The pseudogap in hole-doped cuprates: possible insights from the Kondo effect

J. R. Cooper
Cavendish Laboratory, Department of Physics, University of Cambridge,
J.J. Thomson Avenue, CB3 0HE, United Kingdom
(Dated: December 23, 2021)

The “states-non-conserving” fermion density of states (DOS), deduced from the specific heat of hole-doped cuprates, could arise from a Kondo or heavy fermion-like DOS being suppressed by anti-ferromagnetic spin fluctuations. The large Fermi surface predicted by band theory and observed experimentally, is still expected for zero pseudogap, but with an effective mass corresponding to a Kondo temperature \( T_K \approx 800 \) K. A finite pseudogap could divide it into Fermi arcs. Theoretical results for the asymmetric Anderson model can account for the experimental Wilson ratio.

One of the unsolved questions for cuprate superconductors, in addition to the pairing mechanism and the linear temperature \( (T) \) dependence of the electrical resistivity, is the origin of the pseudogap (PG) in samples with a certain number of holes \( (p) \) per CuO \(_2\) unit. The PG lowers the superconducting condensation energy \[1, 2\], the superfluid density, e.g. \[3\], and an important practical property, the irreversibility field \[4\], where the electrical resistivity first becomes non-zero, even though the zero-field transition temperature initially stays high. Many researchers focus on a \( T^* - p \) “phase diagram” and a \( T^*(p) \) line, ascribed to the onset of a PG, that goes to zero as \( p \) is increased to \( p_{\text{crit}} \). Careful experiments reveal anomalies in several physical properties at, or near \( T^* \), but the values of \( T^* \) and the size of the anomalies are probably too small to account for the PG. For example the small \[3\] charge density wave (CDW) anomalies for under-doped (UD) samples may be linked \[7\] to the Fermi arcs, detected by angle-resolved photoemission (ARPES), that are caused by the pre-existing PG. Abrupt changes in magnetic anisotropy of YBa\(_2\)Cu\(_3\)O\(_{6+x}\) (YBCO) crystals defining a nematic \( T^*(p) \) line \[3\], that also extrapolates to zero at \( p_{\text{crit}} \), are also small - typically 0.5% of the average in-plane spin susceptibility for \( \Delta T \sim 50 \) K. Here we retain the idea, reviewed along with many experimental facts in \[4\], that the PG is best described as a \( T \)-independent energy scale \( E_G \) that varies strongly with \( p \). We argue that it might be caused by low energy anti-ferromagnetic \( (a f) \) spin fluctuations giving a deep dip in a Kondo or heavy fermion-like density of states (DOS) at the Fermi energy \( (E_F) \).

Analysis of the specific heat of YBCO \[10\], Bi\(_2\)Sr\(_2\)CaCu\(_3\)O\(_{6+x}\) (Bi2212) \[2\], at different oxygen levels \( (x) \), and La\(_{2-y}\)Sr\(_y\)CuO\(_4\) (LSCO) \[2\], in terms of a “states-non-conserving” V-shaped fermion DOS \[1\], centered at \( E_F \), in an otherwise flat band, as sketched in Fig. \(\text{a}\), gives intriguing insights. It has some deficiencies because (i) the large thermoelectric power \( (\text{TEP}) \) and its scaling with \( E_G \) \[11\], require electron-hole asymmetry, (ii) the detection of quantum oscillations and Fermi arcs implies that there is a small residual DOS at \( E_F \) and (iii) Y\(^{89}\) NMR Knight shift data for over-doped (OD) Y(Ca)BCO \[12\] show a fall of up to 20% in the spin susceptibility, \( \chi_s(T) \), at higher \( T \) rather than the constant value expected for a flat band, there are also small deviations from flat band behavior for OD Bi2212 \[2\]. On the other hand Cu NMR data for La-doped Bi2201 \[13\] seem to be consistent with a flat band. They also show that the PG is not affected by fields up to 44 T, and that the residual DOS is much larger in Bi2201. The main features of this first-order phenomenological analysis are as follows.

A. \( E_G \) falls linearly with \( p \), as \( J_{af}(1 - p/p_{\text{crit}}) \), disappearing abruptly at \( p_{\text{crit}} = 0.17-0.19 \) \[1\], where \( J_{af}/k_B = 1200-1500 \) K, is similar to the \( af \) interaction \( J_{af}/k_B = 1700 \) K \[14\] in the corresponding parent insulator with \( p = 0 \) and \( k_B \) is Boltzmann’s constant.

B. Plots of \( S_c(T, p) \) vs. \( T \), where \( S_c(T, p) \) is the non-phonon, electronic contribution to the entropy, are approximately linear and parallel at higher \( T \), e.g. Fig. 1(b) of \[2\], and Fig. 5 of \[10\], or Figs. 1(SM) and 2(SM) of \[15\]. \( S_c(T, p) \) curves for \( p < p_{\text{crit}} \) have lower entropy at all \( T \) and there is no sign of the entropy “coming back” up to at least 200 K for Bi-2212 and 300 K for YBCO. This behavior is consistent with a recent comprehensive NMR paper \[16\]. For fermions with an energy \( (E) \)-dependent DOS we expect \( S_c(T) \) as \( T \chi_s(T) \), because both depend on the number of states within the thermal window \( (E - E_F) \lesssim 2k_BT \). \( T \chi_s(T) \) curves for Bi2212 are parallel up to 400 K, see Fig. 13(b) of \[2\] or Fig. 3(SM) \[15\], as are plots of \( T \chi_s(T) \) for \( \text{Y}_{0.8}\text{Ca}_{0.2}\text{Ba}_2\text{Cu}_3\text{O}_{6+x} \), also up to 400 K \[17\], after allowing for an \( x \)-independent Curie or Curie-Weiss term. The difference between a states-conserving CDW- or superconductor-like DOS and a V-shaped gap is shown by the results of model calculations of \( S_c(T) \) in Fig. 4(SM) \[15\].

C. Plots of \( S_c(T_0) \) vs. \( p \) and \( k_BT_0 \chi_s(T_0) \) vs. \( p \) at temperatures \( T_0 \) show anomalously large increases of \( \approx k_B \) and 0.3\( k_BT_0 \) per added hole respectively \[2\], where \( \mu_B \) is the Bohr magneton. For \( p < p_{\text{crit}} \) this is caused by the \( p \)-dependence of the PG and is consistent with the Wilson ratio discussed later. For \( p > p_{\text{crit}} \) they peak near \( p = 0.25 \) before falling again, but this interesting result has only been established experimentally for LSCO \[2\].

D. \( p_{\text{crit}} \) seems to be determined by the product of \( J_{af} \) and the DOS at high energy \[2\]. In contrast to points A to
C this is not expected in the present picture because here $J_{aa}$ in the parent compound and the DOS for $E \gg E_G$ are not related.

It is widely accepted that the Hubbard model describes the basic physics of the cuprates, e.g. [18] and references therein, but it has not been solved rigorously at low $T$ and as far as we know does not account for points A to D above. Similar underlying physics occurs in the Friedel-Anderson model [19] and the Kondo effect [20, 21], where metals such as Cu or Au are alloyed with a small concentration ($c_{imp}$) of magnetic elements (impurities) such as Fe or Mn. For both Kondo alloys and heavy fermion compounds, the electronic specific heat coefficient, $\gamma = dS_c/dT$, is strongly enhanced at low $T$, in Wilson’s solution of the Kondo problem [22] it is given by

$$\gamma_{imp} = c_{imp}\pi^2k_B^2w/(6T_K)$$

(1)

where $T_K$ is the Kondo temperature and $w=0.4128$ is the Wilson number. For impurities with spin 1/2, and $T < T_K$, $\chi_{imp}^s = c_{imp}(g\mu_B)^2w/(4k_BT_K)$, where $g \approx 2$ is the $g$-factor and $\chi_{imp}^s/T_K = R_0 = 2R_0$, in agreement with experiments on some Kondo alloys [21]. Here $R_0 = 3\mu_B^2/(\pi^2k_B^2)$ is the Wilson ratio for non-weakly-interacting fermions.

For Bi2212, $\gamma(200)$ varies from 0.86 to 0.98 mJ/g-at/K$^2$ for 0.95 < $p < 0.22$ [2], and taking $c_{imp}$ to be the concentration of Cu atoms, Eqn. 1 gives a first estimate for $T_K \approx 800$ K. We note that such a single ion Kondo model works well for the heavy fermion compound CePb$_3$ doped with La [23]. For the same range of $p$, $\chi_{s}(200) = 2.18$ to 2.38 $10^{-4}$ emu/mole Bi2212, see footnote 24 giving a Wilson ratio $\chi_s/\gamma = 1.29$ to 1.34$R_0$.

For YBa$_2$Cu$_3$O$_7$, where there is some uncertainty from the Cu-O chain contribution, and in the precise value of $\gamma(300)$, $\chi_s/\gamma \approx 1.2R_0$ [2, 24]. So the experimental data is closer to $R_0$ rather than the Wilson value of 2$R_0$. This is not a limitation because Fig. 10.9 of Ref. 21 shows that for heavy fermion compounds such as CeCu$_2$Si$_2$, CeCu$_6$ and UPt$_3$, $\chi_s/\gamma \approx R_0$, and as explained later, a ratio of 1.3 is consistent with the present picture. Numerical treatments of the $t - J$ model [20], derived from the 2D Hubbard model, give $\chi_s/\gamma \approx R_0$ for 0.1 < $p < 0.2$ and $T < J/k_B$. Large values of $T_K$ do occur for dilute alloys, e.g. AlMn where $T_K = 600$ K at low $T$ falling to 470 K near 300 K because of thermal expansion and the strong volume dependence of $T_K$ [27]. This reduces the $T$-dependence of $\chi_s(T)$ from the $1/(T + T_K)$ law usually found for alloys with lower $T_K$ and less thermal expansion.

Here we consider three localized states, with occupancy 0, 1 and 2 described by the asymmetric single orbital Anderson model. Figs. 1(b) and (c), taken from Fig. 5 of [28], show the spectral density for an isolated ion obtained by numerical renormalization group (NRG) calculations for $E_0 = -U/2$, the symmetric case, and various asymmetric cases, $-E_0/\Delta = 4$, 3 and 2. $|E_0|$ and $U - |E_0|$ > 0 are the energies required to transfer an electron from the singly-occupied state to the Fermi level of the conduction electrons at $E_F$, or from $E_F$ to the doubly-occupied state respectively. $U$, set equal to $4\pi\Delta$ in [28], is the on-site $d - d$ Coulomb repulsion, $\Delta = \pi V^2 N(E)$ [19], where $V$ is the hybridization energy between the $d$ state and the conduction electrons and $N(E)$ is their DOS for a given spin direction at the energy of the virtual bound state. The familiar triple peak structure in Fig. 1(b) has two side peaks arising from the localized level and a peak near $E_F$ from the Kondo resonance. The weak $p$ dependence of $\gamma$ and hence $T_K$ for OD Bi2212 and other OD cuprates implies that $E_0$ and $\Delta$ are also only weakly dependent on $p$.

As originally shown by Kondo [20] the central peak is caused by the effective exchange interaction $-2J_{imp}\Sigma_{imp}\Sigma_{imp}$, with $J_{imp} < 0$, between the spin on the magnetic ion, $\Sigma_{imp}$ and the spin of a conduction electron $\Sigma$. The $S^+_{imp}$ and $S^-_{imp}$ operators do not commute, and because of this, higher order scattering processes contain the Fermi function for the conduction electrons. This leads to the logarithmic Kondo divergence in the scattering rate, responsible for the resistance minimum observed in dilute magnetic alloys and for the formation of the Kondo resonance at $E_F$. It is a many-body effect because it depends on the occupancy of many other conduction electron states.

The Kondo resonance has the same spatial symmetry as the electronic state of the localized level [22, 30]. This is why band-structures of heavy fermion materials, calculated using relatively standard methods, agree with the extremal orbits observed in quantum oscillation experiments, e.g. [31], because the band structures depend on this symmetry. But the effective masses ($m^*$) are often very large, because they are determined by the energy width of the Kondo resonance. For the cuprates this implies that the Fermi surface is large, with the shape predicted by standard band theory, but $m^*$ is larger.

The effects of an applied magnetic field ($H$) and, more generally, an exchange field, are important here. In dilute Kondo alloys, $H$ suppresses the Kondo effect because spin-flip scattering processes become inelastic in a magnetic field [20]. Early tunnelling studies where a fraction of a monolayer of Fe atoms was evaporated on the oxide layer of an Al-Al$_2$O$_3$-Al junction [32] showed evidence for this “hole in the DOS” and it is indeed “non-states-conserving”, see Fig. 9 of [37] shown in Fig. 5(SM) [15]. Experimentally [38, 39] and theoretically [36, 41] this requires $\mu_BH \gtrsim k_BT_K$. A problem here is that such a large value of $H$, $\approx k_BT_K/\mu_B$, is incompatible with the appropriate condition $E_G \approx \mu_BH$ when $p \lesssim p_{crit}$ and $E_G$ is small.

This difficulty is absent for a molecular field in NRG calculations [32] for the two-band Hubbard model plus a ferromagnetic, predominantly Ising, exchange interaction $-2J_{ex}S^z_1S^z_2$ between the localized magnetic mo-
ments and the conduction electrons. Note that these calculations refer to a concentrated heavy fermion-like spin system. For $J_{ex} = 0$ there are two side peaks from the narrow band, i.e. the localized magnetic states, and a central peak from the broad band, equivalent to the conduction electron band here. But as shown in [22] for large enough $J_{ex}$ there is a quadruple peak in the DOS curves, i.e. an extra dip at $E_F$, whose width is $\approx 2J_{ex}$. It is possible that $J_{ex}$ suppresses higher order spin-flip scattering processes, thereby suppressing the DOS at $E_F$. There appears to be no limitation regarding the ratio of $J_{ex}/k_B T_K$, in contrast to the effect of $H$ on Kondo alloys. It is difficult to calculate $\Delta$ from first principles, even for noble-metal hosts, see [21], Chapt. 9.7. We attempted this by calculating the band-structure of the hypothetical compound YBa$_2$Zn$_3$O$_7$, in which all lattice parameters are kept the same as for YBa$_2$Cu$_3$O$_7$, using the Wien2K code [33]. In a sense this is equivalent to experimentalists using Lu compounds with a full $f$-shell as a reference for Yb compounds. The Brillouin zone is sketched in Fig. 6(SM)(a) and dispersion curves shown in Fig. 6(SM)(b) [15]. The Zn $d$-levels are well below $E_F$ as shown by the the DOS vs. energy plots in Fig. 2 and there are approximately 10 $d$-states per Zn atom, i.e. a full $d$ shell as expected from chemical arguments. In-plane states near $E_F$ do have a small amount of $d$ character but as shown in Fig. 2 this is only a few % of the DOS derived from the measured value of $\gamma$. In contrast, band structure calculations for YBa$_2$Cu$_3$O$_7$ using the local density approximation (LDA) give substantial contributions to $N(E_F)$ from the Cu 3$d$ orbitals forming the Cu–O$_2$ plane bands. The full LDA DOS for YBa$_2$Cu$_3$O$_7$ (not shown) agrees well with earlier work [34] and $\gamma$(300) is only moderately enhanced over the LDA value, see Fig. 2. But $N(E_F)$ from the in-plane oxygen bands and the in-plane Zn atoms in YBa$_2$Zn$_3$O$_7$ is much smaller, so in the present picture the measured values of $\gamma$ for YBCO and Bi2212 arise from the Kondo effect - Eqn. 1. Fig. 6(SM)(c) [15] shows the Fermi surface of YBa$_2$Zn$_3$O$_7$, with two quasi-cylindrical sheets of similar size from the bands crossing $E_F$ between S and X and S and Y, and an open surface, from the band crossing $E_F$ between Y and G. From Fig. 6(SM)(b) [15] the average value of $E_F$ for the two quasi-cylindrical bands of the Zn compound is 0.54 eV relative to the top of the band. The average value of $k_F$ in the S-X-Y plane is $0.35\pi/a$ where $a = 0.382$ nm is the in-plane lattice spacing, giving $p$ for the two cylindrical bands which agrees well with the more precise value obtained by integrating the DOS plots in Fig. 2 for the in-plane O(2), O(3) and Zn(2) atoms. Namely $p=2 \times 0.19$/unit cell, in surprisingly good agreement with empirical experimental estimates for YBa$_2$Cu$_3$O$_7$ [35, 37]. It would be interesting to see whether this procedure works for the two-chain compound YBa$_2$Cu$_4$O$_8$.

FIG. 1: Color online. (a) Fermion DOS [1] used to rationalize specific heat data. (b) and (c), taken from [28] with permission, triple peak structure in the spectral density $\rho_0(\omega, T = 0)$ calculated using NRG theory for the single orbital, asymmetric Anderson model with $S_{imp} = 1/2$, for various values of $-E_0/\Delta = 2\pi, 4, 3$ and 2. Here we estimate $-E_0/\Delta = 2.3$ for all the Cu ions in Bi2212. The horizontal arrows show the horizontal arrows show $E-E_0/\Delta = 2$ and 3 from [28]. Here the factor 2 ensures consistency between Eqn. 1 [22] and [28], details are given in [15].

FIG. 2: Color online. Partial DOS vs. energy $E-E_F$; in-plane oxygen orbitals of YBa$_2$Zn$_3$O$_7$, solid black line, $d$ orbitals for Zn(2) atoms in YBa$_2$Zn$_3$O$_7$ and Cu(2) atoms in YBa$_2$Cu$_3$O$_7$, short and long dashed lines respectively. Points show $N(E_F)$ of YBa$_2$Cu$_3$O$_7$ from LDA and from $\gamma$(300). When expressed as states/eV/Cu atom (both spins), $\gamma$(300) for YBa$_2$Cu$_3$O$_7$ gives 3.3±0.5, the same within errors as 2.9±0.2 states/eV/Cu atom (both spins) from $\gamma$(200) of Bi2212.
The complex band structure below $\approx -1$ eV shown in Figs. 2 and 6(SM)(b) prevents us estimating $\Delta$ from first principles, so initially we set $\Delta = 1$ eV, a typical value for 3$d$ impurities in noble metals [19, 20]. But, as explained in more detail in [15], we can account for the experimental value of the Wilson ratio if $E_0/\Delta$ = -2.3, because for this value, linear interpolation of the data in Table 1 of [28] gives the occupancy of the localized level, $n_0 = 0.81$ and an effective moment of 0.81$\mu_B$. This reduces the susceptibility by 0.81 and the Wilson ratio from the value 2 expected for an ion with $S=1/2$ to 1.31, in good agreement with experiment. Theoretical $S_c(T)$ curves were obtained by integrating the specific heat data in Fig. 4 of [28] divided by $T$. For $T_K = 800 K$, $\gamma$ increases by a factor 1.19 between $T = 200 K$ and $T = 0$, so the initial estimate of $T_K = 800 K$ from $\gamma$ (200) of Bi2212 is reduced to 672 K. Theoretical curves for representative samples of Bi2212, one with no PG, $p=0.19$ and one with a PG, are linear over the measured range of $T \lesssim 220 K$ as shown in Fig. 7(SM) [13]. In contrast those for YBa$_2$Cu$_4$O$_6.97$ with no PG and YBa$_2$Cu$_4$O$_6.48$ with $E_G/k_B = 320 K$, in Fig. 8(SM) [13], both having $T_K=775 K$, show deviations from linearity above 200 K. But these do not rule out the present picture because they could arise from a modest decrease in $T_K$ caused by thermal expansion, or because the Cu atoms in the CuO chains and the CuO$_2$ planes are inequivalent.

Hence for Bi2212 $T_K=672 K$ and for $E_0/\Delta = -2.3$, $k_BT_K/\Delta = 0.073$, and $\Delta = 0.79 eV$. The observation of significant $g$-factor anisotropy in YBCO [6] and references therein, supports the present localized picture. Here the concept of valency is meaningful, the Cu$^{2+}$ (d$^9$) state will be dominant and for $-E_0 = 2.3\Delta$ there is a certain admixture of Cu$^{3+}$ (d$^8$). The d$^{10}$ and d$^9$ states are analogous to those in heavy fermion Yb compounds with 0 or 1 hole in the 4f shell, which implies that all the Cu d electrons are contributing to $U$. As shown in Fig. 1(c) the half-width-half-maximum (HWWM) values of the spectral density at $\omega < 0$ for $-E_0/\Delta = 3$ and 2 are $\approx 0.076\Delta$ and 0.135$\Delta$ respectively giving 0.117$\Delta$ for $-E_0/\Delta = 2.3$. The carrier density in the resonance $n_K$, obtained by assuming that it has a Lorentzian shape for $\omega < 0$ and normalizing to the area of the lowest broad peak in Fig. 1(b) that contains $\approx 1$ electron, agrees well with $(1-n_0)/2 = 0.095$. For $\Delta = 0.79 eV$ the HWWM is $T_L=1072 K$. When transformed in the usual way into $k$-space and then by Fourier transform into $\tau$-space, and in the absence of a PG, the carrier density decays as $n_K \exp(-\tau/R)$ with a range $R = k_BT/\Delta$. The resonance is centered at the Fermi level and because of the sharpness of the Fermi surface, the Fourier transform could also give rise to Friedel or Rudermann-Kittel-Kasuya-Yosida type oscillations in $n_K(\tau)$, that could also be important but are not considered here. The Fermi velocity $v_F=1.6 10^7$ cm/sec, estimated for a cylindrical surface containing 1+p holes with $m^* = 5.2m_e$ [43], agrees well with nodal ARPES data for UD and optimally-doped (OP) crystals of the single layer cuprate Hg1201 shown in Fig. 2(b) of [44], 1.7, 2.2 and 2.0 10$^7$ cm/sec for UD70, UD80 and OP98. This agreement supports our statement that $T_K$ and hence $m^*$ vary little with $p$.

The above values of $v_F$ and $T_K$ give $R = 2.83a$. From the previous spatial symmetry argument, $n_K$ will have $|d_{x^2-z^2}|$ symmetry and be larger in the (0,1) and (1,0) directions, where $af$ spin fluctuations cause neighboring spins to be anti-parallel over a certain spatial range, $\xi$. A typical value for $p = 0.1$ is $\xi = 0.9 nm$, i.e. 2-3$a$ [14]. If $\xi = 2a$ then only the 4 nearest neighbors have correlated spins. Their $af$ Kondo screening “clouds” will give a ferromagnetic conduction electron spin polarization at the central site. Including the effect of the PG on both $n_K$ and $R$ (by self-consistently representing the PG by a narrower negative Lorentzian) reduces $n_K$ at the central site by a factor $\approx 2$, giving a magnetic moment from the 4 overlapping Kondo “clouds” of 2 x 0.095 x exp(-1/2.83)$\mu_B$ = 0.133$\mu_B$. It is difficult to prove that this spin polarization is equivalent to $J_{ex}$ in [42], but, for $\chi_s = 2.3 10^{-4}$ emu/mole-Bi2212, a spin polarization of 0.133$\mu_B$/Cu is produced by a field $H$ where $\mu_B H/k_B = 433 K$, so $J_{ex}/k_B = 866 K$. From Fig. 2 of [42] the half energy gap, equivalent to $E_G$ here, is $\approx J_{ex}$, in reasonable agreement with the experimental value $E_G/k_B \approx 600 K$ for $p = 0.1$. Furthermore, the imaginary part of the self energy is unusual and typical of a “bad metal” [42] in agreement with experiments on the cuprates. However, more work is needed to see whether $J_{ex}$ falls linearly with $p$ and whether calculations allowing for asymmetry in the DOS about $E_F$ on a scale of $J_{ex}$, i.e. $E_G$, would account for the strong $p$-dependence of the TEP and its scaling with $k_BT/E_G$ [11, 27], shown in Fig. 9(SM) [13].

Alternatively, in a related picture, the effect of the $af$ fluctuations, i.e. the interactions between neighboring Cu spins, could be considered directly, without appealing to the work of Ref. [42]. These will reduce the entropy of the Cu spins and hence that available for the Kondo resonance. Key theoretical questions are (a) can they cause a deep PG even when the interaction energy ($W$) is much smaller than $k_BT_K$ and (b) does $W$ fall linearly with $p$ and become very small for $p \ge p_{crit}$? Experimental evidence for (b) is given by ratio of the $^{63}Cu$ and $^{17}O$ NMR relaxation rates $T_1^{-1}$ [8] which is a measure of the strength of low frequency spin fluctuations for which the effect of the PG on $\chi_s$ cancels out. It does indeed fall linearly with $p$, approaching zero at $p_{crit}$. Analysis [15] of neutron scattering data shows the same decrease for $af$ spin fluctuations of energy below 50 meV, so on the time scale of the Kondo resonance, with $k_BT_K \approx 60$ meV, they will provide a molecular field that is nearly static.

Within the usual localized picture $W$ arises from superexchange via completely occupied or completely empty oxygen 2$p$ states. For a band with Fermi energy $E_F$, we should consider Wannier functions that
have a lifetime of order $h/E_F$. $E_F$ will increase with $p$, thereby reducing $W$, which could account for point (b) above. As sketched for example in Fig. 13 of [25], only those parts of the large Fermi surface spanned by the $a f$ wave vector $Q \approx (\pi, \pi)$, will be affected by $a f$ fluctuations. This accounts for the Fermi arcs, where $E_G = 0$, seen by ARPES [40] and scanning tunnelling microscopy (STM) [47]. It also gives a strong energy dependence of the scattering rate responsible for the TEP [25], see also [48], and its scaling with $k_BT/E_G$ shown in [25] and Fig. 9(SM) [11, 15].

Evidence for an energy scale $\approx 60$ meV in the many spectroscopic studies of cuprates above $T_c$ would be a good experimental test of the present picture. ARPES data do show structure near 50 meV, e.g. [44] that is presently ascribed to other factors. However, we are dealing with a many-body effect, so comparison of ARPES, STM [47] and optical data [49, 50] for cuprates with those for heavy fermion compounds as well as with theory could be informative. Another test would be the apparent correlation between the $T^4$ term in the electrical resistivity and the superfluid density [51] in OD compounds. One prediction of the present approach is that the pseudogap is the electronic entropy revealed by specific heat measurements [1, 2, 10], we propose that the pseudogap is the electronic entropy revealed by specific heat measurements [1, 2, 10], we propose that the pseudogap is the electronic entropy revealed by specific heat measurements [1, 2, 10].

In summary, guided by the unusual behavior of the electronic entropy revealed by specific heat measurements [1, 2, 10], we propose that the pseudogap is the energy scale over which a Kondo-like enhanced DOS at the Fermi energy is suppressed by $a f$ spin interactions.

The author would like to thank A. Carrington, who also calculated the band structure of YBa$_2$Zn$_3$O$_7$, J. L. Tallon and V. Zlatić for helpful suggestions and discussions and to acknowledge a long and happy collaboration with the late Dr. J. W. Loram.

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