The computation of first order moments on junction trees

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Abstract

We review some existing methods for the computation of first order moments on junction trees using Shafer-Shenoy algorithm. First, we consider the problem of first order moments computation as vertices problem in junction trees. In this way, the problem is solved using the memory space of an order of the junction tree edge-set cardinality. After that, we consider two algorithms, Lauritzen-Nilsson algorithm, and Mauá et al. algorithm, which computes the first order moments as the normalization problem in junction tree, using the memory space of an order of the junction tree leaf-set cardinality.

1 Shafer-Shenoy algorithm

1.1 Potentials and operations

Let $X = (X_u : u \in U)$ be a finite collection of discrete random variables. Let $\Omega_u$ denotes the set of possible values that $X_u$ can take. For $A \subseteq U$ we write $\Omega_A$ for the Cartesian product $\times_{u \in A} \Omega_u$ and write $X_A$ for $\{X_u : u \in A\}$ \footnote{We implicitly assume some natural ordering in sets}. The elements of $\Omega_A$, $A \subseteq U$ are denoted $x_A$ and called the configuration. We adopt the convention that $\Omega_\emptyset$ consists of a single element $x_\emptyset = \diamond$, i.e. $\Omega_\emptyset = \{\diamond\}$.

Let $\Pi_U$ be a set of functions on $\pi_A : \Omega_A \to \mathcal{K}$, where $A \subseteq U$, i.e. $\Pi_U = \{\pi_A : \Omega_A \to \mathcal{K} | A \subseteq U\}$. In the following text, the functions from $\Pi_U$ are called potentials. Let $\otimes$ be a binary operation on $\mathcal{K}$ called combination and let $\downarrow$ denotes the external operation called marginalization which to every $\pi_A : \Omega_A \to \mathcal{K}$ associates $\pi_A^{\downarrow B} : \Omega_{A\cap B} \to \mathcal{K}$, where $A, B \in U$.

We assume that the following Shafer-Shenoy axioms hold for combination and marginalization.

1.2 Shafer-Shenoy axioms

Axiom 1 (Commutativity and Associativity) Let $\pi_A, \pi_B$ and $\pi_C$ be potentials. Then

\[ \pi_A \otimes \pi_B = \pi_B \otimes \pi_A \quad \text{and} \quad \pi_A \otimes (\pi_B \otimes \pi_C) = (\pi_A \otimes \pi_B) \otimes \pi_C. \] (1)

Axiom 1 allows us to use the notation $\pi_A \otimes \pi_B \otimes \pi_C$.

Axiom 2 (Consonance) Let $\pi_A$ be a potential on $A$, and let $A \supseteq B \supseteq C$. Then

\[ (\pi_A^{\downarrow B})^{\downarrow C} = \pi_A^{\downarrow C}. \] (2)
Axiom 3 (Distributivity) Let $\pi_A$ and $\pi_B$ be potentials on $A$ and $B$, respectively. Then

$$(\pi_A \otimes \pi_B)^{\downarrow A} = \pi_A \otimes \pi_B^{\downarrow A}. \quad (3)$$

1.3 Junction Tree

The joint potential $\pi_U : \Omega_U \to \mathcal{K}$ is said factorize on $\mathcal{T}$ with respect to $\otimes$ if there exists potentials $\pi_V : \Omega_V \to \mathcal{K}$ for $V \in \mathcal{V}$, so that we can write $\pi$ as

$$\pi_U = \bigotimes_{V \in \mathcal{V}} \pi_V. \quad (4)$$

In this paper we consider the joint potentials which can be represented with junction tree which is defined as follows.

Definition 1 Let $\mathcal{V} = \{V_1, V_2, \ldots, V_n\}$ be a collection of subsets of $U$ (set of variable indices) and $\mathcal{T}$ a tree with $\mathcal{V}$ as its node set (corresponds to a set of local domains). Then $\mathcal{T}$ is said to be a junction tree if any intersection $V_i \cap V_j$ of a pair $V_i, V_j$ of sets in $\mathcal{V}$ is contained in every node on the unique path in $\mathcal{T}$ between $V_i$ and $V_j$. Equivalently, for any $u \in U$, the set of subsets in $\mathcal{V}$ containing $u$ induces a connected subtree of $\mathcal{T}$.

The set of all neighbors of $A$ is denoted $ne(A)$. We omit the parentheses from the notation when it is not prone to misunderstanding. Hence, $ne(A) \setminus B$ stands for the set of all neighbors of $A$ without $B$, $ne(A) \setminus B, C$ for the set of all neighbors of $A$ without $B$ and $C$ and so on. $V_i \sim V_{i+1}$ denotes that $V_i$ and $V_{i+1}$ are neighbors.

A junction tree is usually drawn with sets $V_i$ as node labels. In the following text the node will be identified with the label. The general procedure for the junction tree building can be found in [1] and [2]. An example of the junction tree which corresponds to chain factorization,

$$\pi_U = \bigotimes_{i=1}^{n} \pi_{V_i}, \quad V_i \sim V_{i+1} \text{ for } i = 1, \ldots, n-1, \quad (5)$$

is given in Fig. 1.

Fig. 1. The junction tree for chain factorization $\pi_U = \bigotimes_{i=1}^{n} \pi_{V_i}, \ V_i \sim V_{i+1}$.

1.4 Problems

The junction tree enables the solution of three important problems:

1. The single vertex problem at node $A$ is defined as the computation of the potential $\psi_A : \Omega_A \to \mathcal{K}$, defined by

$$\psi_A = \pi^{\downarrow A}_U = \left( \bigotimes_{V \in \mathcal{V}} \pi_V(x_V) \right)^{\downarrow A}, \quad (6)$$
2. The **all vertices problem** is defined as the computation of the functions $\psi_A$ for all $A \in \mathcal{V}$;

3. The **normalization problem** is the marginalization of the joint potential $\pi_U$ to the empty set $\emptyset$. Using the consonance of the marginalization $\mathfrak{2}$, it can straightforwardly be solved by the solution of the single vertex problem in arbitrary node $A$:

$$
\pi_U^{\emptyset} = (\pi_U^{\{A\}})^{\emptyset} = \psi_A^{\emptyset}
$$

### 1.5 Local computation algorithm

These problems can efficiently be solved with the Shafer-Shenoy local computation algorithm (LCA). The algorithm can be described as passing the messages over the edges and processing the messages in the nodes of the junction tree.

Messages are passed between the vertexes from $\mathcal{V}$ via mailboxes. All mailboxes are initialized as empty. When a message has been placed in a mailbox, the box is full. A node $A$ in the junction tree is allowed to send a message to its neighbor $B$ if it has not done so before and if all $A$-incoming mailboxes are full except possibly the one which is for $B$-outgoing messages. So, initially only leaves (nodes which have only one neighbor) of the junction tree are allowed to send messages. But as the message passing proceeds, other nodes will have their turn and eventually all mailboxes will be full, i.e., exactly two messages will have been passed along each branch of the junction tree.

The message from $A$ to $B$ is a function $\pi_{A\rightarrow B} : \Omega_{A\cap B} \rightarrow \mathbb{K}$. The passage of a message $\pi_{A\rightarrow B}$ from node $A$ to node $B$ is performed by absorption. Absorption from clique $A$ to clique $B$ involves eliminating the variables $A \setminus B$ from the potentials associated with $A$ and its neighbors except $B$. The structure of the message $\pi_{A\rightarrow B}$ is given by

$$
\pi_{A\rightarrow B} = (\pi_A \otimes \bigotimes_{C \in \text{ne}(A) \setminus B} \pi_{C\rightarrow A})^{\{B\}}.
$$

where $\pi_{C\rightarrow A}$ is the message passed from $C$ to $A$. Since the leaves has only one neighbor, the product on the righthand side is empty and the message can initially be computed as

$$
\pi_{A\rightarrow B} = \pi_A^{\{B\}}.
$$

Suppose we start with a joint potential $\pi_U$ on a junction tree $\mathcal{T}$, and pass messages towards a root clique $R$ as described above. When $R$ has received a message from each of its neighbors, the combination of all messages with its own potential is equal to a decomposition of the $R$-marginal of $\pi_U$.

$$
\pi_U^{\{R\}} = \bigotimes_{V \in \mathcal{V}} \pi_V^{\{R\}} = \pi_R \otimes \bigotimes_{V \in \text{ne}(R)} \pi_{V\rightarrow R},
$$

where $\mathcal{V}$ is vertex-set in $\mathcal{T}$.

Hence, if we want to solve the single vertex problem at node $A$, we need to compute all messages incoming to $A$, while for the all vertices problem we need the messages between all pairs of nodes in the tree.

For the single vertex problem, the algorithm starts at the leaves which send the messages to their neighbors. A node sends a message to a neighbor, once it has received messages from each of its other neighbors. The node $A$ never sends a message. Thus, each message is sent only once
until \( A \) has received the messages from all the neighbors at which point the required marginal
is computed and the algorithm terminates with the total number of computed messages equal
to the number of edges of the tree. Once we have solved the single vertex problem in the node
\( A \), the normalization problem can be solved with (17).

The first part of the algorithm for all vertices problem is similar to the single vertex case. The
messages are sent from leaves toward the tree until a node \( C \) has received the messages
from all the neighbors. After that the messages are sent from \( C \) to the leaves. The algorithm
stops when all leaves receive messages. The total number of computed messages is equal two
times the number of edges in the tree (for any two nodes \( A \) and \( B \) we send the messages
\( \pi_{A \rightarrow B} \) and \( \pi_{B \rightarrow A} \)).

2 First order moments

2.1 Operations on the set of functions

For real-valued functions \( \phi_A : \Omega_A \rightarrow \mathbb{R} \) and \( \phi_B : \Omega_B \rightarrow \mathbb{R} \) the sum, \( \phi_A + \phi_B : \Omega_{A \cup B} \rightarrow \mathbb{R} \) and the product, \( \phi_A \cdot \phi_B : \Omega_{A \cup B} \rightarrow \mathbb{R} \), are respectively defined with:

\[
(\phi_A + \phi_B)(x_{A \cup B}) = \phi_A(x_A) + \phi_B(x_B) \tag{11}
\]

\[
(\phi_A \cdot \phi_B)(x_{A \cup B}) = \phi_A(x_A) \cdot \phi_B(x_B) \tag{12}
\]

for all \( x_A \in \Omega_A \) and \( x_B \in \Omega_B \). The product \( \phi_A \cdot \phi_B \) is, usually, shortly denoted with \( \phi_A \phi_B \).

We define sum-marginal operator \( \sum_{x_C \setminus A} \) for \( A \subseteq C \), which to every real-valued function \( \phi_C : \Omega_C \rightarrow \mathbb{R} \) associates the function \( \sum_{x_C \setminus A} \phi_C : \Omega_A \rightarrow \mathbb{R} \), defined with

\[
\left( \sum_{x_C \setminus A} \phi_C \right)(x_A) = \sum_{x_C \setminus A \in \Omega_{x_C \setminus A}} \phi_C(x_C) \tag{13}
\]

and the marginalization is defined with

\[
\phi_C^A = \sum_{x_{C \setminus A}} \phi_C. \tag{14}
\]

2.2 Definition of first order moments

The joint probability non-negative function of random variable \( X_U \), \( p : \Omega_U \rightarrow \mathbb{R} \) is said to factorize multiplicatively on \( T \) if there exists non-negative real functions \( p_C : \Omega_C \rightarrow \mathbb{R} \) for \( C \in \mathcal{V} \), so that we can write \( p(x_U) \) as

\[
p_U = \prod_{C \in \mathcal{V}} p_C, \tag{15}
\]

Similarly, the function \( h : \Omega_U \rightarrow \mathbb{R} \) is said to factorize additively on \( T \) if there exists real functions \( h_C : \Omega_C \rightarrow \mathbb{R} \) for \( (C \in \mathcal{V}) \), so that we can write \( h(x_U) \) as

\[
h_U = \sum_{C \in \mathcal{V}} h_C \tag{16}
\]
The first order moment potential, \( m_C : \Omega_C \rightarrow \mathbb{R} \), is defined with
\[
m_C = \sum_{x_{U \setminus C}} p_U \cdot h_U.
\] (17)

In the case \( C = \emptyset \), the first order moment potential is simply denoted \( m \),
\[
m = \sum_{x_U} p_U \cdot h_U.
\] (18)

and called the first order moment.

**Example 1** The first order moment potential may be useful for expressing the conditional expectation
\[
E[h_U(X_U)|x_C] = \sum_{x_{U \setminus C}} p(X_U|x_C)h(X_{U \setminus C}, x_C)
\] (19)
for \( C \in \mathcal{V} \). After usage of
\[
p(X_{U \setminus C}|x_C) = \frac{p(X_{U \setminus C}, x_C)}{p(x_C)} = \frac{p(X_{U \setminus C}, x_C)}{\sum_{x_{U \setminus C}} p(X_{U \setminus C}, x_C)}
\] (20)
we have
\[
E[h_U(X_U)|x_C] = \frac{\sum_{x_{U \setminus C}} p(X_{U \setminus C}, x_C)h(X_{U \setminus C}, x_C)}{\sum_{x_{U \setminus C}} p(X_{U \setminus C}, x_C)} = \frac{m_C(x_C)}{p_U^{\downarrow C}(x_C)}.
\] (21)

Consequently, unconditioned expectation equals the first order moment
\[
E[h(X_U)] = \sum_{x_U} p(x_U)h(x_U) = m.
\] (22)

### 2.3 The problem of first order moments computation as all vertices problem

The computation of (18) by enumerating all configurations would require an exponential number of operations with respect to the cardinality of \( \Omega_U \). However, the computational complexity can be reduced using the local computation algorithm which exploits structure of functions given with factorizations (15) and (16). In this case, the marginal values \( p_U^{\downarrow C} \) are computed for all \( C \in \mathcal{V} \) using the local computation over the set of real-valued functions. After that the moment is computed according to equality
\[
m_C = \sum_{C \in \mathcal{V}} \sum_{x_C} h_C p_U^{\downarrow C},
\] (23)
which follows from
\[
m_C = \sum_{x_U} p_U h_U = \sum_{x_U} p_U \sum_{C \in \mathcal{V}} h_C = \sum_{C \in \mathcal{V}} \sum_{x_U} p_U h_C = \sum_{C \in \mathcal{V}} \sum_{x_C} h_C \sum_{x_{U \setminus C}} p_U = \sum_{C \in \mathcal{V}} \sum_{x_C} h_C p_U^{\downarrow C}.
\] (24)

This method requires the storing of marginal values \( p_U^{\downarrow C} \) for all \( C \in \mathcal{V} \), which unnecessary increases the memory complexity. Instead, we can use the local computation algorithms by Lauritzen and Nilsson [3] and Mauá et al. [4], which find the moment as the solution of the normalization problem. In the following section, we consider these two algorithms.
3 First order moments computation using order pair potential algorithms

3.1 Order pair potentials

In our local computation algorithms we represent the quantitative elements through entities called potentials. Each such potential has two parts, as detailed below.

**Definition 2 (Potential)** A potential on \( C \subseteq U \) is a pair \( \pi_C = (p_C, h_C) \) of real-valued functions on \( \Omega_C \), where \( p_C \) is nonnegative.

Thus, a potential consists of two parts - \( p \)-part and \( h \)-part. We call a potential \( \pi_C \) vacuous, if \( \pi_C = (1, 0) \). We identify two potentials \( \pi_C^{(1)} = (p_C^{(1)}, h_C^{(1)}) \) and \( \pi_C^{(2)} = (p_C^{(2)}, h_C^{(2)}) \) on \( C \) and write \( \pi_C^{(1)} = \pi_C^{(2)} \) if \( p_C^{(1)} = p_C^{(2)} \) and \( h_C^{(1)}(x_C) = h_C^{(2)}(x_C) \) whenever

\[
p_C^{(1)}(x_C) = p_C^{(2)}(x_C) > 0,
\]

i.e., two potentials are considered equal if they have identical probability parts and their utility parts agree almost surely with respect to the probability parts.

To represent and evaluate the decision problem in terms of potentials, we define basic operations of combination and marginalization. There are two possible ways to define the operations.

1. Lauritzen-Nilsson algorithm \[3\]
2. Mauá et al. algorithm \[4\]

3.2 Lauritzen-Nilsson algorithm

**Definition 3 (Combination)** The combination of two potentials \( \pi_A = (p_A, h_A) \) and \( \pi_B = (p_B, h_B) \) denotes the potential on \( A \cup B \) given by

\[
\pi_A \otimes \pi_B = (p_A \cdot p_B, h_A + h_B).
\]

**Definition 4 (Marginalization)** The marginalization of \( \pi_C = (p_C, h_C) \) onto \( A \subseteq C \in \mathcal{V} \) is defined by

\[
\pi_C^A = \left( \sum_{\mathcal{X}_{C\setminus A}} p_C, \frac{\sum_{x \in \mathcal{X}_{C\setminus A}} p_C \cdot h_C}{\sum_{x \in \mathcal{X}_{C\setminus A}} p_C} \right)
\]

Here we have used the convention that \( 0/0 \) which will be used throughout.

As shown in Lauritzen and Nilsson \[3\], the operations of combination and marginalization satisfy the properties of Shenoy and Shafer axioms \[5\], and three structured factorizations can be marginalized using the Shafer-Shenoy algorithm.

If the operations are defined in this way and the potentials are set to,

\[
\phi_C = (p_C, h_C)
\]
and the factorizations (15) and (16) hold, then
\[
\pi_U = \bigotimes_{C \in \mathcal{V}} \pi_C = \bigotimes_{C \in \mathcal{V}} (p_C, h_C) = \left( \prod_{C \in \mathcal{V}} p_C, \sum_{C \in \mathcal{V}} h_C \right) = (p_U, h_U).
\]

Accordingly, we have
\[
\pi^\downarrow = \left( \sum_{x_U} p_U, \frac{\sum_{x_U} p_U h_U}{\sum_{x_U} p_U} \right) = (1, m),
\]

where we have used probability condition \(\sum_{x_U} p_U = 1\). Hence, the first order moment potential can be computed using the Shafer-Shenoy local computation algorithm, where the combination and marginalization are defined with (26)-(27). The messages have the form:
\[
\pi_{A \rightarrow B} = (\pi^{(p)}_{A \rightarrow B}, \pi^{(h)}_{A \rightarrow B})
\]

where the \(p\) and \(h\) part are given with
\[
\pi^{(p)}_{A \rightarrow B} = \sum_{x_{A \setminus B}} p_A \prod_{C \in \text{ne}(A) \setminus B} \pi^{(p)}_{C \rightarrow A}
\]
\[
\pi^{(h)}_{A \rightarrow B} = \sum_{x_{A \setminus B}} p_A \prod_{C \in \text{ne}(A) \setminus B} \pi^{(p)}_{C \rightarrow A} \cdot \left( h_A + \sum_{C \in \text{ne}(A) \setminus B} \pi^{(h)}_{C \rightarrow A} \right)
\]

which follows from equations (8), (26), (27), (28) and (43).

**Example 2** Let \(\pi_U\) has the chain factorization
\[
\pi_U = \bigotimes_{i=1}^{n} \pi_{V_i}, \quad V_i \sim V_{i+1} \text{ for } i = 1, \ldots, n - 1,
\]

and let \(\pi_{i \rightarrow (i+1)}\) stands as shorthand for the message \(\pi_{V_i \rightarrow V_{i+1}}\). According to chain factorization \(\text{ne}(V_i) \setminus V_{i+1} = \{V_{i-1}\}\), \(p\) and \(h\) parts of the message reduce to:
\[
\pi^{(p)}_{i \rightarrow (i+1)} = \sum_{x_{V_i \setminus V_{i+1}}} p_{V_i} \pi^{(p)}_{(i-1)\rightarrow i}
\]
\[
\pi^{(h)}_{i \rightarrow (i+1)} = \sum_{x_{V_i \setminus V_{i+1}}} p_{V_i} \pi^{(p)}_{(i-1)\rightarrow i} \cdot \left( h_{V_i} + \pi^{(h)}_{(i-1)\rightarrow i} \right)
\]

### 3.3 Mauá et al. algorithm

**Definition 5 (Combination)** Let \(\pi_A = (p_A, h_A)\) and \(\pi_B = (p_B, h_B)\) be two potentials on \(A\) and \(B\), respectively. The combination \(\pi_A \otimes \pi_B\) of \(\pi_A\) and \(\pi_B\) is the potential on \(A \cup B\) given by
\[
\pi_A \otimes \pi_B = (p_A p_B, h_A p_B + p_A h_B).
\]
Definition 6 (Marginalization) Let \( \pi_C = (p_C, h_C) \) be a potential on \( C \), and let \( A \subseteq C \). The marginalization \( \pi_C^A \) of \( \pi_C \) onto \( A \) is the potential on \( A \) given by

\[
\pi_C^A = \left( \sum_{x_C \in A} p_C, \sum_{x_C \in C \setminus A} h_C \right). \tag{38}
\]

The following lemma can be proven by induction \[6\].

Lemma 1 Let \( N \subseteq V \) and \( \pi_A = (\pi_A^{(p)}, \pi_A^{(h)}) \) be order pair potentials for \( A \in N \). Then,

\[
\bigotimes_{C \in N} \pi_C = \bigotimes_{C \in N} (\pi_C^{(p)}, \pi_C^{(h)}) = \left( \prod_{A \in N} \pi_A^{(p)}, \sum_{A \in N} \pi_A^{(h)} \prod_{B \in N \setminus A} \pi_B^{(p)} \right). \tag{39}
\]

If the operations are defined in this way and the potentials are set to,

\[
\pi_C = (p_C, p_C h_C) \tag{40}
\]

and the factorizations \[15\] and \[16\] hold, then

\[
\pi_U = \bigotimes_{C \in V} \pi_C = \left( \prod_{A \in V} p_A, \prod_{A \in V} p_A \sum_{B \in V} h_B \right) = (p_U, p_U h_U). \tag{41}
\]

Accordingly, we have

\[
\pi_U^\emptyset = \left( \sum_{x_U} p_U, \sum_{x_U} p_U h_U \right) = (1, m), \tag{42}
\]

where we have used probability condition \( \sum_{x_U} p_U = 1 \). Again, the first order moment potential can be computed using the Shafer-Shenoy local computation algorithm, where the combination and marginalization are defined with \[37\] and \[38\]. Like in the Lauritzen-Nilsson algorithm, the messages have the form:

\[
\pi_{A \rightarrow B} = (\pi_{A \rightarrow B}^{(p)}, \pi_{A \rightarrow B}^{(h)}) \tag{43}
\]

but now, according to \[39\] the \( p \)-part and the \( h \)-part of the messages are given with

\[
\pi_{A \rightarrow B}^{(p)} = \sum_{x_{A \setminus B}} p_A \prod_{C \in \text{ne}(A) \setminus B} \pi_{C \rightarrow A}^{(p)}. \tag{44}
\]

\[
\pi_{A \rightarrow B}^{(h)} = \sum_{x_{A \setminus B}} p_A \left( \prod_{C \in \text{ne}(A) \setminus B} \pi_{C \rightarrow A}^{(p)} h_A + \sum_{C \in \text{ne}(A) \setminus B} \pi_{C \rightarrow A}^{(h)} \prod_{D \in \text{ne}(A) \setminus B,C} \pi_{D \rightarrow A}^{(p)} \right). \tag{45}
\]

Note that the \( p \)-parts of the Lauritzen-Nilsson algorithm and the Mauá et al. algorithm are the same. For the trees with large average degree, the \( h \)-parts of messages are more complex in Mauá et al. algorithm, due to repeated multiplications in products in the equality \[45\]. However, Mauá et al. algorithm is simpler for chains as the following example shows.

Example 3 Let \( \pi_U \) has the chain factorization

\[
\pi_U = \bigotimes_{i=1}^{n} \pi_{V_i}, \quad V_i \sim V_{i+1} \text{ for } i = 1, \ldots, n - 1, \tag{46}
\]
and let \( \pi_{i\rightarrow (i+1)} \) stands as shorthand for the message \( \pi_{V_i \rightarrow V_{i+1}} \). According to chain factorization \( \text{ne}(V_i) \setminus V_{i+1} = \{V_{i-1}\} \), \( p \) and \( h \) parts of the message reduce to:

\[
\begin{align*}
\pi_{i\rightarrow (i+1)}^{(p)} &= \sum_{x_{V_i \setminus V_{i+1}}} p_{V_i} \pi_{(i-1)\rightarrow i}^{(p)}, \\
\pi_{i\rightarrow (i+1)}^{(h)} &= \sum_{x_{V_i \setminus V_{i+1}}} p_{V_i} \cdot (\pi_{(i-1)\rightarrow i}^{(p)} h_{V_i} + \pi_{(i-1)\rightarrow i}^{(h)}).
\end{align*}
\]

(47)

\( (48) \)

4 Conclusion

We reviewed some existing methods for the computation of first order moments on junction trees using Shafer-Shenoy algorithm. First, we consider the problem of first order moments computation as vertices problem in junction trees. In this way, the problem is solved using the memory space of an order of the junction tree edge-set cardinality. After that, we considered two algorithms, Lauritzen-Nilsson algorithm, and Mauá et al. algorithm, which computes the first order moments as the normalization problem in junction tree, using the memory space of an order of the junction tree leaf-set cardinality. It is shown, that for trees, the first of them has simpler formulas in comparison to the second one, while the second one is simpler for chains.

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