Design of multiplier-less minimum-phase filters based on sharpening compensated comb filters

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Abstract. Minimum-phase (MP) filters have all zeros inside and/or unit circle. As a consequence, the group delay of an MP system is always less than that of non-minimum phase systems, having the equal magnitude responses. Minimum-phase (MP) filters find applications where it is necessary to have a low group delay, like in communications, speech processing, and predictive coding, among others. This paper presents a novel simple method for the direct design of low-pass minimum-phase (MP) filters. Method is based on design of two compensated combs, using a multiplier-less minimum-phase compensator, and sharpening technique. The first comb defines the stop band and pass band of the MP filter, while the second comb decreases side lobes of the first comb, thus increasing attenuation of the resulting MP filter. Knowing that all zeros of comb filter are on the unit circle, the compensated comb is also a MP filter. Similarly, under the special condition, the sharpening of multiplier-less compensated comb may also result in a MP multiplier-less filter. The benefit of the proposed method is illustrated in the provided design examples.

1. Introduction
Minimum-phase (MP) filters find applications where it is necessary to have a low group delay, like in communications, speech processing, and predictive coding, among others. The zeros of minimum-phase filter are either on the unit circle, or inside the unit circle. Different methods are proposed for MP filter design. The methods can be divided into three principal groups:

- Methods based on the linear-phase prototype filter [1-5],
- Methods based on complex cepstrum [6-8], and
- Direct methods [9-10].

We present here a novel direct method for MP filters design based on the compensated comb filters and sharpening technique. The rest of the paper is organized in the following way. Next section introduces brief review of comb filters, minimum-phase comb compensator, and sharpening technique. Section 3 elaborates the proposed method, and is illustrated with examples. Finally, Section 4 provides conclusion.

2. Brief reviews of comb, compensation filter and sharpening
In this section we present a brief review of comb filters, simple minimum-phase comb compensator recently proposed in literature [11], and sharpening technique [12].

2.1. Comb filters
Comb filters are simple filters with all coefficients equal to unity [13]. The transfer function is given as:

\[
H(z) = \left[ \frac{1}{M} \sum_{i=0}^{M-1} z^{-i} \right]^K,
\]

where \( M \) is the length of the filter, and \( K \) is the number of the cascaded filters.

Comb filter has all zeros on the unit circle, and thus it is a minimum-phase filter. Its zeros are defined in the frequencies \( 2\pi i/M, i=1,\ldots,M \).

The magnitude characteristic is given as:

\[
|H(e^{j\omega})| = \left| \frac{1}{M} \sin(\omega M / 2) \right|^K.
\]

Comb filters are usually used as decimation filters [13]. However, here we will use comb filters as a low pass filters. The comb magnitude characteristic exhibits a pass band droop, and a low stop band attenuation. The goal here is to increase comb attenuations, and decrease the pass band droop. Increasing the comb parameter \( K \) both, the pass band droop and the attenuation increase. However, cascading comb filter with given parameter \( M \) and comb filter with a different parameter \( M_1 \) may result in an increasing of the resulting attenuation, without affecting significantly comb pass band. The value of comb parameter \( M_1 \) is chosen in such a way that the first zero of this comb filter be in the frequency band of the first side lobe of the comb with the parameter \( M \):

\[
\frac{2\pi}{M_1} \approx \frac{3\pi}{M}.
\]

From here:

\[
M_1 = \text{nint} \left( \frac{2}{3} M \right),
\]

where nint(*) is a closest integer to (*).

This idea is illustrated in the following example.

Example 1

We consider two values of \( M \) and in both cases is chosen \( K_1=K_2=1 \). Considering \( M=9 \), from (4) it follows \( M_1=18/3=6 \). Figure 1(a) shows overall magnitude responses of both combs, and their cascade. However, taking \( M=7 \), from (4) the value of \( M_1 \) is closest integer to \( 14/3=4.6667 \), which is equal to 5. Similarly, Figure 1(b) shows the corresponding magnitude responses for this case. Note that in both cases, the first zero of the comb with the parameter \( M_1 \) is in the middle of the first side lobe of the comb with the parameter \( M \).
2.2. Minimum phase comb compensation filter

Here we adopt the simplest minimum phase comb compensator from [11] with the following transfer function:

\[ G(z^M) = 1 + 2^{d-2} - 2^{d-2}z^{-M}, \]  

where \( d \) is integer which depends on the comb parameter \( K \) as shown in Table 1 [11].

| \( K \) | \( d \) |
|-------|-------|
| 2,3   | 0     |
| 4,5   | 1     |
| 6     | 1 or 2|

**Example 2**

Consider comb parameters: \( M=5 \) and \( K=2 \). From Table 1 it follows the parameter \( d=0 \). Figure 2(a) shows the zeros of compensated comb, thus confirming that the filter is a minimum-phase filter. Similarly, Figure 2(b) shows the magnitude responses of comb and compensated comb. Additionally, the pass band zoom is shown to demonstrate the decreasing of the comb pass band droop in the compensated comb.

![Zeros of compensated comb.](image)

![Magnitude responses.](image)

Figure 2. Illustration of comb compensation.

2.3. Sharpening

The sharpening technique is introduced in [12] for simultaneous improvements of both the pass-band and stop-band of a linear-phase FIR filters. In [9] it is shown how this technique can also be used for minimum-phase filters. The sharpening technique uses a polynomial relationship of the form \( H_{sh} = f(H) \) between the amplitudes of the sharpened and the prototype filters, \( H_{sh} \) and \( H \), respectively. We will use here the simple sharpening polynomial: \( H_{sh}=2H-H^2 \).

3. Description of method

The comb parameter \( M \) is defined by the stop band edge frequency \( \omega_s \) of the designed filter:

\[ M = \text{nint}(\frac{2}{\omega_s}), \]  

where \( \text{nint}(\ast) \) is nearest integer of \( \ast \).

For a given \( M \) the pass band edge frequency \( \omega_p \) is:

\[ \omega_p = \frac{\pi}{2M}. \]  

The prototype filter has the following transfer function:

\[ H_p(z) = H_1(z)G_1(z^M)H(z)G(z^M), \]  

where $H_1(z)$ and $H(z)$ are combs with parameters $M_1, K_1$ and $M, K$, respectively and $G_1(z^{M_1})$ and $G(z^{M})$ are the corresponding comb compensators given in (5).

Taking the simplest sharpening polynomial $2H-H^2$, the proposed MP sharpened filter is:

$$F(z) = Sh\{H_p(z)\} = H_p(z)[2 - H_p(z)] = H_p(z)F_1(z),$$

(9)

where $Sh\{\ast\}$ means sharpening of $\ast$, and $H_p(z)$ is given in (8).

The filter (9) is a minimum-phase filter, because the filters $H_p(z)$ and $F_1(z)$ are minimum-phase filters. (In [9] is shown that the filter $F(z)$ is also a minimum-phase filter if the value of 2 is added to the coefficient with index 1 of the filter $-H_p(z)$). The attenuation and absolute value of pass band deviation depend on the values $K_1$ and $K_1$.

Table 2 shows the tentative values of minimum attenuation $A$ and maximum absolute value of passband deviation $\delta$ in terms of the values $K$ and $K_2$, obtained by MATLAB simulation.

Table 2. The values of $A$ and $\delta$ in terms of the values $K$ and $K_1$.

| $K$ | $K_1$ | $A$ [dB] | $\delta$ [dB] |
|-----|-------|----------|--------------|
| 2   | 2     | 45.5     | 0.1          |
| 2   | 3     | 55       | 0.03         |
| 3   | 2     | 62       | 0.01         |
| 2   | 4     | 68       | 0.2          |
| 3   | 3     | 75       | 0.04         |
| 4   | 2     | 75       | 0.2          |
| 3   | 4     | 85       | 0.05         |
| 4   | 3     | 87       | 0.01         |
| 4   | 4     | 100      | 0.4          |

Example 3

We design a minimum-phase filter with $\omega_s=0.3\pi$, and minimum stop band attenuation of 60 dB and the absolute value of maximum pass band deviation of 0.01 dB. From (6) we get $M=7$. From (4) we get $M_1=5$. From Table 1 we chose $K_1=3$, and $K_1=2$. The magnitude response of the designed filter (9) and the pass band zoom are shown in Fig.3.

![Figure 3](image-url)  
(a) Overall magnitude response.  
(b) Pass band zoom.

Figure 3. Magnitude response of the minimum-phase filter from Example 3.

Figure 4(a) shows pole-zero plot of the designed filter $F(z)$, while Figure 4 (b) shows pole-zero plot of the filter $F_1(z)$. Figure 4 thus confirms that the designed filter is a minimum-phase filter, i.e. has all its zeros inside the unit circle.
Example 4

In this example we design a MP filter with $\omega_s=0.23\pi$, while the values of $A$ and $\delta$ are the same as in Example 3 ($A=60$ dB and $\delta=0.01$ dB). From (6) and (4) we get $M=9$ and $M_1=6$. From Table 2 we have: $K=3$ and $K_1=2$. The corresponding magnitude response and pass band zoom are shown in Figure 5.

![Figure 5. Magnitude response of the filter in Example 4.](image)

The pole zero plot of the designed MP filter is shown in Fig.6a. Similarly Figure 6b shows zeros of the filter $F_1(z)$ to demonstrate that the designed filter has all zeros inside the unit circle.

![Figure 6. Pole-zero plots in Example 4.](image)
4. Conclusion

This paper presents a novel method for minimum-phase filters design based on two compensated combs and sharpening technique. The designed parameters are: stop band edge frequency, minimum attenuation in the stop band and absolute value of the maximum pass band deviation. The pass band edge frequency is defined by the stop band edge frequency.

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