Genuine CP-odd Observables at the LHC

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Abstract
We discuss how to construct genuine CP-odd observables at the LHC. We classify the observables according to the even and odd properties under the naive T-transformation ($\hat{T}$). There are two classes of observables of our interests: CP-odd and $\hat{T}$-even; CP-odd and $\hat{T}$-odd. We expect them to have broad applications to many processes in theories beyond Standard Model with CP violation. For the purpose of illustration, we use simple example of $W^+W^-$ production and subsequent decays at the LHC, where the CP violation effects are parameterized by effective CP-violating operators of $WWZ$ coupling. We find significant sensitivity to the CP-odd couplings.

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The non-invariance of fundamental interactions under the Charge-conjugate and parity (CP) transformations still poses a major challenge in physics. The successful parameterization of CP violation in the electroweak Standard Model (SM) fits the observations in the kaon and B systems nearly perfectly. Yet, our mere existence, namely, the matter dominance over antimatter in our observed Universe, can only be explained by new CP violation sources beyond SM. Thus, seeking for new effects of CP violation is always of fundamental importance. CP violation can be non-ambiguously identified only via CP-odd observables, that can be constructed either by comparing two CP-conjugate processes $i \rightarrow f$ and $\bar{i} \rightarrow \bar{f}$, or by observing transitions between CP-odd and CP-even eigenstates.

The Large Hardron Collider (LHC) at CERN will lead us to revolutionary discoveries of new physics beyond the SM, which quite generically introduces new sources of CP violation. The importance of searching for CP-violation effects at the LHC are thus strongly motivated, and cannot be overemphasized. However, it is quite challenging to construct genuine CP-odd observables at the LHC. First, the initial state of the LHC as a $pp$ collider is not a CP eigenstate, in contrast to the neutrality property of an $e^+ e^-$ collider or a $p\bar{p}$ collider. One may have to seek for subprocesses ($q\bar{q}$, $gg$ initial states) or decays of CP eigenstates at the LHC experiments. Second, even with the initial states $g(p_1)\bar{q}(p_2)$ and $g(p_1)g(p_2)$, they form a CP eigenstate only in their center-of-mass (cm) frame which is in general different from the lab frame. These two reference frames are related by a longitudinal boost that is unknown and different event by event. Furthermore, the symmetric beams of $pp$ at the LHC make it impossible to identify the direction of a quark versus an anti-quark on an event-by-event basis, which is often needed when making use of their momenta.

Assuming that CPT transformation is a good symmetry, CP-violation is equivalent to T-violation. Due to the difficulty to directly construct variables that are odd under T-transformation

$$T|\vec{p}, \vec{s}> = < -\vec{p}, -\vec{s}>,$$

attempts have been mainly made in the literature to construct $\hat{T}$-odd observables, where $\hat{T}$ is the “naive” time-reversal transformation, only to change the sign of momenta and spins,

$$\hat{T}|\vec{p}, \vec{s}> = | -\vec{p}, -\vec{s}>,$$

without reversing the initial and final states nor other internal quantum numbers. While a $\hat{T}$-odd observable may not be necessarily a CP-odd observable, the transformation is useful to classify CP-odd observables. In general, there can be two types of phases in an amplitude. One is the CP-violating phase $\theta$ from a fundamental theory, and the other is CP-conserving phase $\delta$ coming from the absorptive part of an amplitude when the intermediate states in the loop become on-shell. General arguments show that CP-odd observables can be either $\hat{T}$-even or $\hat{T}$-odd, which go like

$$CP-odd, \hat{T}-even \propto \sin \delta \sin \theta,$$

$$CP-odd, \hat{T}-odd \propto \cos \delta \sin \theta. \quad (1)$$

The CP-odd and $\hat{T}$-even observables need a sizable CP-conserving phase $\delta$ to show up, while the CP-odd and $\hat{T}$-odd observables do not. On the other hand, in the presence of the CP-conserving phase, the $\hat{T}$-odd observables may fake the CP-violation effects since a CP-even and $\hat{T}$-odd goes like $\sim \sin \delta \cos \theta$. This indicates that caution must be taken when studying only a $\hat{T}$-odd observable without the full information of CP transformation for a state or a process.

There have been significant efforts in the literature to explore the possibility to observe the effects of CP-violation from new physics beyond the SM at the LHC. Examples include that in
the top-quark sector \([4, 5, 6, 7]\), and in SUSY for a scalar top \([8]\). Most of the studies have been
concentrated on \(\hat{T}\)-odd observables \([\hat{1}, \hat{8}]\), with some of them also considering CP-odd observables
in specific context \([\hat{1}, \hat{4}, \hat{6}]\).

In this Letter, we discuss a general approach to construct genuine CP-odd observables at the
LHC. We consider simple but common systems that involve initial state partons. We present the
general discussion in section II and show the illustrating example in section III. The section IV is
devoted to the conclusion.

II. CP-ODD OBSERVABLES AT THE LHC

To construct genuine CP-odd observables at the LHC, we propose to study an exclusive final
state
\[ f\bar{f} + X^0, \]  
(3)
where \(f\) is any charged particle(s) that can be kinematically reconstructed, \(\bar{f}\) is its charge conjugate,
and \(X^0\) is any particle(s) with neutral quantum numbers like charge, baryon and lepton numbers.
With this event specification, the initial states must be from the partons with neutral quantum
numbers like \(q\bar{q}\) or \(gg\).

To avoid the ambiguity due to the longitudinal boost between the partonic center-of-mass frame
and the lab frame, we wish to seek for kinematical observables involving only the quantities that
are invariant under the longitudinal boost, such as transverse components of the momenta, and
the direction of longitudinal momenta difference. The simplest observables are the difference of
the transverse momenta, or equivalently the transverse energies,
\[ p_T^+ - p_T^- \text{ or } E_T^+ - E_T^-, \]  
(4)
where \(p_T = \sqrt{p_x^2 + p_y^2}, E_T = \sqrt{p_T^2 + m_f^2}\), with \(\pm\) specifying the charged particle. This observable is
CP-odd but \(\hat{T}\)-even. Both CP-violating phase \(\theta\) and CP-conserving phase \(\delta\) are needed to generate
such observables. It would be sizable only if there is a large CP-conserving phase shift in the final
state interactions \([\hat{4}]\).

The next commonly used CP-odd variable is the triple product of the three-momenta,
\[ (\vec{p}_f \times \vec{p}_{\bar{f}}) \cdot \hat{p}_q. \]  
(5)
This is a \(\hat{T}\)-odd variable, and is generated by CP-violation in the dispersive amplitude. Since
the momentum direction of the initial quark has the ambiguity with respect to which proton it is from,
this observable cannot be directly used at the LHC. We thus consider a combination
\[ (\vec{p}_f \times \vec{p}_{\bar{f}}) \cdot \hat{p}_q \text{ sgn}(\vec{p}_f - \vec{p}_{\bar{f}}) \cdot \hat{p}_q. \]  
(6)
The factor \((\vec{p}_f - \vec{p}_{\bar{f}}) \cdot \hat{p}_q\) helps keep track of the momentum directions, and it is the sign of it that
is involved in this definition, which is invariant under a longitudinal boost. The advantages of this
variable are (1) it is a CP-odd and \(\hat{T}\)-odd so that no CP-conserving phase is needed for it to be
generated; (2) it involves the quark momentum direction \(\hat{p}_q\) twice so that it renders the specification
of the direction irrelevant. Therefore, we could even simply fix the direction of the initial parton
momentum along with a proton beam \(\hat{z}\), without changing its nature of transformation.

One could view this variable as a dot-product to define a polar angle by \(\cos \Theta\) of the \(\vec{p}_f \times \vec{p}_{\bar{f}}\)
vector with respect to the beam direction \(\hat{z}\). CP violation should thus manifest itself in the angular
distribution $\sigma(\cos \Theta)$. One may thus define a CP asymmetry

$$A_{CP} \equiv \frac{\sigma(\cos \Theta > c_0) - \sigma(\cos \Theta < -c_0)}{\sigma(\cos \Theta > c_0) + \sigma(\cos \Theta < -c_0)},$$

(7)

where $c_0$ is an appropriate selective cutoff for the asymmetry observation. We find that, however, it is more intuitive to consider it as a cross-product in the transverse plane $\vec{p}_{fT} \times \vec{p}_{\bar{f}T} \sim (\vec{p}_f \times \vec{p}_{\bar{f}}) \cdot \hat{p}_q$, so that it defines an azimuthal angle $\Phi$ of $\vec{p}_{\bar{f}T}$ with respect to $\vec{p}_{fT}$ in the range $-180^\circ \leq \Phi \leq 180^\circ$. One can now define an asymmetry

$$A_\Phi \equiv \frac{\sigma(\phi_1 > \Phi > \phi_0) - \sigma(-\phi_1 < \Phi < -\phi_0)}{\sigma(\phi_1 > \Phi > \phi_0) + \sigma(-\phi_1 < \Phi < -\phi_0)},$$

(8)

where, again, $\phi_0$ is an appropriate selective cutoff for the asymmetry observation.

It is common that the CP asymmetries constructed above can take the full angular range with $\phi_0 = 0^\circ$, $\phi_1 = 180^\circ$. It is conceivable that a CP-violating interaction may lead to a shorter period like in $\sin(n\Phi)$. Thus some caution needs to be taken in constructing the asymmetry and in choosing the differential region for $\phi_0$. One could further extend the above discussion to include another factor $\vec{p}_{fT} \cdot \vec{p}_{\bar{f}T} \sim \cos \Phi$. This will make the variable scale like $\sin 2\Phi$, and thus be more sensitive to the phase angle with a shorter period of $\pi$. Keeping this factor may be particularly prudent given the unknown nature of CP-violation theories and the potential complexity for the CP-violating operators. The genuine CP-odd and $\hat{T}$-odd observables can be generalizations of that in Eq. (6)

$$\text{sgn}((\vec{p}_f - \vec{p}_{\bar{f}}) \cdot \hat{z}) \left((\vec{p}_f \times \vec{p}_{\bar{f}}) \cdot \hat{z}\right)^{2m+1} (\vec{p}_{fT} \cdot \vec{p}_{\bar{f}T})^n \sim \sin^{2m+1} \Phi \cos^n \Phi,$$

(9)

with $m, n = 0, 1, 2, ...$

It should be noted that we have chosen a very simple final state as in Eq. (3). Along with the beam direction, there are only three independent momenta, and thus only one triple cross product can be constructed. It is desirable to generalize the situation to include final states with more constructable particles, say $f_1 \bar{f}_1 + f_2 \bar{f}_2 + X^0$. Many more suitable CP-odd observables can be obtained, depending on the underlying interactions and the kinematics. The general construction principle discussed above should still be valid if an initial state particle is involved in the construction.

Let us summarize the points that have guided us for the construction of genuine CP-odd observables for a system involving an initial state parton at the LHC:

1. Consider a set of particles that form a charge-neutral system;

2. Construct kinematical variables that are independent of the longitudinal boost (involving only transverse components of their momenta);

3. For triple products, combine with the direction factor like $\text{sgn}(\vec{p}_f - \vec{p}_{\bar{f}}) \cdot \hat{z}$ to make the variable insensitive to the choice of parton beam direction.

We expect that the above principles should be equally useful in constructing the genuine CP-odd observables for new physics beyond the SM at the LHC:

III. ILLUSTRATING EXAMPLE

We now demonstrate a genuine CP-odd variable discussed above by an explicit process at the LHC. The simplest example of Eq. (3) could be $\ell^+ \ell^- + E_T$. We thus consider the $W^+W^-$ pair
production at the LHC and their subsequent leptonic decays \[ pp \to W^+W^- \to \ell^+\ell^-\nu\bar{\nu}, \] (10)

where we only consider \( \ell = e, \mu \) for the sake of simple experimental identification. In order to explore the effects of CP-violation, we adopt an effective lagrangian to parameterize the \( WWZ \) interaction \[ L_{WWZ} = i g_{Z}^2 (W_\mu^+ W_\nu^+ Z^\nu - W_\mu^+ Z_\nu W_\nu^+) \]

\[ + i \kappa_Z W_\mu^+ W_\nu^+ Z^\nu + \frac{i \lambda_Z}{m_W^2} W_\mu^+ W_\nu^+ Z^{\nu\lambda} - g_4^Z W_\mu^+ W_\nu^+ (\partial^\mu Z^\nu + \partial^\nu Z^\mu) \]

\[ + g_5^Z \varepsilon^{\mu\nu\rho\sigma} (W_\mu^+ \partial_\rho - \partial_\rho W_\mu^+) Z_{\sigma} + i \kappa_Z W_\mu^+ W_\nu^+ \tilde{Z}^{\mu\nu} + \frac{i \lambda_Z}{m_W^2} W_\mu^+ W_\nu^+ \tilde{Z}^{\nu\lambda}, \]

where \( \tilde{Z}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} Z^{\rho\sigma} \). In the SM, \( g_4^Z = \kappa_Z = 1 \), and all the others are zero. We focus on the CP-violating couplings, \( g_4^Z \), \( \kappa_Z \), and \( \lambda_Z \). The most stringent bounds on them are from an LEP-II experiment by DELPHI Collaboration \[ [12] \]

as

\[ g_4^Z = -0.39^{+0.19}_{-0.20}, \quad \kappa_Z = -0.09^{+0.08}_{-0.05}, \quad \lambda_Z = -0.08 \pm 0.07. \]

We will use these non-zero central values in our further illustration.

Our study is based on a Monte Carlo simulation of \( W^+W^- \) pair production and subsequent decays which incorporate the spin correlations. We use the CTEQ6.1M parton distribution functions \[ [13] \]. The following cuts

\[ p_T(\ell) > 25 \text{ GeV}, \quad p_T > 25 \text{ GeV}, \quad |\eta(\ell)| < 2.5, \] (12)

are implemented in our analysis \[ [14] \]. We smear the lepton energy according to

\[ \delta E/E = \frac{a}{\sqrt{E/\text{GeV}}} \oplus b \]

where \( a = 13.4\%, \ b = 2\%, \) and \( \oplus \) denotes a sum in quadrature \[ [14] \].

In practice, we can define an azimuthal angle

\[ \Phi \equiv \text{sgn}((\hat{\ell}^+ - \hat{\ell}^-) \cdot \hat{z}) \sin^{-1}(\hat{\ell}^+ \times \hat{\ell}^-) \cdot \hat{z}, \]

with \(-180^\circ \leq \Phi \leq 180^\circ\). Here the particle names \( (\ell^\pm) \) have been used to denote their momenta. One can now define the asymmetry

\[ A_\Phi \equiv \frac{N_{\Phi>0} - N_{\Phi<0}}{N_{\Phi>0} + N_{\Phi<0}}, \]

where \( N \) is the number of events. A nonzero value of this asymmetry above the statistical error \( \frac{1}{\sqrt{N_{\text{total}}}} \) implies the CP-violation in this process.

In Fig. \[ 1 \] we show the differential distributions for the azimuthal angle \( \Phi \) in the SM and with the anomalous couplings \( g_4^Z, \kappa_Z, \) and \( \lambda_Z \), in the units of fb/bin (18^\circ). With a 100 fb^{-1} integrated luminosity, we expect a large number of events, as indicated by multiplying 100 on the left axis. Significant asymmetry can be visible from the distributions.

\[ \text{The CP-violation effect in this process has been studied using } \hat{T}\text{-odd observable at the LHC } [9]\text{ and CP-odd observables at the Tevatron } [10]. \]
FIG. 1: Differential cross section with respect to the angle $\Phi$ in units of fb/bin (18°) for $pp \rightarrow W^+ W^- \rightarrow \ell^+ \ell^- \nu \bar{\nu}$ at the LHC (14 TeV), with the CP-violating $WWZ$ couplings (a) $g_Z^4 = 0$, $-0.4$, (b) $\tilde{\kappa}_Z = 0$, $-0.1$, and (c) $\tilde{\lambda}_Z = 0$, $-0.1$. 
TABLE I: The number of events for $pp \rightarrow W^+W^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$ ($\ell = e, \mu$) for $\Phi < 0$ and $\Phi > 0$ with and without the CP-violating couplings, at the LHC (14 TeV) with a luminosity of 100 fb$^{-1}$.

|       | $N_{\Phi<0}$ | $N_{\Phi>0}$ | $N_{\Phi>0} - N_{\Phi<0}$ | $A_{\Phi}$ |
|-------|--------------|--------------|----------------------------|------------|
| SM    | 24700 ± 160  | 24700 ± 160  | 0 ± 220                    | $\approx 0.0\%$ |
| $g_4^Z = -0.4$ | 24600 ± 160  | 28600 ± 160  | 3950 ± 230                 | $\approx 7.4\%$ |
| $\kappa_Z = -0.1$ | 25500 ± 160  | 24400 ± 160  | -1130 ± 220                | $\approx -2.3\%$ |
| $\lambda_Z = -0.1$ | 28800 ± 170  | 30500 ± 180  | 1690 ± 240                 | $\approx 2.9\%$ |

Integrating the differential cross-section over $\Phi$, we obtain the asymmetry variable $A_{\Phi}$. We find

$$A_{\Phi}(g_4^Z = -0.4) \approx 7\%, \quad A_{\Phi}(\kappa_Z = -0.1) \approx -2\%, \quad \text{and} \quad A_{\Phi}(\lambda_Z = -0.1) \approx 3\%,$$

respectively. With a luminosity of 100 fb$^{-1}$ at the LHC, the number of events with statistical errors are shown in Table I. We see about 17$\sigma$, 5$\sigma$, and 7$\sigma$, signals of CP violation for cases with $g_4^Z = -0.4$, $\kappa_Z = -0.1$, and $\lambda_Z = -0.1$, respectively.

The asymmetry in Eq. (15) corresponds to the observable in Eq. (6), which is proportional to $\sin\Phi$. As discussed earlier, once the distribution of $\Phi$ is obtained, one can construct an asymmetry in any specific range ($\Phi_0, \Phi_1$) and ($-\Phi_0, -\Phi_1$), as long as it is statistically significant. More complex patterns of CP asymmetry ($\Phi$ distribution) may be counted for with the more general CP-odd observables in Eq. (9). For example, the term in Eq. (9) with $m = 0, n = 1$ is proportional to $\sin\Phi\cos\Phi$. Thus a CP-odd distribution in $\Phi$ leads to an asymmetry in the 1st and the 4th, which is opposite to that of the 2nd and the 3rd quadrats. Thus a collective sum over $\Phi > 0$ (in the 1st and the 2nd) and $\Phi > 0$ (in the 3rd and the 4th) would not lead to an asymmetry due to a cancellation. In such circumstances, it is necessary to consider the asymmetry of $\Phi$ for specific more refined regions.

IV. CONCLUSION

We have discussed how to construct the genuine CP-violating observables at the LHC experiments. We considered simple but common systems that involve initial state partons. We summarized our guiding principles at the end of section II. We classified the observables according to the even and odd properties under the naive T-transformation ($\hat{T}$). There are two classes of observables of our interests: CP-odd and $\hat{T}$-even, with the difference of transverse momenta as a representative as in Eq. (4); CP-odd and $\hat{T}$-odd, involving triple momentum product as in Eqs. (6, 9). We expect them to have broad applications to many processes in theories beyond Standard Model with CP violation. For the purpose of illustration, we use simple example of $W^+W^-$ production and subsequent decays at the LHC, where the CP violation effects are parameterized by effective CP-violating operators of $WWZ$ couplings. We found that using the upper bounds of these couplings allowed by the current collider experiments, the CP asymmetries via these operators can be clearly visible with an observable we constructed at the LHC with an integrated luminosity of 100 fb$^{-1}$ at 14 TeV.

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