Cosmological Vacuum in Unified Theories

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Abstract

The unification of the Einstein theory of gravity with a conformal invariant version of the standard model for electroweak interaction without the Higgs potential is considered. In this theory, a module of the Higgs field is absorbed by the scale factor component of metric so that the evolution of the Universe and the elementary particle masses have one and the same cosmological origin and the flat space limit corresponds to the $\sigma$-model version of the standard model. The red shift formula and Hubble law are obtained under the assumption of homogeneous matter distribution. We show that the considered theory leads to a very small vacuum density of the Higgs field $\rho_{\phi}^{\text{cosmic}} = 10^{-34}\rho_{cr}$ in contrast with the theory with the Higgs potential $\rho_{\phi}^{\text{Higgs}} = 10^{54}\rho_{cr}$.

1. Introduction

The homogeneous scalar field, generating elementary particle masses in the standard model (SM) for electroweak interactions, is based on the Higgs potential. The physical motivation for this potential as a consequence of the first symmetry principles is unclear and there are a number of difficulties in both cosmology (great vacuum density [1], monopole creation [2], domain walls [3]) and the standard model (breakdown of perturbation theory for the expected values of the Higgs mass $m_H \sim 1TeV$ [4, 5, 6]). The present talk is devoted to a unification of Einstein’s theory of gravity with the conformally invariant version of the standard model for electroweak interactions (without the Higgs potential) and to the cosmological consequences of this theory.

2. Model

We consider Einstein’s theory supplemented by the conformal invariant part of SM. The conformal symmetry principle switches off the Higgs potential and introduces the Penrose-Chernikov-Tagirov term $\mathcal{W}$ so that the initial action for our consideration consists of two parts

$$W_{\text{tot}} = W_{SM}^c(g, \phi, W, Z, A, e, \nu) + W_E(g, \phi). \quad (1)$$

The first part $W_{SM}^c$ represents the conformal invariant part of SM and it depends on the entirety of fields with conformal weights ($n$). The second part of the action $W_E(g, \phi)$ represents the Einstein-Hilbert action with the conformally coupled scalar field $\phi$

$$W_E(g, \phi) = \int dt dx^3 \sqrt{-g} \left[ -\frac{1}{6} R(g) (\mu^2 - |\phi|^2) + \partial_\mu |\phi| \partial^\mu |\phi| \right]; \quad (\mu^2 = \frac{3M_{Pl}^2}{8\pi}). \quad (2)$$
We express the total action (1) in terms of the conformal invariant variables \((n) F_c = (n) F a^{-n}\) extracting the space-scale factor \(a = \sqrt[3]{g}^{1/6}; \quad (g_{\mu\nu} = a^2 g^{(c)}_{\mu\nu})\). This choice of variables can be justified by the principle of the conformal invariance of dynamical variables [8].

The action \(W^{c}_{SM}\) does not depend on the scale variable \(a\) due to the conformal invariance. The action \(W_E\) has the symmetric form with respect to the scale factor \(\bar{a} = \mu a\) and the scalar field \(\phi_c\) with \(\sqrt{-g^{(c)}} = N_c\).

\[
W_E = \int dx^4 N_c \left[ -\frac{(4) R(g^{(c)})}{6} (\bar{a}^2 - \phi_{c}^2) + \partial_{\mu} \phi_c \partial^{\mu} \phi_c - \partial_{\mu} \bar{a} \partial^{\mu} \bar{a} \right].
\]  

3. Absorption of the Higgs field by the space metric

The class of physical solutions for the theory \(W\) should be restricted by the condition
\[
(\bar{a}^2 - \phi_{c}^2) \geq 0,
\]  
in the whole four-dimensional space. In the opposite case, the Einstein action changes sign with respect to the matter one, and gravitation converts into antigravitation with a wrong sign of the Newton interaction and negative energy for gravitons. The rough analogy of this restriction is the light cone in special relativity which defines the physically admissible region of the particle motion. The restriction \(\bar{a} = 0, \phi_c = 0\) leads to the symmetric initial data \(\bar{a} = 0, \phi_c = 0\). The symmetric equations (action) and symmetric initial data can lead only to a symmetric class of solutions of the equations of motion and constraints \(\phi_c = \pm \bar{a}\). This solution can be treated as dynamical absorption of the conformal scalar field by the scale factor component of the metric. In terms of the initial scalar field module \(|\phi| = \phi_c / a\) we have only the vacuum value \(|\phi| = \mu\), in contrast with the decomposition of the Higgs field in the potential model. We got accustomed to the decomposition of a scalar field over plane waves treating them as particle-like excitations. In fact, a correct decomposition includes (in addition to plane waves) the zero-mode sector. Here, we face the case when the scalar field looses its particle-like excitations and has only the zero-mode sector formed by the scale factor.

4. Cosmology

In the vicinity of the beginning of the Universe \(\phi_c = \pm \bar{a} = 0\) the total action \(W\) describes only a set of SM massless fields (i.e. radiation). At first, we restrict our consideration to the harmonic excitations of these fields, in the FRW metric. The consistent classical and quantum descriptions of the Universe filled in by these massless harmonic excitations implies two stages: dynamical (D) and geometrical (G) [8].

(D) At the dynamical stage, parameters of the time-reparametrization transformations are completely separated from the sector of physical variables as a result of which the scale factor \(a\) converts into the invariant conformal time of evolution of the Universe [8] and leaves the set of independent physical variables of the theory.

(G) The geometrical stage is the transition to the Friedmann comoving frame of reference connected with the massive dust. The Friedmann observables, in the comoving frame of reference, are constructed by conformal transformation of the dynamical (conformal) variables and coordinates including the Friedmann time interval and distance [8] \(dt_F = a d\eta; D_F = a D_c\).
In the case considered, the Einstein-Friedmann equation \( \delta W^H / \delta N^0_c = 0 \) represents the sum of energy densities of the scale factor \((\rho_{cr})\), scalar field \((\phi)\), and "radiation" \((R)\)

\[-\rho_{cr} + \rho_0^0 + \rho_R = 0.\]

The evolution of the cosmic scale \( a(t_F) \) coincides with the one of the Friedmann Universe filled by radiation. The scalar field \( \phi_c \) repeats this evolution, while the initial scalar field \( |\phi| = \phi_0 / a \) is equal to a constant. The value of this scalar field, which follows from the Weinberg-Salam theory \( \sqrt{g^2/2} |\phi| = m_W \sim 10^2 GeV \), allows us to estimate the value of the relation of energy densities of the scalar field \( (\rho_0^0) \) and the expansion of the Universe \( (\rho_{cr}) \):

\[ \rho_{\phi}^{\text{cosmic}} \sim 10^{-34} \rho_{cr}. \]

Recall that the Higgs potential leads to the opposite situation (see [1])

\[ \rho_{\phi}^{\text{Higgs}} \sim 10^{54} \rho_{cr}. \]

When masses of the SM elementary particles (determined by \( \phi_c(\eta) \)) become greater than their momenta, we should include additional terms in the energy density of the scalar field of the type of \( \rho_\phi = \rho_0^0 - \phi_c < n_f > + \phi_c^2 < n_b^2 > \) associated with the fermion and boson "dusts" at rest. Here \( \rho_\phi, < n_f > \) and \( < n_b^2 > \) are phenomenological parameters which determine the solution to the homogeneous scalar field equations. For the case considered, we have obtained the oscillator-like solution for the conformal scalar field

\[ \phi_c(\eta) = \rho_0^{1/2} \omega_\phi^{-1} \sin(\omega_\phi \eta) + \frac{1}{2} < n_f > \omega_\phi^{-2} (1 - \cos(\omega_\phi \eta)), \quad (\omega_\phi^2 = 1/r_0^2 + < n_b^2 >). \] (5)

If the dust term dominates, the SM-particle masses \( (\phi_c/a) \) become dependent on time. A photon radiated by an atom on an astronomical object (with a distance \( D \) to the Earth) at the time \( t_F - D \) remembers the value of this mass at this time. As a result, the red shift is defined by the product of two factors: the expansion of the Universe space \( (a) \) and the alteration of the elementary particle masses \( (\phi_c/a) \). Finally, we get the red shift \( Z \) and the Hubble law

\[ Z(D) = \frac{\phi_c(t_F)}{\phi_c(t_F - D)} - 1; \quad H_0 = \frac{d\phi_c}{\phi_c dt_F}. \] (6)

Thus, in the theory considered, one and the same function \( (\phi_c) \) describes both the expansion of the Universe and masses of elementary particles. In the flat-space limit \( (4)^R(g) = 0 \), we get the \( \sigma \)-model version of SM without Higgs particles, which is discussed now in order to remove the difficulty of large coupling constant in the Higgs sector.

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