Fuzzy ABC Classification in Inventory Management for a Service Sector Firm

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ABSTRACT

Companies face with the need for evaluation of inventories in terms of multiple criteria in short time in the varying economic conditions. The ABC Analysis clusters these inventories and produces required reports. However, ABC inventory clustering process has weakness in terms of class distinction. The purpose of this study is obtaining strong optimum results in ABC Analysis by fuzzy clustering of inventories. The case is performed in a firm operating in telecommunications sector.

Keywords: ABC Analysis, Fuzzy Clustering C – Means, Classification

INTRODUCTION

There are thousands of stock items in inventory management system. To control all stock items in same level is meaningless and very difficult. To determine to what extent the various stock items in stock to control, they should be classified according to their criticality or value. That determined classes are ranked according to degree of control. Although ABC Analysis has guidelines for class distinction, companies generally set their own distinction point of each class. Therefore, percentages of class A, B and C vary. So the upper and lower limits used in ABC stock classification method does not specify exactly a value. This causes to bounce of a stock item to the upper class with very little difference from its successor in classification. This means sometimes incorrect results or incorrect evaluations. In other words, there is a functional instability in ABC class membership. Actually there is a fuzziness in such cases. This study involves performing stock classification via C-Means method belonging to Fuzzy Clustering then comparing the results with ABC Analysis. The application set in SVS Telecommunication Company which generally works on satellite communications systems.

THE ABC ANALYSIS

Each category shows the amount of money that belongs to it and also its importance. In other words, the stock items who have financially worthwhile amount should be controlled firmer that other items. These stocks are critical. The ABC analysis is used in order to determine the stock categories and degrees of these categories. ABC stock analysis collects stock items under the following three groups (Çokoy, 2013):

Class A: This class has the highest financial volume, not in the auto purchasing process, consists of close control required items. Items in this class forms 80% of the stock investment but constitute 20% of the total number of parts.

Class B: This class has the middle financial volume, being in the auto purchasing process belongs to the authority of the management. Items in this class forms 30-35% of the stock investment but constitute 20% of the total number of parts.

Class C: This class has the lowest financial volume, are in the auto purchasing process. Items in this class forms 50-55% of the stock investment but constitute 5% of the total number of parts.

As shown in Table1 (Çokoy, 2013), class A items are tracked tightly, data is saved properly, safety stock level is low and are reviewed continuously in small quantities. On the other hand,
class B items are tracked normally, data is also saved properly, safety stock level is middle and are reviewed occasionally in middle quantities while class C items are tracked simply, data is saved also simply, safety stock level is high and are reviewed periodically (1-2 year) in high quantities.

### Table 1. ABC classification system properties

| Cluster | Financial percentage | Quantity percentage | Control degree | Record type | Safety stock level | Ordering method |
|---------|----------------------|---------------------|----------------|-------------|--------------------|-----------------|
| A       | 70-80 %              | 10-20 %             | Tight          | Proper      | Low                | Continuous in small quantities occasionally in middle quantities periodically in high quantities |
| B       | 15-20 %              | 30-40 %             | Normal         | Proper      | Middle             |                |
| C       | 5-10 %               | 40-50 %             | Simple         | Simple      | High               |                |

### ABC Application

SVS service firm desires to analyze its 45 stock items by ABC method. Codes of that stocks, their average annual usage and average annual prices is determined. According to ABC operating procedure, annual sum is calculated by multiplication of usage and price for each item. Then each sum is turned to percentages. Percentages are ranked from highest to lowest. Afterward classes are decided in terms of cumulative (for items’ cumulative also).

As in Figure 1, recommended class distinction points (Table 1) not only fractional, but also not together in the same row. So the distinction depends on decision maker’s subjectivity. Some attach importance to financial cumulative while others care quantity. Financial for class A and quantity for class C for instance. In addition, some could increase or decrease class number.

For this application, determined class items are; E3,E8,E5,E4,E1,E0,P0,T0,A6 for class A (distinction points: 79,22% - 20%), A1, P2, B7, P7, U0, E9, A4, E6, B2, A8, P5, B0, P3, A3 for class B (distinction points: 15% - 15%), A7, A5, A9, P6, P3, B5, E7, B4, E2, B8, E10, B3, E12, A2, A0, E11, P1, B6, P8, B1, AR, U1 for class C (distinction points: 2% - 50%).
### Table 1: Cumulative Item %

| Stock ID | Annual Amount of Use % | Cumulative % | Cumulative Item % |
|----------|------------------------|--------------|-------------------|
| E3       | 16.69%                 | 16.69%       | 2.22%             |
| E8       | 13.10%                 | 29.79%       | 4.44%             |
| E5       | 11.86%                 | 41.65%       | 6.67%             |
| E4       | 11.79%                 | 53.44%       | 8.89%             |
| E1       | 5.73%                  | 59.17%       | 11.11%            |
| E0       | 5.35%                  | 64.52%       | 13.33%            |
| P0       | 5.08%                  | 69.60%       | 15.56%            |
| T0       | 5.05%                  | 74.65%       | 17.78%            |
| A6       | 4.57%                  | 79.22%       | 20.00%            |
| A1       | 4.57%                  | 83.79%       | 22.22%            |
| P2       | 4.44%                  | 88.23%       | 24.44%            |
| B7       | 4.24%                  | 92.47%       | 26.67%            |
| P7       | 1.07%                  | 93.54%       | 28.89%            |
| U0       | 1.07%                  | 94.61%       | 31.11%            |
| E9       | 1.06%                  | 95.67%       | 33.33%            |
| A4       | 1.02%                  | 96.69%       | 35.56%            |
| E6       | 1.01%                  | 97.70%       | 37.78%            |
| B2       | 0.39%                  | 98.09%       | 40.00%            |
| A8       | 0.35%                  | 98.44%       | 42.22%            |
| P5       | 0.17%                  | 98.61%       | 44.44%            |
| B0       | 0.12%                  | 98.73%       | 46.67%            |
| P3       | 0.11%                  | 98.84%       | 48.89%            |
| A3       | 0.10%                  | 98.94%       | 51.11%            |
| A7       | 0.09%                  | 99.03%       | 53.33%            |
| A9       | 0.09%                  | 99.12%       | 55.56%            |
| A5       | 0.08%                  | 99.20%       | 57.78%            |
| P6       | 0.08%                  | 99.28%       | 60.00%            |
| P3       | 0.07%                  | 99.35%       | 62.22%            |
| B5       | 0.06%                  | 99.41%       | 64.44%            |
| E7       | 0.06%                  | 99.47%       | 66.67%            |
| B4       | 0.05%                  | 99.52%       | 68.89%            |
| E2       | 0.05%                  | 99.57%       | 71.11%            |
| B8       | 0.05%                  | 99.62%       | 73.33%            |
| E10      | 0.05%                  | 99.67%       | 75.56%            |
| B3       | 0.04%                  | 99.71%       | 77.78%            |
| E12      | 0.04%                  | 99.75%       | 80.00%            |
| A2       | 0.04%                  | 99.79%       | 82.22%            |
| A0       | 0.04%                  | 99.83%       | 84.44%            |
| E11      | 0.03%                  | 99.86%       | 86.67%            |
| P1       | 0.03%                  | 99.89%       | 88.89%            |
| B6       | 0.03%                  | 99.92%       | 91.11%            |
| P8       | 0.03%                  | 99.95%       | 93.33%            |
| B1       | 0.02%                  | 99.97%       | 95.56%            |
| AR       | 0.02%                  | 99.99%       | 97.78%            |
| U1       | 0.01%                  | 100.00%      | 100.00%           |

**Figure 1. ABC Application for SVS items**

### Fuzzy Clustering C-Means Method

Fuzzy C-Means algorithm is the basis for all clustering technique based on the objective function. The algorithm is developed by Bezdek (1974). When the algorithm is finalized, the points in p-dimensional space takes the form a spherical shape which are clusters. These clusters are assumed to be approximately at the same size. The cluster centers represent each
cluster and are called prototypes. As a measure of distance, it uses the Euclidean distance between the center of the clusters and the data (Equation 1).

\[
    d_{ik} = d(x_i, v_k) = \sqrt{\sum_{t=1}^{p} (x_{ji} - v_{jk})^2}
\]  

(1)

To apply this technique, the number of clusters and individual degrees of membership to the cluster must be known in advance. Since it is difficult to know in advance of such parameters, these values can be found by trial and error or by some developed techniques (Erilli, 2014).

Equation (2) shows the objective function of the clustering method. This function is a weighted least squares function. The parameter \( n \), is the number of observations, \( c \) indicates the number of clusters (Sintas et al., 1999).

\[
    J(u,v) = \sum_{j=1}^{n} \sum_{k=1}^{c} u_{jk}^m \left\| x_{ji} - v_{jk} \right\|^2
\]  

(2)

If the function to be minimized for each value of \( c \), in other words, 1st order derivative of \( n \) equalized to 0, the prototype of the algorithm is as Equation (3):

\[
    v_{jk} = \frac{\sum_{j=1}^{n} u_{jk}^m x_{ij}}{\sum_{j=1}^{n} u_{jk}^m} - \frac{\sum_{j=1}^{n} u_{jk}^m}{n}
\]  

(3)

The steps of Fuzzy C-means Algorithm are:

**Step1:** Determination of initial values. The number of cluster \( c \), fuzziness index \( m \), finishing criteria and membership degree matrix \( U \) or \( V \) cluster prototypes are randomly generated.

**Step2:** If \( U \) cluster prototypes are assumed that they are generated randomly, the membership degree matrix is calculated using these values.

**Step3:** \( U \) cluster prototypes are updated according to Step2.

**Step4:** If \( |U^{(t)} - U^{(t-1)}| \leq \varepsilon \), then the iteration is stopped, otherwise it returns to Step 2.

After C-means algorithm is applied, the membership degree is used to decide which individual belongs to which cluster. Each individual is involved to a cluster if its membership is the largest. However, each individual may also enter with a certain degree of membership to other cluster. C-Means Algorithm result is highly dependent on the initial randomly generated values. Therefore, various algorithms have been developed and is still being developed to eliminate problems caused by randomness. C-means updates the cluster centers and membership degrees of each data point by iteration method and moves the cluster centers to the appropriate place in data set. Since the first place of the cluster centers as initially created using the assigned value of the random matrix, C-Means approach will not guarantee optimal results (Sintas et. al. 1999). Performance depends on the starting center spot. For a stronger approach, there are two ways described below.

I. Using an algorithm to identify all centers.

II. Running the C-means repeatedly with different starting centers (Resumption Strategy).
Fuzzy Cluster Validity Index

Cluster Analysis aims to place similar objects to the same group. Many clustering algorithm requires to know the number of clusters in advance. In studies based on actual data; the lack of prior knowledge of the number of clusters of researchers, leads to know whether the number of fuzzy clusters more or less than the number of actual clusters. The process of defining optimal number of clusters is called Cluster Validity.

Thus, the accuracy of the clustering process can be determined after the number of clusters (Erilli, 2009). When given results are in two-dimensional space, cluster number can be decided by interpreting cluster results visually. But the more the number of dimensions increases, the more the visuality become difficult and validity index are needed (Erilli 2014). Consequently, for the value of clustering and for the most suitable clustering planning, two criteria can be mentioned.

Density: Measures the closeness of the cluster members. Variance can be the best example for the density.

Seperation: Shows that how much two sets are seperated from each other. It measures the distance between two clusters (Erilli, 2014).

In the literature, many fuzzy clustering analysis validity index is used (Bezdek, 1974 ve 1981; Rezaee v.d., 1998; Kwon, 1998; Xie ve Beni, 1991). Depending on the number of variables or data structure, suitable clustering validity analysis is used. In this study, the artificial neural network cluster validity index is used.

Artificial Neural Network Based Validity Index

Validity index method is quite difficult technique that can be realized by the traditional programming methods. With enhanced powerful computers and programs, neural networks is regarded as a new branch of science. By this method, firstly, the lowest and the highest number that can be set according to the data is decided. The optimal number of clusters that will be decided should be in that range. Validity index method is proposed by Erilli (2011). The method uses artificial neural networks (ANN) to find most suitable clustering number. ANNs are computing systems that are developed to derive and discover new data by learning process of human brain information inference way automatically without any aid (Öztemel, 2006).

APPLICATION OF FUZZY CLUSTERING TO STOCK MANAGEMENT

The application is performed on NCSS 10 software package by multiplying the average annual amount and the average annual price of stock data, so the annual amount of usage is obtained. Firstly, the cluster number should be determined at fuzzy clustering analysis. ANN cluster validity index values are used. To determine the number of clusters, fuzzy clustering analysis is applied separately to the data from 2 to 10 and the clusters of regions are determined. The process at the software is run by entering 2 to minimum clusters section, entering 10 to maximum clusters section and entering 10 to reported cluster section. And the results can be viewed at summary section. Results are normalized by choosing the Dunn coefficient Fc (U) big and choosing the Kaufman coefficient Dc (U) small. Obtained optimum cluster numbers and values are as in Table2.
Table 2. C-Means Results for SVS items

| CLUSTER | PRODUCT ID | TOTAL STOCK NUMBER | CLUSTER | PRODUCT ID | TOTAL STOCK NUMBER |
|---------|------------|-------------------|---------|------------|-------------------|
| A       | E3         |                   | E       | E6         |                   |
|         | E8         |                   |         | B2         |                   |
|         | E5         |                   |         | A8         |                   |
|         | E4         |                   |         | P5         |                   |
|         | E1         |                   |         | B0         |                   |
|         | E0         |                   |         | P3         |                   |
|         | P0         |                   |         | A3         |                   |
|         | T0         |                   |         | A7         |                   |
|         | A6         |                   |         | A9         |                   |
|         | A1         |                   |         | A5         |                   |
|         | P2         |                   |         | P6         |                   |
|         | B7         |                   |         | P3         |                   |
| B       |           |                   | C       | B5         |                   |
|         | B7         |                   |         | E7         |                   |
|         | P7         |                   |         | B4         |                   |
|         | U0         |                   |         | E2         |                   |
|         | E9         |                   |         | B8         |                   |
|         | A4         |                   |         | E10        |                   |
|         |            |                   |         | B3         |                   |
|         |            |                   |         | E12        |                   |
|         |            |                   |         | A2         |                   |
|         |            |                   |         | A0         |                   |
|         |            |                   |         | E11        |                   |
|         |            |                   |         | P1         |                   |
|         |            |                   |         | B6         |                   |
|         |            |                   |         | P8         |                   |
|         |            |                   |         | B1         |                   |
|         |            |                   |         | AR         |                   |

Comparison of Fuzzy Clustering C-Means Method and ABC Analysis Application Results

Table 3 shows the stock numbers and percentage limits that distinct the clusters. According to that results, A class consists of 9 stocks with 79.22% of entire items (annual usage amount) at ABC analysis while A class consists of 11 stocks with 88.23% at C-means. Item number and distinctive percentage of B class on the other hand is 6 and 15.39% at ABC while 5 and 12.90% at C-means. And the last class C has 30 stock items with 5.39% percentage at ABC while 29 stock items with 3.31% percentage at C-means.

Table 3. ABC and C-Means Comparison

| CLUSTER | ABC | C-MEANS |
|---------|-----|---------|
|         | Stock Number | % Amount Usage Rate | Stock Number | % Amount Usage Rate |
| A       | 9   | %79.22  | 11   | %88.23  |
| B       | 6   | %15.39  | 5    | %12.90  |
| C       | 30  | %5.39   | 29   | %3.31   |

CONCLUSIONS

Uncertainty at cluster memberships in ABC analysis lead to seeking alternative options that enables membership clarification. Since the distinction depends on decision maker’s subjectivity in classic ABC analysis, C-means method could be an option which loads this subjective decision to an autonomous algorithm that artificial neural network based. So, C-Means based software proposes more sensitive results which eliminates unsteadiness. Although results indicates that there is little difference both in stock number and annual usage,
even one item’s cluster is important when the amount is worthy or when the quantity is critical. According to C-means results, two items are added to A cluster, one item from cluster B and cluster C are decreased. ABC results seems to keep %80-20 rule while C-means is concluded at %88 in amount usage.

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## ATTACHMENT 1: NCSS 10 SOFTWARE FUZZY CLUSTERING C-MEANS OUTPUT

### Fuzzy Clustering Report

| Dataset   | Untitled          |
|-----------|-------------------|
| Variables | tutar             |
| Distance Type | Euclidean  |
| Scale Type  | Standard Deviation |

#### Cluster Medoids Section

| Variable | Cluster1 | Cluster2 | Cluster3 |
|----------|----------|----------|----------|
| tutar    | 239019   | 49983,15 | 2391,63  |

#### Membership Summary Section for Clusters = 3

| Row | Cluster Bar | Membership | Memberships | Bar of Memberships | Amount |
|-----|-------------|------------|-------------|--------------------|--------|
| 7   | 1           | 0,8655     | 0,7585      | 0,1762             |
| 8   | 1           | 0,8648     | 0,7573      | 0,1683             |
| 6   | 1           | 0,8494     | 0,7332      | 0,2279             |
| 9   | 1           | 0,8077     | 0,6715      | 0,0076             |
| 10  | 1           | 0,8073     | 0,6710      | 0,0069             |
| 5   | 1           | 0,8047     | 0,6672      | 0,2761             |
| 11  | 1           | 0,7718     | 0,6227      | -0,0549            |
| 12  | 1           | 0,7056     | 0,5430      | -0,1540            |
| 4   | 1           | 0,5216     | 0,3879      | 0,4792             |
| 3   | 1           | 0,5204     | 0,3871      | 0,4790             |
| 2   | 1           | 0,4974     | 0,3750      | 0,4578             |
| 1   | 1           | 0,4505     | 0,3549      | 0,3736             |
| 15  | 2           | 0,9174     | 0,8463      | 0,9752             |
| 14  | 2           | 0,9172     | 0,8460      | 0,9745             |
| 13  | 2           | 0,9168     | 0,8453      | 0,9739             |
| 16  | 2           | 0,9029     | 0,8218      | 0,9626             |
| 17  | 2           | 0,8997     | 0,8165      | 0,9588             |
| 32  | 3           | 0,9881     | 0,9764      | 0,9499             |
| 33  | 3           | 0,9880     | 0,9763      | 0,9499             |
| 31  | 3           | 0,9879     | 0,9761      | 0,9498             |
| 34  | 3           | 0,9876     | 0,9754      | 0,9495             |
| 30  | 3           | 0,9875     | 0,9753      | 0,9495             |

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|   |   |   |
|---|---|---|
| 0,9494 |   |   |
| 35 | 3 | 0,9868 | 0,9738 |
| 0,9488 |   |   |
| 29 | 3 | 0,9865 | 0,9733 |
| 0,9485 |   |   |
| 36 | 3 | 0,9865 | 0,9733 |
| 0,9486 |   |   |
| 37 | 3 | 0,9857 | 0,9718 |
| 0,9479 |   |   |
| 38 | 3 | 0,9840 | 0,9684 |
| 0,9465 |   |   |
| 39 | 3 | 0,9834 | 0,9673 |
| 0,9460 |   |   |
| 28 | 3 | 0,9827 | 0,9660 |
| 0,9453 |   |   |
| 40 | 3 | 0,9808 | 0,9622 |
| 0,9438 |   |   |
| 27 | 3 | 0,9808 | 0,9622 |
| 0,9436 |   |   |
| 41 | 3 | 0,9790 | 0,9587 |
| 0,9423 |   |   |
| 26 | 3 | 0,9782 | 0,9572 |
| 0,9413 |   |   |
| 42 | 3 | 0,9777 | 0,9562 |
| 0,9411 |   |   |
| 25 | 3 | 0,9754 | 0,9519 |
| 0,9389 |   |   |
| 43 | 3 | 0,9743 | 0,9498 |
| 0,9383 |   |   |
| 24 | 3 | 0,9736 | 0,9484 |
| 0,9373 |   |   |
| 44 | 3 | 0,9695 | 0,9406 |
| 0,9341 |   |   |
| 45 | 3 | 0,9657 | 0,9334 |
| 0,9308 |   |   |
| 23 | 3 | 0,9635 | 0,9293 |
| 0,9282 |   |   |
| 22 | 3 | 0,9533 | 0,9104 |
| 0,9188 |   |   |