Magnets and magnetic field around them: what can we learn from simple experiments

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Abstract. Small neodymium magnets are useful tools enabling us to do a lot of experiments usable in physics teaching and learning. Simple experiments with these magnets can be done quickly and at minimal cost. Such experiments are usually used to demonstrate or investigate just qualitative features of magnetic field. However, simple experiments, when combined with small pieces of theory, offer much more than that. In the workshop, the participants went from very simple qualitative observations to experiments that, being still simple and low-cost, enable the measurement of the force between magnets and even to estimate the value of magnetic field $B$ inside magnets and around them. The workshop was led in an inquiry-based style and offered a teaching-learning sequence that teachers can adapt for various levels of schools.

1. Introduction
Magnets are parts of toys with which children play since early childhood. We all have much experience with how magnets behave, we have seen many pictures of field lines around them, and we have learnt this topic in school physics. Despite of this, some relatively simple features of magnets and magnetic fields are not completely clear even to some physics teachers. Here we do not have in mind some well-known misconceptions like “magnets attract all metals” (mentioned, e.g., in [1] and [2]) but somewhat more sophisticated questions like: How large is magnetic inclination? How strong is the field in the vicinity of permanent magnets? How large is the force between magnets? Of course, the answers can be found in textbooks – but can we also “discover” them by simple experiments?

Nowadays, small neodymium magnets are very useful tools enabling us to do a lot of experiments usable in physics teaching and learning at all levels of school. However, quite often simple experiments with magnets presented in various sources are just qualitative, see for example [3]. On the other hand, quantitative experiments described in literature usually require more sophisticated equipment, e.g. [4]. In the workshop, a series of experiments with very simple tools was presented that went from qualitative experiments to quantitative measurements. No magnetometers or other “black boxes” were used but still we were finally able to find the value of magnetic field near a pole of a neodymium magnet. Of course, the experiments were not only “hands-on”; the “minds-on” component was quite important.

The series of experiments described below was previously presented at a few workshops for Czech physics teachers [5] and was adapted according to gained experience. In an informal feedback, teachers stated that it helped them to understand the concepts and features concerning magnets and magnetic field. Also an informal feedback of participants of the workshop at GIREP-ICPE-EPEC 2017 was positive. In the workshop, the participants not only did the experiments by themselves but also built
some very simple tools they can use later for example as an inspiration for their courses for physics teachers.

2. Magnet and Earth’s field

2.1. Magnet in Earth’s magnetic field
Let’s start with one of the simplest experiments. Take a piece of thin thread and click two small flat neodymium magnets on it. Hold one end of a thread and let magnets hang as it is shown in figure 1. (Note: The magnets can be of diameter about 1 cm and thickness 1–2 mm. The thread should be such that the torque by which it tries to turn the magnets is practically negligible.)

![Figure 1. Two small magnets hanging on thread.](image)

Even with such a simple tool we can start our inquiry. The first task is: Observe these hanging magnets. What do you see? How do they behave? Often, the first answer is “They oscillate; they are turning back and forth.” In fact, this is very interesting behaviour and it is worth devoting another whole workshop to. For now, we will concentrate to static situations so we will not pay attention to these oscillations; we will stop them (or, rather, reduce them) either by our fingers or by suitable damping (see below).

The second answer is natural: The poles of magnets point in north-south direction. So, our magnets act like a compass. The third fact one can notice is better seen if we look to the magnets from side, see figure 2. The magnets do not hang vertically, they are slightly tilted.

![Figure 2. Magnets do not hang vertically.](image)

Of course, this is due to inclination of Earth’s magnetic field. The effect is more noticeable in case of thin bar magnets; see the right hand side of figure 2. However, when discussing these experiments students should be warned that the magnets do not show here the right direction of Earth’s magnetic field!

2.2. Make your own simple compass
If we want to use magnets hanging on thread really as a simple compass, it is necessary to damp their oscillation. For this, a small aluminum plate appeared to be quite useful; it can damp the oscillations due to eddy currents. In our case, the plate had a thickness of 3 mm, width 4 cm and length about 7 cm. (The plates were cut from a long flat aluminum bar.) The plate also serves as a base for our compass. A holder at the upper end of a thread can be formed from a piece of thicker wire. A paper compass rose is attached
to a base plate by sticky tape. To be able to measure the angle in which magnets point a short piece of paper can be placed between them which serves as a “compass needle”. (Of course, it is a somewhat strange compass needle because it points in the east-west direction.) The result is shown in figure 3.

![Figure 3. A simple compass.](image)

This construction can be seen just as a children’s toy but as we shall see, it can be used even for some quantitative measurements. Its advantage is that the part sensitive to the magnetic field is quite small so this tool shows the direction of the magnetic field really locally. (At least when compared with some compass with a large compass needle. Anyway, if we want to make it “more local” we can use even smaller magnets.) Even if it is used as just a qualitative “toy” it may be useful for motivation; teachers reported that pupils quite like making this simple tool.

2.3. Magnetic inclination and an approximate model of Earth’s magnetic field

Even when people know that the direction of Earth’s magnetic field is not horizontal, they usually significantly underestimate value of magnetic inclination. Formal research concerning this should yet be done but if you ask people to show by their hands the direction in which the Earth’s magnetic field aims, they typically point in direction that has angle of about 20º to 30º with respect to horizontal direction. (These informal investigations were done in Europe in places where the real values of magnetic inclinations are about 60º.)

A simple model that can clearly show approximate direction of Earth’s magnetic field can be made using just a piece of paper at which the Earth is printed schematically and a small neodymium magnet attached to its centre by a sticky tape, see figure 4.

![Figure 4. A simple 2D model of Earth’s magnetic field.](image)
In our case, the diameter of our model Earth was about 6 cm, and the magnet was about 1.5 cm long. The dimensions are not critical; of course, you can scale the model if you like. As a small “compass needle” showing the direction of magnetic field, a short thin magnet was used tied in its middle to a short piece of thread; the knot was fastened by a piece of sticky tape. In our case, the magnet had length 1 cm and was 2 mm thick.

As shown in figure 4, one can demonstrate that at the equator the inclination is nearly zero and near the pole it reaches 90°. Of course, it should be stressed that the model described here is very rough (in fact, it shows just approximately the dipole component of the field) and the real magnetic field of the Earth is much more complicated. (The values of magnetic inclination and other quantities concerning Earth’s magnetic field can be found for example in [6].) On the other hand, it shows the basic features concerning the shape of magnetic field close to Earth’s surface quite clearly – and students can use it to see the direction of the field even in 3D, i.e. not only at the surface of the paper sheet but also above it.

3. Direction of magnetic field near magnets
From previous experiment, we can naturally proceed to the investigation of field lines of a small magnet, see figure 5.

As figure 5 shows, we can draw the direction of the magnetic field at different points (using a pencil without ferromagnetic parts). The result, at least in the area not too close to the magnet, provides the field lines of a dipole.

An advantage of using a short thin magnet as a “compass needle” is that it shows the direction of a field really locally, practically “in one point”. One technical note: It is useful to hold our “compass needle” at a very short piece of threat, just one or two centimetres, otherwise it is hard to place the needle at the point in which we want to find the direction of the field.

3.1. Direction of a field in a vicinity of a pole of a long magnet
We can use our “compass needle” also to investigate a field near a pole of a long bar magnet as shown in figure 6. Again, we fix the long magnet at a sheet of paper by a sticky tape and draw directions shown by the small magnet.

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Figure 5. Investigation of a field of a small magnet (“nearly dipole”).

Figure 6. Investigation of a field near a pole of a long magnet.
The result strongly resembles a radial field, like the electrostatic field of a point charge. The experiment shows that the magnetic field near a pole of a long magnet is radial. We can ask whether the analogy with electrostatics can be extended further, for example, whether a force between the poles of long thin magnets depends on their distance also as $1/r^2$. This was investigated in the following part of the workshop.

Let’s add two small technical notes: A long cylindrical magnet was composed from shorter ones (in our case, of magnets 2.5 cm long). If we do not investigate the field too close to the places where magnets stick to each other, the field is the same as a field of one long magnet. Also, theoretically it would be necessary to have an infinitely long (and infinitely thin) magnet to obtain a strictly radial field from its pole. In practice, a magnet of length 15 to 20 cm and thickness of 5 mm proved to suit well for this qualitative experiment.

4. Force between poles of long bar magnets and what we can derive from it

To measure the force between poles of long magnets, various tools can be used, of course, for example small digital scales. In the workshop, we used much more low-cost tool: a lever made from a plastic ruler, see figure 7.

A small wooden plate is attached to a ruler in its centre by a piece of double-sided sticky tape. It leans on tips of drawing pins attached to a table by sticky paper tape. (Of course, other ways to make a simple lever can be used.) A bar magnet (composed from shorter ones) is attached to the lever so that its one end is above the rotation axis. In our case, the diameter of the magnet was 5 mm. The weight of the magnet is balanced by a suitable counterbalance, in our case by two coins.

![Figure 7. A lever from a plastic ruler with a bar magnet attached to it.](image)

Then we put a weight of mass $m$ on the other side of the lever (see figure 8 below) and bring a pole of another long magnet above the pole of the magnet on the lever so that the weight of mass $m$ is balanced by a repulsive force between magnetic poles. Then the distance $r$ of the poles is the distance at which the repulsive force is equal to the weight $m \cdot g$.

Note: The advantage of this simple tool is that we can ignore the force at the other end of the magnet at the lever (that above the rotational axis) because its moment of force is zero. Of course, the magnet we bring from above should be long enough or we should consider a correction due to its other pole.

4.1. Does the force between poles depend on direction?

With our tool we can easily demonstrate that the force between poles of long magnets does not depend on direction, see figure 8. If the point where the pole of the upper magnet is stays the same the lever remains balanced independently of the direction of the upper magnet.
Figure 8. A repulsive force between poles of long magnets does not depend on direction.

A technical note: In this experiment, the length of the upper magnet should be about 30 cm or more to make the influence of its other pole negligible.

We can conclude that the field near a pole of a long magnet is really radial; the force between poles of long magnets depends only on their distance.

4.2. How the force between poles depends on their distance

It is possible to make at least semi-quantitative experiment to establish how the force between poles depends on distance. As we can see in figure 9, when the force is 4-times greater (20 mN instead of 5 mN), the distance $r$ is two times smaller (4 cm instead of 8 cm). This agrees with the formula $F \sim 1/r^2$.

Figure 9. Distances between poles of magnets for forces 20 mN (left) and 5 mN (right).

Of course, our simple experiment cannot find the formula but at least it can distinguish between dependences $1/r^n$ for different integer powers $n$.

4.3. Analogy with electrostatics (and how it helps us to find the value of the magnetic field of a magnet)

We can infer from our measurement that the formula for the force between poles of long magnets resembles Coulomb’s law for the force between point charges, $F = \frac{1}{4\pi \mu_0} \frac{Q_1 Q_2}{r^2}$. So we can try to use, as an analogy, the law for the force between poles of very long thin magnets in the form $F = \frac{1}{4\pi \mu_0} \frac{Q_{m1} Q_{m2}}{r^2}$. Here, $\mu_0$ is the permeability of vacuum ($\mu_0 = 4\pi \times 10^{-7}$ Tm/A) and $Q_{m1}$, $Q_{m2}$ are “magnetic charges” of the poles.

Of course, we all know that there are no magnetic monopoles (at least none were found and surely no monopoles reside on poles of magnets). Certainly, this should be stressed to students when discussing
the analogy with electrostatics mentioned above! However, in spite of the fact that there are no real magnetic charges on the poles, the analogy with electrostatics is useful and it will enable us to calculate some useful features of magnets and magnetic field around them. Moreover, we will see in greater detail below how it is possible that a pole of a magnet generates the field as if some “magnetic charge” was present.

(Note: “Coulomb’s law for magnetic poles” was used in some older textbooks and this law was, in fact, experimentally tested by Coulomb himself.)

We can use the “Coulomb’s law for magnetic poles” to establish “magnetic charge” at the poles of magnets quantitatively. We use the same magnets at the lever and above it, so \( Q_{m1} = Q_{m2} \). Therefore, we can derive that \( Q_m = r \sqrt{4\pi \mu_0 F} \). After using values from the measurement described above, we get \( Q_m \approx 2.2 \cdot 10^{-5} \text{ Wb} \).

We can follow the analogy with electrostatics further. The force acting on “magnetic charge” \( Q_m \) in the field with intensity \( H \) is then \( F = Q_m H \). Combining this with the “Coulomb’s law for magnetic poles” gives \( H = \frac{1}{4\pi \mu_0} \frac{Q_m}{r^2} \), so the field \( B = \mu_0 H \) near the pole of the magnet is \( B = \frac{Q_m}{4\pi r^2} \). From this, it follows that \( Q_m = \left(4\pi r^2\right)B \). This resulting formula can be interpreted in a clear and straightforward way. As it is indicated in figure 10, the “magnetic charge” is simply the magnetic flux through a sphere surrounding the pole of the magnet: \( Q_m = \left(4\pi r^2\right)B = SB = \Phi \).

Note: Now it is also clear why the “magnetic charge” is measured in webers.

![Figure 10. Magnetic flux through a sphere around a pole of a long thin magnet.](image)

Now, just one more step is necessary in our teaching-learning sequence. We stressed that there are no real magnetic charges, so no sources of magnetic flux. Therefore, because there is non-zero magnetic flux going out from the pole of our magnet the same magnetic flux has to come into the pole. The only place where the flux could come into the pole is inside the magnet.

Therefore, by measuring the “magnetic charge” \( Q_m \) we, in fact, measured also the magnetic flux inside the magnet. Taking a reasonable assumption that the magnetic field is homogeneous inside the magnet, we can use our previous measurement \( (Q_m \approx 2.2 \cdot 10^{-5} \text{ Wb}) \) to establish the value of the field \( B \) inside the magnet. As \( B \cdot S = \Phi = Q_m \) and the area of cross section of the magnet (of diameter 5 mm) is \( S \approx 2 \cdot 10^{-5} \text{ m}^2 \), we arrive at the result \( B \approx 1.1 \text{ T} \).

According to data provided by suppliers of the magnets, the remanence (remanent magnetisation) \( B_r \) in the magnets should be about 1.35 T. So, our derivation based on a very simple experiment underestimated the value of magnetic field inside the magnets by about 20%. We can say that, as the order of magnitude is concerned, our results are in good agreement with values taken from “technical data” concerning the magnets.
5. Comparison of $B$ generated by a pole of a magnet and Earth’s magnetic field

Another possibility to check our results is to compare the field of the pole of the magnet with the horizontal component of Earth’s magnetic field. To do it, we can use the simple compass built in the workshop. (Now we will use it for quantitative measurement.) It is sufficient to bring a pole of a long magnet to the compass from east or west. When the compass needle is tilted by 45º from its original direction the value of magnetic field from the magnet is the same as a horizontal component of Earth’s magnetic field. From the formulas stated above and from the value of $Q_m$ one can easily derive that it should be when the distance of the pole of our magnet from the compass needle is about 30 cm. The experiment (with a magnet of length about 1 m) confirms this prediction.

6. Conclusions

Formulas derived (or “discovered”) by simple experiments and reasoning described above can be used to calculate forces between poles of long magnets. The results can be applied and tested in other circumstances.

For example, it is clear that the magnetic flux in a magnet which has twice larger diameter is four times greater than that in an original magnet, (In case the remanence is the same.) In the formalism used above we can say that the “magnetic charge” at the pole is now four times greater. So the magnetic field at the same distance from the pole is also four times greater. Moreover, the force acting on the pole of another magnet (with also two-times greater diameter) is 16 times greater – again a fact we can test.

Yet another possibility for generalization: If we take into account the influence of the other pole of the magnet, we can calculate the field generated by short magnets, derive the fact that the force between a short magnet (a dipole) and a pole of a long magnet decreases with distance as $1/r^3$, test this using our lever with a magnet etc. There is a lot of space here for further experiments, reasoning and discussions and it is up to each physics teacher or educator what they use for teaching their students.

Let’s add that the effect we ignored in the present workshop – oscillations of magnets in external magnetic field – was a topic of another workshop led this year with Czech physics teachers. The experience gained from that will be hopefully presented at some further GIREP conference.

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