Realization of quantum operations on photonic qubits by linear optics and post-selection

Holger F. Hofmann and Shigeki Takeuchi
PRESTO, Japan Science and Technology Corporation (JST)
Research Institute for Electronic Science,
Hokkaido University, Sapporo 060-0812
Tel/Fax: 011-706-2648
e-mail: h.hofmann@osa.org

Abstract

One of the greatest difficulties in the applications of single photon polarization states as qubits is the realization of controlled interactions between two photons. Recently, it has been shown that such interactions can be realized using only beam splitters and high efficiency photon detection by post-selecting a well defined part of the results in the output. We analyze these interactions and discuss schemes for qubit operations based on this mechanism.

Keywords:
optical quantum computation, photonic qubits

1 Introduction

Conventionally, the unitary transformations representing controlled quantum gates are implemented by appropriate interactions between physical systems. These interactions should be non-dissipative and involve only the four level Hilbert space of the two interacting qubits. If photonic qubits are realized using the polarization states of individual photons, such an interaction requires a Kerr nonlinearity strong enough to cause a phase change of $\pi$ per photon in the controlling mode, while all interactions with modes other than the two well-defined polarization modes of each photon must be suppressed. However, the suppression of all absorption and scattering processes in the presence of a strong Kerr nonlinearity is very difficult to achieve and requires the development of highly sophisticated new technologies. The implementation of controlled interactions between photonic qubits has therefore been a major obstacle to the realization of quantum computation using photonic qubits. This alternative scheme is based on the insight that the necessary unitary transforms needed for controlled gate operations can also be obtained from a subspace of a total unitary transformation in a much larger Hilbert space. Although the selection of such a subspace corresponds to a limited probability of success for the gate operation, this problem can be compensated by applying various error correction strategies. In optics, this alternative realization of controlled gate operations is especially promising because it allows an implementation of photon-photon interactions using only linear optical elements. In particular, the selection of appropriate subspaces in both the input and the output of a conventional beam splitter is sufficient to obtain basic nonlinear interactions between photons.

2 Properties of the post-selected beam splitter

The action of a beam splitter of reflectivity $R$ on the two input modes given by the operators $\hat{a}_1$ and $\hat{a}_2$ can be described by a unitary transformation $\hat{U}_R$ with the property

$$\hat{U}_R \hat{a}_1 \hat{U}_R^\dagger = \sqrt{R} \hat{a}_1 + i\sqrt{1-R} \hat{a}_2$$

$$\hat{U}_R \hat{a}_1 \hat{U}_R^\dagger = i\sqrt{1-R} \hat{a}_1 + \sqrt{R} \hat{a}_2. \tag{1}$$

While the total number of photons is a conserved quantity of this operation, photons are exchanged between mode 1 and mode 2, so that most of the output results change the photon numbers in the modes. However, the concept of photonic qubits is based on the assumption that the photons can be kept in separate modes. Therefore, the distribution of photons at the beam splitter must be controlled by post-selecting only those output results that conserve the photon number distribution. Since the beam splitter automati-
elements produces a nonlinear effect given by the effective operator $\hat{S}_{nn}$. A nearly unitary result is obtained in the subspace of $R/4$ at an intensity of $\sqrt{R}$ in equation (3). This non-unitary beam splitter operation can be used to compensate amplitude differences between the photon number states of mode 1. However, the most important result is obtained for a single photon input and a single photon detection ($n_2 = 1$). This device produces a nonlinear effect given by the effective operator elements

$$\langle n_1; 0 | \hat{U}_R | n_1; 0 \rangle = \langle n_1 | \hat{S}_{00} | n_1 \rangle = \left( \sqrt{R} \right)^{n_1}. \quad (2)$$

This non-unitary beam splitter operation can be used to compensate amplitude differences between the photon number states of mode 1. However, the most important result is obtained for a single photon input and a single photon detection ($n_2 = 1$). This device produces a nonlinear effect given by the effective operator elements

$$\langle n_1; 1 | \hat{U}_R | n_1; 1 \rangle = \langle n_1 | \hat{S}_{11} | n_1 \rangle$$

$$= \left( \sqrt{R} \right)^{n_1-1} (R - (1 - R) n_1). \quad (3)$$

This matrix element is positive for all $n_1 < R/(1 - R)$ and negative for all $n_1 > R/(1 - R)$. Therefore, equation (3) describes a nonlinear phase change of $\pi$ at an intensity of $R/(1 - R)$ photons. In particular, a nearly unitary result is obtained in the subspace of $n_1 \leq 2$ for $R = 1/4$, where

$$\hat{S}_{11}(R = 1/4) | 0 \rangle = \frac{1}{2} | 0 \rangle \quad \hat{S}_{11}(R = 1/4) | 1 \rangle = -\frac{1}{2} | 1 \rangle \quad \hat{S}_{11}(R = 1/4) | 2 \rangle = -\frac{5}{8} | 2 \rangle. \quad (4)$$

Both the original scheme proposed by Knill, Laflamme and Milburn [1] and the simplified proposal of Ralph, White, Munro and Milburn [2] use reflectivities close to $R = 1/4$ to realize their basic nonlinearity. By choosing a reflectivity slightly higher than $1/4$, the two proposals accomplish a gradual increase in the output amplitude from zero photons to two photons. This increase can then be compensated by linear attenuation. In the proposal of Ralph et al. [2], this is done by using the linear attenuation of the vacuum beam splitter given in equation (4) to compensate an effective linear amplification obtained from the single photon operation of equation (3).

While the basic operation of the beam splitter thus allows the realization of well-defined nonlinear interactions between photons, the overall effort required to adjust the amplitudes of the output is still significant. Therefore, it may be worthwhile to consider applications where this kind of adjustment is less critical.

### 3 Nonlinear filter operation

Instead of compensating the amplitude differences between the photon number states caused by the different matrix elements of $\hat{S}_{11}$, the nonlinear features of these amplitude differences may be exploited directly in order to filter out specific components of the multi photon quantum state [3]. The most fundamental quantum filter is then obtained by the beam splitter with reflectivity $R = 1/2$ - the standard type of beam splitter used in most quantum optics experiments. Its post-selection properties are given by

$$\hat{S}_{11}(R = 1/2) | 0 \rangle = \frac{1}{\sqrt{2}} | 0 \rangle$$

$$\hat{S}_{11}(R = 1/2) | 1 \rangle = 0$$

$$\hat{S}_{11}(R = 1/2) | 2 \rangle = -\frac{1}{2\sqrt{2}} | 2 \rangle. \quad (5)$$

This operation eliminates only the one photon component of the input. Both the vacuum and the two photon component can pass this filter without any loss of quantum coherence.

In order to apply this filtering process to photonic qubits, it is necessary to temporarily transfer the photons of one polarization component of each qubit to the same optical mode. This is usually accomplished by reversible photon bunching in a Mach-Zender geometry. Specifically, we can apply the reversible transformation of an additional pair of beam splitters with reflectivity...
serving coherence between two photon polarization correlations. Unless labeled

...Figure 2: Schematic setup of the quantum filter for input: ones. Represent polarization sensitive beam splitters transmitting H polarized photons and reflecting V polarized ones.

1/2 to realize the following operation on a two mode input:

\[
\begin{align*}
\hat{U}_{1/2}(1, 2) (\hat{S}_{11}(1) \otimes \hat{S}_{11}(2)) \hat{U}_{1/2}(1, 2) |1; 1\rangle &= \frac{1}{4} |1; 1\rangle \\
\hat{U}_{1/2}(1, 2) (\hat{S}_{11}(1) \otimes \hat{S}_{11}(2)) \hat{U}_{1/2}(1, 2) |0; 1\rangle &= 0 \\
\hat{U}_{1/2}(1, 2) (\hat{S}_{11}(1) \otimes \hat{S}_{11}(2)) \hat{U}_{1/2}(1, 2) |1; 0\rangle &= 0 \\
\hat{U}_{1/2}(1, 2) (\hat{S}_{11}(1) \otimes \hat{S}_{11}(2)) \hat{U}_{1/2}(1, 2) |0; 0\rangle &= \frac{1}{2} |0; 0\rangle.
\end{align*}
\] (6)

This operation conserves the photon number in both input modes without requiring measurements on the output of the two modes. By applying this operation e.g. to the H polarized components of two photonic qubits, it is then possible to realize a filter that removes the |H; V\rangle and |V; H\rangle components while preserving coherence between |H; H\rangle and |V; V\rangle. The complete setup is shown in figure 2. By introducing an additional attenuation in the vertically polarized path and by reversing the nonlinear phase shift for the horizontally polarized components, the total filter effect of this setup becomes

\[
\hat{S}_{\text{filter}} = \frac{1}{4} (|H; H\rangle \langle H; H| + |V; V\rangle \langle V; V|).
\] (7)

The successful operation of this filter corresponds to a quantum nondemolition measurement of the relative alignment of HV polarization. This measurement is not sensitive to any local polarization properties. The filter can therefore produce a variety of entangled output states $\hat{S}_{\text{filter}}$. Perhaps the most striking illustration of these nonlocal filter properties is the action on a product state of a right circular polarized photon and a left circular polarized photon,

\[
\begin{align*}
\hat{S}_{\text{filter}} |R; L\rangle &= \\
\hat{S}_{\text{filter}} \left( \frac{1}{\sqrt{2}} (|H\rangle + i |V\rangle) \otimes \frac{1}{\sqrt{2}} (|H\rangle - i |V\rangle) \right) \\
&= \frac{1}{8} (|H; H\rangle + |V; V\rangle) = \frac{1}{8} (|R; L\rangle + |L; R\rangle).
\end{align*}
\] (8)

The filter can thus be used to entangle photons originating from separate and independent sources.

While the quantum gates proposed in [1, 2] and the quantum filter presented here and in [3] allow a reliable implementation of all possible quantum operations, their main disadvantage is that they require not only a large number of beam splitters, but also an additional photon input of one extra photon per photonic qubit, as well as a reliable detector to monitor the post-selection requirement. This effort is necessary so that the post-selection condition can be imposed without requiring any measurements in the output. However, more simple realizations of nonlocal interactions between photonic qubits are possible if the post selection ensuring the presence of exactly one photon in each qubit is performed in the output instead.

4 Phase gate without additional input photons

If post-selection in the output is allowed, it is possible to use both sides of the beam splitter to realize an interaction between photons in modes 1 and 2. If each mode has zero or one photon, the matrix elements for the post-selected interaction are given by

\[
\begin{align*}
\langle 0; 0 | \hat{U}_R | 0; 0 \rangle &= 1 \\
\langle 0; 1 | \hat{U}_R | 0; 1 \rangle &= \sqrt{R} \\
\langle 1; 0 | \hat{U}_R | 1; 0 \rangle &= \sqrt{R} \\
\langle 1; 1 | \hat{U}_R | 1; 1 \rangle &= 2R - 1.
\end{align*}
\] (9)

This diagonal four by four matrix already describes the basic function of a phase gate since the two photon term is an interference between mutual reflection and mutual transmission of the indistinguishable photons. By choosing a reflectivity of $R = 1/3$, the non-unitary
Photon 1 in
 Photon 2 in
total setup for a pair of photonic qubits is then obtained by applying (10) to one of the polarization components of each input qubit. This very compact and symmetric setup is illustrated in figure 3. Its function is described by the operator $\hat{S}_{qpg}$ with

$$\begin{align*}
\hat{S}_{qpg} |V; V\rangle &= \frac{1}{3} |V; V\rangle \\
\hat{S}_{qpg} |V; H\rangle &= \frac{1}{3} |V; H\rangle \\
\hat{S}_{qpg} |H; V\rangle &= \frac{1}{3} |H; V\rangle \\
\hat{S}_{qpg} |H; H\rangle &= -\frac{1}{3} |H; H\rangle.
\end{align*}$$

Since this quantum gate requires no additional photon sources or photon detectors, it appears to be a most promising candidate for the realization of networks for multi-qubit operations.

5 Conclusions

As the examples given above clearly demonstrate, post-selection methods can be very useful in realizing quantum operations that are difficult to obtained from a direct physical interaction in a dissipation free environment. Instead of controlling the Hamiltonian, post-selection applies the dynamics of quantum measurement to realize the desired coherent interactions. It is then possible to employ the same technologies originally developed for an improvement of measurement precision to the evolution of the quantum state. In optics, this method can be applied to solve the problem of realizing strong nonlinear interactions between individual photons. As shown above, the nonlinearity necessary for such interactions is already present in the photon number conserving subspace of a conventional beam splitter. This nonlinearity can be post-selected by reliably controlling the photon numbers in both the input and the output. Effectively, the nonlinearity is implemented by precise photon counting measurements. Using recently developed photon detection technologies, it should then be possible to perform arbitrary quantum operations on photonic qubits with only moderate rates of error.