SIVERS FUNCTION: SIDIS DATA, FITS AND PREDICTIONS*

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The most recent data on the weighted transverse single spin asymmetry \(A_{UT}^{\sin(\phi_h - \phi_S)}\) from HERMES and COMPASS collaborations are analysed within LO parton model; all transverse motions are taken into account. Extraction of the Sivers function for \(u\) and \(d\) quarks is performed. Based on the extracted Sivers functions, predictions for \(A_{UT}^{\sin(\phi_h - \phi_S)}\) asymmetries at JLab are given; suggestions for further measurements at COMPASS, with a transversely polarized hydrogen target and selecting favourable kinematical ranges, are discussed. Predictions are also presented for Single Spin Asymmetries (SSA) in Drell-Yan processes at RHIC and GSI.

1. Introduction

In recent papers¹,² we have discussed the role of intrinsic motions in inclusive and Semi-Inclusive Deep Inelastic Scattering (SIDIS) processes, both in unpolarized and polarized \(\ell p \rightarrow \ell h X\) reactions. The LO QCD parton model computations have been compared with data on Cahn effect³; this allows an estimate of the average values of the transverse momenta of quarks inside a proton, \(k_\perp\), and of final hadrons inside the fragmenting quark jet, \(p_\perp\), with the best fit results: \(\langle k_\perp^2 \rangle = 0.25\, \text{(GeV/c)}^2\), \(\langle p_\perp^2 \rangle = 0.20\, \text{(GeV/c)}^2\).
More detail, both about the kinematical configurations and conventions and the fitting procedure can be found in Ref. [1].

Equipped with such estimates, we have studied transverse single spin asymmetries $A_{UT}^\sin(\phi_+ - \phi_S)$ observed by HERMES collaboration$^6$ and COMPASS collaboration$^7$; that allowed extraction of the Sivers function$^5$

$$\Delta^N f_{q/p}(x, k_\perp) = -\frac{2}{m_p} k_\perp J_{1T}(x, k_\perp),$$  \hspace{1cm} (1)

defined by

$$f_{q/p}(x, k_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p}(x, k_\perp) S \cdot (\hat{P} \times \hat{k}_\perp),$$  \hspace{1cm} (2)

where $f_{q/p}(x, k_\perp)$ is the unpolarized $x$ and $k_\perp$ dependent parton distribution ($k_\perp = |k_\perp|$); $m_p$, $P$ and $S$ are respectively the proton mass, momentum and transverse polarization vector ($\hat{P}$ and $\hat{k}_\perp$ denote unit vectors).

We consider here these whole new sets of HERMES$^6$ and COMPASS$^7$ data and perform a novel fit$^2$ of the Sivers functions. It turns out that the data well constrain the parameters, thus offering the first direct significant estimate of the Sivers functions – for $u$ and $d$ quarks – active in SIDIS processes. The sea quark contributions are found to be negligible, at least in the kinematical region of the available data. Finally, we exploit the QCD prediction$^8$ $f_{q/p}(x, k_\perp)|_{D-Y} = -f_{q/p}(x, k_\perp)|_{DIS}$ and compute a single spin asymmetry, which can only originate from the Sivers mechanism$^{10}$, for Drell-Yan processes at RHIC and GSI. The issue of QCD factorization of SIDIS and Drell-Yan processes was studied in Ref.[9].

2. Extracting the Sivers functions

Following Ref.[1], the inclusive ($\ell p \to \ell X$) unpolarized DIS cross section in non collinear LO parton model is given by

$$\frac{d^2 \sigma_{\ell p \to \ell X}}{dx_a dQ^2} = \sum_q \int d^2 k_\perp f_q(x, k_\perp) \frac{d\hat{\sigma}_{\ell q \to \ell q}}{dQ^2} J(x_a, Q^2, k_\perp),$$  \hspace{1cm} (3)

and the semi-inclusive one ($\ell p \to \ell h X$) by

$$\frac{d^3 \sigma_{\ell p \to \ell h X}}{dx_a dQ^2 dz_h d^2 P_T} = \sum_q \int d^2 k_\perp f_q(x, k_\perp) \frac{d\hat{\sigma}_{\ell q \to \ell q}}{dQ^2} J \frac{z}{z_h} D_h(z, p_\perp),$$  \hspace{1cm} (4)

where

$$J = \frac{s^2}{x_a^2 x^2} \frac{x_0}{x} \left(1 + \frac{x_0 k_\perp^2}{x^2 Q^2}\right)^{-1}, \frac{d\hat{\sigma}_{\ell q \to \ell q}}{dQ^2} = e_q \frac{2\pi\alpha^2}{s^2} \frac{s^2 + u^2}{Q^4}. \hspace{1cm} (5)$$
\( Q^2, x_0 \) and \( y = Q^2/(x_0 s) \) are the usual leptonic DIS variables and 
\( z_h, P_T \) the usual hadronic SIDIS ones, in the \( \gamma^* - p \) c.m. frame; \( x \) and \( z \) 
are light-cone momentum fractions, with (see Ref.\[1\] for exact relationships 
and further detail):
\[
x = x_0 + \mathcal{O} \left( \frac{k_0^2}{Q^2} \right), \quad z = z_h + \mathcal{O} \left( \frac{k_0^2}{Q^2} \right), \quad p_\perp = P_T - z_h k_\perp + \mathcal{O} \left( \frac{k_0^2}{Q^2} \right).
\]

The \( \sin(\phi_h - \phi_S) \) weighted transverse single spin asymmetry, measured 
by HERMES and COMPASS, which singles out the contribution of the 
Sivers function (1), is given by:
\[
A_{UT}^{\sin(\phi_h - \phi_S)} = \left[ \sum_q \int d\phi_S d\phi_h d^2 k_\perp \Delta f_{q/p^\uparrow}^N(x, k_\perp) \sin(\varphi - \phi_S) \cdot \right.
\frac{d\sigma^{\ell q\rightarrow\ell q}}{dQ^2} J \frac{z}{z_h} D_q^h(z, p_\perp) \sin(\phi_h - \phi_S) \bigg] / \\
\left[ \sum_q \int d\phi_S d\phi_h \cdot d^2 k_\perp f_{q/p}(x, k_\perp) \frac{d\sigma^{\ell q\rightarrow\ell q}}{dQ^2} J \frac{z}{z_h} D_q^h(z, p_\perp) \bigg],
\]
where \( \varphi \) is the azimuthal angle of the quark transverse momentum, \( \phi_h \) and 
\( \phi_S \) are the azimuthal angles of produced hadron and polarization vector 
correspondingly. We shall use Eq. (6), in which we insert a parameterization 
for the Sivers functions, to fit the experimental data.

The \( k_\perp \) integrated parton distribution and fragmentation functions 
\( f_q(x) \) and \( D_q^h(z) \) are taken from the literature, at the appropriate \( Q^2 \) values 
of the experimental data \[11,12\].

We parameterize, for each light quark flavour \( q = u, d \), the Sivers function 
in the following factorized form:
\[
\Delta f_{q/p^\uparrow}^N(x, k_\perp) = 2 N_q(x) h(k_\perp) f_{q/p}(x, k_\perp),
\]
where
\[
N_q(x) = N_q x^{a_q}(1 - x)^{b_q} \frac{(a_q + b_q)(a_q + b_q)}{a_q b_q}, \quad h(k_\perp) = \frac{2k_\perp M_0}{k_\perp^2 + M_0^2}.
\]
\( N_q, a_q, b_q \) and \( M_0 \) (GeV/c) are free parameters. \( f_{q/p}(x, k_\perp) \) is the unpolarized 
distribution function. Since \( h(k_\perp) \leq 1 \) and since we allow the constant 
parameter \( N_q \) to vary only inside the range \([-1,1]\] so that \( |N_q(x)| \leq 1 \) for 
any \( x \), the positivity bound for the Sivers function is automatically fulfilled:
\[
\left| \frac{\Delta f_{q/p^\uparrow}^N(x, k_\perp)}{2 f_{q/p}(x, k_\perp)} \right| \leq 1.
\]
We neglect the contributions of sea quark functions and consider only the contributions of $\Delta N f_{u/p}$ and $\Delta N f_{d/p}$, for a total of 7 free parameters:

$$N_u \ a_u \ N_d \ a_d \ b_d \ M_0.$$  \hspace{1cm} (10)

The results of our fits are shown in Figs. 1 and 2.

In Fig.1 we also show predictions, obtained using the extracted Sivers functions (see Table 1), for $\pi^0$ and $K$ production; data on these asymmetries might be available soon from HERMES collaboration.

Table 1. Best fit values of the parameters of the Sivers functions.

| Parameter | Value       |
|-----------|-------------|
| $N_u$     | $0.32 \pm 0.11$ |
| $a_u$     | $0.29 \pm 0.35$ |
| $N_d$     | $-1.00 \pm 0.12$ |
| $a_d$     | $1.16 \pm 0.47$ |
| $b_d$     | $3.77 \pm 2.59$ |
| $M_0$     | $0.32 \pm 0.25$ (GeV/c)$^2$ |
| $\chi^2$/d.o.f. | 1.06 |

Figure 1. HERMES data on $A_{UT}^{\sin(\phi_T - \phi_S)}$ for scattering off a transversely polarized proton target and charged pion production. The curves are the results of our fit. The shaded area spans a region corresponding to one-sigma deviation at 90% CL. Predictions for $\pi^0$ (upper-left panel) and kaon (right panels) asymmetries are also shown.

3. $A_{UT}^{\sin(\phi_T - \phi_S)}$ at COMPASS with polarized hydrogen target

By inspection of Eq. (6) it is easy to understand our numerical results for the $u$ and $d$ Sivers functions. In fact one can see that for scattering off a
Figure 2. COMPASS data on $A_{UT}^{\sin(\phi_h-\phi_S)}$ for scattering off a transversely polarized deuteron target and the production of positively ($h^+$) and negatively ($h^-$) charged hadrons. The curves are the results of our fit. The shaded area spans a region corresponding to one-sigma deviation at 90% CL.

While, for scattering off a deuterium target (COMPASS),

$$\left(A_{UT}^{\sin(\phi_h-\phi_S)}\right)_{\text{deuterium}} \sim \left(\Delta^N f_{u/p}\right) \left(4 D_u^h + D_d^h\right).$$

Opposite $u$ and $d$ Sivers contributions suppress COMPASS asymmetries for any hadron $h$.

However, the COMPASS collaboration will soon be taking data with a transversely polarized hydrogen target. Adopting the same experimental cuts which were used for the deuterium target, the asymmetry is found to be around 5% (see Fig. 3). These expected values can be further increased by properly selecting the experimental data. For example, selecting events with

$$0.4 \leq z_h \leq 1 \quad 0.2 \leq P_T \leq 1 \text{ GeV/c} \quad 0.02 \leq x_B \leq 1,$$

yields the predictions shown in the right panel of Fig. 3. The asymmetry for positively charged hadrons becomes larger, and one expects a clear observation of a sizeable azimuthal asymmetry also for the COMPASS experiment.
Figure 3. Predictions for $A_{U_T}^{\sin(\phi_h - \phi_S)}$ at COMPASS for scattering off a transversely polarized proton target and the production of positively ($h^+$) and negatively ($h^-$) charged hadrons. The plots in the left panel have been obtained by performing the integrations over the unobserved variables according to the standard COMPASS kinematical cuts; results with suggested new cuts, Eq. (13), are presented in the right panel.

4. $A_{U_T}^{\sin(\phi_h - \phi_S)}$ at JLab with polarized hydrogen target

Also JLab experiments are supposed to measure the SIDIS azimuthal asymmetry for the production of pions on a transversely polarised hydrogen target, at incident beam energies of 6 and 12 GeV. The kinematical region of this experiment is very interesting, as it will supply information on the behaviour of the Sivers functions in the large-$x_B$ domain, up to $x_B \simeq 0.6$.

Imposing the experimental cuts of JLab we obtain the predictions shown in Fig. 4. A large and healthy azimuthal asymmetry for $\pi^+$ production should be observed. Similar results have been obtained also in an approach based on a Monte Carlo event generator $^{13}$.

5. Transverse SSA in Drell-Yan processes

Let us now consider the transverse single spin asymmetry,

$$A_N = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-},$$

for Drell-Yan processes, $p^+ p \to \ell^+ \ell^- X$, $p^+ \bar{p} \to \ell^+ \ell^- X$ and $\bar{p}^+ p \to \ell^+ \ell^- X$, where $d\sigma$ stands for $d^4\sigma/dy\,dM^2\,d^2q_T$ and $y$, $M^2$ and $q_T$ are respectively the rapidity, the squared invariant mass and the transverse momentum of the lepton pair in the initial nucleon c.m. system.

In such a case the SSA (14) can only originate from the Sivers function...
and is given (selecting the region with $q_T^2 \ll M^2$, $q_T \simeq k_\perp$) by\(^{10}\)

$$A_N = \left[ \sum_q e_q^2 \int d^2 k_{\perp q} d^2 k_{\perp \bar{q}} \delta^2(k_{\perp q} + k_{\perp \bar{q}} - q_T) \Delta N f_{q/p}(x_q, k_{\perp q}) \right] / \left[ 2 \sum_q e_q^2 \int d^2 k_{\perp q} d^2 k_{\perp \bar{q}} \delta^2(k_{\perp q} + k_{\perp \bar{q}} - q_T) \cdot f_{\bar{q}/p}(x_{\bar{q}}, k_{\perp \bar{q}}) f_{q/p}(x_q, k_{\perp q}) \right],$$

where $q = u, \bar{u}, d, \bar{d}, s, \bar{s}$ and $x_q = \frac{M}{\sqrt{s}} e^y$, $x_{\bar{q}} = \frac{M}{\sqrt{s}} e^{-y}$. Eq. (15) explicitely refers to $p^+ p$ processes, with obvious modifications for $p^1 \bar{p}$ and $\bar{p}^+ p$ ones.

Inserting into Eq. (15) the Sivers functions extracted from our fit to SIDIS data and reversed in sign\(^8\), we obtain the predictions (shown in Fig. 5) for RHIC (left panel) and PAX (right panel) experiment\(^{14}\) planned at the proposed asymmetric $p \bar{p}$ collider at GSI.

Figure 4. Predictions for $A^{\sin(\phi_{\pi^+} - \phi_{\pi^-})}_{UT}$ at JLab for the production of $\pi^+$ and $\pi^-$ from scattering off a transversely polarized proton target.

Figure 5. Predictions for single spin asymmetries in Drell-Yan, $p^+ p \rightarrow \ell^+ \ell^- X$, processes at RHIC (left panel) and GSI (right panel), according to Eq. (15) of the text.
6. Comments and conclusions

The Sivers functions $\Delta^N f_{u/p^+}(x, k_T)$ and $\Delta^N f_{d/p^+}(x, k_T)$ have been extracted using recent HERMES 6 and COMPASS 7 collaborations data on $A_{UT}^{\sin(\phi_h - \phi_S)}$.

A sizeable $h^+$ asymmetry should be measured by COMPASS collaboration once they switch, as planned, to a transversely polarized hydrogen target.

Large values of $A_{UT}^{\sin(\phi_h - \phi_S)}$ are expected at JLab, both in the 6 and 12 GeV operational modes, for $\pi^+$ inclusive production.

We have then used basic QCD relations and computed the single spin asymmetries in Drell-Yan processes. We have used the same Sivers functions as extracted from SIDIS data, with opposite signs. The predicted $A_N$ could be measured at RHIC in $pp$ collisions and, in the long range, at the proposed PAX experiment at GSI 14, in $p\bar{p}$ interactions. It would provide a clear and stringent test of basic QCD properties.

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