Generation of large-scale magnetic fields, non-Gaussianity, and primordial gravitational waves in inflationary cosmology

Kazuharu Bamba$^{1,2,3,*}$

$^1$Leading Graduate School Promotion Center, Ochanomizu University, 2-1-1 Ohtsuka, Bunkyo-ku, Tokyo 112-8610, Japan
$^2$Department of Physics, Graduate School of Humanities and Sciences, Ochanomizu University, Tokyo 112-8610, Japan
$^3$Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan†

Abstract

The generation of large-scale magnetic fields in inflationary cosmology is explored, in particular, in a kind of moduli inflation motivated by racetrack inflation in the context of the Type IIB string theory. In this model, the conformal invariance of the hypercharge electromagnetic fields is broken thanks to the coupling of both the scalar and pseudoscalar fields to the hypercharge electromagnetic fields. The following three cosmological observable quantities are first evaluated: The current magnetic field strength on the Hubble horizon scale, which is much smaller than the upper limit from the back reaction problem, the local non-Gaussianity of the curvature perturbations due to the existence of the massive gauge fields, and the tensor-to-scalar ratio. It is explicitly demonstrated that the resultant values of the local non-Gaussianity and the tensor-to-scalar ratio are consistent with the Planck data.

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* E-mail address: bamba.kazuharu@ocha.ac.jp
† The author’s previous affiliation
I. INTRODUCTION

Galactic magnetic fields on 1–10kpc scale with the strength of $\sim 10^{-6}\text{G}$ and the magnetic fields on 10 kpc–1Mpc scale with those amplitude of $10^{-7}–10^{-6}\text{G}$ in clusters of galaxies have been observed. The origins of cosmic magnetic fields, in particular, such a large-scale magnetic field in clusters of galaxies have not been established yet (for reviews, see, e.g., [1]). There have been proposed various generation mechanisms such as the plasma instability [2, 3], cosmological electroweak and quark-hadron phase transitions [4], cosmic string [5], and primordial density perturbations [6], plus the secondary dynamo amplification mechanism [7]. However, it is difficult to produce the large-scale magnetic fields in these scenarios. It is known that electromagnetic quantum fluctuations generated during inflation are the most natural origin of such large-scale magnetic fields [8], because the coherent scale of magnetic fields can be extended larger than the Hubble horizon at the inflationary stage [9].

The Maxwell theory has its conformal invariance. Moreover, the Friedmann-Lemaître-Robertson-Walker (FLRW) metric describing the homogeneous and isotropic universe, which is supported by observations, is conformally flat\(^1\). Hence, at the inflationary stage, the conformal invariance of the electromagnetic fields has to be broken so that the quantum fluctuations of the electromagnetic fields can be generated [11] and result in the large-scale magnetic fields at the present time [9, 12]. There are several well-known ideas of the breaking mechanism: (i) A non-minimal coupling between the scalar curvature and the electromagnetic fields, which is produced by a one-loop vacuum-polarization effect in quantum electrodynamics in the curved space-time [13]; (ii) A coupling of a scalar field to the electromagnetic fields [14–17]; (iii) The trace anomaly [18].

In this paper, we investigate the generation of large-scale magnetic fields from a sort of moduli inflation inspired by racetrack inflation [19] in the framework of the Type IIB string theory with the so-called Kachru-Kallosh-Linde-Trivedi volume stabilization mechanism [20]. In such a model, the conformal invariance of the hypercharge electromagnetic fields is broken through their coupling to a scalar field and that to an axion-like pseudoscalar field. It should be noted that our present model is still a toy model motivated by the racetrack inflation as well as the so-called axion inflation, where the axion plays a role of the inflaton. The

\(^1\) For the breaking mechanisms of the conformal flatness, see, for instance, [10].
main purpose of this work is that by using the simplified model, we reveal cosmological consequences in the racetrack (or axion) inflation. In Refs. [28, 29], it has been indicated that a coupling of the pseudoscalar inflaton field to the electromagnetic fields can generate the non-Gaussianity of the power spectrum of the curvature perturbations coming from the quantum fluctuations of the inflaton field. Thus, we analyze the non-Gaussianity of the curvature perturbations in the present scenario by following the procedure in Refs. [26, 32]. Moreover, we study the so-called tensor-to-scalar ratio defined by the ratio of the scalar modes of the curvature perturbations to their tensor modes (i.e., the primordial gravitational waves) [21, 22]. It is illustrated that the local non-Gaussianity and tensor-to-scalar ratio in the cosmic microwave background (CMB) radiation with those values smaller than the limits from the Planck satellite [34] can be produced, when the magnetic fields on the Hubble horizon scale whose current strength can avoid the back reaction problem are generated.

The most important meaning of this work is that the explicit values of three cosmological observable quantities, i.e., the large-scale magnetic fields, the local non-Gaussianity, and the tensor-to-scalar ratio are first derived. We should also emphasize the novelty of our present model in comparison with the other recent works on the non-Gaussianity of the curvature perturbations and the tensor-to-scalar ratio in a kind of axion inflation [25, 26, 28, 29, 32]. In our model, a scalar field as well as the axion-like pseudoscalar field couple to the hypercharge electromagnetic field, whereas in the other past models, only the pseudoscalar field couples to the hypercharge electromagnetic field. The existence of such a scalar field coupling to the (hypercharge) electromagnetic field is suggested by the Kaluza-Klein (KK) compactification mechanism [36] for the fundamental higher-dimensional space-time theories including string theories. In fact, these both couplings appears in the framework of the racetrack inflation. Thus, it can be considered that the setting of our model is closer to the realistic one than that in the past related works, although it is still toy model. In addition, there is one more significant advantage that thanks to the coupling of the scalar field to the hypercharge electromagnetic field, in principle, the large-scale magnetic fields with the

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2 Various cosmological consequences of the so-called axion inflation [21, 22] including the generation of large-scale magnetic fields [17, 23, 24] or primordial black holes [25] and observational constraints [26] have also been explored (for a recent review on inflation driven by axion, see [27]).

3 For the non-Gaussianity from magnetic fields, see [33].

4 The recent BICEP2 result [35] on the tensor-to-scalar ratio is also mentioned in Sec. IV C.
current strength enough to explain the observations without any secondary amplification mechanism of galactic dynamo. This point cannot be realized in the past models.

The observational test of this model is the severest, so that it would be very difficult for the model to be viable, because we use the three independent observations of the large-scale magnetic fields, local non-Gaussianity, and tensor-to-scalar ratio. Furthermore, this model is the most general within the fundamental theories which we are considering. Thus, we develop the generic discussions in order not only to extend the theoretical possibility but also to strictly constrain the freedom of the theory. We use the units $k_B = c = \hbar = 1$ and describe the Newton’s constant by $G = 1/M_P^2$, where $M_P = 2.43 \times 10^{18}$ GeV is the reduced Planck mass. In terms of electromagnetism, we adopt Heaviside-Lorentz units.

The paper is organized as follows. In Sec. II, we explain our model action and derive the basic equations. In Sec. III, we investigate the evolution of each field and estimate the current strength of the large-scale magnetic fields. In Sec. IV, we explore the power spectrum of the curvature perturbations, the non-Gaussianity, and the tensor-to-scalar ratio. In Sec. V, conclusions are presented. In Appendix A, we also examine the large-scale magnetic fields, non-Gaussianity, and tensor-to-scalar ratio for the axion (monodromy) inflation, and comparison these results with the ones for a kind of moduli inflation motivated by the racetrack inflation in the previous sections. In Appendix B, the issues of the backreaction and the strong coupling are stated. Moreover, the observational constraints on the field strength of magnetic fields are summarized in Appendix C. Furthermore, cosmological implications related this work are stated in Appendix D.

II. MODEL

We take the Lagrangian given by

$$\mathcal{L} = \frac{M_P^2}{2} R - \frac{1}{4} X F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} g_{\text{ps}} \frac{Y}{M} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) - \frac{1}{2} g^{\mu\nu} \partial_\mu Y \partial_\nu Y - V(Y),$$

$$X \equiv \exp \left( -\frac{\lambda}{M_P} \frac{\Phi}{M_P} \right),$$

Such a kind of the action in Eq. (2.1) has also been studied for a baryogenesis scenario due to the anomaly $^{37,38}$. 

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\[
V(Y) \approx \bar{V} - \frac{1}{2}m^2Y^2,
\]

where \( R \) is the Ricci scalar, \( g_{ps} \) is a dimensionless coupling constant, \( \Phi \) is the canonically normalized field of the scalar field \( X \) with the normalization constant \( \lambda \), \( Y \) is a pseudoscalar field with its canonical kinetic term, and \( M \) corresponds to the decay constant of \( Y \) with a mass dimension. Furthermore, \( F_{\mu \nu} = \nabla_\mu F_\nu - \nabla_\nu F_\mu \) is the field strength of the \( U(1)_Y \) hypercharge gauge field \( F_\mu \), where \( \nabla_\mu \) is the covariant derivative, and \( \tilde{F}^{\mu \nu} \) are the dual field strength of \( F_\mu \). While we do not specify the exact form of scalar potentials \( U(X = X(\Phi)) \), \( V(Y) \) would be expected to have a potential, given by Eq. (2.3) with a normalization factor \( \bar{V} \) and the mass \( m \) of the pseudoscalar \( Y \). The pseudoscalar field \( Y \) couples to the dual of the field strength and, hence, acts as an axion. Throughout our analysis, we assume that inflationary expansion is driven by the potential energy of \( Y \) as in the so-called natural inflation or axion inflation [24, 39, 40].

We take the flat FLRW space-time

\[
ds^2 = -dt^2 + a^2(t)dx^2,
\]

with \( a \) the scale factor. We find that the field equations of \( X \), namely, those of \( \Phi \), and \( Y \) read

\[
\dot{\Phi} + 3H \Phi + \frac{dU(\Phi)}{d\Phi} = 0, \quad \dot{Y} + 3H \dot{Y} + \frac{dV(Y)}{dY} = 0,
\]

where \( H \equiv \dot{a}/a \) is the Hubble parameter and the dot denotes the derivative with respect to the cosmic time \( t \). Using the Coulomb gauge \( F_0(t, x) = 0 \) and \( \partial_j F^j(t, x) = 0 \), we find that the field equation of \( F_\mu \) is described as

\[
\tilde{F}_i(t, x) + \left( H + \frac{\dot{X}}{X} \right) \dot{F}_i(t, x) - \frac{1}{a^2} \partial_j \partial_j F_i(t, x) - \frac{g_{ps}}{M} \frac{1}{aX} \dot{\Phi} \epsilon^{ijk} \partial_j F_k(t, x) = 0,
\]

where the second term within the round bracket ( ) and the fourth term originates from the braking of the conformal invariance of the hypercharge electromagnetic fields.

III. CURRENT STRENGTH OF LARGE-SCALE MAGNETIC FIELDS

In this section, we explore the evolution of the \( U(1)_Y \) gauge field as well as that of the scalar \( X \) and pseudoscalar \( Y \) fields. After that, we estimate the present amplitude of the

\[\text{Here, we have used the fact that the contribution of the hypercharge electromagnetic field is negligible because it exists as a quantum fluctuation during inflation and the amplitude is so small that the squared quantity can be neglected.}\]
large-scale magnetic fields.

A. Scalar and pseudoscalar fields

We assume that inflation is basically driven by the potential of $Y$. In this background, the Friedmann equation is given by $3M_P H^2 = \left[ (1/2) \dot{Y}^2 + V(Y) \right]$. If the so-called slow-roll approximation $\dot{Y}^2/2 \ll V(Y)$ is satisfied, we have $H \approx H_{\inf} = \text{constant}$ with $H_{\inf}$ the Hubble parameter during inflation, so that the exponential inflation can be realized. In this case, the scale factor $a(t)$ can be expressed as $a(t) = a_k \exp \left[ H_{\inf} (t - t_k) \right]$ with $a_k = a(t_k)$, where $t_k$ is the time when a comoving wavelength $2\pi/k$ of the U(1)$_Y$ gauge field first crosses the horizon at the inflationary stage and thus satisfies $k/(a_k H_{\inf}) = 1$. The analytic solution of Eq. (2.5) is given by \[ Y = Y_k \exp \left\{ \frac{3}{2} \left[ -1 \pm \sqrt{1 + \left( \frac{2m}{3H_{\inf}} \right)^2} \right] H_{\inf} (t - t_k) \right\} , \] (3.1) with $Y_k = Y(t_k)$, and therefore we use this solution. In the following, without generality, we take the “+” sign on the right-hand side of this solution. On the other hand, in terms of $X$, in this section we study the case that the concrete dynamics of $X$ during inflation does not influence on the results and only the difference between the initial and final values during inflation is important.

B. U(1)$_Y$ gauge field

1. Quantization

First, we quantize the U(1)$_Y$ gauge field $F_\mu(t, \mathbf{x})$. It follows from the hypercharge electromagnetic part of the action constructed by the Lagrangian (2.1), we find that the canonical momenta conjugate to $F_\mu(t, \mathbf{x})$ read $\pi_0 = 0$ and $\pi_i = X a \dot{F}_i(t, \mathbf{x})$. The canonical commutation relation between $F_i(t, \mathbf{x})$ and $\pi_j(t, \mathbf{x})$ is imposed as

\[ [F_i(t, \mathbf{x}), \pi_j(t, \mathbf{y})] = i \int \frac{d^3k}{(2\pi)^{3/2}} e^{i \mathbf{k}(\mathbf{x} - \mathbf{y})} \left[ \delta_{ij} - \left( k_i k_j / k^2 \right) \right] . \] (3.2)
Here, $k$ is the comoving wave number and its amplitude is expressed as $k = |k|$. This relation leads to the description of $F_i(t, x)$ as

$$F_i(t, x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ \hat{b}(k)F_i(t, k)e^{ik \cdot x} + \hat{b}^\dagger(k)F_i^*(t, k)e^{-ik \cdot x} \right],$$

(3.3)

where $\hat{b}(k)$ and $\hat{b}^\dagger(k)$ are the annihilation and creation operators, respectively, and they obey the relations

$$\left[ \hat{b}(k), \hat{b}^\dagger(k') \right] = \delta^3(k - k'), \quad \left[ \hat{b}(k), \hat{b}(k') \right] = \left[ \hat{b}^\dagger(k), \hat{b}^\dagger(k') \right] = 0.$$  

(3.4)

We also have the normalization condition as

$$F_i(k, t) \dot{F}_j(k, t) - \dot{F}_i(k, t)F_j^*(k, t) = i X_{a} \left[ \delta_{ij} - \left( \frac{k_i k_j}{k^2} \right) \right].$$

(3.5)

2. Set up

We set the $x^3$ axis to lie along the spatial momentum direction $k$ and express the transverse directions as $x^1$ and $x^2$. By using Eq. (2.6) and defining the circular polarizations $F_{\pm}(k, t) \equiv F_1(k, t) \pm iF_2(k, t)$ with the Fourier modes $F_1(k, t)$ and $F_2(k, t)$ of the $U(1)_Y$ gauge field, we acquire

$$\ddot{F}_\pm(k, t) + \left( H_{\text{inf}} + \frac{X}{X} \right) \dot{F}_\pm(k, t) + \left[ 1 \pm \frac{g_{ps} Y}{M X} \left( \frac{k}{a} \right)^{-1} \right] \left( \frac{k}{a} \right)^2 F_\pm(k, t) = 0,$$

(3.6)

During inflation, we solve this equation numerically by following the procedure in Ref. [37], because it is very hard to acquire the analytic solution of Eq. (3.6). Since for the subhorizon scale $k / (aH) \gg 1$, the $F_-(k, t)$ corresponds to the decaying mode, we only examine the evolution of $F_+(k, t)$.

An approximated amplitude $F_+(k, t = t_k)$ at the horizon crossing, where $k / (aH_{\text{inf}}) = 1$, is represented as $[24, 28, 29, 30] F_+(k, t_k) \simeq \left( 1/\sqrt{2k} \right) \left( 1/\sqrt{X(t_k)} \right) (2\xi_k)^{-1/4} \exp(\pi\xi_k - 2\sqrt{2\xi_k})$ with $\xi_k = \xi(t = t_k)$, where

$$\xi \equiv \frac{1}{2} \frac{g_{ps}}{M X} \frac{1}{H_{\text{inf}}},$$

(3.7)

Such amplification originates from the tachyonic instability. Thus, when we numerically calculate Eq. (3.6), we take into account the above amplification factor in the initial conditions. We define the following amplification factor as

$$C_+(k, t) \equiv \frac{F_+(k, t)}{F_+(k, t_k)}.$$  

(3.8)
We estimate the initial amplitude of $F_+(k,t)$, i.e., $F_+(k,t_k)$, by matching with the solution for sub-horizon scales $k/(aH) \gg 1$ at the horizon exit \[37\]. Here, we suppose that in the short-wavelength limit of $k \to \infty$, the amplitude of $F_+(k,t)$ is described by $\left| F_+^{(in)}(k,t) \right| = \left( 1/\sqrt{2k} \right) \left( 1/\sqrt{X(t)} \right)$, where the coefficients of modes have been chosen so that the vacuum can be reduced to the one in the Minkowski space-time in the short-wavelength limit (the so-called Bunch-Davies vacuum \[41\]).

3. Numerical analysis

We estimate the resultant magnetic field strength, provided that during inflation $X$ is approximately regarded as a constant. This means that a dynamical quantity to the hypercharge electromagnetic fields is only the pseudoscalar field $Y$, the case of which has been explored in Refs. \[17, 21, 27–29\]. Indeed, the field strength of the large-scale magnetic fields can be amplified in our model where there exists the coupling between the dilaton field $X$ and the hypercharge electromagnetic fields. In other words, the important quantity to characterize the amplification of the magnetic field is the ratio of the final value of $X$ to the initial one at inflationary stage. As far as the final value of $X$ is concerned, the ordinary electromagnetic field with $X = 1$ has to be recovered till the epoch of the Big Bang Nucleosynthesis (BBN). Hence, in our analysis, we assume that $X$ stays almost constant during inflation but, after inflation, quickly reaches $X(t = t_R) = 1$ at the reheating time $t_R$ due to an appropriate form of $V(\Phi)$.

In Fig. 1, we depict the evolution of $C_+(k,t)$ during inflation with the solid line for $X(t_k) \equiv \exp(\chi_k)$ with $\chi_k = -0.940$, $H_{\text{inf}} = 1.0 \times 10^{10}\text{GeV}$, $m = 2.44 \times 10^9\text{GeV}$, $M = 1.0 \times 10^{-1}\text{MP} = 2.43 \times 10^{17}\text{GeV}$, $\bar{V} = 5.07 \times 10^{-17}\text{MP}^4$, $\xi_k = 2.5590616$, and $g_{\text{ps}} = 1.0$. This is the case (b) in Table I as is shown later. We have numerically solved Eq. (3.6) for $k = a_k H_{\text{inf}}$ mode under the exponential inflation from the initial time at $t = t_k = H_{\text{inf}}^{-1}$ and $C_+(k,t_k) = 1$. We define the values of these parameters by the Cosmic Background Explorer (COBE) \[42\] normalization and Planck data \[43\] on the CMB radiation. In addition, for comparison, we have also plotted the numerical results for the case that $\dot{Y} = 0$ in Eq. (3.6) with the dotted line. Here, the behavior for $\dot{Y} \neq 0$ is quite alike to that for $\dot{Y} = 0$ because the pseudoscalar field $Y$ rolls down its potential very slowly.

We find from Fig. 1 that $C_+(k,t)$ asymptotically approaches a constant within about
FIG. 1: Evolution of $C_+(k, t)$ for $X(t_k) \equiv \exp(\chi_k)$ with $\chi_k = -0.940$, $H_{\text{inf}} = 1.0 \times 10^{10}\text{GeV}$, $m = 2.44 \times 10^9\text{GeV}$, $Y_k = 7.70 \times 10^{-2}M_P = 1.87 \times 10^{17}\text{GeV}$, $M = 1.0 \times 10^{-1}M_P = 2.43 \times 10^{17}\text{GeV}$, $\bar{V} = 5.07 \times 10^{-17}M_P^4$, $\xi_k = 2.5590616$, and $g_{\psi} = 1.0$ (the case (b) in Table I). The solid line shows the case including the dynamics of $Y$, whereas the dotted line depicts that without it, namely, $\dot{Y} = 0$ in Eq. (3.6).

10 Hubble time after the horizon crossing during inflation. This is the significant feature of evolution of $C_+(k, t)$, that is, the amplitude becomes a finite value and does not decay. Therefore, this also contributes the resultant strength of the large-scale magnetic fields. Such a behavior that $C_+(k, t)$ becomes a constant does not depend on the model parameters. This result is also consistent with that in Ref. [28]. The way of determining the values of $m$ and $Y_k$ are explained in the last paragraph of Sec. IV A.

C. Current magnetic field strength

Next, we evaluate the magnetic field strength at the present time. The proper hypermagnetic and hyperelectric fields are expressed with the comoving hypermagnetic fields $B_{Y_i}(t, \mathbf{x})$
and hyperelectric ones $E_{Y_i}(t, \mathbf{x})$, respectively, as \[14\]

$$
B_{Y_i}^{\text{proper}}(t, \mathbf{x}) = \frac{1}{a^2} B_{Y_i}(t, \mathbf{x}) = \frac{1}{a^2} \epsilon_{ijk} \partial_j F_k(t, \mathbf{x}),
$$

(3.9)

$$
E_{Y_i}^{\text{proper}}(t, \mathbf{x}) = a^{-1} E_{Y_i}(t, \mathbf{x}) = -a^{-1} \dot{F}_i(t, \mathbf{x}),
$$

(3.10)

with $\epsilon_{ijk}$ the totally antisymmetric tensor ($\epsilon_{123} = 1$). The energy density of the proper hypermagnetic field in the physical space is obtained by multiplying that in the Fourier space $\rho_{B_Y}(k, t)$ by the phase-space density $4\pi k^3/(2\pi)^3$ as

$$
\rho_{B_Y}(L, t) = \frac{k^3}{4\pi^2} \left[ |B_{Y_+}^{\text{proper}}(k, t)|^2 + |B_{Y_-}^{\text{proper}}(k, t)|^2 \right] X,
$$

(3.11)

where $|B_{Y_{\pm}}^{\text{proper}}(k, t)|^2 = (1/a^2) (k/a)^2 |F_{\pm}(k, t)|^2$, which follows from Eq. (3.9), and $L = 2\pi/k$ is a comoving scale.

The instantaneous reheating stage at $t = t_R$ following inflation occurred much earlier than the electroweak phase transition (EWPT) at $T_{EW} \sim 100\text{GeV}$. It is reasonable to consider that the conductivity of the universe $\sigma_c$ should be very small at the inflationary stage, because few particle present. At the reheating period after inflation, charged particles are created, and therefore $\sigma_c$ increases and would become large enough as ($t \geq t_R$). Hence, when $\sigma_c \gg H$, the hyperelectric fields accelerate the charged particles and dissipate. On the other hand, we find $B_Y \propto a^{-2}$ in the radiation- and matter-dominated stages ($t \geq t_R$) \[14, 16\]. Thus, at a later time after the EWPT when the dilaton reached the true minimum of $X = 1$, the energy density of the hypermagnetic fields $\rho_{B_Y}(L, t)$ reduces to that of the magnetic fields $\rho_B(L, t)$. The expression of $\rho_B(L, t)$ reads \[37\]

$$
\rho_B(L, t) \simeq \frac{1}{8\pi^2} X(t_k) \frac{1}{\sqrt{2\xi_k}} \exp \left[ 2 \left( \pi \xi k - 2 \sqrt{2 \xi k} \right) \right] \left( \frac{k}{a} \right)^4 |C_+(k, t_R)|^2,
$$

(3.12)

where we have imposed $X(t_R) = 1$ and neglected the order unity coefficient factor difference between the magnetic field of U(1)$_Y$ and that of U(1)$_\text{em}$.

We estimate the current strength of the large-scale magnetic fields. We identify a $k$-mode for the present horizon scale $H_0^{-1}$ by setting as $k = 2\pi / (2997.9 h^{-1}) \text{Mpc}^{-1}$ with $h = 0.673$ \[44\], and

$$
H_{\text{inf}} (t_R - t_k) = 45 + \ln \left( \frac{L_k}{\text{Mpc}} \right) + \ln \left\{ \frac{[30/ (\pi^2 g_R)]^{1/12} (\rho^{(Y)}(t_R))^{1/4}}{10^{38/3} [\text{GeV}]} \right\},
$$

(3.13)

under the assumption of instantaneous reheating after inflation \[45\]. In Table \[\square\] we list the parameter sets to reproduce the current strength of magnetic fields of $B(H_0^{-1}, t_0) = \mathcal{O}(10^{-64})$. 


the tensor-to-scalar ratio $[43]$ obtained from the Planck satellite, which are explained in the results are compatible with the observational constraints on the non-Gaussianity $[34]$ and the Hubble horizon scale. We also mention that for all the cases (a)–(f) in Table I, the magnetic field strength is the parameter $\chi^1_{m}$ fluctuations of the $U(1)_Y$ magnetic fields can be amplified by the factor of the ratio of the final value $T_{R}[\text{GeV}] = (1.02 \times 10^{14}, 3.22 \times 10^{13}, 3.22 \times 10^{12}, 3.22 \times 10^{11}, 3.22 \times 10^{10}, 3.22 \times 10^9)$ and $\tilde{V}/M_P^4 = (5.07 \times 10^{-15}, 5.07 \times 10^{-17}, 5.07 \times 10^{-21}, 5.07 \times 10^{-25}, 5.07 \times 10^{-29}, 5.07 \times 10^{-33})$.

| Case | $B(H_0^{-1}, t_0)$ [G] | $H_{\text{inf}}$ [GeV] | $m$ [GeV] | $Y_{k}/M_P$ | $C_+(k, t_R)$ |
|------|-----------------|-----------------|--------|-------------|----------------|
| (a)  | $7.15 \times 10^{-64}$ | $1.0 \times 10^{11}$ | $2.44 \times 10^{10}$ | $7.70 \times 10^{-2}$ | 0.528 |
| (b)  | $7.15 \times 10^{-64}$ | $1.0 \times 10^{10}$ | $2.44 \times 10^{9}$ | $7.70 \times 10^{-2}$ | 0.528 |
| (c)  | $2.33 \times 10^{-64}$ | $1.0 \times 10^{8}$ | $1.0 \times 10^{7}$ | $1.62 \times 10^{1}$ | 0.172 |
| (d)  | $2.33 \times 10^{-64}$ | $1.0 \times 10^{6}$ | $1.0 \times 10^{5}$ | $1.62 \times 10^{1}$ | 0.172 |
| (e)  | $2.85 \times 10^{-64}$ | $1.0 \times 10^{4}$ | $8.0 \times 10^{2}$ | $2.23 \times 10^{1}$ | 0.211 |
| (f)  | $2.85 \times 10^{-64}$ | $1.0 \times 10^{2}$ | 8.0 | $2.23 \times 10^{1}$ | 0.211 |

G at the current Hubble horizon scale, for $X(t_k) = \exp(\chi_k)$ with $\chi_k = -0.940$ and $g_{ps} = 1.0$. It can be seen that for the wide range of the values of $H_{\text{inf}}$ and $m$, that of $C_+(k, t_R)$ is $O(0.1)$. We remark that the most important parameter to determine the size of the resultant magnetic field strength is the parameter $\chi_k$. The essence is that the amplitude of quantum fluctuations of the $U(1)_Y$ fields generated within the Hubble horizon can be a factor of $1/\sqrt{X(t_k)}$ larger than that in the ordinary Maxwell theory. Thus, the energy density of the (hypercharge) magnetic fields can be amplified by the factor of the ratio of the final value of $X(t_R) = 1$ at the inflationary stage to the initial value of $X(t_k)$.

It is one of the important properties in this model that the smaller the value of $X(t_k)$ is, the larger that of the resultant strength of the current magnetic fields $B(H_0^{-1}, t_0)$ on the Hubble horizon scale. We also mention that for all the cases (a)–(f) in Table I, the results are compatible with the observational constraints on the non-Gaussianity $[34]$ and the tensor-to-scalar ratio $[43]$ obtained from the Planck satellite, which are explained in the next section.
IV. POWER SPECTRUM, NON-GAUSSIANITY, AND TENSOR-TO-SCALAR RATIO OF THE CURVATURE PERTURBATIONS

In this section, we study the power spectrum of the curvature perturbations and estimate the non-Gaussianity and tensor-to-scalar ratio, provided that the curvature perturbations generated during inflation originate from only the quantum fluctuations of \( Y \), the inflaton field, and the contribution of the dilaton field \( X \) is negligible because we consider the case in which the energy density of the potential of \( Y \) is much larger than that of \( X \) at the inflationary stage.

A. Power spectrum of the curvature perturbations

First, we explore the power spectrum of the curvature perturbations originating from the quantum fluctuations of \( Y \) corresponding to the inflaton field. It is known that the coupling term between \( Y \) and \( F_{\mu\nu}\tilde{F}^{\mu\nu} \) can lead to the quantum fluctuations \( \delta Y(t, x) \) in terms of \( Y \). These fluctuations satisfy \[ \frac{\partial^2 \delta Y(t, x)}{\partial t^2} + 3H \frac{\partial \delta Y(t, x)}{\partial t} - \nabla^2 \delta Y(t, x) = \frac{g_{ps}}{M} F_{\mu\nu}\tilde{F}^{\mu\nu}. \] (4.1)

The generic solution consists of two parts. One is the solution of the homogeneous equation, namely, the ordinary vacuum fluctuations at the inflationary stage. The other is the particular solution coming from the source term. It is considered that the origin of the latter is the inverse decay of two quanta of the gauge field to the quantum fluctuation of \( Y \). These two terms are independent each other. The power spectrum of scalar modes of the curvature perturbations on hypersurfaces of the uniform density \( \mathcal{R} = -\left(\frac{H}{\dot{Y}}\right)\delta Y \) is defined by the two-point correlation function in the Fourier space \[ <\mathcal{R}_k \mathcal{R}_{k'}^\dagger> \equiv \frac{1}{(2\pi)^3} P_R(k) \delta^3(\mathbf{k} + \mathbf{k}'). \] Thus, the resultant power spectrum becomes \[ P_R(k) \simeq \Delta^2_R \left(\frac{k}{k_s}\right)^{n_s-1} \left(1 + \Delta^2_R f_S(\xi) \exp(4\pi\xi)\right), \] (4.2)

\[ \Delta^2_R = \left(\frac{H_{\text{inf}}}{2\pi}\right)^2 \frac{H_{\text{inf}}^2}{|Y|^2}, \] (4.3)

\[ f_S(\xi) \equiv \begin{cases} 
7.5 \times 10^{-5} \xi^{-6} & \text{for } \xi \gg 1, \\
3.0 \times 10^{-5} \xi^{-5.4} & \text{for } 2 \leq \xi \leq 3.
\end{cases} \] (4.4)
Here, \( k_* = 0.002 \text{ Mpc}^{-1} \). In addition, we have

\[
\dot{Y}(t_R) = \frac{3}{2} \left[ -1 + \sqrt{1 + \left(\frac{2m}{3H_{\text{inf}}}\right)^2} \right] H_{\text{inf}} Y_k \exp \left\{ \frac{3}{2} \left[ -1 + \sqrt{1 + \left(\frac{2m}{3H_{\text{inf}}}\right)^2} \right] (N - 1) \right\},
\]

(4.5)

with \( N \) the number of e-folds, where in deriving Eq. (4.5), we have used Eq. (3.1). Moreover, \( n_s \) denotes the spectral index of \( n_s \) of scalar modes of the curvature perturbations is given by \([26, 47]\)

\[
n_s \simeq 1 - 6\epsilon + 2\eta,
\]

(4.6)

\[
\epsilon \equiv \frac{M_P^2}{2} \left( \frac{V'(Y)}{V(Y)} \right)^2,
\]

(4.7)

\[
\eta \equiv \frac{M_P^2}{2} \frac{V''(Y)}{V(Y)},
\]

(4.8)

where the prime means the derivative with respect to \( Y \) of \( \partial / \partial Y \), and \( \epsilon \) and \( \eta \) are the so-called slow-roll parameters in terms of potential. According to the Planck result \([43]\), the value of the spectral index is estimated with the Planck and Wilkinson Microwave Anisotropy Probe (WMAP) data as \( n_s = 0.9603 \pm 0.0073 \) (95% CL). With the COBE \([42]\) normalization result of the power spectrum of the curvature perturbation \( \Delta^2_R(k) = 2.4 \times 10^{-9} \) at \( k = k_* = 0.002 \text{ Mpc}^{-1} \), which is consistent with the Nine-Year WMAP result \([47]\), and the Planck result of \( n_s = 0.9603 \), for \( H_{\text{inf}} = 1.0 \times 10^{13}\text{GeV} \) and \( \bar{V} = 5.07 \times 10^{-11}M_P^4 \) in Eq. (2.3), from Eq. (4.2) for \( k = 2\pi/(2997.9h^{-1}) \text{ Mpc}^{-1} \) with \( h = 0.673 \) and Eq. (4.3), we have \( m = 2.44 \times 10^{12}\text{GeV} \) and \( Y_k = 7.70 \times 10^{-2}M_P = 1.87 \times 10^{17}\text{GeV} \). We also take \( M = 1.0 \times 10^{-1}M_P = 2.43 \times 10^{17}\text{GeV} \). We remark that also for other various values of \( H_{\text{inf}} \) such as that in Table I by using Eqs. (4.9) and (4.10), those of \( m \) and \( Y_k \) can be derived, as is shown below.

In Fig. 2 for \( X(t_k) \equiv \exp(\chi_k) \) with \( \chi_k = -0.940 \), \( H_{\text{inf}} = 1.0 \times 10^{13}\text{GeV} \), \( M = 1.0 \times 10^{-1}M_P = 2.43 \times 10^{17}\text{GeV} \), \( m = 2.44 \times 10^{12}\text{GeV} \), \( \bar{V} = 5.07 \times 10^{-11}M_P^4 \), \( Y_k = 7.70 \times 10^{-2}M_P = 1.87 \times 10^{17}\text{GeV} \), and \( g_{ps} = 1.0 \), we display the evolution of \( C_+(k, t) \) during inflation with the solid line. This is the case (A) in Tables II and IV presented later. The numerical procedure is the same as the one executed in Fig. II. The qualitative features of the evolution of \( C_+(k, t) \) is equivalent to those shown in Fig. II, namely, \( C_+(k, t) \) becomes close to a constant around the 10 Hubble time after the first horizon crossing during inflation. Even for different values of \( H_{\text{inf}} \), the resultant evolution of \( C_+(k, t) \) is the same as in the case described above.
FIG. 2: Evolution of $C_+(k, t)$. The legend is the same as in Fig. except $H_{\text{inf}} = 1.0 \times 10^{13}\text{GeV}$ and $m = 2.44 \times 10^{12}\text{GeV}$. (the case (A) in Tables [III] and [IV]).

Namely, the value of $C_+(k, t)$ asymptotically approaches an $O(0.1)$ constant value.

We here mention that from the values of the COBE normalization and Planck data, we find

$$f_S(\xi) \exp(4\pi\xi) = \frac{25}{144} \times 10^8, \quad (4.9)$$

$$Y_k = \pm \frac{M_P}{\sqrt{2\beta}} \left[ 4 \frac{\bar{V}}{M_P^2 m^2 \beta^{-2}} + 1 \pm \sqrt{12 \frac{\bar{V}}{M_P^2 m^2 \beta^{-2}} + 1} \right]^{1/2}, \quad (4.10)$$

with

$$\beta \equiv \sqrt{-\frac{(n_s - 1)}{8}}. \quad (4.11)$$

Since $Y$ slow rolls during inflatin, $\xi$ can be considered to be a constant at the infaltinary stage. Therefore, we adopt $\xi \approx \xi_k = \frac{\left( g_{\text{ps}} \dot{Y}(t_k) \right)}{(2M(t_k)H_{\text{inf}})} = \left[ 3Y_k / (4M(t_k)) \right] \left\{ -1 + \sqrt{1 + \left[ 2m / (3H_{\text{inf}}) \right]^2} \right\} \approx [Y_k / (6M(t_k))] \left( m^2 / H_{\text{inf}}^2 \right)$. The last approximate equality can be met for $m/H_{\text{inf}} \ll 1$. Hence, if the values of $n_s$, $\bar{V}$, and $H_{\text{inf}}$ are given, we can determine those of $m$ and $Y_k$. Moreover, in some sense it can be considered that $\bar{V}$ and $M$ are free model parameters. We use the value of $\bar{V}$ derived from the relation

14
\[ \bar{V} = 3H_{\text{inf}}^2M_P^2, \]
which corresponds to the Friedmann equation with \( \dot{Y} = 0 \) at \( Y = 0 \). In this case, we see that in Eq. (4.10), \( \bar{V} / (M_P^2m^2\beta^{-2}) = 3H_{\text{inf}}^2 / (m^2\beta^{-2}) \). We also find the values of \( m \) and \( Y_k \) with Eqs. (4.9) and (4.10). In addition, since the values of \( m \) and \( Y_k \) are real numbers, the values within the square root in Eqs. (4.9) and (4.10) have to be larger than or equal to zero. Thus, we obtain the constraint on \( \bar{V} \) as \( \bar{V} > 2\gamma H_{\text{inf}}^4 \). In the following, we take the “+” sign in front of the right-hand side of \( Y_k \) in Eq. (4.10). Accordingly, for \( m/H_{\text{inf}} \ll 1 \), such cases are reasonable during inflation, we acquire

\[
\begin{align*}
m &\approx \sqrt{6}\xi_k \frac{M}{M_P}X(t_k)H_{\text{inf}}, \\
Y_k &\approx \sqrt{6}M_P \frac{H_{\text{inf}}}{m} = \frac{M_P^2}{\xi_k X(t_k)M},
\end{align*}
\]

where in deriving the last equality in Eq. (4.13), we have used Eq. (4.12). As a result, by obtaining the value of \( \xi_k \) from Eq. (4.9) and substituting it into Eqs. (4.12) and (4.13), we find the approximate values of \( m \) and \( Y_k \). Indeed, from the lower relation for \( 2 \leq \xi \leq 3 \) in Eq. (4.4), we numerically find that a solution of Eq. (4.9) is \( \xi_k = 2.5590616 \). In the following, we evaluate the value of \( m \) with Eq. (4.12) and that of \( Y_k \) with Eq. (4.10).

**B. Non-Gaussianity**

We suppose that the U(1)\(_Y\) gauge field couples to another scalar field, e.g., the Higgs-like field \( \varphi \). In this case, the covariant derivative operating \( \varphi \) is given by \( D_\mu \equiv \partial_\mu + ig'F_\mu \), where \( g' \) is the gauge coupling, and thus the kinetic term of \( \varphi \) becomes \( |D\varphi|^2 \). Provided that the generated gauge field obtains its mass through the Higgs mechanism in terms of \( \varphi \). The quantum fluctuations in terms of mass of the gauge field are produced by those of \( \varphi \), and eventually yield those in the amount of quanta of the generated gauge field. As a result, the generation of the gauge field leads to the perturbations of number of e-folds of inflation \( \delta N \). This produces the local type non-Gaussianity in the anisotropy of the CMB radiation. Such a non-Gaussianity can be calculated by using the \( \delta N \) formalism \[48, 50\], with which the curvature perturbations originating from the quantum fluctuations of \( \varphi \) are derived. When we consider the inflationary model in Ref. \[32\], with using the COBE \[42\], for the model in Ref. \[28, 29\], the equilateral-type non-Gaussianity appears. Since the constraints of the Planck data \[34\] on the local-type non-Gaussianity are stronger than those on the equilateral-type, we here examine the local-type.
Here, \( \Delta N \) is the normalization result of power spectrum of the curvature perturbation \( \Delta^2(k) = 2.4 \times 10^{-9} \) at \( k = k_0 = 0.002 \text{ Mpc}^{-1} \), the local type of a non-Gaussianity \( f_{NL} \) is expressed as

\[
 f_{NL} \approx 1.0 \times 10^{14} \Delta N^3 \frac{\bar{V}^4}{\bar{V}_0^6} \frac{m^2}{H_{inf}^2} .
\]

Here, \( \Delta N_{\text{max}} \) is the maximum value of an extra numbers of e-folds, and \( \xi \) is defined by Eq. (3.7) with \( Y \) in Eq. (4.15).

In Table II, we display numerical results of the local non-Gaussianity \( f_{NL} \) of the curvature perturbations by taking \( \Delta N_{\text{max}} = 1.0 \), \( M = 1.0 \times 10^{-1}M_P = 2.43 \times 10^{17} \text{ GeV} \), \( \bar{V} = 5.07 \times 10^{-11}M_P^4 \) (5.07 \times 10^{-13}M_P^2) for the case (A) (the case (B)), \( g_{fs} = 1.0 \), and \( k = 2\pi/(2997.9h^{-1}) \) \text{ Mpc}^{-1} with \( h = 0.673 \). We note that although there exist cases that the value of \( C_+(k, t_R) \) is negative, we only use the absolute values to estimate the resultant strength of magnetic fields as in Eq. (3.12). According to the Planck satellite [34], the constraint on \( f_{NL} \) is given by \( f_{NL} = 2.7 \pm 5.8 \) (68\% CL). This has been improved very much in comparison with the Seven-Year WMAP analysis \( -10 < f_{NL} < 74 \) (95\% CL) [51]. From Table II, we find that for the case (A), the values of \( f_{NL} \) can be compatible with the data.

| \( f_{NL} \) | \( g^2 \) | \( H_{inf} \) [GeV] | \( m \) [GeV] | \( Y/M_P \) | \( B(H_0^{-1}, t_0) \) [G] |
|---|---|---|---|---|---|
| (A) @ 2.70@ | 1.13 \times 10^{-5} | 1.0 \times 10^{13} | 2.44 \times 10^{12} | 7.70 \times 10^{-2} | 7.15 \times 10^{-64} |
| (B) @ 2.12 \times 10^{8}@ | 1.0 \times 10^{-1} | 1.0 \times 10^{12} | 2.44 \times 10^{11} | 7.70 \times 10^{-2} | 7.15 \times 10^{-64} |

TABLE III: Local type non-Gaussianity of the curvature perturbations. Legend for the case (C) is the same as the case (B) in Table II except \( M = 1.0 \times 10^{-2}M_P = 2.43 \times 10^{16} \text{ GeV} \). In the case (C), we obtain \( C_+(k, t_R) = 0.423 \).

| \( f_{NL} \) | \( g^2 \) | \( H_{inf} \) [GeV] | \( m \) [GeV] | \( Y/M_P \) | \( \chi_k \) | \( B(H_0^{-1}, t_0) \) [G] |
|---|---|---|---|---|---|---|
| (C) 2.12 \times 10^{8} | 1.0 \times 10^{-1} | 1.0 \times 10^{12} | 2.44 \times 10^{11} | 7.70 \times 10^{-2} | 1.36 | 1.81 \times 10^{-64} |
obtained by the Planck satellite, whereas for the case (B), that of $f_{\text{NL}}^{\text{local}}$ is much larger. The upper limits on $f_{\text{NL}}^{\text{local}}$ of less than or equal to $\mathcal{O}(1)$ make the space for our model parameters very small. Indeed, however, there exists a viable room for the parameters such as the case (A) displayed in Table I. The constraint on $f_{\text{NL}}^{\text{local}}$ written above is satisfied by other close values of the parameters. We also demonstrate the case (C) of Table III in which $\Delta N_{\text{max}}$, $\bar{V}$, $g^{\prime 2}$, $g_{\text{ps}}$, and $k$ are the same as in the case (B) of Table II, while the value of $M$ is smaller than that in Table II. Even though the value of $M$ is larger, the value of $f_{\text{NL}}^{\text{local}}$ is not changed. Hence, from the upper limit that $f_{\text{NL}}^{\text{local}}$ is less than or equal to $\mathcal{O}(1)$, we see that the case (C) is not consistent with the observations. Thus, for a region of our model parameters, the resultant non-Gaussianity of the spectrum of the curvature perturbations produced in this scenario can be compatible with the constraint obtained from the Planck satellite.

It is noted that the main feature of the present model is the presence of term of $X(t_k)$, which can make the large-scale magnetic field stronger. The contribution of this factor to the non-Gaussianity $f_{\text{NL}}^{\text{local}}$ in Eq. (4.14) is included through $\xi$ in Eq. (3.7), $m$ in Eq. (4.12), and $Y_k$ in Eq. (4.13).

C. Tensor-to-scalar ratio

In addition to the scalar modes of the curvature perturbations, the tensor modes, i.e., gravitational waves, can be generated. The tensor-to-scalar ratio $r$ is defined by the ratio of amplitude of the tensor modes to that of the scalar modes. For our model, we obtain

$$
r = \begin{cases} 
16\epsilon(t_k) & \text{for } \xi \lesssim 3, \\
7.2\epsilon^2(t_k) & \text{for } \xi \to \infty,
\end{cases}$$

(4.15)

$$
\epsilon(t_k) = \frac{2M_p^2m^4Y_k^2}{(2\bar{V} - m^2Y_k)^2},
$$

(4.16)

where $\epsilon(t_k) = \epsilon(t = t_k)$ in Eq. (4.7), and we have used Eqs. (2.3) and (3.1).

We show the estimations of the tensor-to-scalar ratio $r$ in Tables IV and V. The cases (A) and (B) are the same as those in Table II; that is, the values of $H_{\text{inf}}$, $M$, $m$, $Y_k$, and $\chi_k$ are the same. Similarly, the case (C) is equivalent to that in Table III. We remark that since the value of $H_{\text{inf}}$ and the ratio of $m$ to $H_{\text{inf}}$ in the case (C) is the same value as the case (B), the resultant value of $r$ in the case (C) is also equal to that in the case
TABLE IV: Tensor-to-scalar ratio of the curvature perturbations. Legend is the same as Table II.

|   | $r$       | $H_{\text{inf}}$ [GeV] | $m$ [GeV] | $Y_k/M_P$ |
|---|-----------|------------------------|-----------|-----------|
| (A) | $1.87 \times 10^{-5}$ | $1.0 \times 10^{13}$ | $2.44 \times 10^{12}$ | $7.70 \times 10^{-2}$ |
| (B) | $1.87 \times 10^{-5}$ | $1.0 \times 10^{12}$ | $2.44 \times 10^{11}$ | $7.70 \times 10^{-2}$ |

TABLE V: Tensor-to-scalar ratio of the curvature perturbations. Legend is the same as Table III.

|   | $r$       | $H_{\text{inf}}$ [GeV] | $m$ [GeV] | $Y_k/M_P$ |
|---|-----------|------------------------|-----------|-----------|
| (C) | $1.87 \times 10^{-5}$ | $1.0 \times 10^{12}$ | $2.44 \times 10^{11}$ | $7.70 \times 10^{-2}$ |

(B). We here note that the upper limit found by the Planck satellite is estimated as $r < 0.11 (95\% \text{ CL})$ [43]. Moreover, it is expected that future/current experiments of polarization of the CMB radiation such as POLARBEAR [52] and LiteBIRD [53] can detect $r < 0.01$, and that the future plan of LiteBIRD can does $r < 0.001$ [53]. As a result, when the magnetic fields on the Hubble horizon scale without the back reaction problem are generated at the present time, both the local non-Gaussianity and tensor-to-scalar ratio of the CMB radiation meeting the constraints from the Planck satellite can be produced in a region of the parameters. It is significant to state that to check the effect of the dynamics of the $X$ field, we have also investigated a toy model with the dynamical $X$ field, in which the potential of $X = \exp (-\lambda \Phi/M_P)$ is given by $U(X) = U(\Phi) = \bar{U} \exp \left( -\tilde{\lambda} \Phi/M_P \right)$ with $\lambda$ a dimensionless constant and $\bar{U}$ a constant, and eventually found that qualitatively similar results in terms of the current field strength of the large-scale magnetic fields, the non-Gaussianity $f_{NL}^{\text{local}}$ in Eq. (4.14), and the tensor-to-scalar ratio $r$ in Eq. (4.15) can be acquired.

In addition, it should be noted that the BICEP2 experiment has recently observed the $B$-mode polarization of the CMB radiation with $r = 0.20^{+0.07}_{-0.05} (68\% \text{ CL})$ [35]. There are also several discussions on the way of subtracting the foreground data [54, 55]. Our investigations related to the BICEP2 result on $r$ is presented in Appendix A.

The important property of our model in comparison with the past works is that there exists the term of $X(t_k)$ leading to the strong magnetic fields. This contributes to the tensor-to-scalar ratio $r$ in Eq. (4.15) with $\epsilon(t_k)$ in Eq. (4.16) via $m$ in Eq. (4.12) and $Y_k$ in Eq. (4.13).
In principle, in our model, by the factor $X(t_k)$, the large-scale magnetic fields with its strong amplitude to account for the observational values only through the adiabatic compression without any dynamo amplification mechanism. The reason why we only have small values of the magnetic field strength shown in Tables I–III is that in this work, we attempt to explain three observational quantities, namely, in addition to large-scale magnetic fields, the non-Gaussianity of the curvature perturbations and the tensor-to-scalar ratio. This point is the crucial merit of the present model.

V. CONCLUSIONS

In the present paper, we have explored the generation of large-scale magnetic fields in inflationary cosmology, particularly, a toy model of the so-called moduli inflation, through the breaking of the conformal invariance of the hypercharge electromagnetic fields because of their coupling to both the scalar and pseudoscalar fields appearing in the framework of string theories. We have studied the current magnetic field strength on the Hubble horizon scale in order for it to be compatible with the back reaction problem. Furthermore, we have examined the local non-Gaussianity of the curvature perturbations due to the existence of the massive gauge fields and the tensor-to-scalar ratio. As a consequence, it has been shown that in addition to the magnetic fields on the Hubble horizon scale whose current field strength is consistent with the back reaction problem, the local non-Gaussianity and tensor-to-scalar ratio of the power spectrum of the CMB radiation with those values consistent with the constraints observed by the Planck satellite, i.e., $j_{\text{local}}^{\text{NL}} = \mathcal{O}(1)$ and $r < 0.11(95\% \, \text{CL})$, can be generated for a range of the model parameters.

It should be remarked that one of the most important significance of this work is to present explicit three cosmological observable quantities of the large-scale magnetic fields, the local non-Gaussianity, and the tensor-to-scalar ratio for the first time.

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Appendix A: Axion monodromy inflation

The tensor-to-scalar ratio \( r \) in the moduli inflation model is much smaller than the BICEP2 result\(^8\), although it is still consistent with the Planck data. In this Appendix, we
investigate the axion monodromy inflation model and derive the resultant value of \( r \) in
order to compare it with that in the moduli inflation model.

As a concrete potential in the axion monodromy inflation model, we examine the following
form\(^5\)

\[
V(Y) = AY^q, \quad q = 1, \tag{A1}
\]

with \( A \) a constant. In the axion monodromy inflation model, only the pseudoscalar field \( Y \)
exists and not the dilaton \( \Phi \), i.e., the scalar quantity \( X = 1 \) in Eq. \([2.2]\). Consequently, the
total Lagrangian becomes \( \mathcal{L} \) in Eq. \([2.1]\) with \( X = 1 \) (namely, \( \Phi = 0 \)) and \( V(Y) \) in Eq. \([A1]\)

\(^8\) There have been proposed scalar field models of inflation to realize the BICEP2 result on \( r \), e.g., in
Refs. \([56, 58]\).
instead of that in Eq. (2.3). As the other potential, we can explore the form

\[ V(Y) = \frac{1}{2}m^2Y^2, \]

which follows from the limit \( Y/f \ll 1 \) of the potential \( V(Y) = \lambda^4 (1 - \cos (Y/f)) \) analyzed in Refs. [28, 29].

We study the case of the potential \( V(Y) \) in Eq. (A1). Provided that the slow-roll inflation can be realized, the solution of Eq. (2.5) is given by \( Y = \bar{Y} t \) with \( \bar{Y} = Y = \bar{Y} t, \bar{Y} = -A_3H_{\text{inf}} \).

The field equation of \( F_\mu \) in Eq. (2.6) becomes

\[ \ddot{F}_i(t, x) + H\dot{F}_i(t, x) - \frac{1}{a^2}\partial_j\partial_j F_i(t, x) + \frac{g_{\text{ps}}}{M} \frac{A}{3H_{\text{inf}}} \epsilon^{ijk}\partial_j F_k(t, x) = 0, \]

where we have used Eq. (A3). Moreover, with Eq. (A3), \( \xi \) in Eq. (3.7) reads

\[ \xi = -\frac{1}{6} \frac{g_{\text{ps}}}{M} \frac{A}{H_{\text{inf}}^2}. \]

Clearly, this is not a dynamical quantity but a constant.

By using Eqs. (4.3) with the COBE normalization \( \Delta^2(k) = 2.4 \times 10^{-9} \) and (4.6)–(4.8) with the Planck data \( n_s = 0.9603 \) and providing that \( t \approx H^{-1}_{\text{inf}} \) during inflation, we find \( \epsilon = 6.62 \times 10^{-3} \) and \( A = \left[ 3/(4\sqrt{6\pi}) \right] \times 10^5H_{\text{inf}}^2 \). This value of \( \epsilon \) is realized if \( H_{\text{inf}} = 6.51 \times 10^{15}\text{GeV} \). Moreover, it follows from Eq. (A3) that if \( M = 3.55 \times 10^{18}\text{GeV} \), \( g_{\text{ps}} = 1.0 \), and \( H_{\text{inf}} = 6.51 \times 10^{15}\text{GeV} \), \( |\xi| = 2.98 \). Hence, from Eq. (A5) we obtain \( r = 16\epsilon = 0.106 \). This is the same order of the BICEP2 result. Thus, it is considered that for the axion monodromy inflation model, the tensor-to-scalar ratio compatible with the BICEP2 result can be produced.

**Appendix B: Issues of the backreaction and the strong coupling**

In this Appendix, we explain the issues of the backreaction and the strong coupling. The problem of back reactions by the generation of electromagnetic fields during inflation has been realized [59, 63] (for more recent related works on the relation between the generated gauge fields and inflation, see [64, 69]). It has been pointed out [59] that the amplitude of the current magnetic fields on \( \mathcal{O}(1) \) Mpc scale should be less than \( 10^{-32} \) G. In such a case, the
dynamics of inflation is not disturbed by the back reaction originating from the generation of electromagnetic fields. This means that the strength of the magnetic fields on the Hubble horizon scale should be less than $10^{-35} \text{G}$, which can be derived by $B(k, t) \propto (k/H_{\text{inf}})^{0.8}$ [59]. Throughout this paper, we take parameter sets (for a given coherence scale $\propto k^{-1}$) whose generated magnetic field strength at the present time satisfies this constraint. In addition, the strong coupling problem, the necessity for the very strong gauge coupling to amplify the gauge fields during inflation, has also been indicated in Ref. [59]. Recently, as a solution for this problem, the so-called sawtooth model for the coupling between a scalar field and the $U(1)_Y$ fields has been proposed in Ref. [70]. In this scenario, the behavior of the scalar field is a sawtooth path. We note that in the sawtooth model the current strength about $10^{-16}$ G on 1 Mpc scale can be generated without facing the strong coupling problem as well as the back reaction problem. Furthermore, according to the updated analysis in Ref. [71] with the very recent observational data obtained from the BICEP2 experiment [35], the magnetic field strength on 1 Mpc scale should be less than $10^{-30} \text{G}$. It is quite interesting to apply our analysis on the generation of large-scale magnetic fields and the estimation of power spectrum, non-Gaussianity and tensor-to-scalar ratio of the curvature perturbations to more realistic moduli inflation models such as the racetrack inflation model. One could avoid the strong coupling problem in the sawtooth scenario [70, 71]. Thus, it is an important issue to study whether the sawtooth-like evolution of the dilaton can be realized in the moduli inflation enough to generate the large-scale magnetic fields or not. It may be useful to investigate the racetrack inflation model with positive exponent potential terms, because they induce a quite high potential wall for a large value of the dilaton [72].

Appendix C: Constraints on the cosmic magnetic field strength

In this Appendix, we present the upper bounds of the cosmic magnetic field strength. Regarding the large-scale magnetic fields, from the observations of the CMB radiation, the possible maximum magnetic field strength on 1Mpc scale is $\sim 10^{-9} \text{G}$ [73, 74] and that on the scale larger than the present Hubble horizon is $4.8 \times 10^{-9} \text{G}$ [75]. In addition, it has been calculated in Ref. [76] that by using the data obtained by the polarized radiation imaging and spectroscopy mission (PRISM) [77], the magnetic fields with $\sim 10^{-9} \text{G}$ can be detected. Moreover, there are other methods, such as the 21cm fluctuations of the neutral
hydrogen, the density perturbation parameter of matter $\sigma_8$, the correlation of the curvature perturbations with the magnetic fields, the fifth science (S5) run data obtained from the Laser Interferometer Gravitational-wave Observatory (LIGO), the X-ray galaxy cluster survey by Chandra, the Sunyaev-Zel’dovich (S-Z) survey, and gravitational waves, namely, tensor modes of the curvature perturbations generated during inflation. The upper limits from these observations are consistent with or weaker than those obtained with the CMB radiation data. There have existed generic investigations on the large-scale magnetic fields spectrum from inflation. Also, there have been proposed the lower bounds on cosmic magnetic fields in void regions with the observations of a blazar. On the other hand, for the smaller-scale magnetic fields, there are the upper bounds from the BBN. there are constraints on the magnetic field strength. The constraint on the current strength of the magnetic fields on the BBN horizon scale The upper limit of the magnetic field strength on the Hubble horizon scale at the BBN epoch $\sim 9.8 \times 10^{-5} h^{-1} \text{Mpc}$ with $h = 0.673$, is less than $10^{-6} \text{G}$. Incidentally, intergalactic magnetic fields, the relation between cosmological magnetic fields and blazars, the influence of decay of the cosmic magnetic fields on the CMB radiation, and the secondary anisotropies of the CMB radiation originating from stochastic magnetic fields have been explored. Moreover, constraints on the primordial magnetic fields from the conversion between the CMB photon and graviton, the interaction of the CMB radiation with an axion in the context of the axiverse, trispectrum of the CMB radiation, and the measurement of the Faraday rotation have been proposed.

**Appendix D: Cosmological implications**

In this Appendix, we state cosmological implications obtained from this work. There exists the possibility of baryogenesis coming from the large-scale magnetic fields generated from inflation. These magnetic fields can yield gravitational waves because the space-time is distorted by the existence of the magnetic fields, and eventually the magnetic helicity can be produced. Moreover, the relation between the magnetic helicity and the cosmic chiral asymmetry has been investigated in detail. If the magnetic helicity exists before the cosmological electroweak phase transition, baryon numbers can be produced through the quantum anomaly effect. The coupling of the electromagnetic fields to the
pseudoscalar field can lead to the magnetic helicity, and thus moduli inflation driven by an
axion-like pseudoscalar field can generate not only the large-scale magnetic fields but also
the baryon asymmetry of the universe (for trial scenarios, see, e.g., 37, 38). It has to be sig-
nificant to build a concrete model of inflationary cosmology, in which both cosmic magnetic
fields and baryons can be generated in the framework of fundamental theories such as string
theories describing the physics in the early universe. In addition, a leptogenesis scenario due
to the existence of the primordial magnetic fields has been proposed in Ref. 101. Also, in
Ref. 102, the idea that the dark energy component may be the non-linear electromagnetic
fields has been proposed.

We also state the detectability of cosmic magnetic fields. Current and/or future experi-
ments on the polarizations of the CMB radiation, for example, Planck, QUIET, POLARBEAR,
B-Pol, and LiteBIRD, can detect the large-scale magnetic fields with the current strength \( \sim 4 \times 10^{-11} - 10^{-10} \text{G} \) 97, 107. For the magnetic fields with
the left-handed magnetic helicity, the field strength \( \sim 10^{-14} \text{G} \) on \( \sim 10 \text{Mpc} \) scale can be
observed 108. Further theoretical investigations on the properties of \( B \)-mode polarization
of the CMB radiation has recently been examined in Ref. 109. Furthermore, there have
been appeared various ideas to detect primordial magnetic fields such as future observations
for low-medium redshift 110 and the bias of magnification of lensing effects 111. Since
a number of ways of detecting the cosmic magnetic fields exist as described above, there is
the possibility to examine the physics in both the early- and late-time universe through the
detections of the primordial large-scale magnetic fields, in particular, in the void structures
or the inter-galactic region.

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