A Regression Line for a Laser Doppler Anemometer †

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Abstract: Refrigeration and air conditioning consume 15% of the total generated electricity. Vapor condensation devices need a heat sink which may come in the form of absorption cycles devices. Two fluids, which change phase and concentration, flow through these devices. These changes take place amid a two-phase flow in contact with a solid phase. Hence, an extended study of the velocity profiles across the thin liquid layer is necessary, which is assumed to be conducted by a laser Doppler anemometer. The preliminary studies concerning the calibration of this anemometer are reported.

Keywords: liquid film break down; rivulet; velocity profile; laminar flow; surface tension

1. Introduction

The International Institute of Refrigeration in Paris estimated that 15% of the global electrical energy production is supplied to refrigeration and air conditioning systems; the latter consume about 45% of the energy in households and commercial buildings. However, driving the traditional vapor compression devices needs electricity generated in a thermal power station, which depletes limited fuel resources [1]. A limitation of electrical energy usage enables the absorption cycle devices whose electricity demands are less than 5% of the cooling capacity. These devices may utilize energy from renewable resources or wasted energy with a temperature of at least 80 °C [2]; their main weakness is a great volume in relation to the cooling capacity [3]. These systems consist of the devices through which two fluid substances flow in different phases and concentrations [4]. In order to the most effective, phase and concentration changes need a thin liquid film covering all of the exchange surface. Hence, the process of a film break up into the rivulets is the subject of the investigations that needs acquaintance with the velocity profile; the simplest formula of this profile was derived by Nusselt (1916) (cf. Madejski [5]) as a one dimensional function. Hartley and Murgatroyd [6] discussed two stability criteria: a force balance in the stagnation point and a thermodynamic equilibrium. Bankoff [7] presumed both discharge mass and the sum of the kinetic and surface energies in the liquid film were equal to these in the rivulets. Mikielewicz and Moszyński [8], [9] included in their analysis, the surface energy of the solid-gas phase. Two dimensional velocity profiles for a rivulet on a vertical surface were obtained by El-Genk and Saber [10], as well as by Perazzo and Gratton [11], while Tanasijczuk et al. [12] received a profile for a rivulet hanging from a horizontal plane. Ataki and Bart [13] used the Nusselt profile in the analysis of their experiments. Charogiannis et al. [14] experimentally determined the laminar velocity profile for a Reynolds number (Re) of less than 5.9 and a Kapitza number (Ka) of 14; they also obtained the average velocity profiles for the wavy flow of Re ≈ 5.2, with a forcing frequency between 2 and 6 Hz, as well as for Re = 25 and Re = 27, Ka = 85, with the frequencies 7 and 10 Hz.

The main area of research was the experimental determination of velocity profile in thin liquid layers for a wide range of Reynolds numbers with a laser Doppler anemometer.
(LDA) used as a measuring device. For this reason a calibration of LDA system was necessary and was the subject of the study.

2. Methodology

The experiments are conducted using a system of apparatus [15]. Figure 1 shows the laser beams (1) intersecting in the cross-section with a fully developed flow. They are generated by the laser Doppler anemometer (5) and directed through an optical fiber (3) to the optical system (2) moved to the desired positions by the optical benches system (8) regulated by the controller (6). These devices are controlled by a piece of software installed on the microcomputer (9) and run by a researcher who also fixes the desired flow rates by using the valve (19) and the float meter (20). A stable flow is maintained by the constant height difference between the water levels in the tanks (12) and (21).

![Figure 1. The system of apparatus [15]: 1—laser beams, 2—LDA optical system, 3—optical fiber, 4—return signal wire, 5—laser Doppler anemometer, 6—traverse controller, 7—x,y,z movement signal wires, 8—system of the optical benches with engines, 9—microcomputer, 10—pump, 11—supply pipe, 12—upper tank, 13—inlet pipe, 14—straight coupler, 15—transparent tube, 16—valve, 17—thermometer, 18—outlet pipe, 19—adjustment valve, 20—float meter, 21—bottom tank, 22—overflow pipe.](image)

A benchmark is a laminar flow whose velocity profile is measured 30 times at 31 points across the tube, \( \varnothing 30 \text{ mm} \) (15). The fixed errors \( \delta_{\text{fixed}}(r_i) \), which are the differences between a laminar velocity \( v_i(r_i) \) in a point at the radius \( r_i \) and a mean velocity \( \bar{v}_{\text{LDA}}(r_i) \) from a set of 30 measurements made by the LDA system in this point, and the standard deviation \( \sigma \) for the arithmetic mean, \( \bar{v}_{\text{LDA}}(r_i) \) are determined as follows:

\[
\delta_{\text{fixed}}(r_i) = v_i(r_i) - \bar{v}_{\text{LDA}}(r_i),
\]

\[
\sigma(r_i) = \frac{1}{30} \left( \sum_{i=1}^{30} \left( \bar{v}_{\text{LDA}}(r_i) - v_{\text{LDA}}(r_i) \right)^2 \right)^{1/2}.
\]

Next the overall errors \( \delta_{\text{overall}}(r_i) \) are computed as the square root of the sum of the squares of the fixed error and doubled standard deviation [16]; their inverse squares are the weights \( w_i \):

\[
w_i = \frac{1}{\left( \delta_{\text{overall}}(r_i) \right)^2} = \frac{1}{\left( \delta_{\text{fixed}}(r_i) \right)^2 + (2\sigma(r_i))^2}.
\]
Finally, a weighted regression line slope $a$ and a Pearson’s correlation coefficient $r_{xy}$ are obtained [17]:

$$a = \frac{\sum w_i v_{LDA}(r_i) v_i(r_i)}{\sum w_i [v_{LDA}(r_i)]^2},$$

$$r_{xy} = \frac{\sum w_i r_{LDA}(r_i) v_i(r_i)}{\left\{ \sum w_i [v_{LDA}(r_i)]^2 \sum w_i [p_{lam}(r_i)]^2 \right\}^{1/2}}.$$ (5)

3. Results and Discussion

The regression line is computed from Equation (4) and the experimental results are plotted in Figure 2 where the regression equation and Pearson’s correlation coefficient value are also recorded.

Since the slope of the function is close to 1 and the Pearson’s correlation coefficient value is much higher, then its critical value equals 0.3557 for 29 degrees of freedom at a 95% confidence level and it may be concluded the applied system of apparatus maintains a high measurement quality.

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