Dynamical Wormholes in Higher Dimensions and the Emergent Universe

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We present dynamical wormholes in higher dimensions which admit flat emergent universe (EU) model in an elegant way. The EU model was proposed to get rid of some of the problems of the Big Bang cosmology. It is free from initial singularity with other observed features of the universe. The basic assumption of EU model was that the present universe emerged out from a static Einstein universe. In the paper we study EU model in four and in higher dimensions and proposed that the EU originates from a dynamical wormhole and the throat of the wormhole is the seed of the Einstein Static universe. The shape function obtained here admits closed, open and flat universe in higher dimensions. A class of new cosmological solutions in a higher dimensional flat universe is obtained. We obtain shape functions for flat, asymptotic closed and open universe. The shape function obtained here for asymptotic closed universe is new. It is found that flat EU with a non-linear equation of state (EoS) can be accommodated with dynamical wormhole. The non-linear EoS corresponds to three types of fluids in the universe. The EoS parameter is playing an important role in determining the cosmic fluids. The space-time dimensions determines the rate of change of a particular fluid that varies with the scale factor of a dynamically evolving universe with non-interacting fluids. Considering interaction at time $t > t_0$, among the three types of fluids it is possible to describe the observed universe satisfactorily. In a higher dimensional universe it is found that near the throat null energy condition (NEC) is violated, but away from the throat NEC is found to obey admitting the observed universe for a flat case. Another interesting aspect of the EU model is that it permits late accelerating phase. However, in asymptotic closed or open universe, flat emergent universe can be accommodated with NEC which is obeyed right from the throat to the present epoch. The tension at the throat of the wormhole is estimated which is found to depend on the initial size of the Einstein static universe and dimensions of the universe. It is interesting to note that NEC is not violated to accommodate dynamical wormholes for closed or open universe. Although exotic matter is required at the throat for the flat universe, no exotic matter is required for closed or open universe which encompass the emergent universe.

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I. INTRODUCTION

In recent years, there has been a growing interest to study non-trivial spacetime topologies in Einstein’s General Theory of Relativity (GTR) in understanding different properties of the universe. Lorentzian wormhole solutions are permitted in GTR in various conditions. The wormholes allow topological passage through hypothetical tunnel like bridges connecting two distant regions of a universe or two different universes. In 1957 Wheeler and Misner [1, 2] coined the term wormhole. A number of interesting features of the traversable Lorentzian wormholes led to a spurt in activities in theoretical studies [3, 4]. The Lorentzian wormholes are handles in the spacetime topologies linking widely separated regions of the universe. It was demonstrated that weak energy condition is violated at the throat of the wormholes. Visser [6–9] elegantly constructed traversable wormhole based on surgical modified solution of the Einstein’s field equations. The observed features of the Lorentzian wormhole solution is acceptable for constructing a hypothetical time machines [5, 10]. In the literature [11–13], it is found that astrophysical accretion of ordinary matter could convert wormholes into black holes. Hayward [14] shown that black holes and wormholes are interconvertible structures and stationary wormholes may be the final stage of an evaporating black hole. The stability of the wormhole solutions are analysed in detail in the presence of different types of matter. The wormholes may have played an important role in the early phase of the universe. Considering the end-state of stellar-mass binary black hole mergers in GW150914, a two parameter family of wormholes in a scalar tensor theory, it is shown [15] that one may detect the signatures present in the emitted gravitational waves as they settle down in the post-merger phase from an initially perturbed state.

It is known that wormhole solution in GTR exists when the energy conditions of the matter is violated. The energy-momentum tensor of the matter sector of the Einstein gravity supporting such geometries violates the null energy condition (NEC) at least near the throat region of the wormhole [16–18] for its existence. Such violation of energy condition indicates
the presence of exotic matter. The recent predictions of the astronomical observations is very interesting in the subject as it stirring an elegant discovery that the universe is experiencing a phase of accelerated expansion. This is a great challenge in theoretical physics to explore the fields/matter which can not be understood in the framework of standard model of particle physics. It is known that in describing both the late accelerating phase of the universe and the wormholes one requires a situation allowing the null energy conditions (NECs) to violate. Thus there is an amazing and unexpected overlapping that occurs between these two different topics. The matter sector in this case is considered to be composed of matter different from normal matter. In cosmology to understand the late acceleration of the universe matter sector is modified with dark energy, phantom, Chaplygin gas [19–23]. In the literature, the phantom field which is an exotic matter considered to realize wormhole geometry [24–26]. The matter with property $-p_{\text{eff}}|r=0| > \rho_{\text{eff}}|r=0| > 0$ (where $r(0)$ is the throat radius of the wormhole) is called exotic matter. The exotic matter at the throat of the wormhole signifies that an observer who moves through the throat with a radial velocity approaching the speed of light will observe presence of negative energy density leading to the violation of the null, weak, strong energy conditions. However, it will be interesting if one can construct an exact traversable wormhole without exotic matter.

Euclidean wormholes are important in understanding origin of the universe without singularity which contributes to the Euclidean path integral formulation of quantum gravity and quantum cosmology. A consistent theory of quantum gravity is not yet known. However, the superstring theory is considered to be a promising candidate for quantum gravity which may unify gravity with the other fundamental forces in nature which requires 10 dimensions for its consistent formulation. The idea that spacetime dimensions should be extended from four to higher dimensions originated from the work of Kaluza [27] and Klein [28] (KK) who first tried to unify gravity with electromagnetism. But the old KK-approach does not work well. The higher dimensional theories revived once again in recent decades due to the advent of string theory which led a paradigm shift in higher-dimensional cosmology. It is therefore important to generalize the known results in four dimensions in the framework of Einstein’s GTR in higher dimensions. Lorentzian wormhole solution obtained in a four dimensions with $\mathcal{R} = 0$ is studied in four dimensions [29]. It is shown [30–37] that the NEC, or more precisely the averaged null energy condition can be avoided in certain regions of dynamical wormholes. In this case wormholes in a time-dependent inflationary background [38, 39] are employed to enlarge an initially small and possibly submicroscopic wormhole encompassing an early inflationary universe scenario. Finally, it will continue to be enlarged by the subsequent Friedmann-Robertson-Walker (FRW) phase of expansion. One could perform a similar analysis as used in Ref. (38), replacing the de Sitter scale factor by a FRW scale factor (34–36). In particular, in Ref. (34–35) specific examples for evolving wormholes that exist only for a finite time are considered, and analyzed for a special class of scale factors that permits violation of the weak energy condition (WEC) for wormholes. Higher-dimensional evolving wormholes satisfying the null energy condition was studied by Zangeneh et al. [40], considering a particular class of wormhole solutions corresponding to the choice of a spatially homogeneous Ricci scalar. They explored wormholes with normal and exotic matter and obtained a number of wormhole solutions including those in four dimensions that satisfy the null energy condition. In five dimensions it is found that the solutions satisfy the null energy condition throughout either power law or de Sitter evolution.

Brane world [42, 43], Brans-Dicke theory [44] and in the context of a non-linear sigma model [45]. EU scenario accommodates a late time de Sitter expansion which naturally permits late time acceleration of the universe. The EU scenario is promising from the perspective of offering unification of early as well as late time dynamics of the universe. It may be noted that unification in such emergent universe model lies in the choice of equation of state for the polytropic fluid. A number of issues pertaining to different models of EU have been discussed in the literature [46, 47]. The EU model with a polytropic non-linear equation of state (EoS) [41] gives rise to a flat universe with a composition of three different types of cosmic fluids determined by the EoS parameters $A$ and $B$. In the model one problem was that the fluid contents of the universe is of definite type once EoS parameter $A$ is fixed. The three fluids are identified with exotic matter, dark energy and a barotropic fluid. Later using interactive fluid models it is shown [51, 52] that a viable EU scenario can be realized. The basic need of an emergent universe is the existence of a Einstein’s static Universe phase in the infinitely past time which however emerge out to an expanding universe. But it is not known how a static Einstein universe exists in the infinitely past time. Therefore, it is of interest to explore the origin of an initial static Einstein phase of the universe which is essential to realize EU scenario. The wormhole physics will be employed here. The present paper investigates the possibility and naturalness of expanding wormholes in higher dimensions, which is an important ingredient of the modern theories of fundamental physics, such as string theory, supergravity, Kaluza-Klein, and others. The higher-dimensional spacetimes is an important ingredient of modern theories of fundamental physics. In this context, the existence of higher dimensions may help to construct wormhole solutions that permits EU model. The motivations for the paper is to obtain dynamical wormhole that permits an expanding cosmological model which accommodates emergent universe scenario [41–47] in higher dimensions.

The paper is organised as follows: In sec.II, the
Einstein field equation in higher dimensions is given. In sec. III, we have considered the static spherically symmetric metric to describe the wormhole geometry and the necessary conditions which are to be satisfied making use of shape function obtained from a homogeneous Ricci scalar \((R)\). The field equations are determined for the wormhole metric. In sec. IV, we have considered two possible wormhole shape functions with their features, and in sec. V, the physical analysis has been carried out. The validity of NEC and WEC are studied by plotting graph. In sec. VI, the constraints on the tension and mass density at the throat in Higher dimensions is given. The results are summarised in sec. VI— followed by a brief discussion.

II. THE GRAVITATIONAL ACTION AND SOLUTIONS IN HIGHER DIMENSIONS

The gravitational action in D-dimensions is given by

\[ I = \int d^Dx \sqrt{-g} \left( \frac{1}{2} R + L_m \right) \]  

(1)

where \(R\) is the Ricci scalar curvature and \(L_m\) is the matter Lagrangian, we consider \(c = 8\pi G_D = 1\). Varying the action with respect to the metric, we obtain \(D\)-dimensional Einstein field equation

\[ R_{AB} - \frac{1}{2} g_{AB} R = T_{AB} \]  

(2)

where \(A, B = 0, 1, ..., D - 1\) and \(T_{AB}\) is the matter stress-energy tensor. For an expanding wormhole solutions we use the metric

\[ ds^2 = -dr^2 - S(t)^2 \left[ \frac{dr^2}{1 - \alpha(r)} + r^2 d\Omega_{D-2}^2 \right] \]  

(3)

where \(d\Omega_{D-2}^2 = d\theta_2^2 + \sin^2 \theta_2 d\theta_3^2 + \sin^2 \theta_2 \sin^2 \theta_4 d\theta_4^2 + ... + \sin^2 \theta_2 ... d\theta_{D-2}^2, \) \(S(t)\) represents scale factor, \(\alpha(r)\) is an unknown dimensionless function, defined as \(\alpha(r) = \frac{b(r)}{r}\) where \(b(r)\) denotes the shape function [4, 5]. The metric is a generalization of Friedmann-Robertson metric when \(\alpha(r) = 0\). It may be noted that when the dimensionless shape function \(\alpha(r) \to 0\), the metric reduces to a flat Friedmann-Robertson metric. It approaches to the static wormhole metric as \(S(t) \to constant\). In this case the wormhole form of the metric is preserved with time. Let us consider an embedding of \(t = constant\) and \(\theta_{D-2} = \frac{\pi}{2}\) slices of the space given by eq. (3), in a flat three dimensional Euclidean space with metric becomes

\[ ds^2 = d\bar{z}^2 + d\bar{r}^2 + \bar{r}^2 d\phi^2. \]  

(4)

The metric of the wormhole slice is given by

\[ ds^2 = S(t)^2 \left[ \frac{dr^2}{1 - \alpha(r)} + r^2 d\phi^2 \right]. \]  

(5)

Comparing eq. (4) with eq. (5) one can write

\[ \bar{r} = S(t)r\big|_{t=\text{const.}}, \]  

\[ dr^2 = S^2(t)dr^2\big|_{t=\text{const.}}, \]  

(6)

they may be regarded as rescaling of \(r\)-coordinate on each \(t = constant\) slice [48].

Now one obtains a metric using in eq. (4) which is given by

\[ ds^2 = \frac{d\bar{r}^2}{1 - \alpha(\bar{r})} + \bar{r}^2 d\phi^2, \]  

(7)

where \(\alpha(\bar{r}) = 1\), i.e., \(b(\bar{r})\) attains a minimum at \(\bar{b}(\bar{r}_0) = \bar{r}_0\). Using eq. (6) and eq. (7) one gets

\[ \bar{\alpha}(\bar{r}) = S(t)\alpha(r). \]  

(8)

In the case of a evolving wormhole having same size and shape relative to the \((\bar{z}, \bar{r}, \bar{\phi})\) coordinate system will have that in the initial \((z, r, \phi)\) embedding spacetime considered here. Thus a

\[ \alpha(\bar{r}_0) = 1, \quad \alpha(r) < 1, \quad \alpha'(r) < 0 \]  

(13)

where \(r_0\) represents the wormhole throat.

Using the metric (2) and the energy momentum tensor

\[ T_{\mu \nu} = diag(-\rho, P_r, P_t, P_{\phi}, ..., ...) \]  

Einstein field equations become

\[ \rho(r, t) = \frac{(D - 2)(D - 3)}{2S^2(t)r^2} \alpha(r) + \frac{(D - 2)}{2S^2(t)r} \alpha'(r) \]  

Using eq. (6) and (7) we get

\[ \frac{d^2 \bar{r}(z)}{dz^2} = \frac{1}{S(t)} \left( \frac{\alpha'}{2\alpha^2} \right) = \frac{1}{S(t)} \frac{d^2 \bar{r}(z)}{dz^2} > 0. \]  

(12)
corresponding to the homogeneous Ricci Scalar, \( C \) and for minus sign in eq. (12) where over dot represents derivative with respect to time.

It is evident that although \( k \) is a continuous variable, it leads to the spacetime which is asymptotically FRW and applied the normalization that \( k = 0, -1 \) for Case (I) and \( k = 1 \) for Case (II). In the absence of a cosmological constant the Ricci scalar is obtained if the scale factor is independent of time accommodating the solutions given in Case (I) and Case (II). The dimensionless shape function \( \alpha(r) \) should satisfy the condition mentioned in eq. (13), it is found that the solutions obtained in Case (I) represents flat or open universe for \( k = 0, -1 \) respectively but not the closed universe but in the Case (II) a closed universe \((k = 1)\) is permitted. Thus a new class of solutions in the later case is obtained for a closed universe accommodating wormholes. In this case two closed universes are connected by a wormhole.

A flat universe can be represented by eq. (14)- eq. (16) can be rewritten as

\[
\rho = \rho_{fb}
\]

\[
P_r = - \frac{(D - 2)(D - 3)r_0^{D-3}}{2r^{D-2}S^2} + P_{fb}
\]

\[
P_t = \frac{(D - 3)r_0^{D-3}}{2r^{D-2}S^2} + P_{fb}
\]

\[
P_{fb} = - \frac{(D - 2)S}{S} - \frac{(D - 2)(D - 3)\dot{S}^2}{2S^2}
\]

respectively. It corresponds to a universe with isotropic cosmic fluid.

In the open universe background \((k = -1)\), we get

\[
\rho = \rho_{ob},
\]

\[
P_r = - \frac{(D - 2)(D - 3)(r_0^{D-1} + r_0^{D-3})}{2r^{D-1}S^2} + P_{ob}
\]

\[
P_t = \frac{(D - 3)(r_0^{D-1} + r_0^{D-3})}{2r^{D-1}S^2} + P_{ob}
\]

where \( P_{ob} \) and \( P_{fb} \) corresponds to the open background which are

\[
\rho_{ob} = \rho_{fb} - \frac{(D - 1)(D - 2)}{2S^2},
\]

\[
P_{ob} = P_{fb} + \frac{(D - 2)(D - 3)}{2S^2}
\]

A closed universe background \( k = 1 \) solutions obtained here corresponding to minus sign in eq. (19) is a new set of solutions with wormholes. Thus in this case we can represent following:

\[
\rho = \rho_{ob},
\]
\[ P_r = -\frac{(D - 2)(D - 3)(r_0^{D-1} + r_0^{D-3})}{2r^{D-1}S^2} + P_{cb} \]  
(33)

\[ P_t = \frac{(D - 3)(r_0^{D-1} + r_0^{D-3})}{2r^{D-1}S^2} + P_{cb} \]  
(34)

where \( \rho_{cb} \) and \( P_{cb} \) corresponds to the closed universe background which are

\[ \rho_{cb} = \rho_{fb} - \frac{(D - 1)(D - 2)}{2S^2}, \]  
(35)

\[ P_{cb} = P_{fb} + \frac{(D - 2)(D - 3)}{2S^2}. \]  
(36)

It is evident that wormholes are obtained with anisotropic fluid. In the case of flat universe at a large distance away from the throat it admits isotropic pressure. We get similar picture if the universe is non-flat asymptotically.

### III. WORMHOLES SOLUTIONS

In this section we consider the parameter: \( \alpha(r) = \frac{b(r)}{r} \), where \( b(r) \) is typical shape function which satisfy the following constraints for accommodating wormholes:

- The range of radial coordinate \( r \) is \( r_0 \leq r \leq \infty \), with \( r_0 \) being the throat radius.

- The shape function \( (b(r)) \) satisfies the condition \( b(r_0) = r_0 \), at the throat and away from the throat \( i.e. \) for \( r > r_0 \) it must satisfy the constraint condition

\[ 1 - \frac{b(r)}{r} > 0. \]  
(37)

- For a physical flaring out condition, it must be satisfied by the shape function \( b(r) \) which at the throat of a wormhole solution becomes \( i.e. \) \( b'(r_0) < 0 \).

- For an asymptotic flatness of the spacetime geometry one obtains

\[ \frac{b(r)}{r} \rightarrow 0 \quad as \quad |r| \rightarrow \infty \]  
(38)

It is known that the matter that supports the static wormhole geometry in Einstein’s GTR violates the Null Energy condition (NEC): \( T_{AB}U^A U^B \geq 0 \) where \( U^A \) is a null vector.

1. NEC can be expressed in terms of energy density and pressure as

\[ \rho + P_r \geq 0, \quad \rho + P_t \geq 0. \]  
(39)

The other energy conditions [49] are

2. DEC: \( \rho \geq |P_t| \),

3. WEC : \( \rho \geq 0; \quad \rho + P_t > 0; \)

4. SEC: \( \rho + P_r + 2P_t \geq 0; \)

where \( i = r, t \).

#### A. Flat Background

We consider a non-linear equation of state for matter as follows :

\[ P_r = A\rho - \frac{B}{\rho^\epsilon} \]  
(40)

where \( \epsilon > 0 \) corresponds to modified Chaplygin gas [32]. In a flat background an Emergent Universe model is obtained with a non-linear equation of state [41] which corresponds to \( \epsilon < 0 \). Recently a number of theoretical aspects of EU model with such non-linear part of the EoS is reported, the non-linear term represents the presence of initial viscosity in the universe [50]. In the literature [33], EU model is implemented in a number of theories of gravity and found to work satisfactorily to encompass the observed universe. In the case of a flat background EU scenario in a flat universe is obtained with a non-linear equation of state EoS corresponding to \( \epsilon = -\frac{1}{2} \) which is given by given by

\[ P_r = A\rho - B\sqrt{\rho} \]  
(41)

where \( A \) and \( B \) are arbitrary parameters we use them to determine the composition of matter in the universe which is interesting. Now using the Einstein field eqs. (25) and (26), we get the second order differential equation given by

\[ \frac{\dot{S}}{S} + \delta \frac{\dot{S}^2}{S^2} - \beta \frac{\dot{S}}{S} = 0 \]  
(42)

where we denote \( \delta = \frac{\Delta (D - 1)}{2} \) and \( \beta = \sqrt{\frac{(D - 1)}{(2D - 2)}} \) \( B \) for simplicity. The eq. (42) can be integrated once to obtain

\[ \dot{S} S^\delta = \kappa e^{\beta t} \]  
(43)

where \( \kappa \) is an integration constant. Integrating the above equation once we obtain the solution

\[ S(t) = \left( (1 + \delta) \kappa_0 + \frac{\kappa (1 + \delta)}{\beta} e^{\beta t} \right)^{\frac{1}{\delta}}. \]  
(44)

where \( \kappa_0 \) is constant of integration. We note the following : case (i) If \( B < 0 \), the solution has a singularity and it is not interesting.

case (ii) If \( B > 0 \), the solution describes an emergent universe if \( \delta > -1 \) with positive \( \kappa \) and \( \kappa_0 \).
The solutions given in case (ii) is interesting, it leads to emergent universe scenario with asymptotic closed background. In the emergent universe there is no initial singularity and the universe evolved out from a Einstein static phase in the infinite past i.e. \( t \to -\infty \). It is evident that as \( A \) increases the initial size \( S_0 \) will attain maximum at lower dimension. Thus the value of \( A \) plays here an important role for determining the maximum size of the initial universe in 4-dimensions. Thus one can predict the fluids inside a Higher dimensional universe. In the EU scenario the initial Einstein static phase is an assumption ([54]-[41]). The origin of such phase is not studied. The wormhole physics is used to demonstrate the existence of such a phase of the universe at the throat of the wormhole. In the next section we consider case (ii) with the shape function \( b(r) = r \left( \frac{r_0}{r} \right)^{D-3} \) obtained from eq. (20). It is a positive constant \( b = 2M \) (where \( M \) represents mass) at \( D = 4 \) but \( b(r) \) is a decreasing function of \( r \) for \( D > 4 \). Using eqs. (22) - (24) we represent the following:

\[
\rho + P_r = \rho_{fb} + P_{fb} - \frac{(D-2)(D-3)}{2r^{D-1}S^2} r_0^{D-3}, \quad (45)
\]

\[
\rho + P_t = \rho_{fb} + P_{fb} + \frac{(D-3)}{2r^{D-1}S^2} r_0^{D-1}, \quad (46)
\]

where

\[
\rho_{fb} + P_{fb} = (D - 2) \left( \frac{S_0^2}{S^2} - \frac{\dot{S}}{S} \right). \quad (47)
\]

In the case of a flat emergent universe [41] and the solution obtained here corresponding to eq. (47) we get

\[
\rho_{fb} + P_{fb} \neq 0. \quad (48)
\]

In the case of a power law and de Sitter solutions, one obtains a vanishing value [48], thus it is different in the EU model. Thus new result in emergent universe scenario leads to interesting result which we discuss in sec. It is evident that the Einstein static universe corresponds to \( \dot{S} = 0 \) which leads to \( S_0 = ((1 + \delta)\kappa_0) \frac{\ln \beta}{\ln \beta + \ln (\kappa_0)} \) at infinitely past \( (t \to -\infty) \) corresponding to the wormhole throat for \( D > 2 \). It is found that the size of the initial Einstein static universe varies with dimensions of the universe and \( A \). For a given \( A \) there is a maximum \( S_0 \) which varies for different space-time dimensions. The maximum value however is independent of \( A \). The radial variation of NEC \( \rho + P_r < 0 \) is plotted in Fig. (2), it is violated in the past near the throat but away from the throat NEC is obeyed.

It is also evident that NEC is obeyed at an early time for \( D = 4 \) compared to other dimensions more than four with a given value of \( A \) and \( B \). The time evolution of NEC is plotted in Fig. 3, it is found that violation of NEC disappears earlier for a given \( A \) when \( B \) is increased in \( D = 10 \) dimensions. However, in Fig. 4, it is found that for a given \( B \) when \( A \) is increased the NEC begins to obey at an earlier epoch. A 3D plot of NEC is drawn in Fig. 5 showing the variation with respect to \( t \) and \( r \) at \( D = 4 \) dimensions. The emergent universe at late time asymptotically attains an exponential phase which is

\[
S(t) \sim \left( \frac{\kappa (1 + \delta)}{\beta} \right)^{\frac{1}{\beta}} e^{\left( \frac{\beta}{\kappa (1 + \delta)} \right) t}, \quad (49)
\]

in this case the expansion depends on \( B \) parameter of the EoS only. Thus at late universe we get the following

\[
\rho_{fb} + P_{fb} = 0 \quad (50)
\]
as given by eq. (47) and from eqs. (45) and (46) we get
\[
\rho + P_r = -\frac{(D-2)(D-3)}{2r^{D-1}S^2} r_0^{D-3}, \tag{51}
\]
\[
\rho + P_t = \frac{(D-3)}{2r^{D-1}S^2} r_0^{D-1}. \tag{52}
\]

leading to \( \rho + P_r < 0 \). Consequently although NEC is obeyed as the universe evolves out from wormhole throat, once again at the present epoch NEC and SEC do not obey which accommodates the late time acceleration in the universe.

IV. HIGHER DIMENSIONAL FLAT EMERGENT UNIVERSE

The conservation equation in a higher dimensional universe is given by
\[
\frac{d\rho}{dt} + (D-1)(\rho + P) \frac{\dot{S}}{S} = 0. \tag{53}
\]
Integrating above eq. (53) using the non-linear equation of state: \( P = A\rho - B\sqrt{\rho} \), we get
\[
\rho(S) = \left( \frac{1}{1+A} \right)^2 \left( B + \frac{K}{S^{(D-1)(1+A)}} \right)^2 \tag{54}
\]
where \( K \) is an integration constant. The scale factor is a monotonically increasing function of cosmic time, thus the evolution of the universe can be studied making use of the energy density. Here the energy density depends on the equation of state parameters \( A, B, D \) and an arbitrary integration constant \( K \). It is important to note here that the universe is composed of three different types of fluids. Thus the energy density given by eq. (54) can be expanded and we get
\[
\rho = \left( \frac{B}{1+A} \right)^2 + \frac{2BK}{(1+A)^2} \frac{1}{S^{(D-1)(1+A)}}
\]
TABLE I: Composition of cosmic matter for a choice of $A$ in $D$ dimensions. The value of $\omega_2$ defines the exotic matter (EM) with density $\rho_2$. Dark matter (DM) in addition to barotropic fluid relevant in each case.

| $A$ | $\omega_1$ in $B$ | $\omega_2$ in $(\frac{B}{D-1})^2$ | $\omega_3$ | Composition of fluids |
|-----|-----------------|---------------------------------|----------|----------------------|
| $\frac{1}{3}$ | $\frac{5}{8}S^{-(D-1)}$ | $\frac{2}{3}S^{-(D-1)}$ | $\frac{1}{4}$ | DE, EM, Radiation |
| $\frac{1}{3}$ | $\frac{5}{8}S^{-(D-1)}$ | $\frac{2}{3}S^{-(D-1)}$ | $\frac{1}{4}$ | DE, EM, Cosmic strings |
| $\frac{1}{3}$ | $\frac{5}{8}S^{-(D-1)}$ | $\frac{2}{3}S^{-(D-1)}$ | $\frac{1}{4}$ | DM, EM, Stiff matter |
| $0$ | $\frac{1}{3}S^{-(D-1)}$ | $\frac{1}{2}S^{-(D-1)}$ | $0$ | DE, EM, Dust |

V. INTERACTING COSMIC FLUIDS

We consider an emergent universe with non-linear EoS given by eq. (40), where an interaction among the fluid components. In the conservation eq. (53), assuming an onset of interaction among the cosmic fluids at $t = t_o$, the conservation equations for the energy densities of the fluids now can be written as

$$\frac{d\rho_1}{dt} + (D-1)H(\rho_1 + P_1) = -Q',$$  

$$\frac{d\rho_2}{dt} + (D-1)H(\rho_2 + P_2) = Q',$$  

$$\frac{d\rho_3}{dt} + (D-1)H(\rho_3 + P_3) = Q - Q',$$  

where $H = \frac{\dot{a}}{a}$, $Q$ and $Q'$ represent the interaction terms, which can have arbitrary form, $\rho_1$ represents the dark energy density, $\rho_2$ represents exotic matter, and $\rho_3$ represents normal matter. Here $Q < 0$ corresponds to energy transfer from the exotic matter sector to two other constituents, $Q' > 0$ corresponds to energy transfer from the dark energy sector to the other two fluids, and $Q' < Q$ corresponds to energy loss for the normal matter sector. In the above the limiting case $Q = Q'$ represents that dark energy interacts only with the exotic matter. It is evident that although the above three equations are different, the total energy of the fluid satisfies the conservation equation together which is given by eq. (40). It is possible to construct equivalent effective uncoupled equations described by the following conservation equations:

$$\frac{d\rho_1}{dt} + (D-1)H\left(1 + \omega_1^{eff}\right)\rho_1,$$  

$$\frac{d\rho_2}{dt} + (D-1)H\left(1 + \omega_2^{eff}\right)\rho_2,$$  

$$\frac{d\rho_3}{dt} + (D-1)H\left(1 + \omega_3^{eff}\right)\rho_3,$$

where the effective EoS parameters are given below:

$$\omega_1^{eff} = \omega_1 + \frac{Q'}{(D-1)H\rho_1},$$  

$$\omega_2^{eff} = \omega_2 - \frac{Q}{(D-1)H\rho_2},$$  

$$\omega_3^{eff} = \omega_3 + \frac{Q - Q'}{(D-1)H\rho_3}.$$
Fig. (7) a 3D plot of radial pressure is plotted with radial distance and time and the variation of NEC is plotted in Fig. (8) in $D = 4$ dimensions, it is found that although the radial pressure is negative near the throat of the wormhole the NEC is obeyed. It is found that for asymptotic closed universe, exotic matter is not required.

In Case (II) which admits asymptotic open universe, i.e., $k = 1$ we study the NEC condition to obtain dynamical wormhole which allows an emergent universe scenario as discussed above. The shape function obtained from eq. (21) is given by

$$b(r) = \frac{r^D_0 - 3 + r^D_0 - 1}{r^D - 2} - r^3.$$  \hspace{1cm} (70)

It has the same structure as obtained in closed universe which can be analyzed and in this case also one obtains wormholes without exotic matter.

VI. CONSTRAINTS ON THE TENSION AND MASS DENSITY AT THE THROAT

The constraints on the tension and the mass density can be obtained from the higher dimensional Einstein field equation. At the wormhole throat $b = b_0$ and absence of a horizon, the field equation given by eq. (15) can be used to determine the tension in the throat of the wormhole which is given by

$$\tau = -P_t(r,t) = \frac{(D-2)(D-3)}{16\pi G_D C_D^4 b_0^2 S^2_0(t)}.$$  \hspace{1cm} (72)
where the gravitational coupling constant $G_D = \frac{1}{M_P^D}$. It is evident that the tension depends on the spacetime dimensions and the size of the universe in the Einstein static phase. It may be important to mention here that in the emergent universe the initial size of the universe may be bigger than that one gets in the Planck regime of the FRW-universe. Following the Brane world scenario and higher dimensions [57, 58], it is found that the $M_P G_{4}^{D} \sim 1.22 \times 10^{16}$ TeV, the tension at the throat of the wormhole from which emergent universe that emerged is given by

$$\tau = \frac{(D-2)(D-3)}{16\pi G b_o^2 S_o^2} c^4 \left(10 m/ b_o S_o \right)^2.$$  \hfill (73)

where $S_o = \left(\frac{(D-1)}{2} A k_o \right)^{\frac{D-2}{D-1}-\alpha}$. Thus the equation of state parameter $A$, the arbitrary integration constant $k_o$ and dimensions of the universe $D$ are playing an important role. It is evident that for a large size or the scale factor of the universe ($S_o$), the tension is very small. In four dimensions one gets tension at the throat which is given by

$$\tau = \frac{(D-2)(D-3)}{16\pi G c^{-4} b_o^2 S_o^2} \sim 5 \times 10^{44} \text{dyne/cm}^2 \left(10 m/ b_o S_o \right)^2.$$  \hfill (74)

The tension depends on the initial size of the universe in the case of an Emergent Universe. In emergent universe scenario the size of a Einstein static universe $S_o$ is very big and $D$-dimensional Newtonian constant $(G)$ is large which reduces the tensions considerably, thus it is an important property of a Higher dimensional universe to play an important role for understanding migration from one universe to the other dual space in the case $\frac{\partial \tau}{\partial r} = 0$. For $D = 3$, the effective tension at the throat vanishes, thus it gives rise to an effective phenomena where the tidal force is absent at the throat. Thus an interstellar migration is possible without any hindrance if the space-time dimension is lower than the usual four dimensions. The mass function is $M = \rho V_D$ where the volume is $V_D = \frac{2\pi^{-2(D-1)}}{((D-1)/2)!} r^{D-1}$. It is also evident that wormhole with two open universes connected at the throat can be obtained where NEC is obeyed always.

### VII. DISCUSSION

The wormhole solutions which permits emergent universe models are obtained in Einstein gravity in higher dimensions. In this framework it is shown that the initial Einstein static universe at infinite past in an emergent universe scenario can be represented by a dynamical wormhole throat. The dynamical evolution of the wormhole encompasses the flat emergent universe with all the observed features. Considering a particular class of wormhole solutions corresponding to a spatially homogeneous Ricci scalar we determine the shape functions for the wormhole in a flat, closed and open universe. The cosmological solutions in higher dimensions accommodating emergent universe is obtained here for homogeneous matter with non-linear EoS. We note that EU model can be obtained for flat, asymptotically closed or open universe in higher dimensions. For $D = 4$, the EU model obtained by us [41] is recovered. It is shown that a dynamical wormhole joins the throat of the wormhole with the present universe. The initial Einstein’s static universe required for EU scenario is the throat of the wormhole. It is shown in Fig (1) that the size of the initial static Einstein universe is determined by the EoS parameter $A$. It is also noted that the maximum size of the Einstein static universe does not depend on the spacetime dimensions and geometry of the universe. In Figs. (3) and (4), the variation of NEC is studied in a flat universe with different values of the parameters $A$ and $B$ in $D$ dimensions. It is found that at the throat NEC is violated but away from the throat it is obeyed. In Fig. (5) it is evident that in a four dimensional universe although NEC violates near the throat but it begins to obey NEC near to the throat compared to a universe with dimensions more than four dimensions. In Figs. (6) and (7), we note that the observed universe is obtained in the case of interacting fluids which is determined by the coupling term $\beta$ of the interaction although initially the composition of the universe is determined by $A$. Another interesting solution is that for a closed or open universe dynamical wormholes are permitted with matter that respect the NEC in four and in higher dimensions. It is a new result as earlier cosmological models different from EU models are obtained in 5-dimensions that respect the energy conditions [40]. A new class of wormhole solutions for closed model of the universe is obtained here for dynamical wormhole that leads to flat emergent universe. A flat universe requires exotic matter to begin with near the throat but away from the throat no exotic matter is required for flat emergent universe scenario. A detail numerical analysis is displayed in Table-1 and Table-2. It is evident from the 3D plots in Figs. (8) and (9), that even if the radial pressure is negative in an asymptotically closed universe, NEC is obeyed implying wormhole solution without exotic matter. The tension at the throat of the wormhole is estimated in eq. (73), it is noted that the tension in a 4-dimensional universe depends on the throat of the wormhole ($S_o$). In EU model the size of the static Einstein universe may be large, thus in that case the tension reduces considerably.

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