Inverted Hierarchy of Neutrino Masses Disfavored by Supernova 1987A

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Abstract

We discuss the flavor conversion of supernova neutrinos in the three-flavor mixing scheme of neutrinos. We point out that by neutrino observation from supernova one can discriminate the inverted hierarchy of neutrino masses from the normal one if $s_{13}^2 > \text{a few} \times 10^{-4}$, irrespective of which oscillation solution to the solar neutrino problem is realized in nature. We perform an analysis of data of SN1987A and obtain a strong indication that the inverted mass hierarchy is disfavored unless $s_{13}^2 < \text{a few} \times 10^{-4}$.

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The phenomenon of neutrino flavor transformation which was discovered in atmospheric neutrino observation by Kamiokande and Superkamiokande experiments [1] implies new neutrino properties, tiny masses and unexpectedly large mixing angles. It constitutes, at present, the unique evidence for physics beyond the standard model of particle physics. The persistent discrepancy between the observed and the calculated flux of solar neutrinos [2] provide another evidence in favor of these new neutrino properties. It is worth to note that these robust evidences perfectly fit into the standard three-flavor mixing scheme of neutrinos. Recently it is reported that the third member, the $\tau$ neutrinos, are experimentally detected for the first time [3].

Yet, there still remain many unknowns in the properties of neutrinos. Among other things we have no experimental clue on the question of neutrino mass pattern, i.e., the normal vs. the inverted mass hierarchies, either from any terrestrial experiments or solar and atmospheric neutrino observation. Here, we mean, by normal and inverted mass hierarchies, the mass pattern $m_3 \gg m_1 \sim m_2$ and $m_1 \sim m_2 \gg m_3$, respectively. In our notation $\Delta m^2_{ij} \equiv m^2_j - m^2_i$, and $\Delta m^2_{12}$ and $\Delta m^2_{13} \sim \Delta m^2_{23}$ are the mass squared differences that are related with the solar and the atmospheric neutrino oscillations, respectively.

The issue of normal vs. inverted mass hierarchies is related to the deep question of under what discipline nature organizes the neutrino mass spectrum. If the inverted hierarchy is the case the discipline is clearly quite different from that used to organize the quark sector. Determination of which mass hierarchy is realized should imply powerful constraints on model building of neutrino masses and mixing.

Experimentally the problem of normal vs. inverted mass hierarchies is one of the key issue regarding the question of whether one can measure the absolute neutrino masses. The only imaginable way of detecting neutrino masses of the order of $\sqrt{\Delta m^2_{\text{atm}}} \simeq (0.04 - 0.07)$ eV is the neutrinoless double beta decay experiments, which is of course possible only for Majorana neutrinos. If the normal mass hierarchy is the case, one would have to go down to an order of magnitude smaller value of $\langle m_\nu \rangle$, the observable parameter in the experiments, to $\sim 0.001$ eV [4,5].

In this paper we point out that neutrino observation from a galactic supernova can judge whether the normal or the inverted mass hierarchies is realized in nature, unless the mixing angle $\theta_{13}$ is extremely small, $s^2_{13} < a \text{ few } \times 10^{-4}$. Moreover we perform an analysis of the data of SN1987A in Kamiokande and IMB detectors [6] and obtain a strong indication that it disfavors the inverted mass hierarchy.

Of course, this conclusion must be checked against the direct determination of the sign of $\Delta m^2_{13}$ which will be done in future long-baseline accelerator experiments [7,8]. However, the result we describe in this paper appears to be the unique hint which is available before such experiments are actually done.
We start by summarizing the common knowledges on neutrinos from supernova (SN) [10] and their properties inside neutrinosphere [11–13].

(1) Consideration of energetics of SN collapse indicates that almost all (∼ 99%) of the gravitational binding energy of neutron star is radiated away via neutrino emission. The total energy is estimated to be several ×10^{53} erg, and it is expected that the energy is equipartitioned into three flavors in a good approximation [11,14].

(2) It is discussed that the shape of the energy spectra of various flavor neutrinos can be described by a ”pinched” Fermi-Dirac distribution [15]. The pinched form may be parametrized by introducing an effective ”chemical potential”.

(3) There is no physical distinction between ν_µ and ν_τ and their antiparticles in neutrinosphere. It is because ν_µ and ¯ν_µ are not energetic enough to produce muons by the charged current interactions, and the neutral current cross sections of ν and ¯ν are similar in magnitude. Therefore, we collectively denote them as ”heavy neutrinos” in this paper.

(4) The location of neutrinosphere of heavy neutrinos, ν_µ and ν_τ, is believed to be in deeper place than ¯ν_e and ν_e in SN. It is due to the fact that the heavy neutrinos have weaker interactions with surrounding matter; they interact with matter only via the weak neutral current, whereas, ¯ν_e and ν_e do have additional charged current interactions. Hence, the heavy neutrinos have to have deeper neutrinosphere because their trapping requires higher matter density compared to those required for ¯ν_e and ν_e.

This last feature is of crucial importance for our business. It implies that the heavy neutrinos are more energetic when they are radiated off at neutrinosphere because the temperature is higher in denser region. It may be characterized by the temperature ratios of ν_e and ¯ν_e to ν_{heavy}

\[ \tau \equiv \frac{T_{ν_τ}}{T_{ν_ν}} \approx \frac{T_{ν_ν}}{T_{ν_e}} \approx 1.4 - 2.0 \]

according to the simulation of supernova dynamics which is carried out in Ref. [11,13]. Despite possible slight difference it should be a reasonable approximation of ignoring temperature difference between ¯ν_ν and ν_ν.

We now turn to the the neutrino flavor conversion in supernova (SN), the core matter in our discussion in this paper. In fact, it has a number of characteristic features which makes SN unique among other astrophysical and terrestrial sources.

(i) Because of extremely high matter density inside neutrinosphere all the neutrinos with cosmologically interesting mass range, m_ν ≲ 100 eV, are affected by the MSW effect [16].

*The terminology implicitly assumes that the normal mass hierarchy is the case. Nevertheless, we will use it even when we discuss the inverted mass hierarchy.
(Earlier references on the MSW effect in supernova include Ref. [17].) Consequently, the three neutrino and three antineutrino eigenstates have two level crossings, first at higher (H) density and the second at lower (L) density, inside SN as schematically indicated in Fig. 1. (ii) The key question in the neutrino flavor conversion in SN is whether the H level crossing is adiabatic or not. If it is adiabatic, then the physical properties of neutrino conversion is simply $\nu_e - \nu_{\text{heavy}}$ exchange in the normal mass hierarchy. It should be emphasized that this feature holds irrespective of the possible complexity of the solar neutrino conversion which governs the L resonance. These key features have been pointed out in our earlier paper, Ref. [18].

(iii) The second important question is if the neutrino mass spectrum adopts the normal or inverted mass hierarchies. If the mass hierarchies is of normal (inverted) type, the H level crossing is in the neutrino (antineutrino) channel.

For a recent comprehensive treatment of neutrino flavor conversion in SN in the framework of three-flavor mixing, see Ref. [19].

The last two remarks are crucial in our business. It will allow us to determine which mass hierarchy is realized by analyzing neutrino data from SN without knowing the parameters in the solar neutrino solution. Notice that this statement is valid not only for the MSW but also for the vacuum solar neutrino solutions.

Before going on, let us elaborate (ii) because it is of crucial importance in our argument. The three solid lines at the right most end of the level crossing diagram, Fig. 1a, represent three neutrino mass eigenstates in matter at the core of SN. The neutrino evolution in SN obeys the Schrödinger-like equation

$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} a(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix},$$

where $a(x) = \sqrt{2} G_F N_e(x)$ indicates the index of refraction with $G_F$ and $N_e(x)$ being the Fermi constant and the electron number density, respectively. In (2) $U$ denotes the leptonic flavor mixing matrix, the Maki-Nakagawa-Sakata (MNS) matrix [20].

Because of an extreme density near the neutrinosphere, $\gtrsim 10^{10} \text{ g/cm}^3$, the matter term $\text{diag}[a(x), 0, 0] \equiv H_0$ must be dominant over the other term. One can then formulate perturbation theory in which one takes the matter term $H_0$ as unperturbed part and the other one as perturbation. It is a degenerate perturbation theory and one has to diagonalize the $2 \times 2$ subspace to obtain the zeroth-order wave functions and the first-order correction to the energy eigenvalues. The resulting three matter-mass eigenstates at neutrinosphere in the case of normal mass hierarchy are as follows:
$\nu^\mu_3$ is almost pure $\nu_e$. $\nu^\mu_1$ and $\nu^\mu_2$ are certain superposition of heavy neutrinos, $\nu_\mu$ and $\nu_\tau$, but with negligible $\nu_e$ component.

The left end of Fig. 1a describes the three vacuum mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$. Therefore, if the H resonance is adiabatic, the $\nu_3$ state has the same physical properties, e.g., temperature, as $\nu_e$ inside neutrinosphere. By unitarity, the properties of $\nu_1$ and $\nu_2$ states in vacuum, that is when they get out from the star, must be determined equally accurately by physical properties of $\nu_\mu$ and $\nu_\tau$ inside neutrinosphere. Since there is no physical distinction between $\nu_\mu$ and $\nu_\tau$ there, the properties of $\nu_1$ and $\nu_2$ are not affected by the nature of the L level crossing, adiabatic, moderately nonadiabatic, or nonadiabatic. We suspect that this feature remains true even when the two level crossings come close so that the independent two-resonance approximation may not be completely valid.

This completes the argument to show that if the H resonance is adiabatic the net effect of the neutrino flavor conversion in SN is simply the $\nu_e$-$\nu_{\text{heavy}}$ exchange, irrespective of the nature of the L level crossing.\footnote{one can of course confirm this result by putting $P_H = 0$ in the relevant equations in Ref. \cite{18}, where $P_H$ denotes the nonadiabatic “jump” probability.} This is nothing but the feature that is called as "$\nu_e$-$\nu_\tau$ exchange" in Ref. \cite{18}.

The adiabaticity of the H resonance is guaranteed if the following adiabaticity parameter $\gamma$ is much larger than unity at the resonance point:

$$\gamma \equiv \frac{\Delta m^2 \sin^2 2\theta}{2E \cos 2\theta} \left| \frac{d \ln N_e}{dr} \right|_{\text{res}}^{-1} = \left( \frac{\Delta m^2}{2E} \right)^{1-1/n} \frac{\sin^2 2\theta}{(\cos 2\theta)^{1+1/n}} \frac{r_{\odot}}{n} \left[ \frac{\sqrt{2} G_F \rho_0 Y_e}{m_p} \right]^{1/n},$$

(3)

Here, we assumed that the density profile of the relevant region of the star can be described as $\rho(r) = \rho_0 (r/r_{\odot})^{-n}$ to obtain the second line in the above equation, where $r_{\odot} = 6.96 \times 10^{10}$ cm denotes the solar radius. With the choice $n = 3$ and $\rho_0 \simeq 0.1$ g/cc \cite{21}, we get,

$$\gamma \simeq 0.63 \times \left[ \frac{\sin^2 \theta_{13}}{10^{-4}} \right] \left[ \frac{\Delta m^2}{10^{-3} \text{eV}^2} \right]^{2/3} \left[ \frac{E}{20 \text{ MeV}} \right]^{-2/3},$$

(4)

for the small value of $\theta_{13}$. Since the conversion probability $P$ is approximately given by $P = \exp[-\frac{\pi}{2} \gamma]$, $s_{13}^2 \gtrsim a \text{ few} \times 10^{-4}$ assures adiabaticity in a good accuracy.

Now we notice that the basic elements for the argument toward disfavoring inverted mass hierarchy is actually very simple. Because of (iii), the resonance is in the antineutrino channel if the inverted mass hierarchy is the case as illustrated in Fig. 1b. It means that,

$$\frac{d \ln N_{\bar{e}}}{dr}$$

\cite{19}.
if the H resonance is adiabatic, all the $\bar{\nu}_e$'s at neutrinosphere are converted into heavy antineutrino states, and vice versa. It is also known that if H resonance is adiabatic, final $\bar{\nu}_e$ spectrum at the detector is not affected by the earth matter effect \[19\].

Since the $\bar{\nu}_e$-induced charged current reaction is dominant in water Cherenkov detector, one can severely constrain the scenario of inverted mass hierarchy by utilizing this feature of neutrino flavor transformation in SN. When the next supernova event comes it can be used to make clear judgement on whether the inverted mass hierarchy is realized in nature, a completely independent information from those that will be obtained by the long-baseline neutrino oscillation experiments.

While waiting for the next galactic SN, let us perform an analysis of the data of neutrinos from SN1987A to gain a hint to the problem of the mass pattern which we want to solve. In the following analysis, we assume that $s_{13}$ is large enough, $s_{13}^2 \gtrsim \text{a few } \times 10^{-4}$, to guarantee the adiabaticity of the H resonance.

In fact, very similar analyses have been done by several authors \[23,24\]. We may be able to characterize our work in comparison with theirs in the following way; We formulate the problem in a proper setting of the three-flavor mixing scheme of neutrinos, which is essential for the SN neutrinos. In due course, we will try to clarify how conclusions obtained in earlier works are to be interpreted, or to be conditioned in the three-flavor framework.

We follow Jegerlehner, Neubig and Raffelt \[24\] who employed the method of maximum likelihood. We define the Likelihood function as follows \[24\]:

$$L = C \exp \left( - \int_0^\infty n(E) dE \right) \prod_{i=1}^{N_{\text{obs}}} n(E_i),$$  \hspace{1cm} (5)

where $N_{\text{obs}}$ is the total number of experimentally observed events and the $C$ is some constant which is irrelevant for our purpose of parameter estimation and the determination of confidence regions. Here, $n(E)$ is the expected positron energy spectrum at Kamiokande or IMB detector which is computed taking into account the detector efficiency as well as energy resolution in the same way as in Ref. \[24\]. For a combined analysis of the Kamiokande and

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\[\text{Footnote:}\] The reason is as follows. Let us first note that $\bar{\nu}_3$ state which carry the original $\bar{\nu}_e$ spectrum oscillate very little into $\bar{\nu}_e$ in the earth because $|\Delta m^2_{13}|/E$ is much larger than the earth matter potential and also because $\theta_{13}$ is small \[22\]. Therefore, the oscillation in the earth takes place essentially only between $\bar{\nu}_1$ and $\bar{\nu}_2$, decoupling the $\bar{\nu}_3$ state. It would lead to regeneration of $\bar{\nu}_e$ but it would not give any significant effect for the $\bar{\nu}_e$ component at the detector because both $\bar{\nu}_1$ and $\bar{\nu}_2$ carry original energy spectrum of heavy flavors at the neutrinosphere.
IMB detectors, the likelihood function is defined as the product of the likelihood function for each detector.

We draw in Fig. 2 equal likelihood contours as a function of the heavy to light temperature ratio \( \tau \) on the space spanned by \( \bar{\nu}_e \) temperature and total neutrino luminosity by giving the neutrino events from SN1987A observed by Kamiokande and IMB detectors [3]. In addition to it we introduce an extra parameter \( \eta \) defined by \( L_{\nu_x} = L_{\bar{\nu}_x} = \eta L_{\nu_e} = \eta L_{\bar{\nu}_e} \) which describe the departure from equipartition of energies to three neutrino species and examine the sensitivity of our conclusion against the change in the SN neutrino spectrum. For simplicity, as in Ref. [24], we set the “effective” chemical potential equal to zero in the neutrino distribution functions because we believe that our results would not depend much even if we introduce some non-zero chemical potential.

At \( \tau = 1 \), that is at equal \( \bar{\nu}_e \) and \( \nu_e \) temperatures, the 95 % likelihood contour marginally overlaps with the theoretical expectation [13] represented by the shadowed box in Fig. 2. When the temperature ratio \( \tau \) is varied from unity to 2 the likelihood contour moves to the left, indicating less and less consistency, as \( \tau \) increases, between the standard theoretical expectation and the observed feature of the neutrino events after the MSW effect in SN is taken into account. This is simply because the observed energy spectrum of \( \bar{\nu}_e \) must be interpreted as that of the original one of \( \bar{\nu}_{\text{heavy}} \), in the presence of the MSW effect in the anti-neutrino channel, which implies that the original \( \bar{\nu}_e \) temperature must be lower by a factor \( \tau \) than the observed one, leading to stronger inconsistency at larger \( \tau \).

The solid lines in Fig. 2 are for the case of equipartition of energy into three flavors, \( \eta = 1 \), whereas the dotted and the dashed lines are for \( \eta = 0.7 \) and 1.3, respectively. We observe that our result is very insensitive against the change in \( \eta \).

We conclude that if the temperature ratio \( \tau \) is in the range 1.4-2.0 as the SN simulations indicate, the inverted hierarchy of neutrino masses is disfavored by the neutrino data of SN1987A unless the H resonance is nonadiabatic.

For completeness, we summarize the features of neutrino events that we expect in the three-flavor mixing scheme of neutrinos. They differ depending upon the normal or the inverted mass hierarchies and on the nature of the H and the L level crossings.

**The case of normal mass hierarchy**

There is no level crossing in the antineutrino channel, apart from the possibility that the solar mixing angle is in the “dark side” in the parameter space [20], \( \theta_{12} > \pi/4 \), which we do not consider in this paper. Therefore, as far as \( \theta_{13} \) is not large, which is indicated by CHOOZ result [22], third state \( \nu_3 \) is essentially decoupled from the other antineutrinos and

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§This result has been announced at two international conferences, Dark2000 and NOW2000 [25].
as far as $\bar{\nu}_e$ signal is concerned, the problem can be approximately reduced to the two flavor mixing scheme which is well explored by the previous works [23,24].

The conclusion reached can be summarized as follows: If the small mixing angle (SMA) MSW is the solution to the solar neutrino problem there is only a minor effect because neither the vacuum oscillation nor the earth matter effects are effective because of small $\theta_{12}$. If the large mixing angle (LMA) or low $\Delta m^2$ (LOW) MSW or vacuum oscillation (VO) is the solution, we have a potential trouble because a good fraction of $\bar{\nu}_e$ is transformed into $\bar{\nu}_{\text{heavy}}$ and vice versa. One can repeat the similar analysis as we did for Fig. 2 and would conclude that all of these solutions with large mixing angle are disfavored [23], though less convincingly than the case of inverted mass hierarchy.

Fortunately, the earth matter effect helps us to cure the trouble at least to some extent for the case of LMA MSW solution [23,24]. We present in Fig. 3 the result of the analysis using the same likelihood method. We employ a particular set of parameters of the LMA MSW solution and compare the behavior of the likelihood contours with and without earth matter effect. For simplicity, we set $\theta_{13} = 0$ but our result does not change much as long as the parameter is under the CHOOZ bound [22]. In the present analysis we only deal with the case of equipartition of energy, $\eta = 1$, because we already know that the results are not sensitive to $\eta$. As one observes in Fig. 3, the earth matter effect cures the discrepancy between the likelihood contours and the theoretical expectation to some extent, but not completely.

For the case of LOW MSW as well as VO solutions, there is no significant earth matter effect and the results are essentially the same as was presented in Fig.5 in Ref. [24], and hence we show no plot for these solutions.

There exist interesting effects in the neutrino channel because they have level crossings. We have to discuss three cases separately; the H resonance is (a) adiabatic, (b)moderately nonadiabatic, and (c) nonadiabatic.

In the case (a) the net effect is $\nu_e - \nu_{\text{heavy}}$ exchange independent of the nature of the L resonance, as we have discussed extensively. The characteristic signature of harder spectrum of $\nu_e$ which comes from $\nu_{\text{heavy}}$ in neutrinosphere is:
(i) Enhancement of forward peaking elastic scattering events at high energies which should be observable in water Cherenkov detectors [18], and
(ii) Enhanced oxygen-induced events due to a steep rise of the cross section at energies higher than $\gtrsim 30$ MeV [27], which could be separated from the dominant isotropic $\bar{\nu}_e$ absorption events due to a moderate backward peaking of the events [23,29].

In the case (c) one can disregard the third state and the problem is the pure two-flavor $\nu_e - \nu_{\text{heavy}}$ transformation. Then, if the LMA or LOW MSW is the solution to the solar neutrino problem, there is a significant conversion with similar experimental signatures as
the case (a). We expect that nothing happens in SN in the case of VO solution, and about equal mixture of $\nu_e$ and $\nu_{\text{heavy}}$ (for $\sin^2 2\theta_{12} \simeq 1$) would result due to vacuum oscillations which would also lead similar but somewhat weaker experimental signatures as the case (a). In the case of SMA MSW solution the conclusion depends upon the profile of the outer envelope of progenitor star and it is difficult to draw definitive conclusions.

In the case (b) in which the conversion probability at the H resonance is, say, $\sim 0.5$, then one can expect some similar but weaker effects with the case (a) independent of the solar neutrino solution, and in addition the effects which depend upon the solar solutions.

**The case of inverted mass hierarchy**

The structure of the level crossing is very simple in this case as shown in Fig. 1b. There is the H resonance in the antineutrino channel and the L resonance is in the neutrino channel, since we do not consider the solar parameter which is in the “dark side”. Then, the predictions to the neutrinos are exactly the same as the ones for case (c) in the normal mass hierarchy.

In the antineutrino channel there is the H resonance and we have discussed extensively what happens if it is adiabatic, the case (a). So we only need to discuss the case (b) and (c). If the H resonance is nonadiabatic, the case (c), the effect of neutrino mixing is the same as in the case of normal mass hierarchy. In the case of (b), moderately nonadiabatic case, $\bar{\nu}_e$-$\bar{\nu}_{\text{heavy}}$ transformation occurs with the probability $1 - P_H$, and it would imply the similar but milder effect than that we have obtained with adiabaticity of the H resonance. If the next galactic supernova is detected by Superkamiokande, then we will be able to discriminate the moderately nonadiabatic case from the adiabatic one.

After completion of our work we became aware of the paper by Lunardini and Smirnov [30] in which some points related with our work are mentioned, but with particular emphasis primarily on the detector-dependent earth matter effect.

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FIG. 1. The schematic level crossing diagram for the case of (a) normal and (b) inverted mass hierarchies considered in this work. The circles with the symbol H and L correspond to resonance which occur at higher and lower density, respectively.
FIG. 2. Contours of constant likelihood which correspond to 95.4% confidence regions for the inverted mass hierarchy under the assumption of adiabatic H resonance. From left to right, $\tau \equiv T_{\nu_x}/T_{\bar{\nu}_x} = T_{\nu_x}/T_{\bar{\nu}_x} = 2, 1.8, 1.6, 1.4, 1.2$ and 1.0 where $x = \mu, \tau$. Best-fit points for $T_{\nu_x}$ and $E_b$ are also shown by the open circles. The parameter $\eta$ parametrizes the departure from the equipartition of energy, $L_{\nu_x} = L_{\bar{\nu}_x} = \eta L_{\nu_e} = \eta L_{\bar{\nu}_e}$ ($x = \mu, \tau$), and the dotted lines (with best fit indicated by open squares) and the dashed lines (with best fit indicated by stars) are for the cases $\eta = 0.7$ and 1.3, respectively. Theoretical predictions from supernova models are indicated by the shadowed box.
FIG. 3. Contours of constant likelihood corresponding to 95.4% C.L. for the large mixing angle MSW solution (a) without and (b) with earth matter effect. We have taken mixing parameters as $\Delta m^2 = 3 \times 10^{-5}$ eV$^2$ and $\sin^2 2\theta = 0.8$ for LMA MSW solution.