Constraining the equation of state of the Universe from distant Type Ia supernovae and cosmic microwave background anisotropies

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Accepted 1999 July 21. Received 1999 July 8; in original form 1999 April 26

ABSTRACT

The magnitude–redshift relation for Type Ia supernovae is beginning to provide strong constraints on the cosmic densities contributed by matter, \( \Omega_m \), and a cosmological constant, \( \Omega_\Lambda \), although the results are highly degenerate in the \( \Omega_m–\Omega_\Lambda \) plane. Here we estimate the constraints that can be placed on a cosmological constant or quintessence-like component by extending supernovae samples to high redshift. Such measurements, when combined with constraints from anisotropies in the cosmic microwave background (CMB), could provide an important consistency check of systematic errors in the supernovae data. A large campaign of high-z supernovae observations with 10-m class telescopes could constrain \( \Omega_m \) to an accuracy (1σ) of 0.06 and \( \Omega_\Lambda \) to 0.15. A sample of supernovae at redshift \( z \approx 3 \), as might be achievable with a Next Generation Space Telescope, could constrain \( \Omega_m \) to an accuracy of about 0.02 independently of the value of \( \Omega_\Lambda \). The constraints on a more general equation of state, \( wQ^r \), converge slowly as the redshift of the supernovae data is increased. The most promising way of setting accurate constraints on \( wQ^r \) is by combining high-z supernovae and CMB measurements. With feasible measurements it should be possible to constrain \( wQ^r \) to a precision of about 0.06, if the Universe is assumed to be spatially flat. We use the recent supernovae sample of Perlmutter et al. and observations of the CMB anisotropies to constrain the equation of state in quintessence-like models via a likelihood analysis. The 2σ upper limits are \( wQ^r < -0.6 \) if the Universe is assumed to be spatially flat, and \( wQ^r < -0.4 \) for universes of arbitrary spatial curvature. The upper limit derived for a spatially flat Universe is close to the lower limit \( wQ^r < -0.7 \) allowed for simple potentials, implying that additional fine tuning may be required to construct a viable quintessence model.

Key words: supernovae: general – cosmic microwave background – large-scale structure of Universe.

1 INTRODUCTION

The possible discovery of an accelerating Universe from observations of Type Ia supernovae (Perlmutter et al. 1999; Riess et al. 1999a) has led to a resurgence of interest in the possibility that the Universe is dominated by a cosmological constant (for a recent review see Turner 1999). A number of authors have shown how observations of distant Type Ia supernovae (SN) can be combined with observations of cosmic microwave background (CMB) anisotropies to constrain the cosmological constant and matter density of the Universe (White 1998; Eisenstein et al. 1998; Lineweaver 1998; Garnavich et al. 1998; Efstathiou & Bond 1999; Tegmark 1999; Efstathiou et al. 1999). For example, Efstathiou et al. (1999, hereafter E99) combine the large SN sample of the Supernova Cosmology Project (Perlmutter et al. 1999a, hereafter P99; we will refer to these supernovae as the SCP sample) with a compilation of CMB anisotropy measurements and find \( \Omega_m = 0.25^{+0.18}_{-0.12} \) and \( \Omega_\Lambda = 0.63^{+0.17}_{-0.23} \) (95 per cent confidence errors) for the cosmic densities contributed by matter and a cosmological constant, respectively. These results are consistent with a number of other measurements, including dynamical measurements of \( \Omega_m \), the large-scale clustering of galaxies and the abundances of rich clusters of galaxies (Turner 1999; Bridle et al. 1999; Wang et al. 1999).

It is worth emphasizing that these results (and those presented in Section 4 below) assume that systematic errors in the data are unimportant. At present there is no firm evidence for any significant systematic errors in the supernovae data, but it is extremely difficult to exclude some possibilities, for example extinction by grey dust (Aguirre 1999). There is some tentative evidence that the rise times of distant supernovae are smaller than those of nearby supernovae (Riess et al. 1999b), but it is unclear...
whether this effect, even if real, introduces a systematic error in the peak luminosities.

The observational evidence for an accelerating Universe has stimulated interest in more general models containing a component with an arbitrary equation of state, \( p/\rho = w_0 \) with \( w_0 \geq -1 \). Examples include a dynamically evolving scalar field (see, e.g., Ratra & Peebles 1988 and Caldwell, Dave & Steinhardt 1998, who have dubbed such a component ‘quintessence’; we will refer to this as a ‘Q’ component hereafter) and a frustrated network of topological defects (Spergel & Pen 1997; Bucher & Spergel 1999). In particular, Steinhardt, Wang & Zlatev (1999) have pointed out that for a wide class of potentials, the evolution of a Q-like scalar field follows ‘tracking solutions’, in which the late time evolution is almost independent of initial conditions.

The purpose of this paper is three-fold. First, to illustrate how the constraints on \( \Omega_m \) and \( \Omega_\Lambda \) can be improved by extending the redshift range of the supernovae samples. At low redshifts, the magnitude–redshift relation is degenerate for models with the same value of deceleration parameter \( q_0 = \frac{1}{2} (\Omega_m - 2\Omega_\Lambda) \). This degeneracy can be broken by observing supernovae at redshifts \( \geq 1 \) (see, e.g., Goobar & Perlmutter 1995). Thus, by extending the redshift range of the current supernovae samples it should be possible to set tighter limits on \( \Omega_m \) and \( \Omega_\Lambda \) independently. This is important because there are significant worries that the SN data may be affected by grey extinction, evolution or some other systematic effects. The consistency of SN constraints on \( \Omega_m \) and \( \Omega_\Lambda \) with those derived from CMB anisotropy measurements would provide an important consistency check of systematic errors in the SN data and the interpretation of the CMB data. Secondly, we estimate the accuracy with which a more general Q component with an arbitrary equation of state, \( \Omega_m \) can be constrained. The last two columns give the errors for a sample twice as large as the SCP sample supplemented by 40 supernovae with mean redshifts of 1.0 and 1.5. We adopt a background cosmology with \( \Omega_m + \Omega_\Lambda = 1.0 \) and \( \Omega_\Lambda = 0.53 \) (corrected for a dynamically evolving scalar field). The consistent with \( \Omega_m \) and \( \Omega_\Lambda \) assuming a uniform prior distribution. (The components of the new Fisher matrix after marginalization are given by \( F_{ij} = \frac{1}{2}\exp^{-\frac{1}{2}C_{ij}} \).

In this section we assume that the parameters \( s_k \) are \( \Omega_m \), \( \Omega_\Lambda \) and \( \Omega_\Lambda \) (defined in equation 1). An estimate of the covariance matrix, \( C_{ij} \), for these parameters for a given SN data set is given by the inverse of the Fisher matrix

\[
\mathcal{L} = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp \left[ -\frac{(m_i - m_{i,\text{pred}})^2}{2\sigma_i^2} \right].
\]

The marginalized error on each parameter (given by \( \sqrt{\lambda_{ij}} \)) is listed in Table 1 for several assumed supernova data sets. The column labelled SCP gives the Fisher matrix errors on \( \Omega_m \), \( \Omega_\Lambda \) and \( \Omega_\Lambda \) derived for sample C (56 supernovae) of P99, i.e. assuming the magnitude errors, intrinsic magnitude scatter and redshift distribution of the real sample. The next two columns give the expected errors for the SCP sample supplemented by 20 supernovae with a peak magnitude error of \( \Delta m = 0.25 \) mag and a Gaussian redshift distribution of dispersion \( \Delta z = 0.5 \) and mean redshift \( \langle z \rangle = 1 \) and 1.5. The upper redshift limit is close to the maximum for feasible spectroscopic measurements with 10-m class telescopes (see Goobar & Perlmutter 1995). As these authors comment, ground-based spectroscopy at optical wavelengths becomes prohibitively expensive for supernovae at higher redshifts because of the strong k correction. The last two columns give the errors for a sample twice as large as the SCP sample supplemented by 40 supernovae with mean redshifts of 1.0 and 1.5. We adopt a background cosmology with \( \Omega_m = 0.63 \) and \( \Omega_\Lambda = 0.25 \) as indicated by the joint-likelihood analysis of the SCP sample and CMB anisotropies described in E99.

From Table 1 we see that the Fisher matrix analysis of the SCP sample gives relatively large errors on \( \Omega_m \) and \( \Omega_\Lambda \). The parameters \( \Omega_\Lambda \) and \( \Omega_\Lambda \) are, of course, highly correlated. This is illustrated in Fig. 1 which shows 1\( \delta \), 2\( \sigma \) and 3\( \sigma \) error ellipses in the \( \Omega_\Lambda - \Omega_m \) plane after marginalizing over \( s_k = M \) assuming a uniform prior distribution. (The components of the new Fisher matrix after marginalization are given by \( F_{ij} = F_{ij} - F_{ij}F_{33}/F_{33} \). The points in the figure show the results of Monte Carlo calculations, where we

### Table 1. Fisher matrix errors, \( \Omega_m \) and \( \Omega_\Lambda \).

| \( \langle z \rangle \) | SCP | SCP + 20 SN | 2×SCP + 40SN |
|----------------|------|-------------|---------------|
| \( \delta \Omega_m \) | 0.53 | 0.130 | 0.092 | 0.057 |
| \( \delta \Omega_\Lambda \) | 0.71 | 0.265 | 0.218 | 0.19 | 0.154 |
| \( \delta \Omega_\Lambda \) | 0.056 | 0.053 | 0.049 | 0.035 | 0.035 |

\[
\mathcal{L} = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp \left[ -\frac{(m_i - m_{i,\text{pred}})^2}{2\sigma_i^2} \right].
\]
Figure 1. The dashed lines in each panel show 1σ, 2σ and 3σ likelihood contours in the \( \Omega_\Lambda - \Omega_m \) plane for the SCP distant supernova sample as analysed by E99. The solid contours are derived from the Fisher matrix (equation 4) for the SCP sample supplemented by 20 SN with a mean redshift of \( \langle z \rangle = 1 \) (a) and for twice the SCP sample and 40 SN with \( \langle z \rangle = 1.5 \) (b). The points show maximum-likelihood values of \( \Omega_\Lambda \) and \( \Omega_m \) for Monte Carlo realizations of these samples, as described in the text.

Table 1 show the Fisher matrix analysis of the SN samples including the constraint imposed by equation (5). As is well known, the combination of SN and CMB measurements can break the degeneracy between \( \Omega_\Lambda \) and \( \Omega_m \), and it should be possible to determine these parameters with an error of less than 0.04 with an enlarged supernova sample assuming, of course, that systematic errors are unimportant.

Although the errors on \( \Omega_\Lambda \) from SN measurements alone converge relatively slowly as the redshift range is increased, consistency of the cosmological parameter estimates provides a strong test of systematic errors in the SN data. If we believe that systematic errors are unimportant, and that our interpretation of the CMB anisotropies (in terms of adiabatic CDM-like models) is correct, then current data already constrain \( \Omega_m \) and \( \Omega_\Lambda \) to high precision (see fig. 5 of E99). Consistency requires that the likelihood contours for a high-redshift supernova sample converge to the same answer.

2.2 Constraining \( \Omega_m \) with NGST

Observations of very distant supernovae at \( z \simeq 3 \) may be possible with a Next Generation Space Telescope (e.g. Miralda-Escude & Rees 1997; Madau 1999; Livio 1999). We will not analyse the feasibility of such observations here. Rather, we note from Figs 1 and 2 that the major axes of the error ellipses in the \( \Omega_\Lambda - \Omega_m \) plane tilt and become more vertical as the redshift range of the SN sample is increased. This is because the magnitude–redshift relations for models with very different values of \( \Omega_\Lambda \) and the same \( \Omega_m \) converge at high redshifts. The convergence redshift depends on \( \Omega_m \) and lies at \( z \simeq 2-4 \) for \( \Omega_m \) in the range 0.2–1 (see fig. 1 of Melnick, Terlevich & Terlevich 1999).

This is illustrated by Fig. 3, which shows the 1σ, 2σ and 3σ likelihood contours determined from the Fisher matrix for a sample consisting of twice the SCP sample, 100 SN with \( \langle z \rangle = 1.5 \), \( \Delta z = 0.5 \), and 40 SN with \( \langle z \rangle = 3 \), \( \Delta z = 1 \). As expected, these contours are almost vertical in the \( \Omega_m - \Omega_\Lambda \) plane. A sample of supernovae (or some other distance indicator such as H II galaxies:
Equation of state from SNe and CMB observations

Melnick et al. 1999) at redshifts \( z \), \( z > 3 \) can therefore produce a tight constraint on \( \Omega_m \) independently of the value of \( \Omega_L \).

\[ \Omega_m = 0.9230 - 0.3850 \Omega_A \]

\[ \Omega_B = 0.3850 + 0.9230 \Omega_A \]

\[ 2 \times \text{SCP} + 40 \text{ SN} < z > = 1.5 \]

\[ \Omega_m = 0.9870 - 0.2540 \Omega_A \]

\[ \Omega_B = 0.2540 + 0.9870 \Omega_A \]

\[ 2 \times \text{SCP} + 100 \text{ SN} < z > = 1.5 + 40 \text{ SN} < z > = 3 \]

3 CONSTRAINTS ON AN ARBITRARY EQUATION OF STATE

In this section, we analyse the constraints that SN can place on an arbitrary equation of state. We first consider a constant equation of state. Models of this type (see Bucher & Spergel 1999) include a frustrated network of cosmic strings \( p/\rho = -1/3 \) and a frustrated network of domain walls \( p/\rho = -2/3 \). A constant equation of state is also a good approximation to a \( Q \) component obeying tracker solutions. Tracker solutions are discussed in Section 3.2. Constraints on generalized forms of dark matter with anisotropic stress are discussed by Hu et al. (1999) and will not be considered here.

3.1 Constant equation of state

If we include a \( Q \)-like component with equation of state

\[ p/\rho = w_Q, \]

\[ q^{1999} \text{ RAS, MNRAS} \]

Figure 1 shows that by extending the redshift range one can determine \( \Omega_m \) independently of \( \Omega_L \).

\[ \Omega_m = 0.9230 - 0.3850 \Omega_A \]

\[ \Omega_B = 0.3850 + 0.9230 \Omega_A \]

\[ 2 \times \text{SCP} + 40 \text{ SN} < z > = 1.5 \]

\[ \Omega_m = 0.9870 - 0.2540 \Omega_A \]

\[ \Omega_B = 0.2540 + 0.9870 \Omega_A \]

\[ 2 \times \text{SCP} + 100 \text{ SN} < z > = 1.5 + 40 \text{ SN} < z > = 3 \]

\[ \Omega_m = 0.9230 - 0.3850 \Omega_A \]

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\[ \Omega_m = 0.9230 - 0.3850 \Omega_A \]

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\[ \Omega_m = 0.9870 - 0.2540 \Omega_A \]

\[ \Omega_B = 0.2540 + 0.9870 \Omega_A \]

\[ 2 \times \text{SCP} + 100 \text{ SN} < z > = 1.5 + 40 \text{ SN} < z > = 3 \]
Table 2 lists the results of a Fisher matrix analysis for a $Q$-like component with a constant $w_Q$. Here we have applied the constraints $w_Q \geq -1$ and $\Omega_Q \geq 0$. The upper table gives results for the supernovae magnitude–redshift relation alone assuming a spatially flat Universe with $\Omega_m = 0.25$ and $w_Q = -1$. The constraints on $w_Q$ from a sample such as the SCP data are quite poor and improve relatively slowly as the sample is extended to higher redshift because of a strong degeneracy between $w_Q$ and $\Omega_m$ in the magnitude–redshift relation. This is illustrated in Fig. 4, which shows the analogue of Fig. 2 for $Q$-like models. As the supernovae sample is extended to higher redshift, the likelihood contours narrow but $w_Q$ and $\Omega_m$ remain strongly degenerate.

The situation is dramatically improved by the addition of constraints from CMB anisotropies. The addition of a $Q$-like component affects the location of the Doppler peaks (see Caldwell et al. 1998; White 1998) and, in analogy with equation (5), an accurate determination of the CMB power spectrum imposes the constraint

$$\Delta \Omega_Q = -\frac{\partial \gamma_D / \partial w_Q}{\partial \gamma_D / \partial \Omega_Q} \Delta w_Q - \frac{\partial \gamma_D / \partial \Omega_m}{\partial \gamma_D / \partial \Omega_Q} \Delta \Omega_m. \tag{7}$$

The second panel of Table 2 shows the constraints derived on an arbitrary equation of state by combining supernovae data with the CMB constraint of equation (7). For spatially flat models, the combination of SN and CMB anisotropies constrains $w_Q$ to an accuracy of better than 0.1, sufficient to set tight constraints on the physical parameters of $Q$-like models (for example, whether one requires contrived potentials, see Section 4). However, the constraints on $w_Q$ improve relatively slowly as the SN sample is extended to higher redshift. Similar conclusions apply if the assumption of a spatially flat Universe is relaxed (see the lower panel of Table 2). In that case, the parameters $\Omega_m$ and $\Omega_Q$ can be determined to high precision, but the constraints on $w_Q$ improve slowly as the SN sample is increased. This implies that it is worth analysing the constraints on $Q$-like models with arbitrary spatial curvature using current SN and CMB data (see Section 4.2).

### 3.2 Time-varying equation of state: tracker solutions

In the previous section we have investigated the simplified case of a constant $w_Q$. If, in fact, the $Q$-like component arises from a slowly rolling scalar field evolving in a potential $V(Q)$, the equation of state of the $Q$ component will vary as a function of time. The equations of motion of the $Q$ field can be written in the following compact form (Steinhardt et al. 1999):

$$\frac{V''V}{(V')^2} = \frac{1 + w_Q - \Omega_Q}{2(1 + w_Q)} - \frac{1 + w_Q - 2\Omega_Q}{2(1 + w_Q)} \frac{\ddot{x}}{6 + \dot{x}}$$

$$= \frac{2}{(1 + w_Q) (6 + \dot{x})^2}, \quad x = \frac{(1 + w_Q)}{(1 - w_Q)}$$

where primes denote derivatives with respect to $Q$, $\dot{x} = d\ln x/d\ln a$, $\ddot{x} = d^2\ln x/d\ln^2 a$ and $a$ is the scale factor of the cosmological model. For a wide class of potentials, and almost independently of the initial conditions, the evolution of $Q$ locks on to a tracking solution in which $Q$ and $w_Q$ vary slowly (see Zlatev et al. 1999; Steinhardt et al. 1999). Examples of the evolution of $w_Q$ and $\Omega_Q$ at late times are shown in Fig. 5 for three forms of the potential $V(Q)$. In each case, the evolution of $w_Q$ at $z \lesssim 4$ is well approximated by

$$w_Q = w_Q(\alpha_0) + \alpha \ln(\alpha/a_0),$$

where $\alpha$ is a small number determined from the value of $\dot{x}$ at the present time.

Fig. 6 shows the relations between $\Omega_Q$, $w_Q$ and $\alpha$ at the present time derived from the solutions to equation (8) for the three potentials considered in Fig. 5. The minimum value of $\alpha$ is approximately $-0.14$, reflecting the fact that $Q$ is evolving relatively slowly even at late times.

With the approximation of equation (9), the energy density of the $Q$ component evolves according to

$$\frac{\rho_Q(a)}{\rho_Q(a_0)} = \left(\frac{a}{a_0}\right)^{-3[1+w_Q(\alpha_0)]} \exp\left(-\frac{3}{2} \alpha \ln(\alpha/a_0)^2\right). \tag{10}$$

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Figure 4. As Fig. 2, but for an arbitrary constant equation of state in a spatially flat Universe. The dashed lines in each panel show 1σ, 2σ and 3σ likelihood contours in the $w_Q$–$\Omega_m$ plane for the SCP distant supernova sample as analysed in Section 4 (assuming a constant equation of state). The solid contours are derived from the Fisher matrix for enhanced samples of high-redshift supernovae and the points show maximum-likelihood contours derived from Monte Carlo realizations of these samples.
Equation of state from SNe and CMB observations

847

Note also that with the approximation of equation (9), the tracker equation (8) becomes an algebraic equation relating $V_00/V_0^2$ to $w_Q$, $V_Q$ and $a \approx w_Q V_Q$ in the matter-dominated era).

A small value of $a$, $\approx 0.1$ to $0.2$ cannot be determined accurately from SN and CMB observations because it is highly degenerate with $w_Q$ and $\Omega_m$. As we will show in the next section, the introduction of the parameter $a$ provides a convenient way of testing the sensitivity of constraints on $Q$-like models to the time evolution of $w_Q$.

We note that Huterer & Turner (1999) have recently proposed a prescription for reconstructing the potential of a $Q$-like component directly from the magnitude–redshift relation of Type Ia supernovae. This approach may produce interesting constraints if the field $Q$ is rapidly evolving at late times. For tracker solutions, however, the equation of state changes so slowly that it may be difficult to distinguish the true potential from a perfectly flat one.

4 LIMITS ON THE EQUATION OF STATE FROM TYPE I A SUPERNOVAE AND THE COSMIC MICROWAVE BACKGROUND

4.1 Spatially flat models

In this section, we use current SN and CMB data to constrain the equation of state of the Universe. The analysis closely follows that presented in E99. We use the sample of 56 Type Ia SN of fit C of P99 and adopt the likelihood analysis described by E99 (including a parametric fit to the luminosity–decline rate correlation), modifying the expression for luminosity distance to incorporate the parameters of the $Q$-like model. The CMB data$^1$ that we use are plotted in fig. 1 of E99.

We perform a likelihood analysis for these data assuming scalar adiabatic perturbations, varying the amplitude of the fluctuation spectrum, the scalar spectral index, the physical densities of the CDM and baryons $\Omega_m h^2$, $\Omega_b h^2$, and the Doppler peak location parameter $\gamma_D$. Modifications to the CMB power spectrum arising from spatial fluctuations in the $Q$ component are ignored, as these are negligible in the slowly evolving $Q$ models considered here (see Caldwell et al. 1998; Huey et al. 1999). We integrate over the CMB likelihood assuming uniform prior distributions of the parameters to compute a marginalized likelihood for $\gamma_D$. The likelihood functions for the parameters $w_Q$, $\Omega_Q$ and $\Omega_m$ presented below are constructed from the expression

$^1$ These consist of anisotropy estimates, window functions and estimates of 68 per cent confidence errors computed using likelihood analyses for the COBE, Tenerife, Python, Argo, QMAP, South Pole, Saskatoon, CAT and OVRO experiments (see E99 for further details). We assume that the CMB data points are independent.

$^2$ $h$ is the Hubble constant in units of $100 \text{km s}^{-1} \text{Mpc}^{-1}$.
for the angular diameter distance to the last scattering surface and the probability distribution of $\gamma_D$.

Fig. 7 shows the constraints on $w_Q$ and $\Omega_m$ for spatially flat universes. The different line types show the constraints for three different values of the parameter $\alpha$ characterizing the evolution of $w_Q$, $\alpha = 0$ (solid lines), $\alpha = -0.1$ (dotted lines) and $\alpha = -0.2$ (dashed lines). As described in the previous section, these values span the range found for tracker solutions for a variety of potentials. These rates of evolution are so low that they have very little effect on the likelihood contours. The constraints plotted in Fig. 7 are in very good agreement with those derived by Garnavich et al. (1998) from an analysis of the HZS sample, and with the analysis of the SCP sample (Perlmutter et al. 1999b) and of the combined HZS and SCP samples (Wang et al. 1999). The fact that the constraints are weakly dependent on the size of the SN sample is a consequence of the strong degeneracy between $w_Q$ and $\Omega_m$, discussed in Section 3.2.

Fig. 7(b) shows the results of combining the SN likelihoods with those determined from the CMB. The likelihood peaks at $w_Q = -1$, $\Omega_m = 0.29$. Qualitatively, these results are similar to those of Perlmutter et al. (1999b); the favoured cosmology has an equation of state $w_Q = -1$, and $w_Q$ is constrained to be less than $-0.6$ at the $2\sigma$ level. However, in detail, the constraints in Fig. 7(b) are somewhat less stringent than those of Perlmutter et al., allowing a broader range in $\Omega_m$ ($0.15 \lesssim \Omega_m \lesssim 0.5$ at the $2\sigma$ level). This is because Perlmutter et al. include constraints on the power spectrum of galaxy clustering based on the data compiled by Peacock \& Dodds (1994).\(^3\) In our view this is dangerous because it requires a specific assumption concerning the distribution of galaxies relative to the mass. Qualitatively, for nearly scale-invariant adiabatic models, galaxy clustering imposes a constraint on the parameter combination $\Gamma' = \Omega_m h$ of $0.2 \leq \Gamma' \leq 0.3$, if galaxies are assumed to trace the mass fluctuations on large scales (Efstathiou, Bond \& White 1992; Maddox, Efstathiou \& Sutherland 1996). Combined with measurements of the Hubble constant (for which Perlmutter et al.

\(^3\)Perlmutter et al. (1999b) do not combine the SN and CMB likelihoods but analyse the SN data assuming a spatially flat Universe.

adopt $h = 0.65 \pm 0.05$), galaxy clustering leads to a constraint of $0.25 \lesssim \Omega_m \lesssim 0.5$, partly breaking the degeneracy between $w_Q$ and $\Omega_m$. The combined SN and CMB analysis in Fig. 7(b) provides constraints that are nearly as tight, but are much less model-dependent.

The constraints of Fig. 7(b) place strong limits on $Q$-like models. For tracking solutions, the constraint $w_Q \lesssim -0.6$ excludes steep potentials [e.g. $V(Q) \propto Q^{-\beta}$, with $\beta \gtrsim 2$] and the data clearly favour a standard cosmological term ($w_Q = -1$). These limits on $w_Q$ are very close to the lower limit ($w_Q \approx -0.7$) allowed for `physically well motivated’ tracker solutions (Steinhardt et al. 1999, i.e. smooth potentials with simple functional forms). With a slight improvement of the observations one may be forced to fine tune the shape of the potential to construct a viable quintessence model.

The constraints of Fig. 7(b) are somewhat stronger than those of Wang et al. (1999), who perform a `concordance analysis’ of $Q$-like models using a number of observational constraints, including those from Type Ia supernovae and CMB anisotropies. These authors deduce limits of $-1 \lesssim w_Q \lesssim -0.4$. The difference is caused by the different methods of statistical analysis. The concordance analysis of Wang et al. leads, by construction, to more conservative limits than the maximum-likelihood analysis and is more robust to systematic errors in any particular data set. However, provided systematic errors are negligible in the CMB and SN data sets, the constraints of Fig. 7(b) derived by combining likelihoods should be realistic. These small differences in the upper limits on $w_Q$ are important because they can place significant restrictions on the physics. As stressed in the previous paragraph, the upper limit of $w_Q \approx -0.6$ places strong constraints on tracker models with simple potentials.

4.2 Models with arbitrary spatial curvature

Fig. 8 shows the results of a likelihood analysis of the SN and CMB data, but now allowing arbitrary spatial curvature. We show two projections of the likelihood distributions, marginalizing over $\Omega_Q$ in Figs 8(a) and (c) and over $w_Q$ in Figs 8(b) and (c). The
Figure 8. Analogue of Fig. 5, but for quintessence models with arbitrary spatial curvature. (a) and (b) show marginalized likelihoods in the $w_Q$–$\Omega_m$ and $\Omega_Q$–$\Omega_m$ planes derived from Type Ia supernovae. (c) and (d) show the combined likelihoods for the Type Ia and CMB anisotropies. As in Fig. 5, the solid contours are derived for $\alpha = 0$ and dashed contours for $\alpha = -0.2$.
ACKNOWLEDGMENTS

I thank Richard Ellis, Paul Steinhardt and Roberto Terlevich for useful discussions and PPARC for the award of a Senior Fellowship. I also thank Sarah Bridle, Anthony Lasenby, Mike Hobson and Graca Rocha for allowing me to use their compilation of CMB anisotropy data.

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