Introducing dark energy in the classroom using paradoxes

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Abstract. The concept of dark energy, an active subject in current PHYSICS research, is related to cosmological models and supplies a mechanism to the recently discovered accelerated expansion of the universe. Modeling dark energy is usually done by considering Einstein’s cosmological constant which was introduced by him 100 years ago. In the present work, a speculative discussion on the effects that dark energy could have on everyday life on Earth is provided. It is targeted to first-year college students but arrangements for high school PHYSICS are also possible. Among its strange properties, dark energy acts contrary to gravity and can lead to counterintuitive situations. For instance, if the cosmological constant was big enough, objects left to themselves near the Earth’s surface could lift to the sky instead of falling down to the ground, contradicting the everyday experience. The proposed discussion uses basic concepts of Newtonian PHYSICS in a scenario where the cosmological constant value could counteract Earth’s gravity. It is shown that the problem can be reduced to the analysis of a one-dimensional potential energy diagram which is accessible to a first-year college student. An algebraic version of this problem can turn a modern PHYSICS concept into a classroom exercise on conservation of mechanical energy. The content of this contribution is aimed to the production of resources for different levels of PHYSICS teaching and instruction. Its main focus is not directly related to educational methodologies.

1. Introduction

Predictions in everyday life are possible as we get used to natural phenomena and build our common sense, just as intuition is constructed from experience. However, when dealing with physical science, the phenomena are experienced employing instruments that can probe regions outside the range of our senses and the interpretation one gets from the scientific practice is not always in agreement with the common sense [1]. This could be pointed out as a reason why most students to find physics (and especially its modern concepts) a difficult matter, since it can operate far beyond the usual ordinary understanding of natural phenomena. Sometimes the situation is so contradictory that paradoxes can be formed when contrasting predictions from common sense with the ones made using physical laws based on high sensitive experiments or observations.

Scientific reasoning can roughly be divided in assumptions, deductions and conclusions. A paradox may arise in any of these steps and there are several ways of categorising them [2,3]. They can be used to probe a scientific theory or test its comprehension, since the disassembly of a paradox is only possible with a deeper understanding of the associated subject. As a matter of fact, many branches of physics developed strongly by finding a way out of puzzling or paradoxical situations [4]. Dark energy itself can
be regarded as a solution of a paradox occurring when Newtonian gravity is applied to the cosmos (see below). In the classroom, posing paradoxes can be viewed as an educational tool since they have the potential to generate curiosity which is a driving power for self-motivation and learning.

The paradoxes proposed in this paper bridge gaps between two fields of physics: classical gravity and modern cosmology. The didactic strategy (see section 3) is to present the free fall problem from a mechanical energy conservation point of view and include in this problem the accelerated expansion of the universe through a discussion on Einstein’s cosmological constant [5,6].

2. Discussion

Bringing attention to different subjects in physics classes is sometimes an intricate action. Employing current results from scientific research is a usual strategy to capture the student’s interest during classes and to promote effective teaching. Whenever this happens, students can be confronted with their own view of the world and tends to pursue what is intriguing them. After the discovery of the accelerated expansion of the universe, the media regularly display dark energy to the lay public and that exposure results in a latent interest in the subject that can be explored in the classroom. Since dark energy is a topic related to gravity, a good point to introduce this matter in class is after the presentation of the universal law of gravitation.

The theoretical system composed by the three Newton’s laws of movement together with the universal gravity is tremendously successful in explaining Kepler’s laws of planetary motion, ocean tides and so on. The idea of a static and infinite universe is a subtle suggestion in Newtonian mechanics when the “fixed stars” are chosen as a reference frame for the description of the movement of physical objects. The enormous phenomenological range achieved by Newtonian mechanics brought rapid acceptance to the universal law of gravity which in its most simple modern formulation reads:

\[ F = G \frac{mM}{d^2} \quad (1) \]

where \( d \) is the distance between the centres of the two bodies, \( F \) is the gravitational attraction force between masses \( m \) and \( M \) and \( G \) is the gravitational constant (\( G = 6.67 \times 10^{-11} \) N m\(^2\)/kg\(^2\)). Table 1 shows a comparison of the gravitational attraction between two pairs of gravitationally interacting partners (a) the Sun and Pluto and (b) the Sun and its nearest star, Alpha Centauri, evaluated from equation (1).

| Partner’s mass (kg) | distance (m) | Sun’s mass (kg) | Force (N) |
|---------------------|--------------|----------------|-----------|
| (a) Pluto           | 1.3 \times 10^{22} | 5.9 \times 10^{12} | 2.0 \times 10^{30} | 5.0 \times 10^{16} |
| (b) \alpha - centauri| 2.2 \times 10^{30} | 4.1 \times 10^{16} | 2.0 \times 10^{30} | 15 \times 10^{16} |

Although the stellar distance is much larger than the planetary distance, the gravitational force is greater in case of table 1b because of the compensating effect on the numerator in equation (1) due to the higher stellar mass. One can explain that the planets do not fall on the Sun because of their orbital motion since the combination of force and movement present in the planetary problem permit planets to keep their distance from the Sun. But how to make the picture of a “fixed stars” frame compatible with the non-vanishing gravitational force operating between stars? A possible answer is to consider that the stellar distribution is such that the net force among them is zero, so a static situation could be achieved. In the present case, Alpha Centauri would be pushed by its neighbour stars against the pull by the Sun.
The argument can be repeated indefinitely for consecutive stars, so a static configuration of the stellar system – i.e. the fixed stars – would be possible, at least in principle. In this case, the stellar distribution has to extend to infinity because otherwise, if the universe had boundaries, the stars sitting near the end of the universe would have no possibility to be pushed against the pull from the inner stars and everything would collapse (just as when a hanging blanket slips down to the floor).

Nonetheless, a closer examination brings more difficulties to the application of Newtonian gravity to an infinite cosmic structure since, in such case, there are problems when the inverse square law from a field theory point of view. For instance, if a Gaussian sphere of radius \( r \) is set centred at a given point, the number of field lines crossing the surface of the sphere increases as \( \frac{4\pi r^2}{\rho} \) where \( \rho \) is the mass density of the stellar distribution. Since the surface of the sphere increases as \( 4\pi r^2 \) the density of lines of force, that is, the gravitational field intensity, is given by \( \frac{1}{\rho} \) increasing in direct proportion with \( r \). This means that the field intensity can reach an infinite value as \( r \to \infty \). However, in order to be physically meaningful, the field intensity must remain finite across the cosmos and so the mass density, \( \rho \), representing the stellar distribution, must decay to zero at the infinite so the product \( \rho r \) remains finite [7]. Therefore, if a scalar field is to be defined within Newtonian gravity, there are certain demands the mass density must obey. One can also see this by defining a scalar field, \( u_G \), from the inverse square law in equation (1) letting the mass density be an arbitrary \( \rho(r) \). The following expression:

\[
\begin{align*}
  u_G &= -\frac{G M(r)}{ r} + 4\pi G \int r \rho(r) dr
\end{align*}
\]

where \( M(r) \) is the enclosed mass, defines such a field (see appendix A of [6]). The integral in the last term of equation (2) is divergent unless the mass density decays faster than \( 1/r^2 \) (thus going to zero as \( r \) tends to infinity). This is incompatible with the view of a stellar distribution extending to infinity as pointed out in the previous paragraph, since in this case, the mass density is non zero everywhere. In other words, Newtonian gravity imposes a restriction to the mass density that is in contradiction with the conception of an infinite universe.

The discussion concerning the scalar field can also be done considering the Poisson equation which is written:

\[
\nabla^2 u = 4\pi G \rho
\]

where \( u \) is a scalar field and \( \rho \) its corresponding source. In the present case, \( \rho \) is the mass density reflecting the stellar distribution in the universe, so the right hand of equation (3) should be always different from zero in the picture of an infinite universe (\( \rho = constant \) means an infinite and uniform universe). If in equation (3) one sets \( u = u_G \), the result is zero at infinity, so the left-hand side is zero. Hence, the picture of an infinite stellar distribution brings equation (3) to a mathematical contradiction such as \( 0 = 1 \). Possible ways out from this inconsistency include situations where either \( i \) there is a force balancing gravity, or \( ii \) the universe is dynamical. We know today that the universe is dynamical, but this was not the case until 1930 when Edwin Hubble discovered that the galaxies are moving away from each other by measuring red shifts on spectral lines of distant astronomical objects [8]. A Hubble-like measurement includes the recording of the light distribution emitted by astronomical objects using powerful telescopes and optical systems such as diffraction gratings. The observed differences amongst the spectral distribution of different light sources are attributed to the Doppler effect, meaning the source of the measured light is moving in relation to the observer. In the case of light, the Doppler effect manifests as a red shift, if the source is moving away from the observer, or as a blue shift, if the source is approaching the observer (see figure 1).

Hubble’s law states that the universe is expanding in such a way that the further a galaxy is, the faster it recedes from us. Within a good approximation, Hubble found a proportionality relationship so that \( v = H d \) where \( H \) is the Hubble parameter, having the value of 70.4 km/s Mpc\(^{-1}\), \( v \) is the galaxy receding velocity and \( d \) the corresponding intergalactic distance [9]. Figure 1 shows a potential energy diagram.
for a two-galaxy system showing different scenarios for the galactic dynamics when only Newtonian gravity is involved. If the mechanical energy stored in the system is positive \( (E > 0) \) the final configuration of the system corresponds to the two galaxies flying away from each other with constant and positive velocity. The acceleration is zero since the galaxies have gravitationally escaped from each other and this corresponds to a red shift in a Hubble measurement. In case the mechanical energy is zero \( (E = 0) \), the galaxies will be able to fly apart each other. However, all kinetic energy is transformed in gravitational potential energy so the final configuration is static in this case and there is no shift in the spectral distribution. If the mechanical energy is negative \( (E < 0) \) the system is gravitationally bound and there are two possibilities: a red shift, corresponding to an expansion with diminishing velocity (negative acceleration), or a blue shift, corresponding to a collapse. Note that the only case where the acceleration is positive (in the sense the velocity is increasing) corresponds to a blue shift in a spectral measurement. Hubble-like experiments have been constantly refined since his times and evidence for an accelerated expansion was found quite recently in the supernova cosmology project [10]. So, one has to consider that the receding velocities are positive \( (V > 0) \), corresponding to a red shift, and increasing so the acceleration is positive \( (A > 0) \) (not shown in figure 1).

Figure 1. A potential energy diagram relating the various possibilities for the movement of a two galaxy system. The corresponding accelerations and shifts in the spectral distributions are also shown.

Before Hubble’s law, the infinite and eternal universe view was the dominant one and led Einstein to propose a modification on the cosmological theory to accommodate the idea of an infinite universe. This is the starting point of Einstein’s 1917 paper where he introduces the cosmological constant in the Poisson equation (equation 3 above) [5]. Einstein adds a non-vanishing constant, \( \lambda \), to the field (the left-hand side of equation 3) to compensate for a mass density at infinity. However, in 1917, Einstein did not know Hubble’s law since it was yet to be discovered. The experimental evidence provided by Hubble, that appeared years later, made Einstein regret the input of the cosmological constant in the theory and \( \lambda \) became known as the biggest blunder of his life. However, in recent years, Einstein’s cosmological constant regained attention since it is capable of reproducing the accelerated expansion of the universe and of providing red shifts with positive acceleration (see next section).
3. Application

The modification introduced by Einstein in his 1917 paper, mentioned in the previous section, consists of adding a constant term in the left hand of equation (3) so it reads:

\[ \nabla^2 u + \Lambda c^2 = 4\pi G \rho \]  

(4)

where \( \Lambda \) is the cosmological constant and \( c \) is the speed of light. This is equivalent to generalising the scalar field, such that \( u = u_G + u_\Lambda \) and \( \nabla^2 u_\Lambda = \Lambda c^2 \), which can be interpreted as the addition of an associated force \( \vec{F}_\Lambda \) to the problem (see below and also reference 6). Once the correction is made one can define a \( \Lambda \)-type of potential energy which should come into play in mechanics. This \( \Lambda \)-type form of energy is what is called dark energy in modern cosmology. From the modified Poisson equation (4) one can define a cosmological dark energy scalar field, \( u_\Lambda \), and, by analogy with the gravitational theory, one can define a cosmological dark energy potential energy, \( U_\Lambda = m u_\Lambda \), and, having Newtonian physics in mind, one can define a cosmological “dark force” from the relation \( \vec{F}_\Lambda = [ - dU_\Lambda / dr ] \hat{r} \) [6]. One has that:

\[ \vec{F}_\Lambda = + \frac{1}{3} \Lambda mc^2 r \hat{r} \]  

(5)

Note that the cosmological dark force and the gravity force are in opposite directions. This formulation can be used to input the cosmological constant into school problems at any level of physics education. Conservation of energy and dynamics problems can be modified by the addition of the cosmological potential energy. For instance, the conservation of mechanical energy in this other scenario is given by

\[ E = K + U_G + U_\Lambda = \frac{1}{2} m v^2 - G \frac{Mm}{r} - \frac{1}{6} \Lambda mc^2 r^2 \]  

(6)

For problems near the Earth’s surface, the potential energies in equation (6) can be expanded in a Taylor series about \( r = R + h \), where \( R \) is the Earth radius and \( h \) is the height. Conservation of mechanical energy can be put in the following form:

\[ E = \frac{1}{2} m v^2 + m(g_G - g_\Lambda)h \]  

(7)

where

\[ g_G = G \frac{M}{r^2} \]  

(8)

is the usual acceleration of gravity, and

\[ g_\Lambda = \frac{1}{3} \Lambda c^2 R \]  

(9)

is the cosmological acceleration due to dark energy.

The presence of the “dark force” (equation 5) brings modifications to problems related to gravity such as freefall and projectile movement depending of the cosmological constant value. If the \( \Lambda \) value is set to \( 5.0 \times 10^{-23} \text{m}^{-2} \), the acceleration \( g_\Lambda \) would be equal to the acceleration of gravity on the Earth surface, \textit{i.e.} the usual 9.8 ms\(^{-2}\). That means any object left to itself, a basketball for instance, would simply float instead of falling to the ground since the gravitational and the dark forces would be equal in magnitude. If the ball is thrown up the dark force would lift the ball into the sky indefinitely (figure 2). Freefall would go upwards if the \( \Lambda \) value is slightly higher than the value mentioned above. The estimated \( \Lambda \)-value from the supernova cosmology project is far below that value: one has that \( \Lambda = 10^{-52} \text{m}^{-2} \), and the effect of Einstein’s modification to the Poisson equation is important only at the
cosmic scale. From the supernova data, one can estimate the dark energy density which should be present all over and represents 75% of the universe. It reads [10]:

\[ \rho_\Lambda = \frac{\Delta c^2}{8\pi G} \]  

(10)

**Figure 2.** The modified fall problem by the addition of the dark energy acceleration. The red curve corresponds to a parabolic trajectory driven by gravity alone. The blue curve indicates the ball trajectory in a situation where \( g_\Lambda = g_G \) (see equations 8 and 9).

4. Conclusions

It should be mentioned that modern cosmology uses Einstein’s general relativity to describe the large structure of the cosmos since Newton’s force of gravity is insufficient for that purpose, as seen above. A fundamental difference between these two descriptions is that, in the general relativity view, the cause of motion around a gravitational mass is a space-time distortion, not a force as in Newton’s description [11,12]. When developing general relativity, Einstein took care to make the description of motion provided by general relativity reduce to the one given by Newton so both descriptions agree within the proper limits, *i.e.* weak gravitational fields. Therefore some aspects of general relativity can still be interpreted in classical grounds. In this way, the cosmological constant effect (or dark energy) is to produce what can be interpreted in Newtonian physics as a repulsive force. Even though force is a Newtonian concept, it is still useful in understanding this topic, since force is familiar among students.

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References

[1] Rohrlich F 1987 *From Paradox to Reality*. New York: Cambridge Univ. Press. ISBN 052130749X.

[2] Quine W V 1962 Paradox *Sci. Am.* **206**(4) 84–99. https://www.jstor.org/stable/24937290

[3] Rescher N 2001 *Paradoxes: their roots, range and resolution*. Chicago: Open Court Publishing. ISBN-10: 0812694376

[4] Aharonov Y and Rohrlich D 2005 *Quantum paradoxes*. Weinheim: Wiley - VHS ISBN: 978-3-527-40391-2

[5] Einstein A 2013 *Cosmological considerations on the general theory of relativity*. Translated in Perrett W and Jeffery G B *The principles of relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity*. Dover Publications Inc. (pp. 175)

[6] Pereira J A M 2016 Earth’s gravity and the cosmological constant: a worked example *Eur. J. Phys.* **37** 025602 DOI: 10.1088/0143-0807/37/2/025602

[7] Jammer M 1961 *Concepts of mass in classical and modern physics*. Dover Publications Inc. (pp. 127) ISBN-10: 0486299988

[8] Hubble E 1929 A relation between distance and radial velocity among extra-galactic nebulae *Proc. Nat. Acad. Sci.* **15**(3) 168–73. DOI: 10.1073/pnas.15.3.168

[9] Freedman W L, Madore B F, Gibson B K, Ferrarese L, Kelson D D, Sakai S, Mould J R, Kennicutt R C Jr, Ford H C and Graham J A 2001 Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant *Ap J* **553** 47–72. DOI: 10.1086/320638

[10] Royal Swedish Acad. Sci. (2011) “The accelerating universe” available at http://www.nobelprize.org/nobel_prizes/physics/laureates/2011/advanced-PHYSICS-prize2011.pdf.

[11] Stannard W, Blair D, Zadnik M and Kaur T 2017 Why did the apple fall? A new model to explain Einstein’s gravity *Eur. J. Phys.* **38** 015603. DOI: 10.1088/0143-0807/38/1/015603

[12] Kersting M, Henriksen E K, Bøe M V and Angell C 2018 *Phys. Rev. Phys. Educ. Res.* **14**(1) 010130-1-010130-18. DOI: 10.1103/PhysRevPhysEducRes.14.010130