Cosmological Distances Scale: Cosmology at a Crossroads?

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Abstract. A brief review of the results of identification of the cosmological distance scale based on the redshift in the spectra of extragalactic sources based on the data used for detecting the “acceleration of the expansion of the Universe” is given. The data deviation scattering parameter is about 13 ... 20% of the calculated distance value, which is an order of magnitude greater than the accuracy estimates accepted in a number of studies as the average quadratic deviations of arithmetic averages. In addition, there is a dipole anisotropy in this data. It is shown that among the reasons for the so-called “metrological and scientific deadlock” in cosmology may be the lack of error estimates of the inadequacy of the models used and violations of the logic of statistical inference when identifying them by supernovae of type SN Ia.

1. Introduction
In 1923, Arthur Eddington in a textbook on the mathematical theory of relativity [1] published the results of determining the radial velocities of “spiral nebulae” obtained by Vesto Slipher from 1912 to 1922, almost all of which were associated with the beautiful displacement of $z$ in the spectra of their radiation. The shift, including the “violet” one, was interpreted as a Doppler effect, and the establishment of distances to the “spiral fogs”, which turned out to be galaxies, served as the basis for the construction of non-stationary cosmological models, although at first specialists did not pay attention to the works [2, 3].

A similar situation exists with the equations by Alexander Friedmann in 1922 [4]

$$
\frac{1}{R^2} \left( \frac{dR}{dt} \right)^2 + \frac{2}{R^2} \frac{d^2R}{dt^2} + \frac{c^2}{R^2} - \lambda = 0,
$$

$$
\frac{3}{R^2} \left( \frac{dR}{dt} \right)^2 + \frac{3c^2}{R^2} - \lambda = \kappa c^2 \rho,
$$

$$
\frac{1}{c^2} \left( \frac{dR}{dt} \right)^2 = \frac{1}{R} \left( A - R + \frac{\lambda}{c^2} R^3 \right),
$$

$$
\frac{d\rho}{dt} + \frac{3}{R} \frac{dR}{dt} \left( \rho + \frac{P}{c^2} \right) = 0,
$$

where $R(t)$ is the radius of curvature, $t$ is the cosmological time, $c$ is the speed of light, $\kappa = 8\pi G/c^2$, $\lambda$ and $A$ are constants, $G$ is the gravitational constant, $\rho$ is the density, $P$ is the isotropic pressure. One of the solutions of these equations was the non-stationary cosmological model of the “expanding Universe”. The equations became known after Albert Einstein’s recognition of an error with respect to his stationary cosmological model [5] and anticipated the discovery by Edwin Hubble of the “red shift law” for the photometric distance

$$
D_L = (c/H_0) \cdot z,
$$

(2)
where \( H_0 \) is the Hubble constant, since the relation (2) can be considered as an approximation of the 1st order of the solution of equations (1). Similar equations were independently arrived at by Georges Lemaitre in 1927 [6], Howard Robertson in 1928 [7], and Arthur Walker in 1933 [8].

In 1942, Otto Heckman obtained a 2nd order approximation [9] to solve the equations (1)

\[
D_L \approx (c/H_0) \cdot \left[ z + \frac{1}{2} (1 - q_0) z^2 \right],
\]

where \( q_0 \) is the deceleration parameter and in 1958 Wolfgang Mattig was a strict solution for the “usual” substance [10]:

\[
D = \left[ c/(H_0 q_0^2) \right] \cdot \left[ q_0 z + (q_0 - 1) \cdot \left( \sqrt{2q_0 z + 1} - 1 \right) \right].
\]

In 1960, Fred Hoyle proposed a model to detect the curvature of space [11]

\[
cz = H_0 r + Kr^2,
\]

where \( K \) is the nonlinearity parameter. And two years later, Alan Sandage discovered a deviation from Hubble’s law:

\[
(cz - H_0 r)_{r \sim 10^9 \text{ lightyears}} \sim 10^4 \text{ km} \cdot \text{s}^{-1}[12].
\]

In 1965, the existence of microwave background radiation, predicted in 1948 by George Gammov in the framework of the Big Bang theory, was experimentally confirmed [13] and became a source of data for verification of cosmological models.

In 1966 it turned out [14–16] that the deviation from the Hubble law [12] gives

\[
\frac{K}{c} = 3.73 \cdot 10^{-46}, \quad \text{i.e.} \quad (H_0/c)^2 = 3.38 \cdot 10^{-46} \text{ [km}^2\text{]} \text{ for the model (5), from where}
\]

\[
z \approx (H_0/c) \cdot r + (H_0/c)^2 \cdot r^2
\]

and Hubble radius \( c/H_0 \) acquired the physical meaning of the expanding boundary of the “Universe” at \( q_0 = 3 \), which was consistent with \( q_0 = 2.6 \pm 0.8 \) [17] and the model (4).

By the mid-1970s, with the accumulation of data, the estimates of \( H_0 \) in cosmological models decreased by an order of magnitude, and the estimates of the acceleration parameter were reset to \( q_0 = 0.03 \pm 0.4 \) [17]. Among cosmologists even became a popular joke, which is attributed to Alan Sandage: “There is nothing more variable than the Hubble constant.”

In 1980, General interpolation models with the shape parameter \( \alpha \) were found [14, 18]

\[
z = \left[ 1 + H_0 \cdot r/(\alpha c) \right]^\alpha - 1 \rightarrow D_L = \alpha \cdot (c/H_0) \cdot \left[ (1 + z)^{1/\alpha} - 1 \right],
\]

for small redshifts, they resulted in a model (5) with a free parameter \( K \).

At the beginning of 1990-ies the model of Friedman–Robertson–Walker received new view [19]

\[
D_L = \frac{c}{H_0} \cdot \frac{1 + z}{\sqrt{|\Omega_k|}} \begin{cases} 
\sin \varphi(z), & \Omega_k < 0 \\
\varphi(z), & \Omega_k = 0 \\
\sinh \varphi(z), & \Omega_k > 0
\end{cases}
\]

\[
\varphi(z) = \sqrt{|\Omega_k|} \int_0^z \frac{dx}{\sqrt{(1+x)^2(1+\Omega_M x) - x(2+x)\Omega_A}}, \quad \Omega_k = 1 - \Omega_M - \Omega_A,
\]

2
where $\Omega_M$ is the density of “dark matter”, $\Omega_\Lambda$ is the density of “dark energy”, $\Omega_k$ is the curvature parameter, and $\Lambda$CDM is a model of the microwave background radiation power spectrum as a parameterization of equations (1) in the form of a correlation function of the anisotropy of the rate expressed by the squares of the amplitudes of $a_{lm}$ modes of harmonics for a given number $l$ [20]:

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2 \equiv C_l(H_0, \Omega_b, \Omega_M, \Omega_\Lambda, \Omega_\nu, n,...),$$

(9)

where $\Omega_b$ and $\Omega_\nu$ are densities of baryon matter and massive neutrinos, respectively, and $n$ is an indicator of the adiabatic perturbation spectrum.

From the point of view of the structure of the model of Friedman–Robertson–Walker takes the simplest form for parameter curvature $\Omega_k = 1 - \Omega_M - \Omega_\Lambda = 0$:

$$D_L = 3\left(1 + \frac{z}{H_0}\right) \int_0^z \frac{dx}{\sqrt{(1 + x)^2(1 + \Omega_Mx) - x(2 + x)\Omega_\Lambda}}.$$  

(10)

This corresponds to the “flat Universe” model, which in the $\Lambda$CDM model is related to the first peak of the microwave background radiation fluctuation spectrum at an angle of $\sim 1\degree$ [20].

In 1998, “acceleration of Universe expansion” was found by fitting the parameters of the model (9) by “Riess minimum $\chi^2$” method under the current data on the photometric distances of supernovae SN Ia and the redshifts of their mother galaxies [21]. “Best fitting” at $\Omega_k = 0$ gave the following estimates: $H_0 = 65.2 \pm 1.3 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ for $\Omega_M = 0.24$ [21] and $H_0 = 63.0 \pm 1.3 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ for $\Omega_M = 0.28$ [22], and $\Omega_\Lambda = 0.76$ and $\Omega_M = 0.24$ were accepted.

In 2015, microwave background radiation maps gave $H_0 = 67.8 \pm 1.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [20].

In 2016, the authors of the discovery of “accelerating the expansion of the Universe” according to 300 SN Ia at $z < 0.15$ and 19 Cepheids received an estimate of $H_0 = 73.24 \pm 1.74 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [23], which differs by “3.4σ” from the assessment of Plank project – $H_0 = 67.2 \pm 0.7 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [24], using the model when $\Omega_k = 0$ with the push parameter $j_0$ in the 3rd order approximation [25]:

$$D_L \equiv \left(\frac{c}{H_0}\right) \cdot \left[ z + \frac{1}{2} (1 - q_0)z^2 - \frac{1}{6} (1 - q_0 - 3q_0^2 + j_0)z^3 + O(z^4) \right].$$  

(11)

At the same time, it was stated that the achieved “accuracy of the model (11)” was 2.4%!

However, in the report for 7 years of the WMAP experiment [26] it was noted that the introduction of one or two additional parameters of the $\Lambda$CDM model increases its accuracy by 90...300 %, while the SKO of $H_0$ estimates increases by 1.28...6 times! In the same report for the first 9 years of WMAP experiment [27] $H_0 = 69.7 \pm 2.4 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$, at the same time, according to the final data more accurate measurements for the space probe Planck $H_0 = 67.8 \pm 0.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [28].

By 2017, in cosmology, a situation which the program Manager of the Hubble Space Telescope Wendy Friedman called deadlock [29]: on the background of the General growth trends in the precision of astrophysical measurements, it was noticed a statistically significant discrepancy of estimates of the Hubble constant $H_0$ obtained from measurements microwaves. model the background radiation in the framework of standard $\Lambda$CDM-model, $H_0 = 67.3 \pm 1.2 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ and according to the “ladder distance” to the Cepheids $H_0 = 74 \pm 3 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$.

“To break the current impasse, the steps in the extragalactic distance scale will need to be tested at the percent level” [29].

The similar dynamics of the accuracy of the estimates of the fundamental gravitational constant $G$, which plays an equally important role in cosmology, including in determining the distances to supernovae according to the Chandrasekhar condition, was noticed at the end of the 20th century. Then the confidence intervals of three of the four best definitions of $G$ were not
covered at all [30], and in connection with the analysis of experiments to find neutrino oscillations the problem of “wrong confidence intervals” was considered [31]. In 1998, the Committee on Data for Science and Technology recommended that the new value for the constant of gravity as a weighted average of the experimental results and its standard deviation – $G_{1998} = 6.673(10) \times 10^{-11}$ m$^3$ s$^{-2}$ kg$^{-1}$. This was a “step back” from the 1986 estimate: $6.67259(85)$ m$^3$ s$^{-2}$ kg$^{-1}$. This was followed by:

$$G_{2004} = 6.6742(10); G_{2006} = 6.67428(67); G_{2010} = 6.67384(80)[10^{-11} \text{m}^3 \cdot \text{s}^{-2} \cdot \text{kg}^{-1}].$$

And in 2014 by the method of precision atomic interferometry was obtained unexpec ted accuracy estimate $G = 6.67191(99) \times 10^{-11}$ m$^3$ s$^{-2}$ kg$^{-1}$ [32].

The discrepancy in the definitions for the fundamental constant of gravity Terry Quinn from the International Bureau of weights and measures at a special meeting of the British Academy of Sciences called “metrological and scientific deadlock”.

Since the dependence of the distance $D$ to extragalactic objects on the red shift $z$ in the spectra of their radiation is a random sequence $D(z)$ with a space-time trend, the results of the identification of this trend by isotropic averaging models were actually used as scales of cosmological States. Their accuracy was characterized by “averaged hypothetical estimates of the scattering parameters of the regression function estimates” or, simply, the mean arithmetic mean. However, a full assessment of the accuracy of the scales gives a scale factor – the dependence of the parameter of the distribution of deviations from the distance.

Although the description of models of scales of distances, as a rule, be limited to consideration of characteristics of the provisions, greater interest in the current situation is the probabilistic assessment of the distribution and the scale factor the model scale as a function of distance.

The article analyzes the situation with respect to the accuracy requirements of the scale of cosmological distances at the level of 1 % and considers the problem of its structural-parametric identification based on the Friedman–Robertson–Walker model approximations according to the data [21, 22] on SN Ia supernovae used to detect the “universe expansion acceleration”.

2. Situation analysis

The determination of the Hubble parameter and the identification of models of scales of cosmological States are associated with a number of problems.

First, the General problem is caused by the so-called “Riess minimum $\chi^2$” method, used to approximate the measurement data by models (10) and (11) without checking the fulfillment of the known conditions for the applicability of regression analysis, and first of all – Gaussian. Essentially it was the best fitting Friedman–Robertson–Walker model to a “flat” Universe.

In fact, this “method” is a weighted least squares method with a known set of advantages and disadvantages. For non-Gaussian data, such averaging leads to the fact that the meaning of the $C_l$ coefficients becomes non-obvious [20], and the accuracy estimates are untenable [33]. If we exclude the condition $\Omega_k = 0$, “Riess minimum $\chi^2$” method would correspond to $\Omega_M = 0.72^{+0.44}_{-0.56}$ and $\Omega_\Lambda = 1.48^{+0.56}_{-0.68}$ [21] and $\Omega_M = 0.73$ and $\Omega_\Lambda = 1.32$ [22].

Second, not any SN Ia, but supernovae with refined photometric distances and spectral features are suitable as reference points in the calibration of the scale of cosmological distances. Although the supernova SN Ia are stable stars-Noah value in the maximum luminosity, for what they called “standard candles”, the problem is the unpredictability of the moments of flash and the method of recovering of the luminosity curve.

Third, the estimates of the $\mu$ and redshift $z$ modules had instrumental uncertainty $\sigma_\mu \sim 0.14...0.30$ at $\mu < 44.4$ and $\sigma_z \sim 0.001...0.01$ at $z < 1$ [21, 22]. This inequality in the parametric identification of the model (10) led to the method of weighted least squares and estimates of the type of standard deviation of the arithmetic mean based on the “normal” hypothesis. But even
then it was known that the use of regression analysis without taking into account the sins of inadequacy of mathematical models is not only a common mistake, but also requires compliance with the logic of statistical inference.

Fourth, the metrological examination [34] showed that the truncated Laplace distribution is more plausible by the criterion of the maximum probability of agreement for the distribution of errors of data approximation [21, 22] model (10). Statistical non-homogeneity and anisotropy of the data [21, 22] were observed [35] and earlier in the identification of one-number interpreting models. Their clustering on the transparency windows of the Milky Way allowed to justify the transition from equatorial coordinates to galactic \((l, b)\). As a result, a series decomposition with anisotropy parameters \(\theta_k(l, b)\) of the form [36]

\[
D_L(l, b, z) = \sum_{k=0}^{K} [1 + \theta_k(l, b) \cdot z^k] \quad \text{or} \quad D_L(l, b, z) = \theta_{000} + \sum_{i=0}^{I} \sum_{j=0}^{J} \sum_{k=0}^{K} \theta_{ijk} l^i b^j z^k |_{ijk \neq 000}. \quad (12)
\]

The model proved to be insensitive to the SN 1997ck anomaly \((z = 0.97)\), which confirmed the doubts of the authors of the High-Z SN Search Team and Supernova Cosmology Project in relation to its influence on the final result in the nonlinear Friedman–Robertson–Walker model with zero curvature parameter [21].

Recall data allowed to detect “acceleration of Universe expansion”, held 27 SN Ia at \(z < 0.1245\) and 10 SN Ia at \(z = 0.30...0.97\) [21], and 42 SN Ia at \(z = 0.172...0.830\) [22]. Every element demand-ed special training – breeding types, the recovery curve of luminosity, estimation of the brightness at the maximum and an error-abilities “standard luminosity”, as well as what was not done – an error estimation-abilities of the inadequacy of the theory of mechanism adopted flash and the test data on statistical homogeneity. And in this case it should be not just about the statistical, but strictly speaking, about the compositional homogeneity [36]. The peculiarity of this check is taking into account the structural elements of the model (12), allowing to determine the parameters \(q_0\) and \(H_0\). The fact is that only the parameters \(\theta_{001}\) and \(\theta_{002}\) can be used to determine the corresponding param-eters \(H_0\) and \(q_0\), because the physical meaning of models with anisotropy parameters without rela-tion to the variable \(z\), i.e. at a non-zero point, is not clear.

To the data [21, 22] it is possible to add 33 “pure” (removed from the centers of maternal galax-ies) SN Ia at \(z = 0.010...1.390\) due to obtaining the same result on “acceleration” [37].

Table 1 presents the testing data for compositional homogeneity for the maximum complexity mode at \(K = 3\) samples of SN Ia [21, 22, 37] by the criterion of minimum of the mean error module of inadequacy (MEMI) by the MCMMLS algorithm of maximum compactness method (MCM) [36].

An analysis of the data in Table 1 showed the following.

1. The verification of the three SN Ia Data sources for compositional homogeneity showed that all three sources do not form a homogenous set according to the criterion of the MEMI minimum when representing the model (10) by power series (12) on the red shift and galactic coordinates up to the 3rd order inclusive. The best result is given by the data [21] \(- 11.07 \text{ Mpc}\), but for the data [21, 22, 37] separately due to the imbalance of the supernovae distribution over the distance, this representation gives a nonzero zero point, which is not consistent with the physical meaning of the distance scale. Any combination of sources increases the total MEMI. A full Union describes a zero–point model

\[
D_L(l, b, z)|_{N=112} = -0.21844190l + (5811.1265 + 7.9642649l - 6.4452453b) \cdot z
-1135.2145 \cdot l \cdot z^2 \pm 249.81485.
\]

It is primarily associated with the unbalanced and random nature of the plan of formation samples because of the unpredictability of the moments of occurrence of supernovae.
Table 1. Verification of compositional homogeneity of data on SN Ia* [21, 22, 37].

| No. | Composition | samples | N | Presence of structural elements | \(q_0\) | \(H_0\) km s\(^{-1}\) Mpc\(^{-1}\) | MEMI, Mpc | Total MEMI, Mpc |
|-----|-------------|---------|---|----------------------------------|------|-----------------|---------|-------------|
| 1   | 27          | 27      |   |                                  | -    | \(61.615535\)   | -       | \(11.068886\) |
| 2   | 33          | 33      |   |                                  | -    | \(62.5221431\)  | 76.911354 | -           |
| 3   | 10          | 10      | + |                                  | +    | 82.560642       | -       | -           |
| 4   | 42          | 42      | + |                                  | +    | 257.43247       | -       | -           |
| 5   | 27+33       | 80      | - |                                  | -    | \(74.519903\)   | -       | -           |
| 6   | [27] + 33   | 29 + 33 |   |                                  | -    | \(68.282234\)   | -       | -           |
| 7   | 27+10       | 27      | - |                                  | -    | \(58.518127\)   | -       | -           |
| 8   | [27] + 10   | 27 + 10 |   |                                  | -    | \(11.068886 + 10 \cdot 89.566042\)/17 = 32.26811735 | - | - |
| 9   | 27 + 42     | 92      | - |                                  | -    | \(1.00073861\)  | 52.66543903 | 193.62411 |
| 10  | [27] + 42   | 27 + 42 | + |                                  | -    | \(247.43247 + 193.62411\)/17 = 161.0294928 | - | - |
| 11  | 33 + 10     | 43      | + |                                  | +    | \(-2.15003584\) | 72.65582841 | 136.25981 |
| 12  | 33 + 10     | 33 + 10 | - |                                  | +    | 89.566042 + 33 - 75.911354 = 34 = 81.3752499 | - | - |
| 13  | 33 + 42     | 76      | + |                                  | +    | \(-1.54131159\) | 87.55912376 | 227.31390 |
| 14  | 33 + 42     | 33 + 42 | + |                                  | +    | \(257.43247 + 33 - 75.911354)/7 = 178.883302 | - | - |
| 15  | 42 + 10 + 33 | 70      | - |                                  | +    | \(-3.50605641\) | 94.905931954 | 178.259924 |
| 16  | [27] + 10 + 33 | 37 + 33 |   |                                  | +    | \(58.518127 + 33 \cdot 75.911354)/7 = 68.1320656 | - | - |
| 17  | 27 + 10 + 42 | 37 + 42 | - |                                  | +    | \(-1.01676879\) | 61.43280962 | 209.81485 |
| 18  | [27] + 10 + 42 | 37 + 42 | + |                                  | -    | \(58.518127 + 22 + 257.43247)/17 = 164.2701997 | - | - |
| 19  | 10 + 42 + 33 | 85      | - |                                  | +    | \(-2.4746193\) | 63.47458596 | 261.63760 |
| 20  | 10 + 42 + 33 | 104     | + |                                  | +    | \(-2.4746193\) | 63.47458596 | 261.63760 |
| 21  | 27 + 10 + 42 + 33 | 112  |   |                                  | +    | \(-9.809290\) | \(-9.809290\) | \(-9.809290\) |
| 22  | 27 + 10 + 33 + 42 | 107 + 42 |   |                                  | +    | \(10 \cdot 78.560642 + 10 \cdot 89.566042\)/172 = 185.467917 | - | - |
| 23  | 10 + 42 + 33 + 72 | 85 + 72 |   |                                  | +    | \(27 + 11.068886 + 89.566042\)/172 = 201.2384222 | - | - |
| 24  | 27 + 33 + 42 + 10 | 102 + 10 | + |                                  | +    | \(27 + 11.068886 + 89.566042\)/172 = 201.2384222 | - | - |
| 25  | [27] + 10 + 42 + 33 | 79 + 33 |   |                                  | +    | \(10 \cdot 78.560642 + 10 \cdot 89.566042\)/172 = 185.467917 | - | - |

MCMMLS – the method of least squares in the cross-monitoring scheme for inadequacy errors of MCM.

2. The characteristics of the dependence position \(D_l(l, b, z)\) of the boundary structure of the model of maximal complexity (12) at \(K = 3\) did not reach, except for samples No 4, No 7 and No 20.

3. Elimination of zero-point was possible only for samples No 17 and No 19:

\[
D_l(l, b, z)|_{N = 79} = (4879.9985 + 10.070535 \cdot b) \cdot z + (2854.0151 - 12.452332 \cdot l) \cdot z^2 \pm 249.81485; \\
D_l(l, b, z)|_{N = 85} = 4723.0317 \cdot z + (2945.7644 - 9.6923456 \Delta l) \cdot z^2 \pm 261.63760.
\]

4. In the angular distribution of the red shift SN Ia, the dipole anisotropy associated with the galactic center was found to be statistically significant by the MEMI minimum criterion, in this regard, the Heckman approximation (3) was reduced to the form

\[
D_l = \left(\frac{c}{H_0}\right) \cdot \left[1 + \alpha \cdot b \cdot z + \frac{1}{2} (1 - q_0) \cdot (1 + \beta \cdot l) \cdot z^2\right],
\]

where \(l\) and \(b\) are galactic coordinates, \(\alpha\) and \(\beta\) are anisotropy coefficients [36], and by the criterion of MEMI minimum it was preferable to models (3) and (11).

5. For further analysis, sample No 17 of 79 SN Ia [21, 22] is preferred at a lower value of MEMI.

3. Structural-parametric identification of the scale factor

Perhaps there is no “deadlock in cosmology”. After all, large values of \(H_0\) are obtained by SN Ia for \(z \sim 1\) and less, and smaller values are obtained by microwave background at \(z \gg 1\). Recall that in 2001 the Hubble Space Telescope Key Project recorded the final result:

\[
H_0 = 72 \pm 8 \text{ km s}^{-1} \cdot \text{Mpc}^{-1} = \text{const}
\]

for 56...467 Mpc [38], and later, according to the criterion of the minimum of MEMI characterizing its functional component, a “matching” was found [36]:

\[
H_0(D) = \begin{cases} 
(72.60 \pm 3.82) \text{ km s}^{-1} \cdot \text{Mpc}^{-1}, D \leq 309.5 \text{ Mpc} \\
(65.95 \pm 2.50) \text{ km s}^{-1} \cdot \text{Mpc}^{-1}, D > 391.5 \text{ Mpc}
\end{cases}
\]
and on an interval $391.5...467$ Mpc there were only 2 SN Ia from 36.

In addition, data on SN Ia [21, 22] and data on the same SN Ia from [39] for the interpolation model [6] with respect to the “acceleration of the Universe expansion” lead to opposite conclusions [40] for a banal reason – for a mismatch!

The replacement of the model (10) with the model (11) is of interest, since in the theory of measurement problems the structural-parametric identification of mathematical models of objects is accompanied by the effect of the existence of the model of optimal complexity, when an increase in the number of its parameters leads not to a decrease, but to an increase in the errors of inadequacy.

The results of a more complete structural and parametric identification of model (10) the form of crystals of different orders of Taylor on the criterion of minimum MEMI presented in Table 2.

### Table 2. Friedman–Robertson–Walker model in the approximation by structured series

| Algorithm identification | Code of structure | $D_L(h, l) = \theta_0 + \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5$ | MAD, Mpc | MEMI, Mpc |
|--------------------------|------------------|---------------------------------|---------|---------|
| MCMLMS 0110             | 41.5954450       | 4264.06336                   | 1429.0124 | 0       | 247.4289 | 334.03558 |
| MCMLMM 0110             | 40.6416.844      | 6286.7574                   | 7064.0138 | 4448.270 | 248.4289 | 344.36814 |
| MCMLMS 10111           | 105.71219        | 6286.7574                   | 7064.0138 | 4448.270 | 248.4289 | 344.36814 |
| MCMLMM 01001           | 0                | 6286.7574                   | 7064.0138 | 4448.270 | 248.4289 | 344.36814 |
| MCMLMM 01101           | 0                | 7042.6811                   | 31816.0999 | 18472.841 | 26548.19 | 148876.74 |
| MCMLMM 010011          | 0                | 3099.3246                   | 1522.0401 | 0       | 222.3414 | 226.18255 |

The best for the data [21, 22], as noted in [36], is an anisotropic model

\[
D_L = (c/H_0) \cdot \left[ (1 + 2.027311498 \cdot 10^{-3} \cdot \theta_1 \cdot z + \frac{1}{2} \cdot (1 - q_0) \cdot (1 - 4.491493499 \cdot 10^{-3} \cdot l) \cdot z^2 \right],
\]

where $H_0 = 60.80404234$ km·s$^{-1}$·Mpc$^{-1}$ and $q_0 = -0.14378664$ at MEMI = 247.42842 Mpc (Figure 1a).

Note, the Visser approach (11) to structure code 0110 is not included in the list of models that are optimal according to the criterion of minimum MEMI, in contrast to the Heckman approach (3) to structure code 0110, i.e. the complexity of the model structure did not lead to increased accuracy [36].

Data deviations [21, 22] from models (10), (14) and (15) are given in Table 3.

For data [21] at $H_0 = 65.2$ km·s$^{-1}$·Mpc$^{-1}$ and $\Omega_M = 0.24$ MAD from model (10) $d = 429.34$ Mpc, for data [22] at $H_0 = 63.0$ km·s$^{-1}$·Mpc$^{-1}$ and $\Omega_M = 0.28$–$d = 460.38$ Mpc, and for interpolation model (7) at $\alpha = 0.499160639$ and $H_0 = 77.2924661$ km·s$^{-1}$·Mpc$^{-1}$ $d = 278.32$ Mpc.

In Figure 1b, deviations $\delta_n = D_{Ln} - (c/H_0) \cdot \left[ (1 + \alpha \cdot b_n) \cdot z + 1/2 (1 - q_0) (1 + \beta \cdot l_n) \cdot z^2 \right]$ as a function of $D_{Ln}$ distances at the values of the variables $l_n$, $b_n$ and $z_n$ are represented as a percentage of $D_{Ln}$, and as the result of structural-parametric identification, the scale factor in the class of Taylor formulas of the 4th order is a constant ($d = 10.550125\%$). This means that the random component of the model error (14) is multiplicative and, at the same time, predominant, since the instrumental errors are less than $\sim 0.7\%$.

Table 4 shows the results of identification only the random component of the relative error of the cosmological distance scale and the nonparametric component of the error of the accepted probability distribution without taking into account the functional component of the error of

\[
\text{MAD} = \text{mean absolute deviation.}
\]
Table 3. The distribution of deviations from the characteristics of situation models

| SN   | η [Mpc]     | η [Mpc] | η [Mpc] | η [Mpc] | η [Mpc] | η [Mpc] | η [Mpc] | η [Mpc] | η [Mpc] |
|------|-------------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1995a| 27.425842   | 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842|
| 1995b| 27.425842   | 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842|
| 1995c| 27.425842   | 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842| 27.425842|

Note: The number in parentheses in the lower index for deviations (9) is n is the model number.

The inadequacy of the models of the position characteristics and the instrumental component of the error in the red shift and the module of the photometric distances.

Figure 1. MCM–stat M2 program [35, 33]. Model (14) in coordinates: a) Y = D_L, Mpc, and X3= z; MEMI= 247.42842 Mpc; d = 226.03539 Mpc. b) Y = 100 · [δ_i]/D_L, %, and X1= D_L, Mpc; MEMI=7.2427602 Mpc; d = 7.2247529 Mpc.

The truncated probability distributions – uniform, Laplace, Gauss, Cauchy, Trubtysyn and double exponential with the form parameter “4” – were considered as nonparametric hypotheses concerning random component of error scales. The points of settlement were taken by
an equivalent uniform distribution. The same distribution was adopted for nonparametric component of inadequacy error.

Table 4. Identification results for the distribution of deviations from the position characteristics of the models.

| Model | Distribution of random component | Truncations interval for random component, % | Bias, % | Scale factor, % | Scope for nonparametric component's of inadequacy errors, % | Border of convolutions, % |
|-------|----------------------------------|---------------------------------------------|---------|----------------|-------------------------------------------------------------|---------------------------|
| (9)   | Truncated Cauchy                 | –79.2723; +26.7685                           | 9.322%  | 19.8295        | 13.1461                                                    | 29.4159; +29.7441         |
| (13)  | Truncated Laplace                | –62.1312; +35.4914                           | 0.848%  | 16.4135        | 23.3252                                                    | 72.1487; +48.7991         |
| (14)  | Truncated Cauchy                 | –40.6994; +42.3853                           | 11.57%  | 13.9494        | 10.2204                                                    | 48.7215; +44.3783         |

Figure 2. Random component of relative error: a) model (10); b) anisotropic model (14); c) interpolation model (15).

Note, the hypothesis of Gaussian deviations from the model (11) gives the offset of the drop 2.8% and standard deviation $s = 16.3\%$. But the standard deviation of the arithmetic mean of 1.8 % is close to the “estimate” of 2.4 % [23].

The results of identification by the criterion of minimum span of nonparametric error of inadequacy are presented by convolutions of distributions of components in Figures 2 and 3.

Thus, the anisotropic model (14) based on the Heckman approximation is non-biased and more accurate than the model (11) in the Visser approximation, which illustrates the existence of an optimal complexity model at a given level of instrumental errors in redshift and photometric distance measurements. The interpolation model (15) is more compact, and the model (10) has a displacement of more than 9% and a larger scale factor of Cauchy $\sim 20\%$.

4. Conclusion

The physical meaning of the deadlock in cosmology is related not so much to the divergence of estimates of the Hubble constant, as Wendy Friedman noted, and not so much to the “metrological and scientific deadlock”, according to Terry Quinn, but to the fact that the potential possibilities of the Friedman–Robertson–Walker model in the representation of the scale of cosmological races by the function of one argument, the red shift, are exhausted. Therefore, the idea of testing “distances on an extragalactic scale at the percentage level” is not only unrealistic, but is also associated with an inadequate choice of the accuracy indicator in the form of the “average standard deviation”.

To a large extent this contributed to the use of untested statistical hypothesis of “normality” and use as the dispersion characteristics of the result of identification of the model, not the
dispersion characteristics of the scale as a whole, but only the dispersion characteristics of the actual characteristics of the situation, which is the “root of $N$” times less.

Unfortunately, this is a very common error in the application of regression analysis, when the “result of measurement” is the arithmetic mean of the set of “results of observations”, and the estimate of accuracy is the standard deviation of the arithmetic mean.

The analysis showed that for the models of cosmological distance scales on the red shift axis, using the Friedman–Robertson–Walker model and its representation by Taylor formulas of different orders, the most significant component of the error is the error of inadequacy.

Note that although the statistical analysis of the accuracy of the models of cosmological scales was carried out without taking into account the functional component of the error of the inadequacy of their position characteristics, the fact remains that the scale of cosmological distances on the basis of the red shift of the status of the metric scale does not have any comments.

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