Chiral condensate, susceptibilities, critical coupling and indices in \( \text{QED}_4 \).

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ABSTRACT

We measure chiral susceptibilities in the Coulomb phase of noncompact \( \text{QED}_4 \) in \( 8^4 \), \( 10^4 \) and \( 12^4 \) lattices. The MFA approach allows simulations in the chiral limit which are therefore free from arbitrary mass extrapolations. Using the critical couplings extracted from these calculations, we study the critical behaviour of the chiral condensate, which we find in disagreement with the predictions of logarithmically improved scalar Mean Field theory.
1 INTRODUCTION

The question of the non triviality of QED$_4$ defined at the finite coupling critical point of the theory is long standing [1-4]. While there is agreement on the numerical results obtained by various groups, their interpretation differs markedly. In [3] good agreement is found fitting the equation of state (i.e. critical exponents and the critical coupling from the numerical simulations) with the predictions of a logarithmically improved Mean Field scalar theory, finding, for four flavours, $\beta_c = 0.186(1)$.

The numerical determination of the critical exponents is very sensitive to the exact value of the critical coupling. In [2, 4] different simulations, entirely independent, found perfect agreement with a value of $\beta_c$ larger than 0.186 and critical exponents implying non Mean Field behaviour.

In this paper we present a new precise determination of the critical coupling from the behaviour of the chiral susceptibilities as computed in the Coulomb phase. Chiral susceptibilities (both longitudinal and transverse) are well defined in this phase also in the chiral limit, so that their behaviour can be used to locate the phase transition. In the MFA approach [5], this can be done directly in the chiral limit $m_f = 0$, thus avoiding arbitrary mass extrapolations. Such an approach has been discussed in [6].

The value of the critical coupling so obtained (which agrees with the one reported in [4]) is then used to measure the critical exponent of the chiral condensate, which is compared with the expectations from (logarithmically improved) Mean Field theory [7].

2 THE CHIRAL SUSCEPTIBILITIES

In the MFA approach [5] the light fermion dynamics is expressed through an effective action

$$Z(\beta, m) = \int dEn(E)e^{-6V\beta E-S_f}$$

(1)

where $S_f = \text{ln det} \Delta(E, m)$ is the average of the fermionic determinant over gauge configurations at fixed euclidean energy, and $n(E)$ is the density of states. A modified Lanczos algorithm [8] is used to obtain all the eigenvalues of the fermionic matrix at $m_f = 0$, so that results at different couplings and masses are trivially related to $m_f = 0$ and do not need separate simulations.

In the same way, average of physical operators can be expressed in terms of microcanonical averages of the corresponding operators:

$$\langle O \rangle_{\beta} \leftrightarrow \langle O \rangle_{E}$$

(2)
We use this approach to compute the average value of

\[ O = \frac{2}{V} \sum_\lambda \lambda^2 \]  

which defines both longitudinal and transverse chiral susceptibilities at \( m_f = 0 \) in the symmetric phase of the model; this term diverges at the transition in the thermodynamical limit. In the chirally broken phase, the susceptibilities differ, and cannot be defined if not starting at non zero mass and extrapolating to the massless limit, thus suffering from ambiguities. On the contrary, chiral susceptibility in the Coulomb phase is free from arbitrary mass extrapolations.

We have performed simulations with staggered fermions on lattices of sizes \( L = 8, 10, 12 \), and \( n_f = 0, 2, 8 \). The results we present here are at \( n_f = 4 \); \( L = 10 \) (300 – 400 configurations per energy value) and \( L = 12 \) (60 – 120 configurations). In Fig. 1 we present the inverse massless susceptibility as function of Euclidean energy. The inverse susceptibility shows a clear linear behaviour throughout the whole Coulomb phase. The fermionic effective action is linear in this phase, this implying that the whole effect of dynamical fermions amounts merely to a shift in the coupling \( \beta = \beta_{PG} + r \) with respect to the pure gauge theory, \( r \) depending on the number of flavours, and suggests that the inverse susceptibility can be parametrized in the symmetric phase as

\[ \chi^{-1} = \frac{c (\beta_c - \beta)}{(\beta_c + r) (\beta + r)} \]  

as a function of the coupling (Fig. 2). The critical coupling \( \beta_c \) can be derived directly from the fit of Fig. 2; alternatively a critical energy can be derived from the behaviour of the inverse susceptibility versus energy, and then converted to \( \beta_c \) through the knowledge of the effective action and density of states.

We thus obtain \( \beta_c = 0.2025(2), \quad L = 10 \) (0.2051(2), \( L = 12 \))

### 3 THE CHIRAL CONDENSATE

We use the above derived values of the critical coupling to study the behaviour of the chiral condensate versus mass at the phase transition. Mean Field theory \textit{a la} NJL predicts

\[ \langle \bar{\psi} \psi \rangle \propto m_f \quad (\beta = \beta_c, m_f \to 0) \]  

3
We have tested the above relation in our simulation, in the fermion mass range $0.01 - 0.06$. Due to the fact that we compute all the eigenvalues of the fermionic matrix in the chiral limit, the chiral condensate we derive is essentially known continuously as function of $m_f$.

In Fig. 3 we present the best fit of the Mean Field prediction (Eq. 5) with respect to the data. Mean Field prediction and data are clearly inconsistent, with $\chi^2/(d.o.f) = 57$.

We have fitted the experimental data with the relation
\[
\langle \bar{\psi} \psi \rangle \propto m_f^{1/\delta} \quad (\beta = \beta_c, m_f \to 0)
\] (6)

obtaining very good fits with $\delta = 2.89(2), \quad L = 10$ (2.79(7), $L = 10$) (Fig. 4) in the same mass range.

4 CONCLUSIONS

The use of the MFA approach allows a very precise computation of the massless chiral susceptibility in the symmetric phase of QED$_4$, where, due to the unbroken chiral symmetry, the interchange of the chiral and thermodynamical limit is guaranteed. The inverse chiral susceptibility is to extremely good precision linear in the whole Coulomb phase, this allowing a very precise determination of the critical energy unaffected by finite volume effects near the transition. Through the knowledge of the effective action and density of states this leads to a very precise determination of the critical coupling [3].

We use this information to study the critical behaviour of the chiral condensate at the phase transition as a function of $m_f$, and compare it with the expectations with Mean Field theory a la NJL. We find definite disagreement, while the data are well fitted by a power of the mass, with an inverse power significantly smaller than 3. We remark that the scalar field motivated Mean Field solution advocated by [3] would imply an effective exponent larger than 3, thus a fortiori excluded by our analysis.

If, on the other hand, we take the Mean Field prescription (Eq. 5) and allow for an arbitrary power of the logarithm,
\[
\langle \bar{\psi} \psi \rangle \times \ln q \left( \frac{1}{\langle \bar{\psi} \psi \rangle} \right) \propto m_f
\] (7)

we can find a fit as good as that presented in Fig. 4 with $q = 0.255$. We obviously cannot exclude such a behaviour, at least in the same mass range;
larger interval of masses, towards small ones, where the predictions of Eqs. 6, 7 have to differ, can only be reached in lattices larger than ours.

The numerical simulations quoted above have been done using the Transputer Networks of the Theoretical Group of the Frascati National Laboratories and the Reconfigurable Transputer Network (RTN), a 64 Transputers array, of the University of Zaragoza.

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Figure captions

**Figure 1.** Inverse susceptibility as function of Euclidean Energy, $L = 10$.

**Figure 2.** Inverse susceptibility vs. $\beta$, $L = 10$. The continuous line is a fit with Eqn. 4.

**Figure 3.** Mean Field prediction for the chiral condensate (continuous line) compared with the data; L=10.

**Figure 4.** $m_f^{1/8}$ (continuous line) compared with the data; L=10.
