ON THE $\tau \to (a_1h)^-\nu_\tau$ DECAYS

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The $\tau \to (a_1h)^-\nu_\tau$ decays of the $\tau$-lepton are studied using the method of phenomenological chiral Lagrangians. The expression of weak hadronic currents between pseudoscalar and axial-vector meson states is obtained. Calculated partial widths for these decays are compared with the available experimental data.

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In this paper the $\tau \to (a_1h)^-\nu_\tau$ decays of the $\tau$-lepton are studied using the method of phenomenological chiral Lagrangians (PCL’s) [1]. The main uncertainty in the study of these decay channels is connected with weak hadron currents. Therefore such decay channels are a unique "laboratory" for verification of weak hadron currents between pseudoscalar and axial-vector meson states and investigations of these decays are of interest. Note that the hadron decays of the $\tau$-lepton up to three pseudoscalar mesons in the final state [2,3] and also $\tau \to VP\nu_\tau$ decays [4,5] have been studied in the framework of this method.

In the PCL, the weak interaction Lagrangian, has the form

$$L_W = \frac{G_F}{\sqrt{2}} j^h_{\mu} l^\mu + H.c.,$$

(0.1)

where $G_F \simeq 10^{-5}/m^2_P$ is the Fermi constant,

$l_{\mu} = \bar{u}_{l} \gamma_{\mu} (1 + \gamma_5) u_{\nu_\tau}$ is the lepton current, and hadron currents have the form [1]

$$j^h_{\mu} = J^{1+i2}_{\mu} \cos \Theta_c + J^{4-i5}_{\mu} \sin \Theta_c,$$

where $\Theta_c$ is the Cabibbo angle.

Weak hadron currents between pseudoscalar and axial-vector meson states are obtained by including the gauge fields of these mesons in covariant derivatives [6]:

1
\[ \partial_\mu \rightarrow \partial_\mu + igv_\mu V + iga_\mu A, \]  

(0.2)

here \( v_\mu \) and \( a_\mu \) are the fields of the 1\(^-\) and 1\(^+\) mesons, \( V_i = \lambda_i I/2 \), and \( A_i = V_i \gamma_5 \) are the vector and axial-vector generators of the \( SU(3) \times SU(3) \) group, respectively.

In this method the hadron currents are defined as [6]

\[
i\lambda^i J^i_\mu = F^2 \pi e^{i\xi A} (\partial_\mu + igv_\mu V + iga_\mu A) e^{-i\xi A},
\]

(0.3)

here \( \xi = \frac{1}{F\pi} \lambda^i \varphi^i \), \( F_\pi = 93 \text{ MeV} \), \( \varphi^i \) represent the fields of the 0\(^-\) mesons, and \( g \) is the "universal" coupling constant, which is fixed from the experimental \( \rho \to \pi \pi \) decay width

\[
\frac{g^2}{4\pi} \approx 3.2.
\]

The weak hadron currents between pseudoscalar and axial-vector meson states obtained in this way have the form

\[
J^i_\mu = F_\pi g a^b_\mu \varphi^c f_{bc}. \]

(0.4)

Axial-vector and vector meson currents are defined as

\[
J^i_\mu = m^2 v^i_\mu + m^2 a^i_\mu \frac{g}{g}, \]

(0.5)

where \( m_v \) and \( m_a \) are the masses of vector and axial-vector mesons, respectively.

The strong interaction Lagrangian of axial-vector mesons with vector and pseudoscalar mesons is obtained also by this way and has the form [2]

\[
L_S(1^+, 1^-, 0^-) = -F_\pi g^2 f_{km} a^k_\mu v^l_\mu \varphi^m. \]

(0.6)

The decay amplitudes for these channels can be written as [7]

\[
M(\tau(k_\tau) \to a_1(p) h(p_1) \nu_{\tau}(k_{\nu})) = G_F \epsilon^\lambda_\mu \bar{U}(k_{\nu}) \gamma_\mu [f_1 + g_1 \gamma_5 + \hat{p} (f_2 + g_2 \gamma_5)]
\]

\[
+ \hat{p}_1 (f_3 + g_3 \gamma_5) U(k_\tau),
\]

where \( \epsilon^\lambda_\mu \) is the polarization vector of 1\(^\pm\) mesons, \( f_i \) and \( g_i \) are the form factors that depend on the final state momenta; \( q = k_{\tau} - k_{\nu} = p + p_1 \), and \( k_{\tau}, k_{\nu} \) are the lepton four-momenta (\( \hat{p}_i \equiv p_{\mu_\nu} \gamma^\mu \)).

Using these Lagrangians we calculated the partial widths of the \( \tau \to (a_1 h)^- \nu_{\tau} \) decays by means of the TWIST code [8]. The results are shown in the Table I. In columns
I and II are listed the results without $1^-$ contributions, and with the vector $1^-$-meson contributions, respectively.

These decay channels get contributions from the $\rho(770)$-, $\rho(1450)$-, and $\rho(1700)$- vector intermediate meson states which have widths of 150, 310, and 240 MeV, respectively. Note that the contribution of the $\rho(1450)$- and $\rho(1700)$- mesons to the partial widths dominate those of the $\rho(770)$ ones.

Table I shows that the result obtained for the $\Gamma(\tau^- \to (a_1 h)\nu_\tau) = 0.79 \times 10^{10}\text{sec}^{-1}$ decay channels without taking into account the vector $1^-$ contributions are in good agreement with available experimental data [9]:

$$\Gamma(\tau^- \to (a_1 h)\nu_\tau) < 6.9 \times 10^{10}\text{sec}^{-1}.$$ 

Note that the calculated partial widths with taking into account $1^-$ contributions lie above this experimental value. In these calculations we used, as in Ref.s [2-5], the same $g$-coupling constant, according to Eq. (2), for all the vector intermediate meson states. Indeed, it is a rough approximation and it was more appreciable in study of such $\tau$ lepton rare decays than in Ref.s [2-5]. Therefore, it would be expedient to present $g$-coupling constant in Eq. (2) in a matrix form so that various decay channels have their own coupling constants. Though at present we have shortage of experimental data on the vector intermediate mesons, but there are some theoretical attempts to determine coupling constants of such mesons (see Refs [10,11]). And taking into account corresponding coupling constants in future would allow us to describe these decays more correctly compared to these calculations.

Note that according to Eq.(4) the partial widths of the $\tau^- \to a_1^- \eta \nu_\tau$ and $\tau^- \to a_1^- \eta' \nu_\tau$ decays are equal to zero in the PCL method; as in Ref.[4], these decay channels can be realized via effects of secondary importance [5].

Thus, the expression of weak hadronic currents between pseudoscalar and axial-vector meson states Eq. (4) obtained by including the gauge fields of axial-vector and vector mesons in covariant derivatives allow us to describe the $\tau \to (a_1^- h)^- \nu_\tau$ decays in satisfactory agreement with available experimental data. Determination of corresponding coupling constants in Eq. (4) would allow us to calculate these decay probabilities with high accuracy. Probably, contributions from the vector intermediate mesons which are very sensitive to $g$ would be in satisfactory agreement with the experimental data.

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Table 1. The partial widths (in $sec^{-1}$) for the $\tau^- \rightarrow (a_1 h)\nu_\tau$ decays.

| Decays                  | I       | II       | Experiment [9] |
|-------------------------|---------|----------|----------------|
| $\tau^- \rightarrow a_1^0 \pi^- \nu_\tau$ | $0.39 \times 10^{10}$ | $0.5 \times 10^{11}$ | − − − − |
| $\tau^- \rightarrow a_1^0 K^- \nu_\tau$   | $0.73 \times 10^{5}$  | $0.92 \times 10^{4}$  | − − − − |
| $\tau^- \rightarrow a_1^- \pi^0 \nu_\tau$ | $0.40 \times 10^{10}$ | $0.52 \times 10^{11}$ | − − − − |
| $\tau^- \rightarrow a_1^- \bar{K}^0 \nu_\tau$ | $0.92 \times 10^{5}$  | $0.19 \times 10^{5}$  | − − − − |
Fig.1. Diagrams for the $\tau^- \rightarrow (a_1 h)^- \nu_\tau$ decays, here W and S are the vertices of weak and strong interactions, respectively. (a) is without the pole contribution of $1^-$ mesons and (b) includes these pole contributions.