Robust Non-line-of-sight Imaging with Single Photon Detectors
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Abstract

Imaging objects that are obscured by scattering and occlusion is an important challenge for many applications. For example, navigation and mapping capabilities of autonomous vehicles could be improved, vision in harsh weather conditions or underwater could be facilitated, or search and rescue scenarios could become more effective. Unfortunately, conventional cameras cannot see around corners. Emerging, time-resolved computational imaging systems, however, have demonstrated first steps towards non-line-of-sight (NLOS) imaging. In this paper, we develop an algorithmic framework for NLOS imaging that is robust to partial occlusions within the hidden scenes. This is a common light transport effect, but not adequately handled by existing NLOS reconstruction algorithms, resulting in fundamental limitations in what types of scenes can be recovered. We demonstrate state-of-the-art NLOS reconstructions in simulation and with a prototype single photon avalanche diode (SPAD) based acquisition system.

1 Introduction

The capacity of imaging systems must continue to expand to keep pace with rapidly emerging technologies. Autonomous vehicles, for example, would greatly benefit from improved vision in fog, snow, and other scattering media or from being able to see around corners to detect what lies beyond the next bend or another car. Sensing technology offering such non-line-of-sight (NLOS) capabilities would help make self-driving cars safe and unlock unprecedented potential for other computer vision systems. This is a broad vision towards which this paper makes a significant step.

Two challenges make NLOS imaging difficult. First, the low signal of multiply scattered light places extreme requirements on photon sensitivity of the detectors. Second, large-scale reconstruction algorithms must be developed that robustly infer object reflectance other properties of occluded scenes from indirect light transport. In this paper, we introduce a computational imaging system that addresses both of these challenges. We integrate a detector array of single photon avalanche diodes (SPADs), a picosecond laser, and automatic scanning mechanisms into a computational imager that records the time-of-arrival of individual photons. At the core of our system is a new reconstruction algorithm that analyzes direct and indirect light transport, recorded with the proposed imager, to unveil objects that are neither directly visible by the sensor nor the light source.

As opposed to recent proposals on non-line-of-sight imaging [Kirmeni et al. 2009; Velten et al. 2012; Gupta et al. 2012; Wu et al. 2012; Heide et al. 2014; Buttafava et al. 2015] or object tracking [Pandharkar et al. 2011; Klein et al. 2016; Gariepy et al. 2016], the proposed image formation model accounts for occlusions between higher-order light bounces. The proposed model results in a nonlinear biconvex inverse problem. We derive an algorithm that solves this problem efficiently and demonstrate that it makes NLOS imaging more robust than previously-proposed linear models. Moreover, our approach is significantly faster than attempting to solve the nonlinear problem directly.

The proposed hardware system has a form factor of a conventional camera, the employed single photon detectors are commonly used in commercial lidar systems, and the price of the system is orders of magnitude lower than other non-line-of-sight setups built on streak cameras [Velten et al. 2012] while offering potential reconstruction quality that is significant higher than that achieved with low-cost time-of-flight sensors [Heide et al. 2014]. With the presented work, we take first steps towards making NLOS imaging robust and practical for real-world applications.

We make the following contributions:

• We introduce a nonlinear image formation model for non-line-of-sight imaging that accounts for occlusions in the hidden
light transport.

- We derive a biconvex solver to invert NLOS light transport and show that it achieves significantly higher reconstruction quality than conventional linear models while being computationally efficient.
- We develop a probabilistic model to analyze bounds on NLOS reconstruction accuracy.
- We implement an experimental NLOS acquisition setup using single photon avalanche diodes and a picosecond laser. We adapt our solver to the SPAD-specific image formation.
- We validate the proposed reconstruction algorithms in simulation and with several example scenes that are captured with the prototype SPAD imager.

Overview of Limitations Similar to other non-line-of-sight methods, we make the assumptions that surfaces are Lambertian, that surface normals are known, and we model third-order light bounces. These assumptions may be invalid for volumetric scattering, refractions, and other complex lighting effects.

2 Related Work

Non-line-of-sight Imaging Kirmani et al. [2009] first introduced the idea of “looking around corners” by analyzing the feasibility of reconstructing hidden objects from time-resolved light transport. This concept was demonstrated in practice by Velten et al. [2012] with a system capable of resolving the shape and reflectance of a hidden object. Velten’s hardware setup included a streak camera and a femtosecond laser, which together account for a cost of several hundred thousand dollars. The streak camera provides a theoretical precision of up to 2 ps, which corresponds to a travel distance of 0.6 mm. Correlation-based time-of-flight sensors have also been demonstrated as a low-cost alternative for non-line-of-sight imaging [Kadambi et al. 2013, Heide et al. 2014]. While these systems are about three orders of magnitude less expensive than Velten’s system, they also only offer a very limited temporal resolution. Recently, single photon avalanche diodes (SPADs) have been proposed for NLOS imaging [Buttafava et al. 2015] as a hardware platform that offers a good balance between cost and precision.

NLOS imaging requires a model for the light transport of hidden scene parts as well as a large-scale reconstruction framework. All proposals for NLOS imaging [Kirmani et al. 2009, Velten et al. 2012, Gupta et al. 2012, Wu et al. 2012, Heide et al. 2014] use an image formation model that makes the following assumptions: (1) light bounces at most three times within the scene; (2) the scene contains no occlusions; (3) light reflects isotropically (i.e., surfaces are Lambertian). Under these assumptions, the reconstruction becomes a linear inverse problem. Velten et al. [2012], Gupta et al. [2012], and Buttafava et al. [2015] solved this system using a backprojection algorithm. Due to the fact that this tends to emphasize low frequencies and does not actually solve the inverse problem, the authors proposed to apply an additional edge-enhancing Laplacian filter followed by a nonlinear thresholding operation. Using the same assumptions, Gupta et al. [2012], Wu et al. [2012], and Heide et al. [2014] proposed to solve the inverse problem via large-scale convex optimization.

At the core of this paper is a novel image formation model that accounts for occlusions and an efficient solver for the corresponding nonlinear inverse problem. We demonstrate that occlusions among hidden objects are quite common and severely affect the quality of NLOS reconstructions. We adapt the proposed mathematical model to the unique capabilities of single photon avalanche diodes and demonstrate experimental results with a SPAD imaging prototype.

Single Photon Avalanche Diodes (SPADs) are reverse-biased photodiodes that are operated well above their breakdown voltage (see e.g. [Burri et al. 2016]). Every photon incident on a SPAD has some probability of triggering an electron avalanche which is time-stamped. This time-stamping mechanism usually provides an accuracy of tens to hundreds of picoseconds. SPADs and also avalanche photodiodes (APDs) are commonly used for a wide range of applications in optical telecommunication, fluorescence lifetime imaging, and remote sensing systems (e.g., lidar). Often, these imaging modes are referred to as time-correlated single photon counting [O’Connor 2012].

Recently, SPADs were applied to range imaging [Kirmani et al. 2014, Shin et al. 2016], transient imaging [Gariety et al. 2015] Anonymous [2017] as well as tracking [Gariety et al. 2016] and imaging [Buttafava et al. 2015] non-line-of-sight objects. The work by Buttafava et al. [2015] is closest to ours, but they use a simple linear image formation model that does not account for occlusions of NLOS light transport.

Imaging Through and Around Stuff Other forms of non-line-of-sight imaging have also been demonstrated that do not rely on time-of-flight imaging. For example, Sen et al. [2005] proposed a projector-camera system where the viewpoints of camera and projector could be interchanged. Snapshot looking-around-corners was demonstrated by exploiting correlations that exist in coherent laser speckle [Katz et al. 2014], though this has so far been demonstrated at microscopic scales. Radio and terahertz frequencies were shown to be able to imaging through objects [Adib et al. 2015, Redo-Sanchez et al. 2016]. Tracking hidden objects was also shown to be possible with intensity measurements of conventional cameras to a limited extent [Klein et al. 2016].

3 Forward and Inverse Light Transport

3.1 NLOS Light Transport with Occlusions

The image formation derived in the following is based on the radiosity method [Goral et al. 1984] and models a scene as a collection of piecewise planar surface patches. A Lambertian bi-directional reflection distribution function (BRDF) is assumed for all surfaces. Each patch \(i\) is defined by a position \(x_i\), a surface normal \(n_i\), and a diffuse albedo \(\rho_i\). The radiosity \(b_i\) of patch \(i\) gives the total radiative flux leaving the surface

\[
b_i = e_i + \rho_i \sum_{j=1}^{N} F_{ij} b_j
\]

where \(e_i\) is the emitted energy, \(F_{ij}\) is the geometric form factor between two piecewise planar surface patches spanning areas \(a_{i,j}\).

The form factor is given as

\[
F_{ij} = a_i a_j \cos \phi_j (n_i) \cos \phi_i (n_j) / \pi \Delta_{ij},
\]

where \(\phi_{ij}\) as the binary visibility function between the patches. We use a shortcut notation for \(\cos \phi_j (n_i) = \cos \angle (x_j - x_i, n_i)\) and \(\Delta_{ij} = \|x_i - x_j\|^2\).

Non-line-of-sight imaging is the problem of recovering the albedos of surface patches that are not directly visible by a camera from the measured radiosity of some visible patches. Although the hidden patches are not directly observed, when one of the visible patches is illuminated, even the hidden surfaces scatter light indirectly back to the visible ones. NLOS reconstruction relies on these indirect radiosity components to infer properties of the hidden surface patches.
The Kronecker delta \( \delta \) radiosity of the directly illuminated surface patch located at \( x \) sensor. The visible patches are directly imaged by a detector pixel \( l \) at \( t \). The visible patches are dependent such that \( \tau \) and hence Eq. (3.1) contains two summands (1 here that the light source is not directly illuminating the camera, \( t \)).

In particular, most non-line-of-sight approaches focus on analyzing the 3rd bounce of light transport; that is, light is emitted by an active source towards a visible surface patch, which is indirectly reflected outside the line-of-sight, and then returns to one or more of the visible surfaces. Unrolling the recursive Eq. (1) yields for the radiosity \( b_k \) of the visible patches \( k = 1 \ldots K \)

\[
b_k = e_k + e_k \rho_k \sum_{i=1}^{N} F_{l} F_{l_i} v_{l_i} v_{l_k} \rho_i \tag{3}
\]

Vectorizing the image formation model from Eq. (5) w.r.t. \((k, t)\) in lexicographic order yields the following matrix vector product

\[
s^{(l)} = e_l \delta_{l} t_{+}^{(l)} + e_l D \rho_l \delta_{l} t_{+}^{(l)} F_{l} F_{l_k} v_{l_i} v_{l_k} \cdots \rho
\]

\[
= \tilde{e}_l + e_l D \rho_l \Gamma^{(l)} \rho \tag{6}
\]

Here, \( \tilde{e}_l \) is the direct component of the temporally-resolved transport, which is independent of the hidden volume patches. The indirect component is modeled with a spatio-temporal transport matrix \( \Gamma^{(l)} \in \mathbb{R}^{K T \times M} \). This linear operator maps the vectorized albedos \( \rho \in \mathbb{R}^M \) of the hidden volume patches to temporally-varying radiance. This albedo-vector does not contain the directly observable albedos of the diffuse wall \( p_l \in \mathbb{R}^K \) that are assumed to be known. The operator \( D \) represents a diagonal matrix with the subscript on the diagonal.

A transport matrix \( \Gamma^{(l)} \) can be factorized into four transport components: a temporal sampling matrix \( T^{(l)} \), a visible form-factor matrix \( A^{(l)} \), a hidden form-factor matrix \( N^{(l)} \), and a \( V^{(l)} \) as

\[
\Gamma^{(l)} = \left[ \begin{array}{cc}
    \delta_{l} t_{+}^{(l)} & \circ I \\
    \cdots & \cdots
\end{array} \right] \circ \left[ \begin{array}{c}
    \cos l_i \cos l_k \cos l_i \cos l_k \frac{1}{\pi} x_{l_i} x_{l_k} \\
    \cdots \\
    \cos l_i \cos l_i \cos l_i \cdots \circ \left[ \begin{array}{c}
    v_{l_i} v_{l_k} \\
    \cdots
\end{array} \right]
\end{array} \right]
\]

\[
= T^{(l)} \circ I \left( A^{(l)} \circ N^{(l)}(n) \circ V^{(l)} \right) \tag{7}
\]

The matrix \( T^{(l)} \in \mathbb{R}^{K T \times K} \) contains here the travel time dependent transport coefficients, while all other transport matrices in Eq. (7) are time-independent. \( A^{(l)} \in \mathbb{R}^{K \times M} \) contains the geometric form factors components that are independent of the hidden patches. The matrix \( N^{(l)}(n) \in \mathbb{R}^{K \times M} \) contains all components that are depending on the normals \( n \) of the hidden surface patches. The visibility matrix \( V^{(l)} \in \mathbb{R}^{K \times M} \) includes the accumulated visibility terms, i.e. \( V^{(l)}_{v_{l_i} v_{l_k}} \) model the total visibility along the light path between patches \( l - i - k \). There is a mismatch in the dimensions between \( T^{(l)} \) and \( A^{(l)}, N^{(l)}(n), V^{(l)} \) due to the fact that the former are time-dependent and the latter are not. Hence, the binary matrix \( I \in \mathbb{R}^{K T \times K} \) simply copies the time-independent transport components to all \( T \) time slices.

By illuminating several different visible surface patches \( l = 1 \ldots L \) and measuring the temporally-resolved radiosity reflected to the detector every time, we can derive the following image formation model:

\[
\begin{bmatrix}
    s^{(1)} \\
    s^{(L)}
\end{bmatrix} = \begin{bmatrix}
    T^{(1)} & A \\
    T^{(L)} & A
\end{bmatrix} \circ \begin{bmatrix}
    A^{(1)} & \circ N^{(1)}(n) & \circ V^{(1)} \\
    A^{(L)} & \circ N^{(L)}(n) & \circ V^{(L)}
\end{bmatrix} \rho
\]

\[
\text{where } \mathcal{I} = \left[ I^T \cdots I^T \right]^T \text{ denotes the copy matrix replicated for each of the } L \text{ light source position. For notational simplicity we} \]
assume that the direct components $\ell_i$ are filtered out in time and can be ignored, and the diagonal terms $e_i D_{ii}$ are absorbed in the matrices $A^{(i)}$. For each of the different illumination conditions, the detector measures time-resolved irradiance $s^{(i)} \in \mathbb{R}^{K \times T}$, which is directly proportional to the radiosity of the $K$ visible patches. Each pixel records irradiance information for $T$ discrete time bins at a picosecond scale.

### 3.3 Inverse NLOS Light Transport

#### Backprojection

By ignoring the visibility terms (i.e., $V_{ij} = 1$) and normal directions (i.e., isotropic scatterers), Equation 8 becomes linear. Velten et al. [2012], Gupta et al. [2012], and Butalava et al. [2015] employed a backprojection algorithm to approximate the solution to this linear problem by projecting the measurements into the solution space as $\rho \approx A^{T} s$. Velten et al. proposed to apply a Laplacian filter to the resulting $\rho$ followed by a thresholding operation. Although this approach does not implement a proper inverse method, it does boosts high-frequency image details and was found to be robust to noisy sensor measurements by Velten et al.

#### Linear Model

Several other NLOS reconstruction algorithms [Gupta et al. 2012, Wu et al. 2012, Heide et al. 2014] actually solve the system of linear equations directly, but they make the same assumptions on visibility and normals as the backprojection algorithm. The inverse problem of recovering albedos of hidden surface patches can be expressed as

$$\min_{\rho} \|s - A\rho\|_2^2 + \Gamma(\rho), \quad \text{s.t. } 0 \leq \rho$$

Although the nonnegativity constraints were not directly enforced by all previous proposals, including it generally improves the estimated solution. A prior on the albedos $\Gamma(\rho)$ often helps to further improve the estimated albedos. For example, Heide et al. [2014] used a combination of sparseness and sparse gradients (i.e., total variation) for the unknown albedos.

#### Nonlinear Triconvex Model

With Equation 8 we propose a nonlinear model that accounts for normals of the hidden volume and occlusions between hidden surface patches through factorizing the spatio-temporal light transport. An objective function that would estimate hidden albedos $\rho$, hidden normals $n$ and occlusions $V$ is

$$\min_{\rho, n, V} \|s - (T \circ I(A \circ N(n) \circ V)) \rho\|_2^2 + \Gamma(\rho) + \Lambda(n), \quad \text{s.t. } 0 \leq \rho, V$$

where $\Lambda(n)$ is a prior on the normals $n$, enforcing unit length (i.e., $\|n_i\| = 1 \forall i \in \{1, \ldots, K\}$) and smoothness [Ahmed et al. 2008]. By relaxing the constraint that $V$ must be binary and assuming that the priors are convex, this objective is nonlinear but triconvex. As is standard practice for such problems, we use an alternating least-squares (ALS) approach to solve it. To this end, both the visibility term $V$, the normals $n$, and the hidden surface albedos $\rho$ are initialized with random values. Equation 10 is then solved in an alternating manner by fixing one of these terms and optimizing for the other. This approach is outlined by Algorithm 1.

In the $\rho$-update of Algorithm 1, the system matrix $\tilde{A}_k = (T \circ I(A \circ N(n_k) \circ V_k))$ is fixed for a given iteration $k$. Similar to previously-proposed NLOS solvers (see Eq. 9), this update is a convex nonnegative least squares problem (i.e., $\tilde{A}_k$ takes place of $A$ in Eq. 10) that can be solved by standard approaches, such as the alternating direction method of multipliers (ADMM) [Boyd et al. 2011].

The $V$-update is not quite as intuitive, but it is also convex because we could construct a matrix that absorbs $T, A, N, I$ and $\rho$ and thus also write the equation (Alg. 1 line 3) as a quadratic program (QP). However, the size of this QP is unfortunately very large (the linear operator would be of size $KTL \times KM$), and therefore existing QP solvers (such as MOSEK [Mosek 2010]) that are commonly used for small to medium scale problems (in the order of 10K variables) are intractable. We solve the $V$-update using an efficient projected gradient algorithm that similar to methods used for large-scale non-negative matrix factorization [Lin 2007]. In particular, we solve for $V_k$ using Algorithm 2 to minimize $J(V, n, \rho) = \|s - (T \circ I(A \circ N(n) \circ V)) \rho\|_2^2$ with respect to $V$, the convex box constraint set $C = \{x|0 \leq x \leq 1\}$, and initial guess $x_{init} = V_{k-1}$. The operator $\Pi_C$ in Algorithm 2 represents the projection onto the convex set $C$. To execute this algorithm, we only need to interact with the objective $J$ and its gradient.

The gradient of the quadratic objective with respect to $V$ can be computed as follows

$$\nabla_V J(V, n, \rho) = -2 A \circ N(n) \circ I^T \left( (s - (T \circ I(A \circ N(n) \circ V)) \rho) \right) \circ T.$$

A derivation of the gradient is given in the Supplemental Material. A key benefit of the projected gradient approach is that it can be implemented very efficiently. Note that the visibility term in computer graphics is usually binary. We relax this constraint and allow the term to be continuous to make inverse methods feasible.

In the $n$-update of Algorithm 1 can be solved analogously to $V$, with $C$ as the unit ball. However, a more efficient way to solve this subproblem can be achieved by parameterizing the normals such that the unit constraint is eliminated. This can be achieved by expressing a single normal $n_i$ as

$$n_i(u, v) = [\cos(u) \sin(v), \sin(u) \cos(v), \cos(v)]^T.$$

In this parameterization, it is always $\|n_i(u, v)\| = 1$ and we can solve for the normals as an unconstrained non-linear problem, minimizing $J$ with respect to $n$ using the Quasi-Newton method of your

### Algorithm 1 Triconvex Optimization

1. $V_0 = 1$
2. for $k = 1$ to $N$
   3. $\rho_k \leftarrow \arg \min_{\rho \geq 0} \|s - (T \circ I(A \circ N(n_k) \circ V_k)) \rho\|_2^2 + \Gamma(\rho)$
   4. $V_k \leftarrow \arg \min_{V \geq 0} \|s - (T \circ I(A \circ N(n) \circ V)) \rho\|_2^2$
   5. $n_k \leftarrow \arg \min_{n \geq 0} \|s - (T \circ I(A \circ N(n) \circ V_k)) \rho\|_2^2 + \Lambda(n)$
6. end for

### Algorithm 2 Projected Gradient with First Order Step Size

1. $x_0 = x_{init}, L_0 = 1, \beta = 2$
2. for $i = 0$ to $I$
   3. $v \leftarrow \Pi_C\left(x_i - \frac{1}{\beta} \nabla_x J(x_i)\right)$
   4. while $J(v) - J(x_i) > (\nabla_x J(x_i))^T (v - x_i) + \frac{\beta}{2} \|v - x_i\|_2^2$
      5. $L_i \leftarrow \frac{L_i}{2}$
   6. $v \leftarrow \Pi_C\left(x_i - \frac{1}{\beta} \nabla_x J(x_i)\right)$
7. end while
8. $x_{i+1} \leftarrow v$
9. $L_{i+1} \leftarrow \beta L_i$
10. end for
choice (we use L-BFGS). The gradient of our objective with respect to \( n \) in spherical coordinates is

\[
\nabla n = (\nabla n (n) )^T \cdot \nabla N(n) J(V, n, \rho) \quad \text{with}
\]

\[
\nabla N(n) J(V, n, \rho) = -2 A \circ V \circ J^T \left( \left( s - (T \circ J (A \circ N(n) \circ V) \rho)^T \right) \circ T \right),
\]

\[
\nabla n N(n)_{(k, i)} = \begin{bmatrix}
- \sin(u) \sin(v) & \cos(u) \sin(v) & \cos(v) \\
\cos(u) \cos(v) & \sin(u) \cos(v) & - \sin(v)
\end{bmatrix}
\]

\[
\left( n_i (n_i^T \cdot n_i (u, v)) + n_k (n_k^T \cdot n_i (u, v)) \right).
\]

## 4 Simulated Results

In this section, we assess the proposed solver in simulation and compare it to previously-discussed solutions to the NLOS problem. Figure 3 shows 2D scenes that contain severe (top) and minor occlusions (bottom). Additional scenes can be found in the Supplemental Information. Top-down views of the setups for these experiments are illustrated on the left. Here, a volume of 128 x 128 voxels spanning 2.4 x 2.4 m is simulated to be hidden behind the camera. The camera directly observes a wall in front of it. The 1D sensor has 256 pixels, each recording 256 discrete time bins with a temporal resolution of 82 ps. A transient image is recorded for 5 discrete laser positions on the wall. Figure 4 shows the five transient images recorded by the sensor (top). Note that only the secondary light bounces are visualized and used for the reconstruction; direct reflections from the wall are ignored. The albedo and shape of the wall is assumed to be known. In practice, this could be directly scanned with the system. All measurements are generated with the rendering framework proposed by Jarabo et al. (2014), which includes multi-bounce interactions, occlusions in hidden parts of the scene, and other global illumination effects in the forward simulations.

In both experiments of Figure 3, the backprojection method allows the shape of the hidden object to be roughly made out. However, low-frequency details are amplified and partly occluded objects, such as the top plane in the first example, are completely lost. Following Velten et al. (2012), we apply a Laplacian filter and threshold the result to simulate a filter backprojection. This filtering approach removes some of the low-frequency artifacts but it cannot restore missing information of partly occluded objects and it also does not handle more complex shapes, such as the bunny. The linear solver works well for objects that do not exhibit occlusions. In fact, the linear and the proposed nonlinear solver converge to the same result if there are no occlusions in the scene. However, the linear solver also fails to recover partly occluded scene parts, simply because these are not accounted for in the respective image formation model. The proposed solver accurately recovers partly occluded objects.

We also show convergence plots for all of the 2D simulations in Figure 3. Similar to the trend observed for peak signal-to-noise (PSNR) in Figure 3, we see that the residual of the proposed method is about two orders of magnitude lower than the linear method and four orders of magnitude lower than the backprojection method. We also implement a full L-BFGS nonlinear solver tackling Equation 12 directly without splitting it. The residual achieved with the nonlinear solver drops below that of the linear solver and it may converge to a similar solution than our approach in the limit. However, time to convergence is significantly longer than the proposed factorization and infeasible for large-scale 3D reconstructions.
For this example, the linear method completely misses the first scene in Figure 6 contains two planes that slightly occlude each other. The backprojection method does not recover the target volume in any meaningful way. The filtered backprojection applies a threshold to the backprojected data, but it is only capable of estimating a very rough outline of the hidden scene. Both linear and proposed methods recover the volume more accurately. The second example shows a scene with two planes that are parallel to each other. For this example, the linear method completely misses the occluded scene parts whereas these are accurately recovered with the proposed factorization method. The scene in Figure 6 contains two planes that slightly occlude each other. The backprojection method is capable of even closely approximating the target shape. Self-occlusions pose a challenge for the linear method. Again, the proposed method achieves the best result. Higher-order interreflections and other complex light transport effects that are not modeled by the proposed image formation, however, also place a limit on the quality of these results.

5 Deriving Bounds on NLOS Imaging

Estimating bounds on the confidence of a recovered non-line-of-sight volume is desirable for multiple reasons. First, one could compare different acquisition setups and determine which one of them is likely to result in a better reconstruction. Second, confidence estimation would be required for potential adaptive NLOS approaches to determine what the next-best measurement would be.

We adopt a Bayesian framework to quantify the covariance of the posterior distribution modeling the reconstructed albedos. A brute force approach for this problem could use Markov chain Monte Carlo methods, but this is computationally challenging in our application due to huge size matrix. Thus, we follow Flath et al.’s [2011] approach to estimate the covariance matrix of the posterior $\Lambda_{\text{post}} \in \mathbb{R}^{M \times M}$. For this purpose, we use a Gaussian model for the sensor noise with variance $\sigma^2$, resulting in the covariance matrix $\Lambda_{\text{noise}} = \sigma^2I$, $\Lambda_{\text{noise}} \in \mathbb{R}^{M \times M}$. Then, the pos-
terior is
\[
\Lambda_{\text{post}} = \left( (A \circ (ZV))^T \Lambda_{\text{noise}}^{-1} (A \circ (ZV)) + \Lambda_{\text{prior}}^{-1} \right)^{-1},
\]
where \( \Lambda_{\text{prior}} \in \mathbb{R}^{M \times M} \) is the covariance matrix of the prior. This is modeled as a Laplacian distribution, approximated by the sum of two Gaussians
\[
\Lambda_{\text{prior}} = \Lambda_1 + \Lambda_2 = \sigma_1^2 I + \sigma_2^2 I,
\]
where \( \sigma_1^2 = 27.04 \) and \( \sigma_2^2 = 38.44 \) are variances for two Gaussian distributions approximating the Laplacian prior.

Computing \( \Lambda_{\text{post}} \) requires large matrix inversions, which are infeasible. Similar to Flath et al. [2011], we employ a low-rank approximation to make the process efficient. By observing the diagonal of \( \Lambda_{\text{post}} \) for a particular setup modeled by \( A \circ (ZV) \), we can directly visualize our confidence of our reconstructions. This is illustrated in Figure 7. Here, we show confidence maps of a recovered volume for several different choices of illuminated laser positions. Recovering a hidden volume from measurements taken with only a single laser position will result in varying reconstruction confidence over the volume. Combining the measurements taken for all five different laser spots on the wall (see Fig. 3 top left, for setup) allows for the confidence to be maximized throughout the volume.

As demonstrated in Figure 7, the confidence estimation proposed here allows multiple different acquisition setups to be compared. We believe that this is a viable direction towards adaptive non-line-of-sight imaging, but we leave the development of such a system for future work.

6 NLOS with Single Photon Detectors

6.1 Modeling Single Photon Avalanche Diodes

SPAD detectors time-stamp photon events at a picosecond scale. After a detected event, the SPAD is reset (or quenched) before another event can be recorded. This dead time is usually on the order of a few hundred ns. A SPAD is further characterized by temporal jitter, modeling uncertainty in the time-stamping mechanism using a temporal convolution \( \ast \) between incident photon flux and jitter \( f \). A dark count rate \( d \) [Hz] represents the number of false events that are detected in the absence of photons. Assuming that the length of the emitted laser pulse is significantly smaller than the SPAD’s dead time, at most one of the photons in any pulse can trigger a SPAD event. Note that this is not necessarily the first photon that arrives. The presence or absence of a detected event within a short window is thus a Bernoulli trial.

An ideal photon counter processing the stream of incident photons on a SPAD within a certain time window would thus sample the rate function \( \lambda \) as
\[
\lambda = (f \ast J(\rho, V)) + d. \tag{15}
\]

Usually, the Bernoulli trial is repeated \( F \) times by firing \( F \) laser pulses at a rate that leaves sufficient time for the SPAD to reset between pulses. A low-level counting mechanism, for example implemented by a field programmable gate array (FPGA), accumulates events for a certain “exposure time” and stores the number of detected events per time bin in a histogram \( h \). Assuming that photon events between successive laser pulses are independent, which is the case for the low photon flux observed in NLOS applications, the probability of detecting a certain number of events in a histogram bin can be modeled as a Poisson distribution
\[
h \sim \mathcal{P}(F(\eta \lambda)), \tag{16}
\]
where \( \eta \in [0, 1] \) is the photon detection probability comprised of the quantum efficiency and the avalanche probability of the SPAD. A similar image formation model was recently also used by Shin et al. [2016] and Anonymous [2017]. We refer to these works and the Supplemental Information (SI) for more details on modeling SPAD systems.

6.2 Inverse Single Photon Imaging

Non-line-of-sight image reconstruction from noisy and blurry SPAD histograms is a nonlinear inverse problem embedded in a Poisson deconvolution problem. Using Equation 16, we formulate the reconstruction as a maximum likelihood estimation with non-negativity constraints:
\[
\begin{align*}
\min_{\rho, V} & \quad -\log(p(h|J_f(\rho, V))) + \Gamma(\rho), \\
\text{subject to} & \quad 0 \leq \rho, V
\end{align*}
\]
where \( p(h|\cdot) \) is the likelihood of observing measurements \( h \) for a known scene, \( J_f = f \ast 1 \) is the image formation of the SPAD, and \( \Gamma(\rho) \) is an optional prior on the recovered signal.

Without loss of generality, we replace nonnegativity constraints by the indicator function \( \mathcal{I}_{\mathbb{R}_{\geq 0}}(\cdot) \) and apply a splitting approach that represents the objective as a sum of independent penalty terms with a global consensus enforced by the constraints
\[
\begin{align*}
\min_{\rho, V} & \quad -\log(p(h|z_1)) + \mathcal{I}_{\mathbb{R}_{\geq 0}}(z_2) + \Gamma(z_3) \\
\text{subject to} & \quad \begin{bmatrix} J_f(\rho, V) \\ \rho \\ \rho \end{bmatrix} - \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = 0
\end{align*}
\]
In this formulation, \( z_1 \in \mathbb{R}^{KT} \) and \( z_{2,3} \in \mathbb{R}^M \) are slack variables. Using the Augmented Lagrangian for Equation 19 we derive an iterative update scheme using the alternating direction method of multipliers (ADMM) [Boyd et al. 2011]. This scheme is outlined by Algorithm 3 and derived in the SI. Here, \( z = [z_1^T \ z_2^T \ z_3^T]^T \) and \( u = [u_1^T \ u_2^T \ u_3^T]^T \) is the scaled dual of the Lagrange multiplier. Each of the steps in Algorithm 1 updates one slack variable, or \( \rho \), at a time. We derive closed-form solutions for each of the 3-updates (lines 3–5) in the SI. The \( \rho, V \)-update (line 2) requires a modified version of the biconvex solver (see Sec. 3), which we also derive in the SI.

Algorithm 3 ADMM-based reconstruction
1: for \( \text{iter} = 1 \) to \( \maxiter \)
2: \( \{\rho, V\} \leftarrow \arg\min_{\rho, V} \frac{1}{2} \left\| J_f(\rho, V)^T \rho \rho^T - z + u \right\|_2^2 \)
3: \( z_{1:2} \leftarrow \arg\min_{z_1, z_2} -\log(p(h|z_1)) + \frac{1}{2} \left\| J_f(\rho, V)^T \rho \rho^T + u_1 - z_1 \right\|_2^2 \)
4: \( z_2 \leftarrow \arg\min_{z_2} \mathcal{I}_{\mathbb{R}_{\geq 0}}(z_2) + \frac{1}{2} \left\| \rho + u_2 - z_2 \right\|_2^2 \)
5: \( z_3 \leftarrow \arg\min_{z_3} \Gamma(z_3) + \frac{1}{2} \left\| \rho + u_3 - z_3 \right\|_2^2 \)
6: \( u \leftarrow u + J_f(\rho, V)^T \rho \rho^T - z \)
7: end for

The formulation in Equations 17 and 18 is closely related to that of Anonymous [2017]. Similar to previous approaches to NLOS imaging, Anonymous use a linear system which makes Equations 17, 18 nonlinear but convex (depending on the choice of prior). The nonlinear image formation in our application (i.e.,
Figure 8: A top-down view of our prototype NLOS setup. The illumination optics (including the galvo and laser) is located above the scanning optics (including the second galvo, SPAD, relay lenses, and objective lens). The direction of the laser light (shown in blue) is controlled with a 2-axis galvo (shown here as a 1-axis galvo for simplicity).

\[ J(\rho, V) \] makes the NLOS problem biconvex and thus more difficult. ADMM is only guaranteed to converge for convex objectives with convex constraints. Despite the lack of theoretical convergence guarantees, we observe monotonic and fast convergence of the proposed biconvex solver in practice.

### 6.3 A SPAD-based Imaging Prototype

**Hardware**  
Our NLOS prototype system, shown in Figure 8, is a modified version of the system proposed by Anonymous [2017]. The main components of our system include a picosecond laser (ALPHALAS PICOPOWER-LD-450-50), a 1D SPAD sensor (LinoSPAD) [Burn et al. 2016], a pair of 2-axis scanning galvo mirror systems (Thorlabs GVS012), and several lenses. In comparison to Anonymous [2017], our system offers (1) a larger effective aperture that improves light throughput, and (2) a second galvo mirror system that controls the position of the laser spot.

The picosecond laser pulse has a 300 ps full-width at half-maximum (FWHM), a repetition rate of 50 MHz, a peak power of 450 mW, and a wavelength of 450 nm. We cycle between five laser spots spanning an area of size 0.5 m x 0.5 m on a planar white wall.

The 1D SPAD array has 256 pixels, and each pixel measures a histogram with 256 time bins (a resolution of 78.1 ps per bin). The main objective lens of our system consists of an 8.5 mm focal length objective lens (Computar M8513), followed by a pair of 35 mm focal length lenses (Nikon AF-NIKKOR 35 mm f/2D) to relay the image onto the 1D SPAD array. The second galvo system, located at the aperture plane between the two relay lenses, controls the position of the SPAD pixels.

A total of 320 scanlines produce a transient image with a spatial resolution of 320 x 256.

**Calibration**  
We calibrate the intrinsic and extrinsic parameters of our system by capturing 15 images of a known checkerboard pattern under ambient lighting. The first 5 images measure a checkerboard placed on the wall (to recover its location) and lit by each laser position. The next 5 images measure the checkerboard positioned in front of the wall and lit by the same set of laser positions. The final 5 images capture a checkerboard at various orientations and positions.

MATLAB’s Computer Vision System toolbox uses these images to recover the intrinsic parameters of the system (i.e., focal length, principal point, and distortion parameters) and location of planes and laser spots. The light ray for each laser position is recovered by tracing a line through the 3D locations of corresponding laser spots. The intersection of these 5 light rays is the location of the galvo system.

**NLOS Acquisition**  
Our prototype system captures a transient image for each of the 5 laser spot positions. Each scanline has a total acquisition period of 30 s. Only 1/30th of the 256 SPAD pixels can be active at any given time, resulting in an effective exposure period of \( \frac{30}{256} = 0.117 \) s per pixel. The total acquisition time for each NLOS experiment is approximately \( 30 \times 320 = 9600 \) s = 13.33 h.

### 7 Experimental Results

In this section, we demonstrate the proposed method for measurements acquired with the SPAD imager described in Section 6. Experimental results for two scenes are shown in Figure 10. For each scene, the results of previous approaches to NLOS imaging and the occlusion-based solver are compared. Please see the Supplementary Information (SI) for additional results.

Photographs of the scenes are shown in Figure 10 (center). The camera and laser are placed about 1 m in front of the diffuse wall, which is the only object intersecting with the camera’s cone of vision. The scene objects are placed outside this line-of-sight, at a distance of 0.2 to 0.8 m from the wall. Detailed descriptions of the setup and ground-truth positions and orientation of the scene objects is provided in the SI. We reconstruct a volume of 40 x 40 x 40 voxels spanning 0.7 x 0.7 x 0.7 m centered 0.5 meters away from the wall. The dimensions of our setup are small compared to the effective temporal resolution of our SPAD imaging system, which is around 800 ps (full width at half maximum (FWHM) of the laser impulse response), corresponding to a path length resolution of 24 cm. Figure 9 shows a single streak image for the planar scene (cf. Fig. 10 top). Several limitations of the prototype system are ob-
served in these measurements. First, the temporal resolution is relatively low, exhibited by temporal blur of the signal. Second, partially occluded scene parts (i.e., the second plane) are extremely faint. Longer acquisition times would improve this SNR. Third, the wall used in the physical setup is smaller than that of the simulations, resulting in reduced curvature of the measured hyperbolic streaks. All of these effects currently limit the reconstruction quality.

For the experiment in Figure 10 (top), backprojection and its filtered variant only allow to recover a blob that may include one or more objects, here the entire scene. In contrast, the linear solver allows the first plane to be recovered with a more reasonable quality – the rough shape of the first plane can be observed. However, the linear solver fails to recover the second plane which is partly occluded. The proposed method recovers this occluded plane at a better quality than the other algorithms. Although the effective resolution of the recovered volumes are low, the positions, orientations and rough shape of both planes is correctly reconstructed. Note that both planes are in fact slightly tilted upwards with normals facing the center of the projection plane on the wall. These illustrations convey a limited amount of information, please see the supplemental video for animations of the results presented in Figure 10.

Experimental results for a more complex scene are shown in Figure 10 (bottom). Similar to the results for the planar scenes, the backprojection method is not capable of recovering detail and only provides a rough position of the strongest unoccluded reflectors in the scene. The linear solver allows more detail to be recovered, but it misses scene features in regions that are affected by occluded objects in the scene. For the scene consisting of the two cutout letters (Fig. 10, bottom), the indirect reflections from the occluded “T” affect the unoccluded “X”. This effect is severe due to the low temporal resolution of the SPAD measurements. The proposed factorization approach recovers a rough shape of the unoccluded “X”, as it separates the occluded reflection components from the unoccluded ones. While the position and orientation of the “T” are correctly recovered (see top), all methods fail to recover an accurate shape.

The experimental NLOS results presented in this section exhibit a limited quality. These limitations are primarily caused by the relatively low temporal resolution of the combined laser pulse width and SPAD jitter (FWHM of approx. 800 ps). Modern SPADs are commercially available with temporal jitter < 50 ps and lasers are available with pulse widths in the femtosecond regime. The most critical system parameters than need to be optimized for NLOS applications are the temporal resolution and sensitivity of the system. Both of these factors can be addressed by optimizing the hardware setup, for example using the most sensitive SPADs and higher-power lasers with shorter pulse widths.

8 Discussion

In summary, we develop an image formation model and inverse methods for non-line-of-sight imaging that adequately model and recover occlusions within hidden scene parts. The proposed algorithm is a nonlinear factorization method and it is validated in simulation and physical measurements. We build a prototype time-resolved imaging system using an array of single photon avalanche diodes and a picosecond laser. Arguably, the algorithms and simulations in this paper represent the state-of-the-art in non-line-of-sight imaging.

Modeling Light Transport The image formation model used in this paper builds on previous work in this domain [Velten et al. 2012; Wu et al. 2012; Heide et al. 2014]. We are the first to model occlusion and demonstrate that this has a significant impact on the quality of recovered scenes. However, all of these and also our light transport model make several restricting assumptions: surface normals and BRDFs of hidden objects are assumed to be known; light only scatters once in occluded scene parts. Adopting more
sophisticated light transport models using single scattering (e.g., Baran et al. 2010) or volumetric scattering would likely improve the accuracy of NLOS imaging in general. However, it would most certainly also affect the complexity of associated inverse methods. Recovering surface normals, BRDFs and other scene properties in addition to albedo would be desirable. Naik et al. (2011), for example, took first steps towards NLOS BRDF estimation, but in their proposal the hidden geometry was assumed to be known.

**NLOS Imaging with SPADs** Avalanche photodiodes (APDs) and single photon avalanche diodes (SPADs) are one of the most promising technologies for NLOS imaging and related applications. These detectors are commonly used by remote sensing systems, such as lidars in autonomous vehicles. Further improvements to optimize temporal jitter, reliability, resolution, and cost are likely going to be driven by the remote sensing industry. In the future, novel algorithms taking advantage of specific imaging modalities of these detectors, i.e. the time-resolved aspect, could be directly deployed to fully functional imaging systems that were developed for simple ranging or other remote sensing applications. Although correlation-based time-of-flight sensors seem like a low-cost solution for NLOS imaging, these silicon-based sensors are limited to demodulation frequencies of about 100 MHz. This places severe limits on the temporal imaging resolution and thus on any algorithm using that information, such as NLOS reconstructions. Streak cameras offer some of the most precise temporal light transport information among all discussed NLOS systems, but these cameras are expensive, bulky, and only used for niche applications. SPADs are attractive because they are widely deployed in lidar systems and will continue to rapidly evolve. Finally, interferometric systems are also interesting and potentially offer path length information in the order of the wavelength of light. However, these systems rely on the interference of coherent light reflected from the scene and a reference beam. Such systems are extremely difficult to calibrate, sensitive to tiny vibrations or even changes in room temperature and they cannot necessarily resolved longer-range light paths unambiguously.

**Conclusion** This paper advances the state-of-the-art in NLOS imaging. We strongly believe that the proposed computational imaging system is well aligned with the technological developments of emerging sensing systems used for autonomous vehicles and robotic vision. However, the road to practical use of NLOS systems in-the-wild is a long one with many challenges ahead. Acquisition times must be reduced to allow for real-time capture, imaging systems must work reliably outdoors in the presence of ambient light, and the speed of reconstruction algorithms must be drastically improved. Image quality of captured results is still low, mostly due to the limited temporal precision of available imaging systems. Emerging SPADs, however, have a substantially lower temporal jitter than the prototype and more powerful lasers with shorter pulse widths are becoming readily available. NLOS imaging is a true challenge for computational imaging research, as it requires further improvements in sensors, optics, coded illumination, and reconstruction algorithms. We hope to stimulate future work in this area.

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