An Improved OMP-GA Combined Algorithm for Off-grid DOA Estimation

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Abstract. DOA (Direction of Arrival) is used to determine the direction of the targets and is one of the important technologies in array signal processing. To ensure the accuracy of traditional OMP-based DOA estimation algorithm, the over-complete dictionary needs to contain grids that match the real incident angles. However, for off-grid targets, the estimation accuracy will decrease. In this paper, we propose an improved OMP-based DOA estimation algorithm with genetic algorithm (GA), in which GA is introduced to search incident angle of the off-grid target directly, the subsequent traditional OMP steps are remained and the result got by which is used to filter out the false estimation results that deviate from the true target obtained by GA. Compared with the traditional OMP-based DOA estimation algorithm, this proposed algorithm could achieve higher accuracy. Simulation results verify the effectiveness of this proposed algorithm in the case of low signal-to-noise ratio (SNR) and low number of snapshots.

1. Introduction
DOA estimation could estimate the arrival angles of multiple targets from the received signal, which is used in radar, sonar, communication, etc. [1-2]. Traditional DOA estimation algorithms include MUSIC (Multiple Signal Classification) [3-4] and ESPRIT (Estimating Signal Parameter via Rotational Invariance Techniques) [5], however, since these two subspace-type algorithms are based on the orthogonality of the signal subspace and the noise subspace, high SNR and high number of snapshots are required to ensure the performance of the algorithm [6].

Compressed sensing [7] is a technique for finding the sparse solution of an under-sampling system, in which the sparse recovery algorithms mainly include convex optimization algorithm, greedy algorithm and Bayesian method. As one of the specific implementation algorithms of the greedy algorithm, orthogonal matching pursuit (OMP) algorithm has been widely used in DOA estimation due to its simple structure and fast convergence speed [8-10].

In order to ensure the accuracy of the DOA estimation results based on OMP algorithm, it is necessary to establish an over-complete dictionary containing the target incident angles, which requires a fine grid division [11]. However, fine grid division would cause a strong correlation between adjacent atoms in over-complete dictionary, which is not conducive to problem solving, in the meantime, the computation will increase rapidly. When the grid spacing is not fine enough, the target incident angles might be located between two adjacent grids, which causes off-grid [12-14] problem.

The genetic algorithm (GA) is an optimization algorithm and is proposed according to the evolutionary law of organisms in nature [15-16], which can break through the limitation of region search and perform the global optimization search. In this paper, we propose an OMP algorithm with GA for off-grid targets DOA estimation.
2. Signal model for DOA estimation

Consider an $L$-element uniform linear array located on the X axis as shown in figure 1, whose element spacing is $d$. There are $N$ narrow-band incoherent point targets, whose incident angles are $\theta_i, \theta_l \in [-90°, +90°]$ $i = 1, 2, ..., N$. The snapshot number is $K$, the received signal of the array $\mathbf{X}(t)$ is $L \times K$-dimensional, as shown in equation (1).

$$\mathbf{X}(t) = \mathbf{AS}(t) + \mathbf{N}(t)$$

(1)

$\mathbf{S}(t)$ represents the signal matrix which is $N \times K$-dimensional, $\mathbf{N}(t)$ represents noise matrix which is $L \times K$-dimensional and is assumed to be Gaussian white noise here. $\mathbf{A} = [\mathbf{a}_1(\theta_1), \mathbf{a}_2(\theta_2), ..., \mathbf{a}_n(\theta_n), ..., \mathbf{a}_N(\theta_N)]$, $n = 1, 2, ..., N$ is an $L \times N$-dimensional manifold matrix of array. $\mathbf{a}_n(\theta_n) = [1, e^{-\frac{2\pi}{\lambda} \sin \theta_n}, e^{-\frac{2\pi}{\lambda} \sin \theta_n}, ..., e^{-i\frac{2\pi}{\lambda} \sin \theta_n}]^T$, $l_i = 0, 1, ..., L$, where $\lambda$ is the wavelength.

![Figure 1. DOA estimation array model.](image1)

![Figure 2. Spatial grid division.](image2)

The search grid of $M$ potential incident angles $\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_M$ is generated as shown in figure 2, accordingly an $L \times M$-dimensional over-complete dictionary $\mathbf{D} = [\mathbf{a}_1(\hat{\theta}_1), \mathbf{a}_2(\hat{\theta}_2), ..., \mathbf{a}_m(\hat{\theta}_m), ..., \mathbf{a}_M(\hat{\theta}_M)]$ is constructed, where $\mathbf{a}_m(\hat{\theta}_m) = [1, e^{-\frac{2\pi}{\lambda} \sin \hat{\theta}_m}, ..., e^{-i\frac{2\pi}{\lambda} \sin \hat{\theta}_m}]^T$, $m = 1, 2, ..., M$. $S_1$ is a non-off-grid target and $S_2$ is an off-grid target whose incident angle does not match the grid.

3. OMP-GA combined algorithm

In order to solve the problem that the traditional OMP algorithm can’t obtain the incident angle of off-grid target accurately, this paper proposes an improved OMP-GA combined DOA estimation algorithm.

The step of finding the most matching atoms in the traditional OMP algorithm is replaced by the direct global optimization of the GA to obtain a more accurate result, and the subsequent steps of the traditional OMP algorithm is kept to get a rough estimated result which is close to the real incident angle of the target. Due to the premature convergence of GA, the estimation result obtained by the GA may converge to a wrong angle. Therefore, through the filtering effect of the rough estimation result on the GA estimation result, the false DOA estimation result for every snapshot could be filtered out and the remaining correct estimation angles will be averaged to obtain the final DOA estimation results. The steps of the proposed algorithm are as follows, and the flowchart of the proposed algorithm is shown in figure 3.

- **Step 1:** Initialization. Initial residual $\mathbf{r}^1 = \mathbf{X}(t)$, reconstruction signal $\mathbf{S}^1 = 0$, index set $\Lambda^1 = \emptyset$, atom set $\Theta^1 = \emptyset$, matching angle set $\Theta_{\text{est}}^1 = \emptyset$, iteration counter $n = 1$.

- **Step 2:** Taking $\arg \max_{\hat{m} \in \Theta, m = 1, 2, ..., M} | < \mathbf{a}_m, \mathbf{r}^{n-1} > |$ as the adaptation function. And use GA to obtain the global optimal solution $\mathbf{a}_m$. The angle corresponding to the solution of $\mathbf{a}_m$ in space is $\hat{\theta}_m$, and the footmark of the nearest spatial grid from $\hat{\theta}_m$ is $m$.

- **Step 3:** Update index set $\Lambda^n = \Lambda^{n-1} \cup \{ m \}$, atomic set $\Theta^{n} = [\Theta_{\text{est}}^{n-1} \, \mathbf{a}_m]$ and matching angle set $\Theta_{\text{est}}^{n} = \Theta_{\text{est}}^{n-1} \cup \{ \hat{\theta}_m \}$.

- **Step 4:** Using the least square method to calculate the approximate solution:
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\begin{itemize}
\item \( S^n = (\Theta^n \Theta^n)^{-1} \Theta^n X(t) \).
\item **Step 5**: Update the \( n^{th} \) residual \( r^n = X(t) - \Theta S^n \).
\item **Step 6**: \( n = n + 1 \). Judge whether \( n \) is greater than \( N + 1 \). If it is satisfied, the iteration stops. If it is not satisfied, return to step 1 to continue the loop.
\item **Step 7**: Get the rough estimate angle corresponding to the atom set \( \Theta J^n \). Use the rough estimate angle to filter the matching angle set \( \theta_{est}^n \) to obtain the incident angle of the target.
\end{itemize}

Figure 3. The flow chart of the OMP algorithm.

4. Simulation results
In this section, we set up two experiments to compare the performance of the proposed algorithm and the traditional OMP algorithm under different SNR values and snapshot numbers. The performance of the algorithms is compared through the root mean square error (RMSE), which is defined as follows.

\[
RMSE = \frac{1}{J M} \sum_{m=1}^{M} \sum_{j=1}^{J} (\hat{\theta}_{m,j} - \theta_m)^2
\]  

(2)

Where, \( J \) is the number of Monte Carlo trials, \( M \) is the number of targets, \( \theta_m \) is the real incident angle of the \( m^{th} \) target, and \( \hat{\theta}_{m,j} \) is the estimation result of the \( m^{th} \) target of the \( j^{th} \) experiment. Set the grid spacing of the traditional OMP is 0.1°. The simulations are run on MATLAB 2016a 9.0, the processor is Intel Core i7, and the memory is 8GB. The parameters of the simulations are shown in table 1.
Table 1. Simulation parameters.

| Parameter           | Value               |
|---------------------|---------------------|
| DOAs                | [-20.82°, 50.18°]   |
| SNR values          | 5dB                 |
| Number of elements  | 30                  |
| Number of snapshots | 40                  |
| Element spacing     | Half-wavelength     |
| Number of Monte Carlo trials | 200               |

The simulation results of the proposed OMP-GA algorithm and the traditional OMP algorithm under different SNR values are compared in figure 4, the SNR varies from $-4$ dB to 20 dB, and other conditions remain unchanged.

From figure 4, when SNR is low, the RMSE of the proposed algorithm is much better than traditional OMP algorithm. This is because the traditional OMP algorithm select the most matched atom to the signal-related residual by gradient operation, which tend to false atom selection under low SNR. However, the proposed OMP-GA algorithm uses a global search method to improve the accuracy of atom selection.

When SNR increases, the performance of the two algorithms trends to be consistent. However, due to the existence of the off-grid effect, the accuracy of the traditional OMP algorithm has been always lower than that of the OMP-GA algorithm as the SNR increases.

![Figure 4. RMSE of the DOA estimates versus input SNR.](image)

The simulation results of the proposed OMP-GA algorithm and the traditional OMP algorithm under different snapshot numbers $K$ are compared in figure 5, the $K$ varies from 5 to 50, and other conditions remain unchanged.

From figure 5, when the number of snapshot $K = 5$, the performance of the two algorithms is very close, nevertheless. This is because when approaching the optimal solution, the convergence value of GA is inclined to swing around the optimal point. However, the accuracy of OMP-GA algorithm is still better that the traditional OMP algorithm whose accuracy is limited by the grid division when $K$ is small. As $K$ increases, the difference of the RMSE between the two algorithms is getting larger. That is because OMP-GA could obtain a more accurate result as more estimation results are gathered and the filtering process works better.
From above results, the estimation accuracy the proposed OMP-GA algorithm is better than the traditional OMP algorithm. However, due to the slow iteration speed of the GA algorithm, the computation time of this algorithm will increase dramatically with the increase of snapshots, which is shown in figure 6.

This feature is not beneficial for real-time processing and should be alleviated from the following aspects: (1) By introducing the genetic-disaster algorithm and the immigration algorithm, the problem that GA swings near the optimal solution and converges slowly when it is close to the optimal solution could be solved; (2) The estimation results of the previous snapshots of the received signal could be used as a priori knowledge to generate a good initial group to improve the estimation speed.

5. Conclusion
In order to solve the off-grid problem of traditional OMP-based DOA estimation algorithm, this paper proposes an OMP-GA combined DOA estimation algorithm. On the one hand, GA is introduced to replace the gradual searching process of the matching atom with the global optimization of the incident angle of the target directly. On the other hand, a rough-meshing step of the traditional OMP is retained to filter out estimation results that deviate from the true target value. Simulation results show that the accuracy of the proposed OMP-GA algorithm is better than traditional OMP algorithm in the case of low SNR and low snapshot, but the computation cost of the proposed algorithm will increase rapidly with the number of the snapshot, which should be improved in the future. The proposed algorithm could resolve the off-grid problem of the traditional OMP-based DOA estimation algorithm while a better accuracy could be obtained, which is more useful for targets detection.

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