Analyzing Non-Extensivity of $\eta$-spectra in Relativistic Heavy Ion Collisions at $\sqrt{s_{NN}} = 200$ GeV

Bhaskar De\textsuperscript{1,*}, Goutam Sau\textsuperscript{2†}, S. K. Biswas\textsuperscript{3‡}, S. Bhattacharyya\textsuperscript{4§} & P. Guptaroy\textsuperscript{5¶}

1Department of Physics, Moulana Azad College
8, Rafi Ahmed Kidwai Road, Kolkata - 700013, India.
2 Beramara Ramchandrapur High School,
South 24-Pgs,743609(WB),India.
3 West Kodalia Adarsha Siksha Sadan, New Barrackpore,
Kolkata-700131, India.
4Physics and Applied Mathematics Unit(\textit{PAMU}),
Indian Statistical Institute, Kolkata - 700108, India.
5Department of Physics, Raghunathpur College
Raghunathpur-723133, Purulia, India.

Abstract

The transverse momentum spectra of secondary $\eta$ particles produced in $P+P$, $D+Au$ and $Au+Au$ interactions at $\sqrt{s_{NN}} = 200$ GeV at different centralities have been studied in the light of a non-extensive thermodynamical approach. The results and the possible thermodynamical insights, thus obtained, about the hadronizing process have also been discussed in detail.

Keywords : Relativistic Heavy Ion Collision, Inclusive Cross Section.

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1 Introduction

The multiparticle production in relativistic heavy ion collision provides important information about the hadronizing process in the aftermath of any high energy collision phenomenon. Various phenomenological/mathematical models have, so far, been utilised to analyse the available experimental data to understand the different characteristics and dynamical aspects of this process. Amongst the various existing models, applicability of Tsallis non-extensive thermodynamical approach\[1, 2, 3, 4, 5, 6, 7\], which has already left it’s marked impression in other branches of physics, has recently gained pace in understanding mainly the statistical as well as the thermodynamical characteristics of the production of various secondaries in high energy nuclear interactions at different energies\[8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28\]. We have taken up, here in this paper, the task of analysing the transverse momentum spectra of one of the heavier secondaries, $\eta$-mesons, produced in different nuclear reactions at RHIC-BNL in the light of Tsallis non-extensive thermodynamics. This is in continuation of our previous work\[29\], wherein transverse momentum spectra for the secondaries like $\pi^+$, $\pi^-$, $P$ and $\bar{P}$ were treated with the same approach. As in our previous work\[29\], here also we have confined ourselves only to a particular interaction energy($\sqrt{s_{NN}} = 200$ GeV), so that a comparative study can be made among various nucleus-nucleus systems. Our mention of the $P + P$-interactions in the same breath and bracket with the host of nuclear reactions is to be viewed somewhat loosely; this was done with our eyes cast on the same interaction energy alone. This is specially so when the ‘centrality’-aspects are touched upon and dealt with. In our actual work-procedure we have been attentive to the particularities of each interaction on a case-to-case basis. In the recent past, Biro et al\[19, 20\] analyzed successfully transverse momentum spectra of different identified hadrons, including $\eta$-mesons, produced at RHIC energies, on the basis of both the non-extensive statistical approach and the very essence of quark-coalescence model. However, the study was, probably by choice, confined to the minimum bias $Au + Au$ collisions in central rapidity region. Our present study, however, takes into account (i) the effect of entire longitudinal momentum($p_L$) range, (ii) the role of the same in different nuclear collisions($Au + Au$, $D + Au$ and $P + P$) and (iii) the impact of centrality dependent windows.

The $\eta$ mesons are the heavier species among the various mesonic secondaries produced in high energy nuclear reactions. The formation of such heavy partonic bound states and their interactions in nuclear matter under different conditions throws light on the nature of
the possible hadronic phase transition to a partonic state, with a greater degrees of freedom, formed immediate after the nuclear collisions at relativistic energies. The transverse momentum spectra of such secondaries at different centralities provides significant clues on the mechanism of the formation of such bound states, on their partonic constituents as well as on the thermal and dynamical characteristics of the regions in the partonic matter where they emerged from.

The work is organized as follows: Section 2 presents an outline of the hadronization process. In Section 3 we give some very brief sketch of the nonextensive statistics and the main working formula to be used in our work here. The results are reported in next section(Section 4) with some specific observations made. And the last section is preserved for our conclusions.

2 Non-extensivity and Hadronization Process: An Outline

The evolution of the partonic system created in RHIC-BNL experiments is generally believed by a large section of the theoretical physicists to be best described by hydrodynamics of an “almost ideal fluid”. This approach really gives a fair description of data on the particle transverse momentum spectra and asymmetry of the transverse flow. However, the prospect of this theory and methodology is, at the same time, riddled with and marred by two very serious problems: (i) the ‘HBT puzzle’ and (ii) the ‘puzzle of early thermalization’[30, 31] which we are not going to elaborate here in detail. Besides this, the proposed microscopic mechanisms lack in the detailed knowledge of fragmentation and recombination etc. These deficiencies have prompted the physicists to turn to many different models, often from various different angles, of which the non-extensive thermodynamic approach is an effective one which is, thus, essentially a thermal model. This model assumes the formation of a system which is thermally and chemically in a near-equilibrium state in the hadronic phase and is characterised, in the main, by two thermodynamic variables(observables), for the hadronic phase. These two are ‘temperature’ and ‘non-extensive entropy’ of the system attained finally. The deconfined period of the time evolution dominated by the constituent partons(quarks and gluons) remains hidden: full equilibration generally washes out and destroys large amount of information about the early deconfined phase. Successes of thermal statistical models are shaped and conditioned by the blackening out of information on some intermediate states. And this is an accepted reality.
In the case of full thermal and chemical equilibrium relativistic statistical distributions are generally based on exponential nature of the spectra for the $p_T$-distribution of the produced hadronic secondaries. But, observationally experimental data at SPS and RHIC energies reveal strongly the traits of non-exponential and power-law nature at high-$p_T$ which is also supported by the dictates of the perturbative QCD\cite{32, 33, 34}. So, for the overlap or the transitional regions of the $p_T$-values (low-to-high), a stationary distribution of strongly interacting hadron gas or quark-matter in a finite volume could be considered to have such a distribution with dual nature. In fact, Tsallis distributions\cite{1}, which we have resorted to, satisfy such a criterion\cite{36}.

3 Nonextensive Statistics and Transverse Momentum Spectra

The nonextensive entropy by Tsallis generalized statistics is given by\cite{1},

$$S_q = \frac{1}{q-1} \left( 1 - \sum_i p_i^q \right)$$

where $p_i$ are probabilities associated with the microstates of a physical system with normalization $\sum_i p_i = 1$ and $q$ is the nonextensivity parameter. For $q \to 1$, eqn.(1) gives the ordinary Boltzmann-Gibbs entropy

$$S = -\sum_i p_i \ln p_i$$

The generalized statistics of Tsallis is not only applicable to an equilibrium system, but also to nonequilibrium systems with stationary states\cite{9}. As the name ‘nonextensive’ implies, these entropies are not additive for independent systems. For a system of $N$ independent particles, where particle 1 is in energy state $\epsilon_{i_1}$, particle 2 in energy state $\epsilon_{i_2}$, and so on, the Hamiltonian of the system in nonextensive approach is given by\cite{9},

\footnote{However, the coinage ‘Tsallis distribution’ is not fully justified, because Vilfredo Pareto\cite{35} used much earlier and long ago a power-law probability distribution found in a large number of real-world situations, especially in the field of Economics. And beyond the borders of economics, it is generally referred to as the Bradford distribution, though the high energy physicists, since 1970s, prefer to term this as a “cut-power-law distributions”. In the context of non-extensive thermodynamics alone, this is normally familiar as Tsallis distribution.}
\[ H(i_1, i_2, ..., i_N) = \sum_j \epsilon_{ij} + (q - 1) \beta \sum_{j,k} \epsilon_{ij} \epsilon_{ik} + (q - 1)^2 \beta^2 \sum_{j,k,l} \epsilon_{ij} \epsilon_{ik} \epsilon_{il} + \ldots \] (3)

where \( \beta = 1/T \) is the inverse temperature variable. The above equation clearly indicates that for a non extensive system the total energy is not the sum of the single-particle energies.

The energy associated with a particle, which is a hadron in the present context, denoted by \( j \) in a momentum state \( i \) in a fireball produced in a high energy nuclear collision is given by,

\[ \epsilon_{ij} = \sqrt{p_i^2 + m_j^2} \] (4)

The nonextensive Boltzmann factor is defined as\[9\]

\[ x_{ij} = (1 + (q - 1) \beta \epsilon_{ij})^{-q/(q-1)} \] (5)

with \( q \to 1 \), the above equation approaches the ordinary Boltzmann factor \( e^{-\beta \epsilon_{ij}} \). If \( \nu_{ij} \) denotes the number of particles of type \( j \) in momentum state \( i \), the generalized grand canonical partition function is given by,

\[ Z = \sum_{(\nu)} \prod_{ij} x_{ij}^{\nu_{ij}} \] (6)

The average occupation number of a particle of species \( j \) in the momentum state \( i \) can be written as\[9\]

\[ \bar{\nu}_{ij} = x_{ij} \frac{\partial}{\partial x_{ij}} \log Z = \frac{1}{(1 + (q - 1) \beta \epsilon_{ij})^{q/(q-1)}} \pm 1 \] (7)

where \(-\) sign is for bosons and the \(+\) sign is for fermions.

The probability of observation of a particle of mass \( m_0 \) in a certain momentum state can be obtained by multiplying the average occupation number with the available volume in momentum space\[9\]. The infinitesimal volume in momentum space is given by

\[ dp_x \, dp_y \, dp_z = dp_L \, p_T \, \sin \theta \, dp_T \, d\theta \] (8)

where \( p_T = \sqrt{p_x^2 + p_y^2} \) is the transverse momentum and \( p_z = p_L \) is the longitudinal one. Hence, the probability density \( w(p_T) \) of transverse momenta is obtained by integrating over all \( \theta \) and \( p_L \):
\[ w(p_T) = \text{const.} \, p_T \int_{-\infty}^{+\infty} dp_L \frac{1}{(1 + (q - 1)\beta \sqrt{p_T^2 + p_L^2 + m_0^2})^{q/(q-1)} \pm 1} \] (9)

Since, the integrand in the right hand side of the above equation is an even function of \( p_L \), one can easily write,

\[ w(p_T) = \text{const.} \, p_T \int_{0}^{+\infty} dp_L \frac{1}{(1 + (q - 1)\beta \sqrt{p_T^2 + p_L^2 + m_0^2})^{q/(q-1)} \pm 1} \] (10)

Since the temperature \( T \), which will be called here as Hagedorn temperature and will be denoted as \( T_{\text{eff}} \), is quite small, i.e., \( \beta \sqrt{p_T^2 + p_L^2 + m_0^2} \gg 1 \), one can neglect \( \pm 1 \) in the denominator of the previous equation. Hence, the differential cross section, which is proportional to \( w(p_T) \), can be expressed as,

\[ \frac{1}{\sigma} \frac{d\sigma}{dp_T} \propto \int_{0}^{\infty} dp_L (1 + \frac{(q - 1)}{T_{\text{eff}}} \sqrt{m_0^2 + p_T^2 + p_L^2})^{-q/(q-1)} \] (11)

or, in terms of the transverse mass \( m_T = m_0^2 + p_T^2 \) of the detected secondary,

\[ \frac{1}{\sigma} \frac{d\sigma}{dm_T} \propto \int_{0}^{\infty} dp_L (1 + \frac{(q - 1)}{T_{\text{eff}}} \sqrt{m_T^2 + p_L^2})^{-q/(q-1)} \] (12)

If we put \( x = \frac{p_T}{T_{\text{eff}}} \) and \( u = \frac{m_T}{T_{\text{eff}}} \), the above equation can be written as,

\[ \frac{1}{\sigma} \frac{d\sigma}{dm_T} \propto u \int_{0}^{\infty} dx (1 + (q - 1)u \sqrt{1 + \frac{x^2}{u^2}})^{-q/(q-1)} \] (13)

For large \( x \) the integrand is small and it’s contribution to the integration can be ignored. Hence, for large \( u \) and small \( x \) we can assume, \( \sqrt{1 + \frac{x^2}{u^2}} \approx 1 + \frac{x^2}{2u^2} \). The inclusion of this assumption in the above equation yields

\[ \frac{1}{\sigma} \frac{d\sigma}{dm_T} \propto u(1 + (q - 1)u)^{-q/(q-1)} \int_{0}^{\infty} dx (1 + \frac{(q - 1)x^2}{2u(1 + (q - 1)u)})^{-q/(q-1)} \] (14)

This form of equation is in accordance with the right hand side of eqn(28) given in [8]. So, one can write, following the eqn(32) of the same reference,

\[ \frac{1}{\sigma} \frac{d\sigma}{dm_T} \propto u^{3/2}(1 + (q - 1)u)^{-\frac{q}{q-1} + \frac{1}{2}} \] (15)

or,

\[ \frac{1}{\sigma} \frac{d\sigma}{dm_T} = c_1 \, u^{3/2}(1 + (q - 1)u)^{-\frac{q}{q-1} + \frac{1}{2}} \] (16)
where \( c_1 \) is a normalization constant.

The average multiplicity of the detected secondary in the given rapidity region can be obtained by the relationship

\[
<N> = \frac{1}{\sigma} \int_{m_0}^{\infty} \frac{d\sigma}{d m_T} \, d m_T = c_1 \int_{m_0}^{\infty} u^{3/2} (1 + (q - 1)u)^{-\frac{q}{q-1} + \frac{1}{2}} \, d m_T
\]  

(17)

Hence, the constant \( c_1 \) can be expressed in terms of \( <N> \) by the relationship

\[
c_1 = \frac{<N>}{T_{\text{eff}} \int_{u_0}^{\infty} u^{3/2} (1 + (q - 1)u)^{-\frac{q}{q-1} + \frac{1}{2}} \, du}
\]  

(18)

where \( u_0 = \frac{m_0}{T_{\text{eff}}} \).

Combining eqn(14) and eqn(16) one can write

\[
\frac{1}{\sigma} \frac{d\sigma}{d m_T} = \frac{<N>}{T_{\text{eff}} \int_{u_0}^{\infty} u^{3/2} (1 + (q - 1)u)^{-\frac{q}{q-1} + \frac{1}{2}} \, du} \cdot u^{3/2} (1 + (q - 1)u)^{-\frac{q}{q-1} + \frac{1}{2}} \, du
\]  

(19)

Since,

\[
\frac{1}{\sigma} \frac{d\sigma}{m_T} \frac{dm_T}{d m_T} = \frac{1}{\sigma} \frac{d\sigma}{p_T} \frac{dp_T}{p_T} = \frac{dN}{p_T} \frac{dp_T}{p_T}
\]  

(20)

we can write

\[
\frac{dN}{p_T} \frac{dp_T}{p_T} = \frac{<N>}{T_{\text{eff}}^2 \int_{u_0}^{\infty} u^{3/2} (1 + (q - 1)u)^{-\frac{q}{q-1} + \frac{1}{2}} \, du} \cdot u^{1/2} (1 + (q - 1)u)^{-\frac{q}{q-1} + \frac{1}{2}} \, du
\]  

(21)

Equation(21) provides the working formula for the present analysis.

4 Results

The working formula was applied, in its present form, to obtain a fit to the data on \( \eta \) production in \( P + P \) collision[Fig.1(a)] at \( \sqrt{s_{NN}} = 200 \) GeV and the corresponding parameter-values are given in Table-1, where \( n_o \) denotes the average multiplicity of \( \eta \) produced in \( P + P \) interaction.

However, when the same formula was employed to analyse data from nucleus-nucleus collisions like \( D + Au \) and \( Au + Au \) collisions, the parameters, which had been set free for \( P + P \) collisions, were constrained by the following relationships[24]:

\[
T_{\text{eff}} = T_0 (1 - c(q - 1))
\]  

(22)
\[
\frac{\langle N \rangle}{<N>} - n_0 N_{\text{part}} = c(q - 1)
\]  

with \(c = -\frac{\phi}{Dc_p \rho T_0}\) where \(D\), \(c_p\), \(\rho\), \(T_0\) are respectively the strength of the temperature fluctuations, the specific heat under constant pressure, density, the temperature of the hadronizing system when it is in thermal equilibrium \((q = 1)\) and \(N_{\text{part}}\) is the number of participant nucleons. Eqn.(22) describes the fluctuation in temperature where it is assumed that the effective temperature \(T_{\text{eff}}\) is the outcome of two simultaneous processes: (i) the fluctuation of the temperature around \(T_0\) due to a stochastic process in any selected region of the system and (ii) some energy transfer between the selected region and the rest of the system, denoted by \(\phi\)\[24\].

It is absolutely uncertain whether such energy-transfers could/should be invariably linked up with flow-velocity (normally denoted in the hydrodynamical model-texts as ‘\(u\)’). So, for the sake of calculational simplicity and correctness we assume the factor \(\phi\), for the present, to be independent of any flow-velocity. The fluctuation in multiplicity is described by eqn.(23). The assumption behind this relationship is that if \(N\)-particles are distributed in energy according to Tsallis non-extensive distribution, then their multiplicity will obey Negative-Binomial distribution\[24\].

The fits for \(D + Au\) and \(Au + Au\) collisions obtained on the basis of equation(21) alongwith the constraints given in eqn(22)-(23) are depicted in Fig.1(b)-Fig.1(d). The values of various parameters obtained from the fits are given in Table-2. The values of \(T_{\text{eff}}\) and \(q\) calculated from the fitted parameters are given in tabular form in Table-3 and in graphical format in Fig.2 as a function of participant nucleons. Fig.3 depicts graphically the behaviour of average multiplicity of \(\eta\)-mesons produced per pair of participant nucleons as a function of \(N_{\text{part}}\). The normalized values of \(\langle N \rangle\) for different nucleus-nucleus collisions at \(\sqrt{s_{NN}} = 200\) GeV remain almost constant with respect to \(N_{\text{part}}\) which indicates the linear dependence of \(\langle N \rangle\) on \(N_{\text{part}}\), and hence on the system size.

The obtained values of \(q\) and \(T_{\text{eff}}\), on the average, show centrality-dependences for almost all the collisions. The values of \(T_0\) and \(c\) obtained from different fits show almost constant behaviour according to expectation and the average values of them are found to be 155 MeV and 1.88 respectively. The performance of the present non-extensive approach vis-a-vis the studied experimental data could be rated to be moderately satisfactory, as is seen from the \(\chi^2/ndf\)-values given in the last column of Table-1.
5 Discussion and Conclusions

The centrality dependences of the parameters $q$ and $T_{\text{eff}}$ are quite clear from the Fig. 2. The systematic trend of the non-extensive parameter $q$, in general, is that it decreases with increase in the number of participant nucleons, i.e. with the increase in centrality. On the other hand, the Hagedorn temperature $T_{\text{eff}}$ exhibits completely a different trend, i.e., it increases with the increase in centrality. The obtained values of $q$ and $T_{\text{eff}}$ for $D + Au$, and $Au + Au$ collisions are in agreement with these tendencies.

The contrasting behaviour of $q$ and $T_{\text{eff}}$ with respect to $N_{\text{part}}$ is in accord with the spirit and content of the non-extensive statistics. $q$ is related with the fluctuations in temperature\cite{11, 21}. The high value of $q$ means a high fluctuation in temperature and the system is not in the neighbourhood of its thermal equilibrium. The number of binary collisions in a system with lesser number of participant nucleons is quite low. Hence, the probability of mutual exchange of transverse momenta among the interacting partons in a system involving small $N_{\text{part}}$ is also quite low, which, in turn, keeps the system far away from its thermal equilibrium. On the other hand, an appreciable increment in the number of binary collisions is observed in a system possessing a large number of participant nucleons; and consequently the system can reach quickly to its thermal equilibrium or in the close vicinity of it. So it is quite natural that the fits, obtained with the non-extensive approach, will exhibit a gradual decrement in the values of $q$ and increment in $T_{\text{eff}}$, as one goes from central to peripheral region or from a system with larger $N_{\text{part}}$ to that with smaller ones.

$q$ and $T_{\text{eff}}$ change rapidly when $N_{\text{part}}$ lies in the range $2 \leq N_{\text{part}} \leq 20$. The change is rather slow for high $N_{\text{part}}$. The dependence of $q$ and $T_{\text{eff}}$ on $N_{\text{part}}$ was observed quite strong over the entire $N_{\text{part}}$-region while studying charged pions and proton-antiprotons\cite{29} produced in nuclear collisions at $\sqrt{s_{NN}} = 200$ GeV. This fact is also reflected from the values of $T_0$ and $c$. The values of $T_0$ for charged pions and proton-antiprotons, obtained by Wilk et al\cite{23} on the basis of our previous venture\cite{29}, are 220 MeV and 360 MeV respectively while that of $c$ are 5.7 and 9.4 respectively. But, the values of $T_0$ and $c$, found from the present analysis($T_0 = 155$ MeV and $c = 1.88$), are much closer to the values found from the analysis of pion production in $e^+e^-$ collisions\cite{23}($T_0 = 131 MeV$ and $c = 1.83$); and $T_0 \sim m_\pi$ can be recognized as the critical temperature of phase transition of the hadronizing system in thermal equilibrium\cite{37}. The reason of this discrepancy in the outcome of the two studies could be attributed to the non-inclusion of the constraints, given by eqn(22)-(23), in our previous
analysis involving charged pions and proton-antiprotons. Thus, it is necessary to re-analyse the data on pion, proton and antiprotons produced in RHIC energies in the light of the present model including the constraints applied here, so that a conclusion could be reached. Lastly, but more importantly, there is yet another very interesting and important observation. The Fig.(2a) demonstrates quite convincingly that the parameter $q$ does not seem to tend to unity for even higher participant numbers, so non-extensivity may be present even in the thermodynamical sense and not just as a simulated microcanonical effect.

We are quite aware that so many other topical issues of interest around which we choose not to deal with in the present work as they demand a separate treatment. The properties of energy dependence of the multiplicity-density, inclusive cross-section and of the average transverse momentum behaviour are just a few among the many. They are both important and interesting as there are many reported anomalies and puzzles\cite{38, 39, 40} which would be the subject of our future studies. In fact, we commented in just the preceding paragraph that we would rework in the near future on the pion, kaon and proton-antiproton spectra with the specific concerns of constraining the parameters used by the method of calculation in the non-extensive approach and try to throw light on the problems and puzzles associated with the production characteristics of the secondaries in the high energy collisions under studies.

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Table 1: Values of fitted parameters with respect to experimental data on $\eta$-spectra produced in $P + P$ collision at $\sqrt{s_{NN}} = 200$GeV

| $N_{part}$ | $n_0$     | $q$     | $T_{eff}$(GeV) | $\chi^2/ndf$ |
|------------|-----------|---------|----------------|--------------|
| 2          | 0.075 ± 0.003 | 1.115 ± 0.005 | 0.097 ± 0.006 | 0.691/15     |

Table 2: Values of fitted parameters with respect to experimental data on $\eta$-spectra at different centralities of $Au + Au$ and $D + Au$ collisions at RHIC

| Centrality | $N_{part}$ | $N$   | $c$   | $T_0$(GeV) | $\chi^2/ndf$ |
|------------|------------|-------|-------|------------|--------------|
| $Au + Au$  |            |       |       |            |              |
| 0-20       | 279.9      | 24.8 ± 0.1 | 1.81 ± 0.02 | 0.165 ± 0.003 | 0.949/7   |
| 20-60      | 100.2      | 9.00 ± 0.03 | 1.90 ± 0.03 | 0.160 ± 0.004 | 0.268/7   |
| 60-92      | 14.5       | 1.31 ± 0.01 | 1.82 ± 0.04 | 0.150 ± 0.005 | 0.313/3   |
| Min. Bias  | 109.1      | 9.70 ± 0.03 | 1.83 ± 0.06 | 0.162 ± 0.004 | 0.563/7   |
| $D + Au$   |            |       |       |            |              |
| 0-20       | 15.6       | 1.43 ± 0.01 | 1.90 ± 0.02 | 0.160 ± 0.004 | 0.580/10   |
| 20-40      | 11.1       | 1.02 ± 0.01 | 1.92 ± 0.03 | 0.159 ± 0.005 | 0.553/10   |
| 40-60      | 7.7        | 71 ± 0.01  | 1.88 ± 0.03 | 0.150 ± 0.004 | 0.237/10   |
| 60-88      | 4.2        | 0.39 ± 0.02 | 1.91 ± 0.02 | 0.140 ± 0.006 | 1.657/9    |
| Min. Bias  | 9.1        | 0.84 ± 0.04 | 1.91 ± 0.04 | 0.151 ± 0.001 | 0.560/14   |

Table 3: Calculated values of $q$ and $T_{eff}$ for different participant nucleons($N_{part}$)

| $N_{part}$ | $q$    | $T_{eff}$(GeV) |
|-----------|--------|----------------|
| 2         | 1.115  | 0.097          |
| 4.2       | 1.101  | 0.115          |
| 7.7       | 1.096  | 0.123          |
| 9.1       | 1.100  | 0.121          |
| 11.1      | 1.097  | 0.130          |
| 14.5      | 1.093  | 0.125          |
| 15.6      | 1.096  | 0.131          |
| 100.2     | 1.087  | 0.134          |
| 109.1     | 1.086  | 0.137          |
| 279.9     | 1.085  | 0.140          |
Figure 1: Plots of transverse momentum spectra of $\eta$-mesons produced in $P + P$, $D + Au$ and $Au + Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV at different centralities. The filled symbols represent the experimental data points[41]. The solid curves provide the fits on the basis of nonextensive approach(eqn.(21)).
Figure 2: Plots of the nonextensive parameter $q$ and the effective temperature $T_{\text{eff}}$ as a function of number of participant nucleons in $Au + Au$, $D + Au$ and $P + P$ collisions at $\sqrt{s_{NN}} = 200$ GeV for production of secondary $\eta$-mesons.

Figure 3: Plots of the multiplicity of $\eta$-mesons produced per pair of participant nucleons as a function of number of participant nucleons in $Au + Au$, $D + Au$ and $P + P$ collisions at $\sqrt{s_{NN}} = 200$ GeV.