LOW FREQUENCY QUASI-PERIODIC OSCILLATIONS IN LOW
MASS X-RAY BINARIES AND GALACTIC BLACK HOLE CANDIDATES

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ABSTRACT

We consider the inner regions of accretion disks surrounding black holes and neutron stars and investigate the nonlinear time dependent evolution of thermal-viscous instabilities. The viscous stress is assumed to be proportional to the gas pressure with the viscosity parameter formulated as $\alpha = \min[\alpha_0(h/r)^n, \alpha_{\text{max}}]$, where $h$ is the local scale height, $r$ is the distance from the central compact object, and $n$, $\alpha_0$ and $\alpha_{\text{max}}$ are constants. It is found that the disk is unstable for $\alpha$ sufficiently sensitive to $h$ ($n \gtrsim 1.2$). The instabilities are globally coherent in the entire unstable region of the disk and, depending on the viscosity parameters, the time variability of the mass accretion rates are manifested as periodic or quasi-periodic oscillations. We show that, the low frequency ($\sim 0.04$ Hz) quasi-periodic oscillations (QPOs) discovered recently in some of the black hole candidates (Cyg X-1 and GRO J0422+32) and a low mass X-ray binary (Rapid Burster MXB 1730–335) may be explicable by the thermal-viscous instabilities in accretion disks. The observations of QPOs place constraints on the viscosity parameters and suggest that $(n, \alpha_0) \sim (1.6, 30)$ for the Rapid Burster with a $1.4 M_\odot$ neutron star. In the case of black hole candidates, the dependence of $\alpha$ on $h/r$ is less steep corresponding to $n \sim 1.2 - 1.3$ for black holes less than $10 M_\odot$.

Subject headings:
accretion, accretion disks — instabilities — black hole physics — stars: neutron
1. INTRODUCTION

The quasi-periodic oscillation (QPO) phenomenon in low mass X-ray binaries (LMXBs) and galactic black hole candidates (GBHCs) provides a powerful tool to probe the temporal behavior of these systems. An understanding of the observed luminosity variations may place important constraints on the physics of accretion in these objects. QPO phenomenon in bright LMXBs have been known since the mid 1980s (see Lewin, van Paradijs, & van der Klis 1988; van der Klis 1989). Two distinct kinds of QPOs have been classified, one is the intensity-dependent QPOs characterized by frequencies $\sim 20 - 50$ Hz, and the other is the intensity-independent QPOs with frequencies of $\sim 5 - 10$ Hz. The former QPOs are called horizontal-branch QPOs whereas the latter QPOs are called normal branch QPOs. The horizontal-branch QPO phenomenon has been interpreted in terms of an oscillatory variation in the luminosity due to modulations in the mass accretion rate and the QPO frequencies are identified with a beat frequency corresponding to the difference between the Keplerian orbital frequency at the magnetopause and the rotation frequency of the central neutron star (Alpar & Shaham 1985; Lamb, Shibazaki, Alpar, & Shaham 1985). The normal branch QPOs, on the other hand, have been modeled as a radiation instability in a spherical accretion flow (Fortner, Lamb, & Miller 1989) or as a variation in the vertical structure of an accretion disk (Alpar, Hasinger, Shaham, & Yancopoulos 1992) for accretion rates near the Eddington limit. In contrast to the model for the horizontal branch QPOs, the normal branch QPOs are interpreted in terms of optical depth variations of a nearly constant luminosity.

More recently, another kind of QPOs from X-ray binary sources have been discovered. They are characterized by low frequencies of $\sim 0.04$ Hz in both neutron star and black hole candidate systems. For example, such QPOs have been found in the hump phase of the persistent emission of the Rapid Burster MXB 1730–335 (Lubin et al. 1992, 1993), in the low state of Cyg X-1 (Angelini & White 1992; Vikhlinin et al. 1992a, 1994; Kouveliotou et al. 1992a), and in GRO J0422+32 (Vikhlinin et al. 1992b, Kouveliotou et al. 1992b, Pietsch et al. 1993). While the Rapid Burster is believed to be a neutron star, the other two sources are black hole candidates. If a black hole is involved, the mechanism deemed responsible for the production of these low frequency QPOs may be different from those proposed for the other classes of QPOs (the above models for the horizontal or normal branch QPOs require the central objects to be neutron stars) since such oscillations would then be independent of the nature of the compact object. Thus, it is natural to explore the possibility that disk instabilities are involved in the phenomenon. Among the various models proposed for the general QPO phenomenon is a model suggested by Abramowicz, Szuszkiewicz, & Wallinder (1989) who proposed that thermal and viscous instabilities in accretion disks could be important for QPOs with frequencies of $\sim 1$ Hz for LMXBs and $\sim 0.1$ Hz for GBHCs.

It is well known that the inner regions of viscous accretion disks surrounding black holes or neutron stars may suffer thermal-viscous instabilities when radiation pressure is important (see e.g., Piran 1978). For a standard $\alpha$-model of accretion disks (see Shakura & Sunyaev 1973 ) with the viscous stress, $\tau$, proportional to the total pressure, $p$, i.e., $\tau = -\alpha p$, where $\alpha$ is a constant, disks are thermally and viscously unstable if the pressure is radiation
dominated (Lightman & Eardley 1974; Shakura & Sunyaev 1976). However, if the viscous stress is proportional to the gas pressure, $p_g$, only, i.e., $\tau = -\alpha p_g$, and $\alpha$ is a constant (see, e.g., Lightman & Eardley 1974; Coroniti 1981; Stella & Rosner 1984), then the disk is always secularly stable.

Time dependent calculations of the thermal-viscous instabilities have been carried out to examine the global behavior of the disk (see, e.g., Taam & Lin 1984; Honma, Matsumoto & Kato 1991; Lasota & Pelat 1991). It was revealed that, for the $\tau = -\alpha p$ prescription, the disk is globally unstable and large-amplitude, burst-like fluctuations of the disk luminosity arise if the mass accretion rates are high enough that the pressure in the disk is radiation dominated. However, this strong instability may be reduced or even suppressed by modifying the $\alpha$-viscosity prescription. For example, with $\tau = -\alpha p\beta^m$ (see Abramowicz, Szuszkiewicz & Wallinder 1989; Szuszkiewicz 1990), where $\beta = p_g/p$, the disk is stable for $m = 0.5$ (Taam & Lin 1984; Honma et al. 1991). For $m = 0.25$, the amplitude and the time scale of the fluctuations of the disk luminosity is largely reduced (Honma et al. 1991). The stabilization in the case of $m = 0.5$ or effectively $\tau = -\alpha \sqrt{pp_g}$ in Keplerian disks, is not predicted by local linear analysis. It is due to the advection of energy by material motion in the radial direction (Taam & Lin 1984).

Notwithstanding the lack of a fundamental theory for the viscosity in accretion disks, the two phenomenological viscosity prescriptions, i.e., $\tau = -\alpha p$ and $\tau = -\alpha p_g$, have some theoretical support based on a turbulent viscosity picture (see Shakura & Sunyaev 1973) and magnetic field induced viscosity picture (see, e.g., Lightman & Eardley 1974; Coroniti 1981; Stella & Rosner 1984), respectively. On the other hand, the disk instabilities based upon a viscous stress proportional to the total pressure may be too strong to be relevant to the low frequency QPOs, and disks with the gas pressure prescription are stable and can not introduce any variability within the assumed framework. A possible resolution to this dilemma is to introduce a mixed prescription. In other words, as mentioned above, one may introduce a dimensionless parameter $m$ between 0 and 1. However, there is another possibility, namely, that the viscous parameter, $\alpha$, is not a constant. In fact, a non-constant $\alpha$ is usually required in the studies of accretion disks in dwarf novae. In that case, the disk temperatures are low and radiation pressure is unimportant in comparison with the gas pressure. However, due to the sensitivity of the opacity on the temperature, a thermal instability occurs. These instabilities have been applied to explain the dwarf nova phenomena, and in order to understand both the recurrence time and the duration of the outbursts, different values of the viscosity parameter $\alpha$ are required during the two states. One widely applied formula which can approximate such a variation is the form of $\alpha = \alpha_0 (h/r)^n$ (e.g., Meyer & Meyer-Hofmeister 1983; Duschl & Livio 1989), where $h$ is the local scale height of the disk and $r$ is the distance from the compact object. This form also has a theoretical basis from scaling arguments for magnetic field induced viscosity. In particular, for a mean helicity determined by cyclonic convective motions, Meyer & Meyer-Hofmeister (1983) find $n = 1.5$, whereas for a helicity due to internal waves, Vishniac & Diamond (1992) conclude that $n = 4/3$. In addition, it was shown that, the effective $\alpha$ due to spiral shock waves varies like $(h/r)^{1.5}$, even though its magnitude is very small (see Spruit 1987).
The \((h/r)^n\) dependence has a destabilizing effect (assuming \(n\) is positive) on the disk if it is applied to the high temperature inner regions. Based on the previous nonlinear studies, the \(\tau = -\alpha p\) model is excluded. We apply this form for \(\alpha\) to the most stable model, i.e., \(\tau = -\alpha p_g\). In this case, it has been shown that the disk is locally unstable (Meyer 1986) for \(n \gtrsim 0.75\) in the limit that radiation pressure is dominant. In a recent study, Milsom, Chen, & Taam (1994, hereafter MCT) generalized the results of Meyer (1986) and, furthermore, pointed out that, in this model, unlike the \(\tau = -\alpha p\) prescription, the instability may take a milder form and non-burst-like oscillations may exist in some circumstances, which may be applied to an explanation of the low frequency QPOs.

In this paper, we investigate the global evolution of the thermal-viscous instabilities in accretion disks in a time dependent approach to determine their viability as a mechanism for low frequency QPOs. It is the purpose of this paper to report on the detailed nonlinear time dependent calculations of accretion disks which suffer thermal-viscous instabilities based upon the above modified form of the viscosity prescription. In the next section, the fundamental equations and the approach we adopt are outlined. In §3 a brief review of the local thermal and viscous instability in accretion disks is presented. The detailed numerical results of the global evolution are given in §4, and the implications of our results and their possible applications for interpretation of observations will be discussed in the last section.

2. FORMULATION

We focus only on the thermal-viscous instability which may arise in accretion disks. The axisymmetric inertial-acoustic instabilities, which have a much shorter time scale and may exist possibly in the innermost regions of the disk (see, e.g., Matsumoto, Kato, & Honma 1989; Nowak & Wagoner 1991; Wallinder 1991; Chen & Taam 1993), are not considered here. Non-axisymmetric or self-gravitating effects (see, e.g., Papaloizou & Pringle 1984, 1985, 1987), are also neglected. In other words, we assume a Keplerian disk, which is axisymmetric, non self-gravitating, optically thick and geometrical thin so that it can be described by the vertically integrated equations. In this approximation, the surface density of the disk, \(\Sigma\), at a given cylindrical radius, \(r\), can be obtained by combining the conservation equations of mass and angular momentum. It is given by the standard time dependent mass diffusion equation (Lynden-Bell & Pringle 1974) as

\[
\frac{\partial \Sigma}{\partial t} = 3 \frac{\partial}{r \partial r} \left[ r^{1/2} \frac{\partial}{\partial r} \left( \nu \Sigma r^{1/2} \right) \right],
\]

(1)

where \(t\) is the time, and \(\nu\) is the effective kinematic viscosity. In equation (1), the Newtonian Keplerian angular velocity, \(\Omega = \sqrt{GM/r^3}\), is implicitly assumed, where \(M\) is the mass of the central compact object. Relativistic effects on the gravitational potential are important near the inner edge of the disk, but we anticipate that the long time scale (\(~20\) s) associated with the low frequency QPOs dictates that the region of instability lies at radii sufficiently large compared to the innermost disk that the results we obtain will be insensitive to the neglect of these effects.
The mid-plane temperature, $T$, of the disk is governed by energy conservation which is expressed as

$$C_v \Sigma T \left[ \left( \frac{\partial \ln T}{\partial t} + V_r \frac{\partial \ln T}{\partial r} \right) - (\Gamma_3 - 1) \left( \frac{\partial \ln \Sigma}{\partial t} + V_r \frac{\partial \ln \Sigma}{\partial r} - \frac{\partial \ln H}{\partial t} \right) \right]$$

$$= F^+ - F^- - \frac{2}{r} \frac{\partial (r F_r H)}{\partial r},$$

(2)

where $V_r$, $\Gamma_3$, and $C_v$ are the radial velocity, the adiabatic exponent and the specific heat at constant volume respectively. $H$ is the half-thickness of the disk and is defined as

$$H = \frac{2 p}{\Sigma \Omega^2}.$$

(3)

The terms on the right hand side of equation (2) describe the local heating and cooling processes, where $F^+$ is the viscous dissipation rate, $F^-$ is the cooling rate in the vertical direction, and $F_r$ is the radiative energy flux in the radial direction. The heating rate and viscosity is related by

$$F^+ = \nu \Sigma \left( r \frac{\partial \Omega}{\partial r} \right)^2.$$

(4)

And the energy transport flux in the radial direction is written as

$$F_r = -2 H F^- \frac{\partial \ln T}{\partial r}.$$

(5)

The effective kinematic viscosity is parameterized in terms of the $\alpha$ model and is assumed to be

$$\nu = \frac{2}{3} \alpha c_s h (p_g/p),$$

(6)

where $c_s$ and $h$ are the local sound speed and scale height respectively. The viscosity parameter, $\alpha$, is formulated as

$$\alpha = \min[\alpha_0 (h/r)^n, \alpha_{\text{max}}],$$

(7)

where $\alpha_0$ is a constant and $\alpha_{\text{max}} (\leq 1)$ represents the effect of saturation of the anomalous turbulence.

In the radiative diffusion and vertically average approximation, the viscosity, $\bar{\nu}$, is calculated by using the half-thickness of the disk, $H$, as the scale height $h$ and $\sqrt{2 H p/\Sigma}$ as the sound speed $c_s$. This form of viscosity leads to a viscous stress corresponding to the $\tau = -\alpha p_g$ model in the Keplerian approximation, and it has a corresponding heating rate of

$$\bar{F}^+ = \bar{\nu} \Sigma \left( r \frac{\partial \Omega}{\partial r} \right)^2.$$

(8)
In this approximation, the cooling rate can be expressed as

\[
\bar{F}^- = \frac{4\alpha c T^4}{3\kappa \Sigma}
\]  

(9)

where \(\kappa\) is the sum of electron scattering and free-free opacities.

The heating and cooling rates in the energy equations can also be calculated directly from the detailed vertical structures of the disk. In practice in the time dependent study of dwarf nova accretion disk models, the above simplified heating and cooling rates in equations (8) and (9) are usually not applied directly because they differ from the vertical structure results qualitatively due to the temperature sensitivity of the opacity. Many schemes have been introduced to deal with this situation (see, e.g., Papaloizou, Faulkner & Lin 1983; Mineshige & Osaki 1983; Meyer & Meyer-Hofmeister 1984; Cannizzo, Wheeler & Polidan 1986; Mineshige 1986). For example, Faulkner, Lin & Papaloizou (1983) and Papaloizou, Faulkner & Lin (1983) used a modified approximate expression for the cooling rate. Mineshige & Osaki (1983) tabulated both the cooling and heating rates from detailed vertical structures. To construct the disk vertical structures, four equations (which are, in the steady state approximation, hydrostatic equilibrium, energy transport and energy conservation equations) are required to obtain the solutions of density, temperature and vertical energy flux with respect to the height from the disk mid-plane. In the approach of Mineshige and Osaki (1983), the condition of thermal equilibrium in the vertical direction (i.e., the energy conservation equation) is relaxed, and instead, a relationship between the energy flux and the height is assumed.

For disks surrounding black holes and neutron stars, MCT constructed detailed steady state vertical structures including convection and showed that convection has a stabilizing effect on the disk and changes the disk structure significantly. Therefore, we choose to use the cooling and heating rates from the detailed vertical structure models instead of the simplified rates above. To tabulate the cooling and heating rates, a different approach from that of Mineshige & Osaki (1983) is adopted as follows.

We assume that the departure from thermal equilibrium in the vertical direction has a form of

\[
F^- = \delta F^+,
\]  

(10)

where, \(\delta\) is a dimensionless constant and is introduced to represent the effect of advection of energy by material motion and radiative energy transport in the radial direction. For \(\delta > 1\), the radial energy transport is a heating process, and for \(\delta < 1\), it is a cooling process. The energy equation in the vertical direction becomes

\[
\frac{dF}{dz} = \delta \rho \epsilon_v,
\]  

(11)

where \(z\) is the height from the disk mid-plane, \(F\), \(\rho\) and \(\epsilon_v\) are the total flux, the density and the viscous energy generation rate at height \(z\) respectively. Note that, in the calculation of \(\epsilon_v\), which is \(9/4\nu\Omega^2\), the viscosity is computed with the local adiabatic sound speed and a modified local scale height and they are functions of \(z\) (see MCT). Now, if we identify the
temperature $T(z)$ at the mid-plane as $T_c$, the total flux $F(z)$ at the surface as $F_s$ and the column density $\Sigma(z)$ at the surface as $\Sigma_s$, then we have the following relations for $\Sigma$, $T$, $F^-$ and $F^+$:

$$
\begin{aligned}
\Sigma &= 2\Sigma_s, \\
T &= T_c, \\
F^- &= 2F_s, \\
F^+ &= 2F_s/\delta.
\end{aligned}
$$ (12)

In general, $\delta$ is unknown and $F^-$ and $F^+$ can not be obtained from $\Sigma$ and $T$. However, for a given viscosity prescription and specified radius, it is found that two relations for $F^-$ and $F^+$ exist which are approximately independent of $\delta$. These relations can be formally expressed as

$$
\begin{aligned}
f_1(\ln \bar{F}^-) &= \ln F^-, \\
f_2(\ln \bar{F}^-) &= F^+ / \bar{F}^+.
\end{aligned}
$$ (13)

As an example, $f_1$ and $f_2$ are shown in Figure 1(a) and (b) respectively for a specified set of model parameters and radius. In each plot, there are three curves of $\delta = 0.8, 1.0$ and 1.5 (corresponding to dotted, solid and dashed lines respectively), however, the difference between them is very small. The $f_2$ curve is Z-shaped, and is related to the S-shaped relation between the temperature and surface density of the disk which will be discussed in the following sections.

These two functions are tabulated with respect to $\bar{F}^-$ at specified radii for a given viscosity prescription. For the time dependent evolution, $\bar{F}^-$ and $\bar{F}^+$ are calculated from the two variables $\Sigma$ and $T$ (see eqs. [8] and [9]), and linear interpolation is used to obtain $f_1$ and $f_2$ from the tables to determine the modified values of $F^-$ and $F^+$, i.e.,

$$
\begin{aligned}
F^- &= \exp(f_1), \\
F^+ &= f_2 \bar{F}^+.
\end{aligned}
$$ (14)

The time dependent equations (1) and (2) are solved to calculate $\Sigma$ and $T$ via an explicit method. The accretion disk is divided into 41 grid points distributed equally on logarithmic scale ranging from an inner boundary at $4r_g$ to an outer boundary at $300r_g$, where $r_g$ is the Schwarzschild radius ($r_g = 2GM/c^2$). The initial structure of the disk is given by the thermal equilibrium solutions ($\delta = 1$) of the detailed vertical structures (see MCT). At every grid radius a series of models of steady state vertical structures with different mass accretion rates are constructed, and at the same time, functions of $f_1$ and $f_2$ are tabulated. For the initial values of $\Sigma$ and $T$ with the same steady state mass accretion rate at each grid point, linear interpolation from the vertical structure models of the same grid radius is applied. The boundary conditions are chosen to correspond to zero gradients at the innermost radius and to a fixed temperature and surface density at the outermost radius.

The disk model is determined by the mass of the central object, $M$, the mass accretion rate, $\dot{M}$, and the viscosity parameters, $\alpha_0$, $n$, and $\alpha_{\text{max}}$. We adopt the mass as $1.4M_\odot$ for neutron stars and $10M_\odot$ for black holes. The mass accretion rate is measured in units of
the Eddington limit defined as $\dot{M}_{\text{Edd}} = 4\pi GM/(\kappa_\epsilon \epsilon c)$, where $\kappa_\epsilon$ is the electron scattering opacity and $\epsilon$ is the efficiency for the conversion of rest mass energy into radiation taken to be 1/6 for a neutron star and 1/16 for a black hole.

3. THERMAL AND VISCOS INSTABILITIES

Thermal instability occurs when the heating process is more efficient than the cooling process. In an optically thick and geometrical thin viscous accretion disk, if we consider only the local processes, then the heating is due to the viscous dissipation and the cooling is due to the radiative diffusion (the convection is neglected). The general criterion for thermal instability may be expressed as (see Piran 1978)

$$\frac{\partial \ln \hat{F}^+}{\partial \ln H} \bigg|_\Sigma > \frac{\partial \ln \hat{F}^-}{\partial \ln H} \bigg|_\Sigma.$$  \hspace{1cm} (15)

The local thermal instability of accretion disks for which the viscous stress is given in the form of $\tau = -\alpha_0 (h/r)^n p_g$ has been examined by Meyer (1986) in the limit in which radiation pressure dominates gas pressure and it was shown that thermal instability occurs for $n > 0.75$. In a recent study, MCT generalized the analysis to arbitrary values of the ratio of gas pressure to total pressure ($\beta$) and derived the following condition for thermal instability:

$$\beta < \frac{4n - 3}{3(1 + n)} = \beta_c.$$  \hspace{1cm} (16)

Furthermore, it was shown that this condition is identical to the viscous instability condition which can be inferred from the slope of the relation between the mass accretion rate and surface density at a fixed radius. That is, for $\frac{d\dot{M}}{d\Sigma} > 0$ the disk is locally stable whereas for $\frac{d\dot{M}}{d\Sigma} < 0$ it is locally unstable (see Bath & Pringle 1982). In particular, in the polytropic approximation, the slope is given as (see MCT)

$$\frac{d \ln \dot{M}}{d \ln \Sigma} = -\frac{5 + (5 + n)\beta}{4n - 3 - 3(n + 1)\beta}.$$  \hspace{1cm} (17)

From equation (17), for a given $n$ (less than 6), it is seen that, in the parameter plane of mass accretion rate and surface density, the slope of $\dot{M}(\Sigma)$ relation is positive if the gas pressure is much larger than the radiation pressure, and which corresponds to the stable regime. At the point $\beta = \beta_c$, the slope changes sign from $\frac{d\dot{M}}{d\Sigma} > 0$ to $\frac{d\dot{M}}{d\Sigma} < 0$, and the condition of $\beta < \beta_c$ corresponds to the unstable regime.

In the detailed vertical structures of accretion disks, the above feature is qualitatively recovered. However, another turning point was revealed, which is due to the saturation of the $\alpha$ parameter (see MCT), and, hence, an S-shaped curve is formed in the plane of mass accretion rate and surface density. The upper branch is stable because in that regime ($\frac{d \ln \dot{M}}{d \ln \Sigma} \approx 5/3$) the disk is approximately described by the $\tau = -\alpha_{\text{max}} p_g$ prescription, and no dependence of the viscous parameter on the local scale height exists.
An S-shaped relation of temperature with respect to surface density follows from the relation between $T$ and $\dot{M}$, specifically,

$$\dot{M} \propto F - \propto T^4/\Sigma.$$  \hfill (18)

It is also seen (eq. [17]) that, for larger $n$, $|\frac{d\dot{M}}{d\Sigma}|$ or $|\frac{dT}{d\Sigma}|$ at the middle branch of the S-curve is smaller and so the S-curve becomes steeper, which results in a greater tendency toward instability. This can also be explained through the different sensitivities of the cooling and heating rates on the temperature. For the viscous heating rate, we have, approximately,

$$\frac{\partial \ln \tilde{F}^+}{\partial \ln T} \bigg|_{\Sigma} \approx (2 + n) \frac{4 - 3\beta}{1 + \beta} - \frac{7 - 7\beta}{1 + \beta},$$  \hfill (19)

which becomes larger for larger $n$. On the other hand,

$$\frac{\partial \ln \tilde{F}^-}{\partial \ln T} \bigg|_{\Sigma} \approx 4,$$  \hfill (20)

so the temperature sensitivity of the cooling rate is approximately a constant.

A local thermal-viscous instability at a fixed radius follows from the evolutionary trajectory in the $(T, \Sigma)$ plane. For example, as the mass accretion rate increases while the disk is on the lower stable branch, the surface density increases until the turning point where $\beta = \beta_c$ is reached. The subsequent evolution is expected to lead to heating of the disk until the upper stable branch is reached. The mass flow rate corresponding to this state, however, is higher than that corresponding to the lower stable branch and therefore $\Sigma$ decreases. Eventually, the evolutionary path reaches the upper turning point and the disk undergoes a transition to the lower stable branch, where the mass flow rate is less than the assumed steady state rate. As a result, matter accumulates, thereby increasing $\Sigma$ and the cycle begins anew. To determine whether the disk follows the steady state disk curves, as outlined above, we turn to the description of the global evolution.

4. GLOBAL EVOLUTION

The parameters characterizing each of the model sequences as well as the results of the simulations are summarized in Table 1. Here, $\dot{M}$ is in units of the Eddington value. For the unstable models, the frequency of the oscillation, $f$, and the relative amplitude of the luminosity fluctuation, $\Delta L/L$, are also listed.

4.1 Neutron Stars

The existence of an S-shaped relation between $T$ and $\Sigma$ at a specified radius indicates thermal and viscous instabilities whenever the steady state solutions of $T$ and $\Sigma$ are located at the middle branch. However, these instabilities are local and may be stabilized by nonlinear effects especially if the instability is restricted to only a small region of the disk. Here, we present the results of the time dependent calculation of model sequence 1 ($n = 1.1$) with a
mass accretion rate of $0.3 \dot{M}_{\text{Edd}}$ and a central object of mass equal to $1.4 M_\odot$ to demonstrate the global stabilizing effects. The steady state (i.e., the initial state) is indicated by open circles in the $T$ and $\Sigma$ plane at four specified radii (see Fig. 2). The ratios of gas to total pressure, $\beta$, are also listed for each radius. In the radiative diffusion approximation, for $n = 1.1$, the critical value of the ratio is, $\beta_c = 0.2121$ (see eq. [16]). Thus, the disk is predicted to be unstable in three of the four radii displayed. However, it is seen that, only two open circles in the inner radii are on the unstable middle branches, which confirms the results of MCT that the convection has stabilizing effects and a greater contribution of radiation pressure is required to make the disk locally unstable. The nonlinear calculation shows further that the global effects, i.e., the advection of energy and radiation transport in radial direction, stabilize the entire disk, even though it is locally unstable in the inner regions. As is shown in Figure 2, the evolution models are seen to be approximately the same as that of the initial steady state (the last evolution model is indicated by the cross).

The variation of disk luminosity, calculated by integrating the flux $F^-$ over the area of the disk surface, with respect to time is displayed on Figure 3. Obviously, it evolves to a steady state after a short initial transient stage of less than 10 seconds. Additional calculations have been performed for different mass accretion rates ranging from 0.01 to 1 times the Eddington value, and the disk is always found to evolve to a stable steady state structure. These results do not confirm local analysis which predicts instability for $n \sim > 0.75$ and reveal that stabilization for $n \sim < 1.1$ results from the global effects of non-local energy transport (see also Taam & Lin 1984).

The behavior of the accretion disk is a sensitive function of the parameters characterizing the viscosity prescription (see §3). To strengthen the disk instability, we increase the viscosity parameter index $n$ to 1.3 and keep the other viscosity parameters fixed as that of model sequence 1. We denote that model as sequence 2. The results for $\dot{M} = 0.27 \dot{M}_{\text{Edd}}$ are shown in Figure 4 and Figure 5a. It is seen that, the evolution paths of $T$ and $\Sigma$ at radii $5.53 r_g$ and $11.77 r_g$ (see Figs. 4a, b) are both very narrow loops moving in a counterclockwise direction. The narrow loops represent a weak instability, which can also be seen from the small amplitude periodic oscillations of the disk luminosity (Fig. 5a). In particular, the frequency of the oscillation is about 0.143 Hz and the amplitude is about 22%. At the other two larger radii, $31.10 r_g$ and $59.43 r_g$, the steady state solutions of $T$ and $\Sigma$ are located at the lower branch of the S-curve (see Fig. 4c, d), and they are stable. The instability becomes stronger at higher mass accretion rates ($\dot{M} = 0.3 \dot{M}_{\text{Edd}}$) because radiation pressure becomes more important and the unstable regions of the disk is wider. Therefore, a lower frequency, larger amplitude oscillation results (see Fig. 5b). On the other hand, for mass accretion rates less than $0.21 \dot{M}_{\text{Edd}}$, the disk is eventually stabilized (Fig. 5c).

The low frequency QPOs of about 0.04 Hz from the Rapid Burster imply a relatively small effective $\alpha$ parameter. To obtain a smaller effective $\alpha$ parameter, one can either increase $n$ or decrease $\alpha_0$. However, for a decrease in $\alpha_0$, the surface density increases and, for a given $\dot{M}$, radiation pressure becomes less important in the disk. Instability can only, therefore, occur at higher mass accretion rates. Since the persistent mass accretion rate in the Rapid Burster is probably low ($\lesssim 0.2 \dot{M}_{\text{Edd}}$), a larger $n$ is suggested. Furthermore, the parameter $\alpha_{\max}$
determines the range of mass accretion rates over which the disk is unstable. For smaller $\alpha_{\text{max}}$, the range of mass accretion rates is smaller. The absence of low frequency QPOs during the type II X-ray bursts may require a smaller $\alpha_{\text{max}}$.

In model sequences 5 – 8, $n = 1.6$, $\alpha_0 = 30$ and $\alpha_{\text{max}} = 0.2$ are applied. The larger value of $n$ makes the disk more locally unstable and also results in a wider unstable region of the disk. This effect, in addition to a smaller effective viscosity, leads to a longer viscous time scale. We first examine sequence 5 with $\dot{M} = 0.14 \dot{M}_{\text{Edd}}$. Figure 6 shows the evolution path in the temperature and surface density space at four specified radii. As expected, at the inner radii, the path has larger loops because of a stronger instability. However, the evolution path does not jump vertically from the first turning point to the upper branch. Instead, the trajectory takes an intermediate route between the upper and middle branches (closer to the latter one). The evolution path returns to the lower branch by following a route also closer to the middle branch. On the other hand, it does follow the lower branch approximately because the effects of radial advection are negligible there. At a larger radius, the loop is smaller, and eventually, it dissapears. It should be noticed that, in this case, the unstable region is wider than indicated by the local instability analysis. As seen in Figure 6c, at radius $31.10 r_g$, the initial model is located at the lower stable branch, however, due to the global radial motions, perturbations in the inner regions move outwards beyond this point and the stable region becomes unstable. In this model sequence, the unstable region of the disk is spatially confined inside $40 r_g$.

The light curve resulting from the mass flow modulations in the accretion disk is illustrated in Figure 7a. The period of the oscillations is found to be $\sim 21.5$ s and the amplitude of the fluctuations correspond to $\Delta \frac{L}{L} \sim 67\%$.

To determine the sensitivity of the results to the mass accretion rate, the mass accretion rate was decreased to $\dot{M}/\dot{M}_{\text{Edd}} = 0.12$ in sequence 6. The disk is unstable as evidenced by the luminosity variations illustrated in Figure 7b. The fluctuations are manifested as oscillations and are similar to Figure 7a. It is found that the period has decreased to 11 s, and the relative luminosity amplitude has decreased, to $\sim 20\%$. However, the relation that the frequency of the oscillation decreases and the relative luminosity fluctuation amplitude increases as the mass accretion rate is increased, holds only over a limited range in mass accretion rates for which the disk is unstable. For higher mass accretion rates, the amplitude of the oscillations may decrease and eventually the disk becomes stable due to the saturation of the viscosity parameter, $\alpha$. For a set of viscosity parameters $(n, \alpha_0, \alpha_{\text{max}}) \sim (1.6, 30, 0.2)$, it is found that the accretion disk is stable for $\dot{M} \approx 0.11 \dot{M}_{\text{Edd}}$ (sequence 8) and for $\dot{M} \gtrsim 0.47 \dot{M}_{\text{Edd}}$ (sequence 9).

For a large viscosity parameter index, such as $n = 1.6$, the variation of the disk luminosity may exhibit different evolution patterns in some range of mass accretion rates. One example is shown in Figure 7c where two local maxima can be seen.

### 4.2 Black Hole Candidates

To determine the dependence of the results on the mass of the compact object we increased $M$ to $10 M_\odot$ to model the evolution of an accretion disk surrounding a black hole.
In order to produce oscillations at frequencies $\sim 0.04$ Hz, the effective viscosity parameter, $\alpha$, must be larger than that of the neutron star case since the absolute size of the inner disk is larger. We first keep $\alpha_0 = 30$ unchanged, but use $n = 1.2$ (see model sequences 10–13, Table 1). The instability is moderately mild, as shown by the disk luminosity oscillations in Figure 8a. For example, in the case of sequence 10 with mass accretion rate of $0.076 M_{\text{Edd}}$, the oscillation has a frequency of $0.026$ Hz, and a small amplitude of $\Delta L/L \sim 0.083$. For an increase of mass accretion rate, the frequency of the oscillation becomes even lower (sequence 11). The range of mass accretion rates in which the disk is unstable is about a factor of 3, specifically, $0.07 \lesssim M/M_{\text{Edd}} \lesssim 0.20$ (sequences 12 and 13). In order to obtain a higher frequency, i.e., $0.04$ Hz, $\alpha_0 = 50$ is applied in sequence 14–16. The frequency of the luminosity oscillations of sequence 14 with mass accretion rates $0.076 M_{\text{Edd}}$ is increased to $0.035$ Hz (see Fig. 8b). However, the range of mass accretion rates for which instability is indicated is very narrow in comparison with the model sequences with $\alpha_0 = 30$ (i.e., sequences 12–13), specifically, $0.071 \lesssim M/M_{\text{Edd}} \lesssim 0.084$. The smaller upper limit on the mass accretion rate is due to the larger $\alpha_0$ which makes the viscosity parameter saturate more easily.

The mild instability indicated by $n = 1.2$ suggested that this value of the viscosity parameter index is near the lower limit for an unstable disk. This is confirmed by model sequence 17, which is performed to show that $n = 1.1$ is stable for a $10 M_\odot$ black hole in a wide range of mass accretion rates, specifically, $0.005 \lesssim M/M_{\text{Edd}} \lesssim 2.0$. For a larger $n$, the strength of the instability becomes stronger and for a fixed $\alpha_0$ the period of the oscillation is longer. For a $10 M_\odot$ black hole, the possible modulation frequency of oscillations of the disk luminosity may always be lower than the $0.04$ Hz QPOs observed. Thus, we repeated the simulations of the models with viscosity parameters of $n = 1.2$, $\alpha_0 = 30$, and $\alpha_{\text{max}} = 1.0$, but with a $5 M_\odot$ black hole. In this case, the range of mass accretion rates in which the disk is unstable is narrower than that of a $10 M_\odot$ black hole (see sequences 12–13), specifically, $0.085 \lesssim M/M_{\text{Edd}} \lesssim 0.18$. The frequency of the oscillations, as expected, is higher. An example is given in model sequence 18 with $M = 0.1 M_{\text{Edd}}$, where, the frequency is about $0.05$ Hz.

In the case of a $5 M_\odot$ black hole, a wider range of mass accretion rates in which the disk is unstable can be obtained with $n = 1.3$ (sequence 19–21), and the time scale of the oscillation is in the required range, i.e., $\sim 0.04$ Hz (see Fig. 8c for sequence 19).

5. DISCUSSION

We have demonstrated by global analysis that accretion in a geometrically thin, optically thick disk surrounding either a neutron star or a black hole can be unstable. The instability is thermal in origin and results from the inability of the disk to maintain a local thermal balance whenever radiation significantly contributes to the pressure in the disk. The strength of the instability is determined by the sensitivity of the viscous heating rate to the temperature, and is greater for larger $n$. The instability may be restricted to a relatively narrow spatial extent in the disk, and the nonlinear calculations reveal that the instability results in luminosity oscillations rather than bursts. Such oscillations may exhibit a quasi-periodic behavior if the mass flow rate entering the inner region of the disk is not strictly constant.
The thermal-viscous instability of the accretion disks can be understood through the steady state \( T \sim \Sigma \) relation curve (or effectively, the relation between \( M \) and \( \Sigma \)). The S-shaped relation for \( T(\Sigma) \) exists in slim accretion disk models (see e.g., Abramowicz, Czerny, Lasota & Szuszkiewicz 1988; Szuszkiewicz 1990; Chen & Taam 1993), with its upper stable branch due to radial advection, which is a cooling process. The stabilization occurs only at super-Eddington rates. The S-shaped relation of \( T \) and \( \Sigma \) is also well known in disk models of dwarf novae. In that case, the stable upper branch represents a nearly fully ionized radiative disk structure with the gas pressure dominant. The middle unstable branch is due to the partial ionization of the material, which results in a sensitive temperature dependent opacity. The lower stable branch corresponds to cool non-ionized disk structure. It has been commonly assumed that the evolutionary path in \( T \) and \( \Sigma \) follows the stable lower and upper branches but does not follow the unstable middle branch. Instead, the path makes a vertical transition at constant \( \Sigma \) from the lower turning point to the upper branch, and a downward transition from the upper turning point to the lower branch. This assumption is usually confirmed by the global time dependent calculations (see review by Cannizzo, 1993). However, it is not obvious that is always the case because the evolution path may not necessarily follow the steady state curve, which is a result of the vertical thermal equilibrium. For example, the evolution path will be affected by the radial motion of the disk which could even stabilize it. This has been demonstrated by our results and, in fact, it has been also shown by the calculations of Honma et al. (1991) in the slim disk approximation. Our results show that, disks with viscosity parameter index \( n \approx 1.1 \) are globally stable. Our results also reveal that the evolutionary paths in the \( T \) and \( \Sigma \) plane are loops and the size of the loop reflects the strength of the global instability of the disk.

Even though the time scales of the thermal and viscous instability at different radii of the disk are different, the global evolutions of the disk show that the evolution time scale is not determined locally and the instabilities are globally coherent. To see this, we plot the time variations of the radial distribution of the mass accretion rate of model sequence 19 (see Table 1) in Figure 9. The instability is initiated in the inner region where the contribution of radiation to the total pressure is the greatest and, as a result, a large amount of mass moves inwards at a high radial velocity. This results in a low surface density and a transition wave which propagates outward. This wave terminates at the point where the disk is stable. In the next stage of the evolution the mass in the stable region of the disk diffuses inwards, and the surface density of the inner region begin to increase, as does the temperature. Eventually, the instability is triggered anew.

The unstable behavior is found to lie in a range of mass accretion rates, \( \dot{M}_{\text{c1}} \lesssim \dot{M} \lesssim \dot{M}_{\text{c2}} \), where \( \dot{M}_{\text{c1}} \) and \( \dot{M}_{\text{c2}} \) depends on \( \alpha_0 \) and \( \alpha_{\text{max}} \) for a given \( n \). For larger \( \alpha_0 \) and/or smaller \( \alpha_{\text{max}} \), the difference between \( \dot{M}_{\text{c2}} \) and \( \dot{M}_{\text{c1}} \) decrease. For example, for the model parameters adopted (see Table 1), \( \dot{M}_{\text{c1}} \) and \( \dot{M}_{\text{c2}} \) would be 0.11 and 0.47 \( M_{\text{Edd}} \) for a neutron star and 0.045 and 0.38 \( M_{\text{Edd}} \) for a black hole of 5 \( M_\odot \).

Within the framework of the disk instability model, it is a general property of these instabilities that the frequency of the oscillation is a function of the mass accretion rate, with the frequency increasing for lower source intensity (assuming a direct relation between
intensity and mass accretion rate). Such a behavior appears to have been observed in the Rapid Burster (MXB 1730–335) by Lubin et al. (1992) in the hump immediately following the post dip phase of some Type II bursts. In addition, Lubin et al. (1992) find that the amplitude of the oscillations (∼60%) decreased as the persistent flux declined. This is also consistent with the general trends exhibited by the numerical results in the previous section. Furthermore, the observations suggest that the level of persistent emission in the Rapid Burster corresponds very closely to the minimum accretion rate necessary for instability, $\dot{M}_{\text{crit}}$, since the oscillations disappear as the persistent emission decreases. If we identify the persistent luminosity level to correspond to $\dot{M} \sim 0.14 \dot{M}_{\text{Edd}}$, the low frequency (0.04 Hz) and large amplitude of luminosity oscillations imply a large $n$ (∼1.6) and constrain $\alpha_0$ to be ∼30. This value for $n$ is similar to that inferred for a viscosity based on magnetic dynamo involving non-uniform rotation and cyclonic convection (Meyer & Meyer-Hofmeister 1983). The limit on $\alpha$ (i.e. $\alpha_{\text{max}}$) may be constrained by the absence of these oscillations during the bump phase observed on the decline from a Type II burst (see Lubin et al. 1993). The intensity ratio of the bump phase to the persistent phase limits the range of mass accretion rates to ∼2.5. This suggests that $\alpha_{\text{max}} \lesssim 0.2$.

As shown in §4 the properties of the calculated disk oscillations are sensitive to the form of the viscosity parameters. Hence, the observations of oscillations from other galactic X-ray binary sources can provide additional constraints on the unknown parameters. Low frequency oscillations ∼0.04 Hz have also been observed from the black hole candidate sources Cyg X-1 (Angelini & White 1992; Vikhlinin et al. 1992a, 1994; Kouveliotou et al. 1992a) and GRO J0422+32 (Vikhlinin et al. 1992b; Kouveliotou et al. 1992b; Pietsch et al. 1993), which suggests that $n \sim 1.2 - 1.3$ and $\alpha_0 \sim 30$ for a $5M_\odot$ black hole. These lower values for $n$ are close to those expected for a viscosity based on internal wave driven dynamo (Vishniac & Diamond 1992). Further long term observations, such as those carried out for the Rapid Burster, will be required to provide additional constraints on these parameters as well as on $\alpha_{\text{max}}$.

Finally, we remark that within the framework of the model proposed here one would expect that other X-ray binaries would also exhibit low frequency QPO phenomenon provided that the presence of a magnetosphere about a neutron star does not exclude the presence of the unstable region in the Keplerian disk. In those systems where the accretion disk extends to the compact object the instability may be restricted to a small range of mass accretion rates (as in the Rapid Burster), thus making it difficult to determine whether a given source can exhibit such phenomenon. In this case, transient sources would be ideal candidates for the search of such QPOs since the level of intensity varies over a wide range.

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### Table 1

Model Sequences

| Sequence | $M(M_\odot)$ | $\dot{M}$ | $n$ | $\alpha_0$ | $\alpha_{\text{max}}$ | $f$ (Hz) | $\frac{\Delta L}{L}$ |
|----------|--------------|-----------|-----|------------|----------------|-------|-----------------|
| 1......... | 1.4          | 0.30      | 1.1 | 20         | 1.0            | .... | 0.0             |
| 2......... | 1.4          | 0.27      | 1.3 | 20         | 1.0            | 0.143 | 0.22            |
| 3......... | 1.4          | 0.30      | 1.3 | 20         | 1.0            | 0.125 | 0.31            |
| 4......... | 1.4          | 0.21      | 1.3 | 20         | 1.0            | .... | 0.0             |
| 5......... | 1.4          | 0.14      | 1.6 | 30         | 0.2            | 0.046 | 0.67            |
| 6......... | 1.4          | 0.12      | 1.6 | 30         | 0.2            | 0.093 | 0.20            |
| 7......... | 1.4          | 0.125     | 1.6 | 30         | 0.2            | 0.037 | 0.27            |
|           |              |           |     |            |                | 0.037 | 0.12            |
| 8......... | 1.4          | 0.11      | 1.6 | 30         | 0.2            | .... | 0.0             |
| 9......... | 1.4          | 0.47      | 1.6 | 30         | 0.2            | .... | 0.0             |
| 10........ | 10.0         | 0.076     | 1.2 | 30         | 1.0            | 0.026 | 0.083           |
| 11........ | 10.0         | 0.10      | 1.2 | 30         | 1.0            | 0.019 | 0.31            |
| 12........ | 10.0         | 0.07      | 1.2 | 30         | 1.0            | .... | 0.0             |
| 13........ | 10.0         | 0.20      | 1.2 | 30         | 1.0            | .... | 0.0             |
| 14........ | 10.0         | 0.076     | 1.2 | 50         | 1.0            | 0.035 | 0.056           |
| 15........ | 10.0         | 0.085     | 1.2 | 50         | 1.0            | .... | 0.0             |
| 16........ | 10.0         | 0.071     | 1.2 | 50         | 1.0            | .... | 0.0             |
| 17........ | 10.0         | 0.005–2.0 | 1.1 | 20         | 1.0            | .... | 0.0             |
| 18........ | 5.0          | 0.10      | 1.2 | 30         | 1.0            | 0.05  | 0.12            |
| 19........ | 5.0          | 0.06      | 1.3 | 30         | 1.0            | 0.036 | 0.35            |
| 20........ | 5.0          | 0.045     | 1.3 | 30         | 1.0            | .... | 0.0             |
| 21........ | 5.0          | 0.38      | 1.3 | 30         | 1.0            | .... | 0.0             |
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FIGURE CAPTIONS

Figure 1. Curves of $f_1$ (a) and $f_2$ (b) used to tabulate the cooling and heating rates. The dotted, solid and dashed lines, correspond to the thermal equilibrium departure parameter, $\delta$, of 0.8, 1.0, 1.5 respectively. In this example, the central object is a $10 M_\odot$ black hole, the viscosity parameters ($n$, $\alpha_0$, $\alpha_{\text{max}}$) are (1.2, 30, 1) and the radius is fixed at $10.57 r_g$.

Figure 2. The path of the temperature and the surface density of model sequence 1 (with initial steady state mass accretion rate $\dot{M} = 0.3 \dot{M}_{\text{Edd}}$) at four specified radii (in unit of $r_g$): 5.53(a), 11.77(b), 31.10(c) and 59.43(d). The empty circular points represent the initial steady state model and the cross points correspond to the last evolution model. The S-shaped curves (dashed lines) are the steady state solutions.

Figure 3. The time variations of the bolometric disk luminosity in terms of the steady state value of model sequence 1 with initial steady state mass accretion rate $\dot{M} = 0.3 \dot{M}_{\text{Edd}}$. The disk reaches a stable state after a initial transient stage of less than 10 seconds.

Figure 4. Same as Figure 2 but for model sequence 2. The evolution paths of the temperature and the surface density are small loops at the inner two radii and disappear at the other two larger radii.

Figure 5. The time variations of the disk luminosity in terms of the steady state value. (a)–(c) represent model sequences 2–4 respectively. Note that sequence 2 has a weak periodic oscillations and sequence 4 is stable due to a lower mass accretion rate.

Figure 6. Same as Figure 4 but for model sequence 5. In this model, the evolution paths of the temperature and the surface density at the inner two radii are loops, but they are much larger in comparison with that of sequence 2, indicating a stronger instability. Note that, at radius 31.10 $r_g$ (c), a small loop is present even though the initial model is located at the lower locally stable branch.

Figure 7. The time variations of the disk luminosity in terms of the steady state value. (a)–(c) represent model sequences 5, 6 and 7 respectively.

Figure 8. The time variations of the disk luminosity in terms of the steady state value. (a)–(c) represent model sequences 10, 14 and 19 respectively.

Figure 9. The time variations of the radial distribution of the mass accretion rate of model sequence 19. Note that the variation is global with the same time scale at different radii, but the unstable region is restricted to less than $50 r_g$. 

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