Open String Spectrum in pp-wave background

Chan Yong Park

\textit{a Department of Physics, Sogang University, Seoul 121-742, Korea}

ABSTRACT

In this paper, we consider the D-brane, especially D\textsubscript{--}brane, in the pp-wave background, which has the eight dynamical and the eight kinematical supercharges. Since the pp-wave background has not a SO(8) but a SO(4) \times SO(4) symmetric group, the D-brane world-volume theory has a non-trivial symmetric group which depends on the configuration of D-brane. Here, we will analyze the open string spectrum consistent with the non-trivial symmetric group and, at the low energy limit, classify the field contents of the D-brane world-volume theory which come from the fermionic zero modes of the open string.

PACS numbers: 11.25.-w, 11.25.Uv

November 9, 2018

\*cyong21@hepth.sogang.ac.kr
1 Introduction

Recently, in Ref. [1] it was shown that the pp-wave background is a maximal supersymmetric space like Minkowski, anti-de Sitter(AdS) and de Sitter(dS) space and that the closed string theory on this pp-wave background with the RR five-form flux is exactly solvable [2, 3] in the light-cone gauge. Moreover, this pp-wave background can be obtained from $AdS_5 \times S^5$ by taking the Penrose limit. In according to the AdS/CFT correspondence, it was suggested that the closed string theory in this background has a similar correspondence with the gauge theory with the large R-charge [4]. By many authors it was shown that this conjecture including an interaction terms in the gauge theory [5]-[12] side and the superstring side [13]-[20] is still valid. In ref. [21], the closed superstring field theory is constructed with the similar method used in flat space [22, 23, 24] and the interaction hamiltonians for the closed [21] and open [25] string theories are determined by requiring the closure of the pp-wave background superalgebra .

Furthermore, D-branes in the pp-wave background described by an open string, which is obtained from the type IIB closed string using the boundary conditions [26, 27], was identified and their supersymmetries was classified [28]-[37]. In addition, supersymmetries of the intersection branes is also investigated [38, 39]. In Ref. [34, 35], it was shown that there can be two kinds of D-branes, for example $D_-$-branes, $D_+$-branes, in pp-wave background. These two kinds of branes appear due to the non-trivial open string boundary condition of the mass term caused by the pp-wave metric. Moreover, since the symmetric group of this pp-wave background is given by $SO(4) \times SO(4)$ due to the existence of the five-form flux, each brane having a special configuration exists. This kind of a non-trivial configuration of D-brane induces the non-trivial symmetric group structure of the world-volume theory on the D-brane in the pp-wave background [40]. Usually, in the flat space background the world-volume theory on D-brane becomes SYM (super Yang-Mills) theory which corresponds to the open string theory, especially the fermionic zero modes, at the low energy limit. To describe the world-volume theory having a non-trivial symmetric group, in the first place we consider the open string theory in the pp-wave background and then find the open string spectrum at the low energy limit which becomes field contents of the SYM theory with a non-trivial symmetric group.

In this paper, we will explicitly show that, especially in the $D_-$-brane cases, how we can obtain the field contents of world-volume theory on D-brane from the open string theory in the pp-wave background. The paper is organized as follows. In Sec. 2, we will give a brief review for the open string theory in the pp-wave. In Sec. 3, using the open string spectrum we will show that the field contents of the world-volume theory are obtained from the open string theory at the low energy limit. In Sec. 4, we will conclude this paper with some comments. In the appendix, we will give a shot explanation for the spinor representation.
2 Review of open string in pp-wave background

In the pp-wave background including a constant R-R five-form flux, the metric is given by

\[ ds^2 = 2 dx^+ dx^- - \mu^2 (x^I)^2 (dx^+)^2 + \delta_{IJ} dx^I dx^J, \]  

(2.1)

and

\[ F_{+1234} = F_{+5678} = 2\mu, \]  

(2.2)

where \( \mu \) is an arbitrary constant and index \( I \) runs from 1 to 8. These quantities can be obtained by maximally supersymmetric solutions of the type IIB string theory.

The Green-Schwarz light-cone action in the plane wave background describe massive bosons and fermions. In the light-cone gauge, \( X^+ = \tau \), the action is given by

\[
S = \frac{1}{2\pi \alpha'} \int d\tau \int_0^{2\pi \alpha' p^+} d\sigma \left[ \frac{1}{2} \partial_+ X_I \partial_- X_I - \frac{1}{2} \mu^2 X_I^2 - i \bar{S}(\rho^A \partial_A - \mu \Pi)S \right],
\]

(2.3)

where \( \partial_\pm = \partial_\tau \pm \partial_\sigma \). In this paper, we will set \( \alpha = 2\alpha' p^+ \) for open strings and take the spinors \( S \) as eight two-components Majorana spinors on the world-sheet \( \Sigma \) that transform as positive chirality spinors \( 8 \), under \( SO(8) \):

\[
S^a = \begin{pmatrix} S^{1a} \\ S^{2a} \end{pmatrix}, \quad \bar{S}^a = S^{a T} \rho^\tau,
\]

(2.4)

where

\[
\rho^\tau = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \rho^\sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
\]

(2.5)

The presence of \( \Pi \) in the fermionic action breaks the \( SO(8) \) symmetry into \( SO(4) \times SO(4)' \). From this action Eq.(2.3) we can obtain the equations of motion of the bosonic field \( X^I \) and the fermionic field \( S^a \):

\[
\partial_+ \partial_- X^I + \mu^2 X^I = 0, \]  

(2.6)

\[
\partial_+ S^1 - \mu \Pi S^2 = 0, \quad \partial_- S^2 + \mu \Pi S^1 = 0. \]  

(2.7)

2.1 The bosonic sectors

The Green-Schwarz action for bosonic modes is given by

\[
S_B = \frac{1}{\pi \alpha'} \int d\tau \int_0^{2\pi |p^+|} d\sigma \left[ \frac{1}{2} \partial_+ X_I \partial_- X_I - \frac{1}{2} \mu^2 X_I^2 \right].
\]

(2.8)

Now we pay attention to open strings ending on Dp-branes, especially static Dp-branes for simplest, in plane wave background. For these open strings, Neumann boundary conditions,

\[
\partial_\sigma X^r|_{\partial \Sigma} = 0,
\]

(2.9)
are imposed along directions of the Dp-branes world-volume and Dirichlet boundary conditions,

$$\partial_\tau X^r\big|_{\partial\Sigma} = 0,$$

for remaining transverse coordinates where $X^r$ and $X^r'$ imply the longitudinal and transverse coordinates on Dp-branes respectively. The solutions satisfying equations on motion and boundary conditions are given by

$$X^r(\sigma, \tau) = x_0^r \cos \mu \tau + \frac{p_0^r}{\mu} \sin \mu \tau + i \sum_{n \neq 0} \frac{1}{\omega_n} \alpha_n^r e^{-i \omega_n \tau} \cos \frac{n \sigma}{|\alpha|},
$$

$$X^r'(\sigma, \tau) = x_0'^r(\sigma) + \sum_{n \neq 0} \frac{1}{\omega_n} \alpha_n'^r e^{-i \omega_n \tau} \sin \frac{n \sigma}{|\alpha|},$$

(2.11)

where the zero mode parts, $x_0^r(\sigma)$, represent the positions of D-brane in transverse directions and are given by

$$x_0^r(\sigma) = \frac{x_0^r}{1 + e^{\mu |\alpha| \pi}} \left( e^{\mu \sigma} + e^{\mu (|\alpha| \pi - \sigma)} \right).$$

(2.12)

Using the canonical conjugate momentum defined by

$$P_I \equiv \frac{\partial L}{\partial \partial_\tau X^I},$$

(2.13)

the Hamiltonian is given by

$$\mathcal{H}_B = \frac{1}{2\pi \alpha} \left[ (\pi \alpha)^2 P_I^2 + (\partial_\sigma)^2 + \mu^2 X_I^2 \right],$$

(2.14)

where two phase space coordinates, $X^I$ and $P_I$, satisfy the following commutation relation

$$[X^I(\sigma), P^J(\sigma')] = i \delta^{IJ} \delta(\sigma - \sigma').$$

(2.15)

From the above commutation relation, we can get commutation relations of expanding modes

$$[x_0^r, p_0^s] = i \delta^{rs} \text{ and } [\alpha_n^I, \alpha_m^J] = \omega_n \delta_{n+m,0} \delta^{IJ},$$

(2.16)

where $x_0^r$ and $p_0^s$ are zero modes of $X^r$ and $P^s$ respectively and

$$\omega_n = \text{sign}(n) \sqrt{\mu^2 + n^2 / \alpha^2}.$$

(2.17)

In terms of modes $n$, the Hamiltonian is given by

$$\mathcal{H}_B = \text{sign}(\alpha) \left[ \frac{1}{2} p_0^r + \frac{1}{2} x_0^r + \frac{1}{2} \sum_{n \neq 0} \alpha_n^I \alpha_{-n}^I \right].$$

(2.18)
By introducing new variables for \(n = 0\)

\[
a_0^r = \frac{1}{\sqrt{2}} \left( \frac{p_0^r}{\sqrt{\omega_0}} - i \sqrt{\omega_0} x_0^r \right),
\]

\[
a_0^{r+} = \frac{1}{\sqrt{2}} \left( \frac{p_0^r}{\sqrt{\omega_0}} + i \sqrt{\omega_0} x_0^r \right),
\]

and for \(n > 0\)

\[
a_n^I = \frac{\alpha_n^I}{\sqrt{\omega_n}},
\]

\[
a_n^{I+} = \frac{\alpha_{-n}^I}{\sqrt{\omega_n}}
\]

(2.19)

this Hamiltonian can be rewritten as a harmonic oscillator form

\[
H_B = \text{sign}(\alpha) \frac{1}{2} \sum_{n \geq 0} \omega_n \left( a_n^{I+} a_n^I + a_n^I a_n^{I+} \right),
\]

(2.20)

satisfying the commutation relations of simple canonical form

\[
[a_r^0, a_s^{s^0}] = \delta^{rs}
\]

and \([a_n^I, a_m^{I+}] = \delta^{IJ} \delta_{n,m}\).

Notice that \(a_0^r\) and \(a_0^{s^+}\), which are zero modes of the transverse coordinates, are set to zero. Since the energy contributions of zero modes of the transverse coordinates are absent, the zero point energy of the above Hamiltonian is given by

\[
E_{B,0} = \text{sign}(\alpha) \cdot \frac{(p - 1) \omega_0}{2},
\]

(2.22)

where \((p - 1)\) is the number of coordinates satisfying the Neumann boundary condition except light-cone coordinates. This Eq.(2.23) implies that the bosonic zero point energy depends on a world volume dimension of Dp-brane.

### 2.2 The fermionic sectors

The action for fermionic field is given by

\[
S_F = \frac{1}{\pi \alpha} \int d\tau \int_0^{\pi|\alpha|} d\sigma \left[ S^{1a} \partial^+ S^{1a} - \mu S^{1a} \Pi S^{1a} 
+ S^{2a} \partial^- S^{2a} + \mu S^{2a} \Pi S^{2a} \right],
\]

(2.24)

where the spinor indices \(a\) run from 1 to 8. To describe the open string fermions, we consider the boundary condition which reduces the fermionic degrees of freedom:

\[
S^{2a} \mid_{\partial \Sigma} = \Omega S^{1a} \mid_{\partial \Sigma},
\]

(2.25)
where $\Omega$ is a product of the gamma matrices of the world-volume directions of D-brane. Then, the solutions of equations of motions satisfying the boundary conditions read

\[ S_1(\sigma, \tau) = \cos(\mu \tau) S_0 - \sin(\mu \tau) \Pi S_0 + \sum_{n \neq 0} C_n \left[ \phi_n^1(\sigma, \tau) \Omega S_n + i \rho_n \phi_n^2(\sigma, \tau) \Pi S_n \right], \]

\[ S_2(\sigma, \tau) = \cos(\mu \tau) \Omega^T S_0 - \sin(\mu \tau) \Pi S_0 + \sum_{n \neq 0} C_n \left[ \phi_n^1(\sigma, \tau) \Omega S_n + i \rho_n \phi_n^2(\sigma, \tau) \Pi S_n \right] \] (2.26)

where

\[ \phi_n^1(\sigma, \tau) = e^{-i(\omega_n \tau - n \sigma/|\alpha|)}, \quad \phi_n^2(\sigma, \tau) = e^{-i(\omega_n \tau + n \sigma/|\alpha|)}, \]

\[ \rho_n = \frac{\omega_n - n/|\alpha|}{\mu}, \]

\[ c_n = \frac{1}{\sqrt{1 + \rho_n^2}}. \] (2.27)

Using the canonical momenta defined by

\[ \lambda^A a \equiv \frac{\partial L}{\partial (\partial_\tau S^A a)} = \frac{i}{\pi \alpha} S^A a, \] (2.28)

where $A = 1$ and 2, the Hamiltonian is given by

\[ \mathcal{H}_F = \frac{i}{\pi \alpha} \int_0^{\pi|\alpha|} d\sigma \left[ S^{1a} \partial_\sigma S^{1a} - S^{2a} \partial_\sigma S^{2a} - \mu (S^{1a} \Pi S^{1a} - S^{2a} \Pi S^{2a}) \right] \] (2.29)

At a fixed time $\tau = 0$, the Hamiltonian is written by

\[ \mathcal{H}_F = -\text{sign}(\alpha)2 \left[ i \omega_0 S_0 \Omega S_0 - \sum_{n \neq 0} \omega_n S_n S_{-n} \right]. \] (2.30)

For the non-zero modes ($n > 0$), if we define new variables:

\[ S_{-n} \equiv \frac{b_n^+}{2} \quad \text{and} \quad S_n \equiv \frac{b_n}{2}, \] (2.31)

then the Hamiltonian for the non-zero modes is given by a similar form of the harmonic oscillator

\[ \mathcal{H}_{F,n \neq 0} = \text{sign}(\alpha) \frac{1}{2} \sum_{n > 0} \omega_n \left( b_n^+ b_n - b_n b_n^+ \right). \] (2.32)

Notice that the Hamiltonian of the fermionic zero modes contains $\Omega$, whose form depends on a configuration of Dp-brane. Hence, in the next section, we will describe, case by case, spectrum of the fermionic zero modes.
3 Spectrum of the fermionic zero modes

In pp-wave background space-time, the symmetry group is given by SO(4) × SO(4). To describe an open string, we have to include Dp-brane. This Dp-brane breaks half supersymmetry and also break a symmetry group into a appropriate subgroup according to the Dp-brane configuration. To describe an open string on the Dp-brane world volume, we have to find the representation of an open string under this subgroup. Moreover, at the low energy scale where the stringy excitation modes are excluded, the zero mode states of this open string have one-to-one correspondence to field contents of the SYM theory.

In the bosonic modes of an open string, the existence of a Dp-brane divides coordinates \( X^I \) into two parts: \((p-1)\) Neumann coordinates satisfying \( \partial_\sigma X^r = 0 \), and \((9-p)\) Dirichlet coordinates satisfying \( \partial_\tau X^{r'} = 0 \). As shown in Eq.(2.18), Dirichlet coordinates have no zero modes, so the zero point energy is given by Eq.(2.23).

Now let us consider Hamiltonian of the fermionic zero modes

\[
\mathcal{H}_{F,n=0} = -\text{sign}(\alpha)2i\omega_0 S_0 \Omega \Pi S_0 ,
\]

where \( S_0 \) are the real eight-component spinors. From the fact that \( \Omega \Pi \Omega \Pi = -1 \), an eigenvalue of \( \Omega \Pi \) is given by \(+i\) or \( -i \). Suppose that \( \psi \) are eigenstates of \( \Omega \Pi \) with an eigenvalue \(+i\) or \( -i \), then the fermionic zero modes \( S_0 \) can be described by the appropriate linear combinations of \( \psi \) which are complex spinors and depend on the boundary condition. In the next sections, we will give a full details of these combinations consistent with the boundary condition.

Without considering the boundary condition, the pp-wave background has a SO(4) × S\( \tilde{\text{O}} \)\( (4) \) Lorentz symmetric group, which is caused by a constant RR 5-form flux \( F_{+1234} = F_{+5678} = 2\mu \). This symmetric group can be described by SU(2)\(_L\) × SU(2)\(_R\) × S\( \tilde{\text{U}} \)(2)_\(_L\) × S\( \tilde{\text{U}} \)(2)_\(_R\), which is a bispinor representation (see appendix). Now consider SO(2) × SO(2)’, which are rotations in the 12 and 34 and are generated by \( T_{12} \equiv i\gamma^{12}/2 \) and \( T_{34} \equiv i\gamma^{34}/2 \) respectively, these rotation

| fermions | \( i\gamma^{12} \) | \( i\gamma^{34} \) | \( i\gamma^{56} \) | \( i\gamma^{78} \) |
|----------|----------------|----------------|----------------|----------------|
| \( \psi^1 \) | + | + | + | + |
| \( \psi^2 \) | + | + | - | - |
| \( \psi^3 \) | - | - | + | + |
| \( \psi^4 \) | - | - | - | - |
| \( \psi^5 \) | + | - | + | - |
| \( \psi^6 \) | + | - | - | + |
| \( \psi^7 \) | - | + | + | - |
| \( \psi^8 \) | - | + | - | + |

Table 1: The rotation properties of \( \psi \) in transverse eight-dimension.
generators, \( T_{12} \) and \( T_{34} \), are related with the Cartan generators, \( J_{3L} \) and \( J_{3R} \), of the first \( SU(2)_L \times SU(2)_R \) symmetric group:

\[
J_{3L} = \frac{1}{2}((T_{12} + T_{34})), \\
J_{3R} = \frac{1}{2}((T_{12} - T_{34})).
\]  

(3.2)

At the same way, other Cartan generators, \( \tilde{J}_{3L} \) and \( \tilde{J}_{3R} \), of the second symmetric group \( \tilde{SU}(2)_L \times \tilde{SU}(2)_R \) are given by, in terms of generators \( T_{56} \) and \( T_{78} \) of a rotation group \( \tilde{SO}(2) \times \tilde{SO}(2)' \),

\[
\tilde{J}_{3L} = \frac{1}{2}((T_{56} + T_{78})), \\
\tilde{J}_{3R} = \frac{1}{2}((T_{56} - T_{78})).
\]  

(3.3)

Now define complex fermions:

\[
\psi^{2a-1} \equiv \frac{1}{\sqrt{2}} (S^{2a-1}_0 + iS^{2a}_0), \\
\psi^{2a} \equiv \frac{1}{\sqrt{2}} (S^{2a-1}_0 - iS^{2a}_0),
\]  

(3.4)

where \( a = 1, 2, 3, 4 \), and these fermions \( \psi \) have a proper eigenvalues of generators in \( SO(2) \times SO(2)' \times \tilde{SO}(2) \times \tilde{SO}(2)' \), as shown in the table 1.

The Hamiltonian of the fermionic zero modes can be rewritten as

\[
\mathcal{H}_{F,n=0} = -\text{sign}(\alpha)2i\omega_0 \left[ \psi^{2a-1} (\Omega \Pi)_{2a-1,2b} \psi^{2b} + \psi^{2a} (\Omega \Pi)_{2a,2b-1} \psi^{2b-1} - i \left\{ \psi^{2a-1} (\Omega \Pi)_{2a-1,2b-1} \psi^{2b-1} - \psi^{2a} (\Omega \Pi)_{2a,2b} \psi^{2b} \right\} \right].
\]  

(3.5)

Notice that if we choose \( \psi \) as an eigenstate of \( \Omega \Pi \), then \( \Omega \Pi \) is diagonalized and the second line in Eq.(3.5) becomes zero due to the Pauli exclusion principle.

3.1 For D7-brane

In this section, we will consider D7-brane lying in \( X^0, X^1, \ldots, X^6, \) and \( X^9 \). This D7-brane configuration gives a following symmetric group except light-cone coordinate directions:

\[
SU(2)_L \times SU(2)_R \times \tilde{SO}(2) \times \tilde{SO}(2)' .
\]

Under this symmetric group, fermions \( \psi \) transforms, as shown in the table 2. \( \psi^a \) and \( \psi^{a+2} \) where \( a = 1, 2 \), become a doublet of \( SU(2)_L \) and \( \psi^a \) and \( \psi^{a+2} \) where \( a = 5, 6 \), appear as a doublet of \( SU(2)_R \). This implies that a eight-component Majorana-Weyl spinor, \( 8_s \) in eight-dimensional
fermions \( (J_3^L, J_3^R) \) | \( i\gamma^{56} \) | \( i\gamma^{78} \) \\
\( \psi^1 \) | \(+\frac{1}{2}, 0\) | + | + \\
\( \psi^2 \) | \(+\frac{1}{2}, 0\) | - | - \\
\( \psi^3 \) | \(-\frac{1}{2}, 0\) | + | + \\
\( \psi^4 \) | \(-\frac{1}{2}, 0\) | - | - \\
\( \psi^5 \) | \(0, +\frac{1}{2}\) | + | - \\
\( \psi^6 \) | \(0, +\frac{1}{2}\) | - | + \\
\( \psi^7 \) | \(0, -\frac{1}{2}\) | + | - \\
\( \psi^8 \) | \(0, -\frac{1}{2}\) | - | + \\

Table 2: The eigenvalues of \( \psi \) under a symmetric group, \( SU(2)_L \times SU(2)_R \times \tilde{SO}(2) \times \tilde{SO}(2)' \).

space is divided by two Weyl spinors with their complex conjugate spinors in four-dimensional space, where we use a SO(4) bispinor representation:

\[ 8_s = (2, 1) \otimes (\bar{2}, 1) \otimes (1, 2) \otimes (1, \bar{2}), \]

where \((\bar{2}, 1)\) and \((1, 2)\) are the complex conjugate spinors of \((2, 1)\) and \((1, 2)\), respectively. Therefore, if we choose \((2, 1)\) and \((1, 2)\) as creation operators, \((\bar{2}, 1)\) and \((1, \bar{2})\) are interpreted as annihilation operators. These relations can be explicitly shown by writing a Hamiltonian of fermionic zero modes as a harmonic oscillator form. To do so, notice that in the case of D7-brane, \( \Omega \Pi \) is nothing but \( \gamma^{56} \). Hence, \( \psi \) become eigenstates of \( \Omega \Pi \) with eigenvalues \( \pm i \): \n
\[ \Omega \Pi \psi^{2a-1} = -i\psi^{2a-1} \quad \text{and} \quad \Omega \Pi \psi^{2a} = i\psi^{2a}. \] (3.6) 

Therefore, the Hamiltonian of fermionic zero modes on D7-brane is given by

\[ H_{F,n=0} = \text{sign}(\alpha) 2\omega_0 \left[ \psi^{2a-1} \psi^{2a} - \psi^{2a} \psi^{2a-1} \right]. \] (3.7) 

Introduce new fermionic variables with the following definitions:

\[ b_0^{a+} \equiv 2\psi^{2a-1}, \]
\[ b_0^a \equiv 2\psi^{2a}, \] (3.8) 

where \( \psi \) in \((2, 1)\) and \((1, 2)\) are defined as the creation operators and at the same time, \( \psi \) in \((\bar{2}, 1)\) and \((1, \bar{2})\) are introduced as the annihilation operators. Then, the Hamiltonian can be rewritten as a harmonic oscillator form

\[ H_{F,n=0} = \text{sign}(\alpha) \frac{1}{2} \omega_0 \left[ b_0^{a+} b_0^a - b_0^a b_0^{a+} \right], \] (3.9) 

where \( a \) runs from 1 to 4. Therefore, the total Hamiltonian of an open string ending on Dp-brane is given by

\[ H = \text{sign}(\alpha) \left[ \frac{1}{2} \sum_{n \geq 0} \omega_n \left( a_n^{I+} a_n^I + a_n^I a_n^{I+} + b_n^{a+} b_n^a - b_n^a b_n^{a+} \right) \right]. \] (3.10)
In Eq. (3.10), notice that for \( n = 0 \) \( I \) runs from 1 to \( p - 1 \), which are indices of the Dp-brane world-volume coordinates in a light-cone gauge, and \( a \) runs from 1 to 4. In the case of non-zero modes \( (n \neq 0) \), \( I \) and \( a \) run from 1 to 8. Therefore, the zero point energy of an open string modes is given by

\[
E_0 = \text{sign}(\alpha) \frac{p - 5}{2} \omega_0,
\]

(3.11)

where the zero point energy of non-zero bosonic modes is cancelled by that of fermionic modes, so only zero modes of an open string give a contribution to zero point energy. Note that this zero point energy is exactly the same as that in ref. [35]. In the D7-brane case, the zero point energy is given by \( \text{sign}(\alpha) \omega_0 \).

At the low energy limit, we expect that the world-volume theory on this D7-brane can be described by a Super-Yang-Mills theory and all field contents of this theory can be also obtained from fermionic zero modes of an open string. To obtain field contents of the D7-brane world-volume theory in the pp-wave background which has a \( \text{SO}(4) \times \tilde{\text{SO}}(2) \) symmetric group, we have to consider the Fock space consistent with this symmetric group. Here, we pay attention to the \( \text{SO}(4) \) representation, so all field contents will be described by fields in four-dimensional space. In addition, the rest \( \text{SO}(2) \) symmetric group, which is just rotation group in \( X^5 \) and \( X^6 \) plane, can be considered as an internal symmetric group. To do so, we divide a spinor indices \( a = 1, 2, 3, 4 \) into two parts, \( \alpha = 1, 2 \) and \( \dot{\alpha} = 1, 2 \); here, \( \alpha \) and \( \dot{\alpha} \) imply indices of \( \text{SU}(2)_L \) and \( \text{SU}(2)_R \), respectively. Then the Hamiltonian can be rewritten as

\[
\mathcal{H} = \text{sign}(\alpha) \left[ \frac{1}{2} \sum_{n \geq 0} \omega_n \left( a^+_n a^+_n + a^+_n a^+_n + b^\alpha_n b^\alpha_n - b^\alpha_n b^\alpha_n + b^{\dot{\alpha}}_n b^{\dot{\alpha}}_n - b^{\dot{\alpha}}_n b^{\dot{\alpha}}_n \right) \right].
\]

(3.12)

Now, to construct a Fock space we have to choose an appropriate vacuum which is annihilated by annihilation operators:

\[
0 = b^\alpha_0 | -1, 0 \rangle = b^{\dot{\alpha}}_0 | -1, 0 \rangle,
\]

(3.13)

where \(-1\) and \(0\) in the state imply the charges of the rotation groups \( \tilde{\text{SO}}(2) \times \tilde{\text{SO}}(2)' \). Using these fermionic zero modes and the vacuum state, we can classify the field contents of the world-volume theory, as shown in the table 3.

In table 3, \( A \) and \( \bar{A} \) are identified with complex scalar fields having \(-1\) and \(+1\) charges under the rotation in \( X^5 \) and \( X^6 \) plane, which implies that \( A \) and \( \bar{A} \) are just the linear combinations of the real scalar fields, \( X^5 \) and \( X^6 \). In six-dimensional space, these two fields together with \( A^i \), which is a gauge field in four-dimensional space, become a six-dimensional vector field. The other scalar fields, \( \phi \) and \( \bar{\phi} \) become a singlet under the \( \text{SO}(4) \times \tilde{\text{SO}}(2) \) symmetric group and have \(+\) or \(-\) charge of \( \tilde{\text{SO}}(2)' \).
3.2 For D5-brane

In this section, we will consider a D5-brane lying in \( X^1, X^2, X^3 \) and \( X^8 \). The configuration of this D5-brane breaks a \( \text{SO}(4) \times \text{SO}(4) \) symmetric group into \( \text{SO}(3) \times \text{SO}(3) \sim \text{SU}(2) \times \text{S\( \bar{\text{U}}(2) \) \) in which \( \text{SO}(3) \) or \( \text{S\( \bar{\text{O}}(3) \) \) is the rotation group in \( X^1, X^2, X^3 \) or \( X^5, X^6, X^7 \) directions respectively. Therefore, to find an effective open string theory on a D5-brane world volume, we have to investigate the field contents, especially coming from open string zero modes and transforming under the \( \text{SU}(2) \times \text{S\( \bar{\text{U}}(2) \) \) symmetric group. Here, \( \text{SU}(2) \) and \( \text{S\( \bar{\text{U}}(2) \) \) are considered as a diagonal subgroup of \( \text{SU}(2)_L \times \text{SU}(2)_R \) and \( \text{S\( \bar{\text{U}}(2)\)_L} \times \text{S\( \bar{\text{U}}(2)\)_R} \) respectively. Here, we choose \( X^4 \) and \( X^8 \) as the invariant coordinates under this diagonal subgroup transformation, see Appendix. Then, the Cartan generators \( J_3 = J_{3L} + J_{3R} \) and \( \bar{J}_3 = \bar{J}_{3L} + \bar{J}_{3R} \) are proportional to \( T_{12} \) and \( T_{56} \) respectively. Since \( \gamma^{34} \gamma^{78} = -\gamma^{37} \gamma^{48} \) and \( \Omega \Pi = -\gamma^{48} \), we assume that all fermions have an eigenvalue of \( \gamma^{48} \). Then all fermions living on a D5-brane have the eigenvalues of \( i\gamma^{12}, i\gamma^{37} \) and \( i\gamma^{56} \), so an eigenvalue of \( i\gamma^{48} \) is fixed by the positive chirality of \( \gamma^{1--8} \) acting on all fermions living in the transverse eight-dimensional space-time, see table 4.

As shown in the table 4, note that \( \psi^a \) and \( \psi^{a+4} \), where \( a = 1, 2, 3, 4 \), have the same eigenvalues under \( i\gamma^{12} \) and \( i\gamma^{56} \), so they are distinguished by only the eigenvalue of \( i\gamma^{37} \) or \( i\gamma^{48} \). Here, \( \psi^a \) and \( \psi^{a+2} \), where \( a = 1, 2, 5, 6 \), become elements of the doublet of \( \text{SU}(2) \) and two doublets, for example \( (\psi^1, \psi^3) \) and \( (\psi^2, \psi^4) \), are connected by a \( \text{SU}(2) \) which is an R-symmetry group of N=2 supersymmetry on D5-brane. To obtain a Harmonic oscillator form of the hamiltonian, we have to define all fermions with a positive eigenvalue of \( i\gamma^{48} \) as the creation operators and the rest are defined as the annihilation operators. Then, the Hamiltonian is written as

\[
\mathcal{H} = \text{sign}(\alpha) \left[ \frac{1}{2} \sum_{n \geq 0} \omega_n \left( a_n^{I+} a_n^{I} + a_n^{I} a_n^{I+} + b_n^{A+} b_n^{A} - b_n^{A} b_n^{A+} \right) \right], \tag{3.14}
\]

and the zero point energy \( E_0 \) is given by zero.
| fermions | operators | $i\gamma^{12}$ | $i\gamma^{37}$ | $i\gamma^{56}$ | $i\gamma^{48}$ |
|----------|-----------|----------------|----------------|----------------|----------------|
| $\psi^1$ | $\frac{1}{2}b_0^{1+}$ | $+$ | $+$ | $+$ | $-$ |
| $\psi^2$ | $\frac{1}{2}b_0^1$ | $+$ | $+$ | $-$ | $+$ |
| $\psi^3$ | $\frac{1}{2}b_0^{2+}$ | $-$ | $-$ | $+$ | $-$ |
| $\psi^4$ | $\frac{1}{2}b_0^2$ | $-$ | $-$ | $-$ | $+$ |
| $\psi^5$ | $\frac{1}{2}b_0^3$ | $+$ | $-$ | $+$ | $+$ |
| $\psi^6$ | $\frac{1}{2}b_0^{3+}$ | $+$ | $-$ | $-$ | $-$ |
| $\psi^7$ | $\frac{1}{2}b_0^4$ | $-$ | $+$ | $+$ | $+$ |
| $\psi^8$ | $\frac{1}{2}b_0^{4+}$ | $-$ | $+$ | $-$ | $-$ |

Table 4: The rotation properties of $\psi$ ending on a D5-brane.

| state | representation | energy | field |
|-------|----------------|--------|-------|
| $|0\rangle$ | $1^0$ | $0$ | $\phi$ |
| $b^\alpha+|0\rangle$ | $2^{\frac{1}{2}}$ | $\frac{1}{2}\omega_0$ | $\psi^\alpha$ |
| $b^\beta+|0\rangle$ | $2^{-\frac{1}{2}}$ | $\frac{1}{2}\omega_0$ | $\psi^\beta$ |
| $b^\alpha+b^\beta+|0\rangle$ | $1^1$ | $\omega_0$ | $A$ |
| $b^\alpha+b^\beta+|0\rangle$ | $1^0 \oplus 3^0$ | $\omega_0$ | $A_0 \oplus A^i$ |
| $b^\alpha+b^\beta+|0\rangle$ | $1^{-1}$ | $\omega_0$ | $\tilde{A}$ |
| $b^{\alpha}b^{\beta}b^{\gamma}+|0\rangle$ | $2^{-\frac{1}{2}}$ | $\frac{3}{2}\omega_0$ | $\tilde{\psi}^\alpha$ |
| $b^{\alpha}b^{\beta}b^{\gamma}+|0\rangle$ | $2^{\frac{1}{2}}$ | $\frac{3}{2}\omega_0$ | $\tilde{\psi}^\beta$ |
| $b^{\alpha}b^{\beta}b^{\gamma}b^{\delta}b^{\gamma}+|0\rangle$ | $1^0$ | $2\omega_0$ | $\tilde{\phi}$ |

Table 5: The spectrum of an open string ending on D5-brane.

Here, since the world-volume theory of D5-brane in the light-cone coordinates has a SO(3) symmetric group with a $\tilde{\text{SO}}(3)$ internal R-symmetric group, we will find the field contents in terms of three-dimensional space language. In three-dimensional space, as shown in the table 5., $A^i$ is a vector vector field, which is composed by three components representing the coordinates, $X^1$, $X^2$ and $X^3$. This vector field, $A^i$ transforms as a vector multiplet under SO(3) symmetric group. In addition, $A$ and $\tilde{A}$ are scalar fields whose linear combinations imply the scalar fields of $x^5$ and $x^6$ coordinates. $A^0$ implies $X^7$ field which is a coordinates of target space. This $A^0$ together with $A$ and $\tilde{A}$ become a vector multiplet under a $\tilde{\text{SO}}(3)$ internal R-symmetric group. The linear combinations of the other scalar fields, $\phi$ and $\tilde{\phi}$, represent $X^4$ and $X^8$. In four-dimensional world-volume of D5-brane, the vector field $A^i$ in three-dimensional space and a scalar field $X^8$ become a vector field of D5-brane world-volume theory.
Table 6: The rotation properties of $\psi$ in transverse eight-dimension.

| fermions | operators | $i\gamma^{12}$ | $i\gamma^{34}$ | $i\gamma^{56}$ | $i\gamma^{78}$ |
|----------|-----------|----------------|----------------|----------------|----------------|
| $\psi^1$ | $\frac{1}{2}b^{1+}_0$ | +              | -              | +              | -              |
| $\psi^2$ | $\frac{1}{2}b^{1+}_0$ | -              | +              | +              | -              |
| $\psi^3$ | $\frac{1}{2}b^{2+}_0$ | +              | -              | -              | +              |
| $\psi^4$ | $\frac{1}{2}b^{2+}_0$ | -              | +              | -              | +              |
| $\psi^5$ | $\frac{1}{2}b^{3+}_0$ | -              | -              | +              | +              |
| $\psi^6$ | $\frac{1}{2}b^{3+}_0$ | +              | +              | +              | +              |
| $\psi^7$ | $\frac{1}{2}b^{4+}_0$ | -              | -              | -              | -              |
| $\psi^8$ | $\frac{1}{2}b^{4+}_0$ | +              | +              | -              | -              |

3.3 For D3-brane

Now, we consider the D3-brane lying in $x^1$ and $x^2$ directions. This D3-brane configuration breaks the $SO(4) \times SO(4)$ symmetric group of the pp-wave background into the $SO(2) \times \tilde{SO}(2) \times SU(2) \times SU(2)$ where the first, $SO(2)$ is the Lorentz symmetry group of the D3-brane world-volume theory in the light-cone gauge. The other group, $\tilde{SO}(2) \times SU_L(2) \times SU_R(2)$ is the internal symmetric group representing the R-symmetry. To construct the Fock space, we have to choose the fermion which is an eigenstate of $\Omega \Pi$, otherwise there are extra term in the Hamiltonian, as previously mentioned (see Eq. (3.5)). Since $\Omega \Pi = -\gamma^{34}$, consider the fermions with some quantum numbers under the Lorentz group and the internal group, as shown in the table 5.

In the table 6, each $\psi$ is an one-component Weyl fermion which is the fermionic field in the D3-brane world-volume theory. Now, we define the Cartan generators of $SU_L(2)$ and $SU_R(2)$ internal symmetric groups as $J_{3L}$ and $J_{3R}$:

$$J_{3L} \equiv \frac{1}{2}(T_{56} + T_{78}),$$
$$J_{3R} \equiv \frac{1}{2}(T_{56} - T_{78}).$$

Then, we find that $(\psi^1, \psi^3)$ and $(\psi^2, \psi^4)$ become doublets under $SU_R(2)$ internal group and $(\psi^5, \psi^7)$ and $(\psi^6, \psi^8)$ become doublets under $SU_L(2)$ symmetric group. If we choose $\psi^a$, where $a = 1, 3, 5, 7$, as creation operators and the rest as annihilation operators, the Hamiltonian for the open string modes ending on D3-brane can be rewritten by a harmonic oscillator form:

$$\mathcal{H} = \text{sign}(\alpha) \left[ \frac{1}{2} \sum_{n \geq 0} \omega_n \left( a_n^I a_n^I + a_n^A a_n^A + b_n^A b_n^A - b_n^A b_n^A \right) \right],$$

where in the case of the zero modes ($n = 0$), $I$ runs from 1 to 2 and $A$ from 1 to 4. Therefore, the zero point energy, which is caused by the different numbers of bosonic and fermionic zero modes, is given by $E_0 = -\text{sign}(\alpha) \omega_0$. 

12
By many authors, the open string theory and their superalgebra on the D-brane in the pp-wave background are studied. In this paper, we have exactly shown that the field contents of the SYM theory with the non-trivial background are obtained from the open string theory in the pp-wave background. Note that the world-volume theory is described by a low dimensional representation, which arises due to the non-trivial symmetric group. Like the Ref. [41] where SYM theory as a world-volume theory of D3-brane in the pp-wave background is studied, it has been believed that the world-volume theory on the D-brane in the pp-wave background is SYM theory. However, although it is not clear how in the GS formalism we can obtain the world-volume theory such as the Dirac-Born-Infeld action in the flat space background, the result of this paper gives some evidences that the world-volume theory of D-brane in the pp-wave can be described by a SYM theory.

In the case of the closed string field theory, in the Ref. [16] the interaction Hamiltonian of the three closed strings was obtained and in the Ref. [13] it was shown that this interaction Hamiltonian is a correct one compared with the supergravity calculation. In the open string field theory case, the interaction Hamiltonian was obtained using the similar method of the

| state | (spin) representation | energy | field |
|-------|-----------------------|--------|-------|
| $|1, 0\rangle$ | $(0)^{(1,0,0)}$ | $-\omega_0$ | $\bar{A}$ |
| $b_0^A|1, 0\rangle$ | $\bigoplus \left( \frac{1}{2} \right)^{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}} \bigoplus \left( \frac{1}{2} \right)^{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}} \bigoplus \left( -\frac{1}{2} \right)^{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}} \bigoplus \left( -\frac{1}{2} \right)^{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}}$ | $-\frac{1}{2}\omega_0$ | $\psi^A$ |
| $b_0^{A+}b_0^{B+}|1, 0\rangle$ | $\bigoplus \left( 1 \right)^{(0,0,0)} \bigoplus \left( -1 \right)^{(0,0,0)} \bigoplus \left( 0 \right)^{(0,1,0)} \bigoplus \left( 0 \right)^{(0,-1,0)} \bigoplus \left( 0 \right)^{(0,0,1)} \bigoplus \left( 0 \right)^{(0,0,-1)}$ | $0$ | $A^i$ $\bigoplus \phi^1 \bigoplus \phi^2$ |
| $b_0^{A+}b_0^{B+}b_0^C+|1, 0\rangle$ | $\bigoplus \left( -\frac{1}{2} \right)^{\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}} \bigoplus \left( -\frac{1}{2} \right)^{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}} \bigoplus \left( \frac{1}{2} \right)^{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}} \bigoplus \left( \frac{1}{2} \right)^{\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}}$ | $\frac{1}{2}\omega_0$ | $\bar{\psi}^A$ |
| $b_0^{A+}b_0^{B+}b_0^C+b_0^{D+}|1, 0\rangle$ | $(0)^{(-1,0,0)}$ | $\omega_0$ | $A$ |

Table 7: The spectrum of an open string ending on D3-brane.

Note that the creation operators have $-\frac{1}{2}$ quantum number under $T_{34} \equiv \frac{1}{2}i\gamma^{34}$. Hence, we choose the vacuum which has +1 eigenvalue of $T_{34}$ and represents $|1, 0\rangle$. This vacuum has to be annihilated by the annihilation operators $b_0^A$ and corresponds to the scalar field like $\frac{1}{\sqrt{2}}(X^3 + iX^4)$, which has +1 eigenvalue under SO(2). All field contents coming from the fermionic zero modes are shown in the table 7.

In the table 7, $A$ and $\bar{A}$ are the linear combinations of the $X^3$ and $X^4$ scalar fields. Moreover, $(\phi^1, \bar{\phi}^1)$ and $(\phi^2, \bar{\phi}^2)$ are also the linear combinations of $(X^5, X^6)$ and $(X^7, X^8)$, respectively.

4 Discussion

By many authors, the open string theory and their superalgebra on the D-brane in the pp-wave background are studied. In this paper, we have exactly shown that the field contents of the SYM theory with the non-trivial background are obtained from the open string theory in the pp-wave background. Note that the world-volume theory is described by a low dimensional representation, which arises due to the non-trivial symmetric group. Like the Ref. [41] where SYM theory as a world-volume theory of D3-brane in the pp-wave background is studied, it has been believed that the world-volume theory on the D-brane in the pp-wave background is SYM theory. However, although it is not clear how in the GS formalism we can obtain the world-volume theory such as the Dirac-Born-Infeld action in the flat space background, the result of this paper gives some evidences that the world-volume theory of D-brane in the pp-wave can be described by a SYM theory.

In the case of the closed string field theory, in the Ref. [16] the interaction Hamiltonian of the three closed strings was obtained and in the Ref. [13] it was shown that this interaction Hamiltonian is a correct one compared with the supergravity calculation. In the open string field theory case, the interaction Hamiltonian was obtained using the similar method of the
closed string field theory. To believe that this interaction Hamiltonian is correct, we have to compare with the SYM theory result. So it is very interesting to compare the open string field theory result with that of SYM theory, we expect that the result of this paper has a crucial role in studying that topic.

Acknowledgement: We thank H. S. Yang and B. Lee for helpful discussion. This work was supported by Korea Research Foundation Grant. (KRF-2002-037-C00007).

Appendix

A Spinor representation

In this section, we will give a summary for a symmetry group, especially SO(4) in four-dimensional Euclidean space which was used in constructing spinors on D5-brane world-volume from type IIB string theory, see also a ref [42] for more details. In four-dimensional Euclidean space, the symmetry group leaving the metric invariant is O(4). Let $V$ denote a four component real vector with entries $V^I$. Then, this O(4) symmetry group is composed of SO(4) symmetry group, which is a proper subgroup of O(4), and a parity transformation: reflection of an odd number of directions, e.g. $V^I \rightarrow -V^I$.

Here, we concentrate on a proper subgroup, SO(4). A vector representation of SO(4) is characterized by invariance of a quadratic form under a group of linear transformation:

$$\langle V, V \rangle \equiv G_{IJ} V^I V^J, \quad \text{(A.1)}$$

where $I$ runs from 0 to 3. Since in a Euclidean space, without loss of generality, we can set the metric, $G_{IJ}$ to be equal to an identity matrix $\delta_{IJ}$, so $\langle V, V \rangle$ is just the sum of the squares of $V^I$. Now define

$$V^\alpha \bar{\alpha} \equiv (V^0 I + i V^i \sigma^i)^{\alpha} \bar{\alpha},$$

$$= \begin{pmatrix} V^0 + V^3 & V^2 + V^1 \\ - V^2 + V^1 & V^0 - V^3 \end{pmatrix}, \quad \text{(A.2)}$$

where $i$ runs from 1 to 3. Then, Eq.(A.1) can be rewritten as

$$\det V = \langle V, V \rangle, \quad \text{(A.3)}$$

so a component of a four vector, $V^I$, is given by

$$V^0 = \frac{1}{2} \text{Tr} (V), \quad V^i = -\frac{i}{2} \text{Tr} (V \sigma^i). \quad \text{(A.4)}$$
Due to a reality condition of $V'$, a complex conjugate of $V$ is given by
\[(V^\alpha_\alpha)^* = (\sigma^2)^\alpha_\beta V^\beta_\beta (\sigma^2)_{\bar{\alpha}}^{\bar{\beta}}. \tag{A.5}\]

Now let us consider a linear transformation:
\[V \rightarrow U V U', \tag{A.6}\]

Imposing a reality condition, Eq.(A.5), and an invariance of the above quadratic form, Eq.(A.1), gives two constraints for a linear transformation:
\[
\begin{align*}
\det U &= 1, \\
\sigma^2 U \sigma^2 &= U^*,
\end{align*} \tag{A.7}
\]

and there are also the same constraints for $U'$. These constraints imply that $U$ and $U'$ are just elements of SU(2) group. Note that there is no condition to relate $U$ and $U'$. To distinguish these two SU(2) groups, we denote $U$ and $U'$ as elements of SU(2)$_L$ and SU(2)$_R$, respectively. As a result, a vector of SO(4) can be described by a bi-spinor form as Eq.(A.2), which realizes the (2,2) representation of SU(2)$_L \times$ SU(2)$_R$. Using Eq.(A.1) and Eq.(A.2), we can easily convert a vector representation of SO(4) to a bi-spinor representation, or vice versa.

This SO(4) symmetry group has several subgroups. Clearly, SO(3) group is an subgroup of SO(4) symmetry group, which leaves $V^0$ invariant. Equivalently, this subgroup can be realized as a diagonal subgroup of SU(2)$_L \times$ SU(2)$_R$. The invariance of $V^0$ under a linear transformation Eq.(A.6), gives the following relation
\[UU' = I, \tag{A.8}\]

which characterizes a diagonal subgroup of SU(2)$_L \times$ SU(2)$_R$. Usually, since an element of a Lie group can be written as the following form
\[U = e^{i\Lambda_i \sigma^i}, \tag{A.9}\]

the above relation Eq.(A.8), means that $U'$ must be given by
\[U' = e^{-i\Lambda_i \sigma^i}. \tag{A.10}\]

Now, define the Cartan generators, $J_{3L} \equiv \sigma^3/2$ in SU(2)$_L$ and $J_{3R} \equiv \sigma^3/2$ in SU(2)$_R$. Due to the equivalence of $J_{3L}$ and $J_{3R}$ in a diagonal subgroup, an eigenstate of $J_{3L}$ is also that of $J_{3R}$. Suppose that there is a fermion which is an eigenstate of $J_{3L}$, then this fermion can be characterized by a quantum number of $J_{3L}$ and $J_{3R}$. However, since $J_{3L} - J_{3R} = 0$ a fermion is characterized by a quantum number of $J_3 \equiv J_{3L} + J_{3R}$ only. Finally, to find an appropriate fermion described by spinor representation of a SO(3) subgroup, we have to choose a spinor which is an eigenstate of $J_3$. 

References

[1] M. Blau, J. Figueroa-O’Farrill, C. Hull, and G. Papadopoulos, J. High Energy Phys. 01, 047 (2002), hep-th/0110242; Class. Quant. Grav. 19, L87 (2002), hep-th/0201081.

[2] R. R. Metsaev, Nucl. Phys. B625, 70 (2002), hep-th/0112044.

[3] R. R. Metsaev and A. A. Tseytlin, Phys. Rev. D65, 126004 (2002), hep-th/0202109.

[4] D. Berenstein, J. Maldacena, H. Nastase, J. High Energy Phys. 0204, 013 (2002).

[5] C. Kristjansen, J. Plefka, G. W. Semenoff, and M. Staudacher, Nucl. Phys. B643, 3 (2002), hep-th/0205033.

[6] D. J. Gross, A. Mikhailov, and R. Roiban, Ann. Phys. 30, 31 (2002), hep-th/0205066.

[7] N. R. Constable, D. Z. Freedman, M. Headrick, S. Minwalla, L. Motl, A. Postnikov, and W. Skiba, J. High Energy Phys. 07, 017 (2002), hep-th/0205089.

[8] C.-S. Chu, V. V. Khoze, and G. Travaglini, J. High Energy Phys. 06, 011 (2002), hep-th/0206005.

[9] A. Santambrogio and D. Zanon, Phys. Lett. B545, 425 (2002), hep-th/0206079.

[10] N. Beisert, C. Kristjansen, J. Plefka, G. W. Semenoff, and M. Staudacher, Nucl. Phys. B650, 125 (2003), hep-th/0208178.

[11] N. R. Constable, D. Z. Freedman, M. Headrick, and S. Minwalla, J. High Energy Phys. 10, 068 (2002), hep-th/0209002.

[12] U. Gürsoy, hep-th/0208041; hep-th/0212118.

[13] Y. Kiem, Y. Kim, S. Lee, and J. Park, Nucl. Phys. B642, 389 (2002), hep-th/0205279.

[14] M.-x. Huang, Phys. Lett. B542, 255 (2002), hep-th/0205311.

[15] P. Lee, S. Moriyama, and J. Park, Phys. Rev. D66, 085021 (2002), hep-th/0206065.

[16] M. Spradlin and A. Volovich, J. High Energy Phys. 01, 036 (2003), hep-th/0206073

[17] C.-S. Chu, V. V. Khoze, M. Petrini, R. Russo, and A. Tanzini, hep-th/0208148.

[18] A. Pankiewicz, J. High Energy Phys. 09, 056 (2002), hep-th/0208209.

[19] A. Pankiewicz and B. Stefański, Jr, Nucl. Phys. B657, 79 (2003), hep-th/0210246.
[20] Y. Kiem, Y. Kim, J. Park, and C. Ryou, J. High Energy Phys. 01, 026 (2003), hep-th/0211217.

[21] M. Spradlin and A. Volovich, Phys. Rev. D66, 086004 (2002).

[22] M. B. Green and J. H. Schwarz, L. Brink, Nucl. Phys. B219, 437 (1983).

[23] M. B. Green and J. H. Schwarz Nucl. Phys. B243, 285 (1984).

[24] M. B. Green and J. H. Schwarz, Nucl. Phys. B243, 475 (1984).

[25] B. Chandrasekhar and A. Kumar, J. High Energy Phys. 0306, 001 (2003); B. Stefanski Jr, hep-th/0304114.

[26] N.D. Lambert and P.C. West, Phys. Lett. B459, 515 (1999).

[27] A. Dabholkar and S. Parvizi, Nucl. Phys. B641, 223 (2002).

[28] M. Billo’ and I. Pesando, Phys. Lett. B536, 121 (2002), hep-th/0203028.

[29] A. Dabholkar and S. Parvizi, Nucl. Phys. B641, 223 (2002), hep-th/0203231.

[30] K. Skenderis and M. Taylor, J. High Energy Phys. 06, 025 (2002), hep-th/0205054.

[31] O. Bergman, M. R. Gaberdiel, and M. B. Green, hep-th/0205183.

[32] P. Bain, K. Peeters, and M. Zamaklar, Phys. Rev. D67, 066001 (2003), hep-th/0208038; P.Bain, P.Meessen, M.Zamaklar, Class. Quant. Grav. 20, 913 ((2003).

[33] Y. Hikida and S. Yamaguchi, J. High Energy Phys. 01, 072 (2003), hep-th/0210262.

[34] K. Skenderis and M. Taylor, hep-th/0211011.

[35] K. Skenderis and M. Taylor, hep-th/0212184.

[36] M. R. Gaberdiel and M. B. Green, hep-th/0211122; M. R. Gaberdiel, M. B. Green, S. Schafer-Nameki and A. Sinha, hep-th/0306056.

[37] J. Kim, B. Lee and H. S. Yang, Phys. Rev. D68, 026004 (2003).

[38] K. Cha, B. Lee and H. Yang, hep-th/0307146.

[39] N. Ohta, K. L. Panigrahi and S. Jhingan, hep-th/0306186.

[40] S. Hyun, J. Park and S. Yi, J. High Energy Phys. 0303, 004 (2003), J. High Energy Phys. 0211, 001 (2002).
[41] R.R. Metsaev, Nucl. Phys. B655, 3 (2003).

[42] Y. E. Cheung, Yaron Oz and Zheng Yin, hep-th/0211147.