The Friedberg-Lee Symmetry and Minimal Seesaw Model

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The Friedberg-Lee (FL) symmetry is generated by a transformation of a fermionic field \( q \) to \( q + \xi z \). This symmetry puts very restrictive constraints on allowed terms in a Lagrangian. Applying this symmetry to \( N \) fermionic fields, we find that the number of independent fields is reduced to \( N - 1 \) if the fields have gauge interaction or the transformation is a local one. Using this property, we find that a seesaw model originally with three generations of left- and right-handed neutrinos, with the left-handed neutrinos unaffected but the right-handed neutrinos transformed under the local FL translation, is reduced to an effective theory of minimal seesaw which has only two right-handed neutrinos. The symmetry predicts that one of the light neutrino masses must be zero.

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In trying to understand the properties of neutrinos, Friedberg and Lee [1] proposed a symmetry translating a fermionic field \( q \) to \( q + \xi z \) where \( z \) is an element of Grassmann algebra and \( \xi \) is a complex number. We will call this symmetry the Friedberg-Lee (FL) symmetry. Various applications of the FL symmetries have been studied [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. In this Letter we further study some properties of the FL symmetry being a global or a local symmetry, and apply to neutrino seesaw models. We find that applying the FL symmetry to the whole Lagrangian is dramatically different than applying the same symmetry only to terms related to fermion masses. In the latter case the FL symmetry along a certain direction implies a zero mass eigenstate of fermions, but in the former it implies complete decoupling of the same field in the theory if the fermionic fields have gauge interaction or the FL transformation is local. That is, applying the FL symmetry to \( N \) fermionic fields, we find that the number of independent fields is reduced to \( N - 1 \). Using this property, we find that a seesaw model originally with three generations of left-
and right-handed neutrinos, with the left-handed neutrinos unaffected but the right-handed neutrinos transformed under the local FL translation, is reduced to an effective theory of minimal seesaw which has only two right-handed neutrinos.

The FL symmetry and number of independent fields

Assuming that there are $N$ generations of fermion fields $N^i_R$ with certain gauge charges. Under a FL transformation $N^i_R$ transform as

$$N^i_R \rightarrow N^i_R + \xi_i z,$$

with $z$ an element of the Grassmann algebra, anti-commuting with the field operator $N^i_R$. As an element of the Grassmann algebra, $z$ can be space-time independent or space-time dependent. $z = (z_1, z_2)^T$, with $z_\alpha (\alpha = 1, 2)$ two Grassmann numbers, is a two-component spinor if using two-component theory describing fermionic field. $z$ is a four-component spinor if using four-component theory describing fermionic field. $\xi_i (i = 1, \ldots, N)$ is a particular set of c-numbers, similar to that used in Ref. [2] for quarks. In-equivalent choices of $\xi_i$ say that fermionic fields are translated in different directions in $N$-dimensional space of $(N^1_R, \cdots, N^N_R)$. With a particular set of $\xi_i$ we implement the FL translation of $N^i_R$ only along a specific direction described by a set of $\xi_i$ following Ref. [2].

For a theory having only these $N$ fermionic fields, one can write the renormalizable Lagrangian as the following

$$\mathcal{L}_R = \gamma_{ij} N^i_R \gamma_{\mu} (i D^\mu N^j_R) - \frac{1}{2} \left[ m_{ij} N^{ic}_R N^j_R + H.C. \right] ,$$

(2)

where $i, j = 1, \cdots, N$ and summation over repeated indices is assumed. $\gamma_{ij}$ and $m_{ij}$ are Hermitian and symmetric $N \times N$ matrices, respectively. $N^c_R$ is the charge conjugated field of $N_R$. $D^\mu = \partial^\mu + ig A^\mu$ is the covariant derivative, $A^\mu$ is the gauge field and $g$ is the gauge coupling.

Under the transformation Eq. (1) the Lagrangian transforms as

$$\mathcal{L}_R \rightarrow \mathcal{L}_R + \gamma_{ij} \xi_j \bar{N}^i_R \gamma_{\mu} (i D^\mu z) + \gamma_{ij} \xi_j^* \bar{z} \gamma_{\mu} (i D^\mu N^j_R) + \gamma_{ij} \xi_j^* \xi_j \bar{z} \gamma_{\mu} (i D^\mu z) - \frac{1}{2} \left[ m_{ij} \xi_j \bar{N}^{ic}_R N^j_R + m_{ij} \xi_j^* \xi_j \bar{z} z + H.C. \right]$$

(3)

Requiring that the Lagrangian $\mathcal{L}_R$ to be invariant under the FL symmetry, for the case with $g \neq 0$, implies

$$\gamma_{ij} \xi_j = 0 , \quad m_{ij} \xi_j = 0 .$$

(4)
Both equations imply that the linear combination \( N_R^0 = \sum_i^N N_i R_i / \sqrt{\sum_j \xi_j^* \xi_j} \) is an eigenvector corresponding to zero eigenvalues for \( \gamma_{ij} \) and \( m_{ij} \) matrices. It has been pointed out [3, 8, 17] that if one requires the above equations to be true for an arbitrary set of parameters \( \xi_i \) (a generic FL symmetry), then there are \( N \) number of zero eigenvalues, that is, \( m_{ij} \) must be zero. As have been mentioned before that we follow Ref. [2] to choose FL invariance along a particular direction in \( \xi_i \) parameter space. Therefore there is only one zero eigenvalue for \( m_{ij} \) and also for \( \gamma_{ij} \). Note that the zero eigenvalues in both \( \gamma_{ij} \) and \( m_{ij} \) have the same eigenvector does not mean that the \( \gamma_{ij} \) and \( m_{ij} \) can be, in general, simultaneously diagonalized by unitary transformations in the form \( V^\dagger \gamma V = \hat{\gamma} \) and \( V^T m V = \hat{m} \). Here \( \hat{\gamma} \) and \( \hat{m} \) are diagonal matrices.

If \( g = 0 \), applicable if \( N_R \) is right-handed neutrino, depending on whether the FL transformation is global or local, there are different implications. If the FL is a global symmetry, that is \( z \) is independent of space-time which leads to \( \partial^\mu z = 0 \), the kinetic energy terms are invariant up to terms proportional to total derivatives. There is no constraint on the form of \( \gamma_{ij} \). However, if the transformation is local as discussed in Ref. [7], that is \( \partial^\mu z \neq 0 \), the kinetic terms are not invariant under the FL transformation unless \( \gamma_{ij} \xi_j = 0 \).

If one only applies the FL symmetry to the mass term, regardless whether the FL is global or local, one predicts a zero eigenmass [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. If one applies the FL symmetry to the whole Lagrangian \( L \), the consequences are different. Taking the latter as requirement for the Lagrangian, we find that, if the fermionic fields have gauge interaction or the FL transformation is local, the eigenvector corresponding to the zero eigenvalues of \( \gamma \) and \( m \) matrices completely decouples from the theory. To see this let us work in the basis where \( \gamma \) is in a diagonalized form,

\[
\hat{\gamma} = \begin{pmatrix}
\gamma_1 & 0 & \cdots & 0 & 0 \\
0 & \gamma_2 & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \gamma_{(N-1)} & 0 \\
0 & 0 & \cdots & 0 & 0 \\
\end{pmatrix}
\]
The matrix in this basis must be able to be written, due to Eq. (4), in the following form

\[
m = \begin{pmatrix}
m_{11} & m_{12} & \cdots & m_{1(N-1)} & 0 \\
m_{12} & m_{22} & \cdots & m_{2(N-1)} & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
m_{1(N-1)} & m_{2(N-1)} & \cdots & m_{(N-1)(N-1)} & 0 \\
0 & 0 & \cdots & 0 & 0
\end{pmatrix}.
\] (6)

This implies that when writing in eigenvectors of \( \gamma \), the linear combination \( N^0_R \) does not show up anywhere in the Lagrangian. Assuming that the eigenvectors correspond to the non-zero eigenvalues \( \gamma_i \) are \( \nu^{i1}, \nu^{i2}, \ldots, \nu^{N-1} \), Lagrangian in Eq. (2) is reduced to

\[
\mathcal{L}_R = \gamma_i \bar{\nu}^i_R \gamma_{\mu} (i D^\mu \nu^i_R) - \frac{1}{2} [\tilde{m}'_{ij} \bar{\nu}^{ie}_R \nu^{ij}_R + H.C.] ,
\] (7)

where \( \tilde{m}' \) matrix is the left-upper corner \( (N-1) \times (N-1) \) matrix in Eq. (6).

By a re-scaling of the field \( \nu^i_R = \sqrt{\gamma_i} \nu'^i_R \), one can write the Lagrangian in the usual form

\[
\mathcal{L}_R = \bar{\nu}^i_R \gamma_{\mu} (i D^\mu \nu^i_R) - \frac{1}{2} [\tilde{m} \bar{\nu}^{ie}_R \nu^{ij}_R + H.C.] .
\] (8)

One can further diagonalize \( \tilde{m} = U^T \hat{m} U \), with \( U \) a unitary matrix, to obtain normalized mass eigenstates \( \nu^{mi}_R = U_{ij} \nu^{i}_R \). We finally have

\[
\mathcal{L}_R = \bar{\nu}^{mi}_R \gamma_{\mu} (i D^\mu \nu^{mi}_R) - \frac{1}{2} [\hat{m} \bar{\nu}^{miec}_R \nu^{mij}_R + H.C.] .
\] (9)

The above is a Lagrangian for \( N - 1 \) independent fields. Starting with \( N \) fermionic fields, after imposing the FL symmetry introduced in Eq. (1), the number of independent fields has been reduced by one if the fermionic fields have gauge interaction or the FL transformation is local. If the fermionic fields have no gauge interaction and the FL transformation is global, the number of independent fields is not affected.

One can understand the reduction of the number of fields in a different way as the following. One can build a Lagrangian which is invariant under the FL transformation by using all independent combinations of \( N^i_R \) which do not transform under the FL symmetry as building block. We note that \( \xi_j N^i_R - \xi_i N^j_R \) is manifestly invariant under the FL transformation. They should be naturally used to build \( L \). Because this construction is taking a difference of two fields, out of \( N \) fields only \( N - 1 \) such differences are independent. For example, if one takes \( q_j = \xi_j N^1_R - \xi_1 N^j_R \) as the \( N - 1 \) independent ones, \( \xi_j N^2_R - \xi_2 N^j_R \) can be expressed as
(ξ_2 q_j − ξ_j q_2)/ξ_1. Similarly for other combinations. Imposing the FL symmetry, Eq. (11), to a theory with \( N \) number of fields, only \( N - 1 \) are dynamic fields which are the real physical degrees of freedom in the theory, not all the \( N \) number of the field. The one drops out of the theory is \( N^0_R \) which is the linear combination of \( N^i_R \) in accordance with the FL translation introduced in Eq. (11). In another words, to have a theory having \( N \) number of dynamic fermion fields with a FL symmetry given in Eq. (11), one must start with a theory containing \( N + 1 \) fields.

### The FL symmetry and Seesaw Models

We now study seesaw models with FL symmetry. The simplest seesaw model [18] is the minimal standard model (SM) with additional right-handed neutrinos \( N^i_R \). Experimentally there are three light neutrinos with SM charged current interaction, a successful model for neutrinos must have three left-handed minimal SM lepton doublets. The number of right-handed neutrinos can, in principle, be different than their left-handed ones. But there should be at least two right-handed neutrinos in order to satisfy experimental constraint that two of the light neutrinos are massive. This is the so-called minimal seesaw model. It is sometimes also called the 3+2 seesaw model. This model has some interesting consequences [19, 20, 21], such as a zero mass light neutrino and possible connection of CP violating source for baryon asymmetry and low energy CP violating phases. A more symmetric model is the 3+3 seesaw model in which both the left- and right-handed are three generations. We find that a local FL symmetry can make a passage from a 3+3 seesaw model to a 3+2 minimal seesaw model making the theory with more predictive power. In the following we show this in details.

Particles relevant to our discussions are the three generations of left-handed lepton doublets \( L^i_L = (\nu^i_L, e^i_L)^T \), the three right-handed neutrino singlets \( N^i_R \), and the Higgs doublet \( H = (H^0/\sqrt{2}, H^-)^T \). The transformation properties of these fields are as follows. The left-handed leptons \( L_L \) and Higgs boson \( H \) do not transform under a local FL transformation, but the right-handed neutrinos \( N^i_R \) do:

\[
L_L \rightarrow L_L, \quad H \rightarrow H, \quad N^i_R \rightarrow N^i_R + \xi_i \zeta.
\]

As have been seen from our previous discussions that the local FL symmetry restricts the forms of allowed terms in the Lagrangian, we should pay special attentions to the fields transforming non-trivially under the FL symmetry. To this end we write all renormalizable
terms involving $N_R^i$ in the following for detailed analysis,

$$\mathcal{L} = \gamma_{ij} \bar{N}_R^i \gamma_{\mu} (i \partial^\mu N_R^j) - \frac{1}{2} [m_{ij} \bar{N}_R^i N_R^j + 2 Y'_{ij} \bar{L}_L^i H N_R^j + H.C.]$$ \hspace{1cm} (11)

Again, $\gamma$ is Hermitian, $m$ is symmetric. But there is no constraint on the form of $Y'$ before applying the FL symmetry.

The requirement that $\mathcal{L}$ being invariant under a local FL symmetry constrains the form of $\gamma_{ij}$, $m_{ij}$ and $Y'_{ij}$. Similarly to Eq. (4) we have

$$\gamma_{ij} \xi_j = 0, \quad m_{ij} \xi_j = 0, \quad Y'_{ij} \xi_j = 0.$$ \hspace{1cm} (12)

In general the matrix $Y'$ can be written in the following form

$$Y' = y'_1 u_1 v_1^\dagger + y'_2 u_2 v_2^\dagger + y'_3 u_3 v_3^\dagger,$$ \hspace{1cm} (13)

where $u_i$ are eigenvectors of $Y' Y'\dagger$ and $v_i$ are eigenvectors of $Y'\dagger Y'$. The constraint on $Y'$ in Eq. (12) implies that $v_3 = (\xi_1, \xi_2, \xi_3)^T$ and $y'_3 = 0$. $v_1$ and $v_2$ can be expressed as linear combinations of the other two orthogonal vectors, $(\xi^*_2, \xi^*_1, 0)^T$ and $(\xi^*_3, \xi^*_1, \xi^*_2, (|\xi_1|^2 + |\xi_2|^2)|\xi_3)^T$.

It is interesting to note that the combination: $N'^3_R = \xi_i N_R^i / \sqrt{\xi_j^2 \xi_j}$ is simultaneously the eigenvector of the zero eigenvalue of $\gamma$, $m$ and $Y'$. Choosing the other two orthogonal combinations as:

$$N'^1_R = \frac{\xi^*_2 N_R^1 - \xi^*_1 N_R^2}{\sqrt{|\xi_1|^2 + |\xi_2|^2}},$$

$$N'^2_R = \frac{\xi^*_3 \xi_1 N_R^1 + \xi^*_1 \xi_2 N_R^2 - (|\xi_1|^2 + |\xi_2|^2) N_R^3}{\sqrt{(|\xi_1|^2 + |\xi_2|^2)(|\xi_1|^2 + |\xi_2|^2) + |\xi_3|^2)}},$$ \hspace{1cm} (14)

and re-writing the Lagrangian $\mathcal{L}$ in terms of $N'^i_R$, we find that $N'^3_R$ decouples completely from the theory. We have

$$\tilde{\mathcal{L}} = \tilde{\gamma}_{ij} \bar{N}_R^i \gamma_{\mu} (i \partial^\mu N_R^j) - \frac{1}{2} [\tilde{m}_{ij} \bar{N}_R^i N_R^j + 2 \tilde{Y}'_{ij} \bar{L}_L^i H N_R^j + H.C.]$$ \hspace{1cm} (15)

where $\tilde{\gamma}$ and $\tilde{m}$ are now $2 \times 2$ matrices, and $\tilde{Y}'$ is a $3 \times 2$ matrix.

One then further diagonalizes $\gamma = V^\dagger \tilde{\gamma} V$ to define new fields $\nu'_i = V N'_R$ and re-scale the $\nu'_R$ fields by the square root values of the eigenvalues of $\gamma$, $\gamma_i$, $\nu'_R = \sqrt{\gamma_i} \nu'_R$. Finally one can rewrite the Lagrangian in the standard form

$$\mathcal{L} = \bar{\nu}_R \gamma_{\mu} (i \partial^\mu \nu_R) - \frac{1}{2} [\tilde{\bar{\nu}}_R M \nu'_R + 2 \bar{L}_L Y \nu_R + H.C.]$$ \hspace{1cm} (16)
where $M$ is a $2 \times 2$ matrix and $Y$ is a $3 \times 2$ matrix.

Without the term proportional to $Y$, one can diagonalize $M$ to reduce to Eq. (9). Actually even with non-zero $Y$, one can still diagonalize $M = U^T \hat{M} U$ to have the first two terms in the above equation look like Eq. (9), but the matrix $Y$ needs to be rotated with $\tilde{Y} = YU^\dagger$.

The theory defined by the Lagrangian in Eq. (16) is identical to a theory of three left-handed and two right-handed neutrinos, the minimal seesaw model [19]. The local FL symmetry has reduced right-handed fields by one degree of freedom.

We comment that if the FL symmetry is a global one, there is no constraint on the rank of the $\gamma_{ij}$ matrix. The linear combination $N^R_3$ does not disappear in the kinetic energy terms. Only the mass matrix terms are affected. There is a massless right-handed neutrino in the theory. This is the model considered in Ref. [3, 8].

**Some implications**

We now discuss some implications of the model for right-handed neutrinos to transform under a local FL transformation. After the electro-weak symmetry breaking, that is the Higgs develops a non-zero vacuum expectation value $< H > = v/\sqrt{2}$, the neutrino mass in the basis $(\nu_L, \nu_R)^T$ is given by

$$
\begin{pmatrix}
0 & Y^*v/\sqrt{2} \\
Y^tv/\sqrt{2} & M
\end{pmatrix}.
$$

This leads to the mass matrix $m_\nu$ for left-handed neutrinos to be

$$
m_\nu = -\frac{v^2}{2}Y^*M^{-1}Y^\dagger.
$$

One of the three light neutrinos has zero mass.

It has been previously shown that the minimal seesaw model is consistent with experimental data [19, 20, 21], although the detailed numbers of data have changed [22]. We will not go into details about the phenomenology here, but would like to point out that the zero eigenvalue for the neutrino mass can be traced to the FL symmetry of the theory.

Mathematically one understands why there is an zero eigenvalues by noting that $Y^\dagger$ is a $2 \times 3$ matrix and is rank 2. It has an eigenvector with zero eigenvalue:

$$
Y^\dagger u_3 = 0, \quad m_\nu u_3 = 0
$$

Here $u_3$ is the vector introduced in Eq. (13). It is the eigenvector associated with the $v_3$ vector of the FL symmetry in the right-handed sector.
We note that Eq. (19) implies that after electro-weak symmetry breaking one gets a residual symmetry in the light neutrino mass term. The left-handed neutrinos in the mass term is invariant under the FL-like transformation,

$$\nu \rightarrow \nu + u_3 z,$$

(20)

where $\nu = (\nu^1_L, \nu^2_L, \nu^3_L)^T$. We start with a FL symmetry, Eq. (10), of the full Lagrangian and end up with a residual FL symmetry for the seesaw masses of neutrinos. Note that the original FL symmetry applies to the right-handed neutrinos and the residual FL symmetry applies to left-handed neutrinos which can be traced back to the requirement that $y'_{3a} = 0$ in Eq. (13) dictated by the FL symmetry. The zero mass of a light neutrino is therefore a consequence of the FL symmetry. If the transformation is global, then this residual symmetry also applies to the kinetic energy terms.

It is interesting to note that any mass matrix for fermionic field $\nu$ with a zero eigenvalue, one can define a FL-like transformation related to the associated eigenvector $u$: $\nu \rightarrow \nu + u z$. Under this transformation, the mass term is invariant.

If future experimental data will determine that all three light neutrinos to have non-zero masses, the minimal seesaw needs to be extended. One might wonder if higher order loop corrections can make all three light neutrino masses non-zero. We find that this is not true because in the theory the FL is not broken, the masslessness of one of the neutrinos is true to all orders. To obtain a theory with at least three non-zero mass light neutrinos with FL symmetry imposed on a particular direction in $\xi_i$ parameter space, more fields need to be introduced. In our case since the local FL symmetry always reduce the number of fields by one, we need to start with more than three right-handed neutrinos. For example, starting with 4 right-handed neutrinos, after the reduction discussed before, the $M$ and $Y$ matrices in Eq. (9) become $3 \times 3$ matrices. The resulting theory is a $(3+3)$ seesaw model.

Conclusions

In summary we have studied consequences of the Friedberg-Lee symmetry for seesaw models. We find that if a local FL symmetry is imposed to the full Lagrangian of right-handed neutrinos, one of the right-handed neutrinos completely decouples from the theory. For specific model studies, we begin with a $3 + 3$ seesaw model, which is a model with three generations of left-handed and right-handed neutrinos. After applying a local FL symmetry to the right-handed sector, we arrive at a $3 + 2$ seesaw model, the minimal seesaw model,
which is a model with three generations of left-handed neutrinos and two generations of right-handed neutrinos. In this model one of the light neutrinos has zero mass as a consequence of the FL symmetry. The masslessness of one light neutrino means that there is a FL symmetry in the seesaw mass matrix of the light left-handed neutrinos. This FL symmetry in the seesaw mass matrix of the light left-handed neutrinos is a consequence of the FL symmetry imposed on right-handed sector of neutrinos in the original seesaw model. The FL symmetry can enhance the predictive power of a theory.

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