ON SCALING LAWS AND ALFVÉNIC MAGNETIC FLUCTUATIONS IN MOLECULAR CLOUDS

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Received 1996 April 16; accepted 1996 November 15

ABSTRACT

Under the basic assumption that the observed turbulent motions in molecular clouds are Alfvenic waves or turbulence, we emphasize that the Doppler broadening of molecular line profiles directly measures the velocity amplitudes of the waves instead of the Alfven velocity. Assuming an equipartition between the kinetic energy and the Alfvenic magnetic energy, we further propose the hypothesis that observed standard scaling laws in molecular clouds imply a roughly scale-independent fluctuating magnetic field, which might be understood as a result of strong wave-wave interactions and subsequent energy cascade. We predict that \( \sigma \propto \rho^{-0.5} \) is a more basic and robust relation in that it may hold approximately in any regions where the spatial energy density distribution is primarily determined by wave-wave interactions, including gravitationally unbound regions. We also discuss the fact that a scale-independent \( \sigma_g^2 \) appears to contradict existing one-dimensional and two-dimensional computer simulations of MHD turbulence in molecular clouds.

Subject headings: MHD — ISM: clouds — ISM: kinematics and dynamics — ISM: magnetic fields — turbulence — waves

1. INTRODUCTION

Motions of material within interstellar molecular clouds have long been known to be predominantly turbulent except in small low-mass dense cores. A significant advance in the study of turbulence and cloud support was the finding of a set of scaling relations, i.e., correlations among cloud quantities such as the characteristic radius \( R \), the mean nonthermal velocity dispersion \( \sigma_v \), and the average density \( n_{H_2} = \rho/m \), for various samples of molecular structures ranging from 0.2 to 200 pc in size (Larson 1981; Leung, Kutner, & Mead 1982; Myers 1983; Dame et al. 1986; Scoville & Sanders 1987; Solomon et al. 1987; Falgarone, Puget, & Pérault 1992; Blitz 1993),

\[
\sigma_v \propto R^\alpha, \quad \alpha = 0.5, \quad (1)
\]
\[
n_{H_2} \propto R^{-\gamma}, \quad \gamma = 1, \quad (2)
\]

and

\[
\sigma_g \propto n_{H_2}^{\beta}, \quad \beta = 0.5. \quad (3)
\]

Only two of the above relations are independent; the power-law indices are related by \( \alpha = \beta \gamma \). Since the above values for \( \alpha, \beta, \) and \( \gamma \) are controversial, we will refer to equations (1)–(3) as the standard scaling laws. Largely established on a cloudbound basis (i.e., based on samples of clouds), the physical significance of these scaling laws lies in the natural extrapolation that they are on average a manifestation of self-similar individual clouds satisfying these relations (Larson 1981; Henriksen & Turner 1984), although arguably these correlations can be due to observational selection effects or even artifacts (Kegel 1989; Scalo 1990; Myers 1991; Vázquez-Semadeni, Ballesteros-Paredes, & Rodríguez 1997). In general, observational determination of these scaling relations is based on average properties of clouds and subject to numerous uncertainties. Therefore, these scaling laws are necessarily rough and perhaps of value dimensionally only in a strict sense. On the other hand, the large number of observational studies that do find more or less the same scaling relations indicate that these scaling laws may indeed tell us something physical (Scalo 1987; McKee 1989), at least in a dimensional sense.

Proposed explanations of the scaling laws bifurcate into hydrodynamic and magnetohydrodynamic (MHD) regimes and were critically reviewed by Scalo (1987). Most of them converge on that one of the two independent scaling relations is due to the trend for virial equilibrium. The hydrodynamic approach takes the scaling laws as evidence for energy cascade in strong eddy turbulence over a large range of scales, similar to the Kolmogorov law for incompressible turbulence (Larson 1981), which gives dimensionally \( \sigma_v \sim R^{1/3} \). In the MHD regime, however, a comparably rigorous theoretical approach has been hampered by the lack of theories capable of describing strong compressible MHD turbulence.

2. EVIDENCE FOR A SCALE-INDEPENDENT FLUCTUATING MAGNETIC FIELD

Given a relatively strong interstellar magnetic field and a moderate ionization fraction of \( 10^{-7} \) to \( 10^{-4} \) maintained by penetrating cosmic rays and UV photons, the gas in molecular clouds is expected to behave much like plasma that is subject to numerous instabilities. If one recognizes the existence of a general interstellar magnetic field and turbulence, then it seems difficult to reject the likelihood that the turbulent motions (even if they are initially purely hydrodynamic) further generate MHD waves such as Alfven waves. In fact, Alfven waves have long been proposed as a viable physical process for explaining the observed “turbulence” in molecular clouds (Arons & Max 1975, hereafter AM75; Zweibel & Josafatsson 1983). Identifying (implicitly or explicitly) the turbulent velocity as the Alfven velocity, it has been further proposed that the standard scaling laws can be explained if kinetic energy, energy of the general magnetic field, and gravitational energy are all in approximate equipartition and that the magnetic field strength \( B \) is largely a constant over the scales concerned (Pellegatti Franco, Tarsia, & Quiroga 1985;
Scalo 1987; Myers & Goodman 1988, hereafter MG88; Fleck 1988). The difficulty of understanding such a constant, however, has been noted (Scalo 1987; MG88; Mouschovias & Psaltis 1995); it contradicts not only the “frozen-in” picture for the interstellar magnetic field (cf. Mouschovias 1987) but also the observational evidence for a $B \propto n$ scaling relationship (Troland & Heiles 1986; Heiles et al. 1993).

The Doppler broadening of the spectral line profiles, nevertheless, reflects the velocity amplitudes of the waves (i.e., bulk moving velocity of the particles), not the phase velocity $V_s$. The velocity amplitude of an Alfvén wave packet of scale $\sim R$ is determined by the assumed equipartition between the kinetic energy density and the fluctuating magnetic energy density over the scale $R$ (AM75):

$$\frac{1}{2} \rho \sigma_v^2 \approx \sigma_B^2 \frac{8}{\pi}$$

or

$$\frac{1}{2} \rho \sigma_v^2 = \frac{\delta B}{(4\pi \rho)^{1/2}},$$

where $\delta v = (8 \ln 2)^{1/2} \sigma_v$ and $\delta B = (8 \ln 2)^{1/2} \sigma_B$; Alfvén (1953) was perhaps the first to apply this relation astrophysically (solar granulation). McKee & Zweibel (1995) and Zweibel & McKee (1995) have recently demonstrated this relationship for weak MHD turbulence (when $\delta B$ is much smaller than $B$). Although it is difficult to prove the validity of this relation in the strong compressible turbulence regime, both observational and theoretical studies of solar wind turbulence have confirmed this basic relation, which has become a basic building block of a lot of analyses (cf. Tu & Marsch 1995; Lau & Siregar 1996). Recent MHD turbulence simulations for molecular clouds also verified this energy equipartition (Passot, Vázquez-Semadeni, & Pouquet 1995; Gammie & Ostriker 1996).

Next, let us look at the commonly assumed virial equilibrium for gravitationally bound clouds. Observationally, virial equilibrium is established by comparing the kinetic and the gravitational energy of gas (cf. Larson 1981; Blitz 1993). Theoretically, however, the situation is much less trivial, as the presence of the general magnetic field should also be taken into account (cf. Nakano 1984; Mouschovias 1987; McKee et al. 1993). Naively, the observationally established virial equilibrium seems to underscore the direct role of the general magnetic field, which is consistent with a possibility first raised by Mestel (1965) that the volume and surface terms due to the general magnetic field have opposite signs and thus tend to cancel. A potentially more interesting (and radical) possibility is that turbulent motions can effectively reduce the effective pressure or stress of the magnetic field (Kleeorin, Mond, & Rogachevskii 1996). The gravitational energy density of a gas parcel at radius $R$ from the center of gravity can be written as $U_c = 3CGM(R)/R$, where $C$ is a numerical factor depending mainly on the density distribution and $M(R)$ is the mass inside a radius $R$. The simplest form for the virial equilibrium in accord with observations can be written as

$$CG \frac{M_p}{R} = \rho \sigma_v^2 + \frac{\rho kT}{m},$$

where $\rho$ is the mean mass of the molecules. This is the same approach as adopted by MG88 and Caselli & Myers (1995). Defining a mean density $\rho_m$ so that $M(R) = (4\pi/3)\rho_m R^3$, the left-hand side of equation (5) becomes $CG \rho_wp R^2$. If $\rho_m \sim \rho$ within the errors of observational measurements, equations (4) and (5) lead to

$$\rho = \frac{4}{3} \pi CG \left( \frac{\sigma_B^2}{4\pi m} + \frac{\rho}{kT} \right)^{1/2} R^{-1}. \quad (6)$$

Equations (4) and (6) give

$$\sigma_v = \left( \frac{4\pi}{3} CG \right)^{1/4} \left( \frac{\sigma_B^2}{4\pi} + \frac{\rho}{m} kT \right)^{1/4} R^{0.5}. \quad (7)$$

It is clear that equations (7), (6), and (4) match the standard scaling laws, equations (1), (2), and (3), respectively, if and only if $\sigma_B^2$ is scale-independent and thermal energy is small compared to the nonthermal energy. Since the turbulent pressure in this case is $P_{\text{turb}} \propto \rho \sigma_v^2 = \sigma_B^2/4\pi$, a scale-independent $\sigma_B^2$ is consistent with the consensus that the scaling laws imply an approximate dynamic pressure equilibrium in molecular clouds (cf. Chieze 1987; Fleck 1988; Maloney 1988; Elmegreen 1989; Blitz 1993). For a sample of Galactic molecular clouds associated with H II regions, Solomon et al. (1987) found a velocity dispersion–size relationship $\sigma_v$ (km $s^{-1}$) = 0.7($R/1$ pc)$^{0.5}$. From equation (7), we derive $\sigma_B = 10.6(C/2)^{1/2} \mu G = 24 \mu G$ for $C \sim 0.2$ (uniform density). If these clouds are centrally condensed ($C$ is somewhat larger), the required $\sigma_B$ will be correspondingly smaller. Note, however, that when thermal kinetic energy density is not negligible on small scales, both the density-size and the velocity-size relations will be different from the standard forms. When thermal kinetic energy density dominates, equation (6) yields the classic isothermal $r^{-2}$ density distribution; otherwise the power-law index $\gamma$ for the density-size relation ought to be between 1 and 2, as discussed by MG88, McKee (1989), Fuller & Myers (1992), and Caselli & Myers (1995).

3. WAVE-WAVE INTERACTIONS: AN ANALOGY TO KOLMOGOROV CASCADE

Since observations indicate that the turbulent kinetic energy is often comparable to the energy of the general background magnetic field in some molecular clouds (e.g., MG88; Crutcher et al. 1993, 1994), MHD waves at different wavelengths are expected to have strong mutual interactions and thus become strong MHD turbulence (cf. Myers & Khersonsky 1995). Intuitively, nonlinear wave-wave interactions and consequent energy cascade largely dominate the spatial energy density distribution (cf. Sagdeev & Galeev 1969; Elmegreen 1990; Biskamp 1994; Goldreich & Sridhar 1995), similar to the Kolmogorov energy cascade for hydrodynamic eddy turbulence. Unfortunately, an eigenvalue description that is used for weak MHD turbulence breaks down in this case, leaving essentially no other effective means so far for establishing a working theory which could tell us what the spatial energy density distribution should be for strong compressible MHD turbulence. The theoretical difficulty is clearly witnessed by the recent controversy over the Iroshnikov-Kraichnan theory for incompressible MHD turbulence (e.g., Goldreich & Sridhar 1995; Ng & Bhattacharjee 1996), which is a much more simplified problem than strong compressible MHD turbulence in molecular clouds.

In this case it seems useful to see what observations tell us. Specifically, as argued in the previous section, the observed scaling laws in molecular clouds indicate a scale-independent
fluctuating magnetic energy density $\sigma_B^2$. In terms of the energy spectrum, $P_s$, i.e., energy per unit volume per unit wavenumber as a function of wavenumber $k \sim 1/R$, a scale-independent $\sigma_B^2$ implies a power law $P_s \propto k^{-1}$. What is intriguing is that this $k^{-1}$ spectrum corresponds to equipartition of mode amplitudes (Biskamp 1994) or Rayleigh-Jeans energy equipartition (Sagdeev & Galeev 1969) in the sense that the wave packets of different scales obtain an equal share of the available energy within a unit volume of space. This implies further that wave packets on larger scales would dominate the total amount of energy.

Now, is such a spatial energy density distribution physically plausible? We consider two aspects of the problem. First, the energy. The energy flux of an Alfvén wave packet on a scale $R$ can be written as $S = V_0 \sigma_B^2/4\pi$ (cf. Lau & Siregar 1996). It is evident that an ensemble of interacting wave packets traveling in all directions would yield a net energy transport flux $\Delta S$ only in the direction of the negative spatial gradient for the magnetic pressure or energy density $\sigma_B^2/4\pi$, and thus the developing tendency is to reduce the spatial gradients for $\sigma_B^2$.

Second, a main characteristic of MHD turbulence is the existence of inverse cascade, i.e., energy cascade from small to large scales with the tendency to form self-organized large-scale coherent structures (Pouquet, Frisch, & Léorat 1976; Biskamp 1994), in contrast to the direct energy cascade (from large to small scales) in pure hydrodynamic turbulence. Therefore, it is not unlikely that the interacting wave packets of different scales achieve a net tendency for an equal share of the available energy density due to the effective energy cascade in both the direct and inverse directions while the system is fully turbulent (cf. Biskamp 1994).

One point that appears interesting to us is that if the kinetic energy does remain in equipartition with the fluctuating magnetic energy, then perhaps a hydrodynamic description and a MHD description could lead to comparable physical insights to the turbulence in molecular clouds, at least from an energy point of view. The MHD approach, however, does offer certain advantages. One of these is that MHD waves may effectively transport energy with a high speed and perhaps also a low dissipation rate without necessary bulky flow of gas, and yet achieve a more or less uniform pressure environment (A. Battacharjee 1996, private communication). The disadvantage of the MHD approach, however, is its mathematical complexity.

Given that a scale-independent fluctuating magnetic field is physically plausible and the simplicity with which it is able to explain the observed standard scaling laws as discussed in a previous section is remarkable, we feel obligated to propose this possibility as an empirical hypothesis for further observational and theoretical investigations.

4. DISCUSSION

4.1. $\sigma_B^2$ versus $B$

A fluctuating magnetic field component not only is implied by the observed “turbulence” or hydromagnetic waves (Alfvén 1953; AM75) but is also expected to explain why molecular clouds do not free-fall to the center of mass along the generally ordered magnetic field lines (cf. Shu, Adams, & Lizano 1987). Shu (1991) even suggested the intriguing possibility that an anisotropic fluctuating magnetic field might explain the reported prolateness of low-mass cores. The fluctuating magnetic field $\sigma_B^2$ is, nevertheless, conceptually different from the general magnetic field $B$, even in the extreme case that $\sigma_B^2 \sim B$. This raises the issue as for if the standard scaling laws require a constant general magnetic field $B$ over a large range of scales, as some have discussed (e.g., Pellegratti Franco et al. 1985; Scalo 1987; MG88; Fleck 1988). The answer to this question is negative on the basis of the above discussions, but a constant $B$ would not contradict the scaling laws. A key point is that a constant $B$ does not directly explain the standard scaling laws without invoking $\sigma_B^2$ first, because the Alfvén velocity is not the same as the observed fluctuating turbulent velocity amplitudes. In other words, even if $B$ is a constant over a large range of scales, a separate physical mechanism would still have to be sought to produce a scale-independent $\sigma_B^2$. In practice, however, it is likely that $\sigma_B^2$ becomes comparable to $B$ on large scales. From the empirical $B-n$ relation $B \approx 1.5(n_{H_2}/1\text{ cm}^{-3})^{1/2} \mu G$ (Heiles et al. 1993) and the estimated $\sigma_B \approx 24 \mu G$ (see §2), it can be crudely estimated that $\sigma_B$ becomes comparable to $B$ when $n_{H_2} \approx 260 \text{ cm}^{-3}$, $\sigma_v \approx 2.3 \text{ km s}^{-1}$, and $R \approx 10$ pc. Taking this at its face value, it seems that $\sigma_B$ could be well below $B$, or, equivalently, $\sigma_B$ considerably smaller than the Alfvén velocity on scales smaller than a few parsecs.

4.2. Predictions

Generally speaking, our principal expectation is that regions where the spatial distribution of the turbulent energy density is predominantly determined by wave-wave interactions are more likely to demonstrate the standard scaling laws, while regions where this is not the case are unlikely to do so. Perhaps this is why the recent observational results (Plume et al. 1997) of massive star-forming cores do not agree with the standard scaling laws, as localized energy sources might dominate the energy density distributions over the scales of massive star formation. Further, the velocity-density relation (eq. [3]) among the standard scaling laws should be the most robust physical relation in the sense that it may be expected for any region where magnetic fluctuations become scale-independent. In contrast, the standard density-size and velocity-size relations require not only scale-independent magnetic field fluctuations but also a second physical relationship such as the virial equilibrium.2 This prediction seems supported by the observations of Falgarone et al. (1992), which find a remarkable correlation $\sigma_v \propto n_{H_2}^{0.5}$ for a sample of gravitationally unbound molecular structures, although it is possible that jump shocks may also give rise to this relation (J. Scalo 1996, private communication).

4.3. Comparison with Existing One-dimensional and Two-dimensional MHD Simulations

Our hypothesis for a scale-independent fluctuating magnetic field, however, appears to contradict the recent one-dimensional and two-dimensional computer simulations for strong compressible MHD turbulence in molecular clouds. These simulations report a fluctuating magnetic spectrum (energy per unit volume per unit wavenumber), $P_s \propto k^{-t}$, where $t \approx 2.0–2.3$ for one-dimensional (Gammie & Ostriker 1996) and $t \approx 1.85$ for two-dimensional (Passot et al. 1995). This mag-

\[^1\]Heithausen (1996) has recently reported a remarkable correlation $n \propto R^{-0.8}$ for high-latitude clouds, which made him question whether virial equilibrium is the true reason for an $n \propto R^{-1}$ relation. It is not clear yet, however, whether his assuming equal distance for all the high-latitude clouds would yield the correlations artificially (Leisawitz 1990).
scale-independent fluctuating magnetic field in molecular clouds remain a challenge to MHD simulations, or vice versa. We share the hope with the solar physicists that future three-dimensional MHD simulations will shed light on this important issue in MHD turbulence.

In conclusion, we have proposed the hypothesis that strong wave-wave interactions have led to a scale-independent distribution of fluctuating magnetic energy density and consequently the observed scaling laws in molecular clouds. We suggest that further careful observational studies of interstellar atomic/molecular clouds not in virial equilibrium or not harboring active star-forming activities may serve as valuable direct tests of the basic ideas presented in this article, especially in comparison to the contradicting predictions of recent one-dimensional and two-dimensional computer simulations.

The author gratefully acknowledges useful discussions with Professors Chuan-Sheng Liu, Paul Goldsmith, and Stuart Vogel, and their kind encouragement. He is grateful to John Scalo for a critical review of the manuscript and numerous stimulating discussions. He thanks John Wang, Frank Shu, Pedro Safier, Neal Evans, Paul Goldsmith, Phil Myers, Alyssa Goodman, Bruce Elmegreen, Dick Crutcher, Arieh Königl, Richard Larson, E. Vázquez-Semadeni, Zhiyun Li, and A. Bhattacharjee for useful discussions. This research is supported in part by NSF grant AST9314847 to the Laboratory for Millimeter-Wave Astronomy at the University of Maryland.

REFERENCES

### 1. Alfvén, H. 1953, Cosmical Electrodynamic (Oxford: Oxford Univ. Press), 146

### 2. Arons, J., & Max, C. E. 1975, ApJ, 196, L77 (AM75)

### 3. Biskamp, D. 1994, Phys. Rev. E, 50, 2702

### 4. Blitz, L. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 125

### 5. Caselli, P., & Myers, P. C. 1995, ApJ, 446, 665

### 6. Chieze, J.-P. 1987, A&A, 171, 223

### 7. Crutcher, R. M., Mouschovias, T. Ch., Troland, T. H., & Ciolek, G. E. 1994, ApJ, 427, 839

### 8. Crutcher, R. M., Troland, T. H., Goodman, A. A., Heiles, C., Kazés, I., & Myers, P. C. 1993, ApJ, 407, 175

### 9. Dame, T., Elmegreen, B., Cohen, R., & Thaddeus, P. 1986, ApJ, 305, 892

### 10. Elmegreen, B. G. 1989, ApJ, 338, 178

### 11. Falgarone, E., Puget, J.-L., & Pérault, M. 1992, A&A, 257, 715

### 12. Fleck, R. C. 1988, ApJ, 328, 299

### 13. Fuller, G. A., & Myers, P. C. 1992, ApJ, 584, 523

### 14. Gammie, C. F., & Ostriker, E. C. 1996, ApJ, 466, 814

### 15. Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763

### 16. Heiles, C., Goodman, A. A., McKee, C. F., & Zweibel, E. G. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 279

### 17. Heithausen, A. 1996, A&A, 314, 251

### 18. Henriksen, R. N., & Turner, B. E. 1984, ApJ, 287, 200

### 19. Kegel, W. H. 1989, A&A, 235, 517

### 20. Kleiner, N., Mond, M., & Rogachevskii, I. 1996, A&A, 307, 293

### 21. Larson, R. B. 1981, MNRAS, 194, 809

### 22. Lau, Y.-T., & Siregar, E. 1996, ApJ, 465, 451

### 23. Lehnarts, D. 1990, ApJ, 359, 319

### 24. Leung, C. M., Kuttner, M. L., & Mead, K. N. 1982, ApJ, 262, 583

### 25. Maloney, P. 1988, ApJ, 334, 761

### 26. Marsch, E. 1991, in Physics of the Inner Heliosphere: Particles, Waves and Turbulence, ed. R. Schwenn & E. Marsch (New York: Springer), 159

### 27. McKee, C. F. 1989, ApJ, 345, 782

### 28. McKee, C. F., & Zweibel, E. G. 1995, ApJ, 440, 686

### 29. McKee, C. F., Zweibel, E. G., Goodman, A. A., & Heiles, C. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 327

### 30. McKee, C. F., & Zweibel, E. G. 1999, ApJ, 485, 698

### 31. Mouschovias, T. Ch. 1987, in Physical Processes in Interstellar Clouds, ed. G. E. Morfill & M. Scholer (Dordrecht: Reidel), 453

### 32. Mouschovias, T. Ch., & Psaltis, D. 1995, ApJ, 444, L105

### 33. Myers, P. C. 1989, ApJ, 345, 782

### 34. Myers, P. C., & Goodman, A. A. 1988, ApJ, 329, 392 (MG88)

### 35. Passot, T., Vázquez-Semadeni, E., & Pouquet, A. 1995, ApJ, 445, 536

### 36. Pellegrati Franco, G., Tarsia, R. D., & Quiroga, R. J. 1985, ApSS, 111, 343

### 37. Plume, R., Jaffe, D. T., Evans II, N. J., Martin-Pintado, J., & Gomez-Gonzalez, J. 1997, ApJ, 476, 730

### 38. Pouquet, A., Frisch, U., & Léorat, J. 1976, J. Fluid Mech., 77, 321

### 39. Sagdeev, R. Z., & Galeev, A. A. 1969, Nonlinear Plasma Theory (New York: Benjamin)

### 40. Scalo, J. M. 1987, in Interstellar Processes, ed. D. J. Hollenbach & H. A. Thronson, Jr. (Dordrecht: Reidel), 349

### 41. ———. 1990, in Physical Processes in Fragmentation and Star Formation, ed. R. Capuzzo-Dolcetta, C. Chiòisi, & A. di Fazio (Dordrecht: Kluwer), 151

### 42. Scoville, N. Z., & Sanders, D. B. 1987, in Interstellar Processes, ed. D. J. Hollenbach & H. A. Thronson, Jr. (Dordrecht: Reidel), 21

### 43. Shu, F. H. 1991, in The Physics of Star Formation and Early Stellar Evolution, ed. C. J. Lada & N. D. Kylafis (Dordrecht: Kluwer), 365

### 44. Shu, F. H., Adams, F. C., & Lizano, S. 1987, ARA&A, 25, 23

### 45. Solomon, P. M., Rivolo, A. R., Barrett, J., & Yahil, A. 1987, ApJ, 319, 730

### 46. Troland, T. H., & Heiles, C. 1986, ApJ, 301, 339

### 47. Tu, C.-Y., & Marsch, E. 1995, Space Sci. Rev., 73, 1

### 48. Vázquez-Semadeni, E., Ballesteros-Paredes, J., & Rodríguez, L. F. 1997, ApJ, 474, 292

### 49. Zweibel, E. G., & Josafatsson, K. 1983, ApJ, 270, 511

### 50. Zweibel, E. G., & McKee, C. F. 1995, ApJ, 439, 779