Numerical Analysis of a Rectangular Footing Resting on Two Inclined Layers of Soil

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Abstract. The geotechnical design is related to the amount of settlement and the value of the soil bearing, especially in the case of foundations based on several geological layers. Most of the previous studies concentrated on the analysis and design of foundations rested on the multiple horizontal layers. In the current study, numerical analysis was carried out to assess the behavior of a rectangular footing resting on two inclined layers of clay soil. The upper layer is soft soil underlain by hard soil. The soil layers inclined at different angles 0, 5, 10, 15, and 20° with different thicknesses for the upper weak layer (H) as a ratio between the thickness to the width of the footing (B) comprises H/B= 0.5, 1, 2, 3, and 4. The results showed that the amount of soil bearing decreases and settlement increases with increasing the thickness of the weak upper layer to the extent of H = 2B, after which the effect is almost non-existent. Also, the soil bearing decreases, and the settlement increases with increasing inclination of the subsoil layers, especially at thickness H/B < 2, after which there is no effect. The maximum decrease in soil bearing by an average value of 26.5% when changing the thickness H from 0.5B to 1B. With an increase in H/B more than two or the angle of inclination of the layers less than 10°, their effect is negligible on settlement.

Keywords: Inclined layers; rectangular footing; clay soil; numerical analysis; finite element method.

1. Introduction
The evaluation of the structures is related to the performance of the foundations on which they are rested. One of the most important criteria for designing foundations is the settlement within the permissible limits and the external loads not to exceed the bearing of the soil. Natural soils have been deposited in the layers specified as inhomogeneous layers soil, that the upper layer is weak compared to the bottom layer or vice versa. A square footing was analyzed by Reddy and Kumar [1] in the presence of two soil layers using (Plaxis 2D) software. It was found that changing soil properties causes a change in the deformation pattern according to the thickness of the top layer. By using ABAQUS program to analyze a square footing rested on two layers, the upper sandy layer and the lower clay layer, Saha et al. [2] found that the value of bearing capacity increased with the increase in the thickness of the sandy layer. The optimum value of the thickness of the sandy layer depends on the density of the sandy soil and the strength of the weak clay soils. Mandeel et al. [3] studied the bearing capacity of a square footing found on clay soil underlain by sandy soil and another case opposite condition using the 3D-Plaxis program. It was found that the maximum bearing capacity increases with the increase in the thickness of the upper layer in the case of clay soil over sandy soil and less in the case of clay soil under the sand. In another study Mandeel et al. [4] found that in the case of the two layers being clay soils, the bearing capacity depends mainly on the shear strength of the upper soil and that the size of the footing does not affect the
bearing capacity. A laboratory experiment conducted by Zedan and Abbas [5] on a square footing model based on two layers of sandy upper soil and the lower gypsum soil. The immersion of gypsum soil leads to a decrease in the strength of sandy soil at a thickness of B/2 and this effect decreases by increasing the thickness of the sandy layer.

3D-finite element analysis was performed on a rectangular footing over two-layered clays by Zhu and Michalowski [6]. It stated that the thickness of the weak soil layer and strength ratio were the main factors influencing the determination of the bearing capacity of the soil under the rectangular footing. Abbas [7] used the finite element method to examine the behavior of the shallow footing resting on multi-soil layers. It observed a differential settlement between the center and sides of footing by 4% and 9% of the left and right sides, respectively. Panwar and Dutta [8] conducted a numerical analysis on a rectangular footing resting on two layers. The upper layer is thick sandy soil and the lower layer is loose sand. It was found that the ratio (qo/Wy) increases with increasing the thickness ratio from (H/B=0.5) to (H/B=1.75), and the soil bearing ratio is not affected by increasing the thickness (H) more than 1.75B. Merifield and Naguyan [9] mentioned that the lower stiff layer does not affect the soil bearing when H/B>0.375.

Through the numerical study conducted by Moayed et al. [10] on a ring footing, it was observed that the thickening of the weak upper clay layer decreases the bearing capacity of the soil. Lee et al. [11] concluded that soil bearing does not depend on the shear resistance of the lower soil layer if it is within the critical depth (Hc/D) for a ring footing of diameter D and depth H. Yang et al. [12] examined the effect of several parameters on the behavior of a ring footing rested on sandy soil over clay soil. The parameters included the undrained shear strength of the clay layer, the thickness of the sand layer, the angle of internal friction of sand, and the ratio of the inner diameter of the ring to the outer diameter. Ibrahim [13] studied the effect of two layers of soil, the upper granular and the lower clay saturated weak under the circular footing. Different types of failure appeared in both layers, and the soil bearing depends on the extension of the granular layer below the footing. Misir and Laman [14] showed that the soil bearing was increased by the presence of granular fill under circular footing depending on the thickness of the granular layer. Ninghot and Khan [15] calculated the maximum bearing capacity of a circular footing based on sandy homogeneous soil, and the second is sandy soil composed of two layers of sand (weak over strong). The bearing capacity of soil consisting of two layers at a specific settlement of 50 mm, decreases by increasing the thickness of the weak sand layer to the extent (H = 2B), and then it behaves as a homogeneous soil.

Ramadan and Hussien [16] found that the bearing capacity of the soil for a strip footing placed on top of a sandy layer underneath a layer of clay soil increases with the increase in the relative density of the upper sand layer. Papadopoulou and Gazetas [17] stated that with the increase in the inclination of the load imposed on strip footing resting on two layers of clay, the effect of the second layer decreases. All previous studies focused on the effect of the existence of two horizontal layers on the bearing capacity of the footings rested on them. In this study, the effect of the inclination of the layers under the footing on the bearing capacity of soil and settlement has been considered. The study was performed by a finite element method using the 3D-Plaxis program. Assume that a rectangular footing is rested on soil consisting of two layers. The upper layer consists of soft soil underlain by a hard layer. The study also included several parameters, such as the effect of the layer’s inclination and the thickness of the weak upper layer.

2. Finite element analysis

2.1 Modelling technique

PLAXIS-3D software was adopted in the present study. The model dimensions were selected to prevent the effect of the lateral and the lower boundaries on the deformation of the system [18]. Figure 1 illustrates a schematic diagram of the finite element model. The footing of size 2.5x3.6 m is placed at the top surface of the soil on the center of the model. Figure 2 shows the numerical model's typical finite element mesh and the refinement around the footing region. The total number of models solved was 25.
2.2 Constitutive models and parametric study

In the current study, the behavior of two-layered soils underneath a rigid rectangular footing has been studied. The footing has been modeled as linear elastic materials, while the Mohr-Coulomb model has simulated the soil. The ultimate capacity of footing has been defined from the load-settlement curve according to the (10% B) method [19,20]. The width of footing (B) and the thickness of the upper layer from the center of footing is (H). Different H/B ratios that are 0.5, 1, 2, 3, and 4 and different angles for inclination (i) of sub-soil layers that 0, 5, 10, 15, and 20° have been studied. The properties of the soils and footing used in the finite element analysis are presented in Table 1.

| Property | Unit | Soft clay | Stiff clay | Footing |
|----------|------|-----------|------------|----------|
| Model    |      | Mohr-Coulomb | Mohr-Coulomb | Linear-Elastic |
| Material type | Drained | Drained | Nonporous |
| γ_unsat | kN/m³ | 16 | 17 | 24* |
| γ_sat | kN/m³ | 17 | 19 | --- |
| E | kN/m² | 5000 | 20000 | 3×10⁷ |
| ν | --- | 0.35 | 0.25 | 0.2 |
| c | kN/m² | 10 | 20 | --- |
| ϕ | degree | 20 | 25 | --- |
| ψ | degree | 0 | 0 | 0 |
| R_interface | --- | 1 | 1 | Rigid |

* Depending on ACI Code (318M-11) (E = 4700√f_c′, f_c′ = 25 MPa).
3. Results and discussion
The purpose of this study is to survey the influence of the height of the top layer and the inclination subsoil layer on bearing capacity and settlement of rectangular footing resting on two layers of soil that are soft clay over stiff clay. The ultimate capacity of footing is defined from load-settlement curves according to (10% B) method and the settlement estimated at working applied load of 150 kPa.

3.1. Load-settlement relationship
The load-settlement relationship is illustrated in Figure 3. It is clear that with increasing H/B values, soil bearing capacity decreases. In general, all the values of the layers’ inclination angle (i) showed the same load-settlement trend. Since the thickness of the layer H/B=0.5 gave the greatest resistance, the reason is that the small thickness of the weak upper layer does not significantly affect the bearing of the soil because the stressed bulb reaches the strong bottom layer. With an increase in the thickness of the upper layer to H/B=1, a large difference is noticed. According to Michalowski [21], there exists a so-called critical depth where the strength of the bottom layer does not affect the bearing capacity. The effect of the thickness of the weak upper layer (H) is slightly after H/B=2, where most of the stresses are concentrated within the upper layer, which makes the presence of the strong layer ineffective. Figure 4 shows, the failure pattern remains within the soft upper layer when (H/B=2).

![Load-Settlement relationship](image1)

**Figure 3.** Load-Settlement relationship.

3.2. Variation of bearing capacity with the thickness of upper layer
The variation of the soil bearing (UBC) with changing the thickness of the weak upper layer (H) is shown in Figure 5. It is evident that a sharp dropping in the soil bearing value occurs when the thickness of the upper layer changes from H=0.5B to H=1B. The reductions in the soil bearing are 28.0, 28.4, 26.6, and 23.4% for inclination angle (i) of the subsoil layer of 5, 10, 15, and 20°, respectively.
The soil bearing continues to decrease with the increasing thickness of the weak layer to H=2B, but at a lower rate as the soil bearing decreases by 14.6, 12.7, 9.1, and 11.2% for \( i = 5, 10, 15, \text{ and } 20^\circ \) respectively. A reduction in soil bearing of 9% occurs when the thickness increases from H=1B to H=2B at an inclination angle 15° in comparison to horizontal layers. With an increase in the thickness of the weak layer more than 2B, the bearing capacity slightly affected up to H=4B. This is because the major stresses are distributed in the weak layer in the case of its thickness of 2B or more, thus giving close results. This is referring to the insignificant effect of the hard lower layer on soil bearing when the thickness of the weak upper layer becomes more than 2B. This means that the shear strength of the weak layer's main factor affects the soil bearing when it is (H/B > 2). Meanwhile, Mosadegh and Nikraz [22] found that the bearing capacity of a footing is affected by the properties of both layers if its dimension is relatively large compared to the thickness of the upper layer.

3.3. Variation of settlement with the upper layer thickness

Figure 6 shows a considerable increase in the settlement at working stress of 150 kPa with an increase in the thickness of the weak upper layer to H=1B. Here also, a high rate of increase in settlement appears until the upper layer thickness 2B. The settlement increases by 61.1, 127.5, 142.2, and 146.7%, in the
3.4. Effect of layer inclination on soil bearing

The change of layers inclination angle (i) was studied on soil bearing with a different thickness of the upper soil layer (H). Figure 7 shows the effect of the inclination of the layers on soil bearing (UBC) associated with a smaller thickness, i.e., H ≤ B. As the thickness of the weak upper layer (H) increases to more than 2B, the effect of the inclination of the layers on bearing capacity vanishes. Bearing capacity values converge for all values of H ≥ 2B.

At a value of H=0.5B, the soil bearing decreases from 447 kPa to 385 kPa when i increases from 5° to 20°. The reductions in the bearing capacity are 1.6, 9.2, and 13.9% when the inclination of the subsoil layer increases from 5 to 10, 15°, and 20° respectively. It is less affected in case H=B where the corresponding reductions are 2.2, 7.5, and 8.4%. This is because most stresses dissipate through the thickness of the weak upper layer of more than 2B, after which the increase in thickness of the upper layer does not affect. In the case of thicknesses less than 2B, one part of the stresses is distributed in the upper layer and the other part through the lower layer, and therefore the layers’ inclination has an effect.
3.5. Effect of layer inclination on settlement
Figure 8 shows the variation in settlement with an inclination of the layers under the foundation for inclined angles (i) from 0˚ to 20˚. In general, the settlement increases with the increase in the i of the layers, and the effect is more pronounced at i>10˚, while the effect of the layers’ inclination less than 10˚ is practically minor. The settlement increases by 0.4, 4.3, and 9.0% when increasing the inclination from 5˚ to 10˚, 15˚, and 20˚ in the case of the thickness of the upper layer 2B. As for the case of the thickness of the upper layer 1B, the corresponding increases are 6.2, 11.8, and 13.1%. For the lower thickness of 0.5B, a slight increase in the settlement with the inclination angle of the soil layer vanishes with increases H. In general, it can be said that the effect of the inclination of the layers is associated with the decrease in the thickness of the weak upper layer.

![Figure 8. Variation of settlement with layer inclination.](image)

3.6 Tilt of footing
The variation of the tilt of the footing with the thickness of the weak upper layer (H/B) and with the inclination of the soil layers (i) is shown in Fig. 9. In general, the change in the footings’ tilt for all the angles of inclination of the layers and thickness (H/B) shows similar behavior except in the case of the angle of inclination 10˚ and thickness H/B=0.5, it showed a different behavior.

![Figure 9. Amount of tilt of footing.](image)
The tilt of footing is 1.4, 4.1, 0.9, 2.7, and 3.8° at the inclination angles of the soil layers 0, 5, 10, 15, and 20° respectively. The tilt decreases sharply with the increase in the thickness of the weak upper at H/B=1 to become the corresponding footings’ tilt of 0.5, 1.3, 1.4, 1.1, and 0.5°. Then, a slight change appears with an increase in the H/B greater than 2. It concludes that the thickness of the upper layer (H) greatly influences the tilt of footing, while there is no clear relationship between the tilt and the inclination of the soil layers. It can be attributed to the cause of the tilt of the footing to the changes in the distribution of the pressure under the footing as a result of changing the value of H, as shown in Figure 10. In addition, the tilt is caused due to the increase in the horizontal displacement or sliding of the footing towards the slope of the layers, as shown in Figure 11. In the case of horizontal layers (i=0), a tilt of footing was recorded in the case of H/B≤1, but when the thickness of the upper layer increases, the tilt is null. Reddy and Kumar [1] mentioned the same finding.
4. Conclusions
The following conclusions are made from the results of the present study:

- The ultimate bearing capacity of soft clay over stiff clay decreases with the increase in the thickness of the upper soft soil layer. Insignificant reduction in the bearing capacity beyond the thickness of the upper layer \( H>2B \).
- The maximum decrease in soil bearing is between 23.4% to 28.0% when changing the thickness of the weak upper layer from 0.5B to 1B.
- The ultimate bearing capacity decreases with an increase in the inclination subsoil layers angle for layer thickness up to \( H=2B \), after which insignificant effect.
- The effect of the inclination of the soil layers is greater when the thickness of the upper layer is 0.5B, where the amount of soil bearing decreases by 28% when increasing the inclination from 5° to 20°. The effect of the inclination of the layers decreases when the thickness of the weak upper layer is \( H>B \).
- Increase settlement by increasing the inclination of the subsoil layer angle. The inclination of a layer of more than 10° has a considerable effect on the settlement, which exhibited a maximum increased by 13%.

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