The role of jets as transport barriers in the Earth’s stratosphere

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Abstract. Theoretical results relating to Kolmogorov–Arnold–Moser (KAM) theory, particularly those dealing with the stability of degenerate one-degree-of-freedom Hamiltonian systems under a time-quasiperiodic perturbation, have led to the expectation that, independent of the background Potential Vorticity (PV) distribution, associated with zonal jets should be barriers which inhibit meridional transport. In a recently submitted paper (Beron-Vera et al., 2011) the aforementioned expectation is confirmed based on the analysis of isentropic winds in the lower stratosphere as produced by a comprehensive general circulation model. Concretely, barriers for meridional transport are found to be associated with the (eastward) austral polar night jet, for which the meridional gradient of background PV is very steep, and also for the (westward) boreal summer subtropical jet, for which the background PV is rather uniform. The identification of the meridional transport barriers is based on the computation of Finite-Time Lyapunov Exponents (FTLE), which measures the amount of stretching about fluid particle trajectories. The meridional transport barriers, which are formed by fluid particle trajectories lying on invariant tori, are identified with topologically circular locally minimizing curves or trenches of the backward-plus-forward FTLE field. Results from explicit passive tracer advection experiments and flux computations are also presented, which confirm those inferred using the FTLE diagnostic. In this paper I present results based on the analysis of analytically prescribed stratospheric model winds, which complement the results presented in Beron-Vera et al. (2011) and further allow one to better assess the scope of the FTLE diagnostic.

1. Introduction

In a Potential Vorticity (PV) flow with a steep meridional gradient of background PV, the tendency of the background flow to support Rossby waves has provided sustain for the notion of a “PV barrier” (e.g., McIntyre, 2008). The PV barrier notion has been traditionally used to explain why an eastward zonal jet, which can be associated with a steep meridional gradient of background PV, can behave as a meridional transport barrier. A clear cut example of such behavior is given by the austral polar night jet in the Earth’s lower stratosphere, which prevents ozone-depleted air within the stratospheric polar vortex from spreading to lower latitudes.

The PV barrier notion cannot be applied to westward zonal jets, which, unlike eastward zonal jets, are not possible to be associated with steep meridional gradients of background PV. Yet this does not rule out the possibility that these zonal jets also behave as meridional transport barriers. Examples of westward zonal jets in the lower stratosphere are the boreal and austral summer subtropical jets. Observational evidence suggesting the presence of meridional transport barriers associated with these zonal jets is extensive. The observational evidence comes from...
measurements of the concentration of water vapor, nitrous oxide, and aerosols including those injected into the lower stratosphere by volcanic eruptions, most notably the Mount Pinatubo eruption. A recent review on stratospheric transport including a discussion on transport barriers in the summer subtropical lower stratosphere is provided in Shepherd (2007).

In a recently submitted paper, Beron-Vera et al. (2011) provide evidence of the expectation that, independent of the background PV distribution, associated with zonal jets should be meridional transport barriers. In addition to the above observational evidence, recent application (Rypina et al., 2007a,b; Beron-Vera et al., 2008, 2010) of novel results relating to Kolmogorov–Arnold–Moser (KAM) theory (Arnold et al., 2006) provide support for this expectation. The evidence presented in Beron-Vera et al. (2011) is based on the analysis of isentropic winds in the lower stratosphere as produced by a comprehensive general circulation model. The specific model considered is the Canadian Middle Atmosphere Model (CMAM) (Beagley et al., 1997), which is capable of describing the features of the seasonal circulation in the lower stratosphere most relevant for the study carried out in Beron-Vera et al. (2011). Such features are: the seasonal formation and decay of a polar vortex in the winter hemisphere with an associated eastward zonal jet, at, and a marked PV contrast across, its perimeter; and the seasonal formation and decay of a westward zonal jet in the summer subtropics with mostly uniform background PV distribution associated. Consistent with the KAM theory predictions it is found that associated with both the (eastward) austral polar night jet and the (westward) boreal summer subtropical jet are invariant tori [finite-time generalizations of invariant tori or elliptic Lagrangian Coherent Structures (LCS; cf. Haller & Yuan, 2000) to be more precise] which serve as meridional transport barriers. An additional meridional transport barrier is found to be associated with the (westward) austral summer subtropical jet, while no evidence of a transport barrier in winter polar region is found consistent with the absence of a well-developed and persistent boreal polar night jet in the simulation considered. The identification of invariant tori is carried out using the Finite-Time Lyapunov Exponent (FTLE) diagnostic, which is commonly employed to detect LCS of hyperbolic type (the locally strongest repelling or attracting material fluid curves; cf. Haller, 2011). Unlike hyperbolic LCS, which tend to produce ridges in the backward or forward FTLE field, twistless invariant tori tend to produce topologically circular trenches in the backward-plus-forward FTLE field (Beron-Vera et al., 2010). The transport barrier nature of the identified FTLE trenches is tested by executing explicit passive tracer advection calculations. Also, an estimation is presented of the flux across the identified FTLE trenches, which is found to be consistently small.

In this paper I present results based on the analysis of analytically prescribed stratospheric model winds. Specifically, the model corresponds to an two-dimensional incompressible flow describing a meandering zonal jet perturbed by Rossby-like waves. The results presented complement those presented in Beron-Vera et al. (2011) by further allowing one to better assess the range of validity of the FTLE diagnostic in detecting invariant-tori-like LCS.

2. Background

2.1. Setup

Fluid particle motion in a two-dimensional incompressible flow satisfies

\[
\frac{dx}{dt} = -\frac{\partial \psi}{\partial y}, \quad (1a)
\]

\[
\frac{dy}{dt} = +\frac{\partial \psi}{\partial x}, \quad (1b)
\]

where \(\psi(x, y, t)\) is the streamfunction, \((x, y) \in \mathbb{R}^2\) denotes position, and \(t \in \mathbb{R}\) is time. Coordinate \(x\) is considered to be a zonal coordinate and thus can be regarded as a periodic
coordinate, i.e., \( x/a \cos \vartheta_0 \in \mathbb{T} = \mathbb{R}/2\pi \mathbb{Z} \) where \( a \) is the planet’s mean radius and \( \vartheta_0 \) is a reference latitude. In turn, coordinate \( y \) is considered to be a meridional coordinate. Although \( y \) is formally unbounded, fluid particle motion is assumed to depart from a reference \( y \) by distances which are much smaller than \( a \).

The streamfunction will be assumed to take the form

\[
\psi(x, y, t) = \psi_0(x, y) + \psi_1(x, y, \sigma_1 t, \ldots, \sigma_N t),
\]

(2)

where \( \sigma_n t \in \mathbb{T} \) and thus \( \psi_1 \) depends multiply periodically on time. The existence of a streamfunction of this form can be physically motivated by arguing that the flow is composed of a steady zonal background jet that is perturbed by a superposition of possibly many (e.g., Rossby-like) waves:

\[
\psi(x, y, t) = A_0(y) + \sum_{n=1}^{N+1} A_n(y) \cos[k_n(x - c_n t) + \phi_n].
\]

(3)

Restricting (3) be a multiply periodic function of time might seem to be restrictive. By contrast, the class of perturbations described by the wave superposition in (3) is very large. Indeed, with an appropriate choice of amplitudes and phases, the class of describable perturbations ranges from a deterministic Rossby-like wave packet (small \( N \), narrow spectrum), to an approximation to white noise turbulence (large \( N \), flat spectrum, random phases).

As is customary, the time dependence associated with one of the waves in (3) can be eliminated by viewing the flow in a reference frame uniformly translating in the zonal direction at the speed of that wave. The choice of which wave to absorb into the background flow is arbitrary. In the reference frame moving at speed \( c_{N+1} \) the streamfunction takes the form

\[
\tilde{\psi}(\tilde{x}, y, t) = \tilde{\psi}_0(\tilde{x}, y) + \tilde{\psi}_1(\tilde{x}, y, \sigma_1 t, \ldots, \sigma_N t)
\]

(4a)

where \( \tilde{x} := x - c_{N+1} t \mod 2\pi a \cos \vartheta_0 \) is the wave-comoving zonal coordinate and

\[
\tilde{\psi}_0 = c_{N+1} y + A_0(y) + A_{N+1}(y) \cos(k_{N+1} \tilde{x} + \phi_{N+1}),
\]

(4b)

\[
\tilde{\psi}_1 = \sum_{n=1}^{N} A_n(y) \cos(k_n \tilde{x} - \sigma_n t + \phi_n),
\]

(4c)

where \( \sigma_n := k_n(c_n - c_{N+1}) \).

2.2. KAM theory

System (1) constitutes a nonautonomous one-degree-of-freedom Hamiltonian system with \( \psi \) playing the role of the Hamiltonian and \((y, x)\) the coordinate–conjugate momentum pair. In the wave co-moving frame, this system reduces to the so-called near integrable class, which is the class of systems that KAM theory can tackle. Indeed, a further change of coordinates to so-called action–angle \((I, \theta)\) variables can always be carried out in a piecewise fashion. Such variables provide the most succinct description of the dynamics, and are commonly employed to pose KAM theory results. The canonical transformation \((y, x) \mapsto (I, \theta)\) is defined implicitly as follows:

\[
I = \frac{1}{2\pi} \oint \tilde{X}(y; \tilde{\psi}_0) \, dy,
\]

(5a)

\[
\theta = \frac{\partial G}{\partial I},
\]

(5b)

\[
G(y, I) := \int_0^y \tilde{X}(\xi; \tilde{\psi}_0) \, d\xi,
\]

(5c)
where $\tilde{X}$ is the wave co-moving zonal coordinate of an isoline of $\tilde{\psi}_0$. It is important to realize that associated with each particular fluid particle trajectory is one value of $I$. In other words, $I$ acts as label for a particular fluid particle trajectory. According to transformation (5), the background and perturbation Hamiltonians transform, respectively, as

$$
\tilde{\psi}_0(\tilde{x}, y) = H_0(I),
$$
(6a)

$$
\tilde{\psi}_1(\tilde{x}, y, \sigma_1 t, \cdots, \sigma_N t) = H_1(I, \theta, \sigma_1 t, \cdots, \sigma_N t).
$$
(6b)

The action–angle variables evolve according to:

$$
\frac{dI}{dt} = - \frac{\partial H_1}{\partial \theta},
$$
(6c)

$$
\frac{d\theta}{dt} = \omega(I) + \frac{\partial H_1}{\partial I},
$$
(6d)

where

$$
\omega(I) := H_1'(I).
$$
(6e)

When $H_1 = 0$, the Hamiltonian $H_0$ is a first integral and the motion described by (6) is completely integrable. The latter means that (6) can be solved by quadrature. Trajectories are then seen to lie on one-dimensional tori $\{I = \text{const}\}$. The frequency of the periodic motion along a torus is given by $\omega(I)$.

When $H_1 \neq 0$, the Hamiltonian $H_0 + H_1$ is in general not a first integral. In such a case, the motion obeying (6) is not integrable and trajectories may be irregular (i.e., different than periodic or quasiperiodic). However, under sufficient smoothness assumptions on $H_0$ and $H_1$, if $|H_1|$ is sufficiently small, the frequencies $\{\sigma_n\}$ are sufficiently irrational, and $\omega(I)$ is not a constant, then system (6) admits a measure-theoretically large set of invariant tori close to $\{I = \text{const}\}$ with frequency $\varpi(I)$ close to $\omega(I)$ which vibrate with frequencies $\{\sigma_n\}$. This is an informal statement of a corollary of the KAM theorem proved in Sevryuk (2007).

A few remarks are in order.

(i) Each admitted torus is of maximal dimension. That is, each admitted torus divides the phase space into two disconnected regions with well-defined inside and outside. Thus any trajectory starting in between any two such tori is restricted to remain in between them for ever. In other words, the admitted tori act as barriers that inhibit the meridional transport of passive tracers.

(ii) The condition on the frequency mapping, commonly referred to as a nondegeneracy condition, differs from the standard, so-called Kolmogorov, nondegeneracy condition which requires that the frequency mapping’s twist does not vanish, i.e., $\omega'(I) \neq 0$. The nondegeneracy condition involved in the KAM theorem cited above is a very weak nondegeneracy condition of the Rüssmann type (Rüssmann, 1989), which allows one to prove the existence of many invariant tori even when the Kolmogorov nondegeneracy condition is locally violated, i.e., when $\omega'(I) = 0$ at isolated $I$-values. The importance of this is that such situations are found in the presence of zonal jets: at the core of a zonal jet the frequency of the fluid particle motion has a local maximum, and hence $\omega'(I) = 0$ there.

(iii) Unlike the standard KAM theorem, the nonstandard KAM theorem above does not allow one to relate the frequency along the admitted perturbed tori with that along the unperturbed tori. This prevents one from speaking about “persistence” of invariant tori as in standard KAM theory (Sevryuk, 1995). But this has no consequence for the purposes here as what really matters is that the above KAM theorem guarantees the admittance of invariant tori, and that such invariant tori act as meridional transport barriers.
In Rypina et al. (2007a) it was further argued that tori lying in the vicinity of a twistless torus are very resistant to breaking. This particular form of stability has been referred in Rypina et al. (2007b) to as “strong KAM stability.” The basic rationale for the strong KAM stability is as follows. Torus destruction is caused by the excitation of resonances, which occurs when $\omega(I)/\sigma_n \in \mathbb{Q}$ for some rational $n$. The overlapping of resonances leads to the destruction of tori that are captured in the resonance. The width in action $I$ of a resonance at a twist torus, i.e., for which $\omega'(I) \neq 0$, scales like

$$\delta I_{\text{twist}} \sim |\omega'(I)|^{-1/2},$$

while the distance (in action) between nearby resonances scales like

$$\Delta I \sim |\omega'(I)|^{-1}$$

(Chirikov, 1979). Consequently, when $|\omega'(I)|$ is small the distance between nearby resonances (8) is larger than the width of each individual resonance (7), which reduces the possibility of resonance overlapping and thus torus destruction. In Rypina et al. (2007b) it is shown that at a twistless torus of order $m$, i.e., for which $\omega'(I) = \cdots = \omega^{(m-1)}(I) = 0$, $\omega^{(m)}(I) \neq 0$, where $m > 1$, the resonance width scales like

$$\delta I_{\text{twistless}} \sim |\omega^{(m)}(I)|^{-1/(m+1)}.$$

Clearly, the width of a resonance at a twistless torus (9) is smaller than that at a torus with twist (7). Because resonances are excited at isolated action values and resonance widths are narrower near twistless tori, tori lying in their vicinity are expected to be quite robust. As noted above, $\omega'(I) = 0$ at the core of a zonal jet. The strong KAM stability argument then predicts that invariant tori near the core of a zonal jet are very robust, and that these invariant tori act as meridional transport barriers.

2.3. Model

The specific flow model considered here is given by a perturbed form of the so-called Bickley jet (Drazin & Howard, 1966), which has been studied by various authors (del-Castillo-Negrete & Morrison, 1993; Kovalyov, 2000; Rypina et al., 2007b; Beron-Vera et al., 2010). Specifically, the model has

$$A_0(y) = -UL \tanh \frac{y}{L}, \quad A_n(y) = \varepsilon_n U \text{sech}^2 \frac{y}{L}, \quad k_n = \frac{n}{a \cos \vartheta_0}, \quad \phi_n = 0, \quad n = 1, 2, 3.$$  

The specific choices of the parameters that define the background streamfunction in (4b) are given by: $a = 6371$ km, $\vartheta_0 = -\pi/3$, $U = 62.66$ m s$^{-1}$, $L = 1770$ km, $\varepsilon_3 = 0.3$, $c_3/U = 0.461$. With this choice of parameters, the background streamfunction is characterized by a spatially periodic, meandering, eastward zonal jet which is flanked northward and southward by three stationary eddies. As shown in the left panel of Fig. 1, the structure of $\omega(I)$ near the central jet has a global maximum at the axis of the jet ($I = 0$). The torus $I = 0$ violates the nontwist condition $\omega'(I) \neq 0$, and thus is expected to be quite robust under perturbation as a consequence of strong KAM stability.

The specific choices for the parameters that define the streamfunction perturbation in (4c) are given by: $\varepsilon_1 = 0.075$, $\varepsilon_2 = 0.4$, $c_2/U = 0.205$, and $\sigma_1/\sigma_2 = (\sqrt{5} - 1)/2$. The choice of frequency ratio as the golden mean guarantees that the perturbation depends quasiperiodically on time, which is consistent with the assumptions of the KAM theorem.

With the above specific parameters, the model was employed by Rypina et al. (2007a) to investigate the mechanism by which the (westward) austral polar night jet serves as a barrier to the meridional transport of ozone depleted stratospheric air. With a different choice of parameters, the role of (eastward) summer subtropical jets as transport barriers may also be investigated.
2.4. Transport barrier identification

In Beron-Vera et al. (2010) it was noted that twistless invariant tori produce topologically circular locally minimizing curves or trenches in the field resulting from computing backward-plus-forward FTLE over a lattice of initial fluid particle positions. This provides a means for identifying twistless invariant tori (finite-time generalizations thereof or elliptic LCS to be more precise) in the absence of traditional phase space visualization methods, such as the Poincaré section. Such methods are not available when the time dependence of the velocity field is different than periodic or when the velocity is defined over a finite-time interval, such as the case of numerically generated or observed velocity data or the case of a process that has a finite-time duration.

The FTLE is given by

$$\Lambda^t_\tau (x) := |\tau|^{-1} \ln \| \nabla F_t^{t+\tau}(x) \|.$$ (11)

Here $\| \|$ denotes spectral norm and $F_t^{t+\tau}$ is the flow map that takes a fluid particle position at time $t$ to its new position at time $t + \tau$.

The rationale behind the above identification method is as follows. Because invariant tori are composed of regular trajectories, their infinite-time LE, computed in either forward or backward time, are (typically) zero. For regular trajectories FTLE estimates are $O(|\tau|^{-1} \ln |\tau|)$. For long $|\tau|$ these estimates approach zero, but, in practice, for short $|\tau|$ it can be very difficult to distinguish regular trajectories from irregular trajectories whose LE are small but nonzero. However, this finite-time ambiguity is effectively eliminated for twistless invariant tori, which constitutes the basis for the above identification method. To see this, consider the completely integrable case, i.e.,

$$\frac{dI}{dt} = 0,$$  \hspace{0.5cm} (12a)

$$\frac{d\theta}{dt} = \omega(I),$$  \hspace{0.5cm} (12b)

for which

$$\Lambda = \frac{1}{2|\tau|} \ln \left( \frac{2 + \omega'(I)^2 \tau^2}{2} + \sqrt{\left( \frac{2 + \omega'(I)^2 \tau^2}{4} \right)^2 - 1} \right).$$  \hspace{0.5cm} (13)

As expected $\lim_{\tau \to \pm \infty} \Lambda = 0$. But for finite $\tau$, $\Lambda$ increases as $\omega'(I)$ increases (and vanishes when $\omega'(I) = 0$). For finite $\tau$, the tori for which $|\omega'(I)|$ is maximized produce ridges in the FTLE field. Moreover, $\Lambda \sim |\tau|^{-1} \ln |\omega'(I)\tau|$ for fixed $\omega'(I) \neq 0$ as $\tau \to \pm \infty$, which tends to 0 slowly. This may make it difficult to identify regular trajectories. But this ambiguity is reduced when $|\omega'(I)|$ is small. Twistless tori are thus the ones that can be most easily detected using FTLE as these will tend to produce (topologically circular) trenches of backward-plus-forward FTLE field.

3. Results

The right panel of Fig. 1 shows a backward-plus-forward FTLE field at $t = 0$ computed using $|\tau| = 23$ d. This FTLE reveals a well-defined topologically circular (recall that the wave-comoving zonal coordinate $\tilde{x}$ is periodic) trench meandering about $y = 0$. This trench is indicated by the nearly coincident green and red curves, which correspond to the trenches extracted from the forward and backward FTLE fields, respectively. According to the above described KAM theory results, the trench in the backward-plus-forward FTLE field should correspond to an invariant-torus-like LCS. This follows from the observation that the frequency of the motion along the axis of the meandering jet is maximal (Fig. 1, left panel). That is, the twist of
Figure 1. (left panel) Frequency of the motion as a function of the action within the Bickley jet perturbed with one wave. The action serves as a label for the tori (streamlines) within the resulting steady, but meandering, jet in a frame uniformly translating at the speed of the wave. (right panel) Backward-plus-forward FTLE field for the Bickley jet perturbed with three waves at $t = 0$ with integration time $\tau = \pm 23$ d. The green (red) curve indicates a locally minimizing curve or trench in the backward (forward) FTLE field.

the frequency mapping vanishes along the jet’s axis. This reduces the possibility of resonance overlapping, the mechanism by which invariant tori can be destroyed. That the identified trench behaves, at least approximately, as an invariant-torus-like LCS is demonstrated in the sequel by showing that the flux across the trench is negligibly small.

Let $x_0 = X_{t_0}(s)$, where $s$ varies in a certain domain of $\mathbb{R}$, be a local parameterization of the FTLE trench at time $t = t_0$. The instantaneous area flux per unit length across a point on the moving FTLE trench is equal to the projection of the velocity relative to the moving curve in the normal direction to the curve at that point. Namely,

$$\varphi(X_{t_0}(s), t_0) = \left( u(X_{t_0}(s), t_0) - \frac{dX_{t_0}(s)}{dt_0} \right) \cdot \hat{n}(X_{t_0}(s), t_0),$$  \hspace{1cm} (14)

which vanishes if the FTLE trench is a material curve. Because at any time $t = t_0$ each of the FTLE trenches of interest here is describable by an invertible function $y_0 = \eta(\tilde{x}_0, t_0)$, formula (14) can be readily computed as follows. First, note that

$$\frac{dX_{t_0}(s)}{dt_0} = \alpha(x_0, t_0) \tilde{x} + \left( \alpha(x_0, t_0) \frac{\partial \eta(\tilde{x}_0, t_0)}{\partial \tilde{x}_0} + \frac{\partial \eta(\tilde{x}_0, t_0)}{\partial t_0} \right) \tilde{y},$$  \hspace{1cm} (15)

where $\alpha(x_0, t_0)$ is arbitrary. Second, note that

$$\hat{n}(X_{t_0}(s), t_0) = \frac{\tilde{y} - \partial \eta(\tilde{x}_0, t_0)/\partial \tilde{x}_0 \tilde{x}}{\sqrt{1 + (\partial \eta(\tilde{x}_0, t_0)/\partial \tilde{x}_0)^2}},$$  \hspace{1cm} (16)

As a result,

$$\varphi(X_{t_0}(s), t_0) = v(\tilde{x}_0, \eta(\tilde{x}_0, t_0), t_0) - u(\tilde{x}_0, \eta(\tilde{x}_0, t_0), t_0) \frac{\partial \eta(x_0, t_0)}{\partial \tilde{x}_0} - \frac{\partial \eta(\tilde{x}_0, t_0)}{\partial t_0},$$  \hspace{1cm} (17)
where \( u (v) \) is the zonal (meridional) velocity component. Figure 2 shows evaluations of (17) across trenches in the backward (left panel) and forward (right panel) FTLE fields at \( t = 0 \) for different choices of the integration time \( \tau \). Note that as \( |\tau| \) increases, the area flux per unit length decreases. For all choices of \( \tau \) considered, the resulting flux is smaller compared with the flux across \( y = 0 \), indicated in each panel by the black curve. In particular, for \( |\tau| = 23 \) d the flux is negligibly small, which is in good agreement with the KAM theory prediction.

The reason why the flux calculation with \( |\tau| = 5 \) d produces larger values than that using \( |\tau| = 23 \) (or 11 d) is explained by the fact that for such a short integration time the FTLE technique does not allow one to locate with sufficient precision the invariant-torus-like LCS in question. This can be seen in Fig. 3, which reveals that with \( |\tau| = 5 \) d the trenches in the backward and forward FTLE fields do not coincide in position as precisely as with \( |\tau| = 11 \) or 23 d. Although the mismatch is small, at least to the naked eye, its implication for the localization of the transport barrier associated with the jet can be important. Indeed, isentropic mixing can be considered negligible only up to about 30 d (Haynes, 2005). As a result, \( |\tau| = 23 \) d, which appears to be long enough to achieve a good localization of the transport barrier, may be actually too long to be physically achievable. This imposes a constraint in the ability of the FTLE technique at precisely locating invariant-torus-like LCS. It must remain clear, however, that at least an indication of the presence of such transport barriers, which is nevertheless what may matter in practice, can be attained with small integration time.

4. Conclusions

In this paper results that complement those presented in the recently submitted paper by Beron-Vera et al. (2011) have been presented. In Beron-Vera et al. (2011) evidence is provided for the expectation that, independent of the background Potential Vorticity (PV) distribution, associated with zonal jets should be barriers that suppress the meridional transport of tracers. The expectation is based on novel results from Kolmogorov–Arnold–Moser (KAM) theory. Such novel KAM theory results, reviewed here, suggest that fluid particle motion in the vicinity of the axis of a jet, whether westward (i.e., with a large meridional gradient of PV associated) or eastward (i.e., with a small meridional gradient of PV associated), lies on invariant tori. Such invariant tori are material curves or Lagrangian Coherent Structures (LCS) which prevent meridional transport. The rationale for their existence is that resonance overlapping, the
mechanism that explains the destruction of invariant tori, is reduced near the axis of a jet as a result of the maximization of the frequency of the fluid particle motion. The evidence presented in Beron-Vera et al. (2011) is based on the analysis of stratospheric winds produced by the Canadian Middle Atmosphere Model (CMAM), a comprehensive general circulation model. The results presented here are based on an analytically prescribed model wind. The model represents an meandering zonal jet perturbed by Rossby-like wave modes. The results exemplify the use of identification technique of invariant-tori-like LCS proposed by Beron-Vera et al. (2010). The technique, reviewed here, is based on the computation of the Finite-Time Lyapunov Exponents (FTLE). Invariant-tori-like LCS are identified with topologically circular locally minimizing curves or trenches in the backward-plus-forward FTLE field constructed by computing the FTLE for many initial fluid particle positions. The results from the computation of instantaneous area fluxes per unit length reveal that localization of invariant-tori-like LCS depends on the length of the time interval over which the FTLE is computed. Although long integration time seems to guarantee a good localization, such a good localization may not be achieved when dealing with finite-time records of velocity data or when the processes of interest have a relatively short-time duration. A new identification technique which is not constrained by the integration time is thus desirable. Progress along this line will be reported elsewhere (Haller & Beron-Vera, 2011).

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