Reducing the Number of Changeover Constraints in a MIP Formulation of a Continuous-Time Scheduling Problem

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August 26, 2014

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Abstract

In this paper, we develop a new formulation of changeover constraints for mixed integer programming problem (MIP) that emerges in solving a short-term production scheduling problem. The new model requires fewer constraints than the original formulation and this often leads to shorter computation time of a MIP solver. Besides that, the new formulation is more flexible if the time windows for changeover tasks are given.

Keywords: Short-term scheduling, mixed integer programming model, changeover task

1 Introduction

Scheduling the processes of an industrial plant involves a large number of objects such as processing units, tasks, intermediate and final products (states) and a set of complex relations between them. The production scheduling problem basically consists in selection of a set of tasks to be performed and construction of a schedule complying with the technological requirements and satisfying as much as possible the given demands on a final production.

In earlier studies, a great deal of work was done concerning the modelling of such scheduling problems in a form of Mixed Integer Programs (MIP), see [2, 3, 4, 5, 6]. Solving such a scheduling problem as mixed integer program meets serious difficulties when the number of integer variables and constraints increases. To overcome this, several variants of decomposition scheme were proposed, in which the planning period is split into a sequence of smaller horizons, and in each horizon a short-term MIP problem is solved separately. The shorter the length of horizons, the less products are considered in each of the short-term scheduling problems and the fewer choices are available for optimization.
routine in each horizon. Increasing the length of horizons usually leads to improvement of solution quality but increases the CPU time and memory requirements, which may become prohibitive at some point.

The main objective of the paper is to reduce the number of constraints in MIP formulation of one-horizon short-term scheduling problem proposed in [6]. Reduced number of constraints allows us to increase the size of manageable subproblems and improve the overall performance of decomposition algorithm [1].

2 Problem Statement

A modern chemical production plant is organized as a flexible automated system that contains a number of multipurpose production units and produces a number of products including the final products and intermediate states. The control of the production process involves two main problems: the first one is to choose the most appropriate production plan (the set and the order of reactions to be performed) that can satisfy the market requirements on amount and assortment of the final product, and the second one is to build an optimal production schedule for the chosen plan. These two problems are closely related and for the most effective control they can not be considered separately.

The input data for the basic production scheduling problem \([4, 6]\) consists of:

- The set of production units.
- The set of states, including raw materials, intermediate and final product.
- The set of tasks.
- The suitability table assigning a suitable unit to each task. Here we suppose without loss of generality that each task can be performed only on one unit.
- The minimal and maximal load of each unit when performing a task.
- The production recipes represented in a form of a State Task Network (STN): for each task, a set of states being consumed and produced are given, together with their consumption and production coefficients.
- The processing rates for each task, which is the amount of product produced by a task in one hour.
- The demands and due dates for the final products.

The problem asks for a feasible schedule of tasks. The main objectives are to minimize the underproduced amounts and deviations from the due times for the final products.

In this paper, we consider an industrial plant of a special structure. The principal production processes are performed on the main units, the number of which is relatively small. In addition there are subsidiary units that serve for transportation and storage of the materials. The main units need a changeover task when switching from one production task to another. The durations of the changeover tasks are sequence dependent, i.e. they are defined for the pairs of production tasks and their order. The considered plant has special requirements for the changeover tasks:
(i) No changeovers on main units are allowed between 4 a.m and 7 a.m or between 4 p.m and 7 p.m.

(ii) There are two types of main tasks (the corresponding sets are denoted by $I^{p1}$ and $I^{np1}$), that define two types of changeovers. Let $I^{c1}$ be the set of changeover tasks from $i^1 \in I^{p1}$ to $i^2 \in I^{np1}$. Such tasks may be performed only between 7:30 a.m. and 1:30 p.m. The set of all the rest of changeover tasks on the main units is denoted by $I^{c}$. So for any production unit there are at most two changeover tasks available (one from $I^{c1}$ and one from $I^{c}$).

3 Decomposition Approach

The decomposition approach was proposed in [4] and in [6] it was adapted to the problem under consideration. The whole planning period is split into smaller horizons and a series of scheduling problems corresponding to these horizons is solved sequentially. Two MIP models are used: the upper-level decomposition model (Level-1) determines the time partitioning and assigns the demands to small horizons; the lower-level short-term scheduling model (Level-2) is used to build a schedule in each horizon.

3.1 Upper-Level Decomposition Model (Level-1)

For the considered problem, the planning period (about one month) is split into small horizons with the length of 12 hours. The aim of Level-1 model is to determine how many small horizons will be chosen for the short-term scheduling, and which tasks and states will be considered. The complete model will not be reproduced here because it has many particular details that do not play an important role for our study. Instead, we will only give a list of the most essential conditions that are incorporated in the model:

- If some horizon is selected in the Level-1 model, then all preceding horizons are selected as well (the alternation of selected and non selected horizons is not allowed).
- Product having due dates on some horizon must be included into this horizon.
- If some product is included then all intermediate states corresponding to this product must be included.
- The number of binary variables of the short-term scheduling is bounded by a given tunable parameter.
- The load times of the main units are bounded by a given tunable parameter.

The objective includes the minimization of the number of selected horizons, states, maximization of the demand amount selected and some secondary technical criteria (see the details in [6]). Despite the fact that the Level-1 problem is of mixed integer programming type, the solving time of this problem usually does not exceed several seconds.
3.2 Lower-Level Short-Term Scheduling Model (Level-2)

The MIP model for the short-term scheduling is based on the continuous time concept \[4\]. The aim of the model is to determine the set of operations to perform, their start and finish times, and amounts of produced and consumed states for each task.

The selection of tasks and their sequencing is done by introducing the set of event points \[N = \{1, 2, ..., N_{\text{max}}\}\] for each unit. An event point represents a relative position in the sequence where a task can be assigned. Note that an event point for some unit can be empty, i.e. have no tasks assigned. The binary variable \(w_v(i, n)\) equals 1 iff a task \(i\) is scheduled on the corresponding unit at event point \(n\) (recall that by assumption each task has only one suitable unit). If a task is scheduled at some event point, then we will say that the event point has an active task. The material and timing conditions are modelled by real-valued variables: \(T_s(i, n)\) and \(T_f(i, n)\) give the start and the finish time of task \(i\) at event point \(n\), and \(B(i, n)\) defines the amount of processed material (in case \(w_v(i, n) = 0\) the values of these variables are unimportant).

The system of constraints includes the following conditions:

- For each unit an event point contains at most one task.
- The amount of material processed by a task lies between given minimal and maximal bounds.
- At each event point the produced and consumed amount of some state must be balanced according to production recipes.
- The tasks producing and consuming some intermediate state must be synchronized in time.
- The tasks performing on the same unit must not overlap in time.
- The execution time of a task is defined by its amount.
- The main units may require changeover tasks, i.e. if a task \(j\) is assigned after task \(i\) on the same main unit, then a special changeover task \(i^c\) must be scheduled between them. For each pair of tasks \(i\) and \(j\), the changeover duration is given.

The primary objective is minimization of the non-delivered amount for final products with positive demands (underproduction). A set of secondary objectives can be introduced: minimization of the deviations from the delivery due dates, the overproduced amounts, the number and duration of changeover and processing tasks, and so on.

In this paper, we use the MIP model from \[6\], but reformulate the part concerning the changeover tasks. The new model is equivalent to the original one, but has smaller number of constraints. In what follows, we do not describe the whole set of constraints of the Level-2 model, but only the part that was modified and its new version.

4 Former Model of Changeover Tasks

The following notation will be used below:

- \(M\) is a set of main units.
• $H$ is the length of the planning horizon.
• $I_u$ is the set of tasks that can be performed on unit $u$.
• $C_{time_{i,i'}}$ is a changeover time required to switch a unit from task $i$ to task $i'$.
• $I^p$ is the set of the processing tasks (task of the main units excluding changeover tasks).

The binary variables $wv(i,n)$ equal 1 iff a task $i$ is active on its suitable unit at event point $n$. Let us introduce binary variable $x(i',i,n)$ that equals to 1 iff there is a changeover from task $i'$ occurring at event point $n$ to some other task $i$ ($i \neq i'$) occurring at a later event point $n'$ ($n' > n$) and no other task is active between $n$ and $n'$ on the unit suitable for $i'$ and $i$.

Let us reproduce the constraints (27a)-(28b) from [6].

When task $i'$ is not active at an event point $n$, the variable $x(i',i,n)$ must be zero:

$$x(i',i,n) \leq wv(i',n) \forall u \in M, i, i' \in I_u, i \neq i', C_{time_{i',i}} > 0, n < N_{max}. \quad (1)$$

The changeover from task $i' \in I_u$ to $i \in I_u$ is not activated at event point $n$ if there is a task $i'' \neq i$ at some event point $n' > n$ and all event points in subsequence $n+1, n+2, ..., n'-1$ are empty on unit $u$:

$$x(i',i,n) \leq wv(i',n') + \left(1 - \sum_{i'' \in I^p \cap I_u} wv(i'',n')\right) + \sum_{i'' \in I^p \cap I_u} \sum_{n'' : n' < n'' < n'} wv(i'',n'')$$

$$\forall u \in M, i, i' \in I_u, i \neq i', C_{time_{i',i}} > 0, n < N_{max}, n < n' \leq N_{max}. \quad (2)$$

The changeover from $i'$ to $i$ is activated if task $i'$ is active in event point $n$, task $i$ is active in event point $n' > n$, and there is no active task in event points $n+1, n+2, ..., n'-1$:

$$x(i',i,n) \geq wv(i',n) + wv(i,n') - 1 - \sum_{i'' \in I^p \cap I_u} \sum_{n'' : n < n'' < n'} wv(i'',n'')$$

$$\forall u \in M, i, i' \in I_u, i \neq i', C_{time_{i',i}} > 0, n < N_{max}, n < n' \leq N_{max}. \quad (3)$$

In the following two equations, the changeover task is allocated in event point $n+1$ if there are non-zero values of $x(i',i,n)$. The changeover task is activated at event point $n+1$ in Equation (14) if it is a changeover from a task $i' \in I^p$ to another task $i \in I^p$ or from a task $i' \in I_{np}$ to any other task.

$$wv(i'',n+1) = \sum_{i' \in I^p \cap I_u} \sum_{i \in I^p \cap I_u, i \neq i', C_{time_{i',i}}, > 0} x(i',i,n) + \sum_{i' \in I_{np} \cap I_u} \sum_{i \in I^c \cap I_u, i \neq i'}, C_{time_{i',i}}, > 0 x(i',i,n) \forall u \in M, i'' \in I^c \cap I_u, n < N_{max}. \quad (4)$$
The changeover task is activated at event point \( n + 1 \) in Equation (5) if it is a changeover from a task \( i' \in I_p^1 \) to a task \( i \in I_{np}^1 \).

\[
wv(i'', n + 1) = \sum_{i' \in I_p^1 \cap I_u} \sum_{i \in I_{np}^1 \cap I_u, i' \neq i, Ctime_{i', i} > 0} x(i', i, n)
\]

\( \forall u \in M, i'' \in I_c^1 \cap I_u, n < N_{\text{max}}. \) (5)

The following two constraints correspond to equations (40c) and (40d) in [6] and express the duration of changeover tasks:

\[
T_f(i'', n + 1) - T_s(i'', n + 1) = \sum_{i' \in I_p^1 \cap I_u} \sum_{i \in I_{np}^1 \cap I_u, i' \neq i, Ctime_{i', i} > 0} Ctime_{i', i} x(i', i, n) + \\
\sum_{i' \in I_p^1 \cap I_u} \sum_{i \in I_u, i' \neq i, Ctime_{i', i} > 0} Ctime_{i', i} x(i', i, n) \forall u \in M, i'' \in I_c^1 \cap I_u, n < N_{\text{max}}. \) (6)

\[
T_f(i'', n + 1) - T_s(i'', n + 1) = \sum_{i' \in I_p^1 \cap I_u} \sum_{i \in I_{np}^1 \cap I_u, i' \neq i' , Ctime_{i', i} > 0} Ctime_{i', i} x(i', i, n)
\]

\( \forall u \in M, i'' \in I_c^1 \cap I_u, n < N_{\text{max}}. \) (7)

Note that (2) and (3) contain \( \Theta((N_{\text{max}}^2 |I_u|^2) \) inequalities for each unit \( u \in M \), and each one has \( \Theta(N_{\text{max}} |I_p^1 \cap I_u|) \) summands. The number of binary variables \( x(i', i, n) \) is \( \Theta(N_{\text{max}} |I_u|^2) \). This is acceptable if the number of tasks and event points is small. For large number of tasks and event points, this creates serious difficulties for the MIP solvers like CPLEX due to memory and CPU time requirements. To overcome this problem, the more economical model of changeover tasks was proposed as described in Section 5.

4.1 Time Windows for Changeovers

The changeover blockages between 4 a.m. and 7 a.m. and between 4 p.m. and 7 p.m. are modelled by explicit assignment of the event points to appropriate time intervals. The number of event points is proportional to the duration of the time interval. For example, suppose that the horizon starts at 0:00 and finishes at 12:00 and the total number of event points is \( N_{\text{max}} = 8 \). The forbidden interval for changeover tasks is from 4:00 to 7:00. Then there are two allowed intervals from 0:00 to 4:00 and from 7:00 to 12:00 with the total duration of 9 hours. The number of event points assigned to the first allowed interval is \( \lfloor 4 \cdot 8 / 9 \rfloor = \lfloor 3.55 \rfloor = 4 \), where \( \lfloor \cdot \rfloor \) sign denotes rounding to the closest integer. The other four event points are assigned to the second interval.

5 New Model of Changeover Tasks

Instead of binary variables \( x(i', i, n) \) we use binary variables \( x(i, n) \) assuming that \( x(i, n) \) equals 1 iff the task \( i \) is the first active task on its unit in the event points \( n, n+1, ..., N_{\text{max}} \). In other words, there is \( n' \in \{n, n + 1, ..., N_{\text{max}}\} \) s.t. \( wv(i, n') = 1 \) and all event points
If a task \( i \) is active at event point \( n \) then \( x(i, n) = 1 \):

\[
x(i, n) \geq wv(i, n) \quad \forall u \in M, i \in I^p \cap I_u, n \leq N^{\max}. \tag{8}
\]

If an event point \( n \) contains no active tasks, then \( x(i, n) \) is “copied” from event point \( n + 1 \).

\[
x(i, n) \geq x(i, n + 1) - \sum_{i' \in I^p \cap I_u, i' \neq i} wv(i', n) \quad \forall u \in M, i \in I^p \cap I_u, n < N^{\max}. \tag{9}
\]

If task \( i \) is followed by a different task \( i' \) then a changeover task must be activated at the event point \( n + 1 \). At first we consider the case \( i, i' \in I^{p1} \), so the changeover task must be \( i^c \in I^c \):

\[
wv(i^c, n + 1) \geq \sum_{i' \in I^{p1} \cap I_u, i' \neq i} x(i', n + 1) + wv(i, n) - 1
\]

\[
\forall u \in M, i \in I^{p1} \cap I_u, i^c \in I^c \cap I_u, n < N^{\max}. \tag{10}
\]

In case \( i \in I^{np1}, i' \in I^p \), the changeover task is again \( i^c \in I^c \):

\[
wv(i^c, n + 1) \geq \sum_{i' \in I^p \cap I_u, i' \neq i} x(i', n + 1) + wv(i, n) - 1
\]

\[
\forall u \in M, i \in I^{np1} \cap I_u, i^c \in I^c \cap I_u, n < N^{\max}. \tag{11}
\]

In the last case \( i \in I^{p1}, i' \in I^{np1} \), the changeover task is \( i^{c1} \in I^{c1} \):

\[
wv(i^{c1}, n + 1) \geq \sum_{i' \in I^{np1} \cap I_u} x(i', n + 1) + wv(i, n) - 1
\]

\[
\forall u \in M, i \in I^{p1} \cap I_u, i^{c1} \in I^{c1} \cap I_u, n < N^{\max}. \tag{12}
\]

If a changeover task is allocated then its duration is chosen as \( Ctime_{i,i'} \). We consider three cases again:

\[
Tf(i^c, n + 1) - Ts(i^c, n + 1) \geq \sum_{i' \in I^{p1} \cap I_u, i' \neq i} Ctime_{i,i'} x(i', n + 1) - H(1 - wv(i, n))
\]

\[
\forall u \in M, i \in I^{p1} \cap I_u, i^c \in I^c \cap I_u, n < N^{\max}. \tag{13}
\]

\[
Tf(i^c, n + 1) - Ts(i^c, n + 1) \geq \sum_{i' \in I^{p1} \cap I_u, i' \neq i} Ctime_{i,i'} x(i', n + 1) - H(1 - wv(i, n))
\]

\[
\forall u \in M, i \in I^{np1} \cap I_u, i^c \in I^c \cap I_u, n < N^{\max}. \tag{14}
\]

\[
Tf(i^{c1}, n + 1) - Ts(i^{c1}, n + 1) \geq \sum_{i' \in I^{np1} \cap I_u} Ctime_{i,i'} x(i', n + 1) - H(1 - wv(i, n))
\]
∀u ∈ M, i ∈ Ip1 ∩ Iu, ic ∈ Ic ∩ Iu, n < Nmax. 

(15)

The number of equations in (8)–(15) can be estimated as Θ(Nmax|Iu|) for each unit u ∈ M, the number of binary variables x(i, n) is Θ(Nmax|Iu|). This is by an order of magnitude smaller compared to estimates in Section 4 and allows for solving the problem with larger horizons as in [1].

Constraints (8)–(15) may be combined with the changeover blockage mechanism [6] described in Subsection 4.1. Nevertheless below we propose a more flexible mechanism for blockage of changeover at specific time windows which does not require any explicit assignment of the event points to appropriate time intervals.

5.1 Time Windows for Changeovers

To model the changeover blockages between 4 a.m. and 7 a.m. and between 4 p.m. and 7 p.m., we enumerate all the intervals where changes are allowed and denote them by [CTsk, CTfk], k ∈ K. Here K is the set of indices of changeover intervals. For example, if the planning horizon starts at 0:00 a.m. then the set of changeover intervals is {[0, 4], [7, 16], [19, 28],...}. Now we introduce new binary variables z(i, k, n), which equal 1 iff a changeover task i at event point n is performed in the interval with number k ∈ K.

The following equations ensure that each changeover task is placed in one time interval:

\[ \sum_{k \in K} z(i, k, n) = wv(i, n) \quad \forall i \in Ic, n \leq N_{\text{max}}. \] 

(16)

The last pair of inequalities state that the changeover task is performed in the chosen time interval:

\[ Ts(i, n) \geq CT^s_k \cdot z(i, k, n) \quad \forall i \in Ic, k \in K, n \leq N_{\text{max}}, \] 

(17)

\[ Tf(i, n) \leq CT^f_k + H(1 - z(i, k, n)) \quad \forall i \in Ic, k \in K, n \leq N_{\text{max}}. \] 

(18)

The blockages for the special changeovers from a task of Ip1 type to a task of Inp1 type are modelled the same way, but with a different set of time intervals.

6 Conclusions

A new formulation of changeover constraints for short-term production scheduling problem is proposed. The new model requires significantly less constraints compared to the original formulation, which is important in case of large problem instances where the memory requirements become a limiting factor for MIP solvers.

7 Acknowledgements

Partially supported by Russian Foundation for Basic Research grants 12-01-00122 and 13-01-00862.
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