Numerical methods of Laplace transform inversion in the problem of determination of viscoelastic characteristics of composite materials

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Abstracts.

When designing products made of composite materials intended for operation in difficult conditions of inhomogeneous deformations and temperatures, it is important to consider the viscoelastic properties of the binder and fillers. The article analyzes the relationship between relaxation and creep characteristics. All known creep and relaxation nuclei in the literature are considered. The problem of transformation of creep characteristics into relaxation characteristics and Vice versa is discussed in detail. There is a simple relationship between the creep and relaxation functions in the Laplace image space. However, when returning to the space of the originals, in many cases there are great difficulties in reversing the Laplace transform. Two numerical methods for inverting the Laplace transform are considered: the use of the Fourier series in sine and the method of quadrature formulas. Algorithms and computer programs for realization of these methods are made. It is shown that the operating time of a computer program implementing the Fourier method by sine is almost 2 times less than the operating time of a computer program implementing the quadrature formula method. However, the first method is inferior to the latter method in accuracy of calculations: the relaxation functions and relaxation rates, it is advisable to find the first method, since the computational error is almost indiscernible, and the functions of creep and creep speed, the second way, because for most functions, the result obtained by the second method is much more accurate than the result obtained by the first method.

Keywords: viscoelasticity, relaxation, creep, relaxation and creep nuclei, Laplace transform, Fourier method, quadrature formula method.

1. Introduction

For isotropic viscoelastic materials the defining relations between stress and strain tensors are written as follows

\[ \sigma(t) = G(t)\varepsilon(0) + \int_0^t G(t-\tau) \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau, \]

\[ \varepsilon(t) = J(t)\sigma(0) + \int_0^t J(t-\tau) \frac{\partial \sigma(\tau)}{\partial \tau} d\tau. \]  

where \( G(t) \) and \( J(t) \) are relaxation and creep functions respectively [1-11].

Function \( G(t) \) describes the change in stress over time under constant strain. This process is called stress relaxation [12].

Experiments shows that in this process the stress decreases with time, i.e. the function \( G(t) \) is decreasing and therefore

\[ \frac{dG(t)}{dt} < 0. \]
Function $J(t)$ describes the change in strain over time at constant stress. This process is called creep deformation [12].

Experiments show that in the process of creep at constant stress, the deformation increases, i.e. the function $J(t)$ is increasing and therefore

$$\frac{dJ(t)}{dt} > 0.$$  (3)

2. Analysis of relaxation and creep functions

In elasticity theory, there is a simple relationship between elastic modulus $G$ and malleability $J$ [13-17]

$$GJ = 1.$$  (4)

In the theory of viscoelasticity, there is no such simple connection between relaxation and creep functions. We write the formulas (1) in the image space by Laplace, using the convolution theorem

$$\bar{\sigma}(p) = p\bar{G}(p)\bar{\varepsilon}(p),$$

$$\bar{\varepsilon}(p) = p\bar{J}(p)\bar{\sigma}(p).$$  (5)

From these formulas follows

$$p^2\bar{G}(p)\bar{J}(p) = 1, \quad \bar{J}(p) = \left[p^2\bar{G}(p)\right]^{-1}.$$  (6)

In the Laplace transform theory, the following limit relations take place [2-3]

$$\lim_{t \to 0^+} G(t)J(t) = 1,$$

$$\lim_{t \to \infty} G(t)J(t) = 1.$$  (7)

This means that relations of type (4) in the theory of viscoelasticity take place only in two limiting cases: by $t \to 0^+$ and $t \to \infty$.

3. Analysis of relaxation and creep kernels

Conveniently, the relaxation function $G(t)$ and creep function $J(t)$ to represent in a dimensionless form. For this we denote

$$G(t) = G_0\psi(t),$$

$$J(t) = J_0g(t),$$  (8)

where $[G_n] = \frac{N}{m^2}$, $[J_n] = \frac{m^2}{N}$ and functions $\psi(t)$ and $g(t)$ are dimensionless, function $\psi(t)$ called the relaxation kernel and the function $g(t)$ called the creep kernel.

By virtue of (4) and (7)

$$G_0J_0 = 1,$$

$$\lim_{t \to 0^+} \psi(t)g(t) = 1,$$

$$\lim_{t \to \infty} \psi(t)g(t) = 1.$$  (9)

In the Laplace image space between images of dimensionless functions $\psi(t)$ and $g(t)$ there is a relation of type (6), i.e.

$$p^2\bar{\psi}(p)\bar{g}(p) = 1, \quad \bar{g}(p) = \left[p^2\bar{\psi}(p)\right]^{-1}.$$  (10)
4. Known types of kernels
In the literature [1-12] the following varieties of kernels can be found.

4.1 Maxwell kernels
Maxwell kernels are the simplest of all known kernels in the literature. Table 1 shows Maxwell functions and their images.

| Original | Image |
|----------|-------|
| Relaxation function | $\exp\left[-\left(\frac{t}{\tau}\right)\right]$ |
| Creep function | $1 + \frac{t}{\tau}$ |
| Relaxation rate | $\tau^{-1}\exp\left[-\left(\frac{t}{\tau}\right)\right]$ |
| Creep rate | $\tau^{-1}$ |

4.2 Abel kernels
Table 2 shows Abel functions and their images.

| Original | Image |
|----------|-------|
| Relaxation function | $1 + \sum_{n=1}^{\infty} (-1)^n \Gamma(n\beta) n^{-\beta} \left(\frac{t}{\tau}\right)^n$ |
| Creep function | $1 + \beta^{-1} \left(\frac{t}{\tau}\right)^\beta$ |
| Relaxation rate | $\tau^{-1} \sum_{n=1}^{\infty} (-1)^{n+1} \Gamma(n\beta) n^{-\beta} \left(\frac{t}{\tau}\right)^n$ |
| Creep rate | $\tau^{-\beta} t^{-1}$ |

where $0 < \beta \leq 1$.

Note that when $\beta = 1$ than Abel kernels transformed into Maxwell kernels.

4.3 Rabotnov kernels
Table 3 shows Rabotnov functions and their images.

| Original | Image |
|----------|-------|
| Relaxation function | $1 - \sum_{n=0}^{\infty} (-1)^n \Gamma(n+1) [\gamma(n+1)]^{-\gamma} \left(\frac{t}{\tau}\right)^{\gamma(n+1)}$ |
| Creep function | $1 + \left[\Gamma(1+\gamma)\right]^{-\gamma} \left(\frac{t}{\tau}\right)^\gamma$ |
| Relaxation rate | $\gamma \sum_{n=0}^{\infty} (-1)^n \Gamma(n+1) [\gamma(n+1)]^{-\gamma} \left(\frac{t}{\tau}\right)^{\gamma(n+1)}$ |
| Creep rate | $\gamma \left[\Gamma(1+\gamma)\right]^{-\gamma} \tau^{-\gamma} t^{-1}$ |

where $0 < \gamma \leq 1$.

Note that when $\gamma = 1$ than Rabotnov kernels transformed into Maxwell kernels.
4.4 Rzhanitsyn kernels
Table 4 shows Rzhanitsyn functions and their images.

Table 4. Rzhanitsyn functions and their images

| Original | Images |
|----------|--------|
| Relaxation function | $1 - \gamma(\beta, \frac{t}{\tau})[\Gamma(\beta)]^{-1}$ |
| Creep function | $1 + \sum_{n=1}^{\infty} \gamma(n\beta, \frac{t}{\tau})[\Gamma(n\beta)]^{-1}$ |
| Relaxation rate | $[\Gamma(\beta)]^{-1} \tau^{-\beta} \beta^{-1} \exp\left[-\left(\frac{t}{\tau}\right)^{\beta}\right]$ |
| Creep rate | $\tau^{-1} \exp\left[-\left(\frac{t}{\tau}\right)\sum_{n=1}^{\infty} [\Gamma(n\beta)]^{-1}\left(\frac{t}{\tau}\right)^{n\beta}\right] \quad [(1 + pr)^{\beta} - 1]^{-1}$ |

where $0 < \beta \leq 1$ and $\gamma(\beta, \frac{t}{\tau}) = \int_{0}^{\frac{t}{\tau}} e^{-\xi} \xi^{\beta-1} d\xi$ is an incomplete gamma function [14,15].

Note that when $\beta = 1$ than Rzhanitsyn kernels transformed into Maxwell kernels.

4.5 Kohlrausch kernels
Table 5 shows Kohlrausch functions and their images.

Table 5. Kohlrausch functions and their images

| Original | Images |
|----------|--------|
| Relaxation function | $\exp\left[-\left(\frac{t}{\tau}\right)^{\beta}\right]$ |
| Creep function | $1 + (1 - \beta) \sum_{n=1}^{\infty} \frac{b_n}{n \Gamma[(1 - \beta)n]} \left(\frac{t}{\tau}\right)^{(1-\beta)n}$ |
| Creep function | $(1 - \beta) \tau^{-\beta} \beta^{-1} \exp\left[-\left(\frac{t}{\tau}\right)^{\beta}\right] \sum_{n=1}^{\infty} \frac{b_n}{n \Gamma[(1 - \beta)n]} \left(\frac{t}{\tau}\right)^{(1-\beta)n}$ |

where $0 < \beta \leq 1$, $b_0 = 1$, $b_n = \sum_{k=1}^{n} (-1)^{k-1} \frac{k}{\Gamma[(k-1)\beta]} \frac{\Gamma(1-\beta)k}{(k-1)!} b_{n-k}$, $n = 1, 2, ..., \infty$.

Note that when $\beta = 0$ than Kohlrausch kernels transformed into Maxwell kernels.

4.6 Gavrillac-Negami kernels
Table 6 shows Gavrillac-Negami functions and their images.

Table 6. Gavrillac-Negami functions and their images

| Original | Images |
|----------|--------|
| Relaxation function | $\sum_{n=1}^{\infty} (-1)^{n} \frac{\Gamma(n + \beta)}{\alpha(n + \beta)n! \Gamma[\alpha(n + \beta)] \Gamma(\beta)} \left(\frac{t}{\tau}\right)^{(\alpha(n + \beta))} \quad p^{-1}\left[1 - \left(1 + (pr)^{\beta}\right)^{\alpha(n + \beta)}\right]$ |
| Creep function | $\sum_{n=1}^{\infty} (-1)^{n} \frac{D_n}{\Gamma[\alpha(n\beta + m) + 1]} \left(\frac{t}{\tau}\right)^{(\alpha(n\beta + m))} \quad p^{-1}\left[1 - [(pr)^{\alpha} - 1]^{-1} \sum_{n=1}^{\infty} \frac{D_n}{(pr)^{\alpha n}}\right]$ |
Relaxation rate
\[ r^{-1}(f/\tau)^{\alpha}\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{\Gamma(n+\beta)}{\Gamma(\alpha(n+\beta))} \left(\frac{f}{\tau}\right)^{\alpha n} \left[1+\left(p\tau\right)^{\alpha}\right]^{-\beta} \]

Creep rate
\[ r^{-1}\sum_{n=0}^{\infty} (-1)^n \frac{D_m}{\Gamma(\alpha(n+\beta)+m)} \left(\frac{f}{\tau}\right)^{\alpha(n+\beta)} \left[(1+p\tau)^{\alpha}-1\right]^{-1} \sum_{m=0}^{\infty} (-1)^m \frac{D_m}{(p\tau)^{\alpha m}} \]

where \(0 < \alpha \leq 1, \ 0 < \beta \leq 1, \ D_m = \sum_{k_1=0}^{\gamma} \sum_{k_2=0}^{\gamma} \cdots \sum_{k_{s-1}=0}^{\gamma} \prod_{j=1}^{s-1} a(k_j - k_{j-1}), \ a(n) = \frac{1}{n!} \frac{\Gamma(n+\beta)}{\Gamma(\beta)}, \ k_0 = m, \ k_s = 0.\]

Note that when \(\alpha = 1\) than Gavrillac-Negami kernels transformed into Rzhanitsyn kernels and when \(\alpha = \beta = 1\) than they transformed into Maxwell kernels.

The absence of dimension in all the above functions is due to the parameter \(\tau\), i.e. the relaxation time. The relaxation time is the period of time during which the amplitude value of the disturbance in the unbalanced physical system decreases by a factor of \(e\) times (\(e\) — the base of the natural logarithm) [11-19].

5. Numerical methods for inverting the Laplace transform
In most cases, finding the original function as an analytical function is impossible or, from a practical point of view, impractical. That is why approximate and numerical methods for inverting the Laplace transform have been developed. The next two methods will be considered [20].

5.1 Using Fourier series on sine (1st method)
The method is taken from [20], pp.52-54. Using it, the original function can be written as follows:
\[ f(t) = \sum_{k=0}^{\infty} c_k \sin\left((2k+1)\arccos(e^{-\sigma t})\right). \] (11)

5.2 Method of quadrature formulas with equal coefficients (2nd method)
The method is taken from [20], pp.121-124. According to this method, the original function will take the following form:
\[ f(t) \approx \frac{i^{s-1}}{n\Gamma(s)} \sum_{k=1}^{n} \phi \left(\frac{p_k}{t}\right) \] (12)

6. Results of calculation
Computer programs were written to implement the methods described above. The figures with odd numbers show the graphs obtained for the 1st method, the figures with even numbers – for the 2nd method. In all the above cases, the parameters \(\tau = 10, \ \sigma = 0.01, \ \beta=0.5, \ h = 2, \ s = 1\). Figures 1-2 show the results of the programs that define the Maxwell relaxation function, which is compared with its analytical expression \(\psi(t) = \exp\left[-\left(\frac{f}{\tau}\right)\right]\).

**Figure 1.** Maxwell relaxation function by \(n = 20\), \(\Delta\psi_{cp} \approx 9.35 \times 10^{-6}\)

**Figure 2.** Maxwell relaxation function by \(n = 100\), \(\Delta\psi_{cp} \approx 0.0037\)
Figures 3-4 show the results of the programs that define the Maxwell creep function, which is compared with its analytical expression $g(t) = 1 + \frac{t}{\tau}$.

**Figure 3.** Maxwell creep function by $n = 20$, $\Delta g_{1cp} \approx 0.0983$

Figures 5-6 show the results of the programs that determine the Maxwell relaxation rate, which is compared with its analytical expression $\psi(t) = \tau^{-1} \exp\left[-\left(\frac{t}{\tau}\right)\right]$.

**Figure 5.** Maxwell relaxation rate by $n = 20$, $\Delta \psi_{1cp} \approx 9.35 \times 10^{-7}$

**Figure 6.** Maxwell relaxation rate by $n = 100$, $\Delta \psi_{1cp} \approx 0.0002$

Figures 7-8 show the results of programs that determine the Maxwell creep rate, which is compared with its analytical expression $g_r(t) = \tau^{-1}$.

**Figure 7.** Maxwell creep rate by $n = 20$, $\Delta g_{1cp} \approx 1.04 \times 10^{-6}$

**Figure 8.** Maxwell creep rate by $n = 100$, $\Delta g_{1cp} \approx 5.88 \times 10^{-16}$

Figures 9-10 show the results of the programs that define the Abel relaxation function, which is compared with its analytical expression $\psi(t) = 1 + \sum_{j=1}^{200} \left[-\frac{\Gamma(\beta)}{\beta} \frac{[i \beta \Gamma(i \beta)]^{-1}}{\left(\frac{t}{\tau}\right)^\beta}\right]$.
Figure 9. Abel relaxation function by $n = 20$, $\Delta \psi_{cp} \approx 0.0031$

Figures 11-12 show the results of the programs that define the Abel creep function, which is compared with its analytical expression $g(t) = 1 + \beta^{-1}t^{\beta}e^{-t^{\beta}}$.

Figure 11. Abel creep function by $n = 20$, $\Delta g_{cp} \approx 0.041$

Figures 13-14 show the results of programs that determine the Abel relaxation rate, which is compared with its analytical expression $\psi_i(t) = t^{\beta} \sum_{i=1}^{\infty} (-1)^{i+1} \left[ \Gamma(i\beta) \right] \left[ \Gamma(i\beta) \right]^{-1} \left( \frac{t}{\tau} \right)^{\beta}$.

Figure 13. Abel relaxation rate by $n = 20$, $\Delta \psi_{p} \approx 0.0026$

Figures 15-16 show the results of programs that determine the Abel creep rate, which is compared with its analytical expression $g_i(t) = \tau^{-\beta}t^{\beta-1}$.

Figure 15. Abel creep rate by $n = 100$, $\Delta g_{p} \approx 0.0184$

Figure 16. Abel creep rate by $n = 20$, $\Delta g_{p} \approx 0.0072$
Figures 17-18 show the results of the programs that determine the Rabotnov relaxation function, which is compared with its analytical expression

\[ \psi(t) = 1 - \sum_{i=0}^{200} \frac{(-1)^i}{\beta(i+1) \Gamma(\beta(i+1))} \left( \frac{t}{\tau} \right)^{\beta(i+1)}. \]

**Figure 17.** Rabotnov relaxation function by \( n = 20, \ \Delta \psi_{\text{cp}} \approx 0.0021 \)

**Figure 18.** Rabotnov relaxation function by \( n = 100, \ \Delta \psi_{\text{cp}} \approx 0.002 \)

Figures 19-20 show the results of the work of programs that determine the Rabotnov creep function, which is compared with its analytical expression

\[ g(t) = 1 + \left[ \Gamma(1 + \beta) \right]^{-1} t^\beta \tau^{-\beta}. \]

**Figures 19.** Rabotnov creep function by \( n = 20, \ \Delta g_{\text{cp}} \approx 0.0397 \)

**Figure 20.** Rabotnov creep function by \( n = 100, \ \Delta g_{\text{cp}} \approx 0.0039 \)

Figures 21-22 show the results of the programs that determine the Rabotnov relaxation rate, which is compared with its analytical expression

\[ \psi'(t) = \int \sum_{i=0}^{200} \frac{(-1)^i}{\Gamma(\beta(i+1))} \left( \frac{t}{\tau} \right)^{\beta(i+1)}. \]

**Figure 21.** Rabotnov relaxation rate by \( n = 20, \ \Delta \psi'_{\text{cp}} \approx 0.0028 \)

**Figure 22.** Rabotnov relaxation rate by \( n = 100, \ \Delta \psi'_{\text{cp}} \approx 0.0044 \)

Figures 23-24 show the results of the programs that determine the Rabotnov creep rate, which is compared with its analytical expression

\[ g'(t) = \beta \left[ \Gamma(1 + \beta) \right]^{-1} t^{-\beta-1}. \]
Figures 23-26 show the results of the programs that determine the Rzhanitsyn relaxation function, which is compared with its analytical expression

\[ \psi(t) = 1 - \left[ \Gamma(\beta) \right]^{-1} \sum_{i=0}^{200} \frac{(-1)^i}{i!} (i + \beta)^{-1} \left( \frac{t}{\tau} \right)^{i+\beta}. \]

Figures 27-28 show the results of the programs that determine the Rzhanitsyn creep function, which is compared with its analytical expression

\[ g(t) = 1 + \sum_{i=1}^{N} \gamma \left( \beta, \frac{t}{\tau} \right) \left[ \Gamma(i \beta) \right]^{-1}, \]where \( \gamma(\beta, t/\tau) = \int_{0}^{t/\tau} \xi^{\beta-1} e^{-\xi} d\xi \) is an incomplete gamma function.

Figures 29-30 show the results of the programs that determine the Rzhanitsyn relaxation rate, which is compared with its analytical expression

\[ \psi_1(t) = \left[ \Gamma(\beta) \right]^{-1} \tau^{-\beta} t^{\beta-1} \exp \left[ - \left( \frac{t}{\tau} \right) \right]. \]
Figures 29-30 show the results of programs that determine the Rzhanitsyn creep rate, which is compared with its analytical expression
\[ g_i(t) = t^{-\beta} \exp \left[ -\left( \frac{t}{\tau} \right) \right] \sum_{j=1}^{100} \left[ \Gamma(i\beta) \right]^{-1} \left( \frac{t}{\tau} \right)^{i\beta}. \]

Figures 31-32 show the results of programs that determine the Rzhanitsyn creep rate, which is compared with its analytical expression
\[ \psi(t) = \exp \left[ -\left( \frac{t}{\tau} \right)^{1-\beta}. \right] \]

Figures 33-34 show the results of programs that determine the Kohlrausch relaxation function, which is compared with its analytical expression
\[ \psi(t) = \exp \left[ -\left( \frac{t}{\tau} \right)^{1-\beta}. \right] \]

Figures 35-36 show the results of programs that determine the Kohlrausch creep function, which is compared with its analytical expression
\[ g(t) = 1 + (1-\beta)^{-1} \sum_{i=1}^{100} \frac{b_i}{\Gamma(1-\beta)i} \left( \frac{t}{\tau} \right)^{i(1-\beta)}, \]
where
\[ b_0 = 1, \quad b_i = \sum_{k=1}^{i} (-1)^{i-k} \frac{\Gamma[(1-\beta)k]}{(k-1)!} b_{i-k}, \quad i = 1, 100.\]
Figures 37-38 show the results of programs that determine the Kohlrausch relaxation rate, which is compared with its analytical expression

\[ \psi(t) = (1 - \beta)^{-\tau^{\beta-1}} e^{\beta t} \exp\left(-\left(t/\tau\right)^{1-\beta}\right). \]

Figures 39-40 show the results of programs that determine the Kohlrausch creep rate, which is compared with its analytical expression

\[ g(t) = \tau^{\beta-1} \tau^{-\beta} + \frac{\sum_{i=1}^{100} b_i}{\Gamma[(1-\beta)i]} \left(t/\tau\right)^{1-\beta}, \]

where

\[ b_0 = 1, \quad b_i = \sum_{k=1}^{i} (-1)^{i-k} \frac{\Gamma[(1-\beta)k]}{(k-1)!} b_{i-k}, \quad i = 1, 100. \]

Figures 41-42 show the results of programs that determine the Gavrillic-Negami relaxation function, which is compared with its analytical expression

\[ \psi(t) = \frac{1 - \sum_{i=0}^{100} (-1)^i \frac{\Gamma(i + \beta)}{\alpha(i + \beta)! \Gamma[\alpha(i + \beta)] \Gamma(\beta)} \left(t/\tau\right)^{\alpha(i + \beta)}}{1 - \frac{\Gamma(1 + \beta)}{\alpha(1 + \beta)! \Gamma(\beta)} \left(t/\tau\right)^{\alpha(1 + \beta)}}. \]
Figures 43-44 show the results of programs that determine the Gavrillac-Negami relaxation rate, which is compared with its analytical expression

\[ \psi_0(t) = t^{-1} \left[ \sum_{i=0}^{N} \frac{(-1)^i}{i!} \Gamma(i+\beta) \right] \left( \frac{t}{\tau} \right)^{\alpha(i-\beta)}. \]

Figure 45 shows the result of the program that determines the Gavrillac-Negami creep function

\[ g(t) = 1 + \sum_{i=1}^{N} \sum_{m=0}^{\infty} (-1)^m \frac{D_m}{\Gamma(\alpha(i+\beta)+m+1)} \left( \frac{t}{\tau} \right)^{\alpha(i+\beta+m)}, \]

Figure 46 shows the result of the program that determines the Gavrillac-Negami creep rate

\[ g_0(t) = t^{-1} \sum_{i=1}^{N} \sum_{m=0}^{\infty} (-1)^m \frac{D_m}{\Gamma(\alpha(i+\beta)+m)} \left( \frac{t}{\tau} \right)^{\alpha(i+\beta+m)} \] by using the 1st and 2nd methods, where

\[ D_m = \sum_{k_1=0}^{m} \cdots \sum_{k_{i-1}=0}^{m-k_{i-2}} \prod_{j=0}^{i-2} a(k_j-k_{j+1}), \quad a(i) = \frac{1}{i!} \frac{\Gamma(i+\beta)}{\Gamma(\beta)}, \quad k_0 = m, \quad k_i = 0. \]

For the 1st method \( N = 20 \), for the 2nd method \( N = 100 \).

Figures 45-46 lack an analytical expression for the creep function and creep rate. These functions could not be constructed due to a rather complex recursive formula for the coefficients \( D_m \).
7. Conclusions
After analyzing the results, we can draw the following conclusions:
1) the operating time of a computer program that implements the Fourier sine method (I method) is almost 2 times less than the operating time of a computer program that implements the method of quadrature formulas with equal coefficients (II method);
2) however I method II method is inferior in accuracy calculations: the relaxation functions and relaxation rates, it is advisable to find a I fashion, as the accuracy is almost indistinguishable, and the functions of creep and creep speed – II method, as for most such functions, the result obtained with the II method is much more accurate than the result obtained I method;
3) the Gavrillac-Negami creep functions and creep rate defined in Table 6 could not be constructed due to a rather complex recursive formula for the coefficients $D_m$, however, using both methods, these functions can still be obtained and compared with each other.

For all cases the number of terms of the Fourier series $n = 20$. This choice is due to the fact that with growth $n$ in such cases (except for Maxwell kernels) the values of the coefficients $c_k$ growing rapidly with the growth $k$, because of this, the Fourier series will begin to oscillate very strongly. Therefore, the I method should be used for simpler cases.

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