Abstract.
We compute the amount of inflation required to solve the horizon problem of cosmology in the pre-big-bang scenario. First we give a quick overview of string cosmology as developed by Veneziano and collaborators. Then we show that the amount of inflation in this background solves the horizon problem. We discuss fine-tuning.

The standard cosmological model works well at late times, explaining the red shift, the cosmic microwave background and the cosmic primordial nucleosynthesis, but it has problems associated with the initial singularity, the homogeneity, the isotropy, the flatness, and the large-scale structure. Inflation solve these problems except for the initial singularity. Inflationary models are constrained by demanding a graceful exit, the right amount of reheating and the right amount of large-scale inhomogeneities. This requires fine-tuned initial conditions and inflation potentials. But inflation does not even attempt to solve the initial singularity problem.

A few years ago a stringy cosmology was built with the very basic postulate that the universe did indeed start near its trivial vacuum, solving the initial singularity problem [1].

String theory is the only consistent theory containing quantum gravity. Each of the normal modes of vibration of a quantum string is a conventional particle. At large distances ($\lambda >> 10^{-33}$cm), strings appear as particles. In string theory there are symmetries known as dualities [1] [2] [3] [4] which allow us to find two equivalent solutions to the problem of the initial singularity.

The low-energy effective action for bosonic closed strings is:

$$S = \frac{1}{4\pi\alpha'} \int d^4x \sqrt{|\det g|} e^{-\phi}(R + \partial_\mu\phi \partial^\mu \phi + ...)$$

Where $R$ is the Ricci scalar, $(\alpha')^{-1}$ is the string tension, $g_{\mu\nu}$ is the metric and $\phi$ is the so-called dilaton, a scalar massless particle which may play the role of a inflaton.
The equations of motion for this action in the flat F.R.W. metric

\[ ds^2 = dt^2 - a^2(t)[dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi] \]  

(2)

are invariant under a stringy symmetry known as “scale factor duality”:

\[ t \Rightarrow -t \quad a(t) \Rightarrow \bar{a}(-t) = a^{-1}(-t) \quad \phi \Rightarrow \phi - \ln|g| \]

(3)

We simulate matter with a perfect fluid stress-energy tensor \( T^{\mu}_\nu = \text{diag}(\rho, -p\delta^i_j) \) where \( \rho \) is the energy density and \( p \) is the pressure. Furthermore, we set \( \phi = \text{cte} = \phi_0 \) and \( V(\phi) = 0 \). For radiation we get for \( t > 0 \) our universe in its radiation dominated epoch:

\[ a = \left( \frac{t}{t_0} \right)^{\frac{1}{2}} \quad \phi = \phi_0 \]

(4)

\[ \rho = 3p = \rho_0 \left( \frac{t}{t_0} \right)^{-2} \quad G \sim \alpha'e^{\phi_0} = \text{constant} \]

(5)

Applying the duality transformation, we find for \( t < 0 \):

\[ a = \left( -\frac{t}{t_0} \right)^{-1/2} \quad \phi = \phi_0 - 3\ln\left( \frac{t}{t_0} \right) \]

(6)

\[ \rho = -3p = \rho_0 \left( -\frac{t}{t_0} \right) \quad G \sim \alpha' e^{\phi_0} \left( -\frac{t}{t_0} \right)^{-3} \neq \text{constant} \]

(7)

The universe starts at \( t = -\infty \) with flat empty Minkowski space with zero coupling; Newton’s coupling then starts growing and the universe inflates non-exponentially \( t << 0 \). For \( t >> 0 \) we recover flat Minkowski space with a weak coupling and a decelerated expansion (which corresponds to our universe). This is a description of the evolution of our universe at times well before and after the big bang \( t = 0 \), but what happens during the high curvature regime is, of course, unknown. We may try to simulate it with a time dependent equation of state such as

\[ p = \gamma(t)\rho \]

(8)

where \(-1/3 < \gamma(t) < 1/3\). There is an expanding solution [2] \( (H > 0) \) for all \( t \): the universe dominated by string matter \( (p = -\rho/3) \), starts from flat space \( (H \to 0) \), an unstable solution, with weak coupling \( (e^\phi \to 0) \) regime evolving through an inflationary phase \( [a(t) \sim (-t)^{-1/2}] \) phase with gravitational coupling \( (e^\phi = \text{const.}) \). An analogous solution exists, in which the universe is always contracting \( (H < 0) \) in correspondence with the dual equation of state \( \gamma(-t) \).
What about the other dimensions that string theory allows?

Consider a background in which, during the pre-big-bang phase \( t < 0 \), \( d \) dimensions expand with scale factor \( a(t) \) while \( n \) dimensions shrink with scale factor \( a^{-1}(t) \) with an equation of state \( p = -q = -\rho/(d + n) \).

\[
g_{\mu\nu} = \text{diag}(1, -a^2(t)\delta_{ij}, -a^{-1}(t)\delta_{ab})
\] (9)

The solution is \( a(t) \sim (-t)^{-2/(d+n+1)} \), \( t < 0 \) \cite{2}. This background evolves into a phase of maximal, finite curvature, after which it approaches the dual, decelerated regime \( t > 0 \) in which the internal dimensions are not frozen, but keep contracting like \( a^{-1}(t) \sim t^{-2/(d+n+1)} \) for \( t \to +\infty \). The dilaton vacuum expectation value does not settle down to a finite constant value after the big-bang, but tends to decrease during the phase of decreasing curvature. Such a decrease of \( \phi \) is driven by the decelerated shrinking of the internal dimensions which are not frozen.

**CONSTRAINTS ON INITIAL CONDITIONS**

The condition to solve the horizon problem (in the Einstein frame, thus the tildes) is

\[
d_{\text{HOR}}(t_f) = \tilde{a}(t_f) \int_{t_i}^{t_f} dt'/\tilde{a}(t') > \tilde{a}(t_f)H_0^{-1}/\tilde{a}_0
\] (10)

where \( H_0^{-1} \) is the size of the observed Universe \( (H_0^{-1} \sim 10^{28} \text{cm}) \), \( t_f \) is the time by which the horizon problem is solved and \( t_i \) the time when inflation begins. For us, \( t_i \) and \( t_f \) determines the time range when the pre-big bang description remains valid. Obviously, letting \( t_i \to -\infty \) we see that the horizon problem is solved both for \( k = 0 \) and for \( k = -1 \). Still, it is of some interest to ask how long did the universe have to behave stringily before the big bang in order for it to come out free of flatness and horizon problems from the high curvature epoch (around \( t = 0 \)) \cite{5} \cite{6}.

The amount of expansion required to solve the horizon problem is given by the ratio

\[
Z = \frac{H(t_f)a(t_f)}{H(t_i)a(t_i)}
\] (11)

Experimentally (or rather, observationally), we need

\[
Z > e^{60}
\] (12)

in order to solve the horizon problem for our big universe.

Since our effective actions stops being valid when gravity becomes strongly coupled, we expect the pre big bang inflationary epoch to be over by the time \( t_f \) when


\[ e^{-\phi(t_f)} \gg 1 \]  

(13)

Similarly, the same effective actions remain valid only while the curvature is not too big:

\[ H^{-1}(t_f) \sim (-t_f) \gg l_{st} \]  

(14)

When \( k = 0 \), the amount of inflation is thus

\[ Z = \left( \frac{-t_i}{l_{st}} \right)^{3/2} \]  

(15)

This give us that \( t_i < -10^{17} l_{st} \) in order to get the amount of inflation for succesfully solve the horizon problem.

We conclude from string cosmology that: (a) inflation comes naturally, without ad-hoc fields, (b) initial conditions are natural, (c) the kinematical problems of the standard cosmological model are solved and (d) a hot big bang could be a natural outcome of our inflationary scenario. Furthermore, it can be shown that (e) perturbations do not grow too fast to spoil homogeneity, (f) our understanding of the high curvature (stringy) phase is still poor and (g) the amount of inflation required needs some fine-tuning of initial conditions for \( k = 0 \) and for \( k = -1 \) in order to solve the horizon problem.

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