Odd Scalar Curvature in Anti-Poisson Geometry

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Abstract

Recent works have revealed that the recipe for field-antifield quantization of Lagrangian gauge theories can be considerably relaxed when it comes to choosing a path integral measure $\rho$ if a zero-order term $\nu_\rho$ is added to the $\Delta$ operator. The effects of this odd scalar term $\nu_\rho$ become relevant at two-loop order. We prove that $\nu_\rho$ is essentially the odd scalar curvature of an arbitrary torsion-free connection that is compatible with both the anti-Poisson structure $E$ and the density $\rho$. This extends a previous result for non-degenerate antisymplectic manifolds to degenerate anti-Poisson manifolds that admit a compatible two-form.

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1 Introduction

The main purpose of this Letter is to report on new geometric insights into the field-antifield formalism. In general, the field-antifield formalism \[1, 2, 3\] is a recipe for constructing Feynman rules for Lagrangian field theories with gauge symmetries. The field-antifield formalism is in principle able to handle the most general gauge algebra, \textit{i.e.} open gauge algebras of reducible type. The input is usually a local relativistic field theory, formulated via a classical action principle in a geometric configuration space. In the field-antifield scheme, the original field variables are extended with various stages of ghosts, antighosts and Lagrange multipliers — all of which are then further extended with corresponding antifields; the gauge symmetries are encoded in a nilpotent Fermionic BRST symmetry \[4, 5\]; and the original action is deformed into a BRST-invariant master action, whose Hessian has the maximal allowed rank. The full quantum master action

\[ W = S + \sum_{n=1}^{\infty} \hbar^n M_n \]  

is determined recursively order by order in \( \hbar \) from a consistent set of quantum master equations

\[
\begin{align*}
(S, S) &= 0 , \\
(M_1, S) &= i(\Delta_{\rho} S) , \\
(M_2, S) &= i(\Delta_{\rho} M_1) + \nu_\rho - \frac{1}{2}(M_1, M_1) , \\
(M_n, S) &= i(\Delta_{\rho} M_{n-1}) - \frac{1}{2} \sum_{r=1}^{n-1}(M_r, M_{n-r}) , \quad n \geq 3 .
\end{align*}
\]

Here \((\cdot, \cdot)\) is the antibracket (or anti-Poisson structure), \(\Delta_{\rho}\) is the odd Laplacian and \(\nu_\rho\) is an odd scalar, which become relevant in perturbation theory at loop order 0, 1, and 2, respectively. It has only recently been realized that the field-antifield formalism can consistently accommodate a non-zero \(\nu_\rho\) term, thereby providing a more flexible framework for field-antifield quantization \[6, 7, 8\].

The classical master equation (1.2) is a generalization of Zinn-Justin’s equation \[9\], which allows to set up consistent renormalization (if the field theory is renormalizable). If the theory is not anomalous at the one-loop level, there will exist a local solution \(M_1\) to the next equation (1.3), and so forth. Although the field-antifield formalism in its basic form is only a formal scheme — \textit{i.e.} particularly, it assumes that results from finite dimensional analysis are directly applicable to field theory, which has infinitely many degrees of freedom — it has nevertheless been successfully applied to a large variety of physical models. It has mainly been used in a truncated form of the full set of quantum master eqs. (1.2) – (1.5), where all the following quantities

\[
(S, S), \ (\Delta_{\rho} S), \ \nu_\rho, \ M_1, \ M_2, \ M_3, \ldots ,
\]

are set identically equal to zero. One can for instance mention the AKSZ paradigm \[10, 11\] as a broad example that uses the truncated field-antifield formalism (1.6) to quantize supersymmetric topological field theories \[12, 13, 14, 15\]. Currently, very few scientific works describe solutions with non-zero \(M_n\)’s, primarily due to the singular nature of the odd Laplacian \(\Delta_{\rho}\) in field theory (again because of the infinitely many degrees of freedom). Nevertheless, it should be fruitful to study generic solutions of the full quantum master equation. See the original paper \[1\] for an interesting solution with \(M_1 \neq 0\). Finally, it has in many cases been explicitly checked that the field-antifield formalism produces the same result as the Hamiltonian formulation \[16, 17, 18\]. The formalism has also influenced work in closed string field theory \[19\] and several branches of mathematics. The geometry behind the field-antifield formalism was further clarified in Ref. \[20, 21, 22, 23\].

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In this Letter we shall only explicitly consider the case of finitely many variables. Our main result concerns the odd scalar $\nu^\rho$, which is a certain function of the anti-Poisson structure $E^{AB}$ and the density $\rho$, cf. eq. (6.1) below. It turns out that $\nu^\rho$ has a geometric interpretation as (minus 1/8 times) the odd scalar curvature $R$ of any connection $\nabla$ that satisfies three conditions; namely that $\nabla$ is 1) anti-Poisson, 2) torsion-free and 3) $\rho$-compatible. This is a rather robust conclusion as we shall prove in this Letter that it even holds for degenerate antibrackets. (Degenerate anti-Poisson structures appear naturally from for instance the Dirac antibracket construction for antisymplectic second-class constraints [7, 21, 24, 25].)

2 Anti-Poisson structure $E^{AB}$

An anti-Poisson structure is by definition a possibly degenerate $(2,0)$ tensor field $E^{AB}$ with upper indices that is Grassmann-odd

$$\varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1,$$

(2.1)

that is skewsymmetric

$$E^{AB} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{BA},$$

(2.2)

and that satisfies the Jacobi identity

$$\sum_{\text{cycl. } A,B,C} (-1)^{\varepsilon_A+1}(\varepsilon_C+1) E^{AD}(\partial_D E^{BC}) = 0.$$  

(2.3)

3 Compatible two-form $E_{AB}$

In general, an anti-Poisson manifold could have singular points where the rank of $E^{AB}$ jumps, and it is necessary to impose a regularity criterion to proceed. We shall here assume that the anti-Poisson structure $E^{AB}$ admits a compatible two-form field $E_{AB}$, i.e. that there exists a two-form field $E_{AB}$ with lower indices that is Grassmann-odd

$$\varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1,$$

(3.1)

that is skewsymmetric

$$E_{AB} = -(-1)^{\varepsilon_A\varepsilon_B}E_{BA},$$

(3.2)

and that is compatible with the anti-Poisson structure in the sense that

$$E^{AB}E_{BC}E^{CD} = E^{AD},$$

(3.3)

$$E_{AB}E^{BC}E_{CD} = E_{AD}.$$  

(3.4)

This is a relatively mild requirement, which is always automatically satisfied for a Dirac antibracket on antisymplectic manifolds with antisymplectic second-class constraints [7, 21, 24, 25]. Note that the two-form $E_{AB}$ is neither unique nor necessarily closed. One can define a $(1,1)$ tensor field as

$$P^A_C \equiv E^{AB}E_{BC},$$

(3.5)

or equivalently,

$$P^A_C \equiv E_{AB}E^{BC} = (-1)^{\varepsilon_A(\varepsilon_C+1)}P^C_A.$$  

(3.6)

It then follows from either of the compatibility relations (3.3) and (3.4) that $P^A_B$ is an idempotent

$$P^A_B P^B_C = P^A_C.$$  

(3.7)
4 The $\Delta_E$ Operator

An anti-Poisson structure with a compatible two-form field $E_{AB}$ gives rise to a Grassmann-odd, second-order $\Delta_E$ operator that takes semidensities to semidensities. It is defined in arbitrary coordinates as [7]

$$\Delta_E \equiv \Delta_1 + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{24} + \frac{\nu^{(3)}}{24} + \frac{\nu^{(4)}}{12} + \frac{\nu^{(5)}}{12}, \quad (4.1)$$

where $\Delta_1$ is the odd Laplacian

$$\Delta_1 \equiv \left( -1 \right)^{A} \varepsilon_B \partial_A \rho E^{AB} \partial_B \rho , \quad (4.2)$$

with $\rho = 1$, and where

$$\begin{align*}
\nu^{(1)} &\equiv \left( -1 \right)^{\varepsilon_A} \partial_B E^{AB} \varepsilon_B , \\
\nu^{(2)} &\equiv \left( -1 \right)^{\varepsilon_A \varepsilon_C} \partial_D E^{AB} E_{BC} \partial_A E^{CD} , \\
\nu^{(3)} &\equiv \left( -1 \right)^{\varepsilon_B} \partial_C E^{BC} E^{CD} \partial_A E^{BA} , \\
\nu^{(4)} &\equiv \left( -1 \right)^{\varepsilon_B} \partial_D E^{BC} E^{CD} \partial_D E^{BF} P_F A , \\
\nu^{(5)} &\equiv \left( -1 \right)^{\varepsilon_A \varepsilon_C} \partial_D E^{AB} E_{BC} \partial_D E^{CF} P_F D \\
&= \left( -1 \right)^{\left( \varepsilon_A +1 \right) \varepsilon_B} E^{AD} \partial_D E^{BC} \partial_D E^{AF} P_F B . \quad (4.7)
\end{align*}$$

It is shown in Ref. [7] that the $\Delta_E$ operator defined in eq. (4.1) does not depend on the choice of local coordinates, it does not depend on the choice of compatible two-form field $E_{AB}$, and it does map semidensities into semidensities. Moreover, the Jacobi identity (2.3) precisely ensures that $\Delta_E$ is nilpotent

$$\Delta_E^2 = \frac{1}{2} \left[ \Delta_E, \Delta_E \right] = 0 . \quad (4.8)$$

Earlier works on the $\Delta_E$ operator include Ref. [6, 25, 26, 27, 28, 29].

5 The $\Delta$ Operator

Classically, the field-antifield formalism is governed by the anti-Poisson structure $E^{AB}$, or equivalently, the antibracket

$$(f, g) \equiv \left( f \partial_A \right) E^{AB} \left( \partial_B \rho g \right) = - \left( -1 \right)^{\left( \varepsilon_f +1 \right) \left( \varepsilon_g +1 \right)} (g, f) . \quad (5.1)$$

Quantum mechanically, the field-antifield recipe instructs one to choose an arbitrary path integral measure $\rho$, and to use it to build a nilpotent, Grassmann-odd, second-order $\Delta$ operator that takes scalar functions into scalar functions. It is natural to build the $\Delta$ operator by conjugating the $\Delta_E$ operator (4.1) with appropriate square roots of the density $\rho$ as follows:

$$\Delta \equiv \frac{1}{\sqrt{\rho}} \Delta_E \sqrt{\rho} . \quad (5.2)$$

In this way the $\Delta$ operator trivially inherits the nilpotency property from the $\Delta_E$ operator,

$$\Delta^2 = \frac{1}{\sqrt{\rho}} \Delta_E^2 \sqrt{\rho} = 0 . \quad (5.3)$$

In physical applications the nilpotency (5.3) of $\Delta$ is important for the underlying BRST symmetry of the theory.
6 The Odd Scalar $\nu_\rho$

The odd scalar function $\nu_\rho$ is defined as

$$\nu_\rho \equiv (\Delta 1) = \frac{1}{\sqrt{\rho}} (\Delta E \sqrt{\rho}) = \nu_\rho^{(0)} + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{24} + \frac{\nu^{(4)}}{24} + \frac{\nu^{(5)}}{12}, \quad (6.1)$$

where $\nu^{(1)}$, $\nu^{(2)}$, $\nu^{(3)}$, $\nu^{(4)}$, $\nu^{(5)}$ are given in eqs. (4.3)–(4.7), and the quantity $\nu_\rho^{(0)}$ is given as

$$\nu_\rho^{(0)} \equiv \frac{1}{\sqrt{\rho}} (\Delta_1 \sqrt{\rho}). \quad (6.2)$$

The second-order $\Delta$ operator (5.2) decomposes as

$$\Delta = \Delta_\rho + \nu_\rho, \quad (6.3)$$

where $\Delta_\rho$ is the odd Laplacian (4.2). The nilpotency of $\Delta$ implies that

$$\Delta_\rho^2 = (\nu_\rho, \cdot), \quad (6.4)$$

$$\Delta_\rho \nu_\rho = 0. \quad (6.5)$$

The possibility of a non-trivial $\nu_\rho$ has only recently been observed, cf. Ref. [6, 7, 8]. In the past, the odd scalar term $\nu_\rho$ was not present due to a certain compatibility relation between $E$ and $\rho$, which was unnecessarily imposed, and which (using our new terminology) made $\nu_\rho$ vanish. In terms of the quantum master equation

$$\Delta e^{\frac{i}{\hbar} W} = 0, \quad (6.6)$$

the odd scalar $\nu_\rho$ enters at the two-loop order $O(\hbar^2)$

$$\frac{1}{2}(W, W) = i\hbar \Delta_\rho W + \hbar^2 \nu_\rho, \quad (6.7)$$

which in turn leads to the set of eqs. (1.2) – (1.5).

7 Connection

In the next two Sections 7 and 8 we will briefly state our sign conventions and definitions for the covariant derivative and the curvature in the presence of Fermionic degrees of freedom. A more complete treatment can be found in Ref. [8, 30]. Other references include Ref. [31]. Our convention for the left covariant derivative $(\nabla_A X)^B$ of a left vector field $X^A$ is [30]

$$(\nabla_A X)^B \equiv (\partial_A X)^B + (-1)^{\varepsilon_X(\varepsilon_B + \varepsilon_C)} \Gamma^B_{AC} X^C, \quad \varepsilon(X^A) = \varepsilon_X + \varepsilon_A. \quad (7.1)$$

A connection $\Gamma^B_{AC}$ is called anti-Poisson if it preserves the anti-Poisson structure $E^{AB}$, i.e.

$$0 = (\nabla_A E)^{BC} \equiv (\partial_A E)^{BC} + (\Gamma^B_{AD} E^{DC} - (-1)^{(\varepsilon_B + 1)(\varepsilon_C + 1)}(B \leftrightarrow C)). \quad (7.2)$$

It is useful to define a reordered Christoffel symbol $\Gamma^A_{BC}$ as

$$\Gamma^A_{BC} \equiv (-1)^{\varepsilon_A \varepsilon_B} \Gamma^B_{AC}. \quad (7.3)$$
A torsion-free connection $\Gamma^{A}_{BC}$ has the following symmetry in the lower indices:

$$
\Gamma^{A}_{BC} = -(-1)^{(\varepsilon_B+1)(\varepsilon_C+1)}\Gamma^{A}_{CB} .
$$

A connection $\Gamma^{A}_{BC}$ is called $\rho$-compatible if

$$
\Gamma^{B}_{BA} = (\ln \rho \leftarrow \partial^A_\rho) .
$$

There are in principle two definitions for the divergence $\text{div}X$ of a Bosonic vector field $X$ with $\varepsilon_X = 0$. The first divergence definition depends on the density $\rho$

$$
\text{div}_\rho X \equiv (-1)^{\varepsilon_A} \rho^{-\varepsilon_A} \partial^A_\rho (\rho X^A) ,
$$

while the second definition depends on the connection $\nabla$

$$
\text{div}_\nabla X \equiv \text{str}(\nabla X) \equiv (-1)^{\varepsilon_A} (\nabla_A X)^A = ((-1)^{\varepsilon_A} \partial^A_\rho + \Gamma^{B}_{BA})X^A .
$$

The $\rho$-compatibility condition (7.5) precisely ensures that the two definitions (7.6) and (7.7) coincide, and hence that there is a unique notion of volume [32]. We shall only consider torsion-free connections $\nabla$ that are anti-Poisson and $\rho$-compatible, i.e.

connections that satisfy the above three conditions (7.2), (7.4) and (7.5). Then the odd Laplacian $\Delta_\rho$ can be written on a manifestly covariant form

$$
\Delta_\rho = \frac{(-1)^{\varepsilon_A}}{2} \nabla_A E^{AB} \nabla_B = \frac{(-1)^{\varepsilon_B}}{2} E^{BA} \nabla_A \nabla_B .
$$

8 Curvature

The Riemann curvature tensor is

$$
R^{A}_{BCD} \equiv (-1)^{\varepsilon_A}\varepsilon_B (\partial^D_\rho \Gamma^{A}_{CD}) + \Gamma^{A}_{BE} \Gamma^{E}_{CD} - (-1)^{\varepsilon_B}\varepsilon_C (B \leftrightarrow C) .
$$

(Note that the ordering of indices on the Riemann curvature tensor is slightly non-standard to minimize appearances of sign factors.) The Ricci tensor is

$$
R_{AB} \equiv R^{C}_{AB} = \frac{(-1)^{\varepsilon_C}}{\rho} (\partial^D_\rho \Gamma^{C}_{AB}) - (\partial^D_\rho \ln \rho \partial^B_\rho) - \Gamma^{C}_{AD} \Gamma^{D}_{CB} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}R_{BA} .
$$

9 Odd Scalar Curvature

The odd scalar curvature $R$ is defined as the Ricci tensor $R_{AB}$ contracted with the anti-Poisson tensor $E^{AB}$,

$$
R \equiv R_{AB} E^{BA} = E^{AB} R_{BA} , \quad \varepsilon(R) = 1 .
$$

We now assert that the odd scalar curvature

$$
R = -8\nu_\rho
$$

of an arbitrary connection $\nabla$ that is anti-Poisson, torsion-free and $\rho$-compatible, is equal to (minus eight times) the odd scalar $\nu_\rho$. In particular one sees that the odd scalar curvature $R$ carries no information about the connection $\nabla$ used, and it depends only on $E$ and $\rho$. Equation (9.2) was proven for the non-degenerated case in Ref. [8]. The degenerated case is proven in Appendix A.
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A Proof of the Main Eq. (9.2)

Equation (C.9) in Ref. [8] yields that the odd scalar curvature $R$ can be written as

$$R = -8\nu^{(0)}(\rho) - \nu^{(1)} - \frac{1}{2}R_I,$$

where $\nu^{(0)}(\rho)$, $\nu^{(1)}$ and $R_I$ are defined in eqs. (6.2), (4.3) and (A.2), respectively. Since the expression (A.2) below for $R_I$ only depends on the torsion-free part of the connection, one does in principle not need the torsion-free condition (7.4) from now on. The heart of the proof consists of the following ten "one-line calculations":

$$R_I \equiv \Gamma^A_{BC}(E^{CB}\partial_A^{-}) = \Gamma^A_{BC}((E^{CD}E_{DF}E^{FB})\partial_A^{-}) = 2R_{II} + R_{III},$$

$$R_{II} \equiv \Gamma^A_{BC}D^C_D(\partial^B_D E^{FB}) = -R_{IV} - \nu^{(2)},$$

$$R_{III} \equiv (-1)^{\varepsilon_A(\varepsilon_C + 1)} \Gamma^F_{AB}E^{BC}(\partial^A_D E^{CD})E^{DF} = 2R_{III} + R_V,$$

$$R_{IV} \equiv \Gamma^A_{BC}E^{CD}(\partial^D_D E^{BF})E^F_A = R_{VII} - R_{IV},$$

$$R_V \equiv -(1)^{\varepsilon_A(\varepsilon_C + 1)} \Gamma^F_{AB}P^B_C(\partial^A_D E^{CD})P^D_F = R_{VII} - \nu^{(5)},$$

$$R_{VII} \equiv \Gamma^A_{BC}(E^{CB}\partial_D)P^D_A = 2R_{VII} + R_{IX},$$

$$R_{VIII} \equiv -(1)^{\varepsilon_A(\varepsilon_C + 1)} \Gamma^F_{AB}E^{BC}(\partial^A_D E^{CD})P^D_F = R_{IV} - R_{VIII},$$

$$R_{IX} \equiv -(1)^{\varepsilon_A(\varepsilon_C + 1)} \Gamma^F_{AB}P^B_C(\partial^A_D E^{CD})P^D_F = -R_{X} - \nu^{(4)},$$

$$R_X \equiv -(1)^{\varepsilon_A(\varepsilon_C + 1)} \Gamma^F_{AB}E^{BC}(\partial^A_D E^{CD})E^{DF} = -R_{III} - \nu^{(3)}.$$  

Here we have used the upper compatibility relation (3.3) for the two-form $E_{AB}$ in the second equality of eqs. (A.2), (A.7), (A.8), (A.9) and (A.10); the lower compatibility relation (3.4) for the two-form $E_{AB}$ in the second equality of eq. (A.4); the anti-Poisson property (7.2) for the connection $\nabla$ in the second equality of eqs. (A.3), (A.6), (A.9), (A.10) and (A.11); and the Jacobi identity (2.3) in the second equality of eqs. (A.5) and (A.8). From these ten relations (A.2)–(A.11), the quantity $R_{III}$ can be determined as follows:

$$-R_{III} = R_V = R_{VII} - \nu^{(5)} = (R_{IV} - R_{VIII}) + (R_{IV} + R_{VIII}) = 2R_{IV}$$

$$= R_{VII} = 2R_{VII} + R_{IX} = -2(R_{IV} + R_{VIII}) + (R_{III} + \nu^{(3)} - \nu^{(4)})$$

$$= 2R_{III} + (\nu^{(3)} - \nu^{(4)} - 2\nu^{(5)}),$$

so that

$$R_{III} = \frac{1}{3}(-\nu^{(3)} + \nu^{(4)} + 2\nu^{(5)}).$$

Next, $R_I$ can be expressed in terms of $R_{III}$:

$$\frac{1}{2}R_I = R_{II} + \frac{1}{2}R_{III} = - (R_{IV} + \nu^{(2)}) + \frac{1}{2}R_{III} = R_{III} - \nu^{(2)}.$$
Inserting eqs. (A.13) and (A.14) into eq. (A.1) yields the main eq. (9.2):

$$
R = -8\nu^{(0)} - \nu^{(1)} - \frac{1}{2}R_I = -8\nu^{(0)} + \nu^{(1)} + \nu^{(2)} + \frac{1}{3}(\nu^{(3)} - \nu^{(4)} - 2\nu^{(5)}) = -8\nu^{(0)} .
$$

(A.15)

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