QUANTUM MARKOVIAN KINETIC EQUATION FOR HARMONIC OSCILLATOR

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Specific nonequilibrium states of the quantum harmonic oscillator described by the Lindblad equation have been hereby suggested. This equation makes it possible to determine time-varying effects produced by statistical operator or statistical matrix. Thus, respective representation-varied equilibrium statistical matrixes have been found. Specific mean value equations have been found and their equilibrium solutions have been obtained.

Key words: statistical operator, statistical matrix, Lindblad equation, harmonic oscillator.

1. Lindblad equation

Statistical operator \( \hat{\rho} \) or statistical matrix is basically applied as the quantum mechanics tool, any information of the nonequilibrium process proceeding within the tested system may be gained from [1 - 5]. When the process concerned proceeds within the system which fails interacting with its environment, statistical operator \( \hat{\rho} \) will satisfy Liouville-von Neumann equation as follows:

\[
i \hbar \dot{\hat{\rho}} = \left[ \hat{H}, \hat{\rho} \right].
\] (1.1)

With provision for the fact that the system interacts with any environment, a new equation shall be produced [4 - 16]. Lindblad is the first one who offered the equation describing interaction of the system with a thermostat [10]. This work is devoted to Markovian equation, which hereby describes nonequilibrium quantum harmonic oscillator performance.

We will write the kinetic equation for a quantum harmonic oscillator as follows:

\[
i \hbar \dot{\hat{\rho}} = \left[ \hat{H}, \hat{\rho} \right] + \hbar A \left( \left[ \hat{a} \hat{\rho}, \hat{a}^\dagger \right] + \left[ \hat{\rho}, \hat{a} \hat{a}^\dagger \right] \right) + \hbar B \left( \left[ \hat{a}^\dagger \hat{\rho}, \hat{a} \right] + \left[ \hat{a}^\dagger, \hat{\rho} \hat{a} \right] \right),
\] (1.2)

where

\[
\hat{H} = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right),
\] (1.3)

\( A \) and \( B \) are constants. Operator \( \hat{a} \) is formulated as follows:

\[
\hat{a} = \frac{1}{\sqrt{2} \hbar \omega} \left( \frac{i \hat{p}}{\sqrt{m}} + \sqrt{\kappa} \hat{x} \right).
\] (1.4)

Equation (1.2) is very precise to describe time-varying state of the thermostat-interacted quantum harmonic oscillator and its equilibrium state.

2. Energy representation

Now, we will define the wave functions describing specific energy state \( \varphi_n(x) \). The very functions satisfy the equation as follows:

\[
\hat{H} \varphi_n(x) = E_n \varphi_n(x),
\] (2.1)

where

\[
E_n = \hbar \omega \left( n + \frac{1}{2} \right), \quad n = 0, 1, 2, ...
\] (2.2)
As referred to energy representation, the matrix elements of statistical operator $\hat{\rho}$ will be formulated by the equation as follows:

$$\rho_{nn'} = \int \varphi_n^*(x) \hat{\rho} \varphi_{n'}(x) \, dx.$$  \hfill (2.3)

Wave functions satisfy the following equations

$$\hat{a} \varphi_n = \sqrt{n} \varphi_{n-1}, \quad \hat{a}^+ \varphi_n = \sqrt{n+1} \varphi_{n+1}. \hfill (2.4)$$

With provision for the above formulas the following matrix-formed equation (1.2) is derived:

$$\dot{\rho}_{nn'} = -i \omega (n - n') \rho_{nn'} + A \left( 2 \sqrt{(n+1)(n'+1)} \rho_{n+1,n'+1} - (n + n') \rho_{nn'} \right) +$$

$$+ B \left( 2 \sqrt{n n'} \rho_{n-1,n'-1} - (n + n' + 2) \rho_{nn'} \right). \hfill (2.5)$$

Now, we will write the equation for diagonal elements of statistical matrix $\rho_{nn} = W_n$, where $W_n$ is the probability referred to oscillator state $\varphi_n$. The equation produced has the form as follows:

$$\dot{W_n} = 2 A \left( (n + 1) W_{n+1} - n W_n \right) + 2 B \left( n W_{n-1} - (n + 1) W_n \right). \hfill (2.6)$$

This kinetic equation describes particular harmonic oscillator state transitions. In this case, there may be gained coefficients $A$ and $B$ as follows

$$A = \frac{1}{2} P e^{\frac{1}{2} \beta \hbar \omega}, \quad B = \frac{1}{2} P e^{-\frac{1}{2} \beta \hbar \omega}, \hfill (2.7)$$

where $P$ is probability of transition per unit time; $\beta = 1/kT$ is reciprocal temperature.

Equation (2.6) has specific oscillator state equilibrium distribution, which satisfies the following equation

$$A \left( (n + 1) W_{n+1} - n W_n \right) + B \left( n W_{n-1} - (n + 1) W_n \right) = 0. \hfill (2.8)$$

This equation is solved by the method as follows

$$W_n = (1 - q) q^n \hfill (2.9)$$

under the following condition

$$q = \frac{B}{A} = \exp(-\beta \hbar \omega). \hfill (2.10)$$

### 3. Mean value of coordinate

Mean value $\overline{b}$ assigned by operator $\hat{b}$ is defined as

$$\overline{b} = Tr\left( \hat{b} \hat{\rho} \right). \hfill (3.1)$$

For gaining mean value $\overline{a}$ the respective equation may be derived from formula (1.2). Using the equality of:

$$\hat{a} \hat{a}^+ - \hat{a}^+ \hat{a} = 1, \hfill (3.2)$$

we will get the equation as follows:

$$\overline{a} = (-i \omega - A + B) \overline{a}. \hfill (3.3)$$

Now, we can find the derivatives from mean values $\overline{a}$ and $\overline{b}$. By applying formula (1.4) we will get:

$$\frac{i \overline{p}}{\sqrt{m}} + \sqrt{\kappa} \overline{a} = (-i \omega - A + B) \left( \frac{i \overline{p}}{\sqrt{m}} + \sqrt{\kappa} \overline{a} \right).$$

Then, we will try to equate both the real and imaginary parts of this equation:

$$\begin{align*}
\hat{\overline{a}} &= -(A - B) \overline{a} + \overline{p}/m, \\
\hat{\overline{p}} &= -\kappa \overline{a} - (A - B) \overline{p}.
\end{align*} \hfill (3.4)$$
If we eliminate \( \mathbf{p} \) from this set of equations, we can obtain the mean coordinate equation
\[
\ddot{\mathbf{x}} + 2 (A - B) \dot{\mathbf{x}} + \left( \omega^2 + (A - B)^2 \right) \mathbf{x} = 0.
\] (3.5)

The above equation (3.5) provides the following solution:
\[
\mathbf{x}(t) = (C_1 \cos \omega t + C_2 \sin \omega t) e^{-(A-B)t},
\] (3.6)
where \( C_1 \) and \( C_2 \) are arbitrary constants.

4. Mean oscillator energy

Now, we will find the time derivative from \( a^+ a \). By applying the above equality (3.2) we will produce the following derivative from equation (1.2):
\[
\dot{a}^+ a + a^+ \dot{a} + 2 (A - B) a^+ a = 2 B.
\] (4.1)

We can define harmonic oscillator time-varying energy effects inserting the following formula in equation (4.1):
\[
\overline{a^+ a} = \frac{\hbar \omega}{2}.
\]
Thus, the following differential equation is derived:
\[
\overline{H} + 2 (A - B) \overline{H} = \hbar \omega (A + B) .
\] (4.2)

The solution is given by the equation as follows:
\[
\overline{H}(t) = C e^{-2 (A-B)t} + \frac{\hbar \omega}{2} \frac{A + B}{A - B} ,
\] (4.3)
where \( C \) is an arbitrary constant.

The equation (4.2) has specific stationary solution:
\[
\overline{H} = \frac{\hbar \omega}{2} \frac{A + B}{A - B} .
\] (4.4)

Since constants \( A \) and \( B \) are related (2.7), the stationary solution obeys the formula as follows:
\[
\overline{H} = \frac{\hbar \omega}{2} \frac{e^{\beta \hbar \omega} + 1}{e^{\beta \hbar \omega} - 1} = \frac{\hbar \omega}{2} \cth \frac{\beta \hbar \omega}{2} .
\] (4.5)

If it is assumed that \( T \) tends to zero, than \( \overline{H} = \hbar \omega/2 \). If it assumed that \( T \) increases to infinity, than \( \overline{H} = k T \).

5. Kinetic equation expressed in terms of coordinate and momentum operators

We well express the equation (1.2) in terms of operators \( \hat{x} \) and \( \hat{p} \). For this purpose, we will firstly write the equation (1.2) as follows:
\[
i \hbar \hat{\theta} = \hat{H} \hat{\theta} - \hat{\theta} \hat{H} + i \hbar A \left( 2 \hat{a} \hat{\theta} \hat{a}^+ - \hat{a}^+ \hat{\theta} \hat{a} - \hat{\theta} \hat{a}^+ \hat{a} \right) + i \hbar B \left( 2 \hat{\theta} \hat{a} \hat{a}^+ - \hat{a} \hat{\theta} \hat{a}^+ - \hat{\theta} \hat{a}^+ \hat{a} \right) .
\] (5.1)

Since the energy operator is equal to:
\[
\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\kappa \hat{x}^2}{2} ,
\] (5.2)
we will insert it in equation (5.1) along with formula (1.4) to obtain the following one:
\[
i \hbar \hat{\theta} = \left( \frac{\hat{p}^2}{2m} + \frac{\kappa \hat{x}^2}{2} \right) \hat{\theta} - \hat{\theta} \left( \frac{\hat{p}^2}{2m} + \frac{\kappa \hat{x}^2}{2} \right) -
\[-\frac{i}{2\omega} \left( \frac{1}{m} (\hat{p}^2 \hat{\varrho} - 2 \hat{\varrho} \hat{p} + \hat{\varrho} \hat{p}^2) + \kappa (\hat{x}^2 \hat{\varrho} - 2 \hat{x} \hat{\varrho} \hat{x} + \hat{\varrho} \hat{x}^2) \right) - (A - B) (\hat{x} \hat{\varrho} \hat{p} - \hat{\varrho} \hat{p} \hat{x} + i \hbar \hat{\varrho}) \].

(5.3)

6. Coordinate representation

As referred to coordinate function, the statistical matrix is represented by formula \( \varrho(t, x, x') \). In this case, there shall be produced the following coordinate and momentum operators:

\[ \hat{x} = x, \quad \hat{\varrho} = -i \hbar \partial_x. \]

Using the above values we can write equation (5.3) by the formula as follows:

\[
\partial_t \varrho = \frac{i \hbar}{2m} (\partial_x^2 - \partial_k^2) \varrho - \frac{i \kappa}{\hbar} x \varrho - \frac{A + B}{2\hbar \omega} \left( \frac{\hbar^2}{m} (\partial_x + \partial_k)^2 - \kappa (x - x')^2 \right) \varrho + (A - B) (1 + x \partial_{x'} + x' \partial_x) \varrho.
\]

(6.1)

As concerns statistical matrix physical interpretation, the following formula

\[ w(t, x) = \varrho(t, x, x) \]

is applied for getting specific probability coefficient.

Now, we will add new variables

\[ x_1 = \frac{1}{2} (x + x'), \quad x_2 = x - x'. \]

(6.3)

In this case

\[ \partial_x = \frac{1}{2} \partial_1 + \partial_2, \quad \partial_{x'} = \frac{1}{2} \partial_1 - \partial_2. \]

Referring to statistical matrix \( \varrho(t, x_1, x_2) \) and using the above new variables we will gain the equation as follows:

\[
\partial_t \varrho = \frac{i \hbar}{m} \partial_1 \partial_2 \varrho - \frac{i \kappa}{\hbar} x_1 x_2 \varrho + \frac{A + B}{\hbar \omega} \left( \frac{\hbar^2}{2m} \partial_2^2 \varrho - \frac{\kappa}{2} x_2^2 \varrho + \varepsilon (1 + x_1 \partial_1 - x_2 \partial_2) \varrho \right),
\]

(6.4)

where

\[ \varepsilon = \hbar \omega \frac{A - B}{A + B} = \hbar \omega \text{ th} \frac{\beta \hbar \omega}{2}. \]

(6.5)

In this case

\[ \varrho(t, x, 0) = w(t, x). \]

(6.6)

We will try solution of equation (6.4) as follows:

\[ \varrho(t, x_1, x_2) = \frac{1}{2\pi} \int f(t, k, x_2) e^{i k x_1} dk. \]

(6.7)

Reciprocal transformation

\[ f(t, k, x_2) = \int \varrho(t, x_1, x_2) e^{-i k x_1} dx_1. \]

(6.8)

With provision for equation (6.6) we will obtain:

\[ f(t, 0, 0) = \int \varrho(t, x, 0) dx = \int w(t, x) dx = 1. \]

(6.9)

Thus, in view of function (6.8) the following equation is formed:

\[
\partial_t f = -\frac{\hbar}{m} \partial_k f + \frac{\kappa}{\hbar} x \partial_k f - \frac{A + B}{\hbar \omega} \left( \frac{\hbar^2 k^2}{2m} + \kappa x^2 \right) + \varepsilon (k \partial_k + x \partial_x) f.
\]

(6.10)
This equation has an equilibrium solution which satisfies the both formulas as follows:

\[ -\frac{\hbar k}{m} \frac{\partial}{\partial x} f + \frac{\kappa}{\hbar} x \frac{\partial}{\partial k} f = 0, \]  
(6.11)

\[ \frac{\hbar^2 k^2}{2m} f + \frac{\kappa x^2}{2} f + \varepsilon \left( k \frac{\partial}{\partial k} + x \frac{\partial}{\partial x} \right) f = 0. \]  
(6.12)

We will write the performance equation of the above formula (6.11):

\[ -\frac{m}{\hbar^2 k} \frac{dx}{dk} = \frac{dk}{\kappa x}. \]

This equation has the solution as follows:

\[ \frac{\hbar^2 k^2}{2m} + \frac{\kappa x^2}{2} = \text{const}. \]

This formula implies that the general solution of equation (6.11) takes the form as follows:

\[ f = f(E), \]

where

\[ E = \frac{\hbar^2 k^2}{2m} + \frac{\kappa x^2}{2}. \]

Now, we will insert this function in equation (6.12) to gain the following formula:

\[ \frac{df}{dE} + \frac{f}{2\varepsilon} = 0. \]

With provision for condition (6.9) this equation has the following solution:

\[ f(E) = \exp\left(-\frac{E}{2\varepsilon}\right). \]

Thus, the equilibrium solution of equation (6.10) takes the form as follows:

\[ f(k, x) = \exp\left(-\frac{1}{2\varepsilon} \left( \frac{\hbar^2 k^2}{2m} + \frac{\kappa x^2}{2} \right) \right). \]

(6.13)

We will find the equilibrium statistical matrix by formula (6.7) to obtain the following equation:

\[ \varrho(x_1, x_2) = \frac{1}{2\pi} \int \exp\left(-\frac{1}{2\varepsilon} \left( \frac{\hbar^2 k^2}{2m} + \frac{\kappa x^2}{2} \right) \right) e^{ikx_1} dk. \]

(6.14)

On integrating the following formula is produced:

\[ \varrho(x_1, x_2) = \sqrt{\frac{\alpha}{\pi}} \exp\left(-\alpha x_1^2 - \frac{\sigma^2 x_1^2}{4\alpha} \right), \]

where

\[ \alpha = \sigma \tan \frac{\beta \omega \hbar}{2}, \quad \sigma = \frac{m \omega \hbar}{\hbar}. \]

Using formulas (6.3) we will get the equation as follows:

\[ \varrho(x, x') = \sqrt{\frac{\alpha}{\pi}} \exp\left(-\frac{\alpha (x + x')^2}{4} - \frac{\sigma^2 (x - x')^2}{4\alpha} \right). \]  
(6.15)
Using formula (6.2) we will get respective equilibrium probability coefficient [17]

\[ w(x) = \sqrt{\frac{\alpha}{\pi}} \exp \left(-\alpha x^2 \right). \] (6.16)

7. Momentum representation

As referred to momentum function, the coordinate and momentum operators are represented by the formulas as follows:

\[ \hat{x} = i \hbar \frac{\partial}{\partial p}, \quad \hat{p} = p. \]

In view of momentum representation, the statistical matrix is formulated as \( \varrho(t, p, p') \). This form may be applied for obtaining equation (5.3) as follows:

\[
\partial_t \varrho = -\frac{i}{\hbar m} \left( p^2 - p'^2 \right) \varrho + \frac{i \hbar \kappa}{2} \left( \frac{\partial p^2}{\partial p'} - \frac{\partial p'^2}{\partial p} \right) \varrho - \frac{A + B}{2 \hbar \omega} \left( \frac{1}{m} (p - p')^2 - \kappa \hbar^2 (\partial_p + \partial_{p'})^2 \right) \varrho - (A - B) (1 + p \partial_{p'} + p' \partial_p) \varrho. \] (7.1)

As concerns physical interpretation of statistical matrix \( \varrho(t, p, p') \), the following formula refers to \( w(t, p) = \varrho(t, p, p) \) probability density applied for detecting the state when an oscillator have impulse \( p \).

8. Wigner function

In order to better appreciate the physical significance of various kinetic state summands we will derive the equation for Wigner function \( w = w(t, x, p) \), which is specified as a quantum analog of classical distribution function and may be defined by applying statistical matrix \( \varrho = \varrho(t, x, x') \) specified by the relation as follows:

\[
w(t, x, p) = \frac{1}{2\pi} \int \varrho \left( t, x + \frac{1}{2} \hbar q, x - \frac{1}{2} \hbar q \right) e^{-ipq} dq. \] (8.1)

If the statistical matrix depends on \( x_1 \) and \( x_2 \), then

\[
w(t, x, p) = \frac{1}{2\pi\hbar} \int \varrho(t, x_1 = x, x_2) e^{-ipx_2/\hbar} dx_2. \] (8.2)

Reciprocal transformation

\[
\varrho(t, x_1, x_2) = \int w(t, x_1, p) e^{ipx_2/\hbar} dp. \] (8.3)

Since there is formula (6.9)

\[
\int \varrho(t, x_1, 0) dx_1 = 1,
\]

Wigner function satisfies specific normalization requirement

\[
\int w(t, x, p) dx dp = 1. \] (8.4)

For the purpose of Wigner function, we will derive the following formula from equation (6.4) formulated for statistical matrix \( \varrho(t, x_1, x_2) \)

\[
\partial_t w = -\frac{p}{m} \partial_x w + \kappa x \partial_p w + \frac{A + B}{2 \hbar \omega} \left( \frac{\hbar^2}{2m} \partial_x^2 w + \frac{\hbar^2 \kappa}{2} \partial_p^2 w + \varepsilon (2 + x \partial_x + p \partial_p) w \right). \] (8.5)
The equation produced is much different from its quantum analog of Fokker-Planck equation. Summands containing derivatives \( \partial^2_x w \) and \( \partial^2_p w \) may be interpreted as those describing phase space diffusion. And still, it is necessary to add that it is rather hard to appreciate physical significance of the formula in parentheses that follows coefficient \( \varepsilon \).

Equilibrium solution of equation (8.5) should at the same time refer to the following equations:

\[
- \frac{p}{m} \partial_x w + \kappa x \partial_p w = 0 , \tag{8.6}
\]

\[
\frac{\hbar^2}{2m} \partial^2_x w + \frac{\hbar^2 \kappa}{2} \partial^2_p w + \varepsilon (2 + x \partial_x + p \partial_p) w = 0 . \tag{8.7}
\]

General solution of the equation (8.6) derives by the function as follows:

\[
w = w(E) , \tag{8.8}
\]

where

\[E = \frac{p^2}{2m} + \frac{\kappa x^2}{2} . \tag{8.9}\]

We will insert the above function in equation (8.7). Thus, we will get the following equation:

\[
E \frac{d^2 w}{dE^2} + (1 + \mu E) \frac{dw}{dE} + \mu w = 0 , \tag{8.9}
\]

where

\[\mu = \frac{2}{\hbar \omega} \text{th} \frac{\beta \hbar \omega}{2} . \tag{8.10}\]

Solution of this equation derives the function as follows:

\[w(E) = C e^{-\mu E} . \tag{8.11}\]

Wigner function may be derived by applying formula (8.2), providing that specific equilibrium function (6.14) is inserted in. In view of this we shall obtain the following equation:

\[
w(x, p) = \frac{1}{2\pi \hbar} \sqrt{\frac{\alpha}{\pi}} \int \exp \left( - \alpha x^2 - \frac{\sigma^2 x_2^2}{4\alpha} \right) e^{-ipx_2/\hbar} dx_2 . \tag{8.12}
\]

On integrating we will get the equilibrium function

\[w(x, p) = \frac{\mu \omega}{2\pi} \exp \left( - \mu \left( \frac{p^2}{2m} + \frac{\kappa x^2}{2} \right) \right) . \tag{8.13}\]

This function may be formulated by the equation as follows:

\[w(x, p) = \sqrt{\frac{\alpha}{\pi}} \exp(-\alpha x^2) \sqrt{\frac{\gamma}{\pi}} \exp(-\gamma p^2) , \tag{8.14}\]

where

\[\gamma = \frac{\mu}{2m} . \tag{8.15}\]

Then, we will find the mean value

\[\overline{x^2 p^2} = \int x^2 p^2 w(x, p) dx dp . \tag{8.16}\]

The above computation gives the following formula

\[\overline{x^2 p^2} = \frac{\hbar^2}{4} \left( \frac{e^{\beta \hbar \omega} + 1}{e^{\beta \hbar \omega} - 1} \right)^2 . \tag{8.17}\]
This formula has the following result. If \( T = 0 \), the uncertainty is equal to \( \overline{x^2 - p^2} = \hbar^2/4 \). If \( T \to \infty \), then \( \overline{x^2 - p^2} = m (k T)^2/\kappa \).

9. Conclusion

The equation proposed by Lindblad for the purpose of the statistical operator describing nonequilibrium state of quantum harmonic oscillator is hereby considered. Initially, the statistical matrix equation in energy representation and diagonal matrix element equation have been derived from the equation concerned. Specific formulas appreciating physical significance of Lindblad equation coefficients have been formulated. Then, the mean coordinate equation has been derived to find any general solution. It was demonstrated that the mean coordinate exponentially decays in time. The mean oscillator energy equation has been derived to obtain the general solution and mean equilibrium energy value has been found. Lindblad equation has been formulated by applying coordinate and momentum operators. The coordinate representation statistical matrix equation has been obtained. The equilibrium statistical matrix formula has been derived from this equation. The momentum representation statistical matrix equation has been formulated. Wigner function equation has been obtained and the respective equilibrium state function has been found. Various temperature uncertainty relations have been found by applying Wigner equilibrium function.

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