Rigidity, creep and dynamic strength of plastics for three-dimensional printing of machine parts

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Abstract. Increasingly, parts of complex shapes are made using three-dimensional printing. This technique is also used in the design of equipment at the stage of physical modeling of their structures to determine certain properties of the product as a whole and its individual parts (details) on the corresponding models. The necessary conditions for physical modeling are the geometric and physical similarity of the model and kind. The presence of such proportionality makes it possible to recalculate the experimental results obtained for the model in kind by multiplying each of the determined values by a constant multiplier for all values of this dimension – the similarity coefficient. However, to study the physical characteristics of a product, it is necessary to take into account the mechanical properties of the material of its model. From various sources, you can find the main mechanical characteristics of plastic threads or samples made on a 3-D printer. Their values vary greatly depending on the manufacturing technology of models and are determined under conditions of static loading only, which is not enough to study the stress-strain state of parts that are affected by suddenly applied, long-term and dynamic loads during machine operation. This paper presents the results of a study of the physical and mechanical properties of polylactide (PLA) and polyethylene tereflatate (PET-G), which are used in the manufacture of parts by three-dimensional printing. The specific stiffness of polymers was for PLA 1.61…2.18 MPa·m³/kg, for PET-G – 1.15…1.41 MPa·m³/kg. The specific impact strength for PLA is 751.6…774.2 J·m/kg, for PET-G – 571.6…583.0 J·m/kg. The specific endurance for PLA was 1.75…2.76 kPa·m³/kg, for PET-G – 3.0…3.10 kPa·m³/kg.

1. Introduction

1.1. Relevance

Today, new methods of manufacturing complex parts using three-dimensional printing are being introduced in technology (figure 1).
This technique is also used when designing at the stage of physical modeling of structures of machines and mechanisms, to determine certain properties of the product as a whole and its individual parts (details) on the corresponding models [1].

Physical modeling is a type of experimental study of an object or phenomenon based on its model, which has the same physical nature. Physical modeling is used when it is difficult to carry out full-scale tests of the product, when the size of the object of research or the values of its other characteristics (pressure, temperature, process speed, etc.) are too large (or small), as well as for economic reasons.

A geometrically similar model of a real part can be made using three-dimensional printing, but to study the physical characteristics of the product, it is necessary to take into account the mechanical properties of the model material. From various sources, you can find the main mechanical characteristics of plastic threads or samples made on a 3-D printer: strength limits, relative residual elongation, modulus of elasticity. Their values vary greatly depending on the manufacturing technology of models and are determined for simple types of resistance under static loading, which is not enough to study the stress-strain state of parts that are affected by suddenly applied, long-term and dynamic loads during operation of mechanisms.

1.2. The aim
The aim of this work is to study the stiffness, creep, and dynamic strength of polylactide (PLA) and polyethylene tereflata (PET-G), which are used for manufacturing parts using three-dimensional printing.

The following tasks are set: to determine the mechanical characteristics of PLA and PETG samples under sudden, prolonged static, shock, and cyclic bending.

1.3. Methods
Methods of mechanical testing are regulated by state standards and laboratory studies of mechanical characteristics are carried out in accordance with them [2].

2. Results and discussions
The creep study was performed on cantilever-loaded samples with a working length \( l = 70...100 \text{ mm} \) and a diameter \( d = 10 \text{ mm} \) at a load \( P = 15 \text{ N} \) (figure 2).

The design stresses in the rigid seal for PLA samples were

\[
\sigma = \frac{32Pl}{\pi d^3} = 15.3 \text{ MPa},
\]

for samples from PET-G – \( \sigma = 10.7 \text{ MPa} \).
Figure 2. Stand for the study of creep:
1 – earring with a weight; 2 - investigated sample; 3 – indicator; 4 – bracket.

Then creep diagrams were plotted – the change in the deflection w and the strain rate $dw/dt$ over time $t$. Figure 3 shows the results of a study of the creep of a round PLA sample with a diameter of 10 mm, a working length of 100 mm at a load of 15 N and a temperature of 21-23 °C For a loading time of 5.5 hours (330 min) and figure 4 – for one day of loading (1440 min).

Figure 3. PLA sample creep study at a load of 15 N for the first 330 minutes: a) diagram of creep; b) graph of the strain rate as a function of loading time.
Figures 5 and 6 show the results of a creep study of a round PET-G sample with a diameter of 10 mm and a working length of 70 mm at a load of 15 N and a temperature of 21-23 °C Over a loading time of 6 hours (360 min) and two days (2880 min).

Figure 4. PLA sample creep study at a load of 15 N per day: а) diagram of creep; б) graph of the strain rate as a function of loading time

Figure 5. The study of creep of the sample of PET-G at a load of 15 N in the first 360 minutes: а) diagram of creep; б) graph of the strain rate as a function of loading time
Figure 6. The study of creep of the sample of PET-G at a load of 15 N in two days: a) diagram of creep; b) graph of the strain rate as a function of loading time.

The analysis showed that the materials under study are viscoelastic. The creep rate for the first 0.5...4 minutes is relatively high and is 0.02...0.16 mm/min for PLA samples, 0.01...0.06 mm/min for PET-G samples, after which it sharply decreases and remains small (less than 0.008 mm/min) for tens of hours.

Figure 7 presents a diagram of the relaxation of samples of PLA and PET-G.

Figure 7. Stretch charts: a) for PLA; b) for PET-G.
After removing the external load, the relative residual strain \( \frac{w_{ost}}{l} \) for samples from PLA was \((8.1\ldots8.2) \times 10^{-3}\), for samples from PET-G \( \frac{w_{ost}}{l} = (3.8\ldots3.9) \times 10^{-3}\), the relaxation time, i.e. the time for which the strain decreases by a factor of \( e \), for PLA is 13...14 min, for PET-G – 15...20 min.

*Investigation of viscoelastic properties.* The study of the stiffness of materials was carried out when bending according to the scheme of hinge-supported (figure 8) and cantilever-loaded samples (figure 2).

To determine the modulus of elasticity of samples according to the scheme of a hinged-supported beam with a working length \( l = 63\ldots65\) mm and a diameter \( d = 10.0\ldots10.1\) mm, the approximate formula was used (2)

\[
E = \frac{4l^3P_p}{3\pi d^4w_p}, \tag{2}
\]

where the load \( P_p \) and deflection \( w_p \), corresponding to the limit of proportionality, were determined from point \( A \) in the loading diagrams (figure 9).

![Figure 8. Bending test: 1 – support plates of the machine loading device MP-0.5-1; 2 – indenter; 3 – investigated sample; 4 – centering supports.](image)

![Figure 9. Load-deflection diagrams: a) for PLA; b) for PET-G.](image)
The following values are obtained for the active gripper movement speed of 30 mm/min: for PLA $E = 1.75...1.82$ hPa; for PET-G $E = 1.30...1.35$ hPa.

For the case of a suddenly applied load, studies were performed on cantilever-loaded samples with an estimated length $l = 70...100$ mm and a diameter $d = 10.0...10.1$ mm (see figure 2). The external force was limited by its maximum value found by the expression (3):

$$P_{\text{max}} < \frac{\pi d^3 \sigma_p}{32 l}$$

(3)

Figures 10 and 11 show the graphical relationship between force $P$ and deflection $w$. An approximate formula was used to determine the elastic modulus (4)

$$E = \frac{64 l^3 d P}{3\pi d^4 dw}.$$  

(4)

**Figure 10.** Study of the rigidity of a cantilever-loaded sample with a length of $l = 100$ mm and a diameter of $d = 10$ mm from PLA: a) load-deflection diagram; b) change in sample stiffness at different loading stages.

**Figure 11.** Study of the rigidity of a cantilever-loaded sample with a length of $l = 70$ mm and a diameter of $d = 10$ mm from PETG: a) load-deflection diagram; b) change in sample stiffness at different loading stages.
Studies have shown: for PLA $E = 2.18...2.37$ hPa; for PET-G $E = 1.42...1.62$ hPa. That is, the stiffness under a suddenly applied load for PLA is $1.2...1.3$ times higher and for PET-G $1.1...1.2$ times higher compared to static load.

*Impact tests* were carried out on a KM-30 pendulum copra at a temperature of 21-23 °C. Over a batch of similar samples in the amount of 6...7 pieces with a length of 70.0 mm, a square section with a side of 10.0...10.3 mm (figure 12).

![Figure 12](image12.png)

**Figure 12.** Samples for impact testing: *a*) PLA before and after the test; *b*) PET-G before and after the test.

The value of impact strength $a$ was made: for PLA $a = (815.5...840.0) \times 10^3$ J/m$^2$; for PET-G $- a = (647.1...660.0) \times 10^3$ J/m$^2$.

*Fatigue tests.* For each material, a series of samples with a working length of $l = 100.0...120.0$ mm and a diameter of $d = 10.0...10.1$ mm was tested in an amount of 10 pieces until they were completely destroyed (figure 13) on a UCI-10 M machine (figure 14).

![Figure 13](image13.png)

**Figure 13.** Samples for fatigue testing: *a*) PLA before and after the test; *b*) PET-G before and after the test.

Based on the test results, diagrams of maximum stresses $\sigma$ were plotted as a function of the number of cycles to failure $N_c$ (fatigue curves). The stresses for a cantilever-loaded sample were determined by the formula (5)

$$\sigma = \frac{32P_l l}{\pi d^3}$$

(5),

where $P_l$ – weight of weights.

The fatigue curve was studied using its well-known theoretical representation in the form of an equation (6)

$$\sigma^m N_c = C,$$

(6)

where $m$, $C$ – constant.

When constructing fatigue diagrams for $N_c = 1$, the value of the bending strength was taken.
Figure 14. Fatigue tests on the UCI-10 M machine: a) General view of the installation; 1 – weight; 2 – toggle switch for selecting the speed mode; 3 – a count of the number of revolutions (cycles); 4 – loaded sample; b) destroyed sample.

The study of equation (4) at the control points of the experimental curves (figures 15, 16) allowed us to calculate the limit values of the constants: for PLA \( m = 3.493 \ldots 4.226 \), \( C = (8.67 \ldots 104.15) \cdot 10^7 \) \( Pa^n \); for PET-G \( m = 4.683 \ldots 4.754 \), \( C = (333.02 \ldots 404.18) \cdot 10^7 \) \( Pa^n \). Then, using equation (4), the endurance limits were calculated for a symmetric cycle \( \sigma_1 \), given the base number \( N_b \). In comparison with steel samples, taking \( N_b = 10^7 \) cycles (5), we have: for PLA \( \sigma_1 = 1.9 \ldots 3.0 \) MPa; for PET-G \( \sigma_1 = 3.4 \ldots 3.5 \) MPa.

Figure 15. Fatigue curves for PLA: 1 – for \( m = 3.493 \), \( C = 8.67 \cdot 10^7 \) \( Pa^n \), \( \sigma_1 = 1.9 \) MPa; 2 – for \( m = 4.226 \), \( C = 104.15 \cdot 10^7 \) \( Pa^n \), \( \sigma_1 = 3.0 \) MPa.

Figure 16. Fatigue curves for PET-G: 1 – for \( m = 4.683 \), \( C = 333.02 \cdot 10^7 \) \( Pa^n \), \( \sigma_1 = 3.4 \) MPa; 2 – for \( m = 4.754 \), \( C = 404.18 \cdot 10^7 \) \( Pa^n \), \( \sigma_1 = 3.5 \) MPa.
The results of the research are summarized in a table that shows the specific mechanical characteristics of polymers, carbon steel, and gray cast iron for structural efficiency analysis. Mechanical properties (stiffness, impact strength, and cyclic stress) can be evaluated using dimensionless parameters:

\[
\gamma(E) = \frac{\varepsilon(E) \text{ (polymer)}}{\varepsilon(E) \text{ (metal)}}; \quad \gamma(a) = \frac{a(E) \text{ (polymer)}}{a(E) \text{ (metal)}}; \quad \gamma(\sigma_1) = \frac{\sigma_1 \text{ (polymer)}}{\sigma_1 \text{ (metal)}}
\]

3. Conclusion
The specific stiffness for PLA is 1.61…2.18 MPa·m²/kg, for PET-G – 1.15…1.41 MPa·m²/kg. Evaluation of stiffness of PLA in comparison with steel \( \gamma(E) = 0.06…0.08 \), with cast iron \( \gamma(E) = 0.09…0.10 \). Evaluation of stiffness of the PET-G in comparison with steel \( \gamma(E) = 0.04…0.05 \), with cast iron \( \gamma(E) = 0.06…0.07 \). Stiffness at a sudden applied load for PLA in 1.2…1.3 times and for PET-G in 1.1…1.2 times higher in comparison with static load.

The materials under study are viscoelastic. The creep rate for the first four minutes is relatively high and is 0.02…0.16 mm/min for PLA samples, 0.01…0.06 mm/min for PET-G samples, after which it decreases sharply and remains small (less than 0.008 mm/min) for tens of hours. The relaxation time, i.e. the time during which the strain decreases by a factor of e, for PLA is 13…14 min, for PET-G – 15…20 min.

The specific impact strength for PLA was 751.6…774.2 J/m/kg, for PET-G – 571.6…583.0 J/m/kg. Evaluation of impact strength in PLA in comparison with steel \( \gamma(a) = 2.5…14.6 \), with cast iron \( \gamma(a) = 57.3…127.4 \). Assessment of the strength of the PET-G in comparison with steel \( \gamma(a) = 1.9…11.1 \), with cast iron \( \gamma(a) = 43.2…96.9 \).

The specific endurance for PLA is 1.75…2.76 kPa·m²/kg, for PET-G – 3.0…3.10 kPa·m²/kg. Assessment of the toughness of PLA in comparison with steel \( \gamma(\sigma_1) = 0.03…0.11 \), with cast iron \( \gamma(\sigma_1) = 0.14…0.17 \). PET-G endurance rating compared to steel \( \gamma(\sigma_1) = 0.03…0.19 \), with cast iron \( \gamma(\sigma_1) = 0.15…0.29 \).

| Characteristic and structural efficiency | Material          | Carbon structural steels | Grey cast iron |
|-----------------------------------------|-------------------|--------------------------|----------------|
| Elastic modulus \( E \), GPa           | PLA 1.75 – 2.37   | 200 – 210                | 115 – 160      |
| Impact toughness \( a \), kJ/m²         | PET-G 815.5 – 840.0 | 647.1 – 660.0 | 400 – 2400    | 40 – 100        |
| Endurance limit \( \sigma_1 \), MPa     | 1.9 – 3.0         | 3.4 – 3.5                | 120 – 700      | 70 – 150        |
| Density \( \rho \), kg/m³               | 1085              | 1132                     | 7800           | 6800 – 7400    |
| Specific stiffness \( E/\rho \), MPa·m²/kg | 1.61 – 2.18       | 1.15 – 1.43               | 25.6 – 26.9    | 15.5 – 23.5    |
| Specific endurance \( \sigma_1/\rho \), kPa·m²/kg | 1.75 – 2.76       | 3.0 – 3.10                | 15.38 – 89.74  | 10.30 – 20.27  |
| Specific impact strength \( a/\rho \), J/m/kg | 751.6 – 774.2     | 571.6 – 583.0             | 51.3 – 307.7   | 5.9 – 13.5     |

The results of the research are summarized in a table that shows the specific mechanical characteristics of polymers, carbon steel, and gray cast iron for structural efficiency analysis. Mechanical properties (stiffness, impact strength, and cyclic stress) can be evaluated using dimensionless parameters:

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