The generality of inflation in some closed FRW models with a scalar field

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Abstract

The generality of inflation in closed FRW Universe is studied for the models with a scalar field on a brane and with a complex scalar field. The results obtained are compared with the previously known results for the model with a scalar field and a perfect fluid. The influence of the measure chosen in the initial condition space on the ratio of inflationary solution is described.

1 Introduction

In the recent two decades the dynamics of an isotropic Universe filled with a massive scalar field attracted a great attention. There are several possible dynamical regimes in this system \cite{1}. From the physical point of view the inflationary regime is the most important one \cite{2}, therefore, one of the problems of mathematical cosmology is to describe the set of initial data leading to inflation. This problem for a massive scalar field was studied in \cite{3, 4}. The main result of these studies is that the set of initial conditions required for inflation strongly depends on the sign of the spatial curvature. If it is negative or zero, the scale factor of the Universe $a$ cannot pass through extremum points. In this case all the trajectories starting from a sufficiently large initial value of the scalar field $\varphi_0$, reach a slow-roll regime and experience inflation. If we start from the Planck energy, a measure of non-inflating trajectories for a scalar field with the mass $m$ is about $m/m_P$. From observational reasons, this ratio is about $10^{-5}$ so almost all trajectories lead to the inflationary regime. However, positive spatial curvature allows a trajectory to have a point
of maximal expansion which results in increasing the measure of non-inflating trajectories. In a famous angular parametrization described below this measure is about $\sim 0.3$. 

Hence, the sign of the spatial curvature, which is rapidly "forgotten" after the inflationary regime established, is very important during a possible pre-inflationary period. Another important characteristic of the pre-inflationary era which is also ‘forgotten” during inflation is the possible presence of a hydrodynamical matter in addition to the scalar field. Decreasing even more rapidly than the curvature with increasing scale factor $a$, the energy density of hydrodynamical matter could not affect the slow-rolling conditions and so has a tiny effect on the dynamics if the spatial curvature is non-positive. However, the conditions for extrema of $a$ can change significantly in a closed model, so this model requires special analysis. It was done in [5] where the maximum measure of non-inflationary trajectories in the angular parametrization is found to be $\sim 60\%$ with the significant dependence of this measure on the density of hydrodynamical matter.

In recent years some interesting modification of standard scenario become the matter of investigations. After a famous paper of Randall and Sundrum [6] a lot of effort was done on the idea that our Universe is a boundary (brane) of a manifold with a larger dimensions (bulk). Inflation caused by a massive scalar field on the brane in the case of flat FRW brane metric was considered in [7]. One of the principal features of the brane scenario is the presence of the so called "dark radiation” term in the equations of motion. In the case of FRW brane this term is inversely proportional to the fourth power of the scale factor (as an ordinary radiation matter), so, it rapidly decreases during inflation. On the other hand, we can expect that this term can alter the conditions for inflation on closed FRW brane in a way similar to studied in [5]. This problem is the topic of Sec.2 of the present paper.

Apart from branes, some generalizations of simple scalar field scenario appeared also in the framework of ”standard” cosmology. In this paper we will consider one of them, namely a massive complex scalar field. It is naturally appears in supersymmetry theories. Some problems of quantum cosmology with a complex scalar field were studied in [8, 9]. Recent proposals to use this type of
scalar field to explain dark matter and quintessence were put forward in [10, 11]. We will apply our analysis of generality of inflation to the complex scalar field in Sec. 3.

The results of studies of these three models - a scalar field with a hydrodynamical matter, a scalar field on the brane and a complex scalar field are compared in Sec. 4.

2 A massive scalar field on the brane

In this section we consider a "three–brane universe" in a five dimensional (bulk) space-time. We will consider only purely nonstandard brane dynamics when the matter density on the brane dominates the brane tension. Then the equations describing the cosmological evolution of a closed FRW brane are [12, 13]

\[ \frac{\dot{a}^2}{a^2} = \frac{\kappa^4}{36} \rho^4 + \frac{C}{a^4} - \frac{1}{a^2}, \]

(1)

\[ \frac{a^2}{a^2} + \frac{\ddot{a}}{a} = -\frac{\kappa^4}{36} \rho (\rho + 3p) - \frac{1}{a^2}. \]

Here \( \kappa^2 = 8\pi/m_{5P}^2 \), \( m_{5P} \) is the 5-dimensional fundamental Planck mass which can differ significantly from 4-dimensional Planck mass observing on the brane. For a review of the current status of the brane model see, for example [15].

The equations of motion for a brane universe with a massive scalar field have the form

\[ \ddot{\varphi} + 3 \frac{\dot{a}}{a} \dot{\varphi} + m^2 \varphi = 0, \]

(2)

\[ \frac{\ddot{a}}{a} + 2 \frac{\dot{a}^2}{a^2} + \frac{2}{a^2} = \frac{4\pi^2}{3m_{5P}^2} (m^4 \varphi^4 - \dot{\varphi}^4) + \frac{C}{a^4}, \]

(3)

with a constraint

\[ \dot{a}^2 + 1 = \frac{16\pi^2}{9m_{5P}^2} a^2 \left( \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} m^2 \varphi^2 \right)^2 + \frac{C}{a^2}. \]

(4)

The equations of motion in brane scenario in comparison with those in standard cosmology characterizes by quadratic dependence on energy-momentum tensor and nonlocal effects from the free gravitational field in the bulk, manifesting itself in the presence of a "dark radiation" term (the last term in righthand side of Eqs. (3) and (4)).
The inflationary dynamics on the brane for the flat brane metric was considered in Ref. [7]. The situation appears to be not so different from the standard case, though the slow-roll regime is changed. The slow-roll condition being \( \varphi > m_{5P}/\sqrt{m/m_{5P}} \) in standard cosmology takes the form \( \varphi > m_{5P}/\sqrt{m/m_{5P}} \) in the brane model and, hence, the measure of non-inflationary initial conditions is \( \sqrt{m/m_{5P}} \) instead of \( m/m_{5P} \) in standard scenario. Our goal here is to study the positive curvature case.

First we need to specify a set of initial conditions. It is common to use the hypersurface with the energy density equal to the Planck one (called the Planck boundary) as the initial-condition space. As was pointed out in Ref. [14] this choice is based on the physical considerations of inapplicability of classical gravity beyond the Planck boundary and of the absence of any information from this region. We remind the reader that in the brane scenario the fundamental Planck scale limiting the applications of classical gravity is different from the effective Planck scale on the brane.

To compare our results with those obtained in the standard scenario, we rewrite the constraint equation so that the coefficient in the lefthand side be the same:

\[
\frac{3m_{5P}^2}{8\pi} \left( \frac{a}{a^2} + \frac{1}{a^2} \right) = \frac{2\pi}{3m_{5P}^2} \left( \frac{\dot{\varphi}^2}{2} + \frac{m^2 \varphi^2}{2} \right)^2 + \frac{3m_{5P}^2}{8\pi} \frac{C}{a^4}. \tag{5}
\]

Now, we start from the Planck boundary:

\[
\frac{2\pi}{3m_{5P}^2} \left( \frac{\dot{\varphi}^2}{2} + \frac{m^2 \varphi^2}{2} \right)^2 + \frac{3m_{5P}^2}{8\pi} \frac{C}{a^4} = m_{5P}^4. \tag{6}
\]

To describe the initial data on the Planck boundary it is useful to introduce two dimensionless variables, as in [4]. One of them is the angular parameter \( \phi \) defined as

\[
\dot{\phi}^2 = \left( m_{5P}^2 - \frac{3C}{8\pi a^2} \right)^{\frac{1}{2}} \sqrt{\frac{6}{\pi}} m_{5P}^3 \cos^2 \phi,
\]

\[
m^2 \varphi^2 = \left( m_{5P}^2 - \frac{3C}{8\pi a^2} \right)^{\frac{1}{2}} \sqrt{\frac{6}{\pi}} m_{5P}^3 \sin^2 \phi
\]

and the second one is \( z = \dot{a}/(am) \). A couple \((z, \phi)\) determine the initial point on the Planck boundary completely. The variable \( z \) can vary in a compact region from 0 to \( z_{\text{max}} = \)
Figure 1: The area of initial data on the Planck boundary which do not lead to inflation (gray). Fig. 1(b) corresponds to the situation without a "dark radiation" term, other plots correspond to densities of the "dark radiation" increasing from Fig. 1(a) to Fig. 1(f) (see details in the text). The value of $\phi$ varies in the range $-\pi/2 \leq \phi \leq \pi/2$ and the value of $z$ in the range $0 \leq z \leq z_{\text{max}}$. Initial data below the horizontal line in Fig. 1(f) are physically inadmissible.

$\sqrt{8\pi/3}(m_5 \rho/m)$; the corresponding initial values of the scale factor $a$ varies from $a_{\text{min}} = \sqrt{3/(8\pi m_5^2 \rho)}$ to $+\infty$.

The numerical calculation results are plotted in Fig. 1. The region where inflation is possible is the gray one. The resulting measure of initial data leading to inflation as a function of $C$ is plotted in Fig. 2. Though this measure does not change drastically, the configuration of corresponding regions in initial condition space shown in Fig. 1 undergoes substantial transformation. In particular, for large enough $C$ inflation becomes impossible if the value of initial Hubble parameter $H = \dot{a}/a$ is less than some critical one (or, equivalently, initial spatial curvature is greater than some critical value). We can easily find analytically the minimal value of $C$ when this situation takes place, i.e. when there are no inflation with initial $z = 0$. To do this we remark from Eq.(4) that if we start with some initial energy $E$ with $H = 0$ (i.e. in the extremum of $a$), then possible
initial $\varphi$ lie in the interval
\[
\varphi^2 < \frac{3m_{5\mu}^3}{2\pi m^2 a_{in}} \left(1 - \frac{C}{a_{in}^2}\right)^{1/2},
\] (8)
where initial scale factor $a_{in} = \sqrt{3m_{5\mu}^2/8\pi E}$.

From the other side, the direction of trajectory in the extremum of $a$ depends on the sign of the second derivative $\ddot{a}$. Substituting $\dot{a} = 0$ into Eq. (3) and expressing $\dot{\varphi}$ from the constraint equation (4), we obtain that $\ddot{a}$ at the point $\dot{a} = 0$ is
\[
\ddot{a} = \frac{4C}{a^3} - \frac{5}{a} + m^2 \varphi^2 \frac{4\pi}{m_{5\mu}^3} \left(1 - \frac{C}{a^2}\right)^{1/2}.
\] (9)

We can see that for initial $z = 0$, $\ddot{a} < 0$ (Universe collapses directly from the initial point) when
\[
\varphi^2 < \frac{m_{5\mu}^3}{4\pi m^2 a_{in}} \frac{5 - 4C/a_{in}^2}{(1 - C/a_{in}^2)^{1/2}}.
\] (10)

The intervals (8) and (10) coincide at the critical value of initial scale factor $a_{in} = a_{cr} = \sqrt{2C}$.

It means that if we start from the Planck boundary, then for $C = C_1 = 3/(16\pi m_{5\mu}^2)$ (in this case $a_{cr} = a_{min}$) all trajectories with $z = 0$ go to singularity directly from the initial point and can not reach inflation. For $C > C_1$ it is impossible to reach inflation from some high-curvature initial data independently on the initial $\varphi$.

The curve (8) does not exist for $a_{in} < \sqrt{C}$. If initial $a$ corresponding to a given initial $z$ is less than this value then it is impossible to find any $\phi$, such that the total energy of the system ("dark radiation" plus the scalar field) does not exceed the Planck energy. The Planck boundary is now only part of the initial rectangular area in the coordinates $z$ and $\phi$ (see Fig.4). This situation occurs if $C > C_2 = 3/(8\pi m_{5\mu}^2)$. The ratio of non-inflation trajectories reaches its maximum at $C = C_2$ (the maximum value of this ratio is 65%) and steadily decreases for larger $C$.

We also checked numerically, that another choices of initial energy $E$ lead to qualitatively the same picture with appropriate rescaling of the parameter $C$.

A negative $C$ (Fig.1(a)) does not change the situation significantly in comparison with the case $C = 0$ (Fig.1(b)). The results for positive $C$ are similar to those received in [5] where the degree of generality of inflation in ordinary FRW Universe with massive scalar field and hydrodynamical
Figure 2: The ratio of inflationary solutions in the angular parametrization as a function of the "dark radiation" density. The "dark radiation" parameter $C$ is measured in the units of $3/8\pi m_P$.

matter is analyzed. It is natural, since the $C$-containing term behaves exactly as a hydrodynamical matter with the equation of state $p = \epsilon/3$.

3 A complex scalar field

3.1 Equations of motion

In this section we return to standard FRW cosmology but consider a more complicated version of the scalar field, namely, the massive complex scalar field. The action for this model is

$$S = \int d^4x \sqrt{-g} \left( \frac{m_P^2}{16\pi} R + \frac{1}{2} g^{\mu\nu} \phi_*^* \phi_\mu \phi_\nu - \frac{1}{2} m^2 \phi \phi^* \right)$$

and now the Planck mass $m_P$ has its common sense.

We will use a suitable representation of the complex scalar field

$$\phi = x \exp(i\theta),$$

where $x$ is absolute value of the complex scalar field and $\theta$ is its phase.
Using this representation we can see that the phase variable $\theta$ is cyclical and its conjugate momentum $p_\theta$ is conserved. We call it as a "charge" of the Universe $Q$:

$$p_\theta = Q = a^3 x^2 \dot{\theta}. \quad (12)$$

Expressing the phase $\theta$ through the charge $Q$ we can get the following equations of motion:

$$\frac{m_p^2}{16\pi} \left( \ddot{\theta} + \frac{\dot{\theta}^2}{2a} + \frac{1}{2a^2} \right) + \frac{a^2 x^2}{8} - \frac{m_p^2 a^2}{8} + \frac{Q^2}{4a^6 x^2} = 0, \quad (13)$$

$$\ddot{x} + \frac{3i\dot{x}}{a} + m^2 x - \frac{2Q^2}{a^6 x^3} = 0. \quad (14)$$

Besides we can write down the first integral:

$$\frac{3m_p^2}{8\pi} \left( H^2 + \frac{1}{a^2} \right) = \frac{m_p^2 x^2}{2} + \frac{\dot{x}^2}{2} + \frac{Q^2}{a^6 x^2}. \quad (15)$$

For future use we also find a curve, separating possible maxima and minima of the scale factor. Following the same procedure, as in the previous section we can express $\dot{x}$ from the constraint (15) and substitute it in Eq. (13). This equation gives now for $\dot{a} = 0$

$$\ddot{a} = \frac{4\pi m^2 x^2 a}{m_p^2} - \frac{2}{a}. \quad (16)$$

So, the separating curve is

$$x^2 \leq \frac{1}{\frac{2\pi}{m^2 a^2}} \quad (17)$$

and does not depend on the charge $Q$.\[9]

### 3.2 Inflation in models with a complex scalar field

As in the previous section, we start from the Planck boundary

$$\frac{3m_p^2}{8\pi} \left( H^2 + \frac{1}{a^2} \right) = \frac{m_p^2 x^2}{2} + \frac{\dot{x}^2}{2} + \frac{Q^2}{a^6 x^2} = m_p^4. \quad (18)$$

Then it is necessary to introduce a measure on the initial condition space. We remind the reader that in the case of a real scalar field the usual way to parameterize the Planck boundary
is to introduce two dimensionless variables, \( z = H/m \) and \( \phi \) via the angular parametrization, which in standard cosmology has the form

\[
\frac{m^2 \dot{\phi}^2}{2} + \frac{\dot{\phi}^2}{2} = m_P^4; \quad \frac{m^2 \dot{\phi}^2}{2} = m_P^4 \sin^2 \phi; \quad \frac{\dot{\phi}^2}{2} = m_P^4 \cos^2 \phi.
\]

However, in our case we have additional term in the left part of (18), so we can not use this way. Instead, the second dimensionless coordinate we chose as \( X = x/m_P \).

Two variables, \( z \) and \( X \), completely determine initial conditions on the Planck boundary. We can see from Eq.(15) that possible initial \( X \) for a given \( z \) lie in the interval \([X_{\text{min}}, X_{\text{max}}]\), where

\[
X_{\text{min}} = \frac{m_P^2}{m^2} \left( 1 - \sqrt{1 - \frac{2m^2Q^2}{m_P^8} \left( \frac{8\pi m_P^4}{3} - z^2 m^2 \right)^3} \right)^{1/2},
\]

\[
X_{\text{max}} = \frac{m_P^2}{m^2} \left( 1 + \sqrt{1 - \frac{2m^2Q^2}{m_P^8} \left( \frac{8\pi m_P^4}{3} - z^2 m^2 \right)^3} \right)^{1/2}.
\]

The variable \( z \) as in the previous section varies from 0 to \( \sqrt{8\pi/3(m_P/m)} \).

Numerically found initial conditions leading to inflation for a real scalar field (\( Q = 0 \)) in the coordinates \( z \) and \( x/m_P \) are shown in Fig. 3 (a). The measure of non-inflating trajectories is equal to 0.52. The difference between this value and 0.3 found in \([4]\) arises completely from differences in chosen measures. With growing \( Q \) the picture experiences changes similar to the case studied in the previous section.

If we start with zero \( z \) and some initial \( a \), the maximum and minimum values of the scalar field \( x \) lie on the curve

\[
\frac{3m_P^2}{8\pi a^2} = \frac{m^2 x^2}{2} + \frac{Q^2}{a^6 x^2}.
\]

The separating curve is given by Eq. (17) and crosses the curve (19) at

\[
a^2 = \frac{8\pi^2 Q^2 m^2}{m_P^4}.
\]
Figure 3: The area of initial data on the Planck boundary which does not lead to inflation (black) and leads to inflation (gray). Fig. 3(a) corresponds to the situation without charge, other plots correspond to charges increasing from Fig. 3(b) to Fig. 3(d).
Figure 4: The ratio of inflationary solutions $N$ as a function of charge $Q$. Scalar field mass $m = 1.4 \times 10^{-2} m_P$.

Remembering that the initial value of the scale factor, corresponding to $z = 0$ on the Planck boundary is $a_{\text{min}} = \sqrt{3/8\pi m_P^3}$, we find that for $Q = Q_1 = (1/8\pi) \sqrt{3/2\pi}(m_P/m)$ all trajectories with $z = 0$ fall into a singularity. For $Q > Q_1$ there is a high-curvature region with no way to reach inflation (Fig.3(c)).

As in the previous section, there exists some value of charge for which start from $z = 0$ becomes impossible. It happens if $Q$ exceeds $Q_2 = (3/32\pi) \sqrt{3/\pi}(m_P/m)$ (Fig.3(d)).

It is easy to see, that these values are rather close to each other:

$$\frac{Q_2^2}{Q_1^2} = \frac{9}{8}.$$  \hspace{1cm} (21)

As $Q_1$ and $Q_2$ depend on $m$, this dependence appears also in the curve $N(Q)$, where $N$ is the measure of inflationary solutions. An example for $m = 1.4 \times 10^{-2} m_P$ is plotted in Fig.4. For another $\tilde{m}$ this curve is similar with rescaling $\tilde{Q} \to Q(m/\tilde{m})$.  

11
4 Discussion

In the present paper we have considered two particular examples of a scalar field dynamics on the closed FRW background. Together with the model of a massive scalar field with a hydrodynamical matter studied in the cases, arising from rather different physical models, show several common features.

First of all, the measure of initial conditions leading to inflation remains large enough, not falling below 35%. If we consider a detailed structure of initial conditions for recollapsing universes, we find that depending on the parameter of the model (which can be a charge, a "dark radiation" density or a density of a hydrodynamical matter) there are two situations possible. For small values of the parameter inflation can start from an arbitrary large initial curvatures, or, equivalently, arbitrary small initial scale factors (of course, within the Planck limit). For the parameter bigger than the critical one all trajectories from the high-curvature initial data go back to a singularity instead of inflation. These critical values are $C_1 = 3/16\pi m_P^2$ for a "dark radiation" in the brane model and $Q_1 = (1/8\pi)\sqrt{3/2\pi}(m_P/m)$ for a charge. In the case of the massive scalar field with a perfect fluid which has the equation of state $p = \gamma\epsilon$ we can introduce a constant parameter $D$ which is related to energy density $\epsilon$ as $D = \epsilon a^{3(\gamma+1)}$. In this case $D_1 = (3/8\pi)^{1+p/2}m_P^{2-p}2^{-p/2}$, where $p = 1 + 3\gamma$.

There is also the second critical value of the parameter which is equal to $C_2 = 3/8\pi m_P^2$ for the "dark radiation", $Q_2 = (3/32\pi)\sqrt{3/\pi}(m_P/m)$ for the charge and $D_2 = (3/8\pi)^{1+p/2}m_P^{2-p}$ for the perfect fluid density (the latter value is taken from 5). For bigger value of the parameter the Planck boundary covers only a part of the initial rectangular. The coordinate $z$ varies in the interval $z_{\text{min}} < z < z_{\text{max}}$ with some nonzero $z_{\text{min}}$, corresponding to some minimal possible initial scale factor $a$. Smaller initial $z$ would correspond to smaller initial $a$ and the energy density in "dark radiation", charge or matter terms would exceed the Planck energy which makes these initial conditions physically inadmissible. Numerical data show also that if the parameter exceeds the second critical value the described above range of initial $z$ not leading to inflation independently.
on the second coordinate continues to exist (see Fig.1(f)), though being narrower with increasing value of the parameter.

Looking on the overall measure of the initial condition allowing inflation we can see also a remarkable difference between the brane scenario (Fig.2) and the complex scalar field (Fig.4). In the former case this measure decreases significantly with increasing $C$ up to the second critical value $C_2$, though in the latter case the measure is almost monotonically increasing function of $Q$. In the model of scalar field with a perfect fluid, studied earlier, the situation is similar to the brane case. However, this difference mostly appears because of the different choice of the measure. We remind the reader that in the brane scenario and the perfect fluid case we use the angular parametrization of the initial condition space, and in the complex scalar field case the value $x/m_P$ is chosen for the second coordinate (the first coordinate, $z$, is the same in all three cases). It is possible to use the latter measure for another cases under consideration. The results are presented in Fig.5. For the model with a perfect fluid we can see a very small decrease (about 8%) of the measure in comparison with about 2 times drop of the measure in the angular parametrization. In the brane scenario we can see a rather small decreasing of the measure with the minimum at $C \sim 0.7C_2$. In this point the measure is $\sim 88\%$ of the value at $C = 0$ in comparison with about 1.5 times drop in the angular parametrization.

This difference can be easily understood by comparing of these two parametrizations of the Planck boundary. The important feature of the angular parametrizations is that independently on the possible initial $z$ on the Planck boundary the second coordinate $\phi$ can vary in the same range $[-\pi/2; \pi/2]$. On the other hand, the possible range for $\phi/m_P$ (or $x/m_P$ for the complex scalar field) steadily decreases with increase of the initial curvature (see Eq.(8)). So, the high-curvature initial data, less favorable for inflation give bigger contribution for the measure based on the angular parametrizations than for the measure based on $\phi/m_P$ as the second coordinate.

For parameter bigger than the second critical one the map from one measure to another becomes even discontinuous. The line $z = z_{min}, \varphi \in [-\pi/2; \pi/2]$ in Fig.1(f) (consisting only on non-
Figure 5: The ratio of inflationary solutions in the scalar field parametrization as a function of the "dark radiation" density in the brane scenario (a) and of the radiation matter density in the model with scalar field and a perfect fluid (b). $C$ is measured in the units of $3/8\pi m_{5P}$, $D$ is dimensionless.

inflationary trajectories) maps into a single point (compare Fig.1(f) and Fig.3(d)). So, a definitely nonmonotonic behavior of the measure of inflationary trajectories as a function of the parameter of the model in the case of the angular parametrizations of the Planck boundary can be explained as a measure effect.

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