On one approach to the aggregation of information in the objective-oriented systems

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Abstract. This article discusses aggregation operations for special-type estimates ($d$-estimates). A feature of such estimates is that they allow you to consider the requirements in the form of a threshold value. Generalized $D$-estimates are based on a probabilistic approach. Potential applications are associated with the analysis of objective-oriented systems, and can also be used in estimating models, where for each of the indicators a certain threshold value is set.

1. Introduction

Let us consider the structural representation of the system as a hierarchy of subsystems. Suppose that the objective of the system is to obtain a result of a certain quality. The input of the subsystems of the lower level receives resources, which ultimately are converted into a result, and the quality of this result characterizes the degree of the entire system objective achievement. We note that the concept of a resource is relative: for the subsystems of the $i$-th level, the results of the functioning of the subsystems of the $(i-1)$-th level serve as resources, and the result of the functioning of the subsystem of the $i$-th level can serve as a resource for some subsystem of the $(i+1)$-th level.

Let there be a way to measure the quality of the result $M$ and the quality of the resources $\mu_i$, and $M, \mu_i \in [0, 1]$. We assume that the objective is achieved if the quality of the result $M$ of the system is not worse than the specified threshold value $E$, i.e. $M \geq E$. After the formation of the requirement for the result, a sequential decomposition is performed according to the hierarchy levels of this requirement as a vector of requirements for the result of the operation of the second level subsystems. The process continues until the lower level of the hierarchy is reached, and the requirements for the results of the operation of the subsystems of the lower level induce the requirements for the resources used by the system to obtain the result.

To estimate the degree of inconsistency in the quality of the $i$-th resource $\mu_i$ and the requirement $\varepsilon_i$, which the system has for it in the form $\mu_i \geq \varepsilon_i$, we shall use the estimates of the following form

$$d_i = \frac{\varepsilon_i(1 - \mu_i)}{\mu_i(1 - \varepsilon_i)} \in [0, 1].$$ (1)

We note that for the entire system, a similar estimate can be used

$$D = \frac{E(1 - M)}{M(1 - E)} \in [0, 1].$$ (2)
The estimates of the form (1) and (2), called the difficulty of obtaining the result or the difficulty of achieving the objective, were first introduced in [1]. The concept of "difficulty" arose from intuitive considerations that the lower the quality of the resource and the higher the requirements for the quality of the result, the more difficult it is to obtain the result of a certain quality.

We note that the estimates \( d \) and \( D \) are arranged according to the principle the smaller, the better and, in principle, can be used to determine the degree of inconsistency of the two values \( \mu_i \) and \( \varepsilon_i \), assuming that \( \mu_i \geq \varepsilon_i \). It is easy to notice their connection with the well-known Hilbert metric. These estimates are widely used in various applications [2–8] and have a variety of interpretations [2, 7–9]. In [4], they are used to construct qualitative functions that are analogues of production functions. In [7], the algebra of these estimates was proposed. In [9], the estimates of the form (1) are used to identify the relationship between fuzzy operations represented by rational functions.

In further arguments, these estimates of the form (1) will be called \( d \)-estimates, and for each subsystem there are partial \( d \)-estimates, and, using special aggregation operations, the generalized \( D \)-estimate of the form (2) is computed for the whole system.

We note that these estimates can be used to characterize objective-oriented systems that have set of objectives. Let the \( i \)-th objective of the system be to obtain the result, the quality of which must satisfy the requirement \( \varepsilon_i \) in such a way that \( \mu_i \geq \varepsilon_i \). If \( (d_1, \ldots, d_n) \) is a vector, the component \( d_i \) of which characterizes the degree of non-achievement of the \( i \)-th objective, then the generalized \( D \)-estimate characterizes the degree of non-achievement of the set of objectives of the multi-objective system in general.

The purpose of the paper is to present approaches to the formation of generalized \( D \)-estimates based on aggregation of partial \( d \)-estimates with the use of different principles of aggregation.

2. Materials and methods
2.1. Existing approaches to aggregation of \( d \)-estimates
For aggregation of partial \( d \)-estimates, modern aggregation strategies can be used [10] and the disjunctive strategy is implemented by adding operator, conjunctive — by multiplying operators, and a compromise strategy — by averaging operations, which constitute a numerous class [11, 12].

In [1, 2], we have the introduction of the integral difficulty of obtaining the objective of the form

\[
\bar{D} = 1 - \prod_{i=1}^{n} (1 - d_i). \tag{3}
\]

We note that this operation corresponds to the algebraic sum of partial estimates \( d_i \) and can be used to estimate the degree of achievement of the generalized objective, which is the disjunction of partial objectives.

We can show that in the formula (3)

\[
E = 1 - \prod_{i=1}^{n} (1 - \varepsilon_i), \quad M = \frac{1 - \prod_{i=1}^{n} (1 - \varepsilon_i)}{1 - \prod_{i=1}^{n} (1 - \varepsilon_i / \mu_i)}.
\]

From the above formulas we can see that the generalized estimate of requirements depends only on the requirements for quality of each individual component. A generalized estimate of the quality of the resource is a relative value, which depends not only on the respective estimates
of the quality of individual components, but also on the requirements for the quality of each component.

Formula (3) allows generalization to the case of unequal partial $d$-estimates in the form

$$
\hat{\overline{D}}_\lambda = 1 - \prod_{i=1}^{n} (1 - d_i)^{\lambda_i},
$$

(4)

where $\lambda = (\lambda_1, \ldots, \lambda_n)$, $\lambda_i$ is the weight of the $i$-th objective,

$$
\forall i = 1, n \ (\lambda_i \geq 0), \quad \sum_{i=1}^{n} \lambda_i = 1,
$$

(5)

In [8], it was shown that the generalized estimate of the form (4) is a Cauchys mean value. In [5], the generalized $D$-estimate is a weighted average of the form

$$
\hat{\overline{D}}_\lambda = \sum_{i=1}^{n} \lambda_i d_i,
$$

(6)

where vector $\lambda = (\lambda_1, \ldots, \lambda_n)$ satisfies the condition (5).

The peculiarity of the estimates (4) and (6) is that due to the weighting factors, the priorities of partial objectives can be considered during aggregation.

If the generalized objective is formed by the type of conjunction of partial objectives, then in this case, it is obviously more suitable to use the integral estimate of the form

$$
\hat{\bigotimes} \overline{D} = \prod_{i=1}^{n} d_i.
$$

(7)

We note that for aggregation of $d$-estimates, ordinal weighted aggregation operators — OWA [10] can be used. In this case, the $n$-dimensional OWA-operator associated with the weight vector $W = (w_1, \ldots, w_n)$ that satisfies the condition $\sum_{i=1}^{n} w_i = 1 \ (\forall i = 1, n \ (w_i \in (0, 1]))$ is defined as follows:

$$
D_W = \sum_{i=1}^{n} w_i d'_i,
$$

(8)

where $(d'_1, \ldots, d'_n)$ is the vector obtained from the vector $(d_1, \ldots, d_n)$ by ordering the components by lack of increase.

In the case of (8), to determine the weight vector $W$, quantification functions that implement the principle of fuzzy majority are used [10].

Thus, for the formation of generalized $D$-estimates based on aggregation of partial $d$-estimates, there is a significant arsenal of aggregation operations, most of which belong to the class of averaging operations. On the other hand, when aggregating $d$-estimates, it is advisable to consider the relationship between the values $E$ and $M$, on the one hand, and $(\varepsilon_1, \ldots, \varepsilon_n)$ and $(\mu_1, \ldots, \mu_n)$ on the other. In [2, 9], the approach to solving this problem on the basis of probabilistic interpretation of $d$-estimates is presented.

2.2. Probabilistic interpretation of $d$-estimates

We introduce the following events:

- $A_i$ — the requirement for the quality of the $i$-result is not fulfilled,
- $B_j$ — the requirement for the quality of the $j$-resource is not fulfilled.
If the requirement is not fulfilled, then the result or resource will be called poor-quality result or resource. Let the aggregation of information be carried out from the bottom up, and in each node the answer to the question what is the probability that a good-quality resource will produce a poor-quality result? is determined. Conditional probability $P(A_i/B_j)$ can be expressed by the formula

$$P(A_i/B_j) = \frac{(1 - P(B_j/A_i)) \cdot P(B_j)}{(1 - P(B_j)) \cdot P(B_j/A_i)}$$

where $A_i/B_j$ is the event that consists in the fact that the requirement for the quality of the $i$-th result is not fulfilled, provided that the requirement for the quality of the $j$-th resource is fulfilled.

In accordance with [9], we suppose that $d_{ij} = P(A_i/B_j)$, then there are two variants of probabilistic interpretation $\varepsilon$ and $\mu$:

1) $\varepsilon_j = P(B_j)$ is the probability that the requirement for the quality of the $j$-th resource is not fulfilled; $\mu_{ij} = P(B_j/A_i)$ is the probability that the requirement for the quality of the $j$-th resource is not fulfilled, provided that the requirement for the quality of the $i$-th result is not fulfilled.

2) $\varepsilon_{ij} = P(B_j/A_i)$ is the probability that the requirement for the quality of the $j$-th resource is fulfilled, provided that the requirement for the quality of the $i$-th result is not fulfilled. $\mu_j = P(B_j)$ is the probability that the requirement for the quality of the $j$-th resource is not fulfilled.

The first variant of probabilistic interpretation was originally proposed in [2]. It is easy to show that the inequality $\mu \geq \varepsilon$ holds in both cases.

If we consider the process of functioning of interconnected systems, each of which has its own set of resources and seeks to obtain its result (achievement of its objective), then the following situations are distinguished, which correspond to probabilistic interpretation $\varepsilon$ and $\mu$:

1) the requirement for the quality of the $j$-th resource is the same for all systems and is determined by the value $\varepsilon_j$, while the quality of the $j$-th resource for each $i$-th system ($i$-th result) is the value $\mu_{ij}$;

2) the quality of the $j$-th resource is the same for all systems and is determined by the value $\mu_j$, but the requirements for this quality are different for each system (each result) and are determined by the values $\varepsilon_{ij}$.

3. Results and discussion

3.1. Aggregation based on representation in the form

We shall assume that the quality of the $j$-th resource does not depend on the quality of other resources used to obtain the $i$-th result, therefore $\{B_j\}$ is the set of independent events.

Let the requirement for the quality of the $i$-th result be fulfilled if the requirements for the quality of all resources are fulfilled, i.e. $A_i \subseteq \bigcap_j B_j$, then the probability that the requirement for the quality of the $i$-th result is not fulfilled, provided that all resources are of good-quality, is determined by the formula

$$P\left(A_i/\bigcap_j B_j\right) = 1 - \frac{1 - \prod_j (1 - P(B_j))}{1 - \prod_j (1 - P(B_j/A_i)) \prod_j (1 - P(B_j))}.$$
Generalized estimates corresponding to this probability have the form
\[
D_1(i) = \frac{\prod_j (1 - \mu_{ij}) \left( 1 - \prod_j (1 - \varepsilon_j) \right)}{\prod_j (1 - \varepsilon_j) \left( 1 - \prod_j (1 - \mu_{ij}) \right)} = \frac{\hat{E}_1 (1 - M_1)}{\hat{M}_1 (1 - \hat{E}_1)},
\]
where \( \hat{E}_1 = 1 - \prod_j (1 - \varepsilon_j), \) \( M_1 = 1 - \prod_j (1 - \mu_{ij}), \) and
\[
D_2(i) = \frac{\prod_j \varepsilon_{ij} \left( 1 - \prod_j \mu_{ij} \right)}{\prod_j \mu_{ij} \left( 1 - \prod_j \varepsilon_{ij} \right)} = \frac{\hat{E}_2 (1 - M_2)}{\hat{M}_2 (1 - \hat{E}_2)},
\]
where \( \hat{E}_2 = \prod_j \varepsilon_{ij}, \) \( M_2 = \prod_j \mu_{ij}. \)

We note that \( E \) and \( M \) have the following probabilistic interpretation:

\( \hat{E}_1 = P \left( \bigcup_j B_j \right) \) is the probability that at least one resource does not satisfy the requirement,
\( \hat{M}_1 = P \left( \bigcup_j B_j / A_i \right) \) is the probability that at least one resource is a poor-quality resource, provided that the requirement for the quality of the \( i \)-th result is not fulfilled,
\( \hat{E}_2 = P \left( \bigcup_j B_j / A_i \right) = P \left( \bigcap_j B_j / A_i \right) \) is the probability that the requirement for the quality of all resources are fulfilled, provided that poor-quality result is obtained,
\( \hat{M}_2 = P \left( \bigcup_j B_j \right) = P \left( \bigcap_j B_j \right) \) is the probability that all resources are good-quality resources.

It is easy to show that in both cases \( \hat{M} \geq \hat{E} \).

Now suppose that the requirement for the quality of the \( i \)-th result is fulfilled if the requirements for the quality of at least one resource are fulfilled, that is, \( A_i \subseteq \bigcup_j B_j \). Then the probability that the requirement for the quality of the \( i \)-th result is not fulfilled, provided that the requirements for the quality of at least one resource are fulfilled, is determined by the formula
\[
P \left( A_i / \bigcup_j \overline{B}_j \right) = 1 - \frac{1 - \prod_j P(B_j)}{1 - \prod_j P(B_j / A_i)}.
\]
and the corresponding generalized estimates are determined by the following formulas:

\[ \hat{D}_1(i) = \prod_j \frac{\varepsilon_j (1 - \prod_j \mu_{ij})}{\mu_{ij} (1 - \prod_j \varepsilon_j)} = \frac{\hat{E}_1 (1 - \hat{M}_1)}{\hat{M}_1 (1 - \hat{E}_1)}, \]

where \( \hat{E}_1 = \prod_j \varepsilon_j, \hat{M}_1 = \prod_j \mu_{ij}, \)

\[ \hat{D}_2(i) = \prod_j \frac{(1 - \mu_j) (1 - \prod_j (1 - \varepsilon_{ij}))}{(1 - \varepsilon_{ij}) (1 - \prod_j (1 - \mu_j))} = \frac{\hat{E}_2 (1 - \hat{M}_2)}{\hat{M}_2 (1 - \hat{E}_2)}, \]

where \( \hat{E}_2 = 1 - \prod_j (1 - \varepsilon_{ij}), \hat{M}_2 = 1 - \prod_j (1 - \mu_j). \)

The components \( \hat{E} \) and \( \hat{M} \) have the following probabilistic interpretation:

\( \hat{E}_1 = P \left( \bigcap_j B_j \right) \) is the probability that the requirements for the quality of all resources are not fulfilled,

\( \hat{M}_1 = P \left( \bigcap_j B_j / A_i \right) \) is the probability that all resources are poor-quality resources, provided that the requirement for the quality of the \( i \)-th result is not fulfilled,

\( \hat{E}_2 = P \left( \bigcup_j \bar{B}_j / A_i \right) \) is the probability that at least one resource is a good-quality resource, provided that the \( i \)-th result does not satisfy the requirement,

\( \hat{M}_2 = P \left( \bigcup_j \bar{B}_j \right) \) is the probability that at least one resource is a good-quality resource.

In both cases, \( \hat{E} \) and \( \hat{M} \) are in the relation \( \hat{M} \geq \hat{E} \), which corresponds to the introduced assumptions.

Thus, generalized \( D \)-estimates (9)–(12) are obtained having the structure (2), that is, the same structure as the structure of the partial estimates, and their components allow a meaningful interpretation, which is important for applications.

3.2. Aggregation based on convolution of partial \( d \)-estimates

Let us consider the possibility of using a probabilistic approach to obtain aggregation operations for partial \( d \)-estimates.
Let \( B_1 \) and \( B_2 \) be independent events, \( P(A/B_1) = d_1 \), \( P(A/B_2) = d_2 \). We shall find \( P(A/(B_1 \cap B_2)) \).

\[
P(A \cap (B_1 \cap B_2)) = \frac{P(B_1) \cdot P(A/B_1) \cdot P(B_2/A \cap B_1)}{P(B_2) \cdot P(A/B_2) \cdot P(B_1/A \cap B_2)} = \frac{P(B_1) \cdot P(A/B_1) \cdot P(B_2/A)}{P(B_2) \cdot P(A/B_2) \cdot P(B_1/A)},
\]

because \( A \subset B_j \) (\( j = 1, 2 \)).

On the other hand,

\[
P(A \cap (B_1 \cap B_2)) = P(B_1 \cap B_2) \cdot P(A/(B_1 \cap B_2)),
\]

therefore, we have

\[
P(B_1 \cap B_2) \cdot P(A/(B_1 \cap B_2)) = \frac{P(B_1) \cdot P(A/B_1) \cdot P(B_2/A)}{P(B_2) \cdot P(A/B_2) \cdot P(B_1/A)},
\]

whence, considering the independence of events \( B_j \), we obtain the following two formulas:

1) \( P(A/(B_1 \cap B_2)) = \frac{P(B_1) \cdot P(A/B_1) \cdot P(B_2/A)}{P(B_1 \cap B_2)} = \frac{P(B_2/A)}{P(B_2)} \cdot P(A/B_1), \)

2) \( P(A/(B_1 \cap B_2)) = \frac{P(B_2) \cdot P(A/B_2) \cdot P(B_1/A)}{P(B_1 \cap B_2)} = \frac{P(B_1/A)}{P(B_1)} \cdot P(A/B_2). \)

Then, taking into account the probabilistic interpretation \( \varepsilon \) and \( \mu \), we obtain the following formulas expressing the dependence of the generalized estimate \( \hat{D} \) on the partial estimates \( d_1 \) and \( d_2 \):

\[
\hat{D}(d_1, d_2) = \begin{cases} 
\frac{1 - \mu_2}{1 - \varepsilon_2} \cdot d_1 & \text{or} & \frac{\varepsilon_2}{\mu_2} \cdot d_1, \\
\frac{1 - \mu_1}{1 - \varepsilon_1} \cdot d_2 & \text{or} & \frac{\varepsilon_1}{\mu_1} \cdot d_2.
\end{cases}
\]  

(13)

Since \( d_1 = \frac{\varepsilon_1(1 - \mu_1)}{\mu_1(1 - \varepsilon_1)} \) and \( d_2 = \frac{\varepsilon_2(1 - \mu_2)}{\mu_2(1 - \varepsilon_2)} \), formula (13) can be rewritten in the form

\[
\hat{D}(d_1, d_2) = \begin{cases} 
\frac{\mu_2}{\varepsilon_2} \cdot d_1 d_2 & \text{or} & \frac{1 - \varepsilon_2}{1 - \mu_2} \cdot d_1 d_2, \\
\frac{\mu_1}{\varepsilon_1} \cdot d_1 d_2 & \text{or} & \frac{1 - \varepsilon_1}{1 - \mu_1} \cdot d_1 d_2.
\end{cases}
\]  

(14)

In the general case, we obtain the formula

\[
P(A/(B_1 \cap \cdots \cap B_{n+1})) = \frac{P(B_{n+1}/A)}{P(B_{n+1})} P(A/(B_1 \cap \cdots \cap B_n)),
\]

and then, depending on the probabilistic interpretation, we have the following recurrence relations

\[
\hat{D}_{n+1} = \frac{1 - \mu_{n+1}}{1 - \varepsilon_{n+1}} \cdot \hat{D}_n \quad \text{or} \quad \hat{D}_{n+1} = \frac{\varepsilon_{n+1}}{\mu_{n+1}} \cdot \hat{D}_n.
\]  

(15)

Thus, when increasing the dimension, it is not necessary to use the basic formula. It is enough to multiply the computed generalized estimate by the appropriate factor.
Let us find \( P(A/\overline{B}_1 \cup \overline{B}_2) \), provided that \( B_1 \) and \( B_2 \) are independent, but joint events. Here we have \( P(A/\overline{B}_1) = d_1 \), \( P(A/\overline{B}_2) = d_2 \).

We note that
\[
\overline{B}_1 \cup \overline{B}_2 = (\overline{B}_1 \cap B_2) \cup (B_1 \cap \overline{B}_2) \cup (\overline{B}_1 \cap \overline{B}_2).
\]

With this presentation
\[
P(A/(\overline{B}_1 \cup \overline{B}_2)) = \frac{P(A \cap (\overline{B}_1 \cup \overline{B}_2))}{P(\overline{B}_1 \cup \overline{B}_2)} = \frac{P(A \cap \overline{B}_1 \cap B_2) + P(A \cap B_1 \cap \overline{B}_2) + P(A \cap \overline{B}_1 \cap \overline{B}_2)}{P(\overline{B}_1 \cup \overline{B}_2)}.
\]

This formula is converted to
\[
P(A/(\overline{B}_1 \cup \overline{B}_2)) = \frac{P(\overline{B}_1) \cdot P(B_2/A)}{1 - P(B_1)P(\overline{B}_1)} \cdot P(A/\overline{B}_1) + \frac{P(\overline{B}_2) \cdot P(B_1/A)}{1 - P(B_1)P(\overline{B}_1)} \cdot P(A/\overline{B}_1) + \frac{P(\overline{B}_1) \cdot P(\overline{B}_2/A)}{1 - P(B_1)P(\overline{B}_1)} \cdot P(A/\overline{B}_1),
\]
from which, depending on the probabilistic interpretation, we obtain the following formulas expressing the dependence of the generalized \( D \)-estimate on the partial estimates:

\[
\overset{+}{D}(d_1, d_2) = \left[ \begin{array}{c}
\frac{1 - \varepsilon_1}{1 - \varepsilon_1} \cdot d_1 + \frac{(1 - \varepsilon_2)\mu_1}{1 - \varepsilon_1 \varepsilon_2} \cdot d_2, \\
\frac{\mu_1}{\mu_1 + \mu_2 - \mu_1 \mu_2} \cdot d_1 + \frac{\varepsilon_1 \mu_2}{\mu_1 + \mu_2 - \mu_1 \mu_2} \cdot d_2, \\
\frac{(1 - \varepsilon_1)\mu_2}{1 - \varepsilon_1 \varepsilon_2} \cdot d_1 + \frac{1 - \varepsilon_2}{1 - \varepsilon_1 \varepsilon_2} \cdot d_2, \\
\frac{\mu_1 \varepsilon_2}{\mu_1 + \mu_2 - \mu_1 \mu_2} \cdot d_1 + \frac{\mu_2}{\mu_1 + \mu_2 - \mu_1 \mu_2} \cdot d_2.
\end{array} \right]
\]

(16)

Analyzing the formulas obtained, it can be concluded that in the general case it is impossible to obtain recurrence relations for \( \overset{+}{D} \).

If \( B_1 \) and \( B_2 \) are disjoint events (for example, resources are interchangeable), then
\[
P(A/(\overline{B}_1 \cup \overline{B}_2)) = \frac{P(A \cap (\overline{B}_1 \cup \overline{B}_2))}{P(\overline{B}_1 \cup \overline{B}_2)} = \frac{P(A \cap \overline{B}_1) + P(A \cap \overline{B}_2)}{P(\overline{B}_1) + P(\overline{B}_2)} =
\]
\[
= \frac{P(\overline{B}_1)}{P(\overline{B}_1) + P(\overline{B}_2)} \cdot P(A/\overline{B}_1) + \frac{P(\overline{B}_2)}{P(\overline{B}_1) + P(\overline{B}_2)} \cdot P(A/\overline{B}_2).
\]

Thus, depending on the probabilistic interpretation \( \varepsilon \) and \( \mu \), we have the following formulas:

\[
\overset{+}{D}(d_1, d_2) = \left[ \begin{array}{c}
\frac{1 - \varepsilon_1}{(1 - \varepsilon_1) + (1 - \varepsilon_2)} \cdot d_1 + \frac{1 - \varepsilon_2}{(1 - \varepsilon_1) + (1 - \varepsilon_2)} \cdot d_2, \\
\frac{\mu_1}{\mu_1 + \mu_2} \cdot d_1 + \frac{\mu_2}{\mu_1 + \mu_2} \cdot d_2.
\end{array} \right]
\]

(17)

In this case, the generalized estimate \( \overset{+}{D} \) is a linear combination of partial estimates, and the weighting factors are normalized. As the amount of resources increases, the summand is added, and the coefficients before \( d_j \) change.
4. Conclusion
The paper proposes a special type of aggregation operations that can be used both for objective-oriented systems and in estimation models, if threshold values, which determine the minimum level of manifestation of the corresponding properties, are specified for indicators characterizing a complex object or system. It is important that the partial and generalized estimates characterizing have the same structure. Probabilistic interpretation allows the use of simulation technology and Bayesian networks.

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