Towards QCD-assisted hydrodynamics for heavy-ion collision phenomenology

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Heavy-ion collisions are well described by a dynamical evolution with a long hydrodynamical phase. In this phase the properties of the strongly coupled quark-gluon plasma are reflected in the equation of state (EoS) and the transport coefficients, most prominently by the shear and bulk viscosity over entropy density ratios \( \eta/s(T) \) and \( \zeta/s(T) \), respectively. While the EoS is by now known to a high accuracy, the transport coefficients and in particular their temperature and density dependence are not well known from first-principle computations yet, as well as the possible influence they can have once used in hydrodynamical simulations. In this work, the most recent QCD-based parameters are provided as input to the MUSIC framework. A ratio \( \eta/s(T) \) computed with a QCD based approach is used for the first time [1, 2]. The IP-Glasma model is used to describe the initial energy density distribution, and UrQMD for the dilute hadronic phase. Simulations are performed for Pb–Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV, for different centrality intervals. The resulting kinematic distributions of the particles produced in the collisions are compared to data from the LHC, for several experimental observables. The high precision of the experimental results and the broad variety of observables considered allow to critically verify the quality of the description based on first-principle input to the hydrodynamic evolution.

In ultra-relativistic heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) and at the Large Hadron Collider (LHC) strongly-interacting matter characterised by high energy density and temperature is produced. Under these conditions, the formation of a de-confined state of quarks and gluons, called Quark-Gluon Plasma (QGP), is predicted by Quantum ChromoDynamical (QCD) calculations on the lattice at vanishing [3, 4] and finite net baryon density [5, 6].

In the past decade, hydrodynamic models, including viscosity, have been applied with great success to describe the distribution of soft hadrons produced in heavy-ion collisions at RHIC and the LHC [7–10]. The application of hydrodynamics is based on the assumption of local thermal equilibrium, energy-momentum conservation, as well as baryon number conservation. In this work, we will confront experimental data with a fully integrated state-of-the-art theoretical framework. The IP-Glasma model is used to describe the initial energy density distribution [11, 12], which provides realistic event-by-event fluctuations and non-zero pre-equilibrium flow at the early stage of heavy-ion collisions. Individual collision systems are evolved using relativistic hydrodynamics with non-zero shear and bulk viscosities. As the density of the system drops, fluid cells are converted into hadrons and further propagated microscopically using a hadronic cascade model (UrQMD) [13].

A direct experimental study of the hydrodynamical phase of heavy-ion collisions would reveal the QCD physics reflected in the equation of state as well as the transport coefficients, and in particular in the shear and bulk viscosities. However, most observables measured in experiments are sensitive to the details and uncertainties of all the different phases in the evolution of a heavy-ion collision. These uncertainties range from missing knowledge about the initial state, the details of the kinetic phase of the dynamics to the details of the hadronisation phase. This combination of uncertainties makes it very difficult to reliably pin down physics in a specific phase. Still, in recent years the large wealth of experimental data have been used to provide Bayesian estimates for the transport coefficients [14].

In turn, a first principles determination of this QCD input would remove part of the overall uncertainties of the description of the dynamical evolution of a heavy-ion collision. This would give more reliable access to the physics of the other phases such as the initial state. The QCD input for the hydrodynamic simulations is given by the equation of state (EoS) and the transport coefficients. For the hydrodynamical simulations in the present work we use a state-of-the-art EoS that matches the most recent lattice EoS at zero baryon density [3, 4]. This fixes the crossover temperature to \( T_c \approx 155 \) MeV, determined from the reduced quark susceptibilities [15]. These recent lattice simulations of the equation of state show a relatively small statistical and systematical error, and the remaining uncertainty has no significant impact on the results of the hydrodynamical evolution. Accordingly, at zero baryon chemical potential, we consider the QCD EoS input to the hydrodynamical simulation as settled.

The situation is very different for the transport coefficients, which can be computed from equilibrium real-time correlation functions of QCD. In the present work it is the shear viscosity that follows with the Kubo formula from the spectral function of the energy-momentum tensor [16],

\[
\eta = \lim_{\omega \to 0} \frac{1}{20} \frac{\rho_{\pi \pi}(\omega, 0)}{\omega} . \tag{1}
\]
with the spectral function $\rho_{\pi\pi}$ of the spatial, traceless part $\pi_\mu$ of the energy-momentum tensor. As most quantitative first principle approaches to QCD such as the lattice or functional methods work at imaginary time, a numerical Wick rotation to real time is required for getting access to the transport coefficients from these methods. Such numerical analytic continuation methods come with large statistical and in particular systematic uncertainties. These errors grow large in particular for low frequencies which are specifically relevant for the computation of transport coefficients. Consequently lattice results for the shear viscosity are only available for a few temperatures in pure Yang-Mills theory, see e.g. [17–21]. In general, the computation of transport coefficients from imaginary time data of the respective correlation functions from the lattice and functional methods is intricate.

In the present work we utilise data for the shear viscosity over entropy ratio from a functional diagrammatical approach to QCD transport coefficients put forward in [1, 2]. This approach is based on a diagrammatic real-time representation of the correlation functions, and circumvents the continuation problems discussed above. However, it is subject to other systematic errors as it requires the knowledge of real-time correlation functions of quarks and gluons, and in particular the gluon spectral function $\rho_A$. In [1, 2] the latter has been computed via spectral reconstruction from imaginary time data. While at first sight this brings back the continuation error at small frequencies, the loop representation turns the problematic limit for $\rho_{\pi\pi}$ in (1) into a frequency-integral over products of $\rho_A$. Basically it turns the multiplication with $1/\omega$ into a multiplication with $\omega$, hence suppressing the low frequency regime of the reconstructed $\rho_A$ in the computation of $\eta$.

Still the approach has its own systematic uncertainties: the dominating systematic uncertainty in this approach results from the approximate computation of the real-time correlation functions of quarks and gluons, and in particular the well-known problems at higher temperatures. The reconstruction of the relevant thermal part is performed only on the first few Matsubara frequencies. Typically this leads to an unphysical broadening of the spectral function at large temperatures. In the present case this would lead to a smaller thermal slope of $\eta/s(T)$ at large temperatures $T/T_c \geq 1$.

A second relevant source for the systematic uncertainty is the neglect of higher order diagrams. After a re-summation the diagrammatical representation consists of one- to three-loop diagrams of full correlation functions. Note that the loops here have nothing to do with a perturbative ordering. In [1, 2] it has been argued that the higher order diagrams are subject to a very efficient phase space suppression that originates in the quasiparticle structure of the gluon spectral function. In [2] all one- and two-loop diagrams have been computed. The rapid convergence argument has been checked by computing the dominant three-loop diagrams, the total value of which is in the permille regime. However, while the re-summation works very efficiently it also leads to a global normalisation problem also discussed in [22]. It is related to a standard multiplicative renormalisation. It gives rise to the biggest systematic error in this approach, and will be treated in a future publication.

In the present work we use pure glue $\eta/s(T)$ as computed [1, 2]. We estimate the impact of the combined systematic error by $\eta/s(T) \rightarrow \eta/s(T) + \delta d$ with a temperature-independent shift $d$. The result in [2] is parameterised well with

$$\eta/s(T) = \frac{a}{\alpha_{s, HQ}(cT/T_c)} + \frac{b}{(T/T_c)^{\delta}},$$

(2)

where $a = 0.15, b = 0.14, c = 0.66$ and the scaling coefficients $\gamma = 1.6$ and $\delta = 5.1$. The strong coupling in (2) is a heavy quark effective coupling with a simple analytic form [23]

$$\alpha_{s, HQ}(z) = \frac{1}{\beta_0} z^2 - 1 \frac{z^2}{\beta_0} \log z^2.$$  

(3)

We emphasise that the coupling present in the numerical computation in [1, 2] is that of the underlying functional approach to QCD at finite temperature see [22, 24]. The above fit of $\eta/s(T)$ is depicted in the left panel of Fig. 1. It shows a minimum at $T_{\text{min}} \approx 1.26 T_c$ with a value of 0.14. In our simulations we varied this value with the shift $d \in [-0.06, 0]$ between 0.14 and the AdS value 0.08 for an estimate of the impact of the systematic error of $\eta/s$. Moreover, in [2] a QCD estimate has been provided with $a = 0.2, b = 0.15, c = 0.79$. As the effect of changing $\eta/s(T)$ from the pure glue result to the QCD estimate is covered by the uncertainty estimate obtained from varying the shift parameter $d$ we refrain from discussing the related results separately. We have also checked that a multiplication with a temperature-independent factor has an effect similar to such a shift.

It is remarkable that the fit (2) works so well as it is a direct sum of a hadron resonance gas-type behaviour for $T \lesssim T_c$ and a hard-thermal-loop–type behaviour for $T \gtrsim T_c$. Note however that in the latter non-perturbative information is carried by the running coupling. In any case this indicates a rapid transition in the hydrodynamical evolution from the hadronic phase to the quark-gluon phase. This behaviour is also seen for the gluonic Debye mass [24] in the functional approach underlying the computations in [2] as well as further thermal observables. If this rapid transition is also seen for other transport coefficients it would support the common use of simple fits for transport coefficients. The computation and use of other transport coefficients is left to future work. A respective computation of the bulk viscosity is under way, for recent lattice results see [20]. Due to the diagrammatic similarity of the computation, the systematic uncertainties are correlated. Hence, the multiplicative renormalisation factor drops out in the ratio $\zeta/\eta(T)$.

In a previous publication [25] it has been observed that the bulk viscosity $\zeta/s(T)$ is important for describing
the experimentally observed radial flow and azimuthal anisotropy simultaneously. In the absence of first principle results for $\zeta/s(T)$ we take over the same functional form of $\zeta/s(T)$ as in [26]. It is depicted in the right panel of Fig. 1. The bulk viscosity coefficient shows a maximum at $T = 180 \text{ MeV}$ and starts to decrease almost exponentially as the system cools down. At the maximum, the value of $\zeta/s(T)$ is $\approx 0.3$. The coefficient vanishes in the high temperature limit in the QGP phase whereas in the low temperature limit (hadronic gas), it converges to a finite value equal to 0.03.

\begin{align*}
\left. \frac{\mathrm{d}N_i}{\mathrm{d}p^i} \right|_{T} &= \frac{g_i}{(2\pi)^3} \int_{\Sigma} f_i(x, p) p^\mu d^3 \sigma_\mu, \\
\end{align*}

where $f_i$ is the distribution function of particle $i$, which includes both equilibrium and non-equilibrium contributions $f_i = f_{0,i} + \delta f_i$, integrated in a volume element of the switching hypersurface $\Sigma$, defined by the constant switching temperature $T_{\text{switch}}$. The non-equilibrium distribution function is composed by two terms $\delta f = \delta f_{\text{shear}} + \delta f_{\text{bulk}}$. The same functional forms for $\delta f_{\text{shear}}$ and $\delta f_{\text{bulk}}$ as in [25] have been used,

\begin{align*}
\delta f_{\text{shear}} &= f_0(1 \pm f_0) \frac{\pi^{\mu\nu} p_\mu p_\nu}{2T^2 (e + P)}, \\
\delta f_{\text{bulk}} &= f_0(1 \pm f_0) - \frac{\Pi}{\zeta \tau_1} \frac{1}{3T} \left( \frac{m^2}{E} - (1 - 3c^2_s)E \right),
\end{align*}

where $\pi^{\mu\nu}$ is the shear stress tensor, $\Pi$ is the bulk pressure, and $\tau_1$ is the relaxation time for bulk viscosity.

The bulk viscous correction influences the particle spectra and the flow observables when the expansion rate is large. We want to emphasize that the non-equilibrium corrections of the bulk viscosity to the momentum distribution of hadrons at the moment of switching are still not completely understood from a theoretical point of view and represent a large source of uncertainty in the simulation. Specific effects on measured observables from the bulk viscous correction have been discussed in [29].

In this paper, we study multiplicities of different hadron species, $p_T$-differential spectra and various flow observables in Pb–Pb collisions at 2.76 TeV for the centrality intervals 0–5%, 5–10%, 10–20%, 20–30% and 30–40%.

The IP-Glasma model gave a good description of the centrality dependence of the charged hadron multiplicity [12]. It was observed that charged hadron multiplicities at mid-rapidity as a function of collision centrality for Pb–Pb collisions at 2.76 TeV obtained from the simulations are in good agreement with the experimental measurements performed by ALICE in Pb–Pb collisions at 2.76 TeV [30]. In the left panel of Fig. 2 we also compared our results of identified particle yields at mid-rapidity as a function of collision centrality for pions, kaons and protons with the ALICE measurements [31]. Our calculation agrees with the experimental data within the uncertainties and we observed that also the centrality dependence is well reproduced for identified hadrons.

In the right panel of Fig. 2, the comparison of the mean $p_T$, $\langle p_T \rangle$, for pions, kaons and protons as a function of centrality from our simulations is compared with the ALICE experimental measurements [31]. As it was explained in [25, 32], the bulk viscosity and the UrQMD phase play a fundamental role in the simulation. Without them we would not be able to have a good description of the $\langle p_T \rangle$ and of the $p_T$-differential hadron spectra discussed later. The change in $\langle p_T \rangle$ due to the inclusion of the bulk viscosity is significant for all hadron species (pions, kaons, protons) [25]. Bulk viscosity is essential for a simultaneous description of the multiplicity and $\langle p_T \rangle$ of hadrons when IP-Glasma initial conditions are used. Without the bulk viscosity, the system expands too rapidly, leading to a too large hydrodynamic transverse flow. Bulk viscosity improves the agreement with data by acting as a resistance to expansion, reducing the transverse flow of the system. The $\langle p_T \rangle$ of protons agrees with experimental measurements through all centralities, and the values were observed to increase with respect to the results obtained from the simulations in which UrQMD was not used. This reflects that the more massive protons show a large sensitivity of hydrodynamic radial flow as it was already observed in [32, 33]. The effect of hadronic
rescattering is very similar across centrality classes. Although the pion \( \langle p_T \rangle \) is barely affected by the hadronic rescatterings, it is observed that the \( \langle p_T \rangle \) in more central collisions is slightly overestimated in the simulations.

In the bottom panel of Fig. 2 the integrated charged hadron anisotropic flow coefficients, \( v_{2,3,4}\{2\} \), in Pb–Pb collisions at 2.76 TeV are shown as a function of centrality and are compared with the ALICE measurements [34]. We calculate the flow harmonics \( v_n\{2\} \) using the 2-particle cumulant method within \( 0.2 < p_T < 5.0 \) GeV and \(|\eta| < 0.8\), together with a pseudo rapidity gap \(|\Delta\eta| > 1.0\). The band width in the simulation results represents the statistical uncertainty. It is immediately visible that the \( v_n\{2\} \) coefficients are too low in the simulation, especially when moving to more peripheral collision centrality classes. This is because \( \eta/s(T) \) is likely too large in the region below \( T = 200 \) MeV. Temperature dependent \( \eta/s(T) \) values have been studied previously [14, 35–37]. Besides the difference in the detailed shape, our input values used differ from simple parametrizations mainly in the location of the minimum.

The third and fourth flow harmonic coefficients are observed to be even more suppressed with respect to the second one, also in the most central collision events. This is expected because higher harmonics are more sensitive to the shear viscosity and they get reduced more easily if a too large shear viscosity over entropy ratio is used in the simulations [38].

Figure 3 shows the \( p_T \)-differential flow harmonics \( v_n\{2\}\langle p_T \rangle \) (n=2,3,4) of charged hadrons in 0–5% (left)
and 30–40% (right) Pb–Pb collisions at 2.76 TeV computed in our simulation and compared with the experimental measurements done by ALICE [39]. In agreement with what is discussed for the \( p_T \) integrated \( v_n \)\{2\} measurement, we observe that the results of the simulation tend to underestimate the \( v_n \)\{2\}(\( p_T \)) coefficients in the low transverse momentum region, where the majority of particles is produced, while a better description of the experimental measurements is observed for \( p_T \) larger than 1 GeV/c. Also a better description of the measurements is observed in the 5% most central collisions with respect to the more peripheral case.

Beyond studying and predicting the flow observables for charged hadrons, it is important to check the \( p_T \) spectra of identified hadrons since their mass dependence reflects the radial flow of the expanding system. Fig. 4 shows results for pions, kaons, and protons as a function of \( p_T \). In the top left panel the invariant yields for the 5% most central Pb–Pb collisions are shown, while the yields for the 30–40% centrality interval are shown in the top right plot. The calculations, including the hadronic rescattering, agree reasonably well with the measurements. Tension with data appears, and increases slightly in more peripheral collisions, especially for kaons and for low \( p_T \) pions. The better agreement of the protons with respect to the other hadron species is an additional indication that heavier particles show a larger sensitivity to the radial flow introduced by the hadronic rescattering and BB annihilation in the UrQMD phase: the low \( p_T \) region of the spectra is reduced in the transport phase as a results of shifting more protons to higher \( p_T \). The left and right bottom panels of Fig. 4 show the differential flow harmonics \( v_2 \)\{2\}(\( p_T \)) for pions, kaons, and protons in 0–5% and 30–40% Pb–Pb collisions at 2.76 TeV compared with measurements by ALICE using the 2-particle cumulant method within \( |\eta| < 0.8 \) [40]. The interplay of radial and elliptic flow is expected to lead to a dependence of the \( p_T \)-differential flow on the mass of the particle species. A clear mass ordering is observed when comparing \( v_2 \)\{2\}(\( p_T \)) among different particle species. We observe that the hadronic rescattering has a large effect on the proton \( v_2 \)\{2\}(\( p_T \)). In the 30–40% centrality the proton \( v_2 \)\{2\}(\( p_T \)) is pushed to higher \( p_T \) values with respect to what is observed in data. The balance between the radial and elliptic flow seems to be better described in most central collisions. Even though the elliptic flow around the mean-\( p_T \) is well reproduced, our calculations underestimate the \( v_2 \)\{2\}(\( p_T \)) of pions and kaons at low \( p_T \) for more peripheral collisions. The under estimation of the \( v_2 \)\{2\}(\( p_T \)) is due to the important role played by the out-of-equilibrium correction from bulk viscosity and also due to the possibly too strong shear viscosity over entropy ratio, which suppress the flow coefficients.

The upper (lower) panels of Fig. 5 show our calculations for the identified hadron \( v_3 \)\{2\} and \( v_4 \)\{2\} in the two centrality classes compared with the experimental measurements done by ALICE [41]. Note that in the ALICE data the residual non-flow contributions have been subtracted using information from pp collisions. This is not necessary for our calculations since the related non-flow effects are mainly from resonance decays. We observe that the level of agreement with data is slightly worse for \( v_3 \)\{2\} and \( v_4 \)\{2\} compared to what is observed for \( v_2 \)\{2\}. This is because the higher harmonics, as previously explained, are more sensitive to the shear viscosity over entropy density ratio, and they get substantially reduced in case this transport coefficient implemented in the simulation gets too strong. The overall effect of the hadronic rescattering is similar for \( v_3 \)\{2\} and \( v_4 \)\{2\} compared to \( v_2 \)\{2\}. As expected, the usual mass ordering is observed also for \( v_3 \)\{2\} and \( v_4 \)\{2\}.

Let us now come back to the systematic error of \( \eta/s(T) \). As discussed in (2) the largest uncertainties in the theoretical computations in [1, 2] mainly affect the absolute normalization of \( \eta/s(T) \). In order to test the effect of the absolute normalisation of the \( \eta/s(T) \), we shift the whole distribution according to

\[
\eta/s(T) \rightarrow \eta/s(T) + d, \quad d \in [-0.06, 0].
\]

This covers the conjecture of minimal value of \( 1/4\pi = 0.08 \), the AdS/CFT bound [42]. Note however that the AdS/CFT value is only obtained by reducing the value of \( \eta/s(T) \) at the minimum by a factor 1.75, which is a stretch of the theoretical systematic uncertainty.

We first present the effect of a reduced \( \eta/s(T) \) on the integrated charged hadron anisotropic flow coefficients \( v_{2,3,4} \)\{2\} as a function of centrality, since those are the observables that mainly showed a disagreement with the experimental data, pointing to a too strong \( \eta/s(T) \). In Fig. 6 the results coming from the simulations, using both the standard \( \eta/s(T) \) (solid line) and the shifted one (dashed line), in which the shift \( d \) was choosen to be 0.056, are compared with the experimental measurements. It is observed that with the modified \( \eta/s(T) \) the results of our calculations are in agreement, within the statistical uncertainties, with the experimental measurements by ALICE [34]. The multiplicity, the \( \langle p_T \rangle \) as well as the \( p_T \)-differential spectra are not shown here because it is observed that those observables do not change in the simulations, showing less sensitivity to the \( \eta/s(T) \) with respect to the \( v_n \)\{2\} coefficients.

However, it is worth while to compare also the \( p_T \)-differential flow harmonic coefficients for charged hadrons and for pions, kaons and protons. In the top left panel of Fig. 7 our calculations, using both the standard \( \eta/s(T) \) (solid lines) and the modified one (dashed lines), are compared with the ALICE measurements for \( v_{2,3,4} \)\{2\}(\( p_T \)). Statistical uncertainties in the model calculations are not shown in Fig. 7, but they can be judge from the bin-by-bin fluctuations. Although the flow coefficients obtained with the modified \( \eta/s(T) \) deviate from the data at high \( p_T \), those results show a better agreement with data for \( p_T < 1 \) GeV/c, where most particles are produced. We next turn to identified hadron flow coefficients and also here we note that tensions with ALICE measurements are observed at high \( p_T \) for the \( v_{2,3,4} \)\{2\}(\( p_T \)) of pions,
Figure 3. $v_n(2)(p_T)$ ($n=2,3,4$) measured for charged hadrons in the 0–5% (left) and 30–40% (right) centrality intervals in comparison with ALICE measurements [39].

Figure 4. $p_T$ spectra (upper panels) and $v_n(2)(p_T)$ (lower panels) as a function of the transverse momentum of pions, kaons, and protons in comparison with ALICE measurements [31, 40]. Two centrality classes are considered: 0–5% (left) and 30–40% (right).

kaons and protons as shown in the right top panel Fig. 7 for $v_2(2)$, bottom left panel for $v_3(2)$ and bottom right panel for $v_4(2)$. We note that tension with measurements at high $p_T$ is less worrying than in lower regions of transverse momenta, since the high $p_T$ region is more sensitive to uncertainties in the viscous corrections to the
hadron distribution function ($\delta f$ of the shear and of the bulk). We note that the bulk viscosity significantly affects the entire $p_T$ range of $v_n(p_T)$, mainly via the out-of-equilibrium correction to the distribution function $\delta f$. 

Figure 5. $v_2\{3\}(p_T)$ (upper panels) and $v_2\{4\}(p_T)$ (lower panels) as a function of the transverse momentum of pions, kaons, and protons in comparison with the ALICE measurements [41]. Two centrality classes are considered: 0–5% (left) and 30–40% (right). 

Figure 6. $v_n\{2\}$ coefficients as functions of centrality measured with the default and shifted $\eta/s(T)$. 

In this paper, we compared results from a hybrid model of IP-Glasma initial conditions, shear and bulk viscous hydrodynamics (MUSIC), and microscopic hadronic transport (UrQMD) with a wide range of integrated and differential measurements in Pb–Pb collisions at 2.76 TeV. For the first time the shear viscosity over entropy density ratio as a function of temperature from a functional diagrammatical approach to QCD transport coefficients has been used in a state-of-the-art hydrodynamical framework.

Considering the much larger required statistics in the model calculation, we leave the computation of the differential flow harmonics $v_n(2)(p_T)$ of $\Lambda$, $\Xi$, $\Omega$, and $\phi$ [29, 32, 33], to future work. It will be particularly interesting to study the effects of the microscopic hadronic simulation on strange and multi-strange particles. In the hadronic cascade model a small hadronic cross section is assigned to strange hadrons, and in UrQMD the magnitude of the hadronic re-interaction is related to the number of strange quarks contained in the hadron. Hence the amount of radial flow that the hadrons pick up in the hadronic phase depends on their strange quark content. In addition, the prediction for the $\phi$-meson (containing an $s$ and $\bar{s}$ quark) was observed to be almost not affected by the hadronic phase in the UrQMD, and this was leading to the breaking of the mass ordering when comparing $v_2(p_T)$ of the $\phi$-meson and proton [29]. Due to its small hadronic cross section in UrQMD the $\phi$-meson is rather weakly coupled to the hadronic medium and it decouples from the system almost immediately after hadronization.

On the theory side, a threefold way to QCD transport coefficients is currently pursued. Firstly, the non-perturbative diagrammatic computation of transport coefficients [1, 2] is extended to the computation of bulk viscosity and relaxation time. Moreover, the systematic error is reduced due to a refined computation of the gluon and quark spectral functions [43]. Secondly, the transport coefficients are computed via the Kubo formula from lattice results using a novel lattice approach to imaginary time correlation functions [44]. Thirdly, real-time correlation functions and transport coefficients in QCD are directly computed with non-perturbative functional

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{$v_2(n)(p_T)$ (upper left panel) for charged hadrons and $v_2(2)(p_T)$ (upper right panel), $v_2(3)(p_T)$ (lower left panel) and $v_2(4)(p_T)$ (lower right panel) of pions, kaons, and protons. The 30–40% centrality interval is shown for all panels. Both $\eta/s(T)$ are reported.}
\end{figure}
methods. For an application to QCD, real-time functional methods, see e.g. [45–47], have to be extended for a full numerical application. Such an approach has been set-up in [48–50], and is currently applied to QCD spectral functions. This threefold approach is essential for increasing the reliability in the combined results by reducing the combined systematic uncertainty, such as the absolute normalization of the transport coefficients from the diagrammatic approach used in the present work. The approach to direct real-time correlations also allows for the additional treatment of the non-equilibrium corrections to the thermal distribution functions. If known from first QCD principles, this largely reduces the systematic uncertainty of the hydrodynamic description.

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