Influence of hot end heat exchangers on cascading three pulse tube coolers

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Abstract. Hot end heat exchanger (HHX), an indispensable part in the traditional pulse tube cooler (PTC), rejects the heat generated by dissipation of the acoustic power. The acoustic power, which should have been dissipated at the phase shifters, is delivered to the latter stage cooler in the cascade PTC. Therefore, by removing the HHX, power loss could be decreased. Specifically, in our experiment, after removing HHXs, the cooling power obtained by cascading three PTCs could reach 273.2 W at 233 K under the same working condition, which is 23.6 W more than that of the original structure.

1. Introduction

The demand for liquefied natural gas (LNG) is increasing, due to its advantage of safety during transportation, sufficient resources in storage, and durable applications, it will exceed that for pipeline natural gas by 2035 [1]. The liquefaction temperature of LNG is below 110 K, and the liquefaction process requires cooling power provided by a cooler. The PTC has potential benefits of high reliability and low vibration. Displacer in a stirling cryocooler is replaced by gas piston in a PTC. It's a matter of swings and roundabouts that high reliability is gained at the cost of some acoustic energy dissipated as heat in phase shifters. As a result, the intrinsic efficiency of PTC ($T_c/T_h$) is much lower than Carnot efficiency $T_h/(T_h-T_c)$ at LNG temperature [2], which limits the application of PTC in LNG. Recently, recover the dissipated acoustic power has become a new research trend.

There are three means of acoustic power recovery. The first two ways are by introducing either moving parts [3] or loop configurations [4]. The third one is by using quarter-wavelength PTC, which was proposed by Swift et al. in 2011 [5]. We have proposed a cascade pulse tube cooler capable of energy recovery in 2015 [6] without moving parts or loop configurations being introduced. Theoretical analysis shows a multi-stage cascade PTC could approach Carnot efficiency. The three-stage cascade PTC attains 249.6 W @ 233 K cooling power by experiment when the electrical power input is fixed at 500 W with the 1st, 2nd and 3rd stage refrigerator obtaining 155.8 W, 75.5 W and 18.3 W cooling power, respectively. The cooling efficiency is 37.7% higher than that of a single-stage PTC. Vapor-compression refrigerator is displaced by the cascading three PTCs...
to cool infrared resistance array for there’s no oil in a PTC. The cooling temperature here is set at 233 K, which is a little lower than ambient temperature 300 K.

The HHX is applied to reject the heat generated by dissipated acoustic power at phase shifter. HHX is unnecessary when the dissipated acoustic power is recovered. Pressure drop and irreversible heat transfer are induced as oscillating flow passes through HHX, thus increasing energy loss. Swift et al. [7] had also proposed that the HHX of the pulse tube could be removed to realize energy recovery in his patent. Theoretical analysis indicates that the energy loss will be decreased and the efficiency of a cascade PTC will approach the Carnot efficiency when HHXs are removed.

2. Theoretical analysis

![Figure 1. Schematic diagram of cascading three PTCs.](image)

Theoretically, the refrigeration efficiency of a multi-stage cascade PTC will infinitely approach Carnot efficiency by using several transmission tubes to recover the acoustic power stepwise at the hot end of the former pulse tube. Figure 1 shows the schematic diagram of our three-stage cascade PTC.

2.1. Losses in heat exchanger

There are two main losses in a heat exchanger [8], one is irreversible entropy generation caused by heat transfer temperature difference $\Delta T$, and the other is pressure drop. Irreversible entropy generation caused by $\Delta T$ can be defined as

$$\langle \dot{S}_{irr} \rangle = \frac{1}{\tau} \int_{T_0}^{T_f} \frac{\dot{Q} \Delta T}{T_f} dt = \frac{\dot{Q} \Delta T}{TT_f}$$

Where $\tau$ is the minimum cycle time, $T$ is the external temperature, $T_f$ is the internal fluid temperature and $\dot{Q}$ is the heat flow rejected to the surrounding at ambient temperature. When the external temperature is equivalent to environmental temperature, the acoustic power loss caused by $\langle \dot{S}_{irr} \rangle$ is given as,

$$W_{bat} = T \langle \dot{S}_{irr} \rangle = \frac{\dot{Q} \Delta T}{T_f}$$

![Figure 2. Schematic diagram of a parallel-plate heat exchanger.](image)
The parallel-plate heat exchanger (figure 2) is modeled by the standard linear thermo-acoustic theory with two ideal assumptions: the working fluid is a Newtonian gas, and the length of heat exchanger is much shorter than acoustic wavelength to obtain uniform spatial acoustic field. The gap or plate spacing available to the gas is $2y_0$. The length of the heat exchanger is $\Delta x$. The regulation of wave propagation in the heat exchanger can be described as [9]

$$\frac{dp_i}{dx} = -\frac{i\omega p_m}{(1 - f_v) A_{gas}} U_i$$  \hspace{1cm} (3)

$$\frac{dU_i}{dx} = -\frac{i\omega A_{gas}}{\rho_m a^2} \left(1 + \frac{(\gamma - 1)f_e}{1 + e_a}\right) p_i$$  \hspace{1cm} (4)

where $p_i$ and $U_i$ are the pressure wave and volume flow in the channel, respectively, the parameters in bold represent vectors. Spatially averaged thermal diffusion function is $f_e = \tanh[(1+i)y_0/\delta_e]/[(1+i)y_0/\delta_e]$, spatially averaged viscous diffusion function is $f_v = \tanh[(1+i)y_0/\delta_v]/[(1+i)y_0/\delta_v]$, where $\delta_e$ and $\delta_v$ are thermal penetration depth, viscous penetration depth.

2.2. Calculation results

2.2.1. Pressure drop results in energy loss

![Figure 3](image-url)

Figure 3. Pressure and volume flow amplitude distribution in the parallel-plate heat exchanger. The inlet condition of the 1st stage HHX of pulse tube is known as $|p_i| = 112.4$ kPa, $|U_i| = 0.004$ m$^3$/s. The amplitude distribution of pressure and volume flow changes with the length of the parallel-plate heat exchanger (figure 3), whose half-thickness $y_0$ attained by substituting the inlet conditions into Eqs. (3)-(4) is 0.1 mm. From figure 3, we can see that the volume flow is almost constant along with the length. But the pressure amplitude decreases dramatically from 112.4 kPa to 106.6 kPa when the length $\Delta x$ of the heat exchanger in our experiment is 0.025 m, which is caused by viscous friction. The corresponding energy loss due to pressure drop is 9.5 W. Another 1.8 W energy loss due to pressure drop at the 2nd stage HHX is obtained by the same calculation method. Entrance effects are ignored in the above calculations.
Energy loss due to finite heat transfer temperature difference $\Delta T$ in the 1st, 2nd HHXs are 0.23 W and 0.12 W, respectively, calculated from Sage. To sum up, when the entrance effect that may be the source of minor losses is neglected, a total 11.65 W acoustic power is lost in the two HHXs, and energy loss due to pressure drop is far more than that due to irreversible heat transfer.

2.2.2. Frequency optimization of the three-stage cascade PTC

An existing linear compressor CFIC 2s132 (500 W rated power, 2.5 MPa charging pressure) is used as the pressure wave generator. The operating frequency of the three-stage cascade PTC is optimized by the numerical calculation software Sage. Figure 4 depicts the comparison of cooling capacity of all stages calculated by Sage in the three-stage cascade PTC at cooling temperature of 233 K. Maximum cooling capacity of 271.8 W is obtained @ 60 Hz when the three-stage cascade PTC contains the 1st, 2nd stage HHXs (named “The original” in figure 4a). However, when the two HHXs are removed (figure 4b), about 7.5 W more cooling capacity is obtained @ 60 Hz with the maximum cooling capacity of 279.3 W.

![Figure 4](image-url)

**Figure 4.** Comparison of cooling power: (a) The original structure; (b) Without the 1st and 2nd HHXs.

2.2.3. Phase relation

Figure 5 shows phase diagram of the first stage PTC with different structures. When the two HHXs mentioned above are reserved, mass flow $\dot{m}_{h1}$ at the hot end of regenerator leads 17.7° ahead of the pressure wave $p_1$, while mass flow $\dot{m}_{c1}$ at the cold end of regenerator lags 4.5° behind $p_1$. However, when the two HHXs are removed, mass flow in the 1st stage cooler will lag by a small angle. As a result, after removing the two HHXs, the phase at the midpoint of regenerator
will reach a better condition.

2.2.4. Distribution of energy flow in the three-stage cascade PTC

![Energy flow distribution diagram](image)

**Figure 6.** Calculated energy flow distribution along x axis in the cascading three PTCs.

Effect of the 1st and 2nd HHXs on energy flow is in figure 6. As shown in figure 6a, 21.8 W acoustic power is lost in the 1st HHX. Comparing figure 6a with figure 6b, we find that after the two HHXs are removed, the acoustic power at the inlet of 1st transmission tube increases from 178 W to 196 W, and the acoustic power at the inlet of 2nd transmission tube increases from 80 W to 88 W. Enthalpy flow in the HHXs decreases first and then increases to the original level, which is significantly different from that in the aftercooler. This indicates that the oscillating flow in the HHXs first rejects heat to the HHXs, which does not have a large heat load and then absorbs heat. As for the cascading three PTCs, the HHXs deteriorate the performance of the refrigerator in this calculation.

3. Experimental setup

A three-dimensional model of the cascading three PTCs is shown in figure 7. Main parameters of the cascade three PTCs are listed in table 1. The refrigerator is driven by the CFIC 2s132 model linear compressor. Because the cooling temperature is 233 K, the cold end is insulated by non-vacuum perlite, which is accessible and feasible. While two rhodium-iron resistance thermometers are mounted on the cold end of the primary stage PTC, two platinum resistance thermometers are mounted on the cold end of the secondary stage PTC, and three rhodium-iron resistance thermometers are mounted on the cold end of the tertiary stage PTC, each with accuracy of ±0.1 K. Thermal resistors are uniformly distributed in all stages of cold end and the refrigerating capacity is measured by heat balance calculation. Four 50 Ω resistors capable of providing 200 W heating power are installed in series on the cold end of the primary stage, while another three 50 Ω resistors capable of providing 150 W heating power are installed in parallel on the cold end of the secondary stage, and three 500 Ω resistors capable of providing 150 W heating power are installed in parallel
on the cold end of the tertiary stage. Six pressure sensors are employed (positions shown in figure 7) with P2 to measure static pressure and the others to measure the dynamic pressures. Static pressure sensor is calibrated by Keller Leo2 digital display meter with range of 3.1 MPa and accuracy of 0.1% FS. Real-time data including temperatures and pressures are collected by LabVIEW in a PC while electrical input power of the compressor and quantity of heat applied to cold ends are directly read from the KIKUSUI power supplies.

**Table 1.** Designed parameters of the cascading three PTCs.

| Part name       | Dimension (i.d × length, mm) |
|-----------------|------------------------------|
| The 1st stage   |                              |
| Regenerator     | 53.7 × 37.7                 |
| Pulse tube      | 30.5 × 134.3                |
| Transmission tube I | 14.2 × 7000.0               |
| The 2nd stage   |                              |
| Regenerator     | 47.6 × 48.0                 |
| Pulse tube      | 27 × 150                     |
| Transmission tube II | 10 × 6300.0                |
| The 3rd stage   |                              |
| Regenerator     | 27 × 53                      |
| Pulse tube      | 19.6 × 100                   |
| Inertance tube  | 4 × 1300.0                   |
| Reservoir       | 1 L                          |

**Figure 7.** Experimental setup of cascading three PTCs.

4. Results and discussion

As shown in figure 8, temperatures of the three-stage cascade PTC dropped to the lowest point in 7 hours under the working condition of 60 Hz, 2.5 MPa (mean pressure). With HHXs, the no-load temperature at cold head of the 1st, 2nd and 3rd stage reached 141.4 K, 144.8 K and 197.0 K, respectively. Without the 1st and 2nd stage HHXs, there were minimal differences among cold-head temperatures of the three stages, with the cold-head temperature of the 1st and 3rd stage continued
to drop while that of the 2nd stage began to recover slightly.

When power input was fixed at 500 W and cooling temperature at 233 K, by sweeping frequency of cascading three PTCs with or without the HHXs, the optimum frequency where the maximum cooling capacity was generated all deviated from the design value of 60 Hz (figure 9). Initially, the refrigerator obtained the maximum cooling capacity of 249.6 W at 57 Hz. When the two HHXs were removed, the maximum cooling capacity was 273.2 W at 55 Hz, and the cooling capacity of the 1st, 2nd and 3rd stage was 170.8 W, 80.7 W and 21.7 W, respectively. When power input and cooling temperature was constant, the maximum cooling capacity of the three-stage cascade PTC increased by 23.6 W, and the cooling efficiency rose by 9.46%. The cooling capacity generated without the HHXs increased significantly compared with the theoretical calculations, indicating that discrepancy exists between experiment and theoretical model, which needs further optimization. In the experiment, after removing the HHXs, the temperature of the transmission tube was slightly higher than ambient due to jet flow. We are considering adding flow straighteners to reduce jet flow in the next steps.

Figure 10 and 11 show the theoretical and experimental phase relations of all stages in the three-stage cascade PTC, respectively. In the theoretical measurement, the angle of pressure wave between the 1st and 2nd stage, or between 2nd and 3rd stage was -150°, while the measured phase difference was -157° and -154°, respectively. After removal of the 1st and 2nd HHXs, the pressure amplitude increased slightly.

Figure 8. Comparison of cool down curves. Figure 9. Comparison of cooling power.

Figure 10. Theoretical comparison diagram of phase. Figure 11. Experimental comparison diagram of phase.
5. Conclusion
In contrast with the conventional PTC, which relies on HHX for thermal dissipation, the acoustic power-recovering PTC doesn’t need a HHX from the theoretical point. The interior acoustic power and enthalpy flux distribution of the cascade PTC shows that HHX hardly burdened thermal load in the acoustic power-recovering PTC. On the contrary, the existence of HHX deteriorates the performance of the cooler by increasing the effect of pressure loss. After the removal of the two HHXs, experimental results at 2.5MPa charging pressure, 500 W input power, and 55 Hz frequency indicates the maximum cooling power acquired by cascading three PTCs was 273.2 W @ 233 K, with the 1st, 2nd and 3rd stage refrigerator obtaining 170.8 W, 80.7 W and 21.7 W, respectively. The cooling capacity of the three-stage cascade PTC was improved by 23.6 W, and the efficiency rose by 9.46%.

6. References
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