Coherent imaging with pseudo-thermal incoherent light

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We investigate experimentally fundamental properties of coherent ghost imaging using spatially incoherent beams generated from a pseudo-thermal source. A complementarity between the coherence of the beams and the correlation between them is demonstrated by showing a complementarity between ghost diffraction and ordinary diffraction patterns. In order for the ghost imaging scheme to work it is therefore crucial to have incoherent beams. The visibility of the information is shown for the ghost image to become better as the object size relative to the speckle size is decreased, and therefore a remarkable tradeoff between resolution and visibility exists. The experimental conclusions are backed up by both theory and numerical simulations.

I. HISTORICAL OVERVIEW AND INTRODUCTION

A decade has passed since the first experimental observation of unusual interference fringes in the coincidence counts of photon pairs [1,2]. Signal and idler photons produced by parametric down-conversion (PDC) were spatially separated and in the signal photon arm a double slit was inserted. While no first order interference pattern was visible behind the slit, an interference pattern was observed in the coincidence count by scanning the idler photon detector position. To this phenomenon it was given the name ghost diffraction. Shortly after a ghost image experiment was performed [3], showing a sharp image of an object placed in the signal arm by registering the coincidence counts as a function of the idler photon position.

In the interpretations of the experiments, the quantum nature of the source there employed was emphasized, although the Authors of [3] suggested that “it is possible to imagine some type of classical source that could partially emulate this behavior”. Several years passed before a systematic theory of ghost imaging (GI) started to be developed, and soon a lively debate arose discussing the role of entanglement versus classical correlation in GI schemes. In the first theoretical papers by the Boston group [4], it was claimed that entanglement was a crucial prerequisite for achieving GI, and in particular coherent GI: “the distributed quantum-imaging scheme truly requires entanglement in the source and cannot be achieved using a classical source with correlations but without entanglement.” Soon after, at Rochester University a ghost image experiment was performed exploiting the classical angular correlation of narrow laser pulses [5]. This fueled the debate: which are the features of ghost imaging that truly requires entanglement? The debate was continued by the paper [6], where some of us showed that a classical GI scheme can indeed produce either the object image or the object diffraction pattern, but suggested that both cannot be produced without making changes to the source or the object arm setup. We argued that only entangled beams can give both results by only changing the setup in the reference arm (the one where the object is not present). While by now we know this is not true, at that time it was in partial agreement with [4] and [5]. When the Rochester group recently completed the results by showing that also the object diffraction pattern can be reconstructed using classically correlated beams [7], they had indeed to change the setup (the object location, the lens setup as well as the detection protocol).

Our claim in [4] originated from the fact that only entangled beams can have simultaneously perfect spatial correlation in both the near and the far field (in both position and momentum of the photons), and no classical beams can mimic this [8]. In the same spirit, recent experimental works [7,9,10], brilliantly pointed out a momentum-position realization of the Einstein-Podolsky-Rosen (EPR) paradox using entangled photon pairs produced by PDC. The product of conditional variances in momentum and position was there shown to be below the EPR bound that limits the correlation of any classical (non-entangled) light beam. Based on these results, the Authors of [7] proposed the same EPR bound as a limit for the product of the resolutions of the images formed in the near and in the far-field of a given classical source, and both [7,10] argued that in ghost imaging schemes entangled photons allow to achieve a better spatial resolution than any classically correlated beams.

This was the state of the art, when some of us had an idea leading to a ghost imaging protocol with classical thermal-like beams. Inspired by the fact that the marginal statistics of the signal or idler beam from PDC is of thermal nature, we asked ourselves what would have been the result of splitting a thermal beam on a macroscopic beam splitter and using the two outgoing beams for ghost imaging. Honestly speaking, we expected at that time that this would have lead to identify relevant differences with quantum entangled beams, where the correlation is of microscopic origin. The picture that came out was however rather different. The two output beams of the beam splitter are obviously each a true copy (on a classical level) of the input beam: if there is a speckle at some position in the input beam, then each output beam has also a speckle at the same position. Hence the beam splitter has created beams with a strong spatial correlation between them, while each beam on its own is spatially incoherent. In theoretical works [11] we showed that this correlation is preserved upon propaga-
tion (so it is present both in the near and the far-field planes), and that the beams could therefore be used to perform GI exactly in the same way as the entangled beams from PDC. Actually a very close formal analogy was demonstrated between GI with thermal and PDC beams, which implied that classically correlated beams were able to emulate all the relevant features of quantum GI, with the only exception of the visibility \[11\].

Thus, we actually had to conclude that what we had written in [6] was not wrong but not very correct either. What we failed to recognize is that ghost imaging protocols do not need a perfect correlation at all: with the imperfect (shot-noise limited) spatial correlation of thermal beams both the object image and the object diffraction pattern can be reproduced without making any changes to the source and only changing the reference arm. Moreover the formal analogy between thermal and PDC beams suggested that identical performances with respect to spatial resolution should be achieved by the quantum and classical protocols, provided that the spatial coherence properties of the two sources were similar. This was obviously a controversial result compared to what was published until then [3, 4, 5] and in order to be accepted needed an experimental confirmation. We recently provided this [12] showing high-resolution ghost image and ghost diffraction experiments, performed by using a single source of pseudo-thermal speckle light divided by a beam splitter. As predicted, it was possible to pass from the image by only changing the optical setup in the reference arm, while leaving the object arm untouched. Moreover, the product of spatial resolutions of the ghost image and ghost diffraction experiments was there shown to overcome the EPR bound which was proposed to be achievable only with entangled photons by former literature [2, 3]. The origin of the apparent contradiction with the former literature was there identified, by recognizing that the spatial resolution of GI protocols do not coincide in general with the conditional variance, so that the product of the near and far-field resolution is free from any EPR separability bound.

The idea of using pseudo-thermal light for GI had some enthusiastic followers [13, 14] with proposals for X-ray diffraction [15], some partially converted fans, with experiments characterizing a pseudo-thermal source of photon pairs [16] and using them for realizing a ghost image [17], and some sceptics [18]. The use of pseudo-thermal light in GI schemes inspired also a topic which became of a certain interest, known as "quantum lithography with classical beams", or "sub-wavelength interference with classical beams". The quantum version of this started with the famous paper by Boto et al. [19] claiming that N-photon entangled states could be used for improving the resolution of lithography by a factor of N. A proof-of-principle experiment using N = 2 in the PDC case was provided by [20], where a halving of the period of the interference fringes was observed in a "ghost diffraction" pattern. In [5] some of us observed that the same effect may be observed when thermal-like beams are used, and that in both the entangled and thermal case the sub-wavelength interference relies on a simple geometrical artifact. We therefore questioned whether the Shih experiment really proves Boto's entangled protocol. Sub-wavelength interference using thermal beams was then theoretically discussed in [21], and experimentally demonstrated [22].

In this paper we continue the experimental investigations started in [12]. The main result there established was that high resolution ghost image and ghost diffraction could be achieved with the same classical source, with the product of resolutions well behind the EPR bound proposed by [7]. Here, we shall investigate first of all a fundamental complementarity between coherence and correlation which exists for ghost imaging schemes. Only when the beams are spatially incoherent the correlation functions allows to retrieve information about the object (ghost image or ghost diffraction), while the information disappears as the spatial coherence of the beams increases. This is just the opposite of what happens for direct detection of the light behind the object, where fully coherent information can be obtained only for spatially coherent beams. Thus, the spatial incoherence plays an essential role for realizing ghost imaging, while instead the Hanbury-Brown–Twiss interferometer [23] for determining the stellar diameter relies on coherence gained by propagation. Secondly, we will investigate visibility and signal-to-noise ratio in ghost imaging with thermal light, and highlight a tradeoff between visibility and resolution when reconstructing the information.

The paper is organized as follows: In Sec. II the experiment is described, while Sec. III introduces the formalism and review the formal analogy between classical and quantum ghost imaging. In addition, the relation between visibility and signal-to-noise ratio is discussed. In Sec. IV the spatial coherence properties of the beams are investigated and experimentally characterized. Section V focusses on the ghost diffraction setup and shows the complementarity between coherence and correlation. Section VI focusses on the ghost image, and discusses visibility. In Sec. VII numerical results are presented which provide a more detailed insight into the results of the experiment. Finally, the conclusions are drawn in Sec. VIII.

II. DESCRIPTION OF THE EXPERIMENT

The experimental setup is similar to that of Ref. [12] and is sketched in Fig. 1. The source of pseudo-thermal light is provided by a scattering medium illuminated by a laser beam. The medium is a slowly rotating ground glass placed in front of a scattering cell containing a turbid solution of 3 µm latex spheres. When this is illuminated with a large collimated Nd-Yag laser beam (λ = 0.532 µm, diameter D₀ ≈ 10 mm), the stochastic interference of the waves emerging from the source produces at large distance (z ≈ 600 mm) a time-dependent speckle pattern, characterized by a chaotic statistics and
by a correlation time $\tau_{\text{coh}}$ on the order of 100 ms (for an introduction to laser speckle statistics see e.g. [24]). Notice that the ground glass can be used alone to produce chaotic speckles, whose correlation time depends on the speed of rotation of the ground-glass disk and on the laser diameter, as in classical experiments with pseudo-thermal light [25, 26]. Indeed, in some part of the experiments described in the following it will be used alone. This however presents the problem that the generated speckle patterns reproduce themselves after a whole tour of the disk, which can be partially avoided by shifting laterally the disk at each tour. The turbid solution provides an easy way to generate a truly random statistics of light, because of the random motion of particles in the solution, allowing a huge number of independent patterns to be generated and used for statistics. Notice that the turbid medium cannot be used alone, because a portion of the laser light would not be scattered, thus leaving a residual coherent contribution.

![Diagram](image)

**FIG. 1:** Scheme of the setup of the experiment (see text for details).

At a distance $z_0 = 400$ mm from the thermal source, a diaphragm of diameter $D = 3$ mm selects an angular portion of the speckle pattern, allowing the formation of an almost collimated speckle beam characterized by a huge number (on the order of $10^4$) of speckles of size $\Delta x \approx \lambda z_0 / D_0 \approx 21 \mu m$ [24]. The speckle beam is separated by the beam splitter (BS) into two "twin" speckle beams, that exhibit a high (although classical) level of spatial correlation [11]. The two beams emerging from the BS have slightly non-collinear propagation directions, and illuminate two different non-overlapping portions of the charged-coupled-device (CCD) camera. The data are acquired with an exposure time $(1 - 3 \text{ ms})$ much shorter than $\tau_{\text{coh}}$, allowing the recording of high contrast speckle patterns. The frames are taken at a rate of 1-10 Hz, so that each data acquisition corresponds to uncorrelated speckle patterns.

In one of the two arms (the object arm 1) an object about which we need to extract information is placed. The object plane, located at a distance 200 mm from the diaphragm, will be taken as the reference plane, and referred to as the near-field plane (this is not to be confused with the source near-field, as the object plane is in the far zone with respect to the source). The optical setup of the object arm is kept fixed, and consists of a single lens of focal length $F = 80$ mm, placed at a distance $p_1$ after the object and $q_1 = F$ from the CCD. In this way the CCD images the far-field plane with respect to the object.

We shall consider two different setups for the reference arm 2. In the **ghost-diffraction** setup, the reference beam passes through the same lens $F$ as the object beam, located at a distance $q_2 = F$ from the CCD. In Ref. [12], the spatial cross-correlation function of the reference and object arm intensity distributions was calculated, and showed a sharp reproduction of the diffraction pattern of the object, comparable with the diffraction pattern obtained by illuminating the object with the laser light (see Sec. VI).

In the **ghost-image** setup, without changing anything in the object arm, an additional lens of focal length $F'$ is inserted in the reference arm immediately before $F$. The total focal length $F_2$ of the two-lens system is smaller than its distance from the CCD, being $1/F_2 \approx 1/F + 1/F'$. It was thus possible to locate the position of the plane conjugate to the detection plane, by temporarily inserting the object in the reference arm and determining the position that produced a well focussed image on the CCD with laser illumination. The object was then translated in the object arm. The distances in the reference arm approximately obey a thin lens equation of the form $1/(p_2 - d_1) + 1/q_2 \approx 1/F_2$ [32], providing a magnification factor $m = 1.2$. In Ref. [12] the intensity distribution of the reference arm was correlated with the total photon counts of the object arm showing in this case a well-resolved reproduction of the image of the object (see also Sec. VI). Thus, the setups allows a high-resolution reconstruction of both the image and the diffraction pattern of the object by using a single source of classical light. The passage from the diffraction pattern to the image is performed by only operating on the optical setup of the reference arm, which gives evidence for the the distributed character of the correlated imaging with thermal light.

### III. FORMAL EQUIVALENCE OF GHOST IMAGING WITH THERMAL BEAMS AND THE TWO-PHOTON ENTANGLED SOURCE

The basic theory behind the setup shown in Fig. 1 has been explained in detail in Ref. [11]. The starting point is the input-output relations of the beam splitter

$$b_1(\vec{x}) = ta(\vec{x}) + rv(\vec{x}), \quad b_2(\vec{x}) = ra(\vec{x}) + tv(\vec{x}) \quad (1)$$

where $b_1$ and $b_2$ are the object and reference beams emerging from the BS, $t$ and $r$ are the transmission and reflection coefficients of the BS, $a$ is the boson operator of the speckle field and $v$ is a vacuum field uncorrelated from $a$. The state of $a$ is a thermal mixture, characterized by a Gaussian field statistics, in which any correlation function of arbitrary order is expressed via the second order
correlation function
\[ \Gamma(\vec{x}, \vec{x}') = \langle a(\vec{x})a(\vec{x}') \rangle \] (2)

Since the collection time of our measuring apparatus is much smaller than the time \( \tau_{coh} \) over which the speckle fluctuate, all the beam operators are taken at equal times, and the time argument is omitted in the treatment. Notice that we are dealing with classical fields, so that the field operator \( a \) could be replaced by a stochastic c-number field, and the quantum averages by statistical averages over independent data acquisitions. However, we prefer to keep a quantum formalism in order to outline the parallelism with the quantum entangled beams from PDC.

The fields at the detection planes are given by \( c_i(\vec{x}_i) = \int d\vec{x} h_i(\vec{x}_i, \vec{x})b_i(\vec{x}) + L_i(\vec{x}_i) \), where \( L_i \) account for possible losses in the imaging systems, and \( h_1, h_2 \) are the impulse response function describing the optical setups in the two arms. The object information is extracted by measuring the spatial correlation function of the intensities \( \langle I_1(\vec{x}_1)I_2(\vec{x}_2) \rangle \), where \( I_i(\vec{x}_i) \) are operators associated to the number of photo counts over the CCD pixel located at \( \vec{x}_i \) in the \( i \)-th beam. All the information about the object is contained in the correlation function of intensity fluctuations, which is calculated by subtracting the background term \( \langle I_1(\vec{x}_1) \rangle \langle I_2(\vec{x}_2) \rangle \):

\[ G(\vec{x}_1, \vec{x}_2) = \langle I_1(\vec{x}_1)I_2(\vec{x}_2) \rangle - \langle I_1(\vec{x}_1) \rangle \langle I_2(\vec{x}_2) \rangle. \] (3)

The main result obtained in \[11\] was

\[ G(\vec{x}_1, \vec{x}_2) = |rt|^2 \times \left| \int d\vec{x}_1' \int d\vec{x}_2' h_1^*(\vec{x}_1', \vec{x}_1)h_2(\vec{x}_2', \vec{x}_2)\Gamma(\vec{x}, \vec{x}') \right|^2, \] (4)

Equation (4) has to be compared with the analogous result obtained for PDC beams [5]:

\[ G_{pdc}(\vec{x}_1, \vec{x}_2) = \times \left| \int d\vec{x}_1' \int d\vec{x}_2' h_{1}(\vec{x}_1', \vec{x}_1)h_{2}(\vec{x}_2', \vec{x}_2)\Gamma_{pdc}(\vec{x}_1', \vec{x}_2') \right|^2, \] (5)

where 1 and 2 label the signal and idler down-converted fields \( a_1, a_2 \), and

\[ \Gamma_{pdc}(\vec{x}_1', \vec{x}_2') = \langle a_1(\vec{x}_1')a_2(\vec{x}_2') \rangle \] (6)

is the second order field correlation between the signal and idler (also called biphoton amplitude). As already outlined in \[11\], ghost imaging with correlated thermal beams, described by Eq. (4) presents a deep analogy (rather than a duality) with ghost imaging with entangled beams, described by Eq. (5): (a) both are coherent imaging systems, which is crucial for observing interference from an object, and in particular interference from a phase object; (b) both perform similarly if the beams have similar spatial coherence properties, that is if \( \Gamma \) and \( \Gamma_{pdc} \) have similar properties. They differ in a) the presence of \( h_1^* \) at the place of \( h_1 \), which implies some non-fundamental geometrical differences in the setups to be used and b) the visibility, which can be close to unity only in the in the coincidence count regime of PDC. We define the visibility of the information as

\[ V = \frac{G_{max}}{\langle I_1\rangle_{max}} = \frac{G_{max}}{(\langle I_1\rangle + G_{max})}. \] (7)

In the thermal case \( G(\vec{x}_1, \vec{x}_2) \leq \langle I_1(\vec{x}_1) \rangle\langle I_2(\vec{x}_2) \rangle \) so that the visibility is never above \( \frac{1}{2} \). Conversely, in the PDC case it is not difficult to verify that the ratio \( G_{pdc}/\langle I_1\rangle_{pdc} \) scales as \( 1 + \frac{n}{I_{pdc}} \), where \( \langle n \rangle \) is the mean photon number per mode (see e.g. Ref. \[11\] b). Only in the coincidence-count regime, where \( \langle n \rangle \ll 1 \), the visibility can be close to unity, while bright entangled beams with \( \langle n \rangle \gg 1 \) show a similar visibility as the classical beams. However, despite never being above \( \frac{1}{2} \), in the classical case, we have shown \[11\] \[12\] that the visibility is sufficient to efficiently retrieve information.

The visibility is an important parameter in determining the signal-to-noise ratio (SNR) associated to a ghost imaging scheme (see also \[21\]). Intuitively, one expects that the noise associated to a measurement of \( I_1I_2 \) is proportional to \( \langle I_1 \rangle \langle I_2 \rangle \), being the statistics of thermal nature. This noise also affects the retrieval of the ghost image or of the ghost diffraction in a single measurement, because this is obtained from \( I_1I_2 \) by subtracting the background term. Hence \( SNR \propto G/I_1I_2 \), and the visibility defined by Eq. (7) gives an estimate of the signal-to-noise ratio of a ghost imaging scheme. This picture is confirmed by a more quantitative calculation, not reported here, performed by using the standard properties of Gaussian statistics. By defining \( \Delta G = \sqrt{\langle O^2 \rangle} - \langle O \rangle \), with \( O = I_1I_2 - \langle I_1 \rangle \langle I_2 \rangle \), where \( G := G(\vec{x}_1, \vec{x}_2), \text{ } I_i := I_i(\vec{x}_i), \) we obtained \( \Delta G \approx \sqrt{3\langle I_1I_2 \rangle^2} + 8G\langle I_1 \rangle\langle I_2 \rangle \), where quantum corrections have been neglected. If the visibility is small, as it is often the case, this reduces to \( \Delta G \approx \sqrt{3\langle I_1I_2 \rangle} \), and \( SNR \approx \frac{G}{\sqrt{\langle I_1I_2 \rangle}} \).

Of course, after averaging over \( N \) independent measurements \( SNR(N) = \sqrt{N}SNR \), and if collecting a large amount of samples does not represent the problem, any ghost image/diffraction can be retrieved after a sufficient number of data collections. Hence, if the goal is retrieving information about a macroscopic, stable object, the thermal source represents by far a much better deal than the entangled two-photon source. Needless to say, if the goal is performing a high sensitivity measurement, or using the ghost imaging scheme as a cryptographic scheme where information need to be hidden to a third party, then the issue of SNR becomes crucial, and the two-photon entangled source may turn out to be the only proper choice.

### IV. Spatial Coherence Properties of the Speckle Beams

Relevant for the ghost image and the ghost diffraction schemes are the spatial coherence properties of the
speckle beams in the object near-field plane, and in the far-field plane with respect to the object. These can be investigated by measuring the autocorrelation function of the intensities. In any plane it holds a Siegert-like factorization formula for thermal statistics \(^28\) \(^29\):

\[
\langle : I(\vec{x})I(\vec{x}') : \rangle = \langle I(\vec{x}) \rangle \langle I(\vec{x}') \rangle + \frac{1}{M} |\Gamma(\vec{x}, \vec{x}')|^2, \tag{8}
\]

where \(M\) is the degeneracy factor accounting for the number of temporal and spatial modes detected. Hence, the properties of the field correlation function \(\Gamma\) can be inferred from the measurement of the intensity correlation. In particular we will be interested in the field correlation function at the object near-field plane \(\Gamma_n(\vec{x}, \vec{x}')\), and in the same function at the far-field plane \(\Gamma_f(\vec{x}, \vec{x}')\).

We notice the following equalities, which are a trivial consequence of the BS input-output relations \(^1\):

\[
\langle : I_1(\vec{x})I_1(\vec{x}') : \rangle = |t|^4 \langle I(\vec{x})I(\vec{x}') \rangle
\]

\[
= |t|^2 \langle I(\vec{x}) \rangle |I(\vec{x}') \rangle \tag{9}
\]

\[
\langle : I_2(\vec{x})I_2(\vec{x}') : \rangle = |r|^4 \langle I(\vec{x})I(\vec{x}') \rangle
\]

\[
= |r|^2 \langle I(\vec{x}) \rangle |I(\vec{x}') \rangle \tag{10}
\]

where \(\langle : \rangle\) indicates normal ordering and \(I(\vec{x})\) is the intensity distribution of the speckle beam in the absence of the BS. A part from numerical factors, and from the shot noise contribution at \(\vec{x} = \vec{x}'\), in a given plane the auto-correlation function of each of the two beams coincides with the intensity cross correlation of the two beams.

Frame (b) in this figure is the radial autocorrelation function \(^{10}\), calculated as a function of the distance \(|\vec{x} - \vec{x}'|\), normalized to the product of the mean intensities. The baseline corresponds to the product of the mean intensities while the narrow peak located around \(|\vec{x} - \vec{x}'| = 0\) is proportional to \(|\Gamma_n(\vec{x}, \vec{x}')|^2\), where \(\Gamma_n\) is the second order field correlation function at the near-field plane. This peak reflects the spatial coherence properties of the beams at the object plane. Its width is the near-field coherence length \(\Delta x_n\) and gives an estimate of the speckle size in this plane \(\Delta x_n \approx 2\sigma = 36 \mu m\). Notice that the peak value is slightly smaller than twice the baseline value, giving a degeneracy factor \(M = 1.7\).

Figure 3 shows the instantaneous intensity distribution (a) and the intensity auto-correlation function (b) in the far-field plane, measured in the focal plane of the lens F. The narrow peak in (b) located around \(|\vec{x} - \vec{x}'| = 0\) is now proportional to \(|\Gamma_f(\vec{x}, \vec{x}')|^2\), and its width (the far-field coherence length) gives an estimate of the speckle size in this plane. High-contrast speckles are visible also in the far-field plane. The Van-Cittert Zernike theorem can be again invoked to estimate their expected size, being now the source size represented by the diaphragm diameter \(D\), \(\Delta x_r \propto \lambda F/D \approx 14 \mu m\) \(^{22}\). This is in good agreement with the estimation from the the correlation function, that gives \(\Delta x_r = 2\sigma_f = 14.2 \mu m\). In this case the peak value of the correlation function in frame (b) gives a degeneracy factor \(M = 2.2\). This is slightly higher than in the near field because \(\Delta x_t\) (the size of the spatial mode) is smaller and comparable with the pixel side (6.7 \(\mu m\)).

Because of the identities in Eqs. (9), (10), the cross-correlation \(\langle I_1 I_2 \rangle\) in the near and in the far-field coincides with the auto-correlation plotted in Fig 2 and in Fig 3, respectively. Hence a high degree of mutual spatial correlation is present in both planes, as a consequence of the spatial incoherence of the light produced by our source. The more incoherent is the light (the smaller the speckles with respect to the beam size), the more localized is the spatial mutual correlation function. The more coherent is the source (the larger the speckles with respect to the beam size), the flatter is the spatial mutual correlation function. As it will become clear in the next two
sections, for highly spatially incoherent light, both the ghost diffraction and the ghost image can be retrieved with high resolution. Conversely, in the limit of spatially coherent light no spatial information about the object can be extracted in a ghost imaging scheme, that is from the intensity cross-correlation of the two beams as a function of the pixel position in the reference beam.

Summarizing, two aspects of our experiment are crucial: i) the spatial incoherence of light, and ii) a measurement time \( \ll \tau_{\text{coh}} \). Notice that the presence in the near-field of a large number of small speckles inside a broad beam, implies that the light is incoherent also in the far-field, because \( \Delta x_i \approx 1/D \), while the far-field diameter of the beam \( \approx 1/\Delta x_n \).

V. THE GHOST DIFFRACTION EXPERIMENT: COMPLEMENTARITY BETWEEN COHERENCE AND CORRELATION

In this section we focus on the ghost diffraction setup (Fig. 4 without the lens \( F' \)). The object is a double slit, consisting of a thin needle of 160 \( \mu \)m diameter inside a rectangular aperture 690 \( \mu \)m wide.

In a first set of measurements the source size is \( D_0 = 10 \) mm, and the object is illuminated by a large number of speckles whose size \( \Delta x_n = 36 \) \( \mu \)m is much smaller than the slit separation. The light is spatially incoherent as described in the previous section. The results are shown in the first row of Fig. 4. Frame (a) is the instantaneous intensity distribution of the object beam, showing a speckled pattern, with no interference fringes from the double slit, as expected for incoherent illumination \([27]\).

At a closer inspection, the shape of the speckles resembles

the interference pattern of the double slit, but since these speckles move randomly in the transverse plane from shot to shot, an average over several shots displays a homogeneous broad spot (Fig 4(b)). Frame (c) is a plot of \( G(\vec{x}_1, \vec{x}_2) \) as a function of the reference pixel position \( \vec{x}_2 \), and shows the result of correlating the intensity distribution in the reference arm with the intensity collected from a single fixed pixel in the object arm. Notice that at difference to what was done in \([12]\), no spatial averaging \([30]\) is here employed: this makes the convergence rate slower but the scheme is closer to the spirit of ghost diffraction in which the information is retrieved by only scanning the reference pixel position. The ghost diffraction pattern emerges after a few thousands of averages, and is well visible after 20000 averages. This is confirmed by the data of Fig. 5 which compare the horizontal section of the diffraction pattern from a correlation measurement to that obtained with laser illumination. In

![Fig. 4](image)

![Fig. 5](image)

![Fig. 6](image)
ence fringes are now visible in the instantaneous intensity distribution of the object beam 1 [frame (d)], and become sharper after averaging over some hundreds of shots [frame (e)]. Notice that the shape of the interference pattern is now elongated in the vertical direction, because the light emerging from the small source is not collimated. Horizontal sections of \( \langle I_1 \rangle \), plotted in Fig. 6, show a very good agreement with the diffraction pattern from laser illumination. Instead, no interference fringes at all appear in the correlation function of the intensities in the two arms, when plotted as a function of \( \vec{x}_2 \) [frame (f)]. Notice that in this set of measurements the turbid medium was removed in order to increase the power. This is feasible in this case, because the very small size of the source allows a large number of independent patterns to be generated in a single tour of the glass disk.

Figures 4, 5, 6 evidence a remarkable complementarity between the observation of interference fringes in the correlation function (ghost diffraction), and in the intensity distribution of the object beam (ordinary diffraction). It also shows the fundamental role played by the spatial incoherence of the source in producing a ghost diffraction pattern: the more incoherent is the source, the more the two beams are spatially correlated and the more information about the object is available in the ghost diffraction pattern. The more coherent is the source, the flatter is the spatial correlation function of the two beams and the less information about the object is contained in the ghost diffraction. This is completely analogous to the complementarity between the one-photon and two-photon interference in Young’s double slit experiments with photons from a PDC source [31], which was explained as a complementarity between coherence and entanglement. In our case of thermal beams, the complementarity is rather between coherence and spatial correlation, showing that also in this respect the classical spatial correlation produced by splitting thermal light play the same role as entanglement of PDC photons.

These results can be easily understood by using the formalism developed in Sec. III and in particular by inspection of Eq. 11 for the correlation function of the intensity fluctuations \( G(\vec{x}_1, \vec{x}_2) \). In the limit of spatially coherent light the field correlation function \( \Gamma_n(\vec{x}_1, \vec{x}_2) \) becomes constant in space in the region of interest, and the two integrals in Eq. 11 factorize to the product of two ordinary imaging schemes, showing the diffraction pattern of the object only in the object arm 1. As a result, by plotting the correlation as a function of \( \vec{x}_2 \), no object diffraction pattern can be observed, that is, no ghost diffraction occurs. The same observation can be made with respect to \( \Gamma_{pdc}(\vec{x}_1, \vec{x}_2) \), and \( G_{pdc}(\vec{x}_1, \vec{x}_2) \), explaining thus the analogy between the role of light coherence in the PDC and in the thermal case.

In general, the result of a correlation measurement is obtained by inserting into Eq. 11 the propagators that describe the ghost diffraction setup: \( h_1(\vec{x}_1, \vec{x}_1') = (i\lambda F)^{-1} e^{-\frac{2\pi}{\lambda F} \vec{x}_1, \vec{x}_1'} T(\vec{x}_1') \), with \( T(\vec{x}) \) being the object transmission function, and \( h_2(\vec{x}_2, \vec{x}_2') = (i\lambda F)^{-1} e^{-\frac{2\pi}{\lambda F} \vec{x}_2, \vec{x}_2'} \). We get

\[
G(\vec{x}_1, \vec{x}_2) \propto \int d\vec{\xi} \hat{T} \left( \frac{2\pi}{\lambda F} \vec{x}_1, \vec{\xi} \right)^2 \Gamma_f(\vec{x}_2, \vec{\xi})^2, \quad (11)
\]

where \( \hat{T}(\vec{q}) = \int \frac{d\vec{x}}{2\pi} e^{-i\vec{q} \cdot \vec{x}} T(\vec{x}) \) is the amplitude of the diffraction pattern from the object. The result of a correlation measurement is a convolution of the diffraction pattern amplitude with the second order correlation function in the far-field. Hence the far-field coherence length determines the spatial resolution in the ghost diffraction scheme: the smaller the far-field speckles, the better resolved is the pattern. In the limit of speckles much smaller than the scale of variation of the diffraction pattern.

This is feasible in this case, because the very small size of the source allows a large number of independent patterns to be generated in a single tour of the glass disk.

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Since the amplitude of the object diffraction pattern is involved in Eq. 11, ghost diffraction of a pure phase object can be realized with spatially incoherent pseudothermal beams, a possibility which was questioned in a recent letter [12].

VI. THE GHOST IMAGE: TRADEOFF BETWEEN RESOLUTION AND VISIBILITY

By simply inserting the lens \( F' \) in the reference arm (see Fig. 1), without changing anything in the object arm, we now pass to the ghost image. As it was predicted in [11], and experimentally demonstrated in [12], the result of cross-correlating the intensities of the reference and object arm is now the image of the object, shown e.g. in Fig. 7.

Two issues are important in any imaging scheme: the spatial resolution and the signal-to-noise ratio.

As pointed out in [11, 12], the resolution capabilities of the ghost image setup are determined by the near field coherence length \( \Delta x_n \) (the size of the near-field speckles). This can be easily understood by inserting the propagator \( h_2(\vec{x}_2, \vec{x}_2') = m\delta(\vec{m} \cdot \vec{x}_2 + \vec{x}_2') \), into Eq. 11:

\[
G(\vec{x}_1, \vec{x}_2) \propto \left| \int d\vec{x}_1' \Gamma_n(\vec{x}_1', \vec{x}_2) T^*(\vec{x}_1') e^{i\frac{2\pi}{\lambda F} \vec{x}_1', \vec{x}_1} \right|^2, \quad (13)
\]

which shows that the result of a correlation measurement in this setup is a convolution of the object transmission function with the near-field correlation function \( \Gamma_n \). In the following we shall consider a bucket detection scheme, where the reference beam intensity \( I_2 \) is correlated to the total photon counts in the object arm, that is, in practice to the sum of photo counts over a proper set of pixels. This makes the imaging incoherent [31], because it amounts to measuring

\[
\int d\vec{x}_1 G(\vec{x}_1, \vec{x}_2) = \int d\vec{x}_1' |\Gamma_n(\vec{x}_1', \vec{x}_2)|^2 |T(\vec{x}_1')|^2.
\]

(14)
If we take the limit of spatially coherent light, where \( \Gamma_n(\vec{x}_1, -m\vec{x}_2) \) can be considered as constant over the whole beam size, both Eqs. \((13), (14)\) show that no information about the object image can be obtained by scanning \( \vec{x}_2 \). Conversely in the limit of spatially incoherent light where the speckle size is much smaller than the scale of variation of the object, \( \Gamma_n(\vec{x}_1, -m\vec{x}_2) \approx \delta(\vec{x}_1 + m\vec{x}_2) \), and both Eqs. \((13), (14)\) converge to \( |T(-m\vec{x}_2)|^2 \times \text{const.} \).

Concerning the signal-to-noise ratio, the discussion in Sec. \((3)\) pointed out that it is determined by the image visibility. We have studied the visibility of the ghost image of a double slit in a sequence of measurements where the vertical size of the apertures was progressively reduced, while leaving unchanged their horizontal size and separation. This is shown in Fig. 7, where all the frames display the correlation function measured in a bucket detection scheme as a function of the reference pixel position \( \vec{x}_2 \). Despite the fact that all the frames have been obtained with the same number of averages \( N = 10000 \), the sequence display a remarkable enhancement of the visibility as the object area is reduced. This enhancement can be non-zero for \( \vec{x}_3 \) in a region located around \( \vec{x}_2 \), of area \( A_{coh} \), where \( A_{coh} \) is the coherence area \( \propto \Delta x^2 \). Thus the correlation scales as the coherence area.

\[
\int d\vec{x}_1 G(\vec{x}_1, \vec{x}_2) \propto A_{coh}
\]

Conversely, it is not difficult to see that in the bucket detection scheme

\[
\int d\vec{x}_1 \langle I_1(\vec{x}_1) \rangle = \int d\vec{x}_1 |T(\vec{x}_1)|^2 \langle I_n(\vec{x}_1') \rangle \propto A_{obj},
\]

where \( \langle I_n(\vec{x}_1') \rangle \) is the average intensity distribution of the light illuminating the object, that can be taken as roughly uniform on the object area (the speckles average to a uniform light spot, as shown in Fig. 4b). Hence the ratio of the correlation to the background scales as:

\[
\frac{\int d\vec{x}_1 G(\vec{x}_1, \vec{x}_2)}{\int d\vec{x}_1 \langle I_1(\vec{x}_1) \rangle \langle I_2(\vec{x}_2) \rangle} \propto \frac{A_{coh}}{A_{obj}}.
\]

This formula is reminiscent of role of the mode degeneracy in Eq. \((8)\), and indeed it reflects the fact that in a bucket detection scheme the number of spatial modes detected is proportional to \( A_{obj}/A_{coh} \), which represents a degeneracy factor that reduces the visibility of the correlation with respect to the background. The ratio in Eq. \((17)\) is usually small, so that the visibility of the ghost image roughly coincides with it. Thus the visibility roughly scales as the ratio of the coherence area to the object transmissive area: the larger are the object dimensions with respect to the speckles, that is, the more incoherent is the light illuminating the object, the worse is the visibility of the ghost image. This is confirmed by the plot in Fig. 8, showing how the visibility increases with the inverse of the object area. This rather counter-

![FIG. 7: Reconstruction of the object image via correlation measurements (Fig. 4 with the lens \( F' \) inserted). (a) Cross-correlation of the intensity distribution of the reference arm with the total photo counts in the object arm, as a function of \( \vec{x}_2 \) (statistics over 10000 CCD frames). In the sequence of frames the object area is progressively reduced, and correspondingly an enhancement of the visibility can be observed (b) Horizontal sections of the images in (a), with the correlation normalized to \( \langle I_1 I_2 \rangle \), so that the vertical scales gives the visibility.

![FIG. 8: Visibility of the ghost image as a function of the inverse of the transmissive area of the object, showing an increase of the visibility by reducing the object area.](image-url)
does not prevent from retrieving more complex images (see e.g. Fig. 9), provided that a larger number of data acquisitions are performed.

FIG. 9: Ghost image of the number 4, retrieved from the correlation function after averaging over 30000 shots.

VII. NUMERICAL RESULTS

In this section we use a numerical model for simulating the speckle pattern created by the ground glass and the turbid medium, as to support the results of the previous section. The thermal field is created by generating a noisy field with huge phase fluctuations. We then multiply the noisy field with a Gaussian profile and a sub-

niversary compared to the data obtained by coherent laser illumination of the object. The averages are done over $2 \cdot 10^4$ realizations. In the numerics $\Delta x_n = 34 \mu m$ and $\Delta x_f = 12 \mu m$.

FIG. 10: Comparing a two dimensional numerical simulation of the experiment, by showing the correlation of intensity fluctuations normalized to the product of the beam intensities. (a) The ghost diffraction case $G(\vec{x}_1, \vec{x}_2)/(I_1(\vec{x}_1)/I_2(\vec{x}_2))$. (b) The ghost image case $\int dx_1 G(x_1, x_2)/(I_2(x_2)) \int dx_1/I_1(x_1)$. In both cases the numerics and experiment are real units, and are as reference compared to the data obtained by coherent laser illumination of the object. 

We neglect the temporal statistics, since we assume that the short measurement time of the experiment provides a speckle pattern frozen in time. We should finally mention that a Wigner formalism is used to describe the quantized fields, as described in detail in [5].

Initially, let us briefly show that the numerical simulations are able to describe very precisely the experiment. In Fig. 11 is shown the results of two-dimensional (2D) simulations with all parameters kept as close as possible to the experiment. This includes near-field and far-field speckle sizes, object and aperture sizes, as well as number of realizations. Both the ghost diffraction pattern [Fig. 11(a)] and the ghost image [Fig. 11(b)] show very good agreement with the experimental recorded data (using the small-speckle setup of Secs. IV-VI.IV). We stress that this comparison is not in arbitrary units.

In Sec. VII we showed experimentally the behaviour of the system when using either coherent or incoherent beams to investigate the diffraction properties of an object. In order to investigate better the actual transition from coherent to incoherent illumination of the ob-
ject, we have carried out numerical simulations that are presented in Fig. 11. We have there kept $D = 3 \text{ mm}$ and then for each simulation changed $\Delta x_n$. The simulation only includes one spatial direction (1D), and therefore the coherence properties of the beam is governed by the ratio $\Delta x_n/L_{obj}$, where $L_{obj}$ is the 1D equivalent of $A_{obj}$. Thus, the smaller $\Delta x_n/L_{obj}$ the more incoherent is the beam impinging on the object. For small speckles ($\Delta x_n/L_{obj} \ll 1$) the beams are spatially incoherent, implying a strong spatial correlations between the beams: the ghost diffraction is observed in the correlation [Fig. 11(a)]. In contrast, no diffraction pattern can be observed directly in the object beam [Fig. 11(b)]. As $\Delta x_n$ is increased by generating bigger speckles the beams become more and more spatially coherent ($\Delta x_n/L_{obj} \simeq 1$): the diffraction pattern disappears gradually [Fig. 11(a)], while the diffraction pattern starts to appear from the direct observation of the object beam [Fig. 11(b)]. Figure 11(c) shows what happens for the ghost image during this transition: the incoherence for small $\Delta x_n$ implies that a ghost image of the object can be observed, and it disappears gradually while increasing the coherence.

We saw in Sec. VII that the visibility of the ghost image became better as the object area was reduced, cf. Fig. 5. To investigate this phenomenon in general we show in Fig. 12 how the object size $\Delta x_n$ affects the visibility of the information. The trend we saw in the experiment is repeated in the numerics: in Fig. 12(a) the ghost image visibility increases as the object size decreases because fewer modes are transmitted. In Fig. 12(b) the simulation is repeated for the 1D case with a similar result. However, since in the 1D case much fewer modes are transmitted by the object the visibility is much higher. We have also in Fig. 12 plotted the visibility of the ghost diffraction fringes, and we observe that – in contrast to the ghost image case – the visibility decreases as the object size is decreased. This is is to be expected be-
FIG. 11: 1D numerical simulation of the experiment showing the transition from incoherent to coherent illumination of the object, realized by changing the near-field speckle size $\Delta x_n$. (a) shows the normalized correlation of intensity fluctuations in the ghost diffraction case, while (b) shows the normalized $\langle I_1(x_1) \rangle$ as observed directly in the object arm. (c) shows the correlation of intensity fluctuations in the ghost image case, normalized to the beam intensities $\int dx_1 G(x_1, x_2) / \langle I_2(x_2) \rangle \int dx_1 \langle I_1(x_1) \rangle$. The averages are done over $10^5$ realizations. The object mimics the experimental one, implying $L_{\text{obj}} = 530 \, \mu m$, $\Delta x_f = 12 \, \mu m$.

cause for the diffraction pattern the modes transmitted by the object interfere coherently so $G(\vec{x}_1, \vec{x}_2) \propto A_{\text{obj}}^2$ (for the 2D case). In contrast, for the mean intensity the modes interfere incoherently so $\langle I_1(\vec{x}_1) \rangle \propto A_{\text{obj}}$. Thus $G(\vec{x}_1, \vec{x}_2) / \langle I_1(\vec{x}_1) \rangle \langle I_2(\vec{x}_2) \rangle \propto A_{\text{obj}}$: the bigger the object the better the visibility of the information. We also note that there is basically no difference between the 1D and 2D results for the ghost diffraction visibility. This is because a similar argument can be done for the 1D case showing $G(x_1, x_2) / \langle I_1(x_1) \rangle \langle I_2(x_2) \rangle \propto L_{\text{obj}}$. Finally, we have checked that if the far-field speckle size $\Delta x_f$ is varied and all other parameters are kept fixed, then the visibility of the diffraction fringes increases as $\Delta x_f$ is increased: again a tradeoff between resolution and visibility is found.

FIG. 12: Numerical simulations of the experiment showing how the object size effects the visibility $V$. We kept the speckle sizes constant but varied the width of the two slits. (a) is the 2D case showing $V$ as function of the speckle area relative to the object area. Note that the ghost image visibility has been multiplied by a factor of 30, and that the object length perpendicular to the slits was kept constant. (b) is the 1D case, showing $V$ as function of the speckle size relative to the object length. $\Delta x_n = 34 \, \mu m$ and $\Delta x_f = 12 \, \mu m$.

VIII. CONCLUSIONS

The experimental results reported in this paper confirm that correlated imaging can be performed with a classical thermal source. A remarkable complementarity between spatial coherence and correlation is predicted and confirmed by experiments and numerical simulations. By changing the coherence of the speckle field at the object plane from incoherent to coherent (measured relative to the object dimensions), the object diffraction pattern reconstructed from correlations disappears while it appears in the far field intensity distribution measured in the object arm. We also discussed from a quantitative point of view the issue of the visibility of the correlated imaging scheme. We showed that the visibility of the object image is proportional to the ratio between the object area and the field coherence area at the object plane. This means that a tradeoff between resolution and visibility exists: a better visibility can be obtained only at the expense of a lower resolution and vice versa. However, the experiment clearly shows that a fairly good resolution can be achieved since the problem of low visibility can be overcome by performing a sufficiently large number of averages.

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