Reconnoitering the class distinguishing abilities of the features, 
to know them better

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Abstract

The relevance of machine learning (ML) in our daily lives is closely intertwined with its explainability. Explainability can allow end-users to have a transparent and humane reckoning of a ML scheme’s capability and utility. It will also foster the user’s confidence in the automated decisions of a system. Explaining the variables or features to explain a model’s decision is a need of the present times. We could not really find any work, which explains the features on the basis of their class-distinguishing abilities (specially when the real world data are mostly of multi-class nature). In any given dataset, a feature is not equally good at making distinctions between the different possible categorizations (or classes) of the data points. In this work, we explain the features on the basis of their class or category-distinguishing capabilities. We particularly estimate the class-distinguishing capabilities (scores) of the variables for pair-wise class combinations. We validate the explainability given by our scheme empirically on several real-world, multi-class datasets. We further utilize the class-distinguishing scores in a latent feature context and propose a novel decision making protocol. Another novelty of this work lies with a refuse to render decision option when the latent variable (of the test point) has a high class-distinguishing potential for the likely classes.

Keywords: feature importance, explainability, class distinguishing ability, decision to refuse or render

1. Crux of the work—A toy example

We have images of hand-written digits. Given that we have 10 digits, we define 10 classes or categories. We assign each digit to a unique class – class 0 corresponds to the images with hand-written 0, class 1 corresponds to the images with hand-written 1. We assume that, in an image we will find only one hand-written digit. The task is to classify each image to its respective class. For example, an image with a hand-written 1 should be classified to class 1 and an image with a hand-written digit 2 should be classified to 2. Each image in a sub-figure has four quadrants (top-left, top-right, bottom-left and bottom-right). Each quadrant in an image represents a feature. Now, to better understand the problem being addressed, consider Figure 1. There are six sub-figures in Figure 1, each of which shows 2 images of hand-written digits which we want to distinguish. In subfigures (a) and (d), the full images are visible. In subfigures (b) and (e), we conceal the upper-right quadrants of images in (a) and (d) respectively. In subfigures (c) and (f), we conceal the bottom-left quadrants of images (a) and (d) respectively.

In Case 1, we have to make a choice between 7 and 1. Sub-figure (b) shows that top right quadrant plays a decisive role in making that distinction. Sub-figure (b) shows that the absence of top-right quadrant information makes 7 and 1 indistinguishable to a large extent. If, for a given image categorization, the task is to decide between 1 and 7 and the information from the top-right quadrant is unavailable, the judicious choice would be to refuse to predict a category. On the contrary, a missing information from the bottom right quadrant (subfigure (c)) is not bothersome and does not pose any hindrance in distinguishing 1 and 7.

In Case 2 (subfigures (d)-(f)), we have to make a distinction between 5 and 6. An analysis similar to Case 1 shows that the missing information from the bottom left quadrant (subfigure (f)) is bothersome here.

Case 1 and Case 2 reveal that the information from the top-right quadrant is instrumental in distinguishing 1 and 7, but is not as effective in distinguishing 5 and 6. On the contrary, the feature from the bottom-left quadrant is instrumental in distinguishing 5 and 6, but not very effective in discriminating between 1 and 7. The crux is – the efficaciousness of a feature in a classification task is dependent on the classes which it has
to distinguish. To address this aspect, we need an explainability and quantification (or an explainable quantification) of the class-distinguishing abilities of the features. The basic objective of our work is to obtain the explainability of the features in terms of their class distinguishing capabilities.

2. Introduction

Nowadays, machine learning is an inevitability in our daily life – and its relevance is increasing with each passing day [1–3]. A machine learning model usually learns from prior or available data, generalizes them to learn a model and uses it to make predictions and categorization of an unknown instance [4]. In particular, the research community has been instrumental in tailoring the ML models and real-world datasets for one another to enable the extraction of meaningful information from the data [5–7]. Sincere efforts led to the digital revolution and ML models have become indispensable in various spheres of our lives from medical applications [8–10], construction purposes [11], investigation of automobile crashes [12], finance sector [13] and analyzing student attrition [14].

A key area of application of ML is the automated decision making or decision support systems. To increase the trust and faith of the users in automated decision making, we need to work on the explainability of the decisions and interpretations of the models. The impact of the wrong decisions by a model is a stakeholder in this application. As the automated decision making extends to sensitive domains like clinical decision support systems and finance-banking sectors the wrong decisions of ML model will be critical. Consequently, the velocity of a model’s decision plays a key role in its acceptance. In such a scenario, a human reckoning or an interpretability of the decision making systems is highly desirable to maintain and boost the trust on the systems.

This gave rise to a new need — to study the rationale behind the working of the ML models and explainable machine learning has come up as the call for the day. In general, ML models are perceived as black boxes – where we input some unseen data (that we want to predict), learning goes on inside the opaque black box and we get an output which is the prediction or categorization of the input data [15–17]. To build the credibility and confidence of the end users on the ML models, we will need an interpretable framework where we will have reduced opacity and can explain the causation of the decisions rendered by the models.

One way to understand an ML model is through the features on which it is trained – how each feature is influencing the decision or categorization of the input data [16, 17]. To address this aspect, we need an explainability and quantification (or an explainable quantification) of the feature in distinguishing between class 1 and class 3, we would like to proceed with the contrary, if feature is not that instrumental in distinguishing between class 1 and class 3, we would probably like to deter ourselves from rendering a decision. On the contrary, if feature a not that instrumental in distinguishing between class 1 and class 3, we would like to proceed with the prediction. We incorporate this line of thought into our scheme. We have carried out the experimental study on 5 real-world datasets. The empirical findings indicate that our scheme’s decision on withholding or rendering a decision is mostly right to a considerable extent. This indicates that we require some more intuition into the explainabilities of the features. A more detailed analysis of the features can be more bearing in this regard.

In this work, we present a novel and intuitive protocol of explaining the features through their class-distinguishing abilities.

To the best of our knowledge, we could not find any extant work which has followed this line of thought. We are particularly interested to study the role or capabilities of each feature in discriminating between the different classes for a dataset. It is because – the distinguishing capabilities of the features will influence the distinguishing capability of the model. Hence, if we can explain and estimate the distinguishing capabilities of the features, we will be able to explain the decisions of the models. For each feature, we inspect its distinguishing capability for all possible pair-wise class combinations. For c classes, there can be C2 pairwise classes and we explore and quantify C2 class combinations. Technically, we employ a latent feature framework to obtain the class-distinguishing scores of the features. For example, let us have 4 features a, b, c and d and 3 classes 1, 2 and 3. We are looking for the following answers — How effective is feature a in distinguishing between class 1 and class 3? Is it equally effective in distinguishing between class 1 and class 2 also? We look for similar answers for all features. Once we have the answers, we have explainability on the class-distinguishing capacities of the features. In this paper, we work towards a obtaining a quantitative estimate of the class distinguishing abilities of the features. In our scheme, it is possible to compute the gross importance of each individual feature through their class-distinguishing scores. We may emphasize here that the focus of this work is beyond the gross importance computation and we want to delve into their class-distinguishing aspects.

After addressing the research aspect stated above, we explore another perspective which we believe is intertwined with the one that we have already talked about. The main objective of the second research problem is – validating the correctness of the explainable class distinguishing scores (which we obtain from the first exploration) through a classification task. We use the same framework (operating under latent or missing feature constraint) to design a novel classifier model. The novelty of this classifier is – it will render a decision on a test instance only when it is confident of doing so. It will refuse to render a decision when it is not confident enough. The framework is explained briefly as follows. Let feature a be absent (latent feature) for some test point q. We obtain (have the provision in our framework) an intermediate output from the model which tells us that q either belongs to class 1 or to class 3. The question is – Would we like to classify it? If feature a is instrumental in distinguishing between class 1 and class 3, we would probably like to deter ourselves from rendering a decision. On the contrary, if feature a not that instrumental in distinguishing between class 1 and class 3, we would like to proceed with the prediction.

We have carried out the experimental study on 5 real-world datasets. The empirical findings indicate that our scheme’s decision on withholding or rendering a decision is mostly right.
across all datasets. The difference in accuracies of the two sce-
arios rendering a prediction and withholding the predictions
is a significant figure for all five datasets – the accuracy has
been much more in the case where the model has rendered a
decision than that of the cases where the model withheld the
decision. This in turn validates the explainability of the class-
distinguishing capabilities of the features. We also present case
studies on two datasets – maternal health and contraceptive.
We semantically analyze the features of these datasets and cor-
relate them with our quantitative finding.

The rest of this article is arranged in the following manner.
In Section 3, we describe the extant works in this domain.
We elaborate on the key aspect of this work, class-discernible
scores in the Section 4 followed by a detailed presentation of
the Proposed Approach in Section 5. We describe the Empiri-
cal Setup in Section 6 and discuss the Results and Analysis in
Section 7. We wrap up this article with Conclusion and scope
of Future Work in Section 8.

3. Related Works

The issue which is being addressed in this work is the inter-
pretation of the features in terms of their class distinguishing
capabilities. As mentioned earlier, we could not really find any
work which addresses this matter but a number of researches
have been carried out in feature explainability and interpretabil-
ity in general. We present an account of the prior works in ex-
plainability in machine learning followed by a focus on feature
explainability in the later part.

The inevitability of ML has lead to the requirement of its
explanations which can be understood by the humans. Conse-
quently, explaining the decisions of algorithms which can influ-
ence human lives has come up as the call for the day [25, 26].
Recent researches in diverse domain like property-value pre-
diction, breast cancer profiling [8], fake news detection [27],
glass transition temperature prediction [28], point cloud clas-
sification [29] has particularly focused on explainability of an
algorithm’s decision besides solving the problem. In [30], an
interactive machine learning framework is adopted to cater to
the need for explainability. From a ML perspective, feature ex-
plainability is usually of one of the two types – i) local, or ii) global.
A local explanation deals with the rationale of a model’s
decision on some particular input. On the other hand, a global
explanation usually renders the rationale behind the decision
through a summarization, which is usually independent of any
individual input [31]. A key work in the domain of local ex-
planation is LIME [20] and in the domain of global explanation is
SHAP [21] – these works deal with the importance or con-
tributions of the variables while rendering a decision. Other
notable works are [32]. These methods are model-agnostic and
is not specific to any classifier model. Apart from these, model-
specific research has also been catered to – a deep taylor based
decomposition for neural networks and tree interpreter for ran-
dom forests [33], understanding deep neural network through
prototype and typicality [34]. In [35, 36], explainable classi-
fiers are proposed which is the need of the present times. The
nature of explainability of ML models and the features across
domains. In recent years, a number of works on feature explain-
ability have focused on specific domains like loan underwriting
[15] and business model design and sustainability [37]. Ex-
plainability of features is particularly essential in critical areas
like decision support systems in clinical domain. A few notable
works in this area are – diabetes mellitus prediction [9], non-
communicable disease prediction [38], clinical decision support
system in medical imaging [10], renal mass calcification [39].
[40] has worked towards reducing the opacity of the features
and obtained interpretable features with the assistance from the
prediction of the black-box models.

In this work, we learn the class-distinguishing abilities of
the features through a framework of imputed features. The
paradigm of imputed features has existed in the machine learn-
ing domain for long and it has been perceived as a constraint
in the usual learning [41, 42]. This learning paradigm has also
been used for in some works which deal with explainability.
Some notable ones are on explainable anatomical shape analy-
sis [43], explainable prediction of electrical energy demand
[44] and explainable recommendations [45]. In this work, we
use the paradigm of latent variable or missing feature in a novel
way – to learn the class-distinguishing capabilities of the fea-
tures.

4. Class-distinguishing scores

We are talking about classifiers in this work. We are going
to analyse the class-distinguishing capabilities of the classifier
models. The motivation of this work is – a classifier model
may not be equally good at distinguishing the pairs of possible
classes in a dataset. On a micro-scale, we will analyze the
class-distinguishing capabilities of the features individually.
For example, let there be three possible classes of cat, fish and
boat in a dataset. We may find that the classifier is effective
towards distinguishing between boat and cat but not as good
while distinguishing cat and fish. Instead of providing the ex-
plainability of a model’s decision as a function of the feature
aggregates, we will try to explain and quantitatively capture
the class-distinguishing capabilities of individual features. It
is because – features are the building blocks of a classification
model.

4.1. Classifiers used in this work

We assume the number of features in our data to be γ. We
will require (γ + 1) classifiers in our scheme. We will have
a dedicated classifier for each feature, γ classifiers accounting
from that. The remaining one classifier will be a generic one,
learnt from all the features. Let us denote the classifiers as
M_0, M_{−1}, M_{−2}, . . . , M_{−γ}. We use all γ features to model clas-
sifier M_0. Classifiers M_{−a}, (0 < a ≤ γ) is modelled by imput-
ing or removing feature a. That is, in classifier M_{−a}, a is the
latent feature. From the given dataset, we remove or impute
feature a, and use the remaining (γ − 1) features to train model
M_{−a}, (0 < a ≤ γ).
4.2. Obtaining the class-distinguishing scores

Let us assume that there are \( c \) classes. We will obtain a class-distinguishing score for each pair of classes \( \alpha \) and \( \beta \), \( \alpha \neq \beta, 1 \leq \alpha, \beta \leq c \). Let \( CS_{a}^{\alpha,\beta} \) be the class-distinguishing score of feature \( a \) with respect to classes \( \alpha \) and \( \beta \). We would like to compute the scores from the classification output of \( n \) instances. We denote class, to be the true class of instance \( i \) and \( \text{pred}_{i}^{a} \) denotes the prediction of instance \( i \) obtained from model \( M_{\cdot \cdot a} \).

For each classifier \( M_{\cdot \cdot a}, 0 \leq a \leq \gamma \) and a pair of classes \( \alpha \) and \( \beta \), we will first obtain \( T_{a}^{\alpha,\beta} \), it denotes that number of instances correctly classified to class \( \alpha \) by classifier \( M_{\cdot \cdot a} \). Similarly, \( T_{\beta}^{\alpha,\beta} \) or \( T_{\alpha}^{\beta,\alpha} \) denotes the number of instances correctly classified to class \( \beta \) by classifier \( M_{\cdot \cdot a} \) respectively. \( F_{a}^{\alpha,\beta} \) denotes the number of instances which belong to class \( \alpha \) but has wrongly been classified to class \( \beta \) by classifier \( M_{\cdot \cdot a} \). Similarly, \( F_{\beta}^{\alpha,\beta} \) denotes the number of instances which belong to class \( \beta \) but has wrongly been classified to class \( \alpha \) by classifier \( M_{\cdot \cdot a} \).

\[
T_{a}^{\alpha,\beta} = \sum_{i=1}^{n} \{ \text{pred}_{i}^{a} = \alpha \cap \text{class}_{i} = \alpha \}
\]
\[
T_{\beta}^{\alpha,\beta} = \sum_{i=1}^{n} \{ \text{pred}_{i}^{a} = \beta \cap \text{class}_{i} = \beta \}
\]
\[
F_{a}^{\alpha,\beta} = \sum_{i=1}^{n} \{ \text{pred}_{i}^{a} = \alpha \cap \text{class}_{i} = \beta \}
\]
\[
F_{\beta}^{\alpha,\beta} = \sum_{i=1}^{n} \{ \text{pred}_{i}^{a} = \beta \cap \text{class}_{i} = \alpha \}
\]

Based on these four sets of values, we will compute the class-distinguishing score, \( CS_{a}^{\alpha,\beta} \) for each pair of classes \( (\alpha,\beta), \alpha \neq \beta, 1 \leq \alpha, \beta \leq c \) specific to each feature \( a, 1 \leq a \leq \gamma \). Two intermediate scores, \( \text{pre}_{a}^{\alpha,\beta} \) and \( \text{rec}_{a}^{\alpha,\beta} \) are obtained from the parameters obtained in Equation \( (1) \). We calculate the class-discriminability score, \( CS_{a}^{\alpha,\beta} \) from these two intermediate values. The calculation is substantially similar to that of \( F_1 \). The difference here is, we account for the \textit{true positives} of the two classes together.

\[
\text{Pre}_{a}^{\alpha,\beta} = \frac{T_{a}^{\alpha,\beta} + T_{\beta}^{\alpha,\beta}}{T_{a}^{\alpha,\beta} + T_{\beta}^{\alpha,\beta} + F_{a}^{\alpha,\beta}}
\]
\[
\text{Rec}_{a}^{\alpha,\beta} = \frac{T_{\beta}^{\alpha,\beta} + T_{\alpha}^{\beta,\alpha}}{T_{\beta}^{\alpha,\beta} + T_{\alpha}^{\beta,\alpha} + F_{\beta}^{\alpha,\beta}}
\]
\[
CS_{a}^{\alpha,\beta} = \frac{2 \times \text{pre}_{a}^{\alpha,\beta} \times \text{rec}_{a}^{\alpha,\beta}}{\text{pre}_{a}^{\alpha,\beta} + \text{rec}_{a}^{\alpha,\beta}}
\]

When \( a = 0 \), training happens on the full and complete dataset. We denote the class-distinguishing score for \( (\alpha,\beta) \) of the classifier model trained on the full complete data as \( CS_{a}^{0} \). For \( 1 \leq a \leq \gamma \), we remove feature \( a \) and train the model. Subsequently, while using model \( M_{\cdot \cdot a} \) for predicting a test point, feature \( a \) has to be imputed. As we have said earlier, in a similar fashion, we train \( \gamma \) classifiers \( M_{\cdot \cdot 1}, M_{\cdot \cdot 2}, \ldots, M_{\cdot \cdot \gamma} \), each of which is trained by imputing one feature at a time. We calculate their class distinguishing scores as well. \( CS_{a}^{\alpha,\beta} \) denotes the class-distinguishing score of classifier model \( M_{\cdot \cdot a} \) w.r.t. classes \( (\alpha,\beta) \). For \( c \) classes, the class distinguishing score for each feature will be a \( c \times c \) matrix. The symmetricity of the matrix will depend on whether we select a symmetric score function or not. Let us assume a symmetric score function as of now.

In the next section, we present an detailed explanation of the proposed methodology.

5. Proposed Approach

In this work, we are motivated to estimate the goodness and utility of the features of a dataset. It boils down to — what role each feature is playing in the predictions. The class-distinguishing powers of the features are a good indicator of their predictive powers. To answer this question, we have to engage in a micro-analysis of the features with respect to their class-distinguishing capabilities. In the next subsection, we will introduce and explain a scheme for estimating the predictive capability or goodness of a feature.

5.1. Goodness of features

As in the previous subsection, we will assume that our dataset has \( \gamma \) features and \( c \) classes. Let the features be denoted as \( f_1, f_2, \ldots, f_\gamma \). Let \( \mathcal{D} \) be the full and complete dataset and \( \mathcal{D}_{\cdot \cdot a} \) denote the dataset without feature \( a \). \( \mathcal{D}_{\cdot \cdot a} \) is the entire dataset consisting of all instances without feature \( a \). We will train a model \( M_0 \) with the full and complete dataset \( \mathcal{D} \), evaluate the class-distinguishing scores and use it as yardstick for measuring the goodness and predictive power of each feature. We denote the class-distinguishing score of \( \mathcal{D} \) as \( CS_{\alpha,\beta}^{0} \). To evaluate the class-distinguishing capability of feature \( a \), we will train a classifier model \( M_{\cdot \cdot a} \) with \( \mathcal{D}_{\cdot \cdot a} \). Essentially, we are training classifier model \( M_{\cdot \cdot a} \) without feature \( a \) and we will obtain the class-distinguishing scores from the model. The disparity or difference in the class-distinguishing capability of \( M_0 \) and \( M_{\cdot \cdot a} \) will be indicative of the predictive power of feature \( a \). We will explore this capability for all features \( f_1, f_2, \ldots, f_\gamma \). We denote the class-distinguishing score with respect to \( \mathcal{D}_{\cdot \cdot a} \) for feature \( a \) with \( CS_{a}^{\alpha,\beta} \). The normalized class-distinguishing score for feature \( a \) with respect to classes \( \alpha \) and \( \beta \) is denoted with \( NCS_{a}^{\alpha,\beta} \). Note that \( 1 \leq a \leq \gamma \) where \( \gamma \) denotes the number of features. We compute it as follows.

\[
NCS_{a}^{\alpha,\beta} = CS_{a}^{\alpha,\beta} - CS_{a}^{0}
\]
If $NCS_{a \beta}^\alpha \approx 0$, $\forall a \beta, \alpha \neq \beta$, feature $a$ has no role in differentiating between the classes of a dataset. It is so because the distinguishing capability of the models do not deteriorate without the removal of feature $a$ across all class combinations, which signifies that feature $a$ is rather redundant or unimportant in learning from the dataset.

Alternatively, let us have $NCS_{a \beta}^\alpha \geq \varepsilon$, $\forall a \beta, \alpha \neq \beta$ where $0 < \varepsilon \leq 1$ is a significant amount with respect to the problem. In this particular case, feature $a$ plays a decisive role in differentiating between all pairs of classes.

We explain the basic intuition of our scheme above. The basic idea is – if the class distinguishing reduce change after removal of some feature $a$, feature $a$ is important. In the previous two items, we have assumed for all pairs of classes $\alpha$ and $\beta$, where $\alpha \neq \beta$. But from Figure 1, we have seen that class distinguishing scores of the are specific to the pair of classes. The feature from the top-right quadrant is effective for distinguishing 1 and 7 but not as effective for distinguishing 5 and 6. The reverse holds for the feature from the bottom-left quadrant. Hence, we will need a class-specific (pair wise class) analysis to have a

**Remarks:**

- We can calculate the overall importance of a feature from its normalized class-distinguishing scores for all possible classes. The overall importance of a feature is the cumulative sum of all its normalized class-distinguishing scores. Let $I_{\text{overall}}^a$ be the overall importance of feature $a$.

$$I_{\text{overall}}^a = \sum_{\alpha=1}^{y} \sum_{\beta=1, \alpha \neq \beta}^{y} NCS_{a \beta}^\alpha$$

- The real-world datasets are usually multi-class (more than two classes). In such cases class-distinguishing scores for a feature is usually more than 2 (3 for 3 classes, 6 for 4 classes and so on). We have to look at the normalized scores, $NCS_{a \beta}^\alpha (a \alpha \beta \ v \alpha \beta \ a \neq \beta)$ features to have a detailed picture. We do that in the next subsection.

### 5.2. Class-distinguishing capabilities

Now that we know how to evaluate the importance of each feature, we want to use it to evaluate the class-discriminating capabilities of the features for pair-wise class combinations. $(CS_{a \beta}^\alpha - CS_{a \beta}^\gamma)$ denotes that capability of feature $a$ to discriminate classes $\alpha$ and $\beta$. We have to accumulate this information for all pairs of classes $\alpha$ and $\beta$, where $\alpha \neq \beta$ to get a micro-level information. We will remove each feature (making it a latent variable), and for each feature, we will maintain a matrix to organise this information. We may note that the diagonal elements signify the class-distinguishing scores of one class to itself and it do not bear any relevance (hence we mark them as don't care).

**Note that:** If we consider a symmetric function to calculate the class-distinguishing scores $(CS_{a \beta}^\alpha)$ will be equal to $(CS_{a \gamma}^\beta)$, and the class-distinguishing matrix for each feature will be a symmetric one.

### Further note: The procedure explained above is carried out on the training data. From $(CS_{a \beta}^\alpha - CS_{a \beta}^\gamma)$, we will evaluate the importance of feature $a$ in distinguishing classes $\alpha$ and $\beta$. A higher value indicates that feature $a$ plays a decisive role in distinguishing classes $\alpha$ and $\beta$.

Now, we would like to use this information in a setting where we have to predict test points with some missing features. How do we do that?

### 5.3. Handling the feature importance to make predictions on test points which may have some missing features

Let us ponder over our last statement in the previous subsection.

- We assume that $(CS_{a \beta}^\alpha - CS_{a \gamma}^\beta)$ is a significant value, which manifests the decisive role of feature $a$ in distinguishing classes $\alpha$ and $\beta$.

- The other assumption is — we have to classify a test point $q$ for which feature $a$ is latent variable.

We invoke classifier model $M_{-a}$ to classify $q$. After obtaining the prediction from $M_{-a}$, we find that the two most probable classes for $q$ are $\alpha$ and $\beta$ in some order (either $\alpha$ more probable than $\beta$).

Now that we already know that feature $a$ is decisive in distinguishing between $\alpha$ and $\beta$, will it be a judicious decision to distinguish between the two in absence of feature $a$ information? It is certainly not.

We motivate our next line of action on this thought. While classifying or predicting the test points under latent variable constraint (single latent variable), we will obtain the two most probable classes for each point and also the difference in their likelihood according to the classifier. Let the latent variable be
denoted by \( a \) and we invoke classifier \( M_{\alpha} \) to classify \( q \). Let the two probable classes for \( q \) be \( \alpha \) and \( \beta \) and their respective likelihood for instance \( q \) be \( L_{\alpha}^{\text{a}}(q) \) and \( L_{\beta}^{\text{a}}(q) \) respectively. Let us assume \( L_{\alpha}^{\text{a}}(q) > L_{\beta}^{\text{a}}(q) \).

We use (\( CS_{\alpha}^{\text{a}} - CS_{\beta}^{\text{a}} \)) as the class-distinguishing score of feature \( a \) for distinguishing class \( \alpha \) and class \( \beta \). We desegregate this information from the training phase along with \( L_{\alpha}^{\text{a}}(q) \) and \( L_{\beta}^{\text{a}}(q) \) obtained about \( q \) from the test phase to decide whether we make a judgement about the class of \( q \) or not. Let \( \text{Prediction}(q) \) be the prediction for \( q \).

Note that, in the first part of Equation (1), we compare two types of terms i] (\( L_{\alpha}^{\text{a}}(q) \), \( L_{\beta}^{\text{a}}(q) \)) — which is obtained from the test phase and is dependent on test instance \( q \), and, ii] (\( CS_{\alpha}^{\text{a}} - CS_{\beta}^{\text{a}} \)) — which is obtained solely from the training phase.

The equation signifies that — i] when the difference in likelihood of the two most probable classes is more than the class-distinguishing scores (with respect to those two classes) for the missing feature, the scheme makes the prediction. The class with higher likelihood is predicted, ii] When the difference in likelihood is less than the class-distinguishing scores (which is obtained in the training phase), the scheme refuses to make a prediction. Hence, we will have two principal classes of prediction — i] where we make a prediction or deliver a decision ii] where we refuse to make a decision. We may note that, when we have a test point with all the features, we do not consider the refusal to decision option and we always deliver a decision.

5.4. Were we right to refuse a decision? — and the goodness of the class-distinguishing scores estimated in the training phase

We have the test set denoted by \( D_tr \). It has points with full and complete features (no missing features). We consider test points \( q, q \in D_tr \) and impute its feature \( a \), to form a dataset \( D_{\alpha}^{tr} \), \( a \in \{f_1, f_2, \ldots, f_L\} \). Each point in \( D_{\alpha}^{tr} \) is a point in \( D_tr \) with feature \( a \) removed. We invoke classifier model \( M_{-\alpha} \) to obtain the predictions of points in \( D_{\alpha}^{tr} \).

We consider two classes of cases which we described in the previous subsection and compute their respective accuracies.

- Case 1- where we deliver a decision in spite of having missing features — decisions are made and we compute the accuracies from the decisions (predictions) and true class (category) information of the points. We denote it with \( \text{acc}_{\text{decision}} \).

- Case 2 - where we refuse to deliver a render in presence of a missing feature (because we were not confident of deciding in absence of the feature), we consider the most probable choice of the classifier model as the tentative prediction. We compute the accuracy on the basis of the tentative predictions and the true class (category) information of the points. We denote the accuracy with \( \text{acc}_{\text{reject}} \).

Intuition: We have accuracies from two distinct cases where i] the classifier makes a decision, ii] the classifier is not confident of passing a decision and it refuses a decision.

The decisions to predict or reject are made on the basis of the explainability and class-distinguishing capacities of the features. The explainability or class-discernibility quantification is also a part of the model. Given this scenario, it is logical to deduce that if the model has a good feature explainability, it should be correct about its decision to predict or reject. Or, in other words, the model’s classification accuracy should be more in situations where it is predicting than situations where it is refusing to predict (reject).

6. Experimental Setup

The empirical study is designed to evaluate two primary goals — i] Compute the class distinguishing capabilities (scores) of the features. ii] Validate and check the goodness of the computed scores by employing a missing feature scenario.

We have employed 10 datasets of diverse specifications — i] the number of points in the datasets vary from 336 to 58000, ii] the number of features vary from 7 to 36 and iii] the number of classes vary from 3 to 9.

In addition to that, we perform micro-analysis of the performance of the proposed scheme on credit-r dataset.

Table 3: Description of datasets. \( n \) denotes the number of points in a dataset. \( f \) and \( c \) denotes the number of features and number of classes of a dataset respectively. \( d \) denotes the number of pair-wise combinations of classes to be distinguished by each feature (it is essentially ‘\( C_2 \)’).

| Dataset              | \( n \) | \( f \) | \( c \) | \( d \) |
|----------------------|--------|--------|--------|--------|
| maternal health risk | 1014   | 7      | 3      | 3      |
| contraceptive       | 1473   | 9      | 3      | 3      |
| segment              | 2310   | 19     | 7      | 21     |
| balance              | 625    | 4      | 3      | 3      |
| marketing            | 6876   | 13     | 9      | 36     |

6.1. Data partitioning

Data partitioning is an important component any learning procedure. The scheme presented in this work requires us to delve beyond the usual training-test partitioning where we have a three-way partitioning of data. Before describing the data partitioning, let us recapitulate the assumptions that we have made throughout this work. We assume that a dataset be denoted by \( D \) and there are \( c \) classes, \( \gamma \) features. We train (\( \gamma + 1 \)) classifiers. \( M_0 \) is trained on the full and complete dataset, while \( M_{\gamma i} (1 \leq i \leq \gamma) \) is trained without (permuted) \( i^{th} \) feature. The training phase of our work has two principal elements — i] we train the (\( \gamma + 1 \)) classifiers, data used in this phase is denoted with \( D_tr \) and ii] we evaluate the class-distinguishing capabilities of the features by comparing the performances of the (\( \gamma + 1 \)) classifiers using a test dataset which is also a part of the training phase. We denote this set with \( D_{te} \) (it is essentially the test set of the training phase). The remaining component pertains to the usual test phase and we require a test set on which we evaluate the performance of the proposed learning scheme. We denote this set as \( D_{te} \).

We may note that \( D_{tr}, D_{te} \) and \( D_tr \) are mutually exclusive and
there is no common data points between these sets. Additionally, there are exhaustive with respect to $D$ as well.

\[ D = D_t \cup D_{t \text{re}} \cup D_e, \]
\[ D_t \cap D_{t \text{re}} = \phi \]
\[ D_{t \text{re}} \cap D_e = \phi \]
\[ D_t \cap D_e = \phi. \] 

For all the datasets, we have set the ratio of the number of points in $D_t$, $D_{t \text{re}}$ and $D_e$ to 0.5, 0.3 and 0.2 respectively.

6.2. Training and Testing

- **Training phase**: The model is trained on $D_t$ and the performance scores on $D_{t \text{re}}$ are used to compute the class-distinguishing scores of the features for each pair of classes.

- **Test phase**: The model trained on $D_t$ is used to get the initial predictions of a test point from $D_e$. While making the final decision on rendering or refusing the prediction for a test instance, the class-distinguishing scores obtained via $D_{t \text{re}}$ is also taken into account.

6.3. Metrics used for evaluation

We have evaluated the classification performances on accuracy. Accuracy evaluates the number of correct predictions made by a classifier overall.

6.4. Metrics used for evaluation

Random forest classifier is used in this work for all classification purposes.

7. Results and analysis

The results and analysis of the work is presented in the following manner. Our work has two major analyses. At first, we evaluate the class-distinguishing scores for two real-world datasets — i) Maternal health and ii) Contraceptive. The scores are obtained by our model. We present a semantic analysis and explanation of the features of each dataset in terms of their class-distinguishing abilities (captured through their scores).

The class-distinguishing scores provides an way of computing the overall importance of a feature. SHAP [21] is a notable work on feature importance. We compute the feature importance for these datasets using SHAP and show the outcomes graphically. We also show the overall class-distinguishing scores obtained by our method and show the correspondence between the two.

Secondly, we report the utility of the class-distinguishable scores of our scheme in a classification task.

7.1. Analysis using Maternal health dataset

**Dataset description**: The dataset provides an insight into the risk factors in maternal mortality, the data is collected from rural hospitals in Bangladesh. It reports age and five health parameters of 1014 pregnant women and calculates the risk factor of pregnancy on the basis of these parameters [46]. The health parameters are collected through wearable IoT enabled devices. The parameters reported in this study are — systolic BP, diastolic BP, blood sugar level, body temperature and heart rate. The risk factor is categorized as — low risk (1), medium risk (2) and high risk (3). So, there are three classes and we have $C_3 = 3$ cases of pair-wise class distinctions. It is worth noting that in quite a few cases, the maternal age provided in the dataset was as low as 10 years to as high as 70 years. To operate in a feasible scenario, we have removed all the points where the age was found to be less than 12 years or more than 55 years. The study is carried out on this reduced dataset.

In this section, we will analyse the class-distinguishing scores for each feature. The class-distinguishing scores for each of the features are reported in Figure 6. The bars in the figure indicates the class distinguishing capability of each figure — more the height of a bar, more is the distinguishing capability of a feature with respect to that pair of classes. The class distinguishing scores for a feature can be interpreted as a measure of its importance (specific to pair-wise classes). For example, the blue bar corresponding to the left-most item (age) indicates that — class-distinguishing ability of maternal age in distinguishing between class 1 (low risk) and class 2 (medium risk). Similarly, the green bar corresponding to the left-most item (age) indicates the — class-distinguishing ability of maternal age in distinguishing between class 1 (low risk) and class 3 (high risk). The height of the green bar is more than that of the blue bar, and it indicates that maternal age is more instrumental in distinguishing between low risk and high risk cases than that of low risk and medium risk cases. Let $CS_{t\text{re}}^{(1)}$ be the class distinguishing score with respect to age (feature 1 in this dataset).

- For each feature, the highest class-distinguishable score is obtained in medium risk-high risk cases. For feature 2 (Systolic BP), feature 4 (Blood Sugar) and feature 5 (Body temperature), the score for medium risk-high risk case has surpassed the remaining two cases (low risk-medium risk cases and low risk-high risk cases) by a considerable amount. It is of the significance that — when the classifier predicts high risk (class 3) and medium risk (class 2) as
two most likely classes for a point, the absence of any one of the above three features can lead us to a refusal in prediction (if the classifier’s confidence on the mostly likely class is not very high with respect to the second most probable class). For the remaining three features (Age, Diastolic BP and Heart Rate), this difference is not much pronounced. In these cases, when the classifier predicts high risk and medium risk (in either order) as the two most likely classes and any one of the above three features is absent – the scheme will be confident of rendering a decision even when the likelihood of the two most probable classes varies by a narrow margin. It is because of the low class-distinguishing scores.

- We have expected age (Feature 1) to play a decisive role as a feature for learning the pregnancy. But the outcome that we have obtained is not in congruence with our expectations (with respect to the other features). The class-distinguishing scores of age as a feature is highest for medium risk vs high risk cases, followed by low risk vs high risk cases and the class-distinguishing scores of low risk vs high risk scores is the least.

- The bargraphs of Systolic Blood Pressure (BP) (Feature 2) and Diastolic Blood Pressure (BP) (Feature 3) are of similar nature, with Systolic BP being more informative than Diastolic BP. The respective bars for all three cases is taller for Systolic BP than that of Diastolic BP. The BP parameters’ capability of distinguishing low-risk vs medium-risk cases is more than that of low-risk vs high-risk cases. The class-distinguishing score in medium risk vs high risk case is more pronounced for Systolic BP than that of Diastolic BP. The class-discriminable

- Blood Sugar Level (Feature 4): Blood sugar levels indicates our body’s metabolism rate, which, in turn gives a crucial insight into our overall health. The findings of our study also indicates the same and barplot of Figure 3 indicates that Blood Sugar is key feature for distinguishing the high risk cases from low risk cases and the medium risk cases. The class discriminable scores of this feature is highest for medium risk-high risk cases followed by low risk-high risk cases. It indicates that it will be difficult for the scheme to choose between high risk vs low risk/medium risk cases in absence of the blood sugar information. The class discriminable score for the low risk vs medium risk is not much higher. This indicates that, in absence of Blood Sugar information, the scheme might be able to render a decision for a point when the classifier predicts low risk and medium risk as the two most likely classes.

- Body Temperature (Feature 5): Basal Body temperature’s class-distinguishing capacity is low in low risk vs medium risk and low risk vs high risk cases. But the scores are high in medium risk vs high risk cases which signifies that it plays a crucial role for distinguishing between medium risk and high risk pregnant women.

- Heart Rate (Feature 6): This class-distinguishing capacity of this feature is reported as nominal from Figure 6. The bar of class-distinguishing score for medium risk vs high risk case is slightly better as compared to that of low risk vs medium risk and low risk vs high risk cases. But the absolute values are much less compared to other features.

- Overall feature importance: We show the overall importance of the features obtained via SHAP and the proposed method in Figure 4. In our method, the overall importance of a feature is obtained by summing up its class-distinguishing scores for all possible classes. The outcomes given our scheme is mostly congruent with the outcomes given by SHAP. The top four features are same in both cases. Blood sugar is the most important feature followed by Systolic blood pressure, age and Diastolic blood pressure. In case of SHAP, body temperature is least important feature. But, the proposed method indicates heart rate as the least important feature. However, we can account this difference (very minute) to the randomness associated with a Random Forest Classifier.

7.2. Analysis using Contraceptive

**Dataset description:** This dataset is a subset of the 1987 National Indonesia Contraceptive Prevalence Survey. The goal of this data collection was to understand and correlate the socio-economic and demographic factors with the choice of contraceptive method. The choice of contraceptive were — i] no choice, ii] long-term method and iii] short term method. The data was collected from 1473 non-pregnant or not known to be pregnant women in Indonesia. The data was collected about the socio-economic and demographic situation these served as the features) of each woman and her choice of the contraceptive method (it served as the class).

There are two numerical features (age of wife and number of children ever born) and seven categorical features (wife’s education, husband’s education, wife’s religion, wife is now working or not, husband’s occupation, standard of living index and media exposure). The dataset is multi-class and we have to classify each point (supposedly woman) to three classes – no choice (class 1), long-term method (class 2) and short-term method (class 3). Figure 5 shows the class-distinguishing scores of the features for the pair-wise classes. We discuss the key findings below.

- Wife’s age (Feature 1) shows strong class-discriminable capabilities for two cases — i] no choice vs short term method (classes 1 and 2) and ii] long term method vs short term method (classes 2 and 3). In particular, the class discriminable scores of the latter case is the highest for all cases (all features and all pair-wise classes). The likely explanation for this is the prevalence of long-term contraceptive method among older female (or male), and the prevalence of short-term or reversible methods among younger population (especially if they are undecided about having more...
children in future or losing fertility). Age is not as effective for distinguishing no use vs long term cases. A possible cause may be the prevalence of higher age group in the dataset, for whom the no-use and long-term method are equally likely.

- **Number of children ever born (Feature 4)** has emerged as a key feature from our empirical study. It has good class-distinguishable scores for all three cases of pair-wise classes. This feature shows a particularly strong class-distinguishable score for distinguishing no contraceptive vs long term contraceptive cases. Note that, this feature gives the highest class distinguishing score for distinguishing this pair of classes. It is expected that couples with fewer children (this was back in 1987) would be willing to have more child (hence no contraceptive) and couples with more number of children (or a figure according to their own optimality) will resort to long-term contraceptive methods. Class discernibility score for long term vs short term methods is also a significant figure with respect to this feature. An explanation for this can be – couples with lesser number of children will prefer short term methods while couples with more number of children will prefer long term methods. The bar-graph corresponding to no contraceptive vs short term contraceptive case is also high. The same explanation for the no use-long term case is also applicable here.

- **Wife’s education (Feature 2)** shows weighty class-distinguishing figures for two cases i) no contraceptive and long term contraceptive cases, and ii) short term contraceptive and long term contraceptive cases. In context of 1987, this finding seems pretty reasonable – wife’s education leads to an awareness, which promotes the use of contraceptive. The class-discriminability score of no contraceptive vs short term contraceptive cases is not as high as the other cases. A likely explanation for this may be the prevalence of short term contraceptive method in the lower age group, for whom no contraceptive is also a viable choice (wanting children). The wives from the lower age group is likely to be more educated — leading to two similar choices.

**A technical implication of the above:** Suppose, we want to classify two women, p and q on the basis of all features but one - wife’s education. We find that two most probable classes for each woman from the classifier. Let us assume that for p, the two most probable classes are no contraceptive (class 1) and short term method (class 3). Since wife’s education has good class-distinguishable scores for classifying these two cases (and we do not have access to this information), there is a good chance that the scheme will refuse to render a decision in this case. Let us assume that for q, the two most probable classes are no contraceptive (class 2) and short term method (class 3). Since wife’s education has less class-distinguishable scores for classifying these two cases, there is a good chance that the classifier will not bother about this missing information and the scheme will render a decision.

- **Media exposure (Feature 9):** The class-distinguishing scores of this feature is more for no contraceptive vs long term method and long term method vs short term method cases. Its competence in the remaining case is much
less though. But, we may note that this feature binary-categorical feature is severely imbalanced. That might affect the learning and we might not have a true picture of the socio-economic correlation with the classes.

- **Wife’s religion, Wife’s education, Husband’s occupation and Standard of living, Husband’s education** – the trends of these five features are mostly similar. For each of the feature, the class-discriminability scores are in following order – no contraceptive vs long term cases > long term vs short term cases > no use vs short term cases. In all five of these cases, the class-distinguishing ability of these features is much less in no use vs short term cases with respect to the remaining two cases. The outcome can be accounted to one aspect — these features may have an underlying correlation. We may further note that, of all the features, wife’s religion shows the lowest class-distinguishing scores.

- **Overall feature importance**: We report the overall importance of the features of contraceptive dataset in Figure 6. In Figure (6a) and Figure (6B), we show the importance obtained via SHAP and the proposed method respectively. The findings from the two are partially in congruence with each other, in essence they are largely similar. Wife’s education and No. of children are top two features in either case but SHAP outputs Wife’s education as the most important feature whereas the proposed method’s plot shows No. of children as the most important feature. The same holds for the 3rd most and 4th most important features with respect to the two schemes. According to SHAP, the 3rd and 4th most important features are Standard-of-living...
and Wife’s education respectively. On the contrary, the outcomes from the proposed method indicates just the reverse. The 5th, 6th and 7th features are same for both the methods. We again have a reversal in ranking for the two least important features. We may further note that barring the top two features (obtained in either case), the importances of the remaining features is much lesser.

7.3. Classification and utility of the class-distinguishable scores

In this subsection, we study the utility of the proposed class-distinguishing scores in rendering a decision. We dedicate one figure to each dataset. For each dataset, we report feature-specific performance of the classifiers and plot four bar-graphs for each — i] Plot 1 (blue coloured bar)- all features are present (this serves as the baseline), ii] Plot 2 (orange coloured bar)- a particular feature a is absent, iii] Plot 3 (green coloured bar)- feature a is present and our scheme renders a decision, and iv] Plot 4 (red coloured bar)- feature a is present and our scheme refuses to render a decision. In this particular case, the class with highest probability estimate is chosen as the decision. We plot classification accuracy in all the cases.

In each figure, plot 3 and plot 4 are the outcomes of our scheme. Plot 3 cumulates the cases in which our scheme is confident of rendering a decision. On the contrary, plot 4 cumulates the cases in which our scheme does not want to render a decision. The decision about rendering or refusing a decision is an integral part of the proposed scheme and indicative of the correctness of the explainability given the scheme. We may note that — if we get a difference in accuracies of these two cases, we can say that decision of our scheme bears some significant. To be precise, if the accuracy in plot 3 case is more than the respective accuracy in plot 4 cases, the scheme is right about its decision. This shows that the scheme is refusing a decision which could lead to an incorrect one. We may further note that our scheme has achieved this for all four datasets and across all features. The red bar (plot 4) has much lower height than its corresponding green bar (plot 3) in all the cases. This signifies that our scheme is indeed right about refusing the decision — had it decided, it would have been wrong in the prediction (as indicated from the accuracy). This also indicates the correctness of the class-distinguishable abilities (explainability) of the features — on the basis of which the decision about rendering or refusal has been taken.

8. Conclusion and Future Work

In this work, we analyze and explain the features in micro-scale. We study and quantify their class-distinguishing abilities and obtain a set of feature-specific class-distinguishing scores. We use the computed scores to aid the decision making in latent feature scenarios. A key characteristic of this decision making model is — it can refuse or render a decision (categorization) for a test point. The decision of refusal or rendering is dependent on two things — i] the two most probable classes or categories of the test point, and ii] the latent feature’s ability to distinguish these two likely classes.

The empirical results from the entire procedure indicates the correctness of the class-distinguishing scores as well as its suitability in the latent feature framework. Detailed analyses carried on two real-world datasets also manifests the same. The proposed framework has the provides for the overall importances of the features as well as their class-specific importances. This proposed framework for obtaining feature importance and explainability can aid in the automated decision making of real-world systems. In our future work, we will like to explore two more aspects of real-world data, namely class-imbalance and feature imbalance. We will specifically focus on how they affect the class-distinguishable abilities of the features and the overall feature importances.

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Figure 11: Segment

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