HIGHER-SPIN GAUGE INTERACTIONS FOR MATTER FIELDS IN TWO DIMENSIONS

M.A. VASILIEV

I.E.Tamm Department of Theoretical Physics, Lebedev Physical Institute,
Leninsky prospect 53, 117924, Moscow, Russia

Abstract

1 Introduction

Many of important properties of integrable systems and conformal models in two dimensions originate from underlying infinite-dimensional symmetries. Higher-spin (HS) extensions of the Virasoro symmetry acquired much attention during recent years. In particular, d=2 conformal matter models with gauged HS symmetries have been extensively studied for the cases of $W_N$ [1], $\omega_\infty$ [2] and $W_{1+\infty}$ [3,4] algebras. A d=2 model for pure gauge $W_{1+\infty}$ HS fields was proposed in [5]. The $W_{1+\infty}$ algebra investigated originally in [3] is getting now a wide area of applicability including, e.g., fractional Hall effect [7] and KP hierarchy [8]. In the context of field theory $W_{1+\infty}$-type symmetries were originally introduced as HS symmetries in [3] for the case of d=2, and in [3-4] for d=4. The name $W_{1+\infty}$ was suggested in [4] in the context of the analysis of possible Lie algebra extensions of the Virasoro algebra.

In refs. [3] and [3-4] consistent dynamics of HS gauge fields interacting among themselves and with the lower-spin matter fields in d=4 and d=3 was formulated in terms of gauge fields corresponding to the appropriate versions of $W_{1+\infty}$-type algebras. An interesting problem that remained unsolved for some years was to apply the methods developed for d=3 and d=4 HS problems to d=2 models. The goal of this letter is just to announce a new model which describes HS gauge interactions of boson and fermion matter fields in d=2 along the lines of the approach developed in [3-15]. As expected, the d=2 HS dynamics turns out to be much simpler than that in d=3 and 4. However, the formalism in d=2 has a number of specific properties and does not amount to a straightforward

1 As it often happens in the literature, we use the same name $W_{1+\infty}$ both for the full algebra that admits a non-trivial central extension and for its centerless part which in turn is equivalent to the Moyal bracket [3].
reduction of the higher-dimensional models. In particular, the full set of gauge fields, which is shown below to describe d2 HS-matter interactions, differs from that proposed in \cite{3, 5}, rather corresponding to a current extension of the $W_{1+\infty}$ algebra.

Being a counterpart of the $W_{1+\infty}$ gauge models discussed in \cite{3, 4} in the sense that it is based on bilinear HS currents involving higher derivatives, the presented model has a number of physically important distinctions from the models of \cite{3, 4}. In particular, no vanishing current constraints on the matter fields are present in our model. The model is formulated in an explicitly HS gauge invariant and general coordinate invariant fashion. A natural d2 background is anti-de Sitter (AdS) space-time.

The proposed model is not conformal. Instead, the full equations of motion in this model have a form of some zero-curvature equations and covariant constancy conditions without any additional constraints so that the model turns out to be integrable. This unexpected property is specific for d=2 and allows one to write down a simple action principle for the model.

2 Basic Algebraic Structures

Analogously to the construction developed previously for HS theories in d=3 and 4 (see \cite{10, 11, 13-15} and references therein) the basic algebraic structure is the associative algebra $A$ of power series in the generating elements $\hat{y}_\pm$ and $\hat{z}_\pm$ obeying the commutation relations

\[
[\hat{y}_-, \hat{y}_+] = -2i, \quad [\hat{z}_-, \hat{z}_+] = 2i, \quad [\hat{y}_\alpha, \hat{z}_\beta] = 0 \quad \alpha, \beta = \pm .
\]

Any $\hat{a} \in A$ can be cast into the form

\[
\hat{a} = \sum_{n,m=0}^{\infty} a^{\alpha_1...\alpha_n \beta_1...\beta_m} \hat{y}_{\alpha_1} \ldots \hat{y}_{\alpha_n} \hat{z}_{\beta_1} \ldots \hat{z}_{\beta_m} ,
\]

where the coefficients $a^{\alpha_1...\alpha_n \beta_1...\beta_m}$ are totally symmetric in $\alpha$ and $\beta$ that implies Weyl ordering of the operators $\hat{y}_\alpha$ and $\hat{z}_\beta$. It is also convenient to use the following equivalent set of variables

\[
\hat{u}_\alpha = \frac{1}{2} (\hat{z}_\alpha - \hat{y}_\alpha) , \quad \hat{v}_\alpha = \frac{1}{2} (\hat{y}_\alpha + \hat{z}_\alpha) , \quad [\hat{v}_\pm, \hat{u}_\mp] = \mp i, \quad [\hat{v}_\alpha, \hat{v}_\beta] = [\hat{u}_\alpha, \hat{u}_\beta] = 0 .
\]

Practically, to work with the algebra $A$ it is most useful to use the technics of symbols of operators $[10, 11]$. Namely, given element $\hat{a} (2)$ one introduces its symbol of the form

\[
a = \sum_{n,m=0}^{\infty} a^{\alpha_1...\alpha_n \beta_1...\beta_m} y_{\alpha_1} \ldots y_{\alpha_n} z_{\beta_1} \ldots z_{\beta_m} ,
\]

where $y_\alpha$ and $z_\beta$ are commuting variables. By definition, a star product ($a \ast b$) is the symbol of the operator $\hat{a} \hat{b}$. One can use the following useful formula for the Weyl symbol star product

\[
(f \ast g)(z, y) = (2\pi)^{-4} \int d^2 s d^2 t d^2 p d^2 q \ f(z + s, y + p) g(z - t, y + q) \ exp[i (s_\alpha t^\alpha + p_\alpha q^\alpha)] .
\]

2
An important property of $A$ is that it contains an element $\Pi$ which possesses properties of the projection operator, $\Pi^2 = \Pi$, and behaves as the vacuum vector for the operators $\hat{u}_\pm$ and $\hat{v}_\pm$, i.e. $\hat{v}_\pm \Pi = 0$ and $\Pi \hat{u}_\pm = 0$. It has the following explicit realization in terms of Weyl symbols

$$\Pi = \frac{1}{4} \exp(iz\alpha y^\alpha).$$

(6)

The algebra $A$ is $Z_2$-graded with odd (even) power polynomials in $\{2\}$ considered as odd (even) elements of $A$. This is just the ordinary boson-fermion grading since the indices $\alpha$ and $\beta$ are interpreted as spinor indices in field theoretical applications. The algebra $A$ admits the following unique supertrace operation $\text{str}_A$,

$$\text{str}_A(\hat{a}) = a_0,$$

(7)

where $a_0$ is a constant term in the expansion $\{2\}$. This supertrace operation is a linear mapping obeying the general property

$$\text{str}(\hat{a}\hat{b}) = (-1)^{\pi(\hat{a})\pi(\hat{b})}\text{str}(\hat{b}\hat{a}),$$

(8)

where $\pi(\hat{a})= 0$ or 1 is the parity of $\hat{a}$. Note that this property allows one to construct invariant polynlinear forms with the aid of the supertrace in the standard way as $\text{str}(\hat{a}_1\hat{a}_2\ldots\hat{a}_n)$.

To formulate the $d2$ higher-spin dynamics it is useful to extend the algebra $A$ to $\mathcal{A}$ in the following general way. Given associative algebra $A$ and some projection operator $\Pi \in A$, one defines the algebra $\mathcal{A}$ such that its general element $a \in \mathcal{A}$ is equivalent to a set of four elements of $A$, $a = \{a, \langle a \rangle, \langle a |, \langle |a \rangle \}$, obeying the properties

$$\{a \in \mathcal{A} | a, \langle a \rangle, \langle a |, \langle |a \rangle \in A ; |a\rangle \Pi = |a\rangle, \Pi \langle a | = \langle a|, \langle a \rangle \Pi = \Pi \langle a \rangle = \langle a \}.$$  

(9)

The product law $\circ$ in $\mathcal{A}$ is defined via the product law in $A$ as follows

$$a \circ b = \{ab + |a\rangle\langle b |, a|b \rangle + |a\rangle\langle b |, \langle a|b + \langle a\rangle\langle b |, \langle a||b \rangle + \langle a\rangle\langle b |\}.$$  

(10)

This product law is associative. Note that supertrace operation $\text{str}_A$ in $A$ induces the supertrace operation $\text{str}_A$ in $\mathcal{A}$

$$\text{str}_\mathcal{A}(a) = \text{str}_A(a + \langle a \rangle).$$

(11)

We will use this construction with the projection operator $\{3\}$ to embed all matter and auxiliary fields into the adjoint representation of $\mathcal{A}$.

3 Full Nonlinear Dynamics and Vacuum Solution

In this section we summarize the final results for the full non-linear equations and the action principle of the model and give a vacuum solution which is shown in the subsequent sections to lead to a proper linearized dynamics.
To describe the HS gauge interactions of d2 matter fields \((x^\nu)\) are space-time coordinates \((\nu = 0, 1)\), \(dx^\nu\) are anticommuting differentials, \(d = dx^{\alpha}\frac{\partial}{\partial x^{\alpha}}\) we introduce the gauge one-form \(W(x|z_\alpha, y_\alpha) = dx^\nu W_\nu(x|z_\alpha, y_\alpha)\), and the matter field zero-form \(B(x|z_\alpha, y_\alpha)\) in the adjoint representation of \(A\), i.e.

\[
W = \{W, |W\}, \langle W\rangle, \quad B = \{B, |B\}, \langle B\rangle, \langle B\rangle\} .
\] (12)

The full system of equations for interacting d2 matter fields has a simple form of zero-curvature conditions:

\[
R \equiv dW + W \circ \wedge W = 0, \quad dB + W \circ B - B \circ W = 0 .
\] (13)

These equations can be derived from the B-F type action principle

\[
S = \int_{M_2} str_A(B R) .
\] (14)

The model becomes dynamically non-trivial because the 0-form \(B\) is supposed to have a nonvanishing vacuum value of the form

\[
B_{vac} = \{N, 0, 0, 0\} , \quad N = \frac{1}{4i}\{\hat{z}_-, \hat{z}_+\} .
\] (15)

A physical vacuum value of the gauge 1-form \(W\) is

\[
W_{vac} = \{\omega^{gr}, 0, 0, 0\} , \quad \omega^{gr}(x) = h^+(x)L^+ + h^-(x)L^- + \omega(x)L^0 ,
\] (16)

where

\[
L^\pm = \frac{i}{4}(\hat{\gamma}_\pm)^2 , \quad L^0 = \frac{i}{4}\{\hat{y}_+, \hat{y}_-\}
\] (17)

obey the \(sl_2\) commutation relations,

\[
[L^0, L^\pm] = \pm 2L^\pm , \quad [L^-, L^+] = L^0 .
\] (18)

One-forms \(h^\pm(x) = dx^\nu h^\pm_\nu(x)\) and \(\omega(x) = dx^\nu \omega_\nu(x)\) describe inverse zweibein and Lorentz connection, respectively. The components of the gravitational field-strength two-form,

\[
R^{gr} = d\omega^{gr} + \omega^{gr} \wedge \omega^{gr} = R^+L^+ + R^-L^- + R^0L^0
\] (19)

identify, respectively, with the torsion tensor, \(R^+, R^-\), and with the Riemann tensor, \(R^0\), shifted by a cosmological term \(h^- \wedge h^+\). The vacuum gravitational field is supposed to obey the zero-curvature conditions

\[
R^0 = d\omega + h^- \wedge h^+ = 0 , \quad R^\pm = dh^\pm \pm 2\omega \wedge h^\pm = 0 .
\] (20)

Under the condition that the zweibein \(h^\pm_\nu\) is invertible these conditions imply that the Lorentz connection expresses in terms of \(h^\pm_\nu\) by virtue of the metric postulate while \(h^\pm_\nu\)

\(^2\)Note that the physical vacuum values of the fields \(B\) and \(W\) have nothing to do with the vacuum \(\Pi\) of the algebra \(A\) of auxiliary spinor variables.
describes the two-dimensional AdS space. On the other hand, (20) along with the fact that bilinears in the oscillators \( \hat{y} \) form a subalgebra \( sl_2 \subset A \subset A \) implies that the first of the equations (13) is satisfied. The second one is also true because the vacuum value \( N \) of \( B \) depends only on \( \hat{z} \) and therefore commutes with the background gravitational field (14) due to (1).

HS gauge fields correspond to higher-order terms of the expansion of \( W(x|z_\alpha, y_\alpha) \) in powers of the auxiliary spinor variables. The gauge connection \( W \) and the matter field \( B \) have the standard transformation laws under the HS gauge transformations with the parameter \( \xi(x|z_\alpha, y_\beta) \).

\[
\delta W = d\xi + W \circ \xi - \xi \circ W, \quad \delta B = B \circ \xi - \xi \circ B, \tag{21}
\]

which leave invariant the equations (13) and the action (14). Because of using the exterior algebra formalism, general coordinate invariance is explicit too.

Let us note that a part of the symmetry that acts linearly on physical states consists of the subalgebra spanned by elements commuting with \( B_{vac} \). The gauge parameters of this subalgebra are of the form

\[
\xi = (\xi_{vac}, 0, 0, \langle \xi_{vac} \rangle) \tag{22}
\]

with an arbitrary Abelian parameter \( \langle \xi_{vac} \rangle \) and the parameter \( \xi_{vac} \) of the form

\[
\xi_{vac} = \sum_{n,m,k=0}^\infty \xi_{n,m,k}(x)N^k(\hat{y}_-)^n(\hat{y}_+)^m. \tag{23}
\]

Since \( N \) (15) commutes with the oscillators \( \hat{y}_\pm \), the generating elements of the \( W_{1+\infty} \) algebra, one is left with the non-negative part of the loop extension \( \tilde{W}_{1+\infty} \) of \( W_{1+\infty} \).

The purely topological form of the action (14) is analogous to the topological form of the d2 gravitational action discussed in [17] and to the HS action proposed in [5]. This analogy is not exact however because in the latter models the zero-curvature equations are true in absence of matter and do not describe propagating degrees of freedom while the equations (13) are shown below to describe interactions of propagating scalar and spinor fields. As is demonstrated in the next section, this is possible because of using infinite multiplets of fields.

Another important point is that the non-vanishing vacuum value of the zero-form \( B \) (15) breaks down spontaneously the antisymmetry of the action \( S \) (14) under the transformation \( B \rightarrow -B \). The vacuum value (15) leads effectively to some \( W^2 \) - type terms in the action that opens a way to a proper diagonalization of the action at the linearized level. Practically, a problem of reducing the quadratic part of the action (14) to the standard form is highly involved due to presence of an infinite set of auxiliary fields and will be considered elsewhere.

### 4 Free Matter Equations in d2 AdS Space

In this section we reformulate free equations for matter fields in d2 AdS space described by the zero-curvature conditions (20) in the form of some covariant constantness conditions along the lines of the general “unfolded formulation” approach proposed in [14].
Importance of this reformulation for the analysis below is due to the fact that it is this form to which the full nonlinear equations reduce in the linearized approximation in the sector of matter fields.

Consider the following system of equations

\[ D\phi_n = \alpha(n)h^–\phi_{n+2} + \beta(n)h^+\phi_{n-2}, \]

where \( D \) is the Lorentz covariant derivative,

\[ D\phi_n = d\phi_n + n\omega\phi_n. \]

This system is formally consistent (i.e. the Bianchi identities are satisfied) provided that the numerical parameters \( \alpha(n) \) and \( \beta(n) \) obey the condition

\[ \alpha(n)\beta(n + 2) = \mu + 1/4 n(n + 2) \]

and zero curvature conditions \( (20) \), describing the vacuum AdS geometry, are satisfied. Here \( \mu \) is an arbitrary numerical parameter. Note that the ambiguity in the coefficients \( \alpha(n) \) and \( \beta(n) \), which is not fixed from \( (26) \), is irrelevant and reflects a freedom in the rescaling \( \phi_n \rightarrow \gamma(n)\phi_n \).

Now we observe that the infinite system of equations \( (24) \) is equivalent to the dynamical equations of free d2 fields of arbitrary mass \( m^2 = \mu \). Here boson and fermion fields are described by the set of the fields \( \phi_n \) with \( n \) even and odd, respectively.

To make sure that, e.g., the equations \( (24) \) with even \( n \) are equivalent to the Klein-Gordon equation let us introduce the inverse zweibein \( h^\nu_\pm \) and rewrite the system of equations \( (24) \) in the form

\[ h^\nu_+ D^\nu\phi_n = \beta(n)\phi_{n-2}, \quad h^\nu_- D^\nu\phi_n = \alpha(n)\phi_{n+2}. \]

One observes that these equations with \( n = 0 \) express the fields \( \phi_{\pm2} \) in terms of the first space-time derivatives of \( \phi_0 \). Then the equations \( (27) \) with \( n = \pm 2 \) contain the Klein-Gordon equation and express the fields \( \phi_{\pm4} \) via second space-time derivatives of \( \phi_0 \). Note that although the Klein-Gordon equation appears twice, i.e. both in the first of the equations \( (27) \) with \( n = 2 \) and in the second one with \( n = -2 \), an appropriate combination of these equations vanishes identically due to the Bianchi identities of the original equations \( (24) \) so that, effectively, the Klein-Gordon equation appears only once. Finally, one finds that all higher \( n \) equations in the system \( (27) \) either express the fields \( \phi_m \) with \( m \neq 0 \) via higher derivatives of \( \phi_0 \) or encode all Bianchi identities for these expressions imposing no additional dynamical conditions on the field \( \phi_0 \).

As a result, the system \( (24) \) with even \( n \) turns out to be dynamically equivalent to the original Klein-Gordon equation supplemented with some constraints which express all higher \( \phi_n \) via higher space-time derivatives of the dynamical field \( \phi_0 \). A situation with fermions (\( n \) is odd) is analogous.

5 Perturbative Analysis

To analyze the equations \( (13) \) perturbatively one considers the fields of the form \( W = W_{vac} + w \) and \( B = B_{vac} + b \) where \( w \) and \( b \) denote perturbations. Propagating matter
fields belong to the mutually conjugated components $|b\rangle$ and $\langle b|$ of $b$. The linearized equations (23) in the sector of the matter fields $|b\rangle$ read

$$d|b\rangle + w^{gr}|b\rangle = N|w\rangle. \tag{28}$$

This equation implies, first, that $|w\rangle$ expresses via the matter fields $|b\rangle$ and, second, that it imposes some differential equations on those components of the matter fields which are not proportional to $N$. Let us show that the latter differential equations are just the equations for free matter fields analyzed in the previous section.

The linearized gauge transformation (21) for the field $|b\rangle$ takes the form $\delta|b\rangle = N|\xi\rangle + O(b)$. This implies that the field $|b\rangle$ contains some Higgs part which can be gauged away and a reminder which is to be shown to describe matter fields.

The standard Fock representation for $|b\rangle$ is $|b\rangle = b'(\hat{u}_+, \hat{u}_-)\Pi$. Since $N = \frac{1}{3i}(\hat{\xi}_+, \hat{\xi}_-) = \frac{1}{3i}\{(\hat{u} + \hat{\nu})_+, (\hat{u} + \hat{\nu})_-\}$, the Higgs-type component of the transformation law for $|b\rangle$ allows one to get rid of any polynomial in $\hat{u}_+ \hat{u}_-$ in $b'$. As a result one can chose a gauge with respect to the transform (21) with

$$b'(u_+, u_-) = b'_+(u_+) + b'_-(u_-) + b'_0, \quad b'_+(0) = b'_-(0) = 0. \tag{29}$$

Fields of this form cannot be compensated further by virtue of transformations (21) and therefore can describe some dynamical degrees of freedom. By expanding (29) in powers of $u_{\pm}$ one observes that the structure of the gauge fixed matter field $b'$ (29) is just of the form one expects for d2 matter fields from (24). To work out explicit form of the field equations one has to substitute (23) into (28), decompose the left-hand-side of (28) into a part proportional to $N$ which is compensated by an appropriate choice of $|w\rangle$ and a part depending either only on $u_+$ or only on $u_-$ as in (29) which will impose some equations on $b'$. Let us give the final result for the field equations and the value of the field $|w\rangle = w^l(\hat{u}_+)\Pi$:

$$Db' = \frac{1}{4i}h^+((u_+)^2(b'_+ + b'_0) + iu_+ b'_-(0) - 4\hat{b}'_-(u_-) + \int_0^1 ds (3s + 1)\hat{b}'_-(su_-))$$
$$+ \frac{1}{4i}h^-(u_-)^2(b'_+ + b'_0) - iu_- b'_+(0) - 4\hat{b}'_+(u_+) + \int_0^1 ds (3s + 1)\hat{b}'_+(su_+) \tag{30}$$

$$w^l(u_\pm) = -\omega b^l(u_\pm) + \frac{i}{2}h^+ \left(iu_+ \int_0^1 ds \hat{b}'_-(su_-) - \int_0^1 ds (2s + 1)\hat{b}'_-(su_-)\right)$$
$$+ \frac{i}{2}h^- \left(iu_- \int_0^1 ds \hat{b}'_+(su_+) + \int_0^1 ds (2s + 1)\hat{b}'_+(su_+) \right) \tag{31}$$

where $\hat{f}(x) = \frac{d}{d\mu}f(x)$.

One can check directly that the equation (30) is formally consistent thus corresponding to some particular case of the equations (24) with the coefficients of the form (26). By expanding the function $f^l$ into power series in either $u_+$ or $u_-$ one finds that the coefficients indeed satisfy the condition (24) with $\mu = 3/16$. This value is not occasional. It equals to the value of the $sl_2$ Casimir operator for the realization (16). There is a possibility to generalize the proposed scheme to an arbitrary mass which we will discuss elsewhere [18].
Let us note that the parameter $\mu$ is measured here in units of the inverse radius of the background AdS space-time and therefore tends to zero in the flat limit.

Thus it is shown that the linearized equations for $|b\rangle$ describe properly linearized dynamics for $d^2$ matter fields. Analogously one can analyze the conjugate sector of $\langle b|$ to show that it describes conjugate matter fields.

An important property which we do not prove explicitly here is that all other components in $W$ and $B$ do not carry their own degrees of freedom. This can be shown for example with the aid of the method developed in [14] where it was argued that any system of covariant constantness equations for zero forms cannot describe propagating modes when these zero forms carry some finite-dimensional representations of the space-time symmetry algebra which gives rise to the vacuum gravitational field. Actually, in the model under consideration all components of the zero form $B$ contained in $B$ and $\langle B \rangle$ decompose into a sum of only finite-dimensional representations of the AdS algebra under the adjoint action of the generators (17). We will come back to a more detailed discussion of this point elsewhere [18].

Thus, the matter fields contained in $|b\rangle$ and the conjugated fields $\langle b|$ are the only propagating degrees of freedom in the system. All other fields are either auxiliary or mediate interactions of the matter fields. In particular this is the case for the gravitational field which corresponds to the sector of the $w$ fields quadratic in $\hat{y}_\pm$ and for its HS generalizations corresponding to higher powers in $\hat{y}_\pm$.

6 Conclusions

In conclusion let us summarize some important properties of the proposed equations.

Due to the form of the product law (10) the matter fields contribute quadratically to the equations for the gravitational field and its HS analogues as expected from the matter sources for the gravitational field. A concrete structure of the matter sources is involved and will be given elsewhere.

In the linearized analysis it is shown that the Lorentz symmetry keeps the standard form in the field equations, i.e. Lorentz connection occurs only through the standard Lorentz covariant derivative. This is important and not completely trivial property that can be shown to remain valid in all orders in interactions [18].

The proposed equations (13) have a form of some zero curvature conditions and therefore can be integrated explicitly at least locally

$$W(x) = g^{-1}(x)dg(x), \quad B(x) = g^{-1}(x)B_0g(x), \quad (32)$$

where $g(x)$ is an arbitrary $x$-dependent invertible element of $A$ while $B_0$ is an arbitrary $x$-independent element of $A$. To interpret properly this result one has to keep in mind that there is an infinite collection of component fields in the model due to the auxiliary variables $z_\pm$ and $y_\pm$ so that $B_0$ contains enough degrees of freedom to describe all modes of a relativistic field [14]. Actually, it follows from the analysis of the section 3 that one can analogously integrate the free matter equations in terms of values of the set of fields $\phi_n$ in some point of space-time (see also [14]), which in their turn can be identified with the
fields \( b_{\pm}^I (u_{\pm}, x) \) \(^{(29)}\). The fact that the same phenomenon takes place for the non-linear model is due to the specific HS interaction terms which make the system integrable.

Acknowledgments

I am very grateful to O. Ogievetsky, A. Tseytlin and I. Tyutin for useful discussions. The research described in this publication was made possible in part by Grant # MQM000 from the International Science Foundation. This work was supported in part by the Russian Basic Research Foundation, grant 93-02-15541.

References

[1] C.M. Hull, Phys. Lett. B240 (1990) 110.
[2] K. Schoutens, A. Sevrin and P. Van Nieuwenhuizen, Phys. Lett. B243 (1990) 245; E. Bergshoeff, C.N. Pope, L.J. Romans et al, Phys. Lett. B243 (1990) 350; C.M. Hull, Phys. Lett. B269 (1991) 257.
[3] A.K.H. Bengtsson and I. Bengtsson, Phys. Lett. B174 (1986) 294.
[4] E. Bergshoeff, C.N. Pope, L.J. Romans et al, Mod. Phys. Lett. A5 (1990) 1957.
[5] E.S. Fradkin and V.Ya. Linetsky, Mod. Phys. Lett. A4 (1989) 2635.
[6] V.G. Kac and D.H. Peterson, Proc. Natl. Acad. Sci. USA 78 (1981) 3308; V.G. Kac and A. Radul, Commun. Math. Phys. 157 (1993) 429.
[7] A. Capelli, C. Trugenberger and G. Zemba, Nucl. Phys. B396 (1993) 465; S. Iso, D. Karabali and B. Sakita, Phys. Lett. B296 (1992) 143.
[8] Y. Yamagishi, Phys. Lett. B259 (1991) 436; F. Yu and Y.-S. Wu, Phys. Lett. B263 (1991) 220.
[9] J. Moyal, Proc. Camb. Phil. Soc. 45 (1949) 99.
[10] E.S. Fradkin and M.A. Vasiliev, Dokl. Acad. Nauk. 29 (1986) 1100; Ann. of Phys. 177 (1987) 63.
[11] M.A. Vasiliev, Fortschr. Phys. 36 (1988) 33; Int. J. Mod. Phys. A6 (1991) 1115.
[12] C.N. Pope, L.J. Romans and X. Shen, Phys. Lett. B236 (1990) 173; Phys. Lett. B242 (1990) 401.
[13] M.A. Vasiliev, Phys. Lett. B285 (1992) 225.
[14] M.A. Vasiliev, Class. Quant. Grav. 11 (1994) 649.
[15] M.A. Vasiliev, Mod. Phys. Lett. A7 (1992) 3689.
[16] F.A.Berezin, Mat. Sbornik, 86 (1971) 578; see also F.A.Berezin “The method of Second Quantization”, Nauka, Moscow, 1986 and references therein;
F.A.Berezin and M.S.Marinov, Ann. of Phys. 104 (1977) 336;
F.Bayen, M.Flato, C.Frondal, A.Lichnerowicz and D.Sternheimer, Ann. of Phys. 110 (1978) 61, 111.

[17] R. Jackiw, in: Quantum Theory of Gravity, ed. S. Christensen (Adam Hilger, Bristol 1984) p. 403;
C. Teitelboim, in: Quantum Theory of Gravity, ed. S. Christensen (Adam Hilger, Bristol 1984) p. 327;
A.H. Chamseddine and D. Wyler, Phys.Lett. B228 (1989) 75; Nucl.Phys. B340 (1990) 595.

[18] M.A. Vasiliev, in preparation