Interpretation of One Dimensional Schlumberger Curve Resistivity Data using "Least Square" Inversion

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Abstract. The geoelectric method using Schlumberger configuration is one of the most widely used geophysical methods. In this study, a one-dimensional Schlumberger configuration will be applied to modeling the subsurface structure using two types of data, namely synthetic data and acquisition data in the field. The purpose of this study is to calculate resistivity ($\rho$) and determine the thickness ($d_i$) of the subsurface layer. In order for the model to be obtained to have a high level of accuracy, inversion least square is used. The value of resistivity and layer thickness obtained are very close to the synthetic model used. Comparison of calculation pseudo resistivity curves and observation pseudo resistivity has a small RMS error value which supports the accuracy of the study. This is obtained by giving initial guesses to the resistivity value of the acquisition data. The final results obtained from this study are in the form of five layers with different resistivity values and with a relatively small RMS error value of 1.15.

Keywords: Geoelectric Method, Two-dimensional Schlumberger Configuration, Inverse Modeling, Resistivity.

1. Introduction

In surveying geoelectric methods there are configurations that are often used, including Schlumberger, Wenner, and Dipole-dipole configurations. Schlumberger configurations that have the deepest range compared to other configurations where the potential electrode distance is fixed, but the distance between the current electrodes is changed.

In geophysics generally known models and model parameters are used to characterize a subsurface geological condition. The process of estimating models and model parameters based on the observed data on the surface of the earth to produce a response that matches the observational data or field data is called modeling.

1.1 Electric Potential on the Earth

If a continuous current is channeled into an isotropic homogeneous medium, if $\partial A$ is an element of surface area and $J$ is the electric current density (A / m²), then the magnitude of the current element passing through the surface element [1] is:

$$ \partial I = J \cdot \partial A $$

1.2 Potential by Two Sources
In exploration using geoelectric methods, two pairs of electrodes are commonly used, one pair of current electrodes and one pair of potential electrodes as shown by [2]:

**Figure 1.** The arrangement of two pairs of current and potential electrodes on the surface of the earth.

The potential difference between points P1 and P2 is given by [3]:

\[
\Delta V = \frac{\rho l}{2\pi} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) - \frac{1}{r_3} - \frac{1}{r_4}
\]  

(2)

So, obtained the following equation [1]:

\[
\rho = \frac{2\pi}{\left( \frac{1}{r_1} - \frac{1}{r_2} \right)} \frac{\Delta V}{l}
\]  

(3)

1.3 Potential for Layered Earth Models

For a layered earth model consisting of horizontal layer N (Figure 2) that each layer is homogeneous and isotropic, the separation between one layer and another layer is the boundary plane between two different resistivity media [1].

It is known that if the earth is homogeneous and isotropic then the electrical potential generated by a source point on the surface is [1]:

\[
V_0 = \frac{1}{2\pi} \frac{1}{R} = \frac{1}{2\pi} \frac{1}{(r^2+z^2)^{\frac{3}{2}}}
\]  

(4)

1.4 Kernel Functions and Resistivity Transformation Functions

The initial stage of interpretation of resistivity measurements of the earth's layers is the analytic search for Kernel functions \( K((\lambda)) \), produced [4]:

\[
K_i(\lambda) = \left[ \frac{K_{i+1}(\lambda) + p_i \tanh(\lambda d_i)}{p_i + K_{i+1}(\lambda) \tanh(\lambda d_i)} \right]
\]  

(5)

1.5 Forward Modelling and Inverse Modelling

Forward modeling states the process of calculating "data" which is theoretically observed on the surface of the earth if it is known the price of certain subsurface model parameters. In modeling geophysical data, a model is sought that produces a suitable response or fit with observational data or field data [5]. In Inverse modeling, the model parameters are obtained directly from the data. [6] defines inversion theory as a unity of mathematical and statistical techniques or methods to obtain useful information about a physical system based on observations of the system [5].
Geoelectric data inversion is a complex problem. Therefore, a small change in the data can bring a big change to the model, so it needs to be given the first initial (initial model) to approach like the actual model. This problem can be solved by introducing a damping factor (damping factor) into the equation system \[7\]. This parameter is added to the diagonal of the matrix which helps to increase the level of direct current of eigenvalues so that no eigenvalues can be zero \[8\]. The damped least squares solution is given by the following equation:

\[
\Delta m = (a^T a + \varepsilon^2 I)^{-1} a^T \Delta d
\]

Where \(\Delta m\) is a correction vector parameter, \(\Delta d\) is the data difference vector data, ‘a’ is the Jacobian matrix. ‘I’ is an identity matrix and ‘\(\varepsilon\)’ is called “a” muffled factor (damping factor) \[9\].

Singular Value Decomposition (SVD) can be easily applied to small-scale geophysical problems. It is mathematically strong and numerically stable and also provides other important information about the state of the model and data so as to enable the resolution of the model and the study of covariance \[10\]. Equation (6) can be solved using the SVD scheme in the inversion scheme. A matrix of NxN or NxP matrix a can be factored into products from the other 3 matrices as follows \[9\].

\[
a = USV^T
\]

Where for data N and P are measured parameters. U (NxP) and V (PxP) is respectively data spaces and the eigenvector and S parameters are PxP diagonal matrices which consist of non-zero r of a, with r≤m. This diagonal is included in S (\(\lambda_1, \lambda_2, ..., \lambda_p\)) which is called the singular value of a. This factorization is known as SVD from a \[9\]. If SVD is used in the least square solution damped. Equation (27) is obtained \[9\].

\[
\Delta m = (VS^2V^T + \varepsilon^2 I)^{-1}VSU^T \Delta d
\]

By entering the damping factor into diagonal elements, then inverse it is obtained \[9\]:

\[
(Vdiag(\lambda_j^2 + \varepsilon^2)V^T)^{-1} = Vdiag\left\{\frac{1}{\lambda_j^2 + \varepsilon^2}\right\}V^T
\]

Substitution of equation (8) to equation (9) so that it is obtained:

\[
\Delta m = Vdiag\left\{\frac{1}{\lambda_j^2 + \varepsilon^2}\right\}V^T VSU^T \Delta d
\]

And vector correction parameters can be expressed as:

\[
\Delta m = Vdiag\left\{\frac{\lambda_j}{\lambda_j^2 + \varepsilon^2}\right\} U^T \Delta d
\]

Equation (11) provides a damped least square solution with SVD.

2. Result and Discussion

Results of Synthetic Data Models and Inversion Results

(a) Model 1: Two-Layer Earth Model with \(\rho_1 > \rho_2\)

**Figure 3.** Comparison of observation resistivity curve model and synthetic layer thickness with calculation resistivity curve and inversion layer thickness for the two-layer earth model with \(\rho_1 > \rho_2\).
Table 1. Comparison of apparent resistivity values and thickness of synthetic layers with apparent resistivity values and inversion yield layer thicknesses for two-layer earth models with $\rho_1 > \rho_2$.

| Number of layer | Synthetic $\rho$ (Ω·m) | Inversion $\rho$ (Ω·m) | Difference | Synthetic $d$ (m) | Inversion $d$ (m) | Difference | Error |
|----------------|------------------------|------------------------|------------|------------------|------------------|------------|-------|
| 1              | 320                    | 320                    | 0.00       | 5                | 5                | 0.00       | 0.00  |
| 2              | 120                    | 120                    | 0.00       | -                | -                | -          | -     |

Model 2: Three-Layer Earth Model with $\rho_1 > \rho_2 > \rho_3$

![Figure 4](image)

Figure 4. Comparison of observation resistivity curve model and synthetic layer thickness with calculation resistivity curve and inversion yield layer thickness for the three-layer earth model with $\rho_1 > \rho_2 > \rho_3$.

Table 2. Comparison of apparent resistivity values and thickness of synthetic layers with apparent resistivity values and inversion yield layer thicknesses for three-layer earth models with $\rho_1 > \rho_2 > \rho_3$.

| Number of layer | Synthetic $\rho$ (Ω·m) | Inversion $\rho$ (Ω·m) | Difference | Synthetic $d$ (m) | Inversion $d$ (m) | Difference | Error |
|----------------|------------------------|------------------------|------------|------------------|------------------|------------|-------|
| 1              | 100                    | 100                    | 0.00       | 5                | 4.99             | ±0.01      | 0.01  |
| 2              | 50                     | 50.07                  | ±0.07      | 10               | 9.99             | ±0.01      |       |
| 3              | 10                     | 10.00                  | 0.00       | -                | -                | -          |       |

Model 3: Four-Layer Earth Model with $\rho_1 > \rho_2 > \rho_3 > \rho_4$

![Figure 5](image)

Figure 5. Comparison of observation resistivity curve models and synthetic layer thickness with calculated resistivity curves and inversion yield layer thicknesses for the four-layer earth models with $\rho_1 > \rho_2 > \rho_3 > \rho_4$.

Table 3. Comparison of apparent resistivity values and thickness of synthetic layers with apparent resistivity values and inversion yield layer thicknesses for four-layer earth with $\rho_1 > \rho_2 > \rho_3 > \rho_4$.

| Number of Layer | Synthetic $\rho$ (Ω·m) | Inversion $\rho$ (Ω·m) | Difference | Synthetic $d$ (m) | Inversion $d$ (m) | Difference | Error |
|----------------|------------------------|------------------------|------------|------------------|------------------|------------|-------|
| 1              | 100                    | 100                    | 0.00       | 5                | 5.00             | 0.00       | 0.01  |
| 2              | 80                     | 80.01                  | ±0.01      | 10               | 9.99             | ±0.01      |       |
| 3              | 50                     | 50.01                  | ±0.01      | 15               | 15.00            | 0.00       |       |
| 4              | 30                     | 30.00                  | 0.00       | -                | -                | -          |       |
**Model 4**: Five-Layer Earth Model with $\rho_1 > \rho_2 > \rho_3 > \rho_4 > \rho_5$

![Figure 6](image.png)

**Figure 6.** Comparison of observation resistivity curve models and synthetic layer thickness with calculated resistivity curves and inversion yield layer thickness for the five-layer earth models with $\rho_1 > \rho_2 > \rho_3 > \rho_4 > \rho_5$.

**Table 4.** Comparison of apparent resistivity values and thickness of synthetic layers with apparent resistivity values and inversion yield layer thickness for five-layer earth with $\rho_1 > \rho_2 > \rho_3 > \rho_4 > \rho_5$.

| Number of Layer | Synthetic $\rho$ (Ω.m) | Inversion $d$ (m) | Difference | Synthetic $d$ (m) | Inversion $d$ (m) | Error |
|-----------------|------------------------|------------------|------------|-------------------|------------------|-------|
| 1               | 10                      | 10              | 0.60       | 2                 | 2                | 0.00  |
| 2               | 50                      | 50.08           | ±0.08      | 15                | 15.15            | ±0.15 |
| 3               | 100                     | 100.91          | ±0.91      | 20                | 20.13            | ±0.13 |
| 4               | 20                      | 17.45           | ±2.55      | 25                | 21.74            | ±3.26 |
| 5               | 400                     | 398.86          | ±1.14      | -                 | -                |       |

2.1 Results of Field Data and Inversion Results

Inversion program that has been made in this study was tested using the field data. Located in Bonto Padang Village, Kahu Subdistrict, Bone District, South Sulawesi. From the field data after inverting the data using the program that has been made, the comparison of the results of the observation resistivity curve and the thickness of the layer with the apparent resistivity curve and the thickness of the inversion layer. (Figure 7) are as follows:

![Figure 7](image.png)

**Figure 7.** Comparison of observation resistivity curve model and field data layer thickness with calculation resistivity curve and inversion result layer thickness.

For field data that is processed using the program that has been made obtained the comparison of the apparent resistivity value of the observation and the thickness of the field data layer with the apparent resistivity value of the calculation and the thickness of the inversion result layer. (Table 5) as follows:
Table 5. Comparison of apparent resistivity values and thickness of field data layers with apparent resistivity values and inversion result layer thickness.

| Number of Layer | \( \rho \) (\( \Omega \cdot \text{m} \)) | \( d \) (m) | Error |
|-----------------|-----------------|-----------|------|
| 1               | 9.16            | 6.87      |      |
| 2               | 232.91          | 3.22      |      |
| 3               | 3.66            | 5.60      | 1.15 |

3. Conclusion

The conclusion that can be drawn from this study is that Schlumberger curve interpretation uses least square inversion for some synthetic data models such as a two-layer, three-layer, four-layer, and five-layer models, resulting in actual resistivity and thickness values that are close to synthetic data models. This can be noted from the results of the apparent resistivity curve model and the resistivity synthetic data calculation approaching for the two-layer, three-layer, four-layer and five-layer models with relatively small RMS error values even though by giving an initial guess for the resistivity and thickness values.

This result is quite satisfactory, it can be seen from the matched curve produced between the apparent resistivity curve of the observation and the apparent resistivity of the calculation with a relatively small RMS error value of 1.15.

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