Lifshitz-like space-time from intersecting branes in string/M theory

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Abstract

We construct 1/4 BPS, threshold F-D$p$ bound states (with $0 \leq p \leq 5$) of type II string theories by applying S- and T-dualities to the D1-D5 system of type IIB string theory. These are different from the known 1/2 BPS, non-threshold F-D$p$ bound states. The near horizon limits of these solutions yield Lifshitz-like space-times with varying dynamical critical exponent $z = 2(5 - p)/(4 - p)$, for $p \neq 4$, along with the hyperscaling violation exponent $\theta = p - (p - 2)/(4 - p)$, showing how Lifshitz-like space-time can be obtained from string theory. The dilatons are in general non-constant (except for $p = 1$). We discuss the holographic RG flows and the phase structures of these solutions. For $p = 4$, we do not get a Lifshitz-like space-time, but the near horizon limit in this case leads to an AdS$_2$ space.
1 Introduction

Holographic ideas [1] in the form of gauge/gravity duality [2] have been proved quite useful in recent years to understand the strong coupling behavior of theories without gravity from the weakly coupled gravity theories in one higher space-time dimensions. This general idea is believed to be applicable not only to relativistic theories suitable for QCD (see [3], for some reviews), but also to non-relativistic theories suitable for condensed matter systems (for reviews, see [4]).

Non-relativistic symmetries can be of two types, namely, the Schrödinger symmetry and the Lifshitz symmetry. In both types the time and spaces scale differently breaking the Lorentz invariance. Schrödinger symmetry consists of time and space translations, spatial rotations, Galilean boosts, dilatations or scaling symmetry, a special conformal transformation and a particle number symmetry. On the other hand, Lifshitz symmetry is a much smaller symmetry with only time and space translations, spatial rotations and a scaling symmetry. Gravitational theories having Schrödinger symmetry group which are relevant for strongly coupled condensed matter systems, namely, the fermions at unitarity have been found and they were shown to be easily embedded in string theory [5, 6]. Gravitational theories having Lifshitz symmetry group relevant for certain strongly coupled condensed matter systems at their quantum critical point have also been found [7, 8], however, their embeddings in string theory are not so easy. In recent literature various methods of embedding the Lifshitz space-time into string or M-theory have been reported [9].

In this paper, we report on how Lifshitz-like space-time can be obtained from certain intersecting brane solutions of string/M theory. To be precise, we start from the known intersecting 1/4 BPS D1-D5 threshold bound state solution of type IIB string theory [10]. We apply two successive T-duality transformations to it – first along the common D1-D5 direction to produce D0-D4 bound state and then along one of the D4-brane directions to produce D1-D3 bound state. Note that here D1-branes are transverse to D3-branes and are delocalized. This is a 1/4 BPS, threshold bound state unlike the more familiar D1-D3 bound state which is a 1/2 BPS and non-threshold bound state [11, 12]. An S-duality transformation on this D1-D3 bound state will produce F-D3 bound state which is again a 1/4 BPS, threshold bound state. Next, application of T-duality along D3-brane directions will produce F-D2, F-D1 and F-D0 bound states while the application of T-duality along the common transverse directions of F-strings and D3-branes will produce F-D4 and F-D5 bound states. Thus we obtain all the F-Dp (with 0 ≤ p ≤ 5) bound state solutions of type II string theories. Since these F-Dp solutions are U-dual to D1-D5 system, they are 1/4
BPS and threshold bound states and are different from the known 1/2 BPS, non-threshold F-Dp bound states [13].

The near horizon limits of these intersecting F-Dp solutions yield Lifshitz-like space-time in a suitable coordinate with the dynamical critical exponent \( z = \frac{2(5 - p)}{(6 - p)} \) and the hyperscaling violation exponent \( \theta = p - (p - 2)/(4 - p) \) for \( p \neq 4 \). For \( p = 4 \), the near horizon limit does not yield Lifshitz-like space-time, but gives an AdS_2 space up to a conformal factor. Except for \( p = 1 \), the dilatons in all these solutions are non-constant and as a consequence they produce holographic RG flows. The Lifshitz-like solutions that we just mentioned are valid in certain range of parameters where the effective string coupling and the space-time curvature remain small. However, in other regions, we have to either uplift the solutions to 11 dimensions or M-theory (for type IIA) or go to the S-dual frame (for type IIB). The solutions in other regions also have the structures of Lifshitz-like space-time and we discuss them case by case. For \( p = 4 \), AdS_2 structure is valid in one phase and in other phase we have to uplift the solution to M-theory, where we get an AdS_3 structure, without a conformal factor. Finally, we also discuss a delocalized F-D1 bound state solution whose near horizon limit leads to a completely scale invariant Lifshitz type solution under an asymmetric scale transformation.

This paper is organized as follows. In section 2, we show the construction of 1/4 BPS, threshold F-Dp bound states starting from the known D1-D5 system of type IIB string theory. We then take the near horizon limit on these solutions and show how Lifshitz-like space-time appears. We will start from the 1/4 BPS, threshold D1-D5 bound state of type IIB string theory and then indicate how F-Dp bound state can be obtained from there. We will take the near horizon limit on these solutions and show how Lifshitz-like space-time appears. We

\[ \text{3Such a space-time metric has recently been obtained as a holographic dual of some condensed matter system in [14] [15]. Some aspects of these class of theories have been discussed in [16] [17] [18].} \]

\[ \text{4This concept was introduced in random-field Ising system in [19]. However, in the context of gauge/gravity duality the hyperscaling violation exponent was identified while describing certain compressible metallic states with hidden Fermi surface [14] [15], where the exponent satisfies } \theta = d - 1, \text{ with } d, \text{ the spatial dimensions of the boundary theory. More general gravity solutions with } \theta \text{ not satisfying the relation just mentioned have been discussed in [16] [17] [18].} \]
will discuss some generalities for these solutions.

The string metric and the other field configurations of D1-D5 solution take the following form (see, for example, [10]),

\[
\begin{align*}
\text{ds}^2 &= H_1^4 H_2^4 \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^{4} (dx^i)^2 + H_1^{-1} H_2^{-1} (dx^5)^2 + dr^2 + r^2 d\Omega_3^2 \right] \\
e^{2\phi} &= \frac{H_1}{H_2} \\
A_{[2]} &= (1 - H_1^{-1}) dt \wedge dx^5, \quad A_{[6]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge \cdots \wedge dx^5
\end{align*}
\]  

(1)

In the above, \(H_{1,2}\) are the two harmonic functions given as

\[
H_{1,2} = 1 + \frac{Q_{1,2}}{r^2}
\]

(2)

where \(Q_{1,2}\) are the charges associated with D1-branes and D5-branes. The radial coordinate transverse to D1-D5 system is given as \(r = \sqrt{(x^6)^2 + \cdots + (x^9)^2}\). We note that D1-branes lie along \(x^5\), whereas D5-branes lie along \(x^1, x^2, \ldots, x^5\). The dilaton in general is not constant and we have put the string coupling \(g_s = 1\). \(A_{[2]}\) and \(A_{[6]}\) are the RR 2-form and 6-form which couple to D1-brane and D5-brane respectively. The constant terms in the form fields are added to ensure that the solution is asymptotically flat. But when we take the near horizon limit we generally deal with asymptotically non-flat solutions and in those cases we will ignore the constant terms in the form fields.

We then apply two successive T-duality, first, along \(x^5\) and second, along \(x^4\) to the above solution and we will get a 1/4 BPS, threshold D1-D3 bound state solution of type IIB string theory. The solution has the form,

\[
\begin{align*}
\text{ds}^2 &= H_1^4 H_2^4 \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^{3} (dx^i)^2 + H_1^{-1} (dx^4)^2 + dr^2 + r^2 d\Omega_4^2 \right] \\
e^{2\phi} &= H_1 \\
A_{[2]} &= (1 - H_1^{-1}) dt \wedge dx^4, \quad A_{[4]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge dx^2 \wedge dx^3
\end{align*}
\]

(3)

Note that here D3-branes lie along \(x^1, x^2, x^3\) and are delocalized in \(x^4\), whereas D1-branes lie along \(x^4\) and are delocalized in \(x^1, x^2, x^3\) directions. Also, the transverse radial coordinate is given as \(r = \sqrt{(x^5)^2 + \cdots + (x^9)^2}\) and therefore the harmonic functions have the forms

\[
H_{1,2} = 1 + \frac{Q_{1,2}}{r^3}
\]

(4)

\(A_{[2]}\) and \(A_{[4]}\) are the RR 2-form and 4-form which couple to D1-brane and D3-brane respectively and \(Q_{1,2}\) are the charges associated with them. Although in (3) the 4-form
field has only electrical component, but it should also include the magnetic component
such that the corresponding field-strength is self-dual. But we do not write here its exact
form. We would also like to remark that the known D1-D3 bound state of type IIB string
theory is 1/2 BPS and non-threshold. The solution of the latter type \[11, 12\] also contains
a non-zero NSNS $B$-field, which is absent in the above solution.

Now an S-duality transformation on this D1-D3 bound state will give an F-D3 bound
state, where ‘$F$’ denotes the fundamental string and has the form,

$$
\begin{align*}
    ds^2 &= H_2^\frac{1}{2} \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^{3} (dx^i)^2 + H_1^{-1}(dx^4)^2 + dr^2 + r^2 d\Omega_4^2 \right] \\
    e^{2\phi} &= H_1^{-1} \\
    B_{[2]} &= (1 - H_1^{-1}) dt \wedge dx^4, \quad A_{[4]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge dx^2 \wedge dx^3
\end{align*}
$$

In (5) $H_{1,2}$ has the same form as given in (4), with $Q_{1,2}$ referring to the charges of F-strings
and D3-branes. D3-branes are along $x^1$, $x^2$, $x^3$ and delocalized in $x^4$. F-strings are along
$x^4$ and delocalized in the D3-brane directions. $B_{[2]}$ is the NSNS 2-form which couples to
F-string and $A_{[4]}$ is the RR 4-form which couples to D3-brane. We remark that this F-D3
bound state is 1/4 BPS and threshold unlike the known F-D3 bound state which is 1/2
BPS and non-threshold \[13\].

Applying a series of T-duality transformations on (5) along D3-brane directions we
get F-D2, F-D1 and F-D0\[5\] and along common transverse directions of F-D3 we get F-D4
and F-D5\[6\] bound states. All the F-D$p$, with $0 \leq p \leq 5$, bound state solutions can be
written as,

$$
\begin{align*}
    ds^2 &= H_2^\frac{1}{2} \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^{p} (dx^i)^2 + H_1^{-1}(dx^{p+1})^2 + dr^2 + r^2 d\Omega_4^2 \right] \\
    e^{2\phi} &= \frac{H_2^{\frac{3-p}{2}}}{H_1} \\
    B_{[2]} &= (1 - H_1^{-1}) dt \wedge dx^{p+1}, \quad A_{[p+1]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge \cdots \wedge dx^p
\end{align*}
$$

where the harmonic functions are,

$$
H_{1,2} = 1 + \frac{Q_{1,2}}{r^{6-p}}
$$

\[5\]There is no 1/2 BPS F-D0 bound state as is well-known \[13\]. In that sense this is quite unique in
this case. Existence of such state has been predicted in \[20\].

\[6\]As we are interested in asymptotically flat solutions we do not consider F-D6 bound state. Beyond
that there are no bound states in the massless theories.
with $Q_{1,2}$ representing the charges of F-strings and Dp-branes. From (6) it is clear that Dp-branes lie along $x^1, x^2, \cdots, x^p$ and are delocalized in the F-string direction, whereas F-strings lie along $x^{p+1}$ and are delocalized in the Dp-brane directions. These are 1/4 BPS, threshold bound states and are different from the known 1/2 BPS, non-threshold F-Dp bound states. $B_{[2]}$ is NSNS 2-form and $A_{[p+1]}$ is the $(p + 1)$-form field which couple to F-string and Dp-brane respectively.

The near horizon limit of the above F-Dp solutions amounts to taking $r \to 0$ limit, such that the harmonic functions in (7) can be approximated as

$$H_{1,2} \approx \frac{Q_{1,2}}{r^{6-p}}$$

The radial parameter $r$ is holographically related to the RG flow parameter in the boundary theory and $r \to 0$ corresponds to going to the IR and $r \to \infty$ corresponds to going to the UV. We will further make a coordinate transformation $r \to 1/r$ for convenience and in terms of this new parameter $r \to \infty$ ($r \to 0$) corresponds to going to the IR (UV). In terms of this new $r$ coordinate the metric in (6) reduces in the near horizon limit to,

$$ds^2 = Q_2^2 r^{2-p} \left[ -\frac{dt^2}{Q_1 Q_2 r^{10-2p}} + \frac{\sum_i^p (dx^i)^2}{Q_2 r^{4-p}} + \frac{(dx^{p+1})^2}{Q_1 r^{4-p}} + \frac{dr^2}{r^2} + d\Omega_{7-p}^2 \right]$$

Further introducing a new coordinate $u$ by the relation

$$u^2 = r^{4-p}$$

we can rewrite the metric in (9) and other field configurations of F-Dp solutions from (6) as follows,

$$ds^2 = Q_2^2 u^{2-p} \left[ -\frac{dt^2}{Q_1 Q_2 u^{4(6-p)}} + \frac{\sum_i^p (dx^i)^2}{Q_2 u^{2}} + \frac{(dx^{p+1})^2}{Q_1 u^{2}} + \frac{4}{(4-p)^2} \frac{du^2}{u^2} + d\Omega_{7-p}^2 \right]$$

$$e^{2\phi} = \frac{Q_2^{3-p}}{Q_1} u^{\frac{(6-p)(1-p)}{4-p}}$$

$$B_{[2]} = -\frac{1}{Q_1 u^{2(6-p)}} dt \wedge dx^{p+1}, \quad A_{[p+1]} = -\frac{1}{Q_2 u^{2(6-p)}} dt \wedge dx^1 \wedge \cdots \wedge dx^p$$

Note that the coordinate relation defined in (10) works for all $p$ except for $p = 4$ and so, the field configuration given in (11) is valid for all $p \neq 4$. Therefore, $p = 4$ case needs to be discussed separately. This we will do in the next section where various RG flow and the phase structure will be considered case by case. Because of the relation (10) it is clear that for $p < 4$, $r \to \infty$ implies $u \to \infty$ and corresponds to going to the IR, whereas $r \to 0$
implies \( u \to 0 \) and this corresponds to going to the UV. On the other hand for \( p > 4 \), \( r \to \infty \) implies \( u \to 0 \) and corresponds to going to the IR, whereas, \( r \to 0 \) implies \( u \to \infty \) and corresponds to going to the UV. From (11) we observe that under the following scale transformations

\[
t \to \lambda^{\frac{2(5-p)}{4-p}} t \equiv \lambda^z t, \quad x^{1,2,...,(p+1)} \to \lambda x^{1,2,...,(p+1)}, \quad u \to \lambda u
\]

only the part in the square bracket of the metric remains invariant. However, the full metric changes. Now instead of looking at the full metric, if we compactify the theory on \( S^{7-p} \) and write the reduced metric in Einstein frame it takes the form,

\[
ds_{p+3}^2 = Q_1^2 Q_2 u^2 \frac{2^{p(4-p)-(p-2)}}{(4-p)(p+1)} \left[ -\frac{dt^2}{Q_1 Q_2 u^2} + \sum_i (dx^i)^2 + \frac{(dx^{p+1})^2}{Q_1 u^2} + \frac{4}{(4-p)^2} \frac{du^2}{u^2} \right]
\]

Under the scaling (12) this metric changes as,

\[
ds_{p+3} \to \lambda^{\frac{2(5-p)}{4-p}(p-2)} ds_{p+3} \equiv \lambda^\theta/d ds_{p+3}
\]

where \( z \) in (12) is called the dynamical critical exponent and \( \theta \) in (14) is called the hyperscaling violation exponent. \( d \) is the spatial dimension of the boundary theory which is \( (p+1) \) in this case. We thus find that the near horizon geometries of the F-Dp bound states produce Lifshitz-like theories with dynamical critical exponent \( z \) and hyperscaling violation exponent \( \theta \) having values,

\[
z = \frac{2(5-p)}{4-p}, \quad \theta = p - \frac{p-2}{4-p}
\]

\( z \) takes integer values 3, 4 and 0 for F-D2, F-D3 and F-D5 solutions and \( \theta \) takes the value 2 for the first two cases and 8 for the last case. From (11) we note that the dilaton is constant only for F-D1 solution and for other solutions it varies with \( u \).

Metric of the type given in (11) (or the compactified version of it\(^7\)) has recently been found \([15]\) to be useful in describing some condensed matter system. In fact, it has been observed that some non-Fermi liquid metallic states with hidden Fermi surface can be described by a holographic IR metric with a dynamical critical exponent \( z \) and a hyperscaling violation exponent \( \theta \). However, since a consistent gravity theory must satisfy the null energy condition (NEC), the pairs \( (z, \theta) \) which satisfy NEC given by (16),

\[
(d - \theta)(d(z - 1) - \theta) \geq 0
\]

\[
(z - 1)(d + z - \theta) \geq 0
\]

\(^{7}\)Note that it is the compact metric whose scaling property defines the hyperscaling violation exponent and not the full ten/eleven dimensional metric.
will therefore lead to a consistent dual field theory. It can be easily checked that the pairs 
\((z, \theta)\) obtained in (15) indeed satisfy the NEC (16). Other string theoretic realization of such metric has been obtained in [17]. See also [21] for some other constructions.

Under the scaling (12) the dilaton and the form fields change as,

\[
\phi \rightarrow \phi + \frac{(6 - p)(1 - p)}{2(4 - p)} \log \lambda, \quad B_{[2]} \rightarrow \lambda^{\frac{2 - p}{4 - p}} B_{[2]}, \quad A_{[p+1]} \rightarrow \lambda^{\frac{2 - (p - 2)^2}{4 - p}} A_{[p+1]} \quad (17)
\]

This shows that \(B_{[2]}\) remains invariant under the scaling only for F-D2 solution but \(A_{[p+1]}\) is never invariant. In section 4, we will discuss a case where the full solution will remain invariant under a scale transformation without any hyperscaling violation. Note that the Lifshitz-like solutions we obtained preserve at least a 1/4 space-time SUSY as the intersecting solutions we started out with are 1/4 BPS.

### 3 RG flow & phase structure: case by case study

Since the RG flows and the phase structures are quite different for different values of \(p\), we will study them case by case in this section.

#### 3.1 \(p = 0\): F-D0 case

The near horizon limit in this case gives the following field configurations with a Lifshitz-like space-time (see (11)),

\[
ds^2 = Q_2^\frac{1}{u^2} \left[ - \frac{dt^2}{Q_1 Q_2 u^5} + \frac{(dx^1)^2}{Q_1 u^2} + \frac{1}{4} \frac{du^2}{u^2} + d\Omega_7^2 \right]
\]

\[
e^{2\phi} = \frac{Q_2^\frac{3}{2}}{Q_1} u^\frac{3}{2}
\]

\[
B_{[2]} = - \frac{1}{Q_1 u^3} dt \wedge dx^1, \quad A_{[1]} = - \frac{1}{Q_2 u^3} dt
\]

We have already discussed the scaling properties of this solution in section 2. Here we will discuss its RG flow and the phase structure. Note that the above gravity description is valid when the effective string coupling \(e^\phi\) and the curvature of the space-time remains small. From (18) we find that they amount to the following range of \(u\) where the above gravity description can be trusted,

\[
\frac{1}{Q_2} \ll u \ll \frac{Q_2^3}{Q_1}
\]

(19)
However when \( u \geq Q_1^{2/3}/Q_2 \), the dilaton becomes large and the gravity description breaks down and we have to uplift the solution to eleven dimensions. In eleven dimensions the solution takes the form,

\[
\begin{align*}
\frac{ds^2}{Q_1^{1/3}} &= -\frac{dt^2}{Q_1 Q_2 u^3} + \left(\frac{dx^1}{Q_1 u^2}\right)^2 + \frac{Q_2}{Q_1} \left( dx^{11} - \frac{1}{Q_2 u^3} dt \right)^2 + \frac{1}{4} \frac{du^2}{u^2} + d\Omega_7^2 \\
A_{[3]} &= -\frac{1}{Q_1 u^3} dt \wedge dx^1 \wedge dx^{11}
\end{align*}
\]  

(20)

In M-theory the above solution represents the near horizon limit of a 1/4 BPS, threshold bound state of an M2-brane (along \( x^1, x^{11} \)) with a wave along \( x^{11} \). Note that under the following scale transformation

\[
t \to \lambda^{5/2} t, \quad x^1 \to \lambda x^1, \quad u \to \lambda u, \quad x^{11} \to \lambda^{-1/2} x^{11}
\]

(21)

both the metric and the form field in (20) remain invariant. Thus here also we get a Lifshitz space-time with dynamical critical exponent \( z = 5/2 \) and no hyperscaling violation. However, we have an asymmetric scaling of \( x^1 \) and \( x^{11} \) in this case. The gravity description remains valid when the eleven dimensional metric has small curvature in Planck unit, i.e., \( Q_1 \gg 1 \).

### 3.2 \( p = 1 \): F-D1 case

As we have observed in section 2, the dilaton in this case remains constant and therefore there is no holographic RG flow. The near horizon configuration has the form,

\[
\begin{align*}
\frac{ds^2}{Q_2^{1/3}} &= Q_2^{1/3} \left[ -\frac{dt^2}{Q_1 Q_2 u^{13/2}} + \left(\frac{dx^1}{Q_1 u^2}\right)^2 + \left(\frac{dx^2}{Q_1 u^2}\right)^2 + \frac{4}{9} \frac{du^2}{u^2} + d\Omega_6^2 \right] \\
\frac{e^{2\phi}}{Q_1} &= Q_2 \\
B_{[2]} &= -\frac{1}{Q_1 u^{14/3}} dt \wedge dx^2, \quad A_{[2]} = -\frac{1}{Q_2 u^{14/3}} dt \wedge dx^1
\end{align*}
\]  

(22)

We have discussed the scaling properties of this solution in section 2. Here we note that for the above gravity description to remain valid the effective string coupling \( e^\phi \) and the curvature must remain small. In this case those amount to,

\[
u \gg \frac{1}{Q_2^{4/3}} \gg \frac{1}{Q_1^{4/3}}
\]

(23)

However, for the case \( Q_2/Q_1 \geq 1 \), \( e^\phi \) becomes large and we have to go to the S-dual frame. The S-dual frame configuration representing the near horizon limit of D1-F bound state
will be given as,
\[ ds^2 = Q_2^\frac{1}{2} u^\frac{1}{2} \left[ -\frac{dt^2}{Q_1 Q_2 u^{\frac{1}{4}}} + \frac{(dx^1)^2}{Q_2 u^2} + \frac{(dx^2)^2}{Q_1 u^2} + \frac{4}{9} \frac{du^2}{u^2} + d\Omega_5^2 \right] \]
\[ e^{2\phi} = \frac{Q_1}{Q_2} \]
\[ B_{[2]} = -\frac{1}{Q_2 u^2} dt \wedge dx^1, \quad A_{[2]} = -\frac{1}{Q_1 u^2} dt \wedge dx^2 \]

We again get Lifshitz-like space-time with the same scaling property as the original solution (22). In order to trust the S-dual gravity configuration we must have,
\[ u \gg \frac{1}{Q_1^2} \gg \frac{1}{Q_2^2} \]  

3.3 \( p = 2: \) F-D2 case

The field configurations in the near horizon limit in this case have the form,
\[ ds^2 = Q_2^\frac{1}{2} \left[ -\frac{dt^2}{Q_1 Q_2 u^{\frac{1}{4}}} + \sum_{i=1}^{2} \frac{(dx^i)^2}{Q_2 u^2} + \frac{(dx^3)^2}{Q_1 u^2} + \frac{4}{9} \frac{du^2}{u^2} + d\Omega_5^2 \right] \]
\[ e^{2\phi} = \frac{Q_2^\frac{1}{2}}{Q_1 u^2} \]
\[ B_{[2]} = -\frac{1}{Q_1 u^4} dt \wedge dx^3, \quad A_{[3]} = -\frac{1}{Q_2 u^2} dt \wedge dx^1 \wedge dx^2 \]

The metric has a Lifshitz-like structure and the scaling property of this solution is described earlier. We remark that unlike in other F-Dp cases, here the full metric remains invariant under the scale transformations and so, one might think that this case gives Lifshitz space-time (without any hyperscaling violation), but this is not true. The reason is that the dilaton is not constant. Therefore, when one compactifies the theory on S5, and writes the 5-dimensional metric in the Einstein frame, the resulting metric will not remain invariant under the scaling and will give rise to a hyperscaling violation. Now for the gravity description (26) to remain valid we must impose the conditions that the effective string coupling \( e^\phi \) and the curvature remain small. In this case they amount to the following condition on \( u, \)
\[ u \gg \frac{Q_2^\frac{1}{2}}{Q_1^2}, \quad \text{along with} \quad Q_2 \gg 1 \]  

10
However when \( u \leq Q_1^{1/4}/Q_1^{1/2} \), the effective string coupling \( e^\phi \) becomes large and we have to uplift the solution to M-theory. The uplifted solution has the form,

\[
ds^2 = Q_1^{1/4} Q_2^{1/2} u^{1/2} \left[ -\frac{dt^2}{Q_1 Q_2 u^6} + \frac{\sum_{i=1}^2 (dx^i)^2}{Q_2 u^2} + \frac{(dx^3)^2 + (dx^{11})^2}{Q_1 u^2} + \frac{du^2}{u^2} + d\Omega_5^2 \right]
\]

\[
A_{[3]} = -\frac{1}{Q_1 u^4} dt \wedge dx^3 \wedge dx^{11}, \quad A'_{[3]} = -\frac{1}{Q_2 u^4} dt \wedge dx^1 \wedge dx^2
\]  

(28)

The above configuration represents the near horizon limit of two intersecting M2-branes \([10]\) one along \( x^1, x^2 \) and the other along \( x^3, x^{11} \). Under the scaling \( t \rightarrow \lambda^3 t, \quad x^{1,2,3,11} \rightarrow \lambda x^{1,2,3,11}, \quad u \rightarrow \lambda u \) \n
(29)

the part of the metric in the square bracket remains invariant. However, the metric compactified on S\(^5\) in Einstein frame and the other fields transform as,

\[
ds_6 \rightarrow \lambda^2 ds_6, \quad A_{[3]} \rightarrow \lambda A_{[3]}, \quad A'_{[3]} \rightarrow \lambda A'_{[3]}
\]  

(30)

We thus find that this theory also has a Lifshitz-like structure with the dynamical scaling exponent \( z = 3 \) and the hyperscaling violation exponent \( \theta = 3 \), where this pair of (\( z, \theta \)) satisfies the NEC \([16]\). The gravity description \([28]\) can be trusted for \( u \gg 1/\sqrt{Q_1 Q_2} \).

### 3.4 \( p = 3 \): F-D3 case

In this case the metric having a Lifshitz-like structure and the other field configurations in the near horizon limit are given as,

\[
ds^2 = Q_2^{1/4} u^{1/2} \left[ -\frac{dt^2}{Q_1 Q_2 u^6} + \frac{\sum_{i=1}^3 (dx^i)^2}{Q_2 u^2} + \frac{(dx^4)^2}{Q_1 u^2} + \frac{4du^2}{u^2} + d\Omega_4^2 \right]
\]

\[
e^{2\phi} = \frac{1}{Q_1 u^6}
\]

\[
B_{[2]} = -\frac{1}{Q_1 u^6} dt \wedge dx^4, \quad F_{[5]} = (1 + *) \frac{6}{Q_2 u^5} du \wedge dt \wedge dx^1 \wedge dx^2 \wedge dx^3
\]  

(31)

In \([31]\) instead of the 4-form gauge field we have given the self-dual 5-form field strength which couples to D3-brane. The scaling property of this solution has already been discussed in section 2. Here we note that the gravity description remains valid only when the effective string coupling \( e^\phi \) and the curvature of space-time remain small. In this case we get the following range of \( u \) where both the conditions are satisfied,

\[
\frac{1}{Q_1^3} \ll u \ll Q_2^{1/4}
\]  

(32)
For \( u \leq 1/Q_1^{1/6} \) we have to go to the S-dual frame. The S-dual configurations representing the near horizon limit of D1-D3 bound state have the form,

\[
\begin{align*}
 ds^2 &= Q_1^{\frac{3}{2}} Q_2^{\frac{1}{2}} u^2 \left[ -\frac{dt^2}{Q_1 Q_2 u^8} + \frac{u^2}{Q_2 u^2} \sum_{i=1}^{3} (dx^i)^2 + \frac{(dx^4)^2}{Q_1 u^2} + 4\frac{du^2}{u^2} + d\Omega_4^2 \right] \\
 e^{2\phi} &= Q_1 u^6 \\
 A_{[2]} &= -\frac{1}{Q_1 u^6} dt \wedge dx^4, \quad F_{[5]} = (1 + *) \frac{6}{Q_2 u^5} du \wedge dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \quad (33)
\end{align*}
\]

Under the scaling

\[
t \rightarrow \lambda^4 t, \quad x^{1,2,3,4} \rightarrow \lambda x^{1,2,3,4}, \quad u \rightarrow \lambda u
\]

the Einstein frame metric after an \( S^4 \) compactification and the various fields transform as,

\[
ds_6 \rightarrow \lambda^2 ds_6, \quad \phi \rightarrow \phi + 3 \log \lambda, \quad A_{[2]} \rightarrow \lambda^{-1} A_{[2]}, \quad F_{[5]} \rightarrow \lambda F_{[5]} \quad (35)
\]

We thus find that the S-dual configurations also have a Lifshitz-like space-time with the same dynamical scaling exponent \( z = 4 \) and the hyperscaling violation exponent \( \theta = 2 \) as the original theory and therefore satisfies the NEC \((16)\). The S-dual gravity description can be trusted in the range \( 1/(Q_1 Q_2)^{1/4} \ll u \ll 1/Q_1^{1/6} \).

### 3.5 \( p = 5 \): F-D5 case

The near horizon limit of F-D5 bound state has a Lifshitz-like space-time along with other field configurations given by,

\[
\begin{align*}
 ds^2 &= Q_2^{\frac{1}{2}} u^3 \left[ -\frac{dt^2}{Q_1 Q_2} + \frac{u^2}{Q_2 u^2} \sum_{i=1}^{5} (dx^i)^2 + \frac{(dx^6)^2}{Q_1 u^2} + 4\frac{du^2}{u^2} + d\Omega_2^2 \right] \\
 e^{2\phi} &= \frac{u^4}{Q_1 Q_2} \\
 B_{[2]} &= -\frac{u^2}{Q_1} dt \wedge dx^6, \quad A_{[6]} = \frac{u^2}{Q_2} dt \wedge dx^1 \wedge \cdots \wedge dx^5 \quad (36)
\end{align*}
\]

We have observed the scaling properties of the various fields in section 2. Here we will study its RG flow and the phase structure. From \((36)\) we notice that the above gravity description is valid when \( e^{\phi} \) and the curvature remain small which amounts to the following range of \( u \),

\[
Q_2^{\frac{1}{2}} \ll u \ll (Q_1 Q_2)^{\frac{1}{4}} \quad (37)
\]

However, for \( u \geq (Q_1 Q_2)^{1/4} \), effective string coupling \( e^{\phi} \) becomes large and the gravity description breaks down. In that case we have to go to the S-dual frame. The S-dual
configuration in this case would be given by the near horizon limit of D1-NS5 bound state and has the form,

\[
\begin{align*}
    ds^2 &= Q_1^2 Q_2 u \left[ -\frac{dt^2}{Q_1 Q_2} + \frac{\sum_{i=1}^5 (dx^i)^2}{Q_2 u^2} + \frac{(dx^6)^2}{Q_1 u^2} + 4 \frac{du^2}{u^2} + d\Omega_2^2 \right] \\
    e^{2\phi} &= \frac{Q_1 Q_2}{u^4} \\
    A_{[2]} &= -\frac{u^2}{Q_1} dt \wedge dx^6, \quad H_{[3]} = -Q_2 dx^6 \wedge \epsilon_2 
\end{align*}
\]

where \( \epsilon_2 \) is the volume form of a unit two-sphere. From (38) we find that under the scaling

\[
t \to \lambda^0 t, \quad x^{1,2,\ldots,6} \to \lambda x^{1,2,\ldots,6}, \quad u \to \lambda u
\]

the Einstein frame metric on \( S^2 \) compactification and the various other fields transform as,

\[
ds_8 \to \lambda^4 ds_8, \quad \phi \to \phi - 2 \log \lambda, \quad A_{[2]} \to \lambda^3 A_{[2]}, \quad H_{[3]} \to \lambda H_{[3]} \tag{40}
\]

We therefore find that the theory in the UV also has a Lifshitz-like space-time with the same dynamical scaling exponent \( z = 0 \) and the hyperscaling violation exponent \( \theta = 8 \) as the original theory. This pair of \( (z, \theta) \) satisfies the NEC as we have noted before. The gravity description in this case can be trusted for \( u \gg (Q_1 Q_2)^{1/4} \), where the effective string coupling and the curvature remain small.

### 3.6 \( p = 4 \): F-D4 case

We have mentioned before that \( p = 4 \) case is special since in this case the introduction of new coordinate \( u \) is not possible (see (10)). So, we have to write the near horizon configuration of F-D4 bound state in terms of the original coordinate \( r \). From the general F-Dp solution and using the near horizon limit of the harmonic functions (8) and further making the transformation \( r \to 1/r \), we can write the F-D4 solution in the near horizon limit as,

\[
\begin{align*}
    ds^2 &= \frac{Q_2}{r} \left[ -\frac{dt^2}{Q_1 Q_2 r^2} + \frac{\sum_{i=1}^4 (dx^i)^2}{Q_2 r^2} + \frac{(dx^5)^2}{Q_1 r^2} + \frac{dr^2}{r^2} + d\Omega_3^2 \right] \\
    e^{2\phi} &= \frac{1}{Q_1 Q_2^2 r^3} \\
    B_{[2]} &= -\frac{1}{Q_1 r^2} dt \wedge dx^5, \quad A_{[5]} = -\frac{1}{Q_2 r^2} dt \wedge dx^1 \wedge \cdots \wedge dx^4 \tag{41}
\end{align*}
\]
So, it is clear that we do not get a Lifshitz-like space-time from F-D4, however, the near horizon metric has the structure of AdS$_2$ space up to a conformal factor. The dilaton is non-constant and therefore will produce a holographic RG flow. The above gravity description is valid when the effective string coupling $e^\phi$ and the curvature remain small. In this case these amount to the following range of $r$,

$$\frac{1}{Q_1^2 Q_2^2} \ll r \ll Q_2^2$$

However when $r \leq 1/(Q_1^{1/3} Q_2^{1/6})$, we have to uplift the solution to M-theory. The eleven dimensional metric has the form,

$$ds^2 = Q_1^2 Q_2^2 \left[ -\frac{dt^2}{Q_1 Q_2 r^2} + \sum_{i=1}^4 \frac{(dx^i)^2}{Q_2} + \frac{(dx^5)^2}{Q_1} + \frac{(dx^{11})^2}{Q_1 Q_2 r^2} + \frac{dr^2}{r^2} + d\Omega_3^2 \right]$$

The above solution represents the near horizon limit of intersecting M2-M5 brane meeting on a string where M2-branes are along $x^5$ and $x^{11}$ and M5-branes are along $x^1, \ldots, x^4, x^{11}$. Note that this uplifted solution has the structure of AdS$_3$ space without any conformal factor$^8$. Thus the UV theory has an AdS$_3$ structure. This gravity description can be trusted as long as $Q_1^{1/6} Q_2^{1/3} \gg 1$.

4 A delocalized F-D1 and Lifshitz space-time

In section 3, we noted that among all the F-Dp solutions only F-D2 leads to fully scale invariant ten-dimensional metric in the near horizon limit without any conformal factor. However, the dilaton as well as the RR form field do not remain invariant under the scale transformation. Only the NSNS form field remains invariant. F-D1, on the other hand, leads to constant dilaton in the near horizon limit and therefore remains invariant under the scale transformation. However, the full metric, the NSNS as well as the RR form fields do not remain scale invariant. In this section we will describe a solution which is somewhat in between the two solutions we described, namely, a delocalized F-D1 solution whose near horizon limit will lead to (asymmetric) Lifshitz space-time and the other fields will be invariant under the scale transformation.

To obtain the delocalized solution we start from the F-D2 solution given in (6) for $p = 2$ and then apply T-duality along one of the brane directions of the D2-brane. This

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8This solution has previously been obtained in [22].
will produce a delocalized F-D1 solution. To get its form let us first write F-D2 solution from (6) as,
\[
\begin{align*}
ds^2 &= H_2^{\frac{1}{2}} \left[ -H_1^{-1}H_2^{-1}dt^2 + H_2^{-1} \sum_{i=1}^{2} (dx^i)^2 + H_1^{-1}(dx^3)^2 + dr^2 + r^2d\Omega_5^2 \right] \\
e^{2\phi} &= \frac{H_2^{\frac{3}{2}}}{H_1} \\
B_{[2]} &= (1 - H_1^{-1}) dt \wedge dx^3, \quad A_{[3]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge dx^2
\end{align*}
\]
where the harmonic functions are given as
\[
H_{1,2} = 1 + \frac{Q_{1,2}}{r^4}
\]
(45)
Taking T-duality along \( x^2 \) and then renaming \( x^2 \leftrightarrow x^3 \) we get the delocalized F-D1 solution as,
\[
\begin{align*}
ds^2 &= H_2^{\frac{1}{2}} \left[ -H_1^{-1}H_2^{-1}dt^2 + H_2^{-1}(dx^1)^2 + H_1^{-1}(dx^2)^2 + (dx^3)^2 + dr^2 + r^2d\Omega_5^2 \right] \\
e^{2\phi} &= \frac{H_2}{H_1} \\
B_{[2]} &= (1 - H_1^{-1}) dt \wedge dx^2, \quad A_{[2]} = (1 - H_2^{-1}) dt \wedge dx^1
\end{align*}
\]
(46)
with the harmonic functions having the same form as given in (45). It is clear that the F-strings lie along \( x^2 \) and are delocalized along \( x^1, x^3 \), whereas, D1-branes lie along \( x^1 \) and are delocalized along \( x^2, x^3 \). So, \( x^3 \) is the common delocalized direction. Now if we go to the near horizon limit \( (r \to 0) \) by approximating the harmonic functions by \( H_{1,2} \approx Q_{1,2}/r^4 \) and then make a change of coordinates \( r \to 1/r \), the above solution (46) reduces to
\[
\begin{align*}
ds^2 &= Q_2^{\frac{1}{2}} \left[ -\frac{dt^2}{Q_1Q_2r^6} + \frac{(dx^1)^2}{Q_2r^2} + \frac{(dx^2)^2}{Q_1r^2} + r^2(dx^3)^2 + \frac{dr^2}{r^2} + d\Omega_3^2 \right] \\
e^{2\phi} &= \frac{Q_2}{Q_1} \\
B_{[2]} &= -\frac{1}{Q_1r^4}dt \wedge dx^2, \quad A_{[2]} = -\frac{1}{Q_2r^4}dt \wedge dx^1
\end{align*}
\]
(47)
The solution (47) is invariant under the following scaling,
\[
t \to \lambda^3 t, \quad x^1 \to \lambda x^1, \quad x^3 \to \lambda^{-1}x^3, \quad r \to \lambda r
\]
(48)
\footnote{Note that a localized solution is usually obtained from a delocalized one when we replace the extended source of the delocalized solution by a point source and this is the usual procedure when one takes T-duality. However, instead of changing the source if we keep the extended source the solution remains delocalized \cite{23, 11}.}
We thus find an asymmetric (since $x^3$ transforms differently than $x^{1,2}$) Lifshitz space-time with the dynamical scaling exponent $z = 3$ and no hyperscaling violation. Note that the dilaton and the other form fields also remain invariant under the scale transformation.

The above gravity solution is valid when $e^\phi = \sqrt{Q_2/Q_1} \ll 1$ and also $Q_2^{1/2} \gg 1$. However if $Q_2/Q_1 \gg 1$, we have to go to the S-dual frame. The S-dual solution also has a very similar form, as we have discussed in section 3, with the same scaling property as the original solution.

5 Conclusion

To summarize, in this paper we have shown how to construct 1/4 BPS, threshold F-D$p$ (with $0 \leq p \leq 5$) bound states of type II string theories starting from the well-known D1-D5 system of type IIB string theory by applying two T-duality, an S-duality and then a series of T-duality transformations. The near horizon limits of these solutions (for $p \neq 4$) give rise to Lifshitz-like space-time with the dynamical critical exponent $z = 2(5 - p)/(4 - p)$ and the hyperscaling violation exponent $\theta = p - (p - 2)/(4 - p)$ in a suitable coordinate. We have checked that these values of $(z, \theta)$ satisfy the null energy condition given in (16). As a consistent gravity theory must satisfy the null energy condition in terms of $(z, \theta)$, the pairs which satisfy these conditions will therefore lead to a physically sensible dual field theory. The dilatons are in general non-constant except for $p = 1$ and will generate holographic RG flow. We have given the scaling properties of the dilatons as well as the other form-fields. We have discussed the phase structures of various theories case by case. At different regions of the RG flow parameter, there are different phases. We have analyzed the scaling properties of the theories in other phases and found that they also have Lifshitz-like structure with different dynamical critical exponents and hyperscaling violation exponents. In all cases they also satisfy the null energy condition as discussed in section 3. For $p = 4$, we did not get Lifshitz-like space-time, but the near horizon geometry in this case has an AdS$_2$ structure up to a conformal factor. In the strongly coupled phase the geometry has an AdS$_3$ structure without the conformal factor. We have also discussed a case of a delocalized F-D1 bound state. Here the whole near horizon solution is invariant under an asymmetric scaling of the coordinates. All these solutions discussed here are supersymmetric as they are obtained from 1/4 BPS string states.

The gravity solutions with Lifshitz scaling along with hyperscaling violation, i.e., the type we have discussed in this paper (including the near horizon limit of (F, D2) solution
and its M-theory lift which has $\theta = d - 1$ (see subsection 3.3)) have been used before to model certain strongly interacting condensed matter system with Fermi surface. As at weak coupling it is known that a Fermi surface can be obtained by deforming a relativistic theory with a non-zero chemical potential \cite{21}, it would be interesting to see whether the gravity solutions we have obtained in this paper can also be obtained as some kind of deformation of certain relativistic solutions. Also note that among the various scaling symmetries obtained in sections 2 – 4, the ones discussed in subsection 3.5 (for (F, D5) and its S-dual case in \cite{39}), subsection 3.1 (for M-theory lift of (F, D0) in \cite{21}) and section 4 (for delocalized (F, D1) in \cite{48}) are quite unusual. In \cite{39} we found $z = 0$ which appears to imply that there is no relaxation in time for the system described by the boundary theory. On the other hand for \cite{21} and \cite{48} we found negative scaling exponents for some boundary coordinates and this apparently would imply critical speeding up of the system in those directions. It would be interesting to understand the field theoretic meaning of these scaling symmetries along with the other solutions.

**note added:**

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