CHARMONIUM POLARIZATION IN HIGH ENERGY COLLISIONS

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Abstract

We consider charmonium polarization at high-energy hadron collider Tevatron in the framework of the nonrelativistic QCD (NRQCD) and the $k_T$-factorization approach. The polarization effects are studied for the direct and the prompt production channels. The obtained predictions can be used for test of the Regge limit of QCD and for test of the NRQCD formalism.

High-energy factorization

In the phenomenology of strong interactions at high energy it is needed to describe the QCD evolution of gluon distribution functions in colliding particles starting from the scale $\mu_0$, which controls a nonperturbative regime, to the typical scale of hard scattering processes $\mu \sim M_T = \sqrt{M^2 + |\mathbf{p}_T|^2}$, where $M_T$ is the transverse mass of the produced in a hard process particle. In the region of very high energies $x = \frac{\mu}{\sqrt{S}} \ll 1$. This fact leads to the big logarithmic contributions $\sim (\alpha_s \log(1/x))^n$ in the resummation procedure, which is described by the BFKL \cite{BFKL} evolution equation for an unintegrated gluon distribution function $\Phi(x, |\mathbf{k}_T|^2, \mu^2)$.

In the $k_T$-factorization approach \cite{kt-factorization} hadronic $\sigma(p+p \rightarrow \mathcal{H} + X)$ and partonic $\hat{\sigma}(R+R \rightarrow \mathcal{H} + X)$ cross sections are connected as follows:

$$
\begin{align*}
    d\sigma(p+p \rightarrow \mathcal{H} + X, S) &= \int \frac{dx_1}{x_1} \int d|\mathbf{k}_{1T}|^2 \int \frac{d\varphi_1}{2\pi} \Phi(x_1, |\mathbf{k}_{1T}|^2, \mu^2) \times \\
    &\times \int \frac{dx_2}{x_2} \int d|\mathbf{k}_{2T}|^2 \int \frac{d\varphi_2}{2\pi} \Phi(x_2, |\mathbf{k}_{2T}|^2, \mu^2) \hat{\sigma}(R+R \rightarrow \mathcal{H} + X, \mathbf{k}_{1T}, \mathbf{k}_{2T}),
\end{align*}
$$

where $x_{1,2}$ are the fractions of the proton momenta passed on to the gluons, and $\varphi_{1,2}$ are the angles enclosed between $\mathbf{k}_{1,2T}$ and the transverse momentum $\mathbf{p}_T$ of $\mathcal{H}$, $R$ is considered as reggeized gluon \cite{reggeized}. In a stage of numerical calculation we use the following unintegrated gluon distribution functions $\Phi(x, |\mathbf{k}_T|^2, \mu^2)$: JB \cite{JB}, JS \cite{JS} and KMR \cite{KMR}.

The collinear and the unintegrated gluon distribution functions are formally related as

$$
xG(x, \mu^2) \simeq \int d|\mathbf{k}_T|^2 \Phi(x, |\mathbf{k}_T|^2, \mu^2).
$$

This implies that the cross section (1) is normalized approximately for the parton model cross section, so that when $\mathbf{k}_{1T} = \mathbf{k}_{2T} = 0$ we recover the usual gluon-gluon result for the on-shell gluons.

NRQCD formalism

In the framework of the NRQCD approach \cite{NRQCD} the heavy quarkonium $\mathcal{H}$ production cross section in a partonic process $\hat{\sigma}(a+b \rightarrow \mathcal{H} + X)$ may be presented as a sum of terms
in which the effects of the long and short distances are factorized:

\[ d\hat{\sigma}(\mathcal{H}) = \sum_n d\hat{\sigma}(Q\bar{Q}[n]) \langle \mathcal{O}^\mathcal{H}[n] \rangle. \quad (3) \]

Here \( n \) denotes the set of the color, spin and orbital quantum numbers of the \( Q\bar{Q} \)-pair, \( \hat{\sigma}(Q\bar{Q}[n]) \) is the cross section of the \( Q\bar{Q} \)-pair production with quantum numbers \( n \) and with the equal 4-momenta. The last one can be calculated using the perturbative approach of the QCD as an expansion in small constant of strong interaction \( \alpha_s \) and using the nonrelativistic approximation for the relative motion of the heavy quarks in the \( Q\bar{Q} \)-pair. The nonperturbative transition of the \( Q\bar{Q} \)-pair into the final quarkonium \( \mathcal{H} \) is described by the long distance matrix element \( \langle \mathcal{O}^\mathcal{H}[n] \rangle \). The relevant intermediate states are \([n] = [^3S_1^1, ^3S_1^8, ^1S_0^8, ^3P_1^1, ^3P_1^8] \) for \( \mathcal{H} = J/\psi, \psi' \) and \([n] = [^3P_1^1, ^3S_1^1] \) for \( \mathcal{H} = \chi_{cJ} \), where \( J = 0, 1, 2 \).

**Charmonium production by reggeized gluons**

In this part we obtain squared amplitudes for the charmonium production via the fusion of two reggeized gluons in the framework of the NRQCD. We consider the leading order (LO) in \( \alpha_s \) and \( v \) contributions of the following partonic subprocesses:

\[ R + R \rightarrow \mathcal{H}[^3S_1^1, ^3S_1^8, ^3P_1^1, ^3P_1^8], \quad (4) \]

\[ R + R \rightarrow \mathcal{H}[^3S_1^1] + g, \quad (5) \]

The analysis of the next to leading order (NLO) contribution in the processes of the reggeized gluon-gluon production of the quarkonia in the \( k_T \)-factorization approach is outside the presented paper and it needs a special investigation.

In the case of unpolarized heavy quarkonium production the squared amplitudes for the different \( S \)-wave and \( P \)-wave intermediate states in the fusion of two reggeized gluons have been presented recently in our papers [8].

**Polarization formalism**

In hadronic center-of-mass (CM) reference frame we can define [9, 10] the longitudinal polarization 4-vector for spin-one boson \( (^3S_1, ^3P_1) \) by a covariant way:

\[ \varepsilon^\mu(0) = Z^\mu = \frac{(PQ)P^\mu/M - MQ^\mu}{\sqrt{(PQ)^2 - M^2 S}}, \quad (6) \]

where \( Q = P_1 + P_2, S = Q^2, P_{1,2} \) are colliding hadron 4-momenta. The polarization tensor can be presented as follows:

\[ \mathcal{P}^{\mu\nu} = \sum_{|\lambda|=0,1} \varepsilon^\mu(\lambda)\varepsilon^\nu(\lambda) = -g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2}, \quad (7) \]

\[ \mathcal{P}^{\mu\nu}_0 = \varepsilon^\mu(0)\varepsilon^\nu(0) = Z^\mu Z^\nu, \quad (8) \]

\[ \mathcal{P}^{\mu\nu}_1 = \sum_{|\lambda|=1} \varepsilon^\mu(\lambda)\varepsilon^\nu(\lambda) = \mathcal{P}^{\mu\nu} - \mathcal{P}^{\mu\nu}_0 \quad (9) \]

In the spin-two case \( (^3P_2) \) the polarization tensor \( \mathcal{P}^{\mu\nu}_{|\lambda|} \) reads [11]:

\[ \mathcal{P}^{\mu\nu}_{\rho\sigma} = \frac{1}{6} (2\mathcal{P}^{\mu\nu} - \mathcal{P}^{\mu\nu}_1)(2\mathcal{P}^{\rho\sigma}_0 - \mathcal{P}^{\rho\sigma}_1), \quad (10) \]
\[ \mathcal{P}_1^{\mu\nu\sigma} = \frac{1}{2}(\mathcal{P}_0^{\mu\nu} \mathcal{P}_1^{\nu\sigma} + \mathcal{P}_0^{\mu\sigma} \mathcal{P}_1^{\nu\nu} + \mathcal{P}_0^{\mu\nu} \mathcal{P}_1^{\nu\sigma} + \mathcal{P}_0^{\mu\sigma} \mathcal{P}_1^{\nu\nu}), \]  
\[ \mathcal{P}_2^{\mu\nu\sigma} = \frac{1}{2}(\mathcal{P}_0^{\mu\nu} \mathcal{P}_1^{\nu\sigma} + \mathcal{P}_0^{\mu\sigma} \mathcal{P}_1^{\nu\nu} - \mathcal{P}_0^{\mu\nu} \mathcal{P}_1^{\nu\sigma}). \]

As usual, the spin asymmetry parameter is defined as follows
\[ \alpha(p_T) = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}, \]
where \( \sigma_{L,T} = \sigma_{0,1} = \frac{d\sigma}{dp_T}(p + p \rightarrow J/\psi_{L,T}X) \) For the polarized \( J/\psi \) or \( \psi' \) production via direct channel we can write [12]:
\[ \sigma^{J/\psi,\psi'}_L = \sigma_0^{J/\psi,\psi'}(3S_1^{(1)}) + \sigma_0^{J/\psi,\psi'}(3S_1^{(8)}) + \frac{1}{3}\sigma^{J/\psi,\psi'}(1S_0^{(8)}) + \frac{1}{3}\sigma^{J/\psi,\psi'}(1S_0^{(8)}) + \frac{1}{2}\sigma_1^{J/\psi,\psi'}(P_1^{(1)}) + \frac{2}{3}\sigma_0^{J/\psi,\psi'}(P_2^{(1)}) + \frac{1}{2}\sigma_1^{J/\psi,\psi'}(P_2^{(1)}) \]

As it can be shown [9] for the prompt polarized \( J/\psi \) production one has:
\[ \sigma_{L}^{\text{prompt}} = \sigma_{L}^{J/\psi} + \sigma_{L}^{\chi_{c} \rightarrow J/\psi} + \sigma_{L}^{\psi' \rightarrow J/\psi} + \sigma_{L}^{\psi' \rightarrow \chi_{c} \rightarrow J/\psi} \]

\[ \sigma_{L}^{\chi_{c} \rightarrow J/\psi} = \left[ \frac{1}{3}\sigma^{\chi_{c}}(3P_0^{(1)}) + \frac{1}{3}\sigma^{\chi_{c}}(3S_1^{(8)}) \right] Br(\chi_{c0} \rightarrow J/\psi + \gamma) + \frac{1}{3}\sigma^{\chi_{c}}(3P_1^{(1)}) + \frac{2}{3}\sigma^{\chi_{c}}(3S_1^{(8)}) + \frac{1}{4}\sigma^{\chi_{c}}(3S_1^{(8)})] Br(\chi_{c1} \rightarrow J/\psi + \gamma) + \frac{1}{2}\sigma^{\chi_{c}}(3P_2^{(1)}) + \frac{1}{2}\sigma^{\chi_{c}}(3P_2^{(1)}) + \frac{17}{30}\sigma^{\chi_{c}}(3S_1^{(8)}) + \frac{13}{60}\sigma^{\chi_{c}}(3S_1^{(8)})] Br(\chi_{c2} \rightarrow J/\psi + \gamma) \]

\[ \sigma_{L}^{\psi' \rightarrow J/\psi} = \sigma_{L}^{\psi'} Br(\psi' \rightarrow J/\psi + X) \]

\[ \sigma_{L}^{\psi' \rightarrow \chi_{c} \rightarrow J/\psi} = \frac{1}{3}\sigma^{\psi'} Br(\psi' \rightarrow \chi_{c0}) Br(\chi_{c0} \rightarrow J/\psi + \gamma) + \left( \frac{1}{2}\sigma^{\psi'} + \frac{1}{4}\sigma_{T}^{\psi'} \right) Br(\psi' \rightarrow \chi_{c1}) Br(\chi_{c1} \rightarrow J/\psi + \gamma) + \left( \frac{17}{30}\sigma^{\psi'} + \frac{13}{60}\sigma_{T}^{\psi'} \right) Br(\psi' \rightarrow \chi_{c2}) Br(\chi_{c2} \rightarrow J/\psi + \gamma) \]

**Charmonium production at Tevatron**

We performed fit Tevatron data [13, 14] and obtained sets of nonperturbative matrix elements (NMEs) (see Table 1) [15]. Using these values we calculated parameter \( \alpha(p_T) \) in cases of different production mechanisms. The results for prompt \( J/\psi \) and direct \( \psi' \) are shown in the Fig. 1a and Fig. 1b in compare with Tevatron data [16]. Last one was investigated in the collinear parton mode too [17, 18]. We see that none of these studies were able to prove or disprove the NRQCD factorization hypothesis.
Figure 1a. Polarization parameter $\alpha(p_T)$ for prompt $J/\psi$ production. Curve 1 — the direct production channel, 2 — $J/\psi$ from $\chi_c \rightarrow J/\Psi \gamma$ decays, 3 — $J/\psi$ from $\psi' \rightarrow J/\psi$ decays, 4 — $J/\psi$ from $\psi' \rightarrow \chi_c \rightarrow J/\psi$ decays, 5 — the sum of (1)-(4) terms, 6 — the CSM prediction.

Figure 1b. Polarization parameter $\alpha(p_T)$ for direct $\psi'$ meson production. Curve 1 — the CSM prediction, 2 — the color-octet mechanism prediction, 3 — the direct production channel.

Conclusions

We have obtained analytical formulas for the squared amplitudes of the processes $R + R \rightarrow \mathcal{H}[1, 8]$ and $R + R \rightarrow \mathcal{H}[1] + g$, where $\mathcal{H}$ may be in the polarized state. Using new set of the color-octet NMEs we have predicted $\alpha(p_T)$ for direct $\psi'$ and prompt $J/\psi$. Our predictions are coincide with the collinear parton model calculations rather than with previous $k_T$-factorization results [19].

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References

[1] E. A. Kuraev, L. N. Lipatov, V. S. Fadin, Sov.Phys.JETP 44, 443 (1976) [Zh.Eksp.Teor.Fiz. 71, 840 (1976)]; I. I. Balitsky, L. N. Lipatov, Sov.J.Nucl.Phys. 28, 822 (1978) [Yad.Fiz. 28, 1597 (1978)].

[2] L. V. Gribov, E. M. Levin, M. G. Ryskin, Phys.Rept. 100, 1 (1983); J. C. Collins, R. K. Ellis, Nucl.Phys. B 360, 3 (1991); S. Catani, M. Ciafaloni, F. Hautmann, Nucl.Phys. B 366, 135 (1991).

[3] L. N. Lipatov, Nucl.Phys. B 452, 369 (1995); E. N. Antonov, L. N. Lipatov, E. A. Kuraev, I. O. Cherednikov, Nucl.Phys. B 721, 111 (2005).

[4] J. Blümlein, Report No. DESY 95-121 (1995).

[5] H. Jung, G. P. Salam, Eur.Phys.J. C 19, 351 (2001).

[6] M. A. Kimber, A. D. Martin, M. G. Ryskin, Phys.Rev. D 63, 114027 (2001).
Table 1: NMEs for $J/\psi$, $\psi'$, and $\chi_{cJ}$ mesons from fits in the collinear parton model (PM) and in the $k_T$-factorization approach using the JB [4], JS [5], and KMR [6] unintegrated gluon distribution functions. The CDF prompt data from run I [13] and run II [14] have been excluded from our fit based on the JB gluon density, if these data have been included the $\chi^2$/d.o.f becomes greater than 20.

| $n$ / $n$ | $\langle O_{J/\psi}^{[3]S_1^{(1)}} \rangle$/GeV³ | PM [18] | Fit JB | Fit JS | Fit KMR |
|-----------|---------------------------------|---------|--------|--------|--------|
| $\langle O_{J/\psi}^{[3]S_1^{(8)}} \rangle$/GeV³ | $4.4 \times 10^{-3}$ | $1.5 \times 10^{-3}$ | $6.1 \times 10^{-3}$ | $1.4 \times 10^{-2}$ |
| $\langle O_{J/\psi}^{[1]S_0^{(8)}} \rangle$/GeV³ | $4.3 \times 10^{-2}$ | $6.6 \times 10^{-3}$ | $9.0 \times 10^{-3}$ | $2.8 \times 10^{-2}$ |
| $\langle O_{J/\psi}^{[3]P_0^{(8)}} \rangle$/GeV³ | $2.8 \times 10^{-2}$ | $0$ | $0$ | $0$ |
| $\langle O_{\psi'}^{[3]S_1^{(1)}} \rangle$/GeV³ | $6.5 \times 10^{-1}$ | $6.5 \times 10^{-1}$ | $6.5 \times 10^{-1}$ | $6.5 \times 10^{-1}$ |
| $\langle O_{\psi'}^{[3]S_1^{(8)}} \rangle$/GeV³ | $4.2 \times 10^{-3}$ | $3.0 \times 10^{-4}$ | $1.5 \times 10^{-3}$ | $8.3 \times 10^{-4}$ |
| $\langle O_{\psi'}^{[1]S_0^{(8)}} \rangle$/GeV³ | $6.9 \times 10^{-3}$ | $0$ | $0$ | $0$ |
| $\langle O_{\psi'}^{[3]P_0^{(8)}} \rangle$/GeV³ | $3.9 \times 10^{-3}$ | $0$ | $0$ | $0$ |
| $\langle O_{\chi_c}^{[3]P_0^{(1)}} \rangle$/GeV³ | $8.9 \times 10^{-2}$ | $8.9 \times 10^{-2}$ | $8.9 \times 10^{-2}$ | $8.9 \times 10^{-2}$ |
| $\langle O_{\chi_c}^{[3]P_0^{(8)}} \rangle$/GeV³ | $4.4 \times 10^{-3}$ | $2.2 \times 10^{-4}$ | $4.7 \times 10^{-5}$ |

$\chi^2$/d.o.f = 2.2, 4.1, 3.0

[7] G. T. Bodwin, E. Braaten, G. P. Lepage, Phys.Rev. D 51, 1125 (1995); 55, 5853(E) (1997).
[8] V. A. Saleev, D. V. Vasim, Proc. of the First Int. Workshop HSQCD-2004, 73 (2004).
[9] B. A. Kniehl, J. Lee, Phys.Rev. D 62, 114027 (2000).
[10] M. Beneke, M. Krämer, M. Vänttinen, Phys.Rev. D 57, 4258 (1998).
[11] P. Cho, M. B. Wise, S. P. Trivedi, Phys.Rev. D 51, 2039 (1995).
[12] P. Cho, A. K. Leibovich, Report No. CALT 68-2026 (1995).
[13] CDF Collab., F. Abe et al., Phys.Rev.Lett. 79, 572 (1997); 79, 578 (1997); CDF Collab., T. Affolder et al., Phys.Rev.Lett. 85, 2886 (2000).
[14] CDF Collab., D. Acosta et al., Phys.Rev. D 71, 032001 (2005).
[15] B. A. Kniehl, V. A. Saleev, D. V. Vasim, to be published.
[16] CDF Collab., T. Affolder et al., Phys.Rev.Lett. 85, 2886 (2000).
[17] M. Beneke, M. Krämer, Phys.Rev. D 55, 5269 (1997); A. K. Leibovich, Phys.Rev. D 56, 4412 (1997).
[18] E. Braaten, B. A. Kniehl, J. Lee, Phys.Rev. D 62, 094005 (2000).
[19] F. Yuan, K. T. Chao, Phys.Rev. D 87, 022002 (2001).