Searching for $\tau \rightarrow \mu \gamma$ lepton-flavor-violating decay at super Charm-Tau factory

Hao Zhou, Ren-You Zhang, Liang Han, Wen-Gan Ma, Lei Guo, Chong Chen

Abstract We investigate the possibility of searching the lepton-flavor-violating (LFV) $\tau \rightarrow \mu \gamma$ rare decay at the Super Charm-Tau Factory (CTF). By comparing the kinematic distributions of the LFV signal and the standard model background, we develop an optimized event selection criterion which can significantly reduce the background events. It is concluded that the new $2\sigma$ upper limit of about $1.9 \times 10^{-9}$ on $\text{Br}(\tau \rightarrow \mu \gamma)$ can be obtained at the CTF, which is beyond the capability of Super-B factory in searching $\tau$ lepton rare decay. Within the framework of the scalar leptoquark model, a joint constraint on $\lambda_1\lambda_2$ and $M_{LQ}$ can be derived from the upper bound on $\text{Br}(\tau \rightarrow \mu \gamma)$. With 1000 $fb^{-1}$ data expected at the CTF, we get $\lambda_1\lambda_2 < 7.2 \times 10^{-2}$ ($M_{LQ} = 800 \text{ GeV}$) and $M_{LQ} > 900 \text{ GeV} (\lambda_1\lambda_2 = 9 \times 10^{-2})$ at 95% confidence level.

1 Introduction

The standard model (SM) [1,2] of elementary particle physics provides a remarkably successful description of strong, weak and electromagnetic interactions at the energy scale up to $O(10^7)$ GeV. It is an $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}$ spontaneously broken gauge theory, which also conserves the total baryon number $B$ and the three lepton numbers $L_e$, $L_\mu$, and $L_\tau$, i.e., the lepton flavors, respectively. However, a number of conceptual and experimental difficulties, such as the hierarchy problem, dark matter, and neutrino oscillations, inspired physicists to consider new mechanisms beyond the SM. Many extensions of the SM, such as the supersymmetric models, left–right symmetric models, the little Higgs model with $T$ parity, and leptoquark models, could bring in lepton-flavor-violating (LFV) terms in natural ways and introduce non-zero neutrino masses and rare decays of the charged lepton. As the heaviest lepton, the $\tau$ lepton has more LFV decay modes compared to the $\mu$ lepton. The branching ratios of the $\tau$ lepton LFV decays are predicted at the level of $10^{-10}$–$10^{-7}$ [3–6]. Therefore, searching for $\tau$ LFV decays and improving limits on the branching ratio becomes increasingly important of current and future experiments.

All the LFV decays of $\tau$ lepton, such as $\tau \rightarrow l\gamma$, $\tau \rightarrow lll'$, and $\tau \rightarrow lh$, where $l, l' = e$ or $\mu$ and $h$ is a hadronic system, are sensitive to new physics beyond the SM. Among these modes, the radiative decays $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow e\gamma$ are predicted to have the largest probability close to current experimental upper limits in a wide variety of new physics scenarios. So far the most stringent limits are $\text{Br}(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}$ and $\text{Br}(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$ at 90% confidence level (C.L.), with $\text{Br}(\tau \rightarrow \mu \gamma) < 9.6 \times 10^{-9}$ [7]. To achieve more sensitivity in probing $\tau$ LFV decay, high intensive electron–positron beam facilities are desired. One main project is the well-established upgrade of the B factory, i.e., the Super-B factory, running at energies from open charm threshold to above $\Upsilon(5S)$ resonance with intended accumulated luminosity of 75 $ab^{-1}$. The Super-B factory would provide great opportunity in searching $\tau$ rare decay, where a 90% C.L. upper limit on $\text{Br}(\tau \rightarrow \mu \gamma)$ is expected as $2.4 \times 10^{-9}$ [8]. Another proposal, the so-called Super Charm-Tau factory (CTF), is an $e^+e^-$ collider designed to work in the energy region from 2 to 5 GeV with instant luminosity of $10^{35} \text{ cm}^{-2}\text{s}^{-1}$ [9]. Compared to the Super-B factory, $\tau$ leptons can be copiously produced in pairs at the CTF with center-of-mass (c.m.s.) energies $E_{e^+e^-}$ not far above the $2m_\tau$ threshold, and the radiative background $e^+e^- \rightarrow \tau^+\tau^-\gamma$ is not significant.

In this paper, we investigate the potential of searching the $\tau \rightarrow \mu \gamma$ LFV decay at the CTF and demonstrate its better chance than the Super-B factory. The paper is organized as follows: The LFV signal and dominant background at the CTF are discussed. Then a strategy of experimental event selection to improve signal significance is developed, and an
expected upper limit on $\text{Br}(\tau \to \mu \gamma)$ is presented. Finally, the constraints on the leptoquark model parameters are given as an example of an interpretation of new physics.

2 LFV signal and background at the CTF

At the CTF, the cross section for $e^+e^- \to \tau^+\tau^-$ increases significantly as the increment of the colliding energy from the threshold of $\tau$-pair production ($\sim 3.55 \text{ GeV}$) to the threshold of $D$ meson production ($\sim 3.74 \text{ GeV}$). We set the CTF c.m.s. energy as $3.7 \text{ GeV}$, in order to get $\tau$ lepton pair produced copiously.

The signal under discussion is that one $\tau$ lepton follows LFV decay into a muon and a photon, while the other follows SM decay into a muon and two neutrinos, i.e.,

$$e^+ + e^- \to \tau^+(\mu^+ e^- \nu_\mu) + \tau^-(\mu^- e^+ \bar{\nu}_\mu)$$

and

$$e^+ + e^- \to \tau^+(\mu^+ \nu_\mu) + \tau^-(-\nu_\mu \nu_\tau \nu_\tau)$$

The Feynman diagram for the signal process is as depicted in Fig. 1. Due to the fact that $\sqrt{s} = 0.1(10^{12})$ is sufficiently small, the naive narrow width approximation (NWA) is adopted when calculating the LFV decay $\tau \to \mu \gamma$, and the muon and photon are assumed isotropic in the rest frame of the $\tau$ lepton. However, we do not employ the naive NWA to deal with the SM decay $\tau \to \mu \bar{\nu}_\mu \nu_\tau$ for the LFV signal, and keep the off-shell contribution and spin correlation effect from the potentially resonant intermediate $\tau$ lepton.

In other words, we treat the LFV signal as 3-body production processes, $\sigma(e^+ e^- \to \tau^+ \tau^- \to \mu^+ \nu_\mu \bar{\nu}_\gamma + X)$ and $\sigma(e^+ e^- \to \tau^+ \tau^- \to \mu^+ \nu_\mu \gamma + X)$, followed by sequential 2-body LFV decay $\tau \to \mu \gamma$. Then the cross section of the signal process is as follows and the $\mu$ and $\gamma$ can be factorized as

$$\sigma(e^+ e^- \to \tau^+ \tau^- \to \mu^+ \nu_\mu \bar{\nu}_\gamma)$$

and

$$\times \text{Br}(\tau \to \mu \gamma).$$

Given the efficiency of detector resolution, no requirement on the missing energy $E_t$ is raised by escaping neutrinos is imposed. Thus, the detectable LFV signal at the CTF is comprised of two muons and an isolated photon in the final state as $\mu^+ \mu^- \gamma + X$, where $X$ denotes all the undetected neutrinos and one hard muon is from $\tau$ LFV decay and the other soft one from the standard $\tau$ leptonic decay. Accordingly, the leading background to the LFV signal comes from the $e^+e^- \to \mu^+\gamma$ process, which is depicted in Fig. 2 as the leading order (LO) contribution.

However, due to the smallness of the $\tau$ LFV decay branching ratio, the effect of the SM background that involves four

1 $\tau^{\pm \pm}$ might be off-shell depending on the kinematics of the final $\mu^\pm \nu_\mu (\tau^{\pm \pm})$ system.
Fig. 2 The LO Feynman diagrams for the background process \(e^+e^- \rightarrow \mu^+\mu^-\gamma\)

![Feynman diagrams](image)

Fig. 3 \(\mu_1\gamma\) invariant mass distributions for the LFV signal and the SM background after applying baseline cuts

![Invariant mass distributions](image)

Fig. 4 The trailing muon momentum spectra of the LFV signal and the SM background after applying successive baseline cuts and cut1

![Momentum spectra](image)
the current experimental limit derived at the B factory. For example, with one year data (~1000 fb⁻¹) taken at the CTF, a new 2σ upper bound Br(τ → μγ) < 4.2 × 10⁻⁹ can be obtained, which is about one order of magnitude smaller than the current experimental upper bound of 4.4 × 10⁻⁸; with three year run, the CTF could surpass the proposed Super-B factory in the sensitivity of searching τ LFV decay.

4 Constraints on new physics

New upper bound on Br(τ → μγ) expected at the CTF would constrain new physics beyond the SM. Among all the extensions of the SM, the leptoquark (LQ) model is a promising one to interpret LFV decays and has been extensively studied. In addition to the spin and gauge quantum numbers, the leptoquarks carry both lepton number and baryon number, and the spin-0 and spin-1 particles are called scalar and vector leptoquarks, respectively. The renormalizable and SU(3)C ⊗ SU(2)L ⊗ U(1)Y invariant interactions between scalar leptoquarks and SM fermions are given by the following Lagrangian [11]:

\[
\mathcal{L}_{\text{LQ}} = \begin{bmatrix}
\lambda_0 \overline{q_L} \tau^2 l_L + \lambda_0 \overline{u_R} e_R \\
\lambda_0 \overline{d_R} l_L \cdot \tilde{S}_1 \\
\end{bmatrix} S^a \end{bmatrix} + \begin{bmatrix}
\hat{\lambda}_0 \overline{q_L} \tau^2 l_L + \hat{\lambda}_0 \overline{u_R} e_R \\
\hat{\lambda}_0 \overline{d_R} l_L \cdot \tilde{S}_1 \\
\hat{\lambda}_0 \overline{u_R} e_R + \hat{\lambda}_0 \overline{d_R} l_L \cdot \tilde{S}_1 \\
\hat{\lambda}_0 \overline{d_R} l_L \cdot \tilde{S}_1 \\
\end{bmatrix} S^a \end{bmatrix} + \text{h.c.},
\]

where \( q_L \) and \( l_L \) denote the left-handed \( SU(2)_L \) doublet quarks and leptons of the SM, and \( u_R, d_R, \) and \( e_R \) are the right-handed \( SU(2)_L \) singlet quarks and charged leptons, respectively. We use \( S^a_j \) to denote scalar leptoquark, where the subscript \( j \) can value 0 and 1/2 indicating \( SU(2)_L \) singlet and doublet, respectively, and \( Y \) stands for hyper-
charge. Color and generation indices have been suppressed. τ′ (i = 1, 2, 3) are three Pauli matrices.

In investigating the rare decay τ → µγ, we also require the interactions between the scalar leptoquarks and photon. The photon interactions arise from the following SU(2)L ⊗ U(1)Y invariant kinematic terms of the scalar leptoquarks:

\[ \mathcal{L}_{\text{kinetic}} = (D^\mu S)^\dagger (D_\mu S). \]  

(4.2)

The SU(2)L ⊗ U(1)Y covariant derivative \( D_\mu \) is given by

\[ D_\mu = \partial_\mu - ig \sum_{i=1,2,3} W^i_\mu T^i - i g' \frac{Y}{2} B_\mu, \]

(4.3)

where \( W^i_\mu \) (i = 1, 2, 3) and \( B_\mu \) are the SU(2)L and U(1)Y gauge fields, respectively, and \( T^i \) (i = 1, 2, 3) are the generator matrices for the SU(2)L representation occupied by the scalar leptoquarks. From Eq. (4.2) we obtain the photon interaction for a scalar leptoquark as

\[ \mathcal{L}_{LQ,\gamma} = ie Q_{LQ} \left[ (\partial_\mu S^\dagger) S - S^\dagger (\partial_\mu S) \right] A^\mu, \]

(4.4)

where \( A^\mu \) is the photon field and \( Q_{LQ} \) represents the electric charge of the scalar leptoquark S.

For simplicity, we assume that all the LFV couplings except \( \lambda^L_{ij} \) (i = 1, j = 2, 3), where i and j are quark and lepton-flavor indices, are zero. Therefore, only \( \tau \rightarrow u-LQ \) and \( \mu \rightarrow u-LQ \) couplings are non-zero, and the τ lepton can decay into \( \mu + \gamma \) via quark–leptoquark involved loops at the LO, as shown in Fig. 6.

The LO decay width for the LFV decay process \( \tau \rightarrow \mu \gamma \) is given by

\[ \Gamma(\tau \rightarrow \mu \gamma) = \frac{1}{2} \frac{1}{8 \pi} \frac{1}{m_\tau^2} \sum_{\text{spin}} |\mathcal{M}_{\tau \rightarrow \mu \gamma}|^2, \]

(4.5)

where \( \mathcal{M}_{\tau \rightarrow \mu \gamma} \) is the amplitude for the Feynman diagrams in Fig. 6 and \( \vec{p}_{\mu, \text{cm}} \) is the three-momentum of \( \mu \) in the rest frame of the initial τ lepton. The summation is taken over the spins of the initial and final state particles and the factor \( \frac{1}{2} \) arises from the spin average of the initial state. We compute \( \mathcal{M}_{\tau \rightarrow \mu \gamma} \) by using the related Feynman rules obtained from Eqs. (4.1) and (4.4), and we adopt the Passarino–Veltman reduction method to convert the one-loop amplitude to scalar integrals. The loop divergence is naturally canceled for these four diagrams with no necessity to introduce any counterterm. The LFV decay branching ratio for \( \tau \rightarrow \mu \gamma \) can thus be expressed as

\[ \text{Br}(\tau \rightarrow \mu \gamma) = \frac{9 \alpha_{\text{ew}} (\lambda_1 \lambda_2^2) (m_\tau^2 - m_\mu^2)}{1024 \pi^4 \Gamma_\tau m_\mu^2} \times \left[ (|F_1|^2 + |F_2|^2) (m_\tau^2 + m_\mu^2) - 4 \text{Re}(F_1 F_2^*) m_\tau m_\mu \right], \]

(4.6)

where \( \lambda_1 = \lambda^L_{12} \) and \( \lambda_2 = \lambda^L_{13} \), denoting the \( \mu \rightarrow u-LQ \) and \( \tau \rightarrow u-LQ \) coupling strengths, respectively. The form factors \( F_{1,2} \) are given in the appendix.

Under the simplicity assumption, there are only three parameters of scalar leptoquark determining the \( \tau \rightarrow \mu \gamma \) LFV decay, namely the couplings \( \lambda_1 \lambda_2 \) and the scalar leptoquark mass \( M_{LQ} \). As shown in Eq. (4.6), the τ LFV decay branching ratio is proportional to \( \lambda_1 \lambda_2^2 \) but it is in a much complicated way related to \( M_{LQ} \). The dependence of \( \text{Br}(\tau \rightarrow \mu \gamma) \) as functions of \( \lambda_1 \lambda_2 \) and the leptoquark mass \( M_{LQ} \) is presented in Fig. 7.

According to the dependence, a joint constraint on \( \lambda_1 \lambda_2 \) and \( M_{LQ} \) can be derived from the upper bound on \( \text{Br}(\tau \rightarrow \mu \gamma) \) expected at the CTF, as shown in Fig. 8. One can estimate from the plot the upper bound on \( \lambda_1 \lambda_2 \) and the lower bound on \( M_{LQ} \) for given \( M_{LQ} \) and \( \lambda_1 \lambda_2 \), respectively. For example, with 1000 fb⁻¹ data expected at the CTF, one can get

\[ \lambda_1 \lambda_2 < 7.2 \times 10^{-2}, \quad (M_{LQ} = 800 \text{ GeV}, \quad 95 \% \text{ C.L.}), \]

\[ M_{LQ} > 900 \text{ GeV}, \quad (\lambda_1 \lambda_2 = 9 \times 10^{-2}, \quad 95 \% \text{ C.L.}). \]

(4.7)
Besides the above specified leptoquark interpretation, the \( \tau \) LFV decay can be expressed in a model-independent formalism. An effective vertex \( \tau - \mu - \gamma \) can be introduced in the form of \( \frac{1}{m_\tau} \sigma^{\mu\nu} p_\nu (\sigma_L P_L + \sigma_R P_R) \), where \( \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \), \( P_{L,R} = (1 \mp \gamma^5)/2 \), and \( p_\nu \) is the momentum of the photon [12]. Then the branching ratio for \( \tau \to \mu \gamma \) can be simply expressed in terms of the form factors \( \sigma_L \) and \( \sigma_R \) as

\[
\text{Br}(\tau \to \mu \gamma) = \frac{(m_\tau^2 - m_\mu^2)^3 (|\sigma_L|^2 + |\sigma_R|^2)}{16\pi \Gamma_\tau m_\tau^5}.
\]  

Similarly, a joint upper bound on \( |\sigma_L| \) and \( |\sigma_R| \) can be deduced from the upper bound on \( \text{Br}(\tau \to \mu \gamma) \). As shown in Fig. 9, a more stringent upper bound of \( \sqrt{|\sigma_L|^2 + |\sigma_R|^2} < 5.2 \times 10^{-10} \), much smaller than the current experimental limit, could be derived, if the \( \tau \to \mu \gamma \) LFV decay is not detected with 1000 \( fb^{-1} \) integrated luminosity expected at the CTF.

### 5 Summary

In this paper, we investigate the potential of searching \( \tau \to \mu \gamma \) LFV decay at the CTF. With a center-of-mass energy of 3.7 GeV of electron–positron collisions, \( \tau \) leptons can be copiously produced in pairs at the CTF. The LFV signal processes \( e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\gamma \nu_\mu\bar{\nu}_\mu \) are featured by a detectable final state of \( \mu_1\mu_2\gamma + X \), namely one hard leading \( \mu_1 \) along with a hard photon from \( \tau \) radiative LFV decay and one soft trailing \( \mu_2 \) from the standard \( \tau \) leptonic decay, and leaving the missing energy from escaping neutrinos unmeasured. To improve the significance of the \( \tau \to \mu \gamma \) LFV decay at the CTF, we propose a four-step event selection strategy: an invariant mass window on \( \mu_1\gamma \) system around \( m_\tau \) and a momentum cut on \( \mu_2 \) are imposed to eliminate the dominant \( e^+e^- \to \mu^+\mu^-\gamma \) SM background, and then an energy window and a momentum window on \( \mu_1\gamma \) system are successively applied to significantly suppress the \( e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\gamma \nu_\mu\bar{\nu}_\mu \) subleading SM background. It can be predicted with a couple years of CTF running, new sensitivities on \( \text{Br}(\tau \to \mu \gamma) \), which could surpass current experimental upper bound and those expected at the Super-B factory, can be achieved. The new upper limit on \( \text{Br}(\tau \to \mu \gamma) \) expected at the CTF would cer-
tainly constrain the parameter space of new physics beyond
the SM, either in specific theories as leptoquark theory or in
a model-independent effective formalism.

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Appendix

The form factors $F_1$ and $F_2$ are expressed as

$$F_1 = Q_u \left[ 2C_{00}^{(1)} + m_t^2 \left( C_{11}^{(1)} + C_{12}^{(1)} \right) + m_\mu^2 \left( C_2^{(1)} + C_{22}^{(1)} + C_{12}^{(1)} \right) - m_u^2 C_0^{(1)} - \frac{1}{2} \right]$$

$$-2 Q_{LQ} C_{00}^{(2)} - \frac{m_\mu^2}{m_t^2 - m_\mu^2} \left( B_0^{(1)} + B_1^{(1)} \right)$$

$$+ \frac{m_\mu^2}{m_t^2 - m_\mu^2} \left( B_0^{(2)} + B_1^{(2)} \right),$$

$$F_2 = - Q_u m_\tau m_\mu \left( C_0^{(1)} + C_1^{(1)} + C_2^{(1)} \right) - \frac{m_\tau m_\mu}{m_\tau^2 - m_\mu^2} \left( B_0^{(1)} + B_1^{(1)} - B_0^{(2)} - B_1^{(2)} \right),$$

where

$$B_i^{(1)} = B_i(m_\mu^2, M_{LQ}^2, m_u^2),$$

$$B_i^{(2)} = B_i(m_t^2, M_{LQ}^2, m_u^2),$$

$$C_{j,k}^{(1)} = C_{j,k}(m_t^2, 0, m_\mu^2, M_{LQ}^2, m_u^2, m_\mu^2),$$

$$C_{j,k}^{(2)} = C_{j,k}(m_t^2, m_\mu^2, 0, M_{LQ}^2, m_u^2, M_{LQ}^2),$$

and $M_{LQ}$ is the mass of scalar leptoquark. The definitions of
one-loop 2- and 3-point functions $B_i (i = 0, 1)$ and $C_{j,k}$
($j, k = 0, 1, 2$) are given in Ref. [13].

Note added After submitting this paper we found another
calculation of this LFV signal at the HIEPA [14].

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