Non-commutative superspace from string theory

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Abstract

Turning on background fields in string theory sometimes has an alternative interpretation as a deformation of the target space geometry. A particularly well-known case is the NS-NS two form $B$, which gives rise to space-time non-commutativity. In this note we point out that this phenomenon extends to ten-dimensional superspace when employing a covariant quantization of the superstring, generalizing an observation by Ooguri and Vafa in four dimensions. In particular, we will find that RR field strengths give rise to a non-zero $\{\theta, \theta\}$ anti-commutator, just as in four dimensions, whereas the gravitino yields a non-zero value for $[x, \theta]$. 
1 Introduction

The idea that the coordinates of spacetime do not commute was proposed by Snyder in 1947 [1]. When supersymmetry and supergravity were invented, it was a natural idea to consider also the possibility that the fermionic coordinates of superspace do not commute. In 1982 a model was constructed [2] in which the Dirac bracket for a fermionic point particle was proportional to the coordinates of spacetime

\[ \{\theta^\alpha, \theta^\beta\}_D = \gamma^\alpha_\beta x^m. \] (1)

Due to the composite nature of \( x^m \), it was hoped that this might yield a granular structure of spacetime.

From a more mathematical point of view, non-commuting (super) coordinates arise naturally in non-commutative geometry and in particular in realizations of quantum groups [3]. These realizations employ (anti)-commutators of supercoordinates \( z^A = (x^m, \theta^a) \) that are quadratic in supercoordinates

\[ [z^A, z^B] = R^A_{CD} z^C z^D. \] (2)

Consistency requirements lead to a cubic equation for the matrix \( R \), the Yang-Baxter equation. In order to be able to treat cases like (1), a linear term was added to the right hand side of (2) in [4], and a model was constructed in 1 + 1 dimensional space with two \( \theta \)'s in which (1) and (2) were combined. This model still satisfied the Yang-Baxter equation.

Non-commutative coordinates also appear in string theory. As shown in [5, 6] and elaborated upon in [7], string theory in the presence of an NS-NS \( B \)-field admits in space-time an alternative formulation in terms of non-commutative geometry, where the coordinates satisfy

\[ [x^m, x^n] = i\theta^{mn}, \] (3)

where \( \theta^{mn} = (B^{-1})^{mn} \). Non-commutative field theories can be obtained by taking a suitable scaling limit where \( \alpha' \to 0 \) while scaling the space-time metric as \( g_{ij} \sim \alpha'^2 \) and keeping the \( B \)-field fixed.

A natural next step is to try to obtain non-commutative fermionic coordinates from string theory. Indications that such a structure might be relevant in string theory were recently found e.g. in [8, 9]. There is a rather trivial version of non-commutative superspace in the literature where \( \{\theta, \theta\} \) and \([x, \theta]\)
remain zero, but \([x, x] \neq 0\). This superspace is useful in providing a superfield formalism for certain supersymmetric non-commutative gauge theories, see e.g. \([10]\), but we will be interested in the case where also \(\{\theta, \theta\} \neq 0\) and \([x, \theta] \neq 0\). A general ansatz of this type was already discussed in \([11]\), but our focus will be to obtain such deformations from string theory.

Two of us already considered this problem a few years ago in collaboration with K. Skenderis, in the context of the NSR(spinning) string. However, due to the well-known difficulties of implementing spacetime spinors in the NSR approach, no results were obtained. The Green-Schwarz superstring was also considered, but here the well-known problems with its covariant quantization precluded a sound basis to depart from. Recently, however, a completely covariant quantization of superstrings was developed \([12, 13, 14, 15]\) which covariantizes Berkovits’ pure spinor approach (see e.g. the review \([16]\)). A manifestly super-Poincaré-invariant nilpotent BRST operator with a finite number of ghost fields was obtained, and a new definition of physical states was derived which yields the correct spectrum for the open and closed superstring at the massless and massive level.

Given that there now exists a covariant quantum description of a string model with spacetime supercoordinates \(x^m\) and \(\theta^\alpha\), it is possible to return to the question of the non-commutativity of supercoordinates and base the discussion on a concrete consistent quantum string with manifest spacetime super-Poincaré invariance. Recently, this was done in four dimensions by Ooguri and Vafa \([17]\) using the four dimensional covariant formulation of the superstring given in \([18]\). In particular, they found that the the graviphoton field strength (which sits in the RR sector of the theory) gives rise to non-commutative fermionic coordinates, and that field theories on such spaces are sensitive to higher order topological string amplitudes.

Here we will generalize the calculation of \([x, x]\) and \(\{\theta, \theta\}\) in \([17]\) to ten dimensions. In particular, following \([5, 6, 7]\), we will consider strings in arbitrary constant bosonic and fermionic background fields. The action is obtained by modifying the free action by adding the integrated \((1, 1)\) vertex operator with constant background fields. The general form of this vertex operator was recently obtained in \([13]\). Then we shall invert the kinetic operator to obtain the propagators, and after imposing suitable boundary conditions, we shall derive expressions for the (anti)-commutators of the supercoordinates in a suitable

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1We would like to thank R. Dijkgraaf for informing us of the results in \([17]\).
scaling limit. These expressions depend on the constant background fields. Once we allow these background fields to become $x, \theta$-dependent, and expand them in terms of $x$ and $\theta$, we arrive at relations as in (1) and (2). The $\{\theta, \theta\}$ anti-commutator is given by a bispinor that contains all RR field strengths. In particular, (1) is obtained by taking an axion (the RR pseudoscalar in IIB string theory) which is proportional to $\eta_{\mu\nu}x^\mu x^\nu$.

2 Action

In the formulation of [12, 13, 14, 15] the fields that appear are the superspace coordinates $x^\mu, \theta^\alpha, \bar{\theta}^{\bar{\alpha}}$, the fields $p_\alpha, p_{\bar{\alpha}}$ conjugate to $\theta$, and several ghost fields. We will restrict attention to the type IIB string, so that both $\theta$’s have the same chirality. The massless vertex operators have been described in [13] and they give rise to the linearized field equations of type IIB supergravity. The field equations appear as a set of equations for a set of superfields (see also [19]) which are the entries of the following matrix:

$$\mathcal{M}_{MN} = \begin{pmatrix} A_{\alpha\bar{\beta}} & A_{\alpha n} & A_{\bar{\alpha}\bar{\beta}} \\ A_{m\beta} & A_{mn} & A_{m\bar{\beta}} \\ A^{\alpha\bar{\beta}} & A^{\alpha n} & A^{\bar{\alpha}\bar{\beta}} \end{pmatrix}. \quad (4)$$

We will at first only be interested in vertex operators corresponding to constant fields and/or field strengths. To be precise, we will impose

$$\partial_m A_{\alpha\bar{\beta}} = \ldots = \partial_m A_{\bar{\alpha}\bar{\beta}} = 0. \quad (5)$$

In addition, we will take

$$D_\gamma A^{\alpha\bar{\beta}} = D_{\bar{\gamma}} A^{\alpha\bar{\beta}} = 0. \quad (6)$$

With these assumptions, the integrated vertex operator becomes relatively simple. It is given by

$$\mathcal{V}^{(0,0)} = F^{M}_\bar{z} F^{N}_{\bar{z}} \mathcal{M}_{MN} \quad (7)$$

where

$$F^{M}_\bar{z} = (\partial_\bar{z} \theta^\alpha, \Pi^m_\bar{z}, d_{z\alpha}). \quad (8)$$

The full form of the general integrated vertex operator is quite complicated, but because of the assumptions [5] and [6] the structure simplifies quite a lot and we are only left with (7). We see that all the terms are quadratic in
quantum fields, including the ghosts. We have checked for the open string that requiring the curvature \( f_{mn} \) (the lowest component of the superfield \( F_{mn} \)) and the gaugino \( u^\alpha \) to be constant, and choosing the gauge \( \theta^\alpha A_\alpha(x, \theta) = 0 \), the vertex operator is quadratic in \( x^m, \theta^\alpha, p_\alpha \) and the ghost (except for a tadpole when the gluino is nonvanishing). We expect the same to happen for the closed string. This is very convenient and implies that the whole calculation should be identical to one done in e.g. the Berkovits formalism.

If we use the known expansion of (4) in component fields and the equations of motion for the background fields, and add the integrated vertex operator (7) to the free world-sheet action, the ghost-independent part of the action becomes

\[
S = \frac{1}{4\pi \alpha'} \int d^2z \left[ \partial x^m \bar{\partial} x^n (g_{mn} - 2\pi \alpha' b_{mn}) + p_\alpha \partial \theta^\alpha + \partial \theta^{\dot{\alpha}} \bar{p}_{\dot{\alpha}} \\
+ 2\pi \alpha' \left(p_\alpha \bar{\psi}_m^\dot{\alpha} \bar{\partial} x^m + \partial x^m \bar{\psi}_m^{\dot{\alpha}} \bar{p}_{\dot{\alpha}} - p_\alpha F^{\alpha \dot{\alpha}} \bar{p}_{\dot{\alpha}} \right) \right]
\]

where \( \partial = \frac{1}{2}(\partial_\sigma + i\partial_\tau) \), \( \bar{\partial} = \frac{1}{2}(\partial_\sigma - i\partial_\tau) \), and \( b_{mn} = iB_{mn} \) in Euclidean space, with \( B_{mn} \) real. Furthermore, \( p_\alpha, \bar{p}_{\dot{\alpha}} \) and \( F^{\alpha \dot{\alpha}} \) are antihermitian, while \( \theta^\alpha, \theta^{\dot{\alpha}}, \psi_m^\alpha \), and \( \psi_m^{\dot{\alpha}} \) are hermitian.

As a matter of convention, we put an extra factor of \( 2\pi \alpha' \) in front of the terms that came from the vertex operator.

### 3 Two-point functions

One way to proceed from (9) is to integrate out \( p_\alpha \) and \( \bar{p}_{\dot{\alpha}} \). If we assume that \( F^{\alpha \dot{\alpha}} \) is invertible (this condition can easily be relaxed) one obtains

\[
S = \frac{1}{4\pi \alpha'} \int d^2z \left[ \partial x^m \bar{\partial} x^n (g_{mn} - 2\pi \alpha' b_{mn} + 2\pi \alpha' \psi_m^{\dot{\alpha}} F^{-1 \dot{\alpha} \alpha} \bar{\psi}_n^\alpha) \\
+ \partial \theta^{\dot{\alpha}} F^{-1 \dot{\alpha} \alpha} \bar{\psi}_m^\alpha \bar{\partial} x^m + \partial x^m \bar{\psi}_m^{\dot{\alpha}} F^{-1 \dot{\alpha} \alpha} \partial \theta^\alpha + \frac{1}{2\pi \alpha'} \partial \theta^{\dot{\alpha}} F^{-1 \dot{\alpha} \alpha} \bar{\partial} \theta^\alpha \right]
\]

Notice that we can redefine

\[
\theta^\alpha \rightarrow \theta^\alpha - 2\pi \alpha' \psi_m^\alpha x^m \\
\theta^{\dot{\alpha}} \rightarrow \theta^{\dot{\alpha}} - 2\pi \alpha' \psi_m^{\dot{\alpha}} x^m
\]

which has the effect of removing all gravitino dependence from the action. Without loss of generality we will therefore set the gravitini equal to zero and reinstate them at the end.
The propagators
\[
G^{MN}(z, z') = \begin{pmatrix}
G^{mn}(z, z') & G^{m\beta}(z, z') & G^{m\bar{\beta}}(z, z') \\
G^{\alpha n}(z, z') & G^{\alpha\beta}(z, z') & G^{\alpha\bar{\beta}}(z, z') \\
G^{\bar{\alpha} n}(z, z') & G^{\bar{\alpha}\beta}(z, z') & G^{\bar{\alpha}\bar{\beta}}(z, z')
\end{pmatrix}
\] (12)
are obtained by inverting the field operators \(F_{MN}\) as
\[
F_{MN}G^{NP}(z, z') = -\alpha' \delta^P_M \delta(z - z').
\]
One finds that (with zero gravitini) only \(G^{mn}, G^{\alpha\bar{\beta}}, G^{\bar{\alpha}\beta}\) have sources, and satisfy
\[
\begin{align*}
\partial\bar{\partial} G^{mn}(z, z') &= -\frac{1}{2} \alpha' g^{mn} \delta^2(z - z') \\
\partial\bar{\partial} G^{\gamma\beta}(z, z') &= 2\pi \alpha'^2 F^{\gamma\beta} \delta^2(z - z') \\
\partial\bar{\partial} G^{\gamma\bar{\beta}}(z, z') &= -2\pi \alpha'^2 F^{\gamma\bar{\beta}} \delta^2(z - z').
\end{align*}
\] (13)

The boundary conditions which follow from the Euler-Lagrange field equations for \(\theta^\alpha\) and \(x^n\) read
\[
\begin{align*}
\left. \partial\bar{\partial} G^{mn}(z, z') \right|_{z = \bar{z} = 0} &= 0 \\
\left. (g_{mn} - 2\pi \alpha' b_{mn}) \bar{\partial} x^n - (g_{mn} + 2\pi \alpha' b_{mn}) \partial x^n \right|_{z = \bar{z}} &= 0.
\end{align*}
\] (14)
To obtain the first condition, we assumed that \(\theta^\alpha = \theta^\alpha\) on the boundary; in flat space this follows from the requirement that the action be supersymmetric, and we assume that it continues to hold in the presence of constant background fields. The last boundary condition is the usual boundary condition [7] for the bosonic sector. The equation for \(G^{mn}(z, z')\) in [13] is also the standard one, and therefore the propagator \(G^{mn}(z, z')\) has exactly the form given in eqns (2.3)-(2.5) in [7].

We shall now determine \(G^{\gamma\beta}(z, z')\). From (13) we find
\[
G^{\gamma\beta}(z, z') = P^{\gamma\beta} \ln(z - \bar{z}') + Q^{\gamma\beta} \ln(\bar{z} - z').
\] (15)
To determine \(P^{\gamma\beta}\) and \(Q^{\gamma\beta}\) we use the boundary conditions in (14) to deduce the following relations
\[
\partial G^{\gamma\beta} = \bar{\partial} G^{\gamma\beta}, \quad \partial G^{\bar{\gamma}\beta} = \bar{\partial} G^{\bar{\gamma}\beta}.
\] (16)
Since \(G^{\gamma\beta}(z, z')\) is given by \(\alpha'(2\pi \alpha') F^{\gamma\beta} \ln|z - z'| + P^{\gamma\beta} \ln(z - \bar{z}') + Q^{\gamma\beta} \ln(\bar{z} - z')\), one finds from the first relation in (16)
\[
\alpha' \left. \frac{2\pi \alpha' F^{\gamma\beta}}{z - z'} + \frac{P^{\gamma\beta}}{z - \bar{z}'} - \frac{Q^{\gamma\beta}}{\bar{z} - z'} \right|_{z = \bar{z}} = 0.
\] (17)
Hence $P^{\bar{\beta}\gamma} = 0$ while $Q^{\gamma\bar{\beta}} = \alpha'(2\pi\alpha')F^{\gamma\bar{\beta}}$. In a similar manner one finds from the second relation in (16), using $G^{\bar{\beta}\bar{\gamma}} = P^{\bar{\beta}\gamma} \ln(z - \bar{z}') + Q^{\gamma\bar{\beta}} \ln(\bar{z} - z')$, and $G^{\beta\gamma}(z, z') = -G^{\bar{\gamma}\bar{\beta}}(z', z)$,

$$
\frac{P^{\bar{\beta}\gamma}}{z - \bar{z}'} - \alpha'(2\pi\alpha')F^{\beta\bar{\gamma}} \left|_{z=\bar{z}} \right. = 0. \quad (18)
$$

Hence $P^{\bar{\beta}\gamma} = 0$ while $P^{\bar{\beta}\gamma} = -\alpha'(2\pi\alpha')F^{\beta\bar{\gamma}}$. Since $P^{\bar{\beta}\gamma} = P^{\beta\gamma}$, we find

$$
G^{\gamma\beta}(z, z') = -\alpha'(2\pi\alpha') F^{\beta\bar{\gamma}} \ln \left( \frac{z - \bar{z}'}{\bar{z} - z'} \right), \quad (19)
$$

As in SW this implies that for $z$ and $z'$ approaching the boundary one has

$$
G^{\gamma\beta}(z, z') = \pi i \alpha'(2\pi\alpha') F^{\beta\bar{\gamma}} \frac{1}{2} \epsilon(\tau - \tau') + \text{regular terms}. \quad (20)
$$

Finally, this leads to

$$
\{\theta^\alpha, \theta^{\bar{\beta}}\} = i(2\pi\alpha')^2 F^{\alpha\bar{\beta}}. \quad (21)
$$

Notice that we did not have to take any particular scaling limit to obtain this result. Indeed, the quadratic term in $\theta$ in the action can be interpreted as a pure $B$-term, due to the anti-commutativity of $\theta$.

## 4 Non-commutative superspace

The final result that we have obtained reads

$$
[x^m, x^n] = 2\pi i \alpha' \left( \frac{1}{g + 2\pi\alpha' B} \right)_A^{mn}
$$

$$
\{\theta^\alpha, \theta^{\bar{\beta}}\} = i(2\pi\alpha')^2 F^{\alpha\bar{\beta}}, \quad (22)
$$

where the subscript $A$ denotes the restriction to the antisymmetric part of a matrix. Of course, on the boundary we have to impose that $\theta^\alpha = \theta^{\bar{\alpha}}$ so that it only the symmetric part of $F^{\alpha\bar{\beta}}$ that really appears in (22).

Next, we reintroduce the gravitino. We do this by shifting $\theta$ as explained above. After this shift, we no longer have the boundary condition $\theta^\alpha = \theta^{\bar{\alpha}}$ on the boundary, but instead we will have

$$
\theta^\alpha + 2\pi\alpha' \psi^\alpha_m x^m = \theta^{\bar{\alpha}} + 2\pi\alpha' \psi^{\bar{\alpha}}_m x^m \quad (23)
$$
at the boundary. With nonzero gravitino we find
\[
[x^m, x^n] = 2\pi i\alpha' \left( \frac{1}{g + 2\pi\alpha' B} \right)_A^{mn}
\]
\[
[x^m, \theta^\alpha] = -i(2\pi\alpha')^2 \left( \frac{1}{g + 2\pi\alpha' B} \right)_A^{mn} \psi^\alpha_n
\]
\[
[x^m, \bar{\theta}^\alpha] = -i(2\pi\alpha')^2 \left( \frac{1}{g + 2\pi\alpha' B} \right)_A^{mn} \bar{\psi}^\alpha_n
\]
\[\{\theta^\alpha, \theta^\beta\} = i(2\pi\alpha')^2 F^{\alpha\beta} + i(2\pi\alpha')^3 \psi^\alpha_m \left( \frac{1}{g + 2\pi\alpha' B} \right)_A^{mn} \bar{\psi}^\beta_n.\]
\]

A good scaling limit of this system is (assuming $B$ has maximal rank) to take $\alpha' \to 0$, to scale $g_{mn}$ as $(\alpha')^2$, and to keep fixed
\[
\theta^{mn} = (B^{-1})^{mn}, \quad \Psi^\alpha_m \equiv 2\pi\alpha' \psi^\alpha_m, \quad \bar{\Psi}^{\bar{\alpha}}_m \equiv 2\pi\alpha' \bar{\psi}^{\bar{\alpha}}_m, \quad F^{\alpha\beta} \equiv (2\pi\alpha')^2 F^{\alpha\beta}. \tag{26}
\]
Then the (anti)commutators reduce to
\[
[x^m, x^n] = i\theta^{mn}
\]
\[
[x^m, \theta^\alpha] = -i\theta^{mn} \Psi^\alpha_n
\]
\[
[x^m, \bar{\theta}^\alpha] = -i\theta^{mn} \bar{\Psi}^{\bar{\alpha}}_n
\]
\[\{\theta^\alpha, \theta^\beta\} = iF^{\alpha\beta} + i\Psi^\alpha_m \theta^{mn} \bar{\Psi}^{\bar{\alpha}}_n. \tag{27}
\]
\]

This is our final result for the non-commutative superspace as obtained from string theory.

5 Non-constant background fields

So far we have taken the background fields to be constant. One can in principle also consider non-constant background fields. For commuting coordinates, this was studied in detail by Kontsevich \[20\]. If we put $[x^\mu, x^\nu] = i\theta^{\mu\nu}(x)$, the Jacobi identities impose certain constraints on $\theta^{mn}$. If these Jacobi identities are satisfied, the commutators can be used to define a generalization of the Moyal $\ast$-product.

The relation between the work of Kontsevich and string theory was clarified in \[21\]. From the bosonic string one can extract a topological theory that upon quantization produces the generalized $\ast$-product. This generalized product is therefore present in string theory, but emerges most clearly in a topological
limited. If we do not take the topological limit, the full structure of the theory is more complicated.

It would be interesting to work out corresponding statements for non-commutative superspaces with non-constant (anti)-commutators. Again, Jacobi identities will constrain the space-time fields. In addition, one has to worry whether or not the space-time fields solve the string theory equations of motion (for some discussion, see [22]). Nevertheless, we expect that also in this case, non-commutative superspace with space-time dependent (anti)-commutators describes a sector of string theory in the corresponding background fields. We leave a detailed study of these issues to future research.

6 Conclusions

In conclusion, we have seen that non-commutative superspace appears quite naturally in string theory. There are many interesting problems and generalizations that one may consider.

The world-volume theory for D-branes is described, in superspace, by a $\kappa$-symmetric world-volume theory. Perhaps these can be generalized to non-commutative superspaces. If so, there may also exist a generalization of the Seiberg-Witten map [7] that related the commutative action with explicit background fields to the non-commutative action with no superfields.

For special choices of (space-time dependent) backgrounds, particularly interesting non-commutative superspaces may appear. For example, as discussed in the introduction, one may try to obtain a structure of the form

$$\{\theta^\alpha, \theta^\beta\} = x^\mu \gamma_\mu^{\alpha\beta}$$

which is similar to the bracket obtained in [2]. In view of our results, brackets of the form (29) can appear by considering e.g. an axion configuration (which is the RR scalar) of the form $a \sim \eta_{\mu\nu} x^\mu x^\nu$. This is not an axion background that to our knowledge has any special meaning, and in fact appears rather problematic. Still, it would be interesting to explore this further.

On a more formal level, one would like to understand better the geometrical structure of non-commutative superspace, and which physical theories allow an extension to this space.

It is also important to realize that in the presence of background fields, supersymmetry will generally be broken. In e.g. type IIB string theory on a
Calabi-Yau, the supersymmetry breaking appears through the superpotential \( f G \wedge \Omega \), with \( G \) the RR three-form field strength, and \( \Omega \) the holomorphic three-form \([23, 24, 25]\). We can alternatively study whether supersymmetry is broken directly in non-commutative superspace, by looking at whether the deformation is compatible with global supersymmetry. It would be interesting to understand more directly the relation between these two ways of breaking supersymmetry. Ultimately this may lead to a useful novel mechanism of supersymmetry breaking.

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