The slingshot effect: a possible new laser-driven high energy acceleration mechanism for electrons

Gaetano Fiore\textsuperscript{1,3}, Renato Fedele\textsuperscript{2,3}, Umberto de Angelis\textsuperscript{2,3},
1 Dip. di Matematica e Applicazioni, Università “Federico II”, V. Claudio 21, 80125 Napoli, Italy
2 Dip. di Fisica, Università “Federico II”, Complesso MSA, V. Cintia, 80126 Napoli, Italy
3 I.N.F.N., Sez. di Napoli, Complesso MSA, V. Cintia, 80126 Napoli, Italy

We show that under appropriate conditions the impact of a very short and intense laser pulse onto a plasma causes the expulsion of surface electrons with high energy in the direction opposite to that of propagation of the pulse. This is due to the combined effect of the ponderomotive force and the huge longitudinal field arising from charge separation (“slingshot effect”). The effect should arise also from impact onto gases or other states of matter, provided the pulse is sufficiently intense to cause locally their complete ionization. Its experimental verification seems feasible and, if confirmed, would provide a new acceleration mechanism for electrons, alternative to traditional radio-frequency-based or Laser-Wake-Field ones.

Lasers with power of up to hundreds terawatts can be used to accelerate beams of charged particles injected into a plasma from outside (external injection \cite{1}) or belonging to the plasma itself (self injection \cite{2}). Their typical intensity and pulse duration range from $10^{20}$ to $10^{22}$ Watt/cm\textsuperscript{2} and from sub-pico to femto-seconds, respectively. In these extreme conditions, the ponderomotive effect leads to extreme charge separation corresponding to the maximum plasma electric field, $E_{\text{max}} [\text{V/cm}] \approx \sqrt{n_0 [\text{cm}^{-3}]}$ for equilibrium density $n_0$. Then, for $n_0 \sim 10^{18}$ cm\textsuperscript{-3}, $E_{\text{max}} \sim 1$ GV/cm, while the maximum fields that are produced in the conventional accelerating machines based on Radio-Frequency (RF) cavities are much less (the order of tens MV/m). However, acceleration machines based on external injection can be used only after some previous acceleration stage.

In the self injection scheme\cite{2} the accelerated electrons are those which are naturally (albeit violently) ejected from the plasma itself. Actually, this second acceleration process is not very efficient and reliable in terms of intensity, energy spread and collimation, although in the so-called bubble regime\cite{3,4} few electrons with the record energy of 1 GeV have been observed occasionally. Therefore it appears more suitable as an electron beam source and pre-acceleration device\cite{5} to be used before a subsequent acceleration stage, whether conventional or plasma-based.

In this paper we show that a different acceleration mechanism of plasma electrons can occur under suitable conditions for the pulse length, duration and shape. While in the self injection scheme the plasma electrons are dragged in and ejected forward, in this new “slingshot” mechanism plasma electrons with high energy are expected to be expelled backwards, i.e. in opposite direction w.r.t. to the pulse propagation, shortly after the impact of a suitable ultra-short and ultra-intense laser pulse in the form of a pancake normally onto a plasma. The mechanism is very simple: the plasma electrons in a thin layer - just beyond the surface of the plasma - first are given sufficient electric potential energy by displacement (through the ponderomotive force produced by the pulse) w.r.t. the ions, then are pulled back by the electric force exerted by the latter and leave the plasma. Below we show that the conditions for this to happen are that the pancake is sufficiently thin, its radius is not too small (to avoid trapping of the electrons or even the onset of the bubble regime), and the electromagnetic (EM) field inside is sufficiently intense. As this is based only on the first interaction of the pulse with the first layer of plasma, a reliable analytical description seems possible at least for a low density plasma. We briefly present it in the sequel, referring the reader to\cite{6} for the proof of some essential mathematical results.

The interaction of very intense laser radiation with the boundary of an overdense plasma has been studied in\cite{7,8} using particle-in-cell simulations or (simplified) analytical models. The effect on the EM radiation (resp. transmission and reflection, absorption, conversion from incident femtosecond to reflected attosecond pulses) may be accompanied\cite{7,8} by (temporary or final) emission of nanobunches of electrons backwards; this is the result of a multi-cycle in-out acceleration process (described stochastically, when the hydrodynamic description fails) of the boundary electrons by a quasistationary electromagnetic wave. On the contrary, our acceleration process is induced by a short pulse onto an underdense plasma and produces a single bunch of electrons; moreover, our description is purely hydrodynamical.

We describe the plasma as a fully relativistic collisionless fluid and simplify the Lorentz-Maxwell equations for the “plasma + EM field” system neglecting the dependence on the transverse coordinates before the expulsion of the electrons, i.e. as long as the longitudinal displacement of the electrons is small compared to the radius of the pancake. We thus assume the plasma initially neutral, unmagnetized and at rest with constant electron density $n_0$ in the region $z \geq 0$, and consider a purely
transverse plane electromagnetic pulse

\[ E^\perp(t,z) = \epsilon^\perp(ct-z), \quad B^\perp(t,z) = \hat{z} \wedge \epsilon^\perp(ct-z), \quad t \leq 0 \]

with an arbitrary 'pump' function \( \epsilon^\perp(\xi) \) having support \([0,l]\), propagating from \( z = -\infty \) in the positive \( \hat{z} \) direction. The pulse reaches the plasma surface \( z = 0 \) at \( t = 0 \) (\( c \) stands for the velocity of light, and we use CGS units throughout the paper). As the initial data do not depend on \( x, y \), we can look for solutions of the Lorentz-Maxwell equations depending only on \( t, z \); hence both the magnetic field and the transverse (w.r.t. \( \hat{z} \)) electric field are determined by the transverse gauge potential \( A^\perp(t,z) \) through \( B = B^\perp = \hat{z} \wedge \partial_2 A^\perp, \quad \epsilon E^\perp = -\partial_2 A^\perp \).

Conversely, one can choose \( A^\perp \) as the physical observable \( A^\perp(t,z) = -\int_{-\infty}^t dt' \epsilon E^\perp(t',z) \). Before the impact \( A^\perp \) fulfills the free Maxwell equation and has the form \( A^\perp = \alpha^\perp(ct-z) \), with \( \alpha^\perp(\xi) = -\frac{1}{\epsilon_0} \int_0^\xi \epsilon^\perp(t') dt' \); hence \( \alpha^\perp(\xi) = 0 \) for \( \xi \leq 0 \) and \( \alpha^\perp(\xi) = \alpha^\perp(0) = \text{const} \times \xi \geq l \).

We denote as \( x_\perp(t,X) \) the position time of the electrons’ fluid element initially located at \( X \equiv (X,Y,Z) \); its \( z \)-component \( z_\perp(t,Z) \) does not depend on \( X,y \). The transverse component of the equation of motion \( \frac{d\vec{p}^\perp}{dt} = q \left( E^\perp + \frac{c}{\gamma} \wedge B \right) \) of a particle of charge \( q \) in the above EM field implies \( \vec{p}^\perp + \frac{c}{\gamma} \vec{A} = \text{const} \) along the particle trajectory; as initially \( \vec{p}^\perp = 0 = \vec{A} \) at all \( X \), then along each particle trajectory \( \vec{p}^\perp = -\frac{c}{\gamma} \vec{A}, \) in particular \( \vec{p}_\perp = \frac{c}{\gamma} \vec{A} \) (the subscript \( c \) refers to electrons). We regard the ions as infinitely massive, forming therefore a static, uniform background of positive charge with proton density \( n_p \). Gauss law \( \partial_2 E^\perp = 4\pi e_n (n_p - n_e) \) and the \( z \)-component of the fourth Maxwell equation \( \partial_2 E^\perp = 4\pi e_n, e^\perp \) (\( e^\perp \) is the electrons’ Eulerian velocity) determine the longitudinal electric field \( E^\perp \) at time \( t \) in \( x_\perp(t,X) \) (with \( Z \geq 0 \)) to be

\[ \vec{E}^\perp(t,Z) = E^\perp[t, z_\perp(t,Z)] = 4\pi e_n \left\{ z_\perp(t,Z) \theta[z_\perp(t,Z)] - Z \right\}; \quad (2) \]

where \( \theta \) denotes Heaviside step function. Hence, these electrons feel the electric field \( \vec{F}_\perp^\perp(t,Z) = -e\vec{E}^\perp(t,Z) \), equal to that of a harmonic oscillator (with equilibrium at \( z_e \equiv Z \)) as long as \( z_e > 0 \), and to \( 4\pi e^2 n_0 Z = \text{const} \) when \( z_e \leq 0 \). We denote the potential energy associated to \( \vec{F}_\perp^\perp(z_e, Z) \equiv 4\pi e^2 n_0 \left[ Z - z_e \theta(z_e) \right] \) as \( (3) \)

\[ U(z_e,Z) = 2\pi \gamma e^2 \left[ \theta(z_e) \gamma^2 - 2z_e Z + Z^2 \right]. \]

If the pump \( \epsilon^\perp \) is very large, by continuity we expect the EM field to remain close to the compact-support travelling-wave \([1]\) also for small times after its impact on the plasma. The first effect of this impact is to boost all the reached electrons not only in the \( x, y \) directions through the electric part \( -eE^\perp = -e\epsilon^\perp \) of the Lorentz force, but also in the positive \( z \) direction through the magnetic part \( \vec{F}_\parallel^\parallel \equiv -e(v_e \times \vec{B})^\parallel/c = -\frac{e^2}{2\pi c^2 \gamma} \partial_2 \alpha^\perp(\gamma^2) (ct-z) \) \( [\epsilon^\perp = e_\perp e_\perp^\perp, \text{with a slowly varying } e_\perp(\xi) \geq 0 \text{ of support } [0,l] \] and \( e_\perp^\perp(\xi) \) oscillating with a period \( \lambda \ll l \), the average \( \vec{F}_\parallel^\parallel(t,Z) \) of \( F_\parallel^\parallel \) over \( \lambda \) is the ponderomotive force, proportional to \( \partial_2 \alpha^\perp \). Hence, the first layer of ions remains unshielded and by charge separation generates the slingshot, i.e. the longitudinal electric force \( \tilde{F}_\perp^\perp \) [of harmonic type, see \([2]\) attracting the boosted electrons backwards. Assume \( \alpha^\perp(\xi) \) has a unique maximum point \( \xi_0 \) and let \( t(Z) \) be the time when the maximum reaches the electrons initially located at \( Z \): \( \tilde{F}_\perp^\perp > 0 \) if \( t < t \), \( \tilde{F}_\perp^\perp < 0 \) if \( t > t \). Hence, for \( t > t \) \( \tilde{F}_\perp^\perp \) will add to \( \tilde{F}_\perp^\perp \) to slow down, stop and then boost the electrons back. Let \( \zeta \) be their maximal longitudinal displacement (we can consider \( \zeta \) independent of \( Z \) in a thin layer \([0, Z']\)). The slingshot loading is maximally efficient if \( \tilde{F}_\perp^\perp \tilde{v}_e > 0 \) during all the motion, i.e. if the electrons invert their motion exactly at time \( t = t \). This can be certainly achieved by tuning \( \xi_0 \). In the sequel we ignore the propagation of the (damped) EM pulse further rightwards in the plasma and follow only the backward motion of these electrons, during which for simplicity we underestimate as \( \tilde{F}_\perp^\perp \) the total back-accelerating force, neglecting the backward ponderomotive force of the rest of the incoming wave and of the ‘reflected’ EM wave (see the final remarks). As shown later, the very first layer of electrons is finally expelled in the backward direction with very high energy. We shall call this phenomenon slingshot effect. As \( \vec{p}_\perp = \frac{c}{\gamma} \vec{A} \equiv \alpha \vec{A} \equiv \text{const} \) after the pulse, using energy conservation

\[ H = mc^2 \gamma_e(z_e, Z) + U(z_e, Z) = \sqrt{m^2 c^4 + \vec{p}_{eff}^2 c^2 + 2\pi \gamma_0 e^2 \zeta^2}, \quad (4) \]

we can compute \( \gamma_e \equiv 1/\sqrt{1-v^2_e/c^2} \) as a function of \( z_e, Z \) (here \( \vec{p}_{eff} \) is the final transverse momentum of the electrons). The mechanical energy \( H \) of the surface \((Z = 0)\) electrons after expulsion (\( z_e < 0 \)) is purely kinetic [because \( U(z_e, 0) = 0 \)] and equal to the (constant) rhs. However, as \( U(\infty, Z) = \infty \) for \( Z > 0 \), one would conclude that only the surface electrons can escape to \( z_e = -\infty \) infinity; inner electrons should invert their motion when \( \gamma_e \) reaches its minimal value and then oscillate around \( Z \). We now show that the latter conclusion, consequence of idealizing the transverse plane wave as infinitely-extended, is wrong.

In the realistic case that the laser pulse has the transverse shape of a disk \( \rho^2 \equiv x^2 + y^2 \leq R^2 \) of radius \( R \), the ponderomotive force of the pulse will boost only the electrons approximately in a cylindrical region of the same radius. It will boost and displace longitudinally approximately by \( \zeta \) those near the \( z \)-axis, so that the unshielded ions will exert on them approximately the same backward force as before until their expulsion, provided their way out is not blocked by the electrons initially located near the cylindrical surface (which are first boosted, also outwards, then attracted by the ions towards the \( z \)-axis), i.e. provided the diameter is at least a few times \( \zeta \), say

\[ 2R \gtrsim 2\zeta, \quad (5) \]

which can be seen as a condition for the onset of the
slingshot regime, rather than the bubble regime \cite{4}. We now show that afterwards this force decreases sufficiently fast with |\(z_e \rvert\) to allow a thin layer of electrons, which we now estimate, to escape to infinity. We assume \(p_{\text{ef}}^z = 0\) and consider only the motion of the electrons moving along the \(z\)-axis (\(\rho = 0\)) after the pulse; in other words we consider those electrons which experience the strongest restoring force after expulsion, by symmetry reasons. In fig. 1 up we depict the expected charge distribution of the electrons initially located in \(z = Z \geq 0\) at a time \(t\) shortly after their expulsion. The light blue area is the one occupied only by the electrons, and the arrows show their main directions of motions: the left border, the dashed line and the solid line respectively represent the surfaces \(S_0, S_1, S_2\) occupied by the electrons initially located at \(X'\) with \(\rho^2 = X'^2 + Y'^2 \leq R^2\) and \(Z' = 0, Z, 2Z\) respectively. The red area is positively charged due to an excess of ions. We can certainly bound the real electronic longitudinal force \(F_{\text{el}}^{\text{z}}\) experienced by the electrons moving along the \(z\)-axis as follows:

\[
0 \leq F_{\text{el}}^{\text{z}}(t,Z) = -e\vec{E}_z(t,Z) - e\vec{E}_z^{\dagger}(t,Z) \leq F_{\text{el}}^{\text{z}}(z_e(t,Z),Z).
\]

(6)

Here \(\vec{E}_z(t,Z)\) stands for the part of the longitudinal electric field generated by the electron distribution between \(S_0, S_2\); as the part between \(S_0, S_1\) has by construction the same charge as the part between \(S_1, S_2\), but is more dispersed, it will be \(-e\vec{E}_z(t,Z) \leq 0\). \(-e\vec{E}_z^{\dagger}(t,Z)\) stands for the part of the longitudinal electric force generated by the ions and the remaining electrons [lying at the right of \(S_2\)]; this is certainly smaller than the force \(F_{\text{el}}^{\text{z}}\) generated by the charge distribution depicted in fig. 1 down, where the remaining electrons are located not in their actual positions, but in their original ones \(X'\) farther from \((0,0,z_e)\), so that their repulsive force is smaller. Using cylindrical coordinates \((Z', \rho', \varphi')\) for \(X'\) one easily finds

\[
F_{\text{el}}^{\text{z}}(z_e, Z) = 4\pi n_0 e^2 \left[ 2Z + \sqrt{z_e^2 + R^2} - \sqrt{(2Z - z_e)^2 + R^2} \right] \left[ 2Z + \sqrt{z_e^2 + R^2} - \sqrt{(2Z - z_e)^2 + R^2} \right].
\]

\(F_{\text{el}}^{\text{z}}\) becomes a function of \(t\) only through \(z_e(t,Z)\), is non-negative for \(z_e \leq 0\) and goes to zero as \(z_e \to -\infty\), as it must be. The associated potential energy is \[13\]

\[
U_{\text{el}}(z_e, Z) = 2\pi n_0 e^2 \left[ (z_e - 2Z) \sqrt{(z_e - 2Z)^2 + R^2} - 2Z z_e - z_e \sqrt{z_e^2 + R^2} + R^2 \sqrt{z_e^2 + R^2} \right].
\]

which for each \(Z \geq 0\) is a decreasing function of \(z_e\) with finite left asymptotes \(U_{\text{el}}(-\infty, Z)\). In fig. 2 left we plot \(f \equiv F_{\text{el}}^{\text{z}} / 4\pi n_0 e^2\) and \(u \equiv U_{\text{el}} / 4\pi n_0 e^2\) as functions of \(z_e\) for a few values of \(Z \geq 0\). In fig. 2 right the plots of \(f_r = F_{\text{el}}^{\text{z}} / 4\pi n_0 e^2\), \(u_r \equiv U_{\text{el}} / 4\pi n_0 e^2\) replace those of \(f, u\) for \(z_e \leq 0\).

Let \(K \equiv \pi e^2 n_0 / m c^2\), denote the \(\gamma_e\) determined replacing \(U\) by \(U_r\) in (1) and taking the \(z_e \to -\infty\) limit as

\[
\gamma_e^\infty (Z) = 1 + 2\pi n_0 e^2 \zeta^2 - U_r(-\infty, Z) - K \left[ \frac{\zeta^2 + 2Z^2 - 2Z \sqrt{4Z^2 + R^2 - R^2 \zeta^2}}{R^2} \right].
\]

(7)

and as \(Z_M\) the value of \(Z\) for which \(\gamma_e^\infty (Z) = 1\). As said, by such a replacement we overestimate the restoring backforce experienced by the electrons. Consequently, the real relativistic factor of the electrons will be larger than the above \(\gamma_e\), and the \(M_0 \equiv \pi R^2 n_0 Z_M\) electrons with \(Z \in [0, Z_M]\) will be only part of those escaping to infinity; a lower bound for their final relativistic factor is \(\gamma_e^\infty (Z)\). In fig. 3 we plot \(\kappa \equiv [\gamma_e^\infty - 1] / 2K \zeta^2 \in [0, 1]\), the normalized kinetic energy associated to \(\gamma_e^\infty (Z)\), as a function of \(\gamma \equiv Z / \zeta \in [0, 1]\), for \(R = \zeta (y_M = Z_M / \zeta)\). The plot for \(R = 2\zeta\) does not differ significantly. The fraction of electrons with initial position \(Z' \in [Z, Z + dZ]\) is \(\pi R^2 n_0 dZ = \pi R^2 \zeta n_0 d\zeta\) and, using \(\gamma_e^\infty\) instead of the real final \(\gamma_e\), the fraction with normalized kinetic energy in the interval \([\kappa, \kappa + d\kappa]\) is \(\nu(\kappa) d\kappa\), where \(\nu(\kappa) = -R^2 \zeta n_0 / \kappa(\gamma)(\gamma = \mu(\kappa))\) represents the associated energy spectrum. This is plotted in fig. 3 down [\(\gamma(\kappa)\) stands for the inverse of \(\kappa(y)\)].

It remains to show that the above picture of the boost phase is consistent with the equations of motion of the plasma + EM field system, and to compute \(\zeta\) in terms of the pump. As said, by continuity we expect that for ‘small’ times after the impact the transverse electromagnetic field remains close to the pump (i.e. the influence of the plasma motion on the EM field is still negligible). In Ref. \[9\] we show that this is certainly the case in the
FIG. 2. The rescaled longitudinal electric force $f$ (left, up) and the associated rescaled potential energy $u$ (left, down) in the idealized plane wave case, the rescaled longitudinal electric force $f_R$ (right, up) and the associated rescaled potential energy $u_R$ (right, down) in the case of a pancake of radius $R = 5$, plotted as functions of $z_e$ for $Z = 0, 1, 2, 3$; the horizontal lines in the right down graph are the left asymptotes of $u$ for $Z = 0, 1, 2, 3$.

FIG. 3. $\kappa(y)$ vs. $y \in [0, y_M]$ ($y_M \equiv Z_{M}/\zeta$) (left), and the kinetic energy spectrum $\nu(\kappa)$ in units of $\nu_{max} = \nu(\kappa = 1)$ vs. $\kappa \in [0, 1]$ (right), for $R=\zeta$.

space-time region $0 \leq ct - z \leq \xi_0$, $0 \leq ct + z \ll \frac{2\pi}{\kappa X}$; we also determine in explicit and closed form how the electrons would move if $F_e$ were zero, i.e. under the sole action of the pump, and show that the latter ‘unperturbed’ motion is a good approximation of the real one during the whole forward motion provided [8]

\[ \xi_0 + 2Y(\xi_0) + 2Z \ll \frac{2\pi}{\kappa X}, \quad T(\xi_0) \ll 1, \]

where

\[ Y(\xi) \equiv \frac{1}{2} \int_0^\xi \frac{1}{2} u_e^2(y), \quad V(\xi) \equiv \frac{1}{2} \int_0^\xi Y(y), \quad G(\xi) \equiv \frac{1}{2} \int_0^\xi \frac{1}{2} \left[ u_e^2 \right] y \left[ \frac{e\kappa V_1}{1 - u_e^2} \right] (y), \quad T \equiv G/Y, \]

where $u_e^2 \equiv c\alpha^2/mc^2 = p^2/mc$ is the electrons’ quiver velocity [its mean square in $[0, \xi_0]$ is given by $u_{e,m}^2 = \int_0^{\xi_0} u_e^2 \, dx = 2\sqrt{2}(y)/\xi_0$]; in the unperturbed motion $\gamma_c = 1 + u_e^2/2$, and moreover

\[ \zeta \approx Y(\xi_0). \quad (9) \]

$T(\xi_0)$ gives the relative error between (9) and the real $\zeta$. By (7) the maximum $\gamma$ of the expelled electrons is

\[ \gamma_{CM} = \gamma_c^\infty(0) \approx 1 + 2K\zeta^2 \approx 1 + 2K Y^2(\xi_0). \quad (10) \]

We expect $\gamma, \gamma_{CM}$ to grow with $n_0 \propto \alpha_m^2$ - although at a slower rate - even if condition (8) is not fulfilled; we shall deal with that elsewhere.

Taking $E^2$ into account, the electrons invert their motion when approximately a fraction $\xi_1/l$ of the pulse has overcome them, where $\xi_1$ is determined by

\[ \exp[8KV(\xi_1)] = 1 + p[e\alpha_s(\xi_1)/2\pi mc^2]^2 \quad (\lambda \text{ is the pulse wavelength and } p = 1, 1/2 \text{ resp. for circular, linear polarization}). \]

Fixing $\xi_1 > \xi_0$, this can be solved for $K$:

\[ K \equiv K(\xi_1) = \ln \left\{ 1 + p \left[ e\alpha_s(\xi_1)/2\pi mc^2 \right]^2 \right\}/8V(\xi_1) \quad (11) \]

($K$ is tuned choosing $n_0$). The slingshot loading is efficient if $\xi_1$ near $\xi_0$ makes $K(\xi_1)$ still compatible with (8).

The experimental conditions for the slingshot effect are at hand in several laboratories. Laser pulses of wavelength $\lambda$, length $l \gg \lambda$, for simplicity symmetric around $\xi_0 = l/2$, concentrated onto an area $\pi R^2$, have an energy

\[ E = \frac{dV E^2 + B^2}{8\pi} \sim \frac{(\pi R)^2}{\lambda^2} \int_0^l \xi A^2 \, d\xi = \frac{4(\pi mc^2 R)^2}{(e\lambda)^2} Y(\xi_0). \quad (12) \]
By \( \rho \) \( \chi \simeq \mathcal{E}(e\lambda)^2/4(m\pi e^2 R)^2 \), and \( \rho \) takes the form

\[
R^3 \simeq \frac{\mathcal{E}(e\lambda)^2}{4(m\pi e^2 R)^2}.
\]

To maximize \( \zeta \) and \( \gamma_{em} \) we choose \( R^3 \simeq \text{rhs} \); we obtain

\[
\zeta = R = \left[ \frac{\mathcal{E}(e\lambda)^2}{4(m\pi e^2 R)^2} \right]^{\frac{1}{2}},
\]

(13)

The laser machine at the Flame facility in Frascati can shoot linearly polarized pulses with \( \lambda \simeq 8 \times 10^{-5} \text{cm} \), energy \( \mathcal{E} = 5 \times 10^{16} \text{erg} \), and an approximately gaussian modulating amplitude with width at half height \( l' \simeq 7.5 \times 10^{-4} \text{cm} \) [14, 15]. By \( \rho \) \[13\] this gives \( Y(\hat{\xi}_0) = \zeta = R = 1.4 \times 10^{-3} \text{cm} \). A plasma with \( n_0 \geq 10^{17} \text{cm}^{-3} \) is obtained by ionization from an ultracold gas (typically, helium) jet in a vacuum chamber hit by such an energetic laser pulse as soon as \( \Gamma_1 < 1 \) (the length of the \text{-}z\text{-}interval where \( \Gamma_1 < 1 \) plays the role of pulse length \( l \)). Here \( \Gamma_1 \equiv \sqrt{U_i/\kappa} = 2U_i/mc^2\hat{\xi}_0^2 \) (\( \kappa \) \text{\&} kinetic energy) are the Keldysh parameters (the ionization potentials \( U_i \) are about 24\text{eV}, 54\text{eV} for first and second ionization respectively). The ionization is practically complete and immediate for \( R \lesssim 13 \times 10^{-3} \text{cm} \), because for such field intensities the Keldysh parameter for double ionization reaches values \( \Gamma_1 < 1/100 \ll 1 \) [10, 11] very fast.

Choosing \( \xi_1 - \xi_0 = l'/8 \) in \[11\] gives \( Kl' \simeq 5 \); replacing in the definition of \( T(\xi) \) we find \( T(\xi_0) = 0.2 \), showing that \[12\] is fulfilled. So we adopt \( K = 0.5l' \) as maximal \( K \), leading to \( n_0 = mc^2K/\pi\varepsilon_0 \simeq 10^{18} \text{cm}^{-3} \). Eq. \( \rho \) with \( \zeta = R \) gives \( Z_{\text{opt}} \simeq 3R \); by \[9\], for \( \xi \in [0, Z_{\text{opt}}] \) also condition \[8\] is fulfilled. As outputs we find \( |Q| > e\pi n_0 R^2 Z_{\text{opt}} = 0.3 \pi\varepsilon_0 n_0 R^3 \simeq 1.24 \text{StatC} = 4.1 \times 10^{10} \text{C} \), \( \gamma_{em} \simeq 4.5 \), \( H \simeq 2.3 \text{MeV} \), a maximal longitudinal electric field \( E^\alpha_{\text{opt}} = 4e\pi n_0 \zeta \simeq 8.4 \times 10^9 \text{StatV/cm} \simeq 2.6 \text{GV/cm} \).

As \( n_0 \ll n_{\text{e}} = \pi mc^2/\varepsilon_0 \lambda^2 \simeq 1.7 \times 10^{21} \text{cm}^{-3} \) (\( n_{\text{e}} \) is the critical density), we are indeed dealing with an underdense plasma. As \( 2R \gtrsim 4l' \), we are indeed dealing with a \textit{pancake pulse}. The maximal penetration \( \zeta \) of the ions is obtained from \[9\] replacing \( m \) with their mass; as \( \xi \ll \zeta \), the description of ions as immobile is justified.

If we keep \( R = \zeta \) fixed and decrease \( n_0 \) then \[8\] still holds and by \[10\] \( \gamma_{em} - 1 \) decreases proportionally. This scaling behaviour and the backward direction of expulsion may be used to experimentally discriminate the slingshot effect from LWFA acceleration \[16\].

We expect we can increase \( \gamma_{em} \) by decreasing \( R \) and increasing \( n_0 \) so that \( \zeta \) also decreases and \[5\] remains fulfilled. As this would violate the technical \[8\] condition \[8\], a quantitative estimate is not possible yet.

In the previous model we have done a number of approximations. In particular, we have neglected the negative ponderomotive force of the pulse after that the maximum of the latter has overcome the electrons, as well as the negative ponderomotive force of the ‘reflected EM wave’ generated by the impact of the pulse on the plasma; both add to the electrostatic force to increase the energy of the electrons in the expulsion phase. Besides, we have approximated the transition region from \( n_e = 0 \) to \( n_e = n_0 \) as the surface \( z = 0 \), rather than a thin layer; we expect that the latter would modify the shape of the spectrum in fig. \[3\] but not the main results.

**Acknowledgments.** It is a pleasure to thank L. Gizzi for his detailed information on the Flame facility and his valuable comments and suggestions.

[1] A. Irman, M.J.H. Luttikhof, A.G. Khachatryan, F.A. van Goor, J.W.J. Verschuur, H.M.J. Bastiaens, K.-J. Boller, J. Appl. Phys. 102, 024513 (2007).

[2] C. Joshi, Scientific American 294, 40 (2006).

[3] In the bubble regime the electromagnetic laser radiation is self-trapped in a region where most of the plasma electrons are pushed out. Self-injection of the electrons into the quasi-static plasma bubble can be caused by slow temporal expansion of the bubble. Recent numerical simulations show that this regime is very promising for generating mono-chromatic high-energy electron beams out of low-density plasmas \[4\]. Nevertheless, a more reliable quantitative description is still missing.

[4] S. Kalmykov, S. A. Yi, V. Khudik, and G. Shvets, PRL 103, 135004 (2009).

[5] X. Wang, R. Zgadzaj, N. Fazel, Z. Li, S.A. Yi, X. Zhang, W. Henderson, Y.-Y. Chang, R. Korzekwa, H.-E. Tsai, C.-H. Pai, I. Quevedo, G. Dyer, E. Gaul, M. Martinez, A. C. Bernstein, T. Borger, M. Spinks, M. Donovan, V. Khudik, G. Shvets, T. Ditmire, M. C. Downer, Nature Communications 4, 1988 (2013).

[6] G. Fiore, On plane-wave relativistic electrodynamics in plasmas and in vacuum, arXiv:1312.4665.

[7] V.I. Eremin, A.V. Korzhimanov, A.V. Kim, Phys. Plasmas 17 (2010), 043102.

[8] J.P. Geindre, R.S. Marjoribanks, P. Audebert, Phys. Rev. Lett. 104 (2010), 135001.

[9] A.A. Gonoskov, A.V. Korzhimanov, A.V. Kim, M. Marklund, A.M. Sergeev, Phys. Rev. E84 (2011), 046403.

[10] A. Pukhov, Rep. Prog. Phys. 65 (2002), R1-R55.

[11] If we have chosen the additive constant so that the minima of \( U \) \text{\&} \( \zeta \) are equal to zero for all \( Z \).

[12] If the pulse were weak, one should arrange \( \xi_0 = cT_0/4 \), where \( T_0 = \sqrt{\pi mc^2/n_0} \) is the plasma oscillation period, because in a harmonic motion \( T_0/4 \) is the time necessary to go from the equilibrium position to the maximal displacement, independently of the size of the latter. If the pulse is intense the growing phase has to last much longer [for an estimate see the comments after \[11\].

[13] To perform the work integral we have used the relation \( \sqrt{\xi_0^2 + R^2} = \frac{1}{2} \xi_0 [\sqrt{\xi_0^2 + R^2} + R^2 \sinh^{-1} \frac{\xi_0}{R}] \). We have chosen the additive constant equal to the last three terms so that \( U_n(0, Z) = U(n, Z) \).

[14] D. Jovanović, R. Fedele, F. Tanjia, S. De Nicola, L. A. Gizzi, Eur. Phys. J. D66 (2012), 328.

[15] L.A. Gizzi, et al., Appl. Sci. 2013, 3(3), 559-580; doi:10.3390/app3030559.

[16] W. Lu, et al., Phys. Rev. ST Accel. Beams 10 (2007), 061301.