Magnetic flux penetration and AC loss in a composite superconducting wire with ferromagnetic parts

F Gömöry, M Vojenčiak, E Pardo and J Šouc

Institute of Electrical Engineering, Centre of Excellence CENG, Slovak Academy of Sciences, Dúbravská cesta 9, 842 39 Bratislava, Slovakia

Received 12 August 2008, in final form 20 October 2008
Published 17 February 2009
Online at stacks.iop.org/SUST/22/034017

Abstract

The current distribution and the AC loss in a composite superconducting tape containing a layer of magnetic material is calculated and compared with experiments, showing a very good agreement. The situations of an alternating uniform applied field or a transport current are studied. The newly developed numerical model is an approximation to the critical state model, adapted for applicability to commercial finite element codes that solve the vector potential. A substantial feature of this procedure is that it can be carried out in the case when the critical current density in the superconductor depends on the magnetic field and the magnetic layer material is nonlinear. Additionally, the hysteresis loss in the magnetic material is estimated, based on its measured magnetization loops. Measurements on Bi-2223 multifilamentary tapes covered on edges by nickel confirmed our predictions, showing a substantial ac loss reduction in both the investigated regimes.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The electromagnetic performance of a superconducting wire in AC applications is determined by two main features: the maximum current transported without resistance, and the rate at which the electromagnetic energy is converted into heat. The quantity used to indicate the first characteristic is the critical current, $I_c$. The second property is commonly characterized by the AC loss dependence on transport current amplitude, $I_a$ (transport or self-field loss) and on the amplitude of the applied magnetic field, $B_{\text{max}}$ (magnetization loss). These dependences are a good measure of the tape quality. It has been found experimentally [1] that the performance of a commercial Bi-2223/Ag tape in these two regards can be improved with the help of a ferromagnetic covering of the tape edges. This kind of sample represents a particular instance of a more general case of the wire containing both superconducting (SC) and ferromagnetic (FM) layers. It is therefore of broader interest to investigate to what extent we can theoretically predict the behaviour of such SC–FM composite and optimize its design.

The observed increase of critical current in low background magnetic fields has been explained by numerical simulations. This situation corresponds to the case when all the cross section of the wire is filled with the maximum current density that the material could carry without resistance, i.e. the critical current density, $j_c$. Detailed knowledge of the dependence of $j_c$ on both the magnitude and orientation [2] of the local magnetic field is essential when reliable predictions should be achieved [3]. With these data, the state-of-the-art finite element codes are able to resolve the saturation of the superconducting part of the composite with electrical current. In such static calculations the superconductor is characterized by a nonlinear $j_c(B)$ dependence and the ferromagnetic cover by its nonlinear permeability $\mu(B)$. We have successfully used the empirical expressions

$$j_c(x, y) = \frac{\tilde{j}_c}{1 + \left(\frac{\sqrt{B^2(x, y) + B^2(x, y)} + \frac{B_{\text{c}}}{B_{\text{c}}}}{B_{\text{c}}}\right)^{\beta}}$$

(1)

and

$$\mu(x, y) = \mu_0 \left[1 + \frac{\mu_{r, \text{max}}}{1 + \left(\frac{B(x, y)}{B_{\text{c}}^2}\right)^\alpha}\right]$$

(2)

where $\mu_0 = 4\pi \times 10^{-7}$ H m$^{-1}$ to resolve the distributions of current density and magnetic field in the $xy$ section of the
wire with the longitudinal dimension along the $z$ coordinate of the Cartesian system—see figure 1. Parallel and perpendicular components of the magnetic field, $B_1$ and $B_2$, respectively, are taken with regard to the orientation of the longer dimension of the superconducting core. In the case of high temperature superconductor Bi-2223, this is also the orientation with respect to the $a$–$b$ planes in superconducting grains that is commonly parallel to the wide face of the superconducting tape. Note that, for the complete description of SC properties, the establishing of four parameters $j_{c0}$, $k$, $B_0$ and $\beta$ is necessary and the expression (2) for FM permeability contains three parameters $\mu_{r,\text{max}}$, $B_0$ and $\alpha$.

In contrast to a rather reliable prediction of the critical current, the calculation of AC loss requires us to know how the process of filling the superconductor with (critical) current density develops during a cycle of AC current or AC magnetic field. There have been powerful numerical methods invented to resolve this problem for superconductors of various shapes [4, 5]. In these calculations the superconductor is divided into filamentary elements, and the process of flux penetration is equivalent to filling them with current. The matrix of mutual inductances between elements is calculated in the preparatory part, and further on it governs the evolution of current distribution. Adding an FM layer with constant permeability requires us to modify the calculation of mutual inductances. Using the finite element procedure for this purpose, it was found that the FM layer could significantly modify the way in which magnetic flux penetrates the wire [6] and the loss in the superconducting part can be higher or lower, depending on the exact geometry of the SC–FM composite [7]. This is also the finding from analytical calculations [8].

The main drawback of assuming a constant FM permeability is the impossibility of estimating the loss contribution because of FM magnetic hysteresis. In the attempts published until now, this problem was treated separately from the calculation of the current and field distribution in superconductors [9].

From the practical point of view, the calculation of AC loss based on a commercial numerical code is the procedure that at most could benefit from rapid development of hardware and software. Its availability to cope with nonlinear FM properties is already a standard. A very interesting approach has been recently done by Campbell [10] that resolves the evolution of critical state in a hard superconductor in a way suitable for finite element codes. Following that work, but starting from an alternative physical justification, we have developed a procedure that is particularly suitable for numerical calculations of magnetic flux penetration into the SC–FM composite with nonlinear properties of both superconducting and magnetic parts. The AC loss in the superconductor can be evaluated and a plausible estimation of the loss due to FM hysteresis performed as well. Summing these two components we obtained the total loss that should be compared with experimental data. As will be shown later in this paper, experiments confirmed the validity of our calculation method.

2. Critical state formulation in 2D using the vector potential of magnetic field

In this section, we formulate the theoretical model from which the numerical calculations are developed, taking as a starting point the critical state model. The reader not interested in the physics beyond the proposed procedure can go directly to section 3.

Here we show how our method works in two commonly used arrangements: in the pure transport case, the actual value of the electrical current $I$ carried by the wire is the imposed parameter driving the change of current and field distribution inside the wire. When a transversal AC magnetic field is applied to an insulated wire, the change of its actual value, $B_x$, causes the re-arrangement of the field and current distributions.

The geometry considered here is two-dimensional (2D): an infinite wire along the $z$ axis, with the distribution of current and field in any $x$–$y$ section identical as shown in figure 1. In this geometry, the vector potential of magnetic field, $A$, is the chosen independent variable for electromagnetic calculation. Its curl is the source of magnetic flux density:

$$\vec{B} = \nabla \times \vec{A}$$

and the equation to be resolved is

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \vec{A} \right) = \vec{j}.$$  (4)

The magnetic permeability, $\mu$, as well as the current density, $\vec{j}$, can be functions of the position $\vec{r}$ and of the magnitude of the magnetic field.

The critical state model [11] basically states that in a hard superconductor one can find the local current density with a magnitude that is either zero or equal to the critical current density. Zero current density appears in the part that never experienced any electrical field. The orientation of the current density is controlled by the most recent non-zero electrical field existing locally in the superconductor. The electrical
field accompanying the change of the distribution of electrical current and magnetic field is, in general,

\[ \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \varphi. \]  

Equation (5) is valid for any gauge of the vector potential.

Let us now focus our consideration on superconductors following the critical state model. For simplicity, in this paper we restrict ourselves to the situations when (i) there exists a zone where \( \vec{E} \) vanishes in all the cycle—'neutral zone'—and (ii) the magnetic flux penetrates monotonically from the surface when \( I \) or \( B \) increase monotonically. The first condition is always true except for the case of transporting a current equal to or larger than the critical current, a situation where the vector potential has not changed, and vice versa:

\[ j_{1}(x, y) = \begin{cases} +j_c & \text{if } A_{1}'(x, y) < A_{0}'(x, y) \\ -j_c & \text{if } A_{1}'(x, y) > A_{0}'(x, y) \\ 0 & \text{if } A_{1}'(x, y) = A_{0}'(x, y). \end{cases} \]  

Where in the next step the transport current increases as \( I_{ac,1} \rightarrow I_{ac,2} \), or the applied magnetic field grows as \( B_{ac,1} \rightarrow B_{ac,2} \), the same principle of comparing the local values of vector potential controls the resulting distribution of current density. Next, we use the assumption that the magnetic flux penetrates monotonically in the superconductor for a monotonic increase of \( I_{ac} \) or \( B_{ac} \). Since \( A'(x, y) \) is the magnetic flux per unit length between the neutral zone and the point \((x, y)\), this means that \( A' \) increases monotonically in the initial stage. Therefore

\[ j_{in}(x, y) = \begin{cases} +j_c & \text{if } A'(x, y) < A_p'(x, y) \\ -j_c & \text{if } A'(x, y) > A_p'(x, y) \\ 0 & \text{if } A'(x, y) = A_p'(x, y). \end{cases} \]  

In the following we will analyse the process of a change in transport current or applied field in terms of the vector potential \( A' \). Because the magnetic vector potential as well as the electric field and current density have only the \( z \) components \( A_z, E_z \), and \( j_z \), respectively, we can omit the vectorial notation of these quantities. Moreover, in the infinitely long geometry, the flux per unit length between two points in the \( x-y \) plane is the vector potential difference. Since \( A' \) in the neutral zone is zero, \( A'(x, y, t) \) is the magnetic flux per unit length between the neutral zone and the point \((x, y)\).

1 For a large applied magnetic field (and no transport current) the sample is fully penetrated by current but condition (i) is still fulfilled because there is a boundary between \( j = j_c \) and \( -j_c \), where \( j = 0 \) everywhere, and, consequently, \( E = 0 \).

2 In the most general case, \( (8c) \) contains a uniform time-independent (constant) term due to static electrical current charge distributions. In our case, we do not consider DC transport currents and this contribution to \( E \) vanishes.

2.1. Initial part

Let us now consider the first step in exciting the superconductor from the virgin state, i.e. from \( j = 0 \) everywhere. This means in the transport case that the total current jumps as \( 0 \rightarrow I_{ac,1} \), and in the magnetization case the applied field changes as \( 0 \rightarrow B_{ac,1} \). The values of vector potential locally changed from the original distribution \( A'(x, y) \) to the actual distribution \( A_{1}'(x, y) \). From the expression \( (8c) \) follows that the zero current density would remain in the parts of the superconductor where the vector potential has not changed, and vice versa:

\[ j_{in}(x, y) = j_c \text{ sign}(A_p'(x, y) - A'(x, y)). \]  

As already suggested by Campbell [10]. for the purpose of numerical calculations it is favourable to replace the signum function exhibiting infinite derivatives at zero with a smoother equivalent. The form we have found perfectly plausible for our finite element code is the hyperbolic tangent, with the independent variable scaled by a factor that, on one side, should secure the convergence of calculations and, on the other side, should be compatible with the physical formulation of the problem—see figure 2. Thus the equation we inserted in the finite element calculation to define the current density in the superconductor is

\[ j_{in}(x, y) = j_c \tan \left( \frac{A_p'(x, y) - A'(x, y)}{A_n} \right). \]  

where the choice of the scaling factor \( A_n \) influences the thickness of the borders between parts filled with opposite current densities.

In the case of a monotonic increase considered up to now there is no difference in assuming one single step from the virgin state with \( j = 0 \) everywhere to the maximum current \( I_1 \) (or magnetic field \( B_{max} \)) and the case when the maximum value is reached through a series of positive steps.
The distributions of $A$ current keeps its value from the previous step, with respect to (10) is that for the unchanged value of $j$ superconductor is involved in a very natural way. If this comparison the history of current distribution in a way. Of course the numerical procedure used must allow us to recall the distribution calculated in the previous step in this case. $j$ decreases monotonically in one-half cycle, $j_p$ in equation (13) can be left as $j$ at the $I_{ac}$ or $B_{ac}$ peak and the current distribution for the desired external parameter can be found in one step.

2.3. Boundary conditions

An essential ingredient of the finite element calculation is the proper choice of the boundary condition. We set them on the cylinder concentric with the superconducting tape, with a radius $R$ much larger than the tape width. This cylinder is the boundary box in the finite element calculations in section 3.

For the transport situation, we establish the boundary condition in the following way. In the Coulomb gauge (defined as $\nabla \cdot A' = 0$), the vector potential created by the currents flowing in the wire, $I$, at a distance $R$ much larger than the dimensions defining the cross section is

$$A(R, t) = -\frac{I_0}{2\pi} \ln(R).$$

Therefore, from (8b) we find that

$$A'(R, t) = -\Phi_R(t).$$

where $A_c$ is the vector potential in the neutral zone defined in the Coulomb gauge. An important consequence of (15) is that the vector potential for any point on the cylinder, which conforms to the boundary box, is the same. Since for our gauge conditions in (8), $A'(R, t)$ is the flux per unit length between the neutral zone and the boundary, $\Phi_R$, the boundary condition (15) becomes

$$A'(R, t) = -\Phi_R(t).$$

3. Procedure of finite element calculation

There are various ways to use the proposed approach for the numerical calculations of hysteresis in type II superconductors. Here we describe the procedure that was found reasonably fast and reliable in the case of AC loss calculation in a commercial Bi-2223 tape with an elliptic filamentary zone, covered on its edges by nickel [12]. An optical micrograph of the tape is shown in figure 3 together with the geometrical representation used in the calculations. The layer of nickel was deposited on the edges of the tape by a galvanic process, leaving the central part of the tape uncovered with a width of 1.9 mm. The filamentary zone was replaced by an effective superconducting core with elliptic cross section, defined by the major half-axis $a_{SC} = 1.504$ mm and the minor half-axis $b_{SC} = 0.11$ mm, respectively. The thickness of the Ni layer is 30 $\mu$m. In each half of the tape it is composed of two sheets (upper, lower) inclined by the angle $\delta = \pm 4.3^\circ$ with respect to the long axis, connected at the edge by a half-circular rounding of the same thickness, separated by the distance $s = 80$ $\mu$m from the edge of the superconducting core.
Figure 3. Micrograph of the multifilamentary Bi-2223 wire with the edges covered by a ferromagnetic layer of nickel (left) together with the simplified geometry used in the calculations. The values of parameters defining the geometry are the following: $a_{SC} = 1.504 \text{ mm}$, $b_{SC} = 0.11 \text{ mm}$, $d = 1.9 \text{ mm}$, $t = 30 \text{ μm}$, $s = 80 \text{ μm}$ and $\delta = \pm 4.3^\circ$. The material of the superconducting core is characterized by the effective critical current density given by the expression (1) with parameters $j_{c0} = 1.34 \times 10^8 \text{ A m}^{-2}$, $B_0 = 0.008 \text{ T}$, $k = 0.1$ and $\beta = 0.58$.

In order to establish the parameters of the $j_c(B)$ dependence, the procedure detailed in [13] was followed. It is based on experimental critical current data, $I_c(B)$, taken on the original non-covered tape for the range of applied magnetic fields from 0 to 0.15 T at several inclinations from 0 (parallel field) up to 90° (perpendicular field). For the whole set of experimental conditions, numerical calculations were performed assuming all the central elliptic core is filled with the critical current density defined by the expression (1). The optimal set of parameters in this expression is found at the condition of minimal discrepancy between experimental data and numerical results. In this way, the parameters characterizing the properties of the elliptic superconducting core have been determined as $j_{c0} = 1.34 \times 10^8 \text{ A m}^{-2}$, $B_0 = 0.008 \text{ T}$, $k = 0.1$ and $\beta = 0.58$.

To establish the properties of the ferromagnetic Ni layer deposited on the Ag matrix, a 10 μm layer of Ni was deposited on a 1 mm thick silver plate 10 mm long by the same procedure as used in covering the tape edges. Magnetization of this sample in a parallel magnetic field was measured at 77 K in a SQUID magnetometer [12]. From these data, the dependence of permeability on magnetic field shown in figure 4 (left side) was extracted. In the same plot is given the approximation in the form of the expression (2) with parameters $\mu_{r,\text{max}} = 119$, $B_c = 0.4 \text{ T}$ and $\alpha = 1.3$. As an alternative to this expression, we have used the interpolation of experimental data (symbols) in the calculation of critical state distributions, without an observable difference for the calculated distributions. Other useful information that can be extracted from the magnetization data is the volume density of hysteresis loss in the Ni layer determined by the areas of minor loops. The result of such an analysis based on experimental data is plotted on the right-hand side of figure 4, together with the approximation

$$Q_{\text{Ni}} = Q_{\text{sat}} \times \begin{cases} \left( \frac{B}{B_s} \right)^2 & \text{if } B \leq B_s \\ 1 & \text{if } B > B_s \end{cases} \quad (18)$$

where $B_s = 0.5 \text{ T}$ and $Q_{\text{sat}} = 2750 \text{ J m}^{-3}$.

The wire is placed concentrically in the calculation box that is the infinite cylinder with radius $R$. We used the commercial finite element code [14] that was already found to be able to cope with nonlinearities of the current density and magnetic permeability described by formulae (1) and (2), respectively.

3.1. Applied magnetic field

In the case when the wire is exposed to a magnetic field we use the boundary condition (17). To determine the AC loss, a
series of distributions should be calculated for the finite number of applied magnetic fields corresponding to a cyclic change of the magnetic field.

We found it practical to perform the calculation in the following series of steps: first, the change of applied field from zero to the maximum value of the applied field, $B_x$, is considered in one single step. The choice of vector potential (17) suggests we assume $A'_p(x, y) = 0$ as well as $j_p(x, y) = 0$ in this first calculation step when applying the general equation (12):

$$j_{s, \text{im}}(x, y) = j_c \tanh \left( \frac{-A'(x, y)}{A_n} \right).$$  \hspace{1cm} (19)

In figure 5 are shown four distributions calculated in this way as an example. The direction of current density in the right half is positive (because the vector potential of the applied field is negative there according to the expression (17)) except for the zones with no current where the vector potential is also zero. The value of the scaling factor used in the calculations was $A_n = 10^{-9}$ V s m$^{-1}$. The dependence of $j_c(x, y)$ is given by the expression (1), with the parameters established from experimental data determined for the critical currents in dependence on the magnetic field of various orientations. The permeability $\mu$ in the ferromagnetic free space is given by the interpolated expression (2) while it is equal to the permeability of the vacuum, $\mu_0$, in both the superconductor and the free space. The parameters of the dependence (2) are derived from the magnetization measurement of a sample from the ferromagnetic material. After finishing the first calculation, the distribution of the vector potential at the field amplitude, i.e. at $B_x = B_{\text{max}}$, is stored as $A_{\text{max}}(x, y)$. It is used for the subsequent calculation of the current and field distributions at the values of the applied magnetic field decreasing from $B_{\text{max}}$ down to $-B_{\text{max}}$.

The purely hysteretic nature of the magnetization process is reflected by omitting the time variable in the equations from (9) to (13). The particular choice of magnetic field values used to determine the distributions during the AC cycle therefore does not influence the AC loss evaluation. We found it convenient to use the linear ramp $B_{x,i} = B_{\text{max}}(1 - \frac{i}{N})$ with $i = 1 \ldots N$ and $N$ being the total number of applied fields, $B_{x,i}$, for which the calculation has been performed. There is no background magnetic field applied on top of the AC one, and the downwards half of the AC cycle is representative enough to supply the data for the whole cycle. We found it sufficient to calculate for 20 descending values of the field, i.e. $N = 20$, leading to the hysteresis curve containing 40 points in the whole AC cycle.

The advantage of using one-half of the AC cycle in which the change of the applied field is monotonic is that one can use for the current density in the superconductor a functional dependence quite similar to the one used for the initial part of the cycle:

$$j_{s, \text{down}}(x, y) = j_c \tanh \left( \frac{A_{\text{max}}(x, y) - A'(x, y)}{A_n} \right).$$  \hspace{1cm} (20)

In this expression, the distribution $A_{\text{max}}(x, y)$ calculated at the AC field amplitude $B_{\text{max}}$ is used as the input in the calculation of the distribution at $B_x < B_{\text{max}}$. A suitable choice of the calibration for the vector potential through the boundary condition (17) allowed the reduction of the general expression (10) to this form. The series of distributions calculated in this way for $B_{\text{max}} = 12$ mT and four values of descending field $B_x = (6, 0, -6, -12)$ mT is shown in figure 6. For each distribution we determined the magnetic moment per unit length of the wire, $m$, and the magnetization $M = m/S_{\text{SC}}$, where $S_{\text{SC}}$ is the superconductor’s cross section. Because the applied magnetic field is in the $y$ direction, the magnetization contributing to the AC loss is

$$M(B_{ac}, B_x) = \frac{1}{S_{\text{SC}}} \int_{S_{\text{SC}}} -x j(x, y) dS_{\text{SC}}.$$  \hspace{1cm} (21)

In this expression, we used the result of Brandt and Indenbom [15] who have shown that replacing the factor of 1/2 in the original formula for the magnetic moment is equivalent to accounting for the contribution due to currents closing the current loops at the ends of the wire.

The plot of magnetization data in dependence on the applied field gives the magnetization loops—see figure 7. Evaluating the loop area allows us to compute the volume density of AC loss per AC cycle—in J m$^{-3}$—due to hysteresis of the supercurrent distribution as

$$Q_{\text{mSC}}(B_x) = -\oint M(B_{ac}, B_x) dB_{ac}.$$  \hspace{1cm} (22)

In the case of a ferromagnetic cover without hysteresis of magnetization, this is the total AC loss of the composite material.
wire. Later in this paper we show how the contribution of the hysteretic magnetization loop of the ferromagnetic material can be included in the total loss estimation.

3.2. Transport current

In the calculation of loss in the case of the wire transporting AC current the basic expressions (19) and (20) remain the same, but the boundary condition is (16). This boundary condition means that we impose the magnetic flux between the neutral zone and boundary and then the procedure finds the current that fulfils all the equations. However, it is usually intended to set a certain desired current in the superconductor, not a magnetic flux. The relation between the flux and the current is not straightforward in the considered geometry of circular boundary and elliptic superconducting core. Generally, the larger value of vector potential at the boundary would require a larger transport current in the wire according to the relation (15). We have found it practical to set a transport current approximately to the desired one $I_{\text{des}}$ by setting the vector potential at the boundary of the calculation box using

$$A'(R) = -\frac{\mu_0 I_{\text{des}}}{4\pi} \left( 2 \ln \left( \frac{R}{r_{\text{eq}}} \right) + 1 \right). \quad (23)$$

This expression corresponds to the flux between the centre of a round wire of radius $r_{\text{eq}}$ and the cylinder with radius $R$ if the wire carries a transport current $I_{\text{des}}$ uniformly distributed in its cross section. In our case of a wire with elliptic cross section defined by the semi-axes $a_{\text{SC}}$ and $b_{\text{SC}}$, respectively, we have arbitrarily chosen $r_{\text{eq}} = (a_{\text{SC}} + b_{\text{SC}})/2$. The exact value of $r_{\text{eq}}$ is not important because once the finite element procedure finds the solution with the boundary condition (23), the actual transport current $I_{\text{ac}}$ is found by simply integrating $j$ in the tape cross section. We observed that the result is slightly different from $I_{\text{des}}$ (within 10%)—for this reason the loss curves are always plotted with respect to $I_{\text{ac}}$, not $I_{\text{des}}$.

Similarly as in the magnetization case treated above, we first calculate the distributions at initial increase of the current using the formula (19). In figure 8 are illustrated the results of
such a calculation for the same composite wire as shown in the magnetization regime in figure 5. Current density is of single (positive) polarity because the superconductor is filled with increasing total transport current. Any such distribution can be used as the starting point for calculation of the distributions at the current decrease from $I_a$ to $-I_a$ through the set of intermediate values $I$. Now the expression (20) is to be used for the definition of current density, with $A_{\text{max}}'(x, y)$ stored from the calculation at the AC current amplitude $I_a$. Examples of the current distributions calculated in this way for transport currents decreasing from 50 A are shown in figure 9.

Note that for all these distributions, except the case of the critical current shown in figure 8, there is a visible portion of superconductor free from current. According to the expression (8a) the vector potential in this ‘neutral zone’ is equal to zero. For each transport current distribution, the significant quantity to be evaluated is the magnetic flux calculated for different circumstances given in the previous section, one can predict the effect of the Ni layer on the tape performance. We present here as an illustration the results achieved on one of the prepared samples, with the Ni layer parameters shown in figure 3 and described in detail at the beginning of section 3.

Based on the complete description of the sample geometry, and the properties of the superconductor and ferromagnetic cover, the self-field critical current of the tape at 77 K was calculated with the result that its value should increase from 48 A of the original tape to 56 A when the Ni cover is deposited. In reality, a slightly higher increase to 58 A was observed.

Then, the distributions presented in the previous section have been calculated for the same geometry material properties and compared with experiments carried out by standard methods using a lock-in amplifier. We have checked that for frequencies in the range 36–144 Hz the loss per cycle remains unchanged. This is important because our calculation method assumes a purely hysteretic character of magnetization for both the SC and FM material, and neglects possible eddy currents in the silver matrix of the tape. In the case when a frequency-dependent phenomena would appear, the calculation method could not be used in the present form.

### 4. Comparison with experiments

With the help of distributions of current density and magnetic field calculated for different circumstances given in the previous section, one can predict the effect of the Ni layer on the tape performance. We present here as an illustration the results achieved on one of the prepared samples, with the Ni layer parameters shown in figure 3 and described in detail at the beginning of section 3.

Remember this is the loss due to hysteresis of current distribution in the superconductor, which in our case is influenced by the presence of the ferromagnetic cover on the edges. We did not assume a hysteresis magnetization loop for the FM material in our finite-element-calculation procedure, just its nonlinearity as described by the expression (2). The estimation of FM hysteresis loss is based on the distribution of calculated local magnetic field in FM and the use of formula (18) as explained in the next section.
Figure 10. Hysteresis loops of the magnetic flux surrounding the superconducting wire that transports electrical current. The part proportional to the current has been subtracted from the data presented in the left plot to obtain the plot on the right side.

Figure 11. Effect of Ni cover on the transport losses. Data show that the AC loss measured on the original uncovered tape (open circles) was slightly reduced when the edges have been covered by the Ni layer shown in figure 3 (full triangles). Our numerical method predicts loss reduction in the superconducting core (dashed line with open squares) by nearly one order of magnitude, much more than expected from the ~20% improvement of critical current due to the Ni cover. There is significant additional dissipation due to Ni hysteresis (predicted by the dashed line with open diamonds). Theoretical prediction for the total loss of the composite, plotted by the full line, is in reasonable agreement with experiment.

From the distributions calculated for the transport case the AC loss behaviour can be predicted and compared with the experimental result, as shown in figure 11. Empty circles indicate the transport loss measured on the original tape without ferromagnetic cover. The dashed line with square symbols is the result of AC loss calculation using the formula (25). The estimation for the loss in the Ni cover on edges was carried out following the idea proposed by Grilli et al [9]: assuming that from the magnetic field distribution calculated for the maximum current $I_0$ the local magnetic field in the ferromagnetic material will change monotonically until the state at $-I_0$, one can calculate local Ni loss during such a process (that is one-half of the loss in the whole AC cycle) utilizing the formula (18). Integrating the local loss over the whole volume of the Ni material gives the estimation of AC loss due to FM hysteresis shown by the dashed line with diamond symbols. Theoretical prediction for the total transport loss is given by summing the loss in the superconductor with the Ni loss. The calculated result for our SC–FM composite, given by the solid line in figure 11, is in qualitative agreement with the experimental data (triangles), particularly at currents approaching the critical current.

The same procedure was used to compare the theoretical prediction for the loss in our SC–FM tape exposed to an AC field perpendicular to the long axis of the elliptic core. The result is shown in figure 12, confirming reasonable validity of the theoretical prediction. Interestingly, the cover of edges in the form presented here reduces significantly the AC loss due
to superconductor hysteresis. In spite of the fact that the FM cover is an additional source of hysteresis loss and this loss prevails at low currents or weak AC fields, the total loss of the SC–FM composite is lower than that of the original tape. Because the loss due to Ni hysteresis is the main factor at low currents, there is further room for loss reduction when the FM cover with a narrower hysteresis loop could be used.

5. Conclusions

We have demonstrated both experimentally and by numerical calculation that adding an Ni cover on the edges of a commercial Bi-2223/Ag tape reduces its AC loss in both transport and magnetization regimes.

A numerical procedure simulating the evolution of current and magnetic field distribution during the AC cycle has been developed for the purpose of theoretical prediction. The essential feature of this procedure is the use of the formulation proposed by Campbell [10] for the critical state in a superconductor in terms of the vector potential of the magnetic field. We have developed working formulae relating the current density to the properly calibrated vector potential in a manner compatible with finite element calculations in nonlinear media. In this way, the calculated critical state distributions in the superconductor take into account the modification of local magnetic fields by a nonlinear ferromagnetic material.

With the help of the developed simulation procedure, two scopes not resolved yet can be reached: the theoretical prediction of AC loss in existing superconducting wires containing ferromagnetic parts—like YBCO-coated conductors on a ferromagnetic substrate or MgB$_2$ wires with Ni or Fe in the matrix—can be achieved. Also, optimization of the superconductor-ferromagnetic composite wires can be carried out in order to design wires with improved properties.

Acknowledgments

This work was supported by the Slovak Research and Development Agency under contract no. APVV-51-045605 and the European Commission under contracts FU07-CT-2007-0051 and MRTN-CT-2006-035619.

References

[1] Gómory F, Šouc J, Seiler E, Klinčok B, Vojenčiak M, Alamgir A K M, Han Z and Gu Ch 2007 IEEE Trans. Appl. Supercond. 17 3083–6
[2] Gómory F 2006 Appl. Phys. Lett. 89 072506
[3] Gu C, Alamgir A K M, Qu T and Han Z 2007 Supercond. Sci. Technol. 20 133–7
[4] Brandt E H 1996 Phys. Rev. B 54 4246
[5] Pardo E, Chen D-X, Sanchez A and Navau C 2004 Supercond. Sci. Technol. 17 83
[6] Farinon S, Fabbricatore P, Gómory F, Greco M and Seiler E 2005 IEEE Trans. Appl. Supercond. 15 2867–70
[7] Nakahata M and Amemiya N 2008 Supercond. Sci. Technol. 21 015007
[8] Mawatari Y 2008 Phys. Rev. B 77 104505
[9] Grilli F, Ashworth S P and Civale L 2007 J. Appl. Phys. 102 073909
[10] Campbell A M 2007 Supercond. Sci. Technol. 20 292
[11] Bean C P 1964 Rev. Mod. Phys. 36 31
[12] Gómory F, Šouc J, Seiler E, Vojenčiak M and Granados X 2008 J. Phys.: Conf. Ser. 97 012096
[13] Gómory F, Šouc J, Vojenčiak M and Klinčok B 2007 Supercond. Sci. Technol. 20 S271–7
[14] http://www.comsol.com/
[15] Brandt E H and Indenbom M 1993 Phys. Rev. B 48 12893
[16] Clem J R, Benkraouda M, Pe T and McDonald J 1996 Chin. J. Phys. 34 284
[17] Klinčok B, Gómory F and Pardo E 2005 Supercond. Sci. Technol. 18 694–700