Aging rate of spin glasses from simulations matches experiments

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Experiments on spin glasses can now make precise measurements of the exponent \(z(T)\) governing the growth of glassy domains, while our computational capabilities allow us to make quantitative predictions for experimental scales. However, experimental and numerical values for \(z(T)\) have differed. We use new simulations on the Janus II computer to resolve this discrepancy, finding a time-dependent \(z(T,t_w)\), which leads to the experimental value through mild extrapolations. Furthermore, theoretical insight is gained by studying a crossover between the \(T = T_c\) and \(T = 0\) fixed points.

The study of spin glasses (SGs) [1, 2] has long been a key problem in statistical mechanics, providing ideas that have born fruit in fields as diverse as econophysics, biology or optimization in computer science. From a fundamental point of view, SGs are paradigmatic as the most approachable model for glassy behavior, both experimentally and theoretically. However, despite this relative simplicity, SG experiments and theory have traditionally developed separately, for practical and conceptual reasons. On the one hand, numerical simulations were not long enough to reach experimental times, while experiments were not precise enough or even able to measure key physical quantities. On the other hand, experimental samples are perennially out of equilibrium, while theory mostly focuses on the (unreachable) equilibrium phase.

In a typical experiment, the system is rapidly cooled to a subcritical working temperature \(T < T_c\) and its off-equilibrium evolution (aging) studied. As the waiting time \(t_w\) increases, the size of the glassy domains is seen to grow as \(\xi(t_w) \propto t_w^{1/z(T)}\), with an exponent that is expected to behave as \(z(T) \simeq z(T_c)/T \) [3]. In traditional experiments [4], based on the shift of the peak in the relaxation rate \(S(t_w)\), \(z(T)\) was difficult to measure. Fortunately, the availability of excellent samples with a film geometry has suggested a new approach to the precision measurement of \(z_c = z(T)/T_c\) [5]. The time that \(\xi(t_w)\) needs to saturate to the film thickness relates to the activation energies \(\Delta_{\text{max}}\), vary the film thickness from 9 to 20 nm resulted in the measurement \(z_c \approx 9.62\) [5], very far from the value predicted by numerical simulations \(z_c = 6.86(16)\) [8], \(z_c = 6.80(15)\) [9].

Fortunately, recent theoretical progress makes it feasible to address the above-mentioned disagreement. A key development has been the introduction of the Janus [10] and Janus II [12] computers, which have extended the numerical exploration of the dynamics almost to the experimental scale [8, 13]. In addition, the introduction of quantitative statics-dynamics dictionaries (first based on microscopic quantities [8, 14, 15] and more recently on experimentally measurable features [13]) has clarified the relevance of the equilibrium phase for the off-equilibrium.
dynamics and showed how to extrapolate simulations to the experimental scale. Finally, the (macroscopic) experimental measurement of the size of glassy domains was shown to be consistent with the (microscopic) definition based on correlation functions [13].

Here we resolve the discrepancy in $z_c$ by finding a (very mild) scale dependence in the dynamical exponent $z(T, \xi(t_w))$. We first recognize that time should be traded by length scales. Gentle extrapolations to the relevant experimental scales of 20 nm [5] then reconcile the numerical and experimental measurements. Such a computation has been possible only thanks to new data with unprecedented precision, achieved by reducing the uncertainty due to thermal fluctuations, an issue that was typically neglected in previous numerical work. From the theoretical point of view, our study is based on a characterization of the crossover between critical and low-temperature behavior. This is a very important point, since it resolves a theoretical controversy on how low a temperature must be studied to be free of critical effects, with some authors choosing to work at very low $T$ at the expense of the system sizes that it is possible to equi- 

We consider the standard Edwards-Anderson model [18], defined on a three-dimensional cubic lattice of side $L = 160$, on whose nodes we place spins $S_x = \pm 1$ that interact with their lattice nearest neighbors through

$$\mathcal{H} = - \sum_{\langle x,y \rangle} J_{xy} S_x S_y .$$

For each disorder realization $\{J_{xy}\}$ (a sample), each of the quenched couplings $J_{xy}$ is $\pm 1$ with 50% probability. We shall refer to thin CuMn films [5], where the film thickness of 20 nm translates to a distance of 38 lattice spacings (the typical Mn-Mn distance is 5.3Å).

Our systems are initialized with random orientations for the spins (representing a very high starting temperature) and immediately quenched to the working temperature $T < T_c = 1.102(3)$ [19]. We then follow the evolution with the waiting time $t_w$ (measured in units of full lattice sweeps) at constant temperature. For each sample $\{J_{xy}\}$ we simulate $N_R$ real replicas, evolving with different thermal noise. We estimate our statistical errors with a jackknife method [20] (including fit parameters [21]).

Our basic observable is the spatial autocorrelation of the overlap field (discussed in detail in [22]),

$$C_4(T, r, t_w) = \frac{\langle q^{(a,b)}(x, t_w)q^{(a,b)}(x + r, t_w) \rangle_T}{\langle q^{(a,b)}(x, t_w)q^{(a,b)}(x, t_w) \rangle_T},$$

$$q^{(a,b)}(x, t_w) = S^{(a)}(x, t_w)S^{(b)}(x, t_w). \quad (3)$$

In these equations, the indices $(a,b)$ label the different real replicas; $\langle \cdots \rangle_T$ is the average over the thermal noise [in practice, an average over the $(a,b)$ pairs] and $\langle \cdots \rangle$ is the average over the disorder. In equilibrium simulations, by far the main source of error are the sample-to-sample fluctuations. Therefore, it has been customary to simulate the smallest $N_R$ that permits definitions such as [2] and maximize the number $N_S$ of samples. Instead, we have $N_R = 256$ and $N_S = 16$. This choice, motivated to facilitate future studies of temperature chaos [23], has proven crucial: contrary to our expectations, the increase in $N_R$ has produced a dramatic reduction of statistical errors (see Appendix A for details). As a result, we have been able to follow the decay of $C_4(T, r, t_w)$ over six decades (see inset to Fig. 1). A similar dramatic error reduction with high $N_R$ has also been seen in studies of the Gardner transition in structural glasses [24, 25].

These correlation functions decay with distance as

$$C_4(T, r, t_w) = r^{-\theta} f(r/\xi(T, t_w)),$$

so the growing $\xi$ can be computed through integral estimators [8, 22]:

$$I_k(T, t_w) = \int_0^\infty dr \, r^k C_4(T, r, t_w).$$

Then $\xi_{k+1}(T, t_w) = I_{k+1}(T, t_w)/I_k(T, t_w).$ As in recent work [13, 16, 22, 26] we use $k = 1$ (see [27] for technical details). The resulting $\xi_{12}$ is plotted in Fig. 1 for all our working temperatures.

The numerical [8, 22, 26] and experimental [5] state of the art describes the growth of $\xi_{12}$ with a power law,

$$\xi_{12}(T, t_w) \approx A(T) t_w^{1/z(T)}.$$

However, with our increased precision, [5] is no longer a faithful representation of the dynamics. Indeed, if we switch to $x = \log \xi_{12}$ as independent variable we can interpolate our data as

$$\log t_w(T, \xi_{12}) = c_0(T) + c_1(T) x + c_2(T) x^2.$$
Notice that \( c_2 = 0 \) would reduce to \([5]\), while \( c_2 > 0 \) would indicate a slowing down of the dynamics for increasing \( \xi_{12} \). Indeed, see Fig. 2, we find that \( c_2 \) vanishes only at \( T = T_c \), with \( \xi_c = z(T = T_c) = 6.69(6) \) \([25]\). Of course, \([6]\), useful as an interpolation, is not suitable to extrapolate for longer times than simulated. In order to do that, we need some insight from theory \([29]\).

We can gain much insight into the SG phase by considering the algebraic prefactor in \([4]\), determined by an exponent \( \theta \). At \( T_c \), \( \theta = 1 + \eta \), where \( \eta = -0.390(4) \) \([19]\) is the anomalous dimension. In the SG phase, there are differing expectations for \( \theta \) in the two main theoretical pictures. The droplet description \([30,32]\) expects coarsening domains and therefore \( \theta = 0 \). On the other hand, the replica symmetry breaking (RSB) theory expects space-filling domains where \( C_4 \) vanishes at constant \( r/\xi_{12} \) as \( t_w \) grows. In particular, \( \theta \) is given by the replicon, a critical mode analogous to magnons in Heisenberg ferromagnets (see \([13]\) for a detailed discussion). The best previous numerical study of \( \theta \) \([22]\), found \( \theta = 0.38(2) \), with a small \( T \) dependence that was vaguely attributed to the effect of the critical point.

We can obtain \( \theta \) by noticing that \( I_2(T, \xi_{12}) \propto \xi_{12}^{3-\theta} \). However, again we find that, while \( \theta(T_c) \) is compatible with \( 1 + \eta \), for \( T < T_c \) we actually have \( \theta(T, \xi_{12}) \), slowly decreasing as \( \xi_{12} \) increases (or \( T \) decreases). This may seem an unsatisfactory result, since, in the large-\( \xi \) limit, \( \theta(T, \xi_{12}) \) should tend to a \( T \)-independent constant (possibly zero). The simplest explanation is that low values of \( \xi_{12} \) are affected by the \( T = T_c \) fixed point with \( \theta \approx 0.61 \) [an idea supported by the higher measured \( \theta(T, \xi_{12}) \) for the higher \( T \)], while for \( \xi_{12} \to \infty \) we should see a crossover to the \( T = 0 \) fixed point, with an unknown \( \theta(T = 0) \) (see also \([26]\)).

In analogy with the ferromagnetic phase of the \( O(N) \) model, we can model this crossover in terms of a Josephson length \( \ell_j \) \([24]\). Close to \( T_c \), this should grow as \( \ell_j \propto (T_c - T)^{-v} \), with \( v = 2.56(4) \) \([19]\), while scaling corrections are expected for the lowest temperatures \([54]\).
FIG. 4. Value of the experimental aging rate for SGs $Z_c(T) = z(T, \xi)T/T_c$, extrapolated from our data for values of the coherence length corresponding to thin CuMn films. The main plot considers an ansatz (7) with a finite values of the coherence length, corresponding to thin CuMn films. Notice that critical effects are only visible for $T > T_c$, whose width represents the experimental temperature range. The roughly constant $\xi$ dynamics (8), as in [36]. This is less successful at reproducing the fitting range is minimal, see Appendix D).

Therefore, both (7) and (8) can explain the behavior of the data for the simulated scales. In order to see whether they are useful to explain the experiments we consider the quantity $Z_c(T) = z(T, \xi)T/T_c$, where $z(T, \xi)$ is the derivative of either (7) or (8) at $\xi$. The result is plotted in Fig. 4 (see Appendix D for the full fit parameters). Remarkably, the convergent ansatz of (7) produces an almost constant $Z_c$ in a wide $T$ range, which additionally fits well the experimental value of $Z_c \approx 9.62$. The activated dynamics of (8), on the other hand, is not a good fit for the experimental behavior (inset to Fig. 4).

Using simulations for very large systems with many replicas on Janus II we have found that the growth of the SG coherence length is controlled by a time-dependent $z(T, \xi(t_w))$ exponent. After describing the dynamics as governed by a crossover between a critical and a low-temperature fixed point, we have been able to model this growth quantitatively and to extrapolate to experimental length scales. The resulting exponent is consistent with the most recent experimental measurements for power-law dynamics. In addition, we find clear evidence of non-coarsening dynamics at the experimental scale and find that temperatures $T \lesssim 0.7$ are free of critical effects and therefore safe for numerical studies of the SG phase.

An open question concerns the generality of these results. Indeed, CuMn is a Heisenberg, rather than Ising, SG. However, even the purest Heisenberg system has unavoidable anisotropies, such as Dzyaloshinsky-Moriya interactions [44]. These interactions, though tiny, extend over dozens of lattice spacings, which magnifies their effect. In fact, we know that Ising is the ruling universality must consider possible systematic effects from the fitting range $\xi_1 \geq \xi_1^{\text{min}}$ and the increased statistical error due to our uncertainty in the value of $\theta$. However, see Appendix D these effects have little impact on our final estimates.
class in the presence of coupling anisotropies [45]. We also remark that high-quality measurements on GeMn are excellently fit with Ising scaling laws [7]. Our results also match the most recent and accurate measurements on CuMn [5].

More generally, this study is a clear demonstration of the importance of high-precision results for the investigation of glassiness. Indeed, reducing the errors has shown that the aging rate slows down during the dynamics, contrary to previous findings. A similar change of paradigm might happen for structural glasses.

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Appendix A: Error reduction for high number of replicas

As mentioned in the main paper, the choice of the number of replicas ($N_R$) and samples ($N_S$) was taken with the aim of improving the estimation of observables related to temperature chaos in future work, where it is important to maximize the number of possible overlaps (pairs of replicas) $N_{ov} = N_R(N_R - 1)/2$.

Unexpectedly, this has led to a dramatic increase in precision. Fig. 5 shows the reduction of the statistical error in the correlation function $C_4$ as a function of $1/\sqrt{N_{ov}}$. Moreover, this effect is enhanced as $r$ increases, which leads to a qualitative improvement in the computation of the $I_k(T, r, t_w)$ integrals (Fig. 6).

Appendix B: Controlling finite-size effects

In order to obtain an estimate for $\xi_{k,k+1}(T, t_w) = I_{k+1}(T, t_w)/I_k(T, t_w)$ we need to compute the integrals

$$I_k(T, t_w) = \int_0^\infty dr \ r^k C_4(T, r, t_w). \quad (B1)$$

As discussed in [22,27], the main difficulty in this computation is handling the large-$r$ tail where relative errors $[\Delta C_4(T, r, t_w)/C_4(T, r, t_w)]$ are big. We have to consider two issues: a) how to minimize the statistical errors and b) how to check for finite-size effects (which will appear when $\xi/L$ becomes relatively large).

As explained in more detail in [22,27], our estimate of $I_k(T, t_w)$ is the sum of the numerical integral of our measured $C_4(T, r, t_w)$ up to a self-consistent cutoff and a
from fits that include the first-image term: effects we also consider a second function

\[
\theta
\]

\[A\]

that avoids a pole singularity in the Fourier transform of \(\xi\). For the extrapolating function \(\xi\) there are several choices as to how to implement these

curves are compatible even beyond this cutoff.

The values from the \(L = 256\) simulations are plotted with conventional error bars. Notice that both curves are compatible even beyond this cutoff.

tail contribution estimated with a smooth extrapolating function \( F(r) \sim r^{-\theta} f(r/\xi) \).

In short, the procedure is

1. Obtain \( F(r) \) with fits of \( C_4 \) in a self-consistent region \([r_{\text{min}}, r_{\text{max}}]\) where the signal-to-noise ratio is still good.

2. Integrate \( C_4 \) numerically up to some cutoff and add the analytical integral of \( F(r) \) beyond the cutoff to estimate the tail contribution.

There are several choices as to how to implement these steps, which we have used to control for systematic effects. For the extrapolating function \( F(r) \), we consider first

\[
F_1(r) = A_1 r^{-\theta} e^{-(r/\xi)^{\beta_1}}.
\]

Where \( \theta \) is the replicon exponent discussed in the text and we fit for \( A_1, \beta_1 \) and \( \xi \). This analytical form is motivated by the fact that this is the simplest choice that avoids a pole singularity in the Fourier transform of \( C_4(T, r, t_w) \) at finite \( t_w \). In order to check for finite-size effects we also consider a second function \( F_2(r) \) resulting from fits that include the first-image term:

\[
F_2^\xi(r) = \frac{A_2}{r^\theta} \left[ e^{-(r/\xi)^{\beta_2}} + \frac{e^{-((L-r)/\xi)^{\beta_2}}}{(L-r)^\theta} \right],
\]

so we have a second extrapolating function

\[
F_2(r) = A_2 r^{-\theta} e^{-(r/\xi)^{\beta_2}}.
\]

For these fits we used \( \theta = 0.35 \). However, this value has very little effect on the final computation of \( \xi_{k,k+1} \). We have checked this by recomputing the integrals with \( \theta = 1 \) and \( \theta = 1+\eta \approx 0.61 \) (prediction of the droplet theory and influence of the \( T = T_c \) fixed point respectively). The different choices of \( \theta \) led to a systematic effect smaller than 20% of the error bars in the worst case.

Once we have our two extrapolating functions \( F_1 \) and \( F_2 \) we can combine them with the \( C_4 \) data in several ways:

\[
I^1_r = \int_0^{r_{\text{max}}} dr \ r^k C_4(T, r, t_w) + \int_{r_{\text{max}}}^{\infty} dr \ r^k F_1(r) \quad \text{(B5)}
\]

\[
I^2_r = \int_0^{r_{\text{min}}} dr \ r^k C_4(T, r, t_w) + \int_{r_{\text{min}}}^{\infty} dr \ r^k F_2(r) \quad \text{(B6)}
\]

\[
I^C_r = \int_0^{r_{\text{min}}} dr \ r^k C_4(T, r, t_w) + \int_{r_{\text{min}}}^{\infty} dr \ r^k F_2(r) \,. \quad \text{(B7)}
\]

The difference between \( I^C_r \) and \( I^1_r \) is always under 1% in the error, so choosing between them has no effect in any computation. In contrast, \( I^2_r \) and \( I^1_r \) present measurable differences for long \( t_w \), at least for our highest temperatures, where the faster dynamics allows us to reach higher values of \( \xi_{12}/L \). As a (very conservative) cutoff we have discarded all the \( t_w \) where \( |1 - I^C| \) is larger than 20% of the error bar, thus obtaining a \( \xi_{12}^{\text{max}}(T) \) below which we are assured not to have any finite-size effects in our \( L = 160 \) systems. The reader can find the values in Table I.

As a final check that our data is not affected by finite-size effects, we have compared our \( \xi_{12}(T, t_w) \) with that of [25]. This reference considers shorter simulations but with \( L = 256 \) and 50 samples. As shown in Fig. 7, the \( L = 160 \) and \( L = 256 \) data coincide even beyond our cutoff \( \xi_{12}^{\text{max}} \).

### Appendix C: Josephson Crossover

In this section we will give additional details on the Josephson crossover which describes how \( C_4(T, r, t_w) \) changes from being dominated by the \( T = T_c \) fixed point to being dominated by the \( T = 0 \) behavior as \( T \) and \( t_w \) vary. Assuming that \( \xi(T, t_w) \gg t_3(T) \sim (T - T_c)^{-\nu} \), the

| \( T \) | 0.55 | 0.625 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 |
|---|---|---|---|---|---|---|---|
| \( \xi_{\text{max}} \) | - | - | - | - | 18.1 | 17.3 | 17 |

**TABLE I.** Cutoff values of \( \xi_{\text{max}}(T) \) below which we are guaranteed no finite-size effects in our \( L = 160 \) lattices. For \( T < 0.9 \) the growth of \( \xi_{12} \) is very slow and we never reach the cutoff value.
where $F_k$ and $a_k$ are amplitudes. Finally, we need to eliminate the unknown $\xi$ in favor of the computable $\xi_{12}$, 
\[
\xi_{12}(T, \xi) = \frac{F_2}{F_1} \xi \left[1 + a_1' \left(\frac{\xi}{\ell_3}\right)^{-\theta} + a_2 \left(\frac{\xi}{\ell_3}\right)^{3-\theta} + \ldots\right],
\] (C3)

To estimate this derivative for a given $\xi_{12}$, we fit $\log I_2$ to a quadratic polynomial in $\log \xi_{12}$ in a $[0.75 \xi_{12}, 1.25 \xi_{12}]$ window. We then take the derivative of this polynomial at $\xi^*$. The procedure, as well as the wiggles in the resulting values of $\theta$ due to the extreme data correlation (see Fig. 5) may remind the reader of Fig. 1 in [5].

We have computed a fit to the first two terms in (C5) in the range $0 \leq \ell_1/\ell_{12} \leq 0.33$, resulting in the value of $\theta = 0.30$ reported in the main text.

The previous analysis solves the problem of the crossover between the $T = T_c$ and $T = 0$ fixed points. However, in the framework of the droplet picture one would also need to consider corrections to scaling at the $T = 0$ fixed point. This is precisely what the droplet fit in the main text to $\theta(x) \approx C x^\omega$ does.

**Appendix D: Parameter choices in our fits**

We will discuss separately the choice of $\xi_{12}^{\text{min}}$ for different temperatures and the choice of the value of $\omega$.

1. **Selection of $\xi_{12}^{\text{min}}$ for each temperature**

We have reported fits of our data to three different functional forms

\[ \log t_w = c_0(T) + c_1(T) \log \xi_{12} + c_2(T) \log^2 \xi_{12}, \quad (D1) \]
\[ \log t_w = D(T) + z_{\infty}(T) \log \xi_{12} + E(T) \xi_{12}^{1/3}, \quad (D2) \]
\[ \log t_w = F(T) + z_c \log \xi_{12} + G(T) \xi_{12}^{\phi}, \quad (D3) \]

In these fits we have used $z_c = 6.69$ and $\omega = 0.35$ ($T < T_c$), $\omega = 1.12$ ($T = T_c$), as discussed in the main text. Full results for the fits to (D2) and (D3) can be seen in tables [II] and [IV] for different fitting ranges. We include for both cases the extrapolated values of $z(T, \xi)$.
for the experimental scale (as explained in the main text we use both $\xi_{12} = 38$ and $\xi_{12} = 76$) and for [D2] also the value of the $\xi \to \infty$ aging rate $z_\infty$.

In order to make the choice of fitting range for the values plotted in the paper we have followed two criteria. Firstly we require the parameters of the fit to be stable inside the error when we increase $\xi_{12}$. Secondly, we impose that $\xi_{\min}$ be monotonically increasing in $T$ (with the exception of $T_c$, which has different behavior). Table [II] shows our final choices for $\xi_{\min}(T)$, which is the same for all three fits.

| $T$  | 0.55 | 0.625 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 |
|-----|------|-------|-----|-----|-----|-----|-----|
| $\xi_{\min}^{\xi_{12}}$ | 4    | 5     | 6   | 8   | 8   | 9   | 5   |

TABLE II. Values of $\xi_{\min}^{\xi_{12}}(T)$ determining the common fitting range $\xi_{12} \geq \xi_{\min}^{\xi_{12}}$ for our three different fits of $\log t_w$ as a function of $\log \xi_{12}$.

2. Selection of $\omega$

For our most important result, namely the extrapolation of the aging rate to the experimental scale of $\xi_{12} = 38, 76$, we have repeated our fits with our upper and lower bounds for $\omega = \theta(\xi_{\text{films}})$ (RSB and droplet extrapolations, respectively). The results are completely compatible, as we can see in Table [V].

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[29] A naive explanation for the curvature in $\xi_{12}(T, t_w)$ would be the existence of finite-size effects (see [8]). However, $c_2$ grows as we decrease $T$, while finite-size effects would be controlled by $\xi_{12}/L$, which is smaller for the lower temperatures. See Appendix D for extensive checks that our $L = 160$ are safe.

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TABLE III. Parameters of the fits to (D2) for different fitting ranges $\xi_{12} \geq \xi_{12}^{\text{min}}$. We use $\omega = 0.35$ ($\omega = 1.12$ for $T = T_c$). The fitting range that we choose for our final values is highlighted in boldface.
| $T$ | $\xi_{\text{min}} = 3.5$ | $\xi_{\text{min}} = 4$ | $\xi_{\text{min}} = 5$ | $\xi_{\text{min}} = 6$ | $\xi_{\text{min}} = 7$ | $\xi_{\text{min}} = 8$ | $\xi_{\text{min}} = 9$ |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.55 | 24.07(41) | 24.25(55) | 24.6(11) | 24.7(81) |  |  |  |
| $z(\xi = 38)$ | $z(\xi = 76)$ | 28.86(69) | 29.18(95) | 29.9(19) | 30(15) |  |  |
| $B(T)$ | 13.78(65) | 13.45(92) | 12.8(18) | 18(13) |  |  |  |
| $\Psi$ | 0.3512(92) | 0.355(21) | 0.372(33) | 0.29(24) |  |  |  |
| $\chi^2/\text{dof}$ | 13.3(47)/133 | 6.8(20)/111 | 3.2(15)/73 | 1.7(27)/40 |  |  |  |
| 0.625 | 19.73(22) | 19.72(28) | 19.36(45) | 18.53(77) |  |  |  |
| $z(\xi = 38)$ | $z(\xi = 76)$ | 23.33(38) | 23.31(49) | 22.66(79) | 21.1(13) |  |  |
| $B(T)$ | 10.36(37) | 10.39(52) | 11.3(11) | 14.0(30) |  |  |  |
| $\Psi$ | 0.354(14) | 0.352(12) | 0.334(21) | 0.290(39) |  |  |  |
| $\chi^2/\text{dof}$ | 19(10)/167 | 15(10)/147 | 8.5(33)/114 | 4.5(14)/86 |  |  |  |
| 0.7 | 16.58(22) | 16.44(23) | 16.35(27) | 16.51(32) | 16.55(52) |  |  |
| $z(\xi = 38)$ | $z(\xi = 76)$ | 15.44(33) | 15.48(37) | 15.60(46) | 16.06(59) | 16.31(68) | 16.59(93) |
| $B(T)$ | 4.16(25) | 4.13(29) | 4.01(38) | 3.57(43) | 3.36(48) | 3.13(66) | 3.0(19) |
| $\Psi$ | 0.392(13) | 0.390(19) | 0.395(18) | 0.421(27) | 0.443(31) | 0.447(50) | 0.46(18) |
| $\chi^2/\text{dof}$ | 49(38)/190 | 28(20)/173 | 10.5(83)/144 | 6.3(31)/119 | 5.7(29)/98 |  |  |
| 0.8 | 13.37(18) | 13.39(21) | 13.45(25) | 13.68(31) | 13.80(35) | 13.94(45) | 14.1(17) |
| $z(\xi = 38)$ | $z(\xi = 76)$ | 12.12(30) | 12.31(39) | 12.24(45) | 12.31(53) | 13.12(73) | 13.7(12) |
| $B(T)$ | 2.15(19) | 2.01(23) | 2.07(31) | 2.00(39) | 1.5(31) | 1.18(45) | 1.67(77) |
| $\Psi$ | 0.417(23) | 0.430(34) | 0.431(28) | 0.427(41) | 0.490(51) | 0.546(88) | 0.47(10) |
| $\chi^2/\text{dof}$ | 68(44)/165 | 46(25)/152 | 41(25)/131 | 38(24)/113 | 17(10)/98 | 8.7(45)/84 | 4.8(25)/72 |
| 0.9 | 10.76(17) | 10.86(21) | 10.82(24) | 10.86(27) | 11.23(35) | 11.49(54) | 11.13(56) |
| $z(\xi = 38)$ | $z(\xi = 76)$ | 12.12(30) | 12.31(39) | 12.24(45) | 12.31(53) | 13.12(73) | 13.7(12) |
| $B(T)$ | 2.15(19) | 2.01(23) | 2.07(31) | 2.00(39) | 1.5(31) | 1.18(45) | 1.67(77) |
| $\Psi$ | 0.417(23) | 0.430(34) | 0.431(28) | 0.427(41) | 0.490(51) | 0.546(88) | 0.47(10) |
| $\chi^2/\text{dof}$ | 68(44)/165 | 46(25)/152 | 41(25)/131 | 38(24)/113 | 17(10)/98 | 8.7(45)/84 | 4.8(25)/72 |
| 1.0 | 8.53(15) | 8.54(18) | 8.55(20) | 8.56(30) | 8.59(60) | 8.74(72) | 9.22(18) |
| $z(\xi = 38)$ | $z(\xi = 76)$ | 9.19(27) | 9.20(33) | 9.22(39) | 9.25(58) | 9.3(12) | 9.7(16) |
| $B(T)$ | 0.85(14) | 0.84(18) | 0.83(22) | 0.79(40) | 0.7(10) | 0.5(10) | 1.4(19) |
| $\Psi$ | 0.440(36) | 0.441(51) | 0.444(64) | 0.45(10) | 0.49(25) | 0.55(30) | 0.34(71) |
| $\chi^2/\text{dof}$ | 12.6(95)/137 | 12.1(90)/126 | 10.0(87)/107 | 9.3(83)/91 | 8.2(97)/78 | 0.7(11)/66 | 11(11)/55 |
| 1.1 | 6.684(12) | 6.682(14) | 6.684(11) | 6.672(31) | 6.683(32) | 6.694(31) | 6.712(41) |
| $z(\xi = 38)$ | $z(\xi = 76)$ | 6.684(13) | 6.681(11) | 6.682(14) | 6.674(41) | 6.684(41) | 6.692(31) |
| $B(T)$ | 1.9(10) | 1.71(91) | 0.4(27) | 0.02(64) | 0.0(26) | 1.5(26) | 1.2(10) |
| $\Psi$ | -0.0030(49) | 0.0037(68) | 0.03(15) | 0.29(42) | 0.37(55) | 0.002(16) | 0.023(51) |
| $\chi^2/\text{dof}$ | 34(20)/119 | 33(19)/109 | 27(18)/92 | 25(18)/78 | 23(15)/66 | 21(14)/55 | 11.0(26)/46 |

TABLE IV. Parameters of the fits to $D_3$ for different fitting ranges $12 \geq \xi^\text{min}_{12}$. We use $z_c = 6.69$. The fitting range that we choose for our final values is highlighted in boldface.
| $T$ | $z(T, \xi_{12} = 38)$ | $z(T, \xi_{12} = 76)$ |
|-----|------------------|------------------|
|     | $\omega = 0.35$ | $\omega = 0.28$ | $\omega = 0.35$ | $\omega = 0.25$ |
| 0.55 | 19.80(20) | 20.08(22) | 20.75(24) | 21.41(27) |
| 0.625 | 16.90(19) | 17.07(20) | 17.69(24) | 18.13(27) |
| 0.7 | 14.81(15) | 14.93(16) | 15.54(19) | 15.87(21) |
| 0.8 | 12.73(22) | 12.81(23) | 13.47(30) | 13.71(32) |
| 0.9 | 10.55(25) | 10.61(26) | 11.11(34) | 11.28(37) |
| 1.0 | 8.63(32) | 8.68(33) | 8.98(44) | 9.02(42) |

TABLE V. Comparison of our estimates of the experimental aging rate $z(T, \xi_{12} = \xi_{\text{films}})$ for $\xi_{\text{films}} = 38$ and $\xi_{\text{films}} = 76$ using our lower and upper bounds for $\omega = \theta(\xi_{\text{films}})$. The choice of $\omega$ is immaterial, since even in the worst case (lowest temperatures for $\xi_{\text{films}} = 76$) there is only a two-sigma difference.