Dissipative engineering a tripartite Greenberger-Horne-Zeilinger state for neutral atoms

D. X. Li,1, 2 H. W. Xiao,1, 2 C. Yang,1, 2, * and X. Q. Shao1, 2, †

1 Center for Quantum Sciences and School of Physics, Northeast Normal University, Changchun, 130024, China
2 Center for Advanced Optoelectronic Functional Materials Research, and Key Laboratory for UV Light-Emitting Materials and Technology of Ministry of Education, Northeast Normal University, Changchun 130024, China

(Dated: April 27, 2020)

The multipartite Greenberger-Horne-Zeilinger (GHZ) states are indispensable elements for various quantum information processing tasks. Here we put forward two deterministic proposals to dissipatively prepare tripartite GHZ states in a neutral atom system. The first scheme can be considered as an extension of a recent work [T. M. Wintermantel, Y. Wang, G. Loechard, et al., Phys. Rev. Lett. 124, 070503 (2020)]. By virtue of the polychromatic driving fields and the engineered spontaneous emission, a multipartite GHZ state with odd numbers of atoms are generated with a high efficiency. This scheme effectively overcomes the problem of dependence on the initial state but sensitive to the decay of Rydberg state. In the second scenario, we exploit the spontaneous emission of the Rydberg states as a resource, hence a steady tripartite GHZ state with fidelity around 98% can be obtained by simultaneously integrating the switching driving of unconventional Rydberg pumping and the Rydberg antiblockade effect.

I. INTRODUCTION

Neutral atoms excited to Rydberg states own strong, controllable Rydberg-mediated interactions that make Rydberg-atom systems become one of the most promising and versatile platforms in the fields of quantum information processing [1], quantum optics [2, 3], quantum many-body physics [4–6], and quantum metrology [7–10]. This exotic feature has been intensively explored and several milestones have been put forward. A prominent example is the Rydberg blockade. Benefiting from the significant suppression of the simultaneous excitation for Rydberg atoms, it serves as the backbone not only for a two qubit controlled phase gate [1, 11, 12], but also for entanglement generation [13–16], quantum algorithms [17], quantum simulators [4], and quantum repeaters [18]. On the other hands, an opposite effect, the Rydberg antiblockade[19, 20], also sheds new light on fundamental questions about quantum logic gate [21, 22], preparations of quantum entanglement [23–26], and directional quantum state transfer [27]. It is induced by combining Rydberg interactions with the two-photon detuning to realize the simultaneous excitation of two Rydberg atoms. With the rapid development of quantum information, entanglement in bipartite systems has been well understood and quantified [28]. More and more researchers begin to focus on unleashing the potential of multipartite entanglement in the context of measurement-based quantum computation [29–31], quantum error correction [32, 33], quantum networks [34–36], and condensed matter physics [37, 38]. Compared with bipartite entanglement, multipartite entanglement is more powerful to manifest the nonlocality of quantum physics [28, 39].

As a representative genuine multipartite entanglement, GHZ states [40] enable a new understanding to research the local and realistic worldview further with more refined demonstrations of quantum nonlocality. Besides, they supply efficient manners for large-scale cluster state generation of measurement-based quantum computing [41, 42], quantum metrology [43–45], and high-precision spectroscopy [46, 47]. Therefore, the preparation and measurement of GHZ states via diverse systems have been sought for a long time and remains an attractive field of research. Nowadays, a myriad of theoretical and experimental literatures to generate GHZ states have been proposed [48–51]. Particularly in Ref. [49], the authors presented a dissipative scheme to prepare a GHZ state of three Rydberg atoms in a cavity. Although they united quantum Zeno dynamics with Rydberg antiblockade effect to depress the harmful effect from the cavity, guaranteeing the high quality of a cavity is still a challenge in experiments, and the Rydberg atoms trapped into a cavity pronouncedly increase the experimental difficulties.

Quite recently, integrating the Rydberg interactions and dichromatic driving fields, our group [52] discovered another fantastic effect, unconventional Rydberg pumping (URP), which is ground-state-dependent and differs from the general Rydberg blockade or antiblockade. It will freeze the system consisting of two atoms at the same ground state and excite the system with two atoms at different ground states. The remarkable effect has exhibited the spectacular potential for various quantum information processing tasks, such as the achievement of quantum logic gate and the generation of entangled states. Furthermore, it is a meritorious pillar-stone to perform the autonomous quantum error correction for avoiding the bit-flip error of GHZ states in quantum metrology. Additionally, analogous to the dichromatic driving fields of URP, Wintermantel et al. [53] recently introduced programmable multifrequency couplings in arrays of Rydberg atoms to generalize the Rydberg blockade effect and nonunitarily prepare GHZ states. However, the corresponding system has to be comprised of even numbers of atoms, and the optimal parameters cannot guarantee a unique steady-state solution of system. For instance, the target GHZ state $(|0\rangle^4 + |1\rangle^4)/\sqrt{2}$ or $(|0\rangle^6 - |1\rangle^6)/\sqrt{2}$ cannot be implemented from the initial states in the basis of $\{ |1100\rangle, |0110\rangle, |0011\rangle \}$. The first scheme is considered as an extension of a recent work [T. M. Wintermantel, Y. Wang, G. Loechard, et al., Phys. Rev. Lett. 124, 070503 (2020)]. By virtue of the polychromatic driving fields and the engineered spontaneous emission, a multipartite GHZ state with odd numbers of atoms are generated with a high efficiency. This scheme effectively overcomes the problem of dependence on the initial state but sensitive to the decay of Rydberg state. In the second scenario, we exploit the spontaneous emission of the Rydberg states as a resource, hence a steady tripartite GHZ state with fidelity around 98% can be obtained by simultaneously integrating the switching driving of unconventional Rydberg pumping and the Rydberg antiblockade effect.
where \( \rho_{U_{kj}} \) - and \( \rho_{L} \) correspond to the ground state \( |\ell\rangle \) and a classical laser \( \Omega \) fields, interact with the polychromatic driving fields \( \Omega_{1,2,3} \) and a classical laser \( \Omega_0 \). While the polychromatic driving fields \( \Omega_{1,2,3} \) respectively drive the transitions \( |g\rangle \leftrightarrow |r\rangle \) with detunings \( -\Delta_{1,2,3} \), the classical laser \( \Omega_0 \) resonantly couples the short-lived state \( |e\rangle \) with the Rydberg state \( |r\rangle \).

Supposing the three atoms decay from the short-lived state \( |e\rangle \) to the ground state \( |g\rangle \) with the same spontaneous emission rate \( \Gamma \), the full master equation in the interaction picture can be written as

\[
\dot{\rho} = -i[H, \rho] + \sum_{j=1}^{3} L_j \rho L_j^\dagger - \frac{1}{2} (L_j^\dagger L_j \rho + \rho L_j^\dagger L_j),
\]  

(1)

where

\[
H = \sum_{j,\alpha} \Omega_{\alpha} \sigma_j^{\alpha} e^{-i\Delta_{\alpha} t} + \Omega_0 \sigma_j^{ge} + \text{H.c.} + \sum_{k>j} U_{jk} \sigma_j^{rr} \sigma_k^{rr}.
\]

and \( L_j = \sqrt{\Gamma} \sigma_j^{ge} \). Here \( |x\rangle_j \langle y| \) is parametrized as \( \sigma_j^{xy} \). And \( U_{jk} \) bridges the Rydberg interaction, caused by the dipole-dipole potential or the long-range van der Waals interaction, between the \( j \)- and \( k \)-th Rydberg atoms, which can obey the relation \( U_{12} = U_{23} = U_{13} = U \) through the appropriate adjustments of the interatomic distance and the atomic principal quantum numbers [54, 55]. Since the lifetime of the temporary state \( |e\rangle \) is short, we consider the decay rate \( \Gamma \) is much greater than the coupling strength \( \Omega_0 \), i.e., \( \Gamma \gg \Omega_0 \). And in the limiting condition of \( U \gg \Omega_{0,1,2,3} \), we can reformulate the Hamiltonian in a rotating frame with respect to \( \rho_{0} = \sum_{j>k} \sigma_{j}^{rr} \sigma_{k}^{rr} \).

\[
H_I = H_r + H_e,
\]

(2)

with

\[
H_r = \sum_{j,m,n} 2|m-n|\Omega_{m+n+1} P_{j-1}^{m} P_{j+1}^{n} e^{i(m+n-1)\Delta_{r} t} + \text{H.c.},
\]

\[
H_e = \sum_{j} \Omega_{0} P_{j}^{0} e^{i(m+n-1)\Delta_{r} t} + \text{H.c.},
\]

where \( m, n = 0, 1, P_{j}^{0} = |g\rangle \langle g|, P_{j}^{1} = |r\rangle \langle r| \), periodic boundary conditions of \( j \) is considered, and we have set \( U = \)
\[ \Delta_2 = (\Delta_3 + \Delta_1)/2 \] and \[ \Delta_1 = 2\Omega_2 \] to achieve the Rydberg antibloackade effect. As for the other terms, we have neglected them as the large detuning conditions and the short lifetime of state \(|e\rangle\).

The corresponding operators of atomic spontaneous emission can be simplified as \[ L^{(i)} = \sqrt{\Gamma} P_{j-1}^{00} \sigma_{j+1}^{gr} P_{j+1}^{00}. \] Then we can adiabatically eliminate the state \(|e\rangle\) to obtain an engineered spontaneous emission. For the sake of a clear show about the mechanism, we discard the effective Hamiltonian \(H_r\) and take the Hamiltonian \[ \Omega_0 \sigma_{1+}^{rr} P_{2+}^{00} P_{3+}^{00} + \text{H.c. of } H_s \] and the effective Lindblad operator \(L^{(1)}\) as an example. A reduced master equation reads

\[ \dot{\rho}_c = -i[\Omega_0 \sigma_1^{rr} P_{2}^{00} P_{3}^{00} + \text{H.c.}, \rho_c] + L^{(1)} \rho_c L^{(1)\dagger} - \frac{1}{2}(L^{(1)i} \rho_c + \rho_c L^{(1)\dagger} L^{(1)}). \] (3)

The density operator can be written in the basis of \(|\{ggg, egg, rgg\}\rangle\) as

\[ \rho_c = \begin{pmatrix} \rho_{gg} & \rho_{ge} & \rho_{gr} \\ \rho_{eg} & \rho_{ee} & \rho_{er} \\ \rho_{rg} & \rho_{re} & \rho_{rr} \end{pmatrix}. \] (4)

Substituting it into the Eq. (3), we can obtain a set of coupled equations for the matrix elements

\[ \dot{\rho}_{gg} = \Gamma \rho_{ee}, \] (5)
\[ \dot{\rho}_{ge} = i\Omega_0 \rho_{gr} - \frac{\Gamma}{2} \rho_{ge}, \] (6)
\[ \dot{\rho}_{gr} = i\Omega_0 \rho_{ge}, \] (7)
\[ \dot{\rho}_{ee} = i\Omega_0 (\rho_{er} - \rho_{re}) - \Gamma \rho_{ee}, \] (8)
\[ \dot{\rho}_{er} = i\Omega_0 (\rho_{ee} - \rho_{re}) - \frac{\Gamma}{2} \rho_{er}, \] (9)
\[ \dot{\rho}_{re} = i\Omega_0 (\rho_{re} - \rho_{er}). \] (10)

In the limit of \(\Gamma \gg \Omega_0\), it is reasonable to presume \(\dot{\rho}_{ge} = \dot{\rho}_{ee} = \dot{\rho}_{re} = 0\). We can solve that \(\rho_{ge} = 2i\Omega_0 \rho_{gr}/\Gamma, \rho_{er} = -2i\Omega_0 \rho_{rr}/(\Gamma^2 + 4\Omega_0^2), \) and \(\rho_{re} = 4\Omega_0^2 \rho_{rr}/(\Gamma^2 + 4\Omega_0^2)\). Then the coupled equations of the matrix elements can be rewritten as

\[ \dot{\rho}_{gg} = -\dot{\rho}_{gg} = \Gamma_{\text{eff}} \rho_{rr}, \quad \rho_{gr} = -\frac{\Gamma_{\text{eff}}}{2} \rho_{gr}, \] (11)
\[ \Gamma_{\text{eff}} = 4\Omega_0^2 / \Gamma. \]

Then Eq. (3) can be derived as

\[ \dot{\rho}_c = \Gamma_{\text{eff}} \rho_c L^{(1)i} + \frac{1}{2}(L^{(1)i} \rho_c + \rho_c L^{(1)\dagger} L^{(1)}). \] (12)

where \(L^{(1)} = \sqrt{\Gamma_{\text{eff}}} \sigma_{j+1}^{gr} P_{j+1}^{00} \). The other terms of \(H_e\) and the Lindblad operators \(L^{(2,3)}\) can be simplified via the similar method. Thus, the total system can be equivalent to

\[ \dot{\rho} = -i[H_r, \rho] + \sum_j L^{(j)} \rho L^{(j)i} + \frac{1}{2}(L^{(j)i} L^{(j)i} \rho + \rho L^{(j)i} L^{(j)i}), \]

where \(L^{(j)}_c = \sqrt{\Gamma_{\text{eff}}} P_{j-1}^{00} \sigma_{j+1}^{gr} P_{j+1}^{00}\) is the engineered spontaneous emission.

To further describe the principle of this scheme, we can diagonalize the resonant terms of \(H_r\) and get that

\[
H_r = \sqrt{3\Omega}[|GHZ\rangle \langle E_{1+}| + |GHZ\rangle \langle E_{1-}|]e^{i\Delta_1 t} + \text{H.c.} + \Omega_2 (|E_{1+}\rangle \langle E_{1+}| - 2|E_{1-}\rangle \langle E_{1-}| + |E_{2+}\rangle \langle E_{2+}| - |E_{2-}\rangle \langle E_{2-}| + |E_{3+}\rangle \langle E_{3+}| - |E_{3-}\rangle \langle E_{3-}|),
\]

where we set \(\Omega_1 = \Omega_3 = \Omega\) for simplicity and have abbreviated \(|GHZ\rangle = (|ggg\rangle \pm |rrr\rangle)/\sqrt{2}, \langle E_{1\pm}\rangle = (|rrg\rangle \pm |rgg\rangle \pm |ggr\rangle \pm |grg\rangle \pm |grg\rangle \pm |rgg\rangle)/2, \) and \(\langle E_{2\pm}\rangle = (|rrg\rangle \pm |rgg\rangle \pm |ggr\rangle \pm |grg\rangle \pm |grg\rangle \pm |rgg\rangle)/2\sqrt{2}\). Then we can find that the Hamiltonian of Eq. (12) reveals the dispersive transitions of \(|GHZ\rangle \leftrightarrow |E_{1\pm}\rangle\) with detuning \(\Delta_1 \neq 2\Omega_2\). (Note that for the system consists of even numbers of atoms [53], there is a resonant transition between \(|GHZ\rangle\) or \(|GHZ\rangle\) and a certain dark state in the presence of \(\Delta_1 = 0\).) Once we assume \(\Delta_1 = 2\Omega_2, \Omega_2 \gg \Omega\) and rotate the above Hamiltonian with \(\exp[-2i\Omega_2 t(|E_{1+}\rangle \langle E_{1+}| - |E_{1-}\rangle \langle E_{1-}|)]\), the effective Hamiltonian based on the polychromatic driving fields can amount to

\[ H_{\text{eff}} = \sqrt{3\Omega}|GHZ\rangle \langle E_{1+}| + \text{H.c.} + \Omega_2 (|E_{2+}\rangle \langle E_{2+}| - |E_{2-}\rangle \langle E_{2-}| + |E_{3+}\rangle \langle E_{3+}| - |E_{3-}\rangle \langle E_{3-}|).\]

where the term \(|GHZ\rangle \langle E_{1-}\rangle + \text{H.c.}\) have been omitted as the corresponding large detuning is \(4\Omega_2\) and only the resonant transition of \(|GHZ\rangle \leftrightarrow |E_{1+}\rangle\) remains. Then the effective master equation of the whole system reads

\[ \dot{\rho} = -i[H_{\text{eff}}, \rho] + \sum_j L^{(j)} \rho L^{(j)i} + \frac{1}{2}(L^{(j)i} L^{(j)i} \rho + \rho L^{(j)i} L^{(j)i}), \]

(14)

According to the Eq. (14), the target state \(|GHZ\rangle\) is the unique steady-state solution of this model, i.e., \(H_{\text{eff}}|GHZ\rangle = L_{\text{eff}}|GHZ\rangle = 0\). Therefore, initialized at an arbitrary state, the system can be stabilized at \(|GHZ\rangle\).

### B. Numerical results

In Fig. 2, we plot the dynamical evolution for the populations of the target state \(|GHZ\rangle\) governed by the full master equation Eq. (1) (solid line) and the effective master equation Eq. (14) (empty circles), respectively. The brilliant agreement of the two curves adequately proves the validity of the reduced system. It is significant to forecast and interpret the behaviors of the original system. Furthermore, the populations of \(|GHZ\rangle\) (dashed line) and \(|GHZ\rangle\) are respectively stable at 0.30% and 99.54% with the time just at 200/\(\Omega_2\), which reflects the feasibility and the high efficiency of the first dissipative scheme. The initial state is chosen as a mixed state \(\rho_0 = \sum_{l,m,n} P_l^0 P_m^0 P_n^0 / 8\) \((l, m, n = 0, 1)\). It means the target state is the unique steady state of the whole system, and this is also one of the remarkable features of dissipative entangled-state preparations. Additionally, stimulated by this principle, the present scheme can be generalized to prepare an arbitrary
\[\Omega \neq \Omega_1, \Omega_2 \gg \Omega_1, \Omega_3\] is no longer necessary so long as we select suitable values of \(\Omega', \sigma, \text{ and } \mu\). In Fig. 3(a), we depict the populations of \(|\text{GHZ}_-\rangle\) with the polychromatic driving fields respectively applying the different modulated couplings and the corresponding constant couplings (\(\Omega = \Omega'_1\)). Owing to the Gaussian pulse, when the limiting condition \(\Omega_2 \gg \Omega_1, \Omega_3\) is violated, the populations of \(|\text{GHZ}_-\rangle\) with the former still reach 99.27\% (dashed line), 99.35\% (dash-dotted line), and 99.21\% at \(\Omega_2t = 300\). By contrast, those with the latter markedly decrease to 88.23\% (empty circles), 67.07\% (empty triangles), and 40.29\% (empty squares). On the other hand, in Fig. 3(b), when we choose \(\Omega' = 0.1\Omega_2, \sigma = 90/\Omega_2, \text{ and } \mu = 110/\Omega_2\), the population of \(|\text{GHZ}_-\rangle\) can be raised from 99.46\% (solid line) to 99.81\% (dashed line) at \(\Omega_2t = 250\). This performance manifests that the Gaussian pulse can promote the quality of the target state even though the limiting condition is not violated.

Although the efficiency is excellent, the present scheme is sensitive to the atomic spontaneous emission of the Rydberg state \(|r\rangle\), which can be described by the Lindblad operators \(L_j^\alpha = \sqrt{\gamma}j^\alpha (\gamma \text{ stands for the decay rate}). \) We add \(L_j^\alpha\) with \(\gamma\) just identical to \(0.01\Omega_2\) into the Eq. (1), the population of the target state will steeply descend from 99.54\% to 75.79\% at \(\Omega_2t = 200\), which has been represented by the dash-dotted line in Fig. 2. And this disadvantage is not solved in the Ref. [53], either. Consequently, we devise the second scheme based on the switching driving of URP and the Rydberg antiblockade to change the role of the Rydberg state decay into a useful resource.

\section{III. SCHEME BASED ON SWITCHING DRIVING FIELDS}

Switching driving field is a good candidate to perfectly realize an ideal quantum process that cannot be performed by the natural evolution of systems. This technology has been used to advantage in an enormous amount of ingenious efforts, such as the implementation of quantum logic gates [56–58], the derivation and applications of the Trotter product formula \(\exp\{L\} = \lim_{N \to \infty} \left(\exp\{L/N\}\exp\{(L - L_0)/N\}\right)^N\) [59–61], the preparation of entanglement with trapped ions [62, 63], and so on [64–66]. In this section, we will explicate the second scheme based on the switching driving of URP in detail.

\subsection{A. Physical mechanism and effective dynamics}

For this scheme, the system is constituted by three four-level Rydberg atoms that all encompass two ground states \(|0\rangle\), \(|1\rangle\) (encoded quantum bits), and two Rydberg states \(|r\rangle\), \(|p\rangle\). The corresponding flow chart has been elaborated in Fig. 4. It can be separated into two simultaneous processes to nonunitarily generate tripartite GHZ state \(|\text{GHZ}_-\rangle = (|000\rangle - |111\rangle)/\sqrt{2}\) with an arbitrary initial state. One of the processes uses the switching driving of URP to transform the states with one or two atoms in state \(|0\rangle\) into the subspace.
spanned by $|000\rangle$ and $|111\rangle$, which can be also expanded via 
$\{ |GHZ_+\rangle, |GHZ_-\rangle \}$ as $|000\rangle = (|GHZ_+\rangle + |GHZ_-\rangle)/\sqrt{2}$
and $|111\rangle = (|GHZ_+\rangle - |GHZ_-\rangle)/\sqrt{2}$. In order to stabilize
the system at the target state $|GHZ_-\rangle$, the other process cap-
italizes on the Rydberg antiblockade effect exciting the state
$|++\rangle$ to the Rydberg excited state $|rrr\rangle$, and the stabilization
of $|GHZ_-\rangle = ((|++\rangle + |---\rangle + |+-\rangle + |-+\rangle)/2)
$\pm (\pm = (|0\rangle \pm |1\rangle)/\sqrt{2})$ can be destroyed. Subsequently, the
state $|rrr\rangle$ will further decay to the ground states by the Ryd-
berg state decay. The two simultaneous processes create a
cycle among all states except $|GHZ_-\rangle$ and lead to the system
steady at $|GHZ_-\rangle$ finally.

In Fig. 5(a) and 5(b), we flesh out the atomic levels in
more detail. The process based on the switching driving of
URP composes of the Step 1 and the Step 2 carried out alter-
ately. For the Step 1, there are dichromatic driving fields
with Rabi frequency $\Omega_a$ and $\Omega_b$ resonantly and dispersively
(detuning $-\Delta_1$) driving the transitions $|r\rangle \leftrightarrow |0\rangle$. For the
Step 2, the two lasers are switched to coupling the transi-
itions $|r\rangle \leftrightarrow |1\rangle$ resonantly and dispersively. In the meantime, the
process based on the Rydberg antiblockade effect will con-
tinuously accomplish the transitions $|p\rangle \leftrightarrow |0\rangle$ and $|p\rangle \leftrightarrow |1\rangle$ with
two lasers (Rabi frequencies $\Omega_p$, detunings $-\Delta_2$) regardless
of which Step in action. Moreover, we consider the Ryd-
berg state $|r(p)\rangle$ decays to the ground states with the same rate
$\gamma_r(p)/2$, and in what follows we set $\gamma_r = \gamma_p = \gamma$. In Fig. 5(c),
we also depict the temporal schematic of the alternate opera-
tions to further clarify the scheme. In the interaction picture,
the full master equation for the two steps can be written as

$$
\dot{\rho} = -i[H_{S1} + H_p, \rho] + \mathcal{L}\rho, \quad (15)
$$

and

$$
\dot{\rho} = -i[H_{S2} + H_p, \rho] + \mathcal{L}\rho, \quad (16)
$$

with

$$
H_{S1(2)} = \sum_{j=1}^{3}(\Omega_a + \Omega_b e^{-i\Delta t_j})\sigma_j^{r(1)} + \text{H.c.} + \sum_{j<k} U_{rr}\sigma_j^{rr}\sigma_k^{rr},
$$

$$
H_p = \sum_{j=1}^{3}\sqrt{2}\Omega_p\sigma_j^{pp} e^{-i\Delta t} + \text{H.c.} + \sum_{j<k} U_{pp}\sigma_j^{pp}\sigma_k^{pp},
$$

$$
\mathcal{L}\rho = \sum_{\alpha=1}^{3} \sum_{j=1}^{3} L_j^\alpha \rho L_j^{\alpha\dagger} - \frac{1}{2}(L_j^{\alpha\dagger}L_j^\alpha + \rho L_j^{\alpha\dagger}L_j^\alpha),
$$
where the Rydberg interaction of both atoms at $|r(p)\rangle$ is described by $U_{r(pp)}$, the Rydberg interactions of two atoms occupying different Rydberg states can be ignored by means of regulating the interatomic distance and the atomic principal quantum numbers [54, 55], and the Lindblad operators are $L_{\gamma}^{(1)} = \frac{1}{\Delta} |\alpha\rangle \langle \beta|$, and $L_{\gamma}^{(2)} = \frac{1}{\Delta} |\alpha\rangle \langle \beta|$. For completeness, here we briefly reproduce some results on the URP of Ref. [52] that are essential to understand our computational scheme. Referring to the conditions of the URP, we can take into account the $U_{r} = \Delta_{r}$ and rotate the $H_{S1}$ with $\exp(-i\sum_{j<k} U_{r,\sigma_{\sigma_{j}}^{r}}\sigma_{\sigma_{k}}^{r})$. In the large-detuning regime, the $H_{S1}$ can be divided into

$$H_{S1} = H_{S1}^{1} + H_{S1}^{2} + H_{S1}^{3},$$

with

$$H_{S1}^{1} = \Omega \lambda_{a}(|110\rangle\langle11| + |011\rangle\langle011| + |011\rangle\langle111|) + H.c.,$$

$$H_{S1}^{2} = \frac{\sqrt{3}}{2} \Omega \lambda_{a}(|000\rangle\langle0| + |010\rangle\langle1| + |100\rangle\langle-1|),$$

$$H_{S1}^{3} = \left[\Omega \lambda_{a}(|110\rangle\langle11| + |011\rangle\langle011| + |011\rangle\langle111|) + \sqrt{3} \Omega \lambda_{a}(|111\rangle\langle111| + |000\rangle\langle000|) + |000\rangle\langle000| + |111\rangle\langle111|)

$$e^{\Delta_{r}t} + H.c.,$$

where $|D_{1}\rangle = (|000\rangle + |010\rangle + |100\rangle)\sqrt{3}$, $|D_{2}\rangle = (|000\rangle + |010\rangle + |010\rangle)\sqrt{2}$, $|T_{1}\rangle = (|100\rangle + |110\rangle)\sqrt{2}$, and $|T_{3}\rangle = (|011\rangle + |011\rangle)\sqrt{2}$. To express these interactions visually, we exhibit the corresponding collective three-atom energy levels and transitions of Eq. (17) in Fig. 6. The ground states will be resonantly and dispersively excited to the single excited states except for the ground state $|111\rangle$ which is not evolved via $H_{S1}$. The single excited states $|T_{1}\rangle$, $|T_{2}\rangle$, $|T_{3}\rangle$, and $|D_{1}\rangle$ can be resonantly and dispersively pumped to the corresponding double excited states $|11r\rangle$, $|r1r\rangle$, $|rr1\rangle$, and $|D_{2}\rangle$, where the double excited state $|D_{2}\rangle$ will be further transferred to $|rrr\rangle$ dispersively.

In the limit of $\Delta_{r} \gg \Delta_{a} \gg \Omega_{a}$, $H_{S1}^{3}$ can approximate to the combination between the Stark-shift terms and the equivalent direct transitions from the ground states with three or two atoms at $|0\rangle$ to the corresponding double excited states. Moreover, the Stark-shift terms with the orders of $\Omega_{a}^{2}/\Delta_{a}$ can be canceled out utilizing the other ancillary levels, while the other terms with the orders of $\Omega_{a}^{2}/\Delta_{a}$ and $\Omega_{a}/\Delta_{a}$ can be ignored as $\Omega_{a} \gg \Delta_{a}$. Consequently, $H_{S1}^{3}$ is useless for the scheme.

Then we can rewrite the $H_{S1}^{3}$ by diagonalizing the terms of $\Omega_{a}$, and

$$H_{S1}^{3} = \frac{3}{2} \Omega_{a} |000\rangle\langle000| + |010\rangle\langle010| + |100\rangle\langle100| + H.c.,$$

where $|D_{1}\rangle = (|D_{1}\rangle + |D_{2}\rangle)\sqrt{3}$, $|T_{1}\rangle = (|T_{1}\rangle + |T_{2}\rangle + |T_{3}\rangle)|r1r\rangle\langle r1r| + |r1r\rangle\langle r1r|$, and $|T_{3}\rangle = (|T_{3}\rangle + |rr1\rangle\langle rr1|)/\sqrt{2}$. According to the above equation, we can find that the effective form of $H_{S1}^{3}$ tends to 0 as $\Omega_{a} \gg \Omega_{a}$. In other words, the states with three or two atoms at $|0\rangle$ cannot evolve to others by $H_{S1}^{3}$ since the corresponding detunings $\pm 2\Omega_{a}$ or $\pm \sqrt{2}\Omega_{a}$.

To sum up, the effective Hamiltonian of $H_{S1}$ is $H_{S1}^{eff}$, which is the so-called URP in Ref. [52]. Harnessing the similar recipe, we can obtain the effective form of $H_{S2}$.

$$H_{S1}^{eff} = \Omega_{a}(|100\rangle\langle r00| + |010\rangle\langle0r0| + |100\rangle\langle000| + H.c.)$$

In light of the switching driving of $H_{S1}^{eff}$ and $H_{S2}^{eff}$, it is obvious that the ground states with one or two atoms at $|0\rangle$ can be pumped into the Rydberg excited states $|11r\rangle$, $|1r1\rangle$, $|r11\rangle$, $|r00\rangle$, $|0r0\rangle$, $|00r\rangle$ that will further decay to the ground states via the spontaneous emission, and only the states $|111\rangle$ and $|000\rangle$ are steady at all times. To intuitively verify the validity of these analyses, we have plotted the dynamical evolution for the populations of $|000\rangle$ and $|111\rangle$ governed by the switching driving of $H_{S1}$ and $H_{S2}$ in Fig. 7. After the alternate operations executed $N$ times, the system beginning with a mixed state is stabilized at the subspace spanned by $|000\rangle$, $|111\rangle$, which can be expanded via $\{|G\rangle |G\rangle, \{|H\rangle |H\rangle\}$, the total population trends towards unit, i.e., $P_{000} + P_{111} = 49.79\% + 49.79\% = 99.58\%$ at $\Omega_{a}t = 50000$. It faithfully designates the feasibility of the process based on the switching driving of URP.

Besides, the process taking advantage of the Rydberg antiblockade effect makes the state $|GHZ_{+}\rangle$ unstable by continuously transferring the state $|++\rangle$ to $|rrr\rangle$. It can be
The dynamical evolution for the populations governed by the switching driving of $H_{S1}$ and $H_{S2}$. The initial state is randomly chosen as a mixed state $\rho_0 = \frac{1}{6}(|100\rangle \langle 100| + |010\rangle \langle 010| + |001\rangle \langle 001| + |101\rangle \langle 011| + |101\rangle \langle 101| + |110\rangle \langle 110|)$. The other parameters are $\Omega_\alpha = 0.02\Omega_b$, $\Delta_1 = 300\Omega_b$, $\gamma = 0.01\Omega_b$, and $N = 10$ is the switching number.

Indicated by the Hamiltonian $H_p$. By virtue of the basis $\{|++\rangle, |S_1\rangle, |S_2\rangle, |rrr\rangle\}$ with $|S_1\rangle = (|++\rangle + |+r\rangle + |r+\rangle)/\sqrt{3}$ and $|S_2\rangle = (|+rr\rangle + |r+r\rangle + |rr+\rangle)/\sqrt{3}$, we can simplify $H_p$ as

$$H_p = \sqrt{6}\Omega_p(|++\rangle\langle S_1| + |rrr\rangle\langle S_2|) + 2\sqrt{2}\Omega_p|S_1\rangle\langle S_2| + H.c. - 3\Delta_2|S_1\rangle\langle S_1| + (U_{pp} - 2\Delta_2)|S_2\rangle\langle S_2| + (3U_{pp} - 3\Delta_2)|rrr\rangle\langle rrr|.$$  

When we suppose $U_{pp} = \Delta_2 \gg \Omega_p$, the Rydberg antiblockade effect is satisfied and the effective form of $H_p$ can be equal to

$$H_p^{\text{eff}} = \frac{12\sqrt{2}\Omega_p^3}{\Delta_2} |++\rangle\langle rrr| + H.c.,$$

where we have left out the order of $O(\Delta_2^2/\Delta_2^2)$ and the Stark-shift terms that can be canceled by ancillary levels. Due to the $H_p^{\text{eff}}$, only the state $|++\rangle$ can evolve to the state $|rrr\rangle$, which will spontaneously radiate back to the ground states. Then the state $|\text{GHZ}_+\rangle$ is not stable anymore. Meanwhile, combining the Rydberg state decay and the switching driving of $H_{S1}$ and $H_{S2}$, the target state $|\text{GHZ}_-\rangle$ is turned into the unique steady state of the whole system and the second scheme is finished.

**B. Numerical results**

In Fig. 8, we characterize the fidelity of the target state $|\text{GHZ}_-\rangle$ respectively governed by the full master equation (solid line) and the effective master equation (empty circles) in the interest of exemplifying the correctness for the above derivations, where the effective master equation can be acquired via replacing the $H_{S1(2)} + H_p$ of Eq. (15) (Eq. (16)) with $H_{S1(2)}^{\text{eff}} + H_p^{\text{eff}}$. The empty circles is in full accord with the curve of the original system, which thoroughly certifies the rationality of the reduced system. Moreover, beginning with a mixed state $\rho_0 = \frac{1}{6}(|100\rangle \langle 100| + |010\rangle \langle 010| + |001\rangle \langle 001| + |101\rangle \langle 011| + |110\rangle \langle 110|)$. The other parameters are $\Omega_\alpha = 0.02\Omega_b$, $\Omega_p = \Omega_b$, $\Delta_1 = 300\Omega_b$, $\Delta_2 = 80\Omega_b$, $\gamma = 0.01\Omega_b$, and $N = 64$.

**IV. DISCUSSION AND CONCLUSION**

Here, we succinctly explain the generalization of the scheme based on polychromatic driving fields to prepare an arbitrary multipartite GHZ state with odd numbers of atoms. For example, we consider a five-Rydberg-atom system interacts with polychromatic driving fields $\Omega_{1,2,3}$ and a resonant laser $\Omega_0$. The corresponding atomic energy levels and transitions are the same as those in Fig. 1. But the next-nearest neighbor Rydberg interaction is neglected in the generalized scheme, i.e., the terms of $U_{jk}\sigma_j^r\sigma_k^r$ is replaced with...
are still practicable while needn’t the certain transport time or the tailored initial state. We believe our schemes supply a viable prospect with regard to preparations of multipartite GHZ states.

ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation of China (NSFC) under Grants No. 11774047.

[1] M. Saffman, T. G. Walker, and K. Mølmer, “Quantum information with rydberg atoms,” Rev. Mod. Phys. 82, 2313 (2010).

[2] Y. O. Dudin and A. Kuzmich, “Strongly interacting rydberg excitations of a cold atomic gas,” Science 336, 887 (2012).
[3] J. Lampen, H. Nguyen, L. Li, P. R. Berman, and A. Kuzmich, “Long-lived coherence between ground and rydberg levels in a magic-wavelength lattice,” Phys. Rev. A 98, 033411 (2018).
[4] H. Weimer, M. Muller, I. Lesanovsky, P. Zoller, and H. P. Buechler, “A rydberg quantum simulator,” Nat. Phys. 6, 382–388 (2010).
[5] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papic, “Weak ergodicity breaking from quantum many-body scars,” Nat. Phys. 14, 745 (2018).
[6] A. Piheiro Orioli, A. Signoles, H. Wildhagen, G. Gunter, J. Berges, S. Whitlock, and M. Weidemüller, “Relaxation of an isolated dipolar-interacting rydberg quantum spin system,” Phys. Rev. Lett. 120, 063601 (2018).
[7] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone, “Advances in quantum metrology,” Nat. Photonics 5, 222 (2011).
[8] Adrien Facon, Eva-Katharina Dietsche, Dorian Grosso, Serge Haroche, Jean-Michel Raimond, Michel Brune, and Sébastien Gleyzes, “A sensitive electrometer based on a rydberg atom in a schrödinger-cat state,” Nature 535, 262 (2016).
[9] C. L. Degen, F. Reinhard, and P. Cappellaro, “Quantum sensing,” Rev. Mod. Phys. 89, 035002 (2017).
[10] A. Arias, G. Lochead, T. M. Wintermantel, S. Helmrich, and S. Whitlock, “Realization of a rydberg-dressed Ramsey interferometer and electrometer,” Phys. Rev. Lett. 122, 053601 (2019).
[11] D. Jaksh, J. I. Cirac, P. Zoller, S. L. Rolston, R. Côté, and M. D. Lukin, “Fast quantum gates for neutral atoms,” Phys. Rev. Lett. 85, 2208–2211 (2000).
[12] E. Urban, T. A. Johnson, T. Henage, L. Isenhower, D. Yavuz, T. G. Walker, and M. Saffman, “Observation of rydberg blockade between two atoms,” Nat. Phys. 5, 110 (2009).
[13] M. Saffman and K. Mølmer, “Efficient multiparticle entanglement via asymmetric rydberg blockade,” Phys. Rev. Lett. 102, 240502 (2009).
[14] T. Wilk, A. Gaetan, C. Evellin, J. Wolters, Y. Miroshnychenko, P. Grangier, and A. Browaeys, “Entanglement of two individual neutral atoms using rydberg blockade,” Phys. Rev. Lett. 104, 010502 (2010).
[15] D. D. Bhaktavatsala Rao and K. Mølmer, “Dark entangled steady states of interacting rydberg atoms,” Phys. Rev. Lett. 111, 033606 (2013).
[16] I. I. Beterov, G. N. Hamzina, E. A. Yakshina, D. B. Tretyakov, V. M. Entin, and I. I. Ryabtsev, “Adiabatic passage of radiofrequency-assisted förster resonances in rydberg atoms for two-qubit gates and the generation of bell states,” Phys. Rev. A 97, 032701 (2018).
[17] A.-X. Chen, “Implementation of deutsch-jozsa algorithm and determination of value of function via rydberg blockade,” Opt. Express 19, 2037 (2011).
[18] Y. Han, B. He, K. Heshami, C.-Z. Li, and C. Simon, “Quantum repeaters based on rydberg-blockade-coupled atomic ensembles,” Phys. Rev. A 81, 052311 (2010).
[19] C. Ates, T. Pohl, T. Pattard, and J. M. Rost, “Antiblockade in rydberg excitation of an ultracold lattice gas,” Phys. Rev. Lett. 98, 023002 (2007).
[20] T. Amthor, C. Giese, C. S. Hofmann, and M. Weidemüller, “Evidence of antiblockade in an ultracold rydberg gas,” Phys. Rev. Lett. 104, 013001 (2010).
[21] S. L. Su, H. Z. Shen, Erjun Liang, and Shou Zhang, “One-step construction of the multiple-qubit rydberg controlled-phase gate,” Phys. Rev. A 98, 032306 (2018).
[22] S.-L. Su, Fu-Qiang Guo, L. Tian, X.-Y. Zhu, L.-L. Yan, E.-J. Liang, and M. Feng, “Nondestructive rydberg parity meter and its applications,” Phys. Rev. A 101, 012347 (2020).
[23] M. A. Carr and M. Saffman, “Preparation of entangled and antiferromagnetic states by dissipative rydberg pumping,” Phys. Rev. Lett. 111, 033607 (2013).
[24] S.-L. Su, Y. Tian, H. Z. Shen, H. Zang, E. Liang, and S. Zhang, “Applications of the modified rydberg antiblockade regime with simultaneous driving,” Phys. Rev. A 96, 042335 (2017).
[25] J. Song, C. Li, Z.-J. Zhang, Y.-Y. Jiang, and X. Xia, “Implementing stabilizer codes in noisy environments,” Phys. Rev. A 96, 032336 (2017).
[26] D.-X. Li, X.-Q. Shao, J.-H. Wu, and X. X. Yi, “Dissipation-induced w state in a rydberg-atom-cavity system,” Opt. Lett. 43, 1639 (2018).
[27] D. X. Li and X. Q. Shao, “Directional quantum state transfer in a dissipative rydberg-atom-cavity system,” Phys. Rev. A 99, 032348 (2019).
[28] Ryszard Horodecki, Paweł Horodecki, Michal Horodecki, and Karol Horodecki, “Quantum entanglement,” Rev. Mod. Phys. 81, 865 (2009).
[29] Robert Raussendorf and Hans J. Briegel, “A one-way quantum computer,” Phys. Rev. Lett. 86, 5188 (2001).
[30] Masahito Hayashi and Tomoyuki Morimae, “Verifiable measurement-only blind quantum computing with stabilizer testing,” Phys. Rev. Lett. 115, 220502 (2015).
[31] Mariami Gachechiladze, Ofried Güne, and Akimasa Miyake, “Changing the circuit-depth complexity of measurement-based quantum computation with hypergraph states,” Phys. Rev. A 99, 052304 (2019).
[32] D. Gottesman, “Stabilizer codes and quantum error correction,” ArXiv e-prints 970502 (1997).
[33] D. Schlingemann and R. F. Werner, “Quantum error-correcting codes associated with graphs,” Phys. Rev. A 65, 012308 (2001).
[34] Mark Hillery, Vladimir Bužek, and André Berthiaume, “Quantum secret sharing,” Phys. Rev. A 59, 1829 (1999).
[35] W. McCutcheon, A. Pappa, B. A. Bell, A. McMillan, A. Chailoux, T. Lawson, M. Mafu, D. Markham, E. Diamanti, I. Kerenidis, J. G. Rarity, and M. S. Tame, “Experimental verification of multipartite entanglement in quantum networks,” Nat. Commun. 7, 13251 (2016).
[36] Ahmed Farouk, J. Batle, M. Elhoseny, Mosayeb Naseri, Muzaffar Lone, Alex Fedorov, Majid Alkhambashi, Syed Hassan Ahmed, and M. Abdel-Aty, “Robust general n user authentication scheme in a centralized quantum communication network via generalized ghz states,” Front. Phys. 13, 130306 (2018).
[37] F. Verstraete, V. Murg, and J.I. Cirac, “Matrix product states, projected entangled pair states, and variational renormalization group methods for quantum spin systems,” Adv. Phys. 57, 143 (2008).
[38] Román Orús, “A practical introduction to tensor networks: Matrix product states and projected entangled pair states,” Ann. Phys. 349, 117 (2014).
[39] Jian-Wei Pan, Zeng-Bing Chen, Chao-Yang Lu, Harald Weinfurter, Anton Zeilinger, and Marek Żukowski, “Multiphoton entanglement and interferometry,” Rev. Mod. Phys. 84, 777 (2012).
[40] D. M. Greenberger, M.A. Horne, and A. Zeilinger, “Bell’s theorem, quantum theory, and conceptions of the universe,” (Kluwer Academic, Dordrecht, 1989) pp. 69–72.
[41] Tetsufumi Tanamoto, Yu-xi Liu, Shinobu Fujita, Xuedong Hu, and Franco Nori, “Producing cluster states in charge qubits and flux qubits,” Phys. Rev. Lett. 97, 230501 (2006).
[42] Ying Li, Peter C. Humphreys, Gabriel J. Mendoza, and Simon C. Benjamin, “Resource costs for fault-tolerant linear optical quantum computing,” Phys. Rev. X 5, 041007 (2015).
[43] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone, “Quantum-enhanced measurements: Beating the standard quantum limit,” Science 306, 1330 (2004).

[44] W. Dür, M. Skotiniotis, F. Fröwis, and B. Kraus, “Improved quantum metrology using quantum error correction,” Phys. Rev. Lett. 112, 080801 (2014).

[45] Luca Pezzè, Augusto Smerzi, Markus K. Oberthaler, Roman Schmied, and Philipp Treutlein, “Quantum metrology with nonclassical states of atomic ensembles,” Rev. Mod. Phys. 90, 035005 (2018).

[46] J. J. Bollinger, Wayne M. Itano, D. J. Wineland, and D. J. Heinzen, “Optimal frequency measurements with maximally correlated states,” Phys. Rev. A 54, R4649 (1996).

[47] S. F. Huelga, C. Macchiavello, T. Pellizzari, A. K. Ekert, M. B. Plenio, and J. I. Cirac, “Improvement of frequency standards with quantum entanglement,” Phys. Rev. Lett. 79, 3865 (1997).

[48] F. Reiter, D. Reeb, and A. S. Sørensen, “Scalable dissipative preparation of many-body entanglement,” Phys. Rev. Lett. 117, 040501 (2016).

[49] X. Q. Shao, J. H. Wu, X. X. Yi, and Gui-Lu Long, “Dissipative preparation of steady greenberger-horne-zeilinger states for rydberg atoms with quantum zeno dynamics,” Phys. Rev. A 96, 062315 (2017).

[50] A. Omran, H. Levine, A. Keesling, G. Semeghini, T. T. Wang, S. Ebadi, H. Bernien, A. S. Zibrov, H. Pichler, S. Choi, J. Cui, M. Rossignolo, P. Rembold, S. Montangero, T. Calarco, M. Endres, M. Greiner, V. Vuletić, and M. D. Lukin, “Generation and manipulation of schrödinger cat states in rydberg atom arrays,” Science 365, 570 (2019).

[51] Dong-Xiao Li, Tai-Yu Zheng, and Xiao-Qiang Shao, “Adiabatic preparation of multipartite ghz states via rydberg ground-state blockade,” Opt. Express 27, 20874 (2019).

[52] D. X. Li and X. Q. Shao, “Unconventional rydberg pumping and applications in quantum information processing,” Phys. Rev. A 98, 062338 (2018).

[53] T. M. Wintermantel, Y. Wang, G. Lochead, S. Shevate, G. K. Brennen, and S. Whitlock, “Unitary and nonunitary quantum cellular automata with thermal arrays,” Phys. Rev. Lett. 124, 070503 (2020).

[54] T. G. Walker and M. Saffman, “Consequences of zeeman degeneracy for the van der waals blockade between rydberg atoms,” Phys. Rev. A 77, 032723 (2008).

[55] Y.-M. Liu, X.-D. Tian, D. Yan, Y. Zhang, C.-L. Cui, and J.-H. Wu, “Nonlinear modifications of photon correlations via controlled single and double rydberg blockade,” Phys. Rev. A 91, 043802 (2015).

[56] Anders Sørensen and Klaus Mølmer, “Quantum computation with ions in thermal motion,” Phys. Rev. Lett. 82, 1971 (1999).

[57] Daniel Jonathan and Martin B. Plenio, “Light-shift-induced quantum gates for ions in thermal arrays,” Phys. Rev. Lett. 87, 127901 (2001).

[58] Shi-Biao Zheng, “Quantum logic gates for hot ions without a speed limitation,” Phys. Rev. Lett. 90, 217901 (2003).

[59] K. J. Engel and R. Nagel, One-Parameter Semigroups for Linear Evolution Equations (Springer, New York, 2000).

[60] József Zsolt Bermánd and Juan Mauricio Torres, “Partly invariant steady state of two interacting open quantum systems,” Phys. Rev. A 92, 062114 (2015).

[61] X. X. Li, H. D. Yin, D. X. Li, and X. Q. Shao, “Deterministic generation of maximally discordant mixed states by dissipation,” Phys. Rev. A 101, 012329 (2020).

[62] Klaus Mølmer and Anders Sørensen, “Multiparticle entanglement of hot trapped ions,” Phys. Rev. Lett. 82, 1835 (1999).

[63] Xiao-Qiang Shao, “Engineering steady entanglement for trapped ions at finite temperature by dissipation,” Phys. Rev. A 98, 042310 (2018).

[64] L. Allan and J. M. Eberly, Optical Resonance and TwoLevel Atoms (Dover, New York, 1987).

[65] Fazal Badshah, Muhammad Irfan, Sajid Qamar, and Shahid Qamar, “Coherent control of tunneling and traversal of ultracold atoms through vacuum-induced potentials,” Phys. Rev. A 88, 044101 (2013).

[66] Ran Cheng, Matthew W. Daniels, Jian-Gang Zhu, and Di Xiao, “Ultrafast switching of antiferromagnets via spin-transfer torque,” Phys. Rev. B 91, 064423 (2015).

[67] D. W. Schölenber, A. Eifelf, M. Genkin, S. Whitlock, and S. Wüster, “Quantum simulation of energy transport with embedded rydberg aggregates,” Phys. Rev. Lett. 114, 123005 (2015).

[68] Daniel Barredo, Sylvain de Léséleuc, Vincent Lienhard, Thierry Lahaye, and Antoine Browaeys, “An atom-by-atom assembler of defect-free arbitrary two-dimensional atomic arrays,” Science 354, 1021 (2016).

[69] Manuel Endres, Hannes Bernien, Alexander Keesling, Harry Levine, Eric R. Anschuetz, Alexandre Krajenbrink, Crystal Senko, Vladan Vuletic, Markus Greiner, and Mikhail D. Lukin, “Atom-by-atom assembly of defect-free one-dimensional cold atom arrays,” Science 354, 1024 (2016).

[70] D. W. Schölenber, C. D. B. Bentley, and A. Eifelf, “Engineering thermal reservoirs for ultracold dipolec-dipole-interacting rydberg atoms,” New J. Phys. 20, 013011 (2018).

[71] Alpha Gaétan, Yevhen Miroshnychenko, Tatjana Wilk, Amo- sen Chotia, Matthieu Viteau, Daniel Comparat, Pierre Pillet, Antoine Browaeys, and Philippe Grangier, “Observation of collective excitation of two individual atoms in the rydberg blockade regime.” Nat. Phys. 5, 115 (2009).

[72] Matthias M. Müller, Michael Murphy, Simone Montangero, Tommaso Calarco, Philippe Grangier, and Antoine Browaeys, “Implementation of an experimentally feasible controlled-phase gate on two blockaded rydberg atoms,” Phys. Rev. A 89, 032334 (2014).

[73] Y. Miroshnychenko, A. Gaétan, C. Evelin, P. Grangier, D. Comparat, P. Pillet, T. Wilk, and A. Browaeys, “Coherent excitation of a single atom to a rydberg state,” Phys. Rev. A 82, 013405 (2010).

[74] A Grankin, E Brion, E Bimbard, R Boddeda, I Usmani, A Ourjoumtsev, and P Grangier, “Quantum statistics of light transmitted through an intracavity rydberg medium,” New J. Phys. 16, 043020 (2014).

[75] S. Whitlock, H. Wildhagen, H. Weimer, and M. Weidemüller, “Diffusive to nonergodic dipolar transport in a dissipative atomic medium,” Phys. Rev. Lett. 123, 213606 (2019).