A new kind of metal detector based on chaotic oscillator

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Abstract. The sensitivity of a metal detector greatly depends on the identification ability to weak signals from the probe. In order to improve the sensitivity of metal detectors, this paper applies the Duffing chaotic oscillator to metal detectors based on its characteristic which is very sensitive to weak periodic signals. To make a suitable Duffing system for detectors, this paper computes two Lyapunov characteristics exponents of the Duffing oscillator, which help to obtain the threshold of the Duffing system in the critical state accurately and give quantitative criteria for chaos. Meanwhile, a corresponding simulation model of the chaotic oscillator is made by the Simulink tool box of Matlab. Simulation results shows that Duffing oscillator is very sensitive to sinusoidal signals in high frequency cases. And experimental results show that the measurable diameter of metal particles is about 1.5mm. It indicates that this new method can feasibly and effectively improve the metal detector sensitivity.

1. Introduction
With the deep and extensive research of chaos, applications of chaotic systems in various areas are widely studied. Among these applications, Using Duffing chaotic oscillator to detect weak periodic signals becomes a hot topic recently. Research material shows this detection method is widely applied to many fields, such as vibration measurement, machinery fault diagnosis, etc. To increase the sensitivity of metal detection system, this paper uses this method to detect and extract sinusoidal weak signal of a metal detector [1-5]

Metal detector is a kind of equipment, which can generate signals and then alarm when the products mixed with metal particles or human body to carry metal objects (such as iron, copper, aluminium, etc.) pass by [2,6]. Its detection based on physical principles of electromagnetic, radiation, etc. And its purpose is to protect production equipment, food, personal safety and so on [7]. This paper discusses the metal detectors widely used in industry for food, medicine and other fields based on the principle of electromagnetic induction [6-10].

The fine metal particles mixed in products are very small, so the eddy currents induced by them are thereby very weak. If we can detect the weak signals, namely we can achieve high sensitivity alarm. So sensitivity is a key target when we design a detector. The sensitivity to a great degree depends on weak signal processing technology [6]. The conventional processing technology for a detector is usually using a linear filter to extract signals, but its sensitivity is not very high [3]. So this paper applied Duffing oscillator, a chaotic detection system, to metal detection, for its extreme sensitivity to weak sinusoidal signals and immune to noise in some degree. Here we use Duffing detection system to detect the induced weak sinusoidal signal from the detector’s probe. As far as we know, this method is the first time both here and abroad.
The signal frequency to be detected from the probe is usually over 10KHz [4]. But at present, researchers mainly focus on Duffing detection research about 1Hz, the research data more than 500Hz are rare [1,4,7]. And so about Duffing system phase transition critical value, state determination criterion and so on in high frequency, we have no available material. For this reason, this paper sets the frequency of the system at 31.8KHz (angular speed 10^5 rad/s), which is a frequency generally used in detectors, and also computes Lyapunov characteristics exponents (LCE) of Duffing oscillator in this high frequency, thereby uses LCEs as the quantitative standard to determine the critical phase transition threshold and judge system states. It is more accurate than the method to judge the system state by direct observation of phase diagrams [1,9]. At last, we do experiments on different diameters metal balls respectively. Results indicate that this new method can effectively improve the sensitivity of the detector.

2. Typical structure of a detector
A detector consists of two main parts: probe (coils) and signal processing system. Its common structure is as shown in figure 1 [6]. Three coils are coaxially arranged with the driving coil in the center and two balance coils on each side symmetrically [6]. The size, material and shape of the two balance coils are identical. The tested products pass parallel to the axis through the coils. In operation, the driving coil is sent more than 10kHz sinusoidal current, and so it produces a sinusoidal alternating magnetic field which induces voltages in both balance coils. If there is no metal impurity in the products (metal particle in figure 1), the induced voltages in both balance coils are the same and cancel at the amplifier since the two balance coil reverse series. The detector has no alarm signal issued. But if there is a metal particle mixed in products, the eddy current induced in the particle generates different voltages in two balance coils, and a resultant signal will be given by differential amplifier. If the signal can be detected, the detector will alarm.

The signal processing system consists of a differential amplifier and a Duffing chaotic detection system here, which can distinguish the signal given by the probe and give an alarm signal. In addition, a phase-locked loop (PLL) synthesizer generates sinusoidal signals, which passes through a medium-frequency (MF) power amplifier to drive the driving coil [6]. Besides these parts, the detector also has a signal generating circuit (excitation circuit), and computer control systems, etc [6, 10]. This paper doesn’t discuss them but concentrates on chaotic detection system in detail.

3. Detection theory and LCEs of Duffing oscillator
The common Duffing equation used in signal detection is
Where $\omega$ is the angular speed, $rcos(\omega t)$ is the forced periodic term, $n(t)$ is the white noise, term $-y_1^3 + y_1^5$ is the nonlinear recovery force term, and Ordinary references usually adopt $k=0.5$. If $k$ keeps constant, with the gradual increment of $r$, the system state will experience the interims of chummage trajectory, the bifurcation trajectory, the chaotic state, the chaotic critical state, and finally the large-scale periodic state [1]. By observing the complete difference of phase trajectories between the chaotic state and the large-scale periodic state, we can identify the weak sinusoidal signals [9].

The detection principle is as follows: we use $r_cos(\omega t)$ as the internal reference signal of the system, where $rd$ is the critical threshold when the system will turn from the chaotic state to the large-scale periodic state. If there is no metal particle nearby, namely there is no weak sinusoidal signal, the system continues to be in a chaotic state. If there are metal particles through the probe, it will give a sinusoidal signal, with angular speed $\omega$, thereby the Duffing system will change into large-scale periodic state at once. So by identifying the state transformation, we can know there are metal impurities in products. The approach above still has some problems to be solved.

1. Signal frequency from the detector probe is usually over 10KHz [10]. But the present references are mainly focus on 1Hz or so. Research data more than 500Hz is not available [6].

2. When judge the system state, we only rely on observation of the phase diagram by our eyes directly. It is lack of quantitative precise. Especially due to the impact of noise, the phase diagrams often become deformed. It is difficult for our eyes to distinguish whether or not the system is the chaotic state. And when use this approach to determine the threshold $rd$, we also meet the same problem [1].

To solve the above problems, we set the system frequency at 31.8KHz (angular speed $10^5$ rad/s), and compute the LCEs, which can give precise quantitative standard for state identification and system threshold determination [4]. In addition, LCEs can be feasibly used in automatic identification for computers.

LCEs are unvaried values in describing chaotic system and also important criteria of whether a system is chaos or not. For the Duffing oscillator, it has two LCEs. If its maximum LCE is less than zero, the system is in the periodic state; if it is greater than zero, the system is in the chaotic state [7]. Therefore, by the LCEs, we can clearly know the dynamic instant motion-state of one dynamic system. Their definitions are as follows [1]. To a two dimensional nonautonomous system, its Poircare map is

$$\begin{align*}
x_{n+1} &= X(x_n, y_n) \\
y_{n+1} &= Y(x_n, y_n)
\end{align*}$$

Its Jacobian matrix is [7]

$$
J(x_n, y_n) = \begin{bmatrix}
\frac{\partial X}{\partial x_n} & \frac{\partial X}{\partial y_n} \\
\frac{\partial Y}{\partial x_n} & \frac{\partial Y}{\partial y_n}
\end{bmatrix}
$$

Supposing the points successive mapping from the initial point $P_0(x_0, y_0)$ as $P_1(x_1, y_1), P_2(x_2, y_2), \ldots, P_n(x_n, y_n)$, the Jacobian matrix of the previous ($n-1$)th point is [7]

$$
J_0 = J(x_0, y_0), J_1 = J(x_1, y_1), \ldots, J_{n-1} = J(x_{n-1}, y_{n-1})
$$

Defining $J^{(0)} = J_{n-1} J_{n-2} \cdots J_1 J_0$, the modulo of eigenvalue for $J^{(0)}$ is $j_1^{(0)}, j_2^{(0)}$, and $j_1^{(0)} > j_2^{(0)}$. The definition of the LCEs are [7]

$$
L_1 = \lim_{n \to \infty} \sqrt[4]{j_1^{(n)}}, L_2 = \lim_{n \to \infty} \sqrt[4]{j_2^{(n)}}.
$$
We compute two LCEs by Matlab software in the M-file Module.

4. Simulation

We use Matlab Simulink to make a simulation model as shown in figure 2. This model is not only a Duffing oscillator but also can compute its LCEs. In this model, $k=0.5$, $\omega=10^5\text{rad/s}$, $k_1=0.5\omega$, $k_2=\omega^2$, $k_3=1/\omega$. The outputs $y_1$ is input to Workshop of Matlab, and then it is used to compute LCEs by the M-file Module.

![Simulation Model Diagram](image)

**Figure 2.** Simulation model

To obtain $r_d$ accurately, which is a threshold in the chaotic critical state, is a key step for high sensitivity detection. Here we use LCEs to determine $r_d$.

First, disconnect the white noise module in figure 2, and gradually raise the amplitude of $r$ from 0 to 1, the curves of two LCEs is as shown in figure 3. In the figure 3, the abscissa represents $r$, and the ordinate represents LCEs. From figure 3, it can be seen that, when $r=0.7311118$, the largest LCE changes suddenly from positive to negative. At this point transformation of the system will take place.

![Curves of Two LCEs](image)

**Figure 3.** Curves of two LCEs.

Specific calculation is: when $r=0.7311118$, the largest LCE is $1.861 \times 10^4$ (positive). At this point the system is chaos. Then if only $r$ is raise $10^{-7}$ to 0.7311119, the largest LCE becomes $-2.169 \times 10^4$ (negative). Both LCEs are negative means that the system is in the large scale periodic state. Therefore, we may consider $r=0.7311118$ is the threshold. So 0.7311118 can be set as the amplitude of internal reference signal. If there is an output signal with the amplitude equal or more than $10^{-7}$, the system will change from chaotic state to the large-scale periodic state at once. So by the identification of the system state we can judge whether or not the signal appears. That is to say, if the voltage amplitude induced by the eddy-current in the metal particle is more than $10^{-7}$V, the system can detect it and alarm.
5. Experiments
We use three samples to do experiments in the detector. One driving coil and two balance coil with the
same radius of 30mm is as shown in figure 1. The distances between two coils are 20mm. Sinusoidal
current effective value in the driving coil is 2A and its angular speed is $10^5$ rad/s. Copper balls with
different diameters are as samples. Sample #1 has a diameter of 2mm, sample #2 has a diameter of
1.5mm, and sample #3 has a diameter of 1mm. Their conductivity $\sigma$ and relative magnetic
permeability $\mu_r$ are $5.8 \times 10^7$ S/m and 0.999991 respectively. The distance between the copper ball and the
driving coil center is 15mm. The experiment results is as shown in figure 4, figure 5 and figure 6. In
figure 4(a), figure 5(a) and figure 6(a), the abscissa represents the time and its unit is second, and the
ordinate represents the value of $d\gamma_1/dt$. In figure 4(b), figure 5(b) and figure 6(b), the abscissa
represents the time and its unit is second, and the ordinate represents the value of LCEs.

Experiment 1. When sample 1# is in the detector, the system phase diagram and LCEs are shown in
figure 4. The two LCEs are $-1.812 \times 10^4$ and $-3.188 \times 10^4$ respectively. The maximum LCE is negative
indicates that the system changes from the chaotic state into a large-scale periodic state. So we may
say that the system can detect the weak sinusoidal signal from the probe. Namely the detector
sensitivity is up to at least 2mm.

![Figure 4](image.png)

**Figure 4.** Experiment 1 (a) The phase plane orbits in the large-scale periodic state; (b) Time
evolution of two LCEs.

![Figure 5](image.png)

**Figure 5.** Experiment 2 (a) The phase plane orbits in the large-scale periodic state; (b) Time
evolution of two LCEs.
Figure 6. Experiment 3 (a) The phase plane orbits in the large-scale periodic state; (b) Time evolution of two LCEs.

Experiment 2. When sample 2# is in the detector, the system phase diagram and LCEs are shown in figure 5. Two LCEs are -3.2239×10⁴ and -1.7761×10⁴ respectively. The maximum LCE is negative, indicating that the system changes from the chaotic state into a large-scale periodic state. So we may say that the system can detect the weak sinusoidal signal from the probe. Namely the detector sensitivity is up to at least 1.5mm.

Experiment 3. When sample 3# is in the detector, the system phase diagram and LCEs are shown in figure 6 separately. The two LCEs are 1.7583×10⁴ and -6.7583×10⁴, one of the LCEs is positive and the other is negative, indicating that the system remains in the chaotic state. The results indicate that: although copper impurities are present, the chaotic detection system has a response. Because the diameter of this sample is too small, the eddy current induced in it is too small to make the Duffing system change its state. Moreover, we also do experiments with aluminum and iron balls and also have gotten satisfactory results. The sensitivity for aluminum balls is 1.5mm, and for iron is 2mm.

6. Summary
In terms of improving the sensitivity of metal detectors, this paper adopts Duffing oscillator as a signal detection system for the detector, and uses LCEs for the critical threshold determination and for the system state quantitative judgment. These procedures help to improve the accuracy of the Duffing detection system, thereby improve the sensitivity of the detector. Experimental results show that Duffing chaotic oscillator for metal detection can improve the sensitivity of metal detectors. This application has extensive future.

References
[1] Li Yue, Yang Baojun 2004, Chaotic Oscillator Detection (Beijing: Electronics Industry Press)
[2] Yao Baoheng, Zheng Lianhong, Zheng Lianhong 2006 Journal of Vibration and Shock. vol 25, pp 51-53
[3] Gao Jinzhan 2004, Weak Signal Detection (Beijing: Publishing House of Tsing Hua University)
[4] Wang Yongsheng, Xiao Zicai, Sun Jin, Fang Hongda 2008 Journal of Circuits and Systems. vol 13, pp 132-135
[5] Nie Chunyan, Shi Yaowu 2008 Chinese Journal of Scientific Instrument. vol 22, pp 32-35
[6] Sadao Yamazaki, Negishi 2002 IEEE Trans. Instrum. Meas. vol 51, pp 810-814
[7] Yang Hongying, Ge Bin 2005 Journal of Dalian University. vol 26, pp 15-18
[8] Wang Yongsheng, Ye Hao, Wang Guizeng 2008 Chinese Journal of Scientific Instrument. vol 29, pp 927-932
[9] Li Yue, Xu Kai, Yang Baojun 2008 Acta Physical Sinica. vol 57, pp 3353-3358
[10] Li XiuShan 2005 Journal of Transducer Technology. vol 24, pp 51-53