Majorana fermions in a periodically driven semiconductor-superconductor Heterostructure

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Abstract – We propose a new approach to create Majorana fermions at the edge of a periodically driven semiconductor-superconductor Heterostructure. We calculate the quasi-energy spectrum of the periodically driven Heterostructure by using Floquet’s theory. When the interaction between different Brillouin zones of quasi-energy is neglected, one Majorana fermion with zero quasi-energy can be created at each edge of the Heterostructure when the ratio of driven amplitude and driven frequency is larger than a minimum. Furthermore, Majorana fermions with nonzero quasi-energy emerge when the driven frequency is low and the interaction between different Brillouin zones of quasi-energy needs to be considered. We also discuss the experimental protocol of creating Majorana fermions in the periodically driven Heterostructure.

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Introduction. – Majorana fermions are so unique because a Majorana fermion is its own antiparticle [1–3]. 2n well-separated Majorana bound states can construct n ordinary fermions. Because a Majorana fermion is its own antiparticle, Majorana fermions can be excited without energy, which makes the ground state degenerate, quantum information can be encoded into these degenerate states, which is protected from the decoherence [4]. Braiding Majorana fermions around one another transforms the state to the other degenerate states [5–7], thus quantum information which is encoded in this degenerate state can be manipulated through these transformations. Therefore, Majorana fermions have a great potential in topological quantum computation [4,8–12].

Although there have been many experimental protocols for creating Majorana fermions in static or driven systems. Furthermore, some of the experimental protocols based on a semiconductor-superconductor Heterostructure have been studied in real experiments [34,35], one still does not have enough evidence of the existence of Majorana fermions in such experiments. On the other hand, a semiconductor-superconductor Heterostructure with a periodically driven chemical potential has not been studied before, while such a system can provide Majorana fermions under different conditions. In addition, Majorana fermions with nonzero quasi-energy, which is impossible in static systems, can be found in such a system [25,36–39]. Therefore, it is necessary to find a new approach for creating Majorana fermions in such a real experimental system.

In this paper, we propose a new approach to create Majorana fermions in a periodically driven semiconductor-superconductor Heterostructure. We first discuss a static Heterostructure in which spin-orbit coupling, s-wave pairing field and Zeeman splitting coexist. A system which contains the above effects has been studied before [25,26,40–43]. On this basis, a periodically driven chemical potential is applied to the Heterostructure.
Utilizing Floquet’s theory, we demonstrate that Majorana fermions with zero and nonzero quasi-energy can be created under certain conditions, where the ratio of driven amplitude to driven frequency should be larger than a minimum. Here, because many experiments for finding Majorana fermions in static systems have been performed, we mainly focus on Majorana fermions with zero quasi-energy, which is similar to Majorana fermions in static systems, and just make a simple discussion about Majorana fermions with nonzero quasi-energy. Furthermore, we will discuss the criterion of enough orders of approximation with different parameters.

Periodically driven Heterostructure. — As fig. 1 shows, the Heterostructure we consider is a semiconductor-superconductor nanowire in contact with an ordinary superconductor. While InAs is chosen to be the material of the semiconductor nanowire, a large spin-orbit coupling exists in the Heterostructure. Through the proximity effect between the semiconductor and superconductor, Cooper pairs can leak into the nanowire and a s-wave pairing field emerges (which we take to be real). A magnetic field \( B_x \) is applied along the \( x \)-direction, which produces a Zeeman splitting. Through varying the electron density of the nanowire periodically by applying the alternating gate voltage \( \tilde{V}_{\text{gate}} \), the chemical potential of the Heterostructure changes periodically. For simplify we set \( \hbar = 1 \). The Hamiltonian of the Heterostructure reads

\[
\mathcal{H} = \mathcal{H}_0 + \mathcal{H}(t),
\]

where \( r \) is the coordinate along the wire, \( M \) is the effective mass of the electron, \( \alpha \) (\( i = x, y, z \)) are Pauli matrices. The operator \( \psi_s \) annihilates an electron with spin \( s = \uparrow \). \( \mu(x, t) = \mu(x, t+T) \) is a periodically driven chemical potential which is applied to the Heterostructure with period \( T \). \( \Delta \) is the Zeeman splitting which is provided by the magnetic field \( B_x \), \( \alpha \) is the strength of the spin-orbit coupling, \( \Delta \) is the real s-wave pairing field.

In order to demonstrate that Majorana fermions emerge when a periodically driven chemical potential is applied, we first construct a lattice Hamiltonian that maps onto the continuum Hamiltonian \( \mathcal{H} \) in the low-density limit. This can be done in momentum space by replacing \( p^2 \rightarrow 2(1 - \cos p) \), \( p \rightarrow \sin p \), \( \int dp \rightarrow \frac{2}{\pi} \sum_p \), \( \psi_s \rightarrow \sqrt{L} \psi_s \), \( L \) is the Heterostructure size [44]. Then we obtain the lattice Hamiltonian \( H = H_0 + H(t) \) [44] in real space

\[
H_0 = \sum_j -w(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) -i\alpha \frac{1}{2}(c_j^\dagger \sigma_z c_{j+1} - c_{j+1}^\dagger \sigma_z c_j) + 2w c_j^\dagger c_j + V_x c_j^\dagger \sigma_x c_j + \Delta(c_j^\dagger c_{j+1}^\dagger + \text{h.c.}),
\]

where \( c_j^\dagger \) (\( c_j \)) creates (annihilates) an electron on \( i \) site with spin \( \uparrow \) (up or down), \( w = 1/2Ma^2 \) is the hopping strength and \( \alpha_{\text{lat}} = \frac{1}{a} \alpha_{\text{cont}} \), where \( a \) is the lattice constant. Here it is convenient to introduce the typical parameters with \( M \sim 0.05 M_e \), where \( M_e \) is the bare electron mass, \( \sim 0.1 \text{ eV.A} \), \( V_x \sim 1 \text{ K} \) and \( \Delta \sim 1 \text{ K} \) [26,44]. To simplify, we set the lattice constant \( a = 1 \) and \( w = 1 \) below.

In order to obtain the conditions for creating Majorana fermions in a driven Heterostructure, it is convenient to discuss the conditions for creating Majorana fermions in a static system. In static systems, only Majorana fermions with zero energy can be created [36,37]. We change the Heterostructure to a static Heterostructure by setting \( \mu(x, t) \) to be a constant \( \mu_0 \) [13,26,45]. When the Zeeman splitting \( V_x \) and the pairing field \( \Delta \) vanish, the energy spectrum of the Hamiltonian is shown by the black dashed lines in fig. 2. For arbitrary values of \( \mu_0 \) above the minimum of the energy spectrum, the salient feature of these states is the generic presence of four Fermi points [44].

Now let us consider the situation with Zeeman splitting \( V_x \neq 0 \), in this case, a gap is opened at \( p = 0 \) as shown by the blue lines in fig. 2 where the width of the gap is \( 2V_x \). In this situation, the gap is a chemical potential window. When the chemical potential is in the gap, only two Fermi points exist and we can neglect the upper state of two states which is shown by the blue lines. Turning on a weak s-wave pairing field \( \Delta \), then two Majorana fermions appear at the left and right ends of the wire [26,44]. An analysis of the BDG equation reveals that Majorana fermions exist only when the following condition, \( V_x > \Delta_{\text{eff}} \), is
satisfied, where $\Delta_{\text{eff}} = \sqrt{\Delta^2 + \mu_0^2}$ is the static effective pairing field. The condition is shown by the red lines in fig. 2. When $V_x > \Delta_{\text{eff}}$, which is shown by the lower red line, $\Delta_{\text{eff}}$ is in the chemical potential window and there are two Fermi points which are shown by the green dots, the Majorana fermions emerge at two Fermi points. When $V_x < \Delta_{\text{eff}}$, which is shown by the upper red line, $\Delta_{\text{eff}}$ is out of the chemical potential window and there are four Fermi points which are shown by the black dots, there are no Majorana fermions [26].

**Quasi-energy spectrum of periodically driven Heterostructure.**

Floquet’s theory. As the first step towards calculating the driven Heterostructure, we introduce Floquet’s theory briefly. When a Hamiltonian of the quantum system has a time-periodic dependence, i.e., $H(t) = H(t + T)$ with $T = 2\pi/\omega$, the solution can be described by Floquet’s theory [46]. From Floquet’s theory, we know that the Schrödinger equation with a time-periodic dependent Hamiltonian has a complete set of solutions with the form $|\psi_n(t)\rangle = |u_n(t)\rangle \exp(-i\varepsilon_n t/\hbar)$. $\varepsilon_n$ is quasi-energy which characterizes the Floquet states in a system with the time translational symmetry $t \to t + T$. The periodic function satisfies $|u_n(t)\rangle = |u_n(t + T)\rangle$ with the eigenvalue equation

$$H_{\text{eff}}|u_n(t)\rangle = \varepsilon_n|u_n(t)\rangle,$$

where $H_{\text{eff}} = H - i\hbar\partial_t$ is the Floquet Hamiltonian. Note that the Floquet modes $|u_n(t)\rangle \exp(i\mu t)$ are also the solution of eq. (5), in which the shifted quasi-energy is $\varepsilon_n + m\hbar \omega$. $\hbar \omega$ is similar to the reciprocal lattice vector and we define the width of the Brillouin zone with a sense of time. The integer $m = 0, \pm 1, \pm 2, \ldots$ indexes different Brillouin zones [38,47]. Because of the coupling between the spatial degree of freedom and the temporal degree of freedom, it is convenient to introduce the Floquet basis

$$|\{n_i\}, m\rangle = |\{n_i\}\rangle \exp\left\{ \frac{i}{\hbar} \int_{-\infty}^t dt' \sum_i \mu_i(t') n_i + i m \omega t\right\},$$

(6)

$|\{n_i\}\rangle$ indicates a Fock state with $n_i$ particles on the $i$-th site, $m$ accounts for the Brillouin zones [47], $|\{n_i\}, m\rangle$ consist of an extended Hilbert space of $T$-periodic functions with the scalar product

$$\langle\langle \cdot | \cdot \rangle\rangle = \frac{1}{T} \int_0^T dt \langle\langle \cdot | \cdot \rangle\rangle.$$

(7)

The quasi-energies are obtained by computing the matrix elements of $H_{\text{eff}}$ in the basis (6) with respect to the scalar product (7). To simplify we set $h = 1$ below.

*High driven frequency case.* Let us consider the simplest form of the space-independent driven chemical potential. Because the system we consider is a semiconductor-superconductor Heterostructure, the chemical potential cannot be negative, the driven chemical potential has the form

$$\mu(x, t) = \mu(t) = 2\mu \cos^2 \frac{\omega t}{2},$$

(8)

where $\mu$ is the driven amplitude and $\omega$ is the driven frequency. From eq. (6), eq. (7) and eq. (8) we obtain

$$\langle\langle \{n_i'\}, m'| \{n_i\}, m \rangle\rangle = \delta_{m', m},$$

$$\langle\langle \{n_i'\}, m'| \{n_i\}, m \rangle\rangle = \begin{cases} \frac{1}{T} \int_0^T dt \cdot \exp \left\{ -2\mu \frac{\omega}{2} \sin \omega t - i (m' - m + 2\mu \frac{\omega}{2}) \omega t\right\}, & \langle\langle \{n_i'\}, m'| \{n_i\}, m \rangle\rangle = \\
\frac{1}{T} \int_0^T dt \cdot \exp \left\{ 2\mu \frac{\omega}{2} \sin \omega t - i (m' - m - 2\mu \frac{\omega}{2}) \omega t\right\}. & \langle\langle \{n_i'\}, m'| \{n_i\}, m \rangle\rangle = \end{cases}$$

(9)

where the integrals in eq. (9) can be viewed as a function of $2\mu/\omega$. From the form of integrals in eq. (9), the values of $H_{\text{eff}}(m', m)$ depend on $m' - m$. The diagonal blocks $H_{\text{eff}}(m', m)$ and nondiagonal blocks $H_{\text{eff}}(m', m)$ have the form of $H_{\text{eff}}(0) + m\omega$ and $H_{\text{eff}}(n)$, respectively, where $n = m' - m$.

From eq. (9), we obtain $H_{\text{eff}}$ in real space:

$$H_{\text{eff}} = \begin{pmatrix}
\ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & H_{\text{eff}}^{(0)} & H_{\text{eff}}^{(1)} & H_{\text{eff}}^{(2)} & \ddots \\
\vdots & H_{\text{eff}}^{(-1)} & H_{\text{eff}}^{(0)} + \omega & H_{\text{eff}}^{(1)} & \ddots \\
\vdots & H_{\text{eff}}^{(-2)} & H_{\text{eff}}^{(-1)} & H_{\text{eff}}^{(0)} + 2\omega & \ddots \\
& \ddots & \ddots & \ddots & \ddots
\end{pmatrix},$$

(10)
where the $H^{(0)}_{\text{eff}}$ and $H^{(n)}_{\text{eff}}$ are

$$H^{(0)}_{\text{eff}} = \sum_j -w(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + V_x c_j^\dagger \sigma_x c_j$$

$$-i\frac{\alpha}{2} (c_j^\dagger \sigma_y c_{j+1} - c_{j+1}^\dagger \sigma_y c_j) + 2w c_j^\dagger c_j$$

$$+ J'_n \sum_j \left(-\frac{2\mu}{\omega}\right) \Delta c_j^\dagger c_j + J'_{n+2} \left(\frac{2\mu}{\omega}\right) \Delta c_{j+1} c_{j+1},$$

$$(11)$$

$$H^{(n)}_{\text{eff}} = \sum_j J'_n \sum_j \left(-\frac{2\mu}{\omega}\right) \Delta c_j^\dagger c_j$$

$$+ J'_{n+2} \left(\frac{2\mu}{\omega}\right) \Delta c_{j+1} c_{j+1},$$

$$(12)$$

and the functions $J'_n - 2\mu/\omega (2\mu/\omega)$ and $J'_{n+2} \mu/\omega (2\mu/\omega)$ are

$$J'_n - 2\mu/\omega = \frac{1}{T} \int_0^T dt \cdot \exp\left\{i\left(\frac{2\mu}{\omega}\right) \sin \omega t - i \left(m' - m - \frac{2\mu}{\omega}\right) \omega t\right\},$$

$$(13)$$

$$J'_{n+2} \mu/\omega = \frac{1}{T} \int_0^T dt \cdot \exp\left\{i\left(-\frac{2\mu}{\omega}\right) \sin \omega t - i \left(m' - m - \frac{2\mu}{\omega}\right) \omega t\right\}.$$  

$$(14)$$

In the above matrix, the diagonal block of the Floquet Hamiltonian $H^{(0)}_{\text{eff}} + m\omega$ is the $m$-Brillouin zone of quasi-energy, the nondiagonal blocks $H^{(n)}_{\text{eff}}$ with $m' - m \neq 0$ correspond to the interaction between different Brillouin zones [48]. When the driven frequency $\omega$ is relatively high, the interaction between different Brillouin zone is negligible. In this case, the driven system behaves similar to the static system [48].

Now let us consider the case which neglects $H^{(n)}_{\text{eff}}$ in eq. (10) by choosing a large $\omega$. The parameters are chosen to be $\Delta = 2$, $V_x = 1$, $\mu = 20$. Then we diagonalize $H_{\text{eff}}$ directly. The driven frequency $\omega$ is tuned continuously from 80 to 20, then $2\mu/\omega$ changes from 0.5 to 2. The evolution of the quasi-energy spectrum with the driven frequency is shown by fig. 3. When $\omega$ is lower than about 45, Majorana fermions with zero quasi-energy emerge, and a gap is opened between the ground state and the excited state, which protects Majorana fermions from quantum fluctuation. In this case, the effective Hamiltonian $H_{\text{eff}}$ is similar to an undriven system, but the chemical potential in the static system $\mu_0$ disappears and the condition for creating Majorana fermions is replaced by

$$V_x > |\Delta_{\text{eff}}^d|,$$

$$(15)$$

where the effective pairing field has the form

$$\Delta_{\text{eff}}^d = J'_n \sum_j \left(\frac{2\mu}{\omega}\right) \Delta.$$  

$$(16)$$

With decreasing the driven frequency, $|\Delta_{\text{eff}}|$ also decreases. When the condition (15) is satisfied, Majorana fermions with zero quasi-energy can be created. Utilising eq. (16) and condition (15), the critical frequency $\omega$ which induces Majorana fermions approaches 45, $2\mu/\omega$ approaches 0.9. The result agrees with fig. 3 where Majorana fermions emerge.

**Low driven frequency case.** When the driven frequency is quite low, the interaction between different Brillouin zones cannot be neglected. In this case, the quasi-energy spectrum will change dramatically, and Majorana fermions with nonzero quasi-energy emerge [30].

Due to the low driven frequency, the calculation bases on the no-interaction case have been invalidated. Therefore, we should take into account $H^{(0)}_{\text{eff}}$ in eq. (12) with $n > 0$. The evolution of the quasi-energy spectrum with the driven frequency in real space by considering $H^{(0)}_{\text{eff}}$, $H^{(2,1)}_{\text{eff}}$ and $H^{(2,3)}_{\text{eff}}$ is shown in fig. 4 with $\mu = 2$, $\Delta = 2$, $\alpha = 0.3$ and $V_x = 1$. In fig. 4(A), which is used for comparison with the other panels of fig. 4, only $H^{(0)}_{\text{eff}}$ is considered. Majorana fermions with zero quasi-energy can be created when the driven frequency is turned down to satisfy the condition $V_x > |\Delta_{\text{eff}}^d|$. In this case, a quasi-energy gap between ground state and excited state exists, which protects Majorana fermions from quantum fluctuation. However, when we keep turning down the driven frequency, the gap disappears, this can be explained by the superposition of the quasi-energy spectra of different Brillouin zones. When the interaction between different Brillouin zones is taken into account in fig. 4(B), (C) and (D), the quasi-energy spectra change. First, Majorana fermions with quasi-energy $\varepsilon = \pi/T$ emerge when $\omega$ is tuned from 3.2 to 4.2. Obviously, Majorana fermions emerge because of the interaction between different Brillouin zones. Second, in fig. 4(C) and (D), Majorana fermions with zero quasi-energy emerge when $\omega$ is between 1.4 and 2, while in fig. 4(A) and (B), there are no Majorana fermions. This can also be explained by the interaction between different Brillouin zones. The coupling...
between different Brillouin zones induces the quasi-energy spectrums of different Brillouin zones to couple with each other at \( \varepsilon T = \pm n\pi, \ n = 0, \pm 1, \pm 2, \ldots \), then Majorana fermions can be created by the coupling.

On the other hand, from fig. 4(A) to fig. 4(D), by considering higher orders of \( H_{\text{eff}}^{(n)} \), the quasi-energy spectrums shown in the four panels of fig. 4 have a similar structure when \( \omega \) is high. Nevertheless, the interaction between different Brillouin zones becomes stronger when turning down \( \omega \), and the structure of the quasi-energy spectrum changes when different \( H_{\text{eff}}^{(n)} \) are considered. Furthermore, from fig. 4(C) to fig. 4(D), the quasi-energy spectrums are almost the same even in the low \( \omega \) region. This can be used to determine whether the orders of approximation are enough or not as follows: in a certain region of parameters, we can calculate the quasi-energy spectrums which have been obtained by considering different \( H_{\text{eff}}^{(n)} \). When the quasi-energy spectrums with considering \( H_{\text{eff}}^{(n)} \) and \( H_{\text{eff}}^{(n+1)} \) become stable, we achieve the sufficient order of approximation and obtain a believable result.

**Experimental protocol.** — As fig. 1 shows, a semiconductor nanowire is arranged to contact with a s-wave superconductor. The superconductor is separated from the Si substrate by a SiO\(_2\) layer [49,50]. Through the proximity effect, the Cooper pairs from the superconductor leak into the nanowire. Due to the weak capacitive coupling between the nanowire and the Si substrate, we can apply an alternating gate voltage \( V_{\text{gate}} \) to the Si substrate to vary the electron density in the nanowire, which can be ignored anymore. By taking the interaction between different Brillouin zones into account, Majorana fermions with nonzero quasi-energy can be created at the boundaries of the quasi-energy spectrums of different Brillouin

open a sizable gap without destroying the superconductivity in the superconductor [26].

In order to create Majorana fermions in such a driven Heterostructure, it is necessary to obtain the rough scales of the experimental parameters. From the typical parameters we obtain that the magnetic field \( B_x \) is lower than 0.1 T [26], the driven amplitude \( \mu \) is of the order of 10 K.

In order to find the Majorana fermion in such a system, a bias voltage \( V \) is added between both the sides of the nanowire. Majorana fermions in such a driven system can be found by a quantized conductance sum rule over discrete values of lead bias differing by multiple absorption or emission energies at the driven frequency [51]. That is, if there is a Majorana fermion with quasi-energy \( \varepsilon T = 0 \) or \( \varepsilon T = \pi \), the sum of the differential conductance will peak when \( V T = 0 \) or \( V T = \pm \pi \), respectively. Here, the sum of the differential conductance \( \tilde{\sigma} \) is

\[
\tilde{\sigma}(V) = \sum_n \sigma(V + n\omega),
\]

where \( \sigma(V) \) is the differential conductance which can be measured in experiment [51].

**Conclusion.** — In summary, we propose a new approach to create Majorana fermions in a periodically driven semiconductor-superconductor Heterostructure. By using Floquet’s theory, we calculate the quasi-energy spectrums of the case which neglects the interaction between different Brillouin zones of quasi-energy with a high driven frequency. Then we demonstrate that when the pairing field \( \Delta \) and Zeeman splitting \( V_x \) have certain values, Majorana fermions with zero quasi-energy can be created when the condition \( V_x > \Delta_0 \) is satisfied, in which \( 2\mu/\omega \) should be larger than a minimum. Then, we consider the situation when the driven frequency is low, the interaction between different Brillouin zones cannot be ignored anymore. By taking the interaction between different Brillouin zones into account, Majorana fermions with nonzero quasi-energy can be created at the boundaries of the quasi-energy spectrums of different Brillouin
zones, and Majorana fermions with zero quasi-energy can still exist. Finally, we discuss an experimental protocol for creating Majorana fermions. We hope our work will be useful to the future experimental detection of Majorana fermions.

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