The Holographic Principle and the Early Universe

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Abstract

A scenario is proposed in which the matter-antimatter asymmetry behaves like a seed for the inflationary phase of the universe. The mechanism which makes this scenario plausible is the holographic principle: this scheme is supported by a good prediction of the number of e-folds. It seems that such a mechanism can only work in the presence of a Hagedorn-like phase transition. The issue of the "graceful exit" can be also naturally accounted for.

Key words: Holographic Principle, Inflation, Matter-Antimatter asymmetry.

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1 Introduction

The inflationary scenario [14] [2] [19] [20] has been one of the main achievement in theoretical cosmology of the last decades. It provided many fundamental questions (such as why our universe is flat, homogeneous and isotropic to a very high degree, why we do not observe monopoles or other topological defects, why the primordial perturbations have a flat spectrum and so on; detailed reviews are, for example, [21], [15]) with a natural explanation. All the above questions could be answered in a standard FRW model only assuming very special initial conditions and fine tunings of many kinds. The mechanism which allows to solve such problems is mainly based on a very fast initial expansion of the scale factor of the universe which, in a sense, ”washes out” the inhomogeneities. The standard engine which drives such an expansion is a scalar field, called inflaton, which slowly rolls towards the minimum of its potential. During the slow roll of the inflaton one gets a period of exponential expansion of the universe. It is worth to stress here that, in the inflationary
scenario, what is really needed is the very fast initial expansion (from which it is possible to deduce all the wanted physical predictions by analyzing the evolution of the various kinds of perturbations), the scalar field is the easier way to get it but there is no compelling physical reason which tells that it is a scalar field, and not a vector or a tensor field or a different kind of mechanism, to drive the inflation (see, for example, [21], [15]). Actually, despite its striking successes, the inflationary paradigm still has some problems which can be traced back to the assumption that it is a scalar field to be responsible for the inflationary phase. In particular, it is still not completely clear what is the mechanism which allows a "graceful exit" from the inflationary phase, there is not a commonly accepted potential for the inflaton, the physical origin of the inflaton itself is still unknown and, a priori, it is not lawful to use the classical Einstein equations coupled to the inflaton field, as it is usually done, to study the evolution of the universe in a highly curved regime in which quantum corrections should be expected.

Besides the still unsolved problems of the inflationary scenario (which, on the other hand, do not overshadow its great merits), theoretical cosmology is affected by many unsolved problems. One of the most noticeable is the matter-antimatter asymmetry (detailed reviews are, for example, [17], [11], [9]). At a first glance, the Lagrangian of the Standard Model seems to be unable to explain why in the actual universe there is such an amount of asymmetry between matter and antimatter which enter symmetrically in the interactions. In a seminal paper [24], Sakharov showed that this asymmetry could be the consequence of the presence of baryon number violating processes, CP violations and departure from thermal equilibrium. In fact, the first two conditions can be fulfilled in the Standard Model: the effects are very small but, in the early universe when the temperature was very high, they are significantly enhanced and the third condition could also be met. There is still not a commonly accepted explanation of this asymmetry; at the moment the two most popular models seem to be the Leptogenesis (according to which the weak interactions, converting some lepton number into baryon number, could generate a net baryon and lepton number) and the Affleck-Dine mechanism based on supersymmetry (according to which the scalar supersymmetric partners of quarks and leptons could be responsible for the processes which should give rise to a fulfillment of the Sakharov conditions).

Here, a scenario is proposed in which the matter-antimatter asymmetry is the driving force of the inflationary phase of expansion of the universe. The mechanism which makes this possible is the holographic principle (up to now, the most promising available open window on quantum gravity). From the inflationary point of view, this mechanism also has the advantage of providing a natural explanation of the "graceful exit".

In the first section we will briefly review the physical basis of the holographic
principle. In the second section we will describe a statistical argument which, together with the holographic principle, makes plausible the proposed scenario. Eventually, some conclusions will be drawn.

2 The Holographic Principle

Even if the quantum theory of the gravitational field has not been found yet, in the few examples (such as the AdS/CFT correspondence [23]), in which one can carry on quantum computations in the presence of a gravitational field, the effective number of degrees of freedom is much smaller than the number which one would naively expect on purely Quantum Field Theoretic grounds: the number of degrees of freedom in a space-like region turns out to be proportional to the area of that region. The holographic principle heightens this phenomenon to a basic principle of the would be quantum theory of gravity (see, for example, [7]); the physical basis of such a principle were given in [3], [13], [27] and [26]. Elegant refinements of the original ideas [3], [13], [27] and [26] can be found in [5], [8]. Even if we still have not the final theory of quantum gravity, nevertheless it is possible to argue that the holographic principle could have a prominent role in understanding why the observed value of the cosmological constant is so smaller than the one computed in Quantum Field Theory \(^1\) (henceforth QFT). An intuitive explanation could be that in QFT pairs of degrees of freedom, which are coupled by gravitational interaction to form ”bound states” behaving as single effective degrees of freedom, are counted as distinct. This overcounting could be responsible of the too large cosmological constant obtained in QFT. In a simple classical model [10] it is also possible to find, without using CFT, a direct purely holographic (although qualitative) relation between the cosmological constant and the number of degrees of freedom

\[
\Lambda \sim \frac{\ln N}{N} \lambda
\]  

(1)

where \(\Lambda\) is the cosmological constant, \(N\) is (of the order of) the total number of degrees of freedom of the universe and \(\lambda\) is a characteristic quantum energy density which cannot be determined in a classical model but which, on dimensional analysis grounds, should be of the order of the Planck mass to the fourth power. The above qualitative relation between the cosmological constant and the number of degrees of freedom is in good agreement with the so called \(N\) bound \([6]\) according to which

\[
\Lambda \sim \frac{m_P^4}{N}. \tag{2}
\]

\(^1\) of about 120 orders of magnitude in standard Quantum Field Theory which become 60 orders of magnitude in supersymmetric QFT.
3 Holography and the early universe

Let us recall that, at least immediately after the end of inflation (when it is usually placed the beginning of the baryogenesis [17]), the universe can be described with a very good approximation by its statistical-thermodynamical properties. In particular, the local minima of the free energy

\[ F = H - TS \]

(3)

(where \( H \) is the internal energy, \( T \) the temperature and \( S \) the entropy) play a prominent role in determining the evolution of the universe during and after the baryogenesis. Indeed, internal quantum processes have time scales much smaller than the typical time scale of the cosmological evolution so that there is enough time for the particles filling the universe to reach the thermal equilibrium before the size of the universe changes significantly (after all, the cosmological predictions based on this assumption are in good agreement with observations). This statistical descriptions is inadequate during the inflationary period which is likely to be an out of equilibrium phase. However, a statistical description through the free energy should provide with a detailed picture also immediately before the inflation. It is usually assumed that, before the inflation smoothed out the inhomogeneities, there had been a period in which all the different parts of the universe were causally connected such that thermalization took place ([21], [15]); if this is the case, a description in terms of free energy immediately before the inflation is certainly correct. Of course, in the presence of a big-bang singularity this is not true. In fact, it is commonly believed that the final theory of quantum gravity will resolve the initial singularity: good proposals, for example, are available in loop quantum gravity [4] and string theory [12]. The question is: what mechanism drives the universe out of equilibrium in the inflationary phase and how such an out of equilibrium phase terminates? The following consideration is useful. In the early universe the temperature was very high and in the free energy (3) the second term should had been dominating so that the minimum of the free energy was determined by the maximum of the entropy.

Thus, before the inflation, the particles filling the universe should had tended toward the maximum entropic state. Let us suppose that, at that time, particles and antiparticles entered the microscopic interactions almost symmetrically (as it happens in actual the standard model Lagrangian): this implies that the number of particles should had been almost equal to the number of antiparticles. Is this state with an equal number of particles and antiparticles

\[ \text{It is known that gravity allows irreversible processes to occur without ever reaching any unsurpassable maximum value of the entropy [28]. However the evolution, in the very short time interval we are considering, will be driven mainly by local maxima of the entropy.} \]
the maximum entropic states? The answer is no. Under certain reasonable hypothesis, which will be described below, it is vastly more countenanced a state in which there are only particles. The following argument clarifies this point providing, at the same time, with a promising order of magnitude estimate. Let us suppose that we have to set $N$ bosons and the relative $N$ antibosons in the quantized energy levels of a certain system. Let us divide the quantized energy levels $\varepsilon_j$ in groups labelled by the index $j$. Let us denote the number of states of the $j$–th group with $G_j$ and the number of particles (which need not to be necessarily interpreted as the ”standard” particles of the standard model but simply as the degrees of freedom living, before the inflation, in the universe) living in that group as $N_j$. The sum of the entropy of the particles and the entropy of the antiparticles is a reasonable estimates of the total entropy. As it is shown, for example, in [18], the non equilibrium entropy of the system will be

$$S_{N,N} \sim 2 \sum_j G_j \left[ (1 + \overline{\pi}_j) \ln (1 + \overline{\pi}_j) - \overline{\pi}_j \ln \overline{\pi}_j \right],$$

where $\overline{\pi}_j$ is the mean occupation number of the quantum states:

$$\overline{\pi}_j = \frac{N_j}{G_j},$$

and the upper limit $x$ can be roughly estimated as follows

$$\frac{1}{2} x^2 \sim \sum_j \overline{\pi}_j = \text{total number of particles},$$

In the case in which $N$ bosons and $N$ antibosons are present

$$x \sim \left( \sqrt{N} \right)^\eta$$

where $\eta$, a positive number of order 1, takes into account the uncertainty on procedure of the replacement of discrete sum with an integral. Instead, when we have at our disposal $2N$ bosons $x \sim \left( \sqrt{2N} \right)^\eta$ and the entropy is given by

$$S_{2N} \sim \sum_j G_j \left[ (1 + \overline{\pi}_j) \ln (1 + \overline{\pi}_j) - \overline{\pi}_j \ln \overline{\pi}_j \right],$$

3 In other words, the difference between energies belonging to the same group are assumed to be very small.
4 Of course, if we maximize the above expression (4) for the entropy, taking into account the usual constraints on the total energy and the total number of particles, we get the Bose-Einstein distribution. But, before the inflation, the above assumptions on the total number of particles and total energy could be too strong.
so that

\[
\frac{S_{2N}}{S_{N,N}} \sim \frac{\sum_{j=(\sqrt{N})}^{(\sqrt{2N})} \eta \, G_j \left[ (1 + \pi_j) \ln (1 + \pi_j) - \pi_j \ln \pi_j \right]}{\sum_{j=(\sqrt{N})}^{(\sqrt{2N})} \eta \, G_j \left[ (1 + \pi_j) \ln (1 + \pi_j) - \pi_j \ln \pi_j \right]}. \tag{8}
\]

In the absence of a quantum theory of gravity, to find the explicit expressions of the \(G_j\) and \(x\) is a hopeless task; however we only need the order of magnitude. We should determine how the \(G_j\)'s depend upon the \(\pi_j\)'s.

Some well known features of ”Hagedorn phenomenology” in string theory and (large \(N_C\) SUSY) QCD (see, for example, [16], [22] [1] and references therein) suggest an interesting way toward this goal: it is strongly believed that at the Hagedorn temperature there is a phase transition toward a deconfining phase such that below the Hagedorn temperature the free energy \(F \sim T^n\) (in case, with logarithmic corrections) and above one gets \(F \sim T^{n+1}\), the entropy having a similar behavior. Thus, at the Hagedorn temperature, the exponent of the entropy as a function of the typical energy scale jumps in such a way that, above the transition, the entropy increases faster with the energy scale. The typical energy scale \(\varepsilon_{2N}\), when there are \(2N\) bosons, is higher than the scale \(\varepsilon_{N-N}\) when there are \(N\) bosons and \(N\) antibosons; thus, in the former case, we must impose that the entropy increases faster: this will be the physical motivation behind our assumption Eq. (9). In many string inspired matrix models (which, for example, describe the thermodynamic of a gas of D0-Branes) first order phase transitions are also possible in which the discontinuity of the free energy is of order \(N^2\) (see, for example, [25] and references therein) \(N\) being (of the order of) the number of degrees of freedom: this kind of phase transitions gives rise to Eq. (10) too. Viceversa, one could search for (sufficient) conditions in order to get Eq. (10) (which is at the basis of the main result of the paper Eq. (14)): a good condition would be the existence of a Hagedorn-like (first or second order) phase transition at the scale \(\varepsilon_{N-N}\). It is now apparent an intriguing relation between Hagedorn phase transition, the holographic principle and inflation.

The previous considerations lead us to assume that there exists a critical value \(j_C\) of the label beyond which the increase of the mean number of particles in the group \(j+1\) with respect to the group \(j\) ( \(j > j_C\) ) is less than in the case \(j < j_C\): to be concrete

\[
j \lesssim (\sqrt{N})^\eta \Rightarrow G_j \sim (\pi_j)^\alpha; \quad j \gtrsim (\sqrt{N})^\eta \Rightarrow G_j \sim (\pi_j)^{\alpha+1} \quad \tag{9}
\]

In this case the ratio in Eq. (8) can be roughly estimated replacing the sums with integrals:

\[
\frac{S_{2N}}{S_{N,N}} \sim (\sqrt{2N})^\eta \sim (10^{60})^\eta. \tag{10}
\]
It is now quite clear that, for $2N \sim 10^{120}$ (which should be a reasonable estimates of the number of degrees of freedom in the early universe), the evolution, driven by the entropy, should prefer to have only particles. The observations distinctly tell us that there exists a mechanism to obtain only particles (see, for example, [17]) but now the holographic principle comes into play.

According to the holographic principle, the entropy of the universe should fulfil an equation of the following type

$$S = \alpha A(\partial V) \quad (11)$$

where $\alpha$ is a suitable constant and $A(\partial V)$ is the two dimensional area of the boundary of a (Cauchy) hypersurface of constant time. If, before the inflation, the number of particles is almost equal to the number of antiparticles because they enter almost symmetrically the interactions, then, from Eq. (4), it follows

$$A(\partial V) \sim S^{N/N}. \quad (12)$$

On the other hand, the free energy would drive the universe towards a state in which there are almost only particles (in such a way to achieve a very huge benefit in entropic terms). The holographic principle does not allow this gain immediately: it allows the attainment of the minimum of the free energy only if Eq. (11) is fulfilled. The question is: how much time does the holographic principle employ to boost the value of area in Eq. (12) to the value which allows the attainment of the minimum of the free energy? At this stage of knowledge of quantum gravity it is not possible to give a definite answer to this question; however, since it is strongly believed that the holographic principle is a quantum-gravitational effects, one can assume that the duration of the inflation will be of ”Planck” order

$$\tau_{HP} \lesssim 10^{-30} s. \quad (13)$$

Summarizing: in order to reach the minimum of the free energy without violating the holographic principle, the universe should had expanded, in a interval of time of the order (13), from an area corresponding to an entropy of the order (12) to an area of the order (7). To support the hypothesis proposed here, we have to give an estimate of the number of e-folds $N_E$ predicted in this scenario. This is an easy computation: the number of e-folds $N_E$ is defined as follows

$$N_E = \ln \left( \frac{a_F}{a_I} \right)$$

where $a_I$ and $a_F$ are the scale factor of the universe immediately before and immediately after the inflation. According to the present model, being the linear size of the universe $a$

$$a \sim \sqrt{A(\partial V)},$$
the number of e-folds should be of the order

\[ N_E = \ln \left( \sqrt[2]{\frac{S_{2N}}{S_{N-N}}} \right) \sim \eta 30 \ln 10. \]  \hspace{1cm} (14)

This result is in very good agreement with the commonly accepted value for the number of e-folds which is expected to lie between 60 and 70 (see, for example, [21], [15]). To the authors knowledge, there are not theoretical models in which the number of degrees of freedom of the universe and the number of e-folds (which, in principle, can be determined independently from the observations) can be related in a direct and effective way as in the present scenario: these two numbers are not independent. This result is quite robust: one would get similar estimates for \( N_E \) (which differ of no more than one order of magnitude) by changing slightly \( \eta \) in Eq. (10). It is worth to stress here that the above result (14) has been obtained without using the Einstein equations which, at that time, were likely to be corrected by quantum gravitational effects. Another highly non-trivial benefit of this model is that the problem of the "graceful exit" simply has disappeared: when the area is such that it is possible to attain the minimum of the free energy without violating the holographic principle the inflationary phase terminates and the evolution is again driven by the minima of the free energy. It also appears natural to place the baryogenesis immediately after the end of the inflationary period (as it is usually assumed [17], [11]): the free energy would like the baryogenesis "as soon as possible" because of the entropy gain; the holographic principle prevents this but, when it has been fulfilled, the baryogenesis can freely start. It is interesting to note that, according to the results in [6] (further supported by the analysis in [10]) it is possible to obtain a direct relation between the cosmological constant, via eqs. (2) and (1), the number of degrees of freedom and the number of e-folds which still is in good agreement with observations.

4 Conclusion

In this paper a scenario has been proposed in which the inflationary phase is generated by the matter-antimatter asymmetry. The physical mechanism which makes this possible is the holographic principle. The scheme is the following: in the early universe the temperature was very high so that the entropy should had driven the evolution toward its maximum. At that time, interactions in which particles and antiparticles enter almost symmetrically are not able by themselves to generate the observed asymmetry. On the other hand, a state in which there are only particles is vastly more countenanced from the entropic point of view thus, having at our disposal many microscopical mechanisms which are able to generate the observed asymmetry at high temperatures, one could expect that the evolution freely drives the universe
toward the observed asymmetric state. Now the holographic principle comes into play. According to this principle, which is likely to be a manifestation of quantum gravity, the entropy is tied to the area of the universe. For this reason, it is possible to attain the minimum of the free energy only after that the holographic principle boosted (in a Planckian time scale) the initial area of the universe to a size compatible with the maximum of the entropy. The number of e-fold is in a very good agreement with the commonly accepted one. Moreover, the problem of the "graceful exit" from the inflationary phase has disappeared: the inflationary phase terminates simply when the size of the universe allows the attainment of the minimum of the free energy which is again allowed to drive the evolution. This would also explain in a natural way the fact that, very likely, the baryogenesis started immediately after the end of the inflation. This scheme is likely to work only if one assumes the existence of a Hagedorn-like phase transition. Even if more detailed computations are needed to provide this proposal with further supports, we believe that its merits make the physical basis of this model quite sound.

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