1. Introduction

As international literary sources [1, 2] claim, by the year 2020 and in future, 5G mobile communication systems will be able to provide mobile users with unlimited high-speed access to information at any place and any time. To achieve the set goal, a considerably large variety of applications and devices is needed and networks of mobile communication and broadband wireless access currently have them. Due to this fact, there emerged a necessity to implement long-term technological methods in 5G systems aimed at solving problems of mobile user access and issues of effective link resources utilization. The key long-term technological solutions implemented in the mobile communication systems of the 5th generation are [1, 2]:

- application of evolutional massive (multi-dimensional) multi-antenna MIMO technologies;
- ability to effectively use the modes of dynamic 3D-beamforming. This will allow considerable increasing the signal power for remote users in high frequency bands and improving coverage in ultradense micro- and picocells;
- application of micro-, pico- and femtocells in areas of ultradense user location, which decrease the load on macrocells, with the division of transmission of user traffic and control signals between macro- and microcells in different frequency ranges;
- implementation of the full duplex in common band width (transmission and reception are on the same frequencies);
- application of micro-, pico- and femtocells in areas of ultradense user location, which decrease the load on macrocells, with the division of transmission of user traffic and control signals between macro- and microcells in different frequency ranges;
- implementation of the full duplex in common band width (transmission and reception are on the same frequencies);
– decrease in the level of intra-system interference at the account of using new methods of frequency-space scheduling, coordination of base stations radiation in adjacent cells, improving reception methods;

– new methods of multitone modulation, which provide the increase in spectral efficiency with respect to the OFDM technology.

All this explains the rationale and necessity to use and develop space-time access (STA) methods on the basis of implementation of multi-beam antenna arrays (MBAA). Implementation of the STA methods will increase the effectiveness of radiofrequency resource utilization, link bandwidth and capacity of the channel resources access system.

The main range of problems among the STA tasks with the use of MBAA includes determining signals direction of arrival from subscriber stations (SSs).

The problems of determining signals direction of arrival can be solved using different methods for estimation of antenna array directional characteristics, which have their own effectiveness. The analysis of the given methods concerning their effectiveness and practicability is a rather rational issue when solving the STA problems.

### 2. Literature review and problem statement

The problem of determining the direction of electromagnetic wave (EMW) arrival can both have an independent value (direction finding for radiation sources) and be a composite component of more general problems of radio communication, navigation, etc.

A great number of direction finding techniques have been developed: amplitude, phase, narrow-basis and wide-basis, with Rayleigh resolution, with superresolution (SR), etc.

Estimation methods for direction of arrival (DoA) are considered as a basis of many tasks of telecommunications, including problems of space-time access in mobile communication systems.

On the basis of DoA, in the STA problems it is possible to solve the tasks of setting the main lobe in the DP in the direction of a calling SS during a session. A lot of publications are devoted to the DoA. In [3] the authors consider the problem of estimation for nominal arrival directions of signals from a number of distributed sources using a nested array, which is formed on the basis of the whitening filter. In [4] a new algorithm of finding DoA estimations for several signals correlated in time is proposed. The proposed approach is based on the common diagonalized structure of a set of space-time correlated matrices. In the contrast to subspace methods based on DoA estimates, there is no need to estimate a noise or signal in subspace in their explicit form.

In [5] there is a new method of DoA estimation for coherent signals on the basis of decorrelation using the algorithm, in which every line of a covariance matrix can be used to form a full rank. In [6] the authors are oriented at the problem of DoA estimation using the statistics of the second order. In [7] it is proposed to improve DoA resolution within the scope of the OMP (orthogonal multiplexing with search of match) iteration selection algorithm, where a process of searching for a new DoA estimation in the considered sample is performed at each iteration. In [8] the DoA estimation of several sources of wideband signals on the basis of the maximum likelihood method considering the influence of nonuniform noise has been studied; that has given the possibility to obtain the solution closed to the Cramer-Rao criterion under high signal-to-noise ratio. In [9] the detailed comparison of investigation of operational characteristics for the family of “superresolution” methods is considered to estimate spatial spectrum of Gaussian sources of noise radiation arriving at the antenna array.

In general, in [3–9] analytical approaches in the aspect of DoA estimation are considered.

Within the studied problematics, the authors of this work have conducted a general analysis of methods for determining the direction of arrival of the SS signals. The goal of the analysis is selection of the effective method for determining signals arrival and its practicability in the STA problems in mobile communication systems.

Analyzing publications devoted to this subject [3–9], it is worth mentioning that the relevance of the given problem does not decrease because nowadays there is a deficiency in using frequency, time and code resources. There has emerged an urgent necessity for implementation of additional physical resources, which realize the growth of link throughput and the increase in productivity of a mobile communication system in general. In this aspect, implementation of spatial resource, which possesses a considerable effective potential, is considered to be perspective.

### 3. Goal and tasks of research

The goal of the work is to analyze methods for determining the signals direction of arrival on the basis of the implemented approach for AA field distribution estimation jointly with the estimation of weight vector of SS received signals.

In order to achieve the goal the following tasks have been set:

1. To analyze the effectiveness of antenna array directional characteristics on the basis of different estimation methods.
2. To analyze the technique for estimation of the vector of AA distribution field together with the estimation procedure of weight vector of signals received by SSs from different directions in the AA.
3. To analyze “superresolution” methods when determining the direction of SS signal arrival and estimation of their theoretical resolution.

### 4. Analysis of directional characteristics for antenna arrays in signal arrival estimation

All the methods for determining the signal direction of arrival into the observation point (position of the phase front of electromagnetic wave in the selected coordinate system) rely on certain use of the direction pattern (DP) of the receiving antenna, which is considered as a spatial filter. From the antenna theory it is known [10–12] that any antenna has the maximum and minimum of reception (transmission) from a certain azimuth. Using these features of the DP it is possible to find corresponding directions at the antenna output, which are a normal with respect to the phase wavefront, according to the voltage values of $U_{\text{max}}$ and $U_{\text{min}}$. The specifics of wireless system implementation will make us interested in the directions of azimuthal angles $\beta$. The angles of EMW arrival in the vertical axis $\alpha$ are usually
situated along the horizon and in this case their analysis is less informative.

Thus, if DC \( f(\beta) \), where \( \beta \) is the azimuthal angle, it is needed to turn according to the change in this angle, then the voltage at the antenna output will accordingly change, what brings some useful information due to which the DoA can be determined.

\[
U_{\text{max}} = U_{\text{max}} f(\beta). \tag{1}
\]

For \( N \)-element linear antenna the characteristics of directionality is determined by the expressions [10]:

\[
f_{\Sigma}(\beta) = \sum_{n=1}^{N} I_n \exp\{j k z_n \cos \beta\}, \tag{2}
\]

where \( I_n = \|I_n\| \exp\{j \phi_n\} \) is the complex amplitude of the \( n \)-th element excitation; \( z_n \cos \beta \) is the length difference of paths that come to the origin point.

The reception of SS signals by an antenna of a base station is performed at the main lobe (ML) of DP. It is obvious, that the narrower the ML receiving antenna is, the lesser is the possibility of the impact from undesired signals and interference. The level of a useful signal sent in the direction of the reception point is also growing under narrowing of the ML in transmitting antenna. When the aperture size is \( L/\lambda \), the minimum value of ML width is \( \theta = 0.5 \lambda / L \). According to the specifics of DC \( f(\beta) \), two basic methods of DoA are practically used:

- the method of maximum when the direction finding \( \beta \) is found by the value \( U_{\text{max}} \) (where \( U_{\text{max}} \) is, there is also the direction finding \( \beta \), in addition, \( \phi_n = k z_n \cos \beta_n \) is determined, \( \beta_n \) is the direction finding of the \( n \)-th SS;
- the method of minimum, when the value \( U_{\text{min}} \) is also used.

Both mentioned methods have their advantages and disadvantages. We analyze them as follows.

The method of maximum is implemented rather easily using almost any antenna. Alignment of the reception antenna to a correspondent \( (\beta = \beta_0) \) is nothing but the maximum principle implementation. However, as a rule, the DP position is more blurred in comparison to the position of DP minimum. The resolution, accuracy in the angle \( \beta_0 \) determining under this method is not high and it has an order of the one sixth of the ML width in the DP [13]:

\[
\Delta \beta = 0.2 \theta, \tag{3}
\]

where \( \theta \) is the ML width in the DP at the level of 0.7 or 0.5 under DP according to power.

However, the accuracy can be fixed by changing the parameters (dimension) of the antenna. It is known [13] that the width of antenna ML is connected with its electric length \( l_1 \) and it is determined by the ratio

\[
\theta = k_0 \lambda / l_1, \tag{4}
\]

where \( k_0 = 0.4 + 0.6 \).

Thus, for \( \lambda = 0.15 \) m and aperture radius is \( l = 2 \) m, we receive \( \theta = 8^\circ \). Obviously, such accuracy may not always be enough for practical application.

The resolution of two signals adjacent toward the signal arrival is connected with the implementation of the method of maximum. Fig. 1 shows the DP graphs of two similar antennas in the orthogonal coordinates, which demonstrate the ability of resolution for two adjacent signals with angles of arrival \( \beta_1 \) and \( \beta_2 \).

The resolution of two signals is considered as limited for the given antenna with the characteristics \( f(\beta) \) (Rayleigh), if the angle distance \( \Delta \beta = \beta_2 - \beta_1 \) is so that DC \( f(\beta_2) \) and \( f(\beta_1) \) intersect at the level, which is less than 0.7 (for DC according to voltage) and 0.5 (for DC according to power).

Despite the enormous DC blur in the maximum position, this method is being widely used in practice due to its simplicity and consistency with the resolution tasks, as well as with other tasks (detection, recognition, optimal reception, etc.). However, this method turns out to be the most effective in the complex with the method of minimum (the equisignal-zone method).

![Fig. 1. Illustration for situation of two adjacent signals resolution with arrival angles \( \beta_1 \) and \( \beta_2 \).](image1.png)

In practical work, multi-beam arrays (MBAs) are widely used. These antennas are virtually the analogue of the comb filter, while direction finding is easily determined according to the fact of signal entering into one of the beams (Fig. 2). Such an MBA is usually implemented using a beam-forming Butler matrix [14].

![Fig. 2. An example of a multi-beam array: \( a \) – in the polar coordinate system; \( b \) – in the Cartesian coordinate system](image2.png)
The resolution of SS signals received using the MBA is possible at the angle distances \( \Delta b \), that satisfy the Rayleigh criterion. Considering the specifics of mobile system traffic, there is a necessity to create a large number of beams in a narrow sector of directions according to the number of calling SSs. The more the number of beams is, the more complicated and dimensions arrays are needed, but together with this the resolution of the adjacent signals of SSs is improved. However, such an MBA for the problems of wireless communication will hardly be suitable because there is a great possibility that a signal with the arrival angle \( \beta_i \) (Fig. 2, a) will enter into zero of DP. This can happen, for instance, under relocation of an SS. The MBA with the controlled DC is more constructive, when a user signal is accompanied by a beam from an ML of the DP. Such MBAs include antenna arrays with adaptation to changes in the signal-to-noise ratio (adaptive antenna array – AAA (Fig. 3)).

**The method of minimum** assumes the presence of a DP antenna with zone of DC minimums. Moreover, any antenna of a specific polarization has at least one DP minimum [11]. In addition loop antennas are often used. These antennas are used for practical direction finding; however, they possess several disadvantages, which lead to a shift in DoA estimations due to antenna effect, enormous impact of surrounding objects, small electric length, etc. In practical work, more complex electronically-controlled constructions are more frequently used. Thus, the outputs of two antennas \( A_1 \) and \( A_2 \), the load of which is added to the general load, can have outputs of the corresponding sum or differential directional pattern. The electric scheme of such a construction and the corresponding \( \Sigma \) and \( \Delta \) of diagrams are shown in (Fig. 4).

**In the given array construction there have been created 2 DCs simultaneously: \( f_{\Sigma}(\beta) \) and \( f_{\Delta}(\beta) \) (the method of equi-signal zone). It is obvious that the minimum of DC \( f_{\Delta}(\beta) \) of the equisignal zone is sharper in comparison to the zone of the DC maximum \( f_{\Sigma}(\beta) \).**

Further, we will pay our attention to the methods of determining the direction of SS signals arrival based on using antenna arrays with the electronically-controllable weight vector (WV) due to which it is possible to form the necessary DC \( f(\beta) \) (2).

5. **Analysis of methods for determining signals direction of arrival from subscriber station with estimation of field distribution vector**

When solving the tasks of the STA at the uplink of SS→BS the explicit knowledge about the direction of arrival \( \beta \) is required because this parameter is mandatory as it is included into the access algorithm as an argument. In the backward direction BS→SS, there is a need for knowledge of reversed direction finding because its direct value is impossible to obtain due to the fact that as a rule the subscriber station contains 1, seldom 2 antennas, what prevents creating the DP with high resolution. The estimation of determining direction finding from the SS is difficult due to the constant change of orientation of this station.

Thus, the STA problem of spatial selection and DoA (direction finding) should be solved using BS resources, where these problems are solved on the basis of N-element antenna array. We are interested in distribution of the vector of EMF electric component for the signal at the aperture of the N-element AA.

\[
H = \left[ h_1(t), h_2(t), ..., h_N(t) \right]^T, \quad (5)
\]

where \( H = (N \times L) \) is a matrix;
$$h_i = \{a_1 e^{\jmath \phi_1}, a_2 e^{\jmath \phi_2}, ..., a_N e^{\jmath \phi_N}\}$$

is the N-dimensional vector, which characterizes distribution and spatial structure of the field of the i-th called signal;

$$a_k$$ is a standardized coefficient of gain for the k-th antenna element (AE) in the direction of the i-th signal arrival $$\beta_i$$;

$$\phi_i$$ is the phase shift conditioned by the delay of the i-th signal at the output of the k-th AE with regard to the point accepted as a phase center of the AA. According to (2) the desired estimation of the direction finding is

$$\beta_i = \frac{\arccos \phi_i}{k_z}$$

Thus, the DoA estimation $$\beta_i$$ results in receiving the estimation of the vector $$h_i$$.

$$T$$ is the operation of transposition.

To begin with, let us consider the specifics of the space-time problem at the uplink, when the SS needs the delivery of a communication service and it transmits a ringing signal (RS). The BS receives the RS of the given SS, selects it among many RSS from other SSs and extraneous interference, which get into the ringing frequency band. BS spatial selection is performed by a filter coherent with the broadband signal structure as well as on the basis of the algorithm of adaptation, in particular, using the procedure for WV estimation of the adaptive antenna array [12].

At the downlink, the STA problem is different. Due to the fact that the SS resources are limited, the problem itself is solved at the transmission side, i.e. on the BS. The solution of such a problem results in determining and estimating the arrival direction of the RS and setting a narrow beam of the BS transmitting antenna in the backward direction. In this case, we perform DP synthesis according to the given direction of the RS arrival from the given SS.

Let us obtain the estimations of the N-dimensional vector (5) in the assumption of neglecting the mutual connection between antenna elements on the basis of reception of L-dimensional signals

$$y(t) = Hx(t) + v(t) = W_{\text{opt}} x(t) + v(t).$$

where $$v(t)$$ is the N-dimensional vector of Gaussian thermal noises; $$x(t) = [S_1(t), S_2(t), ..., S_N(t)]$$ is the L-dimensional vector of received signals.

$$W_{\text{opt}} = [w_1, w_2, ..., w_L]^T$$ is the optimal i-dimensional weight vector, which determines amplitude-phase current distribution at the output of the AE to be estimated.

Using (6) we obtain the estimation of H matrix:

$$W_{\text{opt}} = E\{y(t)\},$$

where $$E\{\cdot\}$$ is the estimation on the criterion of the ratio maximum between the SS i-th signal and the total of interfering signals of other SSs and noise (minimum SNIR, a minimum mean square error (MMSE), etc.

According to the WV estimation $$w_i$$ it is possible to select a desired signal and find DoA estimations for the ith signals accepted by the given AA [12].

For the quality criterion of the estimation minimum, which is determined as a mean-square deviation of the received RS from the reference signal for the i-th SS, the optimal value of the WV $$W_{\text{opt}}$$ is found from the solution of the Wiener-Hopf matrix equation:

$$\hat{w}_{\text{MMSE}}^{(i)} = R_{yy}^{-1} R_{yx}^{(i)},$$

where $$R_{yy}$$ is the correlation matrix of SS received signals, including the i-th SS as well as noises $$v(t)$$; $$R_{yx}^{(i)} = \{y(t)x_i(t)\}$$ is the vector of mutual correlation of the reference $$x_i(t)$$ and input $$y(t)$$ signals.

For the criterion of estimation maximum determined as the ratio of the i-th signal to the total of interference signals and noises the WV estimation is:

$$w_{\text{opt}}^{(i)} = \mathbf{BR}_{yx}^{(i)} V,$$

where $$\mathbf{R}_{yx}$$ is the correlation matrix of interference signals and noises; $$V$$ is the control vector corresponding to the signal of the i-th SS; $$\mathbf{B}$$ is the normalizing coefficient.

In the perfect case when choosing the value of $$W_{\text{opt}}^{(i)}$$ there is a suppression of all acting SS ringing signals except for the i-th one and towards the BS of the i-th SS the maximum of DP is set. This maximum can be obtained as a result of created amplitude-phase distribution (APD).

The given DP maximum can be presented as $$F_{\text{max}}(\beta)$$, which provides cumulative yield of the sum-differential block [15]. To obtain F$$_j$$ differential DP, the aperture is divided into two halves and the mutual subtraction of signals of both halves is performed.

The correlation matrix of the received signals $$R_{yy}$$ is directly connected with the vector weight of the AA $$W_{\text{opt}}$$ by the ratio:

$$R_{yy} = W_{\text{opt}}^{(j)} W_{0}\mathbf{v}_{\text{opt}},$$

where $$W_0$$ is the reference vector representing the initial weight vector. During operation this vector changes its orientation in accordance with the change of the signal direction of arrival from the calling subscriber station and it is characterized by (5).

In operation, the AA provides the suppression of all input signals except for the j-th one:

$$h_j = \{a_1 e^{\jmath \phi_1}, a_2 e^{\jmath \phi_2}, ..., a_N e^{\jmath \phi_N}\}.$$ (11)

It is obvious that the signals suppressed at the j-th output include signals from other called stations and interference concentrated at the given part of the frequency band. To solve the STA problem at all ringing SSs we should estimate all the vectors $$h_j, j=1, 2, ..., L$$ at the BS. Thus, the AA of the BS will be used in the L-fold manner to provide the access and processing of all the SS signals.

Let us find the estimation $$\hat{h}_j$$ using the results of the technique given in [16].

The technique for construction of the estimation algorithm $$\hat{h}_j, j=1, 2, ..., L$$.

1. The correlation matrix $$R_{yy}$$ can be obtained on the basis of sample $$y_{ij}, j=1, 2, ..., k-L$$.

$$R_{yx} = \frac{1}{k-L}\sum_{m=1}^{k-L} y_{ij} y_{im}^H.$$ (12)

2. The estimation of the number of signal sources is performed and the values of the matrix eigenvalues are defined $$R_{yy} \lambda_i$$.

3. Form the whitening matrix B of the beam size $$N \times L$$

$$\hat{B} = [(\lambda_1 - \lambda_{\min})(\lambda_2 - \lambda_{\min})...(\lambda_L - \lambda_{\min}) \Lambda_1].$$ (13)
where $\Lambda_i$ is the eigenvector, corresponding to the eigen value of the matrix $R_{xx}$. The eigen values $\lambda_i$ must be arranged in the decreasing manner starting with $\lambda_{\text{max}}$. Moreover, $\lambda_{\text{min}}$ corresponds to the level of thermal noise.

4. Calculate the estimations of the cumulants for the 4th order whitening of the input signals vector

$$ Z(t) = \hat{B}y(t). $$

These cumulants can be expressed via estimations of the corresponding moment functions.

5. Form $N^2 \times N^2$ – cumulant matrix $K_z$.

6. Calculate $\lambda_k(k_2) = \Lambda(\lambda_i(k_2))$ and form $L^2$ of matrices $M_i = \text{vecs}^{-1}\left\{ \Lambda(\lambda_i(k_2)) \right\}$,

where $\Lambda(\lambda_i(k_2))$ is the $N^2$-dimensional eigenvector, $\text{vecs}^{-1}\left\{ \right\}$ is the transformation operator of the $N^2$-dimensional vector into the matrix $N \times N$.

7. Calculate the unitary matrix $\hat{V}$, which provides joint diagonalization for $L^2$ matrices $M_i$ and determine the desired estimation $H = \hat{B}^\dagger \hat{V}$. For such a diagonalization we can use the generalized Givens transformation.

The presented technique is quite difficult to calculate. However, some of these calculations are performed for the benefit of other tasks: ensuring detection, access, anti-jamming, etc. Due to the complexity, this technique does not have practical application. A more constructive approach is the method of equisignal zone, which is widely used in radiolocation.

6. Method for direction of arrival estimation for SS signal using equisignal reception zone

The problem of direction finding measurement $\beta_i$ using the method of search for an equisignal zone consists of two parallel or sequential procedures:

1. Finding the estimation of WVs $w_{\text{eig}}$, which provides the maximum of reception over the ML of DC $f(\hat{\beta})_{\text{max}}$.

2. Finding the differential DC $f(\alpha(\beta))$ by dividing the aperture of the antenna array into two equal parts.

The method of direction finding via an equisignal reception zone is a common technique to determine the angular coordinates of radio sources in radiolocation. The antenna array of the amplitude monopulse direction finder consists of several (two) identical reception links, forming the directional pattern with a small deviation of the main beam from the antenna equisignal direction.

To determine the angular coordinates $\beta_i$ of the radiation source by the amplitude monopulse method we use a comparison of the relative (normalized) $P_i$ powers of signals received simultaneously by two halves (links) of the antenna array, and further the estimation of the radiation source position is reduced to solving a system in the general case of nonlinear equations [13]:

$$ f_i(P_i) = \frac{P_i}{f_i(L_i)} = \frac{P_i}{P_{\text{eig}}}, $$

where $f_i(P_i)$ is the direction pattern over the power of the $i$-th receiving link in the antenna system.

Within the process of solution the $i$ and $j$ links of AAA must be identical.

It is possible to impose the monopulse direction finding algorithms more stringent requirements on the speed and increased accuracy of the reception of information on the angular coordinates of the SS signals. However, the main limitation of these methods is well-known: the presence of multipath propagation conditions or two or more sources with comparable capacities leads to large errors in the measured angular coordinates [14].

The method of superresolution (SR), where the problem of determining the direction of arrival is concomitant is considered as a separate issue.

7. Methods of “superresolution” in determining direction of SS signal arrival

It is worth noting that the successful SR solution requires a higher reception quality (signal-to-noise ratio), more accurate coordinates of the antenna elements (AEs) installation and the design of antennas, known data on the signal statistics and interference, etc. In addition, there are some unpleasant outcomes of decisions, when as a result of the SR method implementation there emerge false marks of the detection of non-existent targets, shifts in the estimates of the direction of arrival. However, superresolution problems are perspective in practice [16], and their application should be given a certain priority.

When implementing the superresolution methods in the spatial domain it is possible to determine the position of the spectral components (directions of arrival) of the analyzed (received) signals.

In works of J. Capon et al., the methods for superresolution of signals separated by an arbitrarily small distance have been developed on the basis of nonlinear spectral analysis techniques.

There is a large number of algorithms that implement the method of SR. They include [16]:

1) methods based on determining the positions of the pseudospectrum $D(\beta)$ local maxima on the scanning in space: Capon, Borgiotti-Lagunas, “thermal noise”.

These methods assume the maximum likelihood estimate of the spatial correlation matrix:

$$ \hat{R} = \hat{X} \hat{X}^\dagger, \quad (15) $$

where $\hat{X} = [x_1, x_2, \ldots, x_k]$ is a sample at the interval $t \in [1..k]$.

Spatial pseudospectrum $D(\beta)$ for $n$ – element system array is determined as

$$ D(\beta) = \pi^{\dagger}(\beta) \hat{R} \pi(\beta), \quad (16) $$

where $a(\beta) = \left[e^{j\omega/b_1}, e^{j\omega/b_2}, \ldots, e^{j\omega/b_n}\right]^\dagger$ is the directing vector for the $n$-th signal of the $n$ – element AAA.

The Capon’s method is implemented in the form:

$$ D_k(\beta) = \left[\pi^{\dagger}(\beta) \hat{R}^{-1} \pi(\beta)\right]^{-1}. \quad (17) $$

The method of “thermal noise” is implemented as:

$$ D_{\text{TN}}(\beta) = \left[\pi^{\dagger}(\beta) \hat{R}^{-2} \pi(\beta)\right]^{-1}. \quad (18) $$

The Borgiotti-Lagunas method is implemented as follows:

$$ D_{BL}(\beta) = D_{\text{TN}}(\beta)/D_k(\beta). \quad (19) $$
2) Methods based on the expansion of the spatial correlation matrix \( R \) over its eigenvectors and \( \mathbf{V}_k \) and \( k \) — eigenvalues.

The MUSIC method is implemented as:

\[
D_{\text{MUSIC}}(\beta) = \frac{1}{n} \left( \mathbf{V}_k^H \mathbf{V}_k^H \mathbf{V}_k \right) \mathbf{V}(\beta)^{-1} \tag{20}
\]

The method of EV (eigenvector) is implemented as follows:

\[
D_{\text{EV}}(\beta) = \frac{1}{n} \left( \sum_{k=1}^{N} \mathbf{V}_k^H \mathbf{V}_k \right) \mathbf{V}(\beta)^{-1} \tag{21}
\]

3) Methods based on the separation of signal and noise spaces.

These methods include ROOT-MUSIC and ESPRIT methods focused on the use of linear equivalent AAs. The ROOT-MUSIC algorithm uses the orthogonality of its own signal and noise vectors, allowing finding the angles of arrival of all the sources in the field of observation within a single computational procedure.

Reverse value of the spatial pseudospectrum is represented as:

\[
D^{-1}(\beta) = \sum_{n=1}^{N} \sum_{m=0}^{N} z^{nm} C_{mn}, \tag{22}
\]

where \( C_{mn} \) is the matrix elements \( R_{nm} \):

\[
z^{nm} = \exp\left\{ j \frac{2\pi}{\lambda} \sin \beta_{nm} \right\}
\]

is the polynomial root

\[
D^{-1}(\beta) = \sum_{n=1}^{N} C_n z^n,
\]

\[
C_n = \sum_{m=1}^{N} C_{mn}
\]

is the total of matrix elements located at the \( l \)-th diagonal

When the roots \( z \) have been found, the direction values \( \beta_{nm} \) can be determined:

\[
\beta_{nm} = \arcsin (\lambda/2\pi) \cdot \arg z^n.
\]

The formulas for finding the polynomial roots according to its coefficient exist only for \( N<5 \). When \( N \) is bigger, it is necessary to apply numerical methods.

Let us study the methods for estimation of direction of signals arrival with superresolution on the basis of the simulation in the MATLAB package.

As a basis for the analysis of superresolution techniques we considered the following techniques: non-adaptive beamforming (NBF), Capon’s method, thermal noise, Borgiotti-Lagunas method, maximum entropy (MME), multisignal classification (MUSIC).

In the model we have used non-correlated signals from 4 point sources with the arrival angles: \(-20^\circ, -10^\circ, 0^\circ, 30^\circ\) and phase: \(\pi/4, \pi/3, \pi/2, \pi\) accordingly. The number of AA = 10, the distance between AEs = d=\(\lambda/2\). The signal-to-noise ratio (SNR): 35; 20 dB.

It follows from the analysis of graphs in Fig. 6, a (SNR=35 dB) that the non-adaptive beamforming method (Fourier) does not allow spatial separation of the signals — as seen, 3 out of 4 signals are in the same resolution element. Other methods have higher signal resolution characteristics and allow estimating the arrival angles of the signals with different accuracy. As it can be seen, the most high-precision method is the MME, but there are false peaks in the spectrum.

With a decrease in the SNR ratio up to 20 dB (Fig. 6, b) the resolution of methods significantly worsens, however, among the considered set of methods, the method of multisignal classification (MUSIC) is the smoothest and it provides a maximum of signals arrival powers from the considered directions.

---

Fig. 6. Graphs of the radiation power on the angle of arrival of uncorrelated signals in superresolution methods:

\(a - \text{SNR}=35 \text{ dB}; \ b - \text{SNR}=20 \text{ dB}\)

To analyze the quality of DoA methods for “superresolution” and Rayleigh methods Fig. 7 shows the dependence of the threshold and objective function of the “superresolution” method in the case of 2 signals resolution with a signal-to-noise ratio of a signal \(h^2=30\text{ dB} \) [17]. The objective function is a minimum standard deviation of DPA in the directions to the signal sources. As a threshold function we assume the degree of depth of signal source suppression from its level to the background of the acting noise.
Obviously, the use of a superresolution method is effective in the case where the threshold function is above the decreasing objective function that involves receiving signals with an SNR of 30 dB. This is consistent with the results of accuracy estimations of methods presented in Fig. 6.

![Fig. 7. The ratios of the threshold and objective functions under superresolution](image)

The calculations show [18] that when SNR of one of the sources is more than 30 dB, the resolution angular distance of the two signals can be \( \Delta \beta \approx 0.1 \theta \).

However, we should bear in mind that the implementation of the “superresolution” method requires rather high energy parameters of the signals that are calculated as 30 dB and above and that is not always possible to achieve in mobile communication lines.

Furthermore, accurate ratios of antenna design are required and there is no guarantee that false artifacts will not appear.

The analysis of the simulation results of the estimation for the spatial resolution of the signals arrival angles from 4 sources of radiation has shown:

1. The classic non-adaptive beamforming method is the easiest in implementation, however, it has a major drawback associated with the restriction of the Rayleigh resolution.

2. The estimation of sources spatial resolution according to Capon’s method is superior to the classical method of beamforming, because it uses every available degree of freedom for the concentration of power produced in the direction of interest direction finding (16). Among the non-parametric methods of continuous analysis such as: “thermal noise”, Borgiotti-Lagunas methods, the Capon’s method has the lowest resolution (Fig. 6, a).

3. The method of multisignal classification (MUSIC) is the limiting case of the Capon’s method when signal-to-noise ratio tends to infinity, which is consistent with a higher resolution of the MUSIC in comparison to the Capon’s algorithm. Analyzing the graphs shown in Fig. 6, a, b, we conclude that the MUSIC method is able to provide a resolution of the incoming signals under sufficient angular diversity of signal sources and at low signal-to-noise ratio values.

4. The simulation results have confirmed the statistical consistency of the proposed methods for determining the signals direction of arrival of the point radiation sources over the space-time sample at the output of the linear equidistant AA of the non-correlated sources.

### 8. Discussion of results of generalized analysis for selection of method to determine signals direction of arrival

Knowledge of direction finding, the directions of SS signals arrival is important practical and useful information that allows to improve the quality of space-time access: the SS signal support is provided under moving, the main lobe in the DP of AA is set in the direction of the calling station, the convergence of adaptive algorithms of the array is facilitated, where the direction finding is used as initial conditions.

In most of the studies, selection of the method to determine the signals direction of arrival especially from nearby sources is based on the method of direct inversion to the sample correlation matrix, which assumes the formation of the sample estimate for the field distribution vector in the AA (14). The study presents a corresponding technique to estimate the vector of AA field distribution, which implements an optimal solution and does not depend on the dispersion of the correlation matrix eigenvalues. However, it is shown that this approach could be hard to implement in practice due to the ill-conditioned inversion of the correlation matrix. The more constructive method is the equisignal zone, which is applied in radiolocation.

The estimation method of equisignal zone with the use of artificial neural networks (ANNs), which allows high-quality processing in real time, is highly constructive and accurate. In addition, together with this, the ANNs method allows specifying the number of received signals and the direction of their arrival and that requires further research in this area.

In the above presented aspects, the conducted analysis is considered to be a continuation of previous studies. A definite advantage of this study is a synthesis of the methods for determining the signals direction of arrival on the basis of the known approaches and methods for estimating the directional characteristics of antenna arrays. The approach presented in the study will allow further definition and selection of the most effective methods to be implemented in the STA problems in mobile communication systems.

### 9. Conclusions

1. There is a large number of methods and algorithms for estimating the signals direction of arrival, but they all result in the methods for estimation of the maximum or minimum of antenna signal reception. In this case, the minimum methods are more accurate in comparison to the maximum methods, what can be justified due to the different steepness of directional patterns in the given areas.

2. The STA problem of spatial selection and estimation of signals direction of arrival is solved using BS resources and the N-element antenna array. Considering the direct functional connection of amplitude-phase distribution of signals over the AA elements with the value of arrival direction, the procedure and method showing the ability to jointly calculate the vector of distribution with the estimate of the weight vector in AA has been presented. However, the computational complexity of this method limits the scope of its application.

3. The methods for determining the SS signal arrival directions using “superresolution” are promising. Under ideal conditions, these methods make it possible to obtain
arbitrarily accurate values of the direction finding; in addition, it is assumed that geometrical and electrodynamic parameters of the AA of the resolved signals themselves are accurately known.

Obviously, in reality, these data have certain dispersion, and the multipath propagation nature in mobile lines leads to shifted estimates and the appearance of false marks. The computational complexity of the methods and the possibility of unexpected errors in the estimation of the direction finding does not allow the implementation of the method for the STA tasks.

4. The algorithm of signal arrival estimation using the sum-differential DP is quicker and easier to implement. Determination of the direction finding is thus carried out by means of the equisignal zone, which allows to obtain a twice higher accuracy in comparison to the Rayleigh resolution. This method seems to be one of the most meaningful in application to the STA problems.

References
1. Skrynnikov, V. G. Future shape 5G [Text] / V. G. Skrynnikov // Telecommunications. – 2013. – Vol. 10. – P. 34–37.
2. 5G Radio Access. Research and Vision [Text]. – Ericsson White Paper, 2013.
3. Wen, C. Estimation of Directions of Arrival of Multiple Distributed Sources for Nested Array [Text] / C. Wen, G. Shi, X. Xie // Signal Processing. – 2016. doi: 10.1016/j.sigpro.2016.07.011
4. Zeng, W.-J. Direction-of-arrival estimation based on spatial–temporal statistics without knowing the source number [Text] / W.-J. Zeng, X.-L. Li, H. C. So // Signal Processing. – 2013. – Vol. 93, Issue 12. – P. 3479–3486. doi: 10.1016/j.sigpro.2013.05.017
5. Qian, C. Direction-of-Arrival Estimation for Coherent Signals Without Knowledge of Source Number Cheng Qian [Text] / C. Qian, L. Huang, W.-J. Zeng, H. C. So // IEEE Sensors Journal. – 2014. – Vol. 14, Issue 9. – P. 3267–3273. doi: 10.1109/j sensors.2014.2327633
6. Chen, H. Direction-of-Arrival Estimation Based on Sparse Recovery with Second-Order Statistics [Text] / H. Chen, Q. Wan, R. Fan, F. Wen // Radioengineering. – 2015. – Vol. 24, Issue 1. – P. 208–213. doi: 10.13164/re.2015.0208
7. Wang, W. High Resolution Direction of Arrival (DOA) Estimation Based on Improved Orthogonal Matching Pursuit (OMP) Algorithm by Iterative Local Searching [Text] / W. Wang, R. Wu // Sensors. – 2013. – Vol. 13, Issue 9. – P. 11167–11183. doi: 10.3390/s130911167
8. Chen, C. Maximum Likelihood DOA Estimation of Multiple Wideband Sources in the Presence of Nonuniform Sensor Noise [Text] / C. Chen, F. Lorenzelli, R. Hudson, K. Yao // EURASIP Journal on Advances in Signal Processing. – 2008. – Vol. 2008, Issue 1. – P. 835079. doi: 10.1155/2008/835079
9. Lekhovytskiy, D. I. Statistical analysis of “superresolving” methods for direction-of-arrival estimation of noise radiation sources under finite size of training sample [Text] / D. I. Lekhovytskiy, Y. S. Shifrin // Signal Processing. – 2013. – Vol. 93, Issue 12. – P. 3382–3399. doi: 10.1016/j.sigpro.2013.03.008
10. Vendik, O. G. Antennas with an electrical scanning [Text] / O. G. Vendik, M. D. Parnes. – Moscow: Radio engineering, 2001. – 352 p.
11. Markov, G. T. Antennas [Text] / G. T. Markov, D. M. Sazonov. – Moscow: Energy, 1975. – 528 p.
12. Monzingo, R. A. Adaptive antenna arrays: Introduction to the theory [Text] / R. A. Monzingo, T. W. Miller. – Moscow: Radio and Communications, 1986. – 486 p.
13. Kukes, I. S. Basics of radio direction finding [Text] / I. S. Kukes, M. E. Starik. – Moscow: Sov. Radio, 1964. – 640 p.
14. Karavaev, V. V. Statistical theory of passive location [Text] / V. V. Karavaev, V. V. Sazonov. – Moscow: Radio and communication, 1987. – 240 p.
15. Sychev, M. N. Spatio-temporal processing of radio signals on the basis of spectral analysis parametricalsekogo [Text] / M. N. Sychev // Antennas. – 2001. – Vol. 1, Issue 47. – P. 28–36.
16. Godara, L. C. Smart Antennas [Text] / L. C. Godara. – CRC Press, 2004. – 457 p.
17. Ratynsky, M. V. An analysis of the characteristics of algorithms with superresolution DF [Text] / M. V. Ratynsky // Radio engineering. – 1992. – Vol. 10-11. – P. 63–66.
18. Grigoryan, D. S. Definition agro sverhrazreshenyya digital radiorum fontes in a tempus ad antenna sobstvennih et sonitus neydeny-_tychnostyah pyrenmih traktov [Text] / D. S. Grigoryan // Radiotechnics. – 2007. – Vol. 8. – P. 43–48.