Extraction of QCD trace anomaly of proton from near threshold $J/\Psi$ photo-production data at JLab

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We present an estimation of the QCD trace anomaly from the near threshold $J/\Psi$ meson and a running strong coupling which includes the nonperturbative effect in low $\mu^2$ region. Although big uncertainty still exists, the experimental data favors a smaller trace anomaly $b = 0.07 \pm 0.17$ based on our approach. The proton mass decompositions at $\mu^2 = 0.41$ GeV$^2$ and $\mu^2 = 4$ GeV$^2$ are also provided based on the extracted trace anomaly.

I. INTRODUCTION

Proton and neutron, collectively referred as nucleons (N), are the main building blocks of the observable universe accounting in mass, however there are still a lot of mysteries about them, such as where do the mass and the spin of the nucleon from? The mass is a fundamental property of the nucleon, and it is thought to be mainly from the dynamical origin. According to the quantum chromodynamics (QCD) theory which governs the strong interaction between quarks, the mass of a nucleon is decomposed into four terms: the quark mass contribution $M_m$, the quark energy contribution $M_b$, the gluon energy contribution $M_g$, and the trace anomaly contribution $M_a$. These mass terms are directly related to the QCD trace anomaly $b$, and the momentum fraction $a$ of all the quarks at $\mu^2$, which are expressed in Eq. (1) [2],

$$
M_q = \frac{3}{4} \left( a - \frac{b}{1 + \gamma_m} \right) M_N, \\
M_g = \frac{3}{4}(1-a)M_N, \\
M_m = \frac{4 + \gamma_m}{4(1 + \gamma_m)} bM_N, \\
M_a = \frac{1}{4}(1-b)M_N,
$$

in which $\gamma_m$ is the quark mass anomalous dimension, which can be calculated in perturbative QCD [3]. The momentum fraction $a$ is defined in Eq. (2), which can be evaluated with parton distribution functions. With decades of worldwide efforts on both experimental and theoretical sides, parton distribution functions are well determined with the global analysis to deep inelastic scattering data. To complete the proton mass decomposition, only the trace anomaly $b$ is less constrained from the experimental data.

$$
a(\mu^2) = \sum_f \int_0^1 x[q_f(x, \mu^2) + \bar{q}_f(x, \mu^2)] dx \quad (2)
$$

Although the proton mass mainly generated from the spontaneous breaking of chiral symmetry, there is still a small part of it comes from the small masses of quarks. The quark mass contribution is calculated with the scalar condensates and the quark masses, which is directly related to the QCD trace anomaly, as,

$$
bM_N = \langle N | m_u \bar{u}u + m_d \bar{d}d| N \rangle + \langle N | m_s \bar{s}s| N \rangle \\
= m_l \langle N | \bar{u}u + d\bar{d}| N \rangle + m_s \langle N | \bar{s}s| N \rangle \\
= \Sigma_{\pi N} + \Sigma_{sN}. \quad (3)
$$

The scalar nucleon matrix element of up and down quarks is also known as $\Sigma_{\pi N}$ term (~ 45 MeV), which can be determined from the $\pi - N$ scattering amplitude at low energy [4–6]. Much less is known about the strange $\Sigma$ term. It is thought to be big since the mass of strange quark is big. However, recent Lattice QCD calculations give the $\Sigma_{\pi N}$ term that is comparable with $\Sigma_{\pi N}$. QCDSF Collaboration predicts $\Sigma_{sN} = 11 \pm 13$ MeV [7], and QCDSF Collaboration predicts $\Sigma_{sN} = 40 \pm 12$ MeV [8].

The QCD trace anomaly is not only a fundamental parameter of gluon field, but also important for us to understand the nucleon mass and the strange $\Sigma$ term. Therefore, how to and to constrain the trace anomaly $b$ in experiment are important in revealing the mysteries of strong interaction of the visible universe.

The low energy scattering between a heavy quarkonium and a nucleon can be used to probe the property of the nucleon. It is not difficult to compute the scattering amplitude using the operator production expansion. In the quarkonium rest frame, if the incident nucleon energy is much smaller than the binding energy between the quark-antiquark pair inside the quarkonium, then only the leading twist gluon field operator is kept in the calculation [9]. In the non-relativistic domain, the heavy
quarkonium is mainly sensitive the chromo-electric part of the gluon field of the nucleon. Since the velocity of the heavy quark inside the heavy quarkonium is small, the chromo-magnetic part is suppressed by powers of velocity. Hence the heavy quarkonium-nucleon scattering amplitude is connected to the strength of the nucleon color field and the quark terms in the energy-momentum tensor [9]. In this work we try to extract the QCD trace anomaly \( b \) from the scattering between the charmonium and the nucleon, based on the theoretical framework in Refs. [9] [10].

II. FORWARD CROSS-SECTION OF \( J/\psi \) PHOTOPRODUCTION ON PROTON

We take the differential cross-section data recently published by GlueX Collaboration at JLab, US, to do the analysis [11]. The cross-section data is on the exclusive \( J/\psi \) photo-production using the real bremsstrahlung photon of 10.72 GeV energy in average. At this energy, \( t_{\text{min}} \) of the exclusive reaction is -0.4361 GeV\(^2\). The differential cross-section as a function of \(-t\) is shown in Fig. 1. A fit with an exponential function (\( d\sigma/dt = d\sigma/dt|_{t=0} \times e^{-kt} \)) is performed to the data, to describe the \( t \)-dependence of the cross-section. \( d\sigma/dt|_{t=0} \) is obtained to be 3.8 ± 1.4 nb/GeV\(^2\), and the exponential slope \( k \) is obtained to be -1.67 ± 0.38 GeV\(^{-2}\). Note that here we use \((\chi^2/ndf) - (\chi^2/ndf)|_{\text{best fit}} = 1\) to find the errors of the parameters.

![Graph of differential cross-section vs. -t, GeV\(^2\)](image)

FIG. 1. This figure shows the differential cross section of \( J/\psi \) photo-production near the threshold, measured by GlueX Collaboration [11]. Only the statistical uncertainties are shown in the plot. The range of the incident photon energy is from 10 GeV to 11.8 GeV.

With the forward cross-section obtained, we simply compare it to the theoretical prediction, in order to find out the value of the trace anomaly \( b \). How the forward cross-section \( d\sigma/dt|_{t=0} \) is related to QCD trace anomaly is discussed in Sec. [11].

III. VMD MODEL AND \( J/\psi \) NEAR THRESHOLD PHOTOPRODUCTION

According to the conventional VMD model, the forward \( J/\psi \) photo-production on nucleon is written as [10],

\[
\frac{d\sigma_{N \rightarrow J/\psi N}}{dt}|_{t=0} = \frac{3\Gamma(J/\psi \rightarrow e^+e^-) (k_{J/\psi N})^2}{\alpha m_{J/\psi}} \frac{2 d\sigma_{J/\psi N \rightarrow J/\psi N}}{dt}|_{t=0},
\]

in which \( k_{J/\psi}^2 = |s-(m_a+m_b)^2|/4s \) denotes the squared center of mass momentum of the corresponding reaction, and \( \Gamma \) stands for the partial decay width of \( J/\psi \) decay. The center of mass energy \( \sqrt{s} \) is 4.58 GeV with the incident photon of 10.72 GeV. The decay width of \( J/\psi \) to electron-positron pair is 5.547 KeV [12]. In Eq. (4), \( \alpha \) is the fine structure constant.

The differential cross-section of \( J/\psi-N \) interaction is given by,

\[
\frac{d\sigma_{J/\psi N \rightarrow J/\psi N}}{dt}|_{t=0} = \frac{1}{64\pi m_{J/\psi}} \frac{1}{(\lambda^2 - m_N^2)^2} |F_{J/\psi N}|^2,
\]

where \( F_{J/\psi N} \) denotes the invariant \( J/\psi-N \) scattering amplitude, and \( \lambda = (p_N p_{J/\psi}/m_{J/\psi}) \) is the nucleon energy in the charmonium rest frame [10]. At low energy of interaction, the amplitude takes the form [9],

\[
F_{J/\psi N} \simeq r_0^3 d_2 \frac{2\pi^2}{27} \left( 2M_N^2 - \left\langle N \right| \sum_{i=u,d,s} m_i q_i \left| N \right\rangle \right)
\]

\[
\simeq r_0^3 d_2 \frac{2\pi^2}{27} (2M_N^2 - 2bM_N^2)
\]

\[
\simeq r_0^3 d_2 \frac{2\pi^2}{27} 2M_N^2 (1 - b).
\]

The sum of \( \Sigma \) terms of the quarks is connected to the trace anomaly \( b \). Note that Kharzeev uses a relativistic normalization of hadron states \( \langle N|N \rangle = 2M_N V \), where \( V \) is a normalization volume [9]. The “Bohr” radius \( r_0 \) of the charmonium in Eq. (6) is given by,

\[
r_0 = \left( \frac{4}{3\alpha_s} \right) \frac{1}{m_c}.
\]

The Wilson coefficient \( d_2 \) in Eq. (6) is defined as [9] [13] [14],

\[
d_2^{(1S)} = \left( \frac{32}{N_c} \right)^2 \frac{\sqrt{\pi}}{\Gamma(n + \frac{3}{2})} \frac{\Gamma(n + \frac{5}{2})}{\Gamma(n + 5)},
\]

in which \( N_c \) is the number of colors. The renormalization scale \( \mu^2 \) is suggested to be the “Rydberg” energy square...
$\epsilon_0^2$ of the bound state of the heavy quark-antiquark pair \cite{9,14}, which is written as,
\begin{equation}
\epsilon_0 = \left(\frac{3\alpha_s}{4}\right)^2 m_c. \tag{9}
\end{equation}

IV. STRONG COUPLING CONSTANT AND THE CHARM QUARK MASS

To avoid the Landau pole of the running strong coupling constant at low $\mu^2$ scale, we take the $\alpha_s$ in an analytic approach \cite{15}, in which the QCD nonperturbative effects are folded into the coupling. The analytic $\alpha_s$ is written as,
\begin{equation}
\alpha_s^{\text{ana}} = \frac{4\pi}{\beta_0} \left( \frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right). \tag{10}
\end{equation}

The term $\Lambda^2/(\Lambda^2 - Q^2)$ is a nonperturbative power law contribution, which cancels Landau pole at $\Lambda^2$.

The QCD running coupling constant can be determined with the global analysis of the deep inelastic scattering data in a wide $Q^2$ range, such as GRV98 \cite{16} and CT14 \cite{17}. We get $\Lambda^2 = 0.0643$ GeV$^2$ in Eq. \cite{10} by matching the CT14(LO)'s $\alpha_s$ at the charm quark threshold $m_c^2 = 1.69$ GeV$^2$. For CT14(LO) analysis, $\alpha_s$ is 0.3719 at 1.69 GeV$^2$. The analytic coupling $\alpha_s^{\text{ana}} = 0.3612$ also matches the GRV98(LO)'s $\alpha_s = 0.3626$, at the chosen charm quark threshold of 1.96 GeV by GRV.

Fig. 2 shows the strong running coupling constant $\alpha_s$ as a function of $\mu^2$.

The charm quark mass in world average is 1.27 GeV by Particle Data Group \cite{12}. The charm quark mass is 1.3 GeV in CT14(LO) analysis \cite{17}. The charm quark mass is 1.4 GeV in GRV98(LO) analysis \cite{10}. In our analysis, these values of the charm quark mass are all used as the different QCD input parameters.

V. TRACE ANOMALY AND PROTON MASS DECOMPOSITION

The strong coupling constant $\alpha_s$ depends on the energy scale $\mu^2 = \epsilon_0^2$. At the same time, in Eq. \cite{9}, the energy scale $\epsilon_0$ also depends on the strong coupling $\alpha_s$. Therefore $\epsilon_0^2$ is fixed by the combination of Eq. \cite{9} and the running strong coupling formula shown in Eq. \cite{10}.

With different charm quark masses, the obtained energy scale $\epsilon_0^2$ and $\alpha_s$ are shown in Table \ref{table:1} with these parameters of strong couplings and the charm quark masses, we calculate the “Bohr” radius $r_0$, and finally get the QCD trace anomaly $b$ applying Kharzeev’s method based on VMD model described in Sec. \ref{sec:III}. The obtained QCD trace anomaly $b$ are listed in Table \ref{table:1}. The formula in Sec. \ref{sec:III} are written in the natural unit. We should not forget the unit conversion in the comparison of the theoretical forward cross-section with the experimental data, 1 GeV$^{-2} = 3.881 \times 10^5$ nb.)

\begin{table}[h]
\centering
\begin{tabular}{cccccc}
\hline
$m_c$ [GeV] & $\epsilon_0^2$ [GeV$^2$] & $\alpha_s(\mu^2 = \epsilon_0^2)$ & $r_0$ [fm] & $b$ & $a$
\hline
1.27 & 0.0937 & 0.654 & 0.316 & 0.05 & 0.618 & 0.861
1.3 & 0.0962 & 0.651 & 0.310 & 0.00 & 0.19 & 0.858
1.4 & 0.105 & 0.641 & 0.293 & -0.20 & 0.22 & 0.846
\hline
\end{tabular}
\caption{The list of the charm quark mass $m_c$. “Rydberg” energy square of heavy quarkonium $\epsilon_0^2$, the strong coupling constant $\alpha_s$, the “Bohr” radius, the QCD trace anomaly $b$, and the total momentum fraction carried by quarks. The uncertainty of $b$ comes from the statistical uncertainties of the cross-section data only. $a$ is calculated by Eq. \cite{9} with the dynamical parton distribution functions \cite{18}.}
\end{table}

$\epsilon_0^2/m_c^2$ from Ref. \cite{14}, i.e. $\epsilon_0^2 = 0.42$ GeV$^2$. If “Rydberg” energy $\epsilon_0$ of the $c\bar{c}$ pair is not necessarily fixed by Eq. \cite{9}, then it is better to calculate $\epsilon_0$ with the experimental data. Thinking about pulling apart a $c\bar{c}$ pair to generate a $D\bar{D}$ pair, a naive method is to estimate “Rydberg” energy $\epsilon_0$ as $m_D + m_{\Psi} - m_{J/\Psi}$ \cite{9}. Using the masses of D meson and $J/\Psi$ meson, $\epsilon_0^{\text{exp}}$ is calculated to be 0.41 GeV$^2$. Around the low energy scale of 0.41 GeV$^2$, the charm quark mass is near the pole mass 1.67 GeV \cite{12}. At these new “Rydberg” energies, the obtained $\alpha_s$, $r_0$, and $b$ are listed in Table \ref{table:1}. The $\chi^2 = (d\sigma/dt|_{t=0} - d\sigma/dt|_{t=0}^{\text{VMD model}})^2 / (\delta^{\text{exp}}_{d\sigma/dt|t=0})^2$ as a function of $b$ is shown in Fig. \ref{fig:3}. If we have forward cross-section data at several photon energies near the threshold, the combined uncertainty of the extracted trace anomaly can be reduced.

With the dynamical parton distribution functions generated from DGLAP equation with parton-parton recombination corrections \cite{18}, we calculate the momentum fraction $a$ carried by all quarks at the “Rydberg” energy $\epsilon_0$ scale of $J/\Psi$, which is listed in Table \ref{table:1} and \ref{table:1}. The momentum fraction carried by all quarks is 0.541 at $\mu^2 = 4$ GeV$^2$ \cite{18}. In principle if the trace anomaly
TABLE II. The list of “Rydberg” energy square of heavy quarkonium $\epsilon_0^2$, the strong coupling constant $\alpha_s$, the charm quark mass $m_c$, the QCD trace anomaly $b$, and the total momentum fraction carried by quarks. The uncertainty of $b$ comes from the statistical uncertainties of the cross-section data only. $a$ is calculated by Eq. (5) with the dynamical parton distribution functions [19].

| $\epsilon_0^2$ [GeV$^2$] | $\alpha_s(\mu^2 = \epsilon_0^2)$ | $m_c$ [GeV] | $r_0$ [fm] | $b$       | $a$       |
|--------------------------|-------------------------------|-------------|-------------|-----------|-----------|
| 0.41                     | 0.494                         | 1.67        | 0.318       | 0.07±0.17 | 0.654     |
| 0.42                     | 0.492                         | 1.67        | 0.320       | 0.08±0.17 | 0.652     |

$b$ is scale-invariant, we can get the proton mass decomposition at any scale $\mu^2$. The proton mass decomposition at $\mu^2 = 0.41$ GeV$^2$ and $\mu^2 = 4$ GeV$^2$ are shown in Fig. 4 for illustrations. In the calculations, the quark mass anomalous dimensions at 0.41 GeV$^2$ and 4 GeV$^2$ are evaluated to be -0.345 and -0.158, respectively [8]. The Lattice QCD calculation gives $M_g = (0.33 \pm 0.04)M_N$, $M_0 = (0.37 \pm 0.05)M_N$, $M_a = (0.23 \pm 0.01)M_N$, and $M_m = (0.09 \pm 0.02)M_N$ at $\mu^2 = 4$ GeV$^2$ [10]. Our result on the proton mass decomposition is close to the result from the Lattice QCD simulation.

VI. DISCUSSIONS AND SUMMARY

We estimate the strange $\Sigma$ term to be around 21 MeV, with the QCD trace anomaly $b = 0.07$ extracted from the current data of near-threshold $J/\Psi$ photo-production and the $\Sigma_{\pi N}$ term determined from experimental measurements of $\pi$-N scattering [11]. The estimated $\Sigma_{\pi N}$ is small based on our analysis in this work. More precise data of the near-threshold heavy quarkonium photo-production and the higher order of theoretical framework are needed. Nonetheless, the current $\Sigma_{\pi N}$ term extracted is more or less consistent with the results of Lattice QCD (11 ± 13 MeV and 40 ± 12 MeV) [11]. In terms of the sum of the $\Sigma$ terms, we give the sum to be $bM_N = 66$ MeV. It is between the Lattice QCD predictions 49 ± 25 MeV [7] and 86 ± 19 MeV [8] from two independent Collaborations.

The trace anomaly is very sensitive to the parameter $r_0$, based on the VMD model we used in the analysis. In theory, the “Bohr” radius $r_0$ depends on the two key QCD inputs – the strong coupling constant and the charm quark mass. Therefore we should look for more experimental constraints on “Bohr” radius $r_0$ of heavy quark-antiquark pair, the strong coupling $\alpha_s$, and the charm quark mass $m_c$. Higher-twist calculations and the systematic uncertainty of the model should be investigated for the next step. So far, the uncertainty of the trace anomaly extracted in this work is definitely huge. More and more statistics are needed on the experimental side for the photo-production of $J/\Psi$ near threshold. Moreover, our analysis also could be tested with the near-threshold exclusive $\Upsilon(1S)$ production using the quasi-real photon on an electron-ion collider machine of low energy [20]. The forward cross-sections of $\Upsilon(1S)$ production at different center-of-mass energy of a real photon and a nucleon are predicted in Table III. Since the binding energy $\epsilon_0$ of $\Upsilon(1S)$ is higher than that of $J/\Psi$, Eq. (6) is valid for a wider kinematical range near the threshold of $\Upsilon(1S)$ production. The chromo-magnetic contribution

![FIG. 3. $\chi^2$ as a function of the QCD trace anomaly using the $J/\Psi$ photo-production data of $E_q \sim 10.72$ GeV at JLab. For the theoretical description of VMD model, $\epsilon_0 = 0.41$ GeV$^2$, $\alpha_s(\epsilon_0) = 0.494$, $m_c(\epsilon_0) = 1.67$ GeV are applied as the inputs.

![FIG. 4. The decompositions of proton mass at $\mu^2 = 0.41$ GeV$^2$ and $\mu^2 = 4$ GeV$^2$, with $b = 0.07$.](image-url)
to Υ(1S)-N interaction is even smaller as the velocity of the bottom quark inside Υ(1S) is even smaller, compared with the J/Ψ − N scattering. Hence the theoretical framework in this work is more suitable for the near threshold Υ(1S) photo-production.

TABLE III. The list of the forward cross-sections of Υ(1S) photo-production near the threshold off proton, with $\epsilon^2 = 1.21$ GeV$^2$, $\alpha_s(\epsilon^2) = 0.397$ and $m_b = 4.18$ GeV. A VMD model is used [9]. $\epsilon^2$ is estimated to be $(m_B + m_P - m_{\Upsilon(1S)})^2$.

| $\sqrt{s_{\gamma N}}$ | $t_{\min}$ | $t_{\max}$ | $r_0$ [fm] | $d\sigma/dt(s, t = 0)$ |
|----------------------|------------|------------|-------------|-------------------|
| 12 GeV               | 0.96 GeV$^2$ | 51 GeV$^2$ | 0.158 fm    | 86 fb/GeV$^2$     |
| 14 GeV               | 0.35 GeV$^2$ | 104 GeV$^2$| 0.158 fm    | 46 fb/GeV$^2$     |
| 16 GeV               | 0.17 GeV$^2$ | 164 GeV$^2$| 0.158 fm    | 27 fb/GeV$^2$     |

Based on the VMD model, Kharzeev also calculate the forward cross-section of J/Ψ photo-production in terms of the total cross-section of J/Ψ − N scattering [10], as

$$\left.\frac{d\sigma_{\gamma N \rightarrow J/\Psi}}{dt}\right|_{t=0} = 3\Gamma(J/\Psi \rightarrow e^+e^-)\alpha_{m_{J/\Psi}}$$

$$\times \left[\frac{1}{16\pi} \frac{[s - (m_N + m_{J/\Psi})^2][s - (m_N - m_{J/\Psi})^2]}{(s - m_N^2)^2}\right] \times (1 + \rho^2)\langle \sigma_{J/\Psi N}^{tot}\rangle^2,$$

in which $s$ is the center-of-mass energy square of $\gamma N$, and $\rho$ is the ratio of the real part and the imaginary part of the amplitude $M_{J/\Psi N}$. The value of $\rho$ is unknown, Sibirtsev et. al. suggest to put it to zero for the first approximation [21]. In the following calculations, we assume $\rho = 0$. With the exploitation of the QCD sum rules [22], the leading approximation of $\sigma_{J/\Psi N}^{tot}$ can be evaluated using the gluon-J/Ψ cross-section [10], which is written as

$$\sigma_{J/\Psi N}^{(0)}(\lambda) = \frac{8\pi}{9} \left(\frac{32}{3}\right) \frac{1}{\alpha_{s}m_{c}^2} \times \int_{\epsilon_{0}^{2}}^{1} dx \left(\frac{x\lambda_{c} - 1}{x\lambda_{c}}\right)^{3/2} g(x, m_{c}^2)$$

where $\epsilon_{0} = 0.642$ GeV is the “Rydberg” energy, and $\lambda$ is the nucleon energy in the J/Ψ rest frame (see Sec. [11]).

From Eq. (12), we see that the J/Ψ near threshold production is also sensitive to the gluon distribution in the large $x$ region of around 0.3 < $x$ < 1. However, the gluon distribution exhibits big uncertainty at large $x$ from the global fits [16, 17]. Fig. 5 shows various gluon distributions from the global analyses of experimental data and a simple parametrization of a scaling gluon distribution. In Kharzeev’s calculation, he choose the gluon distribution to be 2.5$(1-x)^4$ [10]. IMParon gluon distribution is the pure dynamical gluon distribution which lacks of the “valence”-like gluon distribution for the nonperturbative input. The forward cross-sections of the exclusive J/Ψ photo-production are calculated using these gluon distributions. The results are shown in Table IV. The cross-section with Kharzeev’s gluon distribution is around two times smaller than the experimental value 3.8 nb/GeV$^2$. That is also found by GlueX Collaboration, where in their publication they multiply the theoretical cross-section by a factor of 2.3 in order to well compare to the total cross-section data [11]. Nonetheless, the VMD model is a powerful tool in describing the J/Ψ photo-production, considering the approximation that we fix the ratio $\rho$ to be zero in the calculations.

TABLE IV. The list of the PDF sets used, the charm quark mass $m_c$, the strong coupling $\alpha_s(m_c^2)$, the ratio $\rho$ of the real part and the imaginary part of the $J/Ψ - N$ amplitude, and the forward cross-section of J/Ψ photo-production at $E_\gamma = 10.72$ GeV. A VMD model is used [10].

| $g(x, m_c^2)$ | $m_c$ | $\alpha_s(m_c^2)$ | $\rho$ | $d\sigma_{\gamma N}/dt(s, t = 0)$ |
|--------------|------|-----------------|-------|-------------------|
| IMParton16   | 1.27 GeV | 0.375 | 0 | 0.24 nb/GeV$^2$ |
| GRV98(LO)    | 1.27 GeV | 0.375 | 0 | 3.41 nb/GeV$^2$ |
| CT14(LO)     | 1.27 GeV | 0.375 | 0 | 13.6 nb/GeV$^2$ |
| 2.5$(1-x)^4$ | 1.27 GeV | 0.375 | 0 | 2.44 nb/GeV$^2$ |
| IMParton16   | 1.4 GeV | 0.361 | 0 | 0.16 nb/GeV$^2$ |
| GRV98(LO)    | 1.4 GeV | 0.361 | 0 | 2.03 nb/GeV$^2$ |
| CT14(LO)     | 1.4 GeV | 0.361 | 0 | 8.11 nb/GeV$^2$ |
| 2.5$(1-x)^4$ | 1.4 GeV | 0.361 | 0 | 1.78 nb/GeV$^2$ |

From our calculations, potentially the J/Ψ photo-production data near threshold also provides an opportunity to differentiate the various gluon distributions on the market. The gluon distribution by CT14(LO) is a little over-large while the GRV’s gluon distribution is more or less reasonable, judged by the current data and the simplified calculation in this work. We also find that the variable $\rho$ is non-zero, if there is no “valence”-like gluon distribution in the proton [18].

In summary, we have extracted the QCD trace anomaly $b$ from the recently published JLab data. We give a new proton mass decomposition and suggest a small strange $\Sigma_{sN}$ term.
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