MEASURING STRONG AND WEAK PHASES IN TIME-INDEPENDENT B DECAYS

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ABSTRACT

Flavor SU(3) symmetry implies certain relations among B-decay amplitudes to $\pi \pi$, $\pi K$ and $K \bar{K}$ final states, when annihilation-like diagrams are neglected. Using three triangle relations, we show how to measure the weak CKM phases $\alpha$ and $\gamma$ using time-independent rate measurements only. In addition, one obtains all the strong final-state phases and the magnitudes of individual terms describing tree (spectator), color-suppressed and penguin diagrams. Many independent measurements of these quantities can be made with this method, which helps to eliminate possible discrete ambiguities and to estimate the size of SU(3)-breaking effects.
I. INTRODUCTION

In the near future, the study of $B$ meson decays will be a crucial testing ground for the Standard Model (SM) picture of CP violation based on phases in the Cabibbo-Kobayashi-Maskawa (CKM) \[1\] matrix $V_{\alpha i}$, where $i = (d, s, b)$ and $\alpha = (u, c, t)$. Unitarity of the CKM matrix implies the following triangle identity (the “unitarity triangle”)

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 .$$

In the now-familiar Wolfenstein parametrization \[2\], only $V_{ub}$ and $V_{td}$ have non-negligible phases, so that the angles in the triangle are given by $\beta = -\text{Arg}(V_{td})$, $\gamma = \text{Arg}(V_{ub})$, and $\alpha = \pi - \beta - \gamma$ \[3\]. The SM can be tested by independently measuring the three angles $\alpha$, $\beta$ and $\gamma$.

CP violation can occur in the $B$ system if two weak amplitudes contribute to a particular decay. It is necessary that there be both a weak and a strong phase difference between these two amplitudes in order to see an asymmetry between the rates for $B \to f$ and $\bar{B} \to \bar{f}$. One advantage of this type of CP violation is that it can occur in “self-tagging” modes such as those involving charged $B$ decays (e.g. $B^+ \to \pi^0 K^+$). The major disadvantage is that, since the strong phases are unknown, measurements of CP violation in such systems will not yield clean information about the weak CKM phases.

A potentially cleaner class of CP-violating asymmetries involves the decay of neutral $B$ mesons to CP eigenstates such as $J/\psi K_S$ or $\pi^+ \pi^-$. In this case, CP violation arises from the interference between a direct decay amplitude and one which proceeds via $B^0 - \bar{B}^0$ mixing. If one weak amplitude contributes to the direct decay, CKM phase information can be extracted from measurements of these asymmetries independent of strong final-state phases. Such measurements require the ability to obtain time-dependent information and to “tag” the flavor of the decaying $B$ meson, i.e. to know whether it was a $B^0$ or a $\bar{B}^0$ at $t = 0$.

One possible problem in the above program is that there may be more than one weak amplitude – in addition to tree-level decays, “penguin” diagrams may contribute significantly \[4, 5\]. In this case, both types of CP violation are present, which complicates things considerably – it is no longer possible to obtain clean CKM phase information in the same simple way. However, by using isospin relations among several amplitudes, one can separate all the various effects, again obtaining clean CKM phase information. For example, in the case of $B^0 \to \pi^+ \pi^-$, it is necessary to measure the rates for all possible charge states in $B \to \pi \pi$ and $\bar{B} \to \pi \pi$, as well as the time dependence of $B^0(t) \to \pi^+ \pi^-$ \[6\]. A similar analysis can be done for the decays $B \to \pi K$ \[4\].

Given that isospin symmetry is such a useful tool in the measurement of CP violation in the $B$ system, it is only natural to next ask whether considerations of flavor SU(3) symmetry \[8\]-\[12\] can lead to anything interesting. It is the purpose of this Letter to show that indeed they do: by using SU(3) and making some
reasonable approximations it is possible to obtain not only the weak phases \( \alpha \) and \( \gamma \), but also strong phase information and the sizes of different diagrams. This is done through the measurement of several \( B \) decay rates to \( \pi \pi \), \( \pi K \) and \( K \bar{K} \), most of whose branching ratios are of the order of \( 10^{-5} \). No time-dependent measurements are needed. (If time-dependence can be measured, then in addition the weak angle \( \alpha \) can be obtained in other, independent ways.) There are several independent ways to obtain all this information. The use of these gives a possible way to reduce the discrete ambiguities and also tests the SU(3) flavor symmetry.

II. SU(3) RELATIONS AMONG AMPLITUDES

We apply flavor SU(3) symmetry to the decays of \( B \) mesons to pairs of light pseudoscalar mesons \( \pi \pi \), \( \pi K \) and \( K \bar{K} \). The SU(3) amplitudes for these decays, which involve all possible charge states, can be expressed in terms of the following diagrams (see Fig. 1): a “tree” amplitude \( T \) or \( T' \), a “color-suppressed” amplitude \( C \) or \( C' \), a “penguin” amplitude \( P \) or \( P' \), an “exchange” amplitude \( E \) or \( E' \), an “annihilation” amplitude \( A \) or \( A' \), and a “penguin annihilation” amplitude \( PA \) or \( PA' \). Here an unprimed amplitude stands for a strangeness-preserving decay, while a primed contribution stands for a strangeness-changing decay. As noted in Refs. [8, 11, 12], this set of amplitudes is over-complete. The physical processes of interest involve only five distinct linear combinations of these six terms.

The diagrams denoted by \( E \), \( A \) and \( PA \) involve contributions to amplitudes which should behave as \((f_B/m_B)\) in comparison with those from the diagrams \( T \), \( C \) and \( P \) (and similarly for their primed counterparts). This suppression is due to the smallness of the \( B \) meson wavefunction at the origin and should remain valid unless rescattering effects are important. Such rescatterings indeed could be responsible for certain decays of charmed particles, but should be less important for the higher-energy \( B \) decays. In addition the diagrams \( E \) and \( A \) are also helicity suppressed by \((m_u,d,s/m_B)\) since the \( B \) mesons are pseudoscalars. Neglecting the contributions of the above diagrams, we are left with the 6 diagrams \( T \), \( T' \), \( C \), \( C' \), \( P \) and \( P' \). These six complex parameters determine the 13 allowed \( B \) decays to states with pions and kaons, as listed in Table 1. Following the conventions in Refs. [8, 11, 12], we take the \( u \), \( d \), and \( s \) quark to transform as a triplet of flavor SU(3), and the \( -\bar{u}, \bar{d} \), and \( \bar{s} \) to transform as an antitriplet. Thus the \( \pi \)-mesons and kaons form part of an octet and are defined as \( \pi^+ \equiv u\bar{d}, \pi^0 \equiv (d\bar{d} - u\bar{u})/\sqrt{2}, \pi^- \equiv -d\bar{u}, K^+ \equiv u\bar{s}, K^0 \equiv d\bar{s}, K^0 \equiv s\bar{d}, K^{-} \equiv -s\bar{u} \). The \( B \) mesons, which are in the triplet or anti-triplet representation, are taken to be \( B^+ \equiv bu, B^0 \equiv bd, B_s \equiv bs, B^- \equiv -b\bar{u}, \overline{B}^0 \equiv bd \) and \( \overline{B}_s \equiv b\bar{s} \).

The primed and unprimed diagrams are not independent, but are related by CKM matrix elements. In particular, \( T'/T = C'/C = r_u \), where \( r_u \equiv V_{us}/V_{ud} \approx 0.23 \). Assuming that the penguin amplitudes are dominated by the top quark loop [13], one has \( P'/P = r_t \), with \( r_t \equiv V_{ts}/V_{td} \). We therefore have 13 decays described by 3 independent graphs, implying that there are 10 relations among the amplitudes. These can be expressed in terms of 6 amplitude equalities, 3
Table 1: The 13 decay amplitudes in terms of the 8 graphical combinations. The \( \sqrt{2}(B^+ \to \pi^+\pi^0) \) in the \(-(T+C)\) column means that \( A(B^+ \to \pi^+\pi^0) = -(T+C)/\sqrt{2} \), and similarly for other entries. Processes in the same column can be related by an amplitude equality, e.g. the amplitudes for \( B^+ \to K^+\pi^0 \) and \( B^0 \to K^0\pi^0 \) are equal.

| Relation | Combination | Amplitude | Amplitude |
|----------|-------------|-----------|-----------|
| \(- (T + C)\) | \(- (C - P)\) | \(- (T + P)\) | \(P\) |
| \(\sqrt{2}(B^+ \to \pi^+\pi^0)\) | \(\sqrt{2}(B^0 \to \pi^0\pi^0)\) | \(B^0 \to \pi^+\pi^-\) | \(B^+ \to K^+K^0\) |
| \(- (T' + C' + P')\) | \(- (C' - P')\) | \(- (T' + P')\) | \(P'\) |
| \(\sqrt{2}(B^+ \to \pi^0K^+)\) | \(\sqrt{2}(B^0 \to \pi^0K^0)\) | \(B^0 \to \pi^-K^+\) | \(B^+ \to \pi^+K^0\) |

The three independent triangle relations and one quadrangle relation are:

\[
\sqrt{2}A(B^+ \to \pi^+\pi^0) = \sqrt{2}A(B^0 \to \pi^0\pi^0) + A(B^0 \to \pi^+\pi^-),
\]

\[
\sqrt{2}A(B^+ \to \pi^+\pi^0) = \frac{1}{r_u}\sqrt{2}A(B^0 \to \pi^0K^0) + \frac{1}{r_u}A(B^0 \to \pi^-K^+),
\]

\[
\sqrt{2}A(B^+ \to \pi^+\pi^0) = \frac{1}{r_u}\sqrt{2}A(B^+ \to \pi^0K^+) + \frac{1}{r_u}A(B^+ \to \pi^+K^0),
\]

\[
A(B^0 \to \pi^-K^+) + A(B^+ \to \pi^+K^0) = r_u[A(B^0 \to \pi^+\pi^-) + A(B^+ \to K^+\bar{K}^0)].
\]

We have chosen to use only decays of \( B^0 \) or \( B^+ \) mesons in these relations – other decays (of \( B_s \), for example) can be substituted using amplitude equalities. Schematically, these four relations can be written as:

\[
(T + C) = (C - P) + (T + P)
\]

\[
(T + C) = \frac{(C' - P')}{r_u} + \frac{(T' + P')}{r_u}
\]

\[
(T + C) = \frac{(T' + C' + P')}{r_u} - \frac{(P')}{r_u}
\]

\[
(T' + P') - (P') = r_u(T + P) - r_u(P)
\]

**III. MEASURING WEAK AND STRONG PHASES**

The amplitude for \( B \to \pi^+\pi^0 \) decay, given by \(-T + C)/\sqrt{2} \), is pure \( \Delta I = 3/2 \). Hence \( T + C \) has only one term, which we denote by \( A_{1,2}e^{i\phi_2}e^{i\delta_2} \), and we write the triangle relations as:

\[
A_{1,2}e^{i\phi_2}e^{i\delta_2} = (A_Ce^{i\phi_C}e^{i\delta_C} - A_Pe^{i\phi_P}e^{i\delta_P}) + (A_Te^{i\phi_T}e^{i\delta_T} + A_Pe^{i\phi_P}e^{i\delta_P}),
\]

\[
(10)
\]
\[
A_{i=2} e^{i\phi_2} e^{i\delta_2} = (A_C e^{i\phi_C} e^{i\delta_C} - A_P e^{i\phi_P} e^{i\delta_P})/r_u + (A_T e^{i\phi_T} e^{i\delta_T} - A_P e^{i\phi_P} e^{i\delta_P})/r_u,
\]
where the \(\phi_i\) are the weak phases and the \(\delta_i\) are the strong phases. The \(\delta_i\) are chosen such that the quantities \(A_{i=2}, A_T, A_{T'}, A_C, A_{C'}, A_P\) and \(A_{P'}\) are real and positive. By SU(3) symmetry the strong phases for the primed graphs are the same as those for the unprimed ones. Working with the Wolfenstein parametrization [2] of the CKM matrix, it is easy to see that the weak phases of the various amplitudes are: \(\phi_2 = \phi_T = \phi_{T'} = \phi_C = \phi_{C'} = \gamma, \phi_P = -\beta, \) and \(\phi_{P'} = \pi\) (up to corrections of order \(\lambda^2 \approx 0.05\)). Also, \(A_{T'/r_u} = A_T\) and \(A_{C'/r_u} = A_C\). Finally, multiplying through on both sides by \(\exp(-i\gamma - i\delta_2)\), the 3 triangle relations become

\[
A_{i=2} = (A_C e^{i\Delta_C} + A_P e^{i\alpha} e^{i\Delta_P}) + (A_T e^{i\Delta_T} - A_P e^{i\alpha} e^{i\Delta_P}),
\]

\[
A_{i=2} = (A_C e^{i\Delta_C} + A_P e^{-i\gamma} e^{i\Delta_P}/r_u) + (A_T e^{i\Delta_T} - A_P e^{-i\gamma} e^{i\Delta_P}/r_u),
\]

\[
A_{i=2} = (A_T e^{i\Delta_T} + A_C e^{i\Delta_C} - A_P e^{-i\gamma} e^{i\Delta_P}/r_u) + A_P e^{-i\gamma} e^{i\Delta_P}/r_u),
\]

where we have defined \(\Delta_i \equiv \delta_i - \delta_2\). These three triangles and their charge conjugates can be used to determine all weak phases, all strong phase differences, and the sizes of the various diagrams.

Consider first the two triangle relations in Eqs. (14) and (15). These relations define two triangles which share a common base. Each triangle is determined up to a two-fold ambiguity, since it can be reflected about its base. Implicit in these two triangle relations is the relation

\[
A_{i=2} = |T + C| = A_T e^{i\Delta_T} + A_C e^{i\Delta_C}.
\]

Thus both of these triangles also share a common subtriangle with sides \(T + C, C\) and \(T\) as shown in Fig. 2(a). The key point is this: the subtriangle is completely determined, up to a four-fold ambiguity, by the two triangles in Eqs. (14) and (15). This is because both the magnitude and relative direction of \(P'/r_u\) are completely determined by constructing the triangle in Eq. (15). Therefore the point where the vectors \(C\) and \(T\) meet is given by drawing the vector \(P'/r_u\) from the vertex opposite the base [see Fig. 2(a)]. (A similar construction would have given the same point if we had used the vector \(T + P'/r_u\) instead of \(P'/r_u\).) Thus, if we measure the five rates for

\[
B^0 \to \pi^0 K^0 \ (\text{giving } |C - P'/r_u|),
\]

\[
B^0 \to \pi^- K^+ \ (\text{giving } |T + P'/r_u|),
\]

\[
B^+ \to \pi^0 K^+ \ (\text{giving } |T + C + P'/r_u|),
\]

\[
B^+ \to \pi^+ K^0 \ (\text{giving } |P'/r_u|),\text{ and}
\]

\[
B^+ \to \pi^+ \pi^0 \ (\text{giving } |T + C| = A_{i=2}, \ i.e. \ the \ triangle’s \ base),
\]

we can determine \(\Delta_P - \gamma, |T|\) and \(|C|\), up to a two-fold ambiguity and \(\Delta_C\) and \(\Delta_T\).
up to a four-fold ambiguity. As we will discuss later, these discrete ambiguities can be at least partially removed through the knowledge of the relative magnitudes of $|P|$, $|C|$, $|T|$ and $|P'|$, and through independent measurements of the amplitudes and the strong and weak phases.

If we also measure the rates for the CP-conjugate processes of the above decays, we can get more information. These CP-conjugate decays obey similar triangle relations to those in Eqs. (14) and (15). However, recall that under CP conjugation, the weak phases change sign, but strong phases do not. Thus we can perform an identical analysis with the CP-conjugate processes, giving us another, independent determination of $|T|$, $|C|$, $\Delta C$ and $\Delta T$. But, instead of $\Delta P - \gamma$, this time we get $\Delta P + \gamma$. Thus we obtain $\Delta P$ and $\gamma$ separately. Note that it is not, in fact, necessary to measure all 5 CP-conjugate processes. The rate for $B^+ \to \pi^+ \pi^0$ is the same as that for $B^+ \to \pi^+ \pi^0$, since they involve a single weak phase and a single strong phase. Similarly, the rates for $B^+ \to \pi^+ K^0$ and $B^- \to \pi^- K^0$ are equal. Therefore, in order to extract $\gamma$, in addition to the above 5 rates, we need only measure

$$
\overline{B}^0 \to \pi^0 \overline{K}^0 \quad \text{(giving } |\overline{C} - \overline{P}'/r_u|),
$$

$$
\overline{B}^0 \to \pi^+ K^- \quad \text{(giving } |\overline{T} + \overline{P}'/r_u|), \text{ and}
$$

$$
B^- \to \pi^0 K^- \quad \text{(giving } |\overline{T} + \overline{C} + \overline{P}'/r_u|).
$$

To sum up, by measuring the above 8 rates, the following quantities can be obtained: the weak phase $\gamma$, the strong phase differences $\Delta T$, $\Delta C$ and $\Delta P$, and the magnitudes of the different amplitudes $|T|$, $|C|$ and $|P'|$.

A few comments are worth making regarding the above two-triangle construction. First, note that all measurements are time-independent, and that no observation of CP violation is required to obtain the various quantities. This construction extends that of Ref. [11], in which it is observed that the measurement of the sides of the triangle in Eq. (15) and its CP-conjugate can be used to obtain the weak angle $\gamma$. The second point is that most of the decays are self-tagging, the only exceptions being $B^0 \to \pi^0 K^0$ and $\overline{B}^0 \to \pi^0 \overline{K}^0$. However, even at a symmetric $e^+e^-$ machine operating at the $\Upsilon(4S)$, where it is not possible to tag individual $B$’s, these two rates can still be obtained. By measuring the two time-integrated rates for the $B_d^0 \overline{B}_d^0$ pair to decay to the final state $\pi^0 K_S$ plus a semileptonic tag $[(D\ell\nu X)_{\text{tag}}$ or $(\overline{D}\ell\nu X)_{\text{tag}}]$, the two rates can be extracted. Finally, if time-dependent measurements are possible, one can independently measure $\alpha$ through CP violation in neutral $B$ decays to $\pi^0 K_S$. [7]

Now consider the triangle relations in Eqs. (13) and (14). These two triangles share a common base with each other and also with the sub-triangle in Eq. (16) (which still holds). Unlike the previous two-triangle construction, however, the shape of the sub-triangle is not yet fixed. Nevertheless, the point where the vectors $C$ and $T$ meet can still be determined [see Fig. 2(b)]. This point is connected to the apex of the triangle in Eq. (13) by the vector $P$, and to the apex
of the triangle in Eq. (14) by the vector $P'$. Since the magnitudes of the penguin diagrams $P$ and $P'$ are measured by the rates of $B^+ \rightarrow K^+K^0$ and $B^+ \rightarrow \pi^+K^0$, respectively, the meeting point of $C$ and $T$ is determined by the intersection of the two circles of Fig. 2(b). Thus, the sub-triangle is completely determined up to an eight-fold ambiguity. This eight-fold ambiguity correspond to the two possible intersections of the circles, in addition to the 2 two-fold ambiguities caused by reflecting each triangle about its base. Thus by measuring the 7 rates

$$
\begin{align*}
B^+ &\rightarrow K^+K^0 \ (\text{giving } |P|), \\
B^+ &\rightarrow \pi^+K^0 \ (\text{giving } |P'|), \\
B^0 &\rightarrow \pi^0\pi^0 \ (\text{giving } |C - P|), \\
B^0 &\rightarrow \pi^+\pi^- \ (\text{giving } |T + P|), \\
B^0 &\rightarrow \pi^0K^0 \ (\text{giving } |C - P'/r_u|), \\
B^0 &\rightarrow \pi^-K^+ \ (\text{giving } |T + P'/r_u|), \text{ and} \\
B^+ &\rightarrow \pi^+\pi^0 \ (\text{giving } |T + C|),
\end{align*}
$$

we can extract $\Delta_P + \alpha$, $\Delta_P - \gamma$, $\Delta_C$, and $\Delta_T$, up to an eight-fold ambiguity, and $|T|$ and $|C|$ up to a four-fold ambiguity. Through the two quantities $\Delta_P + \alpha$ and $\Delta_P - \gamma$, we can then determine the weak phase $\beta$ (using $\beta = \pi - \alpha - \gamma$), up to discrete ambiguities. As in the first two-triangle construction, all rates are time-independent. What is surprising, perhaps, about this particular construction is that it is not even necessary to measure the CP-conjugate rates in order to obtain $\beta$. The reason is that SU(3) flavor symmetry implies the equality of the strong final-state phases of two different amplitudes, in this case $P$ and $P'$. Subtracting the (strong plus weak) phase of one amplitude from the other then determines a weak phase. Usually, in a given process, without measuring the charge-conjugate rate one can only measure the sum of a weak and a strong phase.

If the CP-conjugate rates are also measured, we can obtain $\Delta_P$, $\alpha$, and $\gamma$ separately. This provides another, independent determination of $|T|$, $|C|$, $\Delta_C$, and $\Delta_T$. As in the first construction, no observation of CP violation is necessary to make such measurements. Again, it is not necessary to measure all the CP-conjugate rates -- only the following four can be different from their counterparts:

$$
\begin{align*}
\bar{B}^0 &\rightarrow \pi^0\pi^0 \ (\text{giving } |\bar{C} - \bar{P}|), \\
\bar{B}^0 &\rightarrow \pi^+\pi^- \ (\text{giving } |\bar{T} + \bar{P}|), \\
\bar{B}^0 &\rightarrow \pi^0K^0 \ (\text{giving } |\bar{C} - \bar{P}'/r_u|), \text{ and} \\
\bar{B}^0 &\rightarrow \pi^+K^- \ (\text{giving } |\bar{T} + \bar{P}'/r_u|).
\end{align*}
$$

If time-dependent measurements can be performed, then mixing-induced CP violation can be seen in neutral $B$ decays to $\pi^+\pi^-$ or $\pi^0K_S$. The measurement of such CP violation, combined with the measurements of the penguin diagram, will
give another, independent value for the angle \( \alpha \), just as in the isospin analysis \( \mathbf{1} \). Finally, the third two-triangle construction uses the triangle relations in Eqs. \( \mathbf{13} \) and \( \mathbf{13} \). This construction is almost identical to the previous one. The only difference is that the decay \( B^0 \to \pi^0 K^0 \) (giving \( |C - P'/ru| \)) is replaced by the decay \( B^+ \to \pi^0 K^+ \) (giving \( |T + C + P'/ru| \)). (This is experimentally preferable since \( B^+ \) decays are self-tagging.) Therefore, while the radius of one of the circles is still \( |P| \) as in Fig. 2(b), the radius of the other circle is given by \( |T + P'/ru| \). Although this construction does not provide new information, it can nevertheless be used as an independent measurement of the weak phases, the strong phase differences, and the size of the various diagrams.

An interesting feature of the last two constructions is that the quadrilateral symbolizing the quadrangle relation of Eq. \( \mathbf{1} \) is contained in the figure. This might seem to imply that it is not an independent relation. In fact, this is not so. What has happened is that the amplitude relation of Eq. \( \mathbf{9} \) has been implicitly replaced by the triangle relation Eq. \( \mathbf{16} \), which is a relation between an amplitude and two graphs.

The three constructions use \( B \) decays to \( \pi \pi, \pi K \) and \( K \bar{K} \) final states. At present, the decays \( B^0 \to \pi^+ \pi^- \) and/or \( \pi^- K^+ \) have been observed, but the two final states cannot be distinguished \( \mathbf{8} \). The combined branching ratio is about \( 2 \times 10^{-5} \). Assuming equal rates for \( \pi^+ \pi^- \) and \( \pi^- K^+ \), which seems likely, the amplitudes \( |T| \) and \( |P'| \) should be about the same size. On the other hand, the amplitude \( |C| \) is expected to be about a factor of 5 smaller: the amplitudes \( |T| \) and \( |C| \) are basically the same as \( |a_1| \) and \( |a_2| \), respectively, introduced in Ref. \( \mathbf{15} \), for which the values \( |a_1| = 1.11 \) and \( |a_2| = 0.21 \) have been found \( \mathbf{16} \). The ratio \( |P|/|T| \) has also been estimated to be small, \( \lesssim 0.20 \). Therefore all the decays used in these constructions should have branching ratios of the order of \( 10^{-5} \), with the exception of \( B \to K \bar{K} \) (\( P \)) and \( B^0 \to \pi^0 \pi^0 \) (\( \sim (C - P) \)), which are probably an order of magnitude smaller.

The knowledge that the amplitudes obey the hierarchy \( |P|, |C| < |T| < |P'/ru| \) will also help in reducing discrete ambiguities. For example, in the first two-triangle construction [Fig. 2(a)], we noted in the discussion following Eq. \( \mathbf{10} \) that the subtriangle can be determined up to a four-fold ambiguity. However, two of these four solutions imply that \( |C| \) and \( |T| \) are both of order \( |P'/ru| \), which violates the above hierarchy. Thus the four-fold ambiguity in the determination of the subtriangle is reduced to a two-fold ambiguity, and the discrete ambiguities in the determination of subsequent quantities such as \( \Delta_P - \gamma, \Delta_C \), etc., are likewise reduced. The ambiguities in the other two constructions can be partially removed in a similar way.

All three two-triangle constructions described above rely on two assumptions. The first is that the diagrams \( A, E \) and \( PA \) (and their primed counterparts) can be neglected. This can be tested experimentally. The decays \( B^0 \to K^+ K^- \) and \( B_s \to \pi^+ \pi^- \) can occur only through the diagrams \( E \) and \( PA \), and \( E' \) and \( PA' \),
respectively. Therefore, if the above assumption is correct, the rates for these two decays should be much smaller than the rates for the decays in Table 1.

The second assumption is that of an unbroken SU(3) symmetry. We know, however, that SU(3) is in fact broken in nature. Assuming factorization, SU(3)-breaking effects can be taken into account by including the meson decay constants $f_\pi$ and $f_K$ in the relations between $B \to \pi\pi$ decays and $B \to \pi K$ decays [10]. In other words, the factor $r_u$ which appears in two of the triangle relations should be multiplied by $f_K/f_\pi \approx 1.2$. One way to test whether this properly accounts for all SU(3)-breaking effects is through the rate equalities in Table 1. Even if it turns out that $f_K/f_\pi$ does not take into account all SU(3)-breaking effects, the large number of independent measurements is likely to help in reducing uncertainties due to SU(3) breaking. For example, note that, not counting the CP-conjugate processes, the last two constructions have six of their seven rates in common. This means that a measurement of only eight decay rates gives two independent measurements of $|T|$, $|C|$, $\Delta_C$, $\Delta_T$, $\Delta_P - \gamma$ and $\Delta_P + \alpha$. In fact, these eight rates already contain the five rates of the first construction [Fig. 2(a)]. Thus we actually have three independent ways of arriving at $|T|$, $|C|$, $\Delta_C$, $\Delta_T$ and $\Delta_P - \gamma$. Including also the CP-conjugate processes, we have a total of 13 $B$-decay rate measurements which give us six independent ways to measure $|T|$, $|C|$, $\Delta_C$ and $\Delta_T$, five ways to measure $\Delta_P$, three independent ways to measure $\gamma$, and two ways to measure $\alpha$. (If time-dependent measurements are possible, there are additional independent ways to measure $\alpha$.) The point is that the three two-triangle constructions include many ways to measure the same quantity. This redundancy provides a powerful way to test the validity of our SU(3) analysis and reduces the discrete ambiguities in the determination of the various quantities.

IV. CONCLUSIONS

We have presented an analysis based on three triangle relations involving $B$ decay amplitudes to $\pi\pi$, $\pi K$ and $K\bar{K}$ which are a consequence of the SU(3) flavor symmetry of the strong interactions and the smallness of annihilation-like diagrams. These relations permit the extraction of the weak phases $\alpha$ and $\gamma$ even if penguin contributions are substantial. In addition, one obtains the strong phase differences and the size of the individual diagrams. No time-dependent measurements are needed, nor is it necessary to observe CP violation to determine these quantities. If time-dependent measurements are possible, additional, independent measurements of $\alpha$ can also be made. Most branching ratios are expected to be of the order of $10^{-5}$, although a few could be an order of magnitude smaller. Interestingly, in some cases our method provides a measurement of CP-violating weak phases without necessarily measuring CP-conjugate processes. All quantities are measured up to certain discrete ambiguities. However, the method includes many independent measurements of the same quantities, which can be used to considerably reduce the discrete ambiguities. Furthermore, this redundancy is likely to be of great help in evaluating the size of SU(3)-breaking effects.
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References

[1] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[2] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[3] For a review, see, e.g., Y. Nir and H. Quinn, Ann. Rev. Nucl. Part. Sci. 42 (1992) 211.

[4] D. London and R. Peccei, Phys. Lett. B 223 (1989) 257; M. Gronau, Phys. Rev. Lett. 63 (1989) 1451; B. Grinstein, Phys. Lett. B 229 (1989) 280.

[5] M. Gronau, Phys. Lett. B 300 (1993) 163.

[6] M. Gronau and D. London, Phys. Rev. Lett. 65 (1990) 3381.

[7] Y. Nir and H. R. Quinn, Phys. Rev. Lett. 67 (1991) 541; M. Gronau, Phys. Lett. B 265 (1991) 389; H. J. Lipkin, Y. Nir, H. R. Quinn and A. E. Snyder, Phys. Rev. D 44 (1991) 1454; L. Lavoura, Mod. Phys. Lett. A 17 (1992) 1553.

[8] D. Zeppenfeld, Z. Phys. C 8 (1981) 77.

[9] M. Savage and M. Wise, Phys. Rev. D 39 (1989) 3346; ibid. 40, 3127(E) (1989); L. L. Chau et al., Phys. Rev. D 43 (1991) 2176.

[10] J. Silva and L. Wolfenstein, Phys. Rev. D 49 (1994) R1151.

[11] M. Gronau, J. L. Rosner, and D. London, Technion preprint TECHNION-PH-94-7, March, 1994, submitted to Phys. Rev. Letters.

[12] M. Gronau, O. F. Hernández, D. London and J. L. Rosner, Technion preprint TECHNION-PH-94-8, April, 1994, submitted to Phys. Rev. D.

[13] T. Inami and C. S. Lim, Prog. Theor. Phys. 65 (1981) 297, 1772(E); G. Eilam and N. G. Deshpande, Phys. Rev. D 26 (1982) 2463.

[14] M. Battle et al. (CLEO Collaboration), Phys. Rev. Lett. 71 (1993) 3922.

[15] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 34 (1987) 103.

[16] M. Neubert et al., in Heavy Flavours, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1992), p. 286.
FIGURE CAPTIONS

FIG. 1. Diagrams describing decays of $B$ mesons to pairs of light pseudoscalar mesons. Here $\bar{q} = \bar{d}$ for unprimed amplitudes and $\bar{s}$ for primed amplitudes. (a) “Tree” (color-favored) amplitude $T$ or $T'$; (b) “Color-suppressed” amplitude $C$ or $C'$; (c) “Penguin” amplitude $P$ or $P'$ (we do not show intermediate quarks and gluons); (d) “Exchange” amplitude $E$ or $E'$; (e) “Annihilation” amplitude $A$ or $A'$; (f) “Penguin annihilation” amplitude $PA$ or $PA'$.

Fig. 2. Triangle relations used to obtain weak phases and strong final-state phase shift differences. The black dot corresponds to the solution for the vertex of the triangle in Eq. (14). (a) Relation based on Eqs. (14) (upper triangle) and (15) (lower triangle). (b) Relation based on Eqs. (13) (lower triangle with small circle about its vertex) and (14) (upper triangle with large circle about its vertex). The relation based on (13) and (15) follows an almost identical construction. One possible set of decay processes which can be used to construct these triangles is given in Eqs. (2), (3) and (4).
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