Mathematical model for engineering tracked vehicle suspension system connections influence assessment

Evgeniy Sarach*, Aleksey Ivanov, Mariya Kurasova and Yaroslav Tkachev

Bauman Moscow State Technical University, Russia

*sarach@bmstu.ru

Abstract. This paper is devoted to designing imitational mathematical model that can help in tracked vehicle, which has connected suspension system, motion smoothness research. In this paper advantages and disadvantages of existing mathematical models are analyzed which allow to evaluate different design choices expediency at designing stage. In order to take suspension system connections into consideration and evaluate the possibility of using the created mathematical model to study transport vehicle motion smoothness the existing rectilinear motion mathematical model, created by BMSTU scientists, was modified. Modeling was carried out in MATLAB Simulink program complex. The results of connected suspension system imitational mathematical model usage are demonstrated on solving the following problem: tracked vehicle motion smoothness comparison in case of using connected and independent suspension system. These results show that connections in suspension system, which were added to decrease chassis loads, do not influence the tracked vehicle motion smoothness in any way. Thus, the modified imitational mathematical model can be used for tracked vehicle, which has connected suspension system, motion smoothness research.

Transport vehicles suspension systems can be classified as independent (individual) and connected (dependent) ones. Independent suspension systems are those, where forces, which act on the vehicle body from its wheels, do not have any visible connections between each other. In connected suspension systems these forces have a visible connection between each other.

Mechanical connections were used in tanks suspension systems of prewar period, such as “Centurion”, “Reno” R-35, “Valentine”, “Vickers” etc. These suspension systems were rather cumbersome and had small strokes, therefore, the connected suspension systems in Russian tank building industry were no longer used after the advent of a reliable independent torsion suspension system.

As a result of pneumohydraulic suspension systems development and suspensions hydraulic connection possibility advent, designers started to use connected suspension systems again. For example, Swedish tank “Stridsvagn 103 (Strv.103)”, which has quite short vehicle base, suspension system’s second and third wheels have independent suspension system in order to decrease longitudinal angular oscillations. The extreme wheels are connected diagonally via a compensation system. Also, extreme wheels connections on each side provide the possibility to regulate vehicle body position in the longitudinal plane.
Nowadays, there is a certain class of tracked vehicles with special suspension systems requirements. These requirements are the same to those made for both transport tracked vehicles, which move at high speed, and traction vehicles, which are used to work with special engineering equipment. These machines are supposed to be used for engineering equipment installation, move on uneven terrain at high average speed, and also should be able to block the suspension system when working with a blade.

Apart from this, due to engineering equipment installation such machines’ suspension systems have a significant uneven load distribution. Machine body gravity center shifts towards equipment position leading to loading the front wheels and unloading rear ones. As a result, bearing assemblies and wheels bandages wear out much faster.

In order to reduce the tracked vehicle wheels load unevenness, it is proposed to use connections in suspension system, uniting several wheels in a group. These connections provide the opportunity to rearrange the loads between the wheels. Thus, the loads on each wheel are reduced.

Serious theoretical transport vehicle dynamics research is impossible without using the computer engineering. Imitational mathematical modeling provides the opportunity to analyze different design choices expediency at designing stage and obtain required information to evaluate developed vehicle future performance.

Different rough terrain driving imitational mathematical models are used to solve problems related to motion smoothness and vibration loads for tracked and wheeled vehicles with independent suspension system [1-3]. They vary in terms of suspension system parts details level, vehicle motion speed calculation and interaction between the chassis elements and the ground representation.

For example, tracked vehicle rectilinear motion speed is set as a gravity center motion speed along the X coordinate in [1]. The spring characteristic and the damping characteristic are set as an array of points. Wheels contact the road in points. Such model has a high calculating speed but cannot be used for tracked vehicle profile pass ability research.

Tracked vehicle motion is carried out via track rewinding modeling in imitational mathematical model, represented in [2]. Wheels interact with road bumps along the circle of their circumference. This model represents the vehicle motion along large terrain roughs adequately and can be used to solve vehicle profile pass ability problems. However, this model has quite low calculation speed.

In [3] a different wheeled vehicle motion model is represented. It is modeled by friction force calculation between the wheel and the ground. In this model spring and damping characteristics are set as pneumohydraulic devices in the suspension system. This model has an acceptable calculation speed and can be used as a basis for tracked or wheeled vehicle connected suspension system model creating for solving problems related to motion smoothness and vibration loads.

In this mathematical model three different coordinate systems are used (Figure 1) due to the object motion equation structure and form.

The first coordinate system \(O_2X_2Y_2Z_2\), which is unrelated, can be used to model a certain set of road and soil motion conditions. The coordinate system starting point \(O_2\) matches the start of the modeled track.

The second coordinate system \(O_1X_1Y_1Z_1\), which is semi-related, has a feature that its starting point \(O_1\) always matches the tracked vehicle gravity center and moves with it in space. \(O_1X_1, O_1Y_1, O_1Z_1\) axis are parallel to respective axis of unrelated coordinate system.

The third coordinate system \(OXYZ\), which is used to mathematically describe tracked vehicle motion, is related because its starting point \(O\) always matches the gravity center while axis match the main vehicle inertia axis.

Tracked vehicle body dynamics equations are determined in the related coordinate system. Therefore, linear \((V_x, V_y, V_z)\) velocity projections and angular \((\omega_x, \omega_y, \omega_z)\) velocity projections on related axis are used as motion parameters.
To determine the tracked vehicle body motion equations body momentum change and body momentum moment change theorems, projected on axis, are used. As result, the equations are as follows:

\[
\begin{align*}
\dot{m}V_x + m(\omega_y V_{cx} - \omega_x V_{cy}) &= G_x + \sum_{i=1}^{N} \sum_{j=1}^{2} (N^x_{ij} + R^x_{ij} + P^x_{ij}) \\
\dot{m}V_y + m(\omega_z V_{cx} - \omega_x V_{cz}) &= G_y + \sum_{i=1}^{N} \sum_{j=1}^{2} (N^y_{ij} + R^y_{ij} + P^y_{ij}) \\
\dot{m}V_z + m(\omega_x V_{cy} - \omega_y V_{cz}) &= G_z + \sum_{i=1}^{N} \sum_{j=1}^{2} (N^z_{ij} + R^z_{ij} + P^z_{ij}) \\
I_x \dot{\omega}_x + \omega_x \omega_x (I_x - I_y) &= \sum_{i=1}^{N} \sum_{j=1}^{2} \left[ M_x (N_{ij}) + M_x (R_{ij}) + M_x (P_{ij}) \right] \\
I_y \dot{\omega}_y + \omega_x \omega_y (I_x - I_z) &= \sum_{i=1}^{N} \sum_{j=1}^{2} \left[ M_y (N_{ij}) + M_y (R_{ij}) + M_y (P_{ij}) \right] \\
I_z \dot{\omega}_z + \omega_x \omega_z (I_x - I_y) &= \sum_{i=1}^{N} \sum_{j=1}^{2} \left[ M_z (N_{ij}) + M_z (R_{ij}) + M_z (P_{ij}) \right]
\end{align*}
\]

where \(\omega_x, \omega_y, \omega_z\) – vehicle body angular velocity vector projections on the axis of mobile coordinate system \(OXYZ\); \(\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z\) - vehicle body angular acceleration vector projections on the axis of mobile coordinate system \(OXYZ\); \(V_{cx}, V_{cy}, V_{cz}\) – point C linear velocity vector projections on the axis of mobile coordinate system \(OXYZ\); \(\dot{V}_{cx}, \dot{V}_{cy}, \dot{V}_{cz}\) - point C linear acceleration vector projections on the axis of mobile coordinate system \(OXYZ\); \(G_x, G_y, G_z\) – gravity vector projections on the axis of mobile coordinate system \(OXYZ\); \(N^x_{ij}, N^y_{ij}, N^z_{ij}\) - perpendicular soil reaction vector projections on the axis of mobile coordinate system \(OXYZ\); \(R^x_{ij}, R^y_{ij}, R^z_{ij}\) - interaction force vector projections on the axis of mobile coordinate system \(OXYZ\); \(M^x_{ij}, M^y_{ij}, M^z_{ij}\) - perpendicular soil reaction momentum projections on the axis of mobile coordinate system \(OXYZ\); \(P^x_{ij}, P^y_{ij}, P^z_{ij}\) - interaction force momentum projections on the axis of mobile coordinate system \(OXYZ\); \(I_x, I_y, I_z\) – vehicle body moment of inertia relative to the axis of mobile coordinate system \(OXYZ\).

During modeling independent suspension system process the load in the spring element can be determined only by relative wheel stroke \(f\):
This equation is different in case pneumohydraulic spring (PHS) is used in the tracked vehicle suspension system. It can be determined by using polytropic as compression process in the spring pneumatic cylinder equation:

\[ P_s = P_c(f). \]

where \( P_0 \) – gas filling pressure in the spring; \( V_0 \) – initial gas chamber volume; \( D \) – spring piston diameter; \( u(f) \) – PHS wheel-rod transmission function, which depends on the wheel stroke; \( n \) – polytropy exponent.

In order to determine the damping force in the suspension system, it is necessary to describe fluid flow through the local liquid resistance equation [4]:

\[ G = \frac{\zeta}{\sqrt{\rho \cdot \zeta}} \cdot \frac{S_2}{\sqrt{p_1 - p_2}}, \]

where \( \zeta \) – local friction losses ratio; \( \rho \) – liquid density; \( S_2 \) – cross-sectional area at the exit from local liquid resistance; \( (p_1 - p_2) \) – pressure drop in the local liquid resistance area.

Skip pings ever al converting stages the local liquid resistance damping force can be described using the following equation:

\[ R = \zeta \cdot \frac{\rho}{2} \cdot S \cdot v_1^2, \]

where equivalent local friction losses ratio is:

\[ \zeta_s = \zeta \cdot \frac{S_1}{S_2}. \]

\( S_1 \) – cross-sectional area at the input of local liquid resistance; \( S \) – area affected by the pressure drop; \( v_1 \) – fluid velocity at the input of local liquid resistance.

If PHS hydraulic losses are presented as a sum of equivalent local friction losses, then the damping force on the rod can be determined by using the equation:

\[ P_{\text{dum}} = \sum \zeta_s \cdot \frac{\rho}{2} \cdot S_n \cdot v_{\text{um}}^2, \]

where \( S_n \) – piston cross-sectional area, \( v_{\text{um}} \) – rod velocity.

PHS throttle system local friction losses equivalent ratio consists of fluid flow narrowing and expansion friction losses in throttle hole, reduced to rod velocity.

Fluid flow narrowing and expansion friction losses can be determined using the following equations [5]:

\[ \zeta_{\text{narrow}} = (1 - S_1 / S_2)^2, \quad \zeta_{\text{exp}} = 0.5(1 - S_2 / S_1). \]

Transition from the damping force on the rod and PHS rod velocity to the force on the wheel and wheel velocity respectively can be carried out via transmission function \( u(f) \).

During PHS modeling process it is important to take compressibility of the fluid, which transmits the force from gas to PHS rod via the piston, into consideration. This whole process is considered in fluid volume model.

Apart from the volume filled with liquid itself, liquid deformation properties are also assigned to the volume, as it is considered in one-dimensional hydraulics [6]. Therefore, the working progress in this volume can be described via liquid volumes change, i.e. via flow non-discontinuity equation which have to include deformation flow (compression-extension flow). Thus, the volume can be described using the equation:

\[ \sum_{i=0}^n Q_i = 0, \text{ including } Q_{\text{exh}} = \frac{V}{E(P)} \cdot \frac{dP}{dt}, \]

where \( E(P) \) – gas-liquid mixture elastic modulus, \( V \) – volume.

\( E(P) \) can be determined via the following equation:
where $V_{a,0}$ and $V_{w}$ – air and liquid volumes at an initial pressure $p_0$; $E_{w}$, $E_{a}$ and $k$ – volumetric liquid resilience modulus for liquid and air at an initial pressure and adiabatic exponent respectively.

The following algorithm is proposed in order to transit from independent suspension system to a connected one in the mathematical model. Pressure sin different springs chambers as well as springs rods forces get equal due to the fact that connections in pneumohydraulic suspension system are implemented via a pipeline system, which connect corresponding PHS chambers (Figure 2). This means that rod force sin each PHS, which were connected by a hydraulic coupling, become equal to the arithmetic mean of the forces on the PHS rods, which existed before the coupling was made.

This rule works only if the wheel, while moving in relation to vehicle body, doesn’t interact with stroke limiter. This means that suspension system “breakdown” should be modeled separately for the connected suspension system. Then forces in stroke limiters should be summed up with forces in suspension system connected elements.

Results of connected suspension system imitational mathematical model work can be demonstrated by solving an applied problem: motion smoothness comparison for vehicles with connected and independent suspension systems.

Tracked vehicle chassis, used for road building equipment installation, with 12 wheels and PHS was the research object. Due to significant gravity center shift towards the vehicle’s front and first and second wheels overload, whilst fifth and sixth wheels were under loaded, 1-3 and 4-6 suspensions were hydraulically combined on each side in order to make chassis loads more even.

The test tracked vehicle parameters are presented in table 1. Wheels suspension system spring characteristics (mathematical dependence of wheel load $P$ from wheel stroke $f$) are shown in Figure 3. In Figure 4 PHS damping characteristics (mathematical dependence of PHS rod load $R$ from rod movement speed $x$) are shown. To transit from the rod load $R_{\text{rod}}$ to the wheel load $R_{\text{wheel}}$ and from the rod speed $x$ to the wheel speed $f$ respectively the following equations can be used:

$$R_{\text{wheel}}(f,\beta) = R_{\text{rod}}(\dot{x})u(\beta); f = \dot{x}/u(\beta),$$

where $u(\beta)$ – PHS wheel-rod transmission function (Fig.5); $\beta$ – rocker rotation angle.
Table 1. Tracked vehicles characteristics

| Parameter                                                                 | Value     |
|---------------------------------------------------------------------------|-----------|
| Sprung mass $M_s$, kg                                                    | 28000     |
| Moments of inertia relative to the axes passing through the sprung vehicle body gravity center: |
| $I_x$, kg·m²                                                             | 21000     |
| $I_y$, kg·m²                                                             | 161000    |
| $I_z$, kg·m²                                                             | 155000    |
| Chassis elements location relative to the vehicle gravity center, m:      |           |
| guiding wheel                                                            | 2.3       |
| 1 wheel                                                                  | 1.56      |
| 2 wheel                                                                  | 0.80      |
| 3 wheel                                                                  | 0.023     |
| 4 wheel                                                                  | -0.75     |
| 5 wheel                                                                  | -1.51     |
| 6 wheel                                                                  | -2.29     |
| driving wheel                                                            | -3.11     |
| Wheel diameter, m                                                        | 0.63      |
| Guiding wheel diameter, m                                                | 0.54      |
| Driving wheel diameter, m                                                | 0.6       |

Figure 3. Suspension system spring characteristics:
- a – for 1-3 suspensions; b – for 4-6 suspensions; 1 – at gas filling temperature of 20°C; 2 – at working temperature of 70°C

Figure 4. PHS damping characteristics:
- a – for 1, 2, 5 and 6 suspensions; b – for 3 and 4 suspensions; 1 – suspension forward stroke; 2 – suspension return stroke
Modeling was carried out in MATLAB Simulink program complex. Speed characteristics were determined for both tracked vehicles (with connected and independent suspension systems) based on the lack of suspension system “breakdown” criteria (Fig. 6). Also, frequency responses based on “shaking” acceleration were determined for both cases (Fig. 7). All the aforementioned characteristics were determined using methods from [1].

Suspension system speed characteristic display mathematical dependence between periodic irregularities heights $h$, which the vehicle can overcome without suspension system “breakdown”, and vehicle speed $V$. There are different speed characteristics graphs for each irregularity length $a$, which are usually equal to vehicle half-base (the distance between the extreme wheels) multiplied. These are the irregularities lengths when the vehicle body longitudinal-angular vibrations resonance usually may occur.

Frequency response display mathematical dependence between “shaking” vertical accelerations amplitude in tracked vehicle driver seat values $\ddot{z}$ and vehicle motion speed while moving along the ground periodic irregularities, which cause the high-frequency disturbances ($h=0.05 \text{ m}; a=0.9…1 \text{ m}$).

Mathematical model calculation results analysis show that periodic irregularities heights, which can be overcome by the vehicle without suspension system “breakdown”, are the same (0.23 m) for both suspension systems in the entire speed range. This fact corresponds to the modern high-speed tracked vehicles standard. “Shaking” acceleration level is acceptable when driving along 1 meter long ground irregularities for both suspension systems in the entire speed range, while for 0.9 m long irregularities it is acceptable only up to speed of 26 km/h.

Thus, it is proved that connections in suspension system do not influence the test tracked vehicle motion smoothness.
**Figure 7.** Tracked vehicle suspension system frequency responses for various irregularities lengths: 1 – 1.0 m; 2 – 0.9 m; a – connected suspension system; b – independent suspension system

**Conclusions**

Wheeled and tracked vehicles rectilinear motion imitational mathematical model, created by BMSTU scientists and modified by this paper authors in order to consider connections in suspension system, can be used for motion smoothness research of transport vehicles, which have connected suspension system.

**References**

[1] Dyadchenko M G Kotiev G O and Naumov V N 2002 *Tracked vehicles suspension system computer calculation fundamentals: Training manual* (BMSTU, Moscow)

[2] Kotiev G O and Sarach E B 2010 *Integrated suspension of highly mobile two-link tracked vehicles* (BMSTU, Moscow)

[3] Zhileykin M M Kotiev G O and Sarach E B 2018 *Transport vehicle systems mathematical modeling: Training manual* (BMSTU, Moscow)

[4] Kotiev G O Smirnov A A and Shilkin V P 2001 *Tracked vehicles suspension system hydraulic devices working processes research: Textbook.* (BMSTU, Moscow)

[5] Butaev D A Kalmykova Z A Subviews L G and others 1981 *Engineering hydraulics tasks collection: a manual for engineering universities* (Mechanical Engineering, Moscow)

[6] Darsh Y A 2004 Small backpressure valves of indirect action characteristics research (*Herald of mechanical engineering* vol 4) pp. 13-15