Overall Evolution of Realistic Gamma-Ray Burst Remnant and Its Afterglow

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Abstract

Conventional dynamic model of gamma-ray burst remnants is found to be incorrect for adiabatic blastwaves during the non-relativistic phase. A new model is derived, which is shown to be correct for both radiative and adiabatic blastwaves during both ultra-relativistic and non-relativistic phase. Our model also takes the evolution of the radiative efficiency into account. The importance of the transition from the ultra-relativistic phase to the non-relativistic phase is stressed.

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The origin of gamma-ray bursts (GRBs) has remained unknown for over 30 years.\textsuperscript{1,2} A major breakthrough appeared in early 1997, when the Italian-Dutch BeppoSAX satellite observed X-ray afterglows from GRB 970228 for the first time.\textsuperscript{3} Since then, X-ray afterglows have been observed from about 15 GRBs, of which ten events were detected optically and five bursts were also detected in radio wavelengths. The cosmological origin of at least some GRBs is firmly established. The so called fireball model\textsuperscript{4,5} is strongly favoured, which is found successful at explaining the major features of the low energy light curves.\textsuperscript{6–9}

In the fireball model, low energy afterglows are generated by ultra-relativistic fireballs, which first give birth to GRBs through internal or external shock waves and then decelerate continuously due to collisions with the interstellar medium (ISM). The dynamics of the expansion has been investigated extensively.\textsuperscript{6–9} Both analytic solutions and numerical approaches are available. It is a general conception that current models describe the gross features of the process very well and further improvements are possible only by considering some details. However, we find that three serious problems are associated with the popular model.
First, it is usually assumed that the expansion is ultra-relativistic. Then for an adiabatic fireball, the evolution of the bulk Lorentz factor is derived to be:

$$\gamma \approx (200 - 400) E_{51}^{1/8} n_1^{-1/8} t^{-3/8},$$

(1)

where $E_{51} = E_0 / (10^{51}\text{erg})$ with $E_0$ the initial fireball energy, $n_1 = n / (1\text{cm}^{-3})$ with $n$ the ISM number density, and $t$ is observer’s time in unit of s. The radius of the blastwave scales as $R \propto t^{1/4}$. Based on Eq.(1), flux density at frequency $\nu$ then declines as $S_\nu \propto t^{3(1-p)/4}$, where $p$ is the index characterizing the power-law distribution of the shocked ISM electrons, $dn_e'/d\gamma_e \propto \gamma_e^{-p}$. These expressions are valid only when $\gamma \gg 1$.

However, optical afterglows from GRB 970228 and GRB 970508 were detected for as long as 190 and 260 d respectively, while in Eq.(1), even $t = 30$ d will lead to $\gamma \sim 1$. It is clear that the overall evolution of the postburst fireball can not be regarded as a simple one-phase process, we should pay special attention to the transition from the ultra-relativistic phase to the non-relativistic phase. This is unfortunately ignored in the literature.

Second, the expansion of the fireball might be either adiabatic or highly radiative. Extensive attempts have been made to find a common model applicable for both cases. As a result, a differential equation has been proposed by various authors,

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M},$$

(2)

where $m$ is the rest mass of the swept-up ISM, $M$ is the total mass in the co-moving frame, including internal energy $U$. Since thermal energy produced during the collisions is $dE = c^2(\gamma - 1)dm$, usually we assume:  

$$dM = \frac{(1 - \epsilon)}{c^2} dE + dm = [(1 - \epsilon)\gamma + \epsilon] dm,$$

(3)

where $\epsilon$ is defined as the fraction of the shock generated thermal energy (in the co-moving
frame) that is radiated. It is putative that Eq.(2) is correct in both ultra-relativistic and non-relativistic phase, for both radiative and adiabatic fireballs.

In the highly radiative case, \( \epsilon = 1, dM = dm \), Eq.(2) reduces to,

\[
\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{ej} + m},
\]

where \( M_{ej} \) is the mass ejected from the GRB central engine. Then an analytic solution is available,\(^{11,13}\) which satisfies \( \gamma \propto R^{-3} \) when \( \gamma \gg 1 \) and \( v \propto R^{-3} \) when \( \gamma \sim 1 \), where \( v \) is the bulk velocity of the material. These scaling laws indicate that Eq.(2) is really correct for highly radiative fireballs. In the adiabatic case, \( \epsilon = 0, dM = \gamma dm \), Eq.(2) also has an analytic solution:\(^{12}\)

\[
M = \left[ M_{ej}^2 + 2\gamma_0 M_{ej} m + m^2 \right]^{1/2},
\]

\[
\gamma = \frac{m + \gamma_0 M_{ej}}{M},
\]

where \( \gamma_0 \) is the initial value of \( \gamma \). During the ultra-relativistic phase, Eqs.(5) and (6) do produce the familiar power-law \( \gamma \propto R^{-3/2} \), which is often quoted for an adiabatic blastwave decelerating in a uniform medium. In the non-relativistic limit (\( \gamma \sim 1, m \gg \gamma_0 M_{ej} \)), Chiang and Dermer have derived \( \gamma \approx 1 + \gamma_0 M_{ej}/m \),\(^{12}\) so that they believe it also agrees with the Sedov solution, i.e., \( v \propto R^{-3/2} \).\(^{14}\) However we find that their approximation is not accurate,\(^{15}\) because they have omitted some first-order infinitesimals of \( \gamma_0 M_{ej}/m \). The correct approximation could be obtained only by retaining all the first and second order infinitesimals, which in fact gives: \( \gamma \approx 1 + (\gamma_0 M_{ej}/m)^2/2 \), then we have \( v \propto R^{-3} \).\(^{15}\) This is not consistent with the Sedov solution!

The problem is serious: (i) It means that the reliability of Eq.(2) is questionable, although it does correctly reproduce the major features for radiative fireballs and even for adiabatic fireballs in the ultra-relativistic limit. (ii) In the non-relativistic phase of the expansion, the fireball is more likely to be adiabatic rather than highly radiative.
However, it is just in this condition that the conventional model fails. So any calculation made according to Eq.(2) will lead to serious deviations in the light curves during the non-relativistic phase.

Third, for simplicity, it is usually assumed that $\epsilon$ is a constant during the expansion. But in realistic case this is not true. The fireball is expected to be highly radiative ($\epsilon = 1$) at first, due to significant synchrotron radiation. In only one or two days, it will evolve to an adiabatic one ($\epsilon = 0$) gradually. So $\epsilon$ should evolve with time.\(^{16}\)

Below, we will construct a new model that is no longer subject to the aforementioned problems.

In the fixed frame, since the total kinetic energy of the fireball is

$$E_K = (\gamma - 1)(M_{ej} + m)c^2 + (1 - \epsilon)\gamma U, \quad \text{Eq.(2)}$$

and the radiated thermal energy is $\epsilon \gamma (\gamma - 1)dm c^2,^{11}$ we have:

$$d[(\gamma - 1)(M_{ej} + m)c^2 + (1 - \epsilon)\gamma U] = -\epsilon \gamma (\gamma - 1)c^2 dm.$$ \hspace{1cm} (7)

For the item $U$, it is usually assumed:

$$dU = c^2(\gamma - 1)dm.\quad \text{Eq.(2)}$$

just in this way. However, the jump conditions\(^{11}\) at the forward shock imply that $U = (\gamma - 1)mc^2$, so we suggest that the correct expression for $dU$ should be:

$$dU = d[(\gamma - 1)mc^2] = (\gamma - 1)dm c^2 + mc^2 d\gamma.$$ Here we simply use $U = (\gamma - 1)mc^2$ and substitute it into Eq.(7), then it is easy to get:\(^{15}\)

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{ej} + \epsilon m + 2(1 - \epsilon)\gamma m}. \quad \text{Eq.(8)}$$

In the highly radiative case ($\epsilon = 1$), Eq.(8) reduces to Eq.(4) exactly. While in the adiabatic case ($\epsilon = 0$), Eq.(8) reduces to:

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{ej} + 2\gamma m}. \quad \text{Eq.(9)}$$

The analytic solution is:

$$(\gamma - 1)M_{ej}c^2 + (\gamma^2 - 1)mc^2 \equiv E_0. \quad \text{Eq.(10)}$$
Then in the ultra-relativistic limit, we get the familiar relation of \( \gamma \propto R^{-3/2} \), and in the non-relativistic limit, we get \( v \propto R^{-3/2} \) as required by the Sedov solution. From these analyses, we believe that Eq.(8) is really correct for both radiative and adiabatic fireballs, and in both ultra-relativistic and non-relativistic phase.

In realistic fireballs, \( \epsilon \) is a variable dependent on the ratio of synchrotron-radiation-induced to expansion-induced loss rate of energy.\(^{16}\) As usual, we assume that in the co-moving frame the magnetic field energy density is a fraction \( \xi_B^2 \) of the energy density \( e' \), \( B'^2/(8\pi) = \xi_B^2 e' \), and that the electron carries a fraction \( \xi_e \) of the energy, \( \gamma_{e,\text{min}} = \xi_e (\gamma - 1) m_p/m_e + 1 \), where \( m_p \) and \( m_e \) are proton and electron masses, respectively. The co-moving frame expansion time is \( t'_\text{ex} = R/(-\gamma c) \), and the synchrotron cooling time is \( t'_\text{syn} = 6 \pi m_e c/\left(\sigma_T B'^2 \gamma_{e,\text{min}} e\right) \), where \( \sigma_T \) is the Thompson cross section. Then we have:\(^{16}\)

\[
\epsilon = \xi_e \frac{t'_\text{syn}^{-1}}{t'_\text{syn}^{-1} + t'_\text{ex}^{-1}}. 
\]

We have evaluated Eqs.(8) and (11) numerically, bearing in mind that:\(^{18}\)

\[
dm = 4\pi R^2 nm_p dR, \tag{12}
\]

\[
dR = v\gamma(\gamma + \sqrt{\gamma^2 - 1})dt. \tag{13}
\]

We take \( E_0 = 10^{52} \text{ erg}, n = 1 \text{ cm}^{-3}, M_{ej} = 2 \times 10^{-5} \text{ M}_\odot \). Figs.(1) and (2) illustrate the evolution of \( \gamma \) and \( R \), where full, dotted, and dashed lines correspond to constant \( \epsilon \) values of 0, 0.5, and 1 respectively. Dash-dotted lines are plot by allowing \( \epsilon \) to vary according to Eq.(11). It is clearly shown that our new model overcomes the shortcomings of Eq.(2). For example, for highly radiative expansion, the dashed lines in these figures approximately satisfy \( \gamma \propto t^{-3/7}, R \propto t^{1/7}, \gamma \propto R^{-3} \) when \( \gamma \gg 1 \), and \( v \propto t^{-3/4}, R \propto t^{1/4}, v \propto R^{-3} \) when \( \gamma \sim 1 \). While for adiabatic expansion, the full lines satisfy \( \gamma \propto t^{-3/8}, R \propto t^{1/4}, \gamma \propto R^{-3/2} \) when \( \gamma \gg 1 \), and satisfy \( v \propto t^{-3/5}, R \propto t^{2/5}, v \propto R^{-3/2} \) when \( \gamma \sim 1 \).
In order to compare with observations, we have also calculated the synchrotron radiation from the shocked ISM. Fig.(3) illustrates R band afterglows from the realistic fireball. We see that after entering the non-relativistic phase, the light curve becomes steeper only slightly, consistent with the prediction made by Wijers et al.\textsuperscript{7} In contrast, Eq.(2) generally leads to a much sharper decline.\textsuperscript{18} In our new model optical afterglows from GRB 970228 are generally well fitted.

To conclude, current researches are mainly concentrated on the ultra-relativistic phase of the expansion of GRB remnants. The popular dynamic model is in fact incorrect for adiabatic fireballs during the non-relativistic phase. This is completely unnoticed in the literature. We have revised the model. Our new model has been shown to be correct in both ultra-relativistic and non-relativistic phase. The revision is of great importance, taking account of the following facts: (i) Optical afterglows lasting for more than 100 – 200 d have been observed from some GRBs, the advent of the non-relativistic phase seems inevitable. (ii) Beaming effects also lead to a steepening in the optical light curve, non-relativistic effects should be considered carefully to tell whether GRB ejecta are beamed or not, which is crucial in understanding the GRB origin. (iii) HI supershells might be highly evolved GRB remnants,\textsuperscript{19,20} to address this question in detail, we should deal with non-relativistic blastwaves. Additionally we suggest that at very late stages, GRB remnants might become highly radiative again, just in the same way that supernova remnants do.\textsuperscript{14} This might occur when the bulk velocity is just several tens kilometers per second.
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Figure Caption

Fig. 1.— Evolution of the bulk Lorentz factor $\gamma$. We take $E_0 = 10^{52}$ erg, $n = 1$ cm$^{-3}$, $M_{ej} = 2 \times 10^{-5}$ M$_{\odot}$. The full, dotted, and dashed lines correspond to $\epsilon = 0$ (adiabatic), 0.5 (partially radiative), and 1 (highly radiative) respectively. The dash-dotted line is plotted by allowing $\epsilon$ to evolve with time.

Fig. 2.— Evolution of the radius. Parameters and line styles are the same as in Fig.1.

Fig. 3.— R band afterglows from a realistic fireball. We take $p = 2.1$, $\xi_e = 1.0$, $\xi_B^2 = 0.01$. The distance $D$ is 18 Gpc. Other parameters are the same as in Fig.1. For the origin of the observational data points, please see Ref.[10].
