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Dirac and normal states on Weyl-von Neumann algebras. (English) [Zbl 1462.81115]  
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Summary: We study particular classes of states on the Weyl algebra \( W \) associated with a symplectic vector space \( S \) and on the von Neumann algebras generated in representations of \( W \). Applications in quantum physics require an implementation of constraint equations, e.g., due to gauge conditions, and can be based on the so-called Dirac states. The states can be characterized by nonlinear functions on \( S \), and it turns out that those corresponding to non-trivial Dirac states are typically discontinuous. We discuss general aspects of this interplay between functions on \( S \) and states, but also develop an analysis for a particular example class of non-trivial Dirac states. In the last part, we focus on the specific situation with \( S = L^2(\mathbb{R}^n) \) or test functions on \( \mathbb{R}^n \) and relate properties of states on \( W \) with those of generalized functions on \( \mathbb{R}^n \) or with harmonic analysis aspects of corresponding Borel measures on Schwartz functions and on temperate distributions.

MSC:

81S10  Geometry and quantization, symplectic methods  
53D50  Geometric quantization  
46L30  States of selfadjoint operator algebras  
81R10  Infinite-dimensional groups and algebras motivated by physics, including Virasoro, Kac-Moody, \( W \)-algebras and other current algebras and their representations  
81R25  Spinor and twistor methods applied to problems in quantum theory  
46F05  Topological linear spaces of test functions, distributions and ultradistributions  
46F10  Operations with distributions and generalized functions

Keywords:

Weyl algebra; quantization with constraints; measures on non-locally compact groups; generalized functions

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