Noncommutativity of the Moving $D2$-brane Worldvolume

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Abstract

In this paper we study the noncommutativity of a moving membrane with background fields. The open string variables are analyzed. Some scaling limits are studied. The equivalence of the magnetic and electric noncommutativities is investigated. The conditions for equivalence of noncommutativity of the $T$-dual theory in the rest frame and noncommutativity of the original theory in the moving frame are obtained.

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1 Introduction

Over the past years there have been attempts to explain noncommutativity on D-brane worldvolume through the study of open strings in the presence of background fields [1, 2]. From the DBI action it is known that on a D-brane electric field cannot be stronger than a critical electric field, while the same is not true for magnetic fields. Also it is known that Lorentz boosts act on electromagnetic backgrounds. This affects the noncommutativity parameter and the effective open string metric. According to these facts, some properties of D-branes with background fields such as decoupling limits and light-like noncommutativity have been studied [3].

Previously we have studied the noncommutativity of a moving $Dp$-brane, with the motion along itself [4]. Now we study the noncommutativity of a moving membrane with electric and magnetic background fields. The motion is parallel or perpendicular to the membrane. For each case, the effective open string variables will be analyzed. We shall observe that for an appropriate magnetic field the open string metric is frame independent. In a special frame the open string metric is proportional to the closed string metric. For both electric and magnetic cases we find decoupling limits, which lead to the definite noncommutative theories. There are frames for the electric and magnetic membranes such that their noncommutativities are proportional to each other.

The effects of $T$-duality on the effective metric and noncommutativity parameter enable us to obtain equivalent noncommutativity structures. That is, we find speeds and background fields for the membrane such that the noncommutativity of the $T$-dual theory becomes equivalent to the noncommutativity of the original theory in the moving frame.

The analysis of Ref.[2] leads to the definitions of the open string metric $G$, the noncommutativity parameter $\Theta$ and the effective open string coupling constant $G_s$,

\[
G^{\mu\nu} = \left( g + 2\pi\alpha' B \right)^{-1} g (g - 2\pi\alpha' B)^{-1})^{\mu\nu},
\]

\[
G_{\mu\nu} = \left( g - 2\pi\alpha' B \right) g^{-1} (g + 2\pi\alpha' B)_{\mu\nu},
\]

\[
\Theta^{\mu\nu} = -(2\pi\alpha')^2 \left( g + 2\pi\alpha' B \right)^{-1} B (g - 2\pi\alpha' B)^{-1})^{\mu\nu},
\]

\[
G_s = g_s \left( \frac{\det G}{\det (g + 2\pi\alpha' B)} \right)^{\frac{1}{2}},
\]

where $g_{\mu\nu}, B_{\mu\nu}$ and $g_s$ are closed string variables.

Note that the effective open string coupling $G_s$ does not change under the Lorentz boosts. Because $g_s$ is the exponential of the scalar field dilation, and the ratio of two determinants also is invariant.
In general a $D2$-brane parallel to the $X^1X^2$-plane has the NS-NS background $B$-field as in the following

\[ B_{\mu\nu} = \begin{pmatrix} 0 & E & E' \\ -E & 0 & b \\ -E' & -b & 0 \end{pmatrix}, \]  

(2)

where $\mu, \nu \in \{0, 1, 2\}$. We shall discuss pure magnetic and pure electric cases. Let the closed string metric of the membrane worldvolume be

\[ g_{\mu\nu} = \begin{pmatrix} -g_0 & 0 & 0 \\ 0 & g_1 & g' \\ 0 & g' & g_2 \end{pmatrix}. \]  

(3)

The paper is organized as follows. In section 2, we study the behavior of effective variables of open string in terms of the background magnetic field and the speed of the membrane. In section 3, the same will be done in the presence of the electric field. In addition, some scaling limits and also equivalence of two noncommutativities will be obtained. In section 4, we study the $T$-duality of the theory and conditions for equivalence of boosted and $T$-dual noncommutativities.

## 2 Magnetic field noncommutativity

For the pure magnetic field (i.e., $E = E' = 0$), the noncommutativity matrix is

\[ \Theta^{\mu\nu} = \theta \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \]  

(4)

where the parameter $\theta$ (i.e., the strength of the noncommutativity) is defined by

\[
\theta = -\frac{(2\pi\alpha')^2b}{g + (2\pi\alpha')^2b^2}, \\
g = g_1g_2 - g'^2.
\]  

(5)

This relation implies that the different magnetic fields $b_\pm = -\frac{1}{2g} \pm \frac{1}{2} \sqrt{\frac{1}{g^2} - \frac{g}{\pi^2\alpha'^2}}$ produce the same noncommutativity on the membrane. Since $g$ is positive, the function $\theta(b)$ has the maximum $\theta_0$ (the minimum $-\theta_0$) at $b = -b_0$, ($b = b_0$) where

\[ b_0 = \frac{\sqrt{g}}{2\pi\alpha'}, \quad \theta_0 = \frac{\pi\alpha'}{\sqrt{g}}. \]  

(6)
Therefore, to obtain a strong noncommutativity we should adopt \( \pm b_0 \) magnetic fields.

Now we proceed to study the expressions for the various geometrical quantities in different frames, appropriate to different states of motion of the brane.

### 2.1 Motion along the \( X^1 \)-direction

Consider the Lorentz transformations on the coordinates \( X^0 \) and \( X^1 \),

\[
\begin{align*}
X'^1 &= \gamma (X^1 - v \sqrt{g_0} X^0), \\
X'^0 &= \gamma (X^0 - \frac{v}{\sqrt{g_0}} X^1),
\end{align*}
\]

where \( \gamma = 1/\sqrt{1 - v^2} \). The effect of these transformations on the matrix (4) is

\[
\Theta'^{\mu \nu} = \gamma \theta \begin{pmatrix} 0 & 0 & -\frac{v}{\sqrt{g_0}} \\ 0 & 0 & 1 \\ \frac{v}{\sqrt{g_0}} & -1 & 0 \end{pmatrix}.
\]

(8)

On the other hand, in the moving frame the noncommutativity parameter has non-zero time-like element. Since for a brane with electric field also there are time-like elements (e.g., see the equation (19)), this implies that the motion along the brane directions is equivalent to an appropriate electric field.

The transformation of the open string metric is

\[
G'_{\mu \nu} = \begin{pmatrix} -\gamma^2 g_0 (1 - g_1 a v^2) & -\gamma^2 v \sqrt{g_0} (1 - g_1 a) & \gamma v \sqrt{g_0} g' a \\ -\gamma^2 v \sqrt{g_0} (1 - g_1 a) & \gamma^2 (g_1 a - v^2) & \gamma g' a \\ \gamma v \sqrt{g_0} g' a & \gamma g' a & g_2 a \end{pmatrix},
\]

(9)

where the parameter \( a \) is defined by

\[
a = 1 + \frac{(2\pi \alpha' b)^2}{g}.
\]

(10)

Since \( g \) is positive we have \( a \geq 1 \). For \( v = 0 \), this metric reduces to the open string metric for the static membrane with magnetic field.

Let the off-diagonal element \( g' \) vanish. In addition, consider the following relation between the magnetic field and the elements of the closed string metric

\[
b = \pm \frac{1}{2\pi \alpha'} \sqrt{g_2 (1 - g_1)}.
\]

(11)

These give the diagonal open string metric

\[
G'_{\mu \nu} = \text{diag}(-g_0, 1, g_2/g_1),
\]

(12)
which is independent of the speed of the membrane. Note that the membrane speed has not
been hidden in the relation (11).

Under the above conditions we have \( \theta = \pm 2\pi \alpha' \sqrt{\frac{1-g_1}{g_2}} \). Therefore, for \( v \to 1, g_1 \to 1 \) (i.e.,
\( b \to 0 \)) and finite \( g_0 \) and \( g_2 \) we can introduce the scaling limit
\[
\gamma \sqrt{1-g_1} = \sigma ,
\]
where \( \sigma \) is a finite constant. This implies that
\[
\begin{bmatrix}
0 & 0 & -\frac{1}{\sqrt{g_0}} \\
0 & 0 & 1 \\
\frac{1}{\sqrt{g_0}} & -1 & 0
\end{bmatrix},
\]
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & g_1 a & g' a \\
0 & g' a & g_2 a
\end{bmatrix}.
\]
Since all elements of \( \Theta^{\mu\nu} \) and \( G'_\mu\nu \) are finite, this is a definite noncommutativity.

### 2.2 Motion along the \( X^3 \)-direction

According to the Lorentz transformations
\[
X'^3 = \gamma (X^3 - v \sqrt{g_0} X^0) ,
\]
\[
X'^0 = \gamma (X^0 - \frac{v}{\sqrt{g_0}} X^3) ,
\]
the noncommutativity parameter \( \Theta^{\mu\nu} \) does not change, while the open string metric \( G_{\mu\nu} \)
transforms to
\[
G'_\mu\nu = \begin{bmatrix}
-\gamma^2 g_0 & 0 & 0 \\
0 & g_1 a & g' a \\
0 & g' a & g_2 a
\end{bmatrix}.
\]
From the relativistic point of view, the motion perpendicular to the brane does not change
the lengths along the brane. Furthermore, since the noncommutativity due to the magnetic
field only has the space-space elements, it is an expected result that \( \Theta^{\mu\nu} \) and the space-space
elements of \( G_{\mu\nu} \) remain unchanged.

For special speeds of the membrane there is \( \gamma^2 = a \), or equivalently
\[
v = \pm \frac{2\pi \alpha' b}{\sqrt{g + (2\pi \alpha' b)^2}} .
\]
These speeds are not greater than the speed of light. In these frames the open string metric
reduces to
\[
G'_\mu\nu = a g_{\mu\nu} .
\]
Therefore, the open string metric is the scaled closed string metric by the scale factor \( a \).
3 Electric field noncommutativity

Now consider an electric field along the $X^1$-direction i.e., $E' = b = 0$. The noncommutativity parameter of this system is

$$\Theta_{\mu\nu} = T(E) \begin{pmatrix} 0 & g_2 & -g' \\ -g_2 & 0 & 0 \\ g' & 0 & 0 \end{pmatrix},$$

(19)

where the function $T(E)$ is

$$T(E) = \frac{E}{g_2 \frac{E^2_0}{E^2} - E^2},$$

$$E_0 \equiv \frac{1}{2\pi \alpha'} \sqrt{\frac{g_0 g}{g_2}}.$$  

(20)

The electric field $E$ only admits space-time noncommutativity. Since all elements of the noncommutativity parameter are proportional to the function $T(E)$, this factor shows the strength of the noncommutativity.

The effective open string coupling is $G_s = g_s \sqrt{1 - \left( \frac{E}{E_0} \right)^2}$. According to the root factor there is $-E_0 \leq E \leq E_0$. Therefore, unlike $\theta(b)$, the function $T(E)$ is one to one i.e., each noncommutativity strength only corresponds to one value of the electric field.

Near the electric field $E_0$, open strings do not interact and the strength of the noncommutativity is infinite. Although at the critical electric field $E_0$ the effective theory of the open string is singular and ill-defined, it is possible to define a space-time noncommutative theory by taking an appropriate scaling limit.

3.1 Motion in the $X^1$-direction

The transformations (7) give the noncommutativity parameter as in the following

$$\Theta^{\mu\nu} = T(E) \begin{pmatrix} 0 & g_2 & -g' \\ -g_2 & 0 & 0 \\ g' & 0 & 0 \end{pmatrix}.$$  

(21)

The open string metric also has the transformation

$$G'_{00} = -\gamma^2 g_0 \left[ 1 - g_1 v^2 - \left( 1 - \frac{g}{g_2} v^2 \right) \frac{E^2}{E_0^2} \right],$$

$$G'_{01} = -\gamma^2 v \sqrt{g_0} \left[ 1 - g_1 - \left( 1 - \frac{g}{g_2} \right) \frac{E^2}{E_0^2} \right],$$
\[ G_{02} = \gamma v g' \sqrt{g_0}, \]
\[ G_{11}' = \gamma^2 \left[ g_1 - v^2 - \left( \frac{g}{g_2} - v^2 \right) \frac{E^2_2}{E_0^2} \right], \]
\[ G_{12}' = \gamma g', \]
\[ G_{22}' = g_2. \] (22)

For \( v = 0 \) this metric reduces to the open string metric of the static membrane with electric field.

Near the electric field \( E = E_0 \) when there is \( g' \to 0 \), for any value of the speed \( v \), all elements of the transformed metric \( G_{\mu\nu}' \), except \( G_{22}' \), go to zero

\[ G_{\mu\nu}' = 0 \quad \text{except } G_{22}', \text{ for } g' \to 0, \quad E \to E_0. \] (23)

To avoid this singularity, we can do the following scaling limit. For \( E \to E_0, \quad g' \to 0 \) and finite \( g_2 \), we should have

\[ \gamma g' = \kappa, \quad \gamma^2 \left( 1 - \frac{E^2}{E_0^2} \right) = \rho, \] (24)

where \( \kappa \) and \( \rho \) are finite constants. These imply that the boost velocity approaches to the speed of light, \( v \to \pm 1 \). Therefore, the metric (22) takes the form

\[
G_{\mu\nu}' = \begin{pmatrix}
-\rho g_0 (1 - g_1) & -\rho \sqrt{g_0} (1 - g_1) & \kappa \sqrt{g_0} \\
-\rho \sqrt{g_0} (1 - g_1) & -\rho (1 - g_1) & \kappa \\
\kappa \sqrt{g_0} & \kappa & g_2
\end{pmatrix}. \] (25)

All elements of this metric are finite. To restore interactions of open strings, the string coupling \( g_s \) can be scaled to infinity i.e., \( g_s \sim \gamma \). This leads to a finite \( G_s \). The noncommutativity parameter \( \Theta_{\mu\nu}' \) near the critical field \( E_0 \) also should be finite. Therefore, we scale \( \alpha' \) to zero as \( \alpha' = \mu \left( 1 - \frac{E^2}{E_0^2} \right) \), where \( \mu \) is finite. In other words, after scaling we have

\[
\Theta_{\mu\nu}' = \frac{2\pi \mu}{\sqrt{g_0 g_2}} \begin{pmatrix}
0 & g_2 & -\kappa \\
-g_2 & 0 & \kappa \sqrt{g_0} \\
\kappa & -\kappa \sqrt{g_0} & 0
\end{pmatrix}, \] (26)

which describes a well defined noncommutativity.
3.2 Motion in the $X^3$-direction

From the transformations (15) we obtain $\Theta'^{\mu \nu} = \gamma \Theta^{\mu \nu}$, where $\Theta^{\mu \nu}$ has been given by the equation (19). The open string metric also becomes

$$G'_\mu \nu = \begin{pmatrix} -\gamma^2 g_0 (1 - \frac{E^2}{E_0^2}) & 0 & 0 \\ 0 & g_1 (1 - \frac{g' g_0}{g_1 g_2} \frac{E^2}{E_0^2}) & g' \\ 0 & g' & g_2 \end{pmatrix}. \quad (27)$$

Again in the limit $E \to E_0$ but arbitrary $v$ and $g'$, we should introduce a scaling limit. Let the elements of the closed string metric $g_0$, $g'$ and $g_2$ be finite. In the limit $E \to E_0$ we can put $\gamma^2 (1 - \frac{E^2}{E_0^2}) = \rho$, which gives

$$G'_\mu \nu = \begin{pmatrix} -\rho g_0 & 0 & 0 \\ 0 & \frac{g'}{g_2} & g' \\ 0 & g' & g_2 \end{pmatrix}. \quad (28)$$

In this limit, the speed $v$ approaches to the speed of light such that $\rho$ to be finite.

To have a finite noncommutativity parameter, the parameter $\alpha'$ should go to zero like $\alpha' = \beta (1 - \frac{E^2}{E_0^2})^{3/2}$, where $\beta$ is another finite constant. Therefore, we obtain

$$\Theta^{\mu \nu} = 2\pi \beta \sqrt{\frac{\rho}{g_0 g_2 g}} \begin{pmatrix} 0 & g_2 & -g' \\ -g_2 & 0 & 0 \\ g' & 0 & 0 \end{pmatrix}. \quad (29)$$

Since $\alpha'$ goes to zero, according to the definition (20), $E_0$ approaches to infinity. In this limit let $E$ be proportional to $E_0$ as in the following

$$E = \pm \sqrt{\frac{1}{2} \left( 1 + \sqrt{1 - 4q^2 \rho} \right)} E_0, \quad (30)$$

where $q$ is an infinitesimal number from the interval $-\frac{1}{2\rho^{1/2}} \leq q \leq \frac{1}{2\rho^{1/2}}$. Using the definition of $\rho$, this equation also can be written in the form $E = q\gamma E_0$. This implies that $q$ goes to zero such that $q\gamma \to \pm 1$. In other words, the field $E$ approaches to infinity like $\gamma^3$,

$$E = -\frac{q\gamma^4}{2\pi \beta \rho^{3/2}} \sqrt{\frac{g_0 g}{g_2}}. \quad (31)$$

For this large electric field, the noncommutativity parameters (19) and (29) are related to each other through the equation

$$\Theta'^{\mu \nu}_{\text{scaled}} = \frac{1}{q} \Theta^{\mu \nu}_E. \quad (32)$$

Since there is $q \neq 1$, the above noncommutativities are equivalent but cannot be equal.
3.3 Equivalence of the electric and magnetic noncommutativities

Consider two parallel membranes which move along the $X^1$-direction. The first is a $D2$-brane with the speed $v$ and the magnetic field $b$. Its corresponding closed string metric has the elements $(-g_0, g_1, g_2 = 0, g')$. The noncommutativity of this brane has been given by the equation (8). The second is $D2'$-brane with the speed $v'$, electric field $E$ and the closed string metric elements $(-g'_0, g'_1, g'_2 = 0, g')$. This system has the noncommutativity parameter (21).

It is possible to have the relation

$$\Theta'_{\mu\nu} E = \eta \Theta'_{\mu\nu} b,$$

where $\eta$ is any real number. This matrix equation leads to the conditions

$$T(E) = \frac{\eta}{g'} \sqrt{\frac{v^2 - 1}{g_0 - g'_0}},$$

$$vv' = \sqrt{\frac{g_0}{g'_0}},$$

where $\theta(b) = \frac{(2\pi\alpha')^2b}{g^2 - (2\pi\alpha'b)^2}$ and $T(E) = -(2\pi\alpha')^2 \frac{E}{g_0 g'}$ with $-\infty < E < +\infty$. On the other hand, electric field also does not have critical value. The first condition compares the strengths of the noncommutativities. The second condition imposes that $g'_0 > g_0$ and the motions should be in the same direction. For the non-zero magnetic field when $v \to \pm 1$ we obtain $E \to \infty$, which is available. For $\eta = 1$ these different systems completely have the same noncommutativity structure.

4 $T$-duality and equivalence of noncommutativities

In the case of toroidal compactification, when $d$-spatial coordinates are compactified on torus $T^d$, the $T$-duality group is $O(d, d; \mathbb{Z})$ [5]. Assume that a $Dp$-brane is wrapped on torus $T^p$. Under the action of a particular element of $O(p, p; \mathbb{Z}) T$-duality group i.e.,

$$T = \begin{pmatrix} 0 & 1_{p \times p} \\ 1_{p \times p} & 0 \end{pmatrix},$$

the background fields have the transformations [6]

$$(g + 2\pi\alpha' B) \to (g + 2\pi\alpha' B) = (g + 2\pi\alpha' B)^{-1}. $$

According to the equations (1) and (36) we obtain

$$G^{\mu\nu} = \tilde{g}^{\mu\nu},$$

$$\Theta^{\mu\nu} = (2\pi\alpha')^2 \tilde{B}^{\mu\nu}. $$

9
That is, the open string metric and the noncommutativity parameter appear as the background fields of the $T$-dual theory of string theory. One can show that the effects of $T$-duality transformations on the open string metric $G_{\mu\nu}$ and on the noncommutativity parameter $\Theta_{\mu\nu}$ are as in the following

$$\tilde{G}^{\mu\nu} = g^{\mu\nu},$$
$$\tilde{\Theta}^{\mu\nu} = (2\pi\alpha')^2 B^{\mu\nu}. \quad (38)$$

Therefore, the background fields of string theory appear as the effective metric and noncommutativity parameter of the effective theory of the $T$-dual theory [7].

Now we find the background fields and the speed of the membrane such that noncommutativity in the moving frame be equivalent to the noncommutativity of the $T$-dual theory in the rest frame i.e.,

$$\Theta'^{\mu\nu} = -\lambda \tilde{\Theta}^{\mu\nu}, \quad (39)$$

where $\lambda$ is a positive constant. This means that, $T$-duality also can act as Lorentz transformations and vice-versa. In other words, noncommutativity parameter in the moving frame is proportional to the background field $B^{\mu\nu}$ in the rest frame. The equations (38) and (39) give the following table for various values of $g'$, $v$ and $B_{\mu\nu}$.

|        | magnetic and $v_1$ | magnetic and $v_3$ | electric and $v_1$ | electric and $v_3$ |
|--------|--------------------|--------------------|--------------------|--------------------|
| $g'$   | $g'$               | $g'$               | 0                  | $g'$               |
| $v$    | 0                  | $v$                | $v$                | 0                  |
| $B_{\mu\nu}$ | $b = \pm \frac{1}{2\pi\alpha'} \sqrt{(\frac{1}{\lambda} - 1)} g$ | $b = \pm \frac{1}{2\pi\alpha'} \sqrt{(\frac{1}{\lambda} - 1)} g$ | $E = \pm E_0 \sqrt{1 - \frac{1}{\lambda}}$ | $E = \pm E_0 \sqrt{1 - \frac{1}{\gamma}}$ |
| $G_s$  | $\frac{g_s}{\sqrt{\lambda}}$ | $\frac{g_s}{\sqrt{\lambda}}$ | $\frac{g_s}{\sqrt{\lambda}}$ | $g_s \sqrt{\frac{2}{\lambda}}$ |

where $v_i$ shows the membrane motion along the $X^i$-direction.

When the equation (39) holds, to find the corresponding $G'_{\mu\nu}$, we should use the equations (9), (16), (22) and (27) and the values of $g'$, $v$, $b$ and $E$ of this table. For example for the magnetic part of the table, the space-space elements of the metric $G'_{\mu\nu}$ are proportional to $g_{\mu\nu}$ with the scale factor $\frac{1}{\lambda}$. For the special case that has been given by the equation (18) we have $\lambda = \frac{1}{\gamma^2}$. According to the equation (38), the equation (18) can be written as

$$G'_{\mu\nu} = \frac{1}{\lambda} \tilde{G}_{\mu\nu}. \quad (40)$$
That is, on the open string metric, Lorentz transformations also act as $T$-duality.

Noncommutativity parameter in the moving frame is $\Theta^{\mu\nu} = -(2\pi\alpha')^2 \lambda B^{\mu\nu}$. For the magnetic case $\Theta^{\mu\nu}$ is similar to the matrix (4), that $\theta$ should be replaced with the factor $\mp 2\pi\alpha' \sqrt{\frac{\lambda - \eta}{\eta}}$. This factor is the strength of the noncommutativity, and for $\lambda = \frac{1}{2}$ has extremum $\mp \frac{\pi\alpha'}{\sqrt{9}}$. For the electric case $\Theta^{\mu\nu}$ is given by the matrix (19) in which the function $T(E)$ should be changed with the factor $\pm \frac{1}{g_2 E_0} \sqrt{\lambda^2 - \eta \lambda}$. For the motion along the $X^1$ and $X^3$ directions $\eta$ is 1 and $\gamma$, respectively.

Note that the equation $\Theta^{\mu\nu} = -\lambda (\hat{\Theta})^{\mu\nu}$, for the membrane with magnetic field, produces the results of the equation (39). The matrix $(\hat{\Theta})^{\mu\nu}$ is noncommutativity parameter of the $T$-dual theory in the moving frame.

5 Conclusions

For the moving membranes with electric or magnetic background fields, we studied the effective variables of open string. For the magnetic membrane we observed the followings. There are two values of the magnetic field that produce the same noncommutativity on the membrane. By choosing an appropriate magnetic field, the open string metric becomes independent of the speed of the membrane. When this magnetic field goes to zero and near the speed of light we obtained a well defined noncommutativity. For the special speeds perpendicular to the membrane, the open string metric is proportional to the closed string metric.

For the pure electric field on the membrane, the strength of the noncommutativity in terms of the electric field is one to one. When the electric field approaches to its critical value, we obtained some definite noncommutative theories from decoupling limits. We found the conditions that the noncommutativity structures of the moving electric and magnetic membranes to be equivalent. Similarly the equivalence of the electric noncommutativities in the rest frame and in the moving frame (with the scaling limit form) was obtained.

The background fields of string theory ($T$-dual of string theory), are effective metric and noncommutativity parameter of the effective $T$-dual theory (the effective theory of string theory). Therefore, we observed that for the special background fields and speeds of the membrane, the noncommutativity of the $T$-dual theory in the rest frame appears like the noncommutativity of the original theory in the moving frame. The open string metric in the moving frame also is equivalent to that one of the $T$-dual theory in the rest frame.
References

[1] A. Connes, M.R. Douglas and A. Schwarz, JHEP 9802(1998)003, hep-th/9711162; M.R. Douglas and C. Hull, JHEP 9802(1998)008, hep-th/9711165; F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, JHEP 9902(1999)016, hep-th/9810072; Nucl. Phys. B576(2000)578, hep-th/9906161; A. Fayyazuddin and M. Zabzine, Phys. Rev. D62(2000)046004, hep-th/9911018; P.M. Ho and Y.T. Yeh, Phys. Rev. Lett. 85(2000)5523, hep-th/0005159; Y.E. Cheung and M. Krog, Nucl. Phys. B528(1998)185, hep-th/9803031; D. Bigatti and L. Susskind, Phys. Rev. D62(2000)066004, hep-th/9908056; A. Schwarz, Nucl. Phys. B534(1998)720, hep-th/9805034; C.S. Chu and P.M. Ho, Nucl. Phys. B550(1999)151, hep-th/9812219; Nucl. Phys. B568(2000)447, hep-th/9906192; V. Schomerus, JHEP 9906(1999)030, hep-th/9903205; D. Kamani, Phys. Lett. B548 (2002)231, hep-th/0210253; Mod. Phys. Lett. A17(2002)2443, hep-th/0212088.

[2] N. Seiberg and E. Witten, JHEP 9909(1999)032, hep-th/9908142.

[3] N. Seiberg, L. Susskind and N. Toumbas, JHEP 0006(2000)021, hep-th/0005040; R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, JHEP 0006(2000)036, hep-th/0005048; J.L.F. Barbon and E. Robinovici, Phys. Lett. B486(2000)202, hep-th/0005073; J. Gomis and T. Mehen, Nucl. Phys. B591(2000)265, hep-th/0005129; O. Aharony, J. Gomis and T. Mehen, JHEP 0009(2000)023, hep-th/0006236; R.G. Cai and N. Ohta, JHEP 0010(2000)036, hep-th/0008119; G.H. Chen and Y.S. Wu, Phys. Rev. D63(2001)086003, hep-th/0006013.

[4] D. Kamani, Europhys. Lett. 57(2002)672, hep-th/0112153.

[5] K.S. Narain, H. Sarmadi and E. Witten, Nucl. Phys. B279(1987)369.

[6] A. Giveon, M. Porrati and E. Robinovici, Phys. Rep. C244(1994)77-202, hep-th/9401139; R.G. Leigh, Mod. Phys. Lett. A4 28(1989)2767.

[7] D. Kamani, Mod. Phys. Lett. A17(2002)237, hep-th/0107184.