1. Introduction

In this paper we consider a task related to verification of models of software and hardware systems. Such systems can be, for example, control systems for airplanes, ships, medical equipment, etc. The price of error in these systems is very high, but they are too complicated for “manually” analysis. Therefore such systems are modeled before implementation. On the stages of design, development, and verification of the models, it is necessary to constantly investigate system safety.

At present, three main methods of system safety assessment [1] are widely used: fault tree analysis, dependency diagram analysis, and Markov analysis. Each method has its own advantages and disadvantages. In this paper, Markov analysis is considered.

Markov analysis works with a Markov chain [2] – a stochastic process, which can be represented as a directed graph with weighted edges. Vertices of Markov chain represent different states, and edges are labeled by probabilities of a transition between states. The main drawback of Markov analysis is a size of Markov chains, which increases exponentially with number of components in the system. In addition, it is necessary to develop an algorithm, that takes system model and translate it to the Markov chain. These problems make Markov analysis not so popular as the other methods, and number of tools that use Markov analysis for complex systems is relatively small. However, such approach has its advantages: Markov analysis allows to look at the entire system, to consider not only causes and probabilities of certain single failure, but also investigate how various failures affect the system in the aggregate. Also Markov analysis, unlike the other approaches, allows to analyze self-recovering systems, since return to operational state is natural for Markov chains.

Thus, the task of development the Markov analysis tool of complex hardware-software systems is quite important and relevant.

2. Context

2.1 AADL and Error Model Annex

Architecture Analysis & Design Language (AADL) [3] is a language, that widely used for describing models of real-time hardware and software systems. Its features include description of both hardware (so-called execution platform) and software components of an analyzed system, and various connections between them. The models, described in AADL, may be used for documentation, for various kinds of analysis and for code generation.

Error Model Annex [4] is an extension of AADL, that allows to simulate appearance and propagation of errors in the system. For each component, a modeller can add a description of component’s behavior states, for example, operational and failed. Transitions between system states are triggered by randomly occurred error events and internal errors propagated from other components. An error propagation condition may depend on certain behavior state of the component, some error events, or error propagated from environment. Each propagated error has its own type, that allows to

Abstract. In this paper we consider Markov analysis of models of complex software and hardware systems. A Markov analysis tool can be used during verification processes of models of avionics systems. In the introduction we enumerate main advantages and disadvantages of Markov analysis. For example, with Markov analysis, unlike other approaches, such as fault tree analysis and dependency diagram analysis, it is possible to analyze models of systems that are able to recovery. The main drawback of this approach is an exponential growth of models size with number of components in analyzed system. It makes Markov analysis barely used in practice. The other important problem is to develop a new algorithm for translating a model of a system to a model suitable for Markov analysis (Markov chain), since the existing solutions have significant limitations on the architecture of analyzed systems. Next we give a brief description of the context – AADL modeling language with Error Model Annex library, MASIW framework, and also give an explanation of Markov analysis method. In a main section we suggest an algorithm for translating a system model into a Markov chain, partially solving the problem of exponential growth of Markov chain. Then follows a description of further steps, and some heuristics that allow to extremely reduce running time of the algorithm.

In this paper we also consider other Markov analysis tools and their features. As a result, we suggest a Markov analysis tool that can be effectively use in practice.

Keywords: Markov analysis; system safety assessment; fault modeling; complex software-hardware system.

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control what is exactly happened in the system. Also transitions between states can be defined implicitly – a state of some component may be a composite state of its subcomponents.

AADL and Error Model Annex together describe not only an architecture, but also error behavior of systems. It becomes possible to evaluate such properties of models as safety, reliability, the availability of its various states and ability to recover from them.

2.2 MASIW

MASIW [5] is an open-source framework for designing and analyzing of integrated modular avionics systems, that use AADL as a modelling language. The project designed as plugins for Eclipse IDE, includes a variety of tools for working with AADL and Error Model Annex models. There is a big number of different analysis tools, for example, a fault tree analysis tool, but there is no Markov analysis tool.

2.3 Markov analysis

Any model subjected to Markov analysis must be represented as a Markov chain. A Markov chain can be represented in the form of a directed graph with vertices containing system states, and edges labeled with intensities of transitions between corresponding states. A Markov chain has the property of Markov process – a probability of a transition to any state depends only on a current state and a moment in time, and previous transitions are unimportant (can be characterized as memorylessness).

Markov models can be divided into models with discrete and continuous time, as well as time-homogeneous (also called stationary) and time-inhomogeneous. In time-homogeneous Markov chains, the intensities of transitions are constant, while in time-inhomogeneous Markov chains they depend on time. In time-homogeneous Markov chains, transitions occur according to the binomial (or fixed) distribution for discrete-time chains, and according to the Poisson distribution for continuous-time chains.

To determine the behavior of an analyzing system, it is necessary to specify a system of differential equations. The equations follow from the Markov chain. For all Markov processes (and a Markov chain, in particular) we have the Kolmogorov-Chapman equation [6]:

\[
p(t+\Delta t) = \sum_{k=1}^{n} P^{(\Delta t)}(S_k/S_i)P^{(t)}(S_j/S_k)
\]

(1)

This equation means that probability of a transition from state \( S_i \) to state \( S_j \) for some time \( t + \Delta t \) is equal to a sum of probabilities of passes into the target state \( S_j \) through all of intermediate states \( S_k \).

Consider a time-homogeneous chain with an intensity of the transition between states \( S_i \) and \( S_j \) equal to \( \lambda(S_i/S_j) \). Then for continuous-time Markov chains, the Kolmogorov-Chapman equation implies a system of differential equations

\[
\frac{dP^{(t)}(S_j/S_i)}{dt} = -\sum_{k=1}^{n} \lambda(S_i/S_k) P^{(t)}(S_j/S_k) + \sum_{k=1}^{n} \lambda(S_k/S_i) P^{(t)}(S_j/S_k)
\]

(2)

And for discrete-time chains, a system of difference equations

\[
\frac{P^{(t+\Delta t)}(S_j/S_i) - P^{(t)}(S_j/S_i)}{\Delta t} = -\sum_{k=1}^{n} \lambda(S_i/S_k) P^{(t)}(S_j/S_k) + \sum_{k=1}^{n} \lambda(S_k/S_i) P^{(t)}(S_j/S_k)
\]

(3)

Denote by \( S_i \) a certain initial state of the system, and consider equations (2)-(3) in case when \( S_i = S_j \). Then, the previous equations takes the following form:

\[
\frac{dP^{(t)}(S_j)}{dt} = -\sum_{k=1}^{n} \lambda(S_i/S_k) P^{(t)}(S_j) + \sum_{k=1}^{n} \lambda(S_k/S_i) P^{(t)}(S_j)
\]

(4)

\[
P^{(t+\Delta t)}(S_j) - P^{(t)}(S_j) = -\sum_{k=1}^{n} \lambda(S_i/S_k) P^{(t)}(S_j) + \sum_{k=1}^{n} \lambda(S_k/S_i) P^{(t)}(S_j)
\]

(5)

In addition, initial conditions appear:

\[
P^{(0)}(S_j) = 1, P^{(0)}(S_j) = 0, i = -2, n
\]

(6)

Thus, we obtain the Cauchy problem [7]. The solution of this problem is a set of probabilistic functions of being a system in a definite state. This is the result of Markov analysis.

In this paper, we consider only the analysis of time-homogeneous Markov chains and models, as the most common ones. However, all results can be applied to time-inhomogeneous models, with the only difference being that intensities of Markov chain transitions depend on time, and they need to be stored as formulas, not as numbers.

3. Problem

The goal of this work is a development and implementation of a Markov analysis tool for complex hardware-software systems models within the MASIW framework. The tool takes input of some system model and a set of time points. At the output, the analyzer provides the probabilities of being the system in each of its possible states at moments of time, defined by user.

The main problem is to create a Markov chain on the basis of the original model. First, we need an algorithm that creates a correct Markov chain corresponding to the input data. Secondly, the result chain should be of acceptable size, so that the program can work for acceptable time in limited memory.

After a construction of a Markov chain, further action reduces to solving a Cauchy problem with a system of linear differential equations. An analytical solution of the Cauchy problem is too complicated, resource-intensive, and result is difficult to comprehend, so we use numerical methods.
The algorithm is completed when all nodes of Markov chain are analyzed, starting from the node corresponding to the initial state of the system.

```java
markovChain.addNode(initialStateNode)
queue.add(initialStateNode)
while not queue.isEmpty() do
  analyzeNode(queue.head())
  queue.add(newNodes)
end while
```

The analysis of each node of Markov chain looks like this: all possible sets of error events are searched, for each of them we calculate a stable state of the system into which the given set leads, and then a new transition (and, if necessary, a new node) is added to the chain.

```java
for each errorEventSet in possibleSets do
  state = currentNode.getState()
  repeat
    watchedStates.add(state)
    state = calculateState(state, errorEventSet)
  until watchedStates.contains(state)
  node = markovChain.addNode(state)
  markovChain.addTransition(
    currentNode, node, errorEventSet.getProbability())
  watchedStates.clear()
end for
```

In the above algorithm, the state of the system is considered stable if we have already reached it before. This correctly handles the case when the state of the system has not changed – we have reached the same state as in the previous step. However, in theory, in a self-recovering systems, cycling may occur if an event with a failure and an event with component recovery occur simultaneously. With this condition, the loop stops, but this situation is not handled correctly. One of the main opportunities for further improvement of the algorithm is to improve the condition for achieving a stable state of the system.

### 4.2 Calculation of new states

In the previous paragraph, a general algorithm for constructing a chain was described, omitting the details of calculating new states of the system. To find out exactly how the system has changed, it is enough to go through all its components, and see what transitions between states are triggered for a given set of events and the current state of the system. The triggered transition is immediately applied to the system, and the algorithm step is completed.
for each componentState in systemState do
    for each compositeState in comp.getCompositeStates() do
        if checkStateExpression(compositeState.getExpression()) then
            systemState.applyTransition(compositeState)
            return
        end if
    end for
    for each transition in comp.getTransitions() do
        if transition.getSource() == compState and checkErrorCondition(transition.getCondition()) then
            systemState.applyTransition(transition)
            return
        end if
    end for
end for

The checkStateExpression and checkErrorCondition functions check whether the transition condition is met. Such conditions can be interpreted as a logical formula, where variables corresponding to components behavior states, error events, and propagated errors, have value of true or false, depending on whether the system is in this state, whether an error event has occurred or whether an error of the specified type has propagated.

As soon as some component of the system changes its state, it means that we obtain a new state of the system, and the step of the algorithm is completed. If none of the transitions is triggered, then the system state has not changed, which is noticed by the algorithm described in the previous section.

### 4.3 Construction and solution of the Cauchy problem

After construction of a Markov chain, the final stage of the Markov analysis of the system is to construct a system of equations and solve the Cauchy problem. As mentioned earlier, each node of the Markov chain generates a differential equation (4) (or similar difference equation (5)). To save memory, it is not necessary to store the system of equations — the equation for any node can be easily constructed dynamically, passing through all transitions entering into this node and outgoing from it.

The resulting Cauchy problem can be solved by a numerical method from the Runge-Kutta [8] family of methods. In the analyzer, two methods are implemented: the Euler method, for discrete-time Markov chains, and the fourth-order Runge-Kutta method, for continuous-time Markov chains. The type of the chain is determined in advance, according to probability distributions of error events. An algorithm for calculating the variation of the function $P(t)$ on each time interval $\Delta t$, taking into account the dynamic construction of the equation (Euler’s method):

for each node in markovChain.getNodes() do
    $i = indexOffset$node
    $res = 0$
    for each transition in node.getInTransitions() do
        $k = indexOffset$transition.getNode()
        $res += transition.getProbability() \times pPrev[k]$
    end for
    for each transition in node.getOutTransitions() do
        $res -= transition.getProbability() \times pPrev[i]$
    end for
    $pCur[i] = pPrev[i] + \Delta t \times res$
end for

Also, the value of the vector of probability functions $P(t)$ is saved at every time point defined by user. As soon as values at each necessary time point are calculated, the algorithm is completed.

### 4.4 Getting Analysis Results

Since number of system states in Markov chain can be very large, the result of analysis in the form of probabilities of being the system in each of them is practically impossible for reading. Considering that each system has its root component, we group all system states according to the states of the root component.

In this case, all the probability functions within the same group are summed up:

for each node in chainNodes do
    $i = indexOffset$node
    state = node.getSystemState().get(rootComp)
    analysisResult.get(state) += p[i]
end for

After this, for each state of the root component, the probability of being the system in a this state at given time points is obtained. This is the desired result of the Markov analysis of the system.

### 4.5 Algorithm acceleration

Despite a partial solution of the problem of exponential growth of Markov chain size, the running time of full version of the algorithm still grows exponentially — due to a thorough search of all possible combinations of error events. Thus, we use some heuristics in the final program, accelerating the algorithm.

First, we limit the search of combinations of events. Since the probability of occurrence of one event is usually extremely small, the situation in which several
The presented tool can be further improved in various ways: adding support for time-inhomogeneous Markov chains, accelerating the work of the algorithm, changing some details of algorithm.

5. Related works
Markov analysis of AADL and Error Model Annex models is usually applied to systems consisting of only one component. Such algorithms do not consider error propagation mechanism and composite states, and limited by root component.

The tool from OSATE [9] framework, created for export AADL model into Markov chain model for PRISM [10] toolset, which provide further steps of Markov analysis, supports only the first nesting level of the component hierarchy and does not support different types of propagated errors. In addition, there were some problems associated with the syntactic correctness of the final PRISM model.

6. Conclusion
In this paper we present a new Markov analysis tool, and in particular, an algorithm for translating AADL and Error Model Annex models into Markov chains. In addition, there were added some improvement for accelerating the algorithm, which make it possible to effectively use the tool in practice.
Стохастические методы анализа комплексных программно-аппаратных систем

Аннотация. В данной работе рассматривается марковский анализ моделей комплексных программно-аппаратных систем. Инструмент марковского анализа может быть использован, в частности, для верификации моделей систем интегральной модульной авионики. Во введении перечисляются основные достоинства и недостатки марковского анализа. К примеру, марковский анализ, в отличие от других подходов — анализа дерева неисправности и анализ алогической схемы, позволяет анализировать модели систем, способных к восстановлению. Основным недостатком данного подхода является экспоненциальный рост размера моделей в зависимости от числа компонентов в анализируемой системе. Это существенно ограничивает возможность применения марковского анализа на практике. Другой важной проблемой является создание нового алгоритма трансляции исходной модели системы в модель, пригодную для марковского анализа (марковскую цепь), так как существующие решения накладывают существенные ограничения на архитектуру анализируемой системы. Далее идет краткое описание контекста, в котором инструмент должен работать — язык моделирования AADL с библиотекой Error Model Annex, набор инструментов MASIW, а также сам метод марковского анализа. В основной части предлагается алгоритм трансляции модели системы в марковскую цепь, частично решающий проблему экспоненциального роста марковской цепи. Затем следует описание дальнейших шагов, а также предлагаются эвристики, позволяющие значительно сократить время работы итоговой программы. В работе также рассматривается существующие инструменты марковского анализа и их недостатки. В качестве результата данной работы предлагается новый инструмент марковского анализа, который может быть эффективно использован на практике.

Ключевые слова: Марковский анализ; оценка безопасности системы; моделирование неисправностей; комплексные программно-аппаратные системы.

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