Finding \( B_c(3S) \) States via Their Strong Decays

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Abstract

The experimentally known \( B_c \) states are all below open bottom-charm threshold, which experience three main decay modes, and all induced by weak interaction. In this work, we investigate the mass spectrum and strong decays of the \( B_c(3S) \) states, which just above the threshold, in the Bethe-Salpeter formalism of and \(^3P_0\) model. The numerical estimation gives 

\[ M_{B_c(3^1S_0)} = 7222 \text{ MeV}, \quad M_{B_c^*(3^3S_1)} = 7283 \text{ MeV}, \quad \Gamma (B_c(3^1S_0) \to B^*D) = 6.190^{+0.570}_{-0.540} \text{ MeV}, \]

\[ \Gamma (B_c^*(3^1S_0) \to BD) = 0.717^{+0.078}_{-0.073} \text{ MeV} \text{ and } \Gamma (B_c^*(3^1S_0) \to B^*D) = 8.764^{+0.801}_{-0.760} \text{ MeV}. \]

Compared with previous studies in non-relativistic approximation, our results indicate that the relativistic effects are notable in \( B_c(3S) \) exclusive strong decays. According to the results, we suggest to find the \( B_c(3S) \) states in their hadronic decays to \( B \) and \( D \) mesons in experiment, like the LHCb.

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The $B_c$ meson family is unique in quark model as its states are composed of heavy quarks with different flavors. The $B_c$ mesons lie intermediate between $(c\bar{c})$ and $(b\bar{b})$ states both in mass and size, while the different quark masses leads to much richer dynamics. On the other hand, the $B_c$ mesons cannot annihilate into gluons or photons and thus they are very stable. The $B_c$ mesons provide an unique window to reveal information about heavy-quark dynamics and can deepen our understanding of both strong and weak interactions.

Although there have been many investigations in the literature [1–15] about the properties of $B_c$ mesons, the excited $B_c$ states, especially above threshold, are rarely explored. The ground state $B_c$ meson was first observed by the CDF Collaboration at Fermilab [16] in 1998, while there was no reported evidence of the excited $B_c$ state until 2014, the ATLAS Collaboration reported a structure with mass of $6842 \pm 9$ MeV [17], which is consistent with the value predicted for $B_c(2S)$. Recently, the excited $B_c(2^1S_0)$ and $B^*_c(2^3S_1)$ states have been observed in the $B_c^+\pi^+\pi^-\pi^-$ invariant mass spectrum by the CMS and LHCb Collaboration, with their masses determined to be $6872.1 \pm 2.2$ MeV and $6841.2 \pm 1.5$ MeV [18, 19], respectively. Since the low-energy photon in the intermediate decay $B^*_c \rightarrow B_c \gamma$ was not reconstructed, the mass of $B^*_c(2^3S_1)$ meson appears lower than that of $B_c(2^1S_0)$.

The successful observation of $B_c(2S)$ states stimulates the interest in searching for $B_c(3S)$ states. Motivated by this, in this work, we calculate the mass spectrum of $B_c(nS)$ states up to $n = 4$ in the framework of Bethe-Salpeter (BS) equation [20]. A long-ranged linear confining potential and a short-ranged one gluon exchange potential are used in our calculation. Our results indicate that the $B_c(3S)$ states lie above the threshold for decay into a $BD$ meson pair. By combining $^3P_0$ model with the calculated relativistic BS wave functions, we investigate the strong decay properties of $B_c(3S)$ mesons. We also estimate the corresponding numbers of events at the Large Hadron Collider (LHC) experimental condition. Although similar topic has been studied in Refs. [3, 8, 11, 13, 14], the relativistic treatment of $B_c(3S)$ exclusive decays is evidently more close to the reality.
In quantum field theory, the BS equation provides a basic description for bound states. The BS wave function of a quark-antiquark bound state is defined as
\[ \chi(x_1, x_2) = \langle 0|T\psi(x_1)\bar{\psi}(x_2)|P\rangle, \] (1)
where \( x_1 \) and \( x_2 \) are the coordinates of the quark and antiquark respectively, \( P \) is the momentum of the bound state, \( T \) denotes the time ordering operator. The wave function in momentum space is
\[ \chi_P(q) = e^{-iP \cdot X} \int d^4x e^{-iq \cdot x} \chi(x_1, x_2), \] (2)
where \( q \) is the relative momentum between the quark and antiquark. The “center-of-mass coordinate” \( X \) and the “relative coordinate” \( x \) are defined as:
\[ X = \frac{m_1}{m_1 + m_2} x_1 + \frac{m_2}{m_1 + m_2} x_2, \quad x = x_1 - x_2, \] (3)
with \( m_1 \) and \( m_2 \) are the masses of the quark and antiquark respectively. Then the bound state BS equation in momentum space reads
\[ S^{-1}_1(p_1)\chi_P(q)S^{-1}_2(-p_2) = i \int \frac{d^4k}{(2\pi)^4} V(P; q, k)\chi_P(k). \] (4)
Here \( S_i(\pm p_i) = \frac{i}{\pm \not{p}_i - m_i} \) denotes the fermion propagator; \( V(P; q, k) \) is the interaction kernel; \( p_1 \) and \( p_2 \) are the momenta of the quark and anti-quark respectively, which can be expressed as
\[ p_i = \frac{m_i}{m_1 + m_2} P + J q, \] (5)
where, \( J = 1 \) for the quark \( (i = 1) \) and \( J = -1 \) for the antiquark \( (i = 2) \). With the definitions \( p_{i\rho} \equiv \frac{p \cdot p_i}{M} \) and \( p_{i\perp}^\mu \equiv p_i^\mu - \frac{p \cdot p_i}{M^2} P^\mu \), the propagator \( S_i(Jp_i) \) can be decomposed as
\[ -iJS_i(Jp_i) = \frac{\Lambda^+_i(q_{\perp})}{p_{i\rho} - \omega_i + i\epsilon} + \frac{\Lambda^-_i(q_{\perp})}{p_{i\rho} + \omega_i - i\epsilon}, \] (6)
where
\[ \Lambda^+_i(q_{\perp}) \equiv \frac{1}{2\omega_i} \left( \frac{\not{p}}{M} \omega_i \pm (\not{p}_{i\perp} + Jm_i) \right), \]
\[ \omega_i \equiv \sqrt{m_i^2 - p_{i\perp}^2}. \] (7)
Under the instantaneous approximation, the interaction kernel in the center of mass frame takes the form \( V(P; q, k)|_{\vec{P}=0} \approx V(q_\perp, k_\perp) \). Then the BS equation can be reduced to

\[
\chi_p(q) = S_1(p_1)\eta_p(q_\perp)S_2(-p_2) ,
\]

with

\[
\eta_p(q_\perp) = \int \frac{d^3k_\perp}{(2\pi)^3} V(q_\perp, k_\perp)\varphi_p(k_\perp) ,
\]

where \( \varphi_p(q_\perp) \equiv i \int \frac{dq\,\chi_p(q)}{2\pi} \) is the 3-dimensional BS wave function. By introducing the notation \( \varphi_p^{\pm\pm}(q_\perp) \) as:

\[
\varphi_p^{\pm\pm}(q_\perp) \equiv \Lambda_1^{\pm}(q_\perp)\frac{p}{M}\varphi_p(q_\perp)\frac{P}{M}\Lambda_2^{\pm}(q_\perp) ,
\]

The wave function can be decomposed as

\[
\varphi_p(q_\perp) = \varphi_p^{++}(q_\perp) + \varphi_p^{+-}(q_\perp) + \varphi_p^{-+}(q_\perp) + \varphi_p^{--}(q_\perp) .
\]

And the BS equation (8) can be decomposed into four equations

\[
(M - \omega_1 - \omega_2)\varphi_p^{++}(q_\perp) = \Lambda_1^+(q_\perp)\eta_p(q_\perp)\Lambda_2^+(q_\perp) ,
\]

\[
(M + \omega_1 + \omega_2)\varphi_p^{--}(q_\perp) = -\Lambda_1^-(q_\perp)\eta_p(q_\perp)\Lambda_2^-(q_\perp) ,
\]

\[
\varphi_p^{+-}(q_\perp) = \varphi_p^{-+}(q_\perp) = 0 .
\]

To solve the BS equation, one must have a good command of the potential between two quarks. According to lattice QCD calculations, the potential for a heavy quark-antiquark pair in the static limit is well described by a long-ranged linear confining potential (Lorentz scalar \( V_S \)) and a short-ranged one gluon exchange potential (Lorentz vector \( V_V \)) [21, 22]:

\[
V(r) = V_S(r) + \gamma_0 \otimes \gamma^0 V_V(r) ,
\]

\[
V_S(r) = \lambda r \frac{1 - e^{-ar}}{ar} + V_0 ,
\]

\[
V_V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} e^{-ar} .
\]
Here, the factor $e^{-\alpha r}$ is introduced not only to avoid the infrared divergence but also to incorporate the color screening effects of the dynamical light quark pairs on the “quenched” potential [23]. The potentials in momentum space are

\[
V(\vec{p}) = V_S(\vec{p}) + \gamma_0 \otimes \gamma^0 V_V(\vec{p}) ,
\]

\[
V_S(\vec{p}) = -\left( \frac{\lambda}{\alpha} + V_0 \right) \delta^3(\vec{p}) + \frac{\lambda}{\pi^2 (\vec{p}^2 + \alpha^2)^2} ,
\]

\[
V_V(\vec{p}) = -\frac{2}{3\pi^2} \frac{\alpha_s(\vec{p}^2)}{\vec{p}^2 + \alpha^2} .
\] (16)

Here, $\alpha_s(\vec{p}^2)$ is the running coupling which defined as

\[
\alpha_s(\vec{p}^2) = \frac{12\pi}{27} \ln \left( a + \frac{\vec{p}^2}{\Lambda_{QCD}^2} \right) .
\] (17)

The constants $\lambda$, $\alpha$, $a$, and $\Lambda_{QCD}$ are the parameters that characterize the potential. In the following, we will employ this potential to both the $B_c$ system and the heavy-light quark system as an assumption.

In the numerical calculation, following well-fitted parameters [24, 25] are used:

\[
\alpha = 0.06 \text{ GeV} , \quad \lambda = 0.21 \text{ GeV}^2 , \quad \Lambda_{QCD} = 0.27 \text{ GeV} ,
\]

\[
m_b = 4.96 \text{ GeV} , \quad m_c = 1.62 \text{ GeV} ,
\]

and $V_0 = 0.1890$ GeV for $^1S_0$ states, $V_0 = 0.1115$ GeV for $^3S_1$ states. Based on the formalism and parameters above, we calculate the masses of $B_c(nS)$ states up to $n = 4$. The numerical results are shown in Table I. For comparison, results obtained from other approaches are also listed.

Our results indicate that the $B_c(3S)$ states lie above the threshold for decay into a $BD$ meson pair. The corresponding OZI-allowed two body decay can be depicted by $^3P_0$ model, where the additional light quark-antiquark pair is assumed to be created from vacuum, as shown in Fig. I. The usual $^3P_0$ model is a non-relativistic model with a transition operator $\sqrt{3}g \int d^3x \bar{\psi}(\vec{x})\psi(\vec{x})$, and it can be extended to a relativistic form $i\sqrt{3}g \int d^4x \bar{\psi}(x)\psi(x)$ [25, 26]. The coupling constant $g$ can be parameterized as $2m_q\gamma$, where $m_q$ is the constitute quark mass and $\gamma$ is a dimensionless parameter which can
TABLE I: Masses (MeV) of $B_c(nS)$ mesons.

| State          | This work | EQ [2] | GI [5] | Lattice [12] | Exp [19] |
|----------------|-----------|--------|--------|--------------|----------|
| $B_c(1^1S_0)$  | 6275(input)| 6264   | 6271   | 6276         | 6271     |
| $B_c^*(1^3S_1)$| 6339(input)| 6337   | 6338   | 6331         | ...      |
| $B_c(2^1S_0)$  | 6853      | 6856   | 6855   | ...          | 6872     |
| $B_c^*(2^3S_1)$| 6920      | 6899   | 6887   | ...          | 6841     |
| $B_c(3^1S_0)$  | 7222      | 7244   | 7250   | ...          | ...      |
| $B_c^*(3^3S_1)$| 7283      | 7280   | 7272   | ...          | ...      |
| $B_c(4^1S_0)$  | 7484      | 7562   | ...    | ...          | ...      |
| $B_c^*(4^3S_1)$| 7543      | 7594   | ...    | ...          | ...      |

FIG. 1: The Feynman diagram of OZI-allowed two-body decay process with a $^3P_0$ vertex.

be extracted from experimental data. Here we take $m_u = 0.305$ GeV, $m_d = 0.311$ GeV, and $\gamma = 0.253 \pm 0.010$. [20]

The transition amplitude for the OZI-allowed two body decay process (with the
momenta assigned as in Fig. 1 can be written as
\[
\langle P_1P_2|S|P \rangle = (2\pi)^4 \delta^4(P - P_1 - P_2) \mathcal{M}
\]
\[
= -ig \int \frac{d^4q_1 d^4q_2}{(2\pi)^4} \frac{d^4q_1 d^4q_2}{(2\pi)^4} \text{Tr}[\chi(q)S_2^{-1}(p_1)(2\pi)^4 \delta^4(p_2 - p_{22})\bar{\chi}(q_2)(2\pi)^4 \delta^4(p_{12} - p_{21})
\times \bar{\chi}(q_1)S_1^{-1}(p_1)(2\pi)^4 \delta^4(p_1 - p_{11})]
\]
\[
= -ig(2\pi)^4 \delta^4(P - P_1 - P_2) \int \frac{d^4q}{(2\pi)^4} \text{Tr}[\chi(q)S_2^{-1}(-p_2)\bar{\chi}(q_2)\bar{\chi}(q_1)S_1^{-1}(p_1)],
\]
where \( q_i = q + (-1)^{i+1}(\frac{m_i}{m_1 + m_2}P_2 - \frac{m_{12}}{m_1 + m_2}P_1) \). The Feynman amplitude takes the form \[25\]:
\[
\mathcal{M} = -ig \int \frac{d^4q}{(2\pi)^4} \text{Tr}[\chi(q)S_2^{-1}(-p_2)\bar{\chi}(q_2)\bar{\chi}(q_1)S_1^{-1}(p_1)]
\]
\[
\approx g \int \frac{d^3q_\perp}{(2\pi)^3} \text{Tr}[\frac{P}{M} \varphi^{++}(q_\perp)\frac{P}{M} \varphi^{++}(q_2)(p_2)\bar{\varphi}^{++}(q_1)].
\]
Note, to get the last line of Eq. \[19\], we have used the fact that the wave function is strongly suppressed when \( |q_\perp| \) is large. With the method developed in Refs. \[28, 29\], the positive energy wave function can be determined by numerically solving Eq. \[12\].

The strong decay widths of \( B_c(3S) \) mesons are shown in Table 11. We obtain \( \Gamma(B_c(3^1S_0) \rightarrow B^*D) = 6.190^{+0.570}_{-0.540} \) MeV, \( \Gamma(B_c^*(3^1S_0) \rightarrow BD) = 0.717^{+0.078}_{-0.073} \) MeV and \( \Gamma(B_c^*(3^1S_0) \rightarrow B^*D) = 8.764^{+0.701}_{-0.700} \) MeV. The total decay widths here are about 5% ~ 10% the widths of Refs. \[13, 14\], where non-relativistic \( 3^3P_0 \) model were used. It’s quite a discrepancy but it’s also understandable for the following reasons:

1. In Refs. \[13, 14\], the coupling constant \( g \) is set to be about 0.264 GeV, while here it’s about 0.155 GeV according to Eq. \( (10) \) of Ref. \[27\]. This lead to a 3 times difference in decay width.

2. In the \( 3^3P_0 \) model with \( \mathcal{H}_{\text{int}} = i\sqrt{3}g\bar{\psi}(x)\psi(x) \), the transition operator contains a factor \( \frac{g}{2\omega_q} \), where \( \omega_q \) is the energy of the created light quark. This factor reduce to \( \frac{g}{2\omega_q} \) in the non-relativistic limit. Since the parameter \( g \) is extracted from the fit of the non-relativistic \( 3^3P_0 \) model to the experimental data, our widths are in fact suppressed by a factor \( \frac{m^2_q}{\omega_q^2} \sim 0.270 \) \[26\]. By multiplying a compensation factor of
TABLE II: Partial widths (Γ) and branching ratios (Br) of $B_c(3S)$ mesons. The number of events (NE) are estimated under the LHC experimental condition.

| Meson            | Decay mode | Γ (MeV)          | Br (%) | NE ($10^8$) |
|------------------|------------|------------------|--------|-------------|
| $B_c^\pm(3^1S_0)$ | $B^\ast D^\pm$ | $2.945^{+0.268}_{-0.254}$ | 47.58  | 2.32        |
| $B_c^\pm(3^1S_0)$ | $B^\ast D^0$   | $3.245^{+0.302}_{-0.286}$  | 52.42  | 2.56        |
| $B_c^\pm(3^3S_1)$ | $B^0 D^\pm$    | $0.198^{+0.022}_{-0.021}$  | 2.09   | 0.30        |
| $B_c^\pm(3^3S_1)$ | $B^\pm D^0$    | $0.519^{+0.056}_{-0.052}$  | 5.48   | 0.79        |
| $B_c^\pm(3^3S_1)$ | $B^\ast D^\pm$ | $3.801^{+0.346}_{-0.328}$  | 40.09  | 5.77        |
| $B_c^\pm(3^3S_1)$ | $B^\ast D^0$    | $4.963^{+0.455}_{-0.432}$  | 52.34  | 7.54        |

about 3.7, the strong decay widths here are comparable with those predicted in Ref. [3].

3. The relativistic effect of the wave functions are non-negligible, as discussed in Ref. [30].

According to non-relativistic quantum chromodynamics factorization formalism [31], the production rates of $B_c(3S)$ mesons can be estimated through

$$\sigma(B_c(3S)) = \sigma(B_c(1S)) \frac{|\Psi_{B_c(3S)}(0)|^2}{|\Psi_{B_c(1S)}(0)|^2},$$  \hspace{1cm} (20)

where $\Psi_H(0)$ is the wave function at the origin for meson $H$. With the $\sigma(B_c(1S))$ predicted in Ref. [32], and the wave functions calculated in Ref. [2], the cross sections for $B_c(3S)$ mesons at the LHC can be estimated as $\sigma(B_c(3^1S_0)) = 4.88$ nb and $\sigma(B_c^*(3^3S_1)) = 14.5$ nb. At an integrated luminosity of 100 fb$^{-1}$, the numbers of $B_c(3^1S_0)$ and $B_c^*(3^3S_1)$ events are $4.88 \times 10^8$ and $1.44 \times 10^9$, respectively. The number of events for different decay channels are also presented in Table III.

In summary, since the relativistic effects are evidently important in $B_c(3S)$ exclusive decays to $B$ and $D$ mesons, we have investigated in this work the mass spectrum and strong decay properties of $B_c(3S)$ states in the framework of BS
equation and relativistic $^3P_0$ model. The numerical estimation gives $M_{B_c(3^1S_0)} = 7222$ MeV, $M_{B_c^*(3^3S_1)} = 7283$ MeV, $\Gamma (B_c(3^1S_0) \to B^*D) = 6.190^{+0.570}_{-0.540}$ MeV, $\Gamma (B_c^*(3^3S_1) \to BD) = 0.717^{+0.078}_{-0.073}$ MeV and $\Gamma (B_c^*(3^3S_0) \to B^*D) = 8.764^{+0.801}_{-0.760}$ MeV. Note, the relativistic correction diminishes the decay widths in previous studies by about one order of magnitude. We also estimate the number of events for different decay channels in the LHC experimental condition. Since a large number of events may be produced in experiment, we suggest to find the the $B_c(3S)$ states in their exclusive strong decays.

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