New Physics in $\Delta L = 2$ neutrino oscillations

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Abstract. We propose a general framework to constrain $\Delta L = 2$ processes by measuring observables associated with neutrino-antineutrino oscillations in $\pi^{\pm}$ decays. First, we use this formalism as a new strategy for detecting the CP-violating phases and the effective mass of muon Majorana neutrinos. Within the generic framework of quantum field theory, we compute the non-factorizable probability for producing a pair of same-charged muons in $\pi^{\pm}$ decays as a distinctive signature of $\nu_\mu - \bar{\nu}_\mu$ oscillations. Using the neutrino-antineutrino oscillation probability reported by MINOS collaboration, a new stringent bound on the effective muon-neutrino mass is derived. Secondly, we interpret the production of the pair of same-charged muons as a result of lepton number violating (LNV) interactions at the neutrino source, which allow us to constrain New Physics.

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INTRODUCTION

As is well known, the Majorana nature of neutrinos can be established via the observation of $\Delta L = 2$ processes [1]. The parameter characterizing the rate of such transitions, the effective neutrino mass $\langle m_{\ell\ell} \rangle \equiv \sum_i U_{\ell i}^2 m_{\nu i}$, involves a combination of neutrino masses, mixings and phases. It turns out that the only way to access the values of Majorana phases is through observables associated to $\Delta L = 2$ transitions [2]. Note, however, that the measurement of the effective electron-neutrino mass in the neutrinoless double beta decay (0$\nu\beta\beta$) experiments can not restrict the two Majorana CP violating phases present in the PMNS mixing matrix [3, 4, 2]. This may be expected since in the ($0\nu\beta\beta$) one measures the lifetime of the decay of two neutrons in a nucleus into two protons and two electrons, which is a CP conserving quantity. Other proposals aiming to gain access to CP-violating phases of Majorana neutrinos using neutrino-antineutrino oscillations were first discussed in [5, 6, 7, 8, 9, 10, 11].

On the other hand, direct bounds on other effective neutrino mass parameters $\langle m_{\ell\ell} \rangle$ from present experimental data are very poor. Currently, the strongest bound for the muon-neutrino case from the $K^+ \rightarrow \pi^- \mu^+ \mu^+$ branching fraction [12] is only $|\langle m_{\mu\mu} \rangle| \leq 0.04$ TeV [13], which leads to a negligible constraint on the neutrino masses and CP violating phases.

In section I we describe the mechanism, based on neutrino-antineutrino oscillation, which would allow to derive a strong bound on the effective Majorana mass of the
muon-neutrino $\langle m_{\mu\mu} \rangle$. In addition, it provides a method for detecting the Majorana neutrino CP violating phases through measuring the CP asymmetry of the $\pi^\pm$ decay where neutrino-antineutrino oscillation take place. Using the preliminary bound on the neutrino-antineutrino oscillation probability reported by the MINOS Collaboration \cite{14}, we derive a bound on $\langle m_{\mu\mu} \rangle$ which improves existing bounds by several orders of magnitude.

In section II we describe the mechanism that would allow us to constrain lepton number violating interactions. In this case we interpret the observation of the final states, same-charged muons at the production and detection of neutrinos, as a result of lepton number violating interactions in pion decays at the neutrino source. Such interactions appear for example in SUSY models with R-Parity violating terms and leptoquark models. In particular in a SUSY model without R-Parity conservation, the radiative contributions are proportional to the R parity couplings $\lambda$ and $\lambda^\prime$, which in general are complex. We attempt to impose constraints over these couplings $\lambda$ and $\lambda^\prime$ from the current bound on neutrino-antineutrino transitions obtained by the MINOS collaboration \cite{14}.

It is worth noting that the probability of a process associated to neutrino oscillation is usually assumed to be factorized into three independent parts: the production process, the oscillation probability and the detection cross section. Here, we adopt the S-matrix amplitude method described in \cite{15}, in order to avoid the usual factorization scheme.

**NEUTRINO-ANTINEUTRINO OSCILLATION**

Let us start by considering a positive charged pion which decays into a virtual neutrino at the space-time location $(x,t)$ together with a positive charged muon. After propagating, the neutrino can be converted into an antineutrino which produces a positive charged muon at the point $(x',t')$ where it interacts with a target, as shown in Fig. 1. For definiteness, we illustrate this process with the production of the neutrino in $\pi^+$ decay and its later detection via its weak interaction with a target nucleon $N$

$$\pi^+(p_1) \rightarrow \mu^+(p_2) + V^\mu_s(p) \leftrightarrow \bar{V}^\mu_t(p) + N(p_N) \rightarrow N'(p_{N'}) + \mu^+(p_1)$$

where the superscript $s(d)$ refers to the virtual neutrino (antineutrino) at the source (detection) vertex. This is a $|\Delta L| = 2$ process. Notice that if two identical anti-muons ($\mu^+(p_2)$ and $\mu^+(p_1)$) are produced at very different space-time locations, well separated in distance and each identified in different detectors, then the total amplitude does not require to be anti-symmetrized.

If one ignores other flavors, the decay amplitude becomes:

$$T_{\nu_{\mu}^{-} - \bar{\nu}_{\mu}}(t) = (2\pi)^4 \delta^4(p_1 - p_N + p_{N'} + p_2 - p_1) (G_F V_{ud})^2 (J_{NN'})_{\mu} f_{\pi}$$

$$\times \sum_i \bar{v}_\mu(p_1) \gamma^\mu (1 + \gamma_5) \rho_1 v(p_2) \times U_{\mu i} U_{\mu i}^* (m_{\nu_i}) e^{-iE_{\nu_i} t} \frac{1}{2E_{\nu_i}},$$

where the relation $v_k = \sum U_{k\alpha} v_\alpha$ between flavor $k$ and mass $\alpha$ neutrino eigenstates has been used. $f_{\pi} = 130.4$ MeV is the $\pi^\pm$ decay constant, and $J_{NN'}$ parametrizes
the interaction with the nucleon. Note that, contrary to the case of neutrino-neutrino oscillations \[15\], only the neutrino mass term survives in this case. For simplicity, one assumes that

\[
(J_{NN'})_\mu = \bar{\nu}_{N'}(p_{N'}) \gamma_\mu [g_V(q^2) + g_A(q^2) \gamma_5] \nu_N(p_N)
\]

(2)

where we keep only the contributions of leading vector \(g_V(q^2)\) and axial-vector \(g_A(q^2)\) form factors, with \(q = p_{N'} - p_N\).

If we neglect terms of \(O(m_\mu/m_{N,N'})\), one obtains

\[
|T_{\nu\mu} - \bar{\nu}_\mu(t)|^2 = (2\pi)^4 \delta^4 (p_1 - p_N + p_{N'} + p_2 - p_1)(GF_{ud})^4 |f_\pi|^2 64 (g_A - 1)^2 m_N m_\mu^2 
\times \sum_{i,j} U_{\mu i} U_{\bar{\nu} j} U_{\bar{\nu} j} e^{-it\Delta E_{Vj}} \frac{m_N m_{\nu j}}{4E_{\bar{\nu} j} E_{\nu j}} (E_2 - E_p) \left( \frac{1}{2} (m_\mu^2 - m_\nu^2) + p_1 \cdot p_2 \right) 
\]

\[- \frac{m_N}{(E_2 - E_p)} G(g_A) p_1 \cdot p_2 - 2m_N F(g_A) \left[ E_2 - E_1 \left( 1 + \frac{m_\mu^2}{m_\nu^2} \right) \right] \]  

(3)

where \(E_2(E_1), E_p\) are, respectively, the initial (final) muon and the pion energies and \(\Delta E_{Vj} = E_{Vj} - E_{\nu j}\). The functions \(F(g_A)\) and \(G(g_A)\) are given by:

\[
F(g_A) = \frac{\gamma_3^+ + 1}{(g_A - 1)^2}, \quad G(g_A) = \frac{\gamma_3^+ + 1}{g_A - 1}. \]

One can easily check that Eq. (3) is not factorizable into (production)\times(propagation)\times(detection) subprocesses due to the terms proportional to \(p_1 \cdot p_2 = E_1 E_2 - |\vec{p}_1||\vec{p}_2| \cos \alpha\), where \(\alpha\) is the angle between the directions of \(\mu^+\) particles. This is an important difference with respect to the case of neutrino-neutrino \((\Delta L = 0)\) oscillations where it was shown in Ref.[15] that the S-matrix formalism reproduces the hypothesis of factorization of the probabilities.

In the following, we shall neglect the \(q^2\)-dependence of the nucleon form factors (namely, we take \(g_V = g_V(q^2 = 0) = 1\) and \(g_A = g_A(q^2 = 0) \approx -1.27\) \[2\]). As is well known \[16\], the cross section of charged current neutrino-nucleon quasielastic scattering is sensitive to the \(q^2\)-dependence of these form factors. However, as long as we confine to the CP rate asymmetry for neutrino-antineutrino oscillations (see below) we expect that the effects of the momentum-transfer dependence of \(g_{V,A}\) will partially cancel in the ratio of oscillation rates. Thus, after integration over kinematical variables, it is possible
to write the rate of the complete process as

$$\Gamma_{\nu_\mu-\bar{\nu}_\mu} = \left| \sum_i U_{\mu i}^2 \frac{m_{\nu_i}}{2E_{\nu_i}} e^{iE_{\nu_i}} \right|^2 \times F(M, \phi),$$

where $F(M, \phi)$ denotes the kinematical function

$$F(M, \phi) = \frac{32\pi}{E_p} (G_F V_{ud})^4 |f_\pi|^2 (g_A - 1)^2 \left( I_4 - m_N G(g_A) I_5 - \frac{1}{2} (m_\mu^2 - m_\pi^2) I_1 \right)$$

$$\times \left( m_\mu^2 - 2m_N F(g_A) \right) \left( m_\mu^2 I_2 - (m_\mu^2 + m_\pi^2) I_3 \right).$$

The functions $I_a$ for $a = 1, \ldots, 4$ can be obtained from the following integral:

$$I_a = \int \frac{d^3 p_2}{2E_2} \frac{d^3 p_1}{2E_1} (E_2 - E_p) f_\alpha \delta(E_p + E_N - E_{N'} - E_1 - E_2),$$

with $f_1 = 1$, $f_2 = E_2$, $f_3 = E_1$, and $f_4 = (p_1 \cdot p_2)$ and $f_5 = (p_1 \cdot p_2)/(E_2 - E_p)$.

There are two interesting limits for this process. At very short times, (as in short-baseline neutrino experiments), $\Gamma_{\nu_\mu-\bar{\nu}_\mu} \simeq \left( \frac{|m_{\mu\mu}|^2}{E_\nu^2} \right) \times F(M, \phi)$, where $\langle m_{\mu\mu} \rangle$ is the effective Majorana mass for the muon neutrino. However in the long time limit (long-baseline neutrino experiments) the oscillation terms cannot be neglected. In the limit of $\theta_{13} = 0$, the Majorana phases $\alpha_{1,2}$ are the only sources of CP violation and hence

$$a_{CP} \simeq \tan [2(\alpha_2 - \alpha_3)] \sin \gamma$$

where $\gamma = \frac{\Delta m^2_{33} L(km)}{2E_\nu(GeV)}$.

Thus, in the case of LBL neutrino experiment like MINOS where the distance $L$ is given by $L = 735$ km and the energy $E_\nu$ is typically around $2 - 3$ GeV, one finds that $\sin \gamma \sim \mathcal{O}(1)$ [17],[18]. Thus, measuring CP asymmetry will be unavoidable indication for large CP violating Majorana phases.

### Application to MINOS results on neutrino-antineutrino oscillations

Recently, MINOS [18] has measured the spectrum of $\nu_\mu$ events which are missing after travelling 735 km. It is these missing events which are the potential source of $\bar{\nu}_\mu$ appearance. In their preliminary analysis, they were able to put a limit on the fraction of muon neutrinos transition to muon anti-neutrinos [14]: $P(\nu_\mu \rightarrow \bar{\nu}_\mu) < 0.026$ (90% c.l.).

Assuming CPT, this limit can be written as $\frac{\Gamma_{\nu_\mu-\bar{\nu}_\mu}}{\Gamma_{\nu_\mu-\bar{\nu}_\mu}} < 0.026$. In the limit of ultrarelativistic neutrinos, $E_\nu \simeq E_\nu (1 + m_\nu^2/2E_\nu)$, keeping only the leading terms in the $m_\nu/E_\nu$ terms, and using our expression for the total rate, we get

$$\left| \sum_i U_{\mu i}^2 m_{\nu i} e^{iE_{\nu i}} \right|^2 \lesssim 0.001 \times E_\nu^2$$

(8)
To illustrate the usefulness of this relation, let us consider the general case of 3 generations. In this case, one finds

\[
0.001 \times E_v^2 \gtrsim \left| \langle m_{\mu\mu} \rangle \right|^2 - 4 \sum_{i>j} \text{Re} \left( U_{\mu i}^2 U_{\mu j}^* \right) m_{\nu i} m_{\nu j} \sin^2 \frac{\Delta m_{ij} L}{4 E_v} - 2 \sum_{i>j} \text{Im} \left( U_{\mu i}^2 U_{\mu j}^* \right) m_{\nu i} m_{\nu j} \sin \frac{\Delta m_{ij} L}{2 E_v} \tag{9}
\]

Assuming that the only phases that appear in the neutrino mixing matrix are the Majorana phases, it is possible to get a bound on the effective muon-neutrino Majorana mass, only depending on the values of the Majorana phases as the oscillation terms cannot be neglected. In such a case, Eq.(9) can be written as:

\[
0.001 \times E_v^2 \gtrsim \left| \langle m_{\mu\mu} \rangle \right|^2 \pm 4 \sin \left( \frac{\gamma_{21}}{2} \right) m_{\nu v_1} m_{\nu v_3} |U_{\mu 1}^2 U_{\mu 3}^*| \sin \left( 2\alpha_2 - 2\alpha_3 \pm \frac{\gamma_{21}}{2} \right) - 4 \sin \left( \frac{\gamma_{21}}{2} \right) m_{\nu v_1} |U_{\mu 1}| |U_{\mu 2}^*| \sin \left( \alpha_1 + \frac{\gamma_{21}}{2} \right) \pm 4 \sin \left( \frac{\gamma_{21}}{2} \right) m_{\nu v_3} |U_{\mu 2}| |U_{\mu 3}^*| \sin \left( \alpha_2 \pm \frac{\gamma_{21}}{2} \right) \tag{10}
\]

where \( \gamma_{ij} = \frac{\Delta m_{ij}^2 (\text{km})}{2 E_v (\text{GeV})} \), and the positive and negative signs refer to normal and inverted hierarchies, respectively.

We can further neglect \( \sin \gamma_{21} \approx 0 \) such that \( m_{\nu v_1} \approx m_{\nu v_2} \) at first order in \( O \left( \frac{\Delta m_{23}^2}{4 E_v} \right) \) for the fixed experimental parameters in MINOS, then Eq.(10) can be written as

\[
0.001 \times E_v^2 \gtrsim \left| \langle m_{\mu\mu} \rangle \right|^2 + A(\alpha_1, \alpha_2) m_{\nu v_1} m_{\nu v_3} \gtrsim \left| \langle m_{\mu\mu} \rangle \right|^2 \tag{11}
\]

where the coefficient \( A(\alpha_1, \alpha_2) \) is a function of the Majorana phases. In Fig. (2) we show the regions in which \( A(\alpha_1, \alpha_2) > 0 \) is satisfied for both cases of normal or inverted hierarchies, and therefore we can find a stringent bound on the effective Majorana mass.

Thus, using \( E_v \approx 2 \text{ GeV} \), one gets the following bound on \( \left| \langle m_{\mu\mu} \rangle \right| \lesssim 64 \text{ MeV} \).

Over the excluded regions it is not possible to get a conservative bound on \( \left| \langle m_{\mu\mu} \rangle \right| \) without making extra assumptions on the neutrino mass matrix, however Eq.(9) can be used to bound Majorana parameters (masses and phases) which appear in \( \left| \langle m_{\mu\mu} \rangle \right| \). As an example, if \( 0 \leq 2(\alpha_3 - \alpha_5) \leq \pi - \gamma/2 \) and assuming that the effective muon-neutrino Majorana mass is dominated by \( m_{\nu v_2} \) and \( m_{\nu v_3} \) (the two flavour limit case) , it is still possible to get the following conservative bound: \( \left| \langle m_{\mu\mu} \rangle \right| \lesssim 109 \text{ MeV} \).

The bound obtained above for \( \left| \langle m_{\mu\mu} \rangle \right| \) is a factor of 3 above the trivial kinematical bound \( m_\pi - m_\mu \approx 34 \text{ MeV} \) that is allowed for the (on-shell) muon neutrino in pion decay. However, this kinematical bound applies only to the effective mass of a lepton-number conserving muon neutrino (\( \pi^+ \to \mu^+ \nu_\mu \)).

A bound on the effective Majorana mass of the muon neutrino, which is independent of the mass hierarchies and Majorana phases, can be obtained using the fluxes of \( \nu_\mu \) and...
and $\bar{\nu}_\mu$ measured with the near detector of the MINOS experiment [19]. Since the near detector is located $L = 1.04$ km away from the target and for neutrino energies above 1 GeV, all oscillatory terms in Eq. (5) are equal to 1. Under the assumption that the excess of $\bar{\nu}_\mu$ events arises from $\nu_\mu \rightarrow \bar{\nu}_\mu$ transitions we get (note the muon-neutrino and muon-antineutrino total cross sections induced by charged currents are flat for neutrino energies above 2 GeV [2]):

$$\left| \langle m_{\mu\mu} \rangle \right|^2 \int \frac{dE_\nu}{E_\nu} \leq \frac{\int (\Phi_{\nu_\mu}^{Obs}(E_\nu) - \Phi_{\nu_\mu}^{MC}(E_\nu))dE_\nu}{\int \Phi_{\nu_\mu}^{Obs}(E_\nu)dE_\nu}.$$  \hspace{1cm} (12)

where $\Phi_{\nu_\mu}^{Obs(MC)}$ denote the observed(expected) fluxes. Using the expected and measured integrated fluxes by the MINOS collaboration for the energy region $5 \leq E_\nu \leq 50$ GeV, we get the following bound: $|\langle m_{\mu\mu} \rangle| \leq 2.7$ GeV , which looks rather poor compared to the value reported above.

### LEPTON NUMBER VIOLATION AT THE NEUTRINO SOURCE

Let us consider next a virtual neutrino(antineutrino) produced together with a positively(negatively) charged muon at the space time location $(x,t)$, it the travels to $(x',t')$ and is detected there because it interacts with a target producing a positively(negatively) charged lepton. We are assuming this $\Delta L = 2$ process is due to NSI interactions at the production vertex. For definiteness, we illustrate this process with production of a antineutrino(neutrino) in the $\pi^+(\pi^-)$ decay and its late detection via its weak interaction with a target nucleon $N$.

$$\pi^+(p_1) \rightarrow \mu^+(p_2) + \nu_\mu^e(p) \leftrightarrow \nu_\mu^c(p) + N(p_N) \rightarrow N'(p_{N'}) + l^+(p_l)$$  \hspace{1cm} (13)
This effective states are not necessarily $\Delta L$ conserving once NSI interactions are introduced. If we assume that neutrinos are left-handed, $\Delta L = 2$ semi-leptonic interactions can be described by the following effective Hamiltonian,

$$
\mathcal{H} = 2\sqrt{2}G_F V_{ud} \left\{ C_1^k \left( \overline{\nu}_k^\alpha \gamma^\mu P_R u \right) \left( \overline{d} \gamma_\alpha P_R u \right) + C_2^k \left( \overline{\nu}_k^\alpha \gamma^\mu P_R u \right) \left( \overline{d} \gamma_\alpha P_L u \right) + C_3^{(3,4)} \left( \overline{\nu}_k^\alpha P_L u \right) \left( \overline{d} P_{(R,L)} u \right) + C_4^{5(3,4)} \left( \overline{\nu}_k^\alpha \gamma_\mu P_R u \right) \left( \overline{d} \sigma^{\alpha\beta} P_{(R,L)} u \right) \right\}, \tag{14}
$$

where $\nu_k$ denotes a neutrino with flavor $k$ and $P_{R,L} = (1 \pm \gamma_5)/2$.

In the following and for simplicity, we consider the case where lepton number violation occurs only at the $\pi^+$ decay vertex. Note that the tensor currents proportional to the $C_{5L(R)}^k$ Wilson coefficients will not contribute to $\pi^+$ decay because it is not possible to generate an antisymmetric tensor from the pion momentum alone. Thus, the only non-vanishing hadronic matrix elements at the production vertex are:

$$
\langle 0 | \overline{\nu}_1^\mu P_R \nu_1^\mu | \pi^+ \rangle = i f_\pi \rho^\mu, \quad \langle 0 | \overline{\nu}_1^\mu P_L \nu_1^\mu | \pi^+ \rangle = -i f_\pi \frac{m_\pi^2}{m_u + m_d}, \tag{15}
$$

where $f_\pi = 130$ MeV is the pion decay constant and $m_{u,d}$ denote the light quark masses.

Again we shall neglect the $q^2$-dependence of the nucleon form factors (namely, we take $g_V = g_V(q^2 = 0) = 1$ and $g_A = g_A(q^2 = 0) \approx -1.27$ [2]). In this case the total rate of the $\Delta L = 2$ is factorized:

$$
|T_{\nu_\mu - \nu_\mu'}(\tau)|^2 \approx \Gamma_\pi \frac{1}{2} \left( A^2 m_\pi^2 + B^2 m_\mu^2 - 2\Re[AB^*] \right) \frac{\rho^\mu}{\rho^\mu}, \tag{16}
$$

with $\tau = (t' - t) > 0$ is the time elapsed from the production to the detection space-time locations of neutrinos. The $A, B$ coefficients are given by,

$$
B_\mu \equiv -i(C_1^{\mu*} - C_2^{\mu*}), \quad A_\mu = \frac{-im_\pi^2}{m_u + m_d} (C_3^{\mu*} - C_4^{\mu*}) \tag{16}
$$

We shall attempt to constrain the LNV parameters with the MINOS preliminary results [14]: $P(\nu_\mu - \nu_\mu') < 0.026$ (90\% c.l.), this is,

$$
(A^2 m_\pi^2 + B^2 m_\mu^2 - 2\Re[AB^*]) \lesssim 0.05 \tag{17}
$$

As an example let us apply the previous formalism to the MSSM without R-Parity conservation, but requiring the conservation of baryon number to ensure the proton is stable. The must superpotential that conserves bayon number is

$$
W_{\Delta L=1} = \lambda_{i,j,k} L_i \cdot L_j \bar{e}_k + \lambda_{i,j,k} L_i \cdot Q_j \bar{d}_k + \mu_L^i L_i \cdot H_u \tag{18}
$$

here the “$\cdot$” is a $SU(2)$ product, therefore $\lambda_{i,j,k}$ is antisymmetric in $i, j$. The summation over the generation indices $i, j, k = 1, 2, 3$ is understood. The bar symbol represents an antiparticle and not the Dirac conjugation. We can expand the Yukawa terms in the
Figure 3. $\Delta L = 2$ SUSY contribution to the $\pi^+ \rightarrow \mu^+ + \nu^\ell$

Lagrangian as

$$L = \lambda_{ijk} \left[ \bar{\nu}_L e_R^i e_L^j + e_L^i \frac{1}{m_d} \bar{e}_L^i v_L^j + \bar{e}_R^k v_L^j c e_L^j - (i \leftrightarrow j) \right]$$

$$+ \lambda'_{ijk} \left[ \bar{\nu}_L d_R^i d_L^j + d_L^i \frac{1}{m_u} \bar{d}_R^j v_L^j + \bar{d}_R^j v_L^j c d_L^j - \bar{d}_R^j d_L^j u_L^j \right]$$

$$- \tilde{m}_L^2 \left[ \frac{1}{m_d} \bar{t}_R^j c \bar{u}_L^j - \bar{d}_R^j e_L^j c u_L^j \right] + h.c.$$ (19)

The corresponding effective hamiltonian (Fig.3) is therefore,

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{ud} \sum_i \left( \langle \bar{\nu} \gamma^\rho P_L \mu \rangle (\bar{d}_R^j P_L u) + C_2 \langle \bar{\nu} P_R \mu \rangle (\bar{d}_R^j P_L u) + C_3 \langle \bar{\nu} P_L \mu \rangle (\bar{d}_R^j P_L u) \right)$$ (20)

The Wilson coefficients can be expressed as $C_i = C_i^{SM} + C_i^{SUSY}$. For $i = 2, 3$ the $C_i^{SM}$ vanish identically, and $C_1 = 1$. In this respect, the Wilson coefficients $C_i$ are given by

$$C_2 = \frac{\sqrt{2}}{G_F V_{ud}} \left( \frac{1}{m_d^2} \right) \left( \lambda_{122}^2 \lambda_{211}^2 + \lambda_{232} \lambda_{311} \right), \quad C_3 = \frac{\sqrt{2}}{G_F V_{ud}} \left( \frac{1}{m_d^2} \right) \sum_{A,B} \lambda_{12A} \lambda_{B11}^* (\delta_{RL})_{AB}$$ (21)

Using our previous results (Eq. 17) for typical slepton masses of 200GeV, we obtain $\sum_{A,B} \lambda_{12A} \lambda_{B11}^* (\delta_{RL})_{AB} \lesssim 10^{-4}$.

CONCLUSIONS

The production of leptons with same charges at the production and detection vertices of neutrinos will be a clear manifestation of $|\Delta L| = 2$ processes. One interesting result is that the time evolution probability of the whole process is not factorizable into production, oscillation and detection probabilities, as is the case in neutrino oscillations [15]. We find that, for very short times of propagation of neutrinos, the observation of $\mu^+ \mu^+$ events would lead to a direct bound on the effective mass of muon Majorana neutrinos. In the case of long-baseline neutrino experiments, the CP rate asymmetry for production of $\mu^+ \mu^+ / \mu^- \mu^-$ events would lead to direct bounds on the difference of CP-violating Majorana phases. Finally, using the current bound on muon neutrino-antineutrino oscillations reported by the MINOS Collaboration we are able to set the bound $\langle m_{\mu\mu} \rangle \lesssim 64$ MeV, which is the first direct limit on the neutrino masses, although it is still several orders of magnitude below current indirect bounds reported in the
literature. Future results from MINOS are expected from the analysis of twice the data set used to get the bound reported so far [14] and quoted in Eq. (12) above. Since current uncertainties in the observed and expected number of $\bar{\nu}_\mu$ events are dominated by statistical errors [14], we could expect only a slight improvement by a factor $1/\sqrt{2}$ on the effective Majorana mass of the muon neutrino. Neutrino factories may improve this bound by more than one order of magnitude.

We also show the possibility to constrain LNV interactions, such as the $\lambda, \lambda'$ terms in the SUSY superpotential with R-Parity violation, for the case in which the observation of the final states, same-charged muons at the production and detection of neutrinos, is a result of non standard lepton number violating interactions at the neutrino source.

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