Investigation of approximate models of experimental temperature characteristics of machines

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Abstract. This work is devoted to the investigation of various approaches to the approximation of experimental data and the creation of simulation mathematical models of thermal processes in machines with the aim of finding ways to reduce the time of their field tests and reducing the temperature error of the treatments. The main methods of research which the authors used in this work are: the full-scale thermal testing of machines; realization of various approaches at approximation of experimental temperature characteristics of machine tools by polynomial models; analysis and evaluation of modelling results (model quality) of the temperature characteristics of machines and their derivatives up to the third order in time. As a result of the performed researches, rational methods, type, parameters and complexity of simulation mathematical models of thermal processes in machine tools are proposed.

1. Introduction

Modern machine-building production uses high-tech equipment - CNC machines. More than 90% of all CNC machines are not precision machines, but there are machines of increased accuracy. At the same time, the need for modern production consists in achieving on such machines the precision according to requirements IT 7 of ISO standard. For machines the main factor determining precision is temperature deformations [1, 2]. To reduce their influence on the precision of processing with the use of non-precision machines is possible only due to the implementation of the systems of compensation of the temperature error in them [3-6]. The system of temperature error compensation assumes carrying out machines thermal tests. To reduce the duration of thermos testing of machines, it is necessary to improve both the experimental procedure and the corresponding mathematical support, which is based on various methods of approximation.

2. Materials and methods

The temperature deformations of machines are correlated with a coefficient close to unity with the temperatures on their working elements [7]; therefore it is possible to investigate the temperature characteristics of machines. As mathematical models that simulate the course of thermal processes, let us apply polynomial models that represent a superposition of power functions with constant coefficients of the following form:

$$T_{mod}(t_j) = \sum_{i=0}^{n} A_i t_j^i,$$

(1)
where \( T_{\text{mod}}(t_j) \) – model temperature values at time \( t_j \); \( n \) – polynomial degree; \( A_i \) – constant coefficients.

The constant coefficients are obtained by solving a constrained optimization problem with the objective function of the form:

\[
f(T(t_j)) = \frac{1}{m} \sum_{j=1}^{m} (T_{\text{mod}}(t_j) - T_e(t_j))^2
\]  

(2)

where \( T_e(t_j) \) – experimental temperature values at time \( t_j \); \( m \) – the number of times at which the temperature was measured.

The solution was performed using the algorithm for minimizing the maximum discrepancies between the experimental and model values of the temperature, calculated in objective function (2) by the least-squares method. In this case, the formal formulation of the optimization problem had the form:

\[
\min_x \max_y f(T(t_j))
\]

(3)

After determining the constant coefficients, the polynomial model was transformed according to the Gorner scheme [8] by taking the variable as a parenthesis, as a result of which formula (1) for calculating the current values takes the form:

\[
T_{\text{mod}}(t_j) = A_0 + t_j(A_{n-2} + t_j(A_{n-1} + t_jA_n))...
\]

(4)

This method is universal, applicable to polynomials of almost any degree and is easy to implement. In addition, it allows one to reduce significantly the computational cost by eliminating the exponentiation operation.

The kinetic and dynamic characteristics of thermal processes, by which one means velocity \( VT(t) \), acceleration \( AT(t) \) and sharpness \( RT(t) \) of the temperature change, respectively [9], were used as additional criteria in assessing the quality of the simulation mathematical models.

3. Experimental research

Experimental research were carried out on a boring-milling and boring machine with CNC 400V manufactured by Ltd ”NPO Stankostroenie”, Sterlitamak. When processing the experimental temperature measurement data \( T(t) \) (Figure 1), it turned out that it is impossible to use them directly. The resulting graph is a piecewise linear step function. Its numerical differentiation by various known methods leads to the result presented in Figure 2. With this, the calculation of derivatives of higher orders with respect to this graph loses its meaning. In this regard, after the initial statistical processing of the experimental results, it is necessary to smooth out the experimental data, while maintaining the specific nature of the changes in the measured parameter. One of the most common methods of smoothing experimental data is the Savitsky-Golay method [10].

At the first stage, the influence of the window width of the smoothing polynomial on the final result was investigated. Figure 3 shows 10 graphs of the kinetic characteristics of \( VT(t) \) obtained by differentiating the initial temperature dependence on the time \( T(t) \) smoothed by the Savitsky-Golay method. The first graph is constructed for a window width of 20 points of experimental values; the last one is 600 points. As one can see, on the first graph there are very significant noise components. At 100 points, the noise components are significantly reduced. In the range from 150 to 250 points, the graphs are almost identical and preserve the specific features of the ongoing thermal process. In the range of 300 points and higher, the smoothing procedure masks specific features of the ongoing thermal process, that is, the sensitivity is reduced. Thus, in order to smooth out the experimental data by the Savitsky-Golay method when studying thermal processes in machines, a window is required whose width should be in the range from 150 to 250 experimental values.
Figure 1. The graph of temperature change $T(t)$

Figure 2. The graph of the change rate of temperature $VT(t)$

Figure 3. The type of graphs of the kinetic characteristic $VT(t)$ as a function of the number of experimental points (window width) used to smooth out the Savitsky-Golay method.

In the second stage, the influence of the degree of the smoothing polynomial was investigated.

Figure 4 shows nine graphs of the kinetic characteristics of $VT(t)$ obtained by differentiating the initial temperature dependence on time $T(t)$, smoothened by the Savitsky-Golay method by polynomials of different degrees.

Let us describe the graphs in detail. At the degree of a polynomial equal to five and above, the specific features of the proceeding thermal process clearly manifest themselves. The sensitivity increases with the increasing degree of the polynomial. Thus, for the construction of an adequate simulation mathematical model of the thermal process, it is necessary to use polynomials of no less than fifth degree when smoothing by the Savitsky-Golay method.
At the third stage, a study was conducted to determine the level of complexity of approximating polynomials sufficient to obtain an adequate simulation mathematical model of the thermal process. Figure 5 shows nine graphs of the kinetic characteristics of VT(t) obtained by differentiating the initial temperature dependence on time T(t), approximated by polynomials of different degrees. As can be seen from the graphs given, with a polynomial degree of five or more, they differ little. Thus, to construct an adequate simulation mathematical model of the thermal process, it is sufficient to use polynomials no lower than the fifth degree.

At the fourth stage of investigation a polynomial thermal process model was made in real time from the moment the first data arrived.
Figure 6. Graphs T(t): a - the experimental result; b – smoothed by the Savitsky-Golay method and approximated by a polynomial; c - approximated by a polynomial in real time; d - the kinetic characteristic of VT(t); e - the dynamic characteristics of AT(t) obtained by differentiating the graph of b; f - the dynamic characteristics of RT(t) obtained by differentiating the graph of b; g - the kinetic characteristic of VT(t); h - the dynamic characteristics AT(t) obtained by differentiating the graph of c; i - dynamic characteristics RT(t) obtained by differentiating the graph of c.

This approach has several advantages. In the first order, it is not necessary to wait for the first 150 experimental values to begin the smoothing procedure by the Savitsky-Golay method. Secondly, it is possible to monitor and evaluate in real time the effect of incoming data on the parameters of the generated model of the thermal process. Thirdly, there is the possibility of parallel construction of secondary mathematical models describing changes in the kinetic and dynamic parameters of the thermal process, increasing the accuracy of the primary model. Figure 6 shows the results of the study. If we consider precisely (Figure 6 a), the original plot and graphs constructed using smoothing (Figure 6 b) and polynomial models (Figure 6 c), do not differ significantly. However, the results of their further mathematical processing are the kinetic curves (Figures 6 d, g) and dynamic (Figures 6 e, f, h, i) characteristics are already different.

For example, it is known a priori from the theory that at the initial moment of time the values of the kinetic and dynamic characteristics VT(t) = AT(t) = RT(t) = 0 are equal to zero. In this case, the smoothing procedure, which requires at the initial stage a certain number of experimental values, yields results that are inconsistent with this (Figures 6 d, e, f). In addition, smoothing distorts and masks significantly the nature of the current heat process at the initial stage of the experiment, which reduces the quality and adequacy of the constructed simulation model.

4. Conclusion
The results of studies carried out on a number of sets of experimental data allow us to formulate the following conclusions:
- for the construction of adequate thermal processes models in machines, a procedure for smoothing the experimental data is needed and the Savitsky-Golay method;
- width of the smoothing window must be in the range from 150 to 250 experimental values applying the Savitsky-Golay method;
- with increasing width of the smoothing window of the Savitsky-Golay method, sensitivity to specific features of the ongoing thermal process decreases;
- for the construction of an adequate simulation mathematical model of the thermal process, it is necessary to use polynomials of no less than fifth degree when smoothing by the Savitsky-Golay method, and the sensitivity of revealing the specific features of the proceeding thermal process increases with the degree of the polynomial;
- to simulate the initial dependencies describing the course of thermal processes in machines in time, it is recommended to use polynomials of not lower than the fifth degree;
- the procedure for approximating the experimental data by polynomials in real time allows one to abandon the procedure of intermediate smoothing by the Savitsky-Golay method and to obtain reliable results;
- the use of kinetic and dynamic characteristics as additional criteria of adequacy makes it possible to improve the accuracy of simulation mathematical models of thermal processes occurring in machines.
- the calculation of the values of this indicator in real time allows a 30% reduction in the duration of full-scale tests of the machine while forming a database of experimental characteristics.

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