The Masses of Gauge Bosons in
$SU(3)_C \times SU(2)_L \times U(1)_{Y'} \times U(1)_{B-L}$ gauge model.

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Abstract

We will present within the context of the supersymmetric models with $SU(3)_C \times SU(2)_L \times U(1)_{Y'} \times U(1)_{B-L}$ gauge symmetry an explanation for the new data on the $W$-boson mass recently presented by the CDF collaboration. We will also study the neutral boson sector of this model.

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1 Introduction

In the the Standard Model (SM), the gauge symmetry is defined as [1] [2]

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$  (1)

and the gauge sector of this model is presented in our Tab.(1).

| Group  | Gauge Bosons | Gauge constant |
|--------|--------------|----------------|
| $SU(3)_C$ | $g^a_m$ | $g_s$ |
| $SU(2)_L$ | $W^i_m$ | $g$ |
| $U(1)_Y$ | $b_m$ | $g_Y$ |

Table 1: Information on fields contents of each vector superfield of this model, where $a = 1, 2, \ldots, 8$ and $i = 1, 2, 3$. 

1
There are two massives gauge boson\(^1\). The charged gauge boson \(W^\pm\)
\[
M_W = \frac{g v}{2},
\]
theoretical estimates indicate the following numerical value for its mass
\[
(M_W)_{\text{SM}} = (80.3505 \pm 0.0077) \text{ GeV}.
\]
however the average value of experimental measurements provides us with
the following data \(^2\)
\[
(M_W)_{\text{EXP}} = (80.4133 \pm 0.0080) \text{ GeV}.
\]
Recently, Fermilab’s CDF collaboration presented its highly precise measurements, with an accuracy of the order of \(\sim 10^{-4}\), for the mass of the \(W\)-boson mass \(^3\)
\[
(M_W)_{\text{CDF}} = (80.4335 \pm 0.0094) \text{ GeV},
\]
this value represents an increase greater than 6 \(\sigma\) in relation to the value predicted by the SM. It is clear that, if this result is proven by other experimental collaborations, we have a strong indication of physics beyond SM \(^5\).

Recently it was proposed this shift can be easily explained by a real triplet Higgs boson \(^6\) \(^7\) and with an extra complex triplet Higgs boson \(^8\) and also at leptoquark model \(^9\). There is an attempt to explain this anomaly in the context of the Minimal \(R\)-Symmetric extension of MSSM, known as MRSSM \(^10\) \(^11\) \(^12\) \(^13\) and also, the use of Two Higgs Doulet Model (2HDM) \(^14\), and there is also a top-down motivated model in which extra sector states from a D3-brane also accommodate this result \(^15\).

Two important problems of today’s particle physics are: explain the masses of neutrinos as well as generate dark matter. Today we know that neutrinos have mass, that their flavors are mixed and also that there is at least one Higgs boson. However, we do not know yet if neutrinos are Majorana or Dirac particles and if there are more neutral scalars. Moreover, right-handed sterile neutrinos i.e., singlets under the SM, have not been observed yet. However, on the other hand, there is strong experimental evidence for the existence of dark matter and it cannot be resolved within the SM.

\(^1\) See our Eq. (6) in App. (A)
\(^2\)
Certainly the most popular extension of the SM is its supersymmetric counterpart called Minimal Supersymmetric Standard Model (MSSM) \[16,17\]. The main motivation to study this model is that it provides a solution to the hierarchy problem by protecting the electroweak scale from large radiative corrections.

Supersymmetric theories (SUSY) has also made several correct predictions \[18\]

- SUSY predicted in the early 1980s that the top quark would be heavy;
- SUSY GUT theories with a high fundamental scale accurately predicted the present experimental value of $\sin^2 \theta_W$ before it was measured;
- SUSY requires a light Higgs boson to exist.

Together these success provide powerful indirect evidence that low energy SUSY is indeed part of correct description of nature. The current status of the search for supersymmetry is presented in reference \[19\].

In this model, as in the SM, neutrinos are massless, but in this model there are interactions that volates Lepton or Baryon Number conservation, which allows us to generate non-zero masses for at least one of the neutrinos as well as have an explanation for the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix is also accommodated \[20\].

Some years ago, it was proposed a model in which $U(1)_{(B-L)}$ is not just a new factor added to the SM gauge symmetry but $U(1)_Y$ is substituted by $U(1)_{Y'} \times U(1)_{(B-L)}$ and the breaking

$$U(1)_{Y'} \times U(1)_{(B-L)} \rightarrow U(1)_Y$$

occurs at the TeV scale. We choose the $Y'$ parameter to obtain the hypercharg $Y$ of the SM through the following expression

$$Y = Y' + (B - L).$$

Moreover, the number of right-handed neutrinos, $N_{iR}$, and their $(B - L)$ (or $Y'$) quantum numbers are free parameters, but the cancellation of the cubic and the linear anomalies implies that at least three right-handed neutrinos must be added to the matter representation content. Explicit solutions for the $(B - L)$ (or $Y'$) parameters show that at least two types of model arise \[21\].
1-) the model with three right-handed neutrinos having \((B - L) = -1\) (known as three identical neutrinos);

2-) the model with two right-handed neutrinos having \((B - L) = -4\) and the third one having \((B - L) = 5\) (known as three non-identical neutrinos).

The Supersymmetric \(U(1)_Y \times U(1)_{B-L}\) model with three identical neutrinos was presented at [22], where it was considered that the sneutrinos\(^2\), both left-handed, \(\tilde{\nu}_i L\), and right-handed, \(\tilde{N}_i R\), have vacuum expectation values (VEV) equal to zero. The supersymmetric model with nonidentical neutrinos is in Ref [23]. In both supersymmetric models we can explain the masses of neutrino as well we have interesting candidate for dark model [22, 23]. There is an interesting model having fractional \((B - L)\) charges [24].

In this article we will relax this hypothesis and show that when we allow our left-handed sneutrinos get VEV, and let’s represent it as \(v_{3L}\), we can explain the new CDF data about \(W\)-boson mass. We will study here the masses of Gauge Bosons in both supersymmetric models mentioned above.

The outline of this paper is as follows. In Sec.2 we study the masses of the gauge bosons in the model with three identical right-handed neutrinos and then we perform the same analyses for the case with three nonidentical right-handed neutrinos. Our conclusions appear in Sec. 4. We will briefly present the MSSMRV to better understand the results of charged bosons and show a previous analysis of the neutral boson sector to better understand the results of our models in App. (A). In the last Appendix we present numerical analysis to understand the mixing in the sector of neutral gauge bosons for three non-identical neutrinos, the App. (B).

2 Model with three identical neutrinos.

The gauge symmetry of the model is given by

\[ SU(2)_L \times U(1)_Y \times U(1)_{B-L}, \]  

the charge operator is defined using the following algebraic expression [21]

\[ \frac{Q}{e} = I_3 + \frac{1}{2} [Y' + (B - L)] . \]  

\(^2\)They are the usual superpartner of neutrinos introduced in SM.
We also assume that the \((B-L)\) and \(Y'\) assignments are restricted to integer numbers, as we mentioned in our introduction.

We introduce the charged leptons\(^3\) as well as the usual MSSM scalars in the following chiral superfields

\[
\hat{L}_i = \left( \begin{array}{c} \hat{\nu}_i \\ \hat{l}_i \end{array} \right)_{L} \sim (1, 2, 0, -1), \quad \hat{E}_{iL} \sim (1, 1, 1, 1), \quad i = 1, 2, 3,
\]

\[
\hat{H}_1 = \left( \frac{\hat{h}_1^+}{\hat{h}_1^0} \right)_{L} \sim (1, 2, +1, 0), \quad \hat{H}_2 = \left( \begin{array}{c} \hat{h}_2^0 \\ \hat{h}_2^- \end{array} \right) \sim (1, 2, -1, 0),
\]

with \(i = 1, 2, 3\), and in parentheses we present the transformations properties under the respective gauge factors \((SU(3)_C,SU(2)_L,U(1)_{Y'},U(1)_{(B-L)})\). We use the standard notation \(\hat{E}_{iL} \equiv (\hat{l}_{iR})^c\). In this model, we also introduce three right-handed neutrinos \(^{22}\)

\[
\hat{\nu}_{iL} \sim (1, 1, -1, 1),
\]

where again \(\hat{\nu}_{iL} \equiv (\hat{\nu}_{iR})^c\) as well as two singlet \(SU(2)_L\) scalars

\[
\hat{\phi}_1 \sim (1, 1, -2, 2), \quad \hat{\phi}_2 \sim (1, 1, 2, -2).
\]

The particle content of each chiral superfields defined above are presented in the Tab.\(^2\), we also show their quantum numbers.

The following neutral scalars in doublet representation get non-zero vacuum expectation value (VEV) denoted as

\[
\langle H_1 \rangle \equiv \frac{1}{\sqrt{2}} \left[ v_1 + \sigma_1^0 + i\varphi_1^0 \right], \quad \langle H_2 \rangle \equiv \frac{1}{\sqrt{2}} \left[ v_2 + \sigma_2^0 + i\varphi_2^0 \right],
\]

\[
\langle \bar{L}_3 \rangle \equiv \frac{1}{\sqrt{2}} \left[ v_3^L + \bar{\nu}_3^R + \nu_3^I \right],
\]

which will be constrained by the relation \((v_1)^2 + (v_2)^2 + (v_3^L)^2 = (246 \text{ GeV})^2\), see our Eq.\(^{19}\) and Fig.\(^3\). The scalars in singlet representation get the following VEV

\[
\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \left[ u_1 + \sigma_4^0 + i\varphi_4^0 \right], \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \left[ u_2 + \sigma_5^0 + i\varphi_5^0 \right].
\]

\(^3\)We are omitting the quarks, if you are interested in them, see \(^{21}\).
Table 2: Particle content in each chiral superfield introduced in Eqs. (10,11,12).

The values of $u_1$ and $u_2$ that give the scale of the $U(1)_{(B-L)}$ symmetry breaking are not fixed, they may have values ranging from TeV to much higher scales.

Concerning the gauge bosons and their superpartners, they are introduced in vector superfields, as we shown in Tab. (3), together with the gauge coupling constant of each group.

Table 3: Information on fields contents of each vector superfield of this model.

The gauge coupling constants are related by the following relation

$$\frac{1}{g_Y^2} = \frac{1}{g_Y'^2} + \frac{1}{g_{BL}^2},$$

(15)

where $g_Y$ is the SM $U(1)_Y$ coupling constant, see Eq. (57). We can also define the $\vartheta$-parameter in the following way

$$\tan \vartheta \equiv \frac{g_{BL}}{g_Y'},$$

(16)
In our Figs. (1,2) and Tab. (4), we show the possible values for $g_{BL}$ and $\tan \vartheta$ as function of $g_{Y'}$ and where we can see that both couplings have the same order of magnitude, that is, they satisfy

$$\mathcal{O}(g_{Y'}) \sim \mathcal{O}(g_{BL}).$$

as showed in [26].

| $g_{Y'}$ | $g_{BL}$ | $\tan \vartheta$ |
|----------|----------|------------------|
| 0.4      | 0.72     | 1.807            |
| 0.485    | 0.506    | 1.042            |
| 0.5      | 0.49     | 0.980            |
| 0.6      | 0.43     | 0.718            |
| 0.7      | 0.40     | 0.577            |
| 0.8      | 0.39     | 0.487            |
| 0.9      | 0.38     | 0.422            |

Table 4: The values for $g_{BL}$ and $\tan \vartheta$ as function of $g_{Y'}$, as defined in our Eqs. (15,16).

We will not write the Lagrangian of the model here, it can be seen in [22].
We calculate the mass of gauge bosons using the following expression

\((\mathcal{D}_m H_1)\dagger (\mathcal{D}^m H_1) + (\mathcal{D}_m H_2)\dagger (\mathcal{D}^m H_2) + (\mathcal{D}_m \tilde{L}_i L)\dagger (\mathcal{D}^m \tilde{L}_i L) + (\mathcal{D}_m \tilde{N}_L)\dagger (\mathcal{D}^m \tilde{N}_L) + (\mathcal{D}_m \phi_1)\dagger (\mathcal{D}^m \phi_1) + (\mathcal{D}_m \phi_2)\dagger (\mathcal{D}^m \phi_2)\). \hspace{1cm} (18)

We get the following expression to the charged gauge bosons masses

\[ M^2_W = \frac{g^2}{4} \left[ (v_1)^2 + (v_2)^2 + (v_3^L)^2 \right] = \frac{g^2 v_1^2}{4} \left[ 1 + \tan^2 \beta + \tan^2 \theta \right]. \hspace{1cm} (19)\]

We define the parameters \( \beta, \theta \) in the following way

\[ \tan \beta = \frac{v_2}{v_1}, \quad \tan \theta = \frac{v_3^L}{v_1}. \hspace{1cm} (20)\]

The parameter \( \beta \) is introduced in the MSSM with \( R \)-Parity conservation \[16, 17\] and also in the MSSM with \( R \)-Parity violation \[20\] and even in this case \[4\], as in this model, we also introduce a new angle \( \theta \). We can also rewrite from Eq. (19) the following relation

\[ \delta M_W = \frac{g v_1}{2} \tan \theta. \hspace{1cm} (21)\]

\[4\] See our Append. (A).
Figure 3: The masses of $W$ gauge boson as function of $v_1$ for $\theta = 0$ rad, and some values of our $\beta$-parameter shown on the box. The value of line red is given by Eq.(3), the green by Eq.(4) and the blue line for Eq.(5).

Our numerical results are shown at Figs.(3,4,5), where it is easy to convince yourself, we can reproduce the experimental values of $W$ for several values of the parameters $\beta$ and $\theta$.

The mass matrix for the neutral gauge boson is given by

$$ (M^2)_{\text{identical}} = \begin{pmatrix} A & -\frac{g_Y v}{4} \left[ (v_1)^2 + (v_2)^2 \right] & -\frac{g_{BL}}{4} \left( v_3^L \right)^2 \\ -\frac{g_Y v}{4} \left[ (v_1)^2 + (v_2)^2 \right] & B & -g_Y g_{BL} \left( u_1^2 + u_2^2 \right) \\ -\frac{g_{BL}}{4} \left( v_3^L \right)^2 & -g_Y g_{BL} \left( u_1^2 + u_2^2 \right) & C \end{pmatrix}, $$

where we have defined

$$ A = \frac{g^2}{4} \left[ (v_1)^2 + (v_2)^2 + \left( v_3^L \right)^2 \right], $$

$$ B = g_Y^2 \left( \frac{(v_1)^2 + (v_2)^2}{4} + u_1^2 + u_2^2 \right), $$

$$ C = g_{BL}^2 \left( \frac{(v_3^L)^2}{4} + u_1^2 + u_2^2 \right). $$

We can show the following analytical results

$$ \det(M^2)_{\text{identical}} = 0, $$

(22)
Figure 4: The masses of $W$ gauge boson in function of $v_1$ for $\theta = (\pi/4)$ rad, and some values of our $\beta$-parameter shown on the box. The value of line red is given by Eq. (3), the green by Eq. (4) and the blue line for Eq. (5).

Figure 5: $\delta M_W$, defined in Eq. (21), as function of the new parameter $\theta$, the black line, for $v_1 = 90$ GeV; the red line $v_1 = 100$ GeV; the green line $v_1 = 110$ GeV to explain the values defined in our Eq. (66). This result is also hold in the case of the MSSM with $R$-Parity violation, see Eq. (19).
\[
Tr(M^2)^{\text{identical}} = A + B + C.
\]  

(24)

Therefore we get the foton, and it is massless, and also two massive gauge boson \( Z \) and \( Z' \) and their masses are
\[
M^2_Z = \frac{1}{8} \left[ (U + N) - \sqrt{(U + N)^2 - (V + O)} \right],
\]
\[
M^2_{Z'} = \frac{1}{8} \left[ (U + N) + \sqrt{(U + N)^2 - (V + O)} \right],
\]

(25)

where we have defined and we also use Eq. (20)
\[
U = 4 \left( g_Y^2 + g_{BL}^2 \right) \left( u_1^2 + u_2^2 \right) + \left( g^2 + g_{Y'}^2 \right) \left( v_1^2 + v_2^2 \right)
\]
\[
= 4v_1^2 \left( g_Y^2 + g_{BL}^2 \right) \left( \tan^2 \xi + \tan^2 \zeta \right) + v_1^2 \left( g^2 + g_{Y'}^2 \right) \left( 1 + \tan^2 \beta \right),
\]
\[
N = \left( g^2 + g_{BL}^2 \right) \left( v_3^L \right)^2 = v_1^2 \left( g^2 + g_{BL}^2 \right) \tan^2 \theta,
\]
\[
V = 16 \left[ g^2 \left( g_Y^2 + g_{BL}^2 \right) + g_{Y'}^2 \right] \left( u_1^2 + u_2^2 \right) \left( v_1^2 + v_2^2 \right)
\]
\[
= 16v_1^2 \left[ g^2 \left( g_Y^2 + g_{BL}^2 \right) + g_{Y'}^2 \right] \left( 1 + \tan^2 \beta \right) \left( \tan^2 \xi + \tan^2 \zeta \right),
\]
\[
O = 4 \left[ g^2 \left( g_Y^2 + g_{BL}^2 \right) + g_{Y'}^2 \right] \left( v_1^2 + v_2^2 + 4u_1^2 + 4u_2^2 \right) \left( v_3^L \right)^2
\]
\[
= 4v_1^2 \left[ g^2 \left( g_Y^2 + g_{BL}^2 \right) + g_{Y'}^2 \right] \left( 1 + \tan^2 \beta \right) \left( \tan^2 \xi + \tan^2 \zeta \right),
\]

(26)

where we have defined the new parameters \( \xi \) and \( \zeta \) in the following way:
\[
\tan \xi = \frac{u_1}{v_1}, \quad \tan \zeta = \frac{u_2}{v_1}.
\]

(27)

The trilinear gauge boson couplings are modified by the following mixing factor [27, 28]
\[
\xi_{Z-Z'} = \left( \frac{M_Z}{M_{Z'}} \right)^2,
\]
\[
M_{Z'} > 3 \text{ TeV},
\]

(28)

this parameter, \( \xi_{Z-Z'} \), must be smaller than \( 10^{-3} \). On the other hand, we still have the following constraint [26]
\[
\frac{M_{Z'}}{g_{BL}} > 6 \text{ TeV}.
\]

(29)
Figure 6: The masses of $Z$ gauge boson in function of $v_1$ and for $\beta = (\pi/4)$ rad and $\zeta = \tan^{-1}(5000/174.1)$, the red by Eq.(4) and the green line for Eq.(62) while blue line for Eq.(65), see the first relation in our Eq.(25).

Figure 7: The masses of $Z'$ gauge boson for $v_1$ and some values of our $\beta$-parameter shown on the box, $\zeta = \tan^{-1}(5000/174.1)$, see the second relation in our Eq.(25) and we see all the values are in agreement with experimental bound given by our Eq.(28).
Figure 8: The masses of $Z$ gauge boson in function of $v_1$ for some values of our $\beta$-parameter shown on the box, $\theta = (\pi/100)$ rad, $\zeta = \tan^{-1}(5000/174.1)$, the red by Eq.(4) and the green line for Eq.(62) while blue line for Eq.(65), see the first relation in our Eq.(25).

Figure 9: The masses of $Z'$ gauge boson for $v_1 = 174.15$ GeV as function of $u_2$ for some values of $\xi$ parameter and some values of our $\beta$-parameter shown on the box, see the second relation in our Eq.(25) and we see all the values are in agreement with experimental bound given by our Eq.(28).
| $u_2$ (TeV) | $M_Z$ (GeV) | $M_{Z'}$ (TeV) | $M_{Z'}/g_{BL}$ (TeV) | $\xi_{Z-Z'}$  |
|---------|---------|---------|---------|---------|
| 3.0     | 91.18   | 2.1     | 4.1     | $1.88 \times 10^{-3}$ |
| 3.5     | 91.18   | 2.5     | 4.8     | $1.38 \times 10^{-3}$ |
| 4.0     | 91.18   | 2.8     | 5.5     | $1.06 \times 10^{-3}$ |
| 4.5     | 91.18   | 3.2     | 6.2     | $8.4 \times 10^{-4}$  |
| 5.0     | 91.18   | 3.5     | 6.9     | $6.8 \times 10^{-4}$  |
| 5.5     | 91.18   | 3.9     | 7.6     | $5.6 \times 10^{-4}$  |
| 6.0     | 91.18   | 4.2     | 8.3     | $4.7 \times 10^{-4}$  |
| 6.5     | 91.18   | 4.6     | 9.0     | $4.0 \times 10^{-4}$  |
| 7.0     | 91.18   | 4.9     | 9.7     | $3.5 \times 10^{-4}$  |

Table 5: The values for $M_Z, M_{Z'}, (M_{Z'}/g_{BL})$ and $\xi_{Z-Z'}$ as function of $u_2$, for the case of $\beta = (\pi/4) \text{ rad}, \theta = (\pi/100) \text{ rad}, \xi = (\pi/3) \text{ rad}$ and we satisfy the bonds of Eqs. (28,29) when $u_2 > 4.5 \text{ TeV}$.

Our numerical result for this boson is presented in our Tab. (5) and Figs. (6,7,8,9).

The superpotential of this model have the following form [22]

$$W = \mu_H \left( \hat{H}_1 \hat{H}_2 \right) + \mu_\phi \hat{\phi}_1 \hat{\phi}_2 + f_{ij} \left( \hat{H}_2 \hat{L}_i \right) \hat{E}_j + f'_{ij} \left( \hat{H}_1 \hat{L}_i \right) \hat{N}_j + f''_{ij} \hat{\phi}_2 \hat{N}_i \hat{N}_j, \quad (30)$$

which generates mass to the model’s charged leptons. We subsequently intend to analyze the masses of these leptons when $\tilde{L}_3$ acquires VEV.

3 The Model with three non-identical Neutrinos

The particle content of this model is as follows: we have the particles described in Eq. (10). In addition to these particles, replacing the three identical neutrinos defined by Eq. (11), we introduce three non-identical right-handed neutrinos [23]

$$\hat{N}_{1R} \sim (1, 1, 5, -5), \quad \hat{N}_{3R} \sim (1, 1, -4, 4),$$

$$\quad (31)$$
where $\beta = 2, 3$. It is necessary to enlarge the scalar doublet sector adding four new scalars

$$\hat{\Phi}_1 = \left( \frac{\hat{\phi}_0}{\hat{\phi}_1} \right) \sim (1, 2, 5, -6), \quad \hat{\Phi}_1' = \left( \frac{\hat{\phi}_1}{\hat{\phi}_0} \right) \sim (1, 2, -5, 6),$$

$$\hat{\Phi}_2 = \left( \frac{\hat{\phi}_0}{\hat{\phi}_2} \right) \sim (1, 2, -4, 3), \quad \hat{\Phi}_2' = \left( \frac{\hat{\phi}_2}{\hat{\phi}_0} \right) \sim (1, 2, 4, -3).$$

(32)

In order to obtain an arbitrary mass matrix for the neutrinos we have to introduce the following additional singlet

$$\hat{\varphi}_1 \sim (1, 1, 8, -8), \quad \hat{\varphi}_2 \sim (1, 1, -10, 10).$$

(33)

Using the VEV defined in our Eq.(13) together with

$$\varphi_1 = \frac{1}{\sqrt{2}} (w_1 + \text{Re}\varphi_1 + i\text{Im}\varphi_1), \quad \varphi_2 = \frac{1}{\sqrt{2}} (w_2 + \text{Re}\varphi_2 + i\text{Im}\varphi_2),$$

$$\Phi_1 = \left( \frac{1}{\sqrt{2}} (u_1 + \text{Re}\phi_1^0 + i\text{Im}\phi_1^0) \right), \quad \Phi_1' = \left( \frac{1}{\sqrt{2}} (u_1' + \text{Re}\phi_1^0 + i\text{Im}\phi_1^0) \right),$$

$$\Phi_2 = \left( \frac{1}{\sqrt{2}} (u_2 + \text{Re}\phi_2^0 + i\text{Im}\phi_2^0) \right), \quad \Phi_2' = \left( \frac{1}{\sqrt{2}} (u_2' + \text{Re}\phi_2^0 + i\text{Im}\phi_2^0) \right).$$

(34)

The mass of charged gauge boson is

$$M_W^2 = \frac{g^2}{4} \left[ (v_1)^2 + (v_2)^2 + (v_3)^2 + (u_1)^2 + (u_2)^2 + (u_1')^2 + (u_2')^2 \right],$$

(35)

now it is hold the following constraint

$$\sum_{i=1}^{2} \left[ (v_i)^2 + (u_i)^2 + (u_i')^2 + (v_3)^2 \right] = (246)^2 \ (\text{GeV})^2.$$ 

(36)

To simplify our numerical analyses we will suppose

$$u_1 = u_2 = u,$$

$$u_1' = u_2' = u',$$

$$w_1 = w_2 = w,$$

(37)

in similar ways as done in [26].
Using Eq. (37) and in similar way as done in MSSM \[16, 17\], we can rewrite this mass expression as

\[
M_W^2 = \frac{g^2}{4} v_1^2 \left[ \tan^2 \beta + 2 \tan^2 \alpha + 2 \tan^2 \gamma + \tan^2 \theta \right],
\]  

(38)

where the parameters \(\beta\) and \(\theta\) are defined in our Eq. (20) and we have defined the following new free parameters:

\[
\tan \alpha = \frac{u}{v_1}, \quad \tan \gamma = \frac{u'}{v_1}.
\]  

(39)

We can also rewrite from Eq. (38) the following relation

\[
\delta M_W = \frac{g v_1}{2} \sqrt{\tan^2 \theta + 2 \tan^2 \alpha + 2 \tan^2 \gamma}.
\]  

(40)

Our results, again, is shown in our Fig. (5) when we take \(\alpha = \gamma = 0\) rad.

In Fig. (10) we shown the behaviour of \(M_W\) as function of \(\alpha\)-parameter and as conclusion we can explain in easy way this experimental values defined in ours Eqs. (3,4,5).

![Figure 10](attachment:figure_10.png)

Figure 10: The Masses of \(W\)- Gauge Bosons, in GeV in function of \(\alpha\)-parameter, see Eq. (38), for \(v_1 = 77.3\) GeV and \(\beta = \gamma = \delta = (\pi/3)\) rad and \(\theta = (\pi/100)\) rad.

The mass matrix for the neutral gauge boson, in the base \((W^3_m, (b_Y)_m, (b_{BL})_m)\), is given by \[20\]
\[ M_{\text{neutral}}^2 = \begin{pmatrix}
g^2(K + P + 2N) & -gg_{Y'}(K + N) & -gg_{BL}(P + N) \\
-gg_{Y'}(K + N) & g^2K & g_{Y'}g_{BL}N \\
-ngg_{BL}(P + N) & g_{Y'}g_{BL}N & g^2_{BL}P
\end{pmatrix} \]

(41)

where

\[
K = \frac{1}{4} \sum_a V_a^2 (Y_a')^2 = \frac{v_2^2}{4} \left[ 1 + \tan^2 \beta + 41 \left( \tan^2 \alpha + \tan^2 \gamma \right) + 164 \tan^2 \delta \right],
\]

\[
P = \frac{1}{4} \sum_a V_a^2 [(B - L)_a]^2 = \frac{v_2^2}{4} \left[ \tan^2 \theta + 45 \left( \tan^2 \alpha + \tan^2 \gamma \right) + 164 \tan^2 \delta \right],
\]

\[
N = \frac{1}{4} \sum_a V_a^2 (Y_a')(B - L)_a = -\frac{v_2^2}{4} \left[ 42 \left( \tan^2 \alpha + \tan^2 \gamma \right) + 164 \tan^2 \delta \right];
\]

(42)

where \(g, g_{Y'}\) and \(g_{BL}\) are the gauge coupling constants are defined in the Tab.(3). The numbers \((Y_a')\) and \((B - L)_a\) being the quantum numbers defined in Tab.(2) and the label \(a\) means all scalars of this model.

It is easy to show

\[
\text{Det} \left( M_{\text{Neutral}}^2 \right) = 0,
\]

\[
\text{Tr} \left( M_{\text{Neutral}}^2 \right) = g^2(K + P + 2N) + g_{Y'}^2K + g_{BL}^2P \equiv R,
\]

(43)

therefore we have one non massive Gauge Boson and it is the photon \(A_m\) plus two massive eigenstates, we will call them as \(Z_{1m}, Z_{2m}\). By another hand the characteristic equation for this mass matrix is

\[ x \left( -T + RX - x^2 \right) = 0, \]

(44)

where \(R\) is defined in our Eq.(43) and we define \(T\) in the following way

\[ T = (KP - N^2) \left[ g^2(g_{Y'}^2 + g_{BL}^2) + g_{Y'}^2g_{BL}^2 \right]. \]

(45)

The massives eigenvalues are given by [26]:

\[
M_{Z_1}^2 = \frac{1}{2} \left[ R - \sqrt{R^2 - 4T} \right],
\]

\[
M_{Z_2}^2 = \frac{1}{2} \left[ R + \sqrt{R^2 - 4T} \right],
\]

(46)
and the mass eigenstates \[26\]:

\[
\begin{align*}
A_m &= \frac{1}{N(y)} \left[ \frac{1}{g} W_m^3 + \frac{1}{g_{Y'}} (b_{Y'})_m + \frac{1}{g_{BL}} (b_{BL})_m \right], \\
Z_m &= \cos \theta_W W_m^3 - \sin \theta_W \sin (b_{Y'})_m - \sin \theta_W \cos \theta_W (b_{BL})_m, \\
Z'_m &= \cos \theta_W (b_{Y'})_m - \sin \theta_W (b_{BL})_m,
\end{align*}
\]

(47)

where the \( \psi \)-parameter is defined at Eq. (16). We can write the two physical massive gauge bosons \(Z_1\) and \(Z_2\) in terms of the usual no physical gauge bosons \(Z\) and \(Z'\) in the following way \[26\]:

\[
\begin{align*}
(Z_1)_m &= \cos \kappa Z_m + \sin \kappa Z'_m, \\
(Z_2)_m &= -\sin \kappa Z_m + \cos \kappa Z'_m,
\end{align*}
\]

(48)

where we have defined

\[
\tan \kappa \equiv \frac{\sqrt{g^2 (g_{BL}^2 + g_{Y'}^2) + g_{BL}^2 g_{Y'}^2 (g_{BL}^2 P - g_{Y'}^2 N - M_{Z_2}^2)}}{g^2 (g_{BL}^2 + g_{Y'}^2) (P + N) + g_{Y'}^2 (g_{BL}^2 (P + N) - M_{Z_2}^2)}. 
\]

(49)

Our numerical results are shown in our Figs. (11, 12, 13) where we can say

- \( Z_{1m} \simeq Z_m \);
- \( Z_{2m} \simeq Z'_m \);
- \( \tan \kappa \simeq 10^{-3} \);

those results are in agreement with \[26\].

The superpotential of this model have the following form \[23\]

\[
W = \mu_H \left( \hat{H}_1 \hat{H}_2 \right) + \mu_{\Phi_1} \left( \hat{\Phi}_1' \hat{\Phi}_1 \right) + \mu_{\Phi_2} \left( \hat{\Phi}_2' \hat{\Phi}_2 \right) + f_{ij} \left( \hat{H}_2 \hat{L}_i \right) \hat{E}_j + f'_{ij} \left( \hat{\Phi}_1' \hat{L}_i \right) \hat{N}_1 + f''_{ij} \left( \hat{\Phi}_2' \hat{L}_i \right) \hat{N}_1 + f'_{\alpha\beta} \hat{\varphi}_2 \hat{N}_\alpha \hat{N}_\beta,
\]

(50)

which generates mass to the model’s charged leptons. We subsequently intend to analyze the masses of these leptons when \( \hat{L}_3 \) acquires VEV.

\(^5\)To understand the eigenvalues see our App. (B).
Figure 11: The Masses of $Z_1$-Neutral Gauge Bosons, in GeV as function of $v_1$, for $\beta = (\pi/3)$ rad, $\theta = (\pi/100)$ rad, $\gamma = \delta = (\pi/3)$ rad, see Eq.(46).

4 Conclusions

We have studied the masses of gauge boson masses in two supersymmetric models, first with three identical right-handed neutrinos and then for the case with three non-identical right-handed neutrinos. We can explain the recent CDF measure of the $W$-boson mass in both models for several values of our $\beta$ and $\theta$ parameters, as shown in our Fig.(5), this results hold for the MSSM with $R$-Parity violation, see Eq.(66), as well for the model with three identical neutrinos, see Eq.(21), and also for three non identical three right handed neutrinos, Eq.(40). Our results for neutral gauge bosons are given in Figs.(6,7,8,9) for the case of three identical neutrinos and in Figs.(11,12) for the three non-identical right and our results are in agreement with current experimental limits. Next we need to study the masses of the leptons as well the masses of the scalar sectors of thoses models.

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Figure 12: The Masses of $Z_2$-Neutral Gauge Bosons, in GeV as function of $v_1$, for $\beta = (\pi/3) \text{ rad}$, $\theta = (\pi/100) \text{ rad}$, $\gamma = \delta = (\pi/3) \text{ rad}$, see Eq.(46) and we see all the values are in agreement with experimental bound given by our Eq.(28).

A Minimal Supersymmetric Standard Model with $R$-Parity violation

In this model, as in MSSM with $R$-parity conservation [16, 17], we will introduce the following chiral superfields for the leptons and usual scalars fields [20]

$$\hat{L}_{iL} = \begin{pmatrix} \hat{\nu}_{iL} \\ \hat{l}_{iL} \end{pmatrix} \sim (1, 2, -1), \quad \hat{E}_{iR} \sim (1, 1, +2), \quad i = 1, 2, 3,$$

$$\hat{H}_1 = \begin{pmatrix} \hat{H}_1^+ \\ \hat{H}_1^0 \end{pmatrix} \sim (1, 2, +1), \quad \hat{H}_2 = \begin{pmatrix} \hat{H}_2^0 \\ \hat{H}_2^- \end{pmatrix} \sim (1, 2, -1). \quad (51)$$

Remember in each chiral superfield we have both spin one-half, fermions and Higgsinos, beyond the scalars, sfermions and Higgs fields. Therefore, the left-handed and right-handed fermions fields have different representation of the gauge group as in the SM. The neutral scalars obtain the VEV defined in our Eq.(13).

Concerning the gauge bosons and their superpartners, they are introduced in vector superfields. See Table[6] the particle content together with the gauge coupling constant of each group.
Figure 13: The $\kappa$-parameter as function of $\alpha$-parameter in rad, for $\beta = (\pi/3)$ rad, $\theta = (\pi/100)$ rad, $\gamma = \delta = (\pi/3)$ rad, see Eq.(49).

| Group       | Vector Superfield | Gauge Bosons | Gaugino | Gauge constant |
|-------------|-------------------|--------------|---------|----------------|
| $SU(3)_C$   | $G^a$             | $g^a_m$      | $\tilde{g}^a$ | $g_s$          |
| $SU(2)_L$   | $W^i$             | $W^i_m$      | $W^i$   | $g$            |
| $U(1)_Y$    | $b$               | $b_m$        | $b$     | $g_Y$          |

Table 6: Information on fields contents of each vector superfield of this model, see our Tab.(1).

The coupling constant $g$ is given by

$$g = \sqrt{\frac{8G_FM_W^2}{\sqrt{2}}} = 0.652673,$$

where the Fermi constant is

$$G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2},$$

the vacuum expectation value can be estimate using the following relation

$$v = \frac{1}{\sqrt{\sqrt{2}G_F}} \simeq 246.22 \text{ GeV.}$$

We get the following expression to the charged gauge bosons masses

$$M_W^2 = \frac{g^2}{4} \left[ v_1^2 + v_2^2 + (v_3^L)^2 \right] = \frac{g^2v_1^2}{4} \left[ 1 + \tan^2 \beta + \tan^2 \theta \right].$$
see Eqs.(19,20).

The mass matrix for the neutral gauge boson is

\[
(M^2)_{\text{RPV}} = \frac{g^2 v^2_1}{4} \left[ 1 + \tan^2 \beta + \tan^2 \theta \right] \begin{pmatrix}
1 & -\tan \theta_W \\
-\tan \theta_W & \tan^2 \theta_W
\end{pmatrix},
\]

(56)

where \(\theta_W\) is the Weinberg angle and it is defined as

\[
\tan \theta_W \equiv \frac{g_Y}{g}, \quad e = g \sin \theta_W = g_Y \cos \theta_W, \quad g_Y = g \left( \frac{M_Z}{M_W} \right) \sqrt{1 - \left( \frac{M_W}{M_Z} \right)^2} \approx 0.350221.
\]

(57)

We can show, from Eq.(56), the following analytical results

\[
\text{det}(M^2)_{\text{RPV}} = 0, \\
\text{Tr}(M^2)_{\text{RPV}} = \frac{g^2 v^2_1}{4 \cos^2 \theta_W} \left[ 1 + \tan^2 \beta + \tan^2 \theta \right] = \frac{M_W^2}{\cos^2 \theta_W}.
\]

(58)

Therefore we get the photon, and it is massless, and also a massive gauge boson \(Z\) and its masses is

\[
M_Z^2 = \frac{M_W^2}{\cos^2 \theta_W},
\]

(59)

and our numerical result for this boson is presented in our Tab.(7) and in our Fig.(14).

The neutral massive gauge boson \(Z\)

\[
M_Z = \frac{M_W}{\cos \theta_W},
\]

(60)

the average value of experimental measurements provides us with the following data [3]

\[
(M_Z)_{\text{EXP}} = (91.1875 \pm 0.0021) \text{ GeV}, \\
\left( \sin \theta_{\text{eff}}^{lep}(Q_{FB}^{had}) \right) = 0.2324 \pm 0.0012, \\
\left( \sin \theta_{\text{eff}}^{lep}(HC) \right) = 0.23143 \pm 0.00025.
\]

(61)
Table 7: The masses of $W$- and $Z$-bosons, see our Eqs.\((55,59)\), when $\beta = \theta = 0\text{rad}$ when we recover the SM formulas given in our Eqs.\((2,60)\). See we get the same $v$ as defined in Eq.\((54)\).

where $\theta_W$ is the Weinberg angle and it is defined in our Eq.\((57)\) and those values are in agreement with

\[
\cos^2 \theta_W\text{ EXP} = \frac{(M_W)^2\text{ EXP}}{(M_Z)^2\text{ EXP}} = 0.778, \quad \sin^2 \theta_W = 0.222. \tag{62}
\]

The result presented in our Eq.\((5)\) together Eq.\((60)\) will imply

\[
(M_Z)_{\text{CDF}} = \frac{(M_W)_{\text{CDF}}}{\cos \theta_W} = 91.21 \text{ GeV}. \tag{63}
\]

We can define

\[
\begin{align*}
\left(\delta M_W^2\right)_{\text{theo}} &= \left(M_W^2\right)_{\text{CDF}} - \left(M_W^2\right)_{\text{SM}} = 13.3451 \text{ GeV}^2, \\
\left(\delta M_W^2\right)_{\text{ave}} &= \left(M_W^2\right)_{\text{CDF}} - \left(M_W^2\right)_{\text{EXP}} = 3.2491 \text{ GeV}^2, \\
\left(\delta M_Z^2\right)_{\text{theo}} &= \frac{\left(\delta M_W^2\right)_{\text{theo}}}{\cos \theta_W\text{ EXP}} = 4.14 \text{ GeV}, \quad \left(\delta M_Z^2\right)_{\text{ave}} = \frac{\left(\delta M_W^2\right)_{\text{ave}}}{\cos \theta_W\text{ EXP}} = 2.04 \text{ GeV}.
\end{align*}
\]

those two last relation imply

\[
\left(M_Z^2\right)_{\text{theo}} = 91.28 \text{ GeV}, \quad \left(M_Z^2\right)_{\text{ave}} = 91.21 \text{ GeV}, \tag{65}
\]

\[
246.21 & 80.3474 & 91.1127 \\
246.211 & 80.3477 & 91.1131 \\
256.212 & 80.3480 & 91.1135 \\
256.213 & 80.3483 & 91.1138 \\
256.214 & 80.3487 & 91.1142 \\
256.215 & 80.3490 & 91.1146 \\
256.216 & 80.3493 & 91.1149 \\
256.217 & 80.3496 & 91.1153 \\
256.218 & 80.3500 & 91.1157 \\
256.219 & 80.3503 & 91.1160 \\
256.220 & 80.3506 & 91.1164
\]
Figure 14: The masses of $Z$ gauge boson in function of $v_1$ for $\theta = 0$ rad and for some values of our $\beta$-parameter shown on the box, the red line, the green by Eq.(11) and the purple line and the green line are defined for Eq.(65).

its value is the same as given by our Eq.(63).

We can write from our Eqs.(55,25) we can show

$$\delta M_W = \frac{g v_1}{2} \tan \theta, \quad \delta M_Z = \frac{\delta M_W}{\cos \theta_W},$$

(66)

our results for those parameters are shown in our Figs.(15), where we shown we can accomodate the measurement of $W$-boson mass for several values of $v_1$ when we fix the new parameter $\theta$.

The superpotential that violate $R$-parity and define MSSMRV is given by

$$W_{MSSMRV} = W_2 + W_{3RC},$$

(67)

$$W_2 = \mu \left( \hat{H}_1 \hat{H}_2 \right) + \sum_{i=1}^{3} \mu_0 i \left( \hat{L}_i \hat{H}_2 \right),$$

$$W_3 = \sum_{i,j,k=1}^{3} \left[ f_{ij}^l \left( \hat{H}_1 \hat{L}_i \right) \hat{E}_{jR} + f_{ij}^u \left( \hat{H}_1 \hat{Q}_i \right) \hat{D}_{jR} + f_{ij}^u \left( \hat{H}_2 \hat{Q}_i \right) \hat{U}_{jR} + \lambda_{ijk} \left( \hat{L}_i \hat{\bar{Q}}_j \right) \hat{E}_{kR} \right] + \lambda'_{ijk} \left( \hat{L}_i \hat{\bar{Q}}_j \right) \hat{D}_{kR}. $$

(68)
Figure 15: $\delta M_Z$ in function of the new parameter $\theta$, the black line, for $v_1 = 90$ GeV; $v_1 = 100$ GeV; $v_1 = 110$ GeV to explain the values defined in our Eq.(66).

B To understand the mixing in neutral gauge bosons in the model with three noidentical neutrinos

In this model we recover the Standard Model when

$$\beta = \theta = \alpha = \gamma = \delta = 0, \text{ rad},$$

in this case our Eq.(41) is given by

$$M^2 = \frac{g^2 v^2}{4} \left( \begin{array}{ccc} 1 & -\left( \frac{g_Y}{g} \right) & 0 \\ -\left( \frac{g_Y}{g} \right) & \left( \frac{g_Y}{g} \right)^2 & 0 \\ 0 & 0 & 0 \end{array} \right),$$

this mass matrix in the superior block $2 \times 2$ is very similar mass matrix of the Standard Model where we exchange $g \rightarrow g_Y$. The eigenvalues for this mass matrix are$^6$

$$M^2_{Z_1} = 0,$$

$^6$See our Eq.(46).
\[ M^2_{Z_2} = \frac{g^2 v^2}{4} \left[ 1 + \left( \frac{g Y'}{g} \right)^2 \right], \]  

(71)

and we can conclude

\[ Z_m = \cos\theta_W W^3_m + c_1 \sin\theta_W b_{Y'} m, \]
\[ Z'_m = c_2 (b_{Y'})_m + c_3 (b_{BL})_m, \]  

(72)

where \( c_1, c_2 \) and \( c_3 \) are the coefficients responsible in mix \( b_{Y'} m \) with \( (b_{BL})_m \).

In this model we recover the MSSM with \( R \) Parity conservation when

\[ \theta = \alpha = \gamma = \delta = 0, \text{ rad}, \]  

(73)

in this case, we again get our Eq.(41) but in this case our Eq.(71) become

\[ M^2_{Z_1} = 0, \]
\[ M^2_{Z_2} = \frac{g^2 v^2}{4} \left[ 1 + \left( \frac{g Y'}{g} \right)^2 \right] (1 + \tan^2 \beta). \]

(74)

In this model we recover the Standard Model with \( R \) Parity violation when Eq.(59) is hold, in this case our Eq.(41) become

\[ M^2 = \frac{g^2 v^2}{4} \begin{pmatrix} (1 + \tan^2 \beta) & -\left( \frac{g \omega}{g} \right) & (1 + \tan^2 \beta) & 0 \\ -\left( \frac{g \omega}{g} \right) & (1 + \tan^2 \beta) & \left( \frac{g \omega}{g} \right)^2 & (1 + \tan^2 \beta) \\ 0 & (1 + \tan^2 \beta) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \]

(75)

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