Stimulated emission of two photons in parametric amplification and its interpretation as multi-photon interference

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Stimulated emission of two photons is observed experimentally in the parametric amplification process and is compared to a three-photon interference scheme. We find that the underlying physics of stimulated emission is simply the constructive interference due to photon indistinguishability. So the observed signal enhancement upon the input of photons is a result of multi-photon interference of the input photons and the otherwise spontaneously emitted photon from the amplifier.

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Stimulated emission, first proposed by Einstein [1] to explain the blackbody radiation spectrum, is the main process in laser operation. It provides the optical gain of an active medium and is responsible for the coherence of laser light [2]. Although the process was studied extensively as an amplification process of a classical wave field as early as in 1955 [3], its effect on the nonclassical state of light was only investigated not long ago [4], especially in the contest of quantum state cloning [5, 6, 7].

Fundamentally, stimulated emission occurs at single-photon level, i.e., it is seen as the creation of an identical photon to an incoming photon. However, the same photon can also be produced even without the incoming photon, due to spontaneous emission. Thus, the existence of the input photon will enhance the production rate, as compared to the case without the input photon. Indeed, in recent study of stimulated emission by single photons, a doubled rate is observed in photon production that is correlated to the input photon [6, 7, 8]. But the above picture is only phenomenological and does not tell the underlying physics. So what fundamental physical principle governs this phenomenon?

If we make a detailed analysis of the enhancement due to stimulated emission, we find that it bears some resemblance to the famous Hanbury Brown-Twiss effect [9], i.e., the photon bunching effect of thermal light [8, 10]: the enhancement factor in both cases is one-fold and the temporal profile is the same. The photon bunching effect, as pointed out by Glauber [11] in 1965, is in essence a two-photon interference effect. This further demonstrates the connection between stimulated emission and the multi-photon interference.

In this letter, we wish to report on an experiment in support of the above claim. In the experiment, we inject a two-photon state into a parametric amplifier and observe an enhancement by a factor of nearly two in the photon production that is correlated to the input photons. Furthermore, we mimic the same phenomenon with a beam splitter, in a similar manor to Ref.[15] for the single-photon input case. These two phenomena can be viewed as a generalized three-photon bunching effect and are a result of three-photon constructive interference due to photon indistinguishability.

To understand the connection between the stimulated emission and multi-photon interference, we consider the two situations in Fig.1. The process of stimulated emission of an \(N\)-photon state is shown in Fig.1(a), where \(N\) photons interact with an atom in an excited state. The atom will emit one photon regardless of the input. Total photon number is \(N + 1\). Assume that the spontaneous emission rate stimulated by one photon is the same. Then since each input photon may stimulate the excited atom, the total rate is then

\[ \text{in stimulated emission} \]

This further demonstrates the connection between stimulated emission and the multi-photon interference.

FIG. 1: Comparison between (a) the stimulated emission and (b) the multi-photon interference.
The extra $R$ is from spontaneous emission. There is an enhancement factor of $N$ in the photon production rate. The case of $N = 1$ was observed in Ref. [6].

In multi-photon interference in Fig.1(b), on the other hand, a single photon and $N$ photons are combined by a 50:50 beam splitter. The probability of detecting all $N+1$ photons in one side is easily calculated to be $(N+1)/2^{N+1}$ (see below). However, when the single photon is distinguishable from the $N$ photons and no multi-photon interference occurs, we find the detection probability is simply $1/2^{N+1}$. The $N+1$ factor is a result of constructive interference of $N+1$ possibilities in detecting the $N+1$ photons. Each possibility corresponds to the situation when the single input photon is detected by a specific detector. (Fig.1 shows two such possibilities in the $N+1$-photon detection.) We add the amplitudes of the $N+1$ possibilities before taking the absolute value for the indistinguishable case but we add the absolute values of the amplitude of each possibility for the distinguishable case. The ratio between the two cases is then $N+1$. The case of $N = 1$ is the Hong-Ou-Mandel photon bunching effect [13]. A similar interference effect was observed by Ou et al. [10] with a $|2,2\rangle$ input state. Notice that the enhancement factor here is the same as the stimulated emission in Fig.1(a). Therefore, the spontaneous emission rate $R$ corresponds to the situation when the input $N$ photons to the atom are distinguishable from the emitted photon by the atom, so that the atom is not influenced by the input photons and only emits spontaneously. This case is exactly the same as the case in Fig.1(b) but when the single photon is distinguishable from the $N$ photons. When the input $N$ photons are indistinguishable from the emitted photon by the atom, constructive multi-photon interference leads to a factor of $N+1$ enhancement in photon detection rate. Thus, the underlying physics in the stimulated emission is the photon indistinguishability that results in multi-photon interference.

In the following, we will use parametric amplifier to study the stimulated emission by multiple photons. Mathematically, any phase insensitive linear amplifier can be modelled as a single mode parametric amplifier, which is described quantum mechanically by [17]:

$$\hat{a}_s^{(out)} = G\hat{a}_s + g\hat{a}_s^\dagger,$$  \hspace{1cm} (1)

where $\hat{a}_s$ corresponds to the internal modes of the amplifier and is the idler mode for the parametric amplifier. It is usually independent of $\hat{a}_g$ and is in vacuum. To preserve the commutation relation, we need $|G|^2 - |g|^2 = 1$.

At microscopic level of atoms, we have a small value of $|g| < < 1$ or $|G| \sim 1$. The unitary evolution operator for Eq. (1) then has the form of

$$\hat{U} \approx 1 + (g\hat{a}_s^\dagger \hat{a}_s^\dagger + h.c.)$$  \hspace{1cm} (2)

With a vacuum input of $|0\rangle$, we have the output state

$$|\Phi_{out}^{(0)}\rangle = \hat{U}|0\rangle \approx |0\rangle + g|1_s\rangle \otimes |1_i\rangle.$$  \hspace{1cm} (3)

The last term gives the spontaneous emission with a probability of $|g|^2$. When the input is a single-photon state $|1_s\rangle \otimes |0_i\rangle$, we have

$$|\Phi_{out}^{(1)}\rangle \approx |1_s\rangle|0_i\rangle + g(\hat{a}_s^\dagger |1_s\rangle) \otimes (\hat{a}_s^\dagger |0_i\rangle) = |1_s\rangle|0_i\rangle + \sqrt{2}g|2_s\rangle \otimes |1_i\rangle.$$  \hspace{1cm} (4)

The probability for the emission from the amplifier is then $2|g|^2$. The extra emission probability of $|g|^2$ is usually attributed to the stimulated emission, which is similar to the photon bunching effect [8, 9, 10, 13].

When the input state is a two-photon state of $|2_s\rangle|0_i\rangle$, we have the output state as

$$|\Phi_{out}^{(2)}\rangle \approx |2_s\rangle|0_i\rangle + g(\hat{a}_s^\dagger |2_s\rangle) \otimes (\hat{a}_s^\dagger |0_i\rangle) = |2_s\rangle|0_i\rangle + \sqrt{3}|3_s\rangle \otimes |1_i\rangle.$$  \hspace{1cm} (5)

The photon emission rate from the amplifier is now three times the spontaneous rate. In fact, it is straightforward to find that, with an $N$-photon state as the input, the rate of photon emission from the amplifier is $N+1$ times the spontaneous emission rate. As the single-photon input case, each fold of increase in the rate can be attributed to the stimulated emission from one individual photon in the input $N$-photon state.

Notice that when the input photons are not in the same mode as the amplifier and thus are distinguishable from the photon emitted by the amplifier, the output state becomes

$$|\Phi_{out}^{(N)}\rangle \approx |0_s\rangle|N_s\rangle|0_i\rangle + g(\hat{a}_s^\dagger |0_s\rangle) \otimes |N_s\rangle \otimes (\hat{a}_s^\dagger |0_i\rangle) = |0_s\rangle|N_s\rangle|0_i\rangle + g|1_s\rangle|N_s\rangle|1_i\rangle,$$  \hspace{1cm} (6)

where $N \geq 1$. So the photon production rate is exactly the same as the spontaneous emission.

The above analysis with the parametric amplifier confirms the previous results obtained from the pictorial argument based on Fig.1. Next, we will demonstrate experimentally the enhancement effect for a two-photon input and compare it with a three-photon interference scheme with a beam splitter.

The experimental arrangement for studying the stimulated emission is sketched in Fig.2. A mode-locked Ti:sapphire laser operating at 780 nm is frequency doubled and the harmonic field serves as the pump field for

![FIG. 2: Schematics for studying the stimulated emission of an input of N-photon state with parametric amplification. SMF: single-mode fiber; IF: interference filter; H,V: horizontal and vertical polarizations; $T_H$: adjustable delay.](image-url)
a parametric amplifier made of a 1-mm long β—Barium Borate (BBO) crystal. The crystal is so oriented that it satisfies the type-II phase matching condition and beamlike fields are generated \[18\]. A small portion is split from the Ti:sapphire laser and serves as the input field to the signal port of the parametric amplifier. The injected coherent field is heavily attenuated down to a rate much less than one photon per pulse. But even so, the coherent state consists of vacuum, one-photon state, two-photon state, and more. So the output state is a superposition of the states in Eqs. (3-5). Therefore, in order to observe the enhancement effect in stimulated emission by a specific number of photons, we need to make a projection measurement to the corresponding states in Eqs. (4, 5). For example, for a two-photon state input, the projection is to the second term in Eq. (6). This is achieved by a four-photon coincidence measurement, as depicted in Fig. 2. Joint measurement with the idler photon is necessary to discriminate against the three-photon contribution directly from the injected coherent field. Photon (in)distinguishability between the input photons and the photon emitted from parametric down-conversion is realized by an adjustable delay \(T_H\) on the coherent injection field. A single-mode fiber (SMF) is used to collect the signal field from the amplifier, in order to ensure a good spatial mode match. Interference filters of bandpass of 3 nm are used for temporal mode cleaning.

The conditions for the situations in Eqs. (3-5) are satisfied by adjusting the delay \(T_H\). When the delay is right, the injected coherent field pulse arrives in time with the pump pulse to the amplifier and the photon emitted by the amplifier is indistinguishable from the incoming photons in the coherent state. But when the delay is either too large or too small, there is no overlap between the coherent pulse and the pump pulse. This is the situation described in Eq. (6). Therefore, as we scan the delay \(T_H\), the four-photon coincidence of A, B, C, D detectors should exhibit a bunching effect with a peak-to-wing ratio close to three. Fig. 3(a) shows the result of the measurement. The error bars are the statistical errors of one standard deviation. The solid curve is a least square fit of the data to a Gaussian of the form

\[
F(T_H) = A \left[1 + ve^{-(T_H-T_0)^2/T^2}\right],
\]

where \(T_0\) is the center position of the peak and \(T_e\) is related to the width of the peak. We obtain \(v_2 = 1.81 \pm 0.15\) as the enhancement factor for the data in Fig. 3(a), which gives 2.81 as the ratio between the peak and the wing. This value is close to the ideal value of three in Eq. (4) for the stimulated emission by two photons.

In the meantime, three-photon coincidences of ABD detectors are also registered and are shown in Fig. 3(b). This measurement corresponds to the second term in Eq. (4) and gives the stimulated emission by one input photon. The Gaussian fit gives an enhancement factor of \(v_1 = 0.88 \pm 0.14\). The peak-to-wing ratio of 1.88 is close to the ideal value of two in Eq. (4).

The reason for the imperfection in the experiment is due to mode mismatch between the input field and the amplifier mode, i.e., mismatch between \(|N\rangle\) and the mode for which the operator \(\hat{a}_s\) represents. Although the spatial mode is matched by the single-mode fiber (SMF in Fig. 2), the temporal mode is hard to match because the temporal coherence of the parametric down-conversion process is very complicated and the fields are not transform-limited even if the pump field is. Nevertheless, we use interference filters to clean up the temporal profile. The full width of the peaks in Fig. 3 is approximately \(2T_e = 660\) ps, close to the coherence time of the interference filters (IF in Fig. 2) of width 3 nm.

Next we consider the situation depicted in Fig. 1(b) where an \(N\)-photon state is superposed with a single-photon state by a 50:50 beam splitter. The output state for the beam splitter is given by \[19\]

\[
|\Phi_{out}^{(BS)}\rangle = \sqrt{\frac{N+1}{2^{N+1}}} |N+1\rangle_1 |0\rangle_2 + \ldots ,
\]

where we only write down the state for which all the \(N+1\) photons exit at one port (port 1) of the beam splitter. On the other hand, if the input \(N\) photons are distinguishable from the single photon from the other input port, they behave like classical particles and follow the Bernoulli distribution. The output state becomes

\[
|\Phi_{out}^{(BS)'}\rangle = \sqrt{\frac{1}{2^{N+1}}} |N\rangle_1 |1\rangle_1 |0\rangle_2 + \ldots .
\]

Therefore, the rate of detecting \(N+1\) photons in port 1 is \(N\) times bigger when the photons are all indistinguishable than when the \(N\) photons are distinguishable from the one photon. As discussed before, this increase stems from a constructive \(N+1\)-photon interference.
weak coherent field is directed to a beam splitter to combine with the signal field (H) from the parametric down-conversion. In order to mimic the stimulated emission process shown in Fig.2, all the experimental parameters such as pump power, the strength of coherent field, etc. are the same as those in Fig.2. We adjust the delay $T_H$ on the coherent field to ensure the temporal overlap between the coherent field and the down-converted photon. When gated on the detection of the V-photon by detector D, the H-field of the down-conversion is in a single-photon state. But because of the complicated dispersion in the parametric down-conversion process, the single-photon state is not transform-limited. Again, interference filters are used to clean up some of the temporal modes but impossible to achieve the perfect match.

We record both the four-photon coincidence of ABCD detectors and the three-photon coincidence of ABD detectors. The former corresponds to the $N = 2$ case in Eq. (9), whereas the latter to the $N = 1$ case. The results are shown in Fig.5. The fitted curves are very similar to Fig.3 but with $v_2 = 1.78 \pm 0.14$ and $v_1 = 0.86 \pm 0.09$.

As can be seen, Figs.3 and 5 show the same result within the statistical errors. This confirms our claim that the underlying physics in stimulated emission is nothing but multi-photon interference. The interference effect is a result of indistinguishability between the input photons and the photon emitted by the amplifier.

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