A Proposal for Experimental Detection of Amplitude nth-Power Squeezing

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Recently, in several theoretical investigations, amplitude nth-power squeezing has been studied with \( n = 2, 3, 4, 5 \). In the present paper, we give a proposal for experimental detection of amplitude nth-power squeezing using ordinary homodyning with coherent light for arbitrary power \( n \) and discuss in detail its theory. The proposed scheme requires only repeated measurements of the factorial moments of number of photons in the light obtained after homodyning, with various shifts of phase of coherent light, and involves no approximations, whatsoever. This has advantage over the method proposed by Shchukin and Vogel [Phys. Rev. A 72, 043808 (2005)] in that our method requires only one beam splitter and only one photodetector, and also lesser number of repetitions of experiment with phase-shifted coherent light.

Keywords: Detection of squeezing, Amplitude nth-power squeezing, Higher-order squeezing, Homodyning, Factorial moments of photon number, Quantum efficiency of photodetector.

I. INTRODUCTION

A single mode light has two components in quadrature and the operators corresponding to these in quantum mechanics are non-commuting \([1–3]\) and satisfy an uncertainty relation. For classical optical fields, which have a non-negative weight function in Sudarshan-Glauber diagonal representation \([4]\), the variances of the quadrature operators have equal lower bounds. Optical fields may have a non-classical feature and variance of one quadrature amplitude may be less than this lower bound at the expense of increased variance for the other. This non-classical feature, called squeezing, was studied earlier in academic interest \([5]\) but its importance has now been understood because of its application to optical communication \([6]\), optical waveguide tap \([7]\), gravitational wave detection \([8]\), interferometric techniques \([9]\), enhancing the channel capacity \([10]\), quantum teleportation \([11]\), quantum dense coding \([12]\), quantum cryptography \([13]\), Nano-displacement measurement \([14]\), optical storage \([15]\) and amplification of signals \([16]\). Generation of squeezed states has been reported in a variety of nonlinear optical processes, e.g., multi-photon absorption \([17]\), degenerate parametric amplifier \([18]\), free electron laser \([19]\), harmonic generation \([20]\), degenerate parametric oscillation \([21]\), degenerate four-wave mixing \([22]\), degenerate hyper-Raman scattering \([23]\), resonance fluorescence \([24]\), the single atom-single field mode interaction \([25]\), superposed coherent states \([26]\), and nonlinear beam splitter \([27]\).

Squeezing is understood by writing the annihilation operator \(\hat{a} \) in terms of the two hermitian quadrature amplitude operators \(\hat{X}_1 \) and \(\hat{X}_2 \) in the form, \(\hat{a} = \hat{X}_1 + i\hat{X}_2 \) or \(\hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger) \), \(\hat{X}_2 = \frac{1}{2}(\hat{a} - \hat{a}^\dagger) \) which gives \(\langle \hat{X}_1, \hat{X}_2 \rangle = \frac{1}{4} \) or \(\langle (\Delta \hat{X}_1)^2 \rangle \langle (\Delta \hat{X}_2)^2 \rangle \geq \frac{1}{16} \), where conical brackets denote expectation values and \(\Delta \hat{X}_{1,2} \equiv \hat{X}_{1,2} - \langle \hat{X}_{1,2} \rangle \). \(\hat{X}_1 \) is said to be squeezed if \(\langle (\Delta \hat{X}_1)^2 \rangle < \frac{1}{4} \) and \(\hat{X}_2 \) is said to be squeezed if \(\langle (\Delta \hat{X}_2)^2 \rangle < \frac{1}{4} \).

Instead of considering \(\hat{X}_1 \) and \(\hat{X}_2 \) one can consider the most general quadrature amplitude operator, \(\hat{X}_\theta = \hat{X}_1 \cos \theta + \hat{X}_2 \sin \theta = \frac{1}{2}([\hat{a}^\dagger e^{i\theta} + \hat{a} e^{-i\theta}]) \). The commutation relation, \(\left[\hat{X}_{\theta}, \hat{X}_{\theta + \pi/2}\right] = \frac{1}{2} \), which gives uncertainty relation, \(\langle (\Delta \hat{X}_\theta)^2 \rangle \langle (\Delta \hat{X}_{\theta + \pi/2})^2 \rangle \geq \frac{1}{16} \), then leads to idea of squeezing of the general component \(\hat{X}_\theta \) when \(\langle (\Delta \hat{X}_\theta)^2 \rangle < \frac{1}{4} \). The non-classical nature of squeezing, is clear from the fact that, for Sudarshan-Glauber diagonal representation \([4]\), \(\rho = \int d^2 \alpha P(\alpha)\langle \alpha \rangle \langle \alpha \rangle \), we have

\[
\langle (\Delta \hat{X}_\theta)^2 \rangle \leq \frac{1}{4} = \int d^2 \alpha P(\alpha) \left| Re\{\langle \alpha - \bar{\alpha} e^{-i\theta} \rangle \right|^2 < 0,
\]

\[
\bar{\alpha} \equiv \int d^2 \alpha P(\alpha) \alpha.
\] (1)

Concepts of amplitude squeezing and phase squeezing were introduced using (i) the uncertainty relation \(\langle (\Delta \hat{N})^2 \rangle \langle (\Delta \hat{\phi})^2 \rangle \geq \frac{1}{4} \), where photon number operator and phase uncertainty were defined by \(\hat{N} = \hat{a}^\dagger \hat{a} \), \(\langle (\Delta \hat{\phi})^2 \rangle = \langle \langle (\Delta \hat{S})^2 \rangle + \langle (\Delta \hat{C})^2 \rangle \rangle \), \(\langle (\Delta \hat{S})^2 \rangle + \langle (\Delta \hat{C})^2 \rangle \rangle \) with \(C + iS = (N + 1)^{-1/2} \hat{a} \), \(\hat{a}^\dagger \hat{a} \), \(\hat{a}^\dagger \hat{a} \) and (ii) the relation \(\langle (\Delta \hat{N})^2 \rangle \geq \langle \hat{N} \rangle \) for classical light, resulting in the definition \(\langle (\Delta \hat{N})^2 \rangle < \langle \hat{N} \rangle \) for amplitude squeezing and \(\langle (\Delta \hat{\phi})^2 \rangle < 1/4 \langle \hat{N} \rangle \) for phase squeezing. The terminology, amplitude and phase squeezing, is widely accepted and is described in text-books \([29]\) and encyclopedia \([30]\).

Squeezing of radiation has been generalized in a number of ways. In the first, Hong and Mandel \([31]\) consider even powers of difference between quadrature amplitudes and its mean values, and the field is called squeezed to order \(2n \) whenever \(\langle (\Delta \hat{X}_\theta)^{2n} \rangle \) is less than its value for any coherent state. This type of squeezing has been studied by several authors \([32]\). Another generalization was introduced by Hillery \([33]\) for the lowest order \((n = 2)\), and by Zhang et al. \([34]\) for \(n > 2 \). These authors consider generalized quadrature operators obtained by separating Hermitian and anti-Hermitian parts of the square or nth-power of the annihilation operator and define squeezing.
Explicitly, in the general case, we define
\[
\hat{X}_\theta^{(n)} = \frac{1}{2}(\hat{a}^n e^{i\theta} + \hat{a}^n e^{-i\theta}),
\]
which gives commutation relation
\[
[\hat{X}_\theta^{(n)}, \hat{X}_{\theta + \pi/2}^{(n)}] = \frac{i}{2} \hat{W}^{(n)},
\]
where
\[
\hat{W}^{(n)} = \sum_{r=1}^{n} r! (n C_r)^2 \hat{a}^{n-r} \hat{a}^{n-r}.
\]
The uncertainty relation,
\[
\langle (\Delta \hat{X}_\theta^{(n)})^2 \rangle \langle (\Delta \hat{X}_{\theta + \pi/2}^{(n)})^2 \rangle \geq \frac{1}{16} \langle \hat{W}^{(n)} \rangle^2,
\]
therefore, leads to squeezing of \( \hat{X}_\theta^{(n)} \) when
\[
\langle (\Delta \hat{X}_\theta^{(n)})^2 \rangle < \frac{1}{4} \langle \hat{W}^{(n)} \rangle.
\]
Since we can write \( \langle (\Delta \hat{X}_\theta^{(n)})^2 \rangle = \langle (\hat{X}_\theta^{(n)})^2 \rangle - \frac{1}{4} \langle \hat{W}^{(n)} \rangle \),
we have \( \langle (\Delta \hat{X}_\theta^{(n)})^2 \rangle = \langle (\Delta \hat{X}_\theta^{(n)})^2 \rangle - \frac{1}{4} \langle \hat{W}^{(n)} \rangle \), where
double dots \( \langle \cdot \rangle \) denote normal ordering of operators.
This tells that Eq. (6), which gives the definition of nth-order squeezing, can also be written in
the simpler form,
\[
\langle (\Delta \hat{X}_\theta^{(n)})^2 \rangle < 0.
\]
This type of squeezing was called amplitude-squared squeezing by Hillery [33] and nth-order
squeezing by Zhang et al. [34]. The spirit in this generalization of squeezing is entirely
different from that of lowest order amplitude squeezing, as in the former the uncertainty
relations for generalized quadrature operators are used while for the latter number-phase uncertainty
relations are used. Just after the publication of papers of Hillery [33] and Zhang et al. [34],
two papers of Zhang [35] used the phrase “Amplitude Nth-Power Squeezing” for such
generalization and since then this phrase has been used invariably in all later publications [36–45].
Amplitude-squared squeezing has been studied for an anharmonic oscillator [36], the interaction
between atom and radiation field [37], mixing of waves [38], Kerr medium [39],
coherent states [40]. Also enhancement of amplitude-squared squeezing is studied by mixing it
with coherent light beam [41]. Amplitude nth-power squeezing for higher-order has been studied by
several authors with \( n = 3 \) [42], 4 [43], 5 [44], and for \( n = k \) [45].

It may be noted that since the Hong and Mandel’s concept of 2nth order squeezing is not based on
any uncertainty relation, 2nth order squeezing can be obtained for both \( \hat{X}_\theta \) and \( \hat{X}_{\theta + \pi/2} \) simultaneously [46]. Simultaneous
squeezing of \( \hat{X}_\theta^{(n)} \) and \( \hat{X}_{\theta + \pi/2}^{(n)} \) is however ruled out because of
the uncertainty relations. Hillery also considered a second type of amplitude squared
squeezing by separating \( \langle \hat{a} - \langle \hat{a} \rangle \rangle^2 \) into its Hermitian and anti-Hermitian
parts. A different type of amplitude nth-power squeezing has been defined by Buek and Jex [47]. Other
generalizations of squeezing involve multi-mode operators and their commutation relations; the
common examples being sum and difference squeezing [48], spin squeezing [49], atomic
squeezing [50], and polarization squeezing [51].

Proposal for experimental detection of ordinary squeezing was given by Mandel [52] using
homodyning with intense light with adjustable phase from a local oscillator and measuring the number
of photons and its square in one of the outputs. Such homodyning of squeezed light converts
squeezing into sub-Poissonian photon statistics [53] and the degree of squeezing is obtained
from measurements of expectation values of photon number operator and its square. Prakash and Kumar
[54] showed a similar conversion of fourth-order squeezing into second-order sub-Poissonian photon
statistics and proposed a balanced homodyne method for detection of fourth-order squeezed light in
the similar fashion. Prakash and Mishra [55] extended the proposal of ordinary homodyning for
experimental detection of amplitude-squared squeezing by measuring higher order
moments of number operator of mixed light with shifted phases. They also studied higher-order
sub-Poissonian photon statistics conditions for non-classicality and discussed its use for the
detection of Hong and Mandels squeezing of arbitrary order. Prakash et al. reported recently [56] an
ordinary homodyne method for detection of second type of amplitude-squared squeezing of Hillery
by measuring the higher-order moments of the number operator in light obtained by homodyning
with intense coherent light. Another proposal for the detection of amplitude-kth-power squeezing
([57] for \( k = 2 \), [58] for general) by the measurements of the moments \( \langle \hat{a}^k \rangle \), \( \langle \hat{a}^{2k} \rangle \)
and \( \langle \hat{a}^k \hat{a}^k \rangle \) has been given on a technique based on balanced
homodyne correlation measurement [57–59]. In this method number of beam splitters and
photodetectors required increase with the increase in order of moments. Also a very large number
of repeated measurements with phase shift \( \varphi \) of the local oscillator are required as evaluation
of Fourier transform of a function of \( \varphi \) is involved. There is no experimental demonstration of any
form of higher-order squeezing in the literature. But proposals for experimental detection of
some higher-order squeezing have been given [54–58].

We present here a proposal for experimental detection of amplitude nth-power squeezing for
arbitrary power \( n \) by using the ordinary homodyne detection method. It may be noted that we
work without taking any approximation whatsoever like replacing operator for coherent state by
\( c \)-number or taking transmittance or reflectance of the beam splitter very small, and
our method requires only one photodetector and only one beam splitter.
II. THE DETECTION SCHEME

Schematic diagram for proposed detection scheme, shown in Fig. 1, contains the experimental outline and also the conceptual meaning of quantum efficiency of the experimental detector [2]. The beam splitter and ideal detector placed inside dotted rectangle model the real photodetector [2]. The signal represented by operator $\hat{a}$ is mixed using beam splitter with signal $be^{i\varphi}$ obtained by shifting by $\varphi$ the phase of signal from a local oscillator represented by operator $\hat{b}$ to give output signal $\hat{c}$ and $\hat{c}'$. One of the output signals, say $\hat{c}$ is detected. If the beam splitter has transmittance $T$ and we write $t = \sqrt{T}$ and as $r = \sqrt{1-T}$ coefficients of transmission and reflection for the amplitudes, respectively, we can write [3]

$$\hat{c} = t\hat{a} + rbe^{i\varphi},$$

with $t$ and $r$ real. Number operator of the mixed light is then

$$\hat{N}_c = \hat{c}^\dagger \hat{c} = (t\hat{a}^\dagger + r\hat{b}^\dagger e^{-i\varphi})(\hat{a} + rbe^{i\varphi}).$$

(9)

In this model [2] of photodetector with quantum efficiency $\eta$, the beam splitter mixes (i) input signal $\hat{c}$ and (ii) vacuum signal $\hat{a}_v$ and one of the outputs $\hat{d}$ given by

$$\hat{d} = \sqrt{\eta}\hat{c} + \sqrt{1-\eta}\hat{a}_v$$

(10)

is detected by an ideal detector having 100% efficiency. It is interesting to note that this model explains that the detected counts, $\langle \hat{d}^\dagger \hat{d} \rangle$, are $\eta$ times the incident number of photons, $\langle \hat{c}^\dagger \hat{c} \rangle$, i.e.,

$$\langle \hat{d}^\dagger \hat{d} \rangle = \eta \langle \hat{c}^\dagger \hat{c} \rangle$$

(11)

which is also obtained directly from Eq. (10). For factorial moments of order $n$, we get similarly

$$\langle \hat{d}^{n\dagger} \hat{d}^n \rangle = \eta^n \langle \hat{c}^{n\dagger} \hat{c}^n \rangle.$$  

(12)

The relationship for $n$-th power of photon number operator is not a simple one. This is one reason why we involve factorial moments of photon number operator and not powers of the photon number operator in our analysis and result, as was done earlier [55].

For the setup under consideration, the observed factorial moments of counts with phase shift $\varphi$ is then

$$M^{(n)}_{\varphi} = \eta^n \langle \hat{c}^{n\dagger} \hat{c}^n \rangle$$

$$= \eta^n \sum_{l,m=0}^{n} n! C^n_m t^{2n-l-m} r^{l+m}$$

$$\langle \hat{a}^{l+n\dagger} \hat{a}^{-m} \hat{b}^l \hat{b}^m \rangle e^{i(m-l)\varphi}. $$

(13)

If the local oscillator gives output in the coherent state $|\beta\rangle$ with the complex amplitude $\beta = |\beta|e^{i\theta_{\beta}}$ and observations are done for $M^{(n)}_{\varphi}$ for $\varphi = k\pi/n$ with $k = 0, 1, ..., 2n - 1$ we can easily find the values of observables

$$P_n = \frac{1}{2n} \sum_{k=0}^{2n-1} M^{(n)}_{\varphi} e^{ik\pi}$$

$$= (\eta tr|\beta\rangle)^n [\langle \hat{a}^{n\dagger} \rangle e^{i\delta_{\beta}} + \langle \hat{a}^{n}\rangle e^{-i\delta_{\beta}}],$$

(14)

$$Q_n = \frac{1}{2n} \sum_{k=0}^{2n-1} M^{(n)}_{\varphi}$$

$$= \eta^n t^{2n} \sum_{l=0}^{n} \langle \hat{c}_l \hat{c}_s \rangle (t^{-1}r|\beta\rangle)^l \langle \hat{a}^{n-l}\rangle \langle \hat{a}^{n-l}\rangle.$$  

(15)

Eq. (14) gives a method for measuring $\langle \hat{X}_x^{(n)} \rangle$ using $\theta_{\beta} = \theta/n$, but a method for measuring $\langle \hat{W}^{(n)} \rangle$ and therefore for moments $\langle \hat{a}^{n-l}\rangle \langle \hat{a}^{n-l}\rangle$ is still to be desired. To achieve this end, we can solve Eq. (15) for factorial moments of photon number operator by iteration. This is done in Appendix and it leads to

$$\langle \hat{a}^{n\dagger} \hat{a}^n \rangle = \eta^n t^{-2n} \sum_{s=0}^{n} \sum_{l=0}^{s-1} K_s(r|\beta\rangle)^2 \langle \hat{a}^{n-l}\rangle \langle \hat{a}^{n-l}\rangle,$$

(16)

where $K_s$ is defined by

$$K_s = \sum_{l=0}^{s-1} K_l(s\hat{c}_l \hat{c}_s)^2$$

(17)

Straight forward calculations lead to

$$P_{2n} - P_{2n}^2 + 2(\eta tr|\beta\rangle)^2 |\langle \hat{a}^{n\dagger} \hat{a}^n \rangle|$$

$$= P_{2n} - P_{2n}^2 + 2 \sum_{s=0}^{n} K_s(r|\beta\rangle)^2 \langle \hat{a}^{n-s}\rangle \langle \hat{a}^{s}\rangle$$

$$= 4(\eta tr|\beta\rangle)^2 [\langle \Delta \hat{X}_x^{(n)} \rangle^2 \cos^2 (n\theta_{\beta} - \theta)$$

$$+ \langle \Delta \hat{X}_x^{(n)} \rangle^2 \sin^2 (n\theta_{\beta} - \theta) + \frac{1}{2} \langle \Delta \hat{X}_x^{(n)} \rangle \Delta \hat{X}_x^{(n)} \rangle \sin 2(n\theta_{\beta} - \theta)]$$

$$\langle \Delta \hat{X}_x^{(n)} \rangle \Delta \hat{X}_x^{(n)} \rangle \sin 2(n\theta_{\beta} - \theta)]$$

$$= \left\{ \begin{array}{ll}
4(\eta tr|\beta\rangle)^2 \langle \Delta \hat{X}_x^{(n)} \rangle^2 \langle \Delta \hat{X}_x^{(n)} \rangle^2, & \text{if } (n\theta_{\beta} - \theta) = 0; \\
4(\eta tr|\beta\rangle)^2 \langle \Delta \hat{X}_x^{(n)} \rangle^2 \langle \Delta \hat{X}_x^{(n)} \rangle^2, & \text{if } (n\theta_{\beta} - \theta) = \frac{\pi}{2}.
\end{array} \right.$$

(18)
This equation shows that amplitude nth-power squeezing for any value of $\theta$, defined by Eqs. (6) or (7), can be detected by measurement of observables $P_n, Q_n, \eta, T$ and $|\beta|$.

III. DISCUSSION OF RESULTS

We explained how $P_n$ and $Q_n$ can be found from measurements of $\langle \hat{N}_c^{(n)} \rangle$ (see Eqs. (14) and (15). Measurement of quantum efficiency $\eta$ for a given detector is somewhat tricky and requires use of spontaneous parametric down conversion [60]. If a photon of a large energy $\hbar \omega$ breaks to create two photons of energies $\hbar \omega_1$ and $\hbar \omega_2$ (with $\omega_1 + \omega_2 = \omega$), the latter two photons should give coincidence counts only ideally. If $N$ photons break and the quantum efficiencies for modes $\omega_1$ and $\omega_2$ are $\eta_1$ and $\eta_2$ (both < 1), the experiment registers counts $N_1 = \eta_1 N, N_2 = \eta_2 N$ and coincidence counts $N_c = \eta_2 \eta N$. The quantum efficiencies are then $\eta_1 = N_c/N_2$ and $\eta_2 = N_c/N_1$.

Once $\eta$ is determined for any detector, the detector can be used to measure $T$ and $|\beta|$ easily.

It should be noted that as $n$ increases the choice of $\theta_\beta$ becomes more sensitive and for detection of $\langle (\Delta \hat{X}_d^{(n)})^2 \rangle$, $\theta_\theta$ is to be set as $\theta/\pi$.

For $n = 1$, i.e., for ordinary squeezing detection was first studied by Mandel [52] (see also references [29] for homodyning). His formula is different from that obtained from Eq. (18) after substituting $n = 1$. This is understandable as Mandel’s result is approximate, and is obtained under the approximation, $|\beta| >> 1/r >> 1$, in our notations. Also, for $n = 2$, i.e., for amplitude-squared squeezing, the results of Prakash and Mishra [55] are different from the results obtained from Eq. (18) as their results are also approximate, the same approximation ($|\beta| >> 1/r >> 1$, in our notations) being used.

Another method for detecting amplitude $k$th-power squeezing ([58]; see also [57]) has been proposed by Shchuinik and Vogel based on measurement of some correlation functions from which the values of $\langle \hat{a}^k \rangle$, $\langle \hat{a}^{2k} \rangle$ and $\langle \hat{a}^{2k} \rangle$ can be inferred. The authors arranged several beam splitters in $d$ levels (rth level having $2^d-1$ beam splitters and $d$ being called the depth) between an entrance beam splitter and a sequence of $2^d$ photodetectors. Since an additional beam splitter is used before the entrance beam splitter, total $2^d + 1$ beam splitters are required. If $k$ satisfies $2^n > k > 2^n - 1$, for obtaining values of $\langle \hat{a}^k \rangle$ and $\langle \hat{a}^{2k} \rangle$ depth $d = n$ is needed, which requires use of $2^n + 1$ beam splitters and $2^n$ photodetectors. For obtaining the values of $\langle \hat{a}^{2k} \rangle$ obviously, depth $n + 1$ and, therefore, $2^{n+1} + 1$ beam splitters and $2^{n+1}$ photodetectors are required. In the method proposed in the present paper, only one beam splitter and only one photodetector is required. The Shchuinik-Vogel method proposes measurement of correlations with several phase shifts $\varphi$ of local oscillator. Since a Fourier transform of correlations over $\varphi$ is required for inferring the values of moments $\langle \hat{a}^k \rangle$ and $\langle \hat{a}^{2k} \rangle$, very large number of repetition of experiments with changed values of $\varphi$ should be required. It may be noted that in the method proposed in the present paper only $4k$ repetitions are required.

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Appendix

We can separate the $\langle \hat{a}^{tn} \rangle$ term on right hand side of Eq. (15) and write

$$\hat{a}^{tn} \hat{a}^{-tn} = \eta^{-nt} t^{-2n} Q_n - \sum_{l=1}^{n-1} K_0 (t C_0)^2 (n C_l)^2$$

$$\langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle$$

with $K_0 = 1$. In the first term in summation on the right hand side we substitute for $\langle \hat{a}^{tn} \rangle$ the expression obtained from Eq. (A.1) using $K_0 (t C_0)^2 = 1$ and this gives

$$\langle \hat{a}^{tn} \rangle = \eta^{-nt} t^{-2n} Q_n + K_1 (t C_0)^2 (n C_l)^2$$

$$\langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle$$

with $K_1 = -K_0 (t C_0)^2$. This can be simplified and written as

$$\langle \hat{a}^{tn} \rangle = \eta^{-nt} t^{-2n} Q_n + K_1 (t C_0)^2 (n C_l)^2$$

$$\langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle$$

The expression obtained for $\langle \hat{a}^{tn} \rangle$ from Eq. (A.1) and simplify, we get

$$\langle \hat{a}^{tn} \rangle = \eta^{-nt} t^{-2n} Q_n + K_1 (t C_0)^2 (n C_l)^2$$

$$\langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle \langle t^{-1} r^t | \beta / \beta \rangle$$

(A.3)
with $K_2 = -[K_0(t_C)_0^2 + K_1(t_C)_1^2]$. If we go on doing similar exercises we get required Eq. (16), where $K_s$ is defined by Eq. (17).

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