Analytical Investigation of In-plane Focusing Surface Plasmon Modes by a Dielectric Lens

Fahimeh Armin, Mir Mojtaba Mirsalehi
Department of Electrical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad 9177948974, Iran
E-mail: mirsalehi@um.ac.ir

Abstract. A dielectric lens placed on a metal-dielectric interface can be used for in-plane focusing of surface plasmon (SP) modes. The propagation behind the lens is usually studied using scalar diffraction theory which is not accurate in nanometer regime and cannot explain the propagation and energy flow of SP modes. Here, we have used vectorial derivation of Huygens-Fresnel principle to calculate the diffracted fields behind the lens in order to study the energy flow of SP modes in a focusing system using the concept of higher order modes.

1. Introduction

Local oscillations of conduction electrons at the interface of a metal and a dielectric will form surface plasmons (SPs), an explanation for what Wood observed in 1902 but was not understood till the publication of Ritchi in 1957 [1]. Since then, plasmonic surface waves have found their ways through various fields, from communications to biosensing [2, 3]. In each field, many interesting structures have been studied and proposed in papers.

In some cases, the use of SPs has made it possible to redesign classical optical components and devices in nanoscale. Zia and Brongersma have simulated the Young double slit experiment by plasmon waves [4] while Hohenau et al. have experimentally studied the transmission of SPs through prisms and lenses made of SiO$_2$ [5]. Shi et al. studied dielectric components made of Al$_2$O$_3$ to manipulate in-plane propagation of surface plasmon modes resulted from coupling of light by a single slit on a silver-air interface [6]. Some papers like the work presented by Bezuse et al. studied the transmission of surface plasmons from diffractive optical components in more details by calculating the phase modulation of these modes using rigorous coupled wave analysis [7].

The aforementioned papers show that it is possible to use dielectric components for guiding and shaping SPs like other electromagnetic waves. However, there are more assumptions to be considered in this case, like the propagation medium which is not loss free. Moreover, SPs are $p$-polarized but inhomogeneous plane waves [8] that means the real and imaginary parts of wavevector, $k$, are not parallel to each other. While analyzing the problem of diffraction from a dielectric lens or other optical components, many papers consider the excitation of scattering or radiation modes behind the dielectric components in which a part of the energy is decoupled from the surface plasmon mode. The concept of excitation of other mode was also studied in other inhomogeneities. For example, Outlan et al. used Gaussian quadrature method to solve the
mode matching integrals in an open waveguide structure to obtain the transmission, reflection, and also scattering coefficients at the boundary [9].

Kim et al. have addressed the issue of losses of the so called scattering modes by adding an air layer between the dielectric lens and metal interface [10]. Bezus et al. used the same solution but with a dielectric layer in which optimizing the height of the layer could significantly decrease the losses.

In this paper, we have studied the transmission and diffraction of surface plasmon modes by a dielectric lens using a vectorial Huygens-Fresnel principal to calculate the diffracted fields and then compared them to higher order plasmonic modes. The rest of the paper is organized as follows. Section 2 is devoted to describe the vectorial Huygens-Fresnel method used for calculation of diffracted fields. The argument of higher order plasmonic modes behind a dielectric lens is presented in Section 3 along with the study of losses imposed by these modes. Finally, Section 4 is dedicated to conclusions.

2. Calculations of diffracted fields

For a surface plasmon mode traveling on a metal-dielectric interface, the wavevector in dielectric medium, in its general form, can be written as

\[ k_{sp} = k_x a_x + k_y a_y + k_z a_z = \alpha_d a_x + i\beta \sin \theta a_y + i\beta \cos \theta a_z, \]  

(1)

where \( \theta \) is the angle of the wavevector with the z axis, providing the metal-dielectric interface is in the yz plane. Also, \( \alpha_d \) and \( \beta \), resulted from the solution of Helmholtz wave equation on the metal-dielectric interface, are

\[ \beta = k_0 \sqrt{\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d}}, \]  

(2)

\[ k_x = \alpha_d = \sqrt{\beta^2 - \epsilon_d k_0^2}. \]  

(3)

In these equations \( k_0 \) is the wavenumber of light in free space and \( \epsilon_d \) and \( \epsilon_m \) are the dielectric constants of dielectric and metal, respectively. Complex dielectric constant of the metal at optical wavelengths results in a complex wavevector for surface plasmon mode and, therefore, constant amplitude and constant phase planes which are not parallel to each other. This is the characteristic of an inhomogeneous plane wave.

Figures 1(a) and 1(b) schematically show the electric and magnetic field components before reaching a dielectric lens. The boundary conditions of the metal-dielectric interface eliminate the existence of transverse electric (TE) modes and therefore only transverse magnetic or TM modes are considered here. As it is assumed in Fig. 1 the surface plasmon mode travels parallel to the optical axis before reaching the lens on its front plane, \( U \) and thus \( \theta = 0 \) in Eq. (1). Dielectric lenses in plasmonic regime cannot be considered as thin lenses. As a result, the phase change that is imposed by the transmission of mode through the lens should be considered in calculation of the diffracted fields behind it.

Figure 1(c) demonstrates the schematic drawing of a thick lens and a ray passing through the lens from the front plane (\( U \)) to the back plane (\( U' \)). The total phase change, \( \Delta \phi \), can be found by calculating the phase change between points \( P_1 \) and \( P_3 \),

\[ \Delta \phi = \beta \Delta z_1 + \beta' \frac{\Delta z_2}{\cos \theta_1} + \beta \frac{\Delta z_3}{\cos \theta_2}. \]  

(4)

As mentioned before, the propagation constant of the surface plasmon mode on the metal-dielectric interface before and after passing through the lens is \( \beta \), but inside the lens, the value
of the propagation constant changes to $\beta^l$, $\Delta z_j$, where $j = 1, 2$ or $3$, denotes the distance along the $z$ axis between successive points $P_j$ and $P_{j+1}$. The angles $\theta_1$ and $\theta_2$ are obtained by using the Snell’s law at the curved interfaces of the lens [11]

$$\theta_1 = \sin^{-1}\left(\frac{y_0}{R_1}\right) + \sin^{-1}\left(\frac{\beta y_0}{\beta^1 R_1}\right),$$

$$\theta_2 = \sin^{-1}\left(\frac{\beta}{\beta^l} \sin(\theta_1 - \sin^{-1}\left(\frac{y_3}{R_2}\right))\right) + \sin^{-1}\left(\frac{y_3}{R_2}\right),$$

where $y_0$ and $y_3$ are the $y$ coordinates of $P_0$ and $P_3$, respectively. Using these equations, the phase delay at the backplane of the lens is obtained as a function of the height from the optical axis.

A vectorial derivation for the diffraction of surface plasmon modes is provided in [12] where the authors have used the surface wave expansion with complex wavevectors. Based on this derivation, they have presented the vectorial Huygens-Fresnel principle for the propagation of surface plasmons on a metal-dielectric interface. Their result is used here to study the propagation of surface plasmon modes behind the dielectric thick lens. After passing through the lens, the surface plasmon field in the dielectric ($x > 0$) can be represented as

$$E(y, z) = \frac{1}{2\pi} \int E(k_y) e^{i\sqrt{\beta^2 - k_y^2} y} e^{ik_y \Delta_0} dk_y.$$  

(7)

Using the concept of the Fourier transform and convolution integral, this equation can be replaced by

$$E(y, z) = \int E_{U', x}(y', z = \frac{\Delta_0}{2}) G(y - y', z) dy',$$

(8)
where
\[ E_{U',x}(y', z = \Delta_0/2) = E_{0x} e^{-i\Delta \phi(y')}, \] (9)
and \( E_{0x} \) is the amplitude of the surface plasmon in the \( x \) direction at the front plane. The other components of the electric field of the TM wave can be calculated using Maxwell’s equations. \( \Delta \phi(y') \) is the phase delay imposed by the lens as a function of the distance from the optical axis on the backplane. Also, in Eq. (8), \( G \) can be calculated as \[ G(y - y', z) = -\frac{1}{2} \left( \frac{\partial}{\partial z} a_x + \frac{k_x}{\beta^2} \frac{\partial^2}{\partial z \partial y} a_y + \frac{k_x}{\beta^2} \frac{\partial^2}{\partial z^2} a_z \right) H_0^{(1)} \left( \beta \sqrt{z^2 + (y - y')^2} \right). \] (10)

Using the equations provided, the diffraction of a surface plasmon mode is calculated as it passes through a convex lens. The normalized magnitude of the electric fields behind the lens is shown in Fig. 1(d) which is in consistence with numerical simulations. The lens is assumed to have similar radius of curvature on both sides equal to \( 5 \mu m \) and a thickness of \( \Delta_0 = 1.75 \mu m \). The lens is made of PMMA with a refractive index of 1.48 and it is placed on the surface of silver. The calculations are done at the wavelength of 780 nm.

3. Higher order modes behind a dielectric lens

In solving the Helmholtz wave equations on a metal dielectric interface (the \( yz \) plane in Fig. 1) the same trend as slab waveguides is usually used and the changes in the \( y \) direction are ignored. Hence the assumption of \( \partial / \partial y = 0 \) results in only one TM polarized mode which is the fundamental mode of the structure, \( TM_0 \). However, without this uniformity assumption, Helmholtz equations reveal a new set of surface modes. Assuming non-uniformity in the \( y \) detection means that wavevector of surface plasmons should have a component in this direction which satisfies the dispersion relation in metal and dielectric, respectively as
\[ \epsilon_d k_0^2 + k_x^2 - k_y^2 - \beta^2 = 0, \] (11)
\[ \epsilon_m k_0^2 + k_x^2 - k_y^2 - \beta^2 = 0. \] (12)

Using the metal-dielectric boundary conditions, it can be shown that these new set of modes have the same propagation constant in the direction normal to the surface (\( x \) axis) and therefore the same confinement in this direction. The propagation constant in the \( y \) direction, is chosen from a continuous spectrum starting from zero for \( TM_0 \) and therefore these modes are higher order plasmonic modes of the metal-dielectric interface. Higher order TM modes form an orthogonal set and therefore any linear combination of them is also a solution of the Helmholtz equation
\[ E_{zd} = \int E_z(k_y) e^{-ik_y y} e^{-i\beta z} dk_y. \] (13)

The resulted fields from the diffraction of surface plasmon on a dielectric lens have a similar form with higher order surface plasmon modes when written as
\[ E(r) = \frac{1}{2\pi} \int E(k_y) e^{i\sqrt{K^2 - k_0^2}} e^{-ik_y y} e^{-ik_x x} dk_y, \] (14)

where \( K \) is
\[ K^2 = \epsilon_d k_0^2 + k_z^2. \] (15)

This shows that passing through a dielectric lens has excited higher order modes.

Although all the TM modes have similar mathematical formulation, they differ in their energy flow directions. The orientation of Poynting vector is schematically shown in Fig. 2. As \( \theta \), the
angle of Poynting vector ($P$) to the surface, increases, the wave behaves more like a radiation mode, transmitting energy away from the surface. On the other hand, modes having smaller values of $\theta$ are more bounded to the surface. In general, the energy flow direction of a TM mode is not in the $xz$ plane and the trajectory of its Poynting vector on the $yz$ plane makes an angle of $\phi$ with the $z$ axis.

Here, we are concerned with the energy loss of the surface and so only focused on $\theta$. The results of the calculation of the Poynting vector orientations are demonstrated in Fig. 3 for different points on a metal-dielectric interface. Using the vectorial Huygens-Fresnel principal and accurate SP phase delay, we have shown that higher order SP modes are excited behind a lens in an in-plane focusing system. This is equivalent to more energy loss from the surface since higher order modes are less bounded to the surface than the fundamental mode. Compared to the fundamental plasmonic mode, higher order modes are less confined to the surface and the orientation of Poynting vector can reach up to $90^\circ$ from the surface in both metal and dielectric which results in extra energy loss from the surface.

4. Conclusions

Although scalar diffraction theory presents a good qualitative description of the diffraction phenomena, it fails to predict the behavior of a thick lens in plasmonic regime. In this paper, equations based on vectorial Huygens-Fresnel theory are used to calculate the phase change imposed by the lens, which can provide more accurate results of the diffracted fields. Analyzing
the obtained equations proves the excitation of higher order plasmon modes behind a dielectric lens placed on the metal-dielectric interface. Excitation of these modes would result in more energy losses since higher order modes are less bounded to the interface. We have monitored these losses from the surface in both dielectric and metal by calculating the orientation of the Poynting vector in different points behind the lens.

References
[1] Ritchie R H 1957 *Phys. Rev.* **106** 874–881
[2] Maier S A and Atwater H A 2005 *J. Appl. Phys.* **98** 011101
[3] Vincenzo A, Roberto P, Marco F, Onofrio M M and Maria Antonia I 2017 *J. Phys. Condens. Matter* **29** 203002
[4] Zia R and Brongersma M L 2007 *Nat. Nano* **2** 426–429
[5] Hohenau A, Krenn J R, Stepanov A L, Drezet A, Ditlbacher H, Steinberger B, Leitner A and Aussenegg F R 2005 *Opt. Lett.* **30** 893–895
[6] Shi W B, Chen T Y, Jing H, Peng R W and Wang M 2017 *Opt. Express* **25** 5772–5780
[7] Bezus E A, Doskolovich L L, Kazanskiy N L, Soifer V A and Kharitonov S I 2010 *J. Opt.* **12** 015001
[8] Polo J A and Lakhtakia A 2011 *Laser Photon. Rev.* **5** 234–246
[9] Oulton R F, Pile D F P, Liu Y and Zhang X 2007 *Phys. Rev. B* **76** 035408
[10] Kim H, Hahn J and Lee B 2008 *Opt. Express* **16** 3049–3057
[11] Saleh B E A and Teich M C 1991 *Fundamentals of photonics* (New York: Wiley)
[12] Archambault A, Teperik T V, Marquier F and Greffet J J 2009 *Phys. Rev. B* **79** 195414
[13] Teperik T V, Archambault A, Marquier F and Greffet J J 2009 *Opt. Express* **17** 17483–17490
[14] Kordi M, Armin F, Malekfar M R, Mirsalehi M M and Shokooh-Saremi M 2017 *Phys. Rev. A* **95** 033824