MESSENGER SNEUTRINOS AS COLD DARK MATTER

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**Abstract**

In models where supersymmetry breaking is communicated into the visible sector via gauge interactions the lightest supersymmetric particle is typically the gravitino which is too light to account for cold dark matter. We point out that the lightest messenger sneutrinos with mass in the range of one to three TeV may serve as cold dark matter over most of the parameter space due to one-loop electroweak radiative corrections. However, in the minimal model this mass range has been excluded by the direct dark matter searches. We propose a solution to this problem by introducing terms that explicitly violate the messenger number. This results in low detection rate for both direct and indirect searches and allows messenger sneutrinos to be a valid dark matter candidate in a wide region of SUSY parameter space.
1 Introduction

Theories in which the effects of supersymmetry (SUSY) breaking are introduced into the “visible” sector via the standard model (SM) gauge interactions have recently received considerable attention. One of the most attractive features of such models is the natural explanation for the smallness of the SUSY-contributions to flavor-changing neutral current (FCNC) phenomena both in the quark and lepton sector as a result of a strongly constrained sparticle spectrum. A complete model of gauge mediated supersymmetry breaking requires three sectors: the “visible” sector containing the minimal supersymmetric standard model (MSSM); the “secluded” sector responsible for SUSY breaking and the messenger sector responsible for the communication of SUSY breaking effects into the visible sector. For the purpose of low energy phenomenology there is no need to specify any details about the secluded sector which in the minimal version provides only a single parameter as explained below. The messenger sector is phenomenologically more important since it provides all soft SUSY breaking parameters for the MSSM. One generally assumes, that the messenger sector contains only complete SU(5) multiplets (i.e. pairs of 5s and 10s) in order to naturally maintain gauge coupling unification (τ-bottom unification on the other hand is problematic). The absence of a Landau-pole below the GUT scale limits the number of additional representations such that \( N = N_5 + 3N_{10} \lesssim 5 \) (this value depends somewhat on the messenger masses assumed to be universal and denoted by \( M \)). The general form of the messenger superpotential is given by

\[
W = \sum_{\Phi} \lambda_{\Phi} S \Phi^3, \tag{1.1}
\]

where \( \Phi \subset 5, 10 \) denotes the messenger superfields, \( S \) is an MSSM singlet belonging to the secluded sector. In order to suppress FCNC effects we have to forbid any renormalizable interactions between messenger fields and MSSM matter fields by means of a symmetry. Hence, all the soft SUSY breaking MSSM parameters are determined by only two free parameters: \( M \equiv \lambda_{\Phi} \langle S \rangle \) and \( \Lambda = F/\langle S \rangle \) with \( F \equiv \langle \partial W_S / \partial S \rangle \) (where \( W_S \) is the superpotential of the secluded sector), and a discrete choice for \( \text{sign}(\mu) \) and \( N \). From here all the low energy parameters are obtained by renormalization group evolution. In addition, there is the Higgs mass parameter \( |\mu| \) determined in the standard fashion by imposing radiative electroweak symmetry breaking. The \( \mu \) parameter is not a soft SUSY breaking parameter and a mechanism has to be introduced in order to give it a phenomenologically allowed value. Such a mechanism typically weakens the predictability of the model by allowing the soft SUSY breaking Higgs mixing parameter \( B_0 \), or the ratio of the Higgs vacuum expectation values (vev) \( \tan \beta \), to become a free parameter.

One of the most important features of gauge mediated models is the fact that the gravitino is very light (its mass decreases with the square of SUSY breaking scale) while its coupling to the SUSY particle spectrum is enhanced. Consequently, the gravitino as the lightest SUSY particle can be an interesting warm and mixed DM candidate, and the lightest neutralino, often heavier than the gravitino, is unstable and no longer a cold dark matter (CDM) candidate. The goal of

\(^{a}\)This sector replaces the so-called “hidden” sector in conventional supergravity (SUGRA) models. The fundamental difference from SUGRA models is the existence of renormalizable interaction in addition to gravity and, hence, a much larger ratio of the visible sector SUSY breaking scale over the secluded sector SUSY breaking scale.

\(^{b}\)Recently, there have also been attempts to combine messenger sector and secluded sector.
this paper is to look for an alternative CDM candidate in the gauge mediated models. In section 2, we first reanalyze the properties of a messenger sneutrino as a CDM candidate. We then discuss the feasibility for its direct and indirect detections in section 3. Our conclusions are presented in section 4. Calculations and formulæ are relegated to three appendices.

2 Messenger Sneutrinos as Cold Dark Matter

Without an interaction of MSSM and messenger sector in the superpotential, the lightest messenger particle is stable and a possible CDM candidate. Several such candidates were investigated in Ref. [7]. In this paper, we will focus on the electrically neutral components of chiral supermultiplets, $\mathbf{5}$ and $\mathbf{\bar{5}}$, which was found to be the most promising possibility [7]. It carries the same gauge quantum numbers as the MSSM scalar neutrinos and will be referred as messenger sneutrinos. We denote the electroweak doublets contained in $\mathbf{5}$ and $\mathbf{\bar{5}}$ by $\Phi$ and $\Phi$, respectively, and define $\Phi^\pm = (\Phi^+ + \Phi^-)/\sqrt{2}$. In this basis the $D$-terms for $\Phi$ and $\Phi$ are

$$D^a = g \left( \Phi^\dagger T^a \Phi + \Phi^\dagger T^a \Phi^+ \right),$$

$$D' = \frac{g'}{2} \left( \Phi^\dagger \Phi + \Phi^\dagger \Phi^+ \right) ,$$

and the mass matrix can be written as

$$V = (\Phi^\dagger, \Phi^\dagger) \left( \begin{array}{cc} M^2 + F & gT^a (D^a) + \frac{g'}{2} (D') \\ gT^a (D^a) + \frac{g'}{2} (D') & M^2 - F \end{array} \right) \left( \begin{array}{c} \Phi^+ \\ \Phi^- \end{array} \right) ,$$

where $M$ is the messenger mass scale. We have assumed that $F$ is real and the electroweak indices have been suppressed. Without loss of generality we set $F \geq 0$.

After electroweak symmetry breaking the $D$ terms acquire a non-zero vev that lifts the mass degeneracy of the neutral and charged components of $\Phi^\pm$. The tree-level masses of the lighter mass eigenstates $\phi^Q$ ($Q = 0, -1$ is the electric charge index) are [1]

$$m_{\phi^Q}^2 = M^2 - \sqrt{F^2 + (T^3 - Q \sin^2 \theta_W) m_\lambda^4 \cos^2 2\beta}.$$ (2.4)

In the SUSY limit (i.e. $F = 0$), the mass squared splitting of the neutral ($Q = 0$) and the charged ($Q = -1$) components is of order the $Z$ mass squared $m_z^2$. With SUSY-breaking and in the limit $F \gg m_z^2$, we see that

$$m_{\phi^-} - m_{\phi^0} = \frac{m_z^4}{16FM} \sin^2 2\theta_W \cos^2 2\beta > 0.$$ (2.5)

This result looks promising: the lightest messenger particle is neutral. Furthermore, in a large part of the interesting parameter space the decay $\phi^- \rightarrow \phi^0 f f'$ is sufficiently fast in order not to disturb big bang nucleosynthesis (BBN). Unfortunately, as pointed out in Ref. [7], this $m_\lambda^4$ suppression does in general not apply to the one-loop radiative corrections due to the SUSY breaking $F$-term. These corrections were found to be dominant and have the wrong sign over most of the parameter space, rendering the lightest messenger sneutrino $\phi^0$ as a CDM candidate questionable. However, we find
that there are large one-loop corrections due to electroweak interactions even in the limit of unbroken SUSY. Numerically, we find that to a good approximation (obtained in the limit $M \gg m_z, F = 0$ and $\tan \beta = 1$),

$$m_{\phi^-} - m_{\phi^0} \simeq \frac{\alpha}{2} m_z \simeq 0.3 \text{ GeV}.$$  \hfill (2.6)

This result dominates over the tree-level term of Eq. (2.5) and is only slightly modified by one-loop SUSY breaking effects for $F \neq 0$. Consequently, the $\phi^-$ decay occurs long before BBN and poses no significant constraint on the parameter space. The detailed results for the various one-loop diagram calculations are given in Appendix A.

We now turn to the CDM relic density obtained from a freeze-out calculation of the messenger sneutrinos. The relic density of any particle is governed by the thermal average of the mean annihilation rate times the relative velocity, $\langle \bar{\sigma} v \rangle$ at the freeze-out temperature, $T_f$. Typically, one finds that $T_f \simeq m_{\phi^0}/20 \gg m_{\phi^-} - m_{\phi^0}$ which implies that both charged and neutral components are present in the thermal bath at $T_f$. Hence, the freeze-out calculation should be most properly performed in the limit of symmetric phase for the four components ($\phi^Q, \phi^{Q\dagger}$). We also ignore the gauge boson masses, the error in this approximation is of order $m_z^2/M^2$ and vanishes in the case that $T_f$ is larger than the critical temperature of the electroweak phase transition. We follow Ref. [13] for the freeze-out calculation and our results are summarized in Appendix B.

In Figure 1, we present our results for the relic density ($\Omega h^2$) versus $m_{\phi^0}$ for $r = M/m_{\phi^0} = 1$ (solid curve) and $r = 10$ (dashed curve). The relic density for $r = 1$ is increased by about 50% for $r = 10$, and would go up by a factor of 2 for $r \to \infty$. For comparison, we have also plotted the results calculated in the broken phase [7], by considering only the lightest messenger sneutrino annihilation. The rather large discrepancy can be understood as follows: the broken phase calculation essentially assumes that all annihilation cross sections are of the same order, characterized by $\phi^0\phi^0$ annihilation. However, notice that $\sigma(\phi\phi \to \text{anything}) = \sigma(\phi^\dagger\phi^\dagger \to \text{anything}) = 0$; and $\sigma(\phi^0\phi^- \to \text{anything})$ is suppressed by one power of $x \equiv T_f/m_{\phi^0}$. As a result, the relic density calculated in the broken phase is underestimated by about a factor 4. Generally speaking, as see in Fig. 1, the sneutrino component of the SU(2) doublet with a mass in the TeV range could be a good CDM candidate. Imposing that the universe is not overclosed by the CDM particles (i.e. $\Omega h^2 < 1$) yields the upper bound on the mass of the lightest messenger sneutrino

$$m_{\phi^0} < 3 \text{ TeV}.$$  \hfill (2.7)

For $m_{\phi^0} \lesssim 1 \text{ TeV}$ it seems inadequate to neglect Higgs and the squark masses. However, from Eq. (B.5) we see that the Higgs and matter fields are only produced in $s$-channel annihilation via intermediate gauge bosons. Hence, their rates arise at first order in $x$ and only contribute a few percent.

3 CDM Particle Detection

The most direct way of detecting the existence of dark matter particles would be to observe the DM particle-nucleus scattering by recording the nuclear recoil in a detector [12]. It is known [14] that the
Figure 1: The relic density of $\phi^0$ as a function of $m_{\phi^0}$ in the limit that the mass of all MSSM particles are much lighter than $m_{\phi^0}$. The two upper curves are for our symmetric phase calculation and the lower ones are for the broken phase.

cross section for the sneutrino-nucleus scattering via the $Z$ exchange is

$$\sigma = \frac{G_F^2 m_{\phi^0}^2 m_A^2}{2\pi (m_{\phi^0} + m_A)^2}[A - 2(1 - 2\sin^2 \theta_W)Z]^2,$$

where $G_F$ is the $\mu$ decay constant, $m_A$ is the nuclei mass, $A$ the atomic mass number and $Z$ the atomic number. The current direct searches put an upper bound on the cross section. In fact, a scalar DM particle with mass 3 TeV or less has been ruled out at a 90% confidence level [15] assuming it accounts for about 30% or more of the local galactic halo density $0.3 \text{ GeV/cm}^3$. This would lead to the exclusion of the whole interesting region of the $m_{\phi^0}$ parameter space in Fig. 1 and results in an unsatisfactory solution for the CDM issue in the minimal version of the gauge mediated model.

To provide a solution to this problem, we propose to introduce a tree-level interaction of the MSSM Higgs sector and messenger sector. Consider the following superpotential ($W$) and soft SUSY breaking terms ($V$)

$$W = \lambda N \Phi H_u + \frac{1}{2} m_N N^2,$$

$$V = \lambda A \Lambda \Phi H_u + \frac{1}{2} B \lambda m_N N^2 + \text{H.c.},$$

where $N$ is a gauge singlet, $H_u$ the Higgs doublet coupled to up-type quarks and $A_\lambda$ and $B_\lambda$ the soft SUSY breaking parameters. The inclusion of an additional singlet $N$ is motivated by the attempt to understand the problem of the $\mu$-parameter [4], namely, why the SUSY invariant $\mu$-parameter is of the same order as the SUSY breaking scale in the visible sector. Eq. (3.2) may also have an effect on electroweak symmetry breaking [14] which deserves further scrutiny. If both $\lambda$ and $m_N$ are non-zero, then the messenger number is explicitly broken. However, the model still respects a $Z_2$ parity which guarantees the stability of the lightest messenger particle.
After EW symmetry breaking, $N$ is no longer a mass eigenstate and mixes with the neutral component of $\Phi$. If we assume that $m_N > M$ and $\lambda^2 \langle H_u \rangle^2 \ll m^2_N - M^2$, then the mixing is small so that our analysis in Sec. 2 is still valid. On the other hand, this mixing has another phenomenologically important consequence: it leads to an additional mass splitting of the CP-even and CP-odd components of the neutral fields

$$m_{\text{CP-even}} - m_{\text{CP-odd}} \simeq \frac{m_N - M}{2m_{\phi^0}} \lambda^2 \langle H_u \rangle^2 \left[ B\lambda m_N \frac{m_N - M}{(m^2_N - m^2_{\phi^0})^2} - \frac{2A\lambda}{(m^2_N - m^2_{\phi^0})} \right]. \quad (3.3)$$

BBN constraints are satisfied if this mass difference is larger than a few MeV. For instance, for $M, m_{\phi^0} = 1 \text{ TeV}$, $m_N = 3 \text{ TeV}$ and $B\lambda, A\lambda = 100 \text{ GeV}$, Eq. (3.3) implies $\lambda \gtrsim O(0.1)$. Since the DM particle coupling to the $Z$ boson requires a CP-even and CP-odd transition, such a large mass difference would prevent DM particle-nuclei scattering via single $Z$ exchange from happening. This is because the initial kinetic energy in the c.m. frame for DM particle-nuclei scattering is typically of order 0.1 MeV (assuming the DM particle velocity is about $10^{-3}c$) and is much smaller than the CP-even and CP-odd mass difference. The dominant contribution to DM particle-nuclei scattering would therefore be the spin-independent Higgs exchange through quarks and quark loops. In analogy to the neutralino-nucleon scattering [17], we find that the cross section for sneutrino-nucleus scattering via Higgs exchanges is

$$\sigma_{\text{nucl}} = \frac{m_A^2}{4\pi (m_{\phi^0} + m_A)^2} \left[ f_p Z + f_n (A - Z) \right]^2, \quad (3.4)$$

where $f_p \simeq f_n$ are the effective sneutrino-nuclei coupling via Higgs exchanges. They are given in Appendix C. Following the discussion in Ref. [17], we estimate the direct search event rate ($R_D$) for scalar neutrino-nuclei scattering by

$$R_D \approx \frac{1.8 \times 10^{11} \text{GeV}^4}{\text{kg day}} \frac{\zeta \sigma_{\text{nucl}}}{m_{\phi^0} m_A}, \quad (3.5)$$

where $\zeta$ describes the suppression due to the nuclear form factor [18].

Another way of detecting the DM particle is by the large scale neutrino detectors via annihilation products from DM particles inside the sun/earth. Following the arguments in Ref. [19], the expected event rate in a neutrino detector induced by $\nu_\mu$ from the $\phi^0 \phi^0$ annihilation in the sun may be estimated by

$$R_{ID} \approx \frac{2.65 \times 10^{39}}{10^4 \text{ m}^2 \text{ yr} \text{ GeV}^{-1}} \frac{m_{\phi^0}}{\text{GeV}} \eta \sum_A \kappa_A \zeta \frac{\sigma_{\text{nucl}}}{\text{cm}^2}, \quad (3.6)$$

where $\kappa_A$ is the relative abundance of element $A$ with respect to hydrogen in the sun. With the nuclear form factor $\zeta$ included, we find that the leading contributions to the DM particle capture are from the elements iron, oxygen and helium. $\eta$ in Eq. (3.6) is the neutrino escape probability in the sun parameterized by [19]

$$\eta = \left[ 1 + \frac{m_{\phi^0}}{5.2 \text{ TeV}} \right]^{-7}, \quad (3.7)$$
which significantly suppresses the $\nu_\mu$ flux for $m_{\phi^0} \geq 1$ TeV. The background rate from the flux of atmospheric neutrinos coming from a pixel around the sun is estimated to be 

$$B_{ID} \approx \frac{0.11}{10^4 \text{ m}^2 \text{ yr}} \left( \frac{m_{\phi^0}}{\text{TeV}} \right)^{-2}.$$  

(3.8)

The scattering cross section given by Eq. (3.4) is rather small. For a 1 TeV scalar neutrino scattering off a nucleon, it is typically of order $10^{-47} \sim 10^{-44}$ cm$^2$, depending on SUSY parameters. Comparing the direct and indirect detection rates given in Eqs. (3.5) and (3.6), we see that the indirect search seems to be more suitable for a heavier DM particle. Especially, the background decreases like $1/m_{\phi^0}^2$. However, the suppression due to $\zeta$ and $\eta$ in Eq. (3.7) becomes increasingly severe for heavier $\phi^0$. Yet, Eq. (3.6) may have overestimated the signal somewhat due to the omission of a correction factor to the capture rate by the sun for a heavier DM particle. The signal rates for the direct and indirect searches are shown in Fig. 2 as a function of $m_{\phi^0}$. The rate for direct search in Fig. 2(a) is for a $^{73}$Ge detector. The lower solid curve corresponds to a high mass limit for the CP-odd Higgs boson so that the dominant contribution is from a SM-like Higgs boson, assuming $m_h = 100$ GeV. The upper solid curve is for an optimal choice of parameters to enhance the signal rate (CP-odd Higgs mass of 80 GeV and $\tan \beta = 50$). In our numerical analysis, the coupling $\lambda$ in Eq. (3.2) has been taken to be unity. We see that the rate is typically of order $10^{-4}$ events/(kg day). It is much below the current experimental sensitivity of about 2 events/(kg day) [20], and is also unreachable by next generation experiments with sensitivity of $10^{-2}$ events/(kg day) [12]. Fig. 2(b) shows the calculated signal rate for the indirect search, with the same sets of SUSY parameters as in (a). Based on Eq. (3.8), the
background rate is calculated and shown by the dotted curve in Fig. 2(b). Again, it is difficult to detect the signal for the indirect search as well.

4 Conclusions

We have considered the possibility that the CDM constitutes of sneutrino-like messenger particles. We found that the neutral components $\phi^0$ of the SU(2) doublet is naturally lighter than the charged component over most of the relevant parameter space due to one-loop electroweak corrections and could serve as a CDM candidate. However, our relic density calculation shows that a significant amount of messenger CDM in the minimal model has been already ruled out by present experimental results. We introduce a mechanism that generates a CP-even–CP-odd mass splitting. This circumvents the constraints from present detection experiments and allows the messenger sneutrino to be a valid CDM candidate in a wide range of SUSY parameter space. Consequently, the detection via direct and indirect searches would be very difficult due to the rather small messenger sneutrino-nucleon scattering cross section.

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Appendix A: $m_{\phi^-} - m_{\phi^0}$ mass difference

At tree-level, the only source of a mass splitting of different members of an SU(2)$_L \otimes$U(1)$_Y$ are a non-zero vev for the SU(2) D-term which is strongly suppressed as shown in Eq. 2.5. In this appendix we present the result for the dominant one-loop contribution to this mass difference. A complete one-loop calculation is rather involved. In particular, it requires a renormalization of $\tan \beta$ which enters already at tree-level. Hence, we consider the case $\tan \beta = 1$ where the corrections to the mass squared difference is finite and given by the difference of the self-energies

$$\Delta m^2 \equiv m_{\phi^-}^2 - m_{\phi^0}^2 = A_{\phi^-\phi^-}(m_{\phi^-}^2) - A_{\phi^0\phi^0}(m_{\phi^0}^2).$$ (A.1)

Furthermore, we neglect terms suppressed by $F/M^2$ (i.e., we set $F = 0$ and as a result $m_{\phi^0} = m_{\phi^-} = M$).

There are three types of diagrams that contribute to the self-energies: the loops involving gauge bosons ($g$), gauginos ($\tilde{g}$), and loops with tri-linear terms arising from the $D$-terms (Higgs field only). Loops with quartic terms arising from the $D$-terms do not generate mass splitting among the different members of the multiplet $\phi$. The results are

$$A_{\phi^0\phi^0}(M^2) = \frac{\alpha}{4\pi \sin^2 2\theta_W} \left[ (4M^2 - m_z^2) B_0(M^2, m_z^2, M^2) - A_0(m_z^2) - A_0(M^2) \right],$$
\[ A_{\phi\phi}(M^2) = \frac{\alpha}{2\pi \sin^2 2\theta_W} \left[ m_\phi^2 B_0(M^2, m_\phi^2, M^2) + A_0(m_\phi^2) + A_0(M^2) \right], \]
\[ A_{H\phi\phi}(M^2) = \frac{-\alpha}{4\pi \sin^2 2\theta_W} m_\phi^2 B_0(M^2, m_\phi^2, M_{H^0}^2), \] (A.2)

where \( \alpha \) is the fine structure constant and \( \theta_W \) the weak mixing angle. In the SUSY limit we have \( m_{H^0} = m_\phi = m_z \). The self-energy for the charged component can be written as
\[ A_{\phi - \phi}(M^2) = \cos^2 2\theta_W A_{\phi\phi\phi}(M^2) + \sin^2 2\theta_W A_{\phi\phi\phi}(M^2)(m_\phi^2 \to 0) \] (A.3)

Hence, we obtain
\[ \Delta m^2 = \frac{\alpha}{4\pi} M^2 \left[ B_0(M^2, 0, M^2) - B_0(M^2, m_\phi^2, M^2) \right]. \] (A.4)

In dimensional regularization, the scalar one and two point functions are given by
\[ 16\pi^2 A_0(m_1^2) = m_1^2 \left( \Delta + 1 - \ln m_1^2 \right), \]
\[ 16\pi^2 B_0(p^2, m_1^2, m_2^2) = \Delta - \int_0^1 dx \ln \left[ x m_1^2 + (1 - x)m_2^2 - x(1 - x)p^2 \right], \] (A.5)

where \( \Delta = 1/\epsilon - \gamma_E + \ln 4\pi \) in \( d = 4 - 2\epsilon \) dimensions, with \( \gamma_E \) the Euler constant. Expanding Eq. (A.4) for \( M \gg m_z \), Eq. (2.6) is recovered.

**Appendix B: Annihilation Rate and Relic Density**

Our calculation for the annihilation rate follows the formalism developed in Ref. [13]. There are four nearly mass-degenerate particles present at the freeze-out temperature \( T_f \), \( \phi^Q \) and \( \phi^{Q\dagger} \) (\( Q = 0, -1 \)). If we assume that there is no messenger number asymmetry then the total relic density is \( n = 4n_\phi^Q = 4n_\phi^{Q\dagger} \) and the rate equation can then be written as
\[ \frac{dn}{dt} = -3Hn - (\bar{\sigma}v_{\text{rel}})n^2, \] (B.1)

where \( H \) is the expansion rate of the universe and
\[ \bar{\sigma} \equiv \frac{1}{8} \sum_{Q,Q'} \sigma_{\phi^Q\phi^{Q'}}. \] (B.2)

Note \( \sigma_{\phi^Q\phi^{Q'}} = \sigma_{\phi^{Q\dagger}\phi^{Q\dagger}} = 0 \).

Following Ref. [13], we first define a Lorentz invariant function
\[ w(s) \equiv \frac{\beta}{64\pi} \int_1^1 \frac{1}{8} \sum_{Q,Q',a,b} |\mathcal{M}(\phi^Q\phi^{Q'} \to ab)|^2 d\cos \theta \] (B.3)

where \( \beta^2 = 1 - 4m_\phi^2/s \) is the squared velocity and the sum over \( a \) and \( b \) symbolizes the sum over all two-body final states. Then
\[ w(s) = \sum_{g,g'=b,w} \left( w^{gg'} + w^{g'g} \right) + w^{s1} + w^{ff}, \] (B.4)
the result of the expansion of the different contributions to $w(s)$ to first order in $\beta^2$ is

\begin{align*}
    w^{ww} &= \frac{g^4}{512\pi} \left( 3 - 2\beta^2 \right), \\
    w^{bw} &= \frac{g^2 g'^2}{512\pi} \left( 3 - 4\beta^2 \right), \\
    w^{bb} &= \frac{g'^4}{512\pi} \left( 1 - \frac{4}{3}\beta^2 \right), \\
    w^{\tilde{w}\tilde{w}} &= \frac{g^4}{512\pi} \left[ \frac{12r}{(1+r)^2} + \frac{4\beta^2}{(1+r)^4} \left( 1 + 7r - 8r^2 + 3r^3 + r^4 \right) \right], \\
    w^{\tilde{b}\tilde{w}} &= \frac{g^2 g'^2}{512\pi} \left[ \frac{12r}{(1+r)^2} - \frac{4r\beta^2}{(1+r)^4} \left( 1 + 6r - 3r^2 \right) \right], \\
    w^{\tilde{b}\tilde{b}} &= \frac{g'^4}{512\pi} \left[ \frac{4r}{(1+r)^2} - \frac{4r\beta^2}{(1+r)^4} \left( \frac{1}{3} + 2r - r^2 \right) \right], \\
    w^{s\bar{s}} &= \sum_s \frac{N_c \beta^2}{4096\pi} \left[ N_w Y^2 g^4 + 6(N_w - 1) g^4 \right], \\
    w^{f\bar{f}} &= 2w^{s\bar{s}}, \quad (B.5)
\end{align*}

where $r = M/m_\phi^0$. The result for the gauge bosons (gauginos) corresponding $U(1)_Y$ and $SU(2)_L$ are denoted by a superscript $b$ and $w$ ($\tilde{b}$ and $\tilde{w}$), respectively. Here, $N_c = 1$ ($3$) for colored singlets (triplets) and $N_w = 1$ ($2$) for $SU(2)_L$ singlets (doublets). The index $s$ ($f$) runs over all MSSM scalar (fermion) fields other than gauge bosons and gauginos.

The thermal average $\langle \sigma v_{\text{rel}} \rangle$ can be expressed by

$$\langle \sigma v_{\text{rel}} \rangle = \frac{1}{m_\phi^0} (A + Bx), \quad (B.6)$$

where $x = T_f / m_\phi^0$, and [13]

$$A = w(4m_\phi^2), \quad B = 3 \left[ 2m_\phi^2 \frac{dw(s)}{ds} - w(s) \right]_{s=4m_\phi^2} \left( \frac{m_\phi}{\text{TeV}} \right)^2 \frac{x^{-1}}{A + \frac{1}{2}Bx}. \quad (B.7)$$

We have ignored the possible contribution to the annihilati on cross sections from the interactions in Eq. (3.2). This implies a lower limit of $\langle \sigma v_{\text{rel}} \rangle$ and hence, an upper limit on the relic density.

It is customary to express the relic abundance in terms of the mass density in units of the critical density $\Omega_\phi = \rho / \rho_c$. It is found that

$$\Omega_\phi h^2 = \frac{8.5 \times 10^{-5}}{\sqrt{g_*}} \left( \frac{m_\phi}{\text{TeV}} \right)^2 \frac{x^{-1}}{A + \frac{1}{2}Bx}, \quad (B.8)$$

where $h$ is the Hubble constant, $g_* \simeq 228.75$ [3]. The value $x$ is related to the freeze-out temperature and can be obtained iteratively by

$$x^{-1} = m_\phi^0 / T_f = \ln \left[ \left( \frac{0.076 M_{Pl}}{\sqrt{g_*} m_\phi^0} (A + Bx) \sqrt{x} \right) \right], \quad (B.9)$$

where $M_{Pl}$ is the Planck mass.
Appendix C: sneutrino-nuclei Elastic Scattering via Higgs exchange

For the model of our interest, the $Z$-exchange contribution between two different sneutrino mass eigenstates is negligible. The leading contribution would therefore be the spin-independent Higgs exchange. We write the effective interaction between a sneutrino ($\phi$) and a nucleon ($N$)

$$\mathcal{L}_{\phi N} = f_N \phi \bar{\phi} \psi_N \psi_N,$$

where $f_N (N = p, n)$ is the effective coupling through Higgs boson exchanges:

$$f_N = m_N (\sum_q K_q + \sum_Q K_Q + \sum_{\tilde{Q}} K_{\tilde{Q}}),$$

where $K_q, K_{\tilde{Q}}$ and $K_{\tilde{Q}}$ are contributions from a light quark $q$, a heavy quark $Q$ and its supersymmetric partner $\tilde{Q}$. In analogy to the neutralino-nucleon interaction [17], we determine the couplings as

$$K_q = f_{T_q} \sum_j \frac{c_j^q c_j^q}{m_{H_j}^2}, \quad K_Q = \frac{2}{27} f_{T_Q} \sum_j \frac{c_j^Q c_j^Q}{m_{H_j}^2}, \quad K_{\tilde{Q}} = \frac{1}{108} f_{T_{\tilde{Q}}} \sum_j \frac{c_j^{\tilde{Q}} c_j^{\tilde{Q}}}{m_{H_j}^2 m_{\tilde{Q}}^2}.$$  

The constant $f_{T_q}$ is the nucleon mass fraction due to a light quark $q$, and $f_{T_{G \rightarrow Z}} = 1 - \sum_{u,d,s} f_{T_q}$. Numerically, we take [21]

for a proton: $f_{T_u} = 0.023, \quad f_{T_d} = 0.034,$  
for a neutron: $f_{T_u} = 0.019, \quad f_{T_d} = 0.041,$

and [22]

$$f_{T_s} = 0.14.$$  

The $H_j^0 \phi \bar{\phi}$ couplings are given by

$$c_{\phi}^{H_j^0} = \frac{1}{2} \lambda^2 v \sin \beta \cos \alpha, \quad c_{\phi}^{H_j^0} = \frac{1}{2} \lambda^2 v \sin \beta \sin \alpha,$$

where $\lambda$ is a model-dependent parameter in Eq. (3.2), $v = 246$ GeV is the Higgs vacuum expectation value and $\beta$ and $\alpha$ are the standard mixing parameters in the SUSY Higgs sector. Finally, $c_j^q, c_j^{Q \rightarrow \tilde{Q}}$ are the couplings of a Higgs boson $H_j$ to quarks and squarks and we have followed the convention in Appendix A of Ref. [17]. In practice, the squark contributions are small and have been neglected.

The cross section for the coherent elastic scattering of $\phi$ off a nuclei $A$ in the non-relativistic limit is thus calculated to be

$$\sigma = \frac{\Sigma |\mathcal{M}|^2}{16\pi (m_\phi + m_A)^2} = \frac{m_A^2}{4\pi (m_\phi + m_A)^2} \left[ f_p Z + f_n (A - Z) \right]^2.$$  

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