Reconstitution of Static Deflections of Suspension Bridge Based on Inclinometer Data

Riqing LAN¹,²*, Yushi WANG² and Qinglei CHI³
¹China IPPR International Engineering CO., LTD, Beijing 100089, China; ²Institute of Geophysics, China Earthquake Administration, Beijing, 100081, China; ³Institute of Engineering Mechanics, China Earthquake Administration, Harbin 150080, China

Abstract. Bridge deflection is a significant index reflecting the rigidity, overall stability, carrying capacity, and seismic performance of bridge structures. However, it is difficult to measure accurately, particularly for large-span bridges with complex deflection curves. Inclinometers are tentatively used in the deflection measurements of highway and railway bridges, and they have been found to be potential high-precision deflection measurement instruments. An effective inclinometer monitoring scheme was designed and used to monitor the static deflections of bridge, and an improved data processing method was proposed through using individual analytical solutions in this study. Based on that, analysis was done to explain why the "5 inclinometers" suggested by previous research is the best choice for the deflection test of the simple supported and the continuous beam bridge, which has not explicitly explained in the previous literature. Through the comparative study of the deflection calculation results of the bridge finite element model, inclinometer is applied to measuring the static deflection of a large-span suspension bridge for the first time, and it is found when the number of the instruments is enough and arrangement of the instrument is reasonable, the deflection of suspension bridge can be tested using inclinometer method. In order to reduce the number of test points and improve the deflection measurement precision, semi-span arrangement scheme of test points is proposed, which is applied in the static deflection test of the Jiangyin Yangtze River Bridge, and the error of static deflections is only 1.13% compared to GPS observations.

1. Introduction
Both the static and dynamic deflections of a bridge are significant indices in bridge health monitoring. These indicators can directly reflect the rigidity, global stability, carrying capacity, and seismic performance of the bridge structures. Currently, steel-wire type displacement sensors, level gauges, electronic total station instruments, photoelectric imaging, global positioning systems (GPS), range cameras, and inclinometers are the main instruments used to measure bridge deflection. The steel wire type displacement sensor is simple and easy to operate, but the deflections values are superposed by the subsidence of the piers and deformation of the bearing supports, so the deflection values always appear much larger than that of actual ones. The testing accuracy of steel wire type displacement sensors is easily influenced by the wind speed and other environmental factors. Furthermore, the method is not applicable for where there is no stationary reference point to fix the steel wire such as cross-river, cross-highway and cross-railway bridges [1]. Level gauges or electronic total station...
instruments are widely used in testing deflection as they have high accuracy. But again, the methods need stationary observation points [2]. Photoelectric or displacement sensors can continuously measure static or dynamic deflections, but both methods are severely affected by external environmental conditions such as rain, fog, and other bad weather besides needing stationary observation points. Currently, photoelectric or displacement sensors are mainly used in small- and medium-sized bridges [3, 4]. Previous studies of the GPS method found remarkable accuracy only within a few millimeters, but a base station located within 1-5 km away from the bridge is required [5, 6]. Range cameras can test the static and the dynamic deflections of a single position, but it also needs a stationary observation, which is difficult to find when the bridge is located on a river or on a sea[7, 8, 9].

A new method to measure bridge deflection by using inclinometers has been proposed [1, 10, 11], which can obtain the real-time dynamic inclination angles and deflection. One advantage is that the method is slightly affected by the environment conditions because the instruments are arranged on the deck directly; the other one is the deflection does not include the subsidence of the piers and the deformation of the bearing supports. Currently, the method is only applied to measuring short-span highway beam bridges with high-precision[10]. But for suspension, cable-stayed, arch bridges and other large-span beam bridges, the deflection curves are generally more complex. To verify whether the inclinometer is suitable for measuring the deflection of a large-span bridge, a large-span suspension bridge is examined in this paper. Main contents and conclusions are as follow: 1) Comparing with the previous studies, the basic function of the deflection is modified based on the analytical solutions of a continuous small-span beam bridge, and then explains why 5 inclinometers are the best choice for testing bridge deflection, which has not explicitly explained in the previous literature; 2) inclinometer is applied to measuring the deflection of a large-span suspension bridge for the first time in the bridge monitoring field.

2. Working Principle of Inclinometer

A bridge will deform under external action as well as the bridge section will rotate with an inclination angle $\theta_1$, $\theta_2$, \ldots, $\theta_n$, which can be measured by an inclinometer distributed at the position of the section, of course, more inclination angle values can be obtained correspondingly distributed along its longitudinal direction. Consequently, the deflection curve of bridge can be acquired through the integration of the inclination angle curve with considering the boundary conditions.

The section plan view of the inclinometer is shown in Figure 1, and its working principle in Figure 2. According to the mechanics principle, the equation of motion of the mass is\[12\]

$$K_1 \ddot{\theta} + b \dot{\theta} + k \theta + Gi = -Mg\theta_1$$

(1)

$$u_0 = K_c X = K_c L \theta$$

(2)

\begin{center}
\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{inclinometer.png}
\caption{Section plan view of the inclinometer}
\end{figure}
\end{center}

\begin{center}
\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{working_principle.png}
\caption{Working principle of the inclinometer [12]}
\end{figure}
\end{center}

where $K_1$ is the moment of inertia($= L_3 L_4 m$); $L_3$ is the equivalent length of the pendulum; $L_4$ is the length which calculated from the moving center of the pendulum to the center of mass block $m$; $b$ is the damping coefficient including air damping; parameter $k$ is the spring stiffness; $M$ is quality of inclinometer; $X$ is the displacement of the capacitance sensor; $G(=BL_1 L_2)$ is the electric constant of
the coil meeting the requirement of the damping ratio; $BL_1$ is the electromechanical coupling coefficient; $L_2$ is the length of the pendulum; $\theta$ is the inclination of the pendulum; $\theta_1$ is vertical inclination of a measuring point; $K_C$ is the sensitivity of the capacitance sensor; $g$ is an electric constant of the self-calibration coil; and $u_0$ is the output voltage. The solution to Eqs. 1 and 2 is[12]

$$\frac{u_0}{\theta_1} = \frac{kL_2Mg}{n^2k_1} \left( \frac{S^2}{n^2} + \frac{2D}{n} (S+1) \right)$$

where $S$ is an operator, and $n$ is natural frequency of vibration. Here, $n^2=k/K_1$, $D$ is the damping constant, $D=G_1^2/2K_1nR$, and $R$ is the resistance of the coil circuit.

3. Reconstitution of Deflection Curve by Analytical Solution

Hou[10,11] proposed the least-square method to calculate the deflection of bridge. It is assumed that $k$ inclinometers are installed along the bridge, and the inclination angles are $\theta_1(t), \theta_2(t), \cdots, \theta_k(t)$ at coordinates $x_1, x_2, \cdots, x_k$ respectively. An appropriate deflection curve $y(x,t)$ is chosen to meet the boundary constraint conditions, which can be described as

$$y(x,t) = A(x) \{a_1(t), a_2(t), \cdots, a_{k-1}(t)\} = \{g_1(x), g_2(x), \cdots, g_{k-1}(x)\}$$

where, $g_j(x)$ is the basis function group related to the coordinates of inclinometers; $A(x)$ is the function which satisfies the boundary conditions; and $a_j(t)$ is the constant coefficient of the basis function $g_j(x)$ at time $t$.

The inclination angles at the position of the inclinometers can be obtained by the derivation of (4), which should be equal to the inclination angle values measured by the inclinometers ($\theta_1, \theta_2, \cdots, \theta_k$) theoretically, that is

$$\{a_1(t), a_2(t), \cdots, a_{k-1}(t)\} \begin{bmatrix} g_1(x) \\ g_2(x) \\ \vdots \\ g_{k-1}(x) \end{bmatrix} + A(x) \begin{bmatrix} g_1(x) \\ g_2(x) \\ \vdots \\ g_{k-1}(x) \end{bmatrix} = \theta_n \{n=1, \cdots, k\}$$

where $A(x)$ and $g_j(x)$ are the first order derivatives of the function $A(x)$ and $g_j(x)$ respectively. There are $k-1$ unknown variants $a_1(t), a_2(t), \cdots, a_{k-1}(t)$ in the above equation set while there are $k$ equations. A group of optimal solutions $a_j^*(t)(j=1, 2, \cdots, k-1)$ can be obtained by using the least square method. To obtain the optimal solutions $a_j(t)$, the following objective function $M(a_1, a_2, \cdots, a_{k-1})$ is constructed:

$$M(a_1, a_2, \cdots, a_{k-1}) = \sum_{j=1}^{k} (\theta(x_j) - \theta_j)^2$$

where

$\theta(x_j)$: is the inclination angle obtained from the fitting deflection curve

$\theta_j$: is the inclination angle measured by the inclinometer

When the objective function is the minimum, the optimal solution is obtained, that is,

$$\frac{\partial M(a_0)}{\partial a_n} = 2 \sum_{j=1}^{k} (\theta(x_j) - \theta_j) \frac{\partial \theta(x_j)}{\partial a_n} = 0 \ (n=1, 2, \cdots, k-1)$$

Substituting this group of optimal solutions into (1), and the deflection curve of a bridge can be acquired:
How to determine the basis function group $g_j(x)$ in formula (8) is one of the key problem. While paper [10-11] did not give the expression of $g_j(x)$.

It is proposed that $g_j(x)$ can be calculated by producing $N$ orthogonal functions [12], that is,

$$
\begin{align*}
\left\{ g_1(x) = x \\
g_2(x) = (x-c_1) \ g_1(x) \\
g_{n+2}(x) = (x-c_{n+1}) \ g_{n+1}(x) - b_n \ g_n(x) \\
c_n = \frac{\int g_n^2(x) \ dx}{\int g_n^2(x) \ dx} \\
b_n = \frac{\int g_{n+1}^2(x) \ dx}{\int g_n^2(x) \ dx} \\
\end{align*}
$$

In paper[12], different number and different arrangement of the instruments are discussed, and through the comparison of a large number of combinations, it concludes that when five instruments are distributed evenly on the bridge, the error is the least, but the paper did not explain the reason that why it is the least. In this paper, the analytical solution of the deflection is adopted to explain it.

For a simply supported beam bridge under the action of a uniform load $q$, the deflection equation with location $x$ is defined as,

$$
y = \frac{q x}{24 EI} \left( l^3 - 2 l x^2 + x^3 \right) 
$$

And for a trapezoidal loads $q_1$ and $q_2$, the deflection equation is,

$$
y = \frac{1}{360 EI} \left[ 3 q_0 \left( \frac{x^5}{5} + 15 q_1 \left( \frac{x^4}{4} - 10 \left( 2 q_1 + q_2 \right) \frac{x^3}{3} + 5 q_2 \right) \right) \right] \quad (11)
$$

where $q_0=q_2-q_1$.

For a continuous beam bridge, such as a two-span or three-span continuous beams under the action of uniform load $q$, the deflection is

$$
y = \frac{q x}{24 EI} \left( 8 l^3 - 4 l x^2 + x^3 \right) - \frac{5q l x}{48 EI} \left( 3 l^2 - x^2 \right) \quad 0 \leq x \leq 2l
$$

where $l$ is the bridge span, and $EI$ is the stiffness of the beam.

From equations (10)-(12), the deflection curve is a power four or power five polynomial with one variable. According to the characteristics of simply supported beams or continuous beams, the deflection is 0 when $x=0$ or $x=l$. Thus, if not considering the settlement of bearing supports and piers, $A(x)$ can be expressed as,

$$
A(x) = y(x-l)
$$

Thus, $g_j(x)$ is a quadratic polynomial for a uniformly-distributed load, and a cubic polynomial for an irregularly-distributed load. Considering the load on the bridges is very complicated, a cubic polynomial needs to determine four unknown coefficients that can fit the deflection curve better, which requires at least five inclinometers. From this, it is clear that 5 inclinometers are the best choice on testing deflections of simply or continuous supported bridges.

In conclusion, for simply or continuous supported bridges, it is highly suggested that $g_j(x)$ should be taking as,
Equations (9) can be converted into Eq. (14), while Eq. (14) can be used to reduce the calculation time due to its simpler form.

4. Case study—simply supported beam test in the laboratory

To verify the accuracy and reliability of Eq. (14), an inverted I-shaped simply supported beam with the length of 6.03m is selected as the test model. 5 inclinometers are used and the layout of them is shown in Figure 3. In the test, a moving load is added on the beam, which is moving from left side to right side, then comes back and stays in the left side of the beam. In order to compare with the result of steel-wire type displacement sensors, a steel-wire type displacement sensor is installed under 3# inclinometer, its main technical performance is shown in Table 1.

The test data of the beam is shown in Figure 4a). The analysis and test results by using Eq. (14) are shown in Figure 4. It can be seen from Figure 4b) that the results of inclinometer are basically consistent with steel-wire type displacement sensor. The maximum deflections of steel-wire type displacement sensor and inclinometer are 4.534mm and 4.598mm, respectively, so the error is only 1.41%.

![Figure 3. Installation position of inclinometers](image)

![Figure 4a. Test data and analysis results](image)

It means that using 5 inclinometers and Eq. (14) can satisfy the engineering requirements. Next, we try to extend the inclinometer test method to the deflection test of a suspension bridge.
5. Static deflection of a suspension bridge

The deflection calculation method for a suspension bridge using inclination angles is studied in this section. The main span of the bridge is 853.44mm long with its tower high at 128m in total. A finite element model of the bridge is generated from commercial software with a concentrated force is applied in the middle of the main span. As shown in Figure 5, the corresponding inclination angles and deflections of each section along the bridge are calculated. To ensure Eq.(14) can be used to calculate the deflection of suspension bridges, it assuming that the inclination angles of the various sections of the bridge are the same as what FEM calculated, from which the deflection of the bridge is computed and compared with the finite element method. If the relative error between the two results is less than 5%, Eq. (14) can be considered an appropriate method to calculate the deflection of a suspension bridge.

Figure 5. Deflection and inclination angle curve of the suspension bridge. a) Deflection curve computed by finite element method (FEM); b) inclination angle curve computed by FEM

As shown in Figure 5b), 5 angular values from 21 uniform distribution testing point numbered from 1 to 21 are chosen to compute deflection curve according to Eq.(14), and the distance between two adjacent points is 42.672m. As shown in Table 2, three representative working conditions are selected.

Table 2. Scheme of the chosen test points

| Working conditions | The five points |
|--------------------|----------------|
| Case 1             | 1, 6, 11, 16, 21 |
| Case 2             | 1, 3, 8, 14, 21 |
| Case 3             | 1, 3, 8, 14, 20 |

According to Eq. (4)-(8) and Eq.(14), the deflection curves of Case 1-3 are computed and shown in Figure 6. All three results fit poorly with those of FEM, it means that Eq. (14) cannot be used to compute the deflections of a suspension bridge directly. Figure 6 also shows that the results are related to the chosen test points, there are large discrepancy between the three different point groups.

Figure 6. Deflection curve of suspension bridge based on five measurement points

Figure 7. Deflection curves of suspension bridge based on six measurement points
(1) Deflection reconstitution of the suspension bridge

Since five test points produce poor results, six points are chosen here to reconstitute the suspension deflection based on angular values. Also, four chosen groups of six-measuring-point combinations listed in Table 3.

Figure 7 shows comparison between four groups of solutions and the FEM. The worst is Case 4 and the best is Case 1. The difference between Case 1 and 4 is very large. For Case 1, the results of its comparison with FEM are presented in Table 4.

Table 3. Scheme of the chosen six test points

| Working conditions | Measurement points |
|--------------------|--------------------|
| Case 1             | 2, 6, 11, 15, 18, 20 |
| Case 2             | 1, 6, 11, 15, 18, 21 |
| Case 3             | 1, 3, 8, 14, 19, 21 |
| Case 4             | 1, 3, 8, 14, 20, 21 |

Table 4. Comparison between FEM and Case 1

| Distance(m) | FEM(mm) | Case1(mm) | Relative error (%) |
|-------------|---------|-----------|--------------------|
| 213.4       | -279.9  | -230.4    | 17.68              |
| 298.7       | 592.2   | 606.9     | 2.48               |
| 384         | 1411.2  | 1336.3    | 5.31               |
| 426.7       | 1534.5  | 1453.6    | 5.27               |
| 725.4       | -469.2  | -436.6    | 6.95               |

In Case 1, the deflection in the middle main span is 1453.6mm. Compared with 1534.5mm which are acquired from FEM, the error is 5.27%, it is still a little greater than 5%.

To improve the accuracy of deflection test, the analytical solution of deflection of suspension bridge is studied. The schematic diagram of deflection calculation of suspension bridge is shown in Figure 8, where $H_g$ is horizontal force generated by dead load $q$, and $H_g=ql^2/8f$, $l$ and $f$ are the horizontal distance of the two pivots of the main cable and sag of the main cable; $H_p$ is horizontal force produced by other external loads $p(x)$; $M$ and $Q$ are bending moment and shear force of section; $\eta$ is deflection of main cable;

(1) Differential equation of deflection curve of stiffening beam

Based on the detachment of the micro-segment $dx$ in Figure 8, after some deduction and simplifications, differential equation of deflection curve of stiffening beam is [13],

$$EI \frac{d^4 \eta}{dx^4} - H \frac{d^2 \eta}{dx^2} = p(x) + H_p \frac{d^2 y}{dx^2} \quad (15)$$

(2) The differential equation of elastic elongation and temperature variation of main cable is [13],

$$H_p \frac{L_k}{E_k A_k} + \alpha L_k \eta = 0 \quad (16)$$

where, $L_k = \int_0^L \frac{dx}{\cos \phi}$; $L = \int_0^L \frac{dx}{\cos \phi}$; $F_\eta = \int_0^L \frac{d \eta}{\cos \phi}$; $\alpha$ is linear thermal expansion coefficient; $f$ is temperature exchange value; $L$ is horizontal distance between anchor points on both ends of the main cable; $\phi$ is inclination angle of main cable; $\Sigma v$ is the sum of side span and middle span of suspension bridge, and for middle span, $v = -d^2 y/dx^2 = 8f/l^2$; for side span, $v_1 = d^2 y_1/dx^2 = 8f_1/l_1^2$. 
According to the principle of the substitutional beam method when bearing an external concentrated force in the middle main span, the above two equations can be solved jointly and the corresponding deflection expression is obtained [13,14],
\[
y = \frac{1}{H} \left( \frac{3}{2} (Lx-x^2) - \frac{q}{h^2} \left[ 1 - \frac{\text{ch} \left( \frac{\beta}{H} (L-x) \right)}{\text{ch} \left( \beta L \right)} \right] \right)
\]  
(17)

After taking the derivative, the inclination angle equation is obtained as,
\[
\theta = \frac{y}{H} \left( q \left( \frac{L}{2} - x \right) - \frac{q}{\beta} \left[ \text{ch} \left( \frac{\beta}{H} (L-x) \right) \right] \right)
\]  
(18)

where \( L \) is the bridge span; \( q \) is the external concentrated force; \( \beta = \sqrt{H/EI} \); \( H \) is the horizontal axis force of the suspension bridge; and \( EI \) is the flexural rigidity.

Combining Eq.(4)-Eq.(8) and Table 3 with Eq.(17) and Eq.(18) (namely improved method), the deflections of the suspension bridge were computed again, and the deflection curves are shown in Figure 9. It is found that the best result is also obtained in Case 1.

(2) Influence of the number of test points

In this study, three working conditions are chosen to clarify the influence of the number of measuring points as listed in Table 5, which are Case1 with 6 points, Case 2 with 9 points, Case 3 with 11 points.

Figure 8. Schematic diagram of deflection calculation of suspension bridge[13]

Figure 9. Deflection curves of suspension bridge based on six test points (by Eq. 17)

Figure 10. Comparison of deflection curves by using Eq. (14) and Eq. (17)
Table 5. Test points of three working conditions

| Working conditions | Measurement points |
|--------------------|--------------------|
| Case 1             | 2, 6, 11, 15, 18, 20 |
| Case 2             | 1, 3, 6, 8, 11, 14, 17, 19, 21 |
| Case 3             | 1, 2, 4, 6, 8, 11, 14, 16, 18, 19, 21 |

According to equations of the improved method and Table 5, the deflection curves were computed and shown in Figure 11.

Figure 11 shows that when eleven measurement points are used, the deflection curve is highly consistent with FEM solution. The deflection of the middle main-span of the suspension bridge is 1528.56mm. Compared with 1534.5mm which calculated from FEM, the error is only 0.387%.

From Figure 10 and Figure 11, the deflection test of suspension bridge can be realized by inclinometer method, but there are some restrictions, for example, it needs enough number of instruments, correct choice of expression and reasonable arrangement of the instrument. In order to improve precision of the test and reduce the number of test points, a semi-span arrangement scheme with only a certain number of instruments is suggested.

(3) semi-span arrangement scheme of test points

For the large-span suspension bridge mentioned above, the test points 1,3,5,7,9,11 on the left semi-span were selected, and the deflection curve were computed shown in Figure 12. The deflection in the middle of the main span is 1538.56mm.

For clarity, some comparison results are listed in Table 6, and the accuracy of the semi-span arrangement scheme satisfies the engineering requirements.

| Layout scheme          | Proposed method(mm) | FEM (mm) | Error (%) |
|------------------------|----------------------|----------|-----------|
| Whole-span             | 6 test points & Eq.(14) | 1453.6   | 5.27      |
|                        | 6 test points & Eq.(17) | 1509.9   | 1.60      |
|                        | 11 test points & Eq.(17) | 1528.6   | 0.387     |
| Semi-span              | 6 test points & Eq.(17) | 1538.6   | 0.264     |

It can be seen from table 6, that only 6 test points can achieve high-precision when adopting semi-span arrangement scheme, while in the whole-span arrangement scheme, 11 test points are needed to achieve the same precision. Therefore, semi-span arrangement scheme can effectively reduce the amount of test points under the requirement of engineering precision.
6. Static deflection measurement of the Jiangyin Yangtze River

(1) the brief introduction of the Jiangyin Yangtze Bridge

Jiangyin Yangtze River Bridge is located between Jiangyin city and Jingjiang city, Jiangsu province. In this large-span suspension bridge, the diameter of two main cables is 0.870m, with the height of the cable support tower is 197m. The main span of the bridge is 1,385m long and 33.8m wide. When the bridge was built, eight sets of kinematic GPS monitoring equipment was installed on two towers and distributed between two towers at 1/4L, 1/2L, and 3/4L (L is the bridge length) to monitor the deformation of the bridge in real time. The bridge was opened on September 28, 1999, and 15 years later in July 2014, it was closed two times for health status monitoring. As one of the measurement term, 5 inclinometers were installed on the bridge at 36m, 164m, 340m, 484m, and 612m from the left tower, as shown in Figure 13.

![Figure 13. Distribution of inclinometers and GPS receivers](image)

(2) Test loading vehicle

The gross weight of the test loading vehicle is 300kN, which includes 60kN of the front axle weight and each 120kN of the middle and rear axle weight. From the middle axle, the distance to the front axle and the rear axle is 3.5m and 1.4m respectively as shown in Figure 14.

![Figure 14. Size of test loading vehicle (cm)](image)

(3) Load arrangement

In the static deflection test, 60 testing loading vehicles are used, which are applied symmetrically in the middle span of the bridge as shown in Figure 15.

![Figure 15. Symmetrical loading in the middle-span (m)](image)
The spots photos of the experiment equipments

Figure 16 shows the spots photos of the experiment equipments.

As shown in Figure 15, under the action of testing loading vehicles, the vertical deformation can be calculated according to Eq.(3)-Eq.(8) and (17). The corresponding comparison results are listed in Table 7 and Figure 17. At the location of GPS1, the results of GPS and the inclinometer are 573.08mm and 561.38mm. The difference is only 11.70mm. In the midspan, the results of GPS and the inclinometer are 1644.72mm and 1626.29mm, the relative error is only 1.13%.

![Figure 16. Test instrument and test vehicle, a) inclinometer; b) test vehicle; c) and d) test equipment installation and connection](image1)

![Figure 17. Deflection curve of GPS and inclinometer method](image2)

| Location | GPS result (mm) | Inclinometer (mm) | Error (%) |
|----------|----------------|------------------|-----------|
| GPS1     | 573.08         | 561.38           | 2.08      |
| GPS2     | 1644.72        | 1626.29          | 1.13      |

Table 7. Comparison between GPS and inclinometer results
From Figure 17 and Table 7, it is proved that semi-span arrangement scheme could achieve a high-precision; besides, it also proved that semi-span can reduce the amount of test points effectively.

7. Conclusion

Based on inclinometer data and analytical solutions, the measuring method of the static deflections of a large-span suspension bridge was improved after analyzing and discussing the measuring results. There are several results that need to be summarized:

1) The analysis proved why "5 inclinometers" in previous researches is the best choice for measuring bridge deflection, which has not been illustrated clearly according to previous literature.

2) According to the analytical solution of the deflection curve of a suspension bridge, an effective arrangement plan of inclinometers was suggested in this study and applied to the deflection measurement for a large-span suspension bridge for the first time.

3) For complex bridges, such as the suspension bridges, more instruments are needed to measure the deflection values accurately. In order to reduce the number of test points and improve the deflection measurement precision effectively, the semi-span arrangement scheme is suggested in this study.

4) By comparing the deflection test results between inclinometer method and GPS in Jiangyin Yangtze River Bridge, the relative error of the deflection is only 1.13% in the midspan of the bridge. The results show that the semi-span arrangement scheme can be applied to the deflection measuring of suspension bridge.

Acknowledgments

This research is supported by the National Key Research and Development Program of China (No. 2017YFC1500400), Beijing Natural Science Foundation (No. 8172049), Major Science and Technology Specialities of SINOMRCH (No. SINOMAST-ZDZX-2017-05).

References

[1] Yang X.S., Hou X.M., Liao Z.P., et al. (2002). A new method for bridge deflection measurement. China Civil Engineering Journal, 35(2): 92-96 (in Chinese)

[2] Psimoulis P.A., and Stiros S.C. (2013). Measuring deflections of a short-span railway bridge using a robotic total station. Journal of Bridge Engineering, 18(2):182-185

[3] Nassif H.H., Gindy M., and Davis J. (2005). Comparison of laser Doppler vibrometer with contact sensors for monitoring bridge deflection and vibration. NDT&E International, 869(38): 213-218

[4] Kwak E., Detchev I., Habib A., et al. (2013). Precise photogrammetric reconstruction using model-based image fitting for 3D beam deformation monitoring. Journal of Surveying Engineering, 139(3): 143-155

[5] Ashkenazi V., and Roberts G.W. (1997). Experimental monitoring of the Humber Bridge using GPS. Proceedings of the Institution of Civil Engineering, 120(4), 177-182

[6] Meng X., Dodson A.H., and Roberts G.W. (2007). Detecting bridge dynamics with GPS and triaxial accelerometers. Engineering Structures, 29(11):3178-3184

[7] Jáuregui D.V., White K.R., Woodward C.B., et al. (2003). Noncontact photogrammetric measurement of vertical bridge deflection. Journal of Bridge Engineering, 8(4): 212-222

[8] Lichti D., and Qi X. (2012). Range camera self-calibration with independent object space scale observations. Journal of Spatial Science, 57(2), 247–257

[9] Chen S., Li X.J., Zhang H., et al. (2015). Vision-based displacement test method for high-rise building shaking table test. Journal of Vibroengineering, 17(8), 4057-4647

[10] Hou X.M., Yang X.S., Liao Z.P., et al. (2002). Bridge Deflection Real Time Measurement. Earthquake Engineering and Engineering Vibration, 22(1): 67-72 (in Chinese)

[11] Hou X.M., Yang X.S, and Huang Q. (2005). Using inclinometers to measure bridge deflection. Journal of Bridge Engineering, 10(5): 564-569
[12] He X.L., Yang X.S., and Zhao L.Z. (2014). New method for high-speed railway bridge dynamic deflection measurement. Journal of Bridge Engineering, 19(7)
[13] Zhao X.D., Cheng X.Y., and Li F. (2007). Bridge design & computation. China Communications Press, (in Chinese)
[14] Jung M.R., Shin S.U., Attard M.M., et al. (2015). Deflection theory for self-anchored suspension bridges under live load. Journal of Bridge Engineering, 20 (7)