Visualization of heat transfer in material for varians of boundary value with Relaxation Iteration Gauss-Seidel method

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Abstract—The research was aimed to know the effect of initial boundary value to the heat propagation rate pattern using iterations over Gauss-Seidel relaxation method and to analyze the exact value of each node descritization profile of test material. This study was an analytical study to fine analytical an numerical solution. Result from this study is that the pattern of variation of the boundary or initial conditions of a material with regard conductivity value remains at steady state the exact value of the smallest are in the same iteration value. The indicates that the value of the thermal equilibrium tend to be at the same iteration. Result from study showed that the pattern of initial boundary values that causes steady state of heat propagation of test material that has smallest exact similar to the iteration value.

1. Introduction

Physics concepts are produced through experiments and theoretical works which are in the form of mathematical equation. Physics is said as the father of technology since the advanced development of physics is in the form of technology, and in turn, physics developed rapidly due to the application of technology in physics, theoretically, physics concepts are described using mathematical modeling in the form of mathematical equation that can be solved analytically or numerically. Physics is closely related to mathematics for theoretical physics often expressed in mathematical notation and mathematical logic can provide a framework in which the laws of physics can be formulated appropriately [1]. One development of the mathematical methods used in the case of the physical and engineering fields is a numerical method [2]. Numerical methods are techniques to solve problems that are formulated mathematically by means of operating a matter of analytic [3].

Heat transfer is one of the physical phenomena that is described partial differential equations Laplace or Poisson [4]. In accordance with the usefulness of mathematics in the fields of physics, the solution partial differential equations can be obtained by several methods. The selection of methods and approaches based on the purpose and complexity of the problems.

Heat transfer is a concept to predict the energy transfer occurs due to a temperature difference between objects or material [5].

The heat energy can not be observed directly but the direction of movement and its effects can be observed and measured, as in the event of conduction heat transfer. Conduction is a process that if two objects or two-part of object is contacted with the other temperature it will pass heat transfer [6].

Heat flows from a higher-temperature object to a lower body temperature. Heat transfer rate that passes through solid objects is proportional to the temperature gradient or temperature difference of unity long [7].
As mentioned before that the rate of heat propagation in a variety of geometric shapes material following the equation of state is presented in the form of mathematical equations in the form of differential equations. Solution method or the propagation model of the flow of heat at a material solved analytically and numerically. The analytically solution by using a systematic calculation and the obtained solution in the form of the exact value [8].

However, some form of differential equations, there is difficulty the applying analytical resolution methods so numerical methods is chosen exam alternative to overcome. Numerical methods are used to solve partial differential equations such as Method Crank- Nicholson, methods Milne, Hamming method and Gauss-Seidel method [9].

In this studies obeys the rate of heat flow that follows the model of partial differential equations are solved using numerical methods of Gauss-Seidel and then subsequently visualized using Mathlab with some variation of the initial boundary value.

Shape of specimens analyzed in this study are presented in Figure 1 with an initial boundary values are follow:

![Figure 1. Model workpiece as two-dimensional metal plate with a specific conductivity value in steady state.](image)

To apply Gauss-Seidel relaxation method, we set variation value of the initial boundary on the completion of iterations over Gauss Seidel relaxation method to observe how the heat propagation rate pattern seen from the distribution of its exact value on each node discretization test material and visualize the propagation of heat profile.

The mathematical equation that govern the heat propagation rate is given as:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + g = 0
\]

With :
- \(T\): temperature
- \(x\): absis
- \(y\): coordinat
- \(g\): \(\frac{q(\Delta x^2)}{k}\)

To solve the above equation, it is used a Taylor series with two independent variables \(T(x, y)\) is a way to add additional variables so that the Taylor series with two independent variables \(T(x, y)\) be [10]:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q(\Delta x^2)}{k} = 0
\]

To get the second derivative can be done in the following way:

1) If the first derivative of the advanced differential form, then the second derivative settled in the form of differential retreat.
2) If the first derivative of the differential form of retreat, then the second derivative settled in the form of forward differential. Now we know the second derivative function of T with respect to x and y, then substituted in equation Poisson equation or temperature distribution, in order to obtain:

\[
\left( \frac{T_{i+1,j} - T_{i,j}}{\Delta x} \right) - \left( \frac{T_{i,j} - T_{i-1,j}}{\Delta x} \right) + \left( \frac{T_{i,j+1} - T_{i,j}}{\Delta y} \right) - \left( \frac{T_{i,j} - T_{i,j-1}}{\Delta y} \right) = 0
\]

\[
\frac{\Delta x}{\Delta y} \frac{\Delta x}{\Delta y} = 0
\]

For the size of \( \Delta x \) and \( \Delta y \) are the same, then the above equation simplifies to:

\[
T_{i+1,j} - 2T_{i,j} + T_{i-1,j} + T_{i,j+1} - 2T_{i,j} + T_{i,j-1} + \frac{q(\Delta x^2)}{k} = 0
\]

or

\[
T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} + \frac{q(\Delta x^2)}{k} = 0
\]

Gauss Seidel equation constructed from the above equation becomes:

\[
T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} + q(\Delta x^2)}{4}
\]

and solving iteratively for \( i = 1 \) to \( n \) and \( j = 1 \) to \( m \). In general, from the heat propagation equation with \( q \) is the rate heat transfer, \( T \) is the temperature distribution at a distance of \( x \) and \( y \), which has a high length \( L \) and \( K \). Therefore the value of \( T \) at the edge of the plate known temperature (boundary conditions) and at the time before propagation, the value at points it is zero (baseline) settlement is counting equation \( x \) and \( y \) \( T \) in particular.

For partial differential equations

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q(\Delta x^2)}{k} = 0 \quad 0 \leq x \leq L \quad dan \quad 0 \leq y \leq K
\]

\[
T(0, y) = 0
\]

\[
T(L, y) = 0
\]

\[
T(x, 0) = 100
\]

\[
T(x, K) = 20
\]

known:

\[
\Delta x = 0.1
\]

\[
\Delta y = 0.1
\]

\[
q = 10000 \text{ Btu/hr ft}
\]

\[
k = 40 \text{ Btu/hr ft}
\]

To find out the solution heat propagation at each point of the Gauss Seidel method, can be done by solving iteratively for \( i = 0 \) to \( n \), and \( j = 1 \) to \( m \).

2. Results and Discussion

Completion of the heat propagation equation solved iteratively with over-relaxation equation. Relaxation parameters can be searched using the equation:

\[
\omega = \frac{1}{1 + \left( \frac{0.1}{0.1} \right)^2 \left( \frac{\Delta x}{\Delta y} \right)^2 + \left( \frac{\Delta y}{\Delta x} \right)^2 \left( \frac{\Delta x}{\Delta y} \right)^2} = \frac{1}{1 + \left( \frac{0.1}{0.1} \right)^2 \left( \frac{3.14}{50} \right)^2 + \left( \frac{0.1}{0.1} \right)^2 \left( \frac{3.14}{50} \right)^2} = 0.93
\]

Thus, \( \lambda \) can be searched using the following equation:

\[
\lambda = \frac{2}{1 + \sqrt{1 - \omega^2}} = \frac{2}{1 + \sqrt{1 - 0.93^2}} = 1.47 = 15
\]

For partial differential equations:
\[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q(\Delta x^2)}{k} = 0 \quad 0 \leq x \leq L \text{ dan } 0 \leq y \leq K \]

using boundary conditions and the same coefficient of relaxation. Furthermore solved iteratively for \(i = 1\) to \(n\) and \(j = 1\) to \(m\) with over-relaxation following equation:

\[ T_{i,j}^{\text{new}} = \lambda T_{i,j}^{\text{new}} + (1 - \lambda)T_{i,j}^{\text{old}} \]

Iteration can be stopped if the relative error has reached the limit. The magnitude of the relative error is defined as:

\[ |(\epsilon_a)_{i,j}| = \left| \frac{T_{i,j}^{\text{new}} + T_{i,j}^{\text{old}}}{T_{i,j}^{\text{old}}} \right| \times 100\% \]

Analytical solutions obtained are as follows: In the initial condition or Iteration 0, the solution in the first iteration by means of Gauss Seidel, then at the point \(T_{1,1}\):

\[ T_{1,1} = \frac{4}{0 + 100 + 0 + 50 + \frac{10000 \times (0.1)^2}{4}} = 37.75 \]

of the equation over-relaxation (\(\lambda = 1.5\)) was obtained

\[ T_{1,1} = 1.5(37.75) + (1 - 1.5)0 = 56.625 \]

for \(T_{2,1}\):

\[ T_{2,1} = \frac{4}{0 + 56.625 + 0 + 50 + 2.5} = 26.906 \]

From over-relaxation of the equation is obtained:

\[ T_{2,1} = 1.5(26.906) + (1 - 1.5)0 = 40.359 \]

iteration followed by Mathlab until iteration on \(T_{7,7}\), point, in order to obtain a value as shown below.

From the results of this iteration is then performed visualization of heat propagation using Mathlab [11] with some variation of the initial boundary conditions. And the results are as follows:

a) \(T(0, y) = 0; T(L, y) = 20; T(x, 0) = 100; T(x, K) = 0\)

**Profile 1**

**Iteration Matrix**

Iteration = 24

Computing times = 0.0042768

| 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0   | 481.716 | 658.982 | 726.063 | 738.412 | 700.602 | 572.574 | 20 |
| 0   | 267.883 | 428.148 | 506.856 | 526.784 | 492.221 | 389.495 | 20 |
| 0   | 161.668 | 278.869 | 346.431 | 369.647 | 351.802 | 293.184 | 20 |
| 0   | 99.920 | 179.693 | 230.353 | 253.569 | 252.165 | 231.435 | 20 |
| 0   | 58.781 | 107.779 | 142.181 | 162.109 | 171.852 | 180.393 | 20 |
| 0   | 27.426 | 50.922 | 68.484 | 80.834 | 92.743 | 118.284 | 20 |
| 0   | 0     | 0     | 0     | 0     | 0     | 0     | 0   |

**Profile 1A**

**Iteration Matrix with Q = 10000; k = 40**

Iteration = 24

Computing times = 0.0076659

| 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0   | 481.979 | 659.382 | 726.524 | 738.873 | 701.203 | 572.837 | 20 |
| 0   | 268.284 | 428.777 | 507.589 | 527.517 | 492.851 | 389.896 | 20 |
| 0   | 162.129 | 279.602 | 347.289 | 370.505 | 352.537 | 293.645 | 20 |
| 0   | 100.381 | 179.963 | 231.211 | 254.426 | 252.898 | 231.897 | 20 |
| 0   | 59.182 | 108.408 | 142.914 | 162.842 | 172.482 | 180.794 | 20 |
| 0   | 27.689 | 51.323 | 68.946 | 81.295 | 93.143 | 118.547 | 20 |
| 0   | 0     | 0     | 0     | 0     | 0     | 0     | 0   |
The number of iterations resulting from this limit value is 24 times. Profile distribution of heat to show the temperature decreases in the same area or exactly on point $T_{7.1}$.

b) $T(0, y) = 100$; $T(L, y) = 20$; $T(x, 0) = 0$; $T(x, K) = 0$

### Profile 2

**Iteration Matrix**

| Iteration = 24 |
|----------------|
| Computing times = 0.0099215 |

The resulting number of iterations 24 times. Second heat distribution profile to show the temperature decreases in the same area or exactly at a node $T_{8.1}$ dan $T_{5.6}$.

When observed more than a second variation of the above limits, namely: 0; 20; 100; 0 and 10; 20; 0; 0 produce the same number of iterations equilibrium which is 24 times. And when the second iteration matrix deductive would generate diagonal matrix with value 0.

| -389.134 | 0 | 218725 | 318364 | 320369 | 218926 |
| -556.536 | -218725 | 0 | 116078 | 145950 | 101043 |
| -618.284 | -318364 | -116078 | 0 | 46311 | 39294 |
| -598.236 | -320369 | -145950 | -46311 | 0 | 9824 |
| -454.290 | -218926 | -101043 | -39294 | -9824 | 0 |

Both matrices show has a positive value on the triangle top and negative on the lower triangle. And both showed the smallest iteration value in the same area.

c) $T(0, y) = 100$; $T(L, y) = 0$; $T(x, 0) = 20$; $T(x, K) = 0$

### Profile 3

**Iteration matrix**

| Iteration = 28 |
|----------------|
| Computing times = 0.14655 |

### Profile 3A

**Iteration Matrix with Q = 10000; k = 40**

| Iteration = 28 |
|----------------|
| Computing times = 0.013056 |

The computing times and iteration number are not significantly different.
The number of iterations resulting from this limit value is 28 times. Profile distribution of heat to show the temperature decreases in the same area or exactly on point $T_{1,1}$

d) $T(0, y) = 100; T(L, y) = 20; T(x, 0) = 0; T(x, K) = 0$

Profile 4
Iteration Matrix
Iteration = 28
Computing times = 0.0096928

|     | 0  | 0   | 0   | 0   | 0   | 0 | 0 | 0 |
|-----|----|-----|-----|-----|-----|---|---|---|
| 20  | 118.284 | 92.743 | 80.834 | 68.484 | 50.922 | 27.426 | 0 | 0 |
| 20  | 180.393 | 171.852 | 162.109 | 142.181 | 107.779 | 58.781 | 0 | 0 |
| 20  | 231.403 | 252.165 | 253.569 | 233.353 | 179.230 | 99.920 | 0 | 0 |
| 20  | 293.184 | 351.804 | 369.647 | 346.431 | 278.890 | 161.688 | 0 | 0 |
| 20  | 389.495 | 492.221 | 526.784 | 506.856 | 428.148 | 267.883 | 0 | 0 |
| 20  | 572.574 | 700.802 | 738.412 | 726.063 | 658.982 | 481.716 | 0 | 0 |
| 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Profile 4A
Matrix iteration with $Q = 10000; k = 40$
Iteration = 28
Computing times = 0.010513

|     | 0  | 0   | 0   | 0   | 0   | 0 | 0 | 0 |
|-----|----|-----|-----|-----|-----|---|---|---|
| 20  | 118.547 | 93.143 | 81.295 | 68.946 | 51.323 | 27.689 | 0 | 0 |
| 20  | 180.794 | 172.482 | 162.842 | 142.914 | 108.408 | 59.182 | 0 | 0 |
| 20  | 231.897 | 252.898 | 253.426 | 231.211 | 179.963 | 100.381 | 0 | 0 |
| 20  | 293.645 | 352.537 | 352.537 | 231.211 | 179.963 | 100.381 | 0 | 0 |
| 20  | 389.896 | 492.851 | 492.851 | 352.537 | 231.211 | 100.381 | 0 | 0 |
| 20  | 572.837 | 701.203 | 701.203 | 492.851 | 352.537 | 100.381 | 0 | 0 |
| 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

The resulting number of iterations 28 times. Second heat distribution profile to show the temperature decreases in the same area or exactly at a node $T_{6,1}$.

When observed more than a second variation of the above limits, namely: 0; 20; 100; 0 and 20; 0; 0; 100 produce the same number of iterations equilibrium which is 28 times. And when the second iteration matrix deductible would generate diagonal matrix with value 0.
Both matrices show a positive value on right side and negative on left side. And both showed the smallest iteration value in the same area.

Results from this study is that the pattern of variation of the boundary or initial conditions of a material with regard conductivity value remains at steady state the exact value of the smallest are in the same iteration value. This indicates that the value of the thermal equilibrium tend to be at the same iteration.

A recommendation for further research is by varying the initial conditions more and include different conductivity.

3. Reference
[1] Alatas, Husen. Buku Pelengkap Fisika Matematika 1. Jakarta.
[2] Hasimi Pane, Ali., 2011, Penyelesaian Numerik Perpindahan Panas Konduksi 2-D Pada Bidang Datar Menggunakan Program MS.Excell dan Engineering Equation Solver, Universitas Sumatra Utara, Medan.
[3] Djojodiharjo, Harijono. 2000. Metode Numerik. Jakarta: PT. Gramedia Pustaka Utama.
[4] Kreyszig, E., 1988, Advance Engineering Mathematics, John Wiley & Sons, Inc.
[5] Holman. 1997. Perpindahan Kalor. Edisi Keenam. Jakarta Erlangga.
[6] Captra, S.C., Canale R.P., 1990, Numerical Methods for Engineering, second edition, McGraw-Hill, New York.
[7] Incropera, F.P., et. al., 1981, Fundamental of Heat Transfer, John Wiley & Sons, Inc.
[8] Kreith, Frank & Arko Prijono. 1986. Prinsip-prinsip Perpindahan Panas. Edisi Ketiga. Jakarta: Erlangga.
[9] James, M.L., et.al., 1993, Applied Numerical Methods for Digital Computation, HarperCollins College Publishers.
[10] Smith, G.D., 1985, Numerical Solution of Partial Differential Equations: Finite Difference Methods, third edition, Oxford University Press.
[11] Setiawan, Agus. 2006. Pengantar Metode Numerik. Yogyakarta: Andi.