Measuring network resilience through connection patterns

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Abstract

Networks are at the core of modeling many engineering contexts, mainly in the case of infrastructures and communication systems. The resilience of a network, which is the property of the system capable of absorbing external shocks, is then of paramount relevance in the applications. This paper deals with this topic by advancing a theoretical proposal for measuring the resilience of a network. The proposal is based on the study of the shocks propagation along the patterns of connections among nodes. The theoretical model is tested on the real-world instances of two important airport systems in the US air traffic network; Illinois (including the hub of Chicago) and New York states (with JFK airport).

Keywords: networks; resilience; paths; weighted arcs; air traffic systems.

1 Introduction

Networks are an important tool used to describe and analyze the structure and dynamical behaviors of several complex systems found in the real world. In particular, network

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representation applies to transportation systems such as airline routes \cite{12} and railroads \cite{23}; large communication infrastructures such as the World-Wide-Web \cite{5} and the Internet \cite{15}; biological systems \cite{4} such as gene and protein interaction networks \cite{22} and many social interaction structures \cite{9}, \cite{16}.

In the framework of networks, the interacting elements are depicted as nodes in the system and their interactions are represented as links connecting the nodes. A relevant issue relies to the behavior of networks with respect to the occurrence of external shocks.

In this respect, it is possible to distinguish the network’s robustness, i.e. the capacity of the network to perform its basic functions even in the event of missing elements (nodes or links) or, to phrase this concept differently, the ability of the system to tolerate strains and to maintain its functionality in some way; and the network’s resilience, i.e. the ability of a system to return to its normal state after a disturbance due to internal or external shocks by altering its processes while continuing to perform \cite{6}, \cite{17}.

Literature on networks resilience is rather wide. At its core, shocks are interpreted as the removal of nodes on the basis of several different criteria (see for example: \cite{3}, \cite{10}, \cite{11}, \cite{17}).

However, engineering applications reveal that shocks can not be associated forcefully to the disappearance of nodes. One might experience the occurrence of a shock as a perturbations whose size proxies the shock’s relevance. An example can be the loss of capacity of an electricity plant due to adverse weather conditions and the corresponding damage of the entire energy system.

The network resilience is considered crucial for the different practical approaches whereby the criticalities related to the eventual failure of nodes and links and by means of overall system tolerance. Disruptive events, whether a natural disaster, intentional attacks or common failures, can have relevant impacts when they lead to the failure of network elements. Indeed, network performances are directly related to their resilience and thus to the abilities of networks in tolerating loss of important elements such as bridges or hubs \cite{11}.

Resilience has been studied across several different network structures and there is now knowledge regarding how specific types of networks react to specific kinds of losses.
Studies about this topic have considered a large variety of structural failures, both induced by attack or naturally occurring, which involve nodes chosen by their central position in the system, as well as those that are random.

However, the current literature on how to measure resilience is very limited [7]; [19]; [26] with most contributions focusing on the remaining part of the network after the damage. For example, some authors measured resilience through the number of node failures and link failures that can occur for a system that is still functioning [20]; others proposed an approach to evaluate the logistics of network resilience based on the redundant resource and reliability of nodes and links [25]; Ash and Newth [1] used an algorithm to evolve the complex systems that are resilient to cascading failure; Wang et al., [24] proposed a measure based on the recovery ability of a system.

In this context, it is therefore relevant to analyze how an external shock that affects a node propagates along the network paths.

The aim of this paper is to introduce a new analytical measure of resilience for weighted networks that can be used when external shocks occur. The idea is to subject the network to something akin to a load test and check how it responds.

Thus, we consider the resilience of a network as its ability to absorb a shock. We introduce the concept of shock and the modality of its propagation, which is assumed to depend strongly on the weights of the arcs. The presence of weights along the links allows us to take into account the relevance given to the length of the paths between the nodes.

The intuitive basic idea is that a shock occurs at a single node and it propagates from there over all the arcs, starting from the involved node. Once the shock reaches the adjacents, it then proceeds to the adjacents of the adjacents, and so on. There is a required condition for the propagation, based on the weights on the arcs. Substantially, the shock propagates only if the propagating arcs have sufficient weight. Moreover, we also assume that an amplifying/damping effect on the shock is in force on the basis of the distance from the originating node. Details will be provided below.

We test the proposed measure of resilience on two empirical networks extracted from the network of US commercial airports: the Illinois state network and the New York state network.
The paper is organized as follows. Section 2 provides preliminaries and notations. Section 3 focuses on the conceptualization of the proposed measure of the resilience. Section 4 shows the analysis of the resilience on two empirical networks. Section 5 offers concluding remarks.

2 Preliminaries and notations

A network is an analytical-geometrical structure which is able to describe a set of interconnected elements in a unified system. The basis of a network is a graph \( G = (V, E) \), where \( V \) is the set of \( n \) nodes (or vertices) and \( E \) is the set of \( m \) links (or arcs). The links formalize the connections among the nodes.

Two generic nodes will be denoted by two integers \( i, j = 1, \ldots, n \). The presence of a link between the nodes \( i \) and \( j \) will be captured through a binary variable \( a_{ij} \) which is 1 or 0 if the link does exist or not, respectively.

In so doing, we can fully identify \( E \) through the adjacency matrix \( A = (a_{ij})_{i,j=1,\ldots,n} \). The proposed setting is grounded on networks which are weighted and oriented. Specifically, two possible arcs connect \( i \) and \( j \): the one from \( i \) to \( j \) and the one from \( j \) to \( i \). Accordingly, we will hereafter denote the arc from \( i \) to \( j \) by \( i \to j \). Moreover, arcs are weighted by positive numbers which proxy the level of interaction among the nodes. We denote the weight of the arc \( i \to j \) by \( w_{ij} \). In general, \( w_{ij} \neq w_{ji} \). By assuming that \( a_{ij} = 0 \) if and only if \( w_{ij} = 0 \), we can replace the matrix \( A \) with the more informative weighted adjacency matrix \( W = (w_{ij})_{i,j=1,\ldots,n} \). Indeed, by employing such a matrix, one can simultaneously identify the existing links and their corresponding weights.

The network is then \( N = (V, W) \).

Two arcs are said to be consecutive if they share exactly one of the two nodes compounding them. Two consecutive arcs form a path of length 2 or, simply, a 2-path. A generic 2-path with starting node \( i_0 \in V \) can be written as \( p^{(2)}_{i_0} = \{i_0 \to i_1 \to i_2\} \), with \( i_0, i_1, i_2 \) being three distinct nodes in \( V \) and \( i_0 \to i_1, i_1 \to i_2 \in E \). Given an integer \( k \geq 3 \), we can extend such a definition to the case of \( k \)-paths in a natural way. A generic \( k \)-path with starting node \( i_0 \in V \) is \( p^{(k)}_{i_0} = \{i_0 \to i_1 \to \cdots \to i_{k-1} \to i_k\} \), where \( i_0, i_1, \ldots, i_{k-1}, i_k \) are distinct nodes in \( V \) and \( i_{h-1} \to i_h \in E \), for each
$h = 1, \ldots, k$.

Under this definition, a 1-path is simply an arc in $E$.

We collect all the $k$-paths of $N$ with starting node $i_0$ in a set $P^{(k)}(i_0)$. All the $k$-paths of the network will be collected in

$$P^{(k)} = \bigcup_{i_0 \in V} P^{(k)}(i_0).$$

(1)

The weight of a $k$-path can be defined by the vector of the weights of the arcs composing it. We denote such a weight by $w(p^{(k)}_{i_0}) = (w_{i_0i_1}, w_{i_1i_2}, \ldots, w_{i_{k-1}i_k})$. A suitable aggregation of the components of the weight of $p^{(k)}_{i_0}$ will be discussed in the next section, in the context of the propagation of the shocks in the network.

We also denote by $\bar{k}$ the length of the longest $k$-path of the network.

3 The measure of the resilience

This section is devoted to the conceptualization of the measure of the resilience of the network.

The resilience of a network describes its ability to absorb a shock. Therefore, in order to provide a definition of resilience, one needs to introduce the concept of shock and also the way in which it propagates in the network.

A shock is an external solicitation of one of the nodes of the network. Such a local occurrence is able to destabilize the entire network by propagating over the other nodes of the network. The destabilization of the network is strongly dependent on how shocks propagate and, of course, on the entity of the shock. Indeed, shocks are not identical, and they can be of different size. We measure the size of a shock by a scalar $\xi \in [0, +\infty)$, with the conventional agreement that the higher the value of $\xi$, the more severe the occurred shock. The case $\xi = 0$ is included for the sake of completeness, and it means no shock.

In this framework, we assume that the propagation of a shock occurring at a given node $i_0$ follows the route traced by the $k$-paths with starting node $i_0$, and depends on the values of the weights of the arcs of the $k$-path and on the distance of the nodes of the $k$-path from the shocked node.

We enter the details.
Consider an integer $k$ and a $k$-path having, as a starting node, a node with a shock, namely $p_{i_0}^{(k)} = \{i_0 \to i_1 \to \cdots i_{k-1} \to i_k\} \in \mathcal{P}^{(k)}(i_0)$. Hypothetically, the shock propagates on a sequential basis: from $i_0$ to $i_1$, from $i_1$ to $i_2$ and so on, till the last node $i_k$. In real practice, the shock propagation is dampened or amplified – it depends on the specific context – as the distance from the starting node grows. As we will see, such a damping/amplifying effect might eradicate/exacerbate the action of the shock at nodes that are far enough from $i_0$.

The way the shock propagates is assumed to be captured by the weight $w(p_{i_0}^{(k)})$. This is a natural requirement, in that the propagation of a shock from a node to another one has necessarily to be related to the entity of the interaction between them.

According to these arguments, we introduce a discount factor $\delta \in [0, +\infty)$, which describes the propagation motion of a shock with size $\xi$ on $p_{i_0}^{(k)}$ in a discrete time framework, as follows:

- at time $t = 0$, the shock with size $\xi_0 = \xi$ occurs at $i_0 \in V$;
- at a generic time $t = h$, with $1 \leq h \leq k$, the shock propagates to $i_h$ with size $\xi_h$, with $\xi_h = [\xi_{h-1} + w_{i_{h-1}i_h}]\delta = [\xi + \sum_{s=1}^{h-1} w_{i_{s-1}i_s}\delta^{h-s+1}]\delta$. If $h < k$, then the shock propagates to node $i_{h+1}$ only if a propagation condition is satisfied (see below). Once the shock ceases to propagate, the propagation motion stops.

Notice that $\delta \in (0, 1)$ is associated to a damping of the distance from the shocked node, while $\delta > 1$ captures an amplifying effect. The case $\delta = 0$ is trivial, and means “no propagation”, while $\delta = 1$ means that the distance from the original node has no impact on the propagation of the shock.

The propagation condition – PC, hereafter – is a requirement to be satisfied in order to avoid the stopping of the propagation motion. In this model, we assume that the shock propagates only if its size is large enough. By recalling that the longest $k$-paths in the network are the ones with $k = \bar{k}$, we introduce a vector of positive scalars $\Gamma = (\gamma_1, \ldots, \gamma_{\bar{k}}) \in \mathbb{R}^{\bar{k}}$.

Given $h = 1, 2, \ldots, k$, we assume that the shock does not propagate to $i_h$ if and only if there exists $s = 0, 1, 2, \ldots, h - 1$ such that $\xi_s < \gamma_s$. 
Given $\Gamma \in \mathbb{R}^k$ and $\xi \in [0, +\infty)$, if the shock is propagated till the last node $i_k$, then we say that $p_{i_0}^{(k)}$ satisfies $(\Gamma, \xi) - PC$ or, simply, $p_{i_0}^{(k)}$ is $(\Gamma, \xi) - PC$. Vector $\Gamma$ is the PC-vector of the network $N$.

Notice that the thresholds $\gamma$’s can be taken as endogenous, and are fixed a priori. In principle, they do not depend on the specific starting node $i_0$. Therefore, we can identify the PC of the entire network by defining the minimal values of the $\gamma$’s serving for the propagation condition. We denote them by $\gamma^*$’s and collect them in $\Gamma^* = (\gamma_1^*, \ldots, \gamma_k^*)$. Vector $\Gamma^*$ could be defined as the critical PC-vector of the network $N$.

Clearly, specific values of $\Gamma$ and $\xi$ determine the set of the $k$-paths which are $(\Gamma, \xi) - PC$. In this respect, fixed $\Gamma \in \mathbb{R}^k$ and $\xi \in [0, +\infty)$, we define

$$P_{\Gamma, \xi}^{(k)} = \bigcup_{i_0 \in V} \left\{ p_{i_0}^{(k)} \in P^{(k)}(i_0) : p_{i_0}^{(k)} \text{ is } (\Gamma, \xi) - PC \right\}. \quad (2)$$

We can define the $\Gamma - \xi$-resilience of the network as the ability of the network to stop the propagation of the shocks when the PC-vector is $\Gamma^*$ and the shocks have initial size $\xi_0 = \xi$.

The sets defined in (1) and (2) can be employed to provide a relative measure of the resilience of $N$, once $\Gamma$ and $\xi$ are fixed. In particular, one can define the measure of the $\Gamma - \xi$-resilience of $N$ – and denote it by $\mu_{(\Gamma, \xi)}(N)$ – as a weighted mean of the relative number of the $k$-paths which are $(\Gamma, \xi) - PC$ as $k$ varies:

$$\mu_{(\Gamma, \xi)}(N) = \sum_{k=1}^k \theta_k \frac{|P_{\Gamma, \xi}^{(k)}|}{|P^{(k)}|}, \quad (3)$$

where the weights are collected in a vector of nonnegative scalars $\Theta = (\theta_1, \ldots, \theta_k)$ such that $\sum_{k=1}^k \theta_k = 1$. By construction, $\mu_{(\Gamma, \xi)}(N) \in [0, 1]$, for each network $N$, $\Gamma \in \mathbb{R}^k$ and $\xi \in [0, +\infty)$. Such a property allows one to compare different networks in terms of their abilities to absorb shocks.

Furthermore, the presence of the weights $\theta$’s allows one to provide different conceptualizations of $\Gamma - \xi$-resilient networks, on the basis of the relevance given to the length of the paths.

The corner cases of $\mu_{(\Gamma, \xi)}(N) = 0$ and $\mu_{(\Gamma, \xi)}(N) = 1$ stand for the minimum and maximum level of $\Gamma - \xi$-resilience, respectively. Indeed, in the former case the shocks with
size ξ are immediately absorbed; in the latter case, any shock of size ξ propagates over all the available paths.

Intuitively, μ_{(Γ,ξ)}(N) decreases with respect to all the components of Γ and increases with respect to ξ, because PC becomes more restrictive if the thresholds γ’s are large or if the size of the shock is small.

4 Applications

Herein we considered the analysis of the resilience on two empirical networks extracted from the network of US commercial airports [13]. The original US commercial airports network has n = 500 nodes denoting airports and m = 2980 directed arcs representing flight connections. In this network, weights are the number of daily flights available on such connections in 2010. The network has both a small-world and scale-free organization [2]. In particular, we present the cases of the airports in Illinois (IL) and New York (NY) states. The selected networks are relatively small. This allows us to overcome the computational complexity problems, since the computation of all the simple paths grows exponentially with the size of the network [18]. However, the case studies regarding the airports in Illinois and New York states can be considered of interest being that the two airport hubs of Chicago (ORD according to International Air Transport Association - IATA - code nome) and New York (JFK) are extremely crucial in US air traffic network.

The subnetwork extracted for IL is composed of 12 nodes and 51 arcs with \( \bar{k} = 4 \) while the NY subnetwork is composed of 21 nodes and 89 connection arcs with \( \bar{k} = 8 \). Figures 1 and 2 show, respectively, the visualizations of relative networks overlapped on the maps of NY and IL states.

The data processing, the network analysis and all simulations are conducted using the software R [21] with the igraph package [14]. The datasets were obtained from the R package tnet authored by Tore Opsahl [http://toreopsahl.com]. Code in the R programming language is available upon request.

Formulas (2) and (3) were implemented in an algorithm which takes as input the network \( N \) under observation and some different settings as follows.

We consider three different settings for Γ, in particular: \( Γ_1 = \{γ_i = 1, i = 1, ..., \bar{k}\} \), i.e.
the propagation thresholds are low and a large number of shocks propagate; more than this, all the shocks propagate when $\delta = 1$; $\Gamma_2 = \{\gamma_i = 1, i = 1, \ldots, \lceil k/2 \rceil \}$ and $\gamma_i = 2^{i - \lfloor k/2 \rfloor + 1}$
for \( i = \lceil \bar{k}/2 \rceil + 1, \ldots, \bar{k} \), i.e shocks propagation becomes more difficult as the distance from the shocked nodes increases; finally, \( \Gamma_3 = \{ \gamma_i = 2^{\lceil \bar{k}/2 \rceil - i + 1} \text{ for } i = 1, \ldots, \lceil \bar{k}/2 \rceil \text{ and } \gamma_i = 1, \quad i = \lceil \bar{k}/2 \rceil + 1, \ldots, \bar{k} \} \), i.e. the propagation is obstructed along shortest simple paths with a decreasing geometric evolution.

Regarding the weights in \( \Theta \), we considered as first setting \( \Theta_1 \) whose \( i \)-th component is \( \theta_i = 1/\bar{k} \), for \( i = 1, \ldots, \bar{k} \); a second is \( \Theta_2 \) with \( \theta_i = 1/2^i \) for each \( i = 1, \ldots, \bar{k} - 1 \) and \( \theta_{\bar{k}} = \theta_{\bar{k}-1} \); a third setting is \( \Theta_3 \) with \( \theta_i = 1/2^{\bar{k}-i+1} \) for each \( i = 2, \ldots, \bar{k} - 1 \) and \( \theta_1 = \theta_2 \).

In the last two settings, coefficients are taken from a geometric progression of decreasing and increasing type, respectively, while the first one is associated to uniform values of \( \theta \)'s.

Finally, for the computation of the \( \mu_1(\Gamma,\xi)(N) \), we considered \( \xi = \{0,1,2,\ldots,10\} \) and \( \delta = \{0,0.1,0.2,\ldots,1\} \), thus we performed 121 computations for each network. The higher values of \( \xi \) were chosen in order to be greater than \( \bar{k} \).

### 4.1 Illinois state network

Figure 3 reports the results for the IL airports for \( \Gamma_1 \) and the three different settings of \( \Theta \). All the cases share the intuitive behavior of the resilience such that for small values of the \( \xi \)'s and \( \delta \)'s, the value of \( \mu_1(\Gamma,\xi)(N) \) is small and increases with respect to \( \delta \) and \( \xi \), becoming equals to 1 when \( \delta \) and \( \xi \) are large enough. However, some remarkable differences emerge.

For cases \( \Theta_1 \) and \( \Theta_3 \), small values of the discount factor leads to hard shock propagation, even if the size of the shock is large. This is not longer true for \( \Theta_2 \), where an enlargement of the shock is able to increase the level of the resilience also when the discount factor is at a low level. This outcome is due to the more prominent role played by the shortest paths in the first and third setting when compared to the second one, where the shortest paths are strongly penalized by the weights. In this respect, one should also consider that the action of the discount is more evident as the distance with the shocked node increases.

Similar arguments can also explain the discrepancies in the rate of growth of the resilience with respect to \( \delta \). Indeed, the increase \( \mu_1(\Gamma,\xi)(N) \) is faster for \( \Theta_3 \), while the lower rate can be observed in the \( \Theta_2 \) case. Specifically, for \( \Theta_3 \) and for each value of \( \xi \), we can identify the resilience as collapsing to a certain degree, with the network passing rapidly
from resilience equal to 1 to very low resilience when \( \delta \) moves slightly below 0.5.

The same collapse of the resilience can be observed for all the cases when the size of the shock goes below 2. Thus, one can argue that there exists a critical threshold for the shock leading to unitary resilience, for each value of the discount factor. This is totally in agreement with \( \Gamma_1 \), which has components with minimum value. In the specific case of the IL airports, it is then associated to the propagation of all the shocks large enough when \( \delta < 1 \) or all the shocks when \( \delta = 1 \).

When we consider \( \Gamma_2 \) (see Figure 4), the \( \gamma \)'s are constant for the first \( \hat{k}/2 \) values, and then they start to grow; thus, we are penalizing longest simple paths. In this case (with \( \Theta_1 \)), the effect of \( \delta \) is more prominent than the one of \( \xi \), and \( \mu_{(\Gamma,\xi)}(N) \) tends to 1 for high values of \( \delta \). Such a trend can be observed also for \( \Theta_2 \) and \( \Theta_3 \). However, \( \Theta_2 \) is much more similar to \( \Theta_1 \) than \( \Theta_3 \), because its longest paths are penalized from the \( \gamma \)'s also in this setting; thus, the damping effect of the second part of \( \Theta_2 \) in the weight vector seems to be hardly noticeable. The penalization of the longest path is also the reason for the absence of an evident collapse effect for \( \Theta_3 \), because shock propagation is impeded on longest paths by \( \gamma \)'s and on shortest paths by \( \theta \)'s.

The third setting of \( \Gamma \) is associated to the penalization of the shortest simple paths, so that the shock does not propagate unless it has a high value and the discount factor is high as well. Figure 5 reports the results, which are similar for all settings of \( \Theta \).

![Figure 3: \( \mu_{(\Gamma,\xi)}(N) \) values in case of IL state airports for \( \Gamma_1 \) and \( \Theta_1 \) (left), \( \Theta_2 \) (center) and \( \Theta_3 \) (right).]
Figure 4: $\mu_{(\Gamma,\xi)}(N)$ values in case of IL state airports for $\Gamma_2$ and $\Theta_1$ (left), $\Theta_2$ (center) and $\Theta_3$ (right).

Figure 5: $\mu_{(\Gamma,\xi)}(N)$ values in case of IL state airports for $\Gamma_3$ and $\Theta_1$ (left), $\Theta_2$ (center) and $\Theta_3$ (right).

4.2 New York state network

In Figure 6 we report the results for $\Gamma_1$ for the NY state airport network. The behavior is very similar to the previous network in both settings of $\Theta$ (see Figure 3) but for $\Theta_3$ the collapse effect of the resilience is very clear for $\delta$ going above 0.4.

When we set $\Gamma_2$ we note that for $\Theta_1$ the behavior is similar to the IL network, even if the level of the resilience of the NY network is below that of IL (compare Figure 7 with Figure 4). When we consider $\Theta_2$, the surface is smoothed in comparison to the same case for the IL network and it is quite similar to the $\Gamma_1$ setting. For $\Theta_3$, the values of $\mu_{(\Gamma,\xi)}(N)$ are equal to 1 only for high values of $\xi$ and $\delta$. Indeed, the action of the $\gamma$’s, which contribute to deteriorating the resilience of the networks at the short paths level, is more effective than that of the $\theta$’s, which reduce the relevance of the short paths in the definition of $\mu_{(\Gamma,\xi)}(N)$.

Comparison results between NY and IL networks can be explained by their dimensions,
being that $\bar{k} = 8$ for the former and $\bar{k} = 4$ for the latter. In the same light, one should interpret also the case of $\Gamma_3$.

Indeed, when considering the NY airports, all the values of $\mu_{(\Gamma,\xi)}(N)$ are equal to 0 and Figures are not shown. In this setting one has $\gamma_1 > 10$, which is the maximum value of the available $\xi$, so that propagation cannot occur.

![Figure 6](image1.png)

Figure 6: $\mu_{(\Gamma,\xi)}(N)$ values in case of NY state airports for $\Theta_1$ and $\Theta_2$ (center) and $\Theta_3$ (right).

![Figure 7](image2.png)

Figure 7: $\mu_{(\Gamma,\xi)}(N)$ values in case of NY state airports for $\Theta_1$ and $\Theta_2$ (center) and $\Theta_3$ (right).

5 Conclusive remarks

The analysis of the resilience of networks and their attitude towards absorbing external shocks is crucial for the various implications related to reliability and functionality of real-world engineering systems.

The aim of this paper is to propose a new measure of network resilience based on the study of the shocks propagation along the patterns of connections among nodes.
In our setting, shocks are assumed to have a positive size and a tendency of not necessarily removing nodes from the network. They propagate in an amplified or dampened way through a suitably defined discount factor, with such a propagation proceeding under the fulfilment of a predefined condition involving the size of the shocks and the weights of the arcs.

The resilience measure is conceptualized as a weighted combination of the cardinality of the sets collecting the paths with different lengths, whereby the weights are able to assign relevance to the short or long paths in the networks.

We test the measure on two real airport systems. The comparison between the empirical instances suggests the incidence of the size of the network on shock propagation, as well as on the resilience. Indeed, for large networks, propagation suffers from difficulty in reaching nodes that are far in respect to the others.

Results state that the resilience is strongly dependent on the weights associated to the short paths, and highlight a prominent role of the discount factor in determining the ability of the network to absorb shocks. This behavior suggests that it is possible to design resilient network, with a strong ability to absorb external shocks, by imposing a propagation pattern on the basis of a certain level of the discount factor.

The proposed resilience measure might then be of usefulness to the engineers for the identification of the vulnerabilities of systems and for designing more resilient infrastructures.

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