Covariant meson-exchange model of the $\bar{K}N$ interaction

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A covariant meson-exchange model of the $\bar{K}N$ interaction within the framework of the Bethe-Salpeter equation is presented. With just one free parameter we are able to get a good description of the available experimental data from below threshold to 300 MeV laboratory momentum.

The construction of nonperturbative models of the $\bar{K}N$ interaction has a long history, dating back to early work such as that of Dalitz, Wong, and Rajasekaran [1]. This model consisted of a coupled-channel Schrödinger equation with static vector meson exchange potentials, and was able to dynamically generate the S-wave $\Lambda(1405)$ resonance. More recent calculations are based on similar ideas, and commonly use a coupled-channel Lippmann-Schwinger equation to iterate a set of potentials to infinite order, with the strengths of the potentials in the various channels constrained by SU(3) symmetry. The strong attraction produced by the $I=0$ $\bar{K}N\to\bar{K}N$ potential results in the dynamical generation of the $\Lambda(1405)$ resonance as an unstable $\bar{K}N$ bound state. Further resonances, including the $\Lambda(1670)$ and $\Sigma(1620)$, have also been found to be formed dynamically [2,3].

Previous works have in general relied upon non-relativistic formulations, or have made use of the (ladder) Bethe-Salpeter equation [4] but solved it in an approximate way. Here we outline a covariant model of low-energy $\bar{K}N$ scattering based on the 4-dimensional Bethe-Salpeter equation, and present some preliminary results.

The multi-channel Bethe-Salpeter equation for the $\bar{K}N$ system is

$$T_{nm}(q', q; P) = V_{nm}(q', q; P) - \sum_k \frac{i}{(2\pi)^4} \int d^4q'' V_{nk}(q', q''; P) G_k(q''; P) T_{km}(q'', q; P),$$

where $m$ ($n$) label the initial (final) states, and $k$ is summed over the included channels ($K^-p$, $\bar{K}^0n$, $\Lambda\pi^0$, $\Sigma^-\pi^+$, $\Sigma^0\pi^0$, $\Sigma^+\pi^-$, $\Lambda\eta$, and $\Sigma^0\eta$). Also, $P = (\sqrt{s}, 0)$ is the total 4-momentum in the center-of-mass (c.m.), while $q$, $q'$ and $q''$ are the relative 4-momenta in the initial, final and intermediate states. The two-body propagator $G_k(q; P)$ is given by the product of the appropriate baryon and pseudoscalar meson propagators. The interaction kernels $V_{nm}$ are constructed from the $s$- and $u$-channel baryon poles and $t$-channel vector meson pole diagrams obtained from the usual SU(3)-symmetric $BBP$, $BBV$, and $PPV$ interaction Lagrangians [5] (here $B$, $P$, and $V$ represent the $J^P = 1/2^+$ baryons, the pseudoscalar mesons, and the vector mesons, respectively). In order to regularize the Bethe-Salpeter equation all the propagators are multiplied by form factors, which are given by

$$f_{B_k}(p^2) = \left( \frac{m_{B_k}^2 - \Lambda^2}{p^2 - \Lambda^2 + i\epsilon} \right)^2, \quad f_{P_k}(p^2) = \left( \frac{m_{P_k}^2 - \Lambda^2}{p^2 - \Lambda^2 + i\epsilon} \right)^2, \quad f_{V_k}(t) = \left( \frac{-\Lambda^2}{t - \Lambda^2 + i\epsilon} \right)^2.$$  

We use the same cutoff mass $\Lambda$ for all particles in order to minimize the number of free parameters.

The method of solution is the same as that described in Ref. [6]. A partial wave decomposition is applied to Eq. (1) which gives a system of coupled 2-dimensional integral equations. The singularities in the relative-energy variables are handled by performing a Wick rotation [6], i.e., the relative-energy integration contour is rotated from the real to the imaginary axis. All of the basic coupling constants in the model are fixed using information from other sources, such as decay widths and
vector meson dominance (VMD), and are not left as free parameters. The only adjustable parameter is the cutoff mass, which we fix by fitting to the $\bar{K}N$ data. The values of the basic coupling constants and the cutoff mass are shown in Table 1.

| Coupling Constants | Value | Expression |
|--------------------|-------|------------|
| $g_{\pi\pi\rho}$  | 6.05  | $\Gamma(\rho^0 \to \pi^+\pi^-)$ |
| $g_{NN\rho}$      | 2.52  | $\Gamma(\rho^0 \to e^+e^-)$ |
| $\kappa_{NN\rho}$ | 3.71  | VMD |
| $g_{NN\omega}$    | 3.4 $g_{NN\rho}$ | $\Gamma(\rho^0 \to e^+e^-)/\Gamma(\omega \to e^+e^-)$ |
| $\kappa_{NN\omega}$ | -0.12 | VMD |
| $f_{NN\pi}/4\pi$  | 0.075 | nucleon-nucleon data |

| $F/(F+D)$ Ratios   | Value | Description |
|--------------------|-------|-------------|
| $\alpha_{PV}$      | 0.4   | semileptonic hyperon decays |
| $\alpha_{\pi}$     | 1.0   | universality |
| $\alpha_{\eta}$    | 0.28  | relativistic SU(6) |

| Cutoff Mass | Value |
|-------------|-------|
| $\Lambda$  | 2.42 GeV |

Table 1: Parameters of the model. The basic coupling constants and the $F/(F+D)$ ratios are fixed: the sources of the values used are given in the right-hand column. Also note that we assume ideal $\phi-\omega$ mixing, and take the physical $\eta$ to be the pure octet state.

The $K^-p$ cross sections are shown in Figure 1, where we find good agreement with the experimental data. The $Y\eta$ channels give non-negligible contributions, even though the energies we consider are well below the $\Lambda\eta$ and $\Sigma\eta$ thresholds.

Figure 1: The first two columns show the cross sections for the six final states. The third and fourth columns show the elastic and charge-exchange differential cross sections, respectively, compared to the experimental data of Mast et al. [8]. The solid lines correspond to the full model, while the $Y\eta$ channels were omitted in the calculations giving the dashed lines.

We now turn to the threshold behavior. For the $K^-p$ scattering length we find $a_{K^-p} = -0.54 + i1.2$ fm, which is consistent with the experimental value of $a_{K^-p} = -0.78 \pm 0.18 + i(0.49 \pm 0.37)$ fm obtained in a kaonic hydrogen experiment [9]. The relative strengths of the different channels at threshold are tightly constrained by the threshold branching ratios $\gamma$, $R_c$, and $R_n$, which are given
Figure 2: The $\Sigma\pi$ mass distribution compared to the experimental data from Hemingway [13]. The solid line is the result of the full calculation, while the dashed line shows the effect of omitting the $Y\eta$ channels.

in Refs. [10, 11] as

$$\gamma = \frac{\Gamma(K^-p \to \Sigma^-\pi^+)}{\Gamma(K^-p \to \Sigma^+\pi^-)} = 2.36 \pm 0.04, \quad R_c = \frac{\Gamma(K^-p \to \text{charged})}{\Gamma(K^-p \to \text{all})} = 0.664 \pm 0.011,$$

$$R_n = \frac{\Gamma(K^-p \to \Lambda\pi^0)}{\Gamma(K^-p \to \text{neutral})} = 0.189 \pm 0.015.$$  

Our values for the branching ratios are $\gamma = 2.14$, $R_c = 0.651$, and $R_n = 0.132$, which are in reasonable agreement with the experimental values. As found previously by Oset and Ramos [12], the $Y\eta$ channels (in particular $\Lambda\eta$) give important contributions to the branching ratios. When the $Y\eta$ channels are neglected, $R_c$ and $R_n$ are essentially unchanged, but $\gamma$ reduces to 1.38.

Finally we consider the energy region below the $\bar{K}N$ threshold, where the $I = 0$ $\Sigma\pi \to \Sigma\pi$ amplitude exhibits the $\Lambda(1405)$ resonance. In Figure 2 we compare the $\Sigma\pi$ mass spectrum of the $\Lambda(1405)$ obtained in our model with experiment, and find good agreement.

To summarize, we have solved the multi-channel Bethe-Salpeter equation for the $\bar{K}N$ system by means of a Wick rotation, and obtained good agreement with the low-energy $\bar{K}N$ experimental data by adjusting a single cutoff mass. Future work will include extending the model to higher energies, and searching for evidence of additional dynamically-generated $S$- and $P$-wave hyperon resonances.

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