The big trip and Wheeler-DeWitt equation

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Abstract Of all the possible ways to describe the behavior of the universe that has undergone a big trip the Wheeler-DeWitt equation should be the most accurate—provided, of course, that we employ the correct formulation. In this article we start by discussing the standard formulation introduced by González-Díaz and Jimenez-Madrid, and show that it allows for a simple yet efficient method of the solution’s generation, which is based on the Moutard transformation. Next, by shedding the unnecessary restrictions, imposed on aforementioned standard formulation we introduce a more general form of the Wheeler-DeWitt equation. One immediate prediction of this new formula is that for the universe the probability to emerge right after the big trip in a state with \( w = w_0 \) will be maximal if and only if \( w_0 = -\frac{1}{3} \).

Keywords Phantom energy · Wheeler-DeWitt equation · Big trip

1 Introduction

In recent years the mathematical community of cosmologists has provided us with a multitude of new amazingly original models, not the least of which is the “Phantom fields” concept, that (among other things) notably postulates the violation of the weak energy condition (WEC) \( \rho > 0 \), \( \rho + p/c^2 > 0 \) (Caldwell et al. 2003; Caldwell 2002), where \( \rho \) is the fluid density and \( p \) is the pressure (for recent reviews, see Bamba et al. 2012; Caldwell and Kamionkowski 2009; Frieman and Turner 2008; Silvestri and Trodden 2009; Li et al. 2011). Such fields, as follows from quantum theory Carroll et al. (2003), should manifest themselves via the scalar field with the negative kinetic term. The thorough investigation asserts that fields with such a property should be anything but fundamental. Still, one cannot guarantee that no Lagrangian with the negative kinetic terms would arise to serve as an effective model. And that, indeed, is exactly what happens in at least some of the supergravity models (Nilles 1984), as well as in the gravity theories with highest derivatives (Pollock 1988) and even in the field string theory (Aref’eva et al. 2006) (in the models describing the fermion NSR-string with regard for GSO-\(-\)sector (see also Sen 2005)). Lately, the “phantom energy” had even managed to penetrate into the domain of yet another “exotic” cosmological model—the brane theory (see, for example, Sahni and Shtanov 2002; Yurov et al. 2006).

Remark 1 Although most theoreticians believe that the phantom energy is nothing but an effective model, this shared notion as of yet remains to be backed up by anything rigorous. In fact, there are some evidence of quite the opposite. One particular example is the phenomena of “crossing of the phantom division line”, first discovered and discussed in Yurov (2003), Elizalde et al. (2004), Yurov and Vereshchagin (2004), and later interpreted in Andrianov et al. (2005). This “crossing” is smooth and it is an inherent property of some of exact solutions of Einstein equations. One can even argue that the smooth (de-)phantomization is a sufficiently general property of Einstein equations. There-