Abstract

Though electrical noise and conduction current are assigned to corpuscle-like electrons drifting in solid matter, this model hardly fits in the Fluctuation-Dissipation framework. However, fluctuations of energy due to displacements of single electrons between two close conductors lead to a discrete noise model looking like continuous in practice. The complex Admittance it uses for the displacement and conduction currents between such conductors, allows a treatment of fluctuations and dissipations of electrical energy that excels the example of Callen and Welton using the complex Impedance of resistors. The new model thus obtained not only unifies the phase noise of electronic oscillators with the linewidth of lasers, but also explains why the shot noise that the drift theory assigns to conduction currents is not observed.

Introduction

Electrical noise is a field where scientists should have a special care in checking that new interpretations of data to support fashion theories do not conflict with basic notions of physics. Although this could seem obvious we will show that the physical connection between the electrical capacitance of Two-Terminal Devices (2TD) like resistors and the Maxwell relaxation time $\tau_d$ measured by such devices is not known today regarding their shot noise. This noise has to do with the passage of electrons of charge $-q$ between two terminals at distance $d$ in space, thus between two conductors offering an electrical capacitance $C$. Hence, any 2TD like resistors, capacitors and L-C resonators will show some $C$ between terminals indicating that the electric field can store electrical energy between them. Because two terminals at distance $d$ is an unavoidable 2TD in electrical noise measurements, we should be aware of the Degree of Freedom (DoF) of its $C$.

Concerning electrical charge, electrons as corpuscles of negative charge $-q$ is a deeply rooted idea that tends to conflict with the charge neutrality notion used in Solid-State devices. We mean the way $10^{22}$ electrons/cm$^3$ are kept in close proximity by an equal amount of positive charge seeking the best mutual screening possible at each spatial point. This trend of electric charge to form dipolar structures in solid matter should be taken into account by the drift theory for conduction current where electrons sensing the electric field of a voltage $V$ between terminals of a resistor to drift accordingly to their charge do not sense the electric field of myriads of charged electrons in close proximity. This “selective sensing” led us to propose a noise model where the charge neutrality is disturbed by impulsive displacements of individual electrons between the terminals of resistors. This is the basis of the impulsive noise model that we are going to consider and to complete in this paper because to our knowledge, it is the first model that fits in the Fluctuation-Dissipation framework of [1] and the model we refer to is that of [2, 3].
I- Electrical capacitance and impulsive noise

The impulsive model of [2, 3] reflects our firm belief that the discreteness of electric charges involved in conduction phenomena should be found at the noise level. Based on thermal Equipartition, it uses two magnitudes varying with time $t$: the voltage $v(t)$ we measure between two terminals and the current $i(t)$ between them that not always can be measured directly. This current can be conduction current linked to the conductivity $\sigma$ of the material between such terminals and displacement current due to its dielectric constant $\varepsilon = \varepsilon_r \varepsilon_0$ that being $\varepsilon_r$ times the vacuum one ($\varepsilon_0$) justifies the capacitance $C$ that always will exist between two close terminals. Although the Maxwell relaxation time ($\tau_d = \varepsilon / \sigma$) for the inner material of a resistor is well-known today, this is not so for the way $\tau_d$ links its resistance $R$ with its capacitance $C$.

We mean the relation $\tau_d = RC$ setting the cut-off frequency of its Johnson noise. This is the voltage noise between terminals of resistors reported in 1928 by Johnson [4], who already used the proper formula to consider their shunting capacitance $C$ in the series circuit of their complex impedance. Very likely, this $C$ led Nyquist to use “conductors of pure resistance $R$” and not resistors as matched loads to end the lossless transmission line he used [5] to show the thermal origin of Johnson noise. However, this awareness of $C$ is lacking today in works like [6] that should show the state of the art with regard shot noise. This work aims at supporting a recent theory where shot noise would come from electrons piling-up “somewhere” in a CdTe resistor where the $\tau_d$ of its inner CdTe is greater than the average transit-time $\tau_t$ that the drift theory assigns to electrons that pass between its terminals as conduction current.

By “somewhere” we mean that no capacitance $C$ is considered in [6], where electrons that use to repel one each other, “agree to pile-up” to give shot noise looking like the well-known $1/f$ resistance noise. Where (or how) do these electrons pile-up? Is there a limit for the charge density piling-up in this way? These questions arise when one tries to keep finite the electrical energy and power linked to proposals like this one or like ours [2, 3], where single electrons randomly passing between terminals of resistors give their Johnson noise. Our proposal that closely tracks the “thermal agitation of electric charge” written in the titles of [4, 5] points out, however, that this agitation takes place in the 2TD where it is measured and not in the “conductors” these titles state. Taking the lonely resistance $R$ we use to calculate the Johnson noise of a resistor in Thermal Equilibrium (TE) we soon realized that this $R$ alone did not allow keeping finite the energy and power associated to the “sudden” passage of an electron between terminals.

By “sudden” we mean that if the electron passes it will pass entirely as a whole, in such a way that electrical measurements will find a null transit-time ($\tau_t=0$) for this passage. Otherwise the charge in transit could be electrically chopped, thus invalidating the role of quantum of charge we assign to the electron in experiments with electrical currents. This led us to consider the capacitive path shunting the Conductance $G = 1/R$ of any resistor that becomes an easy path for such displacements if they charge and discharge the capacitance $C$ between terminals. Let us show this by the circuit of Fig. 1, where a
resistor of resistance $R$ is driven by a generator able to deliver any current provided that the involved energy and power, both are finite and let us study the sudden passage of an electron from terminal B to terminal A across the volume $M=Ap \times d$ of the resistor.

To keep the indivisibility of the electron the generator should give a $\delta$-like current of weight $q$ C (the displaced charge between terminals). Taking null the net charge of each terminal of this 2TD let us drive it by an impulse current $i(t)=q \times \delta(0)$. This way we could think of a corpuscle-like electron that passes between terminals at instant $t=0$ with a null transit-time $\tau$. Keeping neutral both terminals requires current continuity in the closed network of Fig. 1. Thus, for each electron going from terminal B to terminal A across the resistor, an electron must be moved from terminal A to terminal B by the generator. This way the $-q$ C charge leaving terminal B to arrive in terminal A across $M$ makes a closed loop to arrive again in terminal B across the generator.

![Figure 1. One dimensional model of the two-terminal device known as resistor made from material of conductivity $\sigma$ and dielectric constant $\varepsilon$.](image)

The proposed passage requiring conduction current to take place is not possible because it leads to infinite power. This conduction current is not within the resistor, but it is the conduction current $i(t)$ arriving in (and departing from) the generator. This $i(t)$ closing the loop outside the resistor would have an associated magnetic field whose lines of force would encircle the wires (and perhaps the generator) of Fig. 1. The energy linked to this field should be delivered during the null duration of the $i(t)=q \times \delta(0)$ current, thus requiring an infinite power that our generator cannot deliver.

It is worth noting that a conduction current across the resistor is not needed thanks to its $C$ and the magnetic energy term disappears if we enter our generator within the resistor itself to consider electrons suddenly passing between its terminals. This leads to study sudden displacement currents within the resistor, thus sudden fluctuations of the electric field between its terminals without involving magnetic field. A corpuscular picture of this proposal would be that of single electrons suddenly jumping between terminals that charge and discharge the capacitance $C$ of the resistor. Bluntly speaking, we are going...
to consider the random appearance of pure fluctuations of electric field in the resistor giving rise to its Johnson noise. This means impulsive displacement currents in $C$, each of weight $-q\,C$ and since sign is irrelevant, let us consider displacement currents in the form $i(t) = \pm q \times \delta(t)$ taking place in $C$. Section III shows that the null transit-time ($\tau_t = 0$) associated to these currents does not mean that each passage does not take some time $\Delta t$, but that electrical measurements like Johnson noise will not detect this $\Delta t$.

If the resistor only had resistance $R$ between terminals, the electron should pass as a conduction current that being proportional to the voltage between terminals, would need infinite voltage at instant $t=0$ (Ohm’s law). This means infinite power at $t=0$ that the generator cannot deliver. However, any current through $C$ should be displacement current proportional to the change with time of the voltage between terminals, thus not needing a noticeable voltage but a fast varying voltage between terminals. This way electrons using $C$ to pass between terminals make null the power in $R$ and finite the involved power. This is reactive power in $C$ coming from sinusoidal voltage and current terms in quadrature that is finite as it is well-known from myriads of electrons being displaced every day in capacitors.

Hence, the sudden passage of electrons between terminals is possible but at the cost of shunting the resistance $R$ of the resistor by its capacitance $C$ depicted by the two plates of Fig. 1. This $C$ is an easy path for fast-varying currents like those required by the impulsive, thermal agitation of electrons we propose in [2, 3]. The vanishing reactance of $C$ for the components of current and voltage terms whose frequency $f \to \infty$ suggests that the energy an electron needs to suddenly pass between terminals in a resistor is the required energy to charge $C$ with its own charge $q$ displaced between terminals. This way the energy for each passage becomes finite and very small because it only is the energy $U_E$ of a capacitor with charges $-q$ and $+q$ in their plates:

$$U_E = \frac{q^2}{2C} = \frac{q}{2C} \times q$$  \hspace{1cm} (1)

For $C=1$ pF, Eq. (1) gives: $U_E = 8 \times 10^{-8}$ eV that is the thermal energy $kT$ at temperature $T \approx 0.001$ Kelvin degrees ($k$ is the Boltzmann constant). Once known the small and finite value of $U_E$ let us consider the involved power that being reactive power will give rise to fluctuation of electrical energy in this device, but not to dissipation of this energy that means its conversion into another form like phonons or photons that have to leave the admittance (thus the circuit) where the new form of energy no longer can exist.

Bound by a corpuscular view of the electron we could argue that $U_E$ delivered “during” $\tau_t=0$ would mean a power: $P = (U_E/\tau_t) \to \infty$. This is wrong however, because we are facing reactive power making the electrical energy fluctuate in $C$, not active power linked with $G=1/R$. To find $P \to \infty$ we need: a) finite voltage and infinite current or b) finite current and infinite voltage (both at instant $t_i$). Discarding option b) because displacing $q$ in this 2TD gives a voltage $\Delta V = q/C$, let us discard option a) from the 90º phase shift between Fourier components of $i(t)$ and $\Delta V$. This 90º shift prevents the synthesis of finite voltage and infinite current at the same instant because this situation would not correspond to a
null active power. Given that \(i(t)=q \times \delta(0)\) goes to infinite at \(t=0^+\), the voltage \(\Delta V\) must appear later, at \(t=0^+\). This time-ordainment means that the voltage step \(\Delta V=q/C\) that appears in \(C\) is the effect we measure of an impulsive current (cause) already gone when we perceive that it existed by \(\Delta V\). The cause of \(\Delta V\) observed at \(t=0^+\) was current we cannot measure as current at this instant because it already is voltage in \(C\).

In summary: our impulsive model \([2, 3]\) uses the resistor’s admittance formed by its Conductance \(G=1/R\) and its capacitance \(C\) between terminals. This is not bad, however, because this admittance of a parallel circuit is the dual notion of the impedance that Callen and Welton had to use to apply their quantum approach to the electrical noise of a resistor represented by its series circuit \([1]\). Let us say that each random passage of an electron between terminals is a Thermal Action (TA) \([2, 3]\) that causes electrical noise and Eq. (1) gives its associated fluctuation of energy for a discharged \(C\). The null transit time \(\tau_t=0\) deduced from the null rise-time of the voltage step \(\Delta V=q/C\) set in \(C\) by each TA is an intriguing feature at this time that we will explain in Section III.

It is worth noting that TA’s allow electrical energy entering into the resistor from its surrounding thermal bath. For a discharged \(C\) a TA will bring to the resistor the \(U_E\) of Eq. (1) whereas for a charged \(C\), a TA will bring energy to \(C\) or remove energy from \(C\) that will not be \(U_E\) as we will see in Section V. Due to this exchange, the energy in \(C\) will fluctuate around its mean value, which is null because the mean voltage of Johnson noise in TE is zero. Thus, \(U_E\) is the average energy brought to the resistor by each TA and since the instantaneous energy stored in \(C\) is proportional to the square of its voltage between terminals at each instant \(v_n(t)\), thermal Equipartition states that:

\[
\left< \frac{1}{2} C \times v_n(t)^2 \right> = \frac{1}{2} C \times \left< v_n(t)^2 \right> = \frac{kT}{2} \Rightarrow \left< v_n(t)^2 \right> = \frac{kT}{C} V^2
\]  

Eq. (2) gives for the resistor the well-known \(kT/C\) noise of a capacitor of capacitance \(C\) kept in TE at temperature \(T\) as it must be, because this is the device each resistor is from an electrical viewpoint. The mean square voltage \(kT/C\) \(V^2\) between the terminals of a resistor is a key feature of the impulsive noise model of \([2, 3]\) that explains why the Nyquist noise density \(S_f(f)=4kT G\) A²/Hz of a resistor is proportional to its conductance \(G=1/R\) as we will see in the next Section. Considering that \(kT/C\) \(V^2\) is the mean square voltage applied to the \(R\) of the resistor, the active power \(P_R\) associated to this \(R\) will be:

\[
P_R = \frac{kT}{RC} = \lambda \times U_E \Rightarrow \lambda = \frac{2kT}{q^2 R} \text{ TAs/s} \quad (3)
\]

This active power \(P_R\) coming from voltage and current in phase means electrical energy converted into another form (e. g. phonons) that cannot longer exists in the admittance of the resistor, thus power leaving the resistor as heat. It is the familiar power dissipated in the resistor. This requires an equal amount of power entering the resistor in order to keep it in TE at temperature \(T\). If electrical noise was impulsive noise, TAs would bring this power entering the resistor. This is why we have equated the exiting power \(P_R\) to the mean power \(\lambda U_E\) brought to the resistor by a random series of TAs taking place at an average rate \(\lambda\). The second term of Eq. (3) gives this rate that would be as high as
\[ \lambda \approx 3 \times 10^{14} \] Thermal Actions per second for a resistor of \( R = 1 \) k\( \Omega \) in TE at room \( T \). This huge rate makes continuous for practical purposes this impulsive noise model whose discreteness directly explains the phase noise of oscillators [7, 8].

II- Johnson noise: the measurable effect of an elusive current noise

Although Callen and Welton used the series immittance of a resistor in [1], the voltage noise \( v_n(t) \) we measure between its terminals leads to use its parallel immittance to link it with the two type of currents that can exist between them. These are displacement and conduction currents that sharing \( v_n(t) \) become orthogonal currents in frequency domain. This demands an electrical admittance to link them with their common voltage, which is the Johnson noise \( v_n(t) \) of the resistor. Therefore, a good resistor at low frequencies will become a good capacitor at high ones that for a null stray capacitance due to wiring, will have a quality factor given by \( Q(f) = 2\pi f \tau \), which gives the ratio at each frequency \( f \) between its displacement and conduction currents. Fig. 2 shows this \( R-C \) admittance together with the Nyquist noise density \( S(f) = 4kT/R \) A\(^2\)/Hz of a resistor.

This \( S(f) \) that for impulsive noise comes from currents we never measure as currents needs other units. Following [2, 3] thermal activity leads to a charge noise in \( C \) of mean power: \( P_q = 4kT/R \) C\(^2\)/s. Needless to say that A\(^2\)/Hz and C\(^2\)/s have similar dimensions. Each time an electron passes, charge fluctuates by \( q \) C in each terminal. Although the mean displaced charge per unit time is null its mean square value is: \( P_q = 4kT/R \) C\(^2\)/s. This charge noise due to spontaneous displacements of electrons gives rise to phase noise in the otherwise sinusoidal carrier of an oscillator using an L-C resonator with feedback [7, 8]. The 50% of \( P_q \) (2kT/R C\(^2\)/s) is due to TAs and the other 50% is charge noise power linked to discharges of \( C \) through \( G \) called Device Reactions (DR) in [2, 3].

![Figure 2.](image)

**Figure 2.** Typical i-v converter used to convert the noise of density \( S_f = 4kT/R \) A\(^2\)/Hz (Nyquist noise) into a measurable voltage density \( S_V = (4kT/R) \times (R_f)^2 \) V\(^2\)/Hz (see text).

Note that these spontaneous passages of electrons in an L-C resonator are similar to the spontaneous emissions of photons in the Fabry-Perot resonator of a laser causing its line broadening [9]. Linking “monochromatic line” with “pure sinusoidal carrier” and line broadening with phase noise of such a carrier, the phase noise of electronic and optical oscillators becomes a random phase modulation caused by a discrete disturbance of thermal origin they undergo. Given the easiness of this impulsive model to explain the phase noise of L-C oscillators [7, 8], the flicker noise of vacuum devices [10] and the \( 1/f \) excess noise of solid-state devices [11] let us consider the meaning of the way we measure today Johnson and Nyquist noises if electrical noise was this impulsive noise.
Since Johnson noise is a voltage $v_n(t)$ appearing between terminals of resistors, a low-noise amplifier of high input impedance as front-end electronics of a spectrum analyzer allows a proper measurement of its spectrum and its cut-off frequency $f_c$. Thus, this noise is properly measured today, although not everybody is aware of its $f_c$ coming from the complex Admittance where it is born. This unawareness leads to believe that its associated Nyquist noise $S_i(f) = 4kT/R A^2/Hz$ comes from currents that “can be extracted from the resistor” by an $i$-$v$ converter to be converted into voltage noise outside. The $A^2/Hz$ units of $S_i(f)$ suggest it is made from random currents in the resistor that can be “convinced to leave it”; although viewing them as impulsive fluctuations of its electric field between terminals their exit seems unlike. Nevertheless, the idea that $i$-$v$ converters really convert Nyquist noise $S_i(f)$ into a proportional voltage density $S_V(f)$ is widely accepted today. Let us show why by the circuit of Fig. 2.

Fig. 2 shows the basic circuit of an $i$-$v$ converter based on a high-gain voltage amplifier that is negatively feedback by a resistor of $R_F$ ohms connecting its inverting input to its output terminal where the converted noise $v_o$ is measured. To simplify, let the gain of the amplifier be $A_V \to \infty$. This makes null the voltage $v_e$ driving the amplifier no matter the voltage $v_o$ it sets at its output to absorb or to deliver through the $R_F$ resistor those noise currents like $i_n$ going from the resistor to the node A or from the node A towards the resistor. From $v_e \to 0$, no signal current enters the (-) input of the amplifier because $(v_e=0)/(R_{in} \neq 0)=0$ makes null any part of $i_n$ going through its input resistance $R_{in}$. Thus, any current $i_n$ going from the resistor to node A will be deviated through $R_F$ towards the amplifier’s output that will absorb it (or that will deliver it for $i_n$ having the opposed sense). To do these tasks the amplifier will set a voltage $v_o = \pm (R_F \times i_n)$ at its output, being this the reason why this converter gives $(-R_F)$ volts per each amp reaching its input node A, thus having a transresistance gain $A_{V/I} = -R_F V/A$ or $\Omega$.

This basic theory applies to $i$-$v$ converters like those of [12] that were used in [6], where a flat noise density $S_{V0}^{Nyq} = (R_F)^2 \times 4kT/R_{323K} V^2/Hz$ was expected from the flat Nyquist noise $S^{Nyquist}_i = 4kT/R_{323K} A^2/Hz$ of its CdTe resistor at $T=323 K$. Such a flatness however, must vanish at $f_{Nonflat} \approx 20 kHz$ for a resistor of $R=500 M\Omega$, see Fig. 9 of [12]. From this value and Eq. (3) of [12] this limit would be $\approx 43$ kHz for a 233 M$\Omega$ resistor, but when the stray capacitance $C_D=0.5$ pF setting this limit increases by the Maxwell capacitance $C = \tau_d/R = 1.3$ pF of the CdTe resistor with $\tau_d=0.3$ ms, the limit drops down to $f_{Nonflat} \approx 20 kHz$. This could explain the lack of noise data over $f=10 kHz$ in Fig. 2 of [6] and why these authors wrote: “At thermal equilibrium (i.e., zero current), the spectrum is white and takes a value in agreement with Nyquist noise, Eq. (1), as it should be”. “As it should” because the converted noise delivered by the $i$-$v$ converter of Fig. 2 is not white for $f>10 kHz$. Contrarily, it dramatically rises due to the weakening of its feedback caused by $C$.

This weakening increases the noise gain of the converter so that internal noises like the $e_e=1.4nV/\sqrt{Hz}$ given in [12] surpass the aimed $S_{V0}^{Nyq}$ for $f>f_{Nonflat}$. This effect is shown by Eq. (3) of [12]. Thus, the $S_i(f)$ shown in Fig. 2 of [6] is not flat despite the flattening action of the logarithmic units used in its vertical axis. This non-flatness that already is
present at $f=10$ kHz for the expert’s eye, is manifest by a vertically enlarged copy of this figure. Since it proves the existence of $C_{tot}=C+C_D \approx 1.8$ pF, these authors should consider this $C_{tot}$ as a possible place for the piling-up of charge they propose. This omission of $C+C_D$ for charges that supposedly are piling-up led us to exclude [6] from the state of the art in shot noise we were looking for and, given the manifest resistance noise of its Fig. 2 with $1/f$ regions rising proportionally to the square of the current in the CdTe resistor, another reason to discard [6] was its unawareness of the ubiquitous $1/f$ excess noise. Why should be free of $1/f$ excess noise its CdTe resistor?

Leaving aside this interesting question, let us consider the physical meaning of the $S_{V_0}^{Nyq}$ delivered by the $i$-$v$ converter of Fig. 2 if electrical noise was the impulsive noise of [2, 3]. In this case the conversion of impulsive Nyquist noise of spectral density $4kT/R$ $A^2/Hz$ into voltage noise would not be done by the amplifier because the fastest $i$-$v$ converter between two conductors is their capacitance $C$. If this was the case and currents like $i_n$ of Fig. 2 were not impulsive noise currents because these currents already were converted by $C$ into voltage noise appearing in $C$ itself, the question is: Shall we find a noise density $S_{V_0}^{Nyq}=(4kT/R)\times(R_F)^2$ $V^2/Hz$ at the output $v_0$? To say it bluntly: if the actual magnitude driving the $i$-$v$ converter of Fig. 2 was the voltage noise we call Johnson noise, shall we find a density $S_{V_0}^{Nyq}=(4kT/R)\times(R_F)^2$ $V^2/Hz$ suggesting that impulsive noise currents in $C$ really are being deviated through $R_F$ in the way we explained for $i_n$ in Fig. 2?

An impulsive current through $R_F$ in Fig. 2 would require an impulsive voltage at $v_\theta$ that we know the amplifier is unable to give, but we also know that the converted noise density at the output $v_\theta$ of Fig. 2 is: $S_{V_0}^{Nyq}=(4kT/R)\times(R_F)^2$ $V^2/Hz$, as if it could do it. What is the reason for? Before answering this let us note a misleading notion suggested by the small $v_\varepsilon$ set at the input of the $i$-$v$ converter by its feedback. We mean $v_\varepsilon \to 0$ suggesting to neglect currents through $C$ by a similar reason to $(v_\varepsilon=0)/(R_{in}\neq 0)=0$ that we used to discard signal current through $R_{in}$. Nevertheless, the current through $C$ is not due to the value of $v_\varepsilon$ but to its change with time. A fast change of $v_\varepsilon \to 0$ means a high current through $C$, so that electrons can suddenly pass between terminals A and B in Fig. 2 while the voltage at $v_\theta$ remains finite.

The conversion of such displacement current into a small voltage step of $\Delta V=q/C$ volts removes the need for impulsive voltages at $v_\varepsilon$. Since this $i$-$v$ conversion occurs in $C$ we must consider the circuit of Fig. 2 as a voltage amplifier of gain $A_V$ driven by $v_\varepsilon$ that offers a very low input resistance $R^*$ between its terminals A and B. From circuit theory we have: $R^*=R_F/(1+A_V)$ that for typical values ($R_F$ in the $10^5$-$10^6$ ohms range and $10^5<AV<10^6$) drops to $R^*\approx 1\Omega$. However, circuit theory also states that $C^*=(1+A_V)C_F$ is a high capacitance that will shunt $R^*$ because the capacitance $C_F$ shunting $R_F$ can be low (e. g. 0.2 pF) but never null. Thus, the Johnson noise of the resistor is driving the low impedance $Z_{in}$ due to $R^*$ shunted by $C^*$, whose square magnitude is:

$$|Z_{in}|^2 = \frac{(R_F)^2}{1+\left(\frac{f}{f_{conv}}\right)^2} \times \frac{1}{(1+AV)^2} \Omega^2 \quad (4)$$
The cut-off frequency \( f_{\text{conv}} = \frac{1}{2\pi R_F C_F} \) gives the bandwidth (BW) of the i-v converter that for \( R_F = 500 \ \text{k}\Omega \) and \( C_F = 0.2 \ \text{pF} \) would reach \( \text{BW} = f_{\text{conv}} = 1.6 \ \text{MHz} \). Due to this low \( Z_{\text{in}} \) the voltage noise density developed by the resistor between terminals A and B is:

\[
S_V(f) \approx \frac{4kT}{R} \times |Z_{\text{in}}|^2 \times \frac{(R_F)^2}{1 + \left(\frac{f}{f_{\text{conv}}}ight)^2} \times \frac{1}{(1 + A_V)^2} \frac{V^2}{\text{Hz}}
\]  

(5)

The sign \( \approx \) in Eq. (3) means that the resistor admittance whose magnitude falls in the parts per million range of the dominant \( |Y_{\text{in}}| = |1/Z_{\text{in}}| \), has been neglected for simplicity. Thus, the voltage noise density at the output \( v_o \) of the i-v converter will be:

\[
S_V(f) \approx \frac{4kT}{R} \times |Z_{\text{in}}|^2 \times \frac{(R_F)^2}{1 + \left(\frac{f}{f_{\text{conv}}}ight)^2} \times \frac{(A_V)^2}{(1 + A_V)^2} \frac{V^2}{\text{Hz}}
\]  

(6)

For the gain \( A_V \) in the \( 10^5 < A_V < 10^6 \) range, the flat region of Eq. (6) for frequencies below \( f_{\text{conv}} \) is: \( S_V = (4kT/R) \times (R_F)^2 \frac{V^2}{\text{Hz}} \) as expected from Fig. 2. It is worth noting that the term \( 4kT/R \) comes from the electrical equivalence between \( 4kT/R \) \( \frac{V^2}{\text{Hz}} \) shunting \( R \) (Norton source) and \( 4kT \) \( \frac{V^2}{\text{Hz}} \) in series with \( R \) (Thévenin source). A deeper meaning of these densities linked by \( R \) as they are is that they are keeping thermal Equipartition for the DoF linked to the capacitance \( C \) as we wrote in Section I. Thus, the \( 4kT/R \) term appearing in these equations has nothing to do with impulsive currents going as \( i_n \) in Fig. 2 (or in the opposed sense) because such impulsive noise is converted by \( C \) into voltage between terminals before giving rise to any current like \( i_n \).

It is worth noting here the care Johnson had with the \( C \) shunting the resistance \( R_0 \) of their resistors that led him to write these sentences [4]: “In most of the present work the input element Z was a high resistance in parallel with its own shunt capacity and that of its leads and of the input of the amplifier. In such a combination the real resistance component \( R(\omega) \) is related to the pure resistance \( R_0 \) and the capacity \( C \) accordingly to:”

\[
R(\omega) = \frac{R_0}{(1 + \omega^2 C^2 R_0^2)}
\]  

(7)

As it should be known, \( R(\omega) \) is the real part of the complex Impedance (a series notion) for \( R_0 \) shunted by \( C \) (the physical circuit in parallel). Since Eq. (7) was properly used in 1928 to handle noise for \( R_0 \) values up to 5 M\( \Omega \) (see Fig. 4 of [4]) we should consider why this care is lacking in 2004 for a CdTe resistor of \( R_0 = 233 \ \text{M}\Omega \) [6], thus 47 times more sensitive to the shunting action of \( C \) than one of \( R_0 = 5 \ \text{M}\Omega \).

Multiplying Eq. (7) by \( 4kT \) gives the spectrum of Johnson noise of the resistor of Fig. 2 with cut-off frequency \( f_c = 1/(2\pi CR_0) \). Putting charges \( +q \) and \( -q \) in the inner surfaces of the terminals of the \( R-C \) admittance of this resistor as if an electron had suddenly passed between them, the impulse response thus obtained is a voltage pulse of \( q/C \) volts amplitude and null rise-time that decays with lifetime \( \tau = R_0 C \). This decay called Device Reaction (DR) [2] has a spectral content that is a scaled version of the Johnson noise spectrum. Thus, Johnson noise could come from a random series of these decays taking place at some average rate \( \lambda^* \). Considering that these DRs are sensed by the same \( R \) that
senses the Johnson noise, let us obtain the mean rate $\lambda^*$ required to build the Johnson noise spectrum from these individual decays. To do it let us multiply by $\lambda^*$ the spectrum of one of these decays to add in power the $\lambda^*$ decays that would occur during a second.

This sum in power as if these decays were uncorrelated signals gives a spectral density that to be equal to the Johnson noise spectrum of the resistor requires $\lambda^* = \lambda$, thus the rate of TAs given by Eq. (3). This result backing the impulsive model of [2, 3] suggests that the randomness of electrical noise is not affected by the sums in voltage of exponential decays taking place in $C$. We refer to the fact that two consecutive TAs of the same sign in rapid sequence roughly would double the amplitude $\Delta V = q/C$ due to the first TA. This way the accumulated energy unbalance in $C$ roughly would be $4U_k$ and the active power during the relaxation after this couple of consecutive TAs would be four times higher than the power dissipated during the DR following a single TA. This sum in voltage suggests some correlation of DRs that would not allow their sum in power at first sight.

This was a blocking idea for our impulsive model before realizing that because thermal Equipartition must prevail in the device, the randomness of electrical noise must come from its causes (TAs), thus being independent of their cumulative effect in time that is the Johnson noise we measure. If dissipation is enhanced in the device by a sporadic burst of TAs of the same sign, the accumulated fluctuation of energy in the DoF at hand will be surpassing $kT/2$ during this sporadic burst, a situation that thermal activity in the forthcoming time will drive in the opposed sense to keep the mean $kT/2$ we must find for this DoF. This is why $\lambda$ DRs per second added in power give rise to the Johnson noise spectrum of the resistor despite the short term correlations that seem to exist for DRs. Thermal activity setting the rate $\lambda$ and the sign of TAs will keep null the long term correlation of DRs, thus allowing their sum in power to get the Johnson noise spectrum from the impulse responses of the resistor to each displacement of a quantum of charge between its terminals.

Let us give some figures for the Johnson noise of a resistor of $R_\theta = 1$ MΩ shunted by the stray capacitance $C_{\text{stray}} = 0.5$ pF of the measuring setup. Let the Maxwell relaxation time of the material of this resistor be $\tau_d \approx 1$ ns or lower, as it would be for doped silicon used in microelectronics. Hence, $C_{\text{Maxwell}} = \tau_d / R_\theta = 0.001$ pF would be negligible and $C = C_{\text{stray}}$. Therefore, the cut-off frequency of its Johnson noise would be $f_c = 1/(2\pi R_\theta C) \approx 320$ kHz. Accordingly to our impulsive noise model this noise would come from a random series in time of voltage pulses, each with amplitude $\Delta V = q/C = 0.32$ µV and null rise-time that would decay with lifetime $\tau = R_\theta C = 0.5$ µs. From Eq. (3) the average rate of these pulses would be: $\lambda \approx 3 \times 10^{11}$ pulses/second with equal probability for $\Delta V = +0.32$ µV than for $\Delta V = -0.32$ µV (on average). This shows our impulsive model for the Johnson noise of this resistor in TE at $T = 300$ K, thus under open circuit conditions. When it is connected to an $i$-$v$ converter as shown in Fig. 2, Eq. (6) and comments about would explain why this converter would seem to extract its impulsive noise currents and convert them into voltage noise appearing at its output $v_o$. 
Thus, shot noise in macroscopic resistors is what Johnson measured in 1928 [4] by the voltage noise we call today Johnson noise to honor this pioneering scientist. Associated to Johnson noise there is a magnitude we call Nyquist noise to honor the scientist that first showed its thermal origin [5]. Nyquist noise is the average noise density ($A^2$/Hz) derived from Johnson noise ($V^2$/Hz) by circuit theory or the mean power of impulsive charge noise ($C^2$/s) born in the capacitance $C$ of resistors [2, 3]. It is worth noting that Johnson and Nyquist were both aware of the capacity $C$ between terminals of resistors, but “Shot noise in linear macroscopic resistors” is the title of a recent paper [6] whose authors are unaware of $C$. Therefore, the reading of classics like [1, 4, 5] along with today’s works on shot noise is very advisable.

Although the role of $C$ to generate Johnson noise should be clear, let us end this Section with a text showing that the need for capacitance to have electrical noise already was known by Pierce in 1948. It is [13]: “We have conveniently thought of Johnson noise as generated in the resistances in a network. We need not change this concept and say that the voltage and current of (9) and (10) are generated in the capacitance or inductance any more than we would say that the thermal velocities of molecules are generated by the molecules’ mass.” By (9) and (10) Pierce refers to the $kT/C$ and $kT/L$ noises of capacitors and inductors.

From our experience with noise calculations we can find reasons leading Pierce to write this paragraph, but we do not agree with its last sentence. The wrong notion of Johnson noise as generated in the resistances allows technically correct calculations in TE where the mean active power leaving the resistor (dissipation) is equal to the mean reactive power entering it by impulsive currents. However, such a notion conceals that Johnson noise comes from a random series of fluctuations of electrical energy preceding their dissipations as Callen and Welton already foresaw in 1951 [1]. The plural “dissipations” reflects the discontinuities of the active power due to the series of discrete fluctuations taking place randomly in the resistor. Their small size and huge rate $\lambda$, however, show them as a continuous process for practical purposes that we call dissipation associated to $R$ in TE or Joule effect associated to $R$ for a biased resistor out of TE [3].

III- Carriers, charge neutrality and Thermal Actions

The notion of a TA as an electron suddenly displaced in space suggests an event that conflicts with Special Relativity. Due to its non-null mass at rest the electron could not be instantaneously displaced between two points at distance $d$ like the two terminals of a resistor. However, this apparent instantaneous only is a notion we deduce from the null transit time ($\tau_t=0$) that electrical measurements should give to keep the integrity of the displaced electron taken as an indivisible quantum of charge in those experiments like electrical noise and electrical conduction. Thus, this $\tau_t=0$ corresponding to the null rise-time $\tau_r=0$ of the voltage step $\Delta V=q/C$ caused in $C$ by a TA, is a result coming from the system we use to measure electrical noise. If our system is unable to detect any time interval $\Delta t \neq 0$ that the resistor needs to undergo a TA, we should not say that such a TA is instantaneous, but that it looks like instantaneous for the measuring system at hand.
If TAs were truly instantaneous events a burst of many TAs occurring in a vanishing time interval could take place sometimes. However, this possibility was discarded in [3] by a minimum time interval $\Delta t$ that the 2TD resistor would need to be ready for each TA. Although this $\Delta t$ allowed keeping finite the power entering the resistor by TAs, the notion of an electron that seems to pass in a null transit-time $\tau=0$ still is puzzling and needs an added explanation. Thus, let us propose TAs requiring some $\Delta t \neq 0$ to take place but giving rise to voltage steps of amplitude $\Delta V=q/C$ and null rise-time. This leads to “build” the passage of an electron between terminals from two events at least separated by some time interval $\Delta t \neq 0$. This can be accomplished by fluctuations of charge within the volume $M$ of the 2TD, thus involving what in solid-state physics are called carriers.

For this purpose, let us consider that the volume $M=An \times d$ of the 2TD of Fig. 1 is filled with material of conductivity $\sigma$ and electrical permittivity $\varepsilon_r \varepsilon_0$. This leads to find a conductance $G=\sigma A_n/d \Omega^{-1}$ shunting the $C=\varepsilon A_n/d F$ of this 2TD that will be a noisy resistor of $R=d/(\sigma A_n) \Omega$ at low frequencies. Electrical energy exists and fluctuates in its capacitive susceptibility whereas its conductance $G=1/R$ will give rise to the dissipations of electrical energy accounting for the rate of electrical energy that is converted into heat in $M$ (active power). Considering that this resistor contains n-type semiconductor between terminals, let us think of a free carrier involving an electron in the conduction band (CB) of its piece of material. Hereafter, let us consider the quantum description of this electron within the volume $M$ and its time evolution from the ideas given in Chapter 8 of [14].

Let $\Psi_{3D}$ be the delocalized wavefunction that represents this electron within the volume $M$. This $\Psi_{3D}$ departs from a corpuscle-like view of this electron free to move between terminals, but “jailed” in $M$. If this electron took part in a TA, the time evolution of its $\Psi_{3D}$ should describe its passage between terminals in the same form that its description by $\Psi_{0D}$ itself should come from the evolution with time of a $\Psi_{0D}$ locating it in a small volume around a donor atom fixed at some point of $M$ where it was trapped at low $T$. Let us take this non-ionized donor as a hydrogen-like system where the electron at hand is electrically trapped in a kind of spherical dipole at low $T$. We mean the Space Charge Region (SCR) whose charge $+q$ is the impurity ion in its center (impurity core) and whose charge $-q$ is a cloud of negative charge surrounding this core (e. g. the electron described by $\Psi_{0D}$). Considering $r_{cc}(T)$ as an average distance between core and negative cloud at a given $T$ we see that this electron is in one of its Degrees of Freedom (DoF) to store electrical energy by varying $r_{cc}$.

The ionization of this impurity would occur by the absorption of thermal energy enough to break this spherical dipole. The time evolution of $\Psi_{0D}$ to become $\Psi_{3D}$ (a $\Psi_{0D}$ $\rightarrow$ $\Psi_{3D}$ collapse) would represent this ionization process. Since $\Psi_{0D}$ represents one of the DoF of the electron in $M$, the electron must take care of it. This is why from time to time it will be trapped around an impurity core, thus giving rise to the familiar ionization factor $I(T)$ of donor atoms in a bulk region of this semiconductor. Therefore, electrons in $M$ undergoing $\Psi_{0D}$ $\rightarrow$ $\Psi_{3D}$ collapses (ionizations) and $\Psi_{3D}$ $\rightarrow$ $\Psi_{0D}$ collapses (captures) at some average rate would keep $I(T)$ or the concentration $n(T)$ of the free electron gas in
this type of semiconductor. Let us say that this notion of a gas is quite misleading with regard the structure of carriers because they are not freely-moving corpuscles in $M$.

In the same way a dipole was needed to have a negative cloud “wrapping-up” a positive impurity core (to have the electron in a $\Psi_{0D}$ state), another dipolar structure (charge neutrality) will be needed to find it in a $\Psi_{3D}$ state. Therefore, the free carrier resulting from the ionization of a donor impurity will form an extended dipole in the volume $M$ of the resistor to have its negative cloud of charge $-q$ distributed in $M$ (e.g. delocalized in $M$). Thus, the negative cloud of a carrier should be screening “a portion $+q$” of the overall positive charge due to the set of ionized donors at fixed positions in the lattice of this material. This portion should be distributed in $M=Ap\times d$ (uniformly distributed for simplicity). Otherwise, $\Psi_{3D}$ would not be delocalized within $M$ [3]. Thus, free carriers in n-type material would not be the negatively-charged corpuscles we assume in the drift theory for conduction current, but distributed dipoles in $M$ with a fixed charge $+q$ “wrapped” by a deformable cloud of charge $-q$.

This way an electron in the CB would give rise to a carrier structure able to react against external voltages $V$ between terminals, thus able to load energy by deforming its SCR. This loading would be done by displacing the cloud of negative charge from its unperturbed position given by the best screening possible of its portion $+q$ within $M$ (a polarization process). If a voltage $V\neq 0$ exists between terminals the displaced cloud will be pulling the lattice, thus storing elastic energy that will make the lattice vibrate if the electron cloud leaves $M$ to take care of another DoF [3]. This would occur if $\Psi_{3D}$ collapsed into $\Psi_{0D}$ or if $\Psi_{3D}$ collapsed into a two-dimensional wavefunction $\Psi_{2D}$ we are going to describe below. This advance of Joule effect based on reactive carriers, thus departing from the drift theory in use today, avoids the need for its “clever carriers” that sensing the electric field of a voltage $V$ existing between terminals to drift accordingly to their negative charge, do not sense the electric field of myriads of charged carriers in close proximity to drift accordingly too.

From the $\Psi_{0D} \rightarrow \Psi_{3D}$ collapse describing the ionization of a donor or the emission of an electron to the CB and from the $\Psi_{3D} \rightarrow \Psi_{0D}$ collapse representing the capture of an electron of the CB by an impurity core, let us use a two-dimensional $\Psi_{2D}$ to describe electrons as highly located on the inner surface of one of the terminals in Fig. 1. Like $\Psi_{3D}$ and $\Psi_{0D}$, $\Psi_{2D}$ also represents a transient location of the electron that evolves with time (Fig. 8-1 of [14] would sketch this time evolution). Thus, $\Psi_{2D}$ could represent an electron arriving in (or about to leave from) one of the terminals of the resistor and it must exist to account for the DoF linked to the familiar Eq. 1 of a capacitor of $C$ farads with charges $-q$ and $+q$ in its plates. However, the situation we are going to consider is not that with opposed charges $+q$ and $-q$ in each terminal set by a TA, but previous ones leading to such TA.

The first one would be a $\Psi_{3D} \rightarrow \Psi_{0D}$ collapse representing the capture of an electron of the CB by an impurity core at instant $t_1$. This would destroy the carrier structure releasing any elastic energy it could have and leaving “unwrapped” its portion of fixed
charge $+q$, thus a charge density of $+q/M \text{ C/cm}^3$ in the volume $M$. This way the highly localized charge $+q$ of an impurity core is viewed as a delocalized charge in $M$ available for further evolutions.

As it may be guessed, this portion of fixed charge $+q$ left in $M$ would be a strong call for an electron trapped in another impurity core to undergo a $\Psi_{0D} \rightarrow \Psi_{3D}$ collapse that would represent the emission of this electron to the CB. If this collapse took place, we would had replaced the electron of a carrier structure by another, but after releasing to the lattice any elastic energy that could have the former electron from its previous stay in a $\Psi_{3D}$ state. This release of energy is a key feature for Joule effect without drifting carriers as we will see below. As it may be guessed too, the portion of fixed charge $+q$ left in $M$ by the capture of an electron of the CB by an impurity core also would be a strong call for an electron of terminal B in a $\Psi_{2D}$ state to make a $\Psi_{2D} \rightarrow \Psi_{3D}$ transition to become a free carrier in $M$ (an electron in the CB). Reacting to such a call, an electron in a $\Psi_{2D}$ state in terminal B could “wrap-up” the unscreened portion $+q$ left in $M$ by the electron just trapped by the impurity core. This would leave a charge density of $+q/A_P \text{ C}^2/\text{cm}^2$ in the inner surface of terminal B.

If $\Psi_{2D} \rightarrow \Psi_{3D}$ transitions like this one are possible, $\Psi_{3D} \rightarrow \Psi_{2D}$ transitions also should be as it happens for $\Psi_{0D} \rightarrow \Psi_{3D}$ collapses (emissions) and $\Psi_{3D} \rightarrow \Psi_{0D}$ collapses (captures) creating and destroying carriers. Considering an electron in its $\Psi_{0D}$ state around an impurity core, it could acquire enough thermal energy so as to pass to the volume $M$, thus being described by a $\Psi_{3D}$. This $\Psi_{0D} \rightarrow \Psi_{3D}$ collapse would set a charge density in $M$ of $-q/M \text{ C/cm}^3$ due to the negative cloud of this delocalized electron distributed in $M$. This way, the spherical dipole of the impurity atom is broken and its cloud of negative charge described by a $\Psi_{0D}$ closely wrapped around the impurity core, would remain “around” (overall charge neutrality) but at longer distances occupying the whole volume $M$. This means that the electron would be in a $\Psi_{3D}$ state of the CB, but because the impurity core of charge $+q$ just born was not part of the fixed charge that existed before this emission, this electron of the CB would not find its delocalized portion of charge $+q$ to screen. This would be the reason to have a transient charge density of $-q/M \text{ C/cm}^3$ in $M$ “surrounding” the point charge density of overall charge $+q$ of the impurity core.

This electron just emitted to the CB, still “not wrapping” its portion of charge $+q$, could undergo a $\Psi_{3D} \rightarrow \Psi_{2D}$ collapse to become located at instant $t_2$ on the inner surface of terminal A for example, thus setting a charge density of $-q/A_P \text{ C}^2/\text{cm}^2$ on the inner surface of terminal A and leaving neutral the volume $M$. This way the emission of an electron from an impurity atom could give rise to an electronic charge $-q$ on one terminal and the capture of the electron of a carrier by an impurity core could give rise to a charge $+q$ on the opposed terminal. Although from these two events taking place simultaneously the plates of $C$ would become charged with opposed charges $+q$ and $-q$, the required simultaneity makes unlike the birth of TAs in this way for highly ionized donors where $I(T) \rightarrow 1$. This is why we will consider the birth and the destruction of carriers in $M$ involving the transient appearance of charge $-q$ (or charge $+q$) on the inner surface of one of the terminals.
We mean the case of an electron forming a carrier in $\mathbf{M}$ that becomes suddenly located on one of the terminals of the resistor. This means a $\Psi_{3D} \rightarrow \Psi_{2D}$ collapse that must occur from time to time to allow the electron taking care of the DoF linked with Eq. (1) that undergoes the electrical noise of SCRs and capacitive structures [10, 11]. Fig. 3 shows the SCR left in the resistor of Fig. 1 by an electron that being distributed in $\mathbf{M}$ as a carrier suddenly leaves $\mathbf{M}$ to appear on terminal A. This is a dipolar SCR formed by a depleted region of $+q/M \text{C/cm}^3$ in $\mathbf{M}$ and a sheet density of $-q/A_P \text{C/cm}^2$ on the inner surface of the terminal where the electron has just arrived in. This SCR recalling the SCR of a p$^+$-n junction would give rise to an internal barrier within the 2TD, but not to a voltage drop between its terminals. Solving electrostatics the barrier in going from the surface of terminal A (charged with $-q \text{C}$) to terminal B (neutral) is: $\Delta \Phi = q/(2C)$ volts, thus half the voltage step $\Delta V = q/C \text{ V}$. This internal barrier would be the result of a $\Psi_{3D} \rightarrow \Psi_{2D}$ collapse (pass collapse) for an electron that leaving its $\Psi_{3D}$ state in the CB would appear on the inner surface of terminal A.

The counterpart of this pass collapse would be a $\Psi_{2D} \rightarrow \Psi_{3D}$ collapse (age collapse) by which an electron appearing on the surface of one terminal at some instant would pass to occupy one of the states in the CB to form a carrier in $\mathbf{M}$. Changing the sign of the charges in Fig. 3 and solving electrostatics the voltage drop appearing in this case also is $\Delta V/2$ volts. As it may be guessed, the SCR left by a pass collapse in Fig. 3 is a strong call for an electron in a $\Psi_{2D}$ state on terminal B to make a $\Psi_{2D} \rightarrow \Psi_{3D}$ transition to form a free carrier in $\mathbf{M}$ (an electron in the CB). Reacting to such a call, an electron of terminal B in a $\Psi_{2D}$ state could “wrap-up” the unscreened portion $+q$ left in $\mathbf{M}$ by the electron that arrived in terminal A, thus giving rise to an age collapse setting a charge $+q$ on the inner surface of terminal B. This (pass-age) pair of collapses describing the passage of an electron between terminals shows that such passage is not an instantaneous event. This would be a TA coming from the replacement of an electron in the CB by another.

Although two electrons are involved in this TA, the noise we measure will not detect it because the $\Delta V/2$ voltage drop obtained from Fig. 3 for the pass collapse is an internal barrier requiring the sum of the other $\Delta V/2$ voltage drop of an age collapse to appear as the voltage step of $\Delta V = q/C$ volts and null rise-time we measure between terminals at $t=0^+$ once the TA is completed at $t=0$. This $\tau=0$ comes from the fact that what manifests $\Delta V$ between terminals at $t=0^+$ is the displacement current linked to a $\Psi_{2D} \rightarrow \Psi_{3D}$ collapse.
representing the birth of a new carrier in M. This describes a possible TA where the appearance of the measurable voltage between terminals $\Delta V = q/C$ needs the occurrence of the $\Psi_{3D} \to \Psi_{2D}$ collapse illustrated in Fig. 3 for terminal A at instant $t_1$, followed by a $\Psi_{2D} \to \Psi_{3D}$ collapse involving the opposed terminal at instant $t_2$ to give rise to the SCR depicted in Fig. 4.

**Figure 4.** Surface densities of charge left in a resistor when a Thermal Action is completed by an “age” collapse of wavefunctions (see the text).

Thus, the pass and age collapses building a voltage step of $\Delta V = q/C$ volts in C do not have to occur simultaneously. Only a pass collapse at $t_1$, thus preceding in time the age collapse at instant $t_2$ is required. Since $t_1$ and $t_2$ are random instants set by time varying phasors $\exp(\text{i}\omega t)$ of wavefunctions with $\omega = 2\pi(E/h)$ for electrons in different DoFs (thus with different energies $E$, $h$ is Plank constant), the random time taken by a TA to occur would not be null: $(t_2-t_1) \neq 0$, although what we can observe (the null rise-time of its measurable effect $\Delta V$ at instant $t_2$) suggests an electron suddenly passing between plates at this instant. With $\Psi_{3D}$, $\Psi_{0D}$, $\Psi_{2D}$, etc. evolving with time accordingly to their phasors just described and also affected by thermal activity giving rise to the aforementioned collapses, the average value of the $(t_2-t_1)$ intervals: $\Delta t = <(t_2-t_1)>$ should replace the $\Delta t$ we gave in [3] from the time-energy uncertainty principle. Since this TA would involve the destruction of a carrier at $t_1$, any elastic energy it could have would be released to the lattice. Because it also involves the birth of a carrier at $t_2$ in M, the newborn carrier will start to store elastic energy by the process of polarization we have described previously as we are going to consider in the next Section.

**IV- Conductance, dissipation and Joule Effect**

As soon as the voltage step $\Delta V = q/C$ V due to a TA appears between terminals at $t=0^+$, it will start to decay with time-constant $\tau = RC$ due to the conductive path offered by the Conductance $G = I/R$ between terminals of the resistor. This voltage decay, which is the slow effect of a TA we can measure (the Device Reaction to the Thermal Action) is the familiar dielectric relaxation involving an exponentially decaying conduction current (associated to $G$) together with an equal displacement current associated to $C$, opposed in sign to the former. Because $C$ and $R$ are embedded between terminals through $\varepsilon$ and $\sigma$, these two current terms mutually cancel at each instant of time and spatial point in M. Since the net current during this relaxation is null everywhere, magnetic energy does not comes into scene. Given the care we had to avoid magnetic field for each TA in Fig. 1, this is not surprising. If we consider this Maxwell equation:
\[ \nabla \times H = j_v + \frac{\partial D}{\partial t} \quad (8) \]

we could say that the impulse response of the R-C admittance of a resistor has a density of conduction current \( j_v \) linked to \( G = 1/R \) that is cancelled at each instant by an equal but opposed density \( j_D \) of displacement current linked to its \( C \). Eq. (8) giving \( \nabla \times H = 0 \) during a DR agrees with the null magnetic field we associated to its preceding TA.

Let us note that the displacement current density \( j_D = \partial D/\partial t \) during the DR brings back the charge displaced by the preceding TA. Thus, the charge noise in the resistor has an impulsive charge noise due to fast TAs and a charge noise due to slower DRs, in such a way that the charge noise power due to TAs is \( 2kT/R \frac{C^2}{s} \) and the charge noise power associated to DRs also is \( 2kT/R \frac{C^2}{s} \) (both on average). This way the total charge noise power is \( P_q = 4kT/R \frac{C^2}{s} \) as we wrote in Section II. These quite different charge noises produce quite different phase noise in an L-C tank oscillating at its resonant frequency \( f_0 \). Whereas the phase noise close to the “carrier frequency” \( f_0 \) (line broadening) comes from the impulsive charge noise that the electronics of the oscillator loop cannot modify because “it has no time to do it”, the phase noise far from \( f_0 \) (pedestal) has much to do with slow charge noise being modified by the electronics [7, 8].

Contrarily to TAs coming from the behavior of individual electrons, DRs would come from the collective reaction of the whole charge structure of the device giving rise to the slow relaxation of energy unbalances created by previous TAs. This makes possible the slow displacement current accompanying (and cancelling in time and space) the slow decaying conduction current of DRs. We mean that the displacement current decaying with time-constant \( \tau = RC \) during the DR does not come from a single electron slowly moving between terminals. Therefore, if a DR due to the collective behavior of charges in \( M \) is a null current that contains a non-null conduction current, this raises a question linked to the orthogonal character of displacement and conduction currents that share a common voltage between terminals. The question is: if displacement currents producing electrical noise in resistors are electrons that pass between terminals, could conduction currents exist without electron passages other than those existing in TE?

This question comes from a striking feature that is the absence of shot noise associated to conduction currents in macroscopic resistors. As it is well known, the Johnson noise of a resistor in TE is equal (provided heating effects are low) to its voltage noise when it has a conduction current \( I_{SPP} = V/R \) setting \( V \) volts between its terminals. For electrons this is a paradox: why electrons drifting between two terminals do not give rise to shot noise? Let us call the Silent-Parade Paradox (SPP) to this fact where electrons drifting between terminals (thus passing between them) do not give shot noise. For \( V = 1 \) V in the 1M\( \Omega \) resistor whose impulsive noise was described in Section II, its conduction current would be \( I_{SPP} = 1 \mu A \). Let be \( T = 300K \) the temperature of this resistor in TE, thus unbiased. In this case, its Johnson noise spectrum would be this one:

\[ S_V(\omega) = \frac{4kTR_b}{(1+\omega^2C^2R_b^2)} \frac{V^2}{Hz} \quad (9) \]
This is a familiar Lorentzian of cut-off frequency \( f_c = \frac{1}{2\pi R_0 C} \) Hz where \( \omega = 2\pi f \) is the angular frequency and whose flat amplitude \( 4kTR_0 = 1.7 \times 10^{-14} \text{ V}^2\text{Hz} \) uses to allow a checking of the calibration state of our noise meter. Driving this resistor by a current generator of \( I_{\text{SPP}} = 1 \mu\text{A} \) to allow voltage signals exist between terminals, a continuous (dc) voltage \( V = 1 \text{ volts} \) would be set between them (Ohm’s law: \( 1\text{V}/1\text{M}\Omega = 1\mu\text{A} \)). For a typical 0.5 watt resistor, the spectrum analyzer that uses to be ac-coupled to avoid its saturation by this high \( V \) gives (for practical purposes) the Lorentzian of Eq. (9) found in TE. The physical meaning of “0.5 watt resistor” is that due to its size and technology it can safely withstand up to 0.5 watts of active power \( p(t) \) on average. Since \( p(t) \) for this biased resistor is constant with time and has this value: \( P = V \times I_{\text{SPP}} = 1 \mu\text{W} \), it means a mean active power well below the safe limit of 0.5 watts.

Some years ago we would have said that this biased resistor is dissipated 1 microwatt, thus half a million times lower power than its safe limit (1/2 watts) and a low enough power so as to neglect its heating effects in a resistor of this size surrounded by air. Following [2, 3] however, we must say that this resistor is “converting electrical energy into heat at a rate of 1 microwatt” because accordingly to the Fluctuation-Dissipation framework of [1] we will use the word “dissipation” for TE. Thus, the electrical power properly dissipated by this resistor (\( N \) watts) would be the mean square noise voltage of its Johnson noise in TE (its \( kT/C \) noise) divided by its \( R \). For this resistor of \( R = 1 \text{M}\Omega \) shunted by \( C = 0.5 \text{ pF} \) and in TE at \( T = 300\text{K} \) this power is:

\[
N = \frac{kT}{RC} \approx 8 \times 10^{-9} \text{ W} \quad (10)
\]

Thus, this resistor is in TE it is dissipated \( N \approx 8 \text{ nW} \), a power value that is 125 times lower than the 1 microwatt it is converting into heat when it is biased. As we have said, the Johnson noise measured in TE and the voltage noise measured for the biased resistor with \( I_{\text{SPP}} = 1 \mu\text{A} \) (thus out of TE), roughly are the same. Considering that noise comes from Fluctuation-Dissipation processes [1] the noise coming from \( N \approx 8 \text{ nW} \) dissipated in TE and the noise coming from the “dissipation” of \( P = 125N \) should not be equal but quite different. Hence the reason to consider that if \( N \approx 8 \text{ nW} \) is a proper dissipation, but \( P = 125N \) giving the same amount of noise likely is: \( (N + 124N) \), where \( N \) would be a proper dissipation to give the same amount of noise than \( N \) in TE and 124\( N \) would be something else, likely related with \( N \), but different from the noise viewpoint. This is why we distinguish “dissipation” in TE from “conversion into heat” out of TE (Joule effect) although other names could be: noisy dissipation for \( N \) in the resistor in TE and overall dissipation for the biased resistor.

Concerning the SPP, it is well-know that the shot noise density at low \( f \) of a dc current \( I_{\text{SPP}} = 1 \mu\text{A} \) should have this value: \( S_{\text{shot}} = 2qI_{\text{SPP}} A^2/\text{Hz} \), thus \( S_{\text{shot}} = 3.2 \times 10^{-25} \text{ A}^2/\text{Hz} \) in our case. This current noise driving the \( R = 1 \text{ M}\Omega \) of this resistor would give a voltage noise density \( S_{\text{Vshot}} = 3.2 \times 10^{-13} \text{ V}^2/\text{Hz} \) between terminals, thus 19 times higher than its Johnson noise \( 4kTR_0 = 1.7 \times 10^{-14} \text{ V}^2/\text{Hz} \). Despite this factor, the voltage noise measured at low \( f \) in this 1 M\( \Omega \) resistor biased with \( I_{\text{SPP}} = 1 \mu\text{A} \) roughly is: \( S_{\text{VSSP}} = 1.7 \times 10^{-14} \text{ V}^2/\text{Hz} \), thus its Johnson noise. Therefore, carriers drifting between terminals in macroscopic resistors
make a *silent parade* concerning shot noise, hence the SPP acronym we have given to this puzzling fact from the drift theory viewpoint.

To solve this SPP we propose this breaking solution: “*If electrons do not give the shot noise assigned to their net passage between terminals, it likely is because such a net passage does not take place*”. This idea was behind our previous question: can we have conduction currents not requiring electron passages between terminals other than those existing in TE? Our impulsive noise model [2, 3] answers “yes” to this question as we are going to show by considering the *distributed dipole* of a carrier in the volume $M$ of the resistor. This dipole is sketched in Fig. 5-a by a chain of “small grains” of positive charge at fixed positions in the lattice of the material filling $M$. Each of these grains representing the distributed portion of charge $+q$ of the carrier (e. g. its fixed, positive plate) has a shell of *negative charge* around coming from neighbor parts of the cloud of charge $-q$ that is the movable, negative plate of the carrier. This chain of grains and shells of charge aims at representing the distributed dipolar structure of a carrier and *the elastic energy it stores* when it is polarized by the electric field ($E_{\text{Joule}}=V/d$) set in $M$ by an external voltage $V$ between terminals.

![Diagram of dipolar structure](image)

**Figure 5.** Dipolar structure for an electron in the conduction band (carrier): a) unbiased, b) under bias and c) vibrating lattice left by a biased carrier when it disappears from $M$.

Fig. 5-b shows this fast polarization process (recall the meaning of $\tau_d$), where the shells of the negative cloud deformed and displaced by $V$, thus *pulling each point of the lattice towards the anode*. The tilted beam linked to each grain represents the strained lattice...
under such pulling forces. This way, a distributed elastic energy $U_f$ is stored by the carrier structure under the action of the voltage $V$. If the negative cloud (electron) of this biased carrier disappeared from $M$ by a $\Psi_{3D} \rightarrow \Psi_{0D}$ collapse that would be the capture of its electron by an impurity core or by the $\Psi_{3D} \rightarrow \Psi_{2D}$ collapse of a TA, this $U_f$ would be released as soon as the negative shells disappearing from $M$ ceased their pulling forces on each point of the lattice. The lattice recoiling towards its unstrained positions would start to vibrate accordingly to its DoF for this type of energy, thus generating phonons. Let us say here that as soon as a carrier was born it would become polarized by $E_{\text{Joule}}$ in a time interval that being of the order of $\tau_d$, only will be a small fraction of the lifetime expected for these majority carriers in n-type semiconductor.

For a resistor of $R=1$ MΩ we had: $\lambda \approx 3 \times 10^{11}$ TAs per second. Let us considering the resistor of Fig. 1 having $R=1$ MΩ and being made from a beam of length $d=1$ cm whose ending contacts have $A_P=(0.2 \times 0.2)$ cm$^2$. Filling its volume $M=0.04$ cm$^3$ with material doped with $N_d=10^{16}$ shallow donors/cm$^3$, it roughly would contain $n_c \approx 4 \times 10^{14}$ carriers for donor ionization approaching unity. Thus, each carrier would live: $\tau_R=n_c/\lambda \approx 22$ minutes on average before disappearing by a TA or by the capture of an electron of the CB by a donor core. This carrier lifetime ($\tau_R=1333$ s) is orders of magnitude larger that the time a carrier needs to be polarized by the voltage $V$ (a time interval close to $\tau_d$, thus below $10^{-6}$ seconds typically). This lifetime for majority carriers should not be confused with the usually much shorter minority carrier lifetime. Hence, we can assume that each time a carrier is destroyed, it already had the whole elastic energy $U_f$ it can store by its capacity $C_f$ associated to its fixed charge $+q$ screened at each point as much as possible by its movable cloud of charge $-q$.

Let us borrow from [4] the word “capacity” for the mean capacitance $C_f$ that exists between the two opposed charges of the dipolar structure of each carrier taken as two charged plates. This allows thinking of this dipole as two plates with charges $+q$ and $-q$ kept at some “average distance” due to thermal activity [3]. The “exact” value of such distance is not needed to realize that $C_f$ represents another DoF of the electron to store thermal energy. Taking this $C_f$ as a mean capacity with charges $+q$ and $-q$ in its “plates” no matter their actual form and applying equipartition we obtain [3]:

$$\left\langle \frac{1}{2} \times \frac{q^2}{C} \right\rangle = \frac{1}{2} \times \frac{q^2}{\langle C \rangle} = \frac{kT}{2} \Rightarrow \langle C \rangle = C_f = \frac{q^2}{kT} = \frac{q}{V_T} \quad (11)$$

Given the small value of this capacity: $C_f \approx 6$ attoFarads ($C_f \approx 6 \times 10^{-18}$ F) at room $T$, the energy each carrier would load from the field $E_{\text{Joule}}=V/d$ would be very low. This field $E_{\text{Joule}}$ acting on this $C_f$ “made” from two plates with constant charges $+q$ and $-q$ would separate them slightly “along $d$”. This reduction of $C_f$ by the voltage $V$ would increase the electrical energy $q^2/(2C_f)$ of Eq. (11) to account for the energy $U_f$. For truly small changes in $C_f$ as expected for typical voltages in resistors, the energy $U_f$ also should be the electrical energy stored in $C_f$ by the voltage $V$ between terminals, which is:

$$U_f = \frac{1}{2} C_f \times V^2 = \frac{1}{2} \times \frac{q^2}{kT} \times V^2 \quad (12)$$
Therefore, each time a carrier is destroyed in a TA the elastic energy $U_f$ it stored will be released as lattice vibrations (phonons). Multiplying Eq. (12) by the average rate given by Eq. (3) for TAs, we will obtain the average power $P_{phon}$ of elastic energy released to the lattice in the resistor. This power is:

$$P_{phon} = \frac{2kT}{q^2R} \times \frac{1}{2} \times \frac{q^2}{kT} \times V^2 = \frac{V^2}{R} = P_{Joule} \quad (13)$$

Eq. (13) shows that the power of phonons released to the lattice of the resistor having $V$ volts between its terminals is the active power in its admittance that we assign to Joule effect. Since active power is the rate at which electrical energy is converted into another form, the impulsive noise of [2, 3] allows to envisage a way this conversion takes place in solid-state resistors. Moreover, this model also shows that the conduction current $I=V/R$ associated to a resistor of resistance $R$ with voltage $V$ between terminals does not need passages of electrons between terminals other than those impulsive ones that already existed in TE for $V=0$. This way the SPP of the drift theory disappears but at the cost of replacing its corpuscle-like, charge carriers by the distributed dipoles (energy carriers) whose electrons can move between terminals as displacement currents (waves) emulating their passage between terminals. This model also would solve the paradox of charged corpuscles sensing the electric field set by $V$, but not sensing those electric fields of neighbor carriers (charged corpuscles) in close proximity. Finally, let us say that the capacitive way by which we have emulated Joule effect is not far from the way resistance (or better said: dissipation) is emulated in switched capacitor circuits.

**V- Fluctuations under electrical bias**

This Section deals on a subject having to do with the deep-rooted notion of electrons as charged corpuscles in solid matter. Despite the description done in previous Section for a free electron in the CB of n-type semiconductor material, the idea of electrons as negatively-charged corpuscles is hard to leave. Thinking of TAs as sudden “jumps” of corpuscle-like electrons between terminals of resistors, let us consider a biased resistor with $V$ volts between its terminals. Taking each electron as a corpuscle of charge $-q$ jumping between terminals in each TA, it would seem at first sight that those electrons jumping from the cathode to the anode would be “favored” by the $V$ between terminals, whereas those electrons jumping from the anode to the cathode would be “disfavored”. If this was so, the causes of the electrical noise in resistors (e. g. the Thermal Actions in their $C$) could be deeply modified.

We mean that “favoured TAs” where the corpuscle-like electron would lose potential energy $U_{pot}=qV$ eV, would occur “easily” in the same way raindrops easily fall from the clouds on top as they lose potential energy. Contrarily to it, “disfavoured TAs” where the corpuscle-like electron would have to gain a potential energy $U_{pot}=qV$ eV, would not occur with the same easiness as it happens with raindrops that do not use to rise towards the clouds on top gaining potential energy. Although the passage of the electron between terminals as a wave (e. g. as a displacement current or as a fluctuation of electric field) suggests that it would not sense the field $E_{Joule}=V/d$ let us give some ideas
about the way the quite high energy $U_{pot}=qV$ eV associated with the displacement of one electron between the terminals of a biased resistor is handled in the 2TD the resistor is. To begin with, let us recall the small energy $U_E$ of Eq. (1) for an electron passing between terminals of a resistor whose $C$ is discharged.

As we have written, the energy $U_E=8\times10^{-8}$ eV for $C=1$ pF is equal to the thermal energy $kT$ at temperature $T\approx0.001$ Kelvin degrees, but for $V=1$ volt between the terminals of the 1 MΩ resistor used in the examples, the energy $U_{pot}=qV=1$ eV roughly is 12 million times the small energy $U_E$. Despite this enormous difference, the Johnson noise of this resistor in TE and its voltage noise with $V=1$ volts between terminals is the same for practical purposes. This means that the external bias $V$ does not affect the rate of TAs given by Eq. (3) nor their 50% probability for each sign. To say it bluntly, the quantum notion of fluctuation [1] would be independent of the potential energy $U_{pot}=qV=1$ eV we are worried about due to our notion of electrons as charged corpuscles that should sense the electric field $E_{Joule}=V/d$ “during” their passage between terminals whose voltage differs by $V=1$ volt. This was a concern at the time of writing [7, 8] to explain phase noise of electronic oscillators from [2, 3].

Given the good explanation of Leeson results we achieved in [7, 8], let us show the way an electrical energy like $U_{pot}=qV=1$ eV can be released to the lattice (or borrowed from it) each time the packet of elastic energy $U_f$ is released as shown in Fig. 5-c. Using the thermal voltage $V_T=kT/q$ of Eq. (11) we can write Eq. (12) as:

$$U_f = \frac{1}{2}C_f \times V^2 = \frac{1}{2} \times \frac{q}{V_T} \times V^2 \quad (14)$$

Therefore, the fraction of $U_f$ that $U_{pot}=qV$ represents is:

$$\frac{qV}{U_f} = \frac{1}{2}C_f \times V^2 = \frac{2V_T}{V} \quad (15)$$

For $V=1$ volt at room $T$ the potential energy of the $qV$ term roughly would be the 5% of the elastic energy released by each TA. This means that only a small amount of $U_f$ had to be subtracted or added to the energy $U_f$ released by each TA to the lattice. To say it bluntly: the sudden “jump” of an electron from the anode to the cathode of our 1 MΩ resistor biased by $V=1$ volt would be possible because it only would take the 5% of the $U_f$ energy released to the lattice when such TA occurred in TE (thus with $V=0$ volts). This means that a “disfavored” TA like this one only would release 0.95$U_f$ to the lattice whereas the “favored” TA of opposed sign would release 1.05$U_f$ to the lattice. Since favored and disfavored TAs have equal probability to occur and given the huge rate $\lambda$ of Eq. (3) the “dielectric” Joule effect quantified by Eq. (13) would be right.

VI- Conclusions

If electrical noise was impulsive noise we should consider why the capacitance between terminals of resistors is despised today concerning the generation of their Johnson noise. We also should review our notion on Nyquist noise and its conversion into voltage.
noise out of this capacitance or out of the 2TD where it is born. For impulsive noise, the absence of shot noise assigned to conduction currents in resistors no longer would be a paradox, but the expected result of a new way to understand Joule effect where a net flux of charges between two terminals is not required to have conduction current.

If electrical noise was impulsive noise, the spontaneous emissions of electrons between terminals of resonators would make the phase noise of electronic oscillators a mirror of the random phase modulation that spontaneous emissions of photons produces in optical oscillators (lasers). Similarly, the enigmatic resistance noise called “1/f excess noise” would come from the fact that the double layers around (or embedded in) conducting channels are not rigid walls concerning the measured conductance or resistance.

Finally, if electrical noise was impulsive noise, we would have a noise model agreeing with the quantum proposal of Callen and Welton and showing that the “irreversibility” word entitling their work has an interesting meaning. With regard resistors, impulsive noise allows showing that thermal equilibrium appears when the reactive power their capacitance takes from the environment equals (on average) the active power leaving them as heat and that we associate to their conductance.

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