Counting Photons in the Λ-Experiment

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Abstract

Dehmelt’s Λ-experiment for a three-level atom with simultaneously driven strong and weak transition is studied within quantum stochastic calculus approach. The statistics of the emitted photons is found by the method of generating functional of the corresponding two dimensional output counting process. In particular, the average waiting times for a count are calculated.

1 Introduction

In the eighties of last century the series of Dehmelt’s papers [1, 2] became the source of inspiration for many authors who were fascinated by the possibility of observing the fluorescence light emitted by a single confined atom or ion. Dehmelt suggested a very sensitive scheme for detecting very weak transitions in a single trapped ions (or atoms) by exploiting electron shelving effect. The electron shelving effect appears in atom with two transitions: one strong and one weak simultaneously driven. When the electron is shelved on the metastable level, the fluorescence light (resulting due to the intense transition) is switched off. Consequently, the fluorescence light emitted by a single atom exhibits periods of darkness. The length of the dark periods for a three-level system in the $V$-configuration was obtained by Cohen-Tannoudji and Dalibard [3]. These results were confirmed by Barchielli [4] who gave also the physical explanation of these phenomena in the language of quantum stochastic calculus (QSC) [5, 6]. Following [4] we apply the mathematical theory of QSC to describe the Dehmelt experiment for a three-level atom in the Λ-configuration. To find the statistics of the emitted photons we use the theory of the counting processes [7-11]. The generating functional approach of [11] allows us calculate the average waiting times for the counts. Belavkin’s filtering equation for a quantum system under the counting observation, cf. [9] and the literature therein, enables us to study the problem in terms of pure posterior quantum states.

2 The model of experiment within QSC

Let us consider a three-level atom with two transitions: very intense $|1 \rangle \leftrightarrow |0 \rangle$ and very weak one $|0 \rangle \leftrightarrow |2 \rangle$. We will call these transitions for convenience the “blue” and the “red” and assume that they are driven by two lasers.
The Hamiltonian $H$ of the system is given by formula (1):

$$H = \sum_{k=1}^{2} \left[ \frac{1}{2} \Omega_k (|0\rangle\langle k| + |k\rangle\langle 0|) + \Delta_k |k\rangle\langle k| \right],$$

where $\Omega_k$-s represent Rabi frequencies, $\Delta_k$-s are detuning parameters. According to [3], cf. also [6], the dynamics of the system ‘atom plus electromagnetic field’ can be described by quantum stochastic differential equation (QSDE) if we recall to the following physical approximations:

(i) the interaction atom-field is linear in the field operators,

(ii) the rotating-wave approximation (RWA) is made,

(iii) the field spectrum is flat, and the ‘coupling constants’ do not depend on frequency.

Approximation (i). The interaction between an atom and electromagnetic field, in the interaction picture with respect to the free dynamics of the field and in the dipole approximation, takes the form

$$-e \mathbf{r} \cdot \mathbf{E}(\mathbf{r}, t),$$

where $\mathbf{r}$ is the position of the electron, and $\mathbf{E}$ is the quantum electric field.

Approximation (ii). Let $b_j(\omega), b_j^\dagger(\omega), j \in I$, where $I$ is a countable set, denote modal annihilation and creation operators satisfying canonical commutation relations $[b_j(\omega), b_{j'}(\omega')] = \delta_{jj'}(\omega - \omega')$, where $\omega \geq 0$ is a continuous index representing energy, and $j$ is a discrete index. The positive-frequency electric field operator $\mathbf{E}^+(\mathbf{r}, t)$ can be written in terms of $b_j(\omega)$-s as

$$\mathbf{E}^+(\mathbf{r}, t) = \sum_j (2\pi)^{-1/2} \int_0^{+\infty} F_j(\mathbf{r}, \omega) e^{-i\omega t} b_j(\omega) d\omega.$$ 

The explicit form of the coefficients $F_j(\mathbf{r}, \omega)$ is not important for our discussion, we stress only that index $j$ also contains the direction of propagation of photons. Let $\psi_0(\mathbf{r}), \psi_k(\mathbf{r})$ $(k=1, 2)$ denote the wave functions of the considered states. The interaction Hamiltonian in the RWA can be written as
\[ H_{k0} = -e|k\rangle \int \bar{\psi}_k(r) r \cdot E^+(r, t) \psi_0(r) d^3r \langle 0| + \text{hc}. \]

If we introduce
\[ f_j^{k0} := -e \int \bar{\psi}_k(r) r \cdot F_j(r, \omega) \psi_0(r) d^3r, \]
then the interaction Hamiltonian takes the form
\[ H_{k0} = |k\rangle \sum_j (2\pi)^{-1/2} \int_0^{+\infty} e^{-i\omega t} f_j^{k0}(\omega) b_j(\omega) d\omega \langle 0| + \text{hc}. \]

Therefore, in the interaction picture with respect to free dynamics of the atom, one gets
\[ H_{k0} = |k\rangle \langle 0| \sum_j f_j^{k0}(\omega) b_j(t) + \text{hc}, \]

where \( \omega_k, k = 1, 2 \), denote the considered frequencies.

Approximation (iii). We assume that \( f_j^{k0}(\omega) \) is constant in a neighborhood of \( \omega_k \) and zero elsewhere, therefore the expression \( f_j^{k0}(\omega + \omega_k) \) can be replaced with \( f_j^{k0}(\omega_k) \) (coupling constant independent of frequency). The range of integration can be extended from \(-\infty\) to \(+\infty\) according to the fact that the field spectrum is flat. Hence
\[ H_{k0} = |k\rangle \langle 0| \sum_j f_j^{k0}(\omega_k) b_j(t) + \text{hc}, \]

where
\[ b_j(t) = (2\pi)^{-1/2} \int_{-\infty}^{+\infty} e^{-i\omega t} b_j(\omega + \omega_k) d\omega. \] (3)

Let us denote the disjoint sets of indices labeling independent field modes carrying blue and red photons, respectively by \( I_1, I_2 \). Then the coupling systematic operators are given by the expressions
\[ L_j = \left\{ \begin{array}{ll} z_j S_1 = z_j |1\rangle \langle 0| & \text{if } j \in I_1, \\ z_j S_2 = z_j |2\rangle \langle 0| & \text{if } j \in I_2, \end{array} \right. \] (4)

where \( z_j \)-s are complex coupling constants and \( S_j \)-s are lowering atomic operators.

Let us introduce the stochastic differentials of the annihilation and creation processes:
\[ dB(t)_j = B(t + dt) - B(t) = \int_t^{t + dt} b_j(t') dt', \]
\[ dB^\dagger(t)_j = B^\dagger(t + dt) - B^\dagger(t) = \int_t^{t + dt} b^\dagger_j(t') dt'. \] (5)

With the help of these differentials one can represent the dynamical evolution equation for the unitary evolution operator for ‘the system plus electromagnetic field’ in the form of the Ito quantum stochastic differential equation (QSDE) of the form [5, 6]
\[ dU(t) = -KU(t) dt + \sum_{k=1}^{2} \sum_{j \in I_k} \left(L_j dB^\dagger_j(t) - L^\dagger_j dB_j(t)\right) U(t), \quad U(0) = I, \] (6)
where

\[ K = iH + \frac{1}{2} \sum_{k=1}^{2} \sum_{j \in I_k} L_j^\dagger L_j \]  

(7)

and \( H \) is given by the formula (1). With the help of (4) one obtains

\[ K = iH + \frac{1}{2} (\Gamma_1 + \Gamma_2) |0\rangle\langle 0|, \]  

(8)

where

\[ \Gamma_k = \sum_{j \in I_k} |z_j|^2, \quad k = 1, 2 \]  

(9)

and the quantities \( \Gamma_k \) stand for total transition rates for the blue and red transitions.

3 The statistics of the output counting process

Let us consider two-dimensional photon counting process

\[ \hat{N}(t) = [\hat{N}_j(t)]_{j=1}^2 \]  

(10)

with components

\[ \hat{N}_k(t) := \sum_{j \in I_k} \int_0^t \hat{b}_j(t') \hat{b}_j(t') dt', \quad k = 1, 2, \]  

(11)

connected with blue and red photons. Here

\[ \hat{b}_j(t) = U(t)^\dagger b_j(t) U(t), \]  

\[ \hat{b}_j^\dagger(t) = U(t)^\dagger b_j(t) U(t) \]  

(12)

stand for the output field annihilation and creation operators [6], cf. also [7]. These describe the field after interaction with the atom within the time interval \((0, t)\). Therefore the counting process is called the output counting process. We assume that all emitted photons are detected. If the atom is prepared initially in a pure state, then the linear Belavkin filtering equation for the posterior unnormalized wave function takes the form [9]

\[ d\hat{\varphi}(t) = - \left( iH + \frac{1}{2} \Gamma |0\rangle\langle 0| - \frac{1}{2} \Gamma \right) \hat{\varphi}(t) dt + \sum_{j=1}^{2} (S_j - I) \hat{\varphi}(t) d\hat{N}_j(t), \]  

(13)

where

\[ \Gamma := \sum_{k=1}^{2} \Gamma_k. \]  

(14)

QSDE (13) plays the role analogous to that of the Schrödinger equation for unobserved quantum system. It describes the time-evolution of a pure quantum state of the atom evolving according to the trajectory of the counting process. (For the case of observation process of the Wiener type see for instance [13] and the literature quoted therein.)

In order to obtain the average waiting times for the counts we do not need to solve the equation (13). These can be deduced from the statistics of the counting process. It is well-known that the whole statistics of the counting process can be found by solving the differential
equation for the generating (or characteristic) functional of the process. Here we use the results of [11] in which the generating functional approach was applied.

Let us consider the counting trajectory up to \( t, \kappa = ((j_{m_1}, t_1), (j_{m_2}, t_2), \ldots, (j_{m_n}, t_n)) \). The probability density of two or more counts at the same time vanishes. It follows from that the detection of a photon projects the atom into one of the lower states, so the atom must be re-excited before the following photon is detected. To calculate the probability density (with respect to \( \prod_{j=1}^{n} dt_j \)) of counting a photon of type \( j_{m_1} \) at time \( t_1 \), a photon of type \( j_{m_2} \) at time \( t_2 \), \ldots, a photon of type \( j_{m_n} \) at time \( t_n \), where \( t_1 < t_2, \ldots, t_n < t \), and no other photon in the time interval \( (0, t] \) we use the formula [11]

\[
p(\kappa | t) = \prod_{j=1}^{n} \Gamma_{m_j} \langle V(\kappa | t) \psi | V(\kappa | t) \psi \rangle,
\]

where

\[
V(\kappa | t) = e^{-Kt} S_{m_n}(t_n) \cdots S_{m_1}(t_1),
\]

\[
S_{m}(t) = e^{Kt} S_{m} e^{-Kt},
\]

and \( K \) is given by (7).

And hence, if we assume that atom was in the state \(|1\rangle\) at the initial moment \( t = 0 \)

\[
p((j_{m_1}, t_1) | t) = P_{t_1}^t (0 | | k \rangle) \Gamma_k |\langle 0 | e^{-Kt_1} | 1 \rangle|^2,
\]

where the quantity

\[
P_{t_1}^t (0 | | k \rangle) = || e^{-K(t-t_1)} | k \rangle ||^2
\]

is the probability of having no counts within the time-interval \((t_1, t]\) for the atom being in the state \(|k\rangle\) \((k = 1, 2)\) at the instant \( t_1 \).

From (18) we obtain the probability density of one count within the time-interval \((t_0, t]\) conditioned by having a count of a photon of the type \( j_0 \) at \( t_0 \)

\[
p_t((j_{m_1}, t_1) | (j_0, t_0)) = p_{t-t_0}((j_{m_1}, t_1 - t_0) | | k \rangle) \quad k = 1, 2.
\]

The form of the above expression is compatible with fact that after a count of a blue photon the atom is in the \(|1\rangle\) state and after a count of a red photon the atom is in the \(|2\rangle\) state.

The structure of (20) allows us write down the probability density of detecting a blue or red photon at the time \( t_0 + t \) and no other photons in the time-interval \((t_0, t_0 + t]\), conditioned by having a count a blue photon at the instant \( t_0 \)

\[
W_k(t | 1) = \Gamma_k |\langle 0 | e^{-K(t-t_0)} | 1 \rangle|^2, \quad k = 1, 2.
\]

If we assume that a red photon was detected at the moment \( t_0 \) then

\[
W_k(t | 2) = \Gamma_k |\langle 0 | e^{-K(t-t_0)} | 2 \rangle|^2, \quad k = 1, 2,
\]

represents the conditional probability density of detecting a blue or red photon at the instant \( t_0 + t \) and no other photons in the time-interval \((t_0, t_0 + t]\).
4 Existence of dark periods

To describe phenomena occurring in the Λ-experiment we will follow the approach of [3] and [4]. The authors of [3] noticed that the characteristics of dark periods is controlled by the probabilities $P_{t_0+t}^d(0|k)$ for not having any count in the time-interval $(t_0, t_0 + t)$, proceeded by the count of photon at instant $t_0$. Due to (20) one can put for convenience $t_0 = 0$. We will also simplify the notation by writing $P(t|k)$ instead of $P_0^d(0|k)$. To find these probabilities one should calculate probability densities $W_j(t|k)$. For this purpose let us set

$$e^{-Kt|k} = \sum_{j=0}^{2} a_j(t|k)|j\rangle, \quad k = 1, 2. \tag{23}$$

By formulas (21) and (22) one gets

$$W_j(t|k) = \Gamma_j |a_0(t|k)\rangle^2 \quad k = 1, 2, \quad j = 1, 2. \tag{24}$$

The coefficients $a_j(t|k)$ satisfy the equations of motion:

$$\dot{a}_1(t|k) = -\frac{i}{2} \Omega_1 a_0(t|k) - i \Delta_1 a_1(t|k),$$

$$\dot{a}_0(t|k) = -\frac{i}{2} \Omega_1 a_1(t|k) - \frac{i}{2} \Omega_2 a_2(t|k) - \frac{1}{2} \Gamma a_0(t|k),$$

$$\dot{a}_2(t|k) = -\frac{i}{2} \Omega_2 a_0(t|k) - i \Delta_2 a_2(t|k) \tag{25}$$

with the initial conditions:

$$a_j(0|k) = \delta_{j,k}, \quad k = 1, 2 \quad j = 0, 1, 2. \tag{26}$$

The solution to the system (25) can be obtained by using the Laplace transform technique. To develop formulas (24) one needs only a knowledge of

$$a_0(t|1) = -\frac{i}{2} \Omega_1 \left( \frac{z_1 + i \Delta_2}{(z_2 - z_1)(z_3 - z_2)} e^{zt} + \frac{z_2 + i \Delta_2}{(z_1 - z_2)(z_3 - z_2)} e^{zt} + \frac{z_3 + i \Delta_2}{(z_1 - z_3)(z_2 - z_3)} e^{zt} \right),$$

$$a_0(t|2) = -\frac{i}{2} \Omega_2 \left( \frac{z_1 + i \Delta_1}{(z_2 - z_1)(z_3 - z_2)} e^{zt} + \frac{z_2 + i \Delta_1}{(z_1 - z_2)(z_3 - z_2)} e^{zt} + \frac{z_3 + i \Delta_1}{(z_1 - z_3)(z_2 - z_3)} e^{zt} \right),$$

where $z_j$ $(j = 1, 2, 3)$ are the roots of the characteristic equation of the system (25). The others coefficients have an analogous form.

If we assume that

$$\Omega_1 \gg \Omega_2, \quad \Gamma_1 \gg \Gamma_2, \quad \Gamma_1 \gg \Omega_2, \tag{27}$$

then the roots $z_j$ are approximately given by

$$z_1 = -i \Delta_2 + \zeta, \tag{28}$$

$$z_2 \simeq -\frac{1}{4} \Gamma - \frac{1}{2} i \Delta_1 + \frac{1}{4} (\Gamma^2 - 4 \Omega_1^2 - 4 \Delta_1^2 - 4 i \Gamma \Delta_1)^{1/2}, \tag{29}$$

$$z_3 \simeq -\frac{1}{4} \Gamma - \frac{1}{2} i \Delta_1 - \frac{1}{4} (\Gamma^2 - 4 \Omega_1^2 - 4 \Delta_1^2 - 4 i \Gamma \Delta_1)^{1/2}, \tag{30}$$

$$\zeta \simeq \Omega_2^2 (\Delta_2 - \Delta_1) \frac{i(\Delta_1^2 + 4 \Delta_1^2 + 4 \Delta_1 \Delta_2) - 2 \Gamma_1 (\Delta_2 - \Delta_1)}{(\Delta_1^2 + 4 \Delta_1^2 + 4 \Delta_1 \Delta_2)^2 + 4 \Gamma_1^2 (\Delta_2 - \Delta_1)^2}. \tag{31}$$
One can observe that the real parts of the roots $z_j$ are negative for $\Delta_1 \neq \Delta_2$. It can be proved that for $\Delta_1 \neq \Delta_2$

$$\sum_{j=1}^{2} \int_{0}^{+\infty} W_j(t|\langle k \rangle) \, dt = 1 \quad k = 1, 2.$$  \hfill (32)

This means, that the probability that at least one photon is detected in the interval $(0, +\infty)$ is one if $\Delta_1 \neq \Delta_2$ (we have assumed that all emitted photons are detected). To prove (32) let us notice that

$$\sum_{j=1}^{2} \int_{0}^{+\infty} W_j(t|\langle k \rangle) \, dt = 1 - \lim_{t\to+\infty} \text{Tr}[e^{-tK} \langle k | e^{-tK^\dagger}].$$  \hfill (34)

The limit appearing on the right-hand side of (34) vanishes provided the real parts of the roots $z_j$ are negative. Thus, (32) is satisfied if $\Delta_1 \neq \Delta_2$.

Let us now consider the case of $\Delta_1 = \Delta_2 = \Delta$. The condition (35) implies that the difference in photon energies matches the energy separation between the two lower atomic states. One can say that this is the condition for a Raman-type two-photon transition between states $|1\rangle$ and $|2\rangle$. The roots $z_j$ of characteristic equation of the system (25) have now the form

$$z_1 = -i\Delta,$$

$$z_2 = -\frac{1}{4}\Gamma - \frac{1}{2}i\Delta + \frac{1}{4}(\Gamma^2 - 4\Omega_1^2, - 4\Delta^2 - 4i\Gamma\Delta)^{1/2},$$

$$z_3 = -\frac{1}{4}\Gamma - \frac{1}{2}i\Delta - \frac{1}{4}(\Gamma^2 - 4\Omega_1^2, - 4\Delta^2 - 4i\Gamma\Delta)^{1/2}. $$

Therefore, one can get

$$\sum_{j=1}^{2} \int_{0}^{+\infty} W_j(t|\langle 1 \rangle) \, dt = \frac{\Omega_1^2}{\Omega_1^2 + \Omega_2^2},$$  \hfill (39)

and

$$\sum_{j=1}^{2} \int_{0}^{+\infty} W_j(t|\langle 2 \rangle) \, dt = \frac{\Omega_2^2}{\Omega_1^2 + \Omega_2^2}. $$  \hfill (40)

To calculate (39) and (40) we have not taken into account the assumption (27). For $\Omega_1 \gg \Omega_2$, the probability that after emission of a red photon at $t_0 = 0$ at least one photon is emitted in the interval $(0, +\infty)$, is much less than one. This means that in this case dark periods are extremely long or infinite. Moreover, it is worth to notice that from (39) and (40) it follows that there exist the possibility that the atom does not ever emit any photon. In other words, the population can be forever trapped.
Let us now analyse the situation when the condition (35) for equal detunings is not satisfied and assume that a blue photon was counted at the instant $t_0 = 0$. According to (8), (19), (21) one has

$$P(0|1) = 1,$$  \hspace{1cm} (41)

$$\frac{d}{dt} P(t|1) = -2 \sum_{j=1}^{d} W_j(t|1).$$  \hspace{1cm} (42)

Therefore, one gets

$$P(t|1) = 1 - 2 \sum_{j=1}^{d} \int_{0}^{t} W_j(t'|1) dt'.$$  \hspace{1cm} (43)

Due to (32) one can write

$$P(t|1) = 2 \sum_{j=1}^{d} \int_{t}^{\infty} W_j(t'|1) dt'.$$  \hspace{1cm} (44)

Assuming that a red photon was counted at the instant $t_0 = 0$, one gets

$$P(t|2) = 2 \sum_{j=1}^{d} \int_{t}^{\infty} W_j(t'|2) dt'.$$  \hspace{1cm} (45)

Since the expression $1 - P(t|k)$ is the probability of at least one count in $(0,t]$ the quantities

$$p(t|k) := \frac{d}{dt}[1 - P(t|k)] = F_{\text{short}}(t|k) + F_{\text{long}}(t|k)$$  \hspace{1cm} (46)

are probability densities for a waiting time for a count after emission respectively of a blue photon and a red one.

Let us consider the case when a blue photon was emitted at the moment $t_0 = 0$. One can write

$$F_{\text{long}}(t|1) = \frac{\Omega_2^2 \Gamma|z_1 + i\Delta_2|^2 \exp(2\Re(z_1)t)}{4(z_2 - z_1)(z_3 - z_1)^2}.$$  \hspace{1cm} (47)

The roots $z_j$ are now given by the expressions (28)–(31). We assume additionally that $\Delta_1 = 0$. From (27) it follows that

$$|\Re z_1| \ll |\Re z_2,3|.$$  \hspace{1cm} (48)

Let us set

$$\Pi := \int_{0}^{\infty} F_{\text{long}}(t|1) dt,$$  \hspace{1cm} (49)

then by (44) and (46) we obtain

$$\int_{0}^{\infty} F_{\text{short}}(t|1) dt = 1 - \Pi.$$  \hspace{1cm} (50)

With the help of (28)–(31) one gets

$$F_{\text{short}}(t|1) \simeq \frac{\Omega_1^2 \Gamma}{|\Gamma^2 - 4\Omega_1^2|} |z_2 t - z_3 t|^2,$$  \hspace{1cm} (51)

$$F_{\text{long}}(t|1) = \Pi |2\Re(z_1)| \exp[2\Re(z_1)t]$$  \hspace{1cm} (52)

$$\Pi \simeq \frac{\Omega_1^2 \Omega_2^2}{(\Omega_1^2 - 4\Delta_2^2)^2 + 4\Gamma^2 \Delta_2^2}.$$  \hspace{1cm} (53)
Thus, from (27) it follows that $\Pi \ll 1$. We introduce a time delay $\theta$ satisfying the following relation

$$|2\text{Re}(z_{2,3})|^{-1} \ll \theta \ll |2\text{Re}(z_{1})|^{-1}. \quad (54)$$

The time-interval $\Delta t$ between two successive counts is considered as short, if $\Delta t < \theta$, and as long, if $\Delta t > \theta$. The probability of a short waiting time after the emission of a blue photon is

$$P(\Delta t < \theta |1\rangle) = \int_{0}^{\theta} p(t |1\rangle) dt \simeq \int_{0}^{\theta} F_{\text{short}}(t |1\rangle) dt \simeq \int_{0}^{+\infty} F_{\text{short}}(t |1\rangle) dt = 1 - \Pi, \quad (55)$$

and the probability of a long waiting time reads

$$P(\Delta t > \theta |1\rangle) \simeq \int_{0}^{+\infty} F_{\text{long}}(t |1\rangle) dt = \Pi. \quad (56)$$

According to (27), none of these probabilities depend on $\theta$. The short waiting times are distributed with a probability density $(1 - \Pi)^{-1} F_{\text{short}}(t |1\rangle)$ and the long ones with a probability density $\Pi^{-1} F_{\text{long}}(t |1\rangle)$. Hence, the mean duration $T_{\text{short}}$ of the short waiting intervals after the emission of a blue photon has the form

$$T_{\text{short}} = \frac{1}{1 - \Pi} \int_{0}^{+\infty} t F_{\text{short}}(t |1\rangle) dt \simeq \Gamma \Omega^{-2} + 2\Gamma^{-1}. \quad (57)$$

The mean duration $T_{\text{long}}$ of the long waiting intervals after the emission of a blue photon is given by the formula

$$T_{\text{long}} = \frac{1}{\Pi} \int_{0}^{+\infty} t F_{\text{long}}(t |1\rangle) dt = |2\text{Re}z_{1}|^{-1}. \quad (58)$$

Using the assumption (27), one obtains in a similar way

$$F_{\text{short}}(t |2\rangle) \simeq \frac{\Omega_{2}^{2}\Gamma}{4} \left| \frac{z_{2}}{(z_{1} - z_{2})(z_{3} - z_{2})} e^{z_{2}t} + \frac{z_{3}}{(z_{1} - z_{3})(z_{2} - z_{3})} e^{z_{3}t} \right|^{2}, \quad (59)$$

$$F_{\text{long}}(t |2\rangle) = \frac{\Omega_{2}^{2}\Gamma|z_{1}|^{2}}{4(z_{2} - z_{1})(z_{3} - z_{1})^{2}} \exp[2\text{Re}(z_{1})t]. \quad (60)$$

It follows from (27) that $F_{\text{short}}(t |2\rangle)$ is negligible. This proves that short waiting times after a count of a red photon are extremely infrequent. Simple calculation yields an analogous expression for the mean duration of the long intervals as in the case of a blue photon detected at the initial instant $t_{0} = 0$. The difference lies in the fact that in this case long time-intervals play the main role in describing the time evolution of the atom.

### 5 Conclusion

We have shown that there exist periods of darkness in the fluorescence light emitted by a three-level atom in the Λ configuration. But in contrast to the $V$-case, the atom in the Λ-configuration can stay in the coherent states for extremely long or infinite time. This statement is consistent with the corresponding results of [12] and [14]. The dynamics of population trapping in Λ-system has a long history. Let us stress that our main purpose was not to give new physical results, but to show how the theory of the counting processes can be used to find the statistical properties of light emitted in the Λ-experiment.
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