Kane Oscillator

Gashimzade F.M. and Babaev A.M.
Institute of Physics, Azerbaijan Academy of Sciences,
370143 Baku, Azerbaijan, e-mail: semic@lan.ab.az

March 22, 2022

Abstract

The energy spectrum and wave functions for Kane oscillator describing the spectra of electrons, light hole and spin-orbit-splitting bands in a quantum dots with harmonic lateral confinement is found.

1 Introduction

As known, for description of an energy spectrum of quantum dots is used or model "infinite potential barries", or model of harmonic confinement [1]. It was established, that the model of parabolic potential is realistic for describing enough not too large quantum dots. Therefore, within the framework of this model for the standard law dispersion electrons were considered a number of problem of physics of quantum dots, including quantum crystallizations electrons in an external magnetic field [2].

The semiconductor compounds (InAs, GaAs, InSb etc.) on which are created quantum dots now, have a complex energy spectrum described multiband Hamiltonian. In particular, nonparabolicity of a spectrum is possible to take into account within the framework of eight-bands model Kane’s[3]. Such approach was applied in work [3], however because of complexity of the obtained equation, the analysis of its solution was carried out in frameworks rather special approximation.

The mentioned above complexity of the equation arises at a standard way of introduction of parabolic confinement potential through scalar potential.
If introduce the external potential in a minimal way with the substituting[4]:

$$\vec{p} \rightarrow \vec{p} - i\beta \lambda \vec{r}$$

then as shown in [4,5,6,7] for Dirac Hamiltonian the oscillator equation with additional constant term is obtained, which originated from spin-orbital coupling.

Here we have applied the above-stated approach to obtain oscillators equation from the equations invariance under the rotational group [8,9], which directly gives a system of equations for radial functions for any number of considered bands.

The obtained equation we have call Kane oscillator, by as an analogy to Dirac oscillator. For to obtain the Kane’s spectrum from the system of equations (18) [8] let us consider the values \( (j = 1/2, \tau = 0) \), for conduction band, for the heavy-and light hole \( (j = 3/2, \tau = 1) \), and for the spin-orbital-splitting band \( (j = 1/2, \tau = 1) \). The first index characterizes the weight of an irreducible representation and a second one indicates the subspace with the same weight. We have chosen indexes for the states which clearly show that they are created from corresponding s, p states. In order to give a physical meaning to equations we consider the coefficients coupling s, and p states, correspondingly, \( \tau = 0 \), and \( \tau = 1 \) to be nonzero, where \( \tau \) is number of the subspace.

After substituting

$$\frac{d}{dr} \rightarrow \frac{d}{dr} + \lambda \beta r$$

$$\beta = (-1)^{(\tau + \frac{1}{2} - |j_z|)} \delta_{ij}$$

(where \( j_z \) is magnetic quantum number, \( \lambda \) is parameter characterizes a steepness of a well) system of the equations, including also dispersionless band of heavy hole has the form[8,9]:

\[
\begin{align*}
\left\{ -ia & \left[ \frac{d}{dr} + \lambda r + \frac{1 \mp (l_0 + \frac{1}{2})}{r} \right] f_{\frac{3}{2}}^- - \\
& - i\sqrt{2b} \left\{ \left[ \frac{d}{dr} + \lambda r + \frac{5 \pm (l_0 + \frac{1}{2})}{2r} \right] f_{1}^\pm + \frac{\alpha}{r} f_{1}^\mp \right\} + f_{0}^\mp \right\} = 0
\end{align*}
\]
\[ \frac{i\sqrt{2}b}{E} f_0^\pm - f_1^\pm = 0 \] (4)

\[- \frac{i\sqrt{2}b}{E} \left[ \frac{d}{dr} - \frac{\lambda r}{2} \right] f_0^\pm + f_2^\pm = 0 \] (5)

\[- \frac{-ia}{2(E+\Delta)} \left[ \frac{d}{dr} - \frac{\lambda r + 1}{r} \right] f_0^\mp + f_3^\pm = 0 \] (6)

\[ \alpha = \frac{\sqrt{3}}{2} \sqrt{\left( l_0 - \frac{1}{2} \right) \left( l_0 + \frac{3}{2} \right)} \]

Here the following designations are used:

\[ f_0^\pm = f_{l_0}^{0,+1,1/2,0} \pm f_{l_0}^{0,-1,1/2,0} \]
\[ f_1^\pm = f_{l_0}^{0,+1,3/2,1} \pm f_{l_0}^{0,-1,3/2,1} \]
\[ f_2^\pm = f_{l_0}^{0,1,3/2,1} \pm f_{l_0}^{0,1,3/2,1} \]
\[ f_3^\pm = f_{l_0}^{0,1,3/2,1} \pm f_{l_0}^{0,1,3/2,1} \] (7)

As well as:

\[ \frac{C_{1/2,1/2}^{0,1}}{i\chi} = \frac{ia}{E-E_g} \quad \frac{C_{1/2,1/2}^{1,0}}{i\chi} = \frac{ia}{E+\Delta} \quad \frac{C_{1/2,3/2}^{0,1}}{i\chi} = \frac{i\alpha}{E-E_g} \quad \frac{C_{1/2,1/2}^{1,0}}{i\chi} = \frac{ib}{E-E_g} \] (8)

where \( E_g \) is the energy of the bottom of conduction band, \( \Delta \) is the spin-orbit splitting energy. The parameters \( a, b \) are matrix elements of coupling between the conduction and valence bands. The quantities like \( C_{1/2,1/2}^{0,1} \), \( f_{1/2,1/2,0}^{l_0} \) etc. and \( \chi \) are determined in Gelfand et al [8]. The system of equations (1)-(6) are rewritten so that to separate the independent solutions (“even” and ”odd”).

2 The Energy Spectrum

Substituting (4)-(6) in (3) we shall obtain:

\[ \left\{ \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \left( l_0 + \frac{1}{2} \right) \left( l_0 + \frac{1}{2} \pm 1 \right) \frac{1}{r^2} \right\} - \lambda^2 r^2 - 3\lambda \left\{ \frac{a^2}{4(E-E_g)(E+\Delta)} + \frac{2b^2}{E(E-E_g)} \right\} + \right. \]
\[+1 \pm \left[ \frac{a^2}{2(E - E_g)(E + \Delta)} - \frac{2b^2}{E(E - E_g)} \right] \lambda (l_0 + \frac{1}{2} \pm 1) \right \} f_0^\pm = 0 \quad (9)\]

Energy spectrum and the corresponding eigenfunctions are given by:

\[\varphi(E) = 2\lambda (N + \frac{3}{2}) \quad (10)\]

\[\varphi(E) = \frac{4E(E - E_g)(E + \Delta)}{a^2E + 8b^2(E + \Delta)} \pm \lambda \left( l_0 + \frac{1}{2} \pm 1 \right) \frac{2a^2E - 8b^2(E + \Delta)}{a^2E + 8b^2(E + \Delta)} - 3\lambda \quad (11)\]

\[f_{0,n}^\pm = A_{nl_0 \pm \frac{1}{2}} r_{l_0 \pm \frac{1}{2}} \exp \left( -\frac{\lambda r^2}{2} \right) L_{n}^{l_0 \pm \frac{1}{2} + \frac{3}{2}} (\lambda r^2) \quad (12)\]

where \(L_{n}^{l_0 \pm \frac{1}{2} + \frac{3}{2}} (\lambda r^2)\) is an associated Laguerra polynomial, \(N = 2n + l_0 \pm \frac{1}{2}, n = 0, 1, 2, \ldots\) is principal quantum number. The normalization constants are:

\[A_{nl_0 \pm \frac{1}{2}} = \left[ \frac{2\lambda^{l_0 \pm \frac{1}{2} + \frac{3}{2} n!}}{\Gamma(n + l_0 \pm \frac{1}{2} + \frac{3}{2})} \right]^{\frac{1}{2}} \quad (13)\]

\[\lambda = \frac{m_n \omega}{h} \]

The parameters \(a\) and \(b\) are related to the effective mass as follows[11]:

\[\frac{\hbar^2}{2m_n} = \frac{2b^2}{E_g} + \frac{1}{4} \frac{a^2}{E_g + \Delta}; \quad (14)\]

\[\frac{\hbar^2}{2m_{lh}} = \frac{2b^2}{E_g}; \quad (14)\]

\[\frac{\hbar^2}{2m_{sh}} = \frac{1}{4} \frac{a^2}{E_g + \Delta}, \quad (14)\]

where \(m_n, m_{lh}\) and \(m_{sh}\) are the effective masses of electron, light hole and spin-orbit splitting hole, correspondingly.

Case \(a = \frac{2}{\sqrt{3}} P, b = \frac{1}{\sqrt{3}} P\) correspond to one parameters Kane’s model and using (14) we find for
\[ \varphi(E) = \frac{2m_n}{\hbar^2} \left( \frac{E(E - E_g)(E + \Delta)}{E_g(E_g + \Delta)} \frac{E_g + \frac{2}{3} \Delta}{E + \frac{2}{3} \Delta} \mp \frac{\frac{2}{3} \Delta}{E + \frac{2}{3} \Delta} \frac{\hbar \omega}{2} \left( l_0 + \frac{1}{2} \pm 1 \right) - \frac{\hbar \omega}{2} \right) \]  

Using (12) to (4),(5) and (6), we find \( f_1^\pm, f_2^\pm \) and \( f_3^\pm \):  

\[ f_1^\pm = \frac{\sqrt{2} b \alpha}{E} f_{0,n}^\pm \]  

\[ f_2^\pm = \frac{\sqrt{2} b}{E} \left[ \left( \frac{4n - 1 + 2l_0 \pm 1 \mp (l_0 + \frac{1}{2})}{4r} - \lambda r \right) f_{0,n}^\pm - \frac{\sqrt{n (n + \frac{1}{2} + l_0 \mp \frac{1}{2}}}{r} f_{0,n-1}^\pm \right] \]  

\[ f_3^\pm = -\frac{2a}{E + \Delta} \left\{ \left( \frac{2n \mp 1 + l_0 (1 \mp 1)}{2r} - \lambda r \right) f_{0,n}^\mp - \frac{\sqrt{n (n + \frac{1}{2} + l_0 \mp \frac{1}{2}}}{r} f_{0,n-1}^\mp \right\} \]  

The equations (15) describe the spectrum of electrons, light and spin-orbital splitting holes bands.  

As well as in a case Dirac oscillators in ground state energy appears twice more, than for isotropic oscillator of the standard Schrodinger equation.  

The equation (15) can appear useful to the analysis of influence nonparabolicity on a energy spectrum electrons in a quantum dots. Recently this problem is considered in works [10,11] within the framework of model of a infinite potential barrier. The advantage of the given approach consists in simplicity of the analysis of analytical expressions, in comparison with numerical accounts [10,11].  

The authors are grateful to E. Jafarov who addressed their attention to works on Dirac oscillators.  

Reference:  
1. N.E.Kaputkina, Y.E.Lozovik Fiz. Tver.tela v.40,11,1753-1759 (1998)
2. N.E. Kaputkina, Y.E. Lozovik Fiz. Tver. tela v. 40, 9, 2134-2135 (1998)
3. Ö. Darnhofer, U. Rossiler, Rhys. Rev. B 47, 23, 16 020 (1993).
4. Janes P Crawford J. Math. Phys. v. 34, 10, p. 4428-4435 (1993)
5. J. Benitez, R. P. Martinez y Romero Phys. Rev Lett. V. 64, 14 (1990)
6. M. Moshinsky and A. Szezepanik, J. Phys. A 22, L817 (1989)
7. P. A. Cook Lett. Nuovo Cimento 1, 419 (1971)
[8] Gelfand I.M., Minlos R.A., Shapiro Z.Y., Representation of group of rotations and group of Lorenth, Fizmatgiz, 1958.
[9] Lyubarskiy G.Y., Theory of group and its application in physics, Fizmatgiz, 1957.
10. Al. L. Efros and M. Rosen Phys. Rev B 58, 7120-7135 (1998)
11. F. M. Gashimzade, A. M. Babaev, M. A. Bagirov J. Phys.: Condens. Matter 12, 7923-7932 (2000)