Constraints on the parameters of the $V_{CKM}$ matrix at the end of 1997.

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Abstract

A review of the current status of the Cabibbo-Kobayashi-Maskawa matrix ($V_{CKM}$) is presented. This paper contains an update of the results published in [1]. Values of the parameters entering into the constraints, which restrict the range for $\rho$ and $\eta$ parameters, include recent measurements given at 1997 Summer Conferences and progress obtained by lattice QCD collaborations. Experimental constraints imposed by the measurements of $|\epsilon_K|$, $\frac{|V_{ub}|}{|V_{cb}|}$, $\Delta m_d$ and by the limit on $\Delta m_s$, are compatible and do not show evidence for New Physics inside measurements errors. Values for the angles $\alpha$, $\beta$ and $\gamma$ of the C.K.M. triangle have been also obtained:

$$\rho = 0.156 \pm 0.090, \quad \eta = 0.328 \pm 0.054$$

$$\sin 2\alpha = -0.10 \pm 0.40, \quad \sin 2\beta = 0.68 \pm 0.10, \quad \gamma = (64 \pm 12)^\circ$$

Using the analysis of [2], the ratio of the branching fractions for charged and neutral B mesons decaying into $K\pi$ can be predicted with good precision:

$$R_1 = \frac{BR(B^0 \rightarrow \pi^\pm K^\mp)}{BR(B^\pm \rightarrow \pi^\pm K^0)} = 0.89 \pm 0.08$$

Angles $\theta$, $\theta_u$, $\theta_d$ and $\phi$ proposed in the parametrisation [3] of the C.K.M. matrix have been also determined:

$$\theta = (2.30 \pm 0.09)^\circ, \quad \theta_u = (4.87 \pm 0.86)^\circ$$

$$\theta_d = (11.71 \pm 1.09)^\circ, \quad \phi = (91.1 \pm 11.8)^\circ$$

The present value of $\phi$ is compatible with the maximum CP violation scenario advocated in [3].

As there are more constraints than the fitted $\rho$ and $\eta$ parameters two studies have been done.

In the first study, several external measurements or theoretical inputs have been removed, in turn, from the constraints and their respective values have been fitted simultaneously with $\rho$ and $\eta$. Central values and uncertainties on these quantities have been compared with actual measurements or theoretical evaluations. In this way it is possible to quantify the importance of the different measurements and the coherence of the Standard Model scenario for CP violation.

In the other study an additional parameter has been introduced to account for New Physics, inside a given model and limits on the masses of the new particles appearing in this model have been updated by reference to the similar analysis done in [1].
1 Introduction.

In a previous publication [1], uncertainties on the determination of the C.K.M. parameters $\rho$ and $\eta$ corresponding to the Wolfenstein parametrization, have been reviewed [1]. This study was based on measurements and theoretical estimates available at the beginning of 1997. Present results are obtained using new measurements and theoretical analyses available by summer 1997.

| parameters used in [1] | present analysis |
|------------------------|------------------|
| $A = 0.81 \pm 0.04$    | $0.823 \pm 0.033$|
| $|V_{ub}| / |V_{cb}| = 0.08 \pm 0.02$ | unchanged |
| $\Delta m_d = (0.469 \pm 0.019) \text{ ps}^{-1}$ | $(0.472 \pm 0.018) \text{ ps}^{-1}$|
| $\Delta m_s > 8.0 \text{ ps}^{-1}$ at 95\% C.L. | $> 10.2 \text{ ps}^{-1}$ at 95\% C.L.|
| $m_t(m_t) = (168 \pm 6) \text{ GeV/c}^2$ | unchanged |
| $B_K = 0.90 \pm 0.09$ | unchanged |
| $f_{B_d} \sqrt{B_{B_d}} = (200 \pm 50) \text{ MeV}$ | $(220 \pm 40) \text{ MeV}$|
| $\xi = f_{B_s} \sqrt{B_{B_s}} / f_{B_d} \sqrt{B_{B_d}} = 1.17 \pm 0.13$ | $1.10 \pm 0.07$|

Table 1: Values of the relevant parameters entering into the Standard Model expressions for $\Delta m_d$, $|\epsilon_K|$, $|V_{ub}| / |V_{cb}|$ and $\Delta m_d / \Delta m_s$. The second column gives the results, updated with respect to the previous publication [1], which are used in this paper.

The central values and uncertainties for the relevant parameters used in this analysis are given in Table 1.

This analysis (see also [1]) differs from similar studies [3] because recent results from experiments and from lattice QCD have been included. This is of particular importance in the determination of $A$, $B_K$ and of the ratio $\xi$. The constraint from $|\epsilon_K|$ is then really effective because it selects a region in $\eta$ of similar size as the one given by the other constraints. This implies that the compatibility between the measurements of the sides of the C.K.M. triangle (given by the values of $|V_{ub}| / |V_{ub}|$, $\Delta m_d$ and $\Delta m_s$) and a measurement directly related to CP violation (given by the value of $|\epsilon_K|$) is investigated.

In the following sections, the procedures used to obtain these new values have been explained. Section 2 details the evaluation of $A$ through the measurement of the $|V_{ub}|$ element of the C.K.M. matrix. In section 3 improvements in the determination of other important parameters in this analysis are considered. The new limit on $\Delta m_s$ obtained by the LEP experiments is recalled. The present determination of $f_{B_s}$ through the measurement of $f_{D_s}$ and the use of lattice QCD is explained [1]. New results from lattice QCD relating the $B^0_d$ and $B^0_s$ decay constants are reported and finally the value used for $B_K$ is commented.

Using the constraints on $\rho$ and $\eta$ provided by the measurements of $|\epsilon_K|$, $\Delta m_d$, $\Delta m_s$ and $|V_{ub}| / |V_{cb}|$, in the framework of the Standard Model, the region selected in the ($\rho$, $\eta$) plane and the determination of the angles $\alpha$, $\beta$ and $\gamma$ of the unitarity triangle, are analyzed.
in section 4. The parametrization of the C.K.M. matrix proposed in [3] has been also considered and the corresponding parameters have been determined.

As there are more experimental constraints than fitted parameters, the information coming from one of the external parameters ($\Delta m_2, m_t, A, \left| \frac{V_{ub}}{V_{cd}} \right|, B_K$, and $B_{B_{d}}\sqrt{f_{B_{d}}})$ has been removed, in turn, from the fit and the probability density distribution for this parameter, determined by the other parameters and constraints has been determined in section 5.

Another possibility consists in fitting an extra parameter, in addition to $\rho$ and $\eta$, keeping all the constraints. This has been done in section 6, using the model already considered in [1].

2 The measurement of the parameter $A$.

The value of the parameter $A$ is obtained from measurements of $|V_{cb}|$ in exclusive and inclusive semileptonic decays of B hadrons. By definition:

$$|V_{cb}| = A \lambda^2, \text{ with } \lambda = \sin \theta_c. \quad (1)$$

2.1 $|V_{cb}|$ measurement using $B \to D^* \ell^- \bar{\nu}_\ell$ decays.

In this channel, $|V_{cb}|$ is obtained by measuring the differential decay rate $\frac{d \Gamma_{D^*}}{dq^2}$ at maximum value of $q^2$. $q^2$ is the mass of the charged lepton-neutrino system. At $q^2 = q_{\text{max}}^2$, the $D^*$ is produced at rest in the B rest frame and HQET can be invoked to obtain the value of the corresponding form factor: $F_{D^*}(w = 1)$. The variable $w$ is usually introduced; it is the product of the 4-velocities of the B and $D^*$ mesons:

$$w = v_B \cdot v_{D^*} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}, \quad w = 1 \text{ for } q^2 = q_{\text{max}}^2. \quad (2)$$

In terms of $w$, the differential decay rate can be written:

$$\frac{dBR_{D^*}}{dq^2} = \frac{1}{\tau_B^0 48 \pi^3 m_{D^*}^3 (m_B - m_{D^*})^2} \frac{G_F^2}{\sqrt{\pi}} K(w) \sqrt{w^2 - 1} F_{D^*}(w) |V_{cb}|^2 \quad (3)$$

in which $K(w)$ is a kinematic factor.

As the decay rate is zero for $w = 1$, the $w$ dependence has to be adjusted over the measured range, using the previous expression. The form factor is expected to be $w$ dependent:

$$F_{D^*}(w) = F_{D^*}(1)[1 - \hat{\rho}^2(w - 1) + \hat{c}(w - 1)^2 + \Phi(w - 1)^3] \quad (4)$$

The value of the slope at the origin, $\hat{\rho}^2$, is usually also fitted on data and the curvature, $\hat{c}$ is neglected or related to $\rho^2$, using constraints on the $q^2$ dependence of form factors which are due to analyticity properties of QCD spectral functions and unitarity [7]:

$$\hat{c} = 0.66 \rho^2 - 0.11 + \Phi(A/m_b) \quad (5)$$

At LEP the $q^2$ of the reaction is obtained from the measured characteristics of the final state in the decay $B^0_d \to D^{*+} \ell^- \bar{\nu}_\ell$. The B meson direction is measured using the position
of the primary and of the B decay vertices. The neutrino energy is obtained by imposing a global energy-momentum conservation on the whole event to define the energy in each hemisphere and then subtracting the 4-momenta of the measured charged and neutral particles. A typical resolution of 3 GeV is achieved on $\bar{E}_\nu$. The $D^{*+}$ has been exclusively reconstructed or simply the $\pi^*$ from its cascade decay has been used. In the first case, resolutions of 0.07 to 0.1 have been obtained on $w$ whereas, in the other, larger samples of events can be analyzed but with a reduced resolution. In practice, the inclusive analysis does not suffer from larger systematic uncertainties. Experiments have fitted simultaneously $F_{D^*}(1)|V_{cb}|$ and $\hat{\rho}^2$ (Table 2). Results have been combined, including the CLEOII measurement, taking into account common systematics among LEP and CLEO: $BR(D^0 \rightarrow K^-\pi^+)$, $\tau_{B^0_d}$, $BR(D^{*+} \rightarrow D^0\pi^+)$ and $D^{**}$ rate in semileptonic decays and also additional common systematics between LEP experiments as the branching fraction of the $Z^0$ to $b\bar{b}$ pairs and the fraction of $B^0_d$ mesons in $b$ jets. Values given in Table 2 have been updated using common values for all external parameters and include the recent improvements on the uncertainties on these parameters (see Table 3). At present the dominant systematic uncertainty (4% on $|V_{cb}|$) comes from the error on the $D^{**}$ rate in B semileptonic decays.

| Experiment | $(F_{D^*}(1)|V_{cb}|) \times 10^2$ | $\hat{\rho}^2$ |
|------------|---------------------------------|----------------|
| ALEPH      | $31.7 \pm 1.8 \pm 1.9$          | $0.31 \pm 0.17 \pm 0.08$ |
| DELPHI     | $35.7 \pm 2.0 \pm 2.4$          | $0.74 \pm 0.20 \pm 0.17$ |
| OPAL       | $32.5 \pm 1.9 \pm 2.2$          | $0.55 \pm 0.24 \pm 0.05$ |
| LEP average| $33.0 \pm 1.3 \pm 1.8$          | |
| CLEOII     | $35.1 \pm 1.9 \pm 1.9$          | $0.80 \pm 0.17 \pm 0.17$ |
| global average | $33.72 \pm 1.25 \pm 1.56$   | |

Table 2: Measurements of $F_{D^*}(1)|V_{cb}|$ in $B \rightarrow D^*\ell\nu\ell$.

Table 3 shows that the individual LEP experiments have the same sensitivity as published results from CLEO. Using $F_{D^*}(1) = 0.91 \pm 0.06$ [12], the following value is obtained for $|V_{cb}|$:

$$|V_{cb}|(exclusives) = (37.0 \pm 2.2 \pm 2.4) \times 10^{-3}$$ (6)

2.2 $|V_{cb}|$ measurement using inclusive semileptonic decays.

The expression relating the value of $|V_{cb}|$ and the values of the inclusive $b$ lifetime and semileptonic branching fraction can be found in [12]:

$$|V_{cb}| = 0.0419 \sqrt{\frac{BR(B \rightarrow X_c\ell\bar{\nu}\ell)}{0.105}} \sqrt{\frac{1.55}{\tau_B}} (1 \pm 0.015 \pm 0.010 \pm 0.012)$$ (7)

The numbers given at the end of this expression reflect different sources of theoretical uncertainties and have been added linearly in the following. The first uncertainty depends on the evaluation of $\alpha_s^{(5)}(1 \text{ GeV})$, the second error is related to the uncertainty on the value of $m_b$ and the last one reflects uncertainties on the $1/m_Q^2$ and higher power corrections.
Table 3: Values of the parameters used in the present determination of $|V_{cb}|$. The semileptonic branching fraction for the $B^0_d$ meson has been obtained using the inclusive semileptonic branching fraction measurement done at the $\Upsilon(4S)$ \cite{13} and correcting for the contribution of charged $B$ mesons by taking into account the difference between $B^0_d$ and $B^-$ lifetimes.

$$|V_{cb}| (inclusives) = (41.0 \pm 1.1 \pm 1.5) \times 10^{-3}$$

### 2.3 Summary on $|V_{cb}|$ measurements.

The two results obtained for $|V_{cb}|$ have largely uncorrelated experimental and theoretical uncertainties and are compatible, their average is:

$$|V_{cb}| = (40.0 \pm 1.6) \times 10^{-3}$$

which implies:

$$A = 0.823 \pm 0.033$$

If, instead of adding linearly theoretical errors, their quadratic sum is used, the two measurements of $|V_{cb}|$ are marginally compatible within the quoted errors. Applying the PDG recipe to scale the errors \cite{14}, exactly the same result is obtained: $(40.1 \pm 1.6) \times 10^{-3}$.

This value of $|V_{cb}|$ may appear to be precisely known (4% relative error) but, experimentally, only rates which are proportional to $|V_{cb}|^2$ are measured and, for rates, the relative error becomes 8%. There are thus good prospects to improve further on the precision of $|V_{cb}|$ but this requires also a good control of theoretical errors for exclusive and inclusive channels.

In the following analysis, the effect of an uncertainty on $|V_{cb}|$ of $\pm 2 \times 10^{-3}$ has been also examined.

### 3 Improvements in the determination of the other parameters.

#### 3.1 Present limit on $\Delta m_s$.

A new limit on $\Delta m_s$, $\Delta m_s > 10.2 \text{ps}^{-1}$ at 95% C.L., has been provided by the “$B$ Oscillation Working Group” \cite{13}. The sensitivity of present measurements is at $13.0 \text{ps}^{-1}$. The definition of the sensitivity and the way the information on $\Delta m_s$ is used in the constraints have been explained in \cite{14}. 

| Parameter               | Value               | Reference |
|-------------------------|---------------------|-----------|
| $P(b \rightarrow B^0_l)$ | $(39.5^{+1.5}_{-2.0})\%$ | \cite{13} |
| $\text{BR}(D^0 \rightarrow K^- \pi^+)$ | $(3.83 \pm 0.12)\%$ | \cite{14} |
| $\tau_{B^0_d}$         | $(1.57 \pm 0.04)\text{ps}$ | \cite{13} |
| $\text{BR}(B^0_d \rightarrow \ell X)$ | $(10.2 \pm 0.5)\%$ | (see below) |
\section*{3.2 Present value of $f_B$.}

$f_B$ is evaluated from the measurements of $f_{D_s}$ and using the extrapolation from the D to the B sector as predicted by lattice QCD. More details can be found also in \[1\]. The value of $f_{D_s}$ is deduced from the measurements of the branching fractions $D^+_s \rightarrow \tau^+\nu_{\tau}$ and $D^+_s \rightarrow \mu^+\nu_{\mu}$. The different determinations of $f_{D_s}$ \[17\] result in the following average:

$$f_{D_s} = (243 \pm 36) \text{ MeV} \quad (11)$$

In a recent publication, from lattice QCD \[18\], the ratio between the $D^+_s$ and the $B^0_d$ decay constants has been evaluated:

$$\frac{f_{B_d}}{f_{D_s}} = 0.76 \pm 0.07 \quad (12)$$

It results:

$$f_B = (185 \pm 25(\text{exp.}) \pm 17(\text{theo.})) \text{ MeV} \quad (13)$$

and using $B_{B_d} = 1.36 \pm 0.16$ \[1\] it follows:

$$f_{B_d}\sqrt{B_{B_d}} = (220 \pm 40) \text{ MeV} \quad (14)$$

This value is well compatible with the absolute prediction from lattice QCD ($f_{B_d}\sqrt{B_{B_d}} = (200 \pm 50) \text{ MeV}$, which was used in our previous analysis) and also with the value favoured by the measurements of $|\epsilon_K|$, $\Delta m_d$, $|\frac{V_{ub}}{V_{cb}}|$ and the limit on $\Delta m_s$, presented in section 5.

\section*{3.3 Present value for $\xi$.}

Significant improvements have been achieved in the determination of the $\xi$ parameter and several authors agree on a relative precision better than 10\% (see Table 4).

| reference | value          |
|-----------|----------------|
| \[18\]   | 1.10 \pm 0.07  |
| \[19\]   | 1.17 \pm 0.03  |
| \[20\]   | 1.14 \pm 0.08  |

Table 4: Values of the parameter $\xi$ provided by different collaborations working on lattice QCD. Only recent results have been reported.

The value from reference \[18\] has been used in the following.

\section*{3.4 Present value of $B_K$.}

The central value and the uncertainty used for the scaled invariant parameter $B_K$ have not been modified as compared to our previous analysis ($B_K = 0.90 \pm 0.09$ \[1\]). These values are in agreement with recent results obtained by lattice QCD collaborations recently reported \[21\]:

$$B_K = 0.86 \pm 0.09 \quad (15)$$
4 Results with present measurements.

The region of the \((\bar{\rho}, \bar{\eta})\) plane selected by the measurements of \(|\epsilon_K|\), \(\{|V_{ub}|/|V_{cb}|\}\), \(\Delta m_d\) and from the limit on \(\Delta m_s\) has been obtained assuming Gaussian errors or flat probability distributions \(^2\) for the parameters \(B_K, f_{B_d}\sqrt{B_{B_d}}, \xi\) and \(\{|V_{ub}|/|V_{cb}|\}\).

4.1 Measured values of \(\bar{\rho}\) and \(\bar{\eta}\).

Central values and uncertainties on \(\bar{\rho}\) and \(\bar{\eta}\) are given in Table 5.

| fit conditions                  | \(\bar{\rho}\)      | \(\bar{\eta}\)      |
|---------------------------------|----------------------|----------------------|
| Gaussian errors                 | 0.156 ± 0.090        | 0.328 ± 0.054        |
| flat errors                     | 0.166 ± 0.111        | 0.334 ± 0.057        |
| Gaussian errors, \(\sigma(A) = 0.04\) | 0.160 ± 0.094        | 0.341 ± 0.058        |

Table 5: Measured values of the parameters \(\bar{\rho}\) and \(\bar{\eta}\).

Contrary to what is usually claimed \[^{[5]}\]:
- the use of flat distributions for the quantities in which systematic errors are dominant, does not change significantly the results.
- the allowed region for \(\bar{\rho}\) is not symmetric around zero, negative values for \(\bar{\rho}\) being clearly disfavoured:

\[
P_{\rho < 0}^{\text{Gauss}} = 6.7\%, \quad P_{\rho < 0}^{\text{Flat}} = 9.9\%. \quad (16)
\]

The contours corresponding to 68 % and 95 % confidence levels are shown in Figure 1.

4.2 Measured values of \(\sin 2\alpha\) and \(\sin 2\beta\).

It is of interest to determine the central values and the uncertainties on the quantities \(\sin 2\alpha\) and \(\sin 2\beta\) which can be measured directly at future facilities like HERA-B and B factories. Results have been summarized in Table 6.

| fit conditions                  | \(\sin 2\alpha\)      | \(\sin 2\beta\)      |
|---------------------------------|----------------------|----------------------|
| Gaussian errors                 | -0.10 ± 0.40         | 0.68 ± 0.10          |
| flat errors                     | -0.10 ± 0.45         | 0.78 ± 0.08          |
| Gaussian errors, \(\sigma(A) = 0.04\) | 0.02 ± 0.43         | 0.69 ± 0.11          |

Table 6: Measured values of the parameters \(\sin 2\alpha\) and \(\sin 2\beta\).

Determinations of these angles, prior to the present analysis can be found in \[^{[22]}\]. Figure 2 gives the correlation between the measurements of these two quantities and the contours at 68% and 95 % C.L..

\[^{2}\]The considered flat probability distributions have been taken with the same variance as the corresponding Gaussian distributions (their half width is equal to \(\sqrt{3} \sigma\)).
Figure 1: The allowed region for $\rho$ and $\eta$ using the parameters listed in Table 1. The contours at 68% and 95% are shown. The full lines correspond to the central values of the constraints given by the measurements of $|V_{ub}|$, $|V_{cb}|$, $|\epsilon_K|$ and $\Delta m_d$. The dotted curve corresponds to the 95% C.L. upper limit obtained from the experimental limit on $\Delta m_s$.

4.2.1 Measurement of $\sin 2\beta$.

The value of $\sin 2\beta$ is rather precisely determined, with an accuracy already at a level expected after the first years of running at B factories. The situation will improve in the coming years with better measurements of $|V_{cb}|$, with a possible improvement of the sensitivity of LEP analyses on $\Delta m_s$ and with an expected progress from lattice QCD.

4.2.2 Measurement of $\sin 2\alpha$.

In our previous analysis [1], it was concluded that there was no restriction on the domain of variation of $\sin 2\alpha$ between -1 and +1. The present study, see Figure 2, allows to identify a favoured domain for this parameter which is around zero.

4.2.3 Measurement of the angle $\gamma$.

It has been proposed in [23] to restrict the range of variation of the angle $\gamma$ using the measurement of the ratio, $R_1$, of the branching fractions of charged and neutral B mesons into $K\pi$ final states. In the hypothesis that this ratio is below unity, the following constraint has to be satisfied:

$$\sin^2 \gamma < R_1$$

The present result from CLEO [24]:

$$R_1 = \frac{BR(B^0 \rightarrow \pi^\pm K^\mp)}{BR(B^\pm \rightarrow \pi^\pm K^0)} = 0.65 \pm 0.40,$$
Figure 2: The $\sin^2\alpha$ and $\sin^2\beta$ distributions have been obtained using the constraints corresponding to the values of the parameters listed in Table [4]. The contours at 68% and 95% are shown. The $\sin^2\alpha$ and $\sin^2\beta$ distributions are also shown. The dark-shaded and the clear-shaded intervals correspond, respectively, to 68% and 95% confidence level regions.
has a too large uncertainty to be really constraining on $\gamma$. This bound excludes a region which is symmetric around $\gamma = 90^\circ$. In fact, as explained already in section 4.1, negative values of $\rho$ are already excluded and the region around $\gamma = 90^\circ$ has a low probability. These restrictions are clearly apparent in Figure 3 which gives the expected density probability distribution for the angle $\gamma$, which is determined to be: $\gamma = (64 \pm 12)^\circ$.

At present, theorists do not agree on the effects of hadronic interactions on this analysis [25]. But, considering that these effects are under control, the needed experimental accuracy on $R_1$ has been evaluated such that this measurement provides an information on $\rho$ of similar precision as the one obtained at present. The model of [2] has been used in which the authors have studied the variation of $R_1$ with the $\rho$ parameter (Figure 4). The present determination of $\rho$ corresponds to:

$$R_1 = \frac{BR(B^0 \to \pi^\pm K^\mp)}{BR(B^\pm \to \pi^\pm K^0)} = 0.89 \pm 0.08 \text{ (Gaussian errors)}.$$  \hfill (19)

This result indicates the precision which is needed for a direct measurement of the quantity $R_1$ to reduce the present error on the angle $\gamma$ (and similarly on the parameter $\rho$).

### 4.3 Measurement of the angles $\theta$, $\theta_u$, $\theta_d$ and $\phi$.

There are nine possibilities to introduce the CP violation phase into the elements of the CKM matrix [3]. The authors of [3] have argued for a parametrization, based on the observed hierarchy in the values of quark masses. Several theoretical works [26] show that
Figure 4: The ratio $R_1 = \frac{BR(B^\mu (\overline{B}^0) \rightarrow \pi^\pm K^\mp)}{BR(B^{\pm} \rightarrow \pi^\mp K^\pm)}$ as a function of the parameter $\rho$ taken from [3], for $\eta = 0.25$ (lower curve) and $\eta = 0.52$ (upper curve). The horizontal thick lines show the CLEO measurement (with $\pm 1 \sigma$ errors). The shaded vertical band corresponds to the $\pm 1 \sigma$ interval for $\rho$ obtained in the present analysis.

the observed pattern of fermion masses and mixing angles could originate from unified theories with an U(2) flavour symmetry. The authors of [3] have introduced four angles which have simple physical interpretations. $\theta$ corresponds to the mixing between the families 2 and 3. $\theta_{u(d)}$ is the mixing angle between families 1 and 2, in the up(down) sectors. Finally $\phi$ is responsible for CP violation and appears only in the elements of the C.K.M. matrix relating the first two families.

This parametrization is given below:

$$V_{CKM} = \begin{pmatrix}
    s_\mu s_\tau + c_\mu c_\tau e^{-i\phi} & s_\mu c_\tau - c_\mu s_\tau e^{-i\phi} & s_\mu s \\
    c_\mu s_\tau - s_\mu c_\tau e^{-i\phi} & c_\mu c_\tau + s_\mu s_\tau e^{-i\phi} & c_\mu \\
    -s_\mu s & -c_\mu s & c
\end{pmatrix}$$

where $c_\times$ and $s_\times$ stand for $\cos \theta_\times$ and $\sin \theta_\times$ respectively.

The four angles are related to the modulus of the following C.K.M. elements:

$$\sin \theta = |V_{cb}| \sqrt{1 + \frac{|V_{ub}|^2}{|V_{cb}|^2}}$$

$$\tan \theta_u = \frac{|V_{ub}|}{|V_{cb}|}$$

$$\tan \theta_d = \frac{|V_{td}|}{|V_{ts}|}$$

and

$$\phi = \arccos \left( \frac{\sin^2 \theta_u \cos^2 \theta_d \cos^2 \theta + \cos^2 \theta_u \sin^2 \theta_d - |V_{us}|^2}{2 \sin \theta_u \sin \theta_d \cos \theta} \right)$$
The first three equations illustrate the direct relation between the angles $\theta$, $\theta_u$ and $\theta_d$ and the measurements of B decay and oscillation parameters.

The angle $\phi$ has also a nice interpretation because, in the limit of $\theta = 0$ (in practice $\theta \simeq 2^\circ$), the elements $V_{us}$ and $V_{cd}$ have the same modulus, equal to $\sin \theta_c$ ($\theta_c$ is the Cabibbo angle) and can be represented in a complex plane by the sum of two vectors, of respective lengths $\sin \theta_u \cos \theta_d$ and $\sin \theta_d \cos \theta_u$, making a relative angle $\phi$. It can be shown that this triangle is congruent to the usual unitarity triangle [3] and that $\phi \simeq \alpha$.

Using the constraints defined previously, the respective probability distributions for the four angles have been given in Figure 5, and fitted values are summarized in Table 7.

| Angle | measured value | value expected from quark masses [27] |
|-------|---------------|--------------------------------------|
| $\theta$ | $(2.30 \pm 0.09)^\circ$ | | |
| $\theta_u$ | $(4.87 \pm 0.86)^\circ$ | $(3.36 \pm 0.35)^\circ$ |
| $\theta_d$ | $(11.71 \pm 1.09)^\circ$ | $(12.84 \pm 1.27)^\circ$ |
| $\phi$ | $(91.1 \pm 11.8)^\circ$ | $(85 \pm 21)^\circ$ |

Table 7: Fitted values for the angles of the parametrization [3], compared with those obtained using the values of the quark masses as given in [27] evaluated at $Q^2 = M_W^2$ (the values used for the quark masses are: $m_u = 2.35^{+0.42}_{-0.45}$ MeV, $m_d = 4.73^{+0.61}_{-0.67}$ MeV, $m_s = 94.2^{+11.9}_{-13.1}$ MeV and $m_c = 684^{+56}_{-61}$ MeV)

Another interesting aspect of this parametrization of the C.K.M. matrix is its possible interpretation in terms of quark masses [28]:

$$\tan \theta_u = \sqrt{\frac{m_u}{m_c}}, \quad \tan \theta_d = \sqrt{\frac{m_d}{m_s}}.$$ (25)

Using the values for the quark masses given in [27] evaluated at $Q^2 = M_W^2$, the values for the angles $\theta_u$ and $\theta_d$ are given in Table 7. In this interpretation, the angle $\phi$ can be obtained using equations (24) and (25). Present measurements support a value of $\phi$ close to $90^\circ$ which corresponds to the maximal CP violation scenario of [3]. The present analysis indicates that a lower value for $m_u$ is favoured or that the expression relating $\theta_u$ with the $u$ and the $c$ quark masses has to be corrected.

5 Tests of the internal consistency of the Standard Model for CP violation.

Four constraints, three measurements and one limit, have been used until now to measure the values of the two parameters $\overline{\rho}$ and $\overline{\eta}$. It is also possible to remove, from the fit, the external information on the value of one of the constraints or of another parameter entering into the Standard Model expressions for the constraints. Each of these quantities will be considered, in turn, and fitted in conjunction with $\overline{\rho}$ and $\overline{\eta}$. The results will have some dependence in the central values taken for all the other parameters but, the main point in this study, is to compare the uncertainty on a given quantity determined in this
Figure 5: The distributions of the angles $\theta$, $\theta_u$, $\theta_d$ and $\phi$ proposed in the parametrisation [3]. The dark-shaded and the clear-shaded intervals correspond to 68% and 95% confidence level regions respectively. The lines represent the $\pm 1\sigma$ region corresponding to the values of the angles obtained using the values for the quark masses given in [27].
way to its present experimental or theoretical error. This comparison allows to quantify
the importance of present measurements of the different quantities in the definition of the
allowed region in the \((\rho, \eta)\) plane. Results have been summarized in Table 8.

| parameter | Fitted value (Gaussian errors) | Fitted value (flat error dist.) | Present value |
|-----------|-------------------------------|--------------------------------|---------------|
| \(\Delta m_s\) | \((12 \pm 4)\) ps\(^{-1}\) | \((9.5^{+3.3}_{-3.0})\) ps\(^{-1}\) | > 10.2 ps\(^{-1}\) at 95\% C.L. |
| \(|V_{ub}|/V_{cb}| \) | 0.085\(^{+0.037}_{-0.023}\) | 0.090\(^{+0.045}_{-0.026}\) | 0.08 \pm 0.02 |
| \(B_K\) | 0.82\(^{+0.45}_{-0.24}\) | 0.83\(^{+0.47}_{-0.26}\) | 0.90 \pm 0.09 |
| \(f_{B_d}\sqrt{B_{B_d}}\) | \((213^{+21}_{-20})\) MeV | \((214^{+23}_{-22})\) MeV | \((220 \pm 40)\) MeV |
| \(m_t(m_t)\) | \((165^{+52}_{-40})\) GeV | \((166^{+61}_{-42})\) GeV | \((168 \pm 6)\) GeV |
| \(A\) | 0.82\(^{+0.15}_{-0.08}\) | 0.81\(^{+0.17}_{-0.10}\) | 0.823 \pm 0.033 |

Table 8: Fitted values of the different parameters obtained simultaneously with \(\rho\) and \(\eta\) after having removed, in turn, their contribution in the different constraints.

5.1 Expected value for the \(B_s^0\) oscillation parameter, \(\Delta m_s\).

Removing the constraint from the measured limit on the mass difference between the strange B meson mass eigenstates, \(\Delta m_s\), the density probability distribution for \(\Delta m_s\) is given in Figure 6. The present limit excludes already a large fraction of this distribution. Present analyses at LEP are situated in a high probability region for a positive signal and this is still a challenge for LEP collaborations.

5.2 Top mass measurement.

If the information on the top mass measurement by CDF and D0 collaborations is removed, the fitted value for \(m_t(m_t)\) is: \(m_t(m_t) = (165^{+52}_{-40})\) GeV. The present determination of \(m_t\) with a \(\pm 6\) GeV error has thus a large impact on the present analysis.

5.3 Measurement of \(A\).

The central value determined for \(A\) is close to the direct measurement: \(A = 0.82^{+0.15}_{-0.08}\). Direct measurements of \(|V_{cb}|\) are thus important to constrain the allowed region in the \((\rho, \eta)\) plane if their relative error is below 10\%. The density probability distribution for the parameter \(A\) is given in Figure 7.

5.4 Measurement of \(|V_{ub}|/V_{cb}|\).

The central value determined for \(|V_{ub}|/V_{cb}|\) is close to the direct measurement. This indirect measurement shows the importance of having a precision on \(|V_{ub}|/V_{cb}|\) better than 30\%. The density probability distribution for the parameter \(|V_{ub}|/V_{cb}|\) is given in Figure 7.
Figure 6: The $\Delta m_s$ probability distribution obtained with the same constraints as in Figure 1. The dark-shaded and the clear-shaded intervals correspond to 68% and 95% confidence level regions respectively.

5.5 Measurement of $B_K$.

The density distribution for the parameter $B_K$ is given in Figure 6. It indicates that:

- values of $B_K$ smaller than 0.6 are excluded at 93% C.L. (90% C.L. if flat error distributions are considered),

- large values of $B_K$ are compatible with the other constraints over a large domain.

The present estimate of $B_K$, from lattice QCD, with a 10% relative error has thus a large impact for the present analysis.

5.6 Measurement of $f_B\sqrt{B_B}$.

A very accurate value is obtained:

$$f_{B_d}\sqrt{B_{B_d}} = (213^{+21}_{-20})\,MeV$$

(26)

This result is, in practice, in agreement and somewhat more precise than the present evaluation of this parameter (eq. 14). The only possibility to obtain another determination of this quantity with a similar or better accuracy, is to measure $f_{D_s}$ and $f_{D^+}$ at a $\tau$/Charm factory and to use results from lattice QCD on $\frac{f_{D_s}}{f_{D_s}}$ to deduce the value of $f_{B_d}$ [4]. The density probability distribution for the parameter $f_{B_d}\sqrt{B_{B_d}}$ is given in Figure 6.
Figure 7: The $B_K$, $f_{B_d} \sqrt{B_{B_d}}$, $A$ and $|V_{ub}|$ probability distributions obtained with the same constraints as in Figure 4. The dark-shaded and the clear-shaded intervals correspond to 68% and 95% confidence level regions respectively. The points and the errors bars show the central values and the uncertainties for these parameters used in the present analysis.
∆m_{SUSY} , tan β = 1 \hspace{1cm} tan β=1.5 \hspace{1cm} tan β = 5

| \Delta  | m_{SUSY} | > 135 | > 100 | > 80 |
|---------|----------|-------|-------|------|
| 2.36^{+1.09}_{-0.76} | \hspace{1cm} | \hspace{1cm} | \hspace{1cm} | \hspace{1cm} |

Table 9: Results on the ∆ parameter. 95% C.L limits on m_{SUSY} are given in GeV/c^2 unit.

6 Limits for a given New Physics scenario.

It has been shown in [1] that there can be additional contributions to ∆m_d and | \epsilon_K | expected to come from the presence of new physics beyond the Standard Model. The effect of new physics can be parametrized by introducing an extra parameter ∆. This analysis is for the moment quite simple since only one parameter is introduced both in the expressions of ∆m_d and of | \epsilon_K |. The present data can be then fitted using the new expressions for ∆m_d and | \epsilon_K |, fitting ∆ together with the parameters \overline{\rho} and \overline{\eta}, the result is:

\overline{\rho} = 0.147^{+0.09}_{-0.12} \hspace{1cm} \overline{\eta} = 0.328^{+0.071}_{-0.070}

\Delta = 2.36^{+1.05}_{-0.76} \hspace{1cm} (27)

It has to be reminded that the Standard Model predicts: \Delta = 2.55\pm0.15 (in the definition of ∆ the top contribution is included and the error takes into account the uncertainty on the top mass). As in [1] implications of the achieved precision on ∆, can be evaluated in a particular framework of the MSSM extension of the Standard Model in which the stop-right (\tilde{t}) and higgsinos are light, assuming that tan^2 β is lower than \frac{m_t}{m_b} and that the stop-left and the gauginos are heavy. For this study the contribution from the charged Higgs sector has been neglected and the stop-right and the charged higgsino are supposed to have the same mass (generically indicated as m_{SUSY} in the following). The theoretical framework is discussed in [29], results are summarized in Table 9.

Interesting limits on m_{SUSY} can be put for low values of tan β.

7 Conclusions.

The \overline{\rho} and \overline{\eta} parameters have been determined using the constraints from the measurements of |\frac{|V_{ud}|}{V_{cd}|}, |\epsilon_K |, ∆m_d and from the limit on ∆m_s:

\overline{\rho} = 0.156 \pm 0.090, \overline{\eta} = 0.328 \pm 0.054

Contrary to similar studies in this field, which claim a rather symmetric interval of variation for \overline{\rho}, around zero [3], the negative \overline{\rho} region is excluded at about 93% C.L..

Present measurements assume the unitarity of the CKM matrix. In this framework the value of sin2β and sin2α can be also deduced. They are:

sin 2\alpha = -0.10 \pm 0.40, \hspace{1cm} sin 2\beta = 0.68 \pm 0.10

The value of sin2β has an accuracy similar to the one expected after the first years of running at B factories. The region centered on zero is favoured for sin2α in accordance with the anzats of [3].
Values of the angle $\gamma$ larger or equal to $90^\circ$ are disfavoured ($\gamma = 64 \pm 12^\circ$) and present
determinations of $\bar{\rho}$ and $\bar{\eta}$ allow to predict, in a precise way, the value for the ratio (using
the model of $[2]$):

$$R_1 = \frac{BR(B^0 \to \pi^\pm K^\mp)}{BR(B^\pm \to \pi^\pm K^0)} = 0.89 \pm 0.08$$

The other parametrization of the CKM matrix, proposed in $[3]$ has been studied. The
four corresponding parameters, which are angles, have been determined:

$$\theta = (2.30 \pm 0.09)^\circ, \ \theta_u = (4.87 \pm 0.86)^\circ$$

$$\theta_d = (11.71 \pm 1.09)^\circ, \ \phi = (91.1 \pm 11.8)^\circ$$

Present values of $\phi$ agree with the maximal CP violation scenario proposed in $[3]$.

The internal consistency of the Standard Model expectation for CP violation, ex-
pressed by a single phase parameter in the C.K.M. matrix, has been verified by removing,
in turn, the different constraints imposed by the external parameters. No anomaly has
been noticed with respect to the central values used in the present analysis, in agreement
with the small value of the $\chi^2$ of the fit of $\bar{\rho}$ and $\bar{\eta}$ alone. This study has mainly quan-
tified the needed accuracy on the different determinations of these parameters so that
they bring useful constraints in the determination of $\bar{\rho}$ and $\bar{\eta}$. In this respect, present
uncertainties on $m_t$, $|V_{cb}|$ and $B_K$ have important contributions.

Low values of $B_K$, below 0.6, are not compatible with the present analysis at 90% C.L..

$\Delta m_s$ is expected to lie within $1\sigma$ between 8 and 16 ps$^{-1}$. A measurement of this
parameter seems to be possible at LEP.

More accurate measurements, still expected at CLEO and LEP, and more precise
evaluations of non perturbative QCD parameters from lattice QCD, will improve these
results in the coming years, before the start up of B factories. A Tau/Charm factory
providing accurate values for $f_{D^+}$ and $f_{D_s}$ is expected to have important contributions in
this analysis.

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