Astrophysical and cosmological constraints on a scale-dependent gravitational coupling

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Abstract

We study the phenomenological consequences of the recently proposed idea of a running gravitational coupling on macroscopic scales. When applied to the rotation curves of galaxies, we find that their flatness requires the presence of baryonic dark matter. Bounds on the variation of the gravitational coupling from primordial nucleosynthesis and the change of the period of binary pulsars are analysed. We also study constraints on the variations of $G$ with scale from gravitational lensing and the cosmic virial theorem, as well as briefly discuss the implications of such a scenario for structure formation.

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1 Introduction

The flatness of the rotation curves of galaxies and the large structure of the Universe indicate that either the Universe is predominantly made up of dark matter of exotic nature, i.e. non-baryonic, and/or that on large scales gravity is distinctively different from that on solar system scales, where Newtonian and post-Newtonian approximations are valid. The former possibility has been thoroughly investigated on astrophysical as well as on cosmological grounds (see Ref. [1] for a review) and is one of the most active subjects of research in astroparticle physics. The second possibility, however relevant, has drawn little attention so far. This may be essentially due to the fact that until recently no consistent and appealing modification of Newtonian and post-Newtonian dynamics has been put forward. Many of these attempts [2], although consistent with observations, were most often unsatisfactory from the theoretical point of view. Actually, it has been recently shown that under certain fairly general conditions it seems unlikely that relativistic gravity theories can explain the flatness of the rotation curves of galaxies [3]. These conditions however do not exclude the class of generalizations of General Relativity that involve higher-derivatives. Quantum versions of these theories were shown to exhibit asymptotic freedom in the gravitational coupling [4] and one would expect this property to manifest itself mainly on large scales. This possibility would surprisingly imply that quantum effects could actually mimic the presence of dark matter [5], as well as induce other cosmological phenomena [6, 7]. One of the most striking implications of these ideas is the prediction that the power spectrum on large scales would have more power than the one predicted by the $\Omega = 1$ Cold Dark Matter (CDM) Model, in agreement with what is observed by IRAS [8]. Furthermore, due to the increase in the gravitational constant on large scales one finds that the energy density fluctuations grow quicker than in usual matter dominated Friedmann-Robertson-Walker models. Moreover, one can naturally explain with a scale-dependent $G$ the discrepancy between determinations of the Hubble’s parameter made at different scales, as suggested in [6], and recently studied in [9]. Nevertheless, independently of the possible running of the gravitational constant in a higher derivative theory of gravity, it is certainly worthwhile analysing the constraints on the scale-dependence of $G$ from astrophysical and cosmological phenomena, where such an effect would be dominant. On the other hand, in the last few years there has been a revival of Brans-Dicke like theories, with variable gravitational coupling, that has led to a number of phenomenological constraints on possible time variations of $G$. Of course, some of the constraints on $G$ can be written as constraints on $\Delta G$ over scales in which a graviton took a time $\Delta t$ to propagate. For instance, during nucleosynthesis the largest distance that a graviton could have traversed is the horizon distance at that time, i.e. a few light-seconds to a few light-minutes, or $10^{10}$ to $10^{12}$ cm, approximately the Earth-Moon distance. Such a distance is too small for quantum effects to become appreciable, as we will discuss below. However, those effects become important at kiloparsec (kpc) distances and therefore could be relevant for discussing the rotation curves of galaxies. We shall actually show, for a particular theory [6, 7], that the rotation curves of spiral galaxies cannot be entirely explained by the running of $G$, so some amount of baryonic dark matter is required, which is still consistent with the upper bound on baryonic matter coming from primordial nucleosynthesis. This result is generic of theories with a power-law dependence of the gravitational coupling on scale. On the other hand, we could impose bounds on a possible
variation of $G$ from a plethora of cosmological and astrophysical phenomena at large scales, although the lack of precise observations at those scales will make the bounds rather weak. It is nevertheless expected that the increasing precision of future experiments might tell us something about variations in $G$. However, since for most of these phenomena the gravitational constant appears in the factor $GM$, we cannot actually distinguish a variation in $G$ from the existence of some peculiar kind of dark matter. In fact, dark matter seems to us like some kind of ‘ether’, an ad hoc and unobserved medium that permeates space modifying the behaviour of otherwise well known baryonic matter. If the idea of an asymptotically free gravitational coupling is correct, we might be able to get rid of this elusive yet dominating component of the Universe.

Furthermore, a scale dependence of the gravitational constant arises from completely different reasons in the stochastic inflation formalism, as recently explored in [10] and [11]. The scaling behaviour and screening of the cosmological constant was also discussed in the context of the quantum theory of the conformal factor in four dimensions [12], as the theory approaches its infrared fixed point, at distance scales much larger than the horizon size. The way the gravitational constant varies with scale in each case is very different from that of the asymptotically free theories, so it seems worth studying the phenomenological constraints that might rule out one or another.

2 Asymptotic Freedom of the Gravitational Coupling

The main idea behind the results of Refs. [5, 6] is the scale dependence of the gravitational coupling. The inspiration for this comes from the property of asymptotic freedom exhibited by 1-loop higher–derivative quantum gravity models [4]. Since there exists no screening mechanism for gravity, asymptotic freedom may imply that quantum gravitational effects act on macroscopic and even on cosmological scales, a fact which has of course some bearing on the dark matter problem [5] and in particular on the large scale structure of the Universe [6, 7]. It is within this framework that a power spectrum which is consistent with the observations of IRAS [8] and COBE [13] can be obtained and where energy density fluctuations are shown to grow faster than in usual cosmological models [8, 7]. This last feature does bring some hope that the large scale structure may arise from primordial energy density fluctuations entirely amplified by an asymptotically free gravitational coupling and baryonic matter.

Let us now briefly outline this proposal. Removing the infinities generated by quantum fluctuations and ensuring renormalizability of a quantum field theory requires a scale–dependent redefinition of the physical parameters. This was done at the 1-loop level in a higher–derivative theory of gravity in Refs. [4]. Furthermore, the removal of those infinities still leave the physical parameters with some dependence on finite quantities whose particular values are arbitrary. These can be assigned by specifying the value of the physical parameters at some momentum or length scale; once this is performed, variations on scale are accounted for by appropriate changes in the values of the physical parameters as described by the renormalization group equations (RGEs). Thus, the equations of motion in the quantum field theory of gravity should be similar to the
ones of the classical theory, but with their parameters replaced by the corresponding ‘improved’
values, that are solutions of the corresponding RGEs. However, since gravity couples coherently to
matter and exhibits no screening mechanism, quantum fluctuations of the gravitational degrees
of freedom contribute on all scales. One must therefore include the effect of these quantum
corrections into the gravitational coupling, \( G \), promoting it into a scale–dependent quantity.

One-loop quantum gravity models indicate that the coupling \( G(\mu^2/\mu^2_*) \sim r^2/r^2 \) is asymptotically
free, where \( \mu_* \) is a reference momentum, meaning that \( G \) grows with scale [4]. A typical solution
for \( G(r^2/r^2) \) was obtained in Ref. [5] setting the \( \beta \)-functions of matter to vanish and integrating
the remaining RGEs:

\[
G(r^2/r^2) = G_{lab}(r^2_*/r^2_{lab}) \delta(r, r_{lab}),
\]

where \( G_{lab}(r^2_*/r^2_{lab}) \) is the value of \( G \) measured in the laboratory at a length scale \( r_{lab} \), and \( \delta(r, r_{lab}) \)
is a growing function of \( r \), such that it is equal to one at \( r = r_{lab} \). A convenient choice for \( r_* \)
is \( r_* = r_{lab} \). In order for the asymptotic freedom of \( G(\mu^2/\mu^2_*) \) to have an effect on for instance
the dynamics of galaxies and their rotation curves, the function \( \delta(r, r_{lab}) \) should be close to one
for \( r < 1 \text{ kpc} \), growing significantly only for \( r \geq 1 \text{ kpc} \). Naturally, this dependence of \( G \) with
distance has also implications of cosmological nature. A convenient parametrization for \( \delta(r, r_{lab}) \)
from the fit of Ref. [5] in the kpc range is the following:

\[
\delta(r, r_{lab}) = 1.485 \left[ 1 + \beta \left( \frac{r}{r_0} \right)^\gamma \ln \left( \frac{r}{r_0} \right) \right],
\]

where \( \beta \simeq 1/30, \gamma \simeq 1/10 \) and \( r_0 = 10 \text{ kpc} \).

We shall use this fitting in the next Section in our analysis of the rotation curves of galaxies,
and extract from it a prediction for the distribution of baryonic dark matter. However, before
we pursue this discussion let us present some of the ideas developed in Refs. [5-7]. As discussed
above, the classical equations have to be ‘improved’ by introducing the scale dependence of
the gravitational coupling. This method suggests that the presence of cosmological dark matter
could be replaced by an asymptotically free gravitational coupling. Assuming that the Friedmann
equation describing the evolution of a flat Universe is the improved one, then:

\[
H^2(\ell) = \frac{8\pi}{3} G(\frac{a^2}{a^2_\ell}) \rho_m ,
\]

where \( a = a(t) \) is the scale factor, \( H = \dot{a}/a \) is the Hubble parameter, \( \rho_m \) is the density of baryonic
matter, \( \ell \) is the comoving distance and \( \ell_* \) is some convenient length scale.

From Eq. (3) one sees that by construction the present physical density parameter, \( \Omega^\text{phys}_0 \), is
equal to one. However, the quantity which is usually referred to as density parameter is actually:

\[
\Omega_0 = \frac{8\pi}{3} \frac{G\rho_{m_0}}{H^2_{0_*}} ,
\]

where \( H_{0_0} \) is the present Hubble parameter for a given large scale distance, \( r = r_* \), and the value
of the product \( G\rho_{m_0} \) is inferred for different scales with the help of the Virial Theorem. This
leads to \( \Omega_0 \) as a growing function of scale, which is in agreement with observations for a constant
ρ_{m0}. We hence conclude that macroscopic quantum gravity effects do mimic the presence of dark matter.

Furthermore, from Eq. (3) one can clearly see the scale dependence of the Hubble parameter \[6, 7, 9\]. In view of the above arguments, the criticism raised in Ref. [14] concerning the way the effect of the running of the gravitational coupling on \(\Omega_0\) was considered in Ref. [6], seems to be unjustified since, as explained above, \(\Omega^{phys}_0 = 1\). Moreover, as shown in Refs. [6] and [7], the power spectrum resulting from these considerations is similar to that of a low density Cold Dark Matter model with a large cosmological constant [15] which in terms of the density matter:

\[
\Omega^{CDM}_0 = 0.15 \quad \Omega^{\Lambda}_0 = 0.80 \quad \Omega^{Baryons}_0 = 0.05 .
\] (5)

Another popular, although rather ad hoc, possibility to account for the large scale structure of the Universe consists of a mixture of Cold and Hot Dark Matter [16]:

\[
\Omega^{CDM}_0 = 0.65 \quad \Omega^{HDM}_0 = 0.30 \quad \Omega^{Baryons}_0 = 0.05 .
\] (6)

Although these two last possibilities are compatible with COBE [13] and IRAS [8] data one could consider an alternative model where the gravitational coupling is scale-dependent. We stress that the recently discovered evidence for gravitational microlensing by massive astrophysical objects in the Galactic halo [17, 18] implies, at least preliminarily, that a sizeable fraction of the halo is composed by non-luminous baryonic matter. This represents a serious difficulty for the existing models of structure formation since baryonic dark matter is notoriously inefficient as to the amplification of energy density fluctuations. Furthermore, the estimated ratio in density of non-luminous baryonic matter to cold dark matter cannot exceed in those models 1/4 and 1/10, respectively, and reported gravitacional microlensing events by the EROS collaboration [18] are compatible with the halo being entirely composed by Massive Compact Halo Objects (MACHOs). Thus, if it turns out that MACHOs do indeed dominate the halo, then it becomes particularly important to look for alternatives to the existing structure formation models.

3 Rotation Curves of Galaxies

Let us now turn to the discussion of the implications of the fit (2) for the rotation curves of galaxies. It is a quite well established observational fact that the rotation curves of spiral galaxies flatten after about 10 to 20 kpc from their centre, which of course is a strong dynamical evidence for the presence of dark matter and/or of non-Newtonian physics. The rotation velocity of the galaxy is given by the non-relativistic relation,

\[
v^2(r) = \frac{G(r)M(r)}{r},
\] (7)

\footnote{We thank A. Bottino to call our attention to this reference while we were writing our paper.}
which approaches a constant value some distance from the centre, e.g. \( v_0^2 = 220 \text{ km/s} \) for our galaxy. Assuming that the gravitational coupling is precisely Newton’s constant \( G_N \) and imposing that the rotation velocity is constant, using the Virial Theorem at \( r = R \equiv 500 \text{ kpc} \), one finds the standard expression for the mass distribution of dark matter:

\[
M_N(r) = M_N(R) \frac{r}{R}.
\]  

(8)

Assuming instead a running gravitational coupling \( G \), the condition that the rotation velocity is constant yields:

\[
M(r) = \frac{0.673}{1 + \beta \left( \frac{r}{r_0} \right)^\gamma \ln \frac{r}{r_0}} M_N(r).
\]

(9)

Equation (8) reveals after simple computation that the running of the gravitational coupling reduces the amount of dark matter required to explain the flatness of the rotation curves of galaxies by about 44%, assuming that galaxies stretch up to a distance of about 500 kpc. This result is in agreement with Ref. [14], a clear prediction of the dependence of the gravitational coupling with scale and, in particular, of the fit \( G \). Furthermore, since the possibility that the Galactic halo is entirely made up of baryonic dark matter is barely consistent with the nucleosynthesis bounds on the amount of baryons [19], the running of \( G \) is quite welcome since it reduces the required amount of baryonic dark matter in the halo (although not in the bulge). An entertaining hypothesis could be that precisely this effect is responsible for the reduction in the microlensing event rates across the halo in the direction of the Large Magellanic Cloud with respect to those along the bulge of our galaxy, as reported by [20].

4 Bounds on the variation of \( G \) with scale

In this section we constrain the variation of the gravitational coupling given by the fit \( G \) with bounds from primordial nucleosynthesis, binary pulsars and gravitational lensing. We shall also discuss the effect that a scale-dependent \( G \) has on the peculiar velocity field and how future experiments might help resolving such an effect at cosmological distances.

4.1 Primordial nucleosynthesis

As mentioned in the introduction, one could obtain bounds on the variation of the gravitational coupling from observations of the light elements’ abundances in the Universe. Such observations are in agreement with the standard primordial nucleosynthesis scenario (for a review see [21]), but there is still some room for variations in the effective number of neutrinos, the baryon fraction of the universe and also in the value of the gravitational constant. For instance, the predicted mass fraction of primordial \(^4\text{He}\) can be parametrised, in theories with a variable gravitational coupling, in the following way [21, 22],

\[
Y_p = 0.228 + 0.010 \ln \eta_{10} + 0.327 \log \xi,
\]

(10)
where $\eta_{10}$ is the baryon to photon ratio in units of $10^{-10}$ and $\xi$ is the ratio of the Hubble parameter at nucleosynthesis and its present value, itself proportional to the square root of the corresponding gravitational constant. In the fit (10) we have assumed that the effective number of light neutrinos is $N_\nu = 3$ and that the neutron lifetime is $\tau_n = 887$ seconds [23].

By running the nucleosynthesis codes for different values of $G$, Accetta, Krauss and Romanelli [24] were able to find a range of values of the gravitational coupling that were compatible with the observations of the primordial $D$, $^3He$, $^4He$ and $^7Li$ abundances. The range turned out to be rather large, $\Delta G/G = 0.2$ at the 1$\sigma$ level, due to the large statistical and systematic errors of the observations.

This result will now be used to constrain the running of $G$ in an asymptotically free theory of gravity. As mentioned above, in a theory with a scale-dependent gravitational constant, the maximum value of $G$ at a given time is the one that corresponds to the physical horizon distance at that time. During primordial nucleosynthesis, the horizon distance grows from a few light-seconds to a few light-minutes, i.e. less than a few milliparsecs. At that scale we find $\Delta G/G = 0.07$, see Eq. (2), which is much less than the allowed variation of $G$ given in [24]. Therefore, primordial nucleosynthesis does not rule out the possibility of an asymptotically free gravitational coupling. Of course, a light-second is about the distance to the Moon, and there are similar constraints on a variation of $G$ at this scale coming from lunar laser ranging, $\Delta G/G < 0.6$ [23]. As a consequence, a theory where the gravitational constant varies more quickly with scale would be ruled out by observations.

4.2 Binary pulsars

The precise timing of the orbital period of binary pulsars and, in particular, of the pulsar PSR 1913+16, provides another way of obtaining a model-independent bound on the variation of the gravitational coupling [26, 27]. Since the semimajor axis of that system is just about a few light-seconds, the resulting limits on the variation of $G$ can be readily compared with the ones arising from nucleosynthesis. The observational limits on the rate of change of the orbital period, mainly due to gravitational radiation damping, together with the knowledge of the relevant Keplerian and post-Keplerian orbiting parameters, allows one to obtain the following limit [26, 27]:

$$\sigma \equiv \frac{\Delta G}{G} < 0.08\, h^{-1},$$

where $h$ is the value of the Hubble parameter in units of 100 km/s/Mpc. For $h = 0.5$, one obtains $\sigma = 0.16$ which is more stringent than the nucleosynthesis bound, but is still compatible with the fit (2) for $r$ of a few light-seconds.
4.3 Gravitational lensing

Gravitational lensing of distant quasars by intervening galaxies may provide, under certain assumptions, yet another method of constraining, on large scales, the variability of the gravitational coupling. The four observable parameters associated with lensing, namely, image splittings, time delays, relative amplification and optical depth do depend on $G$, more precisely on the product $GM$, where $M$ is the mass of the lensing object. This dependence might suggest that limits on the variability of $G$ could not be obtained before an independent determination of the mass of the lensing object. However, as the actual bending angle is not observed directly, the relevant quantities are the distance of the lensing galaxy and of the quasar. Since these quantities are inferred from the redshift of those objects, they depend on their hand on $G$, on the Hubble constant, $H_0$, and on the density parameter, $\Omega_0$. However, as we have previously seen, a scale-dependent gravitational coupling implies also a dependence on scale of $H_0$ and $\Omega_0$, see Eqs. (3) and (4). This involved dependence on scale makes it difficult to proceed as in Ref. [28], where gravitational lensing in a flat, homogeneous and isotropic cosmological model, in the context of a Brans-Dicke theory of gravity, was used to provide a limit on the variation of $G$:

$$\frac{\Delta G}{G} = 0.2.$$  \hspace{1cm} (12)

Since for this limit $\Omega_0 = 1$ was assumed, while in a scale-dependent model it is achieved via the running of the gravitational coupling, the bound (12) contrains only residual variations of $G$ that have not been already taken into account when considering the dependence on scale of $H_0$ and $\Omega_0$. Of course, for models where the cosmological parameters are independent of scale, the bound (12) can be readily used to constrain the variability of $G$ on intermediate cosmological scales. It is worth stressing that this method, besides being one of the few available where this variability is directly constrained at intermediate cosmological times between the present epoch and the nucleosynthesis era, it is probably the only one which can realistically provide in the near future even more stringent bounds on even larger scales by observing the lensing of light from far away quasars caused by objects at redshifts of order $z \geq 1$.

4.4 Peculiar velocity field

Since we expect the effects of a running $G$ to become important at very large scales, one could try to explore distances of hundreds of Mpc, where the gravitational coupling is significantly different from that of our local scales. That is the realm of physical cosmology: peculiar velocity fields and structure formation. Unfortunately, it is also the realm of large observational uncertainties, which precludes any reasonable detection of the effect we are looking for. However, with the planed future sky surveys like the Sloan survey (SDSS) and powerful telescopes like HST and Keck, one might expect this effect to become observable in the not-so-far future. A possible signature would be a mismatch between the velocity fields and the actual mass distribution, such that at large scales the same mass would pull more strongly. To be more specific, in an expanding universe there is a relation between the kinetic and gravitational potential energy of density perturbations known
as the Layzer-Irvine equation (for a detailed description see Ref. [29]). It is a generalization of the Newtonian Virial Theorem for non-linear gravitational instabilities (although still non-relativistic), and can be written as a relation between the mass-weighted mean square velocity \( \bar{v}^2 \) and the mass autocorrelation function \( \xi(r) \),

\[
\bar{v}^2(r) = 2\pi G \rho_b J_2(r),
\]

(13)

where \( \rho_b \) is the mean local mass density and \( J_2(r) = \int_0^r r \, dr \, \xi(r) \). The galaxy-galaxy correlation function can be parametrized by \( \xi(r) \sim (r/r_0)^{-1.8} \) with \( r_0 = 5h^{-1} \) Mpc, while the cluster-cluster correlation function has the same expression with \( r_0 = 20h^{-1} \) Mpc. This means that the velocity field (13) should be proportional to \( (r/r_0)^{0.2} \), unless the gravitational constant has some scale dependence. So far the relation seems to be satisfied, under huge experimental errors (for a review see Ref. [30]), except perhaps for a discrepancy in the motion of the Local Group towards the Abell cluster, 150\( h^{-1} \) Mpc away, with velocities up to \( v = 689 \pm 178 \) km/s [31], which might indicate some anomaly in the velocity-mass relation. Unfortunately, the errors are so large that it would be naive to infer from this a scale dependence of \( G \). Even worse, phenomenologically there is a proportionality constant between the galaxy-galaxy correlation function and the actual mass correlation function, the so-called biasing factor, which is supposed to be scale dependent and could mimic a variable gravitational constant. However, future sky surveys might be able to constrain more strongly the relation (13) by measuring peculiar velocities with better accuracy at larger distances. It might then be possible to extract the scale-dependence of \( G \).

Another very important area of cosmology in which bounds on a hypothetical scale dependence of \( G \) can be obtained in the forseeable future is the large scale structure of the Universe, i.e. theories of structure formation and evolution. They deal with the largest possible scales, all the way up to the horizon, and thus are presumably the most likely to be sensitive to a strong scale dependence of the gravitational constant. Unfortunately, as mentioned repeatedly before, those are the regions with largest uncertainty errors. However, present and future experiments like COBE, IRAS, the Sloan Digital Sky Survey (SDSS), etc. will soon begin to constrain the existing models of structure formation like cosmic strings, CDM, MDM, etc. and therefore leave room for a possible determination of the scale dependence of \( G \). In particular, as shown in Ref. [6], the amplitude of linearised density perturbations grow more quickly than in general relativity and could be even more important in the non-linear regime, which might help accelerate structure formation in an early epoch without the need of introducing non-baryonic dark matter with \textit{ad hoc} properties. It is certainly worthwhile investigating the relevance of this effect in explaining the large scale structure of the Universe.

\footnote{For a recent review see Ref. [32].}
5 Conclusions

Let us summarise our results and comment on some further directions of investigation. We have seen that the running of the gravitational coupling is compatible with the observational fact that the rotation curves of galaxies are constant provided some amount of baryonic dark matter is allowed, actually about 44% less than what is required for a constant $G$. This could also explain why we see less microlensing events towards the halo than in the direction of the bulge of our galaxy. Failure in reproducing the predicted distribution of baryonic dark matter would signal either that the approach adopted here is unsuitable or that the fit (4) is inadequate, perhaps suggesting alternative scale dependences like those discussed in the introduction. For the purpose of distinguishing between either of them, we have looked for possible bounds on variations of $G$ with scale from primordial nucleosynthesis, lunar laser ranging, variations in the period of binary pulsars, macroscopic gravitational lensing and even deviations in the peculiar velocity flows. Unfortunately, as observational errors tend to increase with the scale probed, we cannot yet seriously constrain an increase of $G$ with scale, as proposed by the asymptotically free theories of gravity. Our study may provide nevertheless a guidance for further efforts in constraining the variability of the gravitational coupling in other cosmological or quantum gravity models.

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