Critical Slowing Down from $T \neq 0$ Wilson RG

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The Thermal Renormalization Group can be employed to study the dynamics of $T \neq 0$ Quantum Field Theories close to second order phase transitions, where neither resummed perturbation theory nor first principle lattice simulations can be employed.

As an example, I discuss the computation of the plasmon damping rate in the scalar field theory from $T \gg T_C$ down to $T_C$. As the critical point is approached, the lifetime of long wavelength thermal fluctuations diverges.

Taking this effect into account, the notion of quasiparticle, and kinetic approaches, can be extended close to the critical regime. The consequences on the scenarios for topological defect formation in the early universe are also discussed.

I. CRITICAL SPEEDING UP VS. CRITICAL SLOWING DOWN

The last few years have seen a dramatic progress in the understanding of the static properties of relativistic quantum field theory (QFT) at high temperature. Perturbative computations, lattice simulations, and the Wilson Renormalization Group, have been successfully applied to many different problems, e.g. to the investigation of the free energy in the scalar theory, in the electroweak standard model, and in QCD. On the other hand, there are many interesting dynamical phenomena, like the generation of the cosmological baryon asymmetry or the formation of topological defects in a second order phase transition for $T \to T_C$. The plasma damping rate is defined as

$$\gamma_k(T) = \frac{\Pi_f(\omega_k,k)}{2\omega_k}, \tag{1}$$

where $\omega_k^2 = k^2 + m(T)^2$, $m(T)$ is the thermal mass and $\Pi_f$ is the imaginary part of the self-energy.

The physical interpretation of the dynamical quantity $\gamma_k(T)$ was given by Weldon long ago (see also [3]): if the plasma is slightly out of thermal equilibrium then $\gamma_k(T)$ gives half the relaxation rate of the quasiparticle distribution function to its equilibrium value,

$$\frac{d\delta n_k}{dt} = -2\gamma_k(T)\delta n_k \tag{2}$$

where $\delta n_k$ is the deviation of the distribution function from equilibrium $\delta n_k = n_k - n_k^{eq}$.

In the following, I will be interested in the behavior of long wavelength fluctuations as the critical temperature is approached, so I will concentrate on the damping rate for vanishing spatial momentum, $\gamma(T) = \gamma_{k=0}(T)$. In perturbation theory, $\gamma(T)$ is a two-loop effect, and is given by (4)

$$\gamma_{p.t.} = \frac{1}{1536\pi} l_q \lambda^2 T^2 \tag{3}$$

where $l_q = 1/m(T) = \left(\frac{3\pi^2}{24}\right)^{-1/2}$ is the Compton wavelength.

In refs. [4] it was shown that the above two-loop result can be reproduced in the classical theory provided that the Compton wavelength $l_q$ is identified with the classical correlation length $l_c$. This can be understood realizing that $\gamma$ probes the theory at scales $\omega = m(T) \ll T$ (if $\lambda$ is perturbatively small), where the Bose-Einstein distribution function is approximated by its classical limit,

$$N(\omega) = \frac{1}{e^{\beta\omega} - 1} \rightarrow \frac{1}{\beta m(T)} \quad \text{if} \quad \beta \omega \ll 1, \tag{4}$$

with $\beta = 1/T$. Considering for instance the 1-1 component of the propagator in the real-time formalism,

$$D_{11} = \mathcal{P} \frac{1}{k^2 - m^2} - 2\pi i \delta(k^2 - m^2) \left(\frac{1}{2} + N(|k_0|)\right), \tag{5}$$

we see that when the loop momenta are $k_0 \ll T$ the ‘statistical’ contribution to the imaginary part of the propagator dominates over the ‘quantum’ one, i.e. $N \gg 1/2$, from temperatures $T \gg T_C$ down to the second order phase transition for $T \to T_C$. The plasmon damping rate is defined as

$$\gamma_k(T) = \frac{\Pi_f(\omega_k,k)}{2\omega_k}, \tag{1}$$

where $\omega_k^2 = k^2 + m(T)^2$, $m(T)$ is the thermal mass and $\Pi_f$ is the imaginary part of the self-energy.

The physical interpretation of the dynamical quantity $\gamma_k(T)$ was given by Weldon long ago (see also [3]): if the plasma is slightly out of thermal equilibrium then $\gamma_k(T)$ gives half the relaxation rate of the quasiparticle distribution function to its equilibrium value,
and the leading order result can be obtained neglecting the $T = 0$ quantum contributions to the loop corrections.

The above argument has been employed to motivate the use of classical equations of motion in the study of the evolution of long wavelength modes in scalar and gauge theories. More recently, this approach has been improved by many authors including the effect of Hard Thermal Loops in the equations of motion for the ‘soft’ modes (see for instance [12]). The separation between hard and soft modes has been made explicit in ref. [8] by introducing a cutoff $\Lambda$ such that $m(T) < \Lambda < T$.

In the $T \gg T_C$ limit we have $m(T)/T \sim (\lambda/24)^{1/2}$, therefore the ‘statistical’ dominance is valid only up to $O(\lambda^{1/2})$ corrections. On the other hand the critical region corresponds, by definition, to $m(T) \to 0$, and the statistical limit is exact in this case.

What does perturbation theory predict in this limit? If we trust the two-loop/classical expression in eq. (4), then we see that $\gamma$ diverges as the correlation length. Physically, this would mean that the lifetime of long wavelength fluctuations gets shorter and shorter as the critical temperature is approached. However, it is well known that (resummed) perturbation theory cannot be trusted close to the critical point, due to the divergence of its effective expansion parameter, i.e. $\lambda T/m(T)$.

A more reliable indication of what’s going on can be obtained from the phenomenological theory of dynamical critical phenomena [9,10] developed by Landau in the fifties. Let’s consider a non-relativistic, classical scalar theory described by an order parameter $\eta(t, r)$ and the free energy

$$\mathcal{F}(\eta) = \int d^4r \left[ \frac{1}{2} (\partial_\eta)^2 + \alpha(T-T_C)|\eta|^2 \right].$$

If $\eta$ is slightly displaced from the minimum of $\mathcal{F}$ (at $\eta = 0$), then it is reasonable to assume that equilibrium will be restored at a rate given by

$$\frac{\partial \eta(t, r)}{\partial t} = -\Gamma \frac{\delta \mathcal{F}}{\delta \eta(t, r)} + \xi(t, r),$$

where $\Gamma$ is a phenomenological parameter which was assumed to be temperature independent in the original formulation of the theory, and $\xi(t, r)$ is a white noise term. Eq. (5) is usually quoted in the literature as the time-dependent Ginzburg-Landau equation. Taking Fourier transform, we see that the $k = 0$ mode vanishes as $\exp(-t/\tau_0)$ with the space-independent relaxation rate given by

$$\tau_0 = 2\gamma(T) = 2\alpha\Gamma(T-T_C),$$

thus vanishing as $T \to T_C$, in open contrast with the divergent perturbative result.

Dynamical critical phenomena have been the subject of intense study by the condensed matter community in the seventies (see the reviews in [10]). The Ginzburg-Landau approach has been improved by renormalization group methods coupled to $\varepsilon$ or $1/N$ expansions. In general, the assumption that transport and kinetic coefficients such as $\Gamma$ in eq. (5) are temperature independent turns out to be false. In any known case, however, the combination $(T-T_C)/\Gamma$ still vanishes. Then, the critical slowing down found in (5) persists, even though with a different power law.

II. WHAT ABOUT 3+1 DIMENSIONAL, RELATIVISTIC, $T \neq 0$, QFT’S?

We have seen that the behavior of $\gamma$ predicted by perturbation theory is exactly opposite to what is obtained by the Landau-Ginzburg approach. On the other hand, the latter is based on quite general and physically reasonable assumptions, while it is well known that perturbation theory is unreliable in the critical region.

In refs. [11,12,1] it was shown that the key effect which is missed by perturbation theory is the dramatic thermal renormalization of the coupling constant, which vanishes in the critical region. This is just a consequence of triviality of $\lambda \phi^4$ theory, which implies that, in absence of any physical IR cut-off, the renormalized coupling at zero external momentum vanishes, even if the ultraviolet cut-off, $\Lambda_{UV}$, is kept fixed. This is true even at $T = 0$, where $\lambda$ vanishes as $1/\log(m/\Lambda_{UV})$ when $m \to 0$. At $T \neq 0$, due to the effective three dimensionality of the theory in the critical region, we have a stronger, linear, dependence, $\lambda(T) \sim m(T)$.

In the framework of Wilson Renormalization Group this can also be understood as follows. The IR regime of the four-dimensional field theory at $T = T_C$ is related to that of the three-dimensional theory at $T = 0$. In particular, the three-dimensional running coupling is obtained from the four-dimensional one by [12]

$$\lambda_{3D}(\Lambda) = \lambda(\Lambda) \frac{T}{\Lambda},$$

where $\Lambda$ is the running parameter. At the critical point, $\lambda_{3D}$ flows in the IR to the Wilson-Fischer fixed point value $\lambda_{3D}^* \neq 0$, so that the four-dimensional coupling vanishes,

$$\lambda(\Lambda) \to \frac{\Lambda}{T} \lambda_{3D}^* \quad \text{(for} \quad T \simeq T_C \text{and} \quad \Lambda \to 0).$$

The critical exponent governing the vanishing of $\lambda$ is the same as that for $m(T)$, so that the ratio $\lambda(T)/m(T)$ goes to a finite value at $T_C$, and a second order phase transition is correctly reproduced.

We will see that the running of the coupling constant for $T \simeq T_C$ is crucial also in turning the divergent behavior of eq. (5) into a vanishing one. As a first rough ansatz one could just replace the tree coupling $\lambda$ with the thermally renormalized one in eq. (5) and readily see that the critical speeding up is indeed turned into the expected slowing down. However, as we will see, this
gives only a qualitatively correct answer, due to the logarithmic singularity of the on-shell imaginary part when \( m(T) \) vanishes.

III. THE TRG

Let me now present in some detail the computation of \( \gamma(T) \) in the TRG framework. This method allows a computation of the damping rate for any value of \( T \), from very high values, where perturbation theory works, to the critical region, where renormalization group methods like those of [1] are necessary to resum infrared divergences. Remarkably, the TRG is applicable also in the intermediate region, in which none of these methods can be employed.

Wilson’s RG idea was originally formulated on the lattice [13], and then implemented in continuum QFT’s by Polchinski [14]. In this framework, it is based on the introduction of an explicit IR cut-off, via the modification of the tree level propagator. Let me briefly recall how it works for the scalar theory at \( T = 0 \) limit, the generating flow equation is obtained

\[ D(p) = \frac{1}{p^2 + m^2} \rightarrow D_\Lambda(p) = D(p)\Theta(p^2 - \Lambda^2), \]

(where \( \Theta \) can be Heavyside’s step function or a smooth cut-off) in the usual expression for the generating functional,

\[ Z_\Lambda[J] = \int D\phi \exp \left[-\left(\frac{i}{2} \phi \cdot D_\Lambda^{-1} \cdot \phi + S[\phi] + J \cdot \phi\right)\right]. \]

\( Z_\Lambda[J] \) generates Green functions in which only the modes with \( p > \Lambda \) have been integrated out. Deriving with respect to \( \Lambda \) the generating flow equation is obtained

\[ \Lambda \frac{\partial}{\partial \Lambda} Z_\Lambda[J] = -\frac{i}{2} \left(\frac{\delta}{\delta J} \delta J \cdot \partial \Lambda^{-1} \cdot \delta J\right) Z_\Lambda[J]. \quad (8) \]

The exact equation (8) provides a non-perturbative definition of the \( T = 0 \) QFT. It interpolates between the ‘bare’ theory, defined at some ultraviolet scale \( \Lambda = \Lambda_{UV} \), and the renormalized one at \( \Lambda = 0 \), in which all quantum fluctuations with momenta \( 0 \leq p \leq \Lambda_{UV} \) are included.

A serious problem of this kind of approach is encountered when gauge theories are considered. Indeed, the momentum cut-off explicitly breaks gauge invariance, leading to \( \Lambda \)-dependent modified Slavnov-Taylor identities (ST) which only in the \( \Lambda = 0 \) limit recover their usual form. Even though in principle one can work with this new form of the ST’s, it is in practice quite non trivial. Moreover, no physical meaning can be assigned to the theory at non-zero value of \( \Lambda \), and the interpretation of \( \Lambda \) as a physical resolution scale is not possible at all.

When it comes to \( T \neq 0 \) QFT’s, two possibilities are on the ground. One is to work in the imaginary time formalism and straightforwardly apply the same steps outlined above, getting flow equations which interpolate between the \( T = 0 \) bare theory and the \( T \neq 0 \) equilibrium QFT [12][13]. As long as one is interested in static quantities in the critical regime of non-gauge theories, this approach works very well.

The other possibility is the TRG [1]. The TRG flow has three unique characteristics: \( i \) it is formulated in the real time; \( ii \) it interpolates between the \( T = 0 \) renormalized QFT and the \( T \neq 0 \) QFT in thermal equilibrium; \( iii \) it is explicitly gauge invariant [13]. All this is achieved by exploiting the fact that the tree level propagator in the real-time formalism (eq. (8)) is made up by the sum of two terms,

\[ D_{11}(p) = \frac{i}{p^2 - m^2 + i\varepsilon} + 2\pi\delta(p^2 - m^2)N(|p_0|) \quad (9) \]

the first one being just the \( T = 0 \) propagator and the second containing all the statistical information. The cut-off is then introduced in the thermal sector only, by modifying the Bose-Einstein distribution function as

\[ N(k_0) \rightarrow N_\Lambda(k_0) = N(k_0)\theta(|k| - \Lambda). \quad (10) \]

Now, as \( \Lambda \rightarrow \infty \) the modified propagator reduces to that of the \( T = 0 \) theory, and the generating functional gives the Green functions of the \( T = 0 \) renormalized QFT (i.e. with all the \( T = 0 \) quantum fluctuations integrated out). By lowering \( \Lambda \), the thermal modes of momenta \( |k| > \Lambda \) are progressively integrated out. Eventually, in the \( \Lambda \rightarrow 0 \) limit, the \( T \neq 0 \) theory in equilibrium is reached. Moreover, since \( N(|p_0|) \) comes along with a delta function forcing the momentum on the mass shell, our cut-off procedure does not break gauge invariance [12].

IV. COMPUTATION OF \( \gamma(T) \)

The flow equations relevant to the computation of the plasmon damping rate are schematically reproduced in Fig. 1. The RHS’s are formally one-loop expression, but they contain exact and \( \Lambda \)-dependent propagators and vertices, represented by a black dot. The empty dot represents the kernel of the evolution equation, which substitutes the full propagator in the corresponding leg. It is given by

\[ K_{\Lambda,ij}(k) = -\rho_\Lambda(k)\varepsilon(k_0)\Lambda\delta(|k| - \Lambda)N(|k_0|)B_{ij} \]

where \( \rho_\Lambda(k) \) is the full, \( \Lambda \)-dependent, spectral function, \( \varepsilon(x) = \theta(x) - \theta(-x) \), and \( B_{ij} = 1 \) with \( i,j = 1,2 \) the thermal indices.

We need to compute the real and imaginary parts of the (1-1) component of the self-energy

\[ \Sigma_{11}(\omega \pm i\varepsilon; k; \Lambda) = \Sigma_{11}^R(\omega; k; \Lambda) \pm i\Sigma_{11}^I(\omega; k; \Lambda) \quad (11) \]

where the real part is the same as that of the self-energy appearing in the propagator, \( \Sigma_{11}^R(k; \Lambda) = \Pi_R(k; \Lambda) \) and
the imaginary part is linked to $\Pi_I$ in (1), by $\Pi_I = \Sigma_{I1}/(1 + 2N)$ [7].

\[
\Lambda \frac{d}{d\Lambda} = \begin{array}{c}
\text{\includegraphics[width=0.2\textwidth]{damping}}
\end{array}
\]

\[
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\text{\includegraphics[width=0.2\textwidth]{damping}}
\end{array}
\]

FIG. 1. Schematic representation of the evolution equations for the two- and four- point functions.

In order to determine the $\Lambda$-dependent four-point function we must consider the system containing also the second equation in Fig. 1. Since the theory has a – possibly spontaneously broken – $Z_2$ symmetry, the trilinear couplings are not arbitrary and will be determined from the mass and the quartic coupling as we will indicate below. All $n$-point vertices with $n > 4$ will be neglected.

The initial conditions for the evolution equations are given at a scale $\Lambda = \Lambda_0 \gg T$ (due to the exponential damping in the Bose-Einstein function, $\Lambda \gtrsim 10T$ will be enough). Here, the effective action of the theory is approximated by

\[
\Gamma_{\Lambda_0}(\Phi) = \int d^4x \left[ \frac{1}{2} (\partial \Phi)^2 - \frac{1}{2} \mu^2_{\Lambda_0} \Phi^2 - \frac{\lambda_{\Lambda_0}}{4!} \Phi^4 \right],
\]

where $\mu^2_{\Lambda_0}$ and $\lambda_{\Lambda_0}$ are the renormalized parameters of the $T = 0$ theory. We are interested to the case in which the $Z_2$-symmetry is broken at $T = 0$, so we will take $\mu^2_{\Lambda_0} < 0$.

Then, we make the following approximations to the full propagator and vertices appearing on the RHS of the evolution equations:

i) the self-energy in the propagator is approximated by a running mass, $\Pi(k; \Lambda) \approx m^2_{\Lambda}$ given by

\[
m^2_{\Lambda} = \mu^2_{\Lambda} \quad (if \mu^2_{\Lambda} > 0), \quad -2\mu^2_{\Lambda} \quad (if \mu^2_{\Lambda} < 0)
\]

ii) the four-point function is approximated as

\[
\Gamma^{(4)}_{\Lambda}(p_i) \simeq -\lambda_{\Lambda} - i\eta_{\Lambda}(p_i),
\]

iii) the three-point function is given by

\[
\Gamma^{(3)}_{\Lambda}(p_i) \simeq 0 \quad (if \mu^2_{\Lambda} > 0), \quad \sqrt{-6\lambda_{\Lambda}\mu^2_{\Lambda}} \quad (if \mu^2_{\Lambda} < 0).
\]

Moreover, all vertices with at least one of the thermal indices different from 1 will be neglected.

Some comments are in order at this point. In perturbation theory, the on-shell imaginary part is given by the two-loop setting sun diagram. The evolution equations of Fig. 1 contain instead only one-loop integrals, so how can an imaginary part emerge? The crucial point here is that the momentum dependence of the imaginary part of the four-point function has to be taken into account (see point ii)). When the latter is inserted into the upper equation, $\Pi_I$ is generated.

As long as the $\Lambda$-dependent action is in the broken phase, trilinear couplings will be present. By neglecting the imaginary part of the self-energy in the propagators on the RHS (point i) above, we lose the contribution to $\Pi_I$ obtained by cutting the second diagram in the RHS of the upper equation of Fig. 1. However, if we restrict ourselves to temperatures $T \geq T_c$, the trilinear couplings will be different from zero only in a limited range of $\Lambda$. They will not contribute in the IR, where the dominant contributions to the on-shell imaginary part emerge (see Fig. 3). The error induced by this approximation can be estimated noticing that the contribution we are neglecting is of the same nature as the one obtained by inserting the last diagram of the second equation in the equation for the two-point function. The latter is taken into account and its effect on the full imaginary part is of the order of a few percent.

We have now a system of four evolution equations for $\Pi_{R}(k; \Lambda) \approx m^2_{\Lambda}, \Sigma_{11}(k; \Lambda), \lambda_{\Lambda}, \text{ and } \eta_{\Lambda}(p_i)$, initial conditions $m^2_{\Lambda_0} = -2\mu^2_{\Lambda_0}, \lambda_{\Lambda_0}, \text{ and } \Sigma_{11}(k; \Lambda_0) = \eta_{\Lambda_0}(p_i) = 0$, respectively. We give, for simplicity, the explicit form of the evolution equations in the symmetric phase only, where the trilinear coupling vanishes:

\[
\Lambda \frac{\partial}{\partial \Lambda} m^2_{\Lambda} = -\frac{\Lambda^3}{4\pi^2} \frac{N(\omega_{\Lambda})}{\omega_{\Lambda}} \lambda_{\Lambda},
\]

\[
\Lambda \frac{\partial}{\partial \Lambda} \lambda_{\Lambda} = -3\frac{\Lambda^2}{4\pi^2} \frac{d}{d\omega_{\Lambda}} \left( \frac{N(\omega_{\Lambda})}{\omega_{\Lambda}} \right) \lambda_{\Lambda}^2,
\]

\[
\Lambda \frac{\partial}{\partial \Lambda} \Sigma_{11}(k; \Lambda) = -\frac{\Lambda^3}{4\pi^2} \frac{N(\omega_{\Lambda})}{\omega_{\Lambda}} \eta_{\Lambda}(k, -k, q_{\Lambda}, -q_{\Lambda}),
\]

\[
\Lambda \frac{\partial}{\partial \Lambda} \eta_{\Lambda}(k, -k, q_{\Lambda}, -q_{\Lambda}) = -\frac{\lambda_{\Lambda}^2}{2} (C_{\Lambda}(k_0 - \omega_{\Lambda}, k) + C_{\Lambda}(k_0 + \omega_{\Lambda}, k)),
\]

where $\omega_{\Lambda} = (\Lambda^2 + m^2_{\Lambda})^{1/2}, k = (k_0, 0), q_{\Lambda} = (\omega_{\Lambda}, |q| = \Lambda)$, and

\[
C_{\Lambda}(q_0, q) = \frac{1}{8\pi} \frac{N(\omega_{\Lambda})}{\omega_{\Lambda}} \frac{\Lambda}{|q|} (J(q_0) + J(-q_0)),
\]

with

\[
J(q_0) = 1 + 2 \theta(\Lambda^2 + q_0^2 - 2\omega_{\Lambda}q_0)^{1/2} - \Lambda
\]

\[
\leq \frac{2\omega_{\Lambda}q_0 - q_0^2 + |q|^2}{2|q|\Lambda} \leq 1,
\]

\[
= 0 \quad \text{otherwise.}
\]

Notice that the subsystem for $m^2_{\Lambda}$ and $\lambda_{\Lambda}$ is closed and can be integrated separately.

We then proceed as follows. Fixing the temperature, we first integrate the subsystem for $m^2_{\Lambda}$ and $\lambda_{\Lambda}$ down to $\Lambda = 0$ in order to find the plasmon mass, $m^2_{\Lambda=0}$. Then we fix the external momentum of $\Pi_I$ on this mass-shell, $k = (m^2_{\Lambda=0}, 0)$ and integrate the full system from $\Lambda = \Lambda_0$ down to $\Lambda = 0$. 

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FIG. 2. Temperature dependence of the coupling constant (solid line), and of the damping rate with the effect of the running of $\lambda$ included (dash-dotted) and excluded (dashed). The values for $\gamma$ have been multiplied by a factor of 10.

In Fig. 2 we plot the results for the damping rate $\gamma$ and the coupling constant at $\Lambda = 0$, as a function of the temperature $T$. The dashed line has been obtained by keeping the coupling constant fixed ($\Lambda$-independent) to its $T = 0$ value ($\lambda = 10^{-2}$), and reproduces the divergent behavior found in perturbation theory (eq. (3)). The crucial effect of the running of the coupling constant is seen in the behavior of the dot-dashed line. For temperatures close enough to $T_C$, the coupling constant (solid line in Fig. 2) is dramatically renormalized and it decreases as $\lambda_{\Lambda=0}(T) \sim t^{\nu}$

where $t \equiv (T - T_C)/T_C$ and we find $\nu \simeq 0.53$. The mass also vanishes with the same critical index. The decreasing of $\lambda$ drives $\gamma$ to zero, but with a different scaling law,

$\gamma_{\Lambda=0}(T) \sim t^\nu \log t$.

The above expression can be understood noticing that the two-loop contribution to $\Pi_I$, computed at vanishing $\omega = m(T)$, goes as $\lambda^2 \log m(T)$. The RG result replaces $\lambda$ with the renormalized coupling, then from (13) we have

$\gamma \sim \frac{\lambda_{\Lambda=0}^3}{m_{\Lambda=0}} \log m_{\Lambda=0} \sim t^{\nu} \log t$.

Taking couplings bigger than the one used in this letter ($\lambda = 10^{-2}$), the deviation from the perturbative regime starts to be effective farther from $T_C$. Defining an effective temperature as $\lambda_{\Lambda=0}(T)/\lambda_{\Lambda=0} \leq 1/2$ for $T_C < T \leq T_{eff}$ we find that $t_{eff}$ scales roughly as $t_{eff} \sim \lambda_{\Lambda=0}$.

In Fig. 3 we plot the running of $\lambda_{\Lambda}(T)$ and $\gamma_{\Lambda}(T)$ for two different values of the temperature. When $T \gg T_C$ most of the running takes place for $\Lambda > \Lambda_{soft}$, so it is safe to stop the running at this scale neglecting the effect of soft loops. When $T \to T_C$ this is not possible any longer, since most of the running of $\lambda$ and $\gamma$ takes place for $\Lambda < \Lambda_{soft}$. Thus, the vanishing of the mass gap forces us to take soft thermal momenta into account.

FIG. 3. $\Lambda$-running of the coupling constant $\lambda$ and of the damping rate $\gamma$ for two different values of $t = (T - T_C)/T_C$. The values of $\gamma$ (dot-dashed lines) have been multiplied by 500.

V. CONCLUSION AND CONSEQUENCES

The world of dynamical critical phenomena in relativistic QFT’s at $T \neq 0$ is still largely unexplored, the reason being that the two main tools employed in the static case, i.e resummed perturbation theory and lattice simulation, are of no use in this context. I think that the TRG is on a much better shape, and can be
properly employed to study time dependent correlation functions, relaxation rates, and transport coefficients at any temperature, from the critical one to much higher (or lower) ones.

In the case of the plasmon damping rate discussed here, the expected critical slowing down of long wavelength fluctuations has been obtained from first principles. The dominant effect has been shown to be the thermal renormalization of the coupling constant, which turns the divergent behavior of the plasmon damping rate into a vanishing one. In the future this work will be extended to non-zero spatial momentum, and to higher orders of approximation of the flow equations in order to check the scaling relations between dynamical critical exponents in a non-trivial way [10].

Before concluding, let me remark two interesting physical consequences of this result.

The first consequence has to do with the notion of quasiparticle. In order for this concept to make sense, a narrow resonance is needed, a good criteria being \( \gamma(T)/m(T) \lesssim 1 \). In perturbation theory this quantity diverges as \( t^{-\nu} \), and one would conclude that it is not possible to talk about quasiparticles around the critical point. In particular, no kinetic description close to the phase transition could be conceivable. The TRG result shows that it is not necessarily the case, as we see in Fig. 4. Now \( \gamma/m \) diverges only as \( 1/\log t \), and it may be less then unity very much closer to the critical point. Indeed, for \( \lambda = 10^{-2} \), the ratio \( \gamma/m(T) \) becomes larger than unity for \( t \lesssim 10^{-3} \) in perturbation theory, whereas the RG result is still \( \gamma/m(T) \approx 0.3 \) at \( t \approx 10^{-9} \).

The second consequence concerns second order phase transitions in the early universe. In a cosmological setting, the increasing lifetime of the fluctuations of the order parameter may modify the dynamics of second order – or weakly first order – phase transitions. Indeed, as the critical temperature is approached, the expansion rate of the universe exceeds the thermalization rate of long wavelength fluctuations, and the latter freeze out. There will be a range of temperatures around \( T_C \) in which the equilibrium expressions for the effective potential or the fluctuation rates cannot be employed. In brief, second order phase transitions in the early universe take place out of equilibrium.

At the electroweak scale this has no practical consequences. Due to the smallness of the expansion rate at that epoch, the temperature interval in which there is departure from equilibrium is too short to have any effect. On the other hand, at higher temperatures, typically for \( T \gtrsim 10^{10} \) GeV, the effect may be very strong and must be taken into account. This would lead to a scenario for the formation of topological defects similar to that proposed by Zurek in [20], and discussed by J. Rivers at this conference. In this scenario, the initial length scale of the network of topological defects is given by the correlation length at the freeze-out, \( \xi_{f.o.} \), whose determination requires in turn the knowledge of the temperature dependence of \( \gamma(T) \), computed here.

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FIG. 4. The ratio \( \gamma/m \) vs. \( T \) as obtained from TRG (dot-dashed line) and without including the running of \( \lambda \) (solid line)

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[1] M. D’Attanasio and M. Pietroni, Nucl. Phys. B472 (1996) 711.
[2] A. Weldon, Phys. Rev. D28 (1983) 2007.
[3] D. Boyanowsky, I.D. Lawrie and D.S. Lee, Phys. Rev. D54 (1996) 4013.
[4] R.R. Parwani, Phys. Rev D45 (1992) 4695; ibid. D48 (1993) 5965 (E).
[5] E. Wang and U. Heinz, Phys. Rev. D55 (1996) 899.
[6] G. Aarts and J. Smit, Pys. Lett. B393 (1997) 395; Nucl. Phys. B511 (1998) 451; W. Buchmüller and A. Jacovác, Phys. Lett. B407 (1997) 39.
[7] M. Gleiser and R.O. Ramos, PRD50 (1994), 2441; C. Greiner and B. Müller, Phys. Rev. D55 (1997) 1026; P.Arnold, D. Son, and L. Yaffe, Phys. Rev. D55 (1997) 6264; E. Iancu, hep-ph/9710543; D. Bödeker, hep-ph/9801430.
[8] W. Buchmüller and A. Jacovác, hep-th/9712093.
[9] N. Goldenfeld, Lectures on Phase Transitions and the Renormalization Group, Frontiers in Physics, Vol. 85, (Addison-Wesley, 1992).
[10] P.C. Hohenberg and B.I Halperin, Rev. Mod. Phys. 49 (1977) 435; J. Zinn-Justin, Quantum Field Theory and Critical Phenomena, International Series of Monographs in Physics, Chapter 35, (Oxford University Press, 1996)
[11] P. Elmfors, Z. Phys. C56 (1992) 601.
[12] N. Tetradis and C. Wetterich, Nucl. Phys. B398 (1993) 659.
[13] K.G. Wilson, Phys. Rev. B4 (1971) 3174, 3148; K.G. Wilson and J.G. Kogut, Phys. Rep. 12 (1974) 75.
[14] J. Polchinski, Nucl. Phys. B231 (1984) 269.
[15] D. Litim, these proceedings; S.B. Liao and M. Strickland, hep-th/9803173; and these proceedings.
[16] M. D’Attanasio and M. Pietroni, Nucl. Phys. B498 (1997) 443.
[17] N.P. Landsman and Ch.G. van Weert, Phys. Rep. 145 (1987) 141.
[18] M. Pietroni, hep-ph/9804351, to appear on Phys. Rev. Lett.
[19] B. Bergerhoff, hep-ph/980549; B. Bergerhoff and J. Reinruber, hep-ph/9809251.
[20] W.H. Zurek Phys. Rep. 276 (1996) 177.