Influence of the long-range of compact Jets in the Aspiration Pipeline during Dustdeposing

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Abstract. There is the relevant issue of cleaning the ducts walls from dust deposits on industrial plants, workshops and production lines equipped with exhaust ventilation systems. Improving the efficiency of the process of cleaning air ducts from dust depends on the long-range of compact jets. The mathematical model is based on the classical solution of the "submerged jet" problem propagating in a stationary medium. The use of this model made it possible with sufficient accuracy to estimate the parameters of the range of compact jets when operating aspiration systems. Studies have shown that the presented model can be used to analyze the state of the pipelines of the suction networks and dust collection plants.

1. Introduction
Ventilation is the set of activities and devices that provide the calculated air in the premises of residential, public and industrial buildings. The production process is accompanied by release of gases and vapors harmful to human health into the air of workplaces. In addition, large amounts of heat, moisture, and dust may be released into the air in production areas, increasing its temperature and humidity, as well as increasing its dustiness [1, 2]. Ventilation is provided to maintain normal air parameters in the premises that meet sanitary, hygienic and technological requirements. Sanitary-hygienic purpose of ventilation is to remove harmful gases, vapors and dust [3, 4].

2. Research problem
Considering the pipeline of the aspiration system, in which the air moves in the turbulent mode, with a certain average velocity $U_1$ (Figure 1). To prevent the accumulation of dust on the pipeline walls, air is additionally supplied through the nozzles to the system in the form of a compact jet with an initial velocity $U_2$. In this case, the vectors $\vec{U}_1$ and $\vec{U}_2$ directed in one direction and parallel to the axis of the pipeline. The purpose is to determine the jet's long range, that is, a function of the maximum speed of the air flow $U_{\text{max}}(z)$ in the pipeline during the nozzle work, as well as the distribution of the flow velocity in the section plane of the pipeline.
3. Solution methods

The work is based on the classical version of the solution of the "submerged jet" problem propagating in a stationary medium with the same physical constants as that of the jet itself. An extremely thin (almost point) source of a jet with an initial impulse (second moment of movement) $J_0$ is considered (Figure 2).

The initial impulse is related to the air flow $Q$ through the expression $J_0 = \rho Q$, ($\rho$ is the air density). The axis Oz is directed along the axis of symmetry of a flat jet. When the external velocity is equal to zero, the equation of motion of a viscous incompressible fluid (air here is considered as an incompressible fluid) will have the following form:

$$
\begin{align*}
\frac{U}{\partial z} + \nu \frac{\partial U}{\partial y} &= \nu \frac{\partial^2 U}{\partial y^2}, \\
\frac{\partial U}{\partial z} + \frac{\partial v}{\partial y} &= 0,
\end{align*}
$$

where $U = \nabla_z$, $v = \nabla_y$, $\nu$ - is the kinematic viscosity of air.

![Figure 2. Scheme to the problem of a flat submerged jet.](image2.png)
The system of equations (1) for the case of a flat laminar submerged jet was solved by H. Schlichting [5].

For a nozzle having a final radius $r_o$, the initial impulse is equal to:

$$J_o = \rho \pi r_o^2 U_o^2,$$

Where $U_o$ - is the relative initial velocity of the jet.

The system of equations (1) for an axisymmetric uncoiled jet having a circular cross section will be written in cylindrical coordinates:

$$\begin{align*}
U \frac{\partial U}{\partial z} + \nu \frac{\partial U}{\partial r} &= \left( \nu \frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} \right), \\
\frac{\partial (rU)}{\partial z} + \frac{\partial (ru)}{\partial r} &= 0.
\end{align*}$$

The solution of system (3) will be self-similar, since equality of each section of the jet is maintained:

$$J = \rho \int 2\pi r \epsilon^2 \partial r = J_o = \text{const},$$

The solution of system (3), with (4) taking into account, is obtained as the distribution of the maximum velocity $U_{\text{max}}$ along the $z$ axis.

$$U_{\text{max}} = \frac{3}{8} \frac{J_o}{\pi \nu z}.$$ 

For a turbulent flow, the problem of the "submerged jet" was solved using the semi-empirical theories of L. Prandt [6], in which the turbulent air flow was compared with the laminar one. In this case, the jet propagation equation has the form:

$$U \frac{\partial U}{\partial z} + \nu \frac{\partial U}{\partial r} + \varepsilon_r \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial U}{\partial r} \right),$$

where $\varepsilon_r$ - is, by analogy with laminar, the coefficient of turbulent viscosity

$$\varepsilon_r = \sigma \sqrt{\frac{J_o}{\rho}},$$

where $\sigma$ - is the empirical coefficient $\sigma \approx 0.21$.

The maximum speed for an asymmetric non-swirl jet spouting from an infinitely small orifice is obtained from (6) with (7) taken into account:

$$U_{\text{max}} = \frac{3}{8\pi \nu} \sqrt{\frac{J_o}{\rho \cdot z}},$$
In the case of a non-swirling turbulent jet flowing from a hole of finite diameter \( d = 2r_o \) with a finite initial flow rate, as well as a swirling turbulent jet, the correction is applied to formula (8):

\[
U_{\text{max}} = \frac{3}{8\pi\rho} \sqrt{\frac{J_o}{\rho}} \left( \frac{1}{z} \left( 1 - \frac{1}{16\sqrt{\pi\sigma} \cdot z} \right) \right),
\]

(9)

4. Results and discussion

Comparison of values \( U_{\text{max}} \) calculated by formulas (8) and (9) showed that deviations of \( U_{\text{max}} \) are observed only at small values of \( z \) (Table 1). The numerical experiment conducted for the values of the constants \( r_o = 0.01 \) m and \( U_o = 150 \) m/s showed that when \( z \approx (1\ldots3)r_o \) relative deviations vary within \( \Delta = 33.33 - 11.11\% \), and when \( z \approx 5r_o \), is \( \Delta \approx 7\% \).

| \( z \), m | 0.01 | 0.02 | 0.04 | 0.10 | 0.20 | 0.40 | 1.00 | 2.00 | 4.00 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| \( \frac{z}{r_o} \) | 1 | 2 | 4 | 10 | 20 | 40 | 100 | 200 | 400 | 1000 |
| \( U_{\text{max}}^{(1)} \) m/s | 150 | 75 | 37.5 | 15 | 7.5 | 3.75 | 1.5 | 0.75 | 0.375 | 0.15 |
| \( U_{\text{max}}^{(2)} \) m/s | 100 | 62.5 | 34.38 | 14.5 | 7.38 | 3.72 | 1.50 | 0.75 | 0.37 | 0.15 |
| \( \Delta \), % | 33.33 | 16.67 | 8.33 | 3.33 | 1.67 | 0.83 | 0.33 | 0.17 | 0.08 | 0.03 |

The value \( U_{\text{max}}^{(1)} \) was calculated by the formula (8), and \( U_{\text{max}}^{(2)} \) - by the formula (9), and the deviation of these values is \( \Delta = \left( \frac{U_{\text{max}}^{(1)} - U_{\text{max}}^{(2)}}{U_{\text{max}}^{(1)}} \right) \cdot 100\% \).

Thus, we have adopted formula (8) for calculating the parameters of a non-swirling turbulent jet flowing from an orifice of a finite diameter.

The velocity distribution \( U(r) \) over the jet section is calculated by the formula:

\[
U(r) = \frac{U_{\text{max}}}{\left[ 1 + \frac{1}{3} \left( \frac{r}{z} \right)^2 \right]^\frac{3}{2}},
\]

(10)

To go to a compact jet propagating in a stream of moving air (medium), consider two inertial reference systems described by cylindrical coordinates. The reference system rOz is stationary and has an origin at the exit of the nozzle. The reference system r'Oz' is tied to the medium and moves with the speed \( U_1 \) along with the air flow along the z axis (Figure 3). At time \( t \), the origin of O and O' are combined, and \( r = r' \). At the same time relative to the system the speed of the main air flow is \( U_2 \). Taking into account that the speed of the jet flowing from their nozzles in the reference system rOz is equal to \( U_2 \), we find the speed of the jet \( U_2' \) relative to the inertial reference system:

\[
U_2' = U_2 - U_1,
\]

(11)
Therefore, if we accept $U_o = U_2$, then the jet having the initial impulse (in the $r'O'z'$ system) $J_o = \rho \pi r_o^2 \left( U_2 \right)^2$ can be considered as a submerged jet propagating in a fixed medium. By substitution (11) in formula (8) by replacing $z$ with $z'$ we get:

$$U_{\text{max}} (z') = \frac{3}{8\pi} \frac{\rho \pi r_o^2 \left( U_2 \right)^2}{\rho} \cdot \frac{1}{z} = \frac{3r_o U_2}{8\sqrt{\pi} \sigma z'} \approx \frac{r_o (U_2 - U_1)}{z'}; \quad z' \in [r_o, \infty),$$

(12)

Here $U_{\text{max}} (z')$ is the maximum speed of the jet relative to the reference system $r'O'z'$. For the transition from $U_{\text{max}} (z')$ to the function $U_{\text{max}} (z)$ a transition from a moving reference system $r'O'z'$ to a stationary $rOz$ is necessary:

$$\begin{cases} U_{\text{max}} (z) = U_{\text{max}} (z') + U_1; \\ z = z' + U_1 T, \end{cases}$$

(13)

where $T$ is the time during which in the $r'O'z'$ system the jet velocity $U_1$ is equal to $U_{\text{max}} (z')$. To determine the value of $T$, we consider the differential:

$$U_{\text{max}} (z') = \frac{dz'}{dt},$$

(14)

then

$$T = \int_0^T dt = \int_{U_{\text{max}} (z')}^{\infty} \frac{dz'}{U_{\text{max}} (z')} = \int_{r_o (U_2 - U_1)}^{\infty} \frac{z' dz'}{2r_o \left( U_2 - U_1 \right)} = \int_{r_o (U_2 - U_1)}^{x} \frac{(z')^2 - r_o^2}{2r_o \left( U_2 - U_1 \right)}.$$  

(15)

By taking into account (15) from formula (13) we get:

$$z = z' + \frac{U_1 \left[ (z')^2 - r_o^2 \right]}{2r_o \left( U_2 - U_1 \right)},$$

(16)
Equation (16) is a quadratic equation with respect to $z'$. We need to find the roots of this equation $z_{1,2}'$.

Since from the condition of the problem $z' > 0$ we have:

$$z' = r_0 \left\{ \sqrt{(U_2 - U_1)^2 + 2\alpha U_1 (U_2 - U_1) + U_2^2 - (U_2 - U_1)^2} \right\},$$

(17)

By substituting (17) into formula (12), and moving from $U_{\text{max}}(z')$ to $U_{\text{max}}(z)$ we get:

$$U_{\text{max}}(z) = U_1 + \frac{U_1 (U_2 - U_1)}{\sqrt{U_2^2 - 2\alpha U_1 (U_2 - U_1) + U_1^2 - (U_2 - U_1)^2}},$$

(18)

The calculation function $U_{\text{max}}(z)$ is given in tabular form (table 2) for the following conditions: $r_o = 0.01$ m and $U_1 = 12$ m/s ($Q_1 = 204$ m$^3$/min); $U_2 = 150$ m/s ($Q_2 = 2.83$ m$^3$/min).

| $z$, m | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.0 | 1.2 | 1.5 | 2.0 |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $U_{\text{max}}(z)$ | 22.7 | 18.6 | 17.0 | 16.2 | 15.6 | 15.6 | 15.3 | 14.9 | 14.4 |

The radial distribution of air velocity $U(r)$ is calculated by the formula:

$$U(r, z) = U(r, z') + U_1,$$

(19)

$$U(r, z') = \frac{U_{\text{max}}(z')}{\left[ 1 + \frac{1}{3} \left( \frac{r}{z'} \right)^2 \right]},$$

(20)

5. Conclusion

The developed mathematical model using the equations systems of H. Schlichting and semi-empirical theories about the "submerged jet" of L. Prandtl allows us to estimate the range of compact jets for cleaning the internal surfaces of the walls of the suction pipelines around the perimeter from dust deposits. The presented mathematical model was used in the development of the device, which eliminates the formation of stagnant and closed zones throughout the dust-gas flow through the duct and provides low aerodynamic resistance of pipelines with dust-cleaning equipment.

References

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