Beyond the black disk limit: antishadow scattering mode

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Abstract

The appearance of the antishadow scattering mode at energies of the Tevatron–Collider is considered on the basis of unitarity and geometrical notions on hadron interactions.

Introduction

One of the most fundamental discoveries in hadron interactions at high energies is the rise of total cross–sections with energy. It is accompanied by the rise of elastic and inelastic cross–sections as well as of the ratio of elastic to total cross–sections.

For the first time the total cross–section rise was observed in $K^+p$–interactions at the Serpukhov accelerator in 1970 and was discovered later also in $pp$–interactions at CERN ISR and at Fermilab in other hadronic reactions.

Since that time a great progress in the experimental and theoretical studies of soft hadronic reactions has been achieved. Quantum Chromodynamics has appeared as a theory of strong interactions. However, dynamics of long distance hadronic interactions is rather far from its complete understanding and much work is needed in this field. The problems here are directly connected with the problems of confinement and chiral symmetry breaking.
Despite of inaplicability of perturbative QCD for the description of long–distance interactions and their obvious nonperturbative character, it is often possible to represent the high–energy amplitude in the model approaches as an expansion in some parameter which depends on the kinematics of the process, e.g. for the case of non–increasing total cross–section the general form of amplitude is

\[ F(s, t) = s \sum_n [\tau(s)]^n \exp \left[ \frac{a(s)t}{n} \right] , \]

where \( \tau(s) \sim 1/\ln s \) is a small parameter at \( s \to \infty \).

This expansion is not valid for the rising total cross–sections. However, it is possible to find another representation in that case with \( t \)–dependent expansion parameter [1]:

\[ F(s, t) = s \sum_{m=1}^{\infty} [\tau(\sqrt{-t})]^m \Phi_m[R(s), \sqrt{-t}], \quad t \neq 0, \]

where

\[ \tau(\sqrt{-t}) = \exp \left( -\frac{\sqrt{-t}}{\mu_0} \right) . \]

and \( \Phi_m[R(s), \sqrt{-t}] \) is an oscillating function of transferred momentum. The both above formulas as well as some other representations may be succesfully used for the phenomenological analysis of the scattering amplitude at high energies.

Thus, by now the theoretical treatment of soft hadronic reactions involves substantial piece of phenomenology and uses various model approaches such as Regge–type, geometrical or QCD–inspired models. They are based on divergent postulates, but their phenomenological parts are similar. In particular, some amplitude \( V(s, t) \) is considered as an input for the subsequent unitarization procedure:

\[ F(s, t) = \Phi[V(s, t)]. \]

To reproduce the total cross–section growth the input amplitude \( V(s, t) \) is usually considered as some power–behaved function of energy. This function taken as an amplitude itself violates unitarity in the direct channel. To obey unitarity in the direct channel the unitarization procedures are to be used.
There are several ways to restore unitarity of the scattering matrix. We are going to consider the two such schemes: based on the use of eikonal and the method of generalized reaction matrix respectively. As it was already mentioned various models for \( V(s, t) \) may be successfully used to provide phenomenological description of high energy hadron scattering. However, in the particular model approaches the important dynamical aspects of interaction are obscured often due to large number of free parameters.

In this brief review we discuss some general properties of hadron scattering, the implications of unitarity and analyticity, in particular, manifestations of the antishadow scattering mode (Section 3). The recent CDF data indicate that the black disk limit is probably already violated at the Tevatron–Collider energy \( \sqrt{s} = 1.8 \text{ TeV} \). Preliminary discussions of geometrical picture and the bounds for observables are given in Sections 1 and 2 to define a frame of the problem.

1 Geometrical Picture

In the collisions of two high energy particles the de Broglie wavelength can be short compared to the typical hadronic size and hence some optical concepts may be used as useful guidelines. We can consider therefore hadron scattering as a collision of two relativistically contracted objects of finite size.

The relevant mathematical tool for the description of high energy hadronic scattering is based on the impact parameter representation for the scattering amplitude. This representation has the following form in the spinless case:

\[
F(s, t) = \frac{s}{\pi^2} \int_0^\infty b db f(s, b) J_0(b\sqrt{-t}).
\]

(1)

Note that for the scattering of particles with non–zero spin the impact parameter representation for the helicity amplitudes has similar form with substitution \( J_0 \rightarrow J_{\Delta\lambda} \), where \( \Delta\lambda \) is the net helicity change between the final and initial states. The impact parameter representation as it was shown in [2] is valid for all physical energies and scattering angles. This representation provides simple semiclassical picture of hadron scattering.

It is often assumed, since the Chou–Yang model was proposed, that the main effect in hadron scattering arises due to overlapping of the two matter distributions. It could be understood by analogy with the Glauber theory
of nuclear interactions: one assumes that the matter density comes from the spatial distribution of hadron constituents and assumes also a zero–range interaction between the constituents. Such contact interaction might result from the effective QCD based for example on the Nambu–Jona-Lasinio model.

The general definition of the interaction radius which is in agreement with the above geometrical picture was given in [3]:

\[ R(s) = l_0(s)/k, \]  

where \( k = \sqrt{s}/2 \) is the particle momentum in the c.m.s. The value for \( l_0(s) \) is chosen provided the contribution to the partial amplitude from the angular momenta \( l > l_0(s) \) are vanishingly small.

In the first approximation one can consider the energy independent intensity and describe the elastic scattering amplitude in terms of the black disk model where it has the form:

\[ F(s, t) \propto iR^2(s) J_1(R(s)\sqrt{-t})R(s)\sqrt{-t}. \]  

Here \( R \sim 1f \) is the interaction radius. The model is consistent with the structure in the differential cross–sections of \( pp \)– and \( \bar{p}p \)–scattering observed at \( t \) near 1 \((GeV/c)^2\).

When neglecting the real part and helicity flip amplitudes the impact parameter amplitude \( f(s, b) \) can be obtained as an inverse transformation according to Eq. [4] with

\[ F(s, t) \propto \sqrt{s\frac{d\sigma}{dt}}(s, t). \]

Thus, one can extract information on the geometrical properties of interaction from the experimental data. The analysis of the experimental data on high–energy diffractive scattering shows that the effective interaction area expands with energy and the interaction intensity — opacity — increases with energy at fixed impact parameter \( b \). Such analysis used to be carried out every time as the new experimental data become available. For example analysis of the data at the ISR energies (the most precise data set on differential cross–section for wide \( t \)–range available for \( \sqrt{s} = 53 \) GeV) shows that one can observe a central impact parameter profile with a tail from the
higher partial waves and some suppression of low partial waves relative to a gaussian. The scattering picture at such energies is close to grey disk with smooth edge which is getting darker in the center with energy.

After these simple geometrical observations we consider the bounds for the experimental observables.

2 Bounds for observables and the experimental data

Bounds for the observables obtained on the firm ground of general principles such as unitarity and analyticity are very important for any phenomenological analysis of soft interactions. However, there are only few results obtained on the basis of the axiomatic field theory.

First of all it is the Froissart–Martin bound that gives the upper limit for the total cross-section:

$$\sigma_{\text{tot}} \leq C \ln^2 s,$$  \hspace{1cm} (4)

where $C = \pi/m^2_\pi$ ($= 60 \text{mb}$) and $m_\pi$ is the pion mass.

Saturation of this bound, as it is suggested by the existing experimental data, imply the dominance of long–distance dynamics. It also leads to number of important consequences for the other observables. For instance, unitarity leads to the following bound for elastic cross-section:

$$\sigma_{\text{el}}(s) \geq c \frac{\sigma_{\text{tot}}^2(s)}{\ln^4 s}.$$ \hspace{1cm} (5)

Therefore, when the total cross-section increases as $\ln^2 s$, elastic cross–section also must rise like $\ln^2 s$. It is important to note here that there is no similar restriction for the inelastic cross–section and as we will see further the absence of such bound allows appearance of the antishadow scattering mode at very high energies.

If one considers a more general case when $\sigma_{\text{tot}} \propto \ln^\gamma s$, then at asymptotic energies one should have

$$\frac{\text{Re} F(s, 0)}{\text{Im} F(s, 0)} \sim \frac{\gamma \pi}{2 \ln s}$$ \hspace{1cm} (6)
and

\[
\frac{\sigma_{\bar{a}}^{\text{tot}}(s) - \sigma_{a}^{\text{tot}}(s)}{\sigma_{\bar{a}}^{\text{tot}}(s) + \sigma_{a}^{\text{tot}}(s)} \leq \ln^{-\gamma/2}(s) \tag{7}
\]

where \(\sigma_{\bar{a}}^{\text{tot}}(s)\) and \(\sigma_{a}^{\text{tot}}(s)\) are the total cross–sections of the processes \(\bar{a} + b \rightarrow X\) and \(a + b \rightarrow X\) correspondingly. In the case of \(\gamma = 2\) the total cross–section difference of antiparticle and particle interaction should obey the following inequality

\[
\Delta \sigma_{\text{tot}}(s) \leq \ln s. \tag{8}
\]

Contrary to the total cross–section behavior the existing experimental data seem to prefer decreasing \(\Delta \sigma_{\text{tot}}(s)\). Possible deviations from such behavior could be expected on the basis of perturbative QCD \[4\] and it was one of the reasons for the recent discussions on the Pomeron counterpart — the Odderon. However, the recent measurements of real to imaginary part ratio for forward \(\bar{p}p\) scattering provide little support for the Odderon. We will not discuss more thoroughly \(\text{Re}F/\text{Im}F\) ratio and will consider for simplicity the case of pure imaginary amplitude.

For pure imaginary scattering amplitude the following inequality takes place for the slope of diffraction cone at \(t = 0\):

\[
B(s) \geq \frac{\sigma_{\text{tot}}^{2}(s)}{18\pi \sigma_{\text{el}}(s)} \tag{9}
\]

which means that when the total cross–section increases as \(\ln^2 s\), the same dependence is obligatory for the slope of diffraction cone. It is stronger shrinkage than the Regge model predicts \(B(s) \sim \alpha' \ln s\).

There is also bound \[5\] for the total cross–section of single diffractive processes. It was obtained in approach where inelastic diffraction as well as elastic scattering are assumed to arise as a shadow of inelastic processes and has the form

\[
\sigma_{\text{diff}}(s, b) \leq \frac{1}{2} \sigma_{\text{tot}}(s, b) - \sigma_{\text{el}}(s, b). \tag{10}
\]

In particular, it was assumed that the diffractive eigenamplitudes in the Good–Walker \[6\] picture do not exceed the black disk limit.

At this point some details of the experimental situation are to be mentioned. Recently the new experimental data for the total and elastic cross–sections, slope parameter of diffraction cone and cross–section of single inelastic diffraction dissociation have been collected in \(\bar{p}p\)–collisions at Fermilab.
We will refer mainly to the recent CDF results for diffractive scattering. In particular these measurements show that

- total cross-section of $p\bar{p}$-interactions is large $\sigma_{tot} = 80.6 \pm 2.3 \text{ mb}$ at $\sqrt{s} = 1.8 \text{ TeV}$ which is consistent with $\ln^2 s$–rise;

- elastic cross-section also has a large value: $\sigma_{el} = 20.0 \pm 0.9 \text{ mb}$ and ratio of elastic to total cross-section $\sigma_{el}/\sigma_{tot} = 0.248 \pm 0.005$;

- the impact parameter scattering amplitude $Imf(s, b = 0) = 0.50 \pm 0.01$.

Comparing these values with the lower energy data one can conclude also that the higher the energy, the larger both absolute and relative probabilities of elastic collisions.

Impact parameter analysis of the data shows that the scattering amplitude is probably beyond the black disk limit $|f(s, b)| = 1/2$ in head-on collisions. The Pumplin bound Eq. 10 is also violated in such collisions and this is not surprising if one recollects the original assumption on the shadow scattering mode.

It should be noted that another experiment at Fermilab E710 [7] gives different values for the cross–sections and therefore the above conclusions should be taken with certain precautions.

However, it seems worth to consider the possibility that the scattering amplitude exceeds the black disk limit in head–on collisions and that the transition to the antishadow scattering mode might occur in the central hadron collisions.

### 3 Antishadow scattering mode

The basic role in our consideration belongs to unitarity of the scattering matrix $SS^+ = 1$ which reflects the probability conservation. In the impact parameter representation Eq. 4 unitarity has a simple form

\[
Imf(s, b) = |f(s, b)|^2 + \eta(s, b)
\]

(11)

where the inelastic overlap function $\eta(s, b)$ is the sum of all inelastic channel contributions. It can be expressed as a sum of $n$–particle production cross–sections at given impact parameter

\[
\eta(s, b) = \sum_n \sigma_n(s, b).
\]

(12)
As it was already mentioned consideration of pure imaginary amplitude is rather good approximation at high energies. Then the unitarity Eq. 11 points out that the elastic scattering amplitude at given impact parameter value is determined by the inelastic processes. Eq. 11 imply the constraint

$$|f(s, b)| \leq 1$$

while the black disk limit presumes inequality

$$|f(s, b)| \leq 1/2.$$  

The equality $$|f(s, b)| = 1/2$$ corresponds to maximal absorption in the partial wave with angular momentum $$l \simeq b \sqrt{s}/2.$$  

The maximal absorption limit is chosen a priori in the eikonal method of unitarization when the scattering amplitude is written in the form:

$$f(s, b) = \frac{i}{2}(1 - \exp[i \omega(s, b)])$$  \hspace{1cm} (13)

and imaginary eikonal $$\omega(s, b) = i \Omega(s, b)$$ is considered. The function $$\Omega(s, b)$$ is called opacity. Eikonal unitarization automatically satisfies the unitarity Eq. 11 and in the case of pure imaginary eikonal leads to amplitude which is always under the black disk limit.

However, unitarity equation has two solutions for pure imaginary case:

$$f(s, b) = \frac{i}{2}[1 \pm \sqrt{1 - 4 \eta(s, b)}].$$  \hspace{1cm} (14)

Eikonal unitarization with pure imaginary eikonal corresponds to the choice of the particular solution with sign minus.

Several models have been proposed for the eikonal function. For instance, Regge–type models lead to the gaussian dependence of $$\Omega(s, b)$$ on impact parameter. To provide rising total cross–sections opacity should have a power dependence on energy

$$\Omega(s, b) \propto s^\Delta \exp[-b^2/a(s)],$$  \hspace{1cm} (15)

where $$a(s) \sim \ln s.$$ In the framework of perturbative QCD–based models the driving contribution to the opacity is due to jet production in gluon–gluon interactions, when

$$\Omega(s, b) \propto \sigma_{jet} \exp[-\mu b],$$  \hspace{1cm} (16)
where $\sigma_{\text{jet}} \sim (s/s_0)^\Delta$. This parameterization leads to the rising total and elastic cross-sections and slope parameter:

$$\sigma_{\text{tot}}(s) \sim \sigma_{\text{el}}(s) \sim B(s) \sim \ln^2 s$$

and the ratio

$$\frac{\sigma_{\text{el}}(s)}{\sigma_{\text{tot}}(s)} \to \frac{1}{2}.$$  

(17)

(18)

Transition to the mode where the scattering amplitude exceeds the black disk limit results in the necessity of considerations of the eikonal functions with non-zero real parts. Then to ensure such transition the real part of eikonal should gain an abrupt increase equal to $\pi$ at some $s = s_0$. The conventional models do not foresee such a critical behavior of the real part of eikonal.

However, it does not mean that the eikonal model itself is in trouble. In particular, the account for the fluctuations of the eikonal \[8\] strongly modifies the structure of the amplitude and reduces it to algebraic form which is similar to that used in another unitarization scheme with the use of the generalized reaction matrix.

This method is based on the relativistic generalization of the Heitler equation \[9\]. The form of the amplitude in the framework of this method is the following:

$$f(s, b) = \frac{U(s, b)}{1 - iU(s, b)}$$

(19)

where $U(s, b)$ is the generalized reaction matrix, which is considered as an input dynamical quantity similar to an eikonal function. Inelastic overlap function is connected with $U(s, b)$ by the relation

$$\eta(s, b) = ImU(s, b)|1 - iU(s, b)|^{-2}.$$  

(20)

Eq. (19) ensures $s$–channel unitarity provided that $ImU(s, b) \geq 0$. Similar form for the scattering matrix was obtained by Feynman in his parton model of diffractive scattering \[10\].

Construction of particular models in the framework of the $U$–matrix approach proceeds with the same steps as it happens for the eikonal function, i.e. the basic dynamics as well as the notions on hadron structure are used
to obtain a particular form for the $U$–matrix. For example, the Regge–pole approach \cite{11} provides the following form for the $U$–matrix:

$$U(s, b) \propto is^\Delta \exp[-b^2/a(s)], \quad a(s) \sim \alpha' \ln s,$$

while the chiral quark model \cite{12} gives the exponential $b$–dependence

$$U(s, b) \propto is^\Delta \exp[-\mu b],$$

where $\mu$ is the constant proportional to the mass of the constituent quarks. We pointed out here only the gross features of these model parameterizations without going into details.

The both parameterizations lead to $\ln^2 s$ rise of the total and elastic cross–sections and slope parameter $B(s)$:

$$\sigma_{tot}(s) \sim \sigma_{el}(s) \sim B(s) \sim \ln^2 s$$

at $s \to \infty$. These results are similar to the results of eikonal unitarization.

However, these two unitarization schemes give different predictions for the inelastic cross–sections and for the ratio of elastic to total cross-section. This ratio in the $U$–matrix unitarization scheme reaches its maximal possible value at $s \to \infty$, i.e.

$$\frac{\sigma_{el}(s)}{\sigma_{tot}(s)} \to 1,$$

which reflects in fact that the bound for the partial–wave amplitude in the $U$–matrix approach is $|f| \leq 1$ while the bound for the case of imaginary eikonal is (black disk limit): $|f| \leq 1/2$.

When the amplitude exceeds the black disk limit (in central collisions at high energies) then the scattering at such impact parameters turns out to have antishadow nature. It means that we should consider in this case the solution of unitarity equation Eq. \cite{11} which has sign plus. In this antishadow scattering mode the elastic amplitude increases with decrease of the inelastic channels contribution.

The shadow scattering mode is often considered as the only possible one. However, it should be noted that the two solutions of unitarity have an equal meaning and the antishadow scattering mode could also be realized at high energies in central collisions. The both scattering modes are achieved in a continuous way in the $U$–matrix approach despite these modes are related with the two different solution of unitarity Eq. \cite{14}.
Let us consider the transition to the antishadow scattering mode \cite{13} in the framework of the $U$–matrix unitarization scheme. With conventional parameterizations of the $U$–matrix in the form of Eq. \ref{eq:21} or Eq. \ref{eq:22} the inelastic overlap function increases with energies at modest values of $s$. It reaches maximum value $\eta(s, b) = 1/4$ at some energy $s = s_0$ and beyond this energy the transition to the antishadow scattering mode occurs. The region of small impact parameters corresponds to this scattering mode when $Imf(s, b) > 1/2$ and $\eta(s, b) < 1/4$. The quantitative analysis of the experimental data \cite{14} has given prediction $\sqrt{s_0} = 2 \text{ TeV}$.

Thus, the function $\eta(s, b)$ becomes peripheral when energy is increasing. At such energies the inelastic overlap function reaches its maximum value at $b = R(s)$ where $R(s)$ is the interaction radius. So, beyond the transition threshold there are two regions in impact parameter space: the central region of antishadow scattering at $b < R(s)$ and the peripheral region of shadow scattering at $b > R(s)$. At $b = R(s)$ the maximal absorption takes place (Fig. 1).

The transition to the antishadow scattering mode at small impact parameters and high energies results also in a relatively slow growth of inelastic cross–section:

$$\sigma_{inel}(s) = 8\pi \int_0^\infty \text{Im}U(s, b)|1 - iU(s, b)|^{-2} \sim \ln s.$$  \hfill (25)

at $s \to \infty$.

It should be noted that appearance of the antishadow scattering mode does not contradict to the basic idea that the particle production is the driving force for elastic scattering. Indeed, the imaginary part of the generalized reaction matrix is the sum of inelastic channel contributions:

$$\text{Im}U(s, b) = \sum_n \bar{U}_n(s, b),$$  \hfill (26)

where $n$ runs over all inelastic states and

$$\bar{U}_n(s, b) = \int d\Gamma_n |U_n(s, b, \{\xi_n\})|^2$$  \hfill (27)

and $d\Gamma_n$ is the $n$–particle element of the phase space volume. The functions $U_n(s, b, \{\xi_n\})$ are determined by the dynamics of $2 \to n$ processes. Thus, the quantity $\text{Im}U(s, b)$ itself is a shadow of the inelastic processes. However,
unitarity leads to self–damping of inelastic channels \[15\] and increase of the function \( \text{Im} U(s, b) \) results in decrease of the inelastic overlap function \( \eta(s, b) \) when \( \text{Im} U(s, b) \) exceeds unity.

At the energies when the antishadow mode starts to develop (it presumably could already occur at energies of the Tevatron–Collider) the Pumplin bound Eq. \[10\] for inelastic diffraction dissociation cannot be applied since the main assumption used under its derivation is no more valid.

The consideration of diffraction dissociation in the framework of the \( U– \)matrix chiral quark model \[16\] shows that \( \sigma_{\text{diff}}(s) \) has a complicated energy dependence: it increases at energies when only the shadow scattering mode exists and decreases when the antishadow scattering mode appears in the central hadron–hadron collisions. Such a behavior of \( \sigma_{\text{diff}}(s) \) reflects the changing energy dependence of \( \eta(s, 0) \).

Conclusions

Thus, loosely speaking the genesis of hadron scattering can be described as a transition from the grey to black disk and finally to black ring with the antishadow scattering in the central region. Such transformations are under control of unitarity of the scattering matrix. Of course, it would be interesting to consider particular physical origin of the antishadow scattering mode. First of all, the existence of such mode points out that new phenomena would be expected at high energies in the central hadronic collisions. Such collisions are usually associated with formation of quark–gluon plasma and disoriented chiral condensate in the interior of interaction region. What are the correlations between these phenomena and the antishadow scattering mode? If there are any, it might be studied in the framework of nonperturbative QCD and in the experiments devoted to measuring observables in soft processes (recent discussions of these problems are given in \[17\]). It seems that the anomalies observed in cosmic ray experiments might also be correlated with development of the antishadow scattering mode in the central hadron collisions.
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Figure 1: The impact parameter dependence of inelastic overlap function at energies $s > s_0$. 