Top quark and QCD physics at $e^+e^-$ linear colliders: recent progress*

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Abstract

I review the studies, which were reported after the last Linear Collider Workshop, on top quark physics and QCD physics at a future $e^+e^-$ linear collider.

1 Introduction

Since the first Linear Collider (LC) Workshop, an enormous amount of studies have been done on top quark physics and QCD physics that can be covered at a future linear collider. At early stages, many new ideas were proposed. More recently, studies are centered toward precise theoretical predictions and simulation studies incorporating more realistic and advanced experimental setups. It indicates a healthy direction of the development, since the role of the top quark and QCD physics within the grand aim of linear collider experiments is to search for new physics through precision studies of top quark properties and through accurate understanding of strong interaction phenomena. It is quite impressive to see how interesting and important progress is still being made.

In this article I summarize the works presented in the top/QCD parallel session at LCWS02. In addition I also summarize some of the important works which

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appeared after the last LC Workshop. The following subjects are reviewed: an updated analysis on parameter determinations at $t\bar{t}$ threshold; an improvement of theoretical predictions for the top threshold cross section; the role of top quark offshellness in the toponium energy level; a new decoupling theorem for the top decay form factors; a kinematical fitting method for top event reconstructions; jet momentum distributions in a flux-tube model. I apologize in advance to those, whose works are not given full justice in my summary.

2 Parameter determinations at top threshold: update

An updated parameter determination study has been carried out in the top quark threshold region [1], which includes the following new aspects: (1) It is based on simultaneous measurements of three physical observables, the top production cross section $\sigma_{t\bar{t}}$, the top momentum $p_t$, and the forward-backward asymmetry of the top quark $A^{t\bar{t}}_{FB}$, which are extracted from the same top quark sample. (2) It performs multiparameter fits (up to four parameters: the top quark $1S$-mass $m_t(1S)$, the strong coupling constant $\alpha_S(M_Z)$, the top width $\Gamma_t$, and the top-Higgs Yukawa coupling $g_{tH}$). (3) Systematic uncertainties are partly included.

Fig. 1 shows the energy dependences of the three observables in the threshold region together with estimated errors corresponding to an integrated luminosity of 300 fb$^{-1}$. The analysis showed that (a) $\sigma_{t\bar{t}}$ is most important for the determination of the above mentioned parameters, (b) $p_t$ reduces correlations of the errors in the parameter determinations considerably, (c) $A^{t\bar{t}}_{FB}$ plays a rather minor role.

For instance, with a 300 fb$^{-1}$ integrated luminosity, a 3-parameter fit was performed and resulted in the following uncertainties of the top mass, strong coupling and top width:

$$\Delta m_t(1S) = 19 \text{ GeV}, \quad \Delta \alpha_S(M_Z) = 0.0012, \quad \Delta \Gamma_t = 32 \text{ MeV}. \quad (1)$$

Particularly the accuracies estimated for $m_t(1S)$ and $\Gamma_t$ are quite impressive: $10^{-4}$ and 2% relative accuracies, respectively. Nevertheless, one should note that presently there remain theoretical uncertainties of order 100 MeV in relating the $1S$-mass $m_t(1S)$ to the $\overline{\text{MS}}$-mass, the latter of which we would like to determine eventually [2]. Thus, the above accuracy of $m_t(1S)$ should be regarded as an accuracy goal with which theorists should predict the relation between the $1S$-mass and the $\overline{\text{MS}}$-mass in the future.
The above errors agree (roughly) with those which can be obtained by scaling the error estimates of previous analyses (e.g. [3]) by square-root of the integrated luminosity. However, the non-trivial point is that in this new analysis a 3-parameter fit was performed, whereas in the previous analyses only one or two parameters were varied while fixing the others. That there is no significant loss of accuracies when the multiparameter fit is performed, shows the smallness of the error correlation thanks to the simultaneous measurements of the physical observables.

A 4-parameter fit was also performed including the top Yukawa coupling in addition, for $M_H = 120$ GeV. The analysis showed that the determination of the top Yukawa coupling is quite challenging in the top threshold region.

3 Improved predicton for $\sigma_{t\bar{t}}$ in the threshold region

In the last two LC Workshops, it was recognized that there exist large uncertainties in the present theoretical prediction for the normalization of the top quark production cross section $\sigma_{t\bar{t}}$ in the threshold region. In order to improve accuracy of the theoretical prediction, Ref. [4] included resummation of logarithms $\alpha_S \times (\alpha_S \log \alpha_S)^n$
into the cross section. This prescription stabilizes the prediction substantially, and it is claimed that the 3% theoretical accuracy is achieved for the normalization of $\sigma_{t\bar{t}}$.

Up to now, a full consensus has not been reached among theorists on this 3% theoretical accuracy. In particular, I find the following two questions relevant: (1) Since $\log \alpha_S$ is not particularly large, if the resummation of logarithms results in a significant effect, one expects that non-logarithmic terms may also contribute with a similar magnitude. Indeed, significance of the non-logarithmic term in the next-to-next-to-leading order (NNNLO) correction has been suggested in [5, 6]. (2) Even if the resummation of logarithms is shown to stabilize the theoretical prediction for the threshold cross section, what is the physical meaning behind it? Answers to these questions may be given by a rather different type of consideration, which is discussed below.

4 Offshellness of $t, \bar{t}$ as an IR cutoff

Let me state briefly a historical background related to this analysis. It is known that the sum of the pole masses of $t$ and $\bar{t}$ and the QCD potential between them, $2m_{\text{pole}} + V_{\text{QCD}}(r)$, contains an $O(\Lambda_{\text{QCD}}^3)$ perturbative uncertainty. Within the potential-NRQCD framework, this uncertainty is absorbed into a non-local gluon condensate [7]. On the other hand, it has been known for a long time that the leading non-perturbative corrections to the resonance energy levels are of order $\Lambda_{\text{QCD}}^4$ because it is proportional to the local gluon condensate $\langle G_{\mu\nu}G^{\mu\nu} \rangle$ [8]. Thus, there is an apparent mismatch in the power of $\Lambda_{\text{QCD}}$ between the two quantities.

It was shown [9] that when the offshellness of $t$ and $\bar{t}$ is incorporated properly, it provides an additional suppression factor of order $\Lambda_{\text{QCD}}/(\alpha_S^2 m_t)$ to the perturbative uncertainty of the resonance energy levels:

$$\delta E_n \sim \Lambda_{\text{QCD}} \times \frac{\Lambda_{\text{QCD}}^2}{(\alpha_S m_t)^2} \times \frac{\Lambda_{\text{QCD}}}{\alpha_S^2 m_t}$$  (2)

Hence, the dimension of the perturbative uncertainty becomes the same as that of the leading non-perturbative correction. Qualitatively the suppression mechanism can be understood as follows. If the offshellness is larger than $\Lambda_{\text{QCD}}$, the rescattering time $\Delta t \sim (\alpha_S^2 m_t)^{-1}$ of $t$ and $\bar{t}$ inside the resonance state becomes shorter than the
hadronization time scale. That is, $t$ and $\bar{t}$ will get distorted before infrared gluons surround them, so that the offshellness acts as an infrared cutoff of the temporal dimension.

One may recall that a large uncertainty of order $\Lambda_{\text{QCD}}$ was inherent in the theoretical prediction for the resonance energy levels when the top quark pole mass had been used conventionally. The uncertainty was suppressed by $\Lambda_{\text{QCD}}^2/(\alpha_S m_t)^2$ when a top quark short-distance mass (such as the MS mass) was used instead. This corresponds to an incorporation of the infrared cutoff of the spacial dimension, namely the fact that the physical size of the $t\bar{t}$ resonance state is much smaller than the typical hadron size. Furthermore additional suppression factor comes from the infrared cutoff in the temporal extent. Correspondingly the series expansions of the energy levels become more convergent when these effects are incorporated, and we may achieve more accurate theoretical predictions.

Conceptually the suppression mechanism by offshellness is clear. It is, however, a non-trivial task to incorporate the offshell suppression effects into theoretical predictions systematically.\footnote{In the usual approach to non-relativistic boundstates, one starts from instantaneous gluon exchange in the leading order and incorporates perturbative corrections to it. Although this is justified for the leading binding kinematics, such treatment spoils the role of offshellness as an infrared cutoff.} It has already been shown that these effects include (as a part of the effects) resummation of ultrasoft logarithms. Hence, it may provide answers to the questions raised in the previous section, although more detailed analyses are still needed.

Closely related to this subject, we note the important progress in the theoretical prediction for $e^+e^- \rightarrow t\bar{t}$ in the threshold region. Theorists are now aiming at calculations of NNNLO corrections to the cross sections. Since the last LC Workshop, important computations have been achieved toward this goal [10, 5].

5 "Decoupling theorem" for top decay form factors

Related to the subject of top quark form factor determinations, a new “decoupling theorem” was found [11]. The theorem can be phrased as follows. Consider a process where two initial particles (“1” and “2”) collide and the top quark is produced in association with some other particle(s) and the top quark decays semi-leptonically:
\[ 1 + 2 \rightarrow t + X \rightarrow b\ell\nu + X. \] In this process we assume presence of general form factors in the top quark decay vertex. There are four of them, \( f_{L,R}^{1,2} \) and \( \bar{f}_{L,R}^{1,2} \): 

\[
\Gamma_{Wtb}^\mu = -\frac{g}{\sqrt{2}} V_{tb} \bar{u}(p_b) \left[ \gamma^\mu (f_1^LP_L + f_1^RP_R) - \frac{i \sigma^\mu_\nu}{M_W} (f_2^LP_L + f_2^RP_R) \right] u(p_t), \tag{3}
\]

where \( P_{L,R} = (1 \mp \gamma_5)/2 \). At tree level of the Standard Model (SM), \( f_1^\ell = 1 \), and all the other form factors are zero. The other parts of the amplitude can be of any form (e.g. anomalous form factors in the top production vertex may be included), as long as it includes the top quark as an intermediate state which has the above decay vertex. Then, the theorem states that the angular distribution of the lepton \( \ell \) from the top quark, defined in the laboratory frame, is independent of the anomalous form factors. Here, the anomalous form factors stand for the deviations of the form factors from the tree-level SM values. This theorem holds under fairly general conditions:

- The initial particles ("1" and "2") can have longitudinal momenta. Hence, the theorem is applicable also to processes at a photon-photon collider or at a hadron collider.
- The mass of \( \ell \) is neglected, but the bottom quark mass can be non-zero.
- Narrow width limits are assumed: \( \Gamma_t/m_t, \Gamma_W/M_W \ll 1 \).
- The theorem is valid up to linear terms in the anomalous form factors, i.e. the terms quadratic in the anomalous form factors are neglected.

In measurements of the top quark form factors at \( e^+e^- \) collider, this theorem provides a useful tool for disentangling the top quark form factors associated with the top quark production and the decay vertices. In fact if we first analyze the angular distribution of leptons from the top quark, this is sensitive only to the top quark production vertex. After determining the form factors in the production vertex, we can use other observables for extracting the decay form factors. Physically this theorem guarantees that the \( \ell \) angular distribution is an ideal analyzer of the top quark spin. This fact has been known within the SM [12]; the above theorem extends this picture to the case where there are small anomalous form factors in the top quark interactions. Since the top quark spin plays a crucial role in the analysis of various top quark properties, this observation can be utilized in many ways. e.g. This theorem has been applied to the analysis of the \( CP \) property of the Higgs boson using the process \( \gamma\gamma \rightarrow H \rightarrow tt \) [13].
6 Kinematical reconstruction in top threshold region

Measurements of the top quark form factors will be carried out both in the open top region and in the top threshold region. So far most of the theoretical analyses as well as simulation studies concerned the open top region due to the complexity of the analysis in the threshold region. It is, however, conceivable that the form factor measurements will be carried out first in the top threshold region. For this reason it has been demanded for some time that serious simulation studies on sensitivities to the top form factors should be performed in the threshold region; it may influence the energy upgrading program of an $e^+e^-$ linear collider.

![Figure 2: Distributions of the difference of the reconstructed and generated energies of leptonically-decayed $W$ bosons.][14]

To facilitate such a simulation study, a likelihood fitting method has been devised for kinematical reconstruction of event profiles, which is particularly tailored to the threshold region [15]. The impact of using this method in the kinematical reconstruction can be seen in the reconstruction of the energy of leptonically decayed $W$’s. Fig. 2 shows the difference of the reconstructed $W$ energies with and without applying the likelihood fitting method. Without the likelihood fitting, reconstructed $W$ energy tends to be larger than the generated value. This is because all the missing momenta, which come from cascade decays in the $b$ jets, are mis-assigned to the neutrino in the $W$ decay. On the other hand, we see a significant recovery of the generated energy after the likelihood fitting. Thus, we expect that the new kinematical fitting method would be useful e.g. in the measurements of the top decay.
form factors.

7 Jet momentum distributions in a flux-tube model

A flux-tube model has been proposed to predict the jet fragmentation properties [16]. It is assumed that the number of jets in a process is determined by the hard processes in perturbative predictions, whereas all the fragmentation inside each jet is predicted by the flux-tube model in momentum space, without any input from perturbative QCD. Based on several assumptions the model is capable of predicting the momentum distribution of hadrons in each jet. The distribution for the two-jet case has been reported in this workshop. It is important to test the prediction by comparison to experimental data.

8 Future programs

As seen above, interesting and important studies on top and QCD physics have been performed since the last LC Workshop. Yet, there remain tasks that have to be done before actual operation of a next-generation $e^+e^-$ linear collider. Among various tasks which will hopefully be done until the next LC Workshop, I would like to stress the following ones:

- Analyses of the top quark form factors both in the open top and threshold regions, together with development of new techniques.
- More accurate theoretical predictions for the relation between the top quark $1S$ mass and the $\overline{MS}$ mass, as well as for the normalization of the top production cross section in the threshold region.
- Analyses of systematic uncertainties in extracting the top quark mass from the top–jet invariant mass.

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