Mathematical model for HIV spreads control program with ART treatment

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Abstract. In this article, using a deterministic approach in a seven-dimensional nonlinear ordinary differential equation, we establish a mathematical model for the spread of HIV with an ART treatment intervention. In a simplified model, when no ART treatment is implemented, disease-free and the endemic equilibrium points were established analytically along with the basic reproduction number. The local stability criteria of disease-free equilibrium and the existing criteria of endemic equilibrium were analyzed. We find that endemic equilibrium exists when the basic reproduction number is larger than one. From the sensitivity analysis of the basic reproduction number of the complete model (with ART treatment), we find that the increased number of infected humans who follow the ART treatment program will reduce the basic reproduction number. We simulate this result also in the numerical experiment of the autonomous system to show how treatment intervention impacts the reduction of the infected population during the intervention time period.

1. Introduction

*Human Immunodeficiency Virus* (HIV) is a virus that could infect lymphocytes in a human body and will result in the decline of human immune system, which well known as *Acquired Immune Deficiency Syndrome* (AIDS). HIV (*Human Immunodeficiency Virus*) contained in the human fluids body who are living with HIV. Even though this virus contains on saliva, tear, and urine, but it does not shown to be at risk of transmitting the infection among human population since the virus levels on these liquids are very low. The spread of this HIV normally spread from unhealthy sex behavior, the use of unsterile syringes, and human blood transfusions who have been infected by HIV to healthy humans [1].

According to World Health Organization (WHO) [2] and United Nations Programme on HIV/AIDS (UNAIDS) [3], in 2016, it is estimated that there are 36.7 million of humans who get infected by HIV/AIDS in all over the world. Most of the HIV-infected human population comes from developing countries, such as South Africa, India, Kenya, Mozambique, Uganda, China, Zimbabwe, Zambia, Malawi, Indonesia, and Thailand. The highest case of HIV spread occurs in the South African country as many as 7 million people are infected with HIV / AIDS [4].

Since 1999, there has been an increase in the number of people living with HIV in a group of people with high-risk behaviors, including commercial sex workers and injecting drug users in several provinces such as DKI Jakarta, Riau, Bali, West Java and East Java. With this reason, therefore these provinces are classified as concentrated level of epidemic areas. Papua province is categorized as a province in a generalized epidemic area. The estimation result of 2009, there are 186.000 people with HIV positive in Indonesia. Humans who get infected with HIV / AIDS...
cannot be cured. The treatment that can be done is only to inhibit the development of viruses in the body through the process of emphasis Anti-Retroviral Therapy (ART). From the Report on HIV Progress Situation in Indonesia until September 2011, it was recorded that the number of PLHA who get Anti-Retroviral Therapy (ART) are 22,843 from 33 provinces and 300 districts cities, with male and female ratio is 3: 1, and the highest percentage in the age group 20-29 years (Ministry of Health, 2011)\(^5\).

Many mathematical models have been developed to understand how HIV infection might spread among the human population, such as a mathematical model that discussed how HIV might spread among the community of injecting drug users (I Mardhiyah, 2012). The authors use a basic reproduction number analysis and equilibrium analysis about the existence and their global stability criterion and also Bendixon-Dulac criterion for the local stability check. They found that HIV will spread among the injecting drug users if and only if the basic reproduction number is larger than one, and the disease will disappear if and only if the basic reproduction number is smaller than one\(^6\). The other authors that also discuss the spread of HIV are (Lindsay Simpson and Abba B. Gumel, 2016) that include Anti-retroviral intervention among the homosexual community. They also using basic reproduction number and the local stability and existence criteria about the equilibrium points to understand how HIV might spread. They found that an infection among human in the homosexual category might suppress with the Anti-retroviral intervention\(^7\).

Some authors explained about benefits and risks treatment with ART for pregnant women (M.G. Fowler, M. Qin, S.A. Fiscus, J.S. Currier, 2016) with randomized-trial data on the risks and benefits of antiretroviral therapy (ART) as compared with zidovudine and single-dose nevirapine to prevent transmission of the human immunodeficiency virus (HIV) in HIV-infected pregnant women with high CD4 counts are lacking, then We randomly assigned HIV-infected women at 14 or more weeks of gestation with CD4 counts of at least 350 cells per cubic millimeter to zidovudine and single-dose nevirapine plus a 1-to-2-week postpartum “tail” of tenofovir and emtricitabine (zidovudine alone); zidovudine, lamivudine, and lopinavir-ritonavir (zidovudine-based ART); or tenofovir, emtricitabine, and lopinavir-ritonavir (tenofovir-based ART). The primary outcomes were HIV transmission at 1 week of age in the infant and maternal and infant safety. Then the conclusion is antenatal ART resulted in significantly lower rates of early HIV transmission than zidovudine alone but a higher risk of adverse maternal and neonatal outcomes\(^8\). The other authors that also discuss enhanced prophylaxis plus antiretroviral therapy for advanced HIV infection in Africa. In sub-Saharan Africa, among patients with advanced human immunodeficiency virus (HIV) infection, the rate of death from infection (including Tuberculosis and Cryptococcus) shortly after the initiation of antiretroviral therapy (ART) is approximately 10%, in this factorial open-label trial conducted in Uganda, Zimbabwe, Malawi, and Kenya, we enrolled HIV-infected adults and children 5 years of age or older who had not received previous ART and were starting ART with a CD4+ count of fewer than 100 cells per cubic millimeter, and the conclusion is among HIV-infected patients with advanced immunosuppression, enhanced antimicrobial prophylaxis combined with ART resulted in reduced rates of death at both 24 weeks and 48 weeks without compromising viral suppression or increasing toxic effects\(^9\).

Different from above model, the spread of HIV among the human population with the ART intervention will be included into the model. We also will include the human behavior about their awareness in following the treatment process. The ART intervention is only given to an infected human in a chronic infected category. The article will be constructed as follows. In the next section, mathematical model construction along with the assumptions descriptions will be discussed. The analysis of the model about the existence and local stability criteria of the model along with the construction of the next-generation matrix will be discussed in section 3. Finally, some numerical simulations and conclusions will be given in section 4 and 5, respectively.
2. Mathematical model construction
Before constructing the model, let us assume that the human population might be separated into seven classes, i.e.:

(i) Susceptible human compartment \( (x_1(t)) \), described as a group of susceptible humans, who might be infected by HIV. They might be infected because of direct contact (free sex), blood transfusion from an infected human, or from the use of a syringe from an infected human.

(ii) Infected human of acute level \( (x_2(t)) \), described as a group of humans who are already infected by HIV, but still in a non-harmful stage. Humans in this category still might perform daily activities like susceptible humans. We assume that this infected compartment does not receive ART treatment since they do not realize that they are already infected with HIV.

(iii) Infected humans of chronic level \( (x_3(t)) \), which includes all infected humans who have shown the symptoms of HIV infection. This group of humans cannot perform daily activities like susceptible humans anymore. Therefore, the ART treatment is implemented in this group of infected humans.

(iv) Infected humans who receive the ART treatment intervention \( (x_4(t)) \), are a group of infected humans at the chronic stage and follow the therapy process with a high level of awareness.

(v) Infected humans who receive ART treatment but with a low level of awareness \( (x_5(t)) \), described as a group of infected humans at the chronic level, following the ART treatment procedure, but with a low awareness.

(vi) Infected humans who have failed in the ART treatment procedure \( (x_6(t)) \). This compartment only increases from the transition of infected humans with the ART treatment intervention with a low level of awareness.

(vii) Infected humans with HIV/AIDS complication \( (x_7(t)) \). This group of infected humans already have AIDS, and their condition might be complicated by other diseases due to the decline of immunity in their body.

The transmission process that involves the seven compartments above is given in Figure 1. The process of interaction between each compartment is given as follows. The recruitment rate only occurs in susceptible human \( x_1 \) with a constant rate \( A \). Each compartment is decreasing due to the natural death rate \( (\mu) \), except for the AIDS human compartment \( (x_7) \) where mortality caused by AIDS has a rate of \( \delta \). Please note that \( \delta > \mu \). Susceptible humans might be infected by direct contact with infected humans with the probability of infection \( \beta_a \) if they are infected by \( x_2 \) compartment, \( \beta_c \) if they are infected by \( x_3, x_4, x_5 \) and \( x_6 \), and \( \beta_s \) if they are infected by an individual in the \( (x_7) \) compartment. Since we assume that more infected human, than it will limit their capability to do an activity in daily life; therefore, we assume that \( \beta_a > \beta_c > \beta_s \). After an incubation period of approximately 3-5 years, individual in compartment \( x_2 \) will be transferred into compartment \( x_3 \). The ART treatment intervention is given to chronically infected humans at a constant rate \( u \), which will separate them into a specific compartment, \( (x_4) \). If individual in the \( (x_4) \) compartment following the treatment process with high-level awareness, it will prevent them becoming an AIDS-infected human. On the other hand, if they do not follow the treatment process obediently, they will transfer into the \( (x_5) \) compartment at a constant rate \( \xi_4 \). If they come back following the treatment process with a high level of awareness, they will come back into the \( (x_4) \) compartment. If individuals in the \( (x_5) \) compartment finally stop following the treatment, they will transfer into the \( (x_6) \) compartment at a constant rate \( \rho_c \). There is a possibility that infected humans who have failed in the treatment process \( (x_6) \) may or may not follow the ART treatment again, with the probability of \( 1 - r_c \) and \( r_c \), respectively. There is also a possibility that if the infected humans in the \( (x_3) \) compartment do not receive an ART treatment, they will be transferred directly into AIDS compartment \( x_7 \).
Figure 1. Transmission diagram of HIV spread with ART treatment.

Based on transmission diagram in Figure 1, the mathematical model of HIV spread with ART treatment now is given by 7-dimensional non linear ordinary differential equation, i.e:

\[
\begin{align*}
\frac{dx_1}{dt} &= A - \frac{\beta_{a} x_1 x_2}{N_x} - \frac{\beta_{c} x_1 (\theta_{h} x_4 + \theta_{l} x_5 + x_3 + x_6)}{N_x} - \frac{\beta_{a} x_1 x_7}{N_x} - \mu x_1 \\
\frac{dx_2}{dt} &= \frac{\beta_{a} x_1 x_2}{N_x} + \frac{\beta_{c} x_1 (\theta_{h} x_4 + \theta_{l} x_5 + x_3 + x_6)}{N_x} + \frac{\beta_{a} x_1 x_7}{N_x} - \gamma_{a} x_2 - \mu x_2 \\
\frac{dx_3}{dt} &= \gamma_{a} x_2 + (1 - r_c) \gamma_{b} x_6 - u x_3 - \gamma_{c} x_3 - \mu x_3 \\
\frac{dx_4}{dt} &= -\mu x_4 + u x_3 - x_4 \xi_4 + x_5 \xi_5 \\
\frac{dx_5}{dt} &= -\mu x_5 - \rho_{c} x_5 + x_4 \xi_4 - x_5 \xi_5 \\
\frac{dx_6}{dt} &= \rho_{c} x_5 - (1 - r_c) \gamma_{b} x_6 - r_c \gamma_{b} x_6 - \mu x_6 \\
\frac{dx_7}{dt} &= r_c \gamma_{b} x_6 - \delta x_7 - \mu x_7 + \gamma_{c} x_3,
\end{align*}
\]

which supplemented with initial condition \( x_i(t = 0) = x_{i0} \) for \( i = 1, 2, \ldots, 7 \). The parameters interpretation of system is given in Table 1.
Table 1. Parameters description.

| Par. | Description                                      | Unit         |
|------|--------------------------------------------------|--------------|
| $A$  | recruitment rate of human                        | human/time   |
| $\beta_a$ | rate of success infection of infected human in acute stage | time         |
| $\beta_c$ | rate of success infection of infected human in chronic stage | time         |
| $\beta_s$ | rate of success infection of infected human in AIDS stage | time         |
| $\theta_h$ | reduction of $\beta_a$ for infected human who undergo with treatment ($x_4$) | -            |
| $\theta_l$ | reduction of $\beta_a$ for infected human who undergo with treatment ($x_5$) | -            |
| $\gamma_a$ | Transition rate from $x_2$ into $x_3$            | 1/time       |
| $\gamma_b$ | Rate of transition for the failure of Treatment ($x_4$ to $x_5$) | -            |
| $\gamma_c$ | Rate of transition from chronic to AIDS infected human | -            |
| $r_c$ | Proportion of human who failed and not repeat the ART treatment | -            |
| $\rho_c$ | Rate of infected human $x_3$ who finally discontinue using ART treatment | 1/time       |
| $u$  | ART treatment rate                               | 1/time       |
| $\xi_4$ | Transition due to decreased awareness for the use of treatment | -            |
| $\xi_5$ | transition due to increased awareness in the use of treatment | -            |
| $\mu$ | Natural death rate                               | 1/time       |
| $\delta$ | Death rate caused by AIDS                        | 1/time       |

3. Analysis of mathematical model

In this section, mathematical analysis of the system will be given to discuss about the existence and local stability criteria of the equilibrium point (numerically and analytically), and also the basic reproduction number of system. We begin with a simpler model in the next subsection, when there is no ART treatment implemented into the model.

3.1. Mathematical model without ART treatment

In this subsection, a mathematical model analysis for a simpler model of system will be discuss. A simplification is taken in the form of when $u = 0$, which means that no more compartment of infected human who undergo with an ART treatment ($x_4, x_5, x_6$). With this simplification, the model in system is simplified into:

$$
\begin{align*}
\frac{dx_1}{dt} &= A - \frac{\beta_a x_1 x_2}{N_x} - \frac{\beta_c x_1 x_3}{N_x} - \frac{\beta_s x_1 x_7}{N_x} - \mu x_1 \\
\frac{dx_2}{dt} &= \frac{\beta_a x_1 x_2}{N_x} + \frac{\beta_c x_1 x_3}{N_x} + \frac{\beta_s x_1 x_7}{N_x} - \gamma_a x_2 - \mu x_2 \\
\frac{dx_3}{dt} &= -\mu x_3 + \gamma_a x_2 - \gamma_c x_3 \\
\frac{dx_7}{dt} &= -\mu x_7 - \delta x_7 + \gamma_c x_3,
\end{align*}
$$

with initial condition $x_{10}, x_{20}, x_{30}$ and $x_{70}$ are given.

There are two equilibrium point of system, i.e the disease free equilibrium when there are no infected human in the system, which given by:

$$
DFE = \left\{ x_1 = \frac{A}{\mu}, x_2 = 0, x_3 = 0, x_7 = 0 \right\}.
$$

3
The other equilibrium is the endemic equilibrium, which represented as a condition when all compartments in system 2 are positive, which given by:

\[ EE = \{ x_1 = \bar{x}_1, x_2 = \bar{x}_2, x_3 = \bar{x}_3, x_7 = \bar{x}_7 \}. \]  

(4)

where:

\[ \bar{x}_1 = \frac{A}{R_0} \]

\[ \bar{x}_2 = \frac{\mu N_x(\mu + \gamma_c)(\mu + \delta)(R_0 - 1)}{k_1} \]

\[ \bar{x}_3 = \frac{\gamma_a N_x(\mu + \delta)(R_0 - 1)}{k_2} \]

\[ \bar{x}_7 = \frac{\gamma_c N_x(R_0 - 1)}{k_3} \]

\[ k_1 = \delta \mu + \delta \beta a_\gamma c + \delta \beta c_\gamma a + \mu^2 \beta a + \mu \beta a \gamma c + \mu \beta c \gamma a + \beta s \gamma a \gamma c \]

\[ k_2 = \delta \mu + \delta \beta a_\gamma c + \delta \beta c_\gamma a + \mu^2 \beta a + \mu \beta a \gamma c + \mu \beta c \gamma a + \beta s \gamma a \gamma c \]

\[ k_3 = \delta \mu + \delta \beta a_\gamma c + \delta \beta c_\gamma a + \mu^2 \beta a + \mu \beta a \gamma c + \mu \beta c \gamma a + \beta s \gamma a \gamma c \]

and

\[ R_0^* = A (\delta \mu + \delta \beta a_\gamma c + \delta \beta c_\gamma a + \mu^2 \beta a + \mu \beta a \gamma c + \mu \beta c \gamma a + \beta s \gamma a \gamma c) \]

\[ \frac{\mu N_x(\mu + \gamma_c)(\mu + \gamma_c)(\delta + \mu)}{R_0} > 1. \]  

(5)

The local stability of the DFE is analyzed using the Jacobian matrix of system 2 which evaluated in DFE. The Jacobian matrix is given by:

\[ J = \begin{bmatrix} -\mu & -\frac{\beta A}{\mu N_x} & \frac{\beta A}{\mu N_x} & \frac{\beta A}{\mu N_x} \\ 0 & \frac{\beta A}{\mu N_x} - \gamma a - \mu & \frac{\beta A}{\mu N_x} & \frac{\beta A}{\mu N_x} \\ 0 & \gamma a & -\mu - \gamma c & 0 \\ 0 & 0 & \gamma c & -\delta - \mu \end{bmatrix}. \]  

(6)

The characteristic polynomial of matrix J is given by:

\[ \lambda^4 h_0 + \lambda^3 h_1 + \lambda^2 h_2 + \lambda h_3 + h_4 = 0. \]  

(7)

where

\[ h_0 = 1 \]

\[ h_1 = -\frac{-\delta \mu N_x - 4 \mu^2 N_x - \mu N_x \gamma a - \mu N_x \gamma c + A \beta a}{\mu N_x} \]
Using the Routh-Hurwitz criterion, all eigenvalues of $J$ will be negative if and only if:

$$
\begin{align*}
\begin{cases}
    h_1 > 0, \\
    h_3 > 0, \\
    h_4 > 0 \\
    h_1 h_2 h_3 > h_4
\end{cases}
\end{align*}
$$

Next, the basic reproduction number of system will be analyzed. Basic reproduction number is defined as the expected number of secondary cases caused by one primary case in a closed population during one infection period. The illustration of basic reproduction number $R_0$ when it is larger than one, smaller than one, or equal to one is given in Figure 2. There are some methods to calculate the basic reproduction number, such as using next-generation matrix, graph theory, etc. In this article, next-generation matrix approach will be implemented to calculate the basic reproduction number of system. Using the next-generation matrix approach implemented to system, the basic reproduction number as the spectral radius of the respected next-generation matrix is given by:

$$
R_0 = \frac{A \left( \delta \mu \beta_a + \delta \beta_a \gamma c + \delta \beta_c \gamma a + \mu^2 \beta_a + \mu \beta_a \gamma c + \mu \beta_c \gamma a + \beta_s \gamma_a \gamma c \right)}{\mu N_x (\mu + \gamma a) (\mu + \gamma c) (\delta + \mu)}.
$$

Let us define the parameters value for system as follows: $A = \frac{1000}{65 \times 365}$, $\mu = \frac{1}{65 \times 365}$, $N_x = 1000$, $\beta_a = \frac{0.025}{1000}$, $\beta_c = 0.75 \beta_a$, $\beta_s = 0.5 \beta_a$, $\gamma_a = \frac{1}{5 \times 365}$, $\gamma_c = \frac{1}{3 \times 365}$. With this parameters value,
the basic reproduction number $R_0$ reach 4.5781 which leads in to the existence of the endemic equilibrium, while the disease free equilibrium became unstable. Also with this parameters, the endemic equilibrium is given by

$$\bar{x}_1 = 164.6141532, \bar{x}_2 = 59.67041757, \bar{x}_3 = 34.22273950, \bar{x}_7 = 0.4462119387.$$ (10)

Now, we want to see how the parameters sensitivities could determine the magnitude of each compartment in the endemic equilibrium. This will be perform with differentiating the variables in endemic equilibrium with the parameters, respectively for $\beta_a, \beta_c, \beta_s, \gamma_a, \gamma_c$ and $\delta$. Do the same thing for variables $x_2, x_3$ and $x_7$. This results is given in Jacobian.

$$J = \begin{bmatrix}
-45921.12681 & -26337.11649 & -343.3955311 & 188056.5074 & 51667.0652 & 6.128373 \\
3280.08045 & 1881.222623 & 24.5285254 & -114552.6545 & -3690.50458 & -0.4377402 \\
1881.222608 & 1078.936408 & 14.06767408 & -3242.81700 & -37937.25243 & -0.2510568 \\
24.5285253 & 14.06767406 & 0.1834208592 & -42.2813513 & -6.0414070 & -6.373893674
\end{bmatrix}.$$ (11)

Based on the matrix $J$, it can be seen that $\gamma_a$ is the most influential parameter that can determine the magnitude of $x_1$ in the endemic equilibrium. Since the sign is positive, it means that larger the transition rate from acutely infected into chronic infected will enlarge total of susceptible human. Using same approach, the order of the most influential parameters in determining the magnitude of the variable $x_1$ at the endemic equilibrium point is $\gamma_a, \beta_c, \beta_4, \beta_c, \beta_s$ and $\delta$. Using the same approach applied to all row of matrix $J$, we find that $\gamma_a$ is the most influential parameter in determining the size of each variable in the endemic equilibrium point.

### 3.2. Mathematical model with ART treatment

In this section, we analyzed the complete model of HIV spread with ART treatment, which given in system (9). Using the same approach with in the previous section, the disease free equilibrium point of system (9) is given by :

$$DFE = \left\{ x_1 = \frac{A}{\mu}, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 0, x_7 = 0 \right\}.$$ (12)

Using the Jacobian matrix approach, the local stability of system (9) is determined by roots of 7 degree polynomial which given by :

$$\lambda^7 a_0 + \lambda^6 a_1 + \lambda^5 a_2 + \lambda^4 a_3 + \lambda^3 a_4 + \lambda^2 a_5 + \lambda a_6 + a_7 = 0,$$ (13)

where :

$$a_0 = 1, a_1 = -\frac{Q_1}{\mu N_x} a_2 = -\frac{Q_2}{\mu N_x} a_3 = -\frac{Q_3}{\mu N_x} a_4 = -\frac{Q_4}{\mu N_x},$$

$$a_5 = -\frac{Q_5}{\mu N_x}, a_6 = -\frac{Q_6}{\mu N_x}, a_7 = \frac{Q_7}{N_x},$$

$$Q_1 = -\mu (\gamma_a + \gamma_c + \gamma_s + u + 7\mu + \delta + \xi \phi + \xi + \mu \phi);$$

$$Q_2 = -21 N_x \mu^3 - 6 N_x (\gamma_a + \gamma_c + \gamma_s + u + \delta + \xi \phi + \xi + \mu \phi) \mu^2 + \left(\begin{array}{c}
-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a \\
+ (-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a)
\end{array}\right) \mu^3 + \left(\begin{array}{c}
-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a \\
+ (-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a)
\end{array}\right) \mu^4 + \left(\begin{array}{c}
-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a \\
+ (-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a)
\end{array}\right) \mu^5 + \left(\begin{array}{c}
-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a \\
+ (-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a)
\end{array}\right) \mu^6 + \left(\begin{array}{c}
-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a \\
+ (-10 \gamma_a - 10 \gamma_s - 10 \mu - 10 \xi \phi \phi \phi \gamma_a)
\end{array}\right) \mu^7.$$
Using the Routh-Hurwitz stability criterion for 7 degree polynomial:

The system will be locally stable if and only if:

\[ a_0, a_1, a_2, a_3, a_4, a_5, a_6 > 0, \]

\[ b_1 = \frac{a_4}{a_0}, b_2 = \frac{a_5}{a_0}, b_3 = \frac{a_6}{a_0} > 0, \]

\[ c_2 = \frac{b_2}{b_1}, c_3 = \frac{b_3}{b_1} > 0, \]

\[ f_1 = c_3 - c_2 > 0. \]
Basic reproduction number ($R_0$)

Next, we want to analyzed the basic reproduction number of the complete system where the ART treatment is involved into the model. We will use the next-generation matrix approach to determine the magnitude of basic reproduction number. First we define the transition matrix of system which given by

$$H = \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 & 0 \\ \gamma_a & M_2 & 0 & 0 & (1 - r_c) \gamma_b & 0 \\ 0 & u & M_3 & \xi_5 & 0 & 0 \\ 0 & 0 & \xi_4 & M_4 & 0 & 0 \\ 0 & 0 & 0 & \rho_c & M_5 & 0 \\ 0 & \gamma_c & 0 & 0 & r_c \gamma_b & M_6 \end{bmatrix}$$

where

$$M_1 = -\gamma_a - \mu$$
$$M_2 = -u - \gamma_c - \mu$$
$$M_3 = -\mu - \xi_4$$
$$M_4 = -\mu - \rho_c - \xi_5$$
$$M_5 = -(1 - r_c) \gamma_b - r_c \gamma_b - \mu$$
$$M_6 = -\delta - \mu$$

and the transmission matrix is given by

$$V = \begin{bmatrix} \beta_a x_1 \frac{N_s}{N_s} & \beta_a x_1 \theta h \frac{N_s}{N_s} & \beta_a x_1 \theta h \frac{N_s}{N_s} & \beta_a x_1 \theta h \frac{N_s}{N_s} & \beta_a x_1 \theta h \frac{N_s}{N_s} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We find that the basic reproduction number as the spectral radius of the next generation matrix $K = E^T VH^{-1} E$ which given by:

$$R_0 = \frac{A(P_1 + P_2 + P_3 + P_4)}{\mu M_6 N_s P_5}$$

where:

$$P_1 = -u M_4 M_5 \beta_c \gamma a \theta h + u M_5 M_6 \beta_c \gamma a \theta h \xi_4 - u M_6 \beta_c \gamma a r_c \rho_c \xi_4$$
With this set of parameters, we find the basic reproduction number when $u$ is 2.

In this section, we will see how the AR T treatment intervention will impact the magnitude of $R_0$ and also impact the dynamic of infected human. Firstly, we will see how the basic reproduction number will change respect to $u$ and $r_c$. Using the set of parameter given by:

$A = 1000/(65\times365)$, $\beta_a = 0.000025$, $\beta_c = 0.008\times0.000025$, $\beta_s = 1.5\times0.000025$, $\gamma_a = 1/(5\times365)$, $\gamma_c = 1/(3\times365)$, $\delta = 0$, $\mu = 1/(65 \times 365)$, $N_x = 1000$, $\xi_4 = 0.75/365$, $\xi_5 = 0.25/365$, $\theta_h = 1$, $\theta_l = 2$, $\rho_c = 0.1/365$, $\gamma_b = 0.5/365$, and substitute it into $R_0$ in [1], we find the function of $R_0$ now as a function of two variables ($f(u, r_c)$). Plot it in the $u-r_c$ plane with various value of $R_0$, the figure is given in Figure 3.

It can be seen that increasing the ART treatment rate ($u$) will reduce basic reproduction number $R_0$, which will lead us into extinction of HIV-AIDS. In the other hand, increasing of transition rate into AIDS compartment ($r_c$) will increase basic reproduction number $R_0$. This means that if the individual who have failed in ART treatment do not re-following the ART treatment will make the HIV-AIDS exist in the field.

The numerical simulation to see how the dynamic of infected population will change respect to the various value of ART treatment ($u$) will be performed in the next simulation. To do this simulation, we use the set of parameters:

$A = 1000/(65\times365)$, $\beta_a = 0.000025$, $\beta_c = 0.008\times0.000025$, $\beta_s = 1.5\times0.000025$, $\gamma_a = 1/(5\times365)$, $\gamma_c = 1/(3 \times 365)$, $\delta = 0$, $\mu = 1/(65 \times 365)$, $N_x = 1000$, $\xi_4 = 0.75/365$, $\xi_5 = 0.25/365$, $\theta_h = 1$, $\theta_l = 2$, $r_c = 0$, $\rho_c = 0.1/365$, $\gamma_b = 0.5/365$. With this set of parameters, we find the basic reproduction number when $u = 0$ is 1.664503135.
Please note that with this set of parameter, it can be seen that this situation represent a situation where no ART treatment is implemented $u = 0$. It can be seen in Figure 4 that the gradation value of ART treatment rate do not impact the change of dynamic on susceptible human, infected human in acute level, and infected human in HIV/AIDS level significantly. Different phenomenon appear in the other four compartment, infected human in chronic, humans in ART treatment and human who failed in ART treatment, when treatment rate success to reduce number of infected human significantly.

![Figure 4. Dynamic of all compartment for various value of u.](image)

5. Conclusions
HIV, or Human Immunodeficiency Virus, attacks the immune system of the human body and will weaken it to attack the disease or infection. There is no medicine to cure people if the infected human already has HIV status. However, early diagnosis and effective handling of the infective people in the early stage with ART treatment will prevent the infected people progressing to the next stage, the AIDS stage.

In this article, a mathematical model to understand ART treatment as an alternative way to control the spread of HIV has been developed. The mathematical model analysis when the ART treatment is not implemented in the model has shown that the transition rate between infected compartment should be reduced to control the spread of HIV. Therefore, implementation of ART treatment is the best way to control the spread of HIV.

Disease-free equilibrium and the form of basic reproduction number as the endemic indicator for the model have been shown analytically. Mathematical analysis to show the sensitivity of ART treatment to reduce the basic reproduction number has been shown, along with a numerical simulation to see how the dynamics of each compartment respond to the ART treatment intervention. We find that ART treatment does not increase the number of susceptible humans, but could reduce the number of infected humans who progress to AIDS status.

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