Muon internal bremsstrahlung: a conventional explanation for the excess $\nu_e$ events in MiniBoone

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We show that the rate of charged-current $\nu_e$ events with a hard internal bremsstrahlung photon is consistent with the excess $\nu_e$ candidate events reported by the MiniBoone and LSND (decay in flight) experiments. Hard photons radiated by the muon leg in charged-current neutrino interactions ($\nu_\mu N \rightarrow \mu \gamma N$) are a significant source of background that should be considered by current and future $\nu_\mu \rightarrow \nu_e$ neutrino oscillations appearance experiments (e.g. LSND, MiniBoone, SuperK, MINOS, T2K and NOVA).

Experimental evidence for oscillations among the three neutrino generations was reported almost a decade ago. The LSND Collaboration [1] has also reported evidence for $\nu_\mu \rightarrow \nu_e$ oscillations in a $\Delta m^2$ and mixing angle region which is not consistent with the atmospheric and solar neutrino oscillation results. This LSND result was obtained from the observation of $\bar{\nu}_e$ events in a $\bar{\nu}_\mu$ beam (with energies between 20 and 55 MeV) that originates from decays of muons at rest [1].

The LSND collaboration also searched for $\nu_e$ candidates in an independent exposure to a beam of $\nu_\mu$'s (with energies in the range 60 to 200 MeV) from pion decay in flight [2]. The observed rate of $\nu_e$ events in the decay in flight sample has been reported to be consistent with the oscillations parameters from the decay at rest results (though with a lower statistical significance).

The MiniBoone experiment was constructed to investigate the origin of the anomalous excess of $\nu_e$ events reported by LSND. The $\nu_\mu$ and $\bar{\nu}_\mu$ beams at MiniBoone are at higher energy (0.2-3.0 GeV)

The first published results [3] from MiniBoone (taken with a $\nu_\mu$ beam) do not confirm the LSND results. The observed yield and energy spectrum of the MiniBoone $\nu_e$ candidates (shown in figure 1) is incompatible with the oscillation parameters in the LSND allowed region. However, the MiniBoone data show an excess of $\nu_e$ candidates at energies below their analysis threshold of 475 MeV. The total excess which is not understood, is about 150 events in a sample of $\approx 400,000\ \nu_\mu$ charged current events, or about 0.03%. When the analysis is extended to lower energy [4] (see in figure 2), the excess of $\nu_e$ events persists.

In this communication we show that the calculated rate of charged-current $\nu_\mu$ neutrino events with a hard internal bremsstrahlung photon is consistent with the excess $\nu_e$ event candidates reported by MiniBoone. Hard photons radiated by the muon leg in charged current $\nu_\mu$ events ($\nu_\mu N \rightarrow \mu \gamma N$) are a significant source of background that should be considered by $\nu_\mu \rightarrow \nu_e$ neutrino oscillations appearance experiments.

The probability for radiation of a hard photon by muons created in a quasielastic $\nu_\mu$ charged current interaction in the MiniBoone energy range is about 1%. The probability to radiate a photon which carries about half of the muon energy is about 0.3%. Such high energy photons appear as electromagnetic (EM) showers in Cerenkov detectors such as MiniBoone. In Cerenkov de-
FIG. 2: The reconstructed neutrino energy \( E_{\nu}^{\text{rec}} \) distribution of excess of \( \nu_e \) from an updated analysis extended to lower energy.

Detectors, muons with energy less than 200 MeV are practically invisible, since they are below Cerenkov threshold. Therefore, 400 MeV muons, which radiates a 200 MeV photon would be invisible in MiniBoone and appear as \( \nu_e \) candidates. Even if a muon is partially visible in MiniBoone because its energy is above Cerenkov threshold, or due to the production of a low light level from the scintillator in oil (about 25%), its signal would most likely be buried within the photon electromagnetic shower. The presence of some of these undetected muons may be inferred in MiniBoone via the observation of a Michel (\( \mu^- \text{DAR} \)) electron from the delayed decay at rest of the muon (\( \mu^- \text{DAR} \)). However, the efficiency for the detection of invisible or partially visible muon events using \( \mu^- \text{DAR} \) photons is not 100% for several reasons:

1. A fraction of the DAR electrons are at very low electron energy.
2. A fraction of the muons decay outside the detector’s active fiducial volume.
3. A fraction of the negatively charged muons are captured by atomic nuclei and undergo internal conversion.

In this communication, we show that the rate of radiative muon events is significant and should be included in the MiniBoone analysis. The details of the inefficiency in the detection of invisible or partially visible muons depend on the specific details of the MiniBoone analysis, and are not addressed in this communication.

The radiative corrections to inclusive charged current neutrino scattering has been calculated by several authors [5] including Kiskis, Barlow and Wolfram, and Bardin et al. In this communication, we use the leading log peaking approximation as derived by De Rújula, Petronzio, and Savoy-Navarro [6].

The contribution from the muon leg to the double differential cross section in leading log approximation can then be written as

\[
\frac{d^2\sigma}{dx dy} = \frac{d^2\sigma_0}{dx dy} + \frac{\alpha}{2\pi} \ln \frac{2M E_{\nu}(1 - y + xy)^2}{m^2_{\mu}} \\
\times \int_0^1 dz \left\{ y \Theta(z - z_m) \frac{d^2\sigma_0}{dx dy}(\tilde{x}, \tilde{y}) - \frac{d^2\sigma_0}{dx dy}(x, y) \right\},
\]

where \( \alpha \approx 1/137 \) is the fine structure constant,

\[
\tilde{E}_{\mu} = \frac{E_{\mu}}{z} \\
\tilde{x} = \frac{xy}{z + y - 1} \\
\tilde{y} = \frac{z + y - 1}{z} \\
z_m = 1 - y + xy,
\]

and \( d^2\sigma_0/dx dy \) is the lowest order Born cross section as shown in figure 3. Here, \( Q^2 = 2M E_{\nu} xy \), \( M \) is the nucleon mass, \( y = (E_{\nu} - E_\mu)/E_{\nu} \), and \( E_{\mu} \) the energy of the outgoing charged muon in the laboratory.
frame. Eq. (1) is expected to be a good approximation for \(\ln(2ME_\nu/m_\nu^2) \gg \ln(1-y)\) [1].

The first term under the integral sign of equation (1) comes from the bremsstrahlung diagram of figure 4 \((\nu_\mu N \rightarrow \nu \gamma N)\). Muons of energy \(E_\mu = E_\mu/y > E_\mu\) radiate photons and contribute to the uncorrected differential cross sections for outgoing muons of energy \(E_\mu\). The case of \(z \rightarrow z_m\) corresponds to the case when the initial muon originates from quasi-elastic scattering. The case of \(z \rightarrow 1\) corresponds to the case where the energy of the radiated photon \(E_\gamma \rightarrow 0\).

For MiniBoone we make the approximation that the cross section is dominated by quasi-elastic scattering from nucleons at rest. In that case, the Born cross section is a delta function at the quasielastic peak.

\[
\frac{d^2\sigma_q}{dx dy} = G(x, y)\delta(x-1) \tag{3}
\]

Where \(G(Q^2)\) is a combination of vector and axial form factors. Integrating equation (1) over the elastic delta function we obtain the probability \(dP(y)/dy\) for a muon (from a quasielastic \(\nu_\mu\) scattering event) with energy \(E_{\mu-q}\) to radiate a hard photon and end up in with energy \(E_\mu = yE_\nu\):

\[
\frac{dP(y)}{dy} = \frac{\alpha}{2\pi} \ln \frac{2ME_\nu z_m^2}{m_\mu^2} \times \frac{1 + z_m^2}{1 - z_m^2} \frac{y}{z_m} \tag{4}
\]

We define \(r = E_\gamma/E_{\mu-q}\). The case of \(y_{\text{low}} = (1 - E_{\mu-q}/E_\nu)\) is when the energy of the radiated photon \(E_\gamma \rightarrow 0 (r = r_{\text{low}} = 0)\). The case of \(y_{\text{high}} = 1 - m_\mu/E_\nu\) is when the momentum of the final state muon is zero. The case of \(r = r_{\text{high}}\) yields the highest possible radiated photon energy \(E_\gamma \rightarrow E_{\mu-q} - m_\mu\). By Integrating equation [4], we obtain the integrated probability \(P(E_{\gamma-min})\) for a muon to radiate a photon with energy \(E_\gamma > E_{\gamma-min}\).

The background for \(\nu_\tau\) events originates from events with a high \(E_\gamma\). In MiniBoone, the total measured energy of an electromagnetic shower for a quasi-elastic events with a high photon energy is the sum of the energy of the photon and the fraction of energy of the muon above Cerenkov threshold. This is given approximately by \(E_{EM} = E_\gamma + \text{Max} (E_{\mu-q} - E_\gamma - 0.2 \text{ GeV}, 0)\).

We now give a few numerical examples. For the case of \(E_\nu = 442 \text{ MeV}, E_{\mu-q} = 414 \text{ MeV}, \text{ and } \theta_\mu = 13 \text{ degrees} (Q^2 = 0.01 \text{ (GeV/c)}^2)\), the probability for radiating a hard photon with an energy which is higher than a typical experimental muon momentum resolution (e.g. 5%) is 0.66%. The probability for radiating a photon with \(E_\gamma > 207 \text{ MeV}^2 \) (i.e. 50% of its energy) is 0.33%. This rate is an order of magnitude larger than the 0.03% rate of excess \(\nu_e\) candidates in MiniBoone. Additional numerical examples at other representative energies are given in Table (I) including both lower energy (e.g. 226 MeV corresponding to the lower energy range at MiniBoone and the upper range of LSND), as well as higher energy muon.

In the MiniBoone experiment the nucleons are bound in carbon. For the case of quasi-elastic \(\nu\) scattering, a muon of mass \(M_1\) (bound in \(C^{12}\)) is scattered to a final state nucleon with mass \(M_2\). Here, \(Q^2 = q^2 - (E_\nu - E_\mu)^2 = -m^2 + 2E_\nu(E_\nu - p_1 \cos \theta_\mu)\), where \(E_\mu\) is the final state lepton energy (muon or electron). MiniBoone calculates the reconstructed energy \(E_{\nu-ex}\) (shown in figure 1) as follows.

\[
E_{\nu-ex} = \frac{2(M_1 - E_B)E_l - (E_B^2 - 2M_1E_B + m_l^2 + \Delta M^2)}{2 ([M_1 - E_B] - E_l + p_1 \cos \theta_\mu)} \tag{5}
\]

MiniBoone uses an average removal energy \(E_R = 37 \text{ MeV}\). Here, \(m_1\) is final state lepton mass, \(p_1\) is the final state lepton momentum, and \(\Delta M^2 = M_1^2 - M_2^2\). For \(\nu_e\) candidates, the final state lepton mass is assumed to be the electron mass (which can be neglected). When calculating the reconstructed energy for the radiative muon background to the \(\nu_e\) candidates, the final state lepton energy and momentum of the lepton is \(E_{\nu-ex}\). Therefore, the reconstructed neutrino energy (shown in figures 1 and 2) is higher than \(E_{EM}\) for electromagnetic showers at larger angles.

Although radiative muon events are a large source of background in \(\nu_\mu \rightarrow \nu_e\) neutrino oscillations appearance experiments, radiative corrections have only a small effect on the overall quasielastic differential cross section at low energies. However, as the precision of the next generation neutrino experiments improves, radiative corrections should be accounted for. It is usual to define \(\delta = P(E_{\gamma-min} = \Delta E_\mu)\) as the radiative correction to quasielastic scattering. Here \(\Delta(E_\mu) = \sigma \times E_{\mu-q}\), where \(\sigma\) is larger than the experimental error on the measurement of the energy of the final state muon. The Born quasielastic cross section \(\frac{d^2\sigma_{\text{quasi}}}{dy}\) is related to the measured quasielastic cross section \(\frac{d^2\sigma_{\text{quasi}}}{dy}\) via the following expression.

\[
\frac{d^2\sigma_{\text{quasi}}}{dy} = \frac{d^2\sigma_{\mu-e}}{dy} \left[1 - \delta(\Delta E_\mu)\right] \tag{6}
\]

The above expression can be used to correct the value of the axial vector mass \(M_A\) extracted from quasielastic neutrino scattering experiments for radiative effects. In general, correcting for radiative effects will increase extracted value of \(M_A\) (since correcting for radiative effects yields a larger correction at larger \(Q^2\)). The values of the radiative corrections for an experiment with a typical muon energy resolution \(\Delta = 0.05 \times E_{\mu-q}\) are also given in Table (I). The updated world average value\[2\] of the axial vector mass \(M_A\) extracted from \(\nu_\mu\) quasielastic
TABLE I: A few numerical examples of the probability for a muon generated quasielastic neutrino scattering to radiate a high energy photon with energy greater than 5% and 50% of the energy of the quasielastic muon $E_{\mu}$. For a typical neutrino energy of 2 GeV, the corrections for radiative effect given in Table (I) implies that this average value should be increased by $\delta M_A \approx 0.002$. For bubble chamber experiments with antineutrinos, the correction depends on the range of $Q^2$ of the analysis.

In summary, we show that the background from $\nu_e N \rightarrow \mu N$ can account for the excess $\nu_e$ candidates in the MiniBoone and LSND (decay in flight) experiments. Hard photons radiated by the muon leg in charged current events are a significant source of background that should be considered by $\nu_e \rightarrow \mu$ neutrino oscillations appearance experiments (e.g. LSND, MiniBoone, SuperK, MINOS, T2K and NOVA).

[1] C. Athanassopoulos et al (LSND) Phys. Rev. C54 2685 (1996).

[2] C. Athanassopoulos et al (LSND) Phys. Rev. C58 2489 (1998).

[3] A.A. Aguilar-Arevalo et al (MiniBoone), Phys. Rev. Lett. 98 231801 (2007); H.A. Tanaka et al (MiniBoone) hep-ex/0707.1115 (2007)

[4] R. Taylor (MiniBoone), presented at Lepton-Photon, 2007
http://chepp.knu.ac.kr/lp07/html/S4/S404_2.pdf

[5] J. Kiskis, Phys. Rev. D58 2129 (1973); Roger J. Barlow and Stephen Wolfram, Phys. Rev. D20 2198 (1979); A.B. Arbusov, D.Yu. Bardin, L.V. Kalinovskaya hep-ph/0407203 JHEP 0506 (2005).

[6] A. De Rújula, R. Petronzio, and A. Savoy-Navarro, Nucl. Phys. B 167, 394 (1979); Gunther Sigl, Phys. Rev. D57 3786 (1998).

[7] A. Bodek, S. Avvakumov, R. Bradford, and H. Budd, hep-ex/0708.1946; A. Bodek, S. Avvakumov, R. Bradford, H. Budd, hep-ex/0709.3538