Measurements of $B$ Decays to Two Kaons

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Abstract

We report measurements of $B$ meson decays to two kaons using 253 fb$^{-1}$ of data collected with the Belle detector at the KEKB energy-asymmetric $e^+e^-$ collider. We find evidence for signals in $B^+ \rightarrow \bar{K}^0K^+$ and $B^0 \rightarrow K^0\bar{K}^0$ with significances of 3.0$\sigma$ and 3.5$\sigma$, respectively. (Charge-conjugate modes included) The corresponding branching fractions are measured to be $\mathcal{B}(B^+ \rightarrow \bar{K}^0K^+) = (1.0 \pm 0.4 \pm 0.1) \times 10^{-6}$ and $\mathcal{B}(B^0 \rightarrow K^0\bar{K}^0) = (0.8 \pm 0.3 \pm 0.1) \times 10^{-6}$. These decay modes are examples of hadronic $b \rightarrow d$ transitions. No signal is observed in the decay $B^0 \rightarrow K^+K^-$ and we set an upper limit of $3.7 \times 10^{-7}$ at 90% confidence level.

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Recent precise measurements of the branching fractions \cite{1} and partial rate asymmetries \cite{2} from the decays $B \rightarrow K \pi, \pi \pi$ provide essential information to understand the $B$ decay mechanism and to probe possible contributions from new physics. The rates for these decays constrain the hadronic $b \rightarrow s$ and $b \rightarrow u$ amplitudes. Here we report results on $B^0 \rightarrow K^0\overline{K}^0$ and $B^+ \rightarrow K^+\overline{K}^0$ decays, which are examples of $b \rightarrow d$ hadronic transitions. We also discuss a search for $B^0 \rightarrow K^+K^-$, which is sensitive to effects of final-state interactions (FSI) \cite{3}. The results are based on a sample of 275 million $B\overline{B}$ pairs collected with the Belle detector at the KEKB $e^+e^-$ asymmetric-energy (3.5 on 8 GeV) collider \cite{4} operating at the $\Upsilon(4S)$ resonance.

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) comprised of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside the coil is instrumented to detect $K^0_L$ mesons and to identify muons (KLM). The detector is described in detail elsewhere \cite{5}. Two different inner detector configurations were used. For the first sample of 152 million $B\overline{B}$ pairs (Set I), a 2.0 cm radius beampipe and a 3-layer silicon vertex detector were used; for the latter 123 million $B\overline{B}$ pairs (Set II), a 1.5 cm radius beampipe, a 4-layer silicon detector and a small-cell inner drift chamber were used \cite{6}.

Charged kaons are required to have a distance of closest approach to the interaction point (IP) in the beam direction (z) of less than 4 cm and less than 0.1 cm in the transverse plane. Charged kaons and pions are identified using $dE/dx$ information and Cherenkov light yields in the ACC. The $dE/dx$ and ACC information are combined to form a $K-\pi$ likelihood ratio, $\mathcal{R}(K/\pi) = \mathcal{L}_K/(\mathcal{L}_K + \mathcal{L}_\pi)$, where $\mathcal{L}_K$ ($\mathcal{L}_\pi$) is the likelihood that the track is a kaon (pion). Charged tracks with $\mathcal{R}(K/\pi) > 0.6$ are regarded as kaons. Furthermore, charged tracks that are positively identified as electrons or muons are rejected. The electron identification uses information composed of $E/p$ and $dE/dx$, shower shape, track matching $\chi^2$, and ACC light yields, while information from the KLM, $dE/dx$ and ACC are combined to identify muons. The kaon identification efficiency and misidentification rate are determined from a sample of kinematically identified $D^{*+} \rightarrow D^0 \pi^+$, $D^0 \rightarrow K^-\pi^+$ decays, where the kaons from the $D$ decay are selected in the same kinematic region as in $B \rightarrow K\overline{K}$ decays. The kaon efficiency is measured to be $(84.24 \pm 0.13)\%$ for Set I and $(82.84 \pm 0.14)\%$ for Set II, while the pion-fake-kaon rates are $(5.40 \pm 0.08)\%$ and $(6.86 \pm 0.11)\%$, respectively.

Candidate $K^0_S$ mesons are reconstructed through the $K^0_S \rightarrow \pi^+\pi^-$ decay. We pair oppositely-charged tracks assuming the pion hypothesis and require the invariant mass of the pair to be within 18 MeV/c$^2$ of the nominal $K^0_S$ mass. Furthermore, the intersection point of the $\pi^+\pi^-$ pair must be displaced from the IP.

Two variables are used to identify $B$ candidates: the beam-constrained mass, $M_{bc} \equiv \sqrt{E_{beam}^2 - p_B^2}$, and the energy difference, $\Delta E \equiv E_B^* - E_{beam}^*$, where $E_{beam}^*$ is the run dependent beam energy and $E_B^*$ and $p_B^*$ are the reconstructed energy and momentum of the $B$ candidates in the center-of-mass (CM) frame, respectively. Events with $M_{bc} > 5.20$ GeV/c$^2$ and $|\Delta E| < 0.3$ GeV are selected for analysis.

The dominant background is from $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum events. Event topology and $B$ flavor tagging information are used to distinguish between the spherically distributed $B\overline{B}$ events and the jet-like continuum backgrounds. We combine a set of modified Fox-Wolfram moments \cite{9} into a Fisher discriminant. A signal/background likelihood is
formed, based on a GEANT-based Monte Carlo (MC) simulation, from the product of the probability density function (PDF) for the Fisher discriminant and that for the cosine of the angle between the $B$ flight direction and the positron beam. The continuum suppression is achieved by applying a requirement on a likelihood ratio $R = \mathcal{L}_s/(\mathcal{L}_s + \mathcal{L}_{qq})$, where $\mathcal{L}_{s(qq)}$ is the signal ($q\bar{q}$) likelihood. Additional background discrimination is provided by $B$ flavor tagging. For each event, the standard Belle flavor tagging algorithm provides a discrete variable indicating the probable flavor of the tagging $B$ meson, and a quality $r$, a continuous variable ranging from zero for no flavor tagging information to unity for unambiguous flavor assignment. An event with a high value of $r$ (typically containing a high-momentum lepton) is more likely to be a $B\overline{B}$ event, and a looser $R$ requirement can be applied. We divide the data into $r > 0.5$ and $r \leq 0.5$ regions. A selection requirement on $R$ for events in each $r$ region of Set I and Set II is applied according to a figure-of-merit defined as $N^\text{exp}/\sqrt{N^\text{exp} + N^\text{qq}}$, where $N^\text{exp}$ denotes the expected signal yields based on MC simulation and the assumed branching fractions, $1 \times 10^{-6}$, and $N^\text{qq}$ denotes the expected $q\bar{q}$ yields from sideband data ($M_{bc} < 5.26 \text{ GeV}/c^2$).

Background contributions from $\Upsilon(4S) \rightarrow B\overline{B}$ events are investigated using a large MC sample, which includes events from $b \rightarrow c$ transitions and charmless decays. After all the selection requirements, no $B\overline{B}$ background is found for the $B^0 \rightarrow K^0\overline{K}^0$ mode. Owing to $K^-\pi$ misidentification, large $B^0 \rightarrow K^+\pi^-$ and $B^+ \rightarrow K^0\pi^+$ feed-across backgrounds appear in the $B^0 \rightarrow K^+K^-$ and $B^+ \rightarrow \overline{K}^0K^+$ modes, respectively. A small charmless three-body contribution is found at low $\Delta E$ values for these two modes.

The signal yields are extracted by performing unbinned two dimensional maximum likelihood (ML) fits to the ($M_{bc}, \Delta E$) distributions. The likelihood for each mode is defined as

$$
\mathcal{L} = \exp \left( - \sum_{s,k,j} N_{s,k,j} \right) \prod_{i} \left( \sum_{s,k,j} N_{s,k,j} \mathcal{P}_{s,k,j,i} \right),
$$

where $s$ indicates Set I or Set II, $k$ distinguishes between events in the $r < 0.5$ and $r > 0.5$ regions, $i$ is the identifier of the $i$-th event, $P(M_{bc}, \Delta E)$ is the two-dimensional PDF of $M_{bc}$ and $\Delta E$, $N_j$ is the number of events for the category $j$, which corresponds to either signal, $q\bar{q}$ continuum, a feed-across due to $K^-\pi$ misidentification, or background from other charmless three-body $B$ decays.

All the signal PDFs ($P_{s,k,j=\text{signal}}(M_{bc}, \Delta E)$) are parametrized by a product of a single Gaussian for $M_{bc}$ and a double Gaussian for $\Delta E$ using MC simulations based on the Set I and Set II detector configurations. The same signal PDFs are used for events in the two different $r$ regions. Since the $M_{bc}$ signal distribution is dominated by the beam energy spread, we use the signal peak positions and resolutions obtained from $B^+ \rightarrow D^0\pi^+$ data ($D^0 \rightarrow K^0_S\pi^+\pi^-$) sub-decay is used for the $K^0\overline{K}^0$ mode, while $D^0 \rightarrow K^+\pi^-$ is used for the other two modes) with small mode dependent correlations obtained from MC. The MC-predicted $\Delta E$ resolutions are verified using the invariant mass distributions of high momentum $D$ mesons. The decay mode $D^0 \rightarrow K^+\pi^-$ is used for $B^0 \rightarrow K^+K^-$, $D^+ \rightarrow K^0_S\pi^+$ for $B^+ \rightarrow K^0\pi^+$ and $D^0 \rightarrow K^0_S\pi^+\pi^-$ for $B^0 \rightarrow K^0\overline{K}^0$. The parameters that describe the shapes of the PDFs are fixed in all of the fits.

The continuum background in $\Delta E$ is described by a linear function while the $M_{bc}$ distribution is parameterized by an ARGUS function $f(x) = x\sqrt{1-x^2} \exp [-\xi(1-x^2)]$, where
$x$ is $M_{bc}$ divided by half of the total center of mass energy $E$. Therefore, the continuum PDF is the product of this ARGUS function and the linear function, where the overall normalization, $\xi$ and the slope of the linear function are free parameters in the fit. These free parameters are $r$-dependent and allowed to be different in Set I and Set II. The background PDFs for charmless three-body $B$ decays for the $K^+K^-$ and $\bar{K}^0K^+$ modes are each modeled by a smoothed two-dimensional histogram, obtained from a large MC sample. The feed-across backgrounds for these two modes from the $K^+\pi^-$ and $K^0\pi^+$ events have $M_{bc} - \Delta E$ shapes similar to the signals with the $\Delta E$ peak positions shifted by $\approx 45$ MeV. The methods to model the $K^+K^-$ and $\bar{K}^0K^+$ signal PDFs are also applied to describe the feed-across background.

When likelihood fits are performed, the yield for each background component ($N_{s,k,j}$ where $j = q\bar{q}$, feed-across, charmless) is allowed to float independently for each $s$ (Set I or Set II), and $k$ bin (low or high $r$ region). For the signal component, the same branching fraction is required by constraining the number of signal events in each $(s,k)$ bin using the measured efficiency in the corresponding $(s,k)$ bin. Table I summarizes the fit results for each mode. We observe $13.3 \pm 5.6 \pm 0.6$ $K^0K^+$ and $15.6 \pm 5.8^{+1.1}_{-0.6}$ $K^0\bar{K}^0$ signal events with significances of $3.0\sigma$ and $3.5\sigma$, respectively. The second errors in the yields are the systematic errors from fitting, estimated from the deviations after varying each parameter of the signal PDFs by one standard deviation, and from modeling the three-body background, studied by excluding the low $\Delta E$ region ($< -0.15$ GeV) and repeating the fit. At each step, the yield deviation is added in quadrature to provide the fitting systematic errors and the statistical significance is computed by taking the square root of the difference between the value of $-2\ln L$ for the best fit value and zero signal yield. The smallest value is chosen to be the significance including the systematic uncertainty.

Figure 1 shows the $M_{bc}$ and $\Delta E$ projections of the fits after requiring events to have $|\Delta E| < 0.06$ GeV and $5.271$ GeV/$c^2 < M_{bc} < 5.289$ GeV/$c^2$, respectively. The feed-across yields are $47.1 \pm 8.7$ in the $K^+K^-$ mode and $16.4 \pm 6.1$ in the $K^0K^+$ mode. The amounts of the feed-across background are consistent with the expectations of $49.1$ $K^+\pi^-$ and $18.8$ $K^0\pi^+$ events, based on MC simulation and measured branching fractions [12]. The MC modeling of the requirement on the likelihood ratio, $R$, is investigated using the $B^+ \rightarrow D^0\pi^+ (D^0 \rightarrow K^0\pi^+\pi^-)$ for $K^0K^0$ and $D^0 \rightarrow K^+\pi^-$ for the others) samples. The obtained systematic errors are $\pm 2.9\%$ for $B^0 \rightarrow K^0\bar{K}^0$ and $\pm 6.8\%$ for the other two modes. The systematic error on the charged track reconstruction efficiency is estimated to be around $1\%$ per track using partially reconstructed $D^*$ events. The resulting $K^0_S$ reconstruction is verified by comparing the ratio of $D^+ \rightarrow K^0_S\pi^+$ and $D^+ \rightarrow K^-\pi^+\pi^+$ yields with the MC expectation. The resulting $K^0_S$ detection systematic error is $\pm 4.5\%$. The final systematic errors are then obtained by quadratically summing the errors due to the reconstruction efficiency and the fitting systematics.

With 275 million $BB$ pairs, we find evidence of $B^+ \rightarrow \bar{K}^0K^+$ and $B^0 \rightarrow K^0\bar{K}^0$ with branching fractions $\mathcal{B}(B^+ \rightarrow \bar{K}^0K^+) = (1.0 \pm 0.4 \pm 0.1) \times 10^{-6}$ and $\mathcal{B}(B^0 \rightarrow K^0\bar{K}^0) = (0.8 \pm 0.3 \pm 0.1) \times 10^{-6}$. These are examples of hadronic $b \rightarrow d$ transitions. Our measurements are consistent with preliminary results reported by the BaBar collaboration and agree with some theoretical predictions [13, 14, 16, 17]. It has been suggested that the branching fraction and CP asymmetry of the mode $B^0 \rightarrow K^0\bar{K}^0$, which originates from the flavor-changing neutral current process $b \rightarrow d\bar{s}s$, may be sensitive to physics beyond the Standard Model [16]. Measurements with larger statistics are needed for this purpose. No signal is
FIG. 1: $M_{bc}$ (left) and $\Delta E$ (right) distributions for $B^0 \rightarrow K^+K^-$ (top) and $B^+ \rightarrow \overline{K}^0K^+$ (middle) and $B^0 \rightarrow K^0\overline{K}^0$ candidates. The histograms show the data, while the curves represent the various components from the fit: signal (dashed), continuum (dotted), three-body $B$ decays (hatched), background from mis-identification (dash-dotted), and sum of all components (solid). In the $K^+K^-$ mode, there is a large contribution from misidentified $K^+\pi^-$ but no significant signal excess. In the $\overline{K}^0K^+$ mode, the signal and misidentified $K^0\pi^+$ contributions are comparable in size. In the $K^0\overline{K}^0$ mode, there is a signal excess but no misidentification background.

observed in $B^0 \rightarrow K^+K^-$ and we set the upper limit of $3.7 \times 10^{-7}$ at the 90% confidence level, using the Feldman-Cousins approach [18] taking into account both the statistical and systematic errors [19].

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TABLE I: Fitted signal yields, reconstruction efficiencies, product of efficiencies and sub-decay branching fractions ($B_s$), branching fractions and significances for individual modes.

| Mode      | Yield | Eff.(%) | Eff.×$B_s$ (%) | $B(10^{-6})$ | Sig. |
|-----------|-------|---------|----------------|--------------|------|
| $K^+K^-$  | 2.5  | 15.5    | 15.5           | < 0.37       | 0.5  |
| $\bar{K}^0K^+$ | 13.3 | 14.5    | 5.0            | 1.0 ± 0.4 ± 0.1 | 3.0  |
| $K^0\bar{K}^0$ | 15.6 | 28.7    | 6.8            | 0.8 ± 0.3 ± 0.1 | 3.5  |

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