Extra Matters Decree the Relatively Heavy Higgs of Mass about 125 GeV in the Supersymmetric Model

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Abstract

We show that the Higgs mass about 125 GeV is easily realized in supersymmetric model with extra matters, simultaneously explaining the anomaly in the muon anomalous magnetic moment and the dark matter density.
1 Introduction

We have not found any convincing and fundamental theory for explaining the observed masses of quarks and leptons. However, it was pointed out, long ago, that if one introduces a pair of extra matters, $10$ and $\bar{10}$, in supersymmetric (SUSY) standard model, the Yukawa coupling $y_t$ for the top quark has a quasi infrared fixed point at $y_t \sim 1$ [1]. This is an extremely interesting observation, since we can understand the observed mass of the top quark as a low-energy prediction of the theory. Furthermore, it has been, recently, pointed out [2] that the extra matters, $10$ and $\bar{10}$, cancels anomalies of a discrete $R$ symmetry in the SUSY standard model, provided that the sum of their $R$ charges is zero. (See also [3].) This is itself very interesting in the LHC phenomenology, since they naturally have a SUSY-invariant mass at the electro-weak scale through the Giudice-Masiero mechanism [4]. The further crucial and important point is that a new Yukawa coupling $y_U$ for the extra matters has also a quasi infrared fixed point at $y_U \simeq 1$ [5] which gives additional large contribution to the mass of the lightest Higgs boson [6, 7], and as a consequence the lightest Higgs boson acquires naturally a relatively large mass of $120 - 130$ GeV [5, 2, 8].

Recently, the ATLAS and CMS experiments have announced the latest results of their searches for Higgs boson. In particular, the ATLAS has found $3.4\sigma$ local excess of the Higgs-like signal near $m_h \simeq 126$ GeV [9]. The CMS has also observed more than $2\sigma$ local excess of the signal at $m_h \simeq 124$ GeV [10]. Although the significances decrease once we take into account the so-called “look-elsewhere effect,” these excesses may be the indication of the existence of the Higgs boson at around $m_h \simeq 125$ GeV. (If we consider the global probability, the excesses are at $2.3\sigma$ by ATLAS and at $1.9\sigma$ by CMS.) If the Higgs mass is as large as $\sim 125$ GeV in the framework of minimal SUSY standard model (MSSM), the mass scale of the superparticles are required to be relatively high ($\gtrsim$ a few TeV) to enhance the Higgs mass via radiative corrections [11]. However, as we have mentioned, this is not the case if there exist extra matters.

In this letter, taking the excesses of Higgs-like signals at around $m_h \simeq 125$ GeV seriously, we consider SUSY standard model with extra matters. We show that the Higgs mass of $m_h \simeq 125$ GeV is easily explained in the model with extra matters if their masses are in a rage of $500$ GeV–$1$ TeV. Surprising is that there is a wide range of parameter space in which not only the Higgs boson mass but also the dark matter density and the anomalous magnetic dipole moment (MDM) of the muon are simultaneously explained. We also show that the gluino is most likely lighter than $1$ TeV which can be tested soon at the LHC.

2 Model

As we have mentioned in the introduction, we consider the SSM with extra matters which are $10$ and $\bar{10}$ representations in $SU(5)_{\text{GUT}}$, which we denote $10'$ and $\bar{10}'$, respectively.\(^\#1\)

\(^\#1\)For another possibility, we can consider the model with three pairs of $5$ and $\bar{5}$; for the enhancement of the Higgs mass, the same number of singlets should be also introduced. In such a case, the quasi fixed point
To discuss the low-energy phenomenology, we decompose the extra matters as $10' = Q + U + E$ and $\bar{10}' = \bar{Q} + \bar{U} + \bar{E}$, where $Q(3, 2, 1/6)$, $U(\bar{3}, 1, -2/3)$, $E(1, 1, 1)$, $\bar{Q}(\bar{3}, 1, -1/6)$, $\bar{U}(3, 1, 2/3)$, and $\bar{E}(1, 1, -1)$ are gauge eigenstates of the standard-model gauge group. (The gauge quantum numbers for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ are shown in the parenthesis.) Then, the superpotential relevant for the discussion of the low-energy phenomenology is

$$W = W^{(MSSM)} + y_U U Q H_u + M_U \bar{U} U + M_Q \bar{Q} Q + M_E \bar{E} E,$$

where $W^{(MSSM)}$ is the superpotential of the minimal SUSY standard model (MSSM), $H_u$ is the up-type Higgs multiplet, $y_U$ is the Yukawa coupling constant for the extra matters, while $M_U$, $M_Q$, and $M_E$ are mass parameters whose origin is assumed to be the Giudice-Masiero mechanism. In our study, we neglect the superpotential of the form $\bar{5}_H \cdot 10' \cdot 10'$ (where $\bar{5}_H$ denotes the Higgs multiplet in $\bar{5}$ representation contains down-type Higgs) because it is not important for the following discussion. In addition, the soft SUSY breaking terms are given by

$$L_{soft} = m_{\tilde{Q}}^2 |\tilde{Q}|^2 + m_{\tilde{Q}}^2 |\tilde{Q}|^2 + m_{\tilde{U}}^2 |\tilde{U}|^2 + m_{\tilde{U}}^2 |\tilde{U}|^2 + m_{\tilde{E}}^2 |\tilde{E}|^2 + m_{\tilde{E}}^2 |\tilde{E}|^2 + (y_U A_{\tilde{U}} \tilde{U} Q H_u + h.c.),$$

where “tilde” is used for superpartners.

The low energy parameters (in particular, the soft SUSY breaking parameters) given above are related to fundamental parameters given at high scale. As in the case of conventional study, we assume that the boundary condition of the low-scale parameters are given at the so-called GUT scale $M_{GUT}$, which we take $M_{GUT} = 2 \times 10^{16}$ GeV. Because the GUT is one of the strong motivation to consider SUSY, we consider a boundary condition which respects $SU(5)$ symmetry. In order to reduce the number of free parameters (as well as to avoid dangerous flavor problems), we adopt the following boundary condition at the GUT scale:

- All the gaugino masses are unified to $m_{1/2}$ at the GUT scale.
- All the matter fields in $\bar{5}$ representation except Higgs have universal SUSY breaking mass-squared parameter, denoted as $m_{\bar{5}}^2$.
- All the matter fields in $10$ and $\bar{10}$ representations have universal SUSY breaking mass-squared parameter, denoted as $m_{10}^2$.
- The up- and down-type Higgses have SUSY breaking mass-squared parameters $m_{5H}^2$ and $m_{\bar{5}H}^2$, respectively.

value of the Yukawa coupling constant relevant for the enhancement of the Higgs mass is smaller compared to the model with $10$ and $\bar{10}$ representations. Even so, a significant enhancement of the Higgs mass may be possible in particular when the mass of the extra matters are relatively low. Detailed discussion on this case will be given elsewhere [12].

#2We neglect the bi-linear SUSY breaking terms for extra matters for simplicity.
• For simplicity, we assume that the SUSY breaking tri-linear scalar couplings vanish at the GUT scale. Notice that the following results are almost unchanged even if we relax this assumption, since the tri-linear scalar couplings become small at quasi infrared fixed points in the present model [5].

Notice that, in the following, the above mass-squared parameters are allowed to be negative, which is important to have a viable low-energy phenomenology as we will see below.

Once the boundary condition is fixed, we solve the renormalization group equation from $M_{\text{GUT}}$ to $M_{\text{SUSY}}$, where $M_{\text{SUSY}}$ is the typical mass scale of the superparticles. We mostly use the one-loop $\beta$-functions. However, the two-loop effects on the gaugino masses (in particular, gluino mass) are significant, so we also take into account $O(\alpha_2^2)$ (as well as $O(\alpha_1^2)$ and $O(\alpha_2^3)$) contributions to the $\beta$-functions of gaugino masses and gauge coupling constants. Then, using the soft SUSY breaking parameters at low energy scale, we solve the conditions of electro-weak symmetry breaking to determine the SUSY invariant Higgs mass (i.e., so-called $\mu$-parameter) as well as bi-linear SUSY breaking Higgs mass parameter (i.e., so-called $B$-parameter); in our analysis, we use the tree-level minimization condition of the Higgs potential.

With the low-energy parameters given above, we calculate experimental observables, in particular, the muon MDM $a_\mu$ and the relic density of the lightest superparticle as well as the the lightest Higgs mass $m_h$. The relevant formula to calculate $a_\mu$ can be found in [13].

For the calculation of the density parameter of the LSP $\Omega_{\text{LSP}}$, we use DarkSUSY package [14].

For the calculation of the Higgs mass, we use the fact that the model is well described by the standard model once the extra matters and superparticles decouple. Then, we denote the potential of the standard-model like Higgs $H_{\text{SM}}$ as

$$V_{\text{SM}} = m_H^2 |H_{\text{SM}}|^2 + \frac{1}{2} \lambda |H_{\text{SM}}|^4,$$

and the Higgs mass is given by

$$m_h^2 = \lambda(m_h)v^2,$$

where $v \simeq 246$ GeV is the vacuum expectation value of the standard-model-like Higgs boson. The quartic Higgs coupling $\lambda$ is determined by the parameters in the supersymmetric model at the mass scale of superpartners and extra matters as

$$\lambda(M_{\text{SUSY}}) = \frac{1}{4}(g_2^2 + g_1^2)\cos^2 2\beta + \delta\lambda',$$

where $g_2$ and $g_1$ are gauge coupling constants for $SU(2)_L$ and $U(1)_Y$, respectively, and $\delta\lambda'$ is the contribution from the extra matters. Then, $\lambda(M_{\text{SUSY}})$ and $\lambda(m_h)$ are related by using the standard-model renormalization group equations. In our analysis, we estimate $\delta\lambda'$ from one-loop effective potential obtained by integrating out the extra matters. Denoting such an effective potential as $\Delta V^{(\text{extra})}$, we obtain

$$\delta\lambda' = \frac{1}{2} \frac{\partial^4 \Delta V^{(\text{extra})}}{\partial H_u^2 \partial H_u^2} \sin^4 \beta.$$
The one-loop contribution of the extra matters to the Higgs potential is given by

$$\Delta V^{(\text{extra})} = \Delta V^{(\text{extra})}_B + \Delta V^{(\text{extra})}_F,$$

(7)

where $\Delta V^{(\text{extra})}_B$ and $\Delta V^{(\text{extra})}_F$ are contributions of bosonic and fermionic loops, respectively. $\Delta V^{(\text{extra})}_B$ is given by

$$\Delta V^{(\text{extra})}_B = \frac{3}{32\pi^2} \text{Tr} \left[ (\mathcal{M}_B^2 + \Delta \mathcal{M}_B^2)^2 \left\{ \ln \left( \frac{\mathcal{M}_B^2 + \Delta \mathcal{M}_B^2}{\mu^2} \right) - \frac{3}{2} \right\} \right],$$

(8)

where

$$\mathcal{M}_B^2 = \text{diag}(M_Q^2 + m_Q^2, M_Q^2 + m_Q^2, M_U^2 + m_U^2, M_U^2 + m_U^2),$$

(9)

and

$$\Delta \mathcal{M}_B^2 = \begin{pmatrix}
y_U^2 |H_u|^2 & 0 & 0 & y_U M_U H_u^* \\
0 & 0 & y_U M_Q H_u^* & 0 \\
y_U M_U H_u & 0 & y_U^2 |H_u|^2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},$$

(10)

while

$$\Delta V^{(\text{extra})}_F = - \Delta V^{(\text{extra})}_B \bigg|_{m_Q^2 = m_Q^2 = m_U^2 = m_U^2 = 0}.$$  

(11)

Notice that we are interested in the case where the tri-linear coupling constants tend to go to the fixed-point values, which are significantly smaller than the soft SUSY breaking mass-squared parameters. Thus, in our analysis, we neglect the effects of the tri-linear coupling constants in the estimation of the Higgs mass.

3 Higgs Mass, Muon MDM, and $\Omega_{\text{LSP}}$

Now we numerically calculate the Higgs mass, the muon MDM, and the relic density of the LSP. Our main purpose is to show that there exists a parameter region where the observed Higgs mass ($\sim 125$ GeV) as well as the muon MDM and the dark matter abundance are simultaneously explained in the present framework. Thus, we fix some of the parameters rather than performing a systematic scan of the full parameter space. (More complete analysis will be given elsewhere [12].) Here, we take $m_{\tilde{b}_H}^2 = m_{\tilde{t}_H}^2 = 0$. Because we would like to enhance the SUSY contribution to the muon MDM, we adopt a large value of $\tan \beta$; numerically, we take $\tan \beta = 50$. In addition, taking account of the quasi fixed point behavior of the Yukawa coupling constant, we take $y_U(M_{\text{SUSY}}) = 1$. (Here, we take $M_{\text{SUSY}} = 1.5$ TeV, which is the typical mass scale of the superparticle in the following analysis. The results given below are quite insensitive to this value.) Furthermore, we approximate $M_Q = M_U$ for simplicity.
Figure 1: Contours are those of constant $m_h$ (123 GeV, · · · , 129 GeV, from below) on $m_{\tilde{\chi}}^2$ vs. $m_{\tilde{\nu}_5}^2$ plane. In the gray-shaded region, the lightest slepton mass becomes smaller than 100 GeV. On the dotted line, the lightest slepton mass becomes equal to the lightest neutralino mass. In the pink (blue) region, the muon MDM becomes consistent with the experimental value at 1$\sigma$ (2$\sigma$) level. Here, we take $\tan\beta=50$, $M_U = 550$ GeV, and $m_{1/2} \simeq 1.1$ TeV (which corresponds to the Bino, Wino, and gluino masses of 170 GeV, 240 GeV, 700 GeV, respectively).

In Figs. 1 and 2 we show the results of our numerical calculations, where we take $m_{1/2} \simeq 1.1$ and 1.3 TeV, respectively. (The predicted Bino, Wino, and gluino masses are 170 GeV, 240 GeV, 700 GeV, and 210 GeV, 310 GeV, 900 GeV, respectively.) In Fig. 1 (Fig. 2), we take $M_U = 550$ GeV (650 GeV); these values of the mass of extra generation quarks are above the present experimental lower bound on the mass of fourth-generation quarks [15]. In the gray-shaded region, the stau becomes lighter than 100 GeV. (In fact, in most of the gray-shaded region, the lighter stau becomes tachyonic.) In addition, on the dotted line, the stau and the lightest neutralino become degenerate. Thus, the stau is the LSP in the region between the dotted line and the gray-shaded region while the lightest neutralino (which is almost purely Bino) is the LSP in the region right to the dotted line. It is notable that the allowed parameter region extends to the region with negative mass-squared parameter at the GUT scale. This is due to the fact that, in the present model, the gaugino masses are more enhanced at high scale compared to the case of the MSSM. Consequently, gaugino contributions to the renormalization group running of the scalar masses are significantly enhanced. For the left-handed sleptons, such an enhancement is so significant that the masses of the left-handed sleptons (at $M_{\text{SUSY}}$) can be positive even if $m_{\tilde{\nu}_5}^2 < 0$. Notice that,
Figure 2: Contours are those of constant $m_h$ (125 GeV, · · ·, 129 GeV, from below) on $m_{\tilde{g}}^2$ vs. $m_{\tilde{10}}^2$ plane. In the gray-shaded region, the lightest slepton mass becomes smaller than 100 GeV. On the dotted line, the lightest slepton mass becomes equal to the lightest neutralino mass. In the pink (blue) region, the muon MDM becomes consistent with the experimental value at 1σ (2σ) level. Here, we take $\tan \beta = 50$, $M_U = 650$ GeV, and $m_{1/2} \simeq 1.3$ TeV (which corresponds to the Bino, Wino, and gluino masses of 210 GeV, 310 GeV, 900 GeV, respectively).

even if some of the scalar masses are negative at the GUT scale, they become positive at the scale orders of magnitude larger than $M_{\text{SUSY}}$. Thus, the tunneling to the unwanted true vacuum is strongly suppressed and irrelevant.

In the figures, the (almost) horizontal lines are contours of constant Higgs mass. As we mentioned earlier, the Higgs mass can be as large as 125 GeV in the present framework. The Higgs mass is more enhanced for larger value of $m_{\tilde{10}}^2$. This is due to the fact that the leading contribution to the Higgs mass from the extra matters is approximately proportional to $\log(m_{\tilde{U}}/M_U)$. This fact also indicates that, if we increase (decrease) $M_U$, the Higgs mass becomes smaller (larger).

In the same figures, we also show the region where the SUSY contribution to the muon MDM $\Delta a_\mu^{(\text{SUSY})}$ well explains the $\sim 3\sigma$ discrepancy between the experimental and standard-model values of the muon MDM [16],

$$\Delta a_\mu^{(\text{SUSY})} = a_\mu^{(\text{exp})} - a_\mu^{(\text{SM})} = (25.9 \pm 8.1) \times 10^{-10},$$

where $a_\mu^{(\text{exp})}$ and $a_\mu^{(\text{SM})}$ are experimental value and standard-model prediction of $a_\mu$. One can see that, in the model with extra matters, the lightest Higgs mass can be $\sim 125$ GeV with sat-
isfying the muon MDM constraint (at the 1σ level). To understand the behavior of the SUSY contribution to the muon MDM, one should note that \( \Delta a_\mu^{(\text{SUSY})} \) is from smuon-neutralino and sneutrino-chargino loop diagrams and that \( \Delta a_\mu^{(\text{SUSY})} \) is approximately proportional to \( \tan \beta \). Thus, in order to make \( \Delta a_\mu^{(\text{SUSY})} \) sizable, at least one of the left- or right-handed smuon should be relatively light. As we have discussed, by taking \( m_{\tilde{b}_L}^2 < 0 \), the left-handed slepton masses can be small even though the renormalization group effect on the sfermion masses is significant. Then, with our present choice of \( \tan \beta = 50 \) and the light slepton masses, we obtain sufficient \( \Delta a_\mu \) to realize Eq. (12).

Next, we discuss the thermal relic density of the LSP. In particular, on the right of dotted line, the lightest neutralino is the LSP, so it is a viable candidate of dark matter. However, in the bulk of such a region, sleptons are much heavier than the lightest neutralino. In addition, the lightest neutralino is (almost) purely Bino. Thus, the pair annihilation cross section of the lightest neutralino is so small that \( \Omega_{\text{LSP}} \) becomes too large to be consistent with the present dark matter density if there is no other annihilation process. As we have mentioned, in the parameter region near the dotted line, the lightest neutralino almost degenerates with the stau and the co-annihilation process becomes important [17, 18]. In particular, if the mass difference between the lightest neutralino and the stau is a few GeV, the relic density of the lightest neutralino becomes consistent with the present dark matter density \( \Omega_c \). We have checked that there indeed exists a contour on which \( \Omega_{\text{LSP}} h^2 = 0.1116 \) so that the lightest neutralino can be dark matter. If we draw such a contour on the figure, it is (almost) indistinguishable from the dotted line, and it crosses the contour of \( m_h = 125 \text{ GeV} \) in the region where the muon MDM anomaly can be explained at 1σ level. A similar situation was also studied in [8], where the simultaneous explanation of \( m_h = 125 \text{ GeV} \), the muon MDM anomaly at 1σ level, and the dark matter density was hardly realized in the so-called mSUGRA model. Compared to [8], our result crucially depends on the fact that we allowed soft SUSY breaking mass-squared parameters to be negative.

Finally, we comment on the gluino mass constraint. In the limit of heavy squark masses, the present experimental lower bound on the gluino mass is \( \sim 700 \text{ GeV} \) [20]. (Here, we have used the constraints on so-called “squark-gluino-neutralino model.”) On the first sample point used in our numerical calculation, the predicted gluino mass is marginally consistent with the bound. In other points, like our second sample point, \( m_h \simeq 125 \text{ GeV} \), the muon MDM, and the dark matter density can be simultaneously explained with heavier gluino mass. Even in such a case, the gluino mass can be well within the reach of future LHC experiment. We comment here that, with very large gluino mass, it becomes difficult to solve the muon MDM anomaly. This is because, adopting larger value of \( m_{1/2} \), the renormalization effect on the SUSY breaking mass-squared parameter of the up-type Higgs boson becomes more negative. Then, in order to realize the proper electro-weak symmetry breaking, a large value of the \( \mu \)-parameter is needed, resulting in the suppression of the muon MDM. Thus, the search for the gluino signal at the LHC is a crucial test of the present scenario. We also note here that the muon MDM can be enhanced if our assumptions on the mass spectrum of superparticles are relaxed. For example, with the gluino mass being fixed, the muon MDM
becomes larger if the Wino and Bino masses are somehow suppressed. Such a mass spectrum is possible if the GUT relation among the gaugino masses is violated; in the product group GUT scenario [21], this may be the case [22].

4 Summary

In this letter, we have argued that the Higgs mass of $\sim 125$ GeV, around which ATLAS [9] and CMS [10] have observed excesses of Higgs-like signals, can be well explained in SUSY models with extra matters. In the MSSM, it is often the case where large values of superparticle masses are preferred to realize such a value of the Higgs mass. However, such a mass spectrum tends to suppress the SUSY contribution to the muon MDM, and also makes the LHC searches for the superparticles difficult. However, in the present model, $m_h \simeq 125$ GeV is realized in the region where the SUSY contribution to the muon MDM can be large enough to explain the $\sim 3\sigma$ discrepancy between the experimental and standard-model values of the muon MDM (at 1$\sigma$). Simultaneously, the relic density of the lightest neutralino can become consistent with the present dark matter density. We have seen that, for the simultaneous explanation of the Higgs mass, the muon MDM, and the dark matter density, some of the SUSY breaking mass-squared parameters is preferred to be negative at the GUT scale. In the parameter region we are interested in, the gluino mass as well as the masses of extra matters can be below $\sim 1$ TeV, so they are within the reach of future LHC experiments.

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