A Minimax Tree Based Approach for Minimizing Detectability and Maximizing Visibility

Zhongshun Zhang¹, Yoonchang Sung¹, Lifeng Zhou¹, Jonathon M. Smereka², Joseph Lee², and Pratap Tokekar¹

¹ Department of Electrical & Computer Engineering, Virginia Tech, USA
{zszhang, yooncs8, lzhou, tokekar}@vt.edu,
² The United States Army Tank Automotive Research, Development and Engineering Center, USA.
{jonathon.m.smereka.civ, joseph.s.lee34.civ}@mail.mil

Abstract. We introduce and study the problem of planning a trajectory for an agent to carry out a scouting mission while avoiding being detected by an adversarial guard. This introduces an adversarial version of classical visibility-based planning problems such as the Watchman Route Problem. The agent receives a positive reward for increasing its visibility and a negative penalty when it is detected by the guard. The objective is to find a finite-horizon path for the agent that balances the trade-off maximizing visibility and minimizing detectability. We model this problem as a sequential two-player zero-sum discrete game. A minimax tree search can give the optimal policy for the agent but requires an exponential-time computation and space. We propose several pruning techniques to reduce the computational cost while still preserving optimality guarantees. Simulation results show that the proposed strategy prunes approximately three orders of magnitude nodes as compared to the brute-force strategy.

1 Introduction

Planning for visually covering an environment is a widely studied problem in robots with many real-world applications, such as environmental monitoring [25, 26], infrastructure inspection [5, 18, 23], precision farming [13, 19], and ship hull inspection [14]. The goal is typically to find a path for an agent to maximize the area covered within a certain time budget or to minimize the time required to visually cover the entire environment. The latter is known as the Watchman Route Problem (WRP) [2,3,6] and is closely related to the Art Gallery Problem (AGP) [17]. The goal in AGP is to find the minimum number of cameras required to see all points in a polygonal environment.

AGP is a classical NP-complete optimization problem [17]. WRP, on the other hand, can be solved optimally for a single agent covering a known 2D
polygonal environment which does not contain any holes (i.e., obstacles) [6]. However, WRP is also NP-complete when the environment contains holes [15]. In this paper, we extend this class of visibility-based coverage problems to adversarial settings.

We consider scenarios where the environment also contains a guard that is actively (and adversarially) searching for the agent. The agent, on the other hand, is tasked with covering the environment while avoiding detection by the guard. This models stealth reconnaissance missions. We consider the version where there is a finite time within which the agent must complete its mission. The objective of the agent is to maximize the total area covered within the given planning horizon while at the same time minimize the number of times it is detected by the guard.

We adopt a game-theoretic approach for this problem where the agent maximizes the total reward collected and the guard minimizes the total reward. The total reward is a weighted combination of a positive and negative reward. The positive reward depends on the specific task at hand. For example, when the task is to scout an environment (Figure 1(a)), the positive reward can be the total area that is scanned by the agent along its path. When the task is to reach a goal position (Figure 1(b)), the positive reward can be the function of the distance to the goal. The agent receives a negative reward whenever it is detected by the guard. The negative reward can also be defined based on the specific application. In this paper, we consider the case where the agent receives a fixed negative reward every time it is detected by the agent. The total reward is a combination of the two reward functions.

Game theory is widely applied in robotics, in particular, for solving pursuit-evasion problems [7, 27]. Pursuit-evasion can largely be classified into two non-cooperative games, i.e., the differential game [8] and combinatorial game [12]. The former considers differential equations to represent the motion of the pursuers/evaders and can be solved using the Hamilton-Jacobi-Isaacs equations. The latter takes a computational geometry approach to represent the given environment and is closer to the approach in this paper. Specifically, ours approach is closer to the visibility-based [10, 22, 24] pursuit-evasion games. However, the main distinction is that in classical pursuit-evasion games, the evader (i.e., the agent in our setting) always evades the pursuer (i.e., the guard) whereas in our setting, the agent can potentially approach the guard in an effort to maximize visibility.

Minimax tree search [9] is a well-known algorithm in game-theory to find an optimal policy in two-player zero-sum discrete games. The optimality is guaranteed by enumerating all possible discrete actions for the two players. Pruning techniques, such as alpha-beta pruning [21], are employed in order to prune away branches from the fully enumeration tree that are guaranteed to not be part of the optimal policy. In addition to employing alpha-beta pruning, we propose four other pruning techniques (Theorems 1-4) using the structural properties of the underlying problem to further reduce the computational expense.
The contributions of this work are as follows: (1) We introduce a new problem of minimizing detectability and maximizing visibility as a sequential two-player zero-sum game between the agent and the guard, (2) We develop four pruning strategies that exploit the characteristics of the proposed problem, (3) We demonstrate the performance of the proposed algorithm through simulations.

The rest of the paper is organized as follows. We begin by describing the problem setup in Section 2. We present a brief introduction to minimax tree search and our proposed pruning strategies in Section 3. The simulation results are presented in Section 4. Section 5 summarizes the paper and outlines future work.

Fig. 1. Two example missions. Maximizing visibility implies maximizing the total reward collected along a finite horizon path while minimizing detectability can be achieved by avoiding the grid cells from where the guard can be seen. Both types of mission can be formulated by assigning different reward functions over a grid-based map.

2 Problem Formulation

We consider a grid-based environment where each cell within the environments is associated with a positive reward. Our key idea is to formulate the proposed problem by appropriately designing the reward function — the agent obtains positive rewards for maximizing visibility (depending on the type of missions) and receives negative rewards when detected by the guard. The reward is used to measure both the detectability of a guard and the visibility of an agent.

In an exploration mission, the positive reward can be a function of the number of previously unseen cells visible from the current agent position (Figure 1-(a)). In a mission where the objective is to reach a goal position, the positive reward can be defined as a function of the (inverse of the) distance to the guard (Figure 1-(b)). The agent receives a negative reward when it is detected by the guard (i.e., when it moves to the same cell as the guard or to a cell that lies with the
guard’s visibility region). At every turn (i.e., the time step), both the agent and the guard can move to one of their neighboring cells (i.e., the action).

(a) The case when the agent is detected by the guard. (b) The case when the agent is not detected by the guard. (c) The agent and the guard move in a grid-based environment.

Fig. 2. We use the visibility polygon to determine whether or not the agent is detected by the guard. A negative penalty will be added if the agent is inside the guard’s visibility (i.e., the blue area) polygon. In a reconnaissance mission, the area of the agent’s visibility polygon (i.e., the red area) is considered as a positive reward. Both the agent and the guard move in the same grid-based environment, as in (c) where the environment is represented by an 8 × 8 grid map.

We make the following assumptions:

– The agent and the guard move in the same grid-based map and can move one edge in one time step.
– Both the agent and the guard know the full grid-based map a priori.
– We assume that the agent and the guard have known sensing ranges (not necessarily the same). In this paper, we assume that both sensing ranges are unlimited for ease of illustration, however, the case of limited sensing range can easily be incorporated.
– The guard has a sensor that can detect the agent when the agent is within its visibility region.
– There is no motion uncertainty associated with the agent or guard actions.
– Although the agent is not aware of which action the guard chooses, the agent can still observe the guard position exactly after each action (even when it is not in the visibility polygon of the agent).

While the last assumption may seem restrictive, there are practical scenarios where it is justified. For example, Bhadauria and Isler [1] describe a visibility-based pursuit-evasion game where police helicopters can always provide the global positions of the evader to the pursuer that is moving on the ground and may not be able to directly see the pursuer.
The agent’s objective can be written as:

$$\max_{\pi_a(t)} \min_{\pi_g(t)} \{ R(\pi_a(t)) - \eta(\pi_a(t), \pi_g(t))P \}.$$  \hspace{1cm} (1)

On the other hand, the objective of the guard is:

$$\min_{\pi_g(t)} \max_{\pi_a(t)} \{ R(\pi_a(t)) - \eta(\pi_a(t), \pi_g(t))P \},$$  \hspace{1cm} (2)

where,

- $\pi_a(t)$ denotes an agent’s path from time step 0 to $t$.
- $\pi_g(t)$ denotes a guard’s path from time step 0 to $t$.
- $R(\pi_a(t))$ denotes the positive reward collected by the agent along the path from time step 0 to $t$.
- $P$ is a constant which gives the negative reward for the agent whenever it is detected by the guard.
- $\eta(\pi_a(t), \pi_g(t))$ indicates the total number of times that the agent is detected from time step 0 to $t$.

For the rest of the paper, we model $R(\pi_a(t))$ to be the total area that is visible from the agent’s path $\pi_a(t)$.

We model this as a sequential two-player zero-sum discrete game between the guard and the agent. In the next section, we demonstrate how to find the optimal strategy for this game and explain our proposed pruning methods.

### 3 Sequential Two-Player Zero-Sum Discrete Game

We refer the agent and the guard as MAX and MIN players, respectively. Even though the agent and the guard move simultaneously, we can also consider this problem as a turn-based game. At each time step, the agent moves first to maximize the total reward, and then the guard moves to minimize the total reward. This repeats for a total of $T$ planning steps.

#### 3.1 Minimax Tree Search

We firstly show how to construct a minimax tree for the given problem. A minimax tree search is a commonly used technique for solving two-player zero-sum games [21]. Each node in the tree stores the position of the agent, the position of the guard, the polygon that is visible to the agent along the path from the root node until the current node, and the number of times the guard detects the agent along the path from the root node to the current node. The tree consists of the following types of nodes:

- **Root node**: The root node contains the initial positions of the agent and the guard.
- **MAX level:** The MAX (i.e., agent) level expands the tree by creating a new branch for each neighbor of the agent’s position in its parent node from the previous level (which can either the root node or a MIN level node). The agent’s position and its visibility region are updated at each level. The guard’s position and the number of times the agent is detected are not updated at this level.

- **MIN level:** The MIN (i.e., guard) level expands the tree by creating a new branch for each neighbor of the guard’s position in its parent node (which is always a MAX level node). The guard’s position is updated at each level. The total reward is recalculated at this level based on the agent’s and guard’s current visibility polygons and the total number of times the agent is detected up to the current level.

- **Terminal node:** The terminal node is always a MIN level node. When the minimax tree is fully generated (i.e., the agent reaches a finite planning horizon), the reward value of the terminal node can be computed.

The reward values are backpropagated from the terminal node to the root node. The minimax policy chooses an action which maximizes and minimizes the backpropagated reward at the MAX and the MIN nodes, respectively.

![Figure 3](image)

**Fig. 3.** A (partial) minimax game tree. The root node contains the initial states of the agent and the guard. Two successive levels of the tree correspond to one time step. The agent moves first to an available position in order to maximize the reward (MAX level). The guard moves subsequently to a neighboring cell to minimize the agent’s reward (MIN level).

Figure 3 illustrates the steps to build a minimax tree that yields an optimal strategy by enumerating all possible actions for both the agent and the guard.
3.2 Minimax Tree Search with Pruning

Enumerating a full minimax tree requires a large amount of computation. To reduce the time needed to generate the full tree, we apply a classical pruning strategy called alpha-beta pruning and propose four new conditions to further prune some part of the tree. With the help of the pruning algorithms, the size of the tree is significantly reduced while still preserving optimality.

**Alpha-Beta Pruning** As a first step in reducing the size of the tree, we use the alpha-beta pruning [20]. The main idea is that if we have explored a part of the tree (i.e., reached a terminal node), we have an upper bound on the optimal minimax value. Consider when exploring a new node, \( n_i \). If the minimax value of the subtree rooted at the new node \( (n_i) \) is greater than the upper bound found so far, that subtree does not need to be explored further. This is because an optimal strategy will never prefer a strategy that passes through the new node \( (n_i) \) since there exists a better action in another part of the tree. Figure 4 illustrates an example of alpha-beta pruning.

![Fig. 4. Example of a minimax tree with the alpha-beta pruning. ▽ and △ nodes represent the agent and guard nodes, respectively. Each node has a number that indicates an available reward. The filled ▽ (colored in red) are pruned after applying the alpha-beta pruning.](image)

**Pruning Strategies** Alpha-beta pruning allows to prune insignificant nodes only after reaching the terminal level. This is preferable when the tree is built in a depth first fashion. However, we can exploit structural properties of this problem to further prune away nodes without needed to explore a subtree fully. We propose four strategies that find and prune redundant nodes before the terminal level is reached.

The first type of pruning (i.e., Theorems 1 and 2) is based on the properties of the given map. Consider the MIN level and the MAX level separately. The
main idea of these pruning strategies is to compare two nodes \( A \) and \( B \) at the same level of the tree, say the MAX level. In the worst case, the node \( A \) would obtain no future positive reward while always being detected at each time step of the rest of the horizon. Likewise, in the best case, the node \( B \) would collect all the remaining positive reward and never be detected in the future. If the worst-case outcome for node \( A \) is still better than the best-case outcome for node \( B \), then node \( B \) will never be a part of the optimal path. It can thus be pruned away from the minimax tree. Consequently, we can save time that would be otherwise spent computing all of its successors.

Note that these conditions can be checked even before reaching the terminal node of the subtrees at \( A \) or \( B \).

Given a node in the minimax tree, we denote the remaining positive reward (unscanned region) for this node by \( F(\cdot) \). Note that we do not need to know \( F(\cdot) \) exactly. Instead, we just need an upper bound on \( F(\cdot) \). This can be easily computed since we know the entire map information \textit{a priori}. The total reward collected by the node \( A \) and by the node \( B \) from time step 0 to \( t \) are denoted by \( R_A(t) \) and \( R_B(t) \), respectively.

**Theorem 1.** Given a time length \( T \), let \( A \) and \( B \) be two nodes in the same MAX level of the minimax tree at time step \( t \). If \( R_A(t) - (T - t)\eta \geq R_B(t) + F(B) \), then the node \( B \) can be pruned without loss of optimality.

**Proof.** In the case of the node \( A \), the worst case occurs when in the following \( T - t \) steps the agent is always detected at every remaining step and collects zero additional positive rewards. After reaching the terminal tree level, the reward backpropagated to node \( A \) will be \( R_A(t) - (T - t)\eta \). For the node \( B \), the best case occurs in the following \( T - t \) steps when the agent is never detected but obtains all remaining positive reward. In the terminal tree level, the node \( B \) collects the reward of \( R_B(t) + F(B) \).

Since \( R_A(t) - (T - t)\eta \geq R_B(t) + F(B) \) and both nodes are at the MAX level, it implies that the reward returned to the node \( A \) is always greater than that returned to the node \( B \). Therefore, the node \( B \) will not be a part of the optimal policy and can be pruned without affecting the optimality.

Similarly, consider that the node \( A \) and the node \( B \) are located in the MIN level. The same idea of Theorem 1 holds as follows.

**Theorem 2.** Given a time length \( T \), let \( A \) and \( B \) be two nodes in the same MIN level of the minimax tree at time step \( t \). If \( R_A(t) + F(A) \leq R_B(t) - (T - t)\eta \), then the node \( B \) can be pruned without loss of optimality.

The proof of Theorem 2 is similar to that of Theorem 1.

The main idea of the second type of pruning strategy \textit{i.e.}, Theorem 3) comes from the past path (or history). If two different nodes have the same agent and guard position but one node has a better history than the other, then the other node can be pruned away.

Here, we denote by \( S^A(\pi(t)) \) and \( S^B(\pi(t)) \) the total scanned region in the node \( A \) and the node \( B \) from time step 0 to \( t \), respectively.
Theorem 3. Given a time length $T$ and $0 < t_1 < t_2 < T$, let the node $A$ be at the level $t_1$ and the node $B$ be at the level $t_2$, respectively such that both nodes are at a MAX level. If (1) the guard’s position stored in the nodes $A$ and $B$ are the same, (2) $S^A(\pi(t_1)) \supset S^B(\pi(t_2))$, and (3) $R^A(t) > R^B(t) + (t_2 - t_1)\eta$, then node $B$ can be pruned without loss of optimality.

Proof. With $0 < t_1 < t_2 < T$, we have the node $B$ appear further down the tree as compared to node $A$. $S^A(\pi(t_1)) \subseteq S^B(\pi(t_2))$ indicates that the node $A$’s scanned area is a subset of the node $B$’s scanned area.

Since the nodes $A$ and $B$ contain the same guard and agent positions, one of the successors of node $A$ contains the same guard and agent positions as node $B$. Since $R^A(t) \geq R^B(t) + (t_2 - t_1)\eta$ and $S^A(\pi(t_1)) \supset S^B(\pi(t_2))$, the value backpropagated from the successor of node $A$ will always be greater than the value backpropagated from the path of node $B$. Furthermore, more reward can possibly be collected by node $A$ since $S^A(\pi(t_1)) \subseteq S^B(\pi(t_2))$. Thus, the node $B$ will never be a part of the optimal path and can then be pruned away.

In most cases, it is undesirable to send the agent to explore an environment where it may collect very little reward. The third type of pruning strategy (Theorem 3) is to consider a scenario where the agent is required to collect at least some desired positive reward $R_d$ from time step $0$ to $T$. We define a constraint function for $R_d$ such that $R(\pi_a(T)) > R_d$. We show that the agent will keep moving rather than stay in one place to hide from the guard if it does not collect enough positive reward in the minimax optimal path.

Theorem 4. Given a desired positive reward $R_d$, if the agent in the node $A$ of the minimax tree satisfies $R^A(\pi_a(T)) < R_d$, then its successor nodes of staying in the same position can be pruned without loss of optimality.

Proof. Theorem 4 can be proved by contradiction. Consider node $A$ that satisfies $R^A(\pi_a(T)) < R_d$. Without loss of generality, we assume that its successor nodes that stays in the same position is a part of the optimal path. Since it is a minimax tree, the MIN level will return the best actions for the guard. Consider the case that the guard also decides not to move. Then, the successor nodes that stay at the same position at the MAX level will never make for a better scenario. As a result, the agent will always fail to achieve the desired positive reward $R_d$.

Therefore, the successor nodes of staying in the same position must not be a part of the optimal minimax path.

Theorem 4 implies the case when we want the agent to at least receive $R_d$ positive reward, then before the agent reaches the desired positive reward $R_d$ it will keep moving regardless of the guard’s position.

3.3 Online Execution of the Tree

Even though the pruning techniques can prune a large amount of the nodes, it is still difficult to find a solution online for large $T$ since the complexity grows
Algorithm 1: The minimax search

```
1 function Minimax(Node, depth, α, β, State)
2     if node is in terminal state
3         return value
4     else if State == MAX level
5         //Recurse for all children of node.
6         for i=1:number of children node
7             V = Minimax(child, depth-1, α, β, MIN)
8             Bestvalue = max(Bestvalue,V)
9             α = max(Bestvalue,α)
10            if (β <= α)
11               break;//alpha-beta pruning
12            if pruning condition
13               break;
14            return value
15     else
16         for i=1:number of children node
17             V = Minimax(child, depth-1, α, β, MAX)
18             Bestvalue = min(Bestvalue,V)
19             β = min(Bestvalue,β)
20            if (β <= α)
21               break;//alpha-beta pruning
22            if pruning condition
23               break;
24         return value
25 Initial ← {S₀}.Map
26 Aᵢ(s), Aᵣ(s) ← Minimax(S₀,1,−∞,∞,MAX)
```

exponentially with $T$. Instead, a suboptimal solution can be obtained by creating a minimax tree over a smaller horizon. The agent can then execute one step of this tree and observe the new position of the agent. This corresponds to a new root node in the tree (at level 3). We extend the tree by two levels to be over a horizon $T$ (as well as expanding nodes at other levels that were pruned but may now possibly be on the optimal path). Figure 5 illustrates the steps of the online path planning with a smaller minimax tree.

4 Simulation

In this section, we evaluate the minimax search technique with the proposed pruning methods in the context of a reconnaissance mission. We assume the visibility range of the agent and the guard are both unlimited (only restricted by the obstacles in the environment) in this simulation. We use the VisiLibity library [16] to compute the visibility polygon for the agent and the guard. First, we evaluate the computational savings due to the proposed pruning techniques by comparing it with the brute force algorithm. Then, we present some online
Fig. 5. Online path planning: applying a smaller minimax tree as a model predictive controller to give a non-myopic one step action at every step.

path planning execution results in various settings. The simulation is executed in MATLAB and the code is available on Github.\(^3\)

We create a 20 × 20 environment as shown in Figure 1. We assume both the agent and the guard are in one of the available grid cells. At every step, they can choose to move to one of the neighboring grid nodes.

We begin by showing the effectiveness of the pruning algorithm by comparing the number of nodes generated by the brute force method and the pruning method. We generate the initial position of the agent and the guard randomly. We find the optimal path for various horizons ranging from \(T = 2\) to \(T = 6\). Therefore, the minimax tree depth ranges from 5 to 13.

The efficiency of the proposed pruning algorithm is presented in Figure 6. Since the effectiveness of our proposed pruning algorithm is highly dependent on the order in which the neighboring nodes are added to the tree first, different results can be achieved by changing the moving order of the minimax tree. Figure 6 shows the median, maximum and minimum of the number of nodes generated. If we enumerate all these nodes by brute force, in the worst case, it takes \(3.05 \times 10^8\) nodes to find the optimal path for a horizon of \(T = 13\). By applying the pruning algorithm, the best case only generates \(2.45 \times 10^5\). Even the worst case, it only needs approximately 1% nodes of the brute force to find the same optimal route.

We use a 13 level minimax (\(T = 6\)) tree to compute an optimal solution. Two results are presented in Figure 7 and Figure 8. With higher negative reward, the agent will tend to avoid the detection from the guard, with a lower negative reward, the agent will choose to explore more area. For instance, in Figure 7, the agent moves behind the obstacle after time step 2, while in Figure 8, the agent continues to explore the new environment after time step 2. Both of the simulations reach an equilibrium after a few steps.

For the suboptimal online path planner, we use a 7 level minimax (\(T = 3\)) tree to compute the actions of the agent with higher negative reward, the agent tends to avoid the detection from the guard. With lower negative reward, the agent

\(^3\) https://github.com/raaslab/ARC-visibility
Fig. 6. Comparison of the number of total nodes generated for the minimax tree. Note that the y axis is in log scale. The red line gives the maximum, median and the minimum number of nodes generated (by randomly choosing one of the neighbors to expand the search tree). The results are for 30 random trials.

Fig. 7. High negative penalty: A 6 steps minimax agent path planning with a higher negative penalty and with an adversarial guard. The negative penalty $P = 25$. The environment scale is an $8 \times 8$ square. The agent starts at (2, 1) and the guards starts at (6, 5). The agent and guard quickly reach an equilibrium after 5 steps, where both the agent and guard continuously move up and down to hide and track each other.
chooses to explore more area to get higher positive feedback. For instance, in Figure 10, the left figure (lower negative penalty) shows the agent keeps exploring the environment, while the right figure (higher negative penalty) shows the agent tries to hide behind the obstacles in the initial steps.

Figures 9 and 10 show simulation results of the online path planner. The agent moves based on the minimax policy. The guard moves based on two pre-defined policies that are not known to the agent. Higher negative penalty $P = 50$ and lower negative penalty $P = 5$ are used to compare the results. Figure 9 gives one preset path for the guard and Figure 10 gives a different guard route path.

In Figure 11, we compare the proposed online minimax path planning with a greedy algorithm. We compute the difference between the reward collected by the minimax and the greedy. We consider 159 instances, one corresponding to each possible starting location for the agent. The guard always starts at $(6,1)$ and follows the path shown in Figure 10. The difference between reward collected by minimax and the reward collected by greedy are sorted from low to high. In most cases, the performance of minimax is better than the greedy algorithm. The reason sometimes the greedy performs better is because the guard is not adversarial. The minimax policy is a conservative one and always assumes the worst-case guard path. Since the guard path in this example is not adversarial, the minimax can lead to a lower reward than a greedy strategy.
Fig. 9. Online path planning experiment 2. At each time step, the agent executes the first control action given by the tree and obtains a measurement of the guard. The guard is not adversarial in this experiment, instead, the guard moves as a preset path which is not known to the agent. The two pictures above is an experiment with a low negative penalty (negative penalty is 5) and the pictures below is the experiment with a high negative penalty (negative penalty is 50). The left side maps show the given agent and guard path from the minimax tree, the right side figures show the accumulated total reward obtained by the agent.

Fig. 10. Online path planning experiment 2. The preset guard path is different from experiment 1. The left side is an experiment with low negative penalty (negative penalty is 5) and the right side is the experiment with high negative penalty (negative penalty is 50). The left side shows the given agent and guard path from the minimax tree, the right side shows the accumulated total reward obtained by the agent.

Fig. 11. The total reward collected by online minmax vs. greedy algorithm.
5 Conclusion and Discussion

We introduce a new problem of maximizing visibility and minimizing detectability in an environment with an adversarial guard. We formulate the problem as a zero-sum game between the agent and the guard. The problem can be solved using a minimax tree to obtain an optimal strategy for the agent (assuming worst-case behavior of the guard). Our main contribution is a set of pruning techniques that reduce the size of the minimax tree while still guaranteeing optimality. Our simulation shows a large number of nodes can be pruned away using the proposed pruning techniques.

The minimax approach is a conservative planner since it assumes the worst-case behavior of the guard. It still outperforms the greedy algorithm, in most instances where the guard moves non-adversarially.

Despite the promising reduction in the game tree, the method can still be time consuming when the planning horizon increases or if the environment becomes large and/or complex. Our immediate work is to further reduce the computational effort using Monte Carlo Tree Search [4], using macro-actions [11], and by exploiting the underlying geometry of the environment.

Acknowledgments

This research was supported in part by the Automotive Research Center (ARC) at the University of Michigan, with funding from government contract DoD-DoA W56HZV-14-2-0001 through the US Army Tank Automotive Research, Development, and Engineering Center.

References

1. Deepak Bhadauria and Volkan Isler. Capturing an evader in a polygonal environment with obstacles. In IJCAI, pages 2054–2059, 2011.
2. Svante Carlsson, Håkan Jonsson, and Bengt J Nilsson. Finding the shortest watchman route in a simple polygon. Discrete & Computational Geometry, 22(3):377–402, 1999.
3. Svante Carlsson and Bengt J Nilsson. Computing vision points in polygons. Algorithmica, 24(1):50–75, 1999.
4. Guillaume Chaslot, Sander Bakkes, Istvan Szita, and Pieter Spronck. Monte-carlo tree search: A new framework for game ai. In AIIDE, 2008.
5. Peng Cheng, James Keller, and Vijay Kumar. Time-optimal uav trajectory planning for 3d urban structure coverage. In Intelligent Robots and Systems, 2008. IROS 2008. IEEE/RSJ International Conference on, pages 2750–2757. IEEE, 2008.
6. Wei-pang Chin and Simeon Ntafos. Optimum watchman routes. Information Processing Letters, 28(1):39–44, 1988.
7. Timothy H Chung, Geoffrey A Hollinger, and Volkan Isler. Search and pursuit-evasion in mobile robotics. Autonomous robots, 31(4):299, 2011.
8. Avner Friedman. Differential games. Courier Corporation, 2013.
9. Sylvain Gelly and Yizao Wang. Exploration exploitation in go: Uct for monte-carlo go. In NIPS: Neural Information Processing Systems Conference On-line trading of Exploration and Exploitation Workshop, 2006.
10. Brian P Gerkey, Sebastian Thrun, and Geoff Gordon. Visibility-based pursuit-evasion with limited field of view. The International Journal of Robotics Research, 25(4):299–315, 2006.
11. Milos Hauskrecht, Nicolas Meuleau, Leslie Pack Kaelbling, Thomas Dean, and Craig Boutilier. Hierarchical solution of markov decision processes using macro-actions. In Proceedings of the Fourteenth conference on Uncertainty in artificial intelligence, pages 220–229. Morgan Kaufmann Publishers Inc., 1998.
12. Volkan Isler, Sampath Kannan, and Sanjeev Khanna. Randomized pursuit-evasion in a polygonal environment. IEEE Transactions on Robotics, 21(5):875–884, 2005.
13. Jian Jin and Lie Tang. Coverage path planning on three-dimensional terrain for arable farming. Journal of Field Robotics, 28(3):424–440, 2011.
14. Ayoung Kim and Ryan M Eustice. Active visual slam for robotic area coverage: Theory and experiment. The International Journal of Robotics Research, 34(4-5):457–475, 2015.
15. Joseph SB Mitchell. Approximating watchman routes. In Proceedings of the Twenty-Fourth Annual ACM-SIAM Symposium on Discrete Algorithms, pages 844–855. SIAM, 2013.
16. K. J. Obermeyer and Contributors. The visilibity library. https://karlobermeyer.github.io/VisiLibity1/.
17. Joseph O’rourke. Art gallery theorems and algorithms. Oxford University Press Oxford, 1987.
18. Tolga Özclas, Shaojie Shen, Yash Mulgaonkar, Nathan Michael, and Vijay Kumar. Inspection of penstocks and featureless tunnel-like environments using micro uavs. In Field and Service Robotics, pages 123–136. Springer, 2015.
19. Cheng Peng and Volkan Isler. View selection with geometric uncertainty modeling. arXiv preprint arXiv:1704.00085, 2017.
20. Stuart Russell and Peter Norvig. Artificial Intelligence: A Modern Approach. Prentice Hall Press, 2009.
21. Stuart J Russell and Peter Norvig. Artificial intelligence: a modern approach. Malaysia; Pearson Education Limited., 2016.
22. Shai Sachs, Steven M LaValle, and Stjepan Rajko. Visibility-based pursuit-evasion in an unknown planar environment. The International Journal of Robotics Research, 23(1):3–26, 2004.
23. Prajwal Shanthakumar, Kevin Yu, Mandeep Singh, Jonah Orevillo, Eric Bianchi, Matthew Hebdon, and Pratap Tokekar. View planning and navigation algorithms for autonomous bridge inspection with uavs. In International Symposium on Experimental Robotics (ISER), 2018. Accepted.
24. Nicholas M Stiffler and Jason M OKane. Complete and optimal visibility-based pursuit-evasion. The International Journal of Robotics Research, 36(8):923–946, 2017.
25. Pratap Tokekar and Volkan Isler. Polygon guarding with orientation. Computational Geometry, 58:97–109, 2016.
26. Pratap Tokekar and Vijay Kumar. Visibility-based persistent monitoring with robot teams. In Intelligent Robots and Systems (IROS), 2015 IEEE/RSJ International Conference on, pages 3387–3394. IEEE, 2015.
27. Zhongshun Zhang and Pratap Tokekar. Non-myopic target tracking strategies for non-linear systems. In Decision and Control (CDC), 2016 IEEE 55th Conference on, pages 5591–5596. IEEE, 2016.