Standard-like Models with Broken Supersymmetry from Type I String Vacua

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Abstract

We construct $D = 4$ Type I vacua with massless content remarkably close to that of the standard model of particle physics. They are tachyon-free non-supersymmetric models which are obtained starting with a standard $D = 4$, $N = 1$ compact Type IIB orientifold and adding the same number of Dp-branes and anti-Dp-branes distributed at different points of the underlying orbifold. Supersymmetry-breaking is felt by the observable world either directly, by gravity mediation or gauge mediation, depending on the brane configuration. We construct several simple three generation examples with the gauge group of the standard model or its left-right symmetric extensions. The models contain a number of $U(1)$ gauge groups whose anomalies are cancelled by a generalized Green-Schwarz mechanism. These $U(1)$'s are broken but may survive as global symmetries providing for a flavour structure to the models. The value of the string scale may be lowered down to the intermediate scale (as required in the gravity mediation case) or down to 1-100 TeV for the non-SUSY models. Thus the present models are the first semirealistic string vacua realizing the possibility of a low string scale. The unbalanced force between the pairs of Dp- and anti-Dp-branes provides for an effect which tends to compactify some of the extra dimensions but no others. This could provide a new mechanism for radius stabilization.
1 Introduction

During the past few years a beautiful picture has emerged regarding the structure of string theory, in which all different supersymmetric theories appear as different points of a general moduli space. Understanding this moduli space is one of the big open questions in this theory. However, one of the few things we do know is that we do not live in such space, since supersymmetry is not a symmetry of nature. Therefore it is mandatory to consider the structure of non-supersymmetric strings.

Recently there has been some progress in understanding non-supersymmetric string models, by studying the dynamics of brane-antibrane systems [1] and other constructions [2, 3, 4]. This study has direct phenomenological interest due to the observation that the fundamental string scale is not necessarily of the order of the Planck mass and can be substantially lower [5, 3, 6]. In particular type I strings may have very low fundamental scale as long as the size of the extra dimensions is large enough. Therefore we may in principle have a nonsupersymmetric type I model with supersymmetry broken only at the electroweak scale, something impossible to realize in the old perturbative heterotic models.

The construction of four-dimensional models from type I theories has been the subject of research only recently [8, 9, 10, 11, 12, 13, 14, 15, 16], since for many years the activity concentrated mostly on heterotic strings. Standard-like models from heterotic strings have been constructed since several years already [17, 18, 19]. Despite the remarkable similarity with the Standard Model of particle physics, they all suffer from different problems. In particular the standard questions of supersymmetry breaking and moduli stabilisation could not be approached from a string theory point of view. On the other hand, supersymmetric type I model building has appeared to be very restrictive and no quasi-realistic models have emerged to date [20]. This difficulty is related to the conditions coming from the cancellation of Ramond-Ramond (RR) tadpoles which are extremely restrictive.

In spite of recent interest in D-brane scenarios with a lowered string scale [21, 7, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31], the studies presented in the literature up to now have used the D-brane techniques more as an inspiration than as an explicit tool to construct new phenomenologically interesting string vacua. A number of phenomenological issues have been discussed but never based on specific semirealistic string vacua with D-branes. One of the purposes of the present article is to fill this gap and provide specific consistent string vacua in which these issues may be addressed.

We present the first standard-like models explicitly constructed from type I strings.
This is a new class of vacua based on $\mathcal{N} = 1$ toroidal Type IIB orientifolds plus the inclusion of branes and anti-branes, generally stuck at different orbifold fixed points [4]. The presence of these brane anti-brane pairs in the vacuum makes substantially easier the construction of tadpole-free models. Supersymmetry is only broken in some subsector of the theory by the presence of anti-D$p$-branes which break supersymmetry. Since these anti-D$p$-branes may be isolated in the bulk of compact space the $N = 1$ sector of the theory may feel SUSY-breaking only in a suppressed manner. We then have explicit string realizations of the gravity mediated scenario. In other models the spectrum is non-supersymmetric or else feels SUSY-breaking by gauge mediation. Our models have three families of quarks and leptons and are remarkably simple. We will present explicit examples with the Standard Model gauge group and also left-right symmetric models as well as models of the Pati-Salam type.

The structure of this paper is as follows. In the next section we give an overview of the orientifold constructions of four-dimensional chiral type I strings, including the recent development of reference [4], where branes and anti-branes are included in a consistent manner, cancelling all tadpoles and projecting out tachyonic states. In section 3 we perform a general treatment of the non-supersymmetric models in the presence of one Wilson line which serves as the basis for our construction. We provide the general massless spectrum for this class of theories and show how $U(1)$ anomalies are cancelled via a generalized Green-Schwarz mechanism along the lines of ref. [32, 33].

In section 4.1 we present explicit examples of quasi-realistic three-generation models where only branes are stuck at fixed points whereas anti-branes live in the bulk. Since the anti-branes break supersymmetry, these are explicit examples of gravity mediated supersymmetry breaking, which then ask for a fundamental string scale to be an intermediate scale [28]. Other example of gravity mediated models in which both branes and anti-branes are stuck at fixed points is provided in section 4.3. Section 4.2 deals with models with branes stuck at some points and anti-branes stuck at different points. There are fields in these models carrying quantum numbers of the anti-branes which can mediate supersymmetry breaking either directly or through gauge interactions. In this case we would need the fundamental scale to be as low as the TeV scale [3, 21, 6].

In chapter 5 we study a number of generic dynamical issues in this class of models. We show how the string scale can be lowered down to the intermediate scale (as required in the gravity mediated models of sections 4.1 and 4.3) or below (as required in the non-SUSY models of section 4.2). We discuss the presence of attractive forces between branes and anti-branes as well as other effects which may lead to a potential for the
moduli \[4\]. We also study the general structure of Yukawa couplings and show how
gauge couplings may naturally unify at the intermediate scale in some particular gravity
mediated models of the class discussed in section 4.1.

Chapter 6 is left for our final conclusions and comments. The appendices provide
for additional models which exemplify some of the issues discussed in the text.

2 Type IIB orientifolds in the presence of anti-Dp-branes

In this section we introduce our notation and projection rules for obtaining the massless
spectrum in \(D = 4\) dimensions. We also comment on different alternatives for building
stable non-supersymmetric vacua. We mainly follow references [14] and [4].

A Type IIB orientifold in four dimensions is obtained when such theory, toroidally
compactified on \(T^6\), is divided out by the joint action of world sheet parity symmetry
\(\Omega\), exchanging left and right movers, and a discrete symmetry group \(G_1\) (like \(\mathbb{Z}_N\) or
\(\mathbb{Z}_N \times \mathbb{Z}_M\)), acting crystalographically on \(T^6\). The \(\Omega\) action can be also accompanied by
extra operations thus leading to a general orientifold group \(G_1 + \Omega G_2\) with \(\Omega h \Omega h' \in G_1\)
for \(h, h' \in G_2\) In what follows we mainly consider \(\mathbb{Z}_N\) twists. Twist eigenvalues,
associated to complex compactified coordinates \(X_a\) where \(a = 1, \ldots, 3\), are encoded into
the vector \(v = \frac{1}{N}(\ell_1, \ell_2, \ell_3)\) where \(\ell_a\) are integers. Such integers are specified by the
number of required unbroken supersymmetries on each movers sector. We choose them
to satisfy \(\ell_1 + \ell_2 + \ell_3 = 0\). A classification for the \(\mathcal{N} = 1\) case may be found in [34].

Orientifolding closed Type IIB string introduces a Klein-bottle unoriented world-sheet.
Amplitudes on such a surface contain tadpole divergences. Tadpoles may be generically
interpreted as unbalanced orientifold planes charges under RR form potentials. Such
unphysical divergences can be eliminated by introducing Dp-branes, carrying opposite
charges.

In this way, divergences occurring in the open string sector cancel up the closed
sector ones and produce a consistent theory. Moreover, sets of branes and antibranes
can be consistently introduced in order to cancel RR tadpoles. Even though the models
are non-supersymmetric (due to the presence of the anti-D-branes), it has been shown
in [4] that models free of tachyons can be easily constructed. Since, in the orientifolds
we are considering, supersymmetry is preserved in the closed string sector, and is only
broken in the open string sector by the presence of the anti-D-branes, most of the
usual techniques in supersymmetric orientifolds hold [35, 8, 10, 11, 14]. Hence, we
focus on the open string sector. Open string states are denoted by \(|\Psi, ab\rangle\chi^{pq}_{ab}\). Here, \(\Psi\) refers to world-sheet degrees of freedom whereas \(a, b\) are Chan-Paton indices associated to the open string endpoints lying on D\(p\)-branes and D\(q\)-branes respectively. \(\chi^{pq}\) is the Chan-Paton, hermitian matrix, containing the gauge group structure information. Analogously, \(\chi^{pq}_{\bar{a}\bar{b}}\) is introduced for open strings ending at D\(\bar{p}\), D\(\bar{q}\)-antibranes ( D\(\bar{p}\) antibrane, D\(\bar{q}\)-brane, etc.). We will denote these situations generically by introducing capital indices \(P = p, \bar{p}\) and \(Q = q, \bar{q}\). The action of an element of the orientifold group on Chan-Paton factors is achieved by a unitary matrix \(\gamma_{g,P}\) such that 
\[
\gamma_{g,P}: \chi^{PQ}_{\lambda_{pq}} \rightarrow \gamma_{g,P}\chi^{PQ}_{\lambda_{pq}}\gamma^{-1}_{g,Q}
\]
(2.1)

Also, generic matrices for the \(\mathbb{Z}_N\) orbifold twist, with \(N = 2M\) (\(N = 2M + 1\)), can be provided. They can be defined as 
\[
\gamma_{1,P} = (\tilde{\gamma}_{1,P}, \tilde{\gamma}^*_{1,P})
\]
with * denoting complex conjugation and where \(\tilde{\gamma}\) is a \(N_P \times N_P\) diagonal matrix given by 
\[
\tilde{\gamma}_{1,P} = \text{diag}(\cdots, \alpha^{NV_j}I_{n^p_j}, \cdots, \alpha^{NV_M}I_{n^p_M})
\]
(2.3)
with \(\alpha = e^{2i\pi/N}\) and \(2N_P = \sum_{j=1}^{M} n^p_j\) is the number of P-branes. The choice \(V_j = \frac{1}{N}\) with \(j = 0, \ldots, M\) corresponds to an action “with vector structure” (\(\gamma^N_{1,P} = 1\)) while \(V_j = \frac{2j-1}{2N}\) with \(j = 1, \ldots, M\) describes an action “without vector structure” (\(\gamma^N_{1,P} = -1\)). In order to compute the spectrum, it proves useful to associate a “shift vector” \(V^P\), with \(V_j\) coordinates, to the twist matrix \(\gamma_{1,P}\). Namely, 
\[
\gamma_{1,P} \rightarrow V^P = (V_1, \ldots, V_M)
\]
(2.4)

The models discussed below are mainly based on \(\mathbb{Z}_3\) orientifolds. The twist matrix and shift vector in such case are 
\[
\tilde{\gamma}_{1,P} = \text{diag}(I_{n^p_0}, \cdots, \alpha I_{n^p_M}) \rightarrow V^P = \frac{1}{3}(0 \ldots 0 1, \ldots 1) = \frac{1}{3} (0^{n^p_0}, 1^{n^p_M})
\]
(2.5)

\footnote{Following the classification introduced in [\ref{36}] for six-dimensional models.}
with $\alpha = e^{2i\pi/3}$. As usual the powers of the entries in the vector $V^P$ refer to the number of times each entry appears. By choosing $\gamma_{\Omega,9}$ and $\gamma_{\Omega,5}$ matrices

$$
\gamma_{\Omega,9} = \begin{pmatrix} 0 & \mathbb{I}_{N_9} \\ \mathbb{I}_{N_9} & 0 \end{pmatrix}; \quad \gamma_{\Omega,5} = \begin{pmatrix} 0 & -i\mathbb{I}_{N_5} \\ i\mathbb{I}_{N_5} & 0 \end{pmatrix}
$$

the consistency constraint (associated to group operation $(\Omega \theta^k)^2 = \theta^{2k}$)

$$
\gamma_{k,P}^* = \gamma_{\Omega,P} \gamma_{k,P} \gamma_{\Omega,P}
$$

is satisfied with the twist matrices defined in (2.3). The requirement of tadpoles cancellation leads to further restrictions on twist matrices. Such constraints have been extensively discussed in the literature for supersymmetric orientifolds. It was shown in [4] that essentially the same constraints are valid, when antibranes are present, if $\text{Tr} \gamma_{k,p}$ is replaced by $(\text{Tr} \gamma_{k,p} - \text{Tr} \gamma_{k,p})$ in the tadpole cancellation equations of the supersymmetric case and the added number of branes and anti-branes is the same. We refer to this point when we construct specific models.

### 2.1 The massless spectrum

The massless spectrum in an array of branes and antibranes was discussed in [3, 4]. Here we follow the discussion in Ref. [4] generalizing it for the case of nonvanishing Wilson lines. In the case of supersymmetric orientifolds, it was shown in Ref. [14] that computation of spectra is greatly simplified if a Cartan-Weyl basis is chosen. Let us recall the basic ingredients of such a construction and indicate how this is easily generalized to uncover the description of systems containing both branes and antibranes. In a Cartan-Weyl basis, Chan-Paton generators are organized into charged generators $\lambda_a = E_a$, $a = 1, \ldots, \dim G_P - \rank G_P$, and Cartan algebra generators $\lambda_I = H_I$, $I = 1, \ldots, \rank G_P$, where $G_P$ is the gauge group on the P-brane. If a vector $H = (H_1, \ldots, H_{\rank G_P})$ is defined, then

$$
[H, E_a] = \rho^a E_a
$$

where the $(\rank G_P)$-dimensional vector $\rho^a$ is the root vector associated to the generator $E_a$. Matrices $\gamma_{1,p}$ (and their powers) represent the action of the $\mathbb{Z}_N$ group on Chan-Paton factors, and they correspond to elements of a discrete subgroup of the Abelian group spanned by the Cartan generators. Thus they can be written as

$$
\gamma_{1,p} = e^{-2i\pi V^P \cdot H}
$$
Generically, invariance of open string massless states under the orientifold group action in $PP$ sectors leads to constraints on Chan-Paton matrices of the form

$$\lambda = \pm \gamma_{\Omega,P} \Lambda^T \gamma_{\Omega,P}^{-1}$$

$$\lambda = e^{2\pi i \gamma_{\Omega,P} \Lambda \gamma_{\Omega,P}^{-1}}$$ (2.10)

The first equation, imposed by $\Omega$ orientifold projection, performs a first selection of allowed $\rho^P$ weight vectors. The second one, required for invariance under orbifold twists, further projects such vectors through the simple constraint $\rho^P \cdot V^P = \frac{k}{N} \mod \mathbb{Z}$. For instance, in $5\bar{5}$ sectors the projections for D5$_s$-branes at fixed points are given by

$$\lambda^{(0)} = \gamma_{\theta,5} \lambda^{(0)} \gamma_{\theta,5}^{-1}$$
$$\lambda^{(i)} = e^{2\pi i v_a} \gamma_{\theta,5} \lambda^{(i)} \gamma_{\theta,5}^{-1}$$ (2.11)

with positive sign for $a = 1, 2$ and negative for $a = 3$. The projections are the same for both bosonic and fermionic states, thus leading to $\mathcal{N} = 1$ supermultiplets. The $\Omega$ projections, with $\gamma_{\Omega,5}$ defined in (2.4), select the $Sp(2N_5)$ root vectors $\rho_S = (\pm 1, \pm 1, 0 \ldots 0)$ and $\rho_L = (\pm 2, 0 \ldots 0)$ (and Cartan generators) when the minus sign in the first equation in (2.10) is present. If there is instead a plus sign, long root vectors are absent. Hence, $\lambda^{(0)}$ constraint on the left

$$\rho^a \cdot V^P = 0 \mod \mathbb{Z}$$ (2.12)

selects the gauge group, a subgroup of $Sp(2N_5)$ whereas matter states correspond to charged generators with

$$\rho^a \cdot V^P = v_a \mod \mathbb{Z}$$ (2.13)

With $v_a = \ell_a/N, a = 1, 2, 3$, the components of the vector that defines the orbifold twist. The adequate $\rho$ vectors must be used in each complex direction.

Recall that in supersymmetric orientifolds with D5-branes (thus with even $\mathbb{Z}_N$ twists) at fixed points, tadpole equations generically require shift vectors “without vector structure” and thus, long root vectors do not contribute to the massless spectrum [14]. However, the situation is different when antibranes are also present. In this case even twists “with vector structure” or odd twist with D5-branes and D$\bar{5}$-antibranes can also appear. For instance, long root vectors complete symmetric representations of unitary groups, not present in supersymmetric cases.

Similar considerations are valid when dealing with $5\bar{5}$ sectors. Exactly the same projections apply to NS states while an extra minus sign appears for R states in the
Ω projections. In mixed sectors, $PQ$ ( $PQ = 59, \overline{59}, \overline{59}$ etc.), with string endpoints lying on different kinds of branes, the subset of roots of $G_P \times G_Q$ of the form

$$\rho_{PQ} = (W_P; W_Q) = (\pm 1, 0, \ldots, 0; \pm 1, 0, \ldots, 0) \quad (2.14)$$

must be considered. In these cases, orientifold projection maps $PQ$ into $QP$ sector and, thus, it imposes no restrictions on the spectrum. The relevant phases for NS and R states in each sector, due to orbifold action, can be read from [4]. Let us consider, for instance, projections in $59$ and $\overline{59}$ sectors. Recall that NS states are labeled by $|s_1, s_2\rangle$ $SO(4)$ spinor weight while R states correspond to $|s_0, s_3\rangle$ spinor weights ($s_j = \pm \frac{1}{2}$) where $s_0$ defines space-time chirality. GSO projection demands $s_1 = s_2$ and $s_0 = s_3$ in $59$ sector, whereas opposite signs are needed in $\overline{59}$ sector. For both, bosons and fermions in $59$ sector we must have

$$\rho_{(59)} \cdot V^{(59)} = \pm \frac{1}{2} (v_1 + v_2) \mod \mathbb{Z} \quad (2.15)$$

with positive and negative signs corresponding to $s_1 = s_2 = \pm \frac{1}{2}$ and $s_0 = s_3 = \pm \frac{1}{2}$ respectively. For NS states in $\overline{59}$ sector we have,

$$\rho_{(\overline{59})} \cdot V^{(\overline{59})} = \pm \frac{1}{2} (v_1 - v_2) \mod \mathbb{Z} \quad (2.16)$$

where positive and negative signs correspond to $s_1 = -s_2 = \pm \frac{1}{2}$. Finally, orbifold projection on R states requires

$$\rho_{(\overline{59})} \cdot V^{(\overline{59})} = \pm \frac{1}{2} v_3 \mod \mathbb{Z} \quad (2.17)$$

with plus and minus signs corresponding to $s_0 = -s_3 = \mp \frac{1}{2}$. Notice that this last condition is similar to (2.15) but there the plus sign corresponds to $s_0 = \frac{1}{2}$, i.e. positive chirality fermions, while here it describes negative chirality ones (we are using $v_1 + v_2 + v_3 = 0$).

### 2.2 Tachyon free non-supersymmetric models

Generic models with branes and antibranes will contain tachyonic states whenever brane-antibrane pairs of the same type coincide. More precisely, tachyons can be present in $9\overline{9}$ sectors or $5_L\overline{5}_L$ sectors, with 5D-branes and $\overline{5}$D-antibranes at the same fixed point $L$ and whenever their Chan-Paton twist matrices do overlap [4].

Once the origin of tachyonic fields is identified different ways for building tachyon free theories open up [4]. Thus, models with branes of only one type (say D9-branes)
and antibranes of the other (5-antibrane) are automatically tachyon free. Another possibility is to consider non-overlapping Chan-Paton twist matrices for the coincident branes and the antibranes. A third way is to place branes and antibranes at different fixed points (or have different Wilson lines, in a T-dual description) of the internal space. It is this last route the one we will mainly pursue in building our models so let us further comment on it.

On the one hand such kind of models indicates an attractive possibility for breaking supersymmetry. Namely, non-supersymmetric antibranes sectors (at some fixed points) could transmit supersymmetry breaking to the supersymmetric brane sector (at different fixed points) through the exchange of bulk fields. However, the situation is more subtle. If branes (or antibranes) are not really stuck at the fixed point, they will be able to move through the bulk as a dynamical brane. Since brane-antibrane systems develop a net attractive force between them, they will eventually come to the top of each other and annihilate into the vacuum. Thus, if brane-antibrane annihilation is completely successful and all antibranes disappear, we will end up with a stable supersymmetric vacuum. In such a case, non-supersymmetric models should be viewed as excitations of a supersymmetric vacuum. Interestingly enough there are situations in which, due to local obstructions, only partial, or not annihilation at all, is allowed. These appear as true stable vacua with both, supersymmetric and non-supersymmetric sectors. In the following sections we will use two different ways for trapping branes:

i. Through tadpole conditions:

Whenever orientifold planes RR twisted fields charges are not vanishing, branes must be locally introduced in order to cancel them. Depending on the specific form of twist matrices, antibranes are also needed for such a cancellation. Even though different configurations of branes and antibranes might be possible, antibranes cannot completely annihilate. Let us briefly illustrate the idea in the $\mathbb{Z}_3$ orientifold example which we further develop in next section. In this case, tadpole cancellation requires

$$\text{Tr} \gamma_{\theta,9} + 3(\text{Tr} \gamma_{\theta,5,L} - \text{Tr} \gamma_{\theta,\bar{5},L}) = -4$$

(2.18)

where $L = 1, \ldots, 9$ denotes the nine orbifold fixed points in the first two complex planes. Also the number of fivebranes and antibranes must be the same. Notice that absence of antibranes implies that neither 5D-branes are present and the supersymmetric orientifold condition \cite{10,11} $\text{Tr} \gamma_{\theta,9} = -4$ is recovered. Notice instead that choosing $\text{Tr} \gamma_{\theta,9} \neq -4$ inevitably demands the presence of branes and/or antibranes at
all fixed points. As mentioned, different arrays may be possible. In some situations adding D5-branes (and no anti-D5-branes) at fixed points may be enough to achieve the twisted tadpole cancellation above. Nevertheless, since the same number of antibranes is required, they must be present in the bulk. Also configurations of branes and antibranes distributed among the fixed points are possible. Examples of such situations are provided in section 4. Recall that, whereas in the supersymmetric case the tadpole conditions have a unique solution, a variety of non-supersymmetric solutions are possible.

ii. By sitting isolated branes at orientifold, but not orbifold fixed points.

Assume that $\mathbb{Z}_3$ orbifold twist generators and $\Omega R$ orientifold action, where $R$ is a $\mathbb{Z}_2$ generator, are present in the orientifold group. This is the case for instance of $\mathbb{Z}_6$ orientifold, where $R$ is a reflection in complex directions $(X_1, X_2)$. This is also realized in the T-dual version of $\mathbb{Z}_3$ with 3 and 7D-branes where we have the element $(-1)^F R_1 R_2 R_3$, with $R_i$ a reflection in $Y_i$ plane. In such a case we could place an isolated antibrane at a $\mathbb{Z}_2$ fixed point, which is not a $\mathbb{Z}_3$ fixed orbifold point. Orbifold invariance requires other two antibranes at each of the $\mathbb{Z}_3$ images. This antibrane triplet is trapped since it would need a $\Omega R$ mirror to leave the orientifold point as dynamical antibrane. Hence, this triplet is a non supersymmetric, bounded configuration, of three antibranes which is stable. Building up a full consistent stable non supersymmetric orientifold model will require that branes are also stuck, otherwise they will be attracted to the antibranes and annihilate. In fact, this appears to be the case for $\mathbb{Z}_6$ orientifold. In section 4.3 we provide a $\mathbb{Z}_3$ example with 3-branes and 3-antibranes. In principle, the above construction could be extended to other orbifold actions like $\mathbb{Z}_3 \times \mathbb{Z}_3$, $\mathbb{Z}_7$ etc.

3 Type I $\mathbb{Z}_3$ vacua with Wilson lines

The introduction of Wilson lines in heterotic orbifold models enormously increased the possible consistent chiral string vacua and allowed for the first explicit construction of quasi-realistic string models in the past decade, including standard-like models with three families of quarks and leptons. The most interesting models, from the phenomenological point of view, happened to be those built from the $\mathbb{Z}_3$
orbifold. One of the reasons for this is that three generations come out very naturally in this construction.

In this section, following [14], we will introduce the effects of Wilson lines on the \( \mathbb{Z}_3 \) Type IIB orientifolds including branes and anti-branes as in ref. [4]. The \( \mathbb{Z}_3 \) orientifold without Wilson lines was already presented in [4]. We will see that Wilson lines substantially increases the versatility of the models, allowing to break the gauge groups in the 9-brane sector and also the possibility to modulate and reduce substantially the matter content from the 5-brane sectors.

Adding one single Wilson line will give us enough freedom to construct phenomenologically interesting three generation models, and the spectrum is simple enough that we can discuss the most general case explicitly.

3.1 Spectrum of the Models

Let us consider the twist matrix \( \gamma_{1,9} = (\tilde{\gamma}_{1,9}, \tilde{\gamma}_{1,9}^* \) as given in 2.2. We can also include a Wilson line \( W = (\tilde{W}, \tilde{W}^*) \) wrapping along direction \( e_1 \) in the first complex plane. The most general explicit form for such matrices can be written as

\[
\tilde{\gamma}_9 = \text{diag} \left( I_{N_0}, I_{N_1}, \alpha I_{N_2}, \alpha I_{N_3}, \alpha I_{N_4} \right) \quad (3.1)
\]

\[
\tilde{W} = \text{diag} \left( I_{N_0}, \alpha I_{N_1}, I_{N_2}, \alpha I_{N_3}, \alpha^2 I_{N_4} \right) \quad (3.2)
\]

with \( \alpha = e^{2i\pi/3} \).

The associated shifts are

\[
V_9 = \frac{1}{3} \left( 0^{N_0}; 0^{N_1}; 1^{N_2}; 1^{N_3}; 1^{N_4} \right) \quad (3.3)
\]

\[
W_9 = \frac{1}{3} \left( 0^{N_0}; 1^{N_1}; 0^{N_2}; 1^{N_3}; 2^{N_4} \right) \quad (3.4)
\]

where, as usual, the power on each of the entries actually means the number of times that number appears in the vector. So each vector has dimension \( N_0 + N_1 + N_2 + N_3 + N_4 = 16 \). Hence the 99 sector gauge group is

\[
SO(2N_0) \times \prod_{s=1}^{4} U(N_s) \quad (3.5)
\]

R-R tadpole cancellation requires

\[
\text{Tr} (W)^k \gamma_{9,9} + 3(\text{Tr} \gamma_{9,5,L} - \text{Tr} \gamma_{9,5,L}) = -4 \quad (3.6)
\]

for \( k = 0, 1, 2 \). Also the total number of branes and antibranes must be the same.
In the first and second complex planes there are nine orbifold fixed points which we label as \((a, i), \ a, i = 0, 1, 2\). We can put several 5-branes or anti 5-branes at each point as long as the tadpole conditions are satisfied. In the general case we can choose to sit \(2n_i^a + 2m_i^a\) 5-branes and \(2p_i^j + 2q_i^j\) \(j \neq i\) anti 5-branes at the fixed points set \(L_0 = \{(0, 0), (0, 1), (0, 2)\}\), which do not feel the action of \(W\). Here the indices \(i, j = 0, 1, 2\) label each of the three fixed points, and the requirement \(i \neq j\) states that we do not put branes and anti-branes at the same point. Similarly, to the fixed points \(L_1 = \{(1, 0), (1, 1), (1, 2)\}\) feeling \(V + W\) we assign \(2n_i^1 + 2m_i^1\) 5-branes and \(2p_i^j + 2q_i^j\) anti 5-branes. Finally at \(L_2 = \{(2, 0), (2, 1), (2, 2)\}\) feeling \(V - W\) we assign \(2n_i^2 + 2m_i^2\) 5-branes and \(2p_i^j + 2q_i^j\) anti 5-branes. We will also consider the possibility of having \(2r\) extra anti 5-branes not attached to any of the fixed points.

Therefore at the \(ith\) fixed point in the set \(L_a, a = 0, 1, 2\) we have:

\[
\gamma_{5,i,a} = \text{diag}(I_{2n_i^a}, \alpha I_{n_i^a}, \alpha^2 I_{n_i^a}) \quad \text{Tr} \gamma_{5,i,a} = 2m_i^a - n_i^a \\
\gamma_{5,j,a} = \text{diag}(I_{2p_i^j}, \alpha I_{q_i^j}, \alpha^2 I_{q_i^j}) \quad \text{Tr} \gamma_{5,j,a} = 2p_i^j - q_i^j
\]

(3.7)

Since the number of branes and antibranes must be the same we must have

\[
\sum_{i,a} (n_i^a + m_i^a) = \sum_{b,j} (p_b^j + q_b^j) + r.
\]

(3.8)

Using the explicit expressions for twist matrices, and

\[
V_9 + W_9 = \frac{1}{3} \left(0^{N_0}; 1^{N_1}; 1^{N_2}; 2^{N_3}; 0^{N_4}\right) \\
V_9 - W_9 = \frac{1}{3} \left(0^{N_0}; 2^{N_1}; 1^{N_2}; 0^{N_3}; 2^{N_4}\right)
\]

(3.9)

the tadpole equations \(3.6\) read \((j \neq i)\):

\[
n_0^i - 2m_0^i = 2p_0^j - q_0^j = 12 - N_2 - N_3 - N_4 \\
n_1^i - 2m_1^i = 2p_1^j - q_1^j = 12 - N_1 - N_2 - N_3 \\
n_2^i - 2m_2^i = 2p_2^j - q_2^j = 12 - N_1 - N_2 - N_4
\]

(3.10)

where we have used that \(N_0 + N_1 + N_2 + N_3 + N_4 = 16\).

The total gauge group is (when all branes are at fixed points) thus

\[
SO(2N_0) \times \prod_{s=1}^{4} U(N_s) \times \prod_{a,i,j \neq i=0}^{2} [Sp(2m_i^a) \times U(n_i^a)] \times [Sp(2p_i^j) \times U(q_i^j)]
\]

(3.11)

Following the method of the previous section we can compute the fermionic spectrum, which is supersymmetric on the branes and non-supersymmetric on the anti-branes.
First, the 99 sector is independent of the brane/anti-brane distribution, and it is quite analogous to what one finds in computing the untwisted sector of heterotic orbifold models. In particular, for the $\mathbb{Z}_3$ orbifold, it comes in three identical copies, associated to each of the three complex planes. This is the origin of the three families in most of our models:

\begin{equation}
\begin{aligned}
\text{Fermions}_+ : \\
3 \left[ (2N_0, N_2) + (N_1, N_3) + (N_1, N_4) + (N_3, N_4) + \pi_2 \right]
\end{aligned}
\end{equation}

Where we write in parentheses pairs of fundamentals of the corresponding $U(N_s)$ groups and $\pi_2$ is the (conjugate) antisymmetric representation of $U(N_2)$. The 59 sectors depend on which shift vector is felt by the corresponding fixed points. Then, for the set $L_0$, the effective shift for the 59 sector is simply $V_9 \times (0^{m_0}; 1^{n_0})$. We then find

\begin{equation}
\begin{aligned}
\text{Fermions}_+ : \\
(2N_0, n_0^i) + (N_1, n_0^i) + (N_1, n_1^i) + (N_2, n_0^i) + (N_3, n_0^i) + (N_4, n_0^i) + (N_2, 2m_0^i) + (N_3, 2m_0^i) + (N_4, 2m_0^i)
\end{aligned}
\end{equation}

For the fixed points on the set $L_1$, the effective shift is now $(V_9 + W_9) \times (0^{m_1}; 1^{n_1})$. The massless fermions fall into the following representations:

\begin{equation}
\begin{aligned}
\text{Fermions}_+ : \\
(2N_0, n_1^i) + (N_1, n_1^i) + (N_2, n_1^i) + (N_3, n_1^i) + (N_4, n_1^i) + (N_1, 2m_1^i) + (N_2, 2m_1^i) + (N_3, 2m_1^i)
\end{aligned}
\end{equation}

Similarly for the $L_2$ set, the effective shift is $(V_9 - W_9) \times (0^{m_2}; 1^{n_2})$, and the massless fermions are:

\begin{equation}
\begin{aligned}
\text{Fermions}_+ : \\
(2N_0, n_2^i) + (N_1, n_2^i) + (N_2, n_2^i) + (N_3, n_2^i) + (N_4, n_2^i) + (N_1, 2m_2^i) + (N_2, 2m_2^i) + (N_3, 2m_2^i)
\end{aligned}
\end{equation}

To finish the supersymmetric part of the spectrum, we should write down the 55 sectors. As explained in the previous section we use here the shift vectors defined from the $\gamma_5$ matrices and obtain:

\footnote{Since all these sectors are supersymmetric there are also massless complex scalar partners transforming exactly in the same way.}
In an analogous way we can also compute the nonsupersymmetric massless spectrum for the anti-brane sectors. For the fermionic part we have to keep in mind that the chirality is opposite from that of the supersymmetric sector, and then the spectrum comes from considering the effective shifts with $q^i_a$ and $p^i_a$ replacing $n^i_a$ and $m^i_a$ respectively. For instance, for the $\mathbf{59}$ sector, the effective shift is $V_9 \times \langle 0^0, 1^0 \rangle$, and so on. We find:

$\mathbf{5}_{L_0} \mathbf{9}$ Fermions$_+$:

$$3 (2m, n)_a^i + 2 (1, \overline{m})_a^i + (1, \overline{m})_a^i$$

$\mathbf{5}_{L_1} \mathbf{9}$ Fermions$_-$:

$$(2N_0, q^0_1) + (\overline{N}_1, q^0_1) + (N_1, q^0_1) + (\overline{N}_2, q^0_1) + (N_3, q^0_1) + (\overline{N}_4, q^0_1) + (N_4, q^0_1) + \overline{(N_2, q^0_1) + (N_3, q^0_1) + (N_4, q^0_1) + h.c}$$

Scalars:

$$(2N_0, 2p^0_1) + (\overline{N}_1, 2p^0_1) + (N_1, 2p^0_1) + \overline{(N_2, q^0_1) + (N_3, q^0_1) + (N_4, q^0_1) + h.c}$$

$\mathbf{5}_{L_2} \mathbf{9}$ Fermions$_-$:

$$(2N_0, q^0_2) + (\overline{N}_1, q^0_2) + (N_1, q^0_2) + (\overline{N}_2, q^0_2) + (N_3, q^0_2) + (\overline{N}_4, q^0_2) + (N_4, q^0_2) + \overline{(N_2, q^0_2) + (N_3, q^0_2) + (N_4, q^0_2) + h.c}$$

Scalars:

$$(2N_0, 2p^0_2) + (\overline{N}_1, 2p^0_2) + (N_1, 2p^0_2) + \overline{(N_2, q^0_2) + (N_3, q^0_2) + (N_4, q^0_2) + h.c}$$
\(5\overline{5}_{La}\) Fermions:
\[
(1^{\pm}1)_{a}^{j} + (1^{\pm}Adj)^{j}_{a}
\]

Fermions_+:
\[
3(2p,q)_{a}^{j} + 2(1,\overline{1})_{a}^{j} + (1,\overline{1})_{a}^{j}
\]

Scalars:
\[
3(2p,q_{a}^{j} + 2(1,\overline{1})_{a}^{j} + (1,\overline{1})_{a}^{j}
\]

As we mentioned, there can also exist branes (or antibranes) not attached to fixed points. For instance, for \(r\) coincident antibranes travelling in the bulk a \(Sp(2r)\) gauge group appears. The \(5\overline{5}\) massless spectrum is

\(5\overline{5}_{bulk}\) Fermions:
\[
(1)
\]

Fermions_+:
\[
2(2r+1)r + (1)
\]

Scalars:
\[
2(1) + ((2r+1)r)
\]

The \(59\) sector contains negative chirality fermions and bosons in
\[
(2N_{0},2r) + [(N_{1},2r) + (N_{2},2r) + (N_{3},2r) + (N_{4},2r) + N_{j} \rightarrow \overline{N}_{j}]
\]
multiplets.\footnote{Recall that the \(r(2r-1)\) dimensional \(Sp(2r)\) antisymmetric representation is actually reducible to a \(r(2r-1)-1\) dimensional representation plus a singlet.}

In the spectrum above we have not indicated the corresponding \(U(1)\) charges. The convention is that a fundamental of \(U(N)\), \(N\) carries charge 1 (no normalized) with respect to \(U(1)\) in \(U(N)\) while \(\overline{N}\) has charge \(-1\). Thus, for instance, \((\overline{N}_{3},n_{i}^{2})\) carries charge \(-1\) with respect to \(U(1)\) in \(U(N_{3})\), and 1 with respect to the \(U(1)\) groups in \(U(n_{i}^{2})\). Also an antisymmetric \(1\) representation carries charge 2 while \(\overline{1}\) has charge \(-2\).

It is straightforward to check that all non-abelian gauge anomalies cancel if tadpole conditions (3.10) are satisfied. For this we have to keep in mind that branes and anti branes do not coexist at a given point so, for instance, if for a given \(i\), we have \(n_{a}^{i} \neq 0\)

\footnote{In the presence of certain discrete Wilson lines there are no massless particles in the \(59\) sectors and hence the \(5\overline{5}\) states act as sort of hidden sectors for the \(99\) particles. See the discussion at the beginning of section 4.1.}
this implies that $q_i = 0$. Also see that multiplicities coming from $9\tilde{5}_{L_a}$ sectors appear with a negative sign since fermions carry opposite chirality.

From this general expressions we can consider special cases which simplify substantially the spectrum. For instance in some models the tadpoles cancel for one or more of the sets $L_a$, so there is no need of introducing 5-branes and antibranes at those points. In those cases the coefficients $m_i^a, n_i^a, p_i^a, q_i^a$ will vanish and also their corresponding gauge groups and matter content. A particularly interesting case is when we set all the antibranes in the bulk, so that only 5-branes are trapped at some of the fixed points $(r = 2 \sum_{i,a} (n_i^a + m_i^a))$. We will see explicit examples of these cases in the following sections.

### 3.2 Anomalous $U(1)$ cancellation and FI-terms

Knowing the general spectrum we can see that there are several $U(1)$ symmetries. As usual in string theory some of these symmetries are anomalous but the anomaly is cancelled by the standard Green-Schwarz mechanism. Unlike the heterotic models, type I models have the peculiarity of having several twisted Ramond-Ramond fields which can participate in the anomaly cancelling mechanism, therefore allowing for the existence of several anomalous $U(1)$’s. In the heterotic case only one $U(1)$ symmetry could be anomalous since there is only one antisymmetric tensor field to cancel it. For example in a model with only 9-branes the coupling of the $U(1)$ fields to the antisymmetric Ramond-Ramond field in the $k$-th twisted sector $B_{k}^{\mu\nu}$ is proportional to [33]:

$$i \text{Tr} \left( \gamma_k \lambda_i \right) B_{k} \wedge F_{U(1)} = (2n_i \sin 2\pi kV_i) \ B_{k} \wedge F_{U(1)} \quad (3.12)$$

where $n_i$ is the rank of the $i$th $U(n)$ group and $V_i$ is the component of the corresponding shift vector overlapping with that $U(n)$. Furthermore, supersymmetry implies that the anomaly cancellation term induces a Fayet-Iliopoulos term of the form:

$$- D_i \sum_f n_i \sin 2\pi kV_i \ M_f \quad (3.13)$$

where the sum runs over the fixed points. $M_f$ is the Neveu-Schwarz partner of the Ramond-Ramond antisymmetric tensor field (which together form a supersymmetric linear multiplet in four dimensions, usually dualized to a standard chiral multiplet). The net effect of the anomaly cancelling and Fayet-Iliopoulos terms is to give a mass [33, 31, 40, 41] to the anomalous gauge field and the multiplet containing $M_f$ and $B_f^{\mu\nu}$. Contrary to heterotic models, in type I models the anomalous $U(1)$’s can survive at
low energies as global symmetries, which could have interesting phenomenologically implications [12].

Let us then identify which of the many $U(1)$ symmetries of our models are anomalous. For definiteness we will concentrate explicitly on the models with only trapped branes (anti-branes on the bulk). First of all we notice that each of the $U(1)$’s is inside a $U(n)$ symmetry. There are four of them coming from the 9-brane sector groups $U(N_s)$, $s = 1, \ldots, 4$ and a maximum of 9 coming from the 5-branes groups $U(n^i_a)$. We need to compute the mixed $U(1) - SU(N_s)^2$, $U(1) - SO(2N_0)^2$, $U(1) - SU(n^i_a)^2$ and $U(1) - Sp(m^i_a)^2$ anomalies, which is then a matrix with 13 rows corresponding to the 4 + 9 $U(1)$’s and 23 columns corresponding to the nonabelian groups $SU(N_s)$, $SO(2N_0)$, $SU(n^i_a)$ and $Sp(2m^i_a)$. It can be labeled as $T^\alpha_\beta_{IJ}$ where the super-indices $\alpha, \beta = 9, 5$ only label the brane origin of the groups. The matrix indices $I, J$ label the anomaly of the $Ith$ $U(1)$ group with the $Jth$ nonabelian group $G_J$. Looking at the spectrum we can simplify the writing of the matrix since for a given fixed point sector $L_a$ the corresponding $U(n^i_a)$ and $Sp(m^i_a)$, $i = 0, 1, 2$, are essentially the same. The anomaly matrix is then an array with the first 4 rows giving the anomalies for the $U(1)$’s of the 9-brane sector and the last three for the $n^i_0, n^i_1, n^i_2$. The 11 columns would then correspond to: the first 4 to $SU(N_s)$, the 5th to $SO(2N_0)$, the next three to $SU(n^i_a)$, $a = 0, 1, 2$ and the last three to $Sp(2m^i_a)$, $a = 0, 1, 2$.

From the spectrum above we find:

$$T^\alpha_\beta_{IJ} = \frac{1}{2}
\begin{pmatrix}
0 & 0 & -3N_1 & 3N_1 & 0 & 0 & -N_1 & N_1 & 0 & N_1 & -N_1 \\
0 & -3N_2 & 0 & 0 & 3N_2 & -N_2 & -N_2 & N_2 & N_2 & N_2 \\
3N_3 & 0 & 0 & -3N_3 & 0 & -N_3 & N_3 & 0 & N_3 & -N_3 & 0 \\
3N_4 & 0 & -3N_4 & 0 & 0 & -N_4 & N_4 & N_4 & 0 & -N_4 & 0 \\
2n^i_0 & -n^i_0 & -n^i_0 & n^i_0 & -3n^i_0 & 0 & 0 & 3n^i_0 & 0 & 0 & 0 \\
-n^i_1 & n^i_1 & -n^i_1 & 2n^i_1 & n^i_1 & 0 & -3n^i_1 & 0 & 0 & 3n^i_1 & 0 \\
-n^i_2 & -n^i_2 & 2n^i_2 & -n^i_2 & n^i_2 & 0 & 0 & -3n^i_2 & 0 & 0 & 3n^i_2 \\
\end{pmatrix}
$$

where the dependence on the ranks of the different $U(n)$ groups reflects the fact that the $U(1)$’s are not normalized. One can check that in the more general case there are up to 9 anomalous $U(1)$’s and 4 non-anomalous $U(1)$’s. The 4 anomaly free combinations have the following general form:

$$b_1 \frac{Q_{N_1}}{N_1} + b_2 \frac{Q_{N_2}}{N_2} + b_3 \frac{Q_{N_3}}{N_3} + b_4 \frac{Q_{N_4}}{N_4} + \cdots$$

(3.14)
where the \( b_i \) are arbitrary real coefficients. A linearly independent set of the four anomaly free \( U(1) \)'s can be written as:

\[
Q_A = Q_{N_4} - Q_{N_3} - Q_{N_1} \\
Q_B = \sum_{a,i=0}^2 Q_{n_a^i} - 3Q_{N_2} \\
Q_C = 3 \sum_{i=0}^2 Q_{n_0^i} - \sum_{a,i=0}^2 Q_{n_a^i} - 3Q_{N_3} - 3Q_{N_4} \\
Q_D = 2 \sum_{i=0}^2 \left( Q_{n_1^i} - Q_{n_2^i} \right) - 2Q_{N_1} + Q_{N_3} - Q_{N_4}
\]

(3.16)

where, to simplify the expressions, we have absorbed the rank in the definition of the charges (\( Q_{N_1}/n_1 \to Q_{N_1} \) and so on)\(^5\). It is straightforward, though tedious, to verify that all cubic \( U(1) \) anomalies cancel for these combinations. \( Q_A \) comes completely from the 9-brane sector and it will be eventually identified with hypercharge in the Standard-like models of the next sections. The 9 anomalous \( U(1) \)'s are then:

\[
Q_{N_2} + Q_{N_3} + Q_{N_4} + 2 \sum_{i=0}^2 Q_{n_0^i}, \\
Q_{N_1} + Q_{N_2} + Q_{N_3} + 2 \sum_{i=0}^2 Q_{n_1^i}, \\
Q_{N_2} - Q_{N_1} + Q_{N_4} + 2 \sum_{i=0}^2 Q_{n_2^i}, \\
Q_{n_1^a} - Q_{n_2^a}; \quad Q_{n_1^a} + Q_{n_2^a} - 2Q_{n_3^a} \quad a = 0, 1, 2
\]

(3.17)

(3.18)

The presence of at most 9 anomalous \( U(1) \)'s is expected because there are only 9 fixed points in the first two complex planes, and hence there are 9 RR twisted fields which can participate in the generalized Green-Schwarz mechanism. The anomalous \( U(1) \)'s will become massive by using the 9 RR twisted field combinations as their longitudinal components so that they become massive vector supermultiplets. As discussed above, Fayet-Iliopoulos terms depending on the twisted NS fields do also appear. Let us write the above 9 anomalous \( U(1) \) generators in the general formula

\[
Q^r = \sum_{s=1}^4 d^r_s Q_{N_s} + \sum_{a,i=0}^2 f_{a,i}^r Q_{n_a^i}
\]

(3.19)

\(^5\)In some particular cases several of the abelian or non-abelian symmetries are not present and the corresponding modifications of the anomaly free and anomalous symmetries have to be done.
where \( r \) goes from 0 to 8 and the coefficients \( d^r_s \) vanish for \( r > 2 \). Then each of the \( U(1)^r \) will have a Fayet-Iliopoulos term \( \xi^r \) given by

\[
\xi^r = \sum_{s=1}^{4} \sum_{\alpha=1}^{N_r} d^r_s \sum_{a,i=0}^{2} \sin[2\pi(V_9 + aW_9)_\alpha] M_{(a,i)}
\]

\[
+ 4 \sin(\pi v_1) \sin(\pi v_2) \sum_{a,i=0}^{2} f^r_{a,i} \sin[2\pi V_5]_{(a,i)} M_{(a,i)}
\]

(3.20)

where the \( M_{(a,i)} \), \( a, i = 0, 1, 2 \) are the twisted NS fields associated to the first two complex planes. The above are nine linear independent combinations of the nine twisted moduli. A nontrivial check of the validity of this expression is that it gives \( \xi = 0 \) for all the anomaly free combinations, as it should. In the effective scalar potential there will be terms coming from the D-terms of the anomalous \( U(1) \)'s which will have the general qualitative structure

\[
V_r = \frac{1}{2} \left( \sum_l q^r_l |\phi_l|^2 + \xi^r \right)^2
\]

(3.21)

where \( \phi_l \) denote scalar fields with charge \( q^r_l \) under \( U(1)^r \). Notice that putting all charged fields to zero and also the twisted modes \( (\xi^r = 0) \) is perfectly consistent. Thus unlike what happens in perturbative heterotic vacua \[44\] the FI-terms do not necessarily trigger further gauge symmetry breaking and the anomalous \( U(1) \)'s may remain as global symmetries at low energies. On the other hand one can also obtain \( V_r = 0 \) for configurations with some non-vanishing vev for some \( \phi_l \) fields by compensating it by appropriately choosing the \( \xi^r \) values. Thus in the specific examples of section 4 we will see how in the (55) sectors there are singlet scalars charged under the anomalous \( U(1) \)'s. They can be given non-vanishing vevs as long as we also give appropriate vevs to some particular combinations of twisted NS fields.

Let us come back now to the issue of \( U(1) \) anomaly cancellation. The anomaly cancelling coefficients have been worked out for the general class of \( \mathbb{Z}_N \) orientifold models and found to take the form \[33\].

\[
A_{I,J}^{\alpha\beta} = \frac{1}{N} \sum_{k=1}^{N-1} C_{k}^{\alpha\beta}(v) n^\alpha_I \sin 2\pi k {V_5}_I^\alpha \cos 2\pi k {V_5}_J^\alpha
\]

(3.22)

Here \( k \) runs over twisted \( \mathbb{Z}_N \) sectors, \( \alpha, \beta \) run over 5,9, (meaning 5- or 9-brane origin of the gauge boson), \( V_I^\alpha \) is the component of the corresponding shift vector \( V_9, V_9 + W_9 \) or \( V_9 - W_9 \), along the entries overlapping with the corresponding group. Notice that since Wilson lines are included, a sum over contributions from different
fixed points is understood in the expression above, each set of fixed points $L_a$ feeling a
different shift vector. Finally

$$
C_k^{\alpha\alpha} = \prod_{a=1}^{3} 2 \sin \pi k v_a \quad \text{for } \alpha = \beta
$$

$$
C_k^{59} = 2 \sin \pi k v_3
$$

(3.23)

Here $v_a = \ell_a/N, a = 1, 2, 3$ specify the orbifold twist on each of the complex planes,
as introduced in the previous section. The same expressions are valid for antibrane-
antibrane sectors but an extra minus sign must be included because of opposite chiral-
ity. For the $\mathbb{Z}_3$ case under consideration we have $C_1^{\alpha\alpha} = -C_2^{\alpha\alpha} = 3\sqrt{3}$ and

$C_1^{59} = -C_2^{59} = -\sqrt{3}$, therefore the contributions of the two sectors $k = 1, 2$ are identi-
cal and we find:

$$
A_{IJ}^{\alpha\beta} = -T_{IJ}^{\alpha\beta}
$$

(3.24)

down therefore anomalies are exactly cancelled as expected. Similarly we can compute the
mixed $U(1)$ anomalies which are also cancelled by the Green-Schwarz mechanism.
Mixed $U(1)$-gravitational anomalies can also be computed. We obtain from the triangle
diagrams: $(0, -9N_2, 0, 0, -3n_1^0, -3n_2^1, -3n_2^2)$ which is exactly cancelled by the general
coefficient $[33]$

$$
A_I^\alpha = \frac{3}{4N} \sum_\beta \sum_k C_k^{\alpha\beta}(v)n_I \sin 2\pi k V_I^\alpha \text{Tr} \left((i^k)^{-1}\right)
$$

(3.25)

where again the $V_I$ is the relevant component of each of the three different shifts.
This cancellation can be considered as a nontrivial consistency check for the whole
construction.

4 Examples of three-generation models

As we discussed in the previous section, untwisted tadpole conditions require the same
number of 5-branes and anti-5-branes in the vacuum. We will consider two general
type of models in turn: i) Models with only 5-branes stuck at fixed points and ii)
Models with both 5-branes and anti-5-branes present at fixed points $[4]$. The first class
of models lead generically to “gravity” mediation of SUSY-breaking. In this case the

$\text{There is a third logical possibility in which only anti-5-branes are stuck at the fixed points and
5-branes live in the bulk. In this case one obtains non-supersymmetric models which are very similar
in their characteristics to the type ii) above. Thus we will not consider this option in the remainder
of this paper.}$
natural value for the string scale is the intermediate scale. The second class of models are explicitly non-supersymmetric. This requires lowering the string scale further, as we discuss in section 5. We will describe these two types in turn.

Using the general formalism of section 3, it is straightforward to look for values for $N_s$ and $n_a^i$, $m_a^i$ that give standard-like models. The groups will generically be much larger than $SU(3) \times SU(2) \times U(1)$, but we can look for models for which some of the gauge sectors are hidden, in the sense that they do not have direct non-abelian couplings with the standard model fields. It turns out to be difficult to accommodate the group of the SM and the necessary left-handed quarks inside a $(55)$ sector and hence we will embed the full standard model group inside the 9-branes sector. We would not get the right spectrum from $SO(2N_0) \times SU(N_2)$ part neither, therefore we are limited to a subgroup of $U(N_1) \times U(N_3) \times U(N_4)$. Furthermore, it is easy to see that the spectrum is invariant under permutations of $N_1, N_3$ and $N_4$, so without loss of generality we can identify say $N_3 = 3, N_4 = 2, N_1 = 1$ to give us the $SU(3) \times SU(2) \times U(1)$ respectively (we may have to redefine fundamental and anti-fundamental representations in some cases). Plugging these values into the tadpole cancellation conditions we find that $N_2$ and $n_a^i, m_a^i, p_a^j, q_a^j$ have to satisfy:

\[
\begin{align*}
7 - N_2 &= n_0^i - 2m_0^i = 2p_0^j - q_0^j \\
8 - N_2 &= n_1^i - 2m_1^i = 2p_1^j - q_1^j \\
9 - N_2 &= n_2^i - 2m_2^i = 2p_2^j - q_2^j 
\end{align*}
\]

(4.1)

Therefore the configurations with the minimum number of 5-branes correspond to

1. $N_2 = 8, N_0 = 2$. The 9-brane group is $U(3) \times U(2) \times U(1) \times U(8) \times SO(4)$. The number of 5-branes or anti 5-branes can be such that $n_0^i = m_0^i = n_2^i = 1$, similar to the previous case or $n_0^i = q_0^i = 1$ with two 5-branes at each of the points $L_2$ and two anti 5-branes at each of the points $L_0$.

2. $N_2 = 7, N_0 = 3$. The 9-brane group is $U(3) \times U(2) \times U(1) \times U(7) \times SO(6)$. We may have four 5-branes at each of the points $L_2$ ($n_2^i = 2$) or two anti 5-branes ($p_2^j = 1$) and two 5-branes at each of the points $L_1$ ($n_1^i = 1$).

3. $N_2 = 9, N_0 = 1$ for which the 9-brane symmetries are $U(3) \times U(2) \times U(1) \times U(9) \times U(1)$. The arrangement of 5-branes and anti 5-branes can be done in at

\footnote{As argued in chapter 6, one may expect that configurations with small number of brane-antibrane pairs in the vacuum to be dynamically preferred with respect to those with large numbers since they contribute to the vacuum energy in a way proportional to their tension.}
least two minimal ways: $n_1^i = m_1^i = m_0^i = 1$, all other $n_a^i, m_a^i, p_a^i, q_a^j$ vanishing. This corresponds to two 5-branes at three fixed points ($L_0$) and four 5-branes at other three fixed points ($L_1$). Alternatively we can set two 5-branes at the points $L_0(m_0^i = 1)$ and two anti 5-branes at each of the points in $L_1$ ($q_1^i = 1$). (we could also choose a more complicated configuration such as $q_0^i = 2, n_1^i = m_1^i = 1$ corresponding to four anti 5-branes at three fixed points ($L_0$) and four 5-branes at each of the other fixed points ($L_1$)).

In each of the three cases the spectrum and 5-branes gauge groups are different implying different phenomenology. We will discuss below the first case ($N_2 = 8$) in detail and leave the other two for the appendix. The three of them are summarized in the tables below.

Another important issue is the way hypercharge is embedded into the gauge group of the models. Hypercharge will be identified with one of the anomaly-free $U(1)$’s in eq.(3.16). The simplest such embedding correspond to $(b_1, b_2, b_3, b_4) = (1, 0, 1, −1)$ which indeed gives the appropriate hypercharge of the left-handed quarks which reside in the (99) sectors of these models. In this case one sees from eqs.(3.16) that the hypercharge generator is purely embedded into the (99) sector without any mixing from (55) sectors.

One can in principle try to identify the hypercharge generator with other combinations which also give the correct hypercharge of left-handed quarks, like $(b_1, b_2, b_3, b_4) = (−1, 0, −2, 1)$. In this case eqs.(3.16) show that the hypercharge generator will mix with the $U(1)$’s coming from 5-branes. This means in particular that the fields in the (55) sector will have (fractional) electric charges. The same will be true in models with $N_1 = 0$. In those models the (99) sector will only contain the left-handed quarks and the $U(1)$ generator giving correctly its hypercharge will necessarily mix with $U(1)$’s coming from 5-branes. In fact, fields coming from the (55) sectors will in general get vacuum expectation values as soon as the blowing-up modes are not strictly zero (see eq.(3.21)). Then only the hypercharge fully embedded in the (99) sector remains unbroken and the other possibilities are not really viable since the corresponding generators are broken. Thus we will identify the physical hypercharge generator with $Q_A$ in (3.16).

A similar analysis can be done if we want a left-right symmetric model $SU(3)_c \times SU(2)_L \times SU(2)_R$, for which we want $N_3 = 3, N_1 = N_4 = 2$. In this case, there are two simple options: $N_2 = 7, N_0 = 2$ which in order to satisfy the tadpole equations require at least $n_2^i = 1$ which means two 5-branes trapped at each of the three fixed points in $L_2$. This is probably the simplest model and we will discuss it in detail next.
The second option requires $N_2 = 8$, $N_0 = 1$ which is a bit more complicated since it requires at least $n^{i}_0 = m^{i}_0 = n^{i}_1 = m^{i}_1 = 1$, which means four 5-branes at each of the six points in $L_0, L_1$, or substituting them by anti 5-branes in the three points in $L_1$ and one of the points in $L_0$. We will not discuss this model in detail, but only present its spectrum in the tables.

If we want to have larger symmetry groups, we may consider a $SU(4) \times SU(2)_L \times SU(2)_R$ model of the Pati-Salam type. For this we then choose $N_3 = 4, N_1 = N_4 = 2$. Again there are two simple solutions. One with $N_2 = 6, N_0 = 2$ requires only $n^{i}_2 = 2$ which is four 5-branes at each of the three fixed points $L_2$. We will discuss this model and its realization in terms of trapped branes and anti branes in the next subsection.

The second option $N_2 = 8, N_0 = 0$ is also very simple, requiring only $m^{i}_1 = m^{i}_0 = 1$ which is two 5-branes at each of the six fixed points $L_0$ and $L_1$. The spectrum can be seen in the tables below. We will write the full massless spectrum in the tables. Symmetries even larger than Pati-Salam, like a standard GUT group (see the example in the appendices) or flipped $SU(5)$ do not seem to have interesting matter content in this class of models.

We will discuss next in some detail several standard-like models, first the class with only branes trapped at the fixed points (gravity mediated) and then the one with both branes and anti-branes trapped at the fixed points with explicit supersymmetry breaking. We summarize the spectrum of the three $SU(3) \times SU(2) \times U(1)$ models, the two $SU(3) \times SU(2)_L \times SU(2)_R$ models and the two Pati-Salam models mentioned above in two tables, one for each class.

### 4.1 Models with only branes stuck at fixed points

In order to cancel twisted tadpoles it is enough to add 5-branes appropriately to the fixed points in the first two complex planes of the underlying $\mathbb{Z}_3$ orbifold. The anti-5-branes will be located generically in the bulk of the first two complex planes. In addition one can always add an appropriate Wilson line in the third complex plane and acting only on anti-5-branes in such a way that they are decoupled both from 9-branes and 5-branes \footnote{This procedure is more transparent if we perform a T-duality transformation in the third complex plane in such a way that 9-branes turn into 7-branes localized at the origin in the third complex plane. The anti-5-branes turn into anti-3-branes. The latter decouple from 7-branes if we locate them away from the origin.}. In this way the anti-5-brane sector (which is the only one which is not supersymmetric) behaves like a sort of "hidden sector" for the "observable"
sector of 9, 5-branes which is supersymmetric. Supersymmetry-breaking effects in this visible sector will be suppressed by the compactification radii providing an example of gravity mediated SUSY-breaking. As we already mentioned one could worry about the presence of attractive forces between branes and antibranes which might render this type of configuration unstable. We postpone the discussion of these effects to chapter 5 and we limit ourselves to the construction of these models (otherwise tachyon-free and free of RR tadpoles).

i) A three generation standard model

Let us consider the $\mathbb{Z}_3$ orientifold with an action of the twist on the Chan-Paton factors given by $N_0 = 2, N_1 = 1, N_2 = 8, N_3 = 3, N_4 = 2$ as mentioned above. We choose to reorder the twist and Wilson lines in order that the Standard Model sector can be clearly distinguished, so we will write:

$$V_9 = \frac{1}{3}(1, 1, 1, -1, -1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1)$$

(4.2)

A quantized Wilson line is added in the first complex plane given by:

$$W_9 = \frac{1}{3}(1, 1, 1, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$

(4.3)

The gauge group from the (99) sector will be $U(3) \times U(2) \times U(1) \times [SO(4) \times U(8)]$. Now, the 9 $\mathbb{Z}_3$ fixed points in the first two complex planes split into three sets of three fixed points each: $(0, m)$, $(1, m)$ and $(-1, m)$ with $m = 0, \pm 1$. The shifts corresponding to each of these three sets are $V$, $V + W$ and $V - W$ respectively. The corresponding value for $\text{Tr} \gamma_{\theta,9}$ are -7, -4 and -1 respectively. Looking at the tadpole cancellation conditions we see that we will be forced to add 5-branes at the fixed points $(0, m)$ with $\text{Tr} \gamma_{\theta,5} = +1$ and also at the fixed points $(-1, m)$ with $\text{Tr} \gamma_{\theta,5} = -1$. No 5-branes have to be added to the points $(1, m)$ since tadpoles cancel without them. Thus the simplest option is to locate 5-branes with twist actions given by:

$$(0, m) \rightarrow \gamma_{\theta,5} = \text{diag} (I_2, \alpha, \alpha^2)$$

$$(-1, m) \rightarrow \gamma_{\theta,5} = \text{diag} (\alpha, \alpha^2)$$

(4.4)

Notice that these 5-branes are stuck at the fixed points, they cannot leave the fixed points without violating the twisted tadpole conditions. Let us then compute the massless spectrum. After an analysis of the $U(1)$’s in the model one can check that
the generator given by $Y$:

$$Y = (-1/3, -1/3, -1/3, -1/2, -1/2, 1, 0, .., 0)$$  \hspace{1cm} (4.5)$$

is anomaly free and can be identified with standard weak hypercharge. One finds the following chiral charged fields from each sector:

(99):

There are chiral multiplets transforming under the gauge group like $[10]$ :

$$3(3, 2, 1/6) + 3(3, 1, -2/3) + 3(1, 2, +1/2) + [3(1, 28) + 3(4, 8)]$$ \hspace{1cm} (4.6)$$

These fields correspond to 3 generations of left-handed quarks and right-handed U-quarks as well as three sets of Higgs fields $H_U$.

(59):

From the 5-branes at the $(-1, m)$ fixed points we get a $U(1)_{U}^3$ gauge group. One then finds chiral multiplets (in three copies, one per fixed point) transforming like:

$$(1, 2, -1/2) + (1, 1, +1) + (3, 1, -1/3) + (3, 1, 1/3) + [(1, 8)' + (4, 1)'_q]$$ \hspace{1cm} (4.7)$$

where the subindex $q$ refers to the $U(1)_{U}^3$ charges. These fields include three standard lepton generations and three sets of vector-like quarks. The $SU(2)$ doublets could also be $H_D$ type of Higgses which have same quantum numbers as left-handed leptons.

From the 5-branes at the $(0, m)$ fixed points we get a $(Sp(2) \times U(1))^3$ gauge group. One then finds chiral multiplets (in three copies, one per fixed point) transforming like:

$$(1, 2, -1/2; 2) + (1, 2, 1/2; 1) + (3, 1, 1/3; 2) + (3, 1, -1/3; 1) + [(1, 8; 2)' + (1, 8)'_r + (4, 1)'_r]$$ \hspace{1cm} (4.8)$$

where the subindex $r$ refers to the $U(1)_{U}^3$ charges. The doublets after the semicolons refer to the $Sp(2)$ groups. These chiral multiplets include a net number of three right-handed D-quarks and three left-handed leptons (or $H_D$ doublets). In addition there are three pairs of vector-like right-handed $D$-quarks and leptons.

(55):

\[ ^9 \text{Notice that this definition corresponds to the } Q_A \text{ generator of eq.(3.16) once one takes into account the flip in sign in the twist vectors } V \text{ and } W \text{ for the } U(2) \text{ and } U(1) \text{ groups compared to the conventions of chapter 3.} \]

\[ ^{10} \text{Properly speaking, using the general formulae of chapter 3 we would have obtained the conjugate states to the ones we display in this chapter 4. We have preferred to list the conjugates so that one recovers the usual conventions of the SM in which it is left-handed quarks which are colour triplets (and not antitriplets).} \]
For those 5-branes sitting at fixed points \((-1, m)\) there is just one singlet chiral field \((1)_{2q}\) associated to the third complex plane. For those 5-branes sitting at fixed points \((0, m)\) there are multiplets transforming under \(Sp(2) \times U(1)\) like:

\[
3(2)_{-r} + (1)_{2r}
\]  

Equation (4.9)

Altogether this model has three standard model quark lepton generations and in addition 6 copies of Higgs-like fields \((H_U + H_D)\) plus 6 vector-like sets of \(D\)-quarks and three vector-like sets of right-handed electrons. Vacuum expectation values of singlets in (55) sector can give masses to many of these extra particles. In particular, as we will mention below, in the (55) sector of 5-branes on the fixed points \((-1, m)\) there are singlets which couple to the extra triplets in that (95) sector. If those singlets get a vev (which may happen in a D-term flat manner if we turn on a vacuum expectation value to the relevant blowing-up fields) those triplets get heavy. The same happens with the extra right-handed leptons in the (59) sectors corresponding to fixed points \((0, m)\). We would be thus left with 3 standard quark-lepton generations plus 6 sets of extra vector-like left-handed leptons and 3 sets of vector-like \(D_R\) quarks. We will see in chapter 5 that in that case gauge coupling unification takes place at the intermediate scale precisely due to the presence of these extra fields.

Let us finally mention a few aspects with regard to the \(U(1)\)'s in this model. Using the results of chapter 3 it is easy to check that there are 7 anomalous \(U(1)\)'s (which then will become massive as usual). There remain 3 anomaly-free \(U(1)\)'s, one of which is the hypercharge defined above. The other two anomaly-free \(U(1)\)'s, which involve a mixture with 5-brane \(U(1)\)'s, will also be spontaneously broken if the singlets in (55) sectors get vev's, as mentioned above. Thus, generically only the standard hypercharge will remain unbroken at low energies.

**ii) A three generation \(SU(3) \times SU(2)_L \times SU(2)_R \times U(1)\) model**

Let us consider now the \(\mathbb{Z}_3\) orientifold with twist action on CP factors given by \(N_0 = 2, N_1 = N_4 = 2, N_2 = 7, N_3 = 3\), again as in the previous example we choose to write explicitly the twist and Wilson lines such that the \(L - R\) model sector looks more transparent:

\[
V_9 = 1/3(1, 1, 1, -1, -1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1)
\]  

Equation (4.10)

A quantized Wilson line is added in the first complex plane given by:

\[
W_9 = 1/3(1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
\]  

Equation (4.11)
The gauge group from the (99) sector will be $U(3) \times U(2) \times U(2) \times [SO(4) \times U(7)]$. Now the nine fixed points in the first two complex planes split again into the same three sets $(0, m), (1, m)$ and $(-1, m)$ which have associated twists $V, V + W$ and $V - W$ respectively. The corresponding value for $\text{Tr} \gamma_{\theta,9}$ are -4, -4 and -1. This means that we will only need to add 5-branes at the points $(-1, m)$ verifying $\text{Tr} \gamma_{\theta,5} = -1$. The simplest option verifying this is adding two 5-branes at each fixed point with $\gamma_{\theta,5} = \text{diag}(\alpha, \alpha^2)$. Again the 5-branes are trapped by tadpole conditions.

Let us now compute the spectrum which in this model is particularly simple. One can check that the $U(1)$ generator defined by:

$$Q_{B-L} = (-2/3, -2/3, -2/3, -1, -1, -1, 0, 0, ..., 0)$$

is anomaly free and can be identified with the standard $B - L$ symmetry of left-right symmetric models. One finds chiral multiplets transforming under the $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ group:

(99):

$$3(3, 2, 1, 1/3) + 3(\bar{3}, 1, 2, -1/3) + 3(1, 2, 2, 0) + [3(1, 21)' + 3(4, 7)']$$

These fields include three standard quark generations plus three sets of Higgs fields. (59):

From the 5-branes at the $(-1, m)$ fixed points we get a $U(1)_5^3$ gauge group. One then finds chiral multiplets (in three copies, one per fixed point) transforming like:

$$(1, 2, 1, -1)_q + (1, 1, 2, +1)_q + (3, 1, 1, -2/3)_{-q} + (\bar{3}, 1, 1, 2/3)_{-q} + [(1, 7)'_q + (4, 1)'_{-q}]$$

The subindex $q$ indicates the charge under the $U(1)_5^3$. These multiplets include three standard generations of leptons and in addition three vectorlike sets of color triplets. (55):

For those 5-branes sitting at fixed points $(-1, m)$ there is just one singlet chiral field $(1)_{2q}$ associated to the third complex plane.

The massless spectrum of this model is remarkably simple. Notice that, like in the previous model the extra colored objects may get massive if the singlets in the (55) sectors get a vev. Then the massless spectrum contains just three standard generations and three sets of Higgs fields.

iii) A three generation $SU(4) \times SU(2) \times SU(2)$ model.
Following similar lines we can consider the following twist action on CP factors and quantized Wilson line in the first complex plane for the $\mathbb{Z}_3$ orbifold $(N_0 = 2, N_1 = N_4 = 2, N_2 = 6, N_3 = 4)$:

\[
V_9 = \frac{1}{3} (1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1) \\
W_9 = \frac{1}{3} (1, 1, 1, 1, 1, 2, 2, 1, 0, 0, 0, 0, 0, 0)
\]  

(4.15)

The gauge group in the 99 sector is $U(4) \times U(2) \times U(2) \times SO(4) \times U(6)$. The sectors feeling the action of $V$ and $V + W$ do not need the addition of 5-branes (nor anti 5-branes) since the corresponding value for $\text{Tr} \gamma_9 = -4$ cancels the tadpoles. On the other hand the sector feeling $V - W$ has $\text{Tr} \gamma_9 = 2$ and we need to add appropriate combinations of 5-branes and anti 5-branes in order to satisfy tadpole cancellation. A configuration of four 5-branes stuck at each of the three fixed points $(-1, m)$ precisely satisfy the tadpole constraint. In this case $\gamma_9 = \text{diag}(\alpha, \alpha, \alpha^2, \alpha^2)$ implying a 5-brane gauge group $U(2)$ on each of the three fixed points. We can identify the first factors of the 9-brane gauge groups as in the Pati-Salam model: $SU(4)_c \times SU(2)_L \times SU(2)_R$. The other factors may be considered as hidden sectors. The spectrum comes again naturally in three families, in a self-explanatory notation:

\[
(99): \\
3 [(4, 2, 1) + (\bar{4}, 1, 2) + (1, 2, 2)] + 3 [(4, \bar{6})' + (1, 15)']
\]  

(4.16)

The observable part includes the complete three families of quarks and leptons, as in the standard Pati-Salam model, plus three copies of $SU(2)_L \times SU(2)_R$ Higgs fields.

\[
(59):
\]

Since we have three identical copies of $U(2)$ on each of the three fixed points, we will get three copies of:

\[
[(4, 1, 1; 2)_{-q} + (\bar{4}, 1, 1; 2)_{-q} + (1, 2, 1; 2)_{q} + (1, 1, 2; 2)_{b}] + [(4, 1; 2)'_{-q} + (1, 6; 2)'_q]
\]

(4.17)

where the last entry on each particle represents its transformation property under the corresponding $SU(2)$ and the subindices the charge with respect to the $U(1)$. Notice that again we have vector-like colored fields in this sector plus three extra doublets of $SU(2)_L \times SU(2)_R$. However, with the usual embedding of hypercharge into the Pati-Salam model their charges are not standard.

\[
(55):
\]

In this sector we only get a triplet plus two singlets under each of the three $U(2)$ factors coming from the 5-branes. The triplet may couple to the vector-like $4 + \bar{4}$ of
the 59 sector to make them massive. Also most of the \( U(1) \) factors are anomalous, their anomaly is cancelled by the Green-Schwarz mechanism and they do not appear as low-energy gauge fields, only probably as remnant global symmetries, as usual in these class of models. There is one nonanomalous \( U(1) \) surviving at low-energy. It is

\[
Q = \frac{1}{2} \left( 1, 1, 1, 1, -2, -2, 2, 2, 0, \ldots, 0 \right)
\]  

(4.18)

All other groups would decouple from the observable sector, leading to a very simple string version of the Pati-Salam model. As in the previous models, we will need an equal number (12) of anti 5-branes in the bulk to cancel the tadpoles, breaking supersymmetry in a truly hidden sector.

### 4.2 Models with both branes and antibranes stuck at fixed points

One can also construct models in which both branes and antibranes are both stuck at (different) fixed points. This possibility has the attractive feature that provides an explanation to the absence of (partial) brane-antibrane annihilation without the need to resort to other dynamical arguments. On the other hand these models are in general non-supersymmetric already at the level of the massless spectrum. There might be in general transitions between the previous type models and the present ones as we will discuss below. In fact we are going to build the models now by simply replacing some of the 5-branes by anti-5-branes in the previous models in such a way that the net total number of 5-branes charge is zero.

**i) A non-SUSY three generation standard model**

Here on the same model of the previous section we can add a different configuration of branes and anti-branes. The (99) sector will be identical and we will still add two 5-branes at each of the three fixed points \((-1,m)\) with \( \gamma_{\theta,5} = (\alpha, \alpha^2) \). Thus, at those fixed points we will have three copies of fermions and complex scalars transforming like \((59)\):

\[
(1, 2, -1/2)_q + (1, 1, +1)_q + (3, 1, -1/3)_{-q} + (\bar{3}, 1, 1/3)_{-q} + [(1, 8)'_q + (4, 1)'_{-q}] \]  

(4.19)

where the subindex \( q \) correspond to the \( U(1)_5 \) charges. We thus have here three sets of right-handed leptons and three sets of left-handed leptons (or \( H_D \) Higgs superfields).

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From the (55) sector one gets one singlet superfield $1_{2r}$ which has couplings to the vectorlike quarks above. Thus the latter may become massive eventually if this singlet gets a vev.

Now we will put two anti-5-branes at each of the three fixed points $(0, m)$ with $\gamma_{\theta, 5} = (\alpha, \alpha^2)$ in such a way that tadpole conditions are fulfilled. There will be a $U(1)_3^\theta$ from the anti-5-branes. From (59) sectors one gets matter fields in three copies transforming like:

\begin{equation}
(\bar{5}9) : \begin{align*}
Fermions : & (\bar{3}, 1, 1/3)_r + (1, 2, -1/2)_r + (1, 1, +1)_{-r} + (1, 1, -1)_{-r} + [(1, \bar{8})'_r + (4, 1)'_{-r}] \\
Scalars : & (\bar{3}, 1, 1/3)_r + (1, 2, -1/2)_r + [\bar{8}'_r] + h.c.
\end{align*}
\end{equation}

where the subindex $r$ refers to the $U(1)_3^\theta$ charges. Notice that now this sector contains the three right-handed $D$-quarks and three left-handed leptons, as well as a set of vector-like right-handed leptons. From the (55) sector one gets a couple of fermions transforming like $1_{2r}$ and one singlet scalar $1_{2r}$, as well as three extra singlet fermions with negative chirality (the would be gauginos of each $U(1)$ coming from the antibranes). The scalar $1_{2r}$ has in general couplings with the vector-like pair of right-handed leptons above which then may become massive eventually.

Notice that the present model is explicitly non-supersymmetric since some of the particles do not have SUSY-partners. On the other hand the number of 5-branes and anti-5-branes is the same and are all stuck at the fixed points. Since there are only $U(1)$’s coming from 5 and anti-5-branes, there will be no gauge group left from those since they are anomalous and will become all massive in the usual way. In this sense this model is simpler than its counterpart in section 4.1. The $U(1)$ symmetries will remain however as effective global symmetries giving a flavour structure to the model.

ii) A non-SUSY $SU(3) \times SU(2) \times SU(2) \times U(1)$ model

Again, on model ii) of previous section we distribute 5-branes and anti-5-branes in a different manner. The (99) sector will be identical but now on two of the fixed points (e.g., $(-1, 0)$ and $(-1, 1)$) we will add two 5-branes with $\gamma_{\theta, 5} = (\alpha, \alpha^2)$ so that tadpoles cancel. These subsectors will be supersymmetric and we will get supermultiplets in two copies transforming like

\begin{equation}
(1, 2, 1, +1)_{q} + (1, 1, 2, -1)_q + (3, 1, 1, -2/3)_{-q} + (\bar{3}, 1, 1, 2/3)_{-q} + [(1, 7)'_{q} + (4, 1)'_{-q}]
\end{equation}
where the subindices denote the $U(1)_5$ charge. Thus from these two fixed points we get two lepton generations (plus vector-like triplets which may become massive through couplings to (55) singlets). Now, on the remaining fixed point $(-1, -1)$ we will add 4 anti-5-branes with $\gamma_{\theta,5} = (I_2, \alpha, \alpha^2)$ so that tadpoles cancel. There will be a $(Sp(2) \times U(1)_5)^3$ from the anti-5-branes. From (59) sectors one gets matter fields in one copy transforming like:

\begin{equation}
Fermions \quad (1, 2, 1, 1; 2) + (1, 2, 1, -1; 1)_r + (1, 1, 2, -1; 2) \\
+ (1, 1, 2, 1; 1)_r + (\bar{3}, 1, 1, 1/3)_{-r} + (3, 1, 1, -1/3)_{-r} \\
+ [(1, \bar{7}; 2)' + (1, 7)_{r}' + (4, 1)'_{-r}]
\end{equation}

\begin{equation}
Scalars \quad [(\bar{3}, 1, 1, 1/3; 2) + (3, 1, 1, -1/3; 2) + (4; 2)'] \\
+ [(1, 2, 1, -1; 1)_r + (1, 1, 2, 1; 1)_r + (7)'_r + h.c.]
\end{equation}

Here we get a third chiral generation of leptons and an extra vector-like set of leptons. Again there are also vector-like triplets which may become massive through scalars in the (55) sector. The latter has fermions transforming like $3(2)_{-r} + 2(1)_{2r}$ and scalars $3(2)_{-r} + (1)_{2r}$ which indeed couple to the above vector-like triplets. It also has one triplet of fermions of the $Sp(2)$ group as well as an extra singlet, both with negative chirality.

Notice that the spectrum of this model is supersymmetric except for one of the lepton generations. The number of 5-branes and anti-5-branes is the same and all are stuck at the fixed points.

\textbf{iii) A non-SUSY $SU(4) \times SU(2) \times SU(2)$ model}

Here on the same model of the previous section we can add a different configuration of branes and anti-branes. If we want all extra branes stuck at orbifold fixed points, the simplest way of distributing the branes and anti-branes is 4 branes on one point, say $(-1, 0)$, and two anti 5-branes at each of the other two points feeling the $V - W$ twist, say $(-1, 1), (-1, -1)$. In those two points we have $\text{Tr} \gamma_{\theta,5} = \text{diag}(1, 1)$ implying a group $USp(2)$ at each of the two fixed points.

The spectrum of this model is as follows. The 99 sector is identical to the one of the previous section, including the three complete families of quarks and leptons. The 59 and 55 sectors include only one copy of the one presented in the previous section. Instead of the two other copies we have now the anti 5-brane sectors which are not supersymmetric.
On each fixed point we will have fermions (with opposite chirality to the ones in the other sectors) transforming as:

\[(1, 2; 1, 2) + (1, 1, 2; 2) + [(1, \bar{6}; 2)']\] (4.23)

where again the last entry on each is the transformation under the corresponding \(Sp(2)\).

The corresponding scalars will belong to the following representations:

\[(4, 1, 1; 2) + (\bar{4}, 1, 1; 2) + (4, 1; 2)']\] (4.24)

in two copies, one per fixed point.

Here there are only two copies of a fermion triplet one for each \(Sp(2)\).

Notice that all non-abelian gauge anomalies cancel in a slightly different manner to the model with only 5-branes stuck. Also, the (59) sector couples directly non-supersymmetric matter to particles charged under the observable gauge group, therefore serving as direct mediators of supersymmetry breaking. This is the main difference between the two models which otherwise are very similar phenomenologically.

### 4.3 A gravity mediated model with stuck branes and antibranes

The gravity mediated models discussed above have non-stuck anti-branes somewhere in the bulk. These antibranes will in general be attracted by the branes stuck at fixed points and the configuration might be unstable. As we discuss in chapter 5, whether this will be the case or not depends on the complete balance of forces of the vacuum but anyway one would like to have some example of gravity mediated model in which all branes and antibranes are stuck. We will provide an example of this type here.

The model we are going to discuss is a \(\mathbb{Z}_3\) orientifold with just 3-branes and anti-3-branes which is just a slight (non-supersymmetric) modification of a supersymmetric model presented in [15]. The orientifold operation will be \(\Omega(-1)^F R_1 R_2 R_3\) which requires the presence of a net number of 3-branes minus anti-3-branes equal to 32 to cancel untwisted tadpoles. Now, the model contains: a) one fixed point (the origin) which is also fixed under the orientifold operation b) other 26 fixed points under the \(\mathbb{Z}_3\) twist which are not fixed under the \(\mathbb{Z}_2\) orientifold projection and c) 63 points which are fixed under the orientifold operation but are not fixed under the \(\mathbb{Z}_3\) action.
At the origin, twisted tadpole conditions require \( \text{Tr} \gamma_{\theta,3} = -4 \) whereas at the other 26 \( \mathbb{Z}_3 \) fixed points one has \( \text{Tr} \gamma_{\theta,3} = 0 \).

We can now locate 11 3-branes at the origin with \( \gamma_{\theta,3} = (1, \alpha I_6, \alpha^2 I_6) \). The gauge group there will be \( SU(5) \times U(1) \) and there will be chiral multiplets transforming like \( 3(10) + 3(\bar{5}) \), i.e., three standard \( SU(5) \) generations. Notice that these 11 branes are stuck because 3-branes can only leave the origin in groups of 6 branes and those remaining at the origin still have to obey the tadpole conditions. Now we are going to add \( 21 + 3n \) 3-branes and \( 3n \) anti-3-branes in such a way that all of them are trapped at some points. In order to do that we recall that there is a simple way to trap 3-branes (or anti-3-branes) at some points. Indeed one can locate one 3-brane at one of the 63 fixed points under \( \Omega(-1)^F R_1 R_2 R_3 \). In order for this configuration to be invariant also under \( \mathbb{Z}_3 \) we need to add one 3-brane at each of the two images of that point under \( \mathbb{Z}_3 \). Now, these three 3-branes are stuck at those orientifold points because they can only travel in groups of 6 in the bulk. So we can locate the \( 21 + 3n \) 3-branes in some of the 63 orientifold points available. In the same way we can locate the \( 3n \) anti-3-branes at some other orientifold points. Now all branes and anti-branes in the model will be stuck and all tadpole conditions are met. The ”observable” \( SU(5) \) sector will only feel the presence of the SUSY-breaking anti-3-branes from closed string exchange and the effective field theory will be that of a gravity mediated model.

Notice that one can stuck anti-3-branes at orientifold fixed points also in the class of \( \mathbb{Z}_3 \) models with 7-branes and 3(\( \bar{3} \)) pairs mentioned above. Indeed, in this case the orientifold operation is \( \Omega(-1)^F R_3 \) and we can locate three anti-3-branes at the three points with coordinates \( (1/2, 0) \), \( (0, 1/2) \) and \( (1/2, 1/2) \) in the 5-th and 6-th dimensions. However we can only trap up to three anti-3-branes in this way whereas we would need a minimum of 6 in the provided examples. One could perhaps find this kind of models in a search more systematic than the one we have made. The example provided in this section is an existence proof of gravity mediated models with all branes and anti-branes stuck.
| (N₀, N₁, N₂, N₃, N₄) (nᵃ, mᵃ) | Gauge Group | 99 sector | 59 sector | 55 sector |
|---------------------------------|------------|-----------|-----------|-----------|
| (2, 1, 8, 3, 2) n₀ = m₀ = n₂ = 1 | [SU(3) × SU(2) × U(1)] × U(1) | 3[(3, 2) + (3, 1) + (1, 2) + (1, 28) + (4, 8)] | 3[(1, 2) + (1, 1) + (3, 1) + (3, 1) + (1, 8′) + (4, 1′) + (3, 1) + (1, 2) + 2(1, 1) + (1, 8′) + (4, 1′)] | 3[(1, 2) + (1, 1) + (3, 1) + (3, 1) + (1, 8′) + (4, 1′)] + (3, 1; 2) + (1, 2, 2) + (1, 8; 2′)] |
| (3, 1, 7, 3, 2) n₁ = 1, n₂ = 2 | [SU(3) × SU(2) × U(1)] × U(1)² | 3[(3, 2) + (3, 1) + (1, 2) + (1, 21) + (6, 7′)] | 3[(3, 1) + (1, 2) + (1, 2) + (1, 1)] + [(1, 7′) + (6, 1′)] + (1, 2; 2) + (1, 1, 2) + (3, 1, 2) + (3, 1, 2) + (1, 7; 2′) + (6, 1; 2′)] | 3[(1, 2) + (1(1) + 3)] |
| (1, 1, 9, 3, 2) m₀ = n₁ = m₃ = 1 | [SU(3) × SU(2) × SU(2)] × U(1) | 3[(3, 2) + (3, 1) + (1, 2) + (2, 9′) + (1, 36′)] | 3[(3, 1; 2) + (1, 2; 2) + (1, 9; 2′)] + (3, 1) + (1, 2) + (1, 2) + (1, 1)] + 3[(2, 1′) + (1, 9′)] + (3, 1; 2) + (1, 2, 2) + (1, 1; 2)] | 3[(2, 1) + 2(1, 1)] |
| (2, 2, 7, 3, 2) n₃ = 1 | [SU(3) × SU(2) × U(1)] × U(1)³ | 3[(3, 2, 1) + (3, 1, 2) + (1, 2) + (1, 21) + (4, 7′)] | 3[(3, 1, 1) + (3, 1, 1) + (1, 2, 1) + (1, 2) + (1, 7)]′ | 3[(1, 2) + (1, 1)] |
| (1, 2, 8, 3, 2) n₀ = n₁ = 1 | [SU(3) × SU(2) × SU(2)] × U(1) | 3[(3, 2, 1) + (3, 1, 2) + (1, 2) + (2, 8′) + (1, 28′)] | 3[(3, 1, 1) + (1, 2, 1) + (1, 1, 2 + (1, 8′)] + (3, 1, 2) + (1, 2, 2) + (1, 8; 2′)] + (3, 1; 2) + (1, 2, 2) + (1, 2; 2) + (1, 2; 2)] | 3[(2, 1) + 1(1)] |
| (2, 2, 6, 4, 2) n₂ = 2 | [SU(4) × SU(2) × SU(2)] × SU(2) | 3[(4, 2, 1) + (4, 2, 2) + (1, 2) + (4, 6′) + (1, 15′)] | 3[(4, 1, 1; 2) + (4, 1, 1; 2) + (4, 1, 1; 2) + (1, 1; 2) + (1, 1; 2) + (1, 1; 2) + (1, 6; 2′)] | 3[(2, 1) + 3] |
| (0, 2, 8, 4, 2) m₀ = n₃ = 1 | [SU(4) × SU(2) × SU(2)] × SU(2) | 3[(4, 2, 1) + (4, 1, 2) + (1, 2) + (4, 6′) + (1, 15′)] | 3[(4, 1, 1; 2) + (4, 1, 1; 2) + (4, 1, 1; 2) + (4, 1, 1; 2) + (8; 2′)] + (4, 1, 1; 2) + (1, 1; 2) + (1, 1; 2) + (8; 2′)] + (4, 1; 1; 2) + (1, 1; 2) + (1, 1; 2) + (8; 2′)] | 3[(2, 1) + 3] |

Table 1: Models with three generations and observable gauge groups like those of the Standard Model, left-right symmetric models and Pati-Salam models. The models are determined by the values of Nₓ, s = 1, ⋯, 4 and nᵃ, mᵃ a, 1 = 0, 1, 2. Only branes are trapped at the orbifold fixed points in these cases. Representations coming from SO(2N₀) × U(N₂) are labeled with a prime. Those coming from each of the 5-brane groups are separated with a semicolon in the 95 sector. In the 55 sector the two entries are the representations of the Sp(2mᵃ) × U(nᵃ) group.
| $(N_0, N_1, N_2, N_3, N_4)$ | 59 sector | 55 sector | 59 sector | 55 sector |
|-----------------------------|-----------|-----------|-----------|-----------|
| $(2, 1, 8, 3, 2)$ | $3[(1, 2) + (1, 1) + (3, 1) + (1, 8') + (4, 1')']$ | $3(1)$ | $f_- : 3[(3, 1) + (1, 2) + 2(1, 1) + (1, 8') + (4, 1')']$ | $f_+ : 6(1), f_- : 3(1)$ |
| $q_0 = n_3^i = 1$ | | | | |
| $(3, 1, 7, 3, 2)$ | $3[(3, 1) + (1, 2) + (1, 1) + (1, 7') + (6, 1')']$ | $3(1)$ | $f_- : 3[(1, 2) + (1, 1) + (1, 7')]'$ | $f_- : 3(3)$ |
| $n_1^i = p_2^j = 1$ | | | | |
| $(1, 1, 9, 3, 2)$ | $3[(3, 1; 2) + (1, 2; 2) + (1, 9; 2)']$ | | $f_- : 3[(3, 1) + (1, 2 + 2) + (1, 1) + (1, 9')]'$ | $f_+ : 3(1), f_- : 3(1)$ |
| $m_0 = q_1^i = 1$ | | | | |
| $(2, 2, 7, 3, 2)$ | $2[(3, 1, 1) + (3, 1) + (1, 2; 1) + (1, 2, 1)] + 2[(4, 1') + (1, 7')]'$ | | $f_- : 3[(3, 1, 1) + (3, 1; 1) + (1, 2, 1)] + (1, 2; 1) + (1, 2, 1) + (1, 7; 2)'$ | $f_+ : 3(2) + 2(1)$ |
| $n_2^{0,1} = p_2^j = q_2^j = 1$ | | | | |
| $(1, 2, 8, 3, 2)$ | $2[(3, 1, 1) + (1, 2; 1) + (1, 1, 2; 2) + (2, 1') + (1, 8')]'$ | $2[3(2) + (1)]$ | $f_- : 3[(3, 1, 1) + (1, 2, 1) + (1, 1, 2) + (2, 1') + (1, 8')]'$ | $f_+ : 3(1) + (1)$ |
| $n_0^i = m_0^{i,1} = m_1^{i,1} = 1$ | | | $3[(3, 1, 1) + (1, 2, 1) + (1, 1, 2; 2) + (2, 1') + (1, 8')]'$ | $f_- : 3(1) + (1)$ |
| $q_1^i = q_0^j = 1$ | | | $3[(3, 1, 1) + (1, 2, 1) + (1, 1, 2; 2) + (2, 1') + (1, 8')]'$ | |
| $(2, 2, 6, 4, 2)$ | $2[(4, 1, 1; 2) + (4, 1, 1; 2) + (1, 2; 1) + (1, 1, 2; 2)] + 3[(4, 1, 2') + (1, 6, 2')]'$ | $2(1) + 3$ | $f_- : 2[(1, 2, 1; 2) + (1, 1, 2; 2) + (1, 6, 2')]'$ | $f_- : 3(2) + 2(3)$ |
| $n_0^j = 2, p_1^j = 1$ | | | | |
| $(0, 2, 8, 4, 2)$ | $2[(4, 1, 1; 2) + (1, 2; 1; 2) + (8, 2')]'$ | | $f_- : (4, 1, 1) + (1, 2, 1) + (1, 1, 2; 2) + (8, 2')'$ | $f_+ : 2[(1) + (2)(3)]$ |
| $m_0^{i,1} = m_1^{i,1} = 1$ | | | | |
| $q_0^j = q_1^j = 2$ | | | | |

Table 2: Models with three generations and observable gauge groups like those of the Standard Model, left-right symmetric models and Pati-Salam models. The models are determined by the values of $N_s, s = 1, \ldots, 4$ the number of trapped 5-branes $n_a, m_a, a, 1 = 0, 1, 2$, and the number of trapped anti 5-branes $p_a^j, q_a^j$. Gauge groups and 99 sectors are as in table 1, except for the groups coming from the anti 5-branes which are \( \Pi_{a,j} Sp(2p_a^j) \times U(q_a^j) \). Representations coming from \( SO(2N_0) \times U(N_2) \) are labeled with a prime. Those coming from each of the 5-brane or anti 5-brane groups are separated with a semicolon in the 59 and 59 sectors. In the non-supersymmetric sectors we specify the fermion spectrum (including chirality) and the scalars also.
5 Scales and dynamics of the vacua

5.1 Lowering the string scale

The Type I vacua in previous sections are non-supersymmetric. The scale of SUSY-breaking is thus essentially the string scale $M_{\text{string}}$. In the second class of models in which some quark-lepton sectors are non-SUSY already at the level of the spectrum, it is clear then that the string scale cannot be much higher than say $M_s \leq 1 - 100$ TeV, otherwise all scalars will get masses in loops and we will have to face the usual gauge hierarchy problem. In the first class of models with anti-5-branes in the bulk and hidden from 9-branes, SUSY-breaking is hidden from the observable world coming from (99) and (59) sectors. Thus SUSY-breaking will be only felt in a suppressed manner and one would guess that there may appear SUSY-partner masses $M_{SB}$ of order $M_{SB} = M_{\text{string}}^2/(M_{\text{Planck}})$ as usual in hidden sector models. Thus, if we want $M_{SB}$ not to exceed the weak scale $M_W$ so that the solution to the hierarchy problem is not spoiled we must require:

$$M_{\text{string}} \leq \sqrt{M_W M_{\text{Planck}}} \propto 10^{11} \text{ GeV} \quad (5.1)$$

Thus in any of the schemes here considered we need to lower the string scale well below the Planck mass, at least down to the intermediate scale.

It is by now well known that one can lower the string scale in generic Type I vacua \cite{3,4,5}. In the $\mathbb{Z}_3$ case at hand it is more appropriate to perform a T-duality transformation along the third compact complex plane and work with 7-branes instead of 9-branes and 3-, anti-3-branes instead of 5-, anti-5-branes. The orientifold operation will be now $\Omega(-1)^F R_3$, where $R_3$ is the reflection operation with respect to the third complex plane. The world-volume of the 7-branes includes the first two complex dimensions and they are located at the origin $X_3 = 0$ in the third complex plane. As in the case of 9-branes, one can add Wilson-lines in the first two complex planes and one can distribute 3-branes or anti-3-branes located anywhere in compact space. The distribution of 3(3)-branes and additions of Wilson lines in the first two complex planes is subject to the tadpole constraints:

$$\text{Tr} (W)^k \gamma_{\theta,7} + 3(\text{Tr} \gamma_{\theta,3,L} - \text{Tr} \gamma_{\theta,3,L}) = -4 \quad (5.2)$$

where $L$ labels the 9 fixed points in the first two complex planes. Thus the model so obtained has exactly the same massless spectrum and interactions as the original one. Now, if we locate the anti-3-branes at points with $X_3 \neq 0$, they will have no overlap.
with the 7-branes and they will act as a SUSY-breaking hidden sector for the (77) and (73) massless fields. This is the T-dual of the class of models considered in section 4.1.

Let us consider the gauge couplings and scales in this class of orientifolds. The Planck mass $M_p$ is related to the compact scales $M_i$, $i = 1, 2, 3$ and 10-dimensional Type I dilaton $\lambda$ by (see e.g. ref.[29]):

$$M_p = \frac{2\sqrt{2}M_4^4}{\lambda M_1 M_2 M_3}$$

(5.3)

and the gauge couplings of the gauge groups from 7-branes and 3-branes are

$$\alpha_7 = \frac{\lambda M_2^2 M_3^2}{2M_s^4}; \quad \alpha_3 = \frac{\lambda}{2}$$

(5.4)

Thus, irrespective of the string scale one has:

$$M_p = \frac{\sqrt{2}M_1 M_2}{\alpha_7 M_3}.$$  

(5.5)

By making $M_3$ much smaller than $M_{1,2} \propto M_s$, one can obtain agreement with the measured values of $M_p$ and $\alpha_7$. In particular, one can set $M_1 = M_2 = M_s$, which leads to $\alpha_7 = \alpha_3 = \lambda/2$. One can now take the string scale at the intermediate scale $M_{1,2} = M_s = 10^{10}$ GeV by choosing $M_3 \propto 1$ TeV. In the same way, one can take $M_s = M_{1,2} = 1$ TeV by taking $M_3 = 10^{-3}$ eV.

One concludes that the string scale, which in this case coincides with the SUSY-breaking scale, may be arbitrarily lowered by making sufficiently large the third compact dimension. Notice that in the present scheme the first two compact dimensions are small, of order the string scale, whereas the third dimension is large. There may be a possible dynamical explanation for this asymmetry in the second class of models discussed in the previous sections where branes and anti-branes are stuck at fixed points in the first two complex compact dimensions. Indeed, as emphasized in ref.[4], in this case one expects that the brane-anti-brane attraction will generate a potential which will tend to shrink the compact radii $r_{1,2}$. This force would not be present for the third compact dimension which can then dynamically grow.

5.2 Stability and Effective Potentials

Another issue we need to address is the stability of these configurations. We know that contrary to most previous considerations of brane/anti-brane systems, there are no tachyons that could destabilize these constructions [1]. Furthermore, some or all of the branes must be trapped in the orbifold fixed points in order to satisfy tadpole
cancellation. For instance, for the situation with only branes stuck at the fixed points, there will be the same number of anti-branes living on the bulk and one may wonder why they do not approach the trapped branes and annihilate. In fact the twisted tadpole cancellation conditions in general forbid this complete annihilation process because some residual branes or antibranes have to remain in the fixed points to cancel the tadpoles. Thus the annihilation can only be partial.

In this connection it is interesting to study the possible relation between the models of section 4.1 and those of section 4.2. One can imagine starting in a configuration with branes stuck at the fixed points and anti-branes moving freely on the bulk, like those of section 4.1. Eventually the right number of anti-branes would get attracted to the right fixed points, annihilating some of the branes at those points in a manner consistent with tadpole cancellation. For instance in the $SU(3) \times SU(2) \times SU(2)$ model, we may start with the configuration with three pairs of stuck 5-branes and six anti 5-branes on the bulk. All of these anti 5-branes may go to one single fixed point, annihilate the two 5-branes there, leaving four anti 5-branes and becoming the corresponding model of section 4.2. In the $SU(4) \times SU(2) \times SU(2)$ model however, the twelve anti 5-branes would have to go, six to one point and six to the second point in order to keep tadpole cancellation. It is interesting to ask which brane configuration is dynamically preferred, if any, since both seem to be connected in some way. This seems to be a complicated dynamical question since several dynamical effects seem to be competing.

Usually since branes and anti-branes carry opposite Ramond-Ramond charge they tend to attract each other\[4\]. However, the presence of the stuck branes also induces a repulsive interaction between them and the anti-branes, due to the total positive brane tension, similar to the effect of a positive cosmological constant in inflationary models, for instance. This extra contribution to the vacuum energy corresponds actually to an effective potential for the dilaton and the moduli fields. In general the exact form of this potential is something beyond our control, since supersymmetry has been broken, there are all sorts of radiative corrections that can contribute as well as standard non-perturbative effects\[11\].

In the case in which both branes and anti-branes are stuck at fixed points (second class of models in the previous sections) naively one could argue that there are competing effects, some of them trying to shrink the first two compact dimensions and other trying to expand them, as we now explain. For the models constructed in terms of 9 and 5-branes, the tension of the 5-branes and anti 5-branes is proportional to $1/\lambda$,\[11\]For some recent discussions of radii stabilisation in the context of brane models see refs.\[45\].
with $\lambda$ the string coupling. Upon compactification the tension will generate a term in the action proportional to $1/M_3^2\lambda$ where $M_3$ is the compactification scale of the third complex direction which the 5-brane is wrapped around. This is precisely the real part of the so-called $T_3$ field of type I compactifications. Therefore we can see that the brane tension induces a linear potential for one of the moduli fields. In the dual version in terms of 7 and 3-branes, which is more appropriate to discuss a lowering of the string scale, this term is only proportional to $1/\lambda$ which is the corresponding dual $T_3$. Recall that in the latter brane configuration, the moduli fields are given by the following expressions [29]:

$$ReS = \frac{2M_4^4}{\lambda M_1^2 M_2^2}; \quad ReT_3 = \frac{2}{\lambda}$$

$$ReT_1 = \frac{2M_4^4}{\lambda M_2^2 M_3^2}; \quad ReT_2 = \frac{2M_4^4}{\lambda M_2^2 M_3^2}$$

The gauge couplings are given by $ReS$ for the groups on the 7-branes and $ReT_3$ for the groups on the 3-branes. Therefore perturbative corrections in the 7-branes field theory would go like $S^{-n}$ whereas non-perturbative corrections would depend on $e^{-S}$. Therefore $S$ seems to have the standard runaway behaviour towards large values whereas $T_3$ has the linear contribution from the brane tension that tends to stabilize it at small values. Thus these two effects tend to drive $M_2^2 M_3^2 = \frac{ReT_3}{ReS}$ towards small values. On the other hand there is in addition the contribution to the vacuum energy from the brane/anti-brane attraction, that, as argued above, tends to drive $M_2^2 M_3^2$ to large values. Thus there seems to be two competing effects in opposite directions. This could lead to two possible outcomes: i) The competing effects lead to stable minimum with fixed $M_{1,2}$ and $\lambda$ or ii) At a given moment it could be more energetically favorable that anti-branes pop out from the fixed points to the bulk and only branes (satisfying tadpole cancellations) remain there. This would mean a transition from the type of models of section 4.2 to those of section 4.1. Without more dynamical information it is difficult to say which type of configurations, either with trapped anti-branes at fixed points or with antibranes away in the bulk is preferred from the effective potential point of view.

All these considerations need further scrutiny in order to be able to extract concrete implications about the effective dynamics of these new brane configurations. We have the advantage over previous discussions that the models are explicit and the effective action is in principle computable, at least in some approximation scheme. There is no need to say that after fixing the dilaton and moduli fields, we would have to be exposed to the standard cosmological constant problem, as in any other treatment of
supersymmetry breaking.

5.3 Yukawa couplings

This is not the place to make a detailed phenomenological analysis of Yukawa couplings which we leave to future work. It is however interesting to present the general structure of Yukawa couplings in this class of theories, which turn out to be quite different to similar heterotic orbifold models. The following renormalizable couplings are generically present:

i) (99)(99)(99) couplings

There are superpotential couplings of the form:

\[ \phi_i^{99} \phi_j^{99} \phi_k^{99}, \quad i \neq j \neq k \neq i \]  

(5.7)

where \( \phi_i^{99}, i = 1, 2, 3 \) are charged chiral fields in the (99) sector associated to the complex plane \( i \). These type of couplings give rise for example to quark Yukawa couplings in the \( SU(3) \times SU(2)_L \times SU(2)_R \times U(1) \) models of chapter 4. These couplings are totally analogous to the ones present for untwisted particles in perturbative heterotic orbifolds. The couplings \( (55)^3 \) have a completely analogous structure.

ii) (59)(99)(95) couplings

The worldvolume of 5-branes includes the third complex plane. Thus there are superpotential couplings of the form:

\[ \psi_a^{59} \psi_a^{59} \phi_3^{99} \]  

(5.8)

where \( a \) labels the fixed point where the 5-brane is localized. Notice that only the (99) chiral fields from the third complex plane have these couplings. In addition these couplings are diagonal in the \( a \) label, i.e., there are no renormalizable couplings involving different fixed points. These type of couplings give rise for example to standard lepton Yukawa couplings in the \( SU(3) \times SU(2)_L \times SU(2)_R \times U(1) \) models of chapter 4.

iii) (55)(59)(95) couplings

In a similar manner there are superpotential couplings of the form

\[ \psi_a^{59} \psi_a^{59} \phi_{3,a}^{55} \]  

(5.9)

in which again \( a \) labels the fixed point. Only the (55) chiral fields in the third complex plane appear in the coupling. For example, there is a coupling of this type between a singlet \( (1)_{2q} \) in the (55) sector and the coloured pair \( (3, 1, 1, -2/3)_{-q} + (\overline{3}, 1, 1, 2/3)_{-q} \) in the same left-right symmetric model of chapter 4. In general the presence of this
type of couplings tends to reduce the number of extra vector-like fields in the massless sector once scalars in the (55) sectors get vevs.

\textit{iv)} $(99)(59)(95)$ couplings

The $(59)$ sectors are not supersymmetric so that some SUSY partners are missing. Again, only the $(99)$ fields in the third complex plane have this type of couplings. There are Yukawa couplings of type:

\begin{align}
\psi_a^{59} \bar{\psi}_a^{59} \phi_{3,a}^{99} \\
\phi_a^{59} \bar{\psi}_a^{59} \psi_{3,a}^{99}
\end{align}

in which now $\phi$ denotes a scalar and $\psi$ a fermion. Examples of these are the lepton Yukawa couplings in the left-right symmetric model in section 4.2.

\textit{v)} $(55)(59)(95)$ couplings

In a similar manner there are Yukawa couplings of the form

\begin{align}
\psi_a^{59} \bar{\psi}_a^{59} \phi_{3,a}^{5} \\
\phi_a^{59} \bar{\psi}_a^{59} \psi_{3,a}^{5}
\end{align}

in which again $a$ labels the fixed point and $\phi(\psi)$ denotes a scalar(fermion). Only the $(55)$ fields in the third complex plane appear in the coupling. For example, there is a coupling of this type between a singlet scalar $(1)_{2r}$ in the $(55)$ sector and the coloured pair of fermions $(3,1,1,-2/3)_{-r} + (3,1,1,2/3)_{-r}$ in the left-right symmetric model of section 4.2.

As we mentioned above, the structure of the $(55)^3$ couplings is analogous to that of the $(99)^3$ couplings. Something similar happens for the couplings of type $(55)^3$, although, of course in this sector the spectrum is not supersymmetric. Still the only Yukawa couplings which do not vanish are those involving fields in three different complex planes. Concerning the sizes of the couplings, $(99)^3$, $(59)(99)(95)$ and $(99)(59)(95)$ couplings are proportional to $(ReS)^{-1/2}$, $S$ being the complex dilaton defined above. $(55)^3$, $(55)^3$, $(55)(59)(95)$ and $(55)(59)(95)$ couplings are proportional to $(T_3)^{-1/2}$ as defined above.

It is interesting to remark that all models present a flavour structure coming from the different 5-brane groups. In the case in which 5 branes give only rise to $U(1)$’s, we have already seen how those $U(1)$’s are broken by the Green-Schwarz mechanism and effective global $U(1)$’s effectively persist. These $U(1)$ symmetries (and possibly larger 5-brane gauge symmetries) may play an important role in ensuring sufficient proton stability. We postpone a thorough analysis of this question for future work.
5.4 Gauge coupling unification

An analysis of this question should be done on a model by model basis and such an analysis goes beyond the scope of this paper. We would like to point out however that in some of the models discussed above there is a tendency to get gauge coupling unification at the intermediate scale. Specifically let us consider the SM orientifold of section 4.1. Notice first that the embedding of the $U(1)$ hypercharge does not have the canonical normalization with $g_1^2/g_2^2 = 3/5$ at the string scale. One rather has $g_1^2/g_2^2 = 3/11$. Then the one-loop renormalization group running from the string scale $M_s$ to the weak scale $M_Z$ gives for the weak angle and the QCD couplings at the weak scale:

$$
\sin^2 \theta_W(M_Z) = \frac{3}{11}(1 + \frac{11\alpha(M_Z)}{6\pi} (b_2 - \frac{3}{11}b_1) \log(\frac{M_s}{M_Z}) )
$$

$$
\frac{1}{\alpha_3(M_Z)} = \frac{3}{11}(\frac{1}{\alpha(M_Z)} - \frac{1}{2\pi} (b_1 + b_2 - \frac{14}{3}b_3) \log(\frac{M_s}{M_Z}) )
$$

(5.12)

In order to perform the running we only have to include the particles which remain massless below $M_s$. In the particular models at hand that depends on whether certain singlet fields in the $(55)$ sectors get vevs and give masses to some vector-like particles. We will consider the most generic case in which indeed those singlets get vevs of order the string scale. Let us assume then that, the three sets of chiral multiplets $(3,1,-1/3)_q + (3,1,1/3)_q$ and also the vector-like right-handed leptons from the $(0,m)$ fixed points get a mass of order $M_s$ due to the couplings of these fields to the singlet scalars in the $(55)$ sectors. Then below the string scale the $\beta$-functions of the $SU(3) \times SU(2) \times U(1)$ interactions are respectively $b_3 = 0$, $b_2 = 6$ and $b_1 = 18$.

Plugging this in the above renormalization group formula one obtains:

$$
M_s = 1.4 \times 10^{12} \text{ GeV} \quad; \quad \sin^2 \theta_W(M_Z) = 0.239
$$

(5.13)

where we have used $\alpha_3(M_Z) = 0.12$. Taking into account that in this simple estimate we have just made a one-loop computation and we have also assumed all light thresholds at $M_Z$ and all heavy thresholds close to $M_s$, these are not unreasonable values, given the uncertainties. In particular, not all fields which get masses through the vevs of $(55)$ scalars need to have identical masses and decouple at the same scale. For example, one can check that if the 3 sets of right-handed vector-like leptons have masses below $M_s$, both the values obtained for $\sin^2 \theta_W$ and $M_s$ decrease accordingly.

In models like those of section 4.2 which are explicitly non supersymmetric, the string scale must be at most of order 1-100 TeV to avoid the hierarchy problem. Thus the usual field theory logarithmic running will not be in general sufficient to achieve
unification at such low string scale. Other approaches like perhaps those of refs. [46, 47, 48, 49] (see also [22, 50, 51, 52]) could be relevant. Meanwhile one must admit that models with only branes at fixed points like the ones in section 4.1 seem to be able to easily accommodate gauge coupling unification as long as the string scale is of order of the intermediate scale which, as we mentioned above, is the natural one if we have gravity mediated SUSY-breaking.

6 Final comments and outlook

In this paper we have constructed the first semirealistic string vacua using D-brane techniques. We have showed that the addition of brane-antibrane pairs on otherwise \( N = 1, D = 4 \) supersymmetric Type I models is a quite versatile technique in order to produce tachyon-free and RR tadpole-free string vacua resembling the standard model of particle physics (or some left-right symmetric generalizations of it).

The vacua constructed are non-supersymmetric but the SUSY-breaking effects may be felt in a suppressed manner in some subsectors of the theory. For example, the sector identified with the SM may be separated in transverse compact space from the SUSY-breaking anti-branes in such a way that only closed strings can transmit SUSY-breaking to the observable physics world. This would correspond to a standard gravity mediated scenario in which SUSY-breaking has to happen at the intermediate scale \( \sqrt{M_W M_{\text{Planck}}} \propto 10^{11} \text{ GeV} \). These are the class of models presented in sections 4.1 and 4.3. Since in this class of theories the SUSY-breaking scale is the string scale one would have to identify \( M_s \) with the intermediate scale also.\(^{12}\) We have explicitly shown how in this class of theories one can indeed lower the string scale to these values and still maintain consistency with the known values of Planck mass and perturbative gauge couplings. We have also shown that in some of the specific examples presented the standard model couplings do indeed join at the intermediate scale. This comes about because of the presence in these models of extra vector-like lepton generations or Higgs doublets.

In other constructions SUSY-breaking may be transmitted to the observable world either by gauge mediation (as in the example in the appendix) or even at the tree level, i.e., one can get some open string sectors with non-SUSY spectra. The latter is the

\(^{12}\)In this setting only closed string states living in the bulk like the dilaton \( S \) and complex moduli \( T_i \) couple simultaneously to branes and antibranes. It is thus reasonable to expect that the SUSY-breaking effects transmitted to the (99) and (59) observable sectors may be parametrized in terms of vev’s for the auxiliary fields of \( S \), and \( T_i \), as proposed in refs. [53, 54].
case of the models presented in section 4.2. In this case the SUSY-breaking (and thus the string scale) should not be above say 1-100 TeV if we want to avoid the standard hierarchy problem.

We have presented specific examples of string vacua with three generations and the gauge group of the standard model or some of their left-right symmetric extensions but one can construct many more along similar lines. The models constructed provide for specific examples in which recent ideas involving a string scale well below the Planck scale may be tested. Their massless spectrum is quite simple compared with analogous perturbative heterotic models constructed in the past. Thus, for example, the left-right symmetric model of section 4.2 contains just three quark-lepton generations and three sets of Higgses in its massless spectrum. We have concentrated on the ZZ$_3$ orientifold because three generations are easily obtained in that case but it should be possible to construct many more examples using other orientifolds.

We have not presented a detailed phenomenological analysis of the specific models presented which we leave for future work. We have however discussed the general structure of $U(1)$ anomaly cancellation and Yukawa couplings which are the required ingredients for such a phenomenological analysis. In general there are several anomalous $U(1)$’s whose anomalies are cancelled by the generalized Green-Schwarz mechanism discussed in ref. [33]. There are associated Fayet-Iliopoulos terms which are controlled by the blowing-up moduli of the orbifold. Generically only the standard hypercharge $U(1)$ remains unbroken down to low energies in the explicit SM orientifolds constructed. The broken $U(1)$ symmetries lead to some flavour structure for the models which deserve further analysis since they very much determine the structure of Yukawa couplings and hence questions like fermion textures and proton stability.

It is perhaps instructive to compare this class of realistic Type I vacua with those first constructed in ref. [38] in the context of ZZ$_3$ heterotic perturbative orbifolds. Models like the ones we have constructed here do not have a perturbative heterotic dual since, e.g., the gauge shifts $V$ and Wilson lines $W$ used do not obey the standard modular invariance constraints. However the models in refs. [38] have a number of similarities (and also important differences) with the ones built here. One of the similarities is that the models constructed with both techniques have a tendency to give rise to the presence of extra vector-like sets of leptons at low energies. However, important aspects like the structure of Yukawa couplings are totally different. In general the Type I constructions are much simpler in most respects: 1) Only the addition of one Wilson line is required in Type I whereas two Wilson lines (leading to a proliferation
of twisted sectors and massless states) are required in the heterotic models. 2) In the heterotic models the necessary presence of a non-vanishing (dilaton-dependent) Fayet-Iliopoulos term makes also the identification of the correct perturbative vacuum quite cumbersome, making a detailed analysis of the effective field-theory D-flat directions necessary. In the Type I case models one can put in principle the FI-terms to zero and no complicated analysis of flat directions is needed. 3) In the Type I case one can lower the string scale down to the intermediate scale and achieve gauge coupling unification. The possibility of lowering the scale is not available in perturbative heterotic models and this makes difficult to achieve gauge coupling unification. 4) In the Type I models here constructed SUSY-breaking (in a hidden sector or not) is a built-in property of the models. In perturbative heterotic models one relies on the possible existence of some hierarchy-generating mechanism like gaugino condensation in order to give rise to SUSY-breaking.

An interesting difference between these models and the perturbative heterotic models is the rank of the gauge group. In perturbative models, the rank cannot exceed 22 and in orbifold models at generic values of the moduli space of the six dimensional compact space, the gauge group has rank 16. In the present models, the rank of the group coming from the 9-branes is also 16. On the other hand, the rank of the groups coming from the 5 and $\bar{5}$ branes may be very large, depending on the values of $N_s$ and it seems to be unlimited. We may easily see this by considering the tadpole conditions (3.10). If for instance the left hand side of those equations vanishes, as in the supersymmetric case, we can have, say $n_1^i = 2m_1^i = q_2^i = 2p_2^i = 2K$ with $K$ arbitrarily large and all equations will be satisfied. This seems to imply that as long as the number of branes and anti branes is equal there is a lot of freedom to satisfy the tadpole conditions and an arbitrary number of branes and anti branes can be added satisfying those conditions, the corresponding rank of the gauge groups increases linearly with $K$ and we have no bound on this value. It was known previously that non perturbative string vacua can have rank much larger than 22. Here, we seem to have explicit models with an unbounded rank, bringing the difference with perturbative vacua to the extreme. However we expect these higher rank models to be unstable and ‘decay’ to the lowest rank models, the reason being the following: if in the example above we can satisfy the tadpole equations with say $n_1^i = m_1^i = 0$. The addition of an arbitrary number of branes that still keeps the conditions satisfied ($n_1^i = 2m_1^i = 2K$) is not necessary for tadpole cancellation therefore the corresponding extra branes (and anti-branes) do not need to be trapped at the fixed points. We then expect that those configurations
will decay to the one with the minimum number of branes required by the tadpole conditions, which are the only ones forced to be trapped at the fixed points. All the extra branes and anti-branes may annihilate each other in the bulk. Recall that the explicit models we have presented require the minimum number of branes and anti-branes.

One open question that one should study more carefully is that of the stability of this class of vacua. As we remarked above, these models are free of tachyons both in the bulk and on the branes. Some of the branes and/or anti-branes are stuck at some of the fixed points of the underlying orbifold due to the tadpole cancellation conditions and hence they are not free to travel to the bulk and give rise to complete brane-antibrane annihilation. The existence of these trapped branes may lead to dynamical effects tending to shrink some of the compact dimensions and no others as discussed in the text.

These are first steps in the search for realistic string vacua with the D-brane techniques. Much work remains to be done in this direction both from the theoretical side, finding new classes of vacua and addressing the important dynamical issues involved in this class of theories, as well as from the phenomenological side, studying the viability of specific examples as well as extracting general properties which could be generic. We hope we have convinced the readers of the interest of such a search.

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7 Appendix

7.1 A model with gauge-mediated supersymmetry breaking

The class of models with 9-branes and 5-branes discussed in chapter 3 allow also for the construction of simple models in which the supersymmetry breaking effects originated from anti-5-branes is transmitted to an “observable” world of 5-branes through loops involving gauge interactions from the 9-branes living in the bulk. In particular, the SUSY chiral multiplets in (59) sectors couple through (99) sector gauge interactions to the non-SUSY (59) sector.

Consider as an example the following $\mathbb{Z}_3$ orientifold. We take the 32 9-branes with twist on CP factors given by:

$$\gamma_{\theta,9} = \text{diag}(I_{18}, \alpha I_7, \alpha^2 I_7) \quad (7.1)$$

Now, since $\text{Tr} \gamma_{\theta,9} = 11$, tadpole cancellation conditions at each of the 9 fixed points in the first two complex planes require:

$$\text{Tr} \gamma_{\theta,5} - \text{Tr} \gamma_{\theta,\bar{5}} = -5 \quad (7.2)$$

Now, we will locate 10 5-branes at each of 4 fixed points with:

$$\gamma_{\theta,5} = \text{diag}(\alpha I_5, \alpha^2 I_5) \quad (7.3)$$

and at each of the remaining 5 fixed points 8 anti-5-branes with

$$\gamma_{\theta,\bar{5}} = \text{diag}(I_6, \alpha, \alpha^2) \quad . \quad (7.4)$$

The complete gauge group is:

$$[U(5)^4]_5 \times [SO(18) \times U(7)]_9 \times [(Sp(6) \times U(1))^5]_\bar{5} \quad (7.5)$$

We can now identify one of the four $SU(5)$’s with the observable physical world. In the (55) sector one has charged chiral multiplets transforming like: $2(10) + 15$ under $SU(5)$. Now, from the (59) sector we will get supermultiplets transforming like $(18, 1; 5) + (1, 7; 5)$ under $SO(18) \times SU(7) \times SU(5)$. Altogether the $SU(5)$ charged spectrum will contain $2(10 + \bar{5}) + (15 + 9(\bar{5})) + 7(5 + \bar{5})$. This is an anomaly-free spectrum with two standard plus one exotic $SU(5)$ generations. This spectrum is totally supersymmetric but it will get non-SUSY corrections in loops coming from their couplings of $SO(18) \times U(7)$ gauge bosons to the non-SUSY (59) sector. Although this particular model is not very realistic, it exemplifies how in this type of vacua one can indeed obtain models with gauge mediated SUSY breaking as advertised in the text.
7.2 A variant SM

One can construct many variations on the models presented in sections 4.1 and 4.2. The $SU(3) \times SU(2) \times U(1)$ model of 4.2 has the right-handed quarks in a non-SUSY sector of the theory. This can be easily modified and e.g., one can get a variation of the model in which only the leptons have a non-SUSY spectrum. Let us first start with the model with only 5-branes at the fixed points and then we will discuss the model with both branes and anti-branes at the fixed points. Take the $\mathbb{Z}_3$ orientifold with an action of the twist on the CP factors given by:

$$V_9 = \frac{1}{3}(1, 1, 1, -1, -1, 0, 0, 0, 1, 1, 1, 1, 1, 1)$$ (7.6)

A quantized Wilson line is added in the first complex plane given by:

$$W_9 = \frac{1}{3}(1, 1, 1, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$ (7.7)

The gauge group from the (99) sector will be $U(3) \times U(2) \times U(1) \times [SO(6) \times U(7)]$. Now, the 9 $\mathbb{Z}_3$ fixed points in the first two complex planes split into three sets of three fixed points each: $(0, m)$, $(1, m)$ and $(-1, m)$ with $m = 0, \pm 1$. The shifts corresponding to each of these three sets are $V, V + W$ and $V - W$ respectively. The corresponding value for $\text{Tr} \gamma_{\theta,9}$ are -4, -1 and 2 respectively. Looking at the tadpole cancellation conditions we see that we will be forced to add 5-branes at the fixed points $(1, m)$ with $\text{Tr} \gamma_{\theta,5} = -1$ and also at the fixed points $(-1, m)$ with $\text{Tr} \gamma_{\theta,5} = -2$. No 5-branes have to be added to the points $(0, m)$ since tadpoles cancel without them. Thus the simplest option is to locate 5-branes with twist actions given by:

$$(1, m) \rightarrow \gamma_{\theta,5} = \text{diag} (\alpha, \alpha^2)$$ (7.8)

One gets the following massless spectrum:

(99):

There are chiral multiplets transforming under the gauge group like (the hypercharge $U(1)$ generator is as in section 4.1):

$$3(3, 2, 1/6) + 3(\bar{3}, 1, -2/3) + 3(1, 2, +1/2) + [3(21)'] + 3(6, 7)']$$ (7.9)

These fields correspond to 3 generations of left-handed quarks and right-handed U-quarks as well as three sets of Higgs fields $H_U$.

(59):
From the 5-branes at the $(1,m)$ fixed points we get a $U(1)_5^3$ gauge group. One then finds chiral multiplets (in three copies, one per fixed point) transforming like:

$$(\bar{3}, 1, 1/3)_q + (1, 1, -1)_q + (1, 2, -1/2)_{-q} + (1, 2, 1/2)_{-q} + [7'_q + 6'_{-q}] \quad (7.10)$$

where the subindex $q$ refers to the $U(1)_5^3$ charges. These fields include three generations of right-handed $D$-quarks and three sets of vector-like doublets. The latter can become massive if the singlets in the $(55)$ sectors get a vev. From the 5-branes at the $(-1,m)$ fixed points we get a $U(2)_5^3$ gauge group. One then finds chiral multiplets (in three copies, one per fixed point) transforming like:

$$(1, 2, -1/2; 2)_r + (1, 1, +1; 2)_r + (\bar{3}, 1, 1/3; 2)_{r'} + (3, 1, -1/3; 2)_{r'} (7.11)$$

$$+ [(7'; 2)_r + 6'_{-r}]$$

where the subindex $r$ refers to the $U(1)_5^3$ charges. This sector includes all leptons and additional weakly interacting particles (as well as vector-like sets of extra color triplets). Now, the $(55)$ sector living on the $(-1,m)$ fixed points contain an adjoint under the $SU(2)$’s which can get a vev along a flat direction and give a mass to the triplet anti-triplet pairs in the corresponding $(59)$ sector. Altogether this model contains at the massless level three standard quark-lepton generations plus three sets of vector-like right- and left-handed extra leptons.

As we did in section 4.2, one can construct a related model with both 5- and anti-5-branes all stuck at the fixed points. The simplest possibility is to replace the 5-branes at the $(-1,m)$ fixed points by anti-5-branes. We put two anti-5-branes with $\gamma_{\theta,5} = (I_2)$. This meets the tadpole conditions and leads to a group $Sp(2)^3$. Now, the spectrum of this model is similar to the previous one but replacing the SUSY spectrum in $(7.12)$ by the following (non-SUSY) one:

$$(59) :$$

$$\begin{align*}
Fermions & : (1, 2, -1/2; 2) + (1, 1, +1; 2) + [(7'; 2)] \quad (7.12) \\
Scalars & : (\bar{3}, 1, 1/3; 2) + (3, 1, -1/3; 2) + [(6'; 2)]
\end{align*}$$

As in the previous situation, this sector provides for the leptons which then are the only non-SUSY sector of the model. The $55$ sector contains only one negative chirality triplet for each of the $Sp(2)$ groups.
7.3 Another Variant SM

Here we will just write the spectrum of the other model discussed at the beginning of section 4 containing $SU(3) \times SU(2) \times U(1)$. $N_2 = 9, N_0 = 1; n_i^1 = m_i^1 = m_i^0 = 1$. The full gauge group is

$$[SU(3) \times SU(2) \times U(1)] \times U(1)^3 \times [SO(2) \times U(9)]' \times Sp(2)^6 \times U(1)^3$$ (7.13)

The hypercharge generator can be defined as in the previous models. The spectrum is

99 sector:

$$3 [(3, 2, 1/6) + (1, 2, 1/2) + (3, 1, -2/3)] + 3 [(2, 9)' + (1, 36)']$$ (7.14)

Again this sector can incorporate the three families of left handed quarks and right handed $U$-quarks together with three sets of Higgs fields $H_U$.

From the 5-branes at the points $L_0$ we have:

55 sector:

$$5 L_0 9$$

$$3 [(3, 1, 1/3; 2) + (1, 2, -1/2; 2) + (1, 9; 2)]$$ (7.15)

Where the index after the semicolon is the $Sp(2)$ representation (one different for each fixed point). Here we have candidate leptons and right handed $D$-quarks, although they are doublets of $Sp(2)$ also. From the 5-branes at the points $L_1$ the massless spectrum is:

55 sector:

$$5 L_1 9$$

$$3 [(3, 1, 1/3) q + (1, 2, -1/2)_{-q} + (1, 2, 1/2) q + (1, 1, -1) q + (2, 1)'_{-q} + (1, 9)'_q] + 3 [(3, 1, -1/3; 2) + (1, 1, -1; 2)] + (1, 9; 2)']$$ (7.16)

Where the subindices $q$ are the charges with respect to each of the three $U(1)$’s coming from the 5-branes. Here we also have candidate leptons and right handed $D$-quarks.

Finally, the massless spectrum of the 55 sector transform under each $Sp(2) \times U(1)$ at the $L_1$ fixed points as: 55$_{L_1}$ sector:

$$3 [(2)_{-q} + 2 (1)_{2q} + (1)_{2q}]$$ (7.17)

Contrary to the previous models, it is not clear how in this model all the extra (unwanted) triplets and doublets can get a mass, therefore this model looks at first
sight less interesting phenomenologically, although a detailed study may reveal ways of making it more realistic. A similar model can be obtained by substituting the branes at the $L_1$ points by four anti branes ($q_i^1 = 1$). Therefore the spectrum is the same except for the $5_{L_1}9$ sector above which is changed by the nonsupersymmetric one (with opposite chirality) in three copies:

\begin{align}
Fermions & : (\bar{3}, 1, 1/3)_r + (1, 2, 1/2)_r + (1, 2, -1/2)_{-r} + (1, 1, +1)_r \\
& + [(2, 1)'_{-r} + (1, 9)']_r \\
Scalars & : (3, 1, -1/3)_{-r} + (1, 1, +1)_r + [(1, 9)']_r + h.c. \quad (7.18)
\end{align}

Where $r$ is the charge with respect to each of the $U(1)$'s coming from the anti branes. The $\bar{5}5_{L_1}$ also replaces the $55_{L_1}$ sector which consists of three fermions and six scalars (singlets) with charge $-2r$. It also includes three extra singlets with negative chirality. Therefore, we may have here both leptons and right handed $D$-quarks in a non-supersymmetric sector.

### 7.4 Continuous Wilson lines. Models with adjoint representations

As we have stressed, in contrast to what happens in supersymmetric models, the addition of antibranes allows for several consistent twist matrices and Wilson lines. In particular continuous Wilson lines can be easily included. This is an interesting possibility for model building since it can be used for reducing the gauge groups ranks and for obtaining adjoint representations. Here we show how continuous Wilson lines can be introduced in $\mathbb{Z}_3$ orientifold discussed in section 3. Let us consider the case $N_1 = N_3 = N_4 = N$, and thus $N_0 + N_2 = 16 - N$ in $\mathbb{Z}_2$ and rewrite the twist matrix and discrete Wilson line $\mathcal{W}$ in terms of $3N \times 3N$ and $(16 - 3N) \times (16 - 3N)$ matrices as $\gamma = (\gamma_{\text{diag}}, \gamma_2)$ where

\begin{align}
\gamma_{\text{diag}} &= \text{diag} (I_N, \alpha I_N, \alpha^2 I_N) \\
\gamma_2 &= \text{diag} (I_{N_0}, \alpha I_{N_2}) \quad (7.19)
\end{align}

and

$$
\tilde{\mathcal{W}} = \text{diag} (\alpha I_N, \alpha I_N, \alpha I_N; I_{N_0}, I_{N_2}) \quad (7.21)
$$

Hence, the gauge group $U(N)^3 \times SO(2N_0) \times U(N_2)$ is obtained, and the massless
spectrum can be read from 3.12 to be

\begin{align}
3[(\mathbf{N}, \mathbf{N}, 1; 1, 1) + (1, \mathbf{N}, \mathbf{N}, 1; 1, 1) + (\mathbf{N}, 1, \mathbf{N}, 1; 1)] + \\
+3[(1, 1, 1; 2N_0, N_2) + (1, 1, 1; 1, a_2)]
\end{align}

(7.22)

(7.23)

which is invariant under the permutations of the \(U(N)\) gauge groups. Moreover, by giving appropriate \(vev's\) to \((\mathbf{N}, \mathbf{N})\) multiplets, \(U(N)^3\) group should break down to the diagonal subgroup \(SU(N)_{\text{diag}} \times U(1)\), leaving three adjoint representations in the spectrum.

In particular an \(SU(5)\), group with three \(24\) massless representations could be obtainable. However, since antisymmetric representations of the \(99\) gauge group cannot appear in crossed sectors, the model is expected to e non chiral.

Such a breaking is easily achieved by considering a continuous Wilson line of the type presented in [14]. Namely,

\[
\tilde{W} = \begin{pmatrix}
W_{\text{diag}} & 0 \\
0 & 1_{N_0 + N_2}
\end{pmatrix}
\]

(7.24)

a \(16 \times 16\) matrix, where \(W_{\text{diag}}\) is a \(3N \times 3N\) matrix (which acts on \(\gamma_{\text{diag}}\)), defined as

\[
W_{\text{diag}} = \begin{pmatrix}
\lambda & a & a \\
a & \lambda & a \\
a & a & \lambda
\end{pmatrix}
\]

(7.25)

where \(a\) and \(\lambda\) are arbitrary complex numbers and each block is a \(n \times n\) matrix. It is straightforward to check that \((\gamma_{\text{diag}}W_{\text{diag}})^3 = cI_{3n}\) where

\[
c = (\lambda^3 - 3a^2\lambda + 2a^3)
\]

(7.26)

and also \(\text{Tr}(\gamma_{\text{diag}}W_{\text{diag}}^k) = 0\) for \(k = 0, 1, 2\). Therefore

\[
\text{Tr} \gamma_9(W)^k = 2N_0 - N_2
\]

(7.27)

for \(k = 0, 1, 2\).

Thus, we see that for \(a \neq 0\) (and \(\lambda/a \neq 1, -2\)) \(a\) can be chosen such that \(c = 1\). Hence, we are still left with a complex continuous parameter defining a continuous Wilson line [54, 14].

\textsuperscript{13} There we must perform the replacement \(\overline{\mathbf{N}}_4 \rightarrow \mathbf{N}_4\), due to the term \(\alpha^2I_{N_4}\) in the twist matrix above.
Performing the usual projections, such a continuous Wilson line achieves the desired breaking to the diagonal group by keeping three adjoints in the \( \mathbf{99} \) spectrum. As mentioned, chiral generations do not appear in this sector.

From (7.27) we see that the contribution to tadpole equation (3.6) comes only from the non rotated piece. Therefore, we have

\[
2N_0 - N_2 + 3(\text{Tr} \gamma_{\theta,5,L} - \text{Tr} \gamma_{\theta,\bar{5},L}) = -4
\]

with \( N_0 + N_2 = 16 - 3v \). This constraint can be satisfied, for instance, by placing five branes at each of the nine \( \mathbb{Z}_3 \) fixed points, satisfying \( \text{Tr} \gamma_{\theta,5,L} = \frac{1}{4}(-4 + 2N_0 - N_2) = -12 + 2N + N_2 \) and the same number of antibranes in the bulk. Other choices, with branes and antibranes stuck at these points are also possible.

For instance, for \( N = 5 \) and choosing \( N_2 = 1 \), tadpole equations are satisfied by placing two five branes with \( \gamma_{\theta,5,L} = (\alpha I_1, \alpha^2 I_1) \) (thus leading to a \( U(1) \) group at each \( \mathbf{55}_{L_4} \) sector) and eighteen antibranes in the bulk.

This choice leads to \( (SU(5) \times U(1))_{\text{diag}} \times U(1) \) gauge group with three \( \mathbf{24} \) adjoint representations in the \( \mathbf{99} \) sector. We also have \( \mathbf{5}_1 + \mathbf{5}_1 \) and \( \mathbf{5}_{-1} + \mathbf{5}_{-1} \) non chiral combinations at each of the nine fixed points, where the subindex indicates the charge with respect to the \( U(1)_{55} \) group at each point.
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