Strongly coupled fixed point in $\varphi^4$ theory

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Abstract – We show explicitly how a fixed point can be constructed in scalar $g\varphi^4$ theory from the solutions to a nonlinear eigenvalue problem. The fixed point is unstable and characterized by $\nu = 2/d$ (correlation length exponent), $\eta = 1/2 - d/8$ (anomalous dimension). For $d = 2$, these exponents reproduce to those of the Ising model which can be understood from the codimension of the critical point. The testable prediction of this fixed point is that the specific heat exponent vanishes. 2d critical Mott systems are well described by this new fixed point.

In 1976, Benzi, Martinelli and Parisi [1] stated at the outset of their paper that: “It is well known that in most of the interesting cases of field theory the perturbation expansion in the coupling constant is useless.” Since then, little progress has enabled controlled computation in the strongly coupled regime where standard perturbative methods break down. For example even in the simplest case of scalar $\varphi^4$ theory, it has proven notoriously difficult [2–6] to establish a fixed point, the terminus of a renormalization group flow, at strong coupling.

This state of affairs is unfortunate because numerous physical systems abound in which the interactions dominate, the normal state of the copper-oxide superconductors and bound states of quarks, two cases in point. In the context of the former problem, the most widely used technique is the dynamical mean-field theory (DMFT) [7] in which the physics of an extended system is boot-strapped from the ultra-local physics of a single site or a cluster. While this procedure gives experimentally accurate results even at low energy, no fundamental principle, such as the variational principle, underlies DMFT. What is peculiar is that even some of the results from the application of the gauge-gravity duality to fermionic matter at finite density [8] have obtained spectral functions that exhibit the ultra-local scaling of DMFT [8]. This coincidence would be more than an accident if perhaps strongly correlated systems, such as Mott insulators, are controlled by an ultra-local fixed point and hence any numerical scheme that builds in local physics must flow to the strongly coupled fixed point.

To access such physics, we must, therefore, treat the interactions in an essentially nonperturbative way. $\varphi^4$ theory, as the fruit-fly model for phase transitions, is a natural system to formulate a general analytic procedure to access strong coupling. The action for classical scalar $\varphi^4$ theory in $d$ dimensions is given by

$$S = -\int d^d x \left\{ (\nabla \varphi)^2 + r \varphi^2 + g \varphi^4 \right\}. \quad (1)$$

In this paper, we develop a nonperturbative treatment of this model and isolate the critical properties at a strong-coupling fixed point.

Nonlinear basis for $\varphi^4$ theory. – Our goal is to encode the interactions in a nontrivial way but still retain a complete representation of the physical system. We will first simplify the action to produce a nonlinear differential equation over the fields. We then solve this equation under periodic boundary conditions and subsequently show that this family of solutions forms a complete basis in $d$-dimensional space and thus consists of “all” solutions to this nonlinear differential equation under periodic boundary conditions. We can use integration by parts on the gradient term to separate out one copy of the field

$$S = -\int d^d x \left\{ -\varphi \nabla^2 \varphi + r \varphi^2 + g \varphi^4 \right\} \quad (2)$$

$$= -\int d^d x \varphi \left\{ -\nabla^2 \varphi + r \varphi + g \varphi^3 \right\} \quad (3)$$

$$= -\int d^d x \varphi \mathcal{N} \varphi, \quad (4)$$

where

$$\mathcal{N} = -\nabla^2 + r + g \varphi^2 \quad (5)$$

is a nonlinear operator involving the field $\varphi$ itself. A simple way to encode the interactions into the degrees of freedom,
at least in part, is to solve a reduced nonlinear eigenvalue problem for $N$

$$\mathcal{N} \varphi_n = -\nabla^2 \varphi_n + r \varphi_n + g \varphi_n^3 = \lambda_n \varphi_n \quad (6)$$

where the reduction is given by neglecting mixing terms generated by the nonlinearity. This reduced equation has a known set of solutions, for $g > 0$, in terms of Jacobi elliptic sine functions

$$\varphi_n = c_n \text{sn}(p_n x + \theta | m_n), \quad (7)$$

$$\lambda_n = p_n^2 + r + \frac{g c_n^2}{2}, \quad (8)$$

$$m_n = \frac{g c_n^2}{2 p_n^2}, \quad (9)$$

$$p_n = \frac{4 K(m_n) n}{L}, \quad (10)$$

where $m$ is the elliptic modulus, $K(m)$ is the complete elliptic integral of the first kind, and $c_n$ is the amplitude of $\varphi_n$. Notice that $p_n$ plays a role similar to the momentum of a noninteracting system in that only integer multiples of $4 K(m) / L$ are allowed due to the periodic boundary. The odd and even solutions are given by $\theta = 0$ and $\theta = K(m)$, respectively. The main goal of this section is to show that these solutions form a complete basis. In fact any set of functions with unique limiting solutions are allowed due to the periodic boundary. The partition function, is separable in the Fourier representation. Taking $\varphi(x) = \sum_n a_n \sin(p_n x) + b_n \cos(p_n x)$ with $p_n = 2 \pi n / L$, we find that

$$Z = \int \mathcal{D}[\varphi] \exp(-S[\varphi])$$

$$= \prod_n d a_n \, d b_n \exp \left\{ -L^2 \frac{1}{2} \sum_n (p_n^2 + r)(a_n^2 + b_n^2) \right\}$$

$$= \prod_n \int d a_n \, d b_n \exp \left\{ -L^2 \frac{1}{2} (p_n^2 + r)(a_n^2 + b_n^2) \right\},$$

and the integral over the product of Fourier amplitudes becomes a product over separated integrals. In order to generate RG equations, one first integrates out some number of “fast” modes from the partition function. In the case that the action separates, this integration is trivial in that we cannot produce terms that connect the remaining degrees of freedom. In such a situation this integration has no effect on the RG equations and the only contribution comes from rescaling the remaining degrees of freedom to regain those lost by integration. Most of the difficulties in calculating RG equations come from the complications involved in the integration step.

Our goal is to identify under what circumstances we can use the remarkable simplification of separability while retaining the generality of a nonlinear basis that encodes the interactions of the system. We find that, in the proximity of the Gaussian point as well as at-s-f yet unexplored strongly coupled fixed point

$$(r, g) \rightarrow (0, 0), \quad (21)$$

$$(r, g) \rightarrow (0, \infty), \quad (22)$$

respectively, the “fast” modes of the action are separable in the nonlinear basis given by eq. (8). Emphasis should be given at this point to the fact that there have been numerous studies on a fixed point at strong coupling in $\varphi^4$ theory [1,9], but these studies all require the bare value of $r \rightarrow -\infty$ as $g \rightarrow \infty$. Thus a study about the point we consider here, $(r, g) \rightarrow (0, \infty)$, is absent from previous studies.

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Transforming the action eq. (6) into the nonlinear basis of functions, eq. (8), we apply a hyperspherical cutoff in these functions \( |n| \leq |n_0| \) resulting in

\[
S = \int d^d x \left\{ \sum_{m,n} (p_m^2 + r) \varphi_m \varphi_n + g \sum_{m,n,o,p} \varphi_m \varphi_n \varphi_o \varphi_p \right\}, \tag{23}
\]

where \( p_m \) is given by eq. (10). The cross-terms in eq. (23) can be complicated in general, but in order to carry out the integration step we will integrate out one cutoff degree of freedom \( c_{n_0} \). The global minimum at both strongly coupled points (21) and (22) corresponds to freedom integration step we will integrate out one cutoff degree of freedom \( c_{n_0} \).

\[
S(c_{n_0}) = \int d^d x \left\{ \sum_n (p_n^2 + r) \varphi_n \varphi_{n_0} + g \sum_{m,n,o,p} \varphi_m \varphi_n \varphi_o \varphi_{n_0} \right\}. \tag{24}
\]

In order to evaluate each integral, we employ the method of steepest descent one degree of freedom at a time through

\[
Z_{n_0} = \int d c_{n_0} \exp \left\{-L^d S_{\varphi_{n_0}} \right\} = \sqrt{\frac{2\pi}{-L^d S_{c_{\min}}}} \exp \left\{-L^d S_{\varphi_{n_0}} \right\}, \tag{25}
\]

where \( c_{\min} \) is the value of \( c_{n_0} \) at the global minimum of \( S_{\varphi_{n_0}} \). We find that the global minimum at both points (21) and (22) corresponds to \( c_{n_0} = 0 \). We can show this by comparing the limiting forms for the gradient term, the \( r \) term, and the \( g \) term at each point. Arbitrarily close to the Gaussian point (21) we can assume \( (|r|, g) \ll (1/L, 1/L) \) with \( L \) the system size, but we are interested in the infinite system limit where \( L \) is arbitrarily large. Thus the \( r \) and \( g \) terms are negligibly small compared to \( p_n^2 = \Lambda^2 \) for finite \( c_n \). Since \( p_n^2 \) is positive, the global minimum is at \( c_{n_0} = 0 \) as desired. For the strongly coupled point (22), we use the same limit for \( r \) as in the Gaussian point since in both cases \( r \to 0 \) but here \( g \to \infty \). The nonlinear basis functions in this limit are square waves as given in eq. (17). Although some of the quartic terms over these waves can produce negative integrals, there are far more terms that integrate to positive values. The terms that are guaranteed positive by symmetry dominate over such pathological terms. Setting \( c_{n_0} = 0 \) would give a trivial result, but we can treat the smallness of \( c_{n_0} \) in the vicinity of these two points as an expansion parameter. Keeping the largest power of \( c_{n_0} \) in each of the three terms we obtain,

\[
S_{c_{\min}} = \int d^d x (p_{n_0}^2 + r) r_{n_0}^2 + g r_{n_0}^4 \tag{27}
\]

\[
= \int d^d x \lambda_{n_0} r_{\min}^2 s^2 (p_{n_0} x | m_{n_0}), \tag{28}
\]

where the last equality shows that the nonlinear basis is separated. Since the “fast” nonlinear degrees of freedom do not mix at these points, integrating them out does not alter the remaining ones.

Renormalization group equations for separated action. Once the integration step is complete, the remaining action must be rescaled. This is initiated by scaling the momentum \( k' = b k \) where \( b > 1 \) and \( k = 2 \pi n / L \). Assuming a general form for \( p \) scaling, namely,

\[
p' = b^{d_p} p, \tag{29}
\]

we look at the gradient term in the action

\[
\int d^d x p'^2 c^2 s^2 n(x, m) \tag{30}
\]

and find that, since the \( n \) part cannot scale, the eigenfunction amplitude scales as

\[
c' = b^{-d_n} c. \tag{31}
\]

We treat this amplitude scaling as analogous to the field scaling from traditional perturbative methods. The constraint on \( m \) given in eq. (9) must scale as well, and since the only parameter free to scale in this equation is \( g \) we use \( m \)-scaling to determine how \( g \) scales as

\[
m' = \frac{g' c^2}{2 dp^2}. \tag{32}
\]

Using eqs. (29), (31) we find that the rescaled form of \( m \) using the notation of eq. (32) is

\[
\frac{g' c^2}{2 dp^2} = \frac{g' c^2}{2 dp^2} b^{d-4d_p} \tag{33}
\]

\[
= \frac{m'}{b^{d-4d_p}}. \tag{34}
\]

A further constraint on \( m \) is given by the periodic boundary condition eq. (10) along with the definition \( k = \frac{2 \pi n}{L} \) resulting in a nontrivial RG equation for \( m \) of the form,

\[
p = 4 K(m) \frac{k}{2 \pi} \tag{35}
\]

\[
p' = \frac{2 K(m') b^{-4d_p}}{b k} \Rightarrow \tag{36}
\]

\[
p' = \frac{K(m') b^{-4d_p}}{K(m)}, \tag{37}
\]

Rearranging the last equality we obtain

\[
m' = b^{d_p} b^{-4d_p} K^{-1}(b^{d_p-1} K(m)). \tag{38}
\]

Absorbing the remaining rescaled terms from the eigenvalue \( \lambda \) into the rescaling of \( r \), we find our final RG equation for \( r \) to be

\[
r' = b^{2d_p} r + b^{2d_p} (m - K^{-1}(b^{d_p-1} K(m))) . \tag{39}
\]
Universal characterization of fixed points in \( d = 2, 3, 4 \). We can use eqs. (38), (39) to identify fixed points (FP) of the theory. At such a point, the RG equations must simplify to \( r' = r \) and \( m' = m \). We find two solutions corresponding to \( m \to 0 \) and \( m \to 1 \), which we denote as the Gaussian (G) FP and the strongly coupled (SC) FP, respectively. This rescaling method does not access the \( d = 3 \) critical point of the theory, so we will not discuss this FP further. Applying these limits to the rescaling equations, we find

\[
\lim_{m \to 0} m'_G = b^{d_p-1}m, \\
\lim_{m \to 0} r'_G = b^{2d_p}r, \\
\lim_{m \to 1} m'_{SC} = b^{d_p+d}m, \\
\lim_{m \to 1} r'_{SC} = b^{2d_p}r.
\]

For the Gaussian fixed point, it is straightforward to see that \( g \to 0 \) as \( m \to 0 \) from eq. (10) using the fact that \( K(m = 0) = \pi^2/2 \) in eq. (9). To find the value of \( g \) for the strongly coupled fixed point, we first use eq. (10) to find that \( \lim_{m \to 1} K(m) \to \infty \) implies that \( \lim_{m \to 1} p \to 0 \). We then solve eq. (9) for \( g \) and assume that the amplitude \( c \) is finite to obtain \( \lim_{m \to 1} g \to \infty \). In both limits, we find that the \( m \)-dependence in eq. (39) vanishes to give the same rescaling equation shown in eq. (40), which leads to \( r^* = 0 \) for both fixed points. Therefore, the fixed points \( (r^*, g^*) \) we identify here correspond to the Gaussian \((0, 0)\) and a new \((0, \infty)\) fixed point at strong \( g \to \infty \) coupling. Using eqs. (40), we immediately find the required values of \( d_p \) for each fixed point as

\[
d_{p,G} = 1, \\
d_{p,SC} = \frac{d}{4}.
\]

Before calculating the power-law exponents for each of these points we characterize them based on the rescaling flows in their vicinity. We do this by choosing a point \( r^* + \delta r \) and \( m^* + \delta m \) near the corresponding fixed point while using the value for \( d_p \) obtained at that fixed point. We then apply eqs. (38), (39) to determine the direction of the resulting RG flows. Taking \( m = 0 + \delta m \) at the Gaussian FP and \( m = 1 - \delta m \) at the SC FP, we find

\[
m'_{G} = b^{4-d}\delta m, \\
m'_{SC} = K^{-1} \left( b^{d_{p,G}} K(1 - \delta m) \right).
\]

As long as \( d < 4 \) we find that for the Gaussian FP \( m' > m \). Since \( b^{d_{p,G}} < 1 \) and \( K(m) \) is a strictly increasing function, we find that for the strongly coupled FP \( m' < m \). Although the equations near the Gaussian FP result in the expected mean-field values, we see that the SC FP equations are marginal for \( d \geq 4 \). In order to address this new FP above the upper critical dimension, we would likely need to include additional cross-terms in the action

![Fig. 1: (Colour online) Flow diagram in the vicinity of Gaussian (0,0) and strongly coupled (0,\( \infty \)) fixed points in \( d = 3 \). Axes are given as \( r \) vs. \( g \) with corresponding values of \( m \) on the left. The dotted line represents the \( m \to 1 \) or \( g \to \infty \) limit. Since flows in both cases are all away from each fixed point both points are unstable.](image)

In order to calculate the exponents for these fixed points, we first use the definition of the correlation exponent \( \nu \) as the inverse of the scaling for \( r \). Recall from eq. (44) that this is made especially simple given that all \( m \)-dependence drops out of eq. (39) for both fixed points. In both cases

\[
\nu = \frac{1}{2d_p},
\]

and the problem reduces to identifying \( d_p \) for each fixed point (see eq. (44)) resulting in

\[
\nu_G = \frac{1}{2}, \\
\nu_{SC} = \frac{2}{d}.
\]

Typically at least two exponents are needed to fully quantify the exponents at a given fixed point. The rest are determined using scaling laws [10]. For the second exponent, we use the definitions of \( \eta \) as the difference between the field scaling at the given fixed point and that at the Gaussian fixed point. This gives \( \nu = 0 \) by definition at the Gaussian point, but we can incorporate this result into a general formula as follows. In our eigensolution, the field scales as the amplitude \( c \) of the eigenfunctions. This scaling is fully determined by \( d_p \) as shown in eq. (31). Setting the Gaussian value to \( d_p = 1 \) we find that

\[
\eta = \frac{d - 2d_p}{2} - \frac{d - 2}{2} \Rightarrow \\
\eta = \frac{1 - d_p}{2} \\
\eta = \frac{1 - \frac{d}{8}}{2}
\]

and for the strongly coupled fixed point, we obtain

\[
\eta = \frac{1 - d}{8}.
\]

Table 1 summarizes all the exponents.
The SC fixed point in $d = 3$ is an unstable one similar to the Gaussian FP as is evident from the flow diagram in fig. 1. Since the exponents are obtained from the exact eigenstates and such states form a complete basis [11], we have exactly characterized the strongly coupled fixed point.

A surprising consequence of the strongly coupled fixed point is that the exponents in $d = 2$ reduce exactly to those of Onsager’s in the 2d Ising model. This implies that the fixed point we have found here should be applicable quite generally to systems in which the interactions dominate. Of course for $d = 2$, it is possible that operators other than $\varphi^4$ are relevant and hence a careful analysis of this system includes higher-order terms. However, the codimension of the Ising critical point is 2 [12] (two relevant directions) and the relevant couplings are in general the quadratic strength $H$. If all of the remaining coupling parameters are found to be irrelevant, then the universality class found $H$ field and $\varphi$ is observed in many non-Fermi liquid systems [16–18] in $d = 2$ reduce exactly to the complete elliptic integral of the first kind,

$$
\int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-m_1 t^2) \cdots (1-m_{i-1} t^2)}}
$$

(58)

Inserting eq. (56) with $i = 3$ into the nonlinear eigenvalue equation for $\varphi^6$ and equating like terms we find that

$$
\frac{g_{\text{var}}^2}{2p^2} = \bar{m},
$$

(59)

$$
-\frac{g_{\text{var}}^4}{3p^2} = \bar{m},
$$

(60)

$$
\lambda = p^2 + r + p^2(m_1 + m_2) \Rightarrow
$$

(61)

$$
\lambda = p^2 + r + \frac{g_{\text{var}}^2}{2} + \frac{g_{\text{var}}^4}{3},
$$

(62)

where $\bar{m} = m_1 + m_2 + m_1 m_2$ and $\bar{m} = m_1 m_2$ and a similarly determined set of solutions obtain for the $\varphi^2$ case. Letting $m_1 \to 1$ and $m_2 \to 0$ while their product $m_1 m_2 \to 0$ leads to the desired FP location where $\bar{m} \to 0$ and $\bar{\varphi} \to 1$. We then generate the rescaling equation analogous to eq. (38)

$$
K_6(\bar{m}, \bar{m}') = b^{d-4}\nu K_6(\bar{m}, \bar{m}),
$$

(63)

and we see that $d_\nu = d/4$ and $d = 2$ gives

$$
K_6(\bar{m}, \bar{m}') = b^{d-4/2}K_6(\bar{m}, \bar{m}),
$$

(64)

so that $K_6$ and $\bar{m}$ are both reduced upon rescaling showing that the $\varphi^0$ term is irrelevant. In general coefficients with $i > 2$ are irrelevant, supporting the claim that this is indeed the $d = 2$ Ising critical point.

The testable prediction of this strongly coupled fixed point is the value of the specific heat exponent. Because of the hyperscaling relation, $2 - \alpha = \nu$, our computed value for $\nu = 2/d$ implies that $\alpha = 0$ as shown in table 1. Consequently, the divergence is at best logarithmic. Two independent systems seem to exhibit this behavior. First, in the pnicotides, a logarithmic divergence of the form $\ln(x - x_c)$ of the specific heat in BaFe$_2$(As$_{1-x}$P$_x$)$_2$ has been seen in low fields [13–15]. A direct measurement of $\alpha$ would be preferable rather than in reference based on the effective mass since the very meaning of a quasiparticle is obscured in the local limit. In addition, care must be taken to distinguish a pure $\ln |T|$ dependence from $T^\alpha \ln |T|$ as is observed in many non-Fermi liquid systems [16–18] in which $\alpha \neq 0$. Second, a recent scaling theory of the finite-temperature Mott transition [19] has predicted that the heat capacity only has a $\ln |T|$ dependence and as a result is well described by the $d = 2$ Ising exponents. What our work clarifies is that $\alpha = 0$ is a generic feature of a strongly coupled fixed point not just the $d = 2$ Ising model. The applicability to Mott criticality is expected as such systems are governed by strong local interactions.

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PWP thanks Sung-Sik Lee for an inspiring discussion at the Isaac Newton Institute from which the idea

Table 1: $\varphi^4$ SC Exponents ($r = 0$, $g \to \infty$).

| FP  | G  | SC  |
|-----|----|-----|
| $\nu$ | $\frac{1}{2}$ | $\frac{3}{7}$ |
| $\eta$ | $0$ | $\frac{1}{2} - \frac{d}{2}$ |
| $\alpha$ | $0$ | $0$ |
| $\beta$ | $\frac{1}{2}$ | $\frac{7}{8} - \frac{3}{2d}$ |
| $\gamma$ | $1$ | $\frac{3}{4} + \frac{1}{4}$ |
| $\delta$ | $3$ | $\frac{9d+12}{8d+12}$ |

where $K_{2i}(\mathbf{m}_n)$ is the hyperelliptic generalization to the complete elliptic integral of the first kind,

$$
\int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-m_1 t^2) \cdots (1-m_{i-1} t^2)}}
$$

(58)

and we see that $d_\nu = d/4$ and $d = 2$ gives

$$
K_6(\bar{m}, \bar{m}') = b^{d-4/2}K_6(\bar{m}, \bar{m}),
$$

(64)

so that $K_6$ and $\bar{m}$ are both reduced upon rescaling showing that the $\varphi^0$ term is irrelevant. In general coefficients with $i > 2$ are irrelevant, supporting the claim that this is indeed the $d = 2$ Ising critical point.

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