 Trap Surface Formation 
in High-Energy Black Holes Collision

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Abstract

We investigate classical formation of a trap surface in $D$-dimensional Einstein gravity in the process of a head-on collision of two high-energy particles, which are treated as Aichelburg-Sexl shock waves. From the condition of the trap surface volume local maximality we deduce an explicit form of the inner trap surface. Imposing the continuity condition on the fronts we obtain a time-dependent solution for the trap surface. We discuss trap surface appearance and evolution.

1 Introduction

Now we see a significant interest to the processes of black hole production in ultra-relativistic particle collisions. A possibility of such production was shown in a series of papers of Amati, Ciafaloni and Veneziano [1] and ’t Hooft [2]. In [3], Aref’eva, Viswanathan and Volovich considered a scenario for similar process by using the Chandrasekhar-Ferrari-Xanthopoulos duality between the Kerr black hole solution and colliding plane gravitational waves.

A new interest to this problem appeared after the proposition of Arkani-Hamed, Dimopoulos and Dvali in [4, 5] for solving the hierarchy problem based on assumption of existence of additional spatial dimensions large compared to the weak scale. The fundamental Planck mass could be of the order of a few TeV which makes plausible observations of effects of extra dimensions existence in future colliders experiments or cosmic rays and astrophysical measurements (see e.g. [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]).

There are some theoretical works [37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48] in which this process was analysed in different aspects.

In [44], Eardley and Giddings developed a trap surface approach [51, 52] in $D$-dimensional Einstein gravity for calculating the cross-section of black hole formation in high-energy collision of two massless particles, which were treated as Aichelburg-Sexl shock waves [49, 50]. In [46], Yoshino and Nambu investigating the same task in the case of a head-on collision used the $T = const$ slicing of space-time in the region between

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the shock-waves to explore horizon formation. However, they obtained the surface that is discontinuous at the shocks and therefore cannot have trapping properties [51, 52].

In this article, we use a similar to [46] space-time slicing to consider the problem of trap surface emergence and evolution. We find a continuous solution for the surface and investigate it’s dynamic.

The paper is organized as follows. We begin in Section 2 by discussing singular and nonsingular forms of the task metric. In Section 3, we formulate an extremum volume method for trap surface finding and obtain explicit expressions describing the inner surface in $D$ space-time dimensions. In Section 4, we find solutions for fronts trap surface and use continuity conditions to determine the whole surface. Here, we present dependencies of surface parameters on time and $D$. Finally, in Section 5, we summarise and analyse our results.

2 Shock waves metric

Coordinate system. We use a Minkowski coordinate system $(\tilde{t}, \tilde{z}, \tilde{x}^i)$ where $\tilde{z}$ is the direction of particles motion and $\tilde{x}^i$, $i = 1 \ldots D - 2$ are transverse coordinates. Let $(\tilde{u}, \tilde{v})$ be light-cone coordinates $(\tilde{t} - \tilde{z}, \tilde{t} + \tilde{z})$. The particles are moving along $\tilde{z}$ axis in the opposite directions with zero impact parameter ($\tilde{x}^i = 0$).

Schwarzschild metric. In the rest-frame of a sole isolated particle with mass $M$ metric is described by $D$-dimensional Schwarzschild solution

$$ds^2 = - \left(1 - \frac{16\pi G_D M}{(D-2)\Omega_{D-2} R^{D-3}}\right) dt^2 + \left(1 - \frac{16\pi G_D M}{(D-2)\Omega_{D-2} R^{D-3}}\right)^{-1} dR^2 + R^2 d\Omega_{D-2}^2,$$

(1)

where $d\Omega_{D-2}^2$ and $\Omega_{D-2}$ are the line element and volume of the unit $(D-2)$-sphere $[\Omega_n = 2\pi^{(n+1)/2}/\Gamma((n+1)/2)]$ and $G_D$ is the $D$-dimensional gravitational constant.

One shock wave metric. The gravitational solution for each of the incoming particles can be found by boosting this solution, taking the limit of large boost and small mass $M$, with fixed total energy $\mu$. The result for a particle moving in the $+z$ direction is the Aichelburg-Sexl [49, 50] metric

$$ds^2 = -d\tilde{u}d\tilde{v} + d\tilde{x}^2 + \Phi(\tilde{x}^i)\delta(\tilde{u})d\tilde{u}^2.$$

(2)

Function $\Phi$ depends only on the transverse radius $\tilde{r} = \sqrt{\tilde{x}^i\tilde{x}_i}$, takes the form

$$\Phi = -2a \ln(\tilde{r}) , \quad D = 4 ,$$

(3)

$$\Phi = \frac{2a^{D-3}}{(D-4)\tilde{r}^{D-4}} , \quad D > 4 ,$$

(4)

where

$$a = \left(\frac{8\pi G_D \mu}{\Omega_D M}\right)^{1/(D-3)}$$

(5)

and satisfies the Poisson equation

$$\frac{\partial^2 \Phi}{\partial \tilde{x}^i \partial \tilde{x}_i} = -16\pi G \mu \delta^{D-2}(\tilde{x}^i).$$

(6)
**Change of coordinates.** It is possible to remove singularity in the metric (2) by introducing new coordinates \((u, v, x^i)\) defined by

\[
\begin{align*}
\bar{u} &= u, \\
\bar{v} &= v + \Phi \theta(u) + \frac{u \theta(u) (\nabla \Phi)^2}{4}, \\
\bar{x}^i &= x^i + u \frac{\nabla_i \Phi(x)}{2} \theta(u)
\end{align*}
\]

(here \(\theta\) is the Heaviside step function). In these coordinates, geodesics and their tangents are continuous across the shock plane at \(u = 0\).

**Two shock waves metric.** Metric (2) is flat everywhere except the null plane \(\bar{u} = 0\) of the shock wave. So in order to obtain a two shock waves metric for time \(\bar{t} < 0\) preceding the collision we can combine it with another similar metric corresponding to the particle moving along \(\bar{v} = 0\) in the \(-z\) direction by matching together the regions of flat space which precede each of two waves.

In coordinates (7), the combined metric of the shock waves becomes

\[
ds^2 = -du dv + (H^u_{ik} H^v_{jk} + H^v_{ik} H^u_{jk} - \delta_{ij}) \, dx^i dx^j
\]

where

\[
H^u_{ij} = \delta_{ij} + \frac{1}{2} \frac{\partial^2 \Phi(x)}{\partial x^i \partial x^j} u \theta(u), \quad H^v_{ij} = \delta_{ij} + \frac{1}{2} \frac{\partial^2 \Phi(x)}{\partial x^i \partial x^j} v \theta(v).
\]

It is convenient to introduce spherical coordinates \((r, \vec{\varphi}) \equiv (r, \varphi^k, k = 1, \ldots, D-3)\) in the transverse space \((x^i)\).

**3 Inner trap surface**

The event of the black hole production in a two particle collision signifies that during the collision a space-time region is formed from which no light ray can go to infinity. The border of this region is a trap surface. More precise definition of the trap surface is following [51]. Trap surface \(T\) is a smooth compact space-like \((D-2)\)-surface with the property that light geodesics which cross it orthogonally converge locally in the future-time direction. In particular, this signifies that a volume of a transferred \((D-2)\)-surface formed by these geodesics points placed on a same distance from the trap surface decreases when the distance increases.

**Slice of space-time.** We shall consider the following slice of space-time:

- region I : \(t = z, \quad t \leq T\),
- region II : \(t = T, \quad T \leq z \leq -T\),
- region III : \(t = -z, \quad t \leq T\).

Here \(T \leq 0\) and the particles collision takes place at \((T = 0, z = 0)\).
**Inner trap surface.** According to this slice, equations describing a trap surface \( \mathcal{M} \) in the region II take the form

\[
t = T = \text{const} , \quad z = \delta f(r) , \quad \delta = \text{sgn} z .
\] (10)

Null normals to this surface are given by

\[
N(\varepsilon, \delta) = \left[ N^t, N^z, N^r, N^\varphi \right] (\varepsilon, \delta) = \left[ 1, -\frac{\varepsilon\delta}{\sqrt{1 + f'^2}}, \frac{\varepsilon f'}{\sqrt{1 + f'^2}}, 0 \right] ,
\] (11)

where \( f' = df/dr \) and \( \varepsilon = \pm 1 \).

A null geodesic which normally crosses the trap surface \( \mathcal{M} \) at \( (T, z_0 = f(r_0) \equiv f_0, r_0, \varphi_0) \) is a straight line described by equations

\[
t = T + \tau, \quad z = \delta f_0 - \tau \frac{\varepsilon\delta}{\sqrt{1 + f'^2}}, \quad r = r_0 + \tau \frac{\varepsilon f'_0}{\sqrt{1 + f'^2}}, \quad \varphi = \varphi_0 .
\] (12)

**Transferred surfaces.** Such geodesics transfer the trap surface \( \mathcal{M} \) on the distance \( \tau \) to the surface \( \mathcal{M}(\varepsilon, \tau) \). A volume \( S(\varepsilon, \tau) \) of \( \mathcal{M}(\varepsilon, \tau) \) is equal to

\[
S(\varepsilon, \tau) = \Omega_{D-3} \int r^{D-3} dl ,
\] (13)

where \( dl \) is an element of \( \mathcal{M}(\varepsilon, \tau) \)-surface generator in \((z, r)\)-plane, which according to \( \text{[1]} \) can be expressed through an initial \( dr_0 \) as

\[
dl = \sqrt{dr^2 + dz^2} = \sqrt{ \left( \frac{dr}{dr_0} \right)^2 + \left( \frac{dz}{dr_0} \right)^2 } dr_0 = \sqrt{1 + f'^2_0} \left[ 1 + \frac{\tau\varepsilon f''_0}{(1 + f'^2_0)^{3/2}} \right] dr_0 .
\] (14)

So \( \text{[1]} \) can be rewritten as

\[
S(\varepsilon, \tau) = \Omega_{D-3} \int \left[ r_0 + \frac{\tau\varepsilon f'_0}{\sqrt{1 + f'^2_0}} \right] r^{D-3} \sqrt{1 + f'^2_0} \left[ 1 + \frac{\tau\varepsilon f''_0}{(1 + f'^2_0)^{3/2}} \right] dr_0 .
\] (15)

For the small \( \tau \) this expression takes the form

\[
S(\varepsilon, \tau) = \Omega_{D-3} \int r_0^{D-3} \sqrt{1 + f'^2_0} \left\{ \frac{(D - 3)f'_0}{r_0} + \frac{f''_0}{1 + f'^2_0} \right\} + \frac{\tau^2(D - 3)}{1 + f'^2_0} \left[ \frac{(D - 4)f'^2_0}{2r_0} + \frac{f'_0 f''_0}{r_0(1 + f'^2_0)} \right] + O(\tau^3) \right\} dr_0 .
\] (16)

**Explicit form of an inner trap surface.** Now we can derive the equation for function \( f(r) \) from the demand that for the small \( \tau \) the volume \( S(\varepsilon, \tau) \) decreases when \( \tau \) increases for both \( \varepsilon = \pm 1 \). The necessary condition is the equality to zero of the \( \tau \)-linear term in the right part of \( \text{[1]} \)

\[
\frac{(D - 3)f'_0}{r_0} + \frac{f''_0}{1 + f'^2_0} = 0 .
\] (17)
Integration of this equation gives the explicit form of an inner trap surface

\[ z = \delta R \int_1^{r/R} \frac{d\rho}{\sqrt{\rho^2(D-3) - 1}}, \tag{18} \]

where \( R = R(T) \) is a trap surface radius at \( z = 0 \). For \( D = 4 \) we have a catenoid

\[ r = R \cosh \left( z/R \right). \tag{19} \]

Using (17), we can rewrite the \( \tau \)-second-order term in (16) in the form

\[
\frac{\tau^2(D-3)}{1 + f''_0} \left[ \frac{(D-4)f'^2_0}{2r^2_0} + \frac{f'_0f''_0}{r_0(1 + f'^2_0)} \right] = -\tau^2 \left( \frac{D-2}{2} \right) \frac{f'^2_0}{1 + f'^2_0},
\]

which clearly demonstrates that the term is negative and consequently (17) is also the sufficient condition. In fact, inner trap surface calculation is equivalent to the soap films or minimal surfaces finding task [53].

**Limitation on \( \tau \) range.** Expression (15) is valid if no self-crossing of \( \mathcal{M}(\varepsilon, \tau) \)-surface takes place. In the case of \( \mathcal{M}(-1, \tau) \), geodesic outgoing from \( (z = 0, r = R) \)-surface must not reach \( (z = 0, r = 0) \)-surface which takes place when \( \tau < R \). In the case of \( \mathcal{M}(1, \tau) \), geodesic outgoing from a vicinity of \( (z = 0, r = R) \)-surface must not cross. Since they cross at \( z = 0 \), then according to (12) \( \tau \) cannot exceed \( \tau_{\text{max}} \)

\[
\tau < \tau_{\text{max}} = \lim_{r \to R} f \sqrt{1 + f'^2_0} = \frac{R}{D - 3}. \tag{21}
\]

The small \( \tau \) limit in (16) corresponds to \( \tau \ll R \).

## 4 Trap surface formation

**Front trap surface in the initial coordinates.** From the point of view of observers in outer areas \((|z| > -t)\) collision takes place at

\[ \bar{u} = +0, \quad \bar{v} = +0. \tag{22} \]

Since the metric in these areas is Minkowskian and there is an axial symmetry, these equations can serve as a definition of a trap surface on the shock waves fronts, viz. in the regions I and II.

**Front trap surface in the new coordinates.** In the new coordinates \((u, v, r, \varphi^k)\), the trap surface in the region I according to (7), (22) is given by

\[ u = +0, \quad v + \Phi(r) - \Phi(r_c) = 0 \tag{23} \]

with the null normals \( n = [n^u, n^v, n^r, n^\bar{\varphi}] \) equal to

\[ n_1 = \left[ 0, 1, 0, \vec{0} \right], \quad n_2 = \left[ \left( \frac{r}{a} \right)^{D-3}, \left( \frac{a}{r} \right)^{D-3}, 1, \vec{0} \right]. \tag{24} \]
Table 1: Dependencies of $T_{\text{min}}$, $R(T_{\text{min}})$, $r_b(T_{\text{min}})$ on $D$.

| $D$ | $T_{\text{min}}/a$ | $R_{\text{min}}/a$ | $r_b(T_{\text{min}})/a$ |
|-----|------------------|------------------|------------------|
| 4   | $-0.43$          | 0.36             | 0.65             |
| 5   | $-0.32$          | 0.47             | 0.76             |
| 6   | $-0.26$          | 0.54             | 0.81             |
| 7   | $-0.22$          | 0.60             | 0.85             |
| 8   | $-0.19$          | 0.64             | 0.87             |
| 9   | $-0.17$          | 0.67             | 0.89             |
| 10  | $-0.15$          | 0.69             | 0.90             |

Similarly, the trap surface in the region II is described by

$$v = +0, \quad u + \Phi(r) - \Phi(r_c) = 0$$ \hspace{1cm} (25)

and

$$n_3 = \left[1, 0, 0, \vec{0}\right], \quad n_4 = \left[\left(\frac{a}{r}\right)^{D-3}, \left(\frac{r}{a}\right)^{D-3}, 1, \vec{0}\right].$$ \hspace{1cm} (26)

Smooth transition from (23) to (25) demands $n_2 = n_4$ when $u = v = 0$. This condition gives the value of the trap surface radius at the moment of collision $r_c$

$$r_c \equiv r_b(T = 0) = a.$$ \hspace{1cm} (27)

**Trap surface continuity.** Demanding trap surface continuity on the border of the regions I and II and taking into account (23) and (18) we obtain equations for $R(T)$ and trap surface radius on the border $r_b(T)$

$$T = \frac{1}{2} \left[\Phi(a) - \Phi(r_b)\right] = -R \int_1^{r_b/R} \frac{d\rho}{\sqrt{\rho^{2(D-3)} - 1}}.$$ \hspace{1cm} (28)

Consideration of the regions II and III border produces the same equations.

In [10] where the same problem was treated one of this condition was replaced by the demand of null normals continuity which leads to discontinuity of the surface on the borders. In our approach, normals $N(1,-1)$ and $n_2$ $[N(1,1)$ and $n_4]$ are not equal on the $(u=0)[(v=0)]$-border. So in our case, the surface is continuous but not smooth on the borders.

**Trap surface dynamic.** Dependencies of $R$ on $T$ for $D = 4, 5, 6, 7, 8, 9, 10$ are shown in Fig[1] Trap surface appears at $t = T_{\text{min}}$ when $r_b = r_b(T_{\text{min}})$. Further $r_b$ increases as $T$ increases (Fig[2] and reaches its maximum value $a$ at the moment of collision $T = 0$. Dependencies of $T_{\text{min}}$, $R(T_{\text{min}})$ and $r_b(T_{\text{min}})$ on $D$ are given in Table[1]
From the condition of a trap surface volume local maximality we deduce the explicit form of the inner trap surface \(13\). The form of the fronts surface is given simply by \(\bar{u} = +0\), \(\bar{v} = +0\) \(22\). The condition of trap surface continuity on the region II borders \(23\) provides the dependencies on \(T\) of \(R\) (trap surface radius at \(z = 0\)) and \(r_b\) (trap surface radius at the shocks) for \(D = 4, 5, 6, 7, 8, 9, 10\) which are shown in Fig.1 and Fig.2.

According to these results, a process of trap surface formation looks like follows. There is no trap surface at time preceding \(t = T_{\text{min}}\). It emerges at \(t = T_{\text{min}}\) with radius on the shocks equal to \(r_b(T_{\text{min}})\). Values of \(T_{\text{min}}\), \(R(T_{\text{min}})\) and \(r_b(T_{\text{min}})\) grow with a raise of \(D\) (see Table 1). Later \(r_b\) and \(R\) increase with time and reach their maximum value \(a\) at the moment of collision.

There are many speculations predicting the end of short distance physics. Calculation of the size of horizon formation in black hole collisions may help us to evaluate a distance at which this end may take place.

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Figure 2: Dependencies of \( r_b \) on \( T \) for \( D = 4, \ldots, 10. \)

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