Observability Analysis for Horizontal Axis Wind Turbines using Empirical Gramian Matrices

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Abstract. This contribution deals with the state observability and parameter identifiability analysis for nonlinear wind turbine control (WTC) systems based on empirical observability Gramian (EOG) matrices. The concepts of observability and Gramian matrices are introduced to investigate the inverse condition number (ICN) and the singular value decomposition (SVD) of the EOG matrices and to evaluate different sensor configurations with respect to their degree of observability. The obtained results are then reviewed for practical plausibility over state estimates, provided by sigma-point Kalman filters (SPKF), in order to relate the observability measures directly to the expectable estimation error. The investigation reveals indeed a correlation between the employed measures for individual state observability and the estimates produced by the nonlinear filters. The joint assessment of ICN, SVD and SPKF results is found as a strong tool when aiming at a holistic and practically relevant observability discussion.

1. Introduction

Wind energy plays an important role in the transition from fossil to renewable energy resources. It is available free-of-charge and almost unlimited. Today, horizontal axis wind turbines (HAWT) with three rotor blades are the most promising concept to harvest this large potential. In order to gain more electrical power from the wind, the dimensions have continuously grown over the last thirty years with the main objective to lower the levelized costs of energy (LCoE) [1]. Due to still strong competition from the conventional energy sectors, the LCoE must drop further for wind energy to prevail. One possibility to do so, is to improve energy yield and component life-time by advanced model-based state-space controllers [2, 3, 4]. These require the complete state information and also the knowledge of relevant model parameters to outperform standard industrial control schemes (cf. [5]).

Therefore, previous work has already focused in detail on the state estimation problem for wind turbines [6] and how to tackle practical issues like real-time capabilities and to explore benefits of advanced sensor configurations [7]. Moreover, noise adaptive filter algorithms have been discussed to cope with time-varying noise statistics and to prevent poor estimation performance [8].

On the contrary, this contribution puts emphasis on the analysis of state observability and parameter identifiability which is conducted using the empirical Gramian approach [9, 10]. For that purpose, the remainder of this paper is structured as follows: Section 2 will briefly explain
the used design model and the realistic simulation environment. Afterwards, Sec. 3 elaborates on the definitions of observability, the theorems and the criteria applied. Sec. 4 will condense the relevant outcomes of the conducted research and present illustrative simulation results. Finally, Sec. 5 will sum up with the conclusions and provide a glimpse on future work.

2. Control-oriented modeling and wind turbine simulation

Control-oriented modeling requires a significant level of abstraction of the real system. Abstraction means that only the relevant dynamics are considered for the problem to be solved. Inevitably, this deliberate simplification introduces modeling errors which the practitioner must be aware of. Control-oriented models are widely used for control design, simulation purposes, state estimation and also for observability analysis. This section introduces the simplified HAWT design model and the realistic simulation model used as reference.

2.1. Design model equations and parameter set

The design models are an indispensable part of internal model-based control schemes or state estimators. The goal-oriented choice of model granularity and system order has quite an importance for real-time applications due to computational requirements.

The nonlinear system considered in this publication is derived from the conservation of angular and linear momentum which indicates that mainly the mechanical dynamics are decisive for WTC. Applied to the wind turbine’s drive-train, this yields

\[ \ddot{\varphi}_g = \frac{\vartheta}{2(\Theta_r + \Theta_g)} R^3 C_M(\lambda, \beta)(v_w - \dot{x}_T)^2 - \frac{i_{gb}}{\Theta_r + \Theta_g} i_{gb} M_g \quad \text{with} \quad \lambda = \frac{\dot{\varphi}_g R}{v_w - \dot{x}_T} \]  

(1a)

where \( \lambda \) is the so called tip-speed-ratio. The second dynamic equation is obtained from the conservation of linear momentum for the nacelle in axial direction. It reads

\[ \ddot{x}_T = \frac{\vartheta}{2 m_T} R^2 C_T(\lambda, \beta)(v_w - \dot{x}_T)^2 - 2\zeta_T \omega_0 \dot{x}_T - \omega_0^2 x_T . \]  

(1b)

Additionally, the instantaneous rotor power \( P_a \) and the electrical generator power \( P_g \) denote

\[ P_a = \frac{\vartheta}{2} R^2 C_P(\lambda, \beta)(v_w - \dot{x}_T)^3 \]  

(2)

\[ P_g = \eta_g i_{gb} \dot{\varphi}_g M_g \]  

(3)

where \( C_P = \lambda C_M \) is the aerodynamic power coefficient. Fig. 1 shows the illustrative contour plots of the aerodynamic characteristics for the widely used 5-MW reference turbine [11] of the NREL (National Renewable Energy Laboratory). Moreover, a complete parameter set for the same turbine is provided in Tab. 1.

Besides the system dynamics, also the available sensors must be described in an output model. Speaking of the state-of-the-art sensor instrumentation, there is first the precise generator speed measurement \( y_1 = \frac{60}{\pi} i_{gb} \dot{\varphi}_g \), then the nacelle acceleration \( y_2 = \ddot{x}_T \) provided by an inertial measurement unit and the generator power measurement \( y_3 = P_g \). The mechanical system states are the generator angular speed \( x_1 = \dot{\varphi}_g \), the nacelle velocity \( x_2 = \dot{x}_T \) and position \( x_3 = x_T \). The main disturbance input is the rotor effective wind speed \( d_1 = v_w \). As unknown and/or time-varying parameters are considered the tower eigenfrequency \( p_1 = \omega_0 \) and the air mass density \( p_2 = \vartheta \). These states, disturbance inputs and parameters are the ones to be estimated/identified in practise. The control inputs are the generator torque \( u_1 = M_g \) and the collective blade pitch angle \( u_2 = \beta \).
Therein, the state vector \( p \) model parameters in Fig. 2. Both simulation models have been run simultaneously with the same wind scenario of modeling errors due to deliberate simplification, comparative simulation results are shown for yaw. Another simulation model is given by the design model in Eqs. (1). To evaluate the effects of fore-aft and side-side motion, blade flap and edge motion, generator and drive-train, nacelle contribution, the FAST8 [12] simulator as well-known and freely available reference is deployed. A simulator is needed to evaluate the performance prior to application in the field. In this order to test new control schemes and/or observers under preferably realistic conditions, a simulator is needed to evaluate the performance prior to application in the field. In this contribution, the FAST8 [12] simulator as well-known and freely available reference is deployed as detailed aero-elastic simulation model, considering the following degrees of freedom: Tower fore-aft and side-side motion, blade flap and edge motion, generator and drive-train, nacelle yaw. Another simulation model is given by the design model in Eqs. (1). To evaluate the effects of modeling errors due to deliberate simplification, comparative simulation results are shown in Fig. 2. Both simulation models have been run simultaneously with the same wind scenario of modeling errors due to deliberate simplification, comparative simulation results are shown in Fig. 2. Both simulation models have been run simultaneously with the same wind scenario.

With the above definitions, the nonlinear wind turbine system can be brought into a generalized state-space representation that denotes as follows:

\[
\begin{align*}
\dot{x} &= f(x, u, d, p) \\
y &= h(x, u, d, p) .
\end{align*}
\]

Therein, the state vector \( x \), the control inputs \( u \), the disturbance inputs \( d \) and the unknown model parameters \( p \) are introduced. The output vector \( y \) contains all measurement variables considered for state estimation. The detailed sensor configurations used for observability analysis are presented in Sec. 4.1.

2.2. Realistic simulation environment

In order to test new control schemes and/or observers under preferably realistic conditions, a simulator is needed to evaluate the performance prior to application in the field. In this contribution, the FAST8 [12] simulator as well-known and freely available reference is deployed as detailed aero-elastic simulation model, considering the following degrees of freedom: Tower fore-aft and side-side motion, blade flap and edge motion, generator and drive-train, nacelle yaw. Another simulation model is given by the design model in Eqs. (1). To evaluate the effects of modeling errors due to deliberate simplification, comparative simulation results are shown in Fig. 2. Both simulation models have been run simultaneously with the same wind scenario.

| \( \zeta_{x} \) | tower/nacelle modal damping | 0.01 | - |
| \( \eta_{h} \) | generator efficiency | 0.944 | - |
| \( \Theta_{e} \) | rotor inertia (low speed side) | \( 3.56 \times 10^{7} \) | kg m² |
| \( \Theta_{g} \) | generator high-speed inertia | 534 | kg m² |
| \( \varrho \) | air mass density | 1.225 | kg/m³ |
| \( \omega_{0} \) | tower/nacelle eigenfrequency | 2.06 | rad/s |
| \( C_{p}^{*} \) | optimum power coefficient | 0.47 | - |
| \( r_{gb} \) | drive-train gear-box ratio | 97 | - |
| \( m_{t} \) | equivalent tower top mass | \( 450 \times 10^{3} \) | kg |
| \( R \) | blade tip radius | 63 | m |
3. Observability definition, measures and problems
The observability analysis deals with the investigation of a given dynamic system whether it is possible to reconstruct the system states \( x(t) \) only from the knowledge of control inputs \( u(t) \) and system outputs \( y(t) \). Such an analysis is one fundamental step prior to estimator design. For linear systems, the Gramian matrix and the Kalman criterion are well-known concepts to perform this evaluation [13, 14, 15]. However, these are not applicable for nonlinear systems without a preceding linearization step.

The analytical investigation for nonlinear control systems is often more demanding, in particular for higher number of states, cf. [16, 17, 18]. Sometimes, it might be a valid approach for low-order systems, as briefly highlighted for wind speed estimation in [7]. Due to this complexity of the analytical procedure, a simplified but still practical approach is desirable.

3.1. Observability definitions
Before measures to assess observability are introduced, one needs to define what is actually meant by saying that a dynamic system is observable. A generally accepted definition of observability reads as follows [18, 19]:

**Definition 3.1 (System Observability) A system is called fully (state) observable if its initial state \( x_0 = x(t_0) \) can be uniquely determined over a finite interval \([t_0, t_e]\) with given input \( u_{[t_0, t_e]} \) and output trajectories \( y_{[t_0, t_e]} \). Otherwise, the system is said to be unobservable.**

Though, some systems may be unobservable but single states are not (which depends on the chosen measurement configuration). Thus, besides Def. 3.1 the definition of observability for single states is useful to distinguish between unobservable and observable states:

**Definition 3.2 (State Observability) A dynamic state \( x_i(t) \) with \( i \in [1, 2, \ldots, n] \), where \( n \) is the number of states, is called observable if and only if its initial value \( x_i(t_0) \) can be exactly...**
determined over a finite time interval \( t \in [t_0, t_e] \) solely from the input trajectories \( u_{[t_0, t_e]} \) and measurement outputs \( y_{[t_0, t_e]} \).

Hence, if the single state of interest is not observable for a given output configuration, then either the model of the system or the sensor configuration must be reformulated.

For nonlinear systems, the following definition of local observability is given in [18]:

**Definition 3.3 (Local Observability of Nonlinear Systems)** A nonlinear system with control is given by

\[
\dot{x} = f(x, u) \quad \text{with} \quad x(t_0) = x_0 \\
y = h(x, u)
\]

where \( x \in D_x \subseteq \mathbb{R}^n \) and \( u \in C_u \subseteq \mathbb{R}^m \) be defined and \( y \in \mathbb{R}^q \) holds. If every initial state \( x_0 \in D_x \) in the neighbourhood \( U \) of a point \( x_p \in D_x \), with \( U = \{ x_0 \in \mathbb{R}^n : ||x_0 - x_p|| < \epsilon \} \), can be reconstructed uniquely from the knowledge of \( u \) and \( y \) for a time interval \( [t_0, t_e < \infty] \) for all \( u \in C_u \), then the system is locally observable for all \( x_p \in D_x \).

Thus, according to Def. 3.3 it is possible that the observability of a nonlinear system depends on the input signal and its excitation characteristics.

### 3.2. Observability Gramian matrices

The Gramian matrices (GM) are used here as practical criteria to assess observability. The analytical approach is described initially, followed by an introduction to the empirical observability Gramians (EOG). Consider at first the following theorem:

**Theorem 3.1 (Linear Gramian Observability Matrix)** The linear time-variant (LTV) dynamic system

\[
\dot{x} = A(t)x + B(t)u \quad \text{with} \quad x(t_0) = x_0 \\
y = C(t)x + D(t)u
\]

is said to be locally observable over a finite time interval \( t \in [t_0, t_e] \) if and only if the analytical observability Gramian matrix

\[
G_{\text{lin}} = \int_{t_0}^{t_e} \Phi^T(t, t_0)C^T(t)C(t)\Phi(t, t_0)dt
\]

is invertible. Otherwise, the system is said to be unobservable.

Therein, \( \Phi(t, t_0) \) represents the state transition matrix as discussed in [14]. Generally speaking, the Gramian matrix \( G_{\text{lin}} \) represents an energy-based measure for observability since it characterizes the generalized transfer of energy

\[
E_{\text{meas}} = \int_{t_0}^{t_e} y^T(t)y(t)dt = x_0^T G_{\text{lin}} x_0
\]

from the initial state \( x_0 \) to the measurement outputs \( y(t) \) [14, 15, 20]. Hence, the GM influences directly which states contribute to the total measurement energy and which not. States that do not contribute significantly to the measurement energy are poorly observable. A drawback of these linear observability GM is their limitation to linear (or at least linearizable) systems.

Fortunately, a more favourable aspect is that the analytical approach from Theo. 3.1 has been extended to nonlinear systems [9, 21, 10]. Moreover, Krener & Ide [17] and Powel & Morgansen [22] proposed the EOG approach as a tool to assess the (local) observability of nonlinear systems with control. This concept avoids the linearization procedure and builds upon nonlinear system simulation only. As an introduction to empirical Gramians, consider the following theorem:
Theorem 3.2 (Empirical Gramian Observability Matrix) The nonlinear controlled system described by
\[
\dot{x} = f(x,u) \quad \text{with} \quad x(t_0) = x_0
\]
\[
y = h(x,u)
\]
is said to be locally observable over a finite time interval \(t \in [t_0, t_e]\) if and only if the empirical observability Gramian matrix
\[
G_O = \sum_{l=1}^{r} \sum_{m=1}^{s} \frac{1}{rsc_m} \int_{t_0}^{t_e} T_l \Psi_{lm}(t) T_l^T dt
\]
has full matrix rank and is therefore invertible. Otherwise, the system is denoted as (locally) unobservable and not all states are uniquely observable at the same time.

Generally speaking, the EOG from Eq. (8) can be interpreted as the sum of output covariance matrices computed for a set of different initial conditions [21]. The reason is that the components \(ij\) of the correlation matrices \(\Psi_{lm} \in \mathbb{R}^{n \times n}\) are computed by
\[
\Psi_{lmij}(t) = (y_{lmi}(t) - y_N(t))^T (y_{lmj}(t) - y_N(t))
\]
where \(y_{lmi}(t)\) is the solution of (7) to an initial condition (perturbed state)
\[
x_{lmi,0} = c_m T_l e_i + x_{N,0}
\]
and an input trajectory \(u_N(t)\). \(y_N(t)\) represents the nominal output and \(x_{N,0} = x_N(t=t_0)\) the initial state of the nominal state trajectory. To determine the set of perturbed states in Eq. (10), a set \(T\) of unitary rotation matrices, a set \(M\) of different perturbation sizes (weights), and a set \(E\) of basis vectors are defined as follows:
\[
T = \{T_1, ..., T_r : T_l \in \mathbb{R}^{n \times n}, T_l^T T_l = I, \quad l = 1, ..., r\}
\]
\[
M = \{c_1, ..., c_s : c_m \in \mathbb{R}, c_m > 0, \quad m = 1, ..., s\}
\]
\[
E = \{e_1, ..., e_n : e_i \in \mathbb{R}^n, \quad i = 1, ..., n\}
\]
With this sets, a region in the state space around the point of interest (including distance and state perturbation direction) can be determined suitably. The above definition of the empirical Gramians Eq. (8) reduces to Eq. (5) in case of linear systems [9, 21].

3.3. Observability measures based on empirical Gramian matrices
Once the EOG matrix \(G_O\) is computed for a given time interval, the observability study is then conducted by analysing the rank, the condition number [17] and the singular values \(\sigma_i\) as well as the singular vectors \(v_i\). In particular, the inverse condition number (ICN) defined by
\[
\text{ICN}(G_O) = \text{cond}^{-1}(G_O) = \frac{\sigma_n(G_O)}{\sigma_1(G_O)} \in [0, 1]
\]
is employed as a measure for the degree of observability of the system, whereby a value closer to 1 indicates a higher observability. \(\sigma_n\) is the smallest singular value and \(\sigma_1\) the largest one.

Taking Eq. (6) as starting point, the energy transfer from the state to the output can be expressed in terms of the singular value decomposition [23] of the EOG as follows:
\[
E_{\text{meas}} = x_0^T G_O x_0 = x_0^T U \Sigma V^T x_0.
\]
Thus, each singular value in $\Sigma$ weights the contribution of each state $x_i$ to the energy function according to observability directions contained in the unitary matrices $U$ or $V$. Since the singular vectors are in general not unit vectors, a direct assignment of each singular value to a state is not readily possible. Though, if only the largest component of each singular vector is assumed to determine the observability direction in state space (and the other ones are omitted), this gives a simplified metric to assess the state observability based on the EOG’s singular values.

4. Simulation results
This section presents the results obtained for an illustrative application example that is relevant for investigation of state-feedback based closed-loop WTC.

4.1. Test scenario and sensor configurations
As realistic test scenario a turbulent wind field with an average wind speed of 12 m/s is chosen as representative scenario, shown earlier in Fig. 2. The turbulent wind scenario is also necessary to be able to investigate the identifiability of critical model parameters since a persistent excitation is necessary. The FAST8 simulation data is used for control inputs $u(t)$ and measurement outputs $y(t)$ and the simplified design model is used as internal model for prediction of state perturbation. In order to use the above model (1) for empirical observability and identifiability analysis, the augmented state vector is defined as:

$$x_a = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_g \\ x_T \\ x_T \\ v_w \\ \omega_0 \\ \rho \end{bmatrix} = \begin{bmatrix} x \\ d \\ p \end{bmatrix}. \quad (14)$$

With this trick, it is possible to investigate state and disturbance observability as well as parameter identifiability with EOG approach simultaneously. The following sensor configurations $y_{(j)}$ with $j \in [1, 2, \ldots, N_{sc}]$ are considered for investigation:

- $y_{(1)} = [x_1, x_2, x_3, x_4, x_5, x_6]^T$
- $y_{(2)} = [x_1, x_2, x_5, x_6]^T$
- $y_{(3)} = [x_1, x_2, x_3, x_4]^T$
- $y_{(4)} = [x_1, x_2, x_3]^T$
- $y_{(5)} = [x_1, x_2, x_4]^T$
- $y_{(6)} = [x_1, x_2, x_6]^T$
- $y_{(7)} = [x_1, x_2]^T$
- $y_{(8)} = [x_1, x_6]^T$
- $y_{(9)} = [x_1]^T$

The generator speed $\dot{\phi}_g = x_1$ is assumed to be known in any configuration because of today’s redundant high precision measurement. Provided that all states are in principle measurable/known, the best possible configuration for state estimation is $y_{(1)}$. This is obviously rarely the case. $y_{(9)}$ represents the least observable sensor configuration. Hence, these two configurations determine the expectable range of the observability results. Configuration $y_{(7)}$ is assumed to be today’s standard and $y_{(5)}$ also incorporates accurate wind information from lidar (not from wind anemometer). Using $y_{(2)}$, the model parameters are compared to $y_{(7)}$ known in addition.

4.2. Assessment of the EOG
For the investigation, a relatively short time window $T_{eog} = 0.5s$, with $N = 10$ samples and a typical sample time of $T_s = 50ms$, has been chosen. Therewith, it is possible to consider fluctuating wind speeds despite modelling them as constant disturbance in the augmented
Figure 3. Observability assessment of EOG using the inverse condition number for 240 small time segments $T_{eog} = 0.5$ s considering the different measurement configurations (the results for $y_{(8)}$ and $y_{(9)}$ are excluded since EOG matrices are rank deficient for both)

system. For each sensor configuration the EOG has been evaluated for the wind trajectory of Fig. 2, what is shown in Fig. 3. The solid lines represent the median values of the ICNs for the studied sensor configurations.

First of all: If $x_2$ is not measured, the EOG is always rank deficient with ICNs $\ll 10^{-16}$. Thus, the complete state vector is not observable. Despite the aerodynamic coupling through the torque Eq. (1a), the measurement of $x_1$ contains obviously just insufficient information on the nacelle dynamics. Hence, including $x_2$ into the sensor configuration provides substantial new information which correlates with an increased measurement energy, cf. Eq. (6). This improves the system’s observability such that all ICNs are well above $10^{-10}$. $y_{(7)}$ represents the configurations with the lowest ICN. Improvement is achieved with every additional sensors incorporated. Adding information about tower position, wind speed, or tower eigenfrequency results in similar increase of observability. Moreover, the output $y_{(3)}$, which gathers complete tower state information combined with knowledge of the wind speed $x_4$, is advantageous for simultaneous identification of both, air density and tower eigenfrequency.

Unfortunately, there are two shortcomings when analyzing the ICN alone. First, it is not always clear which states are responsible for the reduced or increased observability level. And secondly, it is not clear what the actual ICN means in terms of estimation error (see Sec. 4.3).

In order to assess the state observability, the singular values and singular vectors of the EOG must be analyzed in greater detail. The singular vectors constitute the observability directions in state space and thus provide information on the single states. To assign singular values to states, the maximum (absolute) component of the singular vector $v_i$ is used to determine the state $x_j$ that is primarily influenced by the corresponding singular value $\sigma_i$. This constitutes a strong simplification since smaller components are omitted. However, looking at Fig. 4 illustrates that each median singular value corresponds to only one state variable which shows that this assignment works in principle (though roughly simplified) for observable sensor configurations.

In Fig. 4, the assigned singular values for $y_{(4)}$ are almost homogeneous as expected. In contrast, the remaining sensor configurations alternate the distribution of the singular values depending on the sensor information provided. Especially the observability of wind speed, tower eigenfrequency and air density is affected to a notable extent. Looking at the wind speed $x_4$, a decrease of four magnitude orders is observed if no measurement information exists. Excluding also the air density from the sensors reduces the singular value by another two magnitude orders. On the other hand, the singular value associated to the nacelle position $x_3$ varies only slightly.
4.3. State estimation with SPKF

To check plausibility of the above observability measures, a numerical investigation for the state estimation accuracy has been conducted (Fig. 5). A cubature Kalman filter [24] has been chosen as representative SPKF realization. Model errors and erroneous initial state are introduced in order to consider their influence on the state estimates and to obtain meaningful results.

As expected, the results are always very accurate on the first two states. Moreover, simulation results suggest that, without measuring the wind speed, the estimates of both, tower eigenfrequency and air density, may tend to diverge (cf. Fig. 5 right). On the other hand, if both model parameters $x_5$ and $x_6$ are assumed to be known then state estimation is very accurate

\[ y(1) = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T \]
\[ y(2) = [x_1 \ x_2 \ x_5 \ x_6]^T \]
\[ y(3) = [x_1 \ x_2 \ x_3 \ x_4]^T \]
\[ y(4) = [x_1 \ x_2 \ x_3]^T \]
\[ y(5) = [x_1 \ x_2 \ x_4]^T \]
\[ y(6) = [x_1 \ x_2 \ x_6]^T \]
\[ y(7) = [x_1 \ x_2]^T \]

\[ \dot{\phi}_g \text{ in rad/s} \]
\[ \dot{v}_w \text{ in m/s} \]
\[ \omega_0 \text{ in rad/s} \]
\[ x_T \text{ in m} \]
\[ \rho \text{ in kg/m}^3 \]

Figure 4. Median singular values associated to individual state $x_i$ for different measurement configurations (values below $10^{-8}$ are $\ll 10^{-16}$, thus excluded)

Figure 5. Comparison of state estimates from SPKF algorithm for different sensor configurations using FAST8 data including recovery from significant initial estimation error
for the first four states (which emphasizes the relevance of a well-parametrized internal model). Contrary, an erroneous air density $x_6$ affects adversely the estimation error of the nacelle position $x_3$ and the wind speed $x_4$, thus having a significant influence.

4.4. Discussion of results
The final comparison of the above results shall relate observability measure to estimation quality. The left plot in Fig. 6 shows again the median singular values (MSV) from Fig. 4, excluding configurations $y_{(8)}$ and $y_{(9)}$. The right plot compares the normalized root mean-squared errors (NRMSE) obtained for the SPKF results in order to evaluate the filter performance. After computation of RMSE, the vector

$$\bar{x} = -\left[0.4 \text{ rad/s} \quad 0.6 \text{ m/s} \quad 0.8 \text{ m} \quad 10 \text{ m/s} \quad 0.5 \text{ rad/s} \quad 0.7 \text{ kg/m}^3\right]^T,$$

which contains the range of each state variable as negative value, has been used for normalization to compare the results graphically to the observability measures in Fig. 6. The behaviour of MSV and NRMSE is qualitatively similar. That is, the predicted observability distribution from EOG correlates with the estimation error (EE). Though, differences are observed for

- state $x_4$: NRMSE is similar to configurations with measured wind speed $y_{(1)}$, $y_{(3)}$ and $y_{(5)}$ despite a MSV of $10^{-4}$ (which seems to be sufficient for wind estimation). Other than that, a MSV smaller than $10^{-6}$ for $y_{(4)}$ and $y_{(7)}$ leads to a deterioration of EE.
- state $x_6$: NRMSE is similar to configurations with known air density $y_{(1)}$, $y_{(2)}$ and $y_{(6)}$ despite a MSV of $10^{-2}$. However, looking at $y_{(4)}$ we can observe that the same MSV not necessarily results in the same NRMSE.

In summary, the assignment of singular values to state variables requires a simplification of the observability directions. Inevitably, this has an effect on the observability metrics obtained (and possibly produces the deviations seen and discussed above).

Nevertheless, the combined analysis of EOG and NRMSE provides extensive knowledge about the system’s observability and demonstrates that the relationship between state observability and state estimation error is in general not trivial.

5. Conclusions
As concluding remark needs to be said that the above investigation shows principally a correlation between observability measures and the expectable estimation quality. In order to
obtain a better picture with respect to observability of states and its actual estimation accuracy, the combined assessment of ICN, MSV and NRMSE is recommended as a strong set of analysis tools. Eventually, the observability analysis constitutes one relevant aspect for every practical realization of state estimation. It reveals the limits for the investigated estimation problems and also shows what is theoretically possible to reach.

In that context, the present contribution has introduced a promising approach based on empirical Gramian matrices to assess efficiently (local) observability metrics for wind turbine state and parameter estimation. The proposed EOG approach avoids completely the necessity of analytical or numerical linearization since it works with nonlinear system simulation only. In addition, its practical implementation is comparatively straightforward to realize, enabling assessment, even for several sensor configurations and wind scenarios, in a short time.

Furthermore, a tangible example has been discussed where an observation problem for simultaneous evaluation of states, parameters and disturbance inputs has been explored. The EOG approach has been reviewed for plausibility by evaluating the estimation results from a generic SPKF implementation against the chosen observability measures. Future work will point out the usability of this approach for higher order systems including rotor blade dynamics and effects of wind shear. Finally, the EOG approach will be reviewed as part of an online evaluation strategy to detect (at an early stage) situations with low observability.

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