Adaptive Redundant Multiwavelet Packet and its Application for Compound Faults Detection of Mechanical Equipment

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Abstract. It is significant to detect the fault type and assess the fault level as early as possible for avoiding catastrophic accidents. The compound faults detection of mechanical equipment is a challenge task at present. Thereby, it is urgent to develop a novel and effective method for this task. Multiwavelet with two or more base functions and many excellent properties may match two or more features of compound faults. In order to realize the accurate location and detection of the compound faults features, a novel method called adaptive redundant multiwavelet packet (ARMP) is proposed based on the two-scale similarity transforms in this paper. Moreover, the ratio of relative energy at the characteristic frequency of the concerned component is computed to select the sensitive frequency bands of multiwavelet packet coefficients. The proposed method is applied to analyze the compound faults of rolling element bearing with a local spalling on outer race and a slight scrape on the inner race. The results show that the proposed method could enhance the ability of compound faults detection of mechanical equipment.

1. Introduction

The key components of important mechanical equipment always get various faults after a long-term running in the complex and severe conditions such as heavy load, strong impact, high temperature or corrosive environment. Once faults occur in mechanical equipment, they may result in serious accidents and huge economic losses. So it is significant to detect the fault type and assess the fault level as early as possible. However, due to the complexity of equipments and the correlation of structures, several faults often appear at the same time and the features of each fault are coupled together. This kind of failure form is called compound faults. In these situation, mechanical fault detection turns into a challenge task, especially in the operational condition with strong background noise. Thereby, it is urgent to develop a novel and effective method for this task.

Multiwavelet [1] with two or more base functions and many excellent properties may match two or more features of compound faults, which supplies a possibility for the solution of compound faults detection. However, the fixed base functions of multiwavelet transform, which is not related with the vibration signal, may reduce the accuracy of compound faults detection [2]. Moreover, the character that the decomposition results of multiwavelet transform do not own time invariance is harmful to extract the features of periodical impulses [3]. Furthermore, multiwavelet transform only focuses on the multi-resolution analysis in low frequency band, which may leave out the useful features of
compound faults. To overcome these shortcomings, a novel method called adaptive redundant multiwavelet packet (ARMP) is proposed in this paper.

The first step to realize the ARMP is the construction of adaptive multiwavelet. Two-scale similarity transforms (TSTs) [4] is an effective method for constructing the new symmetric biorthogonal multiwavelet base functions with some desirable properties, which can improve the adaptability of the existed multiwavelet. Envelop spectrum entropy [5] effectively reflect the existence of periodical impulses. Based on the TSTs, the adaptive multiwavelet is constructed according to the character of vibration signal by taking the minimum envelop spectrum entropy as the optimization objective. Next, based on the adaptive multiwavelet filters, the matrix low-pass and high-pass filters of adaptive redundant multiwavelet packet can be calculated by inserting zeros between every adjacent pair of elements. Then the vibrations signal is decomposed by the ARMP transform.

In order to realize the accurate location and detection of the compound faults features from the multiwavelet packet coefficients, the ratio of relative energy at the characteristic frequency of the concerned component is computed to select the sensitive frequency bands. The proposed method is applied to analyze the compound faults of rolling element bearing with a local spalling on outer race and a slight scrape on the inner race. Compared with traditional wavelet and multiwavelet methods as well as spectral kurtosis, the results show that the proposed method could enhance the ability of compound faults detection of mechanical equipment.

2. Summary of multiwavelet theory

Multiwavelet bases are generated by two or more mother wavelets. Similar to the scalar wavelet, multi-scaling function vector \( \Phi = [\phi_1, \phi_2, \ldots, \phi_r]^T \) and multiwavelet function vector \( \Psi = [\psi_1, \psi_2, \ldots, \psi_r]^T \) also satisfy the following two-scale matrix refinement equations:

\[
\Phi(t) = \sqrt{2} \sum_{k} H_k \Phi(2t - k) \quad k \in \mathbb{Z}
\]

\[
\Psi(t) = \sqrt{2} \sum_{k} G_k \Phi(2t - k) \quad k \in \mathbb{Z}
\]

Where the coefficients \( \{H_k\} \) and \( \{G_k\} \) are respectively matrix low-pass and high-pass filters. Suppose \( s_{j-1} \) and \( d_{j-1} \) are respectively the vector low frequency coefficients and the vector high frequency coefficients. Then the decomposition and composition of multiwavelet transform are

\[
s_{j-1}(n) = \sum_{k} H_{k-2n}s_j(k) \quad d_{j-1}(n) = \sum_{k} G_{k-2n}s_j(k)
\]

\[
s_j(k) = \sum_{n} H^*_{k-2n}s_{j-1}(n) + \sum_{n} G^*_{k-2n}d_{j-1}(n)
\]

Due to being constructed from translations and dilations of multi-scaling and multiwavelet function vector, multiwavelet can seize the vital signal processing properties of orthogonality, symmetry, short support and vanishing moments simultaneously [6]. Meanwhile, the multiwavelet decomposition and reconstruction is a multi-input and multi-output system. So multiwavelet can express any signal more comprehensive and precise.

3. Adaptive redundant multiwavelet packet

3.1. Construction of adaptive multiwavelet

Strela [4] profoundly studied multiwavelet in the time and frequency domain and put forward TSTs for designing new symmetric multiwavelet with some desirable properties simply and straightforwardly. In this segment, the construction of adaptive multiwavelet base functions is based on the TSTs theory [7]. A series of new symmetric multiwavelet base functions are developed based on TSTs by taking GHM multiwavelet [8] as original multiwavelet. The multi-scaling functions and
multiwavelet functions with the properties of symmetry, short support, orthogonality and with an approximation order 2 are shown in figure 1. After each TSTs, the regularity of the multi-scaling functions increases one and the finite support property is guaranteed [4]. But unfortunately, the orthogonality is destroyed. So the biorthogonal multiwavelet is discussed and designed mainly based on TSTs with the condition of perfect reconstruction.

![Figure 1. Multi-scaling functions Φ₁, Φ₂ and multiwavelet functions Ψ₁, Ψ₂ of GHM multiwavelet.](image)

Let $H(\omega)$ be the symbol of multi-scaling functions $\Phi$ with the approximation order $p \geq 1$, and if $M(\omega)$ is a TSTs matrix which $H(0)$ and $M(0)$ share a common right eigenvector $r$, then the symbol $H_{\text{new}}(\omega)$ of the new multi-scaling functions $\Phi_{\text{new}}$ and the symbol $H_{\text{new}}(\omega)$ of the new dual multi-scaling functions $\Phi_{\text{new}}^*$ can be calculated [4]

$$
H_{\text{new}}(\omega) = \frac{1}{2} M(2\omega) H(\omega) M^{-1}(\omega)
$$

$$
G_{\text{new}}(\omega) = \frac{1}{2} G(\omega) M^{-1}(\omega)
$$

$$
\tilde{H}_{\text{new}}(\omega) = 2(M^{-1})^*(2\omega) \tilde{H}(\omega) M^*(\omega)
$$

$$
\tilde{G}_{\text{new}}(\omega) = 2\tilde{G}(\omega) M^*(\omega)
$$

Besides, the TSTs decreases one approximation order and regularity of the dual multi-scaling functions and meanwhile increases one approximation order and regularity of multi-scaling functions. In order to overcome this shortcoming, another TSTs aiming at the new dual multiwavelet is performed. The two feasible TSTs matrices $M_1(\omega)$ and $M_2(\omega)$ subjected to previous algorithm and theorem are computed as follows [7]. The factors $a$, $b$, $c$, $d$ and $f$ are nonzero constant.

$$
M_1(\omega) = \begin{bmatrix}
a(1 + e^{-i\omega}) & -2\sqrt{2a} \\
b(1 - e^{-i\omega}) & 0
\end{bmatrix}, 
M_2(\omega) = \begin{bmatrix}
c & 0 \\
d(1 + e^{-i\omega}) & f(1 - e^{-i\omega})
\end{bmatrix}
$$

In order to detect and match the fault features, the adaptive multiwavelet is constructed based on the signal by optimally selecting the appropriate parameters $a, b, c, d$ and $f$. One initial example is shown in figure 2 with $a = -1, b = -2, c = -4, d = -6, f = -8$. According to the discussion above, the minimum envelop spectrum entropy is brought into the adaptive multiwavelet construction as the optimization objective and genetic algorithms [9] are utilized as the optimization tool due to the major advantages of flexibility and robustness.
3.2. Redundant multiwavelet transform

Redundant multiwavelet owns the time invariance, which is beneficial to the feature extraction of periodic impulses. Moreover, the redundant multiwavelet transform not only may supply the richer feature information, but also may supply the more precise frequency localization information, which is very beneficial in fault feature extraction. The low-pass and high-pass filters of redundant multiwavelet are computed by padding the multiwavelet low-pass and high-pass filters with the zeros at the corresponding level \( l \). Suppose \( T \) is the operator that alternates an arbitrary sequence with zeros, then, for all integers \( i \),

\[
(Tx)_n = x_i, \quad (Tx)_{n+1} = 0 \quad (6)
\]

Then the filters of redundant multiwavelet can be calculated by the following equations

\[
\begin{align*}
H_i^G &= 0, \quad \text{if } k \text{ is not a multiple of } 2^l \\
H_{2j}^G &= (T^i H)_{2j} = H_i, \quad G_{2j}^G = (T^i G)_{2j} = G_i, \quad \text{otherwise}
\end{align*}
\]

(7)

3.3. The proposed method

Redundant multiwavelet transform only focuses on multi-resolution analysis in low frequency band, which may leave out the useful information on fault features. In order to overcome this shortcoming, redundant multiwavelet packet is conducted based on the low-pass and high-pass filters of redundant multiwavelet. Taking \( P \) stand for the matrix filters \( H^i \) or \( G^i \), the decomposition coefficients of redundant multiwavelet packet transform with two base functions at the level of \( k \) can be obtained

\[
\begin{bmatrix}
C_{j-1,k,1} \\
C_{j-1,k,2}
\end{bmatrix} = \sqrt{2} \sum_{n=0}^{K} P^{k} \begin{bmatrix}
C_{j,k+n,1} \\
C_{j,k+n,2}
\end{bmatrix}, \quad j,k \in \mathbb{Z}
\]

(8)

Figure 3 displays the decomposition stages of the adaptive redundant multiwavelet packet transform with two base functions. The post-processing step used in traditional multiwavelet transform is abandoned in order to get the decoupling features from different output channels of multiwavelet packet coefficients. Moreover, in order to realize the accurate location and detection of the compound faults features from the multiwavelet packet coefficients, the ratio \( r \) of relative energy at the characteristic frequency of the concerned component is computed to select the sensitive frequency band [1].
Figure 3. The decomposition stages of the ARMP transform with two base functions. \( Q(z) \) means the pre-filter in Z-transform. Meanwhile, \( H \) and \( G \) are respectively matrix low-pass and high-pass filters.

\[
\begin{align*}
    r &= \frac{\max \left[ A(f_c) \right]^2}{\sum_{f=0} f \in (f_c - \Delta, f_c + \Delta)} \\
    f_c &= (f_c - \Delta, f_c + \Delta)
\end{align*}
\]  

(9)

Where \( f_c \) means the characteristic frequency. \( A \) is the amplitude of envelope spectrum of multiwavelet packet coefficients and \( \Delta \) is the frequency interval. \( f = 0 \rightarrow f' \) means the range of the frequency band.

The procedure of the proposed method can be summarized in the flow chart as shown in figure 4. Meanwhile, the process of the adaptive redundant multiwavelet packet for the compound faults detection of mechanical equipment can be presented as follows:

1) Compute two-scale similarity transforms matrices \( M_1(\omega) \) and \( M_2(\omega) \) based on the selected multiwavelet
2) Perform two-scale similarity transforms twice to get a series of new symmetric biorthogonal multiwavelet.
3) Generate the optimal adaptive multiwavelet by genetic algorithms. Conduct redundant multiwavelet packet transform with the adaptive multiwavelet base functions.
4) Get the sensitive frequency band and then output the corresponding envelope spectrum of multiwavelet packet coefficients for extracting fault features.

4. Example of engineering application

Figure 5 shows the experimental setup. The type of the rolling element bearing is 552732QT. The acceleration signal was measured at a sampling frequency of 12.8 kHz and the rotational speed is 360r/min. The characteristic frequency of the rolling element defect is 19.4Hz, the outer race defect is calculated to be at 43.3Hz and the inner race defect is 58.7Hz based on the geometric parameters.

The vibration signal with a length of 12800 is displayed in figure 6. Based on the knowledge of bearing kinematics and dynamics, periodic impulses are generated when the rolling elements pass over the damaged area on the out race or inner race. However, from figure 6, such impulse features information is difficult to be distinguished because of the background noise. The frequency spectrum and the directly envelope spectrum of the signal are shown in figure 7. The obvious two peaks in the low frequency area are 43.8Hz and 87.2Hz, which almost are equal to the characteristic frequency of outer race defect and two times frequency. Meanwhile, it is difficult to find the evident frequency information corresponding to the rest part of the testing rolling element bearing.
Figure 4. The flow chart of the proposed method.

Figure 5. The experimental setup.
In order to extract all the faults feature information, the proposed method is introduced to analyze the vibration signal in figure 6. The optimal multiwavelet factors $a, b, c, d, f$ are determined at [4.47, -3.90, 1.37, 0.78, -0.03] by the process in section 3. The vibration signal is performed three-layer decomposition and then we get sixteen of multiwavelet packet decomposition coefficients. The ratio of relative energy at the characteristic frequency of the concerned component is computed to select the sensitive frequency bands and the distribution is shown in figure 8. Due to the larger ratio, we suppose that there should be faults on the outer race and inner race of bearing. The envelope spectrum of three sensitive frequency bands with the maximum ratio of the different component is computed for further validation and shown in figure 9. From figure 9(a), the characteristic frequency of rolling element defect can hardly be found and meanwhile the characteristic frequency of outer race defect as well as the two times frequency can be clearly observed in figure 9(b). Moreover, an obvious peak is 58.2Hz corresponding to the characteristic frequency of inner race defect in figure 9(c). The testing rolling element bearing for electric locomotives with a local rub on outer race and a slight scrape on the inner race is shown in figure 10. The results show that the proposed method has taken good effect on the rolling element bearing compound faults detection.
Figure 9. The envelope spectrum of three sensitive frequency bands with the maximum ratio of the different component. The characteristic frequency of outer race detect is clear in (b) at 43.8Hz and the frequency 58.6 Hz is related with the inner race fault in (c).

Figure 10. The defects of the testing rolling element bearing.

The same experimental signal is respectively processed using spectral kurtosis [10], Db8 scalar wavelet with envelope spectrum, GHM multiwavelet with envelope spectrum for comparison to demonstrate the usefulness of the proposed method. In the all analyzed results from figure 11 to figure 13, the characteristic frequency of outer race defect and inner race defect can’t be extracted together. Compared with the proposed approach, all the methods can’t detect and extract the complete information on fault features.

Figure 11. The analyzed result of the vibration signal using spectral kurtosis: (a) the Kurtogram, (b) the purified signal and (c) the envelope spectrum.
5. Conclusion

Multiwavelet with two or more base functions and many excellent properties may match two or more features of compound faults, which supplies a possibility for the solution of compound faults detection. Considering the advantage of multiwavelet, the new multiwavelet techniques are discussed in this paper for compound faults detection. In order to overcome these following shortcomings that the fixed basis functions is not related with the vibration signal, the decomposition results don’t own time invariance, multiwavelet transform only focuses on the multi-resolution analysis in low frequency band, a novel method called ARMP is proposed based on the two-scale similarity transforms in this paper.

The proposed method is applied to analyze the compound faults of rolling element bearing with a local spalling on outer race and a slight scrape on the inner race. Compared with traditional wavelet and multiwavelet methods as well as spectral kurtosis, the results show that the proposed method could enhance the ability of compound faults detection of mechanical equipment. The results also support the view that multiwavelet with two or more base functions owns the born advantage on the compound faults detection of rotating machinery under non-stationary operations. More methods on the base of multiwavelet theory should be developed for compound faults detection of mechanical equipment in the future.

Acknowledgements

This research is supported financially by the key project of National Natural Science Foundation of China (No.51035007), National Natural Science Foundation of China (No.50975220), National Basic Research Program of China (No.2009CB724405), Important National Science and Technology Specific Projects (No.2010ZX04014-016) and the Specialized Research Fund for the Doctoral Program of Higher Education (No.20110201130001).

References

[1] Wang X, He Z and Zi Y 2010 Adaptive construction of multiwavelet and research on composite fault diagnosis of rolling bearing J. Vibration Engineering 23 438-444
[2] Wang X, Zi Y and He Z 2009 Multiwavelet construction via adaptive symmetric lifting scheme and its applications for rotating machinery fault diagnosis Meas. Sci. Technol. 20 045103
[3] Mallat S 2003 A wavelet tour of signal processing, Second Edition (China: Academic Press) p 50
[4] Strela V 1996 Multiwavelets: theory and application (Cambridge: Massachusetts Institute of
[5] Li H, Zhang Z, Ma X and Wang Z 2008 Investigation on diesel engine fault diagnosis by using Hilbert spectrum entropy Journal of Dalian University of Technology 48 220-224

[6] Daubechies I 1992 Ten Lectures on wavelets (Philadelphia, PA: SIAM) chapters 1, 3-5

[7] Yuan J, He Z and Zi Y 2009 Adaptive multiwavelets via two-scale similarity transforms for rotating machinery fault diagnosis Mech. Syst. Signal Process. 23 1490-1508

[8] Geronimo J S, Hardin D P and Massopust P R 1994 Fractal function and wavelet expansions based on several scaling functions J. Approx. Theory 78 373-401

[9] Hwang S and He R 2006 Improving real-parameter genetic algorithm with simulated annealing for engineering problems Advances in Engineering Software 37 406-418

[10] Antoni J 2006 The spectral kurtosis: a useful tool for characterizing non-stationary signals Mech. Syst. Signal Process. 20 282-307