Contributions of vector-like quarks 
to radiative $B$ meson decay

Mayumi Aoki$^1$, Eri Asakawa$^2$, Makiko Nagashima$^2$, Noriyuki Oshimo$^3$, 
and Akio Sugamoto$^{2,3}$

$^1$Theory Group, KEK, Tsukuba, Ibaraki 305-0801, Japan 
$^2$Graduate School of Humanities and Sciences 
Ochanomizu University, Otsuka 2-1-1, Bunkyo-ku, Tokyo 112-8610, Japan 
$^3$Department of Physics 
Ochanomizu University, Otsuka 2-1-1, Bunkyo-ku, Tokyo 112-8610, Japan

Abstract

We study the decay $B \rightarrow X_s \gamma$ in a minimal extension of the standard model with 
extra up- and down-type quarks whose left- and right-handed components are both 
SU(2) singlets. Constraints on the extended Cabibbo-Kobayashi-Maskawa matrix 
are obtained from the experimental results for the branching ratio. Even if the 
extra quarks are too heavy to be detected in near-future colliders, the branching 
ratio could have a value which is non-trivially different from the prediction of the 
standard model.

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The inclusive decay $B \to X_s \gamma$ is well described by the free quark decays $b \to s \gamma$ and $b \to s \gamma g$, owing to a large mass of the $b$ quark. Since these decays are generated at the one-loop level of the electroweak interactions, the radiative $B$-meson decay is sensitive to new physics beyond the Standard Model (SM) \cite{[1]}, such as the supersymmetric model \cite{[2]}. Its branching ratio could deviate from the prediction of the SM. Or some constraints could be imposed on new physics. Experimentally, the branching ratio has been measured by CLEO \cite{[3]} and ALEPH \cite{[4]} as

$$\text{Br}(B \to X_s \gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}, \quad (1)$$

$$= (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}. \quad (2)$$

These results are consistent with the SM prediction $\text{Br}(B \to X_s \gamma) = (3.29 \pm 0.33) \times 10^{-4}$ \cite{[5]}, though still show room for the contribution of new physics.

The SM is minimally extended by incorporating extra colored fermions whose left-handed components, as well as right-handed ones, are singlets under the SU(2) gauge transformation, with electric charges being 2/3 and/or $-1/3$. In this vector-like quark model (VQM), many features of the SM are not significantly modified. However, the interactions of the quarks with the $W$ or $Z$ boson are qualitatively different from those in the SM. The Cabibbo-Kobayashi-Maskawa (CKM) matrix for the charged current is extended and not unitary. The neutral current involves interactions between the quarks with different flavors. In addition, the neutral Higgs boson also mediates flavor-changing interactions at the tree level. The VQM could thus give sizable new contributions to processes of flavor-changing neutral current (FCNC) \cite{[6,7]} and of $CP$ violation \cite{[8,9]}.

In this paper we study the radiative $B$-meson decay within the framework of the VQM containing one up-type and one down-type extra quarks. The decay receives contributions from the interactions mediated by the $W$, $Z$, and Higgs bosons. The effects by the $Z$ and Higgs bosons have already been studied and found to be small \cite{[4]}. Our study is concentrated on the other effects coming from the $W$-mediated interactions. These interactions give contributions differently from the SM at the electroweak energy scale, since an extra up-type quark is involved and the CKM matrix is not the same as that of the SM. It will be shown that the decay width can be much different from the SM prediction, even if the extra quark is rather heavy. The experimental results for the decay rate thus impose non-trivial constraints on the extended CKM matrix.

We assume that there exist two extra Dirac fermions whose transformation properties are given by $(3, 1, 2/3)$ and $(3, 1, -1/3)$ for the SU(3)×SU(2)×U(1) gauge sym-
metry. The mass terms of the quarks are then expressed by $4 \times 4$ matrices. These mass matrices, which are denoted by $M^u$ and $M^d$ respectively for up- and down-type quarks, are diagonalized by unitary matrices $A^u_L$, $A^u_R$, $A^d_L$, and $A^d_R$ as

\[ A^u_L \, M^u \, A^u_R = \text{diag}(m_{u1}, m_{u2}, m_{u3}, m_{u4}), \]
\[ A^d_L \, M^d \, A^d_R = \text{diag}(m_{d1}, m_{d2}, m_{d3}, m_{d4}). \]

The mass eigenstates are expressed by $u^a$ and $d^a$ ($a = 1 - 4$), $a$ being the generation index, which are also called as $(u, c, t, U)$ and $(d, s, b, D)$.

The interaction Lagrangian for the quarks with the $W$ and Goldstone bosons is given by

\[ \mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{a,b=1}^{4} \bar{u}^a V_{ab} \gamma^\mu \left\{ \frac{1}{2} \left( V^\dagger V \right)_{ab} \frac{1 - \gamma_5}{2} + \frac{1}{3} \sin^2 \theta_W \delta_{ab} \right\} \gamma^\mu d^b W^\dagger_{\mu} \]
\[ + \frac{g}{\sqrt{2}} \sum_{a,b=1}^{4} \bar{u}^a V_{ab} \left\{ \frac{m_{ua} M_W}{2} \left( 1 - \gamma_5 \right) - \frac{m_{db} M_W}{2} \left( 1 + \gamma_5 \right) \right\} d^b G^\dagger_{\mu} \]
\[ + \text{h.c..} \]

Here the $4 \times 4$ matrix $V$ stands for an extended Cabibbo-Kobayashi-Maskawa matrix, which is defined by

\[ V_{ab} = \sum_{i=1}^{3} (A^u_L)_{ai} (A^d_L)_{ib}. \]

It should be noted that $V$ is not unitary:

\[ (V^\dagger V)_{ab} = \delta_{ab} - A^u_{L4a} A^d_{L4b}. \]

The interaction Lagrangian for the down-type quarks with the $Z$, Higgs, and Goldstone bosons is given by

\[ \mathcal{L} = \frac{g}{\cos \theta_W} \sum_{a,b=1}^{4} \bar{d}^a \gamma^\mu \left\{ \frac{1}{2} \left( V^\dagger V \right)_{ab} \frac{1 - \gamma_5}{2} + \frac{1}{3} \sin^2 \theta_W \delta_{ab} \right\} d^b Z^\mu \]
\[ - \frac{g}{2} \sum_{a,b=1}^{4} \bar{d}^a \left( V^\dagger V \right)_{ab} \left\{ \frac{m_{da} M_W}{2} \left( 1 - \gamma_5 \right) + \frac{m_{db} M_W}{2} \left( 1 + \gamma_5 \right) \right\} d^b H^0 \]
\[ + i \frac{g}{2} \sum_{a,b=1}^{4} \bar{d}^a \left( V^\dagger V \right)_{ab} \left\{ \frac{m_{da} M_W}{2} \left( 1 - \gamma_5 \right) - \frac{m_{db} M_W}{2} \left( 1 + \gamma_5 \right) \right\} d^b G^0. \]

Since $V$ is not a unitary matrix, there appear interactions of FCNC at the tree level. The Lagrangians in Eqs. (5) and (8) contain new sources of $CP$ violation [8].
The decay $B \to X_s \gamma$ is approximated by the radiative $b$-quark decays, which are mediated by the $W$, $Z$, and Higgs bosons. The relevant effective Hamiltonian with five quarks is then written as

$$
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left[ \sum_{j=1}^{6} \left\{ C_j(\mu)O_j(\mu) + \tilde{C}_j(\mu)\tilde{O}_j(\mu) \right\} + \sum_{j=7}^{8} C_j(\mu)O_j(\mu) \right],
$$

(9)

where $O_j$, $\tilde{O}_j$ represent operators for the $\Delta B = 1$ transition, with $C_j$, $\tilde{C}_j$ being their Wilson coefficients. The evaluated energy scale is denoted by $\mu$. The four-quark operators induced by the gauge boson interactions are denoted by $O_j (j = 1 - 6)$ [10]. The Higgs boson interactions induce new four-quark operators, which are denoted by $\tilde{O}_j (j = 1 - 6)$. The dipole operators for $b \to s\gamma$ and $b \to sg$ are denoted by $O_7$ and $O_8$, respectively, which are generated by the one-loop diagrams shown in Fig. [1]. Hereafter, we only take the $W$ boson interactions into consideration, since the contributions coming from the $Z$ and Higgs boson interactions are known to be much smaller than the SM contribution.

At the leading order (LO), the Wilson coefficients $C_2$, $C_7$, and $C_8$ have non-vanishing values at $\mu = M_W$, which are given by

$$
C_2(M_W) = V_{32}^* V_{33} + V_{42}^* V_{43} - (V^\dagger V)_{23},
$$

(10)

$$
C_7(M_W) = \frac{23}{36} (V^\dagger V)_{23} - \sum_{a=3}^{4} V_{a2}^* V_{a3} \frac{3}{2} r_a \left\{ \frac{2}{3} I_1(r_a) + J_1(r_a) \right\},
$$

(11)

$$
C_8(M_W) = \frac{1}{3} (V^\dagger V)_{23} - \sum_{a=3}^{4} V_{a2}^* V_{a3} \frac{3}{2} r_a I_1(r_a),
$$

(12)

$$
r_a = \frac{m_{ua}^2}{M_W^2}.
$$

The functions $I_1(r)$ and $J_1(r)$ are defined as [11]

$$
I_1(r) = \frac{1}{12(1-r)^4} (2 + 3r - 6r^2 + r^3 + 6r \ln r),
$$

(13)

$$
J_1(r) = \frac{1}{12(1-r)^4} (1 - 6r + 3r^2 + 2r^3 - 6r^2 \ln r).
$$

(14)

The non-unitarity of the CKM matrix $V$ yields the terms proportional to $(V^\dagger V)_{23}$ for $C_2$, $C_7$, and $C_8$. The Wilson coefficients at $\mu = m_b$ are obtained by solving the renormalization group equations. Using the LO anomalous dimension matrix, the coefficients are given by

$$
C_2(m_b) = \frac{1}{2} (\eta^+ - \eta^-) C_2(M_W),
$$

(15)
\[ C_7(m_b) = \eta \frac{15}{3} C_7(M_W) + \frac{8}{3} (\eta - \frac{16}{23}) C_8(M_W) + \sum_{i=1}^{8} h_i \eta a_i C_2(M_W), \quad (16) \]

\[ C_8(m_b) = \eta \frac{44}{3} C_8(M_W) + \sum_{i=1}^{8} \bar{h}_i \eta a_i C_2(M_W), \quad (17) \]

with \( \eta = \alpha_s(M_W)/\alpha_s(m_b) \) which is set for \( \eta = 0.56 \) in the following numerical study. The constants \( h_i, \bar{h}_i, \) and \( a_i \) are listed in Table 1 [12]. The branching ratio for \( B \to X_s \gamma \) is obtained by normalizing the decay width to that of the semileptonic decay \( B \to X_c e\bar{\nu} \), leading at the LO to

\[ \text{Br}(B \to X_s \gamma) = \frac{6\alpha_{\text{EM}}}{\pi f(z)|V_{23}|^2} |C_7(m_b)|^2 \text{Br}(B \to X_c e\bar{\nu}), \quad (18) \]

with \( z = m_c^2/m_b^2 \) and \( f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z \).

The obtained branching ratio at the LO has non-negligible perturbative uncertainties, which are reduced by taking into account corrections at the next leading order (NLO). For the numerical evaluation, therefore, we incorporate NLO corrections for the matrix elements at \( \mu = m_b \) [13] and the anomalous dimensions [14]. Our calculations follow formulae given in Ref. [3], which also include QED corrections.

The decay width of \( B \to X_s \gamma \) depends on the \( U \)-quark mass \( m_U \) and the CKM matrix elements \( V_{32}^* V_{33}, V_{42}^* V_{43}, (V^\dagger V)_{23} \). The value of \( (V^\dagger V)_{23} \) determines the FCNC interactions at the tree level in Eq. (8), which is constrained from non-observation of \( B \to K \mu^+ \mu^- \) [15] as

\[ |(V^\dagger V)_{23}| < 8.1 \times 10^{-4}. \quad (19) \]

The CKM matrix elements connecting light ordinary quarks, which are directly measured in experiments, have the same values as those in the SM. From the values of \( V_{12}, V_{13}, V_{22}, \text{and } V_{23} \) [15], we obtain a constraint

\[ 0.03 < |V_{32}^* V_{33} + V_{42}^* V_{43} - (V^\dagger V)_{23}| < 0.05. \quad (20) \]

The mass \( m_U \) should be heavier than the \( t \)-quark mass. In principle, the \( U \)-quark mass and the CKM matrix elements are not independent each other, their relations being determined by the mass matrices \( M^u \) and \( M^d \). However, these relations depend on many unknown factors for the mass matrices. Furthermore, the values of \( m_U \) and \( V_{42}^* V_{43} \) are thoroughly unknown phenomenologically except for the above constraints. We therefore take them for independent parameters.

The decay width is mainly determined by the Wilson coefficient \( C_7(m_b) \) as seen from Eq. (18). Expressing explicitly the dependence on the CKM matrix elements,
the coefficient $C_7(m_b)$ in Eq. (13) is written as

$$C_7(m_b) = A_1 (V^\dagger V)_{23} + A_2 V^*_3 V^*_{33} + A_3 V^*_4 V^*_{43}, \quad (21)$$

where $A_3$ is a function of $m_U$ while $A_1$ and $A_2$ are constants. We show the $m_U$ dependency of $A_3$ in Fig. 2, where $A_1$ and $A_2$ are also depicted. For $m_U \gtrsim 200$ GeV, the value of $A_3$ does not vary much with $m_U$ and is comparable with $A_2$. Unless $V^*_4 V^*_{43}$ is much smaller than $V^*_3 V^*_{33}$, the coefficient $C_7(m_b)$ can be predicted differently from the SM value. Although $A_1$ is larger than $A_2$ and $A_3$ in magnitude, the smallness of $(V^\dagger V)_{23}$ makes the term $A_1 (V^\dagger V)_{23}$ less important.

In Fig. 3 we show allowed regions for $V^*_3 V^*_{33}$ and $V^*_4 V^*_{43}$, assuming for simplicity that these values are real. The shaded regions are compatible with the experimental results of both Eq. (2) for $B \to X_s \gamma$ and Eq. (20) for the CKM matrix elements. The regions between the solid lines satisfy the latter. We have taken the $U$-quark mass for $200$ GeV $< m_U < 1$ TeV and $(V^\dagger V)_{23}$ for its maximal value $8.1 \times 10^{-4}$. The branching ratio of $B \to X_s \gamma$ sizably constrains the CKM matrix elements of the VQM. The allowed regions are slightly altered for $(V^\dagger V)_{23} = -8.1 \times 10^{-4}$.

In Fig. 4 the branching ratio of $B \to X_s \gamma$ is depicted as a function of $m_U$ for $V^*_4 V^*_{43} = -0.006, -0.002, 0.004, 0.006$. For definiteness, we put $V^*_3 V^*_{33} = 0.04$ and $(V^\dagger V)_{23} = 8.1 \times 10^{-4}$. The experimental bounds Eqs. (1) and (2) are also shown. For $|V^*_4 V^*_{43}/V^*_3 V^*_{33}| \gtrsim 0.1$, the predicted value is non-trivially different from that of the SM. The branching ratio could have any value within the experimental bounds.

In summary, we have studied the effects of the VQM on the branching ratio for the radiative $B$-meson decay. Among the possible new contributions, the $W$-mediated diagrams yield sizable effects. From the experimental results for the branching ratio, the values of $V^*_3 V^*_{33}$ and $V^*_4 V^*_{43}$ are constrained. These constraints do not much depend on the mass of the extra quark $U$. The VQM could make the branching ratio of $B \to X_s \gamma$ different from the SM prediction. If precise measurements in the near future show a difference between the experimental value and the SM prediction, the VQM may become one candidate for physics beyond the SM.

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Table 1: The values of $h_i$, $\bar{h}_i$, and $a_i$ in Eqs. (16) and (17).

| $i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|-----|----|----|----|----|----|----|----|----|
| $a_i$ | $\frac{14}{23}$ | $\frac{16}{23}$ | $\frac{6}{23}$ | $-\frac{12}{23}$ | 0.4086 | -0.4230 | -0.8994 | 0.1456 |
| $h_i$ | $\frac{626126}{272277}$ | $-\frac{56281}{51730}$ | $-\frac{3}{7}$ | $-\frac{1}{14}$ | -0.6494 | -0.0380 | -0.0186 | -0.0057 |
| $\bar{h}_i$ | $\frac{313963}{363036}$ | 0 | 0 | 0 | -0.9135 | 0.0873 | -0.0571 | 0.0209 |

Figure 1: The diagrams which give contributions to $C_7$ and $C_8$. The photon or gluon line should be attached appropriately.
Figure 2: The values of $A_1$, $A_2$, and $A_3$ in Eq. (21).
Figure 3: The allowed regions for $V_{32}^* V_{33}$ and $V_{42}^* V_{43}$. $(V^\dagger V)_{23} = 8.1 \times 10^{-4}$. 
Figure 4: The branching ratio of $B \rightarrow X_s \gamma$. $V_{42}^* V_{43} = -0.006, -0.002, 0.004, 0.006$, $V_{32} V_{33} = 0.04$, $(V^\dagger V)_{23} = 8.1 \times 10^{-4}$. 