Reliability assessment method for environmental vibrations of a tunnel embedded in layered foundation

Yonggang Xu1, Huiji Guo2*, Honggui Di2

1 Ningbo Rail Transportation Group Co. Ltd., Ningbo 315012, China
2 Key Laboratory of Road and Traffic Engineering of the Ministry of Education, Tongji University, Shanghai 201804, China
*Corresponding author’s e-mail: guohuiji@tongji.edu.cn

Abstract. This paper proposes a reliability assessment method for environmental vibrations of a tunnel embedded in layered foundation under moving train load. Based on wave functions transformation, a vehicle-track-tunnel-soil model for evaluating system dynamic response can be established. Then combined with the Monte Carlo simulation, the reliability assessment of environmental vibrations is realized. The effect of soil parameter uncertainty on the prediction result of environmental vibrations is analyzed via case studies. The results show that the predictions from the deterministic model tend to be conservative, which is less than the superior limit of 95% confidence interval of the uncertainty model predicted outcome. The soil parameter uncertainty has less influence on the distribution of system dynamic response, which mainly affects the system dynamic response amplitude.

1. Introduction
Underground railways play an important role in alleviating the congestion pressure of urban public traffic. As the underground railways are often constructed in densely populated and heavily constructed urban areas, train-induced environmental vibrations have received more and more attention. Therefore the models that can accurately predict environmental vibrations are required[1].

Related scholars have established a series of models for predicting environmental vibrations, mainly including numerical models and analytical (semi-analytical) models [2]. Numerical models mainly include 3D(dimensional) FE(finite element) model [3], 2.5D FE model [4], FE-BE (boundary element) model [5-6], periodic FE-BE model [7], and finite element- infinite element models [8]. Numerical models have the advantage that complex boundary conditions can be considered, but they need a lot of computer resources and computing time, which is not suitable for performing lots of random operations. In recent years, many scholars have proposed analytical (semi-analytical) methods to predict environmental vibrations, including the Eulerian beam model [9], Pip in pip model [10-11], wave surface transformation method model [12-13], and dynamic substructure model in the time domain [14]. Compared with the numerical model, the analytical (semi-analytical) method greatly improves the computational prediction efficiency, which can meet the demands of a large number of calculations. However, the models motioned above cannot consider the effects of system parameters' randomness and variation. There are multiple substructures and numerous stochastic factors involved in the prediction process of environmental vibrations. Based on the existing prediction models, some scholars have analysed the influence of random factors, such as random load and rail irregularity [15-17], on predict results. Foundation soil is the main medium for the propagation of environmental
vibrations, which has great parameter variability due to the influence of deposition sequence and loading history [18]. However, few studies have been reported on the influence of soil parameter uncertainty on predict results. In this study, the vehicle-track-tunnel-soil model is established by wave functions transformation. Then combined with Monte Carlo simulation, the system dynamic response reliability assessment is realized.

2. Deterministic model

The tunnel is modelled as a Flügge cylindrical shell. The foundation soil is modelled as unsaturated layered media, as shown in Fig. 1(a).

![Fig. 1 Foundation soil-tunnel system and coordinate](image)

The vibrational property of foundation soil is described using the unsaturated wave equation[11]:

\[
\begin{align*}
\mu \nabla^2 u + (\lambda + \mu) \nabla (\nabla \cdot u) + M \nabla (\nabla \cdot v) + N \nabla (\nabla \cdot w) &= \rho u' + \rho_i v' + \rho_s w' \\
h_{11} \nabla (\nabla \cdot u) + h_{12} \nabla (\nabla \cdot v) + h_{23} \nabla (\nabla \cdot w) &= \rho u' + \rho_i v' + \rho_s w' \\
h_{21} \nabla (\nabla \cdot u) + h_{22} \nabla (\nabla \cdot v) + h_{23} \nabla (\nabla \cdot w) &= \rho_s u' + \rho_i v' + \rho_s w' \\
h_{23} \nabla (\nabla \cdot u) + h_{22} \nabla (\nabla \cdot v) + h_{23} \nabla (\nabla \cdot w) &= \rho_s u' + \rho_i v' + \rho_s w' \\
\end{align*}
\]

(1)

where the expression and physical significance of each variable can be found in Ref.[13].

The equilibrium relationships of the Flügge cylindrical shell is given by [10]:

\[
\begin{align*}
\frac{\partial u}{\partial z} + \frac{1}{d_i} \frac{\partial u}{\partial \theta} + \frac{d_z^2}{12} \left[ \frac{\partial^4 u}{\partial z^4} + \frac{2}{d_i^2} \frac{\partial^3 u}{\partial z^3 \partial \theta} + \frac{1}{d_i^4} \frac{\partial^2 u}{\partial z^2 \partial \theta^2} \right] - \frac{(1 - \nu) \partial^2 u}{2d_i^2 \partial z \partial \theta^2} - d_i \frac{(1 - \nu)}{Ed} q_i + \frac{\rho_i d_i (1 - \nu)^2}{E} \frac{\partial^2 u}{\partial t^2} = 0 \\
\frac{3(1 - \nu) \partial^3 u}{2d_i^2 \partial z^2 \partial \theta} + \frac{d_i}{2} \frac{\partial u}{\partial \theta} + \frac{(1 - \nu) \partial^2 u}{d_i^2 \partial \theta^2} + d_i \frac{(1 - \nu)}{Ed} \frac{\partial^2 u}{\partial t^2} - \frac{3(1 - \nu) \partial^3 u}{2d_i^2 \partial z^2 \partial \theta} + \frac{d_i}{2} \frac{\partial u}{\partial \theta} + \frac{(1 - \nu) \partial^2 u}{d_i^2 \partial \theta^2} + d_i \frac{(1 - \nu)}{Ed} \frac{\partial^2 u}{\partial t^2} = 0 \\
\frac{3(1 - \nu) \partial^3 u}{2d_i^2 \partial z^2 \partial \theta} + d_i \frac{(1 - \nu)}{Ed} \frac{\partial^2 u}{\partial t^2} - \frac{\rho_i d_i (1 - \nu)^2}{E} \frac{\partial^2 u}{\partial t^2} = 0 \\
d_i \frac{\partial^2 u}{\partial z^2} + \frac{(1 - \nu) \partial^2 u}{2d_i^2 \partial \theta^2} + d_i \frac{(1 - \nu) \partial^2 u}{2d_i^2 \partial z \partial \theta} + \frac{(1 - \nu) \partial^2 u}{12} \frac{\partial^2 u}{\partial z^2 \partial \theta^2} + \frac{(1 - \nu) \partial^2 u}{2d_i^2 \partial z \partial \theta^2} + d_i \frac{(1 - \nu)}{Ed} \frac{\partial^2 u}{\partial t^2} = 0 \\
\end{align*}
\]

(2)

where the expression and physical significance of each variable can be found in Ref.[10].
Next, the structures of vehicles and tracks should be considered. The infinite rail and track slab are simplified as Euler–Bernoulli beams, and the track slabs are coupled to the tunnel via one line of uniform supports, as shown in Fig.1(b).

The train load can be written as:

\[
F = \sum_{n=1}^{\infty} F_n(x-\nu t) e^{i\omega t}
\]

\[
F_n(x-\nu t) = P_{nt_1} \delta(x-\nu t + \sum_{k=1}^{n-1} l_k + l_0) + P_{nt_2} \delta(x-\nu t + w_a + \sum_{k=1}^{n-1} l_k + l_0) + P_{nt_3} \delta(x-\nu t + w_b + \sum_{k=1}^{n-1} l_k + l_0) + P_{nt_4} \delta(x-\nu t + 2w_a + w_b + \sum_{k=1}^{n-1} l_k + l_0)
\]

where the expression and physical significance of each variable can be found in Ref.[10].

The governing equation of the rail beam and track slab beam can be obtained:

\[
\begin{align*}
E_{r} I_{r} \frac{\partial^4 u_{rail}}{\partial x^4} + m_r \frac{\partial^2 u_{rail}}{\partial t^2} + k_r (u_{rail} - u_{slab}) + c_r \left( \frac{\partial u_{rail}}{\partial t} - \frac{\partial u_{slab}}{\partial t} \right) &= f \\
E_{s} I_{s} \frac{\partial^2 u_{slab}}{\partial x^2} + m_s \frac{\partial^2 u_{slab}}{\partial t^2} - k_s (u_{rail} - u_{slab}) - c_s \left( \frac{\partial u_{rail}}{\partial t} - \frac{\partial u_{slab}}{\partial t} \right) + G &= 0
\end{align*}
\]

where the expression and physical significance of each variable can be found in Ref.[10].

Based on wave functions transformation, the rail, track slab, tunnel, and soil medium are coupled by the compatibility of force and displacement at the contact surface, and the vehicle-track-tunnel-soil model for evaluating system dynamic response can be established. The specific boundary assumption and model solving process can be found in Ref.[10,13].

3. Numerical results and discussion

3.1. Validation of the proposed model

To validate the proposed model, the results obtained by the proposed model are compared with those obtained by the existing 2.5D FE–BE model [18]. The parameters for the model are those given in Ref.[20]. The displacement of the points (20m,0m,0m) on the ground surface has been taken for comparison, as shown in Fig.2.

![Fig. 2 Verification result of the method](image-url)
3.2. Reliability assessment

The calculation parameters of the coupling model refer to the existing Ref. [13,19], as shown in Table 1.

| Structure | Parameter | Value | Structure | Parameter | Value |
|-----------|-----------|-------|-----------|-----------|-------|
| Soil      | $K_g$ (kPa) | 100   | Tunnel line | $h_l$ (m) | 10    |
|           | $\rho_s$ (kg·m$^{-3}$) | 1.29 |               | $\beta_l$ | 0.03  |
|           | $\eta_s$ (Pa·s) | $1.5 \times 10^4$ |               |               |       |
|           | $K_l$ (GPa) | 2     | Rail        | $E_{f_l}$ (Pa·m$^4$) | $1 \times 10^7$ |
|           | $\rho_l$ (kg·m$^{-3}$) | 1000  |               | $m_l$ (kg·m$^{-1}$) | 100   |
|           | $\eta_l$ (Pa·s) | $1 \times 10^{-3}$ |               | $k_l$ (N/m$^2$) | $4 \times 10^7$ |
|           | $K_s$ (GPa) | 36    | Slab        | $c_r$ (N·s/m$^2$) | $6.3 \times 10^3$ |
|           | $\rho_s$ (kg·m$^{-3}$) | 2650  |               | $\beta_l$ | 0.02  |
|           | $S_r$ | 0.9   |               |               |       |
|           | $S_{r0}$ | 0.05  |               |               |       |
|           | $\gamma$ | $S_r$ |               |               |       |
|           | $\beta_s$ | 0.04  |               |               |       |
|           | $\kappa$ | $1 \times 10^{-4}$ |               |               |       |
|           | $[\alpha_1, \alpha_2, \alpha_3]$ | $[5 \times 10^{-4}, 0.5, 2]$ |               |               |       |
| Tunnel line | $E$ (Pa) | $5 \times 10^{10}$ | Carriage | $P$ (kN) | 80    |
|           | $\nu$ | 0.3   |               | $w_s$ (m) | 2.5   |
|           | $\rho_l$ (kg·m$^{-3}$) | 2500  |               | $w_b$ (m) | 13.2  |
|           | $d_l$ (m) | 3     |               | $h_b$ (m) | 22.8  |
|           | $d_s$ (m) | 0.25  |               | $l_s$ (m) | 0     |

Note. the meaning of the variables in the table can be found in Ref. [13,19]

On the basis of the above deterministic model, the influence of soil parameter uncertainty on prediction results of environmental vibrations is considered. In this paper, three key parameters of soil are selected [21], including $K_b$ bulk modulus, $n$ void ratio, and $\phi$ internal friction angle, as shown in Table 2.

| Parameter | mean value | standard deviation | Coefficient of variation | Distribution type |
|-----------|------------|---------------------|--------------------------|------------------|
| $K_b$ (MPa) | 36.8 | 5.152 | 0.14 | normal |
| $n$ | 0.4 | 0.024 | 0.06 | normal |
| $\phi$ ($^\circ$) | 27 | 0.81 | 0.03 | normal |

Monte Carlo simulation is used to analyze the influence of soil parameter uncertainty on the prediction results. Figure 4 shows the distribution of vibration displacement amplitude and vibration acceleration amplitude on the ground surface ($x=9$ m) directly above the tunnel axis ($y=0$ m), when the train moves on the tunnel with burial depth $D$ (the diameter of the tunnel) at a speed of 60 km/h. From
Fig. 4, it can be obtained that the soil parameter uncertainty mainly affects the dynamic response amplitude, and its influence on the distribution law of the surface dynamic response is small.

![Fig. 3 Calculated results of surface dynamic response distribution](image)

From the “Urban Regional Ambient Vibration Standards” (GB 10070-88) [22], it is known that the vibration acceleration level can be used as the reference value for environmental vibrations evaluation. Therefore, the following analysis focuses on the vibration acceleration levels, which is defined as:

$$\text{VAL} = 20\log \frac{|a|}{a_0}$$

(5)

where, $a$ and $a_0$ represent the vibration acceleration and the reference acceleration value respectively. $a_0=10^{-6}\text{m/s}^2$.

Figure 5 shows the results of statistical analysis of vibration acceleration levels at the surface observation points (9m, 0m, 0m) and (9m, 0m, 68.4m). The statistical results show that the ground vibration acceleration levels at different locations follow a normal distribution.

![Fig. 4 Statistical results of ground vibration acceleration levels](image)

Monte Carlo simulation is repeated to calculate the environmental vibrations response of the train moving at a different speed (60 km⋅h$^{-1}$, 70 km⋅h$^{-1}$, 80 km⋅h$^{-1}$, 90 km⋅h$^{-1}$, and 100 km⋅h$^{-1}$) for different tunnel burial depths ($h_t$ = 6m, 9m, 12m). The maximum vibration acceleration level at the surface observation point ($h_t$,0m,0m) is analyzed statistically. Figure 6 shows the interval scatter plots with 95% confidence intervals for each case. It can be seen that the vibration acceleration level increases with the increase of train movement speed and decreases with the increase of tunnel burial depth. From the statistical analysis, the coefficients of variation of vibration acceleration levels $h_t$ = 6m, $h_t$ = 9m and $h_t$ = 12m are 0.049, 0.038, and 0.028, respectively. This indicates that the foundation soil acts as the main propagation medium for the dynamic response, and the greater the tunnel burial depth, the greater the influence of the soil parameter uncertainty on the surface environmental vibrations. In addition, comparing the calculation results with those of the deterministic model, it is found that the upper limit
of 95% confidence interval of the calculation results is larger than that of the prediction results of the deterministic model. This phenomenon indicates that the prediction results of the deterministic model are on the conservative side, and the influence of soil parameter uncertainty on the prediction results of environmental vibrations should be considered.

![Fig. 5 The interval of 95% confidence interval of ground vibration acceleration level](image)

### 4. Conclusions

1. Based on wave functions transformation, the vehicle-track-tunnel-soil model for evaluating system dynamic response can be established. The reliability of the deterministic model has been verified by comparing with the existing 2.5D FE-BE model. Based on the Monte Carlo simulation, the analysis of the effect of soil parameter uncertainty on environmental vibrations has been realized.

2. The foundation soil acts as the main propagation medium for the dynamic response, and the greater the tunnel burial depth, the greater the influence of the soil parameter uncertainty on the surface environmental vibrations. The soil parameter uncertainty mainly affects the dynamic response amplitude, and its influence on the distribution law of the surface dynamic response is small.

3. The upper limit of the 95% confidence interval of the calculated results of the uncertainty model is larger than that of the prediction results of the deterministic model, which indicates that the prediction results of the deterministic model are on the conservative side, and the influence of soil parameter uncertainty needs to be considered for environmental vibrations prediction.

### Acknowledgments

The study on which this paper is based was supported by the National Natural Science Foundation of China through the Grant No. 51808405, and the Natural Science Foundation of Shanghai through grant No. 20ZR1459900

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