Standard Model gauge couplings from
gauge–dilatation symmetry breaking

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Abstract: We argue that there is a spontaneously broken rotational symmetry between space-time coordinates and gauge theoretical phases. The dilatonic mode acts as the massive Higgs boson, whose vacuum expectation value determines the gauge couplings.

This mechanism requires that the quadratic divergences, or tadpoles of the three gauge-theory couplings, unify at a certain scale. We verify this statement, and find that this occurs at $\Lambda_u \approx 4 \times 10^9$ GeV.

The tadpole cancellation condition, together with the dilaton self-energy, fixes the value of the unified tadpole coefficient to be $[4 \ln(\Lambda_{\text{cut}}/\Lambda_u)]^{-1}$. The observed values of the coupling constants at $\Lambda_u$ then implies $\Lambda_{\text{cut}} \approx 4 \times 10^{18}$ GeV, which is close to the value of the reduced Planck mass $\tilde{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV. In other words, by assuming a cutoff at $M_{\text{Pl}}$ or $\tilde{M}_{\text{Pl}}$, we are able to obtain predictions for the gauge couplings which agree with the true values to within a few percent.

It turns out that this symmetry breaking can only take place if mass is generated with the aid of some other means such as electroweak symmetry breaking. Assuming dynamical symmetry breaking originating at $\tilde{M}_{\text{Pl}}$, we obtain $M_\chi \approx 10^9$ GeV, which is not unreasonable but somewhat higher than $\Lambda_u$.

The cancellation of an anomaly in the dilaton self-energy requires that the number of fermionic generations equals three.
1 Introduction

One of the fundamental questions of particle physics is that of what determines the gauge couplings, or indeed of what gauge bosons are.

In this paper, we address these questions by extending the Standard Model to a theory in which the gauge bosons arise as effective Nambu–Goldstone modes (denoted Goldstone modes hereafter) of a spontaneously broken symmetry. Provided that the symmetry is broken dynamically [1–3], we can calculate the parameters of the theory by requiring self-consistency.

What can be this broken symmetry?

Goldstone modes in general have the form of the rotation operators of the corresponding broken symmetry, so we ask which rotation is generated by the gauge-field operator $A^i$, and we come to the conclusion that the symmetry is between gauge-theoretical phases and space–time coordinates. This is not unlike Kaluza–Klein theory [4]. However, the structure of space-time itself is not affected in our analysis. Gravity is small and negligible here, and is important only for setting the cutoff scale.

Whenever a symmetry is broken spontaneously, there arises a Goldstone mode for each broken symmetry and a Higgs mode for a preserved symmetry. The preserved symmetry in this case is the approximate scale invariance, or dilatation.
Note that the dilaton in the context of our study is not so much a pseudo-Goldstone mode for broken scaling symmetry [5] as a massive Higgs mode corresponding, somewhat paradoxically, to the preserved scaling symmetry.

The gauge coupling constants are proportional to the inverse of the vacuum expectation value $v_\chi$ of this dilaton.

In this paper, we calculate $v_\chi$ and the dilaton mass $M_\chi$ within the context of dynamical symmetry breaking. These are calculable because $v_\chi^2$ is proportional to the dilaton self-energy whereas $M_\chi$ is fixed by the cancellation condition of the gauge-boson tadpoles. The ratio of the two quantities is more easily calculable than the two quantities separately. This ratio gives us the gauge coupling constants at the $M_\chi$ scale.

In order that this mechanism works, it is necessary that the three tadpoles unify at a scale $\Lambda_u \approx M_\chi$. This is verified phenomenologically. The unification scale turns out to be $\Lambda_u \approx 4 \times 10^7$ GeV, including next-to-leading-order running effects.

Our procedure utilizes an analogous framework that was developed in refs. [6, 7] within the context of chiral symmetry breaking and refs. [8–10] for the case of the Higgs mechanism.

Let us denote the gauge theoretical tadpole coefficients as $c_i$ ($i = 1, 2, 3$), where the tadpole is given as a function of the cutoff scale $\Lambda$ by $\alpha_i c_i \Lambda^2$. Our prediction for the gauge couplings is then

$$ (c_i \alpha_i)^{-1} = 4 \log(\Lambda_{\text{cut}}/\Lambda_u). $$

A natural guess for $\Lambda_{\text{cut}}$ would be $M_{\text{Pl}} = \sqrt{\hbar c / G_N}$ or $\widetilde{M}_{\text{Pl}} = \sqrt{\hbar c / 8\pi G_N}$. Adopting $\Lambda_{\text{cut}} = \widetilde{M}_{\text{Pl}}$ with one order-of-magnitude error estimation on each side, we then obtain

$$ (c_i \alpha_i)^{-1} = 99 \pm 9. $$

(1.2)

The phenomenological value turns out to be

$$ (c_i \alpha_i)^{-1} = 101.9, $$

(1.3)

in good agreement with the prediction.

The disparity between $\Lambda_{\text{cut}}$ and $\Lambda_u$ requires explanation.

In order that the symmetry-broken vacuum is stable, it is necessary that the tadpole of the order parameter, i.e., the dilaton, vanishes. It turns out that if we consider the gauge–dilatation symmetry breaking alone, we can never satisfy this condition, because there is no term that cancels the dilaton self-coupling term. Some other mass-generation mechanism is necessary. The obvious choice would be electroweak symmetry breaking (EWSB).

If the masses of all Standard Model particles are due to the Higgs mechanism, only the Higgs-boson loop contributes to the dilaton tadpole. The tadpole cancellation condition is of the form $\Lambda_{\text{cut}} M_{\text{Higgs}} \sim M_\chi^2$, and therefore the disparity is explained. However, a more detailed calculation based on the assumption of dynamical EWSB at $\Lambda_{\text{cut}} = \widetilde{M}_{\text{Pl}}$ yields $M_\chi = 10^9$ GeV, which is one order-of-magnitude larger than the phenomenological value of $\Lambda_u$. Further investigation into this point will be necessary.

This paper is organized as follows. We write down the effective Lagrangian in section 2. We first discuss the gauge-theory tadpoles phenomenologically in section 3. The
parameters of the Lagrangian are worked out in the context of dynamical symmetry breaking in section 4. The paper is concluded with a summary and brief discussions on future extensions.

2 The Lagrangian

As an introductory remark, when a symmetry is broken spontaneously in general, a Higgs mode and Goldstone modes arise, and they have the following properties:

1. The Goldstone modes are massless (gapless), and their coupling has the form of the broken symmetry operation. This form of coupling is necessary if the Goldstone boson corresponds to the broken part of the symmetry current.

2. The number of Goldstone modes equals the number of broken symmetries.

3. The Goldstone and Higgs fields themselves are connected by the same rotation symmetry.

4. The Higgs mode is massive (finite energy gap), and their coupling has the form of the preserved symmetry operation. This form of coupling is required by the symmetry between Goldstone and Higgs fields.

5. The form factor for Goldstone modes, i.e., the inverse of their coupling strengths, is proportional to the vacuum expectation value (VEV) of the Higgs field.

Our proposal is that the gauge bosons are the Goldstone modes of a certain symmetry. If so, the theory which satisfies the above properties can be written down almost uniquely (unique up to the kinetic terms and the potential). To start off with, from the condition that the vector field $A_\mu^i$ behaves like the rotation operation, we conclude that the symmetry is between gauge-theoretical phases and space-time coordinates.

The four-vector $dx^\mu$ is thus generalized to

$$(dx^\mu, r_0 d\phi^i).$$ (2.1)

Here, $\phi^i$ are the gauge-theoretical phases. $i$ is the index for the generators, e.g. 1 to 8 for QCD. The generalization to multiple gauge groups is trivial. $r_0$ is some number with the dimension of length. This is proportional to the coupling constants $g$.

In Kaluza–Klein theories [4], $r_0$ corresponds to the radius of the compactified dimension. In our picture, $r_0$ is a parameter with no geometrical significance and is fixed dynamically as a function of the energy scale. Even so, the picture of compactified dimensions, depicted in figure 1, is a useful aid. In terms of figure 1, whereas Kaluza–Klein theory deals with oscillations of the cylinder itself, we are dealing with the symmetry operations on the ‘mesh’ that may be imagined to be drawn on the cylinder.

In Kaluza–Klein theory, one starts from a $4 + n$ dimensional theory and then apply compactification to obtain the effective theory in 4 dimensions. Our approach differs in that we do not start from a $4 + n$ dimensional theory. The theory is defined only in 4
Figure 1. An illustration based on Kaluza–Klein-like picture. Gauge phase $\phi^i$ and space-time coordinates $x^\mu$ mix. A rotation in $(x, \phi)$ space maps, for example, vector $A$ to vector $B$.

dimensions, and the parameters of the theory are worked out from the condition of self-consistency as applied to the 4 dimensional theory, which arises because of the spontaneous symmetry breaking.

$x^\mu$ and $\phi^i$ are quite distinct entities even though the rotation between them is a symmetry operation. $\phi^i$ correspond to extra-dimensional rotation angles rather than independent dimensions. As an illustration, take the example of the $n$-sphere as compactified dimensions. The number of coordinates is equal to $n$, whereas the number of rotations among these coordinates is equal to $\frac{1}{2}n(n + 1)$, i.e., equal to the order of the rotational group $\text{SO}(n + 1)$.

The proper time interval for eqn. (2.1) is given by

$$d\tau^2 = dx^2 - r_0^2 (d\phi^i)^2.$$  

(2.2)

The space-time derivative is replaced in the UV by

$$\frac{\partial}{\partial x^\mu} \rightarrow \left( \frac{\partial}{\partial x^\mu} + \frac{1}{r_0} \frac{\partial}{\partial \phi^i} \right).$$  

(2.3)

The rotation between gauge rotation and space-time translation is given by the following transformation, which preserves $d\tau$:

$$\begin{pmatrix}
  dx^\mu \\
  r_0 d\phi^i
\end{pmatrix} \rightarrow (1 - r_0^2 \hat{A}^2)^{-\frac{1}{2}} \begin{pmatrix}
  dx^\mu + r_0^2 \hat{A}^\mu_\nu d\phi^i \\
  r_0 d\phi^i + r_0 \hat{A}^i \cdot dx
\end{pmatrix}.$$  

(2.4)

This is just rotation in $1 + (3 + n)$ space-time-dimensional space (see figure 1) where $n$ is the number of generators. The spontaneous breaking of this symmetry implies that the vacuum chooses an arbitrary $1 + 3$-dimensional time-space direction out of the $1 + (3 + n)$ dimensions. The space-time metric is given by $\eta^{\mu\nu}$.

The preserved symmetry operation is dilatation in the $1 + 3$ space-time. Including dilatation, the above rotation operation can be cast into the following form:

$$\begin{pmatrix}
  dx^\mu \\
  r_0 d\phi^i
\end{pmatrix} \rightarrow \begin{pmatrix} (v_\chi + \chi) \eta^{\mu\nu} A^\mu_\nu \\
  A^i_\nu \Phi^i_j \end{pmatrix} \begin{pmatrix}
  dx^\nu \\
  r_0 d\phi^i
\end{pmatrix}.$$  

(2.5)

This time, $d\tau$ is not necessarily preserved. We then identify $\chi$ with the dilaton ($v_\chi$ is the vacuum expectation value) and $A^i_\mu$ with the gauge fields. $\Phi$ do not become physical modes in our situation because they are not involved in symmetry breaking.
We should clarify in which sense the $A^i_\mu$ field behaves as a Goldstone mode. Goldstone bosons are divergences of a Noether current when the symmetry is spontaneously broken. The Noether current is given, as usual, by the derivative of the Lagrangian with respect to the derivative of the symmetry transformation. That is,

$$J_{\mu i}^\nu = \frac{\partial L}{\partial (\partial a^i_\nu / \partial x^\mu)}.$$  

\(a^i_\nu\) stands for the rotation between \(dx^\mu\) and \(d\phi^i\), or between \((v\chi + \chi)\eta^\mu_\nu\) and \(A^i_\nu\). Note that the two Lorentzian indices \(\mu\) and \(\nu\) are inequivalent.

We must write the (4-dimensional) Lagrangian in a way that respects the symmetry between the dilaton and the gauge fields. This may be achieved by replacing space-time derivatives with

$$D_\mu = (v\chi + \chi)\eta^\nu_\mu \frac{\partial}{\partial x^\nu} - A^i_\mu \frac{\partial}{\partial t_0 \partial \phi^i}.$$  

The negative sign arises because \(\phi^i\) are space-like.

The \(\phi^i\) derivative operator is such that

$$\frac{\partial}{\partial \phi^i} \psi = -i\tau_i \psi,$$  

$$\frac{\partial}{\partial \phi^i} A^j_\mu = -f^i_{jk} A^k_\mu,$$  

$$\frac{\partial}{\partial \phi^i} \chi = 0.$$  

\(\tau\) and \(f^i_{jk}\) are (for QCD) the colour matrices. Note that the definition of \(D_\mu\) is consistent with the usual gauge-theoretical covariant derivative.

This formulation is not without problems. When we try to define \(F_{\mu\nu}\) as the commutator of covariant derivatives, we find that that there is a non-factorizable contribution:

$$\left[(v\chi + \chi)\frac{\partial A^i_\nu}{\partial x^\nu} - (v\chi + \chi)\frac{\partial A^i_\mu}{\partial x^\nu}\right] - \frac{1}{r_0} f^i_{jk} A^j_\mu A^k_\nu.$$  

This will vanish in the limit of soft \(\chi\), in which case we may write

$$F_{\mu\nu} = (v\chi + \chi) \left(\frac{\partial A^i_\nu}{\partial x^\nu} - \frac{\partial A^i_\mu}{\partial x^\nu}\right) - \frac{1}{r_0} f^i_{jk} A^j_\mu A^k_\nu.$$  

In principle, there will be problems with gauge invariance when \(\chi\) is not soft.

Up to the normalization factors, the symmetry-conserved part of the Lagrangian is then written down trivially as

$$\bar{\psi} \gamma^\mu D_\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{3}{2} (v\chi + \chi)^2 \left(\frac{\partial \chi}{\partial x^\mu}\right)^2.$$  

We used the third equation of eqns. (2.10) in the last term. The action \(S\) is \(i\) times the four-dimensional integral of \(L\). This Lagrangian is quite different from interactions that involve the gravitational dilaton (for standard review papers, see refs. [11, 12]).
When $v_\chi = 0$, no equation of motion arises for any field, unlike any other instances of spontaneous symmetry breaking that we know of.

As we are only interested in the case $v_\chi \neq 0$, let us redefine $\psi$ and $A$ so as to absorb $v_\chi$. We also normalize $v_\chi$ and $\chi$ such that they have mass dimension 1. The $\phi^i$ derivative may be replaced by eqns. (2.10). The dimensionless quantity $(r_0 v_\chi)^{-1}$ can be replaced with the gauge coupling strength $g$.

For the fermionic part, the Lagrangian with the appropriate normalization factors is then given by

$$L_f = \bar{\psi} \gamma^\mu \left[ \left( 1 + \frac{\chi}{v_\chi} \right) \frac{\partial}{\partial x^\mu} + ig A^i_\mu \tau_i \right] \psi. \quad (2.14)$$

The bosonic part is written as

$$L_b = T_A + T_\chi - V(A, \chi), \quad (2.15)$$

where

$$T_A = -\frac{1}{4} \left[ \left( 1 + \frac{\chi}{v_\chi} \right) \left( \frac{\partial A^i_\mu}{\partial x^\nu} - \frac{\partial A^i_\nu}{\partial x^\mu} \right) - g f^i_{jk} A^j_\mu A^k_\nu \right]^2, \quad (2.16)$$

$$T_\chi = \frac{3}{2} (1 + \chi/v_\chi)^2 \left( \frac{\partial \chi}{\partial x^\mu} \right)^2, \quad (2.17)$$

and the symmetry breaking potential is given by

$$V(A, \chi) = \frac{\mu_\chi^2}{8v_\chi^2} \left[ 4 (v_\chi + \chi)^2 - (A^i_\mu)^2 - 4v_\chi^2 \right]^2. \quad (2.18)$$

Factors of 4 inside square brackets are due to the trace of $\eta_{\mu\nu}$. Obviously $16 \mu_\chi^2 = 3 M_\chi^2$.

It is worth pointing out here that effective $A^4$ contact terms, which are proportional to $\mu_\chi^2/v_\chi^2$, cancel in the low-energy limit $Q^2 \ll \mu_\chi^2$.

Gauge fixing is problematic. Ordinarily, we enforce transversality by bringing in covariant gauge fixing terms of the form

$$-\frac{1}{2\lambda} \left( \frac{\partial A^i_\mu}{\partial x^\mu} \right)^2 + \text{(ghost term)}. \quad (2.19)$$

However, this does not quite work because the potential $V(A, \chi)$ violates gauge symmetry at scales that are comparable with $M_\chi$. The resolution of this problem requires a study into the structure of $M_\chi$. Our strategy is to relate $M_\chi$ to the gauge theoretical tadpoles. If calculations are done self-consistently, this implies that longitudinal components do cancel. That is, in practical terms, we may adopt the Feynman gauge $\lambda = 1$. This point will be discussed in more detail in section 4.4.

It is a trivial matter to write down the Feynman rules corresponding to our Lagrangian $L = L_f + L_b$. These are shown in figure 2.
Figure 2. The Feynman rules. The dashed lines represent the dilaton $\chi$. For the sake of brevity, we omit the ordinary gauge-theoretical couplings and the non-Abelian self-interaction terms. The colour factors (i.e., $\delta^{AB}$) are omitted. The last diagram applies to any combination of two non-identical gauge bosons, such as two sets of gluons of different colour.

3 Tadpole cancellation in gauge-boson self-energy

If gauge bosons are Goldstone bosons, their masses in the form of the anomalous tadpoles, must vanish. In our framework, the only contributions that can counteract the usual gauge theory contributions are due to the dilatonic contributions which are universal. It follows that the gauge theoretical tadpoles themselves must be universal at a scale $\Lambda_u$ even though the gauge theoretical couplings themselves do not unify.

Before proceeding to calculate the parameters of the theory, let us verify phenomenologically that this statement is indeed true.

The tadpoles arise from diagrams which are shown in figure 3 for the case of QCD.

Figure 3. The usual gauge-theoretical contributions to the vacuum polarization operator. Diagram d represents the ghost contribution. There are, for the case of broken symmetry, also the Higgs and Goldstone-boson contributions which are not explicitly shown.

This is a standard textbook calculation, but we have not found the results explicitly
written out in the standard textbooks. We obtain

\[ \Pi^{\text{tadpole}}_{\mu\nu} = \frac{\alpha \Lambda^2}{8\pi} \eta_{\mu\nu} \left( 8 T_R n_g - 2 C_A + n_{\text{Higgs}} \right), \tag{3.1} \]

for SU(2)_L and QCD. \( n_g = 3 \) is the number of generations. \( n_{\text{Higgs}} \) is 1 for SU(2)_L (and U(1)_Y), and 0 for QCD. \( T_R = 1/2 \), and \( C_A \) is 2 for SU(2)_L and 3 for QCD. \( \Lambda^2 \) is the UV cutoff of space-like \( Q^2 \) integration. A straightforward generalization of this formula makes it applicable also to the case of U(1)_Y.

The factor inside brackets is evaluated to be 6 for QCD, 9 for SU(2)_L and 21 for U(1)_Y without the conventional factor 5/3. Let us denote this as follows:

\[ \Pi^{\text{tadpole}}_{\mu\nu} = c_a \alpha_a \Lambda^2 \eta_{\mu\nu}, \quad c_a = \frac{(21, 9, 6)}{8\pi}. \tag{3.2} \]

In figure 4, we show the inverse tadpole coefficients \((c_a \alpha_a)^{-1}\) as a function of the energy scale. We see that the three tadpoles exhibit unification at a good level. \( c_1 \alpha_1 \) and \( c_2 \alpha_2 \) unify at

\[ \Lambda_u = 3.6 \times 10^7 \text{ GeV}, \tag{3.3} \]

when

\[ (c_a \alpha_a)^{-1} = 101.9. \tag{3.4} \]

The error is small compared with the error in our prediction, with which we shall make comparison.

**Figure 4.** The inverse, \((c_a \alpha_a(\mu))^{-1}\) of the three gauge-theory tadpole coefficients as a function of the energy scale \( \mu \). \( c_a \) are dressed by one-loop anomalous dimensions as explained in the text. The main figure is for central \( \alpha_s(M_Z) = 0.1176 \). The inlay shows the unification region, for \( \alpha_s(M_Z) = 0.1176 \pm 0.0020 \) [14]. The three-loop beta-function coefficients follow ref. [13].
We would like to make a number of remarks on the numbers shown in figure 4.

First, the couplings are calculated at the three-loop order, with the beta-function coefficients of ref. [13]. Second, the value $\alpha_s(M_Z) = 0.1176 \pm 0.0020$ quoted in figure 4 is relatively old [14]. There is more statistical weight on the $e^+e^-$ data in these numbers as compared with the more modern numbers [15] which place more weight on the $\tau$ decay data, and so we would like to think that the former is a more conservative estimate of $\alpha_s(M_Z)$ as it applies to physics at $M_Z$.

Finally, the values of $c_a$ used in figure 4 are not the leading order (one-loop) values which are discussed above. We have dressed $c_a$ by including, partially, the main next-to-leading order (two-loop) effect which is due to the anomalous dimensions.

Fermionic loop contributions are modified by factor $(1 - \gamma_f)$, where

$$\gamma_f = \frac{\partial \ln Z(q^2)}{\partial \ln q^2}.$$  \hspace{1cm} (3.5)

$Z$ is the renormalization factor. The QCD contribution, which is appropriate for quarks, is $\gamma_f = \alpha_s/3\pi$ at the leading order. Gauge-boson loop contributions are modified by factor $(1 - \gamma_v)$, where

$$\gamma_v = \frac{\partial \ln \alpha}{\partial \ln q^2},$$ \hspace{1cm} (3.6)

and this is equal to $b_0\alpha$ at the leading order, where $b_0$ is the first beta-function coefficient.

The values of $c_a$ as shown in figure 4 are dressed by including the $\gamma_v$ factor and the QCD part of the $\gamma_f$ factor. This is a small effect, but helps realize the unification of the tadpole coefficients.

4 Derivation of the parameters

4.1 General remarks

Let us now proceed to calculate the parameters of the theory.

There are 5 unknown parameters, which are $v_\chi$, $\mu_\chi$ and the three gauge couplings $\alpha_a$. There is one additional parameter governing the UV running of $v_\chi$, but let us ignore this for now.

In ordinary instances of dynamical symmetry breaking [8–10], we would expect that $v_\chi$ is calculated from the self-energy integral of the Goldstone bosons, $\mu_\chi$ is determined by the self-energy of the Higgs bosons, and there is an additional constraint from the cancellation of the Higgs boson tadpole. Here the situation is similar, but certain modifications are necessary, namely

1. $v_\chi$ is determined by the dilaton self-energy rather than the gauge boson self-energy.

2. Gauge boson self-energy yields constraints on $\alpha$.

3. The dilatonic tadpole does not vanish by itself. We require an additional mass-generation mechanism. In the minimal framework, EWSB is necessary in order that the gauge bosons exist at all.
4. $\mu_\chi$ does not arise directly from the dilatonic self-energy. It is calculated from the tadpole anomaly of the gauge bosons.

Let us see how this works in practice.

4.2 Dilaton self-energy

First, let us consider dilatonic self-energy. The contributions to $\Sigma$ are as shown in figure 5.

![Figure 5. Dilaton self-energy.](image)

The contributions corresponding to the five diagrams are divergent in general. The divergences are of three forms:

\[
\int \frac{d^4k}{(2\pi)^4} = 0, \quad (4.1)
\]

\[
\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i0} = -\frac{\Lambda^2}{16\pi^2}, \quad (4.2)
\]

\[
\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i0} \left( \frac{1}{k - q} \right)^2 + i0 = -\frac{1}{16\pi^2} \ln \frac{\Lambda^2}{-q^2}. \quad (4.3)
\]

The quartic divergence contains no poles, and so we set it to zero.

The fermionic contribution of figure 5a is evaluated to be

\[
\Sigma_{(a)} = \frac{\#_f \Lambda^2 q^2}{32\pi^2 v_\chi^2}, \quad (4.4)
\]

where $\#_f = 24$ is the number of fermionic degrees of freedom. We have omitted a $q^4$ term with a finite coefficient, as we shall be introducing a counterterm later on.

The contribution of figure 5b is evaluated to be

\[
\Sigma_{(b)} = -\frac{\#_f q^2}{16\pi^2 v_\chi^2} \left( \Lambda^2 + q^2 \ln \frac{\Lambda^2}{-q^2} \right) - \frac{3\#_f \mu_\chi^4}{4\pi^2 v_\chi^2} \left( 2\Lambda^2 + q^2 \ln \frac{\Lambda^2}{-q^2} \right) - \frac{2\#_f \mu_\chi^4}{\pi^2 v_\chi^2} \ln \frac{\Lambda^2}{-q^2}. \quad (4.5)
\]
where \( \#_\nu = 12 \) is the number of vector bosons. The last term is gauge dependent, and we have used the Feynman gauge. However, we shall not be discussing the \( \mu_\chi^2 \) term for the rest of this study. Again we have omitted a \( q^4 \) term.

The contribution of figure 5c is evaluated to be

\[
\Sigma_{(c)} = \frac{\#_\nu \mu_\chi^2 \Lambda^2}{2\pi^2 v_\chi^2}.
\]  

(4.6)

This is in the Feynman gauge.

In the sum of contributions \( \Sigma_{(a-c)} \), we notice that the anomalous \( \Lambda^2 q^2 \) terms cancel, provided \( 2\#_\nu = \#_f \). In our world, assuming that the neutrinos are Dirac particles, this requires that the number of generations is equal to three. It is easy to verify that \( \Lambda^2 \mu_\chi^2 \) terms which are present in \( \Sigma_{(d)} \) and \( \Sigma_{(e)} \) mutually cancel.

The \( \Lambda^2 \mu_\chi^2 \) terms are also anomalous, but we cannot see how they will cancel. On the other hand, \( \Sigma \) by itself does not contain a mass-generation mechanism. That is, \( \mu_\chi^2 \) remains zero if it is zero to start off with. Thus the mass term needs to be inserted by other means, and when this is done, the resultant term must be equal to \( 16 \mu_\chi^2 \). Thus we think it reasonable to drop \( \Lambda^2 \mu_\chi^2 \) term as being unphysical. The terms that are physically significant are therefore

\[
\Sigma_{(a-c)} = -\frac{\#_\nu q^4}{16\pi^2 v_\chi^4} \ln \frac{\Lambda^2}{-q^2} - \frac{3\#_\nu \mu_\chi^2 q^2}{4\pi^2 v_\chi^2} \ln \frac{\Lambda^2}{-q^2}.
\]  

(4.7)

The analogous contributions from figures 5d, 5e are evaluated to be

\[
\Sigma_{(d,e)} = -\frac{q^4}{32\pi^2 v_\chi^4} \ln \frac{\Lambda^2}{-q^2} + \frac{\mu_\chi^2 q^2}{3\pi^2 v_\chi^2} \ln \frac{\Lambda^2}{-q^2},
\]  

(4.8)

when \( q^2 \gg M_\chi^2 \). For \( \#_\nu = 12 \), the latter contributions are one order of magnitude smaller. Furthermore, they will be much more suppressed if the calculation is done self-consistently, because both \( \mu_\chi^2 \) and \( v_\chi^{-2} \), which become functions of the internal momenta, decay rapidly at high energies. Let us therefore neglect eqn. (4.8).

Self-consistency requires that \( \Sigma = -3q^2 \). Note that \( \mu_\chi \) will be fixed by gauge-boson tadpole-cancellation conditions. This implies

\[
v_\chi^2(q^2) = \frac{q^2}{4\pi^2} \ln \frac{-q^2}{\Lambda^2} + \frac{3\mu_\chi^2(q^2)}{\pi^2} \ln \frac{-q^2}{\Lambda^2}.
\]  

(4.9)

The first term is problematic. If \( \Lambda \) in both terms are the same, this will induce a tachyonic pole at \( q^2 = -12\mu_\chi^2 \), and \( v_\chi^2 \) will be negative for large and space-like \( q^2 < -12\mu_\chi^2 \). Since it runs, we cannot subtract it away at all energy scales. The best that can be done is presumably to subtract it away at the symmetry-breaking scale. This implies that \( \Lambda \) will be replaced by \( \Lambda_u \) in the first term, whereas \( \Lambda \) in the second term remains the UV cutoff. Our final expression for \( v_\chi \) reads

\[
v_\chi^2(q^2) = -\frac{q^2}{4\pi^2} \ln \frac{-q^2}{\Lambda_u^2} + \frac{3\mu_\chi^2(q^2)}{\pi^2} \ln \frac{\Lambda_u^2}{-q^2}.
\]  

(4.10)
The second term is the dominant contribution when discussing symmetry breaking, and our results will be derived solely from it. The first term is necessary, nevertheless, to protect both $v_\chi^2$ and $\mu_\chi^2$ from growing negative at high energies.

4.3 Gauge-boson self-energy

The gauge boson tadpoles, which were calculated in section 3, need to be cancelled by the new contributions of the form shown in figure 6.

These contributions are easy to calculate, and we obtain

$$\Pi_{\mu\nu}^{\text{tadpole}} = -\frac{1}{8\pi^2} \eta_{\mu\nu} \int dQ^2 \frac{\mu_\chi^2(Q^2)}{v_\chi(Q^2)} \left( #_v + \frac{1}{16\mu_\chi^2/Q^2 + 3} \right).$$

(4.11)

It is essential that we use the running $\mu_\chi$ and $v_\chi$.

We now compare this with the result of section 3. There should be cancellation at all scales, and so we obtain

$$c_i \alpha_i(Q^2) = \frac{\mu_\chi^2(Q^2)}{8\pi^2 v_\chi^2(Q^2)} \left( #_v + \frac{1}{16\mu_\chi^2/Q^2 + 3} \right).$$

(4.12)

Obviously this will only hold when $c_i \alpha_i$ are approximately universal. We then substitute the second term of eqn. (4.10) to obtain the following prediction:

$$c_i \alpha_i(\Lambda_u^2) = \frac{1}{24 \ln(\Lambda_u^2/\Lambda_{\text{cut}}^2)} \left( #_v + \frac{1}{16\mu_\chi^2/\Lambda_u^2 + 3} \right).$$

(4.13)

By omitting the second term, we obtain the predictions that are quoted in the introduction. By adopting $\Lambda_{\text{cut}} = \tilde{M}_\text{Pl}$ and eqn. (3.3) for $\Lambda_u$, we obtain the remarkably accurate result $(c_i \alpha_i)^{-1} = 100$.

4.4 The behaviour of the dilatonic mass

We would now like to start the discussion of what causes the spontaneous breaking of gauge–dilatation symmetry. First of all, we need to know what physics generates $\mu_\chi^2$.

Consider the $\mu_\chi^2/v_\chi$ interaction term for the dilaton–gauge-boson–gauge-boson vertex that is shown at the bottom left of figure 2. The presence of a tadpole anomaly in the gauge-boson propagator implies that this interaction term arises automatically in amplitudes such as that shown in figure 7.
The one-loop anomalous amplitude which generates $\mu_\chi^2$.

The interaction of the dilaton, at least when it is soft, has the form of the inverse propagator. The insertion of a soft dilaton line in figure 7 therefore removes a fermion line. The result of this procedure is that the induced three-point function will be proportional to the anomalous tadpole. We obtain

$$\frac{4\mu_\chi^2(Q^2)}{v_\chi(Q^2)} = \int_{Q^2} dQ^2 \frac{2c_i\alpha_i(Q^2)}{v_\chi(Q^2)},$$

so that

$$\frac{d\mu_\chi^2(Q^2)}{dQ^2} = -\frac{1}{2}c_i\alpha_i(Q^2).$$

Thus $\mu_\chi^2$ decays at high energy.

This procedure gives us an answer to the problem of gauge invariance. The anomalous tadpole arises from amplitudes such as

$$\int \frac{d^4 k}{4\pi^3} \frac{\alpha}{k^2 + i0} \frac{1}{(k - q)^2 + i0} \text{Tr} [k\gamma_\mu(k - q)\gamma_\nu].$$

The contraction with $q^\nu$ yields

$$\int d^4 k \frac{\alpha}{i\pi^3} \left[ \frac{(k - q)_\mu}{(k - q)^2 + i0} - \frac{k_\mu}{k^2 + i0} \right].$$

Provided that $\alpha$ decays sufficiently fast at large $k^2$, we can replace $k - q$ in the first term with $k$, and the resultant integral will be zero. That is, the mass term will be of the form that kills longitudinal contributions, and therefore the practical recipe will be that we can use the Feynman gauge in our calculations. Note that if $\alpha$ does not decay, eqn. (4.17) will yield $\Lambda^2 q_\mu$. We shall now show that $\alpha(Q^2)$ decays faster than $1/Q^2$.

We substitute the first term of eqn. (4.12) in eqn. (4.15):

$$\frac{d\mu_\chi^2(Q^2)}{dQ^2} = -\frac{3\mu_\chi^2(Q^2)}{4\pi^2v_\chi^2(Q^2)}.$$
However, as $\mu_\chi^2$ reaches zero, the first term of eqn. (4.10) starts to dominate, causing $\mu_\chi^2$ to fall as a power. It should be noted that $\nu_\chi^2(Q^2)$ decreases with scale at first, after which it increases proportionally to $Q^2$. The phase transition is not quite second order in the sense that $\nu_\chi$ does not vanish at $\Lambda_{\text{cut}}$. The ratio of $\mu_\chi^2$ and $\nu_\chi^2$, on the other hand, decreases monotonically, and hence so does $\alpha$.

Since $\mu_\chi^2$ is already small when $\nu_\chi^2$ starts to rise quadratically, a useful approximation would be

$$\mu_\chi^2(Q^2) = \max \left( \mu_\chi^2(\Lambda_u^2) - c_i \alpha_i(\Lambda_u^2) \frac{Q^2 - \Lambda_u^2}{2}, 0 \right).$$  \hspace{1cm} (4.20)

### 4.5 EWSB as the origin of scales

We now consider the tadpole cancellation condition. The relevant diagrams are shown in figure 8a–c. The Higgs boson should not couple to the dilaton directly as it breaks both gauge and scaling symmetries, and so the contribution of figure 8d is zero.

![Figure 8](image_url)

**Figure 8.** The three contributions to the dilatonic tadpole (a–c) and the Higgs boson loop (d) which would have cancelled the massive contributions of diagrams a, b.

One would naively expect that, provided that we can neglect the masses of the fermions, the contribution of diagram a is zero, and diagrams b and c must mutually cancel. However, this cannot work since diagram b will at best give a contribution that has the same sign as that of diagram c:

$$A_{(b)} = \frac{\# \nu_\chi^2}{2\pi^2 v_\chi} \int dQ^2,$$  \hspace{1cm} (4.21)

$$A_{(c)} = \int \frac{\mu_\chi^2(Q^2)}{2\pi^2 \nu_\chi(Q^2)} 16 \frac{dQ^2}{(Q^2)^2/Q^2 + 3}.$$  \hspace{1cm} (4.22)

We conclude that somehow the masses of fermions, gauge bosons and the Higgs boson has a part to play. But this seems unintuitive, since eqn. (4.21) yields a contribution on the order of $\mu_\chi^2 \Lambda^2/v_\chi$, whereas the contribution of figure 8a is of the order of $m_t^2 \Lambda^2/v_\chi$, and is five orders of magnitude smaller.

The answer is that the $\mu_\chi^2/v_\chi$ coupling of figure 8b is induced through an anomaly as discussed in the previous section. If we evaluate $A_{(b)}$ instead as an all-order quantity using a Dyson–Schwinger formalism (bare vertex, dressed propagator), we will obtain zero so long as the gauge bosons are massless. On the other hand, the dilaton is massive, so
the Dyson–Schwinger approach (massless vertex, massive propagator) to $A_{(c)}$ yields

$$A_{(c)}^{\text{DS}} = -\int \frac{\mu_\chi^2(Q^2)}{\pi^2 v_\chi(Q^2)} \frac{dQ^2}{16\mu_\chi^2(Q^2) / Q^2 + 3}. \quad (4.23)$$

Note that because $\mu_\chi^2(Q^2)$ is significant only near the $\Lambda_u$ scale, this integral will be on the order of $\Lambda_u^4 / v_\chi$.

Now let us consider the massive contribution to fermion and boson loops. Provided that all of these masses are generated by the Higgs mechanism, the contributions will necessarily have the form of Higgs–dilaton mixing. If the Higgs-boson itself has a vanishing tadpole, these will all vanish, except for the self-coupling contribution which, had it existed, will have the form of figure 8d. This last contribution will be given by

$$-A_{(d)} = -\frac{3}{32\pi^2 v_\chi} \int M_H^2(Q^2) dQ^2. \quad (4.24)$$

$M_H$ is the mass of the Higgs boson.

Let us assume that EWSB is caused by dynamical symmetry breaking originating at the same cutoff scale $\Lambda_{\text{cut}}$ as gauge–dilatation symmetry breaking. This is natural, because if the latter symmetry breaking requires the former symmetry breaking, the former symmetry breaking will be forced to occur even in the absence of an interaction which grows strong at some large energy scale. This is when the symmetry-broken vacuum is more energetically favourable.

Equation (4.24) is dominated by the region near $\Lambda_{\text{cut}}$. Using the dynamical symmetry breaking hypothesis, it is easy to estimate $M_H^2(Q^2)$ near $\Lambda_{\text{cut}}$. We use the formalism of ref. [8, 10]. In the high-energy limit, eqn. (26) of ref. [10] is approximated by eqn. (44) of ref. [8] and we obtain

$$M_H^2(Q^2) \approx \frac{3}{2v_H^2 \pi^2} m_1^4(\Lambda_{\text{cut}}^2) \ln(\Lambda_{\text{cut}}^2 / Q^2). \quad (4.25)$$

$v_H = 246.22$ GeV is the Higgs condensate. By eqn. (15) of ref. [10] we then obtain

$$M_H^2(Q^2) \approx \frac{3}{128v_H^2 \pi^2} M_H^4 \ln(\Lambda_{\text{cut}}^2 / Q^2). \quad (4.26)$$

$M_H^4$ on the right-hand side refers to the low-energy value $M_H \approx 120$ GeV. Substituting this in eqn. (4.24) yields

$$A_{(d)} \approx \frac{9M_H^4 \Lambda_{\text{cut}}^2}{4096v_H^2 v_\chi \pi^4}. \quad (4.27)$$

The contribution of eqn. (4.23), on the other hand, is evaluated easily using the approximation of eqn. (4.20). We obtain

$$A_{(c)}^{\text{DS}} \approx -\frac{\mu_\chi^2(\Lambda_{\text{cut}}^2)}{3\pi^2 v_\chi} \int \sqrt{1 - Q^2 c_i\alpha_i / 2\mu_\chi^2(\Lambda_u^2)} dQ^2 = -\frac{4\mu_\chi^4(\Lambda_u^2)}{9\pi^2 c_i\alpha_i v_\chi}. \quad (4.28)$$

Let us denote $M_\chi^2 = 16\mu_\chi^2(\Lambda_u) / 3$. The tadpole cancellation condition then yields

$$M_\chi^4 = \frac{9c_i\alpha_i M_H^4 \Lambda_{\text{cut}}^2}{64v_H^2 \pi^4}. \quad (4.29)$$
Using $\Lambda_{\text{cut}} = \tilde{M}_\text{Pl}$ yields $M_\chi \approx 1.3 \times 10^9$ GeV.

Since we expect $M_\chi \approx \Lambda_u$, this prediction is higher than what we had hoped for. We think it significant nevertheless, that a sensible value of $M_\chi$ does emerge from simple considerations of dynamical symmetry breaking.

5 Summary and outlook

5.1 Summary

The presence of a tadpole anomaly suggests that gauge bosons are not fundamental, and that they are Goldstone bosons of some symmetry. We have argued that if this is the case, the symmetry is that of the rotation between the gauge-theoretical phases and space-time coordinates. A dilatonic scalar particle behaves as the Higgs mode.

A necessary condition for this programme is that the three gauge-theory tadpoles unify. We have verified this statement. The unification occurs at a rather low scale of $10^7$ to $10^8$ GeV.

We calculated the parameters of this theory within the dynamical symmetry breaking picture, by imposing self-consistency conditions. The value of the unified coupling turns out to be given by the inverse of the logarithm of the two cutoff scales. Setting the upper cutoff equal to the reduced Planck mass yields a number which agrees with the phenomenological value to within a few %.

The origin of the large difference in the two scales can be explained as being due to EWSB. Without EWSB, there cannot be gauge–dilatation symmetry breaking. By considering the tadpole cancellation condition of the dilaton $\chi$, we predicted $M_\chi \approx 10^9$ GeV. This is somewhat higher than the unification scale, and necessitates further investigation.

5.2 Theoretical outlook

Given that gauge–dilatation symmetry breaking cannot occur without EWSB, dynamical EWSB can proceed without the presence of a strong UV interaction, because one gains fermionic Casimir energy through gauge–dilatation symmetry breaking. As a topic for future study, it will be interesting to analyze how the scale hierarchy between EWSB scale and the Planck scale may arise, similarly to ref. [10] but now including the $10^8$ GeV scale physics.

Another possible topic for future study will be the application of similar ideas to gravitation. If there is a condition on tadpole cancellation that is similar to the present case, we would expect that there arises a light gravitational dilaton with a mass on the order of the electroweak scale. This will be a candidate for dark matter.

In our work, the $\Lambda^2 q^2$ anomaly in the dilatonic self-energy cancels provided that the number of fermionic generations equals three. In the case of gravitation, if similar ideas are applicable, we would like such anomalous terms to survive, so that the form factor will be large and produce the correct value of Newton’s constant.
5.3 Phenomenology

The direct experimental confirmation of our proposal will be difficult. For example, the production of a $\chi$ resonance will require collisions at a centre-of-mass energy of about $10^7$ or $10^8$ GeV.

There are a number of other possibilities:

- The dilatonic contribution to $\gamma \gamma \rightarrow \gamma \gamma$ elastic scattering is suppressed at the amplitude level by $Q^2/v^2_{\chi}$. The Standard Model contributions being suppressed by $\alpha_{\text{EM}}^2$, the two contributions will become comparable at about $10^6$ or $10^7$ GeV.

- The dilaton is associated with conserved scaling symmetry, which is broken by the Higgs mechanism. In other words, the non-conserved trace part of the symmetry current $\sim T_{\mu\nu}$ is due to the Higgs boson, and this condition yields dilaton–Higgs mixing. The mixing angle is of order $v_H M^2_H/(v_{\chi} M^2_{\chi})$, and is therefore tiny when considering low-energy phenomenology.

- As mentioned above, the EWSB-scale gravitational dilaton will be an indirect prediction of our theory but, in reflection, this prediction will hold true even without the presence of a gauge–dilatation symmetry breaking. The lack of such a particle will not rule out gauge–dilatation symmetry breaking either in any way.

- Cosmology will be affected since there will be no propagating fields above the $\Lambda_u$ scale. Gravity will be a possible exception.

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