Inverse Iteration for the Monge-Ampère Eigenvalue Problem

by Farhan Abedin
Lafayette College

Abstract: I will present an iterative method for solving the Monge-Ampère eigenvalue problem: given a bounded, convex domain $\Omega \subset \mathbb{R}^n$, find a convex function $u \in C^2(\Omega) \cap C(\overline{\Omega})$ and a positive number $\lambda$ satisfying
\[
\begin{cases}
\det D^2 u = \lambda |u|^n & \text{in } \Omega, \\
u = 0 & \text{on } \partial \Omega.
\end{cases}
\]

By a result of P.-L. Lions, there exists a unique eigenvalue $\lambda = \lambda_{MA}(\Omega) > 0$ for which this problem has a solution. Furthermore, all eigenfunctions $u$ are positive multiples of each other. In recent work with Jun Kitagawa (Michigan State University), we develop an iterative method which generates a sequence of convex functions $\{u_k\}_{k=0}^\infty$ converging to a non-trivial solution of the Monge-Ampère eigenvalue problem. We also show that $\lim_{k \to \infty} R(u_k, \Omega) = \lambda_{MA}(\Omega)$, where the Rayleigh quotient $R(v)$ is defined as
\[
R(v, \Omega) := \frac{\int_{\Omega} |v| \det D^2 v}{\int_{\Omega} |v|^{n+1}}.
\]

Our method converges for a large class of initial choices $u_0$ that can be constructed explicitly, and does not rely on prior knowledge of the eigenvalue $\lambda_{MA}(\Omega)$. I will also discuss other relevant iterative methods in the literature that motivated our work.