Tests of the nature and of the gluon content of the $\sigma(0.6)$ from $D$ and $D_s$ semileptonic decays

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Abstract
We summarize the different features which show that QCD spectral sum rule analyses of the scalar two- and three-point functions do not favour the $\bar{u}u + \bar{d}d$ interpretation of the broad and low mass $\sigma(0.6)$ and emphasize that a measurement of the $D_s$ semileptonic decays into $\pi\pi$ can reveal in a model-independent way its eventual gluon component $\sigma_B$. The analysis also implies that one expects an observation of the $K\bar{K}$ final states from the $\sigma_B$ which may compete (if phase space allowed) with the one from a low mass $\bar{s}s$ state assumed in the literature to be the SU(3) partner of the observed $\sigma(0.6)$ if the latter is a $\bar{u}u + \bar{d}d$ state.
1 Introduction

The nature of scalar mesons is an intriguing problem in QCD. Experimentally, there are well established scalar mesons with isospin $I = 1$, the $a_0(980)$, with isospin $I = 1/2$ $K^0_0(1410)$ meson, and with isospin $I = 0$, the $f_0$-mesons at 980, 1370 and 1500 MeV \[.\] Besides these resonances there are recent experimental \[1\] and theoretical \[2, 3\] indications for a low lying scalar isoscalar state, the famous $\sigma$. The isoscalar scalar states are very interesting in the framework of QCD since, in this $U(1)_V$ channel, their interpolating operator is the trace of the energy-momentum tensor:

$$
\theta_\mu^a = \theta_g + \theta_q = \frac{1}{4} \beta(\alpha_s)G^2 + \sum_i (1 + \gamma_m(\alpha_s)) m_i \bar{\psi}_i \psi_i ,
$$

where $G^a_{\mu \nu}$ is the gluon field strengths, $\psi_i$ is the quark field; $\beta(\alpha_s)$ and $\gamma_m(\alpha_s)$ are respectively the QCD $\beta$-function and quark mass-anomalous dimension. In the chiral limit $m_i = 0$, $\theta_\mu$ is dominated by its gluon component $\theta_g$, like is the case of the $\eta'$ for the $U(1)_A$ axial-anomaly, explaining why the $\eta'$-mass does not vanish like other Goldstone bosons for $m_i = 0$. In this sense, it is natural to expect that these $I = 0$ scalar states are glueballs/gluonia or have at least a strong glue admixture in their wave function. QCD spectral sum rules (QSSR) are an important analytical tool of nonperturbative QCD and especially well suited to address the question of the quark-gluon mixing since the principal nonperturbative ingredients are the quark condensates, the gluon condensate and the mixed quark-gluon condensate.

In this note we summarize some essential features of previous sum rule analyses and especially point out the relevance of semileptonic $D$ and $D_s$-decays for obtaining information on the gluon content of the scalar mesons.

2 Instantons and tachyonic gluon effects to the $S_2(\bar{u}u + \bar{d}d)$

Masses and couplings of unmixed scalar $\bar{q}q$ mesons and gluonia have been extensively studied in the past and more recently \[4\] reviewed using QSSR within the standard Operator Product Expansion (OPE) of the diagonal two-point correlator:

$$
\psi(q^2) = i \int d^4xe^{iqx}(0)\langle 0 | T J(x) J(0) | 0 \rangle ,
$$

associated to the quark or/and the gluonic currents.

It has been emphasized that the mass of the scalar $S_2 = \bar{u}u + \bar{d}d$ meson is about 1 GeV, in agreement with the one of the observed $a_0(980)$, and with good $SU(2)$ symmetry implying a degeneracy between the isovector $a_0$ and isoscalar state $S_2$, while its width into $\pi\pi$ has been found to be about 100 MeV \[5\].

On the other hand, the mass of the mesons containing a strange quark is above 1 GeV due to $SU(3)$ breaking, which explains successfully the well-known $\phi - \rho$ and $K^* - \rho$ mass splittings \[6\].

Contributions outside the usual OPE do not alter this result. The $1/q^2$-renormalon contribution introduced by a tachyonic gluon mass \[7\] from the linear term of the short distance part of the QCD potential \[8\] are very small. Instanton contributions \[9\] to the scalar current as studied in \[10\] improve the stability for the sum rule of the decay constant, but do not lower the mass of the $S_2$ in a stable way.

In Figure \[1\] we show the results for the mass of the $S_2(\bar{u}u + \bar{d}d)$ as function of the Borel/Laplace parameter $M^2$ for the leading contributions (a), with the inclusion of the two loop corrections and the tachyonic mass contribution (b, dashed line) and also including the instanton contribution (b, solid line). The continuum threshold is 1.4 GeV$^2$ in all cases \[11\].

3 Nature of the $\sigma(0.6)$ and $f_0(980)$

From the previous analysis, one can already conclude that the observed $\sigma$ cannot be a pure $\bar{q}q$ state.

- A QSSR analysis in the gluonium channel, using the subtracted sum rule sensitive to the constant $\psi_G(0) \simeq -16 \beta_2 / \alpha_s G^2 \langle \bar{q}q \rangle$, (where $\beta_2 = -1/2(11 - 2n_f/3)$ and $\langle \bar{q}q \rangle \simeq 0.07$ GeV$^4$ \[12\]) and the unsubtracted sum rule \[13\], requires two resonances for a consistent saturation of the two sum rules, where the lowest mass gluonium $\sigma_B$ should be below 1 GeV.

- A low energy theorem for the vertex $\langle \pi | \Omega_\mu^a | \pi \rangle$ also shows that the $\sigma_B$ can be very wide with a $\pi^+ \pi^-$ width of

\[1\] A more complete spectrum of different scalar mesons from QSSR analysis are given in details in \[14\].

\[2\] A linear term of the potential at all distances has been recently proposed by ‘t Hooft \[15\] as a possible way to solve the confinement problem.

\[3\] This value is analogous to the ones obtained from the QSSR analysis of some other scalar channels \[16\].
about (0.2 – 0.8) GeV corresponding to a mass of (0.7 – 1) GeV and the \( g_{\pi\pi} \) coupling behaves as \( M_2^2 \). A such result shows a huge violation of the OZI rule analogous to the one encountered in the \( \eta' \)-channel.

- From this result, a natural quarkonium-gluonium mixing (decay mixing scheme) has been proposed in the \( I = 0 \) scalar sector to explain the observed spectra and widths of the possibly wide \( \sigma(<1 \text{ GeV}) \) and the narrow \( f_0(0.98) \). The data are well fitted with a nearly maximal mixing angle \( |\theta_S| \approx 40^\circ \), indicating that the \( \sigma \) and \( f_0 \) have equal numbers of quark and gluon each of their wave functions. This mixing scenario also implies a strong coupling of the \( f_0 \) to \( K\bar{K} \) (without requiring to a four-quark \( \bar{s}s(\bar{u}u + \bar{d}d) \) state model) with a strength \( y_{f_0K^+K} = 2y_{f_0\pi^+\pi^-} \), as supported by the data. The physical on-shell \( f_0 \) is narrow (< 134 MeV) due to a destructive mixing, whilst the \( \sigma(7 \sim 1) \) can be (0.4 – 0.8) GeV wide (constructive mixing).

Compared to the four-quark states and/or \( \bar{K}K \) molecules models (see e.g. [16]), this quarkonium-gluonium mixing scenario includes all QCD dynamics based on the properties of the scale anomaly \( \theta_{\mu\mu} \), which comes from QCD first principles. It is certainly interesting to find some further tests of this scenario, which we propose in the following.

### 4 \( D_{(s)} \) semileptonic decays

\( S_2(\bar{u}u + \bar{d}d) \) meson productions

A theoretically very clean way to investigate hadronic resonances is the analysis of semileptonic decays of charmed mesons. Though there is much better statistics for non-leptonic decays, the complicated final state interaction both on the quark and on the hadronic level make the analysis here difficult and poses many puzzles.

- If the scalar mesons were simple \( \bar{q}q \) states, the semileptonic decay width could be calculated quite reliably using the QCD sum rule approach. The relevant diagram is given in Fig. 2, to which nonperturbative contributions are added. This has been done with a good success for the semileptonic decays of the \( D \) and \( D_s \) into pseudoscalar and vector mesons [6]. For the production of a pseudoscalar or scalar \( \bar{q}q \) states several groups [7], [18] predict all form factors to be: \( f_+(0) \approx 0.5 \), where a similar value has been obtained from a completely independent approach [19] based on the constituent quark model. This yields a decay rate:

\[
\Gamma(D \rightarrow S_2\ell\nu) = (8 \pm 3)10^{-16} \text{ GeV},
\]

for \( M_{S_2} \approx 600 \text{ MeV} \).

- However, because of the enigmatic nature of the \( \sigma \), one has also considered, in Fig. 2, the case that the quark-antiquark current occurring in Fig. 2 does not couple to a resonance but rather to an uncorrelated quark-antiquark pair. In that case the decay rate is reduced by a factor 2, but in the spectral distribution nevertheless there is a broad bump visible with a maximum near the presumed \( \sigma \) mass of 600 MeV (see Figure 4 of [20]). Unfortunately even in high statistics experiments the estimated decay rates of the \( D \)-meson are at the edge of observation since the decays into an isocalar are CKM-suppressed due to the \( c-q \) transition at the weak vertex.

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\[\text{This has to be contrasted with the small mass-mixing coming from the off-diagonal two-point function [14].}\]
Scalar gluonium and/or $\bar{s}s$ productions

The diagrams for a semileptonic decay into a gluonium state, are given in Fig. 3. Unfortunately the evaluation of these diagrams is more involved than in the $\bar{q}q$ case. Therefore, we can give only semi-qualitative results which however are model independent.

- The only way to obtain a non-CKM suppressed isoscalar is to look at the semileptonic decay of the $D_s$-meson. As shown in Figs. 2 and 3, the quark $q_1$ is a strange one and an isoscalar $\bar{s}s$ or gluonium state can be formed.

- If the $\bar{s}s$ is relatively light ($< 1$ GeV), which might be the natural partner of the $\bar{u}u + \bar{d}d$ interpreted to be a $\sigma(0.6)$ as often used in the literature, then, one should produce a $KK$ pair through the isoscalar $\bar{s}s$ state. The QSSR prediction for this process is under quite good control [6, 7]. The non-observation of this process will disfavour the $\bar{q}q$ interpretation of the $\sigma$ meson.

- If a gluonium state is formed it will decay with even strength into $\pi\pi$ and a $K\bar{K}$ pairs. Therefore a gluonium formation in semileptonic $D_s$ decays should result in the decay patterns:

$$D_s \rightarrow \sigma B \ell \nu \rightarrow \pi \pi \ell \nu \quad D_s \rightarrow \sigma B \ell \nu \rightarrow K \bar{K} \ell \nu,$$

with about the same rate up to phase space factors. The observation of the semileptonic $\pi\pi$ decay of the $D_s$ would be a unique sign for glueball formation.

- A semi-qualitative estimate of the above rates can be obtained by working in the large heavy quark mass limit $M_c$. Using e.g. the result in [5], the one for light $\bar{q}q$ quarkonium production behaves as:

$$\Gamma(D_s \rightarrow S_{q} (\bar{q}q) \ell \nu) \sim |V_{cq}|^2 G_F^2 M_c^5 |f_+(0)|^2 .$$

- For the $\sigma_B(gg)$ production, we study the $1/M_c$ behaviour of the $WWgg$ box diagram in Fig. 3, where it is easy to find that the dominant (in $1/M_c$) contribution comes from the one in Fig. 3a. Therefore the production amplitude can be described by the Euler-Heisenberg effective interaction:

$$\mathcal{L}_{\text{eff}} \sim \frac{g_W \alpha_s}{p^2 M_c^2} F_{\mu \nu} F^{\mu \nu} G_{\alpha \beta} G^{\alpha \beta} + \text{permutations} + O(\frac{1}{M_c^4})$$

where $g_W$ is the electroweak coupling and $p^2 \simeq M_{\sigma}^2$ is the typical virtual low scale entering into the box diagram. Using dispersion techniques similar to the one used for $J/\Psi \rightarrow \sigma_B \gamma$ processes [21, 12, 3], one obtains, assuming a $D_s$ and $\sigma_B$-dominances:

$$\Gamma(D_s \rightarrow \sigma_B (gg) \ell \nu) \sim |V_{cs}|^2 G_F^2 M_c^3 \frac{1}{M_c^2 M_{\sigma}^2} |\langle 0 | \alpha_s G^2 | \sigma_B \rangle|^2$$
The matrix element $\langle 0 | \alpha_s G^2 | \sigma_B \rangle$ is by definition proportional to $f_\sigma M_\sigma^2$, where $f_\sigma$ is hopefully known from two-point function QSSR analysis [12, 13, 14]. Using $f_\sigma \approx 0.8$ GeV, one then deduce:

$$\Gamma(D_s \to \sigma_B (gg) \ell \nu) \sim \frac{1}{|f_\sigma(0)|^2} \left( \frac{f_\sigma}{M_c} \right)^2 \sim O(1)$$

This qualitative result indicates that the gluonium production rate can be of the same order as the $\bar{q}q$ one contrary to the naive perturbative expectation of the $\alpha_s^2$ suppression rate. This result being a consequence of the OZI-rule violation of the $\sigma_B$ decay.

However, it also indicates that, due to the (almost) universal coupling of the $\sigma_B$ to Goldstone boson pairs, one also expects a production of the $K\bar{K}$ pairs, which can compete with the one from $\bar{s}s$ quarkonium state, and again renders more difficult the identification of the such state $\bar{s}s$ if allowed by phase space.

## 5 Conclusions

After reminding the different features from QCD spectral analysis of the scalar two- and three-point functions which do not favour the $\bar{q}q$ interpretation of the broad and low mass $\sigma(0.6)$, we have emphasized that a measurement of the $D_s$ semileptonic decays into $\pi\pi$ can reveal in a clean, unique and model-independent way the eventual gluon component of the $\sigma$ meson. The analysis also implies that one expects an observation of the $K\bar{K}$ final states from the $\sigma_B$ which can compete (if any) with the one expected from a $\bar{s}s$ state assumed in the literature to be the SU(3) partner of the observed $\sigma(0.6)$ often interpreted as a $\bar{u}u + \bar{d}d$ state.

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