Abstract

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1 Introduction

The study of soft particle production is of particular interest since new information on the physics of confinement can be obtained. An important open question in this subject is whether the soft momentum region is smoothly connected to the hard region, where the purely perturbative description is appropriate, or dramatic changes are present due to the dominance of the hadronization process. In this respect, it is worth noticing that the charged particle inclusive energy spectra measured in $e^+e^-$ annihilation are surprisingly close over the whole momentum range (down to small momenta of a few hundred MeV) to the predictions of the perturbative QCD approach, based on the Modified Leading Log Approximation (MLLA) and the notion of Local Parton Hadron Duality (LPHD) (see also).

In this paper I discuss: a) how to test the validity of the perturbative description in the soft region and b) how to study the sensitivity of experimental data to different aspects of this approach, in particular to coherence effects (see for a discussion of the sensitivity of soft particle production to the running of the QCD coupling). I focus in particular on the study of the invariant density $E dn/dy = dn/dy d^2p_T$ in the limit of vanishing rapidity $y$ and transverse momentum $p_T$ or, equivalently, for vanishing momentum $|\vec{p}| \equiv p$, i.e.,

$$I_0 = \lim_{y \to 0, p_T \to 0} E \frac{dn}{dp} = \frac{1}{2} \lim_{p \to 0} E \frac{dn}{dp}$$

where the factor $1/2$ takes into account that both hemisphere are added in the limit $p \to 0$.

The invariant hadronic density in $e^+e^-$ annihilation is shown to approach a cm$^2$-energy independent value at low particles’ momentum, in agreement with the predictions of the perturbative approach with the inclusion of coherence. This result would then suggest to extend the validity of the theoretical
picture down to the very soft region, thus lending new support to the picture of LPHD, and it would confirm the relevance of coherence effects in soft particle production. To establish this interpretation more definitely, further tests of the perturbative picture are proposed. In particular, new predictions for the yield of the soft radiation in 3-jet events in $e^+e^-$ annihilation are presented. A test of universality of soft particle production in different reactions corresponding to different partonic colour emitters is also discussed.

The results contained in this paper have been obtained in collaboration with Valery A. Khoze and Wolfgang Ochs and have been partially presented elsewhere.

2 The invariant density $E_{\pi p} \frac{dn}{d^3p}$ in $e^+e^-$ annihilation

From the theoretical point of view, the coherence of the gluon radiation requires that very soft partons (with long wave length) are emitted by the total colour current, totally independently of the internal structure of the jet. According to the hypothesis of LPHD, one would then expect that the hadron spectrum at low momentum $p$ should be nearly independent of the jet energy $E_{jet}$, i.e., of the cms energy $E_{\text{cm}}$. The analysis of the inclusive spectra in the variable log $p$, as measured by the TASSO and OPAL Collaborations, suggests indeed that the soft tail of the spectrum is approximately independent of the cms energy in agreement with LPHD expectations. In order to characterize better this phenomenon, it has recently been proposed to study the invariant density $E_{\pi p} \frac{dn}{d^3p}$ at very low particle energies of the order of few hundred MeV. The important prediction of the perturbative QCD approach is that the invariant density approaches in this limit a value independent of cms energy. In the following, experimental analyses and theoretical predictions for the invariant density in the soft region are presented in more details.

2.1 Experimental results for low momentum particles

Let us first consider the invariant density $E_{\pi p} \frac{dn}{d^3p}$ for all charged particles. The angular dependence is actually integrated out in the following, so let us call $E_{\pi p} \frac{dn}{d^3p}$ the quantity $E_{\pi p} \frac{dn}{d^3p}$. Since no direct measurements of this observable have been performed, experimental information can be extracted from the published data on the inclusive momentum spectrum, $dn/d\xi_p$ vs. $\xi_p$, where $\xi_p \equiv \log(1/x_p) = \log(\sqrt{s}/2p)$, using the relation

$$E_{\pi p} \frac{dn}{d^3p} = \frac{E}{4\pi p^3} \frac{dn}{d\xi_p}$$  \hspace{1cm} (2)
where $E^2 = p^2 + m_h^2$ and $m_h$ is an effective particle mass.

There is not a unique prescription for the choice of this effective mass; on one hand, the value of $m_h = 270$ MeV has been used in 14 to give a good description of the moments of the energy spectra with the MLLA formulae. On the other hand, since pions dominate in the soft region, the pion mass would be a natural value for this parameter.

Fig. (1) shows the charged particle invariant density in $e^+e^-$ annihilation as a function of the particle energy $E$ at different cms energies ranging from 3 GeV up to LEP-1.5 cms energy (133 GeV) 12, 13, 15; two different values of $m_h$ are used. It is remarkable that the data from all cms energies tend to converge in the soft limit. The choice of $m_h$ does not modify the gross features of the invariant density, although two different values of the soft limit $I_0$ are observed. More precisely, let us point out that the LEP data seem to tend to a limiting value larger by about 20\% than the data at lower cms energies. This may be due to the overall systematic effect in the relative normalization of the different experiments (in this respect, let us stress that the invariant density is to a better approximation energy independent within sets of data collected at the same detector). Alternatively, a possible physical source of energy dependence

\textbf{Figure 1:} a) Invariant density $E d\eta / d^3 p$ of charged particles in $e^+e^-$ annihilation as a function of the particle energy $E = \sqrt{p^2 + m_h^2}$ at $m_h = 270$ MeV at various cms energies; b) the same as in a), but at $m_h = 138$ MeV.
is given by particles produced via weak decays. They should indeed be added incoherently to the particles produced from the primary quarks and thereby could yield a rise of the soft particle spectrum with increasing energy.

Let us now turn to the invariant density for charged pions, charged kaons and protons. Fig. 2 shows the invariant cross section $E d n / d^3 p$ for $\pi^-$'s, $K^+$'s and $p$'s as a function of the particle energy $E$, as extracted from the inclusive momentum spectra measured at cms energies from 1.6 GeV to 91 GeV according to eq. (2). In this case, the simplest choice for the mass scale $m_i$ is given by the particle mass itself. In all cases the data tend towards an energy-independent limit for $E \to m_i$ ($p \to 0$). Notice that the value of $I_0$ is roughly proportional to $m_i^{-2}$.

2.2 Theoretical predictions on the spectrum shape

The description of the soft region of inclusive energy spectra requires some care in the treatment of kinematics.

Experimental hadronic spectra are usually presented as function of momentum $p$ or $\xi_p = \log(1/x_p)$, and they do not have any kinematical boundary from above. On the other hand, in the MLLA the partons are treated as mass-
less but the transverse momentum is required to be larger than the infrared cutoff $Q_0$, such that $E = p \geq p_T \geq Q_0$. The theoretical predictions are then limited from above, as $\xi \leq Y = \log \sqrt{s/2Q_0}$.

As far as $E \simeq p \gg Q_0$, the different kinematical boundaries can be neglected and the analytical prediction using LPHD is uniquely defined. However, the deeper one goes into the soft region, the more sensitive one becomes to the different kinematical thresholds. In the soft region, there is not a unique procedure for deriving analytical predictions applying LPHD, but one has to introduce some additional assumptions and define practical prescriptions. As a general rule, one may require that the invariant density $Edn/d^3p$ obtained at hadron level by one of these prescriptions approaches a constant limit for $p \to 0$ as observed experimentally. In the following two possible alternative Ansätze are discussed.

It is important to stress that the most important prediction of the perturbative QCD approach, i.e., the energy independence of the invariant density for particles of low momentum, does not depend on the particular Ansatz one uses. This feature will indeed be recovered in both Ansätze we are about to discuss.

**Ansatz 1**

Let us relate hadron and parton spectra in a single $A$-jet in the following way,

$$E_h \frac{dn(\xi_E)}{dp_h} = K_h E_p \frac{dn(\xi_E)}{dp} = K_h D^9_A(\xi_E, Q_0, \Lambda)$$

with $E_h = \sqrt{p_h^2 + Q_0^2} = E_p \equiv E \geq Q_0$, $\xi_E \equiv \xi = \log Q/E$ and $K_h$ a normalization parameter. If hadrons from both hemispheres are added, $K_h$ should be replaced by $2K_h$.

Accordingly, the invariant density $Edn/d^3p$ to be compared with the data in $e^+e^-$ annihilation is given by:

$$E \frac{dn}{d^3p} = \frac{K_h}{4\pi p^2} \frac{2C_F}{N_C} D(\xi, Q_0, \Lambda)$$

with $E = \sqrt{s}/2e^{-\xi}$ and $p^2 = E^2 - Q_0^2$ and $D(\xi, Q_0, \Lambda)$ is the inclusive spectrum predicted for a single gluon jet. $C_F$ and $N_C$ are the colour factors for quark and gluon jets, $C_q = C_F$ and $C_g = N_C$; the ratio $C_F/N_C$ is needed to obtain the spectrum in a single quark-jet and the factor 2 to take both hemispheres into account.

Let us notice that, as far as the theoretical spectrum $D(\xi, Q_0, \Lambda)$ linearly goes to 0 for $p \to 0$, as in MLLA (see below), the invariant density for hadrons...
approaches indeed a finite limit as in [1]

\[ I_0 = K_0 \frac{C_A \beta^2}{8\pi N_C \Lambda Q_0^2} \]  

with \( \beta^2 = 4N_C/b \), \( b \equiv (11N_C - 2n_f)/3 \), and \( \lambda \equiv \log Q_0/\Lambda \), where \( \Lambda \) is the QCD-scale; \( N_C \) and \( n_f \) are the number of colours and of flavours respectively.

In addition, this prescription is particularly suitable for phenomenological analyses of the moments of the inclusive energy spectrum [4]. In this respect, let us notice that a good phenomenological description of the charged particle inclusive spectrum has been obtained with \( Q_0 = 270 \) MeV; this value of \( Q_0 \) will then be adopted in the theoretical calculations.

To obtain the theoretical prediction for the invariant density \( Edn/d^3p \) within this scheme, let us then obtain an explicit solution for the spectrum \( D(\xi, Q_0, \Lambda) \) in the soft limit. Consider first the Double Log Approximation (DLA), in which energy conservation is neglected and only the leading singularities in the parton splitting functions are kept. The evolution equation of the inclusive energy distribution of partons \( p \) originating from a primary parton \( A \) is given by \([5]\):

\[ D_p^A(\xi, Y) = \delta_p^A \delta(\xi) + \int_0^\xi d\xi' \int_0^{Y-\xi} dy' \frac{C_A}{N_C} \gamma_0 (y' + \xi') D_p^g(\xi', y' + \xi') \]  

where \( \xi = \log(1/x) = \log(Q/E) \) and \( Y = \log(Q/Q_0) \) with \( E \) the particle energy and \( Q \) the jet virtuality (\( Q = P\Theta \) for a jet of primary momentum \( P \) and half opening angle \( \Theta \)); \( C_A \) is the respective colour factor for quark and gluon jets; \( \gamma_0 \) denotes the anomalous dimension of multiplicity and is related to the QCD running coupling by \( \gamma_0^2 = 4N_C\alpha_s/2\pi = \beta^2/\log(p_T/\Lambda) \).

Eq. (6) can be solved iteratively; with two iterations one gets:

\[ D_A^g(\xi, Y) = \delta_A^g \delta(\xi) + \frac{C_A}{N_C} \beta^2 \log \left( 1 + \frac{Y - \xi}{\lambda} \right) \times \left[ 1 + \beta^2 \int_0^{Y-\xi} d\tau \log(1 + \frac{\xi}{\lambda}) \right] + \ldots \]  

The term of order \( \beta^2 \), corresponding to a single gluon emission, yields the leading contribution for \( E \rightarrow Q_0 \). It is worth noting that this term does not depend on the cms energy and it is proportional to the colour charge factor of the primary parton. Therefore, we have explicitly found the features of soft radiation which we have qualitatively expected. In addition, the second
iteration provides us with a energy-dependent term, which allows to describe
the rise of the spectrum with increasing $\sqrt{s}$ at particles’ energies around 1
GeV. Let us also point out that the $\xi$-spectrum vanishes in the soft limit
$\xi \to Y$ ($E \to Q_0$) as

$$D_A^\eta(\xi, Y) \sim Y - \xi \sim \log E/Q_0 \sim E - Q_0.$$ 

(8)

In order to get an improved description at nonasymptotic $cms$ energies, it
is important to use the MLLA, which takes into account the exact form of
the parton splitting functions in the evolution equation. The MLLA solution
is in general much more involved than the DLA one, but in the soft limit the
Figure 4: Invariant density $E dn/d^3p$ of charged particles in $e^+e^-$ annihilation as a function of the particle energy $E = \sqrt{p^2 + Q_0^2}$ at $Q_0 = 270$ MeV. Data points at various cms energies from SLAC, TASSO and TOPAZ Collaborations, LEP-1 and LEP-1.5 are compared to MLLA predictions ($\lambda = 0.01, K_h = 0.45$).

expression simplifies and one gets a simple exponential suppression factor:

$$D(\xi, Y, \lambda)|_{\text{MLLA}} = D(\xi, Y, \lambda)|_{\text{DLA}} \exp \left[ -a \int_\xi^Y \frac{\gamma_0^2(y)}{4N_C} dy \right] \quad (9)$$

with $a = 11N_C/3 + 2n_f/3N_C^2$. This solution is valid in the soft limit only, when the difference $|\gamma_0^2(\xi) - \gamma_0^2(Y)|$ is small compared to $\gamma_0^2D$. As a consistency check of eq. (9), the MLLA with fixed coupling has been explicitly solved and eq. (9) has been found to be exactly satisfied in the full energy region.

In order to estimate to what extent the iterative solution gives a good description of the inclusive spectrum, one can consider the inclusive spectrum $D(\xi, Q_0, \Lambda)$ in the limiting case $Q_0 = \Lambda$. In this case, the solution, called Limiting Spectrum, is explicitly known and one can avoid the iterative pro-
procedure. Notice that the Limiting Spectrum does not have the same behaviour near the boundary $\xi \to Y$ as the full solution which follows from the integral equation (8) with eq. (9), as the Limiting Spectrum goes to a constant as $E \to Q_0$. The invariant density computed from the Limiting Spectrum according to eq. (4) shows then a singularity in the soft limit, $p \to 0$.

To illustrate the above analytical results, the predictions for the invariant density $Edn/d^3p$ in the low particle energy region at the two cms energies of 3 and 91 GeV, obtained via eq. (4) in DLA (eq. (7)) and MLLA (eq. (9)) with $\lambda = 0.01$ and in the Limiting Spectrum case are shown in Fig. (3). The normalization of the limiting spectrum is as in fits to $e^+e^-$ annihilation; the normalizations of the DLA and MLLA curves are chosen to approach the limiting spectrum for energies $E \geq 0.5$ GeV.

Both in DLA and in MLLA the invariant density approaches an energy independent value in the soft limit $\xi \to Y$ ($E \to Q_0$). This originates from the soft gluon emission contribution of order $\alpha_s$ which is determined by the total colour charge of the primary partons due to the colour coherence. In this limit the MLLA converges towards the DLA. The MLLA solution is in good agreement with the Limiting Spectrum solution, thus showing that the iterative solution includes indeed the dominant contribution in the soft region.

A quantitative comparison of the theoretical predictions of MLLA with experimental data for the invariant density of all charged particles is shown in Fig. (4). A good agreement is visible, both in the energy independence of the soft limit $I_0$ and in the energy dependence of the invariant density at particle energies above 500 MeV.

It is also possible to compute the predictions of MLLA with fixed coupling. In this case a full solution can be obtained. It is interesting to notice that the model with fixed coupling cannot describe the steep slope at very low momentum, thus suggesting that the running of the coupling is relevant in this region. Further studies of models, where the coupling is frozen below a certain momentum threshold, are in progress (see also [10] for further discussions of the effect of the running of the coupling in inclusive energy spectra).

Ansatz 2

Let us consider the alternative identification

$$\frac{dn}{d\xi_p} = \left(\frac{p}{E}\right)^3 D_{\text{lim}}(\xi_E, Y)$$

where $D_{\text{lim}}(\xi_E, Y)$ indicates the Limiting Spectrum solution. With this Ansatz,
the invariant density for hadrons in $e^+e^-$ annihilation is given by:

$$E \frac{dn}{d^3p} = \frac{K_h}{4\pi E^2} \frac{2C_F}{N_C} D_{\text{lim}}(\xi_E, Y) \quad (11)$$

The preferred value for $Q_0 = \Lambda$ is in this case the pion mass, as natural in the soft region, where most produced particles are indeed pions.

Let us notice that as $p \to 0$, the particle energy $E$ goes to the finite value $Q_0$ and $D_{\text{lim}}$ goes to a constant, $D_{\text{lim}}^0$. One then recovers again a finite value for the soft limit

$$I_0 = \frac{K_h}{4\pi Q_0^2} \frac{2C_F}{N_C} D_{\text{lim}}^0 \quad (12)$$

The invariant densities predicted with this method at different cms energies are shown in Fig. (5). It is important to stress that also with this second
Figure 6: a): invariant density $E d^3n/dp$ of charged particles in $e^+e^-$ annihilation as a function of the particle energy $E = \sqrt{p^2 + m^2}$ with $m_h = 138$ MeV at various cms energies compared with the predictions of eq. (11) with $Q_0 = 138$ MeV and $K_h = 1.125$; b): same as in a), but for charged pions.

Ansatz all curves tend to converge to a common value independent of cms energy at very low particles’ energies.

This approach has been applied both to the invariant density of all charged particles and to the invariant density of pions. Its predictions at different cms energies are compared in Fig. 6 with experimental data for charged particles and charged pions. At large cms energies, a good agreement is found for charged pions in the whole particle energy region and for charged particles in the low particle energy region. However, this prescription cannot properly describe data at low cms energies, i.e., it does not reproduce correctly the cms energy dependence of the data.

3 Further tests of LPHD in 3-jet events in $e^+e^-$ annihilation.

In view of the success of the LPHD picture for describing the invariant density of charged particles in $e^+e^-$ annihilation, it is interesting to look for other independent tests of this picture. In this section, a new analysis which could shed new light on the validity of the perturbative description of the soft region is proposed.

Consider 3-jet events, measure the invariant density of particles produced
into a cone perpendicular to the production plane for different angular configurations of the three jets and extract then the limiting value $I_0$ for each configuration. To get an absolute normalization, look also at the radiation into a cone of the same angle again perpendicular to the primary $q\bar{q}$ direction, but in 2-jet $q\bar{q}$ events and build the ratio

$$R_\perp \equiv \frac{dN_{q\bar{q}g}}{dN_{q\bar{q}}}(13)$$

The proposed observable seems at a first sight similar to the study of the string phenomenon, which concerns the radiation in the plane of 3-jet events; the advantage of studying the radiation in the transverse direction lies in the fact that in this case one can avoid the integration over the $k_T \geq Q_0$ boundaries along the jets. Moreover, the new observable refers to soft production only, and not to the total particle flow integrated over momentum, as in the discussion of the string effect. For the particle flow, the original angular flow should be convoluted with a cascading factor [21], which, however, cancels out when one looks at ratios. The same predictions for the ratio $R_\perp$ should then in principle be valid both for the ratio of the soft particle yields, $I_0$, and for the ratio of the multiplicity flows. However, it would be useful to perform independently both measurements.

Let us now consider the theoretical prediction for the above observable in the framework of the perturbative approach. The calculation only includes the contribution of soft gluon bremsstrahlung emission up to order $\alpha_s$, since, as seen in the study of the invariant density in $e^+e^-$ annihilation, this term should dominate in the soft limit.

The formula for the soft radiation into arbitrary direction $\vec{n}$ from a $q\bar{q}$ antenna pointing in directions $\vec{n}_i$ and $\vec{n}_j$ is given by the usual bremsstrahlung formula [21]:

$$dN_{q\bar{q}} = \frac{dp}{p} d\Omega_{\vec{n}} \frac{\alpha_s}{(2\pi)^2} W^{q\bar{q}}(\vec{n}) \quad , \quad W^{q\bar{q}}(\vec{n}) = 2C_F(\vec{i}\vec{j}) \quad (14)$$

with $(\vec{i}\vec{j}) = a_{ij}/(a_i a_j)$, $a_{ij} = (1 - \vec{n}_i \vec{n}_j)$ and $a_i = (1 - \vec{n}_i \vec{n}_i)$. For the radiation perpendicular to the primary partons $(\vec{i}\vec{j}) = a_{ij} = 1 - \cos \Theta_{ij}$, with $\Theta_{ij}$ the relative angle between the primary partons $i$ and $j$.

The soft gluon radiation in a 3-jet event is given as in eq. (14) with the angular factor

$$W^{q\bar{q}g}(\vec{n}) = N_C[(\vec{g}\bar{q}) + (\vec{q}\bar{g}) - \frac{1}{N_C}(\vec{g}\vec{q})] \quad (15)$$
The soft gluon radiation for a $q\bar{q}$ pair with relative angle $\Theta_{q\bar{q}}$ is given again as in eq. (14), but with the angular factor

$$W_{q\bar{q}}(\Theta_{q\bar{q}}) = 2C_F(1 - \cos \Theta_{q\bar{q}}). \tag{16}$$

which, for 2-jet events in the rest frame, gives $W_{q\bar{q}}(\pi) = 4C_F$.

Correspondingly, the ratio $R_\perp$ of the soft particle yield in 3-jet events to that of 2-jet events in their own rest frame is given by

$$R_\perp \equiv \frac{dN_{q\bar{q}g}^{q\bar{q}}}{dN_{q\bar{q}}^{q\bar{q}}} = \frac{NC}{4CF}[2 - \cos \Theta_{q\bar{q}} - \cos \Theta_{g\bar{q}} - \frac{1}{NC}(1 - \cos \Theta_{q\bar{q}})] \tag{17}$$

Theoretical predictions for $R_\perp$ are presented in Table 1 for three different angular configurations; notice that in the two extreme cases, the soft or collinear primary gluon emission and the parallel $q\bar{q}$ configuration, the expected limiting values are correctly recovered. The particularly simple and interesting situation of Mercedes-type events, where no jet identification is necessary for the above measurement, is also shown. The table also shows the results obtained in the large-$N_C$ approximation, in which the $q\bar{q}g$ event is treated as a superposition of two $q\bar{q}$ dipoles (see, e.g., 22). In this case the expression for the ratio $R_\perp$ simply becomes:

$$R_\perp = \frac{1}{2}[2 - \cos \Theta_{q\bar{q}} - \cos \Theta_{g\bar{q}}] \tag{18}$$

The difference among the full theory and the large-$N_C$ limit can also be investigated by studying the production rate in 3-jet events normalized to

| $\Theta_{gq} = \pi - \Theta_{g\bar{q}}$ (collinear or soft gluons) | $R_\perp$ | $R_\perp$ (large $N_C$) |
|---------------------------------------------------------------|---------|---------------------|
| $\Theta_{gq} = \Theta_{g\bar{q}} = \frac{2}{3}\pi$ (Mercedes) | 1.59    | 1.5                 |
| $\Theta_{gq} = \Theta_{g\bar{q}} = \pi$ (q\bar{q} antiparallel to $g$) | $\frac{NC}{C_F} = 2.25$ | 2                   |

Table 1: Prediction (17) for the ratio $R_\perp = dN_{q\bar{q}g}^{q\bar{q}}/dN_{q\bar{q}}^{q\bar{q}}$ and the large-$N_C$-limit (18) for different angular configurations of the $q\bar{q}g$ events ($\Theta_{q\bar{q}} = 2\pi - \Theta_{q\bar{q}} - \Theta_{g\bar{q}}$).
the sum of rates from the corresponding 2-jet events (dipoles) with opening angle \( \Theta_{qg} \) and \( \Theta_{g\bar{q}} \) respectively, rates which can be extracted experimentally from the analysis of \( q\bar{q}\gamma \) events.

4 Test of universality in different reactions

Let us look at the soft limit of the invariant density

\[
I_0 = \lim_{y \to 0, p_T \to 0} E \frac{dn}{d^3 p} = \frac{1}{2} \lim_{p \to 0} E \frac{dn}{d^3 p}
\]

(19)

in different reactions. This study provides a very direct test of universality in soft particle production; one could indeed directly measure whether the soft particle production is universal, i.e., it is a purely hadronic quantity not related to the underlying partonic processes, or the intensity \( I_0 \) depends on the colour topology of the primary active partons in the collisions process, as predicted in our perturbative approach. For instance, in case of quark exchange the two outgoing jets originate from colour triplet charges and \( I_0 \) should be as in \( e^+e^- \) annihilation; on the contrary, in case of gluon exchange \( I_0 \) should be about twice as large \((N_C/C_F)\). If data would show these features, a further direct support of the dual description of soft particle production in terms of the QCD bremsstrahlung would be achieved.

Unfortunately, an absolute prediction of \( I_0 \) is not possible, since it depends on the normalization factor and the cut-off parameter \( Q_0 \). One should then exploit the energy independence of the soft limit \( I_0 \) in \( e^+e^- \) annihilation and use this value to set a standard scale to be used for the comparison with other processes.

4.1 Quark exchange processes

In case the process goes through quark exchange, i.e., via colour triplet exchange, the soft production intensity \( I_0 \) should be the same as in \( e^+e^- \) annihilation.

A possible realization of quark exchange process is the process \( \gamma\gamma \to q\bar{q} \to 2\)-jets with either the virtuality \( Q^2 \) of the initial photon or the scattering angle photon-jet sufficiently large (see, e.g. \textsuperscript{29}). Another example is deep inelastic scattering at large \( Q^2 \). In this case the current fragments in the Breit frame are expected to have the same characteristics as the quark fragments in one hemisphere of \( e^+e^- \) annihilation (for a QCD analysis, see \textsuperscript{24}), and this is indeed observed for not too small \( Q^2 \). A third case is the production of two W’s which decay in the fully hadronic channel. In this case, since each of
the two W’s decays into 2 jets, the limiting soft particle yield should be twice the yield in $e^+e^- \rightarrow q\bar{q}$, provided the interconnection phenomena are neglected. Alternatively, deviations from the factor 2 could give an estimate of the importance of these effects.

4.2 Gluon exchange processes

In case the process goes through gluon exchange, i.e., via colour octet exchange, the soft production intensity $I_0$ should be roughly twice as large ($N_C/C_F$) as in $e^+e^-$ annihilation. The same factor appears in the theoretical predictions for the ratio of the average multiplicity in gluon vs. quark jets 

experimental analyses of this ratio have obtained values considerably below this limit (see e.g., 4). Since in our case we are not integrating over momentum but we are simply looking at the soft radiation, where some effects, like for instance energy-momentum constraints, are expected to be small, the effects of the different colour charges could be more pronounced than in the study of the multiplicity ratio.

The realization of a $gg$ colour singlet final state is not easy experimentally. It may become available at future colliders at higher energies through the process $\gamma\gamma \rightarrow gg$. An approximate realization of a colour octet antenna has been recently obtained in $e^+e^- \rightarrow q\bar{q}g$, by selecting a configuration with the gluon recoiling against a quasi–collinear $q\bar{q}$ pair (for a recent study of such events, see 30). A process mediated by colour octet exchange is expected to occur also in DIS for $Q^2 \gtrsim \text{few GeV}^2$ and small Bjorken $x$, via the diagrams of photon gluon fusion. Another example is hadron-hadron collision with a particle or jet at moderate $p_T (\gtrsim 1–2 \text{ GeV})$ at small angles so that the overall 2-jet structure is maintained.

4.3 Soft collisions (minimum bias events)

These processes (with initial hadrons or real photons) are not so well understood theoretically as the hard ones but it might be plausible to extrapolate the gluon exchange process towards small $p_T$. Accordingly, the soft radiation is expected to be twice as large as in $e^+e^-$ annihilation. For these reactions, some experimental information is already available. Data at ISR energies show that the soft limit $I_0$ is similar to $e^+e^-$ annihilation at a similar effective energy. On the other hand, $I_0$ roughly doubles when going from $\sqrt{s} \sim 20$ GeV to $\sqrt{s} = 900$ GeV at the collider. If additional incoherent sources can be excluded, such behaviour could indicate the growing importance of one-gluon exchange expected from the perturbative picture. Then a saturation at
\( I_0^{hh}/I_0^{ee} \sim 2 \) and no additional increase for a semihard \( p_T \) trigger would be expected. However, the rise of \( I_0^{hh} \) at collider energies could also result from the incoherent multiple collisions of partons (e.g. \( \gamma p \)) which has recently been postulated in \( \gamma p \) collisions. This unclear situation certainly deserves further studies.

4.4 Rapidity dependence of \( I_0 \) in DIS

According to the previous discussion, different kinematical conditions in \( \gamma p \) collisions select different elementary subprocesses and should then give rise to different yields \( I_0 \)'s. For example, by decreasing \( Q \) at fixed hadronic energy \( W \) one goes from quark exchange process to a gluon exchange process and finally to a soft process. We have seen that different reference frames, for instance the Breit frame, can be useful to select a particular situation. An alternative possibility is provided by the study of the soft particle yield \( I_0 \) at different values of rapidity in one reference frame, for instance the cms frame. Let us consider then the yield \( I_0(y) = \frac{dN}{dydp_T} \bigg|_{p_T \to 0} \) as a function of rapidity \( y \), measured in the cms frame. One would then expect the existence of a “quark plateau” of \( I_0(y) \) in the current region of length \( \Delta y \approx \log(Q/m) \) and height equal to \( e^+e^- \) annihilation near the rapidity corresponding to the Breit frame, i.e., in the direction of the incoming photon, \( y_{Breit} \); moving towards the central region, a transition to a “gluon plateau” in the complementary region is expected to occur. If sufficiently hard gluons are exchanged in the process, \( I_0 \) should then increase but still remain smaller than twice the value in \( e^+e^- \) annihilation. Approaching the fragmentation region, a transition to soft interactions should be observed and a decreasing of \( I_0 \) is likely to occur.

In general, the soft particle yield \( I_0 \) as a function of the rapidity should show a step like behaviour, pointing out the regions where the underlying process is dominated by quark or gluon exchange; a direct indicator of the underlying mechanism would be provided directly by the height of the plateau.

5 Conclusions

The analytical perturbative approach to multiparticle production, based on the Modified Leading Log Approximation and Local Parton Hadron Duality has been shown to successfully describe data on inclusive energy spectra of charged particles in QCD jets. In order to investigate the limitations of this picture and to point out non perturbative effects, this approach has been applied in the soft region.

The invariant densities \( Edn/d^3p \) of all charged particles and identified
particles in $e^+e^-$ annihilation are found to be approximately energy independent at low particle energy over a very large range of c.m.s. energies. The same behaviour is expected in the perturbative approach, as a consequence of the coherence of the soft gluon radiation from all emitters. With proper additional assumptions for the treatment of mass effects, the perturbative predictions reproduce the data also quantitatively, thus extending the phenomenological success of the perturbative approach down to the soft region. This result suggests that the production of hadrons in the soft region, which is known to proceed through many resonance channels, can be simply parametrized through a parton cascade pushed down to a small scale of a few 100 MeV.

The study of transverse radiation in multijet events in $e^+e^-$ annihilation has been proposed to test the sensitivity of the soft particle production to the effective colour charge of the primary emitters. Predictions of the perturbative approach for the soft particle production in different reactions have also been presented. The perturbative approach predicts a breakdown of universality among different reactions, since the soft particle yield reflects in this picture the properties of the underlying partonic process.

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