EXCLUSIVE HADRONIC TAU DECAYS AS PROBES OF NON-SM INTERACTIONS

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BASED ON:
S. GONZÀLEZ-SOLÍS, A. MIRANDA, J. RENDÓN, P. ROIG; PHYS.LETT.B 804 (2020) 135371
S. GONZÀLEZ-SOLÍS AND P. ROIG; EUR. PHYS. J. C79 (2019) 436
R. ESCRIBANO, S. GONZÀLEZ-SOLÍS, M. JAMIN AND P. ROIG; JHEP 1409 (2014) 042
Hadronic tau decays

Tau properties:

- Mass: $m_\tau = 1.77686(12) \text{ GeV}$
- Lifetime: $\tau_\tau = 2.903(15) \times 10^{-13} \text{ s}$

The only lepton heavy enough to decay into hadrons:

- Very rich phenomenology
- Test of QCD and EW interactions

For the test:

- Precise measurements needed
- Hadron reactions: tool to search for New Physics
Test of QCD and Electroweak Interactions

- **Inclusive** decays: $\tau^- \rightarrow (\bar{u}d, \bar{u}s)\nu_\tau$
  
  Full hadron spectra (precision physics)

- **Fundamental SM parameters:**
  
  $\alpha_s(m_\tau), m_s, |V_{us}|$

- **Exclusive** decays: $\tau^- \rightarrow (PP, PPP, \ldots)\nu_\tau$
  
  Specific hadron spectrum (approximate physics)

  Hadronization of QCD currents, study of Form Factors, resonance parameters $(M_R, \Gamma_R)$
**TWO-MESON \( \tau \) DECAYS**

- **Invariant mass distribution** \((\tau^- \rightarrow P^- P^0 \nu_\tau)\)

\[
\frac{d\Gamma}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768 \pi^3} S_{EW}^{\text{had}} C_{PP'}^2 \left( 1 - \frac{s}{M_{\tau}^2} \right)^2 \\
\times \left\{ \left( 1 + \frac{2s}{m_\tau^2} \right) \lambda_{P^- P^0}^{3/2} (s) |F_V^{P^- P^0} (s)|^2 + 3 \frac{\Delta_{P^- P^0}^2}{s^2} \lambda_{P^- P^0}^{1/2} (s) |F_S^{P^- P^0} (s)|^2 \right\},
\]

| Decay channel | Standard Model | Resonances |
|---------------|----------------|-------------|
| \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) | Pion form factor, \((g-2)_\mu\) | \(\rho(770), \rho(1450), \rho(1700)\) |
| \( \tau^- \rightarrow K^- K_S \nu_\tau \) | Kaon form factor, \((g-2)_\mu\) | \(\rho(770), \rho(1450), \rho(1700)\) |
| \( \tau^- \rightarrow K_S \pi^- \nu_\tau \) | \(K\pi\) form factor, \(K_{\ell 3}, |V_{us}|\) | \(K^*(892), K^*(1410)\) |
| \( \tau^- \rightarrow K^- \eta^{(l)} \nu_\tau \) | \(K\pi\) form factor, \(K_{\ell 3}, |V_{us}|\) | \(K^*(1410)\) |
| \( \tau^- \rightarrow \pi^- \eta^{(l)} \nu_\tau \) | isospin violation, 2nd class currents | \(a_0(980)\) |

**Important** experimental activity: BaBar, Belle, Belle-II.
The Pion vector form factor $F_{V}^{\pi}(s)$

- Classic object of low-energy QCD
- How to determine $F_{V}^{\pi}(s)$ experimentally?
  - $e^+e^- \rightarrow \pi^+\pi^-$
  - $\tau^- \rightarrow \pi^-\pi^0\nu_{\tau}$
- What do we know theoretically on the form factor?
  - Its low-energy behaviour: given by ChPT (Gasser&Leutwyler’85)
  - Its high-energy behaviour ($\sim 1/s$): given by pQCD (Brodsky&Lepage’79)
  - For the intermediate energy region: models
Dispersive representation

Dispersion relation with subtractions:

\[ F^\pi_V(s) = \exp \left[ \alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{4m^2_\pi}^{s_{\text{cut}}} ds' \left( \frac{\phi(s')}{(s')^3 (s' - s - i\varepsilon)} \right) \right], \]

Form Factor phase \( \phi(s) \):

- Watson’s theorem:
  \[ \phi(s) = \delta_{\pi\pi\to\pi\pi}(s) \]

Models:
\[ \sum_i c_i \text{BW}(s) \]

Diagram:
- \( \pi \rightarrow \pi \)
- \( \pi \rightarrow \pi \)
- \( \pi \rightarrow \pi \)
- \( \pi \rightarrow \pi \)
- \( \pi \rightarrow \pi \)
- \( \pi \rightarrow \pi \)
- \( \pi \rightarrow \pi \)
- \( \pi \rightarrow \pi \)
Form factor modulus squared

\[ |F_V^\pi(s)|^2 \]

\[ s \text{ [GeV}^2\text{]} \]

| Resonance | Model parameter \((M, \Gamma) [\text{MeV}]\) | Pole position \((M, \Gamma) [\text{MeV}]\) |
|-----------|----------------------------------|----------------------------------|
| \(\rho(1450)\) | 1376(6), 603(22) | 1289(8), 540(16) |
| \(\rho(1700)\) | 1718(4), 465(9) | 1673(4), 445(8) |
### Form Factor Phase

![Graph](image)

**Resonance Model Parameter 
\((M, \Gamma) \text{ [MeV]}\) | Pole Position \((M, \Gamma) \text{ [MeV]}\)**

| Resonance  | Model Parameter  | Pole Position |
|------------|------------------|---------------|
| \(\rho(1450)\) | 1376(6), 603(22) | 1289(8), 540(16) |
| \(\rho(1700)\) | 1718(4), 465(9) | 1673(4), 445(8) |
Low-energy observables

\[ \langle r^2 \rangle_{V}^{\pi} = 6 \alpha_1 \]

González-Solís and Roig, Eur.Phys.J C79 (2019) no.5, 436
**Kaon Vector Form Factor**

\[
\frac{d\Gamma(\tau^\rightarrow K^- K^0 \nu_\tau)}{d\sqrt{s}} = \frac{G_F^2|V_{ud}|^2}{768\pi^3} \left(1 - \frac{S}{M^2_\tau}\right)^2 \left(1 + \frac{2S}{M^2_\tau}\right) \sigma^3_K(S)|F_V^K(S)|^2,
\]

- **Chiral Perturbation Theory** $O(p^4)$

\[
F_V^K(s)|_{\text{ChPT}} = 1 + \frac{2L^r_9}{F^2_\pi} - \frac{S}{96\pi^2 F^2_\pi} \left[A_\pi(S, \mu^2) + \frac{1}{2} A_K(S, \mu^2)\right] = F_V^K(s)|_{\text{ChPT}},
\]

- **Phase dispersive representation with** $\alpha_{1,2}$ **from** $F^\pi_V(s)$

\[
F_V^K(s) = \exp\left[\alpha_1 S + \frac{\alpha_2}{2} S^2 + \frac{S^3}{\pi} \int_{4m^2_\pi}^{s_{\text{cut}}} ds' \frac{\phi_K(S')}{(S')^3(S' - S - i\varepsilon)}\right],
\]

- **Form Factor phase** $\phi_K(s)$:
Fit results to BaBar $\tau^- \rightarrow K^- K^0_S \nu_\tau$ data

1.0 1.2 1.4 1.6
0 20 40 60 80 100

$N dN / dm_{kk_s}$ × $10^3$

$m_{kk_s}$ [GeV]

- BaBar data (2018)
- Our prediction
- Our fit (exponential)
- Our fit (dispersive)

González-Solís and Roig, Eur.Phys.J C79 (2019) no.5, 436
**Kπ Vector Form Factor**

- **RχT with two resonances:** $K^*(892)$ and $K^*(1410)$:

  \[
  \tilde{F}_{V}^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*} \tilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^*'}, \gamma_{K^*'})},
  \]

  \[
  D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n \text{Re} \left[ \tilde{H}_{K\pi}(s) \right] - i m_n \Gamma_n(s),
  \]

  \[
  \kappa_n = \frac{192\pi F_K F_{\pi}}{\sigma_{K\pi}(m_{K^*}^2)} \frac{\gamma_{K^*}}{m_{K^*}}, \quad \Gamma_n(s) = \Gamma_n \frac{s}{m_n^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}(m_n^2)},
  \]

- **We have a phase with two resonances:**

  \[
  \delta_{K\pi}^{K\pi}(s) = \tan^{-1} \left[ \frac{\text{Im} F_{V}^{K\pi}(s)}{\text{Re} F_{V}^{K\pi}(s)} \right],
  \]
Combined fit to $\tau^- \rightarrow K_S \pi^- \nu_\tau$ and $\tau^- \rightarrow K^- \eta \nu_\tau$
### Fit results

- Different choices regarding linear slopes and resonance mixing parameters \( s_{cut} = 4 \text{ GeV}^2 \)

| Fitted value                  | Reference Fit | Fit A   | Fit B   | Fit C   |
|-------------------------------|---------------|---------|---------|---------|
| \( B_{K\pi} \) (%)            | 0.404 ± 0.012 | 0.400 ± 0.012 | 0.404 ± 0.012 | 0.397 ± 0.012 |
| \( (B_{K\pi}^{th}) \) (%)     | (0.402)       | (0.394) | (0.400) | (0.394) |
| \( M_{K^*} \)                 | 892.03 ± 0.19 | 892.04 ± 0.19 | 892.03 ± 0.19 | 892.07 ± 0.19 |
| \( \Gamma_{K^*} \)            | 46.18 ± 0.42  | 46.11 ± 0.42  | 46.15 ± 0.42  | 46.13 ± 0.42  |
| \( M_{K^{*\prime}} \)         | 1305^{+15}_{-18} | 1308^{+16}_{-19} | 1305^{+15}_{-18} | 1310^{+14}_{-17} |
| \( \Gamma_{K^{*\prime}} \)    | 168^{+52}_{-44} | 212^{+66}_{-54} | 174^{+58}_{-47} | 184^{+56}_{-46} |
| \( \gamma_{K\pi} \times 10^2 \) | = \( \gamma_{K\eta} \) | -3.6^{+1.1}_{-1.5} | -3.3^{+1.0}_{-1.3} | = \( \gamma_{K\eta} \) |
| \( \lambda_{K\pi} \times 10^3 \) | 23.9 ± 0.7 | 23.6 ± 0.7 | 23.8 ± 0.7 | 23.6 ± 0.7 |
| \( \lambda_{K\pi} \times 10^4 \) | 11.8 ± 0.2 | 11.7 ± 0.2 | 11.7 ± 0.2 | 11.6 ± 0.2 |
| \( B_{K\eta} \times 10^4 \)   | 1.58 ± 0.10  | 1.62 ± 0.10  | 1.57 ± 0.10  | 1.66 ± 0.09  |
| \( (B_{K\eta}^{th}) \times 10^4 \) | (1.45)       | (1.51)       | (1.44)       | (1.58)       |
| \( \gamma_{K\eta} \times 10^2 \) | -3.4^{+1.0}_{-1.3} | -5.4^{+1.8}_{-2.6} | -3.9^{+1.4}_{-2.1} | -3.7^{+1.0}_{-1.4} |
| \( \lambda_{K\eta} \times 10^3 \) | 20.9 ± 1.5 | = \( \lambda_{K\pi}^\prime \) | 21.2 ± 1.7 | = \( \lambda_{K\pi}^\prime \) |
| \( \lambda_{K\eta}^\prime \times 10^4 \) | 11.1 ± 0.4 | 11.7 ± 0.2 | 11.1 ± 0.4 | 11.8 ± 0.2 |
| \( \chi^2 / \text{n.d.f.} \)   | 108.1/105 ∼ 1.03 | 109.9/105 ∼ 1.05 | 107.8/104 ∼ 1.04 | 111.9/106 ∼ 1.06 |
Most precise determination of the $K^*(1410)$ parameters

$K^*(1410)$ PHYSICAL PARAMETERS

- Boito et al. '09
  $(\tau^- \rightarrow K_S \pi^- \nu_\tau)$

- Boito et al. '10
  $(\tau^- \rightarrow K_S \pi^- \nu_\tau + K_{13})$

- Escribano et al. '13
  $(\tau^- \rightarrow K^- \eta \nu_\tau)$

- This work

$m_{K^*}(1410) \ [\text{MeV}]$
Most precise determination of the $K^*(1410)$ parameters

- **Boito et al. '09**
  \[ \tau^- \to K_S \pi^- \nu_\tau \]

- **Boito et al. '10**
  \[ \tau^- \to K_S \pi^- \nu_\tau + K_{13} \]

- **Escribano et al. '13**
  \[ \tau^- \to K^- \eta \nu_\tau \]

- **This work**

$\Gamma_{K^*(1410)}$ [MeV]

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Escribano, Gonzàlez-Solís, Jamin and Roig, JHEP 1409 (2014) 042
LOW-ENERGY PARAMETERS

Low-energy parameters

\[
\begin{align*}
\lambda_{K\pi}^{'} &= (23.9 \pm 0.9) \times 10^{-3} \\
\lambda_{K\eta}^{'} &= (20.9 \pm 2.7) \times 10^{-3} \\
\lambda_{K\pi}^{''} &= (11.8 \pm 0.2) \times 10^{-4} \\
\lambda_{K\eta}^{''} &= (11.1 \pm 0.5) \times 10^{-4}
\end{align*}
\]

\(\tau^{-} \rightarrow K^{-} \pi^{0} \nu_{\tau}\) (PRD 76 (2007) 051104)
**Tau lepton: SM vs non-SM**

- $2.6\sigma (2.4\sigma)$ LFU deviation from $|g_\tau/g_\mu|(|g_\tau/g_e|)$ in $W^{-} \to \tau^{-}\bar{\nu}_\tau$

- $2.8\sigma$ deviation CP asymmetry in $\tau^{-} \to K_S\pi^{-}\nu_\tau$:

$$A_{CP} = -3.6(2.3)(1.1) \times 10^{-3} \text{ (exp)} \text{ vs } A_{CP} = 3.6(1) \times 10^{-3} \text{ (th)},$$

- $\tau^{-} \to \nu_\tau\bar{u}D \ (D = , d, s)$ as probes on non-SM interactions

$$\mathcal{L}_{CC} = -\frac{G_F V_{uD}}{\sqrt{2}} \left[ (1 + \epsilon_L^T)\bar{\tau}\gamma_\mu (1 - \gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu (1 - \gamma^5)D \\
+ \epsilon_R^T\bar{\tau}\gamma_\mu (1 - \gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu (1 + \gamma^5)D + \bar{\tau}(1 - \gamma^5)\nu_\tau \cdot \bar{u}(\epsilon_S^T - \epsilon_P^T \gamma^5)D \\
+ \epsilon_T^T\bar{\tau}\sigma_{\mu\nu} (1 - \gamma^5)\nu_\tau \bar{u}\sigma^{\mu\nu} (1 - \gamma^5)D \right] + h.c.,$$

- **Garcés** et.al. [JHEP 1712, 027 (2017)]; **Miranda** et.al. [JHEP 1811, 038 (2018)]; **Cirigliano** et.al. [Phys.Rev.Lett. 122 (2019) no.22, 221801]; **Rendón** et.al. [Phys.Rev. D 99, no. 9, 093005 (2019)];

**Gonzàlez-Solís** et.al. [Phys.Lett.B 804 (2020) 135371]
**Strangeness-conserving transitions** ($\Delta S = 0$)

- **One meson** decay $\tau^- \to \pi^- \nu_\tau$ ($G_F \tilde{V}_{uD} = G_F (1 + \epsilon^e_L + \epsilon^e_R) V_{uD}$)

\[
\Gamma(\tau^- \to \pi^- \nu_\tau) = \frac{G_F^2 |\tilde{V}_{u_d}|^2 f^2 \pi m_\tau^3}{16\pi} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \\
\times \left(1 + \delta_{em}^\tau + 2\Delta^\tau\pi + O(\epsilon_i^T)^2 + O(\delta_{em}^\tau \epsilon_i^T)\right),
\]

- **Input:** $f_\pi = 130.2(8)\text{ MeV}$ (FLAG 1902.08191); $\delta_{em}^\tau = 1.92(24)\%$; $|\tilde{V}_{u_d}| = 0.97420(21)$ ($\beta$ decays, PDG).

- **Constraint** for the NP effective couplings:

\[
\Delta^\tau\pi \equiv \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_\pi^2}{m_\tau (m_u + m_d)} \epsilon_P^\tau = (-0.12 \pm 0.68) \times 10^{-2},
\]

- **Errors (hierarchy):** $f_\pi, \text{BR}, \delta_{em}^\tau\pi$
Strangeness-conserving transitions ($\Delta S = 0$)

Partial decay width for two-meson decays

$$
\frac{d\Gamma}{ds} = \frac{G_F^2 |\bar{V}_{ud}|^2 m_T^3 S_{EW}}{384 \pi^3 s} \left(1 - \frac{s}{m_T^2}\right)^2 \lambda^{1/2}(s, m_P^2, m_{P'}^2) \\
\times \left[ (1 + 2(\epsilon_L^T - \epsilon_L^e + \epsilon_R^T - \epsilon_R^e)) X_{VA} + \epsilon_S^T X_S + \epsilon_T^T X_T + (\epsilon_S^T)^2 X_{S^2} + (\epsilon_T^T)^2 X_{T^2} \right],
$$

$$
X_{VA} = \frac{1}{2s^2} \left\{ 3 \left( C_{PP'}^S \right)^2 |F_{0}^{PP'}(s)|^2 \Delta_{PP'}^2 + \left( C_{PP'}^V \right)^2 |F_{0}^{PP'}(s)|^2 \left[ 1 + \frac{2s}{m_T^2} \right] \lambda(s, m_P^2, m_{P'}^2) \right\},
$$

$$
X_S = \frac{3}{s m_T} \left( C_{PP'}^S \right)^2 |F_{0}^{PP'}(s)|^2 \frac{\Delta_{PP'}^2}{m_d - m_u},
$$

$$
X_T = \frac{6}{s m_T} C_{PP'}^V \Re \left[ F_{0}^{PP'}(s) (F_{0}^{PP'}(s))^* \right] \lambda(s, m_P^2, m_{P'}^2),
$$

$$
X_{S^2} = \frac{3}{2 m_T^2} \left( C_{PP'}^S \right)^2 |F_{0}^{PP'}(s)|^2 \frac{\Delta_{PP'}^2}{(m_d - m_u)^2},
$$

$$
X_{T^2} = \frac{4}{s} |F_{0}^{PP'}(s)|^2 \left[ 1 + \frac{s}{2 m_T^2} \right] \lambda(s, m_P^2, m_{P'}^2),
$$
No experimental data

Theoretical assumptions only

\[
\text{Im} F_{PP'}^T(s) = \sigma_{PP'}(s)t_+^*(s)F_{PP'}^T(s),
\]

\[
F_{PP'}^T(s) = F_{PP'}^T(0) \exp \left[ \frac{s}{\pi} \int_{s_{th}}^{s_{cut}} \frac{ds'}{s'} \frac{\delta_{PP'}^{PP'}(s')}{(s' - s - i0)} \right],
\]

- \( s_{th} = (m_P + m_{P'})^2 \): two-meson production threshold

In the elastic region: \( \delta_{PP'}^T(s) = \delta_{PP'}^T(s) \)

We guide the phase to \( \pi \Rightarrow \) asymptotic \( 1/s \) dictated by pQCD

- \( F_{PP'}^T(0) \): ChPT with tensor fields+lattice
Strangeness-conserving transitions ($\Delta S = 0$)

- **Global fit** to one and two meson decays

\[
\chi^2 = \sum_k \left( \frac{\bar{N}^\text{th}_k - \bar{N}^\text{exp}_k}{\sigma^\text{exp}_k} \right)^2 + \left( \frac{BR^\text{th}_{\pi\pi} - BR^\text{exp}_{\pi\pi}}{\sigma^\text{exp}_{\pi\pi}} \right)^2 + \left( \frac{BR^\text{th}_{KK} - BR^\text{exp}_{KK}}{\sigma^\text{exp}_{KK}} \right)^2 + \left( \frac{BR^\text{th}_{\tau\pi} - BR^\text{exp}_{\tau\pi}}{\sigma^\text{exp}_{\tau\pi}} \right)^2
\]

- $\bar{N}^\text{th}_k$: normalized distribution for $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$

\[
\bar{N}^\text{th}_k \equiv \frac{1}{N\text{events}} \frac{dN\text{events}}{ds} = \frac{1}{\Gamma(\epsilon^\tau_i, \epsilon^e_j)} \frac{d\Gamma(s, \epsilon^\tau_i, \epsilon^e_j)}{ds} \Delta^{\text{bin}}
\]

- Data: unfolded distribution measured by Belle (0805.3773)

- **Constraints:**
  - $BR(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau)^\text{exp} = 25.49(9)\%$
  - $BR(\tau^- \rightarrow K^- K^0 \nu_\tau)^\text{exp} = 1.486(34) \times 10^{-3}$
  - $BR(\tau^- \rightarrow \pi^- \nu_\tau)^\text{exp} = 10.82(5)\%$
Strangeness-conserving transitions ($\Delta S = 0$)

**Bounds** for the non-SM effective couplings

\[
\begin{pmatrix}
\epsilon_L^T - \epsilon_L^e + \epsilon_R^T - \epsilon_R^e \\
\epsilon_R^T + \frac{m^2_\pi}{2m_\tau(m_u + m_d)} \epsilon_P^T \\
\epsilon_S^T \\
\epsilon_T^T
\end{pmatrix}
= \begin{pmatrix}
0.5 \pm 0.6^{+2.3}_{-1.8} & 0.2 \pm 0.4 \\
0.3 \pm 0.5^{+1.1}_{-0.9} & 0.1 \pm 0.2 \\
9.7^{+0.5}_{-0.6} & 21.5^{+0.0}_{-0.1} & 0.2 \\
-0.1 \pm 0.2^{+1.1}_{-1.4} & 0.0 \pm 0.1 & 0.2
\end{pmatrix} \times 10^{-2},
\]

**Errors:**

- **i)** Statistic (1st)
- **ii)** Systematic: pion vector form factor (2nd), quark masses (3rd) and tensor form factor (4th)

\[
\rho_{ij} = \begin{pmatrix}
1 & 0.684 & -0.493 & -0.545 \\
1 & -0.337 & -0.372 \\
1 & 0.463 \\
1 & 1
\end{pmatrix},
\]
Strangeness-changing transitions ($|\Delta S| = 1$)

**One meson decay** $\tau^- \rightarrow K^- \nu_\tau$

$$\Gamma(\tau^- \rightarrow K^- \nu_\tau) = \frac{G_F^2 |\tilde{V}_{us}|^2 m_K^3}{16\pi} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 \times \left(1 + \delta_{\text{em}}^{\tau K} + 2\Delta^{\tau K} + O(\epsilon_i^L)^2 + O(\delta_{\text{em}}^{\tau K} \epsilon_i^L)\right),$$

**Inputs:** $f_K = 155.7(7)$ MeV (FLAG 1902.08191); $\delta_{\text{em}}^{\tau K} = 1.98(31)$%; $|\tilde{V}_{us}| = 0.2231(7)$ (PDG).

**Constraint** for the NP effective couplings:

$$\epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_\pi^2}{m_\tau (m_u + m_d)} \epsilon_P^\tau = (-0.41 \pm 0.93) \times 10^{-2},$$

**Errors (hierarchy):** $f_K, |V_{us}|, BR, \delta_{\text{em}}^{\tau K}$
Strangeness-changing transitions ($|\Delta S| = 1$)

Global fit to one and two meson decays

$$\chi^2 = \sum_k \left( \frac{\bar{N}_k^{\text{th}} - \bar{N}_k^{\exp}}{\sigma_{\bar{N}_k^{\exp}}} \right)^2 + \left( \frac{BR_{K\pi}^{\text{th}} - BR_{K\pi}^{\exp}}{\sigma_{BR_{K\pi}^{\exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\text{th}} - BR_{K\eta}^{\exp}}{\sigma_{BR_{K\eta}^{\exp}}} \right)^2 + \left( \frac{BR_{\tau K}^{\text{th}} - BR_{\tau K}^{\exp}}{\sigma_{BR_{\tau K}^{\exp}}} \right)^2$$

$\bar{N}_k^{\text{th}}$: distribution for $\tau^- \rightarrow K_S\pi^-\nu_{\tau}$

$$\bar{N}_k^{\text{th}} \equiv \frac{dN_{\text{events}}}{ds} = \frac{N_{\text{events}}}{\Gamma(e_i^T, e_j^e)} \frac{d\Gamma(s, e_i^T, e_j^e)}{ds} \Delta_{\text{bin}}$$

Data: unfolded distribution measured by Belle (0706.2231)

Constraints:

- $BR(\tau^- \rightarrow K_S\pi^-\nu_{\tau})^{\exp} = 0.404(2)\%$ (Belle)
- $BR(\tau^- \rightarrow K^-\eta\nu_{\tau})^{\exp} = 1.55(8) \times 10^{-4}$ (PDG)
- $BR(\tau^- \rightarrow K^-\nu_{\tau})^{\exp} = 6.96(10) \times 10^{-3}$ (PDG)
**Bounds** for the non-SM effective couplings

\[
\begin{pmatrix}
\epsilon_L^T - \epsilon_L^e + \epsilon_R^T - \epsilon_R^e \\
\epsilon_R^T + \frac{m_\pi^2}{2m_\tau(m_u+m_d)} \epsilon_P^T \\
\epsilon_S^T \\
\epsilon_T^T
\end{pmatrix}
= \begin{pmatrix}
0.5 \pm 1.5 \pm 0.3 \\
0.4 \pm 0.9 \pm 0.2 \\
0.8^{+0.8}_{-0.9} \pm 0.3 \\
0.9 \pm 0.7 \pm 0.3
\end{pmatrix} \times 10^{-2},
\]

**Errors:** Statistic (fit)+systematic (tensor form factor).

\[
\rho_{ij} = \begin{pmatrix}
1 & 0.854 & -0.147 & 0.437 \\
1 & -0.125 & 0.373 \\
1 & -0.055 \\
1
\end{pmatrix},
\]
Global fit to $\Delta S = 0$ and $|\Delta S| = 1$ transitions

- Precision experimental data on kaon decays (FLAG'19, 1902.08191):

$$|V_{us}| f_{K^0}^{K\pi}(0) = 0.2165(4), \quad \left| \frac{V_{us}}{V_{ud}} \right| \frac{f_K}{f_{\pi}} = 0.2760(4),$$

- Correlation between $|V_{us}|$ and $|V_{ud}|$
Global fit to $\Delta S = 0$ and $|\Delta S| = 1$ transitions

- Combination to **one and two meson decays**

\[
\begin{pmatrix}
\epsilon_L^T - \epsilon_L^e + \epsilon_R^T - \epsilon_R^e \\
\epsilon_R^T \\
\epsilon_P^T \\
\epsilon_S^T \\
\epsilon_T^T
\end{pmatrix}
= \begin{pmatrix}
2.9 & \pm 0.6 & \pm 0.6 & \pm 0.0 & \pm 0.4 & \pm 0.2 \\
7.1 & \pm 4.9 & +1.3 & +1.2 & \pm 0.2 & +40.9 \\
-7.6 & \pm 6.3 & +1.9 & +1.7 & \pm 0.0 & +19.0 \\
5.0 & +0.7 & +0.2 & \pm 0.0 & \pm 0.2 & +1.1 \\
-0.5 & \pm 0.2 & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1
\end{pmatrix}
\times 10^{-2},
\]

- Errors: Statistic $\pm V_{CKM}$ $\pm \delta_{em}^{\tau \pi (K)}$ $\pm$ tensor form factor $\pm$ quark masses

\[
A = \begin{pmatrix}
1 & 0.055 & 0.000 & -0.279 & -0.394 \\
1 & -0.997 & -0.015 & -0.022 \\
1 & 0.000 & 0.000 & 0.000 \\
1 & 0.000 & 0.243 & 0.000 \\
1 & 0.000 & 0.243 & 1
\end{pmatrix},
\]
GLOBAL FIT TO $\Delta S = 0$ AND $|\Delta S| = 1$ TRANSITIONS

- Combination to one and two meson decays

$$
\begin{pmatrix}
\epsilon_L^T - \epsilon_L^e + \epsilon_R^T - \epsilon_R^e \\
\epsilon_R^T \\
\epsilon_P^T \\
\epsilon_S^T \\
\epsilon_T^T
\end{pmatrix}
= 
\begin{pmatrix}
2.9 & \pm 0.6 & \pm 0.6 & \pm 0.0 & \pm 0.4 & +0.2 \\
7.1 & \pm 4.9 & +1.3 & +1.2 & \pm 0.2 & +40.9 \\
-7.6 & \pm 6.3 & +1.9 & -1.7 & \pm 0.0 & -14.1 \\
5.0 & +0.7 & +0.2 & \pm 0.0 & \pm 0.2 & +1.1 \\
-0.5 & \pm 0.2 & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1
\end{pmatrix}
\times 10^{-2},
$$

- Comparison with other bounds (assuming LFU):

  ► Semileptonic kaon decays: $\epsilon_S^\mu = -0.039(49) \cdot 10^{-2}$, $\epsilon_T^\mu = 0.05(52) \cdot 10^{-2}$

    [González-Alonso, Martin Camalich JHEP 1612 (2016) 052]

  ► (Excl. and incl.) Tau decays [Cirigliano et al. PRL 122 (2019) no.22, 221801]:

$$
\begin{pmatrix}
\epsilon_L^T - \epsilon_L^e + \epsilon_R^T - \epsilon_R^e \\
\epsilon_R^T \\
\epsilon_P^T \\
\epsilon_S^T \\
\epsilon_T^T
\end{pmatrix}
= 
\begin{pmatrix}
1.0 & \pm 1.1 \\
0.2 & \pm 1.3 \\
-0.6 & \pm 1.5 \\
0.5 & \pm 1.2 \\
-0.04 & \pm 0.46
\end{pmatrix}
\times 10^{-2},
$$
Hadronic $\tau$ decays as a privileged tool for the investigation of QCD...

...but also as a laboratory of New Physics

Hadronic Tau decays as golden modes at Belle-II

SM input: Form Factors from dispersion relations

► (Competitive) Bounds on the NP effective couplings

A lot of interesting physics to be done in the tau sector
The Pion vector form factor $F_V^{\pi}(s)$

- How to determine $F_V^{\pi}(s)$ experimentally?
  
  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ (Belle PRD 78 (2008) 072006) and $e^+e^- \rightarrow \pi^+\pi^-$ (BaBar PRD 86 (2012) 032013)

- What do we know theoretically on the form factor?
  
  ▶ Its low-energy behaviour: given by ChPT (Gasser&Leutwyler’85)
  ▶ Its high-energy behaviour ($\sim 1/s$): given by pQCD (Brodsky&Lepage’79)
  ▶ For the intermediate energy region: models
**Pion vector form factor: ChPT**

\[ \mathcal{O}(p^4) \]

Gasser and Leutwyler, Nucl.Phys.B 250, 517 (1985)

\[ F_{V\pi}^\pi(s) \bigg|_{\text{ChPT}} = 1 + \frac{2L_9^F(\mu)}{F_\pi^2} S - \frac{S}{96\pi^2 F_\pi^2} \left( A_{\pi}(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right), \]

[Graph showing the dependence of \( |F_{V\pi}(s)|^2 \) on \( s \), with data points from Belle (2008) and ChPT at \( \mathcal{O}(p^4) \).]
Pion vector form factor: ChPT with resonances

- Resonance Chiral Theory:

\[
F^\pi_V(s) = 1 + \frac{F_V G_V}{F^2_\pi} \frac{s}{M^2_\rho - s} \Rightarrow \frac{M^2_\rho}{M^2_\rho - s},
\]

- Expansion in s and comparing ChPT and R\(\chi\)T:

\[
F^\pi_V(s) = 1 + \frac{2L^r_9(\mu)}{F^2_\pi} s - \frac{s}{96\pi^2 F^2_\pi} \left( A_\pi(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right),
\]

\[
F^\pi_V(s) = 1 + \left( \frac{s}{M^2_\rho} \right) + \left( \frac{s}{M^2_\rho} \right)^2 + \cdots
\]

- Chiral coupling estimate: \(L^r_9(M_\rho) = \frac{F_V G_V}{2M^2_\rho} = \frac{F^2_\pi}{2M^2_\rho} \approx 7.2 \times 10^{-3}\)

- Combining ChPT and R\(\chi\)T:

\[
F^\pi_V(s) = \frac{M^2_\rho}{M^2_\rho - s} - \frac{s}{96\pi^2 F^2_\pi} \left[ A_\pi(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right].
\]
**Dispersive representation**

- **Unitarity:**
  
  \[
  \text{disc} F_V(s) = 2i\sigma_\pi(s) F_V(s) T_{1*}^1(s) = 2i F_V(s) \sin \delta_1^1(s) e^{-i\delta_1^1(s)},
  \]

  \[
  F_V(s) = \frac{1}{2i\pi} \int_{4M^2_\pi}^{\infty} ds' \frac{\text{disc} F_V(s')}{s' - s - i\varepsilon},
  \]

- **Analytic solution (Omnèès equation):**

  \[
  F_V(s) = P(s) \Omega(s), \quad \Omega(s) = \exp \left\{ \frac{s}{\pi} \int_{4M^2_\pi}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s' - s - i\varepsilon)} \right\},
  \]
Resummation of final-state interactions to all orders (Omnès)

\[ F_\pi^V(s) = P_n(s) \exp \left\{ \frac{s^n}{\pi} \int_{4m^2_\pi}^{\infty} \frac{ds'}{s'^n(s')^{n}} \frac{\delta_1^1(s')}{s' - s - i\varepsilon} \right\}, \]

Get a model for the phase from $\pi\pi \rightarrow \pi\pi$ scattering at $O(p^2)$

\[ T(s) = \frac{s - m^2_\pi}{F^2_\pi} \rightarrow T_1^1(s) = \frac{s\sigma^2_\pi(s)}{96\pi F^2_\pi} \rightarrow \delta_1^1(s) = \sigma_\pi(s)T_1^1(s) = \frac{s\sigma^3_\pi(s)}{96\pi F^2_\pi}, \]

Omnès exponentiation of the full loop function

\[ F_\pi^V(s) = \frac{M^2_\rho}{M^2_\rho - s} \exp \left\{ - \frac{s}{96\pi^2 F^2_\pi} A_\pi(s, \mu^2) \right\}. \]
Incorporation of the (off-shell) $\rho$ width:

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F_\pi^2} \left[ \sigma_\pi(s)^3 \theta(s - 4m_\pi^2) + \sigma_K(s)^3 \theta(s - 4m_K^2) \right],$$

$$F_\pi^V(s) \left|^{1 \text{ res}}_{\text{exp}} \right. = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ -\frac{s}{96\pi^2 F_\pi^2} \text{Re} \left[ A_\pi(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right] \right\}.$$
Incorporation of the $\rho'(1450)$, $\rho''(1700)$

\[
F_V^\pi(s)_{3 \text{ res}} \Bigg|^{\text{expo}} = \frac{M^2 + s (\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M^2 - s - iM \Gamma(s)} \exp \left\{ \text{Re} \left[ -\frac{s}{96\pi^2 F^2} \left( A_\pi(s) + \frac{1}{2} A_K(s) \right) \right] \right\}
\]

\[-\gamma \frac{se^{i\phi_1}}{M^2' - s - iM \Gamma'(s)} \exp \left\{ -\frac{s \Gamma'(M^2_\rho')}{\pi M^3 \Gamma'(M^2_\rho') \Re A_\pi(s)} \right\}
\]

\[-\delta \frac{se^{i\phi_2}}{M^2'' - s - iM \Gamma''(s)} \exp \left\{ -\frac{s \Gamma''(M^2_\rho'')}{\pi M^3 \Gamma''(M^2_\rho'') \Re A_\pi(s)} \right\},
\]

where

\[
\Gamma_{\rho', \rho''}(s) = \Gamma_{\rho', \rho''} \frac{M_{\rho', \rho''}}{\sqrt{s}} \frac{\sigma^3_\pi(s)}{\sigma^3_\pi(M^2_{\rho', \rho''})}.
\]
\( \chi T + \text{ Omnès: Exponential representation} \)

\[ |F_V^\pi(s)_{\text{exp}}|^2 \]

- **Belle (2008)**
- **ChPT at O(\(\rho^4\))**
- **Our fit: exponential representation**

\[
\begin{align*}
M_\rho &= 775.2(4) \text{ MeV}, & \gamma &= 0.15(4), & \phi_1 &= -0.36(24), \\
M_\rho' &= 1438(39) \text{ MeV}, & \Gamma_\rho' &= 535(63) \text{ MeV}, & \delta &= -0.12(4), & \phi_2 &= -0.02(45), \\
M_\rho'' &= 1754(91) \text{ MeV}, & \Gamma_\rho'' &= 412(102) \text{ MeV}, & \chi^2_{\text{dof}} &= 0.92
\end{align*}
\]
RχT + Omnès: Exponential representation

![Graph showing the exponential representation of the resonance](image)

| Resonance | Model parameter \((M, \Gamma) [\text{MeV}]\) | Pole position \((M, \Gamma) [\text{MeV}]\) |
|-----------|-----------------------------------------|------------------------------------------|
| \(\rho(770)\) | 775.2(4) | 762.0(3), 143.0(2) |
| \(\rho(1450)\) | 1438(39), 535(63) | 1366(38), 488(48) |
| \(\rho(1700)\) | 1754(91), 412(102) | 1718(82), 397(88) |
Dispersive representation

- Dispersion relation with subtractions:

\[
F_V^\pi(s) = \exp \left[ \alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{S^3}{\pi} \int_{4m^2_\pi}^{s_{\text{cut}}} ds' \frac{\phi(s')}{(s')^3(s' - s - i\varepsilon)} \right],
\]

- Low-energy observables:

\[
F_V^\pi(s) = 1 + \frac{1}{6} \langle r^2 \rangle_V^\pi s + c_V^\pi s^2 + d_V^\pi s^3 + \cdots,
\]

\[
\langle r^2 \rangle_V^\pi \bigg|_{\text{ChPT}} = \frac{12L_9^r(\mu)}{F_\pi^2} - \frac{1}{32\pi^2 F_\pi^2} \left[ 2 \log \left( \frac{M^2_\pi}{\mu^2} \right) + \log \left( \frac{M^2_K}{\mu^2} \right) + 3 \right],
\]

\[
\langle r^2 \rangle_V^\pi = 6\alpha_1, \quad c_V^\pi = \frac{1}{2} (\alpha_2 + \alpha_1^2), \quad \alpha_k = \frac{k!}{\pi} \int_{4m^2_\pi}^{s_{\text{cut}}} ds' \frac{\phi(s')}{s'^{k+1}}.
\]

- \( s_{\text{cut}} \): cut-off to check stability
Central results

Fit results (central value ± stat fit error ± syst th. error)

\[ \alpha_1 = 1.88(1)(1) \text{ GeV}^{-2}, \quad \alpha_2 = 4.34(1)(3) \text{ GeV}^{-4}, \]

\[ M_\rho = 773.6 \pm 0.9 \pm 0.3 \text{ MeV}, \]

\[ M_{\rho'} = 1376 \pm 6^{+18}_{-73} \text{ MeV}, \quad \Gamma_{\rho'} = 603 \pm 22^{+236}_{-141} \text{ MeV}, \]

\[ M_{\rho''} = 1718 \pm 4^{+57}_{-94} \text{ MeV}, \quad \Gamma_{\rho''} = 465 \pm 9^{+137}_{-53} \text{ MeV}, \]

\[ \gamma = 0.15 \pm 0.01^{+0.07}_{-0.03}, \quad \phi_1 = -0.66 \pm 0.01^{+0.22}_{-0.99}, \]

\[ \delta = -0.13 \pm 0.01^{+0.00}_{-0.05}, \quad \phi_2 = -0.44 \pm 0.03^{+0.06}_{-0.90}, \]

Physical pole mass and width

\[ M_{\rho}^{\text{pole}} = 760.6 \pm 0.8 \text{ MeV}, \quad \Gamma_{\rho}^{\text{pole}} = 142.0 \pm 0.4 \text{ MeV}, \]

\[ M_{\rho'}^{\text{pole}} = 1289 \pm 8^{+52}_{-143} \text{ MeV}, \quad \Gamma_{\rho'}^{\text{pole}} = 540 \pm 16^{+151}_{-111} \text{ MeV}, \]

\[ M_{\rho''}^{\text{pole}} = 1673 \pm 4^{+68}_{-125} \text{ MeV}, \quad \Gamma_{\rho''}^{\text{pole}} = 445 \pm 8^{+117}_{-49} \text{ MeV}, \]
Modulus squared of the pion form factor $s_{\text{cut}} = m_\tau^2, 4 \text{ GeV}^2$
## Dispersive Fits to the Pion Vector Form Factor

- Fits for different values of $s_{\text{cut}}$ and matching at $1 \text{ GeV}$

| Fits | Parameter          | $m_{\tau}^2$ [GeV²] | $4$ (reference fit) | $10$ | $\infty$ |
|------|--------------------|---------------------|---------------------|------|----------|
| Fit 1 | $\alpha_1$ [GeV⁻²] | 1.87(1)             | 1.88(1)             | 1.89(1) | 1.89(1) |
|      | $\alpha_2$ [GeV⁻⁴] | 4.40(1)             | 4.34(1)             | 4.32(1) | 4.32(1) |
|      | $m_\rho$ [MeV]     | $= 773.6(9)$        | $= 773.6(9)$        | $= 773.6(9)$ | $= 773.6(9)$ |
|      | $M_\rho$ [MeV]     | $= m_\rho$          | $= m_\rho$          | $= m_\rho$ | $= m_\rho$ |
|      | $M_\rho'$ [MeV]    | 1365(15)            | 1376(6)             | 1313(15) | 1311(5)  |
|      | $\Gamma_\rho'$ [MeV] | 562(55)            | 603(22)             | 700(6)   | 701(28)  |
|      | $M_\rho''$ [MeV]   | 1727(12)            | 1718(4)             | 1660(9)  | 1658(1)  |
|      | $\Gamma_\rho''$ [MeV] | 278(1)             | 465(9)              | 601(39)  | 602(3)   |
|      | $\gamma$           | 0.12(2)             | 0.15(1)             | 0.16(1)  | 0.16(1)  |
|      | $\phi_1$           | $-0.69(1)$          | $-0.66(1)$          | $-1.36(10)$ | $-1.39(1)$ |
|      | $\delta$           | $-0.09(1)$          | $-0.13(1)$          | $-0.16(1)$ | $-0.17(1)$ |
|      | $\phi_2$           | $-0.17(5)$          | $-0.44(3)$          | $-1.01(5)$ | $-1.03(2)$ |
|      | $\chi^2$/d.o.f    | 1.47                | 0.70                | 0.64     | 0.64     |
Fits for different matching point and with $s_{\text{cut}} = 4$ GeV

| Fits | Parameter       | Matching point [GeV] |   |   |   |
|------|-----------------|----------------------|---|---|---|
| Fit I | $\alpha_1$ [GeV$^{-2}$] | 1.88(1)              | 1.88(1) | 1.88(1) | 1.88(1) |
|      | $\alpha_2$ [GeV$^{-4}$] | 4.35(1)              | 4.35(1) | 4.34(1) | 4.34(1) |
|      | $m_\rho$ [MeV]    | $= 773.6(9)$         | $= 773.6(9)$ | $= 773.6(9)$ | $= 773.6(9)$ |
|      | $M_\rho$ [MeV]    | $= m_\rho$           | $= m_\rho$ | $= m_\rho$ | $= m_\rho$ |
|      | $M_{\rho'}$ [MeV] | 1394(6)              | 1374(8) | 1351(5) | 1376(6) |
|      | $\Gamma_{\rho'}$ [MeV] | 592(19)              | 583(27) | 592(2) | 603(22) |
|      | $M_{\rho''}$ [MeV] | 1733(9)              | 1715(1) | 1697(3) | 1718(4) |
|      | $\Gamma_{\rho''}$ [MeV] | 562(3)               | 541(45) | 486(7) | 465(9) |
|      | $\gamma$         | 0.12(1)              | 0.12(1) | 0.13(1) | 0.15(1) |
|      | $\phi_1$         | $-0.44(3)$           | $-0.60(1)$ | $-0.80(1)$ | $-0.66(1)$ |
|      | $\delta$         | $-0.13(1)$           | $-0.13(1)$ | $-0.13(1)$ | $-0.13(1)$ |
|      | $\phi_2$         | $-0.38(3)$           | $-0.51(2)$ | $-0.62(1)$ | $-0.44(3)$ |
|      | $\chi^2$/d.o.f   | 0.75                 | 0.74     | 0.68     | 0.70     |
**Variant (II): Intermediate States Other Than $\pi\pi$**

- **Fit A**: $\rho' \rightarrow K\bar{K}$ and $\rho'' \rightarrow K\bar{K}$
- **Fit B**: $\rho' \rightarrow K\bar{K}$ + $\rho' \rightarrow \omega\pi$

| Parameter          | Fit A       | Fit B       | reference fit |
|--------------------|-------------|-------------|---------------|
| $\alpha_1$ [GeV$^{-2}$] | 1.87(1)     | 1.88(1)     | 1.88(1)       |
| $\alpha_2$ [GeV$^{-4}$] | 4.37(1)     | 4.35(1)     | 4.34(1)       |
| $m_\rho$ [MeV]     | $= 773.6(9)$ | $= 773.6(9)$ | $= 773.6(9)$ |
| $M_\rho$ [MeV]     | $= m_\rho$  | $= m_\rho$  | $= m_\rho$   |
| $M_{\rho'}$ [MeV]  | 1373(5)     | 1441(3)     | 1376(6)       |
| $\Gamma_{\rho'}$ [MeV] | 462(14)     | 576(33)     | 603(22)       |
| $M_{\rho''}$ [MeV] | 1775(1)     | 1733(9)     | 1718(4)       |
| $\Gamma_{\rho''}$ [MeV] | 412(27)     | 349(52)     | 465(9)        |
| $\gamma$           | 0.13(1)     | 0.15(3)     | 0.15(1)       |
| $\phi_1$           | $-0.80(1)$  | $-0.53(5)$  | $-0.66(1)$    |
| $\delta$           | $-0.14(1)$  | $-0.14(1)$  | $-0.13(1)$    |
| $\phi_2$           | $-0.44(2)$  | $-0.46(3)$  | $-0.44(3)$    |
| $\chi^2$/d.o.f     | 0.93        | 0.70        | 0.70          |

$s_{\text{cut}} = 4$ GeV$^2$
Variant (III)

- Dispersive representation of the pion vector form factor

\[ F_\pi^V(s) = \exp \left[ \frac{s}{\pi} \int_{4m^2_\pi}^{s_{\text{cut}}} ds' \frac{\delta^1_1(s')}{(s')(s' - s - i\varepsilon)} + \frac{s}{\pi} \int_{s_{\text{cut}}}^{\infty} ds' \frac{\delta_{\text{eff}}(s')}{(s')(s' - s - i\varepsilon)} \right] \Sigma(s) \]

- Properties for \( \delta_{\text{eff}}(s) \)

  - \( \delta_{\text{eff}}(s_{\text{cut}}) = \delta^1_1(s_{\text{cut}}) \) and \( \delta_{\text{eff}}(s) \to \pi \) for large \( s \) to recover \( 1/s \)

  \[
  \delta_{\text{eff}}(s) = \pi + \left( \delta^1_1(s_{\text{cut}}) - \pi \right) \frac{s_{\text{cut}}}{s}
  \]

  - Integrating the piece with \( \delta_{\text{eff}}(s) \)

\[
F_\pi^V(s) = e^{1-\frac{\delta^1_1(s_{\text{cut}})}{\pi}} \left( 1 - \frac{s}{s_{\text{cut}}} \right)^{\left( 1 - \frac{\delta^1_1(s_{\text{cut}})}{\pi} \right)} \frac{s_{\text{cut}}}{s} \left( 1 - \frac{s}{s_{\text{cut}}} \right)^{-1}
\]

\[
\times \exp \left[ \frac{s}{\pi} \int_{4m^2_\pi}^{s_{\text{cut}}} ds' \frac{\delta^1_1(s')}{(s')(s' - s - i\varepsilon)} \right] \Sigma(s)
\]

\[
\Sigma(s) = \sum_{i=0}^{\infty} a_i \omega^i(s), \quad \omega(s) = \frac{\sqrt{s_{\text{cut}}} - \sqrt{S_{\text{cut}} - s}}{\sqrt{s_{\text{cut}}} + \sqrt{S_{\text{cut}} - s}}
\]
The resulting fit parameters are found to be

\[
\begin{align*}
& a_1 = 2.99(12), \\
& M_{\rho'} = 1261(7) \text{ MeV}, \quad \Gamma_{\rho'} = 855(15) \text{ MeV}, \\
& M_{\rho''} = 1600(1) \text{ MeV}, \quad \Gamma_{\rho''} = 486(26) \text{ MeV}, \\
& \gamma = 0.25(2), \quad \phi_1 = -1.90(6), \\
& \delta = -0.15(1), \quad \phi_2 = -1.60(4),
\end{align*}
\]

with a \( \chi^2 / \text{d.o.f} = 32.3 / 53 \approx 0.61 \) for the one-parameter fit, and

\[
\begin{align*}
& a_1 = 3.03(20), \quad a_2 = 1.04(2.10), \\
& M_{\rho'} = 1303(19) \text{ MeV}, \quad \Gamma_{\rho'} = 839(102) \text{ MeV}, \\
& M_{\rho''} = 1624(1) \text{ MeV}, \quad \Gamma_{\rho''} = 570(99) \text{ MeV} \\
& \gamma = 0.22(10), \quad \phi_1 = -1.65(4), \\
& \delta = -0.18(1), \quad \phi_2 = -1.34(14),
\end{align*}
\]

with a \( \chi^2 / \text{d.o.f} = 35.6 / 52 \approx 0.63 \) for the two-parameter fit.
Fits for different $s_{\text{cut}}$ and allowing the $\rho$-mass to float

| Fits     | Parameter             | $s_{\text{cut}}$ [GeV$^2$] |
|----------|-----------------------|-----------------------------|
|          | $m^2_\tau$ [GeV$^2$]  | 4 (reference fit)           | 10                          | $\infty$                      |
| Fit 1-$\rho$ | $\alpha_1$ [GeV$^{-2}$] | 1.88(1)                     | 1.88(1)                     | 1.89(1)                       | 1.88(1)                       |
|          | $\alpha_2$ [GeV$^{-4}$] | 4.37(3)                     | 4.34(1)                     | 4.31(3)                       | 4.34(1)                       |
|          | $m_\rho$ [MeV]        | 773.9(3)                    | 773.8(3)                    | 773.9(3)                      | 773.9(3)                      |
|          | $M_\rho$ [MeV]        | $= m_\rho$                  | $= m_\rho$                  | $= m_\rho$                    | $= m_\rho$                    |
|          | $M_\rho'$ [MeV]       | 1382(71)                    | 1375(11)                    | 1316(9)                       | 1312(8)                       |
|          | $\Gamma_\rho'$ [MeV]  | 516(165)                    | 608(35)                     | 728(92)                       | 726(26)                       |
|          | $M_{\rho''}$ [MeV]    | 1723(1)                     | 1715(22)                    | 1655(1)                       | 1656(8)                       |
|          | $\Gamma_{\rho''}$ [MeV]| 315(271)                  | 455(16)                     | 569(160)                      | 571(13)                       |
|          | $\gamma$              | 0.12(13)                    | 0.16(1)                     | 0.18(2)                       | 0.17(1)                       |
|          | $\phi_1$              | $-0.56(35)$                 | $-0.69(1)$                  | $-1.40(19)$                   | $-1.41(8)$                    |
|          | $\delta$              | $-0.09(3)$                  | $-0.13(1)$                  | $-0.17(4)$                    | $-0.17(3)$                    |
|          | $\phi_2$              | $-0.19(69)$                 | $-0.45(12)$                 | $-1.06(10)$                   | $-1.05(11)$                   |
|          | $\chi^2$/d.o.f        | 1.09                        | 0.70                        | 0.63                          | 0.66                          |
Including $\rho'_{(i)} \rightarrow K\bar{K}$, $\rho' \rightarrow \omega\pi$ into the $\rho'_{(i)}$ width

Different matching points with the (elastic) $\pi\pi$ phase shift
Systematic theoretical errors

- Including $\rho'(s) \to K\bar{K}$, $\rho' \to \omega\pi$ into the $\rho'(s)$ width
- Different matching points with the (elastic) $\pi\pi$ phase shift

\[ |F_{V}^{\pi}(s)|^2 \]

Belle data (2008)
Fit 1 (reference fit)
Fit 1–$\rho$
Fit I
Fit A
Fit singularities

González-Solís and Roig, Eur.Phys.J C79 (2019) no.5, 436
## Low-energy observables

| References                        | \( \langle r^2 \rangle_\pi^0 \) (GeV\(^{-2}\)) | \( c_\pi^0 \) (GeV\(^{-4}\)) | Sum rule \( s_{cut} \) (GeV\(^2\)) | Fit Eq. (42) |
|----------------------------------|-----------------------------------------------|-------------------------------|-----------------------------------|--------------|
| Colangelo et al. [55]            | 11.07 ± 0.66                                 | 3.2 ± 1.03                    | 1.52, 1.66, 1.75                  | 1.88 ± 0.01 ± 0.01 |
| Bijnens et al. [32]              | 11.22 ± 0.41                                 | 3.85 ± 0.60                   |                                   |               |
| Pich et al. [6]                  | 11.04 ± 0.30                                 | 3.79 ± 0.04                   |                                   |               |
| Bijnens et al. [33]              | 11.61 ± 0.33                                 | 4.49 ± 0.28                   |                                   |               |
| de Troconiz et al. [56]          | 11.10 ± 0.03                                 | 3.84 ± 0.02                   |                                   |               |
| Masjuan et al. [57]              | 11.43 ± 0.19                                 | 3.30 ± 0.33                   |                                   |               |
| Guo et al. [58]                  | –                                             | 4.00 ± 0.50                   |                                   |               |
| Lattice [59]                     | 10.50 ± 1.12                                 | 3.22 ± 0.40                   |                                   |               |
| Ananthanarayan et al. [60]       | 11.17 ± 0.53                                 | [3.75, 3.98]                  |                                   |               |
| Ananthanarayan et al. [61]       | [10.79, 11.3]                                 | [3.79, 4.00]                  |                                   |               |
| Schneider et al. [48]            | 10.6                                          | 3.84 ± 0.03                   |                                   |               |
| Dumm et al. [7]                  | 10.86 ± 0.14                                 | 3.84 ± 0.03                   |                                   |               |
| Celis et al. [8]                 | 11.30 ± 0.07                                 | 4.11 ± 0.09                   |                                   |               |
| Ananthanarayan et al. [62]       | 11.10 ± 0.11                                 | –                             |                                   |               |
| Hanhart et al. [63]              | 11.34 ± 0.01 ± 0.01                          | –                             |                                   |               |
| Colangelo et al. [39]            | 11.02 ± 0.10                                 | –                             |                                   |               |
| PDG [42]                         | 11.61 ± 0.28                                 | –                             |                                   |               |
| This work                        | 11.28 ± 0.08                                 | 3.94 ± 0.04                   |                                   |               |
### ρ(1450) AND ρ(1700) RESONANCE PARAMETERS

| Reference    | Model parameter $(M_{\rho'}, \Gamma_{\rho'})$ [MeV] | Pole position $(M_{\rho'}, \Gamma_{\rho'})$ [MeV] | Data     |
|--------------|-----------------------------------------------|-----------------------------------------------|----------|
| ALEPH        | $1328 \pm 15, 468 \pm 41$                  | $1268 \pm 19, 429 \pm 31$                  | $\tau$   |
| ALEPH        | $1409 \pm 12, 501 \pm 37$                  | $1345 \pm 15, 459 \pm 28$                  | $\tau + e^+ e^-$ |
| Belle        | $1428(15)(26), 413(12)(57)$               | $1384(16)(29), 390(10)(48)$               | $\tau$   |
| Dumm et. al.’13 | $-$                                    | $1440 \pm 80, 320 \pm 80$               | $\tau$   |
| Celis et. al.’14 | $1497 \pm 7, 785 \pm 51$               | $1278 \pm 18, 525 \pm 16$               | $\tau$   |
| Bartos et. al. | $-$                                    | $1342 \pm 47, 492 \pm 138$              | $e^+ e^-$ |
| Bartos et. al. | $-$                                    | $1374 \pm 11, 341 \pm 24$               | $\tau$   |
| **This work** | $1376 \pm 6^{+18}_{-73}, 603 \pm 22^{+236}_{-141}$ | $1289 \pm 8^{+52}_{-143}, 540 \pm 16^{+151}_{-111}$ | $\tau$   |

| Reference    | Model parameter $(M_{\rho''}, \Gamma_{\rho''})$ [MeV] | Pole position $(M_{\rho''}, \Gamma_{\rho''})$ [MeV] | Data     |
|--------------|-----------------------------------------------|-----------------------------------------------|----------|
| ALEPH        | $= 1713, = 235$                             | $1700, 232$                               | $\tau$   |
| ALEPH        | $1740 \pm 20, = 235$                       | $1728 \pm 20, 232$                       | $\tau + e^+ e^-$ |
| Belle        | $1694 \pm 41, 135 \pm 36^{+50}_{-26}$     | $1690 \pm 94, 134 \pm 36^{+49}_{-28}$     | $\tau$   |
| Dumm et. al.’13 | $-$                                    | $1720 \pm 90, 180 \pm 90$               | $\tau$   |
| Celis et. al.’14 | $1685 \pm 30, 800 \pm 31$               | $1494 \pm 37, 600 \pm 17$               | $\tau$   |
| Bartos et. al. | $-$                                    | $1719 \pm 65, 490 \pm 17$              | $e^+ e^-$ |
| Bartos et. al. | $-$                                    | $1767 \pm 52, 415 \pm 120$             | $\tau$   |
| **This work** | $1718 \pm 4^{+57}_{-94}, 465 \pm 9^{+137}_{-53}$ | $1673 \pm 4^{+68}_{-125}, 445 \pm 8^{+117}_{-49}$ | $\tau$   |
Different resonance mixing contribution than $F_{VV}^\pi(s)$:

\[
F_{VV}^K(s) = \frac{M_\rho^2 + s \left( \tilde{\gamma} e^{i\tilde{\phi}_1} + \tilde{\delta} e^{i\tilde{\phi}_2} \right)}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \text{Re} \left[ -\frac{s}{96\pi^2 F_\pi^2} \left( A_\pi(s) + \frac{1}{2} A_K(s) \right) \right] \right\} \\
-\tilde{\gamma} \frac{s e^{i\tilde{\phi}_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left\{ -\frac{s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_{\pi}^3(M_{\rho'}^2)} \text{Re}A_\pi(s) \right\} \\
-\tilde{\delta} \frac{s e^{i\tilde{\phi}_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left\{ -\frac{s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_{\pi}^3(M_{\rho''}^2)} \text{Re}A_\pi(s) \right\} ,
\]

\[
\Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''} \frac{s}{M_{\rho',\rho''}^2 \sigma_{\pi}^3(M_{\rho',\rho''}^2)} \theta(s - 4m_\pi^2) .
\]

Extract the phase $\tan \phi_{KK}(s) = \text{Im}F_{VV}^K(s)/\text{Re}F_{VV}^K(s)$

Use a three-times subtracted dispersion relation
**COMBINED ANALYSIS OF $F_V^\pi(s)$ AND $\tau^- \to K^- K_S \nu_\tau$**

| Parameter | $s_{\text{cut}} = 4$ [GeV$^2$] |
|-----------|-------------------------------|
| $\alpha_1$ | 1.88(1) | 1.89(1) | 1.87(1) |
| $\alpha_2$ | 4.34(2) | 4.31(2) | 4.38(3) |
| $\tilde{\alpha}_1$ | $= \alpha_1$ | $= \alpha_1$ | 1.88(24) |
| $\tilde{\alpha}_2$ | $= \alpha_2$ | $= \alpha_2$ | 4.38(29) |
| $m_\rho$ [MeV] | 773.6(9) | 773.6(9) | 773.6(9) |
| $M_\rho$ [MeV] | $= m_\rho$ | $= m_\rho$ | $= m_\rho$ |
| $M_{\rho'}$ [MeV] | 1396(19) | 1453(19) | 1406(61) |
| $\Gamma_{\rho'}$ [MeV] | 507(31) | 499(51) | 524(149) |
| $M_{\rho''}$ [MeV] | 1724(41) | 1712(32) | 1746(1) |
| $\Gamma_{\rho''}$ [MeV] | 399(126) | 284(72) | 413(362) |
| $\gamma$ | 0.12(3) | 0.15(3) | 0.11(11) |
| $\tilde{\gamma}$ | $= \gamma$ | $= \gamma$ | 0.11(5) |
| $\phi_1$ | $-0.23(26)$ | $0.29(21)$ | $-0.27(42)$ |
| $\tilde{\phi}_1$ | $-1.83(14)$ | $-1.48(13)$ | $-1.90(67)$ |
| $\delta$ | $-0.09(2)$ | $-0.07(2)$ | $-0.10(5)$ |
| $\tilde{\delta}$ | $= 0$ | $= 0$ | $-0.01(4)$ |
| $\phi_2$ | $-0.20(31)$ | 0.27(29) | $-1.15(71)$ |
| $\tilde{\phi}_2$ | $= 0$ | $= 0$ | 0.40(3) |
| $\chi^2/d.o.f$ | 1.52 | 1.19 | 1.25 |
**Belle** $\tau^- \rightarrow K_S \pi^- \nu_\tau$ MEASUREMENT

$\tau^- \rightarrow K_S \pi^- \nu_\tau$ Belle’s data

**Phys. Lett. B 654 (2007) 65** [arXiv:0706.2231]

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**Unfolded/physical $\tau^- \rightarrow K_S \pi^- \nu_\tau$ Belle’s data**

**Folded/detected $\tau^- \rightarrow K_S \pi^- \nu_\tau$ Belle’s data**

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Precise experimental data:
$\tau^- \rightarrow K_S \pi^- \nu_\tau$ seems to be a good source for determining the $K^*(892)$ resonance parameters

Less precise experimental data.
Our proposal: to add $\tau^- \rightarrow K^- \eta \nu_\tau$ to the fit in order to constraint the $K^*(1410)$ resonance parameters.
Form Factor: Dispersive representation

Dispersion relation with subtractions:

\[ F_{V}^{K\pi}(s) = P(s) \exp \left[ \alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2} \alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi} \int_{s_{K\pi}}^{s_{\text{cut}}} ds' \frac{\delta^{K\pi}(s')}{(s')^3(s' - s - i\omega)} \right], \]

\[ \alpha_1 = \lambda_+', \text{ and } \alpha_1^2 + \alpha_2 = \lambda_+'' \text{ are low energies parameters:} \]

\[ F_{V}^{K\pi}(t) = 1 + \frac{\lambda_+'}{M_{\pi^-}^2} t + \frac{1}{2} \frac{\lambda_+''}{M_{\pi^-}^4} t^2, \]

\[ \text{cut-off to check stability} \]

Parameters to Fit: \( \lambda_+', \lambda_+'', m_{K^*}, \gamma_{K^*}, m_{K^{*'}}, \gamma_{K^{*'}} \)
## Fit results

- Reference fit results obtained for different values of $S_{cut}$

| Parameter | 3.24 | 4 | 9 | $\infty$ |
|-----------|------|---|---|--------|
| $B_{K\pi} (%)$ | 0.402 ± 0.013 (0.399) | 0.404 ± 0.012 (0.402) | 0.405 ± 0.012 (0.403) | 0.405 ± 0.012 (0.403) |
| $B_{K\pi}^{th} (%)$ | 892.01 ± 0.19 | 892.03 ± 0.19 | 892.05 ± 0.19 | 892.05 ± 0.19 |
| $\Gamma_{K\pi}$ | 46.04 ± 0.43 | 46.18 ± 0.42 | 46.27 ± 0.42 | 46.27 ± 0.41 |
| $M_{K^*}$ | $207_{-58}^{+73}$ | $168_{-44}^{+52}$ | $155_{-41}^{+48}$ | $155_{-40}^{+47}$ |
| $\gamma_{K\pi}$ | $23.9 \pm 0.7$ | $24.3 \pm 0.7$ | $24.3 \pm 0.7$ | $24.3 \pm 0.7$ |
| $\chi'_{K\pi} \times 10^3$ | 23.3 ± 0.8 | 11.8 ± 0.2 | 11.7 ± 0.2 | 11.7 ± 0.2 |
| $\chi''_{K\pi} \times 10^4$ | 11.8 ± 0.2 | 11.7 ± 0.2 | 11.7 ± 0.2 | 11.7 ± 0.2 |
| $B_{K\eta} \times 10^4$ | 1.57 ± 0.10 (1.43) | 1.58 ± 0.10 (1.45) | 1.58 ± 0.10 (1.46) | 1.58 ± 0.10 (1.46) |
| $B_{K\eta}^{th} \times 10^4$ | $4.0_{-1.9}^{+1.3}$ | $3.4_{-1.3}^{+1.0}$ | $3.2_{-1.1}^{+0.9}$ | $3.2_{-1.1}^{+0.9}$ |
| $\gamma_{K\eta} \times 10^2$ | 18.6 ± 1.7 | 20.9 ± 1.5 | 22.1 ± 1.4 | 22.1 ± 1.4 |
| $\chi'_{K\eta} \times 10^3$ | 10.8 ± 0.3 | 11.1 ± 0.4 | 11.2 ± 0.4 | 11.2 ± 0.4 |
| $\chi''_{K\eta} \times 10^4$ | 105.8/105 | 108.1/105 | 111.0/105 | 111.1/105 |
| $\chi^2$/n.d.f. | 105.8/105 | 108.1/105 | 111.0/105 | 111.1/105 |
RESULTS OF THE COMBINED $\tau^- \rightarrow K_S\pi^-\nu_\tau$ AND $\tau^- \rightarrow K^-\eta\nu_\tau$ ANALYSIS

$\lambda'_{K\pi} = (23.9 \pm 0.9) \cdot 10^{-3}$
$\lambda'_{K\eta} = (20.9 \pm 2.7) \cdot 10^{-3}$

isospin violation?

$K_{l_3}$

$\tau + K_{l_3}$

$\tau$

ISTRA+ '04
NA48 '04
KLOE '07
KTeV '10
FLAVIANET '10
Boito et al. '10
Bernard '13
Antonelli et al. '13
Moussallam et al. '08
Jamin et al. '08
Boito et al. '09
This work $[K^-\eta]$
This work $[K_S\pi^-]$
RESULTS OF THE COMBINED $\tau^- \to K_S \pi^- \nu_\tau$ AND $\tau^- \to K^- \eta \nu_\tau$ ANALYSIS

$\lambda_{K\pi}'' = (11.8 \pm 0.2) \cdot 10^{-4}$

$\lambda_{K\eta}'' = (11.1 \pm 0.5) \cdot 10^{-4}$
$K\pi$ PHASE
The pion vector form factor: Motivation

- Enters the description of many physical processes

- BaBar measurement of $\tau^- \rightarrow K^- K_\text{S} \nu_\tau$ (PRD 98 (2018) no.3, 032010)
  - good quality data
  - sensitive to $\rho(1450)$ and $\rho(1700)$
  - our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region
Research achievements: Isospin-violating $\tau^- \to \pi^- \eta^{(i)} \nu_\tau$ decays

- Theory predictions: $\text{BR} \sim 1 \times 10^{-5}$ (Escribano’16, Moussallam’14)

- BaBar: $\text{BR} < 9.9 \times 10^{-5}$ 95% CL, Belle: $\text{BR} < 7.3 \times 10^{-5}$ 90% CL

- Challenging for Belle II

BaBar: $\text{BR} < 4 \times 10^{-6}$ 90% CL
SMEFT with dimension 6 operators

- $\tau^- \to \nu_\tau \bar{u} D$ \(D = d, s\)

\[
\mathcal{L}_{CC} = -\frac{G_F V_{ud}}{\sqrt{2}} \left[ (1 + \epsilon^L_\tau) \bar{\tau} \gamma_\mu (1 - \gamma^5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 - \gamma^5) D \\
+ \epsilon^T_R \bar{\tau} \gamma_\mu (1 - \gamma^5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 + \gamma^5) D + \bar{\tau} (1 - \gamma^5) \nu_\tau \cdot \bar{u} (\epsilon^S_\tau - \epsilon^P_\tau \gamma^5) D \\
+ \epsilon^T_T \bar{\tau} \sigma_{\mu\nu} (1 - \gamma^5) \nu_\tau \cdot \bar{u} \sigma^{\mu\nu} (1 - \gamma^5) D \right] + h.c.,
\]

- $\epsilon_i$ \((i = L, R, S, P, T)\) are effective couplings characterizing NP

- Semileptonic kaon decays: $\epsilon^\mu_S = -0.039(49) \cdot 10^{-2}$, $\epsilon^\mu_T = 0.05(52) \cdot 10^{-2}$
  [González-Alonso, Martin Camalich JHEP 1612 (2016) 052]

- (Excl. and incl.) Tau decays [Cirigliano et al. PRL 122 (2019) no.22, 221801]:

\[
\begin{pmatrix}
\epsilon^T_L - \epsilon^e_L + \epsilon^T_R - \epsilon^e_R \\
\epsilon^T_R \\
\epsilon^T_S \\
\epsilon^T_P \\
\epsilon^T_T
\end{pmatrix}
= \begin{pmatrix}
1.0 \pm 1.1 \\
0.2 \pm 1.3 \\
-0.6 \pm 1.5 \\
0.5 \pm 1.2 \\
-0.04 \pm 0.46
\end{pmatrix} \times 10^{-2},
\]
Strangeness-conserving transitions ($\Delta S = 0$)

**One meson decay** $\tau^- \rightarrow \pi^- \nu_\tau$

$$\Gamma(\tau^- \rightarrow \pi^- \nu_\tau) = \frac{G_F^2 |\tilde{V}_{u_d}|^2 f_\pi^2 m_\tau^3}{16\pi} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2$$

$$\times \left(1 + \delta_{e_m}^{\tau\pi} + 2\Delta^{\tau\pi} + O(\epsilon_i^\tau)^2 + O(\delta_e^{\tau\pi}\epsilon_i^\tau)\right),$$

**Constraint for the NP effective couplings (this work):**

$$\epsilon_L^\tau - \epsilon_R^\tau - \epsilon_L^e - \epsilon_R^e - \frac{m_\pi^2}{m_\tau(m_u + m_d)}\epsilon_P^\tau = (-0.12 \pm 0.68) \times 10^{-2},$$

- Errors (hierarchy): $f_\pi, BR, \delta_e^{\tau\pi}$

**Cirigliano et.al. (PRL 122 (2019) no.22 221801)**

$$\epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_\pi^2}{m_\tau(m_u + m_d)}\epsilon_P^\tau = (-0.15 \pm 0.67) \times 10^{-2},$$
Strangeness-conserving transitions ($\Delta S = 0$)

- **One meson decay** $\tau^- \rightarrow \pi^- \nu_\tau$ ($G_F \tilde{V}^e_{ud} = G_F (1 + \epsilon^e_L + \epsilon^e_R) V_{ud}$)

$$
\Gamma(\tau^- \rightarrow \pi^- \nu_\tau) = \frac{G_F^2 |\tilde{V}^e_{ud}|^2 f^2 \pi m^3_\tau}{16\pi} \left(1 - \frac{m^2_\pi}{m^2_\tau}\right)^2 (1 + \delta^\tau_{em} + 2\Delta^\tau\pi + O(\epsilon^\tau_i)^2 + O(\delta^\tau_{em} \epsilon^\tau_i)),
$$

- **Inputs:** $f_\pi = 130.2(8)$ MeV (FLAG 1902.08191); $\delta^\tau_{em} = 1.92(24)$%; $|\tilde{V}^e_{ud}| = 0.97420(21)$ ($\beta$ decays, PDG);

- **Constraint for the NP effective couplings (this work):**

$$
\Delta^\tau\pi \equiv \epsilon^\tau_L - \epsilon^e_L - \epsilon^\tau_R - \epsilon^e_R - \frac{m^2_\pi}{m_\tau (m_u + m_d)} \epsilon^\tau_P = (-0.12 \pm 0.68) \times 10^{-2},
$$

**Errors (hierarchy):** $f_\pi$, $BR$, $\delta^\tau_{em}$

- **$\Gamma(\tau \rightarrow \pi \nu) / \Gamma(\pi \rightarrow \mu \nu)$:** tighter constraints (not used in this work)

$$
\epsilon^\tau_L - \epsilon^e_L - \epsilon^\tau_R - \epsilon^e_R - \frac{m^2_\pi}{m_\tau (m_u + m_d)} \epsilon^\tau_P + \frac{m^2_\pi}{m_\mu (m_u + m_d)} \epsilon^\mu_P = (-0.38 \pm 0.27) \times 10^{-2},
$$
Strangeness-conserving transitions ($\Delta S = 0$)

- Bounds for the non-SM effective couplings

$$\begin{pmatrix}
\epsilon^\tau_L - \epsilon^e_L + \epsilon^\tau_R - \epsilon^e_R \\
\epsilon^\tau_R + \frac{m^2}{2m_\pi(m_u+m_d)} \epsilon^\tau_P \\
\epsilon^\tau_S \\
\epsilon^\tau_T
\end{pmatrix} = \begin{pmatrix}
0.5 \pm 0.6^{+2.3}_{-1.8} \\
0.3 \pm 0.5^{+1.1}_{-0.9} \\
9.7^{+0.5}_{-0.6} \pm 21.5^{+0.0}_{-0.1} \\
-0.1 \pm 0.2^{+1.1}_{-1.4} \pm 0.0 \pm 0.2
\end{pmatrix} \times 10^{-2},$$

- Comparison with other bounds (assuming LFU):

  - Semileptonic kaon decays: $\epsilon^\mu_S = -0.039(49) \cdot 10^{-2}$, $\epsilon^\mu_T = 0.05(52) \cdot 10^{-2}$  
    [González-Alonso, Martin Camalich JHEP 1612 (2016) 052]

  - (Excl. and incl.) Tau decays [Cirigliano et al. PRL 122 (2019) no.22, 221801]:

$$\begin{pmatrix}
\epsilon^\tau_L - \epsilon^e_L + \epsilon^\tau_R - \epsilon^e_R \\
\epsilon^\tau_R \\
\epsilon^\tau_S \\
\epsilon^\tau_P \\
\epsilon^\tau_T
\end{pmatrix} = \begin{pmatrix}
1.0 \pm 1.1 \\
0.2 \pm 1.3 \\
-0.6 \pm 1.5 \\
0.5 \pm 1.2 \\
-0.04 \pm 0.46
\end{pmatrix} \times 10^{-2},$$
GLOBAL FIT TO $\Delta S = 0$ AND $|\Delta S| = 1$ TRANSITIONS

Combination to one and two meson decays

\[
\begin{pmatrix}
\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\
\epsilon_R^\tau \\
\epsilon_P^\tau \\
\epsilon_S^\tau \\
\epsilon_T^\tau
\end{pmatrix}
= \begin{pmatrix}
2.9 & \pm 0.6 & \pm 0.6 & \pm 0.0 & \pm 0.4 & +0.2 & -0.3 \\
7.1 & \pm 4.9 & +1.3 & +1.2 & \pm 0.2 & +40.9 & -14.1 \\
-7.6 & \pm 6.3 & +1.9 & +1.7 & \pm 0.0 & +19.0 & -53.6 \\
5.0 & +0.7 & -0.8 & -0.1 & \pm 0.0 & \pm 0.2 & +1.1 & -0.6 \\
-0.5 & \pm 0.2 & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1 \\
\end{pmatrix} \times 10^{-2},
\]

Our limits on $\epsilon_i$ can be translated into bounds on the NP scale $\Lambda$ through

\[
\Lambda \sim v (V_{uD} \epsilon_i)^{-1/2},
\]

where $v = (\sqrt{2} G_F)^{-1/2} \sim 246$ GeV. Our bounds range $\Lambda \sim 10$ TeV, which are quite restricted compared to the energy scale probed in semileptonic kaon decays $\mathcal{O}(500)$ TeV.
Prospects for tau physics at Belle-II

- Huge amount of **data** to be delivered

- Broad **program** of tau lepton physics:
  - Searches for Lepton Flavor Violation (LFV)
  - CP violation
  - Second Class Currents
  - and much more (Michel parameters, precision $m_\tau$, EDM, ...)

- See "The Belle II Physics Book" (1808.10567)
Searches for charged LFV

- Tau as a tool to probe \textbf{non-SM} interactions:
  - radiative: $\tau^- \rightarrow \ell^- \gamma$
  - leptonic: $\tau^- \rightarrow \ell^- \ell^+ \ell^-$
  - semi-leptonic: $\tau^- \rightarrow \ell^- h(h) \quad (h = P, S, V...)$

- Belle-II will push the current bound forward by at least \textbf{one order of magnitude}!

- Observation of charged \textbf{LFV} would be a clear signal of \textbf{New Physics}
CP VIOLATION IN $\tau \rightarrow K_S\pi^\pm \nu_\tau$

$$A_\tau = \frac{\Gamma(\tau^+ \rightarrow \pi^+ K_S\bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \pi^- K_S\nu_\tau)}{\Gamma(\tau^+ \rightarrow \pi^+ K_S\bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \pi^- K_S\nu_\tau)},$$

- SM prediction: $A_\tau \approx 2\text{Re}(\epsilon) \approx (3.6 \pm 0.1) \times 10^{-3}$ (Bigi, Sanda’05, Grossman, Nir’11)

- Exp. measurement: $(-3.6 \pm 2.3 \pm 1.1) \times 10^{-3}$ (BaBar 2011) $2.8\sigma$ from the SM

- New physics? Very difficult to explain:
  - Charged Higgs, $W_L - W_R$ mixings (Devi, Dhargyal, Sinha’ 2014)
  - Tensor interactions (Rendón, Roig, Toledo 2019)

An improved $A_\tau$ measurement is a priority for Belle II
**Second class currents (SCC) in \( \tau \to \pi \eta \nu_\tau \)**

- SCC: \( J^{PG} = 0^{+-}, 0^{--}, 1^{++}, 1^{--} \) not yet observed!

- In the SM, \( \tau \to \pi \eta \nu_\tau \) decays proceed via SCC with tiny BRs \( \lesssim O(10^{-5}) \) (Escribano, SG-S and Roig, Phys.Rev.D 94 (2016) no.3 034008, Moussallam et.al. ’14)

- Searched for at last-generation B-factories
  - \( BR < 7.3 \times 10^{-5} \) (Belle), \( BR < 9.9 \times 10^{-5} \) (BaBar)

- The observation of SCC via \( \tau \to \pi \eta \nu_\tau \) is a priority at Belle-II

- Clear signal could suggest **New Physics**