Racing a quantum computer through Minkowski spacetime

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Abstract. The Lorentzian length of a timelike curve connecting both endpoints of a computation in Minkowski spacetime is smaller than the Lorentzian length of the corresponding geodesic. In this talk, I will point out some properties of spacetime that allow an inertial classical computer to outperform a quantum one, at the completion of a long journey. We will focus on a comparison between the optimal quadratic Grover speed up from quantum computing and an \( n=2 \) speedup using classical computers and relativistic effects. These results are not practical as a new model of computation, but allow us to probe the ultimate limits physics places on computers.

1. Introduction
Consider a classical computation requiring \( N \) operations each taking time \( \Delta t \), then \( \Delta t N \) becomes the total runtime in a local inertial frame. Indeed, relativistic effects don’t change complexity results inside an inertial frame, and the number \( N \) is agreed on by all observers. However, the total computational time experienced by observers in motion is relative. We will state this as a relation between a polynomial reduction inside the black box model and relativistic mass — this is accomplished in Section 2.2 where Theorem 1.1 is proven.

Theorem 1.1. The minimal relativistic energy \( E (= mc^2) \) required to perform a computation within time \( N \) in an inertial frame and time \( N^{1/n} \) in the frame of an observer \( O \) is given as

\[
E = N^{1-1/n}m_0c^2,
\]

where \( m_0 \) is the rest mass of \( O \), \( \Delta t := 1 \), \( n \) is the order of the sought polynomial reduction and \( N \in \mathbb{N}^*, n \geq 1 \).

We have found a non-linear tradeoff (1) between relativistic energy and reduction of computational runtime \( \Delta t N \). This effect was experimentally observed in 1938 by Ives and Stilwell [1], observation in macroscopic clocks occurred in 1972 [2] with the current state of the art found in [3].

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1.1. Computational Complexity Theory

The resources consumed by an efficient algorithm scale polynomially in the problem size — necessarily from the class \( P \) (of problems known to be efficiently solvable). The most famous open question in Computer Science concerns proving if it is impossible to efficiently solve a complete problem from the class \( NP \) \([4, 5, 6]\).

Although the \( P \neq NP \) question regards general algorithmic complexity, one can consider its physical analogue by asking if the laws of physics allow, even in principle, the existence of a physical process that can be harnessed to speed up the solution to an \( NP \)-complete problem \([7, 8]\). Deterministic query complexity in the black-box model is the ideal framework to address this question. To date, research connecting complexity theory and relativity theory has been focused on the implications the existence of closed time-like curves would have on computation \([9, 10, 11, 12, 13, 14, 15]\).

Consider a classical device computing \( f : \{0,1\}^n \to \{0,1\} :: x \mapsto f(x) \), for a given function defined on its range of inputs for positive integer \( n \). In the case of an unstructured database, one is given \( f \) in a black-box with a promise that \( f \) outputs 1 for a single input \( x' \) and 0 otherwise. Using a classical computer, to determine \( x' \) in the worst case requires \( N := 2^n \) queries of the search space — each taking time \( \Delta t \). We will consider solving such an \( NP \)-complete problem.

2. Einstein’s Computer of 1905

Let \( I \) denote an observer travelling along a geodesic that is the common origin of the coordinate system and let \( O \) denote an observer in relative motion. Consider an event \( 0 \) at which (i) \( O \) passes \( I \); (ii) they synchronize their temporal and spatial orientations and (iii) \( I \) begins the computation.

In the following, we are working in units of ticks (and not roots or powers of seconds, as the slight abuse of notation might suggest). Now consider the general question of relating the proper time interval experienced in \( O \)’s frame as the \( n \)th root of the time interval experienced in \( I \)’s frame. This is done by relating the clocks of \( I \) and \( O \) — we insist that \( t(\tau) = T^n \) ticks and \( \tau(t) = T \) ticks, where \( O \) reaches \( d \) at the event \( (cT^n, d, 0, 0) \) in \( I \)’s frame, and so \( T^n \) and \( T \) are dimensionless constants. This is made possible by the Lorentz factor \( \gamma(u) \) through the temporal relation:

\[
T^n \text{ ticks } = \gamma(u)T \text{ ticks},
\]

where \( u \) is the velocity of \( O \) measured in \( I \)’s frame. For analysis purposes, consider constant \( T > 1, n \geq 1 \) and let \( T^n \) and \( T \) have units of seconds only where applicable, where \( T^{n-1} \) is dimensionless. The velocity \( u \) is now expressed as \( u(T, n) = c\sqrt{1 - T^{2-2n}} \), where we choose an origin \( u \geq 0 \) and note that the large positive constant \( T^{2-2n} \) is the ration of two clocks.

2.1. The Twins get Computers

In 1911, Paul Langevin made Einstein’s 1905 prediction of time-dilation vivid by noting asymmetry in a thought experiment involving twins (\( O \) and \( I \)) — both measure events on \( O \)’s worldcurve. This became known as the twin or clock paradox and was a subject of debate during the first half of the last century \([16, 17]\) and remains a research area today \([16, 17, 18, 19, 20, 21, 22]\). For completeness, let us then state this paradox in terms of our framework.

The twins calculate the time \( (T^n = \Delta t N) \) needed in \( I \)’s frame to perform a computation with the understanding that \( O \) wishes to have the solution in the \( n \)th root of this time \( (T = (\Delta t N)^{1/n} \).
sec), upon returning from a journey. The time of the total trip measured by \( I \) is \( T^n = 2d/u \) and by \( O \) is \( \gamma(u)T = 2d/u \), and the spacetime path is given as (2.1):

**Definition 2.1.** (Spacetime path 2.1) Consider three events: (i) \( O \) starts from rest, reaching a constant velocity within a negligibly short time leaving her twin \( I \) to perform a computation for time \( T^n \); (ii) after journeying for time \( T/2 \) in \( O \)'s frame, and some distance \( d/2 \) in \( I \)'s frame, \( O \) suddenly reverses velocity; (iii) \( O \) arrives back at her starting point, stops, and recovers the result of the computation.

Consider the Lorentz transform of the event where \( O \) reaches \( d \), and hence recover the respective temporal and spatial relations: \( cT^n = \gamma(u)Tc \) and \( d = \gamma(u)Tu \). It is often stated that, “it is possible to travel as far as you like in as short a time as you like, provided the distance \( (d) \) is measured before you set off and the time \( (T) \) is measured along your world-line” (see [23, 24] for instance). The expression (1) now gives the preceding statement computational meaning in terms of a polynomial reduction \( n \) inside the black-box model.

### 2.2. Equivalence among the Polynomial Reduction and Relativistic Mass

We let \( E \) represent energy measured in the frame \( I \), and use \( m_0 \) to denote the rest mass of \( O \). We stated the equivalence among the polynomial reduction and relativistic mass in Theorem 1.1 in Section 1 — an outline of the proof follows:

**Proof.** (Theorem 1.1) The proof relies on the results of Sections 2 and 2.1. From the relation \( E = mc^2 \) it can be established that \( E = mc^2 = T^{n-1}m_0c^2 \) and Theorem 1.1 follows. \( \square \)

**Corollary 2.2.** From (1) it follows that \( n(E, T) = 1 + \log \left( \frac{E}{(m_0c^2)} \right) / \log (T) \).

The 4-momentum of \( O \) measured in the frame \( I \) can now be expressed as \( P = T^{n-1}(E_0/c, p_1, p_2, p_3) \), where \( p_i = T^{n-1}m_0u_i \) is the \( i \)th component of the 3-momentum. When \( E = E_0 = m_0c^2 \) one recovers the classical limit \( n = 1 \). In Minkowski spacetime, it is when the relativistic energy of an observer \( O \) increases past their rest energy in a frame \( I \), that computational gains of order \( n > 1 \) become possible.

### 3. Racing a quantum computer through Minkowski Spacetime

Quantum query complexity broke classical lower bounds on the required number of queries and hence the total time interval \( (\Delta tN) \) to solve certain black-box problems including database search [25, 26]. Let us examine the related speedup using classical computers together with relativistic effects. To recover a Grover speedup [26] for the case of constant velocity requires energy \( E = \sqrt{Nm_0c^2} \), in \( I \)'s frame, where \( m_0 \) is the rest mass of \( O \), \( \Delta t := 1 \) is the single query time and \( N \) gives the total number of items in a search space.

### 4. Conclusion

This study was aimed at understanding what class of computations are made possible or ruled out by the laws of physics [27, 28, 29, 7] in a relativistic setting. We have shown that finite \( n \)th root polynomial reductions in algorithmic run-time are made possible by relativistic effects. The runtime improvement is predicted by Einstein’s theory of relativity and the connection to computation was explored by considering polynomial reductions inside the black-box model.
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References
[1] Ives H E and Stilwell G R 1938 J. Opt. Soc. Am. 28 215–226
[2] Hafele J C and Keating R E 1972 Science 166–168
[3] Reinhardt S et al 2007 Nature Physics 3 861–864
[4] Papadimitriou C 1994 Computational Complexity (1st ed.) (Addison Wesley)
[5] Sipser M 1996 Introduction to the Theory of Computation (International Thomson Publishing)
[6] Garey M and Johnson D 1979 Computers and Intractability: A Guide to the Theory of NP-Completeness (W.H. Freeman and Co. New York, NY, USA)
[7] Aaronson S 2005 SIGACT News 36 30–52 ISSN 0163-5700
[8] Kitaev A, Shen A and Vyalyi M 2002 AMS, Graduate Studies in Mathematics 47
[9] Geroch R P and Hartle J B 1986 Found. Phys. 16 533–550
[10] Deutsch D 1991 Phys. Rev. D 44 3197–3217
[11] Etessi G and Nemeti I 2002 Int. J. Theo. Phys. 41 341–370
[12] Nemeti I and David G 2006 App. Math. Comp. 178 118–142
[13] Brun T 2003 Foundations of Physics Letters 16 245–253
[14] Bacon D 2004 Phys. Rev. A 70 032309
[15] Aaronson S and Watrous J 2008 Closed timelike curves make quantum and classical computing equivalent (Preprint quant-ph/0808.2669)
[16] Darwin C G 1957 Nature 180 976–977
[17] McCrea W H 1957 Nature 179 909–910
[18] Jarett K and Cover T 1981 IEEE Trans. Inf. Theory 27 152–159
[19] Barrow J D and Levin J 2001 Phys.Rev. A 63 044104
[20] Minguzzi E 2005 Am. J. Phys 73 876
[21] Iorio L 2005 Found. Phys. Lett. 18 1
[22] Minguzzi E 2006 Found. Phys. Lett. 19 353
[23] Taylor E and Wheeler J 1992 Spacetime Physics (W.H.Freeman and Co Ltd)
[24] Woodhouse N 2007 Special Relativity, 2nd ed (Springer)
[25] Bennett C, Bernstein E, Brassard G and Vazirani U 1997 SIAM J. Comp. 5 1510–1523
[26] Grover L K 1997 Phys. Rev. Lett. 79 325
[27] Deutsch D 1985 Proc. Roy. Soc. Lon. A 400 97–117
[28] Lloyd S 2000 Nature 406 1047–1054
[29] Yao A C 2003 J. ACM 50 100–105