Electronic Processes at the Breakdown of the Quantum Hall Effect

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Microscopic processes giving the energy gain and loss of a two-dimensional electron system in long-range potential fluctuations are studied theoretically at the breakdown of the quantum Hall effect in the case of even-integer filling factors. The Coulomb scattering within a broadened Landau level is proposed to give the gain, while the phonon scattering to give the loss. The energy balance equation shows that the electron temperature \( T_e \) and the diagonal conductivity \( \sigma_{xx} \) exhibit a bistability above the lower critical electric field \( E_{c1} \). Calculated values of \( E_{c1} \) as well as \( T_e \) and \( \sigma_{xx} \) at \( E_{c1} \) are in agreement with the observed values in their orders of magnitude.

**KEYWORDS:** integer quantum Hall effect, nonlinear transport, theory

In the quantum Hall effect (QHE), the diagonal conductivity \( \sigma_{xx} \) is vanishingly small in the low-current regime. When the current is increased up to a critical value, \( \sigma_{xx} \) increases by several orders of magnitude within a narrow range of the current and the QHE breaks down. In spite of extensive studies on the breakdown of QHE, the mechanism of the breakdown has not been fully understood. In many samples, \( \sigma_{xx} \) increases discontinuously at the critical current and exhibits a hysteresis as a function of the current, which suggests that the breakdown of QHE belongs to the nonequilibrium phase transition. The phases below and above the transition can be considered to be homogeneous in a large class of samples, since the critical current is proportional to the sample width. In this paper, we study theoretically the nonequilibrium phase transition between homogeneous states at the breakdown of the quantum Hall effect in the case of even-integer filling factors.

Among a variety of theories proposed for the mechanism of the breakdown of QHE, the hot-electron theory, which showed the existence of the hysteretic \( \sigma_{xx} \) and reproduced the observed abrupt change of \( \sigma_{xx} \) with the current density, employed an observed result to obtain the energy gain \( j \cdot E = \sigma_{xx} E^2 \) (\( j \): current density, \( E \): electric field) and did not study the microscopic process of \( \sigma_{xx} \). Studies were made for the process of \( \sigma_{xx} \) by considering an inter-Landau-level phonon scattering. The calculated \( \sigma_{xx} \), however, had no hysteresis as a function of the electric field and the calculated critical electric field was at least one order of magnitude larger than the observed one. The microscopic process of the energy gain and the energy dissipation remains unsolved.

In this paper we propose a Coulomb scattering within a Landau level in a slowly-varying potential as the dominant electronic process giving the energy gain. The potential in the plane of the two-dimensional electron system (2DES) is fluctuating due to ionized donors in the layer several hundred Å above 2DES, and its importance in the breakdown of QHE has been discussed in the literature. Fluctuations are in a scale of \( l_{ch} = 0.1 \mu \text{m} \) (\( l_{ch} \) is the distance between a potential hill and a neighboring valley) and have a standard deviation of 20meV before the screening. The screening does not completely wash out the fluctuations in strong magnetic fields due to the discrete nature of the energy spectrum. The screened potential has a reduced width equal to the Landau level separation and, even in a filling factor of \( 2N(N = 1, 2, \cdots) \), electrons (holes) populate in the \( N + 1 \)th (\( N \)th) Landau level (we assume the spin degeneracy). Therefore the energy dissipation occurs within a Landau level and it is much larger than that due to inter-Landau-level scatterings. In such a slowly-varying potential in strong magnetic fields, closed orbits are formed around its hills and valleys, and their typical size is of the order of \( l_{ch} \) and much larger than the magnetic length \( l \) since \( l_{ch} \sim 10l \) in \( B = 5 \text{T} \). A hill orbit at the center of the Landau level is in close proximity to neighboring valley orbits and hoppings between these orbits along the electric field are the most dominant process of the energy gain. We propose a Coulomb scattering as the dominant mechanism of a hopping, in which an electron hops from a hill (valley) orbit to a valley (hill) orbit and, at the same time, one of other electrons is excited (relaxed). The dominant process for the energy loss, on the other hand, is given by acoustic-phonon deformation-potential scatterings, again, within the Landau level. In our calculations, (1) we consider only the activation transport at the Landau-level center and neglect the tunneling transport at the Landau-level edge, which limits the quantitative validity of our calculations to the higher \( T_e \) branch with the larger \( \sigma_{xx} \). We also assume (2) the vanishing lattice temperature: \( T_e = 0 \), and (3) even-integer filling factors. Our calculations show that the electron temperature \( T_e \) and \( \sigma_{xx} \) exhibit a bistability above the lower critical electric field \( E_{c1} \), giving a hysteresis as a function of \( E \). Calculated orders of magnitude of \( E_{c1} \) as well as \( T_e \) and \( \sigma_{xx} \) at \( E_{c1} \) agree with the experimental ones.
In a steady state, the energy balance equation

\[ P_G(T_e, E) = P_L(T_e, T_c), \]

holds for the energy gain \( P_G \) and the loss \( P_L \), and \( P_G \) is related with the diagonal conductivity \( \sigma_{xx} \) by

\[ P_G(T_e, E) = j \cdot E = \sigma_{xx}(T_e)E^2. \]

We assume, following the previous works, that \( \sigma_{xx} \) depends on \( E \) only through \( T_e(E) \), which is consistent with the experiments. The electron temperature is obtained as a function of \( E \) from the energy balance equation.

The electron distribution function \( f(\varepsilon) \) is given by

\[ f(\varepsilon) = \frac{1}{\exp[(\varepsilon - \mu)/k_BT_e] + 1}. \]

Since the filling factor is an even integer, \( 2N \), and the spin splitting is much smaller than \( k_BT_e \), the chemical potential \( \mu \) is at the middle point between the \( N \)th and the \( N+1 \)th Landau levels, and the electron distribution in the \( N+1 \)th Landau level and the hole distribution in the \( N \)th Landau level for both spins are described by the same function. In the following the zero of energy is taken at the center of the \( N+1 \)th Landau level and \( \mu = -\hbar\omega_c/2 \) with \( \omega_c \) the cyclotron frequency. The electron (hole) occupation of current-carrying states around the \( N+2 \)th (\( N-1 \)th) Landau-level center, \( f(\hbar\omega_c) \), is neglected, which means that we restrict our calculations to a lower-\( T_e \) range of the higher-\( T_e \) branch.

![Fig. 1. A Coulomb scattering among one hill orbit \( \varepsilon_3 \) and three valley orbits \( \varepsilon_1, \varepsilon_2, \varepsilon_4 \) in a slowly-varying potential \( V \). \( \Delta \mu \) is the chemical potential difference between the hill and the valley.](image)

The proposed electronic process for the energy gain is a Coulomb scattering, as illustrated in Fig. 1, from a hill orbit \( \varepsilon_3 \) to a valley orbit \( \varepsilon_4 \) with an excitation from \( \varepsilon_1 \) to \( \varepsilon_2 \) (we use the energy to label an orbit since we consider processes within one hill and one valley). We consider the excitation at the closest valley and hill from the hopping electron. Excitations at larger distances are negligible since the Coulomb matrix element, which is approximately the interaction between a point charge and a dipole, decreases with the distance \( d \) as \( d^{-2} \) and the transition rate as \( d^{-4} \). The guiding-center distance \( \Delta X \) corresponding to the energy difference \( \Delta \varepsilon = \varepsilon_2 - \varepsilon_1 = \varepsilon_3 - \varepsilon_4 \) is \( \Delta X = \Delta \varepsilon/eE_l \) with \( E_l \) the local electric field strength due to the slowly-varying potential. The transition rate is appreciable only when \( \Delta X \sim l \) or smaller, since the wavefunction overlap integral \( S(\Delta X) \) (along the perpendicular direction) decreases with \( \Delta X \) as

\[ S(\Delta X) = \exp(-\Delta X^2/(4l^2)) = \exp(-\Delta \varepsilon^2/4\bar{v}^2), \]

where \( \Delta \varepsilon = \Delta \varepsilon/\hbar\omega_c \) and \( \bar{v} = vE_l/\hbar\omega_c \). Inter-Landau-level scatterings are neglected here since the corresponding \( S(\Delta X) \) is much smaller: \( S(\Delta X) = 10^{-11} \) at \( \bar{v} = 0.1 \) for a quasi-elastic phonon scattering with \( eE_l\Delta X = \hbar\omega_c \) and \( S(\Delta X) = 10^{-3} \) for the most dominant Coulomb scattering with \( \Delta \varepsilon = \hbar\omega_c/2 \) (the transition rate is proportional to \( S(\Delta X)^2 \) for phonon scatterings, and to \( S(\Delta X)^4 \) for Coulomb scatterings). The change of electronic states due to the applied electric field is neglected since \( E_{c1}/E_l \sim 0.1 \).

The transition rate \( W_{12,34}^c \) of the hill-to-valley Coulomb scattering is

\[ W_{12,34}^c = \frac{2\pi}{\hbar}|\langle \varepsilon_1 \varepsilon_3 | H_c | \varepsilon_2 \varepsilon_4 \rangle|^2 \delta(\varepsilon_1 + \varepsilon_3 - \varepsilon_2 - \varepsilon_4) \]

where \( H_c \) is the Coulomb interaction: \( H_c = e^2/|r_n - r_h| \) with \( \epsilon \) the dielectric constant. The matrix element of \( H_c \) is given by

\[ \langle \varepsilon_1 \varepsilon_3 | H_c | \varepsilon_2 \varepsilon_4 \rangle = \frac{e^2}{\epsilon \hbar c} S(\Delta X)^2 f_L I_{\text{orb}}. \]

\( f_L \) is the fraction in length of the orbit \( \varepsilon_4 \) in which the distance to the orbit \( \varepsilon_3 \) is around \( \Delta X \). \( I_{\text{orb}} \) represents the dependence on orbital configurations. When the orbits \( \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \) are circles with the common center, \( I_{\text{orb}} \) is a function of the radius \( r_1 \) of orbit \( \varepsilon_1 \) as well as \( \Delta X \) (\( r_1 = l_{\text{orb}}/2 \)), and the square root of the average of \( r_{1}^2 \) over \( r_1 \) and \( \Delta X \) is estimated to be 0.4.

The energy gain \( P_G \) per unit area per unit time is given by

\[ P_G = \frac{N_e^2}{2\epsilon \hbar c} (P_{Gi})_i \]

to the energy gain \( P_{Gi} \) in the \( i \)th pair of a hill and a valley within the \( N+1 \)th Landau level with either spin. The coordination number \( N_e \) is the average number of valleys to which an electron hops from a hill. \( N_i = 4 \) is the number of possible states for each orbit with different spins and Landau indices. The energy gain due to a single hill-to-valley hopping is \( \varepsilon_G = eE\Delta \varepsilon r \) with \( \Delta r \) the difference between their position vectors. The chemical potential is assumed to be constant within each hill and each valley and the difference between the hill and the valley to be \( \varepsilon_G \). \( P_{Gi} \) is given by

\[ P_{Gi} = \sum_{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4} f_1(1-f_2)f_3(1-f_4)W_{12,34}^c \varepsilon_G - \sum_{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4} (1-f_1)f_2(1-f_3)f_4W_{12,34}^c \varepsilon_G \]

with \( f_n = f(\varepsilon_n) \) (\( n = 1, 2, 4 \)) and \( f_3 = f(\varepsilon_3 - \varepsilon_G) \). We make approximations that \( f(\varepsilon_1) \sim f(\varepsilon_2), f(\varepsilon_3 - \varepsilon_G) \sim f(\varepsilon_4) \), and \( f_3 \sim f(\varepsilon_3 - \varepsilon_G) \).
with $\varepsilon_G - f(\varepsilon_3) - \varepsilon_G f'(\varepsilon_3)$, and $f'(\varepsilon_3) \sim f'(0)$, by assuming $k_B T_e \gg \Delta \varepsilon \sim \hbar \omega_c$ and $k_B T_e \gg \varepsilon_G$, which hold approximately in the higher $T_e$ branch around $E_{c1}$ if we use the value of $T_e(E_{c1})$ and $E_{c1}$ estimated below. Then $P_{Gi}$ becomes

$$P_{Gi} = \sum_{\varepsilon_1, \varepsilon_2, \varepsilon_3} f_1(1-f_1)[-f'(0)]W_{12,34}^c \varepsilon_2^2.$$

In averaging $P_{Gi}$, we change the summation to the integration over the energy $\varepsilon$. We assume that the density of states $\rho(\varepsilon)$ is slowly varying so that $\rho(\varepsilon_1) \sim \rho(\varepsilon_2)$ and $\rho(\varepsilon_3) \sim \rho(\varepsilon_4) \sim \rho(0)$. With use of $\varepsilon_G i = (eEl\nu)^2/2$, we obtain

$$P_G = A_G \tilde{E}^2 f(0) \left| 1 - f(0) \right| I_f$$

with

$$A_G = \frac{8}{\pi^2 h^2} \left( \frac{e^2}{\alpha} f_L \langle \text{orb} \rangle \right)^2 \tilde{\rho}(0)^2 N_c,$$

$$\tilde{E} = eEl/\hbar \omega_c,$$

$$I_f = -\int_{\mu}^{\infty} d\varepsilon \tilde{\rho}(\varepsilon)^2 f'(\varepsilon).$$

$\tilde{\rho}(\varepsilon)$ is normalized to be unity when integrated within a Landau level and $\tilde{\rho}(\varepsilon)$ is a dimensionless density of states: $\tilde{\rho}(\varepsilon) = \rho(\varepsilon) \hbar \omega_c$.

As the most dominant process of the energy loss at vanishing lattice temperature, we consider the acoustic-phonon emission due to the deformation potential. The rate $W_{12}^p$ of the phonon emission with a transition from $\varepsilon_1$ to $\varepsilon_2$ is

$$W_{12}^p = \frac{2\pi}{\hbar} \sum_q \left| C_q^i \right|^2 \left| \langle \varepsilon_1 | \exp(\mathbf{i} \mathbf{q} \cdot \mathbf{r}) | \varepsilon_2 \rangle \right|^2 \delta(\varepsilon_1 - \varepsilon_2 - \hbar \omega_q)$$

where $q = (q_x, q_y, q_z)$ and $\omega_q = \epsilon_i q$ are the phonon wavevector and angular frequency, respectively, with $\epsilon_i$ the group velocity of the longitudinal acoustic mode. $C_q = qD(h/2\rho_m V \omega_q)^{1/2}$ with $D$ the deformation potential, $\rho_m$ the density, and $V$ the volume of the sample. The matrix element is calculated to give

$$\int dq_x dq_y \left| \langle \varepsilon_1 | \exp(\mathbf{i} \mathbf{q} \cdot \mathbf{r}) | \varepsilon_2 \rangle \right|^2 = \frac{(2\pi)^{3/2}}{L_p} \frac{S(\Delta X)^2}{(1 + q_x^2/b_x^2)^3}.$$

with $L_p$ the perimeter of the orbit. We have used the Fang-Howard wavefunction along $z$ (perpendicular to the plane): $\phi_0(z) = (b\sqrt{2})^{1/2} z \exp(-b_z/2)$. We here put $q_z = q$ since $q/(q_x^2 + q_y^2)^{1/2} \sim \hbar \omega_c/\epsilon_i \sim 3$ at $B=5T$. We also assume that orbits are circles with radius $r_0$ and average the transition rate over $r_0$.

The energy loss $P_L$ per unit area per unit time is

$$P_L = \frac{N_l}{2\nu_{vh}} (P_{Li}) i$$

with

$$P_{Li} = \sum_{\varepsilon_1 > \varepsilon_2} f_1(1-f_2)W_{12}^p \cdot (\varepsilon_1 - \varepsilon_2).$$

Using the approximations used already in the calculation of $P_G$: $f(\varepsilon_1) \sim f(\varepsilon_2)$ and $\rho(\varepsilon_1) \sim \rho(\varepsilon_2)$, we obtain

$$P_L = A_L \tilde{T}_e I_f$$

with $	ilde{T}_e = k_B T_e/\hbar \omega_c$ and

$$A_L = \frac{4\sqrt{2} l_{vh}}{\pi^5/2 h^3} (D\beta)^2 \tilde{\epsilon}^3 I_p$$

with $\beta = (u_0/l)(\omega_c/\nu_0)$, $u_0^2 = \hbar/\rho_m c l^2$, and $\omega_0 = c_l/l$. $I_p$ is defined by

$$I_p = \int_0^\infty dx \frac{x^2}{(1 + ax^2)^2} \exp(-x^2/2).$$

with $a = \nu_0/c_l b$.

Phonon scatterings (absorptions and emissions) also give an electron hopping between a hill and a valley and contribute to the energy gain. However, the energy gain due to phonon scatterings is shown to be $10^{-3}$ of that due to Coulomb scatterings even at $T_L = T_e$.

![Fig. 2. (a) The energy gain $P_G$ at three values of the electric field $E$ and the loss $P_L$ (both divided by $A_L I_f$) as a function of the electron temperature $T_e$. Points of intersection indicate steady states. (b) $T_e$ and $\sigma_{xx} / \sigma_{0} I_f$ as a function of $E$ (both have the lower branch of $T_e = 0$ and $\sigma_{xx} = 0$). $P_G(E) = \frac{P_L(E)}{T_e(E)}$ if the weak $T_e$ dependence of $I_f$ is neglected.](image-url)
cal field is $E_{c2} = \infty$, since $P_G = P_L = 0$ at $T_c = 0$ in the present approximations of $T_L = 0$ and no tunneling conduction. $T_c$ in steady states as a function of $E$ is given by the equation $P_G = P_L$ which is simplified to

$$\frac{A_G}{A_L}E^2 = g(\bar{T}_c) \equiv \bar{T}_c[\exp(1/2\bar{T}_c) + \exp(-1/2\bar{T}_c) + 2],$$

and is plotted with $\sigma_{xx}(E) = P_G/E^2$ in Fig. 2(b). At the critical point, $\bar{T}_{ec} \equiv \bar{T}_c(E_{c1}) = 0.32$, and $A_GE_{c1}^2/A_L = 2.2$.

The estimate of the critical electron temperature is free from ambiguities in values of $A_G$ and $A_L$ and is given by $T_{ec} = 0.32\hbar\omega_c/k_B$. Unfortunately, there exists no direct experimental estimate of $T_c$. Indirect estimates have been made from the energy balance equation using the observed temperature dependence of $\sigma_{xx}$ at $E \ll E_{c1}$ and a calculated energy loss. We use the equation $\rho(\varepsilon) = \text{const}$, and assume that $\rho(\varepsilon) = \text{const}$. Then we obtain

$$E_{c1}(\text{theory}) = 20\text{V/cm} \quad (B = 5T),$$
$$E_{c1}(\text{ref.}) = 65\text{V/cm} \quad (B = 4.7T),$$
$$E_{c1}(\text{ref.}) = 40\text{V/cm} \quad (B = 3.8T),$$
$$E_{c1}(\text{ref.}) = 100\text{V/cm} \quad (B = 7.6T).$$

The order of magnitude agrees between the theory and the experiments. Since there are large ambiguities in values of $l_{vh}/l, N_c$, and $f_L$, the discrepancy between the theory and the experiments is within the limitation of accuracy ($E_{c1} \propto N_{c}^{-1/2}$ and $N_c$ takes a smaller value in a sparse network of conduction channels between hills and valleys, which is probable in disordered potentials). Note that $E_{c1}$ in the real system is larger than the present theoretical estimate, since we have neglected fluctuations around the homogeneous steady state.

The estimate of the energy dissipation at $E = E_{c1}$ is given by $P_G(T_{ec}, E_{c1}) = P_L(T_{ec})$, and depends on $A_L$ and $I_f$. Theoretical and experimental estimates of $P_G$ at $E_{c1}$ are

$$P_{Gc}(\text{theory}) = 5 \times 10^{-2}\text{Js}^{-1}\text{cm}^{-2} \quad (B = 5T),$$
$$P_{Gc}(\text{ref.}) = 1.5 \times 10^{-2}\text{Js}^{-1}\text{cm}^{-2} \quad (B = 4.7T),$$
$$P_{Gc}(\text{ref.}) = 0.8 \times 10^{-2}\text{Js}^{-1}\text{cm}^{-2} \quad (B = 3.8T),$$
$$P_{Gc}(\text{ref.}) = 0.6 \times 10^{-2}\text{Js}^{-1}\text{cm}^{-2} \quad (B = 7.6T).$$

The agreement in $P_{Gc}$ is poorer than in $E_{c1}$, possibly because $P_{Gc}$ depends stronger on $A_L$.

In conclusion, we have considered the Coulomb scattering between localized orbits as an electronic process for the energy dissipation in the breakdown of the quantum Hall effect in the presence of slowly-fluctuating potentials. Compared to the previous theories based on inter-Landau-level phonon scatterings, we have obtained better agreements with the experiment in the value of the lower critical electric field, and in the existence of the bistability.

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Fig. 1

\[ \Delta \mu = eE \cdot \Delta r \]

\[ V(r) \]

\[ l_{vh} \]

\[ \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 \]
Fig. 2

(a) $P_G(E = 1.2E_{c1})$, $P_G(E = E_{c1})$, $P_G(E = 0.8E_{c1})$

(b) $\sigma_{xx} / \sigma_0 I_f$

$(A_G / A_L)^{1/2} \tilde{E}$