Higher moments on strangeness fluctuation using PNJL model

Paramita Deb

Department of Physics, Indian Institute of Technology Bombay, Powai, Mumbai-400076, India

Amal Sarkar

Department of Physics, NRF iThemba LABS, Cape Town, South Africa

Raghava Varma

Department of Physics, Indian Institute of Technology Bombay, Powai, Mumbai-400076, India

(Dated: September 26, 2018)

PACS numbers: 12.38.AW, 12.38.Mh, 12.39.-x

I. INTRODUCTION

The strongly interacting matter is supposed to have a rich phase structure at finite temperature and density. While our Universe at present epoch contains a significant fraction of color singlet hadrons, color non-singlet states especially quarks and gluons may have been prevalent in the few microseconds after the Big bang. One of the fundamental goals of the heavy ion collision experiments is to map the QCD phase diagram and to locate the critical end point (CEP), where the first order phase transition from hadronic state to quark gluon plasma (QGP) phase becomes continuous [1, 2]. Presently, neither the existence nor the exact location of the critical point is known in spite of the heavy ion collision experiments being carried out at Relativistic Heavy Ion Collider (RHIC) at BNL and Super Proton Synchrotron (SPS) at CERN. Further, it is equally important to choose the correct experimental observables that will help locate the critical point. In the heavy ion collision experiments, formation of QGP is followed by expansion and attainment of freeze out characterized by cessation of all chemical and kinetic interactions. The detected particles at the freeze out condition can lead to the location of the freeze out point. Thus to locate the critical point, experiments are being conducted to bring the freeze out point close to the critical point by varying the collision center of mass energy $\sqrt{s}$. Therefore, there is a need to select the suitable experimental observables such as fluctuations of the conserved quantities that are sensitive to the proximity of the freeze-out point. The fluctuations of an experimental observable is defined as the variance and higher non-Gaussian moments of the event-by-event distribution for an experimental observable of each event in an ensemble of many events. These fluctuations result in a long range correlation length $\xi$; maximal value $\approx 1.5 - 3 \text{ fm}$ [3]. Hence the non-monotonic behavior of these fluctuations could be the signature of the critical point [4]. As different particles correspond to different conserved quantum numbers like baryon number ($B$), electric charge ($Q$) and strangeness ($S$); an event-by-event analysis of fluctuation of these can help locate the critical end point.

The QGP matter formed in the heavy ion collision experiments has a finite volume depending on the size of the colliding nuclei, center of mass energy and collision centrality. Several efforts have been made to estimate the finite volume during the freeze-out for different centrality measurement of HBT radii [5]. These results suggest that the volume increases with the centrality during freeze out and it is estimated to be $2000 \text{ fm}^3$ to $3000 \text{ fm}^3$. Theoretically the effects of finite volume have been addressed by many models such as non-interacting bag model [6], chiral perturbation theory [7, 8], Nambu–Jona-Lasinio (NJL) model [9, 10], linear sigma model [11, 12] and by the first principle study of pure gluon theory on space time lattices [13, 14]. Specifically, in a $1 + 1$ dimensional NJL model the finite size effect of a dense baryonic matter has been described by the induction of a charged pion condensation phenomena.
Recently, this has been extended to Polyakov loop Nambu–Jona-Lasinio (PNJL) model where it was observed that as the volume decreases, critical temperature for the crossover transition decreases. For lower volumes, CEP is shifted to a domain with higher chemical potential ($\mu$) and lower temperature ($T$) [16, 17]. It is quite evident that broadly both the Lattice calculations and QCD-based models indicate that the fluctuation of strongly interacting matter at zero density show significant volume dependence which might be relevant to study the formation of fireball in heavy-ion collision.

Further, both the Lattice QCD results [18, 24] and the QCD inspired models [16, 23, 36] show that the net conserved quantum numbers ($B$, $Q$ and $S$) are related to the conserved number susceptibilities ($\chi_x = \langle (\delta N_x)^2 \rangle / VT$ where $x$ can be either $B$, $S$ or $Q$ and $V$ is the volume). Close to the critical point, models also predict that the distributions of the conserved quantum numbers to be non-Gaussian and susceptibilities to diverge causing both skewness ($s$) and kurtosis ($\kappa$) to deviate where $s\sigma = (\chi_3^2/\chi_2^2)$ and $\kappa\sigma^4 = \chi_4^2/\chi_2^2$. These quantities are much more sensitive (skewness $\sim \xi^{4,5}$ and kurtosis $\sim \xi$) to the correlation length and they can provide much better handle for location of CEP. Moreover, the higher order coefficients become increasingly sensitive in the vicinity of phase transition. For example, in a 2 flavor QCD model it has been shown that the baryon number fluctuation ($\chi_B$) increases with temperature and its fourth moments attains a maxima in the phase transition region from low to high temperature [27]. Similarly, fluctuations have been also computed with respect to the quark chemical potential ($\mu$) in the Polyakov loop coupled quark-meson (PQM) model [35] and its renormalized group improved version, 2 flavor PNJL model with three-momentum cutoff regularization [38]. Fluctuations and the correlations of conserved charges have also been studied in higher flavor PNJL model [16, 17, 32, 39] with or without finite volume effects as well as simplistic lattice QCD [40]. Recently, a realistic continuum limit calculation [41–43] for the lattice QCD data has been performed and the 3 flavor PNJL model parameters have been reconsidered [44]. This re-parametrization has indeed resulted in a very good quantitative agreement between the model and the lattice data at finite temperature and zero density region. The second order and fourth order susceptibilities of the baryon number were found to be in reasonable quantitative agreement with the lattice data. For electric charge susceptibilities there were some disagreement for the temperature less than the critical temperature. However, in order to understand the QCD phase diagram and find the critical end point one need to explore the finite density region. Current work will emphasis on the 3 flavor finite volume finite density PNJL model [16, 17] with six-quark and eight-quark interactions. Subsequently, the method to calculate the correlations of conserved charges in PNJL model has been elaborated. Finally, the variation of skewness ($s$), kurtosis ($\kappa$) of strangeness fluctuations ($\chi_S$) and higher moments of cross-correlations with collision energy has been determined.

II. THE PNJL MODEL

We shall consider the 2+1 flavor PNJL model with six quark and eight quark interactions. In the PNJL model the gluon dynamics is described by the chiral point couplings between quarks (present in the NJL part) and a background gauge field representing Polyakov Loop dynamics. The Polyakov line is represented as,

$$L(\bar{x}) = \mathcal{P} \exp[i \int^\beta_0 d\tau A_4(\bar{x}, \tau)]$$

where $A_4 = i A_0$ is the temporal component of Euclidean gauge field ($\bar{A}, A_4$), $\beta = \frac{1}{T}$, and $\mathcal{P}$ denotes path ordering. $L(\bar{x})$ transforms as a field with charge one under global Z(3) symmetry. The Polyakov loop is then given by $\Phi = (T \tau, L^1)/N_c$, and its conjugate by, $\bar{\Phi} = (T \tau, L^1)/N_c$. The gluon dynamics can be described as an effective theory of the Polyakov loops. Consequently, the Polyakov loop potential can be expressed as,

$$\mathcal{U}'(\Phi[A], \bar{\Phi}[A], T) = \frac{\mathcal{U}(\Phi[A], \bar{\Phi}[A], T)}{T^4} = -\bar{\kappa} \ln(\mathcal{J}[\Phi, \bar{\Phi}])$$

where $\mathcal{U}(\phi)$ is a Landau-Ginzburg type potential commensurate with the Z(3) global symmetry. Here we choose a form given in [26],

$$\mathcal{U}(\Phi, \bar{\Phi}, T) = -b_3(T)\Phi\bar{\Phi} - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2,$$
where
\[ b_2(T) = a_0 + a_1 \exp(-a_2 T / T_0) T_0 / T \]  
Equation (4)

\( b_3 \) and \( b_4 \) being constants. The second term in eqn. (2) is the Vandermonde term which replicates the effect of SU(3) Haar measure and is given by,
\[ J[\Phi, \bar{\Phi}] = (27/24\pi^2) \left[ 1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2 \right] \]
The corresponding parameters were earlier obtained in the above mentioned literature by choosing suitable values by fitting a few physical quantities as function of temperature obtained in LQCD computations. The set of values chosen here are listed in the table I [14].

| Interaction  | \( T_0 (MeV) \) | \( a_0 \)  | \( a_1 \)  | \( a_2 \)  | \( b_3 \)  | \( b_4 \)  | \( \kappa \) |
|-------------|-----------------|---------|---------|---------|---------|---------|---------|
| 6-quark     | 175             | 6.75    | -9.0    | 0.25    | 0.805   | 7.555   | 0.1     |
| 8-quark     | 175             | 6.75    | -9.8    | 0.26    | 0.805   | 7.555   | 0.1     |

**TABLE I:** Parameters for the Polyakov loop potential of the model.

For the quarks we shall use the usual form of the NJL model except for the substitution of a covariant derivative containing a background temporal gauge field. Thus the 2+1 flavor the Lagrangian may be written as,
\[
\mathcal{L} = \sum_{f=u,d,s} \bar{\psi}_f i D^\mu \psi_f - \sum_{f} m_f \bar{\psi}_f \psi_f + \sum_{f} \mu_f \gamma_0 \bar{\psi}_f \psi_f + \frac{g_S}{2} \sum_{a=\lambda,\Sigma} \left[ (\bar{\psi}\lambda^a \psi)^2 + (\bar{\psi}\gamma_5 \lambda^a \psi)^2 \right] \\
- g_D [d \bar{\psi}_f P_L \psi_f + d \bar{\psi}_f P_R \psi_f] \\
+ 8g_1[(\bar{\psi}_f P_R \psi_m)(\bar{\psi}_m P_L \psi_f)]^2 + 16g_2[(\bar{\psi}_f P_R \psi_m)(\bar{\psi}_m P_L \psi_f)(\bar{\psi}_m P_L \psi_f)] \\
- U[(\bar{\Phi}[\Lambda], \bar{\Phi}[\Lambda], T)]
\]
where \( f \) denotes the flavors \( u, d \) or \( s \) respectively. The matrices \( P_{L,R} = (1 \pm \gamma_5)/2 \) are respectively the left-handed and right-handed chiral projectors, and the other terms have their usual meaning, described in details in Refs. [16, 31, 32]. This NJL part of the theory is analogous to the BCS theory of superconductor, where the pairing of two electrons leads to the condensation causing a gap in the energy spectrum. Similarly in the chiral limit, NJL model exhibits dynamical breaking of \( SU(N_f)_L \times SU(N_f)_R \) symmetry to \( SU(N_f)_V \) symmetry \( (N_f \) being the number of flavors). As a result the composite operators \( \bar{\psi}_f \psi_f \) generate nonzero vacuum expectation values. The quark condensate is given as,
\[
\langle \bar{\psi}_f \psi_f \rangle = -iN_c \mathcal{L}_{y \to x+} (tr \mathcal{S}_f(x-y)),
\]
where trace is over color and spin states. The self-consistent gap equation for the constituent quark masses are,
\[
M_f = m_f - g_S \sigma_f + g_D \sigma_{f+1} \sigma_{f+2} - 2g_1 \sigma_f (\sigma_0^2 + \sigma_d^2 + \sigma_s^2) - 4g_2 \sigma_f^2
\]
Equation (6)
where \( \sigma_f = \langle \bar{\psi}_f \psi_f \rangle \) denotes chiral condensate of the quark with flavor \( f \). Here if we consider \( \sigma_f = \sigma_u \), then \( \sigma_{f+1} = \sigma_d \) and \( \sigma_{f+2} = \sigma_s \). The expression for \( \sigma_f \) at zero temperature \( (T = 0) \) and chemical potential \( (\mu_f = 0) \) may be written as [32],
\[
\sigma_f = -\frac{3M_f}{\pi^2} \int_0^\Lambda \frac{p^2}{\sqrt{p^2 + M_f^2}} dp,
\]
Equation (7)
\( \Lambda \) being the three-momentum cut-off. This cut-off have been used to regulate the model because it contains couplings with finite dimensions which leads to the model to be non-renormalizable.
Due to the dynamical breaking of chiral symmetry, $N^2_f - 1$ Goldstone bosons appear. These Goldstone bosons are the pions and kaons whose masses, decay widths from experimental observations are utilized to fix the NJL model parameters. The parameter values have been listed in table II. Here we consider the $\Phi$, $\bar{\Phi}$ and $\sigma_f$ fields in the mean field approximation (MFA) where the mean field are obtained by simultaneously solving the respective saddle point equations.

Now that the PNJL model is described for infinite volumes we discuss how we implement the finite volume constraints. Ideally one should choose the proper boundary conditions – periodic for bosons and anti-periodic for fermions. This would lead to a infinite sum over discrete momentum values $p_i = \pi n_i/R$, where $i = x, y, z$ and $n_i$ are all positive integers and $R$ is the lateral size of the finite volume system. This implies a lower momentum cut-off $p_{\text{min}} = \pi/R = \lambda$. One should also incorporate proper effects of surface and curvatures. In this first case study we shall however take up a number of simplifications listed below:

- Surface and curvature effects have been neglected.
- The infinite sum will be considered as an integration over a continuous variation of momentum albeit with the lower cut-off.
- Any modifications to the mean-field parameters due to finite size effects will not be considered. Thus the Polyakov loop potential as well as the mean-field part of the NJL model would remain unchanged.

The thermodynamic potential for the multi-fermion interaction in MFA of the PNJL model can be written as,

$$\Omega = U'[\Phi, \bar{\Phi}, T] + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2} \sigma_u \sigma_d \sigma_s + 3 \frac{g_1}{2} (\sigma_f^2)^2 + 3g_2 \sigma_f^4 - 6 \int_0^\Lambda d^3p \frac{e^{\frac{p_i}{T}}}{(2\pi)^3} E_{pf} \Theta(\Lambda - |p|)$$

$$- 2 \sum_f \int_0^\infty d^3p \frac{e^{\frac{p_i}{T}}}{(2\pi)^3} \ln \left[ 1 + 3(\Phi + \bar{\Phi} e^{-\frac{E_{pf}^-}{T}}) e^{-\frac{E_{pf}^-}{T}} + e^{-3\frac{E_{pf}^-}{T}} \right]$$

$$+ \ln \left[ 1 + 3(\Phi + \bar{\Phi} e^{-\frac{E_{pf}^+}{T}}) e^{-\frac{E_{pf}^+}{T}} + e^{-3\frac{E_{pf}^+}{T}} \right]$$

(8)

where $E_{pf} = \sqrt{p^2 + M_f^2}$ is the single quasi-particle energy, $\sigma_f^2 = (\sigma_u^2 + \sigma_d^2 + \sigma_s^2)$ and $\sigma_f^4 = (\sigma_u^4 + \sigma_d^4 + \sigma_s^4)$. In the above integrals, the vacuum integral has a cutoff $\Lambda$ whereas the medium dependent integrals have been extended to infinity. The eight quark interaction in the Lagrangian stabilize the vacuum. In the present study we have considered PNJL model with 6-quark and 8-quark interactions for two sets of finite volume system with lateral size $R = 2\, fm$ and $R = 4\, fm$. Thus we have four sets of parameter sets (a) PNJL-6-quark for $R = 2\, fm$, (b) PNJL-6-quark for $R = 4\, fm$, (c) PNJL-8-quark for $R = 2\, fm$ and (d) PNJL-8-quark for $R = 4\, fm$.

### A. Taylor expansion of pressure

The freeze-out curve $T(\mu_B)$ in the $T - \mu_B$ plane and the dependence of the baryon chemical potential on the center of mass energy in nucleus-nucleus collisions can be parametrized by \[7\]

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4$$

(9)

where $a = (0.166 \pm 0.002) \, GeV$, $b = (0.139 \pm 0.016) \, GeV^{-1}$, $c = (0.053 \pm 0.021) \, GeV^{-3}$ and

$$\mu_B(\sqrt{s_{NN}}) = d/(1 + e\sqrt{s_{NN}})$$

(10)
with \( d, \, e \) given in Table 1 in [48]. The ratio of baryon to strangeness chemical potential on the freeze-out curve shows a weak dependence on the collision energy

\[
\frac{\mu_S}{\mu_B} \sim 0.164 + 0.018\sqrt{\Delta N_N}
\]

(11)

The pressure of the strongly interacting matter can be written as,

\[
P(T, \mu_B, \mu_Q, \mu_S) = -\Omega(T, \mu_B, \mu_Q, \mu_S),
\]

(12)

where \( T \) is the temperature, \( \mu_B \) is the baryon (B) chemical potential, \( \mu_Q \) is the charge (Q) chemical potential and \( \mu_S \) is the strangeness (S) chemical potential. From the usual thermodynamic relations the first derivative of pressure with respect to quark chemical potential \( \mu_q \) is the quark number density and the second derivative corresponds to the quark number susceptibility (QNS).

Minimizing the thermodynamic potential numerically with respect to the fields \( \sigma_u, \sigma_d, \sigma_s, \Phi \) and \( \Phi \), the mean field value for pressure can be obtained using the equation (12) [32]. The scaled pressure obtained in a given range of chemical potential at a particular temperature can be expressed in a Taylor series as,

\[
\frac{P(T, \mu_B, \mu_Q, \mu_S)}{T^4} = \sum_{n=i+j+k} \frac{c_{i,j,k}^B Q S}{\mu_B (\mu_B)^i} \frac{\partial^i}{\partial (\mu_B)^i} \frac{\partial^j}{\partial (\mu_Q)^j} \frac{\partial^k}{\partial (\mu_S)^k} \bigg|_{\mu_Q, \mu_S = 0}
\]

(13)

where,

\[
c_{i,j,k}^B Q S(T) = \frac{1}{i!j!k!} \frac{\partial^i}{\partial (\mu_B)^i} \frac{\partial^j}{\partial (\mu_Q)^j} \frac{\partial^k}{\partial (\mu_S)^k} \bigg|_{\mu_Q, \mu_S = 0}
\]

(14)

where \( \mu_B, \mu_Q, \mu_S \) are related to the flavor chemical potentials \( \mu_u, \mu_d, \mu_s \) as,

\[
\mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q, \quad \mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q, \quad \mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S
\]

(15)

In this work we evaluate the correlation coefficients up to fourth order which are generically given by;

\[
X_{i,j}^Y = \frac{1}{i!j!} \frac{\partial^{i+j}}{\partial (\frac{\mu_B}{T})^i} \frac{\partial^j}{\partial (\frac{\mu_Q}{T})^j}
\]

(16)

where, \( X \) and \( Y \) each stands for \( B, \, Q \) and \( S \) with \( X \neq Y \). To extract the Taylor coefficients, first the pressure is obtained as a function of different combinations of chemical potentials for each value of \( T \) and fitted to a polynomial about zero chemical potential using the gnu-plot fit program [49]. Stability of the fit has been checked by varying the ranges of fit and simultaneously keeping the values of least squares to \( 10^{-10} \) or even less. At low temperature fluctuations of a particular charge are dominated by lightest hadrons carrying that charge. The dominant contribution to \( \chi_B^2 \) at low temperatures comes from protons (lightest baryon), while \( \chi_S^2 \) receives leading contribution from kaons (lightest strange hadron) and \( \chi_Q^2 \) from pions (lightest charged hadron). Since, pion is lighter than proton and kaon, magnitude of \( \chi_Q^2 \) is more than that of \( \chi_B^2 \) and \( \chi_S^2 \).

### B. Results

The experimental results for volume independent cumulant ratios of net-kaon distributions are presented for the first time for all BES energies \( \sqrt{s_{NN}} = 7.7, \, 11.5, \, 14.5, \, 19.6, \, 27, \, 39, \, 62.4 \) and \( 200 \text{GeV} \) for top central and peripheral collisions. We have presented our results for four set of parameters of PNJL model with six-quark and eight-quark interactions. Also we have compared our results with the recent experimental result and HRG model results [48]. The ratios of charge fluctuations for different moments have been considered as they are independent of definitions of the interaction volume and also are more sensitive to produce correlation length.

Figure (1) shows the variation of \( C_3/C_{2s} \) and \( C_4/C_{2s} \) fluctuation with respect to different collision energies for PNJL model with four-quark and six-quark interactions with finite volume system with \( R = 2 \text{fm} \) and \( R = 4 \text{fm} \). As we increase the temperature \( C_3/C_{2s} \) decreases quantitatively. Also for higher collision energy it decreases for each temperature. \( C_4/C_{2s} \) has similar features as \( C_3/C_{2s} \). The values of \( C_4/C_{2s} \) becomes higher for smaller collision
energies and gradually decreases with increasing energy. For higher temperature the value decreases quantitatively. We have compared our results with the recent experimental data from STAR and with the Hadron Resonance gas model (HRG) results. In recent experimental data no significant deviation has been found with respect to the Poisson expectation value within statistical and systematic uncertainties for both the moments [57]. For the skewness ratio our model results for a particular temperature $T = 130\,\text{MeV}$ are very near to the Poisson expectation value. The results from the PNJL model are in good agreement with the experimental results. For the collision energy $\sqrt{s} < 27\,\text{GeV}$, there is an enhancement of fluctuation for PNJL model. Also in case of STAR results, there is a deviation from Poisson expectation value. Although the results for both skewness and kurtosis have qualitative similarities for both PNJL and HRG model, the values have quantitative differences.

We now set out to present the results for correlations among different conserved charges. In QGP, as baryon number as well as electric charge are carried by different flavors of quarks, a strong correlation is expected between B-Q, Q-S as well as B-S correlations. Also it is expected that the heavier particle will interact with the sigma field more strongly than the lighter particle. So it is important to study the different freeze-out stages of the produced QGP medium. On the other hand, in the hadronic sector presence of baryons and mesons would generate an entirely different type of correlations between these quantities. Hence these correlations are expected to show changes across the freeze-out which are characteristics of the changes in the relevant degrees of freedom.

Let us consider the baryon-strangeness (BS) correlation. In figure 2 the leading order BS correlation is shown for 4 sets of PNJL model. The correlation normalized to the baryon number fluctuations are given by

$$C_{BS} = -\frac{\chi_{BS}}{\chi_{SS}} = -\frac{1}{2} \frac{c_{11}^{BS}}{c_{22}^{SS}}$$

$$C_{SB} = -\frac{\chi_{BS}}{\chi_{BB}} = -\frac{1}{2} \frac{c_{11}^{BS}}{c_{22}^{BB}}$$

where we have used the notation $\chi_{XY} = \frac{\partial^{2}P}{\partial\mu_{X}\partial\mu_{Y}}$ and $\chi_{XX} = \frac{\partial^{2}P}{\partial\mu_{X}^{2}}$. Since $C_{BS}$ has entirely different behavior in the hadron gas and in QGP, it can be a reasonable diagnostic tool for identifying the nature of matter formed in heavy-ion collisions. Figure 2 represents the baryon and strangeness correlation normalized to the fluctuation of baryon number for all 4 parameter sets of PNJL model. As the temperature increases, the value of the ratio $C_{SB}$ decreases. For smaller collision energy, the value of the fluctuation is large compared to the higher collision energy. With the increase

FIG. 1: (Color online) Kurtosis (right panel) and skewness (left panel) of strangeness fluctuations for different PNJL parameter sets and comparison with recent STAR data and HRG model data. PNJL 6 quark data are plotted with closed symbols • and 8 quark data are plotted with open symbols ◦. $R = 2\,\text{fm}$ data are denoted by straight line in all red symbols – and $R = 4\,\text{fm}$ data are denoted by dotted lines in all blue symbols −−. The temperature scheme for different plots are as follows: $T = 100\,\text{MeV}$ as square □, $T = 130\,\text{MeV}$ as circle ◦, $T = 150\,\text{MeV}$ as up triangle △, $T = 170\,\text{MeV}$ as down triangle ▽ and $T = 200\,\text{MeV}$ as rhombus ◽. Black star ⋆ is denoted as HRG data. Green circles ◦ describe 70-80 percent peripheral collision and magenta circles ◦ are denoted as 0 - 5 percent collision in recent STAR preliminary result.
in collision energy, the value of the baryon chemical potential decreases. So the baryon fluctuation decreases with the decrease of baryon chemical potential. Thus the baryon number and the strangeness correlation is much larger than the baryon fluctuations. For all parameter sets of PNJL model the plots show similar enhancement of fluctuation at lower collision energy.

FIG. 2: (Color online) $\chi_{BS}^{11}/\chi_B^2$ correlations for different PNJL parameter sets. PNJL 6 quark data are plotted with closed symbols • and 8 quark data are plotted with open symbols ◦. $R = 2 fm$ data are denoted by straight line in all red symbols − and $R = 4 fm$ data are denoted by dotted lines in all blue symbols −−. The temperature scheme for different plots are as follows: $T = 100 MeV$ as square □, $T = 130 MeV$ as circle ◦, $T = 150 MeV$ as up triangle △, $T = 170 MeV$ as down triangle ▽ and $T = 200 MeV$ as rhombus ◊. Black star ⋆ is denoted as HRG data.

Now we show the behavior of some fourth order correlations - $\chi_{13}^{BS}/\chi_B^2$, $\chi_{31}^{BS}$. We have plotted the correlations for the four sets of PNJL parameters for different temperatures. Figure 3 represents the $-\chi_{BS}^{13}/\chi_B^2$ and $\chi_{BS}^{31}$ correlations. The value of $-\chi_{BS}^{13}/\chi_B^2$ is higher for lower collision energy for 6q PNJL model. For PNJL model with 8q interaction, correlation has qualitative similarity as HRG model. At low collision energy the value is low compared to the higher collision energy region. But they have quantitative difference than HRG data. $\chi_{BS}^{13}$ value increases with increasing temperature and for higher temperature the values are lower for low collision energy region.

We now turn to baryon-charge (BQ) correlation. In case of electric charge, fluctuations multiple charged hadrons have larger contribution in higher moments which results in characteristic deviations of the kurtosis and skewness. In figure 4 the leading order baryon-charge correlation has been normalized by $\chi_B^2$. For lower collision energy there is an enhanced fluctuation for all sets of PNJL model. The fluctuation increases with increasing temperature which indicates the transition region and also it is more for smaller volume system.

In fig. 5 the fluctuation increases for lower collision energy for system with $R = 2 fm$. The value of the higher order correlation increases quantitatively with temperature. But for the finite volume system with $R = 4 fm$, the situation is different. The value of $\chi_{BQ}^{13}$ correlation is higher near the transition temperature, but at lower collision energy the value decreases. In fig. 5 there is an enhanced fluctuation at lower collision energy near the transition temperature. The value of the fluctuation is low qualitatively for higher temperature.

Now we will discuss the leading order and higher order charge-strangeness correlations. As in the case of the baryon number, the charge is also strongly correlated to strangeness through strange quarks. At lower collision energy and near transition temperature there is an enhanced fluctuation at all temperature in fig. 6. The value of QS correlation increases quantitatively with the temperature. $\chi_{BS}^{13}$ increases as we increase the temperature and for lower collision energy. They have similar behavior as for the BS correlations. Therefore the BS and QS correlations can be used complimentary to understand the state of affairs in heavy-ion collisions.

III. SUMMARY

We have discussed properties of net kaon fluctuations in nuclear matter within PNJL model. We have considered the ratio of fourth order moment to second order moment (kurtosis) and the third order moment to the second order...
FIG. 3: (Color online) $\chi_{13}^{BS}$ (left panel) and $\chi_{31}^{BS}$ (right panel) correlations for different PNJL parameter sets. PNJL 6 quark data are plotted with closed symbols • and 8 quark data are plotted with open symbols ◦. $R = 2$ fm data are denoted by straight line in all red symbols − and $R = 4$ fm data are denoted by dotted lines in all blue symbols −−. The temperature scheme for different plots are as follows : $T = 100$ MeV as square □, $T = 130$ MeV as circle ◦, $T = 150$ MeV as up triangle △, $T = 170$ MeV as down triangle ▽ and $T = 200$ MeV as rhombus ◦. Black star ⋆ is denoted as HRG data.

moment (skewness) of strangeness fluctuations. We have also focused on the cross correlations related to baryon number, strangeness and electric charge conservation. All the correlations were obtained by fitting the pressure in a Taylor series expansion around the finite baryon, charge and strangeness chemical potentials. The baryon, charge and strangeness chemical potentials are obtained from the freeze-out curve which depends on the collision energies in the BES scan at the heavy-ion collision experiment. The results are shown for PNJL model with 6 quark and 8 quark interactions and two finite volume systems with lateral size $R = 2$ fm and $R = 4$ fm.

Skewness and kurtosis of strangeness fluctuation in PNJL model have similar features along the collision energy of heavy ion experiments. As we increase the temperature both skewness and kurtosis value decreases quantitatively. For collision energy less than 27 GeV, the value of kurtosis and skewness are higher. The recent experimental
observations show no significant deviation from Poisson expectation value for both the observables. However there is small deviations for skewness and kurtosis for low collision energy. Similarly in PNJL model we have found an enhancement of fluctuations for low collision energy less than 27 GeV. Also near the transition temperature the skewness ratio is very near to the Poisson expectation value.

The various correlators have been discussed to understand the matter created in the heavy ion collision experiments. The leading order coefficients can be most useful for identifying if the QGP is formed, while the higher order coefficients could identify the crossover region. We have noted a qualitative similarity of the leading order correlators of BS and QS with HRG model data. However they have a quantitative differences. The $\chi_{13}^{11}$ has large fluctuations at lower collision energies which differs from HRG model data both qualitatively and quantitatively.
FIG. 7: (Color online) $\chi_{QS}^{13}$ (left panel) and $\chi_{QS}^{31}$ (right panel) correlations for different PNJL parameter sets. PNJL 6 quark data are plotted with closed symbols • and 8 quark data are plotted with open symbols ◦. $R = 2\, fm$ data are denoted by straight line in all red symbols − and $R = 4\, fm$ data are denoted by dotted lines in all blue symbols −−. The temperature scheme for different plots are as follows: $T = 100\, MeV$ as square □, $T = 130\, MeV$ as circle ◦, $T = 150\, MeV$ as up triangle △, $T = 170\, MeV$ as down triangle ▽ and $T = 200\, MeV$ as rhombus ◊.

For higher order correlators containing strangeness $\chi_{BS}$ and $\chi_{QS}$ show similar behavior near low collision energy region. All the higher order cross correlations show increase or decrease of fluctuation at low collision energy. This might indicate the location of critical region in heavy-ion collision experiment.

The study of various equilibrium thermodynamic measurements of the correlators using PNJL model would be helpful in determining the finite temperature finite density behavior of the hadronic sector. Comparison of PNJL results with the experimental value will ensure the understanding of the physics behind the critical region and to locate the critical point in the strongly interacting matter.

Acknowledgments

P.D would like to thank Indian Institute of Technology Bombay for financial support. The part of the work has been published in the Few Body System. 59 (2018) no. 4, 55 of the Light Cone Conference 2017.

[1] J. Adams et.al., Nucl. Phys. A 751 102–183 (2005).
[2] M. M. Aggarwal et. al., Phys. Rev. C 82 024905 (2010).
[3] B. Berdnikov and K. Rajagopal, Phys. Rev. D 61 105017 (2000).
[4] M. Stephanov, K. Rajagopal and E. Shuryak, Phys. Rev. Lett. 81:4816–4819 (1998).
[5] D. Adamova et. al., Phys. Rev. Lett. 90, 022301 (2003).
[6] H.-T. Elze and W. Greiner, Phys. Lett. B 179, 385 (1986).
[7] M. Luscher, Commun. Math. Phys. 104, 177 (1986).
[8] J. Gasser and H. Leutwyler, Phys. Lett. B 188, 477 (1987).
[9] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); 124, 246 (1961).
[10] O. Kiriyama and A. Hosaka, Phys. Rev. D 67, 085010 (2003).
[11] G. Shao, L. Chang, Y. Liu and X. Wang, Phys. Rev. D 73, 076003 (2006).
[12] J. Braun, B. Klein and P. Piazecki, Eur. Phys. Jr. C 71, 1576 (2011).
[13] J. Braun, B. Klein and B.-J. Schefer, Phys. Lett. B 713, 216 (2012).
[14] A. Gopie and M. C. Ogilvie, Phys. Rev. D 59, 034009 (1999).
[15] A. Bazavov and B. A. Berg, Phys. Rev. D 76, 014502 (2007).
[16] A. Bhattacharyya, P. Deb, S. K. Ghosh, R. Ray, S. Sur, Phys.Rev. D 87 054009 (2013).
[17] A. Bhattacharyya, R. Ray and S. Sur, Phys. rev. D 91 051501 (2005).
[18] G. Boyd et. al., Nucl. Phys. B 469 419 (1996).
[19] J. Engels et. al., Nucl. Phys. B 530 307 (1999).
[20] Z. Fodor and S.D. Katz, Phys. Lett. B 534 87 (2002); Z. Fodor, S.D. Katz, and K.K. Szabo, Phys. Lett. B 568 73 (2003).
[21] C.R. Allton, S. Ejiri, S.J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, Ch. Schmidt, and L. Scorzato, Phys. Rev. D 66 074507 (2002); C.R. Allton, S. Ejiri, S.J. Hands, O. Kaczmarek, F.Karsch, E. Laermann, and Ch. Schmidt, Phys. Rev. D 68 014507 (2003); C.R. Allton, M. Doring, S. Ejiri, S.J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, and K. Redlich, Phys. Rev. D 71 054508 (2005).
[22] P. de Forcrand and O. Philipsen, Nucl. Phys. B642 290 (2002); B673 170 (2003).
[23] Y. Aoki, Z. Fodor, S.D. kat, and K.K. Szabo, Phys. Lett.B 643 46 (2006); Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, and K.K.Szabo, Nature (London) 443 675 (2006).
[24] M. Ciminale, R. Gatto, N.D. Ippolito, G. Nardulli, and M. Ruggieri, Phys. Rev. D 63 054023 (2006); M. Ciminale, R. Gatto, N.D. Ippolito, G. Nardulli, and M. Ruggieri, Phys. Rev. D 66 096003 (2006).
[25] K. Fukushima, Phys. Rev. D 77 114028 (2008).
[26] C. Ratti, M.A. Thaler, and W. Weise, Phys. Rev. D 73 014019 (2006).
[27] R.D. Pisarski, Phys. Rev. D 62 111501 (2000); A. Dumitru and R.D. Pisarski, Phys. Lett. B 504 282 (2001); 525 95 (2002); Phys. Rev. D 66 096003 (2002).
[28] K. Fukushima, Phys. Rev. D 79 074015 (2009).
[29] H. Hansen, W.M. Alberico, A. Beraudo, A. Molinari, M. Nardi, and C. Ratti, Phys. Rev. D 75 065004 (2007).
[30] S. K. Ghosh, T.K. Mukherjee, M.G. Mustafa, and R. Ray, Phys. Lett. B 73 114007 (2002); S.K. Ghosh, T.K. Mukherjee, M.G. Mustafa, and R. Ray, Phys. Rev. D 77 094024 (2008).
[31] A.A. Osipov, B. Hiller, and J. da Providencia, Phys. Lett. B 634, 48 (2006); A.A. Osipov, B. Hiller, V. Bernard, and A.H. Bliu, Ann. Phys. (N.Y.) 321, 2504 (2006); A.A. Osipov, B. Hiller, A.H. Blin, and J. da Providencia, Ann. Phys. (N.Y.) 322, 2021 (2007); B. Hiller, J. Moreira, A.A. Osipov, and A.H. Bliu, Phys. Rev. D 81, 116005 (2010).
[32] S. Roessner, C. Ratti, and W. Weise, Phys. Rev. D 75, 034007 (2007); C. Sasaki, B. Friman, and K. Redlich, Phys. Rev. D 75, 074013 (2007).
[33] P. Deb, A. Bhattacharyya, S. Datta, and S.K. Ghosh,Phys. Rev. C 79 055208 (2009); A. Bhattacharyya, P. Deb, S. K.Ghosh, R. Ray, Phys. Rev. D 28 014021 (2010); A. Bhattacharyya, P. deb, A. Lahiri, R. Ray, Phys. Rev. D 82 114028 (2010); A. Bhattacharyya, P. Deb, A. Lahiri, R. Ray, Phys. Rev. D 83 014011 (2011).
[34] Y. Aoki, Z. Fodor, S.D. kat, and K.K. Szabo, Phys. Lett.B 643 46 (2006); Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, and K.K.Szabo, Nature (London) 443 675 (2006).
[35] B. J. Schaefer and J. Wambach, Phys. Rev. D 75, 085015 (2007), B.J. Schaefer, J.M. Pawlowski, and J. Wambach, Phys. Rev. D 76, 074023 (2007), B.J. Schaefer, M. Wagner, and J. Wambach, Phys. Rev. D 81, 074013 (2010), J. Wambach, B.J. Schaefer, and M. Wagner, Acta Phys. Polon. Supp. 3 , 691 (2010); V. Skokov, B. Friman, E. Nakano, K. Redlich, and B. J. Schaefer, Phys. Rev. D 82, 034029 (2010).
[36] S. R. Roessner, C. Ratti, and W. Weise, Phys. Rev. D 75, 034007 (2007); C. Sasaki, B. Friman, and K. Redlich, Phys. Rev. D 75, 074013 (2007); S. Mukherjee, M.G. Mustafa, and R. Ray, Phys. Rev. D 75, 094015 (2007).
[37] A. Bazavov et. al., Phys. Rev. D 85 054003 (2012).
[38] A. Bazavov et. al., Phys. Rev. D 90 094030 (2014).
[39] S. Borsanyi et. al., Phys. Lett. B 730 99 (2014).
[40] A. Bazavov et. al., Phys. Rev. D 90 094030 (2014).
[41] A. Bazavov et. al., Phys. Rev. D 90 094030 (2014).
[42] A. Andronic, P. B. Munzinger and J. Stachel, Phys. Lett. B 673 142–145 (2009).
[43] J. Cleymans, H. Oeschler, K. Redlich and S. Wheaton, Phys. Rev.C 73 034905 (2006).
[44] S. Borsanyi et. al., Phys. Lett. B 730 99 (2014).
[45] A. Andronic, P. B. Munzinger and J. Stachel, Phys. Lett. B 673 142–145 (2009).
[46] J. Cleymans, H. Oeschler, K. Redlich and S. Wheaton, Phys. Rev.C 73 034905 (2006).
[47] F. Karsch and K. Redlich, Phys. Lett. B 695 136-142 (2011).
[48] http://www.gnuplot.info/