Search for $b\bar{b}b\bar{b}$ tetraquark decays in 4 muons, $B^+B^-$, $B^0\bar{B}^0$ and $B^0_s\bar{B}^0_s$ channels at LHC

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Abstract

We perform a quantitative analysis of the $b\bar{b}b\bar{b}$ tetraquark decays into hidden- and open-bottom mesons and calculate, for the first time, the $b\bar{b}b\bar{b}$ tetraquark total decay width. On the basis of our results, we propose the $b\bar{b}b\bar{b} \rightarrow B^+B^- (B^0\bar{B}^0)(B^0_s\bar{B}^0_s) \rightarrow l^+l^- + X$ decays as the most suitable channels to observe the $b\bar{b}b\bar{b}$ tetraquark states, since the calculated two-lepton cross section upper limit, $\sim 39$ fb, is so large as to be potentially detectable with the 2018 LHCb sensitivity, paving the way to the observation of the $b\bar{b}b\bar{b}$ tetraquark in the future LHCb upgrade. The $4\mu$ signal for the ground state, $J^{PC} = 0^{++}$, is likely to be too small even for the upgraded LHCb, but it may not be hopeless for the $J^{PC} = 2^{++}$ fully-bottom state.
I. INTRODUCTION

The hypothetical existence of hadronic states with more than the minimal quark content \((q\bar{q} \text{ or } qqq)\) was proposed by Gell-Mann in 1964 \cite{Gell-Mann} and Zweig \cite{Zweig}, followed by the construction of a quantitative model for two quarks plus two antiquarks by Jaffe \cite{Jaffe}, to describe the lightest scalar mesons. Recent years have seen considerable growth in the observations of four valence quark states that cannot be included in the well-known systematics of mesons made up of quark-antiquarks, \(Z(4248), Z(4430), \text{etc.}\). Similar particles have also been found in the bottom sector, \(Z_b(10610)\) and \(Z_b(10650)\), observed by the Belle collaboration \cite{Belle} (see \cite{BelleRev} or \cite{BelleRev2} for recent reviews).

The first predictions of a fully-bottom, \(b\bar{b}b\bar{b}\) tetraquark below the \(2\Upsilon\) threshold were made in Refs. \cite{Amoroso,Colangelo}, and were supported by more recent contributions \cite{Beneke1,Beneke2,Beneke3,Beneke4,Beneke5,Beneke6,Beneke7,Beneke8,Beneke9}. Theoretically, \(J^{PC} = 0^{++}\) is expected for the \(b\bar{b}b\bar{b}\) ground-state.

In 2018, LHCb performed a search for \(b\bar{b}b\bar{b}\) decaying into four-muons in the mass range \(17.5-20\) GeV, but no significant excess was found, leading to the 95\% CL upper limit \cite{LHCb}\n
\[
\sigma(pp \rightarrow T) \times B(T \rightarrow \Upsilon(1S)\mu^+\mu^-) \times B(\Upsilon(1S) \rightarrow \mu^+\mu^-) < 20 \text{ fb.} \tag{1}
\]

Ref. \cite{Amoroso2} estimates the \(J^{PC} = 0^{++}\), fully-bottom tetraquark decay width into \(\Upsilon\mu^+\mu^-\) to be in the range: \(10^{-3} - 10\) MeV \cite{Amoroso2}. Ref. \cite{Beneke10} gives a total decay width of 1.2 MeV, a partial decay width into four-leptons in the range: \(2.4 \cdot 10^{-3} - 2.4 \cdot 10^{-7}\) MeV with a branching ratio in the range: \(2 \cdot 10^{-3} - 2 \cdot 10^{-7}\).

In this letter, assuming the mass of the \(J^{PC} = 0^{++}\) fully-bottom tetraquark to lie below the \(2\eta_b\) threshold, we present for the first time a calculation of decay widths and branching ratios of the main, hidden- and open-bottom channels. Our results are as follows.

The total width is expressed as: \(\Gamma(T(J = 0^{++}) = 7.7\) MeV \(\cdot \xi\), where \(\xi\) is the ratio of the overlap probabilities of the annihilating \(b\bar{b}\) pairs in \(T\) and \(\Upsilon\), respectively. Following Ref. \cite{Beneke10}, we estimate:

\[
\xi_{\text{th}} = \frac{|\Psi_T(0)|^2}{|\Psi_{\Upsilon}(0)|^2} \sim 1.1_{-0.9}^{+0.9} \rightarrow \Gamma(T) = 8.5\) MeV (best guess) \tag{2}
\]

Decay rates are all proportional to \(\xi\) so that the branching fractions are uniquely determined; these are reported in Table \ref{table:branching_ratios}. In particular, we find

\[
B(T \rightarrow 4\mu) = 7.2 \cdot 10^{-7}. \tag{3}
\]
The result (3) is not far from the lower limit of the range in [17], the reason being that the total tetraquark width is in the order of the $\eta_b$ rather than of the $\Upsilon(1S)$ width, which gives a $\mathcal{O}(10^{-3})$ suppression.

With (3), we obtain a realistic estimate of the cross section for $p+p \rightarrow T \rightarrow \Upsilon + \mu^+\mu^- \rightarrow 4\mu$. Our result is about 350 times lower than the 95% CL upper limit quoted in [1]. On the other hand, the calculated cross section for the tetraquark strong decays into two $B_q\bar{B}_q$ mesons, $(q = u, d, s, c)$ is large enough, see Tab. [III] to be potentially detectable in the future LHCb upgrade [19].

We repeated the calculation for the $J^{PC} = 2^{++}$, fully-bottom tetraquark, assuming it lay below the $2\eta_b$ threshold. The $J = 2$ tetraquark is produced in $p + p$ collisions with a statistical factor of 5 with respect to the spin 0 state; the decay $T \rightarrow \eta_b + \text{light hadrons}$ is suppressed. However, annihilation into two vector mesons $M^*_q\bar{M}_q^*$ takes place at a greater rate than for $0^{++}$. Branching fractions of $J^{PC} = 2^{++}$ are listed in Tab. [I] and, with (2),

$$\Gamma(T(J = 2^{++}) = 12 \text{ MeV (best guess)} \quad \text{(4)}$$

| $[bb][\bar{b}\bar{b}]$ | $\eta_b+ \text{ any}$ | $B_q\bar{B}_q$ (q = u, d, s, c) | $B^*_q\bar{B}^*_q$ | $\Upsilon_b+ \text{ any}$ | $\Upsilon_b + \mu^+\mu^-$ | $4\mu$ |
|-------------------------|----------------------|-----------------|----------------|-----------------|----------------------|--------|
| $J^{PC} = 0^{++}$       | 0.65                 | 0.022           | 0.066         | 1.2 $\times$ 10$^{-3}$ | 2.9 $\times$ 10$^{-5}$ | 7.2 $\times$ 10$^{-7}$ |
| $J^{PC} = 2^{++}$       | 0                    | 0               | 0.25          | 3.4 $\times$ 10$^{-3}$ | 8.3 $\times$ 10$^{-5}$ | 20 $\times$ 10$^{-7}$ |

TABLE I: Branching ratios of $J^{PC} = 0^{++}$ and $2^{++}$ fully-bottom tetraquarks, masses below $2\eta_b$ threshold, assuming $S$-wave decay.

II. DETAILS OF THE CALCULATION

The starting point is the Fierz transformation, which brings $b\bar{b}$ together [5]:

$$T(J = 0) = \left((bb)_3^1 (\bar{b}\bar{b})_3^1 \right)^0_1 = -\frac{1}{2} \left( \sqrt{\frac{1}{3}} \left((bb)_1^1 (bb)_1^1 \right)^0_1 - \sqrt{\frac{2}{3}} \left((bb)_8^1 (bb)_8^1 \right)^0_1 \right) +$$

$$+ \frac{\sqrt{3}}{2} \left( \sqrt{\frac{1}{3}} \left((bb)_1^0 (\bar{b}\bar{b})_1^0 \right)^0_1 - \sqrt{\frac{2}{3}} \left((bb)_8^0 (\bar{b}\bar{b})_8^0 \right)^0_1 \right). \quad \text{(5)}$$
quark bilinears are normalised to unity, subscripts denote the dimension of colour representations, and superscripts the total spin. Similarly, for the $J = 2$ tetraquark, one finds:

$$\mathcal{T}(J = 2) = \left| (bb)_{3}^{1} (\bar{b}\bar{b})_{3}^{1} \right|_{1}^{2} = \left( \frac{1}{3} \left| (bb)_{1}^{1} (bb)_{1}^{1} \right|_{1}^{2} - \sqrt{\frac{2}{3}} \left| (bb)_{8}^{1} (\bar{b}\bar{b})_{8}^{1} \right|_{1}^{2} \right). \quad (6)$$

We describe the $\mathcal{T}$ decay as due to individual decays into lower mass states of one of the $b\bar{b}$ pairs in (5), described as follows.

1. The colour singlet, spin 0 pair decays into 2 gluons, which are converted into confined, light hadrons (i.e. not containing $b$ flavour) with a rate of the order of $\alpha_{S}^{2}$; taking the spectator $b\bar{b}$ pair into account, this decay leads to: $\mathcal{T} \to \eta_{b} + \text{light hadrons}$.

2. The colour singlet, spin 1 pair decays into 3 gluons, which are converted into confined light hadrons with a rate of the order of $\alpha_{S}^{3}$, leading to: $\mathcal{T} \to \Upsilon + \text{light hadrons}$; final states $\Upsilon + \mu^{+}\mu^{-}$ and $4\mu$ are also produced.

3. The colour octet, spin 1 pairs annihilate into one gluon, which materializes into a pair of light quark flavours, $q = u, d, s, c$, the latter recombine with the spectator pair to produce a pair of lower-lying, open-beauty mesons $B_{q}\bar{B}_{q}$ and $B_{q}^{*}\bar{B}_{q}^{*}$, with a rate of the order of $\alpha_{S}^{2}$.

4. The colour octet, spin 0 pairs annihilate into a pair of lighter quarks (necessary to neutralize the colour of the spectator $b\bar{b}$ pair) with amplitude of the order of $\alpha_{S}^{3}$ and with a rate of the order of $\alpha_{S}^{4}$, which we neglect.

The total $\mathcal{T}$ decay is the sum of these individual decay rates, which are obtained from the simple formula [21]

$$\Gamma((bb)_{c}^{*}) = |\Psi(0)_{\mathcal{T}}|^{2} v \sigma ((bb)_{c}^{*} \to f) \quad (7)$$

$|\Psi(0)_{\mathcal{T}}|^{2}$ is the overlap probability of the annihilating pair, $v$ the relative velocity and $\sigma$ the spin-averaged annihilation cross section in the final state $f$ [1]. The spectator $b\bar{b}$ pair, given the lack of extra energy, appears as $\eta_{b}$ or $\Upsilon$ on the mass shell, or combines with the outgoing

[1] Our method of calculation is borrowed from the theory of $K$ electron capture, where an atomic electron reacts with a proton in the nucleus to give a final nucleus and a neutrino. The rate is computed from formula (7), in which $|\psi(0)|^{2}$ is the overlap probability of the electron to the proton in the nucleus and $\sigma$ the electron-proton weak cross section. The other electrons in the atom act as spectators, and rearrange later into a stable atom, after emission of radiation, with unit probability.
$qar{q}$ pair into an open-beauty meson pair. Our results are valid in the situation where the
tetraquark mass is just below the $2\Upsilon$ threshold and each non-relativistic pair has mass very
close to $2m_b$.

We normalise the overlap probabilities to that of $|\Psi_{\Upsilon}(0)|^2$, which can be derived from the
$\Upsilon$ decay rate into lepton pairs. Eq. (7) applied to this case gives:

$$\Gamma(\Upsilon \rightarrow \mu^+ \mu^-) = Q_b^2 \frac{4\pi\alpha^2}{3} \frac{4}{m_{\Upsilon}^2} |\Psi_{\Upsilon}(0)|^2, \ (Q_b = -1/3).$$

It is useful to connect with the Vector Meson Dominance parameter [22] defined by

$$J^\mu(x) = \bar{b}(x)\gamma^\mu b(x) = \frac{m_{\Upsilon}^2}{f} \Upsilon^\mu(x)$$

$f$ being a pure number. One obtains [20]

$$|\Psi_{\Upsilon}(0)|^2 = \frac{m_{\Upsilon}^3}{4f^2}; \ f = 13.2; \ |\Psi_{\Upsilon}(0)|^2 \sim 1.2 \text{ GeV}^3.$$  

**Numerical results.** From Eq. (7), the contribution to the $T$ decay rate of the colour
singlet, spin 0 decay is

$$\Gamma_0 = \Gamma(T \rightarrow \eta_b + \text{light hadrons}) = 2 \cdot \frac{1}{4} \cdot |\Psi(0) T|^2 v\sigma((b\bar{b})^0_1 \rightarrow 2 \text{ gluons})$$

$$= \frac{1}{2} \Gamma(\eta_b) \cdot \xi = 5 \text{ MeV} \cdot \xi$$

the factor 2 arises because of the two $(b\bar{b})^0_1$ pairs, $1/4$ is the spectroscopic coefficient in (5)
and we have approximated

$$|\Psi(0) T|^2 v\sigma((b\bar{b})^0_1 \rightarrow 2 \text{ gluons}) \sim \Gamma(\eta_b) = 10 \text{ MeV}. \ (11)$$

In a similar way, we obtain

$$\Gamma_1 = \Gamma(T \rightarrow \Upsilon + \text{light hadrons}) = 2 \cdot \frac{1}{12} \cdot |\Psi(0) T|^2 v\sigma((b\bar{b})^1_1 \rightarrow 3 \text{ gluons}) =$$

$$= \frac{1}{6} \Gamma(\Upsilon) \cdot \xi = 9 \text{ keV} \cdot \xi$$

$$\Gamma_2 = \Gamma(T \rightarrow \Upsilon + \mu^+ \mu^-) = B_{\mu\mu} \Gamma_1 = 0.22 \text{ keV} \cdot \xi$$

$$\Gamma_4 = \Gamma(T \rightarrow 4\mu) = B_{\mu\mu}^2 \Gamma_1 = 5.5 \cdot 10^{-3} \text{ keV} \cdot \xi$$

Finally, we consider the annihilation of the $(b\bar{b})^1_8$ in light quark pairs. This is illustrated
in Fig. [1]. Open circles represent the insertion of quark bilinears, and black dots the QCD
vertices. Colour matrices and normalizations are explicitly indicated. The numerical factor
associated to the traces of the colour matrices along fermion closed paths, $C$ (the Chan-Paton factor \[23\]) gives the effective coupling constant of the process, $\alpha_{\text{eff}} = C \alpha_S$, which is what replaces $Q_b \alpha$ in Eq. (8). From Fig. 1 we read: $C = \sqrt{2}/3$ and we find:

$$\Gamma_5 = \Gamma(\mathcal{T} \to M(b\bar{q}) + M(q\bar{b})) = 2 \cdot \frac{1}{6} \cdot \frac{2}{9} \cdot \left( \frac{4\pi \alpha_S^2}{3} \frac{4}{m_T^2} \right) |\Psi(0)|^2 \cdot \xi$$

(14)

The factor 2 arises from the two choices of the annihilating bilinear\(^2\). We have inserted the spectroscopic factor of the spin 1 colour octet from \([5]\) and the Chan Paton factor. In parenthesis $v\sigma(b\bar{b} \to q\bar{q})$. Using Eq. (10), $\alpha_S = 0.2$ and massless $q$, we obtain

$$\Gamma_5 = \frac{8\pi}{81} \left( \frac{\alpha_S}{f} \right)^2 m_T \cdot \xi = 0.67 \text{ MeV} \cdot \xi$$

(15)

and

$$\Gamma(\mathcal{T}) = \Gamma_0 + \Gamma_1 + 4\Gamma_5 = 7.7 \text{ MeV} \cdot \xi$$

(16)

A non-vanishing mass of the final quark brings a negligible correction even for the charm. Eq. (15) refers to the total decay rate into pseudoscalar and vector meson pairs. We can separate the two rates according to the following argument. In non-relativistic notation, the spin-colour structure of the final state after annihilation corresponds to the operator ($\mathcal{O}$ is normalized to unit norm, see \([5]\)):

$$\mathcal{O}_{\text{fin}} = \frac{1}{4\sqrt{2}} \sum_A \left( b_C \lambda^A \sigma_2 \sigma b \right) \cdot \left( q_C \lambda^A \sigma_2 \sigma q \right)$$

(17)

Using the appropriate Fierz-rearranging relations, one sees that\(^3\) $B_q B^*_q$ and $B_q B^*_{\bar{q}}$ pairs are

\(^2\) given the symmetry of the tetraquark, we may call $b_1$ the annihilating $b$ quark and pair it, in Eq. (5), to either $\bar{b}_1$ or $\bar{b}_2$.

\(^3\) Fierzing colours produces $b\bar{q}$ and $q\bar{b}$ bilinears in colour singlets and colour octet; one may argue that gluons from the vacuum will screen colour octet charges \([24, 25]\).
produced in the spin combination, \( \frac{1}{2} \left[ (B_q B_q^* + (B_q \cdot B_q^*) \right] \) and the rate in \( \bar{b}q + \bar{q}b \) is shared between pseudoscalar and vector mesons in the ratio 1 : 3.

III. THE VALUE OF \( |\Psi_T(0)|^2 \)

A value for \( |\Psi_T(0)|^2 \) can be obtained from the calculation in Refs. [14, 26]. Constituent coordinates are defined as

\[
\begin{align*}
x, y & : \text{antiquarks; } z, 0 : \text{quarks} \\
x, y & : \text{antiquarks; } z, 0 : \text{quarks}
\end{align*}
\]

One defines the Jacobi coordinates

\[ \xi_1 = x - y; \xi_2 = z; \xi_3 = x + y - (z + 0) \]  

The \( T \) wave function is a product of normalized gaussians with parameters \( \beta_1 = \beta_2 = 0.77 \text{ GeV} \), \( \beta_3 = 0.60 \text{ GeV} \), obtained by minimising the expectation of the Hamiltonian of Ref. [14, 26]. The equality \( \beta_1 = \beta_2 \) is due to Charge conjugation invariance. By elementary integrations, one can obtain the wave function squared in the variable \( x \), i.e. the separation of an antiquark from the quark in the origin, or in the variable \( \ell = \frac{1}{2} \xi_3 \), i.e. the separation of the centers of gravity of quarks and antiquarks.

| Particle | Method | \( |\Psi(0)|^2 \) (GeV\(^3\)) | \( \sqrt{<R^2>} \) (fm) |
|----------|--------|------------------|-----------------|
| \( \Upsilon \) | Eq. (10) | 1.2 | 0.13, using Eq. (11) |
| \( \Upsilon \) | Eq. (22) | 0.16 | 0.25 |
| \( \Upsilon \) | Ref. [26] | 0.24 (gaussian w.f.) | 0.22 |
| \( T \) | \( x \), Eq. (18) | 0.094 | 0.30 |
| \( T \) | \( \ell \), Eq. (11) | 0.31 | 0.20 |

TABLE II: Estimates of overlap probability and radius, for \( \Upsilon \) and \( T \).

One finds

\[
|\Psi_T(x)|^2 = \left( \frac{\gamma}{\pi} \right)^{3/2} \cdot e^{-\gamma x^2}, \quad \sqrt{\gamma} = \sqrt{\left( \frac{4 \beta_1^2 \beta_2^2}{\beta_1^2 + 2 \beta_2^2} \right)} = 0.81 \text{ GeV};
\]

\[
|\Psi_T(\ell)|^2 = \left( \frac{2 \beta_3}{\pi} \right)^{3/2} \cdot e^{-4 \beta_3^2 \ell^2}
\]

For gaussian wave function there is a fixed relation \( |\Psi(0)|^2 = \left( \frac{3}{2 \pi R^2} \right)^{3/2} \).
To compute $|\Psi_T(0)|^2$, we follow Ref. [14]. We obtain the wave function

$$|\Psi_T(x)|^2 = \left(\frac{\beta^2}{\pi}\right)^{3/2} e^{-\beta^2 x^2}, \quad \beta = 0.96 \text{ GeV}$$

(21)

$$|\Psi_T(0)|^2 = 0.159 \text{ GeV}^3; \quad R_T = \sqrt{\frac{3}{2\beta^2}} = 0.252 \text{ fm.}$$

(22)

Tab. II reports the results obtained from different methods. As the table shows, the overlap probability from $\Upsilon$ leptonic decay is substantially larger than the one obtained in the gaussian model, which, for the radius, agrees reasonably with the independent evaluation of Ref. [26]. The discrepancy underlines the need to estimate $\xi = |\Psi_T(0)|^2/|\Psi_T(0)|^2$ by means of the same method for the numerator and denominator.

With the two definitions of the radius in Eqs. (19) and (20), for $T$, and with Eq. (22) for $\Upsilon$, we find $\xi(x) = 0.58 < \xi < 1.95 = \xi(\ell)$. A good compromise is the geometrical mean, with the previous result used as an error estimate:

$$\xi_{th} = \sqrt{\xi(x)\xi(\ell)} = 1.1^{+0.9}_{-0.5}$$

(23)

Branching ratios do not depend on $\xi$ and are not affected by this error.

IV. TETRAQUARK CROSS SECTION IN THE 4 $\mu$ AND $B_F^{(s)}B_F^{(s)}$ CHANNELS

By combining Eqs. (13) and (16) we obtain a very low branching fraction for $T \rightarrow 4\mu$:

$$B_{4\mu} = B(T \rightarrow 4\mu) = 7.2 \times 10^{-7}$$

(24)

The cross section upper-limit obtained from (24) is

$$\sigma_{tho.}(T \rightarrow 4\mu) \leq \sigma(pp \rightarrow 2\Upsilon)B_{4\mu} = \begin{cases} 0.049 \text{ fb}, \text{ with } \sigma(pp \rightarrow 2\Upsilon) \simeq 69 \text{ pb} & [27] \\ 0.056 \text{ fb}, \text{ with } \sigma(pp \rightarrow 2\Upsilon) \simeq 79 \text{ pb} & [28] \end{cases}$$

(25)

We observe that $\sigma(pp \rightarrow 2\Upsilon) \simeq 69(79) \text{ pb}$ is the two-$\Upsilon$ production cross section measured by CMS at 8 TeV [27] (13 TeV [28]).

According to (25), the four-muon tetraquark cross section is far below the current LHCb capabilities, the upper limit in Eq. (25) being more than 350 times lower than the CL of the 95% LHCb upper limit of $\simeq 20 \text{ fb}$ quoted in Eq. (1).
Summing over the four light flavours, we see from Tab. I that decays into meson pairs account for about 35% of $\mathcal{T}$ decays. Decay into $B^+B^-$ mesons may provide a promising channel to discover the 4$b$ tetraquark. In Tab. I we report the tetraquark open-bottom branching fractions and the upper limits to the tetraquark two-lepton cross section, $\sigma_{\text{theo.}}(\mathcal{T} \rightarrow 2B_q \rightarrow 2l)$, calculated as

$$
\begin{align*}
\sigma_{\text{theo.}}(\mathcal{T} \rightarrow 2B_f \rightarrow 2l) &= \sigma(pp \rightarrow \mathcal{T}) Br(\mathcal{T} \rightarrow 2B_q) \left[ Br(B_q \rightarrow l + \bar{\nu} + X) \right]^2 \\
&\leq \sigma(pp \rightarrow 2\Upsilon) Br(\mathcal{T} \rightarrow 2B_q) \left[ Br(B_q \rightarrow l + \bar{\nu} + X) \right]^2
\end{align*}
$$

Decays such as $B_f^+ \rightarrow l + \bar{\nu} + X$ occur by means of intermediate electromagnetic decays, for example $B_f^+ \rightarrow B_f + \gamma \rightarrow l + \bar{\nu} + X + \gamma$. For this reason, the excited open-bottom channels are not suited to the search for tetraquark states. Upper limits for $\mathcal{T}$ production and decay into into $B^+B^-, B^0\bar{B}^0$ and $B^0_s\bar{B}^0_s$ are reported in Tab. III.

TABLE III: Upper limits of two- and four-lepton cross sections via $\mathcal{T}$ production calculated using as inputs the two-$\Upsilon$ production cross sections measured by CMS at 8 TeV [27] and 13 TeV [28].

| Decay Channel | Predicted two-lepton cross section upper limit (fb) | 8 TeV | 13 TeV |
|---------------|--------------------------------------------------|-------|--------|
| $\mathcal{T}(J = 0) \rightarrow B^+B^- (B^0\bar{B}^0, B^0_s\bar{B}^0_s) \rightarrow 2l + \ldots$ | 0.022 | 34 | 39 |
| $\mathcal{T}(J = 0) \rightarrow 4\mu$ | $7.2 \cdot 10^{-7}$ | 0.049 | 0.056 |
| $\mathcal{T}(J = 2) \rightarrow 4\mu$ | $20 \cdot 10^{-7}$ | 0.14 | 0.16 |

In conclusion, we propose the $B^+B^-, B^0\bar{B}^0$ and $B^0_s\bar{B}^0_s$ channels in the search for full-bottom tetraquarks in future LHCb upgrades. The $4\mu$ signal produced by the $J^{PC} = 2^{++}$ tetraquark may not be hopeless.

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