Threshold Resummation Effects in the Associated Production of Chargino and Neutralino at Hadron Colliders

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Abstract

We investigate the QCD effects in the associated production of the chargino and the neutralino, \( \tilde{\chi}^\pm_1 \) and \( \tilde{\chi}^0_2 \), in the Minimal Supersymmetric Standard Model (MSSM) at both the Fermilab Tevatron and the CERN Large Hadron Collider (LHC). We include the next-to-leading order (NLO) QCD corrections (including supersymmetric QCD) and the threshold resummation effects. Our results show that, compared to the NLO predictions, the threshold resummation effects can increase the total cross sections by 3.6% and 3.9% for the associated production of \( \tilde{\chi}^+_1 \tilde{\chi}^0_2 \) and \( \tilde{\chi}^-_1 \tilde{\chi}^0_2 \) at the LHC, respectively, and by 4.7% for those of \( \tilde{\chi}^+_1 \tilde{\chi}^0_2 \) at the Tevatron. In the invariant mass distributions the resummation effects are significant for large invariant mass. The threshold resummation reduces the dependence of the total cross sections at the LHC (Tevatron) on the renormalization/factorization scales to 5% (4%) from up to 7% (11%) at NLO.

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I. INTRODUCTION

The search for new physics beyond the Standard Model (SM), especially supersymmetry (SUSY), is one of the objectives at the CERN Large Hadron Collider (LHC). Many calculations have been carried out based on the Minimal Supersymmetric Standard Model (MSSM), a version of SUSY. Phenomenologically SUSY predicts many new particles; e.g., the superpartners of the SM particles. Specifically, in the MSSM, there are squarks, gluino, sleptons, charginos, neutralinos and more Higgs bosons in addition to the SM particles. Besides squarks and gluinos, perhaps the most interesting new particles are the four neutralinos $\tilde{\chi}_i^0 (i = 1, 2, 3, 4)$ and the two charginos $\tilde{\chi}_j^\pm (j = 1, 2)$, which are the mass eigenstates of the superpartners of the Higgs and gauge bosons, since the lightest chargino $\tilde{\chi}_1^\pm$ and the two lightest neutralinos ($\tilde{\chi}_1^0, \tilde{\chi}_2^0$) can be lighter than the squarks and the gluino in most of the parameter space. In most of the MSSM parameter regions the associated production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ is the main source of trilepton events. In Refs. [1, 2] the trilepton signal was investigated for the Fermilab Tevatron in the Minimal Supergravity Model (mSUGRA) at leading order (LO). If the leptonic decays of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ are the dominant decay modes the signal to background ratio can be quite large after suitable cuts. Because the trilepton signal is also quite sensitive to the SUSY parameters, it is potentially also a sensitive probe of the SUSY parameters. In fact, the trilepton signal from $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ is now being searched for by the D0 Collaboration [3] at the Tevatron. So far no excess has been observed above the expected SM background, but the results have been used to constrain the masses. A plan for searching for the trilepton signal from chargino and neutralino has also been presented by the Compact Muon Solenoid (CMS) Collaboration [4] at the LHC. Therefore, high precision theoretical predictions for the associated production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ are very important for the forthcoming experiments at the LHC.

The next-to-leading order (NLO) SUSY QCD corrections to the process $pp \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0$ in mSUGRA was first investigated in Refs. [5, 6] where infrared singularities were dealt with using the dipole subtraction method [7]. Also, the NLO SUSY QCD and SUSY electroweak (EW) corrections to this process in the general MSSM were calculated in Ref. [8]. In the following we further investigate the QCD effects on this process, including the NLO SUSY QCD corrections and, in addition, the next-to-leading-logarithmic (NLL) threshold resummation effects in mSUGRA using the most recent SM parameters [9, 10] at both the Tevatron...
and the LHC.

The paper is organized as follows: In Sec. II we present the analytic results at fixed order. In Sec. III we briefly summarize the threshold resummation formalism and derive the expressions for the resummed cross sections. In Sec. IV the numerical results are presented and discussed. Sec. V contains a brief summary of the conclusions. The SUSY vertexes involved in our calculations are summarized in Appendix A. The abbreviations for the Passarino-Veltaman integrals are defined in Appendix B. The standard matrix elements and the explicit expressions for the form factors are summarized in Appendix C.

II. CALCULATIONS AT FIXED ORDER

For hadron colliders the total cross section for the hadronic process,

\[ A + B \rightarrow \tilde{\chi}_1^\pm + \tilde{\chi}_2^0 + X, \]

(1)
can be factorized into the convolution of the parton distribution functions and the parton cross section,

\[ \sigma(S) = \sum_{a,b} \int dx_a\,dx_b\, f_{a/A}(x_a,\mu_f) f_{b/B}(x_b,\mu_f) \hat{\sigma}_{ab}(\hat{s} = x_a x_b S, \alpha_s), \]

(2)

where \( \mu_f \) is the factorization scale, \( f(x,\mu_f) \) is the parton distribution function (PDF) and \( \hat{s} \) is the parton center of mass energy. A and B both refer to protons at the LHC and proton and antiproton at the Tevatron, respectively. The parton cross section \( \hat{\sigma} \) is given by

\[ \hat{\sigma} = \frac{1}{2\hat{s}} \int \sum |\mathcal{M}|^2 dPS^{(n)}, \]

(3)

where \( \sum \) indicates the summation over final states and the average over initial states and \( \int dPS^{(n)} \) represents the phase space integration.

For simplicity, in this section we only present the expressions for the subprocess

\[ u + \bar{d} \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_2^0. \]

(4)

The other processes are given by similar expressions.

A. LEADING ORDER CALCULATION

The LO Feynman diagrams are shown in Fig.1. Considering the light quarks as massless, at LO the production of \( \tilde{\chi}_1^+ \tilde{\chi}_2^0 \) proceeds mainly via an s-channel exchange of a W boson, a
FIG. 1: The tree level Feynman diagrams for chargino and neutralino associated production.

t-channel exchange of a down-type squark, and a u-channel exchange of an up-type squark.

The LO amplitude is

\[ \mathcal{M}_0 = \mathcal{M}_{s0} + \sum_{k=1}^2 (\mathcal{M}_{t0}^k + \mathcal{M}_{u0}^k), \]

with

\[ \mathcal{M}_{s0} = -\frac{D_L (A_R M_{1R} + A_L M_{1L})}{\hat{s} - M_W^2}, \]

\[ \mathcal{M}_{t0}^k = \frac{(M_{2k}^{LR} a_{k2}^d + M_{2k}^{LL} b_{k2}^d) C_k^U}{\hat{t} - M_{d_k}^2}; \]

and

\[ \mathcal{M}_{u0}^k = \frac{(M_{3k}^{RL} a_{k2}^u + M_{3k}^{RR} b_{k2}^u) C_k^V}{\hat{u} - M_{u_k}^2}, \]

where \( D_L \equiv g_W v_u / \sqrt{2}, A_L, A_R, a_{k2}^d, b_{k2}^d, a_{k2}^u, b_{k2}^u, C_k^U \) and \( C_k^V \) are the coefficients appearing in the SUSY couplings and their explicit expressions are given in Appendix A. The standard matrix elements \( M_n^{ab} \) are given in Appendix C. The LO amplitude and all of the NLO calculations in this paper are carried out in t’Hooft-Feynman gauge. \( \hat{s}, \hat{t} \) and \( \hat{u} \) are the Mandelstam variables defined as

\[ \hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 - p_3)^2, \quad \hat{u} = (p_1 - p_4)^2. \]

In order to simplify the expressions, we further introduce the following modified Mandelstam variables:

\[ \hat{t}' = \hat{t} - M_{\tilde{\chi}_1^+}^2, \quad \hat{u}' = \hat{u} - M_{\tilde{\chi}_1^0}^2. \]

After the \( D \)-dimensional phase space integration, the LO parton differential cross sections are given by

\[ \frac{d^2 \hat{\sigma}^B}{d\hat{t}' d\hat{u}'} = \frac{1}{16\pi \hat{s}^2 \Gamma(1 - \epsilon)} \left( \frac{4\pi \mu^2 \hat{s}}{\hat{t}' \hat{u}' - \hat{s} M_{\tilde{\chi}_1^+}^2} \right)^\epsilon \Theta(\hat{t}' \hat{u}' - \hat{s} M_{\tilde{\chi}_1^+}^2) \Theta[\hat{s} - (M_{\tilde{\chi}_1^+} + M_{\tilde{\chi}_2^0})^2] \]

\[ \times \delta(\hat{s} + \hat{t} + \hat{u} - M_{\tilde{\chi}_1^+}^2 - M_{\tilde{\chi}_2^0}^2) \sum |\mathcal{M}_0|^2, \]
where $\epsilon = (4 - D)/2$ and the $\Theta$ function is the Heaviside step function. The explicit expression for $\sum |M_0|^2$ is

$$
\sum |M_0|^2 = \frac{1}{6} \left\{ - \sum_{r,s=1}^2 \frac{2 \delta M_{\chi_1 x}^2 M_{\chi_2 x}^2 a_r a_s C_U^r C_V^s}{(\hat{t} - M_{\chi_1}^2)(\hat{u} - M_{\chi_2}^2)} + \sum_{s=1}^2 \left[ (\hat{a}_s^2)^2 + (\hat{b}_s^2)^2 (C_U^s)^2 (\hat{t} - M_{\chi_1}^2)(\hat{t} - M_{\chi_2}^2) \right] (\hat{u} - M_{\chi_2}^2)^2 \right. 
$$

$$
+ \sum_{s=1}^2 \left[ (\hat{a}_s^2)^2 + (\hat{b}_s^2)^2 (C_U^s)^2 (\hat{t} - M_{\chi_1}^2)(\hat{u} - M_{\chi_2}^2) \right] (\hat{u} - M_{\chi_2}^2)^2 \right. 
$$

$$
+ \frac{2 D_L A_R (\hat{t} - M_{\chi_1}^2)(\hat{t} - M_{\chi_2}^2)}{\hat{s} - M_W^2} \sum_{s=1}^2 \left( \hat{t} - M_{\chi_1}^2 \right) - \frac{2 D_L A_R (\hat{t} - M_{\chi_1}^2)(\hat{t} - M_{\chi_2}^2)}{\hat{s} - M_W^2} \sum_{s=1}^2 \left( \hat{u} - M_{\chi_2}^2 \right) 
$$

$$
+ \frac{2 C_U^0 (\hat{t} - M_{\chi_1}^2)(\hat{t} - M_{\chi_2}^2)}{\hat{s} - M_W^2} \sum_{s=1}^2 \left( \hat{t} - M_{\chi_1}^2 \right) + \frac{2 C_U^0 (\hat{t} - M_{\chi_1}^2)(\hat{t} - M_{\chi_2}^2)}{\hat{s} - M_W^2} \sum_{s=1}^2 \left( \hat{u} - M_{\chi_2}^2 \right) 
$$

$$
+ \frac{A_R^0 D_L^0 (\hat{t} - M_{\chi_1}^2)(\hat{t} - M_{\chi_2}^2) + A_L D_L^0 (\hat{t} - M_{\chi_1}^2)(\hat{t} - M_{\chi_2}^2)}{\hat{s} - M_W^2} \sum_{s=1}^2 \left( \hat{t} - M_{\chi_1}^2 \right) + \frac{8 A_L A_R D_L^0 \delta M_{\chi_1}^2 M_{\chi_2}^2}{\hat{s} - M_W^2} \sum_{s=1}^2 \left( \hat{t} - M_{\chi_1}^2 \right) \right\}. 
$$

The LO total cross section at the hadron colliders is obtained by convoluting the parton cross section with the PDFs in the hadrons A and B:

$$
\sigma^B = \int d\chi_1 d\chi_2 [f_{u/A}(\chi_1, \mu_f) f_{\bar{d}/B}(\chi_2, \mu_f) + (A \leftrightarrow B)] \hat{\sigma}^B, 
$$

where $\hat{\sigma}^B$ is the Born cross section for $u \bar{d} \rightarrow \chi_1^+ \chi_2^-$. Obviously, the LO results are finite and free of singularities.

**B. NEXT-TO-LEADING ORDER CALCULATION**

The NLO QCD (including SUSY QCD) corrections for the production of $\chi_1^\pm \chi_2^\mp$ consist of the virtual corrections, generated by loop diagrams of colored particles, and the real corrections with the radiation of a real gluon or a massless (anti)quark. For both virtual and real corrections, we will first give the results in the dimensional regularization scheme (DREG)\[11\], in which, to restore supersymmetry, we modify the Yukawa coupling at the one loop level \[\[5, 6, 12, 13\] :

$$
g_W(q\bar{q}\chi) = g_W(1 - \frac{\alpha_s}{6\pi}).
$$

We will show the results in the dimensional reduction scheme (DRED)\[14\] and compare the two schemes.
1. VIRTUAL CORRECTIONS

The Feynman diagrams for the virtual corrections are shown in Fig. 2. In the calculations of the virtual corrections we used the computer program package FormCalc [15] to generate the one loop amplitudes and the self energies. The unrenormalized amplitudes for the virtual corrections are given by

\[ \mathcal{M}_V = \sum_{n=1}^{24} \sum_{a,b=L,R} (f_{QCDVn}^{ab} + f_{SUSYVn}^{ab}) M_n^{ab}, \]  

where the explicit expressions for the standard matrix elements \( M_n^{ab} \) and the form factors \( f_{QCDVn}^{ab} \) and \( f_{SUSYVn}^{ab} \) are given in Appendix C. The ultraviolet (UV) divergence in the amplitude for the QCD corrections can be expressed as

\[ \mathcal{M}_V^{QCD}_{UV} = \frac{\alpha_s}{4\pi} \frac{C_F}{4\pi} \left\{ \mathcal{M}_{s0} + \sum_{k=1}^{2} M_{l0}^k \frac{1}{t - M_{d_k}^2} \left\{ -\sum_{r=1}^{2} S_{k r}^{\ddagger} S_{k r} M_{u r}^2 + 4M_g^2 - 2t \right\} \right. 
\]

\[ + \left. \sum_{k=1}^{2} \mathcal{M}_{u0}^k \frac{1}{\bar{u} - M_{u_k}^2} \left\{ -\sum_{r=1}^{2} S_{k s}^{\ddagger} S_{k s} M_{u s}^2 + 4M_g^2 - 2\bar{u} \right\} \right\}, \]

and the UV divergence in the amplitude for the SUSY QCD corrections is

\[ \mathcal{M}_V^{SUSY}_{UV} = \frac{\alpha_s C_F}{4\pi} \left\{ \mathcal{M}_{s0} + \sum_{k=1}^{2} M_{l0}^k \frac{1}{t - M_{d_k}^2} \left[ -\sum_{r=1}^{2} S_{k r}^{\ddagger} S_{k r} M_{u r}^2 + 4M_g^2 - 2t \right] \right. 
\]

\[ - \left. \sum_{k=1}^{2} \sum_{r=1}^{2} C_{l r}^c (M_{l}^{R})_s^a \frac{q_{k s}^{\ddagger} q_{k s} M_{u s}^2}{(t - M_{d_k}^2)(\bar{u} - M_{d_k}^2)} \sum_{s=1}^{2} S_{k s}^{\ddagger} S_{k s} M_{u s}^2 \right\}, \]  

where \( C_F = 4/3 \) and \( S_{ij}^{\ddagger} = R_{i1}^{\ddagger} R_{j1}^{\ddagger} - R_{i2}^{\ddagger} R_{j2}^{\ddagger} \). \( R^{\ddagger} \) is the 2 \times 2 matrix shown below, and is defined to transform the squark \( \tilde{q} \) current eigenstates to the mass eigenstates:

\[ \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = R^{\ddagger} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}, \quad R^{\ddagger} = \begin{pmatrix} \cos \theta_{\tilde{q}} & \sin \theta_{\tilde{q}} \\ -\sin \theta_{\tilde{q}} & \cos \theta_{\tilde{q}} \end{pmatrix}, \]

with \( 0 \leq \theta_{\tilde{q}} < \pi \), by convention. Correspondingly, the mass eigenstates \( M_{\tilde{q}_1} \) and \( M_{\tilde{q}_2} \) (with \( M_{\tilde{q}_1} \leq M_{\tilde{q}_2} \)) are given by

\[ \begin{pmatrix} M_{\tilde{q}_1}^2 & 0 \\ 0 & M_{\tilde{q}_2}^2 \end{pmatrix} = R^{\ddagger} M_{\tilde{q}}^2 (R^{\ddagger})^\dagger, \]  

with 

\[ 0 \leq \theta_{\tilde{q}} < \pi, \] by convention. Correspondingly, the mass eigenstates \( M_{\tilde{q}_1} \) and \( M_{\tilde{q}_2} \) (with \( M_{\tilde{q}_1} \leq M_{\tilde{q}_2} \)) are given by

\[ \begin{pmatrix} M_{\tilde{q}_1}^2 & 0 \\ 0 & M_{\tilde{q}_2}^2 \end{pmatrix} = R^{\ddagger} M_{\tilde{q}}^2 (R^{\ddagger})^\dagger, \]  

with 

\[ 0 \leq \theta_{\tilde{q}} < \pi, \] by convention.
\[ \tilde{M}_q^2 = \begin{pmatrix} M_{\tilde{q}L}^2 & a_q M_q \\ a_q M_q & M_{\tilde{q}R}^2 \end{pmatrix}, \tag{20} \]

with

\[ M_{\tilde{q}L}^2 = M_Q^2 + M_q^2 + M_Z^2 \cos 2\beta (f_{3L}^2 - e_q \sin^2 \theta_W), \tag{21} \]

\[ M_{\tilde{q}R}^2 = M_D^2 + M_q^2 + M_Z^2 \cos 2\beta e_q \sin^2 \theta_W, \tag{22} \]

\[ a_q = A_q - \mu \tan \beta. \tag{23} \]

FIG. 2: The Feynman diagrams for the virtual corrections to chargino and neutralino associated production.
Here $\hat{M}_q^2$ is the squark mass matrix. $M_{\tilde{q},\tilde{\tilde{q}}}$ and $A_q$ are soft SUSY breaking parameters and $\mu$ is the Higgsino mass parameter. $I^q_{3L}$ and $e_q$ are the third component of the weak isospin and the electric charge of the quark $q$, respectively.

In order to remove the UV divergences above we renormalized the wave functions of the (s)quarks and the masses of squarks, adopting the on-shell renormalization scheme [16]. And the squark mixing matrix must also be renormalized. Denoting $M_{\tilde{q},0}$, $\tilde{q}_s$ and $q_0$ as the bare squark mass, the bare squark wave function, and the bare quark wave function, respectively, the relevant renormalization constants are then defined as

$$M_{\tilde{q},0}^2 = M_{\tilde{q}}^2 + \delta M_{\tilde{q}}^2,$$

$$\tilde{q}_s = (1 + \frac{1}{2} \delta \tilde{Z}_{ss}^q) \tilde{q}_s + \frac{1}{2} \delta \tilde{Z}_{sr}^q \tilde{q}_r,$$

and

$$q_0 = (1 + \frac{1}{2} \delta Z_L^q P_L + \frac{1}{2} \delta Z_R^q P_R) q.$$ 

After calculating the self energy diagrams in Fig.2, we obtain the explicit expressions for the above renormalization constants:

$$\delta M_{\tilde{q}}^2 = - \frac{\alpha_s C_F}{4\pi} \left\{ A_0(M_{\tilde{q}}^2) + 4M_{\tilde{q}}^2(B_0 + B_1)(M_{\tilde{q}}^2, 0, M_{\tilde{q}}^2) + 4A_0(M_{\tilde{g}}^2) + 4M_{\tilde{q}}^2 B_1(M_{\tilde{q}}^2, 0, M_{\tilde{g}}^2) - \sum_{k=1}^2 S_{sk}^q S_{ks}^q A_0(M_{\tilde{q}}^2) \right\},$$

$$\delta \tilde{Z}_{ss}^q = \frac{\alpha_s C_F}{\pi} \left\{ [B_0 + B_1 + M_{\tilde{q}}^2 (B_0' + B_1')](M_{\tilde{q}}^2, 0, M_{\tilde{q}}^2) + (B_1 + M_{\tilde{q}}^2 B_1')(M_{\tilde{q}}^2, 0, M_{\tilde{g}}^2) \right\},$$

$$\delta \tilde{Z}_{sr}^q = \frac{\alpha_s C_F}{2\pi (M_{\tilde{q}}^2 - M_{\tilde{g}}^2)} \sum_{k=1}^2 S_{sk}^q S_{kr}^q A_0(M_{\tilde{q}}^2), \quad (s, r = 1, 2, \ s \neq r),$$

$$\delta Z_L^q = - \frac{\alpha_s C_F}{2\pi} \sum_{k=1}^2 (R_{k1}^q)^2 (B_0 + B_1)(0, M_{\tilde{q}}^2, M_{\tilde{g}}^2),$$

and

$$\delta Z_R^q = - \frac{\alpha_s C_F}{2\pi} \sum_{k=1}^2 (R_{k2}^q)^2 (B_0 + B_1)(0, M_{\tilde{q}}^2, M_{\tilde{g}}^2),$$

where $B'_i = \partial B_i / \partial p^2$ and $A_0$ and $B_i$ are the one-point and two-point integrals [17], respectively. Since we will factorize the collinear singularities into the parton densities, as will be discussed below, the $\overline{\text{MS}}$ scheme for the renormalization of the initial quark wave functions
should be used here. However, the initial quark renormalization constants have no finite terms, except in the SUSY QCD corrections which are irrelevant for the PDF’s. Therefore, the on-shell renormalization scheme is equivalent to the $\overline{MS}$ scheme for initial quark renormalization.

As for the renormalization of the squark mixing matrix, the counterterm for the squark mixing matrix $R\tilde{q}$ is defined as

$$R\tilde{q} \rightarrow R\tilde{q} + \delta R\tilde{q}, \quad (32)$$

where the counterterm $\delta R\tilde{q}$ can be fixed by requiring that the counterterm $\delta R\tilde{q}$ cancels the antisymmetric part of the wave function corrections\[18\]. The squark mixing matrix $R\tilde{q}$ counterterm can be written as

$$\delta R_{\tilde{q}}^{sr} = \frac{1}{4} \sum_{k=1}^{2} (\delta \tilde{Z}_{sk}^{q} - \delta \tilde{Z}_{ks}^{q}) R_{\tilde{q}}^{kr}. \quad (33)$$

The corresponding counterterms for the virtual amplitudes are given by

$$\mathcal{M}_C = \frac{1}{2}(\delta Z_{L}^{u} + \delta Z_{L}^{d})\mathcal{M}_{s0} + \frac{1}{2} \delta Z_{L}^{u} \sum_{s=1}^{2} \mathcal{M}_{s0}^{s} + \frac{1}{2} \sum_{s=1}^{2} \frac{(M_{L}^{LR}\alpha_{s}^{d}\delta Z_{L}^{d} + M_{L}^{LR}\beta_{s}^{d}\delta Z_{R}^{d})C_{U}^{s}}{t - M_{d_{s}}^{2}}$$

$$+ \frac{1}{4} \sum_{s=1}^{2} \frac{(1}{t - M_{d_{s}}^{2}} + \frac{1}{t - M_{d_{k}}^{2}})(M_{3}^{RL}\alpha_{s}^{d}\delta Z_{L}^{d} + M_{3}^{RR}\beta_{s}^{d}\delta Z_{R}^{d})C_{U}^{s}$$

$$+ \frac{1}{2} \delta Z_{L}^{u} \sum_{s=1}^{2} \mathcal{M}_{s0}^{s} + \frac{1}{2} \sum_{s=1}^{2} \frac{(M_{3}^{RL}\alpha_{s}^{d}\delta Z_{L}^{d} + M_{3}^{RR}\beta_{s}^{d}\delta Z_{R}^{d})C_{V}^{s}}{u - M_{u_{s}}^{2}}$$

$$- \frac{1}{2} \sum_{s=1}^{2} \sum_{k=1}^{2} C_{U}^{s}(a_{k2}^{d}M_{2}^{LR} + b_{k2}^{d}M_{2}^{RL})\left[\frac{\delta \tilde{Z}_{sk}^{d}}{t - M_{d_{k}}^{2}} + \frac{\delta \tilde{Z}_{ks}^{d}}{t - M_{d_{k}}^{2}}\right]$$

$$- \frac{1}{2} \sum_{s=1}^{2} \sum_{k=1}^{2} C_{V}^{s}(b_{k2}^{d}M_{2}^{LR} + a_{k2}^{d}M_{2}^{RL})\left[\frac{\delta \tilde{Z}_{sk}^{d}}{u - M_{u_{k}}^{2}} + \frac{\delta \tilde{Z}_{ks}^{d}}{u - M_{u_{k}}^{2}}\right]$$

$$+ \sum_{s=1}^{2} \frac{C_{U}^{s}M_{2}^{LR} + C_{V}^{s}M_{2}^{RR}}{(t - M_{d_{s}}^{2})^{2}}(a_{s2}^{d}M_{2}^{LR} + b_{s2}^{d}M_{2}^{RL}) + \sum_{s=1}^{2} \frac{C_{U}^{s}M_{2}^{LR} + C_{V}^{s}M_{2}^{RR}}{(u - M_{u_{s}}^{2})^{2}}(b_{s2}^{d}M_{2}^{RR} + a_{s2}^{d}M_{2}^{RL}),$$

and can be factorized:

$$\mathcal{M}_C = \sum_{n=1}^{3} \sum_{a,b=L,R} (f_{QCDCn}^{ab} + f_{SUSYCn}^{ab})M_{n}^{ab}. \quad (35)$$

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The explicit expressions for the form factors $f_{QCD}^{ab}$ and $f_{SUSY}^{ab}$ are presented in Appendix C. In Eq. (35), the UV divergences in QCD and SUSY QCD corrections are given by

$$M_{C}^{QCD}_{UV} = -\frac{\alpha_s C_F}{4\pi \epsilon} \left\{ M_{s0} + \sum_{k=1}^{2} M_{t0}^{k} \frac{\hat{t} + 2M_{d_k}^{2}}{\hat{t} - M_{d_k}^{2}} + \sum_{k=1}^{2} M_{u0}^{k} \frac{\hat{u} + 2M_{d_k}^{2}}{\hat{u} - M_{d_k}^{2}} \right\},$$

(36)

and

$$M_{C}^{SUSY}_{UV} = \frac{\alpha_s C_F}{4\pi \epsilon} \left\{ - M_{s0} + \sum_{k=1}^{2} M_{t0}^{k} \frac{1}{\hat{t} - M_{d_k}^{2}} \left[ \sum_{r=1}^{2} S_{kr}^{\bar{u}} S_{kr}^{d} M_{d_r}^{2} - 4M_{d_r}^{2} - \hat{t} + 3M_{d_k}^{2} \right] + \sum_{k=1}^{2} M_{u0}^{k} \frac{1}{\hat{u} - M_{d_k}^{2}} \left[ \sum_{r=1}^{2} S_{kr}^{\bar{u}} S_{kr}^{d} M_{d_r}^{2} - 4M_{d_r}^{2} - \hat{u} + 3M_{d_k}^{2} \right] + \sum_{k=1}^{2} \sum_{r=1}^{2} C_{U}^{(M_{2LR}^{k})} a_{k}^{\bar{u}} \hat{t} + M_{2LR}^{k} b_{k}^{d} \frac{\hat{t} - M_{d_r}^{2}}{(\hat{t} - M_{d_k}^{2})(\hat{t} - M_{d_k}^{2})} \sum_{s=1}^{2} S_{kr}^{\bar{u}} S_{kr}^{d} M_{d_s}^{2} + \sum_{k=1}^{2} \sum_{r=1}^{2} C_{V}^{(M_{3LR}^{k})} a_{k}^{\bar{u}} \hat{u} + M_{3LR}^{k} b_{k}^{d} \frac{\hat{u} - M_{d_r}^{2}}{(\hat{u} - M_{d_k}^{2})(\hat{u} - M_{d_k}^{2})} \sum_{s=1}^{2} S_{kr}^{\bar{u}} S_{kr}^{d} M_{d_s}^{2} \right\}.$$

(37)

respectively. From Eqs. (16), (17), (36) and (37), we obtain

$$\left( M_{V} + M_{C} \right)^{QCD}_{UV} = \frac{\alpha_s C_F}{4\pi \epsilon} \frac{3}{2} \sum_{k=1}^{2} (M_{t0}^{k} + M_{u0}^{k}),$$

(38)

and

$$\left( M_{V} + M_{C} \right)^{SUSY}_{UV} = -\frac{\alpha_s C_F}{4\pi \epsilon} \frac{3}{2} \sum_{k=1}^{2} (M_{t0}^{k} + M_{u0}^{k}).$$

(39)

The UV divergences above cancel, as they must. The renormalized amplitude at one-loop order is UV convergent

$$\left( M_{V} + M_{C} \right)_{UV} = 0,$$

(40)

but it still contains infrared (IR) divergences:

$$M_{V} \bigg|_{IR} = \frac{\alpha_s}{4\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_{0}^{2}}{\hat{s}} \right)^{\epsilon} \left( \frac{-2}{\epsilon^2} - \frac{4}{\epsilon} \right) M_{0},$$

(41)

$$M_{C} \bigg|_{IR} = \frac{\alpha_s}{4\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_{0}^{2}}{\hat{s}} \right)^{\epsilon} \frac{1}{\epsilon} M_{0},$$

(42)

and

$$\left( M_{V} + M_{C} \right) \bigg|_{IR} = \frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_{0}^{2}}{\hat{s}} \right)^{\epsilon} \left( \frac{A_{2}^{V}}{\epsilon^2} + \frac{A_{1}^{V}}{\epsilon} \right) M_{0}. $$

(43)

Here $A_{2}^{V} = -C_{F}$ and $A_{1}^{V} = -\frac{3}{2} C_{F}.$
The $\mathcal{O}(\alpha_s)$ virtual corrections to the differential cross section can be expressed as

$$
\frac{d^2\hat{\sigma}^V}{d\hat{t}d\hat{u}'} = \frac{1}{16\pi\hat{s}^2(1-\epsilon)} \left( \frac{4\pi\mu_s^2\hat{s}}{\hat{t}'\hat{u}' - \hat{s}M_{\tilde{\chi}^+_1}^2} \right) ^\epsilon \Theta(\hat{t}'\hat{u}' - \hat{s}M_{\tilde{\chi}^+_1}^2)\Theta[\hat{s} - (M_{\tilde{\chi}^+_1} + M_{\tilde{\chi}^0_2})^2] 
\times \delta(\hat{s} + \hat{t} + \hat{u} - M_{\tilde{\chi}^+_1}^2 - M_{\tilde{\chi}^0_2}^2)2Re\left[ \sum (\mathcal{M}_C + \mathcal{M}_V)M_0^2 \right].
$$

Equation (44)

The IR divergences in Eq.(44) include both the soft and collinear divergences, which cancel after adding the real emission corrections and absorbing divergences into the redefinition of PDF's [19], as will be discussed below.

2. REAL CORRECTIONS

The Feynman diagrams for the real emission corrections are shown in Figs. 3 and 4.

FIG. 3: The Feynman diagrams for the real corrections without squark resonances in chargino and neutralino associated production.
FIG. 4: The Feynman diagrams for the real corrections with squark resonances in chargino and neutralino associated production.

After calculating the relevant Feynman diagrams the amplitudes for the real gluon emission process

\[ u(p_1) + \bar{d}(p_2) \rightarrow \tilde{\chi}^+_1(p_3) + \tilde{\chi}^0_2(p_4) + g(p_5) \]  

and the real massless (anti)quark emission processes

\[ u(p_1) + g(p_2) \rightarrow \tilde{\chi}^+_1(p_3) + \tilde{\chi}^0_2(p_4) + d(p_5) \]  

and

\[ \bar{d}(p_1) + g(p_2) \rightarrow \tilde{\chi}^+_1(p_3) + \tilde{\chi}^0_2(p_4) + \bar{u}(p_5) \]  

can be written as

\[ \mathcal{M}_{RG} = \sum_{n=1}^{23} \sum_{a,b=L,R} f_{RGn}^{ab} M_{Gn}^{ab} \]  

and

\[ \mathcal{M}_{RQ} = \sum_{n=1}^{21} \sum_{a,b=L,R} f_{RQn}^{ab} M_{Qn}^{ab} \]  

respectively. The explicit expressions for the form factors \( f_{RGn}^{ab} \) and \( f_{RQn}^{ab} \) and the standard matrix elements \( M_{Gn}^{ab} \) and \( M_{Qn}^{ab} \) in Eqs.(48) and (49) are given in Appendix C.

The phase space integration for the real corrections will produce soft and collinear singularities, which can be conveniently isolated by slicing phase space into different regions using suitable cutoffs. We used the two-cutoff phase space slicing method\[20\], which introduces two arbitrarily small cutoffs, \( \delta_s \) and \( \delta_c \), to decompose the three-body phase space into three regions.
The parton level cross section for real gluon emission $\hat{\sigma}^R$ contains both the soft and the collinear singularities and, in general, can be written as

$$\hat{\sigma}^R = \hat{\sigma}^S + \hat{\sigma}^{HC} + \hat{\sigma}^{HC},$$

(50)

where $\hat{\sigma}^S$ and $\hat{\sigma}^{HC}$ are the contributions from the soft and the hard collinear regions, respectively, and $\hat{\sigma}^{HC}$ is the hard noncollinear part. The explicit forms are described below.

In the soft limit the energy of the emitted gluon is small, i.e. $E_5 \leq \delta_s \sqrt{s}/2$, and the squared amplitude $\sum |M_{RG}|^2$ can simply be factorized into the squared Born amplitude times an eikonal factor $\Phi_{eik}$:

$$\sum |M_{RG}|^2 \rightarrow \text{soft} (4\pi\alpha_s\mu_r^2) \sum |M_0|^2 \Phi_{eik},$$

(51)

where the eikonal factor $\Phi_{eik}$ is given by

$$\Phi_{eik} = C_F \frac{\hat{s}}{(p_1 \cdot p_5)(p_2 \cdot p_5)}.$$  

(52)

The phase space in the soft limit also be factorizes:

$$dPS^{(3)}(ud \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 g) \rightarrow \text{soft} dPS^{(2)}(ud \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0) dS,$$

(53)

where $dS$ is the integration over the phase space of the soft gluon which is given by

$$dS = \frac{1}{2(2\pi)^{3-2\epsilon}} \int_0^{\delta_s \sqrt{s}/2} dE_5 E_5^{1-2\epsilon} d\Omega_{2-2\epsilon}.$$  

(54)

The parton level cross section in the soft region can then be expressed as

$$\hat{\sigma}^S = (4\pi\alpha_s\mu_r^2) \int dPS^{(2)} \sum |M_0|^2 \int dS \Phi_{eik}.$$  

(55)

Using the approach in Ref.[20], after integration over the soft gluon phase space, Eq.(55) becomes

$$\hat{\sigma}^S = \hat{\sigma}^B \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_r^2}{\hat{s}} \right)^\epsilon \right] \left( \frac{A_S^2}{\epsilon^2} + \frac{A_S^1}{\epsilon} + A_S^0 \right),$$

(56)

with

$$A_S^2 = 2C_F, \quad A_S^1 = -4C_F \ln \delta_s, \quad A_S^0 = 4C_F \ln^2 \delta_s.$$  

(57)

In the hard collinear region, $E_5 > \delta_s \sqrt{s}/2$ and $-\delta_c \sqrt{s} < \hat{u}_{1,2} \equiv (p_{1,2} \cdot p_5)^2 < 0$, the emitted hard gluon is collinear to one of the partons. As a consequence of the factorization theorems [21], the squared amplitude for the gluon emission process (45) can be factorized
into the product of the squared Born amplitude and the Altarelli-Parisi splitting function \[22\]
for \( u(\bar{d}) \to u(\bar{d})g \),
\[
\sum |\mathcal{M}_{\text{RG}}|^2 \xrightarrow{\text{collinear}} (4\pi\alpha_s\mu_r^2) \sum |\mathcal{M}_0|^2 \left( \frac{-2P_{uu}(z, \epsilon)}{z\hat{u}_1} + \frac{-2P_{dd}(z, \epsilon)}{z\hat{u}_2} \right). \tag{58}
\]
Here \( z \) denotes the fraction of the momentum carried by parton \( u(\bar{d}) \) with the emitted gluon carrying a fraction \((1 - z)\) and \( P_{ij}(z, \epsilon) \) are the unregulated splitting functions in \( D = 4 - 2\epsilon \) dimensions for \( 0 < z < 1 \), which are related to the usual Altarelli-Parisi splitting kernels \[22\] as follows:
\[
P_{ij}(z, \epsilon) = P_{ij}(z) + \epsilon P'_{ij}(z).
\]
Explicitly
\[
P_{uu}(z) = P_{\bar{d}d}(z) = C_F \frac{1+z^2}{1-z} + C_F \frac{3}{2}\delta(1-z), \tag{59}
\]
and
\[
P'_{uu}(z) = P'_{\bar{d}d}(z) = -C_F(1-z) + C_F \frac{1}{2}\delta(1-z). \tag{60}
\]
The three-body phase space can also be factorized in the collinear limit and, for example, in the limit \(-\delta_c \hat{s} < \hat{u}_1 < 0\) it has the following form \[20\]:
\[
dPS^{(3)}(u\bar{d} \to \tilde{\chi}_1^+ \chi_2^0 g) \xrightarrow{\text{collinear}} dPS^{(2)}(u\bar{d} \to \tilde{\chi}_1^+ \chi_2^0; \hat{s}' = z\hat{s}) \frac{(4\pi)^\epsilon}{16\pi^2\Gamma(1-\epsilon)} dzd\hat{u}_1[1-(1-z)\hat{u}_1]^{-\epsilon}. \tag{61}
\]
Here the two-body phase space is evaluated at a squared parton-parton energy \( z\hat{s} \). The three-body cross section in the hard collinear region is then given by \[20\]
\[
\lim_{\epsilon \to 0} d\sigma^{HC} = \hat{a}_B \left[ \frac{\alpha_s}{4\pi} \frac{\Gamma(1-\epsilon)}{(1-2\epsilon)} \left( \frac{4\pi\mu_r^2}{\hat{s}} \right)^\epsilon \right] \left( -\frac{1}{\epsilon} \right) \delta(1-z) \frac{1}{2}\delta(1-z) \hat{s} \frac{1}{z} \left( \frac{1}{z} \right)^{-\epsilon} d\hat{s} d\hat{u}_1 d\hat{u}_2,
\]
where \( f(x) \) is a bare PDF.

After subtracting the soft and collinear region of the phase space, the remaining hard non-collinear part \( \hat{a}^{\text{NC}} \) is finite and can be numerically computed using Monte-Carlo integration techniques \[23\]. The result and can be written in the form
\[
\lim_{\epsilon \to 0} d\hat{a}^{\text{NC}} = \frac{1}{2\hat{s}} \sum |\mathcal{M}_{\text{RG}}|^2 dPS^{(3)}, \tag{63}
\]
where \( dPS^{(3)} \) is the hard noncollinear region of the three-body phase space.

In addition to real gluon emission, other real emission corrections to the inclusive cross section for \( A + B \to \tilde{\chi}_1^+ \tilde{\chi}_2^0 \) at NLO involve the processes with an additional massless (anti)quark,
in the final state. Since the contributions from real massless (anti)quark emission contain initial state collinear singularities, we also need to use the two-cutoff phase space slicing method [20] to isolate these collinear divergences. But we only split the phase space into two regions since there are no soft divergences. Consequently, using the approach in Ref. [20], the cross sections for the processes with an additional massless (anti)quark in the final state can be expressed as

$$d\sigma^{\text{add}} = \sum_{(\alpha=u,d)} \hat{\sigma}^C(g_\alpha \rightarrow \bar{X}_1 X_2 + X)[f_{g/A}(x_1)f_{\alpha/B}(x_2) + (A \leftrightarrow B)]dx_1dx_2$$

$$+ d\hat{\sigma}^B\left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)} \left(\frac{4\pi\mu^2}{\bar{s}}\right)^\epsilon \right] \left(-\frac{1}{\epsilon}\right) \delta_c^{-\epsilon}[P_{ug}(z, \epsilon)f_{g/A}(x_1/z)f_{d/B}(x_2)$$

$$+ f_{u/A}(x_1)P_{dg}(z, \epsilon)f_{g/B}(x_2/z) + (A \leftrightarrow B)]d\frac{z}{z}\left(1-\frac{z}{z}\right)^{-\epsilon}dx_1dx_2,$$

where

$$P_{ug}(z) = P_{dg}(z) = \frac{1}{2}[z^2 + (1-z)^2],$$

$$P'_{ug}(z) = P'_{dg}(z) = -z(1-z).$$

The first term in Eq. (64) represents the noncollinear cross section. The parton cross section $\hat{\sigma}^C$ can be written in the form

$$\hat{\sigma}^C = \int \frac{1}{2\bar{s}}\sum |M_{RQ}|^2 d\overline{PS}^{(3)},$$

where $d\overline{PS}^{(3)}$ is the three-body phase space in the noncollinear region. The second term in Eq. (64) represents the collinear singular cross sections.

3. MASS FACTORIZATION AND NLO TOTAL CROSS SECTIONS

As mentioned above, after adding the renormalized virtual corrections and the real corrections the parton level cross sections still contain collinear divergences. These can be absorbed into a redefinition of the PDF’s at NLO, using mass factorization [22]. In practice this means that first we convolute the parton cross sections with the bare PDF’s $f_{\alpha/H}(x)$ ($H = A, B$) and then use the renormalized PDF’s $f_{\alpha/H}(x, \mu_f)$ to replace $f_{\alpha/H}(x)$. In the $\overline{\text{MS}}$ convention the scale-dependent PDF’s $f_{\alpha/H}(x, \mu_f)$ are given by [20]

$$f_{\alpha/H}(x, \mu_f) = f_{\alpha/H}(x) + \sum_\beta \left(-\frac{1}{\epsilon}\right) \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)} \left(\frac{4\pi\mu^2_f}{\mu^2_f}\right)^\epsilon \right] \int_x^1 \frac{dz}{z} P_{\alpha\beta}'(z)f_{\beta/H}(x/z).$$
This replacement produces a collinear singular counterterm which, when combined with the hard collinear contributions, gives, as in Ref. [20], the $\mathcal{O}(\alpha_s)$ expression for the remaining collinear contribution:

$$d\sigma^{\text{coll}} = \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_F^2}{s} \right)^\epsilon \right] \left\{ \tilde{f}_{u/A}(x_1, \mu_f) f_{d/B}(x_2, \mu_f) + f_{u/A}(x_1, \mu_f) \tilde{f}_{d/B}(x_2, \mu_f) \right\} + \left[ A_{sc}^{u} + A_{sc}^{d} \right] f_{u/A}(x_1, \mu_f) f_{d/B}(x_2, \mu_f) + (A \leftrightarrow B) \} \hat{\sigma}^B dx_1 dx_2,$$

where

$$A_{sc}^{u} = C_F (4 \ln \delta_s + 3), \quad A_{sc}^{d} = A_{sc}^{u} \ln(\frac{\hat{s}}{\mu_F^2}),$$

$$\tilde{f}_{\alpha=(u,d)/H}(x, \mu_f) = \sum_{\beta=g,\alpha} \int_x^{1-\delta_s \delta_{\alpha\beta}} \frac{dy}{y} f_{\beta/H}(x/y, \mu_f) \tilde{P}_{\alpha\beta}(y),$$

with

$$\tilde{P}_{\alpha\beta}(y) = P_{\alpha\beta}(y) \ln(\delta_{\epsilon} \frac{1-y}{y} \frac{\hat{s}}{\mu_F^2}) - P'_{\alpha\beta}(y).$$

Finally, the NLO total cross section for $A + B \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$ in the $\overline{\text{MS}}$ factorization scheme is

$$\sigma^{NLO} = \sum_{\alpha,\beta=u,d} \int dx_1 dx_2 \left\{ \left[ f_{\alpha/A}(x_1, \mu_f) f_{\beta/B}(x_2, \mu_f) \right] (\hat{\sigma}^B + \hat{\sigma}^V + \hat{\sigma}^S + \hat{\sigma}^{\text{HC}}) \right\} + \sigma^{\text{coll}}$$

$$+ \sum_{\alpha=u,d} \int dx_1 dx_2 \left[ f_{\beta/A}(x_1, \mu_f) f_{\alpha/B}(x_2, \mu_f) + (A \leftrightarrow B) \right] \hat{\sigma}^C (g\alpha \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 X).$$

Note that the expression above contains no singularities for $2A_{V} + A_{S}^{u} = 0$ and $2A_{V} + A_{S}^{d} + A_{sc}^{u} = 0$.

### 4. ON-SHELL SUBTRACTION

In the massless (anti)quark corrections there is resonance production of squarks, which actually corresponds to squark and gaugino production at the LO followed by squark decay to a gaugino and a quark, as shown in Fig. 4. We used the method in Ref. [13] to subtract their contributions. For example, consider a representative process

$$u + g \rightarrow \tilde{u} + \tilde{\chi}_2^0, \quad \tilde{u} \rightarrow \tilde{\chi}_1^+ + d,$$

with

$$u + g \rightarrow \tilde{u} + \tilde{\chi}_2^0, \quad \tilde{u} \rightarrow \tilde{\chi}_1^+ + d,$$
which is shown as the first Feynman diagram in Fig. 4. Using the Breit-Wigner propagator 
\[ \frac{1}{(p^2 - m^2 + i m \Gamma)} \], the squared resonance matrix elements can be expressed as
\[ |\mathcal{M}|^2 = \frac{f(Q^2)}{(Q^2 - M_0^2)^2 + M_0^2 \Gamma_0^2}, \] (75)
where \( Q^2 = (p_{\chi_1^+} + p_d)^2 \). After subtracting the contributions due to resonance production
the squared matrix element is
\[ |\mathcal{M}|^2 = \frac{f(Q^2)}{(Q^2 - M_0^2)^2 + M_0^2 \Gamma_0^2} - \frac{f(M_0^2)}{(Q^2 - M_0^2)^2 + M_0^2 \Gamma_0^2}. \] (76)
This subtracted result avoids double counting and makes the numerical calculation more
stable since the resonance peaks are subtracted before the phase space integration. The
dependence on the squark widths will be discussed in Sec. IV.

5. NLO TOTAL CROSS SECTIONS IN BOTH DREG AND DRED SCHEMES

In our calculations we used the DREG scheme. However, this scheme is not appropriate
for SUSY models because it violates supersymmetry. To restore supersymmetry we modified
the Yukawa coupling at the one loop level as shown in Eq. (14).

The real corrections and NLO total cross sections in the DREG scheme have been given
above. Next we show the corresponding results in the DRED scheme. The contributions
from soft gluon emission remain the same, but in addition to the the modified Yukawa
couplings those from hard collinear gluon emission and massless (anti)quark emission are
also different. These differences arise from the splitting functions and the PDF’s.

First, note the LO amplitude in the DREG scheme with modified Yukawa couplings
(DREGM) is different from that in the DRED scheme:
\[ \mathcal{M}_{0}^{DREGM} - \mathcal{M}_{0}^{DRED} = -\frac{\alpha_s C_F}{4\pi} \sum_{k=1}^{2}(\mathcal{M}_{t0}^{k} + \mathcal{M}_{u0}^{k}). \] (77)

Here, and below, the LO amplitudes and cross sections in the right hand side of equations
are all in 4-dimensions, and their Yukawa couplings are not modified. Calculating the virtual
corrections in the DRED scheme one finds that \( \delta Z_L^f, \delta Z_R^f, \delta \tilde{Z}_{ij}^q \) and \( \delta M_{\tilde{q}}^2 \) remain the same
as in the DREG scheme. Thus
\[ \mathcal{M}_{C}^{DREGM} - \mathcal{M}_{C}^{DRED} = 0. \] (78)
However, the unrenormalized amplitudes $M_V$ differ:

$$M_{V,DREGM} - M_{V,DRED} = -\frac{\alpha_s C_F}{4\pi} M_{s0}. \quad (79)$$

From Eqs. (77), (78) and (79), one finds the following relations:

$$(M_0 + M_V + M_C)^{DREGM} - (M_0 + M_V + M_C)^{DRED} = -\frac{\alpha_s C_F}{4\pi} M_0, \quad (80)$$

$$(\hat{\sigma}^B + \hat{\sigma}^V)^{DREGM} - (\hat{\sigma}^B + \hat{\sigma}^V)^{DRED} = -\frac{\alpha_s C_F}{2\pi} \hat{\sigma}^B + O(\alpha_s^2). \quad (81)$$

Second, note the splitting functions in the DRED scheme have no dependence on $\epsilon$:

$$P_{ij}(z,\epsilon)^{DRED} = P_{ij}(z). \quad (82)$$

From Eqs. (69) and (82), one finds

$$(\sigma^{coll})^{DREGM} - (\sigma^{coll})^{DRED} = -\frac{\alpha_s}{2\pi} \left\{ \sum_{\alpha} \int_{x_1}^{1-\delta_s \delta_{\alpha\alpha}} \frac{dy}{y} f_{\alpha/A}(x_1/y, \mu_f) P_{\alpha A}(y) f_{\bar{d}/B}(x_2, \mu_f) 
$$

$$+ \sum_{\alpha} \int_{x_2}^{1} \frac{dy}{y} f_{\alpha/B}(x_2/y, \mu_f) P_{\alpha B}(y) f_{\bar{u}/A}(x_1, \mu_f) 
$$

$$+ (A \leftrightarrow B) \right\} \hat{\sigma}^B dx_1 dx_2 + O(\alpha_s^2). \quad (83)$$

Third, note the PDF’s in the DRED and DREG schemes are related [24]:

$$f_{\alpha/A,B}(x, \mu_f)^{DREG} = f_{\alpha/A,B}(x, \mu_f)^{DRED} + \frac{\alpha_s}{2\pi} \sum_{\beta} \int_{x}^{1} \frac{dy}{y} P_{\alpha \beta}(x/y) f_{\beta/A,B}(y, \mu_f)^{DRED}. \quad (84)$$

Substituting into the formula for the Born cross section we obtain an additional difference at $O(\alpha_s)$ arising from the PDF’s:

$$(\sigma^B)^{DREGM} - (\sigma^B)^{DRED} = \frac{\alpha_s}{2\pi} \left\{ \sum_{\alpha} \int_{x_1}^{1} \frac{dy}{y} f_{\alpha/A}(x_1/y, \mu_f)^{DRED} P_{\alpha A}(y) f_{\bar{d}/B}(x_2, \mu_f)^{DRED} 
$$

$$+ \sum_{\alpha} \int_{x_2}^{1} \frac{dy}{y} f_{\alpha/B}(x_2/y, \mu_f)^{DRED} P_{\alpha B}(y) f_{\bar{u}/A}(x_1, \mu_f)^{DRED} 
$$

$$+ (A \leftrightarrow B) \right\} \hat{\sigma}^B dx_1 dx_2. \quad (85)$$

Finally note that Eqs. (83) and (85) are very similar except for the limits on the integral over $y$. Substituting Eqs. (81), (83) and (85) into Eq. (73), we obtain the following relations
for the NLO total cross sections in two schemes:

\[
\left(\sigma^{NLO\,DREGM}\right) - \left(\sigma^{NLO\,DRED}\right) = \frac{\alpha_s}{2\pi} \left\{ \sum_\alpha \int_{1-\delta_{\alpha\alpha}}^1 dy \frac{f_{\alpha/A}(x_1/y, \mu_f) P'_{\alpha\alpha}(y) f_{\bar{d}/B}(x_2, \mu_f)}{y} \right.
\]

\[
+ \sum_\alpha \int_{1-\delta_{\bar{d}\alpha}}^1 dy \frac{f_{\alpha/B}(x_2/y, \mu_f) P'_{\alpha\bar{d}}(y) f_{d/A}(x_1, \mu_f)}{y} \right.
\]

\[
+ (A \leftrightarrow B) \left\{ \hat{\sigma}^B dx_1 dx_2 - \frac{\alpha_s C_F}{2\pi} \sigma^B + O(\alpha_s^2) \right\}.
\]

Using the explicit expressions, including the \(\epsilon\) dependence, for the splitting functions \(P'\), one finds

\[
\left(\sigma^{NLO\,DREGM}\right) - \left(\sigma^{NLO\,DRED}\right) = O(\alpha_s^2).
\]

Therefore, the NLO total cross sections in the two schemes are the same at NLO.

III. THRESHOLD RESUMMATION

Here we briefly summarize the basic formalism for threshold resummation Refs. [25, 26, 27, 28, 29, 30, 31, 32]. Using pair inclusive (PIM) kinematics the invariant mass differential cross section can be written as

\[
\omega(S, Q^2) = \sum_{a,b} \int_{\tau}^1 dz \int d\tau_a d\tau_b f_{\tau_a/A}(x_a, \mu_f) f_{\tau_b/B}(x_b, \mu_f) \delta(z - \frac{Q^2}{x_a x_b S}) \hat{\omega}_{ab}(z, Q^2),
\]

where

\[
\tau = \frac{Q^2}{S}, \quad \omega = \frac{d\sigma}{dQ^2}, \quad \hat{\omega}_{ab} = \frac{d\hat{\sigma}_{ab}}{dQ^2}, \quad z = \frac{Q^2}{s},
\]

and \(Q^2\) is the invariant mass of the chargino and neutralino. The differential cross section \(\hat{\omega}_{ab}\) contains large logarithmic terms \(\alpha_s^n [\ln^m (1-z)/(1-z)]_+\), which come from the incomplete cancelation between real gluon emission and virtual gluon corrections. In the region \(z \approx 1\) \((Q^2 \approx \hat{s})\) these large logarithms have to be resummed to all orders in \(\alpha_s\).

In order to calculate the hard-scattering function \(\hat{\omega}_{ab}\) we consider the IR regularized cross section for parton-parton scattering which factorizes:

\[
\omega_{ab} = \int_{\tau}^1 dz \int d\tau_a d\tau_b \phi_{\tau_a/a}(x_a, \mu_f) \phi_{\tau_b/b}(x_b, \mu_f) \delta(z - \frac{Q^2}{x_a x_b S}) \hat{\omega}_{ab}(z, Q^2),
\]

where \(\phi_{\tau_a/a}\) and \(\phi_{\tau_b/b}\) are the flavor diagonal parton distributions in partons. Using a Mellin transformation with respect to \(\tau\) the convolution in Eq. (90) can be simplified as the product

\[
\hat{\omega}_{ab}(N) = \hat{\phi}_{\tau_a/a}(N) \hat{\phi}_{\tau_b/b}(N) \hat{\omega}_{ab}(N),
\]
where

\[ \tilde{\omega}(N) = \int d\tau \tau^{N-1}\omega(\tau), \]  
(92)

\[ \tilde{\phi}_{a/a}(N) = \int dx x^{N-1}\phi_{a/a}(x), \]  
(93)

\[ \tilde{\phi}_{b/b}(N) = \int dx x^{N-1}\phi_{b/b}(x), \]  
(94)

and

\[ \tilde{\omega} = \int dz z^{N-1}\omega(z). \]  
(95)

Here the large logarithmic terms in \( \tilde{\omega}_{ab} \) turn out to be \( \alpha_s^n \ln^m N \) and \( z \to 1 \) corresponds to \( N \to \infty \). The next step is to resum the logarithms of \( N \).

In order to separate the soft gluon effects from the short distance hard scattering we can factorize the differential cross section into the form

\[ \tilde{\omega}_{ab}(N) = \tilde{\psi}_{a/a}(N)\tilde{\psi}_{b/b}(N) \sum_{IJ} H_{IJ}^{ab} \tilde{S}_{JI}(\frac{Q}{N \mu_f}), \]  
(96)

where \( I, J \) are color indices, \( H_{IJ} \) describes the short distance hard scattering and \( \tilde{S}_{JI} \) is a soft gluon function associated with noncollinear soft gluons. The explicit definitions of \( H_{IJ} \) and \( \tilde{S}_{JI} \) can be found in Ref. [25]. The \( \psi \)'s are the center-of-mass parton distribution functions in which the universal collinear singularities associated with the initial partons are absorbed.

From Eq. (91) and Eq. (96) we have

\[ \tilde{\omega}_{ab} = \frac{\tilde{\psi}_{a/a} \tilde{\psi}_{b/b}}{\tilde{\phi}_{a/a} \tilde{\phi}_{b/b}} Tr[H \tilde{S}]. \]  
(97)

After resumming the terms with the \( N \) dependence we obtain the exponentiated differential cross section in the space of moments [25, 26]:

\[ \tilde{\omega}_{ab}^{EXP} = \exp \left[ \sum_i E^{(f_i)}(N) \right] \exp \left[ \sum_i 2 \int_{\mu_f}^Q \frac{d\mu'}{\mu'} \gamma_i(\alpha_s(\mu'^2)) \right] \times \exp \left[ \sum_i 2d_{\alpha_s} \int_{\mu_r}^{Q/N} \frac{d\mu'}{\mu'} \beta(\alpha_s(\mu'^2)) \right] \times \exp \left[ \int_{Q/N}^Q \frac{d\mu'}{\mu'} (\Gamma_{SB}^{ab}(\alpha_s(\mu'^2)) \right] \times \exp \left[ \int_{Q/N}^Q \frac{d\mu'}{\mu'} \Gamma_{SB}^{ab}(\alpha_s(\mu'^2)) \right] \times P \exp \left[ \int_{Q/N}^Q \frac{d\mu'}{\mu'} (\Gamma_{SB}^{ab}(\alpha_s(\mu'^2)) \right] \times \exp \left[ \int_{Q/N}^Q \frac{d\mu'}{\mu'} \Gamma_{SB}^{ab}(\alpha_s(\mu'^2)) \right], \]  
(98)
where \( d_{\alpha_s} \) is a constant and its definition is given in Ref. [27]. \( P \) and \( \bar{P} \) denote path ordering in the same sense as the integration variable \( \mu' \) and in the opposite sense, respectively. The first exponent in Eq. (98) resums the collinear and soft gluon emission from initial partons in the hard scattering and is given in the modified minimal subtraction (MS) scheme by

\[
E^{(f_i)}(N) = - \int_0^1 \frac{dz}{1-z} z^{N-1} \left\{ \frac{d\mu'^2}{\mu'^2} A^{(f_i)}[\alpha_s(\mu'^2)] + \frac{1}{2} \nu^{(f_i)}[\alpha_s((1-z)^2 Q^2)] \right\},
\]

with

\[
A^{(f_i)}(\alpha_s) = C_f \left( \frac{\alpha_s}{\pi} + \frac{1}{2} K \left( \frac{\alpha_s}{\pi} \right)^2 \right),
\]

\[
K = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f,
\]

\[
\nu^{(f_i)} = 2 C_f(\alpha_s/\pi) \left[ 1 - \ln(2\nu^{(f_i)}) \right],
\]

\[
v^{(f_i)} = (\hat{\beta}_i \cdot \hat{n})^2/|\hat{n}|^2,
\]

\[
\hat{\beta}_i = p_i \sqrt{2/s},
\]

where \( \hat{\beta}_i \) is the particle velocity, \( \hat{n} \) is the axial gauge vector, \( N_c \) is the number of colors, and \( n_f \) is the flavor number of light quarks. \( C_f = C_F = (N_c^2 - 1)/(2N_c) \) for initial quarks and \( C_f = C_A = N_c \) for initial gluons. In Eq. (98) \( \gamma_i \) is the anomalous dimension of \( \psi \) and is given at one loop by \( \gamma_q = 3C_F \alpha_s/4\pi \) for quarks and \( \gamma_g = \beta_0 \alpha_s/\pi \) for gluons. The \( \beta \) function is defined as

\[
\beta(\alpha_s) = \frac{1}{2} \mu \frac{d\ln g}{d\mu} = - \sum_{n=0}^{\infty} \beta_n \left( \frac{\alpha_s}{\pi} \right)^{(n+2)},
\]

with

\[
\beta_0 = (11C_A - 2n_f)/12,
\]

\[
\beta_1 = (17C_A^2 - 5C_A n_f - 3C_F n_f)/24.
\]

\( \Gamma_S \) in Eq. (98) is the soft anomalous dimension matrix [26], which can be derived from the eikonal diagrams as shown in Fig. 3, and is given by

\[
\Gamma_S = \frac{\alpha_s C_F}{2\pi} \left[ -2 \ln 2 - \ln(v_u v_d) + 2 - 2\pi i \right].
\]

Next Eq. (98) at NLL can be written in the simplified form

\[
\tilde{\omega}^{NLL}_{ab} = \tilde{\omega}_0 c_{ab}(\alpha_s) \exp \left[ X(N, \alpha_s) \right],
\]

21
\[ \bar{\omega}_0 = \int d\tau \tau^{-1} \omega_0, \quad (110) \]

\[ \bar{\omega}_0 = \frac{\hat{\sigma}_0}{dQ} \]

\[ X(N, \alpha_s) = g_1(\lambda) \ln \bar{N} + g_2(\lambda), \quad (111) \]

\[ g_1(\lambda) = \frac{C_F}{\beta_0} \left[ 2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda) \right], \quad (112) \]

\[ g_2(\lambda) = \frac{C_F \beta_1}{\beta_0} \left[ 2\lambda + \ln(1 - 2\lambda) + \frac{1}{2} \ln^2(1 - 2\lambda) \right] - \frac{C_F K}{2 \beta_0^2} \left[ 2\lambda + \ln(1 - 2\lambda) \right] \]

\[ + \frac{C_F}{\beta_0} \left[ 2\lambda + \ln(1 - 2\lambda) \right] \ln Q^2 / \mu_f^2 - \frac{C_F}{\beta_0} 2\lambda \ln Q^2 / \mu_f^2, \quad (113) \]

where \( \omega_0 = d\hat{\sigma}_0 / dQ^2 \) is the Born differential cross section, \( \bar{N} = N \exp(\gamma_E) \), \( \gamma_E \) is the Euler constant, and \( \lambda = \beta_0 \alpha_s \ln \bar{N} / \pi \).

The function \( C_{ab} \) can be expanded as

\[ C_{ab}(\alpha_s) = 1 + \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n C_{ab}^{(n)} \quad (114) \]

By matching the moments of the NLO cross section \[30\] we obtain the function \( C_{ab} \) in Eq.(109) at the NLO. The contributions to the first term \( C_{ab}^{(1)} \equiv C_{ab}^{(1)} \alpha_s / \pi \) in the expansion above come from the constant terms in the moments of the differential cross section, which are primarily the coefficients of the \( \delta(1 - z) \) terms in the differential cross sections. The other terms come from the Mellin transformations of the logarithms depending on \( z \).

As shown in Eqs.(38), (39) and (40) in Sec. II, the divergences from QCD and SUSY QCD corrections cancel each other. Therefore, combining the contributions from real gluon
corrections and PDF renormalization, the QCD contributions to $C_{ab}^{(1)}$ are given by

$$C_{ab}^{(1)} = \frac{2 \text{Re}(\tilde{\mathcal{M}}_{V}^{QCD} \tilde{M}_0)}{|\tilde{M}_0|^2} + \frac{\alpha_s C_F}{2\pi} \left\{ \frac{2}{\epsilon^2} + \frac{1}{\epsilon} \left( 2 \ln \frac{4\pi \mu^2}{Q^2} - 2\gamma_E + 3 \right) \right.$$  

$$+ \left( \ln \frac{4\pi \mu^2}{Q^2} - \gamma_E \right)^2 + 3 \ln \frac{4\pi \mu^2}{Q^2} + \frac{\pi^2}{6} - 3\gamma_E + 3 \ln \frac{Q^2}{\mu^2}$$  

$$+ 4 \ln^2 N - 4 \ln \frac{Q^2}{\mu^2} \ln N + \frac{4 \ln N - 2 \ln \frac{Q^2}{\mu^2}}{N} \right\},$$  

(115)

with

$$\tilde{\mathcal{M}}_{V}^{QCD} = \sum_{n=1}^{24} \sum_{a,b=L,R} \left[ f_{QCDV_n}^{ab} + f_{QCDC_n}^{ab} + \left( f_{SUSYV_n}^{ab} \right|_{UV} + f_{SUSYCN}^{ab} \right|_{UV} \right] M_n^{ab}.$$  

(116)

Here the terms of order $\mathcal{O}(\ln \tilde{N}/N)$ and $\mathcal{O}(\ln(Q^2/\mu^2)/N)$ are included.

In order to more completely include the behavior of the full towers of logarithms, Eq.(109) is modified:

$$\tilde{\omega}^{NLL}_{ab} = \tilde{\omega}_0 \exp[C_{ab}^{(1)}(\alpha_s)] \exp[X(N, \alpha_s)].$$  

(117)

To obtain the physical cross section we perform the inverse Mellin transformation,

$$\omega(\tau) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} dN \tau^{-N} \tilde{\omega}(N).$$  

(118)

where the "minimal prescription" is used.

To improve the convergence of the integration in Eq.(118) we adopt the methods in Ref.[29]. First, we rotate the contour by an angle $\phi$ with respect to the real axis and parameterize it in the form

$$N = C + z \exp^{\pm i\phi},$$  

(19)

where the upper (lower) sign applies to the upper (lower) half plane $0 \leq z \leq \infty$ ($\infty \geq z \geq 0$).

Then we rewrite the inverse transformation convolution:

$$\omega(\tau) = \frac{1}{2\pi i} \int_{C_N} dN \tau^{-N} \sum_{a,b} [(N - 1) \tilde{f}_a(N)] [(N - 1) \tilde{f}_b(N)] \frac{\tilde{\omega}_{ab}(N)}{(N - 1)^2},$$  

(120)

where $C_N$ represents the modified contour. The inverse Mellin transformation of $\tilde{\omega}_{ab}/(N - 1)^2$,

$$\mathcal{Y}_{ab}(z) = \frac{1}{2\pi i} \int_{C_N} dN z^{-N} \frac{\tilde{\omega}_{ab}(N)}{(N - 1)^2},$$  

(121)
is well behaved near the region \( z \approx 1 \) due to the suppression by the factor \( 1/(N - 1)^2 \). The inverse Mellin transformation of \( (N - 1)f_i(N) \) is then

\[
\frac{1}{2\pi i} \int_{C_N} dN x^{-N} (N - 1)f_i(N) = -\frac{d}{dx}[xf_i(x)] = \mathcal{F}_i(x).
\] (122)

Finally,

\[
\omega(\tau) = \sum_{a,b} \int_{\tau}^{\infty} \frac{dz}{z} \int_{\tau/z}^{1} \frac{dx}{x} \mathcal{F}_a(x) \mathcal{F}_b(\frac{\tau}{xz}) \mathcal{Y}_{ab}(z).
\] (123)

And after integrating over the invariant mass \( Q^2 \) in the differential cross section and inserting the terms ignored in the Mellin transformation we obtain the resummed total cross section

\[
\sigma^{\text{RES}} = \sigma^{\text{NLO}} + \left[ \sigma^{\text{NLL}} - \sigma^{\text{NLL}} \bigg|_{\alpha_s=0} - \alpha_s \left( \frac{\partial \sigma^{\text{NLL}}}{\partial \alpha_s} \right)_{\alpha_s=0} \right].
\] (124)

### IV. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical calculations the following SM input parameters were chosen:

\[
M_t = 170.9 \text{GeV}[10], \quad \alpha(M_Z)^{-1} = 127.918, \quad \alpha_s(M_Z) = 0.1176,
\]

\[
M_W = 80.403 \text{GeV}, \quad M_Z = 91.1876 \text{GeV}.
\] (125)

The masses of the light quarks were neglected. The running QCD coupling \( \alpha_s \) was evaluated at the two-loop level and the CTEQ6.5M PDF’s were used to calculate the various cross sections, either at LO or at NLO. As for the renormalization and factorization scales, we chose \( \mu_r = \mu_f = Q_{\tilde{\chi}^0, \tilde{\chi}^\pm} \equiv \sqrt{(p_{\tilde{\chi}^0}^2 + 2m_0^2)} \), unless specified otherwise.

Using the program package SPheno the MSSM spectrum, including the widths of the squarks, was calculated in the mSUGRA scenario in which there are five input parameters: the ratio of Higgs-field vacuum expectation values (VEV’s) \( \tan \beta \), the common scalar mass \( m_0 \), the common gaugino mass \( m_{1/2} \), the trilinear coupling \( A_0 \), and the sign of the Higgs mixing parameter \( \mu \). The value of \( A_0 \) does not significantly affect our numerical results so we put \( A_0 = 0 \) and, based on the analysis in the literature, focused on \( \mu > 0 \).
TABLE I: The dependence of the chargino mass $M_{\tilde{\chi}^\pm}$ and the neutralino mass $M_{\tilde{\chi}^0_2}$ on the top quark mass $M_t$. The masses were calculated using SPheno [35] for $\tan\beta = 5$, $m_0 = 200\text{GeV}$, and $m_{1/2} = 250\text{GeV}$.

| $M_t$/GeV | $M_{\tilde{\chi}^\pm}$/GeV | $M_{\tilde{\chi}^0_2}$/GeV |
|----------|-----------------|-----------------|
| 170.9    | 188.506         | 189.915         |
| 176.1    | 190.609         | 191.779         |
| 180.1    | 191.978         | 193.003         |

Table II shows the dependence of the chargino mass $M_{\tilde{\chi}^\pm}$ and the neutralino mass $M_{\tilde{\chi}^0_2}$ on top quark mass $M_t$. We see that the chargino and neutralino masses depend slightly on top quark mass due to the fact that they are calculated using the SUSY renormalization group evolution (RGE). The explicit expressions for the total cross sections for the associated production of a chargino and a neutralino are independent of $M_t$ as shown in Sec. II and Sec. III. Thus the top quark mass $M_t$ only enters in the SUSY RGE.

TABLE II: The NLO total cross sections for different squark widths $\Gamma(\tilde{q})$ using and not using on-shell subtraction, respectively.

| $\Gamma(\tilde{q})/\Gamma_0(\tilde{q})$ | $\sigma'_{\text{NLO}}(pp \to \tilde{\chi}^\pm_1\tilde{\chi}^0_2)/pb$ | $\sigma_{\text{NLO}}(pp \to \tilde{\chi}^\pm_1\tilde{\chi}^0_2)/pb$ |
|-----------------|-----------------|-----------------|
| 2               | 0.734           | 0.650           |
| 1               | 0.825           | 0.653           |
| 0.5             | 1.006           | 0.661           |

Table II shows the NLO total cross sections for $\tilde{\chi}^\pm_1\tilde{\chi}^0_2$ production at the LHC, using on-shell subtraction, $\sigma_{\text{NLO}}(pp \to \tilde{\chi}^\pm_1\tilde{\chi}^0_2)$, or not, $\sigma'_{\text{NLO}}(pp \to \tilde{\chi}^\pm_1\tilde{\chi}^0_2)$, for different squark widths $\Gamma(\tilde{q})$, assuming $\tan\beta = 5$, $m_0 = 200\text{GeV}$ and $m_{1/2} = 250\text{GeV}$. The squark widths, which were calculated using SPheno [35], are $\Gamma_0(\tilde{q})$. Table II shows that the variation in $\sigma'_{\text{NLO}}(pp \to \tilde{\chi}^\pm_1\tilde{\chi}^0_2)$ is about 26% while the variation in $\sigma_{\text{NLO}}(pp \to \tilde{\chi}^\pm_1\tilde{\chi}^0_2)$ is only about 2%. Obviously, using on-shell subtraction reduces the dependence on the squark widths.

To present the resummation effects we defined the following quantities:

$$\delta K = \frac{\sigma_{\text{RES}} - \sigma_{\text{NLO}}}{\sigma_{\text{NLO}}}, \quad (126)$$

$$\delta K_d = \frac{d\sigma_{\text{RES}} - d\sigma_{\text{NLO}}}{d\sigma_{\text{NLO}}}.$$
These represent the threshold resummation effects relative to the NLO cross sections.

We present the numerical results for both $\tilde{\chi}_1^+\tilde{\chi}_2^0$ and $\tilde{\chi}_1^-\tilde{\chi}_2^0$ production at the LHC, but show only those for $\tilde{\chi}_1^+\tilde{\chi}_2^0$ production at the Tevatron since these cross sections are different at the LHC but the same at the Tevatron.

In Fig.15 we chose $\tilde{\chi}_1^+\tilde{\chi}_2^0$ production at the LHC as an example to show that it is reasonable to use the two-cutoff phase space slicing method in the NLO calculations, i.e. the dependence of the NLO predictions on the arbitrary cutoffs $\delta_s$ and $\delta_c$ is indeed very weak, as shown in Ref.[20]. Here $\sigma_{\text{other}}$ includes the contributions from the Born cross section and the virtual corrections, which are cutoff independent. Both the soft plus hard collinear contributions and the hard noncollinear contributions depend strongly on the cutoffs. However, these two contributions in $(\sigma_{\text{soft}} + \sigma_{\text{hardcoll}} + \sigma_{\text{virtual}}$ and $\sigma_{\text{hardnon-coll}})$ nearly cancel, especially for the cutoff $\delta_s$ between $5 \times 10^{-5}$ and $10^{-3}$, where the final results for $\sigma_{\text{NLO}}$ are almost independent of the cutoffs and very near 7.1pb. Therefore, we will take $\delta_s = 10^{-4}$ and $\delta_c = \delta_s/100$ in the numerical calculations below.

Using the same parameters we reproduced the results in Ref.[5] as shown in Fig.16, which provides a check on our calculations. However, our results are not exactly the same as the results in Ref.[5] because the masses calculated using SPheno[35] are different from those in Ref.[3].

Fig.17 shows the total cross sections as a function of $\tan \beta$, assuming $m_{1/2} = 150$GeV, for $m_0 = 200$GeV and 1000GeV. The general shapes of the cross sections are similar. The main difference is that the absolute values of the total cross sections are different. $\tilde{\chi}_1^+\tilde{\chi}_2^0$ production at the LHC has the largest cross section. In general, the total cross sections at the LHC are a few pb while those at the Tevatron are hundreds of fb. Fig.17 also shows that the total cross sections for large $\tan \beta (> 10)$ are almost independent of $\tan \beta$ while those for small $\tan \beta (< 10)$ decrease with the increasing $\tan \beta$ especially for $m_0 = 200$GeV. We note that the contributions from the resummation effects do not change the shapes of the curves very much.

With the same parameters as in Fig.17 the resummation effects $\delta K$ are presented in Fig.18 as a function of $\tan \beta$ for $m_0 = 200$GeV and 1000GeV. Note that $\delta K$ is almost independent of $\tan \beta$ for large $\tan \beta (> 10)$ and there are larger resummation effects for $m_0 = 1000$GeV than for $m_0 = 200$GeV. However, $\delta K$ at the LHC decreases with the increasing $\tan \beta$ for $m_0 = 200$GeV and $\tan \beta < 10$. Fig.18 shows that $\delta K$ at the LHC for $\tilde{\chi}_1^-\tilde{\chi}_2^0$ production is
larger than that for $\tilde{\chi}_1^+\tilde{\chi}_2$ production and $\delta K$ at the Tevatron is larger than at the LHC. For $m_0 = 1000\text{GeV}$ the resummation effects can reach about 4% at the LHC and about 4.7% at the Tevatron.

Fig.19 shows the total cross sections as a function of $m_{1/2}$ assuming $m_0 = 200\text{GeV}$ and $\tan \beta = 5$. As $m_{1/2}$ varies from 150GeV to 250GeV $M_{\tilde{\chi}_2^0}$ increases from 101GeV to 190GeV and $M_{\tilde{\chi}_1}$ increases from 406GeV to 599GeV, respectively. And the total cross sections decrease rapidly with the increasing of $m_{1/2}$. For example, when $m_{1/2} > 240\text{GeV}$ the total cross sections are less than 1pb and 100fb at the LHC and Tevatron, respectively. Note that the total cross section for $\tilde{\chi}_1^+\tilde{\chi}_2^0$ production at the LHC is the largest while $\tilde{\chi}_1^+\tilde{\chi}_2$ production at the Tevatron is the smallest.

With the same parameters as in Fig.19 the resummation effects $\delta K$ are shown in Fig.20 as a function of $m_{1/2}$. At the LHC the resummation effects increase with the decreasing of $m_{1/2}$, reaching 3.3% for $m_{1/2} = 150\text{GeV}$. At the Tevatron $\delta K$ increases with the increasing $m_{1/2}$, reaching 4.9% for $m_{1/2} = 250\text{GeV}$. We also find that the smallest resummation effects $\delta K$ at the Tevatron for $m_{1/2} = 150\text{GeV}$ are about 3.8%, which is larger than that at the LHC for all values of $m_{1/2}$.

Fig.21 shows the total cross sections as a function of $m_0$ assuming $m_{1/2} = 150\text{GeV}$ and $\tan \beta = 5$. As $m_0$ varies from 100GeV to 1000GeV, $M_{\tilde{\chi}_1^+}$ increases from 96GeV to 116GeV and $M_{\tilde{\chi}_2^0}$ increases from 100GeV to 117GeV, respectively. The total cross sections decrease with the increasing $m_0$ for $m_0 > 300\text{GeV}$. However, note that the total cross section for $\tilde{\chi}_1^+\tilde{\chi}_2^0$ production at the LHC is independent of $m_0$ for $m_0 < 300\text{GeV}$.

With the same parameters as in Fig.21 the resummation effects $\delta K$ are presented in Fig.22 as a function of $m_0$. The resummation effects increase at both the LHC and the Tevatron as $m_0$ increases. For $m_0 = 1000\text{GeV}$ the resummation effects reach 3.9% at the LHC and 4.7% at the Tevatron. The total cross sections increase rapidly with the increasing $m_0$ for $m_0 < 500\text{GeV}$ while they become independent of $m_0$ when $m_0 > 900\text{GeV}$.

Figs.23 and 24 show the total cross section for $\tilde{\chi}_1^+\tilde{\chi}_2^0$ production at the LHC and the Tevatron, respectively, as functions of the renormalization scale $\mu_r$ and the factorization scale $\mu_f$, and for $\mu_r = \mu_f$, assuming $m_{1/2} = 250\text{GeV}$, $m_0 = 200\text{GeV}$ and $\tan \beta = 5$. The $\mu_r$ dependence in the LO cross sections at both colliders is increased by the NLO corrections and the $\mu_r$ dependence is slightly decreased by the resummation effects. The $\mu_f$ dependence in the LO cross sections at the LHC (Tevatron) is decreased by the NLO corrections and is
further increased (decreased) by the resummation effects. However, setting $\mu_f = \mu_r = \mu_{\text{scale}}$, the resummation effects reduce the scale dependence at NLO. In fact, from Fig. 23 it can be seen that the renormalization/factorization scale dependence of the total cross sections at the LHC(Tevatron) is reduced to 5% (4%) with the threshold resummation from up to 7% (11%) at NLO.

Figs. 25 and 26 present the differential cross sections as a function of the invariant mass $Q_{\tilde{\chi}_1^\pm \tilde{\chi}_2^0}$ assuming $m_{1/2} = 150\text{GeV}$ and $\tan \beta = 5$ for $m_0 = 200\text{GeV}$ and $1000\text{GeV}$, respectively. We see that the maximum in the differential cross section occurs at about $Q_{\tilde{\chi}_1^\pm \tilde{\chi}_2^0} = 230\text{GeV}$ and $280\text{GeV}$ for $m_0 = 200\text{GeV}$ and $1000\text{GeV}$, respectively, and the differential cross sections decrease rapidly with the increasing $Q_{\tilde{\chi}_1^\pm \tilde{\chi}_2^0}$. The NLO corrections change the shapes of the differential cross sections, especially for $Q_{\tilde{\chi}_1^\pm \tilde{\chi}_2^0} < 300\text{GeV}$. The threshold resummation effects enhance the NLO differential cross sections more at moderate values of $Q_{\tilde{\chi}_1^\pm \tilde{\chi}_2^0}$ and much less so at low or high values of $Q_{\tilde{\chi}_1^\pm \tilde{\chi}_2^0}$.

Fig. 27 shows $\delta K_d$ as a function of the invariant mass. In general, after slightly decreasing, $\delta K_d$ increases more rapidly for $m_0 = 200\text{GeV}$ than that for $m_0 = 1000\text{GeV}$. The resummation effects are significant for large invariant mass. For example, for $m_0 = 200\text{GeV}$, $\delta K_d$ is larger than 18% and 35% at the LHC and Tevatron for $Q_{\tilde{\chi}_1^\pm \tilde{\chi}_2^0} > 5000\text{GeV}$ and $Q_{\tilde{\chi}_1^\pm \tilde{\chi}_2^0} > 1200\text{GeV}$, respectively. However, in general, $\delta K$ is only a few percent as shown in Figs. 18, 20 and 22.

V. CONCLUSION

In conclusion, we have calculated the QCD effects in the associated production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ in the MSSM within the mSUGRA scenario at both the Tevatron and the LHC, including the NLO SUSY QCD corrections and the NLL threshold resummation effects. Our results show that, compared to the NLO predictions, the threshold resummation effects can increase the total cross sections by 3.6% and 3.9% for the associated production of $\tilde{\chi}_1^+ \tilde{\chi}_2^0$ and $\tilde{\chi}_1^- \tilde{\chi}_2^0$ at the LHC, respectively, and 4.7% for the associated production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ at the Tevatron. In the invariant mass distributions the resummation effects are significant for large invariant mass. The renormalization/factorization scale dependence of the total cross sections at the LHC (Tevatron) is reduced to 5% (4%) with threshold resummation from up to 7% (11%) at NLO.
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APPENDIX A

In this appendix we summarize\[37\] the SUSY vertexes involved in our calculations.

1. The chargino-neutralino-W vertex is

\[ \mathcal{L}_{\tilde{\chi}^i \tilde{\chi}_j^0 W} = -\overline{\tilde{\chi}_i^+} \gamma^\mu (\hat{A}^{ij}_L P_L + \hat{A}^{ij}_R P_R) \tilde{\chi}_j^0 W^\mu - \overline{\tilde{\chi}_j^0} \gamma^\mu (\hat{A}^{ij}_L P_L + \hat{A}^{ij}_R P_R) \tilde{\chi}_i^+ W^-_\mu, \]  

(A1)

with

\[ \hat{A}^{ij}_L = g_W \left( \frac{V_{i2}Z_{j4}}{\sqrt{2}} - V_{i1}Z_{j2} \right), \]  

(A2)

\[ \hat{A}^{ij}_R = -g_W \left( \frac{U_{i2}Z_{j3}}{\sqrt{2}} + U_{i1}Z_{j2} \right), \]  

(A3)

where \( g_W = e / \sin \theta_W \), \( P_L = (1 - \gamma_5)/2 \) and \( P_R = (1 + \gamma_5)/2 \). \( \theta_W \) is the weak mixing angle. \( Z \) is the neutralino mixing matrix while \( V \) and \( U \) are the chargino mixing matrixes. The chargino index is \( i(=1, 2) \) and the neutralino index is \( j(=1, 2, 3, 4) \). Also we define \( A_L = \hat{A}^{12}_L \) and \( A_R = \hat{A}^{12}_R \).

![FIG. 6: The Feynman rules for the chargino-neutralino-W vertex.](image)

2. The chargino-quark-squark vertex is

\[ \mathcal{L}_{\tilde{\chi}^i q^i \bar{q}} = -\bar{C}_U^d u P_R \tilde{\chi}_i^+ d_s - \bar{C}_U^u \overline{\tilde{\chi}_i^+} d_s - \bar{C}_V^d P_R \tilde{\chi}_i^+ \overline{\bar{u}}_s - \bar{C}_V^u \overline{\tilde{\chi}_i^+} \bar{u}_s - \bar{C}_V^d \overline{\tilde{\chi}_i^+} P_L d_s \]  

(A4)
with
\[
C^{usi} = g_W V_{ud} U_{i1} R^{s1}_{u}, \quad (A5)
\]
\[
C^{vis} = g_W V_{ud} V_{i1} R^{s1}_{v}, \quad (A6)
\]
where \( V_{ud} \) is the \((u,d)\) component of the CKM matrix and \( R^{s} \) is the squark mixing matrix. \( s(=1,2) \) is the index of the relevant squarks in the mass eigenstates and \( i(=1,2) \) is the chargino index. We define \( C^{s}_U = \hat{C}^{usi} \) and \( C^{s}_V = \hat{C}^{vis} \).

FIG. 7: The Feynman rules for the chargino-quark-squark vertex.

3. The neutralino-quark-squark vertex is
\[
\mathcal{L}_{\tilde{\chi}^0_{i}\tilde{q}\tilde{q}} = -\tilde{\chi}^0_{j}(a_{s j}^{\tilde{q}} P_L + b_{s j}^{\tilde{q}} P_R) q\tilde{q}_s - \bar{q}(a_{s j}^{\tilde{q}} P_R + b_{s j}^{\tilde{q}} P_L) \tilde{\chi}^0_{j} \tilde{q}_s, \quad (A7)
\]
where
\[
a_{s j}^{\tilde{q}} = \sqrt{2} g_W R^{\tilde{q}}_{s1} [(e_q - I_{3L}^{q}) \tan \theta_W Z_{j1} + I_{3L}^{q} Z_{j2}], \quad (A8)
\]
and
\[
b_{s j}^{\tilde{q}} = -\sqrt{2} g_W e_q \tan \theta_W R^{\tilde{q}}_{s2} Z_{j1}. \quad (A9)
\]
eq and \( I_{3L}^{q} \) is the electric charge and the third component of the weak isospin of the left-handed quark \( q \).
4. The squark-Higgs vertex is

\[ \mathcal{L}_{\tilde{q}\tilde{q}H(G)} = -D^{mn}_H \tilde{u}_m \tilde{d}_n H^- - D^{mn}_H \tilde{u}_m \tilde{d}_n H^+ - D^{mn}_G \tilde{u}_m \tilde{d}_n G^- - D^{mn}_G \tilde{u}_m \tilde{d}_n G^+, \tag{A10} \]

where

\[ D^{mn}_H = g_W V_{ud} R_{m1}^\tilde{u} R_{n1}^\tilde{d} \sin(2\beta) M_W/\sqrt{2}, \tag{A11} \]

and

\[ D^{mn}_G = -g_W V_{ud} R_{m1}^\tilde{u} R_{n1}^\tilde{d} \cos(2\beta) M_W/\sqrt{2}. \tag{A12} \]

\( m \) and \( n \) are the indices of the relevant squarks in the mass eigenstates.

5. The squark-W vertex is

\[ \mathcal{L}_{\tilde{q}\tilde{q}W} = i D^{mn}_W [\tilde{u}_m (\partial^\mu \tilde{d}_n) - (\partial^\mu \tilde{d}_n) \tilde{u}_m] W^_\mu + i D^{mn}_W [\tilde{u}_m (\partial^\mu \tilde{d}_n) - (\partial^\mu \tilde{d}_n) \tilde{u}_m] W^+_\mu, \tag{A13} \]
where
\[ D_{\nu \mu} = g_W V_{u \nu} P_{m1} P_{n1} / \sqrt{2}. \] (A14)

\[ \tilde{u}_m \quad W^-_\mu \quad -i D_{\nu \mu} (p + k)^\mu \]
\[ \tilde{d}_n \quad W^+_\mu \quad -i D_{\nu \mu} (p + k)^\mu \]

FIG. 10: The Feynman rules for the squark-W vertex. \( p \) and \( k \) are the four-momenta of \( \tilde{u}_m \) and \( \tilde{d}_n \) in direction of the charge flow, respectively.

6. The chargino-neutralino-Higgs vertex is
\[
\mathcal{L}_{\tilde{\chi}^+ \tilde{\chi}^0 H(G)} = - \overline{\tilde{\chi}^+_i} [(\tilde{C}^H_R)_{ij} P_L + (\tilde{C}^H_L)_{ij} P_R] \tilde{\chi}^0_j H^+ - \overline{\tilde{\chi}^0_j} [(\tilde{C}^G_R)_{ij} P_L + (\tilde{C}^G_L)_{ij} P_R] \tilde{\chi}^+_i G^- \\
- \overline{\tilde{\chi}^+_i} [(\tilde{C}^H_L)_{ij} P_L + (\tilde{C}^H_R)_{ij} P_R] \tilde{\chi}^0_j H^- - \overline{\tilde{\chi}^0_j} [(\tilde{C}^G_L)_{ij} P_L + (\tilde{C}^G_R)_{ij} P_R] \tilde{\chi}^+_i G^-, \]
\] (A15)

where
\[
(\tilde{C}^H_L)_{ij} = g_W \cos \beta (V_{i1} Z_{j4} + \frac{V_{i2}}{\sqrt{2}} \tan \theta_W Z_{j1} + Z_{j2}), \]
\] (A16)
\[
(\tilde{C}^H_R)_{ij} = g_W \sin \beta (U_{i1} Z_{j3} - \frac{U_{i2}}{\sqrt{2}} \tan \theta_W Z_{j1} + Z_{j2}), \]
\] (A17)
\[
(\tilde{C}^G_L)_{ij} = g_W \sin \beta (V_{i1} Z_{j4} + \frac{V_{i2}}{\sqrt{2}} \tan \theta_W Z_{j1} + Z_{j2}), \]
\] (A18)

and
\[
(\tilde{C}^G_R)_{ij} = -g_W \cos \beta (U_{i1} Z_{j3} - \frac{U_{i2}}{\sqrt{2}} \tan \theta_W Z_{j1} + Z_{j2}). \]
\] (A19)

We define \( C^H_L = (\tilde{C}^H_L)_{12}, \ C^H_R = (\tilde{C}^H_R)_{12}, \ C^G_L = (\tilde{C}^G_L)_{12}, \) and \( C^G_R = (\tilde{C}^G_R)_{12}. \)

7. The SUSY QCD sector of the four-squark vertex is
\[
\mathcal{L}_{qqqq} = -\frac{1}{2} q_S^2 T^\alpha_{rs} T^\alpha_{tu} S^\beta_{ij} S^\gamma_{kl} q^\gamma_{jr} q^\gamma_{lt} q_{ku} \]
\] (A20)

where
\[
S^\alpha_{ij} = R^\alpha_{i1} R^\alpha_{j1} - R^\alpha_{i2} R^\alpha_{j2}. \]
\] (A21)
\[ \bar{\chi}_i^+ \] $\quad$ \[ H^- : -i[(\hat{C}_L^H)_{ij}P_L + (\hat{C}_R^H)_{ij}P_R] \] $\quad$ \[ \bar{\chi}_j^0 \]

\[ \bar{\chi}_i^+ \]$\quad$ \[ G^- : -i[(\hat{C}_L^G)_{ij}P_L + (\hat{C}_R^G)_{ij}P_R] \]$\quad$\[ \bar{\chi}_j^0 \]

\[ \bar{\chi}_i^+ \]$\quad$ \[ H^+ : -i[(\hat{C}_R^H)_{ij}P_L + (\hat{C}_L^H)_{ij}P_R] \]$\quad$ \[ \bar{\chi}_j^0 \]

\[ \bar{\chi}_i^+ \]$\quad$ \[ G^+ : -i[(\hat{C}_R^G)_{ij}P_L + (\hat{C}_L^G)_{ij}P_R] \]$\quad$ \[ \bar{\chi}_j^0 \]

FIG. 11: The Feynman rules for the chargino-neutralino-Higgs vertex.

\[ \alpha \quad \text{and} \quad \beta \quad \text{represent the flavors of the relevant squarks. Here} \quad i, \quad j, \quad k \quad \text{and} \quad l \quad \text{are the relevant squark indices.} \quad r, \quad s, \quad t \quad \text{and} \quad u \quad \text{are the color indices of the relevant squarks.} \]

\[ \bar{\tilde{q}}_k^\beta \quad \bar{\tilde{q}}_l^\beta \]

\[ u \quad t \]

\[ s \quad r \]

\[ \bar{\tilde{q}}_i^{\alpha'} \quad \bar{\tilde{q}}_j^{\alpha'} \]

FIG. 12: The Feynman rules for the SUSY QCD interaction of the four-squark vertex.

8. The squark-gluon vertex is

\[ \mathcal{L}_{q\tilde{q}g} = ig_S T_{rs}^a \delta_{ij} G_\mu^a [\bar{q}_j^a (\partial^\mu \tilde{q}_i^s) - (\partial^\mu \bar{\tilde{q}}_i^s) \tilde{q}_j^a]. \] (A22)

Here \( i \) and \( j \) are the indices of the relevant squarks in the mass eigenstates. \( r \) and \( s \) are the color indices of the relevant squarks.

9. The quark-squark-gluino vertex is

\[ \mathcal{L}_{q\tilde{q}g} = -\sqrt{2} g_S T_{rs}^a [\bar{q}_j^a (R_{i1}^b P_R - R_{i2}^b P_L) \tilde{q}_i^a \bar{\tilde{q}}_s^a + \tilde{g}^a (R_{i1}^b P_L - R_{i2}^b P_R) \tilde{q}_r \bar{q}_s^a]. \] (A23)

Here \( i \) is the mass eigenstate index and \( s \) is the color index of the squark.
\[ g S T a r s \delta_{ij}(p + k) \]

FIG. 13: The Feynman rules for the squark-gluon vertex. \( p \) and \( k \) are the relevant four-momenta of squark \( \tilde{q} \) in direction of the charge flow.

\[ -i \sqrt{2} g S T a \tilde{q}i (R_{L1}P_L - R_{L2}P_R) \]

\[ -i \sqrt{2} g S T a \tilde{q}i (R_{R1}P_R - R_{R2}P_L) \]

FIG. 14: The Feynman rules for the quark-squark-gluino vertex.

APPENDIX B

In this appendix, for simplicity, we introduce the following abbreviations for the Passarino-Veltman two-point integrals \( B_{i} \), three-point integrals \( C_{i(j)} \), and four-point integrals \( D_{i(j)} \), which are defined as in Ref. [17] except that we use internal masses squared as arguments:

\[
\begin{align*}
B_i^{a} &= B_i(M_{\tilde{t}_1}^2, 0, M_{\tilde{d}_a}^2), \\
B_i^{b} &= B_i(M_{\tilde{u}_1}^2, 0, M_{\tilde{d}_a}^2), \\
B_i^{c} &= B_i(\hat{t}, 0, M_{\tilde{d}_a}^2), \\
B_i^{d} &= B_i(M_{\tilde{t}_2}^2, 0, M_{\tilde{u}_a}^2), \\
B_i^{e} &= B_i(\hat{u}, 0, M_{\tilde{u}_a}^2), \\
B_i^{f} &= B_i(M_{\tilde{t}_2}^2, 0, M_{\tilde{d}_a}^2), \\
B_i^{g} &= B_i(\hat{t}, 0, M_{\tilde{d}_a}^2), \\
B_i^{h} &= B_i(M_{\tilde{t}_1}^2, 0, M_{\tilde{u}_a}^2), \\
B_i^{i} &= B_i(\hat{u}, 0, M_{\tilde{u}_a}^2), \\
B_i^{j} &= B_i(\hat{t}, 0, M_{\tilde{d}_a}^2), \\
B_i^{k} &= B_i(\hat{u}, 0, M_{\tilde{d}_a}^2).
\end{align*}
\]
Passarino-Veltman integrals can be reduced \[^{38}\] to the scalar functions \(^{15}\). And the explicit expressions for these singular scalar functions have been calculated previously in a different context \[^{39}\]. The remaining IR finite functions can be calculated by LoopTools \[^{15}\].

\[
B_i^1 = B_i(\hat{s}, 0, 0),
\]

\[
B_i^m = B_i(M_{x_2}^2, 0, M_{\hat{u}_m}^2),
\]

\[
B_i^a = B_i(\hat{u}, 0, M_g^2),
\]

\[
B_i^p = B_i(0, M_{\hat{u}_k}^2, M_g^2),
\]

\[
B_i^q = B_i(0, M_{\hat{d}_k}^2, M_g^2),
\]

\[
B_i^s = B_i(M_{\hat{d}_s}^2, 0, M_g^2),
\]

\[
B_i^t = B_i(M_{\hat{u}_s}^2, 0, M_g^2),
\]

\[
B_i^u = B_i(M_{\hat{d}_s}^2, 0, M_g^2),
\]

\[
C_i^a = C_i(0, \hat{s}, 0, 0, 0, 0),
\]

\[
C_i^b = C_i(0, M_{\chi_1}^2, \hat{t}, 0, 0, M_{\hat{d}_s}^2),
\]

\[
C_i^c = C_i(0, M_{\chi_2}^2, \hat{t}, 0, 0, M_{\hat{d}_s}^2),
\]

\[
C_i^d = C_i(\hat{s}, M_{\chi_2}^2, M_{\chi_1}^2, 0, 0, M_{\hat{u}_s}^2),
\]

\[
C_i^e = C_i(\hat{s}, M_{\chi_1}^2, M_{\chi_2}^2, 0, 0, M_{\hat{d}_s}^2),
\]

\[
C_i^f = C_i(0, M_{\chi_1}^2, \hat{u}, 0, 0, M_{\hat{u}_s}^2),
\]

\[
C_i^g = C_i(0, M_{\chi_2}^2, \hat{u}, 0, 0, M_{\hat{u}_s}^2),
\]

\[
C_i^h = C_i(\hat{t}, 0, M_{\chi_1}^2, 0, M_g^2, M_{\hat{u}_m}^2),
\]

\[
C_i^i = C_i(0, \hat{s}, 0, M_g^2, M_{\hat{u}_s}^2, M_{\hat{d}_m}^2),
\]

\[
C_i^j = C_i(\hat{u}, M_{\chi_1}^2, 0, M_g^2, 0, M_{\hat{d}_m}^2),
\]

\[
C_i^k = C_i(\hat{u}, M_{\chi_1}^2, 0, M_g^2, 0, M_{\hat{d}_m}^2),
\]

\[
C_i^l = C_i(\hat{u}, M_{\chi_2}^2, 0, M_g^2, 0, M_{\hat{d}_m}^2),
\]

\[
C_i^m = C_i(\hat{t}, M_{\chi_2}^2, 0, M_g^2, 0, M_{\hat{d}_m}^2),
\]

\[
D_i^a = D_i(0, \hat{s}, M_{\chi_1}^2, \hat{t}, 0, M_{\chi_2}^2, 0, 0, 0, M_{\hat{u}_m}^2),
\]

\[
D_i^b = D_i(0, \hat{s}, M_{\chi_2}^2, \hat{u}, 0, M_{\chi_1}^2, 0, 0, 0, M_{\hat{u}_m}^2),
\]

\[
D_i^c = D_i(\hat{u}, M_{\chi_2}^2, \hat{s}, 0, 0, M_{\chi_1}^2, M_g^2, 0, M_{\hat{u}_m}^2, M_{\hat{d}_m}^2),
\]

\[
D_i^d = D_i(\hat{t}, 0, \hat{s}, M_{\chi_2}^2, M_{\chi_1}^2, 0, 0, M_g^2, M_{\hat{u}_s}^2, M_{\hat{d}_m}^2).
\]

Many of the above functions contain soft and/or collinear singularities, but all the Passarino-Veltman integrals can be reduced \[^{38}\] to the scalar functions \(B_0\), \(C_0\) and \(D_0\). And the explicit expressions for these singular scalar functions have been calculated previously in a different context \[^{39}\]. The remaining IR finite functions can be calculated by LoopTools \[^{15}\].
APPENDIX C

In this appendix we collect the explicit expressions for the nonzero form factors in Eqs.\((15)-(35)\). The standard matrix elements in Eqs.\((15)-(35)\) for the subprocess
\[
u(p_1) + d(p_2) \rightarrow \bar{\chi}_1^+(p_3) + \bar{\chi}_2^0(p_4),
\]
\(\text{(C1)}\)
are defined as follows:
\[
\begin{align*}
M_{1}^{ab} &= \bar{v}_2 P_a \gamma^\mu u_1 \bar{u}_4 P_b \gamma_\mu v_3, \\
M_{2}^{ab} &= \bar{v}_1 P_a v_3 \bar{v}_2 P_b u_4, \\
M_{3}^{ab} &= \bar{v}_3 P_a v_3 \bar{u}_4 P_b u_1, \\
M_{4}^{ab} &= \bar{v}_2 P_a v_4 \bar{u}_3 P_b u_1, \\
M_{5}^{ab} &= \bar{v}_2 P_a v_3 \bar{u}_4 P_b \hat{p}_2 u_1, \\
M_{6}^{ab} &= \bar{v}_2 P_a v_3 \bar{u}_4 P_b \hat{p}_2 \hat{p}_3 u_1, \\
M_{7}^{ab} &= \bar{v}_2 P_a v_3 \bar{u}_4 P_b \hat{p}_3 u_1, \\
M_{8}^{ab} &= \bar{v}_2 P_a v_4 \bar{u}_3 P_b \hat{p}_2 u_1, \\
M_{9}^{ab} &= \bar{u}_4 P_a u_1 \bar{v}_2 P_b \hat{p}_1 v_3, \\
M_{10}^{ab} &= \bar{u}_3 P_a u_1 \bar{v}_2 P_b \hat{p}_1 v_4, \\
M_{11}^{ab} &= \bar{v}_2 P_a \hat{p}_1 \gamma^\mu v_3 \bar{u}_4 P_b \hat{p}_2 \gamma_\mu u_1, \\
M_{12}^{ab} &= \bar{u}_4 P_a \gamma^\mu u_1 \bar{v}_2 P_b \hat{p}_1 \gamma_\mu v_3, \\
M_{13}^{ab} &= \bar{u}_3 P_a \gamma^\mu u_1 \bar{v}_2 P_b \hat{p}_1 \gamma_\mu v_4, \\
M_{14}^{ab} &= \bar{v}_2 P_a \hat{p}_1 \gamma^\mu v_4 \bar{u}_3 P_b \hat{p}_2 \gamma_\mu u_1, \\
M_{15}^{ab} &= \bar{v}_2 P_a \gamma^\mu v_3 \bar{u}_4 P_b \hat{p}_2 \gamma_\mu u_1, \\
M_{16}^{ab} &= \bar{v}_2 P_a \gamma^\mu v_3 \bar{u}_4 P_b \gamma_\mu u_1, \\
M_{17}^{ab} &= \bar{v}_2 P_a \gamma^\mu v_3 \bar{u}_4 P_b \gamma_\mu u_1, \\
M_{18}^{ab} &= \bar{v}_2 P_a \gamma^\mu v_4 \bar{u}_3 P_b \gamma_\mu u_1, \\
M_{19}^{ab} &= \bar{v}_2 P_a \gamma^\mu \gamma^\nu v_3 \bar{u}_4 P_b \gamma\gamma_\nu u_1, \\
M_{20}^{ab} &= \bar{v}_2 P_a \gamma^\mu \gamma^\nu v_4 \bar{u}_3 P_b \gamma\gamma_\nu u_1, \\
M_{21}^{ab} &= \bar{v}_2 P_a u_1 \bar{u}_4 P_b v_3, \\
M_{22}^{ab} &= \bar{v}_2 P_a u_1 \bar{u}_4 P_b \hat{p}_1 v_3, \\
M_{23}^{ab} &= \bar{u}_4 P_a v_3 \bar{v}_2 P_b \hat{p}_3 u_1, \\
M_{24}^{ab} &= \bar{v}_2 P_a \hat{p}_3 u_1 \bar{u}_4 P_b \hat{p}_1 v_3,
\end{align*}
\]
where \(a\) and \(b\) are the left-hand index \(L\) or right-hand index \(R\), while \(u_i = u(p_i)\) and \(v_i = v(p_i)\) are the spinors of the particle with momentum \(p_i\).
The nonzero form factors in Eqs. (15)-(35) are the following:

\begin{align*}
    f^{RR}_{QCDV1} &= \frac{A_1 D_L \alpha_s}{3 \pi (s - M_W^2)} (-2 B_0^f + 2 \delta C_0^a + 4 C_{00}^a + 2 \delta C_1^a + 2 \delta C_2^a + 1) \\
    f^{RL}_{QCDV1} &= \frac{A_1 D_L \alpha_s}{3 \pi (s - M_W^2)} (-2 B_0^f + 2 \delta C_0^a + 4 C_{00}^a + 2 \delta C_1^a + 2 \delta C_2^a + 1) \\
    f^{LR}_{QCDV2} &= \sum_{s=1}^2 \frac{d_{s2}^2 C_{s2}^s \alpha_s}{3 \pi (t - M_{d_s}^2)^2} \left\{ (t - M_{d_s}^2) B_0^f + 4 t B_0^e + 4 t B_1^e + (t - M_{d_s}^2) \left[ \left( \hat{s} + 4 t + \hat{u} - M_{d_s}^2 \right) + A_0(M_{d_s}^2) \right] \right\} \\
    f^{LL}_{QCDV2} &= \sum_{s=1}^2 \frac{d_{s2}^2 C_{s2}^s \alpha_s}{3 \pi (t - M_{d_s}^2)^2} \left\{ (t - M_{d_s}^2) B_0^f + 4 t B_0^e + 4 t B_1^e + (t - M_{d_s}^2) \left[ \left( \hat{s} + 4 t + \hat{u} - M_{d_s}^2 \right) + A_0(M_{d_s}^2) \right] \right\} \\
    f^{RR}_{QCDV3} &= \sum_{s=1}^2 \frac{d_{s2}^2 C_{s2}^s \alpha_s}{3 \pi (u - M_{d_s}^2)^2} \left\{ (u - M_{d_s}^2) B_0^f + 4 u B_0^e + 4 u B_1^e + (u - M_{d_s}^2) \left( B_0^f + 2(\hat{u} - M_{d_s}^2) C_{d2}^f + 2(\hat{u} - M_{d_s}^2) C_{d2}^f - (\hat{s} + \hat{t} + \hat{u} - M_{d_s}^2) \right) + A_0(M_{d_s}^2) \right\}
\end{align*}
\[ f_{QCDV}^{RL} = -\sum_{s=1}^{2} \frac{a_s^d C_s^p \alpha_s}{3\pi (t - M_{d_s}^2)} \left( B_0^a + 2(\hat{t} - M_{x_1}^2) C_0^b + 2(\hat{t} - M_{x_1}^2) C_1^b + (3\hat{t} - M_{x_1}^2) C_2^b \right. \\
- 2(\hat{t} - M_{d_s}^2) \left( -C_0^c + \hat{s} D_0^a + \hat{s} D_1^a + (M_{x_1}^2 - \hat{u}) D_{13} + (2\hat{s} + \hat{t} + \hat{u} - M_{x_1}^2) D_2^a \right. \\
+ (\hat{s} + \hat{t} - M_{x_1}^2) D_{23} + 2\hat{s} D_3^a + (\hat{s} + \hat{t}) D_{33} \right) \] 
\[ f_{QCDV}^{RR} = -\sum_{s=1}^{2} \frac{b_s^d C_s^p \alpha_s}{3\pi (t - M_{d_s}^2)} \left( B_0^a + 2(\hat{t} - M_{x_1}^2) C_0^b + 2(\hat{t} - M_{x_1}^2) C_1^b + (3\hat{t} - M_{x_1}^2) C_2^b \right. \\
- 2(\hat{t} - M_{d_s}^2) \left( -C_0^c + \hat{s} D_0^a + \hat{s} D_1^a + (M_{x_1}^2 - \hat{u}) D_{13} + (2\hat{s} + \hat{t} + \hat{u} - M_{x_1}^2) D_2^a \right. \\
+ (\hat{s} + \hat{t} - M_{x_1}^2) D_{23} + 2\hat{s} D_3^a + (\hat{s} + \hat{t}) D_{33} \right) \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{a_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} (D_{12}^b + D_{13}^b + D_{23}^b + D_{33}^b) \] 
\[ f_{QCDV}^{RL} = \sum_{s=1}^{2} \frac{b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} (D_{12}^b + D_{13}^b + D_{23}^b + D_{33}^b) \] 
\[ f_{QCDV}^{RR} = -\sum_{s=1}^{2} \frac{2a_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RL} = -\sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2a_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RL} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = -\sum_{s=1}^{2} \frac{2a_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RL} = -\sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = -\sum_{s=1}^{2} \frac{2a_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{LL} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \] 
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{2b_s^s M_{x_2}^2 C_s^p \alpha_s}{3\pi} \]
\[ f_{QCDV}^{RL} = \sum_{s=1}^{2} \frac{a_{s2}^{s} C_{V}^{s} D_{12}^{b} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{a_{s2}^{s} M_{\chi_{1}^{0}} C_{V}^{s} D_{23}^{b} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{LR} = \sum_{s=1}^{2} \frac{b_{s2}^{s} M_{\chi_{1}^{0}} C_{V}^{s} D_{23}^{b} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{RR} = -\sum_{s=1}^{2} \frac{a_{s2}^{s} M_{\chi_{1}^{+}} C_{U}^{s} D_{12}^{a} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{LR} = -\sum_{s=1}^{2} \frac{b_{s2}^{s} M_{\chi_{1}^{+}} C_{U}^{s} D_{12}^{a} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{RR} = \sum_{s=1}^{2} \frac{b_{s2}^{s} M_{\chi_{2}^{0}} C_{V}^{s} D_{13}^{a} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{LR} = \sum_{s=1}^{2} \frac{b_{s2}^{s} M_{\chi_{2}^{0}} C_{V}^{s} D_{13}^{a} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{RR} = -\sum_{s=1}^{2} \frac{a_{s2}^{s} M_{\chi_{1}^{+}} M_{\chi_{2}^{0}} C_{V}^{s} D_{23}^{b} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{LR} = -\sum_{s=1}^{2} \frac{b_{s2}^{s} M_{\chi_{1}^{+}} M_{\chi_{2}^{0}} C_{V}^{s} D_{23}^{b} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{RL} = -\sum_{s=1}^{2} \frac{a_{s2}^{s} M_{\chi_{1}^{0}} M_{\chi_{2}^{0}} C_{V}^{s} D_{13}^{a} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{LR} = -\sum_{s=1}^{2} \frac{b_{s2}^{s} M_{\chi_{1}^{0}} M_{\chi_{2}^{0}} C_{V}^{s} D_{13}^{a} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{RL} = -\sum_{s=1}^{2} \frac{a_{s2}^{s} M_{\chi_{2}^{0}} C_{U}^{s} D_{13}^{a} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{LR} = -\sum_{s=1}^{2} \frac{b_{s2}^{s} M_{\chi_{2}^{0}} C_{U}^{s} D_{13}^{a} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{RR} = -\sum_{s=1}^{2} \frac{b_{s2}^{s} C_{V}^{s} D_{23}^{b} \alpha_{s}}{3\pi} \]
\[ f_{QCDV}^{LR} = -\sum_{s=1}^{2} \frac{a_{s2}^{s} C_{V}^{s} D_{23}^{b} \alpha_{s}}{3\pi} \]
\[ f_{QCDV20}^{RL} = \sum_{s=1}^{2} \frac{a_{s}^{\tilde{d}} C_{U}^{s} D_{00}^{s} \alpha_{s}}{3\pi} \]

\[ f_{QCDV20}^{LL} = \sum_{s=1}^{2} \frac{b_{s}^{\tilde{d}} C_{U}^{s} D_{00}^{s} \alpha_{s}}{3\pi} \]

\[ f_{SUSY1}^{RR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2\alpha_{s} R_{s1}^{\tilde{d}} R_{m1}^{\tilde{d}}}{3\pi(\tilde{s} - M_{d}^{2})} (a_{s}^{\tilde{d}} C_{U}^{m} D_{00}^{s}(\tilde{s} - M_{W}^{2}) - 2A_{R} C_{00}^{i} D_{W}^{m}s) \]

\[ f_{SUSY1}^{LR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2\alpha_{s} R_{s2}^{\tilde{d}} R_{m2}^{\tilde{d}}}{3\pi(\tilde{s} - M_{d}^{2})} (a_{s}^{\tilde{d}} C_{U}^{m} D_{00}^{s}(\tilde{s} - M_{W}^{2}) - 2A_{R} C_{00}^{i} D_{W}^{m}s) \]

\[ f_{SUSY1}^{RL} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2\alpha_{s} R_{s2}^{\tilde{d}} R_{m1}^{\tilde{d}}}{3\pi(\tilde{s} - M_{d}^{2})} (2A_{L} D_{W}^{ms} C_{00}^{i} + a_{m2}^{d} C_{V}^{d} D_{00}^{s}(\tilde{s} - M_{W}^{2})) \]

\[ f_{SUSY1}^{LL} = -\sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2\alpha_{s} R_{s1}^{\tilde{d}} R_{m2}^{\tilde{d}}}{3\pi(\tilde{s} - M_{d}^{2})} (2A_{L} D_{W}^{ms} C_{00}^{i} + a_{m2}^{d} C_{V}^{d} D_{00}^{s}(\tilde{s} - M_{W}^{2})) \]

\[ f_{SUSY2}^{RR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{\alpha_{s}}{3\pi(\tilde{s} - M_{d}^{2})} \left\{ 2M_{s} a_{s}^{\tilde{d}} m_{\tilde{x}_{1}}^{2} (\tilde{s} - M_{d}^{2}) (C_{1}^{h} + C_{2}^{h}) R_{m1}^{\tilde{d}} C_{V}^{m} \right. \]

\[ + C_{U}^{s} \left[ 2b_{s2}^{\tilde{d}} (\tilde{s} - M_{d}^{2}) \left( B_{0}^{s} + (\tilde{s} + 2\tilde{t} + \tilde{u} - M_{2}^{2} + M_{\tilde{x}_{1}}^{2}) C_{1}^{m} + (2\tilde{s} + 2\tilde{t} + 2\tilde{u} - 2M_{\tilde{x}_{1}}^{2}) \right) \right. \]

\[ - \left. M_{\tilde{x}_{2}}^{2} C_{2}^{m} \right) R_{m1}^{\tilde{d}} C_{2}^{m} + a_{m2}^{d} \left\{ 4(R_{m1}^{\tilde{d}} R_{s1}^{d} + R_{m2}^{\tilde{d}} R_{s2}^{d}) (\tilde{s} B_{1}^{d} + A_{0}^{m}) - \sum_{n=1}^{2} D_{m}^{mnns} A_{0}(M_{d}^{2}) \right. \]

\[ + 2M_{s} M_{\tilde{x}_{2}}^{2} (\tilde{s} - M_{m}^{2}) (C_{1}^{m} + C_{2}^{m}) R_{m1}^{\tilde{d}} R_{s1}^{d} \} \}

\[ f_{SUSY2}^{LL} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{\alpha_{s}}{3\pi(\tilde{s} - M_{d}^{2})} \left\{ 2M_{s} b_{s2}^{\tilde{d}} M_{\tilde{x}_{1}}^{2} (\tilde{s} - M_{d}^{2}) (C_{1}^{h} + C_{2}^{h}) R_{m1}^{\tilde{d}} C_{V}^{m} \right. \]

\[ + C_{U}^{s} \left[ 2a_{s2}^{\tilde{d}} (\tilde{s} - M_{d}^{2}) \left( B_{0}^{s} + (\tilde{s} + 2\tilde{t} + \tilde{u} - M_{2}^{2} + M_{\tilde{x}_{1}}^{2}) C_{1}^{m} + (2\tilde{s} + 2\tilde{t} + 2\tilde{u} - 2M_{\tilde{x}_{1}}^{2}) \right) \right. \]

\[ - \left. M_{\tilde{x}_{2}}^{2} C_{2}^{m} \right) R_{m1}^{\tilde{d}} C_{2}^{m} + b_{m2}^{d} \left\{ 4(R_{m1}^{\tilde{d}} R_{s1}^{d} + R_{m2}^{\tilde{d}} R_{s2}^{d}) (\tilde{s} B_{1}^{d} + A_{0}^{m}) - \sum_{n=1}^{2} D_{m}^{mnns} A_{0}(M_{d}^{2}) \right. \]

\[ + 2M_{s} M_{\tilde{x}_{2}}^{2} (\tilde{s} - M_{m}^{2}) (C_{1}^{m} + C_{2}^{m}) R_{m1}^{\tilde{d}} R_{s1}^{d} \} \}

\[ f_{SUSY2}^{RR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{a_{s}^{\tilde{d}} C_{V}^{s} \alpha_{s} R_{m2}^{\tilde{d}} R_{s1}^{d}}{3\pi(\tilde{s} - M_{d}^{2})} (B_{0}^{s} + \tilde{t} C_{1}^{h} + M_{\tilde{x}_{1}}^{2} C_{2}^{h}) \]

\[ f_{SUSY2}^{LR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{b_{s}^{\tilde{d}} C_{V}^{s} \alpha_{s} R_{m2}^{\tilde{d}} R_{s1}^{d}}{3\pi(\tilde{s} - M_{d}^{2})} (B_{0}^{s} + \tilde{t} C_{1}^{h} + M_{\tilde{x}_{1}}^{2} C_{2}^{h}) \]

\[ f_{SUSY3}^{RR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{\alpha_{s}}{3\pi(\tilde{s} - M_{d}^{2})} \left\{ -2M_{s} R_{m2}^{\tilde{d}} R_{s2}^{d} C_{V}^{m} (\tilde{s} - M_{d}^{2}) (C_{0}^{l} + C_{1}^{l}) R_{m2}^{\tilde{d}} R_{s2}^{d} C_{V}^{m} \right. \]

\[ + 2M_{s} R_{m2}^{\tilde{d}} R_{s2}^{d} C_{V}^{m} (\tilde{s} - M_{d}^{2}) (B_{0}^{s} + (\tilde{s} - \tilde{t} - \tilde{u} + M_{2}^{2} + M_{\tilde{x}_{1}}^{2} + M_{\tilde{x}_{2}}^{2}) C_{0}^{l} \]

\[ + (\tilde{s} - \tilde{t} + M_{2}^{2} C_{1}^{m} + b_{s2}^{d} C_{V}^{m} (4(M_{2} B_{0}^{s} + \tilde{t} B_{1}^{d}) (R_{m1}^{\tilde{d}} R_{s1}^{d} + R_{m2}^{\tilde{d}} R_{s2}^{d}) \right. \]

\[ - \sum_{n=1}^{2} D_{m}^{mnns} A_{0}(M_{d}^{2}) - 2M_{s} M_{\tilde{x}_{1}}^{2} (\tilde{s} - M_{d}^{2}) (C_{0}^{l} + C_{1}^{l}) C_{U}^{m} R_{m1}^{\tilde{d}} R_{s1}^{d}) \]
\[ f_{\text{SUSY3}}^{RL} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{\alpha_s}{3\pi(\tilde{u} - M_{\tilde{u}}^2)(\tilde{u} - M_{\tilde{u}}^2)} (-2M_3 a_{m2}^\alpha M_{\chi_2}^0(\tilde{u} - M_{\tilde{u}}^2)(C_0^i + C_1^i) R_{m1} \tilde{R}_{s1} C_V^i + 2b_{m2}^\alpha R_{m1} \tilde{R}_{s2} C_V^i(\tilde{u} - M_{\tilde{u}}^2)(B_0^m + (-\tilde{s} - \tilde{t} + \tilde{u} + M_{\tilde{u}}^2 + M_{\chi_2}^0) C_0^i + (-\tilde{s} - \tilde{t} + M_{\chi_1}^2 + M_{\chi_2}^0) C_1^i) + a_{s2}(\tilde{C}_V^m(4(M_2^2 B_0^m + \tilde{u} B_k)(R_{m1} \tilde{R}_{s1} + \tilde{R}_{m2} R_{s2})) - 2D_{mn} A_0(M_{\tilde{u}}^2)) - 2M_3 M_{\chi_1}^2(\tilde{u} - M_{\tilde{u}}^2)(C_{0k}^i + C_{1k}^i) C_V^m R_{m1} \tilde{R}_{s1})) \]

\[ f_{\text{SUSY3}}^{LL} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2b_{m2}^\alpha C_U m_{R_{m2} R_{s1}}}{3\pi(\tilde{u} - M_{\tilde{u}}^2)} (B_0^h + M_2^2 C_{0k}^i + \tilde{u} C_{1k}^i) \]

\[ f_{\text{SUSY21}}^{RR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2M_3 C_{U} R_{s2} R_{m1}}{3\pi(\tilde{s} - M_{\tilde{u}}^2)(\tilde{s} - M_{\tilde{u}}^2)} ((-C_{L}^G(\tilde{s} - M_{\tilde{u}}^2)) D_{G}^{ms} - C_{L}^H D_{H}^{ms}(\tilde{s} - M_{\tilde{u}}^2)) C_0^i + (\tilde{s} - M_{\tilde{u}}^2)(M_{\tilde{u}} A_L - M_{\tilde{u}} A_{R})(C_0^i + 2C_2^i) D_{W}^{ms} + a_{m2}^\alpha M_{\chi_1}^0 C_{V}^i (D_{1}^d + D_{2}^d) \]

\[ f_{\text{SUSY21}}^{RR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2M_3 C_{U} R_{s2} R_{m1}}{3\pi(\tilde{s} - M_{\tilde{u}}^2)(\tilde{s} - M_{\tilde{u}}^2)} ((-C_{R}^G(\tilde{s} - M_{\tilde{u}}^2)) D_{G}^{ms} - C_{R}^H D_{H}^{ms}(\tilde{s} - M_{\tilde{u}}^2)) C_0^i - (\tilde{s} - M_{\tilde{u}}^2)(M_{\tilde{u}} A_L - M_{\tilde{u}} A_{R})(C_0^i + 2C_2^i) D_{W}^{ms} + a_{m2}^\alpha M_{\chi_1}^0 C_{V}^i (D_{1}^d + D_{2}^d) \]

\[ f_{\text{SUSY21}}^{LR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2M_3 C_{U} R_{s2} R_{m1}}{3\pi(\tilde{s} - M_{\tilde{u}}^2)(\tilde{s} - M_{\tilde{u}}^2)} ((-C_{L}^G(\tilde{s} - M_{\tilde{u}}^2)) D_{G}^{ms} - C_{L}^H D_{H}^{ms}(\tilde{s} - M_{\tilde{u}}^2)) C_0^i + (\tilde{s} - M_{\tilde{u}}^2)(M_{\tilde{u}} A_L - M_{\tilde{u}} A_{R})(C_0^i + 2C_2^i) D_{W}^{ms} + (\tilde{s} - M_{\tilde{u}}^2) a_{m2}^\alpha M_{\chi_1}^0 C_{V}^i (D_{1}^d + D_{2}^d) \]

\[ f_{\text{SUSY21}}^{LR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2M_3 C_{U} R_{s2} R_{m1}}{3\pi(\tilde{s} - M_{\tilde{u}}^2)(\tilde{s} - M_{\tilde{u}}^2)} ((-C_{R}^G(\tilde{s} - M_{\tilde{u}}^2)) D_{G}^{ms} - C_{R}^H D_{H}^{ms}(\tilde{s} - M_{\tilde{u}}^2)) C_0^i - (\tilde{s} - M_{\tilde{u}}^2)(M_{\tilde{u}} A_L - M_{\tilde{u}} A_{R})(C_0^i + 2C_2^i) D_{W}^{ms} + (\tilde{s} - M_{\tilde{u}}^2) a_{m2}^\alpha M_{\chi_1}^0 C_{V}^i (D_{1}^d + D_{2}^d) \]

\[ f_{\text{SUSY21}}^{LL} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2M_3 C_{U} R_{s2} R_{m1}}{3\pi(\tilde{s} - M_{\tilde{u}}^2)} ((-C_{L}^G(\tilde{s} - M_{\tilde{u}}^2)) D_{G}^{ms} - C_{L}^H D_{H}^{ms}(\tilde{s} - M_{\tilde{u}}^2)) C_0^i - (\tilde{s} - M_{\tilde{u}}^2)(M_{\tilde{u}} A_L - M_{\tilde{u}} A_{R})(C_0^i + 2C_2^i) D_{W}^{ms} + a_{m2}^\alpha M_{\chi_1}^0 C_{V}^i (D_{1}^d + D_{2}^d) \]

\[ f_{\text{SUSY22}}^{RR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2M_3 C_{U} R_{s2} R_{m1}}{3\pi(\tilde{s} - M_{\tilde{u}}^2)(\tilde{s} - M_{\tilde{u}}^2)} (a_{m2}^\alpha C_{V}^m (D_{0}^d + D_{1}^c + D_{2}^c + D_{3}^c)(\tilde{s} - M_{\tilde{u}}^2) - 2A_{R}(C_0^i + C_1^i + C_2^i) D_{W}^{ms}) \]

\[ f_{\text{SUSY22}}^{LR} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2M_3 C_{U} R_{s2} R_{m1}}{3\pi(\tilde{s} - M_{\tilde{u}}^2)(\tilde{s} - M_{\tilde{u}}^2)} (a_{m2}^\alpha C_{V}^m (D_{0}^d + D_{1}^c + D_{2}^c + D_{3}^c)(\tilde{s} - M_{\tilde{u}}^2) - 2A_{R}(C_0^i + C_1^i + C_2^i) D_{W}^{ms}) \]

\[ f_{\text{SUSY22}}^{RL} = \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2M_3 C_{U} R_{s2} R_{m1}}{3\pi(\tilde{s} - M_{\tilde{u}}^2)(\tilde{s} - M_{\tilde{u}}^2)} (a_{m2}^\alpha C_{V}^m (D_{0}^d + D_{1}^c + D_{2}^c + D_{3}^c)(\tilde{s} - M_{\tilde{u}}^2) - 2A_{R}(C_0^i + C_1^i + C_2^i) D_{W}^{ms}) \]
\[
\begin{align*}
 f_{\text{SU SY}23}^{RR} &= \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2 \alpha_s R^s_{11 \lambda m} \lambda^d_{\bar{s}_2}}{3 \pi} (a^d_{\bar{s}_2} M_{\chi^c_U} \tilde{D}^c_{13} + a^d_{m2} M_{\chi^c_V} \tilde{C}^s_{13}) \quad (D^d_{13} + D^d_{23} + D^d_3 + D^d_{33}) \\
 f_{\text{SU SY}23}^{RL} &= \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2 \alpha_s R^s_{12 \lambda m} \lambda^d_{\bar{s}_2}}{3 \pi} (a^d_{\bar{s}_2} M_{\chi^c_U} \tilde{D}^c_{13} + a^d_{m2} M_{\chi^c_V} \tilde{C}^s_{13}) \quad (D^d_{13} + D^d_{23} + D^d_3 + D^d_{33}) \\
 f_{\text{SU SY}23}^{LR} &= - \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2 \alpha_s R^s_{11 \lambda m} \lambda^d_{\bar{s}_2}}{3 \pi} (a^d_{\bar{s}_2} M_{\chi^c_U} \tilde{D}^c_{13} + a^d_{m2} M_{\chi^c_V} \tilde{C}^s_{13}) \quad (D^d_{13} + D^d_{11} + D^d_{22} + D^d_{23}) \\
 f_{\text{SU SY}23}^{LL} &= - \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2 \alpha_s R^s_{12 \lambda m} \lambda^d_{\bar{s}_2}}{3 \pi} (a^d_{\bar{s}_2} M_{\chi^c_U} \tilde{D}^c_{13} + a^d_{m2} M_{\chi^c_V} \tilde{C}^s_{13}) \quad (D^d_{13} + D^d_{11} + D^d_{22} + D^d_{23}) \\
 f_{\text{SU SY}24}^{RR} &= \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2 \alpha_s R^s_{11 \lambda m} \lambda^d_{\bar{s}_2}}{3 \pi} (D^d_{11} + D^d_{12} + D^d_{13}) \\
 f_{\text{SU SY}24}^{RL} &= \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2 \alpha_s R^s_{12 \lambda m} \lambda^d_{\bar{s}_2}}{3 \pi} (D^d_{11} + D^d_{12} + D^d_{13}) \\
 f_{\text{SU SY}24}^{LR} &= \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2 \alpha_s R^s_{11 \lambda m} \lambda^d_{\bar{s}_2}}{3 \pi} (D^d_{11} + D^d_{12} + D^d_{13}) \\
 f_{\text{SU SY}24}^{LL} &= \sum_{m=1}^{2} \sum_{s=1}^{2} \frac{2 \alpha_s R^s_{12 \lambda m} \lambda^d_{\bar{s}_2}}{3 \pi} (D^d_{11} + D^d_{12} + D^d_{13}) \\
 f_{\text{QCDC2}}^{QR} &= - \sum_{s=1}^{2} \frac{4 M^2_{d_s} C^s \alpha_s a^d_{\bar{s}_2}}{3 \pi (\bar{t} - M^2_{d_s})^2} (B^u_{\bar{t}} + B^u_{\bar{t}}) \\
 f_{\text{QCDC2}}^{QL} &= - \sum_{s=1}^{2} \frac{4 M^2_{d_s} C^s \alpha_s b^d_{\bar{s}_2}}{3 \pi (\bar{t} - M^2_{d_s})^2} (B^u_{\bar{t}} + B^u_{\bar{t}}) \\
 f_{\text{QCDC3}}^{QR} &= - \sum_{s=1}^{2} \frac{4 M^2_{d_s} b^d_{\bar{s}_2} C^s \alpha_s}{3 \pi (\bar{u} - M^2_{d_s})^2} (B^u_{\bar{t}} + B^u_{\bar{t}}) \\
 f_{\text{QCDC3}}^{QL} &= - \sum_{s=1}^{2} \frac{4 M^2_{d_s} a^d_{\bar{s}_2} C^s \alpha_s}{3 \pi (\bar{u} - M^2_{d_s})^2} (B^u_{\bar{t}} + B^u_{\bar{t}}) \\
 f_{\text{SU SY}1}^{RR} &= \sum_{k=1}^{2} \frac{A R D^L \alpha_s}{3 \pi (\bar{s} - M^2_{\bar{t}})} (R^u_{1k} B^u_{\bar{t}} + R^d_{1k} B^d_{\bar{t}} + B^u_{\bar{t}} R^u_{1k} + B^d_{\bar{t}} R^d_{1k}) \\
 f_{\text{SU SY}1}^{RL} &= \sum_{k=1}^{2} \frac{A R D^L \alpha_s}{3 \pi (\bar{s} - M^2_{\bar{t}})} (R^u_{1k} B^u_{\bar{t}} + R^d_{1k} B^d_{\bar{t}} + B^u_{\bar{t}} R^u_{1k} + B^d_{\bar{t}} R^d_{1k}) \\
 f_{\text{SU SY}2}^{RR} &= \sum_{k=1}^{2} \left\{ \sum_{s=1}^{2} \sum_{k=1}^{2} \frac{C^s \alpha_s a^d_{m2}}{3 \pi (\bar{t} - M^2_{d_s})(\bar{t} - M^2_{d_s})^2} (R^u_{1k} R^d_{1k} - R^u_{1k} R^d_{1k}) \right\}_{A_0(M^2_{d_s})} \\
 &- \frac{C^s \alpha_s a^d_{m2}}{3 \pi (\bar{t} - M^2_{d_s})^2} \left[ (\bar{t} - M^2_{d_s}) \sum_{k=1}^{2} (R^u_{1k} B^u_{\bar{t}} + R^d_{1k} B^d_{\bar{t}} + R^u_{1k} B^u_{\bar{t}} + R^d_{1k} B^d_{\bar{t}}) \right] + 4 M^2_{d_s} B^u_{\bar{t}} 
\end{align*}
\]
\[ f_{SU SY C}^{LL} = \frac{2}{3\pi (\bar{t} - M_{d_{3}}^2)^2} \left[ \sum_{k=1}^{2} (\bar{t} - M_{d_{3}}^2) \left( \sum_{m=1}^{2} C_{U}^{\alpha_{s}} b_{m}^{2} \sum_{k=1}^{2} (R_{k_{1}}^{R} R_{s_{1}}^{R} - R_{k_{2}}^{R} R_{s_{2}}^{R}) A_{0}(M_{d_{3}}^2) \right) \right] \]

\[ f_{SU SY C}^{RR} = \frac{2}{3\pi (\bar{t} - M_{d_{3}}^2)^2} \left[ \sum_{k=1}^{2} (\bar{t} - M_{d_{3}}^2) \left( \sum_{m=1}^{2} C_{U}^{\alpha_{s}} b_{m}^{2} \sum_{k=1}^{2} (R_{k_{1}}^{R} R_{s_{1}}^{R} - R_{k_{2}}^{R} R_{s_{2}}^{R}) A_{0}(M_{d_{3}}^2) \right) \right] \]

The standard matrix elements in Eq. (48) for the subprocess

\[ u(p_{1}) + \bar{d}(p_{2}) \rightarrow \bar{\chi}_{1}^{+}(p_{3}) + \bar{\chi}_{2}^{0}(p_{4}) + g(p_{5}), \quad (C2) \]

are defined as follows:

\[ M_{G1}^{ab} = \bar{v}_{1}^{m} P_{a} v_{3}^{m} \bar{u}_{2}^{m} P_{b} v_{4}[\epsilon \cdot (p_{1} - p_{3})](T^{x})_{mn}, \]

\[ M_{G2}^{ab} = \bar{v}_{2}^{m} P_{a} v_{4} u_{3}^{m} P_{b} u_{4}[\epsilon \cdot (p_{1} - p_{3})](T^{x})_{mn}, \]

\[ M_{G3}^{ab} = \bar{v}_{2}^{m} P_{a} v_{4} u_{3}^{m} P_{b} u_{4}[\epsilon \cdot (p_{1} - p_{3})](T^{x})_{mn}, \]

\[ M_{G4}^{ab} = \bar{u}_{1}^{m} P_{a} v_{3}^{m} \bar{v}_{2}^{m} P_{b} \epsilon^{x} u_{1}^{m}(T^{x})_{mn}, \]

\[ M_{G5}^{ab} = \bar{u}_{1}^{m} P_{a} v_{4} u_{3}^{m} \bar{v}_{2}^{m} P_{b} \epsilon^{x} u_{1}^{m}(T^{x})_{mn}, \]

\[ M_{G6}^{ab} = \bar{v}_{1}^{m} P_{a} v_{3}^{m} \bar{u}_{2}^{m} P_{b} \epsilon^{x} v_{4}(T^{x})_{mn}, \]

\[ M_{G7}^{ab} = \bar{v}_{2}^{m} P_{a} v_{4} u_{3}^{m} \bar{u}_{2}^{m} P_{b} \epsilon^{x} v_{4}(T^{x})_{mn}, \]

\[ M_{G8}^{ab} = \bar{v}_{2}^{m} P_{a} v_{3}^{m} \bar{u}_{4}^{m} P_{b} \epsilon^{x} u_{1}^{m}(T^{x})_{mn}, \]

\[ M_{G9}^{ab} = \bar{v}_{2}^{m} P_{a} v_{4} u_{3}^{m} \bar{u}_{4}^{m} P_{b} \epsilon^{x} v_{3}(T^{x})_{mn}, \]

\[ M_{G10}^{ab} = \bar{v}_{2}^{m} P_{a} v_{4} u_{3}^{m} \bar{u}_{4}^{m} P_{b} \epsilon^{x} v_{3}(T^{x})_{mn}, \]
The nonzero form factors in Eq. (48) are the following:

\[ M_{G11}^{ab} = v_2^m P_a \hat{f}^x u_1^m \bar{u}_4 P_b \hat{p}_1 v_3(T^{x})_{mn}, \]
\[ M_{G12}^{ab} = v_2^m P_a \hat{f}^x u_1^m \bar{u}_4 P_b \hat{p}_2 v_3(T^{x})_{mn}, \]
\[ M_{G13}^{ab} = v_2^m P_a \gamma^\mu u_1^m \bar{u}_4 P_b \gamma_\mu v_3[\hat{f}^x \cdot (p_1 + p_2)](T^{x})_{mn}, \]
\[ M_{G14}^{ab} = \bar{u}_4 P_a u_1^m \bar{v}_2^m P_b \hat{f}^x \hat{p}_1 v_3(T^{x})_{mn}, \]
\[ M_{G15}^{ab} = \bar{u}_4^m P_a v_3 \bar{v}_2^m P_b \hat{f}^x \hat{p}_1 v_4(T^{x})_{mn}, \]
\[ M_{G16}^{ab} = \bar{u}_4^m P_a v_3 \bar{v}_2^m P_b \hat{f}^x \hat{p}_2 v_4(T^{x})_{mn}, \]
\[ M_{G17}^{ab} = \bar{u}_4 P_a u_1^m \bar{v}_2^m P_b \hat{f}^x \hat{p}_4 v_3(T^{x})_{mn}, \]
\[ M_{G18}^{ab} = \bar{v}_2^m P_a v_4 \bar{u}_3 P_b \hat{f}^x \hat{p}_2 u_1^m(T^{x})_{mn}, \]
\[ M_{G19}^{ab} = \bar{v}_2^m P_a v_4 \bar{u}_3 P_b \hat{f}^x \hat{p}_4 u_1^m(T^{x})_{mn}, \]

where \( x, m, \) and \( n \) are color indices for gluons, up quarks and down quarks, respectively.

The nonzero form factors in Eq. (48) are the following:

\[ f_{LR1}^{LR} = \sum_{s=1}^{2} \frac{2g_s \hat{d}_s \hat{C}_s^t}{(\hat{t} - M^2_{d_s})(\hat{t}_{24} - M^2_{d_s})} \]
\[ f_{LR1}^{LL} = \sum_{s=1}^{2} \frac{2g_s \hat{b}_s \hat{C}_s^t}{(\hat{t} - M^2_{d_s})(\hat{t}_{24} - M^2_{d_s})} \]
\[ f_{LR1}^{RL} = \sum_{s=1}^{2} \frac{2g_s \hat{a}_s \hat{C}_s^t}{(\hat{t} + \hat{t}_{14} - M^2_{\chi^1_s} - M^2_{\chi^2_s})(\hat{t}_{24} - M^2_{d_s})} \]
\[ f_{RR1}^{LR} = \sum_{s=1}^{2} \frac{2g_s \hat{b}_s \hat{C}_s^t}{(\hat{t} + \hat{t}_{14} - M^2_{\chi^1_s} - M^2_{\chi^2_s})(\hat{t}_{14} - M^2_{d_s})(M^2_{\chi^2_s} - \hat{u})} \]
\[ f_{RR1}^{RL} = \sum_{s=1}^{2} \frac{2g_s \hat{a}_s \hat{C}_s^t}{(\hat{t} + \hat{t}_{14} - M^2_{\chi^1_s} - M^2_{\chi^2_s})(\hat{t}_{14} - M^2_{d_s})(M^2_{\chi^2_s} - \hat{u})} \]
\[ f_{RR1}^{RG} = \frac{2D_L g_s(A_R M_{\chi^1_s} - A_L M_{\chi^2_s})}{(M^2_{W_s} - \hat{s}_{34})(\hat{t} + \hat{t}_{14} - M^2_{\chi^1_s} - M^2_{\chi^2_s})} \]
\[ f_{RR1}^{RG} = \frac{2D_L g_s(A_L M_{\chi^1_s} - A_R M_{\chi^2_s})}{(M^2_{W_s} - \hat{s}_{34})(\hat{t} + \hat{t}_{14} - M^2_{\chi^1_s} - M^2_{\chi^2_s})} \]
\[ f_{RR1}^{RG} = -\sum_{s=1}^{2} \frac{2g_s \hat{b}_s \hat{C}_s^t}{(\hat{t} + \hat{t}_{24} - M^2_{\chi^1_s} - M^2_{\chi^2_s})(\hat{t}_{14} - M^2_{d_s})} \]
\[ f_{RR1}^{RG} = -\sum_{s=1}^{2} \frac{g_s \hat{a}_s \hat{C}_s^t}{(\hat{t} + \hat{t}_{24} - M^2_{\chi^1_s} - M^2_{\chi^2_s})(\hat{t}_{14} - M^2_{d_s})} \]
\[ f_{RR1}^{RG} = -\sum_{s=1}^{2} \frac{g_s \hat{a}_s \hat{C}_s^t}{(\hat{t} + \hat{t}_{24} - M^2_{\chi^1_s} - M^2_{\chi^2_s})(\hat{t} - M^2_{d_s})} \]
\[
\begin{align*}
J^{LL}_{RG0} &= -\sum_{s=1}^{2} \frac{g_s b_s^2 C_{U}^{s} M_{\chi_{1}^{0} \chi_{2}^{0}}}{(s + t + t_{24} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2}) (t - M_{d_{s}}^{2})} \\
J^{RR}_{RG7} &= \sum_{s=1}^{2} \frac{g_s a_s^{d} C_{U}^{s} M_{\chi_{1}^{0}}^{2}}{(s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2}) (t_{24} - M_{d_{s}}^{2})} \\
J^{LR}_{RG7} &= \sum_{s=1}^{2} \frac{g_s b_s^{d} C_{U}^{s} M_{\chi_{1}^{0}}^{2}}{(s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2}) (t_{24} - M_{d_{s}}^{2})} \\
J^{RR}_{RG8} &= \sum_{s=1}^{2} \frac{g_s a_s^{d} C_{V}^{s} M_{\chi_{0}^{0}}^{2}}{(s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2}) (\hat{u} - M_{\hat{u}_{s}}^{2})} \\
J^{RL}_{RG8} &= \sum_{s=1}^{2} \frac{g_s b_s^{d} C_{V}^{s} M_{\chi_{0}^{0}}^{2}}{(s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2}) (\hat{u} - M_{\hat{u}_{s}}^{2})} \\
J^{RR}_{RG9} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{R} D_{L} g_{s}} \\
J^{RL}_{RG9} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{L} D_{L} g_{s}} \\
J^{RR}_{RG10} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{R} D_{L} g_{s}} \\
J^{RL}_{RG10} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{L} D_{L} g_{s}} \\
J^{RR}_{RG11} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + \hat{u} + t_{24} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{R} D_{L} g_{s}} \\
J^{RL}_{RG11} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + \hat{u} + t_{24} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{L} D_{L} g_{s}} \\
J^{RR}_{RG12} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{R} D_{L} g_{s}} \\
J^{RL}_{RG12} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{L} D_{L} g_{s}} \\
J^{RR}_{RG13} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{R} D_{L} g_{s}} \\
J^{RL}_{RG13} &= \frac{(M_{W}^{2} - \hat{s}_{34}) (s + t + t_{14} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2})}{2 A_{L} D_{L} g_{s}} \\
J^{RR}_{RG14} &= \sum_{s=1}^{2} \frac{g_s b_s^{d} C_{V}^{s} M_{\chi_{1}^{0} \chi_{2}^{0}}}{(s + \hat{u} + t_{24} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2}) (t_{14} - M_{d_{s}}^{2})} \\
J^{LR}_{RG14} &= \sum_{s=1}^{2} \frac{g_s a_s^{d} C_{V}^{s} M_{\chi_{1}^{0} \chi_{2}^{0}}}{(s + \hat{u} + t_{24} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2}) (t_{14} - M_{d_{s}}^{2})} \\
J^{LR}_{RG15} &= \sum_{s=1}^{2} \frac{g_s a_s^{d} C_{U}^{s} M_{\chi_{1}^{0} \chi_{2}^{0}}}{(s + \hat{u} + t_{24} - M_{\chi_{1}^{0}}^{2} - M_{\chi_{2}^{0}}^{2}) (t - M_{d_{s}}^{2})}
\end{align*}
\]
\[ f_{RL}^{15} = - \sum_{s=1}^{2} \frac{g_s b_{s2}^d C_U^s}{(s + \hat{u} + \hat{t}_{24} - M_{x_1}^2 - M_{x_2}^2)(\hat{t} - M_{d_s}^2)} \]
\[ f_{RL}^{16} = \sum_{s=1}^{2} \frac{g_s a_{s2}^d C_U^s}{(s + \hat{u} + \hat{t}_{24} - M_{x_1}^2 - M_{x_2}^2)(\hat{t} - M_{d_s}^2)} \]
\[ f_{RL}^{17} = \sum_{s=1}^{2} \frac{g_s b_{s2}^u C_V^s}{(s + \hat{u} + \hat{t}_{24} - M_{x_1}^2 - M_{x_2}^2)(\hat{t}_{14} - M_{\bar{u}_s}^2)} \]
\[ f_{RL}^{17} = \sum_{s=1}^{2} \frac{g_s a_{s2}^u C_V^s}{(s + \hat{u} + \hat{t}_{24} - M_{x_1}^2 - M_{x_2}^2)(\hat{t}_{14} - M_{\bar{u}_s}^2)} \]
\[ f_{RL}^{18} = \sum_{s=1}^{2} \frac{g_s b_{s2}^d C_U^s}{(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{t}_{24} - M_{d_s}^2)} \]
\[ f_{RL}^{18} = \sum_{s=1}^{2} \frac{g_s b_{s2}^u C_V^s}{(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{t}_{24} - M_{d_s}^2)} \]
\[ f_{RL}^{19} = - \sum_{s=1}^{2} \frac{g_s a_{s2}^d C_U^s}{(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{t}_{24} - M_{d_s}^2)} \]
\[ f_{RL}^{19} = - \sum_{s=1}^{2} \frac{g_s b_{s2}^u C_V^s}{(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{t}_{24} - M_{d_s}^2)} \]
\[ f_{RL}^{20} = - \sum_{s=1}^{2} \frac{g_s b_{s2}^u C_V^s}{(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{u} - M_{\bar{u}_s}^2)} \]
\[ f_{RL}^{20} = - \sum_{s=1}^{2} \frac{g_s a_{s2}^u C_V^s}{(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{u} - M_{\bar{u}_s}^2)} \]
\[ f_{RL}^{21} = \sum_{s=1}^{2} \frac{g_s b_{s2}^u C_V^s}{(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{u} - M_{\bar{u}_s}^2)} \]
\[ f_{RL}^{21} = \sum_{s=1}^{2} \frac{g_s a_{s2}^u C_V^s}{(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{u} - M_{\bar{u}_s}^2)} \]
\[ f_{RL}^{22} = \frac{A_R D_L g_s (2s + \hat{t} + \hat{u} + \hat{t}_{14} + \hat{t}_{24} - 2(M_{x_1}^2 + M_{x_2}^2))}{(M_{W}^2 - s_{34})(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{t} + \hat{u} + t_{24} - M_{x_1}^2 - M_{x_2}^2)} \]
\[ f_{RL}^{22} = \frac{A_R D_L g_s (2s + \hat{t} + \hat{u} + \hat{t}_{14} + \hat{t}_{24} - 2(M_{x_1}^2 + M_{x_2}^2))}{(M_{W}^2 - s_{34})(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{t} + \hat{u} + t_{24} - M_{x_1}^2 - M_{x_2}^2)} \]
\[ f_{RL}^{23} = \frac{A_R D_L g_s (2s + \hat{t} + \hat{u} + \hat{t}_{14} + \hat{t}_{24} - 2(M_{x_1}^2 + M_{x_2}^2))}{(M_{W}^2 - s_{34})(s + \hat{t} + \hat{t}_{14} - M_{x_1}^2 - M_{x_2}^2)(\hat{t} + \hat{u} + t_{24} - M_{x_1}^2 - M_{x_2}^2)} \]
\[ f_{RG23}^{LR} = \frac{A_L D_L g_s (2 \hat{s} + \hat{t} + \hat{u} + \hat{t}_{14} + \hat{t}_{24} - 2(M_{\chi_1}^2 + M_{\chi_2}^2))}{(M_{12}^2 - \hat{s}_{34})(\hat{s} + \hat{t}_{14} - M_{\chi_1}^2 - M_{\chi_2}^2)(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

The standard matrix elements in Eq. (49) for the subprocesses

\[ u(p_1) + g(p_2) \rightarrow \tilde{\chi}_1^+(p_3) + \tilde{\chi}_2^0(p_4) + d(p_5), \quad (C3) \]

and

\[ \bar{d}(p_1) + g(p_2) \rightarrow \tilde{\chi}_1^+(p_3) + \tilde{\chi}_2^0(p_4) + \bar{u}(p_5), \quad (C4) \]

are defined as follows:

\[ M_{Q1}^{ab} = \bar{u}_4 P_a u_1^m \bar{u}_5^n P_b v_3(T^x)_{mn}, \]

\[ M_{Q2}^{ab} = \bar{v}_4^m P_a v_3 \bar{u}_5^n P_b v_4(T^x)_{mn}, \]

\[ M_{Q3}^{ab} = \bar{u}_3 P_a u_1^m \bar{u}_5^n P_b v_4(T^x)_{mn}, \]

\[ M_{Q4}^{ab} = \bar{u}_4 P_a \hat{p}_2 v_3 \bar{u}_5^n P_b \hat{\epsilon} \epsilon u_1^m(T^x)_{mn}, \]

\[ M_{Q5}^{ab} = \bar{u}_4 P_a \hat{\epsilon} \epsilon v_3 \bar{u}_5^n P_b \hat{p}_2 u_1^m(T^x)_{mn}, \]

\[ M_{Q6}^{ab} = \bar{u}_4 P_a \gamma^{\mu} v_3 \bar{u}_5^n P_b \gamma_{\mu} u_1^m(T^x)_{mn}, \]

\[ M_{Q7}^{ab} = \bar{u}_5 P_a v_4 \bar{u}_3 P_b \hat{\epsilon} \epsilon \hat{p}_2 u_1^m(T^x)_{mn}, \]

\[ M_{Q8}^{ab} = \bar{u}_5 P_a \hat{\epsilon} \epsilon v_3 \bar{u}_3 P_b \hat{\epsilon} \epsilon \hat{p}_2 u_1^m(T^x)_{mn}, \]

\[ M_{Q9}^{ab} = \bar{u}_4 P_a u_1^m \bar{u}_5^n P_b \hat{\epsilon} \epsilon \hat{p}_2 v_3(T^x)_{mn}, \]

\[ M_{Q10}^{ab} = \bar{v}_1^m P_a v_3 \bar{u}_5^n P_b \hat{\epsilon} \epsilon \hat{p}_2 v_4(T^x)_{mn}, \]

\[ M_{Q11}^{ab} = \bar{u}_4 P_a \gamma^{\mu} v_3 \bar{u}_5^n P_b \hat{\epsilon} \epsilon \hat{p}_2 \gamma_{\mu} u_1^m(T^x)_{mn}, \]

\[ M_{Q12}^{ab} = \bar{v}_1^m P_a v_4 \bar{u}_3 P_b v_5^m(T^x)_{mn}, \]

\[ M_{Q13}^{ab} = \bar{v}_1^m P_a v_3 \bar{u}_4 P_b v_5^m(T^x)_{mn}, \]

\[ M_{Q14}^{ab} = \bar{v}_1^m P_a \hat{p}_2 v_5^m \bar{u}_4 P_b \hat{\epsilon} \epsilon v_3(T^x)_{mn}, \]

\[ M_{Q15}^{ab} = \bar{v}_1^m P_a \hat{\epsilon} \epsilon v_3 \bar{u}_4 P_b \hat{p}_2 v_3(T^x)_{mn}, \]

\[ M_{Q16}^{ab} = \bar{v}_1^m P_a \gamma^{\mu} v_5^n \bar{u}_4 P_b \gamma_{\mu} v_3(T^x)_{mn}, \]

\[ M_{Q17}^{ab} = \bar{u}_4 P_a v_5^m \bar{v}_1^m P_b \hat{\epsilon} \epsilon \hat{p}_2 v_3(T^x)_{mn}, \]

\[ M_{Q18}^{ab} = \bar{u}_3 P_a v_5^m \bar{v}_1^m P_b \hat{\epsilon} \epsilon \hat{p}_2 v_4(T^x)_{mn}, \]

\[ M_{Q19}^{ab} = \bar{v}_1^m P_a v_4 \bar{u}_3 P_b \hat{\epsilon} \epsilon \hat{p}_2 v_5^m(T^x)_{mn}, \]

\[ M_{Q20}^{ab} = \bar{v}_1^m P_a v_3 \bar{u}_4 P_b \hat{\epsilon} \epsilon \hat{p}_2 v_5^m(T^x)_{mn}, \]

\[ M_{Q21}^{ab} = \bar{u}_4 P_a \gamma^{\mu} v_3 \bar{v}_1^m P_b \hat{\epsilon} \epsilon \hat{p}_2 \gamma_{\mu} v_5^m(T^x)_{mn}, \]

The nonzero form factors in Eq. (49) are the following:

\[ f_{RRQ1}^{RR} = \sum_{s=1}^{2} \frac{2g_s b_{s2}^2 C_V^s}{\hat{t}_{14} - M_{us}^2} \left( \frac{-(\hat{\epsilon} \cdot (p_1 - p_4))\hat{s} - (\hat{\epsilon} \cdot p_1)\hat{t}_{14} + (\hat{\epsilon} \cdot p_1)M_{ts}^2}{\hat{s}(\hat{s} + \hat{u} + \hat{s}_{34} - 2M_{\chi_1}^2 - M_{\chi_2}^2 + M_{us}^2)} - \frac{[\hat{\epsilon} \cdot (p_1 - p_3 - p_4)]}{\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2} \right) \]
\[ f_{RQ1}^{LR} = \sum_{s=1}^{2} \frac{2g_{s}a_{s2}^{\hat{u}}C_{V}^{s}}{t_{14} - M_{d_{s}}^{2}} \left( \frac{[\epsilon^{x} \cdot (p_{1} - p_{3} - p_{4})]\hat{s} - (\epsilon^{x} \cdot p_{1})\hat{t}_{14} + (\epsilon^{x} \cdot p_{1})M_{u_{s}}^{2}}{\hat{s}(t + \hat{u} + \hat{s}_{34} - 2M_{x_{1}}^{2} - M_{x_{2}}^{2} + M_{u_{s}}^{2})} - \frac{[\epsilon^{x} \cdot (p_{1} - p_{3} - p_{4})]}{\hat{s}(\hat{t}_{14} + \hat{t}_{24} - M_{x_{1}}^{2} + 2M_{x_{2}}^{2} + M_{d_{s}}^{2})} \right) \]

\[ f_{RQ2}^{LR} = \sum_{s=1}^{2} \frac{2g_{s}a_{s2}^{\hat{d}}C_{U}^{s}}{t - M_{d_{s}}^{2}} \left( \frac{[\epsilon^{x} \cdot (p_{1} - p_{3} - p_{4})]}{\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_{1}}^{2} - M_{x_{2}}^{2} - M_{d_{s}}^{2}} - \frac{[\epsilon^{x} \cdot (p_{1} - p_{3})]}{\hat{s}_{34} + t_{14} + \hat{t}_{24} - M_{x_{1}}^{2} - 2M_{x_{2}}^{2} + M_{d_{s}}^{2}} \right) \]

\[ f_{RQ2}^{LL} = \sum_{s=1}^{2} \frac{2g_{s}a_{s2}^{\hat{d}}C_{U}^{s}}{t - M_{d_{s}}^{2}} \left( \frac{[\epsilon^{x} \cdot (p_{1} - p_{3} - p_{4})]}{\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_{1}}^{2} - M_{x_{2}}^{2} - M_{d_{s}}^{2}} - \frac{[\epsilon^{x} \cdot (p_{1} - p_{3})]}{\hat{s}_{34} + t_{14} + \hat{t}_{24} - M_{x_{1}}^{2} - 2M_{x_{2}}^{2} + M_{d_{s}}^{2}} \right) \]

\[ f_{RQ3}^{LR} = \sum_{s=1}^{2} \frac{2g_{s}(\epsilon^{x} \cdot p_{1})a_{s2}^{\hat{d}}C_{U}^{s}}{\hat{s}(\hat{s}_{34} + \hat{t}_{14} + \hat{t}_{24} - M_{x_{1}}^{2} + 2M_{x_{2}}^{2} + M_{d_{s}}^{2})} \]

\[ f_{RQ3}^{LL} = \sum_{s=1}^{2} \frac{2g_{s}(\epsilon^{x} \cdot p_{1})b_{s2}^{\hat{d}}C_{U}^{s}}{\hat{s}(\hat{s}_{34} + \hat{t}_{14} + \hat{t}_{24} - M_{x_{1}}^{2} + 2M_{x_{2}}^{2} + M_{d_{s}}^{2})} \]

\[ f_{RQ4}^{RR} = \frac{2\hat{A}_{R}D_{L}g_{s}}{s(M_{d_{s}}^{2} - \hat{s}_{34})} \left( \frac{[\epsilon^{x} \cdot (p_{1} - p_{3} - p_{4})]}{\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_{1}}^{2} - M_{x_{2}}^{2} - M_{d_{s}}^{2}} - \frac{(\epsilon^{x} \cdot p_{1})}{\hat{s}} \right) \]

\[ f_{RQ5}^{RR} = \frac{2\hat{A}_{R}D_{L}g_{s}}{\hat{s}(\hat{s}_{34} - M_{d_{s}}^{2})} \left( \frac{[\epsilon^{x} \cdot (p_{1} - p_{3} - p_{4})]}{\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_{1}}^{2} - M_{x_{2}}^{2} - M_{d_{s}}^{2}} - \frac{(\epsilon^{x} \cdot p_{1})}{\hat{s}} \right) \]

\[ f_{RQ6}^{RR} = \frac{2\hat{A}_{R}D_{L}g_{s}}{\hat{s}_{34} - M_{d_{s}}^{2}} \left( \frac{[\epsilon^{x} \cdot (p_{1} - p_{3} - p_{4})]}{\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_{1}}^{2} - M_{x_{2}}^{2} - M_{d_{s}}^{2}} - \frac{(\epsilon^{x} \cdot p_{1})}{\hat{s}} \right) \]

\[ f_{RQ7}^{RL} = \sum_{s=1}^{2} \frac{2g_{s}a_{s2}^{\hat{u}}C_{U}^{s}}{\hat{s}(\hat{s}_{34} + \hat{t}_{14} + \hat{t}_{24} - M_{x_{1}}^{2} - 2M_{x_{2}}^{2} + M_{d_{s}}^{2})} \]

\[ f_{RQ7}^{LR} = \sum_{s=1}^{2} \frac{2g_{s}b_{s2}^{\hat{u}}C_{U}^{s}}{\hat{s}(\hat{s}_{34} + \hat{t}_{14} + \hat{t}_{24} - M_{x_{1}}^{2} - 2M_{x_{2}}^{2} + M_{d_{s}}^{2})} \]

\[ f_{RQ8}^{RR} = \sum_{s=1}^{2} \frac{2g_{s}b_{s2}^{\hat{u}}C_{V}^{s}}{\hat{s}(t + \hat{u} + \hat{s}_{34} - 2M_{x_{1}}^{2} - M_{x_{2}}^{2} + M_{u_{s}}^{2})} \]

\[ f_{RQ8}^{RL} = \sum_{s=1}^{2} \frac{2g_{s}a_{s2}^{\hat{u}}C_{V}^{s}}{\hat{s}(t + \hat{u} + \hat{s}_{34} - 2M_{x_{1}}^{2} - M_{x_{2}}^{2} + M_{u_{s}}^{2})} \]

\[ f_{RQ9}^{RR} = \sum_{s=1}^{2} \frac{2g_{s}b_{s2}^{\hat{u}}C_{V}^{s}}{(\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_{1}}^{2} - M_{x_{2}}^{2})(\hat{t}_{14} - M_{u_{s}}^{2})} \]

\[ f_{RQ9}^{LR} = \sum_{s=1}^{2} \frac{2g_{s}a_{s2}^{\hat{u}}C_{V}^{s}}{(\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_{1}}^{2} - M_{x_{2}}^{2})(\hat{t}_{14} - M_{u_{s}}^{2})} \]
\[ f^{LR}_{RQ10} = \sum_{s=1}^{2} \left( g_s a_{s2}^\dagger C_U \right) \frac{(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)(\hat{t} - M_{d_s}^2)}{s(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{LL}_{RQ10} = \sum_{s=1}^{2} \left( g_s b_{s2}^\dagger C_U \right) \frac{A_R D_L g_s(\hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)}{s(\hat{s}_W - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{RR}_{RQ11} = \frac{A_L D_L g_s(\hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)}{s(\hat{s}_W - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{LR}_{RQ11} = - \frac{A_R D_L g_s(\hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)}{s(\hat{s}_W - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{RL}_{RQ12} = \sum_{s=1}^{2} \left( 2g_s a_{s2}^\dagger C_U \right) \frac{\left( \left[ e^x \cdot (p_1 - p_3 - p_4) \right] \hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2 \right)}{s(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{LL}_{RQ12} = \sum_{s=1}^{2} \left( 2g_s b_{s2}^\dagger C_U \right) \frac{\left( \left[ e^x \cdot (p_1 - p_3 - p_4) \right] \hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2 \right)}{s(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{RR}_{RQ13} = \sum_{s=1}^{2} \left( 2g_s b_{s2}^\dagger C_U \right) \frac{-\left[ e^x \cdot (p_1 - p_3) \right] \hat{s} - \hat{t}(e^x \cdot p_1) + (e^x \cdot p_1) M_{u_s}^2}{s(\hat{s}_W - \hat{s}_{34} + \hat{t}_{14} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2) - \hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2} \]

\[ f^{RL}_{RQ13} = \sum_{s=1}^{2} \left( 2g_s a_{s2}^\dagger C_U \right) \frac{-\left[ e^x \cdot (p_1 - p_3) \right] \hat{s} - \hat{t}(e^x \cdot p_1) + (e^x \cdot p_1) M_{e_s}^2}{s(\hat{s}_W - \hat{s}_{34} + \hat{t}_{14} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2) - \hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2} \]

\[ f^{RR}_{RQ14} = \frac{2A_R D_L g_s}{(\hat{s}_W - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{RL}_{RQ14} = \frac{2A_L D_L g_s}{(\hat{s}_W - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{RR}_{RQ15} = \frac{2A_R D_L g_s}{(\hat{s}_W - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{RL}_{RQ15} = \frac{2A_L D_L g_s}{(\hat{s}_W - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2)} \]

\[ f^{RR}_{RQ16} = \frac{2A_R D_L g_s}{\hat{s}_W - \hat{s}_{34}} \frac{\left( e^x \cdot p_1 \right) \hat{s}}{\hat{s}} - \frac{\left[ e^x \cdot (p_1 - p_3 - p_4) \right]}{\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2} \]

\[ f^{RL}_{RQ16} = \frac{2A_L D_L g_s}{\hat{s}_W - \hat{s}_{34}} \frac{(e^x \cdot p_1) \hat{s}}{\hat{s}} - \frac{\left[ e^x \cdot (p_1 - p_3 - p_4) \right]}{\hat{s} + \hat{u} + \hat{t}_{24} - M_{\chi_1}^2 - M_{\chi_2}^2} \]

\[ f^{RR}_{RQ17} = - \sum_{s=1}^{2} \frac{g_s b_{s2}^\dagger C_V s}{\hat{s}(\hat{s}_3 + \hat{t}_{14} + \hat{t}_{24} - M_{\chi_1}^2 - 2M_{\chi_2}^2 + M_{u_s}^2)} \]

\[ f^{LR}_{RQ17} = - \sum_{s=1}^{2} \frac{g_s a_{s2}^\dagger C_V s}{\hat{s}(\hat{s}_3 + \hat{t}_{14} + \hat{t}_{24} - M_{\chi_1}^2 - 2M_{\chi_2}^2 + M_{u_s}^2)} \]
\[ f_{RQ18}^{LR} = \sum_{s=1}^{2} \frac{g_s a_s^g C_s^U}{\hat{s}(\hat{t} + \hat{u} + \hat{s}_{34} - 2M_{x_1}^2 + M_{x_2}^2 + M_{d_s}^2)} \]
\[ f_{RQ18}^{LL} = \sum_{s=1}^{2} \frac{g_s b_s^g C_s^U}{\hat{s}(\hat{t} + \hat{u} + \hat{s}_{34} - 2M_{x_1}^2 + M_{x_2}^2 + M_{d_s}^2)} \]
\[ f_{RQ19}^{RL} = \sum_{s=1}^{2} \frac{g_s a_s^d C_s^U}{(\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_1}^2 + M_{x_2}^2)(\hat{t}_{14} - M_{d_s}^2)} \]
\[ f_{RQ19}^{LL} = \sum_{s=1}^{2} \frac{g_s b_s^d C_s^U}{(\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_1}^2 + M_{x_2}^2)(\hat{t}_{14} - M_{d_s}^2)} \]
\[ f_{RQ20}^{RR} = -\sum_{s=1}^{2} \frac{g_s b_s^d C_s^V}{(\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_1}^2 + M_{x_2}^2)(\hat{t} - M_{u_s}^2)} \]
\[ f_{RQ20}^{RL} = -\sum_{s=1}^{2} \frac{g_s a_s^d C_s^V}{(\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_1}^2 + M_{x_2}^2)(\hat{t} - M_{u_s}^2)} \]
\[ f_{RQ21}^{RR} = \frac{A_R D_{Lg_s}(\hat{u} + \hat{t}_{24} - M_{x_1}^2 + M_{x_2}^2)}{\hat{s}(M_{W}^2 - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_1}^2 + M_{x_2}^2)} \]
\[ f_{RQ21}^{LR} = \frac{A_L D_{Lg_s}(\hat{u} + \hat{t}_{24} - M_{x_1}^2 + M_{x_2}^2)}{\hat{s}(M_{W}^2 - \hat{s}_{34})(\hat{s} + \hat{u} + \hat{t}_{24} - M_{x_1}^2 + M_{x_2}^2)} \]

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FIG. 15: The dependence of the total cross sections for the associated production of $\tilde{\chi}^+ \tilde{\chi}^0$ at the LHC on the cutoff $\delta_s$, assuming $m_0 = 200$GeV, $m_{1/2} = 150$GeV, $\tan \beta = 5$ and $\delta_c = \delta_s/100$. 

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FIG. 16: The dependence of the total cross sections on the renormalization/factorization scale with the same parameters chosen as in Fig.2 of Ref.\cite{5}.
FIG. 17: The total cross sections as a function of $\tan \beta$ for the associated production of $\tilde{\chi}_\pm \tilde{\chi}_0$ at the two colliders assuming $m_{1/2} = 150\text{GeV}$, $m_0 = 200\text{GeV}$ and 1000GeV, $A_0 = 0$ and $\mu > 0$. 
FIG. 18: $\delta K$, defined as $\delta K = (\sigma^{RES} - \sigma^{NLO})/\sigma^{NLO}$, as a function of $\tan \beta$ for the associated production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ at the two colliders assuming $m_{1/2} = 150\text{GeV}$, $m_0 = 200\text{GeV}$ and $1000\text{GeV}$, $A_0 = 0$ and $\mu > 0.$
FIG. 19: The total cross sections as a function of $m_{1/2}$ for the associated production of $\tilde{\chi}^\pm_1\tilde{\chi}^0_2$ at the two colliders assuming $m_0 = 200\text{GeV}$, $\tan \beta = 5$, $A_0 = 0$ and $\mu > 0$. 
FIG. 20: $\delta K$, defined as $\delta K = (\sigma^{\text{RES}} - \sigma^{\text{NLO}})/\sigma^{\text{NLO}}$, as a function of $m_{1/2}$ for the associated production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ at the two colliders assuming $m_0 = 200\text{GeV}$, $\tan \beta = 5$, $A_0 = 0$ and $\mu > 0$. 
FIG. 21: The total cross sections as a function of $m_0$ for the associated production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ at the two colliders assuming $m_{1/2} = 150$GeV, $\tan \beta = 5$, $A_0 = 0$ and $\mu > 0$. 

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FIG. 22: $\delta K$, defined as $\delta K = (\sigma^{RES} - \sigma^{NLO}) / \sigma^{NLO}$, as a function of $m_0$ for the associated production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ at the two colliders assuming $m_{1/2} = 150\,\text{GeV}$, $\tan \beta = 5$, $A_0 = 0$ and $\mu > 0$. 
FIG. 23: The dependence of the total cross sections for $\tilde{\chi}_1^+\tilde{\chi}_2^0$ production on the factorization scale (a), the renormalization scale (b), and both scales equal (c) at the LHC assuming $m_{1/2} = 200\text{GeV}$, $m_0 = 200\text{GeV}$, $\tan\beta = 5$, $A_0 = 0$ and $\mu > 0$. $\mu_0 = (m_{\tilde{\chi}_1^0}^2 + m_{\tilde{\chi}_2^0}^2)/2$. $R = \sigma(\mu_{\text{scale}})/\sigma(\mu_0)$.

FIG. 24: The dependence of the total cross sections for $\tilde{\chi}_1^+\tilde{\chi}_2^0$ production on the factorization scale (a), the renormalization scale (b), and both scales equal (c) at the Tevatron assuming $m_{1/2} = 200\text{GeV}$, $m_0 = 200\text{GeV}$, $\tan\beta = 5$, $A_0 = 0$ and $\mu > 0$. Here $\mu_0 = (m_{\tilde{\chi}_1^0}^2 + m_{\tilde{\chi}_2^0}^2)/2$ and $R = \sigma(\mu_{\text{scale}})/\sigma(\mu_0)$. 60
FIG. 25: The invariant mass differential cross sections for the associated production of $\tilde{\chi}^\pm \tilde{\chi}^0$ at the two colliders assuming $m_{1/2} = 150\text{GeV}$, $\tan\beta = 5$, $m_0 = 200\text{GeV}$, $A_0 = 0$ and $\mu > 0$. 

...
FIG. 26: The invariant mass differential cross sections for the associated production of $\tilde{\chi}_1^\pm$ at the two colliders assuming $m_{1/2} = 150\text{GeV}$, $\tan\beta = 5$, $m_0 = 1000\text{GeV}$, $A_0 = 0$ and $\mu > 0$. 
FIG. 27: The dependence of $\delta K_d$, defined as $\delta K_d = (d\sigma^{\text{RES}} - d\sigma^{\text{NLO}})/d\sigma^{\text{NLO}}$, on the invariant mass for the associated production of $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ at the two colliders assuming $m_{1/2} = 150\text{GeV}$, $\tan \beta = 5$, $m_0 = 200\text{GeV}$ and $1000\text{GeV}$, $A_0 = 0$ and $\mu > 0$. 
\[
\sigma(pp \rightarrow \tilde{\chi}_1^0, \tilde{\chi}_2^0) \quad [\text{pb}]
\]
\[
\sqrt{S} = 14\text{TeV}
\]

- \(\text{LO}\)
- \(\text{NLO}\)
- \(\text{RES}\)

CTEQ

MRST

\(M_{\tilde{\mu}_1}\)

\(m_0\)
$\chi \sim \chi \sim \to K(p p) \delta$

$\sqrt{S} = 14\text{TeV}$

CTEQ

MRST
\[ \chi \sim 1 + \chi \rightarrow K(pp) \delta \]

\[ M_{\tilde{u}_1} = 14 \text{TeV} \]

\[ \sqrt{S} = 14 \text{TeV} \]

\[ \delta K(pp \rightarrow \chi_1 \chi_2^0) [\%] \]

- CTEQ
- MRST
