Analysis of Students’ Error in Proving Convergent Sequence using Newman Error Analysis Procedure

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Abstract. Mathematics in college demands students to think on high-level. One of the courses that demands high-order thinking is Real Analysis. In this course, there are mathematical proofs that often become students’ difficulties so that there are too many errors in the process. The purpose of this research was to identify students’ errors in proving convergent sequences. Students were classified into high-ability, moderate-ability, and low-ability. One student from each ability was taken randomly and interviewed to obtain data about student’s errors. The subjects of this study were 3 students of the Mathematics Department Universitas Sebelas Maret Surakarta. The type of student’s error is based on Newman Error Analysis Procedure that includes reading, comprehension, transformation, process skill, and encoding error. Based on the results, (1) high-ability student made error in process skill, (2) moderate-ability student made errors in transformation and process skill, (3) low-ability student made errors in transformation, process, and encoding.

1. Introduction
Mathematics is a study that linked to many aspects of human life. Mathematics is taught in every level of education in order to expand logical, analytical, systematical, critical, and creative thinking ability and ability to work in a team. Mathematics should be taught to the students because it is always used in all facts of life [1]. Mathematics plays a significant role in developing human thinking to be more creative and help to analyse real life problems [2]. Mathematics also plays big role to develop human thinking and trains the reasoning which used in analysis and problem solving [3]. Problem-solving in mathematics learning context has a meaning as a process and skill that is required for obtaining the solution to mathematics problem [4]. By learning problem solving in mathematics, students can acquire ways of thinking and confidence in unfamiliar situations that will help them outside the classroom [5]. Therefore, mathematical learning which occurs in every level of education has an important role to determine students success. Mathematical learning encourage students to expand their ability and implement it in daily life. Success measurement of mathematical learning can be measured and observed based on comprehension level, subject mastery, achievements of the study, and students ability to apply mathematics in the daily life [6]. Spesifically, mathematical learning in college asks students not only to memorize and apply formulas, but also expanding their way of thinking to the higher order, e.g., analyse, synthesize, and evaluate. Abilities that must be achieved in mathematics of 21st century were high order thinking, communication, creativity and innovation, also problem solving and confidence [7]. In reality, mathematical learning is still implementing one-way lecture. Students are only taking notes of what has been delivered by the lecturer and can’t expand their high order thinking. Once a problem which have higher difficulties was given, students will face some difficulties to solve it because students are used to memorize the formula instead of expand their thinking ability.
Difficulties in mathematics can be detected from several symptoms. One of the symptoms is that students show low learning achievement. The other symptom is students have learned but the results are not in line with the effort. Students faced difficulties in algebra understanding and algebra manipulation [8]. Common difficulties in algebra are text understanding and equation modelling [9]. Difficulties of mathematical learning had several characteristics, which were difficulty to understand mathematical symbols, mistakes in the using of mathematical language, disability to memorize definitions and theorems, lack of abstract thinking, lack of ability to identify and use algorithm to solve problems, and imperfect study.

These difficulties were faced by Mathematics students of Universitas Sebelas Maret Surakarta in Real Analysis subject. In the Mathematics Department of Universitas Sebelas Maret, Calculus along with Logic and Set were the prerequisites of Real Analysis subject. Real Analysis develops students to think in a hierarchy and rationally deductive and express the result in a logical and systematical writing. Calculus main focuses are calculating and applying formulas but in Real Analysis, students must prove a given problem and expand their ability to analyse. Proving is a sequence of logical arguments that explain a statement is true. Application of known facts such as definitions, lemmas, theorems, or axioms was needed in proving process. Students were trained to explain a statement is true or false and communicate it in logic and systematic language. There are four stages of mathematical proving, namely understand the problem, devise a plan, carry out the plan, and look back [10].

One example of proving in Real Analysis is proving convergent sequence. A number of sequence is convergent if that sequence is monotonic and bounded. Both requirements is needed in proving convergent sequence. In reality, students often face difficulties in the proving process. Proving becomes a fear among the students. The difficulties in proving process can lead students to do some errors. Most commonly used method for identifying students’ difficulty in mathematics is error analysis. Examples of errors that often occurs in the students: (1) no comprehension of things that have been known, (2) no clue of where to start in proving, (3) lacking ability to do algebra manipulation, and (4) can’t deliver the idea into systematical language. An analysis is needed to find out the types of errors which made by students and the factors that cause those errors. Error analysis is the process of reviewing students’ item responses to identify a pattern of misunderstanding [11]. Through analysis of the errors, type and the cause of the errors will be obtained which can be a clear and detailed picture in the Real Analysis learning process, especially in proving convergent sequence.

Newman Error Analysis Procedure is a method of analyzing errors in the troubleshooting process. Newman proposed five activities to find the errors of the students in solving the given problem or called Newman’s error categories, namely reading errors, comprehension errors, transformation errors, process skills errors, and encoding errors [12]. Therefore, the aim of this research is to identify students’ errors in proving convergent sequence. In this research, there are five types of errors were used based on Newman Error Analysis Procedure [13] and presented in Table 1.

| Type of Errors          | Criteria(s)                                                                 |
|------------------------|----------------------------------------------------------------------------|
| Reading Error          | Students couldn’t read a key word or a symbol correctly.                   |
| Comprehension Error    | Students have read all the words in the problem accurately but didn’t understand the overall problem or specific terms within the problem. |
| Transformation Error   | Students understood what the problem requires but unable to identify the operations which are needed to solve the problem. These include: |
|                        | • Incorrect steps which is use of steps that are not associated with any operations. |
|                        | • Missing steps where steps necessary to complete a procedure are missing.  |

Table 1. Types of errors based on Newman Error Analysis Procedure.
2. Methods
This research was conducted at Mathematics Department of Universitas Sebelas Maret. This research is a qualitative research. Students were classified into three groups of abilities namely high-ability, moderate-ability, and low-ability where each group consists of 6 students. The classification was based on their values in Calculus and Logic and Set because Calculus and Logic and Set are prerequisites in Real Analysis. The subjects of this research were 3 students of fifth semester of Mathematics Department Universitas Sebelas Maret. The selection of the research subject was based on: (1) subjects have received a lesson about real number sequence; (2) subjects have taken the test of about real number sequence and made errors in proving it; and (3) subjects have communication skill in both oral and written.

The data were collected through essay test and the task-based interview. The main instrument at this research was the researcher. The secondary instrument at this research were the test about real number sequence and the guide of interview. The analysis was conducted on the high-ability student, moderate-ability student, and low-ability student. Based on the test result and interview, the errors in every ability level can be concluded.

The data of this research must be valid so that researchers conduct time triangulation. Data validity from time triangulation can be seen when there are no significant differences in different time from the obtained data. The data analysis technique includes the following activities: (1) data reduction, (2) presentation of data, and (3) drawing conclusions and verification.

3. Results and Discussions
Essay test were used to gather data. After the data from essay test were obtained, one student was taken randomly from each ability and the interview process was conducted on the chosen subject. All of three subjects were being asked to prove convergent sequence that defined by

\[
a_1 = \frac{3}{2}; \quad a_{n+1} = 2 - \frac{1}{a_n}, \quad n \geq 1.
\]

Requirements to prove convergent sequence are proving that the sequence is monotonic and bounded. Based on the test results given to the students, researchers obtained data of students who made errors in proving convergent sequence.

Student who had low-ability made some errors in proving convergent sequence. He could answer that the sequence is convergent. His answer was shown in Figure 1.

![Figure 1. Low-ability student’s solution.](image_url)
made conclusion. It is according to student’s statement which goes “I don't know how to prove it. I'm just trying to substitute and if the value goes to a point, then that sequence is convergent.” The errors of low-ability student based on the errors in the written test and interview results are presented in Table 2.

| Type of Errors         | Interview Results                                                                 |
|------------------------|----------------------------------------------------------------------------------|
| Reading Error          | Subject have read the given problem correctly.                                   |
| Comprehension Error    | Subject have read all the words in the problem accurately and he understood the overall problem. |
| Transformation Error   | Subject didn't understand what the problem requires. He couldn't explain the needed theorems about convergent sequence. He didn’t understand the definition of monotonic and bounded sequence. |
| Process Skill Error    | Subject could not prove monotonic and bounded sequence with mathematical induction. He only tried to prove by substituting the value of n to the sequence formula. |
| Encoding Error         | He concluded that the sequence is convergent by looking at the decreasing pattern. However, he couldn’t determined the convergence value of the sequence. |

Student who had moderate-ability answered the problem, but she didn’t use mathematical induction. She also substituted the value of n. Her answer was shown in Figure 3.

Based on the interview, moderate-ability student only understood one requirement to prove convergent sequence. She said that "The requirement to prove a convergent sequence is prove that this sequence is monotonic. However, subject couldn’t executed the proving process with mathematical induction. She concluded if the sequence is decreasing and monotonic, then that sequence is convergent. It is according to student’s statement which goes “I only substituted the value of n. If the sequence is
decreasing and monotonic, it's enough to prove that it is convergent too, isn't it?” She also concluded the convergent value based on the limit. The errors of moderate-ability student based on the errors in the written test and interview results are presented in Table 3.

| Type of Errors      | Interview Results                                                                 |
|---------------------|-----------------------------------------------------------------------------------|
| Reading Error       | Subject have read the given problem correctly.                                    |
| Comprehension Error | Subject understood the overall problem. She understood the main goal of the problem. |
| Transformation Error| Subject understood one of the requirement to prove convergent sequence. She knew the theorems about monotonic sequence. However, she couldn’t explain theorem about bounded sequence. |
| Process Skill Error | Student couldn’t executed mathematical induction to prove a monotonic sequence. She only tried to prove by substituting the value of \( n \). She also didn’t prove bounded sequence because she didn’t understand that theorem. |
| Encoding Error      | She concluded that this sequence is convergent because it is decreasing and monotonic. She concluded the convergence value too. |

Student who had high-ability answered the problem. He was able to use mathematical induction but that steps were wrong. His answer was shown in Figure 3.

![Figure 3](image-url)

High-ability student understood the requirement to prove convergent sequence. He proved that the sequence is bounded and monotonic. He used mathematical induction but his steps stopped when \( n = k \). He didn’t prove for \( n = k + 1 \). Based on the interview result, he couldn't find his mistakes in mathematical induction. It is according to student’s statement which goes “I have proven that the
sequence is convergent with mathematical induction. There is no wrong step because it has been proven for \( n = k \).” The errors of high-ability student based on the errors in the written test and interview results are presented in Table 4.

| Type of Errors             | Interview Results                                                                 |
|----------------------------|----------------------------------------------------------------------------------|
| Reading Error              | Subject have read the given problem correctly.                                  |
| Comprehension Error        | Subject understood the overall problem. She understood the main goal of the problem. |
| Transformation Error       | Subject understood the requirement of convergence sequence is the sequence must be monotonic and bounded. |
| Process Skill Error        | Subject knew that mathematical induction must be used to prove monotonic and bounded sequence. He was not able to use mathematical induction to prove bounded and monotonic sequence correctly. |
| Encoding Error             | He concluded that this sequence is convergent because it is monotonic and bounded. He concluded the convergence value too. |

Based on the above description of the subjects, it can be concluded that all of three subjects understood the problem and did not have error in phase reading and comprehension. They understood what have been known. They also knew what is the main goal of the problem. Student who had low-ability made error in transformation. Subject didn’t understand the requirement of convergent sequence. Subject didn’t understand the concept in the problem [14]. This is why subject made error in process skill too. Subject didn’t use mathematical induction. He only substituted the value of \( n \) and could not determine the convergent value. He also made error in encoding.

Student who had moderate-ability made errors in transformation and process skill too. Subject only knew one requirement of convergent sequence. She only proved monotonic sequence and didn’t use mathematical induction. She didn’t prove that the sequence is bounded, but she could calculate the convergent value. Student who had high-ability also made error in process skill because he couldn’t use mathematical induction correctly. The students’ errors in proving convergent sequence have similarity and difference to other researches. Therefore, the results of this study enrich theories of students’ errors in proving convergent sequence.

4. Conclusion

Based on the research result, it can be concluded that (1) high-ability student made error in process skill, (2) moderate-ability student made errors in transformation and process skill, (3) low-ability student made errors in transformation, process, and encoding. It’s showed that all of three subjects still encountered some errors in the proving process. They understood what is known and what is asked, but they weren’t used to do mathematical induction correctly because they didn’t fully understand how to use mathematical induction which has been taught in Logic and Sets. Subjects also didn’t fully understand the related theorems which can be used to prove convergent sequence. These errors which made by the subjects were caused by lack of understanding in the subject or prerequisite of Real Analysis. Students did not understand the purpose of Real Analysis because it is different from other courses that are more concerned with calculation.

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