STUDY ON CONSTITUTIVE MODEL OF ELASTOPLASTIC BEHAVIOR FOR SWELLING BUFFER MATERIAL

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A new constitutive model of swelling soil material is proposed to evaluate its mechanical characteristics. In the case of the compacted bentonite material, nonlinear behavior appears more dominantly than ordinary clay materials in the consolidation test. The original Cam-clay model cannot describe these characteristics observed in the compacted bentonite materials. Therefore, a new modeling that strain increment consists of linear summation of the swelling component and the elast-plastic component is introduced to describe the nonlinear behavior in the unloading stage. From the result of the simulation of the triaxial compression test, it is demonstrated that the stress pass and the stress and strain relationship are well simulated by the new model.

Key Words: high-level radioactive waste, geological repository, buffer material, bentonite, elastoplastic behavior, Cam-clay model

1. INTRODUCTION

Soil materials containing large quantities of swelling clay minerals such as montmorillonite are generally known as "swelling soil materials". Such materials swell considerably when they absorb water. Tunnel construction in swelling ground results in construction safety and efficiency being greatly affected by the swelling pressure and swelling-induced displacements. No methods are, however, available for accurately evaluating the mechanical behavior of swelling soil materials. Few analyses have therefore been made using a constitutive model incorporating swelling in relation to tunnel excavation in swelling ground that is expected to exhibit complex mechanical behavior. Developing an appropriate constitutive model for highly swelling soil materials would make possible efficient support planning and construction management for excavation in swelling ground based on preliminary analysis.

In deep geological disposal of high-level radioactive waste in the field of nuclear power, bentonite-mixed material is considered a candidate for a buffer material in artificial barriers because it swells considerably and seals itself effectively in the case of cracking. Buffer materials are required to support overpack or alleviate stress induced in the overpack due to the creep displacement of the rock for a long time. It is therefore necessary to accurately predict the mechanical behavior of a buffer material designed to designated specifications and to confirm that the above requirements are satisfied and that safety is ensured against the failure of overpack or buffer material. At present, however, no optimal mechanical evaluation methods are available. Evaluation using a highly accurate constitutive model is required.

As one of the existing researches for evaluating the mechanical behavior of swelling soil materials in relation to radioactive waste disposal, Borgesson has made simulation analysis of a model test on the effect of faulted rock on the bentonite buffer materials, using the Drucker-Prager model. Takaji et al. have predicted the effect of settlement of overpack due to its weight using the Sekiguchi-Ohta model while
predicting the long-term behavior of artificial barriers, and have predicted the effects of overpack swelling due to corrosion and of creep deformation of rock using a modified version of Cam-clay model \(^3\). In relation to the swellability of buffer materials, Komine has proposed an equation representing the correlation between swelling strain and swelling pressure for the compressed bentonite based on the diffusion double layer theory, and accurately reproduced the results of swelling tests \(^4\). None of these studies, however, have verified the validity of the evaluation model used for predicting composite mechanical behavior due to shear and dilation. The authors have presented a modified version of Cam-clay model that can accurately evaluate laboratory test results for compressed bentonite buffer materials \(^6\),\(^7\). The constitutive model that the authors presented in the study aligns elastoplastic and swelling elements in series and assumes that the total volumetric strain is the sum of elastoplastic strain and swelling strain. In buffer materials with predominant swelling strain, therefore, elastoplastic strain may be sometimes almost nil. Even in such a case, the buffer material exhibits elastoplastic behavior. Thus, the model does not represent actual behavior. Then, an attempt was made to build a model that aligns elastoplastic and swelling elements in parallel. This study has focused as a swelling soil material on compressed bentonite that is used as a buffer material for high-level radioactive waste repository. Relatively large quantities of laboratory element test data are available on compressed bentonite used as buffer materials in the disposal of high-level radioactive waste. The applicability of a constitutive model developed for such compressed bentonite has been examined.

2. MECHANICAL CHARACTERISTICS OF COMPRESSED BENTONITE BUFFER MATERIAL

(1) Specifications for buffer material

Fig. 1 shows a compressed bentonite buffer material applied to a high-level radioactive waste repository. Table 1 lists specifications for the buffer material proposed by Japan Nuclear Cycle Development Institute \(^5\).

| Type of bentonite          | KUNIGEL-V1  |
|---------------------------|-------------|
| Silica sand content       | 30%         |
| Silica sand component     | No.3 : No.5 = 1 : 1 |
| Dry density               | 1.6 Mg/m³   |

(2) Consolidation of buffer material

To identify the consolidation of the compressed bentonite buffer material designed to the specifications shown in Table 1, a consolidation test was conducted \(^8\). Specimens had a diameter of 60 mm and a height of 20 mm. The specimen was compressed under lateral restraining conditions and supplied with water while restraining vertical displacement. Then, vertical reaction equivalent to water-absorbing swelling pressure was measured. When vertical reaction became constant, it was assumed that saturation was complete. The condition at the point was used as the initial condition for consolidation test. Loads were applied in eight steps until 19.6 MPa was reached (consolidation phase). Loads were then reduced in four steps to 0.54 MPa, the initial swelling pressure (unloading phase). Loads were applied again in four steps up to 19.6 MPa (re-loading phase). The completion of consolidation and swelling in each step in the consolidation, unloading and re-loading phases was determined using the 3T method based on the relationship between vertical displacement and logarithmic time.

Fig. 2 shows consolidation test results for three specimens under the same conditions. The figure shows that the consolidation of the buffer material is nearly in proportion to the logarithm of consolidation pressure. Consolidation of the buffer material is more nonlinear than that of ordinary clay in the unloading phase and exhibits a loop in the re-loading phase.

(3) Shear capacity of buffer material

To identify the shear capacity of compressed bentonite buffer material designed to the specifications in Table 1, an undrained triaxial compression test
was conducted \(^8\). Specimens with a diameter of 50 mm and a height of 100 mm were made by supplying water to compressed bentonite mixed with silica sand in a saturated cell under a pressure of 0.2 MPa and saturating the bentonite for about two months. The rate of axial strain during loading was set at 0.01%/min, lower than 0.05%/min applicable to ordinary clay \(^9\).

**Fig. 3** shows relationships between normalized deviator stress \(q\) and normalized mean effective stress \(p'/p_0'\) at varying levels of consolidation pressure \(p_0\) obtained in the consolidated undrained triaxial compression test on a saturated buffer material. The figure shows that the stress path was generally drawn as a single curve owing to normalization and that the mean limit state coefficient \(M\), or \(q/p'\), was 0.63.

**Fig. 4** shows relationships between the normalized deviator stress \(q\) and normalized deviator strain \(\varepsilon_d\) at varying levels of consolidation stress \(p_0\). The figure shows that the relationship between the normalized deviator stress and deviator strain is represented by a single curve regardless of consolidation stress.

Then, the shear capacity of buffer material can be represented uniformly based on the concept of effective stress and that an yield function can be set using normalized deviator stress and mean effective stress.

## 3. APPLICABILITY OF CAM-CLAY MODEL

Based on the consolidation and shear capacity of buffer material discussed above, the following points were revealed. (i) The relationship between void ratio \(e\) and mean effective stress \(\log p\) during the consolidation of compressed bentonite buffer material is represented generally by a straight line. (ii) The paths of normalized deviator stress and mean effective stress in undrained shear tests can be represented generally by single curves. (iii) The stress path in undrained shear tests forms a curve with a relatively low curvature as excess pore water pressure starts increasing in initial stages due to dilatancy. (iv) The relationship between normalized deviator stress and strain is represented generally by a single curve.

As a result, it has been found that the Cam-clay model, which has a good track record in the evaluation of ordinary clay, is applicable to the mechanical evaluation of compressed bentonite buffer material.

The equilibrium condition of plastic work in the Cam-clay model and the yield condition obtained based on Calladine’s assumption are expressed by the following equations \(^3\):

\[
\frac{q}{Mp} + \ln \frac{p'}{p_y} = 0
\]

where, \(M\) is the limit state coefficient and \(p_y'\) is the hardening parameter. Total strain \(\varepsilon\), the sum of
elastic strain $\varepsilon^e$ and plastic strain $\varepsilon^p$, is assumed to be expressed by

$$\varepsilon' = \varepsilon^e + \varepsilon^p$$  \hfill (2)

Suppose the relationship between void ratio $e$ and mean effective stress $\log p'$ is represented by a straight line either in the consolidation or in the unloading phase, then the relationship between plastic volumetric strain $\varepsilon_v^p$ and $p'_v$ is expressed by

$$\varepsilon^p_v = \frac{\lambda - \kappa}{\nu} \ln \frac{p'_v}{p_0}$$  \hfill (3)

where, $\lambda$ is the compressive index, $\kappa$ is the swelling index and $e_0$ is the initial void ratio. From equations (1) and (3), the yield function $f$ can be expressed by

$$f = \frac{q}{Mp} + \ln \frac{p'_v}{p_0} - \frac{1 + e_0}{\lambda - \kappa} \varepsilon_v^p$$  \hfill (4)

If it is assumed that a flow rule is applied, the normalized stress path obtained in a consolidated undrained triaxial compression test is obtained by equation (5) because the test is conducted under an undrained condition ($dE_v = 0$). The relationship between deviator stress and strain is obtained by

$$q' = \frac{M}{1 - \nu} \ln \frac{p'_0}{p}$$  \hfill (5)

$$\varepsilon_d = \frac{2\kappa(1 + \nu)}{\nu(1 + e_0)(1 - 2\nu)} \left\{ \frac{q}{p'} - \frac{\lambda - \kappa}{2\lambda M} \left( \frac{q}{p'} \right)^2 \right\}$$  \hfill (6)

where, $\nu$ is the effective Poisson's ratio of the soil skeleton.

The stress path obtained in the consolidated undrained triaxial test on the buffer material and the results of consolidation test were used to set $\lambda = 0.114$, $\kappa = 0.08$ and $M = 0.63$. Consolidation and triaxial compression tests were simulated. Fig. 5 and 6 show the results of simulation. These results indicate that the stress-strain relationship obtained in the triaxial compression test deviates greatly from laboratory test results and that simulation is not accurate in the unloading phase of consolidation test.

4. NONLINEARITY OF BUFFER MATERIAL IN THE UNLOADING PHASE

One reason for inaccurate evaluation of mechanical behavior of buffer material by the Cam-clay model may be nonlinearity in the unloading phase of consolidation test. Then, whether water-absorbing swelling of bentonite induces nonlinearity or not was examined.

In relation to the water-absorbing swelling of compressed bentonite, Komine et al. have conducted laboratory tests on the quantity and pressure of water-absorbing swelling using materials of varying dry densities and bentonite contents, and accurately reproduced test results based on the diffusion double layer theory using the following equations 4).

$$p_s = \frac{1}{\text{CEC}} \sum_{i=1}^{n} \left[ \frac{1}{k'} \frac{1}{M_g} \right] \left( f_i \right) \left( f_{d_i} \right) \left( f_{a_i} \right)$$  \hfill (7)

$$f_{i} = 2nkT(\cosh u_i - 1) \times 10^{-3}$$  \hfill (7-a)

$$u_i = 8 \tanh^{-1} \left[ \exp \left( -\eta_i d_i \right) \tanh \left( \frac{z_i}{4} \right) \right]$$  \hfill (7-b)
\[ \eta_i = \sqrt{\frac{2 n V_i^2 e^2}{e_i k T}} \]  

\[ z_i = 2 \sinh^{-1}\left( \frac{96.5 \times \text{EXC} \left( \frac{1}{8 \Delta n k T} \right)}{S} \right) \]  

\[ (f_a)_i = \frac{A_h}{24\pi} \left[ \frac{1}{d_i^3} + \frac{1}{(d_i + t)^3} - \frac{2}{(d_i + \frac{t}{2})^3} \right] \times 10^{-3} \text{(kPa)} \]  

\[ e^{+}_{\alpha} = e_{0} + \frac{e_{\alpha, max}}{100} (e_{0} + 1) \times \left\{ 1 + \left( \frac{100}{C_m - 1} \right) \frac{\rho_m}{\rho_{mn}} + \left( \frac{100}{\alpha - 1} \right) \frac{100}{C_m} \frac{\rho_m}{\rho_{sand}} \right\} \times 100 \]  

\[ e_0 = \frac{\rho_{solid} - \rho_{\alpha}}{\rho_{\alpha}} - 1 \]  

\[ \rho_{solid} = \frac{100}{C_m - 1} \frac{\rho_m}{\rho_{mn}} + \frac{100}{\alpha - 1} \frac{100}{C_m} \frac{\rho_m}{\rho_{sand}} \]  

\[ d_i = \frac{e^{+}_{\alpha}}{100}\left[ \left( R_{\alpha, i} \right) + \left( R_{\alpha, \text{in}} \right) \right] \text{(m)} \]  

\[ n = \frac{n_0 \times N_{\alpha}}{1 + e^{+}_{\alpha} / 100} \]  

\[ S = \frac{C_m}{100} S_m + \left( 1 - \frac{C_m}{100} \right) S_{\text{nm}} \text{ (m}^2\text{/g)} \]  

Where,

- \( P_s \): Water-absorbing swelling pressure of bentonite buffer material (kPa)
- \((f_i)_i\): Repulsive force caused by exchangeable cation \( i \) (kPa), where \( i \) indicates an exchangeable cation, \( \text{Na}^+, \text{Ca}^{2+}, \text{K}^+ \) or \( \text{Mg}^{2+} \). \( f_i \) below indicates the same.
- \((f_a)_i\): Attraction caused by exchangeable cation \( i \) (kPa)
- \( \text{EXC}_i \): Exchange capacity of exchangeable cation \( i \) (mequiv./g)
- \( \text{CEC} \): Cation exchange capacity (mequiv./g)
- \( d_i \): 1/2 of distance between crystal layers of exchangeable cation \( i \)
- \( v_i \): Valence of exchangeable cation \( i \)
- \( e' \): Electron charge (= 1.602 \times 10^{-19} \text{C})
- \( k \): Boltzmann constant (= 1.38 \times 10^{-23} \text{J/k})
- \( T \): Absolute temperature (K)
- \( n \): Ion concentration of buffer material pore water (mol/m^3)
- \( n_0 \): Initial ion concentration of buffer material pore water (mol/m^3)
- \( \varepsilon_c \): Dielectric constant of pore water (C^2 J^-1 m^-1)

\( A_h \): Hamaker constant (= 2.2 \times 10^{-20} \text{J for montmorillonite})

\( t \): Thickness of montmorillonite crystal layer (=9.60 \times 10^{-10} \text{m})

\( e_{\alpha, max} \): Maximum swelling ratio of buffer material (%)

\( e_0 \): Initial void ratio of buffer material

\( C_m \): Montmorillonite content of bentonite (%)

\( \rho_{\alpha} \): Initial dry density of buffer material (Mg/m^3)

\( \alpha \): Bentonite content of buffer material (%)

\( \rho_m \): Soil particle density of montmorillonite (Mg/m^3)

\( \rho_{\text{nm}} \): Soil particle density of mineral other than montmorillonite (Mg/m^3)

\( \rho_{\text{sand}} \): Sand particle density (Mg/m^3)

\( R_{\alpha, i} \): Unhydrated radius of exchangeable cation \( i \) between montmorillonite crystal layers (m)

\( N_A \): Avogadro’s number (= 6.023 \times 10^{23} \text{)}

\( S \): Specific surface area of bentonite (m^2/g)

\( S_m \): Specific surface area of montmorillonite (m^2/g)

\( S_{\text{nm}} \): Specific surface area of mineral other than montmorillonite (m^2/g)

Fig.7 compares the relationship between water-absorbing swelling pressure and void ratio obtained by the above evaluation equations with the relationship between consolidation pressure and void ratio obtained in tests using three pieces of specimen.

Table 2 lists constants used for analysis.

The figure shows that the consolidation pressure-void ratio curve gradually approaches the curve proposed by Komine (4) in the area where consolidation pressure lowers in the unloading phase, and that the consolidation pressure-void ratio curve is similar to the swelling pressure-void ratio curve in the consolidation phase. The swellability of buffer material may therefore be greatly responsible for the nonlinearity in the unloading phase in the consolidation test.

5. CONSTITUTIVE MODEL CONSIDERING SWELLING

(1) In-line and parallel models

In-line or parallel model is available as a constitutive model of swelling soil material (Fig. 8). The in-line model represents in series elastic and plastic elements composed of an elastic spring and a slipping element, and a swelling element based on the diffusion double layer theory. The total strain is assumed to be the sum of elastoplastic strain and swelling strain. Equal forces act on each component of the model. In the parallel model, elastoplastic and swelling elements are represented in parallel. Elastoplastic strain is always equal to swelling strain. The total force is the sum of forces acting on the elastoplastic and swelling elements. Van der Waals
Table 2 Parameters of Komine’s equations.

| Item                                                                 | Symbol | Unit   | Value          |
|----------------------------------------------------------------------|--------|--------|----------------|
| Silica sand content                                                  | W      | %      | 30             |
| Bentonite content                                                   | X      | %      | 70             |
| Void ratio of buffer material at the start of consolidation         | e_o   |        | 0.68           |
| Void ratio of buffer material at the start of unloading             | e_u   |        | 0.28           |
| Dry density of buffer material at the start of consolidation        | d_o   |        | 1.6            |
| Dry density of buffer material at the start of unloading             | d_u   |        | 2.1            |
| Soil particle density of montmorillonite                            | d_m   |        | 1.73           |
| Soil particle density of mineral other than montmorillonite         | d_NM  |        | 2.81           |
| Sand particle density                                                | d_s   |        | 2.66           |
| Specific surface area of montmorillonite                            | S_m   | m²/g   | 810.0          |
| Specific surface area of mineral other than montmorillonite         | S_NM  | m²/g   | 0.0            |
| Montmorillonite content                                             | C_m   | %      | 48.0           |
| Cation exchange capacity                                            | CEX   | mequiv./g | 0.732         |
| Dielectric constant of pore water                                   | ε      | C²/(J.m) | 6.83 × 10⁻¹⁰ |
| Ion concentration of buffer material pore water                     | n_o   | mol/m³ | 45.0           |
| Temperature                                                         | T      | K      | 295.0          |
| Ratio exchange capacity                                            | R_0   | mm     | 0.406          |
| Unhydrated radius                                                   | R_u   | mm     | 0.098          |
| Volume of ion                                                       | V      |        | 1              |
| Ratio exchange capacity                                            | R_0   | mm     | 0.287          |
| Unhydrated radius                                                   | R_u   | mm     | 0.1115         |
| Volume of ion                                                       | V      |        | 2              |
| Ratio exchange capacity                                            | R_0   | mm     | 0.009          |
| Unhydrated radius                                                   | R_u   | mm     | 0.133          |
| Volume of ion                                                       | V      |        | 3              |
| Ratio exchange capacity                                            | R_0   | mm     | 0.00           |
| Unhydrated radius                                                   | R_u   | mm     | 0.0835         |
| Volume of ion                                                       | V      |        | 2              |

It was also assumed from the result of analysis of nonlinearity in the unloading phase of consolidation test on buffer material that swelling function \( \phi \) was expressed by

\[
\phi = a \ln \left( \frac{P'}{P_0} + b \right) \tag{10}
\]

It was made clear based on the above assumptions that the in-line model provides for the evaluation of nonlinearity during unloading in a consolidation test and for fairly accurate evaluation in an undrained triaxial compression test.

(3) Parallel model

Fig. 9 shows the upward parallel translation of the swelling pressure-void ratio curve that was calculated using the Komine model to the curve based on the mean value obtained in three consolidation tests. The figure shows that upward parallel movement of the swelling pressure-void ratio line causes the line to nearly fit the curve representing the results of consolidation test in the loading phase. Thus, the quantity of change in void ratio corresponding to the change in consolidation pressure obtained in the consolidation test is nearly identical to the quantity of change in void ratio corresponding to the change in swelling pressure that is equivalent to the change in consolidation pressure. In the in-line model described above, therefore, the stress acting on the elastoplastic element is the same as that on the swelling element. The total volumetric strain then
becomes the same as the swelling strain with the elastoplastic strain being nil. Thus, no elastoplastic behavior takes place. This is in conflict with actual phenomena. In the parallel model, however, no conflict occurs even where the swelling strain becomes the same as the total volumetric strain, and both elastoplastic behavior and swelling from a viewpoint of clay science can be evaluated simultaneously. In actual buffer materials, the distance between water layers and void distribution in a soil particle vary greatly. Each soil particle causes electrochemical effects. Then, it was assumed that average electrochemical effects and the elastoplastic behavior of skeleton can be aligned in parallel.

In the parallel model, total strain, elastoplastic strain and swelling strain are equal to one another, and can be expressed by equation (11).

$$\varepsilon'_{v} = \varepsilon_{v}^{p} = \varepsilon_{v}^{p} + \varepsilon_{v}^{p}$$  \hspace{1cm} (11)

The mean principal stress is the sum of mean principal stress acting on the skeleton and swelling pressure and expressed by equation (12). Of effective stresses $\sigma_{ij}$, only average stress is assumed to be affected by swelling pressure. Then, equations (13) and (14) become effective.

$$p' = \rho' + p_{sw}$$  \hspace{1cm} (12)

$$\sigma_{ij} = \delta_{ij} \rho' + \delta_{ij} p_{sw}$$  \hspace{1cm} (13)

$$\rho' = \hat{\rho}'$$  \hspace{1cm} (14)

where, $\rho'$ is the mean effective stress acting on the soil material skeleton (an elastoplastic element), $\delta_{ij}$ is the stress acting on the soil material skeleton and $p_{sw}$ is the water-absorbing swelling pressure.

Once equation (12) becomes effective, the mean principal stress acting on the soil material skeleton at a given void ratio can be expressed by

$$\rho' = p' - p_{sw}$$  \hspace{1cm} (15)

Suppose that the swelling pressure-void ratio relationship is reversible and as shown by the Komine model, then the relationship between the mean principal stress acting on the skeleton and void ratio based on the assumption of $K_0 \approx 1$ will be as shown in Fig. 10 either in the unloading or in the re-loading phase. Fig. 9 and 10 show that consolidation pressure is lower than swelling pressure at the same void ratio near the point of minimum consolidation pressure in the unloading phase of consolidation test. $\rho'$ becomes negative and compression occurs in the skeleton, resulting in disagreement with the physical phenomenon. Actually, however, $\rho'$ is not likely to become negative because compressed bentonite with a high plasticity index is used in the specimen in the consolidation test, because friction between the ring and specimen cannot be ignored and because there is a variance in swelling between the Komine model and the specimen. Thus, consolidation test results are expected to gradually approach the swelling pressure-void ratio curve. Fig. 9 and 10 show that the compression index $\lambda$ for consolidation pressure acting on the skeleton is nearly equal to the compression index $\lambda'$ for consolidation pressure and that the behavior during unloading is represented by a straight line.

As described earlier, the upward parallel translation of the swelling pressure-void ratio curve causes it to nearly fit the consolidation pressure-void ratio curve in the loading phase. The e-logsw relationship is therefore represented approximately by a straight line. Then, the following relationship is assumed to be effective.
\[ e_i^v = e_i^p = \frac{c_i}{1 + c_0} \ln \frac{p_{sw}}{p_{sw0}} \]  

(16)

where, \( c_i \) is the water-absorbing swelling index and \( p_{sw0} \) is the swelling pressure at the initial void ratio. For the compressed bentonite buffer material, \( c_i \approx \lambda \).

becomes effective. Then, the relationship between swelling pressure and total volumetric strain is expressed by equation (17), which is parallel to the relationship between mean principal stress and total volumetric strain during consolidation.

\[ e_i^v = e_i^p = \frac{\lambda}{1 + e_0} \ln \frac{p_{sw}}{p_{sw0}} \]  

(17)

If the soil material skeleton is in compliance with the elastoplastic constitutive rule of the Sekiguchi-Ohta model, the yield function is expressed by equation (18) using plastic volumetric strain as the hardening parameter.

\[ f = \frac{\lambda - \bar{k}}{1 + e_0} \ln \frac{p'}{p_0} + D \hat{\eta}^* = e_i^p \]  

(18)

where, \( \hat{\eta}^* \) is a parameter related to deviator stress expressed in equation (19) and \( S^d \) is the deviator stress acting on the soil material skeleton.

\[ \hat{\eta}^* = \frac{3}{2} \begin{vmatrix} \hat{s}_{ij} - \hat{s}_{ij} \\ \hat{p} \hat{p}_0 \end{vmatrix} \]  

(19)

Applying a related flow rule, the incremental plastic strain is finally expressed by

\[ \dot{e}_i^p = \hat{e}_i^p = \frac{3}{3 + \frac{3}{2} \hat{s}_{ij} \hat{p} + \frac{3}{2} \hat{s}_{ij} \hat{p}_0} \left( \frac{\delta_{ij}^*}{\eta^*} \right) \]  

(20)

The incremental elastic strain is expressed by

\[ \dot{e}_i^e = \frac{\hat{k}}{3(1 + e_0)} \left( \hat{p} \delta_{ij}^* + \hat{s}_{ij} \right) \]  

(21)

Swelling index \( \hat{k} \) is assumed to vary according to stress because the gradients are different in the unloading and re-loading phases of the consolidation test, which are generally considered to be in the elastic area. Where stress acts so that the soil material skeleton in the plastic area satisfies yield function \( f \) expressed by equation (18), the typical mean principal stress \( \hat{p} \) is identical to the mean principal stress acting on the skeleton where \( \hat{p} \) is null in the yield function. \( \hat{p} \) in Fig. 10 is interchangeable with \( \hat{p}^* \).

\[ \hat{p}^* = \hat{p}_0 \exp \left( \frac{1 + e_0}{\lambda - \hat{k}} \right) \]  

(22)

\( \hat{k} \) is assumed to vary as shown below. \( \hat{k} \) is set using the consolidation test results and the Komine model (Fig. 10).

(During unloading: \( \hat{p}^* = 0 \))

\( \hat{k} = \hat{k}_1 \)

(At the start of re-loading: \( \hat{p}^* \geq 0, \hat{p}^* < \hat{p}^* \))

\( \hat{k} = \hat{k}_2 \)

(0.1 \hat{k}_1)

(During re-loading: \( \hat{p}^* \geq 0, \hat{p}^* \geq \hat{p}^* \))

\( \hat{k} = \hat{k}_3 \)

Typical mean principal stress at the point of gradient change during re-loading is obtained by

\[ \hat{p}^* = \hat{p}_0 \exp \left( \frac{\hat{p}^* - e_i^*}{\hat{k}_3} \right) \]  

(23)

where, \( \hat{p}^* \) is the typical mean principal stress at the start of unloading and \( e_i^* \) and \( e_i^* \) are void ratios at the start of re-loading and unloading, respectively. The stress-strain relationship of the soil material skeleton is obtained by equations (24) through (27) based on equations (20) and (21).

\[ \dot{\hat{p}}_i = \hat{D}_{ijkl} \hat{e}_{kl} \]  

(24)

\[ \hat{D}_{ijkl} = \hat{D}_{ijkl} - \hat{D}_{ij} \left( f_{ij} \hat{D}_{ijkl} + \hat{\mu} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \right) \]  

(25)

\[ \hat{D}_{ijkl} = \hat{\lambda} \delta_{ij} \delta_{kl} + \hat{\mu} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \]  

(26)

\[ f_{ij} = \frac{\hat{f}}{\tilde{\lambda}} \]  

(27)

where, \( \hat{\lambda} \) and \( \hat{\mu} \) are Lame's constants and are given by

\[ \tilde{\lambda} = \frac{3\nu \left( 1 + e_0 \right)}{(1 + \nu)} \hat{p}' \]  

\[ \hat{\mu} = \frac{3\left( 1 - 2\nu \right) \left( 1 + e_0 \right)}{2(1 + \nu)} \hat{p}' \]  

(28)
Compression test analysis model

Fig. 11 One-element plane-strain analysis model.

In relation to water-absorbing swelling, the swelling stress-strain relationship is obtained based on equation (16) as shown below.

\[ \dot{\varepsilon}_n = \frac{\lambda}{1 + \varepsilon_0} \frac{\dot{p}_{sw}}{p_{sw}} \]  

(29)

Then, the swelling stress in the effective stress in equation (13) is expressed by

\[ \delta_{ij} \dot{p}_{sw} = D_{ijkl}^{sw} \dot{\varepsilon}_{kl} \]  

(30)

\[ D_{ijkl}^{sw} = \frac{1 + \varepsilon_0}{\lambda} p_{sw} \delta_{ij} \delta_{kl} \]  

(31)

As a result, the relationship between effective stress and strain is expressed by equations (32) and (33) based on equations (24) through (30) and (13).

\[ \sigma_{ij}^{e} = D_{ijkl} \dot{\varepsilon}_{kl} \]  

(32)

\[ D_{ijkl} = \hat{D}_{ijkl} + D_{ijkl}^{sw} \]  

(33)

6. SIMULATION ANALYSIS USING A PARALLEL MODEL

Laboratory tests on the buffer material were simulated using a parallel model considering water-absorbing swelling that was described in 5.3. For analysis, DACSAR, an FEM analysis code of the Sekiguchi-Ohta model developed by Iizuka et al. was enhanced so as to comply with the constitutive model described in 5.3 11). Fig. 11 shows a one-element plane-strain model used for analysis. Table 3 lists constants used for analysis.

Fig. 12 Results of analysis of consolidation test. (up to unloading)

Fig. 13 Stress paths in consolidated undrained triaxial compression test.

being tracked down.

Fig. 13 shows stress paths in the consolidated undrained triaxial compression test. Evaluation was relatively accurate except for the behavior in the initial stages of loading. Fig. 14 shows test and simulation results for stress-strain relationship. The peak strength and strain were nearly in agreement with the test results. The behavior in the initial stages of loading was, however, not simulated accurately as in Fig. 13 that shows stress paths. This may be be cause the test results involved time dependency while the analysis was made treating specimens as elastoplastic bodies. In the future, therefore, constitutive models will be developed that could evaluate time dependency as well. As a result, it has been found that the elastoplastic behavior of the buffer material, a swelling soil material, can be estimated fairly accurately using a parallel model that takes water-
absorbing swelling into consideration.

7. CLOSING REMARK

A model that aligns swelling and elastoplastic elements in parallel was built as a constitutive model applicable to highly swelling soil materials. The applicability of the model could be presented based on the results of a laboratory test on a compressed bentonite buffer material. In the future, applicability to swelling ground will be examined.

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