Clutter clipping in radars with quasicontinuous mode of transmission and reception of signals with pseudorandom amplitude- and phase-shift keying

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Abstract. This paper considers the target locating against the clutter background in radars with quasicontinuous mode of transmission and reception of signals with pseudorandom amplitude- and phase-shift keying. The clutter clipping is proposed to increase the signal-to-clutter-plus-noise ratio (SCNR). The mathematical model of the time-distributed clutter clipping is presented. The modelling example of a pinpoint target locating using correlation processing of a return signals is given with and without clutter clipping. Given example shows the effectiveness of clutter clipping. The estimation of SCNR in presence of clutter clipping is derived using the distribution histogram of the received signal amplitudes. The clutter clipping efficiency depending on the parameters of the probe signal is investigated.

1. Introduction
Clipping is a very effective method of pulse noise amplitude normalizing. It is usually used to mitigate the effects of a pulse noise in different wireless communication systems [1,2].

In this paper we investigate the clutter clipping in radars with quasicontinuous mode of transmission and reception of signals with pseudorandom amplitude- and phase-shift keying (APSK) [3,4].

The phase-shift keying pulse interval of the probing APSK signal is random and the phase-shift keying law changes from pulse to pulse. The coded pulse transmission interval is not adjusted to the instrumental radar range and is much shorter than the signal length. Coded pulses in the quasicontinuous signal can follow each other without time gaps, or have several of pause intervals between them. The random amplitude shift-keying law makes APSK signals different from other types of pulse trains, such as signals with pulse repetition frequency jittering, staggered pulse repetition interval [5-8].

The ambiguity function (AF) of such APSK signal has a thumbtack shape. Due to such AF shape there is no delay or Doppler frequency shift ambiguity.

It should be mentioned that return signals partially overlap in time, because the probing pulse interval is much shorter than the instrumental radar range and the reception of the return signal is performed during the pauses in probing signal. This creates an additional noise background, which makes it difficult to detect weak signals.

In quasicontinuous mode the time overlap of the return valid signal and high-power clutter is determined by the pseudorandom APSK envelope. Time intervals with no high-power returns occur in additive mixture of return signals. Clipping of input signal normalizes the high-power signal level, that interfere with weak valid APSK signal. A part of valid signal pulses is also suppressed during clipping. However, valid signal pulses that do not clipped can be coherently processed. As a result of
the clipping, the signal-to-clutter-plus-noise ratio (SCNR) after the correlation processing can increase.

The gain in SCNR depends on many factors: the distribution of the clutter power by delay, the valid signal power, the parameters of the APSK signal.

In this article the SCNR with clutter clipping in a radar with quasicontinuous mode of wideband signals transmission and reception is estimated. The mathematical modelling of the linear sum of APSK signal, clutter and noise allows us to construct a distribution histogram of the amplitude values of the received signal before and after the clipping. In a realistic interference situation, the distribution of interference intensity is complex. The example of pinpoint target detection using clipping is shown. The obtained probability of the received signal clipping makes it possible to form SCNR equation.

The SCNR estimate after correlation processing of the APSK return signal is shown with and without clipping. In addition, when operating with single antenna for transmission and reception, the estimate of the SCNR should take in to account the energy loss due to the receive path blanking.

Analysis of the SCNR dependence on the APSK signal parameters is carried out in this article on a simpler example. The interference in the model of the return signal was formed as a linear sum of three signals:

1) The valid signal with the complex envelope \( s(t) = A_c u(t - \tau_c) \exp(j2\pi F_c t + \varphi_c) \), where \( A_c \) - amplitude, \( \tau_c \) - delay, \( F_c \) - Doppler frequency shift and \( \varphi_c \) - random initial phase ;
2) The clutter, that is formed by the targets in the radar’s coverage area:

\[
\tilde{\xi}(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(\tau, F) \sum_{n=0}^{N-1} w_n u_n(t + \tau - n T_b) \exp(j2\pi F t) d\tau dF
\]

where the statistical averaging \( |\rho(\tau, F)|^2 \) determines the distribution density of the total interference power along delay \( \tau \) and the Doppler frequency shift \( F \);
3) The noise \( \eta(t) \) with the normal distribution law and power \( P_0 \) in the signal band.

When radar is operating with single antenna for transmission and reception, the receive path is blanked during the transmission of probing pulse by the signal

\[
x_i(t) = \sum_{n=0}^{N-1}(1-x_n) \cdot u_n(t - n \cdot T_b)
\]
The complex envelope of the processed signal is described as $s(t) = [s(t) + \xi(t) + \eta(t)] \cdot x(t)$.

We assume that the clutter component $\xi(t)$, with a delay, equal to the valid signal delay $\tau_s$, has low power and differs from the valid signal by the Doppler frequency shift.

Typically, the power of the valid signal is significantly lower than the total interference power, but, at the same time, it can exceed the noise power $P_0$. In this case, the received signal envelope is defined by the distribution of a clutter total power along the delay $p_\xi(\tau) = \int |\rho(\tau, F)|^2 dF$.

In some cases, we have the high-level regions in $p_\xi(\tau)$. It leads to the instantaneous amplitude pulsed bursts of the received signal complex envelope $|s(t)|$. The pulse character of the complex envelope $|s(t)|$ makes it possible to apply received signal clipping, providing a vast reduction of the clutter power at the input of correlation receiver.

Let $U_O$ be the clipping threshold of the received signal. After clipping the complex envelope $s(t)$ transforms to

$$s_O(t) = \begin{cases} U_O \cdot \exp\left[j \cdot \arg(s(t))\right], & |s(t)| \geq U_O \\ s(t), & |s(t)| < U_O \end{cases}$$

where $\arg(s(t))$ is the instantaneous phase of the received signal complex envelope.

The signal is processed by a multichannel delay $\tau_m = m \cdot \tau_s$, $m = 1, 2, 3, ..., m_{\max}$ and the Doppler frequency shift $F_v = v/T$, $v = 0, \pm 1, \pm 2, ..., \pm v_{\max}$ correlation receiver. The output response at the $(m, v)$ processing channel is described by

$$R(\tau_m, F_v) = \int_0^1 s_O(t) u^*(t - \tau_m) \exp(-j2\pi F_v t) dt,$$

where * indicates the complex conjugation operation. When the clipping is not applied we must take $s_O(t) = s(t)$ in equation (6).

The detection reliability of the valid signal with $\tau_s$ time delay and $F_v$ Doppler frequency shift after the clutter clipping is determined by the SCNR at the $(\tau_m, F_v)$ processing output, where $\tau_m = \tau_s$, $F_v = F_c$.

When a modelling of a synthesis and processing of the APSK signal is performed in a presence of clutter, SCNR is estimated as the ratio of the response in time-frequency channel $\tau_m = \tau_s$, $F_v = F_c$ of the correlation receiver to the RMS value of $R(\tau_m, F_v)$.

$$q = \frac{\sum_{m=1}^{m_{\max}} \sum_{v=-v_{\max}}^{v_{\max}} R^2(\tau_m, F_v)}{\sum_{m=1}^{m_{\max}} \sum_{v=-v_{\max}}^{v_{\max}} R^2(\tau_m, F_v)}$$

![Figure 1. Ternary keying sequence $w_n$ of APSK signal.](image_url)
The number of elementary pulses $N$ of the APSK signal can reach several hundreds of thousands. Typically, the interference signal $\xi(t)$ is created by a large number of reflectors. The correlation receiver also contains several thousands of time-frequency channels. Therefore, a large number of computations is required to obtain the SCNR estimation for the APSK signal synthesis and processing model. In the following sections, this article describes the SCNR estimation and modelling results to confirm its correctness.

3. SCNR estimation with applied clipping

Let the APSK signal has $k_s=1$.

In clutter clipping mode the estimation of the SCNR averaged over all correlation processing channels can be obtained on the basis of the probability $D(|s(t)|\geq U_0)$ of complex envelope amplitude $|s(t)|$ exceeding the clipping level $U_0$. In case of discrete observation with sampling interval $\tau_s=nT_n$, \(n=0..N-1\), the value of $D$ is estimated by the ratio $D=K_{\xi>U_0}/N$, where $K_{\xi>U_0}$ is the number of clipping threshold excess cases. The probability that the clipping threshold is not exceeded equals ($1-D$).

In quasicocontinuous mode return signals are received during the transmission gaps, what leads to the loss of a part of the return signals, including those exceeding the clipping threshold. As a result of the receive path keying by the signal $x_\nu(t)$, the number of clipping threshold excess cases $K_{\xi>U_0}$ decreases.

If we use the pseudorandom keying sequence, the probability of APSK signal pulse transmission is determined by duty cycle $C$. $x_\nu=1$. The probability that the receive path is opened in the $n$-th time delay equals $\Lambda=1-C$. The $(CA)$ product defines the probability of the arrival of the valid signal with $m$-th time delay in the $m$-th range processing channel. All range channels have the same statistical characteristics.

In the absence of clipping threshold excesses during $T$, every range channel will coherently accumulate (CNA) pulses in average. Because of the time overlap with the clutter pulses that exceed the clipping threshold, the number of coherently accumulated valid signal pulses decreases to the value $[CNA(1-D)]$. The power of the valid signal at the correlation receiver output matches the estimate on the basis of

$$P_c = A_c^2 \left[ CNA (1-D) \right]^2$$

The power of the signal $s(t)$ pulses over the clipping threshold becomes equal $U_0^2$. They interfere with the detection of the valid signal with power less than $U_0^2$ during the correlation processing. The valid signal pulses, which exceed the clipping threshold, are suppressed. The probability of reception of the clipped clutter in the $m$-th range channel is determined as the product of the clipping threshold excess probability $D$ by the probability of reference signal non-zero value.

The signal $s(t)$ pulses that do not exceed the clipping threshold contain both valid signal pulses, noise and clutter and zero-amplitude pulses formed as a result of receive path keying. Let’s now estimate their contribution to the response power at the range channel output, tuned to receive the valid signal.

Let us denote the power of clutter pulses, that not exceed the clipping threshold, by $P_{\xi<U_0}$. The reception probability of such clutter pulses at the range channel, tuned to the valid signal reception, is the same as for noise and is determined by the $[CA(1-D)]$.

As a result, the clutter and noise power after clipping and correlation processing is estimated by

$$P_{\xi=U_0} = CNDU_0^2 + CNA (1-D) \left( P_{\xi<U_0} + P_0 \right)$$

Taking into account equation (8) and equation (9) the SCNR after clutter clipping and correlation processing is defined by

$$q = \frac{P_c}{P_{\xi=U_0}} = \frac{A_c^2 CNA^2 (1-D)^2}{U_0^2 D + \left( P_{\xi<U_0} + P_0 \right) \Lambda (1-D)}$$

The clipping threshold in equation (10) should ensure the maximum reduction of the clutter power with the minimum power loss of the valid signal.
An especial case when the interference $\xi(t)$ is the linear sum of return signals from $M$ pinpoint targets, $k=1..M$, with different delays $\tau_k$ and Doppler frequency shifts $F_k$,

$$\xi(t) = \sum_{k=1}^{K_k} \rho(\tau_k, F_k) \sum_{n=0}^{N-1} w_n u_n(t + \tau_k - nt_b) \cdot \exp(j2\pi F_k t) \quad (11)$$

Let the signals of the linear sum have the same power, such that $p_\xi(\tau_k) = |\rho(\tau_k, F_k)|^2 > U_o^2$ for any $k$-th signal in the linear sum $\xi(t)$. In this case, any pulse of the signal $\xi(t)$ will exceed the clipping threshold. Therefore, $P_{\xi < U_o} = 0$ for such interference $\xi(t)$.

The duty cycle $C$ determines the probability $|u(t- \tau_k)| = 1$ for any $k$. The probability that $|u(t- \tau_k)| = 0$ equals to $(1-C)$ for any $k$. The probability that the total signal amplitude will be zero is $(1-C)^M$. This expression also determines the probability $(1-D)$ that no pulse of the interference $\xi(t)$ exceeds the clipping threshold. Then $D = 1-(1-C)^M = 1-A^M$ is the probability of exceeding the clipping threshold for this interference.

The equation (10) transforms to

$$q = \frac{A^2CN_{A}^{2(M+1)}}{U_o^2 \left( 1-A^M \right) + P_o \Lambda^{(M+1)}} \quad (12)$$

If $k_x > 1$ and $M$ interference signals occupy a continuous range of delays of width $M_\tau$, then expression (12) can be converted to

$$q = \frac{A^2CN_{A}^{2(M+1)/k_x}}{U_o^2 \left( 1-A^{M/k_x} \right) + P_o \Lambda^{(M+1)/k_x}} \quad (13)$$

The equation (13) takes into account time overlap of the $M$ signals.

4. Modelling results

4.1. Modelling results for the clipping of the interferences that are distributed over a delay

The distribution function $p_\xi(\tau)$ of a clutter total power along the delay is depicted in figure 2.

![Figure 2. Clutter power distribution along the delay.](image)

The phase and amplitude keying law of the probe signal is described by a ternary sequence from the figure 1.

The envelope $|s_c(t)|$ of the additive mixture of the valid signal, clutter and noise before and after blanking in the receive path and clipping is pictured in figure 3a and 3b respectively. The same figure 3b outlines the valid signal envelope $|s_c(t) - x(t)|$, contained in the processed signal $s(t)$. 

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Figure 3. The envelope of the additive mixture of the valid signal, clutter and noise: a) before clipping; b) after clipping.

Comparing the difference between $|s_O(t)|$ and $|s_c(t) - x_r(t)|$, we can note, that the valid signal level is similar to the clutter pulses level after clipping, although the valid signal power before clipping was 44 dB lower than the total clutter power $A_p > \int p_1(\tau) d\tau > N$, and 14 dB higher that its own noise. It became possible to detect the valid signal due to a partial time overlap.

The result of the response function simulation $R(\tau_m, F_v)$ of the signal $s(t)$ without clipping is shown in figure 4a and with clipping – in figure 4b. Note that the normalization of $R_{\text{max}}$ corresponds to the maximum value of $R(\tau_m, F_v)$ obtained by the correlation processing of $s(t)$ without the clipping.

If the clutter is not clipped, then for a given APSK signal spectrum width, its duration $T=Nt_b$ is insufficient to detect the valid signal against a noise background. The peak to average sidelobe ratio of the ambiguity function of the APSK signal of length $N=2048$ equals $10\lg(N)=33.1$ dB. This value is 11 dB less than the clutter-to-signal ratio at the input of the correlation receiver for the given example. The valid signal is not detected (figure 4a).

After the clipping the clutter power at the input of the correlation receiver decreases. The valid signal power loss is minor and the signal can be detected.

The lack of information about the valid signal power and its SNR allows us to propose the clipping threshold according to expression

$$U_0 = \left[ \int_0^T u_{\text{mask}}(t) \left| s(t) \right|^2 dt / \int_0^T u_{\text{mask}}(t) dt \right]^{1/2},$$

(14)
where

\[ u_{\text{mask}}(t) = \begin{cases} 1, & |s(t)| < P_s \\ 0, & |s(t)| \geq P_s \end{cases} \]

is a mask signal, \( P_s = \frac{1}{T_0} \int_0^T |s(t)|^2 \, dt \).

Expression (14) determines the clipping threshold by the RMS value of the received signal without the high-power interference overshoots.

The amplitude distribution histograms of the additive mixture of depicted in figure 3 realizations of \( s(t) \) and \( s_0(t) \) before and after clipping are given in figure 5. The clipping threshold equals \( U_0 \) calculated according equation (14). The peak value at zero point equals to 0.2 and reproduces the keying of the receive path. The clipping threshold excess probability is 0.19.

Figure 5. Amplitude values distribution histogram.

The mathematical simulation showed that after clutter clipping the RMS signal level at the multichannel correlation receiver output reduced by 19 dB (figure 4b). The RMS signal level is set by the detection threshold. As a result, the SCNR after clipping was 18 dB, which corresponds with the estimate from the equation (10).

Let’s note that in shown example when we decrease the clipping threshold to the noise level, the valid signal is also affected by clipping. At the same time the clipping threshold excess probability increases to 0.58. The signal level at the multichannel correlation receiver output is further reduced by 16.3 dB and the SCNR is increased to 22 dB.

4.2 Modelling results for the clipping of M pinpoint targets of the same power

The gain in SCNR is determined by the ratio of SCNR values after correlation processing of signals with and without clutter clipping.

\[
g = \frac{\text{SCNR}_{\text{with clipping}}}{\text{SCNR}_{\text{without clipping}}} \tag{15}\]

In the following section we study the change in the gain in SCNR, depending on the APSK signal parameters.

Let the interference signal \( \xi(t) \) be the linear sum of the return signals from \( M \) pinpoint targets. Signals in \( \xi(t) \) have the same power and different delays. The valid signal delay differs from the interference delays in \( \xi(t) \). The valid signal power equals to the noise power \( P_0 = A_c^2 \).

It is known that the signal power at the output of the time-frequency channels of the correlation receiver is proportional to the level of its AF. The APSK signal AF has the thumbtack shape. The integral sidelobe level of the AF is determined by the time-bandwidth product of the APSK signal \( T(1/t_b) = N \) [3].

Let us set the total interference power that \( \text{SCNR}_{\text{without clipping}} = 1 \). It is necessary that the interference power at the correlation receiver output channel, which is tuned to the valid signal \( (t_m = t_v, F_i = F_c) \), be
equal to the valid signal power. Therefore, SCNR\text{without\ clipping}=1 when the power of each term of $\xi(t)$ equals to $p_{\xi}(r_{\xi})=A_{c}^{2}N/M$, $k=1..M$, where $A_{c}^{2}$ is the valid signal power. The valid signal is not detected.

Let us set the clipping threshold according to the noise level $U_{0}=3\sqrt{P_{0}}$. After the correlation processing with clutter clipping, the amount of gain $g$ will determine SCNR\text{with\ clipping}.

Simulation of the correlation processing of the linear sum of the valid signal, $M$ interference signals of the same power and noise with applied clipping was carried out for different parameters $N$, $C$ and $k_{x}$ of the APSK signal. The gain in SCNR $g$ was estimated as a function of the number of interference signals $M$. The $g(M)$ curves are presented in figures 6-8.

Figure 6. The $g(M)$ for APSK signals with $C=0.14$, $k_{x}=1$ and varying $N$.

Figure 7. The $g(M)$ for APSK signals with $N=2^{16}$, $k_{x}=1$ and varying $C$.

Figure 8. The $g(M)$ for APSK signals with $N=2^{16}$, $C=0.14$ and varying $k_{x}$.

When $C$, $k_{x}$ and $M$ are constant, increasing of the APSK signal length increases the SCNR value (see Figure 6 with $g(M)$ for APSK signals with $C=0.14$, $k_{x}=1$ and varying $N$). For $C=0.14$ and $k_{x}=1$, the SCNR increases by 12 dB with increase of the $N$ by 16 times. Also, the number of the interference increase for the $g(M)=14$ dB: $M=4$ for $N=2^{12}$ and $M=20$ for $N=2^{20}$. With increase of the interference number $M$, no separate interference pulses are detected in return signal envelope. The SCNR decreases. For large $M$, the processed signal becomes similar to noise. The efficiency of clutter clipping disappears. The valid signal is not detected. Thus, the clutter clipping can be efficiently used when it is necessary to detect a target with small effective radar cross-section (RCS) against a background of an object with big effective RCS.
The lower the probability $C$ of pulses transmission of the probing APSK signal, the higher the number of the interference signals, when the valid signal is still detected (see Figure 7 with $g(M)$ for APSK signals with $N=2^{10}$, $k_x=1$ and varying $C$). This is critically important when $M$ pinpoint targets are distributed over the delay. Simulation has shown that the maximum number of possible interference returns should not exceed $2/C$.

The increase of the length of the phase-shifted pulse $k_x$ of the probing APSK signal makes it possible to increase the SCNR for a fixed number $M$ of interference returns (see Figure 8 with $g(M)$ for APSK signals with $N=2^{10}$, $C=0.14$ and varying $k_x$). This is especially important when the interference occupies a long range of delays.

Thus, the variation of the APSK signal parameters $C$ and $k_x$ makes it possible to optimize the parameters of the probe signal to achieve the necessary efficiency of the clutter clipping under the given interference situation.

5. Conclusion
The simulation results of a linear sum of APSK signal returns have demonstrated the presence of pulse overshoots in the signal envelope. These pulse overshoots come from the high-power returns and prevent the weak signals detection. To increase the SCNR of processed signal, it is suggested to perform the clipping before the correlation processing. The clipping threshold was set by the RMS value of the received signal without the high-power overshoots. Comparison of the simulation results with and without the clipping demonstrated the clipping efficiency for detecting weak valid signals against a background of high-power clutter.

The derived expression for the SCNR after the signal correlation processing with the clipping helps to carry out the optimization of the probing APSK signal parameters. The simulation showed that with increasing the interferrence length in time, to increase the SCNE it is necessary to stretch the probing signal pulses length $k_xt_b$ and decrease the probability of pulse transmission $C$. The probing APSK signal parameter optimization will ensure the high clutter immunity of the radar with quasicontinuous mode of APSK signal transmission and reception.

6. References
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