Causes of Irregular Energy Density in $f(R, T)$ Gravity

Z. Yousaф\(^1\) * Kazuharu Bamba\(^2\) † and M. Zaeem ul Haq Bhatti\(^1\) ‡

\(^1\) Department of Mathematics, University of the Punjab,
Quaid-i-Azam Campus, Lahore-54590, Pakistan
\(^2\) Division of Human Support System,
Faculty of Symbiotic Systems Science,
Fukushima University, Fukushima 960-1296, Japan

Abstract

We investigate irregularity factors for a self-gravitating spherical star evolving in the presence of imperfect fluid. We explore the gravitational field equations and the dynamical equations with the systematic construction in $f(R, T)$ gravity, where $T$ is the trace of the energy-momentum tensor. Furthermore, we analyze two well-known differential equations (which occupy principal importance in the exploration of causes of energy density inhomogeneities) with the help of the Weyl tensor and the conservation laws. The irregularity factors for a spherical star are examined for particular cases of dust, isotropic and anisotropic fluids in dissipative and non-dissipative regimes in the framework of $f(R, T)$ gravity. It is found that as the complexity in the matter with the anisotropic stresses increases, the inhomogeneity factor has more correspondences to one of the structure scalars.

Keywords: Relativistic systems; Instability; Spherical systems.

PACS: 04.40.Cv; 04.40.Dg; 04.50.-h

Report number: FU-PCG-12

*zeeshan.math@pu.edu.pk
†bamba@sss.fukushima-u.ac.jp
‡mzaeem.math@pu.edu.pk
1 Introduction

The influence of modification in gravity theories have gained significant attention due to the motivation in both high energy physics and cosmology. Since there exist observational evidences of the accelerating universe \([1, 2, 3, 4, 5]\) but still some predictions and compelling theoretical work about the expansion of the universe is under consideration. After the successful detection of the gravitational waves, it is still possible that the cosmological constant added by Einstein in his field equations can describe the accelerating phase of the universe. However, the unnatural fine tuning problem favors the possibility of dark side in the universe due to the dark dynamical effects. To test the viability of any gravitational theory, the most significant way is to match their predictions with real object’s motion.

The modification in the Einstein’s theory to discuss the dark effects involves generalization in the Lagrangian of Einstein-Hilbert action. The simplest generalization is to use function \(f(R)\) instead of Ricci scalar in the action. However, in order to include the matter contents, a simplest generalization is to replace \(R\) with \(f(R, T)\), where \(T\) represents the trace of stress-energy tensor. It is noted that such addition in the Lagrangian can be observed as the addition of new degrees of freedom. The equation of motion emerging from such kind of Lagrangian will differ from the Einstein’s one. In that case, it would be possible to eliminate the cosmological constant to describe the acceleratory phase of the universe. Such Lagrangians have much significance to study the dark energy (DE) and dark matter (DM) problems and much attention has never been made along this direction (for reviews on the late-time cosmic acceleration, i.e., dark energy problem, and modified gravity theories, see, e.g., \([6, 7, 8, 9, 10]\)).

Theories involving curvature matter coupling have attained significant attention to explore the enigma of cosmic evolution and other cosmological aspects. A geometry matter coupled system results in existence of extra force due to non-geodesic motion of test particles and such systems in the setting of Lagrangian for \(f(R, T)\) have been introduced by \([11]\). It has been observed that the extra force vanishes if one used the specific form of the Lagrangian for usual matter (e.g. \(L_m = p\)) for non-minimally coupled \(f(R)\) theories \([12, 13]\), however, the extra force does not vanish for matter geometry coupled system. The \(f(R, T)\) gravity theory is considered as a useful candidate to study the acceleratory behavior during the cosmic expansion which is not only due to the scalar-curvature part but includes the matter components as well. This theory is also considered as a useful candidate among the modified gravities which is based on the non-minimally curvature matter coupling. This theory came into with the background that the cosmological constant could be considered as the trace dependent function, i.e., \(\Lambda(T)\) gravity. This is done to make the interaction between the usual cosmic matter and DE which is supported by some modern cosmological data \([14]\).
Faulkner et al. [15] explored the $f(R)$ theory and try to equate it with the scalar tensor theories with two classes of models: massive and Chameleon $f(R)$ models. In recent years, it has been observed that most of the models proposed in $f(R)$ gravity does not satisfy the weak-field solar system constraints [16]. Harko et al. [11] extended the Einstein’s standard relativity theory to $f(R, T)$ and investigated that higher-curvature theories can assist enough to resolve the flatness issue in the rotation curves of galaxies. The field equations for some specific models with explicit $f(R, T)$ configuration has also been presented. Reddy et al. [17] discussed the the Bianchi type III universe model with perfect matter configuration in the background of $f(R, T)$ gravity to study the early universe. Adhav [18] has found some interesting Bianchi-I universe model in this theory.

Sharif and Yousaf [19] examined the stability of isotropic compact objects framed within $f(R, T)$ gravity and found relatively more stable and compact objects than that observed in $f(R)$ gravity. Sun and Huang [20] analyzed cosmic evolution in $f(R, T)$ gravity by means of redshift fluctuations against distance modulus and confronted good fitting numerical plots consistent with the observational data of astronomy. Baffou et al. [21] investigated dynamical evolution along with the stability of power-law and de Sitter cosmic models against linear perturbation. They concluded that such models can be considered as a competitive dark energy candidate. Alves et al. [22] explored the physical behavior of gravitational waves in different formalisms of $f(R, T)$ gravity models and showed that the gravitational wave spectrum have strong dependence on $f(R, T)$ model. Alhamzawi and Alhamzawi [23] explored a considerable contribution of $f(R, T)$ gravity on gravitational lensing and found comparable results with those already exists in literature, thereby suggesting the viability of this theory. Recently, Yousaf and Bhatti [24] observed more restricted unstable Newtonian and post-Newtonian regimes in $f(R, T)$ gravity (as compared to $f(R)$ gravity) for the locally anisotropic collapsing stellar model.

The emergence of curvature singularity for the stellar systems has been discussed in $f(R)$ gravity theory [25, 26]. Houndjo [27] performed the cosmological reconstruction of $f(R, T)$ gravity examining the transition of matter dominated epoch to the late time accelerated regime. Alvarenga et al. [28] formulated energy conditions depending upon the attractive nature of gravity using Raychaudhuri equation. Also, they investigated the viability of some particular $f(R, T)$ gravity models using these energy conditions. Azizi [29] carried out the possibility whether static spherically symmetric traversable wormhole geometries (which are basically the exotic cosmic models) exist in $f(R, T)$ gravity.

In the study of accelerated universe expansion, the viscosity effects due to matter configurations are quite vital and appearing as the only non-adiabatic way in FRW models. Bulk viscosity disburse negative pressure offering thus providing a platform for negative pressure indicating repulsive gravity. During the particle creation and formation of galaxies and clusters in the early universe, neutrinos decouple from the cosmic fluid and viscosity arises in
the system \cite{30}. Naidu et al. \cite{31} studied the cosmological model with FRW metric in the presence of viscosity in $f(R,T)$ gravity. Reddy et al. \cite{32} investigated the Kaluza-Klein universe model in the presence of viscosity with the background of $f(R,T)$ modified gravity theory. Sharif and his collaborators \cite{33} have explored some physical processes with shear-free as well as expansion and expansion-free self-gravitating collapsing objects. Kiran and Reddy \cite{34} determined the solutions of field equations in $f(R,T)$ gravity theory for Bianchi type III spatially homogeneous model.

Nojiri and Odintsov \cite{35} claimed that inflationary modified high degrees of freedom quantities boosts up the evolution of Schwarzschild-de Sitter black hole anti-evaporation in classical background. Farinelli et al. \cite{36} discussed equilibrium state of the hydrostatic celestial objects and concluded that wide range of compact objects exists in the nature of modified gravity. Guo et al. \cite{37} investigated dynamical behavior of spherical relativistic collapse in modified gravity. Albareti et al. \cite{38} analyzed homogeneous cosmological models through Raychaudhuri expressions and produce some viability constraints coming due to modified gravity theory expansion regimes of the universe. Hason and Oz \cite{39} observed extended configurations of Jeans instability condition for the relativistic systems for normal and super fluids.

During the evolution of star model, a large amount of radiations emit in the form of photons and neutrinos which gradually increases as the evolution proceeds. The radiating energy can be characterized in two approximations i.e., diffusion and free-streaming approximation. The diffusion limit is applicable when the typical length of the object is greater than the mean free path of the particles responsible for the motion of energy. In that case, the dissipation is described by a heat flow type vector while in the other case it is characterized by an outflow of null fluid. Herrera et al. \cite{40} investigated that the energy density should be inhomogeneous if the system is involving with zero expansion condition in non-dissipative fluid background. Herrera \cite{41} explored some factors for a self-gravitating spherical star which are important to describe the irregularities in the matter distribution. Sharif and his collaborators \cite{42} have also explored some factors describing the inhomogeneous density distribution for self-gravitating objects with different matter configurations.

In a recent paper, Yousaf et al. \cite{43} have formulated some dynamical variables by splitting the Riemann tensor into its constituent trace and trace-free scalar parts in $f(R,T)$ theory. They have also discussed the evolution of shear and expansion using the Raychaudhuri equation. This paper is organized in the following manner. In the next section, we will provide some basic equations including the action of this framework and equation of motion. In section 3, modified field equations, some kinematical and dynamical quantities as well as modified Ellis equations are formulated for the construction of our analysis in a systematic way. Section 4 explores the irregularity factors with some particular cases dissipative and non-dissipative matter distribution. Finally, we conclude our results in the last section.
2 \( f(R, T) \) Gravity and Spherical Systems

The notion of \( f(R, T) \) gravity as a possible modifications in the gravitational framework of GR received much attention of researchers. This theory provides numerous interesting results in the field of physics and cosmology like plausible explanation to the accelerating cosmic expansion \([6, 8, 9]\). The main theme of this theory is to use an algebraic general function of Ricci as well trace of energy momentum tensor in the standard EH action. It can be written as \([11]\)

\[
S_{f(R,T)} = \int d^4 x \sqrt{-g} [f(R, T) + L_M],
\]

where \( g, T \) are the traces of metric as well as standard GR energy-momentum tensors, respectively while \( R \) is the Ricci scalar. There exists variety \( L_M \) in literature which corresponds to particular configurations of relativistic matter distributions. Choosing \( L_M = \mu \) (where \( \mu \) is the system’s energy density) and making variation in the above equation with \( g_{\alpha\beta} \), the corresponding \( f(R, T) \) field equations are given as follows

\[
G_{\alpha\beta} = T_{\alpha\beta}^{\text{eff}},
\]

where

\[
T_{\alpha\beta}^{\text{eff}} = \left[ (1 + f_T(R, T)) T_{\alpha\beta}^{(m)} - \mu g_{\alpha\beta} f_T(R, T) - \left( \frac{f(R, T)}{R} - f_R(R, T) \right) \right] \frac{R}{2} +

+ \left( \nabla_\alpha \nabla_\beta + g_{\alpha\beta} \Box \right) f_T(R, T) \frac{1}{f_R(R, T)}
\]

is a non-standard energy-momentum tensor representing modified version of gravitational contribution coming from \( f(R, T) \) extra degrees of freedom while \( G_{\alpha\beta} \) is an Einstein tensor. Further, \( \nabla_\alpha \) represents covariant derivation while \( f_T(R, T), \Box, f_R(R, T) \) indicate \( \frac{df(R, T)}{dT}, \nabla_\alpha \nabla^\alpha \) and \( \frac{df(R, T)}{dR} \) operators, respectively.

We consider a spherical relativistic self-gravitating non-rotating and non-static system. The metric of whose can be expressed with the help of the following diagonal form form

\[
ds^2 = -A^2(t, r) dt^2 + B^2(t, r) dr^2 + C^2 d\theta^2 + C^2 \sin^2 \theta d\phi^2.
\]

It is assumed that this system is filled with shearing viscous, locally anisotropic and radiating fluid. This fluid can be indicated through the following configurations of the mathematical form

\[
T_{\alpha\beta} = P_\perp h_{\alpha\beta} + \mu V_\alpha V_\beta + \Pi \chi_\alpha \chi_\beta + \varepsilon l_\alpha l_\beta + q(\chi_\beta V_\alpha + \chi_\alpha V_\beta) - 2\eta \sigma_{\alpha\beta},
\]

where \( P_\perp \) is the tangential pressure, \( \Pi \equiv P_r - P_\perp, P_r \) is a fluid pressure along the radial direction. \( \varepsilon \) is a radiation density, \( q_\beta \) is a vector controlling heat dissipation, \( \sigma_{\alpha\beta} \) is a tensor
controlling shearing viscosity while $\eta$ is its coefficient. Further, $h_{\alpha\beta}$ is the projection tensor defined as follows

$$h_{\alpha\beta} = g_{\alpha\beta} + V_\alpha V_\beta$$

The vectors $U^\gamma$, $V^\gamma$ and $\chi^\gamma$ represent null four-vector, fluid four-velocity and radial unit four-vector, respectively. Under co-moving coordinates, these four-vectors can evaluated as $V^\gamma = \frac{1}{\Lambda} \delta^\gamma_0$, $\chi^\gamma = \frac{1}{B} \delta^\gamma_0 + \frac{1}{\Lambda} \delta^\gamma_1$, $q^\gamma = q(t,r) \chi^\gamma$. Moreover, they obey

$$V_\beta V^\beta = -1, \quad \chi_\beta \chi^\beta = 1, \quad \chi_\beta V^\beta = 0, \quad l^\beta V_\beta = 0, \quad l^\beta l_\beta = 0.$$  

The scalar variable controlling expansion and contraction of matter distribution is known as expansion scalar. This can be obtained through $\Theta = V^\alpha V_\alpha$ mathematical expression. For Eq. (3), it is found as follows

$$\Theta = \frac{1}{A} \left( \dot{B} B^{-1} + 2 \dot{C} C^{-1} \right), \quad \sigma = -\frac{1}{A} \left( \dot{C} C^{-1} - \dot{B} B^{-1} \right),$$  

where over dot notation stands for temporal partial derivation.

The $f(R,T)$ field equations (2) for spherical non-static interior (3) are found as

$$G_{00} = \frac{A^2}{f_r} \left[ \mu + \varepsilon - \frac{R}{2} \left( \frac{f}{R} - f_R \right) + \psi_{00} \right],$$  

$$G_{01} = \frac{AB}{f_r} \left[ -(1 + f_T)(q + \varepsilon) + \psi_{01} \right],$$  

$$G_{11} = \frac{B^2}{f_r} \left[ \mu f_T + (1 + f_T)(P_r + \varepsilon - \frac{4}{3} \eta \sigma) + \frac{R}{2} \left( \frac{f}{R} - f_R \right) + \psi_{11} \right],$$  

$$G_{22} = \frac{C^2}{f_r} \left[ (1 + f_T)P_\perp + \frac{2}{3} \eta \sigma \right] + \mu f_T + \frac{R}{2} \left( \frac{f}{R} - f_R \right) + \psi_{22} \right],$$  

where

$$\psi_{00} = 2 \partial_t f_R + \left( \frac{\dot{B}}{B} - 2 \frac{A'}{A} + 2 \frac{\dot{C}}{C} \right) \partial_r f_R + \left( A^2 \frac{B'}{B} - 2 AA' - 2 A^2 \frac{C'}{C} \right) \frac{\partial_r f_R}{B^2},$$

$$\psi_{01} = \partial_t \partial_r f_R - \frac{A'}{A} \partial_t f_R - \frac{\dot{B}}{B} \partial_r f_R,$$

$$\psi_{11} = \partial_{rr} f_R - \frac{B^2}{A^2} \partial_t f_R + \left( B^2 \frac{A'}{A} - 2 B^2 \frac{\dot{C}}{C} - 2 B \right) \frac{\partial_r f_R}{A^2},$$

$$\psi_{22} = \frac{C^2}{f_r} \left[ (1 + f_T)P_\perp + \frac{2}{3} \eta \sigma \right] + \mu f_T + \frac{R}{2} \left( \frac{f}{R} - f_R \right) + \psi_{22} \right].$$
\[ + \left( \frac{A'}{A} + 2 \frac{C'}{C} - 2 \frac{B'}{B} \right) \partial_r f_R, \]

\[ \psi_{22} = -C^2 \frac{\partial \mu f_R}{A^2} + \frac{C^2}{A^2} \left( \frac{\dot{A}}{A} - 3 \frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right) \partial_t f_R + \frac{C^2}{B^2} \left( \frac{C'}{C} + \frac{A'}{A} - \frac{B'}{B} \right) \partial_r f_R. \]

Here, the prime stands for radial partial differentiation.

Now, we are interested to evaluate expressions that would be helpful to study the dynamical phases of spherical anisotropic radiating and shearing viscous interiors in \( f(R, T) \) gravity. It is seen that in this gravitational theory, the divergence of energy momentum tensor is non vanishing and is found to be

\[ \nabla^\alpha T_{\alpha\beta} = \frac{f_T}{(1 - f_T)} \left[ (\Theta_{\alpha\beta} + T_{\alpha\beta}) \nabla^\alpha \ln f_T - \frac{1}{2} g_{\alpha\beta} \nabla^\alpha T + \nabla^\alpha \Theta_{\alpha\beta} \right]. \tag{10} \]

The divergence of \( f(R, T) \) energy-momentum tensor gives the following couple of equations of motion

\[ \dot{\mu} \left( \frac{1 + f_T + f_R f_T}{f_R (1 + f_T)} \right) - \frac{\mu}{f_R} \partial_t f_R - \left\{ \frac{f - R f_R}{2} \right\}_{,0} + \left( \frac{\psi_0}{A^2} \right)_{,0} - \frac{q'}{A} \]

\[ \times \left( \frac{1 + f_T}{f_R} - \frac{\ddot{q}}{A^2} \left\{ \frac{AB(1 + f_T)}{f_R} \right\}_{,1} + \frac{1}{A^2} \left( \frac{\psi_0}{f_R} \right)_{,1} - \frac{B \dot{B}}{A^2 f_R} \right) \}

\[ + \varepsilon + (1 + f_T) \left( \frac{\dot{P}_r + \varepsilon + \frac{4}{3} \eta \sigma}{A^2} \right) + \left( \frac{\psi_0}{A^2} \right)_{,1} + \left\{ (1 + f_T) \mu \right\}_{,0} + \mu f_T + (1 + f_T) \left( \frac{\psi_0}{A^2} \right)_{,1} + \frac{1}{1 + f_T} \left\{ (2 \mu + \varepsilon) \partial_t f_T + \left( \frac{\dot{\mu} + \dot{T}}{2} \right) f_T \right\} = 0, \tag{11} \]

\[ \frac{(1 + f_T)}{f_R} \left\{ \frac{\dot{P}_r - \frac{4}{3} \eta \sigma'}{f_R} \right\} + \frac{\mu' f_T}{f_R} + \frac{1}{1 + f_T} \left( \frac{\dot{P}_r - \frac{4}{3} \eta \sigma - \mu}{A^2} \right) \partial_r f_T - \frac{A}{B f_R} \ddot{q}' \]

\[ \times (1 + f_T) - \left\{ \frac{AB(1 + f_T)}{f_R} \right\}_{,0} \frac{1}{B^2} + \frac{1}{B^2} \left( \frac{\psi_0}{f_R} \right)_{,0} + \frac{\mu \partial_r f_T}{f_R} - \frac{\mu f_T}{f_R^2} \partial_r f_R \]

\[ + \frac{1}{f_R} \left( \frac{\dot{P}_r - \frac{4}{3} \eta \sigma}{f_R} \right) \left\{ \partial_r f_T - \frac{(1 + f_T)}{f_R} \partial_r f_R \right\} + \left( \frac{f - R f_R}{2} \right) \left( \frac{\psi_1}{B^2} + \frac{\psi_0}{A^2} \right)_{,1} - \frac{2 C C'}{B^2 f_R} \}

\[ \times \left\{ \mu f_T + \dot{\mu} + (1 + f_T) \left( \frac{\dot{P}_r - \frac{4}{3} \eta \sigma'}{f_R} \right) + \frac{\psi_0}{A^2} + \frac{2 C}{B^2 f_R} \right\} (1 + f_T) \left( \frac{\dot{P}_r}{f_R} \right) = 0. \]
\[-P_\perp - 2\eta \sigma + \frac{\psi_{11}}{B^2} - \frac{\psi_{22}}{C^2}\} - \frac{f_T}{(1 + f_T)} \left( \mu' + \frac{\partial_r T}{2} \right) + \left( \frac{\dot{B}}{B} + \frac{\dot{A}}{A} + \frac{2C\dot{C}}{A^2} \right) + 2B\dot{B} \right) \left\{ \frac{A}{B} \frac{(1 + f_T)}{f_R} \tilde{q} - \frac{\psi_{01}}{B^2 f_R} \right\} = 0. \quad (12) \]

The matter content within the spherical collapsing stellar geometry can be defined through the general Misner-Sharp formula [44]. This is obtained as

\[m = \left\{ \frac{\dot{C}^2}{A^2} - \frac{C'^2}{B^2} + 1 \right\} \frac{C}{2}, \quad (13)\]

Before calculating its variations among adjacent surfaces of spherical radiating fluid configurations, we first introduce some useful operators. The operators corresponding to proper and radial derivations are found as

\[D_T = \frac{1}{A} \frac{\partial}{\partial t}, \quad D_C = \frac{1}{C'} \frac{\partial}{\partial r}. \quad (14)\]

The relativistic velocity associated with the spherical stellar structure can be found by using above mentioned proper derivative operator. This turns out to be

\[U = D_T C = \frac{\dot{C}}{A}. \quad (15)\]

Now, we define \(E\) as a ratio \(\frac{C'}{B}\). From Eqs. (13) and (15), one can obtain

\[E = \sqrt{1 + U^2 - \frac{2m(t, r)}{C}}. \quad (16)\]

Using field equations and above two equations, the radial mass variations is found as

\[D_C m = \frac{C^2}{2f_R} \left[ \bar{\mu} - \frac{R}{2} \left( \frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} + \frac{U}{E} \left\{ (1 + f_T)\tilde{q} - \frac{\psi_{01}}{AB} \right\} \right], \quad (17)\]

whose integration yields

\[m = \frac{1}{2} \int^C_0 \frac{C^2}{f_R} \left[ \bar{\mu} - \frac{R}{2} \left( \frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} + \frac{U}{E} \left\{ (1 + f_T)\tilde{q} - \frac{\psi_{01}}{AB} \right\} \right] dC, \quad (18)\]

where over bar indicates the addition of radiation density in the corresponding variable quantity. The particular combinations of radiating matter parameters, \(f(R, T)\) higher curvature
terms and energy density can be achieved via Misner-Sharp mass formulation. This can be obtained, after using Eq. (18), and is found as follows

\[
\frac{3m}{C^3} = \frac{3}{2C^3} \int_0^r \left[ \bar{\mu} - \frac{R}{2} \left( \frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} + \frac{U}{E} \left\{ (1 + f_T)\bar{q} - \frac{\psi_{01}}{AB} \right\} C^2C' \right] dr.
\] (19)

This equation can be recast to obtain a scalar related with the tidal forces acting on the anisotropic radiating shearing viscous spherical stellar system

\[
E = \frac{1}{2f_R} \left[ \bar{\mu} - (1 + f_T)(\Pi - 2\eta\sigma) - \frac{R}{2} \left( \frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} - \frac{\psi_{11}}{B^2} + \frac{\psi_{22}}{C^2} \right] - \frac{3m}{C^3},
\] (20)

where \(E\) is the Weyl scalar. It so happened that Weyl scalar can be decomposed into its magnetic and electric constituents. The magnetic part of the Weyl scalar is zero for spherical matter distribution. However, electric part do exists. The scalar, \(E\), is associated with this component of the Weyl tensor. In this way, \(E\) describes the gravity effects coming due to tidal forces in cosmos. Equation (20) has related tidal forces with the structural properties of the fluid configurations and \(f(R, T)\) extra curvature terms. Equation (20) has been evaluated by taking regular distribution of fluid contents at the central point, i.e., \(m(t, 0) = 0 = C(t, 0)\).

3 Expansion-free Condition and \(f(R, T)\) Ellis Equations

In this section, we shall evaluate expansion-free constraint and then discuss its meaning in the interpretation mysterious dark universe. We then consider viable and well-consistent \(f(R, T)\) gravity model. We then proceed forward our analysis by evaluating well-known Ellis equations. The expansion-free equation can be achieved by equating expansion scalar to zero. Thus, Eq. (5) yields

\[
\frac{\dot{B}}{B} = -\frac{2\dot{C}}{C},
\] (21)

which upon integration gives

\[
B = \frac{h}{C^2},
\] (22)

where \(h\) is an arbitrary integration radial function.

Gravitational collapse is the phenomenon that takes place in this accelerating expanding cosmos, when the state of hydrostatic equilibrium of a celestial body is destroyed. If a stellar object is massive enough such that the gas pressure is insufficient to support it against gravitational forces then, the star undergoes gravitational collapse giving birth to new stars. It is
important to stress that any self-gravitating stellar body would subject to gravitational collapse once it bears through inhomogeneous and irregular surface energy density. Therefore, in the collapse of self-gravitating relativistic fluids, the role of energy density inhomogeneity has gained much significance. If the fluid of relativistic celestial interiors is expansion-free, then this study may gain even more attention.

The expansion-free condition has produced several interesting results at galactic and cosmological scales. Skripkin [45] noticed the captivating process of cavity emergence within non-radiating ideal relativistic matter field. It is seen from the literature that in null expansion evolution, the innermost boundary surface of the interior fluid configuration slides away from the central point, thus conceiving vacuum Minkowskian core [46]. The nullity of $\Theta$ is sufficient but not a necessary constraint that guarantee the cavity emergence. The scenario of cavity emergence have been explored in the literature [47] under some kinematical constraints other than $\Theta = 0$. Di Prisco et al. [48] and Sharif and Yousaf [49] studied core formation within the relativistic celestial locally anisotropic configurations after its central explosion and demonstrated some expansion-free relativistic solutions.

The possible implementations of null expansion condition is anticipated for those astronomical settings where a Minkowskian core is probably to be appear. In addition to this, during the process of gravitational collapse, whenever the expansion-free matter moves inside to reach the central point, there will be a strong shear scalar blowup. Joshi et al. [50] claimed that the apparent horizon formation could be delayed due to the effects originated from strong shear of the collapsing system. This suggests the emergence of naked singularity (NS). Therefore, the study of expansion-free relativistic interiors could provide an uncomplicated platform for the analysis of NS appearance. NS is a spacetime singularity that can be observed directly by a distant observer. It represents the formation of extremely high curvature and strong gravity regions and could provides a source of gravitational waves. Are black holes (BHs) and NS observationally distinguished from each other? In this perspective, Virbhadra et al. [51] gave a very useful mathematical tool to understand the NS physics.

Virbhadra and Ellis [52] established that one can observationally differentiate NS from BHs by analyzing the corresponding characteristics of gravitational lensing (GL). Claudel et al. [53] demonstrated that any photon relativistic spherical body could be around the BH only if it obeys a reasonable energy condition. For the observational study of cosmic censorship hypothesis, GL could provide a reliable direction. In this context, it is seen from [54] that BH and NS of the same symmetry and Arnowitt-Deser-Misner mass yield variety of different images of the same source of light. Further, time of image delays because of GL by a BH is greater than that of NS. This asserts that one can get smaller time delays by choosing extreme values of nakedness variables. NS could also provide images with negative delays of time [55].

Due to zero expansion, matter sources could be effective for the voids explanation. Voids
are, so called, underdense regions incorporating substantial amount of information on the cosmological environment [56]. Voids offers a reliable guide to discuss the cosmic structure appearance at large scales. In comparison with GR, they are more rich in modified gravity [57]. Wiltshire [58] claimed that the actual picture of cosmos constitutes sponge-like structural bodies in which voids have a dominant role. Further, some cosmological indications asserts that about 40 − 50% volume of the today cosmos is endowed with cosmological voids with a scale $30h^{-1}\text{Mpc}$, where $h$ is the non-dimensional Hubble parameter, $H_0 = 100h \text{ km sec}^{-1}\text{Mpc}^{-1}$.

The evident relevance of such mentioned effects could reinforces the interest of the problem mentioned in this paper.

The fascinating phenomenon of accelerating cosmic expansion could be described by taking into account extended gravity models involving curvature matter coupling, like $f(R, T)$ gravity. For theoretically and cosmologically consistent $f(R, T)$ gravity, the choice of $f(R, T)$ function is very crucial. We are considering the following particular $f(R, T)$ model form

$$f(R, T) = f_1(R) + f_2(T),$$

This model form does involve direct minimal curvature matter coupling. This could be assumed as a possible correction in the well-known $f(R)$ gravity. Here, we take a linear choice of $f_2$ due to which some striking outcomes can be observed on the basis of non-trivial coupling as compared to $f(R)$ gravity. Thus, we assume $f_2(T) = \nu T$, where $\nu$ is a constant. The Lagrangian with this background of $f_2$ has broadly been examined by many relativistic astrophysicists. Harko et al. [59] obtained some cosmic solutions depicting clear accelerating expanding picture of the universe framed within $f_2 = \nu T$.

Now, we are interested to take a physical feasible generic Ricci invariant function. These may give birth to the existence of some new spherical models. A cosmological viable model needs to obey the big-bang nucleosynthesis, radiation as well as matter dominated regimes. Also, they should expect to allow cosmological perturbations consistent with cosmic restrictions emerging from anisotropies in cosmic microwave background. In this realm, we consider power law Ricci scalar corrections, i.e., $f(R) = R + \lambda R^n$, where $\lambda \in \mathbb{R}^+$ with $\mathbb{R}^+$ is the set of positive real numbers and $n$ is a constant. Depending upon the selection of $n$, this model has some physical descriptions. For instance, this model, for $n = 2$, could depicts exponential behavior of the early cosmic expansion as proposed by Starobinsky [60]. Such $f(R)$ model could draw dark matter (for $\lambda = \frac{1}{6M}$ [61] with $M = 2.7 \times 10^{-12}\text{GeV}$ [7]) and DE effects in the gravitational theory. Furthermore, gravity induced under $n = 2$ [62] and $n = 3$ [64] support the existence of more massive compact objects as compared to GR. The $f(R)$ tanh corrections have also been investigated in the study of stellar collapse [63]. However, the negative $n$ values could helps to explain dynamics of stellar object in the presence of late time accelerating cosmic expansion corrections [6].
In order to delve with the survival of the regular energy density over the dissipative spherical celestial object, we now calculate couple of well-known equation by following the procedure introduced by Ellis [65]. These expressions in the background of dark source $f(R, T)$ corrections can be found by using Eqs.(6)-(9), (13), (14), (20) and (23) as

$$\left[ \mathcal{E} - \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ \bar{\mu} - (1 + \nu)(\bar{\Pi} - 2\eta\sigma) - \frac{(1 - n)}{2} \lambda R^n - \frac{\nu}{2} T + \frac{\varphi_{00}}{A^2} \right\} - \frac{\varphi_{11}}{B^2} + \frac{\varphi_{22}}{C^2} \right]_0, = \frac{3\dot{C}'}{C} \left[ \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ \bar{\mu} + (1 + \nu) \left( P_\perp - \frac{2}{3} \eta\sigma \right) + \mu\nu \right\} + \frac{3AC'}{2BC(1 + n\lambda R^{n-1})} \left\{ (1 + \nu)\bar{q} - \frac{\varphi_{01}}{AB} \right\}, \right.$$

$$\left[ \mathcal{E} - \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ \bar{\mu} - (1 + \nu)(\bar{\Pi} - 2\eta\sigma) - \frac{(1 - n)}{2} \lambda R^n - \frac{\nu}{2} T + \frac{\varphi_{00}}{A^2} \right\} - \frac{\varphi_{11}}{B^2} + \frac{\varphi_{22}}{C^2} \right]' = -\frac{3C'}{C} \left[ \mathcal{E} + \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ (1 + \nu)(\bar{\Pi} - 2\eta\sigma) + \frac{\varphi_{11}}{B^2} \right\} - \frac{3BC'}{2AC(1 + n\lambda R^{n-1})} \left\{ (1 + \nu)\bar{q} - \frac{\varphi_{01}}{AB} \right\}, \right.$$

where $\varphi_{ii}$ encapsulate $f(R, T)$ extra degrees of freedom involved in the evolution of shearing viscous radiating spherical body. These quantities can be evaluated by considering Eqs.(6)–(9) and (23) accordingly.

### 4 Irregularities in the Dynamical System

In this section, we shall calculate some irregularity factors that causes the appearance of irregularities over the surface of stellar spherical system with $f(R, T)$ background. The system enters in the collapsing window once celestial surface suffers energy density inhomogeneities. Therefore, the understanding of the system’s collapsing nature is directly related to the exploration of irregularity factors. For this purpose, we assume that our stellar spherical relativistic system is in complete homogenous phase. We shall take some specific choices of matter fields framed within dark source terms coming from $f(R, T)$ model. As $f(R, T)$ field equations are highly non-linear, therefore we would confine ourself at the constant values of trace of stress-energy tensor as well as cosmological Ricci scalar. These are represented by putting over tilde over the respective quantities. We shall also calculate irregularity factors for those spherical relativistic interior that continue their evolutions by establishing central Minkowskian cavity. This would be achieved by taking expansion-free condition
in the corresponding equations. We shall classify our investigation in two scenarios, i.e., dissipative/radiating and non-dissipative/non-radiating systems as follows:

4.1 Non-radiating Matter

Here, we deal with adiabatic non-interacting, ideal and locally anisotropic forms of relativistic matter distributions coupled framed within $f(R,T)$ background.

4.1.1 Non-interacting Relativistic Particles

This subsection addresses geodesically moving non-interacting fluid configurations. So, we take all pressure gradients effects to be zero $\hat{P}_r = 0 = P_\perp = \hat{q}$ along with $A = 1$. Then $f(R,T)$ Ellis equations (24) and (25) reduce to

$$[\mathcal{E} - \frac{1}{2(1 + n\lambda\tilde{R}^{n-1})}\left\{\mu - \frac{(1-n)}{2}\lambda\tilde{R}^{n} - \frac{\nu^T}{2}\right\}]_{,0} = \frac{3\tilde{C}'}{C}\left[\frac{1}{2(1 + n\lambda\tilde{R}^{n-1})}\right]_{\mu(1+\nu)} - \mathcal{E},$$

$$\left[\mathcal{E} - \frac{1}{2(1 + n\lambda\tilde{R}^{n-1})}\left\{\mu - \frac{(1-n)}{2}\lambda\tilde{R}^{n} - \frac{\nu^T}{2}\right\}\right]' = -3\frac{C'}{C}\mathcal{E}$$

Using Eqs.(5), (11) and (23) in above equations, we have

$$\dot{\mathcal{E}} + \frac{3\tilde{C}'}{C}\mathcal{E} = \frac{\mu(1+\lambda)}{2(1 + n\lambda\tilde{R}^{n-1})}\left[\frac{3\tilde{C}'}{C} + \frac{(1+\nu)B^2C^2}{\{1 + (1 + n\lambda\tilde{R}^{n-1})(1 + \nu)\}}\right]\Theta, \hspace{1cm} (26)$$

$$\mathcal{E}' + \frac{3C''}{C}\mathcal{E} = \frac{\mu'}{2(1 + n\lambda\tilde{R}^{n-1})}. \hspace{1cm} (27)$$

It can be seen from Eq.(27) that, if $\mu' = \mu(t)$ then

$$\mathcal{E} = 0,$$

thereby indicating that existence of Weyl scalar is directly proportional to the existence of regular energy density of non-interacting self-gravitating particles. This is the very result as found in GR by many relativistic astrophysicists. Thus, we conclude that $f(R,T)$ extra curvature terms has not altered or disturb the Weyl curvature role in the conformally flat solutions of dust relativistic cloud. Now, we solve Eq.(26) to investigate that which quantities
are infact making impact over the contribution of Weyl scalar in $f(R, T)$ gravity. The solution of Eq.(26) yields

$$E = \frac{(1 + \nu)}{2(1 + n\lambda \hat{R}^{n-1})C^3} \int_0^t \left[ 3C^2 \dot{C} + \frac{(1 + \nu)B^2C^5}{\{1 + (1 + n\lambda \hat{R}^{n-1})(1 + \nu)\} \Theta} \right] \mu dt. \quad (28)$$

This shows that Weyl scalar for dust particles in $f(R, T)$ model is directly related with the temporal integrals of energy density and expansion scalar. If we take null expansion scenario, then above equation gives

$$E = \frac{3(1 + \nu)}{2(1 + n\lambda \hat{R}^{n-1})C^3} \int_0^t \mu C^2 \dot{C} dt. \quad (29)$$

The relativistic systems that are evolving by encapsulating Minkowskian core in the universe should satisfy the above constraint in order to enter in the inhomogeneous phase. In other words, for the regular distribution of dust expansion-free particles in $f(R, T)$ gravity, one needs to take Eq.(29) to be zero.

4.1.2 Isotropic Fluid

Here, we consider the case of ideal self-gravitating fluid in the environment of $f(R, T)$ gravity. The extended Ellis equations (24) and (25) give rise to the following set of differential equations

$$\left[ E - \frac{1}{2(1 + n\lambda \hat{R}^{n-1})} \left\{ \mu - \frac{(1 - n)}{2} \lambda \hat{R}^n - \frac{\nu}{2} \hat{T} \right\} \right] \bigg|_0 = \frac{3C^2 \dot{C}}{C} \left[ \frac{1}{2(1 + n\lambda \hat{R}^{n-1})} \right],$$

$$\left[ (\mu + P)(1 + \nu) \right] - E, \quad (30)$$

$$\left[ E - \frac{1}{2(1 + n\lambda \hat{R}^{n-1})} \left\{ \mu - \frac{(1 - n)}{2} \lambda \hat{R}^n - \frac{\nu}{2} \hat{T} \right\} \right]' = -3\frac{C'}{C} E. \quad (31)$$

Equations (5) and (12) provide

$$\dot{E} + \frac{3\dot{C}}{C} E = \frac{(1 + \lambda)(\mu + P)}{2(1 + n\lambda \hat{R}^{n-1})} \left[ \frac{3\dot{C}}{C} + \frac{(1 + \nu)B^2C^2}{\{1 + (1 + n\lambda \hat{R}^{n-1})(1 + \nu)\} \Theta} \right], \quad (30)$$

$$E' + \frac{3C'E}{C} E = \frac{\mu'}{2(1 + n\lambda \hat{R}^{n-1})}. \quad (31)$$
It is seen from the second of above equation that energy density will be regular as long as $\mathcal{E} = 0$. However, the solution of Eq. (30) yields

$$\mathcal{E} = \frac{(1 + \nu)}{2(1 + n\lambda R^{n-1})C^3} \int_0^t \left[ 3C^2\dot{\mathcal{C}} + \frac{(1 + \nu)B^2C^5}{1 + (1 + n\lambda R^{n-1})(1 + \nu)} \Theta \right] (\mu + P)dt. \quad (32)$$

This indicates that the influence of tidal forces is controlled by the linear combination of system energy density and locally isotropic pressure gradient. This also highlights the importance of expansion scalar in the modeling of homogeneous spherical geometry coupled with isotropic matter configurations in the presence of $f(R, T)$ corrections. However, if we eliminate this scalar with the help of Eq. (21), then we have

$$\mathcal{E} = \frac{3(1 + \nu)}{2(1 + n\lambda R^{n-1})C^3} \int_0^t (\mu + P)C^2\dot{\mathcal{C}}dt. \quad (33)$$

This suggests that pressure gradient has increased the impact of tidal forces over the isotropic spherical stellar interior. Further, $f(R, T)$ corrections tends to reduce the influence of Weyl scalar due to its non-attractive nature.

4.1.3 Anisotropic Fluid

This subsection is aimed to extend our previous work. Here, we introduce effects of anisotropic stresses, thus $\Pi \neq 0$ in our analysis. In this realm, $f(R, T)$ Ellis equations (24) and (25) take the following forms

$$\left[ \mathcal{E} - \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ \mu - (1 + \nu)\Pi \left( \frac{1 - n}{2} \lambda \tilde{R}^n - \frac{\nu}{2T} \right) \right\} \right]_0 = \frac{3\dot{\mathcal{C}}}{C}$$

$$\times \left[ \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ (\mu + P_\perp)(1 + \nu) \right\} - \mathcal{E} \right],$$

$$\left[ \mathcal{E} - \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ \mu - (1 + \nu)\Pi - \frac{(1 - n)}{2} \lambda \tilde{R}^n - \frac{\nu}{2T} \right\} \right]' = -3 \frac{C'}{C}$$

$$\times \left[ \mathcal{E} + \frac{(1 + \nu)\Pi}{2(1 + n\lambda R^{n-1})} \right],$$

which can be manipulated, after using Eq. (12), in the following forms

$$\left[ \mathcal{E} + \frac{(1 + \lambda)\Pi}{2(1 + n\lambda R^{n-1})} \right]_0 + 3 \left[ \mathcal{E} + \frac{(1 + \lambda)\Pi}{2(1 + n\lambda R^{n-1})} \right] \frac{\dot{\mathcal{C}}}{C} = \frac{3[\mu + (1 + \nu)P_r]C}{2C(1 + n\lambda R^{n-1})}$$
\[
+ \frac{(1 + \nu)^2(1 + n\lambda \tilde{R}^{n-1})^{-1}}{2A\{1 + (1 + n\lambda \tilde{R}^{n-1})(1 + \nu)\}} \left[\frac{(\mu + P_r)B^2C^5\Theta + \frac{2CI\hat{C}}{A}}{C}\right],
\]

\[
\left[\mathcal{E} + \frac{(1 + \lambda)\Pi}{2(1 + n\lambda \tilde{R}^{n-1})}\right] + 3 \left[\mathcal{E} + \frac{(1 + \lambda)\Pi}{2(1 + n\lambda \tilde{R}^{n-1})}\right] \frac{C'}{C} = \frac{\mu'}{2(1 + n\lambda \tilde{R}^{n-1})}.
\]

It is well-known from the working of several relativistic astrophysicists that in GR \[66\] as well in \(f(R, T)\) \[43\], one can break Riemann tensor into couple of tensors, namely, \(X_{\alpha\beta}\) and \(Y_{\alpha\beta}\). The trace-less part of \(X_{\alpha\beta}\) yields the following following (for details please see \[43\])

\[
X_{TF} = -\mathcal{E} - \frac{(1 + \lambda)\Pi}{2(1 + n\lambda \tilde{R}^{n-1})}.
\]

It is seen that some terms involved in Eqs.\((34)\) and \((35)\) has the same configurations as that of the trace-less part of the 2nd dual of Riemann curvature tensor mentioned in Eq.\((33)\). In this context, Eqs.\((34)\) and \((35)\) can be recasted as

\[
\dot{X}_{TF} + \frac{3X_{TF}\dot{C}}{C} = -\frac{3[\mu + (1 + \nu)P_r]\dot{C}}{2C(1 + n\lambda \tilde{R}^{n-1})} - \frac{(1 + \nu)^2(1 + n\lambda \tilde{R}^{n-1})^{-1}}{2A\{1 + (1 + n\lambda \tilde{R}^{n-1})(1 + \nu)\}} \times \left[(\mu + P_r)B^2C^5\Theta + \frac{2CI\hat{C}}{A}\right],
\]

\[
X'_{TF} + \frac{3X_{TF}C'}{C} = -\frac{\mu'}{2(1 + n\lambda \tilde{R}^{n-1})}.
\]

The second of above equation points out if \(\mu' = 0\) the \(X_{TF} = 0\) and vice versa. This indicates \(X_{TF}\) as an entity supervising inhomogeneities in the energy density of the anisotropic spherical fluids. This results supports the consequences of \[43\]. Now, we are interested to find out that to which factors this \(X_{TF}\) further depends, in the presence of dark source terms due to \(f(R, T)\) gravity. The solution of Eq.\((37)\) yields

\[
X_{TF} = \frac{-3}{2(1 + n\lambda \tilde{R}^{n-1})C^3} \int_0^t \left[\mu + (1 + \nu)P_r\right] C^2 \dot{C} dt
- \frac{(1 + \nu)^2(1 + n\lambda \tilde{R}^{n-1})^{-1}}{2A\{1 + (1 + n\lambda \tilde{R}^{n-1})(1 + \nu)\}} \times \int_0^t \left[(\mu + P_r)B^2C^5\Theta + \frac{2CI\hat{C}}{A}\right] dt.
\]
This points out the importance of pressure anisotropy and expansion scalar in the modeling of regular energy density of the celestial spherical geometry in \( f(R, T) \) gravity. Now, using Eq. (21) and Eq. (22), we get

\[
X_{TF} = -\frac{-3}{2(1 + n\lambda R^{n-1})C^3} \int_0^t [\mu + (1 + \nu)P_r]C^2 \dot{C}dt
- \frac{(1 + \nu)^2(1 + n\lambda R^{n-1})^{-1}}{A^2\{1 + (1 + n\lambda R^{n-1})(1 + \nu)\}} \int_0^t C^2 \dot{C}dt.
\]

(40)

This provides that inhomogeneity factor, i.e., \( X_{TF} \) depends upon anisotropic pressure gradients in the scenario of \( f(R, T) \) gravity. Since we know that in the null expansion stellar body, the central point is covered by another metric appropriately joined with the rest of the matter distributions.

### 4.2 Radiating Shearing Viscous Non-Interacting Particles

This subsection discusses the irregularity factors in the realm of dissipation with both free streaming and diffusion limits, but with a special case of viscous particles. Therefore, we consider \( P_r = 0 = P_\perp \) in the matter field and the evolution is characterized by geodesics. This assumption is well established in the background of some theoretical developments. Then, Eqs. (24) and (25) give

\[
\left[ \mathcal{E} - \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ \mu - 2(1 + \nu)\eta \sigma - \frac{(1 - n)}{2} \lambda \tilde{R}^n - \frac{\nu \tilde{T}}{2} \right\} \right]_0 = 3\dot{C}/C
\]

\[
\times \left[ \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ \tilde{\mu} - \frac{2}{3}(1 + \nu)\eta \sigma + \mu \nu \right\} - \mathcal{E} \right] + \frac{3A(1 + \nu)\bar{q}C'}{2 BC(1 + n\lambda R^{n-1})},
\]

(41)

\[
\left[ \mathcal{E} - \frac{1}{2(1 + n\lambda R^{n-1})} \left\{ \tilde{\mu} + 2(1 + \nu)\eta \sigma - \frac{(1 - n)}{2} \lambda \tilde{R}^n - \frac{\nu \tilde{T}}{2} \right\} \right]' = -3C'/C
\]

\[
\times \left\{ \mathcal{E} + \frac{(1 + \nu)\eta \sigma}{(1 + n\lambda R^{n-1})} \right\} - \frac{3B(1 + \nu)\bar{q}\dot{C}}{2 AC(1 + n\lambda R^{n-1})}.
\]

(42)

It has been investigated from the above equation that the quantity which is controlling irregularities is \( \Psi \) defined as follows

\[
\Psi \equiv \mathcal{E} + \frac{(1 + \nu)}{C^3(1 + n\lambda R^{n-1})} \left[ \eta \int_0^r \left( \sigma' - \frac{3C'}{C} \right) C^3 dr - \frac{3}{2} \int_0^r B\dot{C}qC^2 dr \right]
\]

(43)
Thus, if we there is a regular configurations of energy density i.e., \( \mu' = \mu(t) \), then \( \Psi = 0 \), and vice versa. Thus in order to enter in the homogeneous window by the radiating dust cloud, it should vanish the above quantity \( \Psi \). It can be seen that \( \Psi \) is controlled shearing viscosity and heat flux. Making use of Eqs. (3) and (12) in Eq. (31), the \( \Psi \) evolution equation is found as follows

\[
\dot{\Psi} - \frac{\dot{\Omega}}{C^3} = \frac{(1 + \nu)^2(1 + n\lambda\tilde{R}^{n-1})^{-1}}{2\{1 + (1 + n\lambda\tilde{R}^{n-1})(1 + \nu)\}} \left[ \tilde{q}'B + B^2C^2 \left( \bar{\mu} + \frac{2}{3}\eta \sigma \right) \Theta + B \right. \\
\times \left. \left( \varepsilon + \frac{2}{3}\eta \sigma \right) \dot{B} \right] + \frac{\varepsilon}{2(1 + n\lambda\tilde{R}^{n-1})} \left\{ 1 - \frac{(1 + \nu)}{\{1 + (1 + n\lambda\tilde{R}^{n-1})(1 + \nu)\}} \right\} \\
+ \frac{\eta(1 + \nu)}{(1 + n\lambda\tilde{R}^{n-1})} \left( \dot{\sigma} + \frac{\dot{C}}{C} \right) + \frac{3\dot{C}}{2C(1 + n\lambda\tilde{R}^{n-1})}(\varepsilon + \mu(1 + \nu)) - \frac{3\dot{C}}{C}\Psi \\
+ \frac{(1 + \nu)\bar{q}}{2(1 + n\lambda\tilde{R}^{n-1})} \left\{ \frac{3C'}{C} - \frac{(1 + \nu)}{\{1 + (1 + n\lambda\tilde{R}^{n-1})(1 + \nu)\}} \left( \frac{B'}{B} - \frac{CC'}{B^2} \right) \right\},
\]

whose solution leads to

\[
\Psi = \frac{1}{C^3} \int_0^t \left[ \dot{\Omega} + \left\{ \frac{(1 + \nu)^2(1 + n\lambda\tilde{R}^{n-1})^{-1}C^3}{2\{1 + (1 + n\lambda\tilde{R}^{n-1})(1 + \nu)\}} \left[ B^2C^2 \left( \bar{\mu} + \frac{2}{3}\eta \sigma \right) \Theta + \tilde{q}'B \right. \right. \\
\left. \times \left. \left( \varepsilon + \frac{2}{3}\eta \sigma \right) \dot{B} \right] \right\} + \frac{\varepsilon}{2(1 + n\lambda\tilde{R}^{n-1})} \left\{ 1 - \frac{(1 + \nu)}{\{1 + (1 + n\lambda\tilde{R}^{n-1})(1 + \nu)\}} \right\} \\
+ \frac{\eta(1 + \nu)}{(1 + n\lambda\tilde{R}^{n-1})} \left( \dot{\sigma} + \frac{\dot{C}}{C} \right) \right\} \\
+ \frac{(1 + \nu)\bar{q}}{2C^3(1 + n\lambda\tilde{R}^{n-1})} \left\{ \frac{3C'}{C} - \frac{(1 + \nu)}{\{1 + (1 + n\lambda\tilde{R}^{n-1})(1 + \nu)\}} \left( \frac{B'}{B} - \frac{CC'}{B^2} \right) \right\} \right]
\]

For expansion-free condition, the inhomogeneity factor for the viscous dissipative system is found as follows

\[
\Psi = \frac{1}{C^3} \int_0^t \left[ \dot{\Omega} + \left\{ \frac{(1 + \nu)^2(1 + n\lambda\tilde{R}^{n-1})^{-1}C^3}{2\{1 + (1 + n\lambda\tilde{R}^{n-1})(1 + \nu)\}} \left[ \frac{h\bar{q}'C^2}{C^3} \left( \varepsilon + \frac{2}{3}\eta \sigma \right) \right. \right. \\
\left. \times \left. \left( \dot{\sigma} + \frac{\dot{C}}{C} \right) \right] \right\} + \frac{\eta(1 + \nu)}{(1 + n\lambda\tilde{R}^{n-1})} \right\} \\
\times \left( \dot{\sigma} + \frac{\dot{C}}{C} \right) \right\} \\
+ \frac{(1 + \nu)\bar{q}}{2C^3(1 + n\lambda\tilde{R}^{n-1})} \left\{ \frac{3C'}{C} \right\}
\]
\[-\frac{(1 + \nu)}{(1 + (1 + n\lambda R^{n-1})(1 + \nu))} \left\{ \frac{h'}{h} - \frac{2C'}{C} - \frac{C^5 C'}{h^2} \right\} \]  

(46)

This asserts the importance of matter parameters as the irregularity factor has some correspondence with the fluid variables, especially shearing viscosity, heat flux, as well as structural properties of the system.

5 Conclusions

In this paper, we have studied the impact of modified gravity on the distribution of matter configuration for a self-gravitating spherical star. The disturbance in the hydrostatic equilibrium of a celestial object leads to homogeneous or inhomogeneous matter state. We have taken into consideration the spherically symmetric source in the gravitational field of $f(R, T)$ gravity. The geometry is filled with imperfect fluid due to anisotropic stresses and dissipation which is designed in both limits i.e., diffusion and free-streaming limit. We have constructed the modified field equations and corresponding dynamical equations using conservation laws. Some kinematical and dynamical quantities are formulated to explain the evolutionary development of such objects. The mass function using the Misner-Sharp [44] approach is calculated for our spherical object and the curvature tensor as well as the Weyl tensor are explored in this framework. It is found that the Weyl tensor have its constituent tensor like its electric and magnetic parts. Its magnetic part vanishes due to symmetry of the under considered problem while only its electric part exist.

It is well-established fact that the Weyl tensor is responsible for the emergence of tidal forces which makes the object to be more inhomogeneous with the evolution of time. In our case, the spherical system suffers the inhomogeneous states due the presence of its electric part only. We have established an explicit expression between the Weyl tensor and the matter variables like heat flux and anisotropic stresses etc., which holds a significant importance in the light of Penrose’s proposal [67]. Penrose provided the idea of relationship between the Weyl tensor with inhomogeneities in energy density and isotropic pressure, however, such a link is no longer valid in the presence of anisotropies. In this manuscript, we have established such a relation between the Weyl tensor and fluid parameters in the background of higher order curvature terms emerging due to $f(R, T)$ gravitational field.

We have also disintegrated the curvature tensor into its constituent parts using the comoving coordinates. These are found to be structure scalars as already obtained in the framework of $f(R)$ gravity. These scalars have gained significant importance in the light of Newtonian and general relativistic star models. It is also observed that these scalars are used to find the solutions of the Einstein field equations [68]. Moreover, these scalars are also used to discuss the irregular distribution of matter density. It is still unclear that how different
physical factors emerging in fluid configuration can affect the production of inhomogeneities in energy density. Here, we have found some factors creating the irregularities in the matter distribution with $f(R, T)$ extra curvature ingredients. Our analysis will strictly depend upon two differential equations emerging from the explicit expression of Weyl tensor with matter variables and the mass function. These equations are carried out by using the Ellis’s procedure as adopted in his paper. We have constructed our analysis to demonstrate the inhomogeneities in two regimes i.e., with dissipative and non-dissipative fluid. The results obtained in the particular cases of dust, isotropic and anisotropic matter are given as follows:

(i) In the absence of dissipative effects and with dust cloud and $f(R, T)$ dark source terms, we found that the evolutionary motion of the celestial bodies will be homogeneous if the Weyl scalar is zero with extra curvature terms of the theory. In other words, the Weyl tensor due to its electric part and the impact of modified gravity makes the system more inhomogeneous in the gravitational arrow of time. This result can be seen from Eqs. (28) and (29), i.e., if we have a homogeneous matter distribution then Weyl tensor and dark source term should vanish.

(ii) By increasing the complexity in the previous case with isotropic pressure, we observed the same factors creating the irregularities in the density distribution but in the presence of isotropic pressure.

(iii) For non-dissipative anisotropic fluid model, it is found that a linear combination of matter profile is now responsible for the emergence of density inhomogeneity. Further, we have examined that such linear combination corresponds to one of the structure scalars as obtained in Eq. (40).

(iv) For dissipative dust cloud case, we have factor controlling the density distribution as a combination of geometrical and physical variables in the background of $f(R, T)$ gravity theory as obtained in Eq. (45) and (46).

We mention that this study can be generalize to study the density inhomogeneity to $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity. All of our results reduce to general relativity if we take $f(R) = R$.

Acknowledgments

We would like to sincerely appreciate the kind encouragements on this work of Professor Muhammad Sharif. This work was partially supported by the JSPS Grant-in-Aid for Young Scientists (B) # 25800136 and the research-funds presented by Fukushima University (K.B.).
References

[1] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999); A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998).

[2] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. **192**, 18 (2011); G. Hinshaw et al. [WMAP Collaboration], *ibid.* **208**, 19 (2013); P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO]; arXiv:1502.02114 [astro-ph.CO]; P. A. R. Ade et al. [BICEP2 Collaboration], Phys. Rev. Lett. **112**, 241101 (2014); P. A. R. Ade et al. [BICEP2 and Planck Collaborations], *ibid.* **114**, 101301 (2015); P. A. R. Ade et al. [BICEP2 and Keck Array Collaborations], *ibid.* **116**, 031302 (2016).

[3] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004); U. Seljak et al. [SDSS Collaboration], *ibid.* **71**, 103515 (2005).

[4] D. J. Eisenstein et al. [SDSS Collaboration], Astrophys. J. **633**, 560 (2005).

[5] B. Jain and A. Taylor, Phys. Rev. Lett. **91**, 141302 (2003).

[6] S. Nojiri and S. D. Odintsov, eConf C **0602061**, 06 (2006) [Int. J. Geom. Meth. Mod. Phys. 4, (2007) 115] [hep-th/0601213].

[7] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).

[8] S. Nojiri and S. D. Odintsov, Phys. Rep. **505**, 59 (2011).

[9] S. Capozziello and M. D. Laurentis, Phys. Rep. **509**, 167 (2011).

[10] S. Capozziello and V. Faraoni, *Beyond Einstein Gravity* (Springer, Dordrecht, 2010); K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, Astrophys. Space Sci. **342**, 155 (2012); A. de la Cruz-Dombriz and D. Sáez-Gómez, Entropy **14**, 1717 (2012); A. Joyce, B. Jain, J. Khoury and M. Trodden, Phys. Rept. **568**, 1 (2015); K. Koyama, arXiv:1504.04623 [astro-ph.CO]; K. Bamba, S. Nojiri and S. D. Odintsov, arXiv:1302.4831 [gr-qc]; K. Bamba and S. D. Odintsov, arXiv:1402.7114 [hep-th]; Symmetry **7**, 220 (2015) arXiv:1503.00432 [hep-th]].

[11] T. Harko, F. S. N. Lobo, S. Nojiri and S. D. Odintsov, Phys. Rev. D **84**, 024020 (2011).

[12] O. Bertolami, C. G. Boehmer, T. Harko and F. S. N. Lobo, Phys. Rev. D **75**, 104016 (2007).
[13] T. Harko and F. S. N. Lobo, Eur. Phys. J. C 70, 373 (2010).

[14] N. J. Poplawski, arXiv:gr-qc/0608031v2 (2006).

[15] T. Faulkner, M. Tegmark, E. F. Bunn and Y. Mao, Phys. Rev. D 76, 063505 (2007).

[16] T. Chiba, Phys. Lett. B 575, 1 (2003); A. L. Erickcek, T. L. Smith and M. Kamionkowski, Phys. Rev. D 74, 121501 (2006).

[17] D. R. K. Reddy, R. Santikumar and R. L. Naidu, Astrophys. Space Sci. 342, 249 (2012).

[18] K. S. Adhav, Astrophys. Space Sci. 339, 365 (2012).

[19] M. Sharif and Z. Yousaf, Astrophys. Space Sci. 354, 471 (2014).

[20] G. Sun, Y-C. Huang, arXiv:1510.01061v1 [gr-qc].

[21] E. H. Baffou, A. V. Kpadonou, M. E. Rodrigues, M. J. S. Houndjo and J. Tossa, Astrophys. Space Sci. 356, 173 (2015).

[22] M. E. S. Alves, P. H. R. S. Moraes, J. C. N. de Araujo and M. Malheiro, arXiv:1604.03874v1 [gr-qc].

[23] A. Alhamzawi and R. Alhamzawi, Int. J. Mod. Phys. D 25, 1650020 (2016).

[24] Z. Youaf and M. Z. Bhatti, Eur. Phys. J. C 76, 267 (2016) [arXiv:1604.06271 [physics.gen-ph]].

[25] E. V. Arbuzova and A. D. Dolgov, Phys. Lett. B 700, 289 (2011).

[26] K. Bamba, S. Nojiri and S. D. Odintsov, Phys. Lett. B 698, 451 (2011).

[27] M. J. S. Houndjo, Int. J. Mod. Phys. D. 21, 1250003 (2012).

[28] F. G. Alvarenga, M. J. S. Houndjo, A. V. Monwanou and J. B. C. Orou, J. Mod. Phys. 4, 130 (2013).

[29] T, Azizi, Int. J. Theo. Phys. 52, 3486 (2013).

[30] B. L. Hu, L. J. Fang, and R. Ruffini, Advances in Astrophysics World Scientific, Singapore (1983).

[31] R. L. Naidu, et al., Glob. J. Sci. Front. Res. A 13, (2013).
[32] D. R. K. Reddy, R. L. Naidu, K. D. Naidu and T. R. Prasad, Astrophys. Space Sci. 346, 261 (2013).

[33] M. Sharif and Z. Yousaf, Mon. Not. R. Astron. Soc. 440, 3479 (2014); J. Cosmol. Astropart. Phys. 06, 019 (2014); Astropart. Phys. 56, 19 (2014); Astrophys. Space. Sci. 355, 317 (2015); Eur. Phys. J. C 75, 194 (2014); Sharif, M. and Bhatti, M.Z.: J. Cosmol. Astropart. Phys. 10, 056 (2013); Astrophys. Space Sci. 347, 337 (2013); ibid. 352, 883 (2014); ibid. 355, 389 (2015).

[34] M. Kiran and D. R. K. Reddy, Astrophys. Space Sci. 346, 521 (2013).

[35] S. Nojiri and S. D. Odintsov, Class. Quantum Grav. 30, 125003 (2013).

[36] R. Farinelli, M. De Laurentis, S. Capozziello and S. D. Odintsov, Mon. Not. R. Astron. Soc. 440 2894 (2014).

[37] J. Guo, D. Wang and A. V. Frolov, Phys. Rev. D. 90, 024017 (2014).

[38] F.D. Albareti, J. A. R. Cembranos, A. de la Cruz-Dombriz and A. Dobadob, J. Cosmol. Astropart. Phys. 03, 012 (2014).

[39] I. Hason and Y. Oz, Eur. Phys. J. C 74, 3183 (2014).

[40] L. Herrera, G. Le Denmat and N. O. Santos, Phys. Rev. D 79, 087505 (2009).

[41] L. Herrera, Int. J. Mod. Phys. D. 20, 1689 (2011).

[42] M. Sharif and M. Z. Bhatti, Mod. Phys. Lett. A 27, 1250141 (2012); ibid. 29, 1450094 (2014); ibid. 1450129; M. Sharif and Z. Yousaf, Astrophys. Space Sci. 352, 321 (2014); ibid. 354, 431 (2014); ibid. 357, 49 (2015); Eur. Phys. J. C 75, 58 (2015); Gen. Relativ. Gravit. 47, 48 (2015); Can. J. Phys. 29, 1450094 (2014).

[43] Z. Yousaf, K. Bamba and M. Z. Bhatti, Phys. Rev. D 93, 064059 (1960) [arXiv: 1603.03175 [gr-qc]].

[44] C. W. Misner and D. H. Sharp, Phys. Rev. 136, B571 (1964).

[45] V. A. Skripkin, Soviet Physics-Doklady 135, 1183 (1960).

[46] L. Herrera, N. O. Santos and A. Wang, Phys. Rev. D 78, 084026 (2008).

[47] L. Herrera, N. O. Santos and G. Le Denmat, Class. Quantum Grav. 27, 135017 (2010).
[48] A. Di Prisco, L. Herrera, J. Ospino, N. O. Santos and V. M. Vinâ-Cervantes, Int. J. Mod. Phys. D 20, 2351 (2011).

[49] M. Sharif and Z. Yousaf, Can. J. Phys. 90, 865 (2012); ibid. Int. J. Mod. Phys. D 21, 1250095 (2012).

[50] P. S. Joshi, N. Dadhich and R. Maartens, Phys. Rev. D 65, 101501 (2002).

[51] K. S. Virbhadra, D. Narasimha, and S. M. Chitre, Astron. Astrophys. 337, 1 (1998).

[52] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D 65, 103004 (2002).

[53] C. M. Claudel, K. S. Virbhadra and G. F. R. Ellis, J. Math. Phys. 42, 818 (2001).

[54] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D 62, 084003 (2000); K. S. Virbhadra, Phys. Rev. D 79, 083004 (2009).

[55] K. S. Virbhadra and C. R. Keeton, Phys. Rev. D 77, 124014 (2008).

[56] R. Van de Weygaert and E. Platen, Int. J. Mod. Phys. Conf. Ser. 1, 41 (2011).

[57] B. Li, G. B. Zhao and K. Koyama, Mon. Not. R. Astron. Soc. 421, 3481 (2012); M. Sharif and Z. Yousaf, Phys. Rev. D 88, 024020 (2013).

[58] D. L. Wiltshire, arXiv:0712.3984.

[59] T. Harko, et al., Phys. Rev. D 89, 123513 (2014).

[60] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).

[61] J. A. R. Cembranos, Phys. Rev. Lett. 102, 141301 (2009); J. Phys. Conf. Ser. 315, 012004 (2011).

[62] Z. Yousaf and M. Z. Bhatti, Mon. Not. Roy. Astron. Soc. 458, 1785 (2016).

[63] M. Z. Bhatti and Z. Yousaf, Eur. Phys. J. C 76, 219 (2016).

[64] A. V. Astashenok, S. Capozziello and S. D. Odintsov, J. Cosmol. Astropart. Phys. 12, 040 (2013).

[65] G. F. R. Ellis, Gen. Relativ. Gravit. 41, 581 (2009).

[66] L. Herrera, J. Ospino and A. Di Prisco, Phys. Rev. D 84, 107502 (2011).
[67] R. Penrose, *General Relativity, An Einstein Centenary Survey*, Ed. (Cambridge: Cambridge University Press, 1979)

[68] L. Herrera, A. Di Prisco and J. Ospino, Gen. Relativ. Gravit. **44**, 2645 (2012); M. Sharif and M. Z. Bhatti, Gen. Relativ. Gravit. **44**, 2811 (2012); M. Sharif and Z. Yousaf, Astrophys. Space Sci. **357**, 49 (2015).