Attack RMSE Leaderboard: An Introduction and Case Study

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Abstract

In this manuscript, we briefly introduce several tricks to climb the leaderboards which use RMSE for evaluation without exploiting any training data.

1 Introduction

Root-Mean-Squared-Error (RMSE) is no longer widely used for evaluation in data challenges because of its easy-to-hack properties. To be more specific, given any vector \( \hat{y} \in \mathbb{R}^n \) and the true labels \( y \in \mathbb{R}^n \), where \( n \) is the number of desired label, we can easily obtain the inner product \( \langle \hat{y}, y \rangle \), which can be used to extract the information of the true labels \( y \).

There are other kinds of vulnerable evaluations such as log-loss [Whitehill, 2017]. For classification problems, there is also Monte-Carlo style attacks such as [Hardt, 2015].

2 Notations

Denote the desired vector of the true labels as \( y \in \mathbb{R}^n \), where \( n \) is the number of labels. \( \hat{y} \in \mathbb{R}^n \) is the vector of submitted labels. After each submission, the data challenge platform (such as Kaggle) will evaluate the submitted labels and provide the RMSE oracle which is defined as follows:

**Definition 1.** Root-Mean-Squared-Error (RMSE):

\[
\text{RMSE}(\hat{y}) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2} = \sqrt{\frac{1}{n} \| \hat{y} - y \|^2},
\]

where \( \| \cdot \| \) is the \( \ell_2 \)-norm.

For simplicity, we denote \( y^2 = \| y \|^2 \), \( \hat{y}^2 = \| \hat{y} \|^2 \), \( \text{SE}(\hat{y}) = \| \hat{y} - y \|^2 \). Furthermore, we denote the mean values as \( m = \frac{1}{n} \sum_{i=1}^{n} y_i \), \( \hat{m} = \frac{1}{n} \sum_{i=1}^{n} \hat{y}_i \), and the segmental mean values as \( m_{i:j} = \frac{1}{j-i+1} \sum_{k=i}^{j} y_k \), \( \hat{m}_{i:j} = \frac{1}{j-i+1} \sum_{k=i}^{j} \hat{y}_k \).

3 Assumptions

Globally, we take the following assumptions:

**Assumption 1.** The labels are upper-bounded and lower-bounded: \( y_i \in [a, b] \) for \( \forall i \in [n] \).

The above assumption is easily satisfied in most data challenges. The constants \( a \) and \( b \) can be easily inferred/estimated from the background information of the challenges.
4 Methodology

In this section, we introduce several tricks to climb the leaderboards.

For most of the tricks, the basic ideas are very similar: we first submit \( \hat{y} = 0 \), the RMSE oracle \( \sqrt{\frac{1}{n} \| \hat{y} - y \|^2} = \sqrt{\frac{1}{n} \| 0 - y \|^2} = \sqrt{\frac{1}{n} \| y \|^2} \) will then give us \( y^2 \). Once we obtain \( y^2 \), for any \( \hat{y} \in \mathbb{R}^n \), we can obtain the inner product \( \langle \hat{y}, y \rangle \) via the decomposition \( \text{SE}(\hat{y}) = y^2 + y^2 - 2 \langle \hat{y}, y \rangle \). Note that \( y^2 \) and \( \hat{y}^2 \) are known, and \( \text{SE}(\hat{y}) \) can be inferred from the RMSE oracle. Thus, we obtain \( \langle \hat{y}, y \rangle = \frac{1}{2} \{ y^2 + y^2 - \text{SE}(\hat{y}) \} \). In the rest of this manuscript, we assume that \( \langle \hat{y}, y \rangle \) is already known for any \( \hat{y} \).

4.1 Mean-value Attack

The idea is very simple. For any segment of indices \( i, \ldots, j \), we take \( \hat{y}' = [0, \ldots, 1, \ldots, 1, \ldots, 0] \).

Hence, we obtain the mean value \( m_{i:j} = \frac{1}{j-i+1} \sum_{k=i}^{j} y_k = \frac{1}{j-i+1} \langle \hat{y}', y \rangle \). Then, given any submission \( \hat{y} \), we improve the evaluation by using Algorithm 1.

Algorithm 1 Mean-value improvement

**Input:** Any segment of indices \( \{i, \ldots, j\} \), the corresponding submitted labels \( \hat{y} = \hat{y}_{i:j} \) and the mean value \( \hat{m} = m_{i:j} \), and the lower bound \( a \) and upper bound \( b \).

**Output:** The improved segment of submission \( \hat{y} \).

Use any solver (e.g. quadprog[2017]) to solve the following quadratic programming with constraints:

\[
\begin{align*}
\min_{\hat{y}} & \quad \frac{1}{2} \| \hat{y} - \bar{y} \|^2, \\
\text{s.t.} & \quad 1^T \hat{y} = (j - i + 1) \hat{m}, \\
& \quad a \mathbf{1} \leq \hat{y} \leq b \mathbf{1}.
\end{align*}
\]

Note that when the size of the segment is 1, the inner product is the exact value of the corresponding true label. Furthermore, we obtain better evaluation with more submissions for different segments.

4.2 Linear-regression Attack

Assume that the budget of submissions (the remaining number of submissions) is \( r \leq n \). We somehow generate a matrix \( A \in \mathbb{R}^{n \times r} \) composed of \( r \) columns of bases, where each column is linearly independent or orthogonal to one another. Note that some or all of the columns of \( A \) can be different submissions. We can generate a submission better than any of these submissions, which is a linear combination of these columns \( Ax \), where \( x \in \mathbb{R}^r \). The optimal \( x \) is obtained by solving the following least-square linear regression problem:

\[
\min_{x} \frac{1}{2} \| Ax - \hat{y} \|^2.
\]

Because we have the full control of the matrix \( A \), we can easily make it of full rank (e.g. with linearly independent columns), which implies that \( A^\top A \) is invertible. Note that the above regression problem has the closed-form solution: \( x = (A^\top A)^{-1} A^\top y \). The \( j \)th element of the vector \( A^\top y \) is simply the inner product \( \langle A_{:,j}, y \rangle \), which can be easily obtained as mentioned before, where \( A_{:,j} \) is the \( j \)th column of \( A \). Hence, we generate the new submission \( Ax \).

Furthermore, we can easily infer the RMSE of the new submission, which is \( \sqrt{\frac{1}{n} \{ x^\top A^\top Ax + y^2 - 2x^\top (A^\top y) \}} \), where \( x, A, y, \) and \( A^\top y \) are all known.
4.3 Finite-label Attack

Ideally, this kind of attack can generate perfect labels (exactly $y$) with one single submission (additional to the submission of obtaining $\hat{y}^2$). However, such attack also requires much stronger conditions as follows:

**Assumption 2.** There are only finite number of different values for the true labels, which means that the labels are discrete.

Together with the assumption that the labels are bounded, we can easily transform the original problem into a new problem where the labels are integers lying in the range $[0, c]$, where $c$ is also an integer. Then, we take $\hat{y} = [1, c, c^2, \ldots, c^{n-1}]$. In other words, $\hat{y}_i = c^{i-1}$ for $\forall i \in [n]$. Using the resulting inner product $\langle \hat{y}, y \rangle$, we can obtain all the true labels via Algorithm 2.

**Algorithm 2** Perfect submission

| Input: $\hat{y}_i = [1, c, c^2, \ldots, c^{n-1}]$, $p = \langle \hat{y}, y \rangle$. |
| Output: Transformed true labels $y_i$. |

for $i = 1, \ldots, n$ do

- $y_i \leftarrow \text{mod}(p, c)$.
- $p \leftarrow \lfloor p/c \rfloor$.

end for

Note that this attack also requires that the $c$ is small enough so that $c^n$ does not overflow. However, we can also infer a segment of length $n' < n$ of the labels so that $c^{n'}$ does not overflow.

**Case study: Restaurant Revenue Prediction [Kaggle, 2015]** In this data challenge, any true label satisfies $y_i \in \{0, 1\}$. And, the submission can be any real number. Thus, we simply take $c = 2$. Then, the perfect submission can be obtained as follows:

1. Submit $\hat{y} = 0$. Obtain $y^2$.
2. Submit $\hat{y} = [1, 2, 2^2, \ldots, 2^{n-1}]$. Obtain $p = \langle \hat{y}, y \rangle$.
3. Use Algorithm 2 to obtain the perfect submission.

Thus, we only need 2 submissions to get ranked as top-1 (0-RMSE) of the leaderboard!

5 Conclusion

RMSE as evaluation is highly vulnerable. Enjoy climbing the leaderboard!

References

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