**TESS Photometry of the Eclipsing $\delta$ Scuti Star AI Hydrae**

Jae Woo Lee$^1$, Kyeongsoo Hong$^2$ and Martti H. Kristiansen$^{3,4}$

$^1$Korea Astronomy and Space Science Institute, Daejeon 34055, Republic of Korea
$^2$Institute for Astrophysics, Chungbuk National University, Cheongju 28644, Republic of Korea
$^3$DTU Space, National Space Institute, Technical University of Denmark, Elektrovej 327, DK-2800 Lyngby, Denmark
$^4$Brorfelde Observatory, Observator Gyldenkernes Vej 7, DK-4340 Tølløse, Denmark

*E-mail: jwlee@kasi.re.kr

Received ; Accepted

**Abstract**

AI Hya has been known as an eclipsing binary with a monoperiodic $\delta$ Scuti pulsator. We present the results from its TESS photometry observed during Sector 7. Including our five minimum epochs, the eclipse timing diagram displays the apsidal motion with a rate of $\dot{\omega} = 0.075^{+0.031}_{-0.031}$ deg year$^{-1}$, which corresponds to an apsidal period of $U = 4800^{+2000}_{-2000}$ years. The binary star model represents that the smaller, less massive primary component is 427 K hotter than the pulsating secondary, and our distance of $612^{+36}_{-36}$ pc is in good agreement with the Gaia distance of $644^{+26}_{-26}$ pc. We subtracted the binary effects from the observed TESS data and applied a multifrequency analysis to these residuals. The result reveals that AI Hya is multiperiodic in its pulsation. Of 14 signals detected, four ($f_1$, $f_2$, $f_3$, $f_6$) may be considered independent pulsation frequencies. The period ratios of $P_{\text{pul}}/P_{\text{orb}} = 0.012^{−0.021}_{+0.012}$ and the pulsation constants of $Q = 0.30^{−0.52}_{+0.30}$ days correspond to $\delta$ Sc pulsations in binaries. We found that the secondary component of AI Hya pulsates in both radial fundamental $F$ modes ($f_2$ and $f_3$) and non-radial $g_1$ modes with a low degree of $\ell = 2$ ($f_1$ and $f_6$).

**Key words:** asteroseismology — binaries: eclipsing — stars: fundamental parameters — stars: individual
1 Introduction

Eclipsing binaries (EBs) are a primary source of empirical measurements of stellar properties. In particular, detached double-lined EBs provide accurate determinations of absolute masses, radii, and luminosities from the simultaneous analysis of radial velocities (RVs) and light curves (Hilditch 2001; Torres et al. 2010). The fundamental data are used to test stellar evolution models and to determine geometric distance to the binary systems. EBs with pulsating components are extremely valuable, because more reliable modeling of stellar pulsations allows the exploration of internal structure and rotation. Asteroseismic studies are dependent on external determinations of surface gravity and effective temperature, which can be independently measured from binary modeling. When pulsating stars are present in EBs, the synergy between them greatly improves our understanding about stars.

To develop this subject, we have been looking for pulsating stars in EBs and studying their physical properties, utilizing Kepler satellite photometry and ground-based spectroscopy (e.g., Lee & Park 2018; Lee et al. 2020). Recently, the 2-min cadence observations of the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015) have proved very helpful for asteroseismology of the pulsating EBs showing multiperiodic oscillation with low amplitudes (Lee et al. 2019; Antoci et al. 2019). This work focuses on AI Hya (TIC 455178154, BD+002259, TYC 196-626-1; $T_p = +9.035$; $V_T = +9.40$, $(B-V)_T = +0.45$), which is known to be an eccentric detached EB with a binary orbital period of about 8.29 days (Lause 1938; Busch 1970).

From the uvby light curves, Jørgensen & Grønbech (1978) obtained the photometric elements of AI Hya, which indicate this system to be an eccentric EB with an orbital eccentricity of $e = 0.230$ and an argument of periastron of $\omega = 245.6$ deg. Further, they reported that the program target is a pulsating EB with a period of 0.13803 days and a semi-amplitude of 0.01 mag and that the pulsation corresponds to a first overtone radial mode of $\delta$ Sct stars. The double-lined RVs of the binary star were measured by Popper (1988). He derived the velocity semi-amplitudes of $K_1 = 90.1$ km s$^{-1}$ and $K_2 = 83.1$ km s$^{-1}$, and calculated the absolute parameters of each component combined with the photometric results of Jørgensen & Grønbech (1978). On the other hand, Khaliullin & Kozyreva (1989) carried out incomplete photoelectric
observations of the primary and secondary eclipses, and obtained an apsidal motion rate of \( \dot{\omega} = 0.029 \pm 0.049 \text{ deg year}^{-1} \) based on the comparison with the light curves of Jørgensen & Grønbech (1978). Here, we present and discuss the binary and pulsational properties of AI Hya using the high-precision space photometry from the TESS mission.

2 Observations

AI Hya was observed by the TESS mission during Sector 7 in 2-min cadence mode. The observations were collected by camera 1 from January 8 to February 1 2019 (BJD 2,458,491.63 – 2,458,516.09). There is no plan to observe the binary star in other sectors during the TESS primary mission\(^1\). We obtained the simple aperture photometry (SAP) data from MAST\(^2\) and omitted seven obvious outliers using visual inspection and the 5\(\sigma\) criterion. A total of 16,397 individual measurements were used for this work. The raw SAP data were detrended and normalized by fitting a second-order polynomial to the outside-eclipse light curve (Lee et al. 2017). We converted the normalized fluxes to magnitudes by requiring a TESS magnitude of +9.035 at maximum light. In Figure 1, the full time-series data of AI Hya is presented as magnitude versus BJD. We can see that there are short-period oscillations in the primary minima, as well as in outside eclipses. This variability implies that the secondary component is a pulsating star as suggested by Jørgensen & Grønbech (1978).

3 Binary Modeling

3.1 Eclipse Timings

From the TESS times-series data, we obtained five mid-eclipse times and their errors with the method of Kwee & van Woerden (1956) based on observations during each minimum. This method requires no assumptions about the light curve morphology except for symmetrical eclipses and does not depend on binary parameters. Our minimum epochs are listed in Table 1 together with those collected from the literature, where Min I and Min II represent the primary and secondary minima, respectively. Because the literature data were given as HJD alone, we transformed their time stamps into TDB-based BJD by using online applets\(^3\) (Eastman et al. 2010). The \( O - C_1 \) timing residuals of AI Hya were computed using the linear ephemeris of Kreiner et al. (2001):

\(^1\) https://heasarc.gsfc.nasa.gov/cgi-bin/tess/webtess/wtv.py
\(^2\) https://archive.stsci.edu/
\(^3\) http://astroutils.astronomy.ohio-state.edu/time/
\[ C_1 = \text{BJD} 2,441,726.1433 + 8.2896735E, \]  
(1)

where the reference epoch BJD 2,441,726.1433 was converted from HJD 2,441,726.1428. The eclipse timing diagram constructed with ephemeris (1) is displayed in Figure 2, in which the filled and open symbols are Min I and Min II, respectively.

As shown in this diagram, the timing residuals for the primary eclipses are approximately 180° out of phase with those for the secondary eclipses. Because our program target is an eccentric EB, the eclipse timing variation may originate from apsidal motion. This can be represented by the ephemeris-curve equation of Giménez & Bastero (1995), which has five independent variables \( (T_0, P_s, e, \dot{\omega}, \omega_0) \). The Levenberg-Marquardt technique (Press et al. 1992) was used to evaluate the apsidal motion elements. At this point, the initial values of \( e \) and \( \omega_0 \) were taken from the light curve solutions given in Section 3.2. As a consequence, we measured the observed rate of apsidal motion of \( \dot{\omega} = 0.075 \pm 0.031 \text{ deg year}^{-1} \) and hence its long period of \( U = 4800 \pm 2000 \text{ years} \). The result is plotted in Figure 2 and summarized in Table 2. The \( O - C_2 \) residuals from the best-fitting elements are given in the fifth column of Table 1.

To phase the \( \text{TESS} \) observations of AI Hya, we introduced all primary times of minima into a linear least-squares fit and found the following ephemeris:

\[ \text{Min I} = \text{BJD} 2,458,496.34577(16) + 8.2896499(12)E. \]  
(2)

The 1σ-errors for the coefficients of the equation are given in the parentheses. The observed light curve phased with ephemeris (2) are plotted as gray circles in the top panel of Figure 3 as a normalized flux versus orbital phase.

### 3.2 Light Curve Synthesis

The \( \text{T} \text{ESS} \) light curve of AI Hya presents almost flat light maxima and seems to display a total eclipse at primary minimum. Further, the secondary minimum is displaced to an orbital phase of about 0.447. These mean that AI Hya belongs to the class of eccentric detached EBs. To get the binary parameters of the program target, all \( \text{T} \text{ESS} \) observations were modeled using version 2007 of the Wilson-Devinney binary code (Wilson & Devinney 1971, van Hamme & Wilson 2007; hereafter W-D). As the binary system is well detached, we used the W-D code in mode 2 (Wilson & Biermann 1976). In this article, the subscripts 1 and 2 denote the primary and secondary stars being eclipsed at Min I and Min II, respectively.

In this synthesis, we fixed the spectroscopic mass ratio of \( q = 1.084 \pm 0.012 \) measured by Popper (1988). The effective temperature of the more massive secondary component was initialized to be \( T_{\text{eff,2}} = 6860 \text{ K} \), according to the color index of \( (b-y) = +0.24 \) (Jørgensen &
Grønbech 1978; Popper 1988) and the color-temperature relations in Pecaut & Mamajek (2013). The logarithmic bolometric \((X, Y)\) and monochromatic \((x, y)\) limb-darkening coefficients were taken from the values of van Hamme (1993). The bolometric albedos \((A)\) and the gravity-darkening exponents \((g)\) were set to be standard values of \(A_1 = 1.0\) and \(g_1 = 1.0\), and \(A_2 = 0.5\) and \(g_2 = 0.32\). In Table 3, the parentheses signify the fitted parameters and their errors. The differential correction program was iterated until the corrections to the free parameters were lower than their standard deviations.

The binary modeling was carried out in a method analogous to that for the TESS target TIC 309658221 (Lee et al. 2020). The model parameters for the observed TESS data are given as Model 1 in columns (2)–(3) of Table 3. The synthetic light curve appears as the blue solid curve in the top panel of Figure 3, and the corresponding residuals are displayed in the middle panel of the figure. To reduce pulsation effects in the binary parameters, we removed the pulsation frequencies discussed in the next section from the observed data. The pulsation-subtracted TESS data were solved with the W-D binary code using the Model 1 parameters as initial values. The result is presented as Model 2 in columns (4)–(5) of Table 3, and displayed in Figure 3. This synthesis indicates that the hotter but less massive primary fills 36 \% of its inner Roche lobe, while the larger secondary star fills 46 \%. Here, the fill-out factor is defined as \(\Omega_{\text{in}}/\Omega_{1,2}\). For the error estimates in the adjusted parameters, we divided the TESS data into 15 subsets and separately modeled them (cf. Koo et al. 2014). Then, the standard deviations of each parameter were computed, and the 1\(\sigma\)-values were adopted as the parameter errors presented in Table 3.

4 Pulsational Characteristics

The light curve residuals of AI Hya, obtained by subtracting the binary effects from the observed TESS data, are illustrated in Figure 4 as magnitude versus BJD. We can clearly see the multiperiodic oscillations on a timescale of hours in the eclipse-subtracted data. Considering our absolute parameters presented in the following section, both components of AI Hya reside inside the \(\delta\) Sct and \(\gamma\) Dor instability strips in the Hertzsprung-Russell (HR) diagram (e.g., Lee 2016). Jørgensen & Grønbech (1978) reported that the larger and more massive secondary star is pulsating at a frequency of \(f_0 = 7.2448\) day\(^{-1}\). The TESS time-series data exhibit total eclipses and oscillations at the primary minima. These indicate that the smaller primary star is completely eclipsed by the cooler secondary and the latter component is the main source of the light variations.
To search for the pulsation frequencies of AI Hya, we performed a multifrequency analysis of the out-of-eclipse light residuals. The PERIOD04 software package (Lenz & Breger 2005) was used to compute the amplitude spectrum up to the Nyquist frequency \( f_{Ny} \approx 360 \text{ day}^{-1} \). According to the successive and simultaneous prewhitening described by Lee et al. (2014), we calculated the signal to noise amplitude ratio (S/N) for each frequency peak and detected 14 significant frequencies by adopting the empirical threshold of S/N > 4.0 (Breger et al. 1993). The results from this process are listed in Table 4. The amplitude spectra for AI Hya before and after prewhitening the first 3 frequencies, and then after all 14 frequencies, are displayed in the top to bottom panels in Figure 5, respectively. The synthetic curve computed from the 14-frequency fit is presented as a solid line in the lower panel of Figure 4.

As shown in Figure 5 and Table 4, almost all significant signals for AI Hya lie in the frequency range of 5–16 day\(^{-1}\), but we could find no credible periodicity near the \( f_0 \) frequency detected by Jørgensen & Grønbech (1978). Within the frequency resolution \( 1.5/\Delta T = 0.061 \text{ day}^{-1} \), where \( \Delta T = 24.5 \text{ days} \) is the observation time span (Loumos & Deeming 1978), we carefully identified possible combination frequencies and orbital harmonics. The results are presented in the last column of Table 4. The low-frequency signal of \( f_4 = 0.1205 \text{ day}^{-1} \) appears to be the orbital frequency of \( f_{orb} = 0.1206 \text{ day}^{-1} \), which could be due to insufficient removal of the eclipses in the observed data. Located in the typical range of 4–80 day\(^{-1}\) for \( \delta \text{ Sct stars} \) (Breger 2000), the four frequencies of \( f_1, f_2, f_3, \) and \( f_6 \) may be independent pulsations originating from the more massive secondary component.

5 Discussion and Conclusions

In this article, we report the TESS photometry of the double-lined EB system AI Hya exhibiting total eclipses and multiperiodic oscillations. The binary modeling represents that the eclipse timings vary due to apsidal motion and that our target star is in an eccentric-orbit, detached configuration with parameters of \( e = 0.234, \, \omega = 250.0 \text{ deg}, \, i = 89.23 \text{ deg}, \, T_{\text{eff},1} = 7291 \text{ K}, \) and \( T_{\text{eff},2} = 6864 \text{ K} \). The primary and secondary components fill their inner Roche lobe by 36\% and 46\%, respectively. Combining our Model 2 parameters for the TESS data and the spectroscopic elements of Popper (1988), we derived the absolute dimensions for AI Hya given in Table 5. The temperature error was assumed to be about 200 K, which is the difference between Popper’s temperatures and ours. The luminosities and the bolometric magnitudes were obtained by applying the solar values of \( T_{\text{eff}, \odot} = 5,780 \text{ K} \) and \( M_{\text{bol}, \odot} = +4.73 \). For the absolute visual magnitudes (\( M_V \)), we used the bolometric corrections (BCs) taken from the
expression given by Torres (2010) between log $T_{\text{eff}}$ and BC. Using the apparent magnitude of $V = +9.36$ and the interstellar extinction of $A_V = 0.097$ (Schlafly & Finkbeiner 2011), we obtained the distance to the AI Hya system of 585±34 pc. This is consistent with 644±26 pc calculated by the inverted parallax from Gaia DR2 (1.554±0.063 mas; Gaia Collaboration et al. 2018) and 632±25 pc estimated by the geometric parallax (Bailer-Jones et al. 2019) within their errors.

Using the PERIOD04 program, we applied a multifrequency analysis to the eclipse-subtracted light residuals and examined possible harmonic and combination frequencies. As a result, 14 frequencies with S/N $> 4.0$ were found, four ($f_1$, $f_2$, $f_3$, and $f_6$) of which may be considered independent pulsations. The pulsation periods for these frequencies are in the range of $P_{\text{pul}} = 0.100$−0.173 days. The ratios between the pulsation and orbital periods are $P_{\text{pul}}/P_{\text{orb}} = 0.012$−0.021, which is within the upper limit of 0.09 for δ Sct-type pulsators in binaries (Zhang et al. 2013). Liakos & Niarchos (2017) presented a possible $P_{\text{orb}} - P_{\text{pul}}$ relation in the binary star systems with orbital periods below 13 days. In case of AI Hya, $f_2$ and $f_3$ match well their correlation for the δ Sct stars in detached EBs, while $f_1$ and $f_6$ do not conform to the empirical relation. From the well-known relation of $Q = P_{\text{pul}} \sqrt{\rho/\rho_\odot}$ and the mean density $\rho_2$ given in Table 5, we got the observed pulsation constants of $Q_1 = 0.048$ days, $Q_2 = 0.030$ days, $Q_3 = 0.032$ days, and $Q_6 = 0.052$ days, corresponding to classical δ Sct pulsations. Then, we compared the $Q$ values with the theoretical models predicted by Fitch (1981) for 2.0 $M_\odot$. The $f_2$ and $f_3$ frequencies might be related to the radial fundamental $F$ modes, while the $f_1$ and $f_6$ could be regarded as the non-radial $g_1$ modes with a low degree of $\ell = 2$. AI Hya is a double-lined EB with a multiperiodic δ Sct component, so it should be a good candidate for asteroseismology in binaries. Long-term multiband photometric monitoring of the pulsating EB will help to identify the pulsation modes and to inspect the eclipse timing variation in detail.

**Acknowledgments**

This paper includes data collected by the TESS mission, which were obtained from MAST. Funding for the TESS mission is provided by the NASA Explorer Program. We thank the TESS team for its support of this work. This research has made use of the Simbad database maintained at CDS, Strasbourg, France, and was supported by the KASI grant 2020-1-830-08. K.H. was supported by the grants 2017R1A4A1015178 and 2019R1I1A1A01056776 from the National Research Foundation (NRF) of Korea.

**References**

Antoci, V., et al. 2019, MNRAS, 490, 4040
Bailer-Jones, C. A. L., Rybizki, J., Fouesneau, M., et al. 2018, AJ, 156, 58
Breger, M. 2000, in ASP Conf. Ser. 210, Delta Scuti and Related Stars, ed. M. Breger, & M. H. Montgomery (San Francisco: ASP), 3
Breger, M., et al. 1993, A&A, 271, 482
Busch, H. 1970, Mitteilungen der Bruno-H.-Bürgel-Sternwarte, Hartha, Heft 3, 1970
Eastman, J., Siverd, R., & Gaudi, B. S. 2010, PASP, 122, 935
Fitch, W. S. 1981, ApJ, 249, 218
Gaia Collaboration, et al. 2018, A&A, 616, A1
Giménez, A., & Bastero, M. 1995, Ap&SS, 226, 99
Hilditch, R. W. 2001, An Introduction to Close Binary Stars (Cambridge: Cambridge Univ. Press)
Hübscher, J., Braune, W., & Lehmann, P. B. 2013, IBVS, 6048, 1
Jørgensen, H. E., & Grønbech, B. 1978, A&A, 66, 377
Kallinger, T., Reegen, P., & Weiss, W. W. 2008, A&A, 481, 571
Khaliullin, Kh. F., & Kozyreva, V. S. 1989, Ap&SS, 155, 53
Kizilirmak, A., & Pohl, E. 1974, IBVS, 937, 1
Koo, J.-R., et al. 2014, AJ, 147, 104
Kreiner, J. M., Kim, C.-H., & Nha, I.-S. 2001, An Atlas of $O-C$ Diagrams of Eclipsing Binary Stars (Krakow: Wydawn. Nauk. Akad. Pedagogicznej)
Kwee, K. K., & van Woerden, H. 1956, BAN, 12, 327
Lause, F. 1938, AN, 266, 237
Lee, J. W. 2016, ApJ, 833, 170
Lee, J. W., & Park, J.-H. 2018, MNRAS, 480, 4693
Lee, J. W., Hong, K., Kim, S.-L., & Koo, J.-R. 2017, ApJ, 835, 189
Lee, J. W., Hong, K., Koo, J.-R., & Park, J.-H. 2020, AJ, 159, 24
Lee, J. W., Kim, S.-L., Hong, K., Lee, C.-U., & Koo, J.-R. 2014, AJ, 148, 37
Lee, J. W., Kristiansen, M., & Hong, K. 2019, AJ, 157, 223
Lenz, P., & Breger, M. 2005, Comm. Asteroseismology, 146, 53
Liakos, A., & Niarchos, P. 2017, MNRAS, 465, 1181
Loumos, G. L., & Deeming, T. J. 1978, Ap&SS, 56, 285
Nagai, K. 2006, VSOLJ Variable Star Bull., 44, 5
Pecaut, M. J., & Mamajek, E. E. 2013, ApJS, 208, 9
Popper, D. M. 1988, AJ, 95, 190
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes (Cambridge: Cambridge Univ. Press), Chapter 15
Ricker, G. R., et al. 2015, JATIS, 1, 014003
Schlafly, E. F., & Finkbeiner, D. P. 2011, AJ, 737, 103
Torres, G. 2010, AJ, 140, 1158
Torres, G., Andersen, J., & Giménez, A. 2010, A&ARv, 18, 67
Van Hamme, W. 1993, AJ, 106, 209
Van Hamme, W., & Wilson, R. E. 2007, ApJ, 661, 1129
Wilson, R. E., & Biermann, P. 1976, A&A, 48, 349
Wilson, R. E., & Devinney, E. J. 1971, ApJ, 166, 605
Zhang, X. B., Luo, C. Q., & Fu, J. N. 2013, ApJ, 777, 77
Fig. 1. Black circles present the *TESS* time-series data of Al Hya separated at intervals of 8.3 days. The green lines are the sum of the two model curves computed from the Model 2 parameters of Table 3 and the 14-frequency fit of Table 4, respectively. They are offset by +0.1 mag for clarity.
Fig. 2. Eclipse timing diagram of AI Hya. The filled and open symbols represent the primary and secondary times of minimum light, respectively. The circles and squares denote the literature and *TESS* eclipse timings, respectively. The dotted lines are computed with the apsidal motion elements in Table 2.
Fig. 3. Binary light curve before (gray circle) and after (black circle) subtracting the pulsation signatures from the observed TESS data. The blue and red lines are computed with the Model 1 and Model 2 parameters in Table 3, respectively. The corresponding residuals from the fits are plotted in the middle and bottom panels in the same order as the light curves. In the bottom panel, some features visible during the times of both eclipses may come from insufficient removal of the pulsations in the orbital phases, caused by using only out-of-eclipse data in the frequency analysis.

Fig. 4. Light curve residuals after removing the binarity effects from the observed TESS data. The lower panel presents a short section of the residuals marked by the inset box in the upper panel. The synthetic curve is computed from the 14-frequency fit to the whole data except for the times of both eclipses.
Fig. 5. Amplitude spectra before (top panel) and after pre-whitening the first three frequencies (middle) and all 14 frequencies (bottom) from the PERIOD04 program for the outside-eclipse residuals. In all panels, the red vertical lines represent the location of $7.2448 \text{ day}^{-1}$ previously detected by Jørgensen & Grønbech (1978) but undetected in the $TESS$ data.
Table 1. Observed photoelectric and CCD times of minimum light for AI Hya.

| BJD (2,400,000+) | Error | Epoch | $O - C_1$ | $O - C_2$ | Min | References                      |
|------------------|-------|-------|-----------|-----------|-----|--------------------------------|
| 41,411.3685      | -38.0 |       | +0.23279  | -0.00623  | I   | Kizilirmak & Pohl (1974)       |
| 41,721.7305      | -0.5  |       | -0.26796  | +0.00042  | II  | Jørgensen & Grønbech (1978)    |
| 41,726.3877      | 0.0   |       | +0.24440  | +0.00621  | I   | Jørgensen & Grønbech (1978)    |
| 53,406.5007      | ±0.007 | 1409.0 | +0.20744  | +0.00043  | I   | Kreiner et al. (2001)          |
| 53,435.0565      | ±0.0011 | 1412.5 | -0.25062  | -0.00093  | II  | Nagai (2006)                   |
| 55,963.4110      | ±0.0011 | 1717.5 | -0.24654  | -0.00091  | II  | Hübscher et al. (2013)         |
| 58,491.76077     | ±0.00016 | 2022.5 | -0.24118  | +0.00038  | II  | This article (TESS)            |
| 58,496.34588     | ±0.00022 | 2023.0 | +0.19309  | -0.00008  | I   | This article (TESS)            |
| 58,500.05661     | ±0.00005 | 2023.5 | -0.24102  | +0.00053  | II  | This article (TESS)            |
| 58,508.34027     | ±0.00022 | 2024.5 | -0.24103  | +0.00050  | II  | This article (TESS)            |
| 58,512.92494     | ±0.00023 | 2025.0 | +0.19280  | -0.00032  | I   | This article (TESS)            |

Table 2. Apsidal motion elements of AI Hya.

| Parameter     | Value                      |
|---------------|----------------------------|
| $T_0$ (BJD)   | 2,441,726.086±0.017        |
| $P_s$ (day)   | 8.289672±0.000015          |
| $P_a$ (day)   | 8.289711±0.000017          |
| $e$           | 0.241±0.083                |
| $\omega_0$ (deg) | 247±8                   |
| $\dot{\omega}$ (deg year$^{-1}$) | 0.075±0.031 |
| $U$ (year)    | 4800±2000                 |
Table 3. Binary parameters of Al Hya.

| Parameter              | Model 1<sup>a</sup> | Model 2<sup>b</sup> |
|------------------------|----------------------|----------------------|
|                        | Primary   | Secondary | Primary   | Secondary |
| $T_0$ (BJD)            | 2,458,496.09280(48) |          | 2,458,496.09319(14) |          |
| $P_{\text{orb}}$ (day) | 8.29070(31)       |          | 8.28994(10)        |          |
| $q$                    | 1.084         |          | 1.084               |          |
| $e$                    | 0.2317(23)     |          | 0.2343(9)           |          |
| $\omega$ (deg)        | 249.73(21)     |          | 249.96(9)           |          |
| $i$ (deg)              | 89.314(29)     |          | 89.234(18)          |          |
| $T_{\text{eff}}$ (K)   | 7309(70)       | 6854(60) | 7291(33)            | 6864(28) |
| $\Omega$              | 11.280(19)     | 8.759(17) | 11.241(5)           | 8.816(3) |
| $\Omega_{\text{in}}$  | 3.885          |          | 3.885               |          |
| $A$                    | 1.0           | 0.5      | 1.0                 | 0.5      |
| $g$                    | 1.0           | 0.32     | 1.0                 | 0.32     |
| $X$, $Y$               | 0.643, 0.249   | 0.635, 0.249 | 0.643, 0.249    | 0.635, 0.249 |
| $x$, $y$               | 0.618, 0.268   | 0.620, 0.296 | 0.617, 0.270   | 0.620, 0.296 |
| $L/(L_1+L_2)$          | 0.3780(6)      | 0.6220   | 0.3805(3)           | 0.6195   |
| $r$ (pole)             | 0.1012(3)      | 0.1441(23)| 0.1017(1)           | 0.1431(2) |
| $r$ (point)            | 0.1017(3)      | 0.1461(24)| 0.1022(1)           | 0.1451(2) |
| $r$ (side)             | 0.1013(3)      | 0.1445(23)| 0.1018(1)           | 0.1435(2) |
| $r$ (back)             | 0.1017(3)      | 0.1457(23)| 0.1021(1)           | 0.1447(2) |
| $r$ (volume)<sup>c</sup> | 0.1014(3)  | 0.1448(23) | 0.1019(1)           | 0.1438(2) |
| $\sum W(O-C)^2$       | 0.0041         |          | 0.0017              |          |

<sup>a</sup>Result from the observed data.

<sup>b</sup>Result from the pulsation-subtracted data.

<sup>c</sup>Mean volume radius.
Table 4. Results of the multiple frequency analysis for AI Hya.

| Frequency (day$^{-1}$) | Amplitude (mmag) | Phase (rad) | S/N$^b$ | Remark$^c$ |
|------------------------|------------------|-------------|---------|------------|
| $f_1$                  | 6.2406±0.0002    | 4.76±0.16   | 2.58±0.10 | 51.41      |
| $f_2$                  | 9.9064±0.0012    | 1.21±0.19   | 1.26±0.47 | 10.76      |
| $f_3$                  | 9.2539±0.0017    | 0.93±0.20   | 2.71±0.62 | 8.07       |
| $f_4$                  | 0.1205±0.0007    | 1.37±0.12   | 4.25±0.26 | 18.98      |
| $f_5$                  | 9.3682±0.0012    | 1.31±0.20   | 5.20±0.44 | 11.30      |
| $f_6$                  | 5.783±0.0017     | 0.69±0.15   | 4.92±0.63 | 7.91       |
| $f_7$                  | 5.554±0.00015    | 0.77±0.14   | 3.92±0.55 | 9.07       |
| $f_8$                  | 5.6379±0.0016    | 0.70±0.15   | 0.82±0.61 | 8.25       |
| $f_9$                  | 9.8441±0.0023    | 0.66±0.19   | 1.30±0.85 | 5.88       |
| $f_{10}$               | 12.9322±0.0024   | 0.55±0.16   | 3.36±0.88 | 5.71       |
| $f_{11}$               | 9.3183±0.0022    | 0.73±0.20   | 3.09±0.79 | 6.32       |
| $f_{12}$               | 9.204±0.0029     | 0.54±0.19   | 3.37±1.05 | 4.79       |
| $f_{13}$               | 12.716±0.0028    | 0.48±0.17   | 4.54±1.02 | 4.89       |
| $f_{14}$               | 15.463±0.0023    | 0.42±0.12   | 3.99±0.84 | 5.98       |

$^a$Uncertainties were calculated according to Kallinger et al. (2008).

$^b$Calculated in a range of 5 day$^{-1}$ around each frequency.

$^c$Possible harmonic and combination frequencies.

Table 5. Absolute parameters of AI Hya.

| Parameter      | Popper (1988) | This Paper |
|----------------|---------------|------------|
|                | Primary       | Secondary  | Primary       | Secondary  |
| $M$ ($M_\odot$) | 1.98±0.04     | 2.15±0.04  | 1.97±0.03     | 2.14±0.03  |
| $R$ ($R_\odot$) | 2.77±0.02     | 3.92±0.03  | 2.81±0.02     | 3.96±0.04  |
| log $g$ (cgs)  | 3.85±0.01     | 3.58±0.01  | 3.84±0.01     | 3.57±0.01  |
| $\rho$ ($\rho_\odot$) | 0.089±0.002 | 0.034±0.001 |
| $T_{\text{eff}}$ (K) | 7096±65       | 6699±62   | 7291±200      | 6864±200   |
| $L$ ($L_\odot$)  | 17.4±0.8      | 27.5±1.3  | 20±2          | 31±4       |
| $M_{\text{bol}}$ (mag) | 1.48±0.12     | 0.99±0.13  |
| $BC$ (mag)       | 0.03±0.01     | 0.03±0.01  |
| $M_V$ (mag)      | 1.53±0.04     | 1.05±0.04  | 1.45±0.12     | 0.97±0.13  |
| Distance (pc)    | 575±15        |            | 585±34        |            |