Effect of Surface Texture Distribution Parameters on Hydrodynamic Lubrication and Numerical Optimization

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Abstract. Surface texture technology has been proven to be an effective way to improve the lubrication performance of friction surfaces. In order to obtain the optimal surface texture distribution parameters, taking the spherical crown pit texture model as the research object, and selecting the unequal rectangular control unit to establish the surface texture distribution model with different horizontal and vertical distribution distances (densities). According to the principle of fluid dynamic pressure lubrication, a two-dimensional Reynolds equation based on the N-S equation is established and solved by the multigrid method to study the effect of texture distribution parameters on the surface hydrodynamic lubrication under plane motion. The results show that when the texture unit \(L_x\) is 3.4r and the aspect ratio \(L_y / L_x\) is 0.82, the optimal oil film bearing capacity can be obtained. When the control unit of the boundary pit is appropriately increased so that the left and right sides of the pit at the boundary are about 3.4r, the oil film pressure stability can be effectively improved.

Key words: Surface Texture; micro-pits; Fluid Hydrodynamic Lubrication; Numerical Optimization.

1. Introduction
In recent years, surface texture technology has been proved to be an effective means to improve the tribological properties of friction pairs [1]-[3]. The surface texture technique is to process micro-pits with certain size parameters, geometric shapes and arrangement on the friction surface. When the friction pairs move relatively, the wedge-shaped effect of the micro-pits causes additional fluid dynamic pressure between the opposite moving surfaces. Therefore, the friction pairs are hardly in direct contact, thereby generating hydrodynamic lubrication [4], which increases the oil film bearing capacity and reduces the friction and wear.

Studies have shown that the geometry and parameters of surface texture are the main factors affecting the tribological properties of the friction pair. The theoretical and experimental methods are used to optimize reasonable surface texture geometric parameters and distribution spacing, which can make the texture surface produce the best oil film bearing capacity and lubricating performance. Ji [5] et al. studied the effects of surface textures in the form of parabolic grooves, rectangular grooves and triangular grooves on hydrodynamic pressure. The results show that the geometry, area density, groove depth and direction of the groove have a significant effect on the hydrodynamic pressure. Yu Haiwu [6]...
et al. selected the cylindrical micro-pits to study the influence of texture area ratio, depth, diameter, depth-diameter ratio on the lubrication performance of the texture surface, and obtained the optimal geometric parameters of the micro-pits. Wang Xiaolei [7] et al. established a dynamic hydrodynamic lubrication model of elliptical micro-pits based on Reynolds equation, and analyzed the influence of inclination, length-to-minor axis ratio, depth, area ratio and equivalent radius of elliptical pit on the average dimensionless lubrication film pressure. At home and abroad, the surface texture has been studied from the aspects of geometric parameters and distribution forms, but few studies have been carried out to take the square texture unit [8]-[10] as the object and research on the unequal rectangle texture. The influence of the horizontal and vertical distribution of the pit texture on the average oil film pressure is also rarely studied.

Therefore, in this paper, we selected the unequal-edge control unit for the spherical crown-shaped pit texture, simulated the different lateral and longitudinal distribution of the pit. Solving the Reynolds equation of the control area based on the multi-grid method to calculate the optimal distribution distance of the surface micro-pits. Finally, we selected a 5 × 5 texture model to analyze the influence of pit distribution on the peak value of oil film pressure on the texture centerline, and the boundary pit distribution was optimized to effectively reduce the fluctuation of oil film pressure peaks.

2. Establishment of mathematical model

The pressure distribution of the surface texture changes periodically. In order to improve the calculation efficiency, a single pit control unit can be selected as the calculation domain. Since the spherical crown-shaped pits texture has good isotropic property, and can obtain better lubrication performance than cylindrical, triangular, diamond texture [11]-[12], the control unit of the spherical cone-shaped single pit texture model is selected as the research object in this paper. Assuming that the upper surface is a smooth surface, the lower surface is a surface with a spherical crown-shaped pit distribution, the upper surface moves parallel to the lower surface at a certain speed. Take the pit depth as \( h_p \), the pit radius as \( r \), and the control unit as a rectangle of \( L_x \times L_y \), It is also assumed that the two surfaces of the friction pair are completely separated by a layer of lubricating film, that is, the minimum oil film thickness is \( h_0 \), and the texture model is as shown in figure 1.

![Figure 1. Texture model diagram](image)

In order to eliminate the difference between the values, reduce the singularity of the calculation matrix, reduce the calculation error, and make the numerical calculation results not limited by the unit, increase the versatility of the solution, and the dimension in the above equation needs to be dimensionless processed. The dimensionless method can be defined as follows:

\[
x = \frac{x}{Xr}, \quad y = \frac{y}{Yr}, \quad h = \frac{h}{Hh_0}, \quad \rho = \frac{P}{P_0}
\]

(1)

The Reynolds equation and the film thickness equation that can be obtained after the dimensionless:
\[
\frac{\partial}{\partial X} \left( H^3 \frac{\partial P}{\partial X} \right) + \frac{\partial}{\partial Y} \left( H^3 \frac{\partial P}{\partial Y} \right) = 6\Lambda \frac{\partial H}{\partial X}
\]
(2)

\[
H = \left\{ \begin{array}{ll}
1 & + \sqrt{\left( h_p^2 + r^2 \right)^2 - \left( \eta^2 + \xi^2 \right)^2} - \left( \frac{r^2 - h_p^2}{2h_p} \right) \right\}/h_p, \quad \left( \eta^2 + \xi^2 \right) \leq r^2 \\
1 & + \left( \eta^2 + \xi^2 \right) > r^2
\end{array} \right.
\]
(3)

Where \( P_0 \) is the ambient pressure, \( X, Y \) is the dimensionless coordinate of the control region node, \( H \) is the dimensionless oil film thickness, \( P \) is the dimensionless oil film pressure, and \( \Lambda = \frac{6U\mu r}{h_p P_0} \) is defined as the working condition coefficient of the friction pair motion.

3. Model solution

3.1. Numerical solution

In solving the Reynolds equations of two-dimensional partial differential equations, the equations need to be discretized and transformed into nonlinear equations for solving. The finite difference method [13] discrete governing equations can be used to obtain the difference equations:

\[
A P_{r+1,j} + BP_{r-1,j} + CP_{r,j+1} + DP_{r,j-1} - EP_{r,j} = F
\]
(4)

Where

\[
A = \frac{H_{r+1,j}^3 + H_{r-1,j}^3}{2\Delta X^2}, \quad B = \frac{H_{r+1,j}^3 + H_{r-1,j}^3}{2\Delta X^2}, \quad C = \frac{H_{r,j+1}^3 + H_{r,j-1}^3}{2\Delta Y^2}, \quad D = \frac{H_{r,j+1}^3 + H_{r,j-1}^3}{2\Delta Y^2}, \quad F = \Lambda \frac{H_{r+1,j}^3 - H_{r-1,j}^3}{2\Delta X}
\]

\[
E = \frac{H_{r+1,j}^3 + H_{r,j+1}^3 + H_{r,j+1}^3 + H_{r+1,j}^3 + 2H_{r,j}^3 + H_{r+1,j}^3}{2\Delta Y^2}
\]

Using the four-layer grid, the multi-grid method of the W loop [14] and the super-relaxation iterative method, the oil film pressure distribution on the discrete nodes of the control region can be solved. Use equation (5) to calculate the dimensionless average oil film pressure \( P_{av} \) in the control area, and use it as an index to evaluate the lubrication performance of friction pairs. Then change the geometric parameters of the surface pits respectively to study the change of the dimensionless average oil film pressure.

\[
P_{av} = \frac{1}{LX \cdot LY} \iint \Omega P(X, Y)dXdY
\]
(5)

3.2. Parameter selection

The flow of lubricating oil between the friction pairs follows the Navier-Stokes equation. The Reynolds equation widely used in lubrication calculations is a simplified form of the N-S equation on the lubrication problem, and it ignores the influence of the inertia term and the pressure gradient in the oil film thickness direction. However, when the oil film thickness and the Reynolds number are too large, the above effects are not negligible for the calculation of the surface texture lubrication calculation model. Therefore, it is necessary to consider the validity of the Reynolds equation when selecting the texture parameters, reduce the influence of the inertia term and the pressure gradient in the thickness direction of the oil film on the calculation results, and reduce the calculation error of the equation compared with the N-S equation. Two key factors for evaluating the effectiveness of surface texture dynamic pressure lubrication calculations [16]: ratio of oil film thickness to texture feature length \( \delta \) and reduced Reynolds number \( r_e \).
\[ \delta = \frac{h}{L}, \quad Re = \frac{\rho U h}{\mu}, \quad r_e = \left( \frac{h}{L} \right) Re \]  \tag{6}

Where \( L \) is the length dimension of the calculation unit in the \( x \) direction, that is, the texture feature length \( L_X \), and \( Re \) is the Reynolds number. For the spherical crown pit, the evaluation criteria for the validity of the Reynolds equation \([15]-[16]\) are as follows.

\[
\begin{cases}
\delta(h / L) \leq 0.022 \text{ and } r_e \leq 0.20 & \text{yes} \\
\text{others} & \text{no}
\end{cases} \tag{7}
\]

According to the above evaluation criteria, the texture parameters can be shown in the following table:

| Parameter                        | Numerical |
|----------------------------------|-----------|
| Pit depth \( h_p \)/um          | 5         |
| Pit radius \( r \)/um           | \( \geq 115 \) |
| Minimum oil film thickness \( h_0 \)/um | 5         |
| Control unit side length \( L \)/um | \( > 2r \) |
| Lubricating oil dynamic viscosity \( \mu \)/Pa·s | 0.0132 |
| Lubricating oil density \( \rho \)/(kg·m\(^{-3}\)) | 840       |
| Movement speed \( U \)/((m·s\(^{-1}\))) | 2         |

### 4. Results and discussion

#### 4.1. Influence of pit distribution on the dimensionless mean oil film pressure

In order to more intuitively represent the change of the oil film bearing capacity of the control area with the pitch distribution of the pit, the side length of the control unit was changed to simulate the variation of the pitch distribution of the pit. Considering the validity of the Reynolds equation, the minimum oil film thickness \( h_0 \) is 5 \( \mu \)m, the pit depth \( h_p \) is 5 \( \mu \)m, and the pit radius \( r \) is 115, 125, 135 and 145 \( \mu \)m respectively. Figure 2 below shows the curve of the dimensionless average oil film pressure \( P_{av} \) along with the length of the control unit \( r \). It can be seen from the figure that the average oil film pressure first increases rapidly and then decreases slowly with the increase of the side length of the control unit. When the side length \( L \) of the control unit is about 3.4 times of the pit radius, that is, the pit area occupancy rate is 0.272 and the pit spacing \( L \) is 3.4\( r \), the average oil film pressure can reach the maximum.

![Figure 2](image_url)

**Figure 2.** The relationship between the dimensionless average oil film pressure and the pit distribution distance \( (h_0=5, \ h_p=5, \ U=2, \ \delta \leq 0.021, \ r_e \leq 0.013) \)
The above describes the change in the average oil film pressure when the control unit is a square unit with equal length and width, that is, the horizontal and vertical distribution intervals of the pits are the same. In addition, the bearing capacity of the oil film should be considered when the horizontal and vertical distribution intervals of the surface pits are not equal, investigate whether it can produce greater oil film pressure.

Therefore, we selected the fixed control unit width $L_x=3.4r$ and different pit radius $r$, simulated the oil film pressure condition when the lateral and longitudinal distribution pitch (density) of the pit is different, by changing the length $L_y$ of the control unit. $L_x$ is the length of the control unit in the lubricant flow direction, and $L_y$ is the length of the control unit in the lubricant normal direction. As shown in figure 3 a), the variation trend of the average oil film pressure $P_{av}$ is basically the same as that of the square equilateral control unit under different pit radii. The average oil film pressure first increases and then decreases with the increase of the side length $L_y$ of the control unit. There is an optimal $L_y$, so that the oil film pressure reaches the maximum value. The difference is that the maximum value of the average oil film pressure appears at $L_y/2.8r$, that is at $L_y/L_x$ is about 0.82, and at this time, a larger average oil film pressure is obtained, compared to the case where the $L_y/L_x$ is 1.

Selecting the pit radius $r$ to be 115 um, selecting several sets of different control unit wide sides $L_x$, and then changing the control unit aspect ratio, to study the variation of the dimensionless average oil film pressure with the control unit aspect ratio under the wide side $L_x$ of different control units. The relationship between the dimensionless average oil film pressure and the control unit aspect ratio is shown in figure 3 b). Obviously, under different pit widths, there is an optimal aspect ratio to maximize the oil film bearing capacity. However, the optimal aspect ratios of the different width control units are slightly different. When $L_x=3.4r$, the optimal aspect ratio $L_y/L_x$ is about 0.82, and when $L_x$ is 3$r$, 4$r$, 5$r$ respectively, the optimal value is 0.9, 0.8, 0.7 or so, that is, $L_y$ is 2.7$r$, 3.2$r$ and 3.5$r$ respectively, and under the optimal aspect ratio, the dimensionless average oil film pressure is the largest when $L_x=3.4r$.

Considering the influence of the depth of the texture pit and the speed of the lubricating oil on this optimal value, the oil film pressure curve at different pit depths and the oil film pressure curve at different speeds are shown in figure 3 c) d), which can be seen in different. Under the texture depth and lubricating oil speed, the optimal aspect ratio of the all texture unit is about 0.82, and the pit depth and lubricating oil speed have no effect on the optimal aspect ratio.
In order to further verify that the oil film bearing capacity can be obtained when the lateral and longitudinal distribution of the pits are not equal, a $5\times5$ texture model was selected to calculate the dimensionless average oil film pressure. The oil film pressure distribution on the textured surface is shown in figure 4. Here, the lateral distribution pitches (i.e. $L_x$) of the pits are respectively selected as $3r$, $3.4r$, $4r$, $5r$. Comparison of the dimensionless average oil film pressure, when the horizontal and vertical distribution spacing is equal ($L_x=L_y$) and unequal ($L_y/L_x$ are 0.9, 0.82, 0.8, 0.7 respectively), as is shown in figure 5. It can be seen that when the $L_y$ is appropriately adjusted so that the lateral distribution of the pits is not equal, a larger oil film bearing capacity can be obtained. In the four cases, all of the bearing capacity of the oil film in the control area is improved, and the dimensionless average oil film pressure is the largest when $L_x=3.4r$ and $L_y/L_x=0.82$. Therefore, it can be concluded that when the texture unit $L_x=3.4r$ and the aspect ratio $L_y/L_x$ is 0.82, the lateral distribution pitch of the pits is $3.4r$, and the longitudinal distribution pitch is $2.8r$, the optimal oil film bearing capacity can be obtained.
4.2. Influence of pit distribution on oil film pressure stability under synergistic action

When studying the surface texture of a single pit, it is only necessary to study the influence of the geometric parameters of the pit on the bearing capacity and friction of the oil film in the control area. When studying the texture model of multi-pit distribution, since the pressure at the entrance of the texture will be affected by the pressure at the exit of the previous texture, the peak pressure of the texture and the exit pressure will also change accordingly, which will affect the oil film pressure of the next texture. The main factor affecting the pressure fluctuation of the oil film is spacing and form of the pit distribution.

In this paper, the oil film pressure peak $P_{\text{max}}$ was used as the evaluation index, a 5 × 5 texture model was selected to study the fluctuation of oil film pressure in the direction of lubricant speed. Wherein the pit radius $r$ is 115 um, the pit depth $h_p$ is 5 um, the minimum gap $h_0$ is 5 um, and the pit control unit $L_x$ is $3.4r$, $L_y$ is $2.8r$, and the aspect ratio $L_y/L_x$ is 0.82. Since the pressure of the oil film perpendicular to the velocity direction is symmetrical, we took the oil film pressure generated by five pits on the centerline of the calculation domain for analysis, and obtained the pressure distribution map of the dimensionless oil film of the five-pit of the centerline of figure 6. It can be seen that the change in oil film pressure peak occurs mainly at the boundary unit. This is due to the interaction between the textures. The influence of the texture on the oil film pressure in the control area will extend to the control area of the next texture. This effect accumulates in the direction of the lubricant speed, result in the influence area of the pit at the texture boundary is suppressed, and such oil film pressure fluctuation causes the entire control region to exert a tipping force on the friction surface, results in high-precision friction surface vibration and even motion instability.
Since the pit influence area at the texture boundary is suppressed, the peak fluctuation of the oil film pressure mainly occurs at the boundary unit. Therefore, reducing the degree of change in the peak value of the oil film pressure by appropriately increasing the width of the pit unit at the boundary of the control region. Increasing the length $L_x$ of the pit unit along the oil film velocity direction at the boundary to compensate the reduction of the pit influence region at the boundary. The pits in the boundary control unit $L_x$ were respectively $4.5r$ and $5r$, and the center arrangement and the offset arrangement were selected. The texture model is shown in Figure 7 b) c).

\[\text{Figure 7. Texture model of boundary} \quad \text{Figure 8. Center line five-pit oil film pressure peak comparison}\]

And the maximum oil film pressure $P_{\text{max}}$ generated by the five pits on the center line of the calculation domain was calculated, as shown in figure 8, it can be seen that when the pit control unit at the boundary is not biased, the oil film pressure peak change amount is significantly reduced, but the peak pressure of the oil film on the left side is increased by the influence of the boundary pit control unit. When the pits are offset in the control unit, that is, the spacing between the pits on the left side is appropriately reduced, the negative effect is significantly weakened, and it can be seen that the oil film pressure distribution effect is better produced by $L_x=5r$ compared with $L_x=4.5r$. At this time, the distance between the two sides of the boundary pit is almost equal to about $3.4r$, the maximum oil film pressure generated by the five pits on the center line of the calculation domain tends to be consistent, which effectively reduces the fluctuation of the oil film pressure peak.

5. Conclusion
In this paper, for the spherical crown-shaped pit texture, the dimensionless average oil film pressure is selected as the evaluation index of dynamic pressure lubrication performance to study the influence of the surface texture distribution interval on fluid dynamic pressure lubrication. The following conclusions summarize the results of the present study:

1) The dimensionless average oil film pressure in the control area first increases and then decreases gradually with the increase of the side length of the pit control unit. When the control unit side length $L=3.4r$, that is, the pit distribution spacing is $3.4r$, the oil film pressure reaches a maximum value.

2) When considering the case where the length and width of the control unit are not equal, that is, the influence of the unequal horizontal and vertical distribution spacing of the pits on the average oil film pressure. When the texture unit $L_x=3.4r$ and the aspect ratio $L_y/L_x$ is 0.82, the lateral distribution pitch of the pits is $3.4r$, and the longitudinal distribution pitch is $2.8r$, and the optimal oil film bearing capacity can be obtained. Compared with the distribution of pits with equal horizontal and vertical spacing, the oil film pressure has been further improved.

3) When the texture model is a multi-pit distribution, the oil film pressure peak between the pits fluctuates due to the synergistic effect between the pits, especially the pressure of the boundary pit oil film is suddenly reduced. When the control unit of the boundary pit is increased to $L_x=5r$, and the center
of the pit is appropriately offset so that the spacing between the two sides of the boundary pit is substantially equal to about $3.4r$, the oil film pressure of the pit at the boundary can be effectively raised to make the oil film pressure more stable.

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References
[1] Etsion & L. Burstein (1996) A Model for Mechanical Seals with Regular Microsurface Structure, Tribology Transactions, 39:3, 677-683.
[2] Martin Z, Ivan K, Martin H, et al. EFFECT OF SURFACE TEXTURING ON EHD LUBRICATION FILMS UNDER TRANSIENT SPEED CONDITION[C].//ASME/STLE international joint tribology conference 2011 : Nanotribology ...2012:187-189.
[3] Etsion I, Kligerman Y and Halperin G. Analytical and experimental investigation of laser-textured mechanical seal faces [J]. Tribology Transactions, 1999, 42 (3): 511-516.
[4] Hamilton DB, Walowitz JA, Allen CM. A theory of lubrication by microirregularities. Journal of Basic Engineering 1966; 88(1):177–185.
[5] JI J H, FU Y H and BI Q S. Influence of Geometric Shapes on the Hydrodynamic Lubrication of a Partially Textured Slider With Micro-Grooves[J]. Journal of Tribology, 2014, 136(4):041702-1-041702-8.
[6] YU H W, WANG X L, SUN Z et al. Theoretical Analysis on Hydrodynamic Lubrication of Cylinder Micro-dimple Surface Texture[J]. JOURNAL OF NANJING UNIVERSITY OF AERONAUTICS & ASTRONAUTICS, 2010, 42(2):209-213.
[7] SUN Z Y. A Study about The Impact of Ellipse Micro-dimple on Hydrodynamic Lubrication Performance [D]. Nanjing University of Aeronautics and Astronautics, 2016.
[8] Shinkarenko A, Kligerman Y and Etsion I. The effect of surface texturing in soft elastohydrodynamic lubrication [J]. Tribology International, 2008, 42(2).
[9] Etsion I. Modeling of surface texturing in hydrodynamic lubrication [J]. Friction, 2013, 1(3).
[10] LIU Y, FU L D, LU Y and ZHAN C C. Hydrodynamic Lubrication Performance on Hydraulic Cylinder Piston Surface by Rhombus Texture[J]. Machinery Design & Manufacture, 2017(04):40-42.
[11] Uddin, M. S. Ibatan, Tom and Shankar, S. Influence of surface texture shape, geometry and orientation on hydrodynamic lubrication performance of plane-to-plane slider surfaces [J]. LUBRICATION SCIENCE, 2017, 29(3):153-181.
[12] HE X, LIAO W L and WANG G R. Evaluation Method and Experimental Verification of Hydrodynamic Lubrication Performance of Different Texturing [J]. Lubrication Engineering, 2018, 43(02):53-59.
[13] WANG X, YU H, DENG H, et al. The effect of dimple shapes on friction of parallel surfaces [J]. Proceedings of the Institution of Mechanical Engineers, Part J. Journal of engineering tribology, 2011, 225(8):693-703. LIU Chaoqun. Multigrid Method and Its Application in Computational Fluid Dynamics [M]. Beijing: Tsinghua University Press, 1995: 1-40.
[14] LIU C Q. Multigrid Method and Its Application in Computational Fluid Dynamics [M]. Beijing: Tsinghua University Press, 1995: 1-40.
[15] HAN J, FANG L, SUN J P, GE S R and ZHU Hua. On the Validation of Reynolds Equation for Hydrodynamics Lubrication Simulation of Textured Surface [J]. Tribology, 2014, 34(04):348-356.
[16] MA C B, ZHU H and SUN J J. Applicable Equation Study of Lubrication Calculation of Surface Texture Based on CFD Analysis [J]. JOURNAL OF MECHANICAL ENGINEERING, 2011, 47(15): 95-100+106.