Hadron energy response of the Iron Calorimeter detector at the India-based Neutrino Observatory

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Abstract

The results of a Monte Carlo simulation study of the hadron energy response for the magnetized Iron CALorimeter detector, ICAL, proposed to be located at the India-based Neutrino Observatory (INO) is presented. Using a GEANT4 modeling of the detector ICAL, interactions of atmospheric neutrinos with target nuclei are simulated. The detector response to hadrons propagating through it is investigated using the hadron hit multiplicity in the active detector elements. The detector response to charged pions of fixed energy is studied first, followed by the average response to the hadrons produced in atmospheric neutrino interactions using events simulated with the NUANCE event generator. The shape of the hit distribution is observed to fit the Vavilov distribution, which reduces to a Gaussian at high energies. In terms of the parameters of this distribution, we present the hadron energy resolution as a function of hadron energy, and the calibration of hadron energy as a function of the hit multiplicity. The energy resolution for hadrons is found to be in the range 85% (for 1GeV) – 36% (for 15 GeV).

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1 Introduction

The India-based Neutrino Observatory (INO) [1] is a planned underground laboratory in Southern part of India. The primary focus in the first phase of the program is to study atmospheric neutrinos with a magnetized iron calorimeter (ICAL) detector similar in concept to the design of the MONOLITH detector [2]. The detector is designed to observe neutrino (and anti-neutrino) interactions in the GeV range. Recent measurements of the mixing angle $\theta_{13}$ in reactor experiments [3, 4, 5] will enable ICAL to pin down the mass ordering of neutrinos by separate measurements of $\nu_\mu$ and $\bar{\nu}_\mu$ interactions, exploiting the matter effects in the Earth.

The magnetized ICAL detector will consist of three identical modules with 151 layers of iron plates interspersed with Resistive Plate Chamber (RPC) detectors, and will be approximately 50 kt in mass. Atmospheric neutrinos (anti-neutrinos) interact with the iron target in the detector through quasi-elastic (QE), resonance (RS) and deep inelastic scattering (DIS) processes, producing charged leptons in charged-current (CC) interactions, with a set of possible final hadrons, typically one pion and a nucleon in RS, and multiple hadrons in DIS interactions. At these energies, coherent interactions and the interactions with the electrons in the detector are rare and are ignored. ICAL is expected to have good charge identification efficiency and good tracking and energy resolution for muons produced in CC interactions. The reach of ICAL for determining the mass hierarchy of neutrinos and measuring the atmospheric mixing parameters using only the information on muon energy and direction has been presented in [6, 7].

While the information on muon energy and direction is crucial for the physics goals of ICAL, the detector is also sensitive to hadrons, and the additional information from the response of the detector to hadrons can only enhance its physics reach. Recently it has been shown that with the inclusion of hadron energy information, the analysis of atmospheric neutrino events at ICAL will significantly improve the determination of the neutrino mass hierarchy from this analysis [8]. Also, hadrons are the only signature of neutral current (NC) events in the detector. While NC events are not affected by active neutrino oscillations, they are sensitive to active–sterile oscillations; in addition, the NC rates are altered by the contribution of tau-neutrinos (generated by active oscillations) through the hadronic decay of taus produced in CC interactions. Hence, in order to understand these events, it is important to characterise the behaviour of single and multiple hadrons in the detector.

The simulation study performed for estimating the hadron energy resolution for the ICAL detector is reported here. The hadrons consist mainly of pions (both neutral and charged, and about 85% of the events on the average), kaons, and also nucleons, including the recoil nucleon which cannot be distinguished from the remaining hadronic final state. The neutral pion decays immediately, giving rise to two photons, while the charged pions propagate and develop into a cascade due to strong interactions. A visualization of a neutrino DIS event with large hadron energy component in the simulated ICAL detector (using the VICE event display package [9]) is shown in Fig. 1. The main uncertainty in determining the incident neutrino energy comes from the uncertainty in estimating the energy of these hadrons. For the neutrino-nucleon interaction $\nu_\mu N \rightarrow \mu X$, the incident neutrino energy is given by

$$E_\nu = E_\mu + E_{\text{hadrons}} - E_N,$$  \hspace{1cm} (1)
where $E_N$ is the energy of the initial nucleon which is taken to be at rest, neglecting its small Fermi momentum. It can be seen from Eq. (1) that any uncertainty in measuring the hadron energy will directly affect the determination of incoming neutrino energy. Note that the visible hadron energy depends on factors like the shower energy fluctuation, leakage of energy, and invisible energy loss mechanisms, which in turn affect the energy resolution of hadrons.

The analysis to obtain the hadron energy resolution is done in two parts. First, the charged pions of fixed energies are generated via Monte Carlo and propagated through the ICAL detector at fixed energies. This is used to calibrate the detector for the number of hits at fixed energies. Next, the multiple hadrons produced through neutrino interactions are considered, and the parameter $E'_{\text{had}}$, defined as

$$E'_{\text{had}} = E_\nu - E_\mu,$$

(2)

is used to calibrate the detector response. Here, all types of hadrons contribute to the energy, though it is dominated by pions at the energies of a few GeV.

This paper describes the complete simulation chain of ICAL events which includes event generation through the event generator or particle gun, full detector simulation by propagating the events through the virtual detector and data analysis to assess the detector's capability to estimate hadron energy. In section 2, the hit distributions of charged hadrons, in particular, pions, in the ICAL detector are discussed. This is done by a Monte-Carlo (MC) generator that generates hadrons at fixed energies up to 15 GeV. The hits are then analysed for determining the detector response. The energy and its resolution are estimated using the hit multiplicity information of events generated with appropriate position- and angle-smearing. In section 3, the response of the detector to multiple hadrons generated in atmospheric neutrino-nucleus interactions is studied using the neutrino event generator NUANCE [10] to obtain the net hadron energy resolution as a function of $E'_{\text{had}}$. In section 4, the calibration of $E'_{\text{had}}$ using the detector hit information is described. Our results are summarized in section 5.
2 Energy response to fixed-energy hadrons

The ICAL detector is simulated using the GEANT4 package [11, 12]. The input parameters for the simulation – like the gas mixture, conducting coating, efficiency, etc – correspond to those for a 12-layered stack of glass RPC detectors (without the Iron absorbers) [13] that is being run under stable conditions for several years. Iron layers, 5.6 cm thick, are interleaved with the active RPCs with a 2 mm gas gap. The spacing between two consecutive RPCs is 9.6 cm. When a charged particle propagates through the ICAL detector, hits in the X and Y strips of the RPC layers are recorded. The layer number provides the z-coordinate. Thus the full position information is available, with a precision of 1.96 cm in the x and y directions and 2 mm in the z direction. In this study we have neglected the hits due to edge effects and noise, and these will be taken care of once the data from the prototype detector becomes available in future. A check using the data [14] obtained from the cosmic muon study with the prototype mentioned above indicates that the hadron energy resolutions are not much affected by the edge effects. For more details about the GEANT4 based simulation of the detector geometry and the nature of hits generated, see Ref. [15], which also discusses the muon energy response. The focus of this section will be the analysis of fixed-energy single-pion hit distributions.

A muon usually leaves one or two hits per layer and so the hits from both X and Y strips can easily be combined to obtain the number of hits and their position coordinates \((x, y)\) in a given layer. However, in the case of hadron shower there are multiple hits per layer, and combining X and Y strip hits leads to some false count of hits (ghost hits). To avoid the ghost hit counts, the energy calibration may be done with counts from either X or Y strips. The variables x-hits and y-hits store the number of hits in the X and Y strips of the RPC, respectively. The maximum of x-hits or y-hits is stored as the variable “orig-hits”. In Fig. 2 the comparisons of these three types of hit variables for \(\pi^\pm\) of energy 3 GeV are shown. As is clear from Fig. 2 any of the variables x-, y- or orig-hits could have been used in the analysis. The variable orig-hits has been chosen as the unbiased parameter here. It is also observed that the detector response to the positively and negatively charged pions is identical, so we shall not differentiate between them henceforth.

Figure 2: The comparison of the distributions of x-hits, y-hits and orig-hits for \(\pi^-\) (left) and \(\pi^+\) (right) of energy 3 GeV.
Fixed-energy single pion events in the energy range of 1 to 15 GeV were generated using the particle gun. Unless otherwise specified in this section, the total number of events generated for each input energy value is 10000. Each event is randomly generated over a volume $2 \, \text{m} \times 2 \, \text{m} \times 2 \, \text{m}$ in the central region of the ICAL detector. In addition, since there is very little impact of the magnetic field on the showers produced by hadrons, the hadron direction is uniformly smeared over zenith angle $0 \leq \theta \leq \pi$ and azimuth of $0 \leq \phi \leq 2\pi$. (The angles are denoted with respect to a reference frame, where the origin is taken to be the center of the detector, the $z$-axis points vertically up, while the plates are horizontal in the $x$-$y$ plane.) This serves to smear out any angle-dependent bias in the energy resolution of the detector by virtue of its geometry which makes it the least (most) sensitive to particles propagating in the horizontal (vertical) direction.

In Fig. 3, the hit distributions in the detector for pions, kaons, and protons at various energies in the range of 1 to 15 GeV are shown. It is observed that for all these hadrons the hit patterns are similar, though the peak position and spread are somewhat dependent on the particle ID. Hence the detector cannot distinguish the specific hadron that has generated the shower. The large variation in the number of hits for the same incident particle energy is partly due to angle smearing and more dominantly due to the strong interaction processes with which hadrons interact with the detector elements. The exception is $\pi^0$, which decays almost immediately into an $e^+ e^-$ pair; the fewer number of hits and the narrower hit distribution in this case reflects the nature of electromagnetic interactions of this pair with the predominantly iron target.

Figure 3: The hit distributions at various energies (angle-averaged) for $\pi^\pm$, $\pi^0$, $K^\pm$ and protons propagated from vertices smeared over the chosen detector volume.
Since hadrons produced in neutrino interactions with ICAL are primarily charged pions, the focus in this section is on the detector response to charged pions. A more general admixture of different hadrons is considered in the next section.

### 2.1 Analysis of the charged pion hit pattern

The charged pion hit distributions at sample values of $E = 3, 8$ GeV are shown in Fig. 4. Typical patterns show a mean of roughly 2 hits per GeV as seen from the figure, but with long tails, so the distribution is not symmetric. In addition, several events yield zero hits in the detector at lower energies; such events are virtually absent at higher energies.

![Figure 4: The hit distributions at 3 GeV (left) and 8 GeV (right), for pions propagating in the detector, starting from randomized vertices over a volume of $2 \, m \times 2 \, m \times 2 \, m$ in the detector. The red curve denotes a fit to the Vavilov distribution.](image)

A good fit is obtained with the Vavilov distribution function for all energies, as is illustrated in Fig. 4. This distribution (see Appendix A) is described by the four parameters $P_0$, $P_1$, $P_2$ and $P_3$. The energy dependence of these parameters is shown in Fig. 5. This may be used directly for reconstructing the hit distribution at any given energy. Note that the Vavilov distribution reduces to a Gaussian distribution for $P_0 \geq 10$. In this analysis it is observed to happen at energies greater than 6 GeV, as can be seen from Fig. 5. At lower energies, it is necessary to use the full Vavilov distribution.

The mean $\bar{n}(E)$ of the number of hits from the Vavilov fit at different energies is shown in the left panel of Fig. 6. It increases with increasing pion energy, and saturates at higher energies. It may be approximated by

$$\bar{n}(E) = n_0 [1 - \exp(-E/E_0)]$$

where $n_0$ and $E_0$ are constants. This fit has to be interpreted with some care, since $n_0$ and $E_0$ are sensitive to the energy ranges of the fit. The value of $E_0$ is found to be $\sim 30$ GeV when a fit to the energy range 1–15 GeV is performed. Since the energies of interest for atmospheric neutrinos are much less than $E_0$, Eq. (3) may be used in its approximate linear form $\bar{n}(E) \approx n_0 E/E_0$. A fit to this linear form is also shown in Fig. 6.
Figure 5: The parameters $P_0$, $P_1$, $P_2$ and $P_3$ of the Vavilov fit to the hit multiplicity, as functions of pion energy, for fixed-energy charged pions.

Since in the linear regime ($E \ll E_0$) one has

$$\frac{n(E)}{n_0} = \frac{E}{E_0},$$

(4)

The energy resolution may be written as

$$\frac{\sigma}{E} = \frac{\Delta n(E)}{\bar{n}(E)},$$

(5)

where $(\Delta n)^2$ is the variance of the distribution. In the rest of the paper the notation $\sigma/E$ will be used for energy resolution, and Eq. (5) will be taken to be valid for the rest of the analysis.

The energy resolution of pions may be parameterized by

$$\frac{\sigma}{E} = \sqrt{\frac{a^2}{E} + b^2},$$

(6)

where $a$ and $b$ are constants. The energy resolutions for charged pions functions of pion energy are shown in Fig. 6. The parameters $a$ and $b$ extracted by a fit to Eq. (6) over the pion energy range 1–15 GeV are shown in the right panel of Fig. 6.
3 Energy response to hadrons produced by atmospheric neutrinos

The previous section contained an analysis of the energy resolution with single pion events. But in reality there are multiple hadrons produced in the atmospheric neutrino interactions. We analyze the charged-current (CC) $\nu_\mu$ interactions in the detector via quasi-elastic (QE), resonance, and deep inelastic scattering (DIS) processes. QE dominates at $E_\nu \sim 1$ GeV, and contains no hadron in the final state except for the recoil nucleon. Resonance events at a few GeV contain an additional hadron, typically a pion. As the energy increases, DIS events that contain multiple hadrons in the final state dominate.

Both atmospheric neutrino ($\nu_\mu$) and anti-neutrino ($\bar{\nu}_\mu$) events in ICAL are generated using the neutrino event generator NUANCE (v3.5) [10]. The hadrons produced in these interactions are primarily pions, but there are some events with kaons (about 10%) and small fraction of other hadrons as well. As discussed earlier, it is not possible to discern one hadron from the other in the “shower pattern” of hadron hits. However, since the hit distribution of various hadrons are similar to each other (see Fig. 3), and the NUANCE generator is expected to produce a correct mixture of different hadrons at all energies, it is sufficient to determine the hadron energy resolution at ICAL through an average of NUANCE events, without having to identify the hadrons separately.

A total of 1000 kt-years of “data” events (equivalent to 20 years of exposure with the 50 kton ICAL module) were generated with NUANCE. The events were further binned into the various $E'_\text{had}$ energy bins and the hit distributions (averaged over all angles) in these bins are fitted to the Vavilov distribution function. The energy dependence of the Vavilov fit parameters is shown in Fig. 7. This information can be used directly to simulate the hadron energy response of the detector for physics analysis. The mean values (Mean$_{\text{Vavilov}}$) of these distributions as a function of $E'_\text{had}$ are shown in the left panel of Fig. 8. As expected, these are similar to the mean values obtained earlier with fixed energy pions. Since the mean hits grow approximately linearly with energy, the same...
Figure 7: The parameters $P_0$, $P_1$, $P_2$ and $P_3$ of the Vavilov fit to the hit multiplicity, as functions of $E'_\text{had}$, from NUANCE data. These parameters can be directly used to reconstruct the hit distribution pattern. The bin widths are indicated by horizontal error bars.

The linearized approximation used in section 2.1 can be used to obtain the energy resolution $\sigma/E = \Delta n/n$. The energy resolution as a function of $E'_\text{had}$ is shown in Fig. 8. The energy resolution ranges from 85% (at 1 GeV) to 36% (at 15 GeV).
The effective energy response obtained from the NUANCE-generated data is an average over the mixture of many hadrons that contribute to hadron shower at all energies. The fractional weights of different kinds of hadrons produced in neutrino interactions may, in principle, depend upon neutrino oscillations. In addition, the relative weights of events with different energy that contribute in a single energy bin changes because neutrino oscillations are energy dependent. Events with oscillations using the best-fit values of standard oscillation parameters (mixing angles and mass-squared differences) were also generated. The resolutions obtained without and with oscillations are very close to each other. Thus, the hadron energy resolution can be taken to be insensitive to oscillations.

4 Hadron energy calibration

When the actual ICAL detector starts collecting the data, the only available observable for the hadrons is the hit multiplicity. Therefore, it is imperative to calibrate this hit multiplicity with the hadron energy in the simulation. To this end, the hadrons from simulated NUANCE “data” were divided into “hit\_n” bins, where hit\_n corresponds to n number of hadron hits. The distributions of these energies were obtained for each bin as shown in Fig. [9]. Even here, a good fit was obtained for the Vavilov distribution function at all hit multiplicities. The Mean\textsubscript{Vavilov} and \( \sigma_{\text{Vavilov}} \) obtained from the fit were used to produce the calibration plot presented in Fig. [10]. From the hit multiplicity of the hadron shower in any event in ICAL, the hadron energy can be estimated using this information. This can then further be used to reconstruct the energy and direction of the incident neutrinos. The details of this reconstruction will be discussed elsewhere [17].
5 Summary and Conclusions

The ICAL detector is expected to be a good tracking detector for the muons produced in charged-current atmospheric $\nu_\mu$ events. However, in order to reconstruct the neutrino energy, the estimation of the energy of the hadrons produced in such interactions is crucial. Recently, a physics simulation study of neutrino mass hierarchy determination with ICAL has shown that the inclusion of hadron energy information appreciably improves the sensitivity of ICAL to this important issue that is still unresolved \cite{8}. Reconstructing the hadron energy and directions is also the only way to investigate the neutral current events, which can provide information on tau neutrinos, and on phenomena like active-sterile oscillations. It is therefore important to characterize the behaviour of hadrons in the detector.

A GEANT4-based simulations framework of the ICAL detector has been employed to explore the energy calibration of hadrons from the hadron hit pattern and to obtain their energy resolutions in the region of interest to atmospheric neutrino oscillations. The results of this simulation would be crucial for the studies that analyze the reach of the ICAL detector for determining the neutrino mixing parameters.
The hadron events of interest in the ICAL detector primarily contain charged pions. The hit pattern of pions and kaons in the detector is similar; hence it is not possible to separate different hadrons in the detector. Similarly, neutrino-nucleus interactions produce events with multiple hadrons in the final state (generated by the NUANCE neutrino generator), whose energies cannot be reconstructed individually. However, the total energy deposited in hadrons can be determined by a calibration against the hit multiplicity of hadrons in the detector.

The hit patterns in single and multiple hadron events are roughly similar, and may be described faithfully by a Vavilov distribution. Analyses, first with fixed-energy pions, and later with a mixture of hadrons from atmospheric $\nu_\mu$ interaction events, show that a hadron energy resolution in the range $85\%$ (at 1 GeV) – $36\%$ (at 15 GeV) is obtainable for hadrons produced in charged-current neutrino interactions. The parameters of the Vavilov fit presented here as a function of hadron energy can be used for simulating the hadron energy response of the detector, in order to perform physics analyses that need the hadron energy resolution of ICAL. We also present the calibration for the energy of the hadron shower as a function of the hit multiplicity. This analysis will be improved upon by incorporating edge effects and noise in a later study, after data from the prototype detector is available.

The results in this paper will allow us to reconstruct the total visible energy in NC events. Combined with the information on the muon energy and direction in the CC events, it will allow one to reconstruct the total neutrino energy in the CC events. ICAL will be one of the largest neutrino detectors sensitive to the final state hadrons in neutrino interactions, and its potential for extracting hadronic information needs to be fully exploited.

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A The Vavilov probability distribution function

As has been observed from Fig. 3, the distribution of hit multiplicity in the detector, obtained for fixed-energy charged pions (and hadrons in general) is asymmetric, particularly at lower energies. The Vavilov probability distribution function is found to be a suitable one to represent the hit distributions.
The Vavilov probability density function in the standard form is defined by
\[ P(x; \kappa, \beta^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \phi(s)e^{xs}ds , \tag{A.1} \]
where
\[ \phi(s) = e^{Ce^{\psi(s)}}, \quad C = \kappa(1 + \beta^2\gamma), \tag{A.2} \]
and
\[ \psi(s) = s\ln\kappa + (s + \beta^2\kappa) \cdot \left[ \int_0^1 \frac{1 - e^{-st/\kappa}}{t} dt - \gamma \right] - \kappa e^{-s/\kappa}, \tag{A.3} \]
where \( \gamma = 0.577\ldots \) is the Euler’s constant. The parameters mean and variance \((\sigma^2)\) of
the distribution in Eq. (A.1) are given by
\[ \text{mean} = \gamma - 1 - \ln\kappa - \beta^2; \quad \sigma^2 = \frac{2 - \beta^2}{2\kappa}. \tag{A.4} \]

For \( \kappa \leq 0.05 \), the Vavilov distribution may be approximated by the Landau distribution,
while for \( \kappa \geq 10 \), it may be approximated by the Gaussian approximation, with the

We have used the Vavilov distribution function \( P(x; \kappa, \beta^2) \) defined above, which is
also built into ROOT, as the basic distribution for the fit. However the hit distribution
itself is fitted to the modified distribution \((P_4/P_3) P((x - P_2)/P_3; P_0, P_1)\), to account
for the x-scaling \((P_3)\), normalization \(P_4\) and the shift of the peak to a non-zero value, \(P_2\).
Clearly \( P_0 = \kappa \) and \( P_1 = \beta^2 \). The modified mean and variance are then
\[ \text{Mean}_{\text{Vavilov}} = (\gamma - 1 - \ln P_0 - P_1) P_3 + P_2; \quad \sigma^2_{\text{Vavilov}} = \frac{(2 - P_1) P_2}{2P_0} P_3. \tag{A.5} \]

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