Scheduling to Optimize Sojourn Time of Successful Jobs

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Abstract—Deep neural networks training jobs and other iterative computations frequently include checkpoints where jobs can be canceled based on the current value of monitored metrics. While most of existing results focus on the performance of all jobs (both successfully completed and canceled), in this work we explore scheduling policies that improve the sojourn time of successful jobs, which are typically more valuable to the user. In the single-server case where all jobs are available for scheduling simultaneously, we prove that optimal schedules do not preempt jobs, even when preemption overhead is negligible. Based on this, we develop a scheduling policy that minimizes the sojourn time of successful jobs asymptotically. Through an extensive numerical study, we show that this policy performs better than existing alternatives even when the number of jobs is finite. For more realistic scenarios with multiple servers and dynamic jobs arrivals, we propose an online approach based on our single-server scheduling policy. Through an extensive simulation study, using real-world traces, we demonstrate that this online approach results in better average sojourn time for successful jobs as compared to existing techniques.

Index Terms—scheduling, sojourn time optimization, multi-stage jobs, early termination

I. INTRODUCTION

Machine learning jobs are becoming an increasingly important component of data center workloads due to the adoption of deep learning [1], together with larger models and datasets this results in compute-intensive training jobs of several hours or days. Notably, multiple machine learning jobs are often scheduled as part of an exploratory process, where the user (or an AutoML system [2]) trains a machine learning model repeatedly to adjust its structure (e.g., to vary the number of layers and channels in a convolutional neural network), to tune hyperparameters (e.g., learning rate and momentum of stochastic gradient descent), or to experiment with different data augmentation strategies. Many of these jobs are often unsuccessful, especially during the initial phase of the process, when broadly different variants are trained in order to characterize the search space; several metrics, such as training and validation loss or accuracy, and their changes between training iterations, can be used to detect and terminate unsuccessful jobs, freeing resources to process more promising variants. Previous work proposed time-slicing across multiple jobs to terminate unsuccessful jobs (e.g., jobs with predicted final accuracy lower than 30% [3]), or scheduling strategies driven by training metrics (e.g., a resource assignment that maximizes the total loss reduction predicted for all jobs [4]).

Motivated by applications corresponding to jobs with possible termination, in this paper, we consider the general problem of scheduling jobs that can be terminated early at predefined checkpoints. For each job, we assume that a discrete size distribution is known from historical data: the largest size of the distribution represents the full duration of the job, while other size values correspond to checkpoints where unsuccessful jobs are terminated. Our goal is to schedule these jobs on a single server in order to minimize the mean sojourn time of successful jobs.

This goal is not considered by existing results which, instead, focus on the minimization of mean sojourn time of all jobs (successful and unsuccessful); for instance, in our setting, the scheduling policy selecting, at each time, the job with maximum Gittins index [5], [6] (computed from service time distributions) or, equivalently, minimum SR index [7], would minimize the mean sojourn time of all jobs. Notably, this policy requires job preemption to switch between jobs when their Gittins indices are updated. We note that optimizing the mean sojourn time of all jobs (e.g., using the algorithms mentioned above) often results in suboptimal expected sojourn times for successful jobs. For instance, Figure 1 depicts the mean sojourn time of successful jobs (computed numerically) as a function of the number of jobs in a single server system. From this figure we observe that an algorithm that optimizes the sojourn time of all jobs produces a significantly higher mean sojourn time of successful jobs, as compared to an algorithm that optimizes the mean sojourn time of successful jobs. We explore such quantitative comparisons in detail in Section IV. Here, we note that this motivates the need for scheduling algorithms that distinguish between successful and unsuccessful jobs.

The contributions of our work are as follows. In a single server system where jobs arrive at the same time, our analysis shows that sojourn time of successful jobs can be reduced by favoring promising jobs (Section II). We prove that, in contrast with known results for the sojourn time of all jobs, optimal policies that minimize sojourn time of successful jobs do not use job preemption; based on this result, we define a non-preemptive scheduling policy (with linear complexity)
and prove its optimality when the number of jobs grows to infinity (Section III). Through an extensive numerical study, we compare our policy with alternative approaches to illustrate the advantageous performance of our approach even for a finite number of jobs (Section IV).

Finally, we consider more realistic scenarios with multiple servers and jobs arriving dynamically over time. In this setting, we propose an online approach based on our asymptotically optimal scheduling policy and show, through extensive simulations using real-world traces, achieved performance improvements (Sections V and VI).

II. PROBLEM DEFINITION

We consider a queue where $N$ jobs arrive over time. The exact size of each job $i = 1, \ldots, N$ is unknown, but it is modeled by a discrete random variable taking one of $M_i$ values $0 < x_{i,1} < x_{i,2} < \cdots < x_{i,M_i}$; we denote by $p_{i,j} \in (0,1)$ the probability that job $i$ terminates when service time reaches size $x_{i,j}$, with $\sum_{j=1}^{M_i} p_{i,j} = 1$ for all $i = 1, \ldots, N$. Job $i$’s arrival time is denoted by $a_i$.

In our applications of interest, the largest possible size $x_{i,M_i}$ corresponds to the successful completion of job $i$, while other sizes $x_{i,j}$ for $1 < j < M_i$ represent the early termination of job $i$ after its $j$-th checkpoint. Note that checkpoints are set at deterministic fractions of a job.

The system contains $W$ homogeneous servers. We assume that jobs can be scheduled using a preemptive, non-anticipating policy which assigns at most one server to each job: when service reaches a checkpoint, the job terminates with probability given by its size distribution. For example, if job $i$ is initially selected, its first stage with size $x_{i,1}$ is completed: with probability $p_{i,1}$, the job terminates after this stage; with probability $1 - p_{i,1}$, the job continues. If job $i$ continues, its remaining size values are $x_{i,2} < \cdots < x_{i,M_i}$, with probabilities $p_{i,j}/(1 - p_{i,1})$ for $j = 2, \ldots, M_i$; when it is selected to run again by the scheduler, the second stage is completed in $x_{i,2}$ time units. This process continues until all jobs terminate (at some checkpoint) or complete their last stage successfully.

Let $C \subseteq \{1, \ldots, N\}$ be the set of successful jobs and $t_i$ be the time of job $i$’s completion. Then, the goal of our scheduler is to minimize

$$E \left[ \frac{\sum_{i \in C} t_i - a_i}{|C|} \right]$$

i.e., the expected sojourn time of successful jobs.

III. SPECIAL CASE: SINGLE SERVER WITHOUT ARRIVALS

In this section we focus on a special but important case of the problem defined in the previous section, where there is only one server and all jobs arrive at the same time (i.e., $W = 1$ and $a_i = 0$ for all $i = 1, \ldots, N$). Below we present our theoretical results: first, we show that existing algorithms do not provide optimal solutions for this problem; then we characterize schedules minimizing response time of successful jobs; lastly, we develop a scheduling algorithm that is optimal when the number of jobs is large.

A. Ineffectiveness of Existing Approaches

As mentioned in Section II when there is only one server and all jobs arrive at the same time, the expected sojourn time of all jobs is minimized when the next job is selected by ranking jobs according to their Gittins index [5] or, equivalently, their SR rank [7]. Specifically, the SR rank policy selects job $i$ with minimum rank

$$r(i) = \min_{1 \leq j \leq M_i} \left( \sum_{k=1}^{j} x_{i,k} p_{i,k} \right) + x_{i,j} \left( 1 - \sum_{k=1}^{j} p_{i,k} \right)$$

When job preemption is not allowed at checkpoints, the mean sojourn time of all jobs is minimized by selecting the job with the shortest expected remaining processing time (SERPT), which is given by

$$E_{\text{OPTIMAL}} = 0.75 \cdot 0.4 \cdot (10 + 16)/2 + 0.25 \cdot 0.4 \cdot 7 + 0.75 \cdot 0.6 \cdot 10 = 9.1.$$
B. Preemption of Jobs in an Optimal Schedule

We begin by characterizing an optimal schedule, which allows us to narrow the search for an optimal algorithm.

Theorem III.1. In a schedule minimizing the expected sojourn time of successful jobs, jobs are not preempted: each job runs until either all of its stages complete, or it terminates early.

Proof. Assume instead that there is a schedule minimizing the expected sojourn time of successful jobs, where job \( i \) is the last job being preempted; let \( j \) be the last job preempting \( i \), and let \( s_j \) and \( s_i \) indicate the last interleaving stages of \( j \) and \( i \), respectively, as illustrated in Fig. 2 (each job in the figure may terminate after any of its stages). Then, the modified schedule where all stages of \( i \) before \( s_i \) are moved between \( s_j \) and \( s_i \) (so that \( i \) is not preempted, as illustrated in the bottom half of the figure) has strictly lower expected sojourn time of successful jobs: (1) if \( j \) terminates early after any of its stages, its sojourn time is increased, but it does not contribute to the mean sojourn time of successful jobs; (2) if \( i \) completes successfully, its sojourn time does not change in the modified schedule; (3) sojourn time does not change for other jobs completing before the first stage or after the last stage of \( i \) in the original schedule; (4) for other jobs completing or terminating before the first and last stage of \( i \) in the original schedule, sojourn time is reduced. Since, by hypothesis, \( i \) is the last job being preempted, at least one other job \( k \) has nonzero probability of terminating successfully between the first and last stage of \( i \) in the original schedule; the expected sojourn time of successful jobs is then strictly lower in the modified schedule, contradicting the hypothesis of an optimal schedule where jobs are preempted.

Theorem III.1 significantly reduces scheduling complexity, since it allows us to schedule entire jobs until completion or termination, instead of individual job stages. As a result, only \( N! \) schedules need to be considered, instead of \( \left( \sum_{M_1,M_2,...,M_N} \right) \).

C. Order of Adjacent Jobs in an Optimal Schedule

Let \( S = (J_1, J_2, ..., J_N) \) be a permutation of \( 1, 2, ..., N \), which we use to identify a schedule. We use \( E[S] \) to denote the expected sojourn time of successful jobs \( E[\sum_{i \in C} t_i / |C|] \)

\( ^1 \)Note that, this would not be true if we were optimizing the sojourn time of all jobs rather than successful jobs.

when jobs are scheduled according to \( S \). Our objective is to find the optimal schedule \( S^* \) that satisfies:

\[ S^* = \arg \min_S E[S] \]

where \( G_N \) is the set of all permutations of \( 1, 2, ..., N \). First, we establish a property of adjacent jobs in an optimal schedule.

Theorem III.2. Consider two adjacent jobs \( i \) and \( j \) (where \( i \) comes before \( j \)) of an optimal schedule \( S^* \) and let

\[ R_{i,j}^N(d) = \frac{\sum_{k=1}^{M_d-1} p_{d,k} x_{d,k}}{p_{d,M_d}} + \alpha_{i,j}(N) \cdot x_{d,M_d} \]

and

\[ \alpha_{i,j}(N) = \frac{\sum_{l=2}^{N} Q_{i,j}(l-1)}{\sum_{l=1}^{N-1} Q_{i,j}(l-1)} \]

where \( Q_{i,j}(l) \) is the probability of exactly \( l \) remaining \( N - 2 \) jobs succeeding. Then, \( R_{i,j}^N(i) < R_{i,j}^N(j) \).

Proof. Without loss of generality, we assume that \( i = 1, j = 2 \), and \( S^* = (1, 2, ..., N) \), i.e., \( i \) and \( j \) are the first and second job of the optimal schedule \( S^* \). We invert jobs 1 and 2 in \( S^* \) to obtain \( S^* = (2, 1, ...) \) and we evaluate \( E[S^*] - E[S^*] \) to compare the two schedules.

Let \( t_1, t_2, ..., t_N \) denote the execution times of jobs 1 through \( N \), i.e., \( t_k \in \{x_{k,1}, x_{k,2}, ..., x_{k,N}\}, k = 1, 2, ..., N \). Then, we have:

\[ E[S^*] = E[S^*|t_1 = x_{1,1}, t_2 = x_{2,2} \cdot p_{1,1}\cdot p_{2,2} \cdot \cdot \cdot + \]

\[ E[S^*|t_1 = x_{1,1}, t_2 < x_{2,2} \cdot p_{1,1}\cdot (1 - p_{2,2}) + \]

\[ E[S^*|t_1 < x_{1,1}, t_2 = x_{2,2} \cdot (1 - p_{1,1})\cdot p_{2,2} \cdot + \]

\[ E[S^*|t_1 < x_{1,1}, t_2 < x_{2,2} \cdot (1 - p_{1,1})\cdot (1 - p_{2,2}) \]

The four terms on the right-hand side of Eq. (5) correspond to the four cases where: (1) both jobs 1 and 2 succeed, (2) job 1 succeeds but job 2 terminates early, (3) job 1 terminates early while job 2 succeeds, and (4) both jobs terminate early. Similarly:

\[ E[S^*] = E[S^*|t_1 = x_{1,1}, t_2 = x_{2,2} \cdot p_{1,1}\cdot p_{2,2} \cdot + \]

\[ E[S^*|t_1 < x_{1,1}, t_2 = x_{2,2} \cdot (1 - p_{1,1})\cdot p_{2,2} \cdot + \]

\[ E[S^*|t_1 = x_{1,1}, t_2 < x_{2,2} \cdot p_{1,1}\cdot (1 - p_{2,2}) + \]

\[ E[S^*|t_1 < x_{1,1}, t_2 < x_{2,2} \cdot (1 - p_{1,1})\cdot (1 - p_{2,2}) \]

Computing \( E[S^*] - E[S^*] \) directly is tedious. However, the computation can be significantly simplified through the following two observations.

1) The impact of jobs 1 and 2 on sojourn times of other successful jobs is the same in schedules \( S^* \) and \( S^* \), because the sojourn time of any other job \( k \neq 1,2 \) always includes \( t_1 \) and \( t_2 \), regardless of which job executes first.

2) If both job 1 and 2 terminate early, then their order does not matter, i.e., \( E[S^*|t_1 < x_{1,1}, t_2 < x_{2,2}] = E[S^*|t_1 < x_{1,1}, t_2 < x_{2,2}] \) because their sojourn times do not contribute to \( E[S^*] \) or \( E[S^*] \), and the sojourn times of other successful jobs are not affected.

Using these observations, we only need to consider the first
three terms of the right-hand side of Eq. (5). Note that in the following, for clarity of presentation, we drop the subscript of $Q(l)$, since it always refers to $Q_{1,2}(l)$.

For the first term:

$$E[S^*|t_1 = x_1, M_1, t_2 = x_2, M_2] \cdot p_{1, M_1, 2, M_2} = \left( \sum_{l=2}^{N} \frac{x_1, M_1 + x_2, M_2}{l} Q(l - 2) + C_1 \right) \cdot p_{1, M_1, 2, M_2}$$

(7)

where $l$ is the number of successful jobs and $C_1$ contains contributions to mean sojourn time of successful jobs that are due to execution times and success probabilities of jobs other than jobs 1 and 2. That is, job 1’s execution time will contribute $l$ times to the final result, while job 2’s execution time will contribute $l - 1$ times.

Similarly, for the second and third terms we have

$$E[S^*|t_1 < x_1, M_1, t_2 = x_2, M_2] \cdot (1 - p_{1, M_1})p_{2, M_2} = \sum_{l=1}^{N-1} \sum_{k_1=1}^{M_1 - 1} \frac{x_1, k_1 + x_2, M_2}{l} Q(l - 1) \cdot p_{1, k_1, p_{2, M_2}} + C_2 \cdot (1 - p_{1, M_1})p_{2, M_2}$$

(8)

and

$$E[S^*|t_1 = x_1, M_1, t_2 < x_2, M_2] \cdot p_{1, M_1}(1 - p_{2, M_2}) = \sum_{l=1}^{N-1} \sum_{k_2=1}^{M_2} \frac{x_1, M_1 + (l - 1)x_2, k_2}{l} Q(l - 1) \cdot p_{1, M_1, p_{2, k_2}} + C_3 \cdot p_{1, M_1}(1 - p_{2, M_2})$$

(9)

Here $C_2$ and $C_3$ also contain contributions to mean sojourn time of successful jobs that are due to execution times and success probabilities of jobs other than jobs 1 and 2.

Similarly, we can derive components of Eq. (5), with the only difference being that subscripts 1 and 2 are swapped since the positions of jobs 1 and 2 are inverted in $S^*$, i.e.:

$$E[S^*|t_1 = x_1, M_1, t_2 = x_2, M_2] \cdot p_{1, M_1, 2, M_2} = \left( \sum_{l=2}^{N} \frac{(l - 1)x_1, M_1 + x_2, M_2}{l} Q(l - 2) + C_1 \right) \cdot p_{1, M_1, 2, M_2}$$

(10)

$$E[S^*|t_1 < x_1, M_1, t_2 = x_2, M_2] \cdot (1 - p_{1, M_1})p_{2, M_2} = \sum_{l=1}^{N-1} \sum_{k_1=1}^{M_1 - 1} \frac{(l - 1)x_1, k_1 + x_2, M_2}{l} Q(l - 1) \cdot p_{1, k_1, p_{2, M_2}} + C_2 \cdot (1 - p_{1, M_1})p_{2, M_2}$$

(11)

$$E[S^*|t_1 = x_1, M_1, t_2 < x_2, M_2] \cdot p_{1, M_1}(1 - p_{2, M_2}) = \sum_{l=1}^{N-1} \sum_{k_2=1}^{M_2} \frac{x_1, M_1 + (l - 1)x_2, k_2}{l} Q(l - 1) \cdot p_{1, M_1, p_{2, k_2}} + C_3 \cdot p_{1, M_1}(1 - p_{2, M_2})$$

(12)

Note that Eqs. (7) and (10) share the same $C_1$ due to Observation 1. The same is true for $C_2$ and $C_3$.

Now we can compute $E[S^*] - E[S^*]$ by adding Eqs. (7) to (9) and then subtracting Eqs. (10) to (12), which gives:

$$E[S^*] - E[S^*] = \sum_{l=2}^{N} \frac{Q(l - 2)}{l} \left( (x_1, M_1 - x_2, M_2) \cdot p_{1, M_1, 2, M_2} + \sum_{l=1}^{M_1 - 1} \frac{x_1, k_1 \cdot p_{1, k_1, p_{2, M_2}}}{l} - \sum_{l=1}^{M_2 - 1} \frac{x_2, k_2 \cdot p_{1, M_1, p_{2, k_2}}}{l} \right) + \sum_{l=1}^{N-1} \frac{Q(l - 1)}{l} \sum_{k_1=1}^{M_1 - 1} x_1, k_1 \cdot p_{1, k_1, p_{2, M_2}} - \sum_{l=1}^{N-1} \frac{Q(l - 1)}{l} \sum_{k_2=1}^{M_2 - 1} x_2, k_2 \cdot p_{1, M_1, p_{2, k_2}}$$

$$= p_{1, M_1, 2, M_2} \left( \sum_{l=1}^{N-1} \frac{Q(l - 1)}{l} \sum_{k_1=1}^{M_1 - 1} x_1, k_1 \cdot p_{1, k_1, p_{2, M_2}} + \sum_{l=2}^{N} \frac{Q(l - 2)}{l} \cdot x_1, M_1 - \sum_{l=2}^{N} \frac{Q(l - 2)}{l} \cdot x_2, M_2 \right)$$

(13)

If $R_{1,2}^N(2)$ is smaller than $R_{1,2}^N(1)$, then switching the positions of jobs 1 and job 2 in the schedule reduces the expected sojourn time of successful jobs.

Finally, we observe that the earlier assumption of having jobs 1 and 2 occupy the first and second positions in the schedule can be removed since Eq. (13) is derived based only on the assumption that they are adjacent.

Unfortunately, the two values we compare in Eq. (13) ($R_{1,2}^N(i)$ and $R_{1,2}^N(j)$) are challenging to use in a scheduling algorithm because for different $(i, j)$ job pairs the corresponding $Q_{i,j}(l)$ values are different, i.e., $R_{i,j}^N(d)$ is not only a function of $i$ but also a function of $j$, and job $i$’s characterization changes depending on which other job $j$ it is paired with. For example, if we consider jobs 1 and 2, then the coefficient $\alpha_{1,2}(N)$ contains $p_{3,1}, p_{3,2}, ..., p_{3, M_3}$. However, if we compare jobs 1 and 3, then the coefficient $\alpha_{1,3}(N)$ will not contain $p_{3,1}, p_{3,2}, ..., p_{3, M_3}$. This is in contrast with quantities such as Gittins index [5] or SR rank [7]. Consequently, Theorem III.2 only establishes a way to determine the order of two jobs in an optimal schedule when we already know that these jobs are adjacent, and it cannot be used directly to determine the order of arbitrary job pairs.

D. Asymptotically Optimal Scheduling Algorithm

We now focus on the behavior of $\alpha_{i,j}(N)$ as the number of jobs $N$ grows to infinity. We assume that job success probabilities $p_{i,M_i}$ are i.i.d. random variables with a probability density function (PDF) satisfying mild assumptions. The next lemma shows that $\alpha_{i,j}(N) \rightarrow 1$ as $N \rightarrow \infty$ for all $i$ and $j$.

**Lemma III.3.** Assume that the success probabilities $p_{i,M_i}$ for $i = 1, 2, \ldots, N$ are i.i.d. random variables distributed according to a PDF $f(x)$ such that $\beta = \int_0^{\infty} x^2 f(x)dx$ satisfies $1 < \beta < \infty$. Then, $\lim_{N \rightarrow \infty} \alpha_{i,j}(N) = 1$. 
Proof. Since \( p_{k,M_k}, k \notin \{i,j\} \) are i.i.d. random variables with PDF \( f(x), p_{k,M_k}/(1 - p_{k,M_k}), k \notin \{i,j\} \) are also i.i.d. and the law of large numbers gives for \( N \to \infty \)
\[
\frac{1}{N-2} \sum_{k \notin \{i,j\}} p_{k,M_k} \frac{1}{1 - p_{k,M_k}} \int_0^1 x \frac{1}{1 - x} f(x) dx = \beta
\] (14)
i.e., for any \( \epsilon > 0 \), we can find \( N_1 \) such that when \( N > N_1 \)
\[
\left| \frac{1}{N-2} \sum_{k \notin \{i,j\}} p_{k,M_k} \frac{1}{1 - p_{k,M_k}} - \beta \right| < \epsilon .
\] Then, we can rewrite \( Q_{i,j}(1) \) as:
\[
Q_{i,j}(1) = \sum_{k \notin \{i,j\}} p_{k,M_k} \prod_{l \notin \{i,j,k\}} (1 - p_{l,M_l})
= \prod_{k \notin \{i,j\}} (1 - p_{k,M_k}) \cdot \left( \sum_{k \notin \{i,j\}} p_{k,M_k} \right) \left( \frac{1}{1 - p_{k,M_k}} \right)
= \prod_{k \notin \{i,j\}} (1 - p_{k,M_k}) \cdot \left( \frac{N - 2}{1} \right) \beta + \delta_1
\] (15)
where \( \delta_1 \triangleq \sum_{k \notin \{i,j\}} p_{k,M_k} \frac{1}{1 - p_{k,M_k}} - (N - 2)\beta \). From Eq. (14), \( |\delta_1| < (N - 2)\epsilon \) when \( N > N_1 \).

For the same \( \epsilon \), we can find \( N_1 \) for \( l = 2,3, \ldots, N-2 \) such that \( |\delta_2| < \left( \frac{N-2}{l} \right) \epsilon \) when \( N > N_1 \), e.g.,
\[
Q_{i,j}(2) = \sum_{k_1,k_2 \notin \{i,j\}} p_{k_1,M_1} p_{k_2,M_2} \prod_{k \notin \{i,j,k_1,k_2\}} (1 - p_{k,M_k})
= \prod_{k \notin \{i,j\}} (1 - p_{k,M_k}) \cdot \left( \sum_{k_1,k_2 \notin \{i,j\}} p_{k_1,M_1} \frac{1}{1 - p_{k_1,M_1}} \right) \frac{p_{k_2,M_2}}{1 - p_{k_2,M_2}}
= \prod_{k \notin \{i,j\}} (1 - p_{k,M_k}) \cdot \left( \frac{N - 2}{2} \right) \beta^2 + \delta_2
\] (16)
where \( \delta_2 \triangleq \sum_{k_1,k_2 \notin \{i,j\}} p_{k_1,M_1} p_{k_2,M_2} \frac{1}{1 - p_{k_1,M_1}} \frac{1}{1 - p_{k_2,M_2}} - \left( \frac{N-2}{2} \right) \beta^2 \).

More generally, for \( l = 1,2, \ldots, N-2 \),
\[
Q_{i,j}(l) = \prod_{k \notin \{i,j\}} (1 - p_{k,M_k}) \cdot \left( \frac{N - 2}{l} \right) \beta^l + \delta_i
\] (17)
By applying Eqs. (15) to (17) to Eq. (4), we have
\[
\alpha_{i,j}(N) = \frac{\sum_{l=2}^N Q_{i,j}(l-2)}{\sum_{l=1}^{N-2} Q_{i,j}(l-1)}
= \prod_{k \notin \{i,j\}} (1 - p_{k,M_k}) \sum_{l=2}^N \frac{1}{l} \left( \frac{N-2}{l-2} \beta^{l-2} + \delta_{l-2} \right)
= \prod_{k \notin \{i,j\}} (1 - p_{k,M_k}) \sum_{l=1}^{N-1} \frac{1}{l} \left( \frac{N-2}{l-1} \beta^{l-1} + \delta_{l-1} \right)
= \frac{N}{N-1} \left( \frac{N-2}{N-1} \right) \beta^{l-1} + \delta_{l-1}
\] (18)
For the numerator of Eq. (13), note that
\[
\sum_{l=2}^N \frac{1}{l} \left( \frac{N-2}{l-2} \right) \beta^{l-2} = \sum_{l=2}^N \frac{l - 1}{N(N-1)} \left( \frac{N}{l} \right) \beta^{l-2}
= \frac{1}{N(N-1)\beta^2} \sum_{l=2}^N \frac{N}{l} \beta^l - \frac{N}{l} \beta^l
= N\beta(\beta+1)^{N-1} - (\beta+1)^{N+1} N(N-1)\beta^2,
\] (19)
and
\[
|\sum_{l=2}^N \delta_{l-2}| < \frac{N}{l} \sum_{l=2}^N \left( \frac{N-2}{l} \right) \beta^{l-2} \epsilon = \frac{(N2^{N-1}-2N+1)\epsilon}{N(N-1)}.
\] (20)
Thus when \( N \to \infty \) and \( \beta > 1 \), from (19) and (20) we can see that \( \sum_{l=2}^N \delta_{l-2} / \sum_{l=2}^N \frac{N}{l} \beta^{l-2} \to 0 \).
Similarly for the denominator, we have:
\[
\sum_{l=1}^{N-1} \frac{1}{l} \left( \frac{N-2}{l-1} \right) \beta^{l-1} = \frac{N\beta(\beta+1)^{N-1} - (\beta+1)^{N+1} \beta^{N-1}}{(N-1)\beta} \to 1.
\] (21)
and \( \sum_{l=1}^{N-1} \delta_{l-1} / \sum_{l=1}^{N-1} \frac{N}{l} \beta^{l-1} \to 0 \). By applying Eq. (19), Eq. (21) and the limiting results we can see that when \( N \to \infty \), \( \alpha_{i,j}(N) \) converges to
\[
\frac{\sum_{l=2}^N \frac{N}{l} \beta^{l-2} \beta^{l-1}}{\sum_{l=1}^{N-1} \frac{N}{l} \beta^{l-1}} = \frac{N\beta(\beta+1)^{N-1} - (\beta+1)^{N+1} \beta^{N-1}}{(N-1)\beta} \to 1.
\] (22)
Theorem III.3 states that \( \alpha_{i,j}(N) \to 1 \) as \( N \to \infty \), independently of \( i \) and \( j \). As a result, for the quantity \( R_{ij}^N(i) \) used in Theorem III.2 to decide the order of two adjacent jobs, we have that:
\[
\lim_{N \to \infty} R_{ij}^N(i) = \frac{\sum_{k=1}^M p_{k,M_k} \delta_{i,k} p_{i,M_i}}{p_{i,M_i}} \triangleq R(i)
\] (23)
where we define \( R(i) \) as the rank of job \( i \). Note that \( R(i) \) depends only on the size distribution of job \( i \), and it is equal to the ratio between the expected job size and the probability that job \( i \) completes successfully.

Our scheduling algorithm, which we call RANK, computes the rank of each job using Eq. (23) and then sorts the jobs in ascending rank order to assign them to the server. These operations can be performed with \( O(\sum_{k=1}^M M_k + N \log N) \) complexity, where the resulting schedule is asymptotically optimal as \( N \to \infty \), which is described in the following result.

**Theorem III.4.** The schedule generated by algorithm RANK is asymptotically optimal for \( N \to \infty \).

*Proof. Denote by \( S_{RANK} \) the schedule generated by algorithm RANK and assume that \( N \) is sufficiently large. Suppose that \( S_{RANK} \) is not optimal and that there exists some other optimal schedule \( S_{OPT} \) with smaller expected sojourn time of successful jobs. Now consider the job with the smallest rank. Without loss of generality, let’s assume that it is job 1. Job 1 must be the first job in \( S_{OPT} \). Otherwise suppose that job 2 is right before job 1 in \( S_{OPT} \). That cannot happen since,\n
according to Theorem III.2 and Lemma III.3 if we schedule job 1 before job 2, then the expected sojourn time of successful jobs will decrease. Using a similar argument, we can state the following. Whenever a higher rank job is scheduled before a lower rank job, then the schedule must contain two adjacent jobs, \( k \) and \( m \) with \( k \) before \( m \), such that \( k \)'s rank is higher than \( m \)'s rank. We can then decrease the expected sojourn time by swapping these adjacent jobs. That is, all jobs should be placed according to ascending rank order in schedule \( S_{OPT} \). That makes \( S_{OPT} \) exactly the same as \( S_{RANK} \). 

E. Discussion

The assumptions made to obtain our results are satisfied in most applications of interest. First, our policy is asymptotically optimal for \( N \to \infty \), but we can assume \( N \) to be very large, since large-scale machine learning systems often include hundreds or thousands of multi-stage jobs running at any given time. Our second assumption is that the final success probabilities are independently drawn from a distribution; since machine learning jobs are typically submitted by different teams under different configurations, their final success probabilities can be reasonably modeled as being drawn independently from a distribution, and the density function of this distribution can be approximated through job statistics. Finally, we assume that \( 1 < \beta < \infty \) in Theorem III.3. This assumption is not restrictive: when \( p_{i,M_{i}} < 1, i = 1,2, \ldots, N \), we have \( \beta < \infty \), no matter how close \( p_{i,M_{i}} \) is to 1. \( \beta > 1 \) is also reasonable: for example, it is satisfied when job success probability is greater than 0.5, or when half of the jobs succeed with probability greater than \( 2/3 \).

We also observe that our rank metric \( R(i) \) in Eq. (23) can be interpreted as a special case of the "smallest expected processing time to cost ratio" policy in [9], since jobs are ranked according to the ratio between expected processing time and success probability, which represents our expectation of whether they contribute to the cost metric in Eq. (1); but, in contrast with the setting in [8], the number of successful jobs \( C \) in Eq. (1) is a random variable, resulting in a different optimization goal, thus complicating proofs in this section.

IV. NUMERICAL RESULTS

In the previous section, we presented an algorithm that minimizes the expected sojourn time of successful jobs when \( N \) grows to infinity. In this section, we provide numerical results illustrating that our algorithm performs well even when \( N \) is not large.

A. Experimental setup

We use generated workloads under a variety of settings to evaluate the performance of our algorithm and to compare it with existing scheduling approaches.

1) Experimental Methodology: For each setting, the experiments are performed as follows. (i) We generate possible job sizes \( x_{i,k}, i = 1, \ldots, N, k = 1, \ldots, M_{i} \) and probabilities \( p_{i,k}, i = 1, \ldots, N, k = 1, \ldots, M_{i} \) for all \( N \) jobs according to workload settings described in Section IV-A2. (ii) For each algorithm (ours and the existing scheduling algorithms described in Section IV-A3), we compute job schedules using that algorithm and then we evaluate the resulting expected sojourn time of successful jobs by iterating over all possible combinations of job successes (weighted by probabilities \( p_{i,k} \)); the results are recorded as output of one trial. (iii) We repeat steps (i) and (ii) over multiple trials, at least 50000 times for each experiment. Then we use the results of these trials to compute metrics described in Section IV-A4.

2) Workloads: Initially we generate workloads with 2-stage jobs. We then study the impact of the number of stages on algorithm performance in Section IV-C.

To evaluate the performance of our algorithm under a broad range of settings, we consider workloads that vary in the following three dimensions.

The first dimension is the number of jobs \( N \). In our experiments, we vary \( N \) from 3 to 17.

The second dimension considers the distribution of final success probabilities \( p_{i,M_{i}}, i \in \{1,2, \ldots, N \} \). These quantities are used to compute \( \alpha_{i,j}(N) \) in Eq. (18). Thus, it is important to understand their impact on the performance of our algorithm. In our study we look at three different distributions. The first is a uniform distribution \( U[0.00001,0.99999] \). The second and third have probability mass functions as defined in Tables I and II respectively. For simplicity, we refer to these as distribution I (Table I) and distribution II (Table II).

The third dimension is the distribution of job stage sizes. Understanding the impact of job distribution sizes on our algorithm is important as they also contribute to \( \alpha_{i,j}(N) \) in Equation (18). We consider three different distributions for job size stages: uniform, exponential and Weibull with shape factor of 0.5. Intuitively, these three distributions provide a range of possible job sizes, from light tail to heavy tail.

| Value | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Prob. | 0.2 | 0.15 | 0.1 | 0.05 | 0 | 0.05 | 0.1 | 0.15 | 0.2 |

Table I: Final Success Prob. Dist. for Workload Set 2

| Value | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Prob. | 0.025 | 0.05 | 0.15 | 0.35 | 0.15 | 0.1 | 0.05 | 0.025 |

Table II: Final Success Prob. Dist. for Workload Set 3

More specifically, we generate five sets of workloads with different combinations of success probability distributions and job stage size distributions. These combinations are summarized by Table III

| Workload set | 1 | 2 | 3 | 4 | 5 |
|--------------|---|---|---|---|---|
| Job size dist. | uniform | uniform | uniform | exp | Weibull |
| Success prob. dist. | uniform | dist. I | dist. II | uniform | uniform |

Table III: Workloads description

Intuitively, workload sets 1, 2, and 3 represents three different scenarios, where the final success probabilities are (1) uniformly distributed between 0.00001 and 0.99999, (2) with high probability are around 0.5, and (3) with high probability are either small (0.1) or large (0.9). In all these three workloads, the stage sizes are uniformly distributed. Conversely, workload sets 4 and 5 represent scenarios where
the final success probabilities are uniformly distributed but the job sizes are non-uniformly distributed.

3) Baseline Algorithms: To better demonstrate the performance of our approach and to demonstrate the need to tailor the scheduling algorithm to successful jobs (as compared to all jobs), we also include the following four baseline algorithms in our experiments:

- **OPTIMAL**: The optimal schedule obtained through an exhaustive search (only for \(N \leq 8\), as for larger \(N\) this is too expensive).
- **RANDOM**: Execute jobs in random order.
- **SERPT**: Sort and execute jobs based on expected remaining processing time, in ascending order.
- **SR**: Sort jobs based on SR rank \(\frac{1}{2}\) (equivalent to Gittins’ index) and schedule them in ascending rank order.

Note that, by design, RANDOM will execute each job until success or failure (i.e., without preemption) before selecting a different job for execution. This is reasonable since we know from Theorem III.1 that preemption of jobs only increases expected sojourn time of successful jobs. Note also that SERPT will naturally continue executing a job until its completion or early termination. The only baseline algorithm in our experiments that might preempt jobs is SR; including SR in the experiments allows us to demonstrate whether such an approach can deliver good performance given our objective of minimizing mean sojourn time of successful jobs.

4) Performance Metrics: Let there be \(Z\) job groups (or trials) for each experiment, denoted as \(G_1, G_2, ..., G_Z\) (as described above). Let \(E_{ALG}[G_i], i = 1, 2, 3, ..., Z\) be the expected sojourn time of successful jobs for job group \(G_i\) under algorithm \(ALG\), where \(ALG\) can be one of the baseline algorithms described above or our algorithm RANK.

For each workload set we use two performance metrics to evaluate scheduling algorithms. The first metric is the average expected sojourn time of successful jobs over all workload groups, i.e., \(E_Z^2_{ALG} = \sum_{i=1}^{Z} E_{ALG}[G(i)]/Z\). The second metric is the competitive ratio; we consider the maximum, 95th and 75th percentiles of competitive ratio (referred to as \(p95\) and \(p75\) below) for each algorithm as compared to the optimal. Specifically, let \(CR^i_{ALG} = E_{ALG}[G(i)]/E_{OPTIMAL}[G(i)]\) for \(ALG \in \{RANK, RANDOM, SERPT, SR\}\) and \(i = 1, 2, ..., Z\); here, we focus on \(CR^\text{max}_{ALG} = \max_i CR^i_{ALG}\), \(CR^95_{ALG} = P^5\%_{i} CR^i_{ALG}\), and \(CR^75_{ALG} = P^75\%_{i} CR^i_{ALG}\).

The rationale behind using these two metrics is that the first metric tells us the average performance of each algorithm, whereas the second metric shows how well an algorithm performs as compared to optimal in the worst case as well as the 75th and 95th percentile, giving us an overview of the range of the algorithm’s performance. A good algorithm should exhibit low competitive ratio, without being particularly sensitive to the input. That is, its competitive ratios over the entire workload set should also be low.

B. Results

Our numerical results for the first metric (average expected sojourn time of successful jobs) on different workloads and number of jobs \(N\) are depicted in Figures 3-7 for each of the workload sets described in Table III.

From these figures we can observe that our algorithm (RANK) results in lower average expected sojourn time for successful jobs, with respect to the baseline algorithms (except, of course, for OPTIMAL) for all \(N\) values, regardless of the distribution of final success probabilities or job size distributions. In all cases our algorithm results in nearly optimal performance. To better illustrate how close the results are, we report the mean sojourn time values for RANK and OPTIMAL in Table IV through Table VIII, which correspond to Fig. 4.
to Fig. 7. From Table V, we observe that RANK produces results that are at most 0.2% greater than the corresponding results of OPTIMAL for all three workload sets.

Table IV: Average expected sojourn time: workload set 1

| Number of jobs | 3   | 4   | 5   | 6   | 7   | 8   |
|----------------|-----|-----|-----|-----|-----|-----|
| OPTIMAL        | 1.219 | 1.515 | 1.784 | 2.043 | 2.299 | 2.540 |
| RANK           | 1.221 | 1.518 | 1.786 | 2.045 | 2.301 | 2.542 |

Table V: Average expected sojourn time: workload set 2

| Number of jobs | 3   | 4   | 5   | 6   | 7   | 8   |
|----------------|-----|-----|-----|-----|-----|-----|
| OPTIMAL        | 1.237 | 1.537 | 1.816 | 2.083 | 2.347 | 2.601 |
| RANK           | 1.238 | 1.540 | 1.818 | 2.085 | 2.349 | 2.603 |

Table VI: Average expected sojourn time: workload set 3

| Number of jobs | 3   | 4   | 5   | 6   | 7   | 8   |
|----------------|-----|-----|-----|-----|-----|-----|
| OPTIMAL        | 1.236 | 1.538 | 1.818 | 2.087 | 2.343 | 2.607 |
| RANK           | 1.238 | 1.540 | 1.821 | 2.088 | 2.345 | 2.609 |

We can also observe from these figures that the SR rank based algorithm is not well-suited for our task. Its performance is not only worse than our algorithm (RANK), but also worse than SERPT for all workload sets as well as N values; the smallest gap between the performance of the SR rank based algorithm and our algorithm is 20.5%, with the maximum gap being over 30%.

Numerical results for the second metric (maximum, p95 and p75 of competitive ratios) on different workload sets and number of jobs N are reported in Tables IX to XIII. In these tables smaller numbers indicate better algorithm performance.

We observe that the maximum competitive ratios differ significantly across different algorithms as well as different workload sets. The results of RANDOM are the worst for all workload sets, with all having values greater than 3.0. This is not surprising as RANDOM performs no optimization. In the case of workload set 5, RANDOM results in a very high maximum competitive ratio for some N values. In the extreme case of N = 4, the maximum competitive ratio is over 400. This makes sense because job stage lengths in workload set 5 are generated using a heavy tail distribution. As a result some job stages are quite large as compared to others. Since RANDOM might schedule those jobs first, it can experience very high mean sojourn times.

SR performs better than RANDOM but not as well as SERPT or our algorithm (RANK) in workloads 1 through 4 for all N values. For these four workloads the competitive ratios of SR are less than 6.548. However, SR’s performance is similar to that of RANDOM when running on workload set 5. The main cause here is that SR preempts jobs and thus introduces an increase in sojourn times of successful jobs.

SERPT performs better than RANDOM or SR. For all N values and workload sets, its maximum competitive ratios are between 2.014 and 3.267.

Finally, our algorithm (RANK) has the lowest maximum competitive ratios for all values of N and all workload sets, with maximum competitive ratios between 1.025 and 1.118.

Similar conclusion can be drawn for p95 and p75 results. For p95, RANDOM still has the worst performance although the values are much lower as compared to maximum values for workload set 5 (3.797 vs 415.039). SR performs better than RANDOM, and SERPT performs better than SR. RANK performs best with the maximum p95 value of 1.012 over all workload sets. For p75, RANK’s results is at most 1.001, performing best over all algorithms and workload sets.

In summary, using numerical examples over a broad range of workloads we highlighted that our algorithm (RANK) results in the best performance on average expected sojourn time of successful jobs and maximum competitive ratios, irrespective of success probability distributions, job stage size distributions, or number of jobs.

**C. The impact of number of job stages**

Intuitively, the number of stages of a job has should not affect the performance of RANK since our algorithm does preempt jobs. Here, we validate this intuition using an experiment where we vary the number of jobs stages and consider the effect on RANK’s performance. Specifically, we generate workloads with a fixed final success probability distribution \(U[0.00001, 0.99999]\), a fixed stage size distribution \(U[0, 1]\), and a fixed number of jobs \(N = 5\). The only parameter we

We observe that the maximum competitive ratios differ significantly across different algorithms as well as different workload sets. The results of RANDOM are the worst for all workload sets, with all having values greater than 3.0. This is not surprising as RANDOM performs no optimization. In the case of workload set 5, RANDOM results in a very high maximum competitive ratio for some N values. In the extreme case of N = 4, the maximum competitive ratio is over 400. This makes sense because job stage lengths in workload set 5 are generated using a heavy tail distribution. As a result some job stages are quite large as compared to others. Since RANDOM might schedule those jobs first, it can experience very high mean sojourn times.

SR performs better than RANDOM but not as well as SERPT or our algorithm (RANK) in workloads 1 through 4 for all N values. For these four workloads the competitive ratios of SR are less than 6.548. However, SR’s performance is similar to that of RANDOM when running on workload set 5. The main cause here is that SR preempts jobs and thus introduces an increase in sojourn times of successful jobs.

SERPT performs better than RANDOM or SR. For all N values and workload sets, its maximum competitive ratios are between 2.014 and 3.267.

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Similar conclusion can be drawn for p95 and p75 results. For p95, RANDOM still has the worst performance although the values are much lower as compared to maximum values for workload set 5 (3.797 vs 415.039). SR performs better than RANDOM, and SERPT performs better than SR. RANK performs best with the maximum p95 value of 1.012 over all workload sets. For p75, RANK’s results is at most 1.001, performing best over all algorithms and workload sets.

In summary, using numerical examples over a broad range of workloads we highlighted that our algorithm (RANK) results in the best performance on average expected sojourn time of successful jobs and maximum competitive ratios, irrespective of success probability distributions, job stage size distributions, or number of jobs.
all jobs succeeded with probability 1 is NP-complete. The problem is even harder if we assume that the scheduler has no job arrival times information. This is further complicated when jobs’ service times (and corresponding distributions) are not known in advance. Thus, we propose the following heuristic approach for online scheduling:

- When a new job arrives: if there is a server available, assign it to the job; otherwise, we add the job to a queue.
- When a server completes the next stage of a job, it selects the job with the smallest rank among all jobs currently in the queue as well as the job being served (unless it completed successfully or terminated early).

Note that, to implement this approach efficiently, we can maintain a priority queue to store all jobs, using the rank of each job as its priority; jobs can be added or removed from the queue with \( O(\log N) \) complexity, while the next job can be selected in constant time.

The intuition behind this approach is simple: without looking into the future, a server acts as if it is the only server in the system and serves jobs in a greedy manner. This approach is clearly not optimal, as illustrated by the following simple example. Consider a single server system with two jobs. Job 1 has two possible sizes \( x_{1,1} = 4 \) and \( x_{1,2} = 10 \) with probability \( p_{1,1} = 0.4 \) and \( p_{1,2} = 0.6 \). Job 2 has two possible sizes \( x_{2,1} = 1 \) and \( x_{2,2} = 2 \) with probability \( p_{2,1} = 0.2 \) and \( p_{2,2} = 0.8 \). Finally, their arrival times are \( a_1 = 0, a_2 = 2 \). Our approach will first schedule the first stage of Job 1 at time 0. Then at time 4 it will change to serve all stages of Job 2. After that it will finish the remaining stages (if any) of Job 1. However, one can verify that a better schedule is to have the server wait till time 2, then serve all stages of Job 2, and then serve all stages of Job 1. Nevertheless, in Section [V] we show, using a simulation study, that our heuristic outperforms other online algorithms.

### VI. Simulation Results

In this section we evaluate our online approach (presented in Section [V]) using a simulation study with real-world traces.
A. Data Set and Methodology

We use real-world job execution traces provided in [10] which include execution logs for a total of 117,325 jobs over a 75 day period in late 2017 at Microsoft. Of all these jobs we pick a total of 109,967 jobs for our simulations: these jobs contain at least one valid attempt that maps to stages in our setting. (The remaining small fraction of jobs either have zero attempts, or a bad start or end timestamps. Thus, these are excluded from our simulations). The majority of jobs (95,345 out of 109,967) have only one attempt; there exists one valid job with as many as 7 attempts. The breakdown of jobs by the number of attempts can be found in Table XV.

| Number of Attempts | Count     | Percentage |
|--------------------|-----------|------------|
| 1                  | 95188     | 86.595%    |
| 2                  | 5465      | 4.972%     |
| 3                  | 1674      | 1.523%     |
| 4                  | 554       | 0.866%     |
| 5                  | 6574      | 5.981%     |
| 6                  | 67        | 0.061%     |
| 7                  | 1         | 0.001%     |

Table XV: Distribution of number of attempts: Microsoft trace

In addition, jobs in the trace are marked as being in one of three different categories: passed, failed, or killed. Out of all jobs considered, about 75% (82,445 out of 109,967) are in the passed category, 15% (16,927 out of 109,967) failed, and 10% (10,595 out of 109,967) were killed.

We map the attempts of jobs in the traces to stages in our formulation. The length of a stage is exactly the execution time of the corresponding attempt. The idea here is that each attempt has a start and an end timestamp, corresponding to the time period when the job is in service during this attempt. At the end of each attempt, the scheduler may decide to continue to serve this job (move to the next attempt) or complete this job with as many as 7 attempts. The breakdown of jobs by the number of attempts can be found in Table XV.

To understand the impact of job success rates, in addition to the original traces, we also generate two synthetic datasets using the statistics obtained from the original traces. In these two data sets (referred to as synthetic data set I and II, respectively), the number of job stages and stage lengths, as well as the interarrival times, follow the same distribution as in the original Microsoft traces; the only difference is that job success probabilities are set to be \( p_M = 0.5 \) and \( p_M = 0.25 \) in data sets I and II, respectively.

Our discrete event simulator, implemented in Go [11], allows us to experiment with different scheduling algorithms as well as vary the number of servers. To better understand the performance of our approach relative to existing techniques, we also compare its performance against the following approaches to rank jobs:

- **FIFO**: Jobs are served in FIFO manner.
- **SERPT**: Similar to our approach mentioned in Section V but jobs are sorted by ERPT to select the next job.
- **SR**: Similar to our approach but jobs are sorted by the SR rank to select the next job.

B. Results

| Number of Servers | FIFO       | SERPT     | RANK      | SR       |
|-------------------|------------|-----------|-----------|----------|
| 5                 | 3.807E8    | 1.214E7   | 1.031E7   | 1.679E7  |
| 10                | 1.906E8    | 5.553E6   | 4.705E6   | 8.196E6  |
| 20                | 9.266E7    | 2.511E6   | 2.115E6   | 3.759E6  |
| 50                | 3.428E7    | 6.553E5   | 5.345E5   | 1.378E5  |
| 80                | 1.979E7    | 3.198E5   | 2.516E5   | 7.303E5  |
| 100               | 1.499E7    | 2.320E5   | 1.830E5   | 5.685E5  |
| 200               | 5.545E6    | 7.7890E5  | 5.324E5   | 2.064E5  |
| 300               | 2.551E6    | 3.8412E5  | 3.0332E0  | 8.3271E9 |

Table XVI: Mean delay for Microsoft traces

| Number of Servers | FIFO       | SERPT     | RANK      | SR       |
|-------------------|------------|-----------|-----------|----------|
| 5                 | 3.892E8    | 3.870E7   | 3.078E7   | 4.048E7  |
| 10                | 2.909E8    | 1.725E7   | 1.394E7   | 2.521E7  |
| 20                | 1.403E8    | 7.919E6   | 5.781E6   | 7.675E6  |
| 50                | 5.053E7    | 1.908E6   | 1.1548E6  | 2.144E6  |
| 80                | 2.609E7    | 9.061E5   | 6.797E5   | 9.814E5  |
| 100               | 2.061E7    | 5.894E5   | 4.344E5   | 6.562E5  |
| 200               | 5.686E6    | 1.232E5   | 8.6452E5  | 1.306E5  |
| 300               | 9.217E5    | 4.9136E4  | 4.0815E2  | 4.9487E3 |

Table XVII: Mean delay for Synthetic data set I

| Number of Servers | FIFO       | SERPT     | RANK      | SR       |
|-------------------|------------|-----------|-----------|----------|
| 5                 | 5.938E8    | 3.531E7   | 2.695E7   | 4.192E7  |
| 10                | 2.948E8    | 1.583E7   | 1.211E7   | 1.973E7  |
| 20                | 1.428E8    | 6.671E6   | 3.119E6   | 9.240E6  |
| 50                | 5.178E7    | 1.861E6   | 1.394E6   | 2.197E6  |
| 80                | 2.905E7    | 8.550E5   | 6.253E5   | 1.392E5  |
| 100               | 2.148E7    | 5.553E5   | 4.055E5   | 1.043E5  |
| 200               | 6.393E6    | 1.128E5   | 9.2343E2  | 2.283E5  |
| 300               | 1.490E6    | 48550.119 | 43588.312 | 73602.013|

Table XVIII: Mean delay for Synthetic data set II

Average delay results for different scheduling approaches with varying number of servers using different data sets are given in Tables XVI to XVIII. We observe that:

- **FIFO** performs far worse than any other scheduling approach. This is expected given its lack of optimization.
• Our approach outperforms the other policies under all settings and data sets. For example, it outperforms SERPT (the second best) by 15.2% to 26.4% (depending on the number of servers) on the Microsoft trace.

• Using Gittin’s index does not result in performance comparable with our approach under all server settings.

• Job success probability has a qualitative impact on the performance of our approach. For instance, we observe that with a lower success probability, our approach performs better as compared to SERPT, e.g., in the case of 20 servers, our approach outperforms SERPT by ≈ 15.8% on the Microsoft traces, which has a job success probability close to 0.75. In the case of synthetic data sets I and II, where job success probabilities are 0.5 and 0.25, the improvements are 19.7% and 23.3%, respectively. This effect is expected: as shown in Section III-A, other approaches optimize response time of all jobs; when job success probability is high, such goal is similar to optimizing response time of successful jobs (as in our algorithm). In contrast, when job success probability is lower, our algorithm can find better schedules by distinguishing between successful and terminated jobs.

In summary, the online approach proposed in Section V performs better than other online approaches when using real-world job execution traces as well as our synthetic data sets.

VII. RELATED WORK

Job scheduling has been studied extensively since the 1950s. A vast literature exists on this topic; for instance, [12] and [13] provide a comprehensive survey of the literature on the theory and applications of scheduling in the past several decades.

The works that focus on scheduling preemptive jobs on a single server, such as [6] and [7], are most relevant to ours since in our work, at each checkpoint, the scheduler can switch to another job, i.e., preempt the job currently in service. As discussed earlier, our work differs from previous efforts as the algorithms developed in existing literature do not address the problem of optimizing mean sojourn time of successful jobs.

Another set of scheduling works that is related to our effort, e.g., such as [14] and [15], considers a single server with failures. However, in these are server failures rather than job failures. Moreover, the main objective in this line of work is to optimize throughput or reliability of the system where the server is unreliable. Often, in these works, e.g., such as [14], preventative maintenance is introduced to improve reliability of the system. In contrast, our work considers job termination and a different objective (namely minimizing mean sojourn time of successful jobs).

On the system side, works such as [16], [17] and [18] propose different scheduling frameworks for deep learning workloads with various targets. However, none of these works focus on optimizing sojourn times of successful jobs, which is the focus of our work.

Lastly, there are works, e.g., such as [19], that consider jobs with variable execution times. However, these works focus on objectives such as minimizing weighted sum of sojourn times of all jobs; this is in contrast to our goal of minimizing the mean sojourn time of successful jobs.

VIII. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper we studied the problem of scheduling multi-stage jobs with early terminations. Given the objective of optimizing expected sojourn time of successful jobs (i.e., jobs that complete their final stage rather than terminate early), we showed that when there is only one server and all jobs arrive simultaneously, preemption is not needed, even if it is allowed. In addition, we presented an asymptotically optimal scheduling algorithm, as well as numerical results showing that its performance is close to optimal even under finite settings.

We also provided an online approach for the scenario where there are multiple servers and jobs arrive over time and illustrated the benefits of this approach through simulations using real-world and synthetic traces.

Our future directions in this early termination setting include: (1) exploration of the online problem with the goal of developing a scheduling algorithm with a provable performance bound and (2) exploration of scheduling algorithms with reduced knowledge of job size distributions.

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