Bondi on spherically symmetric accretion

Philip J. Armitage

1 Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11790, USA
2 Center for Computational Astrophysics, Flatiron Institute, 162 Fifth Avenue, New York, NY 10010, USA

ABSTRACT

Hermann Bondi’s 1952 paper “On spherically symmetrical accretion” is recognized as one of the foundations of accretion theory. Although Bondi later remarked that it was “not much more than an examination exercise”, his mathematical analysis of spherical accretion on to a point mass has found broad use across fields of astrophysics that were embryonic or non-existent at the time of the paper’s publication. In this non-technical review, I describe the motivations for Bondi’s work, and briefly discuss some of the applications of Bondi accretion in high energy astrophysics, galaxy formation, and star formation.

Key words: accretion, accretion discs—hydrodynamics—history and philosophy of astronomy

A black hole of mass $M$ is immersed in gas, which at large distances is at rest with uniform density $\rho_{\infty}$ and sound speed $c_{\infty}$. At what rate does the black hole swallow the gas in its vicinity? A Newtonian version of this apparently simple problem was posed and answered by Bondi (1952). If spherical symmetry is maintained, and there is no feedback on to the flow, the accretion rate is,

$$\dot{M} = \frac{4\pi\lambda (GM)^2}{c_{\infty}^3} \rho_{\infty},$$

where $G$ is Newton’s gravitational constant, and the prefactor $\lambda$ depends upon the adiabatic index of the gas. Bondi’s paper with this result is one of the most influential to have been published in Monthly Notices of the Royal Astronomical Society.

Hermann Bondi (1919-2005) was a mathematical physicist, cosmologist, and relativist. He made key contributions to our understanding of the physical nature of gravitational waves (Bondi et al. 1962), and was one of the originators of the steady-state cosmological model (Bondi & Gold 1948). Later in life he held a series of high-level posts in public service, including as the second Director General of the European Space Research Organization, a predecessor of today’s European Space Agency (ESA). Although he was one of the pre-eminent experts of the day in general relativity, the stimulus for his 1952 paper on accretion had nothing to do with black holes or neutron stars, whose existence and importance for astronomical phenomena would not be recognized for more than another decade. Rather, he was extending a line of research that had been started by Fred Hoyle and Raymond Lyttleton, into the accretion of gas by stars moving through the Interstellar Medium (ISM) at supersonic speeds (Hoyle & Lyttleton 1939). Hoyle and Lyttleton, in turn, were motivated by hypotheses and astronomical problems of their time that may have a quixotic flavour to modern readers. They had suggested that Solar luminosity variations, sourced by changes in the rate of accretion from the ISM, might be the cause of ice ages, and that massive O and B stars could survive for the age of the Galaxy by continually accreting fresh hydrogen fuel. These ideas had stimulated debate (Gamow 1940) but had not found broad favour among the astronomical community. Undeterred by the lukewarm reception to their ideas, a few years later Hoyle suggested that Bondi return to the problem with the goal of putting it “on a proper mathematical basis”*. Bondi did just that (Bondi & Hoyle 1944), and the process by which a gravitating object accretes gas as it moves through a surrounding medium is now known as Bondi-Hoyle-Lyttleton accretion.

The problem Bondi turned to in 1951 was to calculate the rate of accretion from a uniform medium on to a Newtonian point mass, in the limit where the accreting object is at rest relative to the surrounding gas. Although this is a more symmetric situation than Bondi-Hoyle-Lyttleton accretion, it requires a proper treatment of the hydrodynamics of the inflow, which can be ignored in the simplest description of highly supersonic Bondi-Hoyle-Lyttleton flows. Nonetheless, it did not detain Bondi for long. He solved the problem in the course of just a few days, and later commented that it was

* Interview of Hermann Bondi by David DeVorkin on March 20, 1978, Niels Bohr Library & Archives, American Institute of Physics, College Park, MD, USA.
a very simple analysis that was “not much more than an examination exercise” (Bondi 1990). That description may be overly modest—at least today few instructors would dare to ask for a derivation of Bondi accretion sight unseen—but it is certainly true that the technical difficulties of the calculation would not have fazed mathematical physicists of earlier generations. Bondi’s achievement was as much in appreciating that the problem needed solving, as it was in carrying through the calculation. Once he had the solution it was Lyttleton who persuaded him that it was substantial enough to merit publication.

Early citations to Bondi (1952) focused on whether the model had anything interesting to say about the problems that had motivated Hoyle, Lyttleton and Bondi from the start. It was soon determined beyond doubt that accretion had nothing to do with Solar problems such as the origin of the Sun’s corona. As early as 1951, Biermann (1951) had deduced that the properties of cometary ion tails implied that charged particles were flowing radially outward from the Sun, and drawing on these observations Eugene Parker developed the first models of the Solar wind (Parker 1958). The simplest Solar wind models are closely related to the isothermal limit of Bondi accretion, with outflow replacing inflow, and boundary conditions that are specified as \( r \to 0 \) rather than \( r \to \infty \). Bondi accretion and the Parker wind are both examples of transonic flows, in which gas accelerates from subsonic to supersonic speeds as it passes through a sonic point. The two problems are mathematically so similar as to be taught jointly in many courses on astrophysical fluid dynamics.

The role of accretion in the formation and evolution of massive stars took more time, and several observational breakthroughs in infrared and mm-wave astronomy, to elucidate. The original idea that Galactic O and B stars can be almost indefinitely rejuvenated by accretion is wrong, though in a broader context of course these (and all other) stars form as a result of accretion. Strikingly little was known empirically about star formation in the mid-twentieth century. Carbon monoxide, the most important tracer of molecular gas, was not observed astronomically until 1970 (Wilson et al. 1970), with early surveys of its distribution within the Milky Way following a few years later (Scoville & Solomon 1975). These discoveries were steps toward the modern understanding of star formation taking place within molecular clouds, whose density is far higher than the relatively diffuse phases of the ISM that were known earlier. The physics of the initial collapse of dense molecular gas to form stars necessarily involves the self-gravity of the gas (Larson 1969), which is not included in Bondi’s solution. That said, there are a number of scenarios in which the “late” accretion of gas by stars (or their discs) within a young stellar cluster could have observable consequences (e.g. Bonnell et al. 2001; Throop & Bally 2008; Bastian et al. 2013). Accretion from a turbulent magnetized medium onto a moving star is a complex problem (Burleigh et al. 2017), but at heart it is a variation on Bondi-Hoyle-Lyttleton accretion.

Extension of Bondi’s work, to include additional physical effects that matter in specific environments, started immediately and continues to this day. The second ever citation to Bondi (1952) was a prescient paper by Mestel (1954) entitled “The influence of stellar radiation on the rate of accretion”, that foreshadows what is now a vast literature on the role that feedback plays in accretion processes. Study of the collisionless limit of spherical accretion—which might be appropriate for example for a primordial star interacting with dark matter—started even prior to work in the fluid approximation (Eddington 1926), and has been successively extended and improved (Danby & Camm 1957; Begelman 1977). Analogues of Bondi accretion appropriate for radiation dominated fluids (Begelman 1978), and for inflows vulnerable to thermal instability and the formation of multiple phases (Mościbrodzka & Praga 2013), have been considered and, in more recent years, simulated.

It took the discovery of neutron stars and stellar-mass black holes in X-ray binaries, together with the realization that quasars and other Active Galactic Nuclei are powered by gaseous inflow toward supermassive black holes, to demonstrate the broad importance of accretion as an astrophysical process. Whether in binary systems or galactic nuclei, the surrounding gas invariably has too much angular momentum to accrete spherically. Accretion discs, whether of the geometrically thin flavour found in luminous sources (Lynden-Bell 1969; Shakura & Sunyaev 1973; Lynden-Bell & Pringle 1974) or the radiatively inefficient type present in our Galactic Centre (Rees et al. 1982; Narayan & Yi 1994; Yuan & Narayan 2014), are therefore responsible for most of the observed properties of these systems. Even given this contemporary understanding of the primacy of discs, Bondi’s paper remains one of the key foundations of accretion theory. The Bondi accretion rate represents a basic, order of magnitude estimate, for how rapidly gas at large distances can be supplied to a gravitating object. Another foundation from the same era is the Eddington limit, which represents (also only approximately) the rate at which an accreting object can accept gas before radiation pressure overwhelms gravity and disrupts the inflow. The interplay between the Bondi and Eddington accretion rates—and the basic fact that the former scales as \( M^2 \) whereas the latter is linear in \( M \)—lies at the heart of open questions such as how the first supermassive black holes formed (Inayoshi et al. 2019).

Many of the recent citations to Bondi (1952) are in papers using numerical simulations to follow the growth of supermassive black holes in galactic nuclei. The Bondi formula expresses the accretion rate in terms of the density and sound speed of gas at large distances from the accreting object, where “large” means distances much greater than the Bondi radius.

\[
\dot{M} = \frac{2\pi^2 \rho_0}{c_s^2} \frac{2GM}{r_B^2},
\]

where the thermal energy of the the accreting gas matches its gravitational potential energy. For the supermassive black hole at the centre of the Milky Way, \( r_B \approx 0.05 \text{ pc} \), so the Bondi radius is very small compared to galactic scales, which are measured in kpc. The Bondi radius is larger than the scale of the event horizon, however, by a comparably large factor of the order of \((c/c_{\text{esc}})^2 \approx 10^5\), where \( c \) is the speed of light). This scale separation has led to the use of the Bondi formula as a part of sub-grid models for black hole accretion in galactic and cosmological-scale numerical simulations. The need and motivation for these models arises from the fact that black hole accretion releases tremendous
It is amusing to note that the theoretical study of accretion was initiated, in part, by speculations about one of the systems where accretion is not taking place—the present-day Sun. Despite this false start, accretion is now recognized to be a process that cuts across disparate fields of astrophysics, including star and planet formation, common envelope evolution, X-ray binaries and Active Galactic Nuclei (Frank et al. 2002). Feedback from supermassive black holes means that the importance of accretion spreads further, into the formation and evolution of galaxies and galaxy clusters. Bondi’s contribution to accretion theory, at a time when many of these fields were embryonic or non-existent, was to understand mathematically perhaps its simplest manifestation.

ACKNOWLEDGMENTS

My thanks to Mitch Begelman and Daniel Proga for their comments on a draft of this essay.

REFERENCES

Baganoff F. K., et al., 2003, ApJ, 591, 891
Bastian N., Lamers H. J. G. L. M., de Mink S. E., Longmore S. N., Goodwin S. P., Gieles M., 2013, MNRAS, 436, 2398
Begelman M. C., 1977, MNRAS, 181, 347
Begelman M. C., 1978, MNRAS, 184, 53
Biermann L., 1951, Z. Astrophys., 29, 274
Blandford R. D., Begelman M. C., 1999, MNRAS, 303, L1
Bondi H., 1952, MNRAS, 112, 195
Bondi H., 1990, Science, Churchill and me. The autobiography of Hermann Bondi, Master of Churchill
Bondi H., Gold T., 1944, MNRAS, 108, 252
Bondi H., Hoyle F., 1944, MNRAS, 104, 273
Bondi H., van der Burg M. G. J., Metner A. W. K., 1962, Proceedings of the Royal Society of London Series A, 269, 21
Bonnell I. A., Bate M. R., Clarke C. J., Pringle J. E., 2001, MNRAS, 323, 755
Booth C. M., Schaye J., 2009, MNRAS, 398, 53
Burleigh K. J., McKee C. F., Cunningham A. J., Lee A. T., Klein R. I., 2017, MNRAS, 468, 717
Cuadra J., Nayakshin S., Springel V., Di Matteo T., 2006, MNRAS, 366, 558
Danby J. M. A., Camm G. L., 1957, MNRAS, 117, 50
Dashyan G., Choi E., Somerville R. S., Naab T., Quirk A. C. N., Hirschmann M., Ostriker J. P., 2019, MNRAS, 487, 5889
Eddington A. S., 1926, The Internal Constitution of the Stars
Frank J., King A., Raine D. J., 2002, Accretion Power in Astrophysics: Third Edition
Gamm G., 1940, Nature, 146, 97
Hoyle F., Lyttleton R. A., 1939, Proceedings of the Cambridge Philosophical Society, 35, 405
Inayoshi K., Visbal E., Haiman Z., 2019, arXiv e-prints, p. arXiv:1911.05791
Larson R. B., 1969, MNRAS, 145, 271
Lynden-Bell D., 1969, Nature, 223, 690
Lynden-Bell D., Pringle J. E., 1974, MNRAS, 168, 603
Marrone D. P., Moran J. M., Zhao J.-H., Rao R., 2007, ApJ, 654, L57
Mestel L., 1954, MNRAS, 114, 437
Mocibrodzka M., Praga D., 2013, ApJ, 767, 156
Narayana R., Yu L., 1994, ApJ, 428, L13
Parkin E. N., 1958, ApJ, 128, 664
Rees M. J., Begelman M. C., Blandford R. D., Phinney E. S., 1982, Nature, 295, 17
Ressler S. M., Quataert E., Stone J. M., 2018, MNRAS, 478, 5344
Scolville N. Z., Solomon P. M., 1975, ApJ, 199, L105
Shakura N. I., Sunyaev R. A., 1973, A&A, 500, 33
Springel V., Di Matteo T., Hernquist L., 2005, MNRAS, 361, 776
Thropo H. B., Bally J., 2008, AJ, 135, 2380
Waters T., Aukutapla A., Proga D., Johnson J., Li H., Smitd J., 2020, MNRAS, 491, L76
Wilson R. W., Jefferts K. B., Penzias A. A., 1970, ApJ, 161, L43
Yuan F., Narayana R., 2014, ARA&A, 52, 529

MNRAS 000, 1–77 (0000)