Traffic dynamics in scale-free networks with limited packet-delivering capacity

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We propose a limited packet-delivering capacity model for traffic dynamics in scale-free networks. In this model, the total node’s packet-delivering capacity is fixed, and the allocation of packet-delivering capacity on node \( i \) is proportional to \( k_i \phi \), where \( k_i \) is the degree of node \( i \) and \( \phi \) is an adjustable parameter. We have applied this model on the shortest path routing strategy as well as the local routing strategy, and found that there exists an optimal value of parameter \( \phi \) leading to the maximal network capacity under both routing strategies. We provide some explanations for the emergence of optimal \( \phi \).

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I. INTRODUCTION

Since the discovery of small-world effect by Watts and Strogatz [1] and scale-free property by Barabási and Albert [2], the structure and dynamics of complex networks have attracted growing interest and attention from the physics community [3–7]. Due to the increasing importance of large communication networks such as the Internet and WWW, information traffic on complex networks has drawn more and more attention [8–32]. The ultimate goal of studying these large communication networks is to control the traffic congestion and improve the efficiency of information transportation.

Researchers have proposed some models to mimic the traffic on complex networks by introducing packets generating rate \( R \) as well as randomly selected sources and destinations of each packet [9–13]. In these models, the capacity of networks is measured by a critical generating rate \( R_c \). At this critical rate, a continuous phase transition from free flow state to congested state occurs. In the free-flow state, the numbers of created and delivered packets are balanced, leading to a steady state. While in the jammed state, the number of accumulated packets increases with time due to the limited delivering capacity or finite queue length of each node. It has been found that both network structure and packet routing strategy can influence the capacity and efficiency of information transportation.

The node packet-delivering capacity, that is, the number of packets a node can forward to other nodes in each time step, is assumed to be a constant or proportional to node’s degree in most of previous works. Obviously more packet-delivering capacity can help to alleviate traffic congestion, but the extending of packet-delivering capacity will bring economic and technique pressure. So the question arises: how to rationally allocate the limited packet-delivering capacity onto nodes in order to maximize the networks capacity? In the following, we will explore this question in scale-free networks.

The paper is organized as follows: In Section 2, the traffic model is introduced. The simulation results are presented and discussed in Section 3. The conclusion is given in Section 4.

II. THE MODEL

Recent studies indicate that many communication networks such as the Internet and WWW are heterogeneous with degree distribution following the power-law distribution \( P(k) \sim k^{-\gamma} \). In this paper, we use the well-known Barabási-Albert (BA) scale-free network model [2] as the physical infrastructure to study information traffic flow. The BA model can be constructed as follows: starting from \( m_0 \) fully connected nodes, a new node with \( m \) edges is added to the existing graph at each time step according to preferential attachment, i.e., the probability \( \Pi_i \) of being connected to the existing node \( i \) is proportional to the degree \( k_i \).

Once the network is generated, it remains fixed, and the traffic dynamics is modeled on top of it as follows: at each time step, there are \( \hat{R} \) packets generated in the system, with randomly chosen sources and destinations. All the nodes act as both hosts and routers and node \( i \) can deliver at most \( C_i \) packets per time step towards their destinations. Once a packet arrives at its destination,
it will be removed from the system. The queue length of each node is assumed to be unlimited and the FIFO (first in first out) discipline is applied at each queue [9,10].

Packets can be delivered according to different routing strategies. In this paper, we considers the network traffic in the cases of both the shortest path and local routing strategy. The local routing strategy [17] can be described as follows. Each node performs a local search among its neighbors. If the packet’s destination is found within the searched area, i.e., among the node’s immediate neighbors, it is delivered directly to its target. Otherwise, it is forwarded to a neighbor node \( i \), according to the probability:

\[
\Pi_i = \frac{k_i^\alpha}{\sum_j k_j^\alpha},
\]

where the sum runs over the neighbors (searched area) of the searching node, \( k_i \) is the degree of node \( i \) and \( \alpha \) is an adjustable parameter. The average packet-delivering capacity of the network is:

\[
\langle C \rangle = \frac{1}{N} \sum_{i=1}^{N} C_i.
\]

Based on economic and technique considerations, it’s significative to investigate how to allocate packet-delivering capacity onto nodes when \( \langle C \rangle \) is fixed. Since scale-free network is heterogeneous, packet-delivering capacity can be allocated in the form of:

\[
C_i = N\langle C \rangle \frac{k_i^\phi}{\sum_{j=1}^{N} k_j^\phi},
\]

where \( \phi \) is an adjustable parameter. For \( \phi > 0 \) (\( \phi < 0 \)), nodes with higher (smaller) degrees have larger packet-delivering capacity. When \( \phi = 0 \), all nodes have the same packet-delivering capacity. Noting that \( C_i \) may be an integer plus a fractional part, the fractional part is implemented as the probability of delivering additional packets in a time step.

### III. SIMULATION RESULTS

In order to characterize the network capacity, we use the order parameter presented in Ref. [8]:

\[
\eta(R) = \lim_{t \to \infty} \frac{1}{R} \frac{\langle \Delta N_p \rangle}{\Delta t},
\]

where \( \Delta N_p = N_p(t + \Delta t) - N_p(t) \), \( \langle \cdot \cdot \cdot \rangle \) indicates the average over time windows of width \( \Delta t \), and \( N_p(t) \) represents the number of data packets within the network at time \( t \). With increasing packet generation rate \( R \), there will be a critical value of \( R_c \) that characterizes the traffic phase transition from free flow to a congested state. When \( R < R_c \), \( \langle \Delta N_p \rangle = 0 \) and \( \eta(R) = 0 \), corresponding to the case of free-flow state. However, for \( R > R_c \), \( \eta(R) \) is a constant larger than zero, the packets will continuously pile up within the network and the system will collapse ultimately. Therefore \( R_c \) is the maximal generating rate under which the system can maintain its normal and efficient functioning. Thus the overall capacity of the system can be measured by \( R_c \).

Figure 1 reports the order parameter \( \eta \) versus generating rate \( R \) for different parameter \( \phi \) under the shortest path root strategy. One can see that, for all different \( \phi \), \( \eta \) is approximately zero when \( R \) is small; it suddenly increases when \( R \) is larger than the critical point \( R_c \). It is clear to find that the capacity of the system is not the same for different \( \phi \).

Figure 2 shows \( R_c \) versus \( \phi \) for different \( \langle C \rangle \) under the shortest path routing strategy. Interestingly, we find \( R_c \) is not a monotonic function of \( \phi \). There exists an optimal value of \( \phi \) (positive) corresponding to the largest \( R_c \), which means neither the uniform allocation nor the extremely uneven distribution can maximize network capacity. The emergence of optimal \( \phi \) can be explained by betweenness centrality (BC) distributions in scale-free network [33–35]. The BC of a node \( v \) is defined as:

\[
g(v) = \sum_{s \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}},
\]

where \( \sigma_{st} \) is the number of shortest paths going from \( s \) to \( t \) and \( \sigma_{st}(v) \) is the number of shortest paths going from \( s \) to \( t \) and passing through \( v \). BC gives an estimate of the traffic load on nodes when packets are forwarded following their shortest paths. For scale-free networks it has been shown that relationship between betweenness centrality and degree obeys power-law form: \( g(k) \sim k^\mu \), and large-degree nodes endure much heavier traffic load than that of small-degree nodes. Figure 3 shows that the exponent \( \mu = 1.33 \) when the network parameters are \( m_0 = m = 3 \), \( N = 1000 \). Interestingly, we find this value of exponent is approximately equal to the optimal \( \phi_{opt} = 1.3 \) observed in Fig. 2.
FIG. 1: The order parameter $\eta$ versus $R$ for different $\phi$ under the shortest path root strategy. Average packet-delivering capacity of the network is $\langle C \rangle = 3$ and the network parameters are $N = 1000$, $m_0 = m = 3$.

FIG. 2: The critical $R_c$ versus $\phi$ for different $\langle C \rangle$ under the shortest path routing strategy. The network parameters are $N = 1000$, $m_0 = m = 3$.

FIG. 3: Log-Log plot of betweenness centrality $g(k)$ versus degree $k$. The network parameters are $N = 1000$, $m_0 = m = 3$. The fitted line has a slope $\mu = 1.33$. 
FIG. 4: Evolution of queue length $n(k)$ for different degree $k$ under the shortest path routing strategy. $N = 1000$, $m_0 = m = 3$, $\langle C \rangle = 3$. The smallest degree $k = 3$ and the largest degree $k = 95$ in the BA network. (a) $R = 25 > R_c = 18$ for $\phi = 0.0$ and (b) $R = 100 > R_c = 47$ for $\phi = 2.5$.

FIG. 5: The critical $R_c$ versus $\phi$ for different $\langle C \rangle$ under the local routing strategy ($\alpha = 0$). The network parameters are $N = 1000$, $m_0 = m = 4$.

To understand why $\phi_{\text{opt}} = \mu$ results in the maximal network capacity under the shortest path routing strategy, we investigate the queue length of a node $n(k)$ as a function of its degree $k$ in the congested state ($R > R_c$). The queue length of a node is defined as the number of packets in the queue of that node. For $\phi$ is small, i.e., $\phi = 0$, large-degree nodes do not get enough packet-delivering capacity while small-degree nodes have redundant packet-delivering capacity which exceeds their actual load. As shown in Fig. 4(a), the queue length of large-degree nodes becomes longer and longer while at the same time small-degree nodes almost have no packets on their queue. Contrarily, if $\phi$ is very large, i.e., $\phi = 2.5$, most of packet-delivering capacity is allocated to a few large-degree nodes and many small-degree nodes have too little packet-delivering capacity to fully dispose the load on them. As a result, packets continuously pile up on small-degree nodes (see Fig. 4(b)). In order to make full use of limited packet-delivering capacity and avoid congestion on a few nodes, the load distribution should be consistent with the packet-delivering capacity distribution, that is, $\phi_{\text{opt}} = \mu$. This average effect results in the maximal network capacity.

Next we investigate the behavior of $R_c$ versus $\phi$ for different $\langle C \rangle$ under the local routing strategy. As shown in Fig. 5, there also exists an optimal value of $\phi$ corresponding to the largest $R_c$. For $\alpha = 0$, $\phi_{\text{opt}} = 1$. The optimal value of $\phi$ corresponding to different $\alpha$ is shown in Fig. 6(a). It’s found that $\phi_{\text{opt}} \approx 1 + \alpha$ for the local routing strategy. According to the analysis in Ref. [17], the relationship between the queue length and degree is a power-law form: $n(k) \sim k^{1+\alpha}$ in the free flow state. To maximize the network capacity, the relationship between the packet-delivering capacity and degree also obeys the same power-law form. Furthermore, we study the maximum $R_c$ as a function of $\alpha$ (Fig. 6(b)). One can find that there also exists nonmonotonous
FIG. 6: (a) The optimal value of $\phi$ for different $\alpha$ under the local routing strategy. The line is the theoretical prediction. (b) The maximum $R_c$ as a function of $\alpha$. The network parameters are $N = 1000$, $m_0 = m = 4$. The average packet-delivering capacity of the network $\langle C \rangle = 8$. Behavior with a peak at about $\alpha = 0.2$.

IV. CONCLUSION

In conclusion, we have investigated how to rationally allocate packet-delivering capacity onto nodes in the BA scale-free network when the sum of all nodes’ packet-delivering capacity is fixed. A tunable parameter is introduced, governing node’s packet-delivering capacity based on its degree. Interestingly, we find there exists an optimal value of parameter $\phi$ leading to the maximal network capacity. We provide some explanations for the emergence of optimal $\phi$ by investigating betweenness centrality distribution in the shortest path routing strategy and the queue length distribution in the local routing strategy. Our work may be helpful for designing realistic communication network.

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