Late time acceleration in a deformed phase space model of dilaton cosmology

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Abstract

The effects of noncommutativity on the phase space of a dilatonic cosmological model is investigated. The existence of such noncommutativity results in a deformed Poisson algebra between the minisuperspace variables and their momenta conjugate. For an exponential dilaton potential, the exact solutions in the commutative and noncommutative cases, are presented and compared. We use these solutions to address the late time acceleration issue of cosmic evolution.

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1 Introduction

Noncommutativity between space-time coordinates was first introduced by Snyder [1], and in more recent times a great deal of interest has been generated in this area of research [2, 3]. This interest has been gathering pace in recent years because of strong motivations in the development of string and M-theories. However, noncommutative theories may also be justified in their own right because of the interesting predictions they have made in particle physics, a few examples of which are the IR/UV mixing and non-locality [4], Lorentz violation [5] and new physics at very short distance scales [5]-[7]. Noncommutative versions of ordinary quantum [8] and classical mechanics [9, 10] have also been studied and shown to be equivalent to their commutative versions if an external magnetic field is added to the Hamiltonian. A different approach to noncommutativity is through the introduction of noncommutative fields [11], that is, fields or their conjugate momenta are taken as noncommuting. These effective theories can address some of the problems in ordinary quantum field theory, e.g. regularization [11] and predict new phenomenon, such as Lorentz violation [12], considered as one of the general predictions of quantum gravity theories [13].

In cosmological systems, since the scale factors, matter fields and their conjugate momenta play the role of dynamical variables of the system, introduction of noncommutativity by adopting the approach discussed above is particularly relevant. The resulting noncommutative classical and quantum cosmology of such models have been studied in different works [14]. These and similar works have opened a new window through which some of problems related to cosmology can be looked at and,
hopefully, resolved. For example, an investigation of the cosmological constant problem can be found in [15]. In [16] the same problem is carried over to the Kaluza-Klein cosmology. The problem of compactification and stabilization of the extra dimensions in multidimensional cosmology may also be addressed using noncommutative ideas in [17].

In recent years many efforts have been made in cosmology from string theory point of view [18]-[21]. In the pre-big bang scenario, based on the string effective action, the birth of the universe is described by a transition from the string perturbative vacuum with weak coupling, low curvature and cold state to the standard radiation dominated regime, passing through a high curvature and strong coupling phase. This transition is made by the kinetic energy term of the dilaton, a scalar field with which the Einstein-Hilbert action of general relativity is augmented, see [22] for a more modern review.

In this Letter we deal with noncommutativity in a dilaton cosmological model with an exponential dilaton potential and to facilitate solutions for the case under consideration, we choose a suitable metric. Our approach to noncommutativity is through its introduction in phase space constructed by minisuperspace fields and their conjugate momenta. Indeed, in general relativity formulation of gravity in a noncommutative geometry of space-time is highly nonlinear and setting up cosmological models is not an easy task. Here our aim is to study some aspects regarding the application of noncommutativity in cosmology, i.e. in the context of a minisuperspace reduction of the dynamics. Since our model has two degrees of freedom, the scale factor $a$ and the dilaton $\phi$, with a change of variables, we have a set of dynamical variables $(u, v)$, which are suitable candidates for introducing noncommutativity in the phase space of the problem at hand. We present exact solutions of commutative and noncommutative cosmology and show that commutative model cannot describe the late time acceleration while the noncommutative counterpart of the model clearly points to a possible late time acceleration.

2 The Model

In $D = 4$ dimension lowest order gravi-dilaton effective action, in the string frame, can be written as [23]

$$S = -\frac{1}{2\lambda_s^2} \int d^4x \sqrt{-g} e^{-\phi} \left( R + \partial_\mu \phi \partial^\mu \phi + V(\phi) \right),$$

where $\phi$ is the dilaton field, $\lambda_s$ is the fundamental length parameter of string theory and $V(\phi)$ is the dilaton potential. In the string frame our fundamental unit is the string length $l_s$, and thus the Planck mass, which is the effective coefficient of the Ricci scalar $R$, varies with the dilaton. One can also write the action in the Einstein frame, for which the fundamental unit is the Planck length. Since the Planck length is more appropriate for our purpose, we prefer to work in the Einstein frame. In [21], it is shown in detail that action (1) in the Einstein frame takes the form

$$S = -\frac{M_p^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right),$$

where now all quantities in the action are in the Einstein frame. We consider a spatially flat FRW space-time which, following [24], is specified by the metric

$$ds^2 = -\frac{N^2(t)}{a^2(t)} dt^2 + a^2(t) \delta_{ij} dx^i dx^j.$$  

Here $N(t)$ is the lapse function and $a(t)$ represents the scale factor of the universe. The square of the scale factor dividing the lapse function turns out to simplify the calculations and makes the Hamiltonian quadratic. Now, it is easy to show that the effective Lagrangian of the model can be written in the minisuperspace $Q^A = (a, \phi)$ in the form

$$\mathcal{L} = \frac{1}{N} \left( -\frac{1}{2} a^2 \dot{a}^2 + \frac{1}{2} a^4 \dot{\phi}^2 \right) - N a^2 V(\phi).$$
The momenta conjugate to the dynamical variables are given by
\[ p_a = \frac{\partial L}{\partial \dot{a}}, \quad p_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{1}{N} a^4 \dot{\phi}, \]
leading to the following Hamiltonian
\[ H = \frac{1}{2} G^{AB} P_A P_B + U(Q^A) = \frac{N}{2a^2} \left( -p_a^2 + \frac{1}{a^2} p_\phi^2 \right) + Na^2 V(\phi). \]
Now, it is easy to see that the corresponding minisuperspace has the following minisuper metric
\[ G_{AB} dQ^A dQ^B = \frac{a^2}{N} \left(-da^2 + a^2 d\phi^2\right). \]
To apply the deformed commutators to the dynamical variables in a minisuperspace which is represented by a curved manifold with a minisuper metric given by (7), in a natural way, one has to deal with some difficulties which the most important of them is the ambiguity in the ordering of factors \( Q \) and \( P_Q \). Therefore, the above minisuperspace does not have the desired form for introducing noncommutativeness among its coordinates. To avoid the physical difficulties and simplify the model, consider the following change of variables \( Q^A = (a, \phi) \rightarrow q^A = (u, v), [25] \)
\[ u = \frac{a^2}{2} \cosh \alpha \phi, \quad v = \frac{a^2}{2} \sinh \alpha \phi, \]
where \( \alpha \) is a positive constant. In terms of these new variables the Lagrangian (4) takes the form
\[ L = \frac{1}{2N} \left( \dot{v}^2 - \dot{u}^2 \right) - 2N (u - v) e^{\alpha \phi} V(\phi). \]
From now on, we choose an exponential potential
\[ V(\phi) = \frac{V_0}{2} e^{-\alpha \phi}, \]
which simplifies the last term in the Lagrangian (9) leading to
\[ L = \frac{1}{2N} \left( \dot{v}^2 - \dot{u}^2 \right) - NV_0 (u - v), \]
with the corresponding Hamiltonian becoming
\[ H = N \mathcal{H} = N \left[ -\frac{1}{2} p_u^2 + \frac{1}{2} p_v^2 + V_0 (u - v) \right]. \]
Thus, in the minisuperspace constructed by \( q^A = (u, v) \), the metric, for a constant \( N \), is Minkowskian and represented by
\[ \bar{G}_{AB} dq^A dq^B = N \left(-du^2 + dv^2\right). \]
Now, we have a set of variables \( (u, v) \) endowing the minisuperspace with a Minkowskian metric and hence this set of dynamical variables are suitable candidates for introducing noncommutativity in the phase space of the problem at hand. The preliminary setup for describing the model is now complete. In what follows, we will study the classical cosmology of the minisuperspace model described by Hamiltonian (12) in commutative and noncommutative frameworks.

### 3 Cosmological dynamics

The classical solutions of the model described by Hamiltonian (12) can be easily obtained. Since our aim here is to compare the commutative solutions with their noncommutative counterparts, in what follows we consider commutative and noncommutative classical cosmologies, and compare the results with each other.
3.1 Commutative case

As is well known for a dynamical system with phase space variables \( (x_i, p_i) \), the Poisson algebra is described by the following Poisson brackets

\[
\{x_i, x_j\} = \{p_i, p_j\} = 0, \quad \{x_i, p_j\} = \delta_{ij},
\]

where \( x_i(i = 1, 2) = u, v \) and \( p_i(i = 1, 2) = p_u, p_v \). Therefore, the equations of motion become (in \( N = 1 \) gauge)

\[
\begin{align*}
\dot{u} &= \{u, N\mathcal{H}\} = -p_u, \quad \dot{p}_u = \{p_u, N\mathcal{H}\} = -V_0, \\
\dot{v} &= \{v, N\mathcal{H}\} = p_v, \quad \dot{p}_v = \{p_v, N\mathcal{H}\} = V_0,
\end{align*}
\]

Equations (15) and (16) can be immediately integrated to yield

\[
\begin{align*}
u(t) &= \frac{1}{2} V_0 t^2 - p_{0u} t + u_0, \quad p_u(t) = -V_0 t + p_{0u}, \\
v(t) &= \frac{1}{2} V_0 t^2 + p_{0v} t + v_0, \quad p_v(t) = V_0 t + p_{0v}.
\end{align*}
\]

Now, these solutions must satisfy the zero energy condition, \( \mathcal{H} = 0 \). Thus, substitution of equations (17) and (18) into (12) gives a relation between integration constants as

\[
p_{0v}^2 - p_{0u}^2 = 2V_0(v_0 - u_0).
\]

Equations (17) and (18) are like the equation of motion for a particle moving in a plane with its acceleration components equal to \( V_0 \), while \( -p_u(t) \) and \( p_v(t) \) play the role of its velocity components.

Finally, using equation (8), the scale factor and dilaton take the following forms

\[
a(t) = \sqrt{2} \left[ -V_0(p_{0u} + p_{0v}) t^3 + 3V_0(u_0 - v_0) t^2 - 2(u_0 p_{0u} + v_0 p_{0v}) t + (u_0^2 - v_0^2) \right]^{1/4},
\]

\[
\phi(t) = \frac{1}{2\alpha} \ln \frac{V_0 t^2 + (p_{0u} - p_{0v}) t + (u_0 + v_0)}{-(p_{0u} + p_{0v}) t + (u_0 - v_0)}.
\]

The limiting behavior of \( a(t) \) and \( \phi(t) \) in the early and late times is then as follows

\[
\begin{cases}
a(t) \sim t^{1/4}, & \phi(t) \sim \text{const.}, \quad t \ll 1, \\
a(t) \sim t^{3/4}, & \phi(t) \sim \ln t, \quad t \gg 1.
\end{cases}
\]

We see that in the usual commutative phase space of our model the scale factor has a decelerated expansion in early times while it also undergoes a decelerated phase in its late time evolution due to a constant and growing with time dilatonic field, respectively. The evolution of the universe based on (22) begins with a big-bang singularity at \( t = 0 \) and follows the power law expansion \( a(t) \sim t^{3/4} \) at late time of cosmic evolution (which is not consistent with late time observations) while the dilaton has a monotonically increasing behavior coming from a constant value and logarithmically blows up at late time. We shall see in the next subsection how this classical picture will be modified by introducing the deformed phase space model.

3.2 Deformed phase space model

An important ingredient in any model theory related to the quantization of a cosmological setting is the choice of the quantization procedure used to quantize the system. The most widely used method has traditionally been the canonical quantization method based on the Wheeler-DeWitt equation, which is nothing but the application of the Hamiltonian constraint to the wavefunction of the universe. A particularly interesting but rarely used approach to study the quantum effects is to
introduce a deformation in the phase space of the system. It is believed that such a deformation of phase space is an equivalent path to quantization, in par with other methods, namely canonical and path integral quantization [26]. This method is based on the Wigner quasi-distribution function and Weyl correspondence between quantum mechanical operators in Hilbert space and ordinary c-number functions in phase space. The deformation in the usual phase space structure is introduced by Moyal brackets which are based on the Moyal product. However, to introduce such deformations it is more convenient to work with Poisson brackets rather than Moyal brackets. From a cosmological point of view, models are built in a minisuperspace. It is therefore safe to say that studying such a space in the presence of deformations mentioned above can be interpreted as studying the quantum effects on cosmological solutions.

Let us now proceed to study the behavior of the above model in a deformed phase space framework such that the minisuperspace variables do not (Poisson) commute with each other. In general, noncommutativity between phase space variables can be understood by replacing the usual product with the star product, also known as the Moyal product law between two arbitrary functions of position and momentum, as

\[ (f \star g)(x) = \exp \left[ \frac{1}{2} \alpha^{ab} \partial_a \partial^b \right] f(x_1)g(x_2) \big|_{x_1=x_2=x} , \]

such that

\[ \alpha_{ab} = \begin{pmatrix} \theta_{ij} & \delta_{ij} + \sigma_{ij} \\ -\delta_{ij} - \sigma_{ij} & \beta_{ij} \end{pmatrix} , \]

where the $2 \times 2$ matrices $\theta$ and $\beta$ are assumed to be antisymmetric and represent the noncommutativity in coordinates and momenta, respectively. Also, $\sigma$ can be written as a combination of $\theta$ and $\beta$. With this product law, the $\alpha$-deformed Poisson brackets can be written as

\[ \{f, g\}_\alpha = f \star g - g \star f . \]

Upon a simple calculation we have

\[ \{x_i, x_j\}_\alpha = \theta_{ij}, \quad \{x_i, p_j\}_\alpha = \delta_{ij} + \sigma_{ij}, \quad \{p_i, p_j\}_\alpha = \beta_{ij} . \]

It is worth noting at this stage that in addition to noncommutativity in position variables, we have also considered noncommutativity in the corresponding momenta. Such a noncommutativity can be motivated by string theory corrections to the Einstein gravity. This should be interesting since its existence is in fact due essentially to the existence of noncommutativity on the space sector and it would somehow be natural to include it in our considerations. In this work we consider a deformed phase space in which $\theta_{ij} = -\theta \epsilon_{ij}$ and $\beta_{ij} = -\beta \epsilon_{ij}$, where $\epsilon_{ij}$ are the Levi-Civita symbols.

Now, consider the following transformation on the classical phase space $\{u, v, p_u, p_v\}$

\[ \begin{cases} \hat{v} = v - \frac{\theta}{2} p_u , & \hat{u} = u + \frac{\theta}{2} p_v , \\ \hat{p}_v = p_v + \frac{\beta}{2} u , & \hat{p}_u = p_u - \frac{\beta}{2} v . \end{cases} \]

It can easily be checked that if $\{u, v, p_u, p_v\}$ obey the usual Poisson algebra (14), then

\[ \{\hat{v}, \hat{u}\}_p = \theta , \quad \{\hat{u}, \hat{p}_u\}_p = \{\hat{v}, \hat{p}_v\}_p = 1 + \sigma , \quad \{\hat{p}_v, \hat{p}_u\}_p = \beta , \]

where $\sigma = \frac{1}{4} \theta \beta$. These commutation relations are the same as (26). Consequently, for introducing noncommutativity, it is more convenient to work with Poisson brackets (28) than $\alpha$-star deformed Poisson brackets (26). It is important to note that the relations represented by equations (26) are defined in the spirit of the Moyal product given above. However, in the relations defined in (28), the variables $\{u, v, p_u, p_v\}$ obey the usual Poisson bracket relations so that the two sets of ordinary and deformed Poisson brackets represented by relations (26) and (28) should be considered as distinct.
With the noncommutative phase space defined above, we consider the Hamiltonian of the noncommutative model as having the same functional form as equation (12), but in which the dynamical variables satisfy the above deformed Poisson brackets, that is

$$\hat{H} = N\hat{H} = N\left[\frac{1}{2}(-\hat{p}_u^2 + \hat{p}_v^2) + V_0(\hat{u} - \hat{v})\right].$$

In physical point of view this is because that the Hamiltonian constraint is the result of time reparametrization invariance which remains even when the noncommutativity is turned on. Using the transformations (27), we can write the Hamiltonian without the hat variables, that is, those that satisfy the usual commutation relations, as

$$H = N\mathcal{H} = N\left[\frac{1}{2}(p_v^2 - p_u^2) + \frac{1}{8}\beta^2(u^2 - v^2) + \frac{1}{2}\beta(up_v + vp_u) + \frac{V_0}{2}\theta(p_v + p_u) + V_0(u - v)\right].$$

The equations of motion (again in $N = 1$ gauge) can now be written easily with respect to Hamiltonian (30)

$$\begin{align*}
\dot{u} &= \{u, H\}_P = -p_u + \frac{1}{2}\beta v + \frac{V_0}{2}\theta, \\
\dot{v} &= \{v, H\}_P = p_v + \frac{1}{2}\beta u + \frac{V_0}{2}\theta, \\
\dot{p}_u &= \{p_u, H\}_P = -\frac{1}{4}\beta^2 u - \frac{1}{2}\beta p_v - V_0, \\
\dot{p}_v &= \{p_v, H\}_P = \frac{1}{4}\beta^2 v - \frac{1}{2}\beta p_u + V_0,
\end{align*}$$

Eliminating the momenta from the above equations, we get

$$\begin{align*}
\ddot{v} - \beta\dot{u} - V_0(1 - \sigma) &= 0, \\
\ddot{u} - \beta\dot{v} - V_0(1 - \sigma) &= 0.
\end{align*}$$

We see that the deformation parameters appear as a coupling constant between equations of motion for $u$ and $v$. Let us define the new variables $\eta = v + u$ and $\zeta = v - u$, in terms of which we have

$$\begin{align*}
\ddot{\eta} - \beta\dot{\eta} - 2V_0(1 - \sigma) &= 0, \\
\ddot{\zeta} + \beta\dot{\zeta} &= 0.
\end{align*}$$

Integrating these equations yields

$$\begin{align*}
\eta(t) &= A'e^{\beta t} - \frac{2V_0}{\beta}(1 - \sigma)t + C', \\
\zeta(t) &= B'e^{-\beta t} + D',
\end{align*}$$

where $A', B', C', D'$ are integration constants. Going back to the variables $v$ and $u$, we obtain

$$\begin{align*}
v(t) &= Ae^{\beta t} + Be^{-\beta t} - \frac{V_0}{2\beta}(1 - \sigma)t + C, \\
u(t) &= Ae^{\beta t} - Be^{-\beta t} - \frac{V_0}{2\beta}(1 - \sigma)t + D.
\end{align*}$$

Here, $A, B, C, D$ are linear combinations of the prime constants. The requirement that the deformed Hamiltonian constraints (30) should hold during the evolution of the system leads to the following relation between these integrating constants
$$2\beta A B + \frac{V_0}{\beta} (1 - \sigma) (C - D) = 0. \quad (36)$$

To proceed further, we can choose a particular set of constants, say $B = -A$. Now, $v(t)$, $u(t)$, and Hamiltonian constraint take the form

$$\begin{align*}
v(t) &= 2A \sinh(\beta t) - \frac{V_0}{\beta} (1 - \sigma) t + C, \\
u(t) &= 2A \cosh(\beta t) - \frac{V_0}{\beta} (1 - \sigma) t + D, \\
2\beta A^2 &= \frac{V_0}{\beta} (1 - \sigma)(C - D).
\end{align*} \quad (37)$$

Now, let us return to the variables $a(t)$ and $\phi(t)$ using the transformation (8), in terms of which we obtain the corresponding deformed classical cosmology as

$$\begin{align*}
a(t) &= 2 \left[ (1 + \beta t) A^2 + \left(D - \frac{V_0}{\beta} (1 - \sigma) t\right) A \cosh(\beta t) - \left(C - \frac{V_0}{\beta} (1 - \sigma) t\right) A \sinh(\beta t) + \frac{1}{4} (D^2 - C^2) \right]^{1/2}, \\
\phi(t) &= \frac{1}{2a} \ln \left( \frac{2A e^{\beta t} - \frac{2V_0}{\beta} (1 - \sigma) t + D + C}{2A e^{-\beta t} + D - C} \right).
\end{align*} \quad (38)$$

As in the previous section, we may express the early and late time limiting behavior of the scale factor. The scale factor $a$ in these two regimes takes the form

$$\begin{align*}
a(t) &\sim \left[ \frac{V_0}{\beta^3} (1 - \sigma) t \right]^{1/4}, \quad t \ll 1, \\
a(t) &\sim e^{\frac{\beta t}{2}}, \quad t \gg 1.
\end{align*} \quad (39)$$

This means that the scale factor begins again from a big-bang singularity and then behaves as the usual commutative case (22) in the early times. However, the differences between the commutative and noncommutative cases are manifest in the late time behavior where in the noncommutative case with $t \to \infty$ the scale factor becomes an exponentially function, for which (3) represents a de-Sitter metric with a cosmological constant $\Lambda = \frac{3}{16} \beta^2$. Therefore, for late times the behavior of the scale factor becomes exponential, pointing to the existence of a deformation parameter in the corresponding phase space of cosmological setting which can be interpreted as a candidate for dark energy. Taking the relation for $\Lambda$ given above, the value of the deformation parameter $\beta$ can be estimated. If the cosmological constant is taken as $\Lambda \sim 10^{-52} m^{-2}$, a generally accepted value determined by observation, the deformation parameter is found to be $\beta \sim 10^{-26} m^{-1}$, which is comparable with the upper bounds for such noncommutative parameters resulting from theoretical predictions and experimental data obtained in the field theory and quantum gravity [27].

We can also compute the deceleration parameter $q = -\frac{a \ddot{a}}{a^2}$ for these two models. As is well known the deceleration parameter indicates by how much the expansion of the universe is slowing down. If the expansion is speeding up, for which there appears to be some recent evidence, then this parameter will be negative. In figure 1, we have shown the approximate behavior of the scale factor $a(t)$ and the deceleration parameter $q(t)$ for typical values of the parameters in both commutative and noncommutative regimes. It is seen that the behavior of two scale factors is the same in the early times and the minor differences may be interpreted by the fact that the early time behavior of the commutative scale factor will be determined only by the initial conditions, see (20), while for the noncommutative one, the quantum effects are presence as well, see the first relation of (39). On the other hand, as is clear from the figure, in contrast to the commutative case for which the deceleration parameter is positive in all times, for the deformed phase space case we have negative acceleration (positive deceleration parameter) at early times so that the universe decelerates its expansion in this era, and positive acceleration (negative deceleration parameter) for late times which means that the
universe currently accelerates its expansion. Therefore, the late time acceleration, suggested by recent supernova observation, can be addressed by considering a deformed phase space in the corresponding cosmological set up.

4 Conclusions

In this letter we have studied the effects of noncommutativity in phase space, on classical cosmology of a dilaton model with an exponential dilaton potential. Motivation of such study is that the construction of an effective theory is usually the result of the observation that the corresponding full theory is too complicated to deal with. This, for example, is true in describing the quantum effects on cosmology since the full theory is immensely difficult to handle. Introduction of a deformed phase space can be interpreted as one such effective theory. In the present study, in the case of commutative phase space, the evolution of the classical universe is like the motion of a particle (universe) moving on a plane with a constant acceleration. We saw that in this case the universe decelerates its expansion both in early and late times of cosmic evolution, in contrast to the current observation data. We have investigated the possibility of having a late time accelerated phase of the universe, suggested by recent supernova observation, in the context of a deformed phase space model of string cosmology action. Indeed, we have presented a scenario in which cosmic acceleration occurs late in the history of the universe due to introduction of noncommutativity in phase space, on classical cosmology of a dilaton model with an exponential dilaton potential. We have found that while the usual classical model cannot support this acceleration, a classical model with noncommutative phase space variable can drive this late time acceleration.

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