Hadronic Weak Decays of Heavy Mesons and Nonfactorization

Hai-Yang Cheng
Institute of Physics, Academia Sinica
Taipei, Taiwan 11529, Republic of China

Abstract

The parameters $\chi_{1,2}$, which measure nonfactorizable soft gluon contributions to hadronic weak decays of mesons, are updated by extracting them from the data of $D, B \to P P, V P$ decays ($P$: pseudoscalar meson, $V$: vector meson). It is found that $\chi_2$ ranges from $-0.36$ to $-0.60$ in the decays from $D \to K \pi$ to $D^+ \to \phi \pi^+$, $D \to K^* \pi$, while it is of order 10% with a positive sign in $B \to \psi K$, $D \pi$, $D^* \pi$, $D \rho$ decays. Therefore, the effective parameter $a_2$ is process dependent in charm decay, whereas it stays fairly stable in $B$ decay. This implies the picture that nonfactorizable effects become stronger when the decay particles become less energetic after hadronization. As for $D, B \to VV$ decays, the presence of nonfactorizable terms in general prevents a possible definition of effective $a_1$ and $a_2$. This is reinforced by the observation of a large longitudinal polarization fraction in $B \to \psi K^*$ decay, implying $S$-wave dominated nonfactorizable effects. The nonfactorizable term dominated by the $S$-wave is also essential for understanding the decay rate of $B^- \to D^{*0} \rho^-$. It is found that all nonfactorizable effects $A_{nf}^f/A_{fK}^f$, $A_{nf}^f/A_{fB}^f$, $A_{nf}^f/A_{fD}^f$ ($nf$ standing for nonfactorization) are positive and of order 10%, in accordance with $\chi_2(B \to D(D^*)\pi(\rho))$ and $\chi_2(B \to \psi K)$. However, we show that in $D \to K^* \rho$ decay nonfactorizable effects cannot be dominated by the $S$-wave. A polarization measurement in the color- and Cabibbo-suppressed decay mode $D^+ \to \phi \rho^+$ is strongly urged in order to test if $A_{nf}^f/A_2$ plays a more pivotal role than $A_{nf}^f/A_1$ in charm decay.
1. Introduction

It is customary to assume that two-body nonleptonic weak decays of heavy mesons are dominated by factorizable contributions. Under this assumption, the spectator meson decay amplitude is the product of the universal parameter $a_1$ (for external $W$-emission) or $a_2$ (for internal $W$-emission), which is channel independent in $D$ or $B$ decays, and hadronic matrix elements which can be factorized as the product of two independent hadronic currents. The universal parameters $a_1$ and $a_2$ are related to the Wilson coefficient functions $c_1$ and $c_2$ by

$$a_1 = c_1 + \frac{1}{N_c} c_2, \quad a_2 = c_2 + \frac{1}{N_c} c_1,$$

with $N_c$ being the number of colors. It is known that the bulk of exclusive nonleptonic charm decay data cannot be explained by this factorization approach [1]. For example, the predicted ratio of the color-suppressed mode $D^0 \to \bar{K}^0\pi^0$ and color-favored decay $D^0 \to K^-\pi^+$ is in violent disagreement with experiment. This signals the importance of the nonfactorizable effects.

The leading nonfactorizable contribution arises from the soft gluon exchange between two color-octet currents

$$O_c = \frac{1}{2}(\bar{q}_1 \lambda^a q_2)(\bar{q}_3 \lambda^a q_4),$$

where $(\bar{q}_1 \lambda^a q_2)$ stands for $\bar{q}_1 \gamma_\mu (1 - \gamma_5) \lambda^a q_2$. For $M \to PP, VP$ decays ($P$: pseudoscalar meson, $V$: vector meson), the nonfactorizable effect amounts to a redefinition of the parameters $a_1$ and $a_2$ [2],

$$a_1 \to c_1 + c_2 \left( \frac{1}{N_c} + \chi_1 \right), \quad a_2 \to c_2 + c_1 \left( \frac{1}{N_c} + \chi_2 \right),$$

where $\chi_1$ and $\chi_2$ denote the contributions of $O_c$ to color-favored and color-suppressed decay amplitudes respectively relative to the factorizable ones. For example, for $D_s^+ \to \phi \pi^+$, $D^+ \to \phi \pi^+$ decays,

$$\chi_1(D_s^+ \to \phi \pi^+) = \frac{\langle \phi \pi^+ | \frac{1}{2}(\bar{u}\lambda^a d)(\bar{s}\lambda^a c)|D_s^+ \rangle}{\langle \phi \pi^+ | (\bar{u}d)(\bar{s}c)|D_s^+ \rangle}_f,$$

$$\chi_2(D^+ \to \phi \pi^+) = \frac{\langle \phi \pi^+ | \frac{1}{2}(\bar{u}\lambda^a c)(\bar{s}\lambda^a s)|D^+ \rangle}{\langle \phi \pi^+ | (\bar{u}c)(\bar{s}s)|D^+ \rangle}_f.$$

The subscript $f$ in Eq.(4) denotes a factorizable contribution:

$$\langle \phi \pi^+ | (\bar{u}d)(\bar{s}c)|D_s^+ \rangle_f = 2m_\phi f_\pi (\bar{e}^* \cdot p_{D_s}) A_0^{D_s \phi}(m_{\pi}^2),$$

$$\langle \phi \pi^+ | (\bar{u}c)(\bar{s}s)|D^+ \rangle_f = m_\phi f_\phi (\bar{e}^* \cdot p_{D}) F_1^{D \pi}(m_{\pi}^2).$$

\footnote{Note that our definition of $\chi_1$ and $\chi_2$ is different from $r_1$ and $r_2$ defined in [3] by a factor of 2.}
where $\varepsilon_\mu$ is the polarization vector of the $\phi$ meson, and we have followed Ref.[4] for the definition of form factors. The nonfactorizable contributions have the expressions

$$
\langle \phi \pi^+ | \frac{1}{2} (\bar{u} \lambda^a d) (\bar{s} \lambda^a c) | D_s^+ \rangle = 2m_\phi f_\pi (\varepsilon^* \cdot p_{D_s}) A_0^{nf}(m_\pi^2),
$$

$$
\langle \phi \pi^+ | \frac{1}{2} (\bar{u} \lambda^a c) (\bar{s} \lambda^a s) | D^+ \rangle = m_\phi f_\phi (\varepsilon^* \cdot p_\phi) F_1^{nf}(m_\phi^2),
$$

with the superscript $nf$ referring to nonfactorizable contributions. It is clear that

$$
\chi_1(D_s^+ \to \phi \pi^+) = \frac{A_0^{nf}(m_\pi^2)}{A_0^{D_s \phi}(m_\pi^2)}, \quad \chi_2(D^+ \to \phi \pi^+) = \frac{F_1^{nf}(m_\phi^2)}{F_1^{D \pi}(m_\phi^2)}.
$$

That is, $\chi$ simply measures the fraction of nonfactorizable contributions to the form factor under consideration.

Although we do not know how to calculate $\chi_1$ and $\chi_2$ from first principles, we do anticipate that [3]

$$
|\chi(B \to PP)| < |\chi(D \to PP)| < |\chi(D \to VP)|,
$$

based on the reason that nonperturbative soft gluon effects become more important when the final-state particles move slower, allowing more time for significant final-state interactions after hadronization. As a consequence, it is obvious that $a_{1,2}$ are in general not universal and that the rule of discarding $1/N_c$ terms [5], which works empirically well in $D \to K\pi$ decay, cannot be safely extrapolated to $B \to D\pi$ decay as $|\chi(B \to D\pi)|$ is expected to be much smaller than $|\chi(D \to K\pi)| \sim -\frac{1}{3}$ (the c.m. momentum in $D \to K\pi$ being 861 MeV, to be compared with 2307 MeV in $B \to D\pi$) and hence a large cancellation between $1/N_c$ and $\chi(B \to D\pi)$ is not expected to happen. The recent CLEO observation [6] that the rule of discarding $1/N_c$ terms is not operative in $B \to D(D^*)\pi(\rho)$ decays is therefore not stunning. Only the fact that $\chi(B \to D\pi)$ is positive turns out to be striking.

Unlike the $PP$ or $VP$ case, it is not pertinent to define $\chi_{1,2}$ for $M \to VV$ decay as its general amplitude consists of three independent Lorentz scalars:

$$
A[M(p) \to V_1(\varepsilon_1, p_1)V_2(\varepsilon_2, p_2)] \propto \varepsilon^\mu_\mu(\lambda_1)\varepsilon^\nu_\nu(\lambda_2)(\hat{A}_1 g^{\mu\nu} + \hat{A}_2 p^\mu p^\nu + i\hat{V} e^{\mu\nu\alpha\beta} p_1 p_2),
$$

where $\hat{A}_1$, $\hat{A}_2$, $\hat{V}$ are related to the form factors $A_1$, $A_2$ and $V$ respectively. Since $a \text{ priori}$ there is no reason to expect that nonfactorizable terms weight in the same way to $S$-, $P$- and $D$-waves, namely $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V$, we thus cannot define $\chi_1$ and $\chi_2$. Consequently, it is in general not possible to define an effective $a_1$ or $a_2$ for $M \to VV$ decays once nonfactorizable effects are taken into account [7]. In the factorization approach, the fraction of polarization, say $\Gamma_L/\Gamma$ ($L$: longitudinal polarization) in $B \to \psi K^*$ decay, is independent
of the parameter \(a_2\). As a result, if an effective \(a_2\) can be defined for \(B \to \psi K^*\), it will lead to the conclusion that nonfactorizable terms cannot affect the factorization prediction of \(\Gamma_L/\Gamma\) at all. It was realized recently that all the known models in the literature in conjunction with the factorization hypothesis fail to reproduce the data of \(\Gamma_L/\Gamma\) or both [8,9]. Evidently, if we wish to utilize nonfactorizable effects to resolve the puzzle with \(\Gamma_L/\Gamma\), a key ingredient will be the nonexistence of an effective \(a_2\) for \(B \to \psi K^*\).

In short, there are two places where the factorization hypothesis can be unambiguously tested: (i) To extract the parameters \(a_1\) and \(a_2\) from the experimental measurements of \(M \to PP, VP\) to see if they are process independent. (ii) To measure the fraction of longitudinal polarization in \(M \to VV\) decay and compare with the factorization prediction. Any failure of them will indicate a breakdown of factorization.

The purpose of the present paper is threefold. (i) The parameters \(\chi_1\) and \(\chi_2\) have been extracted in Ref.[3] (see also [10]). Here we wish to update the values of \(\chi_{1,2}\) using the \(q^2\) dependence of form factors suggested by QCD-sum-rule calculations and other theoretical arguments. (ii) It was recently advocated by Kamal and Sandra [7] that the assumption that in \(B \to \psi K^*\) decay the nonfactorizable amplitude contributes only to \(S\)-wave final states, namely \(A_1^{nf} \neq 0, A_2^{nf} = V^{nf} = 0\), will lead to a satisfactory explanation of the data of \(\Gamma(B \to \psi K^*)/\Gamma(B \to \psi K)\) and \(\Gamma_L/\Gamma\). We would like to show that this very assumption is also essential for understanding the ratio \(B(B^- \to D^{*0}\rho^-)/B(\bar{B}^0 \to D^{*+}\rho^-)\), which cannot be explained satisfactorily in previous work. (iii) Contrary to the \(B\) meson case, we will demonstrate that the assumption of \(S\)-wave dominated nonfactorizable terms does not work in \(D \to VV\) decay.

2. Nonfactorizable contributions in \(D, B \to PP, VP\) decays

Because of the presence of final-state interactions (FSI) and the nonspectator contributions (\(W\)-exchange and \(W\)-annihilation), it is generally not possible to extract the nonfactorization parameters \(\chi_{1,2}\) except for a very few channels. Though color-suppressed decays, for example, \(D^0 \to \bar{K}^0(K^{*0})\pi^0(\rho^0)\) are conventionally classified as Class II modes [11], color-flavored decay \(D^0 \to K^-\pi^+\) will bring some important contribution to \(D^0 \to \bar{K}^0\pi^0\) via FSI. This together with the small but not negligible \(W\)-exchange amplitude renders the determination of \(a_2\) from \(D^0 \to \bar{K}^0\pi^0\) impossible. Therefore, in order to determine \(a_1\) and especially \(a_2\) we should focus on the exotic channels e.g. \(D^+ \to \bar{K}^0\pi^+, \pi^+\pi^0\), and the decay modes with one single isospin component, e.g. \(D^+ \to \pi^+\phi, D^+_s \to \pi^+\phi\), where nonspectator contributions are absent and FSI are presumably negligible.
We next write down the relations between $\chi_{1,2}$ and form factors

$$\chi_1(D \rightarrow \bar{K}\pi) = \frac{F_0^{nf}(m_2^2)}{F_0^{DK}(m_2^2)}, \quad \chi_2(D \rightarrow \bar{K}\pi) = \frac{F_0^{nf}(m_K^2)}{F_0^{D\pi}(m_K^2)},$$

$$\chi_1(D \rightarrow \bar{K}^*\pi) = \frac{A_0^{nf}(m_2^2)}{A_0^{DK^*}(m_2^2)}, \quad \chi_2(D \rightarrow \bar{K}^*\pi) = \frac{F_1^{nf}(m_K^2)}{F_1^{D\pi}(m_K^2)},$$

$$\chi_1(D \rightarrow \bar{K}\rho) = \frac{F_1^{nf}(m_2^2)}{F_1^{DK}(m_2^2)}, \quad \chi_2(D \rightarrow \bar{K}\rho) = \frac{A_0^{nf}(m_K^2)}{A_0^{D\rho}(m_K^2)},$$

$$\chi_1(D_s^+ \rightarrow \phi\pi^+) = \frac{A_0^{nf}(m_2^2)}{A_0^{D_s\phi}(m_2^2)}, \quad \chi_2(D^+ \rightarrow \phi\pi^+) = \frac{F_1^{nf}(m_2^2)}{F_1^{D\pi}(m_2^2)}.$$  (10)

It is clear that only the three form factors $F_0$, $F_1$ and $A_0$ entering into the decay amplitudes of $M \rightarrow PP$, $VP$. A consideration of the heavy quark limit behavior of the form factors suggests that the $q^2$ dependence of $F_1$ ($A_2$) is different from that of $F_0$ ($A_0$ and $A_1$) by an additional pole factor [12]. Indeed, QCD-sum-rule calculations have implied a monopole behavior for $F_1(q^2)$ [13-16] and an approximately constant $F_0$ [15]. With a dipole form factor $A_2$, as shown by a recent QCD-sum-rule analysis [16], we will thus assume a monopole behavior for $A_0$.

Unlike the decays $D^+ \rightarrow \pi^+\phi$, $D_s^+ \rightarrow \pi^+\phi$ which are described by a single quark diagram, we cannot extract $\chi_{1,2}$ from the data of $D^+ \rightarrow \bar{K}^0\pi^+$, $\bar{K}^0\rho^+$, $\bar{K}^*\rho^+$ alone without providing further information. For example, the decay amplitude of $D^+ \rightarrow \bar{K}^0\pi^+$ reads

$$A(D^+ \rightarrow \bar{K}^0\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} [a_1(m_D^2 - m_K^2) f_{\pi} F_0^{DK}(m_2^2) + a_2(m_D^2 - m_\rho^2) f_{K} F_0^{D\pi}(m_K^2)],$$  (11)

which consists of external $W$-emission and internal $W$-emission amplitudes. We will therefore make a plausible assumption that $\chi_1 \sim \chi_2$ so that $\chi(D \rightarrow \bar{K}\pi)$ can be determined from the measured rate of $D^+ \rightarrow \bar{K}^0\pi^+$. Since the extraction procedure is already elucidated in Ref.[3], here we will simply present the results (only the central values being quoted) followed by several remarks

$$\chi_2(D \rightarrow \bar{K}\pi) \simeq -0.36,$$

$$\chi_2(D \rightarrow \bar{K}^*\pi) \simeq -0.61,$$

$$\chi_2(D^+ \rightarrow \phi\pi^+) \simeq -0.44,$$  (12)

where we have used the following quantities:

$$c_1(m_c) = 1.26, \quad c_2(m_c) = -0.51,$$

$$f_{\pi} = 132 \text{ MeV}, \quad f_K = 160 \text{ MeV}, \quad f_{K^*} = 220 \text{ MeV}, \quad f_{\phi} = 237 \text{ MeV},$$

$$F_0^{DK}(0) = F_1^{DK}(0) = 0.77 \pm 0.04 \ [17], \quad F_0^{D\pi}(0) = F_1^{D\pi}(0) = 0.83 \ [18],$$

$$A_1^{DK^*}(0) = 0.61 \pm 0.05, \quad A_2^{DK^*}(0) = 0.45 \pm 0.09 \ [17], \quad \Rightarrow A_0^{DK^*}(0) = 0.70,$$  (13)
and the Particle Data Group [19] for the decay rates of various decay modes.

Several remarks are in order. (i) As pointed out by Soares [10], the solutions for \( \chi \) are not uniquely determined. For example, the other possible solution for \( \chi_2(D \to K\pi) \) is \(-1.18\). To remove the ambiguities, we have assumed that nonfactorizable corrections are small compared to the factorizable ones. (ii) Assuming \( A_{0}^{D_{K}}(0) = A_{0}^{D_{K}^*}(0) \), we find from the decay \( D^+ \to \bar{K}^0 \rho^+ \) that \( \chi(D \to \bar{K}\rho) \approx -1.5 \), which is unreasonably too large. We do not know how to resolve this problem except for noting that thus far there is only one measurement of this decay mode [20]. (iii) To determine \( \chi_1(D^+_s \to \phi\pi^+) \) requires a better knowledge of the form factor \( A_{0}^{D_{\phi}} \) and the branching ratio of \( D^+_s \to \phi\pi^+ \). Unfortunately, a direct measurement of them is still not available. Assuming \( A_{0}^{D_{\phi}}(0) \approx A_{0}^{D_{K}^*}(0) \) and \( B(D^+_s \to \phi\pi^+) = (3.5 \pm 0.4)\% \) [19], we get \( \chi_1(D^+_s \to \phi\pi^+) \approx -0.60 \). So in general nonfactorizable terms are process or class dependent, and satisfy the relation \( |\chi(D \to PP)| < |\chi(D \to VP)| \) as expected. (iv) Since \( \chi_2(D \to \bar{K}\pi) \) is close to \(-\frac{1}{3}\), it is evident that a large cancellation between \( 1/N_c \) and \( \chi_2(D \to \bar{K}\pi) \) occurs. This is the dynamic reason why the large-\( N_c \) approach works well for \( D \to \bar{K}\pi \) decay. However, this is no longer the case for \( D \to VP \) decays. The predicted branching ratios in \( 1/N_c \) expansion are

\[
\begin{align*}
B(D^+ \to \bar{K}^{*0}\pi^+) &= 0.3\%, & B(D^+ \to \bar{K}^0\rho^+) &= 16\%, \\
B(D^+ \to \bar{K}^{*0}\rho^+) &= 17\%, & B(D^+ \to \phi\pi^+) &= 0.4\%,
\end{align*}
\]

(14) to be compared with data [19]

\[
\begin{align*}
B(D^+ \to \bar{K}^{*0}\pi^+ \text{expt}) &= (2.2 \pm 0.4)\%, & B(D^+ \to \bar{K}^0\rho^+ \text{expt}) &= (6.6 \pm 2.5)\%, \\
B(D^+ \to \bar{K}^{*0}\rho^+ \text{expt}) &= (4.8 \pm 1.8)\%, & B(D^+ \to \phi\pi^+ \text{expt}) &= (0.67 \pm 0.08)\%.
\end{align*}
\]

(15)

Consider the decay \( D^+ \to \bar{K}^{*0}\pi^+ \) as an example. Its amplitude is given by

\[
A(D^+ \to \bar{K}^{*0}\pi^+) = \sqrt{2}G_FV_{cs}V_{ud}[a_1f_{\pi}m_{K}\cdot A_{0}^{D_{K}^*}(m_{\pi}^2) + a_2f_{K}\cdot m_{K}\cdot F_{1}^{D\pi}(m_{K}^2)].
\]

(16)

Since the interference is destructive and \( f_{K}\cdot F_{1}^{D\pi} > f_{\pi}A_{0}^{D_{K}^*} \), a large \( |a_2| \) is needed in order to enhance the branching ratio of \( D^+ \to \bar{K}^{*0}\pi^+ \) from 0.3% to 2.2%. (Note that \( a_1 \) is relatively insensitive to the nonfactorizable effects.) This in turn implies a negative \( (\chi_1 + \chi_2) \) and hence \( \chi_2(D \to \bar{K}\pi) < -\frac{1}{3} \). Therefore, we are led to conclude that the leading \( 1/N_c \) expansion cannot be a universal approach for the nonleptonic weak decays of the meson. However, the fact that substantial nonfactorizable effects which contribute destructively with the subleading \( 1/N_c \) factorizable contributions are required to accommodate the data of charm decay means that, as far as charm decays are concerned, the large-\( N_c \) approach greatly improves the naive factorization method in which \( \chi_{1,2} = 0 \); the former approach amounts to having a universal nonfactorizable term \( \chi_{1,2} = -1/N_c \).
We next turn to $B \to D(D^*)\pi(\rho)$ decays. Though both nonspectator and FSI effects are known to be important in charm decays, it is generally believed that they do not play a significant role in bottom decays as the decay particles are moving fast, not allowing adequate time for FSI. This gives the enormous advantage that it is conceivable to determine $a_1$ and $a_2$ separately from $B \to D(D^*)\pi(\rho)$ decays. Using the heavy-flavor-symmetry approach for heavy-light form factors and assuming a monopole extrapolation for $F_1$, $A_0$, $A_1$, a dipole behavior for $A_2$, $V$, and an approximately constant $F_0$, as suggested by QCD-sum-rule calculations and some theoretical arguments [21], we found from the CLEO data that [21]

\begin{align}
  a_1(B \to D^{(*)}\pi(\rho)) &= 1.01 \pm 0.06, \\
  a_2(B \to D^{(*)}\pi(\rho)) &= 0.23 \pm 0.06. 
\end{align}  \tag{17}

Taking $c_1(m_b) = 1.11$ and $c_2(m_b) = -0.26$ leads to

\begin{align}
  \chi_1(B \to D^{(*)}\pi(\rho)) &\simeq 0.05, \\
  \chi_2(B \to D^{(*)}\pi(\rho)) &\simeq 0.11.  \tag{18}
\end{align}

Since $(\chi_{1,2} + \chi_{1,2}) = (a_{1,2} - c_{1,2})/c_{2,1}$ and $|c_2| < < |c_1|$, it is clear that the determination of $\chi_1$ is far more uncertain than $\chi_2$: it is very sensitive to the values of $a_1$, $c_1$ and $c_2$.

We see from (18) that nonfactorizable effects become less important in $B$ decays, as what expected [see (8)]. However, a positive $\chi_2(B \to D(D^*)\pi(\rho))$, which is necessary to explain the constructive interference in $B^- \to D^0(D^{*0})\pi^- (\rho^-)$ decays, appears to be rather striking.

A recent light cone QCD-sum-rule calculation [22] following the framework outlined in [23] fails to reproduce a positive $\chi_2(B \to D\pi)$. This tantalizing issue should be resolved in the near future.

For $B \to \psi K$ decays, we found [21]

\begin{align}
  |a_2(B^- \to \psi K^-)| &= 0.235 \pm 0.018, \\
  |a_2(B^0 \to \psi K^0)| &= 0.192 \pm 0.032. \tag{19}
\end{align}

The combined value is

\begin{align}
  a_2(B \to \psi K) &= 0.225 \pm 0.016,  \tag{20}
\end{align}

where its sign should be positive, as we have argued in [21]. (It was advocated by Soares [10] that an analysis of the contribution of $B \to \psi K$ to the decay $B \to K\ell^+\ell^-$ can be used to remove the sign ambiguity of $a_2$.) It follows that

\begin{align}
  \chi_2(B \to \psi K) &= \frac{F_1^{\psi K}(m_{\psi}^2)}{F_1^{BK}(m_{\psi}^2)} \simeq 0.10,  \tag{21}
\end{align}

\begin{footnote}
Contrary to the charmed meson case, the variation of $a_{1,2}$ from $B \to D\pi$ to $D^*\pi$ and $D\rho$ decays is negligible (see Table IV of [21]).
\end{footnote}
which is in accordance with $\chi_2(B \to D^{(*)}\pi(\rho))$.

Finally, it is very interesting to note that, in contrast to charm decays, the large-$N_c$ approach is even worse than the naive factorization method in describing $B \to D(D^{*})\pi(\rho)$ decays as $\chi_2(B \to D^{(*)}\pi(\rho))$ is small but positive.

3. Nonfactorizable contributions in $B \to \psi K^*$, $D^*\rho$ decays

As stressed in the Introduction, in general one cannot define $\chi_{1,2}$ and hence an effective $a_{1,2}$ for $M \to VV$ decays unless the nonfactorizable terms weight in the same manner in all three partial waves. It was pointed out recently that there are two experimental data, namely the production ratio $R \equiv \Gamma(B \to \psi K^*)/\Gamma(B \to \psi K)$ and the fraction of longitudinal polarization $\Gamma_L/\Gamma$ in $B \to \psi K^*$, which cannot be accounted for simultaneously by all commonly used models within the framework of factorization [8,9]. The experimental results are

$$R = 1.74 \pm 0.39 [6], \quad \frac{\Gamma_L}{\Gamma} = 0.78 \pm 0.07,$$

where the latter is the combined average of the three measurements:

$$\left(\frac{\Gamma_L}{\Gamma}\right)_{B \to \psi K^*} = \begin{cases} 0.97 \pm 0.16 \pm 0.15, & \text{ARGUS [24]}; \\ 0.80 \pm 0.08 \pm 0.05, & \text{CLEO [6]}; \\ 0.66 \pm 0.10^{+0.08}_{-0.10}, & \text{CDF [25]}. \end{cases}$$

Irrespective of the production ratio $R$, all the existing models fail to produce a large longitudinal polarization fraction [8,9]. This strongly implies that the puzzle with $\Gamma_L/\Gamma$ can only be resolved by appealing to nonfactorizable effects. However, if the relation $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V$ holds, then an effective $a_2$ can be defined for $B \to \psi K^*$ and the prediction of $\Gamma_L/\Gamma$ will be the same as that in the factorization approach as the polarization fraction is independent of $a_2$. Consequently, nonfactorizable terms should contribute differently to $S$-, $P$- and $D$-wave amplitudes if we wish to explain the observed $\Gamma_L/\Gamma$.

The large longitudinal polarization fraction observed by ARGUS and CLEO suggests that the decay $B \to \psi K^*$ is almost all $S$-wave. To see this, we write down the $B \to \psi K^*$

---

3 An interesting observation was made recently in [26] that the factorization assumption in $B \to \psi K(K^*)$ is not ruled out and the data can be accommodated by the heavy-flavor-symmetry approach for heavy-light form factors provided that the $A_1(q^2)$ form factor is frankly decreasing. To our knowledge, a decreasing $A_1$ with $q^2$ is ruled out by several recent QCD-sum-rule analyses (see e.g. [16]). Using the same approach for heavy-light form factors but the $q^2$ dependence of form factors given in [21], we found that $R = 1.84$ and $\Gamma_L/\Gamma = 0.56$ [21]. Evidently, the factorization approach is still difficult to explain the observed large polarization fraction.
amplitude
\[ A[B(p) \to \psi(p_1)K^*(p_2)] = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left( c_2 + \frac{c_1}{3} \right) f_\psi m_\psi \varepsilon^*_\mu(\psi) \varepsilon^*_\nu(K^*) [\hat{A}_1 g^{\mu\nu} + \hat{A}_2 p^\mu p^\nu] + iV_\epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}, \tag{24} \]

with
\[ \hat{A}_1 = (m_B + m_{K^*}) A^{BK^*}(m_\psi^2) \left[ 1 + \kappa \frac{A^{nf}(m_\psi^2)}{A^{BK^*}(m_\psi^2)} \right], \]
\[ \hat{A}_2 = -\frac{2}{(m_B + m_{K^*})} A^{BK^*}(m_\psi^2) \left[ 1 + \kappa \frac{A^{nf}(m_\psi^2)}{A^{BK^*}(m_\psi^2)} \right], \tag{25} \]
\[ \hat{V} = -\frac{2}{(m_B + m_{K^*})} V^{BK^*}(m_\psi^2) \left[ 1 + \kappa \frac{V^{nf}(m_\psi^2)}{V^{BK^*}(m_\psi^2)} \right], \]

and \( \kappa = c_1/(c_2 + \frac{1}{3}c_1) \). It is easily seen that we will have an effective \( a_2 = c_2 + c_1(\frac{1}{3} + \chi_2) \) if the nonfactorizable terms happen to satisfy the relation \( A^{nf}_1/A^{nf}_1 = A^{nf}_2/A^{nf}_2 = V^{nf}/V = \chi_2 \). The decay rate of this mode is of the form
\[ \Gamma(B \to \psi K^*) \propto (a - b\tilde{x})^2 + 2(1 + c^2\tilde{y}^2), \tag{26} \]

where
\[ a = \frac{m_B^2 - m_\psi^2 - m_{K^*}^2}{2m_\psi m_{K^*}}, \quad b = \frac{2m_B^2 p_c^2}{m_\psi m_{K^*}(m_B + m_{K^*})^2}, \quad c = \frac{2m_B p_c}{(m_B + m_{K^*})^2}, \]
\[ \tilde{x} = \frac{A^{BK^*}(m_\psi^2) + \kappa A^{nf}(m_\psi^2)}{A^{BK^*}(m_\psi^2) + \kappa A^{nf}(m_\psi^2)}, \quad \tilde{y} = \frac{V^{BK^*}(m_\psi^2) + \kappa V^{nf}(m_\psi^2)}{A^{BK^*}(m_\psi^2) + \kappa A^{nf}(m_\psi^2)}, \tag{27} \]

with \( p_c \) being the c.m. momentum. The longitudinal polarization fraction is then given by
\[ \frac{\Gamma_L}{\Gamma} = \frac{(a - b\tilde{x})^2}{(a - b\tilde{x})^2 + 2(1 + c^2\tilde{y}^2)}. \tag{28} \]

If the decay is an almost \( S \)-wave, one will have \( \Gamma_L/\Gamma \sim a^2/(a^2 + 2) = 0.83 \). Since \( \kappa >> 1 \), \( \tilde{x} \) \((D\text{-wave})\) and \( \tilde{y} \) \((P\text{-wave})\) can be suppressed by assuming that, as first postulated in [7], in \( B \to \psi K^* \) decay the nonfactorizable amplitude contributes only to \( S \)-wave final states; that
is, \[ A_1^{nf} \neq 0, \quad A_2^{nf} = V^{nf} = 0. \] (29)

The rational for this assumption is given in [7].

With the assumption (29), the branching ratio followed from (24) is

\[ \mathcal{B}(B \to \psi K^*) = 0.0288 \left( c_2 + \frac{c_1}{3} \right) A_1^{BK^*}(m_{\psi}^2) \left[ (a\xi - bx)^2 + 2(\xi^2 + c^2y^2) \right] \] (30)

with

\[ x = \frac{A_2^{BK^*}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}, \quad y = \frac{V^{BK^*}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}, \quad \xi = 1 + \frac{\kappa A_1^{nf}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}, \] (31)

where uses of \(|V_{cb}| = 0.040\) and \(\tau(B) = 1.52 \times 10^{-12}\)s have been made. It follows that

\[ \frac{\Gamma_L}{\Gamma} = \frac{(a\xi - bx)^2}{(a\xi - bx)^2 + 2(1 + c^2y^2)}. \] (32)

We use the measured branching ratio \(\mathcal{B}(B \to \psi K^*) = (0.172 \pm 0.030)\% [6]\) to determine the ratio \(A_1^{nf}(m_{\psi}^2)/A_1^{BK^*}(m_{\psi}^2)\), which is found to be

\[ \frac{A_1^{nf}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)} \simeq 0.08, \] (33)

which we have used \(A_2^{BK^*}(m_{\psi}^2) = 0.41, A_2^{BK^*}(m_{\psi}^2) = 0.36, V^{BK^*}(m_{\psi}^2) = 0.72 [21]\) and discarded the other possible solution \(A_1^{nf}/A_1^{BK^*} = -0.22\) for its “wrong” sign, recalling that \(F_1^{nf}/F_1^{BK}\) is positive [cf. Eq.(21)]. The predicted longitudinal polarization fraction is \(\Gamma_L/\Gamma = 0.73\), which is in accordance with experiment.

The assumption of negligible nonfactorizable contributions to \(P\)- and \(D\)-waves also turns out to be essential for understanding the decay rate of \(B^- \to D^{*0}\rho^-\) or the ratio \(R_4 \equiv \mathcal{B}(B^- \to D^{*0}\rho^-)/\mathcal{B}(B^0 \to D^{*+}\rho^-)\). The issue arises as follows. In Ref.[21] we have determined \(a_1\) and \(a_2\) from \(B \to D\pi, D^*\pi, D\rho\) decays and obtained a consistent ratio \(a_2/a_1\) from \(NF\)-effects. A different approach for nonfactorizable effects adopted in Ref.[27] amounts to \(A_1^{nf} = A_2^{nf} = 0\) and \(V^{nf} \neq 0\). It follows from Eq.(28) that

\[ \frac{\Gamma_L}{\Gamma} = \frac{(a - bx)^2}{(a - bx)^2 + 2(1 + c^2y^2)}, \]

with \(\bar{y} = (V^{BK^*}(m_{\psi}^2) + \kappa V^{nf}(m_{\psi}^2))/A_1^{BK^*}(m_{\psi}^2)\) and \(x\) being defined in (31). It is clear that in order to get a large longitudinal polarization fraction one needs a negative \(V^{nf}/V\) ! Using the numerical values \(a = 3.164, b = 1.304, x = 0.89,\) we find \((\Gamma_L/\Gamma)_{max} = 0.67\). The prediction \(\Gamma_L/\Gamma = 0.65\) given by [27] is one standard deviation from experiment (22).
channel to channel: $0.24 \pm 0.10$, $0.24 \pm 0.14$, $0.21 \pm 0.08$ (see Table IV of [21]). Assuming factorization, we get $a_2/a_1 = 0.34 \pm 0.13$ from $B \to D^*\rho$ decay, which deviates somewhat from above values. In the presence of $S$-wave dominated nonfactorizable contributions, it is no longer possible to define an effective $a_1$ and $a_2$ for $B \to D^*\rho$ decay. Therefore, the quantities to be compared with are $A_1^{nf}/A_1$ in $B \to D^*\rho$ decay and $\chi_2$ in $B \to D\pi, D^*\pi, D\rho$. A straightforward calculation yields (see [21] for the factorizable case)

$$R_4 = \frac{\tau(B^-)}{\tau(B^0)}\left(1 + 2\eta \frac{H_1}{H} + \eta^2 \frac{H_2}{H}\right), \quad (34)$$

with

$$\begin{align*}
H &= (\hat{a} \hat{c} - \hat{b}\hat{\hat{x}})^2 + 2(\hat{c^2} + \hat{\hat{y}}^2), \\
H_1 &= (\hat{a} \hat{c} - \hat{b}\hat{\hat{x}})(\hat{a} \hat{c}' - \hat{\hat{y}}\hat{\hat{x}'}) + 2(\hat{c \hat{c}'} + \hat{\hat{y}}\hat{\hat{y}'}) , \\
H_2 &= (\hat{a} \hat{c}' - \hat{\hat{y}}\hat{\hat{x}'})^2 + 2(\hat{c^2} + \hat{\hat{y}}^2), \\
\eta &= \frac{m_D(m_B + m_\rho)}{m_\rho(m_B + m_{D^*})} \frac{f_{D^*} A_1^{B1\rho}(m_D^2) c_2 + \frac{1}{3} c_1}{f_\rho A_1^{BD^*}(m_\rho^2) c_1 + \frac{2}{3} c_2}, \\
\hat{\hat{c}} &= 1 + \frac{c_2}{c_1 + \frac{2}{3} c_2} \frac{A_1^{nf}(m_\rho^2)}{A_1^{BD^*}(m_\rho^2)}, \\
\hat{\hat{c}'} &= 1 + \frac{c_1}{c_2 + \frac{2}{3} c_1} \frac{A_1^{nf}(m_D^2)}{A_1^{BD^*}(m_D^2)},
\end{align*}$$

(35)

where $\hat{a}$, $\hat{b}$, $\hat{c}$ are obtained from $a$, $b$, $c$ respectively in (27), $\hat{x}$, $\hat{y}$ from $x$, $y$ in (31) by replacing $\psi \to D^*$, $K^* \to \rho$, and $\hat{\hat{x}'}$, $\hat{\hat{y}'}$ are obtained from $\hat{\hat{x}}$, $\hat{\hat{y}}$ respectively by replacing $D^* \leftrightarrow \rho$; for instance $\hat{x}' = A_2^{B\rho}(m_D^2)/A_1^{B\rho}(m_D^2)$. Assuming $A_1^{nf}/A_1^{BB} \sim A_1^{nf}/A_1^{B\rho}$ and fitting (34) to the experimental value $R_4 = (1.68 \pm 0.35)\%$ [6], we get

$$\frac{A_1^{nf}(m_D^2)}{A_1^{B\rho}(m_D^2)} \sim \frac{A_1^{nf}(m_\rho^2)}{A_1^{BD^*}(m_\rho^2)} \simeq 0.12. \quad (36)$$

We see that the $S$-wave dominated nonfactorizable effect in $B \to \psi K^*$ and $B \to D^*\rho$ decays is of order $10\%$, consistent with $\chi_2(B \to \psi K)$ and $\chi_2(B \to D(D^*)\pi(\rho))$.

4. Nonfactorizable contributions in $D \to \bar{K}^*\rho$ decay

We have shown in the previous section that $S$-wave dominated nonfactorizable terms are needed to explain the large longitudinal polarization fraction observed in $B \to \psi K^*$ and the ratio $\mathcal{B}(B^- \to D^{*0}\rho^-)/\mathcal{B}(\bar{B}^0 \to D^{*+}\rho^+)$). However, we shall see in this section that the assumption (29) is no longer applicable to $D \to \bar{K}^*\rho$ decay. An experimental measurement of $D^+ \to \bar{K}^{*0}\rho^+$ and $D^0 \to \bar{K}^{*0}\rho^0$ by Mark III [28] shows that (i) the decay $D^+ \to \bar{K}^{*0}\rho^+$ is a mixture of longitudinal and transverse polarization consistent with a pure
and (ii) $D^0 \to K^{*0} \rho^0$ is almost all transverse, requiring a cancellation between the longitudinal $S$-wave and $D$-wave.

We first consider the decay $D^+ \to K^{*0} \rho^+$, whose amplitude is given by

$$A(D^+(p) \to K^{*0}(p_1)\rho^+(p_2)) = \frac{G_F}{\sqrt{2}} V_{us}^{*} V_{ud}^{*} (K^*) \xi_{\rho}^{*}(\rho) [\tilde{A}_1 g^{\mu\nu} + \tilde{A}_2 p^{\mu} p^{\nu} + i\tilde{V} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}], \quad (38)$$

where

$$\tilde{A}_1 = \left( c_1 + \frac{c_2}{3} \right) f_{\rho} m_{\rho} (m_D + m_{K^*}) \left( 1 + \frac{c_2}{c_1 + \frac{1}{3} c_2 A_1^{DK^*}(m_{\rho}^2)} \right) A_1^{DK^*}(m_{\rho}^2)$$

$$+ \left( c_2 + \frac{c_1}{3} \right) f_{K^*} m_{K^*} (m_D + m_{\rho}) \left( 1 + \frac{c_1}{c_2 + \frac{1}{3} c_1 A_1^{D\rho}(m_{K^*}^2)} \right) A_1^{D\rho}(m_{K^*}^2), \quad (39)$$

and $\tilde{A}_2 (V)$ is obtained from $\tilde{A}_1$ with the replacements $A_1 \to A_2 (A_1 \to V)$, $(m_D + m_{K^*}) \to -2/(m_D + m_{K^*})$ and $(m_D + m_{\rho}) \to -2/(m_D + m_{\rho})$. Since $A_1^{nf}/A_1^{DK^*}$ and $A_1^{nf}/A_1^{D\rho}$ are expected to be negative [see Eq.(12)], it is obvious that if nonfactorizable terms are dominated by the $S$-wave, it will imply a more severe destructive interference in the $S$-wave amplitude than in $P$- and $D$-wave amplitudes, in contradiction to the observation that this decay is almost all $S$-wave. The branching ratio is calculated to be

$$B(D^+ \to \bar{K}^{*0} \rho^+) = 0.10 \left| \left( c_1 + \frac{1}{3} c_2 \right) A_1^{DK^*}(m_{\rho}^2) \right|^2 \left( H' + 2\eta'H_1' + \eta^2 H_2' \right), \quad (40)$$

with the expressions of $\eta'$, $H'$, $H_{1,2}'$ analogous to $\eta$, $H$, $H_{1,2}$ in (35). A fit of (40) to the Mark III data for the branching ratio (37) gives rise to (assuming $A_1^{nf}/A_1^{DK^*} \sim A_1^{nf}/A_1^{D\rho}$)

$$\frac{A_1^{nf}(m_{\rho}^2)}{A_1^{DK^*}(m_{\rho}^2)} \sim \frac{A_1^{nf}(m_{K^*}^2)}{A_1^{D\rho}(m_{K^*}^2)} \approx -0.98, \quad (41)$$

which is uncomfortably too large. Moreover, the $P$-wave branching ratio is predicted to be $2.0 \times 10^{-2}$, in disagreement with experiment [28]

$$B(D^+ \to \bar{K}^{*0} \rho^+)_{P\text{-wave}} < 0.5 \times 10^{-2}. \quad (42)$$

It thus appears to us that an almost $S$-wave $D^+ \to \bar{K}^{*0} \rho^+$ implies that

$$\left| \frac{A_2^{nf}}{A_2^{DK^*}(\rho)} \right|, \left| \frac{V^{nf}}{V^{DK^*}(\rho)} \right| \leq \left| \frac{A_1^{nf}}{A_1^{DK^*}(\rho)} \right|. \quad (43)$$

The other measurement by E691 [29] disagrees severely with Mark III on the branching ratio

$$B(D^+ \to \bar{K}^{*0} \rho^+) = \left\{ \begin{array}{ll} (4.8 \pm 1.2 \pm 1.4)\%, & \text{Mark III [28];} \\ (2.3 \pm 1.2 \pm 0.9)\%, & \text{E691 [29].} \end{array} \right. \quad (37)$$

Recall that model calculations tend to give a very large branching ratio of 17% [see Eq.(14)].

A fit to the E691 measurement (37) for the branching ratio yields an even larger value: $A_1^{nf}/A_1^{DK^*} \sim A_1^{nf}/A_1^{D\rho} \approx -1.41$. 

\[12\]
Taking $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V = \chi(D \to \bar{K}\pi)$ as an illustration, we obtain

$$\chi(D \to \bar{K}\pi) \approx -0.65$$

and $B(D^+ \to \bar{K}^*\rho^+)_{P\text{-wave}} = 2.0 \times 10^{-3}$, which are certainly more plausible than before.

Another indication for the failure of the $S$-wave dominated hypothesis for nonfactorizable effects comes from the decay $D^0 \to \bar{K}^0\rho^0$, where $\bar{K}^0$ and $\rho^0$ are completely transversely polarized, implying a large $D$-wave which is compensated by the longitudinal $S$-wave. Recall that the factorizable $D \to VV$ amplitudes have the sailent feature:

$$|S\text{-wave amplitude}| > |P\text{-wave amplitude}| > |D\text{-wave amplitude}|.$$  

Since the color-suppressed $D$-wave amplitude of $D^0 \to \bar{K}^0\rho^0$ is proportional to $[1 + c_1/(c_2 + \frac{1}{3}c_1)](A_2^{nf}/A_2^{D\rho})$, a large $D$-wave thus indicates a negative $A_2^{nf}/A_2$ and

$$\left|\frac{A_1^{nf}}{A_1}\right| < \left|\frac{A_2^{nf}}{A_2}\right|, \quad \text{or} \quad \frac{A_1^{nf}}{A_1} \approx 0, \quad \frac{A_2^{nf}}{A_2} \neq 0.$$  

Therefore, we see that nonfactorizable terms in charm decay are consistently to be negative [cf. Eqs.(12) and (44)]. Unfortunately, at this point we cannot make a further quantitative analysis due to unknown final-state interactions and $W$-exchange contributions. A measurement of helicities in $D^0 \to \bar{K}^0\rho^0$, $D^+ \to \phi\rho^+$ will be greatly helpful to pin down the issue. In particular, the color- and Cabibbo-suppressed mode $D^+ \to \phi\rho^+$ is very ideal for this purpose since it is not subject to FSI and nonspectator effects. A polarization measurement in this decay is thus strongly urged (though difficult) in order to test if $A_2^{nf}/A_2$ plays a more essential role than $A_1^{nf}/A_1$ in charm decay.

5. Discussion and conclusion

The factorization assumption for hadronic weak decays of mesons can be tested on two different grounds: (i) to extract the effective parameters $a_1$ and especially $a_2$ from $M \to PP, VP$ decays to see if they are process independent, and (ii) to measure helicities in $M \to VV$ decay. Using the $q^2$ dependence of form factors suggested by QCD-sum-rule calculations and by some theoretical arguments, we have updated our previous work. It is found that $a_2$ is evidently not universal in charm decay. The parameter $\chi_2$, which measures the nonfactorizable soft-gluon effect on the color-suppressed decay amplitude relative to the factorizable one, ranges from $-\frac{1}{3}$ to $-0.60$ in the decays from $D \to \bar{K}\pi$ to $D^+ \to \phi\pi^+$, $D \to \bar{K}\pi\pi$. By contrast, the variation of $a_2$ in $B \to \psi K$, $B \to D(D^*)\pi(\rho)$ is negligible and nonfactorizable terms $\chi_2(B \to \psi K)$, $\chi_2(B \to D^{(*)}\pi(\rho))$ are of order 10% with a positive sign. The pattern for the relative magnitudes of nonfactorizable effects

$$|\chi(B \to PP, VP)| < |\chi(D \to PP)| < |\chi(D \to VP)|$$

13
is thus well established. This means that nonperturbative soft gluon effects become more important when the final states are less energetic, allowing more time for final-state interactions. This explains why $a_2$ is class ($PP$ or $VP$ mode) dependent in charm decay, whereas it stays fairly stable in $B$ decay.

Taking factorization as a benchmark, we see that the nonfactorizable terms necessary for describing nonleptonic $D$ and $B$ decays are in opposite directions from the factorization framework. On the one hand, the leading $1/N_c$ expansion, which amounts to a universal $\chi = -\frac{1}{3}$, improves the naive factorization method for charm decays. On the other hand, the naive factorization hypothesis works better than the large-$N_c$ assumption for $B$ decays because nonfactorizable effects are small, being of order 10%. The fact that $\chi$ is positive makes it even more clear that the large-$N_c$ approach cannot be extrapolated from $D$ to $B$ physics. Theoretically, the next important task for us is to understand why $\chi$ is negative in $D$ decay, while it becomes positive in $B$ decay.

As for $M \to VV$ decay, a priori effective $a_{1,2}$ cannot be defined since, as pointed out by Kamal and Sandra, its amplitude (factorizable and nonfactorizable) involves three independent Lorentz scalars, corresponding to $S$, $P$ and $D$ waves. This turns out to be a nice trade-off for solving the puzzle with the large longitudinal polarization fraction $\Gamma_L/\Gamma$ observed in $B \to \psi K^*$, which cannot be accounted for by the factorization hypothesis or by nonfactorizable effects weighted in the same way in all three partial waves, namely $A_{1n}^f/A_1 = A_{2n}^f/A_2 = V_{nf}/V$. A large $\Gamma_L/\Gamma$ can be achieved if $B \to \psi K^*$ is almost all $S$-wave, implying that nonfactorizable contributions are dominated by the $S$-wave. The same assumption is also needed for understanding the ratio $B(B^+ \to D^{*0}\rho^-)/B(\bar{B}^0 \to D^{*+}\rho^-)$. We found that all nonfactorizable terms $A_{1n}^{nf}/A_{1}^{BKK}$, $A_{1n}^{nf}/A_{1}^{B\rho}$, $A_{1n}^{nf}/A_{1}^{BD}$ are of order 10% consistent with $\chi_2(B \to D(D^*)\pi(\rho))$ and $\chi_2(B \to \psi K)$.

Surprisingly, the assumption of $S$-wave dominated nonfactorizable effects is not operative in $D \to \bar{K}\rho$ decay, which exhibits again another disparity between $B$ and $D$ physics. We found that $A_2^{nf}/A_2$ should play a more pivotal role than $A_1^{nf}/A_1$ in charm decay. We thus urge experimentalists to measure helicities in the color- and Cabibbo-suppressed decay mode $D^+ \to \phi\rho^+$ decay to gain insight in the nonfactorizable effects in $D \to VV$ decay.

ACKNOWLEDGMENT

This work was supported in part by the National Science Council of ROC under Contract No. NSC84-2112-M-001-014.
REFERENCES

1. See e.g. H.Y. Cheng, *Int. J. Mod. Phys.* A4, 495 (1989).

2. N. Deshpande, M. Gronau, and D. Sutherland, *Phys. Lett.* 90B, 431 (1980); M. Gronau and D. Sutherland, *Nucl. Phys.* B183, 367 (1981).

3. H.Y. Cheng, *Phys. Lett.* B335, 428 (1994).

4. M. Wirbel, B. Stech, and M. Bauer, *Z. Phys.* C29, 637 (1985).

5. A.J. Buras, J.-M. Gérard, and R. Rückl, *Nucl. Phys.* B268, 16 (1986).

6. CLEO Collaboration, M.S. Alam et al., *Phys. Rev.* D50, 43 (1994).

7. A.N. Kamal and A.B. Santra, Aberta Thy-31-94 (1994).

8. M. Gourdin, A.N. Kamal, and X.Y. Pham, *Phys. Rev. Lett.* 73, 3355 (1994).

9. R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, DAPNIA/SPP/94-24, LPTHE-Orsay 94/15 (1994).

10. J.M. Soares, TRI-PP-94-78 (1994).

11. M. Bauer, B. Stech, and M. Wirbel, *Z. Phys.* C34, 103 (1987).

12. Q.P. Xu, *Phys. Lett.* B306, 363 (1993).

13. C.A. Dominguez and N. Paver, *Z. Phys.* C41, 217 (1988); A.A. Ovchinnikov, *Sov. J. Nucl. Phys.* 50, 519 (1989); *Phys. Lett.* B229, 127 (1989); V.L. Chernyak and I.R. Zhitnitski, *Nucl. Phys.* B345, 137 (1990); S. Narison, *Phys. Lett.* B283, 384 (1992); V.M. Belyaev, A. Khodjamirian, and R. Rückl, *Z. Phys.* C60, 349 (1993); P. Colangelo, BARI-TH/93-152 (1993).

14. P. Ball, V.M. Braun, and H.G. Dosch, *Phys. Lett.* B273, 316 (1991); *Phys. Rev.* D44, 3567 (1991); P. Ball, *Phys. Rev.* D48, 3190 (1993).

15. P. Colangelo and P. Santorelli, *Phys. Lett.* B327, 123 (1994).

16. K.C. Yang and W-Y.P. Hwang, NUTHU-94-17 (1994).

17. M. Witherell, in *Proceedings of the XVI International Symposium on Lepton-Photon Interactions*, Ithaca, 10-15 August 1993, eds. P. Drell and D. Rubin (AIP, New York, 1994).
18. L.L. Chau and H.Y. Cheng, *Phys. Lett.* **B333**, 514 (1994).

19. Particle Data Group, *Phys. Rev.** D50*, 1173 (1994).

20. Mark III Collaboration, J. Adler *et al.*, *Phys. Lett.* **B196**, 107 (1987).

21. H.Y. Cheng and B. Tseng, IP-ASTP-21-94, hep-ph/9409408 (revised) (1994).

22. I. Halperin, TECHNION-PHY94-16 (1994).

23. B. Blok and M. Shifman, *Nucl. Phys.** B389*, 534 (1993).

24. ARGUS Collaboration, H. Albrecht *et al.*, DESY 94-139 (1994).

25. CDF Collaboration, FERMILAB Conf-94/127-E (1994).

26. M. Gourdin, Y.Y. Keum, and X.Y. Pham, PAR/LPTHE/95-01 (1995).

27. C.E. Carlson and J. Milana, WM-94-110 (1994).

28. Mark III Collaboration, D. Coffman *et al.*, *Phys. Rev.** D45*, 2196 (1992).

29. E691 Collaboration, J.C. Anjos *et al.*, *Phys. Rev.** D46*, 1941 (1992).