A UNIVERSAL DENSITY PROFILE FOR DARK AND LUMINOUS MATTER?

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ABSTRACT

We explore similarities in the luminosity distribution of early-type galaxies and the mass profiles of $\Lambda$CDM halos. The spatial structure of these systems may be accurately described by a simple law in which the logarithmic slope of the density varies as a power of the radius: the Sérsic law. We show that this law provides a significantly better fit to a set of high-resolution $\Lambda$CDM halos than a three-parameter generalization of the Navarro-Frenk-White profile. We discuss possible reasons why the same law should describe dark and luminous systems that span a range of over 7 decades in mass.

Subject headings: dark matter — galaxies: elliptical and lenticular, cD — galaxies: structure

1. INTRODUCTION

The Sérsic (1968) law,

$$\ln \left( \frac{\Sigma}{\Sigma_*} \right) = -b(X^{1/n} - 1),$$

relating the two-dimensional (projected or surface) density, $\Sigma$, and the dimensionless radius, $X = R/R_*$, is often fitted to the luminosity profiles of elliptical galaxies and to the bulges of disk galaxies. The parameters of the fit include the Sérsic index $n$ as well as the constant $b$, which is normally chosen so that $R_*$ is the radius containing one-half of the projected light: $b = b(n) \approx 2n - 0.324$ (Ciotti & Bertin 1999).

In a recent series of papers, A. Graham and coworkers have shown that the Sérsic law provides a remarkably good fit to the luminosity profiles of stellar spheroids, from dwarf elliptical to the most luminous elliptical galaxies (Graham 2001, 2002; Graham & Guzmán 2003; Graham et al. 2003; Trujillo et al. 2004). The fits apply over 2–3 decades in radius and often extend down to the innermost resolvable radius. Deviations from the best-fitting Sérsic law are typically on the order of 0.05 mag rms. The Sérsic index $n$ is found to correlate with galaxy absolute magnitude (Graham & Guzmán 2003), and also with other structural parameters such as $R_*$ (Caon et al. 1993; Graham & Guzmán 2003). Setting $n = 4$ gives the de Vaucouleurs (1948) law, which approximates luminous elliptical galaxies, and $n = 1$ is the exponential law, which approximates the luminosity profiles of dwarf elliptical galaxies.

The density profiles of the dark matter halos formed in $N$-body simulations of hierarchical clustering have traditionally been fitted to a rather different class of functions, essentially broken power laws (Navarro et al. 1996, 1997; Moore et al. 1999). However, the most recent simulations (Power et al. 2003; Reed et al. 2005) suggest that halo density profiles are better represented by a function with a continuously varying slope. Navarro et al. (2004) proposed the fitting function

$$\frac{d \ln \rho}{d \ln r} = -2(r/r_\gamma)^{\alpha},$$

where $r_{\gamma}$ is the radius at which the logarithmic slope of the space density is $-2$ and $\alpha$ is a parameter describing the degree of variation of the slope. The corresponding density profile is

$$\ln (\rho/\rho_{\gamma}) = -(2/\alpha)(x^n - 1)$$

with $x \equiv r/r_\gamma$. Remarkably, this is precisely the same functional form as equation (1)—with the difference that Navarro et al. fitted equation (3) to the space density of dark matter halos, while equation (1) applies to the projected luminosity density of galaxies.

Nevertheless, the connection is intriguing, and a number of questions spring to mind. Does Sérsic’s law fit the surface densities of dark matter halos as well as it fits the luminosity distributions of galaxies? We show here (§ 3) that the answer is “yes”: the same fitting function provides an equally good description of the projected densities of both dark and luminous spheroids. In § 4, we ask whether it is most appropriate to fit the Sérsic law to the space or projected densities of dark matter halos, and whether these functions are better fits than other three-parameter functions. Section 5 contains some speculations about why a single density law should describe dark and luminous systems over such a wide range in mass.

2. METHOD

We constructed nonparametric estimates of the space and projected density profiles of the 19 $\Lambda$CDM halo models in Navarro et al. (2004) and compared them with a number of fitting functions, including the Sérsic law (eq. [1]), the deprojected Sérsic law (defined as the spherical density law whose spatial projection is eq. [1]), and a generalized, three-parameter NFW profile (Navarro et al. 1996, 1997) profile, which may be expressed as

$$\frac{d \ln \rho}{d \ln x} = -\gamma + 3x$$

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with $x = r/r_\gamma$. The NFW profile has $\gamma = 1$ and $r = r_\gamma$; the Moore et al. (1999) profile has a similar functional form with inner slope $\gamma = 1.5$. 

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Fig. 1.—(a) Nonparametric estimates of the surface density profiles of the 19 halo models. Profiles of the D and C models have respectively been shifted downward and upward by 0.75 in the logarithm. (b) Deviations of the best-fitting Sérsic model from \( \Sigma(R) \). Fit parameters are given in Table 1.

Details of the numerical simulations are described in Navarro et al. (2004). Four halos are “dwarf” sized \((M \approx 10^{10}\ M_\odot)\), seven are “galaxy” sized \((M \approx 10^{12}\ M_\odot)\), and eight are “cluster” sized \((M \approx 10^{15}\ M_\odot)\). We adopt the notation of that paper (D = dwarf, G = galaxy, C = cluster) in what follows.

Nonparametric estimates of the space and projected density profiles, \( \hat{\rho}(r) \) and \( \hat{\Sigma}(R) \), were constructed using the spherically symmetrized kernels defined by Merritt & Tremblay (1994). Each N-body point was replaced by a kernel of the form

\[
K_i(r, \, r_i, \, h_i) = \frac{1}{2(2\pi)^{3/2}} \left( \frac{r_i}{h_i^3} \right)^{-1} e^{-r_i^2/2h_i^2} \sinh (r_i/h_i^2) \text{,}
\]

(5)

\[
K_e(R, \, R_i, \, h_i) = \frac{1}{2\pi} e^{-/(R_i^2 + R^2/h_i^2)} I_0(RR_i/h_i^2) \text{,}
\]

(6)

with \( h_i \) the width of the kernel associated with the \( i \)th particle and \( I_0 \) the modified Bessel function. The projected radii \( R_i \) were obtained from the N-body radii \( r_i \) by assigning each particle a random position on a sphere of radius \( r_i \). Density estimates were computed on a grid of 100 radial points spaced logarithmically from \( r_{\text{conv}} \) to \( r_{200} \) (these radii are defined below). We followed standard practice (Silverman 1986) and first computed a pilot estimate of the density by means of a nearest-neighbor scheme and then allowed the \( h_i \) to vary as a power \( \delta \) of this pilot estimate.

When fitting one of the parametric functions defined above to \( \hat{\rho} \) or \( \hat{\Sigma} \), we computed the density estimates on a grid in \( (h_i, \, \delta) \) \((h_i \text{ is the geometric mean of the } h_i \) to see which choice of kernel parameters minimized the residual of the fit; typically there was a broad range of \((h_i, \, \delta)-\text{values over which the best-fit parameters and their residuals were nearly constant. Below we state the rms deviations between the “measured” profile \( \Sigma(R) \) and the best-fitting parametric model \( \Sigma(R) \) in terms of magnitudes, denoted by \( \Delta \mu(R) = -2.5 \log \frac{\Sigma(R)R^2}{\Sigma(R)R} \).

We followed the practice from Navarro et al. (2004) of only constructing density estimates in the radial range \( r_{\text{conv}} \leq r \leq r_{200} \), where \( r_{\text{conv}} \) is the radius beyond which the halo mass distribution is considered robust to errors or approximations associated with the simulations (particle softening, relaxation, etc.) and \( r_{200} \) is the virial radius, that is, the radius within which the mean density contrast is 200 times the critical density. Table 2 of Navarro et al. (2004) gives values of \( r_{200} \) and \( r_{\text{conv}} \) for all halo models.

3. DARK MATTER HALOS AS SÉRISIC MODELS

With few exceptions, modeling of the luminosity profiles of galaxies is done in projected space. We therefore began by analyzing the surface density profiles of the dark halos. Nonparametric estimates of \( \hat{\Sigma}(R) \) for the 19 halos are shown in Figure 1a, and Figure 1b plots the deviations from the best-fitting Sérsic model (eq. [1]). Table 1 gives the best-fitting \( n \) and \( \Delta \mu \). The mean Sérsic index \( 3.11 \pm 0.49 \) (D), \( 3.00 \pm 0.17 \) (G), 2.38 \pm 0.24 (C), possibly indicating a (weak) trend toward decreasing curvature (lower \( n \)) in the profiles of halos of increasing mass.

The \( \Delta \mu \)-values average 0.043 (D), 0.048 (G), and 0.048 (C). For comparison, Caon et al. (1993) found \( \Delta \mu \approx 0.05 \) in a sample of 45 E and SO galaxies, and Trujillo et al. (2004) find a mean \( \Delta \mu \) of 0.09 in a sample of 12 elliptical galaxies without cores. The radial range over which luminous galaxies are fitted varies from \( \sim 1.5 \) to \( \sim 3.5 \) decades, comparable on average to the \( \sim 2 \) decades characterizing our dark matter halos. While the noise properties are different for the two types of data, the particle numbers in our halo models are small enough (\( \sim 10^6 \)) to con-
Fig. 2.—(a) Nonparametric estimates of the space density of the 19 dark halos. The vertical normalization is arbitrary. (b–d) Deviations in magnitudes of three parametric models from $\hat{\rho}(r)$: (b) deprojected Sérsic model; (c) eq. (3); (d) generalized NFW model, eq. (4). Best-fit parameters are given in Table 1.

tribute nonnegligibly to $\Delta \mu$. We conclude that the Sérsic law fits dark halos as well as, and possibly even better than, it fits luminous galaxies.

4. WHICH FUNCTION FITS THE SPACE DENSITY BEST?

Navarro et al. (2004) showed that equation (3) provides a good fit to the spatial density profiles of dark halos. We showed above (§ 3) that the Sérsic law (eq. [1]) is a good fit to the surface density profiles of dark halos. An obvious inference is that a deprojected Sérsic law should provide a good fit to the space density. Here we ask which function—equation (3) or a deprojected Sérsic law—gives a better fit to $\rho(r)$. We also consider the quality of fit of another three-parameter function, the generalized NFW profile presented in equation (4).

When fitting deprojected Sérsic profiles to the dark halos, we define $n_x$ to be the Sérsic index of the projected function; hence, $n_x$ should be close to the index $n$ derived when fitting a Sérsic law to the surface density (and the two would be equal if the halo’s surface density were precisely described by Sérsic’s law). When reporting fits to $\rho(r)$ with equation (3), we define $n \equiv \alpha^{-1}$ with $\alpha$ the shape parameter of equation (2).

The results are shown in Figure 2 and Table 1. Mean values of $\Delta \mu$ for the three fitting functions (deprojected Sérsic, eq. [3], generalized NFW) are $(0.043, 0.047, 0.073)$ for the dwarf halos, $(0.054, 0.055, 0.060)$ for the galaxy halos, and $(0.061, 0.063, 0.060)$ for the cluster halos. Thus, the two Sérsic functions are almost indistinguishable in terms of their goodness of fit: at least over the radial range available, a deprojected Sérsic profile with index $2.5 \leq n_x \leq 3.5$ can be approximated by a Sérsic profile with $n$ in the range $4.5 \leq n \leq 7.5$. Both functions provide a significantly better fit to $\rho(r)$ than the generalized NFW profile in the case of the dwarf halos, and the two Sérsic functions perform at least slightly better than NFW for the galaxy halos. No single function is preferred when fitting $\rho(r)$ for the cluster halos.

Another way to compare the halo density profiles with Sérsic’s law is by means of the radial dependence of the slope. Figure 3 shows nonparametric estimates of the logarithmic slope, $\frac{\partial \log \rho}{\partial \log r}$, for the dark halos; slopes were computed by direct differentiation of the kernel density estimates, using a larger kernel width to compensate for the greater noise generated by the differentiation. Equation (2) predicts a straight line on this plot. That is a reasonable description of Figure 3. The value of $\frac{\partial \log \rho}{\partial \log r}$ in the G and C halos reaches about $-1$ at the innermost radii, consistent with the asymptotic power-law inner behavior of an NFW profile. No obvious convergence to a power law (constant logarithmic slope) is seen in Figure 3, and it is likely that simulations with improved resolution may lead to even shallower slopes at smaller radii (Navarro et al. 2004).
5. WHAT DOES IT MEAN?

Figure 4 shows Sérac’s $n$ (derived from fits to the surface density) as a function of mass for our dark halos and for a sample of early-type galaxies. There is overlap at $M = 10^{10} M_\odot$, the mass characteristic of “dwarf” halos and giant elliptical galaxies. However, the galaxies exhibit a much wider range of $n$-values, extending to $n < 0.5$ in the case of dwarf elliptical galaxies. A natural interpretation is that $n$ is determined by the degree to which (dissipationless) merging has dominated the evolution. The nearly exponential ($n \approx 1$) profiles of dE galaxies are similar to those of disk galaxies, suggesting that dissipation played a critical role in their formation. Luminous elliptical galaxies are the end products of many mergers, the most recent of which are likely to have been gas-poor, and have de Vaucouleurs–like profiles ($n \approx 4$). This view is supported by numerical simulations (Scannapieco & Tissera 2002; Eliche-Moral et al. 2005) that show how exponential profiles are converted into de Vaucouleurs–like profiles by means of repeated mergers.

A thornier question is why a law such as Sérac’s should fit dark or luminous spheroids in the first place. Sérac’s law with $2 \leq n \leq 4$ has an energy distribution that is roughly Boltzmann, $N(E)\,dE \propto e^{-E} \,dE$, and it is sometimes loosely argued that this “maximum entropy” state is a result of the mixing that accompanies violent relaxation or merging (Binney 1982; Merritt et al. 1989; Ciotti 1991). With regard to dark halos, Taylor & Navarro (2001) have shown that the dependence of phase-space density on radius is well approximated by a power law whose corresponding inner density profile has the shallowest slope. This can again be interpreted as an indication that the halos are well mixed. While our study does not shed a great deal of light on this question, it does suggest that the scale-free property $d\ln \rho/d\ln r \propto r^{\alpha}$ is the feature that links dark and luminous spheroids and that this property may be a hallmark of systems that form by means of gravitational clustering.

We have shown that the fitting function that best describes luminous galaxies, the Sérac law, is an equally good fit to dark halos. We have not shown that the Sérac law is a good fit in an absolute sense to either sort of system. But given that dark and luminous density profiles are not pure power laws, a three-parameter law such as Sérac’s is as parsimonious a description as one can reasonably expect. Future work should explore whether other three-parameter fitting functions can describe dark or luminous systems better than Sérac’s law can.

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