SCHWINGER-DYSON EQUATIONS FOR FERMIIONS SELF-ENERGY IN THE ELECTROWEAK-SCALE RIGHT-HANDED NEUTRINO MODEL

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Abstract: The dynamical formation of condensates is studied in the framework of Schwinger-Dyson approach. Two types of condensates are considered here: that which is produced by the exchange of the fundamental Higgs double \( \Phi_{2m} \) between two mirror quarks and the other which is produced by the exchange of the fundamental Higgs triplet \( \tilde{\chi} \) between two right-handed neutrinos. The conditions for which fermion bilinear condensates can get formed were analytically found. Mirror quark and right-handed neutrino condensates can be determined through solutions of Schwinger-Dyson equations. Vacuum expectation values of the aforementioned fundamental Higgs are related to the condensate scale. In addition, the threshold for the new strong interactions is found with no fine-tuned requirement.

Keywords: Condensate, Schwinger-Dyson equation, Electroweak Symmetry Breaking, EW-scale \( \nu_R \)

1 Introduction

The dark matter and neutrino oscillation data [1] are the two typical experiments demonstrating Standard Model (SM) to be an incomplete theory. The underlying question of why neutrinos have (tiny) masses is one of the motivations for the physics beyond the SM.

The most popular mechanism to explain these tiny neutrino masses is see-saw mechanism [2] in which sterile right-handed (RH) neutrinos are SM singlets. However, one cannot directly probe a heavy RH singlet neutrino in any laboratory experiments. Can RH neutrino be non-sterile and interact directly with weak bosons? Ref. [3] proposed such a model called the EW-scale \( \nu_R \) model in which RH neutrinos are non-sterile and detectable because their masses are proportional to the electroweak (EW) scale \( \Lambda_{EW} \sim 246 \text{ GeV} \). Regarding the EW precision test and the Large Hadron Collider (LHC) discovery of the 125-GeV SM-like Higgs boson, the EW-scale \( \nu_R \) model is remarkably viable [4, 5]. More detailed discussion of this model can be found in [6-11].

In this work, we investigate solutions to the Schwinger-Dyson (SD) equations for RH neutrinos and mirror quarks self-energy. This study has important consequences for Dynamical Electroweak Symmetry Breaking (DEWSB) in the EW-scale \( \nu_R \) model which detailed discussion is in progress. The SM Higgs mechanism can be stated as theoretically unsatisfactory that remains the fundamental question of what is the underlying physics responsible for Electroweak Symmetry Breaking (EWSB). There have been many interesting models proposed to deal with this problem such as Supersymmetry, Technicolor, Top-condensates, Little Higgs, etc. [12]. Among these, there is a class of composite Higgs models in which a fermion-

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antifermion bound state would be regarded as the agent of DEWSB. The main idea of this direction is that a heavy fermion would participate in the condensates. The top quark, the heaviest fermion in the SM could not be the available candidate for this scenario because the resulting dynamical top mass required to be heavier than $m_t \sim 171 \text{GeV}$. Moreover, top-condensate models provide a fine-tuned picture of DEWSB since there are many orders of magnitudes between the EW scale $\Lambda_{EW}$ and the momentum cutoff for the new strong interactions, i.e. $O(10^{16} \text{GeV})$. This leads to the construction of several interesting generalizations of the top-condensate model which attempt to avoid these obstacles [13-21].

Ref. [19] proposed a model in which condensation of a heavy fourth generation fermion was assumed to drive EWSB. As in [19], the SD equation for the fermion self-energy was studied. It was shown that, above the critical values $\alpha_Y > \alpha_Y^c = \frac{\pi}{2}$, the solutions to the aforementioned equation have the form of bound states. Condensate of the form $\langle \bar{t}'_L t'_R \rangle$ can be written in terms of the $t'$ self-energy. This framework would avoid the fine-tuned scenario since the physical cutoff scale is of the order of $O(\text{TeV})$. DEWSB in the EW-scale $\nu_R$ model is based on this scenario. It goes without saying that the study of the SD equations for fermions in the EW-scale $\nu_R$ model plays a crucial role in studying DEWSB.

The organization of this paper will be as follows. First, we give a brief review of the EW-scale $\nu_R$ model. Next, in section 3, the SD equations for RH neutrinos and mirror quarks self-energy will be written. In section 3, we will first find solutions to the SD equations and the critical values of Yukawa couplings of RH neutrinos and mirror quarks for which the condensate states occur. We will then determine the condensate scale. The physical cutoff scale will then be defined. We conclude in section 4.

## 2 Review of EW-scale $\nu_R$ model

The essential elements of the model presented in [3] and its extended version [5] can be reviewed as below.

- The Gauge group 
  \[ SU(3)_C \times SU(2)_W \times U(1)_Y. \]

- The $SU(2)_L \times U(1)_Y$ fermion content
  - $SU(2)_L$ lepton doublets 
    \[ \text{SM: } l_L = (\nu_L, e_L) ; \text{Mirror: } l'_R^M = (\nu'_R^M, e'_R^M), \]
    for the SM left-handed lepton doublet and for the RH mirror lepton doublet, respectively.
  - $SU(2)_L$ lepton singlets 
    \[ \text{SM: } e_R ; \text{Mirror: } e'_L^M, \]
    for the SM RH lepton singlet left-handed mirror lepton singlet, respectively.

Similarly, for the quarks, we have
- **SU(2)$_L$ quark doublets**

  \[
  \text{SM: } q_L = (u_L, d_L); \text{Mirror: } q_R^M = (u_R^M, d_R^M),
  \]

  for the SM left-handed quark doublet and for the RH mirror quark doublet

- **SU(2)$_L$ quark singlets**

  \[
  \text{SM: } u_R, d_R; \text{Mirror: } u_L^M, d_L^M,
  \]

  for the SM RH lepton singlet left-handed mirror lepton singlet, respectively.

- **Higgs content**

  - **Higgs doublets**

    \[
    \Phi_2 = \left( \phi_2^+, \phi_2^0 \right) = \left( 1, \frac{Y}{2} = \frac{1}{2} \right),
    \]

    \[
    \Phi_{2M} = \left( \phi_{2M}^+, \phi_{2M}^0 \right) = \left( 1, \frac{Y}{2} = \frac{1}{2} \right).
    \]

  - **Higgs triplets**

    \[
    \bar{\chi} = \frac{1}{\sqrt{2}} \vec{\tau} \cdot \vec{\chi} = \begin{pmatrix}
    \chi^+ \\
    \chi^0 \\
    -\frac{1}{\sqrt{2}} \chi^+
  \end{pmatrix} = \left( 1, \frac{Y}{2} = 1 \right),
    \]

    \[
    \xi = \begin{pmatrix} \xi^+, \\
    \xi^0 \\
    \xi^-
  \end{pmatrix} = \left( 1, \frac{Y}{2} = 0 \right),
    \]

    \[
    \chi = \begin{pmatrix}
    \chi^0 \\
    \chi^- \\
    \xi^0 \\
    \chi^+ \\
    \xi^- \\
    \chi^0^*
  \end{pmatrix}.
    \]

  - **Higgs singlet**

    \[
    \phi_S = \left( 1, \frac{Y}{2} = 0 \right).
    \]

Higgs triplets such as in Eq. (4) were considered earlier in various contexts [22].

- **The Yukawa interactions**

In the case of SM fermions, we have the usual Yukawa interactions

\[
\mathcal{L}_{YSM} = -g_{SM} \bar{\psi}_L \phi \psi_R + \text{h.c.},
\]

For mirror fermions, we consider the terms

\[
\mathcal{L}_{e^M} = -g_{e^M} \bar{\psi}_L^M \Phi_2 e^L_R + \text{h.c.},
\]

\[
\mathcal{L}_d^M = -g_{d^M} \bar{q}_R^M \Phi_{2M} q^M_L - g_{d^M} \bar{q}_R^M \Phi_{2M} q^M_L + \text{h.c.},
\]

\[
\mathcal{L}_v_R = g_{v^M} \sigma_2 \tau_2 \tilde{\chi}_R^M,
\]

\[
\mathcal{L}_{SI} = -g_{SI} l^M_R \phi_S + \text{h.c.},
\]
\[ \mathcal{L}_{Sq} = -g_{Sq} q^M_R q_L \phi_S - g_{Sq} q^M_R \phi_S^* + \text{h.c.} \]  

(12)

One obtains Majorana and Dirac masses for the RH neutrinos through Eqs. (10) and (11), respectively. Since the top quark could not provide a viable picture of condensate formation, then we do not study the SD equations for SM fermions self-energy. Besides, from recent studies of \( \mu \to e\gamma \) [1], \( g_{Sl} \) is constrained to be less than \( 10^{-3} \) and if we assume \( g_{Sl} \approx g_{Sq} \approx g_{Sq}' \), then Eqs. (11) and (12) will not be considered in the framework of SD approach.

3 The SD approach to condensate formation in the Higgs-Yukawa system

We discuss in this section the conditions for which the formation of condensate states occurs. As stressed in [19], if the Yukawa couplings are sufficiently large, one obtains a heavy fourth-generation fermion condensate. This state will then dynamically break the EW symmetry. We will derive the critical Yukawa couplings in the EW-scale \( \nu_R \) model based on what presented in [19].

As mentioned in [3, 5], the vacuum expectation values (VEVs) of both Higgs triplets and doublets are responsible for the full EWSB. Mirror quarks and RH neutrino condensates will be considered in the DEWSB scenario. In particular, the mirror quarks condense through the exchange of the fundamental Higgs doublet \( \Phi_2 \) and the RH neutrinos condense through the exchange of the fundamental Higgs triplet \( \tilde{\chi}_0 \). In the following analysis, we then have made the following assumption

- In our discussion of condensate formation, the fundamental Higgs fields are assumed to be massless. These fields then have no VEV at tree level.
- To preserve custodial \( SU(2)_D \) symmetry which will be discussed in the framework of DEWSB, non-zero expectation values for quark bilinear in the EW-scale \( \nu_R \) model are assumed to satisfy the conditions: \( \langle \bar{U} L^M R \rangle = \langle \bar{D} D^M R \rangle \). This condition is satisfied by assuming \( g_u^M = g_d^M = g_q^M \).
- Yukawa couplings of mirror leptons would not be sufficiently large to form condensates.

3.1 The solutions to the SD equations for RH neutrinos and mirror quarks self-energy

We begin by considering the case of Higgs triplet. The Yukawa Lagrangian given in Eq. 9 can be written as

\[ \mathcal{L}_{\nu_R} = g_{M1} \sigma_2^M \sigma_2 \tilde{\chi}_0 \nu_R + g_{M2} \sigma_2 \tilde{\chi}_1 \nu_R + g_{M3} \sigma_2 \tilde{\chi}_2 \nu_R \]  

(13)

The SD equation for the \( \nu_R \) self-energy with Feynman diagram shown in Fig. 1a is given by the third term of Eq. (13)
Fig. 1. Graph contributing to the right-hand side of the SD equation for the (a) RH neutrino self-energy $\Sigma_{\nu_R}$, (b) mirror quark self-energy $\Sigma_q$

$$\Sigma_{\nu_R}(p) = \frac{g^2}{(2\pi)^4} \int d^4q \frac{1}{(p - q)^2} \frac{\Sigma_{\nu_R}(q)}{q^2 + \Sigma_{\nu_R}^2(q)}.$$  \hspace{1cm} (14)

By converting Eq. (14) to a differential equation, and setting $\alpha_{\nu_R} = \frac{g}{(2\pi)^4}$, one obtains

$$\Sigma_{\nu_R}(p) = -\left(\frac{\alpha_{\nu_R}}{\alpha_{\nu_R}^c}\right) \frac{\Sigma_{\nu_R}(p)}{p^2 + \Sigma_{\nu_R}^2(p)}.$$  \hspace{1cm} (15)

where the critical coupling $\alpha_{\nu_R}^c$ is

$$\alpha_{\nu_R}^c = \pi,$$  \hspace{1cm} (16)

with the following boundary conditions

$$\lim_{p \to 0} p^4 \frac{d\Sigma_{\nu_R}(p)}{dp^2} = 0,$$  \hspace{1cm} (17a)

$$\lim_{p \to \Lambda} p^2 \frac{d\Sigma_{\nu_R}(p)}{dp^2} + \Sigma_{\nu_R}(p) = 0.$$  \hspace{1cm} (17b)

If one introduces a variable $t$ and a function $u(t)$ by

$$p = e^t,$$  \hspace{1cm} (18)

$$\Sigma_{\nu_R}(p) = e^t u(t - t_0),$$  \hspace{1cm} (19)

then Eq. (15) becomes

$$\frac{d^2u}{dt^2} + 4 \frac{du}{dt} + 3u + \frac{\alpha_{\nu_R}}{\alpha_{\nu_R}^c} \frac{u}{1 + u^2} = 0.$$  \hspace{1cm} (20)

The boundary conditions turn into

$$\lim_{t \to t_0^+} (u + 3u) = 0,$$  \hspace{1cm} (21a)

$$\lim_{t \to \infty} (u + u) = 0.$$  \hspace{1cm} (21b)
with \( t_\Lambda = \log \Lambda \) and \( \Lambda \) being the momentum cutoff. Here the parameter \( t_0 \) is defined as \( t_0 = \ln \Sigma(0) \) when the normalization is taken to be \( e^t u(t) \to 1 \) as \( t \to -\infty \).

As the momentum is adequately large, this equation is worthy of

\[
\frac{d^2 u}{dt^2} + 4 \frac{du}{dt} + \left[ 3 + \left( \frac{\alpha_{v_R}}{\alpha_{\tilde{v}_R}} \right) \right] u \approx 0,
\]

(22)

In addition, \( \alpha_{v_R} = \alpha_{\tilde{v}_R}^c \) is called a critical point since in Ref. [23] there are two classes of asymptotic solutions at large momentum for different values of \( \alpha \)

\[
\Sigma_{v_R}(p) \sim p^{-1+\sqrt{1 - \frac{\alpha_{v_R}}{\alpha_{\tilde{v}_R}}}}, \quad \alpha_{v_R} \leq \alpha_{\tilde{v}_R}^c,
\]

(23)

\[
\Sigma_{v_R}(p) \sim p^{-1} \sin \left( \sqrt{\frac{\alpha_{v_R}}{\alpha_{\tilde{v}_R}^c}} - 1 \ln p + \delta \right),
\quad \alpha_{v_R} > \alpha_{\tilde{v}_R}^c,
\]

(24)

with \( \delta \) being a constant phase. In the case of strong Yukawa coupling solution which satisfies the boundary conditions given in Eq. 21, one obtains

\[
\Sigma_{v_R}(0) = \Lambda e^{1-n\pi/\sqrt{\alpha_{\tilde{v}_R} - 1 - \delta}}, \quad n = 1, 2, \ldots
\]

(25)

It turns out that the solution with \( n = 1 \) yields the largest fermion self-energy and hence corresponds to the vacuum. Eq. 25 can be rewritten for \( n = 1 \) as follows

\[
\delta_0 = \ln \left( \frac{\Lambda}{\Sigma_{v_R}(0)} \right) + \frac{\pi}{\sqrt{\alpha_{\tilde{v}_R} - 1}} - 1,
\]

(26)

where \( \delta_0 \) denotes the phase corresponding to the vacuum solution.

A similar task holds for Higgs doublet \( \Phi_{2M} \). SD equations for mirror quark self-energy with relevant Feynman diagrams drawn in Fig. 1b read

\[
\Sigma_{q^M}(p) = 2 \times \frac{g_{q^M}^2}{(2\pi)^4} \int d^4q \frac{1}{(p-q)^2} \frac{\Sigma_{q^M}(q)}{q^2 + \Sigma_{q^M}(q)}.
\]

(27)

Together with the assumption that \( \Sigma_{q^M}(p) = \Sigma_{d^M}(p) = \Sigma_{u^M}(p) \), one finds the critical coupling for mirror quark

\[
\alpha_{q^M}^c = \frac{\pi}{2}.
\]

(28)

Above this value, the solution to the SD equation for mirror quark also has the form of bound states.

### 3.2 The condensate scale

The condensate of mirror quark can be written in terms of its self-energy, in particular,
\[ \langle q_L^M q_R^M \rangle = -\frac{3}{4\pi^2} \int d^4q \frac{\Sigma_{q^M}(q)}{q^2 + \Sigma_{q^M}^2(q)} \]
\[ \approx -\frac{3}{\pi^2} \left( \frac{\alpha_{q^M}^c}{\alpha_{q^M}} \right) \Lambda \Sigma_{q^M}^2(0) \sin \left[ \sqrt{\frac{\alpha_{q^M}}{\alpha_{q^M}^c}} - 1 \right]. \]  

(29)

Likewise, the condensate state of RH neutrino would be
\[ \langle \nu_R^T \sigma_2 \nu_R \rangle \approx -\frac{1}{\pi^2} \left( \frac{\alpha_{\nu_R}^c}{\alpha_{\nu_R}} \right) \Lambda \Sigma_{\nu_R}^2(0) \sin \left[ \sqrt{\frac{\alpha_{\nu_R}}{\alpha_{\nu_R}^c}} - 1 \right]. \]

(30)

Since condensates given in Eqs. (29) and (30) carry the EW quantum number \((SU(2)_L\) doublet), they will spontaneously break the EW symmetry. Then these condensates are directly related to \(\nu_\chi\) and \(\nu_{\Phi_{2M}}\), namely
\[ \langle q_L^M q_R^M \rangle \sim O(-\nu_{\Phi_{2M}}^3), \]
\[ \langle \nu_R^T \sigma_2 \nu_R \rangle \sim O(-\nu_{\chi}^3), \]

(31)

(32)

where it will be seen later that \(\nu_\chi\) and \(\nu_{\Phi_{2M}}\) are the VEVs of \(\chi\) and \(\Phi_{2M}, 24-26\), respectively. The two quantities given in Eqs. (31) and (32) will be the agents for the mass generation of fundamental Higgs, and in consequence dynamically drive the EWSB in the EW-scale \(\nu_R\) model.

### 3.3 The physical cutoff scale

There are several ways to estimate at what scale new strong interactions have to set in, using unitarity [24-26]. These references have shown that the scale of new physics which breaks the EW symmetry is approximately of the order of \(O(\text{TeV})\). It is quite motivating, that the symmetry breaking mechanism can be probed with collider machines, such as the LHC or the proposed International Linear Collider (ILC) which can reach the TeV scale. We will discuss in this subsection whether the momentum cutoff \(\Lambda\) in the condensate scenario satisfies what have been demonstrated in Refs. [24-26].

As energy increases, the Yukawa couplings \(\alpha_{\nu_R}, \alpha_{q^M}\) become efficiently large, reaching the critical values, \(\alpha_{\nu_R}^c = \pi, \alpha_{q^M}^c = \frac{\pi}{2}\), then
\[ \sin \left[ \sqrt{\frac{\alpha_{\nu_R}}{\alpha_{\nu_R}^c}} - 1 \right] \sim \sqrt{\frac{\alpha_{\nu_R}}{\alpha_{\nu_R}^c}} - 1. \]  

(33)

In order to have the relations found in Eqs. (31) and (32), one must have the constraints
\[ \frac{\alpha_{\nu_R}}{\alpha_{\nu_R}^c} - 1 \sim \Lambda_{EW}, \]
\[ \frac{\alpha_{q^M}}{\alpha_{q^M}^c} - 1 \sim \Lambda_{EW}. \]

(34)

(35)
If the momentum cutoff $\Lambda \sim 10^{16}\text{GeV}$, then $\frac{\alpha_{\nu R}}{\alpha_{qM}} \approx \frac{\alpha_{qM}}{\alpha_{qM}} \approx 1 + 10^{-28}$. This gives rise to the fine-tuned case. If the physical cutoff $\Lambda \sim O(\text{TeV})$, there is no fine-tuned requirement. One easily imagines that the EW-scale $\nu_R$ model would provide the TeV-scale DEWSB scenario.

4 Conclusion

As with the 4th-generation scenario of [19], we have presented in this manuscript the properties of condensate states of RH neutrinos and mirror quarks in the model of EW-scale $\nu_R$. The solutions to the SD equations for the aforementioned fermions self-energy were analytically found. It was shown that when the Yukawa couplings exceed the critical values found in Sec. 3.1, in particular, $\alpha_{\nu R} = \pi, \alpha_{qM} = \frac{\pi}{2}$, the RH neutrinos and mirror quarks condensate states occur through the exchange of the fundamental Higgs triplet $\chi$ and fundamental Higgs doublet $\Phi_{2M}$, respectively. Condensates of the forms $\langle \nu_R^T \sigma_2 \nu_R \rangle$ and $\langle \bar{q}_L \Phi_2 M q_R M \rangle$ are written in terms of the $\nu_R$ and $q_M$ self-energy and their scales are related to the VEVs of $\chi$ and $\Phi_{2M}$. Moreover, section 3.3 stated that there is no such fine tuning if the momentum cutoff is of the order of $O(\text{TeV})$. It is not too unreasonable to speculate that the TeV-scale DEWSB scenario in the case of the EW-scale $\nu_R$ model would be the natural outcomes of the study of condensates. This issue will be discussed in not-too-distant future.

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