Well-dominated graphs without cycles of lengths 4 and 5

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Abstract

Let $G$ be a graph. A set $S$ of vertices in $G$ dominates the graph if every vertex of $G$ is either in $S$ or a neighbor of a vertex in $S$. Finding a minimal cardinality set which dominates the graph is an $\text{NP}$-complete problem. The graph $G$ is well-dominated if all its minimal dominating sets are of the same cardinality. The complexity status of recognizing well-dominated graphs is not known. We show that recognizing well-dominated graphs can be done polynomially for graphs without cycles of lengths 4 and 5, by proving that a graph belonging to this family is well-dominated if and only if it is well-covered.

Assume that a weight function $w$ is defined on the vertices of $G$. Then $G$ is $w$-well-dominated if all its minimal dominating sets are of the same weight. We prove that the set of weight functions $w$ such that $G$ is $w$-well-dominated is a vector space, and denote that vector space by $\text{WWD}(G)$. We prove that $\text{WWD}(G)$ is a subspace of $\text{WCW}(G)$, the vector space of weight functions $w$ such that $G$ is $w$-well-covered. We provide a polynomial characterization of $\text{WWD}(G)$ for the case that $G$ does not contain cycles of lengths 4, 5, and 6.

Keywords: vector space, minimal dominating set, maximal independent set, well-dominated graph, well-covered graph

1 Introduction

1.1 Definitions and Notations

Throughout this paper $G$ is a simple (i.e., a finite, undirected, loopless and without multiple edges) graph with vertex set $V(G)$ and edge set $E(G)$.

Cycles of $k$ vertices are denoted by $C_k$. When we say that $G$ does not contain $C_k$ for some $k \geq 3$, we mean that $G$ does not admit subgraphs isomorphic to $C_k$. It is important to mention that these subgraphs are not necessarily induced. Let $\mathcal{G}(C_1, \ldots, C_{i_k})$ be the family of all graphs which do not contain $C_{i_1}, \ldots, C_{i_k}$. 

1
Let $u$ and $v$ be two vertices in $G$. The distance between $u$ and $v$, denoted $d(u,v)$, is the length of a shortest path between $u$ and $v$, where the length of a path is the number of its edges. If $S$ is a non-empty set of vertices, then the distance between $u$ and $S$, denoted $d(u,S)$, is defined by:

$$d(u,S) = \min\{d(u,s) : s \in S\}.$$  

For every $i$, denote

$$N_i(S) = \{x \in V(G) : d(x,S) = i\},$$
and

$$N_i[S] = \{x \in V(G) : d(x,S) \leq i\}.$$  

We abbreviate $N_1(S)$ and $N_1[S]$ to be $N(S)$ and $N[S]$, respectively. If $S$ contains a single vertex, $v$, then we abbreviate $N_i(\{v\})$, $N_i(\{v\})$, $N(\{v\})$, and $N[\{v\}]$ to be $N_i(v)$, $N_i[v]$, $N(v)$, and $N[v]$, respectively.

For every vertex $v \in V(G)$, the degree of $v$ is $d(v) = |N(v)|$. Let $L(G)$ be the set of all vertices $v \in V(G)$ such that either $d(v) = 1$ or $v$ is on a triangle and $d(v) = 2$.

### 1.2 Well-Covered Graphs

A set of vertices is independent if its elements are pairwise nonadjacent. Define $D(v) = N(v) \setminus N_2(v)$, and let $M(v)$ be a maximal independent set of $D(v)$. An independent set of vertices is maximal if it is not a subset of another independent set. An independent set is maximum if $G$ does not admit an independent set with a bigger cardinality. Denote $i(G)$ the minimal cardinality of a maximal independent set in $G$, where $\alpha(G)$ is the cardinality of a maximum independent set in $G$.

The graph $G$ is well-covered if $i(G) = \alpha(G)$, i.e. all maximal independent sets are of the same cardinality. The problem of finding a maximum cardinality independent set in $G$ is NP-complete. However, if the input is restricted to well-covered graphs, then a maximum cardinality independent set can be found polynomially using the greedy algorithm.

Let $w : V(G) \rightarrow \mathbb{R}$ be a weight function defined on the vertices of $G$. For every set $S \subseteq V(G)$, define $w(S) = \Sigma_{s \in S} w(s)$. The graph $G$ is $w$-well-covered if all maximal independent sets are of the same weight. The set of weight functions $w$ for which $G$ is $w$-well-covered is a vector space [2]. That vector space is denoted $WCW(G)$ [1].

Since recognizing well-covered graphs is co-NP-complete [14], finding the vector space $WCW(G)$ of an input graph $G$ is co-NP-hard. Finding $WCW(G)$ remains co-NP-hard when the input is restricted to graphs with girth at least 6 [12], and bipartite graphs [12]. However, the problem is polynomially solvable for $K_{1,3}$-free graphs [11], and for graphs with a bounded maximal degree [12]. For every graph $G$ without cycles of lengths 4, 5, and 6, the vector space $WCW(G)$ is characterized as follows.
Theorem 1 [10] Let $G \in \mathcal{G}(\hat{C}_4, \hat{C}_5, \hat{C}_6)$ be a graph, and let $w : V(G) \rightarrow \mathbb{R}$. Then $G$ is $w$-well-covered if and only if one of the following holds:

1. $G$ is isomorphic to either $C_7$ or $T_{10}$ (see Figure 1), and there exists a constant $k \in \mathbb{R}$ such that $w \equiv k$.

2. The following conditions hold:
   - $G$ is isomorphic to neither $C_7$ nor $T_{10}$.
   - For every two vertices, $l_1$ and $l_2$, in the same component of $L(G)$ it holds that $w(l_1) = w(l_2)$.
   - For every $v \in V(G) \setminus L(G)$ it holds that $w(v) = w(M(v))$ for some maximal independent set $M(v)$ of $D(v)$.

![Figure 1: The graph $T_{10}$](image)

Recognizing well-covered graphs is a restricted case of finding $WCW(G)$. Therefore, for all families of graphs for which finding $WCW(G)$ is polynomial solvable, recognizing well-covered graphs is polynomial solvable as well. Recognizing well-covered graphs is co-NP-complete for $K_{1,4}$-free graphs [3], but it is polynomially solvable for graphs without cycles of lengths 3 and 4 [7], for graphs without cycles of lengths 4 and 5 [8], or for chordal graphs [13].

1.3 Well-Dominated Graphs

Let $S$ and $T$ be two sets of vertices of the graph $G$. Then $S$ dominates $T$ if $T \subseteq N[S]$. The set $S$ is dominating if it dominates all vertices of the graph. A dominating set is minimal if it does not contain another dominating set. A dominating set is minimum if $G$ does not admit a dominating set with smaller cardinality. Let $\gamma(G)$ be the cardinality of a minimum dominating set in $G$, and let $\Gamma(G)$ be the maximal cardinality of a minimal dominating set of $G$. If $\gamma(G) = \Gamma(G)$ then the graph is well-dominated, i.e. all minimal dominating sets are of the same cardinality. This concept was introduced in [6], and further studied in [9]. The fact that every maximal independent set is also a minimal dominating set implies that

$$\gamma(G) \leq i(G) \leq \alpha(G) \leq \Gamma(G)$$

for every graph $G$. Hence, if $G$ is not well-covered, then it is not well-dominated.
Theorem 2 [6] Every well-dominated graph is well-covered.

In what follows, our main subject is the interplay between well-covered and well-dominated graphs.

Problem 3 WD
Input: A graph $G$.
Question: Is $G$ well-dominated?

It is even not known whether the WD problem is in NP [5]. However, the WD problem is polynomial for graphs with girth at least 6 [6], and for bipartite graphs [6]. We prove that a graph without cycles of lengths 4 and 5 is well-dominated if and only if it is well-covered. Consequently, by [6], recognizing well-dominated graphs without cycles of lengths 4 and 5 is a polynomial task.

Let $w : V(G) \rightarrow \mathbb{R}$ be a weight function defined on the vertices of $G$. Then $G$ is $w$-well-dominated if all minimal dominating sets are of the same weight. Let $WWD(G)$ denote the set of weight functions $w$ such that $G$ is $w$-well-dominated. It turns out that for every graph $G$, $WWD(G)$ is a vector space. Moreover, if $G$ is $w$-well-dominated then $G$ is $w$-well-covered, i.e., $WWD(G) \subseteq WCW(G)$.

Problem 4 WW $D$
Input: A graph $G$.
Output: The vector space of weight functions $w$ such that $G$ is $w$-well-dominated.

Finally, we supply a polynomial characterization of the WW $D$ problem, when its input is restricted to $G(C_4, C_5, \hat{C}_6)$.

2 Well-Dominated Graphs Without $C_4$ and $C_5$

A vertex $v$ is simplicial if $N[v]$ is a clique. In [8] the family $F$ of graphs is defined as follows. A graph $G$ is in the family $F$ if there exists $\{x_1, \ldots, x_k\} \subseteq V(G)$ such that $x_i$ is simplicial for each $1 \leq i \leq k$, and $\{N[x_i] : 1 \leq i \leq k\}$ is a partition of $V(G)$. Well-covered graphs without cycles of lengths 4 and 5 are characterized as follows.

Theorem 5 [8] Let $G \in G(\hat{C}_4, \hat{C}_5)$ be a connected graph. Then $G$ is well-covered if and only if one of the following holds:

1. $G$ is isomorphic to either $C_7$ or $T_{10}$.
2. $G$ is a member of the family $F$.

Actually, under the restriction $G \in G(\hat{C}_4, \hat{C}_5)$, the families of well-covered and well-dominated graphs coincide.
Theorem 6 Let $G \in \mathcal{G}(\overline{C}_4, \overline{C}_5)$ be a connected graph. Then $G$ is well-dominated if and only if it is well-covered.

Proof. By Theorem 5 if $G$ is not well-covered then it is not well-dominated.

Assume $G$ is well-covered, and it should be proved that $G$ is well-dominated. One can verify that $\gamma(C_7) = \Gamma(C_7) = 3$, and $\gamma(T_{10}) = \Gamma(T_{10}) = 4$. Therefore, $C_7$ and $T_{10}$ are well-dominated.

By Theorem 5 it remains to prove that if $G$ is a member of $F$ then it is well-dominated. There exists $\{x_1, \ldots, x_k\} \subseteq V(G)$ such that $x_i$ is simplicial for each $1 \leq i \leq k$, and $\{N[x_i] : 1 \leq i \leq k\}$ is a partition of $V(G)$. Define $V_i = N[x_i]$ for each $1 \leq i \leq k$. Let $S$ be a minimal dominating set of $G$. It is enough to prove that $|S| = k$. The fact that $S$ dominates $x_i$ implies that $S \cap V_i \neq \Phi$. Assume on the contrary that there exists $1 \leq i \leq k$ such that $|V_i \cap S| \leq 2$. Let $S' \subseteq S$ such that $|S' \cap V_i| = 1$ for each $1 \leq i \leq k$. Clearly, $S'$ dominates the whole graph, which is a contradiction. Therefore, $|S| = k$, and $G$ is well-dominated. \hfill \blacksquare

If $G \notin \mathcal{G}(\overline{C}_4)$, then Theorem 5 does not hold. Let $n \geq 3$. Obviously, $K_{n,n} \in \mathcal{G}(\overline{C}_5)$, and the cardinality of every maximal independent set of $K_{n,n}$ is $n$. Therefore, $K_{n,n}$ is well-covered. However, there exists a minimal dominating set of cardinality 2. Therefore, $K_{n,n}$ is not well-dominated.

If $G \notin \mathcal{G}(\overline{C}_5)$, then Theorem 5 does not hold. Let $G$ be comprised of three disjoint 5-cycles, $(x_1, \ldots, x_5), (y_1, \ldots, y_5), (z_1, \ldots, z_5)$, and a triangle $(x_1, y_1, z_1)$. Clearly, $G \in \mathcal{G}(\overline{C}_4)$, and every maximal independent set contains 2 vertices from each 5-cycle. Hence, the cardinality of every maximal independent set is 6, and $G$ is well-covered. However, $G$ is not well-dominated because it contains a minimal dominating set of cardinality 7: $\{x_1, x_2, x_5, y_3, y_4, z_3, z_4\}$.

3 Weighted Well-Dominated Graphs

Theorem 7 Let $G$ be a graph. Then the set of weight functions $w : V(G) \rightarrow \mathbb{R}$ such that $G$ is $w$-well-dominated is a vector space.

Proof. Obviously, if $w_0 \equiv 0$ then $G$ is $w_0$-well-dominated.

Let $w_1, w_2 : V(G) \rightarrow \mathbb{R}$, and assume that $G$ is $w_1$-well-dominated and $w_2$-well-dominated. Then there exist two constants, $t_1$ and $t_2$, such that $w_1(S) = t_1$ and $w_2(S) = t_2$ for every minimal dominating set $S$ of $G$. Let $\lambda \in \mathbb{R}$, and let $w : V(G) \rightarrow \mathbb{R}$ be defined by $w(v) = w_1(v) + \lambda w_2(v)$ for every $v \in V(G)$. Then for every minimal dominating set $S$ it holds that

$$w(S) = \sum_{s \in S} w(s) = \sum_{s \in S} (w_1(s) + \lambda w_2(s)) = \sum_{s \in S} w_1(s) + \lambda \sum_{s \in S} w_2(s) = t_1 + \lambda t_2,$$

and $G$ is $w$-well-dominated. \hfill \blacksquare

For every graph $G$, we denote the vector space of weight functions $w$ such that $G$ is $w$-well-dominated by $WWD(G)$.

Let $G$ be a graph, and let $w : V(G) \rightarrow \mathbb{R}$. Denote $mDS_w(G), MDS_w(G), mIS_w(G), MIS_w(G)$ the minimum weight of a dominating set, the maximum
weight of a minimal dominating set, the minimum weight of a maximal independent set, and the maximum weight of an independent set, respectively.

The fact that every maximal independent set is also a minimal dominating set implies that
\[
mDS_w(G) \leq mIS_w(G) \leq MIS_w(G) \leq MDS_w(G)
\]
for every graph \(G\) and every weight function \(w\) defined on its vertices.

If \(mIS_w(G) = MIS_w(G)\) then \(G\) is \(w\)-well-covered, and if \(mDS_w(G) = MDS_w(G)\) then \(G\) is \(w\)-well-dominated. Theorem 2 is an instance of the following.

**Corollary 8** For every graph \(G\) and for every weight function \(w : V(G) \rightarrow \mathbb{R}\), if \(G\) is \(w\)-well-dominated then \(G\) is \(w\)-well-covered, i.e., \(WWD(G)\) is a subspace of \(WCW(G)\).

Figure 2: An example of the definition of \(L^*(G)\). In this graph, \(G\), it holds that \(L^*(G) = \{y_1, \ldots, y_{11}\}\) and \(L(G) \setminus L^*(G) = \{x\}\). Let \(w : V(G) \rightarrow \mathbb{R}\). By Theorem 11 \(G\) is \(w\)-well-covered if and only if \(w(y_2) = w(y_3)\) and \(w(v) = w(M(v))\) for every \(v \in V(G) \setminus L(G)\). By Theorem 12 \(G\) is \(w\)-well-dominated if and only if \(G\) is \(w\)-well-covered and \(w(x) = 0\).

Let \(L^*(G)\) be the set of all vertices \(v \in V(G)\) such that either

- \(d(v) = 1\);

or

- the following conditions hold:
  - \(d(v) = 2\);
  - \(v\) is on a triangle, \((v, v_1, v_2)\);
Every maximal independent set of \( V(G) \setminus N_2[v] \) dominates at least one of \( N(v_1) \cap N_2(v) \) and \( N(v_2) \cap N_2(v) \).

Note that \( L^*(G) \subseteq L(G) \) (see Figure 2). Moreover, \( v \in L(G) \setminus L^*(G) \) if and only if the following conditions hold:

- \( d(v) = 2 \)
- \( v \) is on a triangle, \( (v, v_1, v_2) \).
- There exists a maximal independent set of \( V(G) \setminus N_2[v] \) which dominates neither \( N(v_1) \cap N_2(v) \) nor \( N(v_2) \cap N_2(v) \).

**Theorem 9** Let \( G \in \mathcal{G}(C_4, \hat{C}_5, \hat{C}_6) \) be a connected graph, and let \( w : V(G) \to \mathbb{R} \). Then \( G \) is \( w \)-well-dominated if and only if one of the following holds:

1. \( G \) is isomorphic to either \( C_7 \) or \( T_{10} \) (see Figure 7), and there exists a constant \( k \in \mathbb{R} \) such that \( w \equiv k \).

2. The following conditions hold:
   
   (a) \( G \) is isomorphic to neither \( C_7 \) nor \( T_{10} \).
   (b) For every two vertices, \( l_1 \) and \( l_2 \), in the same component of \( L(G) \) it holds that \( w(l_1) = w(l_2) \).
   (c) \( w(v) = 0 \) for every vertex \( v \in L(G) \setminus L^*(G) \).
   (d) For every \( v \in V(G) \setminus L(G) \) it holds that \( w(v) = w(M(v)) \) for some maximal independent set \( M(v) \) of \( D(v) \).

**Proof.** The following cases are considered.

**Case 1:** \( G \) IS ISOMORPHIC TO EITHER \( C_7 \) OR \( T_{10} \). If there does not exist a constant \( k \in \mathbb{R} \) such that \( w \equiv k \), then by Theorem [1] \( G \) is not \( w \)-well-covered. By Corollary [3] \( G \) is not \( w \)-well-dominated. Suppose that \( w \equiv k \) for some \( k \in \mathbb{R} \).

If \( G \) is isomorphic to \( C_7 \), then the cardinality of every minimal dominating set is 3. Hence, \( mDS_w(C_7) = MDS_w(C_7) = 3k \), and \( G \) is \( w \)-well-dominated. If \( G \) is isomorphic to \( T_{10} \), then the cardinality of every minimal dominating set is 4. Hence, \( mDS_w(T_{10}) = MDS_w(T_{10}) = 4k \), and \( G \) is \( w \)-well-dominated.

**Case 2:** \( L(G) = V(G) \).

In this case \( G \) is a complete graph with at most 3 vertices. In that case the cardinality of every minimal dominating set is 1. Therefore, \( G \) is \( w \)-well-dominated if and only if there exists a constant \( k \in \mathbb{R} \) such that \( w \equiv k \). In this case \( mDS_w(G) = MDS_w(G) = k \).

**Case 3:** \( L(G) \neq V(G) \) and \( G \) is isomorphic to neither \( C_7 \) nor \( T_{10} \).

Let \( N(L^*(G)) = \{ v_1, ..., v_k \} \).

Then

\[
L^*(G) \subseteq \bigcup_{1 \leq i \leq k} D(v_i) \subseteq L(G).
\]

For each \( 1 \leq i \leq k \) let \( V_i = \{ v_i \} \cup D(v_i) \).
Assume that Condition 2 holds. Let \( S \) be a minimal dominating set of \( G \). Then \( S \cap V_i \) is either \( v_i \) or \( M(v_i) \). Hence, \( w(S \cap V_i) = w(v_i) \) for every \( 1 \leq i \leq k \).

Let \( 1 \leq i < j \leq k \). Then \( V_i \cap V_j \subseteq L(G) \setminus L^*(G) \). Hence, \( w(x) = 0 \) for each \( x \in V_i \cap V_j \). The fact that \( w(v) = 0 \) for every \( v \in V \setminus N[L(G)] \) implies that

\[
\begin{align*}
w(S) &= w(S \setminus \left( \bigcup_{1 \leq i \leq k} V_i \right)) + \sum_{1 \leq i \leq k} w(S \cap V_i) - \sum_{1 \leq i \leq j \leq k} w(S \cap V_i \cap V_j) = \\
&= 0 + \sum_{1 \leq i \leq k} w(v_i) - 0 = \sum_{1 \leq i \leq k} w(v_i).
\end{align*}
\]

Hence,

\[
mDS_w(G) = MDS_w(G) = \sum_{1 \leq i \leq k} w(v_i),
\]

and \( G \) is \( w \)-well-dominated.

Assume that \( G \) is \( w \)-well-dominated. Then, by Corollary 8, \( G \) is \( w \)-well-covered. By Theorem 1, Conditions 2a, 2b and 2d hold. It remains to prove that Condition 2c holds as well. Let \( v \in L(G) \setminus L^*(G) \). We prove that \( w(v) = 0 \).

Let \( N(v) = \{v_1, v_2\} \), and let \( S \) be a maximal independent set of \( G \setminus N_2[v] \) which dominates neither \( N(v_1) \cap N_2(v) \) nor \( N(v_2) \cap N_2(v) \). For each \( 1 \leq i \leq 2 \) let \( S_i \) be a maximal independent set of \( (N(v_i) \cap N_2(v)) \setminus N(S) \). Define \( T_i = S \cup S_{2-i} \cup \{v\} \) for \( i = 1, 2 \). Define \( T_3 = S \cup S_1 \cup S_2 \cup \{v\} \) and \( T_4 = S \cup \{v_1, v_2\} \). Clearly, \( T_1, T_2, T_3 \) and \( T_4 \) are minimal dominating sets of \( G \).

For each \( i = 1, 2 \) the fact that \( w(T_i) = w(T_3) \) implies \( w(S_i \cup \{v\}) = w(v_i) \). Therefore \( w(S_i) + w(v) = w(v_i) \). The fact that \( w(T_3) = w(T_4) \) implies \( w(S_1 \cup S_2 \cup \{v\}) = w(\{v_1, v_2\}) \). Therefore, \( w(S_1) + w(S_2) + w(v) = w(v_1) + w(v_2) \). Hence, \( w(S_1) + w(S_2) + w(v) = w(S_1) + w(S_2) + 2w(v) \). Thus \( w(v) = 0 \).

**Corollary 10** \( \dim(WWD(G)) = |L^*(G)| \) for every graph \( G \in \mathcal{G}(\widehat{C}_4, \widehat{C}_5, \widehat{C}_6) \).

**Corollary 11** Suppose \( G \in \mathcal{G}(\widehat{C}_4, \widehat{C}_5, \widehat{C}_6) \). If \( L^*(G) = L(G) \), then \( WWD(G) = WCW(G) \). Otherwise, \( WWD(G) \subsetneq WCW(G) \).

Combining Corollaries 10 and 11 with Algorithm 20 from 11 we obtain the following.

**Corollary 12** If \( G \in \mathcal{G}(\widehat{C}_4, \widehat{C}_5, \widehat{C}_6) \), then

\[
|L^*(G)| = \dim(WWD(G)) \leq \dim(WCW(G)) = \alpha(G[L(G)]) .
\]

Theorem 9 does not hold if \( G \not\in \mathcal{G}(\widehat{C}_6) \). Let \( G \) be the graph with two edge disjoint 6-cycles, \((v_1, \ldots, v_6)\) and \((v_6, \ldots, v_{11})\). Clearly, \( G \in \mathcal{G}(\widehat{C}_4, \widehat{C}_5) \) and \( L(G) = L^*(G) = \emptyset \). However, the vector space \( WWD(G) \) is the set of all functions \( w : V(G) \to \mathbb{R} \) which satisfy

1. \( w(v_1) = w(v_2) = -w(v_4) = -w(v_5) \)
2. \( w(v_7) = w(v_8) = -w(v_{10}) = -w(v_{11}) \)
3. \( w(v_3) = w(v_6) = w(v_9) = 0 \)
4 Future Work

The main findings of the paper stimulate us to discover more cases, where the \( WD \) and/or \( WWD \) problems can be solved polynomially.

We have proved that if \( G \in G(\tilde{C}_4, \tilde{C}_5) \), then \( G \) is well-dominated if and only if it is well-covered. It motivates the following.

**Problem 13** Characterize all graphs, which are both well-covered and well-dominated.

We have also shown that if \( G \in G(\tilde{C}_4, \tilde{C}_5, \tilde{C}_6) \) and \( L^*(G) = L(G) \), then \( WCW(G) = WWD(G) \). Thus one may be interested in approaching the following.

**Problem 14** Characterize all graphs, where the equality \( WCW(G) = WWD(G) \) holds.

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