A HMM-based and multi-indicator driven network model for complex electromechanical system reliability analysis with considering reliability fluctuation

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Abstract. In this paper, a new network model is developed for better expression of the interaction of a complex electromechanical system. The developed model can reflect the current reliability fluctuation followed by system state disturbances. The Failure Mode and Effect Analysis (FMEA) data and indicator data from detection and monitoring is collected and processed for the structure and parameters of this network model. Hidden Markov Model (HMM) is used for transforming indicators into the probability of fault causes. The recorded fault statistic data is considered as the prior knowledge, and the current indicator data can be used to renew the prior probability of component failure and system malfunction. With this model, the current component failure probability and system malfunction probability can be obtained.

1. Introduction

Complex electromechanical system such as wind turbine generator and naval diesel engine is composed of many interactional components, and usually each component has more than one fault cause. With these characters, the reliability analysis of the complex electromechanical system is more difficult. However, an effective reliability analysis model for complex electromechanical system is of great importance both in safety and economy.

For an accurate analysis of complex electromechanical system reliability, many researchers have made their efforts in recent years. FMEA is the basic of most existing reliability analysis models, and many improved FMEA methods have been proposed to achieve more convincing analysis results [1,2]. With these improved FMEA methods, the importance of each component is sorted, and these methods have been widely applied to real areas, including gear box and engine [3,4]. Considering the hierarchical relationships in a system, reliability analysis models such as Fault tree [5,6], Petri net [7], Small-world network and Bayesian Network have been adjusted and applied to specific systems [8,9], these models can give clear expression of a complex electromechanical system with their strong model presentation ability, and are also widely researched and applied. To extract the distribution feature of fault data, many researchers studied various distribution functions and optimized these functions [10-12]. With their unremitting efforts, a number of verified reliability data fitting models can be selected for our work. However, all these models mentioned above has a deficiency that they use only the recorded statistics to build the reliability analysis model, and no actual reliability fluctuations are considered. This means a plenty of detection data and monitoring data are not utilized for an accuracy estimation of the current system reliability. To make the system analysis model more practical, HMM...
has been studied and used widely for estimating the current state of a component or system [13,14], and application has been achieved in many fields such as oil hydraulic pump and bearing.

In this paper, a reliability analysis model for complex electromechanical system is developed with considering the reliability fluctuation. Aiming at the character of complex electromechanical system, a specific network with six layers is constructed. The occurrence probability of each fault cause can be estimated with the indicator data by using HMM. The diffuse relationship between network model nodes can be derived from FMEA data. On account of the integration of the recorded reliability statistics and current indicator data, the fault probability of each component and the whole system with considering the current state fluctuation can be obtained with this proposed model. There are four sections in this paper, the first section is introduction, which gives an overview of the background. The second section illustrates the model proposed and the method for data collection and processing. The third section gives a demonstration of the proposed method, and the method is applied to a fictitious system. In the last section, a conclusion of this paper is made.

2. Data and model
The proposed reliability analysis model is shown in Figure 1. This network model has six layers, including indicator layer, fault cause layer, connection layer 1, component layer, connection layer 2, and system layer. The node $S$ in system layer represents the event that the system is down due to malfunction (including automatic shutdown caused by the fault and shutdown due to the maintenance measures that must be taken after the fault is found). The node $C_n$ in component layer represents the event that component $n$ (a system have $N$ components in total) is in failure. The node $F_m$ in fault cause layer represents the event that fault cause $n$ (a component have $M$ fault causes in total) happens. Indicator layer consists of the detection data and monitoring data, which can be a reflection of the current state of the system. In these two connection layers, the propagation paths that fault causes make component failure and component failure make system malfunction are demonstrated.

![Figure 1. Proposed reliability analysis model.](image-url)
2.1. Data collection and notation
This part gives the parameters needed for the proposed model. From the recorded FMEA data, all parameters can be obtained. Here the three steps are given.

(1) The probability of each component failure in the case of system malfunction, the probability of each failure cause happens under a certain component failure, the probability of relative component failure under a certain component failure, and the probability of relative failure cause happens under a certain failure cause happens are given by equations (1), (2), (3) and (4):

\[
P(C_n | S) = \frac{q(C_n, S)}{q(S)} \quad (1)
\]

\[
P(F_m | C_n) = \frac{q(F_m, C_n)}{q(C_n)} \quad (2)
\]

\[
P(C_n | C_n) = \frac{q(C_n, C_n)}{q(C_n)} \quad (3)
\]

\[
P(F_m | F_m) = \frac{q(F_m, F_m)}{q(F_m)} \quad (4)
\]

where the function \( q \) represents the occurrence frequency of a certain event; \( n_1 \) is the component related to component \( n \); \( m_1 \) is the fault cause related to fault cause \( m \).

(2) According to the fault or failure records, the time data of system malfunction, component failure and failure cause occurrence are extracted respectively, and the probability density curve of system malfunction time, component failure time and failure cause happen time, which are represented as \( f_S(t) \), \( f_C(t) \) and \( f_F(t) \), can be obtained by fitting the extracted data under an appropriate distribution model such as Weibull distribution. Then the prior probability of system malfunction, component \( n \) failure and fault cause \( m \) happen at time \( t \) can be given by equations (5), (6) and (7).

\[
P(S) = f_S(t) \quad (5)
\]

\[
P(C_n) = f_C(t) \quad (6)
\]

\[
P(F_m) = f_F(t) \quad (7)
\]

(3) According to the Bayesian formula, the probability of component \( n \) failure under the condition of fault cause \( m \) happen and the probability of system malfunction under the condition of component \( n \) failure can be calculated with equations (8) and (9).

\[
P(C_n | F_m) = \frac{P(F_m | C_n) \cdot P(C_n)}{P(F_m)} \quad (8)
\]

\[
P(S | C_n) = \frac{P(C_n | S) \cdot P(S)}{P(C_n)} \quad (9)
\]

2.2. HMM-based indicator processing
Here indicator data is seen as the reflection of the current system state fluctuation, and the fault cause can be divided into three states: high occurrence probability (with occurrence probability in \((T_h, 1])\), middle occurrence probability (with occurrence probability in \([T_m, T_h]\)), and low occurrence probability (with occurrence probability in \([0, T_m]\)). Then HMM is applied to estimate the current state of each fault cause from the indicator data.

2.2.1. Transform the indicator data into fault cause occurrence probability. For continuous variable indicator, Gaussian HMM can be applied to estimate the probability that the fault cause under each
state. In a Gaussian HMM, each state has corresponding mean and variance of a specific indicator variable (represented as mean vector and variance vector or covariance matrix), and these states can transform from one to another with a particular probability (represented as state transform probability matrix). Given the current indicator data, Gaussian HMM can obtain the probability that the fault cause under each state with these parameters.

For finite discrete variable indicator, Multinormial HMM can be applied to find the probability. The parameter of Multinormial HMM includes state transform probability matrix and observation matrix (represent the observation probability of an indicator data under each state). Similar to continuous variable indicator, the probability that the fault cause under each state can be obtained.

From the above, the probability that the fault cause under each state can be obtained, here give $P_h$ as the probability of high occurrence probability state, give $P_m$ as the probability of middle occurrence probability state, and give $P_l$ as the probability of low occurrence probability state. Then, the current probability of fault cause $m$ can be given by equation (10).

$$P_{new}(F_m) = P_h \cdot T_h + P_m \cdot \frac{T_h + T_m}{2} + P_l \cdot \frac{T_m}{2}$$  \hspace{1cm} (10)

### 2.2.2. Other traits of HMM for indicator processing

HMM can make a comprehensive estimation basing on more than one indicator. For a Gaussian HMM, the variance matrix can be covariance matrix if it has more than one indicator. This ability of synthesizing multi-feature gives HMM a more accuracy estimation result.

(1) If a fault cause can be reflected by many indicators, which both have continuous indicator and discrete indicator, Multinormial HMM and Gaussian HMM can be used to estimate the probability basing on corresponding indicator respectively, and then the evidence fusion theory can be applied to get the final probability.

(2) Gaussian HMM has a hypothesis that the indicator variable under each state follow the Gaussian distribution. If the real variable not follow this distribution, then Gaussian mixture HMM can be used to approximate the real distribution by combining multiple Gaussian distributions.

(3) There are two methods to get the HMM parameters: given by expert or train from the recorded indicator data (using the expectation maximization algorithm to obtain the corresponding parameter which maximizes the occurrence probability of training observation sequence).

### 2.3. Multi-indicator driven reliability analysis model

In this section, the whole reliability analysis model is established with concerning the propagation relationship between model nodes, and the current probability of component failure and system malfunction can be renewed to a more reasonable value.

#### 2.3.1. Determination of connection layer node interaction

In connection layer 1, the interaction between different fault causes is considered, and the posterior probability of component $n$ failure caused by different failure causes is combined, in order to get the posterior probability of component $n$ failure (the node of next layer).

The in-degree and out degree of node $f_m$ is defined in equations (11) and (12) respectively:

$$\lambda_{in}(F_m) = \sum_{m_i} P(F_m | F_{m_i})$$  \hspace{1cm} (11)

$$\lambda_{out}(F_m) = \sum_{m_i} P(F_{m_i} | F_m)$$  \hspace{1cm} (12)

where $m_i$ is the fault cause related to fault cause $m$.

Defining the scaling factor $\gamma \in [0,1]$, then the influence coefficient that concerning the interaction relationship between fault causes can be obtained by equation (13).
\[
\alpha(F_m) = \frac{\gamma + \lambda_{out}(F_m)}{\gamma + \lambda_{in}(F_m)}
\]  

(13)

Here \(\alpha(F_m)\) represent the effect that failure cause \(m\) to other failure causes. The larger the \(\lambda_{out}(F_m)\) is, the greater effect failure cause \(m\) have to other failure causes; and the larger the \(\lambda_{in}(F_m)\) is, the greater effect failure cause \(m\) receive from other failure causes. Considering that the collected fault or failure records are the result of the interaction of different fault causes, the parameter \(\alpha(F_m)\) can separate the influence of different fault causes on component failure and be used for the calculation of component failure probability. In equation (13), \(\gamma\) adjust the contribution that the interaction of nodes give to the calculated probability of component failure, the contribution is biggest when \(\gamma\) equals 0 and smallest when \(\gamma\) equals 1.

2.3.2. Computation of component failure probability and system malfunction probability. Combining the parameter obtained in section 2.1 and the current probability of fault cause \(m\) obtained from section 2.2, the posterior probability that component \(n\) failure caused by failure cause \(m\) can be derived from equation (14).

\[
P_{new}(F_m, C_n) = P(C_n | F_m) \cdot P_{new}(F_m)
\]  

(14)

For the instance that each combination in \(M\) failure causes may lead to the component failure, all the mutually independent combinations of failure causes can be listed, and the posterior probability of component failure can be computed by adding the conditional probability of each combination. Each failure cause may happen or not happen, so \(2^M\) conditions has to be considered. But actually, there is only few condition has a relatively high occurrence rate in all these \(2^M\) conditions. However, these \(2^M\) conditions add a great computational complexity, and it is difficult to meet the requirements of so many parameters with actual fault statistics. So the simplified algorithm is used in this model. \(P(F_m | C_n)\) is considered as the weight that fault cause \(m\) to component \(n\) failure. The weighted sum of component failure probability fluctuation due to each failure cause is calculated, as the renewed fluctuation of component failure probability. If more than one fault cause happened simultaneously and led to the component failure, the sum of \(P(F_m | C_n)\) for all fault cause would be bigger than 1, and the excess is considered as the common effect that fault causes give to component failure probability. Then the posterior probability that component \(n\) failure under the effect of \(M\) fault causes in time \(t\) can be calculated with equation (15).

\[
P_{new}(C_n) = \sum_{m=1}^{M} \left( \alpha(F_m) \cdot P_{new}(F_m, C_n) \cdot P(C_n) \right) \cdot P(F_m | C_n) + P(C_n)
\]

\[
= \sum_{m=1}^{M} \left\{ \frac{\gamma + \lambda_{out}(F_m)}{\gamma + \lambda_{in}(F_m)} \cdot P(C_n | F_m) \cdot P_{new}(F_m) - P(C_n) \right\} \frac{q(F_m, C_n)}{q(C_n)} + P(C_n)
\]

\[
= \sum_{m=1}^{M} \left\{ \frac{\gamma + \sum_{n=1}^{N} \frac{q(F_{m,n} | F_m)}{f_{C_n}(t)} \cdot \frac{f_{C_n}(t)}{f_{C_n}(t)} \cdot \frac{q(F_{m,n} | C_n)}{q(C_n)} \cdot P_{new}(F_m) - f_{C_n}(t) \right\} \frac{q(F_m, C_n)}{q(C_n)} + f_{C_n}(t)
\]

(15)

The calculation method from the component layer to the system layer is the same as that from the fault cause layer to the component layer, so the probability of system malfunction in time \(t\) can be given directly by equation (16).

\[
P_{new}(S) = \sum_{n=1}^{N} \left\{ \frac{\gamma + \sum_{n=1}^{N} \frac{q(C_{n1} | C_n)}{f_{C_n}(t)} \cdot f_{C_n}(t)}{\gamma + \sum_{n=1}^{N} \frac{q(C_{n1} | C_n)}{q(C_n)}} \cdot P_{new}(C_n) - f_{S}(t) \right\} \frac{q(C_{n1}, S)}{q(S)} + f_{S}(t)
\]

(16)
3. Method demonstration

To make a clear demonstration of the proposed reliability analysis method, a simple fictitious model is constructed. As Figure 2 shows, this model represents a system with two components $C_1$ and $C_2$, and each component has one failure cause. For component $C_1$, $I_1$ is the indicator of failure cause $F_1$, for component $C_2$, $I_2$ is the indicator of failure cause $F_2$.

![Figure 2. Demonstration model.](image)

Before computing the probability of system malfunction, some parameters are given (all these parameters are made-up just for the demonstration of the developed model):

$q(S) = 78; q(C_1) = 43; q(C_2) = 80; q(F_1) = 56; q(F_2) = 87; q(C_1, S) = 8; q(C_2, S) = 74; q(C_1, C_2) = 4; q(F_1, C_1) = 43; q(F_2, C_2) = 80$

(17)

Assuming that the hazard function of each model node ($S, C_1, C_2, F_1, F_2$) is a bathtub curve, and follow the format of equation (18) that given in the references [15,16]:

$$f(t) = \frac{2v(1-\frac{t+u}{b})(1-(1-\frac{t+u}{b})^2)^{v-1}}{b^v-1-(1-\frac{t+u}{b})^2})^{v-1}$$

(18)

Where $v$, $u$ and $b$ are parameters. Let $v_S$, $b_S$ and $u_S$ be the parameters of $f_S(t)$, and make the same expression for other nodes. Then the parameters in equation (18) are given:

$v_S = 0.3; b_S = 1300; u_S = 70$
$v_{C_1} = 0.5; b_{C_1} = 1200; u_{C_1} = 60$
$v_{C_2} = 0.11; b_{C_2} = 1170; u_{C_2} = 30$
$v_{F_1} = 1.5e-3; b_{F_1} = 1500; u_{F_1} = 100$
$v_{F_2} = 3.2e-2; b_{F_2} = 1400; u_{F_2} = 100$

(19)

Substitute the parameters above into equation (17), the hazard function of each node is show in Figure 3.
For transforming the value of two indicators $I_1$ and $I_2$ to the occurrence probability of failure causes $F_1$ and $F_2$, two HMMs are built with parameters in Table 1. With the built HMM, 1000 samples were generated for each indicator by the sample function of hmmlearn package in python. Taking the samples generated as the indicator data, the probability of failure cause occurrence, component fault and system malfunction can be renewed with equation (15) and equation (16). Then, with substituting the given parameters into these two equations, the posterior probability of $C_1$ failure, $C_2$ failure and system malfunction are calculated and shown in Figure 4, Figure 5 and Figure 6. Obvious reliability fluctuation can be seen from these three figures. The fluctuation comes from the effective information contained in indicator data, and these data make the estimated hazard rate more approximate to the real current state.

Table 1. Parameters of HMMs.

|                      | $I_1$                      | $I_2$                      |
|----------------------|---------------------------|---------------------------|
| Start probability vector | $[1 \ 0 \ 0]$            | $[1 \ 0 \ 0]$            |
| State transform probability matrix | $\begin{bmatrix} 0.995 & 0.003 & 0.002 \\ 0.99 & 0.007 & 0.003 \\ 0.98 & 0.018 & 0.002 \end{bmatrix}$ | $\begin{bmatrix} 0.989 & 0.006 & 0.005 \\ 0.991 & 0.005 & 0.004 \\ 0.99 & 0.003 & 0.007 \end{bmatrix}$ |
| Mean vector          | $[3 \ 7 \ 9]$             | $[0 \ 0.5 \ 1]$           |
| Variance vector      | $[5 \ 0.1 \ 0.05]$        | $[0.3 \ 0.2 \ 0.1]$       |
| $T_h$                | 0.01                      | 0.02                      |
| $T_m$                | 0.006                     | 0.007                     |
Figure 4. Posterior hazard function of C₁.

Figure 5. Posterior hazard function of C₂.

Figure 6. Posterior hazard function of system.
4. Conclusion
This paper develops a reliability analysis model for complex electromechanical system with considering the reliability fluctuation. The developed model with the network structure can better express the interaction of fault causes and components in system. By combining the historical statistic data and the current monitoring data, the reliability analysis results concluded with the developed model can be more accurate, and more consistent with the real state of system. With the computational equation derived in this model, the current probability of component failure and system malfunction can be calculated. The results of method demonstration give an intuitive representation of the reliability fluctuation. This is consistent with the truth that the reliability changes according to the current state of the system. With the ability of capturing the current hazard rate, the proposed model can be widely expanded to fault tracing, preventative maintenance and other applications. Beyond that, there are still some issues need a further research, such as how to get more effective and reasonable parameters. Focus on this issue, the detailed parameter determination method might be generalized in the near future.

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