Neutrino and cosmic-ray emission from multiple internal shocks in gamma-ray bursts

Mauricio Bustamante1,2,3, Philipp Baerwald4,5, Kohta Murase4,5,6 & Walter Winter2

Gamma-ray bursts (GRBs) are short-lived, luminous explosions at cosmological distances, thought to originate from relativistic jets launched at the deaths of massive stars. They are among the prime candidates to produce the observed cosmic rays at the highest energies. Recent neutrino data have, however, started to constrain this possibility in the simplest models with only one emission zone. In the classical theory of GRBs, it is expected that particles are accelerated at mildly relativistic shocks generated by the collisions of material ejected from a central engine. Here we consider neutrino and cosmic-ray emission from multiple emission regions since these internal collisions must occur at very different radii, from below the photosphere all the way out to the circumburst medium, as a consequence of the efficient dissipation of kinetic energy. We demonstrate that the different messengers originate from different collision radii, which means that multi-messenger observations open windows for revealing the evolving GRB outflows.
Gamma-ray bursts (GRBs) are violent outbursts of energy distributed over cosmological distances. Most of the energy is detected as gamma rays during the so-called prompt phase, lasting from a few seconds to several hundred seconds (see refs 1–3 for reviews). The common view is that relativistic jets are ejected from a central engine, triggered by a collapsing star or a neutron star merger, in the direction of the observer. The inhomogeneity in the jets naturally leads to internal shocks, at which charged particles can be accelerated. In the classical GRB scenario, the observed gamma-ray emission is attributed to synchrotron radiation from non-thermal electrons. It is natural to expect that protons are accelerated as well, and GRBs have also been considered as a possible candidate class for the origin of the ultra-high-energy cosmic rays (UHECRs)

\[ R_{\gamma} \] becomes transparent to gamma-ray emission. Possible subphotospheric contributions enhance the detectability. We predict a minimal neutrino flux per flavour at the level of \( E^2 \sim 10 \) GeV cm\(^{-2}\) sr\(^{-1}\) s\(^{-1}\) for the contribution from beyond the photosphere, with a spectral shape similar to the original theoretical prediction. However, in striking contrast to earlier approaches, this prediction turns out to hardly depend on model parameters such as the Lorentz boost, the baryonic loading or the variability timescale.

Results

Dynamical burst model. To demonstrate neutrino and cosmic-ray emission from various \( R_{\gamma} \), we follow the internal collision model of ref. 37; see Fig. 1 for illustration. We set out a number \( N_{kb} \) of shells from a central engine with equal initial kinetic energies but a spread in the bulk Lorentz factor, around \( \Gamma_0 \), of

\[ \ln \left( \frac{\Gamma_{0,kb}}{\Gamma_0 - 1} \right) = 0.2x, \]

where \( \Gamma_{0,kb} \) is the initial Lorentz factor of the \( k \)-th shell and \( x \) follows a Gaussian distribution \( P(x)dx = e^{-x^2/2} \sqrt{2\pi}dx \). Note that a large dispersion \( \Delta r > 0.1 \) is required to achieve high efficiencies as, we have explicitly tested, since the energy dissipation is proportional to the difference between the Lorentz factors of the colliding shells. The shells are assumed to be emitted with an uptime of the emitter \( \delta t_{\text{eng}} \) followed by an equally long downtime, which is an input of the simulation. The variability timescale \( t_v \) will be obtained after running the simulation from the light curve as an output, with a value that is typically similar to \( \delta t_{\text{eng}} \). For simplicity, we have assumed constant uptime and downtime, but ref. 24 explored a scenario where \( \delta t_{\text{eng}} \) is different for each emitted shell and follows a log-normal distribution; post simulation, it is possible also in this case to infer a variability timescale for the whole burst. While the shells evolve, their widths, masses and speeds (that is, their Lorentz factors) are assumed to be constant, and their mass
density decreases $\propto r^{-2}$, with $r$ the radial distance to the emitter. Because of the different speeds of the shells, a shell will collide with another and merge into a new one; see Fig. 1. During the burst evolution, shells may collide several times. We assume that after a collision the new shell immediately cools by prompt radiation of the internal energy into gamma rays, cosmic rays and neutrinos. Derivations of the properties of the newly formed shell are given in refs 37,39 and are maintained in the simulations presented here. Our results match the analytical predictions for the dissipation of modest-amplitude fluctuations from refs 40,41. Note that we simplify the evolution of the internal shocks in our choice of 5.5/C2, as well as $\delta t_{\text{eng}}$ is the uptime of the central emitter. The shells have a spread in the bulk Lorentz factor, but initially equal bulk kinetic energies. The shells propagate, collide and merge (marked by the shell coloured purple) as soon as they meet other shells (multiple collisions are allowed), whereupon their masses, widths and speeds change. The energy dissipated in the collision is assumed to be radiated away immediately.

Figure 1 | Illustration of the internal collision model of gamma-ray bursts used in this study. A set of $N_{\text{sh}}$ shells with equal energies, widths and separations $l = d = c\delta t_{\text{eng}}$ are emitted from a central engine, where $\delta t_{\text{eng}}$ is the uptime of the central emitter. The shells have a spread in the bulk Lorentz factor, but initially equal bulk kinetic energies. The shells propagate, collide and merge (marked by the shell coloured purple) as soon as they meet other shells (multiple collisions are allowed), whereupon their masses, widths and speeds change. The energy dissipated in the collision is assumed to be radiated away immediately.

Simulation results. The light curve of the simulated burst is shown in Fig. 2a as a black curve. Although we show the light curves for only two representative simulations in this study (the aforementioned one and another one with $N_{\text{sh}} = 100$ and $\delta t_{\text{eng}} = 0.1$ s, in Fig. 2b), we will present a more detailed parameter space study in a future work (Bustamante et al., manuscript in preparation). We do not investigate effects of the spectral evolution during the dynamical time for one collision46, as we imply that taking into account contributions from multiple shells is more relevant, like in the case of gamma rays39. Note that, although we do not calculate hadronic cascades, their feedback on neutrino spectra is unimportant, given the value of the baryonic loading factor used in this work.

We show in Fig. 3 the neutrino fluence (a), maximal proton energy (b) and maximal gamma-ray escape energy (c) for each collision (dot) as a function of $R_C$. The maximal proton energies are obtained from comparing acceleration, dynamical, synchrotron loss and photohadronic (for protons) timescales. As a result, we find that the collisions are spread between about 10^{16} km and our choice of 5.5 \times 10^{14} km for the deceleration radius46, where outflows terminate by the external shock into the circumburst medium. Most collisions occur around 10^{10} km—slightly above the estimate from the geometry equation (1), $R_C \approx 1.6 \times 10^{10}$ km. Red dots mark collisions in the neutron escape model regime (optically thick to $p\gamma$ interactions) and blue empty circles, collisions in the direct proton escape regime.

Black squares mark subphotospheric collisions, that is, those for which the Thomson optical depth is larger than unity. The optical depth is obtained by calculating the proton number density from the masses of the shells and assuming that the electron number density is as high as the proton density, which is expected for an electrically neutral plasma. In reality, however, the electron and positron densities may be somewhat higher if there is a significant non-thermal contribution from electron–positron pair production. The obtained photospheric radius...
$R_{ph} \approx 2 \times 10^8 \text{km}$ is somewhat larger than the conventional expectation calculated using the dissipated energy in gamma rays ($R_{ph} \approx 3 \times 10^7 \text{km}$). This estimate is affected by the efficiency of the conversion from kinetic energy into dissipated energy, which is roughly 25% in our cases. However, the more important reason is the significant baryonic loading: since most of the energy is dissipated into protons, the masses of the shells have to be upscaled to match the required energy output in gamma rays ($10^{53}$ erg), which leads to larger radii of the photosphere because of higher electron densities. It can therefore be expected that the large baryonic loadings that are needed to describe the UHECR observations$^{25,34}$ will lead to larger fractions of subphotospheric collisions.

We find that the obtained range of collision radii is large, from under the photosphere out into the deceleration radius, since dissipation occurs for a wide range of $R_C$, especially when the spread of the Lorentz factor $\Gamma_{\text{f}}$ is large$^{8,0,1}$. Note that $\lesssim 12\%$ of collisions occurs under the photosphere for the chosen parameter set, altogether 118 out of the total 990 collisions, but most of the energy dissipation occurs at large radii $> 10^{10} \text{km}$. In the internal shock model, gamma-ray emission should be produced beyond the photosphere, so we only consider collisions beyond the photosphere in the following, unless noted otherwise. This is conservative, since the baryonic loading may be smaller under the photosphere$^{19}$ and particle acceleration becomes inefficient for radiation-mediated shocks$^{50}$. The ratio of total energy emitted as neutrinos via optically thin internal shocks to the total energy emitted by these collisions as gamma rays is 4.8% for this representative parameter set.

Most importantly, Fig. 3a demonstrates that neutrinos are dominantly produced at small collision radii $R_C \lesssim 10^9 \text{km}$, close to the photospheric radius $R_{ph} \approx 2 \times 10^8 \text{km}$. This result can be understood as follows. In each collision, the emitted gamma-ray energy, $E_{\gamma}^{\text{iso}} \lesssim \tilde{R}_{\gamma \rightarrow \text{sh}}$, is a fraction of the total dissipated energy. The pion production efficiency (fraction of proton energy going into produced pions) at the photon spectral break $\epsilon_{\gamma, \text{break}}$, which is neglecting spectral effects, can be approximated as

$$f_{\gamma} \approx \frac{\epsilon_{\gamma, \text{break}}}{4\pi R_C^{-2} \frac{\tilde{R}_{\gamma \rightarrow \text{sh}}}{\Gamma_{\text{m}}}}.$$  

Here $\Gamma_{\text{m}}$ is the Lorentz factor of the merged shell, $\sigma_{\gamma p}$ is the photopion cross section, and $\epsilon_{\gamma, \text{break}} \approx 0.2$ is the fraction of proton energy going into the pion per interaction. Since the internal shock model predicts$^{40} E_{\gamma}^{\text{iso}} < \tilde{R}_{\gamma \rightarrow \text{sh}} \propto R_C^{-1}$ for $0 < q \lesssim 2/3$, we expect $f_{\gamma} \propto R_C^{-2 - q \epsilon_{\gamma, \text{break}}^{-1}}$. Hence, since $\Gamma_{\text{f}}$ has to be sufficiently large

---

**Figure 2** | Two simulated light curves. The curves for the energy flux of gamma rays (solid, black) and neutrinos (dotted, red) are built from the collisions of shells output by an engine emitting shells with equal kinetic energies, with $t_{\text{obs}}$ the time in the observer’s frame. The light curves in (b) correspond to a simulation with $N_{\text{sh}} = 1,000$ (100), $N_{\text{coll}} = 990$ (91), $\delta_{\text{long}} = 0.01$ s (0.1 s) and $t_c = 0.06$ s (0.66 s). A redshift of $z = 2$ was assumed to produce these light curves.

**Figure 3** | Neutrino fluence and maximum proton and gamma-ray energies for each internal collision. (a) Muon-neutrino fluence ($\nu_\mu + \bar{\nu}_\mu$ in the observer’s frame), (b) maximal proton energy (in the source frame, for ideal $\epsilon_{\gamma} = 1$ acceleration) and (c) maximal allowed gamma-ray energy (in the source frame, where $\epsilon_{\gamma, \text{break}}(E_{\gamma, \text{max}}) = 1$) as a function of the collision radius. Each dot represents one collision: red filled dots represent collisions where cosmic rays mainly escape as neutrons (optical thickness to $\gamma$ interactions larger than unity), blue empty circles represent collisions where cosmic-ray leakage dominates over the neutron escape model, and black squares denote subphotospheric collisions or collisions where this picture cannot be maintained (that is, where the Thomson optical depth is large). In (b), the ultra-high energy range for cosmic rays, above $10^{16} \text{GeV}$, is shown as a green band; the downward-pointing arrow marks the approximate energy above which adiabatic energy losses dominate. In (c), the energy ranges that can be reached by the Fermi-GBM (pink), Fermi-LAT (blue) and Cherenkov Telescope Array (CTA) (green) instruments are illustrated as coloured bands. Collisions in which protons with energies above $10^6 \text{GeV}$ are able to escape are marked as upward-pointing arrows.
for efficient energy dissipation ($A = 1$ in the simulations in the present study), neutrino production is typically dominated by collisions at radii around the photosphere. UHECR protons come from collisions in the range $10^{8.5} \text{ km} \leq R_C \leq 10^{10} \text{ km}$; see Fig. 3b. First the maximal proton energy increases with collision radius as (close to the peak) synchrotron losses limit the maximal proton energy, and the magnetic field drops with $R_C$. The peak occurs where adiabatic losses take over, and the decline comes from a decrease of the acceleration timescale for dropping magnetic fields; the expressions for the different energy-gain and energy-loss timescales can be found, for example, in refs 9,31. Note that the UHECRs come from two different components dominating at different collision radii: for $R_C \leq 10^{8.5} \text{ km}$, neutron escape dominates and for $R_C \geq 10^{8.5} \text{ km}$, protons directly escaping from the source dominate—which are obviously not related to strong neutrino production; see Fig. 3a. In the chosen example, the main contribution to cosmic rays actually comes from direct escape. Finally, Fig. 3c illustrates that high gamma-ray energies, which can only be observed in Cherenkov Telescope Array (CTA) or other next-generation imaging atmospheric Cherenkov telescopes, come from large collision radii $R_C \geq 10^9 \text{ km}$ since for lower radii the optical depth for $\gamma\gamma$ interactions is too high. As a consequence, neutrinos, cosmic rays and Fermi Large Area Telescope (LAT)/CTA gamma rays probe different emission radii. Neutrinos are useful to probe dissipation at small radii, including subphotospheric dissipation. For dissipation at large radii, where heavy nuclei survive, the TeV gamma-ray diagnostics of a GRB would be useful.\textsuperscript{35}

There has been some evidence that the composition of UHECRs is heavy.\textsuperscript{51} Initial studies such as refs 52–54 concluded that heavy nuclei cannot survive inside GRBs: photodisintegration on fireball photons would break them up into lighter nuclei and protons. Anchordoqui et al.\textsuperscript{55} calculated the neutrino emission from the injection of both protons and nuclei and found that the latter cannot survive in internal shocks; however, only collisions at very small radii, around $10^8 \text{ km}$, were considered. It has been argued that the typical collision radius is much larger (see, for example, ref. 43 and references therein) and that heavy nuclei can be largely loaded in GRB jets\textsuperscript{6,57}. Therefore, acceleration of nuclei to ultra-high energies and their survival against photodisintegration are possible, provided $R_C$ is large enough.\textsuperscript{25,58,59} In Fig. 4, we show that this is indeed the case for our simulations. The figure shows the maximum energy to which iron nuclei ($A = 56$, $Z = 26$) can be accelerated at each of the collisions. The energy is a factor of 26 higher than for protons (compare with Fig. 3b), where its absolute magnitude is a consequence of the assumed acceleration efficiency. Here the photodisintegration timescale has been calculated using the approximation in ref. 25. Triangles (blue) and circles (red) represent collisions in which the maximum energy is limited, respectively, by the break-up of the nucleus due to photodisintegration and by adiabatic losses. Even though photodisintegration losses dominate up to $\sim 10^9 \text{ km}$, after which adiabatic losses take over, maximum energies well within the UHE band can be achieved at the turning point, where most of the UHECR emission would come from. Note that this turning point is about a factor of five higher in $R_C$ than for protons (compare arrows in Figs 3b and 4), which means that UHECR nuclei on average reach their peak energy at higher $R_C$ than UHECR protons. UHECR nuclei may also escape directly at the highest energies, but there is no such thing as neutron escape. It is therefore expected that nuclei come from somewhat larger collision radii than protons at the highest energies, where the radiation densities are too low to break up the nuclei. Since the actual energy output of heavy nuclei depends on the nuclear loading (that is, an additional assumption is required), we do not show their energy output explicitly in the following.

To obtain an even more quantitative statement of how much energy is released as a function of collision radius, we show in Fig. 5 binned distributions for the prompt gamma rays, neutrinos and cosmic-ray protons, which are all directly calculable within our model. Figure 5a shows the energy output per bin, while Fig. 5b shows the fraction of energy in each bin compared with the total, for each particle species. We note that the energy per messenger per bin is obtained as a product of energy released per collision, and the number of collisions occurring per $R_C$-bin; especially the latter number is important to get the proper weighing of $R_C$. The result confirms the above observations: the neutrino production is dominated by small values of $R_C$ just beyond the photosphere from within a relatively narrow region $R_C \approx 10^{8.5}–10^9 \text{ km}$, the cosmic-ray production by intermediate $R_C \approx 10^9–10^{10} \text{ km}$ and the prompt gamma-ray emission is, in fact, dominated by large $R_C$, at around $10^{10}–10^{11.5} \text{ km}$—compatible with what is typically expected in the literature.\textsuperscript{24} These results have significant implications: our knowledge of the prompt phase of GRBs is obtained from gamma rays, of course, and, consequently, $R_C$ is derived from gamma-ray observations. This collision radius is, however, not the one to be used for neutrino or cosmic-ray calculations. It is therefore conceivable that multi-zone predictions are different from the naive one-zone expectation based on the gamma-ray emission radius. One can also read off from Fig. 5 that a significant amount of energy in UHECRs is transported away by direct escape, unrelated to neutrino production, which may affect the predicted neutrino flux if normalized to the observed UHECRs, as in, for example, ref. 34.

We show in Fig. 6 the predicted quasi-diffuse neutrino spectra from collisions beyond the photosphere as thick orange curves for three different values of $\Gamma_0$, where Fig. 6b corresponds to our standard assumptions. Note that the neutrino fluence per burst has been rescaled to a quasi-diffuse flux prediction by assuming 667 (identical) bursts per year and is significantly below the current diffuse neutrino signal reported by IceCube at the level of
\[ R_C \approx 10^{8.5} \text{ km, while proton escape dominates above this radius.} \]

The rough value of the photospheric radius and the assumed radius of the circumburst medium are indicated as dashed lines.

\[ \text{Figure 5} \ | \ \text{Energy dissipated beyond the photosphere.} \ \text{We consider energy dissipated in (prompt) gamma rays, neutrinos (all flavours) and CR protons (UHECRs from } 10^{10} \text{ to } 10^{12} \text{ GeV). Energies are binned as a function of the collision radius. (a) Absolute energy values; (b) the fraction of energy output normalized to one for each messenger. Neutron escape dominates the cosmic-ray emission below } R_C \approx 10^{8.5} \text{ km, while proton escape dominates above this radius. The rough value of the photospheric radius and the assumed radius of the circumburst medium are indicated as dashed lines.} \]

\[ 10^{-8} \text{ GeV cm}^{-2} \text{s}^{-1} \text{ sr}^{-1} \text{ flux}^{60}. \] The dashed curves correspond to the standard assumption that all collisions occur at the same radius, derived from gamma-ray observations. To generate these curves, we use the parameters \( N_{\text{coll}}, \Gamma, \) and \( T \) obtained from the simulation assuming identical shells with a collision radius obtained from equation (1) \( (R_C \approx 10^{9.2} \text{ km in Fig. 6b}). \) The reference flux in Fig. 6b is significantly lower than the prediction in ref. 16. In that reference, the same parameters as in the IceCube analysis\(^\text{61} \) were used for comparison, implying that \( R_C \approx 1.9 \times 10^{9} \text{ km}. \) That is about one order of magnitude smaller than the \( R_C \) used here; cf. equation (3) for its impact on the neutrino flux. The reference flux in Fig. 6a is comparable to ref. 16.

We first of all find that the neutrino spectra from collisions beyond the photosphere (thick orange curves) all exhibit the same flux level quite independently of \( \Gamma \) (and even of \( \Gamma \)), as we have explicitly tested. The expected neutrino flux per flavour is at the level of \( E^{3/2} \approx 10^{-11} \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}, \) peaking between \( 10^{5} \) and \( 10^{7} \text{ GeV}. \) This contribution can be regarded as a minimal prediction for the neutrino flux, as it can be inferred from gamma-ray observations and hardly depends on the parameters. Note that this flux is probably outside the sensitivity of the existing IceCube experiment, but it will provide a target for the optimization of the planned high-energy volume upgrade. There is a significant qualitative difference to conventional models such as refs 7,15, for which the pion production efficiency contains a factor \( \Gamma^{-4} \) coming from the collision radius estimate in equation (1) applied to equation (3). However, the optical thicknesses to Thomson scattering and photodisproportion interactions both scale \( \propto R_C^{-2}, \) which leads to the following estimate for the pion production efficiency at the photosphere independent of \( \Gamma \) (ref. 10):

\[ f_{p}^{\text{th}} \sim 5 \times \frac{\Gamma}{0.25} \times \frac{C_{0}}{0.1} \times \frac{1 \text{ keV}}{C_{\gamma,\text{break}}} \]  

Here \( \epsilon_{\gamma} \) is the fraction of the dissipated energy going into photons and \( \epsilon \) is the dissipation efficiency (ratio between dissipated and
From burst to burst increase, we show six different realizations of the predicted flux. The independence on the model parameters implies that the neutrino prediction above model parameters (see thin solid curves in Fig. 6), the neutrino flux prediction is relatively robust. The neutrino prediction above the photosphere hardly depends on the baryonic loading $\epsilon_n/E_s$ as well, as long as most of the energy is dissipated into protons. Increasing the baryonic loading in equation (5) is compensated by a correspondingly smaller $\epsilon_n$ in equation (4). As a result, the neutrino flux is roughly independent of $\epsilon_n/E_s$—which we have explicitly tested numerically.

We have also tested that this prediction does not depend on the variability timescale of the burst: Fig. 7 shows predictions for two different values of the emitter uptime $\delta t_{\text{eng}} = 0.1\,\text{s}$ (a) and $1\,\text{s}$ (b), where the fixed $N_{\text{sh}} = 1000$ leads to a longer burst duration $T$. Clearly, the quasi-diffuse flux coming from the simulations is independent of the value of $t_v$, as expected from equation (4). This markedly contrasts with the standard numerical prediction, in which larger variability timescales unavoidably imply lower particle densities at the source and, therefore, a reduced neutrino production. In Fig. 7c, $\delta t_{\text{eng}} = 0.1\,\text{s}$ with a reduced number of shells $N_{\text{sh}} = 100$ is chosen, corresponding to the light curve in Fig. 2b, that has a similar duration as the light curve for our standard simulation, but fewer pulses. In this case, the obtained result depends on the actual realization of the $\Gamma$-distribution, as only a few collisions dominate the neutrino flux and lead to strong variations—six different realizations of the $\Gamma$-distribution are shown in Fig. 7c. As expected, our conclusions hold for a sufficiently large sample of GRBs centred around our average prediction. We expect that these fluctuations become more severe for even fewer pulses from fewer collisions, as it has been studied for the contributions from different bursts in section 2 of ref. 47. The independence on the model parameters implies that the predicted flux $E^{2}J \sim 10^{-11}$ GeV cm$^{-2}$ sr$^{-1}$ s$^{-1}$ is very robust.

The only exception may be increasing $E_{\text{iso}}$ (see equation (5)) or the baryonic loading, which may in fact be required to match the injected energy needed to describe UHECR observations; see section 2 in ref. 34.

The photon spectra can still be approximated by the Band function up to a Thomson optical thickness of 10 or so$^{62,63}$, which occurs under the photosphere. This means that we can extrapolate our assumptions to below the photosphere to some degree. In the most extreme case, all energy may be dissipated into neutrinos, whereas the energy of neutrons, protons and gamma rays is reconverted into kinetic energy—this is, however, very speculative, as nonthermal particle acceleration may not occur efficiently$^{10}$. We show the corresponding subphotospheric extrapolations for the neutrino spectra as highly uncertain shaded regions in Fig. 6, corresponding to the contribution to the black squares in Fig. 3. Since the photospheric radius increases with decreasing $\Gamma$, the number of subphotospheric collisions increases with it, and their contribution in Fig. 6a can be much higher than in Fig. 6b (and in Fig. 6c much lower). As a consequence, the subphotospheric extrapolation may even reach the current sensitivity limit, and can be already constrained with current data. However, note again that this extrapolation is highly uncertain, as gamma-ray data cannot be used to obtain information about below the photosphere.

Finally, we show the ‘neutrino light curve’ for our standard parameter set in Fig. 2a as a dotted (red) curve; Fig. 2b shows it for a simulation with fewer collisions and longer emitter uptime, corresponding to the neutrino spectra in Fig. 7c. It can be clearly seen that the neutrino flux is typically much lower than the gamma-ray flux, except in some rare cases where the collision occurs close to the photosphere. Furthermore, the variation of the neutrino flux is larger due to the strong dependence of the pion production efficiency on $R_C$. One qualitative prediction that could help neutrino searches is that neutrinos are more likely to be associated with gamma-ray spikes that are pulses with very short variability timescales.

**Discussion**

In summary, we have studied neutrino, gamma-ray (at different energies) and cosmic-ray production in an evolving GRB outflow. We have demonstrated that they are produced at different collision radii. Consequently, the typical emission radius derived from prompt gamma rays cannot be directly applied to neutrino and UHECR production, and the GRB will look very different in different energy ranges.

**Figure 7** | Quasi-diffuse muon-neutrino spectra from simulations with alternative parameter sets. The $\nu_\mu + \bar{\nu}_\mu$ neutrino spectra in this figure should be compared with the ones obtained using our standard parameter set, Fig. 6b ($\delta t_{\text{eng}} = 0.01\,\text{s}$, $N_{\text{sh}} = 1000$ and $\Gamma_0 = 500$). (a,b) Two larger values of $\delta t_{\text{eng}}$ for $N_{\text{sh}} = 1000$, which leads to a longer burst duration $T$. (c) Using $\delta t_{\text{eng}} = 0.1\,\text{s}$ with a reduced number of shells $N_{\text{sh}} = 100$ (corresponding to the light curves in Fig. 2b), that is, $T$ is similar to the original example, but the light curve is less spiky because of fewer collisions. Since in that case the statistical fluctuations from burst to burst increase, we show six different realizations of the predicted flux.
from the point of view of different messengers. This concept is well known from conventional astronomical observations, where astrophysical objects look very different in different wavelength bands.

The neutrino spectra derived from gamma-ray observations are dominated by the emission close to the photosphere at $R_C \approx 10^{25} - 10^{27}$ km, as the pion production efficiency depends on the collision radius in a nonlinear way. UHECR protons have been shown to be produced at intermediate collision radii $R_C \approx 10^{25} - 10^{27}$ km, where the magnetic fields are high enough for efficient acceleration, but not so high that synchrotron losses limit the maximal proton energies. We have taken into account two possibilities for UHECR escape: emission as neutrons, which are not magnetically confined, and emission as protons from the edges of the shells—the dominant mechanism in each collision depends on the parameters of the colliding shells. Since the neutrons come from photohadronic interactions, their production dominates at smaller collision radii, where the $\gamma\gamma$ optical depth is higher, whereas protons tend to be directly emitted at large radii. Heavier nuclei can also survive for sufficiently large collision radii; their actual contribution depends on the nuclear loading. The main energy in gamma rays is deposited between around $10^{10}$ and $10^{11.5}$ km, compatible with earlier estimates. In particular, gamma rays at the highest energies, such as in the energy range only accessible to CTA, cannot come from collision radii $\lesssim 10^8$ km as the photon densities are too high there to let them escape.

For the quasi-diffuse neutrino flux prediction, we have identified two distinctive contributions. Above the photosphere, gamma-ray observations can be used to infer the pion production efficiency, which leads to a neutrino flux per flavour $\nu^2 \sim 10^{-11}$ GeV cm$^{-2}$ sr$^{-1}$ s$^{-1}$ for an assumed isotropic energy of $10^{52}$ erg emitted in gamma rays. Especially, there is no strong dependence on the Lorentz boost $\Gamma$, in contrast to conventional one-zone models, as both the photosphere and the pion production efficiency scale with the collision radius in the same way. This is the minimal neutrino flux that one would expect in stacking analyses based on the actual gamma-ray observations, such as ref. 14. There is also a significantly milder dependence on the baryonic loading, as this parameter changes the photosphere of the model at the same time that it rescales the neutrino flux. The prediction hardly depends on the time variability or number of pulses in the GRB light curve within a certain time window either. However, if the overall number of pulses is low, these will only come from a very small number of collisions, which means that large statistical fluctuations of the neutrino flux from burst to burst are expected even for the same parameter values. In that case, our observations have to be instead interpreted for a large enough ensemble of bursts. Note that the chosen isotropic energy and baryonic loading may not be sufficient to describe UHECR observations, see section 2 of ref. 34 for a detailed discussion, which will need to be addressed in a future study.

The neutrino flux is significantly lower than earlier predictions because (a) we have explicitly excluded subphotospheric contributions, (b) large photospheric radii have been obtained as a consequence of significant baryonic loadings (10) and the moderate energy dissipation efficiency of the fireball (25%) and (c) only a small number of collisions beyond the photosphere occurs at radii where the neutrino production efficiency is high. This expected ‘minimal’ flux is beyond the sensitivity of the current IceCube experiment, but could be reached in future high-energy extensions. No gamma-ray information from deep below the photosphere can be directly obtained, and the neutrino production in that regime is more speculative. In principle, however, a high-energy extension of the detector could also constrain the subphotospheric neutrino production.

Our results imply that model-dependent studies of the multi-messenger connection, such as a GRB stacking analysis of neutrino fluences, can be improved and give a stronger case for testing the hypothesis that UHECRs originate from GRBs. Compared with the one-zone model, some additional assumptions need to be made for the distribution of the collision radii. In particular, the width of the initial distribution of the bulk Lorentz factor $\Gamma$, with which the shells are set out by the central engine, turns out to be the key additional parameter. It can in principle be obtained from comparing the light curves between simulation and observation. On the other hand, we have the advantage that the uncertainty in $R_C$, which is the key issue in the standard model, disappears, as a collision radius distribution is now predicted by the theory. While we expect that the bulk Lorentz factor distribution has to be broad in some way to maintain a high dissipation efficiency, it remains to be studied how the results change for qualitatively different distributions. There should also be new opportunities stemming from our results: different messengers can be used to study different regions of an evolving GRB outflow. For instance, direct neutrino and gamma-ray observations in CT, or a single GRB would open windows to very different regions of the GRB.

During completion of this work, ref. 59 appeared, which shares some common aspects.

References

1. Piran, T. The physics of gamma-ray bursts. Rev. Mod. Phys. 76, 1143–1210 (2004).
2. Mészáros, P. Gamma-Ray Bursts. Rep. Prog. Phys. 69, 2259–2322 (2006).
3. Zhang, B. Gamma-ray bursts in the Swift era. Chin. J. Astron. Astrophys. 7, 1–50 (2007).
4. Milgrom, M. & Usov, V. Possible association of ultrahigh-energy cosmic ray events with strong gamma-ray bursts. Astrophys. J. Lett. 449, L37–L40 (1995).
5. Waxman, E. Cosmological gamma-ray bursts and the highest energy cosmic rays. Phys. Rev. Lett. 75, 386–389 (1995).
6. Vietri, M. On the acceleration of ultrahigh-energy cosmic rays in gamma-ray bursts. Astrophys. J. 453, 883–889 (1995).
7. Waxman, E. & Bahcall, J. N. High-energy neutrinos from cosmological gamma-ray burst fireballs. Phys. Rev. Lett. 78, 2292–2295 (1997).
8. Aartsen, M. G. et al. (IceCube Collaboration). Evidence for high-energy extraterrestrial neutrinos at the IceCube detector. Science 342, 1242836 (2013).
9. Murase, K. & Nagataki, S. High energy neutrino emission and neutrino background from gamma-ray bursts in the internal shock model. Phys. Rev. D 73, 063002 (2006).
10. Murase, K. Prompt high-energy neutrinos from gamma-ray bursts in the photospheric and synchrotron self-compton scenarios. Phys. Rev. D 78, 101302 (2008).
11. Baerwald, P., Hümmer, S. & Winter, W. Magnetic field and flavor effects on the gamma-ray burst neutrino flux. Phys. Rev. D 83, 067303 (2011).
12. Zhang, B. & Kumar, P. Model-dependent high-energy neutrino flux from gamma-ray bursts. Phys. Rev. Lett. 110, 121101 (2013).
13. Aartsen, M. G. et al. (IceCube Collaboration). First observation of PeV-energy neutrinos with IceCube. Phys. Rev. Lett. 111, 021103 (2013).
14. Abbasi, R. et al. (IceCube Collaboration). An absence of neutrinos associated with cosmic-ray acceleration in γ-ray bursts. Nature 484, 351–353 (2012).
15. Guetta, D., Hooper, D., Álvarez-Muiz, J., Halzen, F. & Reuveni, E. Neutrinos from individual gamma-ray bursts in the BATSE catalog. Astropart. Phys. 20, 429–455 (2004).
16. Hümmer, S., Baerwald, P. & Winter, W. Neutrino emission from gamma-ray burst fireballs, revised. Phys. Rev. Lett. 108, 231101 (2012).
17. Li, Z. Note on the normalization of predicted GRB neutrino flux. Phys. Rev. D 85, 027301 (2012).
18. He, H.-N. et al. Icecube non-detection of GRBs: constraints on the fireball properties. Astrophys. J. 792, 29 (2012).
19. Halzen, F. & Hooper, D. High-energy neutrino astronomy: the cosmic ray connection. Rep. Prog. Phys. 65, 1025–1078 (2002).
20. Piran, T. Gamma-ray bursts and the fireball model. Phys. Rep. 314, 575–667 (1999).
21. Zou, Y.-C. & Piran, T. Lorentz factor constraint from the very early external shock of the gamma-ray burst ejecta. Mon. Not. R. Astron. Soc. 402, 1854–1862 (2010).
22. Nakar, E. & Piran, T. Time scales in long GRBs. Mon. Not. R. Astron. Soc. 331, 1041–1048 (2002).
23. Bhat, P. N. Variability time scales of long and short GRBs, Proceedings of the 7th Huntsville Gamma-Ray Burst Symposium (GRB 2013), 14–18 April, 2013, Nashville, Tennessee. Preprint at http://arxiv.org/abs/1307.7618 (2013).
24. Nakar, E. & Piran, T. Gamma-ray burst light curves—another clue on the inner engine. *Astrophys. J. Lett.* 572, L139–L142 (2002).

25. Murase, K., Ioka, K., Nagataki, S. & Nakamura, T. High-energy cosmic-ray nuclei from high- and low-luminosity gamma-ray bursts and implications for multi-messenger astronomy. *Phys. Rev. D* 78, 023005 (2008).

26. Wang, X.-Y. & Dai, Z.-G. Prompt TeV neutrinos from dissipative photospheres of gamma-ray bursts. *Astrophys. J. Lett.* 691, L67–L71 (2009).

27. Murase, K., Kashiya, K. & Mészáros, P. Subphotospheric neutrinos from gamma-ray bursts: the role of neutrons. *Phys. Rev. Lett.* 111, 131102 (2013).

28. Bartos, I., Beloborodov, A. M., Hurley, K. & Márka, S. Detection prospects for GeV neutrinos from collisionally heated gamma-ray bursts with IceCube/DeepCore. *Phys. Rev. Lett.* 110, 241101 (2013).

29. Gao, S., Asano, K. & Mészáros, P. High energy neutrinos from dissipative photospheric models of gamma ray bursts. *J. Cosmol. Astropart. Phys.* 1211, 058 (2012).

30. Rees, M. J. & Mészáros, P. Dissipative photosphere models of gamma-ray bursts and x-ray flashes. *Astrophys. J.* 628, 847–852 (2005).

31. Baerwald, P., Bustamante, M. & Winter, W. UHECR escape mechanisms for protons and neutrons from GRBs, and the cosmic ray-neutrino connection. *Astrophys. J.* 768, 186 (2013).

32. Rachan, J. P. & Mészáros, P. Photophobia driven neutrinos from transients in astrophysical sources. *Phys. Rev. D* 58, 123005 (1998).

33. Mannheim, K., Protheroe, R. J. & Rachan, J. P. On the cosmic ray bound for models of extragalactic neutron production. *Phys. Rev. D* 63, 023003 (2001).

34. Guetta, D., Spada, M. & Waxman, E. On the neutrino flux from gamma-ray bursts: time-dependent simulations. *Mon. Not. R. Astron. Soc.* 375, 275–286 (1996).

35. Aartsen, M. G. et al. (IceCube Collaboration). Observation of high-energy astrophysical neutrinos in three years of IceCube data. *Phys. Rev. Lett.* 113, 101101 (2014).

36. Abbasi, R. et al. (IceCube Collaboration). Limits on neutrino emission from gamma-ray bursts with the 40 string IceCube detector. *Phys. Rev. Lett.* 106, 141101 (2011).

37. Pe’er, A. Temporal evolution of thermal emission from relativistically expanding plasma. *Astrophys. J.* 682, 463–473 (2008).

38. Beloborodov, A. M. Radiative transfer in ultra-relativistic outflows. *Astrophys. J.* 737, 68 (2011).

39. Abbasi, R. G. et al. (IceCube Collaboration). IceCube-Gen2: a vision for the future of neutrino astronomy in Antarctica, Preprint at http://arxiv.org/abs/1412.5106 (2014).

**Acknowledgements**

We thank V. Mangano, E. Waxman and B. Zhang for discussion and comments. This work is supported by NASA through Hubble Fellowship Grant No. 51310.01 awarded by the STScI, which is operated by the Association of Universities for Research in Astronomy Inc., for NASA, under Contract No. NAG 5-26555 (K.M.). M.B. and W.W. would also like to acknowledge support from DFG grants WI 2639/3-1 and WI 2639/4-1, and the ‘Helmholtz Alliance for Astroparticle Physics HAP’, funded by the Initiative and Networking Fund of the Helmholtz Association. P.B. acknowledges support from NASA grant NNX13AH50G. M.B., K.M. and W.W. would like to thank the Kavli Institute for Theoretical Physics at UCSB for its hospitality during the development of part of this work. This research was supported in part by the National Science Foundation under Grant No. NSF PHY11-25915.

**Author contributions**

All authors contributed to all aspects of this work, discussed the results and commented on the manuscript.

**Additional information**

Competing financial interests: The authors declare no competing financial interests.

Reprints and permission information is available online at http://npg.nature.com/reprintsandpermissions/

How to cite this article: Bustamante, M. et al. Neutrino and cosmic-ray emission from multiple internal shocks in gamma-ray bursts. *Nat. Commun.* 6:6783 doi: 10.1038/ncomms7783 (2015).