Effect of exact thermal pairing on nuclear level density

Le Thi Quynh Huong\textsuperscript{1,2,3}, Nguyen Quang Hung\textsuperscript{1}, Le Tan Phuc\textsuperscript{1,3}, and Nguyen Hoang Tung\textsuperscript{1,3}

\textsuperscript{1} Institute of Fundamental and Applied Sciences, Duy Tan University, 3 Quang Trung, Danang city, Vietnam
\textsuperscript{2} Department of Natural Science and Technology, University of Khanh Hoa, Nha Trang City, Khanh Hoa Province, Vietnam
\textsuperscript{3} Faculty of Physics and Engineering Physics, Ho Chi Minh University of Science, Ho Chi Minh City, Vietnam

E-mail: lethiquynhhuong@ukh.edu.vn (L.T.Q.H)

Abstract.
Nuclear level density is studied within a microscopic approach, which is derived based on the exact thermal pairing of the pairing Hamiltonian for the truncated single-particle levels around the Fermi surface in combination with the finite-temperature independent-particle model for the single-particle levels outside the truncated space. The numerical calculations are carried out for \textsuperscript{170,171,172}Yb isotopes, whose experimental data are available. The results obtained show that the exact thermal pairing is indeed very important for the valid description of nuclear level density in the low and intermediate regions of excitation energy.

1. Introduction
Nuclear level density (NLD), which was first introduced by Hans Bethe in 1936, is defined as the number of excited levels per unit of excitation energy \cite{1}. This quantity has been investigated within various experimental and theoretical studies such as slow neutron resonances at the neutron binding energy and cumulative numbers of discrete levels at low excitation energy \cite{2,3}. The NLD plays an important role in the understanding of several properties of atomic nuclei such as pairing correlation, nuclear temperature, entropy, heat capacity, etc \cite{4}.

Theoretically, the NLD has been extensively calculated within a number of phenomenological and microscopic models. Two effective phenomenological models, which have been popularly employed in the study of NLD, are the back-shifted Fermi gas (BSFG) \cite{5} and constant temperature model (CTM) \cite{6}. These models are both derived based on the Fermi gas picture in which nucleons are supposed to be the non-interacting Fermi particles moving in a square-well potential. To describe the shell and pairing effects, several empirical parameters extracted from the fitting to the experimental data are introduced, for example, the back-shifted energy, shell correction, level density parameter, and pairing energy. Consequently, the quality of NLD predictions depends on the reliability of systematic estimates of those empirically adjusted parameters. When no experimental data is available, the results obtained within those methods are certainly not reliable. In this case, the microscopic models should be employed instead of the phenomenological ones.
Within the microscopic approaches, the effects of superfluid pairing, deformation, vibration, rotation, and shell structure, which are altogether important for the description of NLD, should be either microscopically or empirically included. The NLD can be well reproduced within the finite-temperature shell model Monte Carlo (SMMC) [7]. However, this method is very time-consuming, especially when it is applied to heavy nuclei. Other two microscopic approaches, which have been widely used to estimate the NLD, are the Hartree-Fock Bardeen-Cooper-Schrieffer (HFBCS) [8] and Hartree-Fock-Bogoliubov plus combinatorial method (HFBC) [9]. Although, these two approaches are considered as the most microscopic approaches to NLD up to date, they both violate the conservation of particle number, which is important for the description of light and neutron/proton-rich nuclei [8, 9, 10, 11, 12]. Therefore, in order to have a good description of the experimental data, the NLDs obtained within these approaches have to be renormalized using two phenomenological parameters, whose values are extracted from the fitting to the existed data, namely the NLD data obtained from the analysis of the cumulative numbers of discrete levels at low excitation energy and the data calculated from the average neutron resonance spacing at the neutron binding energy (See e.g., Eq. (9) of Ref. [13]).

The present paper will review our recent microscopic approach to the NLD proposed in Ref. [14]. This approach is derived based on the exact thermal pairing solution to avoid the particle-number conserving problem for the truncated single-particle levels around the Fermi surface in combination with the independent-particle model at finite temperature for the levels outside the truncated space. Moreover, more details on the method, which have not written in Ref. [14] because of its limitation on the number of pages, will be presented in the present paper. In particular, we will present and discuss the results of the thermodynamic quantities such as total energy, heat capacity, free energy and consequently explain the contribution of these quantities to the prediction of the NLD. The comparison of the NLD obtained within our method with that obtained within the independent-particle model at finite temperature (without pairing) and the HFBCS will also be additionally discussed within the present paper.

2. Formalism

2.1. Exact solution of the nuclear pairing at finite temperature (EP)

The paring Hamiltonian describes a system of $N$ particles with single-particle energies $\epsilon_j$ interacting via a monopole pairing interaction $G$ as [15]

$$H_p = \sum_{jm} \epsilon_j (a^\dagger_{jm} a_{jm} + \tilde{a}^\dagger_{jm} \tilde{a}_{jm}) - G \sum_{jm,j'm'} a^\dagger_{jm} \tilde{a}^\dagger_{jm} a_{j'm'} \tilde{a}_{j'm'} , \quad (1)$$

with $\tilde{a}_{jm} \equiv (-1)^{j-m}a_{j-m}$.

In Eq. (1), $a^\dagger_{jm}(a_{jm})$ is the creation (annihilation) operator of a nucleon with angular momentum $j$, spin projection $m_j$, and energy $\epsilon_j$, whereas $\tilde{a}^\dagger_{jm}(\tilde{a}_{jm})$ is the creation (annihilation) operator of a nucleon with angular momentum $j$ but spin projection -$m_j$ having the opposite direction.

By using the quasi-spin operators of the form

$$L_j^- = \frac{1}{2} \sum_m \tilde{a}_{jm} a_{jm} , \quad (2)$$
$$L_j^+ = (L_j^-)^\dagger = \frac{1}{2} \sum_m a^\dagger_{jm} \tilde{a}^\dagger_{jm} , \quad (2)$$
$$L_j^z = \frac{1}{2} \sum_m (a^\dagger_{jm} a_{jm} - \frac{1}{2}) = \frac{1}{2} (N_j - \Omega_j) , \quad (3)$$

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the Hamiltonian (1) can be rewritten as [15]
\[ H = \sum_j \epsilon_j \Omega_j + 2 \sum_j \epsilon_j L_j^+ L_j^- + G \sum_{jj'} L_j^+ L_{j'}^- , \]  
(4)
where \( N_j = \sum_m a_{jm}^\dagger a_{jm} \) is the particle-number operator and \( \Omega_j = j + 1/2 \) is the level degeneracy.

It is noticed that the operators \( L_j^+ , L_j^- , \) and \( L_j^z \) close an SU(2) algebra of angular momentum. Therefore, the pairing problem with the Hamiltonian (1) can be solved exactly by diagonalizing directly a matrix, which is constructed based on the subsets of basis states \( | \{ s_j \} , \{ N_j \} \rangle \).

The latter are expressed in terms of the partial occupation number \( N_j \equiv L_j^z / \Omega_j \) and the partial seniority \( s_j \equiv \Omega_j - 2L_j \) (the number of unpaired particles) of the single-particle orbital. Using these basis states, the diagonal and off-diagonal elements of the Hamiltonian matrix are given as [15]
\[ \langle \{ s_j \} , \{ N_j \} | H | \{ s_j \} , \{ N_j \} \rangle = \sum_j \left( \epsilon_j N_j + \frac{G}{4} (N_j - s_j) (2\Omega_j - s_j - N_j + 2) \right) , \]
\[ \langle \{ s_j \} , ...N_j + 2 , ...N_{j'} - 2 , ... | H | \{ s_j \} , ...N_j , ...N_{j'} , ... \rangle = \frac{G}{4} [(N_{j'} - s_{j'}) (2\Omega_{j'} - s_{j'} - N_{j'} + 2) (2\Omega_j - s_j - N_j) (N_j - s_j + 2)]^{1/2} . \]

Diagonalizing the matrix (5) for each total seniority \( S = \sum_j s_j \), one obtains the exact eigenvalues \( \mathcal{E}_S \) and eigenvectors \( f_j^S \). These exact eigenvalues and eigenvectors are then used to construct the canonical partition function at finite temperature \( T \) as [16]
\[ Z(T) = \sum_S 2^S \exp \left( -\frac{\mathcal{E}_S}{T} \right) . \]

Consequently, all the thermodynamic quantities such as total energy \( E \), heat capacity \( C \), free energy \( F \), paring gap \( \Delta \), and single-particle occupation numbers are calculated based on the partition function as
\[ F = -T \ln Z(T) , S = \frac{\partial F}{\partial T} , \]
\[ E = F + TS , C = \frac{\partial E}{\partial T} , \]
\[ \Delta = \sqrt{-GE_{pair}} , E_{pair} = E - 2 \sum_j \left[ \epsilon_j - \frac{G}{2} f_j \right] f_j , \]
\[ f_j = \frac{1}{Z(T)} \sum_S 2^S f_j^S \exp \left( -\frac{\mathcal{E}_S}{T} \right) . \]

2.2. Nuclear level density within the exact paring plus independent-particle model (IPM) at finite temperature (EP+IPM)

Due to the limitation in the size of the matrix to be diagonalized, the EP is carried out only for a limited number of levels around the Fermi surface where pairing is mostly affected. To account for the single-particle levels outside this truncated space, we have employed the independent-particle model at finite temperature (IPM) [17, 18, 19], which treats the nucleons inside the nucleus similar to independently moving particles, that is, without pairing interaction. The occupation numbers of these independent nucleons are approximated well by the Fermi-Dirac distribution as
\[ f_j = \frac{1}{1 + \exp \left( \frac{\epsilon_j - \lambda}{T} \right)} , \]
(11)
where the chemical potential $\lambda$ is obtained by solving the equation for the particle number

$$N = 2 \sum_j f_j ,$$

with $N$ being the number of neutrons or protons.

The total EP+IPM partition function is then written as \[17, 18, 19\]

$$\ln Z'_{EP+IPM} = \ln Z'_{EP} + \ln Z'_{IPM} - \ln Z'_{IPM,tr} ,$$

where $Z'_{EP} = Ze^{E_0/T}$ is the excitation partition function with respect to the ground-state energy $E_0$, $Z'_{EP+IPM}$ is the EP+IPM total partition function, $Z'_{EP}$ is the EP partition function (6) for the truncated single-particle levels, and $Z'_{IPM}$ and $Z'_{IPM,tr}$ are the partition functions calculated within the IPM for the whole and truncated single-particle levels, respectively. Knowing the total partition function, one can easily calculate all the thermodynamic quantities by using Eqs. (7) and (8).

The density of states at a given excitation energy $E^*$ calculated from the inverse Laplace transformation of the partition function is defined as \[12\]

$$\omega(E^*) = \frac{e^S}{T \sqrt{2\pi C}} ,$$

where the excitation energy $E^*(T) = E(T) - E(T = 0)$, the total energy $E(T)$, the entropy $S$ and the heat capacity $C$ are obtained from the total partition function (13). The total NLD can be calculated from the state density as

$$\rho(E^*) = \frac{\omega(E^*)}{\sqrt{2\pi \sigma}} ,$$

where $\sigma$ is the spin cut-off parameter. In axially deformed nuclei, this spin cut-off parameter contains the perpendicular $\sigma_\perp$ and parallel $\sigma_\parallel$ components, whose explicit formulas can be taken from the empirical forms as \[5, 20\]

$$\sigma_\perp^2 \approx 0.015 A^{5/3} T , \quad \sigma_\parallel = \sigma_\perp \sqrt{(3 - 2\beta_2) / (3 + \beta_2)} ,$$

with $A$ and $\beta_2$ being the mass number and the quadrupole deformation parameter.

To take into account the vibrational and rotational degree of freedom, which are not present in the pairing Hamiltonian (1), the NLD (15) is then multiplied with the vibrational $k_{vib}$ and rotational $k_{rot}$ enhancement factors, whose empirical formulas are given as \[20, 21, 22\]

$$k_{vib} = \exp \left( 0.0555 A^{2/3} T^{4/3} \right) , \quad k_{rot} = \frac{\sigma_\perp^2 - 1}{1 + \exp \frac{E^* - U_C}{D_C}} + 1 ,$$

with $D_C = 1400 \beta_2^2 A^{-2/3}$ and $U_C = 120 \beta_2^2 A^{1/3}$ \[20\].

The final total NLD obtained within the EP+IPM is \[23\]

$$\rho(E^*) = k_{rot} k_{vib} \frac{\omega(E^*)}{\sqrt{2\pi \sigma_\parallel}} .$$
3. Numerical results and discussion

The NLDs are calculated within the EP+IPM for $^{170,171,172}$Yb. The results obtained are then compared with the available experimental data given by the Oslo group [24, 25]. The single-particle spectra $\epsilon_j$ are taken from the axially deformed Woods-Saxon (WS) potential including the spin-orbit and Coulomb interactions. The parameters of the WS potential are taken from Ref. [26]. The quadrupole deformation parameters $\beta_2$ are adjusted to reproduce the experimental ground-state properties including the nuclear binding energy and charge radii. The pairing interaction parameter $G_{N(Z)}$ is as usual adjusted so that the pairing gap $\Delta_{N(Z)}$ obtained within the EP+IPM at $T=0$ fits the experimental odd-even mass difference. The exact diagonalization of the pairing Hamiltonian is carried out for 12 single-particle levels, in which 6 levels are above and 6 levels are below the Fermi surface. The entire single-particle spectra, which are used for the EP+IPM calculation, contain all the doubly degenerate levels from bottom to a $N=126$ closed shell.

In Figs. 1(a1)-1(a3), it is clearly seen that the exact neutron (solid lines) and proton (dotted lines) pairing gaps decrease with increasing $T$ and remain finite even at $T=3$ MeV, well above the critical temperature $T_C \approx 0.57\Delta(T=0)$, where the pairing gap collapses within the conventional finite-temperature BCS (FTBCS) theory [27, 28, 29, 30, 31]. In $^{171}$Yb, a slight increase in the neutron gap (solid lines in Fig. 1(a2)) is seen at low $T < 0.5$ MeV because of the blocking effect causing by the odd neutron [32].

The excitation energies $E^*$ as functions of $T$ obtained within the IPM (dotted lines), EP+IPM (solid lines) in comparison with those obtained within the HFBC (dashed lines), HFBCS (dash dotted lines) are shown in Figs. 1(b1)-1(b3). At low $T < 0.7$ MeV, $E^*_{EP+IPM}$ is always lower than those obtained within other three approaches, whose predictions almost coincide. This difference is due to the fact that the pairing is treated exactly within the EP+IPM while it is only approximated within the HFBC and HFBCS or even is not included as the case of the IPM. In contrast, at $T > 0.7$ MeV, $E^*_{EP+IPM}$ is higher than $E^*_{IPM}$, $E^*_{HFBC}$, and $E^*_{HFBCS}$ because in this temperature region, which is above $T_C$, the pairing gaps obtained within both the HFBC and HFBCS are zero, whereas the EP+IPM gaps still remain finite as shown in Figs. 1(a1)-1(a3). Moreover, it is also seen in that $E^*_HFBC$ at $T > 0.7$ MeV is always the lowest one. The reason is that the HFBC is constructed based on the partition function of the incoherent 1 particle - 1 hole (1p1h) state densities built on top of the HFB single-particle spectra. These 1p1h states are certain not enough for the high temperature and/or excitation energy regions. For the latter, all the possible couplings of particle and hole states such as 2p2h, 3p3h, etc., as well as particle-particle (pp) and hole-hole (hh) should be included. Meanwhile, since the EP+IPM is derived based on the exact CE partition function obtained from the direct diagonalization of the matrix elements of the Hamiltonian, which include all the possible couplings between the ph, pp, and ph states, $E^*_{EP+IPM}$ is always higher than $E^*_HFBC$ in the high-temperature region.

Shown in the Figs. 1(c1)-1(c3) and 1(d1)-1(d3) are the entropy $S$ and heat capacity $C$ obtained within the IPM and EP+IPM. Due to the presence of the pairing in the entire temperature region, $S_{EP+IPM}$ is always lower than $S_{IPM}$ in which no pairing is included, whereas $C_{EP+IPM}$ is lower (higher) than $C_{IPM}$ at low (high) temperature region. These results indicate that the existence of exact thermal pairing tends to make the system more stable than the cases without pairing.

The results of the thermodynamic quantities shown in Fig. 1 strongly affect the corresponding NLD $\rho$ depicted in Fig. 2. In fact, Fig. 2 shows that without pairing, $\rho_{IPM}$ strongly overestimates the experimental data, whereas with pairing being included, $\rho_{EP+IPM}$, $\rho_{HFBCS}$, and $\rho_{HFBC}$ are almost close to the experimental data. Especially, $\rho_{EP+IPM}$ (solid lines) almost coincides with the experimental data in the excitation energy region below the neutron binding energy $E^* < B_n$. $\rho_{EP+IPM}$ also coincides with $\rho_{HFBC}$ for both negative and positive parities. The latter are the NLDs obtained after the normalization to the known data as mentioned.
Figure 1. Exact neutron and proton pairing gaps $\Delta$ [(a1)-(a3)], excitation energy $E^*$ [(b1)-(b3)], entropy $S$ [(c1)-(c3)], and heat capacity $C$ [(d1)-(d3)] as functions of $T$ obtained within the IPM, EP+IPM as well as the HFBCS and HFBC (negative and positive parities) for $^{170,171,172}$Yb. In Figs. [(b1)-(b3)], the predictions within the HFBCS and HFBC are taken from RIPL-2 [33] and RIPL-3 [34] databases, respectively.
Figure 2. Total level densities $\rho$ as function of $E^*$ obtained within the EP+IPM [14] in comparison with predictions of the IPM, HFBCS [33] and HFBC for the positive and negative parities [34] and the experimental data [24, 25] for $^{170}$Yb (a), $^{171}$Yb (b), and $^{172}$Yb (c).
above. It is also observed in the insets of Fig. 2 that $\rho_{HFBC}$ is much lower than $\rho_{EP+IPM}$ in the high energy region $E^* > B_n$. The reason of this discrepancy is certainly due to the fact that in the high temperature region, the excitation energy obtained within the HFBC, which is constructed based on only the 1p1h state densities, is much lower than that obtained within the EP+IPM as discussed above. These results indicate that the HFBC predictions for the NLD in the high-excitation energy region are still in question. In this case, the EP+IPM predictions seem to be more reliable.

The advantage of the EP+IPM approach proposed here is that it is rather simple, based only on a Woods-Saxon mean-field potential with a set of global parameters for all nuclei. Beyond the Woods-Saxon mean field, the proposed approach uses only two parameters, namely, the monopole pairing interaction parameters $G_N$ (for neutron) and $G_Z$ (for proton), which are fixed at $T = 0$ in order to reproduce the experimental odd-even mass formula at $T = 0$. This means that all of the parameters used in the EP+IPM calculation are fixed at $T = 0$ and no fitting parameters at finite temperature and/or excitation energy are employed, ensuring the microscopic nature of this approach.

4. Conclusions

The present paper reviews our recent microscopic approach published in Ref [14], which offers a valid and reliable description of total nuclear level density. The approach is derived based on the exact solutions of thermal pairing problem in combination with the temperature-dependent independent-particle model. The very good agreement between the results obtained within this approach and the experimental NLD data of $^{170,171,172}$Yb isotopes has shown that exact pairing is indeed very important for the description of total nuclear level density in both regions of low and high excitation energies. The advantages of this approach are its microscopic nature and the absence of parameter fitting at finite excitation energies as well as the very short computation time. The latter takes only less than 5 min even for a heavy nucleus, and therefore all the numerical calculations can be performed on a PC.

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References

[1] Bethe H A 1936 An Attempt to Calculate the Number of Energy Levels of a Heavy Nucleus Phys. Rev. 50 332
[2] Rauscher T and Thielemann F K 2000 Astrophysical reaction rates from statistical model calculations At. Data Nucl. Data Tables 75 1
[3] Rauscher T, Thielemann F K and Kratz K L 1997 Nuclear Level Density and the Determination of Thermonuclear Rates for Astrophysics Phys. Rev. C 56 1613
[4] Ericson T 1960 The statistical model and nuclear level densities Adv. Phys. 9 425
[5] Dilg W, Schantl W, Vonach H and Uhl M 1973 Level density parameters for the back-shifted fermi gas model in the mass range 40 \textless A \textless 250 Nucl. Phys. A 217 269
[6] Gilbert A and Cameron A G W 1965 A composite nuclear-level density formula with shell corrections Can. J. Phys. 43 1446
[7] Alhassid Y, Liu S and Nakada H 1999 Particle-number reprojection in the shell model Monte Carlo method: Application to nuclear level densities Phys. Rev. Lett. 83 4265
[8] Demetriou P and Goriely S 2001 Microscopic nuclear level densities for practical applications Nucl. Phys. A 695 95
[9] Hilaire S and Goriely S 2006 Global microscopic nuclear level densities within the HFB plus combinatorial method for practical applications Nucl. Phys. A 779 63
[10] Ring P and Schuck P 1980 \textit{The nuclear many-body problem} (Heidelberg: Springer Verlag)
[11] Brink D M and Broglia R A 2005 \textit{Nuclear superfluidity: pairing in finite systems} (England: Cambridge University Press)
[12] Bohr A and Mottelson B R 1975 \textit{Nuclear Structure, Vol. I: Single-Particle Motion} (New York: Benjamin)
[13] Goriely S, Hilaire S and Koning A J 2008 Improved microscopic nuclear level densities within the Hartree-Fock-Bogoliubov plus combinatorial method Phys. Rev. C 78 064307
[14] Hung N Q, Dang N D and Huong L T Q 2017 Simultaneous Microscopic Description of Nuclear Level Density and Radiative Strength Function Phys. Rev. Lett. 118 022502
[15] Volya A et al 2001 Exact solution of the nuclear pairing problem Phys. Lett. B 509 37
[16] Hung N Q and Dang N D 2009 Exact and approximate ensemble treatments of thermal pairing in a multilevel model Phys. Rev. C 79 054328
[17] Alhassid Y, Bertsch G F and Fang L 2003 Nuclear level statistics: Extending shell model theory to higher temperatures Phys. Rev. C 68 044322
[18] Hung N Q and Dang N D 2010 Canonical and microcanonical ensemble descriptions of thermal pairing within BCS and quasiparticle random-phase approximation Phys. Rev. C 81 057302
[19] Hung N Q and Dang N D 2010 Thermodynamic properties of hot nuclei within the self-consistent quasiparticle random-phase approximation Phys. Rev. C 82 044316
[20] Junghans A R et al 1998 Projectile-fragment yields as a probe for the collective enhancement in the nuclear level density Nucl. Phys. A 629 635
[21] Ignatyuk A V 1983 The Statistical Properties of the Excited Atomic Nuclei (Moscow: Energoatomizdat)
[22] Iljinov A S 1992 Phenomenological statistical analysis of level densities, decay widths and lifetimes of excited nuclei Nucl. Phys. A 543 517
[23] Capote R et al 2009 RIPL- Reference Input Parameter Library for Calculation of Nuclear Reactions and Nuclear Data Evaluations Nucl. Data Sheets 110 3107
[24] Schiller A et al 2001 Critical temperature for quenching of pair correlations Phys. Rev. C 63 021306(R)
[25] Agvaanluvsan U et al 2004 Level densities and γ-ray strength functions in 170,171,172Yb Phys. Rev. C 70 054611
[26] Cwiok S et al 1987 Single-particle energies, wave functions, quadrupole moments and g-factors in an axially deformed woods-saxon potential with applications to the two-centre-type nuclear problems Comput. Phys. Commun. 46 379
[27] Frauendorf S et al 2003 Temperature-induced pair correlations in clusters and nuclei Phys. Rev. B 68 024518
[28] Sheikh J A, Palit R and Frauendorf S 2005 Reappearance of the pairing correlations at finite temperature Phys. Rev. C 72 041301(R)
[29] Mang H J, Rasmussen O and Rho M 1966 Comparisons of BCS Nuclear Wave Functions with Exact Solutions. II. Particle-Number-Conserving Method Phys. Rev. 141 941
[30] Dang N D and Hung N Q 2008 Pairing within the self-consistent quasiparticle random-phase approximation at finite temperature Phys. Rev. C 77 064315
[31] Hung N Q and Dang N D 2008 Pairing in hot rotating nuclei Phys. Rev. C 78 064315
[32] Hung N Q, Dang N D and Huong L T Q 2016 Improved treatment of blocking effect at finite temperature Phys. Rev. C 94 024341
[33] Belgya T et al 2006 Reference Input Parameter Library (RIPL-2) (online) Available at: https://www-nds.iaea.org/RIPL-2/ (Accessed 14 December 2017)
[34] Capote R et al 2009 Reference Input Parameter Library (RIPL-3) (online) Available at: https://www-nds.iaea.org/RIPL-3/ (Accessed 14 December 2017)