Research on Surface Modeling of Turbo Vane of Vehicle Based on NURBS

Xiaoyun Zhao1,2*, Xuding Song1 and Yueling Zhao1

1Ministry of Education Key Laboratory for Technology and Equipment of Highway Construction, Chang’an University, Xi’an Shanxi, China;
2School of Intelligent Manufacturing, Taizhou University, Taizhou Zhejiang, China

*Corresponding author email: zhaoxy@tzc.edu.cn

Abstract. In order to verify the induced inverse matrix equation for the control vertices of quartic NURBS curve and to improve the construction accuracy and fairness of complex twisted surface, this work constructed the bi-quartic NURBS surface of turbo vane of vehicle with NURBS technology in MATLAB. The bi-quartic NURBS vane of turbo was reconstructed in CATIA. The results were compared with the bi-cubic NURBS surface of vane in the same data, and the bi-cubic NURBS surface were constructed with the current inverse matrix equation for the control vertices of cubic NURBS curve. It shows that the constructed surface of vane with the generalized method of modeling of quartic NURBS surface has better accuracy and fairness through analyzing the error and fairness. The work provides the reference for the accurate modeling and the high-accuracy, high-efficiency NC machining of turbo vane in the engineering application.

Keywords: Quartic NURBS technology; Free-form surface modeling; Free-form surface analysis; Construction of turbo vane.

1. Introduction

Charging turbine of vehicle is an important part in modern automotive applications. After installing a turbocharger in the automotive engine system, it not only improves the power performance of vehicle and the economic performance of fuel oil, but also reduces the emission of harmful waste gas and the noise of engine[1-2]. The waste gas turbo vane has a typical free-form surface characterized by thin wall, with the high machining accuracy requirement on its shape, position and size. Therefore, carrying out accurate geometric modeling on turbo vane is the key to its high-precision, high-efficiency and high-speed machining. At present, because of the powerful functionality and unified data representation of non-uniform rational B-spline (NURBS) technology, NURBS modeling technology has become the key technology of complex surface modeling[3-5]. NURBS techniques have advantages including obvious geometrical visualization, local continuity and good controllability in modeling, are extensively applied in multiple fields[6-8]. In NURBS modeling technique, the key points include the inverse solution of the control vertex, the parameterized method, the construction of node vector, and the construction of curve and surface equations, which directly affecting the accuracy and fairness of the surface construction.

References [9-11] studied the matrix representation and algorithm of NURBS curves and surfaces. References [12-13] studied the problem of bi-quartic NURBS modeling of complex surfaces, proposing the corresponding matrix expressions, constructing the bi-cubic and bi-quartic NURBS surface models of aircraft blades, carrying out the surface analysis. However, it did not adopt the efficient, stable inverse matrix equation for the control vertices computation of quartic NURBS curve.
References [14-16] analyzed the inverse computation of the control vertices of curves and surfaces, and proposed the inverse matrix equation for the control vertices of cubic NURBS curve, according to the principle of affine transformation. The inverse matrix equation for the control vertices of quartic NURBS curve had not been proposed. The application of the inverse matrix equation for the control vertices of quartic NURBS curve based on affine transformation principle has not been reported in related materials. In order to carry out a detailed research on NURBS surface modeling technology and improve the accuracy and fairness of surface modeling, the work combined affine transformation principle and geometric recursion technique to get the new inverse matrix equation for the control vertices of quartic NURBS curve, analyzed the model of the complex surface. The bi-quartic NURBS vane was constructed with the data of turbo vane of vehicle measured by the three-coordinate measuring machine, and the accuracy and surface fairness were analyzed with that of the bi-cubic NURBS vane, which was constructed by using the same data and method. The traditional design method of integral vane wheel can no longer meet the higher technical requirements of various parts with complex surface in precision or in the subsequent NC machining[17-19].

In order to improve the modeling precision and fairness of the integral turbine vane for the high-precision digital design and high-efficiency NC machining, the surface modeling of turbo vane was summarized, for promoting the high-precision digital design and high-efficiency NC machining of the waste gas turbo.

2. Quartic NURBS Surface Construction

First, the data of turbo vane were collected based on reverse engineering. A vane of the selected turbo wheel was measured by the bridge type three-coordinate measuring machine Daisy686. Fifteen sections, selected as the cross sections, were denoted as u directions; 12 data points were collected on each cross section, with 15 sets of data points being collected. Through MATLAB programming, the cross-section curves of the vane were drawn in MATLAB (see figure 1).

2.1. Inverse Matrix Equation for the Control Vertices of Quartic NURBS Curve

Based on the affine transformation principle and the B-spline geometric recursive construction technology, the general expression for inverse computation of the control vertices of quartic NURBS curve was deduced as follows:

\[
(\Delta_j + \Delta_{j+1})p_{j,1} = \frac{(\Delta_{j,1})^3}{\Delta_{j,3} + \Delta_{j,1} + \Delta_j + \Delta_{j,1}}d_{1,1}^{d_{1,1}} + \frac{\Delta_{j,1}(\Delta_{j,2} + \Delta_{j,1} + \Delta_j)}{\Delta_{j,3} + \Delta_{j,1} + \Delta_j + \Delta_{j,1}}d_{1,2}^{d_{1,2}} + \frac{\Delta_j(\Delta_{j,3} + \Delta_{j,2} + \Delta_{j,1})}{\Delta_j + \Delta_{j,1} + \Delta_{j,2} + \Delta_{j,3}}d_{1,3}^{d_{1,3}}
\]

According to the above expression, the inverse matrix equation for the control vertices of quartic NURBS curve was obtained as:
\[ \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \vdots & \vdots & \vdots \\ a_n & b_n & c_n \end{bmatrix} \begin{bmatrix} a_{i1} \\ a_{i2} \end{bmatrix} = \begin{bmatrix} d_i \\ e_i \end{bmatrix} \]

where,
\[ a_{i1} = \frac{(\Delta_{i1})^2}{\Delta_1 + \Delta_{i1} + \Delta_{i2} + \Delta_{i3}}, \quad b_{i1} = \frac{\Delta_{i3}(\Delta_1 + \Delta_{i1} + \Delta_{i2})}{\Delta_1 + \Delta_{i1} + \Delta_{i2} + \Delta_{i3}}, \quad c_{i1} = \frac{\Delta_{i3}(\Delta_1 + \Delta_{i1} + \Delta_{i2} + \Delta_{i3})}{\Delta_1 + \Delta_{i1} + \Delta_{i2} + \Delta_{i3}}, \]
\[ e_{i1} = (\Delta_{i3} + \Delta_{i4}) p_{i1} \quad (i = 2, 3, \ldots, n) \]
\[ a_1 = b_1 = c_{n2} = 1, \quad b_1 = c_1 = a_2 = c_2 = a_{n2} = b_{n2} = 0, \]
\[ e_1 = p_0 + \frac{\Delta_1}{4} p_0', \quad e_2 = p_0 + \frac{\Delta_2 + \Delta_3}{4} p_0', \quad e_{n1} = p_n + \frac{\Delta_{n3}}{4} p_n'. \]

\( p_0' \) and \( p_n' \) represent the tangent vectors of the head end and the tail end, respectively.

2.2. Construction of bi-quartic NURBS Surface of Turbine Vane

For the obtained data points of the turbine vane, the modeling technique of bi-quartic NURBS surface was adopted for the construction. In simple, a set of data was used to make constructive specification as follows. Table 1 shows the selected data points. All the Z coordinates are identical in the value of -17.2004mm.

| No. | \( x_i (\text{mm}) \) | \( y_i (\text{mm}) \) | No. | \( x_i (\text{mm}) \) | \( y_i (\text{mm}) \) |
|-----|----------------------|----------------------|-----|----------------------|----------------------|
| 1   | -7.6144              | 17.5920              | 7   | -4.1123              | 14.8033              |
| 2   | -7.2719              | 17.3772              | 8   | -3.6905              | 14.3606              |
| 3   | -6.6009              | 16.9193              | 9   | -3.4137              | 14.0483              |
| 4   | -5.9458              | 16.4285              | 10  | -2.8430              | 13.3935              |
| 5   | -5.3198              | 15.9201              | 11  | -2.1792              | 12.8179              |
| 6   | -4.7049              | 15.3758              | 12  | -1.6906              | 12.5076              |

The node vector was constructed using cumulative chord length parameterization. The constructed node vector is as follows:
\( U = [0, 0, 0, 0, 0, 0.0516, 0.1552, 0.2596, 0.3624, 0.4671, 0.5722, 0.6502, 0.7034, 0.8142, 0.9262, 1, 1, 1, 1, 1, 1]. \)

The boundary conditions are the two tangent vectors of the curve. The two tangent vectors were calculated as follows: \( p_0 (0.8472, -0.5313, 0) \) and \( p_n' (-0.8442, -0.5361, 0) \).

With the inverse computation matrix expression for control vertices, the control vertices for quartic NURBS curve were obtained through MATLAB programming (see table 2).
Table 2. Reversely calculated control vertices $d_i$ for quartic NURBS curve

| No. | $x_i$ (mm) | $y_i$ (mm) | $z_i$ (mm) | No. | $x_i$ (mm) | $y_i$ (mm) | $z_i$ (mm) |
|-----|------------|------------|------------|-----|------------|------------|------------|
| 1   | -7.6144    | 17.5920    | -17.2004   | 9   | -4.1376    | 14.8338    | -17.2002   |
| 2   | -7.6035    | 17.5851    | -17.2004   | 10  | -3.7084    | 14.3829    | -17.2004   |
| 3   | -7.5706    | 17.5645    | -17.2004   | 11  | -3.3650    | 13.9948    | -17.2007   |
| 4   | -7.0732    | 17.2554    | -17.2004   | 12  | -2.8450    | 13.3792    | -17.2001   |
| 5   | -6.5827    | 16.9096    | -17.2002   | 13  | -2.3890    | 12.9304    | -17.2004   |
| 6   | -5.9315    | 16.4243    | -17.2023   | 14  | -1.7062    | 12.5175    | -17.2004   |
| 7   | -5.3177    | 15.9154    | -17.1887   | 15  | -1.6906    | 12.5076    | -17.2004   |
| 8   | -4.7180    | 15.3960    | -17.2022   | 16  | -1.6800    | 12.5076    | -17.2004   |

The data of the remaining 14 sets of control vertices were obtained by using the same method. According to the equation for constructing quartic NURBS curve in references [12-13], through programming, the 15 quartic NURBS curves in U (transverse) direction were obtained in MATLAB (see figure 2). The above 15 sets of control vertices in U direction were taken as the data points in V (longitudinal) direction, and the control vertices in V direction were obtained using the same method. A total of 15 sets of control vertices were obtained. Then, according to the equation for constructing quartic NURBS curve in references [12-13], through programming, the 15 quartic NURBS curves in V (longitudinal) direction were obtained in MATLAB (see figure 3).

![Quartic NURBS curves of vane in V direction](image1)

![Bi-quartic NURBS surface of vane](image2)

Finally, according to the matrix equation of the bi-quartic NURBS surface given in references [12-13], through programming, the bi-quartic NURBS surface of the turbine vane was obtained in MATLAB (see figure 4).

2.3. Construction of bi-quartic NURBS Surface Body of Turbine Vane

The data of bi-quartic NURBS surface of turbine vane in MATLAB was imported into CATIA. The bi-quartic NURBS surface body of turbine vane was constructed with skinning method in the reverse engineering module (see figure 5). In the end, figure 6 shows the constructed integral vane body.

![bi-quartic surface body of turbine vane](image3)

![bi-quartic integral vane body](image4)
3. Cubic NURBS Surface Construction
For contrastive analysis, the Bi-cubic NURBS surface of turbine vane was constructed with the same data.

3.1. Inverse Matrix Equation for the Control Vertices of Cubic NURBS Curve
According to reference[3], the current inverse matrix equation for the control vertices of cubic NURBS curve is as follows:

\[
\begin{bmatrix}
    a_1 & b_1 & c_1 \\
    a_2 & b_2 & c_2 \\
    \vdots & \vdots & \vdots \\
    a_{n+1} & b_{n+1} & c_{n+1} \\
\end{bmatrix}
\begin{bmatrix}
    d_1 \\
    d_2 \\
    \vdots \\
    d_{n+1} \\
\end{bmatrix}
= \begin{bmatrix}
    e_1 \\
    e_2 \\
    \vdots \\
    e_{n+1} \\
\end{bmatrix}
\]

(3)

where

\[
a_i = \frac{(\Delta i)^2}{\Delta i + \Delta i+1 + \Delta i+2},
b_i = \frac{\Delta i^2}{\Delta i + \Delta i+1 + \Delta i+2} + \frac{\Delta i+2}{\Delta i+1 + \Delta i+2 + \Delta i+3},
c_i = \frac{(\Delta i)^2}{\Delta i+1 + \Delta i+2 + \Delta i+3},
e_i = (\Delta i+1 + \Delta i) p_i (i = 0,1,\ldots,n),
\]

\[
a_{i+1} = c_{i+1} = 1, \quad b_{i+1} = c_{i+1} = a_{i+1} = b_{i+1} = 0, \quad e_{i+1} = p_0 + \frac{\Delta i}{3} p_i, \quad e_{i+1} = p_0 - \frac{\Delta i+1}{3} p_i.
\]

\(p_0\) and \(p_n\) represent the tangent vector of the head end and the tail end, respectively.

3.2. Construction of bi-cubic NURBS Surface of Turbine Vane
According to the obtained data points of the turbo vane, the modeling technique of bi-cubic NURBS surface was adopted for the construction. Similarly, a set of data was used to make constructive specification below.
The selected data points are exactly the same as that in table 1.
The node vector was also constructed by using cumulative chord length parameterization. The constructed node vectors is as follows:
\[U = [0, 0, 0, 0, 0.0516, 0.1552, 0.2596, 0.3624, 0.4671, 0.5722, 0.6502, 0.7034, 0.8142, 0.9262, 1, 1, 1, 1]\]
The boundary conditions are the two tangent vectors of the curve. The two tangent vectors were the same as the above: \(p_0 (0.8472, -0.5313, 0)\) and \(p_n (0.8442, -0.5361, 0)\).

With the inverse computation matrix expression for control vertices, the control vertices for cubic NURBS curve were obtained through MATLAB programming (see table 3).

Table 3. Reversely calculated control vertices \(d_i\) for cubic NURBS curve

| No. | x_i(mm) | y_i(mm) | z_i(mm) | No. | x_i(mm) | y_i(mm) | z_i(mm) |
|-----|---------|---------|---------|-----|---------|---------|---------|
| 1   | -7.6144 | 17.5920 | -17.2004| 8   | -4.1641 | 14.8730 | -17.2182|
| 2   | -7.5998 | 17.5829 | -17.2004| 9   | -3.7300 | 14.4071 | -17.1977|
| 3   | -7.0499 | 17.2237 | -17.1764| 10  | -3.3162 | 13.9408 | -17.2021|
| 4   | -6.6323 | 16.9520 | -17.2100| 11  | -2.8270 | 13.3551 | -17.1995|
| 5   | -5.9352 | 16.4237 | -17.1937| 12  | -2.3760 | 12.9200 | -17.2004|
| 6   | -5.3200 | 15.9380 | -17.2170| 13  | -1.7114 | 12.5208 | -17.2004|
| 7   | -4.6969 | 15.3704 | -17.1908| 14  | -1.6906 | 12.5076 | -17.2004|
The data of the remaining 14 sets of control vertices were obtained by using the same method. According to the equation for constructing cubic NURBS curve in references [12-13], through programming, the 15 cubic NURBS curves in U (transverse) direction were obtained in MATLAB (see figure 7).

![Figure 7. Cubic NURBS curves of vane in U direction](image1)

![Figure 8. Cubic NURBS curves of vane in V direction](image2)

The above 15 sets of control vertices in U direction were taken as the data points in V (longitudinal) direction, and the control vertices in V direction were obtained using the same method. A total of 14 sets of control vertices obtained. Then, according to the equation for constructing cubic NURBS curve in references [12-13], through programming, the 14 cubic NURBS curves in V (longitudinal) direction were obtained in MATLAB (see figure 8).

Finally, according to the matrix equation of the bi-cubic NURBS surface given in references [12-13], through programming, the bi-cubic NURBS surface of the turbine vane was obtained in MATLAB (see figure 9).

![Figure 9. bi-cubic NURBS surface of vane](image3)

![Figure 10. bi-cubic surface body of turbine vane](image4)

3.3. Construction of bi-cubic NURBS Surface Body of Turbine Vane

The data of bi-cubic NURBS surface of turbine vane in MATLAB was imported into CATIA. The bi-cubic NURBS surface body of turbine vane was constructed with skinning method in the reverse engineering module (see figure 10).

4. Analysis of the Constructed NURBS Surface

4.1. Error Analysis of the Constructed NURBS Surface

First, the adopted data points measured by the three-coordinate measuring machine were used as the original data to analyze model error. The selected set of specified data was analyzed. In MATLAB, the constructed quartic NURBS curves were compared with the cubic NURBS curves (see figure 11 and figure 12).
When the values of x and y were equal, the differences in z value between the cubic NURBS curves, and the quartic NURBS curves with the original curves were calculated. The values were the model errors of the quartic NURBS curves and the cubic NURBS curves respectively. With the same method, the quartic NURBS curves obtained from the other 14 sets of data were compared with the cubic NURBS curves. The model errors were calculated. By considering all the data with mathematical statistics, it could obtain the model errors of the constructed bi-quartic NURBS surface and bi-cubic NURBS surface.

Table 4 shows that the errors of the constructed bi-quartic NURBS surface and bi-cubic NURBS surface of the turbo vane are all relatively small, so the precision of the constructed complex free-form surface are all relatively higher. In addition, the difference between the constructed surface and theoretical surface of bi-quartic NURBS is less than that of bi-cubic NURBS in terms of the maximum error and the average error. The root-mean-squared error of the bi-quartic NURBS surface is also less than that of the bi-cubic NURBS surface.

**Table 4.** Error analysis results of bi-cubic NURBS surface and bi-quartic NURBS surface

|                     | maximum error (μm) | minimum error (μm) | average error (μm) | root-mean-squared error (μm) |
|---------------------|--------------------|--------------------|--------------------|-----------------------------|
| bi-cubic NURBS surface | 11                 | 0                  | 1.4228             | 3.0                         |
| bi-quartic NURBS surface | 8.1                | 0                  | 1.2994             | 2.3                         |

4.2. Fairness Analysis of the Constructed NURBS Surface

The fairness of free-form surfaces usually refers to its smoothness degree. The fairness of surfaces affects the performance and appearance of surface parts, as well as their manufacturability. Therefore, carrying out fairness analysis of part surface is a key link in the digital design and manufacture of free-form parts. This work analyzed the constructed bi-quartic NURBS surface and bi-cubic NURBS surface by the comparative change of the curvature of section curves at the same location of the surface.

For the quartic NURBS curve and cubic NURBS curve constructed by the data of the selected set, figure 13 shows the contrast of the two curvatures.
From the figure 13, the minimum value of the curvature of cubic NURBS curve is -0.112 mm\(^{-1}\), with the maximum value of 0.096 mm\(^{-1}\). The minimum value of the curvature of quartic NURBS curve is -0.087 mm\(^{-1}\), with the maximum value of 0.067 mm\(^{-1}\). Therefore, the curvature change of quartic NURBS curve is smaller than that of cubic NURBS curve.

By using the same method, curvature contrasts were analyzed on the section curves in the same position of the constructed bi-cubic NURBS surface and the bi-quartic NURBS surface. The results are roughly the same. Mathematical statistics were used to obtain the average curvatures of the section curves on the bi-cubic NURBS surface and the bi-quartic NURBS surface. According to all the section curves in the same position of the bi-cubic NURBS surface and the bi-quartic NURBS surface, the values are 0.0447 mm\(^{-1}\) and 0.0367 mm\(^{-1}\), respectively.

It can be found that the fairness of bi-quartic NURBS surface is higher than that of the bi-cubic NURBS surface. Therefore, the constructed bi-quartic NURBS surface has higher smoothness than that of the bi-cubic NURBS surface.

5. Conclusions

Based on the basic theories and technical methods of NURBS curves and surfaces, the inverse matrix equations for the control vertices of the newly deduced quartic NURBS curve and that for the common cubic NURBS curve were adopted to construct the surface of the turbo vane. Through contrastive analyses of the error and fairness of the constructed surface of the turbo vane, the main conclusions are as follows:

1. The errors of both the bi-quartic and bi-cubic NURBS surfaces of the turbo vane are relatively small, so the precisions of the constructed complex free-form surface are relatively higher.
2. Compared with the bi-cubic NURBS surface of the turbo vane, the bi-quartic NURBS surface has smaller maximum error, minimum error and root-mean-squared error. Meanwhile, the constructed bi-quartic NURBS surface has higher fairness than that of the bi-cubic NURBS surface.
3. The integral model of the turbo vane of vehicle constructed in CATIA proves that the bi-quartic NURBS modeling technique is a high-precision and effective method for reconstructing the surface of turbo vane.

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