The analysis of students’ answers in solving ratio and proportion problems

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Abstract. The ratio and proportion is one of the basic concepts in mathematics. This concept is not only used to solve problems in mathematics but also in other disciplines and in daily lives. So, students should have proficiency in solving ratio and proportion problems. During solving ratio and proportion problems, among students may have difference answers. Based on the students’ answer and prediction of didactical situation, lesson plan and anticipation of didactical situation would be done by teacher. The purpose of this study was to analyze the various answers of students as well as to see the mistakes made by students in solving ratio and proportion problems. The method used in this study was a descriptive qualitative. The subject were ten students of junior high school in South Tangerang, Indonesia. Written tests related to ratio and proportion problems would be done by the subject and to be continued with depth interviewing. The result of this study are that almost students apply formula although without understand why the formula be used and the concept of ratio and proportion. These results can be used as a reference in implementing the learning of the ratio and proportion.

1. Introduction
Mathematics contains materials that require students to think and develop their ways of thinking. Mathematics is known as a structured and systematic matter. It means that the parts of mathematics are structured hierarchically and intertwined in close functional relation [1]. Mathematics is a knowledge that has a relation among concepts, so that it is important for students to understand the concepts they learned in order that students do not have difficulties in learning new concepts.

One of the basic concepts in mathematics is the ratio and proportion. Lobato & Ellis argues that students need to understand the concepts of ratio and proportion if they want to succeed during the grade 6-8 and in their later math learning [2]. In the curriculum of Indonesia, the ratio and proportion is studied by students from the elementary school and re-studied in grade 7 [3]. The Common Core State Standards for Mathematics (CCSSM) states that in the grade 7, students should have the ability to analyze the relation of the ratio and proportion and use them in solving concrete problems and mathematical problems [2]. The concept of ratio and proportion is one of the concepts used on other concepts and even subjects outside mathematics [4].

The study of ratios and proportions found that the ability of 5th grade students to solve the problem of ratio and proportion is still low. While grade 7 students tend to memorize the context associated with the topic taught in solving the problem. They have not really understand the basic concepts of the rules used and still made mistakes in applying mathematical concepts. In addition, another study said that
students’ error is not only influenced by the teaching methods used by teacher but is also influenced by other factors, so the teacher must also understand the students’ different ways in solve a problem [5-7].

Besides to need to master the teaching materials, teachers also need to have other knowledge related to the students. One of them by anticipating the various possibilities of students responses in such a way that in the implementation of the learning process can create a didactic and pedagogical situation that can support the learning process [8]. Research on teachers’ professional development suggest that effective teachers need to pay attention to students’ thinking about mathematical tasks [9]. Harel represented mathematical knowledge by the dual construct of ways of thinking and understanding. That duality can help to reveal new insights for researchers about student conceptual change and to create a learning path that explains how students understand concepts from different levels [10].

By understanding the different ways of thinking and answers of students about a similar concept, teachers can make predictions of students’ answers in their lesson plans so that teachers can anticipate the various possible situations that occur during the learning. Teachers’ anticipation in the learning process can also reduce the students’ difficulties in understanding the concept they learned. This study aims to describe and analyze various answers and to know the mistakes made by students in solving problems consisting of the concept of the ratio and proportion. The questions of this research are: 1) What are the students’ answer in solving the problems of the ratio and proportion 2) What are the students’ mistake in solving the problems of the ratio and proportion 3) Do the students’ answer match the predictions that were made.

2. Method

This research was a descriptive qualitative research through the written test and interview. The subjects of this study were 10 students at one of the junior high school in the city of South Tangerang, Indonesia. Students had learned about ratio and proportion concepts before being tested and interviewed. Each student was given three questions taken from the Indonesia national exam and the students’ textbook. Firstly, predictions of student answers were made to the given problem. After students had completed the problem, some students were interviewed to support the answer analysis of students.

3. Results and discussion

3.1. Analysis of problem 1

Problem 1 is a question consisting of the concept of ratio and direct proportion that is the scale presented in the form of room sketch then the students were asked to determine the actual area from the room sketch. This problem is an Indonesia national exam of mathematics for junior high school in 2016 [11]. Based on observations from the data, only three of ten students can answer the question correctly. Some samples of students’ answer were presented in Figure 1 and Figure 2.

![Figure 1](image1.png)

Figure 1. Some students’ correct answer on problem 1.
From Figure 1, it can be shown that the students’ answer are correct. They could understand the given problem, by first determining the actual length and the actual width then determining the actual area. Nevertheless, the three students solved the problem with different steps only one student could communicate the answers quite coherently and clearly with the concept of ratio and proportion and with that of the cross multiplication rule. The other two solved problems with formulas and direct calculations. From the interviews, the both students only remember the way or formula that was learned to solve the problem, even though they did not know where the formula was obtained. These three answers are consistent with the predictions of the researcher, but the researcher did not make the exact predictions like the answers presented in the last picture where the student presents the answer by describing the length and width of the room sketch and then directly calculating it.

![Figure 1. Students' correct answer on problem 1.](image1)

Figure 2. Some students’ incorrect answer on problem 1.

The answers of the other students can be shown in Figure 2. In the Figure, it shown that these answers are incorrect. Students solved the problem in a similar way that is by first determining the area of the room sketch then calculating the actual area with the given scale. This means that students did not understand the concept of the scale. Aside from the concept that is less understood by students, it is clear that students did not well understand the cross multiplication. Students knew that the cross multiplication could be used to determine the unknown value of a proportion. However, students did not really know why the cross multiplication was used. The evidence is clear from the following completion steps:

\[
\frac{1}{18} = \frac{500}{1} = \frac{9000}{1}
\]

Langrall and Swafford in their research suggested that introducing the cross multiplication procedures should be postponed until students have the opportunity to build the informal knowledge and to develop their understanding of the essentials of proportional reasoning. Once the concept of the proportion of students developed, the student can be introduced by the cross multiplication as an efficient way to determine the unknown value of a proportion [12]. The National Council of Teachers of Mathematics (NCTM) revealed that writing in mathematics could also assist students in consolidating their thinking, since students can reflect on their work and clarify their thoughts on the idea [13], but from the above it is clear that students are less concerned about things, the evidence is shown from the following completion steps:

\[
6 \times 3 = 18 = \frac{18}{1} \cdot \frac{1}{500}
\]
Other student errors can be seen from students’ inaccuracy in writing the unit of the area. The five student answers in accordance with the predictions of researchers that is calculating the area of the room sketch then calculating the actual area with the proportion or the direct way. Two other students solved the problem not exactly as predicted by the researcher that is students determined the actual area with the formula and operated the calculation with the form $\frac{18}{1}$ instead of the form of $18 \times 500$.

3.2. Analysis of problem 2
Problem 2 is a question consisting of the concept of ratio and direct proportion. This problem is in the mathematics curriculum book of the curriculum 2013 with revision [14]. In this case, the student must determine the quantity of each novel from the ratio of two types of novels (drama and mystery) and the quantity of the both types of novels, understand the meaning of the ratio of 1:1, and determine the quantity of novels that must be purchased so that both types of novels have a ratio of 1:1. Based on observations from the data, eight students can answer correctly and in a similar way. So there are only two different answers from students in solving this problem. The sample of students’ answer are presented in Figure 3 and Figure 4.

![Figure 3. Some students’ correct answer on problem 2.](image)

From Figure 3, it can be shown that the students’ answer are correct. They can understand the meaning of a ratio of 1:1 from both novel types that are both types of novels must have the same quantity. When counting the quantity of both novel types, students directly calculated them with memorized rules. Based on the results of interviews with students, students said that they did not know where the formulas/rules came from. Students only remembered the knowledge that is if the quantity of the two objects was known then the problem solving is by summing their ratios. This indicated that students solved problems only by the memorizing. This made learning less meaningful for students. As Ausubel said that if students only learn by remembering without relating to other things then the learning process and the learning outcome is only a rote and not meaningfull at all [15]. Two of the eight student answers can be predicted by the researchers that is counting directly without a clear step and determining the quantity of drama and mystery novels in the following way:

The quantity of drama novels $= \frac{7}{7+5} \times 72 = 42$ and the quantity of mystery novels $= \frac{5}{7+5} \times 72 = 30$

Therefore, the quantity of the mystery novel that has to be bought is $42 - 30 = 12$ pieces, so that the ratio of both novel types is 1:1.

![Figure 4. Student’ incorrect answer on problem 2.](image)
Figure 4 shows that a student solved the problem with the proportion but he did not make proportion exactly. When trying to solve the problem, he had difficulty in the calculation and did not continue it. This maybe happened because students got results in the form of non-integer in the final calculation. This student answer was beyond the predictions of the researcher. The last another student did not answer this problem.

3.3. Analysis problem 3

Problem 3 is a question consisting of the concept of inverse proportion. This problem is an Indonesia national exam of mathematics for junior high school in 2005 [16]. In this case, students were asked to determine the quantity of cakes of each child that was received, if the cakes were distributed to ten children, whereas if it was distributed to 30 children, each child got six cakes. Based on the observation of the data, nine students could answer the problem correctly while one student could not answer correctly, but his steps is correct. The sample of students’ answer are presented in Figure 5.

![Figure 5. Some students’ answer on problem 3.](image)

Figure 5 shows that the left figure is correct and the right one is wrong. Nine students solved this problem with a similar step as predicted by the researcher, that is the students first determined the quantity of cakes are \(6 \times 30 = 180\) then determined the quantity of cakes that were obtained by each child is \(\frac{180}{10} = 18\).

From the nine students, one of them initially solved the problem with the concept of the proportion, but the student wrote the wrong proportion that is \(\frac{6}{10} = \frac{30}{x}\) so that the calculation process got the wrong result. When interviewed, this student did not remember the concept of the inverse proportion so that he made such that proportion. This indicated that students did not well understand the problem of direct and inverse proportions. Students did not understand the relation between the quantities, but only remembered the problem that was given by teachers and the solution. The last one student could solve the problem with the concept of inverse proportion. However, in the process he mistook in distinguishing the variables that he wrote with the mark of the multiplication so that an error occurred in the completion step. Students also still made mistakes in the operation of division that is \(\frac{10x}{10} = 1\).

4. Conclusion

Based on the results of the analysis, some predictions match with students’ answers. Students generally still use the memorizing rather than understanding when solving problems. Students still make mistakes in calculations and in determining relationships between quantities. Teachers are expected to emphasize the meaningful learning for students rather than just memorizing. In addition, teachers should make predictions and anticipations of students’ answers in developing lesson plans.

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