Single production of the doubly charged $Higgs$ boson via $e\gamma$ collision in the $Higgs$ triplet model

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Abstract

The $Higgs$ triplet model ($HTM$) predicts the existence of a pair of doubly charged $Higgs$ bosons $H^{\pm\pm}$. Single production of $H^{\pm\pm}$ via $e\gamma$ collision at the next generation $e^+e^-$ International Linear Collider ($ILC$) and the Large Hadron electron Collider ($LHeC$) is considered. The numerical results show that the production cross sections are very sensitive to the neutrino oscillation parameters. Their values for the inverted hierarchy mass spectrum are larger than those for the normal hierarchy mass spectrum at these two kinds of collider experiments. With reasonable values of the relevant free parameters, the possible signals of the doubly charged $Higgs$ bosons predicted by the $HTM$ might be detected in future $ILC$ experiments.

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1. Introduction

During the past decade, neutrino oscillation experiments have provided us with very convincing evidence that neutrinos are massive particles mixing with each other [1]. This exciting breakthrough gives a strong motivation for physics beyond the standard model (SM), since the SM itself only contains three massless neutrinos [2]. Various ways to go beyond the SM have been proposed in order to accommodate this observation. The Higgs triplet model (HTM) [3, 4] is a simple extension of the SM with an SU(2)$_L$-triplet Higgs boson of hypercharge $Y = 2$ whose vacuum expectation value (VEV) $\nu_\Delta$ provides Majorana neutrino masses without introducing right-handed neutrinos.

The HTM [3, 4] generates a Majorana neutrino mass via the product of a triplet Yukawa coupling $h_{ij}$ and a triplet VEV $\nu_\Delta$. This model predicts the existence of seven physical Higgs bosons which are two CP-even neutral bosons $h^0$ and $H^0$, a CP-odd neutral one $A^0$, a pair of single charged bosons $H^\pm$, and a pair of doubly charged Higgs bosons $H^{\pm\pm}$. These new particles can produce rich phenomenology at present and in future high energy collider experiments. Since charge conservation prevents doubly charged scalars from decaying to a pair of quarks, $H^{\pm\pm}$ carry the lepton number violation and would give rise to a distinctive experimental signal, which has lower background. This fact has lead to many studies involving the doubly charged Higgs bosons $H^{\pm\pm}$ in the literature. For example, it has been shown that such particles can be produced with sizable rates at hadron colliders via the processes $q\bar{q} \rightarrow H^{++}H^{--}$ [5, 6] and $q\bar{q}' \rightarrow H^{\pm\pm}H^{\mp}$ [5, 7, 8]. Using these production mechanisms, discovery of $H^{\pm\pm}$ at the Large Hadron Collider (LHC) has been studied in Refs. [9, 10].

There is a direct connection between the Yukawa coupling $h_{ij}$ and the neutrino mass matrix which gives rise to phenomenological prediction for processes which depend on $h_{ij}$ [11]. The branching ratios of the lepton flavor violating (LFV) decay processes $l_i \rightarrow l_jl_ll_k$ and $l_i \rightarrow l_j\gamma$ contributed by the doubly charged Higgs bosons $H^{\pm\pm}$ are strong depend on $h_{ij}$ in the context of the HTM. Recently, it has been shown [12] that the current upper experimental limits for some of these LFV decay processes can give serve constraints on the parameter space of $h_{ij}$ in the HTM. Taking into account these constraints, in
this paper, we consider single production of $H^\pm\pm$ via $e\gamma$ collision at the next generation $e^+e^-$ International Linear Collider (ILC) [13, 14] and the Large Hadron electron Collider (LHeC) [15]. Although single production of the doubly charged Higgs bosons via $e\gamma$ collision has been studied in Refs. [16], in this paper, we will focus our attention on the effects of the neutrino oscillation parameters on its production cross section and see whether the neutrino mass hierarchy can be disentangled via this kind of production processes at the ILC and LHeC. Our numerical results show that the production cross sections at these two kinds of high energy colliders are very sensitive to the neutrino oscillation parameters. The production cross section of the process $e\gamma \rightarrow e^+H^--$ for the inverted hierarchy mass spectrum is significantly larger than that for the normal hierarchy mass spectrum. In the sizable range of the parameter space of the HTM, their values can be large enough to be detected in the future ILC experiments.

Our paper is organized as follows. The essential features of the HTM are briefly reviewed in section 2. In this section, we also summarized the constraints on the HTM from some LFV decay processes. The effective production cross sections for the subprocess $e\gamma \rightarrow l^+H^--$ at the ILC and LHeC are calculated in sections 3 and 4. Our conclusions and discussions are given in section 5.

2. The essential features of the HTM

The Higgs triplet model (HTM) [3, 4] is one of the appealing scenarios for generating neutrino masses without the introduction of a right-handed neutrino. In this model, a complex $SU(2)_L$ triplet scalar with the hypercharge $Y = 2$ is introduced to the SM, which can be parameterized by

$$\triangle = \begin{pmatrix} \triangle^+/\sqrt{2} & \triangle^{++} \\ \triangle^0 & -\triangle^+ / \sqrt{2} \end{pmatrix}. \quad (1)$$

A nonzero VEV value $<\triangle^0>$ gives rise to the following mass matrix for neutrinos

$$m_{ij} = 2h_{ij} <\triangle^0> = \sqrt{2}h_{ij}\nu_\triangle. \quad (2)$$

The symmetric complex matrix $h_{ij}$ ($i, j = e, \mu, \tau$) is the Yukawa coupling strength.
interaction of the triplet scalar with lepton doublet \( L_i \equiv (\nu_{iL}, l_{iL})^T \) is given by

\[
\mathcal{L} = -h_{ij}[v_{iL}^T C v_{jL} \Delta^0 - \frac{1}{\sqrt{2}}(v_{iL}^T C l_{jL} + l_{iL}^T C v_{jL}) \Delta^+ - l_{iL}^T C l_{jL} \Delta^{++}] + h.c.,
\]

(3)

here \( C \) is the charge conjugation operator.

The neutrino mass matrix \( m_{ij} \) can be diagonalized by the Maki-Nakagawa-Sakata (MNS) matrix \( V_{MNS} \) [17]. Then the couplings \( h_{ij} \) can be written as following

\[
h_{ij} = \frac{m_{ij}}{\sqrt{2}\nu_{\Delta}} \equiv \frac{1}{\sqrt{2}2\nu_{\Delta}}[V_{MNS}^{diag}(m_1, m_2 e^{i\varphi_1}, m_3 e^{i\varphi_2})V_{MNS}^T]_{ij}
\]

(4)

with

\[
V_{MNS} = \begin{pmatrix}
    c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\
    -c_{12}s_{13}s_{23}e^{i\delta} - c_{23}s_{12} & c_{12}c_{23} - e^{i\varphi_1}s_{13}s_{23} & c_{13}s_{23} \\
    s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -c_{23}s_{12}s_{13}e^{i\delta} - c_{12}s_{23} & c_{13}c_{23}
\end{pmatrix}
\times diag(e^{i\varphi_1/2}, 1, e^{i\varphi_2/2}).
\]

(5)

Where \( m_1, m_2, m_3 \) are the absolute masses of the three neutrinos. The phase \( \delta \) is the Dirac CP-violating phase, \( \varphi_1 \) and \( \varphi_2 \) are referred to as the Majorana phases [4, 18] and there are \( 0 \leq \varphi_1, \varphi_2 < 2\pi \). \( s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij} \) with \( 0 \leq \theta_{ij} \leq \pi/2 \), in which \( \theta_{ij} \) are the mixing angles. The explicit expression forms for \( h_{ij} \) have been given in Refs. [8, 9].

After the electroweak symmetry breaking, there are seven physical massive \textit{Higgs} bosons \((H^{\pm \pm}, H^{\pm}, h^0, H^0, A^0)\) left in the spectrum for the \textit{HTM}. The doubly charged \textit{Higgs} bosons \( H^{\pm \pm} \) are entirely composed of the triplet scalars \( \Delta^{\pm \pm} \), while the remaining eigen states are mixtures of the doublet and triplet scalars. Such mixing is proportional to the triplet \( VEV \) \( \nu_{\Delta} \), which is generally small even if \( \nu_{\Delta} \) assumes its largest value of a few GeV. Thus, \( H^{\pm}, H^0, A^0 \) are predominantly composed of the triplet scalars, while the \textit{SM-like Higgs} boson \( h^0 \) is mainly composed of the doublet scalars.

It is well known that the doubly charged \textit{Higgs} bosons \( H^{\pm \pm} \) have contributions to the \textit{LFV} decays \( l_i \to l_j \gamma, l_j \to l_k l_l \) [19]. In the \textit{HTM}, their branching ratios depend on the neutrino mass matrix parameters, the \( VEV \) value \( \nu_{\Delta} \), and the mass \( m_{H^{\pm \pm}} \) of the doubly charged Higgs bosons [12]. The current experimental upper limits for some of these \textit{LFV}

\[ \begin{pmatrix}
    c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\
    -c_{12}s_{13}s_{23}e^{i\delta} - c_{23}s_{12} & c_{12}c_{23} - e^{i\varphi_1}s_{13}s_{23} & c_{13}s_{23} \\
    s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -c_{23}s_{12}s_{13}e^{i\delta} - c_{12}s_{23} & c_{13}c_{23}
\end{pmatrix}
\times diag(e^{i\varphi_1/2}, 1, e^{i\varphi_2/2}).
\]
decay processes can give severe constraints on the relevant free parameters. In the HTM, the LFV decay process $\mu \to e\gamma$ proceed via exchange of $H^{\pm\pm}$ and $H^\pm$ at one-loop. In the case of ignoring the electron mass in the final state and all lepton masses in the loop, the branching ratio $Br(\mu \to e\gamma)$ can be written as [12]

$$Br(\mu \to e\gamma) \approx \frac{27\alpha |(h^+ h)_{e\mu}|^2}{64\pi G_F^2 m_H^4},$$

where $\alpha = 1/137$ is the fine structure constant, $G_F = 1.17 \times 10^{-5} GeV^{-2}$ is the Fermi constant. In above equation, it has been assumed $m_{H^{\pm\pm}} = m_{H^\pm} = m_H$. The current experimental upper limit for the LFV process $\mu \to e\gamma$ is $Br^{exp}(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ [20], then we have

$$| (h^+ h)_{e\mu} |^2 \leq 2.6 \times 10^{-9} \left( \frac{m_H}{200 GeV} \right)^4.$$ 

Using Eq.(4), there is

$$2\nu_\Delta^2 (hh^+)_{ij} = m_1^2 \delta_{ij} + [V_{MNS}diag(0, \Delta m^2_{21}, \Delta m^2_{31})V_{MNS}^+]_{ij},$$

where $\Delta m^2_{21} = m_2^2 - m_1^2$ and $\Delta m^2_{31} = m_3^2 - m_1^2$. Since the sign of $\Delta m^2_{31}$ is undetermined at present, distinct patterns for the neutrino mass hierarchy are possible. $\Delta m^2_{31} > 0$ is referred to as normal hierarchy (NH) with $m_1 < m_2 < m_3$ and $\Delta m^2_{31} < 0$ is defined as inverted hierarchy (IH) with $m_3 < m_1 < m_2$. Combing Eq.(7) and Eq.(8), there is the following relation

$$2\nu_\Delta^2 \geq 1.96 \times 10^4 \left( \frac{200 GeV}{m_H} \right)^2 [V_{MNS}diag(0, \Delta m^2_{21}, \Delta m^2_{31})V_{MNS}^+]_{e\mu}.$$ 

In our following numerical estimation, we will consider this constraint on the triplet $VEV \nu_\Delta$ to calculate the cross sections for single production of the doubly charged Higgs boson $H^{-\pm}$ via $e\gamma$ collision at the ILC and LHCC.

3. Single production of the doubly charged Higgs boson $H^{-\pm}$ at the ILC

The LHC can generate very massive new particles and will essentially enlarge the possibilities of testing for new physics effects, while the ILC is also need to complement
the probe of the new particles with detailed measurement. For the ILC, the center of mass (c.m.) energy $\sqrt{s} = 0.5 - 1 TeV$ and the typical integrated luminosity $L_{int} = 0.5 - 1 ab^{-1}$ is currently being designed [13, 14]. An unique feature of the ILC is that it can be transformed to $\gamma\gamma$ or $e\gamma$ collision with the photon beam generated by laser-scattering method. The effective luminosity and energy of the $\gamma\gamma$ and $e\gamma$ collisions are expected to be comparable to those of the ILC. In some scenarios, they are the best instrument for discovery of the new physics signatures. In this section, we will consider single production of the doubly charged Higgs boson $H^{--}$ predicted by the HTM via $e\gamma$ collision.

\[ (a) \quad \gamma \gamma e^- + e^- \rightarrow l^+ + \gamma \rightarrow l^+ + H^{--} \]

\[ (b) \quad \gamma \gamma e^- + e^- \rightarrow l^+ + H^{--} \]

\[ (c) \quad \gamma \gamma e^- + e^- \rightarrow l^+ + H^{--} \]

Figure 1: The Feynman diagrams for the partonic process $e^-(p_1) + \gamma(p_2) \rightarrow l^+(p_3) + H^{--}(p_4)$ in the HTM.

From discussions given in section 2, we can see that the doubly charged Higgs boson $H^{--}$ can be produced via $e\gamma$ collision associated with a positive lepton. The relevant Feynman diagrams are shown in Fig.1, in which $l^+$ denotes the lepton $e^+, \mu^+$ or $\tau^+$. For the process $e^-(p_1) + \gamma(p_2) \rightarrow l^+(p_3) + H^{--}(p_4)$, the renormalization amplitude can be written as

\[
M = M_a + M_b + M_c = \frac{2eh_{el}}{(p_1 + p_2)^2} u^T(p_3)C^{-1}P_L(g_1 + p_2)\gamma^\mu u(p_1)\varepsilon_\mu(p_2)
- \frac{4eh_{el}}{(p_2 - p_4)^2 - m_H^2} u^T(p_3)C^{-1}P_L u(p_1)(2p_4 - p_2)^\mu \varepsilon_\mu(p_2)
- \frac{2eh_{el}}{(p_1 - p_4)^2} u^T(p_3)\gamma^\mu (g_1 - p_4)C^{-1}P_L u(p_1)\varepsilon_\mu(p_2).
\]  

(10)

Here $P_L = (1 - \gamma_5)/2$ is the right-handed projection operator and $\varepsilon_\mu(p_2)$ is the polarization vector of the photon $\gamma$. The explicit expressions for the Yukawa coupling constants $h_{ee}$, $h_{e\mu}$, and $h_{e\tau}$ can be written as [8, 9]

\[
h_{ee} = \frac{1}{\sqrt{2}\nu_\Delta} [m_1 e^{-i\delta_1} c_{12} c_{13}^2 + m_2 s_{12} c_{13}^2 + m_3 e^{(2\delta - \phi_2)} s_{13}^2],
\]  

(11)
\[
\begin{align*}
\eta_{e\mu} &= \frac{1}{\sqrt{2\nu_\Delta}}[m_1 e^{-i\varphi_1} c_{12} c_{13} (-s_{12}s_{23} - e^{-i\delta} c_{12}s_{13}s_{23}) + m_2 s_{12} c_{13} (c_{12} s_{23} - e^{-i\delta} s_{12}s_{13}s_{23})] + m_3 e^{i(\delta - \varphi_2)} s_{13} c_{13}s_{23}], \\
\eta_{e\tau} &= \frac{1}{\sqrt{2\nu_\Delta}}[m_1 e^{-i\varphi_1} c_{12} c_{13} (s_{12}s_{23} - e^{-i\delta} c_{12}c_{23}s_{13}) + m_2 c_{13}s_{12} (-c_{12}s_{23} - e^{-i\delta} s_{12}s_{13}s_{23})] + m_3 e^{i(\delta - \varphi_2)} s_{13} c_{13}s_{23}].
\end{align*}
\]

Figure 2: At the ILC, the production cross section \(\sigma_1\) for the subprocess \(e\gamma \to e^+H^{--}\) as a function \(m_0\) for different values of the \(m_H\) in the \(NH\)(a) and \(IH\)(b) cases.

After calculating the cross section \(\hat{\sigma}(\hat{s})\) for the subprocess \(e\gamma \to l^+H^{--}\), the effective cross section \(\sigma_1(s)\) at the ILC with the c.m. energy \(\sqrt{s} = 1\) TeV can be obtained by folding the cross section \(\hat{\sigma}(\hat{s})\) with the photon distribution function \(f_{\gamma/e}(x)\) \[21\]

\[
\sigma_1(s) = \int_{m_H^2/s}^{0.83} \hat{\sigma}(\hat{s}) f_{\gamma/e}(x) dx,
\]

where \(x = \hat{s}/s\), in which \(\sqrt{s}\) is the c.m. energy of the subprocess \(e^-\gamma \to l^+H^{--}\). In above equation, we have neglected the mass of the lepton \(l^+\) due to \(m_H >> m_{l^+}\).

It is obvious that the production cross section \(\sigma_1\) is dependent on the neutrino oscillation parameters, the triplet \(VEV\) \(\nu_\Delta\), and the mass parameter \(m_H\). Direct searches for \(H^{\pm\pm}\) have been performed at \(LEP\) \[22\], Tevatron \[23\], and \(HERA\) \[24\] via the production mechanisms \(e^+e^- \to H^{++}H^{--}\) and \(q\bar{q} \to H^{++}H^{--}\). Both of these production
mechanisms are depend on only one unknown parameter $m_H$. The lower mass limits for $m_H$ from the Tevatron searches are $m_H > 110\text{GeV} \sim 150\text{GeV}$ [23]. In this paper, we take $m_H$ as a free parameter and assume that its value is in the range of $300\text{GeV} \sim 600\text{GeV}$.

The current experimental upper limit for the LFV process $\mu \rightarrow e\gamma$ has given severe constraint on the triplet $\nu\nu\nu\nu$, as shown in Eq.(9). In our numerical estimation, we take the minimal value given by Eq.(9). In this case, the production cross section $\sigma_1$ is only dependent on $m_H$ and the neutrino oscillation parameters.

There are nine neutrino oscillation parameters: three neutrino masses $m_i$ (or $m_{ij}$), three mixing angles $\theta_{ij}$, the Dirac phase $\delta$ and two Majorana phases $\varphi_1$ and $\varphi_2$. According to the current constraints on the neutrinos and mixing parameters from neutrino oscillation experiments [25], we use the following values in this article:

\[
\begin{align*}
\Delta m^2_{21} &= m^2_2 - m^2_1 \simeq 7.59 \pm 0.20 \left(\begin{array}{c}
+0.61 \\
-0.69 
\end{array}\right) \times 10^{-5}\text{eV}^2, \\
\Delta m^2_{31} &= m^2_3 - m^2_1 = -2.36 \pm 0.11(\pm 0.37) \times 10^{-3}\text{eV}^2 \text{ for IH}, \\
\Delta m^2_{31} &= m^2_3 - m^2_1 = 2.46 \pm 0.12(\pm 0.37) \times 10^{-3}\text{eV}^2 \text{ for NH,} \\
\theta_{12} &= 34.4 \pm 1.0 \left(\begin{array}{c}
+3.2 \\
-2.9 
\end{array}\right)^\circ, \\
\theta_{23} &= 42.8^{+4.7}_{-2.9} \left(\begin{array}{c}
+10.7 \\
-7.3 
\end{array}\right)^\circ.
\end{align*}
\]

Figure 3: Same as Fig.2 but for the subprocess $e\gamma \rightarrow \mu^+H^-$. 
\[ \theta_{13} = 5.6_{-3.7}^{+3.0}(\leq 12.5) \circ. \]  

Information on the mass \( m_0 \) of the lightest neutrino, the Dirac and Majorana phases cannot be obtained from neutrino oscillation experiments. To simply our paper, we will take \( \delta = 0 \) and \( \varphi_1 = \varphi_2 = 0 \) to calculate the cross sections for single production of \( H^{-} \) in the cases of \( NH \) \( (m_0 = m_1 < m_2 < m_3) \) and \( IH \) \( (m_0 = m_3 < m_1 < m_2) \). For the free parameter \( m_0 \), we will assume its value in the range of \( 0.001 \text{eV} \leq m_0 \leq 1 \text{eV} \).

\[
\theta_{13} = 5.6_{-3.7}^{+3.0}(\leq 12.5) \circ. \tag{16}
\]

From above discussions we can see that the production cross section for the process \( e\gamma \rightarrow l^+H^- \) depends on two free parameters \( m_0 \) and \( m_H \). In our numerical estimation, we take the cross section \( \sigma_1 \) as a function of the lightest neutrino mass \( m_0 \) for three values of \( m_H \). Our numerical results are summarized in Fig.2, Fig.3, and Fig.4, which correspond the subprocesses \( e\gamma \rightarrow e^+H^- \), \( e\gamma \rightarrow \mu^+H^- \), and \( e\gamma \rightarrow \tau^+H^- \), respectively. One can see from these figures that the value of the production cross section \( \sigma_1 \) increases as \( m_H \) increases, which is because the minimal value of the triplet VEV \( \nu_\Delta \) is proportional to the factor \( 1/m_H \). In both of the NH and IH cases, the production cross section \( \sigma_1(\mu H) \) for the subprocess \( e\gamma \rightarrow \mu^+H^- \) is approximately equal to that for the subprocess \( e\gamma \rightarrow \tau^+H^- \) and their values are smaller than 16 fb in most of the parameter space of the HTM. This is due to \( h_{\mu\mu} \approx h_{\mu\tau} \), which reflects the neutrino mixing patterns. However, for the subprocess \( e\gamma \rightarrow e^+H^- \), it is not this case. For the NH spectrum, its cross section
$\sigma_1(eH)$ is larger or smaller than the cross section $\sigma_1(\mu H)$ (or $\sigma_1(\tau H)$), which depends on the mass $m_0$ of the lightest neutrino. For the IH spectrum, the cross section $\sigma_1(eH)$ is always larger than the cross section $\sigma_1(\mu H)$ (or $\sigma_1(\tau H)$). For $0.001eV \leq m_0 \leq 1eV$ and $300GeV \leq m_H \leq 600GeV$, the values of the cross sections for production of $H^{-}$ associated with a positron $e^+$ via $e\gamma$ collision at the ILC with $\sqrt{s} = 1TeV$ are in the ranges of $1.7fb \sim 17.9fb$ and $34.8fb \sim 72.6fb$ corresponding to the NH and IH cases, respectively.

Ref. [9] has shown that, for $\nu_\Delta < 0.1MeV$, the leptonic decays of $H^{\pm\pm}$ are the dominant decay modes, while the decay $H^{\pm\pm} \rightarrow W^{\pm}W^{\pm}$ is negligible. In the context of the HTM, the leptonic decay channels $H^{\pm\pm} \rightarrow l^{\pm}l^{\pm}$ have been extensively studied in Refs. [11, 26] and the direct connections between their branching ratios and the triplet Yukawa couplings $h_{ij}$ (i.e. the neutrino oscillation parameters) are discussed in these papers. For the leptonic decays of $H^{\pm\pm}$, the possible decay modes are $ee$, $\mu\mu$, $\tau\tau$, $e\mu$, $e\tau$ and $\mu\tau$. In the case of $\delta = 0$ and $\varphi_1 = \varphi_2 = 0$, $\mu\mu$ and $\tau\tau$ are the dominate decay modes, their branching ratios are roughly equal to each other for the NH spectrum and $m_1 < 0.1eV$. While for the IH mass spectrum and $0.001eV \leq m_3 \leq 0.1eV$, the doubly charged Higgs boson $H^{-}$ mainly decays to $ee$ and its branching ratio is $32\% \leq Br(H^{-} \rightarrow ee) \leq 53\%$. Thus, for the final states generated by the subprocess $e\gamma \rightarrow e^+H^-$, we consider the three

![Figure 5: The number of the $\overline{\tau}\mu\mu$ (a) and $\overline{\tau}ee$ (b) events at the ILC.](image-url)
leptons $\tau\mu\mu$ and $\tau ee$ for the NH and IH cases, respectively. The number of the $\tau\mu\mu$ and $\tau ee$ events at the ILC with $\sqrt{s} = 1\text{TeV}$ and the integrated luminosity $L_{\text{int}} = 500\text{fb}^{-1}$ are shown in Fig. 5, where we have taken $m_0$ in the range of $0.001\text{eV} \sim 0.1\text{eV}$. From this figure, one can see that there will be several hundreds and up to ten thousands three lepton events to be generated per year in future ILC experiments. It is obvious that this is not the realistic number of the observed events. To take into account detector acceptance we should impose appropriate cuts. For example, the observed leptons must carry a minimum energy and respect a suitable rapidity cut, the angles of the observed leptons relative to the beam, $\theta_e, \theta_\mu$, must be restricted to some ranges [27]. Detailed analysis is needed, which is beyond the scope of this paper.

It is well known that, at the ILC, the LFV processes can provide extremely clear signatures and are experimentally interesting. The leptonic decays of the doubly charged Higgs bosons give rise to same-sign lepton pairs, which can generate distinct experimental signals. The three lepton states $\tau\mu\mu$ and $\tau ee$ generated by the subprocess $e\gamma \rightarrow e^+H^{- -}$ are almost free of the SM backgrounds at the ILC, which have been investigated in Ref. [27]. Thus, as long as the triplet VEV $\nu_\Delta$ is enough small, the doubly charged Higgs boson $H^{- -}$ predicted by the HTM might be detected in future ILC experiments.

4. Single production of the doubly charged Higgs boson $H^{- -}$ at the LHeC

Recently, the high-energy $ep$ collision has been considered at the LHC, which is called the LHeC [15]. For the LHeC, the energy $E_p$ of the incoming proton is given by the LHC beam, and the energy $E_e$ of the incoming electron is in the range of $50 - 200\text{GeV}$, corresponding to the c.m. energies of $\sqrt{s} = 2\sqrt{E_pE_e} \approx 1.18 - 2.37\text{TeV}$. Its anticipated integrated luminosity is at the order of $10 - 100\text{fb}^{-1}$ depending on the energy of the incoming electron and the design. Several studies have been performed to discuss the possibility of detecting the Higgs boson at the LHeC [28]. In this section, we consider single production of the doubly charged Higgs boson $H^{- -}$ via $e\gamma$ collision at the LHeC.

It is well known that the equivalent photon approximation (EPA) can be successfully to describe most of the processes involving photon exchange [29]. A significant fraction of pp collision at the LHC will involve quasi-real photon interactions occurring at energies
well beyond the electroweak energy scale. The quasi-real photons emitted by the incoming protons have low virtuality $Q^2$ and scattered with small angles from the beam pipe. The photon spectrum depending on virtuality $Q^2$ and energy $E_\gamma$ can be described by the following relation \[30, 31\]

$$dN = \frac{\alpha}{\pi} \frac{dE_\gamma}{E_\gamma} \frac{dQ^2}{Q^2} \left[ (1 - \frac{E_\gamma}{E_p})(1 - \frac{Q^2_{\text{min}}}{Q^2}) F_E + \frac{E_\gamma^2}{2E_p^2} F_M \right]$$ (17)

with

$$Q^2_{\text{min}} = \frac{m_p^2 E_\gamma^2}{E_p (E_p - E_\gamma)}, \quad F_E = \frac{4 m_p^2 G_E^2 + Q^2 G_M^2}{4 m_p^2 + Q^2},$$ (18)

$$G_E^2 = \frac{G_M^2}{\mu_p^2} = (1 + \frac{Q^2}{Q_0^2})^4, \quad F_M = G_M^2, \quad Q_0^2 = 0.71 \text{GeV}^2. \quad (19)$$

Where $E_p$ is the energy of the incoming proton which is related to the photon energy by $E_\gamma = \xi E_p$ and $m_p$ is the mass of the proton. Thus, $\xi$ is the proton momentum fraction carried by the photon. The magnetic moment of the proton is $\mu_p^2 = 7.78$, $F_E$ and $F_M$ are functions of the electric and magnetic form factors. Electromagnetic form factors are steeply falling as a function of $Q^2$. Then, the $Q^2$ integrated photon flux can be written as

$$f(E_\gamma) = \int_{Q^2_{\text{min}}}^{Q^2_{\text{max}}} \frac{dN}{dE_\gamma dQ^2} dQ^2,$$ (20)

where $Q^2_{\text{max}} \approx 2 - 4 \text{GeV}^2$. Since the contribution to the above integral formula is very small for $Q^2_{\text{max}} > 2 \text{GeV}^2$, in our numerical estimation, we will take $Q^2_{\text{max}} \approx 2 \text{GeV}^2$.

At the LHeC, the effective production cross section $\sigma_2(lH)$ for the subprocess $e\gamma \rightarrow l^+ H^{-}$ can be written as

$$\sigma_2(lH) = \int_{(m_H + m_l)^2/s}^{\xi_{\text{max}}} \sigma(\hat{s}) f(\xi E_p) d\xi,$$ (21)

where $\hat{s} = 4 E_e E_\gamma = \xi s$ with $E_e = 140 \text{GeV}$ and $E_p = 7 \text{TeV}$. To detect the intact proton or its loss energy, the forward proton detectors are needed. The ATLAS forward physics (AFP) collaboration will have forward detectors with an acceptance of $0.0015 < \xi < 0.15$ [31]. The acceptance of the CMS–TOTEM forward detectors can reach $0.0015 < \xi < 0.5$ [32]. In our numerical estimation, we will take $\xi_{\text{max}} = 0.5$ and $\xi_{\text{min}} = (m_H + m_l)^2/s$, which corresponds the c.m. energy of the subprocess $e\gamma \rightarrow l^+ H^-$, $\sqrt{\hat{s}} \geq m_H + m_l$.  


Our numerical results are summarized in Fig.6 and Fig.7, which plot the production cross sections for the subprocesses \( e\gamma \rightarrow e^+H^{--} \) and \( e\gamma \rightarrow \mu^+H^{--} \) at the \( LHeC \) as functions of the lightest neutrino mass \( m_0 \) for three values of the mass \( m_H \) of the doubly charged Higgs boson. In these figures we have shown our results for the \( NH \) and \( IH \) cases. Our calculation shows that, at the \( LHeC \), the effective production cross section for the subprocess \( e\gamma \rightarrow \mu^+H^{--} \) is roughly equal to that for the subprocess \( e\gamma \rightarrow \tau^+H^{--} \), which is similar with the conclusion at the \( ILC \). Thus, we have not given the contours for the subprocess \( e\gamma \rightarrow \tau^+H^{--} \). From these figures, one can see that the production cross section of the subprocess \( e\gamma \rightarrow l^+H^{--} \) at the \( LHeC \) is smaller than that for the \( ILC \).

For \( 0.001eV \leq m_0 \leq 1eV \) and \( 300GeV \leq m_H \leq 600GeV \), the values of the production cross section \( \sigma_2(eH) \) and \( \sigma_2(\mu H) \) are in the ranges of \( 0.048fb \sim 0.51fb \) and \( 0.095fb \sim 0.43fb \), respectively, for the \( NH \) spectrum, while are in the ranges of \( 0.97fb \sim 2.03fb \) and \( 0.11fb \sim 0.45fb \), respectively, for the \( IH \) spectrum. If we assume the integrated luminosity \( L_{int} = 100fb^{-1} \) of the \( LHeC \) with \( \sqrt{s} = 1.96TeV \), which corresponds \( E_e = 140GeV \) and \( E_p = 7TeV \), then there will be several and up to hundreds \( e^+H^{--} \) events to be generated per year.

The process \( ep \rightarrow e^+H^{--} + X \) with \( H^{--} \) decaying to \( ee \) can give rise to number
of signal events with same-sign electron pair and an isolated positron, which is almost free of the SM backgrounds. Thus, it is also possible to detect the signatures of the doubly charged $Higgs$ boson $H^{-+}$ via its production associated with a positron at the LHeC. Certainly, detailed confirmation of the observability of the signals generated by the HTM at the LHeC experiments would require Monte-Carlo simulations of the signals and backgrounds, which is beyond the scope of this paper.

5. Conclusions and discussions

Doubly charged $Higgs$ bosons ($H^{±±}$) appear in some popular new physics models beyond the SM, which can accommodate neutrino masses. This kind of new particles have distinct experimental signals through their decay to same-sign lepton pairs. Their observation in future high energy collider experiments would be a clear evidence of new physics. Thus, searching for $H^{±±}$ is one of the main goals of current and future high energy collider experiments.

The HTM is one of the attractive new physics models, in which a $Higgs$ triplet is introduced and the neutrino masses can be obtained by the product of the triple $VEV$ $\nu_\Delta$ and the triplet Yukawa coupling $h_{ij}$. This model predicts the existence of the doubly charged $Higgs$ bosons ($H^{±±}$). In this paper, we consider production of $H^{-+}$ associated with a positive lepton $l^+$ via $e\gamma$ collision at the ILC and LHeC. The production cross

Figure 7: Same as Fig.6 but for the subprocess $e\gamma \rightarrow \mu^+ H^{-+}$. 

sections are dependent on the neutrino oscillation parameters, the triplet VEV $\nu_\Delta$, and the mass parameter $m_H$. In our numerical estimations, we use the lower bound on $\nu_\Delta^2$ given by the LFV process $\mu \rightarrow e\gamma$, assume $\delta = 0$ and $\varphi_1 = \varphi_2 = 0$, and take the experimental measurement values for the parameters $s_{ij}$, $c_{ij}$, and $m_{ij}$. In this case, the production cross sections depend only on two free parameters, the mass parameter $m_H$ and the lightest neutrino mass $m_0$. From our numerical results, we can obtain following conclusions.

1. The cross section for production of the doubly charged Higgs boson $H^{--}$ associated with a positive lepton $l^+$ at the ILC is generally larger than that at the LHeC. The production cross sections for the IH mass spectrum are larger than those for the NH mass spectrum at these two kinds of collider experiments.

2. Because of $h_{e\mu} \simeq h_{e\tau}$, the production cross section of the subprocess $e\gamma \rightarrow \mu^+ H^{--}$ is roughly equal to that of the subprocess $e\gamma \rightarrow \tau^+ H^{--}$ at both the ILC and LHeC experiments. For the IH mass spectrum, the cross section $\sigma(eH)$ is always larger than $\sigma(\mu H)$, while for the NH mass spectrum, $\sigma(eH)$ is larger or smaller than $\sigma(\mu H)$ which depends on the value of the lightest neutrino mass $m_0$.

3. For $0.001eV \leq m_0 \leq 1eV$ and $300GeV \leq m_H \leq 600GeV$, the values of the cross section $\sigma(eH)$ at the ILC with $\sqrt{s} = 1TeV$ are in the ranges of $1.7fb - 17.9fb$ and $34.8fb - 72.6fb$ corresponding with the NH and IH cases, respectively. While their values at the LHeC with $\sqrt{s} = 1.97TeV$ are in the ranges of $0.048fb - 0.51fb$ and $0.97fb - 2.03fb$ for the NH and IH cases, respectively.

4. For the leptonic decays of $H^{--}$, the process $e\gamma \rightarrow l^+ H^{--}$ can produce three lepton final state, which is almost free of the SM backgrounds. In the context of the HTM, this process can only generate number of three lepton events at the LHeC, while, with reasonable values of the relevant free parameters, it can give rise to large number of three lepton events at the ILC. For instance, for $0.001eV \leq m_0 \leq 0.1eV$ and $300GeV \leq m_H \leq 600GeV$, the subprocess $e\gamma \rightarrow l^+ H^{--}$ with $H^{--}$ decaying to $ee$ can generate $8.0 \times 10^3 - 1.17 \times 10^4 \, \text{ee}$ events at the ILC experiment with $\sqrt{s} = 1000GeV$ and the integrated luminosity $L_{\text{int}} = 500fb^{-1}$ in the IH mass spectrum. Thus, the possible
signals of $H^{--}$ predicted the HTM might be detected via this process in the future ILC experiments.

In our numerical estimation, we have taken the minimal value of the triplet VEV $\nu_\Delta$ demanded by the current upper experimental bound for the LFV process $\mu \rightarrow e\gamma$. However, the constraint on the HTM from the process $\mu \rightarrow ee\bar{\tau}$ is generally stronger than that from the process $\mu \rightarrow e\gamma$. For example, Ref. [12] has shown that, for $\varphi_1 = \varphi_2 = 0$ and $\sin^2 2\theta_{13} = 0$, the processes $\mu \rightarrow ee\bar{\tau}$ and $\mu \rightarrow e\gamma$ demand $\nu_\Delta m_H \geq 400 eV \cdot GeV$ and $\nu_\Delta m_H \geq 100 eV \cdot GeV$, respectively. It is obvious that the minimal value of $\nu_\Delta$ demanded by $\mu \rightarrow e\gamma$ is smaller than that for $\mu \rightarrow ee\bar{\tau}$. Thus, if we use the constraints on the free parameters of the HTM coming from the process $\mu \rightarrow ee\bar{\tau}$, the production cross sections and the number of the signal events will be decreased at least by one order of magnitude. However, as long as $\nu_\Delta < 10^{-5} GeV$, there will be several $\tau ee$ events to be generated at the ILC.

It has been shown [26] that the Majorana phases $\varphi_1$ and $\varphi_2$ have large effects on the branching ratios $Br(H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm)$. Thus, $\varphi_1$ and $\varphi_2$ also have effects on the cross section for production of the doubly charged Higgs boson $H^{--}$ associated with a positive lepton $l^+$. If we vary the values of $\varphi_1$ and $\varphi_2$, the numerical results for the cross sections $\sigma(eH)$, $\sigma(\mu H)$, and $\sigma(\tau H)$ are also changed. However, our physical conclusions are not changed.

Certainly, the doubly charged Higgs boson $H^{++}$ can also be singly produced at the ILC and the LHeC via the charge-conjugation processes of the corresponding processes for $H^{--}$. Similar to above calculation, we can give the values of the production cross sections for $H^{++}$. Thus, the conclusions for $H^{--}$ also apply to $H^{++}$.

If the doubly charged Higgs bosons $H^{\pm\pm}$ are indeed found at the LHC, various consistency checks will have to be performed. One of the important tasks is to exact calculate and precise measure the branching ratios $Br(H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm)$ and see whether their values are consistent with a neutrino mass matrix from oscillation data. We expect that our work will be helpful to test the Higgs triplet mechanism for neutrino masses and further to distinguish different new physics models.
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