Supersymmetry breaking by constant superpotentials and O’Raifeartaigh model in warped space

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Abstract

Supersymmetry breaking together by constant boundary superpotentials and by the O’Raifeartaigh model is studied in a warped space model. It is shown that the contribution of constant boundary superpotentials enables the moduli of chiral supermultiplets to be stabilized and that the vacuum at the stationary point has zero cosmological constant in a wide region of parameters.

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1 Introduction

Supersymmetric models with extra dimensions has attracted interest. Since extra dimensions and supersymmetry have not been discovered, these must be invisible at low scales. One of the simple ways to compactify extra dimensions and break supersymmetry simultaneously is the Scherk-Schwarz mechanism. It is known that supersymmetry breaking by the Scherk-Schwarz mechanism [1][2] is equivalent to the supersymmetry breaking by constant superpotentials in flat bulk space [3]–[7]. These two scenarios generate the same mass spectrum.

The equivalence between supersymmetry breakings by the Scherk-Schwarz mechanism and constant superpotentials has been discussed also in warped space [8]–[22], particularly in the Randall-Sundrum background [23]. Here the more fundamental question has been examined, i.e., whether supersymmetry can be broken by the Scherk-Schwarz mechanism in Randall-Sundrum background. From the viewpoint of supergravity, the answer seems to be negative. This statement means that the Scherk-Schwarz mechanism cannot be only the source of supersymmetry breaking in Randall-Sundrum background. On the other hand, the degrees of freedom of the Scherk-Schwarz twist may exist in Randall-Sundrum background if other supersymmetry-breaking sources are taken into account [17]. Thus it would be important to clarify the effects of Scherk-Schwarz twists or constant superpotentials in systems with additional sources.

In the previous papers [21][24], we have shown that a warped space model with a constant boundary superpotential is an efficient model both to break supersymmetry and to stabilize the radius when a hypermultiplet, a compensator and a radion multiplet are taken into account. We presented possible additional supersymmetry-breaking sources of $F$-term and $D$-term to cancel the cosmological constant. The resulting soft scalar mass, gravitino mass and radion mass as well as zero cosmological constant all gave evidence that this model is phenomenologically viable. In this model, the sectors of constant superpotentials and additional supersymmetry breaking are decoupled. It would be worth to work with systems where these sectors are coupled because if constant superpotentials are allowed only in the case with supersymmetry breaking in additional sectors, they may be mixing each other.

In this Letter we study supersymmetry breaking in a warped space model with constant boundary superpotentials, a hypermultiplet, a compensator, a radion multiplet and boundary chiral supermultiplets. Equations of motion are solved together for these fields. We take into account the mass parameter $c$ for the hypermultiplet and a superpotential of O’Raifeartaigh model [25] for the boundary chiral supermultiplets. If the hypermultiplet is decoupled, the model reduces to ordinary O’Raifeartaigh model. There is a flat direction of the chiral supermultiplets. In the presence of the hypermultiplet, it is shown that the flat direction is lifted due to the mixing of the equations of motion. Then we show that a modulus of the hypermultiplet remains unfixed for zero constant superpotentials and that it is stabilized in the case with nonzero constant superpotentials for large negative $c$. In other words, the additional stabilization of moduli can be developed when the sector of constant superpotentials is coupled to a system with spontaneous supersymmetry breaking.
2 Model

We consider a five-dimensional supersymmetric model of a single hypermultiplet and three chiral supermultiplets on the Randall-Sundrum background \cite{23} whose metric is

\[ ds^2 = e^{-2R\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2, \quad \sigma(y) \equiv k|y|, \]  

(2.1)

where \( R \) is the radius of \( S^1 \) of the orbifold \( S^1/Z_2 \), \( k \) is the curvature of the five-dimensional Anti-de-Sitter (AdS\(_5\)) space, and the angle of \( S^1 \) is denoted by \( y \) (\( 0 \leq y \leq \pi \)). In terms of superfields for four-dimensional \( N = 1 \) supersymmetry, our Lagrangian is \cite{3, 21}

\[
\mathcal{L} = \int d^2\theta d^2\bar{\theta} \frac{1}{2} \varphi(T + T') e^{-(T + T')\sigma} (\Phi^\dagger \Phi + \Phi'^\dagger \Phi'^c - 6M_5^3) \\
+ \int d^2\theta \left[ \varphi e^{-3T\sigma} \left\{ \Phi'^\dagger \left[ \partial_y - \left( \frac{3}{2} - c \right) T' \sigma' \right] \Phi + W_c \right\} + \text{H.c.} \right] \\
+ \delta(y) \left[ \int d^2\theta d^2\bar{\theta} \varphi^\dagger \varphi (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_3^\dagger \Phi_3) + \left\{ \int d^2\theta \varphi^3 W(\Phi_1, \Phi_2, \Phi_3) + \text{H.c.} \right\} \right],
\]

(2.2)

where the compensator chiral supermultiplet \( \varphi \), and the radion chiral supermultiplet \( T \) are denoted as

\[ \varphi = 1 + \theta^2 F_\varphi, \quad T = R + \theta^2 F_T, \]

(2.3)

respectively and the chiral supermultiplets representing the hypermultiplet are denoted as

\[ \Phi = \phi + \theta^2 F, \quad \Phi^c = \phi^c + \theta^2 F^c. \]

(2.4)

The \( Z_2 \) parity is assigned to be even for \( \Phi \) and odd for \( \Phi^c \). The derivative with respect to \( y \) is denoted by ‘, such as \( \sigma' \equiv d\sigma/dy \). The five-dimensional Planck mass is denoted as \( M_5 \). We assume the constant (field independent) superpotential localized at the fixed points \( y = 0 \) and \( y = \pi \),

\[ W_c \equiv 2M_5^3 (w_0 \delta(y) + w_\pi \delta(y - \pi)), \]

(2.5)

where \( w_0 \) and \( w_\pi \) are dimensionless constants. In the Lagrangian \( (2.2) \), the last line is the O’Raifeartaigh model coupled to the compensator. The three chiral supermultiplets are denoted as

\[ \Phi_i = \phi_i + \theta^2 F_i, \quad i = 1, 2, 3. \]

(2.6)

which are confined at \( y = 0 \). The superpotential \( W \) is given by

\[ W(\Phi_i) = \lambda (\Phi_1^2 - \mu^2) \Phi_2 + m \Phi_1 \Phi_3 \]

(2.7)

where \( \lambda, \mu, m \) are real parameters.

As the part of the Lagrangian \( (2.2) \) containing auxiliary components is relevant to extra dimensions, we extract the part

\[ \mathcal{L}_{\text{aux}} = \frac{1}{2} e^{-2R\sigma} (2RF^\dagger F + F_T F_T^\dagger \phi + F_1^\dagger F_\phi^\dagger) \]

(2.8)
where we have defined

\[ \frac{1}{2} e^{-2R_\sigma} (2R_\phi \phi \phi + F_T(\phi \phi - 3M_5^3)) (F_\phi^\dagger - F_\sigma^\dagger) \] + (\phi \leftrightarrow \phi^c)\)

+ e^{-2R_\sigma} R(\phi \phi + \phi^c \phi^c - 6M_5^3) (F_\phi^\dagger - F_\sigma^\dagger) (F_\phi - F_\sigma)

\[ + 3e^{-3R_\sigma} (F_\phi - F_\sigma) \left\{ \phi^c \left[ \frac{3}{2} - c \right] R_\sigma' \right\} \phi + W_c \]

+ e^{-3R_\sigma} \left\{ F^c \left[ \frac{3}{2} - c \right] R_\sigma' \right\} \phi + \phi^c \left[ \frac{3}{2} - c \right] R_\sigma' \phi \right\} + H.c. \]

\[ + \delta(y) \left[ |F_\phi|^2 |\phi_i|^2 \right. + \left. \left\{ F_\phi \phi_i F_i^\dagger + 3F_\phi W + W_i F_i + H.c. \right\} \right] \]

(2.8)

where the summation over \( i \) is taken. The derivatives of the superpotential are denoted as \( W_i \equiv \partial W/\partial \phi_i \), \( i = 1, 2, 3 \). The Lagrangian (2.8) gives the following equations of motion for auxiliary fields:

\[ F = -\frac{e^{-R_\sigma}}{R} \left[ -\frac{1}{6M_5^3} \phi \phi \partial_y \phi \phi - \frac{1}{3M_5^3} \phi \phi \partial_y \phi \phi^\dagger - \frac{1}{6M_5^3} \phi \phi \phi \phi \phi \left( \frac{9}{2} - c \right) R_\sigma' \right], \tag{2.9} \]

\[ F^c = -\frac{e^{-R_\sigma}}{R} \left[ -\frac{1}{6M_5^3} \phi \phi \partial_y \phi \phi - \frac{1}{3M_5^3} \phi \phi \partial_y \phi \phi^\dagger - \frac{1}{6M_5^3} \phi \phi \phi \phi \phi \left( \frac{9}{2} - c \right) R_\sigma' \right], \tag{2.10} \]

\[ F_\phi = -\frac{e^{-R_\sigma}}{R} \left[ -\frac{1}{6M_5^3} \phi \phi \partial_y \phi \phi - \frac{1}{3M_5^3} \phi \phi \partial_y \phi \phi^\dagger - \frac{1}{6M_5^3} \phi \phi \phi \phi \phi \left( \frac{9}{2} - c \right) R_\sigma' \right], \tag{2.11} \]

\[ F_T = -\frac{e^{-R_\sigma}}{R} \left[ -\frac{1}{6M_5^3} \phi \phi \partial_y \phi \phi - \frac{1}{3M_5^3} \phi \phi \partial_y \phi \phi^\dagger - \frac{1}{6M_5^3} \phi \phi \phi \phi \phi \left( \frac{9}{2} - c \right) R_\sigma' \right], \tag{2.12} \]

\[ F_i = -\frac{e^{-R_\sigma}}{R} \phi_i \left[ -\frac{1}{6M_5^3} \phi \phi \partial_y \phi \phi - \frac{1}{3M_5^3} \phi \phi \partial_y \phi \phi^\dagger - \frac{1}{6M_5^3} \phi \phi \phi \phi \phi \left( \frac{9}{2} - c \right) R_\sigma' \right], \tag{2.13} \]

where we have defined

\[ r \equiv \phi \phi + \phi^c \phi^c - 6M_5^3. \]

In Eq.(2.9) a partial integration has been performed. The Lagrangian (2.8) are written as

\[ \mathcal{L}_{\text{aux}} = e^{-3R_\sigma} \left[ (\partial_y \phi \phi + \left( \frac{3}{2} - c \right) R_\sigma' \phi^c) F + (\partial_y \phi - \left( \frac{3}{2} - c \right) R_\sigma' \phi) F^c \right] \]
\[
+ \left( 3\phi^c (\partial_y \phi - \left( \frac{3}{2} - c \right) R\phi') + 3W_c + (3W - \phi_i W_i) \delta(y) \right) F_\phi
- \left( 3\sigma \phi^c (\partial_y \phi - \left( \frac{3}{2} - c \right) R\phi') + 3\sigma W_c + \phi^c \left( \frac{3}{2} - c \right) \sigma' \phi \right) F_T \right] - |W_i|^2 \delta(y) \tag{2.14}
\]

with \( F, F^c, F_\phi, F_T \) given in Eqs. (2.9)–(2.12).

### 3 Moduli and potential

#### 3.1 The O’Raifeartaigh sector (no hypermultiplet)

We begin with examining moduli in the part of the O’Raifeartaigh model coupled to the compensator. If the hypermultiplet is absent, the Lagrangian (2.14) becomes

\[
\mathcal{L}_{aux} = e^{-R\sigma} \frac{4\sigma(1 - R\sigma)}{6M_5^2} |3W - \phi_i W_i|^2 (\delta(y))^2 - |W_i|^2 \delta(y) = - |W_i|^2 \delta(y) \tag{3.1}
\]

where we used \( \sigma(\delta(y))^2 = 0 \). The Lagrangian (3.1) is the same as in the O’Raifeartaigh model without the compensator. The solution of \( \phi_1 \) is

\[
\phi_1 = \begin{cases} 
0 & \text{for } \mu^2 > m^2/(2\lambda^2) \\
\pm \sqrt{\mu^2 - m^2/(2\lambda^2)} & \text{for } \mu^2 < m^2/(2\lambda^2) 
\end{cases} \equiv \phi_1. \tag{3.2}
\]

The other fields \( \phi_2, \phi_3 \) only need to satisfy a single equation

\[
2\lambda \phi_1 \phi_2 + m \phi_3 = 0, \tag{3.3}
\]

and one (or two) of \( \phi_2 \) and \( \phi_3 \) is undetermined.

From Eqs. (3.2), (3.3) and (3.1), the potential is obtained as

\[
V = - \int_0^\pi dy \mathcal{L}_{aux} = |\lambda (\phi_1^2 - \mu^2)|^2 + |m \phi_1|^2 \geq 0 \tag{3.4}
\]

where \( \phi_1 \) is given in (3.2). In order to be consistent with the Randall-Sundrum background, the parameters \( \lambda \) and \( m \) must be zero.

In this pure boundary chiral supermultiplet case, the compensator has no effects on the potential and the background solution.

#### 3.2 Mixing of the two sectors \( (W_c = 0) \)

We next examine the background, potential and moduli in the case with nonzero hypermultiplet and chiral supermultiplets. In the model without chiral supermultiplets, we found that the hypermultiplet solution for \( W_c = 0 \) is \[21\]

\[
\phi = N_2 \exp \left[ \left( \frac{3}{2} - c \right) R\sigma \right] \equiv \phi, \tag{3.5}
\]

\[
\phi^c = 0 \tag{3.6}
\]

where \( N_2 \) is an overall complex constant for the flat direction \( \phi \). From this situation, it is one possibility that a simplest nontrivial solution may exist for \( W_c = \phi^c = 0 \) even with chiral supermultiplets. In this section, we consider the case \( W_c = \phi^c = 0 \).
From the Lagrangian (2.14), the equations of motion \((W_c = \phi^c = 0)\) are

\[
(\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma \phi) \left[-\left(\frac{3}{2} - c\right) R\sigma - e^{4R\sigma} \partial_y \{(\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma \phi)e^{-4R\sigma}\}\right]
+ |3W - \phi_i W_i|^2 (\delta(y))^2 \frac{(1 - 2R\sigma)^2}{r^2} \phi = 0
\]

for \(\phi^i\),

\[
(3W - \phi_i W_i)\delta(y) \left[-\frac{1}{3M_5^2} \partial_y \phi^i + \frac{1}{6M_5^2} \phi^i \left(\frac{9}{2} - c\right) R\sigma' - \frac{3}{r} \partial_y \phi^i + \frac{1}{r} \phi^i \left(\frac{3}{2} - c\right) R\sigma'\right] = 0
\]

for \(\phi^{c\dagger}\),

\[
(3W - \phi_i W_i)(\delta(y))^2 \left(-\frac{1}{R} \right) \left(-\frac{1}{6M_5^2} - \frac{1}{r}\right) (m\phi_3)^\dagger
- ((2\lambda \phi_1 \phi_2 + m\phi_3)(2\lambda \phi_2)^\dagger + \lambda(\phi_1^2 - \mu^2)(2\lambda \phi_1)^\dagger + m\phi_1 m^\dagger)\delta(y) = 0
\]

for \(\phi_1^\dagger\),

\[
(3W - \phi_i W_i)(\delta(y))^2 \left(-\frac{1}{R} \right) \left(-\frac{1}{6M_5^2} - \frac{1}{r}\right) (m\phi_1)^\dagger - (2\lambda \phi_1 \phi_2 + m\phi_3)(2\lambda \phi_1)^\dagger\delta(y) = 0
\]

for \(\phi_2^\dagger\),

\[
(3W - \phi_i W_i)(\delta(y))^2 \left(-\frac{1}{R} \right) \left(-\frac{1}{6M_5^2} - \frac{1}{r}\right) (m\phi_3)^\dagger - (2\lambda \phi_1 \phi_2 + m\phi_3)m^\dagger\delta(y) = 0
\]

for \(\phi_3^\dagger\).

These equations of motion reduce to

\[
2\lambda \phi_1 \phi_2 + m\phi_3 = 0, \quad -2\lambda \mu^2 \phi_2 + m\phi_1 \phi_3 = 0, \quad \lambda(\phi_1^2 - \mu^2)(2\lambda \phi_1)^\dagger + m\phi_1 m^\dagger = 0, \quad \partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma \phi = 0,
\]

where \(3W - \phi_i W_i = -2\lambda \mu^2 \phi_2 + m\phi_1 \phi_3\). The first equation above is the same as Eq. (3.3). The second equation gives an additional constraint to \(\phi_i\). As a result, there are four equations to determine the four variables \(\phi_i\) and \(\phi\). We find the solution (for \(W_c = 0\))

\[
\phi = \phi, \quad \phi^c = 0, \quad \phi_1 = \phi_1, \quad \phi_2 = 0, \quad \phi_3 = 0.
\]

(3.7)

Thus the fields \(\phi_i\) are all determined unlike no hypermultiplet case in the previous section. Still \(\phi\) includes unfixed \(N_2\) and \(R\).

From the solution (3.7) and the Lagrangian (2.2), the potential itself is seen to be the same as in the O’Raifeartaigh model which is given in Eq.(3.4). In this potential, the moduli \(N_2\) and the radius \(R\) are not stabilized. In other words, even if the two sectors have been mixed, there still exist moduli for the \(W_c = 0\) case.

### 3.3 Moduli stabilization with \(W_c \neq 0\)

Now we study the case with a nonzero constant superpotential \(W_c\). We assume \(|w_0| \sim |w_x| \equiv w \ll 1\) and work out perturbative solutions of the equations of motion for \(\phi, \phi^c\) and \(\phi_i\) similarly to analysis in [21]. To allow possible discontinuities of the \(Z_2\)-odd field \(\phi^c\) across the fixed points \(y = 0\) and \(y = \pi\), we define

\[
\phi^c(x, y) \equiv \hat{\epsilon}(y) \chi^c(x, y), \quad \hat{\epsilon}(y) \equiv \begin{cases} +1, & 0 < y < \pi \\ -1, & -\pi < y < 0 \end{cases}
\]

(3.8)
where $\chi^c(x, y)$ is a parity even function with possibly nonvanishing value at $y = 0, \pi$. Up to $O(w)$, the solution for $\phi$ is found to be $\phi = \hat{\phi}$ which is given in Eq.(3.5). Using this solution $\phi = \hat{\phi}$ and examining $(\delta(y))^2$ terms in the equation of motion for $\phi^\dagger$ derived from the Lagrangian (2.14), we find that

$$3W - \phi_i W_i \propto w_0,$$

which is of order of $O(w)$. As for singular terms, we use the following identity valid as a result of a properly regularized calculation:

$$\delta(y) (\epsilon(y))^2 = \frac{1}{3} \delta(y), \quad \delta(y - \pi) (\epsilon(y))^2 = \frac{1}{3} \delta(y - \pi). \quad (3.9)$$

This respects the relation $2\delta(y) = de(y)/dy$.

From the Lagrangian (2.14), the equation of motion for $\phi^\dagger$ is identical to Eq.(3.7) up to the first order of $w$. The equation of motion for $\phi^c \dagger$ up to $O(w)$ is

$$\begin{align*}
&(-\partial_y \phi^c + \left(\frac{3}{2} + c\right) R\sigma' \phi^c) \left[\left(\frac{3}{2} + c\right) R\sigma' - \frac{1}{3M_5^2} \phi \partial_y \phi^\dagger - \frac{1}{6M_5^3} \phi^\dagger \phi \left(\frac{9}{2} - c\right) R\sigma'\right] \\
&- e^{4R\sigma} \partial_y \{(-\partial_y \phi^c + \left(\frac{3}{2} + c\right) R\sigma' \phi^c)e^{-4R\sigma} \left[-1 + \frac{1}{6M_5^3} \phi^\dagger \phi\right]\} \\
&+(\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma' \phi) \left[\frac{1}{3M_5^2} \phi \partial_y \phi^\dagger - \frac{1}{6M_5^3} \phi^\dagger \phi \left(\frac{9}{2} - c\right) R\sigma'\right] \\
&- e^{4R\sigma} \partial_y \{(\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma' \phi)e^{-4R\sigma} \frac{1}{6M_5^3} \phi^\dagger \phi\} \\
&+(3\phi^c(\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma' \phi)) \left[-\frac{1}{3M_5^2} \partial_y \phi^\dagger + \frac{1}{6M_5^3} \phi^\dagger \phi \left(\frac{9}{2} - c\right) R\sigma'\right] \\
&- 3(1 - 2R\sigma) \rho \partial_y \phi^\dagger + \frac{1 - 2R\sigma}{r} \phi^\dagger \left(\frac{3}{2} - c\right) R\sigma' \\
&- e^{4R\sigma} \partial_y \{(3\phi^c(\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma' \phi) + 3W_\epsilon + (3W - \phi_i W_i)\delta(y))e^{-4R\sigma} \left[-\frac{1}{6M_5^3} \phi^\dagger \phi\right]\} \\
&-(3\phi^c(\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma' \phi) + \phi^c \left(\frac{3}{2} - c\right) R\sigma' \phi) \frac{1}{r} \left[6\partial_y \phi^\dagger - 2\phi^\dagger \left(\frac{3}{2} - c\right) R\sigma'\right] \\
&= 0 \quad (3.10)
\end{align*}$$

where $\delta(y) = 0$ is used and $r = \phi^\dagger \phi - 6M_5^3 + O(w^2)$ should be taken. The equation of motion for $\phi^\dagger_1$ up to $O(w)$ is

$$\begin{align*}
&(-\partial_y \phi^c + \left(\frac{3}{2} + c\right) R\sigma' \phi^c) \frac{\phi}{6M_5^2} (m\phi_3)^\dagger \delta(y) + (\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma' \phi) \frac{\phi^c}{6M_5^2} (m\phi_3)^\dagger \delta(y) \\
&+(3\phi^c(\partial_y \phi - \left(\frac{3}{2} - c\right) R\sigma' \phi) + 3W_\epsilon + (3W - \phi_i W_i)\delta(y))(-\frac{1}{6M_5^3} - \frac{1}{r}) (m\phi_3)^\dagger \delta(y) \\
&-\phi^c \left(\frac{3}{2} - c\right) R\sigma' \frac{1}{r} (m\phi_3)^\dagger 2\delta(y) \\
&-((2\lambda\phi_1 \phi_2 + m\phi_3)(2\lambda\phi_2)^\dagger + \lambda(\phi_1^2 - \mu^2)(2\lambda\phi_1)^\dagger + m\phi_1 m^\dagger)\delta(y) = 0. \quad (3.11)
\end{align*}$$

The equation of motion for $\phi^\dagger_2$ up to $O(w)$ is

$$\begin{align*}
&(-\partial_y \phi^c + \left(\frac{3}{2} + c\right) R\sigma' \phi^c) \frac{\phi}{6M_5^2} (-2\lambda\mu^2)^\dagger \delta(y)
\end{align*}$$
\[ + (\partial_y \phi - \left( \frac{3}{2} - c \right) R\sigma' \phi) \frac{\phi^c}{6M_5^3} (-2\lambda \mu^2) \delta(y) \]
\[ + (3\phi^c(\partial_y \phi - \left( \frac{3}{2} - c \right) R\sigma' \phi) + 3W_c + (3W - \phi_i W_i) \delta(y)) \left( -\frac{1}{6M_5^3} - \frac{1}{r} \right) (-2\lambda \mu^2) \delta(y) \]
\[ - \phi \phi^c \left( \frac{3}{2} - c \right) R\sigma' \frac{1}{r} (-2\lambda \mu^2)^{\dagger} \delta(y) - (2\lambda \phi_1 \phi_2 + m \phi_3)(2\lambda \phi_1)^{\dagger} \delta(y) = 0. \tag{3.12} \]

The equation of motion for \( \phi^c_3 \) up to \( \mathcal{O}(w) \) is
\[ (-\partial_y \phi^c + \left( \frac{3}{2} + c \right) R\sigma' \phi^c) \frac{\phi}{6M_5^3}(m \phi_1)^{\dagger} \delta(y) + (\partial_y \phi - \left( \frac{3}{2} - c \right) R\sigma' \phi) \frac{\phi^c}{6M_5^3}(m \phi_1)^{\dagger} \delta(y) \]
\[ + (3\phi^c(\partial_y \phi - \left( \frac{3}{2} - c \right) R\sigma' \phi) + 3W_c + (3W - \phi_i W_i) \delta(y)) \left( -\frac{1}{6M_5^3} - \frac{1}{r} \right)(m \phi_1)^{\dagger} \delta(y) \]
\[ - \phi \phi^c \left( \frac{3}{2} - c \right) R\sigma' \frac{1}{r} (m \phi_1)^{\dagger} 2\delta(y) - (2\lambda \phi_1 \phi_2 + m \phi_3)m^{\dagger} \delta(y) = 0. \tag{3.13} \]

From the equations of motion for \( \phi^\dagger \) and \( \phi^{c\dagger} \), it is seen that the hypermultiplet has the same bulk solutions as in the model without the boundary chiral supermultiplets. The solutions are given for generic values of the bulk mass parameter \( c \neq 3/2, 1/2 \) as \[21\]
\[ \phi = \frac{\phi}{r}, \quad \phi^c = \frac{\hat{e}}{X} \frac{(X + 1)^{(5/2-c)/(3-2c)}}{(X + 1)^{(1-2c)/(3-2c)}} \left[ c_1 + c_2(X + 1)^{(1-2c)/(3-2c)} \right] \tag{3.14} \]
where \( c_1 \) and \( c_2 \) are constants of integration. We have changed a variable from \( y \) to a dimensionless variable \( X \equiv \frac{\phi^\dagger}{\phi}(6M_5^3) - 1 \). The remaining parts of the equations of motion give boundary conditions. The \( \partial_y \delta(y), \partial_y \delta(y - \pi) \) terms of the equation of motion for \( \phi^{c\dagger} \) gives rise to the boundary conditions
\[ \chi^c \left|_{y=0} = - \left( \frac{1}{2X} \phi^{\dagger} \left( w_0 + \frac{3W - \phi_i W_i}{6M_5^3} \right) \right) \right|_{y=0}, \tag{3.16} \]
\[ \chi^c \left|_{y=\pi} = \left( \frac{1}{2X} \phi^{\dagger} w_{\pi} \right) \right|_{y=\pi}. \tag{3.17} \]

From the equations of motion for \( \phi^\dagger_2 \) and \( \phi^\dagger_3 \), the boundary conditions are
\[ 2\lambda \phi_1 \phi_2 + m \phi_3 = 0, \tag{3.18} \]
\[ (-\partial_y \phi^c + \left( \frac{3}{2} + c \right) R\sigma' \phi^c) \delta(y) + (6M_5^3 w_0 + 3W - \phi_i W_i) \delta(y))^2 \left( -\frac{\phi^{\dagger}}{r} \right) \]
\[ - \phi \chi^c \left( \frac{3}{2} - c \right) R\sigma' \frac{6M_5^3}{r} 2\delta(y) = 0. \tag{3.19} \]

In Eq.(3.19), the \( (\delta(y))^2 \) terms give the same boundary condition as Eq.(3.16). The other terms lead to
\[ \delta(y) \hat{e} \left[ -\partial_y \chi^c + \left( \frac{3}{2} + c \right) R\sigma' \chi^c - \chi^c \left( \frac{3}{2} - c \right) R\sigma' \frac{12M_5^3}{r} \right] = 0. \tag{3.20} \]

This equation gives the boundary condition for \( \chi^c \) at \( y = 0 \). Eq.(3.16) gives the boundary condition for \( (3W - \phi_i W_i) \) rather than for \( \chi^c \) at \( y = 0 \). The boundary condition for \( \chi^c \) at \( y = \pi \) is given by Eq.(3.17). Finally, the equation of motion for \( \phi^\dagger_1 \) becomes
\[ \phi_1 = \phi_1^{\dagger} \tag{3.21} \]
subject to the boundary conditions (3.18) and (3.19).

The boundary conditions given above are solved in the following. Firstly we calculate the constants of integration \( c_1 \) and \( c_2 \). Substituting the bulk solution (3.15) into the boundary conditions (3.17) and (3.20) one obtains

\[
c_1 + c_2 \hat{N}^{(2c-1)/(3-2c)} \frac{1-2c}{3-2c} = 0,
\]

\[
c_1 + c_2 (\hat{N} e^{(3-2c)Rk\pi})^{(2c-1)/(3-2c)} (\hat{N} e^{(3-2c)Rk\pi} - \frac{2}{3-2c}) = \frac{N_2^\dagger e^{(3/2-c)Rk\pi} w_\pi}{2(\hat{N} e^{(3-2c)Rk\pi})^{(5/2-c)/(3-2c)}},
\]

where we defined a dimensionless parameter \( \hat{N} \equiv |N_2|^2/(6M_5^3) \). These equations are solved as

\[
c_1 = \frac{(2c-1)N_2^\dagger \hat{N}^{-(5/2-c)/(3-2c)} e^{-Rk\pi}}{e^{(2c-1)Rk\pi} ((3-2c)\hat{N} e^{(3-2c)Rk\pi} - 2) + 2c - 1 \frac{2}{2}},
\]

\[
c_2 = \frac{(3-2c)N_2^\dagger \hat{N}^{-(3/2+c)/(3-2c)} e^{-Rk\pi}}{e^{(2c-1)Rk\pi} ((3-2c)\hat{N} e^{(3-2c)Rk\pi} - 2) + 2c - 1 \frac{2}{2}}.
\]

The coefficients \( c_1 \) and \( c_2 \) are independent of \( w_0 \). Lastly, from Eqs. (3.16), (3.18) and (3.21), \( \phi_i \) are solved as

\[
\phi_1 = \phi_1,
\]

\[
\phi_2 = -\frac{6M_5^3}{2\lambda (\mu^2 + \phi_1^2)} \left( \frac{3-2c)(1-\hat{N}) e^{-Rk\pi} w_\pi}{e^{(2c-1)Rk\pi} ((3-2c)\hat{N} e^{(3-2c)Rk\pi} - 2) + 2c - 1 \frac{2}{2}} - w_0 \right),
\]

\[
\phi_3 = \frac{6M_5^3 \phi_1}{m(\mu^2 + \phi_1^2)} \left( \frac{3-2c)(1-\hat{N}) e^{-Rk\pi} w_\pi}{e^{(2c-1)Rk\pi} ((3-2c)\hat{N} e^{(3-2c)Rk\pi} - 2) + 2c - 1 \frac{2}{2}} - w_0 \right),
\]

where \( \phi_1 \) is given in Eq. (3.21). Obviously, the solutions (3.14), (3.15) and (3.22)–(3.26) include the result in the previous section where \( w_0 = w_\pi = 0 \). As in the previous section, \( \phi_i \) are determined unambiguously.

Here we would like to stress that dependence of the above solutions on \( w_0, w_\pi \) are different from that of the case decoupled to boundary chiral supermultiplets. When boundary chiral supermultiplets are absent, the boundary conditions for \( \chi^c \) are given in Eq. (3.17) and Eq. (3.16) with zero \( (3W - \phi_i W_i) \). At \( y = 0 \) the boundary condition includes \( w_0 \). Then \( c_1 \) and \( c_2 \) depend on \( w_0 \) and \( w_\pi \). Even for zero \( w_\pi \), there exists a nontrivial solution for \( \phi^c \). On the other hand, when the boundary chiral supermultiplets are coupled, Eq. (3.16) is interpreted as a boundary condition for \( \phi_i \) or more concretely for \( (3W - \phi_i W_i) \). The three fields \( \phi_i \) are solved for the three equations (3.16), (3.18) and (3.21). The boundary conditions for \( \chi^c \) are Eqs. (3.17) and (3.20). They do not include \( w_0 \). Thus the coefficients \( c_1 \) and \( c_2 \) are independent of \( w_0 \). In obtaining a nontrivial solution for \( \phi^c \), it is required that \( w_\pi \) is at least nonzero.

We have solved the equations of motion. We can now calculate the potential. By inserting the solutions into the Lagrangian (2.8) and integrating over the extra dimension \( y \), we obtain the potential as a function of the radius \( R \) and the complex normalization parameter \( N_2 \)

\[
V = \frac{k}{2M_5^3} \int_0^\pi dy \left\{ -2c_1^2 \hat{N}^{5/2-2c+2/(3-2c)} e^{(5/2-c)(5/2-2c) + 2)R\sigma} \right\}
\]
where \( W \equiv W_c + (W - \phi_i W_i / 3) \delta(y) \) and \( c_2 \) is given in Eq. (3.23) for generic values of \( c \). This form of the potential is similar to the decoupled model [21]. Performing integration and using the boundary conditions (3.17) and (3.20) lead to the potential

\[
V \approx -N_k^2 k w_0^2 \left[ \frac{3}{2} + c + (3 - 2c) \left( -\frac{5}{2} + 2c - \left( 3(N e^{(3-2c)R_k} - 1) \right)^{-1} \right) \right] \frac{\phi_i^4}{2} e^{-4R_k} \\
+ |\lambda(\phi_i^2 - \mu^2)|^2 + |m \phi_i|^2
\]  

(3.27)

where \( \tilde{W} \equiv W_c + (W - \phi_i W_i / 3) \delta(y) \) and \( c_2 \) is given in Eq. (3.23) for generic values of \( c \). This form of the potential is similar to the decoupled model [21]. Performing integration and using the boundary conditions (3.17) and (3.20) lead to the potential

\[
V \approx -N_k^2 k w_0^2 \left[ \frac{3}{2} + c + (3 - 2c) \left( -\frac{5}{2} + 2c - \left( 3(N e^{(3-2c)R_k} - 1) \right)^{-1} \right) \right] \frac{\phi_i^4}{2} e^{-4R_k} \\
+ |\lambda(\phi_i^2 - \mu^2)|^2 + |m \phi_i|^2
\]  

(3.28)

where we defined \( \tilde{w}_0 \equiv w_0 + (3W - \phi_i W_i / 6M_5^2) \). Using Eqs. (3.22)–(3.26) for \( c_1, c_2, \phi_i \), we find the potential

\[
V = \frac{(6M_5^2)k w_0^2}{4} \left\{ -2(3 - 2c) \frac{\hat{N}^7/2 - 2c + (1/2 - c) / (3 - 2c) \ e((3 - 2c)^2 - 3) R_k}{e((3 - 2c) e^{(3 - 2c) R_k} - 2) + 2c - 1} \\
+ \hat{N}(1 - \hat{N}) \left( \frac{3 - 2c}{e((3 - 2c) R_k) / (3 - 2c) e^{(3 - 2c) R_k} - 2) + 2c - 1} \right)^2 \\
\times \left[ -4c^2 + 12c - 6 + \frac{3 - 2c}{3(1 - \hat{N})} - 2 \frac{\hat{N}^5/2 - 2c + (1/2 - c) / (3 - 2c)}{3(1 - \hat{N})} \right] \\
+ |\lambda(\phi_i^2 - \mu^2)|^2 + |m \phi_i|^2
\]  

(3.29)

This potential is independent of \( w_0 \) as it is seen from the fact that \( \tilde{w}_0 \) is proportional to \( w_\pi \) subject to Eq. (3.10).

We move on the stabilization of the radius \( R \) and the modulus \( N_2 \). For simplicity, we consider the case where \( -c \gg 1 \) and the constant \( N_2 \) is real. Then the potential becomes

\[
V \approx -(6M_5^2)k w_0^2 e^2 \left( \frac{\hat{N}^2 - N_2^2}{c^2} e^{-2R_k} + \hat{N} - 1}{(N - 1)(N - e^{2c R_k})} \right) + |\lambda(\phi_i^2 - \mu^2)|^2 + |m \phi_i|^2
\]  

(3.30)

We need to require the stationary condition for both modes \( R \) and \( N_2 \)

\[
\frac{\partial V}{\partial R} = 0 \quad \text{and} \quad \frac{\partial V}{\partial N} = 0.
\]  

(3.31)

The former condition \( \partial V / \partial R = 0 \) leads to

\[
e^{-2R_k} \approx \frac{(\hat{N} - 1)^2}{N^2}
\]  

(3.32)

whereas the latter condition gives

\[
0 \approx \left( \frac{N^3 + \hat{N}^{-2c}}{2c^2} \right) \frac{(\hat{N} - 1)^2}{N^2} + c(\hat{N}^2 + \hat{N} - 1)e^{2c R_k}.
\]  

(3.33)
From Eqs. (3.32) and (3.33), we find that the stationary condition is satisfied for infinite radius and that the modulus $N_2$ is stabilized at

$$N_2 \approx \sqrt{6M_5^3}. \quad (3.34)$$

It is important to notice that this stabilization originates from the terms proportional to $w_\pi^2$ in the potential (3.30). Only when a constant superpotential at $y = \pi$ is nonzero, the modulus of the hypermultiplet is stabilized. At the stationary point, the potential is

$$V \approx -(6M_5^3)kw_\pi^2c^2 + |\lambda(\phi_1^2 - \mu^2)|^2 + |m\phi_1|^2 \quad (3.35)$$

which can be zero dependently on the parameters.

4 Conclusion

We have studied supersymmetry breaking in a warped space model with constant boundary superpotentials, hypermultiplet, boundary chiral supermultiplets, compensator and radion multiplet. We have presented the classical background solution and have shown that all of the fields are determined unambiguously.

Dependence of the potential on $w_0$ and $w_\pi$ is significantly different from that of the potential in the model without the mixing between bulk and brane field equations \[21\]. The potential (3.29) is independent of $w_0$. In the situation we have considered where the boundary chiral supermultiplets are only at $y = 0$, the constant superpotential at $y = \pi$ is required to be nonzero to stabilize the modulus of the hypermultiplet.

For large negative $c$, we have shown that the modulus of the hypermultiplet is stabilized at a finite value and that the radius is infinite. It would be worth mentioning that large $|c|$ is closely related to flat space limit. The bulk mass parameter $c$ should have large magnitude in order to take a proper flat space limit $k \to 0$ as seen from the Lagrangian (2.2). In Ref. \[21\], we found that there is a similarity of hypermultiplet mass spectrum between flat space case and $k \to 0$ limit of warped space case with fixed $ck$. Infinite radius that we have obtained for large negative $c$ might be analogous to disappearance of the potential over $R$ in flat space case.

The radius stabilization has been studied also in the AdS$_4$ background where Scherk-Schwarz supersymmetry breaking is formulated. In models with nonzero superpotentials \[18\] \[20\], it has been found that hypermultiplets give positive contributions to the radion potential, contrary to the negative contributions from the gravity multiplet. This provides various patterns of radion potential. It would be interesting to study such a model with mixing of equations of motion for bulk and brane fields.

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