World-Volume Description of M2-branes Ending on an M5-brane and Holography

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Abstract

We consider world-volume description of M2-branes ending on an M5-brane. The system can be described either as a solitonic solution of the M5-brane field equations or in terms of an effective string propagating in 6-dimensions. We show that the zeroth order scalar scattering amplitudes behave similarly in both pictures. The soliton solution appears to have a horizon-like throat region. Due to the underlying geometric structure of the M5-brane theory, modes propagating near the horizon are subject to a large red-shift. This allows one to define a decoupling limit and implies a holographic duality between two theories which do not contain dynamical gravity.
1 Introduction

There is now considerable evidence that quantum gravitational theories in \(d\)-dimensions on \(AdS\) spaces are dual to conformal gauge theories defined in \(d−1\) dimensions \([1, 2, 3]\). This is a concrete realization of the holographic principle \([4, 5]\) which states that in quantum gravity, physics within a volume can be described in terms of a theory without gravity on the boundary of that volume. The holographic principle is usually associated with the presence of dynamical gravity in the bulk, and indeed the original motivation was to explain the area dependence of the black hole entropy. In this letter, we will argue that this principle can be generalized to world-volume theories of branes, which indicates that holography is mainly related to diffeomorphism invariance acting as a gauge symmetry. Although, world-volume theories do not contain dynamical gravity, there is still the notion of the induced geometry and gravity. A possible holographic duality of this type, between the 6-dimensional open membrane (OM) theory on \(AdS_3 \times S^3\) and \(\mathcal{N} = (4,4)\) superconformal field theory in 2-dimensions, has been proposed in \([6]\) which motivated this paper.

We consider the \((2,0)\) theory in 6-dimensions \([7, 8]\) which is associated with the M5-brane dynamics. The theory is conformally invariant; there is no (dimensionless) coupling constant and no length scale \([9]\). There are 16 real supersymmetries and the supermultiplet consists of a self-dual 2-form potential, 5 scalars and chiral spinors. The moduli space labeling the vacua of the theory is given by \(\mathcal{R}^{5K}/W\), where \(K\) is the number of coincident M5-branes and \(W\) is a discrete group \([9]\). The fundamental degrees of freedom are presumably related to self-dual, tensionless string excitations in 6-dimensions, and thus it is hard to give a field theoretic definition of the theory. On the other hand, it can be obtained from M-theory by taking a decoupling limit in the presence of M5-branes, which gives a dual description (for large number of coincident branes) in terms of 11-dimensional supergravity on \(AdS_7 \times S^4\).

The covariant field equations of an interacting (2,0) supermultiplet in 6-dimensions coupled to 11-dimensional supergravity background has been found by Howe and Sezgin in \([10]\), which describe M5-brane dynamics in M-theory. Since the conformal (2,0) theory in 6-dimensions is also related to M5-brane dynamics, it is natural to expect a possible connection to the equations of \([10, 11]\). Because the conformal (2,0) theory is not an ordinary field theory and has stringy excitations, a connection with a field theory can only be established at “large” distances. Therefore, one should first find a way of breaking the conformal invariance in order to introduce a length scale in the theory.

The hint for how to break the conformal invariance comes from the 11-dimensional origin of the theory. It is well known that, in M-theory, an M2-brane can end on an M5-brane \([12, 13, 14, 15]\). This M2-brane looks like a (self-dual) string in 6-dimensional world-volume and preserves half of the supersymmetries of the M5 brane theory. Thus, the states associated with this self-dual
string are stable, world-volume BPS states. The mass per unit length of such a string would diverge, since it is the end of an infinitely long M2-brane. However, it is possible to obtain finite energy configurations simply by cutting the M2-brane at a finite length. In 11-dimensions this has the interpretation of placing another parallel M5-brane for M2-brane to end. One can now take a decoupling limit, where two M5-branes are still separated from each other, but the mass of the stretched membrane is fixed. The resulting theory on the M5-brane is the (2,0) theory decoupled from the bulk, however the conformal invariance is explicitly broken by the energy scale of the stretched membrane. This is similar to breaking of the conformal invariance of the $\mathcal{N} = 4$, $U(N)$, Super Yang-Mills (SYM) theory by separating a group of D3-branes from others (i.e., by Higgsing). In this paper, we assume that the (2,0) theory can be approximated by a field theory for distances larger than the scale of the self-dual string tension, and the dynamics is governed by the covariant equations obtained in [10], where the 11-dimensional space is chosen to be flat.

To support such a picture, the equations of the field theory should admit a self-dual string background as a solitonic solution. This is indeed the case; as shown in [16] there is a supersymmetric self-dual string solution which has an infinite mass per unit length. This is in agreement with the interpretation of the string being the ending of a semi-infinite membrane. As we will discuss in a moment, formally it is possible to incorporate finite energy configurations simply by cutting the solution, which can be thought of placing another parallel M5-brane for M2-brane to end. Similar solutions describing fundamental-branes or D-branes ending on D-branes have been found in [17, 18].

In the (2,0) theory (and in Dirac-Born-Infeld (DBI) theories in general), fluctuations around a background configuration are characterized by the so called Boillat metric [19, 20]. As we will discuss, the Boillat metric for the self-dual string looks like a black-string metric which has a singular horizon. Singularity of the horizon is associated with the bad behaviour of the self-dual string solution at the origin. On this background, one can consider a limit in which the asymptotic region decouples from the "near horizon" region. As in the case of AdS/CFT duality, this is possible thanks to a redshift factor related to the underlying geometric structure of the theory. After taking the limit, the resulting background geometry can only be trusted for some region of the radial parameter. This is very similar to non-conformal supergravity duals of D-brane world-volume theories [21].

A possible holographic dual for this system can be obtained by using an effective string theory in 6-dimensions coupled to (2,0) tensor multiplet. The world-sheet action for this string should have $\mathcal{N} = (4,4)$ supersymmetry. Although the complete action is not known, one can easily

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To be more precise as in the case of $\mathcal{N} = 4$, $U(N)$ SYM, one should first consider the equations of coincident M5-branes and break conformal invariance by giving vacuum expectation values to some scalars. Since the world-volume equations of the coincident M5-branes are not known, we assume that at low energies, when the two parallel M5-branes are well separated, one can consider the dynamics of each one independently.
determine the bosonic part (for a single string). In this action, the coupling of the string world-sheet to the bulk (2,0) multiplet is determined by the tension of the self-dual string. This implies that, in the low energy limit that we will consider, the string action decouples from the bulk (2,0) theory. The decoupled theory can be identified with the ”near horizon” region of the self-dual string soliton, which would imply a holographic duality.

The plan of the paper is as follows. In section 2, we discuss the (2,0) equations, the self-dual string solution and different possible descriptions of the system. In section 3, we define a low energy limit which would imply a holographic duality between two theories. We conclude in section 4.

2 The self-dual string solution

Let us start with a brief description of the equations of [10], which are based on super-embedding of a 6-dimensional world-volume superspace into 11-dimensional target superspace. The equations are invariant under both bosonic and fermionic reparametrizations of the world-volume superspace. The former can be regarded as the usual diffeomorphism invariance and the latter is called the $\kappa$-symmetry. The world-volume manifold has no specified metric structure, but it has an induced geometry defined by the embedding map.

One can use super-reparametrization invariance of the world-volume to choose the so called static gauge, where 6 of the 11 bosonic, and 16 of the 32 fermionic embedding functions are identified with the bosonic and fermionic world-volume coordinates respectively. This leaves 5 scalars, a self-dual three-form and 16 fermionic fields as physical degrees of freedom. After setting fermions to zero, the bosonic field equations are given as [11]:

\begin{align}
G^{MN} \nabla_M \nabla_N X^{a'} &= 0, \\
G^{MN} \nabla_M H_{NPQ} &= 0,
\end{align}

where $M, N = 0, 1, ..5$, $X^{a'}$'s ($a' = 1, ..5$) represent transverse fluctuations of the M5-brane and $H_{MNP}$ is the curl of the 2-form potential. The covariant derivative $\nabla_M$ is constructed using $g_{MN}$, which is the pull-back of the 11-dimensional metric into world-volume by the map $X^{a'}$. The metric $G_{MN}$ is related to $g_{MN}$ by $E_M^A = e_M^B (m^{-1})_B^A$, where $E_M^A$ and $e_M^A$ are vielbeins of $G_{MN}$ and $g_{MN}$ respectively, and $m_{AB}^D$ is defined as

$$m_{AB}^D = \delta_A^B - 2 h_{ACD} h^{BCD},$$

and $H_{MNP} = E_M^A E_N^B E_P^C m_{E}^D m_{C}^E h_{ADE}$. Although, $H_{MNP}$ satisfies a non-linear self-duality condition, $h_{ABC}$ is linearly self-dual, and in general it cannot be written as the curl of a two-form.
The equations (1) and (2) admit a supersymmetric self-dual string solution [16], where the M5-brane is embedded into 11-dimensional flat Minkowski space. The solution is given by

\[ X^{5'} = l_s h; \]  
\[ X^{i'} = \text{const. for } i' = 1, 2, 3, 4 \]  
\[ H_{01m} = \frac{l_s}{4} \partial_m h, \]  
\[ H_{mnp} = \frac{l_s}{4} \epsilon_{mnpq} \delta^{5'} \partial_{5'} h \]

where \( \delta^{mn} \partial_m \partial_n h = 0 \) and thus

\[ h = 1 + \frac{q l_s^2}{r^2}. \]

In 6-dimensions, \((x^0, x^1)\) parametrize the string world-sheet, and \(y^m\) are the transverse coordinates. \(X^{5'}\) is one of the coordinates perpendicular to the M5-brane world-volume and together with \((x^0, x^1)\) it parametrizes the M2-brane ending on the M5-brane. The embedding functions \(X^{a'}\), and the coordinates on the world-volume are chosen to have dimension of length, and the components of the tensor \(H_{MNQ}\) are dimensionless. The length scale \(l_s\) and charge \(q\) in (8) are related to the tension and the charge of the self-dual string.

The mass per unit length of the solution can be calculated by first performing a double dimensional reduction [16], which gives a 0-brane in 5-dimensions. Then, the energy is obtained by using the DBI expression, and as shown in [16, 17], it turns out to be infinite. This is consistent with the interpretation that the self-dual string is the ending of a semi-infinite M2-brane. From the world-volume point of view, this infinity is associated with the \(r \to 0\) behaviour of the solution, and it is similar to the self-energy divergence of the Coulomb field in electromagnetism.

This shows that, the solution can only be trusted for \(r > \epsilon\) and should be modified (or possibly the field equations can no longer be trusted) for \(r < \epsilon\), where \(\epsilon\) is a small length scale. For the modified solution, the energy should be finite. This has the interpretation of placing another parallel M5-brane for M2-brane to end. Without knowing this modification, one could simply cut the space at \(r = \epsilon\). The energy would then have an \(\epsilon\) dependence and thus \(\epsilon\) acts as a regulator in the theory. (Following [22], one can sharpen these suggestions by considering a test M5-brane in a supergravity M5-brane background. In this case, the world-volume theory admits finite energy solutions corresponding to M2-branes stretched between the test and the source M5-branes [22].)

It is possible to relate the length scale \(l_s\) of the (2,0) theory and the critical length \(\epsilon\) to the 11-dimensional planck length \(l_p\) and the separation of the M5-branes \(L\). Since the solution should be modified for \(r < \epsilon\), it is natural to assume that the M2-brane ends on the second parallel M5-brane at \(r = \epsilon\). Noting that the field \(X^{5'}\) represents the transverse spike of the M2-brane,

\footnote{We thank D. Mateos for pointing out this reference to us.}
one finds \( L = X^5(\epsilon) \) (or at least \( L \) and \( X^5(\epsilon) \) have the same order of magnitude). On the other hand, the energy (per unit length) of an M2-brane with length \( L \) is \( T_2 L \), where \( T_2 \) is the M2-brane tension. This can be identified with the tension \( T_s \) of the self-dual string, and thus \( 1/l_s^2 \sim T_s = T_2 L \). Since \( T_2 \sim 1/l_p^3 \), \( l_s \) can be determined in terms of \( l_p \) and \( L \). For our purposes, we are interested in a configuration where \( \sqrt{q} l_s \gg \epsilon \). It is easy to see that this corresponds to \( L \gg l_p \). In this case \( l_s^2 = l_p^3/L \) and \( \epsilon^4 = q^2 l_p^9/L^5 \).

At low energies, the dynamics of the self-dual string should be described by an effective action involving the Goldstone modes of the solution, which contains 4 bosonic and 4 fermionic zero modes. Although, we do not know the exact form of the action, it is possible to determine the bosonic part and its coupling to the bulk (2,0) fields.

In [23], it was shown that there is a static membrane solution on the M5-brane background of \( D = 11 \) supergravity which can be interpreted as the orthogonal intersection of an M2-brane with an M5-brane. In the same paper, the dynamics of the membrane boundary on M5-brane was shown to be governed by the Nambu-Goto action for a string moving in a 6-dimensional Minkowski space. To include non-trivial 6-dimensional bulk excitations, one should consider a generalization of this action. The fluctuations and causal structure around a non-trivial background of M5-brane fields are characterized by the Boillat metric which is given by [20]

\[
C^{AB} = \frac{1}{K} \left[ (1 + \frac{4}{3} H^2) g^{AB} - 4 (H^2)^{AB} \right] \tag{9}
\]

where

\[
K = \sqrt{1 + \frac{2}{3} H^2}, \tag{10}
\]

and all contractions are performed with the metric \( g_{AB} \). Since the zero modes of a soliton can be viewed as fluctuations around that soliton, it is natural to assume that the effective self-dual string, which represents the Goldstone modes, couples to the Boillat metric \( C_{AB} \). Therefore, one can write the following bosonic action

\[
S_{\text{string}} = \frac{1}{l_s^2} \int d^2 \sigma \sqrt{-\text{Det} C} + \frac{q}{l_s^2} \int \tilde{B}, \tag{11}
\]

where \( B \) is the 2-form potential for the self-dual 3-form and \( \tilde{C} \) and \( \tilde{B} \) are pull-backs of the \( C_{AB} \) and \( B_{AB} \) to the world-sheet.

In this paper, we are mainly interested in how this string action couples to the scalars of the (2,0) theory. This is important for understanding the emission, absorption or scattering of scalars by the effective string. To see the leading order couplings we first split 6-dimensional space \( (z^A) \) into \( (x^\mu, y^m) \), where \( \mu, \nu = 0, 1 \) and \( m, n = 2, \ldots, 5 \), and use reparametrization invariance to identify the first two coordinates with the world-volume coordinates of the string. We then

\[ C_{AB} \text{ is defined to be the inverse of } C^{AB}. \]
expand the string action (11) to obtain

\[ S_{\text{string}} = \frac{1}{2l_s^2} \int d^2x \left[ (\partial_{\mu}X^a)^2 + (\partial_{\mu}y^m)^2 \right] + S' \]  

(12)

where \( y^m(x) \) are string world-sheet scalars and \( S' \) starts with the quartic couplings between \( \partial_{\mu}X^a \) and \( \partial_{\mu}y^m \) which have the following structure:

\[ S' \sim \frac{1}{l_s^2} \int d^2x \left[ \partial X \partial X \partial X \partial X + \partial X \partial X \partial y \partial y + \partial y \partial y \partial y \partial y + .... \right] \]  

(13)

To include the M5-brane dynamics, this string action should be coupled to the M5-brane action in the bulk. The leading order quadratic terms of such an action read

\[ S_{\text{bulk}} = \frac{1}{l_s^6} \int d^6z \left[ (\partial A \partial A X^a')^2 + (H_{ABC} H^{ABC}) + .... \right] \]  

(14)

The total action governing the system is equal to \( S_{\text{total}} = S_{\text{bulk}} + S_{\text{string}} \).

Compared to the usual p-brane actions coupled to supergravity scalars, \( S_{\text{total}} \) has some interesting features. The most striking difference is that there is an extra kinetic term for the bulk scalars which appears on the world-sheet action (12). Also, the structure of the interaction terms in (13) indicates that the bulk scalars are scattered from the string rather than being absorbed or emitted. This is mainly related to the fact that the bulk scalars are themselves Goldstone modes of an M5-brane which represent transverse fluctuations, and thus only the terms containing even number of \( \partial_{\mu}X \) can appear in the interactions. All these are also consistent with the fact that contrary to the p-branes of supergravity theories which are black objects, the self-dual string is a soliton solution of a field theory.

To see the zeroth order effect of the extra kinetic terms for the bulk scalars, we ignore the interaction terms in \( S_{\text{total}} \) and obtain the following field equations for \( X^a' \)

\[ \partial_A \partial^A X^a' = 0, \]  

(15)

\[ \partial_{\mu} \partial^\mu X^a'|_{y=0} = 0, \]  

(16)

where we assume that the string world-sheet is located at \( y^m = 0 \). For the localized modes that do not intersect the string, equation (16) is satisfied identically and equation (15) gives usual massless modes propagating in the bulk. Also the other modes in the bulk should obey (16), which implies

\[ X^a' = e^{i(-\omega t + k_1 x)} \sin(k_m y^m), \]  

(17)

As in the case of type IIB supergravity, one can write an action and then impose the self-duality of the 3-form field at the level of field equations.

The field equations are \( [\partial_{A} \partial^{A} + \delta(y) \partial_{\mu} \partial^{\mu}] X^a' = 0 \). For smooth fields each term should vanish separately which gives (15) and (16).

\( e^{i(-\omega t + k_1 x + k_m y^m)} \) gives usual massless modes, which are also eigenvectors of the operator \( \partial_{\mu} \partial^{\mu} \) with non-zero eigenvalues. Therefore, (16) is equivalent to the Dirichlet boundary conditions \( X^a' = 0 \) at \( y = 0 \).
where $\omega^2 = k_1^2 + k_m k_m$. Thus, these modes scatter from the string and form standing waves along transverse directions. This is the zeroth order contribution to the scattering amplitude, and higher order corrections can be calculated using the interaction terms in $S'$. 

On the other hand, in finding the field equations, if one restricts the field space so that they have non-zero support only on $y^m = 0$ hyperplane, then only $S_{\text{string}}$ gives non-zero contribution to the variation of $S_{\text{total}}$, which implies

$$\partial_\mu \partial^\mu X^{a'} = 0. \quad (18)$$

Since $X^{a'}$'s are assumed to have non-zero support only on $y^m = 0$, this represents a massless mode purely localized on the string. It is also possible to see the existence of this extra sector of modes from the path integral approach. In such a functional integral, evaluating $S_{\text{bulk}}$ for the scalars with non-zero support only on the string world-sheet, would give no contribution. However, $S_{\text{string}}$ gives non-zero contribution to the path integral, where the scalars also have the standard kinetic term. This indicates existence of an extra massless sector in the theory.

Let us try to see the validity of the effective string picture by comparing how scalars are scattered in the self-dual string solution picture. Here we analyze the following scalar fluctuations around (4)-(7)

$$X^{i'} = \text{const.} + \phi^{i'}, \quad \text{for} \quad i' = 1, 2, 3, 4. \quad (19)$$

and find that for all $i'$ they obey the same equation

$$\eta^{\mu\nu} \partial_\mu \partial_\nu \phi + f^{-1} \delta^{mn} \partial_m \partial_n \phi = 0, \quad (20)$$

where

$$f = 1 + l_s^2 \delta^{mn} \partial_m h \partial_n h = 1 + \frac{R^6}{r^6}, \quad (21)$$

and $R^6 = 4q^2 l_s^6$. Considering a spherically symmetric s-wave, near the throat region the constant term in (21) can be ignored. In that case, one can define a new radial coordinate by $\hat{r} \sim 1/r^2$ and equation (20) becomes

$$(\partial_x^2 + \partial_t^2 - \partial_{\hat{r}}^2) \phi = 0. \quad (22)$$

This is nothing but the free-wave equation in the $t, x, \hat{r}$ plane. As in the case of DBI theories, near the self-dual string, the solution is so singular that one reaches another asymptotic region. From (4), we recognize that near $r = 0$, $\hat{r} \sim X^5(r)$, and thus these waves can be considered to be propagating on the M2-brane. In the effective string picture, this corresponds to the extra sector that we pointed out following equation (18). We see that the effective string picture nicely captures the second asymptotic region.

Next, we consider the scattering of scalars from the self-dual string solution. Since the potential term is so singular that near $r = 0$ there is another asymptotic region, we expect a perfect
reflection at very low energies. Considering a wave of the form $\phi(r, t) = e^{i\omega t}\phi(r)$ we obtain a one-dimensional scattering problem given by

$$\left(\frac{d^2}{dz^2} + k(z)\right)\phi = 0$$

where $z = (R\omega)^3/(r\omega)^2$ and

$$k(z) = \frac{1}{4}[1 + \frac{R^3\omega^3}{z^4}].$$

We would like to consider the problem in the limit $l_s \to 0$ or $R\omega \to 0$ which corresponds to a low energy scattering. To be able to understand the structure of the potential in this limit, following [17], we define a new coordinate suggested by the WKB approximation

$$x = \int_{z}^{0} dz' \sqrt{k(z')} \kappa,$$

where $\kappa = R\omega$. The coordinate $x$ extends from $-\infty$ to $+\infty$. Introducing a properly-normalized (in the sense of WKB) wave-function

$$\phi = [k(z)]^{-1/4} \hat{\phi}$$

equation (23) becomes

$$\left(-\frac{d^2}{dx^2} + V(x) - 1\right) \hat{\phi} = 0, \quad V(x) = \frac{a\kappa^3}{z^3k(z)^3} + \frac{bk^6}{z^8k(z)^3},$$

where $a$ and $b$ are two positive numbers. This is a one-dimensional quantum mechanical scattering problem with the potential $V(x)$. As $x \to \pm\infty$, $V(x) \to 0$ and $V(x)$ is finite at $x = 0$.

Let us try to see the form of the potential for low energy scattering when $l_s \to 0$, which implies $\kappa \to 0$. By a scaling argument it is easy to see that $\int dx\kappa V(x)$ is independent of $\kappa$. Furthermore, $V(x)$ is everywhere positive and when $\kappa = 0$, $V(x)$ is zero except at $x = 0$. So, in the limit $\kappa \to 0$, $V(x)$ becomes a delta function. The scaling shows that

$$V(x) \sim \frac{1}{\kappa} \delta(x), \quad \kappa \to 0.$$  

The potential turns out to have the same limiting behaviour compared to strings ending on D-branes and in this limit, one finds a perfect reflection [17]. Recall that in the effective string picture in the same limit ($l_s \to 0$), we had found perfectly reflecting modes [17] from the string. Thus, to the lowest order in perturbation theory, the effective string picture nicely captures the scalar scattering amplitudes on the string soliton.

### 3 The low energy limit

We now consider M-theory in the presence of two M5-branes and $N$ parallel M2-branes stretched between them. In 6-dimensions, this corresponds to (2,0) theory in the presence of $N$ coincident self-dual strings. At low energies, this system has two different descriptions:
i) from the world-volume theory point of view; one can consider the self-dual string solution,

ii) from the effective string picture; one can consider the string action in 6-dimensions describing the fluctuations of the \( N \) coincident self-dual strings.

We would like to consider a limit where we let the tension of the self-dual string \( l_s \to 0 \), and keep \( L \) fixed, which corresponds to a low energy limit. Since \( l_p^3 = L l_s^2 \), we also have \( l_p \to 0 \). Therefore, 11-dimensional bulk excitations decouple from the system. In the world-volume theory we would like to keep the energy of coincident strings finite. As pointed out in [16], the string soliton might represent a D-string of (2,0) theory since the “fundamental” strings do not carry any \( H \)-charge [15, 24]. Therefore, to keep the energy of the coincident strings finite, one should assume \( r/l_s^2 = \text{fixed} \) where \( r \) is the separation between parallel self-dual strings.

Let us now apply the above limit to both descriptions of the system. The solution describing \( N \) coincident self-dual strings is given by (4)-(7) where the harmonic function \( h \) is

\[
h = 1 + \frac{Nql_s^2}{r^2}. \tag{29}
\]

In taking a low energy limit we need to be careful how we define the energy since the background solution induces a geometry. Of course far from the string the induced geometry is flat and the observer is located near that flat region. To determine how the observer sees the string solution we look at the behaviour of small fluctuations. In the previous section, we found that the scalar perturbations obey equation (24). However, (24) is nothing but the massless scalar equation on the background

\[
ds^2 = f^{-1/2}(-dt^2 + dx^2) + f^{1/2}(dr^2 + r^2 d\Omega_3^2), \tag{30}
\]

which looks like a black-string in 6-dimensions.\footnote{For \( N \) coincident strings, \( R \) in (21) is given by \( R^6 = 4q^2 N^2 l_s^6 \).} We conclude that, although the theory does not have any dynamical gravity, the induced geometry of the solution (4)-(7) is such that the observer far from the string sees it as a black-string background.

The same geometry can also be obtained by calculating the Boillat metric (9) corresponding to the self-dual string background. Actually this is not surprising, since the Boillat metric characterizes fluctuations around a non-trivial background. This further indicates that a complete analysis of the scalar and 3-form fluctuations should reflect the geometry of (30).

It is important to remember that, the self-dual string soliton is defined for \( r > \epsilon \). Therefore, the above geometry can only be trusted for this region. On the other hand, the metric (30) has a horizon at \( r = 0 \) which turns out to be a singular surface. As we will see shortly the horizon is conformal to \( AdS_3 \times S^3 \). Although the solution is defined before reaching the horizon, there is still a large red-shift factor since \( \epsilon \) can in principle be chosen very small (but still non-zero)
by moving the parallel M5-branes apart (thus letting $L$ to be very large). This property of the (2,0) theory makes it different from other ordinary field theories defined on flat spaces.

We now consider the low energy limit on the self-dual string solution. Remembering that we define the energies with respect to the observer located at $r \to \infty$ and taking into account the red-shift factor in the metric (30), the low energy limit (which is equivalent to taking $l_s \to 0$ with $r/l_s^2$ fixed\(^{10}\)) gives the geometry

$$ds^2 = \frac{r^3}{2Nq}(-dt^2 + dx^2) + \frac{2Nq}{r^3}dr^2 + \frac{2Nq}{r^2}d\Omega_3^2,$$

(31)

where we set $l_s = 1$. This is conformal to AdS$_3 \times S^3$ and the conformal factor is $1/r$. Now, recall that the geometry is defined only for $r > \epsilon$. On the other hand, the field equations break down at scales smaller than $l_s$. Looking at the the curvature of (31) one finds that it is small compared to $l_s$ when $r \ll Nq$. Therefore, for large number of coincident strings, the low energy limit on the self dual string can be characterized by the geometry of (31), where the radial coordinate is restricted to $\epsilon < r \ll Nq$. Note that the curvature gets smaller for larger $Nq$.

Let us now consider the same low energy limit in the effective string picture. For $N$ coincident self-dual strings one should find the non-abelian generalization of (12). This is a common problem for DBI theories which has not been solved yet. On the other hand, from (12) and (14), after properly normalizing the bulk scalars, one finds that the coupling of the string world-sheet to the bulk scalars and the extra kinetic terms of the bulk scalars on the world-sheet vanishes as $l_s \to 0$. This indicates that in the low energy limit of the non-abelian generalization of (12) one ends up with two different theories; a non-trivial theory living on the string-world sheet and a decoupled theory in the bulk. Similar to the well-known AdS/CFT duality, we started with two different descriptions of the same system, and after taking the limit we obtained two decoupled pieces in each description. Identifying the two non-trivial pieces one can claim a duality between (2,0) theory on the background (31) and a two-dimensional field theory on the string. The structure of (31) indicates that the string action is not conformal and for some range of the renormalization group flow one can use the dual geometry. Also, since the 6-dimensional space has 3 compact and 3 non-compact directions, (2,0) theory is effectively compactified to 3-dimensions. Thus, the suggested duality is a holographic duality in the sense that a 3-dimensional theory (having diffeomorphism gauge invariance and presumably tensionless strings) is dual to a 2-dimensional field theory.

At this point, we need to be more explicit on what we mean by saying (2,0) theory on the space defined by equation (31). Since (31) does not have conformal symmetry, there is a length scale in the theory. Thus, at low energies, we assume that the (2,0) theory is described by the equations of (10). Since these equations are intrinsically 11-dimensional, the most natural thing is to start

\(^{10}\)The situation is very similar to the D-string of the Type IIB theory discussed in [2].
with a non-trivial background corresponding to M2-branes ending on M5-branes, obtain the near horizon geometry (which corresponds to a low energy description) and write down equations of \[10\] on this geometry. Unfortunately, to the best of our knowledge, no explicit supergravity solution describing M2-branes ending on an M5-brane exists in the literature. On the other hand, the solution of M5-M2 brane intersection is known. Choosing the harmonic functions of the branes equal, the near horizon geometry of this intersection becomes \(AdS_3 \times S^3 \times R^4 \times R\) (supported by non-trivial 4-form fields). This should correspond to a conformal fixed point of the dual theory. It would be interesting to find out branes ending on branes type of solutions (which would presumably have a throat region given by \(31\)) and give a precise meaning to the formulation of (2,0) theory on the background of \(31\).

4 Conclusions

In this paper, we considered the world-volume description of M2-branes ending on an M5-brane and defined a low energy limit which would imply a holographic duality between two theories which do not contain gravity. The duality could be established since the system has two different descriptions. This is indeed an essential ingredient in all examples of holography. Furthermore, in taking a low energy limit with respect to the observer at infinity, one needs a mechanism for energies to be red-shifted. Otherwise, in the low energy limit everything would become trivial. In gravitational theories, existence of a horizon is responsible for this red-shift. In our case, although we do not have a dynamical gravity, there is a hidden geometric structure. This defines a background geometry for fluctuations around a field configuration and for the self-dual string solution this geometry turns out to have a throat region. Therefore, energies near the throat region are red-shifted and after the low energy limit is taken one still ends up with non-trivial excitations.

The dualities proposed in this paper and in \(6\) are examples of holography where dynamical gravity does not play any role. However, as pointed out in the introduction, (2,0) theory is not an ordinary field theory. Despite the absence of a dynamical gravity, in many aspects it is similar to a gravitational theory (since it is related to tensionless strings). In the formulation of \(10\), diffeomorphism invariance for both bosonic and fermionic fields plays a crucial role. Presumably, in quantum theory, due to the diffeomorphism invariance acting as a gauge symmetry, the degrees of freedom in the bulk may have a holographic description as in the case of gravitational theories.

As in \(3\), one can also consider open membrane (OM) theory in the same context. In \(3\), OM metric was considered in the large field limit, whereas in \(23, 24\) the complete form was found to be

\[
(G_{OM})_{AB} = \frac{\left[ (2K^2 - 1) - 2K^2 \sqrt{1 - K^{-2}} \right]^{1/6}}{K} \left( g_{AB} + 4 H_{AB}^2 \right),
\]

where \(K\) is given by \(10\). Evaluating \(32\) for the self-dual string solution \(4\)-\(7\), one obtains
the following metric
\[ ds^2 = f^{-2/3}(-dt^2 + dx^2) + f^{1/3}(dr^2 + r^2d\Omega_3^2), \]
(33)
where \( f \) is given by (21). Although the solution (4)-(7) is singular at \( r = 0 \) and thus valid for \( r > \epsilon \), it turns out that (33) is regular even at \( r = 0 \). Indeed, \( r = 0 \) is a horizon and near horizon geometry is \( AdS_3 \times S^3 \) without any conformal factor. In (33), a low energy limit is defined so that the near horizon region is decoupled from the asymptotic region. Thus, one has OM theory defined on \( AdS_3 \times S^3 \) which would be dual to a 2-dimensional field theory. The dual \( AdS_3 \) geometry indicates that this theory is defined at a conformal fixed point. The 2-dimensional field theories considered in this paper and in [6] should be the same (since they both describe collective motion of the same soliton). This theory should have a renormalization group flow so that at an isolated conformal fixed point it has a dual \( AdS_3 \) OM theory. When the conformal invariance is broken, (33) describes physics for some interval in the renormalization group flow. It would be interesting to verify such a picture. For that, one needs to find the complete \( N = (4,4) \) supersymmetric action for \( N \) coincident self-dual strings in 6-dimensions.

Finally, there is also a supersymmetric 3-brane soliton of the M5-brane field equations [27]. One may try to find the background geometry corresponding to this 3-brane and define a low energy limit which would imply a holographic duality. It turns out, the Boillat metric for the 3-brane does not have any horizon or throat region. It is a smooth metric which interpolates between \( R^6 \) and \( R^5 \times S^1 \). Thus, it is not possible to define a decoupling limit in this case. The 3-brane in 6-dimensions has two transverse directions. Therefore, this object requires a special treatment (like D7-brane of IIB theory) since it is not a trivial task to define an asymptotic region.

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