A finiteness proof for the Lorentzian state sum spinfoam model for quantum general relativity

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Abstract

We show that the normalized Lorentzian state sum is finite on any triangulation. It thus provides a candidate for a perturbatively finite quantum theory of general relativity in four dimensions with Lorentzian signature.

1 Introduction

In [1], a state sum, or spinfoam model, for discretized Euclidean signature general relativity was proposed. The model was made finite by passing from the representations of $\text{so}(4)$, the Euclidean Lorentz algebra, to its q-deformation at a root of unity. In [2], a better motivated normalization for the model in [1] was derived, and it was conjectured that with this new normalization the model was finite on any finitely triangulated manifold without passing to the quantum group. The finiteness conjecture was proved in [3]. The same normalization was obtained, from a different perspective, in [4].

In [5], a Lorentzian signature version of the state model in [1] was proposed. It was suggested that the model could be made finite by passing to the representations of the quantum Lorentz algebra $\mathfrak{g}$. In [6], a change of normalization, similar to the one that made finite the Euclidean model, was proposed for the Lorentzian model. The purpose of the present paper is to prove that the
Lorentzian signature model of \( \mathbb{R} \) with the normalization of \( \mathbb{R} \) is in fact finite for any finite triangulation of a 4-manifold.

This result is remarkable. The model is an integral over infinite domains of terms each of which is the trace of an operator on certain infinite dimensional representations of the Lorentz algebra, the balanced unitary ones. If we adopt the point of view of \( \mathbb{R} \) that the sum on a specific triangulation should be interpreted as a term in a perturbation series \( \mathbb{R} \), \( \mathbb{R} \), \( \mathbb{R} \), this result implies that—up to the issue of singular triangulations discussed below—the theory is finite order by order, since there are only a finite number of triangulations with a given number of 4-simplices. For a theory of four dimensional Lorentzian quantum general relativity, perturbative finiteness is significant. Finiteness on nondegenerate triangulations is proven here with mathematical rigor. The proof relies on technology developed in \( \mathbb{R} \).

At the moment our proof does not work for degenerate triangulations, and we do not know if these give finite contributions or not. This will require a more delicate analysis of the relevant integrals. In the Euclidean context, singular triangulations are finite \( \mathbb{R} \).

The finiteness of the model is the result of several mathematical surprises. The finiteness of the individual \( 10J \) symbols comes about by representing them as multiple integrals on hyperbolic space, then making the extremely simple regularization of omitting any single integration. This regularization does not break any of the symmetries. This procedure does not work for spin nets for arbitrary unitary representations of the Lorentz algebra, but only for the balanced ones \( \mathbb{R} \). The finiteness of the complete state sum on any finite triangulation, which we prove in this paper, depends then on a rather delicate relationship between asymptotic estimates for the relativistic spin nets which go into it.

We recall the definition of the model in Section 2, we prove its finiteness in Section 3, and discuss the result in Section 4.

2 The model

We do not review any of the derivations or the motivations of the model here, but merely recall its definition. We refer the reader to the bibliography for a thorough introduction.

The principal series unitary representations of the Lorentz algebra \( so(3,1) \) are denoted \( R(k, \rho) \), where \( k \) is an integer and \( \rho \) is a nonnegative real number. The balanced representations are those with \( k = 0 \); they are just labelled with \( \rho \).

We construct the state sum model by taking a (fixed) nondegenerate finite triangulation of a 4-manifold, with or without boundary. We label each 2-simplex \( f \) of the triangulation with a balanced representation of the Lorentz algebra, or, more simply, with a positive real parameter \( \rho_f \). The state sum is
then given by the expression

\[ Z = \int_{\rho_f=0}^{\infty} d\rho_f \prod_f \rho_f^2 \prod_e \Theta_4(\rho_e^1 \ldots \rho_e^4) \prod_v I_{10}(\rho_v^1 \ldots \rho_v^{10}). \]  

(1)

The integration is over the labels of all internal faces (the faces not belonging to the boundary). The three products run over the the 2-simplices \( f \), the 3-simplices \( e \) and the 4-simplices \( v \) of the triangulation, respectively (the choice of the letter refers to corresponding dual elements: faces, edges and vertices.) The labels \( (\rho_e^1, \ldots, \rho_e^4) \) are the ones of the four 2-simplices adjacent to the 3-simplex \( e \). The labels \( (\rho_v^1, \ldots, \rho_v^{10}) \) are the ones of the ten 2-simplices adjacent to the 4-simplex \( v \).

The functions \( \Theta_4 \) and \( I_{10} \) are defined as traces of recombination diagrams for the balanced representations, regularized as explained in [5]. The function \( \Theta_4 \) is given by the diagram in Figure 1. It was discovered to play a role in the model in [6] and its evaluation is in [6]. The function \( I_{10} \) is given by the diagram in Figure 2. These traces are called relativistic spin networks. As shown in [6], these relativistic spin networks can be explicitly expressed as multiple integrals.
on the upper sheet $H$ of the 2-sheeted hyperboloid in Minkowski space. To this purpose, we define the projector kernel

$$K_\rho(x, y) = \frac{\sin(\rho d(x, y))}{\rho \sinh(d(x, y))}$$  

(2)

where $d(x, y)$ is the hyperbolic distance between $x$ and $y$. Then the trace of a recombination diagram is given by a multiple integral of products of $K$'s. More precisely, by one integral over $H$ per each node, of the product on one kernel per each link. The integral is then normalized by dropping one of the integrations. By Lorentz symmetry, the result is independent of the point not integrated over. Thus in particular $\Theta_4$ and $I_{10}$ are given by

$$\Theta_4(\rho_1, \ldots, \rho_4) = \frac{1}{2\pi^2} \int_H K_{\rho_1}(x, y) \ldots K_{\rho_4}(x, y) dy$$  

(3)

and

$$I_{10}(\rho_1, \ldots, \rho_{10}) = \frac{1}{2\pi^2} \int_{H^4} \prod_{i \leq j = 1, 5} K_{\rho_{i,j}}(x_i, x_j) \, dx_1 dx_2 dx_3 dx_4.$$  

(4)

Equations (1–4) define the state sum completely. For a four dimensional manifold with boundary, (1) gives a function of the boundary labels. These functions can be interpreted as three-geometry to three-geometry transition amplitudes, computed to a certain order in a perturbative expansion. They can be viewed as the (in principle) observable quantities of a quantum theory of gravity, as explained in [12].

Each term in the sum (1) is a multiple integral on an unbounded domain. The functions $\Theta_4$ and $I_{10}$ were shown to be bounded in [11]. Therefore convergence is a question of sufficiently rapid decay at infinity. Since we are assuming a nondegenerate triangulation, each 2-simplex in (1) appears in at least three distinct 3-simplices and at least three distinct 4-simplices. Therefore, by power counting, we need a combined power law decay at infinity adding to more than 1 in the combination of the $\Theta_4$ and $I_{10}$ factors for each $\rho$ separately. As we show below, the $\Theta_4$ factor has a power law decay with exponent $3/4$, while the $I_{10}$ has a decay with exponent $3/10$. The first exponent seems to be sharp, the second can probably be strengthened but not by very much. We now supply a rigorous proof of finiteness.

3 The proof

We begin with two results by Baez and Barrett in [11], which we state without proof.

**Lemma 1.** (Baez-Barrett) $\Theta_4$ and $I_{10}$ are bounded.

This follows immediately from Theorems 1, 2 and 3 of [11].
Lemma 2. (Baez-Barrett) If \( n \geq 3 \), the integral
\[
J(x_1, \ldots, x_n) = \int_H |K_{\rho_1}(x, x_1)K_{\rho_2}(x, x_2) \cdots K_{\rho_n}(x, x_n)|
\]
converges, and for any \( 0 < \epsilon < 1/3 \) there exists a constant \( C > 0 \) such that for any choice of the points \( x_1, \ldots, x_n \),
\[
J \leq C \exp \left( -\frac{n-2-\kappa \epsilon}{n(n-1)} \sum_{i<j} r_{ij} \right)
\]
where \( r_{ij} \) is the hyperbolic distance \( d(x_i, x_j) \) between \( x_i \) and \( x_j \).

This result, (Lemma 5 in reference [11]) is one of the fundamental tools in the proof of Lemma 1.

Lemma 3. The \( \Theta_4 \) relativistic spin network satisfies the following bound
\[
|\Theta_4| \leq \frac{1}{\rho_1 \rho_2 \rho_3}
\]
for any arbitrary triple \( \rho_1 \rho_2 \rho_3 \).

Proof. From the fact that \( |K_\rho| \leq 1 \) we have that \( |\Theta_4| \leq |\Theta_3| \), where \( \Theta_3 \) corresponds to the evaluation of the spin network obtained by dropping an arbitrary link from the original one.
\[
\Theta_3 = \frac{1}{2\pi^2} \int H K_{\rho_1}(x, y)K_{\rho_2}(x, y)K_{\rho_3}(x, y) dy
\]
As it is shown in [5],
\[
\Theta_3 = \frac{2}{\pi\rho_1 \rho_2 \rho_3} \int_0^\infty \sin \rho_1 r \sin \rho_2 r \sin \rho_3 r \sinh r \, dr
\]
\[
= \frac{1}{4\rho_1 \rho_2 \rho_3} \left( \tanh \left( \frac{\pi}{2} (\rho_1 + \rho_2 - \rho_3) \right) + \tanh \left( \frac{\pi}{2} (\rho_3 + \rho_1 - \rho_2) \right) \\
+ \tanh \left( \frac{\pi}{2} (\rho_2 + \rho_3 - \rho_1) \right) - \tanh \left( \frac{\pi}{2} (\rho_1 + \rho_2 + \rho_3) \right) \right)
\]
From this we have
\[
|\Theta_3| \leq \frac{1}{\rho_1 \rho_2 \rho_3}
\]
and (5) follows.

Corollary 1. For any subset of \( \kappa \) elements \( \rho_1 \ldots \rho_\kappa \) out of the corresponding four representations appearing in \( \Theta_4 \) the following bounds hold
\[
|\Theta_4| \leq \frac{C_\kappa}{\prod_{i=1}^\kappa \rho_i^{\alpha_\kappa}} \quad \text{where} \quad \alpha_\kappa = \begin{cases} 
1 & \text{for } \kappa \leq 3 \\
\frac{3}{4} & \text{for } \kappa = 4
\end{cases}
\]
for some positive constant \( C_\kappa \).
Proof. The case \( \kappa = 3 \) corresponds to equation (6). For \( \kappa < 3 \) we observe that in the definition of \( \Theta_3 \) in (6) we can obtain a bound containing \( \kappa \) different \( \rho \)'s in the denominator by bounding \( 3 - \kappa \) of the three \( K_{\rho} \)'s by \( \text{sinh} r \). For the case \( \kappa = 4 \) we can write four inequalities as in the previous Lemma choosing different triplets. Multiplying the four inequalities each representation appears repeated three times so we obtain the exponent \( \frac{4}{3} \) in the bound. \[ \Box \]

**Lemma 4.** The tetrahedron amplitude

\[
I_6 = \int_{H^3} dx_2 dx_3 dx_4 K_{\rho_{12}}(x_1, x_2)K_{\rho_{13}}(x_1, x_3)K_{\rho_{14}}(x_1, x_4) K_{\rho_{23}}(x_2, x_3)K_{\rho_{24}}(x_2, x_4)K_{\rho_{34}}(x_3, x_4). \tag{8}
\]

(see Figure 3) satisfies

\[
I_6 \leq \frac{K}{\rho_1\rho_2\rho_3}, \tag{9}
\]

for a constant \( K \), and any choice of three non vanishing colorings \( \rho_1, \rho_2, \) and \( \rho_3 \) in the same triangle.

Proof. We study the integral

\[
I = \int_{H^3} dx_2 dx_3 dx_4 |K_{\rho_{12}}(x_1, x_2)K_{\rho_{13}}(x_1, x_3)K_{\rho_{14}}(x_1, x_4) K_{\rho_{23}}(x_2, x_3)K_{\rho_{24}}(x_2, x_4)K_{\rho_{34}}(x_3, x_4)|. \tag{10}
\]

for any choice of numbers \( \rho_{ij} \geq 0 \) for \( 1 \leq i < j \leq 4 \) and a point \( x_1 \in H \). First we integrate out \( x_4 \) using Lemma 2, obtaining

\[
I \leq C \int_{H^2} dx_2 dx_3 e^{-\frac{1}{6}(1-3\epsilon)(r_{12}+r_{13}+r_{23})} |K_{\rho_{12}}(x_1, x_2)K_{\rho_{13}}(x_1, x_3)K_{\rho_{23}}(x_2, x_3)|. \tag{11}
\]
where \( r_{ij} = d(x_i, x_j) \). We can bound the previous expression by

\[
I \leq \frac{C}{\rho_{12}\rho_{13}\rho_{23}} \int_{H} dx_2 \frac{e^{-\frac{1}{6}(1-3\epsilon)r_{12}}}{\sinh r_{12}} \int_{H} dx_3 \frac{e^{-\frac{1}{6}(1-3\epsilon)(r_{13}+r_{23})}}{\sinh r_{13}\sinh r_{23}}.
\]  (12)

Let’s concentrate on the \( x_3 \) integration. In order to do so we use a coordinate system in which two of the coordinates are

\[
k = \frac{1}{2}(r_{13} + r_{23}), \quad \ell = \frac{1}{2}(r_{13} - r_{23}),
\]

while the third is the angle \( \phi \) between \( x_3 \) and a given plane containing the geodesic between \( x_1 \) and \( x_2 \). The ranges of these coordinates are

\[
r/2 \leq k < \infty, \quad -r/2 \leq \ell \leq r/2, \quad 0 \leq \phi < 2\pi,
\]

where we set \( r = r_{12} \). In terms of this coordinates the measure \( dx_3 \) becomes

\[
dx_3 = 2 \sinh r_{13} \sinh r_{23} \sinh r \, dk \, d\ell \, d\phi.
\]

In terms of this coordinates (12) becomes

\[
I \leq \frac{2C}{\rho_{12}\rho_{13}\rho_{23}} \int_{H} dx_2 \frac{e^{-\frac{1}{6}(1-3\epsilon)r}}{\sinh r} \int_{0}^{2\pi} d\phi \int_{r/2}^{\infty} dk \int_{-r/2}^{r/2} d\ell \frac{e^{-\frac{1}{6}(1-3\epsilon)k}}{\sinh r}.
\]

Finally if we put in the form of the measure \( dx_2 \), i.e. \( dx_2 = \sinh^2 r \, dr \, d\Omega \) (where \( d\Omega \) is the measure of the unit sphere), we can complete the integration to obtain the sought for bound, namely

\[
I \leq \frac{16\pi^2 C}{\rho_{12}\rho_{13}\rho_{23}} \int_{0}^{\infty} dr e^{-\frac{1}{6}(1-3\epsilon)r} \int_{r/2}^{\infty} dk \, r e^{-\frac{1}{6}(1-3\epsilon)k} = \frac{16\pi^2 C}{3(1-3\epsilon)\rho_{12}\rho_{13}\rho_{23}} \int_{0}^{\infty} dr \, r \, e^{-\frac{1}{6}(1-3\epsilon)r},
\]  (13)

which concludes the proof.

**Lemma 5.** The 4-simplex amplitude \( I_{10} \) satisfies the following bound:

\[
I_{10} \leq \frac{K}{\rho_1\rho_2\rho_3},
\]

for some constant \( K \).

**Proof.** The 4-simplex amplitude \( I_{10} \) corresponds to introducing four additional \( K_\rho \) in the multiple integral (8) together with an additional integration corresponding to the four new edges and the additional vertex respectively. Using Lemma 2 this additional integration can be bounded by a constant, so that after using Lemma 4 we have

\[
I_{10} \leq \frac{K}{\rho_1\rho_2\rho_3}
\]

for any arbitrary triple \( \rho_1\rho_2\rho_3 \) in the same triangle. \( \square \)
Lemma 6. \(I_{10}\) satisfies also the following bounds:

\[
I_{10} \leq \frac{K_1}{\rho_1 \rho_2}, \quad I_{10} \leq \frac{K_2}{\rho_1},
\]

where \(K_1\), and \(K_2\) are constant and \(\rho_1\) and \(\rho_2\) are in the same triangle.

Proof. We observe that a different bound can be obtained for \(I_6\) containing respectively two or one representations in the denominator if we bound either two or one of the three \(K_\rho\) in (11) by \(\frac{1}{\sin h}\) instead of just taking absolute value. The integration on the right still converges (see [11]). \(\square\)

Using Lemma 3 and 4 it is easy to prove the following corollary.

Corollary 2. For any subset of \(\kappa\) elements \(\rho_1 \ldots \rho_\kappa\) out of the corresponding ten representations appearing in \(I_{10}\) the following bounds hold

\[
|I_{10}| \leq \frac{K_\kappa}{\left(\prod_{i=1}^{\kappa} \rho_i\right)^{10}}
\]

for some positive constant \(K_\kappa\).

Theorem 1. Given a non singular triangulation, the state sum partition function \(Z\) is well defined, i.e., the multiple integral in (7) converges.

Proof. We divide each integration region \(\mathbb{R}^+\) into the intervals \([0, 1)\), and \([1, \infty)\) so that the multiple integral decomposes in a finite sum of integrations of the following types:

i. All the integrations are in the range \([0, 1)\). We denote this term \(T(F, 0)\),
where \(F\) is the number of 2-simplices in the triangulation. This term in the sum is finite by Theorem 1.

ii. All the integrations are in the range \([1, \infty)\). This term \(T(0, F)\) is also finite since using Corollary 1 and 2 for \(\kappa = 4\), and \(\kappa = 10\) respectively we have

\[
T(0, F) \leq \prod_f \int_{\rho_f = 1}^{\infty} d\rho_f \rho_f^{2 - \frac{1}{2} h_n - \frac{1}{2} h_n} \leq \left( \int_{\rho_f = 1}^{\infty} d\rho_f \rho_f^{-\frac{42}{40}} \right)^F < \infty
\]

iii. \(m\) integrations in \([0, 1)\), and \(F - m\) in \([1, \infty)\). In this case \(T(m, F - m)\) can be bounded using Corollaries 1 and 2 as before. The idea is to choose the appropriate subset of representations in the bounds (and the corresponding values of \(\kappa\)) so that only the \(m - F\) representations integrated over \([1, \infty)\) appear in the corresponding denominators. Since this is clearly possible, the \(T(m, F)\) terms are all finite.

We have bounded \(Z\) by a finite sum of finite terms which concludes the proof. \(\square\)
4 Conclusion

Given the history of attempts to quantize general relativity, the finiteness of the Lorentzian state sum is remarkable, and came to us as a surprise. The model is the result of a number of choices made from physical or geometrical arguments. Altering one of those choices seems to generally have the effect of destroying the finiteness. An example is the uncertainty, in the older $15J$ formulation, as to whether to sum independently on the internal spins in tetrahedra or to require them to be equal. The version of the model studied here has $\Theta_4$ terms which can be thought of as sums of $6J$ terms, or alternatively as diagrams which have the effect of forcing the two internal labels to be equal. Thus finiteness seems to impose a choice here, the other version would almost certainly diverge. Similarly, without the balanced constraint of [1], the model would also, most likely, be divergent. That constraint can be deduced from geometrical thinking or from the Plebansky formulation of general relativity; both derivations are unrelated to the representation theory which ensued, so it is interesting that it plays a critical role in finiteness.

In order to turn the model into a complete theory, it is necessary to handle the limit as the number of simplices in a triangulation goes to infinity. It is possible to imagine several plausible approaches to this. The simplest would be to cut off the number of triangulations and regularize, hoping for a good limit. The conjecture of quantum self censorship of [13] might play a role, either in this model or in an extended one with matter terms added, in ensuring that such a limit would exist; the intuition being that any new information in a sufficiently large triangulation would fall into its Schwarzschild radius, and hence not affect physically observable quantities.

An alternative to just studying the limit of larger triangulations is to sum over triangulations, or suitably extended triangulations [8, 9, 10]. From this perspective, the state sum we have studied here is a term in the Feynman expansion of an auxiliary field theory [7]. The field theory fixes the relative weights in the sum over extended triangulations, which includes also a sum over space-time topologies. In this perspective, the result in this paper is the finiteness of Feynman integrals. In such a context, however, singular triangulations appear as well, an issue not addressed here. Finiteness on singular triangulations requires us to get a power decay law above $3/4$ for the $I_{10}$ integrals. We do not know if this is possible or not. The question of including manifolds with conical singularities, or equivalently, more general 2-complexes, is related to this. The possibility of discrete geons is therefore open.

Another question which we think deserves to be investigated is the finiteness of the variant of the model considered in [14], in which timelike as well as spacelike balanced representations are used, and in which the discreteness characteristic of the canonical theory [12] reappear.

More generally, the results presented here emerge from the comparison of two different ways of viewing spinfoam models: the quantum geometric and field theoretic ones. We believe that comparing this approach with the techniques and the results from the other approaches to the construction of spinfoam models,
such as the ones of Reisenberger [16], Freidel and Krasnov [17] and Iwasaki [18], is likely to be productive as well.

The suggestion that quantum geometry is in some sense discrete is an old idea in quantum gravity: it can be traced all the way to Einstein [19]. We view the categorical algebraic elegance of the current model in which such discreteness is realized, and its close relationship to TQFTs, as particularly attractive. The structural similarity between categorical state sums and the Feynman vacuum, emphasized in [13], is also suggestive. Because of the finiteness proof given here, it is now possible to explore the consequences of these ideas and the physical content of this model by exact calculation.

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