The Simulation of Magneto-Mechanical Properties of Magnetorheological Elastomers

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Abstract. Magnetorheological elastomers is an intelligent material which can sense and judge the external magnetic field and change its mechanical properties. In this paper, functional carbon nanotubes are added to polyurethane magnetorheological elastomers. According to the micro-mechanical finite element calculation, the mechanical parameters of the equivalent matrix composed of carbon nanotubes and polyurethane elastomers are obtained. Multi-scale simulation analysis of the magneto-mechanical properties of isotropic magnetorheological elastomers with different volume fractions of carbon nanotubes in tensile, compressive and shear modes using magnetic coupling software. The simulation results show that the magnetorheological elastomer with larger volume fraction of carbon nanotubes has higher storage modulus in the three modes than the magnetorheological elastomer with smaller volume fraction of carbon nanotubes.

1. Introduction

The research of mechanical analysis of magnetorheological elastomer (MRE), is the coupling research of the magnetic-particle under the magnetic field force and the Shear or tension force. In recent years, many scholars have studied on the magneto-mechanical properties of MRE. In 1999, Ginder [1] used dipole model and finite element method to calculate and analyze the shear stress-strain relationship of MRE under different magnetic fields. In 2004, Y. Shen [2], according to the characters of the MRE particle chains and the nonlinearity of matrix materials, established a model for the magneto-mechanical modulus of MRE. In 2007, Kallio [3] studied on the mechanical properties of MRE in compression mode, the results showed that the compression modulus of anisotropic MRE formed by magnetic particles is greater than that of isotropic MRE. Zhu [4], calculated the magnetic field distribution, shear modulus, relative permeability and other parameters in magnetorheological elastomers by finite element method. Sun [5, 6] carried out numerical simulation on the mechanical properties of magnetorheological elastomers with or without external magnetic field through RVE based on periodic boundary conditions, then compared and analyzed the result with the theory. Zhao [7], based on the Meso-Mechanics RVE, studied on the magneto-mechanical of mechanical properties and magneto-mechanical shearing of multi-chain MRE by theoretical analysis and numerical simulation, the results showed that the initial compressive stress of MRE decreases with the application of magnetic field, and the shear modulus increases with the application of magnetic field.
In this paper, carbon nanotubes were added into the magnetorheological elastomer of polyurethane. Since the geometrical size of carbon nanotubes was nanoscale and carbonyl iron powder (CIP) was micron scale, meso-mechanics and finite element analysis theory were used to conduct multi-scale simulation analysis on the mechanical properties of MRE to study the impact of carbon nanotubes on the mechanical properties of MRE.

2. Analysis of Mechanical Properties of MWCNTs/ Polyurethane Elastomer Equivalent Matrix

In this paper, the process of the multi-walled carbon nanotube model constructed by figure 1. The length of MWCNTs $L_{\text{MWCNT}} = 12 \mu \text{m}$, The outside diameter of MWCNTs is 10nm, inner diameter is 4nm. MWCNTs and the polyurethane elastomer are isotropic materials, The performance parameters of the material are $E_{\text{MWCNT}} = 1\text{TPa}$, $\nu_{\text{MWCNT}} = 0.3$, $E_{\text{PU}} = 39.4\text{MPa}$, $\nu_{\text{PU}} = 0.42$.

![MWCNTs molecular model](image1) (a) MWCNTs molecular model (b) frame structure model (c) Cylindrical continuum model

**Figure 1.** The evolution process of MWCNTs.

In recent years, studies have found that MWCNTs can change the mechanical properties of PU. With the increase of the dosage of MWCNTs, the stiffness, strength and elasticity of polyurethane elastomer (PU) increase. The relevant experimental results also showed that, when MWCNTs are added to the composite material, agglomeration will occur when the volume fraction of MWCNTs is 1:1 to the specific gravity of the other filler, and all properties of the composite material will decline. According to the previous research results, carbonyl iron powder with volume fraction of 18% is selected in this paper to study the influence of carbon nanotubes with volume fraction of 4%, 8% and 12% on the mechanical properties of magnetorheological elastomers. First, establish the equivalent matrix two-dimensional representative volume unit (RVE) composed of MWCNTs/PU, and uses carbon nanotubes as the reinforcement of polyurethane matrix. The model size of RVE was selected as $160\text{nm} \times 160\text{nm} \times 160\text{nm}$, and the number of particles of carbon nanotubes was calculated as 4, 6 and 10 respectively. The model is shown in figure 2.

![Equivalent models of carbon nanotubes of different volumes](image2)

**Figure 2.** Equivalent models of carbon nanotubes of different volumes.

The position vector of RVE is $x$, the volume is $V$, take a point, the stress is $\sigma(x)$, the strain is $\varepsilon(x)$. Thus, the average stress of the volume element is:

$$
\bar{\sigma} = \frac{1}{V} \int_V \sigma(x) \, dV
$$  (1)
The average strain is:

\[ \overline{\varepsilon} = \frac{1}{V} \int_{V} \varepsilon(x) \, dV \]  

(2)

Under small deformation, the effective stiffness matrix of the composite material is:

\[ \overline{\sigma}_{ij} = C_{ij}^{*} \overline{\varepsilon}_{ij} \]  

(3)

The \( \overline{\sigma}_{ij} \) is the average stress of the composite material, \( \overline{\varepsilon}_{ij} \) is the average strain of the composite material. According to the equation (1-3), the effective stiffness matrix or effective elastic modulus of RVE can be obtained.

\[ \overline{\sigma} = \frac{1}{V} \sum_{i=1}^{n} \sigma_{i} v_{i} \]  

(4)

\[ \overline{\varepsilon} = \frac{1}{V} \sum_{i=1}^{n} \varepsilon_{i} v_{i} \]  

(5)

\( V \) is the volume of RVE, \( \sigma_{i}, \varepsilon_{i}, v_{i} \) is the corresponding RVE unit which called i unit, its stress, strain and volume.

\( C_{ij}^{*} \) is a 6 by 6 matrix, about the isotropic material, the unknown quantities are only \( C_{11}^{*} \) and \( C_{12}^{*} \). The solution is to set a set of boundary load conditions, using volume averaging method to calculate the six average stress component and the six average strain, then according to the equations the effective stiffness matrix \( C_{ij}^{*} \), of RVE, can be solved. The flexibility stiffness matrix \( S_{ij}^{*} \), can be obtained by inverting the effective stiffness matrix, \( S_{ij}^{*} = C_{ij}^{* \text{-1}} \). The effective mechanical elastic modulus of composite materials can be obtained.

\[ \begin{bmatrix} \bar{\sigma}_{XX} \\ \bar{\sigma}_{YY} \\ \bar{\sigma}_{ZZ} \\ \bar{\tau}_{YZ} \\ \bar{\tau}_{XY} \\ \bar{\tau}_{XZ} \end{bmatrix} = \begin{bmatrix} C_{11}^{*} & C_{12}^{*} & C_{12}^{*} & 0 & 0 & 0 \\ C_{12}^{*} & C_{11}^{*} & C_{12}^{*} & 0 & 0 & 0 \\ C_{12}^{*} & C_{12}^{*} & C_{11}^{*} & 0 & 0 & 0 \\ 0 & 0 & 0 & (C_{11}^{*} - C_{12}^{*})/2 & 0 & 0 \\ 0 & 0 & 0 & (C_{11}^{*} - C_{12}^{*})/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (C_{11}^{*} - C_{12}^{*})/2 & 0 \end{bmatrix} \begin{bmatrix} \bar{\varepsilon}_{XX} \\ \bar{\varepsilon}_{YY} \\ \bar{\varepsilon}_{ZZ} \\ \bar{\gamma}_{YZ} \\ \bar{\gamma}_{XY} \\ \bar{\gamma}_{XZ} \end{bmatrix} \]  

(6)

In this paper, carbon nanotubes are equivalent to cylinders and the volume fraction of carbon nanotubes is overestimated, but it does not have a great impact on the subsequent analysis. The results show that, adding carbon nanotubes to polyurethane elastomers, when the content of carbon nanotubes between 0% to 12%, as the volume fraction of carbon nanotubes increases, the young's modulus of matrix is observably increased. Therefor, appropriate carbon nanotubes can obviously enhance the mechanical properties of matrix materials. The Young's modulus and Poisson ratio has been shown in table 1.

| \( E_{1} \) | \( E_{2} \) | \( E_{3} \) | \( v_{1} \) | \( v_{2} \) | \( v_{3} \) |
|---|---|---|---|---|---|
| 201MPa | 368MPa | 549MPa | 0.37 | 0.36 | 0.35 |

Table 1. Young's modulus and Poisson ratio.
3. Numerical Simulation of Polyurethane Magneto-mechanical Properties

The parameters of the magnetic - mechanical coupling numerical simulation model of polyurethane MRE with carbon nanotubes are shown in table 1 and table 2.

Table 2. Parameters of the numerical simulation model of polyurethane MRE with carbon nanotubes.

| Parameter                  | Value          |
|----------------------------|----------------|
| RVE Square length          | 68um           |
| Magnetic field intensity   | 1T             |
| Air                        |                |
| Young's modulus            | 210GPa         |
| Poisson's ratio            | 0.3            |
| Density                    | 7900kg/m³      |
| Relative permeability      | 5000           |
| Radius                     | 8um            |

4. Magnetic Compression Simulation of Polyurethane MRE

Figure 3 shows the compressive deformation and strain diagram of MRE.

Figure 3 is the stress cloud diagram of isotropic MRE with different volume fraction carbon nanotubes under magneto-compression. When added carbon nanotubes to ordinary MRE, the elastic modulus and stiffness of the equivalent matrix increase.

Figure 4 shows the stress-strain curves of isotropic MRE with different volume fractions of carbon nanotubes under magneto-compression. It is clearly shown in the figure that the initial stress curve of MRE decreases to a negative value after the magnetic field is applied, it shows that under the action of magnetic field, magnetorheological elastomer is subjected to the force opposite to the magnetic field. This shows that under the action of magnetic field, magnetorheological elastomer is subjected to the force opposite to the magnetic field.

Figure 4 is the magneto-strictive module-strain diagram of MRE of carbon nanotubes with different volume fractions under different field intensities drawn according to the magneto-strictive stress-strain curve according to figure 4. As shown in figure 5, under the same strain, the magneto-strictive modulus
of MRE with field intensity is always greater than that without field intensity; Whether carbon nanotubes are added, the magneto-strictive modulus of MRE increases with the increase of compressive strain.

**Magnetic shear simulation of polyurethane MRE**

Figure 6 is the stress cloud diagram of isotropic MRE with different volume fraction carbon nanotubes under magneto-induced shear condition. First, it is shown that the comparison of the stress diagram that the external magnetic force and the external shear force are caused by the external magnetic field, it makes the surface stress of ferromagnetic particles obviously greater than that of elastomers, particularly, the surface stress of ferromagnetic particles in the same direction as the magnetic field is obviously greater than that of ferromagnetic particles perpendicular to the magnetic field. Even if the external magnetic field and external pressure are the same, however, the maximum stress value of MRE with a large volume fraction of carbon nanotubes added is larger than that of MRE with a small volume fraction of carbon nanotubes added.

![Figure 6. Shear deformation and stress map of MRE.](image)

Figure 7 shows the simulation of magneto-strictive shear stress-strain curves of MRE with different volume fractions of carbon nanotubes. Firstly, it can be seen from the figure that the magneto-strictive stress increases with the increase of strain value, and the initial magneto-strictive stress increases with the application of external magnetic field. When the external magnetic field is applied, the magneto-strictive stress-strain curve starts from non-zero. Secondly, under the same shear stress condition, the strain value of magnetorheological elastomers with small volume fraction added carbon nanotubes was smaller.

After applying external magnetic field to MRE, the magneto-strictive modulus of MRE decreases, while the magneto-compression and shear modulus of MRE increase. Figure 8 shows the magneto-strictive module-strain curves of MRE with different volume fractions of carbon nanotubes.

![Figure 7. The shear stress-strain curves.](image)  ![Figure 8. The shear module-strain curves.](image)

**Conclusion**

This paper is based on Meso-Mechanics theory and finite element analysis software, simulated and analyzed the mechanical properties of MRE with different volume fractions of carbon nanotubes in the presence or absence of magnetic field at multiple scales. The results show that: In the compression state, the stress value of MRE decreases with the application of external magnetic field, magneto-strictive modulus increases; In the shear state, the stress value of MRE decreases with the application of external magnetic field, and the magneto-induced shear modulus increases; As the volume fraction of carbon nanotubes increases, the stress value and energy storage modulus of MRE increase, indicating that the
addition of carbon nanotubes can improve the stiffness of MRE and optimize the mechanical properties of MRE.

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