Divergence of dynamical conductivity at certain percolative superconductor-insulator transitions

Yen Lee Loh, Rajesh Dhakal, John F Neis and Evan M Moen

Department of Physics and Astrophysics, University of North Dakota, Grand Forks, ND 58201, USA
E-mail: yenleeloh1@gmail.com

Received 11 June 2014, revised 25 October 2014
Accepted for publication 28 October 2014
Published 24 November 2014

Abstract
Random inductor–capacitor (LC) networks can exhibit percolative superconductor-insulator transitions (SITs). We use a simple and efficient algorithm to compute the dynamical conductivity \( \sigma(\omega, p) \) of one type of LC network on large \((4000 \times 4000)\) square lattices, where \( \delta = p - p_c \) is the tuning parameter for the SIT. We confirm that the conductivity obeys a scaling form, so that the characteristic frequency scales as \( \Omega_1 \propto |\delta|^{\nu_z} \) with \( \nu_z \approx 1.91 \), the superfluid stiffness scales as \( \Upsilon \propto |\delta|^t \) with \( t \approx 1.3 \), and the electric susceptibility scales as \( \chi_E \propto |\delta|^{-s} \) with \( s = 2\nu_z - t \approx 2.52 \). In the insulating state, the low-frequency dissipative conductivity is exponentially small, whereas in the superconductor, it is linear in frequency. The sign of \( \text{Im} \sigma(\omega) \) at small \( \omega \) changes across the SIT. Most importantly, we find that right at the SIT \( \text{Re} \sigma(\omega) \propto \omega^{t/\nu_z - 1} \propto \omega^{-0.32} \), so that the conductivity diverges in the DC limit, in contrast with most other classical and quantum models of SITs.

Keywords: superconductor-insulator transition, dynamical conductivity, percolation, disorder, circuit, network, quantum phase transition

One of the most important questions concerns the AC conductivity in the ‘collisionless DC’ limit \([22, 23]\), \( \sigma^* = \lim_{\omega \to 0} \lim_{T \to 0} \sigma(\omega, T) \). It is often claimed that at the SIT \( \sigma^* \) is finite and takes a universal value of the order of \( \sigma_Q = \frac{4e^2}{h} \), but there are large discrepancies between the ‘universal’ values from various studies, and it is not clear whether there really is a universal value \([1, 4, 16, 23, 24–31]\). In this paper we consider a coarse-grained superconductor-insulator composite (e.g. millimeter-sized superconducting particles deposited on an insulating substrate) where the SIT is governed by classical percolation. Then the kinetic inductances and mutual capacitances of the superconducting grains can be represented by inductors and capacitors (see appendix A for justification). We find that for one of the simplest inductor-capacitor network models, \( \sigma(\omega) \) diverges as \( \omega \to 0 \), so that \( \sigma^* \) is infinite (figure 1)! This is surprising and important, especially in light of prior work on classical models \([32]\). Furthermore, it ties in well with recent studies \([33–36]\) to

\[ \sigma^{\text{DC}} = \lim_{T \to 0} \lim_{\omega \to 0} \sigma(\omega, T). \]
complete a bigger picture. For disordered Josephson junction arrays (JJAs) that include Coulomb blockade effects, those studies found that $\sigma^* \approx 0.4(4e^2/h)$ for zero disorder ($p = 0$) whereas $\sigma^* \approx 0.5(4e^2/h)$ for moderate disorder ($p = 0.373$). According to our work, $\sigma^* = \infty$ at a percolative SIT, corresponding to an extremely disordered JJA ($p = 0.5$). This suggests a monotonic trend in $\sigma^*(p)$.

There have been several high-profile studies [19, 21, 37] giving detailed measurements of $\sigma(\omega)$ across a finite-temperature superconductor-normal transition. Thin NbN films had a large fluctuation regime: for a film with $T_c = 3.14$ K, finite-frequency superfluid response persisted up to $T = 6$ K. A careful scaling analysis indicated a 'fundamental breakdown of the standard Cooper-pair fluctuation scenario' due to phase fluctuations. The ultimate goal is to measure $\sigma(\omega)$ across the superconductor-insulator transition. This has not yet been achieved, partly because it is difficult to characterize insulating films by tunneling. Thus, theoretical predictions are crucial in guiding experiments. In this paper we point out that $\text{Im} \sigma$ has a sign change across the SIT, which would be hidden in traditional plots of $\log|\text{Im} \sigma(\omega)|$ [19, 37]. We elucidate the general structure of $\sigma(\omega)$, addressing the static limit, characteristic frequency, reactive response, and low-frequency contributions from rare regions in the insulator and from Goldstone modes in the superconductor.

1. Model

We focus on the $L_{ij}C_i$ model (see figure 2), which contains capacitances-to-ground $C_i = C_0$ on every site $i$ and inductances $L_{ij} = L_0$ along bonds $ij$ with probability $p$. This can be thought of as a model of spherical superconducting grains each of capacitance $C_0 = 4\pi\varepsilon_0 R$, connected by superconducting links obeying the London equation $\frac{dJ_i}{dt} = \frac{\pi\varepsilon_0 J_i L_0}{m} E$. The latter are represented by kinetic inductances $L_0$ (since $L_0\frac{dI}{dt} = V$ for an inductor). The $L_{ij}C_i$ model is almost the same as a Josephson junction array, except that it lacks Coulomb blockade effects resulting from charge quantization. The dynamical electromagnetic response $\chi = -\delta j/\delta A$, conductivity $\sigma = \delta j/\delta E$, and electric susceptibility $\chi_E = \delta P/\delta E$ can be defined in the usual way for a 2D system.

2. Methods

Previous authors have attacked similar problems numerically using a matrix formalism [38], transfer matrix methods [39–41], and the Frank–Lobb bond propagation algorithm [42, 43]. In this study we employ a variant of the equation-of-motion method [44], which is much simpler, more general, and more efficient. Our approach is the theoretical analogue of Fourier transform nuclear magnetic resonance (FT-NMR) spectroscopy, where the frequency response is inferred from the free induction decay signal—the impulse response in the time domain. We apply a transient uniform electric field $E(t) = \delta(t)$, evolve currents and voltages according to the dynamical Kirchhoff equations, record the uniform component of the current $I_0(t)$, and extract the dissipative conductivity $\text{Re} \sigma(\omega)$ using a fast Fourier transform. The discretization error in the time evolution enters entirely in the form of systematic phase error, which we eliminate by a suitable transformation of the frequency variable. This procedure can be shown to be formally equivalent to Chebyshev methods such as the kernel polynomial method [45–47]. The only source of error is the finite duration of the simulation, which leads to a finite frequency resolution. We use a Kaiser window function that gives $\text{Re} \sigma(\omega)$ with sidelobe amplitude below $\Delta\sigma \approx 10^{-8}$ and main lobe width $\Delta\omega \approx 15\omega_{\text{max}}$ (where $M$ is the number of timesteps). This prevents exponentially small tails in the spectrum from being contaminated by spectral leakage. We compute $\text{Im} \sigma(\omega)$ using a Kramers–Kronig transformation. We estimate the superfluid stiffness $\chi(\omega)$ from the weight in the lowest-frequency bin, and the electric susceptibility from $\chi_E = \int_{-\infty}^\infty d\omega 2\text{Re} \sigma(\omega)/\omega^2$. See appendices B and C for details.

We simulated $4000 \times 4000$ lattices for $40000$ timesteps (i.e. $40000$ Chebyshev moments), giving a resolution of
\(\Delta \omega \approx 0.0015\) after windowing and rebinning. We found that larger (6144 \(\times\) 6144) lattices gave no discernible differences. This is because the finite-frequency (\(\omega \gtrsim 0.005\)) dynamical response is dominated by clusters considerably smaller than the simulation size, and is thus relatively insensitive to finite-size effects. See appendix D.

3. Results

We quote angular frequency \(\omega\) in units of \(1/\sqrt{L_0 C_0}\) and 2D conductivity (sheet conductance) \(\sigma\) in units of \(1/\sqrt{L_0 C_0}\). Color plots of \(\text{Re} \sigma(\omega, p)\) and \(\text{Im} \sigma(\omega, p)\) are shown in figure 3. The data contain a wealth of interesting information that we list below.

(a) Superfluid stiffness: On the superconducting side of the SIT (\(p > p_c\)), the superfluid stiffness scales as \(\Upsilon(p) \sim \delta^d\) where \(\delta = p - p_c\) and \(t \approx 1.30\), as shown in figure 4. This agrees with results for resistor networks [48,49].

(b) Characteristic frequency: For 2D percolation the correlation length diverges as \(\xi \sim |\delta|^{-\nu_\xi}\) with \(\nu = 4/3\) [50]. This suggests that there is a characteristic (angular) frequency \(\Omega \sim \delta^{\nu_\xi} \sim |\delta|^{\nu_\xi}\) where \(\nu_\xi\) is a dynamical critical exponent. Indeed, figure 3 suggests that most of the spectral weight in \(\text{Re} \sigma(\omega)\) occurs above a frequency \(\Omega \sim \delta^{\nu_\xi}\), forming a shape analogous to a ‘quantum critical’ fan.

(c) Divergent conductivity: At the SIT (\(p = p_c\)), \(\text{Re} \sigma(\omega)\) does not tend to a finite limit as in many other systems, but instead it diverges! This is illustrated in figure 1. We find a very good fit to a power law of the form \(\text{Re} \sigma(\omega) \approx \omega^{-\nu}\) where \(\nu \approx 0.32(1)\).

(d) Scaling collapse: Based on observations (a) and (b), we postulate the scaling form \(\sigma(\omega, \delta) = \omega^{1/(\nu z)} f(\omega^{1/(\nu z)} \delta)\). This mandates that \(\sigma(\omega) \propto \omega^{1/(\nu z)}\) at the SIT. Comparing with (c), we see that we must have \(t/vz - 1 = -\alpha\), so that \(\nu z = t/(1 - \alpha) \approx 1.91\). Indeed, we find that both \(\text{Re} \sigma\) and \(\text{Im} \sigma\) collapse onto single curves for \(\nu z \approx 1.91\), as shown in figure 5. The details of \(f(x)\) will be reported elsewhere.

Figure 3. Dynamical conductivity for the \(L_{ij} C_{ij}\) model for \(p = 0.02, 0.04, \ldots, 0.40, 0.41, \ldots, 0.60, 0.62, \ldots, 0.98\) for single realizations on a 4000 \(\times\) 4000 lattice. Red, green, and blue indicate insulating (capacitive), metallic (resistive), and superconducting (inductive) behavior respectively. The SIT occurs at the percolation threshold (\(p_c = 0.5\)). The dissipative conductivity \(\text{Re} \sigma(\omega, p)\) has a delta function in the superconductor (\(p > p_c\)), visible as a thin green line; most of the remaining weight occurs above a characteristic frequency \(\Omega \propto (p - p_c)^{\nu_\Omega}\), forming a fan shape reminiscent of quantum criticality. The reactive conductivity \(\text{Im} \sigma(\omega, p)\) changes sign from capacitive to inductive as \(p\) increases.

Figure 4. Static response functions of \(LC\) networks as a function of superconducting bond fraction \(p\). The insulating state is characterized by a finite electric susceptibility \(\chi_E\), which diverges at the percolation transition. The superconducting state is characterized by a finite superfluid stiffness \(\Upsilon\), which vanishes at the transition. Near the percolation threshold \(p\), the superfluid stiffness scales as \(\Upsilon \sim (p - p_c)^{1.30}\). For the \(L_{ij} C_{ij}\) model near \(p_c\), \(\chi_E \approx (p - p_c)^{-1.30}\) due to duality. For the \(L_{ij} C_{ij}\) model near \(p_c\), \(\chi_E \sim (p - p_c)^{-2.52}\).

(e) Electric susceptibility: On the insulating side of the SIT (\(p < p_c\)), the scaling form dictates that the electric susceptibility must scale as \(\chi_E \sim |\delta|^{-s}\) with \(s = 2\nu z - t \approx 2.52\). Indeed, figure 4 shows a good fit to this power law.

(f) Low-frequency dissipation: The characteristic frequency \(\Omega\) does not correspond to a hard gap in the spectrum. In the insulating state, large rare regions contribute exponentially small weight to \(\text{Re} \sigma\) all the way down to zero frequency, as shown in the bottom panel of figure 1. This is a ramification of Griffiths–McCoy–Wu physics [51–53] in systems with quenched disorder. The superconducting state has linear low-frequency dissipation \(\text{Re} \sigma \sim \omega\), as shown in the top panel of figure 1. We believe this is due to the excitation of acoustic ‘transmission-line’ modes that is permitted in the presence of disorder. (It is also reminiscent of [54].)
verified that scaling collapse is good for 10 beyond the dynamic range shown above. 

\[ \nu_z \]

extracted using scaling collapse of \( \sigma(p, \omega) \). We have studied the percolative superconductor-insulator transition in two-dimensional classical LC networks, in particular, the \( L_{ij}C_i \) model. We used an efficient algorithm to compute \( \sigma(\omega, p) \) on large lattices (4000 × 4000 sites). We find the critical exponents \( t \approx 1.30 \) (in agreement with results on resistor networks), \( \nu_z \approx 1.91 \), and \( s = 2\nu_z - t \approx 2.52 \). We have extracted the complex-valued scaling function. In the insulating state, the low-frequency dissipative conductivity is exponentially small, whereas in the superconductor, it is linear in frequency. The sign of \( \Im \sigma(\omega) \) at small \( \omega \) changes across the SIT. Most surprisingly, right at the SIT, \( \Re \sigma(\omega) \) diverges as \( \omega \to 0 \). Our results form an important baseline to which to compare simulations of more complicated models such as XY, Bose–Hubbard, and Fermi–Hubbard models, thus allowing one to separate the effects of quenched disorder, quantum phase fluctuations, and pairbreaking physics.

4. Importance of on-site capacitances

In previous studies of the dynamical conductivity of classical systems near percolation, the insulating (capacitive) elements were placed along bonds, in series with the superconducting (inductive) elements [32, 38]. We have studied an LC model of this type, which we call the \( L_{ij}C_{ij} \) model (right panel in figure 2). The dynamical conductivities of the \( L_{ij}C_i \) and \( L_{ij}C_{ij} \) models are compared in figure 6. We find that for the latter model at percolation (\( p = 0.5 \)), \( \sigma(\omega) \) tends to a finite value \( \sigma^* \) as \( \omega \to 0 \). This is a consequence of self-duality [55], which implies that \( \nu_z = t \approx 1.3 \). Also, \( \Re \sigma \) indicates exponentially small low-frequency dissipation in the superconducting state (as opposed to \( \Re \sigma \propto \omega \) for the \( L_{ij}C_i \) model).

The \( L_{ij}C_i \) model includes on-site capacitances, which are absent from the \( L_{ij}C_{ij} \) model. In general, one expects that any real system of superconducting grains will have finite on-site capacitances, literally or in a ‘renormalization group’ sense. (All previous studies of bosonic models of SITs [29, 30] included finite on-site capacitances; in fact, the \( L_{ij}C_i \) model is the limiting case of a JJA when the charging energy is negligible.) Thus our results suggest that classical percolative SITs should generically have a divergent conductivity.

5. Conclusions

We have studied the percolative superconductor-insulator transition in two-dimensional classical LC networks, in particular, the \( L_{ij}C_i \) model. We used an efficient algorithm to compute \( \sigma(\omega, p) \) on large lattices (4000 × 4000 sites). We find the critical exponents \( t \approx 1.30 \) (in agreement with results on resistor networks), \( \nu_z \approx 1.91 \), and \( s = 2\nu_z - t \approx 2.52 \). We have extracted the complex-valued scaling function. In the insulating state, the low-frequency dissipative conductivity is exponentially small, whereas in the superconductor, it is linear in frequency. The sign of \( \Im \sigma(\omega) \) at small \( \omega \) changes across the SIT. Most surprisingly, right at the SIT, \( \Re \sigma(\omega) \) diverges as \( \omega \to 0 \). Our results form an important baseline to which to compare simulations of more complicated models such as XY, Bose–Hubbard, and Fermi–Hubbard models, thus allowing one to separate the effects of quenched disorder, quantum phase fluctuations, and pairbreaking physics.

Acknowledgment

We gratefully acknowledge M Swanson, M Randeria and P Karki for useful discussions.

Appendix A. Modeling superconductors and inductors by lumped circuit elements

We first review the relationship between Ohmic conductance and conductivity. First, consider a solid metal tube of length \( l \), cross-sectional area \( A \), and DC conductivity \( \sigma \). The potential difference and electric field are related by \( V = EL \), whereas the current and current density are related by \( I = JA \).

\[ G = \frac{I}{V} = \frac{J}{A} = \frac{\sigma}{l}. \quad (A1) \]

Second, consider a solid superconducting tube of length \( l \) and cross-sectional area \( A \). Suppose the superconductor carries a small uniform current density, much smaller than the critical value. In this regime the superconductor satisfies the London equation \( \frac{\partial j}{\partial t} = \Upsilon E \), where \( \Upsilon \) is the superfluid stiffness (conventionally expressed as a superfluid density \( n_s \)), with \( \Upsilon = \frac{n_s c s}{\sigma \omega} \). Then, the superconducting tube behaves as an inductor of ‘kinetic’ inductance \( L \) such that

\[ L^{-1} = \frac{\partial j}{\partial t} = \frac{(\partial j)A}{EI} = \Upsilon \frac{A}{l}. \quad (A2) \]

Indeed, the low-frequency dynamical conductivity of a superconductor (\( \Im \sigma \sim \omega^{-1} \)) is the same as the low-frequency admittance of an inductor (\( \Im G \sim \omega^{-1} \)). Thus a superconductor can be represented by an inductor provided that nonlinear effects are negligible (i.e. phase fluctuations are small compared to \( \pi \)). (In this paper we are not concerned with high frequencies, where \( \sigma(\omega) \) in a real superconductor is dominated by the Mattis–Bardeen contribution coming from two-quasiparticle excitations.) Finally, consider a solid insulating tube of the same geometry containing a uniform polarization \( P \) and electric field \( E \). The surface polarization charge is \( \sigma_{pol} = \epsilon_0 P = \chi_e E \), where \( \chi_e \) is the electric susceptibility of the material. This gives a naive estimate of the capacitance,

\[ C = \frac{Q}{V} = \frac{\sigma_{pol} A}{EI} = \chi_e \frac{A}{l}. \quad (A3) \]
Figure 6. Stacked plots of Re \( \sigma(\omega) \) for \( L_{ij}C_i \) model (left column) and \( L_{ij}C_{ij} \) model (right column). At the percolation threshold \( (p = 0.5) \) at low frequency \( (\omega \rightarrow 0) \), \( \sigma(\omega) \) diverges as \( \omega^{-0.32} \) for the \( L_{ij}C_i \) model, whereas it tends to a constant \( \sigma^* \) for for the \( L_{ij}C_{ij} \) model.

In reality the surface charges produce an induced electric field that partially opposes the applied electric field, so that the capacitance of a parallel-plate capacitor is \( C = (1 + \chi_e)\frac{A}{L} \). In fact, two electrodes separated by vacuum still have a finite capacitance. Even a single electrode can have a finite capacitance to ground; for example, a sphere of radius \( a \) has \( C = 4\pi\varepsilon_0 a \) to ground. Also note that real metals and insulators do not respond instantaneously to an applied voltage, because of the inertia of the electrons; this can be modeled by inserting a kinetic inductance in series with the resistor or capacitor.

In principle, superconducting grains deposited on an insulator can be treated by dividing the system into small cubes (or other finite elements) and going from there to a lumped element model. However, for elucidating universal critical behavior, it is better to study simple models with well-defined disorder distributions, as we do here.

Appendix B. Algorithm description for \( L_{ij}C_i \) model

The dynamical Kirchhoff equations for the \( L_{ij}C_i \) model are simply

\[
\begin{align*}
\dot{V}_i &= C_i^{-1} \sum_j I_{ji} \\
\dot{I}_{ji} &= L_{ij}^{-1}(V_i - V_j)
\end{align*}
\]

where \( V_i \) is the current on node \( i \) and \( I_{ji} \) is the current along link \( ji \). A detailed simulation algorithm is as follows.

Set up a square lattice with periodic boundary conditions with \( N = N_x N_y \) nodes, where \( x = 0, 1, 2, \ldots, N_x - 1 \) and \( y = 0, 1, 2, \ldots, N_y - 1 \). Initialize the node capacitances \( C_{xy} \) and the link inductances. \( L_{xy}^x \) is the inductance on the link \((x, y)\)–\((x + 1, y)\), and \( L_{xy}^y \) is the inductance on the link \((x, y)\)–\((x, y + 1)\).

Identify the highest possible eigenfrequency according to Gerschgorin’s theorem (which is \( \sqrt{8} \) for this model). For safety, choose \( \omega_{\text{max}} \) to be slightly greater than \( \sqrt{8} \), and set the timestep to \( \tau = 2/\omega_{\text{max}} \). Initialize node voltages and link currents:

\[
\begin{align*}
I_{xy}^x|_{t=0} &= 1/L_{xy}^x \\
I_{xy}^y|_{t=0} &= 0 \\
V_{xy}|_{t=0} &= \frac{\tau}{2C_{xy}^z}(I_{x-1,y}^x - I_{x,y}^x + I_{x,y-1}^y - I_{x,y}^y) |_{t=0}
\end{align*}
\]

Evolve system using leapfrog algorithm:

\[
\begin{align*}
I_{xy}^x_t &= I_{xy}^x|_{t-\tau} + \frac{\tau}{L_{xy}^x}(V_{xy} - V_{x+1,y})|_{t-\tau} \\
I_{xy}^y_t &= I_{xy}^y|_{t-\tau} + \frac{\tau}{L_{xy}^y}(V_{xy} - V_{x,y+1})|_{t-\tau} \\
V_{xy}|_{t+\tau} &= V_{xy}|_{t-\tau} + \frac{\tau}{C_{xy}^z}(I_{x-1,y}^x - I_{x,y}^x + I_{x,y-1}^y - I_{x,y}^y) |_{t-\tau}
\end{align*}
\]

Record the total \( x \)-current at each timestep, which is an estimate of the real-time conductivity:

\[
I(t) = \sum_{xy} I_{xy}^x(t).
\]
Appendix C. Spectral estimation

Given the time series \( I_m(t = mt) \) for \( m = 0, 1, 2, \ldots, M - 1 \), multiply it by a Kaiser window function \( W_0 \) involving modified Bessel functions \( I_\lambda(x) \) with a suitable width parameter \( \beta = 24 \),

\[
W_m = I_\lambda \left( \beta \sqrt{1 - \left( \frac{m}{M - 1} \right)^2} \right) / I_\lambda(\beta).
\]

(Appendix C.1)

Perform a type-III discrete cosine transform to obtain the weights \( \sigma_b \) for \( b = 0, 1, 2, \ldots, M - 1 \):

\[
\sigma_b = \frac{1}{N} \left\{ I_0 + 2 \sum_{m=1}^{M-1} \left[ \cos \left( \frac{\pi m (b + \frac{1}{2})}{M} \right) \right] W_m I_m \right\}.
\]

(Appendix C.2)

In a naive implementation of the equation-of-motion method, \( \{\sigma_b\} \) represent the integrals of Re \( \sigma(\omega) \) over bins delimited by the frequencies \( \omega_b = \frac{2\pi b}{M} \). As explained in the text, the phase error can be eliminated by writing \( \omega = \frac{2}{\tau} \sin \frac{\omega_b}{2} \). Therefore \( \{\sigma_b\} \) can be interpreted as weights in \( N \) unequal bins covering the interval \( \omega \in [0, \omega_{\text{max}}] \):

\[
\sigma_\omega \approx \int_{\omega_b}^{\omega_b+0.01} d\omega \, \text{Re} \, \sigma(\omega),
\]

\[
\omega_b = \frac{2}{\tau} \sin \frac{\pi b}{M} \quad (m = 0, 1, 2, \ldots, M).
\]

(Appendix C.3)

For the Kaiser window with a width parameter \( \beta = 24 \) as defined above, the spectral leakage function has a central lobe of width \( \beta/\pi \approx 15 \) bins, and the ratio of central- to side-lobe amplitude is about \( 10^5 \). Thus, if the true Re \( \sigma(\omega) \) contains a delta function at some frequency, the numerically calculated weights \( \sigma_\omega \) will have the delta function spread over about 15 consecutive bins. To save memory, and for later convenience, it makes sense to resample the spectrum into approximately \( M/15 \) equal bins. This should be done in a way that preserves the sum rule. Thus, we calculate the cumulative distribution function \( C(\omega_b) \) for \( b = 0, 1, 2, \ldots, M \); interpolate this and evaluate \( C(\omega) \) at equally spaced frequencies \( \omega_j = 0, 1, 2, \ldots, [M/15] \); and take differences to give the new weights [56]. The final result can be quoted, roughly speaking, as an estimate of Re \( \sigma(\omega) \) with frequency resolution (horizontal error bars) \( \Delta \omega \approx \frac{15 \omega_{\text{max}}}{M} \) and ‘vertical error bars’ \( \Delta \sigma \approx 10^{-8} \). Figure 7 illustrates the result of this procedure for a test spectrum.

Other than the Kaiser window, there are other possible window functions such as the Dolph–Chebyshev window (which is optimal but slightly more complicated to set up), and the window function derived by Storck and Wolff [57].

Appendix D. Finite-size effects

As mentioned in the text, the dynamical response \( \sigma(\omega) \) is mainly determined by finite-sized clusters that are smaller than the system sizes in this study. Thus, finite-size errors are practically negligible, as shown in figure 8.

References

[1] Haviland D B, Liu Y and Goldman A M 1989 Phys. Rev. Lett. 62 2180
[2] Adams P 2004 Phys. Rev. Lett. 92 067003
[3] Hebard A F and Paalanen M A 1990 Phys. Rev. Lett. 65 927
[4] Bollinger A T, Dubuis G, Yoon J, Pavuna D, Misewich J and Bozovic I 2011 Nature 472 458
