Metrological characterization of the pulsed Rb clock with optical detection

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Abstract

We report on the implementation and metrological characterization of a vapour-cell Rb frequency standard working in a pulsed regime. The three main parts of the clock, physics package, optics and electronics, are described in detail in this paper. The prototype is designed and optimized to detect the clock transition in the optical domain. Specifically, the reference atomic transition, excited with a Ramsey scheme, is detected by observing the interference pattern on a laser absorption signal.

The metrological analysis includes the observation and characterization of the clock signal and the measurement of frequency stability and drift. In terms of Allan deviation, the measured frequency stability is as low as $1.7 \times 10^{-13} \tau^{-1/2}$, $\tau$ being the averaging time, and reaches the value of a few units of $10^{-15}$ for $\tau = 10^4$ s, an unprecedented result for a vapour-cell clock. We discuss the physical effects leading to this result in this paper with particular care to laser and microwave noises transferred to the clock signal. The frequency drift, probably related to temperature, stays below $10^{-14}$ per day, and no evidence of flicker floor is observed.

We also mention some possible improvements that in principle would lead to a clock stability below the $10^{-13}$ level at 1 s and to a drift of a few units of $10^{-15}$ per day.

(Some figures may appear in colour only in the online journal)

1. Introduction

Atomic clocks based on vapour-cell technology are receiving considerable attention from basic and applied research. This is basically due to the need in many current activities for a stable frequency reference delivered by a compact, low cost and reliable device. While the current lamp-pumped Rb clocks show good performances and reliability for both in-orbit and on-ground applications, a number of innovative clock schemes have been proposed exhibiting the capabilities to improve the stability of vapour-cell based clocks to the level of the passive hydrogen maser (PHM) or even better [1–3]. Also, practical realizations of such schemes are expected to impact marginally on the size, mass and power consumption budgets of currently operating Rb clocks.

In this paper, we present the implementation and metrological characterization of a pulsed optically pumped (POP) Rb frequency standard based on a vapour-cell–microwave cavity arrangement. Our prototype exhibits frequency stability (standard Allan deviation) $\sigma_y(\tau) \leq 2 \times 10^{-13} \tau^{-1/2}$, remaining below $10^{-14}$ up to integration times $\tau \approx 10^5$ s and with a relative frequency drift below $10^{-14}$/day. Proper engineering of such a clock would then be extremely interesting not only in a variety of technological applications such as radio-navigation systems and synchronization of telecommunication networks but also in basic research as a local oscillator (LO) for primary frequency standards.

The great potential of the POP scheme in a Rb vapour-cell with buffer gas has been demonstrated in several works, see, for example, [2, 4]. It is based on the time separation of the three phases (preparation, interrogation and detection) that usually rule the operation of an atomic clock (see figure 1). Specifically, an intense laser pulse initially prepares the atomic sample, producing a population imbalance in the two ground-state hyperfine levels of $^{87}$Rb. The atoms are then interrogated with a couple of microwave pulses resonant with the clock transition (6.834 GHz) and separated by a time $T$ (Ramsey interaction). Finally, a detection window is enabled in order to detect the atoms that have made the transition. The main feature of this
In particular, the POP Rb maser exhibits an Allan deviation approach has been extensively studied in our previous works. In other words, in the pulsed scheme the mutual influence of laser and microwave signals is avoided and, accordingly, the coupling between microwave and optical coherences turns out to be negligible. In this regime, the atoms behave most likely as a pure two-level system, as, e.g., in an atomic fountain. Moreover, the Ramsey interrogation technique allows one to observe a clock signal with a very narrow linewidth that, in addition, is nearly insensitive to any working parameter, such as laser and/or microwave power.

The detection can take place either in the microwave domain (passive maser) or in the optical domain. The first approach has been extensively studied in our previous works. In particular, the POP Rb maser exhibits an Allan deviation of $\sigma_y(\tau) = 1.2 \times 10^{-12} \tau^{-1/2}$ for averaging times $\tau$ up to $10^5 \text{s}$, and reaches the value of $5 \times 10^{-15}$ after drift removal at $\tau = 50 000 \text{s}$ [12], a result very close to a PHM performance. In the case of optical detection, the laser is again turned on during the detection phase but it is used as a probe: duration and intensity are in general reduced from those used in the pumping phase.

Qualitative physical arguments lead to the consideration that the detection of the clock transition through the laser absorption signal offers some advantages in comparison with the maser approach. First, due to the fact that optical photons carry more energy than microwave photons, a higher signal-to-noise ratio is expected and, provided other noise sources are controlled, also better short-term frequency stability. Moreover, the microwave cavity is used in the interrogation phase to couple the microwave signal to the atoms and does not play any role during the detection process. This means that a high-$Q$ cavity, mandatory in the case of the POP maser, is not required anymore, with great benefit for medium–long-term performances since cavity-pulling turns out to be negligible.

The system described in this paper was designed and optimized to detect the clock transition in the optical domain. The characterization reported here includes an analysis of the clock frequency dependence on the variation of the working parameters and the evaluation of the respective conversion factors. In order to reduce those sensitivities, some technical solutions, described in the paper, were adopted in the clock design. Particular care is devoted to the effects limiting the frequency stability performances, such as phase noise of the interrogating microwave signal and laser noise sampled during the detection process.

In the paper we will often refer to the theory reported in [2]. That theory is based on a multilevel atomic model where the full Zeeman manifold of the ground and excited states is taken into account. The model also considers the dynamics induced among the Zeeman sublevels by the relaxation processes such as buffer gas, spin-exchange and cell-wall collisions. (The theory has been developed for the $D_1$ transition as the optical pumping line; however, most of the conclusions can also be extended to the $D_2$ line).

We point out that the multilevel model is needed to provide an accurate evaluation of the pumping efficiency and of the clock signal features, such as contrast and signal-to-noise ratio. The naive three-level model (two ground-state clock levels and one excited state) gives in fact only a qualitative description of the atomic sample behaviour inside the cell and fails when a quantitative comparison between theory and experiment is made. Confident of the good agreement between theory and experiments already proved in the case of the maser approach, we extended the model to the optical detection to identify the values of the working parameters, such as operation temperature, laser power and microwave power, that offer the best clock performances. Here we show that the multilevel model is also suitable to reproduce well the experimental results in this case.

This paper is organized as follows. We describe in the following sections the implementation and characterization of the three main parts of the prototype: physics package (section 2), optics (section 3) and electronics (section 4). In section 5 we recall the principle of operation of the POP clock, discussing the optimization of some working parameters. Section 6 is devoted to the characterization of the entire apparatus working as a clock, reporting the sensitivity to the working parameters and the respective conversion factors, the drift and frequency stability. On the basis of the achieved results and theoretical predictions, in the last section we propose some possible improvements that can possibly lead to a frequency stability of $\sigma_y(\tau) \approx 5 \times 10^{-14} \tau^{-1/2}$, down to the $10^{-16}$ region, maintaining the frequency drift of a few units of $10^{-15}$ per day.
2. Physics package

Figure 2 shows a schematic of the physics package. It has a layer structure and from the innermost one to the outermost the following components can be recognized:

- quartz cell;
- microwave cavity;
- C-field solenoid;
- internal heater;
- first magnetic shield;
- external heater;
- two external magnetic shields.

Before describing the physics package components, we point out that the physics package has been designed in order to reduce the inhomogeneities in the active atomic medium. In fact, according to theory [2], the spatial inhomogeneity results in the resonance frequencies of the Rb atoms depending on their position in the cell. The observed clock frequency is then an average of such frequencies and, in addition, it changes with laser intensity, simulating an off-resonant light shift. In this context, the spatial distribution of the cavity electromagnetic mode plays a fundamental role since it introduces a dependence of its linewidth on the atomic density and laser intensity. Taking into account the cell size, the buffer gas composition and the working temperature (66°C, see section 2.4), it is possible to estimate the relaxation rates of atomic population and coherence in the ground state $\gamma_1 = 360 \text{s}^{-1}$, $\gamma_2 = 300 \text{s}^{-1}$, respectively. The relaxation rate of the excited state turns out to be $\Gamma^+ = 3 \times 10^3 \text{s}^{-1}$ and the cell optical length $\xi = 17$ for the D$_2$ optical transition.

As is well known, the buffer gas induces a shift in the clock transition [16, 17], which in our case is about 171 Hz Torr$^{-1}$. The expected shift is then of the order of 4.3 kHz and is in good agreement with the measured value.

2.2. Cavity

The microwave cavity has a cylindrical geometry with internal diameter $2a = 52$ mm and internal length $d = 49$ mm. It is made of molybdenum and resonates on the electromagnetic mode TE$_{011}$ at the $^{87}\text{Rb}$ ground-state hyperfine frequency (6.834 GHz). The cavity is designed with mode chokes in the end-caps to suppress the degenerate electromagnetic mode TM$_{111}$. The relative size of the cell-cavity arrangement was optimized, and comes from a trade-off between two opposite requirements: (a) the atomic sample should fill a small portion of space at the centre of the cavity to ensure a good uniformity of the microwave field experienced by the atoms; (b) a sufficiently large cell is required to ensure a large number of interacting atoms. The aspect ratios $2a/d \approx 2R/L \approx 1$ satisfy both these requirements.

The cavity loaded quality factor is $Q_L \leq 1000$ and is mainly defined by the condensation of Rb films on the inner surface of the cell. The cavity coupling parameter is $\beta \approx 0.05$.

The cell-cavity arrangement (see figure 3) employed here has been characterized in more detail in [18] (in that paper it corresponds to the cavity system called C$_2$). That cavity is equipped with both a PIN diode (used for $Q$-switching technique) and a GaAs varactor diode. In the experiments reported in this paper we used only the latter for electronic tuning of the cavity.

2.3. Quantization magnetic field

The cavity is placed inside an Al cylinder that holds a solenoid generating a C-field aligned with the cavity axis ($\hat{z}$-axis). The quantization magnetic field $B_0$ is around 1.5 $\mu$T. This value ensures a negligible Rabi pulling frequency shift due to $\pi$-transitions with $m_F \neq 0$ and avoids the generation of Zeeman coherences in the two ground-state manifolds.
2.4. Heaters and thermal sensitivity

Temperature fluctuations can be transferred to the clock transition mainly through the buffer gas; specifically, for a thermally uniform cell and for our temperature-compensated buffer gas, the expected relative frequency sensitivity is \( \frac{\Delta \nu / \nu}{\Delta T} \leq 1 \times 10^{-11} \, \text{C}^{-1} \) around \( T = 65^\circ \text{C} \) [19]. However, dynamical temperature inhomogeneities in the cell due to the stems worsen the above sensitivity to the measured value of \( \frac{\Delta \nu / \nu}{\Delta T} \approx 1 \times 10^{-10} \, \text{C}^{-1} \).

This sensitivity implies that the temperature should be stabilized at a level of 100 µK in order to reach a frequency stability in the \( 10^{-14} \) range. To keep the temperature stable at that level and at the same time maintain the required gradient between the stems and the cell body, two active controls are implemented.

The first temperature-controlled element is the same Al cylinder (innermost thermal shield) used to sustain the C-field solenoid; a heater is in fact wrapped on it. The heater is ac-driven (≈40 kHz) to avoid the generation of spurious magnetic fields, and works at a temperature of \( T_{\text{int}} = 68^\circ \text{C} \) during clock operation.

The cylinder is placed inside a mu-metal magnetic shield which in turn is housed in a second Al cylinder supporting the external heater. This second heater may be dc- or ac-driven and its operating temperature is \( T_{\text{ext}} = 66^\circ \text{C} \).

Figure 4 shows that both the temperatures are stable at the level of 100 µK for integration times up to one day.

2.5. Magnetic shields and vacuum enclosure

The physics package is completed by two magnetic shields; each magnetic shield rejects the external longitudinal magnetic fluctuations by a factor of 10, so the overall shielding factor is around 1000 along the \( z \)-axis. Under quiet geomagnetic conditions, this shielding factor is sufficient to guarantee a stable magnetic field at the level of 10 pT. The shields’ magnetic noise is negligible in our set-up.

Figure 3. The cell-cavity system. It is possible to recognize the two stems of the cell (a), the photodiode (b), the NTC sensors (c), the tuning screw (d) and the microwave cable (e).

Figure 4. Temperature stability of the internal (cavity) and external (stem) thermal shields as measured by two NTC sensors used as monitors.

The physics package of figure 2 and the trans-impedance amplifier (see section 4) are housed in a vacuum enclosure to isolate the apparatus from the environmental fluctuations. The vacuum chamber is equipped with an optical window to couple the laser beam to the physics package and a vacuum feed-through for electrical and microwave cables. As explained in [19], the environmental pressure fluctuation is responsible for a change in the refractive index of air: 1 Pa gives rise to a relative variation of the cavity resonance frequency of \(-2.6 \times 10^{-9} \) (17 Hz Pa\(^{-1}\)) and then to clock frequency instability via the residual cavity-pulling effect. Humidity in air is also responsible for the change in the resonance frequency [18]. The vacuum is maintained below \( 10^{-3} \) Pa using an ion pump; this value guarantees the operation of the physics package under vacuum conditions also from a thermal point of view.

3. Optics

We made experiments with lasers exciting both \( D_1 \) (795 nm) and \( D_2 \) (780 nm) optical lines [20]. In this paper we report only the data referring to the set-up operating at 780 nm that offers the best clock performances, as will also be explained in section 6.

The laser source is a DFB diode with a linewidth of about 20 MHz limited by the current noise of the power supply. It is frequency stabilized through a third harmonic frequency control loop to the crossover transition \( F = 1 \rightarrow F' = 1, 2 \) via a saturated absorption signal. The locking dip is observed in a reference cell containing isotopically enriched \(^{87}\text{Rb} \). The laser is linearly polarized and propagates along the cell \( z \)-axis; therefore, due to selection rules, the atoms see the light as a combination of \( \sigma^+ \) and \( \sigma^- \) polarizations. The main part of the laser beam is sent to the physics package through an acousto-optic modulator (AOM) that acts as an optical switch for the pulsed operation operating in double pass configuration at an RF frequency of 85 MHz. In this way, the laser frequency matches the absorption frequency of the atoms in the cell that becomes red-shifted by the buffer gas pressure (this shift
is $\approx -6.8\, \text{MHz/Torr}^{-1}$ and in our cell is about $-170\, \text{MHz}$). Moreover, the AOM also acts as an optical switch for the pulsed operation. The laser power extinction ratio $P_L^{\text{in}} / P_L^{\text{out}}$ is $45\, \text{dB}$.

At the entrance of the cell the laser delivers a full power of 15 mW; the laser beam shape is Gaussian (TEM$_{00}$) and, after collimation, the waist is $w_0 \approx 15\, \text{mm}$. The angular fluctuations of the linearly polarized laser amount to $5\, \mu\text{rad}$ (rms) over a bandwidth of 10 kHz, as measured at the physics package entrance. An aspheric lens with a diameter of 11 mm and a focal length of 8 mm focuses the laser transmitted through the cell onto the detection photodiode placed outside the cavity.

In the following, in order to compare the experimental results with the theoretical predictions, we will express the laser intensity in terms of the pumping rate defined as $\Gamma_p = \omega_R^2/2\Gamma_1^*$, where $\omega_R$ is the (angular) optical Rabi frequency. The pumping rate is related to the laser intensity $I_L$ through the relation

$$\Gamma_p = \frac{Z_0}{\Gamma^*} \left( \frac{d_e}{\hbar} \right)^2 I_L,$$

where $d_e$ is the electric dipole moment of the optical transition, $Z_0$ is the impedance of free space and $\hbar$ is the reduced Planck constant. From equation (1), it results that an intensity of 1 mW cm$^{-2}$ corresponds to a pumping rate $\Gamma_p \approx 10^{5}\, \text{s}^{-1}$ for the D$_2$ transition.

We also measured the relative intensity noise (RIN) of the laser; its noise spectral density is

$$S_I(f) = (8 \times 10^{-12}\, f^{-1} + 3 \times 10^{-15})\, \text{Hz}^{-1},$$

$f$ being the Fourier frequency expressed in hertz. This measurement will be discussed later in the section devoted to the analysis of clock performances.

4. Electronics

Figure 5 shows the clock scheme from an electronic point of view. The architecture is similar to that we developed for the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the POP maser [21], with some important differences. In optical detection the laser is not only used to pump the atoms but also to detect the clock resonance; the atomic signal provided by the photodetector is processed by a trans-impedance amplifier to detect the clock resonance; the atomic signal provided by the PO

The phase-noise power spectral density $S_{\phi}(f)$ was measured versus a second identical chain with the homodyne technique and, referred to a 10 MHz carrier, turns out to be

$$S_{\phi}(f) = (1.26 \times 10^{-17} + 4 \times 10^{-15}\, f^{-1} + 3.1 \times 10^{-13}\, f^{-2} + 3.1 \times 10^{-12}\, f^{-3})\, \text{rad}^2\, \text{Hz}^{-1}$$

where $f$ is the Fourier frequency. The $S_{\phi}(f)$ behaviour is reported in figure 6(a). It is possible to measure the noise contribution due to the synthesis chain only. To do that, we implemented a tight PLL locking of the OCXOs of the two synthesis chains; in this way, the noise of the LOs is cancelled out. The result in terms of Allan deviation (time domain) is plotted in figure 6(b).
Figure 6. (a) Phase-noise power spectrum of the synthesis chain and OCXO referred to a 10 MHz carrier; (b) frequency stability of the synthesis chain with a measure bandwidth of 500 Hz; the measure is limited by the mixer used in the PLL.

4.2. Trans-impedance amplifier

A 15 mm² silicon photodiode followed by a trans-impedance amplifier of gain 1.6 V mA⁻¹ is used to detect the light transmitted through the cell (≈1 mW) during the detection stage. It is able to operate with power levels up to 10 mW avoiding, at the same time, a degradation of the clock stability. The noise performance of this circuit is only limited by the thermal noise of the feedback resistor thanks to the use of an operational amplifier based on ultra-low-noise bipolar junction transistor (BJT) technology.

4.3. Digital board

The pulsed operation is performed by a dedicated digital board where a single FPGA coordinates the operation of the two DDSs and of the ADC: (1) DDS1 generates the baseband version of the two microwave pulses of the Ramsey scheme (see figure 5); DDS2 drives the AOM during the pumping and the detection phases, (3) ADC is the front-end of the lock-in amplifier. The frequency loop controller of the LO is also implemented in the FPGA. The working parameters are set by a PC that is used to monitor the operation, but it is not a part of the running clock.

5. Operation and timing

Before reporting the results obtained with the system described above, we recall the principle of operation of the POP clock. As mentioned in the introduction, the POP clock operation separates in time the following three phases: (1) laser optical pumping, (2) microwave interrogation and (3) detection of the clock transition. Figure 7 shows a typical timing sequence made explicit in the case of optical detection; \( t_p \) is the pumping time, \( t_1 \) is the duration of each microwave pulse (Rabi time), \( T \) represents the time interval where the atoms are allowed to evolve freely (Ramsey time) and \( t_d \) is the detection time in which the clock transition is observed through the laser absorption signal.

The optimization of the timing sequence in the POP scheme is not easy from a mathematical point of view since many parameters (laser power, atomic density, microwave field uniformity, etc) contribute to the clock signal, and often they appear entangled among them. However, some basic requirements suggested by the theory reported in [2] provide the guidelines to optimize the clock signal and then the stability performances.

First, the laser pulse during the pumping phase should produce a large population inversion among the two clock levels and, at the same time, should effectively destroy the microwave coherence to avoid residual light shift; both the conditions are satisfied when \( \Gamma_p t_p \gg 1 \) [2].

As regards the Ramsey pulses, the microwave power is adjusted in order to maximize the contrast of the central Ramsey fringe; in terms of pulse area this corresponds to \( b_e t_1 \approx \pi/2 \), where \( b_e \) is the angular Rabi frequency associated with the microwave field.

The Ramsey time is basically limited by the relaxation phenomena taking place inside the cell; in particular, a Ramsey time \( T \) of the order of \( \gamma_s^{-1} \) is a good trade-off between a large clock signal and a narrow fringe.
In the detection phase the laser pulse should minimally perturb the atoms, being mainly used as a probe: intensity and duration are reduced consequently.

Provided the previous points are satisfied, $t_p$, $t_1$ and $t_d$ should be as short as possible in order to reduce the Dick effect (see equation (10)) that is inversely proportional to the duty cycle $T/T_C$, $T_C$ being the cycle duration: $T_C = t_p + 2t_1 + T + t_d$.

Once the previous requirements are fulfilled, the values of the working parameters are finely adjusted to minimize the frequency instability of the clock.

6. Results

6.1. Ramsey fringes and contrast

Figure 8 shows Ramsey fringes as detected by the photodiode at the exit of the cell. The laser power during the pumping period is $P_{\text{pump}} = 10\, \text{mW}$ while during the detection time it is $P_{\text{det}} = 1.5\, \text{mW}$. The figure refers to an internal temperature of $T_{\text{int}} = 63.5\, \text{°C}$ and an external temperature of $T_{\text{ext}} = 54\, \text{°C}$. Although the optimum clock operation temperature is higher, we observed the Ramsey fringes also in the low-temperature regime in order to verify the behaviour predicted by the theory.

The other main working parameters are indicated in the caption of the figure. Under these operating conditions, the central fringe of the Ramsey pattern exhibits a contrast of 15% and a linewidth, expressed in terms of full-width at half-maximum (FWHM), of $\Delta v_{1/2} = 120\, \text{Hz}$, according to the well known relation $\Delta v_{1/2} = 1/2T$.

The best clock performances were obtained at a higher working temperature ($T_{\text{int}} = 68.5\, \text{°C}$ and $T_{\text{ext}} = 66.5\, \text{°C}$); the corresponding Ramsey fringes are reported in figure 9. In this case we used $P_{\text{pump}} = 15\, \text{mW}$ and $P_{\text{det}} = 1\, \text{mW}$, corresponding approximately to pumping rates of $8.5 \times 10^3\, \text{s}^{-1}$ and $5.6 \times 10^4\, \text{s}^{-1}$, respectively.

The different Rabi envelopes of the curves reported in figures 8 and 9 are predicted by theory and can be explained well in terms of the values assumed by relaxation rates $\gamma_1$ and $\gamma_2$ in the two temperature regimes, as already observed in the case of microwave detection [2]. A comparison between theory and experiments has already been reported in [20].

We point out that the atomic density and the other cell parameters (relaxation rates, etc) appear to be tightly connected to the external temperature rather than to the internal one. This can be explained taking into account that the system is under vacuum: thermal convection between the cavity and the cell is absent while irradiation and conduction through the small cell-cavity contacts are negligible. The thermal behaviour is in fact driven by the diffusive motion of the buffer gas particles towards the stems and backwards to the cell body; in this way, the atoms thermalize to a temperature closer to that of the external thermal shield.

In the configuration of figure 9 the cycle time is $T_C = 4.65\, \text{ms}$ and the duty cycle is $T/T_C \approx 0.7$, a condition particularly favourable to reduce the Dick effect (see equation (10)).

The central fringe of the Ramsey pattern shows a contrast of 28% and a linewidth of $\approx 150\, \text{Hz}$. The contrast of the central fringe was characterized in detail in the high-temperature regime. In fact, it plays a key role not only to determine the signal level but also to reduce some noise sources, as will be shown later (see equations (8) and (10)).

Figure 10 shows the contrast versus the pumping parameters ($t_p$ and $\Gamma_p$). It is observed that, as expected, the pumping process is initially characterized by an exponential trend, after which a saturation value is reached. This level is strongly related to the maximum population imbalance which is generated in the ground state during the pumping process. In figure 10 and the following figures the white square corresponds to the working point when the system operates as a clock.

Figure 11 shows the contrast behaviour when measured versus the detection parameters (we call $\Gamma_p$ the laser pumping rate during the detection period). The contrast behaviour can be explained taking into account that by increasing the detection time and/or the detection laser power, the laser light...
does not act simply as a probe but gives rise to a pumping process in the atomic sample with a consequent degradation of the clock signal.

The contrast was also measured versus the microwave power. The maximum of figure 12 corresponds to $\pi/2$ pulses. Definitely, from previous figures it turns out that around typical operating conditions the observed contrast is 28%–30%; we will discuss later (section 6.3) how the contrast affects the clock frequency stability.

### 6.2. Residual laser and microwave sensitivities

The pulsed approach allows one, in principle, to uncouple the laser pumping phase from the successive microwave interrogation, in such a way that the atomic response is insensitive to any laser fluctuation. However, due to the inhomogeneity in the atomic sample a residual coupling between optical and microwave signals exists and the clock frequency still exhibits a dependence on the laser power. Specifically, we measured the following sensitivity of the clock frequency to the laser power fluctuations $\Delta P_L$:

$$\frac{\Delta \nu}{\nu} = \frac{\Delta P_L}{P_L} \approx -6 \times 10^{-14}/\%.$$  \hspace{1cm} (4)

Although it is not possible to totally exclude a residual off-resonant light-shift contribution, we mainly attribute this effect to a position shift that makes the resonant frequency of Rb atoms dependent on their spatial location within the cell.

We also measured the clock frequency sensitivity to the laser frequency fluctuations $\Delta \nu_L$ and we observed a residual
resonant light-shift of

\[
\frac{\Delta \nu / \nu}{\Delta t} \approx 1.5 \times 10^{-14} \text{ MHz}^{-1}. \tag{5}
\]

It is worth mentioning that the sensitivities to the laser parameters are orders of magnitude lower than those observed in continuously operating vapour-cell clocks.

Although it is in residual form, the clock frequency suffers from a cavity-pulling shift. We recall that in pulsed operation, in the limit of small cavity detuning, the cavity-pulling shift may be written as

\[
\Delta \nu_{cp} = \frac{4}{\pi} \frac{Q_t}{Q_a} c(\theta) \Delta \nu_C \tag{6}
\]

where \(Q_a\) is the atomic quality factor, \(\Delta \nu_C\) is the cavity detuning and \(c(\theta)\) is a function of the microwave pulse area \(\theta = b_t t_I\) that is minimized when \(\theta \approx \pi/2\) [19]. We measured a relative clock frequency change of \(-2 \times 10^{-13}\) when \(\Delta \nu_C = 300\) kHz; for our cavity in Mo this corresponds to \(\Delta \nu_{cp}/\Delta T_{int} = 3 \times 10^{-14} \text{ C}^{-1}\), a negligible value in our case.

Actually, equation (6) suggests that cavity-pulling has another component related to the microwave pulse area. Figure 13 shows the relative clock frequency versus \(\theta\); the measurement was carried out by changing the microwave power at a fixed value of \(t_I\) (\(t_I = 400\) µs).

From figure 13, the sensitivity of the clock frequency to the variation of \(\theta\) may be expressed as

\[
\frac{\Delta \nu_{cp}/\nu}{\Delta \theta/\theta} \approx 5.5 \times 10^{-14}/\% \tag{7}
\]

This sensitivity may impact the medium–long-term frequency stability and an active system to stabilize the microwave power has then been implemented.

6.3. Frequency stability performances

Figure 14 shows the clock frequency stability when the LO is locked on the central fringe of the Ramsey pattern of figure 9.

The figure also reports the stability of the reference frequency standard (a H-maser filtered by a BVA quartz). The figure refers to a sample of \(10^5\) data, corresponding to about 12 days of measurement. At 1 s, the Allan deviation is as low as \(1.7 \times 10^{-13}\), a record result for a vapour-cell clock. As expected, the clock signal is affected by white frequency noise and consequently the Allan deviation scales as \(\tau^{-1/2}\), \(\tau\) being the integration time, and reaches the value of \(6 \times 10^{-15}\) after \(\tau \approx 2000\) s. A drift of \(-8 \times 10^{-15}\)/day is removed from the data. The kink around \(\tau \approx 200\) s is synchronous with the laboratory air-conditioning cycle (electronics and optics of this prototype are in fact in open air). For our cell the He permeation effect, which has a time constant of about 200 days, is considered extinguished and we attribute the frequency drift mainly to temperature. In particular, the observed value corresponds to a temperature drift \(\leq 100\) µK day^{-1}.

To give a physical insight into the effects leading to this result, we start analysing the factors limiting the short-term frequency stability of the POP clock.

First, we consider the shot-noise associated with the detection of optical photons that provides the ultimate theoretical stability limit. In terms of Allan deviation, this noise contribution can be expressed as

\[
\sigma_y^m(\tau) = \frac{1}{\pi Q_a R_{sn}} \sqrt{\frac{T_C}{\tau}} \tag{8}
\]

where \(Q_a\) is the quality factor of the atomic resonance (\(Q_a = 4.4 \times 10^7\)) and \(R_{sn}\) is the signal-to-noise ratio, which can be written as [2]

\[
R_{sn} = C \sqrt{\eta_q N_p} \tag{9}
\]

In equation (9), \(C\) is the contrast of the central Ramsey fringe, \(\eta_q\) is the quantum efficiency of the photodetector and \(N_p\) is the number of photons reaching the photodetector. Under our operating conditions, it turns out that \(R_{sn} \approx 20 000\) and \(\sigma_y^m(\tau) = 2 \times 10^{-14} \tau^{-1/2}\).

We also evaluated the noise contribution due to the trans-impedance amplifier (ti) that introduces an additive white
frequency noise. It was measured in the absence of laser light and gives a contribution of \( \sigma_1^y(\tau) = 4 \times 10^{-14} \tau^{-1/2} \).

In frequency standards working in pulsed operation a well-known issue is the phase noise of the microwave interrogating signal that is transferred to the atomic signal through the Dick effect [23]:

\[
\sigma_y^{LO}(\tau) = \left( \sum_{k=1}^{\infty} \sin^2 \left( k\pi \frac{T}{T_C} \right) S_y^{LO}(kT_C)^2 \right)^{1/2} \tau^{-1/2}
\]

where \( S_y^{LO}(f) \) is the power spectral density of the microwave fractional frequency fluctuations and \( f_C = 1/T_C \) (\( f_C \approx 200 \text{ Hz} \) in our case). With the performances reported in section 4.1, the Dick effect results in \( \sigma_y^{LO}(\tau) = 7 \times 10^{-14} \tau^{-1/2} \).

Another effect limiting the short-term stability of the POP clock in the optical detection mode is the amplitude fluctuations of the laser probe through which the clock signal is observed (see also [24, 25]). Since this additive noise is sampled only during the detection time, its contribution to the clock Allan deviation can be written in a form similar to the Dick effect as

\[
\sigma_y^{AM}(\tau) = \frac{1}{C Q_d} \left( \sum_{k=1}^{\infty} \sin^2 \left( k\pi \frac{T}{T_C} \right) S_y^{AM}(kT_C) \right)^{1/2} \tau^{-1/2}
\]

where \( S_y^{AM}(f) \) is the power spectral density of the fractional intensity fluctuations of the probe signal reaching the photodetector. It contains both the laser RIN transferred at the output of the cell (AM-AM) and the laser frequency noise converted into amplitude fluctuations (PM-AM). We have measured \( S_y^{AM}(f) = 2 \times 10^{-11} \text{ Hz}^{-1} \) for \( 100 \text{ Hz} < f < 1 \text{ kHz} \) which reveals an excess of laser frequency noise converted into AM fluctuations at the cell output. Equation (11) then gives the following value for the Allan deviation: \( \sigma_y^{AM}(\tau) = 1.2 \times 10^{-13} \tau^{-1/2} \). With a narrower laser linewidth (of the order of 1 MHz or less), the contribution from PM–AM fluctuations reduces [25] and \( \sigma_y^{AM} \) can be lowered by up to one order of magnitude.

It is worth mentioning that, since this is an additive noise, it scales as the contrast; this means that the larger the contrast the lower is this noise contribution. The shot-noise limit improves only slightly, increasing the contrast; in fact, in equation (8) the contrast appears joint to the laser background signal (included in \( N_p \)) and \( R_{an} \) turns out, at the end, to be weakly sensitive to the working parameters when varied around their typical values [2].

From the previous evaluation, it turns out that the sum of all the previous contributions leads to \( 1.6 \times 10^{-13} \) at 1 s, very close to the measured value; moreover, the short-term clock stability is mainly limited by the laser noise.

It may be useful at this point to relate the contrast \( C \) to a few important physical parameters to explain some experimental results and some design criteria adopted in this work.

From the theory reported in [2], the contrast may be expressed as

\[
C \propto \frac{\hbar \omega \delta_0 (d_e/h)^2 N_a}{\Gamma^+} S_{\Delta} e^{-\gamma t} \tag{12}
\]

where \( \omega_0 /2 \pi \) is the laser frequency, \( N_a \) the number of Rb atoms on the interaction zone of section 4.1, and \( \Delta \), the hyperfine population difference after the pumping pulse. This relation was obtained for a thin atomic medium, under collision-limited operating conditions and in the probe detection limit (\( T_p \delta \tau_\Delta \ll 1 \)). From equation (12), it turns out that \( C \propto d_e^2 \) so that, at least in the phase of detection of the clock transition, the D2 line is better than D1 due to its higher electric dipole moment [26]. Moreover, \( C \) is nearly independent of \( \Gamma^+ \) and \( d_e \) in the probe regime, as is experimentally observed in figure 11.

Finally, to achieve a high contrast the Ramsey time \( T \) should be of the order of \( \approx \gamma^{-1} \) and \( \Delta \), as large as possible.

Table 1 lists the instability sources and their impact on the short-term clock stability.

| Instability source          | \( \sigma_y(\tau) \) |
|-----------------------------|----------------------|
| Shot-noise limit            | \( 2 \times 10^{-14} \tau^{-1/2} \) |
| Dick effect (microwave)     | \( 7 \times 10^{-14} \tau^{-1/2} \) |
| Laser noise (PM-AM)         | \( 12 \times 10^{-14} \tau^{-1/2} \) |
| Trans-impedance noise       | \( 4 \times 10^{-14} \tau^{-1/2} \) |
| Total \( \sqrt{\sum \sigma_y^2(\tau)} \) | \( 16 \times 10^{-14} \tau^{-1/2} \) |

Regarding the medium–long-term performances, the observed behaviour after 2000 s of integration time is mainly due to thermal fluctuations, probably related to thermal bridges between the external environment and the physics package. A high level of correlation between the clock frequency and the laboratory temperature was in fact measured. Medium–long-term effects are discussed in more detail in [19].

To show that the clock signal is not affected by any flicker noise coming from the laser and/or the electronics we refer to the plot of figure 15. This graph is obtained by selecting a subset of 50,000 points from the same run of figure 14; this subset corresponds to the period where the temperature was more stable in the 12-day run. The plot shows that under quiet environmental conditions the clock stability may reach values below the \( 10^{-15} \) level. We are then confident that with better engineering of the system, based on a more refined thermal model, the stability of figure 15 can become a typical performance for \( 1 \leq \tau \leq 10^4 \text{ s} \).

7. Conclusions

We have reported in this paper the implementation and metrological characterization of a pulsed optically pumped Rb clock based on the optical detection mode. The design of the clock followed the guidelines of the theory developed in [2]. That theory, already successfully tested for the maser approach, proved its validity also in this case: physical behaviour and frequency stability performances are correctly predicted.

Specifically, the short-term stability currently turns out to be limited by the laser noise transferred to the amplitude of the clock signal. First, this noise contribution may be lowered by a factor of two using a better laser power supply and up to a factor of ten with a 1 MHz linewidth laser. In addition, since
this noise scales as the contrast, it is possible to adopt the total pumping technique in order to increase the number of pumped atoms and then the contrast [27], according to equation (10). With these improvements it is reasonable to reduce $\sigma_N$ to the level of $3 \times 10^{-14}$ at 1 s or even better.

The other main instability source comes from the phase noise of the interrogating microwave. Thanks to the spectral characteristics of ultra-low-noise 100 MHz quartz oscillators and proper adjustment of the timing sequence, the Dick effect contribution can be reduced to $3 \times 10^{-14}$ at 1 s. Definitely, the overall clock frequency stability can achieve the value of $5 \times 10^{-14}$ at 1 s.

As regards integration times in the medium–long-term period, the performances appear to be currently limited by the instability and non-homogeneity of the cell temperature. However, once this issue is solved, we are confident that the clock can reach, at least under laboratory conditions, the region of $10^{-16}$, with a drift below $5 \times 10^{-15}$/day.

Indeed, limiting ourselves to considering the results reported in this paper, we point out that they already represent a record achievement for a vapour-cell clock.

To make more evident the importance of the result we attained, we show in figure 16 a comparison between the POP clock frequency stability and the ground tests of the GALILEO’s clocks [28]. It is observed that our clock exhibits a frequency stability even better than the performance of the PHM, with a physics package that can be more compact and lighter.

In addition, the result of figure 15 turns out to be particularly remarkable if we think that it is achieved with a hot atomic sample. A similar short-term stability ($2.2 \times 10^{-13}$ at 1 s using a cryogenic oscillator as LO) is reported in [1] where a cold-atom vapour-cell clock is described. However, that value is very close to the ultimate stability limit given by the atomic shot noise, while in our case the shot-noise limit is still far from being reached, as just discussed.

Definitely, the POP clock combines typical features of vapour-cell devices, such as compactness, reliability and low power consumption with the high-frequency stability performances of H-masers. All these features make the POP clock with optical detection very attractive for many technological applications and particularly suitable for space-oriented engineering.

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