Safety Model Checking with Complementary Approximations

(Jianwen Li, Shufang Zhu, Yueling Zhang, Geguang Pu, and Moshe Y. Vardi.

Rice University
Houston, TX, USA
lijwen2748@gmail.com

East China Normal University
Shanghai, China
saffiechu@gmail.com

East China Normal University
Shanghai, China
yueling671231@163.com

Abstract

Formal verification techniques such as model checking, are becoming popular in hardware design. SAT-based model checking techniques such as IC3/PDR, have gained a significant success in hardware industry. In this paper, we present a new framework for SAT-based safety model checking, named Complementary Approximate Reachability (CAR). CAR is based on standard reachability analysis, but instead of maintaining a single sequence of reachable-state sets, CAR maintains two sequences of over- and under-approximate reachable-state sets, checking safety and unsafety at the same time. To construct the two sequences, CAR uses standard Boolean-reasoning algorithms, based on satisfiability solving, one to find a satisfying cube of a satisfiable Boolean formula, and one to provide a minimal unsatisfiable core of an unsatisfiable Boolean formula. We applied CAR to 548 hardware model-checking instances, and compared its performance with IC3/PDR. Our results show that CAR is able to solve 42 instances that cannot be solved by IC3/PDR. When evaluated against a portfolio that includes IC3/PDR and other approaches, CAR is able to solve 21 instances that the other approaches cannot solve. We conclude that CAR should be considered as a valuable member of any algorithmic portfolio for safety model checking.

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1 Introduction

Model checking is a fundamental methodology in formal verification and has received more and more concern in the hardware design community [2, 10]. Given a system model M and a property P, model checking answers the question whether P holds for M. When P is a linear-time property, this means that we check that all behaviors of M satisfy P, otherwise a violating behavior is returned as a counterexample. In the recent hardware model checking competition (HWMCC) [5], many benchmarks are collected from the hardware industry. Those benchmarks are modeled by the aiger format [3], in which the hardware circuit and properties (normally the outputs of the circuit) to be verified are both included. For safety checking, it answers the question whether the property (output) can be violated by feeding the circuit an arbitrary (finite) sequence of inputs. In this paper, we focus on the topic of safety model checking.

Popular hardware model checking techniques include Bounded Model Checking (BMC) [4], Interpolation Model Checking (IMC) [15] and IC3/PDR [6, 8]. BMC reduces the search to a sequence of SAT calls, each of which corresponds to the checking in a certain step. The saturability of one of such SAT calls proves the violation of the model to the given property. IMC combines the use of Craig Interpolation as an abstraction technique with the use of BMC as a search technique. IC3/PDR starts with an over-approximation, gradually then refined to be more and more precise [6, 8]. All of the three approaches have proven to be highly scalable, and are today parts of the algorithmic portfolio of modern symbolic model checkers, e.g. ABC [7].

We present here a new SAT-based model checking framework, named Complementary Approximate Reachability (CAR), which is motivated both by classical symbolic reachability analysis and by IC3/PDR as an abstraction-refinement technique. While standard reachability analysis maintains a single sequence of reachable-state sets, CAR maintains two sequences of over- and under-approximate reachable-state sets, checking safety and unsafety at the same time. While IC3/PDR also checks safety and unsafety at the same time, CAR does this more directly by keeping an over-approximate sequence for safety checking, and an under-approximate sequence for unsafety checking. To compute these sequences, CAR utilizes off-the-shelf Boolean-reasoning techniques for computing Minimal
Unsat (MUC) [14], in order to refine the over-approximate sequence, and Minimal Satisfying Cube (i.e., partial assignment) [18], in order to extend the under-approximate sequences. In contrast, IC3/PDR uses a specialized technique, called generalization, to compute Minimal Inductive Clauses (MIC) [6]. Thus, IC3/PDR computes relatively-inductive clauses to refine the over-approximate state sequence, while CAR does not. Because of this difference, CAR and IC3/PDR are complementary, with CAR faster on some problem instances where refining by non-relatively-inductive clauses is better, and IC3/PDR faster on others where refining by relatively-inductive clauses is better.

To evaluate the performance of CAR, we benchmarked it on 548 problem instances from the 2015 Hardware Model-Checking Competition, and compared the results with IC3/ PDR. The results show that while the performance of CAR does not dominate the performance of IC3/PDR, CAR complements IC3/PDR and is able to solve 42 instances that IC3/PDR cannot solve. When evaluated against a portfolio that includes IC3/PDR, BMC, and IMC, CAR is able to solve 21 instances that the other approaches cannot solve. It is well known that there is no "best" algorithm in model checking; different algorithms perform differently on different problem instances [1], and a state-of-the-art tool must implement a portfolio of different algorithms, cf. [7]. Our empirical results also support the conclusion that CAR is an important contribution to the algorithmic portfolio of symbolic model checking.

The paper is organized as follows. Section 2 introduces preliminaries, while Section 3 describes the framework of CAR. Section 4 describes the implementation of the framework, Section 5 introduces experimental results, and Section 6 discusses and concludes the paper. Missing proofs are in Appendix.

2 PRELIMINARIES

2.1 Boolean Transition System, Safety Verification and Reachability Analysis

A Boolean transition system SyS is a tuple \((V, I, T)\), where \(V\) is a set of Boolean variables, and every state \(s\) of the system is in \(2^V\), the set of truth assignments to \(V\). \(I\) is a Boolean formula representing the set of initial states. Let \(V'\) be the set of primed variables (a new copy) corresponding to the variables of \(V\), then \(T\) is a Boolean formula over \(V \cup V'\), denoting the transition relation of the system. Formally, for two states \(s_1, s_2 \in 2^V\), \(s_2\) is a successor state of \(s_1\), denoted as \((s_1, s_2) \in T\), if \(s_1 \cup s_2' = T\), where \(s_2'\) is a primed version of \(s_2\).

A path (of length \(k\)) in \(SyS\) is a finite state sequence \(s_1, s_2, \ldots, s_k\), where each \((s_i, s_{i+1})\) (\(1 \leq i \leq k - 1\)) is in \(T\). We use the notation \(s_1 \rightarrow s_2 \rightarrow \ldots \rightarrow s_k\) to denote a path from \(s_1\) to \(s_k\). We say that a state \(t\) is reachable from a state \(s\), or that \(s\) reaches \(t\), if there is a path from \(s\) to \(t\). Moreover, we say \(t\) is reachable from \(s\) in \(i\) steps (resp., within \(i\) steps) if there is a path from \(s\) to \(t\) of length \(i\) (resp., of length at most \(i\)).

Let \(X \subseteq 2^V\) be a set of states in \(SyS\). We define \(R(X) = \{s'| (s, s') \in T \text{ where } s \in X\}\), i.e., \(R(X)\) is the set of successors of states in \(X\). Conversely, we define \(R^{-1}(X) = \{s| (s', s) \in T \text{ where } s' \in X\}\), i.e., \(R^{-1}(X)\) is the set of predecessors of states in \(X\). Recursively, we define \(R^0(X) = X\) and \(R^i(X) = R(R^{i-1}(X))\) for \(i > 0\). The notations of \(R^{-1}(X)\) is defined analogously.

Given a Boolean transition system \(SyS = (V, I, T)\) and a safety property \(P\), which is a Boolean formula over \(V\), the system is called safe if \(P\) holds in all reachable states of \(SyS\), and otherwise it is called unsafe. The safety checking asks whether \(SyS\) is safe. For unsafe systems, we want to find a path from an initial state to some state \(s\) that violates \(P\), i.e., \(s \in \neg P\). We call such state reachable to \(\neg P\) a bad state, and the path from \(I\) to \(\neg P\) a counterexample.

In symbolic model checking (SMC), safety checking is performed via symbolic reachability analysis. From the set \(I\) of initial states, we compute the set of reachable states by computing \(\Gamma(I)\) for increasing values of \(i\). We can compute the set of states that can reach states in \(\neg P\), by computing \((\Gamma^{-1}(\neg P))\) for increasing values of \(i\). The first approach is called forward search, while the second one is called backward search. The formal definition of these two approaches are shown in the table below.

| Basic | Forward | Backward |
|-------|---------|----------|
| \(I_0\) = \(I\) | \(I_{i+1} = R(I_i)\) | \(I_{i+1} = R^{-1}(B_j)\) |
| Induction: | \(I_{i+1} \subseteq \bigcup_{j \leq i} F_j\) | \(B_{i+1} \subseteq \bigcup_{j \leq i} B_j\) |
| Terminate: | \(f_i \land \neg P \neq \emptyset\) | \(B_i \land \neg \emptyset \neq \emptyset\) |
| Check: | \(f_i \neq \emptyset\) | \(B_i \neq \emptyset\) |

For forward search, the state set \(F_i\) is the set of states that are reachable from \(I\) in \(i\) steps. This set is computed by iteratively applying \(R\). To find a counterexample, forward search checks at every step whether one of the bad states has been reached, i.e., whether \(F_i \cap \neg P \neq \emptyset\). If a counterexample is not found, the search will terminate when \(F_i + 1 \subseteq \bigcup_{j \leq i} F_j\). For backward search, the set \(B_i\) is the set of states that can reach \(\neg P\) in \(i\) steps. The workflow of backward search is analogous to that of forward search. Note that forward checking of \(SyS = (V, I, T)\) with respect to \(P\) is equivalent to backward checking of \(SyS^{-1} = (V, \neg P, T^{-1})\) with respect to \(\neg I\), where \(T^{-1}\) is simply \(T\), with primed and unprimed variables exchanged.

2.2 Notations

Each variable \(a \in V\) is called an atom. A literal \(l\) is an atom \(a\) or a negated atom \(\neg a\). A conjunction of a set of literals, i.e., \(l_1 \land l_2 \land \ldots \land l_k\), for \(k \geq 1\), is called a cube. Dually, a disjunction of a set of literals, i.e., \(l_1 \lor l_2 \lor \ldots \lor l_k\), for \(k \geq 1\), is called a clause. Obviously, the negation of a cube is a clause, and vice versa. Let \(C\) be a set of cubes (resp., clauses), we define the Boolean formula \(f(C) = \bigvee_{C \in C} \phi\) (resp., \(f(C) = \bigwedge_{C \in C} \phi\)). For simplicity, we use \(C\) to represent \(f(C)\) when it appears in a Boolean formula; for example, the formulas \(\phi \land C\) and \(\phi \lor C\), abbreviate \(\phi \land f(C)\) and \(\phi \lor f(C)\).

A cube (/clause) \(C\) can be treated as a set of literals, a Boolean formula, or a set of states, depending on the context it is used. If \(C\) appears in a Boolean formula, for example, \(C \Rightarrow \phi\), it is treated as a Boolean formula. If we say a set \(C_1\) is a subset of \(C_2\), then we treat \(C_1\) and \(C_2\) as literal sets. If we say a state \(s\) is in \(C\), then we treat \(s\) as a set of states.

We use \(s(x)/s'(x')\) to denote the current/primed version of the state \(s\). Similarly, we use \(\phi(x)/\phi'(x')\) to denote the current/primed version of a Boolean formula \(\phi\). For the transition formula \(T\), we use the notation \(T(x, x')\) to highlight that it contains both current and primed variables. Consider a Boolean formula \(\phi\) whose alphabet is \(V \cup V'\) and is in the conjunctive normal form (CNF). If \(\phi\) is satisfiable, then there is a full assignment \(A \in 2^{V \cup V'}\) such that \(A \models \phi\). Moreover, there is a partial assignment \(A^P \subseteq A\) such that for every
full assignment \( A' \supseteq A^0 \) it holds that \( A' \models \phi \). In the following, we use the notation \( pa(\phi) \) to represent a partial assignment of \( \phi \), and use \( pa(\phi)_{|V} \) to represent the subset of \( pa(\phi) \) achieved by projecting variables only to \( V \). On the other hand, if \( \phi \) is unsatisfiable, there is a Minimal Unsat Core (MUC) \( C \subseteq \phi \) (here \( \phi \) is treated as a set of clauses) such that \( C \) is unsatisfiable and every \( C' \subseteq C \) is satisfiable. In the following, we use the notation \( muc(\phi) \) to represent such a MUC of \( \phi \), and use \( muc(\phi)_{|V} \) to represent the subset of \( muc(\phi) \) achieved by projecting clauses only to \( c' \). Since \( c' \) is a cube, \( muc(\phi)_{|c'} \) is also a cube.

3 THE FRAMEWORK OF CAR

We present here a variant of standard reachability checking, in which the set of maintained states is allowed to be approximate. The new approach is named Complementary Approximate Reachability, abbreviated as CAR. As in standard reachability analysis, CAR also enables both forward and backward search. In the following, we introduce the forward approach in detail; the backward approach can be derived symmetrically.

3.1 Approximate State Sequences

In standard forward search, described in Section 2, each \( F_i \) is a set of states that are reachable from \( I \) in \( i \) steps. To compute elements in \( F_{i+1} \), previous SAT-based symbolic-model-checking approaches consider the formula \( \phi = F_i(x) \land T(x, x') \), and use partial-assignment techniques to obtain all states in \( F_{i+1} \) from \( \phi \) (by projecting to the prime part of the assignments). Since the set of reachable states is computed accurately, maintaining a sequence of sets of reachable states from \( I \) enables to check both safety and unsafety. However in Forward CAR, two sequences of sets of reachable states are necessary to maintain: 1) \((F_0, F_1, \ldots) \) is a sequence of overapproximate state sets, which are supersets of reachable states from \( I \). 2) \((B_0, B_1, \ldots) \) is a sequence of under-approximate state sets, which are subsets of reachable states to \( \neg P \). Under the approximations, the first sequence is only sufficient to check safety, and the second one is then required to check unsafety. The two state sequences are formally defined as follows.

**Definition 3.1.** For a Boolean system \( Sys \) and the safety property \( P \), the over-approximate state sequences \((F_0, F_1, \ldots, F_i) \) (i ≥ 0), which is abbreviated as \( F \)-sequence, and the under-approximate state sequence \((B_0, B_1, \ldots, B_k) \) (k ≥ 0), which is abbreviated as \( B \)-sequence, are finite sequences of state sets such that:

| Basic                      | Inductive                      |
|----------------------------|-------------------------------|
| \( F_0 = I \)              | \( B_0 = \neg P \)            |
| \( F_j \subseteq P(0 \leq j) \) | \( B_{j+1} \subseteq R^{-1}(B_j) \) (j ≥ 0) |

For each \( F_i \) (i ≥ 0), we call it a frame. We also define the notation \( S(F) = \bigcup_{0 \leq j < i} F_j \) is the set of states in the \( F \)-sequence, and \( S(B) = \bigcup_{0 \leq j \leq k} B_j \) is the set of states in the \( B \)-sequence.

Note that the \( F \)- and \( B \)-sequence are not required to have the same length. Intuitively, each \( F_{i+1} \) is an over-approximate set of states that are reachable from \( F_i \) in one step, and \( B_{i+1} \) is an under-approximate set of states that are reachable to \( B_i \) in one step. As we mentioned in Section 2, we overload notation and consider \( F_i \) to represent (1) a set of states, (2) a set of clauses and (3) a Boolean formula in CNF. Analogously, we consider \( B_i \) to be (1) a set of states, (2) a set of cubes and (3) a Boolean formula in DNF.

The following theorem shows that, the safety checking is preserved even if \( F_i(i \geq 0) \) becomes over-approximate.

**Theorem 3.2 (Safety Checking).** A system \( Sys \) is safe for \( P \) if and only if there is \( i \geq 0 \) and an \( F \)-sequence \((F_0, F_1, \ldots, F_i) \) such that \( F_{i+1} \subseteq \bigcup_{0 \leq j \leq i} F_j \).

Theorem 3.2 is insufficient for unsafety checking, as \( B_{i+1} \subseteq \bigcup_{0 \leq j \leq i} F_j \) to has to prove false for every \( i \geq 0 \). On the other hand, the unsafety checking condition \( \exists i \in F_i \cap \neg P \neq \emptyset \) in the standard forward reachability is not correct when \( F_i \) becomes over-approximate. Our solution is to benefit from the information stored in the \( B \)-sequence.

**Theorem 3.3 (Unsafety Checking).** For a system \( Sys \) and the safety property \( P \), \( Sys \) is unsafe for \( P \) if there is \( i \geq 0 \) and a \( B \)-sequence \((B_0, B_1, \ldots, B_i) \) such that \( I \cap B_i \neq \emptyset \).

Besides, since CAR maintains two different sequences, exploring the relationship between them can help to establish the framework. The following property shows that, the states stored in \( F \)- and \( B \)-sequences are unreachable when the system \( Sys \) is safe for the property \( P \).

**Property 1.** For a system \( Sys \) and the safety property \( P \), \( Sys \) is safe for \( P \) if there is an \( F \)-sequence such that \( S(F) \cap R^{-1}(S(B)) = \emptyset \) for every \( B \)-sequence.

Property 1 suggests a direction that how we can refine the \( F \)-sequence and update the \( B \)-sequence. That is to try to make the states in these two sequences unreachable. More details are shown in the next section.

We have established the Forward CAR framework, and presented the theoretical guarantee for both safety and unsafety checking. Note that symmetrically, Backward CAR performs the same framework on \( Sys^{-1} = (V, \neg P, T^{-1}) \) with respect to \( \neg T \), where \( T^{-1} \) is simply \( T \) with primed and unprimed variables exchanged.

3.2 The Framework

Unlike the standard forward reachability, which computes all states in \( F_{i+1} \) from the single formula \( F_i(x) \land T(x, x') \), Forward CAR computes elements of \( F_{i+1} \) from different SAT calls with different inputs. Each SAT call gets as input a formula of the form \( F_i(x) \land T(x, x') \land c'(x') \), where the cube c is in some \( B_j \) and \( c' \) is its primed version. If the formula is satisfiable, we are able to find a new state which is in \( B_{j+1} \); otherwise we prove that \( c \land R(F_i) = \emptyset \), which indicates \( F_{i+1} \) can be refined by adding the clause \( \neg c \). The following lemma shows the main idea of computing new reachable states to \( \neg P \) and new clauses to refine \( F_i \).

**Lemma 3.4.** Let \((F_0, F_1, \ldots) \) be an \( F \)-sequence, \((B_0, B_1, \ldots) \) be a \( B \)-sequence, cube \( c_1 \in B_j(j \geq 0) \) and the formula \( \phi = F_i(x) \land T(x, x') \land c_1'(x')(0 \leq i) \):

1. If \( \phi \) is satisfiable, there is a cube \( c_2 \) such that every state \( t \in c_2 \) is a predecessor of some state in \( c_1 \) and \( t \in F_i \). By updating \( B_{j+1} = B_{j+1} \cup \{c_2\} \), the sequence is still a \( B \)-sequence.

2. If \( \phi \) is unsatisfiable, \( c_1 \land R(F_i) = \emptyset \). Moreover, there is a cube \( c_2 \) such that \( c_1 \Rightarrow c_2 \) and \( c_2 \land R(F_i) = \emptyset \). By updating \( F_{i+1} = F_{i+1} \cup \{\neg c_2\} \), the sequence is still an \( F \)-sequence.

In the lemma above, Item 1 suggests to add a set of states rather than a single one to the \( B \)-sequence, and similarly Item 2 suggests to...
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relatively inductive, while clauses from MUC cannot guarantee.
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IC3/PDR requires the
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Theorem 3.5. Given a system Sys and the safety property P, the
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Algorithm 1 Implementation of explore in CARchecker
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Require: F-sequence: (F_0, F_1, . . . , F_m (m \geq 0));
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B-sequence: (B_0, B_1, . . . , B_n (n \geq 0));
F
Ensure: A counterexample is returned or assert S(F) \cap R^{-1}(S(B)) = \emptyset;
F
1: for n \geq i \geq 0 do
F
2: \textbf{while } \phi = F_m(x) \land T(x,x') \land B_i(x') \text{ is satisfiable do}
F
3: \textbf{if } dfscheck (c, m - 1, n + 1) returns unsafe with a counterexample \textbf{then}
F
4: \quad \textbf{return } unsafe with a counterexample
F
5: \textbf{end if}
F
6: \textbf{end while}
F
7: \textbf{end for}
F
8: Let updated B-sequence be (B_0, B_1, . . . , B_{n'} (n' \geq n \geq 0));
F
9: \textbf{Assert } \forall_{0 \leq m \leq n'} F_i(x) \land T(x,x') \land \forall_{0 \leq j \leq n'} B_j(x') \text{ is unsatisfiable;}
F
it more complex to run IC3/PDR in a backward mode. Indeed, the
evaluation of IC3/PDR in the backward mode is still an open topic.
CAR also maintains an under-approximate state sequence (B-
sequence) to check unsafety, while IC3/PDR checks unsafety “on-
the-fly”. Other papers also introduced multiple state sequences.
The approach of “Dual Approximated Reachability” maintains two
over-approximate state sequences to check safety in both forward
and backward directions [16]. In contrast, we maintain two comple-
mentary (over- and under-)approximate state sequences to check
safety and unsafety at the same time. In [12], states reachable from
initial states are maintained to help to handle proof-obligation gen-
eration. In contrast, the B-sequence keeps states that reach bad
states. In PD-KIND [13], the idea of keeping both over- and under-
approximate (F- and B-) state sequences is also introduced, and the
B-sequence is used to refine the F-sequence as well. However, CAR
utilizes the F-sequence for safety checking and B-sequence for
unsafety checking, while PD-KIND utilizes the F-sequence for
unsafety checking and another “induction frame” has to be intro-
duced for the safety checking. Moreover, CAR and PD-KIND use
very different underlying techniques: CAR uses MUC and partial
assignment, while PD-KIND uses interpolation and generalization.

4 IMPLEMENTATION
In this section, we introduce the implementation of CAR in our
hardware model checker CARchecker. We implement both Forward
and Backward CAR in CARchecker, but we introduce only the implementation of For-
ward CAR here to save paper space. (Actually backward CAR is
implemented symmetrically.)

Main Procedure We only show details on Step 3b of the frame-
work, which is clear in theory, but leaves some choices in algorithmic
level. We should highlight that, our implementation of Step 3b in CARchecker is not the only option, and readers can find an
alternative easily: We aim to make CAR a general framework rather
than a specific algorithm.
We name Step 3b in the framework the procedure explore, whose
implementation is shown in Algorithm 1. Recall that the task of
3.3 Related Work
There are two main differences between CAR and IC3/PDR. First,
IC3/PDR requires the F-sequence to be monotone, while CAR does not.
Because CAR keeps the F-sequence non-monotone, it does not
require the push and propagate processes, which are necessary
in IC3/PDR. A drawback for CAR is that additional SAT calls are
needed to check safety, i.e. to find i > 0 such that F_{i+1} \subseteq \bigcup_{0 \leq j \leq i} F_j holds. In IC3/PDR, since the F-sequence is monotone, it is easy to
find such i that F_i = F_{i+1} syntactically.
Another main difference between CAR and IC3/PDR is the way
they refine the F-sequence. CAR utilizes the off-the-shelf MUC
techniques, while IC3/PDR puts more efforts to compute MIC such
that the refined F-sequence is still monotone. Moreover, MIC are
relatively inductive, while clauses from MUC cannot guarantee.
As a result, CAR and IC3/PDR refines the F-sequence by different
kinds of clauses, and thus perform differently. Although computing
relatively-inductive clauses is proved to be efficient in IC3/PDR,
we show in the experiments that, CAR complements IC3/PDR on
the instances that computing relatively inductive clauses is not
conducive for efficient checking.
It is trivial to apply the framework of CAR in both forward
and backward directions by simply reversing the direction of the
model. Indeed, our implementation of CAR runs the forward and
backward modes in parallel. Although in theory it is also possible
to run IC3/PDR in backward mode, there is a technical issue that
must be addressed. In most IC3/PDR implementations, the initial
states I is considered as a single cube. This helps to save a lot of
SAT calls in the process of generalization, in which the computed
clause c must satisfy I \land c is unsatisfiable. (When I is a cube it is
reduced to checking the containment of \neg c \subseteq I.) But usually the set
of unsafe states cannot be expressed by a single cube, which makes
the
Table 1: The Framework of Forward CAR

|   |   |
|---|---|
| 1 | Initially, set \( B_0 = \neg P, F_0 = I \); |
| 2 | If \( F_0 \cap B_0 \neq \emptyset \) or \( R(F_0) \cap B_0 \neq \emptyset \), return unsafe with counterexample; |
| 3 | For \( i \geq 1 \), |
|   | (a) Set \( F_i := P \); |
|   | (b) while \( S(F) \cap R^{-1}(S(B)) \neq \emptyset \) |
|   | (i) Let \( j \) be the minimal index such that \( F_j \cap R^{-1}(B_k) \neq \emptyset \) for some \( k \geq 0 \); |
|   | (ii) If \( j = 0 \), return unsafe with counterexample; |
|   | (iii) Let cube \( c_1 = pa(F_j(x)) \cap T(x, x') \cap B_k'(x') \) \( \subseteq \emptyset \) (From 3(b) i \( \subseteq \emptyset \) must exist); |
|   | (iv) Set \( B_{k+1} := B_{k+1} \cup \{ c_1 \} \) if \( B_{k+1} \) exists, otherwise set \( B_{k+1} = \{ c_1 \} \); |
|   | (v) Let \( \phi = F_{j-1}(x) \cap T(x, x') \cap c_1'(x') \); |
|   | (vi) If \( \phi \) is satisfiable, let \( c_2 = pa(\phi) \) then assert \( c_2 \not\subseteq R^{-1}(S(B)) \) and set \( B_{k+2} := B_{k+2} \cup \{ c_2 \} \) if \( B_{k+2} \) exists, otherwise set \( B_{k+2} = \{ c_2 \} \); |
|   | (vii) If \( \phi \) is unsatisfiable, let \( c_2 = muc(\phi) \) then assert \( c_2 \not\subseteq F_j \) and set \( F_j := F_j \cup \{ \neg c_2 \} \). |
|   | (c) If \( 30 \leq j \leq i \cdot F_j \subseteq \bigcup_{0 \leq m \leq j-1} F_m \), return safe; |
|   | (d) Set \( i := i + 1 \); |

\( explore \) is to either find a counterexample, or make sure \( S(F) \cap R^{-1}(S(B)) = \emptyset \) is true. Here \( explore \) is designed to enumerate the \( B \)-sequence from the end to the beginning (\( n \to 0 \)), and once finding a cube \( c \subseteq F_m \cap R^{-1}(S(B)) \) it invokes the procedure \( dfscheck \) (Line 4), which applies trivial backward search, either to confirm a counterexample or to learn the knowledge that \( c \) can be removed from \( F_m \). If a counterexample is never returned in the IF block at Line 4, \( F_m \) is repeatedly refined (inside \( dfscheck \)) and finally the formula \( \phi \) becomes unsatisfiable to terminate the while loop at Line 2. Finally we use the assertion at Line 10 to guarantee \( S(F) \cap R^{-1}(S(B)) = \emptyset \) must be true at the end of Algorithm 1.

The procedure \( dfscheck \) uses a depth-first strategy to search back from the input cube \( c \) to the initial states in \( F_0 \). Starting from \( c \), \( dfscheck \) searches its predecessors which are also in \( F_{index} \). If such predecessor \( c_1 \) is found, \( dfscheck \) recursively searches the predecessor of \( c_1 \) which are also in \( F_{index-1} \) (Line 10). Once a predecessor in \( F_0 \) is found, a counterexample can be returned (Line 7). Otherwise if no predecessor of \( c \) can be found in \( F_{index} \), \( c \) is then removed from \( F_{index+1} \) (Line 14). Note that the IF block at Line 15 is either to find a counterexample which is longer than the \( F \)-sequence, just as IC3/PDR does, or to make \( c \subseteq S(B) \) empty, which is a necessary condition to keep \( S(F) \cap R^{-1}(B_{binindex}) = \emptyset \).

The implementation of \( explore \) here is to provide a depth-first search (DFS) strategy to find new bad states to enlarge the \( B \)-sequence and refine the \( F \)-sequence. However, since the search problem has been well-studied in other areas, such as AI, better strategies may be found to replace DFS. We leave this conjecture to our future work.

**Heuristics** Two kinds of heuristics are implemented in Forward CAR. First we apply an optimized partial-assignment technique. Consider the checking formula \( \phi = \left( F_i(x) \cap T(x, x') \cap c'(x') \right) \) where \( F_i \) is the \( i \)-th frame and \( c \) is a cube representing a set of bad states. Assume the formula is satisfiable, there is a full assignment \( A \) and a state \( t \in c \) and a predecessor \( s \) of \( t \) can be extracted, that is, \( A = s \cup t \). By using standard partial-assignment technique with the inputs \( (A, \phi) \), we obtain \( PA \subseteq A \) and two cubes \( ps, pt \) such that \( PA = ps \cup pt \), \( ps \subseteq s \) and \( pt \subseteq t \). Here \( ps \) is the result of \( pa(\phi) \). However based on the structure of formula \( \phi \), we are able to get a better \( ps \) by setting \( pt = c \). (In practice, the size of \( pt \) is normally larger than \( c \).) To achieve that, we apply the partial-assignment technique on

Algorithm 2 Implementation of \( dfscheck \) in CARchecker

**Require:** \( F \)-sequence: \( (F_0, F_1, \ldots, F_m \ (m \geq 0)) \); \( B \)-sequence: \( (B_0, B_1, \ldots, B_n \ (n \geq 0)) \); \( c \): a cube; \( index \): the current index of \( F \)-sequence; \( binindex \): the current index of \( B \)-sequence;

**Ensure:** A counterexample is returned or \( S(F) \cap R^{-1}(B_{binindex}) = \emptyset \) is asserted;

1. **while** true **do**
2. \( \phi = F_{index}(x) \cap T(x, x') \cap c'(x') \);
3. **if** \( \phi \) is unsatisfiable **then**
4. break;
5. **end if**
6. Let \( c_1 = pa(\phi) \) and update \( B_{binindex+1} = B_{binindex+1} \cup \{ c_1 \} \);
7. **if** \( index = 0 \) **then**
8. return unsafe with a counterexample
9. **end if**
10. **if** \( dfscheck \left( c_1, \neg bindex - 1, bindex + 1 \right) \) returns a counterexample **then**
11. return unsafe with a counterexample
12. **end if**
13. **end while**
14. Let \( c_2 = muc(\phi) \) and update \( F_{index+1} = F_{index+1} \cup \{ \neg c_2 \} \);
15. **if** \( index + 1 < m \) (Note: \( m \) is the largest index in the \( F \)-sequence) **then**
16. **if** \( dfscheck \left( c, \neg index + 1, binindex \right) \) returns unsafe with a counterexample **then**
17. return unsafe with a counterexample
18. **end if**
19. **end if**
20. Assert \( \bigvee_{0 \leq i \leq m} F_i(x) \cap T(x, x') \cap B_{binindex}'(x') \) is unsatisfiable;
the inputs \((A, \phi_c)\), where \(\phi_c = F_i(x) \land T_c(x, x') \land c'(x')\) and \(T_c \subseteq T\) contains only necessary constraints for \(c\). Then the resulting \(pt = c\) can be guaranteed, and the corresponding \(ps\) is the optimized result. Actually, we consider this optimization as another implementation of the so-called “ternary simulation”, which is widely used in circuit simulation. It is integrated in the state-of-the-art model checker ABC [7] to accelerate the performance.

As in PDR [8], we also keep a special set of states \(F_{\infty}\), which are proved to be unreachable from initial states in any steps. We check the satisfiability of formula \(T(x, x') \land c'(x)\), where \(c\) is a cube. If the formula is unsatisfiable, \(muc(c)\) represents the set of states that do not have predecessors. We call such states the dead states. Obviously, dead states can be added into \(F_{\infty}\), as they are unreachable from initial states in any steps. We call this heuristics the dead-states-detect. Surprisingly, when we search from \(\neg p\) (bad states), it is quite often that dead states are detected. Although extra SAT calls are necessary for detecting dead states, it really pays off to successfully block those states in any frame \(\geq 1\).

It should be mentioned that, neither ternary simulation nor dead-states-detect is useful for Backward CAR. Based on the particular circuit structure, the ternary simulation cannot be performed reversely. And when we search from \(l\) (good states) in Backward CAR, very few dead states can be detected. As a result, applying the dead-states-detect in Backward CAR will only slow down the checker.

5 EXPERIMENTS

Experimental Setup In this section, we report the results of our empirical evaluation. Our (C++) model checker CARchecker runs CAR in both Forward and Backward modes, using Minisat [9] and Muser2 [14] as the SAT and MUC engines. The tool implements the algorithm from [18] to extract partial assignments. The performance of CARchecker is tested by evaluating it on the 548 safety benchmarks from the 2015 Hardware Model Checking Competition [5].

We first compared the performance of CAR with that of IC3/PDR, as implemented in the state-of-the-art model checker ABC [7] (the “pdr” command in ABC with default parameters). It should be noted that, there are many variants of IC3/PDR implementations currently, in which many heuristics are applied to the original one [11]. We choose ABC as the reference implementation for comparison because it is a standard model checker integrating several kinds of SAT-based model checking techniques. Moreover, a portfolio of modern model checkers consists of IC3/PDR, BMC (Bounded Model Checking), and IMC (Interpolation Model Checking), so we also run the experiments of BMC and IMC in ABC to explore the contribution of CAR compared to an existing portfolio (we used the “bmc2” and “int” commands in ABC with default parameters).

We run the experiments on a compute cluster that consists of 2304 processor cores in 192 nodes (12 processor cores per node), running at 2.83 GHz with 48GB of RAM per node. The operating system on the cluster is RedHat 6.0. When we run the experiments, each tool is run on a dedicated node, which guarantees that no CPU or memory conflict with other jobs will occur. Our tool CARchecker can run CAR in Forward mode, Backward mode or combined mode, which returns the best result from either of the approaches. In our experiments, memory limit was set to 8 GB and time limit (CPU time) to 1 hour. Instances that cannot be solved within this time limit are considered unsolved, and the corresponding time cost is set to be 1 hour. We compare the model-checking results from CARchecker with those from ABC on all benchmarks, and no discrepancy is found.

Experimental Results We show first overall performance comparison among different approaches in Figure 1. Neither Forward CAR nor Backward CAR by itself is currently competitive with IC3/PDR. The reasons are as follows. First, the implementation of CARchecker does not utilize the power of incremental SAT computing, since the clauses to be added to the SAT solver are from the output of MUC solvers; We do not know of a way to combine them incrementally. In contrast, incremental SAT calling is an important feature in IC3/PDR. Secondly, ABC is a mature tool, incorporating many heuristics, while CARchecker has only been in development for a few months so it is not be surprising that ABC performs better. We believe that the performance of CARchecker can be improved in the future.
Nevertheless, CAR is able to compete with IC3/PDR when combining the Forward and Backward modes. In Figure 1, the plotted line for “Combined CAR” is obtained from the best results which selected from either Forward or Backward CAR. Combined CAR solves a total number of 288 instances, while IC3/PDR solves a total number of 271 instances. Moreover, 42 instances are solved only by Combined CAR. We view the advantage of running CAR in both directions as one of the contributions of this paper; it remains to be seen whether this would also be an advantage for IC3/PDR.

A major point we wish to demonstrate in this section, is that Forward CAR complements IC3/PDR. In Figure 1, the plotted line for “Forward CAR + IC3/PDR” shows the best results from Forward CAR and IC3/PDR. This combination outperforms IC3/PDR by solving 21 more instances. These 21 instances are solved by Forward CAR on checking safe models, see the results in Fig. 3. In contrast, in Figure 2 we see that the portfolio of Forward CAR combined with IC3/PDR does not win more instances than IC3/PDR on checking unsafe models (the two lines are plotted together), while in Figure 3 the improvement is obvious.

As a concrete example, consider a real instance "6s24.aig" in the benchmark, which is solved quickly by Forward CAR but times out by IC3/PDR. Forward CAR adds the clause \(2956\) (2956 is an input id in the model) into \(F_{\infty}\) in Frame 1 because the cube \(\sim 2956\) is detected to represent a set of dead states. Moreover, it also detects the clause \(\sim 2956\) must be added to \(F_1\) as the states represented by the cube (2956) are one step unreachable from \(I\). As a result, Forward CAR quickly decides in Frame 1 that this model is safe, because both clauses \(2956\) and \(\sim 2956\) cannot meet in \(F_1\) (recall that \(F_{\infty} \subseteq F_1\)). For IC3/PDR, although it can detect that the clause \(2956\) should be added to \(F_{\infty}\), it cannot add the clause \(\sim 2956\) into \(F_1\), because it is not a relatively inductive clause, i.e. \(F_1(x) \land T(x, x') \land (\sim 2956')(x')\) is not true. As a result, computing relatively-inductive clauses is less conductive than computing non-relatively-inductive clauses to check this benchmark. So Forward CAR is able to complement IC3/PDR in such similar instances.

Furthermore, if we consider important parameters that influence the performance, e.g., number of clauses and number of frames, we get more positive results. Note that comparing the number of SAT calls between Forward CAR and PDR is not too informative, since Forward CAR also contains MUC calls. So fewer SAT calls in Forward CAR does not mean lower cost. Figure 4 and Figure 5 shows respectively the scatter plots between Forward CAR and IC3/PDR on number of clauses and number of frames. From the figures, Forward CAR does not generate more clauses or more frames than IC3/PDR. In detail, 172 (175) instances are solved with fewer clauses (frames) by Forward CAR than IC3/PDR, comparing with that 121 (118) instances are solved with fewer clauses (frames) by IC3/PDR than Forward CAR. Generally speaking, the number of clauses and frames are positively correlated to the overall performance, which indicates that Forward CAR should be competitive with IC3/PDR, once CARchecker is as optimized as ABC.

Backward CAR complements both Forward CAR and IC3/PDR on unsafe models. As shown in Figure 2, Backward CAR solves more unsafe cases (80) than IC3/PDR (72) and Forward CAR (51). Moreover, the combination of the three approaches solves 17 more unsafe cases than Forward CAR and IC3/PDR. It is surprising to see that the performance of Forward CAR is much worse than that of Backward CAR (51 vs 80), and all solved cases by Forward CAR are also solved by Backward CAR. Forward CAR searches from bad states (\(\sim P\)) and the goal states are in \(I\), while Backward CAR searches from \(I\) and the goal states are in \(\sim P\). A conjecture is that the state space of \(\sim P\) is much larger than that of \(I\), thus causing the overapproximate search to the states in \(\sim P\) to be easier. Although Backward CAR solves more unsafe cases than IC3/PDR, there are several cases that can be solved by IC3/PDR but cannot be solved by Backward CAR. We leave further comparison between IC3/PDR and Backward CAR to future work.

Finally, we explore the contribution of CAR to the current SAT-based model-checking portfolio, which includes BMC, IMC and IC3/PDR. Figs. 1 shows the plots on the combinations IC3/PDR+BMC+IMC, IC3/PDR+BMC+IMC+Forward CAR, and IC3/PDR+BMC+IMC+Combined CAR. Forward CAR add 19 solved instances (all safe models) to the combination of IC3/PDR+BMC+IMC, and Backward CAR solves another two (1 safe and 1 unsafe model). Although BMC solves the most unsafe cases (116), there are three unsafe instances solved only by IC3/PDR and one unsafe instance solved only by Backward CAR. For safe models, the number of solved instances only by IC3/PDR, IMC, Forward CAR and Backward CAR are 13, 12, 19, 1, respectively.

In summary, we conclude from our experimental results that 1) Forward CAR complements IC3/PDR on checking safe models; 2) Backward CAR complements IC3/PDR on checking unsafe models; 3) Running CAR in both directions improves the performance, and 4) CAR contributes to the current portfolio of model checking strategies. We expect this conclusion to be strengthened as the development of CARchecker matures.

6 CONCLUDING REMARKS
CAR is inspired by IC3/PDR, but it differs from it in some crucial aspects. A main difference between CAR and IC3/PDR is that CAR does not require the \(F\)-sequence to be monotone. Also CAR uses a different strategy (MUC) to refine the \(F\)-sequence than IC3/PDR. Furthermore, CAR combines its over-approximate and under-approximate searches in both forward and backward modes.
Due to these differences, CAR and IC3/PDR have different performance profiles. Our experiments show that IC3/PDR and CAR complement each other. The fact that our new tool, after a few months of development, outperforms mature tools that have been under development for many years over a non-negligible fraction of the benchmark suite, speaks to the merit of the new approach.

Furthermore, the area of SAT-based model checking is still a very active research topic. Many improvements to IC3/PDR have been proposed since the first published paper [6]. For example, a recent development is the combination of IC3/PDR with IMC which consists of IC3/PDR, BMC and IMC. We argue therefore CAR is a promising approach for safety model checking.

Figure 4: Comparison on clauses.

Figure 5: Comparison on frames.



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A PROOF OF THEOREM 1

Proof. (⇐) Let $S = \bigcup_{0 \leq i \leq j} F_i$. According to Definition 3.1, if $F_i \subseteq S$ is true, $R(S) = R(S \cup F_{i+1}) \subseteq (S \cup F_{i+1}) = S$. So $S$ contains all reachable states from $I$. Also we know $P \supseteq F_i (0 \leq i \leq k)$, so $P \supseteq S$ holds. That means $S \cap \neg P$ is empty, and thus all reachable states from $I$, which are included in $S$, are not in $\neg P$. So the system $Sys$ is safe for $P$. 


B PROOF OF THEOREM 2

Proof. (⇒) Assume the system $S_{ys}$ is safe for $P$, then all reachable states from $I$ are in $P$. Let $S \subseteq P$ be the set of reachable states from $I$. Now let $F_0 = I, F_1 = S \not\subseteq P$, and according to Definition 3.1 we know that $\delta = (F_0, F_1, F_2)$ is an $F$-sequence satisfying $F_2 \subseteq F_0 \cup F_1$.

(⇐) If $I \cap B_i \neq \emptyset$ and according to Definition 3.1, we can find a path $p = s_0 \rightarrow s_1 \rightarrow \cdots \rightarrow s_i$ in $S$ such that $s_0 \in I \cap B_i$ and $s_j \in B_{i-j}$ for $1 \leq j \leq i$. Hence, we have that $s_i \in B_0 = \neg P$, which means $\rho$ is a counterexample. So $S_{ys}$ is unsafe for $P$.

(⇒) If $S_{ys}$ is unsafe for $P$, there is a path $\rho$ from $I$ to $\neg P$. Let the length of $\rho$ be $n + 1$, and state $j(0 \leq j \leq n)$ on the path is labeled as $\rho[j]$. Now we construct the $B$-sequence $(B_0, B_1, \ldots, B_i)$ in the following way: Let $i = n$ and $B_j = \{\rho[i-j]\}$ for $0 \leq j \leq i$. So $B_i = \{\rho[0]\}$ satisfying $B_i \cap I \neq \emptyset$ (because $\rho[0] \in I$).

C PROOF OF PROPERTY 1

Proof. (⇒) Theorem 3.2 shows if $S_{ys}$ is safe for $P$ there is an $F$-sequence and $n \geq 0$ such that $F_{n+1} \subseteq \bigcup_{i=0}^{n} F_i$. Let $S = \bigcup_{i=0}^{n} F_i$. We have proven that $R(S) \subseteq S$, i.e. $S$ is the upper bound of $S$. So $S(F) = \bigcup_{i=0}^{n} F_i = S$ for all $i \geq n$. On the other hand, since we consider arbitrary $B$-sequence, we set $S(B)$ to its upper bound: the set of all reachable states to $\neg P$. In this situation, $R^{-1}(S(B)) \subseteq S(B)$. Now if $S(F) \cap R^{-1}(S(B)) \neq \emptyset$, assume that $s \in S(F) \cap R^{-1}(S(B))$. So $s$ is reachable to $\neg P$. Assume $s$ is reachable to $t \in \neg P$ and from the definition of $S(F)$ we know $t \in S(F)$ too. However, this is a contradiction, because $S(F) \cap \neg P$ is empty based on the constraint $S(F) \subseteq P$. So $S(F) \cap R^{-1}(S(B)) = \emptyset$ is true.

(⇐) Since both $S(F)$ and $S(B)$ have an upper bound, we can set them to their upper bounds. That is, set $S(F)$ to contain all reachable states from $I$, and $S(B)$ to contain all reachable states to $\neg P$. Because $R^{-1}(S(B))$ has the same upper bound with $S(B), S(F) \cap R^{-1}(S(B)) = \emptyset$ indicates that $I$ is not reachable to $\neg P$. So $S_{ys}$ is safe for $P$.

D PROOF OF LEMMA 1

Proof. (1) Since $\phi$ is satisfiable, there exists a partial assignment $A^\prime$, which is a set of literals, of $\phi$. Now by projecting $A^\prime$ to the part only contains current variables, i.e. $A^\prime|_x$, we set $c_2 = A^\prime|_x$. Let $t \in c_2$ and $s \in c_1$ are two states. From the definition of partial assignment, we know that $t(x) \cup s'(x')$ is a full assignment of $\phi$. So $t$ is a predecessor of $s$. Moreover, since $F_1$ consists of only current variables, $t(x) \cup s'(x') \models F_1(x)$ implies $t \models F_1$. So $t \in F_1$ is true. Before updating $B_{j+1}$, we know that $B_{j+1} \subseteq R^{-1}(B_j)$. And since $c_2 \subseteq R^{-1}(c_1)$ and $c_1 \subseteq B_j$, so $c_2 \subseteq R^{-1}(B_j)$. Hence, $B_{j+1} \cup \{c_2\} \subseteq R^{-1}(B_j)$. That is, $B_{j+1}$ is under-approximate. Finally, for every other $B_j(1 \leq k \neq j + 1), B_k \subseteq R^{-1}(B_{k-1})$ is true. From Definition 3.1, the sequence is still a $B$-sequence.

(2) From the definition $R(F_i) = \{s(t,s) \in T \text{ and } i \in F_i\}$, we know for every state $s \in R(F_i)$ there is a state $t \in F_i$ such that $t(x) \cup s'(x') \models F_i(x) \land T(x_2, x')$. If there is a state $s \in c_1$ such that $s \in R(F_i)$, we know that there is $t \in F_i$ such that $t(x) \cup s'(x')$ is an assignment of $\phi$, making $\phi$ satisfiable. But this is a contradiction. So $c_1 \cap R(F_i) = \emptyset$. Moreover, since $\phi$ is unsatisfiable, there is a cube $c_2$ such that $c_1 \models c_2$ and $F_i(x) \land T(x, x') \land c_2'$ is still unsatisfiable. So $c_2 \cap R(F_i) = \emptyset$ is also true, which implies that $\neg c_2 \ni R(F_i)$, i.e. states represented by $\neg c_2$ includes all those in $R(F_i)$. So $F_{i+1} \cup \{\neg c_2\} \supseteq R(F_i)$ is true, which means $F_{i+1}$ is still over-approximate. For every other $F_k(1 \leq k \neq i + 1)$, they remains over-approximate. As a result, we prove that the sequence is still an $F$-sequence.

E PROOF OF THEOREM 3

We first show that the assertions in the framework are always true, and then prove the while loop in Step 3b can finally terminate.

Lemma E.1. The assertions in Step 3(b)vi and 3(b)vii are always true.

Proof. We first prove the assertion in Step 3(b)vi is true. From Step 3(b)ii we know that $F_i \cap R^{-1}(S(B)) = \emptyset$, so $(c_2 \subseteq F_i) \cap R^{-1}(S(B)) = \emptyset$ is also true. Thus the assertion $c_2 \not\subseteq R^{-1}(S(B))$ is true. For the assertion in Step 3(b)vi, first from Step 3(b)ii we know that $c_1 \cap F_j = \emptyset$, so $(c_2 \not\subseteq c_1) \cap F_j = \emptyset$ is also true. Thus the assertion $\neg c_2 \not\subseteq F_j$ is true.

Informally speaking, Lemma E.1 guarantees that adding $c_2$ to $B_{k+1}$ increases strictly the states in $R^{-1}(S(B))$ (recall that each $B_i$ is in DNF), while adding $\neg c_2$ to $F_j$ decreases strictly the states in $F_j$ and $F_j$ is in CNF. The assertions help to prove the termination of Step 3b.

Lemma E.2. Step 3b in the framework will finally terminate.

Proof. Assume that $(S(F) \cap R^{-1}(S(B)) = \emptyset$ never holds, and $j > 0$ in the framework is always true. So there is always a formula $\phi = F_{j-1}(x) \land T(x, x') \land c_2'(x')$ such that the $F_i$- or $B$-sequence can be updated (see Step 3(b)vi and 3(b)vii). Moreover, Lemma E.1 guarantees that the size of each $F_i$ decreases strictly while the size of $R^{-1}(S(B))$ increases strictly. However, since the sizes of $F_i$ and $R^{-1}(S(B))$ are bounded, they cannot be updated forever. As a result, either $j = 0$ must be finally true, or $S(F) \cap R^{-1}(S(B)) = \emptyset$ finally holds.

Now we are ready to prove Theorem 3.

Proof. First of all, if the framework returns, Lemma 3.4 guarantees the $F_i$- and $B$-sequences are preserved under the framework. Hence, the correctness is guaranteed by Theorem 3.3 and Theorem 3.2.

Now we prove the framework will finally return. First Theorem 3.5 guarantees the while loop in the framework can always terminate, with unsafe or $S(F) \cap R^{-1}(S(B)) = \emptyset$ is true. As a result, the loop body on each $i$ can finally terminate. Now we prove the loop on $i$ can also terminate. If $S_{ys}$ is unsafe for $P$ with a counterexample of length of greater than one, the framework will return finally according to Item 3(b)ii. It is because the size of $S(B)$ is bounded, so the size of $R^{-1}(S(B))$ is also bounded. Moreover, Lemma E.1 guarantees the size of $R^{-1}(S(B))$ keeps growing in each $i$, thus $R^{-1}(S(B))$ will finally contain all reachable states to $\neg P$ for some $i \geq 0$, which will include an initial state in $I = F_0$. On the other hand, if $S_{ys}$ is safe for $P$, since $S(F)$ is also bounded, and Lemma E.1 guarantees the size of each $F_i$ keeps decreasing, so $S(F)$ will finally contain
only the set of reachable states from $I$ for some $i$. At that time, our framework will return according to Item 3c based on Theorem 3.2.