Weak Phases and CP Violation

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1. INTRODUCTION

At the end of the second millennium, thirty five years after the discovery in 1964 of CP violation in $K \rightarrow \pi^+\pi^-$ [1], theoretical interpretations within the Kobayashi-Maskawa framework [2] of CP non-conservation in $K$ decays involved large hadronic uncertainties. This situation has changed dramatically through progress made in the past five years by the BaBar and Belle detectors operating at SLAC and KEK. Theoretical ideas proposed between twenty five and fifteen years ago and developed subsequently to measure the weak phases $\beta$, $\alpha$ and $\gamma$ through CP asymmetries in $B^0 \rightarrow J/\psi K_S$ [3], $B^0 \rightarrow \pi^\pm\pi^\mp$, $\rho^+\rho^-$ [4] and $B^+ \rightarrow DK^+$ [5] were applied experimentally, thereby improving greatly our confidence in the Kobayashi-Maskawa mechanism of CP violation. The purpose of this presentation is to describe this remarkable progress, applying simple equations (instead of $\chi^2$ fits) to most recent data in order to obtain the current values of $\beta, \alpha$ and $\gamma$.

Two major targets of high statistics experiments studying $B$ and $B_s$ decays in $e^+e^-$ and hadron colliders are (1) achieving great precision in Cabibbo-Kobayashi-Maskawa (CKM) parameters, and (2) identifying potential inconsistencies by overconstraining these parameters. For instance, the phase $2\beta$ measured in time-dependent CP asymmetries of $B^0$ decays via $b \rightarrow c\bar{c}s$ may be tested also in $b \rightarrow sq\bar{q}$ ($q = u, d, s$) penguin-dominated decays [6,7] which are susceptible to effects of physics beyond the Standard Model [8].

Section 2 reviews the current status of precision determinations of the weak phases $\beta$, $\alpha$ and $\gamma$, while Section 3 compares measurements of $\sin 2\beta$ in $b \rightarrow c\bar{c}s$ and in penguin-dominated decays. A way of identifying New Physics in the latter decays through direct CP asymmetries in $B \rightarrow K\pi$ is discussed in Section 4, and Section 5 concludes with a few remarks about future prospects.

2. PRECISION TESTS FOR $\beta$, $\alpha$, $\gamma$

Semileptonic $B$ decays, $B^0$-$\bar{B}^0$ mixing, $B_s$-$\bar{B}_s$ mixing, and $\epsilon_K$ constrain indirectly the three angles of the unitarity triangle in an overall fit combining theoretical and experimental errors [9],

$\beta = (23\pm 4)^\circ$, $\alpha = (99\pm 13)^\circ$, $\gamma = (58\pm 13)^\circ$. (1)

We quote symmetric $1\sigma$ errors using CKMfitter Group pre-summer 2005. A crucial question confronting certain measurements of CP asymmetries in $B$ decays is do they agree with these values and can they reduce the above errors?

2.1. The phase $\beta$

The classical way of determining $\sin 2\beta$ in time-dependent CP asymmetries [3] is based on interference between $B^0$-$\bar{B}^0$ mixing and a $b \rightarrow c\bar{c}s$ decay amplitude which carries a single weak phase at a very high precision [10]. The world averaged value before Summer 2005 [11], $\sin 2\beta = 0.726\pm 0.037$, corrected by a recent Belle measurement [12], $\sin 2\beta = 0.652\pm 0.039\pm 0.020$, becomes

$\sin 2\beta = 0.687 \pm 0.032$.

(2)
A twofold ambiguity in $\beta (\beta \to \pi/2 - \beta)$ in the range $0 < \beta < \pi/2$ may be resolved by measuring the sign of $\cos 2\beta$. A transversity analysis of $B^0 \to J/\psi K^*$ \cite{13} implying $\cos 2\beta = 2.72^{+0.50}_{-0.79} \pm 0.27$ excludes at 86% confidence level a negative value of $\cos 2\beta$. A recent time-dependent Dalitz analysis of $D^0 \to K_S \pi^+ \pi^-$ in $B^0$ decays to $D^0$ and a light neutral meson \cite{14} improves the confidence level to 97%. Thus, we conclude $\beta = (21.6 \pm 1.3)^\circ$, in agreement with (1), and implying an overall average, $\beta = (21.7 \pm 1.2)^\circ$.

$|\beta| = (23.2 \pm 0.9)^\circ$ was obtained this summer by CKEfitter \cite{9}, using a constraint from $|V_{ub}/|V_{cb}|$ in which a very small error ($< 5\%$) was assumed.

2.2. The phase $\alpha$ in $B \to \pi\pi, \rho\rho, \rho\pi$

2.2.1 $B \to \pi\pi$

The amplitude for $B^0 \to \pi^+\pi^-$ contains two terms \cite{17}, conventionally denoted “tree” ($T$) and “penguin” ($P$) amplitudes, involving a weak phase $\gamma$ and a strong phase $\delta$:

$$A(B^0 \to \pi^+\pi^-) = |T| e^{i\gamma} + |P| e^{i\delta}. \quad (3)$$

Time-dependent decay rates, for an initial $B^0$ or a $\bar{B}$, are given by \cite{7}:

$$\Gamma(B^0(t) \to \pi^+\pi^-) \propto e^{-\Gamma m t} \sum_{S\pm} \sin \Delta m t, \quad (4)$$

$$S_{\pm} = \frac{2 \text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \quad C_{\pm} = 1 - |\lambda_{\pi\pi}|^2 \quad (5)$$

$$\lambda_{\pi\pi} = e^{-2i\beta} \frac{A(B^0 \to \pi^+\pi^-)}{A(B^0 \to \pi^+\pi^-)}. \quad (6)$$

The measurables, $\Gamma_{\pi\pi}, S_{\pm}$ and $C_{\pm}$ are insufficient for determining $|T|, |P|, \delta$ and $\gamma$. One uses additional information obtained from an isospin amplitude triangle for $B$ decays,

$$A(\pi^+\pi^-)/\sqrt{2} + A(\pi^0\pi^0) - A(\pi^+\pi^0) = 0, \quad (7)$$

and a similar one for $\bar{B}$.

$$A(\pi^-\pi^-)/\sqrt{2} - A(\pi^0\pi^0) + A(\pi^-\pi^0) = 0, \quad (8)$$

This bound is improved by measuring $C_{00} = -A_{CP}(\pi^0\pi^0)$, the direct asymmetry in $B^0 \to \pi^0\pi^0$.

Current measurements $|B_{+0}| = 5.0 \pm 0.4$, $B_{00} = 1.45 \pm 0.29$, $S_{+} = 0.50 \pm 0.12$, $C_{+} = -0.37 \pm 0.10$, $C_{00} = -0.28 \pm 0.29$, imply $\alpha_{\text{eff}} = (106 \pm 5)^\circ$, $|\delta| < 36^\circ$. A second solution at $\alpha_{\text{eff}} = 164^\circ$ is excluded by (1). A positive sign, $\Theta \geq 0$, is determined \cite{16} by two properties, $|P/T| \leq 1$, $|\delta| \leq \pi/2$, which are confirmed experimentally. This leads to a solution $\alpha = (88 \pm 18)^\circ$ using isospin symmetry alone. [A value $\alpha = (99 \pm 18)^\circ$ \cite{17}, obtained by relating $B \to \pi\pi$ and $B \to K\pi$, does not contain SU(3) breaking and will not be used.] We stress that the condition $\Theta \geq 0$ should also be applied in $\chi^2$ likelihood fits for $B \to \pi\pi$ \cite{9}.

2.2.2 $B \to \rho\rho$

Angular analyses of the pions in $\rho$ decays have shown that $B^0 \to \rho^+\rho^-$ is dominated by longitudinal polarization, $f_{\perp}^- = 0.978^{+0.014}_{-0.021}$ \cite{18}, $0.951^{+0.033}_{-0.029} +0.029$ \cite{19}. This simplifies the study of CP asymmetries in these decays to the level of studying asymmetries in $B^0 \to \pi^+\pi^-$. As long as a nonzero asymmetry $C_{00}$ has not been measured, an advantage of $B \to \rho\rho$ over $B \to \pi\pi$ is $B(\rho^0\rho^0)/B(\rho^+\rho^-) \sim B(\pi^0\pi^0)/B(\pi^+\pi^-)$, implying a stronger bound on $|\delta|$ in $B \to \rho\rho$.

Using $|B_{\rho\rho}| = 26.2^{+3.6}_{-2.9} < 1.1$, and taking averages for CP asymmetries from \cite{18} and \cite{19}, $S_L = -0.21 \pm 0.23, C_L = -0.02 \pm 0.17$, one finds $\alpha_{\text{eff}} = (96 \pm 6)^\circ, |\delta| < 11^\circ(1\sigma)$, implying $\alpha = (96 \pm 13)^\circ$. This is currently the most precise single determination of $\alpha$.

2.2.3 $B \to \rho\pi$

A time-dependent Dalitz analysis of $B^0 \to \pi^+\pi^-\pi^0$ involving interference of $B^0 \to \rho^+\rho^-\pi^0$ provides twenty seven mutually dependent measurable parameters including $\alpha$ \cite{21}. Limited statistics leads to a large statistical error, and potential contributions from $S$-wave $\pi\pi$ resonance states and excited $\rho$ meson states lead to model-dependent uncertainties. An analysis by the Babar collaboration obtained $\alpha = (113^{+29}_{-27} \pm 6)^\circ$.

\footnote{Branching ratios will be quoted in units of $10^{-6}$.}
Alternatively, one may apply flavor SU(3) symmetry to quasi two-body $B^0 \to \rho^\pm \pi^\mp$, using time-dependent decay rates [24],
\[
\Gamma(B^0(t) \to \rho^\pm \pi^\mp) \propto e^{-\Gamma t} \times \left[1 + (C \pm \Delta C) \cos \Delta mt - (S \pm \Delta S) \sin \Delta mt\right].
\] (9)

As in $B \to \pi^+\pi^-$, one defines $\alpha_{\text{eff}}$ which equals $\alpha$ in the limit of vanishing penguin amplitudes [24],
\[
4\alpha_{\text{eff}} \equiv \arcsin\left[\frac{(S + \Delta S)/\sqrt{1 - (C + \Delta C)^2}}{\sqrt{1 - (C - \Delta C)^2}}\right] + \arcsin\left[\frac{(S - \Delta S)/\sqrt{1 - (C - \Delta C)^2}}{\sqrt{1 - (C + \Delta C)^2}}\right].
\] (10)

In the SU(3) limit, the difference $|\alpha_{\text{eff}} - \alpha|$ is bounded by ratios of decay rates for $B \to K^+\pi$ and $B \to K\rho$ and decay rates for $B \to \rho^\pm \pi^\mp$. The bound [24], $|\alpha_{\text{eff}} - \alpha| < 13^\circ$, can be reduced by a factor two under very mild assumptions about (small) ratios of penguin and tree amplitudes and about a strong phase difference [10].

Taking averages from Refs. [25] and [29],
\[
C = 0.30 \pm 0.13, \quad \Delta C = 0.33 \pm 0.13,
\]
\[
S = -0.04 \pm 0.17, \quad \Delta S = -0.07 \pm 0.18, \quad (11)
\]
one finds $\alpha_{\text{eff}} = (92 \pm 8)^\circ$ and therefore $\alpha = (92 \pm 8 \pm 8)^\circ$. The second error follows from the SU(3) bound in which a 30% uncertainty is included. To be conservative, we add the experimental and theoretical errors linearly, $\alpha = (92 \pm 16)^\circ$.

2.2.4 Averaged $\alpha$

Combining the values of $\alpha$ from $B \to \pi\pi$ and $B \to \rho\rho$, one finds an average $\alpha = (93 \pm 11)^\circ$. The average becomes $(97 \pm 8)^\circ$ when including the two values of $\alpha$ obtained from $B \to \rho\pi$. This direct determination agrees with Eq. (1) representing all other CKM constraints, and is more precise than this indirect value. Combining these two values we find an overall average
\[
\alpha = (98 \pm 7)^\circ. \quad (12)
\]

In comparison, two global fits combining all constraints and using different methods for error estimates obtain [9] $\alpha = (98.1^{+6.3}_{-7.0})$ and [27], $(97.9 \pm 6.0)^\circ$.

2.2.5 Isospin breaking corrections in $\alpha$

The overall determination $\alpha = (98 \pm 7)^\circ$, equivalently $\gamma = (60 \pm 7)^\circ$ when using $\beta = (22 \pm 1)^\circ$, relies in part on isospin symmetry. At this precision one must consider isospin breaking corrections caused by the charge and mass differences of $u$ and $d$ quarks. Here we will summarize briefly the results of a recent study of isospin violating effects in $\alpha$ [28], updating an earlier analysis [29].

Effects due to the different charges of the $u$ and $d$ quarks have been calculated model-independently and process-independently [30] by noting that the $\Delta I = 3/2$ electroweak penguin (EWP) operator in the effective Hamiltonian for $b \to d\bar{q}q$ is proportional to the $\Delta I = 3/2$ current-current operator (contributions of EWP operators with small Wilson coefficients $c_7$ and $c_8$ are neglected). The calculated EWP correction to $\alpha$ in $B \to \pi\pi$ and $B \to \rho\rho$ is negative, $\Delta_{\text{EWP}} = \left[3(c_9 + c_{10})/2(c_1 + c_2)\right]|\sin(\beta + \alpha)|\sin\alpha/\sin\beta$ = $-(1.7 \pm 0.3)^\circ$, and should be included in the extracted value using $\alpha = \alpha_{\text{eff}} - \theta + \delta_{\text{EWP}}$.

Effects caused by $\pi^0$ mixing with $\eta$ and $\eta'$ are parametrized in terms of mixing parameters $\epsilon$, $\epsilon'$ of order 0.01 [31]. In an SU(3) symmetry expansion their leading effect on the isospin triangle [7] is multitipling $A(B^+ \to \pi^+\pi^0)$ by $1 - e_0$, where $e_0 = \epsilon\sqrt{2/3} + \epsilon'\sqrt{1/3} = 0.016 \pm 0.003$. Using measured branching ratios for $B^+ \to \pi^+\eta$ and $B^+ \to \pi^+\eta'$, one finds a stringent upper limit on the effect of $\pi^0\eta\eta'$ mixing on $\alpha$ [28], $|\delta_{\text{mix}}^\eta\eta'| < 1.4^\circ$. Additional $\Delta I = 5/2$ corrections are hard to calculate, however are expected to introduce another uncertainty at this level [32].

Thus, while a known negative correction of $-(1.7 \pm 0.3)^\circ$ is negative, $\Delta_I = 5/2$ contributions should be included in the isospin-extracted value of $\alpha$ in $B \to \pi\pi$ and $B \to \rho\rho$, an uncertainty at this level remains in $B \to \pi\pi$ from other isospin breaking terms. The extraction of $\alpha$ in $B \to \rho\rho$ involves two additional corrections, from $p\omega$ mixing [28] and from the $\rho$ width [33]. Both effects can be included in the extraction of $\alpha$ by adequate measurements of $\pi\pi$ invariant mass distributions.

2.3. The phase $\gamma$ in $B^+ \to DK^+$

The processes $B^+ \to D^{(*)}K^{(*)+}$ and their charge conjugates provide a way for determining $\gamma$ with high precision [5]. The neutral $D$ meson can be either a $D^0$ from $\bar{b} \to \bar{c}u\bar{s}$ or a $D^0$ from $\bar{b} \to \bar{u}c\bar{s}$. Every hadronic state $f$ accessible to $D^0$
decay is also accessible to $D^0$ decay. An interference between the two channels leading to a common state $fK^+$ involves the weak phase difference $\gamma$. This phase can be determined by decay rate measurements for $B^+ \to fK^+$ and $B^- \to fK^-$. Effects from $D^0$-$D^0$ mixing are negligible [44].

Since the original suggestion of fifteen years ago several variants have been proposed studying mainly three classes of states $f$: CP eigenstates (e.g. $K^+K^-$) [3], flavor states (e.g. $K^-$) [34] and multi-body states (e.g. $K_S\pi^+\pi^-$) [36]. In $B^+ \to DK^+$, $D \to f$ the three variants involve a common ratio of amplitudes, $A(B^+ \to D^0K^+)/A(B^+ \to D^0K^+) \equiv re^{i\delta}e^{i\gamma}$, $r \sim 0.1 - 0.2$, differing by their complex ratio, $A(D^0 \to f)/A(D^0 \to f)$. This ratio is exactly $\pm 1$ and about $\lambda^{-2}$ in the first two variants, and depends on the point in a Dalitz plot in the third variant.

The limiting factor of the method is the small value of $r$, for which the current upper limit (at 90% C.L.) [37] $r < 0.18$ approaches estimates [38]. A nonzero value of $r$ may be measured soon. The corresponding ratio of amplitudes in self-tagged $B^0 \to DK^{*0}$ is expected to be larger than in $B^+ \to DK^+$. If that is the case also for a ratio $r^*$ in $B^+ \to DK^{**}$ then one may be able to observe soon a difference between two ratios, $R_+^* \equiv B(DCP^+K^{*+})/B(DCP^-K^{*+})$ which grows linearly with $r^*$. This is a key step towards measuring $\gamma$ using CP eigenstates [38]. Recent measurements [39], $R_+^* = 1.96 \pm 0.40 \pm 0.11$, $R_-^* = 0.65 \pm 0.26 \pm 0.08$, implying a difference at 2.6$\sigma$, illustrate the need for somewhat higher statistics.

The variant which involves the largest statistics studies multi-body Cabibbo-allowed $D$ decays [36]. Defining $m_{\perp}^2 = (p_{KS} + p_{\pi \pm})^2$, one writes

$$A(B^+ \to (K_S\pi^+\pi^-)_K^+) = f(m_{\perp}^2, m_+^2) + re^{i(\delta + \gamma)}f(m_+^2, m_0^2),$$

replacing $m_+ \leftrightarrow m_-$, $\gamma \rightarrow -\gamma$ in $B^-$ decay. The function $f$ is obtained by modeling separately measured flavor-tagged $D^0 \to K_S\pi^+\pi^-$ as a sum of about twenty resonant and nonresonant contributions [40][41]. This introduces a certain ambiguity [42] and a model-dependent uncertainty in the analysis. Fitting the $B^+ \to (K_S\pi^+\pi^-)_K^+$ rates for a given function $f$ to the parameters $r$, $\delta$, $\gamma$, one then determines the three parameters.

Averaging [40] $\gamma = (68^{+14}_{-15} \pm 13 \pm 11)^0$ and [41] $\gamma = (67 \pm 28 \pm 13 \pm 11)^0$ combining $DK, D^*K, DK^{*0}$, one finds $\gamma = (68 \pm 18)^0$. The first errors ($14^0$ and $28^0$) depend inversely on $r$ (or $r^*$), showing the importance of fixing $r$ (or $r^*$), a key ingredient also in using CP-eigenstates of $D$. The last errors ($11^0$) from modeling $f$ may be reduced by studying at CLEO-c $D_{CP} \rightarrow K_S\pi^+\pi^-$, which determines strong phases in $D$ decays [43].

2.4. Consistency between $\alpha$, $\beta$ and $\gamma$

The direct measurements of the weak phases, $\beta = (21.6 \pm 1.3)^0, \alpha = (97 \pm 8)^0$ (in $B \rightarrow \pi\pi, \rho\rho, \rho\pi$) and $\gamma = (68 \pm 18)^0$ (in $B \rightarrow D^{(*)}K^{(*)}$), imply $\alpha + \beta + \gamma = (187 \pm 20)^0$. Since $\alpha$ in $B \rightarrow \pi\pi, \rho\rho, \rho\pi$ is defined as $\pi - \beta - \gamma$, the question posed by the sum is actually whether the above measurements of $\beta$ and $\gamma, \beta + \gamma = (90 \pm 18)^0$, agree with the measurement $\beta + \gamma = (83 \pm 8)^0$ in $B \rightarrow \pi\pi, \rho\rho, \rho\pi$.

The sum $\alpha + \beta + \gamma$ measured in this way does not check the unitarity of the $3 \times 3$ CKM matrix which is violated in models with additional quarks. The sum is unaffected by New Physics in $B^0\overline{B}^0$ mixing, which contributes equally with opposite signs to $\beta$ and $\alpha$, nor is it affected by New Physics in $\Delta I = 1/2 b \rightarrow d\overline{q}q$ amplitudes which are eliminated in the isospin method [3].

The sum would be affected by New Physics in $\Delta I = 3/2 b \rightarrow d\overline{q}q$ transitions. This could lead to nonzero CP asymmetries in $B^+ \rightarrow \pi^+\pi^0$ (currently [11] $A_{CP}(\pi^+\pi^0) = 0.01 \pm 0.06$) and $B^+ \rightarrow \rho^+\rho^0$, where the Standard Model predicts a vanishing asymmetry including EW contributions [20]. Other probes of $\Delta I = 3/2$ New Physics in $B \rightarrow \pi\pi$ are discussed in [44]. The sum $\alpha + \beta + \gamma$ could also be affected by CP violation in $D^0\overline{D}^0$ mixing, which can be tested directly in $D^0$ decays.

3. sin$2\beta$ IN $b \rightarrow sq\overline{q}$ DECAYS

In a class of penguin-dominated $B^0$ decays into CP-eigenstates, including the final states $f = \eta'K_S, \phi K_S, \pi^0 K_S, f_0 K_S, \rho^0 K_S, \omega K_S, K^{+}K^{-}K_S, K_SK_SK_S, K_S\pi^0\pi^0$, decay amplitudes contain two terms: a penguin amplitude, $p_f$, involving a dominant CKM factor $V_{ub}V_{cs}$, and another term, $c_f$, with a smaller CKM factor $V_{ub}V_{us}$. The CP asymmetry involves two terms, $S_f \sin \Delta m_t -$
\[ C_f \cos \Delta m t, \] for which expressions were derived sixteen years ago \[4] for a final state of CP eigenvalue \( \eta_f, \) and for a small value of \( \xi_f \equiv |c_f|/|p_f|, \)
\[ C_f \approx 2 \xi_f \sin \gamma \sin \delta_f, \] (14)
\[ \Delta S_f \equiv -\eta_f S_f - \sin 2\beta \approx 2 \xi_f \cos 2\beta \sin \gamma \cos \delta_f. \]
For fixed \( \beta, \) these equations describe a circle,
\[ C_f^2 + (\Delta S_f / \cos 2\beta)^2 = 4 \xi_f^2 \sin^2 \gamma, \] (15)
on which points are parametrized by \( \delta_f, \) the strong phase difference between \( c_f \) and \( p_f. \)

Predictions for \( C_f \) and \( \Delta S_f \) require knowing the hadronic quantities \( \xi_f \) and \( \delta_f. \) A precise knowledge of \( \xi_f \) and \( \delta_f \) is crucial for claiming evidence for physics beyond the Standard Model in the relevant asymmetry measurements. This question has been studied using two major approaches, flavor SU(3) \[44\] and QCD factorization \[40\]. A third approach is based on final state rescattering \[47\]. I will describe the first two methods sketching their predictions.

Flavor SU(3) has been applied in two ways. In one type of study, decay rates and CP asymmetries have been correlated successfully for a wide variety of charmless \( B \) decays involving two light pseudoscalars (\( P \)) \[45\] and a pseudoscalar and a vector meson (\( V \)) \[49\]. This led to predictions for the magnitudes and signs of \( C_f \) and \( \Delta S_f \), which may involve uncertainties at a level of 30% from SU(3) breaking. Note that the sign of \( \Delta S_f \) is predicted to be positive under a very mild assumption, \( |\delta_f| < \pi/2, \) which holds for several final states including \( \pi^0 K_S \) and \( \eta K_S \) \[48\].

In a more conservative approach, SU(3) has been used to relate the amplitudes \( p_f \) and \( c_f \) to linear combinations of corresponding amplitudes in strangeness conserving decays \[50\]. The resulting prediction for a given final state \( f \) is that the point \( (C_f, \Delta S_f / \cos 2\beta) \) must lie within a circle of a given radius. In this approach the signs of \( C_f \) and \( \Delta S_f \) are unpredictable.

QCD factorization has been applied to \( B \rightarrow PP \) and \( B \rightarrow VP \) by expanding decay amplitudes in \( 1/m_b \) and \( \alpha_s \) \[51\]. Since strong phases are suppressed in this expansion, one expects \( \Delta S_f > 0 \) in most cases. Uncertainties include corrections from nonperturbative charm-penguin contributions (also interpreted as long distance final state interactions), and \( 1/m_b \) terms which may be large. This ambitious approach fails, for instance, in \( B \rightarrow K^0 \pi, \) where predicted branching ratios are consistently lower than the data by a factor two to three.

A sample of predictions, \( \Delta S_f = 0.10 \pm 0.05, \ 0.03 \pm 0.02, \ 0.03 \pm 0.02, \) for \( f = \pi^0 K_S, \ \eta K_S, \ \phi K_S, \) respectively, is common to flavor SU(3) and QCD factorization. Asymmetry measurements \[11\] updated by recent studies \[12, 32\] are consistent with these predictions and with predictions or bounds on other asymmetries. Errors must be reduced by at least a factor two before claiming evidence for New Physics.

4. NEW PHYSICS IN \( A_{CP}(B \rightarrow K \pi) \)

An independent test for New Physics in \( b \rightarrow s q \bar{q} \) \((q = u, d)\) has been proposed recently in terms of a sum rule among four \( B \rightarrow K \pi \) CP asymmetries \[53\]. The sum rule, obeyed by CP rate differences, \( \Delta_{ij} \equiv \Gamma(B \rightarrow K^+ \pi^-) - \Gamma(B \rightarrow K^- \pi^+), \) reads
\[ \Delta_{+-} + \Delta_{0+} = 2(\Delta_{++} + \Delta_{00}). \] (16)
This relation, reminiscent of a similar sum rule among partial decay rates \[54\], is more precise than relations omitting \( \Delta_{++} \) \[55\] and \( \Delta_{00} \) \[53\]. It is expected to hold in the Standard Model within a few percent. A proof of the sum rule will now be sketched discussing briefly its implication.

The first step of the proof is based on isospin considerations, neglecting subleading \( \Delta I = 1 \) electroweak penguin contributions to decay amplitudes \[56\]. A dominant \( \Delta I = 0 \) penguin term with CKM factor \( V_{ub}V_{us} \) is common to the four \( K \pi \) decay amplitudes, up to a factor \( 1/\sqrt{2} \) in processes involving a \( \pi^0. \) This common term interferes in CP rate differences with tree amplitudes involving a CKM factor \( V_{ub}V_{us}. \) The difference between the left and right-hand sides of (16) consist of an interference with a superposition of amplitudes which vanishes by isospin \[57\].

The remaining terms in (16) consist of subleading electroweak penguin amplitudes interfering with tree amplitudes involving \( V_{ub}V_{us}. \) This interference vanishes in the flavor SU(3) and heavy quark limits. Here one is using a proportionality
relation between the strangeness changing $\Delta I = 1$ EWP operator and the $\Delta I = 1$ current-current operator in the effective Hamiltonian 20,25, and a property, $\text{Arg}(C/T) \sim O(A_{QCD}/m_b, \alpha_s(m_b))$, of the ratio of color-suppressed and color-favored tree amplitudes 59. Terms which are both sub-leading and symmetry breaking are estimated to be a few percent of $\Delta_{+-}$ and are negligible.

Using measured asymmetries in $B \to K^+\pi^-$, $K^+\pi^0, K^0\pi^+$, and the four $K\pi$ branching ratios 11, one predicts 55 $A_{CP}(K^0\pi^0) = -0.17 \pm 0.06$, to be compared with the current value, $A_{CP}(K^0\pi^0) = 0.02 \pm 0.13$. Testing New Physics requires reducing the experimental error by at least a factor two. The sum rule may be violated by an anomalous $\Delta I = 1$ EWP-like operator.

5. CONCLUSIONS

CP asymmetries measured in a number of $B$ decays, involving a variety of interferences, support the hypothesis that the dominant origin of CP violation is a single phase in the CKM matrix. These measurements have reduced the error on $\beta$ to $\pm 1^{\circ}$, and the error on $\alpha$ (or $\gamma$) to $\pm 7^{\circ}$. A correction $\Delta_{EW}^{CP} = -1.7^c$ must be included.

Reducing the error in $\alpha$ depends on improved measurements of $A_{CP}(\pi^0\pi^0)$ predicted to be large and positive 55, and on improved upper bounds on $B(\rho^0\bar{\rho}^0)$, for which a nonzero value may be measured soon. Isospin breaking effects on $\alpha$ caused by $\rho$-$\omega$ mixing and by the $\rho$ width should be studied in $\pi\pi$ mass distributions in $B \to pp$. A precision determination of $\gamma$ in $B \to D^{(*)}K^{(*)}$, by combining CP-eigestates, flavor states and multibody decays of $D^0$, may be achieved soon.

Time-dependent CP asymmetries in $b \to s$ decays converge to Standard Model predictions. Testing New Physics in these decays requires precise calculations of small amplitudes and strong phases. A test using the sum rule for all $B \to K\pi$ asymmetries requires reducing the error in $A_{CP}(K^0\pi^0)$ which is experimentally challenging.

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