Research and Application on Mathematical Model of Ship Maneuvering Motion under Shallow water effect

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Abstract. When ships sail on the inland river in mountainous areas, the limited water depth can frequently lead to shallow water effect. To some extent, safety and maneuverability of shipping will be impacted. Consequently, it's necessary to consider the shallow water effect when studying the ship ascending rapids in mountainous rivers. By considering the impact of shallow water effect, a three freedoms maneuvering mathematical model based on shallow water effect was established in this paper on the basis of two-dimensional water flow mathematical model. Application of the model was implemented on the regulation of the waterway in Suoluo rapids section of the Lancang River to obtain ship navigating parameters (helm angle, drift angle, heading angle, ship's speed and speed against the water) for comparative analysis. Results show that the ship can reduce the impact of shallow water effect by adjusting the rudder angle and reducing the ship speed when navigating in shallow and narrow channel. The obtainment of the voyage line diagram by the simulation of ship's maneuverability test validates that the ship can ascending rapids through self-propulsion after the river regulation.

1. Introduction

At present, there are many studies on the mathematical model of ship maneuvering motion at home and abroad[1-3]. Li Liangwei and Du Jianye use the mathematical model of ship motion to simulate the collision environment of ships in marine environment[4]. Li Anbin and Liu Xiaoping navigate through the mathematical model of ship motion. Simulation of the hydrodynamic force between the ship and the bridge piers caused the change of the ship's movement posture due to the different sailing speeds[5]. Xiao Zhongming, Zhang Faming and others used dynamic overlapping grid technology to realize the ship's navigation posture change[6], and verified the feasibility of the method in the numerical simulation of ship maneuverability.

With the rapid development of the economy and comprehensive transportation system, in order to adapt to the continuous improvement of contemporary new types of ships to super-large ships and high-speed cruises, the development of inland water transport in mountainous areas has received more and more attention. The development of inland waterways is the foundation for ensuring shipping safety and economic development in the west China[7]. In mountain rivers, the characteristics of waterways such as steep slopes, rapid currents, narrow waterways, and shallow water depths are prominent, and it is
necessary to consider the shallow water effect. Nowadays, more and more attention has been paid to the development of water transportation in inland waterways, and it is necessary to study the maneuverability of ships in mountainous inland waterways considering the shallow water effect. When a ship sailing in shallow water reaches, the speed of the water around the hull increases and the resistance increases. In severe cases, it will cause the ship to absorb the bottom, which is called the shallow water effect of the ship.

Based on a depth integrated two-dimensional mathematical model, this paper establishes a mathematical model of ship maneuvering motion, combined with the regulation of the waterway of Suoluo rapids, considering the shallow water effect. Characteristics of ship maneuvering motion was analyzed. Results show that shallow water effect is of great significance to ship navigation safety. This study can provide effective suggestions for mountain channel regulation, and has engineering significance and can be used in practical application.

2. Ship maneuvering mathematical model under shallow water effect

2.1. Basic equations

Ship maneuvering mathematical model could develop application base on water flow model which provides it with basic flow field distribution information. The actual motion of the ship processes complex and variable circumstances because of its six freedoms. The effect of the ship's heaving, pitch and rolling will be negligible, since there is an exclusive situation that the ship move in the horizontal plane. The main consideration is the motion of the ship in three degrees of freedom (advance, transverse shift and cycle). The ship motion can be described on the basis of Newton's second principle.

\[
\begin{align*}
X_0 & = m \dot{x}_{\text{ref}} \\
Y_0 & = m \dot{y}_{\text{ref}} \\
N & = I_z \dot{\psi}
\end{align*}
\]  \hspace{1cm} (1)

In Eq. (1), \(X_0\) is the component of the resultant external force acting on the ship in the \(x_0\)-axis directions; \(Y_0\) is the component of the resultant the external force acting on the ship in the \(y_0\)-axis directions; \(N\) is the gyroscopic moments about the vertical center of gravity of shaft; \(m\) is the quality of the ship; \(\psi\) is the heading angle of the ship; \(x_{0G}, y_{0G}\) is the coordinate of the ship's center of gravity \(G\) at time \(t_0\); \(I_z\) is the mass moment of inertia of the ship around the \(Z\) axis.

2.2. Ship maneuvering mathematical model

2.2.1. Motion equation of deep-water ship in still water

Considering additional inertia moment, additional momentum and additional moment of momentum decomposition formula, fluid inertial force and moment of inertia decomposition formula, assuming that the ship is symmetrical, the equation of motion of a deep-water ship in still water can be derived as:

\[
\begin{align*}
(m + m_{11}) \dot{v}_x - (m + m_{22}) v_y r & = X + (m + m_{11}) \dot{v}_y + (m + m_{22}) v_x r \\
(m + m_{11}) \dot{v}_y - (m + m_{22}) v_x r & = Y + (m + m_{66}) \dot{r} \\
(I_{zz} + m_{11}) \dot{r} & = N
\end{align*}
\]  \hspace{1cm} (2)

In Eq. (2), \(v_x, v_y\) is the relative speed of the water flow and the hull; \(r\) is the heading angular velocity of the hull; \(X\) and \(Y\) are the resultant forces acting on the ship’s \(x\)-axis and \(y\)-axis directions; \(N\) is the resultant moment about the vertical axis of the ship's center of gravity; \(m_{11}, m_{22}\) are the additional mass in the \(x\)-axis direction of the hull and the \(y\)-axis direction of the hull; \(m_{66}\) is the additional moment of inertia in the \(Z\)-axis direction of the hull.

2.2.2 Motion equation of deep-water ship in moving water

When the ship is navigating under dynamic water conditions, the effect of unsteady and non-uniform flow needed to be considered. By deducing the formula, the deep-water ship motion equation in moving water can be written as:

\[
\begin{align*}
(m + m_{11}) \dot{u}_x & = X(v_x, v_y, r) + (m + m_{22}) V_F \sin (\psi_F - \psi) \\
(m + m_{22}) \dot{u}_y & = Y(v_x, v_y, r) - (m + m_{11}) V_F \cos (\psi_F - \psi) \\
(l_z + m_{66}) \dot{r} & = N(v_x, v_y, r)
\end{align*}
\]  \hspace{1cm} (3)
In Eq. (3), $V_f$ is the absolute velocity and direction of water flow; $u_x$, $u_y$ are the origin speed of the attached coordinate; $V_x$ and $V_y$ are ship's speed to water; is heading angle.

2.3. Equation of ship motion in shallow water
Based on the mathematical model of deep-water ship maneuvering motion, considering the shallow-water effect, modifications are made for the hydrodynamic derivatives that are affected by the shallow-water effect. Then the ship motion equation in shallow water can be written as:

$$
(m+m_1)v_x-(m+m_2)v_y=r X
(m+m_2)v_y+(m+m_1)v_x=r Y
(I +m_1f_x)f_y=N-Y_Hx_c
$$

(4)

In Eq. (4), $v_x$, $v_y$ is the relative speed of the hull and the current, see 2.2 for other parameters.

2.4. Algorithm of other forces on ship in shallow water
Regarding the added mass, additional inertia moment, longitudinal fluid dynamic, dynamic moment, propulsion thrust, torque, rudder force suffered by the ship in shallow water, the relevant forces and parameters can be calculated in accordance with Jia Xin-le, Yang Yan-sheng[9].

3. Model validation

3.1. Water flow model validation
After constructing a two-dimensional mathematical model of water flow, tests need to ensure its accuracy. The test section is selected from the end of the Lancang River Nuozhadu Reservoir to Boundary Monument No. 243. When the low water level ($Q=1635m^3/s$), the measured velocity data of the boundary marker section and the measured water level of the left bank section are compared and analyzed together with the numerical simulation results. The difference between the two is very small, which indicates that the measured value is more consistent with the simulated value.

![Water level verification and flow rate verification diagram](Q=1635m^3/s)

3.2. Ship manoeuvrung mathematical model validation.
The ship type 500t(Total length:52.6m; Depth of model: 2.70m; Design draft: 1.95m; Standard discharge:594.2t) was selected in this research to calculate the hydrostatic straight line navigation performance and hydrostatic cyclone performance in order to ensure the accuracy of experiment, which is equivalent to testing the propulsion performance and ship steering performance. The mathematical model was used to calculate out the ship's hydrostatic straight line navigation performance and hydrostatic cyclone performance.
Fig. 2. Propulsion and Gyroscopic Performance Calculations for 500t Class Cargo freighters

The simulation results show that the speed, track and heading of the ship in a straight line in still water are reasonable[10], and the mathematical model of ship motion can be used in the research of more practical problems.

4. Application of a mathematical model of ship maneuvering motion considering shallow water effect

4.1. Reach survey

The Suoluo rapids studied in this paper belongs to the Lancang River Basin, located between 94° ~ 102°E longitude and 21°20′ ~ 33°40′N latitude. The Lancang River generally flows from north to south. Suoluo rapids is located 65km downstream of Jinghong and belongs to the alluvial rapids at the mouth of the river during the dry season. The width of the river where the beach is located is 70m, which is relatively narrow. The water flow of this beach mainly drains from the upper left bank, and the main flow is blocked when it reaches the location of the rock pan and floodplain bayonet on the left bank, resulting in a steep slope and poor flow regime. According to the measured data of this beach in May 2016, the maximum surface velocity is 4.05m/s, and the maximum surface drop is about 7.8‰. The Suoluo rapids section itself is relatively narrow. During the dry season, the existence of rock disks and shallow floodplains on the left bank of the beach will compress the main channel and make the navigation width insufficient. Therefore, it is difficult for ships to sail upstream on the beach.

4.2. Regulation scheme

In response to the obstruction characteristics and evolution of this beach, the improvement way is to take measures to clear the rock pans in the beach risk, set choke point in the shallow flooded, and intercept rocks and sand from tributaries to widen the channel for improving flow patterns. Additionally, it is necessary to increase the flow-water area, slow down the flow velocity and smooth the water surface gradient.

Fig. 3. Layout of channel regulation
4.3. Simulation of ship maneuvering motion in Shallow water effect

4.3.1. Introduction to Numerical Simulation Program. The test layout can be roughly divided into two parts. Firstly, the two-dimensional numerical simulation of water flow based on the actual topographical river section data of Suoluo rapids was simulated and obtain flow field data to provide flow field conditions for the numerical simulation of ship maneuvering motion. Secondly, based on the mathematical model of deep-water ship maneuvering motion, a mathematical model of shallow-water ship maneuvering motion is established. The mathematical model of shallow-water ship maneuvering motion is combined with flow field data to simulate the ship's beach movement. Then through the program output the changes of various parameters of the ship in the motion engineering, and draw the track line.

4.3.2. Simulation process. When the flow field is Q=1635m$^3$/s, set the initial speed to 5.45m/s and the initial heading angle to -21° (the angle between the bow direction and the true north direction, clockwise is positive), and the ship is self-propelled considering the shallow water effect. The upper beach motion simulation, the entire Suoluo rapids section simulated sailing for 600s. After the rectification of the stream section of the Suoluo rapids, the ship can sail to the beach by itself after adjusting the rudder angle. The range of various parameters of the ship's self-propelled beach access can be seen in Figure 5. The rudder angle is -6 to 2°, the drift angle is -4.42 to 4.14°, and the opposite shore speed is 2.30 to 5.45 m/s.

![Fig.4.Ship's trajectory map of Suoluo rapids section (shallow water effect)](image)

![Fig.5.The 500t representative ship maneuvering parameter map of Suoluo rapids](image)

5. Conclusion
This paper combines the mathematical model of ship maneuvering motion under the shallow water effect on the basis of two-dimensional currents, and applies it to the navigating of Suoluo rapids. Finally, the following conclusions and recommendations are obtained:

In this paper, the mathematical model of ship maneuvering motion established based on the two-dimensional mathematical model of water flow provides accurate flow field information for the ship maneuvering motion in the mountainous inland waterway.

Relying on the two-dimensional flow field and surrounding environment of the Suoluo rapids
section of the Lancang River, after the river section is renovated, the impact of shallow water effects on ship navigation is considered. The mathematical model of ship motion and simulation are used to output ship manipulation related parameters, which can be adapted. In the mountainous inland waterway, and provide reliable real-time simulation, for its navigation, and provide accurate maneuvering parameters for ship navigation.

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