot \mathbf{B} = 0, \\
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \tilde{x}^0},
\]  

which can be expressed as the following forms:

\[
\nabla \cdot \mathbf{E} = c_s J^0, \\
\nabla \times \mathbf{B} = \mathbf{J} + \frac{1}{c_s^2} \frac{\partial \mathbf{E}}{\partial \tilde{x}^0}, \\
\nabla \cdot \mathbf{B} = 0, \\
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \tilde{x}^0},
\]  

where the electromagnetic fields \( \mathbf{E} \) and \( \mathbf{B} \) as well as the gravigeometry-medium electromagnetic fields \( \mathbf{E} \equiv \mathbf{D} \) and \( \mathbf{B} \equiv \mathbf{H} \) in the special background medium all have the same unit since the ratio \( c_s \) is a dimensionless constant in the present definition.

Let us now examine the Lorentz covariance of gravigeometry-medium Maxwell equations under the global Lorentz transformation of group symmetry SO(1,3) in globally flat Minkowski spacetime, i.e.,

\[
x^\mu \rightarrow x'^\mu = L^\mu_{\nu} x^\nu, \quad \mathbf{A}_\mu(x) \rightarrow A'_\mu(x') = L_\mu^\nu \mathbf{A}_\nu(x), \\
\hat{\chi}^{\nu\sigma}(x) \rightarrow \hat{\chi}'^{\nu\sigma}(x') = L_\mu^\nu L_\sigma^\rho \hat{\chi}^{\rho\nu}(x).
\]  

For simplicity, we should consider the gravigeometry-medium Maxwell equations in special background medium under the following global Lorentz boost transformation \( L^\nu_{\nu'} \):

\[
\hat{\mathbf{L}}^{00} = \gamma - 1, \quad (\hat{\mathbf{L}})^{00} = -\gamma \beta^0 = (\hat{\mathbf{L}})^{00}, \\
\hat{\mathbf{L}}^{ij} = (\gamma - 1) \frac{\beta^i \beta^j}{\beta^2} = (\hat{\mathbf{L}})^{ij}, \quad \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}, \\
\mathbf{\beta} \equiv (\beta^1, \beta^2, \beta^3), \quad \beta^2 \equiv \mathbf{\beta} \cdot \mathbf{\beta} \equiv \beta_1^2 + \beta_2^2 + \beta_3^2,
\]  

where \( \mathbf{\beta} \) is regarded as a boost vector.
To obtain the gravigeometry-medium electromagnetic field in the boosted new coordinate reference frame, it is useful to present the gravimetric of the special background medium in the boosted new coordinate reference frame as follows:

\[ \hat{\mathbf{x}}^{	ext{00}} = \gamma^2 c^{-2} (1 - \beta^2 c^2), \]
\[ \hat{\mathbf{x}}^0 = \hat{\mathbf{x}}^0 = -\gamma^2 c^{-2} (1 - c^2 \beta^2), \]
\[ \hat{\mathbf{x}}^{\alpha\beta} = \eta^{\alpha\beta} + \gamma^2 c^{-2} (1 - c^2 \beta^2) \beta^\alpha \beta^\beta, \]

which enables us to figure out from eqs. (276) and (278) the following gravigeometry-medium electromagnetic field in the boosted new coordinate reference frame:

\[ \dot{\mathbf{E}}'(x') = \gamma^2 (1 - \beta^2 c^2) \epsilon^{-1} \mathbf{E}'(x') - \gamma^2 (c^2 - 1) c_\alpha \beta \times \mathbf{B}'(x') \]
\[ - \gamma^2 (1 - c^2 \beta^2) \epsilon / \mu_s \times \mathbf{B}'(x'), \]
\[ \dot{\mathbf{B}}'(x') = \left[ 1 - 2 \gamma^2 \beta^2 (c^2 - 1) \right] c_\alpha \mathbf{B}'(x') \]
\[ - \gamma^2 c_\alpha \beta \cdot \mathbf{E}'(x') \]
\[ + 2 \gamma^2 (c^2 - 1) c_\alpha \beta \cdot \mathbf{B}'(x') \beta, \]

which may be rewritten into the following forms:

\[ \mathbf{D}'(x') \equiv \dot{\mathbf{E}}'(x') = \gamma^2 \left( 1 - \beta^2 / \epsilon / \mu_s \right) \epsilon \mathbf{E}'(x') \]
\[ - \gamma^2 (c_\alpha \mu_s - 1) \epsilon / \mu_s \times \mathbf{B}'(x') \]
\[ - \gamma^2 (1 - 1 / \epsilon / \mu_s) \epsilon / \mu_s \times \mathbf{E}'(x') \beta, \]
\[ \mathbf{H}'(x') \equiv \dot{\mathbf{B}}'(x') = \left[ 1 - 2 \gamma^2 \beta^2 (c_\alpha \mu_s - 1) \right] \epsilon / \mu_s \mathbf{B}'(x') \]
\[ - \gamma^2 (1 - 1 / \epsilon / \mu_s) \epsilon / \mu_s \times \mathbf{E}'(x') \]
\[ + 2 \gamma^2 (1 - 1 / \epsilon / \mu_s) \epsilon / \mu_s \times \mathbf{B}'(x') \beta. \]

Correspondingly, the current density in the boosted new coordinate reference frame is found to be

\[ \mathcal{J}'(0)(x') = \gamma \left[ \mathcal{J}^0(0)(x') - c_\alpha \beta \cdot \mathbf{J}(x') \right] \]
\[ = \gamma \left[ \mathcal{J}^0(0)(x') - \frac{1}{\sqrt{\epsilon / \mu_s}} \beta \cdot \mathbf{J}(x') \right] \]
\[ \mathcal{J}'(x') = c_\alpha \mathbf{J}(x') - \gamma \mathcal{J}^0(0)(x') \beta + (\gamma - 1) c_\alpha \beta \cdot \mathbf{J}(x') \beta \]
\[ = \frac{1}{\sqrt{\epsilon / \mu_s}} \mathbf{J}(x') - \gamma \mathcal{J}^0(0)(x') \beta \]
\[ + (\gamma - 1) \frac{1}{\sqrt{\epsilon / \mu_s}} \beta \cdot \mathbf{J}(x') \beta. \]

For the case \( \beta^2 \ll 1 \), when keeping only the leading term of \( \beta_i \) with \( \gamma \approx 1 \), we arrive at the following simplified relations:

\[ \mathbf{D}'(x') \equiv \dot{\mathbf{E}}'(x') \approx c_\alpha \mathbf{E}'(x') - (c_\alpha \mu_s - 1) \frac{1}{\mu_s} \beta \times \mathbf{B}'(x'), \]
\[ \mathbf{H}'(x') \equiv \dot{\mathbf{B}}'(x') \approx \frac{1}{\mu_s} \mathbf{B}'(x') - \frac{1}{\epsilon / \mu_s} \epsilon \beta \times \mathbf{E}'(x'), \]

and

\[ \mathcal{J}'^0(0)(x') \approx \mathcal{J}^0(0)(x') - \frac{1}{\sqrt{\epsilon / \mu_s}} \beta \cdot \mathbf{J}(x'), \]
\[ \mathcal{J}'(x') \approx \frac{1}{\sqrt{\epsilon / \mu_s}} \mathbf{J}(x') - \mathcal{J}^0(0)(x') \beta, \]

where \( \mathcal{J}^0(0)(x') \) and \( \mathbf{J}(x') \) are free current density. It can be verified explicitly that the gravigeometry-medium electromagnetic field in the boosted new coordinate reference frame does satisfy the gravigeometry-medium Maxwell equations obeying the general coordinate covariance, i.e.,

\[ \mathbf{V}' \cdot \mathbf{D}' = \mathcal{J}'^0, \]
\[ \mathbf{V}' \times \mathbf{H}' = \mathbf{J}' - \frac{\partial \mathbf{D}'}{\partial x'^0}, \]
\[ \mathbf{V}' \cdot \mathbf{B}' = 0, \]
\[ \mathbf{V}' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial x'^0}. \]

In conclusion, we have demonstrated how the gravigeometry-medium Maxwell equations in the special background medium obey the general coordinate covariance by explicitly making the global Lorentz boost transformation into a new coordinate system with a constant boost vector \( \beta \). Actually, when the boost vector \( \beta \) becomes local \( \beta = \beta(x') \), the gravigeometry-medium Maxwell equations in such a boosted new reference frame remain valid. This is because the above derivation does not depend on whether the Lorentz boost transformation is global or local. In general, the gravigeometry-medium Maxwell equations for the gravigeometry-medium electromagnetic field hold in any motional coordinate reference frame due to the general coordinate covariance of electrodynamics.

### 11.5 Gravigauge-mediated electrodynamics in special background medium and spin gauge covariance of generalized Maxwell equations in any spinning reference frame

To show the spin effect arising from the interaction of spin gravigauge field \( \Omega^{\mu\nu}(x) \), it is useful to examine the gravigauge-mediated Maxwell equations in the special background medium and demonstrate explicitly the spin gauge covariance under the spin boost transformation of spin gauge symmetry SP(1,3) in locally flat gravigauge spacetime.

From the simple gravimetric form of special background medium shown in eq. (287), the corresponding dual gravigauge field \( \hat{\chi}^i_a \) is simply given by

\[ \hat{\chi}^0_0 = c_1 \epsilon \eta_0^0, \quad \hat{\chi}^i_0 = 0 = \hat{\chi}^0_a, \quad \hat{\chi}^i_a = \eta_1^a, \]

with \( \alpha = (0, \alpha) \) and \( \mu = (0, i) \) \( (a, i = 1, 2, 3) \). In this simple case, the background gravigauge field strength vanishes, and
the gravigauge-dressed electromagnetic field gets the following simple forms:
\[ \tilde{E} = c_s^{-1} E, \quad \tilde{B} = B. \] (304)

The gravigauge Maxwell equations presented in eq. (235) are simplified to be
\[ \tilde{\nabla} \cdot \tilde{E} = j^0, \quad \tilde{\nabla} \times \tilde{B} = \mathbf{J} + \partial_0 \tilde{E}, \quad \tilde{\nabla} \times \tilde{E} = -\partial_0 \tilde{B}, \quad \tilde{\nabla} \cdot \tilde{B} = 0, \] (305)
where the gravicoordinate derivatives in such a special background medium are given by
\[ \tilde{\nabla} = \nabla, \quad \partial_0 = c_s^{-1} \partial_0 \equiv c_s^{-1} \frac{\partial}{\partial x^0}. \] (306)

Let us now consider the following spin gauge transformation in locally flat gravigauge spacetime:
\[ \chi_a^{(\prime)}(x) \rightarrow \tilde{\chi}_a^{(\prime)}(x) = \Lambda_a^{\prime b}(x) \tilde{\chi}_b^{(\prime)}(x), \quad \delta_a \rightarrow \tilde{\delta}_a^{(\prime)} = \Lambda_a^{\prime b}(x) \delta_b, \] (307)
\[ F^{ab}(x) \rightarrow F^{\prime ab}(x) = \Lambda_a^{\prime b}(x) \Lambda_b^{\prime c}(x) F^{cd}(x), \quad J^a(x) \rightarrow J^{\prime a}(x) = \Lambda_a^{\prime c}(x) J^c(x), \]
with the spin boost transformation given explicitly as follows:
\[ \Lambda_a^{\prime b}(x) \equiv \eta_a^{\prime b} + \Lambda_a^{\prime b}(x), \]
\[ (\tilde{\Lambda})_{\alpha 0} = \cosh \sigma - 1, \quad (\tilde{\Lambda})_{\alpha 0} = -c_s \sinh \sigma = (\Lambda)_{\alpha 0}, \] (308)
\[ \tilde{\zeta} \equiv (\tilde{\zeta}_1, \tilde{\zeta}_2, \tilde{\zeta}_3), \quad \tilde{\zeta}^2 \equiv \tilde{\zeta}_1^2 + \tilde{\zeta}_2^2 + \tilde{\zeta}_3^2 = 1, \quad \sigma \equiv \sigma(x), \quad \zeta_a \equiv \zeta_a(x), \quad \zeta \equiv \zeta(x). \]
The gravigauge-dressed electromagnetic field in the spin boosted new reference frame has the following relations:
\[ \tilde{E}^{\prime}(x) = \cosh \sigma \tilde{E} + \sinh \sigma \tilde{\zeta} \times \tilde{B} - (\cosh \sigma - 1) \tilde{\zeta} \cdot \tilde{E} \tilde{\zeta}, \]
\[ \tilde{B}^{\prime}(x) = (1 - 2 \cosh (\sigma - 1)) \tilde{B} - \sinh \sigma \tilde{\zeta} \times \tilde{E} - 2 \cosh (\sigma - 1) \tilde{\zeta} \cdot \tilde{B} \tilde{\zeta}. \] (309)
The corresponding relations for the current density and gravicoordinate derivative are found to be
\[ j^0(x) = \cosh \sigma j^0 - \sinh \sigma \tilde{\zeta} \cdot \mathbf{J}, \quad j^1(x) = J^1 + [-j^0 \sinh \sigma + (cosh - 1)\tilde{\zeta} \cdot \mathbf{J}] \tilde{\zeta}, \] (310)
and
\[ \tilde{\nabla} \tilde{\zeta} = \tilde{\nabla} - \tilde{\zeta} \sinh \sigma \tilde{\nabla} \tilde{\zeta} = \nabla - c_s^{-1} \sinh \sigma \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \tilde{\zeta} \cdot \nabla \right), \]
\[ \delta_0^{\prime} = \cosh \sigma \delta_0 - \sinh \sigma \tilde{\zeta} \cdot \tilde{\nabla} \tilde{\zeta} = c_s^{-1} \cosh \sigma \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \tilde{\zeta} \cdot \nabla \right). \] (311)

It can be verified that in the spin boosted new reference frame, the gravigauge field strength in the special background medium becomes no vanishing and gets the following form:
\[ F_{\alpha \beta}^{\prime}(x) = -\Lambda_a^{\prime \alpha}(\partial_x \Lambda_b^{\prime \beta} - \partial_y \Lambda_c^{\prime \beta}) \equiv -\Omega_{\alpha \beta}^{\prime}(x), \] (312)
which results from a pure spin gauge field $\Omega_{\alpha \beta}^{\prime}(x)$ after the spin gauge transformation $\Lambda_a^{\prime b}(x)$. To be more explicit, the electromagnetic-like gravigauge fields are given as follows:
\[ \tilde{E}_a^{\prime} = -[\delta_0 \sigma + \sinh \sigma (\tilde{\zeta}_0 \cdot \tilde{\zeta})] \zeta_a - \sinh \sigma [\delta_0 \zeta_a + \sinh \sigma (\tilde{\zeta}_0 \cdot \tilde{\zeta})] \cdot \zeta' \]
\[ \tilde{B}_a^{\prime} = -c_s^{-1} \sinh \sigma (\frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \tilde{\zeta} \cdot \nabla \tilde{\zeta}). \]

The gauge covariance of gravigauge-mediated Maxwell equations in the special background medium can be expressed into the following forms in the spin boosted new reference frame:
\[ \nabla \cdot \tilde{E} - c_s^{-1} \sinh \sigma \tilde{\zeta} \cdot \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \tilde{\zeta} \cdot \nabla \right) \tilde{E}' = 0, \quad \tilde{\chi}_a^{\prime \beta}(x) \rightarrow \tilde{\chi}_a^{\prime \beta}(x) = \Lambda_a^{\prime b}(x) \tilde{\chi}_b^{\prime \beta}(x), \]
\[ (\tilde{\Lambda})_{\alpha 0} = \cosh \sigma - 1, \quad (\tilde{\Lambda})_{\alpha 0} = -c_s \sinh \sigma = (\Lambda)_{\alpha 0}, \] (308)
\[ \tilde{\zeta} \equiv (\tilde{\zeta}_1, \tilde{\zeta}_2, \tilde{\zeta}_3), \quad \tilde{\zeta}^2 \equiv \tilde{\zeta}_1^2 + \tilde{\zeta}_2^2 + \tilde{\zeta}_3^2 = 1, \quad \sigma \equiv \sigma(x), \quad \zeta_a \equiv \zeta_a(x), \quad \zeta \equiv \zeta(x). \]
\[ \nabla \times \mathbf{B} - c_s^{-1} \sinh \sigma \mathbf{s} \times \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{B} \]
\[ - \dot{A}' \times \mathbf{B}' + \mathbf{B} \cdot \nabla \mathbf{B}' \]
\[ = J' + c_s^{-1} \cosh \sigma \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{E}' \]
\[ - \dot{A}_0 \mathbf{E}' + \mathbf{B}' \cdot \mathbf{E}', \]
\[ \nabla \cdot \mathbf{E}' - c_s^{-1} \sigma \mathbf{s} \cdot \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{B}' \]
\[ + 2 \dot{A}' \cdot \mathbf{B}' + 2 \mathbf{E}' \cdot \mathbf{B}' = \mathbf{B} \cdot \mathbf{E}', \]
\[ \nabla \times \mathbf{E}' - c_s^{-1} \mathbf{s} \times \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{B}' \]
\[ + \mathbf{E}' \times \mathbf{E}' - \mathbf{B}' \cdot \mathbf{E}', \]
\[ = -c_s^{-1} \cosh \sigma \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{B}' \]
\[ - 2 \dot{A}_0 \mathbf{B}' + \mathbf{B}' \cdot \nabla \mathbf{B}', \]

where both the gravitgage-dressed electromagnetic fields and electromagnetic-like gravigauge fields in the gravigauge-mediated Maxwell equations are located at the same point \( x^0 \) in the coordinate spacetime as the spin gauge transformation operates on the locally flat gravigauge spacetime and Dirac spinor field in Hilbert space.

For a special case that the spin transformation is taken to be a global transformation, namely the boost parameters \( \sigma \) and \( \varsigma_a \) are all constant parameters, the electromagnetic-like gravigauge fields get to be vanishing. As a consequence, the above gravigauge-mediated Maxwell equations in the globally spin boosted new reference frame are simplified to be

\[ \nabla \cdot \mathbf{E}' - c_s^{-1} \sinh \sigma \mathbf{s} \times \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{E}' = J^0, \]
\[ \nabla \times \mathbf{B}' - c_s^{-1} \mathbf{s} \times \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{E}' \times \mathbf{B}' \]
\[ = J' + c_s^{-1} \cosh \sigma \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{E}', \]
\[ \nabla \cdot \mathbf{B}' - c_s^{-1} \mathbf{s} \cdot \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{E}' \cdot \mathbf{B}' = 0, \]
\[ \nabla \times \mathbf{E}' - c_s^{-1} \mathbf{s} \times \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{E}' \times \mathbf{B}' \]
\[ = -c_s^{-1} \cosh \sigma \left( \frac{\partial}{\partial x^0} - c_s \tanh \frac{\sigma}{2} \mathbf{s} \cdot \nabla \right) \mathbf{B}'. \]

Therefore, we come to the observation that the gravigauge-mediated Maxwell equations for gravigauge-dressed electromagnetic fields in locally flat gravigauge spacetime maintain the spin gauge covariance with hidden general coordinate invariance, and the gravigauge-medium Maxwell equations for gravigauge-medium electromagnetic fields in coordinate spacetime keep the general covariance of coordinates with hidden spin gauge invariance.

### 12 Discussion and conclusions

In this paper, we have begun from the fact that the electron and charged leptons and quarks as basic constituents of matter in SM are actually Weyl fermions although they appear as Dirac fermions in electromagnetic interaction. This is because their weak interactions in SM produce the maximal parity violation. Based on such an observation, we have represented such a Dirac fermion in four-dimensional Hilbert space as the superposition of left-handed and right-handed Weyl fermions and treated it as a chirality-based Dirac spinor in the chiral spinor representation of eight-dimensional Hilbert space. We have demonstrated that such a chirality-based Dirac spinor possesses the inhomogeneous spin symmetry \( \text{WS}(1,3) \) in eight-dimensional Hilbert space instead of the usual spin symmetry \( \text{SP}(1,3) \) in four-dimensional Hilbert space. Such an inhomogeneous spin symmetry group \( \text{WS}(1,3) \) is a semi-direct product group \( \text{WS}(1,3)=\text{SP}(1,3)\rtimes W_{1,3} \) with \( \text{SP}(1,3) \) being the spin group and \( W_{1,3} \) referred to as \( W_c \)-spin group, which realizes an internal Poincaré-type group (or internal inhomogeneous Lorentz-type group) in eight-dimensional Hilbert space. It has been shown that the \( W_c \)-spin symmetry group emerges as a translation-like chiral-type spin symmetry group and the electric charge symmetry group \( U(1) \) appears as an internal Abelian group symmetry for the chirality-based Dirac spinor formulated in eight-dimensional Hilbert space.

Following along the gauge invariance principle which states that the laws of nature should be independent of the choice of local field configurations, we have gauged the inhomogeneous spin symmetry \( \text{WS}(1,3) \) and electric charge symmetry \( U(1) \) for the chirality-based Dirac spinor in eight-dimensional Hilbert space to be local symmetries. Meanwhile, the inhomogeneous Lorentz group symmetry (or Poincaré group symmetry) \( \text{PO}(1,3) \) remains to be global symmetry in four-dimensional Minkowski spacetime of coordinates. The inhomogeneous spin gauge symmetry \( \text{WS}(1,3) \) as well as electric charge gauge symmetry \( U(1) \) lead to the introduction of spin gauge field and \( W_c \)-spin gauge field as well as electromagnetic gauge field. It has been demonstrated that the translation-like \( W_c \)-spin invariant-gauge field provides a gravigauge field as a basic gravitational field, which reveals the nature of gravity and brings on the gravitational origin of spin gauge symmetry. So such an inhomogeneous spin gauge symmetry \( \text{WS}(1,3) \) naturally leads to an internal Poincaré-type gauge symmetry. Furthermore, based on the displacement correspondence between the gravivector field in Hilbert space and coordinate vector in Minkowski spacetime through the gravigauge field, a locally flat gravigauge spacetime spanned by the gravigauge basis emerges to form a fiber bundle structure of biframe spacetime, where the glob-
ally flat Minkowski spacetime appears as base spacetime and the locally flat gravigauge spacetime behaves as a fiber. By noticing the intriguing property that the gravigauge field plays a role as a Goldstone-like boson, we are able to define all gauge fields and field strengths in locally flat gravigauge spacetime, where the gravitational effects are shown to be characterized by the emergent non-commutative geometry of locally flat gravigauge spacetime.

Based on the action principle of path integral formulation, we have presented various formalisms of gauge and scaling invariant actions for the chirality-based Dirac spinor in eight-dimensional Hilbert space, which has been shown to possess the maximal joint symmetry $PO(1,3)\times WS(1,3)\times U(1)$ and generate a hidden general linear group symmetry $GL(1,3,R)$. It has been verified that when the basic symmetry of chirality-based Dirac spinor in eight-dimensional Hilbert space is gauged to be local symmetry via the gauge invariance principle, the symmetry of coordinate system automatically turns out to possess a hidden general linear group symmetry $GL(1,3,R)$, which indicates that the laws of nature should be independent of the choice of coordinate systems and meanwhile brings on the genesis of Riemann geometry in curved Riemannian spacetime. Such a feature enables us to demonstrate the gauge-gravity and gravity-geometry correspondences and obtain the gauge-geometry duality relation within the framework of gravitational quantum field theory.

From the least action principle, we have derived the generalized Dirac equation to characterize the gravitational relativistic quantum theory. In particular, the quadratic gravigauge Dirac equation is derived in locally flat gravigauge spacetime to show the gravitational effect. The gauge-type gravitational equations of gravigauge field have been obtained to describe the gravidynamics in biframe spacetime and also in locally flat gravigauge spacetime. Meanwhile, we have investigated in detail the geometric gravidynamics based on the inhomogeneous spin gauge symmetry and discussed its new effects beyond the Einstein theory of general relativity. The equation of motion for the spin gauge field has also been derived to describe the spinodynamics.

In particular, we have made a detailed analysis on the various formalisms of electrodynamics via deriving different generalized Maxwell equations in the presence of gravitational interactions, such as the gravimegometry-medium electrodynamics by redarding the gravimetric field as the medium, the gravimegometry-mediated electrodynamics in locally flat gravigauge spacetime and the gravimetric-gauge-mediated electrodynamics in dynamic Riemannian spacetime. In analyzing the gravimetric-gauge-mediated Maxwell equations, we have shown that it is useful to introduce the electric-like and magnetic-like gravimetric-gauge fields. In general, we have explicitly demonstrated that the dynamics of all basic fields should maintain the general coordinate covariance in curved Riemannian spacetime and the spin gauge covariance in locally flat gravigauge spacetime, so the gravimegometry-medium Maxwell equations hold in any motional reference frame and the gravimegometry-mediated Maxwell equations validate in any spinning reference frame. We have also presented a full discussion on the generalized Maxwell equations in special background medium and verified explicitly the general coordinate covariance in motional reference frame and the spin gauge covariance in spinning reference frame.

It is straightforward to extend the present analyses to the standard model by including the gravitational interaction, which enables us to provide a unified description on four basic forces, i.e., electromagnetic, weak, strong and gravitational interactions, within the framework of gravitational quantum field theory. Meanwhile, such an extended standard model with inhomogeneous spin gauge symmetry is expected to result from a more fundamental theory, such as the hyperunified field theory [55-59] which is found to be governed by inhomogeneous hyperspin gauge symmetry $WS(1,18)$ and inhomogeneous Lorentz-type symmetry (or Poincaré-type symmetry) $PO(1,18)$ in 19-dimensional hyperspacetime. In particular, the foundation of the hyperunified field theory [58-60] has turned out to be laid by two simple guiding principles, i.e., maximum locally entangled-qubits motion principle and gauge invariance principle.

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