QUANTUM POINCARÉ RECURRENCES IN MICROWAVE IONIZATION OF RYDBERG ATOMS

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Abstract

We study the time dependence of the ionization probability of Rydberg atoms driven by a microwave field. The quantum survival probability follows the classical one up to the Heisenberg time and then decays inversely proportional to time, due to tunneling and localization effects. We provide parameter values which should allow one to observe such decay in laboratory experiments. Relations to the $1/f$ noise are also discussed.

The pioneering experiment of Bayfield and Koch performed in 1974 [1] attracted a great interest to ionization of highly excited hydrogen and Rydberg atoms in a microwave field. The main reason of this interest is due to the fact that such ionization requires the absorption of a large number of photons (about $20-70$) and can be explained only as a result of the appearance of dynamical chaos and diffusive energy excitation in the corresponding classical system. Such classical diffusive ionization requires many microwave periods and quantum interference effects can suppress this diffusion leading to quantum localization of chaos [2].

More recent experiments [3] with alkali Rydberg atoms showed an algebraic time dependence of the survival probability: $P(t) \propto t^{-\alpha}$, with $\alpha \approx 0.5$. This result cannot be explained within the picture of diffusive ionization in the domain of classical chaos. The origin of the slow algebraic decay was attributed to the underlying structure of classical mixed phase space composed by integrable islands surrounded by chaotic components: Chaotic trajectories can be trapped in the vicinity of the hierarchy of regular islands [4] and this slows down the ionization process.

Recent studies of quantum Poincaré recurrences for the standard map model in the semiclassical regime [5] showed that quantum $P(t)$ follows the classical decay up to the Heisenberg time $t_H$, which is determined by inverse level spacings. For $t > t_H$, the quantum survival probability starts to decay inversely proportional to time ($\alpha = 1$) and becomes much larger than the classical one. The power $\alpha = 1$ is due to the exponentially
The Hamiltonian reads

\[ H = \frac{p^2}{2} - \frac{1}{r} + \epsilon z \sin(\omega t), \]  

where \( \epsilon \) and \( \omega \) are the strength and frequency of the microwave field, measured in atomic units. The quantum evolution is numerically simulated by the one-dimensional (1d) model of a hydrogen atom and by the 3d model for atoms initially prepared in states extended along the field direction and with magnetic quantum number \( m = 0 \). In the 1d model the motion is assumed to take place along the field direction (\( z \)-axis, with \( z \geq 0 \)).

In order to compare classical and quantum dynamics it is convenient to use the scaled field strength \( \epsilon_0 = \epsilon n_0^4 \) and frequency \( \omega_0 = \omega n_0^3 \), which completely determine the classical dynamics. The classical limit corresponds to \( \hbar_{\text{eff}} = \hbar/n_0 \rightarrow 0 \), at constant \( \epsilon_0, \omega_0 \). In classical mechanics diffusive ionization takes place for fields above the chaos border: \( \epsilon_0 > \epsilon_c \approx 1/(49\omega_0^{1/3}) \) [2]. Quantum interference effects can suppress this diffusion leading to quantum localization of chaos [3]. Such dynamical localization leads to a quantum probability distribution \( f_N \) exponentially localized in the number of absorbed photons \( N_{\phi} = (E - E_0)/\omega \) (\( E \) electron energy, \( E_0 = -1/2n_0^2 \)): \( f_N \propto \exp(-2|N_{\phi}|/\ell_{\phi}) \), with localization length \( \ell_{\phi} = 3.3\epsilon_0^2\omega_0^{-10/3}n_0^2 \) [3].

We introduce an absorption border for levels with \( n \geq n_c \); like this the number of photons required to ionize the atom is \( N_c = (n_0/2\omega_0)(1 - n_0^2/n_c^2) \). Such border occurs in real laboratory experiments, for example as a consequence of unavoidable stray electric fields experienced by the Rydberg atoms during their interaction with the microwave. The absorption border \( n_c \) can be varied in a controlled way via a static electric field \( \epsilon_s \), the static field ionization border being \( \epsilon_s n_c^4 \approx 0.13 \).

In Fig. [4] we show a realistic case \((n_0 = 60, \epsilon_0 = 0.1, \omega_0 = 2.6)\) in which, initially, classical and quantum probabilities decay in a very similar way and only after approximately \( 5 \times 10^2 \) microwave periods the quantum survival probability starts to decay more slowly \((P(t) \propto 1/t)\) than the classical one which decays as \( 1/t^\alpha \), with \( \alpha \approx 2.15 \). The comparison of quantum simulations for the 1d hydrogen atom model and the 3d dynamics is shown in the inset of Fig. [4]. It demonstrates that both dynamics give very close results, confirming that the essential physics is captured by the 1d model. Actually, due to Coulomb degeneracy, the slow motion in the orbital momentum \( l \) acts as an adiabatic perturbation on the \( n \) motion and as a result the excitation in \( n \) is well described by the 1d model [3]. We put the absorption border near the initial state \((n_c = 64)\) in order to have \( \rho_c = \ell_{\phi}/N_c \approx 3.5 > 1 \). In this way the probability can go out very easily and the \( 1/t \) probability decay is observed after a short transient time of
The order of 20 microwave periods. On the contrary, when $\rho_c < 1$, strong fluctuations around the $1/t$ decay take place [6]. This is analogous to the huge (log-normally distributed) conductance fluctuations in a disorder solid with localization length smaller than the sample size.

In order to confirm that the algebraic probability decay is related to localization effects and to the sticking of classical trajectories and of quantum probability near the integrable islands in the phase space, we show in Fig. 2 the time evolution of the survival probability distribution in the phase space of action-angle variables $(n, \theta)$ for the 1d model. In the classical case $3 \times 10^6$ orbits are initially homogenously distributed in the angle $\theta$ on the line $n = n_0 = 60$, corresponding to the initial quantum state with principal quantum number $n_0 = 60$. After 50 microwave periods, the classical distribution of non ionized orbits shows a fractal structure which surrounds the stability islands (Fig. 2 top left). At larger times this distribution approaches more and more closely the boundary critical invariant curves (Fig. 2 top right). One of them confines the motion in the region with $n > n_b \approx n_0(\epsilon_c/\epsilon_0)^{1/5} \approx 41$ where $n_b$ determines the classical chaos border for given $\epsilon_0$. Other invariant curves mark the critical boundaries around internal stability islands (for example at $n \approx 55$, corresponding to $\omega n^3 \approx 2$). This phase space metamorphosis is associated with a change in the exponent of the power law decay of classical Poincaré recurrences (the crossover occurs at $t \approx 5 \times 10^2$). In the quantum case the value of $\hbar_{\text{eff}}$ is not sufficiently small to resolve the fractal structure at small scales. However, the Husimi function (constructed in a way similar to Ref. 7) shows similarities with the classical probability distribution at $t = 50$.

Figure 1: Survival probability $P(t)$ as a function of the interaction time $t$ (in units of microwave periods) for $\epsilon_0 = 0.1$, $\omega_0 = 2.6$, $n_0 = 60$, $n_c = 64$: quantum (solid line) and classical (dashed line, ensemble of $3 \times 10^6$ trajectories) 1d hydrogen model. The straight lines have slopes 1 and 2.15, the latter coming from a fit of the classical decay for $5 \times 10^2 < t < 2 \times 10^4$. Inset: quantum survival probability for the 1d model (solid line) and the 3d hydrogen atom (dot-dashed line).
Figure 2: Classical density plot (top) and Husimi function (middle and bottom) in action-angle variables \((n, \theta)\), with \(30 \leq n \leq 63.5\) (vertical axis) and \(0 \leq \theta < 2\pi\) (horizontal axis), for the 1d model in the case of Fig. 1. Top left: \(50 \leq t \leq 60\); top right: \(2 \times 10^3 \leq t \leq 10^4\); Husimi functions are averaged in a small time interval \(\delta t = 10\) to decrease fluctuations, near \(t = 50\) (middle left), \(t = 10^3\) (middle right), \(t = 10^4\) (bottom left) and \(t = 10^5\) (bottom right). Bright regions correspond to maximal density.
Figure 3: Quantum (solid line) and classical (dashed line) average action $<n>$ as a function of the interaction time $t$ for the case of Fig. 1. The straight lines indicate the position of the stability islands for $\omega n^3 \approx 1$ ($n \approx 55$) and $\omega n^3 \approx 2$ ($n \approx 44$).

(Fig. 2 middle left). At longer times, the diffusion towards the boundary at $n_b$ is slowed down due to localization effects and penetration of the quantum probability inside the classical integrable islands. At $t = 10^3$ (Fig. 2 middle right) the quantum probability is concentrated in a layer around $n \approx 49$. Due to localization effects, the Husimi function does not change significantly for a very long interaction time (compare with Fig. 2 bottom left at $t = 10^4$). Eventually the probability starts to penetrate very slowly inside the main island at $n \approx n_b$ (see Fig. 2 bottom right at $t = 10^5$). Therefore tunneling and localization effects are responsible for the slow $1/t$ decay of quantum survival probability seen in Fig. 1. The chaos border starts to influence the dynamics only after a very long interaction time because its distance from the starting line $n = n_0$ is much greater than the localization length $\ell_\phi$: $\rho_b = \ell_\phi/|N_b| \approx 0.35 < 1$, where $N_b = (n_0/2\omega_0)(1 - n_b^2/n_0^2)$.

Fig. 3 shows the average action $<n>$ of the non-ionized electrons as a function of the interaction time. The diffusion towards the chaos border is slowed down in the quantum case and different regimes are clearly visible: tunneling inside the islands for $\omega n^3 \approx 2$ (for $t < 10^2$), localization of the wave packet (for $10^3 < t < 3 \times 10^4$) and tunneling inside the main island at $\omega n^3 \approx 1$ (for $t > 10^5$).

Notice that the probability decay $P(t)$ is related to correlations decay via $C(t) \propto tP(t)$ [4]. In the case of $\alpha = 1$ this implies that correlations do not decay. The Fourier transform of $C(t)$ gives the spectral density $S(\omega)$ of the effective noise produced by the dynamics:

$$S(\omega) = \int C(t) \exp(i\omega t) dt \propto 1/\omega.$$ \hspace{1cm} (2)

This shows that the spectral noise associated with the quantum Poincaré recurrences with $\alpha = 1$ scales like $S(\omega) \propto 1/\omega$. A similar behavior of noise has been observed in
various scientific phenomena \[8\], for example in the luminosity of stars, the velocity of undersea currents, the flow rate of the river Nile, the magnetization of spin glasses and the resistance of different solid state devices \[9\]. It is known as $1/f$ noise and usually extends over several orders of magnitude in frequency, indicating a broad distribution of time scales in the system. In the case of quantum Poincaré recurrences this property stems from the exponentially broad distribution of escape times from some regions of the phase space, due to tunneling and localization effects.

In summary, we have shown that the survival probability for Rydberg atoms in a microwave field decays, up to the Heisenberg time $t_H$, in a way similar to the classical probability. For $t > t_H$ the quantum probability starts to decay slower than the classical one, with the exponent of the algebraic decay $\alpha = 1$. We have given parameter values which should allow one to observe quantum Poincaré recurrences in microwave experiments with Rydberg atoms. Indeed, the thermal beams used with alkali Rydberg atoms allow one to vary the interaction time by orders of magnitude up to $10^5$ microwave periods \[3\]. Therefore the results of the present paper show that theoretical and experimental studies of chaotic Rydberg atoms still represent a challenge for fundamental research of quantum chaos.

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