Numerical simulation of melting of scattering ice in a single-phase Stephan problem

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Abstract. A numerical study of the formation of the temperature field and the melting rate of an ice layer scattering radiation, located on an opaque substrate during radiation heating was carried out. To solve the radiation part of the problem, a modified mean fluxes method was used, taking into account the volume absorption and scattering of radiation into the medium, as well as the selective nature of the radiation source. The effect of the spectral volumetric properties of ice on the melting and temperature increase of the non-irradiated side is shown. Comparison of calculation results with experimental data shows satisfactory agreement.

1. Introduction

Ice, as well as snow represent translucent media in which heat is transported together by radiation and thermal conductivity. Krass and Merzlikin [1] describes early studies of thermophysical processes in the snow and ice cover. In [2] and [3], the current state of theoretical and numerical simulation in the snow-ice mass at solar irradiation, when snow and ice are considered as absorbing and scattering media, is presented. The ice melting simulation is based on Stefan's problem for a translucent medium. The authors of [4–6] carried out the validation of the solution of the single-phase Stefan problem in a translucent medium using the experimental data [7] as an example of melting of pure, non-scattering ice. Seki et al. [7] performed calculation and experimental investigation in a climatic chamber at a constant temperature of 0°C under the action of radiation of two types of lamps (halogen and with nichrome filament). The ice layer was on a vertical opaque substrate. Thus, the radiation melting of pure (non-scattering) ice and ice scattering radiation was simulated under the conditions of an incident short-wave and long-wave radiation flux. In the mathematical model of the process, the authors neglected the presence of a water film formed on the ice surface and the calculation was carried out in a single-phase formulation of the Stefan problem. A comparison of the melting rate of the ice layer and the heating of the non-irradiated ice surface showed satisfactory agreement between the experimental and numerical results. When calculating radiation heat transfer in [7], the fitting parameters and direct integration according to the Bouguer law were used. For scattering ice, the authors of [7], unfortunately, did not indicate the nature of radiation scattering, the albedo, and the scattering indicatrix.

The aim of this work is to further develop the methodology of [4–6], taking into account the bulk optical properties of a translucent medium. The mathematical model and computational algorithm described in [4–6] are used. The selectivity of the radiation source, as well as the selective volumetric absorption and scattering of radiation, are taken into account. The results of the numerical calculation are compared with the experimental data given in [7].
2. Formulation of the problem and methods of solution

Figure 1 shows a geometrical scheme of the problem in which a layer of scattering ice of thickness $L_0$ is located on a vertical opaque substrate and is in a medium with a constant temperature $T_\infty$. The right surface of the flat ice layer is illuminated by a lamp with filament an incandescent temperature of 3200 K filament with a constant incident radiation flux of $*E_\infty = 4648.88$ W·m$^{-2}$. The spectral composition of the radiation source is similar to [6]. The authors of [3, 7] indicate that in the radiation wavelength range from 0.3 to 1.2 μm (hereinafter referred to as the short-wavelength range), radiation scattering significantly prevails over absorption, and in the wavelength range from 1.2 μm and higher (long-wavelength range) on the contrary, the absorption of radiation in ice significantly prevails over scattering.

![Figure 1. Geometrical scheme of the problem.](image)

It is assumed that the boundary surfaces of the ice layer diffusely absorb, reflect and transmit radiation, so that $A_1 + R_1 + D_1 = 1$, where $A_1$, $R_1$, $D_1$ are the absorption, reflective and transmission hemispherical abilities of the ice surface, $i = 1, 2$. The validity of Kirchhoff's law is also assumed: $A_i = \varepsilon_i$. The left surface of the ice layer on the substrate is maintained at a constant temperature of $T_{\text{sub}} = 260.15$ K, which coincides with the initial temperature of the flat ice layer $T(x,0)$, the temperature inside the chamber is $T_\infty = 273.15$ K [7].

The solution to the problem is carried out in two stages. At the first stage, we consider radiation-conductive heat transfer is considered, which continues until the right surface of the ice layer $T(L_0,t)$ reaching the phase transition temperature $T_f$. At the second stage, the Stefan problem is solved with a fixed value of the temperature of the right boundary $T(L(t),t) = T_f$, and on the irradiated surface it is assumed that a film of water flows off under the action of gravity. We believe that the film temperature of the $T_{\text{fil}}$ is higher than the temperature of the phase transition of ice and, therefore, the boundary condition on the irradiated surface takes into account its own radiation and convective heat transfer with the water film.

The non-stationary energy equation in a flat ice layer with temperature $T(x,t)$ taking into account the energy transfer by radiation is written as follows:

$$c_p \rho \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T(x,t)}{\partial x} - E_i(x,t) \right), \quad 0 < x < L(t). \tag{1}$$

Here $c_p$ is the heat capacity at constant pressure, $\rho$ is the density, $\lambda$ is the thermal conductivity, and $E_i(x,t) = E_i^-(x,t) - E_i^+(x,t)$ is the flux density of the resulting radiation.

The boundary conditions for equation (1) at the first stage of the process are written as follows:
Here $|E_{\text{res,2}}| = A_k \left( E'_k(x,t) + E''_k \right) - e_x \sigma_o T^4(x,t)$. According to [7], the left surface of the substrate is maintained at a constant temperature $T_{\text{sub}}$. On its right surface, on the border with ice, heat transfer is carried out by heat conduction and radiation. The right boundary of the ice is exposed to radiation from a radiation source; cooling associated with convection is also taken into account. Equations (1) and (2) are supplemented by the initial condition: $T(x,0) = T_{\text{sub}}$.

At the second stage of the process, the temperature of the surface of the right boundary, when $x = L(t)$, is fixed: $T(x,t) = T_f$. The boundary condition (2) is transformed into the Stefan condition with allowance for a thin film of water formed on the surface. We assume that the water film is isothermal, and the temperature difference over its thickness is negligible:

$$\lambda \frac{\partial T}{\partial x} + h(T_{\text{fil}} - T_x) - |E_{\text{res,fil}}| = \rho L \frac{\partial L}{\partial t}$$

(3)

Where $|E_{\text{res,fil}}|$ has the following form:

$$|E_{\text{res,fil}}| = A_k \left( E'_k(x,t) + E''_k \right) - e_x \sigma_o \left( T^4(x,t) - T_{\text{fil}}^4 \right)$$

at $x = L(t)$.

(4)

Here, $T_f = 273.15$ K is the melting temperature of ice, $T_{\text{fil}} = 277.15$ K is the temperature of the water film, $\gamma$ is the latent heat of the phase transition. In condition (3), heat transfer from the outer surface of the water film is taken into account, in (4), the natural radiation of the film and the right surface is considered.

The assumption that there is a thin film of water on the ice surface does not contradict the single-phase approximation of the Stefan problem, since radiation is not absorbed in the film itself and it is taken into account only in an additional boundary condition on the interface. The thermal problem is solved only in the ice on a vertical substrate.

The density of radiation fluxes $E^\pm(x,t)$ included in equations (1) – (4), reduced to the dimensionless form $\Phi^\pm = E^\pm / (4 \sigma \theta^4)$ and $\Phi = \sum_j (\Phi^+_j - \Phi^-_j)$, are determined from the solution of the radiation transfer equation in a flat layer of a radiating, absorbing, and scattering medium with a known temperature distribution over the layer, $j$ is the number of the spectral band [2, 4].

As in the previous works of the authors, the calculation of radiation transfer is carried out using a simple and accurate modified mean fluxes method [2, 6]. According to this approach, the integral and differential equation of radiation transfer reduces to a system of two nonlinear differential equations for a plane layer of a translucent absorbing medium. The differential analog of the transport equation for hemispherical fluxes $\Phi^+_j$ taking into account radiation scattering is presented in the form [2, 6]:

$$\frac{d}{d \tau} \left( \Phi^+_j - \Phi^-_j \right) + \left( m^+_j \Phi^+_j - m^-_j \Phi^-_j \right) = n^2 \Phi_0$$

$$\frac{d}{d \tau} \left( m^+_j \Phi^+_j - m^-_j \Phi^-_j \right) + \left( 1 - \omega_j \right) \left( \Phi^+_j - \Phi^-_j \right) = 0$$

(5)

The boundary conditions for the system of equations (5) in dimensionless variables are written as follows [2]:

$$\frac{d}{d \tau} \left( \Phi^+_j \left( \Phi^+_j - \Phi^-_j \right) + \left( m^+_j \Phi^+_j - m^-_j \Phi^-_j \right) = n^2 \Phi_0 \right)$$

$$\frac{d}{d \tau} \left( m^+_j \Phi^+_j - m^-_j \Phi^-_j \right) + \left( 1 - \omega_j \right) \left( \Phi^+_j - \Phi^-_j \right) = 0$$
\[ \tau_{j,i} = 0; \Phi_{j}^{*} = (1-R_{2})\Phi_{j}^{0} + \left(1-\frac{n^2}{n^2_{j}}\right)\Phi_{j}^{0} + R_{2}\frac{n^2}{n^2_{j}}\Phi_{j}^{0}; \]

\[ \tau_{j,i} = \alpha_{j}L(t): \Phi_{j}^{t} = \epsilon_{j}\frac{\Phi_{j}^{0}}{4} + R_{j}\Phi_{j}^{0}; \]

\[ \tau_{j,i} = \alpha_{j}L(t)+\infty: \Phi_{j,i}^{*} = \Phi_{j}^{*}. \]  

In (6), the selective radiation source \( \Phi_{0}^{*} = E_{\infty}^{*}/(4\sigma_{\infty}T_{\infty}^{4}) \) is taken into account. Here \( \Phi_{0} = n^2B_{c}/(4\sigma_{0}T_{c}^{4}) \) is the dimensionless flux density of equilibrium radiation, \( B_{c} \) is the Planck function, \( n \) is the refractive index of ice, \( n^{*} \) is the refractive index of the environment, and \( \tau_{j} = \alpha_{j}L(t)/(1-\epsilon_{j}) \) is the spectral optical thickness of the layer at the moment of time \( t \), \( \epsilon_{j} = \beta_{j}/(\beta_{j}+\alpha_{j}) \) is single scattering spectral albedo, \( \beta_{j} \) is spectral scattering coefficient. The values of the coefficients \( m^{*}, l^{*} \) are determined from the recurrence relation obtained using the formal solution of the radiation transfer equation, where \( j \) is the number of the spectral band [2, 4]. Layer I refers to ice, layer II refers to the external space (Fig. 1). The solution of the boundary value problem is carried out similar to [4 – 6].

### 3. Result analysis

Below is an analysis of the results of numerical simulation of a vertically layer of a radiation-scattering ice with the following physical parameters: \( L_{0} = 0.045 \) m is an initial ice thickness, \( T_{\text{sub}} = 253.15 \) K is the temperature of the left boundary of the substrate and the initial temperature of the substrate and ice, the temperature of the atmosphere inside the chamber is maintained at a constant value of \( T_{w} = 273.15 \) K, equal to the melting ice temperature \( T_{j} \), a constant density of the incident radiation flux is \( E_{\infty}^{*} = 4648.88 \) W·m\(^{-2}\). The following thermophysical properties of ice scattering radiation are accepted: thermal conductivity \( \lambda = 1.87 \) W·m\(^{-1}\)·K\(^{-1}\); thermal diffusivity \( \alpha = 1.31 \times 10^{-7} \) m\(^2\)·s\(^{-1}\); and latent heat of the phase transition \( \gamma = 335 \) kJ·kg\(^{-1}\). Optical parameters are: the refractive index of ice \( n = 1.31 \), that of air \( n^{*} = 1 \); reflection coefficients \( R_{1} = 0.97 \) and \( R_{2} = 0.063 \); and the boundary emissivity \( \epsilon_{i} = 1 - R_{i} \). The spectral characteristics are presented in table 1.

Three parameters varied in the calculations: the heat transfer coefficient \( h \), the emissivity \( \epsilon_{2} \) of the irradiated ice surface and the albedo in the long-wavelength part of spectrum \( \omega_{2} \). At the first stage, \( h = 17.17 \) W·m\(^{-2}\)K\(^{-1}\) and \( \epsilon_{2} = 0.97 \). At the second stage, \( h = 80 \) W·m\(^{-2}\)K\(^{-1}\) and approximately corresponds to the conditions of heat transfer in [7], \( \epsilon_{2} = 0.5 \). The above parameter values were obtained in the numerical experiments and correspond to conditions on very rough surfaces [7]. Values of \( \omega_{2} \) are given in the table.

**Table 1.** Table 1. Spectral dependences of ice parameters and radiation source.

| \( j \) | \( \nu_{i}, 10^{14} \) Hz | \( \lambda_{j}, \mu m \) | \( \alpha_{s}, m^{-1} \) | \( \omega_{i} \) | \( E_{\infty}^{*}, W\cdot m^{-2} \) |
|---|---|---|---|---|---|
| 1 | 9.09–2.02 | 0.33 – 1.2 | 0.001 | 0.999 | 2073 |
| 2 | 2.02 – 1.18 | 1.2 – infinite | 1 | 0.1/0.4/0.8 | 1883 |
Figure 2. Calculated temperature field (1 – beginning of heating, 2 – melting beginning, 3 – end of melting).

Figure 3. Temperature rise of the left border of ice at different albedo values $\omega_2$ and comparison with data [7] (1 – experimental data, calculation for $\omega_2 = 0.1$ (2), 0.4 (3) 0.8 (4) and pure ice (5)).

Figure 2 shows the calculated field of ice temperature at the stages of heating and subsequent melting at an albedo of $\omega_1=0.999$ and $\omega_2=0.1$ is the most characteristic values for ice scattering radiation. Hereinafter, the curves between 1 and 2 refer to the first stage, 2 and 3 to the second. At the heating stage, the temperature curves are not monotonous; at the boundaries, the influence of the optical properties of the surface is noticeable, as well as (on the right), the influence of the source radiation. At the second stage, the temperature curves become monotonic and at the end of the calculations, with a decrease in the thickness of the medium, turn out to be linear.

Figure 3 shows the temperature rise of the left ice boundary with time and comparison with experimental data [7]. The simulation was carried out at three albedo values $\omega_2$: 0.1, 0.4, and 0.8, as well as for non-radiation scattering ice ($\omega_1 = \omega_2 = 0$). The albedo values of 0.4 and 0.8 are accepted for a numerical experiment; in reality, such parameters are not observed in ice in this spectral range. For the pure ice, the thermophysical parameters are taken, as well as for scattering ice. The calculated curves of temperature increase and the corresponding experimental data are approximately consistent with each other, but have a different character. The calculated lines have a pronounced increase at time $t<20$ min and then stop, reaching a quasistationary state. Experimental points show monotonous growth throughout the entire melting process. A curve closer to the experimental data is obtained at 0.4 (figure 3, line 3); for other albedo values, the difference in the curves is noticeable. In [7], the authors also failed to achieve complete agreement between the calculations and the experiment. This indicates the need for the correct measurement of radiation scattering parameters in an ice sample.

Conclusions
A physical and mathematical model of ice melting taking into account radiation scattering is presented. In the calculations of the radiation part, the modified mean flux method was used, taking into account the volumetric selective absorption and scattering, as well as the selective nature of the radiation source. It was shown that the melting rate is more dependent on the albedo of the short-wavelength part of the spectral range, while the increase in temperature of the left part is more dependent on the long-wavelength spectral range. A comparison of the results with experimental data shows satisfactory agreement with the calculations, however, the model requires refinement taking into account the real characteristics of anisotropic radiation scattering in the medium.
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