On simulation of the photon wave function to explain the Young's experiment and prospects for its use in quantum cryptography

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Abstract. The method developed in previous works for building the photon wave function in configuration representation is briefly described. The simulation of such function (wave packet) appropriate to femtosecond laser radiation, using the distribution of Gauss of photon momenta in the state of a single laser impulse is expounded. By other appropriate simulated wave function of photon the light interference in Young's experiment is explained within the frame of quantum mechanics without the attraction of transition probability amplitudes used in approach of quantum electrodynamics. This allows us considering interference phenomena for photons and particles from the same basis of the quantum-mechanical principle of superposition employed to the wave function built in the configuration space. Using the building apparatus of photon quantum mechanics, the Malus law postulated in quantum theory is justified. The relevance of new experiments to study the interaction of quantum objects between themselves and the physical vacuum, which would reveal the origins of the mathematical description of quantum states using the wave function of these quantum objects, is substantiated. It is suggested that the ideas of constructing the wave function of a free photon in the coordinate representation can find relevance in the field of quantum cryptography.

1. Introduction

It is well known that Young's experiment in historical terms was the first decisive argument in favor of revising the corpuscular point of view on the nature of light and the transition to a wave concept of it. However, a hundred years after Young's experiment, the corpuscular nature of light received a new impetus after Einstein explained the laws of the photoelectric effect based on the quantum nature of light. About a couple of decades later, after a series of fundamental experiments and the development of quantum mechanics, the conception of wave-particle dualism of light and microparticles of matter arose. In accordance with this conception, microparticles and photons can exhibit both particle and wave properties in various experimental sets.

After the basic interpretations of quantum mechanics, such as Copenhagen and Feynman's, were formulated, it was assumed for a long time that wave-particle dualism, although essential in terms of stating the properties of quantum objects, was essentially only of historical interest. However, this concept has recently gained relevance due to advance of new directions and applications of quantum physics, such as quantum computing and quantum cryptography. The concept of quantum
entanglement of the states of two or more particles began to be widely discussed and applied in them. In this regard, the relevance of quite methodological questions of quantum theory itself has revived.

In particular, in the current situation, as a result of experimental verification of the of Bell’s [1], Leggett’s [2], Leggett-Garg’s [3] inequalities, experiments with “delayed choice” and also quantum “eraser”, it was assumed that quantum systems do not satisfy the concepts of “classical realism”, and the choice between “quantum realism” and “quantum anti-realism” has not yet been made. In our opinion, in discussing such questions, the conception of the microparticles wave function plays an important role, with which some real “metamorphoses” like collapse as it happens when a quantum system transitions from a state described by a superposition of basic wave functions to a state described by one specific basis function, implemented as a result of a certain measurement. For particles having mass, the wave function may be used in coordinate and momentum representations, and the transition from one representation to another is easily carried out by means of a three-dimensional Fourier transform. In quantum electrodynamics, to describe interference experiments equivalent to Young’s experiment with two slits, for instance, with use the interferometer of Mach-Zehnder, the amplitudes of transition are successfully applied in lieu of wave functions, both for individual particles with mass and for photons [4, 5], which are introduced into the experimental setup, knowingly one by one.

Translated to Young’s equivalent experiment, the results of all experiments are the same: when on the first screen the both slots are unblock, an interference fringes are observed on second screen. In this event, “in metaphysical language” it is customary to imagine that photon and particle of a substance seems to exhibit properties of wave, penetrating two slots at the same time and, in this way, interfering with itself each time. Nonetheless, if you try to discover through which slot a photon (or particle) specifically penetrates, the interference fringes are disturbed.

Despite that the quantum approach correctly describes all experiments, the transition of photon perceived from a metaphysical perspective as a particle, and, especially, the transition of particle of matter “simultaneously” through both open slots, contravenes “reasonable insight”. However, in this instance it’s generally accepted that a quantum object manifests itself as a wave, but not as a corpuscle.

When only one slit is open, it is believed that the “same” quantum object manifests itself as a particle. Along with that, somehow, “it is usually forgotten” about that the particle (or photon), showing its supposedly “corpuscularity”, is nevertheless a quantum entity, hence it should be characterized by the wave function satisfying the respective quantum equation, while this entity “travels” from "its" only one open slot to the next screen, and later falls into certain spot of it. Such (each) hit is a likelihood quantum event, and it therefore must be characterized by the appropriate wave function in the coordinate representation.

Thus, “from a metaphysical point of view”, an unresolved mystery arises, how can a quantum object, such as a microparticle of matter or a photon associated with a corpuscle (since it is never divided either by energy or momentum into two or more parts), successfully “pass through both slits”? In our opinion, it could assumed that not the quantum object itself, but its wave function “passes through two slits simultaneously”. This wave function would interfere with itself, “forming” the hitting quantum object probability density into some point of the second screen. Of course, a quantum object must always pass across exclusively one slot, independently of whether one slot or two are open.

However, another, one might say, more important and global problem arises here, namely, about that the wave function of quantum object itself is not a physical entity, since experiment measures solely the probability density, that is, the modulus of the wave function if it is a one-component, but the “global” phase is not measured, although experiments have appeared (see, for example, [6]) in which its certain “local” phase is measured. Moreover, the structure of the wave function can consist from several complex functions, such as, for example, from two - for nonrelativistic particles with spin 1/2, from four complex functions - for relativistic particles with the such spin, or, say, of six complex functions - as for a photon (see below). Hence, for each complex component it would be required to measure its modulus and the “global” phase, which, obviously, is completely unrealistic. However, even in the case when the wave function of a particle consists of only one complex component, it is,
nevertheless, only a mathematical tool, and not a real physical object, since it does not have such “internal” characteristics of physical objects as mass, electric charge, etc., this wave function does not have even some “dynamic” characteristics such as energy-momentum.

How can something that does not exist in nature propagate in space, evolve, “seep” through slits, or, for example, “experience a collapse”? This is the main, highly significant question. Its solution, certainly, goes beyond the scope of orthodox quantum mechanics. Research the answer to this question, actually, will set the direction of the physics development in future. However, there is a second, though not so global, but still relevant at the moment question, straightly connected with the wave function of photon. This function in configuration representation, obviously, also enjoys the “right” to “allegedly penetrate” contemporaneously through both open slots, just like the “wave function” of particles with mass “seems to do” it.

But the question about wave function of the photon in configuration representation can be built has not yet been fully resolved, therefore, without this function the photon “even has nothing” to “seep” through the slits, for example. The purpose of this article is to show that the corresponding photon wave function in configuration space can not only be built, but also may be applied to explain the light interference in Young’s or similar experience.

However, after demonstrating such possibilities of the photon wave function, we can say that the description of the process of transferring each of the two entangled photons to large distances at realization the quantum cryptography may be considered from the point of view the evolution in the space and time of photon wave function in the coordinate representation. Such an evolution should differ significantly from the evolution of a quickly spreading free wave packet, taking into account specially implemented boundary conditions. In our opinion, this circumstance leads to a further increase in the relevance of the single-particle (and two-particle) wave function of the photon (photons) in the coordinate representation from the point of view of its practical application, in particular when implementing safe data transmission.

2. Photon wave function in coordinate representation

Beginning with the publication [7], which prohibited (see [8, 9] for more details) the possibility of building the photon wave function in configuration representation, it was customary to use it only in the momentum representation, for example, in quantum electrodynamics, despite the obvious fact from the general apparatus of quantum mechanics, according to which the wave function can always be obtained in any representation, if it is known in at least one. The cause for this situation is known to be the zero mass of the photon. However, we are convinced that the photon wave function (“wave packet”) in configuration space is also may be built if this function is intended to point out probability of photon detection at some point, rather than its localization, such as, for instance, of an electron in an atom. Works in which the understanding of wave function of photon shifted the emphasis on probability density of its detection at a given spatial point, appeared around the mid-90s of the last century (see, for example, [10]), and the ideas about the possibility of describing the photon states using the wave function normalized to a unit probability were first proposed in [11]. The development of ideas about the possibility of building the wave function of photon in configuration space was also facilitated by the creation of sources [12, 13] and detectors [14, 15] of single photons. In [16–21] the single-photon wave function got ensuing expansion in theoretical foundation.

In [22–24], to illustrate a single-photon wave function, the propagation in space-time of a packet of free waves characterizing the state of a photon in accordance with laser radiation was simulated for laser pulses with a duration of 80 fs, central length of wave 10 μm, and with Gaussian distribution over the momentum (of photon) presented in this the packet. As a result of the simulation, In summary accounting we can say that the simulation showed an interesting evolutionary process of the spreading photon's wave packet, namely: its spatial form turns from primary almost “ball-shaped” into a certain “cone-like” shape reminiscent the Cherenkov radiation portrayal, because the periphery portions of the probability density of this packet fall behind the central part traveling in its vanguard almost exactly with light velocity c in vacuum.

The evolution of this photon wave packet generally corresponds to the evolution of quantum-mechanical packets describing the free propagation of particles with mass, since this packet is a pure
wave function of photon in configuration representation, satisfying all the formal requirements of quantum mechanics. In particular, for it the notion of group velocity, employed in classical electrodynamics, is not used, but in accordance with quantum mechanics for this packet, one can count the mathematical expectation (mean value) of coordinate in which the photon is detected every time and, therefore, to establish the mean travel speed of this packet in the space. It was discovered that this mean speed set by means of single alone parameter specifying the photon momentum distribution in the state of this wave packet is noticeably less of light speed \( c \), due to its spreading. In [24], this circumstance is explained in the scope of the built photon quantum mechanics (see [19, 21]) and is consistent with the result of a not so long ago carried out experience [25] revealed the “reduce” of group light velocity in Bessel and Gaussian beams, and which was as if explained in the framework of classical electrodynamics, but attached to separate photons. The explanation of this “reduction of the speed of light”, given in [24], as a whole does not also contradict the common ideas expressed in [26, 27] etc., leaning on general concepts of classical electrodynamics and quantum mechanics.

Here we give a brief scheme of the building and interpretation of photon wave function in the configuration representation according to [19, 21].

This building is founded on the compound of classical electrodynamics and quantum mechanics applying principle of correspondence. Maxwell's equations in Majorana's format [23] for complex vectors \( \xi = E + iH \) and \( \eta = E - iH \) constituted of the electric (\( E \)) and magnetic (\( H \)) fields intensities, accepted as initial (in Gaussian System):

\[
ih \frac{\partial \xi}{\partial t} = c(\hat{s}\hat{p})\xi, \quad ih \frac{\partial \eta}{\partial t} = -c(\hat{s}\hat{p})\eta, \quad (\hat{p}\xi) = 0, \quad (\hat{p}\eta) = 0,
\]  

(1)

where \( \hat{p} = -i\hbar \hat{\nabla} \) is operator of the particle momentum; \( c \) – light speed in vacuum; \( \hat{s} \) – photon spin operator in vector representation:

\[
\hat{s} = e_x \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} + e_y \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} + e_z \begin{pmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = i \begin{pmatrix} 0 & -e_z & e_y \\ e_x & 0 & -e_x \\ -e_y & e_x & 0 \end{pmatrix}.
\]  

(2)

In column matrix form the vectors \( \xi \) and \( \eta \) are written as

\[
\xi = \begin{pmatrix} E_x + iH_x \\ E_y + iH_y \\ E_z + iH_z \end{pmatrix}; \quad \eta = \begin{pmatrix} E_x - iH_x \\ E_y - iH_y \\ E_z - iH_z \end{pmatrix}
\]

(3)

and should be considered as independent (see [12], p. 81). For bivector \( \Phi_{bv} = \begin{pmatrix} \xi \\ \eta \end{pmatrix} \) it's needful to solve an equation being the generalization of Dirac equation for massless particle with spin \( s=1 \) in “standard” or “bivector” representation. In last case the equation has an appearance

\[
ih \frac{\partial \Phi_{bv}}{\partial t} = \hat{H}_{bv} \Phi_{bv} \text{ or } \quad ih \frac{\partial}{\partial t} \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} e \left( \hat{s}\hat{p} \right) & 0 \\ 0 & -e \left( \hat{s}\hat{p} \right) \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix},
\]

(4)

Thus jointly with the equation (4) solution the purpose is set to find the eigenvalues and eigenfunctions \( \Phi_{bv} \) of the complete set of mutually commuting operators

\[
\left\{ \hat{E} = ih \frac{\partial}{\partial t}; \quad \hat{H}_{bv} = c \left( \hat{\alpha}_{bv} \hat{p} \right); \quad \hat{p} = -i\hbar \hat{\nabla}; \quad \hat{\lambda} \right\},
\]

(5)

where the matrix \( \hat{\alpha}_{bv} \), the operators of helicity \( \hat{\lambda} \) and of spin \( \hat{\lambda} \) of photon in bivector representation are equal to
\[ \hat{\alpha}_{bv} = \begin{pmatrix} \hat{s} & 0 \\ 0 & -\hat{s} \end{pmatrix}, \quad \hat{\lambda} = \frac{\langle \hat{\mathbf{p}} \rangle}{p} = \frac{1}{p} \begin{pmatrix} \langle \hat{\mathbf{p}} \rangle & 0 \\ 0 & \langle \hat{\mathbf{p}} \rangle \end{pmatrix}, \quad \hat{S} = \begin{pmatrix} \hat{s} & 0 \\ 0 & \hat{s} \end{pmatrix}. \]  

(6)

This task has the following solution [19, 21]:

1) The photon states with the *positive* energy \( E^{(+)}(k) = \hbar k c = +pc \) corresponding to the helicity \( \lambda = \pm 1 \), are described by the orthonormal bivectors

\[
\Phi_{bv; k, \pm 1}^{(+)}(r, t) = \begin{pmatrix} \xi_{k, \pm 1}(r, t) \\ 0 \end{pmatrix} = \frac{\langle \rho \rangle e_{\pm 1}(k)}{(2\pi)^{3/2}} e^{i(kr - kct)} \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

(7)

\[
\Phi_{bv; k, -1}^{(+)}(r, t) = \begin{pmatrix} \eta_{k, -1}(r, t) \\ 0 \end{pmatrix} = \frac{\langle \rho \rangle e_{-1}(k)}{(2\pi)^{3/2}} e^{i(kr - kct)} \begin{pmatrix} 0 \\ 1 \end{pmatrix},
\]

(8)

accordingly, where \( \langle \rho \rangle \) is measurement unit (oersted) of quantities \( \xi \) and \( \eta \).

2) The (hypothetical) photon states with the *negative* energy \( E^{(-)}(k) = -\hbar k c = -pc \) corresponding to the helicity \( \lambda = \mp 1 \), are described by the orthonormal bivectors

\[
\Phi_{bv; k, -1}^{(-)}(r, t) = \begin{pmatrix} \xi_{k, -1}(r, t) \\ 0 \end{pmatrix} = \frac{\langle \rho \rangle e_{-1}(k)}{(2\pi)^{3/2}} e^{i(kr + kct)} \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

(9)

\[
\Phi_{bv; k, +1}^{(-)}(r, t) = \begin{pmatrix} \eta_{k, +1}(r, t) \\ 0 \end{pmatrix} = \frac{\langle \rho \rangle e_{+1}(k)}{(2\pi)^{3/2}} e^{i(kr + kct)} \begin{pmatrix} 0 \\ 1 \end{pmatrix},
\]

(10)

where \( e_{\pm 1}(k) = [e_f(k) + i\lambda e_{H}(k)]/\sqrt{2} \) are the vectors of polarization; \( e_f \), \( e_{H} \) the real unit mutually orthogonal vectors composing the right-handed triad with \( n = k/k \) for certain vector \( k = p/h \):

\[
|e_f| = |e_{H}| = 1; \quad (e_f n) = (e_{H} n) = (e_f e_{H}) = 0; \quad e_{H} = [n \times e_f],
\]

(11)

\[
\vec{n} = [e_f \times e_{H}] = i\lambda [e_{\lambda} \times e_{\lambda}^+ ] = \lambda e_{\lambda}(k) \hat{s} e_{\lambda}(k),
\]

(12)

that provides the orthonormality of \( e_{\lambda} \) and ensures the needed relations *if* vector \( e_f \) *is not changed at change of direction of vector* \( n \), namely:

\[
e_{\lambda}^+ e_{\lambda} = \delta_{\lambda'\lambda}; \quad e_{\lambda}(n) = e_{-\lambda}(-n); \quad e_{\lambda}(k) = e_{-\lambda}(k) = e_{\lambda}(-k).
\]

(13)

Formulas (13) provide the orthonormality relations of bivectors (7) – (10):

\[
\int d^3 r \bigg[ \Phi_{bv; k, \lambda'}^{(\pm)}(r, t) \bigg]^* \Phi_{bv; k, \lambda}^{(\pm)}(r, t) = \langle \rho \rangle^2 \delta_{\lambda'\lambda} \delta(k' - k),
\]

(14)

\[
\int d^3 r \bigg[ \Phi_{bv; k, \lambda'}^{(\mp)}(r, t) \bigg]^* \Phi_{bv; k, \lambda}^{(\mp)}(r, t) = 0.
\]

(15)

By next step it’s postulated that the single-photon states with positive and negative energy, set initially by vectors \( \mathbf{E} \) and \( \mathbf{H} \), represent a superposition of bivectors which are the plane monochromatic waves (7) – (10):

\[
\Phi^{(\pm)}_{bv}(r, t) = \Phi_{bv; 1}^{(\pm)}(r, t) + \Phi_{bv; 2}^{(\pm)}(r, t) \equiv \int B(k, \pm 1) \Phi_{bv; k, \pm 1}^{(\pm)}(r, t) d^3 k + \int \{ B(\pm k, \mp 1) \}^* \Phi_{bv; k, \mp 1}^{(\pm)}(r, t) d^3 k.
\]

(16)

Employing the determined bivectors (7)–(10) satisfying the orthonormality and completeness conditions, how it can be shown, we have the opportunity to introduce the distribution of spatial energy density of the photon in state (16) and the wave function \( \Psi^{(\pm)}(r, t) \) of this state, which satisfies to the normalization condition to unit of total probability:
\[ \rho_E^{(\pm)}(r,t) = \frac{1}{8\pi} \left[ \Phi_{bv}^{(\pm)}(r,t) \right]^* \Phi_{bv}^{(\pm)}(r,t) = \frac{1}{8\pi} \sum_\lambda \left[ \Phi_{bv;\lambda}^{(\pm)}(r,t) \right]^* \Phi_{bv;\lambda}^{(\pm)}(r,t) \]

\[ = \frac{1}{8\pi} \left\{ \left[ \xi_{\pm 1}^{(\pm)}(r,t) \right]^* \xi_{\pm 1}^{(\pm)}(r,t) + \left[ \eta_{\pm 1}^{(\pm)}(r,t) \right]^* \eta_{\pm 1}^{(\pm)}(r,t) \right\} = \frac{1}{8\pi} \left\{ \left| \xi_{\pm 1}^{(\pm)}(r,t) \right|^2 + \left| \eta_{\pm 1}^{(\pm)}(r,t) \right|^2 \right\} \]

\[ = \frac{1}{8\pi} \left\{ \left[ F_{\xi,\pm 1}(r,t) \right]^2 + \left[ H_{\xi,\pm 1}(r,t) \right]^2 + \left[ F_{\eta,\pm 1}(r,t) \right]^2 + \left[ H_{\eta,\pm 1}(r,t) \right]^2 \right\}; \quad (17) \]

\[ \Psi^{(\pm)}(r,t) = \int b(k,\pm 1) \Psi_{k,\pm 1}^{(\pm)}(r,t) d^3k + \int [b(-k,\pm 1)]^* \Psi_{k,\pm 1}^{(\pm)}(r,t) d^3k, \quad (18) \]

where

\[ b(k,\lambda) = \frac{(Oe)}{\sqrt{8\pi\hbar c}} B(k,\lambda); \quad \Psi_{k,\lambda}^{(\pm)}(r,t) = \frac{1}{(Oe)} \Phi_{bv; k,\lambda}^{(\pm)}(r,t). \quad (19) \]

Thus the \( \Psi^{(\pm)}(r,t) \) and dimensionless functions \( \Psi_{k,\lambda}^{(\pm)}(r,t) \) satisfy the orthonormality relations

\[ \int d^3r \left[ \Psi_{k,\lambda}^{(\pm)}(r,t) \right]^* \Psi_{k',\lambda'}^{(\pm)}(r,t) = \delta_{\lambda',\lambda} \delta(k' - k); \quad (20) \]

\[ \int d^3r \left[ \Psi^{(\pm)}(r,t) \right]^* \Psi^{(\pm)}(r,t) = \int d^3r \rho_p^{(\pm)}(r,t) = 1. \quad (21) \]

The wave function \( \Psi^{(\pm)}(r,t) \) of photon in the form of packet (18) characterized by both positive and negative energy satisfies the equation (4), from which we can get the continuity equation for probability density of \( \rho_p^{(\pm)}(r,t) \), normalized per unit, and flux density \( j_p^{(\pm)}(r,t) \) of probability – of detect the photon near the point \( r \) at the timepoint \( t \):

\[ \frac{\partial \rho_p^{(\pm)}(r,t)}{\partial t} + \text{div} j_p^{(\pm)}(r,t) = 0, \quad (22) \]

where

\[ \rho_p^{(\pm)}(r,t) = \left[ \Psi^{(\pm)}(r,t) \right]^* \Psi^{(\pm)}(r,t), \quad j_p^{(\pm)}(r,t) = \mathbf{e} \left[ \Psi^{(\pm)}(r,t) \right]^* \hat{\mathbf{a}}_{bv} \Psi^{(\pm)}(r,t). \quad (23) \]

The wave function in the momentum representation corresponds to the wave function (18), namely

\[ \Psi^{(\pm)}(k,t) \equiv \left| \Psi^{(\pm)} \right> = \frac{1}{(2\pi)^{3/2}} \int e^{-ikr} \Psi^{(\pm)}(r,t) d^3r \]

\[ = e^{ikrt} \left\{ b(k,\pm 1) e_{\pm 1}(k) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) + [b(-k,\mp 1)]^* e_{\mp 1}(k) \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \right\}. \quad (24) \]

If in (18) or (24) the coefficients \( b(k,\lambda) \) are given, it’s possible to carry out the average values of all physical quantities and their uncertainties in free photon state. For instance, the energy average value of photon turns out

\[ \overline{E^{(\pm)}} \equiv \left< \Psi^{(\pm)} \left| \hat{E} \right| \Psi^{(\pm)} \right> = \sum_\lambda \sum_{\lambda'} \left( \overline{\Psi^{(\pm)} \hat{E} \Psi^{(\pm)}} \right) \]

\[ = \int (\pm \hbar c) \left\{ |b(k,\pm 1)|^2 + |b(-k,\mp 1)|^2 \right\} d^3k = \int E^{(\pm)}(k) \rho_p^{(\pm)}(k) d^3k. \quad (25) \]

This formula is an expression that can also be written in the framework of classical electrodynamics using classical statistical physics. This expression also coincides with the result
obtained by applying formulas (17). This fact realizes, obviously, the quantum-mechanical correspondence principle, which allows us interpreting the wave packet (18) as the wave function of the photon in a free state.

Taking into account (13), the basis vectors \( \xi_{k,\lambda}(r,t) \) and \( \eta_{k,\lambda}(r,t) \), also being in turn eigenfunctions of the energy, momentum and helicity \( \lambda = (\hat{\mathbf{p}}/\hbar) \) (in vector representation), satisfy to the orthonormality relations

\[
\int d^3r \, \xi_{k,\lambda}^+(r,t) \xi_{k',\lambda'}(r,t) = \delta_{\lambda',\lambda} \delta(k' - k)(\text{Oe})^2, \tag{26}
\]

\[
\int d^3r \, \eta_{k,\lambda}^+(r,t) \eta_{k',\lambda'}(r,t) = \delta_{\lambda',\lambda} \delta(k' - k)(\text{Oe})^2. \tag{27}
\]

They immediately provide the orthonormality of relations (14), (15).

The relations (26), (27) make it possible to decompose any vectors and bivectors (see also (16)) in the corresponding bases:

\[
\xi(r,t) = E(r,t) + i H(r,t) = \xi_{+1}^+(r,t) + \xi_{-1}^-(r,t) \tag{28}
\]

\[
= \left[ B(k,+,1) \xi_{k,+1}^+(r,t) d^3k + \int B(k,-1) \xi_{k,-1}^-(r,t), \right]
\]

\[
\eta(r,t) = E(r,t) - i H(r,t) = \eta_{+1}^+(r,t) + \eta_{-1}^-(r,t) \tag{29}
\]

\[
= \left[ B(-k,-1)] \eta_{k,-1}^+(r,t) d^3k + \int [B(-k,+,1)] \eta_{k,+1}^-(r,t) d^3k, \right] \tag{30}
\]

\[
\Phi^{(\pm)}_{bv}(r,t) = \begin{pmatrix} E_{\xi,\pm 1}^{(\pm)}(r,t) + i H_{\xi,\pm 1}^{(\pm)}(r,t) \\ E_{\eta,\pm 1}^{(\pm)}(r,t) - i H_{\eta,\pm 1}^{(\pm)}(r,t) \end{pmatrix} = \begin{pmatrix} \xi^{(\pm)}_{\pm 1}(r,t) \\ \eta^{(\pm)}_{\pm 1}(r,t) \end{pmatrix}. \tag{32}
\]

From (28–31) it follows that if the electromagnetic field is initially set by some classical field intensities \( E \) and \( H \), the single-photon state corresponding to this field is impossible to be determined directly by quantities \( \xi \) and \( \eta \), because unobserved negative energy contributes into decompositions (28–31).

So, as a matter of fact, postulating, we can say that along with (18), (24) the single-photon state may be characterized by the bivector (16), (32) with a plus sign for real photons and with a minus sign for hypothetical photons with negative energy.

Thus, the bivector (16), (32) also plays essential role in description of the single-particle state of a photon. In particular, this statement can be explained by the next reasoning.

We can denote the intensities offering contributions to integrals (16), (29), (31), according to the following relations:

\[
E(r,t) = E_{\xi,\pm 1}^{(\pm)}(r,t) + E_{\eta,\mp 1}^{(\pm)}(r,t) = E_{\eta,\pm 1}^{(\pm)}(r,t) + E_{\eta,-1}^{(\pm)}(r,t), \tag{33}
\]

\[
H(r,t) = H_{\xi,\pm 1}^{(\pm)}(r,t) + H_{\eta,\mp 1}^{(\pm)}(r,t) = H_{\eta,\pm 1}^{(\pm)}(r,t) + H_{\eta,-1}^{(\pm)}(r,t). \tag{34}
\]

Then from (7)–(10), (13), (32), (33), (34) important connections follow:

\[
E_{\xi,\pm 1}^{(\pm)}(r,t) = E_{\eta,\mp 1}^{(\pm)}(r,t), \quad H_{\xi,\mp 1}^{(\pm)}(r,t) = H_{\eta,\pm 1}^{(\pm)}(r,t), \tag{35}
\]

\[
\left[ \xi_{\mp 1}^{(-)}(r,t) \right]^{*} = \eta_{\pm 1}^{(\pm)}(r,t), \quad \left[ \eta_{\pm 1}^{(-)}(r,t) \right]^{*} = \xi_{\mp 1}^{(\pm)}(r,t). \tag{36}
\]

Together with the formulas (33)–(34) associating with the decompositions (29), (31), the relations (35), (36) give (see [19, 21]) the superposition principle for the intensities \( E \) and \( H \):
\[ E(r,t) = E^{(+)}_{\phi+1}(r,t) + E^{(+)}_{\phi-1}(r,t) \quad \text{and} \quad H(r,t) = H^{(+)}_{\phi+1}(r,t) + H^{(+)}_{\phi-1}(r,t). \] (37)

3. The Young’s interference experiment

When the problem of wave-particle dualism is considered, touching, for example, the interpretation of interference in Young’s experiment, then the wave function in the configuration space is used to explicate the wave properties of the particles with mass. A like pattern of light diffraction and interference is elucidated by attributing the classical electrodynamics built on the Maxwell’s equations, or the postulated Feynman probability amplitudes. In the case of Young’s experiment, for instance, an explanation of the interference result in the framework of classical electrodynamics reduces to finding the difference of phases of monochromatic waves radiated by two both slots falling at a given point \( P \) of second screen. A like situation arises with the Feynman probability amplitudes.

In [23, 29–32] the idea was put forward that such a phase \( \varphi = kr - kct \) is present in each addendum of wave photon function (18). If the emitting is sufficiently monochromatic, then in formula (23) for the probability density, evidently, the single term arises proportional to the cosine of phase difference \( \delta = \varphi_1 - \varphi_2 = k(r_1 - r_2) \) of these waves emitted by two slots. It also gives an explication of presence of the interference fringes, likewise the explanation which takes place in classical electrodynamics.

Consider this explication in more detail. We put the coefficients \( b(k, \pm 1) \) equal

\[ b(k, \pm 1) = b(k, \pm 1) = \frac{\alpha}{\sqrt{\pi}} \exp \left[ -\alpha^2 \left( k - k_0 \right)^2 \right], \] (38)

which realize a propagating spherical wave corresponding to the photon state with zero middle momentum and the average its modulus \( \hbar k_0 \). Choice (38) forms a delta-like function, which makes it possible to select a monochromatic wave in the decomposition of a plane wave (18) and, as well as, gives a rather simple analytical formula for the photon wave function.

We write the factor \( \exp(ikr) \) in (7)–(10), (18), first assuming \( \varphi_r = \pi/2 \) in it, subject to the geometry of the Young experiment, and then expand this factor on the Maclaren series near the small parameter \( \cos \theta_r \) in absolute value, given that in this experiment the values of the angle \( \theta_r \) are in area \( \theta_r = \pi/2 \):

\[ e^{ikr} = e^{i(kx + ky + kz)} = e^{ikr \sin \theta \sin \varphi (1 + ikr \cos \theta \cos \varphi)} \] (39)

where \( \theta_r \), and above-mentioned \( \varphi_r \), determine the radius vector \( r \) in spherical coordinate system of the configuration space; so the Cartesian coordinates of this vector in the made approximation are equal to

\[ x = r \sin \theta \cos \varphi_r \approx 0, \quad y = r \sin \theta_r \sin \varphi_r \approx r, \quad z = r \cos \theta_r. \] (40)

The demands (11)–(13) are satisfied, for example, for the following polarization vectors [29–32]:

\[ e_I(k) = \begin{pmatrix} 1 - (1 - \cos \theta) \cos^2 \varphi \\ - (1 - \cos \theta) \sin \varphi \cos \varphi \\ - \sin \theta \cos \varphi \end{pmatrix}, \quad e_H(k) = \begin{pmatrix} - (1 - \cos \theta) \sin \varphi \cos \varphi \\ \cos \theta + (1 - \cos \theta) \cos^2 \varphi \\ - \sin \theta \sin \varphi \end{pmatrix} \] at \( 0 \leq \theta \leq \frac{\pi}{2} \), (41)

\[ e_I(k) = \begin{pmatrix} 1 - (1 + \cos \theta) \cos^2 \varphi \\ - (1 + \cos \theta) \sin \varphi \cos \varphi \\ \sin \theta \cos \varphi \end{pmatrix}, \quad e_H(k) = \begin{pmatrix} (1 + \cos \theta) \sin \varphi \cos \varphi \\ \cos \theta - (1 + \cos \theta) \cos^2 \varphi \\ - \sin \theta \sin \varphi \end{pmatrix} \] at \( \frac{\pi}{2} < \theta \leq \pi \), (42)

where the Cartesian components of the vectors \( e_{I,H}(k) \) in the configuration space are specified, expressed in terms of the spherical coordinates of vector \( k \) in momentum space.

Substituting the (41), (42), (39) into (7)–(10), (18) and performing integration over angles \( \theta \) and \( \varphi \), we obtain
\[ \Psi^{(\pm)}(r, t) = \int_{0}^{\infty} \frac{kb(k, \pm 1)}{r \sqrt{\pi}} e^{ikz} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} \sin kr \\ \pm \cos kr \cos \theta_r \\ \mp \cos kr \cos \theta_r \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \sin k_0 r \\ \pm \cos k_0 r \cos \theta_r \\ \mp \cos k_0 r \cos \theta_r \end{pmatrix} \right] dk, \] (43)

where we neglected the terms proportional to \(1/r^2\), keeping in mind the radiation in the wave zone. In (43) the coefficient \(b(k, \pm 1)\) has not yet been specified in a concrete form. Substituting instead of it the expression (38) at \(\alpha \rightarrow \infty\), that finally corresponds to the selection of the monochromatic wave by this method, we find

\[ \Psi^{(\pm)}(r, t) = \frac{k_0 e^{ik_0 z}}{r} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} \sin k_0 r \\ \pm \cos k_0 r \cos \theta_r \\ \mp \cos k_0 r \cos \theta_r \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \sin k_0 r \\ \pm \cos k_0 r \cos \theta_r \\ \mp \cos k_0 r \cos \theta_r \end{pmatrix} \right]. \] (44)

Multiplying (44) by the necessary factor, independent of the coordinates and time, we can restore the correct dimension of the photon wave function, which is disturbed by the parameterization (38). Taking into account it, let us write down the wave function of a photon passing “through both holes” at the first screen in Young’s experiment, as the sum of two terms, each of which has the form (44):

\[ \Psi^{(\pm)}(r, t) = \Psi^{(\pm)}_1 \left( r_1 + \frac{d}{2}, t \right) + \Psi^{(\pm)}_2 \left( r_2 - \frac{d}{2}, t \right) = \]

\[ + A e^{i k_0 z} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} \sin k_0 r_1 \\ \pm \cos k_0 r_1 \cos \theta_{r_1} \\ \mp \cos k_0 r_1 \cos \theta_{r_1} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \sin k_0 r_1 \\ \pm \cos k_0 r_1 \cos \theta_{r_1} \\ \mp \cos k_0 r_1 \cos \theta_{r_1} \end{pmatrix} \right] \]

\[ + A e^{i k_0 z} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} \sin k_0 r_2 \\ \pm \cos k_0 r_2 \cos \theta_{r_2} \\ \mp \cos k_0 r_2 \cos \theta_{r_2} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \sin k_0 r_2 \\ \pm \cos k_0 r_2 \cos \theta_{r_2} \\ \mp \cos k_0 r_2 \cos \theta_{r_2} \end{pmatrix} \right]. \] (45)

where \(A\) is the real normalization constant; the angles \(\theta_{r_1}, \theta_{r_2}\) are counted from the spatial axis \(z\), now directed along the line connecting the both holes (in the first screen) connected by the vector \(d\); \(r_1\) and \(r_2\) are the distances from the slots to interference observation point \(P\) on second screen (apart from the first screen at a distance \(\ell\)).

Having written down the probability density of photon detection according to formula (23), it is easy to verify that the interference result is determined by the interference term, which after transformations and neglect of the summand including the product \(\cos \theta_{r_1} \cos \theta_{r_2}\) is reduced to

\[ \rho_{\text{int}} = \frac{4A^2}{r_1 r_2} \left[ \sin(k_0 r_1) \sin(k_0 r_2) + \cos(k_0 r_1) \cos(k_0 r_2) \right]. \] (46)

In the geometry of the Young’s experiment, considered usually in classical electrodynamics, it is assumed that \(r_1 + r_2 \approx 2\ell\), \(r_2 - r_1 = \Delta\), where \(\Delta\) is the optical path difference (in vacuum or air) of the rays emanating from both holes. In the same approximation \(\Delta \approx zd/\ell\), where \(z\) is the coordinate of the point \(P\) on the second screen, measured from the (average) symmetry line of the interference pattern. Taking these relations into account, the interference term (46) assumes exactly the same form as in classical electrodynamics:

\[ \rho_{\text{int}} = \frac{4A^2}{r_1 r_2} \cos(k_0 \Delta) = \frac{4A^2}{r_1 r_2} \cos \delta, \] (47)
where $\delta = 2\pi \Delta / \lambda_0$ is the phase difference of two interfering rays from viewpoint of classical electrodynamics, $k_0 = 2\pi / \lambda_0$ is the wave number of the corresponding (almost) monochromatic radiation.

We note that in the same approximation as in (46), the photon detection probability density on the second screen from only one open hole (in first screen) is determined by the only one corresponding term (45):

$$\rho_1 = \frac{2A^2}{r_1}, \quad \rho_2 = \frac{2A^2}{r_2^2}. \quad (48)$$

Thus, introducing into the work the wave function of the photon in the configuration representation, we can explain the wave phenomena on a single basis for all particles and light. This turns out especially actually when the experience is put with the participation of photons radiated, specifically, one by one.

4. Justification of the Malus law in the framework of photon quantum mechanics

In classical electrodynamics, the Malus law follows from the wave nature of light, according to which its intensity $I$ is proportional to the square of the amplitude of the electric field of a plane monochromatic wave, and the amplitude of the transmitted wave equal

$$E = E_0 \cos \alpha, \quad (49)$$

where $E_0$ is the amplitude of the linearly polarized light wave incident on the polarizer, $\alpha$ is the angle between the planes of polarization of the incident wave and the polarizer. Since $I_0 = E_0^2$, $I = E^2$, then

$$I = I_0 \cos^2 \alpha, \quad (50)$$

This law is well tested in practice and is widely used in “classical” devices, such as photometers and spectrophotometers.

However, at the turn of the century, new experiments with single and entangled photons appeared - when checking Bell’s inequalities (and the like), performing quantum teleportation, computing, and cryptography. In these experiments, instead of the intensity in (50), the probability of the passage of a photon with a certain polarization through the polarizer is used; that is, instead of (50) in quantum calculations, the postulated relation

$$W = \cos^2 \alpha, \quad (51)$$

where the probability of absorption by the polarizer is assumed to be equal $\sin^2 \alpha$.

Independent quantities $\xi^{(\pm)}_{k, \pm 1}$ and $\eta^{(\pm)}_{k, \pm 1}$ defined in (7)–(10) are composed of Cartesian projections of real vectors $E$ and $H$, according to the formulas

$$\xi = E_x + iH_y, \quad \eta = E_y - iH_x. \quad (52)$$

Therefore, for quantities $\xi^{(\pm)}_{k, \pm 1}$ and $\eta^{(\pm)}_{k, \pm 1}$, according to (29), (31), (37) the following formulas hold:

$$\xi^{(\pm)}_{k, \pm 1}(r, t) = E^{(\pm)}_{\xi, k}(\pm; r, t) \pm iH^{(\pm)}_{\xi, k}(\pm; r, t); \quad \eta^{(\pm)}_{k, \pm 1}(r, t) = E^{(\pm)}_{\eta, k}(\pm; r, t) \mp iH^{(\pm)}_{\eta, k}(\pm; r, t). \quad (53)$$

State (18) corresponds to classical field strengths

$$E(r, t) = E^{(+)}_{x+1}(r, t) + E^{(-)}_{x-1}(r, t); \quad H(r, t) = H^{(+)}_{x+1}(r, t) + H^{(-)}_{x-1}(r, t), \quad (54)$$

where the terms are found as the real and imaginary parts of the quantities

$$\xi^{(+)}_{x+1}(r, t) = E^{(+)}_{x+1}(r, t) + iH^{(+)}_{x+1}(r, t) = \int B(k, +1) \xi^{(+)}_{k, x+1}(r, t) d^3k, \quad (55)$$
\[ \eta^{(+)}(r, t) = E^{(+)}_{\eta, (-)}(r, t) - i H^{(+)}_{\eta, (-)}(r, t) = \int B^*(-k, -1) \eta^{(+)}_{k, (-)}(r, t) d^3k . \] (56)

Instead of (18), a linearly polarized wave normalized to a given probability stream density can be used in the form of a superposition of functions \( \Psi_{k, (-)}(r, t) \). If a photon moves along the \( z \) axis and its polarization is set, from the view point of classical electrodynamics, by the \( xz \)-oscillation plane, then its wave function

\[ \Psi_{k, (-)}^{(lin)}(r, t) = \frac{A}{|E|} \left( \frac{\eta_{k, (-)}^{(+)}(r, t)}{\eta_{k, (-)}^{(+)}(r, t)} \right) = A \Psi_{k, (-)}^{(+)}(r, t) + A \Psi_{k, (-)}^{(+)}(r, t) . \] (57)

At \( A = 2\pi^{1/2} E_0 / |E| \) from formulas (52)–(54), (57) we can see that the constant \( E_0 \) is equal to the amplitude of a linearly polarized wave:

\[ E_{k, z}^{(lin)}(r, t) = E_{k, r}^{(+)}(r, t) + E_{k, r}^{(+)}(r, t) = \begin{pmatrix} E_0 \cos(\omega t - z) \\ 0 \end{pmatrix} . \] (58)

Then defined by formulas (23) the probability density and probability stream density to detect a photon in the wave incident on the polarizer are

\[ \rho_{m, k_{z}}^{(+)} = \left[ \Psi_{k_{z}}^{(lin)}(r, t) \right]^* \Psi_{k_{z}}^{(lin)}(r, t) = \frac{E_0^2}{|E|^2} , \quad j_{m, k_z}^{(+)} = c \rho_{m, k_z}^{(+)} = \frac{c E_0^2}{|E|^2} . \] (59)

The amplitude of classical transmitted wave is determined by relation (51). Bearing this in mind, it can be shown, similarly to (57), that the wave function of transmitted photon is

\[ \Psi_{k, z_{out}}^{(lin)}(r, t) = A \cos \alpha \left[ e^{-i \alpha} \Psi_{k, z_{out}}^{(+)}(r, t) + e^{i \alpha} \Psi_{k, z_{out}}^{(+)}(r, t) \right] . \] (60)

Therefore, analogously to (59), for the photon passing through the polarizer we obtain the probability densities and its stream equal

\[ \rho_{out, k_z}^{(+)} = \frac{E_0^2}{|E|^2} \cos^2 \alpha , \quad j_{out, k_z}^{(+)} = \frac{c E_0^2}{|E|^2} \cos^2 \alpha . \] (61)

Then, the ratio of the probability flux density (61) to detect a photon in the transmitted polarizer wave to the probability flux density (59) of its detection in the incident one is exactly equal to the right-hand side of (51).

5. Conclusion

The results of simulation the propagation of the wave function of free photon allow us arguing about the opportunity of single-photon interpretation of interference and diffraction phenomena of light. In particular, the Young’s experiment, usually discussed in the language of classical electrodynamics, may obviously also be explicated within the ordinary quantum mechanics, without secondary quantization or using postulated Feynman probability amplitudes. Moreover, the above justification of the Malus law postulated in quantum electrodynamics contributes to the theoretical confirmation of some heuristic methods of modern quantum theory. This circumstance significantly weakens the problem of the duality of microparticles and light and expands the scope of the photon quantum mechanics in the framework of primary quantization. However, we emphasize that a photon we’re seeing is not a particle in the sense of understanding as one of the point particles of the of elementary particle physics standard model. A photon is some quasiparticle arising in a physical vacuum as a propagation result in it of a certain spin wave, mechanism of which must be examined taking into account the structure of the vacuum at Planck distances [33, 34].
However, the nature of the propagation of light in the vacuum must be closely related to some physical reality, which also must take place for particles of matter, the distribution of which and the interaction with each other should be thus determined by the physical vacuum properties. The study of these properties requires new experiments.

6. References

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