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Determination of the threshold values of orthotropic bi-material notches

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Abstract

In the present contribution the joint of two orthotropic materials is investigated as the singular stress concentrator and it is modelled as an orthotropic bi-material notch. Such places in constructions are usually responsible for the crack initiation and consequently for the final failure of the construction. Within the paper the procedure for the determination of the threshold value of generalized stress intensity factor is presented. Knowledge of the threshold value is necessary for the reliable assessment of general singular stress concentrations.

Keywords: Bi-material notches; generalized singular stress concentrators; stability criterion; orthotropic materials;

1. Introduction

In practical engineering structures or in parts of electronic devices joints of different materials occur (e.g. layered composite materials, constructions with protective surface layers, thermal barriers). They enable achievement of properties which could not be attained by means of homogeneous materials. In the case of composite materials, parts of the joints often exhibit orthotropic material properties. As fatigue damage of such constructions usually starts at the surface of the structure, places of material discontinuities at the surface are in our scope of interest.

The stress field in closed vicinity of such material joints has singular character and complicated form. In comparison to a crack in homogeneous media, in the case of bi-material joints, the stress singularity exponent is different from 1/2 and can generally be complex. The stress is mostly characterized by more singular terms and at the same time each singular term covers combination of both normal and shear modes of loading.

Such stress concentrators preclude any application of the fracture mechanics approaches originally developed for a crack in isotropic homogeneous materials, so the assessment of such singular stress concentrators becomes topical \cite{1} - \cite{4}. The orthotropic material properties seriously complicate procedures for assessment of bi-material notch stability. Most such discontinuities can be mathematically modelled as bi-material notches composed of two orthotropic materials. The edge of a protective coating, a free edge stress singularity or other shapes of notches can...
be modelled for varying angles $\alpha_1$ and $\alpha_2$, see figure 1. In the contribution the orthotropic bi-material notch is analysed from the perspective of generalized linear elastic fracture mechanics, i.e. the validity of small-scale yielding conditions is assumed. We further assume ideal adhesion at the bi-material interface and the notch radius $R\to0$ (the sharp bi-material notch tip).

![Fig. 1. Bi-material orthotropic notch with corresponding polar and cartesian coordinate systems](image)

The contribution aims to suggest and present a procedure for the determination of the fatigue crack initiation direction from an orthotropic bi-material notch. Further the critical loading conditions are estimated. The approach is based on knowledge of the stress distribution in place of the concentration. Combined analytical and numerical approaches are employed for the stress field determination. The criterion of the maximum tangential stress (MTS) and the criterion of the strain energy density factor (SEDF) are modified and adapted to particularities of the nature of the stress concentrator.

2. Stress distribution

The necessary step for the crack initiation assessment is detailed knowledge of the stress distribution. Within plane elasticity of anisotropic media the Lekhnitskii-Eshelby-Stroh (LES) formalism based on [5]-[7] can be used. Complex potentials satisfying the equilibrium and the compatibility conditions as well as the linear stress-strain dependence and given boundary conditions are the basis for the determination of stress and deformation fields. In the case of general plane anisotropic elasticity all the components of the stress and deformation tensors have to be considered. In the case of orthotropic materials symmetry in the stiffness and compliance matrices occur. Thus the stress and strain tensor is significantly reduced. According to the LES theory for an orthotropic material, the relations for deformations and stresses can be written as follows:

$$u_i = 2 \text{Re}\{A_j f_j(z_j)\}$$
$$\sigma_{zi} = 2 \text{Re}\{L_{ij} f_j'(z_j)\}, \quad \sigma_{li} = -2 \text{Re}\{L_{ij} \mu_j f_j'(z_j)\}$$ (1)

where $\text{Re}$ denotes the real part of the complex expression, $\mu_j = \mu_j' + i\mu_j''$ are the eigenvalues (complex numbers) of the elastic constants, $z_j = x + \mu_j y$ and for matrices $A_{ij}$ and $L_{ij}$ holds:

$$A = \begin{bmatrix} s_{11}\mu_1^2 + s_{12} & s_{11}\mu_2^2 + s_{12} \\ s_{12}\mu_1 + s_{22}/\mu_1 & s_{12}\mu_2 + s_{22}/\mu_2 \end{bmatrix}, \quad L = \begin{bmatrix} -\mu_1 & -\mu_2 \\ 1 & 1 \end{bmatrix}$$ (2)
where \( s_{ij} \) are the elastic compliances. Introducing the stress function

\[
\phi = [\phi], \quad \phi_i = 2 \text{Re} \{ L_{ij} f_j(z_j) \} \quad (i, j = 1, 2),
\]

the radial and tangential stresses can be expressed as:

\[
\sigma_{rr} = n \cdot t_r, \quad \sigma_{r\theta} = m \cdot t_r, \quad \sigma_{\theta\theta} = m \cdot t_\theta.
\]

where \( t_i \) is the traction vector, \( n_i \) and \( m_i \) are the normal and tangential vectors to the selected curve. If the comma denotes differentiation, one can write

\[
t_r = -\frac{1}{r} \phi_\theta, \quad t_\theta = \phi_r, \quad n^T = [\cos \theta, \sin \theta], \quad m^T = [-\sin \theta, \cos \theta].
\]

In the case of the studied notch, the potential \( f_j(z_j) \) has the following form:

\[
f = H \left( z_i^\delta \right) v,
\]

where \( H \) is the generalized stress intensity factor (it can generally be complex), \( v_i = v_i' + iv_i'' \) is a complex eigenvector corresponding to the eigenvalue \( \delta \) where \( 1 - \delta = 1 - (\delta' + i\delta'' \) represents the exponent of the stress singularity at the notch tip. Eigenvector \( v_i \) and eigenvalue \( \delta \) are the solution of the eigenvalue problem leading from the prescribed notch boundary and compatibility conditions. The expression \( \left\langle z_i^\delta \right\rangle \) is a diagonal matrix for which \( \left\langle z_i^\delta \right\rangle = \text{diag}[z_1^\delta, z_2^\delta]. \)

In most practical cases, as well as in the cases studied in the paper, there are two singular terms corresponding to two stress singularity exponents \( 1-\delta_1 \) and \( 1-\delta_2 \). Note that for the final determination of the stress field in the bi-material notch vicinity the generalized stress intensity factors have to be estimated by means of numerical approaches. Their values result from a numerical solution for a certain construction with given material properties, geometry and boundary conditions, see also [8].

3. Crack initiation direction

3.1. Mean value of the tangential stress

The stress field around a bi-material notch inherently covers combined normal and shear modes of loading. For mixed mode fields a crack may grow along the interface or at a certain angle \( \theta_0 \) with the interface into material I or II. In the present paper where the two orthotropic materials are assumed as perfectly bonded, only crack propagation into materials I or II will be supposed. Erdogan and Sih [9] proposed and Smith et al. [10] modified the MTS theory in a study on the slant crack under mixed mode I/II loading, see also [1]. This criterion states that the crack is initiated in the direction \( \theta_0 \) where the circumferential stress \( \sigma_{\theta\theta} \) at some distance from the crack tip has its maximum and reaches a critical tensile value. The local maximum of the tangential stress \( \sigma_{\theta\theta} \) in the case of a bi-material orthotropic notch depends on the radial distance \( r \) from the notch tip. In order to suppress the influence of the distance \( r \), the mean value of the tangential stress is evaluated over a certain distance \( d \):

\[
\bar{\sigma}_{\theta\theta}(\theta) = \frac{1}{d} \int_0^d \sigma_{\theta\theta}(r, \theta) dr = \frac{1}{d} \sum_{k} \left( m_k (d, \theta, \delta_k) + m_k (d, \theta, \delta_k) \right).
\]

Inserting the equations (3), (5), and (6) into (7) one gets for material \( m = I, II \)

\[
\bar{\sigma}_{\theta\theta}(\theta) = H_1 F_{\theta\theta1m}(\theta) + H_2 F_{\theta\theta2m}(\theta),
\]
where

\[
F_{\theta \theta_{0m}}(\theta) = -2 \sin \theta \left\{-d^{S_{-1}} R_{1}^{S_{\theta}} e^{-S_{\theta}^{\Psi_1}} \left[ (\mu'_{1} v'_{1} - \mu''_{1} v''_{1}) \cos (\delta'_{1} \ln d + \delta''_{1} \ln R_{1} + \delta'_{1}^{\Psi_1}) \right] \\
- (\mu'_{1} v'_{1} + \mu''_{1} v''_{1}) \sin (\delta'_{1} \ln d + \delta''_{1} \ln R_{1} + \delta'_{1}^{\Psi_1}) \right\} \\
-d^{S_{-1}} R_{2}^{S_{\theta}} e^{S_{\Psi_1}} \left[ (\mu'_{2} v'_{2} - \mu''_{2} v''_{2}) \cos (\delta'_{2} \ln d + \delta''_{2} \ln R_{2} + \delta'_{2}^{\Psi_1}) \right] \\
- (\mu'_{2} v'_{2} + \mu''_{2} v''_{2}) \sin (\delta'_{2} \ln d + \delta''_{2} \ln R_{2} + \delta'_{2}^{\Psi_1}) \right\} \\
+ 2 \cos \theta \left\{ d^{S_{-1}} R_{1}^{S_{\theta}} e^{-S_{\theta}^{\Psi_1}} [v'_{1} \cos (\delta'_{1} \ln d + \delta''_{1} \ln R_{1} + \delta'_{1}^{\Psi_1})] \\
- v''_{1} \sin (\delta'_{1} \ln d + \delta''_{1} \ln R_{1} + \delta'_{1}^{\Psi_1}) \right\} \\
-d^{S_{-1}} R_{2}^{S_{\theta}} e^{-S_{\Psi_1}} [v'_{2} \cos (\delta'_{2} \ln d + \delta''_{2} \ln R_{2} + \delta'_{2}^{\Psi_1})] \\
- v''_{2} \sin (\delta'_{2} \ln d + \delta''_{2} \ln R_{2} + \delta'_{2}^{\Psi_1}) \right\} \right\} \\
(9)
\]

\[
R_{2}^{2} = (\cos \theta + \mu' \sin \theta)^{2} + (\mu'' \sin \theta)^{2}, \\
(10)
\]

\[
\Psi_{j} = \begin{cases} 
0 & \text{for } \theta = 0 \\
\arccot((\cos \theta + \mu' \sin \theta) / \mu'' \sin \theta) & \text{for } \theta \in (0, \pi) \\
\arccot((\cos \theta + \mu' \sin \theta) / \mu'' \sin \theta) - \pi & \text{for } \theta \in (-\pi, 0) \\
-\pi & \text{for } \theta = -\pi 
\end{cases} \\
(11)
\]

The distance \( d \) has to be chosen in dependence on the mechanism of a rupture, e.g. as a dimension of a plastic zone or as a size of material grain. The distance \( d \) can also be chosen by means of the theory of critical distances, see [2]. The mean value of the tangential stress is determined in dependence on the polar angle \( \theta \). The potential direction of crack initiation is determined from the maximum of the mean value of tangential stress in both materials

\[
\left( \frac{\partial \sigma_{\theta \theta}}{\partial \theta} \right)_{\theta_0} = 0, \quad \left( \frac{\partial^{2} \sigma_{\theta \theta}}{\partial \theta^{2}} \right)_{\theta_0} < 0. \\
(12)
\]

The crack initiation angle \( \theta_0 \) is independent of the absolute values of GSIFs and depends only on their ratio \( \Gamma_{21} = H_{2} / H_{1} \). Generally, the value of the angle \( \theta_0 \) depends on the averaging distance \( d \). The distance \( d \) has to be chosen depending on the corresponding rupture mechanism and the microstructure of the material.

3.2. Mean value of the strain energy density factor

Similarly, the crack initiation direction can be derived via the mean value of the generalized strain energy density factor (GSED) defined in [11]. The mean value of GSED:

\[
\Sigma_{m}(\theta) = \frac{1}{d} \int_{0}^{d} \Sigma_{m}(r, \theta) dr, \\
(13)
\]

where

\[
\Sigma_{m}(r, \theta) = \frac{1}{4\pi} \left( r^{2S_{-1}} H_{1}^{2} U_{1m} + r^{2S_{-1}} H_{2}^{2} U_{2m} + 2r^{S+2S_{-1}} H_{1} H_{2} U_{12m} \right). \\
(14)
\]
\[ U_{1m} = s_{11m}F_{111m}^2 + s_{22m}F_{221m}^2 + 2s_{12m}F_{112m}F_{221m} + s_{66m}F_{121m}^2, \]
\[ U_{2m} = s_{11m}F_{112m}^2 + s_{22m}F_{222m}^2 + 2s_{12m}F_{112m}F_{222m} + s_{66m}F_{122m}^2, \]
\[ U_{12m} = s_{11m}F_{111m}F_{112m} + s_{22m}F_{221m}F_{222m} + s_{12m}(F_{112m}F_{221m} + F_{111m}F_{222m}), \]
\[ F_{11km} = \sqrt{8\pi} \left[ R_{1m}^{\delta_1} e^{-\delta_1 \psi_{m}} \left[ \left( \mu_{1m}^2 - \mu_{1m}^n \right) (v_{1m}^2 \delta_k^r - v_{1m}^n \delta_k^s) - 2\mu_{1m}^n \mu_{1m}^s \left( v_{1m}^e \delta_k^e + v_{1m}^s \delta_k^s \right) \right] \cos(\Theta_{1km}) \right. \]
\[ + \left( \mu_{1m}^2 - \mu_{1m}^n \right) (v_{1m}^2 \delta_k^r - v_{1m}^n \delta_k^s) - 2\mu_{1m}^n \mu_{1m}^s \left( v_{1m}^e \delta_k^e + v_{1m}^s \delta_k^s \right) \sin(\Theta_{1km}) \right] \]
\[ + R_{2m}^{\delta_1} e^{-\delta_1 \psi_{m}} \left[ \left( \mu_{2m}^2 - \mu_{2m}^n \right) (v_{2m}^2 \delta_k^r - v_{2m}^n \delta_k^s) - 2\mu_{2m}^n \mu_{2m}^s \left( v_{2m}^e \delta_k^e + v_{2m}^s \delta_k^s \right) \cos(\Theta_{2km}) \right. \]
\[ + \left( \mu_{2m}^2 - \mu_{2m}^n \right) (v_{2m}^2 \delta_k^r - v_{2m}^n \delta_k^s) - 2\mu_{2m}^n \mu_{2m}^s \left( v_{2m}^e \delta_k^e + v_{2m}^s \delta_k^s \right) \sin(\Theta_{2km}) \right], \]
\[ F_{22km} = \sqrt{8\pi} \left[ R_{1m}^{\delta_1} e^{-\delta_1 \psi_{m}} \left[ (v_{1m}^2 \delta_k^r - v_{1m}^n \delta_k^s) \cos(\Theta_{1km}) + (v_{1m}^e \delta_k^e - v_{1m}^s \delta_k^s) \sin(\Theta_{1km}) \right] \right. \]
\[ + \left( v_{1m}^2 \delta_k^r - v_{1m}^n \delta_k^s) \cos(\Theta_{2km}) + (v_{1m}^e \delta_k^e - v_{1m}^s \delta_k^s) \sin(\Theta_{2km}) \right] \]
\[ F_{12m} = -\sqrt{8\pi} \left[ R_{1m}^{\delta_1} e^{-\delta_1 \psi_{m}} \left[ \left( \mu_{1m}^2 (v_{1m}^2 \delta_k^r - v_{1m}^n \delta_k^s) - 2\mu_{1m}^n \mu_{1m}^s \left( v_{1m}^e \delta_k^e + v_{1m}^s \delta_k^s \right) \right) \cos(\Theta_{1km}) \right. \]
\[ + \left( v_{1m}^2 \delta_k^r - v_{1m}^n \delta_k^s) - 2\mu_{1m}^n \mu_{1m}^s \left( v_{1m}^e \delta_k^e + v_{1m}^s \delta_k^s \right) \sin(\Theta_{1km}) \right] \]
\[ + R_{2m}^{\delta_1} e^{-\delta_1 \psi_{m}} \left[ \left( \mu_{2m}^2 (v_{2m}^2 \delta_k^r - v_{2m}^n \delta_k^s) - 2\mu_{2m}^n \mu_{2m}^s \left( v_{2m}^e \delta_k^e + v_{2m}^s \delta_k^s \right) \right) \cos(\Theta_{2km}) \right. \]
\[ + \left( v_{2m}^2 \delta_k^r - v_{2m}^n \delta_k^s) - 2\mu_{2m}^n \mu_{2m}^s \left( v_{2m}^e \delta_k^e + v_{2m}^s \delta_k^s \right) \sin(\Theta_{2km}) \right], \]
\[ \Theta_{1km} = \delta_k^r \ln r + \delta_k^s \ln R_{1m} + (\delta_k^r - 1) \psi_{1m}. \]

For the case of real eigenvalues \( \delta_k = \delta_k^r \), the mean value \( \Sigma_m(\theta) \) is expressed as:

\[ \Sigma_m = \frac{H}{4\pi} \left[ \frac{d^{2\delta_1 - 1}}{2\delta_1^2} U_{1m} + \frac{d^{2\delta_1 - 1}}{2\delta_1^2} U_{1m}^2 + 2 \frac{d^{2\delta_1 - 1}}{\delta_1^2 + \delta_1^2} - \Gamma_{1m} \right] \]

Note that in case of generally complex eigenvalues \( \delta_k = \delta_k^r + i\delta_k^s \), numerical integration of (13) is advantageous due to complicated dependences of \( U_{1m}, U_{2m}, \) and \( U_{12m} \) on \( r \).

To find the minimum of \( \Sigma_m(\theta) \) and consequently the crack initiation direction in materials I or II, following conditions have to be determined:

\[ \left( \frac{\partial \Sigma_m}{\partial \theta} \right)_{\delta_0} = 0, \left( \frac{\partial^2 \Sigma_m}{\partial \theta^2} \right)_{\delta_0} > 0. \]
The direction of potential crack initiation determined from the maximum of \( \overline{\sigma}_{\theta M}(\theta) \) or form the minimum of the \( \overline{\tau}_m(\theta) \) can exist in both material I in the interval \((0; \omega_1)\) and material II in the interval \((-\omega_2; 0)\). If there are more than one direction of possible crack initiation, it is necessary to consider all of them. The direction \( \theta_0 \) enters to a stability criterion, and thus it is the necessary step in the reliable assessment of general singular stress concentrations.

4. Threshold values of generalized stress intensity factors

The threshold values of GSIFs used in stability criterion of the orthotropic bi-material notches define the loading conditions above that a fatigue crack is initiated in the tip of the singular stress concentrator. Both the criterion of the mean value of the tangential stress and the criterion of the mean value of generalized strain energy density factor are utilised. In further we assume the cyclic loading with the minimum stress \( \sigma_{\min} = 0 \). Thus the range of the (generalized) stress intensity factor corresponds to the values under maximum loading \((\Delta K = K_{\text{max}}, \Delta H = H_{\text{max}})\). Note that the subscript \((\text{max})\) is then omitted.

4.1. Mean value of the tangential stress

The determination of the threshold value of GSIF based on the average stress calculated across a distance \( d \) from the wedge tip is presented. The value of the average stress \( \overline{\tau}_V \) corresponding to the bi-material notch is calculated for the direction of \( \theta_0 \) and it is compared with the critical stress \( \bar{\tau}_{\theta C} \) corresponding to the crack [12]. Assuming the direction of crack propagation \( \theta_0 = 0 \) in homogeneous material under mode I of loading, for the critical stress \( \bar{\tau}_{\theta C} \) can be obtained

\[
\overline{\tau}_{\theta C} = \frac{2K_{\text{th},m}}{\sqrt{2\pi d}}. \tag{20}
\]

To find the relation between \( H(\tau_{\text{appl}}) \) and \( H_{\text{th}}(M_{\text{th}}) \), let us consider the fact that the ratio of the values \( H_1 \) and \( H_2 \) is constant for a given bi-material configuration and boundary conditions and does not depend on the absolute value of the applied stress \( \tau_{\text{appl}} \) and it holds

\[
\Gamma_{21} = \frac{H_2}{H_1} = \frac{H_{\text{th},b}}{H_{\text{th},h}} \left[ m^{\delta_2-n} \right] \tag{21}
\]

This assumption is justified because when changing the value of the applied stress, only the absolute values of GSIFs change, but their ratio is constant even for the threshold values \( H_{\text{th},b}/H_{\text{th},h} \). Inserting the ratio \( \Gamma_{21} \) and the threshold value \( H_{\text{th},h} \) into the relation (8) we get the critical value of the average tangential stress for a bi-material notch. Following the assumption of the same mechanism of a rupture in both cases (crack and notch) we can compare it with the relations for a crack (20) and obtain an expression for \( H_{\text{th},m} \) value corresponding to material \( m \):

\[
H_{\text{th},m} = \frac{2K_{\text{th},m}}{\overline{\tau}_{\theta M}(\theta_0)} + \frac{\Gamma_{21}F_{\theta 2}(\theta_0)}{\overline{\tau}_{\theta M}(\theta_0)} \tag{22}
\]
4.2. Mean value of the strain energy density factor

The same strategy is applied to the estimation of the threshold value of GSIF via the strain energy density factor. Proceeding from the suggestion of the same mechanism of rupture and consequently the same value of average generalized SEDF corresponding to threshold conditions in the case of a bi-material notch and a crack \( \sum_{m,th} = S_{m,th} \) one can determine the threshold value \( H_{1,th} \). The threshold value of the SEDF of the crack in homogeneous orthotropic media is defined as:

\[
S_{m,th} = \frac{K_{1,th}^2 k_m}{4\pi\mu_{m}^*}.
\]

(23)

where \( k_m \) holds \( k_m = 1 - 2\nu_m^* \) for plane strain and \( k_m = (1 - \nu_m^*) / (1 + \nu_m^*) \) for plane stress,

\[
\mu_{m}^* = \frac{E_{m}^*}{2(1 + \nu_{m}^*)}.
\]

(24)

The elastic constants denoted with the star respect the orthotropic nature of particular material component and they cover the direction of supposed crack initiation \( \theta_0 \). If the \( x \) axis is parallel to the bi-material interface, i.e. the direction \( \theta = 0 \), then:

\[
E_{m}^* = \sqrt{\frac{E_x^2 E_y^2}{E_y^2 - (E_x^2 - E_y^2) \cos^2 \theta_0}}, \quad \nu_{m}^* = \sqrt{\frac{\nu_x^2 \nu_y^2}{\nu_y^2 - (\nu_x^2 - \nu_y^2) \cos^2 \theta_0}}
\]

(25)

Considering the SEDFs for the cases of the bi-material notch and the crack under the threshold condition \( \sum_{m,th} = S_{m,th} \), the threshold value of GSIF is given from comparing relations (18) and (23):

\[
H_{1,th, m} = K_{1,th, m} \sqrt{\frac{k_m}{\mu_{m}^* \left( \frac{d^2 \delta_{1,-1}}{2 \delta_1^2} U_{1m} + \frac{d^2 \delta_{2,-1}}{2 \delta_2^2} \Gamma_2^2 U_{2m} + 2 \frac{d^2 \delta_{1,1}}{\delta_1^2 + \delta_2^2} \Gamma_2 U_{12m} \right)^2}},
\]

(26)

where \( K_{1,th, m} \) is the threshold value of the material \( m \).

4.3. Stability criterion

It is evident that the threshold values \( H_{1,th, m} \), see (22) and (26), depend on the threshold value \( K_{1,th, m} \) that is a material characteristic. The threshold values are gained for the directions \( \theta_0 \) of assumed crack initiation that is in the case of a bi-material notch ascertained from the criterion of maximum tangential stress or minimum strain energy density factor, see chapter 3. Because of that fact the normal loading mode is predominant. Thus the comparison with crack propagation characterized by the threshold value \( K_{th} \) is justified.

Finally the stability criterion can be suggested in the following form:

\[
H_{1}(\sigma_{app}) < H_{1,th}(K_{th})
\]

(27)

The fatigue crack is not initiated in the tip of a bi-material notch if the value \( H_1 \) of GSIF is lower than its threshold value \( H_{1,th} \). The value \( H_1 \) is determined from a numerical solution of a given bi-material configuration with the given boundary conditions, geometry, and material properties of the bi-material wedge \([8],[13],[14]\). The threshold value \( H_{1,th} \) is given by the relation (22) or (26).
5. Conclusion

The procedure for the determination of conditions of fatigue crack initiation from an orthotropic bi-material notch based on the knowledge of the stress distribution has been presented. Two approaches utilizing the mean value of the tangential stress and the mean value of the generalized strain energy density factor has been shown. The expression for the distribution of the controlling magnitudes is derived as a function of the generalized stress intensity factors $H_1$ and $H_2$.

It is concluded that for the estimation of the crack initiation direction $T_0$ both existing stress singular terms have to be taken into account, and $T_0 = T_0 (H_2/H_1)$. The determination of the initial crack propagation angle is one of the necessary steps for service-life evaluation of constructions containing compound materials. The procedure makes it possible to assess into which material component and in which direction the initiated crack will propagate. Consequently, the crack initiation conditions are estimated via comparison of GSIF $H_1$ and its threshold value $H_{1\text{th}}$. The threshold values $H_{1\text{th}}$ depend on the values of $K_{1\text{th}}$ which is a common material characteristic. The advantage of this approach is that no new material property needs to be measured. The presented methods of determination of the conditions for fatigue crack initiation can be used to increase the reliability of service-life estimation of structures made of composite materials.

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