Medium effects on heavy-flavour observables in high-energy nuclear collisions

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Outline

- Heavy flavor in elementary collisions:
  - observables to verify our understanding of pQCD,
  - a benchmark to quantify medium-effects in the AA case;
  - a source of background for experiments like ICECUBE (e.g. atmospheric neutrinos from charm decays)
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- **Heavy flavor in heavy-ion collisions:**
  - From the understanding of the parton-medium interaction...
  - to the tomography of the produced matter ($T(x)$, $\epsilon(x)$, transport coefficients...)
  - or vice versa!

Transport calculations (e.g. relativistic Langevin equation) represent the theoretical tools to study HF in-medium propagation.
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Transport calculations (e.g. relativistic Langevin equation) represent the theoretical tools to study HF in-medium propagation.

- HF observables in small systems (p-A and d-A): initial-state and possible hot-medium effects. Of relevance also for high-energy neutrino experiments?
The strategy we employed to simulate the initial $Q\bar{Q}$ production is to interface the output of a NLO event-generator for the hard process with a parton-shower describing the Initial and Final State Radiation and modeling other non-perturbative processes (intrinsic $k_T$, MPI, hadronization).

This provides a fully exclusive information on the final state.
FONLL vs POWHEG+PS

**FONLL**

- It is a *calculation*
- It provides NLL accuracy, resumming large $\ln(p_T/M)$
- It includes processes missed by POWHEG (hard events with light partons)

**POWHEG+PS**

- It is an *event generator*
- Results compatible with FONLL
- It is a more flexible tool, allowing to address a wider set of observables (e.g. $Q\bar{Q}$ correlations, jets...)
Heavy quark production in pQCD: some references

- For a **general introduction**: M. Mangano, hep-ph/9711337 (lectures);
- For **POWHEG**: S. Frixione, P. Nason and G. Ridolfi, JHEP 0709 (2007) 126;
- For **FONLL**: M. Cacciari, M. Greco and P. Nason, JHEP 9805 (1998) 007.
- For a **systematic comparison** (POWHEG vs MC@NLO vs FONLL): M. Cacciari *et al.*, JHEP 1210 (2012) 137.
Heavy flavour: experimental observables

- $D$ and $B$ mesons (e.g. $D^0 \rightarrow K\pi$, $B^0 \rightarrow J/\psi K^*0$ ...)
- Non-prompt $J/\psi$'s ($B \rightarrow J/\psi X$)
- Heavy-flavour electrons (or muons), from the decays
  - of charm ($e_c$)
    $$D \rightarrow X\nu e$$
  - of beauty ($e_b$)
    $$B \rightarrow D\nu e$$
    $$B \rightarrow D\nu e \rightarrow X\nu e\nu e$$
    $$B \rightarrow DY \rightarrow X\nu eY$$
- B-tagged jets
HF production in *pp* collisions: results

- Besides reproducing the inclusive **D-meson** $p_T$-spectra\(^1\)
- and the heavy-flavour **electrons**

\(^1\)W.M. Alberico *et al*, Eur.Phys.J. C73 (2013) 2481
Besides reproducing the inclusive $D$-meson $p_T$-spectra\textsuperscript{1} and the heavy-flavour electrons, ...the POWHEG+PYTHIA setup allows also the comparison with $D−h$ correlation data, which start getting available.

\textsuperscript{1} W.M. Alberico \textit{et al}, Eur.Phys.J. C73 (2013) 2481
HF in p-p collisions: a summary

- A setup based on a NLO pQCD hard-event generator (POWHEG) + a Parton-Shower stage simulated with PYTHIA is able to reproduce, within the systematic uncertainties, the experimental data.

- Such an approach provides a richer information on the final state wrt other schemes (e.g. FONLL): this can be of interest for further studies like azimuthal correlations or b-tagged jets.

- Theoretical calculations, in particular at low-$p_T$, are affected by large systematic uncertainties, arising from the choice of:
  - the renormalization and factorization scales $\mu_R$ and $\mu_F$, usually taken $\sim m_T$,
  - the quark masses $m_c$ (varied from 1.3 to 1.7 GeV) and $m_b$,
  - the PDF set, in particular due to the poorly constrained gluon contribution at low-$x$.

For charm at low-$p_T$ uncertainties can be $\sim 100\%$: a limitation in the evaluation of the background for neutrino studies.
HF in AA collisions
Heavy-ion collisions: exploring the QCD phase-diagram

QCD phases identified through the order parameters

- Polyakov loop $\langle L \rangle \sim e^{-\beta \Delta F_Q}$ energy cost to add an isolated color charge
- Chiral condensate $\langle \bar{q}q \rangle \sim$ effective mass of a “dressed” quark in a hadron

Region explored at LHC: high-\(T\)/low-density (early universe, \(n_B/n_\gamma \sim 10^{-9}\))

- From QGP (color deconfinement, chiral symmetry restored)
- to hadronic phase (confined, chiral symmetry breaking\(^2\))

NB $\langle \bar{q}q \rangle \neq 0$ responsible for most of the baryonic mass of the universe: only \(\sim 35\) MeV of the proton mass from \(m_{u/d} \neq 0\)

\(^{2}\)V. Koch, Aspects of chiral symmetry, Int.J.Mod.Phys. E6 (1997)
Heavy-ion collisions: a typical event

- Valence quarks of participant nucleons act as sources of strong color fields giving rise to particle production;
- Spectator nucleons don’t participate to the collision;

*Almost all the energy and baryon number carried away by the remnants*
Soft probes (low-\(p_T\) hadrons): collective behavior of the medium;

Hard probes (high-\(p_T\) particles, heavy quarks, quarkonia): produced in hard pQCD processes in the initial stage, allow to perform a tomography of the medium
In *non-central collisions* particle emission is not azimuthally-symmetric!
Hydrodynamic behavior: elliptic flow

- **In non-central collisions** particle emission is not azimuthally-symmetric!

- The effect can be quantified through the **Fourier coefficient** $v_2$

  $$\frac{dN}{d\phi} = \frac{N_0}{2\pi} (1 + 2v_2 \cos[2(\phi - \psi_{RP})] + \ldots)$$

  $$v_2 \equiv \langle \cos[2(\phi - \psi_{RP})]\rangle$$

- $v_2(p_T) \sim 0.2$ gives a modulation 1.4 vs 0.6 for in-plane vs out-of-plane particle emission!
Elliptic flow: physical interpretation

- Matter behaves like a fluid whose *expansion is driven by pressure gradients*.

\[
(\epsilon + P) \frac{dv^i}{dt} \approx - \frac{\partial P}{\partial x^i} \quad \text{(Euler equation)}
\]

- Spatial anisotropy is converted into momentum anisotropy;
- At freeze-out *particles are mostly emitted along the reaction-plane*.
- It provides information on the EOS of the produced matter (Hadron Gas vs QGP) through the *speed of sound*: \( \nabla P = c_s^2 \nabla \epsilon \)
The medium is opaque: jet-quenching

The nuclear modification factor

\[ R_{AA} \equiv \frac{(dN^h/dp_T)^{AA}}{\langle N_{coll} \rangle (dN^h/dp_T)^{pp}} \]

quantifies the suppression of high-\(p_T\) hadron spectra
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Hard-photon \(R_{AA} \approx 1\)

- supports the Glauber picture (binary-collision scaling);
- entails that quenching of inclusive hadron spectra is a final state effect due to in-medium energy loss.
An important fraction of events display a \emph{huge mismatch} in $E_T$ between the leading jet and its away-side partner.

Possible to observe event-by-event, without any analysis!
Di-jet imbalance at LHC: looking at the event display

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Heavy Flavour in the QGP: the conceptual setup

- Description of soft observables based on hydrodynamics, assuming to deal with a system close to local thermal equilibrium (no matter why);
- Description of jet-quenching based on energy-degradation of external probes (high-$p_T$ partons);
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- Description of heavy-flavour observables requires to employ/develop a setup (transport theory) allowing to deal with more general situations and in particular to describe how particles would (asymptotically) approach equilibrium.
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NB At high-$p_T$ the interest in heavy flavor is no longer related to thermalization, but to the study of the mass and color charge dependence of jet-quenching (not addressed in this talk)
Why are charm and beauty considered heavy?

- $M \gg \Lambda_{\text{QCD}}$: their initial production (as shown!) is well described by pQCD
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Note for realistic temperatures \( g \sim 2 \), so that one can wonder whether a charm is really "heavy", at least in the initial stage of the evolution.
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- $M \gg gT$, with $gT$ being the *typical momentum exchange* in the collisions with the plasma particles: many soft scatterings necessary to change significantly the momentum/trajectory of the quark.
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Heavy quarks as probes of the QGP

A realistic study requires developing a *multi-step setup*:

- **Initial production**: pQCD + possible nuclear effects (nPDFs, \( k_T \)-broadening) → QCD event generators;
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- **Description of the background medium** ($T(x)$, $u^\mu(x)$, *local* value of *transport coefficients*...) $\rightarrow$ hydrodynamics;

Dynamics in the medium $\rightarrow$ transport calculations;

Final decays ($D \rightarrow X \nu$, $B \rightarrow X J/\psi$, ...).
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- **Dynamics in the medium** $\rightarrow$ transport calculations;
- **Hadronization**: not well under control (fragmentation in the vacuum? recombination with light thermal partons?)
  - An item of interest in itself (*change of hadrochemistry* in AA)
  - However, *a source of systematic uncertainty for studies of parton-medium interaction*;
A realistic study requires developing a *multi-step setup*:

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- **Final decays** ($D \rightarrow X\nu e$, $B \rightarrow X J/\psi...$)
Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t, x, p)$:

$$\frac{d}{dt} f_Q(t, x, p) = C[f_Q]$$

- **Total derivative** along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v \frac{\partial}{\partial x} + F \frac{\partial}{\partial p}$$

Neglecting $x$-dependence and mean fields: $\partial_t f_Q(t, p) = C[f_Q]$

- **Collision integral**:

$$C[f_Q] = \int d\mathbf{k} \left[ w(p + k, k) f_Q(p + k) - w(p, k) f_Q(p) \right]$$

$w(p, k)$: HQ transition rate $p \rightarrow p - k$

For results based on BE see e.g. Frankfurt and Catania-group papers.
From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange* (Landau)

\[
C[f_Q] \approx \int dk \left[ k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(p, k) f_Q(t, p)]
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\(^4\)B. Svetitsky, PRD 37, 2484 (1988)
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Expanding the collision integral for *small momentum exchange*\(^4\) (Landau)

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The Boltzmann equation reduces to the *Fokker-Planck equation*

\[
\frac{\partial}{\partial t} f_Q(t, p) = \frac{\partial}{\partial p^i} \left\{ A^i(p) f_Q(t, p) + \frac{\partial}{\partial p^j} [B^{ij}(p) f_Q(t, p)] \right\}
\]

where (verify!)

\[
A^i(p) = \int dk k^i w(p, k) \quad \rightarrow \quad A^i(p) = A(p) \, p^i
\]

\[
B^{ij}(p) = \frac{1}{2} \int dk k^i k^j w(p, k) \quad \rightarrow \quad B^{ij}(p) = \hat{p}^i \hat{p}^j B_0(p) + (\delta^{ij} - \hat{p}^i \hat{p}^j) B_1(p)
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Expanding the collision integral for \textit{small momentum exchange} \footnote{B. Svetitsky, PRD 37, 2484 (1988)} (Landau)

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where (verify!)

\[ A^i(p) = \int dk \ k^i w(p, k) \quad \rightarrow \quad A^i(p) = A(p) \hat{p}^i \] \textit{friction}

\[ B^{ij}(p) = \frac{1}{2} \int dk \ k^i k^j w(p, k) \quad \rightarrow \quad B^{ij}(p) = \hat{p}^i \hat{p}^j B_0(p) + (\delta^{ij} - \hat{p}^i \hat{p}^j) B_1(p) \] \textit{momentum broadening}

Problem reduced to the \textit{evaluation of three transport coefficients}
The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark: the **Langevin equation**

\[
\frac{\Delta p^i}{\Delta t} = -\eta_D(p)p^i + \xi^i(t),
\]

with the properties of the noise encoded in

\[
\langle \xi^i(p_t)\xi^j(p_{t'}) \rangle = b^{ij}(p_t)\delta_{tt'} \quad b^{ij}(p) = \kappa_{\parallel}(p)\hat{p}^i\hat{p}^j + \kappa_{\perp}(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)
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\[ \langle \xi^i(p_t)\xi^j(p_{t'}) \rangle = b^{ij}(p_t) \frac{\delta_{tt'}}{\Delta t}, \quad b^{ij}(p) \equiv \kappa_{\parallel}(p)\hat{p}^i\hat{p}^j + \kappa_{\perp}(p)(\delta^{ij} - \hat{p}^i\hat{p}^j) \]

Transport coefficients to calculate:

- **Momentum diffusion** \( \kappa_{\perp} \equiv \frac{1}{2} \frac{\langle \Delta p^2 \rangle}{\Delta t} \) and \( \kappa_{\parallel} \equiv \frac{\langle \Delta p^2 \rangle}{\Delta t} \);

- **Friction** term (dependent on the discretization scheme!)

\[ \eta_D^{\text{Ito}}(p) = \frac{\kappa_{\parallel}(p)}{2TE_p} - \frac{1}{E_p^2} \left[ (1 - v^2) \frac{\partial \kappa_{\parallel}(p)}{\partial v^2} + \frac{d - 1}{2} \frac{\kappa_{\parallel}(p) - \kappa_{\perp}(p)}{v^2} \right] \]

fixed in order to assure approach to equilibrium (Einstein relation).
A first check: thermalization in a static medium

For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution\(^5\)

$$f_{MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T K_2(M/T)}, \quad \text{with} \quad \int d^3p \, f_{MJ}(p) = 1$$

(Test with a sample of $c$ quarks with $p_0 = 2 \text{ GeV/c}$)

\(^5\)A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)
The realistic case: expanding fireball

Within our **POWLANG** setup (**POWHEG**+**LANG**evin) the HQ evolution in heavy-ion collisions is simulated as follows

- $Q\bar{Q}$ pairs initially **produced with** the **POWHEG-BOX** package (with nPDFs) and **distributed** in the transverse plane **according to** $n_{\text{coll}}(x_\perp)$ from (optical) Glauber model;

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\(^6\)P. Romatschke and U.Romatschke, Phys. Rev. Lett. **99** (2007) 172301 and **ECHO-QGP**, L. Del Zanna *et al.*, Eur.Phys.J. **C73** (2013) 2524.
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- $Q\bar{Q}$ pairs initially produced with the POWHEG-BOX package (with nPDFs) and distributed in the transverse plane according to $n_{\text{coll}}(x_\perp)$ from (optical) Glauber model;

- update of the HQ momentum and position to be done at each step in the local fluid rest-frame:
  - $u^\mu(x)$ used to perform the boost to the fluid rest-frame;
  - $T(x)$ used to set the value of the transport coefficients with $u^\mu(x)$ and $T(x)$ fields taken from the output of hydro codes\(^6\);

- Procedure iterated until hadronization

\(^6\)P. Romatschke and U.Romatschke, Phys. Rev. Lett. \textbf{99} (2007) 172301 and ECHO-QGP, L. Del Zanna et al., Eur.Phys.J. C\textbf{73} (2013) 2524.
The Langevin equation provides a link between *what is possible to calculate in QCD (transport coefficients)* and *what one actually measures* (final $p_T$ spectra).

**Evaluation of transport coefficients:**

- **Weak-coupling** hot-QCD calculations\(^7\)
- **Non perturbative approaches**
  - Lattice-QCD
  - AdS/CFT correspondence
  - Resonant scattering

\(^7\)Our approach: W.M. Alberico *et al.*, Eur.Phys.J. C71 (2011) 1666
Transport coefficients: perturbative evaluation

It's the stage where the various models differ!

We account for the effect of $2 \rightarrow 2$ collisions in the medium

Intermediate cutoff $|t^*| \sim m_D^2$ separating the contributions of

- hard collisions ($|t| > |t^*|$): kinetic pQCD calculation
- soft collisions ($|t| < |t^*|$): Hard Thermal Loop approximation (resummation of medium effects)

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$^8$Similar strategy for the evaluation of $dE/dx$ in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).
Transport coefficients $\kappa_{T/L}(p)$: hard contribution

\[
\kappa_{T/L}^{g/q(\text{hard})} = \frac{1}{2} \frac{1}{2E} \int_k \frac{n_B/F(k)}{2k} \int_{k'} \frac{1 \pm n_B/F(k')}{2k'} \int_{p'} \frac{1}{2E'} \theta(|t| - |t|^*) \times \\
\times (2\pi)^4 \delta^{(4)}(P + K - P' - K') \left| \overline{M}_{g/q}(s, t) \right|^2 q_{T/L}^2 
\]

where: $(|t| \equiv q^2 - \omega^2)$
Transport coefficients $\kappa_{T/L}(p)$: soft contribution

When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.
Transport coefficients $\kappa_{T/L}(p)$: soft contribution

When the exchanged 4-momentum is soft the t-channel gluon feels the presence of the medium and requires resummation. The *blob* represents the *dressed gluon propagator*, which has longitudinal and transverse components:

$$\Delta_L(z, q) = \frac{-1}{q^2 + \Pi_L(z, q)}, \quad \Delta_T(z, q) = \frac{-1}{z^2 - q^2 - \Pi_T(z, q)},$$

where medium effects are embedded in the HTL gluon self-energy.
One consider the non-relativistic limit of the Langevin equation:

\[ \frac{dp^i}{dt} = - \eta \mathcal{D} p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t)\xi^j(t') \rangle = \delta^{ij}\delta(t - t')\kappa \]

Hence, in the $p \to 0$ limit:

\[ \kappa = \frac{1}{3} \int_{-\infty}^{+\infty} dt \langle \xi^i(t)\xi^i(0) \rangle_{\text{HQ}} \approx \frac{1}{3} \int_{-\infty}^{+\infty} dt \langle F^i(t)F^i(0) \rangle_{\text{HQ}}, \]

\[ \equiv D^>(t) \]
Lattice-QCD transport coefficients: setup

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In the static limit the force is due to the color-electric field:

\[ \mathbf{F}(t) = g \int d\mathbf{x} Q^\dagger(t, \mathbf{x}) t^a Q(t, \mathbf{x}) \mathbf{E}^a(t, \mathbf{x}) \]
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In the **static limit** the force is due to the **color-electric field**:

\[ F(t) = g \int dx Q^\dagger(t, x)t^a Q(t, x)E^a(t, x) \]

In a thermal ensemble \( \sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1 - e^{-\beta\omega})D^>(\omega) \) and

\[ \kappa \equiv \lim_{\omega \to 0} \frac{D^>(\omega)}{3} = \lim_{\omega \to 0} \frac{1}{3} \frac{\sigma(\omega)}{1 - e^{-\beta\omega}} \sim \frac{1}{3} \frac{T}{\omega} \sigma(\omega) \]
The spectral function $\sigma(\omega)$ has to be reconstructed starting from the *euclidean electric-field correlator*

$$D_E(\tau) = -\frac{\langle \text{Re Tr}[U(\beta, \tau)gE^i(\tau, 0)U(\tau, 0)gE^i(0, 0)]\rangle}{\langle \text{Re Tr}[U(\beta, 0)]\rangle}$$

according to

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)} \sigma(\omega)$$
Lattice-QCD transport coefficients: results

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$$D_E(\tau) = -\frac{\langle \text{Re Tr}[U(\beta, \tau)gE^i(\tau, 0)U(\tau, 0)gE^j(0, 0)]\rangle}{\langle \text{Re Tr}[U(\beta, 0)]\rangle}$$

according to

$$D_E(\tau) = \int_0^{+\infty} d\omega \frac{\cosh(\tau - \beta/2)}{2\pi} \frac{\cosh(\beta \omega/2)}{\sinh(\beta \omega/2)} \sigma(\omega)$$

One gets (arXiv:1409.3724)

$$\kappa/T^3 \approx 2.4(6) \text{ (quenched QCD, \textit{cont. lim.})}$$

$\sim$3-5 times larger than the perturbative result (W.M. Alberico et al, EPJC 73 (2013) 2481). Challenge: approaching the continuum limit in full QCD (see Kaczmarek talk at QM14)!
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From quarks to hadrons

In-medium hadronization may affect the $R_{AA}$ and $v_2$ of final D-mesons due to the collective flow of light quarks. We tried to estimate the effect through this model interfaced to our POWLANG transport code:

- At $T_{\text{dec}}$ c-quarks coupled to light $\bar{q}$'s from a local thermal distribution, eventually boosted ($u_{\text{fluid}}^\mu \neq 0$) to the lab frame;
- Strings are formed and given to PYTHIA 6.4 to simulate their fragmentation and produce the final hadrons ($D + \pi + \ldots$)

In the following we display results from Eur.Phys.J. C75 (2015) no.3, 121
From quarks to hadrons: effect on $R_{AA}$ and $v_2$

Experimental data display a peak in the $R_{AA}$ and a sizable $v_2$ one would like to interpret as a signal of *charm radial flow and thermalization* at the partonic level (see the hydrodynamic spectra).
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However, *comparing transport results with/without the boost* due to $u^\mu_{fluid}$, at least part of the effect might be due to the radial and elliptic flow of the light partons from the medium picked-up at hadronization.
It is possible to perform a systematic study of different choices of

- Hadronization scheme (left panel)
- Transport coefficients (weak-coupling pQCD+HTL vs non-perturbative l-QCD) and decoupling temperature (right panel)
Experimental data for central (0–20%) Pb-Pb collisions at LHC display a strong quenching, but – at least with the present bins and $p_T$ range – don’t show strong signatures of the bump from radial flow predicted by “thermal” and “transport + $Q\bar{q}_{\text{therm}}$-string fragmentation” curves.
$D$ meson $R_{AA}$: in-plane vs out-of-plane

One can study di $R_{AA}$ in- and out-of-plane in non-central (30−50%) Pb-Pb collisions at LHC:

- Data better described by weak-coupling (pQCD+HTL) transport coefficients;
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- Data better described by weak-coupling (pQCD+HTL) transport coefficients;
- $Q\bar{q}_{\text{therm}}$-string fragmentation describes data slightly better than in-vacuum independent Fragmentation Functions.
Concerning $D$-meson $v_2$ in non-central (30−50%) Pb-Pb collisions:

- $Q\bar{q}_{\text{therm}}$-string fragmentation routine significantly improves our transport model predictions compared to the data;

- HTL curves with a lower decoupling temperature display the best agreement with ALICE data
HF in small systems
(p-Pb and d-Au collisions)

Recent POWLANG results displayed in JHEP 1603 (2016) 123
Hydrodynamic behavior in small systems?

- Long-range rapidity correlations in high-multiplicity p-Pb (and p-p) events: collective flow?
Long-range rapidity correlations in high-multiplicity p-Pb (and p-p) events: collective flow?

Evidence of non-vanishing elliptic flow $v_2$ (and mass ordering) in d-Au and p-Pb.
Heavy-flavor in elementary collisions
Heavy Flavor in heavy-ion collisions
Hot-medium effects in small systems?

Hard observables in p-A collisions: no medium effect?

No evidence of medium effects in the nuclear modification factor
- neither of jets
Hard observables in p-A collisions: no medium effect?

No evidence of medium effects in the nuclear modification factor

- neither of jets
- nor of charged particles

NB Current lack of a p-p reference at the same center-of-mass energy source of systematic uncertainty
Hard and soft probes: different sensitivity to the medium

The quenching of a high-energy parton is described by the pocket formula

$$\langle \Delta E \rangle \sim C_R \alpha_s \hat{q} L^2 \sim T^3 L^2$$

with a strong dependence on the temperature and medium thickness.
Hard and soft probes: different sensitivity to the medium

The quenching of a high-energy parton is described by the pocket formula

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with a strong dependence on the temperature and medium thickness. If one believes that also in p-A collisions soft physics is described by hydrodynamics ($\lambda_{\text{mfp}} \ll L$), then starting from an entropy-density profile

$$s(x, y) \sim \exp \left[ -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right]$$

and employing the Euler equation (for $v \ll 1$) and $Tds = d\epsilon$

$$(\epsilon + P) \frac{d}{dt} \vec{v} = -\vec{\nabla}P \quad \xrightarrow{\vec{\nabla}P = c_s^2 \vec{\nabla}\epsilon} \quad \partial_t \vec{v} = -c_s^2 \vec{\nabla} \ln s$$

whose solution and mean square value over the transverse plane is

$$v^i = c_s^2 \frac{x^i}{\sigma^2_i} t \quad \xrightarrow{\vec{x}/y} \quad \vec{v}^{x/y} = c_s^2 \frac{t}{\sigma^{x/y}}$$

The result has a much milder temperature dependence ($c_s^2 \approx 1/3$) wrt $\hat{q}$ and, although the medium has a ($\approx 3$ times) shorter lifetime, radial flow develops earlier, due to the larger pressure gradient.
HF in small systems: experimental indications

So far, experimental data don’t allow one to draw firm conclusions

- HF electrons in central d-Au collisions at RHIC: \( R_{AA} \gtrsim 1 \)
- D-meson in p-Pb at LHC: \( R_{AA} \approx 1 \) over a wide \( p_T \)-range;
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How to reconcile the two observations?
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Medium modeling: event-by-event hydrodynamics

Event-by-event fluctuations (e.g. in the nucleon positions) modeled by Glauber-MC calculation leads to an initial *eccentricity* (responsible for a non-vanishing elliptic flow)

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s(x) = \frac{K}{2\pi\sigma^2} \sum_{i=1}^{N_{\text{coll}}} \exp \left[ -\frac{(x - x_i)^2}{2\sigma^2} \right] \quad \rightarrow \quad \epsilon_2 = \frac{\sqrt{(y^2 - x^2)^2 + 4xy^2}}{x^2 + y^2}
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One can consider an *average background* obtained summing all the events of a given centrality class rotated by the *event-plane* angle \(\psi_2\).
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Initial and Final-State effects

The final result comes from the interplay of initial and final-state effects:

- nPDF's (shadowing and anti-shadowing)
- $k_T$-broadening in nuclear-matter
- energy-loss in the hot-medium
- in-medium hadronization via recombination
We display our predictions, with different initializations (source smearing) and transport coefficients (HTL vs IQCD), compared to

- **HF-electron** $R_{dAu}$ by PHENIX at RHIC (left panel)
- **D-mesons** $R_{pPb}$ by ALICE at the LHC (right panel)
We also predict a non-vanishing $v_2$ of charmed hadrons, arising mainly from the elliptic flow inherited from the light thermal partons.
A number of experimental challenges or theoretical questions remain to be answered:

- Charm measurements down to $p_T \to 0$ (flow and total cross-section)
- $D_s$ and $\Lambda_c$ measurements (change in hadrochemistry and total cross-section)
- Beauty measurements in AA via exclusive hadronic decays
- Higher flow-harmonics
- $Q\bar{Q}$ correlations in AA
- Charm in p-A collisions: which relevance for high-energy atmospheric neutrinos?