Long Josephson junctions with exciton-polariton condensates

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We demonstrate the possibility to build stable Josephson $\pi$-junction stripes with exciton-polariton condensates. The stability of the $\pi$-junction between arbitrary long polariton stripes is achieved at low pumping by balancing the snaking instability with counter-propagating flows towards the junction. Not dissimilar from a dark soliton, the instability becomes relevant at high pumping leading to formation of vortex dipoles. The resulting structures can be stabilised to produce static lattices of Josephson vortices in straight and ring geometries. Our results build towards realization of quantum technological applications based on the Josephson effect at room temperature.

Introduction.—Observation [1–3] of Bose-Einstein condensates (BECs) of exciton-polaritons (polaritons) in semiconductor microcavities [4], including formation of macroscopic quantum coherence at room temperature [5–7], offer a new platform for realizing quantum technological applications. The driven-dissipative polariton superfluids, however, are essentially different from their equilibrium counterparts such as liquid helium or ultra-cold atoms. Apart from the composite character of polaritons as mixed states of excitons and photons, and their finite lifetime in microcavity, a crucial difference is in the presence of condensate currents even in steady states. These currents usually originate from non-uniform pumping that is necessary to excite the polariton BEC, but they can appear spontaneously as well [8]. When the pump-dissipation balance is nonlocal, the flows of particles appear from regions of positive to negative balance [9]. The polariton supercurrents can lead, for instance, to the generation of quantized vortices (loop currents) due to inhomogeneities of the samples [10].

The steady-state polariton flows play a fundamental role in the distinct dynamics of topological defects in polariton condensates. The steady-state currents depend sensitively on the particular geometry of the polariton pumping scheme, which in turn can lock the resulting stationary configuration of vortex lattices [11]. Even low-intensity flows are able to drag a single vortex out of the system [9, 12], and more complicated arrangements have to be designed to achieve the dynamical stability of these defects [13]. The design of excitation scheme is also of key importance in recently proposed polariton simulators and graphs [14, 15]. Underpinning the locking of relative phases, the basic element of polariton simulators and networks is the Josephson junction of two condensates [16, 17]. The Josephson junctions are frequently formed with a phase difference of $\pi$ between the condensates [18–20]. In this case, the order parameter has a node in between the small-sized condensates, while the dark soliton, or 1D curve of zero density, should be imprinted for large-sized condensates forming a long Josephson junction (LJJ). LJJ are significant in their ability to host Josephson vortices [21] that are strictly localised within the junction. The long lifetime and form-stability of dark solitons in equilibrium BECs can be exploited for qubit operations by using the soliton profile as a non-harmonic potential for a two-level system [22]. LJJs in polariton BECs offer exciting prospects for realizing the ac Josephson effect [23] at room temperature without the need for cryogenic refrigeration.

In spite of the vast technological importance, the emergence and stability of LJJs with polariton condensates is not comprehensively understood. In an incoherently pumped homogeneous polariton BEC, a stationary dark soliton (LJJ) is always unstable to any small perturbation, which leads to acceleration and blending of the dark soliton with the background [24]. In a one-dimensional (1D) waveguide geometry, experimental realization of dark solitons in polariton fluids has been achieved recently [25]. More recently, Josephson vortices have been experimentally observed to emerge from a phase twist imposed at the boundary of a polariton condensate [26]. An imprinted $\pi$ phase difference between resonant lasers beams situated at two spatially separated spots was observed to produce at low pumping power a robust LJJ in the phase of a two-dimensional (2D) polariton cloud. Further increase of the non-resonant-pump power (and, equivalently, of the polariton density) was observed to lead to the decay of the domain wall into stable vortex dipoles, in a similar way as Josephson vortices (or fluxons) emerge in LJJs between superconductors in the presence of external magnetic fields [27].

In this Letter, we demonstrate how it is possible to form arbitrarily long stable Josephson junctions (dark solitons) and the vortex chains on demand, by exploiting the out-of-equilibrium nature of polariton condensates and the steady-state currents. We show that, contrary to their equilibrium counterparts, and contrary to the homogeneous polariton condensate, there is a parameter regime where the dark soliton between two stripe condensates is stable. The snaking instability is suppressed by the counter-propagating flows towards the soliton line. The vortex instability remains important nonetheless, and it can be initiated at certain range of parameters and excitation conditions, leading to formation of stable...
the reservoir density. Here where the rate of amplification $g \gamma \pi \sigma \kappa r$ of polaritons and condensate polaritons, $g > 0$, represents polariton-polariton interactions, $g_R$ represents interactions between the reservoir polaritons and condensate polaritons, $\gamma$ defines the loss rate of polaritons from microcavity, $P_0$ represents polariton-polariton interactions, $\gamma_R$ and $\gamma$ are the loss rate of reservoir polaritons.

We consider condensate excitation in ring geometry, which attracts much interest recently, and assume that the pumping consists of two static concentric rings with a Gaussian profile, a spacing of $d = 20 \mu m$, and situated equidistantly from a middle circle of radius $21.5 \mu m$. The pumping intensities are (a) $hP_0 = 58 \text{ meV/\mu m}^2$ and (b) $hP_0 = 59 \text{ meV/\mu m}^2$. The lifetime of states in (a) and (b) exceeds 1 ns, and the vortex chain in (b) is stable with the vortices locked in place.

1D lattices of vortex-antivortex dipoles.

**Model.**—To incorporate the dynamical balance of pumping and loss, we use a coupled mean-field description for the macroscopic wavefunction of the BEC $\psi(r, t)$ and density of the reservoir polaritons $n_R(r, t)$,

\[
\frac{i\hbar}{\partial t} \psi = \left\{ -\frac{\hbar^2}{2m} \nabla^2 + g |\psi|^2 + \left( g_R + i\frac{\hbar}{2} \right) n_R - i\frac{\hbar}{2} \right\} \psi, \tag{1a}
\]

\[
\frac{\partial n_R}{\partial t} = P - \left( \gamma_R + r |\psi|^2 \right) n_R, \tag{1b}
\]

where the rate of amplification $n_R$ of the condensate from reservoir-induced stimulated scattering is linear in the reservoir density. Here $m$ is the effective mass of polaritons, $g > 0$ represents polariton-polariton interactions, $g_R$ represents interactions between the reservoir polaritons and condensate polaritons, $\gamma$ defines the loss rate of polaritons from microcavity, $P$ is the pumping rate of polaritons into the reservoir, and $\gamma_R$ is the loss rate of reservoir polaritons.

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The resulting system (2) describes a long Josephson junction along the $y$-direction characterised by the complex coupling $J\phi y \sigma \kappa r$ and density of the reservoir polaritons $n_R(r, t)$, with $r$ representing the radial width of the ring and $d$ the spacing of the rings. The pumping intensities are (a) $hP_0 = 58 \text{ meV/\mu m}^2$ and (b) $hP_0 = 59 \text{ meV/\mu m}^2$. The lifetime of states in (a) and (b) exceeds 1 ns, and the vortex chain in (b) is stable with the vortices locked in place.

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by

$$\omega = -\frac{\eta}{2} \pm \sqrt{\frac{\hbar k^2}{2m} \pm \omega_J} \left( \frac{\hbar k^2}{2m} + \frac{2g_n}{\hbar} \pm \omega_J \right) - \frac{\eta^2}{4},$$

where $\eta = \gamma/(1 + \gamma R n_R/\gamma n)$, and the ± sign inside the square root accounts for the symmetric (−) and anti-symmetric (+) states. As can be seen, analogously to conservative systems, the contribution of $-\omega_J$ to the dispersion involves dynamical instabilities responsible for the production of vortices either for the anti-symmetric state when Re$[J] > 0$, or for the symmetric state when Re$[J] < 0$.

For comparison with experimental parameters, we set $\hbar \gamma = 1$ meV, as energy unit, and $\gamma^{-1} = 0.66$ ps and $t_\gamma = \sqrt{\hbar/m \gamma} = 0.77 \mu m$ as time and length scales. In these units, we use $g = 4\pi \times 0.0054$, $g_R = 0.085$, $r = 0.17$, and $\gamma R = 10$ in all our numerical calculations. Implementing the double Gaussian-stripe pump we choose $d = 20$ and $\sigma = 6$, and consider a range of pumping intensities $P_0 \in [65, 90]$ above the threshold for the generation of the polariton condensate. For homogeneous pumping the threshold is $P_{th} = \gamma R / r \approx 58.8$. Figure 2 depicts the steady states at low $P_0 = 67$ [panels (a)] and intermediate $P_0 = 81$ [panels (b)] pumping intensities. The anti-symmetric states [top panels in (a) and (b)] present a nodal line at $x = 0$ and will be referred to as dark solitons. Both the symmetric states [bottom panels] and generally the anti-symmetric states show off-center density dips associated with steep phase gradients. These shoulders are situated symmetrically around $x = 0$, and will be referred to as gray solitons due to the non-zero background polariton current at their locations. As it can be seen, the gray solitons develop deeper density depletions for increasing pumping. For the considered ratio $2\sigma/d = 0.6$, the analysis of linear excitations (Fig. 3) shows that the anti-symmetric states are dynamically stable in the range $P_0 \in [65, 82]$, whereas the symmetric states are unstable. This happens in spite of the fact that, for given pumping, both configurations alternate in hosting the maximum population of anti-symmetric states (either symmetric or anti-symmetric) for a given pumping intensity $P_0$. Figure 3 collects our results for the linear stability spectrum from the numerical solution of the Bogoliubov
as we elaborate below.

The snaking instability can produce the decay of solitons in polariton fluids, and hence the appearance of vortices, if the pumping is high enough (non-shaded range in Fig. 3). However, the character of the instability is different from that of equilibrium systems. As depicted in the top right panel of Fig. 3(a), for \( P_0 = 85 \), the unstable modes present both a maximum and a minimum wavenumber, whereas there is no such minimum in equilibrium BECs. The maximum unstable wavenumber \( k_{\text{max}} \) allows for preventing the snaking instability by choosing a short enough system with \( L_y < k_{\text{max}}^{-1} \). The minimum unstable wavenumber, on the other hand, constraints the type of bifurcated vortex states and so the minimum number of vortices emerging from the solitons.

Our numerical results show that the unstable modes are exponentially localized along the axial \( x \)-direction, around the lowest density lines of the solitons, and excite standing waves along the transverse direction that depend on the system geometry. During the soliton decay, the vortices can be seen to emerge from the nodal points of the unstable standing waves producing \( N_v = 2kL_y \) vortices (\( N_v/2 \) vortex dipoles). In fact, the emerging stable configurations comprise of vortex dipoles aligned along the original solitons. Figure 1 shows a neat example of 8 vortex dipoles uniformly distributed on the initial position of a single ring dark soliton (without off-center solitons in this case for \( 2\sigma/d = 0.77 \)). Analogously, Figure 4(b) depicts the resulting configuration from the decay of a straight anti-symmetric state at \( P_0 = 85 \) (corresponding to the top right panel of Fig. 3) after seeding a perturbation with \( k = 6 \times 2\pi/L_y \). The vortex-dipole cores cannot be clearly discerned in this case, and the emerged low density waves are better described as solitonic lumps, or also as the Jones-Robert solitons [32]. In both cases, either vortex dipoles or solitonic lumps, the inward currents towards the junction permit the static arrangement of these otherwise moving non-linear waves, which fly away from the junction in conservative BECs (see e.g. Ref. [33]).

Multistability is another consequence of the combination of inward currents and multiple unstable channels. The appearance of unstable purely imaginary modes as a function of increased pumping (Fig. 3) is associated with the bifurcation of a new family of stationary vortex states inheriting the nodal configuration of the unstable mode. Although concerning bifurcations the scenario is analogous to its conservative counterpart [34], a crucial difference is the existence of multiple dynamically stable configurations between the new vortex states; a situation that manifests more clearly with strong pumping with more instability channels. For instance, Fig. 4(c) shows another stable pattern of lumps from the decay of the anti-symmetric state at \( P_0 = 85 \), in this occasion as a result of feeding a perturbation with \( k = 7 \times 2\pi/L_y \). At higher pumping, more complex configurations arise combining lumps and vortex dipoles, as can be seen in Fig. 4(d) for \( P_0 = 90 \).

**Conclusions.**—We have shown how arbitrarily long,
dynamically stable Josephson junctions (dark solitons) can be generated on-demand in exciton-polariton semiconductor microcavities. Decay of the junction into a stable locked array of vortex dipoles can also take place at higher pumping intensity. Out of equilibrium, the dynamics and stability of the Josephson junction and the associated Josephson vortices are thus under complete control. A prominent outlook of our results concerns the dynamics of a linear LJJ array [35], and the implementation of dynamically stable weak links in ring geometry.

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