B-spline fitting of sunglasses lens contour based on adaptive sampling

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Abstract. To solve the problems of low accuracy and low efficiency of B-spline fitting in sunglasses lens contour cutting, a sampling method based on the characteristic function of a weighted combination of arc length and curvature square root was proposed, and then perform B-spline fitting according to the sampling points to improve the accuracy and efficiency. First, read and identify the sunglasses lens data in the DXF file, and disperse it to obtain a series of coordinate points; combine the defined feature function for uniform sampling, and adjust the parameter weight in the feature function to obtain a more reasonable sampling point for the sunglasses lens contour; Then use the LU decomposition and interpolation method to obtain the B-spline curve through all the sampling points; the maximum absolute error (MAE) and the root-mean-square error (RMSE) are used as the accuracy criteria, by increasing the number of sampling points iteratively until the accuracy is satisfied to construct a reasonable fitting curve.

1. Introduction
In the CNC machining of sunglasses lenses, it is necessary to carry out trajectory fitting planning for its contour. And the fitting process is usually based on the DXF format file generated by the AutoCAD in traditional CNC machining. The software reads the lens profile data composed of multiple arcs stored in the DXF file, and then manually select points are used to obtain control points for B-spline curve fitting, finally, combined with camber parameter of the sunglasses lens to obtain three-dimensional fitting curve. This traditional method requires manual selection of points with technician experience, which has low efficiency and uncontrollable fitting accuracy. For this reason, it is necessary to use the method of adaptive curve sampling [1], so this paper proposes to use the characteristic function weighted by arc length and square root of curvature to select high-quality points for B-spline fitting.

The two most important factors in the curve sampling are the distribution and number of sampling points. It is commonly used in uniform sampling, which determines the distribution of sampling points according to specific parameter quantity, such as parameter, arc length. Uniform parameter sampling and uniform arc length sampling[2], the former is to calculate the points on the curve according to the parameter value, and the latter distributes the points on the curve according to the equal arc length. They are simple and easy to use, so they are the most common. But they all ignore that the actual shape of the curve needs to be taken into account by introducing new sampling variations. A more efficient uniform sampling strategy requires sampling in combination with the shape information of the curve, which is achieved by defining a characteristic function that reflects the change in curvature. A characteristic
function based on the square of curvature and arc length[3], and the other characteristic function based on curvature and arc length[4] are proposed. They considered the complexity of the curve, and the results also showed that the sampling and fitting of the curve can be performed well in the processing of two-dimensional data. For three-dimensional space data, there is also a geometric quantity of torsion. For this reason, Ling Haiya et al. proposed a sampling algorithm based on the inherent geometrical quantities of arc length, curvature and torsion [5], which has a good effect on the sampling and fitting of spatial curves.

The contour of the sunglasses is a spatial curve, and the contour data of the sunglasses stored in the DXF file is a two-dimensional arc data composed of circle center, radius, starting angle, ending angle, and the camber parameter of the lens. To reconstruct the spatial contour of the sunglasses information, disperse the arc data with equal arc length and combine the camber parameter of the lens to obtain three-dimensional data. Discrete three-dimensional points have limitations in the use of torsion. For sunglasses lens data, the larger curvature is often the larger torsion. Therefore, the algorithm in this paper does not use the torsion to construct the characteristic function, but defined a characteristic function of a weighted combination of arc length and the square root of curvature for sampling the lens data, and it can adjust the parameter weights according to the lens data of different shape characteristics to obtain more reasonable sampling data points for B-spline fitting.

2. Data point sampling and B-spline curve fitting

2.1. B Spline curve

A kth-order B-spline curve is defined by

\[ P(t) = \sum_{i=0}^{n} N_{i,k}(t) P_i, t \in [0,1] \]  

(1)

Where \( \{P_i\} \) is the control points, and \( \{N_{i,k}(t)\} \) is the kth-order B-spline basis function defined on the knot vector \( \text{Node} = \{t_i\} i=0,\ldots,n+k \). According to the algorithm defined by De Boor[6], the B-spline basis function defined as

\[
N_{i,k}(t) = \begin{cases} 
1, & u \in [t_i, t_{i+1}] \\
0, & \text{else}
\end{cases}
\]

\[
N_{i,k}(t) = \frac{(t-t_i)N_{i,k-1}(t) + (t_{i+k+1}-t)N_{i+1,k-1}(t)}{t_{i+k+1}-t_i}, \quad t \in [t_i, t_{i+1}]
\]

(2)

Formula (2) agrees that 0/0=0, k is the degree of B-spline, the \( \text{Node} = \{t_i\} \) is a non-decreasing sequence. When the control point \( \{P_i\} \) is given and the node vector is determined, the point on the curve with the parameter \( t_i \) can be defined according to the above recurrence formula, thereby defining the kth B-spline curve.

2.2. Sampling and fitting of data points

The initial lens data are stored in the DXF file in the form of multiple arcs, which is two-dimensional data information. Before the lens cutting trajectory planning, we need to map the data to three-dimensional space and then fit it into a B-spline curve. By extracting the layer information in the DXF file to analyze the lens data, n segments of arc data can be obtained, and then disperse it to obtain the data \( \{P_{i,j}\}_{j=1}^{M} \), and then sample the data \( \{P_{i,j}\}_{j=1}^{N} \), which can vividly describe the shape of the arc curve from the data \( \{P_{i,j}\}_{j=1}^{M} \). The high-quality point sampling method should be where the curvature of the initial curve is larger, the number of samples is denser [7], so the characteristic function \( K(t_i) \) that combines the arc length and the square root of the curvature is used for uniform point sampling, and it is defined as:
\[ K(t_i) = \omega_i \frac{\sigma(t_i)}{\sigma(M)} + (1-\omega_i) \frac{l(t_i)}{l(M)} , \quad t_i \in [1, M] \]  
\[ \sigma(t_i) = \int_0^u \sqrt{\rho(u)} \, dl(u) = \sum_{i=1}^{M} \left( \sqrt{\rho_{1,i}} + \sqrt{\rho_{2,i}} \right) (t_i - t_{i-1}) / 2 \]  

Where \( \omega_i \) is the weight of the characteristic function \( K(t) \). To overcome the curvature integral, the formula (4) is to integrate the curvature. As the data are quite dense, Newton-Cotes formula is employed to calculate the integration[8]. The curvature \( \rho_j \) can be calculated by combining the arc radius \( R \) in each arc data.

After obtaining a uniformly sampled point set according to the characteristic function, interpolation or progressive iteration approximation(PIA) method[9] can be used for B-spline fitting through the sampled point. Considering the curve obtained by the PIA will not pass these samples, the cubic B-spline interpolation algorithm is used to interpolate these sampling points to form the lens contour[10]. And the specific sampling and fitting steps are as follows:

Step 1: Construct the feature function, and set a small initial sampling point number \( N \). Each sampling point has the same feature amount. For the sunglasses lens contour is closed, we can get the formula:

\[ K(t_{i+1}) - K(t_i) = 1/N \]

And then obtain the sampling points \( \{P_{t_i}\} \) \( j=1,...,M \).

Step 2: According to the sampling points construct a sequence \( \{K(t_j)\} \), and the \( C^2 \) continuity requirement of the closed curve connection, to increase the boundary conditions of the curve[11] for constructing a node vector sequences \( \text{Node}=\{t_j\}_{j=0}^{N+k+3} \).

Step 3: Using the algorithm proposed by DeBoor-Cox [6], adding constraints according to the sampling points \( \{P_{t_i}\} \) and using the Thomas method which is optimized based on the LU Factorization method[12], to obtain the control points of the B-spline curve more quickly.

Step 4: According to the formula(1), calculate the data sequence \( \{C_{j}\}_{j=1}^{M} \) at the corresponding node.

Step 5: Compare the errors between the original data sequence and the B-spline data sequence obtained by interpolation, and continuously adjust during the period to obtain the best \( \omega_i \) under the number of sampling points, for getting the minimum errors fitting curve.

Step 6: Repeat iteratively to increase the sampling point \( N \) until the error meets the accuracy requirement, and then output the B-spline fitting curve.

The errors used for comparison above are the formula (5) root mean square error (RMSE) and formula (6) maximum absolute error (MAE).

\[ \text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \| C(t_i) - P(t_i) \|^2} \]  
\[ \text{MAE} = \max_{i=1}^{M} \| C(t_i) - P(t_i) \|_2 \]  

3. Curve sampling analysis and application

3.1. Sampling and fitting analysis of contour points of sunglasses

Sample and fit the data of the two groups of sunglasses (data 1 has a smaller curvature change, and data 2 has a larger curvature change). Three sampling methods are investigated: uniform arc length sampling[2], uniform sampling based on curvature and arc length[4], and the sampling method in this paper.

Figure 1(a) is the XY plane diagram of the lens data 1 read from DXF, and each arc is connected between "**". Set the following error conditions: the maximum RMSE is 0.02mm, and the maximum MAE is 0.1mm. It needs to sample 20 data points to meet the fitting accuracy requirements by the method proposed in this paper, and The sampled three-dimensional side view effect is shown in Figure
Figure 1(b) and figure 1(c) show the sampling of 20 points by the other two sampling methods, it is possible to observe that the reconstruction with the two methods has a bigger error compared with the methods in this article. Reconstruction with the uniform arc length sampling has the worst performance at the corner of sunglasses lens, and for the uniform sampling based on curvature and arc length methods, it occurs where the points are too far from each other. Table 1 records the RMSE and MAE fitted by the three sampling methods under this sampling.

![Figure 1. Lens data 1 and the three kinds of sampling fitting](image)

| Error   | Sampling, 20 points |
|---------|---------------------|
| RMSE    | 0.0658              | 0.0389              | 0.0193              |
| MAE     | 0.4507              | 0.2430              | 0.0879              |

Figure 2(a) is shown the XY plane diagram of it, the curvature of the entire lens profile is reflected in the large changes at the corners of the lens. Through the proposed sampling method, the same fitting accuracy as the lens data 1 is achieved, that is, the RMSE is 0.02mm, the MAE is 0.1mm, and 22 data points need to be sampled for fitting. Figure 2(b), (c), (d) respectively show the lens data 2, using the above three sampling methods to fit the three-dimensional side view effect diagram of 22 sampling points, Table 2 records the RMSE and MAE fitted by the three sampling methods under the sampling.

![Figure 2. Lens data 2 and the three kinds of sampling fitting](image)

| Error   | Sampling, 22 points |
|---------|---------------------|
| RMSE    | 0.0200              | 0.0193              | 0.0175              |
| MAE     | 0.1000              | 0.0879              | 0.0794              |

For the glasses lens data 2, Figure 2(a) is shown the XY plane diagram of it, the curvature of the entire lens profile is reflected in the large changes at the corners of the lens. Through the proposed sampling method, the same fitting accuracy as the lens data 1 is achieved, that is, the RMSE is 0.02mm, the MAE is 0.1mm, and 22 data points need to be sampled for fitting. Figure 2(b), (c), (d) respectively show the lens data 2, using the above three sampling methods to fit the three-dimensional side view effect diagram of 22 sampling points, Table 2 records the RMSE and MAE fitted by the three sampling methods under the sampling.


| Error     | Sampling, 22 points |
|-----------|---------------------|
| RMSE      | 0.0734              |
| MAE       | 0.4518              |

(a) Lens data 2

(b) Uniform arc

(c) Uniform arc and curvature

(d) this essay

**Figure 2.** Lens data 2 and the three kinds of sampling fitting

Through analysis, it can be seen that in the three-dimensional data fitting of sunglasses lens, the uniform arc length sampling [2] will have large errors at the corners with large curvature changes. Based on the sampling method weighted by arc length and curvature and the sampling method in this paper, the maximum sampling error occurs at the place where the curvature is small, and too far distance between the sampling points causes the error to be too large. In short, this section has shown that the reconstruction based on the proposed sampling method outperforms the other sampling models.

### 3.2. Practical application of sun lens contour sampling and fitting

Using this method in the engineering practice of sunglasses lenses data fitting. First, set the initial number of sampling points \( N \) and the maximum root mean square error (RMSE). Program will select high-quality sampling points according to the given characteristic function and adjusts the weight of the function, iteratively increases the number of sampling points \( N \) until a B-spline fitting curve that meets the accuracy requirements is constructed.

The adaptive sampling and fitting are implemented on the sunglasses lens data 1, the input initial sampling point \( N \) is 15, and the maximum allowable RMSE is 0.005. Figure 3 shows the process of
sunglasses lens data 1 that iteratively increases the number of sampling points until the accuracy meets the requirements, the sampling number increased from 15 to 30. The horizontal axis is the iteratively increasing sampling points, and the vertical axis (red) is each the most reasonable weight of arc length under the number of sampling points, and RMSE of the corresponding fitting (blue). It can be seen that the most reasonable weight of the function is around 0.6. And as the number of sampling points increases, the curve fitting error gradually decreases. The final B-spline curve obtained by sampling fitting meets the accuracy requirements, and the number of sampling points is 30. This curve fitting of such a small number of sampling points can be better used in the CNC machining of sunglasses lens contour.

Figure 3. Sampling and fitting process of lens data 1

4. Conclusion
An adaptive sampling method is proposed for the contour sampling and fitting of sunglasses lens in CNC machining. After discretizing the sunglasses lens data stored in the DXF file, considering the two inherent geometric factors (arc length and curvature) of the lens, the characteristic function of the sunglasses lens data sampling is determined. In the uniform sampling process using the feature function, the weight $\omega_1$ in the feature function is continuously adjusted to obtain more reasonable and higher-quality sampling points. In the actual application process, through the preset fitting RMSE, iteratively increase the number of sampling points $N$ until the accuracy requirements are met, which can realize the adaptive sampling fitting of the sunglasses lens contour, and improve the fitting quality and efficiency in the CNC machining.

5. References
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