Peculiar behaviors of excited modes in harmonic chains with correlated disorder

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Abstract.
The localization behaviors of harmonic chains with correlated random masses are theoretically investigated. We show that the specific long-range correlation in a mass sequence gives rise to the occurrence of peculiar localized eigenmodes at high frequencies as well as an anomalous behavior in level statistics in vibrational spectrum.

1. Introduction
Wave propagation in disordered media has attracted much attention in many branches of condensed matter physics [1]. Disorder induces spatial localizations of collective excitations, which are largely enhanced in reduced dimensions. Particularly interesting are one-dimensional (1D) structures, where all eigenstates are known to be exponentially localized for arbitrarily small amount of disorder. This kind of localization stems from interference between excited waves; hence, it commonly occurs in a wide variety of disordered systems such as elastic [2], electron [3], and optical [4] ones.

During the last decade, much studies have been accumulated on the localization properties of 1D systems. The recent numerical findings that specific correlation in disorder induces delocalization of electron eigenstates in 1D tight-binding models [5] has again stimulated much interest in this subject. This result was later confirmed by analytic calculations [6], and also experimentally by employing semiconductor superlattices [7] and microwave guides [8].

Compared to electron systems, the closely related vibrational-localization phenomena have remained to be explored. Despite of certain similarities, there are two intrinsic differences between electron and vibrational systems. First, unlike the electron case, vibrational systems have no negative eigenvalue due to the mechanically-stable conditions. Second, the lower bound of spectrum in vibrational systems (zero-frequency mode) must be extended due to the infinitesimal translational invariance. Due to these properties, vibrational systems endowed with correlated disorder are expected to produce new localization phenomena in 1D structures.

In the present work, we investigate the vibrational excitations in harmonic chains subject to long-range correlated random masses with a power spectrum $S(k) \propto k^{-\alpha}$. Spatial profiles of eigenmodes for $\alpha > 1$ are evaluated using direct diagonalization method, which reveals that long-range correlation in the mass landscape yields non-trivial localized modes at high frequencies. Furthermore, long-range correlation in random masses turns out to show an anomalous behavior.
in level statistics of vibrational spectrum. The relevance of our findings to graded elastic systems discussed in Ref. [9] is also considered.

2. Correlated random mass sequences
We consider a harmonic chain of $N$ random masses connected by $N$ linear springs with an equal force constant $K$. The equation of motion reads $m_j\dddot{u}_j = K(2u_j - u_{j-1} - u_{j+1})$, where $m_j$ is the mass at $j$th site and $u_j$ is the displacement of the $j$th mass. The sequence of random masses $\{m_j\}$ is determined by the formula [10]:

$$m_j = c_{N,\alpha} \sum_{k=1}^{N/2} k^{-\alpha/2} \cos \left( \frac{2\pi j k}{N} + \phi_k \right),$$  \hspace{1cm} (1)

where $k$ is the wave number of the modulations on the random mass landscape and $\phi_k$ are $N/2$ random phases uniformly distributed in the interval $[0, 2\pi]$. The normalization constant $c_{N,\alpha}$ is chosen to give a unit mass variance $\langle m_j^2 \rangle - \langle m_j \rangle^2 = 1$. In addition, we shift all masses generated by the relation (1) to have an average value $\langle m_j \rangle = 5$ in order to avoid the cases with negative masses. The resulting mass landscape becomes spatially long-range correlated, which is described by the power-law decay of the Fourier transform of the two-point correlation function: $\mathcal{F}(\Delta m_j \Delta m_\ell) \equiv S(k) \propto k^{-\alpha}$, with the definition $\Delta m_j = m_j - \langle m_j \rangle$. The case $\alpha = 0$ corresponds to the uncorrelated random mass sequence; as $\alpha$ increases, the mass landscape become relatively smooth due to spatial correlation. A typical profile for $\alpha = 3$ is depicted in Fig. 1(a).
3. Vibrational eigenmodes

We employ the direct diagonalization technique to obtain the explicit form of eigenmodes for various values of $\alpha$. Figures 1 (b)-(d) show three examples of eigenmodes for fixed $\alpha = 3.0$ and $N = 1000$ associated with the mass distribution plotted in Fig. 1 (a). It follows that the amplitude of the low-frequency mode labeled by (b) is uniformly distributed over the whole system, while the other two high-frequency modes are localized. The occurrence of extended modes at low frequencies is consistent with the earlier work done by de Moura et al. [11]; they suggested that for $\alpha > 1$, the model exhibits a continuum phase of extended modes in the low-frequency region. The critical frequency $\omega^2_c$ that separates an extended phase from a localized one is thought to depend on the value of $\alpha$, which is yet to be clarified quantitatively.

Notable fact is that in high frequency region, there are two distinct classes of localization. The one consists of strongly-localized eigenmodes as depicted in Fig. 1 (c), whose localization length $\xi (\ll N)$ is unchanged as $N$ increases so that the ratio $\xi/N$ vanishes in the limit $N \to \infty$. It thus belongs to the usual Anderson localization that is caused by interference effect. The other class is established by weakly-localized modes as shown in Fig. 1 (d). Amplitudes of these eigenmodes are confined in a light mass region indicated by arrows in Fig. 1 (a). Interestingly, the latter kind of localization has to be distinguished from the former one, since $\xi$ grows almost linearly with $N$ so that $\xi/N \sim 1$ independently of $N$. This implies that $\xi$ may diverge as $N \to \infty$ while keeping the localized condition $\xi < N$, which is completely different from the behavior of Anderson-type localization. We have confirmed that such the non-trivial localized modes are observed for any $\alpha > 2.0$, provided the system size $N$ is sufficiently large.

4. Level statistics

We next consider the statistical properties of squared eigenfrequencies $\omega^2_i$, which is described by the distribution function $P(s)$ of the eigenvalue-spacings denoted by $s_i = \omega^2_i - \omega^2_{i-1}$. It is known that $P(s)$ follows the Poisson distribution $P(s) \propto e^{-s/s_0}$ for localized states, while it takes the Wigner-Dyson form $P(s) \propto s e^{-s^2/a}$ for extended states (Here $s_0$ and $a$ are appropriate constants). Hence, the behavior of $P(s)$ gives information as to localization properties of disordered systems.

Figure 2 (a) shows the numerical results of $P(s)$ for $\alpha = 0.5$ with various values of $\omega$; averaging over 1000 samples is taken for each $\omega$. Though the curves in Fig. 2(a) seem close to Wigner-Dyson form, we have numerically confirmed that they approach monotonically to the Poisson form with increasing $N$. This implies that all eigenmodes for $\alpha = 0.5$ are spatially localized.
Similar behavior can be observed for any $\alpha < 1.0$. It is thus concluded that the localization properties of the systems for $\alpha < 1.0$ is described by the conventional Anderson theory.

In contrast, $P(s)$ for larger $\alpha$ exhibits a quite different behavior that looks neither the Poisson- nor the Wigner form. This is demonstrated in Fig. 2 (b), where $\alpha = 3.0$ is chosen for the sake of comparison with the eigenmodes profiles shown in Figs. 1 (b)-(d). We see that in Fig. 2 (b), the plots of $P(s)$ has a sharp peak at $s \sim 1$ for all values of $\omega$, regardless of whether the eigenmodes in question are localized or extended. Furthermore, we have found that the magnitude of the peak at $s \sim 1$ becomes enlarged as $N$ increases. At present, it remains unclear what causes such an anomalous behavior of $P(s)$. We comment that similar results of $P(s)$ were obtained by Carpena et al. regarding 1D electronic systems with long-range correlated disorder [12], which suggested the possibility of a new class of universality occurred in 1D disordered structures.

5. Discussions

The occurrence of weakly-localized eigenmodes at high frequencies is attributed to the presence of a continuum light-mass region in a long-range correlated random mass landscape. When the parameter $\alpha$ is large enough, spatial fluctuation of mass landscape becomes smooth as seen from the formula (1). This smoothness of landscape results in the occurrence of a light-mass region (e.g., $700 < i < 900$ in Fig. 1 (a)), in which light masses accumulate densely around a particular site. Then, vibrational amplitudes of some high-frequency modes are confined to the light-mass region, which gives rise to the weakly-localized eigenmodes. Since the spatial extent of the light-mass region for a fixed $\alpha$ grows almost linearly with $N$, the localization lengths $\xi$ of these eigenmodes also develop linearly with $N$, and thus possibly diverges in the limit $N \rightarrow \infty$. Emphasis is made on the fact that, since $\xi$ increases linearly with $N$, certain algorithms to measure $\xi$ (e.g., the transfer matrix method [3]) are out of use for the current issue.

Before closing, we mention the relevance of the current issue to the studies of graded elastic chains done by Xiao et al [9]. They considered harmonic chains subject to gradual modulation in mass sequences, and found that a new kind of delocalization transition. Such a graded harmonic chain corresponds to the limiting case of our model with $\alpha \gg 1$. Hence, comparison between the two systems would provides further insight into the effect of long-range correlated disorder.

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