EFFECTIVE FIELD THEORY OF MULTIPARTICLE CORRELATIONS

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ABSTRACT

We present an effective field theory of multiparticle correlations based on analogy with Ginzburg-Landau theory of superconductivity. We assume that the field represents particle density fluctuations, and show that in the case of gaussian-type effective action there are no higher-order correlations, in agreement with the recent data. We predict that two-particle correlations have Yukawa form. We also present our results for the two-dimensional and one-dimensional two-particle correlations (i.e. cumulants) as projections of our theory to lower dimensions.

1. Introduction

Recent experimental measurements of fluctuations in transverse energy in ultrarelativistic heavy-ion collisions have provided evidence that nuclear constituents scatter and produce particles coherently [1]. The observed fluctuations in the deposited energy and multiplicity are remarkably large and can not be described by an independent-collision model [2]. The same conclusion has been reached when multiparticle density fluctuations in different phase space regions have been studied via factorial moments, defined as

\[ F_q(\delta y) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m - 1) \ldots (n_m - q + 1) \rangle}{\langle n_m \rangle^q}, \]

where \( M \) is the number of rapidity bins \( (\delta y = Y/M) \) and \( n_m \) is the number of particles in the \( m^{th} \) bin [3]. In the past few years, these moments have been measured for different targets and projectiles at energy of 200GeV/nucleon [4]. They were found to increase with decreasing bin size indicating nonstatistical fluctuations and being incompatible with the predictions of the standard classical hadronization models embedded in the existing Monte Carlo models [4]. In addition, the observed effect,

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sometimes referred to as the “intermittency effect”, can not be accounted for by the superposition of independent nucleon-nucleon collisions, even when rescattering and geometrical effects are included [3].

The possibility of creating the new form of matter, the quark-gluon plasma, in high-energy heavy-ion collisions have inspired intensive theoretical work on identifying the unambiguous QGP signal. The observation of the unusually large multiparticle density fluctuations have created a new excitement in the field, especially as a possibility of pointing towards the onset of the phase transition from quark-gluon plasma to hadronic matter. Phase transitions in QCD at high temperatures are of general interest – they are directly relevant to cosmology, since such a phase transition occurred throughout the universe during the early moments of the big bang and a first order phase transition could have altered primordial nuclear abundances. Unfortunately, up to now there are no conclusive predictions for detecting the quark matter in heavy-ion collisions and there is no theory to describe the observed “intermittency” phenomenon [5].

2. Multiparticle Correlations in Heavy-Ion Collisions

Multiparticle correlations in three “dimensions” are usually measured in the following way. A given total interval $\Omega_{\text{tot}} = \Delta Y \Delta \phi \Delta P$ is subdivided into $M^3$ bins of side lengths $(\Delta Y/M, \Delta \phi/M, \Delta P/M)$. With $n_{klm}$ the number of particles in bin $(k, l, m)$ and $n^{[q]} = n!/(n - q)!$ the “vertical” factorial moment is

$$F_v^q(M) = \frac{1}{M^3} \sum_{k,l,m=1}^M \frac{\langle n^{[q]}_{klm} \rangle}{n_{klm}^q} = \frac{1}{M^3} \sum_{k,l,m=1}^M \int_{\Omega_{klm}} \prod_i d^3x_i \rho_q(x_1 \ldots x_q) \int_{\Omega_{klm}} d^3x \rho_1(x) q^q. \quad (2)$$

The second equality illustrates how the factorial moment can be written in terms of integrals of the correlation function [6]. The alternative “horizonatal” factorial moment is often preferred for three-dimensional analysis; this form, while being much more stable, has the drawback that it depends on the shape of the one-particle distribution function $\rho_1$.

To measure true particle correlations, known as cumulants, the trivial background must be subtracted. The first two cumulants are

$$C_2(x_1, x_2) = \rho_2(x_1, x_2) - \rho_1(x_1)\rho_1(x_2) \quad (3)$$

$$C_3(x_1, x_2, x_3) = \rho_3(x_1, x_2, x_3) - \sum_{\text{perm}} \rho_1(x_1)\rho_2(x_2, x_3) + 2\rho_1(x_1)\rho_1(x_2)\rho_1(x_3).$$

By integrating these relations over each bin, one can derive equations for integrated cumulants $K_q^q = \int \prod dx_k C_q / (\int dx \rho_1(x))^q$ in terms of the above vertical factorial moments [7],

$$K_2^q = F_2^v - 1, \quad K_3^q = F_3^v - 3F_2^v + 2 \quad \text{etc.} \quad (4)$$

Whenever there are no true correlations, these cumulants become zero. It was found that in the case of heavy-ion collisions, there are only two-particle correlations: while
$K_2$ is positive, the values of $K_3$, $K_4$ and $K_5$ have been found to be consistent with zero [8]. First found in terms of one-dimensional rapidity data, this has been confirmed by measurements by NA35 in two and three dimensions [9]. Corresponding findings for other nuclei and energies were published before [10].

3. Effective Field Theory of Multiparticle Correlations

We have seen that particles produced in high-energy heavy-ion collisions exhibit only two-particle correlations, indicating that perhaps higher-order correlations are washed out by rescattering of the initially correlated particles. Presently, there is no theory that describes this phenomena. Recently, we have proposed a three-dimensional statistical field theory of density fluctuations which has these features [11,12]. This model was formulated in analogy with the Ginzburg-Landau theory of superconductivity. The large number of particles produced in ultrarelativistic heavy-ion collisions justifies the use if a statistical theory of particle production. The formal analogy with the statistical mechanics of a one-dimensional “gas” was first pointed out by Feynman and Wilson and was later further developed by Scalapino and Sugar [13] and many others [5]. The idea is to build a statistical theory of the macroscopic observables by imagining that the microscopic degrees of freedom are integrated out and represented in terms of a few phenomenological parameters and by postulating that this theory will eventually be derived from a more fundamental theory, such as QCD.

While in the G-L theory of superconductivity the field (i.e. the order parameter) represents superconducting pairs, in the particle production problem, the relevant variable is the density fluctuation. We define a random field $\Phi$ as a function in a three-dimensional space spanned by $(y, \phi, p_\perp)$. Throughout, $p_\perp$ will be implicitly divided by a constant scale $P$ so that it is dimensionless. Since we are not looking for a phase transition, we omit the quartic term and start with the functional [11]

$$F[\Phi] = \int_0^P dy \int_{-p/2}^{p/2} d^2 p_\perp \left[ a^2 \left( \frac{\partial^2 \Phi}{\partial y^2} \right) + a^2 \left( \nabla_{\hat{p}_\perp} \Phi \right)^2 + \mu_\perp^2 \Phi^2 \right]. \quad (5)$$

Taking the appropriate functional derivative, we find for the functional (5) the three-dimensional form of the two-point function

$$\langle \Phi(\vec{x}_1)\Phi(\vec{x}_2) \rangle = \frac{1}{8\pi a^2} \frac{e^{-R/\xi} R}{R} , \quad (6)$$

where $\xi = a/\mu$ and $R \equiv [(y_1 - y_2)^2 + p_{1\perp}^2 + p_{2\perp}^2 - 2p_{1\perp}p_{2\perp}\cos(\phi_1 - \phi_2)]^{1/2}$. Further, we define $\Phi(\vec{x})$ as the fluctuation at the point $\vec{x}$ of the particle density for a given event, $\hat{\rho}_1(\vec{x})$, above/below the mean single particle distribution $\rho_1$ at that point:

$$\Phi(\vec{x}) = \frac{\hat{\rho}_1(\vec{x})}{\rho_1(\vec{x})} - 1. \quad (7)$$

Through these definitions, we find that $\langle \Phi(\vec{x}_1)\Phi(\vec{x}_2) \rangle = k_2(\vec{x}_1, \vec{x}_2)$ and that all higher order cumulants become exactly zero, $k_{q \geq 3} = 0$. By means of the specific form of
the functional (5) and the definition of $\Phi$ as a fluctuation, we take account of the experimental facts in this regard. What is not experimentally certain and is to be tested is whether the second order correlations obey the Yukawa form (6).

4. Projections of Multiparticle Correlations (i.e. Cumulants) to Lower Dimensions

The second reduced cumulant $k_2 \propto e^{-R/\xi}/R$ can be compared to data only after a suitable integration over its variables. For three dimensions, the vertical integrated cumulant is given by $K_2^v(\delta y, \delta \phi, \delta p) = F_2^v - 1 = M^{-3} \sum_{k,l,m=1}^M K_2^v(k,l,m)$, (always taking $\bar{\vec{x}} \equiv (y, \phi, p_\perp)$), with

$$K_2^v(k, l, m) = \frac{\int_{\Omega_{klm}} d^3\vec{x}_1 d^3\vec{x}_2 C_2(\vec{x}_1, \vec{x}_2)}{\left[ \int_{\Omega_{klm}} d^3\vec{x} \rho_1(\vec{x}) \right]^2} = \int_{\Omega_{klm}} d^3\vec{x}_1 d^3\vec{x}_2 \frac{k_2(\vec{x}_1, \vec{x}_2) \rho_1(\vec{x}_1) \rho_1(\vec{x}_2)}{\left[ \int_{\Omega_{klm}} d^3\vec{x} \rho_1(\vec{x}) \right]^2},$$

i.e. the integration of $k_2$ involves a correction due to the shape of the one-particle three-dimensional distribution function $\rho_1(\vec{x})$. Eq. (8) as it stands is exact; horizontal versions have also been derived [12]. A first test of our model would therefore be to see if Eq. (4) or its horizontal equivalent obeys the data in $(y, \phi, p_\perp)$.

The theoretical $k_2(\vec{x}_1, \vec{x}_2)$ is further tested by comparing to factorial cumulant data of lower dimensions. For example, in $(y, \phi)$, the cumulant is $K_2^v(\delta y, \delta \phi) = M^{-2} \sum_{lm} K_2^v(l, m)$ with $p_\perp$ integrated over the whole window $\Delta P$,

$$K_2^v(l, m) = \int_{\Omega_m} dy_1 dy_2 \int_{\Omega_l} d\phi_1 d\phi_2 \int_{\Delta P} dp_1 dp_2 \frac{k_2(\vec{x}_1, \vec{x}_2) \rho_1(\vec{x}_1) \rho_1(\vec{x}_2)}{\left[ \int_{\Omega_m} dy \int_{\Omega_l} d\phi \int_{\Delta P} dp \rho_1(\vec{x}) \right]^2}.$$

Cumulants of other variable combinations and lower dimensions are obtained analogously. With these relations it is thus possible, given any three-dimensional theoretical function $k_2$ (or $r_2$), to compute factorial cumulants and moments for any combination of its variables. Doing this for different variables serves as a strong test of the theoretical function as the moments probe its different regions.

In Figures 2-3 (see Ref. 12) we present our results for the vertical and horizontal factorial moments. In our calculation of the projections, we have made the following approximations: We factorize the one-particle distribution into its separate variables: $\rho_1(\vec{x}) = \langle N \rangle_{\Omega_1} g(y) h(\phi) f(p_\perp)$, where the three distributions $g$, $h$ and $f$ are separately normalized over their respective total intervals $\Delta Y$, $\Delta \Phi$ and $\Delta P$. The azimuthal distribution is taken as flat, $h(\phi) = 1/\Delta \Phi$. We use the full experimental parametrization for $f(p_\perp)$ provided by NA35. The choice of two parameters $\alpha$ and $\xi$ in Figures 2-3 [12] are given only as an illustration. They should be determined by comparison with three-dimensional data. Once they are fixed, the two-dimensional and one-dimensional projections are genuine predictions of the theory.

In summary, we have presented a three-dimensional effective field theory of multiparticle correlations, which gives no higher-order correlations, in agreement with the recent heavy-ion data. In our theory, two-particle correlations have Yukawa form and the corresponding integrated cumulants have singular behavior for small
regions of phase space. This prediction seems to be in qualitative agreement with the recent NA35 data [14]. In addition, we have shown that once the parameters $a$ and $\xi$ are determined from comparison with three-dimensional data, our theory gives genuine predictions for the two-dimensional and one-dimensional cumulants. It will be interesting to see whether all our predictions are confirmed with recent NA35 data presented at this conference [14].

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6. References

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