Abstract

We investigate the pure penguin decays $B \to \pi \phi$ in the Standard Model (SM) and in the Constrained Minimal Supersymmetric Standard Model (CMSSM) using the QCD factorization approach and consider the Sudakov effects in the twist-3 contribution. We find $\text{Br}(B^- \to \pi^- \phi) = (1.95 - 5.70) \times 10^{-9}$ in SM and $(1.1 - 2.4) \times 10^{-8}$ in CMSSM with large $\tan \beta$ which is about one order of magnitude larger than that in SM.

PACS numbers: 13.25.Hw, 12.38.Bx
1 Introduction

One of charmless two-body nonleptonic decays of B mesons, the process $B \to \phi\pi$, is interesting because it is a pure penguin process and, in particular, there are no annihilation diagram contributions the importance of which is still in dispute $[1, 2]$. It is sensitive to new physics due to all contributions arising from the penguin diagrams. The calculation of the hadronic matrix element relevant to the process is relatively reliable because of no contributions coming from diagrams of annihilation topology. Therefore, we shall investigate the process in both SM and MSSM.

The study of exclusive processes with large momentum transfer in the perturbative QCD (PQCD) has been extensively carried out and it is shown that the application of PQCD to them is successful $[3]$. The key point to apply PQCD is to prove that the factorization, the separation of the short-distance dynamics and long-distance dynamics, can be performed for those processes. Recently, two groups, Li et al. $[4, 1]$ and BBNS $[2]$, have made significant progress in calculating hadronic matrix elements of local operators relevant to charmless two-body nonleptonic decays of B mesons in the PQCD framework. In the letter we shall use BBNS’s method (QCD factorization) to calculate the hadronic matrix element of operators relevant to the decay $B \to \phi\pi$.

The decay $B \to \phi\pi$ has been studied by several people $[5, 6, 7, 8, 9]$. The naive factorization or BSW model $[10]$ is used in calculating the hadronic matrix elements in Refs. $[5, 6]$. The modified perturbative QCD approach $[4]$, instead of using BSW model, is used in Ref. $[7]$. In Ref. $[8]$, the QCD improved factorization (simply, QCD factorization)$[2]$ is used and only the leading twist contribution is included. The numerical result of the branching ratio (Br) in SM given in Ref. $[8]$ is about an order of magnitude larger than that in Refs. $[5, 6]$. That is, ”non-factorizable” contributions (including vertex, penguin, and hard spectator scattering corrections), which are the $O(\alpha_s)$ corrections to the leading order result, dominate over the leading order result. In Ref. $[9]$ the same QCD factorization is used and the twist-3 contributions are included. However, the Br given in Ref. $[9]$ is $(3-8) \times 10^{-10}$ which is of the same order of or even smaller than that in Refs. $[5, 6]$. Considering these disagreements, it is necessary to do a calculation of Br in SM using the QCD factorization. We carry out such a calculation in SM first and then in Constrained MSSM. The difference between our calculation and that in $[9]$ is how to calculate the twist-3 contributions. The authors in Ref. $[9]$ follow BBNS’s approach, i.e., to introduce a phenomenological parameter instead of the integral containing end point singularity $[11]$. Indeed, the solution of the end point singularity in investigating form factors of mesons is known for a long time $[12]$. That is, to retain quarks’ transverse momenta in both the hard scattering kernels and distribution amplitudes of mesons and to include the Sudakov suppression $[13]$ make the integral convergent and computable. Thus, there is no any phenomenological parameter introduced. In the letter we shall use the method to calculate the twist-3 effects. Our numerical result of the Br($B^\pm \to \pi\phi$) in SM using QCD factorization approach(QCDF) is Br($B^- \to \pi^-\phi$) = (1.95 – 5.70) $\times 10^{-9}$. We have also calculated the Br in Constrained MSSM in order to see supersymmetric (SUSY) effects on the decay. The numerical result in Constrained MSSM with large
\[ \tan \beta = \text{Br}(B^- \to \pi^- \phi) = (1.1 - 2.4) \times 10^{-8} \text{ depending on the choice of some relevant parameters.} \]

The \( \Delta B = 1 \) effective weak Hamiltonian in SM is given by

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{\mu=\tau} \lambda_{\mu} \left( C_1 Q_1^\mu + C_2 Q_2^\mu + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}, \tag{1}
\]

where \( \lambda_{\mu} = V_{\mu b} V_{\mu d}^* \), \( Q_{1,2}^\mu \) are the left-handed current-current operators arising from \( W^- \) boson exchange, \( Q_{3,6,7} \) and \( Q_{7,10} \) are QCD and electroweak penguin operators, and \( Q_{7\gamma} \) and \( Q_{8g} \) are the electromagnetic and chromomagnetic dipole operators, respectively. Their explicit expressions can be found in, e.g., Ref. \[11\]. Follow BBNS approach \[2\], the hadronic matrix elements of local operators \( Q_i \) at the leading order of the heavy quark expansion can be written as

\[
\langle \pi(p) | \phi(q) | Q_i | B(p) \rangle = F_0^{B \to \pi}(q^2) \int_0^1 dv T^I(v) \Phi_{\phi}(v) \\
+ \int_0^1 d\xi dudv T^{II}(\xi, u, v) \Phi_{B}(\xi) \Phi_{\pi}(u) \Phi_{\phi}(v) \tag{2}
\]

where \( \rho_M \) (\( M = \phi, \pi, B \)) are light-cone distribution amplitudes of the meson \( M \), \( T^I_i \) and \( T^{II}_i \) are hard scattering kernels.

2 \( \alpha_s \) order corrections of hadronic matrix elements

The \( \alpha_s \) order hard scattering kernels in Eq.(4) can be obtained by calculating the diagrams in Fig.(1). Substituting the kernels into Eq.(4), we get

\[
\langle \pi | \phi | Q_i | B \rangle_{\alpha_s \text{order}} = \frac{\alpha_s C_F}{4\pi N} F_i \langle \pi | \phi | Q_{i-1} | B \rangle_{\text{tree}} \tag{3}
\]

where \( F_i = F \) for \( i = 4, 10 \), \(-F-12\) for \( i = 6, 8 \) and 0 otherwise with

\[
F = -12 \ln \frac{\mu}{m_b} - 18 + f^I_\phi + f^{II}_\phi. \tag{4}
\]

In Eq.(4)

\[
f^I_\phi = \int_0^1 dx g(x) \Phi_\phi(x), \tag{5}
\]

\[
g(x) = 3 \frac{1 - 2x}{1 - x} \ln x - i3\pi \tag{6}
\]

is the contribution from the diagrams (a)-(d), the vertex corrections, and \( f^{II}_\phi \) presents the contribution from the hard spectator scattering diagrams (e) and (f) which is of real
Figure 1: Order $\alpha_s$ corrections to the hard-scattering kernels $T^I$ and $T^{II}$. 

non-factorization contribution. If we ignore the transverse momenta of partons, like that in Eq.(6),

$$f^{II}_\phi \propto \int d\xi dudv \left[ \frac{\phi_B(\xi)}{\xi} \frac{\phi_\pi(u)}{u} \frac{\phi_\parallel(v)}{v} + \frac{2\mu_\pi}{m_B} \frac{\phi_B(\xi)}{\xi} \frac{\phi_\pi(u)}{u} \frac{\phi_\parallel(v)}{v} \right],$$

where the distribution amplitudes of $\pi, \phi, B$ mesons can be found in Refs. [17, 18, 19]. When $u \to 0$, the twist-3 contribution, the next term of Eq.(6), will lead to divergence due to the end-point singularity. Therefore, it is necessary to consider the transverse momentum $k_T$ effect and include the Sudakov suppression factors to eliminate the end-point singularity [13]. After including the transverse momenta of partons, Eq.(7) changes into

$$f^{II}_\phi \propto \int d\xi dudv d^2k_T d^2k_1 d^2k_2 \left[ \frac{-um_B^2 \phi_B(\xi) \phi_\pi(u)}{[\xi um_B^2 + (k_T - k_{1T})^2][-uvm_B^2 + (k_T - k_{1T} + k_{2T})^2]} \right]$$
in the end-point region. By a straightforward calculating, we obtain the hard spectator scattering contribution

\[
mm^2 f \frac{m_B^2}{2m_B^2 + (k_T - k_{1T}^2)^2} f_B(\xi) \frac{\phi(a)}{\phi(0)}(v) \left[ \xi u m_B^2 + (k_T - k_{1T} + k_{2T})^2 \right].
\]

(8)

The $k_T$ resummation of large logarithmic corrections to $B, \phi$ and $\pi$ meson distribution amplitudes leads to the presence of the exponentials $S_B, S_\phi$ and $S_\pi$ respectively [20].

\[
S_B(t) = \exp \left[-s(\xi t, b) - 2 \int_{1/b}^t \frac{d\mu}{\mu} \gamma(\alpha_s(\mu^2)) \right],
\]

\[
S_\phi(t) = \exp \left[-s(v t, b_2) - s((1 - v) t, b_2) - 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma(\alpha_s(\mu^2)) \right],
\]

\[
S_\pi(t) = \exp \left[-s(u t, b_1) - s((1 - u) t, b_1) - 2 \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma(\alpha_s(\mu^2)) \right],
\]

(9)

with the quark anomalous dimension $\gamma = -\alpha_s/\pi$. The variables $b, b_1,$ and $b_2$, corresponding to the parton transverse momenta $k_T, k_{1T},$ and $k_{2T}$ respectively, represent the transverse extents of the $B, \pi$ and $\phi$ mesons, respectively. The expression for the exponent $s$ is referred to [21, 24, 23]. The above Sudakov exponentials decrease so fast in the large $b$ region that the $B \to \pi\phi$ hard amplitudes remain sufficiently perturbative in the end-point region. By a straightforward calculating, we obtain the hard spectator scattering contribution

\[
f_{\phi\pi}^{H} = \frac{4\pi^2}{N} \frac{f_\pi f_B}{F_+^{B\to\pi}(m_\phi^2)} \int d\xi dv \int bdb_2 db_2 \times \left\{ -um_B^4 P_B(\xi, b)P_\pi(u, b)P(\xi, b_2)K_0(-i\sqrt{uv}m_Bb_2)
\right.
\]

\[
\times \left[ \theta(b_2 - b)I_0(\sqrt{\xi u m_B b_2})K_0(\sqrt{\xi u m_B b_2}) + \theta(b - b_2)I_0(\sqrt{\xi u m_B b_2})K_0(\sqrt{\xi u m_B b_2}) \right]
\]

\[
-2uvu m_B^5 P_B(\xi, b)P_\pi(u, b)P(\xi, b_2) \frac{b_2}{6K_0(-i\sqrt{uv}m_Bb_2)}
\]

\[
\times \left[ \theta(b_2 - b)I_0(\sqrt{\xi u m_B b_2})K_0(\sqrt{\xi u m_B b_2}) + \theta(b - b_2)I_0(\sqrt{\xi u m_B b_2})K_0(\sqrt{\xi u m_B b_2}) \right] \}
\]

(10)

where $f_\pi (f_B)$ is the pion ($B$) meson decay constant, $m_B$ the $B$ meson mass, $F_+^{B\to\pi}(m_\phi^2)$ the $B \to \pi$ form factor at the momentum transfer $m_\phi^2$, $\xi$ the light-cone momentum fraction of the spectator in the $B$ meson, $K_i, I_i, \phi_i$ are modified Bessel functions of order $i$ and $P_B, P_\pi, P_\phi$ are $B, \pi, \phi$ corrected meson amplitudes with the exponentials $S_B, S_\phi$ and $S_\pi$ respectively [20]. As noted in Refs. [13], although the twist-3 contribution is power suppressed it is numerically comparable with the twist-2 contribution due to the chirally-enhanced factor $m_B^2(\mu)/m_B(\mu)[\hat{m}_u(\mu) + \hat{m}_d(\mu)]$. From Eq. (10), we obtain that the twist-3 contribution to $f_{H\pi}$ is numerically about the fourth of the twist-2 contribution to $f_{H\pi}$. It is worth to note that the numerical result of the twist-2 contribution to $f_{H\pi}$ obtained from Eq. (10) is almost completely the same as that obtained without including the Sudakov factor, as expected.
3 The branching ratio in SM

The effective Hamiltonian (1) results in the following matrix element for the decay

$$\langle \pi \phi | H_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle \pi \phi | T_p | B \rangle ,$$  \hspace{1cm} (11)

where

$$T_p = a_3(\pi \phi) (\bar{d} \bar{b})_{V-A} \otimes (\bar{s} s)_{V-A} + a_5(\pi \phi) (\bar{d} \bar{b})_{V-A} \otimes (\bar{s} s)_{V+A} + a_7(\pi \phi) (\bar{d} \bar{b})_{V-A} \otimes \frac{3}{2} \epsilon_s (\bar{s} s)_{V+A} + a_9(\pi \phi) (\bar{d} \bar{b})_{V-A} \otimes \frac{3}{2} \epsilon_s (\bar{s} s)_{V-A} .$$  \hspace{1cm} (12)

The symbol $\otimes$ indicates that the matrix elements of the operators in $T_p$ are to be evaluated in the factorized form $\langle \pi \phi | j_1 \otimes j_2 | B \rangle \equiv \langle \pi | j_1 | B \rangle \langle \phi | j_2 | 0 \rangle$. The $O(\alpha_s)$ corrections, including the nonfactorizable corrections corresponding to the diagrams (e) and (f) in Fig. 1, of hadronic matrix elements are, by definition, included in the coefficients $a_i$. Collecting the results in the above section, we have

$$a_3(\pi \phi) = C_3 + \frac{1}{N} C_4 + \frac{\alpha_s}{4\pi} C_F \frac{NF_4}{N} ,$$  \hspace{1cm} (13)

$$a_5(\pi \phi) = C_5 + \frac{1}{N} C_6 + \frac{\alpha_s}{4\pi} C_F \frac{NF_6}{N} (-F - 12) ,$$  \hspace{1cm} (14)

$$a_7^p(\pi \phi) = C_7 + \frac{1}{N} C_8 + \frac{\alpha_s}{4\pi} C_F \frac{NF_8}{N} (-F - 12) + \frac{\alpha_{em}}{9\pi} P_{em}^p (C_1 + 3C_2) ,$$  \hspace{1cm} (15)

$$a_9^p(\pi \phi) = C_9 + \frac{1}{N} C_{10} + \frac{\alpha_s}{4\pi} C_F \frac{NF_{10}}{N} C_10 F + \frac{\alpha_{em}}{9\pi} P_{em}^p (C_1 + 3C_2) ,$$  \hspace{1cm} (16)

which has form of $A + B \alpha_s$. In order to keep our calculation consistent, we use LO Wilson coefficients which contribute to $B$ and NLO Wilson coefficients which contribute to $A$. In Eqs. (13,14) $C_F = (N^2 - 1)/(2N)$ ( $N = 3$ is the number of colors ), and $P_{em}^p$ arises from electroweak penguin contributions, Fig. 1(g), and is given by

$$P_{em}^p = \frac{10}{9} - 4 \int_0^1 duu(1-u) \ln \left( \frac{m^2 - q^2 u(1-u)}{\mu^2} \right) .$$  \hspace{1cm} (17)

From Eq.(11), the decay amplitude for $B^- \to \pi^- \phi$ is

$$A(B^- \to \pi^- \phi) = \sqrt{2} A(B^0 \to \pi^0 \phi) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[ a_3 + a_5 - \frac{1}{2} (a_7^p + a_9^p) \right] f_\phi m_\phi F^B_{+ \to \pi} \left( m_\phi^2 \right) \frac{2}{2} \epsilon_\phi \cdot p_B ,$$  \hspace{1cm} (18)
The relevant Wilson coefficients in NDR scheme are showed in Table 1. For the other parameters, we use

\[ f_B = 0.190 \text{ GeV}, \quad f_\pi = 0.131 \text{ GeV}, \]
\[ f_\phi = 0.237 \text{ GeV}, \quad f_\phi^T = 0.215 \text{ GeV}, \quad P_{B \to \pi}^{B} (m_\phi^2) = 0.30 \] (19)

and Wolfensein parameters fitted by Ciuchini as \[ A = 0.819, \quad \lambda = 0.224, \]
\[ \bar{\rho} = \rho (1 - \lambda^2/2) = 0.224, \quad \bar{\eta} = \eta (1 - \lambda^2/2) = 0.317. \] (20)

The numerical results of the branch ratio at different scales are showed in Table 2.

### Table 1: Wilson coefficients \( C_i \) in the NDR scheme. Input parameters are \( \Lambda^{(5)}_{MS} = 0.225 \text{ GeV}, \ m_t(m_t) = 167 \text{ GeV}, \ m_b(m_b) = 4.2 \text{ GeV}, \ M_W = 80.4 \text{ GeV}, \alpha = \frac{1}{129}, \) and \( \sin^2 \theta_W = 0.23. \)

| NLO     | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) | \( C_6 \) |
|---------|----------|----------|----------|----------|----------|----------|
| \( \mu = m_b/2 \) | 1.137    | -0.295   | 0.021    | -0.051   | 0.010    | -0.065   |
| \( \mu = m_b \)   | 1.081    | -0.190   | 0.014    | -0.036   | 0.009    | -0.042   |
| \( \mu = 2m_b \)  | 1.045    | -0.113   | 0.009    | -0.025   | 0.007    | -0.027   |
| NLO     | \( C_7/\alpha \) | \( C_8/\alpha \) | \( C_9/\alpha \) | \( C_{10}/\alpha \) | \( C_{eff}^{7\gamma} \) | \( C_{eff}^{8g} \) |
| \( \mu = m_b/2 \) | -0.024   | 0.096    | -1.325   | 0.331    |         |         |
| \( \mu = m_b \)   | -0.011   | 0.060    | -1.254   | 0.223    |         |         |
| \( \mu = 2m_b \)  | 0.011    | 0.039    | -1.195   | 0.144    |         |         |

| LO      | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) | \( C_6 \) |
|---------|----------|----------|----------|----------|----------|----------|
| \( \mu = m_b/2 \) | 1.185    | -0.387   | 0.018    | -0.038   | 0.010    | -0.053   |
| \( \mu = m_b \)   | 1.117    | -0.268   | 0.012    | -0.027   | 0.008    | -0.034   |
| \( \mu = 2m_b \)  | 1.074    | -0.181   | 0.008    | -0.019   | 0.006    | -0.022   |
| NLO     | \( C_7/\alpha \) | \( C_8/\alpha \) | \( C_9/\alpha \) | \( C_{10}/\alpha \) | \( C_{eff}^{7\gamma} \) | \( C_{eff}^{8g} \) |
| \( \mu = m_b/2 \) | -0.012   | 0.045    | -1.358   | 0.418    | -0.364   | -0.169   |
| \( \mu = m_b \)   | -0.001   | 0.029    | -1.276   | 0.288    | -0.318   | -0.151   |
| \( \mu = 2m_b \)  | 0.018    | 0.019    | -1.212   | 0.193    | -0.281   | -0.136   |

### 4 Br in Constrained MSSM

It has been shown that the neutral Higgs bosons (NHBs) do make significant contributions to leptonic and semileptonic rare B decays in Constrained MSSM with large \( \tan \beta \) [25, 26, 27]. For \( b \to d s \bar{s} \), it is expected that the similar enhancement of \( Br \) will happen since the mass of the strange quark is the same order as or a little of larger than
Table 2: Branch ratios of $B^- \to \pi^- \phi$ at scale $m_b/2$, $m_b$ and $2m_b$.

|        | $\mu = m_b/2$ |        | $\mu = m_b$ |        | $\mu = 2m_b$ |
|--------|----------------|--------|-------------|--------|-------------|
|        | NF  | QCDF | NF  | QCDF | NF  | QCDF |
| Br in SM | 2.47 $\times 10^{-9}$ | 1.95 $\times 10^{-9}$ | 6.53 $\times 10^{-10}$ | 4.51 $\times 10^{-9}$ | 2.52 $\times 10^{-9}$ | 5.70 $\times 10^{-9}$ |
| Br in CMSSM | 2.4 $\times 10^{-8}$ | 6.53 $\times 10^{-10}$ | 1.6 $\times 10^{-8}$ |        | 1.1 $\times 10^{-8}$ |        |

that of muon. In the section we will calculate the Br of $B \to \pi \phi$ in the large tan $\beta$ case of the Constrained MSSM.

In addition to Eq.(1), we have [24, 25]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}(-\lambda_t) \sum_{i=11,\ldots,16} C_i Q_i + \text{h.c.}, \quad (21)$$

where $Q_{11}$ to $Q_{16}$, the neutral Higgs penguins operators, are given by

$$Q_{11} = (\bar{d}b)_{S+P} \sum_q (\bar{q}q)_{S-}, \quad Q_{12} = (\bar{d}_ib_j)_{S+P} \sum_q (\bar{q_j}q_i)_{S-},$$
$$Q_{13} = (\bar{d}b)_{S+P} \sum_q (\bar{q}q)_{S+P}, \quad Q_{14} = (\bar{d}_ib_j)_{S+P} \sum_q (\bar{q_j}q_i)_{S+P},$$
$$Q_{15} = \bar{d}\alpha_{\mu}(1 + \gamma_5)b \sum_q \bar{q}\sigma_{\mu}(1 + \gamma_5)q,$$
$$Q_{16} = \bar{d}_i\alpha_{\mu}(1 + \gamma_5)b \sum_q \bar{q_j}\sigma_{\mu}(1 + \gamma_5)q_i. \quad (22)$$

Here $(\bar{q}_1q_2)s_{S+P} = \bar{q}_1(1 \pm \gamma_5)q_2$. Then Eq.(11) is extended to

$$\langle \pi \phi | \mathcal{H}_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle \pi \phi | \mathcal{T}_p + \mathcal{T}_{p\text{new}} | B \rangle, \quad (23)$$

where the term $\mathcal{T}_{p\text{new}}$ arises from the neutral Higgs contributions, given by

$$\mathcal{T}_{p\text{new}} = \tau_{\phi}(\mu) [a_{11}(\pi \phi) + a_{13}(\pi \phi)](\bar{d}b)_{V+A} \otimes (\bar{s}s)_{V-A}. \quad (24)$$

The Wilson coefficients $C_{Q_i}$ (i=11, ..., 16) in Eq.(21) is calculated in the same way as that in Ref.25, 26, 27 and results are

$$C_{Q_{11}}(M_W) = -\frac{\alpha_{em}}{4\pi} \frac{m_b m_s \tan^3 \beta}{4 \sin^2 \theta_w M_W^2 \lambda_t} \sum_{i=1, k=1}^{6} U_{i1} T_{U1K} K_{mb} \left[ -\sqrt{2} V_{i1}(T_{ULK})^*_{ks} + V_{22}(T_{UR\tilde{m}_u K})^*_{ks} \right]$$
$$\times (r_H + r_A) \sqrt{x_{x^1_i}} f_{B_0} \left( x_{x^1_i}, x_{\bar{u}_k} \right) + O(\tan^2 \beta), \quad (25)$$

$$C_{Q_{13}}(M_W) = -\frac{\alpha_{em}}{4\pi} \frac{m_b m_s \tan^3 \beta}{4 \sin^2 \theta_w M_W^2 \lambda_t} \sum_{i=1, k=1}^{6} U_{i2} T_{U1K} K_{mb} \left[ -\sqrt{2} V_{i1}(T_{ULK})^*_{ks} + V_{22}(T_{UR\tilde{m}_u K})^*_{ks} \right]$$
$$\times (r_H - r_A) \sqrt{x_{x^1_i}} f_{B_0} \left( x_{x^1_i}, x_{\bar{u}_k} \right) + O(\tan^2 \beta), \quad (26)$$

$$C_{Q_{i}}(m_W) = 0, \quad i = 12, 14, 15, 16, \quad (27)$$
where the definitions of various symbols are the same as those in Ref. [26]. In calculating $C_{Q_i(m_W)}$ the following values of relevant parameters are used:

\[
\begin{align*}
&\tan \beta = 60, \quad m_{h^0} = 110 \text{ GeV}, \quad m_{H^0} = 150 \text{ GeV}, \\
&m_{H^-} = 200 \text{ GeV}, \quad m_b = 4.2 \text{ GeV}, \quad M_2 = 320 \text{ GeV}, \quad \mu = 270 \text{ GeV}, \\
&m_{t_1} = 120 \text{ GeV}, \quad \theta_{t_1} = -\pi/4, \quad m_s = 0.11 \text{ GeV}.
\end{align*}
\]  

(28)

At the low scale we get

\[
C_{Q_{12}} + C_{Q_{14}} = 0.139, 0.0897, 0.0565 \text{ for } \mu = m_b/2, m_b, 2 m_b.
\]  

(29)

$a_i$ in Eq. (24) in MSSM read as

\[
a_{11,13} = -\frac{\alpha_s}{4\pi} \frac{C_F}{N} (f_s^I + f_s^{II}) C_{Q_{12,14}},
\]  

(30)

(31)

where

\[
\begin{align*}
f_s^I &= -2 \int_0^1 du [\ln^2 u + 2 \ln u - 2 \text{Li}_2(u)] \phi_s(u) \\
&\quad + 2 \int_0^1 du \int_0^{m_b} dk \int db \ln \left[ \frac{4k^2}{m_b^2} + u^2 + u \right] J_0(bk) P_s(u, b)
\end{align*}
\]

(32)

\[
\begin{align*}
f_s^{II} &= \frac{2\pi m_B f_T f_B}{F_{B^+\rightarrow\pi^+} m_{\phi}^2} \int [du][db] \delta^2 (b_1 + b_2) b \mathcal{P}_B(\xi, b) \mathcal{P}_s(v, b_2) \\
&\quad \times [\mu_p(u + v) \mathcal{P}_p(u, b_1) + m_B(\xi - v) \mathcal{P}(u, b_1)] \\
&\quad \times \left[ \theta(b_2 - b) I_0(b\sqrt{u\xi m_B}) K_0(b\sqrt{u\xi m_B}) + \theta(b - b_2) I_0(b\sqrt{u\xi m_B}) K_0(\sqrt{-u v m_b b_2}) \right].
\end{align*}
\]  

(33)

We found in numerical results that the last terms of the distribution amplitude $\phi_s(u)$ of $\phi$ meson make main contributions to $a_{11}$.

The amplitude for for $B^- \rightarrow \pi^- \phi$ now is given as

\[
A(B^- \rightarrow \pi^- \phi) =
\]

\[
\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[ a_3 + a_5 - \frac{1}{2} (a_7^p + a_9^p) + r^\phi (a_{11} + a_{13}) \right] \times f_\phi m_\phi F_{B^+\rightarrow\pi} (m_\phi^2) 2 \epsilon^\phi \cdot p_B
\]  

(34)

where

\[
r^\phi (\mu) = \frac{m_B}{4\epsilon \cdot p_B} \frac{f_T^T}{f_\phi}.
\]  

(35)

\[\text{In CMSSM the mass spectrum and mixing of sparticles and Higgs bosons can be calculated given a set of values of a few parameters at the high (GUT or Planck) scale. Here, in stead of scanning the parameter space, we take reasonable values of relevant parameters for the sake of simplicity.}\]
In calculating $a_i (i=3,5,7,9)$ we should use the relevant Wilson coefficients in CMSSM. However, we still use their SM values in numerical calculations for simplicity because the contributions of SUSY only modify them in a few percents in most part of the parameter space including the values given above, Eq. (28) [28]. The numerical result of the Br is shown in Table 2.

5 Summary

In summary, we have studied the pure penguin process $B^- \to \pi^- \phi$ using QCD factorization approach, in particular, calculated the twist-3 contribution by including the Sudakov effects. We find $\text{Br}(B^- \to \pi^- \phi) = (1.95 - 5.70) \times 10^{-9}$ in SM, which is roughly in agreement with that in Ref. [7]. Comparing with the naive factorization (NF) result which is $(0.7 - 3) \times 10^{-9}$, the QCD factorization result (to the $O(\alpha_s)$) is less sensitive to the decay scale, as can be seen from Table 2. Actually, as noticed in Ref. [29], the coefficients $a_i$ given in Eq. (13-16) are scale independent to the $O(\alpha_s)$, which can be demonstrated by using the LO anomalous dimension matrix of relevant operators. However, from Table 2 one can see that there still is the significant scale dependence. Because we use the NLO Wilson coefficients the significant scale dependence comes mainly from the $O(\alpha_s)$ corrections of hadronic matrix elements. Indeed the $O(\alpha_s)$ corrections of hadronic matrix elements depend heavily on the scale (see, Eq.(4), which contains the factor $12 \ln[\mu/m_b]$).

In order to decrease the scale dependence it is expected to calculate the $\alpha_s^2$ order corrections of hadronic matrix elements. We have also calculated the Br in Constrained MSSM with large $\tan \beta$ and the result is $\text{Br}(B^- \to \pi^- \phi) = (1.1 - 2.4) \times 10^{-8}$. That is, the Br can be enhanced by one order of magnitude at most compared with that in SM, which is still far below the Babar experimental bound $B_r(B^0,\pm \to \pi^0,\pm \phi) < 5.6 \times 10^{-7}$ at 90% CL [30].

Acknowledgment

The work was supported in part by the National Nature Science Foundation of China. One of the authors (Chao-Shang Huang) would like to thank C.S. Lam for discussions and Phys. Dept., McGill University where the paper was written for the warm hospitality.

References

[1] Y.Y. Keum, H.N. Li and A.I. Sanda, Phys. Lett. B504 (2001) 6; Phys. Rev. D63 (2001) 054008.

\footnote{In NF case the Br at $\mu = 2 m_b$ is close to that at $\mu = m_b/2$ while the amplitude at $\mu = 2 m_b$ is close to that at $\mu = m_b/2$ in magnitude but with the opposite sign.}
[2] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. **83** (1999) 1914; Nucl. Phys. **B579** (2000) 313.

[3] F. Farrar and D. Jackson, Phys. Rev. Lett. **43** (1979) 246; S. Brodsky and G. Lepage, Phys. Rev. Lett. **43** (1979) 545, Phys. Lett. **B87** (1979) 359, Phys. Rev. **D22** (1980) 2157; A. Efremov and A. Radyushkin, Phys. Lett. **B94** (1980) 245; A. Duncan and A. Mueller, Phys. Rev. **D21** (1980) 1636; C.-S. Huang, Phys. Energ. Fortis Et Phys. Nucl. **4** (1980) 761, in Chinese. (An english translation was published in Chinese Phys. **2** (1982) 179.)

[4] H.N. Li and H. Yu, Phys. Rev. Lett. **74** (1995) 4388; H.N. Li and T. Yeh, Phys. Rev. **D56** (1997) 1615.

[5] R. Fleischer, Phys. Lett. **B321** (1994) 259.

[6] A. Deandrea et al., Phys. Lett. **B320** (1994) 170; G. Kramer, W.F. Palmer, and H. Simma, Z. Phys. **C66** (1995) 429; D.S. Du and L.B. Guo, Z. Phys. **C75** (1997) 9; A. Ali, G. Kramer, and C.-D. Lü, Phys. Rev. **D58** (1998) 094009.

[7] B. Melić, Phys. Rev. **D59** (1999) 074005

[8] S. Bar-Shalom, G. Eilam and Y.-D. Yang, [hep-ph/0201244](http://arxiv.org/abs/hep-ph/0201244).

[9] D. Du, H. Gong, J. Sun, D. Yang and G. Zhu, Phys. Rev. **D65** (2002) 094025.

[10] M. Bauer, B. Stech and M. Wirbel, Z. Phys. **C34** (1987) 103.

[11] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Nucl. Phys. **B606** (2001) 245; M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. **B612** (2001) 25; S.W. Bosch and G. Buchalla, Nucl. Phys. **B621** (2002) 459.

[12] B. V. Geshkenbein and M. V. Terentyev, Phys. Lett. **B117** (1982) 243; C. S. Huang, Commun. Theor. Phys. **2** (1983) 1265; B. V. Geshkenbein and M. V. Terentyev, Sov. J. Nucl. Phys. **39** (1984) 554; F.-G. Cao, Y.-B. Dai and C.-S. Huang, Eur. Phys. J. **C11** (1999) 501.

[13] Sudakov, V.V., Sov. Phys. JETP **30** (1956) 87.

[14] M.Ciuchini *et al.*, J. High Energy Phys. **07** (2001)13.

[15] W.N. Cottingham, H. Mehrban and I.B. Whittingham, Preprint [hep-ph/0102012](http://arxiv.org/abs/hep-ph/0102012).

[16] G.Buchalla, A.J.Buras, and M.E.Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125

[17] V.M. Braun and I.E. Filyanov, Z. Phys. C **44** (1989) 157; Z. Phys. C **48** (1990) 239.

[18] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. B **529** (1998) 323

[19] S. Descotes-Genon and C.T. Sachrajda, Nucl. phys. **B625** (2002) 239.
[20] T.-H. Chen, Y.-Y. Keum and H.-n. Li, Phys. Rev. D 64 (2001) 112002.
[21] J.C. Collins and D.E. Soper, Nucl. Phys. B193 (1981) 381.
[22] J. Botts and G. Sterman, Nucl. Phys. B325 (1989) 62.
[23] H.-n. Li and G. Sterman Nucl. Phys. B381 (1992) 129.
[24] Y.-B. Dai, C.-S. Huang and H.-W. Huang, Phys. Lett. B390 (1997) 257.
[25] C.-S. Huang and Q.-S. Yan, phys. Lett. B442 (1998) 209; C.-S. Huang, W. Liao, and Q.-S. Yan, Phys. Rev. D59 (1999) 01701.
[26] C.-S. Huang, L. Wei, Q.-S. Yan and S.-H. Zhu, Phys. Rev. D 63 (2001) 114021.
[27] C. Bobeth, T. Ewerth, F. Krüger and J. Urban, Phys. Rev. D 64 (2001) 074014; For more references on the subject, see, e.g., C.-S. Huang, Nucl. Phys. Proc. Suppl. 93 (2001) 73 and hep-ph/0210314.
[28] C.-S. Huang, Q.-S. Yan, hep-ph/9906493, unpublished.
[29] H.-Y. Cheng, K.-C. Yang, Phys. Rev. D 63 (2001) 074011.
[30] For details, see A. Telnov, http://dpf2002.velopers.net/talks_pdf/362talk.pdf.