Chapter 3
Modeling State Interventions

3.1 A Control Theory Model of Disease Control

In the context of the modern nation state, the ecology of infectious diseases cannot be described by interacting populations alone, as much of the modeling literature implicitly presumes (Wallace and Wallace 2016). Modern states incorporate elaborate public health bureaucracies tasked with either containing or eliminating pathogen outbreaks. States are thus highly cognitive entities at the institutional level. It is then appropriate, indeed arguably necessary, to reconsider vector-borne infection from a control theory perspective.

Cognition, following the model of Atlan and Cohen (1998), involves choosing one of many possible responses to a stimulus. Choice reduces uncertainty in a formal manner, a reduction that implies the existence of an information source (Wallace 2015, 2017).

We assume that the underlying two-dimensional disease ecosystem is in explosive mode, i.e., either sudden ecosystem “smoothing” has occurred or a change in socioeconomic policy has imposed cross-effect noise, as in Fig. 2.4 or Eqs. (2.9) and (2.10). Thus the system becomes unstable, and corrective policy must be chosen and imposed, implying not only “control information” exists but that it is effective.

We enter the realm where information and control theories intersect. More precisely, the data rate theorem (DRT) (Nair et al. 2007) establishes the minimum rate at which externally supplied control information must be provided for an inherently unstable system to maintain stability. The first approximation assumes a linear expansion near a nonequilibrium steady state, in which actors of a dynamic system produce each other at a constant distribution. An $n$-dimensional vector of system parameters at time $t$, say $x_t$, determines the state at time $t + 1$ according to the model of Fig. 3.1 as
Fig. 3.1 A linear expansion near a nonequilibrium steady state of an inherently unstable control system, for which \( x_{t+1} = Ax_t + Bu_t + W_t \). \( A, B \) are square matrices, \( x_t \) the vector of system parameters at time \( t \), \( u_t \) the control vector at time \( t \), and \( W_t \) a white noise vector. The data rate theorem states that the minimum rate at which control information must be provided for system stability is \( H > \log|\det(A^m)| \), where \( A^m \) is the subcomponent of \( A \) having eigenvalues \( \geq 1 \). This is interpreted as asserting that the rate of control information must exceed the rate at which the unstable system generates topological information.

\[
x_{t+1} = Ax_t + Bu_t + W_t
\]  

(3.1)

\( A \) and \( B \) are fixed \( n \times n \) matrices, \( u_t \) is the vector of control information, and \( W_t \) is an \( n \)-dimensional vector of white noise. The data rate theorem under such conditions states that the minimum control information rate \( H \) necessary for system stability is determined by the relation

\[
H > \log|\det(A^m)| = a_0
\]  

(3.2)

where, for \( m \leq n \), \( A^m \) is the subcomponent of \( A \) having eigenvalues \( \geq 1 \). The right-hand side of Eq. (3.2) is interpreted as the rate at which the system generates “topological information.”

The essence of the DRT is the onset of instability if the inequality of Eq. (3.2) is violated. Here we will use the rate distortion theorem (RDT) in conjunction with the stochastic self-stabilization theorem to examine in more detail the dynamics leading to control failure under increasing noise.

We examine the manner in which a control signal \( u_t \) emitted by the control source producing information at the rate \( H \) at time \( t \) is expressed in the system response \( x_{t+1} \). Assume it is possible to deterministically retranslate an observed sequence of system outputs \( X^i = x_1^i, x_2^i, \ldots \) into a sequence of possible control signals that is written as \( \hat{U}^i = \hat{u}_0^i, \hat{u}_1^i, \ldots \), and compare that sequence with the original control
sequence $U^i = u^i_0, u^i_1, \ldots$. The difference between them has a particular value under some chosen distortion measure:

$$d(z^n, \hat{z}^n) = \frac{1}{n} \sum_{i=1}^{n} d(z_i, \hat{z}_i)$$

(3.3)

so that the distortion for a sequence is the average of the per symbol distortion of the elements of the sequence. (See Cover and Thomas (2006) for details.)

An average distortion can then be defined as

$$D = \sum_j p(U^j) d(U^j, \hat{U}^j)$$

(3.4)

where $p(U^j)$ is the probability of the sequence $U^j$ and $d(U^j, \hat{U}^j)$ is the distortion between $U^j$ and the sequence of control signals that has been deterministically reconstructed from the system output.

According to the rate distortion theorem, for certain classes of channels, there exists a rate distortion function, $R(D)$, that determines the minimum channel capacity necessary to keep the average distortion below some fixed real number limit $D \geq 0$. Based on Feynman’s (2000) interpretation of information as a form of free energy, it becomes possible to construct a Boltzmann-like pseudo-probability in the “temperature” of the control information rate $H$ as

$$dP(R, \kappa, H) = \frac{\exp[-R/\kappa H]dR}{\int_0^\infty \exp[-R/\kappa H]dR}$$

(3.5)

where the dimensionless quantity $\kappa$ parameterizes the degree to which the control signal produced at the rate $H$ is actually effective. Higher values of $\kappa H$ must necessarily be associated with greater system channel capacity and hence less distortion between what the control instructs and the actual system response.

The denominator can be interpreted as a statistical mechanical partition function, and it becomes possible to define a “free energy” Morse function (Pettini 2007) $F$ as

$$\exp[-F/\kappa H] = \int_0^\infty \exp[-R/\kappa H]dR = \kappa H$$

(3.6)

so that $F(\kappa H) = -\kappa H \log[\kappa H]$.

See Chap. 5 for an introduction to Morse theory.

Then, assuming $H$ is fixed, an “entropy” can be defined as the Legendre transform of $F$ in $\kappa$,

$$S = F(\kappa) - \kappa \frac{\partial F}{\partial \kappa} = \kappa H$$

(3.7)
The Onsager approximation of nonequilibrium thermodynamics (de Groot and Mazur 1984) can now be invoked, based on the gradient of $S$ in $\kappa$ at the fixed rate of control information $H$ so that a stochastic Onsager equation can be written describing the dynamics of $\kappa$ as

$$
d\kappa_i = \mu \partial S / \partial \kappa_i dt + \sigma g(\kappa_i) dW_i = \mu H dt + \sigma g(\kappa_i) dW_i, \quad (3.8)
$$

$\mu$ is a coefficient indexing the attempt by the system – here, the modern state – to penetrate the inevitable “fog-of-war” confusion surrounding, in this case, regulation during normal times and intervention during an emergency. The second term, in $\sigma g(\kappa_i)$, is a white noise volatility measure, representing the density of that fog. Recall that larger $\kappa H$ will be associated with higher $R$ and hence smaller distortion between intent and effect.

We have reduced the two-dimensional vector-borne pandemic to a one-dimensional control theory problem that falls under an inversion of the simplest version of the stochastic stabilization theorem, i.e., sufficiently large system “noise” $\sigma$ can drive $\kappa$ to zero, a converse of the previous argument, as small $\kappa H$ will violate the strictures of the DRT, releasing the pandemic outbreak.

The appearance of the log term in Eq. (3.2) suggests the possibility of a nuanced — if heuristic — approach that applies the Itô chain rule to $\log[\kappa]$ using Eq. (3.8), providing a more detailed picture of system failure. Since the log is a concave function, Jensen’s inequality (Cover and Thomas 2006) implies that, in terms of the expectation $E$, we again have

$$
\log \left[ E \left( x \right) \right] \geq E \left( \log [x] \right) \quad (3.9)
$$

so the procedure produces a lower limit for the expectation of $\kappa$, remembering that sufficient noise ‘tests the limit’, driving it essentially to zero.

We assume $g(\kappa) = \sqrt{\kappa^2 + \alpha^2}$ in Eq. (3.8), so that there is residual volatility even for small $\kappa$. This produces the stochastic differential equation:

$$
d\kappa_i = \mu H dt + \sigma \sqrt{\kappa^2 + \alpha^2} dW_i, \quad (3.10)
$$

Expanding $\log[\kappa_i]$ using the Itô chain rule gives a relation

$$
d\kappa(t) / dt = \mu H - \frac{1}{2} \kappa(t) \sigma^2 - \frac{1}{2} \frac{\sigma^2 \alpha^2}{\kappa(t)} \quad (3.11)
$$

where the terms in $\sigma$ represent the typical Itô correction factor. Recall that $d \log(\kappa) / dt = (1/\kappa) dx/dt$. We are, then, calculating the dynamics of $E(\log[\kappa])$ and making a heuristic application of Jensen’s inequality to estimate a lower limit on $E(\kappa)$.

There are two nonequilibrium steady-state (nss) solutions characterizing lower-limit dynamics. Some exploration using the DEploy tool of the computer...
algebra system Maple shows the larger solution is stable but declines with increasing $\sigma$. The smaller solution increases monotonically with $\sigma$ but is not stable, either collapsing to zero or rising to the larger NSS. The expectations satisfy inequalities

$$E(\kappa_{La}) \geq \frac{\mu H + \sqrt{H^2 \mu^2 - \alpha^2 \sigma^2}}{\sigma^2} \quad (3.12)$$

$$E(\kappa_{Sm}) \geq \frac{\mu H - \sqrt{H^2 \mu^2 - \alpha^2 \sigma^2}}{\sigma^2}$$

This result represents a parsing of stability conditions beyond the stochastic stability theorem. However, if $H^2 \mu < \alpha^2 \sigma^2$, the two branches coalesce, and no stability is possible. Thus we recover a further necessary condition for stability as

$$H > \frac{\alpha \sigma^2}{\mu} \quad (3.13)$$

Violation of this condition drives the “effectiveness” parameter $\kappa$ to zero, suggesting, in the next section, another iteration of the data rate theorem.

3.2 A Cognitive Model of Disease Control

The data rate theorem argument we presented in the previous section suggests a greater probability that a stabilized pathogen system will transition to unstable behavior if the temperature analog $\kappa H$ falls below a critical value.

We can extend the perspective to more complicated patterns of phase transition via the “cognitive paradigm” of Atlan and Cohen (1998), under which a system, such as the modern nation state exercising public health power, is considered cognitive if it compares incoming signals with a learned or inherited picture of the world, then actively chooses a response from a larger set. Choice, as we described in the previous section, implies the existence of an information source, as it reduces uncertainty in a formal way (Wallace 2015, 2017).

In the section above, we introduced the notion government public health policy engages in such group cognition. Given such a “dual” information source – government vs disease – associated with such an inherently unstable cognitive system of interest, an equivalence class algebra can be constructed by choosing different system origin states and defining the equivalence of subsequent states at a later time by the existence of a high-probability path connecting them to the same origin state.

Disjoint partition by equivalence class, analogous to orbit equivalence classes in dynamical systems, defines a symmetry groupoid associated with the cognitive process (Wallace 2015, 2017). Groupoids are generalizations of group symmetries.
in which there is not necessarily a product defined for each possible element pair (Weinstein 1996). An example would be the disjoint union of different groups. See Chap. 5 for an introduction to groupoid theory.

Nation states are comprised of functional nodes – different agencies, different competing interests – which must connect and collaborate to solve a societal problem or remain disconnected and fail to act (Chappell 2018). How the nodes connect determines whether group cognition can be turned onto a public health problem.

The equivalence classes across possible origin states define a set of information sources dual to different cognitive states available to the inherently unstable cognitive system. These create a large groupoid, with each orbit corresponding to a transitive groupoid, whose disjoint union is the full groupoid. Each subgroupoid is associated with its own dual information source, and larger groupoids must have richer dual information sources than smaller.

Let \( X_{G_i} \) be the system’s dual information source associated with groupoid element \( G_i \). We next construct a Morse function (Pettini 2007) in a standard manner, using \( T \equiv \kappa H \) as a temperature analog.

Let \( H(X_{G_i}) \equiv H_{G_i} \) be the Shannon uncertainty of the information source associated with the groupoid element \( G_i \). Define another Boltzmann-like pseudo-probability as

\[
P[H_{G_i}] = \frac{\exp[-H_{G_i}/T]}{\sum_j \exp[-H_{G_j}/T]} \quad (3.14)
\]

where the sum is over the different possible cognitive modes of the full system.

Another “free-energy” Morse function \( F \) can then be defined as

\[
\exp[-F/T] = \sum_j \exp[-H_{G_j}/T]
\]

\[
F = -T \log \left[ \sum_j \exp[-H_{G_j}/T] \right] \quad (3.15)
\]

As a consequence of the underlying groupoid-generalized symmetries associated with high-order cognition, as opposed to simple control theory, it is possible to apply an extension of Landau’s version of phase transition (Pettini 2007). Landau argued that spontaneous symmetry breaking of a group structure represents phase change in physical systems, with the higher energies available at higher temperatures being more symmetric. The shift between symmetries is highly punctuated in the “temperature” index \( T = \kappa H \) – but in the context of groupoid rather than group symmetries.

Based on the analogy with physical systems, there should be only a few possible phases, with highly punctuated transitions between them as the fog-of-war
“temperature” $T$ decreases, either as the effects of “friction” become manifest, decreasing $\kappa$, or by decline in the rate of control information $H$.

For nonergodic systems the groupoids become trivial, associated with the individual high-probability paths for which an $H$-value may be defined, although that cannot be represented in the form of a Shannon “entropy” (Khinchin 1957, p. 72).

The policy implications orbit about the institutional control parameter $\kappa H$. Minimal erosion in the parameter – a variance in regulation here, a corrupt blind eye there, tolerating some minor intimidation of local dissent or an outright campaign of murdering environmental activists (Global Witness 2014) – can trigger a very sudden collapse in disease control. Under such a regime, enforcement efforts, once the disease carrier – here a mosquito population – has escaped, need to be ramped up to an extreme level just to get atop a spreading outbreak that has broken across thresholds in populations infected and geographic space. Such punctuated dynamics may well involve considerable hysteresis, the complication of long-trailing effects across the system, once initial barriers to infection fall.

The implication here is that good governance, protected agricultural commons, farmer autonomy, conservation agroecology, urban public health, community resilience, and biocontrol are foundationally integrated (Hanspach et al. 2017; Rotz and Fraser 2015; Wallace et al. 2018).

### 3.3 A Ratchet Mechanism

Disease outbreaks are rarely about the pathogen or clinical outcomes alone in cause or effect. An outbreak can drive the very conditions that brought about its initial emergence into a new phase or degrade a government’s capacity to respond.

Recall the derivation of Eq. (3.6), in terms of the rate distortion function of the channel connecting the controller with the system under control:

$$
\exp[-F / \kappa H] = \int_0^\infty \exp[-R / \kappa H] dR = \kappa H \equiv T
$$

We suppose that $T$ is fixed, but $T \to T + \Delta, 0 \leq \Delta \ll T$. Thus

$$
\exp[-F / (T + \Delta)] = T + \Delta
$$

and we can define a new “entropy” as $S = F(\Delta) - \Delta \partial F / \partial \Delta$ leading to the stochastic differential equation:

$$
d\Delta = \frac{\mu \Delta}{T + \Delta} dt + \sigma \Delta dW_t \approx \frac{\mu}{T} \Delta dt + \sigma \Delta dW_t
$$

(3.18)
where $\mu$ is a new ‘diffusion coefficient’ and $\sigma$ a new ‘noise’ parameter and $dW_t$ represents the usual white noise, and we have used the condition that $0 \leq \Delta \ll T$.

Applying the stochastic stabilization theorem,

$$\lim_{t \to \infty} \frac{\log|\Delta|}{t} \to 0$$

(3.19)

unless

$$\frac{\mu}{T} > \frac{1}{2}\sigma^2$$

$$T < \frac{2\mu}{\sigma^2}$$

(3.20)

This places an upper limit on $T = \kappa H$. More importantly, it establishes the possibility of a public health analog to an economic ratchet.

Suppose “structural adjustment” or other neoliberal policy triggers a reduction in the ability of a polity to implement public health measures, so that $T$ declines in the face of endemic or episodic disease. The resulting rise in disease prevalence and incidence then causes social disintegration that increases $\sigma$ so that the disease outbreak itself interferes with the ability to carry out needed public health interventions. $T$ then declines even further, $\sigma$ rises again, the outbreak becomes more severe, and a race to the bottom ensues. Hemorrhagic fevers, SARS, and MERS-CoV offer classic nosocomial examples, but such ratchets can extend beyond the hospital setting and across the social fabric, from mosquito control to the very function of the state (Ho et al. 2003; De Waal 2003; Fisher-Hoch 2005; Whiteford and Hill 2005; Shin et al. 2017).

### 3.4 Effectiveness, Efficiency, and Their Synergism

The argument above can be made more precise by abducting an approach from the Arrhenius treatment of reaction rates, where the rate at which resources are provided is the temperature analog. Using this method we can examine the classic conflict between efficiency and effectiveness so often commented on in the management science literature. In addition, we can study their synergistic interaction.

“Public health” is the result of collaboration across a number of institutional entities in the control theory sense leading to Eqs. (3.5) and (3.6), indexed as $j = 1 \ldots n$. Each entity consumes resources at some rate $M_j$ under an overall constraint $M - \sum_j M_j = 0$, and we are interested in the response rate of each entity, above some “action trigger threshold.” For such entities the response rate will be proportional to the probability that the channel rate distortion function connecting entity to outcome is greater than some “action threshold” $R_j^0$.
The effectiveness and efficiency of a particular entity can then be expressed as the two quantities

\[ \exp\left[-R_j^0 / M_j \right] \]
\[ \exp\left[-R_j^0 / M_j \right] / M_j \]  

(3.22)

Short-term goals in public health surround effectiveness, while long-term plans are confronted with the need to maximize efficiency, i.e., the expression

\[ \sum_j \exp\left[-R_j^0 / M_j \right] / M_j \] under the overall constraint on resources provided to public health measures, \( M - \sum_j M_j = 0 \).

Paying dues to the economists, we first examine efficiency. Let

\[ L \equiv \sum_j \frac{\exp\left[-R_j^0 / M_j \right]}{M_j} + \lambda \left(M - \sum_j M_j\right) \]  

(3.23)

where \( \lambda \) is the Lagrange undetermined multiplier.

The gradient equations determining the maximum of efficiency under the resource constraint are then

\[ \frac{R_j^0 \exp\left[-R_j^0 / M_j \right]}{M_j^2} - \frac{\exp\left[-R_j^0 / M_j \right]}{M_j} = \lambda \]

\[ M = \sum_j M_j \]  

\[ \partial L / \partial M = \lambda \]  

(3.24)

where, abducting arguments from physical theory, \( \lambda \) is taken as an inverse response temperature. Figure 3.2 shows a single term for \( R_j^0 = 0.5 \) over the range \( 0 \leq M \leq 1 \).

It is easy to show that \( \lambda = 0 \) for \( M_j = R_j^0 \).

For negative response temperature, i.e., \( \partial L / \partial M = \lambda < 0 \), the individual \( M_j \) can become unconstrained, closely analogous to the excited state of a “pumped” physical system like a laser. As a consequence, if the disease outbreak “gets inside the command decision loop” of the bureaucratic entities constituting the “public health” response, so that \( \lambda \) becomes negative, then resource demands cannot be met under
the constraint relation, and the infection outbreak will proliferate until it burns out or becomes high-level endemic.

Efficiency is only one aspect of public health intervention. In the face of a spreading pandemic with high morbidity and mortality, effectiveness becomes paramount. Then we must optimize

\[
L = \sum_j \exp\left(-R_j^0 / M_j\right) - \lambda \left[\sum_j M_j - M\right]
\]

leading to the relation \(R_j^0 \exp[-R_j^0 / M_j] / M_j^2 = \lambda\) as shown in Fig. 3.3. Here, \(\lambda\) is never negative, but small values imply unconstrained demand for resource \(M_j\), an impossible condition.

Again, while Fig. 3.2 may apply to long-time strategic time scales, Fig. 3.3 is more appropriate to tactical “do-or-die” time frames.

More generally, strategy, the long-time frame, and tactics, the immediate challenges, will become synergistic, leading to optimization of the product term:

\[
\frac{\exp[-R_j^0 / M_j]}{M_j} \exp[-R_j^0 / M_j] - \frac{\exp[-2R_j^0 / M_j]}{M_j}
\]

\[\text{(3.26)}\]
This gives a functional form exactly like Fig. 3.2 but with the node $\lambda = 0$ at $2R_j^0$ instead of $R_j^0$.

The implication, at both tactical and strategic scales, is that sufficient “structural adjustment” or unconstrained agroecological exploitation in the face of stagnant “public health” resources can allow disease outbreaks to proceed more rapidly than a weakened state bureaucracy can respond. It appears an obvious point often lost: diseases are more than objects on which we act. Some are capable of degrading the very public health capacity they attract.

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