An analytical method for J-resistance curve based on a new fracture criteria and finite element aided testing method

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Abstract. J-resistance curve of ductile material is important to describe the ability of a crack structure or material to resist fracture. Experimental and theoretical methods are developed to obtain J-resistance curve of metallic material. However, there are rare reports to predict J-resistance curve directly by tensile test on ductile materials. In this paper, an analytical method is proposed to predict the J-resistance curve for metallic material. Based on a new method, true stress-strain curve up to failure of ductile material is obtained. Thus, fracture threshold in different constrains are calculated using different kinds of specimens. A simple logarithmic-relationship between $\sigma_1$ and $\sigma_1^*$ (Yao-Cai criteria) is proposed. The new fracture criterion and full-range constitutive relationship are then used to describe the fracture behaviour near the crack tip. Stress triaxiality and critical fracture first-principle stress around crack tip are obtained. Therefore, fracture procedure is divided into two parts: crack tip blunting and steady crack growth. The relationship of J-integral and crack growth amount $R$ is established respectively according to the blunting line equation and HRR stress field. The predicted results for test specimens (CT, SENT, CIET) with different initial crack length of the A508-3 steel coincide well with the test results. The results show that stress triaxiality at crack tip is a typical characteristic feature following the constraint effect and the Yao-Cai criteria is recommended for the fracture threshold of ductile material.

1. Introduction

The J-integral proposed by Rice [1] is one of the most widely used fracture parameters. Research in ductile fracture and acquisition of ductile fracture toughness JIC is important to structural integrity assessment issues, forming analysis of cutting, punching problems and damage accident analysis.

The experimental method for J-R curve of ductile material has been established over the years. Such as unloading compliance method, electric potential drop method, and load separation theory [2]. However, the test procedure needs relatively long time and a great deal of material is required to obtain the J-R curve to characterize the in-plane effect of crack tip constraint. To solve the problem, some theoretical methods were put forward, such as finite element analysis (FEA) method, J-Q and J-A2 parameter prediction method, cohesive method and micromechanical method.

FEA method for fracture toughness of ductile materials is the most common manner which has been conducted by many researchers. Acharyya. S [3], Benseddiq N [4], Xue Z [5] and Hutchinson [6] calculated the fracture toughness of ductile by GTN constitutive model, but the mesh size varies. There are no standard size for meshing in FEA to calculate the J-$\Delta$a curve at present. The J-Q and J-A2 approaches have been frequently applied to describe the constraint effect of the crack tip [7-10]. By the
method above, a set of $J-\Delta a$ curve is obtained. However, this type of test needed to determine the fracture toughness. Cohesive and micromechanical methods [11-12] are new approaches developed in recent years. Worth to emphasis that the accuracy of these methods needs verification.

Analytical approach to crack growth resistance was initiated by McClintock [13] first proposed an analytical method to obtain fracture toughness of a mode III crack. This approach culminated in the work of Rice and coworkers [14] continued McClintock’s work for mode I crack. In this mode, solutions for the crack opening profile behind the advancing tip were obtained. The refined numerical calculations of SHAM [15] were used to establish the relation of the opening profile on $K$ and $\Delta a$. A propagation criterion was imposed on the solution requiring that the near-tip opening profile be invariant once propagation is under way. The outcome of the approach was the prediction of the crack growth resistance curve, K ($\Delta a$). However, the $J-\Delta a$ prediction method is rare reported.

In this study, a new method [16-19] is proposed to acquire the true stress-strain curve of the A508-3 steel up to failure using a notched round bar specimen. According to the analysis of failure mechanism of ductile materials, the relationship between fracture threshold and stress triaxiality at the fracture is studied by the tensile tests applying different kinds of specimens. Based on the true stress-strain curve for ductile materials, the Yao-Cai fracture criterion [20] of the A508-3 steel is obtained. By dividing the crack propagation procedure into two parts, the relationship between $J$-integral and blunting propagation value $\Delta a_b$, steady propagation value $\Delta a_t$ was obtained, respectively. In plane-strain condition, stress distribution in front of crack tip is assumed to be satisfying the HRR stress field. Thus, the quadratic function relationship $J$-integral and $\Delta a$ is obtained.

2. Research conditions

2.1. Material and test system

The servo-hydraulic universal testing machine 809 MTS, the static/dynamic 632.12C-21 MTS extensometer (25 mm gauge length, 50% measuring range), the centering grips system and the Crack Opening Displacement 632.02F-20 MTS extensometer (5 mm gauge length, 4mm measuring range) were employed for the experiment. Tensile test was conducted under displacement control mode at velocity of 0.02 mm/s. An analysis of the Young’s modulus determined in four directions of the material showed that the test error did not exceed 0.5%. It was possible by means of centering grips (the alignment system).

| Material chemical | C (%) | Mn (%) | Si (%) | P (%) | S (%) | Ni (%) | Cr (%) | Mo (%) |
|-------------------|-------|--------|--------|-------|-------|--------|--------|--------|
| A508-3 steel      | 0.25  | 1.12-1.58 | 0.15-0.37 | <0.018 | <0.005 | 0.57-1.03 | 0.1-0.25 | 0.4-0.6 |

| Material chemical | V (%) | Cu (%) | Al (%) | Ti (%) | Co (%) | B (%) | Nb (%) | Ca (%) |
|-------------------|-------|--------|--------|--------|--------|-------|--------|--------|
| A508-3 steel      | ≤0.02 | ≤0.15  | ≤0.025 | ≤0.015 | ≤0.25  | ≤0.003 | ≤0.01  | ≤0.015 |

The A508-3 steel employed in reactor pressure vessel was investigated. Tab. 1 shows the chemical composition of the A508-3 steel. Uniaxial true stress-true strain curve before necking of the steel examined was captured in a tensile test on a smooth round bar specimen. This characteristic can be described by Ramberg-Osgood relationship model,

\[
\begin{align*}
\varepsilon &= \varepsilon_y + \varepsilon_p \\
\varepsilon_y &= E\sigma \\
\varepsilon_p &= \frac{\sigma - \sigma_0}{\sigma_0} + \alpha\left(\frac{\sigma - \sigma_0}{\sigma_0}\right)^n
\end{align*}
\]

where, $\sigma_0=(R_{0.2}+R_m)/2$ is flow stress, $R_{0.2}$ is yield stress, $R_m$ represents ultimate tensile strength,
$\varepsilon_0 = \sigma_0/E$ is flow strain, and $E$ is Young’s modulus. All mechanical parameters from equation (1) are listed in table 2. Figure 1 shows the tensile curves of the A508-3 steel before necking. The material is assumed to be isotropic, and it satisfies the basic assumption of elastic-plastic mechanical theory.

Table 2. Mechanical parameters of the A508-3 steel.

| Materials and specimens | $E$ /MPa | $R_{p0.2}$ /MPa | $R_m$ /MPa | $\sigma_0$ /MPa | $\varepsilon_0$ /% | $\alpha$ | $n$ |
|-------------------------|----------|-----------------|------------|----------------|-----------------|----------|-----|
| A508-3 steel-1#         | 195300   | 505             | 634        | 570            | 0.292           | 2.47     | 8.46 |
| A508-3 steel-2#         | 195200   | 485             | 643        | 564            | 0.289           | 2.68     | 8.40 |

Figure 1. The uniaxial tensile test results for the A508-3 steel.

2.2. **Test specimens**

Fracture criterion for the A508-3 steel was obtained applying the specimens shown in figure 2. The true stress-strain curve was elaborated by notched bar specimen (NRB). Figure 3 shows the specimens for fracture toughness and $J$-$R$ curve representing different constraint. Compact tension (CT) specimen contained $a/W=0.5$ and $a/W=0.7$, while $a/W=0.5$ expressed the single edge-notched bending (SENB) and small-sized C-shaped inside edge-notched tension (CIET) specimens.

As shown in the figure 3, compared with CT and SENB specimen, CIET specimen is a kind of small sized components. Less material consumption is needed and CIET specimen satisfies loading separation assumption in a wide range of crack size [21]. Thus, loading separation method was used to obtain the fracture toughness of the A508-3 steel by CT, SENB and CIET specimen.
2.3. Fatigue method for true stress-strain curve up to failure of the A508-3 steel (Please renumber the figures submitted in this section)

Figure 2 shows the basic procedure of FAT method. The true stress-strain curve and its Ramberg-Osgood model are shown in figure 5, where the strain hardening exponent is equal to \( n=8.40 \) and the yield offset is equal to \( \alpha=2.68 \).

As demonstrated in the figure 4, the true stress-strain curve coincides well with the uniaxial tensile results, but when after necking, the results differ. Thus, the true stress-strain curve was used to simulate the stress field at crack tip of CT specimen, as shown in figure 6.

Compared with the HRR stress field,

\[
\sigma_{ij}(r, \theta) = \sigma_0 \left| \frac{J}{\alpha \sigma_0 \varepsilon_n^* I_n r} \right|^{\frac{1}{n+1}} \sigma_{\theta\theta} \theta = 0^\circ
\]

(2)
2.4. Stress mode fracture criterion for the A5085-3 steel on the Yao-Cai model

A stress mode fracture criterion based on FAT method has been put forward [20] to obtain the stress mode fracture criterion of ductile materials. As shown in figure 7, the critical fracture first-stress in a large range of stress triaxiality is obtained. Yao-Cai criterion (equation 3) describing the relationship between the fracture first-principle stress and stress triaxiality is then established.

The full-range constitutive relationship simulating result coincide well, which verifies the validity of FAT method.
As shown in figure 6, Yao-Cai criterion for the A508-3 steel has a simple logarithmic law, Where, 
\[
\bar{\sigma}_f = \ln(\sigma^*_{f}) + \bar{S}
\]
\[
\bar{S} = \sigma_{f} \bigg|_{\sigma_f=1} \sigma_{f}^{U} \bigg|_{\text{Uniaxial plane stress}}
\]
\[
\bar{\sigma}_f = \sigma_{f} \bigg|_{\text{Uniaxial plane stress}}
\]
Therefore, critical fracture first-stress in any stress triaxiality can be obtained. For example, for CT specimen with \(a/W=0.5\), the stress triaxiality value \(\sigma^*_f\) at crack tip equals to 3.0. The critical fracture first-stress \(\sigma_f=2097\text{MPa}\), which is much larger than fracture first-stress in notched round bar specimen.

2.5. Experiments of J-R curve for the A508-3 steel
According to the ASTM E1820-15a, fracture tests were finished. Loading-separation method was used to obtain the fracture toughness and J-R curve by a use CT, SENB and CIET specimens. Figure 8 shows the test results of different kinds of specimens. As we can see in the figure 7, the J-R curve of SENB specimens with \(a/W=0.5\) assume to be the highest fracture toughness. The lowest fracture toughness is obtained from CIET specimens with \(a/W=0.64\). The difference is because the constraint at crack tip of the specimens is various. The J-Q and J-A2 approaches are generally used to characterize crack tip constraint, but in this study, stress triaxiality is applied to describe the constraint effect.

3. Analytical Method for J-Resistance curve
The crack propagation procedure is divided into two parts: crack tip blunting and steady crack growth. The relationship of J-integral and crack growth amount \(R\) is established respectively.

3.1. Crack tip blunting
Crack tip blunting is described completely by Landes.et.al [22]. A linear relationship exist between crack stretch zone width \(\Delta a_B\) and J-integral,
\[ J = \frac{2.5\sigma_0}{d_n^*} \Delta a_B = \frac{2.5E}{d_n^*} \Delta a_B \]  

(4)

where, \( d_n^* \) is material related parameter, which can be referred from literature [22],

\[
\begin{aligned}
 d_n^* &= D_n \left( \frac{\sigma_0}{E} \right)^{n-1} \\
 D_n &= 0.787 + 1.554n - 2.45n^2 + 16.952n^3 - 38.206n^4 + 33.13n^5 \\
 \sigma_0 &= R_{p0.2} 1.0' \\
 t &= \left[ n / (n + 1) \right] \log \left( \frac{E\varepsilon_y}{R_{p0.2}} \right) \\
 R_{p0.2} / R_m &= \left[ 1 / (1 + \varepsilon_y) \right] \left[ (2.718 / n) \ln(1 + \varepsilon_y) \right]^n \\
 \varepsilon_y &= R_{p0.2} / E + 0.002
\end{aligned}
\]

(5)

The equation is then simplified as follows,

\[ J = 3.75R_m \Delta a_B \]  

(6)

When necking occurs,

\[ \varepsilon_n = \frac{1}{n} \]  

(7)

Therefore, the ultimate tensile strength \( R_m \) can be obtained,

\[ R_m = \sigma_0 \left[ \frac{1}{\alpha} \left( \frac{1}{n\varepsilon_0} - \frac{\sigma}{\sigma_0} \right) \right]^\frac{1}{n} \]  

(8)

The blunting line equation is improved,

\[ J = 3.75\sigma_0 \left[ \frac{1}{\alpha} \left( \frac{1}{n\varepsilon_0} - \frac{\sigma}{\sigma_0} \right) \right]^\frac{1}{n} \Delta a_B \]  

(9)

3.2. Steady crack growth mechanism

The steady crack growth procedure for mode I crack is shown in figure 9. The stress state at crack tip is equivalent to tensile processing of a series of notched-round bar specimens. Therefore, the step-by-step steady crack growth occurs in front of the crack tip. The propagation amount has gradually increasing trend. So we assume the \( J \)-integral as the driving energy for crack growth.

CT specimen with \( a/W=0.5 \) was taken as an example to show the prediction method in detail.

Figure 10 shows the evolution law of stress triaxiality when the loading increases. As we can see, the stress triaxiality at a certain region in front of crack tip is almost constant.

We take the distribution law of stress triaxiality when \( b / aJ=63 \) as the representative law of the whole loading procedure. The critical fracture stress envelope is then obtained. As shown in figure 11, the Y-stress increases as \( J \)-integral increasing. Thus, a critical \( J \)-integral \( J_0 \) is existing, under which the maximum Y-stress reaches critical value:

\[
\sigma_{yy}^{\max} = \sigma_f = \sigma_0 \left| \beta = 0^\circ \right. = \ln(\sigma_{yy}^{*\max}) + \bar{S}
\]

(10)
As shown in the figure, the critical \( J_0 = 222 \text{ MPa-mm} \), the maximum stress triaxiality \( \sigma^*_{\text{max}} = 3.1 \) and the maximum Y-stress \( \sigma_{yy_{\text{max}}} = 2121 \text{MPa} \). Therefore, the propagation value \( D_0 \) is obtained, according to the HRR field:

\[
\sigma_{yy_{\text{max}}} = \sigma_0 \left( \frac{J_0}{\alpha \sigma_0 \varepsilon_0 I_0} \right)^{1/(n+1)} \left. \frac{1}{\sigma_{\theta \theta}} \right|_{\theta = 0^\circ} \quad (11)
\]

\[
D_0 = \frac{J_0}{\alpha \sigma_0 \varepsilon_0 I_0} \left( \frac{\sigma_{yy_{\text{max}}}}{\sigma_0 \sigma_{\theta \theta}} \right)^{-(n+1)} \quad (12)
\]

For CT specimen with \( a/W = 0.5 \) of A508-3 steel, \( D_0 = 0.0577 \text{mm} \). After the first-step propagation, we assume \( J \)-integral as the driving energy of crack propagation. The increment value of \( J \)-integral remains unchanged every step:

\[
J = i J_0 \quad (13)
\]

When the crack propagation reaches the \( i \) step, the stress distribution still satisfies the HRR field after
the maximum value, although the stress redistribute. According to the HRR field, the crack propagation value \( D_i \) is obtained:

\[
D_i = \frac{J_i}{\alpha \sigma_0 \varepsilon_0 I_n} \left( \frac{\sigma_{yy \text{max nominal}}}{\sigma_0 \sigma_\theta} \right)^{-\theta = 0^n} \tag{14}
\]

where, \( \sigma_{yy \text{max nominal}} \) represents the average critical stress in \([0~l]\) region, as shown in figure 8. For CT specimen with \( a/W=0.5 \) of A508-3 steel, the average stress triaxiality \( \sigma^*_\text{nominal}=2.9 \), as shown in figure 11. Therefore, the fracture critical Y-stress is obtained as follows:

\[
\sigma_{yy \text{nominal}} = \phi(\sigma^*_\text{nominal}) = \ln(\sigma^*_\text{nominal}) + \frac{S}{n} \sigma_{yy \text{nominal}} < \sigma_{yy \text{nominal}} = 2096 \text{MPa} \tag{15}
\]

![Figure 12. Prediction and experimental J- \( \Delta a \) of CT specimen with \( a/W=0.5 \).](image)

The total steady crack propagation value \( \Delta a_t \) is calculated as below,

\[
\Delta a_t = \sum_{i=0}^{n} D_i(i) = \sum_{i=0}^{n} \frac{J_i}{\alpha \sigma_0 \varepsilon_0 I_n} \left( \frac{\sigma_{yy \text{max nominal}}}{\sigma_0 \sigma_\theta} \right)^{-\theta = 0^n} \tag{16}
\]

3.3. Theoretical equation for J- \( \Delta a \)

When \( J<J_0 \), only blunting propagation occurs, but when \( J \) reaches \( J_0 \), blunting propagation and steady propagation occur at the same time. Therefore, the total crack growth value can be obtained,

\[
\Delta a = \begin{cases} 
\Delta a_B = \frac{J_i}{3.75\sigma_0 \left[ \frac{1}{\alpha} \left( \frac{1}{n\varepsilon_0} - \frac{\sigma^*}{\sigma_0} \right) \right]^n} 
& J \leq J_0 \\
\Delta a_j + \Delta a_B = \sum_{i=0}^{n} \frac{J_i}{\alpha \sigma_0 \varepsilon_0 I_n} \left( \frac{\sigma_{yy \text{max nominal}}}{\sigma_0 \sigma_\theta} \right)^{-\theta = 0^n} + \frac{J_i}{3.75\sigma_0 \left[ \frac{1}{\alpha} \left( \frac{1}{n\varepsilon_0} - \frac{\sigma^*}{\sigma_0} \right) \right]^n} & J > J_0
\end{cases} \tag{17}
\]
Combined with equation (12), \( i = J/J_0 \), the relationship between \( J \)-integral and \( \Delta a \) is obtained as follows.

\[
J = \begin{cases} 
3.75\sigma_0 \left[ \frac{1}{\alpha} \left( \frac{1}{n\varepsilon_0} - \frac{\sigma}{\sigma_0} \right) \right]^{\frac{1}{n}} \Delta a_B \\
\sqrt{B^2 + 4\alpha \Delta a - \beta} \\
\frac{2\alpha}{\Delta a}
\end{cases}
\]

\[
\Delta a \leq \frac{J_0}{3.75\sigma_0 \left[ \frac{1}{\alpha} \left( \frac{1}{n\varepsilon_0} - \frac{\sigma}{\sigma_0} \right) \right]^{\frac{1}{n}}}
\]

\[
\Delta a > \frac{J_0}{3.75\sigma_0 \left[ \frac{1}{\alpha} \left( \frac{1}{n\varepsilon_0} - \frac{\sigma}{\sigma_0} \right) \right]^{\frac{1}{n}}}
\]

As we can see in the equation, \( J \)-integral and \( \Delta a \) have a simple quadratic function relationship. All the parameters are obtained by tensile tests on smooth round bar and notched round bar specimens. The Yao-Cai criteria is obtained in tensile tests using notched round bar and center circle sheet specimens. \( J_0 \) is calculated by FEA simulation based on the full-range constitutive relationship and Yao-Cai criteria. Therefore, an analytical method for \( J \)-Resistance curve of ductile materials is established.

4. Prediction results of the analytical method for \( J \)-resistance curve

4.1. Prediction results of CT specimen with \( a/W=0.5 \)

Figure 12 shows the prediction result of CT specimen with \( a/W=0.5 \). As shown in the figure, the prediction result coincides well with the experimental results.

4.2. Prediction results of CT specimen with \( a/W=0.5 \) and \( a/W=0.7 \)

Figure 13 shows the stress triaxiality distribution in front of the crack tip of CT specimen with \( a/W=0.5 \) and \( a/W=0.7 \). Table 3 shows the parameters for \( J \)-\( \Delta a \) curve of the two types of specimens. The stress triaxiality value of CT specimen with \( a/W=0.5 \) is larger than the specimen with \( a/W=0.7 \). Therefore, the constraint for CT specimen with \( a/W=0.5 \) is larger than in the case of the specimen with \( a/W=0.7 \). The critical \( Y \)-stress is also different. Stress triaxiality represents the in-plane constraint effect for fracture toughness. Figure 14 shows the prediction results and experimental results, both coincide well.
Figure 13. Stress triaxiality distribution in front of crack tip of the two types of specimens.

Figure 14. Prediction and experimental $J - \Delta a$ of CT specimen with $a/W=0.5$ and $a/W=0.7$.

Table 3. parameters for $J - \Delta a$ curve of the two types of specimens.

| CT specimen | $D_0$/mm | $\sigma^*_\text{nominal}$/MPa | $\sigma_{\text{ymax}}$/MPa | $J_0$/MPa $\cdot$ mm |
|------------|----------|-----------------|----------------|----------------|
| $a/W=0.5$  | 0.057    | 2.97            | 2096           | 222           |
| $a/W=0.7$  | 0.055    | 2.48            | 1971           | 156           |

4.3. Prediction results of CT, SENB and CIET specimens with $a/W=0.5$

The prediction $J - \Delta a$ curve for three types of specimens are shown in figure 15. As we can see, the analytical method can preferable predict the $J - \Delta a$ curve of different constraint types of specimens.

Figure 15. Prediction and experimental $J - \Delta a$ of CT, SENT and CIET specimen with $a/W=0.5$.

5. Discussions and conclusion

Based on the true stress-strain curve of ductile materials, the Yao-Cai fracture criterion for the A508-3 steel is obtained. By dividing the crack propagation procedure into two parts, the relationship between $J$-integral and blunting propagation value $\Delta a_b$, steady propagation value $\Delta a_c$ was obtained, respectively. In plane-strain condition, the stress distribution satisfies the HRR stress field. Thus, the quadratic function relationship $J$-integral and $\Delta a$ is obtained. The analytical method can be concluded as follows,
• Finish the tensile experiment on various types of specimens;
• Calculate the full-range constitutive relationship basing on tensile test on notched-round specimen;
• Calculate Yao-Cai criteria employing data from tensile experiment on notched-round and flat with a hole specimens;
• Establish the basic parameters for FEA from the full-range constitutive relationship;
• Predict the \( J-\Delta a \) curve of ductile material by equation (17), where \( J_0 \) is obtained by FEA. Therefore, by finish a small number of tensile tests on ductile material, the \( J-\Delta a \) curve for different constraint can be obtained. This can reduce material consumption.

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