Kelvin-Helmholtz instability of magnetized plasmas with surface tension and dust particles

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Abstract. We investigate the linear Kelvin-Helmholtz (K-H) instability of two streaming magnetized plasmas considering the effects of surface tension and suspended dust particles. The usual magnetohydrodynamic (MHD) equations are considered for the present configuration including the effects of suspended dust particles and surface tension. The general dispersion relation is obtained using the normal mode analysis by applying the appropriate boundary conditions. The dispersion relation is modified due to the presence of surface tension, magnetic field and suspended dust particles for three-dimensional perturbations. It is found that the effect of magnetic field appears in the dispersion relation due to consideration of three-dimensional perturbations of the present problem of K-H instability with suspended dust particles and surface tension. It is also found that the condition of K-H instability is affected by the presence of suspended dust particles, surface tension and magnetic field. Numerical calculations have been performed to show the effect of magnetic field, mass concentration and relaxation frequency of suspended dust particles on the growth rate of K-H instability. It is found that magnetic field, mass concentration and relaxation frequency of suspended dust particles have stabilizing influence on the growth rate of K-H instability.

Introduction
The Kelvin-Helmholtz (K-H) instability arises at the plane interface of two superposed fluids when they flow over each other with a relative velocity. It is of much importance in discussions of many phenomena in different media viz. solar and magnetosphere dynamics and astrophysical jet simulation, the magnetic confinement, tokamak edge plasma and in astrophysical plasmas [1,2]. Chandrasekhar [3] has investigated the problem of K-H instability taking the effects of surface tension, rotation, variable viscosity, magnetic field and many other important parameters. He has given the conditions of K-H instability and K-H stability with these effects. Along with this, the presence of surface tension may have application in laboratory and cosmical plasmas with region of different plasma densities and temperature [4]. Alterman [5] has investigated the fundamental K-H instability including the effects of rotation and surface tension of two superposed plasmas. Bhatia and Steiner [6] have explored the problem of K-H discontinuity of two superposed plasmas considering the effect of collision with neutral particles and magnetic field both transverse and longitudinal to the direction of the flow. Sharma and Spanos [7] have investigated the K-H instability of two streaming fluids in a porous media considering the effect of surface tension. The K-H instability of partially ionized plasma in porous medium including effect of collisions, magnetic field and surface tension is investigated by Sunil [8]. Recently, Shadmehri and Downes [9] have discussed the K-H instability in a weakly ionized bounded medium. Watson et al. [10] have studied the K-H instability of weakly ionized unbounded medium.
Thus we find that the investigation of the problem of K-H instability is a current subject of further research. 

In the recent spacecraft observations we find that suspended dust particles play comprehensive role in the dynamics of weather. Along with this, Michael [11] has investigated the K-H instability of a dusty gas using Stokes drag force formula. El-Sayed [12] has discussed the linear hydromagnetic K-H instability of fluid particle flow in oldroydian viscoelastic porous media taking the effects of suspended particles and uniform magnetic field. Sanghvi and Chhajlani [13] have discussed the problem of hydromagnetic K-H instability including suspended dust particles and finite Larmor radius (FLR) corrections. El-Sayed [14] has reviewed and discussed the hydromagnetic transverse K-H instability of two highly viscous fluid flows with suspended particles.

Thus from the above discussed problems, in the present paper the effects of surface tension, suspended dust particles and magnetic field on K-H instability of two streaming plasmas are investigated.

**Linearized perturbation equations of the problem**

We consider two semi-infinite homogeneous fluid plasmas separated by a plane interface of negligible thickness at \( z = 0 \). Let the mixture of the magnetized plasmas and suspended dust particles streaming together in the presence of a uniform external magnetic field \( B(\hat{B}, \hat{B}, 0) \) with flow velocity \( U(0, 0, 0) \) and a downward gravitational field \( g(0, 0, -g) \) (see figure 1).

![Figure 1. Schematic diagram of the configuration.](image)

Let \( N \) denote the number density of the suspended dust particles. It is supposed that the net effect of the suspended dust particles on the fluid plasmas is equivalent to extra body force. The relevant linearized MHD equations of the configuration can be written as

\[
\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla \delta \rho + g \delta \rho + KN (\nu - q) + \frac{1}{4\pi} (\nabla \times \delta B) \times \mathbf{B} \\
+ \sum T_i \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \delta x, \delta (z - z_i),
\]

(1)
where \( q(u, v, w), v, \delta B(\delta B_1, \delta B_2, \delta B_3), \delta p \) and \( \delta \rho \) denote the perturbations in the fluid velocity, velocity of suspended dust particles, magnetic field, fluid pressure and density respectively. \( \tau = m/k \) is the relaxation frequency of the suspended dust particles.

**Dispersion relation**

Assuming all the perturbed quantities vary as
\[
\exp(i k x + n y + m z + i t).
\]  

On simplifying (1)-(7) with the help of (8) we get the following relation for \( w \)
\[
[D(p \, D) - k^2 \rho \, w]/[n + ik x + \alpha \rho (n + i k x)U] + (B_x k_x + B_y k_y)^2/(4\pi (n + i k x)U)(D^2 - k^2)w + gk^2[Dp - k^2/\beta \sum \delta(z - z_s)]w/(n + ik x)U = 0,
\]
where \( D = d/\beta z \) and \( \alpha \rho = mN/\rho \) is the mass concentration of the suspended dust particles.

Consider the case of two superposed fluids of densities \( \rho_1 \) (lower fluid) and \( \rho_2 \) (upper fluid), separated by horizontal boundary at \( z = 0 \). Let the velocities of streaming of the two fluids be \( U_1(U_1, 0, 0) \) and \( U_2(U_2, 0, 0) \) then, in each of the two regions of constant densities, (9) becomes
\[
(D^2 - k^2)w = 0.
\]

Since \( w \) must be bounded both when \( z \to -\infty \) (in the lower fluid) and \( z \to +\infty \) (in the upper fluid), the solutions of (10) can be written as
\[
w_1 = A(n + ik x)U_1 \exp(kz) \quad (z < 0),
\]
\[
w_2 = A(n + ik x)U_2 \exp(-kz) \quad (z > 0),
\]
where \( A \) is the constant.

Following Chandrasekhar [3], the boundary conditions across the interface of the two fluids are

i. The normal component of the velocity is continuous, we get
\[
(w_1/(n + ik x)U_1) = (w_2/(n + ik x)U_2).
\]

ii. The total pressure should be continuous. This condition can be obtained by integrating (9) across the interface \( z = 0 \).

iii. The normal component of the magnetic field is continuous. This reduces to condition (i).

To satisfy the boundary conditions, integrating (9) across the interface \( z = 0 \), we obtain
measuring the mass concentration and the relaxation time of the suspended dust particles. If the effect of the suspended dust particles enters into the dispersion relation (15) through two parameters tension on the hydromagnetic K-H instability of two streaming magnetized plasmas. It is clear that the two plasmas are moving relative to each other with same flow velocity having equal densities i.e. the condition of K-H instability we reduce this dispersion relation for special case by assuming that the FLR corrections in that case. If we ignore the effect of transverse magnetic field then dispersion relation (15) reduces to Sunil [8] excluding the effect of porosity in that case with same behavior of collision effect taken by them and suspended dust particles taken by us in the present problem. Thus the present result is the improvement of the problem of K-H instability of two streaming magnetized plasmas flowing over each other with uniform velocities including the effect of suspended dust particles.

\[
\Delta_0 \left[ \rho D w \left[ n + i k_x U + \frac{\alpha_0 (n + i k_x U)}{\tau (n + i k_x U) + 1} \right] \right] + \frac{(B_x k_x + B_y k_y)^2}{4\pi} \Delta_0 \left[ \frac{D w}{(n + i k_x U)} \right] 
\]

(14)

\[
+ g k^2 \left[ \Delta_0 (p) - \frac{k^2 T}{g} \right] \left[ \frac{w}{(n + i k_x U)} \right] = 0,
\]

where \( \Delta_0 (f) \) is the jump that a quantity \( f \) experiences at the interface \( z = 0 \) and \( \left[ w / (n + i k_x U) \right]_{z=0} \) is the unique value that this quantity has at \( z = 0 \).

Using the values of \( w_I \) and \( w_J \) from (11) and (12) in (14), we obtain the following dispersion relation

\[
n^2 + 2 i n k_i \left( \beta_i U_1 + \beta_2 U_2 \right) - k_i^2 \left( \beta_i U_1^2 + \beta_2 U_2^2 \right) + 2(\beta_A + k_i V_B)^2
\]

\[
+ \frac{\alpha_i \beta_i (n + i k_x U_1)^2}{\tau (n + i k_x U_1) + 1} + \frac{\alpha_2 \beta_2 (n + i k_x U_2)^2}{\tau (n + i k_x U_2) + 1} - g k \left[ \beta_i - \frac{k^2 T}{g(p_1 + p_2)} \right] = 0,
\]

(15)

where \( \alpha_1 = mN / \rho_1, \alpha_2 = mN / \rho_2, \beta_1 = \rho_1 / (\rho_1 + \rho_2), \beta_2 = \rho_1 / (\rho_1 + \rho_2), \beta_A = \rho_A / (\rho_1 + \rho_2), \beta_B = \rho_B / 4\pi (\rho_1 + \rho_2) \).

The dispersion relation (15) represents the influence of the suspended dust particles and surface tension on the hydromagnetic K-H instability of two streaming magnetized plasmas. It is clear that the effect of the suspended dust particles enters into the dispersion relation (15) through two parameters \( \alpha_0 \) and \( \tau \) measuring the mass concentration and the relaxation time of the suspended dust particles. If we ignore the effects of suspended dust particles, surface tension and transverse magnetic field \( (B_x = 0 \text{ or } V_B = 0) \) then dispersion relation (15) reduces to Chandrasekhar [3]. These results are also identical to Bhatia and Steiner [6] if we remove the effect of surface tension and consider the similar behavior of collision effect taken by them and suspended dust particles taken by us in the present problem. In the absence of transverse magnetic field, surface tension and considering perturbation only in \( y \)-direction \( (k_y = 0) \), dispersion relation (15) reduces to Sanghvi and Chhajlani [13] excluding the FLR corrections in that case. If we ignore the effect of transverse magnetic field then dispersion relation (15) reduces to Sunil [8] excluding the effect of porosity in that case with same behavior of collisions taken by Sunil [8] and suspended dust particles taken in the present problem. Thus the present result is the improvement of the problem of K-H instability of two streaming magnetized plasmas with magnetic field both in transverse and longitudinal directions, and surface tension with suspended dust particles.

**Discussion**

The dispersion relation given by (15) is complex and in order to study the effects of all the parameters on the condition of K-H instability we reduce this dispersion relation for special case by assuming that the two plasmas are moving relative to each other with same flow velocity having equal densities i.e. \( U_1 = -U_2 = U \) and \( \rho_1 = \rho_2 \), therefore \( \beta_1 = \beta_2 = 1/2 \). Thus we obtain the simple dispersion relation by introducing the relaxation frequency of suspended dust particles \( f_r (= 1/\tau) \). We get

\[
n^2 + f_r (2 + \alpha_0) n^2 + \left[ f_r^2 (1 + \alpha_0) + 2(\beta_A V_A + k_i V_B)^2 + T k^3 / \rho \right] n^2
\]

\[
+ \left[ 4(\beta_A V_A + k_i V_B)^2 + k_i^2 U^2 (\alpha_0 - 2) \right] f_r n
\]

\[
+ \left\{ f_r^2 + k_i^2 U^2 \right\} \left[ 2(\beta_A V_A + k_i V_B)^2 - k_i^2 U^2 + T k^3 / 2 \rho \right] - \alpha_0 f_r^2 k_i^2 U^2 = 0.
\]

Equation (17) gives the dispersion relation for K-H instability of two streaming magnetized plasmas flowing over each other with uniform velocities including the effect of suspended dust
particles and surface tension. In absence of transverse magnetic field \( (V_B = 0) \) and considering perturbation only in y-direction \((k_y = 0)\) this dispersion relation reduces to El-Sayed [14] excluding the effect of viscosity and FLR corrections in that case. Hence the present results are the improvement of El-Sayed [14] with magnetic field and surface tension in absence of viscosity and FLR corrections.

The condition of K-H instability can be easily obtained from the constant term of (17), we get

\[
\left( f_s^2 + k^2 U^2 \right) \left[ 2(k_x V_A + k_y V_B)^2 + \frac{Tk^3}{2\rho} \right] < k_x^2 U^2 \left[ f_s^2 (1 + \alpha_0) + k_x^2 U^2 \right]
\]

Equation (18) represents the condition of K-H instability of two streaming plasmas of same densities and flow velocities including the effect of magnetic field, suspended dust particles and surface tension. The system will obviously be unstable for the condition given by (18). In absence of suspended dust particles and magnetic field the condition (18) reduces to Sharma and Spanos [7] excluding porosity with same densities of the fluids in that case. Thus the condition of K-H instability is modified due to the presence of suspended dust particles and magnetic field without porosity.

We now find the derivative of the growth rate of the unstable K-H mode \((n_u)\) with increasing relaxation frequency of suspended dust particles \((f_s)\). Thus from (17) we get

\[
\frac{dn_u}{df_s} = \frac{-n_u^2 (2 + \alpha_0) + 2 f_s (1 + \alpha_0) n_u + n_u [4(k_x V_A + k_y V_B)^2 - k_x^2 U^2 (2 - \alpha_0)]}{4n_u^2 + 2n_u f_s (2 + \alpha_0) + 2n_u [f_s^2 (1 + \alpha_0) + 2(k_x V_A + k_y V_B)^2] + Tk^3 / \rho}.
\]

In writing (19) we have assumed the fact that \(\alpha_u\) cannot exceed 1. Let us now consider the inequalities

\[
n_u^2 (2 + \alpha_0) + 2 f_s (1 + \alpha_0) n_u + n_u [4(k_x V_A + k_y V_B)^2] + 2 f_s [Tk^3 / 2\rho + 2(k_x V_A + k_y V_B)^2] \geq k_x^2 U^2 [n_u (2 - \alpha_0) + 2 f_s (1 + \alpha_0)]
\]

and

\[
4n_u^2 + 3n_u f_s (2 + \alpha_0) + 2n_u [f_s^2 (1 + \alpha_0) + 2(k_x V_A + k_y V_B)^2] + Tk^3 / \rho
+ 4 f_s (k_x V_A + k_y V_B)^2 \geq k_x^2 U^2 (2 - \alpha_0),
\]

The above two inequalities show the effects of magnetic field, surface tension and suspended dust particles on the conditions of K-H instability and K-H stability. These conditions can be easily reduced with old results. In the absence of magnetic field and surface tension these inequalities reduce to Sanghvi and Chhajlani [13]. Thus we find that magnetic filed and surface tension both modifies these inequalities. If both upper signs of (20) and (21) are satisfied simultaneously we find that \(dn_u / df_s\) is negative and if the upper and lower signs or vice versa hold then \(dn_u / df_s\) turns out to be positive. Thus we find that suspended dust particles and magnetic field reduce the growth rate of the considered system of K-H instability.

In order to study the effect of magnetic field, relaxation frequency and mass concentration of suspended dust particles on the growth rate of K-H instability, we write dispersion relation (17) in dimensionless form by considering perturbations along the direction of flow (i.e. \(k_x = k_x\) and \(k_y = 0\)).

We have taken some dimensionless parameters in terms of flow velocity. Thus in the absence of surface tension (17) is written in dimensionless form as

\[
n^* \left[ n^* f_s^2 (2 + \alpha_0) + n^* [2V_A^2 + f_s^2 (1 + \alpha_0)] + n^* f_s^2 (4V_A^2 + \alpha_0 - 2)
+ [(2V_A^2 - 1) + f_s^2 (1 + \alpha_0) - \alpha_0 f_s^2] \right] = 0,
\]

where \(n^* = n / kU\), \(f_s^* = f_s / kU\) and \(V_A^* = V_A / U\).
In figures 2 and 3, we have depicted the growth rate of K-H instability versus the relaxation frequency of suspended dust particles \((n^* \text{ v/s } f_s^*)\) for different values of magnetic field and mass concentration of suspended dust particles respectively. From the figures we find that on increasing the value of magnetic field, the growth rate of K-H instability and peak value both decreases. It is further noticed that on increasing relaxation frequency of suspended dust particles i.e. size of suspended dust particles \((f_s = 6\pi\mu a / m)\), the growth rate decreases. A similar behavior of the mass concentration is

**Figure 2.** The growth rate of K-H instability (positive real roots of \(n^*\)) versus relaxation frequency of suspended dust particles \((f_s^*)\) for different values of magnetic field \((V_{\lambda}^*)\). The value of \(a_0\) is taken 0.5 to be constant.

**Figure 3.** The growth rate of K-H instability (positive real roots of \(n^*\)) versus relaxation frequency of suspended dust particles \((f_s^*)\) for different values of mass concentration of suspended dust particles \((\alpha_0)\). The value of \(V_{a}^*\) is taken 0.5 to be constant.
found on the growth rate of K-H instability. In this case the peak value is unaffected by the presence of mass concentration of suspended dust particles. Hence magnetic field, mass concentration and relaxation frequency of suspended dust particles all have stabilizing influence on the growth rate of the linear K-H instability.

Conclusions
In the present paper, a linear analysis of the effect of suspended dust particles on the problem of K-H instability is carried out in the presence of surface tension and uniform magnetic field both longitudinal and transverse to the direction of flow using three-dimensional perturbations of the system. The medium is assumed to be incompressible and certain simplifying assumptions have been made for the motion of the suspended dust particles in the fluid plasmas. A general dispersion relation is obtained for such a medium using the appropriate boundary conditions. We find that the effect of magnetic field and surface tension appears in the dispersion relation due to the consideration of three-dimensional perturbations of the system. The condition of K-H instability is also affected by the presence of magnetic field, suspended dust particles and surface tension due to consideration of three-dimensional perturbations of the problem. The change in growth rate with respect to relaxation frequency of the suspended dust particles is presented and conditions of K-H stability and K-H instability are discussed which depend upon magnetic field, surface tension and suspended dust particles. From the curves it is found that magnetic field, relaxation frequency and mass concentration of suspended dust particles all have stabilizing influence on the growth rate of the linear K-H instability.

Thus in the present paper we have studied the effects of surface tension and suspended dust particles on the K-H instability of two streaming magnetized incompressible plasmas.

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