The Hawking temperature in the context of dark energy

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Abstract – An emergent-gravity metric incorporating k-essence scalar fields \( \phi \) having a Born-Hufeld-type Lagrangian is mapped into a metric whose structure is similar to that of a blackhole of large mass \( M \) that has swallowed a global monopole. However, here the field is not that of a monopole but rather that of a k-essence scalar field. If \( \phi_{\text{emergent}} \) are the solutions of the emergent-gravity equations of motion under cosmological boundary conditions at \( \infty \), then for \( r \to \infty \) the rescaled field \( \tilde{\phi}_{\text{emergent}} \) has exact correspondence with \( \phi \). The Hawking temperature of this metric is given by

\[
T_{\text{emergent}} = \frac{\hbar^2}{2GMK}(1 - K)^2 = \frac{k}{\pi \sigma M^2}(1 - K)^2,
\]

taking the speed of light \( c = 1 \). Here \( K = \tilde{\phi}_{\text{emergent}}^2 \) is the kinetic energy of the k-essence field \( \phi \) and \( K \) is always less than unity, \( k_B \) is the Boltzmann constant.

The motivation of this work is to show that the Hawking temperature \( T_{\text{emergent}} \) is modified in the presence of dark energy. This is realised in the following scenario: Dynamical solutions of the k-essence equation of motion changes the metric for the perturbations around these solutions [8,9]. The perturbations propagate in an emergent spacetime with metric \( \tilde{G}^{\mu \nu} \) different from (and also not conformally equivalent to) the gravitational metric \( g^{\mu \nu} \). \( \tilde{G}^{\mu \nu} \) now depends on \( \phi \). Now, if a global monopole falls into a Schwarzschild blackhole the resulting metric is different from the Schwarzschild case and the blackhole carries the global monopole charge. Barriola and Vilenkin obtained solutions for Einstein equations outside the monopole core [10]. We determine the k-essence scalar field configurations \( \phi \) for which the metric \( \tilde{G}^{\mu \nu} \) becomes conformally equivalent to the Barriola-Vilenkin (BV) metric. However, in our case the global monopole charge is now replaced by the kinetic energy of the k-essence scalar field \( \phi \). Thus we show that BV-type metrics can also result from k-essence theories and not necessarily from global monopoles only. It should be mentioned that monopoles are akin to topological defects and there exist substantial and well-known literature on the subject. Recent interesting developments in this area can be found in [11].

So one can calculate the Hawking temperature \( T_{\text{emergent}} \) for such a metric and this is obviously different from that of the Schwarzschild case. Moreover, if \( \phi_{\text{emergent}} \) are the solutions of the emergent gravity equations of motion for \( r \to \infty \), then the rescaled field \( \tilde{\phi}_{\text{emergent}} \) has exact correspondence with the k-essence scalar field \( \phi \) configurations for which the BV metric is realised. \( M \) is the mass of the BV blackhole and \( M \) is very large, i.e., \( M \gg \frac{G}{M} \), where \( \delta \sim \frac{1}{\eta} = \frac{1}{M} \) is the monopole core size. Here the global monopole Lagrangian is

\[
L = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{2} \lambda (\phi^a \phi^a - \eta^2)^2
\]

where \( \phi^a \) is a triplet of scalar fields (\( a = 1, 2, 3 \)) and the global \( O(3) \) symmetry is spontaneously broken to \( U(1) \). \( \eta \sim 10^{16} \text{ GeV} \) is the typical grand unification scale. This

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result is phenomenologically interesting in the context of Belgiojoro et al.’s [12] demonstration of spontaneous emission of photons in a gravitational analogue experiment for Hawking radiation. Accordingly, the plan of the paper is as follows. In the next section we give a brief review of emergent gravity. In the following section we show how an emergent-gravity metric incorporating k-essence scalar fields $\phi$ and having a Born-Infeld–type Lagrangian can be mapped into a Barriola-Vilenkin metric and how these field configurations are related to solutions of the emergent-gravity equations of motion for $r \to \infty$. The Hawking temperature is calculated in the subsequent section. The phenomenological consequences are then discussed in the context of the analogue gravity experiments of Belgiojoro and the last section is the conclusion.

**Emergent gravity.** – The k-essence scalar field $\phi$ minimally coupled to the gravitational field $g_{\mu\nu}$, has action

$$S_k[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} L(X, \phi),$$

where $X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$. The energy momentum tensor is

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S_k}{\delta g^{\mu\nu}} = L_X \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} L,$$

and is physically meaningful only when $1 + 2X L_X > 0$.

We first carry out the conformal transformation $\tilde{G}_{\mu\nu} \equiv \frac{-1}{L_X} G_{\mu\nu}$, with $c_2^2(X, \phi) \equiv (1 + 2X L_X)^{-1}$ sound speed. Then the inverse metric of $G_{\mu\nu}$ is

$$G_{\mu\nu} \equiv \frac{L_X}{c_2^2} \left[ g_{\mu\nu} - c_2^2 \frac{L_X}{L_X} \nabla_\mu \phi \nabla_\nu \phi \right].$$

A further conformal transformation $\hat{G}_{\mu\nu} \equiv \frac{c_2}{L_X} G_{\mu\nu}$ gives

$$\hat{G}_{\mu\nu} = g_{\mu\nu} - \frac{L_X}{L_X + 2XL_X} \nabla_\mu \phi \nabla_\nu \phi.$$

Note that one must always have $L_X \neq 0$ for the sound speed $c_2^2$ to be positive definite and only then eqs. (1)–(4) will be physically meaningful. This can be seen as follows. $L_X = 0$ implies that $L$ does not depend on $X$ so that in eq. (1), $L(X, \phi) \equiv L(\phi)$. So the k-essence Lagrangian $L$ becomes pure potential and the very definition of k-essence fields becomes meaningless because such fields correspond to Lagrangians where the kinetic energy dominates over the potential energy. If there are no derivatives of the field in the Lagrangian then there is no question of identifying a kinetic energy part. Also, the very concept of minimally coupling the k-essence field $\phi$ to the gravitational field $g_{\mu\nu}$ becomes redundant, eq. (1) meaningless and eqs. (4)–(6) ambiguous.

For the non-trivial configurations of the k-essence field $\partial_\nu \phi \neq 0$ and $G_{\mu\nu}$ is not conformally equivalent to $g_{\mu\nu}$. So this k-essence field has properties different from canonical scalar fields defined with $g_{\mu\nu}$ and the local causal structure is also different from those defined with $g_{\mu\nu}$. Further, if $L$ is not an explicit function of $\phi$ then the equation of motion (3) is replaced by

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_k}{\delta \phi} = G^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0.$$

We shall take the Lagrangian as $L = L(X) = 1 - V \sqrt{1 - 2X}$. This is a particular case of the BI Lagrangian $L(X, \phi) = 1 - V(\phi) \sqrt{1 - 2X}$ for $V(\phi) = V = \text{const}$ and $V < \text{kinetic energy of } \phi$, i.e. $V(\phi)^2$. This is typical for the k-essence field where the kinetic energy dominates over the potential energy. Then $c_2^2(X, \phi) = 1 - 2X$. For scalar fields $\nabla_\mu \phi = \partial_\mu \phi$. Thus (6) becomes

$$\hat{G}_{\mu\nu} = g_{\mu\nu} - \partial_\mu \phi \partial_\nu \phi.$$

The rationale of using two conformal transformations now becomes clear. The first transformation is used to identify the inverse metric $G_{\mu\nu}$. The second conformal transformation realises the mapping onto the metric given in (8) for the Lagrangian $L(X) = 1 - V \sqrt{1 - 2X}$.

Consider the second conformal transformation $\hat{G}_{\mu\nu} \equiv \frac{c_2}{L_X} G_{\mu\nu}$. Following [8] the new Christoffel symbols are related to the old ones by

$$\Gamma_{\mu\nu}^\alpha = \Gamma_{\mu\nu}^\alpha + (1 - 2X)^{-1/2} c_2^2 \Gamma_{\mu\nu}^\gamma G_{\gamma\nu}(1 - 2X)^{-1/2} + G_{\mu\nu} \partial_\gamma(1 - 2X)^{1/2} + \delta_{\mu\gamma} \partial_\nu X + \delta_{\nu\gamma} \partial_\mu X.$$

Note that the second term on the right-hand side is symmetric under exchange of $\mu$ and $\nu$ so that the symmetry of $\hat{\Gamma}$ is maintained. The second term has its origin solely to the k-essence Lagrangian and this additional term signifies additional interactions (forces). The geodesic equation in terms of the new Christoffel connections $\hat{\Gamma}$ now becomes

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma_{\mu
u}^\alpha \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0.$$

**Mapping on to the Barriola-Vilenkin–type metric.** – Taking the gravitational metric $g_{\mu\nu}$ to be Schwarzschild, $\partial_t \phi \equiv \phi$, $\partial_r \phi \equiv \phi'$ and assuming that the k-essence field $\phi(r, t)$ is spherically symmetric one has

$$\hat{G}_{00} = g_{00} - (\partial_t \phi)^2 = 1 - 2GM/r - \phi^2,$$

$$\hat{G}_{11} = g_{11} - (\partial_r \phi)^2 = -(1 - 2GM/r)^{-1} - (\phi')^2,$$

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\[ \bar{G}_{22} = g_{22} = -r^2, \]
\[ \bar{G}_{33} = g_{33} = -r^2\sin^2\theta, \]
\[ \bar{G}_{01} = \bar{G}_{10} = -\dot{\phi}_0. \]

(9)

For the Schwarzschild metric, \( g_{00} = (1 - 2GM/r); g_{11} = -(1 - 2GM/r)^{-1}; g_{22} = -r^2; g_{33} = -r^2\sin^2\theta; g_{ij} (i \neq j) = 0. \) So the emergent-gravity line element becomes

\[ ds^2 = (1 - \frac{2GM}{r} - \dot{\phi}_0^2)dt^2 - (1 - \frac{2GM}{r})^{-1} + (\dot{\phi}_0^2)dr^2 - 2\dot{\phi}_0'dtdr - r^2d\Omega^2, \]

(10)

where \( d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2. \)

Making a co-ordinate transformation from \((t, r, \theta, \phi)\) to \((\omega, r, \theta, \phi)\) such that [13]:

\[ d\omega = dt - \left( \frac{\dot{\phi}_0}{1 - \frac{2GM}{r} - \dot{\phi}_0^2} \right) dr. \]

(11)

Then (10) becomes

\[ ds^2 = (1 - \frac{2GM}{r} - \dot{\phi}_0^2)d\omega^2 - \left[ \frac{(\dot{\phi}_0^2)}{1 - \frac{2GM}{r}} + \frac{1}{(1 - \frac{2GM}{r})} + (\dot{\phi}_0^2) \right] dr^2 - r^2d\Omega^2; \]

(12)

(12) will be a blackhole metric if \( \bar{G}_{00} = \bar{G}_{11}^{-1} \), i.e.,

\[ \dot{\phi}_0^2 = (\dot{\phi}_0)^2(1 - \frac{2GM}{r})^2. \]

(13)

Let us assume a solution to (13) of the form \( \phi(r, t) = \phi_1(r) + \phi_2(t) \). Then (13) reduces to

\[ \dot{\phi}_0^2 = (\phi_1)^2(1 - \frac{2GM}{r})^2 = K, \]

(14)

\( K \neq 0 \) is a constant (\( K \neq 0 \) means the k-essence field will have non-zero kinetic energy). The solution to (14)

\[ \phi(r, t) = \phi_1(r) + \phi_2(t) = \sqrt{\frac{K}{r + 2GM\ln(r + 2GM)}} + \sqrt{K} t \]

(15)

with \( \phi_1(r) = \sqrt{K}[r + 2GM\ln(r + 2GM)]; \phi_2(t) = \sqrt{K} t, \) and we have taken an arbitrary integration constant to be zero. So the line element (12) reduces to

\[ ds^2 = \left( 1 - \frac{2GM}{r} - K \right) d\omega^2 - \frac{1}{1 - \frac{2GM}{r} - K} dr^2 - r^2d\Omega^2 \]

(16)

and this is the Barriola-Vilenkin blackhole which represents the situation where a global monopole carrying charge \( K = \dot{\phi}_0^2 = \text{const} \) has fallen into a Schwarzschild blackhole. It should be noted that \( K \) has to be always less than unity because if \( K \) is greater than unity the signature of the metric (16) becomes ill defined. This is easily seen: for \( K > 1, \bar{G}_{00} \) is negative while \( \bar{G}_{11} \) is positive. However, it should also be noted that \( K \gg V \). This is a requirement for k-essence fields where the kinetic energy dominates over the potential energy. Therefore, we have \( K < 1 \) and \( V \ll K \).

So the metric components are

\[ \bar{G}_{00} = g_{00} - (\partial_0^2)^2 = (1 - \frac{2GM}{r} - K), \]
\[ \bar{G}_{11} = g_{11} - (\partial_r^2)^2 = -(1 - \frac{2GM}{r} - K)^{-1}, \]
\[ \bar{G}_{22} = g_{22} = -r^2; \]
\[ \bar{G}_{33} = g_{33} = -r^2\sin^2\theta. \]

(17)

In the context of global monopoles [10], the above metric has been shown to satisfy the Einstein field equations. Thus we have shown that this metric can also arise from an emergent-gravity scenario with k-essence scalar fields and the global monopole charge is now replaced by the constant kinetic energy of the k-essence field.

Now it is also known that the solutions to the emergent-gravity equations of motion (7) for the scalar field under cosmological boundary conditions are given by [9] \( \phi_{\text{emergent}}(t,r) = \text{const}[t + r + 2GM\ln(\frac{r - GM}{r}) - 2GM - K]\)

\[ F(r) = \frac{r}{r - 2GM} \left[ \sqrt{\frac{A - 2GM}{A - (A - 1)r}} - 1 \right] \]

where \( A \) is a constant. Substituting this solution in (13) and taking the limit \( r \to \infty \) and ignoring terms of \( O(\frac{1}{r^2}) \) and higher gives

\[ \lim_{r \to \infty}(\phi_{\text{emergent}}')^2(1 - \frac{2GM}{r})^2 = K(2GM - 1)^2. \]

(18)

Therefore for \( r \to \infty \), the rescaled field \( \phi_{\text{emergent}}(t,r) \) has exact correspondence with the k-essence scalar field \( \phi \) which satisfies the blackhole metric condition (13).

**Hawking temperature.** – Let us calculate the Hawking temperature for this metric (16) using the tunnelling formalism as outlined in [14] which corrects for the factor of two in the Hawking temperature as often mentioned, e.g., in [15]). Going over to the Eddington-Finkelstein coordinates \((v, r, \theta, \phi)\)

\[ v = \omega + r^*, \quad u = \omega - r^*, \]
\[ r^* = \frac{r}{1 - K} + \frac{2GM}{(1 - K)^2} \ln[(1 - K)r - 2GM], \]

we get

\[ ds^2 = (1 - 2GM/r)dv^2 - 2dvdr - r^2d\Omega^2. \]

(19)

By analogy with the Schwarzschild case a massless particle in the background of \( G_{\mu\nu} \) is described by the Klein-Gordon equation

\[ \hbar^2(-\bar{\Delta})^{-1/2}\partial_\mu(\bar{\Delta}^{1/2}\partial_\mu\Psi) = 0. \]

(21)

One expands

\[ \Psi = \exp\left(-\frac{i}{\hbar}S + \ldots\right), \]

(22)

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temperature of the emitted photons in the laboratory reference frame [16,17]. However, Belgiorno et al. also pointed out that the dielectric medium in which the RIP is created will always be dominated by optical dispersion and therefore the spectrum will not be that of a perfect blackbody and that in any case only a limited spectral portion of the full spectrum will be observable. To show this last point they described the RIP as a perturbation induced by the laser pulse on top of a uniform, dispersive background refractive index $n_0$, i.e., $n(z,t,\omega) = n_0(\omega) + \delta n f(z-\nu t)$, where $\omega$ is the optical frequency, $f(z-\nu t)$ is a function bounded by 0 and 1, that describes the shape of the laser pulse. In the reference frame co-moving at velocity $\nu$ with the RIP, the event horizon in a 2D geometry is defined by $c/\nu = n$ which admits solutions only for RIP velocities satisfying the inequality [17]: $1/n_0(\omega)+\delta n < \frac{\nu}{c} < 1/n_0(\omega)$. This predicts an emission spectrum with well-defined boundaries and it is precisely this feature of the spectral emission that is peculiar to analogue Hawking radiation. In the experiment a clear photon emission was registered in the wavelength window predicted by the last inequality, the emitted radiation was unpolarized, and the emission bandwidth increased with the input energy. The Bessel pulse intensity evolution along the propagation direction $z$ was estimated analytically from the input energy. By fitting the measured spectra with Gaussian functions the bandwidth was estimated as a function of input energy and Bessel intensity. Using the fused silica dispersion relation the authors obtained the bandwidth and the $\delta n$ as a function of input energy and Bessel peak intensity (at $z = 1$ cm where measurements were performed). There was a clear linear dependence which was in qualitative agreement with the fact that the emission bandwidth was predicted to depend on $\delta n$ which in turn is a linear function $\delta n = n_2 I$ of the pulse intensity $I$. The slope of the linear fit was in good agreement with the tabulated value [18,19]. Therefore there is also an agreement at the quantitative level between the measurements and the model based on Hawking-like radiation emission.

In this context, we propose that a different Hawking temperature should give a different set of values for the above-mentioned phenomenological parameters i.e. the slope $n_2$ and also the inequality $\frac{1}{n_0(\omega)+\delta n} < \frac{\nu}{c} < \frac{1}{n_0(\omega)}$ etc. because the intensity of photon emission should depend on the relevant blackbody temperature. Therefore, Belgiorno et al.’s gravitational analogue experiment has scope of being further enhanced into testing the existence of other cosmological entities like dark energy. In order to include the effects of dark energy, the RIP method must be accordingly modified. At a basic level this means that the effect of the presence of the constant $K$ in the metric must be included. For example, if we consider the Schwarzschild metric, the Belgiorno et al.’s experiment currently has $K = 0$. So the experimental situation has to move over to a scenario which can mimic $K \neq 0$, i.e., $0 < K < 1$.

Here some aspects need to be clarified. First, note that in (16) if we take $M \sim M_{\text{monopole core}}$, i.e. $M$ is
very small and negligible, we have $ds^2 = (1 - K)d\omega^2 - \frac{1}{(1 - K)^2} dr^2 - r^2 d\Omega^2$ and rescaling $r$ and $\omega$ one has $ds^2 = dw^2 - dr^2 - (1 - K)r^2 d\Omega^2$ and this is the metric of the global BV monopole and describes a space with a deficit solid angle, i.e. the area of a sphere of radius $r$ is not $4\pi r^2$ but $(1 - K)4\pi r^2$, $K < 1$. Such spaces are not asymptotically flat, but asymptotically bound.

Secondly, we are dealing with the BV blackhole, i.e. $M \gg M_{\text{monopole core}}$, i.e. $2GM/r = \frac{2M}{r}$ is not negligible, i.e. for $r \sim \delta$ where $\delta$ is the monopole core size. Here also for $r \to \infty$ the metric (16) is not strictly asymptotically flat owing to the presence of $K$.

If the Schwarzschild metric is the reference then one has to move over from an asymptotically flat metric to one which is not exactly asymptotically flat but rather asymptotically bound. This aspect can be incorporated into the Belgiorno et al.’s analogue gravity experiment in the following way in order for the results to be compatible to the existence of an analogue event horizon corresponding to a Hawking temperature $T_{\text{emergent}}$. Here we draw heavily from ref. [17]. Consider the wave equation for a perturbation of a full nonlinear electric field propagating in a nonlinear Kerr medium where for simplicity the electric field has been replaced by a scalar field $\phi$, (eq. (1) of ref. [17]):

$$\frac{n^2(x_l - vt)}{c^2} \partial^2_t \Phi - \partial^2_x \Phi - \partial^2_y \Phi - \partial^2_z \Phi = 0,$$

where all coordinates are in the lab frame. The suffix $l$ is omitted from $y, z$ because they are not involved in the boost relating the lab frame with the pulse frame. $n(x_l - vt)$ is the refractive index that accounts for the propagating RIP in the dielectric. The RIP is propagating with a constant velocity $v$. The analogue Hawking temperature (eq. (13) of ref. [17]) for the blackhole horizon $(x_+)$ is

$$T_+ = \frac{\gamma^2 v^2 c^2}{2\pi k_B c} \left[ \frac{dn}{dx} \right]_{x_+},$$

where $\gamma$ is the boost, $v$ is the constant velocity of the RIP, $k_B$ is the Boltzmann constant and $x_+$ denotes the blackhole horizon. Note that in order to have $T_{\text{emergent}} = (1 - K)^2 T_H$ one can have either of the following scenarios:

**Case 1:**

Realise an experimental situation where

$$\gamma^2 \to \gamma_1^2 = (1 - K)^2 \gamma^2. $$

This would imply

$$v_1^2 = \frac{v^2 - c^2 K(2 - K)}{(1 - K)^2}. $$

As $v_1^2$ must be positive, one should ensure that

$$K(2 - K) < \frac{v^2}{c^2}. $$

All this should also be made consistent with

$$\frac{c}{v_1} = n_1 |_{x_+} = n_{10} + k_1 \eta_1,$$

where the parameter $\eta_1 \ll 1$ and $k_1$ is the normalised intensity of the pulse at the blackhole horizon taking values in the interval (0, 1).

**Case 2:**

Realise an experimental situation with

$$n \to n_2 = (1 - K)^2 n.$$

Obviously the RIP propagation velocity $v$ must be changed to some new value $v_2$ together with a new consistency condition

$$\frac{c}{v_2} = n_2 |_{x_+} = n_{20} + k_2 \eta_2.$$ 

Therefore, a realisation of photon emission (similar to Belgiorno et al.’s original experiments) corresponding to a blackbody temperature $T_{\text{emergent}}$ in any of the above-described two scenarios will be compatible with a theory of $k$-essence fields in an emergent-gravity metric in the same way as the original Belgiorno experiment, though not proving the existence of blackhole horizons, however, proves the relation between blackhole horizons and the Hawking radiation.

**Conclusion.** – We have described here a situation where a BV-type metric can also result from scenarios with $k$-essence scalar fields. The Hawking temperature for such a metric has a dark energy component and we have indicated a phenomenological scenario (in the context of Belgiorno et al.’s gravitational analogue experiment [12,17]) where there is scope for this to be tested.

Our work is in the domain of an emergent-gravity metric $G^{\mu\nu}$ incorporating $k$-essence scalar fields $\phi$ having a Born-Infeld–type Lagrangian. We first determine the scalar field configurations for which $G^{\mu\nu}$ is mapped into a BV-type metric. We then show that if $\phi_{\text{emergent}}$ be solutions of the emergent-gravity equations of motion under cosmological boundary conditions at $\infty$, then for $r \to \infty$ the rescaled field $\tilde{\phi}_{\text{emergent}} = \frac{T_{\text{emergent}}}{2GM-1} \phi \big|_{r=\delta}$ has exact correspondence with $\phi$ with $\phi(r, t) = \phi_1(r) + \phi_2(t)$. The Hawking temperature of the resulting BV-type metric is found to be $T_{\text{emergent}} = (1 - K)^2 T_H$. Here $K = \frac{\delta_k^2}{2}$ is the kinetic energy of the $k$-essence field $\phi$ and $K$ is always less than unity. We have then indicated why certain phenomenological parameters in Belgiorno’s analogue gravity experiment will be modified because of the difference of the Schwarzschild metric from that of a BV-type metric. Finally, we described certain analogue gravity experimental situations with which the compatibility of our theoretical considerations may be tested.

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