Two photon decay of the pseudoscalars, the chiral symmetry breaking corrections

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Abstract
The extrapolation of the decay amplitudes of the pseudoscalar mesons into two photons from the soft meson limit where it is obtained from the axial-anomaly to the mass shell involves the contribution of the $0^-$ continuum. These chiral symmetry breaking corrections turn out to be large. The effects of these corrections on the calculated $\pi^0$ decay rate, on the values of the singlet-octet mixing angle and on the ratios $f_8/f_\pi$ and $f_0/f_\pi$ are discussed. The implications for the transition form factors $\gamma\gamma^* \rightarrow \pi^0, \eta, \eta'$ are also evaluated and confronted with the available experimental data.

1 Introduction

Current Algebra and PCAC (partial conservation of the axial-vector current) together with the ABJ axial-anomaly [1] expresses the two-photon decay rate of the pseudoscalar mesons ($P = \pi^0, \eta, \eta'$) in terms of the coupling constants $f_{\pi^0}, f_0$ and $f_8$ and of the singlet-octet mixing angle $\theta$

$$\langle 0 \mid A_\alpha \mid \pi^0(p) \rangle = -2i f_{\pi^0} p_\alpha$$
$$\langle 0 \mid A_\alpha^{(8)} \mid \eta(p) \rangle = 2i f_8 \cos \theta.p_\alpha \quad , \quad \langle 0 \mid A_\alpha^{(0)} \mid \eta(p) \rangle = -2i f_0. \sin \theta.p_\alpha$$
$$\langle 0 \mid A_\alpha^{(8)} \mid \eta'(p) \rangle = 2i f_8 \sin \theta.p_\alpha \quad , \quad \langle 0 \mid A_\alpha^{(0)} \mid \eta'(p) \rangle = 2i f_0. \cos \theta.p_\alpha$$

with

$$f_{\pi^0} \simeq f_{\pi^+} = 92.4\text{MeV}$$

and where the axial-vector currents are given in terms of the quark fields.

$$A_\alpha = (\overline{\psi}\gamma_\alpha\gamma_5 u - 7\gamma_\alpha\gamma_5 d) \quad , \quad A_\alpha^{(8)} = \frac{1}{\sqrt{3}}(\overline{\psi}\gamma_\alpha\gamma_5 u + 7\gamma_\alpha\gamma_5 d - 2\overline{\psi}\gamma_\alpha\gamma_5 s)$$
A_α^{(0)} = \sqrt{\frac{2}{3}}(\bar{\pi}_\alpha \gamma_5 u + \bar{\sigma}_\alpha \gamma_5 d + \bar{\pi}_\alpha \gamma_5 s) \quad (1.2)

Unlike \( f_\pi, f_0 \) and \( f_8 \) are not directly related to any physical process.

SU(3) breaking enters through singlet-octet mixing and through the deviation of the ratio \( \frac{f_8}{f_\pi} \) from unity. Another symmetry breaking effect originates from SU(3)×SU(3) breaking: the decay rate is obtained in the soft meson limit from the ABJ anomaly [1] and the extrapolation to the mass-shell involves corrections \( O(m_\pi^2) \) which are expected to be small for the \( \pi^0 \) but which are not necessarily so for the \( \eta \) and \( \eta' \). These corrections to the PCAC limit, which arise from the 0− continuum are similar to the corrections to the Goldberger-Treiman relation [2]. Attempts to estimate these corrections have been undertaken [3], [4], [5]. In [3] and [4] only the contribution of the high energy part of the \( O^- \) spectrum was considered and in [5] only the low energy part was used. These three calculations are moreover heavily model dependent.

It is the purpose of the present work to provide an estimate of the PCAC corrections which enter in the evaluation of the two photon decay rates of the pseudoscalars under the sole assumptions that the V-A-A vertex is given by the quark triangle graph in the deep euclidean region and that the main contribution to SU(3)×SU(3) breaking arises from the energy interval

\[ 1\text{GeV}^2 \leq s \leq 2\text{GeV}^2 \]

of the \( O^- \) continuum.

In section 2 we give details of the calculation and the resulting constraints on the values of \( f_8, f_0 \) and the mixing angle \( \theta \) are discussed and finally an evaluation of the implications of our results on the transition form factors \( \gamma^* \gamma \to P \) are presented in section 3 and confronted with the available experimental data.

## 2 The chiral symmetry breaking corrections

Consider the three-point function

\[ T_{\alpha\mu\nu}(p, q_1, q_2) = 2\pi i \int \int dxdy \exp(-iq_1x + ipy)(0 | T A_\alpha(y) V_\mu(x) V_\nu(0) | 0) \]

\[ = \frac{1}{\pi} \frac{1}{(p^2 - m_\pi^2)} \epsilon_{\mu\nu\lambda\sigma} q_1^\lambda q_2^\sigma p_\alpha F(p^2, q_1^2, q_2^2) + ... \quad (2.1) \]

where the dots represent other tensor structures and where \( V_{\mu,\nu} \) denotes the electromagnetic current

\[ V_\mu = \frac{2}{3} \pi\gamma_\mu u - \frac{1}{3} \sigma\gamma_\mu d - \frac{1}{3} \pi\gamma_\mu s \quad (2.2) \]
The pion pole contribution having been isolated in expression (2.1).

In the soft $\pi$ limit and with both photons on mass-shell, the axial-anomaly [1] yields

$$F(0, 0, 0) = 1$$  (2.3)

The rate of the decay $\pi^0 \rightarrow 2\gamma$ provides a measurement of

$$F(m^2_\pi, 0, 0) = 1 + \Delta_\pi, \Delta_\pi = O(m^2_\pi)$$  (2.4)

Equation (2.1) is used to define the off-shell symmetric amplitude

$$F(s, t) = F(s = p^2, t = q^2_1 = q^2_2)$$  (2.5)

In the deep euclidean region $F(s, t) = F^{QCD}(s, t)$, $F^{QCD}(s, t)$ includes perturbative and non-perturbative contributions

$$F^{QCD}(s, t) = F_p(s, t) + F_{np}(s, t)$$  (2.6)

The perturbative part $F_p(s, t)$ is obtained from the quark triangle graph, the contribution of which takes a particularly simple form in the symmetric case [6]

$$F_p(s, t) = -2(s - m^2_\pi) \int_0^1 dx dy \frac{x^2 y^2}{(x^2 s - t) + t}, \tau = 1 - x$$  (2.7)

The non-perturbative part $F_{np}(s, t)$ arises from the contribution of the vacuum condensates

$$F_{np}(s, t) = \frac{b_1}{t^2} + \frac{b_2}{t^3} + \frac{b_3}{s t} + ...$$  (2.8)

where

$$b_1 = -\frac{\alpha^3}{g} \langle \frac{G^2}{\pi} \rangle, \quad b_2 = -\frac{64}{27} \pi^4 \langle \alpha_s (qq)^2 \rangle, \quad b_3 = -b_1$$  (2.9)

For $t$ fixed, $F(s, t)$ is an analytic function of the complex variable $s$ with a cut on the positive real axis which extends from $s = 9m^2_\pi$ to $\infty$ [7].

A dispersion relation between $F(s = m^2_\pi, t)$ and $F(0, t)$ is obtained from the integral

$$\frac{1}{2\pi i} \int_c \frac{ds}{s(s - m^2_\pi)} F(s, t)$$  (2.10)

where $c$ is the closed contour in the complex plane consisting of a circle of large radius $R$ and two straight lines lying immediately above and immediately below the cut. Cauchy’s theorem yields then

$$F(m^2_\pi, t) = F(0, t) + \frac{m^2_\pi}{2\pi i} \int_c \frac{ds}{s(s - m^2_\pi)} F(s, t)$$  (2.11)
The integral in the equation above consists of two parts: an integral of the discontinuity of $F(s, t)$ over the cut, which provides the main contribution and an integral over the circle of radius $R$ where $F^{\text{QCD}}(s, t)$ provides a good approximation to $F(s, t)$ except possibly in the vicinity of the positive real axis. Little is known about the integrand over the cut which consists of the contribution of the axial-vector ($1^+$) and pseudoscalar ($0^-$) intermediate states. The contribution of the low energy region (the three pion states) is suppressed by loop factors and amounts to little [8]. We expect the major part of the contribution to originate from the $a_1(1260)$ and $\pi'(1300)$ bumps i.e from the range $1\text{GeV}^2 \leq s \leq 2\text{GeV}^2$. In order to eliminate this contribution, we shall consider, instead of integral (2.10), the following modified integral

$$
\frac{1}{2\pi i} \int_c \frac{ds}{(s - m_1^2)} (\frac{1}{s} - a_0 - a_1.s).F(s, t)
$$

In the equation above, the coefficients $a_0$ and $a_1$ will be chosen so as to annihilate the integrand at $m_1^2 = m_{a_1}^2 = 1.56\text{GeV}^2$ and at $m_2^2 = m_{\pi'}^2 = 1.70\text{GeV}^2$, i.e

$$
a_0 = \frac{1}{m_1^2} + \frac{1}{m_2^2}, \quad a_1 = -\frac{1}{m_1^2}.m_2^2
$$

This choice reduces the integrand to only a few percent of its initial value over the interval $1\text{GeV}^2 \leq s \leq 2\text{GeV}^2$. It will also considerably reduce the contribution to the integral over the circle near the positive real axis. The contribution of the continuum having thus been drastically reduced, we neglect it.

Cauchy’s theorem now yields

$$
F(m_2^2, t) \approx F(0, t) + a_0 m_2^2 F(m_2^2, t) + a_1 m_2^4 F(m_2^2, t) + \frac{1}{2\pi i} \int_c \frac{ds}{(s - m_2^2)} (\frac{1}{s} - a_0 - a_1.s).F(s, t)
$$

(2.14)

The integral being now carried over the circle of radius $R$.

In order to obtain $F(s, 0)$ we proceed in a similar fashion. $F(s, t)$ is an analytic function of the complex variable $t$ except for a cut on the positive real axis extending from $t = 4m_2^2$ to $\infty$. In the low $t$ region $F(s, t)$ is dominated by the $\rho - \omega$ double and single poles,

$$
F(s, t) = \frac{c_1(s)}{(t - m_2^2)^2} + \frac{c_2(s)}{(t - m_3^2)^2} + ... \quad (2.15)
$$

Consider now the integral

$$
\frac{1}{2\pi i} \int_{c'} \frac{dt}{t} \frac{(t - m_2^2)^2}{(t - m_3^2)^2}.F(s, t)
$$

(2.16)

where $c'$ is a contour similar to $c$ in the complex $t$ plane. The double and single vector meson poles having been removed and for an appropriate choice of $R'$
the major contribution to the integral (2.16) comes from the integral over the circle so that

$$F(s,0) \approx \frac{1}{m_\rho^4} \int_{R'} \frac{dt}{2 \pi i} \left( t - m_\rho^2 \right)^2 t . F(s,t)$$

(2.17)

It follows from equations (2.14) and (2.17) that

$$F(m_\pi^2,0).(1 - a_0.m_\pi^2 - a_1.m_\pi^4)$$

\[= F(0,0) + \frac{m_\pi^2}{(2 \pi i)^2} \int \int \frac{ds dt}{t(s - m_\pi^2)} (t - m_\rho^2)^2 (\frac{1}{s} - a_0 - a_1.s). F^{QCD}(s,t) \]

(2.18)

The integral above being carried over the circles of large radii $R$ and $R'$ we have replaced $F(s,t)$ by $F^{QCD}(s,t)$ in the integrand. Inserting eqs (2.6) - (2.8) in eq. (2.18) gives

$$F(m_\pi^2,0).(1 - a_0.m_\pi^2 - a_1.m_\pi^4)$$

\[= F(0,0) + \frac{m_\pi^2}{9.m_\rho^2} + \frac{m_\pi^2}{m_\rho^2}. \left( \frac{R}{5} - \frac{1}{3}(R' - 2.m_\rho^2 \ln \frac{R'}{R - R'}) \right) \]

\[+ a_0. \frac{m_\pi^2}{m_\rho^2}. R. \left( \frac{R}{10} - \frac{2}{3}. m_\rho^2 \right) - a_1. \frac{m_\pi^2.R^2}{m_\rho^2}. \left( \frac{R}{15} - \frac{m_\pi^2}{3} \right) - \frac{m_\pi^2}{m_\rho^2}. (a_0 + a_1.m_\pi^2). b_1 \]

(2.19)

The non-perturbative contributions to equation (2.19) turn out to be negligible.

The decays $\eta \rightarrow 2\gamma$ and $\eta' \rightarrow 2\gamma$ are treated in a similar fashion. The axial-vector currents which project on the $\eta$ and $\eta'$ states are, respectively,

$$A_\alpha^\eta = (\frac{A_\alpha^{(s)}}{f_\pi}. \cos \theta - \frac{A_\alpha^{(0)}}{f_0}. \sin \theta) \quad , \quad A_\alpha^\eta' = (\frac{A_\alpha^{(s)}}{f_\pi}. \sin \theta + \frac{A_\alpha^{(0)}}{f_0}. \cos \theta) \quad (2.20)$$

These currents we use in the definition of the 3-point function equation (2.1). The results for the two photon decay rates of the pseudoscalars are then:

$$\Gamma(\pi^0 \rightarrow 2\gamma) = \frac{\alpha^2.m^3_{\pi^0}}{64 \pi^3.f_{\pi^+}^2}. (1 + \Delta_{\pi}). \frac{f^2_{\pi^+}}{f_{\pi^0}} \quad (2.21)$$

$$\Gamma(\eta \rightarrow 2\gamma) = \frac{\alpha^2.m^3_{\eta}}{192 \pi^3.f_{\pi^+}^2}. (1 + \Delta_{\eta}). \left( \frac{\cos \theta}{f_\pi} - 2\sqrt{2} \frac{\sin \theta}{f_0} \right)^2 \quad (2.22)$$
\[ \Gamma(\eta' \rightarrow 2\gamma) = \frac{\alpha^2 m_{\eta'}^3}{192\pi^3 f_{\eta'}^2} (1 + \Delta_{\eta'})^2 \left( \frac{\sin \theta}{F_8} + 2\sqrt{2} \frac{\cos \theta}{F_0} \right)^2 \]  

(2.23)

With the notation \( F_{0,8} = \frac{f_{0,8}}{f_{\pi}} \) and

\[ F(m_{\pi}^2, 0, 0) = (1 + \Delta_{\pi}) \]

\[ F(m_{\eta'}^2, 0, 0) = \frac{1}{\sqrt{3}} (1 + \Delta_{\eta}) \quad , \quad F(m_{\eta'}^2, 0, 0) = \frac{2\sqrt{2}}{3} (1 + \Delta_{\eta'}) \]  

(2.24)

\( \Delta_{\pi} \) is obtained from equation (2.19). The same equation with the negligible non-perturbative part omitted yields \( \Delta_{\eta} \) and \( \Delta_{\eta'} \) when \( m_{\pi} \) is replaced by \( m_{\eta,\eta'} \) and when the strange quark mass is neglected.

\( R' \), the duality radius in the \( \rho \)-meson channel is usually taken to be \( R' \approx 1.5\text{GeV}^2 \) in the literature [9]. The value of \( R \) should be large enough to include the contribution of the pseudoscalar excitations but not too large in order not to invalidate the approximations at hand. Moreover \( \Delta_{\rho} \) should be stable against small variations in \( R \).

For the parameters \( m_1 \) and \( m_2 \) we take

\( m_1^2 = m_{a_1(1260)}^2 = 1.56\text{GeV}^2 \quad , \quad m_2^2 = m_{\pi(1300)}^2 = 1.70\text{GeV}^2 \)  

(2.25)

as discussed previously.

In the \( \eta \) and \( \eta' \) channels two pseudoscalar excitations, \( \eta(1295) \) and \( \eta(1440) \) as well as two axial-vectors, \( f_1(1285) \) and \( f_1(1420) \) dominate the 0\(^{-}\) and 1\(^{+}\) continua and couple to both \( \eta \) and \( \eta' \) mesons with unknown strengths. Because \( \eta(1295) \) and \( f_1(1285) \) on the one hand and \( \eta(1440) \) and \( f_1(1420) \) on the other hand are practically degenerate in mass it suffices to take

\[ m_1^2 = 1.66\text{GeV}^2 \quad , \quad m_2^2 = 2.04\text{GeV}^2 \]  

(2.26)

for both \( \eta \) and \( \eta' \).

With these choices expression (2.19) passes through a maximum for \( R \approx 2\text{GeV}^2 \) in the \( \pi \) channel and \( R \approx 2.25\text{GeV}^2 \) in the \( \eta, \eta' \) channels., values which we adopt because of the stability criteria stated above. Numerically then, we find

\( \Delta_{\pi} \approx 0.47 \quad , \quad \Delta_{\eta} \approx 0.77 \quad , \quad \Delta_{\eta'} \approx 6.0 \)  

(2.27)

The relative theoretical error on the numbers above is estimated to be of the order of \( \alpha_s(2.5 \text{GeV}^2)/\pi=12\% \). The chiral symmetry breaking corrections are thus seen to be quite large. For the \( \pi \) the theoretical value of the width is increased from \( \Gamma_{\pi^0 \rightarrow 2\gamma} = 7.74 \text{eV} \) to

\[ \Gamma_{\pi^0 \rightarrow 2\gamma} = (8.25 \pm 0.09) \text{eV} \]  

(2.28)
if we take $f_{\pi^+}/f_{\pi^0} = 1$. Another correction to the chiral limit could originate from $\pi^0, \eta$ and $\eta'$ mixing. It has been estimated in chiral perturbation theory [11] and would increase the decay rate by another 1.045 factor to

$$\Gamma_{\pi^0 \to 2\gamma} = (8.60 \pm .10) \text{ eV} \quad (2.29)$$

The current world average experimental value is

$$\Gamma_{\pi^0 \to 2\gamma} = (7.75 \pm .60) \text{ eV} \quad (2.30)$$

The large dispersion of the experimental results however suggests that the quoted error is underestimated [11]. The outcome of the PRIMEX experiment at Jefferson Lab [13], which aims at a precision of 1.5% in the measurement of the $\pi^0$ decay rate, is thus eagerly expected in order to clarify the situation.

In order to extract information from our results on the $\eta$ and $\eta'$ more input has to be added.

An analysis of the $\eta, \eta'$ mesons mass matrix together with chiral perturbation theory [10] yields for the mixing angle $\theta = -19.5^\circ$, this value and the experimentally measured decay rates, when inserted in equations (2.22), (2.23) and (2.27) imply

$$F_8 = 1.29 \pm .09 \quad , \quad F_0 = 4.70 \pm .45 \quad (2.31)$$

Our result for $F_8$ is consistent with that obtained from chiral perturbation theory, $F_8 = 1.25$ [10], the value obtained for $F_0$ is quite larger than the ones appearing in the literature. It is worth noting that a pure gluon component present in $\eta, \eta'$ would yield the value appearing in eq.(2.31) as an upper limit for $F_0$ [4].

It is unfortunate that $F_8$ and $F_0$ are not directly linked to any physically measurable quantity. Data exist however on the transitions $\gamma\gamma^* \to P$ [12]. We shall examine in the next section the implications of our results on the corresponding form factors.

3 Form factors of the transitions $\gamma\gamma^* \to \pi^0, \eta, \eta'$

In the deep euclidean region we have [6]

$$F_p(s, Q^2, t) = -2(s - m_P^2) \int_0^1 dx x\pi \int_0^1 dy \left( y^2 (x\pi s + xQ^2 + \pi q^2) - xq^2 + \pi t \right)$$

$$t = q_1^2, Q^2 = -q_2^2 \text{ and } s = p^2 \text{ as before. } \text{In order to be able to compare with experiment, we need to evaluate } F(m_P^2, Q^2, 0). \text{ The method used in the preceding section, modified to take into account the fact that we have now only a single pole in the vector meson channel now yields}\n
$$F(m_P^2, Q^2, 0)$$
\[\begin{align*}
&= -\frac{1}{m_\rho^2} \frac{1}{(2\pi i)} \int ds dt \frac{(t - m_\rho^2)}{t} \left( \frac{1}{s - m_\rho^2} - a_0 - a_1 s \right) \cdot F^{QCD}(s, Q^2, t) \\
&\quad \text{an expression valid for large } Q^2. \text{ Inserting expression (3.1) in the equation above, it takes some straightforward algebra in the complex plane to evaluate } F(m_\pi^2, Q^2, 0). \text{ The deep inelastic limit } Q^2 \to \infty \text{ is readily obtained}
\end{align*}\]

\[Q^2 F_{\pi^0}(m_\pi^2, Q^2, 0) \to \frac{R_3 - (m_1^2 + m_2^2)R_2}{(m_1^2 - m_2^2)} (m_2^2 - m_1^2) \quad (3.3)\]

The corresponding expressions for the \(\eta\) and \(\eta'\) have to be multiplied by the factors \(\frac{1}{\sqrt{3}} \left( \frac{\cos \theta}{F_8} - \frac{2\sqrt{2} \sin \theta}{F_0} \right)\) and \(\frac{1}{\sqrt{3}} \left( \frac{\sin \theta}{F_8} + \frac{2\sqrt{2} \cos \theta}{F_0} \right)\) respectively. Numerically then

\[\frac{Q^2 F_{\eta^0}(m_\eta^0, Q^2, 0)}{8\pi^2 f_\pi^2} \to .81 \quad (3.4)\]

\[\frac{Q^2 F_{\eta}(m_\eta^2, Q^2, 0)}{8\pi^2 f_\pi^2} \to (1.34 \pm .12) \frac{1}{\sqrt{3}} \left( \frac{\cos \theta}{F_8} - \frac{2\sqrt{2} \sin \theta}{F_0} \right) \quad (3.5)\]

\[\frac{Q^2 F_{\eta'}(m_\eta'^2, Q^2, 0)}{8\pi^2 f_\pi^2} \to (3.72 \pm .54) \frac{1}{\sqrt{3}} \left( \frac{\sin \theta}{F_8} + \frac{2\sqrt{2} \cos \theta}{F_0} \right) \quad (3.6)\]

From the experimental widths and the values of \(\theta, F_0, F_8\) given by equation (2.31) we get

\[\frac{Q^2 F_{\eta^0}(m_\eta^0, Q^2, 0)}{8\pi^2 f_\pi^2} \to .72 \pm .07 \quad (3.7)\]

\[\frac{Q^2 F_{\eta'}(m_\eta'^2, Q^2, 0)}{8\pi^2 f_\pi^2} \to .98 \pm .14 \quad (3.8)\]

The outputs of equations (3.4), (3.7) and (3.8) together with the experimental data [12] are shown on the figures

4 Conclusion

The chiral symmetry breaking corrections to the decays \(\pi^0, \eta, \eta' \to 2\gamma\) have been estimated in a model independant way and found to be quite large. In the case of \(\pi^0 \to 2\gamma\) the theoretical value is considerably increased and this lends a great significance to the upcoming PRIMEX experiment [13].

In addition the form factors of the transitions \(\gamma^* \gamma \to \pi^0, \eta, \eta'\) were evaluated and shown to compare favourably with the available experimental data.
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Figure Captions:

Fig 1: The form factors of the transitions:
  a) $\gamma\gamma^* \rightarrow \pi^0$,  b) $\gamma\gamma^* \rightarrow \eta$,  c) $\gamma\gamma^* \rightarrow \eta'$ as obtained from eq. (3.2).
The vertical lines represent the data as taken from [12].
\[ Q^2 F_{\pi}(0, Q^2)/8\pi f_\pi^2 \]
$Q^2 F_1(0,Q^2)/(8\pi f_\pi^2)$
\[ Q^2 F_{\eta}^{\prime}(0,Q^2) / 8\pi f_{\pi}^2 \]

fig. 1c