Non-Markov dynamics and phonon decoherence of a double quantum dot charge qubit

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Abstract

In this paper we investigate decoherence times of a double quantum dot (DQD) charge qubit due to it coupling with acoustic phonon baths. We individually consider the acoustic piezoelectric as well as deformation coupling phonon baths in the qubit environment. The decoherence times are calculated with two kinds of methods. One of them is based on the quasidiabatic propagator path integral (QUAPI) and the other is based on Bloch equations, and two kinds of results are compared. It is shown that the theoretical decoherence times of the DQD charge qubit are shorter than the experimental reported results. It implies that the phonon couplings to the qubit play a subordinate role, resulting in the decoherence of the qubit.

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I. INTRODUCTION

Solid state qubits are considered to be promising candidates for realizing building blocks of quantum information processors because they can be scaled up to large numbers. The double quantum dot (DQD) charge qubit is one of these qubits. Two low-energy charge states are used as the local states $|0\rangle$ and $|1\rangle$ in the qubit. The qubit can be controlled directly via external voltage sources. There are some effective schemes to prepare the initial states and read out the final states of the qubit. So it is considered that decoherence may be the central impediment for the qubit to be taken as the cell of quantum computer. Finding out the primary origin or the dominating mechanism of decoherence for the qubit is a basal task for overcoming the impediment. Experimentally, many attempts for detecting the decoherence time of this kind of qubit have been performed. The decoherence has also been investigated theoretically. In 2000, Fedichkin et al. investigated the Born-Markov type electron-phonon decoherence at large times due to spontaneous phonon emission of the quantum dot charge qubits. Recently, Vorojtsov et al. studied the error coherence of the DQD charge qubit by using Born-Markov approximation. But, as it has been pointed out, the use of the Born-Markov approximation is inappropriate at large tunneling amplitudes. The method is expected to become increasingly unreliable at DQD with larger inter-dot tunneling amplitudes. Wu et al. investigated the decoherence in terms of a perturbation treatment based on a unitary transformation. The Born-Markov approximation has not been used in the method but it neglects some terms of the effective Hamiltonian with high excited states. This kind of processing introduces an new approximation which has not been estimated to the affects of the dynamics. Fedichkin et al. studied the error rate of DQD charge qubit with short-time approximation. This method is accurate enough in adequate short time. But the decoherence in a moderately long time is also interesting. Recently, Thorwart et al. investigated the decoherence of the DQD charge qubit in a longer time with a numerically exact iterative quasidiabatic propagator path integral (QUAPI). This method is proved valid in investigating the qubit decoherence. In Ref. Thorwart et al. considered the coupling of longitudinal piezoelectric acoustic phonons with the investigated qubit and neglected the contribution of the deformation acoustic phonons to decoherence. These two kinds of phonons may constitute two kinds of different coupling baths in the environment of the qubit. We call the former the piezoelectric coupling phonon bath (PCPB) and the latter the deformation coupling phonon bath (DCPB). Comparing Thorwart’s result and the reported experimental value they found that the theory predicts the decoherence time of the DQD charge qubit is two orders of magnitudes smaller than the experimental one. Thus, Thorwart et al. conclude that the piezoelectric coupling phonon decoherence is a subordinate mechanism in decoherence of the DQD charge qubit. Recently, Wu et al. gave the spectral density functions of PCPB as well as DCPB. Then how about the DCPB to the decoherence of the DQD charge qubit? In other words, is the decoherence of the DQD charge qubit induced by DCPB also subordinate? In this paper we shall use an iterative tensor multiplication (ITM) scheme derived from the QUAPI to study the decoherence times of the DQD charge qubit not only in PCPB but also in DCPB. In order to validate if our result is in accordance with Thorwart’s result we at first investigate the decoherence times of the qubit in PCPB. Then we shall investigate the decoherence times of the qubit in another bath, DCPB, which will show that the influence of the DCPB to the decoherence of the DQD charge qubit is also subordinate because it results in a shorter decoherence time than the experimental value of $1$ ns.

II. MODELS

The DQD charge qubit consists of left and right dots connected through an interdot tunneling barrier. Due to Coulomb blockade, at most one excess electron is allowed to occupy the left and right dot, which defines two basis vectors $|0\rangle$ and $|1\rangle$. The energy difference $\varepsilon$ between these two states can be controlled by the source-drain voltage.
Neglecting the higher order tunneling between leads and the dots, the effective Hamiltonian in the manipulation process reads

\[ H_{\text{eff}} = \hbar T_r \sigma_x + \hbar \sum_q \omega_q b_q^\dagger b_q + \hbar \sigma_z \sum_q (M_q b_q^\dagger + M_q^* b_q). \]

Here, \( T_r \) is the interdot tunneling, \( \sigma_x \) and \( \sigma_z \) are Pauli matrix, \( b_q^\dagger \) (\( b_q \)) are the creation (annihilation) operators of phonons, \( \hbar \omega_q \) is the energy of the phonons, and \( M_q = C_q / \sqrt{2m_q \omega q \hbar} \) where \( C_q \) are the classical coupling constants of qubit-phonon system. We call the collective coupling phonons to the qubit in the environment a phonon bath. In order to obtain the reduced density matrix of the qubit in the system, one should know the coupling coefficients \( M_q \), but in fact we need not know the details of each \( M_q \) because all characteristics of the bath pertaining to the dynamics of the observable system are captured in the spectral density function

\[ J(\omega) = \sum_q |M_q|^2 \delta(\omega - \omega_q). \]

It is pointed out that the spectral density of PCPB is

\[ J^{pz}(\omega) = g_{pz}\omega \left( 1 - \frac{\omega_d}{\omega_d^2} \sin^2 \frac{\omega}{\omega_d} \right) e^{-\frac{\omega^2}{2\sigma_d^2}}. \]

Here, \( \omega_d = s/d \) and \( \omega_l = s/l \), where \( d \) denotes the center-to-center distance of two dots, \( l \) the dot size, \( s \) the sound velocity in the crystal, and

\[ g_{pz} = \frac{M}{\pi^2 g^3} \left( \frac{6}{35} + \frac{8}{x \cdot 35} \right). \]

Here, \( M \) is the piezoelectric constant, \( g \) is the density of the crystal, and \( x \) is the rate of transverse to the longitudinal of sound velocity in the crystal, (see for example Refs. 11, 12). As in Refs. 11, 12 in this paper we set the sound velocity in the GaAs crystal \( s \approx 5 \times 10^3 \) m/s. With the parameters of GaAs in Ref. 19, Wu et al. 11 propose a value \( g_{pz} \approx 0.035 \) (ps)\(^{-2} \). The spectral density of DCPB is

\[ J^{df}(\omega) = g_{df}\omega^3 \left( 1 - \frac{\omega_d}{\omega_d^2} \sin^2 \frac{\omega}{\omega_d} \right) e^{-\frac{\omega^2}{2\sigma_d^2}}, \]

where

\[ g_{df} = \frac{\Xi^2}{8\pi^2 g^3 s^5}. \]

Here, \( \Xi \) is the deformation potential. In the same paper, Wu et al. also propose a value \( g_{df} \approx 0.029 \) (ps)\(^{-2} \). One can investigate the dynamics and then the decoherence of the open qubit with the help of the definite spectral density functions of the baths. Before investigations of decoherence of the DQD charge qubit we introduce an optimal numerical path integral method, the ITM method in the following section.

### III. QUAPI AND ITM

In the following, we firstly review the QUAPI and then the ITM scheme. Suppose the initial state of the qubit-bath system has the form

\[ R(0) = \rho(0) \otimes \rho_{\text{bath}}(0), \]

where \( \rho(0) \) and \( \rho_{\text{bath}}(0) \) are the initial states of the qubit and bath. The evolution of the reduced density operator of the open qubit

\[ \tilde{\rho}(s''', s'' ; t) = \text{Tr}_{\text{bath}} \langle s'' | e^{-iHt/h} \rho(0) \otimes \rho_{\text{bath}}(0) e^{iHt/h} | s''' \rangle, \]

is given by

\[ \tilde{\rho}(s''', s'' ; t) = \sum_{s_0} \sum_{s_1} \cdots \sum_{s_{N-2}} \sum_{s_{N-1}} \sum_{s_0'} \cdots \sum_{s_{N-1}'} e^{iH_{\text{env}}t/2h} \rho_{\text{bath}}(0) e^{iH_{\text{env}}t/2h} | s_0 \rangle | s_0' \rangle \sum_{s'} \langle s'' | e^{-iH_{\text{env}}t/2h} \langle s_1 | e^{iH_{\text{env}}t/2h} \langle s_2 | e^{iH_{\text{env}}t/2h} \cdots \langle s_{N-1} | e^{iH_{\text{env}}t/2h} | s'''. \]

where the influence functional is

\[ I(s_0^+, s_1^+, \ldots, s_{N-1}^+, s_0^-, s_1^-, \ldots, s_{N-1}^-; \Delta t) = \text{Tr}_{\text{bath}} \left[ e^{-iH_{\text{env}}t'/2h} e^{-iH_{\text{env}}t'/2h} \cdots e^{iH_{\text{env}}(s_0^+) \Delta t/2h} e^{iH_{\text{env}}(s_0^-) \Delta t/2h} \cdots e^{iH_{\text{env}}(s_{N-1}^-) \Delta t/2h} e^{iH_{\text{env}}(s_{N-1}^+) \Delta t/2h} \right]. \]

Here, \( H_0 \) is a reference Hamiltonian that in general depends on the coordinate and momentum of the system. In the qubit system, it usually depends on Pauli matrix of \( \sigma_x \) and \( \sigma_z \). The \( H_{\text{env}} \) is defined as \( H_{\text{env}} = H - H_0 \). In our system we set \( H_0 = hT_r \sigma_x \).

The discrete path integral representation of the qubit density matrix contains temporal nonlocal terms

\[ I(s_0^+, s_1^+, \ldots, s_{N-1}^+, s_0^-, s_1^-, \ldots, s_{N-1}^-; \Delta t) \]

which denotes the process being non-Markovian. With the quasiadiabatic discretization of the path integral, the influence functional, Eq. (8) takes the form

\[ I = \exp \left\{ -\frac{i}{\hbar} \sum_{k=0}^{N} \sum_{k'=0}^{k} \left( s_k^+ - s_{k'}^+ \right) \langle \eta_{kk'} s_{k'}^+ - \eta_{kk'}^* s_k^- \rangle \right\}. \]

where \( s_k^+ = s'' \) and \( s_k^- = s' \). The coefficients \( \eta_{kk'} \) can be obtained by substituting the discrete path into the Feynman-Vernon expression. Their expressions have been shown in Ref. 13. Thus, the influence functional can be expressed with a product of terms corresponding
to different $\Delta k$ as

$$I = \prod_{k=0}^{N} I_0 (s^\pm_k) \prod_{k=0}^{N-\Delta k} I_{\Delta k} (s^\pm_k, s^\pm_{k+\Delta k}) \prod_{k=0}^{N-\Delta k_{\max}} I_{\Delta k_{\max}} (s^\pm_k, s^\pm_{k+\Delta k_{\max}}).$$

(10)

Here, $\Delta k = k - k'$, where $k'$ and $k$ are points of discrete path integral expressions, (see Ref. [15]) and

$$I_0 (s^\pm_i) = \exp \left\{-\frac{1}{\hbar} \left( s^+_i - s^-_i \right) (\eta_i s^+_i - \eta^*_i s^-_i) \right\},$$

$$I_{\Delta k} (s^\pm_i, s^\pm_{i+\Delta k}) = \exp \left\{-\frac{1}{\hbar} \left( s^+_i - s^-_i \right) \left( \eta_{i+\Delta k} s^+_i - \eta^-_{i+\Delta k} s^-_i \right) \right\}, \Delta k \geq 1.$$  

(11)

The length of the memory of the time can be estimated by the following bath response function

$$\alpha^x (t) = \frac{1}{\pi} \int_0^\infty d\omega J^x (\omega) \left[ \coth \left( \frac{\beta \hbar \omega}{2} \right) \cos \omega t - i \sin \omega t \right].$$  

(12)

Here, the superscript $x$ denotes the bath type, $\beta = 1/k_B T$, where $k_B$ is the Boltzmann constant, and $T$ is the temperature. It is shown that when the real and imaginary parts behave as the delta function $\delta (t)$ and its derivative $\delta' (t)$, the dynamics of the reduced density matrix is Markovian. However, if the real and imaginary parts are broader than the delta function, the dynamics is non-Markovian. The broader the Re$[\alpha^x (t)]$ and Im$[\alpha^x (t)]$ are, the longer of the memory time will be. The broader the Re$[\alpha^x (t)]$ and Im$[\alpha^x (t)]$ are, the more serious the Markov approximation will distort the practical dynamics. In Fig.1 we plot the Re$[\alpha^{px} (t)]$ and Im$[\alpha^{px} (t)]$ of the PCPB and in Fig.2 we plot the Re$[\alpha^{dy} (t)]$ and Im$[\alpha^{dy} (t)]$ of the DCPB.

**Fig.1,**

**Fig.2**

We see that the memory times are about $\tau_{\text{mem}}^{px} = 1 \times 10^{-11}$ s for PCPB and $\tau_{\text{mem}}^{dy} = 2 \times 10^{-11}$ s for DCPB (where the points beyond $\pm 1 \times 10^{-11}$ s have not been plotted for clearly distinguishing the Re$[\alpha^{dy} (t)]$ and Im$[\alpha^{dy} (t)]$ in the same figure). Due to the nonlocality, it is impossible to calculate the reduced density matrix by Eq. (7) in the matrix multiplication scheme. However, the short range nonlocality of the influence functional implies that the effects of the nonlocality should drop off rapidly as the “interaction distance” increases. In the ITM scheme the interaction can be taken into account at each iteration step. The reduced density matrix at time $t = N\Delta t$ ($N$ even) is given as

$$\tilde{\rho} (s^\pm_N, N\Delta t) = A^{(1)} (s^\pm_N; N\Delta t) I_0 (s^\pm_N),$$

where

$$A^{(1)} (s^\pm_{k+1}; (k + 1)\Delta t) = \int ds^\pm_k \exp \left( -iH_0 \Delta t / \hbar \right) I_{\Delta k} (s^\pm_k, s^\pm_{k+\Delta k}).$$

Here,

$$A^{(1)} (s^\pm_{k+1}; (k + 1)\Delta t) = \prod_{n=k}^{k+\Delta k_{\max}-1} K \left( s^\pm_k, s^\pm_{n+1} \right) I_0 (s^\pm_n) I_1 (s^\pm_n, s^\pm_{n+1})$$

and

$$A^{(\Delta k_{\max})} (s^\pm_0, s^\pm_1, ..., s^\pm_{\Delta k_{\max}-1}; 0) = \langle s^\pm_0 | \rho_s (0) | s^-_0 \rangle,$$

where

$$K \left( s^\pm_k, s^\pm_{k+1} \right) = \langle s^\pm_k | \exp (-iH_0 \Delta t / \hbar) | s^\pm_{k+1} \rangle.$$  

(12)

In the ITM scheme a short-time approximation instead of the Markov approximation is used. The approximation makes an error of the short-time propagator in order $(\Delta t)^3$, which is small enough as we set the time step $\Delta t$ very small. It is shown that when the time step $\Delta t$ is not larger than the characteristic time of the qubit system, which can be calculated with $1/T_\text{rel}$, the calculation is accurate enough [20]. In particular, the scheme does not discard the memory of the temporal evolution, which may be appropriate to solve the decoherence of qubit. In the following section we shall use the ITM scheme to study the decoherence times of the DQD charge qubit in PCPB and DCPB.

## IV. DECOHERENCE OF DQD CHARGE QUBIT

To measure effects of decoherence one can use the entropy, the first entropy, and many other measures, such as maximal deviation norm, etc. (see for example Ref. [20]). However, essentially, the decoherence of a open quantum system is reflected through the decays of the off-diagonal coherent terms of its reduced density matrix. The decoherence is in general produced due to the interaction of the quantum system with other systems with a large number of degrees of freedom, for example the devices of the measurement or environment. The decoherence time denoted by $\tau_2$ measures the time of the initial coherent terms to their $1/e$ times, namely, $\rho_i (n, m) \frac{\tau_2}{e}$
\[ \rho_f(n,m) = \rho_i(n,m)/e. \] Here, \( n \neq m \), and \( n, m = 0 \text{ or } 1 \) for qubits. In this paper, we investigate the decoherence times via directly describing the evolutions of the off-diagonal coherent terms, instead of using any measure of decoherence. In our following investigations, we suppose the temperature \( T = 30 \text{ mK} \) and the cut-off frequency of the bath modes \( \omega_C = 5 \text{ (ps)}^{-1} \). We set the initial state of the bath \( \rho(0) = \frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle) \), which is a pure state and it has the maximum coherent terms, and the initial state of the environment is \( \rho_{\text{bath}}(0) = \prod_k e^{-\beta M_k}/\text{Tr}(e^{-\beta M_k}). \) In the calculations, we set \( \omega_d = 0.02 \text{ (ps)}^{-1} \), \( T_c = 0.1\omega_l \) according to Ref. [11], and two kinds of cases \( \omega_l = 0.5 \text{ (ps)}^{-1} \) and \( \omega_l = 0.7 \text{ (ps)}^{-1} \) are calculated.

Decoherence time obtained from ITM scheme: In the following, at first, we use the ITM scheme investigating the decoherence time of the DQD charge qubit. The evolutions of the coherent elements of the reduced density matrix of the DQD charge qubit in PCPB and DCPB are plotted in Figs. 3 and 4. Here, we simply choose \( \Delta k_{\text{max}} = 1 \) and \( \Delta t = 1 \times 10^{-11} \text{ s} \) for PCPB and \( \Delta t = 2 \times 10^{-11} \text{ s} \) for DCPB in the ITM scheme. These choices of the time steps are feasible as we consider that it should be not smaller than the memory times of the baths, because the latter is about \( \tau^{\text{mem}}_{\text{pzz}} \approx 1 \times 10^{-11} \text{ s} \) for PCPB and \( \tau^{\text{mem}}_{\text{pzz}} \approx 2 \times 10^{-11} \text{ s} \) for DCPB (see Figs. 1 and 2). It is also appropriate as we consider that the time steps should not be larger than the characteristic time of the qubit, because the characteristic time of the qubit is about \( 2 \times 10^{-11} \text{ s} \).

\[ \text{Fig. 3}, \quad \text{Fig. 4} \]

Helped with detailed numerical analyses, we can obtain that the decoherence times of the DQD charge qubit in PCPB are about \( \tau^{\text{pzz}}_2 \approx 97 \text{ ps} \) when \( \omega_l = 0.7 \text{ (ps)}^{-1} \) and \( \tau^{\text{pzz}}_2 \approx 118 \text{ ps} \) when \( \omega_l = 0.5 \text{ (ps)}^{-1} \). Similarly, we can obtain that the decoherence times of this qubit in DCPB are about \( \tau^{\text{df}}_2 = 1.04 \text{ ps} \) when \( \omega_l = 0.7 \text{ (ps)}^{-1} \) and \( \tau^{\text{df}}_2 = 3.5 \text{ ps} \) when \( \omega_l = 0.5 \text{ (ps)}^{-1} \). It is shown that the DCPB behaves more destructively than the PCPB does to the coherence of the DQD charge qubit. A further calculation shows that the decoherence time will increase with the decreasing of \( T_c \).

Decoherence time calculated on Bloch equations: It is well known that the decoherence time can be calculated based on Bloch equations. In the following, we calculate the decoherence time of the DQD charge qubit in PCPB and DCPB with the Bloch equation method. In this method, the relaxation and dephasing times can be evaluated from the spin-bosonic model with Bloch equations [17, 18]. For our model, they are [22]

\[ \tau^{-1}_1 = \tau^{-1}_2 = \frac{1}{2\hbar} J(\omega_0) \coth(\beta\hbar\omega_0/2), \]

where \( \omega_0 = 2T_c \) is the natural frequency of the DQD charge qubit. By using the parameters of the DQD charge qubit and PCPB bath as above, we can calculate the decoherence times with this method as \( \tau^{\text{pzz}}_2 \approx 122.3 \text{ ps} \) [when \( \omega_l = 0.7 \text{ (ps)}^{-1} \)] and \( \tau^{\text{pzz}}_2 \approx 192.2 \text{ ps} \) [when \( \omega_l = 0.5 \text{ (ps)}^{-1} \)]. Similarly, we can obtain the decoherence times of the DQD charge qubit in the DCPB with this method as \( \tau^{\text{df}}_2 \approx 3.18 \text{ ps} \) [when \( \omega_l = 0.7 \text{ (ps)}^{-1} \)] and \( \tau^{\text{df}}_2 \approx 12.6 \text{ ps} \) [when \( \omega_l = 0.5 \text{ (ps)}^{-1} \)]. It is shown that the decoherence times obtained from the ITM scheme are shorter than those obtained on Bloch equations. We suggest that the differences are derived from the different choices of approximation schemes. The Bloch equations are in general derived from the Markov approximation which discards the memory of baths in the derivation of dynamical evolution. The decoherence of the qubit obtained on Bloch equations is similar to the “resonant decoherence” [21] obtained from the Fermi golden rule. It is not accurately equal to the actual decoherence except that the “nonresonant decoherence” very small.

Decoherence time derived from the quality factor: We like to compare our results obtained from the ITM scheme based on QUAPI with Thorwart’s results which are also obtained from QUAPI. Thorwart et al. [14] investigated the PCPB case and they obtained the quality factor instead of the decoherence time. By using a set of parameters of the DQD charge qubit and the PCPB they obtained the quality factor of the qubit as \( Q_{\text{pzz}} = 336 \), which corresponds to decoherence time \( \tau^{\text{pzz}}_2 = Q_{\text{pzz}}\pi/\omega_{\text{pzz}} \approx 115.9 \text{ ps} \), where \( \omega' = \omega_0 + \Delta \omega \), and \( \Delta \omega \) is the bath-induced shift [22] in the natural frequency \( \omega_0 = 2T_c \). From Fig.1 of Ref. [11] we see \( \Delta \omega_{\text{pzz}} \approx 1.75\omega_c \) and \( \Delta \omega_{\text{df}} \approx 1.65\omega_c \). Their used parameters \( T_l = 10 \text{ mK} \), \( T_c \approx 0.07 \text{ (ps)}^{-1} \) have a little difference from ours. But we have calculated that the difference does not result in much decoherence time departure. It is meant that our results is in accordance with Thorwart’s result. On the other hand, from our decoherence time \( \tau^{\text{df}}_2 \approx 3.5 \text{ ps} \) of the qubit in DCPB we can obtain its quality factor \( Q_{\text{df}} = \tau^{\text{df}}_2\omega_{\text{df}}/\pi \approx 8 \).

V. DISCUSSIONS AND CONCLUSIONS

In this paper we investigated the decoherence times of the DQD charge qubit in PCPB and DCPB with the ITM scheme based on QUAPI. The decoherence times are also calculated based on Bloch equations. The results derived from the two kinds of methods are compared to each other. It is shown that the latter are longer than the former. We think this results from the different choices of approximation schemes because the Markov approximation used in the latter method discards the memory of
the baths. On the other hand, Hayashi et al. have detected that the decoherence time of the DQD charge qubit is about 1 ns as $T_c \sim 0.07 \text{(ps)}^{-1}$. The exact ITM theoretical decoherence times are two orders of magnitude and five orders of magnitude smaller than the experimental value even when we consider the DQD charge qubit in independent PCPB and DCPB. These can finally and without accident lead to the conclusion that the phonon decoherence is a subordinate mechanism in the DQD charge qubit. In general, besides the phonon couplings’ decoherence, the qubit can also result in decoherence from electromagnetic fluctuations (with Ohmic noise spectrum), cotunneling effect, background charge fluctuations (with $1/f$ noise spectrum), and so on. To find out the dominating mechanism of the DQD charge qubit decoherence and to the best of our abilities to suppress the central decoherence resources are important challenges in the quantum computation field.

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**VI. FIGURE CAPTIONS**

Fig.1: Real (line) and imaginary (short lines) parts of the response function of the piezoelectric coupling phonon bath (PCPB). Here, we set the temperature $T = 30 \text{ mK}$, and $\omega_d = 0.02 \text{ (ps)}^{-1}$, $\omega_l = 0.5 \text{ (ps)}^{-1}$, $g^{(2)} = 0.035 \text{ (ps)}^{-2}$.

Fig.2: Real (line) and imaginary (short lines) parts of the response function of the deformation coupling phonon bath (DCPB). Here, we set $g^{(2)} = 0.029 \text{ (ps)}^{-2}$, and other parameters are same as those in Fig.1.

Fig.3: The evolutions of the off-diagonal elements of the reduced density matrix for the DQD charge qubit in PCPB when $\omega_l = 0.5 \text{ (ps)}^{-1}$ (line) and $\omega_l = 0.7 \text{ (ps)}^{-1}$ (short lines). Here, the cutoff frequency is $\omega_c = 5 \text{ (ps)}^{-1}$, other parameters are same as those in Fig.1. The initial state of the qubit and environment are described in the text.

Fig.4: The evolutions of the off-diagonal elements of the reduced density matrix for the DQD charge qubit in DCPB when $\omega_l = 0.5 \text{ (ps)}^{-1}$ (line) and $\omega_l = 0.7 \text{ (ps)}^{-1}$ (short lines). Here, the parameters are same as those in Figs.2 and 3.
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Fig.2
\[ \omega = 0.5 \text{ (ps)}^{-1} \]

\[ \omega = 0.7 \text{ (ps)}^{-1} \]

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Fig.3
