Mirror skin effect and its electric circuit simulation

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We analyze impacts of crystalline symmetry on the non-Hermitian skin effects. Focusing on mirror symmetry, we propose a novel type of skin effects, a mirror skin effect, which results in significant dependence of energy spectrum on the boundary condition only for the mirror invariant line in the two-dimensional Brillouin zone. This effect arises from the topological properties characterized by a mirror winding number. We further reveal that the mirror skin effect can be observed for an electric circuit composed of negative impedance converters with current inversion where switching the boundary condition significantly changes the admittance eigenvalues only along the mirror invariant lines. Furthermore, we demonstrate that extensive localization of the eigenstates for each mirror sector result in an anomalous voltage response.

Introduction. – Topological properties [1–3] of the systems have become a central issue in condensed matter systems because of their remarkable ubiquity. The topological phenomena can be observed even for classical systems (e.g., photonic systems [4–6], mechanical systems [7–11], electric circuits [12–13] etc.), which are mathematically described by an eigenvalue problem. Among the extensive studies of the topological physics, one of the significant progresses is the discovery of the topological crystalline insulators [14–15] which has elucidated that the topological properties can be enriched by the crystalline symmetry. A prime example of the topological crystalline insulators is SnTe [16–17] where the mirror Chern number topologically protects two surface Dirac cones. Nowadays, it is found that crystalline symmetry may also induce the higher-order topological insulators/superconductors [18–27].

Along with the above significant progresses, non-Hermitian topological systems have been extensively studied, which has discovered a variety of novel phenomena [28–30]. The platform of non-Hermitian systems includes photonic systems [31–35], open quantum systems [36–38] as well as equilibrium systems where the quasi-particles have finite lifetime [39–44]. An important difference of non-Hermitian topological systems is that there exist two types of gaps [45], a line gap [46] and a point gap [47]. The line gap topology indicates the presence of the Hermitian counterpart. The point gap topology protects non-Hermitian degeneracies in the bulk [48–55], such as exceptional points [56–57], symmetry-protected exceptional rings/surfaces [58–62] etc. On these band touching points, the non-Hermitian Hamiltonian becomes non-diagonalizable. Other unique phenomena induced by the non-Hermiticity can be observed for the system with boundaries [58–63]. For non-Hermitian system with the line gap, topological invariants for the complex Brillouin zone predict the edge states [59–60–63]. Furthermore, the topological aspect of the non-Hermitian skin effect has been elucidated; the winding number characterizes the non-Hermitian skin effect of class A (no symmetry) [64–66]. The mathematically rigorous proof of the above relation has been obtained in Ref. 66 for class A. In addition, the $\mathbb{Z}_2$ skin effect with time-reversal symmetry has been proposed [66].

The above progresses for Hermitian and non-Hermitian systems lead us to the following issue; understanding impacts of crystalline symmetry on non-Hermitian topological properties which is crucial because a variety of non-Hermitian topological phenomena are expected as is the case for Hermitian systems. In particular, it is expected that the interplay between crystalline symmetry and non-Hermiticity may result in a novel type of skin effects. In spite of the significance of the above issue, there are few works addressing effects of crystalline symmetry on non-Hermitian skin effects.

Therefore, in this letter, we analyze effects of mirror symmetry on non-Hermitian skin effects, shedding new light on the interplay between crystalline symmetry and non-Hermitian topology. Our analysis discovers a novel type of skin effects, a mirror skin effect which results in significant dependence of energy spectrum on the boundary condition only along mirror invariant lines in the two-dimensional Brillouin zone. We also elucidate that a mirror winding number characterizes this skin effect. We verify the mirror skin effect by numerically diagonalizing a tight-binding model with the mirror winding number taking one. Furthermore, by making use of the ubiquity of the topological phenomenon, we theoretically suggest that the mirror skin effect can be observed for an electric circuit composed of negative impedance converters with current inversion (see Fig. 1). In this system, switching the boundary conditions drastically changes the impedance for the mirror invariant lines, which serves as a distinct evidence of the mirror skin effect for the electric circuit.

Theory of mirror skin effect. – Let us first elucidate that the topology protected by mirror symmetry induces a novel skin effect.

For comparison, we start with a brief review of the ordinary skin effect for symmetry class A. Consider a two-dimensional system under the periodic boundary condition for the $x$-direction which can be regarded as a set
of one-dimensional systems aligned along the \(x\)-direction in the momentum space. When the winding number \(\nu_{\text{tot}}(k_x)\) takes a finite value for the subsystem with a given momentum \(k_x\), the skin effect occurs; the energy spectrum significantly changes by switching the boundary condition for the \(y\)-direction, [i.e., the periodic boundary condition (PBC) to the open boundary condition (OBC)]. This relation can be understood with topological deformation; each subsystem for given momentum \(k_x\) is topologically deformed into the Hatano-Nelson model showing the skin effect. The above fact indicates that the ordinary skin effect of class A is induced by the winding number \(\nu_{\text{tot}}(k_x)\). (For later use, we call it total winding number.)

In contrast to the ordinary skin effect mentioned above, the mirror skin effect elucidated below occurs even when the total winding number is zero for arbitrary momenta \(k_x\). In the rest of this paper, we assume \(\nu_{\text{tot}}(k_x) = 0\) unless otherwise stated. Firstly, we note that the presence of mirror symmetry results in an additional topological invariant. Consider the Hamiltonian which is invariant under applying the mirror operator \(M_x\):

\[
M_x H(k) M_x^{-1} = H(M_x k),
\]

\[
M_x = U_m P_z,
\]

where \(P_z\) flips the momentum \(k := (k_x, k_y) \rightarrow M_x k := (-k_x, k_y)\). \(U_m\) is an unitary matrix satisfying \(U_m^2 = 1\). Along the mirror invariant line specified by \(k_x^\pm\), the Hamiltonian can be block-diagonalized for the plus and the minus sectors of the operator \(M_x\). Thus, besides the total winding number \(\nu_{\text{tot}}\), the following mirror winding number can be defined

\[
\nu_M = (\nu_+ - \nu_-)/2.
\]

Here, \(\nu_\pm(k_x^\pm)\) denotes the winding number computed with the block-diagonalized Hamiltonian \(H_\pm(k_x^+, k_y)\) for each sector

\[
\nu_\pm(k_x^\pm) = \int \frac{dk_y}{2\pi i} \partial_{k_y} \log \det [H_\pm(k_x^+, k_y) - E_{pg}],
\]

where \(E_{pg}\) is the reference energy for the point gap. We note that the total winding number is computed with \(\nu_{\text{tot}}(k_x^\pm) = \nu_+(k_x^\pm) + \nu_-(k_x^\pm)\) for the mirror invariant lines.

The mirror winding number taking a nontrivial value results in a skin effect; in spite of \(\nu_{\text{tot}} = 0\), the energy eigenvalues significantly depend on the boundary condition for the mirror invariant line in the Brillouin zone. We call this skin effect mirror skin effect because the mirror symmetry protects the topological properties.

In the following, we verify that the mirror winding number results in the above significant dependence by numerically analyzing a tight-binding model. The Hamiltonian reads,

\[
H(k) = [2t(\cos k_x + \cos k_y) - \mu] \rho_0 + i\Delta \sin k_x \rho_3 + i\Delta \sin k_y \rho_2,
\]

where \(\rho_i\) (\(i = 1, 2, 3\)) are the Pauli matrices and \(\rho_0\) is the \(2 \times 2\) identity matrix. The above Hamiltonian preserves the mirror symmetry with \(M_x = \rho_2 P_z\). Therefore, for \(k_x^\pm = 0\) or \(\pi\), the Hamiltonian can be block-diagonalized with \(\rho_2\). For \(k_x^\pm = 0\) (\(k_x^\pm = \pi\)), the mirror winding number takes \(\nu_M = 1\) with \(E_{pg} = 2t - \mu\) \((E_{pg} = -2t - \mu)\), while the total winding number is zero for the arbitrary value of \(k_x\).

In Fig. 2 the energy spectrum of the Hamiltonian \((4)\) are plotted for \((t, \mu, \Delta) = (1, 2, 1.8)\) at \(k_x = 0, \pi/6, \pi/2, \pi\). The data denoted with blue (orange) dots represent the energy eigenvalues for the PBC (OBC) along the \(y\)-direction, respectively. Figure 2(a) indicates that the energy spectrum under the PBC form a circle enclosing the origin of the complex plane which is consistent with the relation, \(\nu_M = 1\) for \(k_x = 0\). Imposing the OBC along the \(y\)-direction significantly changes the spectrum; energy eigenvalues are aligned along the real axis \((i.e., \text{Im} E_n \sim 0\) with \(n = 1, 2, \cdots, \dim H\)). This striking dependence of the energy spectrum is a signal of the skin effects. Here, we note that the mirror symmetry plays an essential role; away from the mirror invariant line, the spectra obtained for the two distinct boundary conditions coincide with each other [see Figs. 2(b) and (c)]. At \(k_x = \pi\), the mirror symmetry is preserved which again induces the skin effect [see Fig. 2(d)].

In association with the significant change of the energy eigenvalues, the eigenvectors shows extensive localization. Figure 3 plots amplitude of the right eigenvectors \(|\langle i_y | \Psi_{nR} \rangle|^2\) for \(k_x = 0, \pi/6\). Here, \(|\Psi_{nR}\rangle\) denotes the right eigenvector of the Hamiltonian \((4)\) \((i.e., H|\Psi_{nR}\rangle = |\Psi_{nR}\rangle E_n\) with \(n = 1, \cdots, \dim H\)) and \(i_y\) labels the sites along the \(y\)-direction. We note that the eigenstates are extended in the bulk under the PBC.
Mirror skin effect in an electric circuit.— Before detailed proposal for the implementation of the circuit showing the mirror skin effect, let us briefly review how an electric circuit mimics a generic tight-binding Hamiltonian. Consider an electric circuit where the voltage $V_a(\omega)$ is applied at nodes $a = 1, 2, \cdots$ with angular frequency $\omega$. In this case, based on the Kirchhoff’s law, the current $I_b(\omega)$ at node $b$ is given by

$$I_b(\omega) = \sum_a J_{ba}(\omega)V_a(\omega). \quad (5)$$

Thus, the admittance matrix $J_{ba}(\omega)$ serves as a Hamiltonian for the corresponding tight-binding model, which means that topological phenomena can also be observed for electric circuits \cite{13}. For instance, the Su-Schrieffer-Heeger model can be realized for an electric circuit composed only with capacitors and inductors. The energy conservation of the electric circuit implies the Hermiticity of the matrix $J_{ba}(\omega)$ up to the global phase factor $i$.

Now, let us discuss how to experimentally verify the mirror skin effect for electric circuits. In order to implement a circuit showing the mirror skin effect, we need to reproduce non-Hermitian terms of the Hamiltonian [i.e., the second and the third term of Eq. (3)], which can be accomplished by employing the negative impedance converters \cite{68, 69}. Specifically, we propose that an electric circuit shown in Fig. 1 serves as a platform of the mirror skin effect. The corresponding admittance matrix is given by

$$
\begin{pmatrix}
I_A(\omega, k) \\
I_B(\omega, k)
\end{pmatrix} = J(\omega, k) \begin{pmatrix}
V_A(\omega, k) \\
V_B(\omega, k)
\end{pmatrix}, \quad (6a)
$$

$$J(\omega, k) = i\omega [-2C_0(\cos k_x + \cos k_y)\rho_0 + (4C_0 + \frac{1}{\omega^2 L_0})\rho_0 + 2iC_1 \sin k_x\rho_3 + 2iC_1 \sin k_y\rho_1], \quad (6b)$$

in the momentum space. $I_\alpha(\omega, k)$ and $V_\alpha(\omega, k)$ $(\alpha = A, B)$ denotes the Fourier transformed current and voltage, respectively \cite{70, 71}. The detailed derivation of Eq. (6) is given in Sec. S2A of the supplemental material \cite{67}. This model preserves the mirror symmetry with $M_\pi = \rho_1 P_\pi$. In addition, its topology is characterized by the mirror winding number taking $\nu_M(k^*_+ = 0, \pi)$ for the parameter set summarized in caption of Fig. 3. We note that the relation $\nu_{tot} = 0$ holds for arbitrary momentum $k$. We mention here that the circuit elements of the above parameters are commercially available. Numerical data elucidating the above topological properties are shown in Sec. S2B of the supplemental material \cite{67}.

In the following, we see that the above model shows the mirror skin effect. For the electric circuit, the skin effect can be experimentally observed by measuring the admittance eigenvalues $j_n$ with $n = 1, \cdots, \dim J$ [i.e., eigenvalues of the admittance matrix \cite{68}]. One can access the

Based on the above significant difference of energy spectra and the eigenstates, we can conclude that a non-trivial value of the mirror winding number $\nu_M$ results in the mirror skin effect. We furthermore propose that this mirror skin effect can be observed for an electric circuit composed of negative impedance converters (see Fig. 1), the details of which are discussed in the rest of this paper.

![Image](image-url)
admittance eigenvalues by the impedance measurement $[J^{-1}_{ab}(\omega)]$ [59]. When the skin effect occurs, the admittance spectrum significantly depends on the boundary condition as we have seen in Fig. 2(a). Figure 4(a) [(b)]

\[\text{Re}(j_{n})\ [\Omega^{-1}] \quad \text{Im}(j_{n})\ [\Omega^{-1}]\]

\text{layer A}\quad \text{layer B}

\begin{align*}
\text{I}_{\text{in}} &= \text{I}_{\text{L}} \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad 14 \\
0.00 & \quad 0.02 & \quad 0.04 & \quad 0.06 & \quad 0.08 \\
|V_{i_{y}}A| &
\end{align*}

shows the admittance spectrum for each case of the boundary condition for $k_{x} = 0$ ($k_{x} = \pi/6$), respectively. Blue (orange) dots denote the admittance eigenvalue $j_{n}$ under the periodic (open) boundary condition for the $y$-direction with $L_{y} = 14$. The periodic boundary condition is imposed for the $x$-direction. The data are obtained for $L_{0} = 120 \mu\text{H}$, $C_{0} = 47\text{nF}$, $C_{1} = 33\text{nF}$, and $f = 117.4\text{kHz}$, where $f$ denotes frequency $\omega = 2\pi f$. (c) [(d)]: Voltage profile of layer A for the case where the current with plus (minus) sector of reflection is fed. (e) Illustration of setup to observe the anomalous voltage response with $I_{\text{in}} = I_{+L}$. For the above parameter sets, the fed current is $I_{L} = 0.0001\text{A}$ with the fed voltage $V_{s} = 0.002\text{V}$ for $C_{s} = 0.009\text{mF}$, and $L_{s} = 27\mu\text{H}$.

The above results indicate that the mirror skin effect can be observed for the circuit shown in Fig. 4. Specifically, it induces significant change of admittance eigenvalues [Figs. 3(a) and(b)] and the anomalous voltage response [Figs. 3(c) and(d)] both of which can be observed in experiments.

\textbf{Summary.—} In this letter, we have analyzed interplay between mirror symmetry and skin effects, shedding new light on crystalline symmetry and non-Hermitian topology. Our analysis has clarified a novel type of skin effects, a mirror skin effect which results in the significant dependence both of the energy spectrum and the states on the boundary condition only along mirror invariant lines in the two-dimensional Brillouin zone. The topological characterization of this skin effect can be done with the mirror winding number. The mirror skin effect has been verified by numerically diagonalizing a tight-binding Hamiltonian with the mirror winding number taking one. Furthermore, we have proposed how to implement the electric circuit for the experimental observation of the mirror skin effect. In this system, switching the boundary condition significantly changes the admittance eigenvalues, which serves as a distinct evidence of the mirror skin effect.

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Supplemental Materials:
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S1. AMPLITUDE OF THE RIGHT EIGENSTATES OF EQ. (4)

In Fig. 3(a), we have seen that the eigenstates \(|\Psi_{nR}\rangle\) \(n = 1, 2, \ldots\) are extensively localized around the boundary. Here, we show that the boundary condition is essential for the above extensive localization. Figure S1 plots the amplitude of the eigenstates under the PBC along the \(y\)-direction. Figure S1(a) [(b)] shows data for \(k_x = 0\) \((k_x = \pi/6)\), respectively. In these figures, we can see that the states are delocalized for the PBC.

\[ |\langle i_y | nR \rangle^2| \]

FIG. S1. (Color Online). (a) [(b)]: Amplitude of the right eigenvectors of the Hamiltonian \((4)\) for \((t, \mu, \Delta) = (1, 2, 1.8)\) at \(k_x = 0\) \((\pi/6)\), respectively. The data is obtained under the PBC both for the \(x\)- and \(y\)-direction. The number of the sites along the \(y\)-direction is set to \(L_y = 20\).

S2. DETAILS OF ELECTRIC CIRCUIT

A. Derivation of Eq. (6b)

We start with explaining how the negative impedance converters with current inversion induces the non-Hermitian terms. The element shown in Fig. S2 responses as \([68, 69]\)

\[
\left( \begin{array}{c}
I_{in} \\
I_{out}
\end{array} \right) = i\omega C_1 \left( \begin{array}{cc}
-1 & 1 \\
-1 & 1
\end{array} \right) \left( \begin{array}{c}
V_{in} \\
V_{out}
\end{array} \right),
\]

where the vectors \((I_{in} \ I_{out})^T\) and \((V_{in} \ V_{out})^T\) represent the current and the voltage illustrated in Fig. S2, \(C_1\) denotes the capacitance. This can be seen as follows. We can tune the current and voltage as shown in Fig. S2. In this case, based on the Kirchhoff’s law, we obtain

\[
I_{in} = (i\omega C_a + R_a^{-1})(V_{in} - V_a),
\]

\[
I_{out} = (i\omega C_a + R_a^{-1})(V_{out} - V_a),
\]

\[
I_{out} = i\omega C_1(V_{out} - V_{in}).
\]

where \(R_a\) and \(C_a\) represent the resistance and the capacitance, respectively. \(V_a\) and \(I_a\) denote the current and the voltage as illustrated in Fig. S2, respectively. \(\omega\) denotes the angular frequency. Solving these equations yields Eq. (S1). Here, we note that current \(I_{in}\) has the opposite sign; the ordinary capacitor responses as

\[
\left( \begin{array}{c}
I_{in} \\
I_{out}
\end{array} \right) = i\omega C_0 \left( \begin{array}{cc}
1 & -1 \\
-1 & 1
\end{array} \right) \left( \begin{array}{c}
V_{in} \\
V_{out}
\end{array} \right),
\]

with capacitance \(C_0\). The above additional sign plays an essential role in the non-Hermitian hopping.
FIG. S2. (Color Online). Sketch of the circuit elements. We can tune the amplifier (denoted by triangle) so that the current does not flow into it. Here, the voltage at each nodes is also illustrated.

Now, let us consider the circuit illustrated in Fig. 1. Connecting intra-layer nodes by the negative impedance converters with current inversion [Eq. (S1)] yields

\[
I_{AR_i} = i\omega C_1(-V_{AR_i} - e_x + V_{AR_i} + e_x),
\]

(S4a)

\[
I_{BR_i} = -i\omega C_1(-V_{BR_i} - e_x + V_{BR_i} + e_x),
\]

(S4b)

where \( I_{\alpha R_i} \) (\( V_{\alpha R_i} \)) denotes the current and the voltage of the node specified with \( \alpha \) and \( R_i \), respectively. \( R_i \) is the position vector at site \( i \). \( \alpha = A, B \) specifies the layer. \( e_\mu (\mu = x, y) \) denotes the unit vector for each direction.

Applying the Fourier transformation, we obtain

\[
\begin{pmatrix}
I_{Ak} \\
I_{Bk}
\end{pmatrix}
= -2i\omega C_1 \sin k_x \rho_3 \begin{pmatrix}
V_{Ak} \\
V_{Bk}
\end{pmatrix},
\]

(S5)

with

\[
I_\alpha(\omega, k) = \frac{1}{\sqrt{L_x L_y}} \sum_a e^{i k \cdot R_a} I_{\alpha a}(\omega, k),
\]

(S6a)

\[
V_\alpha(\omega, k) = \frac{1}{\sqrt{L_x L_y}} \sum_a e^{i k \cdot R_a} V_{\alpha a}(\omega, k).
\]

(S6b)

Therefore, we can see that connecting intra-layer nodes by the elements defined in Eq. (S1) yields the third term of Eq. (6b). In a similar way, we can see that connecting inter-layer nodes with the elements (S1) yields the fourth term of Eq. (6b); in the real-space, we obtain

\[
I_{AR_i} = i\omega C_1(-V_{BR_i} - e_x + V_{BR_i} + e_x),
\]

(S7a)

\[
I_{BR_i} = i\omega C_1(-V_{AR_i} - e_x + V_{AR_i} + e_x),
\]

(S7b)

which yields

\[
\begin{pmatrix}
I_{Ak} \\
I_{Bk}
\end{pmatrix}
= -2i\omega C_1 \sin k_y \rho_1 \begin{pmatrix}
V_{Ak} \\
V_{Bk}
\end{pmatrix},
\]

(S8)

Concerning the other circuit elements illustrated in Fig. 1, we obtain

\[
I_{\alpha R_i} = -i\omega C_0 \sum_{\mu = x, y} (V_{\alpha R_i} - e_\mu - V_{\alpha R_i} + e_\mu - V_{\alpha R_i} + e_\mu - V_{\alpha R_i}) + (i\omega L_0)^{-1} V_{\alpha R_i},
\]

(S9)

which yields

\[
I_{\alpha k} = \left[ -2i\omega C_0 (\cos k_x + \cos k_y - 2) + (i\omega L_0)^{-1} \right] V_{\alpha k}.
\]

(S10)

Summing up the contributions [Eqs. (S5), (S8), and (S10)], we end up with the admittance matrix (6b).
B. Topological properties

Here, we confirm that topological invariants take \((\nu_{\text{tot}}, \nu_M) = (0, -1)\) for \(k_x = 0, \pi\). Firstly, we note that the winding number for each sector [Eq. (3)] can be rewritten as

\[
\nu_{\pm} = \int \frac{dk_y}{2\pi} \sum_n \partial_{k_y} \arg(E_{\pm n(k_y)}} - E_{pg}),
\]

where \(E_{\pm n} (n = 1, 2, \ldots, \dim H_{\pm})\) denotes the eigenvalue of the non-Hermitian Hamiltonian \(H_{\pm}\).

Thus, by analyzing the momentum dependence of the argument of admittance eigenvalues, we obtain the topological invariants \(\nu_{\text{tot}}\) and \(\nu_M\).

In Fig. S3, the momentum dependence of the argument is plotted. Figure S3(a) indicates that the winding numbers take \((\nu_+, \nu_-) = (-1, 1)\) for \(k_x = 0\) with \(E_{pg} = 0\). In a similar way, we can confirm that the winding numbers take \((\nu_+, \nu_-) = (-1, 1)\) for \(k_x = 0\) with \(E_{pg} = 0\).

\[
\begin{align*}
\text{Fig. S3. (Color Online). (a) [(b)]: Argument of the admittance eigenvalue } j(k_y) \text{ for } k_x = 0 \text{ (}(b) k_x = \pi/6) \text{. Blue circles (orange triangles) represent the data for the plus and minus sector of the mirror operator } M_x. \text{ The data are obtained for } L_0 = 82\mu\text{H, } C_0 = 47n\text{F, } C_1 = 33n\text{F, and } f = 130kHz, \text{ respectively.}
\end{align*}
\]