The part of the proton spin $\Sigma$ carried by $u, d, s$ quarks is calculated in the framework of the QCD sum rules in the external fields. The operators up to dimension 9 are accounted. An important contribution comes from the operator of dimension 3, which in the limit of massless $u, d, s$ quarks is equal to the derivative of QCD topological susceptibility $\chi'(0)$. The comparison with the experimental data on $\Sigma$ gives $\chi'(0) = (2.3 \pm 0.6) \times 10^{-3} \text{GeV}^2$. The limits on $\Sigma$ and $\chi'(0)$ are found from selfconsistency of the sum rule, $\Sigma = 0.33 \pm 0.20, \chi'(0) = (2.4 \pm 0.5) \times 10^{-3} \text{GeV}$.

The values of $g_A = 1.37 \pm 0.10$ and $g_8^a = 0.65 \pm 0.15$ are also determined.

We calculated the value of $\Sigma$ – the part of nucleon spin carried by three flavours of light quarks $\Sigma = \Delta u + \Delta d + \Delta s$, where $\Delta u, \Delta d, \Delta s$ are the parts of nucleon spin carried by $u, d, s$ quarks. On the basis of the operator product expansion (OPE) $\Sigma$ is related to the proton matrix element of the flavour singlet axial current $j_{\mu 5}^0$

$$2ms_\mu \Sigma = \langle p, s | j_{\mu 5}^0 | p, s \rangle,$$

where $s_\mu$ is the proton spin 4-vector, $m$ is the proton mass. The polarization operator

$$\Pi(p) = i \int d^4xe^{ipx} \langle 0 | T \{ \eta(x), \bar{\eta}(0) \} | 0 \rangle$$

was considered, where $\eta(x)$ is the current with proton quantum numbers. It is assumed that the term $\Delta L = j_{\mu 5}^0 A_\mu$ where $A_\mu$ is a constant singlet axial field, is added to QCD Lagrangian. In the weak axial field approximation $\Pi(p)$ has the form

$$\Pi(p) = \Pi^{(0)}(p) + \Pi^{(1)}(p) A_\mu.$$

$\Pi^{(1)}(p)$ is calculated in QCD by OPE at $p^2 < 0, |p^2| \gg R_c^{-2}$, where $R_c$ is the confinement radius. On the other hand, using dispersion relation, $\Pi^{(1)}(p)$ is represented by the contribution of the physical states, the lowest of which is the proton state. The contribution of excited states is approximated as a continuum and suppressed by the Borel transformation. The desired answer is obtained by equalling of these two representations.
An essential ingredient of the method is the appearance of induced by the external field vacuum expectation values (v.e.v). The most important of them in the problem at hand is

$$\langle 0 | j^{\mu}_{\nu 5} | 0 \rangle_A \equiv 3 f_0^2 A_{\mu} \quad (4)$$

dimension 3. The constant $f_0^2$ is related to QCD topological susceptibility. We can write

$$\langle 0 | j^{\mu}_{\nu 5} | 0 \rangle_A = \lim_{q \to 0} i \int d^4x e^{iqx} \langle 0 | T \{ j^{\mu}_{\nu 5}(x), j^{0}_{\mu 5}(0) \} | 0 \rangle_A \equiv \lim_{q \to 0} P_{\mu\nu}(q) A_{\nu} \quad (5)$$

The general structure of $P_{\mu\nu}(q)$ is

$$P_{\mu\nu}(q) = - P_L(q^2) \delta_{\mu\nu} + P_T(q^2)(-\delta_{\mu\nu} q^2 + q_{\mu} q_{\nu}) \quad (6)$$

Because of anomaly there are no massless states in the spectrum of the singlet polarization operator $P_{\mu\nu}$ even for massless quarks. $P_{T,L}(q^2)$ also have no kinematical singularities at $q^2 = 0$. Therefore, the nonvanishing value $P_{\mu\nu}(0)$ comes entirely from $P_L(q^2)$. Multiplying $P_{\mu\nu}(q)$ by $q_{\mu} q_{\nu}$, in the limit of massless $u,d,s$ quarks we get

$$q_{\mu} q_{\nu} P_{\mu\nu}(q) = - P_L(q^2) q^2 = N_c^2 (\alpha_s/4\pi)^2 i \int d^4x e^{iqx} \times$$

$$\langle 0 | T G_{\mu\nu}(x) \tilde{G}_{\mu\nu}(x), G_{\lambda\sigma}(0) \tilde{G}_{\lambda\sigma}(0) | 0 \rangle, \quad (7)$$

where $G_{\mu\nu}$ is the gluonic field strength, $\tilde{G}_{\mu\nu} = (1/2) \varepsilon_{\mu\nu\lambda\sigma} G_{\lambda\sigma}$. At $q^2 \to 0$, we have

$$f_0^2 = -(1/3) P_L(0) = \frac{4}{3} N_c^2 \chi(0), \quad (8)$$

where $\chi(q^2)$ is the topological susceptibility

$$\chi(q^2) = i \int d^4x e^{iqx} \langle 0 | T Q_5(x), Q_5(0) | 0 \rangle \quad (9)$$

$$Q_5(x) = (\alpha_s/8\pi) G_{\mu\nu}(x) \tilde{G}_{\mu\nu}(0), \quad (10)$$

$\chi(0) = 0$ if there is at least one massless quark.

In ref. the sum rule, expressing $\Sigma$ in terms of $f_0^2$ (4) or $\chi'(0)$ was found. The OPE up to dimension $d = 7$ was performed.
However, the accuracy of the calculation was not good enough for reliable calculation of $\Sigma$ in terms of $f_0^2$; the necessary requirement of the method – the weak dependence of the result on the Borel parameter was not well satisfied.

In this paper we improve the accuracy of the calculation by going to higher order terms in OPE up to dimension 9 operators. Under the assumption of factorization – the saturation of the product of four-quark operators by the contribution of an intermediate vacuum state – the dimension 8 v.e.v.’s are accounted (times $A_\mu$):

$$q \langle 0 \mid \bar{q} \sigma^{\alpha\beta}(1/2)\lambda^\alpha G_{\alpha\beta}^\mu q \cdot \bar{q} \mid 0 \rangle = m_0^2 \langle 0 \mid \bar{q} q \mid 0 \rangle^2$$

(11)

where $m_0^2 = 0.8 \pm 0.2$ GeV$^2$ was determined in the sum rule for $\Sigma$ is given by

$$\Sigma + C_0 M^2 = -1 + \frac{8}{9\lambda_N^2} e^{m^2/M^2} \left\{ a^2 L^{4/9} + +6\pi^2 f_0^2 M^4 E_1 \left( \frac{W^2}{M^2} \right) L^{-4/9} + 
\right.$$}

$$+14\pi^2 h_0 M^2 E_0 \left( \frac{W^2}{M^2} \right) L^{-8/9} - \frac{1}{4} \frac{a^2 m_0^2}{M^2} - \frac{1}{9} \pi\alpha_s f_0^2 \frac{a^2}{M^2} \right\}$$

(12)

Here $M^2$ is the Borel parameter, $\lambda_N$ is defined as $\lambda_N^2 = 32\pi^4 \lambda_N^2 = 2.1$ GeV$^6$, $\langle 0 | \eta | p \rangle = \lambda_N v_p$, where $v_p$ is proton spinor, $W^2$ is the continuum threshold, $W^2 = 2.5$ GeV$^2$, $h_0 = 3 \cdot 10^{-4}$ GeV$^4$.

$$a = - (2\pi)^2 \langle 0 | \bar{q} q | 0 \rangle = 0.55 \text{ GeV}^3$$

(13)

$$E_0(x) = 1 - e^{-x}, \quad E_1(x) = 1 - (1 + x)e^{-x}$$

$L = \ln(M/\Lambda)/\ln(\mu/\Lambda)$, $\Lambda = \Lambda_{QCD} = 200$ MeV and the normalization point $\mu$ was chosen $\mu = 1$ GeV. When deriving (12) the sum rule for the nucleon mass was exploited what results in appearance of the first term, -1, in the right hand side (rhs) of (12). This term absorbs the contributions of the bare loop, gluonic condensate as well as $\alpha_s$ corrections to them and essential part of terms, proportional to $a^2$ and $m_0^2a^2$. The values of the parameters, $a, \lambda_N^2, W^2$ taken above were chosen by the best fit of the sum rules for the nucleon mass (see Appendix B) performed at $\Lambda = 200$ MeV. The unknown constant $C_0$ in the left-hand side of (12) corresponds to the contribution of inelastic transitions $p \rightarrow N^* \rightarrow p$ (and in inverse order). The sum rule (12) as well as the sum rule for the nucleon mass is reliable in the interval of the Borel parameter $M^2 0.85 < M^2 < 1.4$ GeV$^2$. The $M^2$-dependence of the rhs of (12) at $f_0^2 = 3 \times 10^{-2}$ GeV$^2$ is plotted in Fig.1a. The complicated expression in...
rhs of (12) is indeed an almost linear function of \( M^2 \) in the given interval! The best values of \( \Sigma = \Sigma^{fit} \) and \( C_0 = C_0^{fit} \) are found from the \( \chi^2 \) fitting procedure

\[
\chi^2 = \frac{1}{n} \sum_{i=1}^{n} [\Sigma^{fit} - C_0^{fit} M_i^2 - R(M_i^2)]^2 = \text{min},
\]

where \( R(M^2) \) is the rhs of (12).

The values of \( \Sigma \) as a function of \( f_0^2 \) are plotted in Fig.1b together with \( \sqrt{\chi^2} \).

In our approach the gluonic contribution cannot be separated and is included in \( \Sigma \). The experimental value of \( \Sigma \) can be estimated as \( \Sigma = 0.3 \pm 0.1 \).

Then from Fig.1b we have \( f_0^2 = (2.8 \pm 0.7) \times 10^{-2} \text{ GeV}^2 \) and \( \chi'(0) = (2.3 \pm 0.6) \times 10^{-3} \text{ GeV}^2 \).

The error in \( f_0^2 \) and \( \chi' \) besides the experimental error includes the uncertainty in the sum rule estimated as equal to the contribution of the last term in OPE (two last terms in Eq.12) and a possible role of NLO \( \alpha_s \) corrections.

From \( \chi^2 \) fit, i.e. from the requirement of the selfconsistency of the sum rule, allowing the deviation of \( \chi^2 \) by a factor 1.5 from its minimal value, we find: \( \Sigma = 0.33 \pm 0.20 \), \( \chi'(0) = (2.4 \pm 0.5) \times 10^{-3} \text{ GeV}^2 \).

From the sum rule analogous (12) it is possible to find \( g_A^8 \) – the proton coupling constant with the octet axial current, which enters the QCD formula for \( \Gamma_{p,n} \).

The \( M^2 \)-dependence of \( g_A^8 + C_8 M^2 \) is presented in Fig.1a and the best fit according to the fitting procedure (14) at \( 1.0 \leq M^2 \leq 1.3 \text{ GeV}^2 \) gives

\[
g_A^8 = 0.65 \pm 0.15, \quad C_8 = 0.10 \text{ GeV}^{-2} \quad \sqrt{\chi^2} = 1.2 \times 10^{-3}
\]

The obtained value of \( g_A^8 \) within the errors coincides with \( g_A^8 = 0.59 \pm 0.02 \) found from the data on baryon octet \( \beta \)-decays under assumption of strict SU(3) flavour symmetry.

A similar sum rule with the account of dimension 9 operators can be derived also for \( g_A \) – the nucleon axial \( \beta \)-decay coupling constant. It is an improvement of the sum rule found in [13] and has the form

\[
g_A + C_A M^2 = 1 + \frac{8}{9\lambda_N^2} e^{m^2/M^2} \left[ a^2 L^{4/9} + 2\pi^2 m_1^2 f_\pi M - \frac{1}{4} a^2 \frac{m_0^2}{M^2} + \frac{5}{3} \pi \alpha_s f_\pi^2 \frac{a^2}{M^2} \right]
\]

The \( M^2 \) dependence of \( g_A - 1 + C_A M^2 \) is plotted in Fig.1a, lower curve. The best fit gives

\[
g_A = 1.37 \pm 0.10, \quad C_A = -0.088 \text{ GeV}^{-2}, \quad \sqrt{\chi^2} = 1.0 \times 10^{-3}
\]
Figure 1: a.) The $M^2$-dependence of $\Sigma + C_0 M^2$ at $f_0^2 = 3 \times 10^{-2} \text{ GeV}^2$, $g_A + C_8 M^2$, and $g_A - 1 + C_A M^2$. b.)$\Sigma$ (solid line) and $\sqrt{\chi^2}$ (crossed line) as a functions of $f_0^2$.

in comparison with the world average $g_A = 1.260 \pm 0.002$.

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