Theoretical and experimental studies of the radiative properties of plasma and their applications to temperature diagnostics of Z-pinch plasma

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Abstract. Important features of the theoretical model known as the ion model of plasma, which is used for quantum mechanical calculations of radiative opacity, are discussed. Reliability of ion-model results was tested with experiment, where measurements of X-pinch radiation energy yield for two exploding wire materials, NiCr and Alloy 188 were made. Theoretical estimations of radiative efficiency were compared with experimental results, and ion-model calculations agree well with the experimental data. Subsequently, the theoretical approach has been applied for theoretical and experimental studies of radiative and gas dynamic properties of plasma at high energy density. As it was found, the theoretical approach can be used for temperature diagnostics of Z-pinch plasma. Calculations of the spectral brightness were made for W plasma radiation at the temperatures 1 and 1.2 keV and the densities 1 and 2 g/cc.

1. Introduction

Theoretical and experimental studies of the radiative and gas dynamic properties of plasma are performed widely in the framework of general investigations of matter at high temperature and density [1]. The theoretical part of these studies includes following items: (a) gas dynamics [2–4], (b) photon transport processes [5–7], (c) equation of state [8–11], (d) the radiative opacity [12]. The radiative opacity represents an important part of the study.

Different theoretical models were created last years to calculate the Rosseland and Planck mean free paths, the spectral coefficients for x-ray absorption and other plasma characteristics. Comparative analysis of these models has been carried out based upon the density-functional theory to estimate different physical approximations, which were used in the framework of these models [13]. The general set of self-consistent field equations that describe the state of the whole ensemble of plasma atoms and ions has been obtained. The set contains (1) the Hartree–Fock equations for all atoms and ions with different electron configurations, (2) the unbound electron densities in phase space for each atom and ion, (3) the Gibbs distribution, and (4) the electroneutrality condition.
The important feature of this set is the general coupling of all equations for all plasma atoms and ions, including exited states, and, at first sight, the set cannot be solved because of huge number of equations. Therefore, until recently, further physical approximations were used to simplify these equations.

On the other hand, any simplification of the equations leads to a restricted range of plasma temperature and density, over which the model can be applied [13]. Thus, the Thomas–Fermi model (TF) [14] can be used only for very high temperature plasmas. The main feature of the Hartree–Fock–Slater model (HFS) [15,16] is its average atom approximation, a fictitious atomic system with non-integer numbers of bound electrons in the atomic shells. These numbers are calculated using the Fermi–Dirac formula. As the temperature of the plasma decreases, this model cannot provide accurate enough results. The limitation is connected with the application of perturbation theory, which is used to calculate the properties of a real atomic or ionic system. The detail configuration accounting (DCA) model [17] uses the Hartree–Fock equations for real atomic and ionic systems and the Saha method for calculating their concentrations. This approach cannot be used for strongly coupled plasmas because of a peculiarity of the Saha method.

It is necessary to keep in mind that high power laser pulse can produce intense shock wave and intense radiation wave in a laser target. As a result, strongly coupled plasma can be produced. It can lead to considerable deviation of atomic and ionic characteristics from the characteristics, which were calculated or measured for ideal plasma. A modern version of DCA, which was called as detailed level accounting (DLA) model [18], uses experimental data from different data bases to improve the mentioned characteristics. It should be noted that application of experimental data, which were measured at the ideal plasma conditions, and applied to strongly coupled plasmas calculations in the framework of DLA model can considerably increase the error bar of the DLA model results.

Thus, one can indicate the range of plasma temperature and density over which the mentioned models cannot provide accurate enough results. The solution of the general set of self-consistent field equations that describe the state of the whole ensemble of plasma atoms and ions is the way to solve the problem in general.

The ion model of plasma has been developed to achieve this aim [19]. Although more complicated than previous models, the set of equations was solved for “pure” substances. Subsequently, it was applied to compound chemical compositions. As a result, reliable quantum mechanical calculations of radiative opacity became possible over a wide range of plasma temperature and density.

Subsequently, the ion model of plasma has been applied widely for theoretical and experimental studies of radiative and gas dynamic properties of plasma at high energy density.

In the main, the ion model results, namely the spectral coefficients for x-ray absorption, the Rosseland and Planck mean free paths and other characteristics, were used to solve the radiative gas dynamic equations [20]. But, as it was found, the theoretical approach can be also used for temperature diagnostics for plasma of large nuclear charge \((Z)\) materials, which usually used in Z-pinch [2,5].

2. Temperature diagnostics of high-\(Z\) element plasmas in Z-pinch

A very complicated problem is connected with temperature diagnostics of high-\(Z\) element plasma, which is produced in Z-pinch. Exploding wire in Z-pinch produces so-called “hot spot”, which represents a tiny drop of plasma with high temperature and density. On the first stage, the x-ray radiation intensity is not so big because of high density. On the second stage, the drop extends, and the temperature increases by means of electromagnetic field energy. The diagnostics aim is estimation of this temperature. It should be noted that x-ray radiation intensity increases with temperature increase.
Traditional method of temperature diagnostics are based on the spectral line shape analysis for materials the wire made of. Relative short interval of photon energy is considered in experiment to study in detail the spectral line intensity and broadening. As usual, the intervals are used, where spectral lines are well distinguishable [21]. Then, theoretical calculations are performed at different temperatures and densities and the results are compared with experiment to determine plasma parameters.

The approach has two important restrictions. The first of them is connected with the nuclear charge of wire material. If $Z$ increases, the calculations become very complicated. The second one is connected with plasma temperature. If the temperature exceeds 4–6 keV, the spectral lines become indistinguishable for some materials.

The modern method of temperature diagnostics we propose has not the mentioned restrictions. One can perform calculation of spectral brightness of x-ray radiation on the wide range of photon energy. Since the total yield of radiation increases with temperature increasing, one can hope to determine the energy interval, where spectral brightness is changed considerably even at little increase of plasma temperature. The interval can be determined with calculations, which use the ion model of plasma.

It should be noted that working time of radiative plasma properties calculations increases with $Z$ increasing. Therefore initial calculations were performed for relatively simple material, namely, for molybdenum. The results confirm possibility of this scientific approach [22]. Now we present the calculations for considerably more complicated material, namely, for tungsten. The calculations of the spectral brightness were made using the ion model for W plasma radiation at the temperatures 1 and 1.2 keV and the densities 1 and 2 g/cc.

The normal density of W is 19.35 g/cc, and the value of density 2 g/cc approximately corresponds to the normal density decrease with factor of ten. This value of density is achieved at the second stage of hot spot burning we mentioned above, and coincides with the stage of high radiation intensity.

Geometric size of radiating plasma plays an important part in Z-pinch experiments. Experimental measurements show the size of radiation source is from 0.7 to 2.7 $\mu$m for different experiments. We used the radiating ball approximation for our calculations, and the radius of radiating ball was $R_b = 1 \mu$m. The second approximation we used is the approximation of optically thin plasma. The approximation can be applied if geometric size of radiating plasma does not exceed the Planck mean free path $L_P$ [21]. To confirm applicability the optically thin plasma approximation, the Planck mean was calculated for mention above values of plasma temperatures $T$ and densities $D$ using the ion model of plasma.

The results are presented in table 1.

The results confirm applicability the optically thin plasma approximation for $R_b = 1 \mu$m.

### Table 1. Calculated Planck mean free path for tungsten.

| $T$, keV | $D$, g/cc | $L_P$, $\mu$m |
|----------|-----------|---------------|
| 1        | 1         | 4.92          |
| 1.2      | 1         | 10.3          |
| 1        | 2         | 1.88          |
| 1.2      | 2         | 4.34          |
The spectral brightness of x-ray radiation $j$ (W/cm$^2$/eV/ster) calculated for W plasma at the temperatures $T = 1$ keV (thin line) and $T = 1.2$ keV (bold line), and the density 1 g/cc.

The spectral brightness of x-ray radiation was calculated using the formula:

$$j_{\nu}(E) = \frac{1}{4\pi} J_{\nu}(E) \frac{R_6}{3}, \quad J_{\nu} = cK(E)DU_{\nu}. \quad (1)$$

Here $K(E)$ (cm$^2$/g) is the spectral coefficients for x-ray absorption of photons with energy $E = h\nu$ and frequency $\nu$, $D$ is the plasma density (g/cc), $c$ is the light velocity, $h$ is the Planck constant. The spectral density of equilibrium radiation is [21]:

$$U_{\nu}(E) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(h\nu/T) - 1}. \quad (2)$$

The spectral brightness of x-ray radiation $j(E)$ (W/cm$^2$/eV/ster) was calculated for W plasma (figure 1) at the temperatures $T = 1$ keV (thin line) and $T = 1.2$ keV (bold line), and the density 1 g/cc. The similar calculations was made at the temperatures $T = 1$ keV (thin line) and $T = 1.2$ keV (bold line), but for the density 2 g/cc (figure 2).

Let us consider figure 1 and the photon energy range $E$ on the interval from 17 to 35 keV. One can see that function $j(E)$ value, which was calculated at the temperature $T = 1$ keV (thin line) for $E$ value near 20 keV, increases into more than 20 times for the $j(E)$ value, which was calculated at the same $E$ and the temperature $T = 1.2$ keV (bold line). Thus, the function $j(E)$ variation achieved the factor more than 20 whereas temperature variation was only 20%. Similar variation is achieved for the density 2 g/cc (figure 2). This fact can be used for temperature diagnostics of high-Z element plasma.
Figure 2. The same as in figure 1, but for the density 2 g/cc.

3. Conclusions
The modern method of temperature diagnostics is proposed for plasma of large nuclear charge (Z) materials, which is used in Z-pinch. The method is based on calculation of spectral brightness of x-ray radiation on the wide range of photon energy to determine the energy interval, where spectral brightness is changed considerably even at little increase of plasma temperature. The theoretical model known as ion model of plasma can be used to this end. This approach can be applied even in case when traditional method of temperature diagnostics cannot be used. One can hope that modern methods can be useful for theoretical and experimental applications.

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