The Intensity Modulation of the Fluorescent Line by a Finite Light Speed Effect in Accretion-powered X-Ray Pulsars

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Abstract

The X-ray line diagnostic method is a powerful tool for an investigation of plasma around accretion-powered X-ray pulsars. We point out an apparent intensity modulation of emission lines, with their rotation period of neutron stars, due to the finite speed of light (we call this effect the “finite light speed effect”) if the line emission mechanism is a kind of reprocessing, such as fluorescence or recombination after ionization by X-ray irradiation from pulsars. The modulation amplitude is determined by the size of the emission region, which is in competition with the smearing effect by the light crossing time in the emission region. This is efficient if the size of the emission region is roughly comparable to that of the rotation period multiplied by the speed of light. We apply this effect to a symbiotic X-ray pulsar, GX 1+4, where a spin modulation of the intense iron line of which has been reported. The finite light speed effect can explain the observed intensity modulation if its fluorescent region is the size of \( \sim 10^{12} \) cm.

Key words: pulsars: individual (GX 1+4) – stars: neutron – X-rays: binaries

1. Introduction

Many of the accretion-powered X-ray pulsars exhibit prominent iron emission lines from neutral atoms or ions. A detailed investigation of these emission lines is useful for the diagnostics of their emission mechanism. The identification of emission lines (the element and its ionization state) and their observed variability in the line parameters with time and with source luminosity provide important information on the line emission mechanism and their emission regions. Several possible line emission regions have been considered, such as an inhomogeneous and clumpy stellar wind from the companion star (Sako et al. 2002; Wojdowski et al. 2003), the surface or the photosphere of the companions star, and the accreting matter around the Alfvén shell (Basko 1980; Kohmura et al. 2001; Vrtilek et al. 2005).

Bright neon and oxygen emission lines (OVII) from 4U 1626-67 were first reported by Angelini et al. (1995), which is a low-mass X-ray binary pulsar with a 7.7 s spin period. The flux modulation of the oxygen line, by a factor of up to 4, was reported by Beri et al. (2015), supporting the warped accretion-disk origin (Schulz et al. 2001). Hercules X-1 is another X-ray pulsar with a spin period of 2.3 s. Its iron line shows an intensity modulation with its spin period (Choi et al. 1994; Zane et al. 2004; Vasco et al. 2013). Zane et al. (2004) and Choi et al. (1994) reported that the intensity modulation of the emission line is roughly represented by a sinusoidal function and the bottom phase, which corresponds to the peak phase of the pulse shape of the hard X-ray continuum. But Vasco et al. (2013) reported the disappearance of the iron line flux around the peak phase of the pulse shape of the hard X-ray continuum and speculated a hollow cone geometry for the accretion column. For these fast rotators, the emission region is expected to be located either near the vicinity of the neutron star or at the inner part of the accretion disk.

The iron fluorescent line is notable in the spectrum of GX 301-2, of which the the spin period is \( \sim 700 \) s, but the question of the fluorescent region is of vigorous debate. Endo et al. (2002) analyzed the line shape of the intense iron emission line from GX 301-2, and they found a significantly broader width of the line than that estimated based on the terminal velocity of the stellar wind. They concluded that the iron lines originate within \( 10^{10} \) cm of the continuum emission source, i.e., inside of the accretion radius. Fürst et al. (2011) reported a significantly small pulse fraction of the iron emission lines but a strong pulse-to-pulse variation in its intensity. They investigated the correlation of the continuum flux and the iron emission line flux and showed that the iron emission region is not far from the X-ray source, which is consistent with the result of Endo et al. (2002). On the other hand, Suchy et al. (2012) reported results from a Suzaku observation, and they argued that the line flux did not change significantly throughout the pulse phase and concluded that the iron fluorescence region was greater than \( \sim 700 \) lt-s (\( \sim 2 \times 10^{13} \) cm) from the neutron star, although there is a tendency toward higher fluxes around phase 0.6–1.0, where the pulse profile of the continuum X-rays shows the second peak, with an amplitude of \( \sim 10\% \).

The origin of emission lines in GX 1+4, which is a symbiotic X-ray pulsar with a \( \sim 150 \) s spin period, was discussed by Kotani et al. (1999) and Yoshida et al. (2017; hereafter Paper I). Based on the ionization state of the iron atoms, the absorption column density, and the estimated X-ray luminosity derived from an Advanced Satellite for Cosmology and Astrophysics (ASCA; Tanaka et al. 1994) observation of GX 1+4, Kotani et al. (1999) suggested that the line emitting region in GX 1+4 consisted of low ionization matter that extended to greater than \( 10^{12} \) cm from the neutron star. This radius was estimated with the assumption that the matter was distributed homogeneously. The size of the emitting region of \( r > 3.4 \times 10^{12} \) cm in radius was estimated with a Suzaku observation by adopting a similar method. It was also suggested that the size of the fluorescent region can be reduced to \( \sim 10^{11} \) cm by introducing an inhomogeneity of the matter (see Paper I). Paper I also reported that the iron emission line shows its intensity modulation, with an amplitude of \( 7 \pm 1\% \), according to the pulse phase.

In this paper, we point out the importance of the following two effects in the discussion of the observed time variation of the line flux. If the fluorescent lines are emitted from such a...
large region, then the observed time variation should be smeared with the light crossing time of the region, which is roughly 100 s, for the size of $3.4 \times 10^{12}$ cm. On the other hand, we should take into account an apparent intensity modulation of the fluorescent lines by an effect of the finite light speed, as described in the next section.

2. Finite Light Speed Effect

We are now considering the fluorescent lines from matter that is exposed to X-rays from a X-ray pulsar, of which the emission profile is not spherically symmetric. Therefore, the fluorescent region, which is illuminated by X-rays from the X-ray pulsar, changes according to the rotation of the neutron star. As a consequence, the fluorescent region appears to show an apparent movement to the observer. The apparent movement affects the observed intensity per unit time of the fluorescent lines, since the speed of light is finite, even if the circumstellar matter around the neutron star is homogeneous and uniform, and the intensity of X-rays from the neutron star stays constant. If the fluorescent region is going away from an observer with a velocity of $v_{\text{los}}$, along the line of sight, then the emitted photons at the fluorescent region in 1 s are observed in $(1 + v_{\text{los}}/c)$ s by the observer. Therefore, the intensity of the fluorescent line observed in 1 s decreases with a factor of $(1 + v_{\text{los}}/c)^{-1}$. On the other hand, the intensity observed in 1 s increases by a factor of $(1 - v_{\text{los}}/c)^{-1}$ at the moment when the emission region is approaching to the observer. This phenomenon is similar to Doppler boosting, but the wavelength of the fluorescent lines does not change because the gas in the fluorescent matter is not moving, but the region of the fluorescent is changing. We call this the finite light speed effect.

As mentioned in the previous section, the fluorescent region of GX 1+4 is thought to extend to greater than $10^{12}$ cm. Since the pulse period of GX 1+4 is $\sim 150$ s, the speed of the change of the fluorescent region is comparable to the speed of light. Therefore, a significant effect of this finite light speed effect can be expected.

2.1. Model Calculation and Results

We demonstrated the finite light speed effect using the Monte Carlo simulation. Figure 1 shows a schematic drawing of a configuration of the simulation. In the simulation, a neutron star is located at the center of spherically uniform and homogeneous matter. We assume that X-rays are emitted at magnetic poles and that the magnetic axis is assumed to be tilted with an angle of $\beta$ from the rotational axis. The inclination angle of the rotational axis is denoted by an angle $i$ between the rotation angle and the line of sight. The intensity of the photons from a magnetic pole is assumed to be proportional to $\cos \theta$, where $\theta$ is the angle of direction of each photon from the magnetic axis. Although the illuminated region is indicated by the conic geometry for simplicity, the X-rays from the pulsar are emitted with the above given intensity distribution from the pole in our calculation. Based on the optical depth of the absorbing circumstellar material, the X-ray photons will be absorbed with an exponential probability. Once the absorption occurs, a fluorescent photon is immediately emitted, which can be detected by the observer. We calculate the propagation time of the individual photons from the magnetic pole to the observer. By assuming a constant emission rate of the photons from the magnetic pole, we generate an expected light curve by enumerating the arrival time of the photons.

We calculated six cases of the radius of the circumstellar matter, $r = 1 \times 10^{11}, 3 \times 10^{11}, 1 \times 10^{12}, 3 \times 10^{12}, 1 \times 10^{13}$, and $3 \times 10^{13}$ cm, with a spin period of $P = 150$ s, $\beta = 45^\circ$, and $i = 90^\circ$. We folded the six generated light curves on the spin period by assuming the time origin to be a time when a direct photon from a magnetic pole was detected by the observer. The intensities of the folded light curves were normalized as the average intensity to be unity and were plotted as a function of the spin phase in Figure 2. We can see the intensity modulation by the finite light speed effect. The amplitude of the intensity modulation becomes larger with the increasing radius of the circumstellar matter up to $1 \times 10^{12}$ cm, while, in the cases of $r \geq 3 \times 10^{12}$ cm, the amplitude gradually becomes smaller than that of $1 \times 10^{12}$ cm, as the radius becomes larger. We can also see phase shifts as a change of the radius of the circumstellar matter, $r$, where a larger phase delay is seen in the larger radius.

In Figure 3, the modulation amplitude as a function of $\beta$ is plotted for the six cases of the radius of the matter for $i = 90^\circ$. We can recognize that the modulation amplitude becomes larger when increasing the angle of $\beta$ from $0^\circ$ to $90^\circ$. The dependence on the radius of matter is again found, as shown in Figure 2 (i.e., the amplitude is the largest at the radius of $1 \times 10^{12}$ cm).

If $i = 90^\circ$, then the velocity of the apparent movement of the center of the fluorescent region along the line of sight, $v_{\text{los}}$, is calculated as $v_{\text{los}} \sim 2\pi (r/2) \sin \beta \sin(2\phi)/P$ for an optically thin case, where $\phi (=0.0-1.0)$ is the phase of the rotation of the fluorescent region from the origin defined as the direction of the observer. Therefore, $v_{\text{los}}$ becomes large at a large $\beta$ and at a

![Figure 1. Geometry used in the Monte Carlo simulation. The neutron star is located at the center of the circumstellar matter with the radius of $r$, which is represented by the largest yellow circle. The blue and red arrows indicate the rotational axis and the magnetic axis, respectively. The blue cone, whose bottom is a magnetic pole, shows the fluorescent region at a certain phase. The red and blue circles with dashed lines indicate the trajectories of the magnetic pole and the magnetic axis, respectively. The size of the neutron star is enlarged to make it easier to see.](image-url)
The obtained folded light curves were normalized by dividing by the average fluorescent intensity.

The competition effect is smeared with the light crossing time. This smearing determines the amplitude of the modulation and causes phase shifts.

We calculated the arrival time to the observer for each photon, considering six cases for a radius of the circumstellar matter, \( r = 1 \times 10^{11}, 3 \times 10^{11}, 1 \times 10^{12}, 3 \times 10^{12}, 1 \times 10^{13}, \) and \( 3 \times 10^{13} \) cm, and the spin period, \( P = 150 \) s. The mean free path of the circumstellar matter was fixed at the optical depth of the neutral iron K-edge, as derived in Paper I with Suzaku observations. The simulation was conducted for different angles of \( i \) and \( \beta \) in a range of \( 0^\circ \) to \( 90^\circ \) with a \( 5^\circ \) step, respectively.

Figure 4 shows the calculated amplitude of the intensity modulation of the fluorescent line on two-dimensional maps of the angles \( i \) and \( \beta \) for each radius. The calculated amplitude of the time variation for \( r = 1 \times 10^{11} \) cm is less than 7\% in any combination of the angles, while in cases of \( r \geq 3 \times 10^{11} \) cm, a modulation amplitude of the 7\% level can be seen in some combinations of \( i \) and \( \beta \). The solid red lines indicate the acceptable region by the uncertainty (1\%) of the observed amplitude, which were obtained from Suzaku observations (see Paper I).

The combination of the two angles, \( i \) and \( \beta \), can also be restricted by an interpretation of the pulse profile. The pulse profile of GX 1+4 in lower energy range (up to 10 keV) has a prominent sharp dip, as shown in Figure 5(b). This sharp dip is interpreted as an obscuration or eclipse of X-ray emitting region at a vicinity of a neutron star surface by the accretion column to the magnetic poles of the pulsar (Galloway et al. 2000). Thus, the hot spot (i.e., magnetic pole) of the pulsar being at a base of the accretion column should face in the direction along the line of sight at least once a period. Therefore, \( i \) should be equal to \( \beta \), with a certain acceptable range. We estimated an angle of accretion column viewed from a magnetic pole from the duration of the dip in the pulse profile. The dip was fitted by a Gaussian function and we got a width of the dip as \( 0.096 \pm 0.005 \) (FWHM) in a unit of phase. This value is equivalent to \( \theta_{\text{FWHM}} = 34.56 \pm 0.2^\circ \) in angle. We therefore estimated the acceptable range as \( i - 17^\circ \leq \beta \leq i + 17^\circ \). In Figure 4, the solid green lines indicate \( i = \beta \) and the dotted green lines indicate the acceptable range between \( i \) and \( \beta \).

The finite light speed effect constrains the modulation phase of the fluorescent line with regard to the pulse shape of the continuum emission. In the case of GX 1+4, the sharp dip is considered to occur when a direct photon from a magnetic pole is detected by the observer. In Figure 5, the intensity modulation of iron \( K_{\alpha} \) emission line of GX 1+4 obtained from Suzaku and five typical calculated modulation samples, with an amplitude of approximately 7\% that satisfy \( i = \beta \), are plotted as a function of the pulse phase. The simulated modulation curve with \( r = 1 \times 10^{12} \) cm can reasonably reproduce the data modulation, but those with \( r \geq 3 \times 10^{12} \) cm are mismatched with the data plots because of their inconsistent phase shifts. On the other hands, the modulation with an amplitude of 7\% can not be reproduced with \( r \leq 1 \times 10^{11} \) cm (see Figure 4).

2.2. Application to GX 1+4

We used the simulation framework as described in the previous section to mimic the intensity modulation of the fluorescent line observed in the symbiotic X-ray pulsar GX 1+4. Using Suzaku observations, Paper I found that the iron \( K_{\alpha} \) emission line shows the intensity modulation according to the pulse period, with an amplitude of 7 ± 1\%, peaking at around 0.7–1.1 in the phase.

At a radius of \( 1 \times 10^{11} \) cm, the light crossing time becomes \( \sim 300 \) s, which is roughly two times larger than the spin period. Therefore, the intensity modulation by the finite light speed effect is smeared with the light crossing time. This smearing decreases the amplitude of the modulation. The competition between the finite light speed effect and the smearing effect determines the amplitude of the modulation and causes phase shifts and waveform-variations, as we can see in Figure 2.
In conclusion, even if the fluorescent matter is homogeneous and uniform, the observed intensity modulation can be explained by the matter extending up to \(1 \times 10^{12}\) cm from the neutron star. This distance is much larger than the co-rotation radius and the Alfvén radius, even for the extremely strong magnetic field of \(10^{14}\) G.

Here, we discussed only one fluorescent region, though the emission from the neutron star arises from its two poles. Several previous works suggest the existence of an accretion disk in GX 1+4. Chakrabarty & Roche (1997) argued that the optical emission lines suggest the existence of thermal ultraviolet radiation from an accretion disk. Rea et al. (2005) discussed a reflection by a torus-like accretion disk. Galloway et al. (2001) interpreted the dip in the pulse profile as being due to the eclipse of the X-ray emitting region by the accretion curtain, which is formed by an accretion flow along with the magnetic field from the inner edge of the accretion disk, showing a schematic figure of accretion geometry. The size of the accretion disk is expected to be \(1 \times 10^{12}\) cm from its orbital period (Pereira et al. 1999), and thus the fluorescent emission from the other side of the neutron star may be hidden by the accretion disk.

### 3. Discussion

The finite light speed effect causes an intensity modulation of the fluorescent lines with the rotation period of X-ray pulsars. This is due to the apparent movement of the fluorescent region, which is exposed to X-rays from a pulsar. The finite light speed effect thus can be expected to act across the X-ray pulsar source population. Therefore, for an interpretation of the intensity modulation of the fluorescent lines from X-ray pulsars, this effect should be taken into account. This effect...
is efficient if the size of the fluorescent matter is comparable to the pulse period multiplied by the speed of light, or, in other words, the apparent movement speed of the fluorescent region is comparable to the speed of light. In the case of GX 1+4, the previously estimated fluorescent size ($\sim 10^{12}$ cm) by Kotani et al. (1999), if true, matches to the above condition and a relatively strong effect can be expected. Actually, the intensity modulation was detected and could be explained by this finite light speed effect.

However, in practice, the matter is not spherically symmetric, and there should be various causes of the time variation of the fluorescent line. In the case of the accretion-powered X-ray pulsars, its circumstellar condition will change as a function of its binary orbital phase. Several authors have reported the variations of the emission line parameters depending on the orbital phase of the binary system containing an X-ray pulsar, such as Centaurus X-3, GX 301-2, and Vela X-1 (Ebisawa et al. 1996; Endo et al. 2002; Naik et al. 2011; Odaka et al. 2013). On the other hand, the modulation due to the finite light speed effect should not change with the orbital phase. Therefore, the observation of the spin phase modulation at the various orbital phase would help to distinguish them.

It might be possible that the emission lines from photo-ionized plasma, which is exposed by a variable X-ray sources, also show a similar effect by the finite speed of light. In this case, we have to consider the smearing effect by a recombination timescale, as well as the light crossing time of the emission region. Also, even in pulsars with short spin periods, such as Hercules X-1 and 4U 1626-67, if the emission region is comparable to the pulse period multiplied by the speed of light and if the line photons are created by the fluorescence or by the photo-ionization, then we have to examine the finite light speed effect for the discussion of the line flux modulation.

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**References**

Angelini, L., White, N. E., Nagase, F., et al. 1995, ApJL, 449, L41
Basko, M. M. 1980, AdA, 87, 330
Beri, A., Paul, B., & Dewangan, G. C. 2015, MNRAS, 451, 508
Chakrabarty, D., & Roche, P. 1997, ApJ, 489, 254
Choi, C. S., Nagase, F., Makino, F., et al. 1994, ApJ, 437, 449
Ebisawa, K., Day, C. S. R., Kallman, T. R., et al. 1996, PASJ, 48, 425
Endo, T., Ishida, M., Masai, K., et al. 2002, ApJ, 574, 879
Fürst, F., Suchy, S., Kreykenbohm, I., et al. 2011, A&A, 535, A9
Galloway, D. K., Giles, A. B., Greenhill, J. G., & Storey, M. C. 2000, MNRAS, 311, 755
Galloway, D. K., Giles, A. B., Wu, K., & Greenhill, J. G. 2001, MNRAS, 325, 419
Kohmura, T., Kitamoto, S., & Torii, K. 2001, ApJ, 562, 943
Kotani, T., Dotani, T., Nagase, F., et al. 1999, ApJ, 510, 369
Naik, S., Paul, B., & Ali, Z. 2011, ApJ, 737, 79
Odaka, H., Khangulyan, D., Tanaka, Y. T., et al. 2013, ApJ, 767, 70
Pereira, M. G., Braga, J., & Jablonksi, F. 1999, ApJL, 526, L105
Rea, N., Stella, L., Israel, G. L., et al. 2005, MNRAS, 364, 1229
Sako, M., Kahn, S. M., Paerels, F., et al. 2002, in High Resolution X-ray Spectroscopy with XMM-Newton and Chandra, ed. G. Branduardi-Raymond (London: MSSL), 36
Schulz, N. S., Chakrabarty, D., Marshall, H. L., et al. 2001, ApJ, 563, 941
Suchy, S., Fürst, F., Pottschmidt, K., et al. 2012, ApJ, 745, 124
Tanaka, Y., Inoue, H., & Holt, S. S. 1994, PASJ, 46, L37
Vasco, D., Staubert, R., Klochkov, D., et al. 2013, A&A, 550, A111
Vrtilek, S. D., Raymond, J. C., Boroson, B., & McCray, R. 2005, ApJ, 626, 307
Wojdowski, P. S., Liedahl, D. A., Sako, M., Kahn, S. M., & Paerels, F. 2003, ApJ, 582, 959
Yoshida, Y., Kitamoto, S., Suzuki, H., et al. 2017, ApJ, 838, 30
Zane, S., Ramsay, G., Jimenez-Garate, M. A., Willem den Herder, J., & Hailey, C. J. 2004, MNRAS, 350, 506