The Relationship between Executive Function and the Conservation of Quantity in Early Childhood Cognitive Processes from the Viewpoint of the Prefrontal Cortex

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ABSTRACT

Conservation and executive function (EF) are important early childhood skills; however, knowledge about their relationship is scarce. Hence, in this study, this relationship is investigated, and a comparison is conducted between the Piagetian conservation and EF tasks to obtain the total hemoglobin (mMmm) for the left and right brain activity in the prefrontal cortex of children, using a functional near-infrared spectroscopy. As a case study, a survey was conducted in the home of a four-year-old child (boy) in Japan in 2020. His home was selected as the primary place of investigation owing to the spread of the coronavirus disease 2019 infection. The researcher was an expert in pedagogy and psychology. The following hypothesis could be formulated from the case studies analyzed herein: From the viewpoint of the brain activity in the prefrontal cortex, there may be a link between the conservation of quantity and EF. The preservation of quantity is closely related to mathematics education. This hypothesis can thus expand the scope of research and practice in the fields of psychology, education, and mathematics education. Furthermore, the results may help psychologists, psychiatrists, teachers, parents, and others involved in the development of children sublimate the value of their concept of conservation and improve their support methods.

Keywords: conservation of quantity, executive functions, Piagetian conservation task, cerebral blood flow (total hemoglobin), prefrontal cortex

INTRODUCTION

As conservation is an important cognitive ability for young children, research findings have been accumulated in psychology over the years. According to Piaget (1952), and Piaget and Inherder (1974), conservation involves logical thinking that allows us to understand that the quantity of things remains the same even if the shape changes. The acquisition of conservation plays an important and direct role in the cognitive development of children. This aspect can be observed, for example, from the fact that mathematics in elementary schools is treated as national education content (e.g., Ministry of Education, Culture, Sports, Science, and Technology [MEXT], 2018). Therefore, it can be perceived that preservation is closely related to mathematics education. Although a certain amount of robust knowledge has been accumulated, little research has been conducted recently.

Conversely, in recent years, executive function (EF) has increasingly been pointed out as important during early childhood. EF is goal-oriented and involves thinking, behavioral, and emotional control (Moriguchi, 2015); it is broadly classified into emotional hot and thinking cool functions (Zelazo & Carlson, 2012). Several studies have also been conducted on developmental stages and tasks of developmental stages [e.g., marshmallow test and dimensional change card sort (DCCS) task] (Figner et al., 2010; Mischel, 2014; Moriguchi, 2015; Zelazo, 2006). Moreover, EF is related to the prefrontal cortex (Burgess & Stuss, 2017; Stuss & Alexander, 2000). Furthermore, inhibitory control mechanisms have a significant effect on cognitive development and life and are associated with the prefrontal cortex (Bjorklund & Harnishfeger, 1995; Bjorklund & Kipp, 2002; Dempster, 1992; Harnishfeger & Bjorklund, 1993). Although there are numerous studies on children with developmental disorders (Barkley & Murphy, 2010; Biederman et al., 2004; Seidman et al., 2001), only a few studies exist on support methods for all children because EF and the prefrontal cortex were observed to be unaffected by experience and environment (Moffitt et al., 2011; Moriguchi, 2015). On the other hand, spending time in less-structured activities is said to influence EF (Barker et al., 2014).

Thus, is the conservation of quantity associated with the EF of noncognitive skills?
With respect to conservation, several studies have focused on the factors of cognitive development from the information processing theory perspective; this theory emphasizes the limitations of processing at each developmental stage among children and the role of the strategies and procedures used to exceed them (Vauclair, 2004). On the basis of the information processing theory, failure to understand number conservation can be explained. For instance, when a child sees a sequence of numbers, schemas are evoked based on the rule that length-equals-number, as the sequences that appear long contain many numbers. Moreover, children cannot control this instinct (Case, 1998; Houdé et al., 2011; Poirel et al., 2012). This breakthrough requires an improvement in the information processing capability (short-term memory), and this improvement emerges from automation and brain maturation (Case, 1978; Siegler, 1991; Vauclair, 2004). These theories are deemed related to EF. In fact, in recent years, some studies have revealed that, with respect to the number of Piagetian tasks, the prefrontal cortex activity is involved in the information processing theory (Borst et al., 2013; Houdé et al., 2011; Poirel et al., 2012), and a relationship prevails between the prefrontal cortex activity and the EF (Houdé & Borst, 2014).

Moreover, few papers have revealed a direct relationship between EF and conservation. However, some research findings have found that nonconserved children also detect the error (De Neys, Lubin, & Houdé, 2014), physicality improves the cognitive development of conservation (Lozada & Carro, 2016), accelerated cognitive development of conservation can occur (Watanabe, 2017), and the development of conservation by attachment can occur (Watanabe, 2019). Such studies may suggest that if EF improves (matures), it may lead to the acquisition of the concept of conservation. It can be said that such research is useful not only for finding relationships but also for supporting conservation and EF acquisition.

This study examines whether the following hypothesis could be proven by case studies: From the viewpoint of the brain activity in the prefrontal cortex, there may be a link between the conservation of quantity and EF.

Hence, this relationship is investigated, and a comparison is conducted between the Piagetian conservation and EF tasks to obtain the total hemoglobin (mMmm) for activities in the left and right sides of the, particularly the prefrontal cortex, of children during each performance. Practical research and case study approaches were employed because they are useful in investigating child support. This study suggests that the presented hypothesis can expand the scope of research and practice in the fields of psychology and education. In particular, it is thought to have a significant impact on mathematics education. For example, it is possible to find new value of the conservation concept, namely, the value of EF. Furthermore, scientific approaches can be suggested and formulated to support EF. In addition, renewing the results of previous research on the conservation concept, such as the acquisition stage, is possible.

Note that case studies are conducted to study the individuality of one or a few cases. The usefulness of such studies has already been demonstrated (George et al., 2005; Yin, 2017). In practical research, a researcher directly supports and practices at the clinical site while directly working with the targets (Maebara, Ichikawa, & Shimoyama, 2001). Case/practical studies correspond to actual individuals and thus are essential in the study of children because they can quickly respond to problems and aid in deducing novel hypothesis findings.

**MATERIALS and METHODS**

**Study Design**

I compared and examined the function of the prefrontal cortex in a conservation task and tasks related to other EF (DCCS task, Stroop task, and marshmallow test) on the basis of the measurement of the cerebral blood flow [total hemoglobin (Hb)] on both sides [mMmm)] through a practice/case study.

**Target**

The target sample was a four-year-old child, who is the most fully formed and real-life example of rapport and the most representative example in a real clinical setting. The reason for selecting a four-year-old was not because of his age but because conservation is not yet acquired at this age; however, it was at the approaching acquisition stage. Therefore, I focused on the acquisition stage rather than the age. During the experiment, I gave a written and verbal explanation to the child’s parents and obtained a consent form. In addition, I provided a report to the Ethics Committee of the Kwansei Gakuin University Committee for Regulations for Behavioral Research with Human Participants.

**Ethical Considerations**

This study has been performed in accordance with The Code of Ethics of the World Medical Association (Declaration of Helsinki). It is approved by Approval Number: 2020-05 and 2020-06, Approval Date: June 9, 2020, and May 26, 2020. Written informed consent was obtained for experimentation with human subjects. The privacy rights of human subjects were safeguarded.

**Experimental Design**

I measured the blood flow of the brain activity in the prefrontal cortex as the child performed the activities in each task. As it is difficult to set a rest period in the early childhood, if there was a period of less than 0.1 (mMmm) within 10 s after the
commencement of the experiment, rest was considered to have been performed, and the measured value was validated. Total hemoglobin change (left/right subtracted) was adopted as the measured value. The baseline correction was set to 0 at the beginning, and the average brain activity during the task was examined \((\Sigma f(x)-\min f(x))/{\text{total number of milliseconds}}, x: \text{time}, f(x): \text{total hemoglobin}\). Total hemoglobin was acquired every millisecond.

**Tasks**

- As for the EF in early childhood, the marshmallow test (Mischel, 2014) was employed for hot EF, and the DCCS (Zelazo, 2006), and Stroop tasks (Stroop, 1935) were used for cool EF, as the EF can be determined through these tasks (Zelazo & Carlson, 2012).
- Conservation has been related to the neural contribution of a bilateral prefrontal network (Houdé et al., 2011). During the acquisition stage, the number, length, liquid, and mass tasks are mastered approximately by the age of 6–7 years old, and the weight and volume tasks are mastered approximately by the age of 9–10 years old (Goswami, 1998).
- Zelazo (2006) had proposed the DCCS task. In brain science, the activity and involvement in the right lower prefrontal cortex region are clear. The post-switch task is mastered by those 4–5 years old, and the mixture (border) task is mastered by those ≥6 years old (Moriguchi, 2015).
- In the case of children, the Stroop task (Stroop, 1935) is related to the brain activation in the lateral cortex of the left prefrontal cortex. The task can be performed approximately by the age of four and onward (Schroeter et al., 2004). In addition, the correct answer rates and processing speed tend to improve as the child grows (Prevo & Diamond, 2005).
- Mischel (2014) proposed the marshmallow test. In brain science, the involvement with the prefrontal cortex, particularly the left side of the lateral prefrontal cortex, has been pointed out, and the passage rate of four-year-old children is approximately one-third (Figner et al., 2010; Mischel, 2014).

On the basis of these facts, I observed the brain activity of the prefrontal cortex during each task, compared the developmental stages, and inferred the relationship between conservation and EF.

**Cerebral blood flow measurement**

When targeting children, it is necessary to reduce the burden and have a degree of freedom. Therefore, I employed HOT-2000 (NeU, Japan), which is a two-channel functional near-infrared spectroscopy (fNIRS) measurement device. As the degree of freedom was high and the t-Hb could be measured, it was possible to examine the prefrontal cortex.

**Methods**

**Protocol for each task**

Conservation, DCCS, and Stroop tasks were performed wherein the researcher and the child sat facing against each other on a desk, and the experimental tools were placed between them. Further, the marshmallow test was performed, and the duration of the experiment and the durations before and after the child wore the measuring device on his head were recorded.

- During the conservation task, the general protocol (Piaget, 1952; Piaget & Inherder, 1974) was used. Length, number, volume, mass, and weight were used in the given order; however, in some cases, the order was changed, or the questions were omitted.

For example, during the length task, three straws were used. First, as a comparative problem, long and short straws were placed parallel to each other on the desk in between the questioner and the child. Then, the questioner asked the following question: “Which one is longer?” and recorded the child’s answer. Later, two straws of the same length were placed parallel to each other, and the child’s answer was recorded for the following questions asked by the questioner: “Which is longer? Is it the same?” The straw on the child’s side was shifted to the right by approximately one-third; the questioner asked, “Which is longer? Is it the same?” and recorded the child’s answer. Upon answering this question, the questioner further asked, “Why?” and recorded the child’s answer. If there was no answer, then the questioner directly proceeded to the next step.

In the number task, 10 toy coins were used; first, as a comparison problem, three and five coins were placed in parallel and in rows between the questioner and the child. The questioner asked, “Which is more?” and recorded the child’s answer. Next, five coins each were placed in parallel rows. The questioner asked, “Which is more? Are they the same?” and recorded the child’s answer. The coins on the child’s side were opened one by one, and the questioner asked, “Which is more? Are they the same?” Subsequently, the child’s answer was recorded. The questioner also asked, “Why?” and recorded the child’s answer. If there was no answer, then the questioner directly proceeded to the next step.

Three glasses were used in the liquid task. First, as a comparison problem, different amounts of water were put in two identical glasses that were placed side by side. The questioner asked, “Which is more?” and recorded the child’s answer. Next, the same amount of water was put in two identical glasses and placed between the two. The questioner asked, “Which is more? Are they the same?” and recorded the child’s answer. One of the two identical glasses was replaced with a long and narrow glass, and the water in that one glass was transferred to the narrow glass. The questioner asked, “Which is more? Are they the same?” The child’s answer was then recorded. The questioner also asked, “Why?” and recorded the child’s answer. If there was no answer, then the questioner directly proceeded to the next step.

Clay was used in the mass task. Two clay balls of the same size were presented between the questioner and the child. The questioner asked, “Which is more? Are they the same?” and recorded the child’s answer. One clay ball was compressed and flattened, and the questioner asked, “Which is more? Are they the same?” Subsequently, the child’s answer was recorded. The questioner also asked, “Why?” and recorded the child’s answer. If there was no answer, then the questioner directly proceeded to the next step.
Clay was also used in the weight task. Two clay balls of the same size were presented between the questioner and the child. The questioner asked, “Which is heavier? Are they the same?” and recorded the child’s answer. One clay ball was compressed and flattened, and the questioner asked, “Which is heavier? Are they the same?” The child’s answer was then recorded. The questioner also asked, “Why?” and recorded the child’s answer. If there was no answer, then the questioner directly proceeded to the next step.

The volume task used clay, cylinders, and water. Two clay balls of the same size and two same-sized cylinders with the same amount of water were presented between the questioner and child. Subsequently, the questioner asked, “How does water change when one clay ball is put in water?” If the child answered that the water will increase (the bulk will increase or water’s level crease high, etc.), they proceeded to the next step. Further, the questioner put one clay ball in the water and asked, “If the other clay was put in the other cylinder, which water are higher (increase)? Or are they the same?” The child’s answer was then recorded. Subsequently, the questioner did not put the clay in the water. Therefore, only the clay that was first put in the water remained in the water. Next, the questioner compressed and flattened the clay ball that he had in his hand and asked, “If the clay put in the cylinder, which water are higher (increase)? Or are they the same?” The child’s answer was then recorded. The questioner also asked, “Why?” and recorded the child’s answer. If there was no answer, then the questioner directly proceeded to the next step.

During the DCCS task, the pre-switch, post-switch, and mixture tasks (border task) were performed in the given order. During the pre-switch task, I prepared the target and classification cards. For example, the target cards displayed a red car and blue star, and the classification cards displayed a blue car and red star, whose dimensions were opposite in color and shape. Further, the child classified six classification cards (three cards each) in accordance with one dimension (shape or color). During the post-switch task, the same six classification cards (three cards each) were classified in a different dimension. During the mixture task (border task), six cards with frames (three cards each) were added to the classification cards, and in total, 12 cards were classified. During the classification, when the frame was used, the color (or shape) was employed, and when no frame was used, the shape (or color) was employed. I named these games as the “color” and “shape” games (for colors and shapes, respectively). As a set of cards, I conducted the pattern of “blue/red or car/star” twice, and the pattern of “purple/yellow and triangle/beetle” was conducted once.

- The Stroop task was performed with color names by using PowerPoint. To begin with, there was a rest period of 30 s wherein I explained the task to the child. Next, as a pre-task, the character string was displayed in the same color as the displayed color name (e.g., “Green” displayed in green) on a slide. During the task, 15 questions were asked in 60 s. The task and the blank slides were displayed for 3 and 1 s, respectively. Then, there was a description or rest period of 15 s. As a post-switch task, a character string written in a color different from the displayed color name (e.g., “Green” displayed in red) was displayed, and even during this task, 15 questions were asked in 60 s. The color sets were “red/blue/yellow/green,” “red/blue/orange/green,” “pink/blue/orange/green,” and “pink/blue/brown/green.”
- The marshmallow test followed the general protocol (Mischel, 2014), i.e., a child sat on a chair in an almost empty room. Then, I placed some sweets and a bell on the desk in the room and told the child that if he did not eat the sweets for 15 minutes, he would get more sweets; however, if he ate them, he would get nothing. Moreover, if he could not stand it, he could ring the bell. I left the room thereafter.

All the basic interactions for each of the abovementioned tasks were the same.

**THEORY/CALCULATION**

The purpose of this study is to test a novel hypothesis; therefore, it is a pilot and an exploratory case study. The new hypothesis is that conservation concepts and EF are closely related in early childhood. This hypothesis is tested by measuring the brain activity using fNIRS in the prefrontal cortex. The study was analyzed the conservation concepts tasks, such as number, length, liquid, mass, weight, and volume tasks, along with the EF tasks, such as the DCCS task were used for evaluating the cognitive shifts, the Stroop task was used for evaluating the working memory, and the marshmallow test was used for evaluating the inhibitory functions. The brain activity (total Hb in the left and right prefrontal cortex) in each task and the legitimate numbers were measured; the average value of each task was then calculated. The means were compared, and a test of the difference of the means (t-test) was calculated using IBM SPSS Statistics ver.26.0.0.0.

**RESULTS**

*Table 1* illustrates the acquisition stages of conservation.
Table 1. Acquisition Stage of Conservation

| Conservation | 1: Pass | 0: Failed |
|--------------|--------|----------|
| Age in Months | Length | Number | Liquid | Mass | Weight | Volume |
| 48           | 0      | 0       | 0      | 0    | 0      | 0      |
| 49           | 0      | 0       | 0      | 0    | 0      | 0      |
| 50           | 0      | 0       | 0      | 0    | 0      | 0      |
| 51           | 0      | 0       | 0      | 0    | 0      | 0      |
| 52           | 1      | 1       | 0      | 1    | 0      | 0      |
| 53           | 0      | 1       | 0      | 0    | 0      | 0      |
| 54           | 1      | 1       | 0      | 0    | 0      | 1      |
| 55           | 0      | 1       | 0      | 1    | 0      | 1      |

At the age of 48 months, length, number, liquid, and mass tasks were not acquired. By observing the subsequent process, it can be considered that weight and volume tasks had not yet been acquired. Some acquisition began at the age of 52 months but was unsteady since then. At the age of 55 months, number, length, mass, and volume tasks were close to acquisition. Although the child’s answers were incorrect for the questions related to weight and liquid tasks, he answered them immediately with confidence; this aspect suggested that he had not yet entered in the conflict stage.

The correct numbers of the DCCS and Stroop tasks and the waiting intervals for the marshmallow test are shown in Table 2.

Table 2. Correct Numbers of the Dimensional Change Card Sort (DCCS) and Stroop Tasks and Waiting Intervals for the Marshmallow Test

| DCCS Task | Correct Numbers | Age in Months |
|-----------|----------------|--------------|
| Pre-Switch Task Q:6 | Post-Switch Q:6 | Mixture Q:12 |
| 54 (The first time) | 6 | 6 | 8 |
| 54 (Second time) | 6 | 6 | 12 |
| 55 (Third time) | 6 | 6 | 12 |

| Stroop Task | Age in Months | Pre-Switch Task Q:15 | Post-Switch Task Q:15 |
|-------------|--------------|----------------------|----------------------|
| 55 (The first time) | 15 | 14 |
| 55 (Second time) | 15 | 15 |
| 55 (Third time) | 15 | 15 |
| 55 (Fourth time) | 15 | 15 |

| Marshmallow Test | Age in Months | Minutes |
|------------------|--------------|---------|
| 54 (The first time) | 14 min (15 min) |
| 54 (Second time) | 15 min |
| 54 (Third time) | 15 min |
| 55 (Fourth time) | 15 min |

The DCCS task was performed once a day; it was performed twice at the age of 54 months and once at the age of 55 months. At the first instance, the mixture task failed and was evaluated as lacking acquisition, i.e., the EF was in the process of being acquired.

The Stroop task was employed because the child could sufficiently read the character strings. Therefore, the speed of the correct answers is often used as an index. However, at the first instance, a wrong answer was recorded, and thus, a complete acquisition stage was not achieved.

The marshmallow test was conducted once on three different days at the age of 54 months. Further, it was conducted once at the age of 55 months. At the first instance, the test lasted for 14 min. However, the bell rang after 14 min, but because the child did not come out of the room, I entered the room in 15 min. The reason for ringing the bell was that the brain science device had fallen. Hence, the measurement could be recorded for only approximately 6 min, indicating the difficulty of recording the brain science measurements in early childhood.

The total Hb (left/right of the brain activity) for each conservation task is shown in Table 3.

Table 3. Total Quantity of Hemoglobin (mMmm) (Left/Right of the Brain Activity) for Each Task

| Total Hb (Left/Right of the Brain Activity) | Ave. |
|-------------------------------------------|------|
| Tasks | Left | Right |
| Length | 0.366254 | 0.376765 |
| Number | 0.421835 | 0.385188 |
| Liquid | 0.26547 | 0.283228 |
| Mass | 0.358198 | 0.365493 |
| Weight | 0.356107 | 0.23325 |
| Volume | 0.609176 | 0.527289 |
| DCCS Post-Switch Task | 0.474081 | 0.361162 |
| DCCS Mixture Task | 0.816453 | 0.581659 |
| Stroop Task | 0.230852 | 0.243633 |
| Marshmallow Test | 1.149012 | 0.758741 |

Note. DCCS: Dimensional change card sort.
First, the higher the cerebral blood flow is, the more active the brain is. Hence, I estimated the difficulty in the order of the average value of the total quantity of hemoglobin in the child’s brain. The results of the activity of the left and right brains were in descending order, i.e., volume, number, length, mass, and weight. From these results, it can be estimated that it is the volume that has the highest degree of difficulty. This is consistent with behavioral evidence. With respect to the weight and the liquid that are considered difficult, the measurements recorded for them were low, but the child had given a wrong answer without being at conflict. From these results, I could infer that brain activity does not increase without conflict, even if the difficulty level is high.

On the basis of simple average values, the results of the activity of the left and right brains were in descending order, i.e., the marshmallow test, DCCS mixture task, DCCS post-switch task, and Stroop task. It can be observed that this aspect almost matches the acquisition stage, that is, the task difficulty level.

Table 4 illustrates the results of a significant difference in the t-test owing to the difference between the mean values for the left and right brain activities in each task.

| Table 4. T-Test Values for Each Task |
|-------------------------------------|
| **Left** | **Length** | **Number** | **Liquid** | **Mass** | **Weight** | **Volume** | **DCCS Post-Switch** | **DCCS Mixture** | **Stroop** | **Marshmallow** |
|---------|------------|------------|------------|---------|------------|-----------|---------------------|----------------|-----------|----------------|
| Length  | –5.330     | 2.981      | –5.892     |         |             |           |                     |                |           |                |
| Number  | –4.325     | 3.424      | –5.354     |         |             |           |                     |                |           |                |
| Liquid  | –3.500     | 3.424      | –5.892     |         |             |           |                     |                |           |                |
| Mass    | –5.797     | 0.002''    | –5.354     |         |             |           |                     |                |           |                |
| Weight  | –3.814     | 0.011'     | –3.600     |         |             |           |                     |                |           |                |
| Volume  |             |            | –2.788     |         |             |           |                     |                |           |                |
| DCCS    |             |            | –2.834     |         |             |           |                     |                |           |                |
| Post-Switch |         |            | –4.160     |         |             |           |                     |                |           |                |
| DCCS    |             |            | –2.834     |         |             |           |                     |                |           |                |
| Mixture |             |            | –4.160     |         |             |           |                     |                |           |                |
| Stroop  |             |            | –5.648     |         |             |           |                     |                |           |                |
| Marshmallow |         |            | –5.648     |         |             |           |                     |                |           |                |

| **Right** | **Length** | **Number** | **Liquid** | **Mass** | **Weight** | **Volume** | **DCCS Post-Switch** | **DCCS Mixture** | **Stroop** | **Marshmallow** |
|-----------|------------|------------|------------|---------|------------|-----------|---------------------|----------------|-----------|----------------|
| Length    | –2.346     | 0.044''    | –3.853     |         |             |           |                     |                |           |                |
| Number    | –3.409     | 0.008''    | –4.988     |         |             |           |                     |                |           |                |
| Liquid    | –3.558     | 0.024''    | –4.758     |         |             |           |                     |                |           |                |
| Mass      | –2.721     | 0.026'     | –2.826     |         |             |           |                     |                |           |                |
| Weight    | –3.558     | 3.030      | –4.758     |         |             |           |                     |                |           |                |
| Volume    |             | 0.029'     | –2.834     |         |             |           |                     |                |           |                |
| DCCS      |             |            | –4.160     |         |             |           |                     |                |           |                |
| Post-Switch |         |            | –4.160     |         |             |           |                     |                |           |                |
| DCCS      |             |            | –4.160     |         |             |           |                     |                |           |                |
| Mixture   |             |            | –5.648     |         |             |           |                     |                |           |                |
| Stroop    |             |            | –5.648     |         |             |           |                     |                |           |                |

| Marshmallow |         |            | –5.648     |         |             |           |                     |                |           |                |

**Note.** * = < 0.05, ** = < 0.01; DCCS: Dimensional change card sort**

With respect to the cool EF, by observing the DCCS mixture task, the left side or the right side exhibited significant differences, except for volume task and marshmallow test. On the basis of this finding, it can be deduced that there is no difference between the DCCS mixture and volume task in the EF. Conservation (length, number, volume, mass, etc.) acquired at the early stage was such that the brain activity was similar to the DCCS post-switch task. It can be considered that the volume task, which was difficult, stimulated the brain activity that is similar to the DCCS mixture task.

With respect to the cool EF task, the brain activity in the prefrontal cortex became more active as the acquisition stage became more difficult. Thus, the hot EF problem was clearly higher than that of the cool.

As for the conservation task, the brain activity in the prefrontal cortex increased as the acquisition stage became more difficult. When compared to the acquisition/passage stage with EF, the trends in the brain activity could be deduced.
DISCUSSION

The results indicate the following. Regarding the brain activity in the prefrontal cortex, there may be a link between the conservation of quantity and EF. In other words, the results of the case studies confirm that we can propose the abovementioned hypothesis (from the viewpoint of the brain activity in the prefrontal cortex, there may be a link between conservation of quantity and EF) to be novel.

The prefrontal cortex activity has been observed in a number of Piagetian tasks (Borst et al., 2013; Houdé et al., 2011; Poirel et al., 2012). Previous studies only included knowledge about the relationship between the “conservation of number” and EF (Houdé et al., 2011). My results revealed that not only number conservation but also the conservation of quantities was associated with the prefrontal cortex. Moreover, the results of the tendency of the brain activity of the EF and conservation are almost in agreement with the findings of behavioral evidence of conservation and EF (Figner et al., 2010; Goswami, 1998; Mischel, 2014; Moriguchi, 2015; Schroeter et al., 2004). These findings are evidence of my novel hypothesis and can be said to be an important suggestion that bridges the research gap in psychology and mathematics education. Conservation had previously been firmly established as logical thinking and was a theory. The results of this study developed the novel hypothesis of the involvement of EF, and this is a noteworthy finding that opens a new door for psychological and mathematics education research. Furthermore, the importance of acquiring conservation is not limited to logical thinking, but there is a possibility that can be extended to EF; this is a novel finding as a pedagogical study. Furthermore, the method of acquiring conservation has been discussed in different contexts and in terms of the environmental information, but it is necessary to discuss it in terms of EF. In psychology, brain science, and pedagogy, it can be said that this aspect creates a perspective for future research. I also found that the difficulty of acquiring the EF was closely associated with the brain activity in the prefrontal cortex. Therefore, in an actual field, by applying this aspect and using a certain task related to the brain activity, it is possible to measure how much activity of the EF is performed. This aspect provides the essential direction for the future to support the EF in the brain science field, which has been unexplored. This aspect also holds significance in pedagogy. It shows that knowledge can be imparted while measuring EF and conservation in real time, which is useful for new research in educational engineering aiming to build an effective learning support system. Notably, such findings, which are difficult to determine in the research community and are hardly established, are deduced through practical and case studies.

The limitations of the study include the following. First, it was a case study. Needless to say, in this regard, it is desirable to have considerable data that is in line with other case studies. However, case studies cannot be used to determine causation but for the development of a novel hypothesis. In addition, as there are individual differences in development, finding only a part of the evidence is a significant achievement in the fields of educational psychology, mathematics education, and even school settings. Additionally, as it was a practical study, it revealed the objectivity of the data. However, as this is the actual style in the clinical setting, it can be said that it is the most objective research. Furthermore, there exists a problem in measuring the cerebral blood flow of the EF using an evidence-based approach. However, it is not realistic to study a child several times using multiple channels. Moreover, it is considered extremely burdensome for children. Therefore, considering research and educational ethics, human rights, and fNIRS—which does not burden the two channels—is the optimal policy.

In the future, if my hypothesis regarding the relationship between EF and the concept of conservation for infants can be proved using brain measurement equipment such as fNIRS, then it could greatly contribute to psychology, neuropsychology, and school settings in particular. It is imperative that we work on it soon; only then is this concert practice/study possible, as shown in this study.

Finally, as of November 2020, COVID-19 is raging worldwide. However, practice and case studies are easily possible even in such circumstances. In fact, this study was conducted at home.

CONCLUSION

Regarding the results, the relationship between the conservation of numbers and EF is consistent with the results of previous studies. In other words, it can be observed that this measurement method may be effective. The results of measurements of this study suggested a possible association between the conservation of quantity and EF. As a feature at that time, it is also suggested that the brain activity may be activated when the difficulty level of the task is hit. This avenue of research has the potential to be developed in the future.

As conservation, which has been dealt with so far, is actually associated with the EF, it can also contribute to the development of methods that show the relationship between other cognitive abilities and EF, and it can be said that this is a vital achievement in fields related to child support, such as mathematics education, psychology, brain science, and pedagogy. The results also show that education can be performed while measuring EF in real time by using fNIRS, which is useful for new research in the fields of educational engineering and educational psychology in terms of aiming to build a learning support system that is related to effective EF. In addition, the results may help psychologists, psychiatrists, teachers, parents, and others involved in the development of children to sublate the value of their concept of conservation and improve their support methods. Case studies cannot be used to determine causation but for the development of a novel hypothesis. Therefore, a model (theoretical content and supportive method) that is responsive, appropriate, and novel for clinical sites (school/home) could be used to develop a novel hypothesis because this study was a practice study/case study.
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