Non-linear evolution of the BAO scale in alternative theories of gravity

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The scale of Baryon Acoustic Oscillations (BAO) imprinted in the matter power spectrum provides an almost-perfect standard ruler: it only suffers sub-percent deviations from fixed comoving length due to non-linear effects. We study the BAO shift in the large Horndeski class of gravitational theories and compute its magnitude in momentum space using second-order perturbation theory and a peak-background split. The standard prediction is affected by the modified linear growth, as well as by non-linear gravitational effects that alter the mode-coupling kernel. For covariant Galileon models, we find a 14-45% enhancement of the BAO shift with respect to standard gravity and a distinct time evolution depending on the parameters. Despite the larger values, the shift remains well below the forecasted precision of next-generation galaxy surveys. Models that produce significant BAO shift would cause large redshift-space distortions or affect the bispectrum considerably. Our computation therefore validates the use of the BAO scale as a comoving standard ruler for tests of general dark energy models.

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I. MOTIVATION

One of the most exciting promises of modern cosmology is the possibility of testing fundamental physics using the largest scales available to observation [1]. Among other developments, the signatures of Baryon Acoustic Oscillations (BAO) have provided an invaluable test of models for cosmic acceleration through their imprint in the Cosmic Microwave Background [2] and the distribution of Large Scale Structure (LSS) [3] using either galaxies [4-8], the Lyman-α forest or quasars [9, 10] (see Ref. [11-13] for reviews). To an excellent approximation, the BAO signal in the LSS provides a comoving standard ruler that traces the expansion of the universe and probes the onset of cosmic acceleration.

Non-linear corrections are known to introduce a small departure from the perfect standard ruler behavior, systematically shifting the BAO scale towards smaller values at low redshift. This effect has been well studied for cold dark matter cosmologies with a cosmological constant using perturbation theory [14-21] and simulations [22-24] (for earlier works see [25, 26]). The result is that the BAO scale imprinted in the matter distribution shrinks by approximately 0.3% at redshift zero [22, 24]. However, this value relies on the assumption that gravity is Newtonian in the scales of interest.

Little attention has been devoted to the non-linear BAO evolution in more general theories of gravity. Since the shift in the BAO scale is comparable to the sub-percent level of precision expected by forthcoming galaxy surveys [27] that aim to test such theories, it will be necessary to understand the effects of non-standard gravity on the BAO scale to correctly interpret the data in the next generation of dark energy experiments.

II. PEAK-BACKGROUND SPLIT COMPUTATION OF THE BAO SHIFT

Sherwin and Zaldarriaga have explained the BAO shift in terms of the effect of long modes on the short scale power spectrum [21]. In their picture, large overdense regions undergo less overall expansion, reducing the size of the physical BAO scale with respect to the average (see also [28]). This effect is not compensated by underdense regions, because cosmic structures in overdense regions undergo more growth and give a larger contribution to the power spectrum. Therefore, local differences in expansion and growth lead to a net shortening of the comoving BAO scale, causing a small departure from the standard ruler behavior. Alternatively, the shift of the BAO scale can also be understood as arising from contributions to the power spectrum which are off-
phase with respect to the linear prediction [16, 18].

The BAO shift can be studied by comparing the non-linear power spectrum to a rescaled version of the linear one [18]:

\[ P(k) \approx P_{11}(k/\alpha) = P_{11}(k) - (\alpha - 1)kP'_{11}(k) + \cdots, \tag{1} \]

where the shift can be read from the coefficient of the second term (more sophisticated templates are actually used to obtain the BAO scale from data, but we will stick to this description for simplicity). Here and below the power spectrum is defined as \( \langle \delta(\vec{k})\delta(\vec{k}')\rangle \equiv (2\pi)^3\delta_D(\vec{k} + \vec{k}')P(k) \) and \( P_{nm} \propto \langle \delta_n \delta_m \rangle \), where we expand the density contrast as \( \delta = \delta_1 + \delta_2 + \cdots \).

One can compare Eq. (1) with the prediction from standard perturbation theory

\[ P(k) = (P_{11} + \cdots + P_{1n}) + (P_{22} + \cdots + P_{nn}) \]. \tag{2} \]

All \( P_{1n} \) contributions are proportional to \( P_{11}(k) \) and thus do not contribute to the second term in Eq. (1) [18]. Only the mode-coupling terms (second parenthesis) do contribute to the shift, with the first of such contributions given by

\[ P_{22}(k) = \int \frac{d^3q}{(2\pi)^3} \left[ F_2(\vec{k} - \vec{q}, \vec{q}) \right]^2 P_{11}(\vec{k} - \vec{q})P_{11}(\vec{q}). \tag{3} \]

Here \( F_2 \) is the second-order symmetrized mode-coupling kernel [29]. The rest of the computation in the peak-background split approximation proceeds by expanding in \( k/q \), integrating with a cutoff at \( k_{BAO} \) and extracting the coefficient of \( kP'_{11}(k) \) from the result (see [21] for further details). The long modes with \( q \ll k \sim k_{BAO} \) describe the effect of the large fluctuations on the smaller scales.

The computation can be generalized to alternative theories of gravity by noting that the structure of the kernel \( F_2 \) is preserved on sub-horizon scales, but each term acquires a time-dependent coefficient \( C_i(t) \)

\[ F_2(p, q) = C_0 + C_1\mu \left( \frac{p}{q} + \frac{q}{p} \right) + C_2 \left( \mu^2 - \frac{1}{3} \right), \tag{4} \]

which reduces to the standard constant values, \( C_0 = 17/21, C_1 = 1/2 \) and \( C_2 = 2/7 \), under matter domination in the case of standard gravity (we drop the time-dependence for notation convenience). Explicit computations in the sub-horizon, quasi-static limit of Horndeski theories with non-relativistic matter determine that the modifications to the kernel coefficients are not independent [30, 31] and satisfy

\[ C_1 = \frac{1}{2}, \quad C_0 + \frac{2}{3}C_2 = 1. \tag{5} \]

The BAO shift can be read by plugging the generalized kernel (4), into the mode-coupling power spectrum (3), expanding to leading order in \( q/k \), performing the integration and comparing with Eq. (1). This generalizes the result of Ref. [21] to

\[ \alpha - 1 = \frac{2}{5} \left( 2C_0 - \frac{1}{2} \right) \langle \delta_1^2 \rangle, \tag{6} \]

where Eqs. (5) have been used to write the result in terms of the monopole \( C_0 \). In the above expression the integration over the momentum leads to the long mode variance

\[ \langle \delta_1^2 \rangle \equiv \int_0^{k_{BAO}} \frac{dq}{(2\pi)^3} 4\pi q^2 P_{11}(q,t) \approx \sigma_{r_s}^2(t), \tag{7} \]

where we use a cut-off at BAO scale, well estimated by the sound horizon at the drag epoch \( k_{BAO} \sim 1/r_s(z_d) \) [32]. Following Ref. [21], we use the square of the variance of the density field on a sphere of radius \( r_s(z_d) \) for the computation of the BAO shift: \( \langle \delta_1^2 \rangle \approx \sigma_{r_s}^2(t) \). This gives a slight underestimate with respect to the shift measured in simulations of standard cosmology, but we expect comparison among models to be accurate.

LSS surveys observe galaxies, which are known to be biased with respect to the underlying matter distribution. The effects of non-linear density-halo bias can be parameterized as

\[ \delta_h(x) = b_1\delta(x) + \frac{1}{2}b_2\delta^2(x) + \cdots. \tag{8} \]

where the bias parameters \( b_1, b_2 \) relate the matter and the halo overdensity (the above expansion should hold on large scales). The halo-halo power spectrum generalizing Eq. (2) reads

\[ P_h(k) = b_1^2 (P_{11} + P_{22}) + b_1b_2P_{\delta_h^2} + \cdots, \tag{9} \]

where second-order terms that do not involve mode coupling have been omitted as they do not contribute to the BAO shift. The first parenthesis contains the linear and mode-coupling matter power spectrum renormalized by the linear bias. The second term mixes second order corrections with non-linear bias, and affects the BAO shift. This term is given by

\[ P_{\delta_h^2} = \int \frac{d^3q}{(2\pi)^3} 2F_2(\vec{k} - \vec{q}, \vec{q})P_{11}(\vec{k} - \vec{q})P_{11}(\vec{q}), \tag{10} \]
where the only difference with Eq. 3 is that the kernel appears linearly. As before, one can compute the leading order expansion in $q/k$ and compare with a rescaled version of the linear halo power spectrum (cf. Eq. 1 multiplied by $b^2$). Identifying the $kP_{11}'(k)$ term yields the contribution to the shift, which now reads

$$\alpha - 1|_h = \left( \frac{4}{5}C_0 - \frac{1}{5} + \frac{2b_2}{3b_1} \right) \langle \delta^2 \rangle . \tag{11}$$

This result generalizes equation 6. Note that the relations 3 for the perturbation theory kernels make the bias contribution to the shift independent of the theory of gravity, i.e. of any departure in the value of $C_0$. For this reason we will not consider non-linear bias in the next section.

III. BAO SHIFT IN ALTERNATIVE THEORIES OF GRAVITY

We focus our analysis on theories within the Horndeski Lagrangian [33], which contains many examples of interest for cosmology including Brans-Dicke, $f(R)$, Chameleons, Kinetic Gravity Braiding and covariant Galileons. Horndeski’s theory also contains the characteristic interactions that appear in consistent theories of massive gravity and higher dimensional theories, and it is thus expected to effectively describe some of their distinctive features [34].

Although our analysis is general, for the sake of simplicity we will present results for a covariant Galileon model [44] (see also [45, 46]). We fix the Galileon Lagrangian parameters and the cosmological parameters to the best-fit models obtained by Barreira et al. [47] (without massive neutrinos), which have zero cosmological constant. We noticed that the quintic galileon model we present has a gradient instability in the tensor sector. However we decided to include it in our analysis since it is a good fit for the data and has interesting properties at second-order in perturbation theory. Indeed, as we shall see it produces large modifications of the dark matter kernel, which can be detected by studying the bispectrum with current surveys [48, 49].

Table I. Density contrast, mode coupling kernel monopole and BAO shift in the matter power spectrum for reference model and selected Galileon models at redshift zero (cf. Fig. 1). Values in parenthesis indicate the relative deviation with respect to a Cosmological Constant model. Tracer bias can be added using Eq. 11.

| Model     | $\sigma_{r_s}$ | $2C_0$ | $\alpha_k - 1$ [%] |
|-----------|----------------|--------|-------------------|
|           |                 |        |                   |
| $\Lambda$ | 0.067           | 1.62   | 0.20              |
| Cubic     | 0.071 (7%)      | 1.61 (-0.4%) | 0.23 (14%)       |
| Quartic   | 0.073 (9%)      | 1.58 (-2%) | 0.23 (15%)       |
| Quintic   | 0.071 (7%)      | 1.92 (19%) | 0.29 (45%)       |

Simpler scalar-tensor theories (such as Brans-Dicke, Chameleons or $f(R)$) lead to very constrained modifications and can not produce large contributions to the Kernel [31]. For such models the only sizeable contributions to the BAO shift stem from the enhancement of linear growth, and would thus be in conflict with measurements of LSS clustering. If one further demands that these theories are screened in the Galaxy or the Solar system, the range of the scalar force is too short to even affect cosmological scales in the linear regime [50]. An exception is given by non-universal couplings to matter: Coupled dark matter models can significantly increase the shift of the BAO scale [51].

The generalized Sherwin-Zaldarriaga formula 6 depends on the theory of gravity in two ways: a correction from linear physics, given by $\sigma_{r_s}^2$, and a modification of the mode coupling kernel 4, given by $C_0$. We compute the evolution of the background, the linear power spectrum and the density contrast $\sigma_{r_s}^2$ using a modified version of the CLASS code [52, 53] based on the general description of Horndeski perturbations presented in [54] (see also [55]). The computation of the non-linear corrections to the kernel follows the approach of ref. [31] (see also [56]) by taking the sub-horizon approximation and the quasi-static approximation (valid for covariant Galileons on the scales of interest [57]). We will also assume that the scale at which the model becomes strongly coupled is smaller than the BAO scale. This is indeed the case for cubic and quartic galileon models, as suggested by a comparison between fully non-linear and linearized N-body simulations for Galileons [58, 59].

The results for the BAO interesting quantities at redshift zero are presented in Table I. All the models considered tend to increase the density contrast $\sigma_{r_s}$.

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1 We will not consider viable extensions of Horndeski’s theory [35, 36], nor full theories containing interacting gravitons [37, 39]. See Refs. [40, 41] for reviews on the cosmology of alternative theories of gravity.
due to an enhanced effective force of gravity and the different background expansion. The non-linear corrections to $C_0$ are highly dependent on the model parameters, acquiring positive and negative sign and ranging from sub-percent in the cubic, percent in the quartic, and becoming fairly large in the quintic example.

The time evolution of the BAO shift and the mode-coupling corrections are displayed in Figure 1. Departures with respect to standard gravity occur only at low redshift and become largest in the accelerated era when the scalar field energy density drives the cosmic expansion. Besides this general trend, each model is characterized by a specific time dependence. Our results allow to distinguish between a very soft non-linear regime in which the mode coupling is mostly determined by interactions of the matter fluid (standard gravity, cubic model) and large non-linear effects, as in the case of the quintic model, with the quartic case being an intermediate example. This is a consequence of the non-linear gravitational interactions introduced in Horndeski's theory.

**IV. DISCUSSION**

Our results show an enhancement of the BAO shift with respect to the standard prediction and provide the first estimate of this effect for modified gravity.

It is possible to compare the predicted shift to the forecasted sensitivity of next-generation galaxy surveys. Let us focus on measurements on the BAO scale transverse to the line of sight $\theta_{\text{BAO}} = r_s/D_A$, where $D_A$ is the comoving angular diameter distance (comparison with line-of-sight BAO yields similar results). One can compare the two sources of uncertainty

$$\frac{\Delta D_A}{D_A} = \frac{D_A}{r_s} \Delta \theta + \frac{\Delta r_s}{r_s},$$

where the first term is the observational error (assuming known $r_s$) and the second term is the systematic error induced by the shift, $\Delta r_s/r_s = \alpha - 1$.

Weinberg et al. have provided an example forecast for an all-sky BAO survey in which the expected error ranges from 2.8% at $z = 0.15$ to 0.1% $z \gtrsim 3.5$ (these data can be found in Table 2 of Ref. [12]).

Figure 1 compares both terms in Eq. (12) and shows that the BAO shift is well below the precision for all the examples considered at any fiducial redshift. Note that the forecasted precision is mainly limited by survey volume, implying that more sophisticated observational setups will not be able to reduce the errors considerably. More realistic forecasts based on specific surveys lead to lower precision (see Ref. [27]).

It is very unlikely that models more general than the ones considered here can lead to sufficiently large

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2 Their forecast also assumes density field reconstruction improvements in the non-linear damping by a factor of 2. Since this procedure has not been validated for general theories of gravity, we take the forecasted precision as an optimistic bound.
shifts to bias the BAO scale measurements while remaining compatible with other observations. The theoretical prediction, Eq. (6), allows one to identify two contributions to the shift: the modified linear growth and the non-linear gravitational effects that modify the mode-coupling kernel. Any theory of gravity with a very large shift requires a considerable enhancement of at least one of these contributions, which can be probed by observables other than BAO.

A large departure of $\sigma_r$, would be ruled out by redshift space distortions or other clustering measurements. Similarly, large corrections to the mode coupling kernel would induce large distortions in the bispectrum (note that Eq. (5) implies that a $\gtrsim 23.5\%$ increase in $C_0$ would change of sign in the quadrupole term in $F_2$). We emphasize that these non-linear gravitational effects are exclusive of fully fledged Horndeski theories (cf. quartic and quintic example Galileons considered here) and very suppressed in simpler scalar-tensor theories (e.g. Brans-Dicke, $f(R)$) or cubic theories (e.g. our cubic example, Kinetic Gravity Braiding [60] and limits of extra-dimensional theories [61]). Most works on higher order perturbation theory for modified gravity have focused on the latter type of models [30, 56, 62–64].

Our findings validate the use of BAO measurements as a comoving standard ruler for current and next-generation LSS surveys, at all redshifts of interest and even for the most extreme theories of gravity. There are several refinements that can improve our calculation, such as including higher order perturbation theory corrections. Other developments would be necessary in order to better connect these results with observations, such as the inclusion of more sophisticated bias models (which has been shown to affect the magnitude and time evolution [24]) and redshift-space distortions (which typically increase the magnitude of the shift relative to real space). Finally, extending our result would allow to confirm the validity of density field reconstruction [65, 66] of BAO for general theories of gravity.

Despite possible refinements, the smallness of the effects ensures the validity of our conclusions regarding the shift.

These are some initial steps in understanding the interplay between extended theories of gravity and the BAO scale imprints on the distribution of large scale structure. Further work should address other aspects of LSS and BAO in general theories of gravity in order to optimize the performance and model independence of the next-generation of dark energy experiments. This will ultimately shed light on the optimal strategy to test gravitational physics using future LSS surveys and learn more on the connections between fundamental physics and cosmology.

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3 Another possibility is that a modification of gravity enhances the BAO shift by producing a large value of the non-linear halo bias $b_2/b_1$. Such a possibility would however rely on the details of halo formation and its study would require methods other than cosmological perturbation theory.

4 The validity of BAO reconstruction schemes has been ar-

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