Analyzing chaotic effect of reinforced concrete pile driving in cohesive soil

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Abstract. This paper presents the chaos phenomenon in pile driving. Firstly, the pile wave equation that Smith put forward is used for painting Poincare section, and then the basic features of chaos are obtained. By analyzing the Poincare section, a point with which the ratio of penetration to velocity is the largest is discovered, and that is to say energy consumption is the lowest at this point. Moreover, the relationship between hammer weight and dropping height is proved to satisfy the construction experience of light blow with heavy hammer through curve fitting. So according to the theory, the value of impact force and initial value can change to get the hammer weight and dropping height for different square and circular piles under the lowest energy consumption. The results show it is better to use square piles during driving, and the relationships between square and circular piles with different cross-sections and its impact force on pile top, hammer weight and dropping height satisfy a certain function equation, respectively. Hence, the corresponding hammer weight and dropping height can be selected to save energy consumption and reduce the cost during construction according to their functional relation.

1. Introduction
In 1931, D.V.Isaacs[1] proposed the fluctuation phenomenon of pile driving for the first time, and he brought pile shaft resistance R’ into the classical one-dimensional wave equation.In 1962, E.A.L.Smith[2] used finite difference method to solve the one-dimensional wave equation, this method was widely used in practice because of the rapid development of electronic computer.Take et al.[3] analysed the pile stress wave produced by using numerical calculation method.A numerical model for the prediction of free field vibrations due to vibratory and impact pile driving was presented by Masoumi and Degrande[4].An axisymmetric finite difference numerical model was developed having solid elements for both pile structure and the soil media surrounding and below the pile by Shahram Feizee Masouleha and Kazem Fakharian[5]. Y. Guo and co-workers treated the pile driving as multi-body dynamic contacts and used the penalty function method and three-dimensional model of finite-element method to obtain the dynamic process of pile driving[6]. Accuracy of predicting pile capacities have been investigated by pile driving formulas[7]. The main work of chaos theory is to study nonlinear dynamic system, and the chaos theory reveal the simple rules that may be hidden in a seemingly random phenomenon [8]. So the chaotic phenomena also exist in piling process due to its nonlinear dynamical system, but there is no report about chaotic motion of piling.

In this paper, the Poincare section is plotted in MATLAB by using the classical one-dimensional wave equation proposed by Smith, and it proves that the chaotic phenomena exist in the process of piling. By analyzing the Poincare section, the required hammer weights and dropping heights under the lowest energy consumption are obtained by changing the initial value of equation and impact force.
on the pile top. Based on these results, the values of hammer weights and dropping heights in construction are calculated for square and circle piles with different cross-sections, and the relationship is got from their fitting curve. So it provides theoretical basis for the required construction equipment of piling.

2. Material and Methods

2.1 Smith’s one dimensional wave equation

The equivalent model of the wave equation is established by applying Smith method, and then the displacement function is only related to time. To make the two-dimensional function \( S(x, t) \) into an one-dimensional function \( S(t) \)[9], the equation that satisfy displacement functions is as follows:

\[
m_i \frac{d^2 S_i(t)}{dt^2} + \eta_i \frac{dS_i(t)}{dt} + k_i S_i(t) = F_i(t)
\]

Where \( F_i(t) \) = axial force acting on the i-th segment at the moment of t.

\( \frac{dS_i(t)}{dt} \) = vibration velocity.

\( \eta_i \) = friction factors of the soil around the pile.

The first item of the left in equation (1) is inertia reaction force of the segment, the second is lateral soil resistance and the third is elastic reaction force.

The initial conditions are given as:

\[
\begin{align*}
S_i(0) &= 0 \\
S'_i(0) &= \frac{nm_i}{nm_i + m} \sqrt{2gh_i}
\end{align*}
\]

Where \( m_i \) = hammer weight

\( \xi = \text{effective coefficient of hammer, } \xi = 0.9. \)

\( h_\text{=} \text{the height between hammer and pile top.} \)

\( S'_i(0) \) is obtained by using principle of conservation of momentum at first blow.

2.2 Do the transformation of variable for the wave equation

Do the transformation [10] for equation (1) and assume:

\[
s_i = S_i, \quad s_1 = \frac{dS_i}{dt}
\]

The follow equation can be got by substituting equation (3) in equation (1).

\[
\begin{align*}
\frac{ds_1}{dt} &= s_1 \\
\frac{ds_2}{dt} &= F(t) - \eta_i \cdot s_2 - k_i \cdot s_1 - m_i
\end{align*}
\]

Where \( m_i = A_i \rho_i \), the mass of pile for the i-th segment.

\( A_i \cdot l\cdot \rho_i \) = the area of cross-section, the length, and the density of material for the segment, respectively. The density of concrete pile can be \( 2.4 \times 10^3 \text{ kg/cm}^3 \).

\( k_i = \frac{A_i \rho_i E}{l} \text{ t/m, the equivalent spring coefficient.} \)

\( E= \text{the young's modulus of material, } E=3.2 \times 10^3 \text{ kg/cm}^2. \)

3. Results

Dividing reinforced concrete pile of \( 25 \text{cm} \times 25 \text{cm} \times 8 \text{m} \) into 8 segments along the length direction of pile and studying driving efficiency. And assuming the impact load \( F=1500 \text{kN} \) at first blow. In clay soil, \( \eta = 0.16. \)

The blow frequency for diesel hammer is about 40-80 times per minute, so the blow period is \( T=1.2 \text{s}. \) And secondly, because the sample interval needs to satisfy the condition of \( \Delta t \leq t_c = NC \)[11],
the wave velocity \( C \) in reinforced concrete is taken as 3600m/s, and \( N \) is the length of each segment. The displacement curve, phase trajectory and Poincare section of piling are obtained according to the period and the sample interval when initial value is \((2, 0)\), respectively. And they are as shown in figure 1~3.

In figure 1, penetration of pile decreases with time in one period of piling. The phase plane diagram within one cycle is not a closed orbit in figure 2. In figure 3, there are a lot of points with fractal structures. The phase trajectory curve of three periods and the Poincare section for the initial value of \((2.5, 0)\) are shown in Figure 4 and 5, respectively. They show it is sensitive to initial values, and it has the complex fractal structure and self similar. The basic characteristics of chaotic motion are obtained according to contrasting figure 2 with Figure 4, and figure 3 with figure 5: randomness, self-similarity and ergodicity. From the three aspects above, the emergence of chaos in the process of piling is proved.

Figure 1. The time-path curve of pile driving.

Figure 2. Phase trajectory curve of pile driving within one cycle.

Figure 3. The Poincare section of initial value \((2.0,0)\).
4. Discussion

4.1 The calculation process of required hammer weights and dropping heights for the piling.

The maximum ratio of displacement to velocity is discovered from figure 3 and figure 5, namely when it has the maximum penetration, the value of vibration rate is the minimum, and energy consumption is the lowest. And moreover the corresponding hammer weights and dropping heights can be calculated on the basis of $v = \sqrt{\frac{gh}{\varepsilon}}$ and the initial condition. Therefore, the values of hammer weights and dropping heights in each case can be got through changing the impact force $F$ of pile top and initial value for different cross-sections. After studying, the corresponding hammer weights and dropping heights of the maximum value of $S/v$ are the parameters that should be selected in construction for precast reinforced concrete piles, and energy consumption is the lowest right now. The construction experience of light blow with heavy hammer while piling[12] is attested by fitting the curve between hammer weights and $S/V$ in the foregoing analysis(Figure 6). Therefore, the values of hammer weights and dropping heights are reasonable by analyzing of chaotic motion.

4.2 The relationships between cross-sectional dimensions and its corresponding impact forces, hammer weights and dropping heights

For square and circular piles, the curves between cross-sectional dimensions and its required impact forces, hammer weights and dropping heights are shown in figure 7~9. In order to make sure that the energy consumption of piling machine is the lowest during construction, the relationships between cross-sectional dimensions of square piles and the impact forces of pile top and dropping heights are in accord with the curve of power function, while fulfilling the exponential relationship with hammer weights. And for circle piles, when energy consumption is the lowest, the curve between cross-sectional dimensions and impact forces of pile top is nearly exponential function, and cross-sectional
dimensions and the hammer weights and dropping heights satisfy the polynomial curve. The suitable of hammer weights and dropping heights can be chosen by the above-mentioned relationships during the construction of precast reinforced concrete piles. Thus it has the trait of great adaptability to guide the construction.

Figure 6. The relationship between hammer weight and s/v.

Figure 7. The relationship between impact force and sectional dimension in the minimum power consumption.

Figure 8. The relationship between hammer weight and sectional dimension in the minimum power consumption.
5. Conclusions
(1) The chaotic phenomena arise in the process of piling.
(2) In order to make the energy consumption of pile driver is the lowest, the appropriate hammer weights and dropping heights can be selected on the basis of their functional relation.
(3) On the basis of the practical situation of construction site, the eigenvalue of the pile and the soil can change to get the optimal hammer weights and dropping heights for other types of pile in different soil conditions, so it can save energy consumption and guide the construction.

Nowadays, chaotic analysis has been applied in more and more areas, and it has a very important significance for guiding engineering practice. Study on chaotic phenomena is mainly used for predicting the future state by establishing non-linear statistical model. Therefore, the impact of chaos effect to piling can be studied, and then control or strengthen the emergence of chaos.

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References
[1] D.V. Issacs, Transaction of the Institution of Engineers, Australia XII, 312-323(1931)
[2] E. A. L. Smith, Journal of the Soil Mechanics & Foundations Division, 86, 35-61 (1960)
[3] W.A. Take, A.J. Valsangkar, M.F. Randolph, Computers and Geotechnics, 25, 57-74 (1999)
[4] H.R. Masoumi, G. Degrande, Journal of Computational and Applied Mathematics, 2, 503-511 (2008)
[5] Shahram Feizee Masouleha, Kazem Fakharian, Computers and Geotechnics, 3(35), 406-418 (2008)
[6] Y. Guo, J. P. Hu, L. Y. Zhang, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 5, 1153-1161 (2011)
[7] M. Ali, M. Selamat, S. Yii, et al, Geotechnical and Geological Engineering, 3, 351-361 (2011)
[8] E. N. Lorenz, J Atmos Sci, 20, 130-141 (1963)
[9] L. Lei, Pile Dynamics (Metallurgical Industry Press, 2000)
[10] Y. Zhong, Discuss chaos and fractal (Peking University Press, 2010)
[11] R. Liu, R. Zhuo, S. Yan, et al, Rock and Soil Mechanics, 25, 383-387 (2004)
[12] L. Xie, Sichuan Building Science, 3, 13-19 (1984)