Numerical simulation of thin film flowing down a vertical uniformly heated plane using the regularized weighted residuals model

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Abstract. With the developed three-dimensional Regularized Weighted Residuals Model (RM), discussion on its accuracy and simulation work are conducted. It is shown that for small to moderate Reynolds number, the accuracy of RM decreases with increasing Marangoni number and decreasing Biot number of the plane. The streamwise error ratios are larger than spanwise ones and reach a maximum when Reynolds number equals 3, which is due to underestimated Marangoni instability. The increase of Reynolds number or introducing streamwise most unstable disturbance makes film duration time longer. With increasing Reynolds number, the early stage of wave evolution becomes faster and ripples are more intensive; the final pattern is more unstable and disordered. Heat transfer efficiency reaches two peaks with increasing Reynolds number, one due to the interaction between thermal condition and flow rate, the other due to film no-rupture. Introducing streamwise most unstable disturbance makes the early stages of wave evolution longer and wave amplitude smaller in later stages. Unless Reynolds number is small enough, heat transfer efficiency decreases with the introduction of streamwise most unstable disturbance.

1. Introduction

Film flowing is widely used in industry due to is high efficiency of heat and mass transfer with small flow rate, for example, passive containment cooling system for AP1000, the typical nuclear reactor of generation III. Interaction of hydrodynamic instability and Marangoni instability results in rivulet structure formation aligned with the mean flow[1], which on the one hand enhances heat and mass transfer with increasing free surface area and local Reynolds number, on the other hand leads to film rupture and dry-out. There are several typical numerical models for thin falling liquid film, such as Benney equation (BE)[2], Integral-boundary-layer model(IL)[3], and regularized weighted residuals model (RM)[4]. RM is simpler than IL and more accurate than BE. Scheid et al.[5] used RM under specific temperature (ST) condition of the plane to simulate film evolution and found formation of rivulet structure at small Reynolds number. Compared to the ST condition, uniform heat flux (HF) condition is more realistic and closer to experimental and engineering situation. Scheid[6] first established two-dimensional model under HF condition. Wang et al.[7] further developed three-dimensional model under HF condition and discussed the influence of different initial disturbances on wave evolution and heat transfer at small Reynolds number.

This paper uses RM under HF condition to discuss its accuracy through linear stability analysis and the influence of increasing Reynolds number and introduction of the most unstable streamwise disturbance on wave evolution and heat transfer through numerical simulation.
2. Theory model

Figure 1. Sketch of the thin liquid film flowing down an infinite vertical plate.

An incompressible Newtonian viscous thin liquid film driven by gravity $g$ flows down an infinite vertical plate, as shown in figure 1. $y = h(x, z, t)$ is the free surface. The uniform heat flux from plate is $Q_w$, while $Q_{loss}$ is the heat loss through the plate to ambient atmosphere. The average film thickness is $\overline{h}_N$.

The dimensionless relation of the variables is as following,

$$\left(\frac{u}{u_v}, \frac{w}{\rho v^2/\gamma_l^2}, \frac{T}{T_v}, \frac{h}{l_v}, \frac{q_v}{\beta}, \frac{\gamma\Delta T_l}{\rho v^2}, \frac{\overline{q}_N}{v}, k\right) = f(Pr, \Gamma, Bi, Bi_w, \beta, Ma, Re, k)$$

Where, $u_v = (vg \sin \beta)^{1/2}$ is velocity scale; $l_v = (v^2/g \sin \beta)^{1/2}$ is the length scale; $Pr = \nu \chi / \nu$ is Prandtl number, which reflects the ratio of the momentum diffusivity to the thermal diffusivity; $\Gamma = \alpha l_v / \rho v^2$ is Kapitza number, which reflects the ratio of surface tension force to inertial force; $Bi = al_v / \lambda$ is the Biot number of film surface, which reflects the heat transfer rate from film to atmosphere; similarly, $Bi_w = al_v / \lambda$ is the Biot number of plate, which reflects the heat transfer rate through plate; $Ma = \gamma\Delta T_l / \rho v^2 = \gamma q_w / \lambda p g l_v \sin \beta$ is Marangoni number, which is temperature difference related with heat flux; $Re = \overline{u}_v \overline{h}_N / v$ is Reynolds number, which is the dimensionless flow rate; $k$ is dimensionless disturbance wavenumber.

The governing equations for this physical model include continuity equation, Navier-Stokes (N-S) equation and energy equation. The boundary conditions include velocity no slip and no penetration on the solid wall, the heat transfer between the heated wall and the liquid film satisfying the energy conservation, the free interface satisfying the continuous boundary conditions of motion and stress and Newton's cooling law. Using the Shkadov scale[9] and the same deduction idea of Scheid's[6], get the RM model with four equations[8], which are shown as following:

$$\nabla h + \partial_x q_1 + \partial_z q_1 = 0$$

$$\delta \nabla q_1 = \delta(9 \frac{q_1^2}{h} \frac{\partial}{\partial h} \frac{\partial}{\partial h} - \frac{17}{7} \frac{\partial}{\partial h} \frac{\partial}{\partial h}) + \left(1 - \frac{\delta}{70} q_v \frac{\partial}{\partial h} + M \frac{\delta}{56} \frac{\partial}{\partial h}\right)^{-1} \left(9 \frac{\partial q_v}{\partial h} \frac{\partial}{\partial h} + 9 \frac{\partial q_v}{\partial h} \frac{\partial}{\partial h} + 9 \frac{\partial q_v}{\partial h} \frac{\partial}{\partial h}\right)$$

$$+ \frac{5}{6} - \frac{5}{2} h - \frac{5}{6} \frac{\partial}{\partial h} + (1 - \frac{\delta}{70} q_v \frac{\partial}{\partial h} + M \frac{\delta}{56} \frac{\partial}{\partial h})$$

$$+ \left[4 \frac{q_1}{h^2} \frac{\partial}{\partial h} \frac{\partial}{\partial h} + \frac{3}{4} \frac{q_1}{h^2} \frac{\partial}{\partial h} + 13 \frac{q_1}{h^2} \frac{\partial}{\partial h} - 23 \frac{q_1}{h^2} \frac{\partial}{\partial h} + \frac{7}{2} \frac{q_1}{h^2} \frac{\partial}{\partial h}\right].$$

(3)
\[ \begin{align*} 
\delta \partial_t q_{\perp} &= \frac{9}{7} q_{\perp}^2 \partial_z h - \frac{17}{7} q_{\perp} \partial_z q_{\perp} - \frac{8}{7} q_{\perp} \partial_z \dot{q}_{\perp} - \frac{9}{7} q_{\perp} \partial_z q_{\parallel} + \frac{9}{7} q_{\perp} \partial_z \dot{q}_{\parallel} + \frac{5}{2} \frac{q_{\parallel}}{h^2} - 6 \partial_z \dot{q}_{\parallel} h \\
+ e^2 W e \left[ \frac{5}{6} \partial_z \delta h + \frac{5}{4} M a o + \eta \left[ \frac{9}{7} q_{\perp} \partial_z \dot{q}_{\perp} - \frac{9}{2} q_{\perp} \partial_z h - \frac{9}{2} q_{\parallel} \partial_z \dot{q}_{\parallel} + \frac{13}{16} \partial_z \dot{q}_{\parallel} h \right] \right] \\
\frac{43}{16} \partial_z q_{\parallel} \partial_z h - \frac{73}{16} \partial_z \dot{q}_{\parallel} - \frac{3}{4} q_{\perp} (\partial_z h)^2 + \frac{13}{4} q_{\perp} \partial_z \dot{q}_{\parallel} h + \frac{23}{16} \partial_z \dot{q}_{\parallel} h - \frac{3}{4} q_{\parallel} + \frac{7}{2} \partial_z q_{\parallel} \right] 
\end{align*} \]

\[ \begin{align*} 
F - B \theta &= \text{Pr} \frac{1}{2} \frac{h F_{sec} + \theta + \frac{3}{8} q (F_\perp + F_{\parallel})}{h} - \frac{5}{8} F_{sec} + q \dot{\theta} h + \frac{3}{8} \left( \frac{F_{sec} + F_{\parallel}}{h} \right) - \frac{5}{8} F_{sec} + q \dot{\theta} h + \frac{3}{8} \left( \frac{F_{sec} + F_{\parallel}}{h} \right) \\
+ (F - B \theta) \left( \frac{h F_{sec} + \theta + \frac{3}{8} q (F_\perp + F_{\parallel})}{h} - \frac{5}{8} F_{sec} + q \dot{\theta} h + \frac{3}{8} \left( \frac{F_{sec} + F_{\parallel}}{h} \right) - \frac{5}{8} F_{sec} + q \dot{\theta} h + \frac{3}{8} \left( \frac{F_{sec} + F_{\parallel}}{h} \right) \right) = 0 
\end{align*} \]

Where, \( h \) is local thickness of film, \( q_{\parallel} \) and \( q_{\perp} \) represents streamwise and spanwise local flow rate, \( \theta \) the film surface temperature, \( t \) is time. \( F = (1 + B_w \theta)/(1 + B_w h) \) is effective heat flux, \( h_N = \frac{\Gamma h}{\eta} \) is modified Nusselt film thickness; \( We = \Gamma/h^2 \) is Weber number, which reflects the ratio of surface tension pressure to the viscous normal stress generated by gravity at the film surface; \( \epsilon = We^{1/3} \) is film parameter; \( B = h_N Bi \) is the modified Biot number of free surface; \( B_w = h_N Bi_w \) is the modified Biot number of plate; \( \delta = 3cRe \) is the reduced Reynolds number; \( M = \epsilon Ma / h_N \) is the reduced Marangoni number; \( \zeta = c \) is reduced inclination number.

In this paper, the fluid is considered as water at 20°C and 1.0 atm, thus \( \Gamma = 3375, Pr = 7 \); according to realistic heat transfer coefficient, assume \( Bi = 0.12, Bi_w = 0.6 \), as done by Scheid et al.[10]. This paper focuses on the vertical case, so \( \zeta = 0 \).

3. Linear stability analysis

Substituting the disturbance \( h = 1 + \delta h e^{ik_x x + k_z z - i \omega t} \), \( q = 1/3 + \delta q e^{ik_x x + k_z z - i \omega t} \), \( p = 0 + \delta p e^{ik_x x + k_z z - i \omega t} \), \( \theta = 1/(B + B_w (1 + B)) + \delta \theta e^{ik_x x + k_z z - i \omega t} \) (\( \delta h, \delta q, \delta p, \delta \theta \) is the amplitude of disturbance) into the linearized RM equations, the dispersion relation among the wavenumber \( k_x, k_z \) and angular frequency \( \omega \) is got. This section considers temporal mode. The imaginary part of \( \omega_\perp, \omega_\parallel \) is the temporal growth rate.

Using Orr–Sommerfeld equation (OS) which is based on small perturbation assumption and linearization from N-S equation as benchmark, the accuracy of RM is discussed through linear stability analysis with small to moderate \( Re \). Wang[7] found that the relationship between characteristic wavenumbers with RM fitted Scheid’s[5] results well. Further accuracy analysis about growth rate are needed.

Define error ratio and maximum error between the growth rates predicted by OS and RM as following:

\[ ER = \left( \frac{\omega_{\parallel, OS} - \omega_{\parallel, RM}}{\omega_{\parallel, OS}} \right) \times 100\%, MER = \left( \frac{\omega_{\parallel, max, OS} - \omega_{\parallel, max, RM}}{\omega_{\parallel, max, OS}} \right) \times 100\% \]

Figure 2. \( ER \) and growth rate versus wavenumber in streamwise direction.

(a) \( Re = 1.25, Ma = 2 \)
(b) \( Re = 10, Ma = 2 \)
(c) \( Re = 1.25, Ma = 20 \)
than 1%, indicating the accuracy of RM from small to moderate increasing maximum at small when \( Bi \) smaller than 10% before the direction, increasing indicating error is mainly from thermal instability. Spanwise prediction is more sensitive to linearly with \( Ma \) and the growth rate; spanwise relationship between spanwise \( k \) makes \( RM \) smaller, indicating the more scattered rivulets structure; and increasing \( Ma \) leads to contrary results, indicating more intensive rivulet structures. When \( Ma \) is small and \( Re \) increases, RM is in great agreement with OS. When \( Re \) is small and \( Ma \) increases, the streamwise \( ER \) increases obviously and \( ER > 0 \), but still smaller than 10% before the \( k_m \), indicating that \( ER \) increase is mainly from underestimating the influence of Marangoni instability on the growth rate; spanwise \( ER \) keeps small.

Figure 3. \( ER \) and growth rate versus wavenumber in spanwise direction.

From figure 2 and figure 3, with increasing \( k \), there exists the most unstable wavenumber \( (k_m) \) which makes growth rate reaches maximum. As Scheid et al. [5] concluded, \( \lambda_{iw} = 2\pi/k_{z, \omega} \), which reflects the relationship between spanwise \( k_m \) and the linear prediction of rivulet wavelength. In streamwise direction, increasing \( Re \) and \( Ma \) lead to larger \( k_m \). In spanwise direction, increasing \( Re \) makes \( k_m \) smaller, indicating the more scattered rivulets structure; and increasing \( Ma \) leads to contrary results, indicating more intensive rivulet structures. When \( Ma \) is small and \( Re \) increases, RM is in great agreement with OS. When \( Re \) is small and \( Ma \) increases, the streamwise \( ER \) increases obviously and \( ER > 0 \), but still smaller than 10% before the \( k_m \), indicating that \( ER \) increase is mainly from underestimating the influence of Marangoni instability on the growth rate; spanwise \( ER \) keeps small.

Figure 4. \( MER \) of the RM versus \( Ma \).

Spanwise \( MER \) keeps smaller than streamwise \( MER \) and not exceeding 2%. \( MER \) increases nearly linearly with \( Ma \), and the slope varies with \( Re \). Note that \( MERs \) and their increase rates achieve the maximum at \( Re = 3 \) in streamwise direction, while those in spanwise direction decreases with the increasing \( Re \), as shown in figure 4. According to figure 5, when \( Ma = 0 \), streamwise \( MER \) keeps smaller than 1%, indicating the accuracy of RM from small to moderate \( Re \). For each \( Ma \) except 0, streamwise \( MER \) reaches a maximum at \( Re = 3 \), which is consistent with the conclusion above.

The influence of \( Bi_w \) is further conducted and the results are shown in figure 6 and table 1.

Table 1. \( MER \) versus \( Bi_w, Re = 6, Ma = 20, Bi = 0.12 \).

| \( Bi_w \) | 0.12 | 0.6 | 1 | 2 |
|-----------|------|-----|---|---|
| \( MER_\% \) | 13.7987 | 9.0727 | 7.0510 | 4.5169 |
| \( MER_\% \) | 1.9525 | 0.5903 | 0.3759 | 0.1946 |

Figure 6. \( ER \) of the RM as a function of the wavenumber.

From figure 6 and table 1, it can be seen that the larger the \( Bi_w \) is, the better accuracy of RM is, also indicating error is mainly from thermal instability. Spanwise prediction is more sensitive to \( Bi_w \), indicating the importance of thermal condition for spanwise disturbance growing.

Overall, with small to moderate \( Re \), the accuracy of RM decreases with increasing \( Ma \) or decreasing \( Bi_w \) and becomes lowest around \( Re = 3 \). Spanwise \( ER \) and \( MER \) keep small, while streamwise ones are small when \( Ma \) is small enough or \( Re \) is large enough. Streamwise \( ER \) keeps positive, indicating that
RM underestimates the growth rate, due to underestimating Marangoni effect corresponding to thermal conditions. The following three-dimensional simulation is conducted with $Ma=20$, $Bi_w=0.6$, when the linear analysis error is acceptable.

4. Three-dimensional numerical simulation

RM under ST is validated by Liu et al.[11]. It is reasonable to assume that RM has similar characteristics according to Trevelyan’s[12] simulation work. In this section, periodic boundary conditions in streamwise and spanwise directions are set. The computational domain is discretized with $M\times N=128 \times 128$ grid points. Using Fourier spectral method[13] to calculate spatial derivatives and keep the first 2/3 of the Fourier modes. Grid independence tests are performed by increasing $M$ and $N$ until the change deformation energies is less than 2%. When the thickness of the film $h < 3 \times 10^{-3}$, the film is adjudged to have ruptured and the computation is terminated, as done in the previous research[8].

The simulation uses initial parameters shown as following, $q_i = 1/3$, $q_\perp = 0$, $\theta = 1 / (B + B_w(1 + B))$, $L_x = 8 \times 2\pi/k_x$, $L_z = 6 \times 2\pi/k_z$, $k_0 = 1 + A_x \cos(8 \times 2\pi x / L_x) + A_z \cos(6 \times 2\pi z / L_z) + A_{noise} \tilde{r}(x, z)$.

Where, $L_x \times L_z$ is computational domain. $A_x, A_z, A_{noise}$ represents the amplitude of disturbance; $\tilde{r}$ is the random function with range [-1, 1], which represents random disturbance.

The wave deformation energy of streamwise and spanwise direction are defined as:

$$
E_x(t) = \frac{1}{MN} \sum_{j=1}^{N} \left( \sum_{m=1}^{M/2+1} \left| (a_m(x_j, t)) \right|^2 \right)^{1/2},
E_z(t) = \frac{1}{MN} \sum_{j=1}^{N} \left( \sum_{m=1}^{N/2+1} \left| (b_m(x_j, t)) \right|^2 \right)^{1/2},
$$

(7)

Here use the ratio of $H$ to $H_N$ to describe the heat transfer compared to stationary film,

$$
E_{HT} = \frac{H}{H_N} = \frac{A_x}{A_N} \bar{\theta} = A' \bar{\theta}(B + B_w(1 + B))
$$

(8)

Where, $\bar{\theta}$ and $\theta_N$ are dimensionless average surface temperature of wavy film and Nusselt film respectively. According to previous research[8], the wave evolution process can be divided into five stages. In stage I, $E_x$ and $E_z$ increase rapidly, due to the disturbance growth in both directions under hydrodynamic and Marangoni effect. The ripples are almost parallel to spanwise direction; in stage II, $E_x$ and $E_z$ are almost the same and keep constant, when wave breaks into short, caterpillar-shaped pieces; in stage III, $E_x > E_z$ and they both increase. Wave streaks tend to be parallel to streamwise direction; in stage IV, $E_x$ decreases and $E_z$ increases continuously, their difference becomes larger. During this period, in spanwise direction, the wave grows due to Marangoni effect self-acceleration; in streamwise direction, the curvature induced capillary force tends to flatten the wave. Rivulet structures appears; in stage V, both $E_x$ and $E_z$ increases and $E_x > E_z$. Solitary waves riding on rivulet appears.

Previous research[8] found that the increase of $Re$ and introduction of streamwise most unstable disturbance ($A_z$) can make the film lasting longer. This section further discuss their influence on wave evolution and heat transfer.

4.1. Wave evolution

4.1.1. The effect of increasing $Re$. The deformation energy and wave patterns evolution with time are shown as following, while $Re$ increases from 1.25 to 6, $Ma=20$, $k_x=0.3891$, $k_z=0.2997$, $A_x=A_z=0$. 


Figure 7. Deformation energies as a function of time. Letters refer to evolution stages. $Ma=20$.

Figure 8. Wave evolution final patterns, $Ma=20$ and under random disturbance.
The extrema of film thickness are given in the brackets.
The final patterns above show that rivulet wavelength increases with increasing $Re$, which is consistent to the prediction of section 3.

Figure 9. Wave patterns comparison at $t=930$ under random disturbance.
The extrema of the film thickness are given in the brackets. $Ma=20$.

Figure 10. Wave patterns comparison at stage I under random disturbance.
The extrema of the film thickness are given in the brackets. $Ma=20$.

Over all, with $Ma=20$ and under random disturbance, as $Re$ increases, according to figure 9 and figure 10, the evolution from stage I to stage II becomes faster and the ripples are more intensive at first stage;
the film duration time becomes longer, final wave patterns are more unstable and disordered with more fast and small fluctuation and rivulet wavelength becomes longer.

4.1.2. Effect of introduction of streamwise most unstable disturbance. With $Ma=20$, introducing $A_x$ leads to deformation energies development in both directions being different, especially in stage I. Notes that when $Re=3$, $k_x=0.5033$, $k_z=0.2729$; when $Re=4$, $k_x=0.5604$, $k_z=0.2642$.

According to figure 11, compared with the case only under random disturbance, $E_x$ grows faster and $E_z$ slower in longer stage I. Smaller difference between $E_x$ and $E_z$ indicates the smaller amplitude of wave before rupture. The longer duration time of film is mainly due to the corresponding longer stage I and stage II.

![Figure 11. Deformation energies as a function of time under different disturbances. $Ma=20$.](image)

4.2. Heat transfer

Comparison of heat transfer with increasing $Re$ and introduction of $A_x$ are conducted.

![Figure 12. Comparison of heat transfer change. $Ma=20$.](image)

According to the figure 12(a), with increasing $Re$, the film flow lasts longer, but the max $E_{HT}$ decreases. In stage I, rapider and larger increase of heat transfer occurs, which is induced by more intensive ripples and quicker development during this period. In stage II, the increase of amplitude, which is beneficial for heat transfer, is slower. Accordingly, $E_{HT}$ almost keeps constant. With the same $Ma=20$, the larger the $Re$ is, the longer time it takes for Marangoni instability to dominate and the weaker its relative effect is. Therefore, after the stable stage, with larger $Re$, the increase of heat transfer becomes later and slower.

Here define $E_{HT}$ to measure the average heat transfer efficiency over film duration time,

$$E_{HT} = \frac{\int_{t_0}^{t_f} E_{HT}(t) \, dt}{t_f - t_0}$$  \hspace{1cm} (9)

**Table 2.** Average heat transfer with $Ma=20$, random disturbance.

| $Re$ | 1.25 | 3 | 4 | 6 |
|------|------|---|---|---|
| Rupture time | 5165 | 6195 | 10275 | 20000~ |
| $(E_{HT} - 1) \times 10^3$ | 5.03 | 5.25 | 5.06 | 6.15 |

As $Re$ increases, the heat transfer efficiency reaches peaks at $Re=3$ and $Re=6$. While the film flow with $Re=6$ is fully developed and doesn’t rupture until the presupposed time limit, its heat transfer
efficiency will continuously increases to maximal $E_{HT}$ given more time. The comparison of the heat transfer efficiency with $Re=3$ and $Re=6$ given the same time is as following,

$$E_{HT, Re=6} = 1.00613 > E_{HT, Re=3} = 1.00525$$

According to figure 12(b), (c), compared with the case only under random disturbance, when $Re=1.25$, introducing $A_x$ doesn’t have much effect on the delay of stages IV and V, and the increase of duration time is pretty little. The suppression of Marangoni instability is not obvious and it even dominate in latter stage and corresponding heat transfer increases; but when $Re$ increases to 3, the suppression of Marangoni instability is strengthened. Although the film rupture occurs later, but the increase of heat transfer is slower and less than that of the cases under random disturbance in later stages. The comparison of average heat transfer efficiency with $Re=3$ is shown as following,

$$E_{HT, ran} = 1.00525 > E_{HT, ran+stream} = 1.00374$$

Subscript “ran” means in random disturbance case, “ran+stream” means in random disturbance superposed with streamwise most unstable disturbance case. It notes that the longer duration time of film is due to the corresponding longer stage I and stage II. The duration time of later stages almost don’t change, so with $Re=4$, the heat transfer of the first two stages are compared as shown in figure 12(d).The average heat transfer during the first two stages is shown as following,

$$E_{HT, ran} = 1.00235 > E_{HT, ran+stream} = 1.00185$$

Overall, unless $Re$ is small enough, the introduction of $A_x$ is not beneficial for heat transfer.

5. Conclusion

Through the linear analysis, it is found that the accuracy of RM decreases with increasing $Ma$ and decreasing $Bi$ and becomes lowest in the vicinity of $Re = 3$. The accuracy in spanwise direction is better than that in streamwise direction. $ER$ and $MER$ in streamwise direction are small when $Ma$ number is small enough or $Re$ is large enough. Streamwise $ER$ keeps positive, indicating that RM underestimates the growth rate, due to the underestimate of Marangoni effect.

Through three-dimensional numerical simulation, as $Re$ increases, the ripples in early stage are more intensive and evolution becomes faster; the difference between deformation energies in two directions in later stages is smaller; the final pattern is more unstable and disordered; film duration time becomes longer; average heat transfer efficiency arrives peak at $Re=3$ and $Re=6$. Film is fully developed at $Re=6$ and overall heat transfer efficiency is best. Introducing streamwise most unstable disturbance leads to longer stage I, during which $E_x$ grows faster and $E_z$ grows slower; longer film duration time is due to longer early evolution stages; when $Re=1.25$, compared with only under random disturbance, the influence on the competition of hydrodynamic instability and Marangoni instability is small and heat transfer efficiency is enhanced, but when $Re$ increases, introduction of streamwise most unstable disturbance strengthens the suppression of Marangoni instability, which is not beneficial to overall heat transfer efficiency.

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