BEYOND THE CHAMELEON MECHANISM

D. F. MOTA\textsuperscript{1} and D. J. SHAW\textsuperscript{2}

\textsuperscript{1} Institut f"ur Theoretische Physik, Universit"at Heidelberg, D-69120 Heidelberg, Germany
\textsuperscript{2} School of Mathematical Sciences, Queen Mary, University of London, London E1 4NS, UK

As a result of non-linear self-interactions, in chameleon theories where the field couples to matter much more strongly than gravity does, the fifth force between two bodies with thin-shell is independent of their coupling to the field. As a consequence the bounds on the coupling coming from terrestrial tests of gravity, measurements of the Casimir force and those constraints imposed by the physics of compact objects, big-bang nucleosynthesis and measurements of the cosmic microwave background anisotropies can be exponentially relaxed.

Stringent experimental limits on the properties of light scalar fields coupled to matter can be relaxed if the scalar field theory in question possesses a chameleon mechanism\textsuperscript{11}. This mechanism provides a way to suppress the forces mediated by the scalar fields via non-linear field self-interactions. A direct result of these self-interactions is that the mass of the field is no longer fixed but depends on the ambient density of matter. The properties of these scalar fields therefore change depending on the environment; for this reason such fields have been dubbed chameleon fields. We consider theories where the chameleon field, $\phi$, has a self-interaction potential given by: $V(\phi) = M^4 (\beta \rho/M)^n$. If $M \sim 2 \times 10^{-3}$ eV $\approx (0.1 \text{ mm})^{-1}$ the chameleon may play the role of dark energy\textsuperscript{2}. The equation of motion for $\phi$ is:

\[ -\Box \phi = V_{,\phi}(\phi) + \beta \rho/M_{\text{pl}}, \]

where $\rho$ is the energy density of matter and $\beta$ parametrizes the strength of the coupling to matter relative to that of gravity; $c = h = 1$; $M_{\text{pl}} = 1/\sqrt{8\pi G}$. The right hand side of Eq. (1) vanishes when $\phi = \phi_c(\rho)$:

\[ \phi_c(\rho) = M \left( \beta \rho/(nM_{\text{pl}}M^3) \right)^{-\frac{1}{n+1}}. \]

For $\phi_c(\rho)$ to be real when $\beta \rho > 0$, we need either $n \geq 0$ or for $n$ to be negative and even; and $n \neq 0, -2$ for the theory to be non-linear. The mass of small perturbations about $\phi = \phi_c$ is $m_c = \sqrt{V_{,\phi\phi}(\phi_c)} = \sqrt{n(n+1)}M |M/\phi_c|^{n/2+1}$.

A body is said to have a thin-shell if $\phi$ is approximately constant everywhere inside the body, except in a thin-shell near the surface of the body where large changes in its value occur. Deep inside a body with a thin-shell $\phi$ is constant, and so we might expect $\phi = \phi_c(\rho)$. The effective chameleon mass, $m_{\text{eff}}$, in the body would then be given by $m_{\text{eff}} = m_c(\rho)$. The thin-shell effect is in fact the reason why chameleons, which couple to matter with the similar strength as gravity does, $\beta \sim \mathcal{O}(1)$, evade experimental bounds on fifth forces and WEP violations. This effect suppresses the $\phi$-force between large objects. This is because experiments, in reality, probe only an effective coupling given by: $\beta_{\text{eff}} = 3\beta \cdot \frac{\Delta R}{R}$. Where $R$ is the radius of an object and $\Delta R$ is the size of the (thin) region at the surface of the body where all variations in $\phi$ occur\textsuperscript{11}. However,
the original analysis\[1\] was only valid for a coupling of gravitational strength, $\beta \leq O(1)$, and required that Eq. \[1\] could be linearized. In fact, the non-linear nature of the potential plays actually a fundamental role. This is particularly apparent if one considers the strong coupling regime, $\beta \gg 1$, and becomes clear when determining the effective, large-scale behaviour of the chameleon\[2\] Eq. \[1\] defines the microscopic, or particle-level, field theory for $\phi$, whereas in most cases we are interested in the large scale or coarse grained behaviour of $\phi$. The effect of the non-linearities on the averaging is to limit the averaged value of $m_\phi$ to be smaller than some critical value, $m_\text{crit}$\[3\] $m_\text{crit}$ is a macroscopic quantity but it depends only on the microscopic properties of the body and the index $n$. It is independent of $\beta$ and $M$\[3\] Modelling the body as being composed of particles of radius $R_p$ separated by an average distance $d_p$, the macroscopic mass of the chameleon in the body can be shown to be $m_\text{eff} = \min (m_c(\rho), m_\text{crit})$, where:

$$m_\text{crit} \approx \sqrt{3|n+1|d_p^{-1}(R_p/d_p)^{4(n+4)/3}}, \quad n \neq -4,$$

where $q(n) = \min(1, (n+4)/(n+1))$. Whenever $m_\text{eff} = m_\text{crit}$ is it because the individual particles that make-up the body have themselves developed thin-shells. This critical behaviour emerges from the requirement that non-linear effects are negligible outside of the particle from $r \gtrsim R_p$ to $r = d_p$: this implies a maximal value of $m_\text{eff}$, i.e. $m_\text{crit}$, that depends only on $R_p$, $d_p$ and $n$. The $n$ appears as it determines precisely when linearized theory breaks down\[3\].

The $\beta$-independent critical behaviour is also seen in the $\phi$-force between two bodies. The onset of this critical behaviour is linked to the emergence of a thin-shell. A body of radius $R$ and density $\rho_c$ in a background of density $\rho_b \ll \rho_c$ has a thin-shell if:

$$m_\text{eff} R \geq \sqrt{3|n+1|}\left|1 - (\rho_c/\rho_b)^{1/(n+1)}\right|^{1/2}, \quad n \neq -4. \quad (2)$$

The existence of a thin-shell is essentially due to non-linearities being strong near the surface of a body but weak in other regions. When $n > 0$, $(\rho_c/\rho_b)^{1/(n+1)} \gg 1$ and so the thin-shell condition, eq.\[2\], depends greatly upon the background density. The same is not true when $n \leq -4$ since here $(\rho_c/\rho_b)^{1/(n+1)} \ll 1$. Therefore $n > 0$ theories can behave differently in space-based experiments than they do in laboratory ones, because the thin-shell condition is more restrictive in low-density background of space than it is in the lab\[3\]. In contrast, there will be no great difference between the predictions of $n \leq -4$ theories for space and ground based tests.

The existence of a thin-shell in the test-masses used in experimental searches for deviations from general relativity is vital if we are to evade their bounds. Whereas the force between two non-thin-shelled bodies with separation $r$ is $2\beta^2(1 + m_b r)e^{-m_b r}$ times the gravitational force between them ($m_b$ is the chameleon mass in the region between the bodies), the force between two bodies, of masses $M_1$ and $M_2$, with thin-shells is found to be independent of the coupling $\beta$\[3\]. It is for this reason that strong couplings, $\beta \gg 1$, are allowed. When $d \gg R_1, R_2$, where $R_1$ and $R_2$ are the respective radii of the two bodies, this force is found to be $\alpha_{12}$ times the strength of gravity, where for $n \neq -4$:

$$\alpha_{12} = \frac{S(n, m_b)M_1^2(1 + m_b r)e^{-m_b r}}{M_1^2M_2}(M^2R_1R_2)^{q(n)},$$

where $S(n, m_b)$ is $(3/|n|)^{2/[n+2]}$ for $n < -4$, whereas for $n > 0$ it equals $(n(n+1)M^2/m_b^2)^{2/(n+2)}$. This $\beta$-independence was previously noted for $\phi^4$ theory\[4\].

The $\beta$-independence can be understood as follows: just outside a thin-shelled body, the $V_\phi$ in eq. \[1\] is large and negative ($\sim O(-\beta \rho/M_\text{pl})$), and so $\phi - \phi_b$ decays very quickly. At some point $\phi - \phi_b$ reaches a critical value, $\delta \phi_\text{crit}$, that is small enough so that non-linearities are no longer important. Since this all occurs outside the body, $\delta \phi_\text{crit}$ can only depend on the size of the body, the choice of potential $(M, n)$ and the value (and hence mass) of $\phi$ in the background.
This $\beta$-independence is of great of importance if one wishes to design an experiment to detect the chameleon through WEP violations. Since the $\phi$-force is independent of the coupling, $\beta$, for bodies with thin-shells, any microscopic composition dependence in $\beta$ will be hidden on macroscopic length scales. The only ‘composition’ dependence in $\eta$ comes from the bodies and their dimensions ($R_1$ and $R_2$). If we measure the differential accelerations of two test masses, $M_1$ and $M_2$, of radii $R_1$ and $R_2$ towards a third body, mass $M_3$ and radius $R_3$, then the Eötvos parameter is $\eta = \alpha_{13} - \alpha_{23}$. Taking the third body to be the Sun or the Moon, experimental searches for WEP violations currently limit $\eta \leq 10^{-13}$. In most of these searches, although the composition of the test-masses is different, they have the same mass ($M_1 = M_2$) and the same size ($R_1 = R_2$). Therefore, if the test-masses have thin-shells we have $\eta = 0$ and no WEP violation will be detected. The only implicit dependence of this result on $\beta$ is that the larger the coupling is, the more likely it is that the test-masses will satisfy the thin-shells conditions. Hence, to detect a chameleon field through WEP violations, the test-masses must either not satisfy the thin-shell conditions or have different masses and/or dimensions.

Using two spherical test bodies both with a mass of $10^{12} g$, where one is made entirely of copper and the other of aluminium. The strongest bounds on chameleon fields would then come from measuring the differential acceleration of these bodies towards the Moon. We indicate in FIG.1 the restrictions that finding $\eta \leq 10^{-13}$ in such an experiment would place on these chameleon theories. The Moon is a better choice of attractor than the Earth or the Sun for such experiments since $\alpha_{13}$ is proportional to $M_2^2/M_1 M_3$ and so the smaller mass of the test-bodies, $M_1$, and the attractor, $M_3$, the larger $\eta$ will be compared to gravity. The corollary of this result is that if we are unable to detect $\phi$ in lab-based, micro-gravity experiments where both $M_1$ and $M_2 \sim O(10^g)$ then the $\phi$-force between larger objects, would also be undetectably small. For this reason measurements of the differential acceleration of the Earth and Moon towards the Sun, e.g. lunar laser ranging, are not competitive with lab-based experiments. Future, space-based tests of WEP promise to be able to detect $\eta$ up to a precision of $10^{-18}$; we indicate on FIG.1 how such tests would improve the constraints.

The $\phi$-mediated force will also produce effective corrections to the $1/r^2$ behaviour of gravity, which are constrained by the Eöt-Wash experiment which probes gravity over separations $d \geq 0.1\text{mm}$, with the bound being $\alpha_{12} \leq 10^{-2}$. For a chameleon theory to satisfy this bound we need the tests masses to have thin-shells. In this scenario $d$ is small compared to the size of test-masses ($d < R_1, R_2$) and so the previous formula for $\alpha_{12}$ does not apply. When the mass of the chameleon inside the test masses, $m_\phi$, obeys $m_\phi d \gg 1$ (as is the case for $\beta \geq 1$) we find that the $\phi$-force is $\alpha$ times the strength of gravity, where $\alpha_{12}$ is

$$5 \times 10^{-4} \left( \frac{M}{(0.1\text{mm})} \right)^{2(n+1)/(n+2)} \left( \frac{\sqrt{2}B \left( \frac{1}{2}, \frac{1}{2} + \frac{1}{n} \right)}{|n|d/0.1\text{mm}} \right)^{\frac{2}{n+2}},$$

where $B(p, q)$ is the beta function. We note that $\alpha$, as before, is independent of $\beta$.

In this experiment a uniform $d_s = 10\mu\text{m}$ thick BeCu membrane is placed between the test masses to shield electromagnetic forces. For $O(1)$ values of $\beta$ or $M \sim (0.1\text{mm})^{-1}$ this sheet does not have a thin-shell and makes little difference to the analysis. For slightly larger values of $\beta$ however it will develop a thin-shell. Taking the chameleon mass inside the sheet to be $m_s$, the effect of this membrane is to attenuate $\alpha_{12}$ by a factor of $\exp(-m_s d_s)$. The larger $\beta$ becomes, the larger $m_s$ is and the less restrictive this bound becomes.

The prospect that light scalar fields with couplings $\beta \gg 1$ could be allowed was exciting and opens the door to many interesting and testable effects. But to be taken seriously we must also consider bounds coming from astrophysical constraints, such as the stability and mass-radius relationship of white dwarfs and neutron stars as well as bounds coming from BBN and the CMB anisotropies. These bounds can be summarized as requiring $|\beta \phi/M_{pl}| \leq 0.1$ over the
whole universe since the BBN epoch\cite{23}. In figure\ref{fig:bounds} we show the bounds on chameleon models from the astrophysical and cosmological observations.

In summary, chameleon fields strongly coupled to matter can be dark energy candidates and avoid at the same time stringent gravity experiments and cosmological bounds. The reason is due to the surprising result that the chameleon force between two bodies with thin-shell is independent of their coupling to the field \( \phi \), and that as a result the bounds on the coupling, \( \beta \), can be exponentially relaxed. When the chameleon plays the role of dark energy the strongest upper bounds on \( \beta \) probably come from particle colliders and \( 200\text{GeV} \leq M_{pl}/\beta \leq 10^{15}\text{GeV} \) is allowed for all \( n \). If \( M_{pl}/\beta \sim 1\text{TeV} \) we might even hope to see chameleon production at the LHC; Planned space-based tests such as STEP, MICROSCOPE and SEE, promise improved precision and, when \( n > 0 \) there is also still the possibility that WEP violations in space can be stronger than the level already ruled out by laboratory based experiments.

Acknowledgments

DFM and DJS acknowledge many discussions with P. Brax, C. van de Bruck, A.C. Davis, J. Khoury and A. Weltman. DFM and DJS are funded by Humboldt Foundation and STFC.

References

1. J.Khoury, A.Weltman, Phys.Rev. D\textbf{69}, 044026 (2004); Phys.Rev.Lett. 93, 171104 (2004)
2. P.Brax, C.van de Bruck, A.C.Davis, J.Khoury, A.Weltman, Phys.Rev. D\textbf{70}, 123518 (2004)
3. D.F.Mota, D.J.Shaw, Phys.Rev. D\textbf{75}, 063501 (2007); Phys.Rev.Lett. \textbf{97}, 151102 (2006).
4. B.Feldman, A.E.Nelson, JHEP \textbf{0608}, 002 (2006)
5. H.Gies, D.F.Mota, D.J.Shaw, Phys.Rev. D\textbf{77}, 025016 (2008); P.Brax, C.van de Bruck, A.C.Davis, D.F.Mota, D.Shaw, Phys.Rev. D \textbf{76}, 124034 (2007); P.Brax, C.van de Bruck, A.C.Davis, D.F.Mota, D.J.Shaw, Phys.Rev. D\textbf{76}, 085010 (2007); S.Das, N.Banerjee, \texttt{arXiv:0803.3936} [gr-qc]; A.W.Brookfield et al., Phys. Rev. D\textbf{73}, 083515 (2006); A.E.Nelson, J.Walsh, \texttt{arXiv:0802.0762} [hep-ph]; P.Brax, C.van de Bruck, A.C.Davis, Phys.Rev.Lett. \textbf{99}, 121103 (2007); B.Li, J.D.Barrow, D.F.Mota, Phys. Rev. D \textbf{76}, 104047 (2007); D.A.Easson et al., JCAP \textbf{0802}, 010 (2008); S.Nojiri, S.D.Odintsov, Mod. Phys. Lett. A \textbf{19}, 1273 (2004); S.Capozziello, S.Tsujikawa, \texttt{arXiv:0712.2268} [gr-qc]; D.F.Mota et al., Mon. Not. R. Astron. Soc. 382, 793-800 (2007), \texttt{arXiv:0708.0830} [astro-ph]; H.Wei, R.G.Cai, Phys. Rev. D \textbf{71}, 043504 (2005).