Mixed Convection and Thermally Radiative Flow of MHD Williamson Nanofluid with Arrhenius Activation Energy and Cattaneo–Christov Heat-Mass Flux

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In this paper, we explored the impact of thermally radiative MHD flow of Williamson nanofluid over a stretchy plate. The flow in a stretchy plate is saturated via Darcy–Forchheimer relation. Cattaneo–Christov heat-mass flux theory is adopted to frame the energy and nanoparticle concentration equations. Additionally, the mass transfer analysis is made by activation energy and binary chemical reaction. Activation energy is invoked through the modified Arrhenius function. The intention of the current investigation is to enhance the heat transfer rate in industrial processes. The non-Newtonian nanofluids have more prominent thermal characteristics compared to ordinary working fluids. The governing models are altered into ODE models, and these models are numerically solved by applying the MATLAB bvp4c algorithm. The graphical and tabular interpretations have scrutinized the impact of sundry distinct parameters. The fluid speed escalates for enhancing the Richardson number, and it falls off for higher values of the Weissenberg number. It is noticed that the fluid temperature declines for higher values of the Brownian motion parameter and it grows for larger values of the thermophoresis parameter. The activation energy enriches the heat transfer gradient and suppresses the local Sherwood number. Additionally, the more significant heat transfer gradient occurs in heat-absorbing nonradiative viscous nanofluid and a smaller heat transfer gradient occurs in heat-generating radiative Williamson nanofluid. Also, we noticed that a higher heat transfer gradient appears in the Fourier model than in the Catteneo–Christov model. In addition, the comparative results are confirmed and reached an outstanding accord.

1. Introduction

Cooling and heating procedures are essential in many industries, and fluids make this process. The effectual cooling techniques are essential for cooling a higher thermal system in a short time. However, ordinary fluids such as ethylene glycol, engine oil, and water have poor thermal conductivity and do not fulfill the demand for powerful heat transfer cooling agents. Considering the needs of modern industry, including microelectronics, chemical production, and power generation plants, we need to establish a new type of fluids that will be efficient in cooling thermal systems. Nanofluid is a fluid consisting of nanoparticles (nanosized particles) such as oxides, nitrides, carbides, and metals stably and uniformly suspended in a base fluid. These fluids overcome the difficulty of the base fluids and act as an agent of efficient cooling.
The nanofluid flow on a stretchy sheet was reported by Khan and Pop [1]. They noticed that the fluid temperature grows when the quantity of thermophoresis parameter is greater. Kuznetsov and Nield [2] addressed the natural convective flow of nanofluid on a plate. They noted that the heat transfer rate becomes less in the presence of the Brownian motion parameter. Goyal and Bhargava [3] derived the numerical solution of viscoelastic nanofluid on a sheet under velocity slip condition. Their outcomes clearly show that the thermophoresis parameter leads to deceleration in the fluid temperature. The Titania nanofluid flow in a cylindrical annulus was illustrated by Mebarek-Oudina [4]. The problem of bioconvective flow of MHD tangent hyperbolic nanofluid subject to Newtonian heating was solved by Shafiq et al. [5]. They detected that the nanoparticle concentration suppresses when rising the thermophoresis parameter. Mabood et al. [6] illustrate the consequence of MHD flow of hybrid nanofluid on a wedge with thermal radiation. They proved that the fluid velocity uplifts when enriching the magnetic field parameter.

In nature, heat transference occurs due to the temperature difference between one body to another body or within the same body. In the past, the heat transfer phenomenon was mostly addressed by using “Fourier’s law of heat conduction.” However, this law is not sufficient to express the fundamental characteristic of heat transfer. That is, each part of the entire object having an initial disturbance. In general, there is no material satisfying this property. To overcome this complication, Cattaneo [7] incorporated the thermal relaxation in Fourier’s theory which implements the heat transport is identical to the propagation of thermal waves with normal speed. Christov [8] upgraded the Cattaneo model by recommending the thermal relaxation time with upper convected Oldroyd’s derivatives for the frame-invariant formation. The time-dependent flow of nanofluid with Cattaneo–Christov double diffusion was examined by Ahmad et al. [9]. They noticed that the thermal relaxation parameter declines the fluid temperature. Reddy and Kumar [10] delivered the impact of Cattaneo–Christov heat flux of micropolar fluid on carbon nanotubes. The 2D incompressible flow of heat-generating/-absorbing Oldroyd-B fluid with Cattaneo–Christov heat flux on an uneven stretching sheet was portrayed by Ibrahim and Gadisa [11]. They proved that the heat flux relaxation time parameter leads to thinning the thermal boundary layer thickness. Kumar et al. [12] analyzed the significance of Cattaneo–Christov flow on a cone. They detected that the smaller heat transfer gradient occurred in the wedge than the cone for varying the thermal relaxation time parameter. Some recent developments on this concept are collected in [13–19].

The convective fluid flows on a porous medium play a vital role in many science and engineering systems. Some examples are crude oil production, heat exchanger layouts, groundwater systems, grain amassing, nuclear waste disposal, warm insurance outlining, fossil fuels beds, and many others. Darcy developed a semiempirical equation that uses in low porosity and low-velocity conditions. These empirical equations were not sufficient for a larger Reynolds number. In this situation, Forchheimer [20] was developed a new model named as the Darcy–Forchheimer model, which includes the square velocity term in the Darcian model. Pal and Mondal [21] derived the numerical solution of MHD fluid flow on Darcy–Forchheimer porous medium. They discovered that the mass transfer gradient accelerates for more availability of local inertia parameters. The dual solution of forced convective stagnation-point flow on a Darcy–Forchheimer porous medium over a shrinking sheet was derived by Bakar et al. [22]. They achieved that the fluid temperature declines in the first solution and enhances the second solution when raising the porosity parameter. Meraj et al. [23] inspected the Darcy–Forchheimer flow of Maxwell fluid with Cattaneo–Christov heat flux theory. They acknowledged that the thermal boundary layer thickness becomes high for a larger quantity of the porosity parameter. The Darcy–Forchheimer flow of H2O-based CNTs on rotating disk was studied by Hayat et al. [24]. They found that the fluid velocity is a nonincreasing function of the porosity parameter. Latest improvements for these concepts are collected in [25–35].

In recent decades, many researchers are willing to study the chemical reactions and activation energy because they have more industrial applications. Few applications are fog formation, fibrous insulation, thermal oil recovery, cooling of nuclear reactors, etc. The mixed convective flow of Carreau nanofluid flow with activation energy was illustrated by Javed et al. [36]. Zaib et al. [37] explored the consequences of a binary chemical reaction and activation energy of a nonlinear radiative flow of Casson nanofluid on a Darcy–Brinkman porous medium. They detected that the thickening of the solutal boundary layer thickness when raising the activation energy parameter. The impact of activation energy of an electrically conducting Carreau nanofluid flow in a stagnation point was discussed by Hsiao [38]. Time-dependent MHD natural convective flow with Arrhenius activation energy was analyzed by Maleque [39]. He noticed that the activation energy is enhancing the nanofluid concentration. Mabood et al. [40] portray the outcomes of Arrhenius activation energy effect on micropolar fluid on a thin needle. They concluded that the Sherwood number decelerates for upsurging values of the activation energy parameter. The results of Arrhenius activation energy of a tangent hyperbolic fluid was revealed by Kumar et al. [41]. A variety of studies on this direction was found in [42–45].

The primary objective of this paper is to portray the 2D Darcy–Forchheimer radiative flow of Williamson nanofluid with subject to activation energy and heat absorption. The thermophoresis and Brownian motion effects are taking into account. The energy and mass equation models are constructed via Cattaneo–Christov heat-mass flux theory. The Darcy–Forchheimer flow of radiative Williamson nanofluid with activation energy and Cattaneo–Christov dual flux was not examined yet. So, we fill this gap and will give a significant contribution to the existing investigations. Generally, Williamson nanofluid has a wide range of usages in biological engineering; especially, it is used for computing
the heat and mass transmission through the vessels in blood and hemodialysis, see [46]. The impact of pertinent parameters of the governing model of velocity, temperature and nanofluid concentration, local skin friction, local Nusselt number, and local Sherwood number are examined in terms of tables, charts, and figures.

2. Mathematical Formulation

We exhibit the steady mixed convective flow of 2D Williamson nanofluid on a Darcy–Forchheimer porous medium over a stretchy sheet. Let \( x \)-axis is considered in the flow direction and \( y \)- is perpendicular to the flow. The uniform magnetic effect \( B_0 \) is applied in the \( y \)-direction and the induced magnetic effect excluded becomes a small quantity of Reynolds number. The fluid temperature and nanofluid concentration nearby the boundary is \( T_w \) and \( C_w \) which is larger than the ambient fluid temperature \( T_\infty \) and concentration \( C_\infty \), respectively. The Cattaneo–Christov model replaced Fourier’s heat conduction law. The consequences of activation energy and binary chemical reaction are considered for our study. In addition, the fluid is heat consumption/generating. Under the above considerations, the governing flow problems are (see [47])

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{\nu} \frac{\partial u}{\partial y} - \frac{\nu}{k_1} u - \frac{C_b}{\sqrt{k_1}} u^2 - \frac{\sigma B_0^2 u}{\rho_f} + \frac{1}{\rho_f} \left[ (1 - C_\infty) \rho_f \beta (T - T_\infty) - \left( \rho_p - \rho_f \right) (C - C_\infty) \right] g,
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \lambda_f \Omega_T = \frac{16 \sigma^2 \nu^3 \rho C_p}{3k^2} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho_f C_p} (T - T_\infty) + \tau \left[ D_T \frac{\partial^3 T}{\partial y^3} + \frac{2D_T}{T_\infty} \frac{\partial T}{\partial y} \right] t,
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + \lambda_c \Omega_c = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - k_f (C - C_\infty) \left( \frac{T}{T_\infty} \right)^n \exp \left( \frac{-E_a}{kT} \right),
\]

where

\[
\Omega_T = \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + u^2 \frac{\partial T}{\partial x} + v^2 \frac{\partial T}{\partial y} + 2uv \frac{\partial^2 T}{\partial x \partial y} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y},
\]

\[
\Omega_c = \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + u^2 \frac{\partial C}{\partial x} + v^2 \frac{\partial C}{\partial y} + 2uv \frac{\partial^2 C}{\partial x \partial y} + \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y}.
\]

All symbols are defined in the nomenclature part.

With the boundary conditions,

\[
at y = 0: \quad u = U_w(x) = ax, \quad v = -V_w, \quad T = T_w, \quad D_B \frac{\partial C}{\partial y} + D_T \frac{\partial T}{\partial y} = 0,
\]

as \( y \rightarrow \infty: \quad u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty.
\]

Define
\[ \Psi = y \frac{\tilde{a}}{\sqrt{\psi}} \]
\[ u = xa f'(\Psi), \]
\[ v = -\sqrt{\tilde{u}} f(\Psi), \]
\[ \theta(\Psi) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \]
\[ \phi(\Psi) = \frac{C - C_{\infty}}{C_{\infty}}. \]

The corresponding ODE’s are

(4)

\[ f''' - f f'' + We f'' f''' - \lambda f' - Fr f''^2 - M f' + Ri (\theta - N r \phi) = 0, \]
\[ \frac{1}{Pr} \left( 1 + \frac{4}{3} R \right) \theta'' + f \theta' - \Gamma_c \left( f f' \theta' + f^2 \theta'' \right) + H g \theta + N b \theta' \phi' + N t \theta'^2 = 0, \]
\[ \frac{1}{Sc} \phi'' + f \phi' - \Gamma_c \left( f f' \phi' + f^2 \phi'' \right) + \frac{1}{Sc} \left( \frac{N t}{N b} \theta'' - \sigma^* \left( 1 + \delta \theta \right) \phi \exp \left( \frac{-E}{1 + \delta \theta} \right) \right) = 0. \]

(7)

All parameters are defined in the nomenclature part.

(5)

(6)

The corresponding boundary conditions are

(8)

At \( \Psi = 0 \): \( f(\Psi) = f w, f'(\Psi) = 1, \theta(\Psi) = 1, N b \phi'(\Psi) + N t \theta'(\Psi) = 0, \)

As \( \Psi \rightarrow \infty \): \( f'(\Psi) \rightarrow 0, \theta(\Psi) \rightarrow 0, \phi(\Psi) \rightarrow 0. \)

The dimensionless form of wall shear stress, heat, and mass flux are expressed as

(9)

3. Numerical Solutions

The ODE models (5)–(7) with associative conditions (8) are numerically solved by implementing MATLAB bvp4c procedure. In this regard, first, we change the 2nd and 3rd order ODE into a system of first-order ODE.

Let \( f = y_1, f' = y_2, f'' = y_3, \theta = y_4, \theta' = y_5, \phi = y_6, \) and \( \phi' = y_7. \)
The system of first-order ODEs is as follows:

\[
\begin{align*}
y_1' &= y_2, \\
y_2' &= y_3, \\
y_3' &= \frac{y_2 - y_1 y_3 + \lambda y_2 + Fr y_2^2 + M y_2 - R l[y_4 - N r y_4]}{1 + W}, \\
y_4' &= y_5, \\
y_5' &= \frac{-y_1 y_3 + Fr y_1 y_3 y_5 - H g y_4 - N b y_3 y_2 - N t y_2}{1 + Pr (1 + 4/3 R)} - \Gamma y_1', \\
y_6' &= y_7, \\
y_7' &= \frac{-y_1 y_2 + \Gamma y_3 y_2 y_2 - (1/Sc)(N t) y_2}{1 + Pr (1 + 4/3 R)} - \Gamma y_1', \\
y_8' &= y_9, \\
y_9' &= \frac{-y_1 y_2 + \Gamma y_3 y_2 y_2}{1 + Pr (1 + 4/3 R)} - \Gamma y_1', \\
y_{10}' &= y_{11}.
\end{align*}
\]

under the boundary conditions,

\[
\begin{align*}
y_1 (0) &= f w, \\
y_2 (0) &= 1, \\
y_2 (\infty) &= 0, \\
y_4 (0) &= 1, \\
y_4 (\infty) &= 0, \\
N b y_7 (0) + N t y_3 (0) &= 0, \\
y_6 (\infty) &= 0.
\end{align*}
\]

The numerical procedure needs initial calculation with tolerance \(10^{-6}\).

4. Results and Discussion

This section scrutinises the consequences of pertinent parameters on velocity, temperature, nanofluid concentration, skin friction coefficient, local Nusselt number, and local Sherwood number with a fixed quantity of Prandtl and Schmidt numbers. Table 1 provides the comparison of our numerical results and Mustafa et al. [47] results. We achieved that our results are exactly matched with Mustafa’s results. The estimation of \(We, \lambda, Fr, M, Ri,\) and \(Nr\) on skin friction coefficient, local Nusselt number, and local Sherwood number was presented in Table 2. We noticed that the surface shear stress accelerates when enhancing the \(We\) and \(Ri\) values, and it decelerates for heightening the quantity of \(\lambda, Fr, M,\) and \(Nr\) values. The heat transfer gradient grows when growing the values of \(Ri\) and it diminishes when upgrading the \(We, \lambda, Fr, M,\) and \(Nr\) values. Quite the opposite results are attained in the local Sherwood number. Table 3 describes the impact of \(H g, R, \Gamma y, N b,\) and \(N t\) on local Nusselt number. It is detected that the heat transfer gradient upturns when upturning the values of \(R, \Gamma y,\) and \(N b.\) On the contrary, it decimates for enhancing the quantity of \(H g\) and \(N t.\) The variations of local Sherwood number for different values of \(\sigma^*, E, n, \Gamma y, N b, N t,\) and \(\delta\) were illustrated in Table 4. We found that the mass transfer gradients decelerate for the small quantity of \(\sigma^*,\) and after that, it enriches for higher magnitudes. The LSN develops when developing the values of \(n, N b,\) and \(\delta,\) and the opposite trend was obtained for the more presence of \(E, \Gamma y,\) and \(N t.\)

Figures 1(a) and 1(b) explain the impact of \(We\) and \(M\) on DFF and NDFF in velocity profile. We found that the fluid velocity decelerates for rising the values of \(We\) and \(M.\) Physically, a higher Weissenberg number leads to enriching the fluid relaxation time, and this causes to slow down the motion of the fluid particles. The higher magnitude of the magnetic field parameter develops the fluid resistance and this causes to suppress the motion of the fluid particles. Also, we have seen that higher momentum boundary layer thickness occurs in NDFF compared to the DFF. The changes of fluid velocity for diverse values of \(\lambda, Fr\) on WNF and VNF were presented in Figures 2(a) and 2(b). We noticed that the fluid velocity reduces when enriching the \(\lambda\) and \(Fr\) values. The thickness of the momentum boundary layer is lessened in WNF compared to VNF. Figures 3(a) and 3(b) show the variations of velocity profile for disparate values of \(f w\) and \(Ri.\) We achieved that the fluid velocity declines when escalating the \(f w\) values, and it grows when increasing the \(Ri\) values. Also, we have seen that the higher momentum boundary layer thickness occurs in NDFF compared to the DFF. The temperature distribution for diverse quantity of \(f w\) and \(H g\) was illustrated in Figures 4(a) and 4(b). We detected that the fluid temperature diminishes when heightening the \(f w\) values and upturns for developing the \(H g\) values. The larger values of the heat generation parameter enhance the fluid thermal state and this leads to enhance the fluid temperature. The thicken boundary layer occurs in DFF and radiation compared to NDFF and without radiation, respectively. Figures 5(a) and 5(b) depict the \(N t\) and \(N b\) consequences of temperature distribution.
Table 1: Comparison of local Nusselt number when \( We = \lambda = Fr = R = \Gamma_c = Hg = \Gamma_s = 0, M = Nr = 0.5, Sc = 5, \) and \( \delta = 1 \) by Mustafa et al. [47].

| Pr  | Nt  | E  | \( \sigma^{**} \) | n  | Ri  | \( Nu/\sqrt{Re} \) |
|-----|-----|----|-----------------|----|-----|------------------|
|     |     |    |                 |    |     | [47] | Present         |
| 2.0 | 0.5 | 1.0 | 1.0             | 0.5| 0.5 | 0.706605        | 0.706604        |
| 4.0 |     | 0.5 | 0.935952        | 0.935955       |
| 7.0 |     | 1.0 | 1.132787        | 1.132788       |
| 10.0| 0.5 | 1.0 | 1.257476        | 1.257482       |
| 5.0 | 0.5 | 0.5 | 1.426267        | 1.426269       |
|     | 0.7 | 1.0 | 1.013939        | 1.013938       |
| 5.0 | 0.5 | 0.7 | 0.846943        | 0.846928       |
|     | 1.0 | 0.5 | 0.649940        | 0.649939       |
| 5.0 | 1.0 | 0.5 | 0.941201        | 0.941209       |
| 5.0 | 2.0 | 1.0 | 1.013939        | 1.013943       |
|     | 4.0 | 0.5 | 1.064551        | 1.064563       |
| 5.0 | 1.0 | 0.5 | 1.114549        | 1.114191       |
|     | 2.0 | 0.5 | 0.926282        | 0.926281       |
| 5.0 | 2.0 | 0.5 | 0.798671        | 0.798669       |
| 10.0| 0.5 | 1.0 | 1.145304        | 1.145301       |
| 5.0 |     | 1.0 | 1.013939        | 1.013938       |
| 5.0 |     | 2.0 | 0.964286        | 0.964285       |
| 5.0 |     | 5.0 | 0.886830        | 0.886830       |
| 5.0 | 0.5 | 2.0 | 1.145304        | 1.145301       |
| 1.0 |     | 1.0 | 1.114549        | 1.114191       |
| 2.0 |     | 1.0 | 1.056704        | 1.056706       |
| 5.0 | 2.0 | 1.0 | 1.032281        | 1.032280       |
| 1.0 |     | 1.0 | 1.032281        | 1.032280       |
| 2.0 |     | 1.0 | 1.154539        | 1.154538       |
| 5.0 |     | 1.0 | 1.215937        | 1.215938       |

Table 2: Numerically obtained values of skin friction coefficient, local Nusselt number, and local Sherwood number for various values of \( We, \lambda, Fr, M, Ri, \) and \( Nr. \)

| \( We \) | \( \lambda \) | \( Fr \) | \( M \) | \( Ri \) | \( Nr \) | \( 1/2Cf/\sqrt{Re} \) | \( Nu/\sqrt{Re} \) | \( Sh/\sqrt{Re} \) |
|---------|------------|------|-----|------|------|-----------------|-----------------|-----------------|
| 0.0     | 0.2        | 0.4  | 0.5 | 0.6  | 0.5  | -1.369910       | 1.736070        | -1.041642       |
| 0.1     | 0.3        | 0.2  | 0.5 | 0.6  | 0.5  | -1.336353       | 1.731626        | -1.038976       |
| 0.2     | 0.4        | 0.3  | 0.5 | 0.6  | 0.5  | -1.299006       | 1.726528        | -1.035917       |
| 0.3     | 0.5        | 0.2  | 0.5 | 0.6  | 0.5  | -1.255948       | 1.720440        | -1.032264       |
| 0.4     | 0.6        | 0.1  | 0.5 | 0.6  | 0.5  | -1.150084       | 1.740831        | -1.044499       |
| 0.5     | 0.7        | 0.0  | 0.5 | 0.6  | 0.5  | -1.150084       | 1.740831        | -1.044499       |
| 0.6     | 0.8        | 1.0  | 0.5 | 0.6  | 0.5  | -1.150084       | 1.740831        | -1.044499       |
| 0.7     | 0.9        | 2.0  | 0.5 | 0.6  | 0.5  | -1.150084       | 1.740831        | -1.044499       |
| 0.8     | 1.0        | 3.0  | 0.5 | 0.6  | 0.5  | -1.150084       | 1.740831        | -1.044499       |
| 0.9     | 1.1        | 4.0  | 0.5 | 0.6  | 0.5  | -1.150084       | 1.740831        | -1.044499       |
### Table 3: Numerically obtained values of local Nusselt number for the various values of $Hg$, $R$, $\Gamma_T$, $Nb$, and $Nt$.

| $Hg$ | $R$  | $\Gamma_T$ | $Nb$ | $Nt$ | $Nu/\sqrt{Re}$ |
|------|------|------------|------|------|----------------|
| -0.5 | 0.5  | 0.1        | 0.5  | 0.5  | 1.726528       |
| -0.3 | 0.5  | 0.1        | 0.5  | 0.5  | 1.542341       |
| 0.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.205077       |
| 0.3  | 0.5  | 0.1        | 0.5  | 0.5  | 0.871873       |
| 0.5  | 0.5  | 0.1        | 0.5  | 0.5  | 0.349241       |
| -0.5 | 0.0  | 0.1        | 0.5  | 0.5  | 1.438110       |
| 0.3  | 0.0  | 0.1        | 0.5  | 0.5  | 1.619396       |
| 0.6  | 0.0  | 0.1        | 0.5  | 0.5  | 1.776956       |
| 1.0  | 0.0  | 0.1        | 0.5  | 0.5  | 1.962378       |
| -0.5 | 0.5  | 0.0        | 0.5  | 0.5  | 1.701132       |
| 0.1  | 0.5  | 0.0        | 0.5  | 0.5  | 1.726528       |
| 0.2  | 0.5  | 0.0        | 0.5  | 0.5  | 1.753300       |
| 0.3  | 0.5  | 0.0        | 0.5  | 0.5  | 1.775803       |

### Table 4: Numerically obtained values of the local Sherwood number for various values of $\sigma^{**}$, $E$, $n$, $\Gamma_C$, $Nb$, $Nt$, and $\delta$.

| $\sigma^{**}$ | $E$  | $n$  | $\Gamma_C$ | $Nb$ | $Nt$ | $\delta$ | $Sh/\sqrt{Re}$ |
|---------------|------|------|------------|------|------|----------|----------------|
| 0.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -1.031660      |
| 0.7           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -1.034681      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -1.031771      |
| 2.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -1.027447      |
| 1.0           | 1.0  | -1.0 | 0.1        | 0.5  | 0.5  | 1.0      | -1.028634      |
| 1.0           | 1.0  | -0.5 | 0.1        | 0.5  | 0.5  | 1.0      | -1.035293      |
| 1.0           | 1.0  | 0.0  | 0.1        | 0.5  | 0.5  | 1.0      | -1.026188      |
| 1.0           | 1.0  | -1.0 | 0.0        | 0.5  | 0.5  | 1.0      | -1.077603      |
| 1.0           | 1.0  | 0.0  | 0.0        | 0.5  | 0.5  | 1.0      | -1.03243       |
| 1.0           | 1.0  | 0.0  | 0.0        | 0.5  | 0.5  | 1.0      | -1.029591      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -1.034323      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.517626      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.345282      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.259035      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.216786      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.194959      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.276326      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -1.035867      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -1.03423       |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.345282      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.259035      |
| 1.0           | 1.0  | 0.5  | 0.1        | 0.5  | 0.5  | 1.0      | -0.216786      |
We exposed that the fluid temperature develops for increasing the $N_t$ values and the opposite trend was obtained for $Nb$ values. Also, we noticed that the reaction rate leads to suppressing of the thermal boundary layer thickness. The effects of $Hg$ and $R$ on temperature distribution were plotted in Figures 6(a) and 6(b). We noted that the fluid temperature raises for rising the $Hg$ and $R$ values. In addition, we found the larger thermal boundary layer thickness attains in the FHF model compared to the CCHF model. Figures 7(a) and 7(b) portray the consequences of $fw$ and $Nt$ for nanofluid concentration profile. We concluded that the nanofluid concentration enhances near the plate and falling-off away from the plate. The $N_t$ values lead to enriching the nanofluid concentration boundary layer thickness. The nanofluid concentration distribution for different values of $Nb$ and $\Gamma_C$ were shown in Figures 8(a) and 8(b). These figures clearly show that the nanofluid concentration is an increasing behavior for $\Gamma_C$ and quite the opposite occurs for $Nb$ values. Also, we noticed that the reaction rate leads to suppressing of the nanofluid concentration boundary layer thickness.

Figure 1: The velocity distribution for diverse quantity of $We$ (a) and $M$ (b).

Figure 2: The velocity distribution for diverse quantity of $\lambda$ (a) and $Fr$ (b).
The skin friction coefficient on WNF and VNF on DFPM and NDFPM with \(f_w = -0.4\) and \(f_w = 0.4\) was plotted in Figures 9(a) and 9(b). We proved that the larger surface shear stress \((-0.707201)\) occur in NDFF of WNF with injection case and smaller surface shear stress \((-1.36991)\) occur in DFF of VNF with the suction case.

Figures 10(a) and 10(b) provide the skin friction coefficient on WNF and VNF on a DFPM and NDFPM with \(R_i = 0.0\) and \(R_i = 1.0\). We concluded that the larger surface shear stress \((-0.875878)\) occur in the DFF of WNF with the presence of \(R_i\) and smaller surface shear stress \((-1.61002)\) occur in the DFF of VNF with the absence of \(R_i\). The local Nusselt number on WNF and VNF with \(R = 0.0\), \(R = 1.0\), \(H_g = -0.4\), and \(H_g = 0.4\) with the CC model and the FF model was plotted in Figures 11(a) and 11(b) and Figures 12(a) and 12(b). In the CC model, the larger heat transfer gradient \(1.97227\) occurs in heat-absorbing radiative viscous nanofluid, and a smaller heat transfer gradient \(0.718399\) occurs in a heat-generating radiative viscous nanofluid. In the FF model, the larger
heat transfer gradient (3.2986) occurs in heat-absorbing nonradiative viscous nanofluid, and a smaller heat transfer gradient (0.646887) occurs in heat-generating radiative Williamson nanofluid. Figures 13(a) and 13(b) provide the local Sherwood number on various combinations of $E$ and $n$ of a CC, FF flow of WNF and VNF. The larger mass transfer gradient (−1.0171) occurs in FF without activation energy, and a smaller mass transfer gradient (−1.03809) occurs in the CC model with $n = −0.5$. 

**Figure 5:** The temperature distribution for diverse quantity of $Nt$ (a) and $Nb$ (b).

**Figure 6:** The temperature distribution for diverse quantity of $Hg$ (a) and $R$ (b).
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Figure 7: The nanofluid concentration distribution for diverse quantity of $fw$ (a) and $Nt$ (b).

Figure 8: The nanofluid concentration distribution for diverse quantity of $Nb$ (a) and $\Gamma_C$ (b).
Figure 9: The skin friction coefficient on WNF and VNF on a DFPM and NDFPM with $fw = -0.4$ (a) and $fw = 0.4$ (b).

Figure 10: The skin friction coefficient on WNF and VNF on a DFPM and NDFPM with $Ri = 0.0$ (a) and $Ri = 1.0$ (b).

Figure 11: The local Nusselt number on various combinations of $Hg$ and $R$ of a CCHF flow of WNF and VNF.
5. Conclusions

This analysis clearly shows the consequences of thermal radiation of a Darcy–Forchheimer flow of Williamson nanofluid on a stretchy plate with a magnetic field. The energy and nanoparticle concentration equations were framed with Catteneo–Christov heat-mass flux theory. Additionally, the mass transfer analysis is made by activation energy and binary chemical reaction. The governing PDE problems were converted into ODE problems by applying suitable variables, and these equations were solved using MATLAB bvp4c algorithm. The salient outcomes of the current analysis are outlined as below:

(i) The fluid velocity decelerates when enhancing the Williamson fluid, magnetic field, and porosity parameters, and it accelerates by increasing the Richardson number.

(ii) The fluid temperature accelerates when strengthening the heat generation/absorption radiation and thermophoresis parameters, and it declines when increasing the Brownian motion parameter.

(iii) The fluid concentration suppresses when increasing the Brownian motion parameter, and it enhances when escalating the thermophoresis and mass relaxation time parameters.

(iv) The smaller SFC occurs in the non-Darcy–Forchheimer flow of Williamson nanofluid.

(v) The larger heat transfer gradient exists in viscous nanofluid without radiation.
(vi) The larger Sherwood number attains in the Fourier mass flux model without activation energy.

The skin friction coefficient on WNF and VNF on a DFPM and NDFPM with $Ri = 0.0$ and $Ri = 1.0$ was plotted in Figures.

**Nomenclature**

$a$: Constant  
$B_0$: Magnetic field strength  
$C$: Fluid concentration  
$C_{p0}$: Drag coefficient  
$C_P$: Specific heat (Jkg$^{-1}$K$^{-1}$)  
$C_v$: Wall concentration  
$C_{\text{co}}$: Ambient fluid concentration  
$D_{\text{gr}}$: Mass diffusivity  
$D_T$: Thermophoretic diffusion coefficient  
$E = (E_o/\kappa T_{\text{co}})$: Nondimensional activation energy  
$E_a$: Activation energy  
$Fr = (C_{p0}/\sqrt{K_2})$: Forchheimer number  
$f_i$: Nondimensional velocity  
$f_w = (V_w/\sqrt{av})$: Suction/injection parameter  
$g$: Acceleration due to gravity  
$Gr_x = ((\beta(1-C_{\text{co}}))/(T_w-T_{\text{co}})x^3)/\gamma^2$: Local Grashof number  
$Hg = (Q/\rho_f C_f a)$: Heat generation/absorption parameter  
$k$: Thermal conductivity  
$k_f$: Permeability of porous medium  
$k^*$: Mean absorption coefficient  
$k_r$: Reaction rate  
$M = (\sigma B_0^2/\rho_f a)$: Hartmann number  
$n$: Fitted rate  
$Nb = (\tau D_{\text{gr}} C_{\text{co}}/\gamma)$: Brownian diffusion parameter  
$N_r = ((\rho_p - \rho_w)C_{\text{co}}/((\rho_f - \beta(1-C_{\text{co}}))/(T_w-T_{\text{co}})))$: Buoyancy ratio parameter  
$N_t = (\tau D_T(T_w-T_{\text{co}}))/T_{\text{co}})$: Thermophoresis parameter  
$Pr = (\nu/\alpha)$: Prandtl number  
$Q$: Heat generation/absorption coefficient  
$R = (4a^3 T_{\text{co}}^3/\kappa k^*)$: Thermal radiation  
$Re_x = (U_w x/\nu)$: Local Reynolds number  
$Re = (Gr_x/Re_x^2)$: Richardson number  
$Re_x = ((\beta(1-C_{\text{co}}))/(T_w-T_{\text{co}}))a^2 x$: Schmith number  
$Sc = (\nu/D_{\text{gr}})$: Fluid temperature (K)  
$T_{\text{co}}$: Ambient temperature (K)  
$T_w$: Wall temperature (K)  
$u$ and $v$: Velocity components (ms$^{-1}$)  
$U_w$: Stretching surface velocity (ms$^{-1}$)  
$We = (\Gamma x^2/2a^2/v)$: Weissenberg number  
$x$ and $y$: Direction coordinates (m)

**Greek symbols**

$\alpha$: Thermal diffusivity (m$^2$ s$^{-1}$)  
$\beta$: Thermal expansion coefficient  
$\delta = (T_w-T_{\text{co}})/T_{\text{co}}$: Temperature difference parameter  
$\Gamma$: Williamson parameter  
$\Gamma_T = (a\lambda_T)$: Solute relaxation parameter  
$\Gamma_C = (a\lambda_C)$: Local porosity parameter  
$\lambda_T$: Relaxation time of mass flux  
$\lambda_C$: Relaxation time of heat flux  
$\nu$: Nondimensional viscosity (m$^2$s$^{-1}$)  
$\phi$: Non-dimensional nanofluid concentration  
$\rho_f$: Fluid density (kgm$^{-3}$)  
$\sigma$: Electrical conductivity  
$\sigma^*$: Stefan Boltzmann constant (Wm$^{-2}$K$^{-4}$)  
$\tau$: Dimensionless reaction rate  
$\theta$: Nondimensional temperature

**Abbreviations**

AC: Activation energy  
CCHF: Cattaneo – Christov heat flux  
DF (F): Darcy – Christov (flow)  
FHF: Fourier heat flux model  
HA: Heat absorption  
HG: Heat generation  
NAC: Nonactivation energy  
NDF (F): Non – Darcy – Forchheimer (flow)  
NDFPM: Nonradiative Williamson nanofluid  
NR: Negative fitted rate  
NRVNF: Nonradiative viscous nanofluid  
NRWNF: Nonradiative Williamson nanofluid  
PFR: Positive fitted rate  
RVNF: Radiative viscous nanofluid  
RWNF: Radiative Williamson nanofluid  
WNF: Williamson nanofluid

**Data Availability**

The data used to support the findings of the study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Authors’ Contributions**

All authors contributed equally to this work. All the authors have read and approved the final version manuscript.

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