In this lecture I discuss jet-shape distributions and describe how from jet evolution one may design Monte Carlo simulations which are used in the analysis of short distance distributions in $e^+e^-$-annihilation, lepton-hadron and hadron-hadron collisions.

1 Introduction

A major difficulty is encountered in dealing with perturbative QCD: in computations one uses quarks and gluons but only hadrons are observed. Actually, there is no obvious conflict with colour confinement since it is impossible to compute distributions with quarks or gluons in initial or final states, they are divergent due to collinear and infrared singularities. Even, with this general impediment, there are ways to make predictions for hadronic distributions. In more than thirty years since the discovery of asymptotic freedom$^1$ an enormous amount of information on QCD at short distance has been obtained by perturbative (PT) methods. For instance, collinear (and infrared) divergences are present in DIS structure functions (hadron in initial state) or in $e^+e^-$ fragmentation functions (hadron in final state). However these divergences factorize so that one is able to compute the evolution$^2$ of these distributions in the hard scale and makes predictions in terms the distribution at a given scale.

In principle, the only possibility to make absolute predictions (i.e. without involving phenomenological inputs except for $\Lambda_{\text{QCD}}$) is to compute inclusive short distance distributions such as $e^+e^-$ total hadronic cross section in which all PT coefficients are finite (however, as I will recall, difficulties come from non-convergence of PT expansions). Using inclusive observables one can reach a “complete” description of the emission of QCD radiation. To give examples of the amount of information obtained in jet-physics studies I discuss in the following jet-shape distributions and describe how from jet evolution one may design Monte Carlo simulations.

2 Jet-shape observables

In all hard processes ($e^+e^-$-annihilation, lepton-hadron and hadron-hadron collisions) one may introduce a large variety of jet-shape observables. Example in $e^+e^-$ are thrust $T$ and broadening $B$

$$T = 1 - \frac{\sum_h p_{ht} e^{-|\eta_h|}}{Q}, \quad 2B = \sum_h \frac{p_{ht}}{Q},$$

(1)

with $Q$ the hard scale (total $e^+e^-$ center of mass energy). The sum is over all emitted hadrons with $p_{ht}$ transverse momentum and $\eta_h$ rapidity with respect to the thrust axis (which maximizes $T$). It is clear from these examples that different jet-shape observables characterize different aspects of radiation (transverse momentum contributes uniformly to $B$ and mostly at large angles to $T$). In general one finds that jet-shape observables ($V = \tau, B, C, D, K_{\text{out}}, \rho, y_{ij}$, etc) are small for most events. In the hard process under consideration, $V \rightarrow 0$ corresponds to the exclusive limit in which the minimum number of hadrons are emitted (two for $T$ or $B$ in $e^+e^-$) so that QCD radiation is characterized by jets around these primary hadrons.

A quantitative description of jet radiation can be obtained by studying the fully inclusive hadron distributions for the various jet-shape observables $V$

$$\Sigma(V) = \sum_n \int \frac{d\sigma_n}{\sigma_{\text{tot}}} \Theta \left( V - \sum_n v_h \right).$$

$^1$Talk given at “QCD at Cosmic Energies,” Erice, 29 August-5 September, 2004.
In the PT study of $\Sigma(V)$ one replaces the sum of hadron momenta with the one of quarks and gluons. Is it possible to assume that the PT distributions represent the hadron distributions? A positive answer is suggested by the property that in general these inclusive observables are collinear finite (given $\vec{p}_i$, $\vec{p}_j$ which are collinear, then $V$ is the same if one replaces them with the single momentum $\vec{p}_i + \vec{p}_j$) and infrared finite (given $\vec{p}_i$, $\vec{p}_j$ with $|\vec{p}_i| \ll |\vec{p}_j|$ then $V$ is the same if $\vec{p}_i$ is neglected). Therefore, assuming that hadrons are made of collinear or soft partons, $V$ remains the same if one replaces hadrons with partons: hadron flow $\sim$ parton flow. In these cases all PT coefficients of $\Sigma(V)$ are finite. One has

$$\Sigma_{PT}(V) = a(V) \alpha_s^{n_0} + b(V) \alpha_s^{n_1} + \cdots$$  \hspace{1cm} (3)

with $\alpha_s = \alpha_s(Q)$ in $\overline{\text{MS}}$ scheme and $n_0$ depending on the number of jets involved ($n_0 = 1$ for 2-jet observable such as $T$ or $B$). In the exclusive limit $V \to 0$ all PT coefficient diverge due to collinear and infrared singularities. For $V \neq 0$ these singularities cancel but for small $V$ they leave large logarithmic terms. A reliable PT calculation of $\Sigma(V)$ at small $V$, where it is large, requires the resummation of the most important enhanced terms. They are organized as follows

$$\ln \Sigma_{PT}(V) = \sum_{n=1}^{\infty} \left\{ d_n \alpha_s^n L^{n+1} + s_n \alpha_s^n L^n + \cdots \right\}, \quad L = \ln V. \hspace{1cm} (4)$$

In order to control the scale of the logarithms one needs to resum at least $d_n$ (double logs, DL) and $s_n$ (single logs, SL) terms. To obtain a complete PT prediction for all values of $V$ one has to match in $\Sigma_{PT}(V)$ both resummed (4) and the exact (3) expressions. Although resummation procedures are now well established, to reach the required SL accuracy one needs very complex tools (Mellin or/and Fourier transforms, asymptotic estimations of integrals, etc). Recently a numerical program$^3$ is available that performs automate resummations for jet-shape distributions in $e^+e^-$, DIS and hadron-hadron collisions.

Before describing the structure of Feynman diagrams contributing to the enhanced terms in (4) I will recall the non-convergence$^8$ of PT expansion and its relation to the large distance region of confinement.

### 2.1 Power corrections

Various physical facts are at the origin of the non convergence of PT expansions. In general they imply the presence of power corrections to PT results. The fact which is phenomenologically most important$^5$ is that the running coupling is involved at any scale smaller than $Q$. For example, the average value of $V$ is given by

$$\langle V \rangle = \int_0^Q \frac{dk_t}{k_t} \alpha_s(k_t) \cdot V(k_t/Q), \hspace{1cm} (5)$$

where the virtual momentum $k_t$ in the Feynman diagrams runs into the large distance region (although the observable is at short distance). Since the observable is collinear and infrared finite, one has $k_t^{-1}V(k_t/Q) \sim Q^{-1}$ for $k_t \to 0$. Here however the coupling enters the confinement region. Mathematically this PT difficulty is reflected into the fact that, although all PT coefficients in $\alpha_s(Q)$ are finite, the expansion is non-convergent (renormalon singularity). To make a quantitative prediction one has to deal with the large distance region for the running coupling. There are various prescriptions for this.

One prescription$^6$ consists in introducing a non-perturbative (NP) parameter given by the integral of the running coupling in the large distance region and expressing (5) as

$$\langle V \rangle = \langle V \rangle_{PT}^N + \frac{\mu_I}{Q} \left\{ C_V \alpha_0(\mu_I) + \sum_{n=1}^{N} A_V^0 \alpha_s^n \right\}, \quad \alpha_0(\mu_I) = \int_{\mu_0}^{\mu_I} \frac{dk_t}{\mu_I} \alpha_s(k_t), \hspace{1cm} (6)$$

with $\mu_I$ a short distance scale ($\mu_I$-independence is ensured by renormalization group) and $C_V$ a known constant depending on the observable. Renormalons in the $N$-order PT expressions $\langle V \rangle_{PT}^N$ are canceled by the sum over the known coefficients $A_V^0$. The contribution from the NP parameter $\alpha_0(\mu_I)$ is suppressed by inverse powers in the hard scale. This power correction is detectable$^5$ even at LEP energies. Similar NP contributions are found in the PT study of the distribution $\Sigma(V)$ and also here one needs to introduce the NP parameter.
The effect here is in general a power correction “shift” in the argument of the PT result $\Sigma_{\text{PT}}(V)$. A consequence of this prescription is that the same NP parameter $\alpha_0(\mu_I)$ enters all jet-shape observables and then one can study is phenomenological consistence. In general one finds\(^5\) that for the various quantities the fitted values of $\alpha_0(\mu_I)$ varies within about 20%.

Higher power corrections are present and maybe important. Here one needs more general prescriptions\(^7\) with introduction of shape functions to modulate large distance contributions. These prescriptions allows one to describe the distributions at low values of $V$.

2.2 Structure of PT contributions

To understand the features of QCD radiation one needs to consider how PT resummation is obtained via factorization of (universal) collinear and infrared singularities. As pointed out by Dasgupta and Salam\(^8\), the situation is different for global and non-global jet-shape observables.

**Global jet-shape observables.** Here the full phase space of emitted hadrons is considered. Examples in $e^+e^-$ are $T$ and $B$ in (1). In these cases the DL and SL contributions in (4) are due to gluon bremsstrahlung emission off the primary quark-antiquark. These collinear and/or infrared enhanced contributions factorize and are resummed by linear evolution equations leading to Sudakov form factors. Therefore, after factorization of collinear and infrared singularities (including soft gluon coherence) QCD radiations appears as produced by “independent” gluon emission. Gluon branching (into two gluons or quark-antiquark pair) enters only in reconstructing the running coupling as function of transverse momentum.

The fact that here the branching component does not contribute (within SL accuracy) can be understood as a result of real-virtual cancellations of singularities. Indeed, in the collinear limit, the transverse momentum of an emitted gluon is equal to the sum of transverse momenta of its decaying products. Therefore, if one measures the total emitted transverse momentum, as in broadening for instance, it is enough to consider the contributions of bremsstrahlung gluons (independent emission similar to QED). Further branching does not contribute due to unitarity (real-virtual cancellation).

**Non-global jet-shape observables.** Here only a part of the phase space of emitted hadrons is considered. The best known example is the Sterman-Weinberg distribution\(^9\) of energy recorded inside a cone around a jet. A simpler example in $e^+e^-$ is the distribution in energy recorded outside a cone around the thrust axis.

\[
\Sigma(E_{\text{out}}) = \sum_n \int \frac{d\sigma_n}{\sigma_{\text{tot}}} \Theta \left( E_{\text{out}} - \sum_{\text{out}} k_{ti} \right).
\]

Since the jet region is excluded, there are no collinear singularities to SL accuracy and the resummed PT contributions come from large angle soft emission. Here resummation is more complex but informative then in the previous cases\(^8,^{10,11}\). Both bremsstrahlung and branching components contribute

The bremsstrahlung component resums contributions from gluons emitted in the recorded region outside the cone. This gives a Sudakov function exponentially decreasing with the SL function

\[
\tau = \int_{E_{\text{out}}}^Q \frac{dk_t}{k_t} \frac{N_c \alpha_s(k_t)}{\pi}.
\]
to branch in order to generate decaying products entering the recoder region. Here real-virtual cancellation is incomplete and virtual enhanced contributions are dominating thus leading to a strong suppression in the distribution which asymptotically turns out to be Gaussian in $\tau$.

The described behaviour has been obtained by numerical\(^8\) and analytical\(^10\) studies based on the multi-soft gluon emission distributions\(^12\). I describe this calculation which will be used also to introduce Monte Carlo simulations for jet physics.

## 3 Monte Carlo simulation

I start by describing the derivation of the evolution equation\(^10\) used to compute non-global jet-shape observables. One introduces the generating functional for all multi-soft gluon distributions\(^12\) (known only in the planar approximation of large $N_C$) and shows that it satisfies a branching evolution equation corresponding to a Markov process which can be numerically implemented into a Monte Carlo simulation program. To do this one needs to include proper cutoff for collinear and infrared singularities. From generated events one computes the described behaviour has been obtained by numerical\(^8\) and analytical\(^10\) studies based on the multi-soft gluon emission distributions\(^12\). I describe this calculation which will be used also to introduce Monte Carlo simulations for jet physics.

### 3.1 Generating functional (soft and planar limit)

The starting point is the amplitude for the emission of $n$-soft gluon $k_1, \ldots, k_n$ off a primary $q\bar{q}$ pair of momentum $p, \bar{p}$. It is represented as a sum of Chan-Paton factors and the colour ordered amplitudes coefficients

$$\mathcal{M}_n(p\bar{p}q_1 \cdots q_n) = \sum_{\text{perm.}} \{\lambda^a_1, \ldots, \lambda^a_n\} \beta \gamma_1 M_n(i_1 \cdots i_n). \quad (8)$$

From the factorization of the softest emitted gluon $q_n$

$$M_n(\cdots \ell n \ell' \cdots) = g_s M_{n-1}(\cdots \ell \ell' \cdots) \cdot \left(\frac{q_{\ell}^a}{q_{\ell} q_n} - \frac{q_{\ell'}^a}{q_{\ell'} q_n}\right), \quad (9)$$

one deduces a recurrence relation and computes all colour amplitudes in strong energy ordering (leading order in soft limit). Summing over colour and polarization indices, the distribution is given, to leading $N_c$-order, by

$$|\mathcal{M}_n|^2 = \frac{1}{n!} \prod_i \frac{N_c}{\omega_i^2} \sum_{\text{perm.}} W_{ab}(i_1 \cdots i_n), \quad W_{ab}(1 \cdots n) = \frac{(ab)}{(a_1 \cdots (q_n b)}, \quad (10)$$

where $(ij) = 1 - \cos\theta_{ij}$ and the emission is off a general $ab$-dipole. Similar approximations give the multi-soft gluon distributions in pure Yang-Mills theory. It is interesting that this multi-soft gluon distribution coincides with the square of the exact MHV colour amplitude discussed by Parke-Taylor\(^13\).

To study arbitrary jet-shape distributions one introduces a source function $u(q)$ for each soft gluon

$$\Sigma_{ab}(E, u) = \sum_n \frac{1}{n!} \int \frac{d\sigma_n}{\sigma_{\text{tot}}} \prod_i u(q_i) = \sum_n \int \prod_i \left\{ \frac{d\Omega_{\alpha_i}}{4\pi} u(q_i) \alpha_i \right\} \cdot W_{ab}(1, \cdots, n), \quad (11)$$

with $\alpha_i = N_c \alpha_s / \pi$ and $E = Q$. This functional summarizes the full information for the soft gluon emission. It involves only real emission distribution so that it is infrared and collinear divergent. Virtual corrections will be included later at the same accuracy in the soft limit.

The evolution equation is obtained by using the factorization structure of multi-soft gluon distribution

$$W_{ab}(1, \cdots, n) = \frac{(ab)}{(a\ell)(\ell b)} W_{a\ell}(1, \cdots, \ell - 1) \cdot W_{\ell b}(\ell + 1, \cdots, n), \quad (12)$$
with \( q_t \) one of the soft gluons. Taking \( q_t \) as the hardest (soft) gluon one obtains

\[
E \partial_E \Sigma_{ab} = \int \frac{d\Omega q}{4\pi} \tilde{\alpha}_s w_{ab}(q) [u(q) \Sigma_{aq} \cdot \Sigma_{qb} - \Sigma_{ab}] , \quad w_{ab}(q) = \frac{(ab)}{(aq)(qb)} \tag{13}
\]

The negative terms in the integrand corresponds to the virtual corrections (via Cauchy integration). They are included in the scheme in which, for the fully inclusive case of \( u(q) = 1 \), they completely cancel against the real contribution (\( \Sigma(E,1)=1 \)). Both the real emission branching (first term in the integrand) and the virtual correction are collinear and infrared singular. For inclusive observables, (i.e. for suitable sources \( u(q) \)) these singularities cancel.

### 3.2 Monte Carlo simulation, soft gluons at large angles

The evolution equation (13) can be formulated as a Markov process and then numerically solved by Monte Carlo simulations\(^8\). Here the basic system is the \( ab \)-dipole which, emitting a soft gluon \( q \), branches into the two dipole \( aq \) and \( qb \). To construct a Monte Carlo process\(^9\) one rewrites (13) by splitting real and virtual corrections. To do so it is necessary to introduce a cutoff \( Q_0 \) in transverse momentum (the argument of \( \alpha_s \)). The Sudakov form factor

\[
\ln S_{ab}(E) = -\int_{Q_0}^{E} \frac{dE_q}{E_q} \int \frac{d\Omega q}{4\pi} \tilde{\alpha}_s w_{ab}(q) \cdot \theta(q_{ab}-Q_0) , \quad q_{tab} = \frac{2E^2_q}{w_{ab}(q)} , \tag{14}
\]

is the solution of (13) in which the real emission piece in the integrand is neglected. Here \( q_{tab} \) is the transverse momentum of \( q \) with respect to the \( ab \)-dipole. Then one introduces the probability for dipole branching: \( (ab) \to (aq)(qb), \ E \to E_q \)

\[
dP(E_q, \Omega_q) = \left\{ \frac{dE_q}{E_q} \frac{S_{ab}(E)}{S_{ab}(E_q)} \right\} \left\{ \frac{d\Omega q}{4\pi} \tilde{\alpha}_s w_{ab}(q) \right\} \cdot \theta(q_{ab}-Q_0) . \tag{15}
\]

Here the first factor selects \( E_q \) and the second the direction \( \Omega_q \) of the emitted soft gluon. The branching does not take place if the momentum \( q_{tab} \) is below the cutoff. Each emitted dipole then undergoes successive branchings with decreasing energy until no further branching is permitted by the cutoff. Each Monte Carlo run generates an “event” with soft gluon emitted above the cutoff. Unitarity of probabilities ensures, in fully inclusive distribution \( (u(q)=1) \), the complete cancellation of real-virtual contributions. Computing jet-shape observables one has cancellations of collinear and soft singularities with residual ln \( V \)-contributions and power corrections in \( Q_0/(VE) \). The accuracy reached in the calculation of jet-shape observable is based on the fact that here one uses (planar) multi-gluon emission distributions. This implies that for jet-shape distributions one has included both DL and SL of soft origin. SL terms of collinear origin are missing so that here the \( g \to gg \) splitting function

\[
P_{g\to gg}(z) = N_c \left( \frac{1}{z} + \frac{1}{1-z} + z(1-z) - 2 \right) , \tag{16}
\]

includes only the infrared singular pieces for \( z \to 0 \) or \( z \to 1 \). Together with the non-soft pieces of the splitting function also quark branching channels are missed here.

An additional crucial missing element for a realistic jet-emission simulation is the fact that no dipole momentum recoil is here taken into account. However this branching formulation correctly accounts for soft emission also at large angle which are contributions missed in the present Monte Carlo simulation. It would be then important to include recoil in (15).

\(^{a}\)A Monte Carlo program based on dipole branching similar to what is described here has been constructed by L. Lönnblad\(^{14}\).
3.3 Improved Monte Carlo simulation

The most accurate available Monte Carlo simulations\textsuperscript{15,16,14} are based on a branching algorithms\textsuperscript{12,17} which resum (for instance for jet-shape distributions) DL and some of the most important SL terms. In particular soft gluon coherence is included (angular ordering). Continuous upgradings of Monte Carlo codes are underway which account for new theoretical, phenomenological and experimental results. I list here\textsuperscript{6} some of the major features and recent (or future) developments.

Full collinear singularity structures are included. Branching is in general formulated as successive parton emission\textsuperscript{15,16} so that recoil and splitting functions for all parton branching (\(g \rightarrow gg, g \rightarrow q\bar{q}, q \rightarrow qg\)) are naturally included. A major development in the structure of branching is expected including resummation of soft radiation at large angles. Partial results are however available\textsuperscript{18}.

Parton branching discussed in this lecture involves only final state emission. Similar branching holds for initial state radiation needed in hadron-lepton or hadron-hadron collisions. However in the small-\(x\) region (i.e. \(Q^2 \ll s\)) there are peculiar differences. Here soft gluons are both emitted and exchanged partons. For soft exchanged gluons angular ordering requires\textsuperscript{19} additional virtual corrections of non-Sudakov type (connected to gluon “Reggeization”). This brings one into the domain of “small-\(x\) physics” in which an accurate branching algorithms has still to be found. Partial results are however available\textsuperscript{20}.

Using factorization structure of collinear and infrared singularities the same branching structure holds for all hard processes. It is then possible to construct a single Monte Carlo code for \(e^+e^-\) annihilation, lepton-hadron DIS and hadron-hadron (large \(E_T\)). Moreover, such a factorization structure allows one to include in the QCD Monte Carlo code also non-QCD processes (Electro-weak, beyond the standard model, gravity,...). Such Monte Carlo codes are then very useful instruments for quantitative study of “new-physics” scenarios.

Major recent developments in the Monte Carlo codes are systematic attempts\textsuperscript{21} to account for NLO and NNLO exact results including heavy flavour processes.

3.4 From partons to hadrons

The above description of the Monte Carlo code refers to the generation of events with emission of partons (possibly together with non-QCD particles). As noted before, due to the presence of collinear and infrared singularities these emission processes require a cutoff \(Q_0\). The question is then how to go from partons to hadrons and how this “affects and distorts” the QCD radiation. Hadronization in the Monte Carlo code is based on the property of preconfinement\textsuperscript{22}: after successive branchings, partons are emitted in clusters of colour singlets of mass of order \(Q_0\). It is then natural to convert these colour singlet clusters into hadrons without “affecting and distorting” the QCD radiation within a scale of order \(Q_0\). Preconfinement may be basis for the property that parton flow \(\sim\) hadron flow

The basis for preconfinement is again the collinear and infrared structure of QCD. It can be explained as follows. In the large \(N_C\) limit, one may follow the colour line of partons in the successive branchings. The colour line of an emitted quark (or quark part of a gluon) ends into the colour line of an emitted antiquark (or antiquark part of a gluon). Due to this colour connection, they form a colour singlet. If two partons are colour connected, no emission takes place along the colour line which connects them. So virtual corrections are dominating and the distribution in the mass of the two colour connected partons is suppressed by a Sudakov form factor. As a result the mass of the two parton system is of order \(Q_0\).

4 Final considerations

Since 1973 exact and resummed PT calculations produced enormous inside on QCD radiation and its phenomenological “evidence”. A clear sign of this success is given by the Monte Carlo codes which summarize

\textsuperscript{6}Of course the following list does not really account for all the work done in the field. It reflects my personal view of the important points.
many QCD results and are used for the analysis of data in all hard processes (within known theoretical accuracy). Their phenomenological success is also an indication that it is “reasonable” to assume that parton flow $\sim$ hadron flow.

In PT studies it is possible to circumvent the difficulties of colour confinement and hadronization via factorization properties or “reasonable” phenomenological assumption. However this is not satisfactory on the theoretical point of view. There are indications that the problem of colour confinement could be approached by a dual formulation of QCD in terms of extended objects. Lattice QCD calculations indicate\(^{23}\) that QCD vacuum is populated by extended low dimensionality objects responsible for confinements. String theory provides in principle a basis for this study. Here there are attempts\(^{24}\) to develop new ideas for PT calculations. Also Regge behaviour of scattering amplitudes is studied\(^{25}\) and the language has some similarity with the “Poneron” pre-QCD topological expansion\(^{26}\). However the natural questions are how NP formulations could account for the enormous PT “evidence” and how they could inspire modeling hadronization.

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