Determination of optimal quantities of different types of driers for addition in the batches of paint formulation

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Abstract

Paint is generally considered as a mixture of pigment, binder (vehicle), solvent, driers and other additives. Though the basic function of driers is to decrease the drying time after application, it significantly affects the gloss and clarity of paint coatings too. In a batch of manufactured paint usually three kinds of driers (surface drier, through drier and auxiliary drier) are used. An Indian paint manufacturing company identifies that determination of optimal quantities for different types of driers for addition in a batch of solvent-based paint production is a major quality problem for the organization. This paper describes how the management of the company resolved the quality problem by carrying out properly planned statistical studies. Using rotatable central composite design (RCCD), response surface methodology (RSM) and goal programming (GP) technique optimal quantities of the three types of driers for a batch are determined and implemented. The results show that the paint quality improved substantially.

Keywords: Rotatable central composite design, Response surface methodology, Multiple response optimization, Goal programming

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1. Introduction

The paint and coatings industry produces a huge variety of products that protect, preserve, and also beautify the objects to which they are applied. Typical products include architectural coatings (e.g. house paints), industrial coatings (e.g. automotive finishes, wood furniture and fixture finishes), and special purpose coatings (e.g. traffic paints, roof coatings) (Weiss, 1997). Paint is generally considered as a mixture of pigment, binder (vehicle), solvent, driers and other additives. Driers and other additives are added in small concentrations (0.2-10%). The pigment provides the coating with colour, opacity and a degree of durability. Binders are components which form a continuous phase, hold the finely ground pigment in the dry film and cause it to adhere to the surface to be coated. For viscosity adjustment, solvents are required. Driers and other additives (thickeners, antifoams, dispersing agents, catalysts etc.) perform a special function or give a certain property to the coating. The basic purpose of driers is to hasten drying of paint after application. The drying of alkyd paint is essentially an autoxidation process catalysed by transition metal salts. Rotstein et al. (1998) have considered the problem of selecting appropriate technologies and capabilities for the design of paint manufacturing plants, and have provided an optimal manufacturing strategy. Overbeek et al. (2003) have described a new technology to achieve prolonged open time and wet edge time in a water-borne paint. Gunde et al. (2003) have studied the influence of state of the dispersion of paint on optical properties of coatings. Adonyi et al. (2008) have proposed a framework for effective scheduling of a large-scale paint production system. Karakas and Celik (2012) have studied the effect of quantity and size distribution of calcite filler on the quantity of water borne paints.

Driers are typically metal soaps of carboxylic acids. Driers function via a homogenous catalysis pathway and are typically deactivated when phase separation of the catalyst occurs, and the drying process is then inhibited (Gorkum and Bouwman, 2005). Driers are generally classified by function namely surface drier, through drier and auxiliary drier. Surface driers induce drying
from the top down. Through driers promote curing beneath the surface of the coating and auxiliary driers inhibit phase separation of the surface and through driers from the polymeric phase (Soucek et al., 2012). The most commonly used surface driers are octoates of cobalt and manganese; through driers are octoates of lead, alluminium and zirconium; and auxiliary driers are octoates of calcium, potassium and lithium. The autoxidative cured coatings usually have a mixture of driers consisting of surface, through and auxiliary driers (Gorkum and Bouwman, 2005). Although the basic function of driers is to decrease the drying time after application, driers have a considerable influence on the final properties of the coating (Soucek et al., 2012). In addition to the drying time, driers also affect drying quality, gloss and clarity of the coated surface. Whereas gloss and clarity are the measurable quality characteristics, drying quality is an ordered categorical quality characteristics. The gloss is measured by determining the amount of reflected light given off a painted surface after 48 hours whereas clarity is assessed by measuring turbidity of clear liquid in the paint surface after 48 hours. On the other hand, the drying process takes place by rapidly adding consecutive layers of paint and the extent of drying taken place in 24 hours, assessed subjectively, is considered as the drying quality.

In recent years, substantial amount of research have been carried out aiming to understanding the chemical mechanism of drying/autoxidation process, identifying environmental friendly driers and increasing efficiency of driers. Hein (1999) has presented several drier systems that are suitable for high-solids coatings, and discussed the advantages and disadvantages of various drier systems. Weissenborn and Motiejauskaite (2000) have found that the main reason for loss of dry upon storage is adsorption of the cobalt (drier) on pigment surfaces as a precipitated hydroxide. Deffar and Soucek (2001) have found that synergy exists between the driers and sol-gel precursors with respect to the autoxidation process. Mallegol et al. (2002) have observed that more uniform crosslinking can be achieved by using a combination of surface and through driers. Oyman (2002) have investigated the oxidation process of different model compounds (including methyl oleate (MO), ethyl linoleate (EL) and methyl linoleate (MLn) in emulsions under the influence of an emulsifiable cobalt drier. Wu et al. (2004) have found that two chelating ligands 2-aminomethylpyridine and 2-hydroxymethylpyridine are better than, or at least comparable with 2, 2′-bipyridine in promoting the drying capability of manganese 2-ethylhexagonate. Micciche et al. (2005) have developed an environment friendly iron-based drier as replacement for Co-driers in alkyd paints. Micciche et al. (2005) studied the structure and actual mechanism by which driers catalyze the oxidation process. Bouwman and Gorkum (2007) have compared the performance of commercial cobalt driers and manganese driers and developed new drier catalysts based on manganese. Korbahti et al. (2007) investigated the electrochemical oxidation of water-based paint wastewater. Brinckmann and Stephan (2011) have investigated the drying process of water-based paints used in automotive industry. Sheoran et al. (2011) have proposed a new application of lensless Fourier transform digital holographic interferometry for studying the drying process and detection of cracking/disbanding of the painted surface by observing the dynamics of interference phase maps. Erben et al. (2012) have shown that acryl-substituted ferrocenes can be a good alternative to cobalt-based driers for solvent-borne alkyd binder. Shaaban et al. (2012) have developed a technically and economically feasible process for manufacturing of manganese octoate, which can be an efficient and environmentally safe primary paint drier. No work is found in literature concerning the optimum quantities of different kinds of driers for the paint formulation.

This article originates when an Indian paint manufacturing companies identifies that determination of appropriate quantities of different types of driers that are added in the thinning tank is a major quality problem for its solvent-based paint production and decides to resolve the problem using statistical studies. The paint manufacturing company uses a combination of three kinds of drier, namely Cobalt octoate (as surface drier), Lead octoate (as through drier) and Calcium octoate (as auxiliary drier). The process engineers of the company identified that the drying quality, gloss as well as clarity of the coated surface of the manufactured solvent-based paints could be improved by appropriately selecting the addition quantities for the three kinds of driers. Therefore, the management of the company decided to initiate statistical studies for determination of the optimal addition quantities for the three types of driers. In this study, first experimentations are carried out using rotatable central composite design (RCCD). Then the mathematical relationship of each quality characteristic (response variable) with the input variables (i.e. driers quantities) is established using response surface methodology (RSM). Finally, goal programming (GP) technique is applied to determine the optimal quantities of the three kinds of driers for using in the paint formulation. Confirmatory experiments reveal that drying quality as well as gloss and clarity of the manufactured solvent-based paint improved substantially by using the optimal quantities of the three kinds of driers.

2. The Paint Manufacturing Process and the Problem

The flow chart of the paint manufacturing process is shown in Figure 1. The required quantities of binders, pigments, fillers and additives (except driers) for a batch are fed into the feed hopper. These materials are then transferred to the mixing machine where the input ingredients are thoroughly mixed. The batch masses are conveyed to the next operation, i.e. milling, where grinding and further mixing take place. After milling, the paint is transferred to the next operation, where it is thinned and tinted in agitated tanks in presence of solvents, plasticizers and driers. The liquid paint is strained into a transfer tank or directly into the hopper of the filling machine. Centrifuges, screens, or pressure filters are used to remove non-dispersed pigments as sludge. The paint is poured into cans or drums, labeled, packed, and moved to storage.
In the Indian paint manufacturing industry, the process engineers were interested to know the optimal quantities for Cobalt octoate, Lead octoate and Calcium octoate that should be added into the thinning tank in a batch of production for achieving the best quality with respect to drying quality, gloss and clarity of the coated surface of the manufactured paints. Thus, the input variables for the current problem were as follows:

a) Quantity of Cobalt octoate (drier 1) to be added (x1)

b) Quantity of Lead octoate (drier 2) to be added (x2) and

c) Quantity of Calcium octoate (drier 3) to be added (x3)

It was required to determine the optimal quantities of the three input variables with respect to the following three quality characteristics (response variables):

a) Gloss after 48 hours (y1): The lower bound of y1 is 80 units

b) Clarity after 48 hours (y2): The lower bound of y2 is 80 units

c) Drying quality after 24 hours (y3): The categorical value of y3 is required to be at least ‘Good’.

It was planned to tackle the problem by establishing first the relationship between the response and input variables based on experimental data, and then to optimize the response variables using goal programming approach.

3. Experimental Design and the Experimentation

Box and Hunter (1957) introduced the concept of rotatability, which is very important in response surface second-order designs. A design is said to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design center. Central composite design is a two-level full factorial with $F=2^k$ factorial points, augmented with additional $n_0$ centre points, which are set at the mid-point of each factor range, and $2k$ axial points. Axial points are located at a specific distance $\alpha = \left(\frac{F}{2}\right)^{\frac{1}{4}}$ from the design centre in each direction in each axis. The factorial points represent a first-order model, while centre points, set at the midpoint of each factor range, provide information about the existence of curvature. In addition, axial points allow estimation of the pure quadratic properties of the model. Pictorial representation of a RCCD of three factors is given below in Figure 2.
The advantage of a RCCD is that one can build a second order (quadratic) model for the response variable without needing to use a complete three-level factorial experiment. Moreover, it can provide information about the existence of curvature in a system. Therefore, it was decided to prepare the experimental layout using RCCD. Following the principles of RCCD, the final design matrix was obtained by augmenting the $2^3 = 8$ factorial points with $2 \times 3 = 6$ axial points and 3 centre points. The selected values of the three types of driers (henceforth will be termed as factors only) at different design points (levels) are shown in Table 1. Table 1 also shows the coded values at different design points for these factors. Accordingly experimental layout was prepared and the experimentation was carried out. During actual experimentation other constituents of the paint were kept at their existing level of operation and the experimental runs were carried out in a random manner with each run being replicated only once. Table 2 shows the experimental layout along with the observed response values corresponding to different trials.

### Table 1. Selected and coded values of the factors at different design points (levels)

| Level | $x_1$ | $x_2$ | $x_3$ | Coded values |
|-------|-------|-------|-------|--------------|
| Axial | 1.95  | 0.66  | 1.16  | -1.682 -1.682 -1.682 |
| Low   | 4     | 1     | 1.5   | -1 -1 -1 |
| Center| 7     | 1.5   | 2     | 0 0 0 |
| High  | 10    | 2     | 2.5   | 1 1 1 |
| Axial | 12.05 | 2.34  | 2.84  | 1.682 1.682 1.682 |

Note: $X_1 = \frac{x_1 - 7.0}{3.0}$, $X_2 = \frac{x_2 - 1.5}{0.5}$, and $X_3 = \frac{x_3 - 2.0}{0.5}$.

### Table 2. Experimental layout and the observed responses

| Standard order | Run order | Coded variables | Observed responses |
|----------------|-----------|-----------------|--------------------|
| 1              | 6         | -1 -1 -1        | Fair 81.5 73 |
| 2              | 11        | -1 -1 1         | Fair 79.9 70 |
| 3              | 17        | -1 1 -1         | Excellent 81.4 78 |
| 4              | 1         | -1 1 1          | Excellent 87.2 82 |
| 5              | 4         | 1 -1 -1         | Bad 81.7 78 |
| 6              | 8         | 1 -1 1          | Bad 82.6 87 |
| 7              | 5         | 1 1 -1          | Good 83.4 68 |
| 8              | 15        | 1 1 1           | Good 90 79 |
| 9              | 2         | -1.682 0 0      | Good 78.9 68 |
| 10             | 3         | 1.682 0 0       | Bad 83.5 75 |
| 11             | 12        | 0 -1.682 0      | Very Bad 75.5 75 |
| 12             | 13        | 0 1.682 0       | Good 86.1 73 |
| 13             | 16        | 0 0 -1.682      | Excellent 87 76 |
| 14             | 9         | 0 0 1.682       | Excellent 88.8 90 |
| 15             | 14        | 0 0 0           | Fair 85.1 73 |
| 16             | 7         | 0 0 0           | Fair 83.2 74 |
| 17             | 10        | 0 0 0           | Good 84 74 |

4. Modeling the Response Surfaces

Response surface methods are useful where several independent variables influence a dependent variable. Conventionally, the independent variables are assumed to be continuous and controllable by the experimenter. However, in the current problem, the response variable ‘drying quality ($y_c$)’ is observed to be categorical. Therefore, using Nair’s (1986) scoring scheme the categorical values for $y_c$ are transformed first into pseudo-observations appropriately, which are considered as the observed response values for further analysis. A brief description of the Nair’s method is given in Appendix 1. As there was only one observation for each
experimental run, the method was used to detect the location effect only. The location pseudo-observation \( (y_3) \) for each of the observed categorical responses for the 17 experimental combinations is given in Table 3.

| Standard order | Observed categorical value for drying quality \( (y_c) \) | Pseudo-observation for location \( (y_3) \) |
|----------------|-----------------------------------------------|------------------------------------------|
| 1              | Fair                                         | -0.5247                                  |
| 2              | Fair                                         | -0.5247                                  |
| 3              | Excellent                                    | 1.3641                                   |
| 4              | Excellent                                    | 1.3641                                   |
| 5              | Bad                                          | -1.2592                                  |
| 6              | Bad                                          | -1.2592                                  |
| 7              | Good                                         | 0.4197                                   |
| 8              | Good                                         | 0.4197                                   |
| 9              | Good                                         | 0.4197                                   |
| 10             | Bad                                          | -1.2592                                  |
| 11             | Very bad                                     | -1.6789                                  |
| 12             | Good                                         | 0.4197                                   |
| 13             | Excellent                                    | 1.3641                                   |
| 14             | Excellent                                    | 1.3641                                   |
| 15             | Fair                                         | -0.5247                                  |
| 16             | Fair                                         | -0.5247                                  |
| 17             | Good                                         | 0.4197                                   |

The assumed response models for the three response variables, i.e. \( y_1, y_2 \) and \( y_3 \) are given below:

\[
y_k = \beta_0 + \sum_{i=1}^{3} \beta_i X_i + \sum_{i=1}^{3} \beta_{ii} X_i^2 + \sum_{i=1}^{3} \sum_{j=i+1}^{3} \beta_{ij} X_i X_j + \epsilon, \quad k = 1, 2, 3.
\]

where \( y_k (k = 1,2,3) \) is the expected response value of the \( k \)th response variable, \( \beta_0 \) is the intercept, \( \beta_i \) represent the first order coefficients, \( \beta_{ii}'s \) are pure quadratic coefficients, \( \beta_{ij}'s \) \( (i \neq j) \) are mixed quadratic coefficients and \( \epsilon \) is random error component with mean zero and variance \( \sigma^2 \).

The model parameters were estimated using the method of ordinary least squares. The models were developed using Design-Expert software (2006). The model-building procedure was identical for each response and throughout the analysis the level of significance was taken as 5%, except in case of lack of fit (LOF) where level of significance was taken as 10%. Linear, two-factor interaction (2FI), quadratic and cubic models were fitted and selection of the final model was based on significance of sequential model sum of squares, LOF test and examination of the model summary statistics. Summary statistics (Myers and Montgomery, 1995) used were \( R^2 \), Adjusted \( R^2 \), PRESS (Predicted error sum of square) and \( R^2 \) predicted. In addition, normal probability plots of studentized residuals and plots of studentized residuals versus predicted response were also examined to determine the adequacy of the least squares fit.

### 4.1 Response model for gloss \( (y_1) \)

Table 4 shows the model-building summary for the response variable \( y_1 \). The upper panel of this table summarizes the statistical tests on the polynomial terms in the model. It can be noted that the F-value corresponding to the quadratic model is the maximum, thereby implying that the quadratic model is an adequate fit. It was further observed that the full cubic model is aliased, as the design does not contain enough runs to fit a cubic model. The middle panel gives the LOF tests for each model. This panel too shows that the quadratic model has insignificant LOF. Above findings in conjunction with the adjusted \( R^2 \) and predicted \( R^2 \) statistics of the quadratic model, shown in the lower panel of Table 4, indicate that the quadratic model may be the best fit. The finally selected quadratic model is given below:

\[
\hat{y}_1 = 84.09 + 1.13 X_1 + 2.50 X_2 + 1.08 X_3 - 0.98 X_1^2 - 1.12 X_2^2 + 1.39 X_3^2 + 0.24 X_1 X_2 \\
+ 0.41 X_1 X_3 + 1.64 X_2 X_3 
\] (1)
Table 4. Model-building summary for gloss ($y_1$)

| Source                  | Sum of squares | Degrees of freedom | Mean sum of squares | F-value | Prob. > F |
|-------------------------|----------------|--------------------|---------------------|---------|-----------|
| Mean vs. Total          | 1.186E+05      | 1                  | 1.186E+.05          |         |           |
| Linear vs. Mean         | 118.61         | 3                  | 39.54               | 5.25    | 0.0136    |
| 2FI vs. Linear          | 23.26          | 3                  | 7.75                | 1.04    | 0.4170    |
| Quadratic vs. 2FI       | 64.82          | 3                  | 21.61               | 15.39   | 0.0018    |
| Cubic vs. Quadratic     | 7.87           | 4                  | 1.97                | 3.01    | 0.1963    |
| Residual                | 1.96           | 3                  | 0.65                |         |           |
| TOTAL                   | 1.188E+05      | 17                 | 6987.93             |         |           |

Lack of fit (LOF) tests

| Source                  | Sum of squares | Degrees of freedom | MS         | F-value | Prob. > F |
|-------------------------|----------------|--------------------|------------|---------|-----------|
| Linear                  | 96.10          | 11                 | 8.74       | 9.60    | 0.0980    |
| 2FI                     | 72.83          | 8                  | 9.10       | 10.00   | 0.0940    |
| Quadratic               | 8.01           | 5                  | 1.60       | 1.76    | 0.4007    |
| Cubic                   | 0.14           | 1                  | 0.14       | 0.16    | 0.7310    |
| Pure Error              | 1.82           | 2                  | 0.91       |         |           |

Model summary statistics

| Source                  | Standard deviation | $R^2$ | Adjusted $R^2$ | Predicted $R^2$ | PRESS |
|-------------------------|-------------------|-------|----------------|-----------------|-------|
| Linear                  | 2.74              | 0.5478| 0.4434         | 0.1623          | 181.39|
| 2FI                     | 2.73              | 0.6552| 0.4484         | 0.2652          | 159.09|
| Quadratic               | 1.18              | 0.9546| 0.8962         | 0.6974          | 65.52 |
| Cubic                   | 0.81              | 0.9909| 0.9517         | 0.8364          | 35.43 |

Table 5 shows the results of analysis of variance (ANOVA) of the finally selected model. Normal probability plot of studentized residuals and plot of predicted values versus studentized residuals for gloss ($y_1$) are shown in Figures 3 and 4 respectively. Both the residual plots are satisfactory and indicate that there is no apparent violation of the assumption that the errors are approximately normally distributed with constant variance.

Table 5. Results of ANOVA for the fitted model

| Source                  | Sum of Squares | Degrees of freedom | Mean sum of squares | F-value | Prob. > F |
|-------------------------|----------------|--------------------|---------------------|---------|-----------|
| Model                   | 206.70         | 9                  | 22.97               | 16.36   | 0.0007    |
| $X_1$                   | 17.45          | 1                  | 17.45               | 12.43   | 0.0097    |
| $X_2$                   | 85.28          | 1                  | 85.28               | 60.74   | 0.0001    |
| $X_3$                   | 15.88          | 1                  | 15.88               | 11.31   | 0.0120    |
| $X_1^2$                 | 10.79          | 1                  | 10.79               | 1.69    | 0.0276    |
| $X_2^2$                 | 14.14          | 1                  | 14.14               | 10.07   | 0.0156    |
| $X_3^2$                 | 21.79          | 1                  | 21.79               | 15.52   | 0.0056    |
| $X_1X_2$                | 0.45           | 1                  | 0.45                | 0.32    | 0.5885    |
| $X_1X_3$                | 1.36           | 1                  | 1.36                | 0.97    | 0.3576    |
| $X_2X_3$                | 21.45          | 1                  | 21.45               | 15.28   | 0.0058    |
| Residual                | 9.83           | 7                  | 1.40                |         |           |
| LOF                     | 8.01           | 5                  | 1.60                | 1.76    | 0.4007    |
| Pure Error              | 1.82           | 2                  | 0.91                |         |           |
| TOTAL                   | 216.53         | 16                 |                     |         |           |
Figure 3. Normal probability plot of studentized residuals for gloss ($y_1$)

Figure 4. Plot of studentized residuals versus predicted values of gloss ($y_1$)

Figure 5 presents three two-dimensional contour plots for gloss ($y_1$) in terms of the coded variables with one factor kept fixed at zero. In the plot of drier 1 (Cobalt octoate) vs. drier 2 (Lead octoate), the stationary point is a point of maximum response and from the contours it is evident that to meet the requirement for gloss ($y_1$) the levels of both drier 1 and drier 2 should be high. But the other two plots fail to give any such direction as both of them represent second order systems with saddle points.
4.2 Response surface for clarity ($y_2$) and drying quality ($y_3$)

Similar analysis were carried out for establishing the best fitted response surface models for the response variables $y_2$ and $y_3$, checking adequacy of assumptions and finding out the optimization directions. The established models for $y_2$ and $y_3$ that gives a reasonable fit to the data based on adjusted $R^2$, predicted $R^2$, PRESS and LOF statistic are given in equations 2 and 3 respectively.
\[
\hat{y}_2 = 73.63 + 1.52X_1 - 0.32X_2 + 3.26X_3 - 0.64X_1^2 + 0.24X_2^2 + 3.42X_3^2 - 4.37X_1X_2
+ 2.38X_1X_3 + 1.13X_2X_3
\]  
\[R^2 = 0.9713, \text{ Adjusted } R^2 = 0.9345, \text{ Predicted } R^2 = 0.7798, \text{ PRESS } = 125.74\]  
\[
\hat{y}_3 = -0.20 - 0.45X_1 + 0.78X_2 - 0.10X_1^2 - 0.18X_2^2 + 0.53X_3^2 - 0.052X_1X_2
\]  
\[R^2 = 0.9469, \text{ Adj. } R^2 = 0.8787, \text{ Pred. } R^2 = 0.7841, \text{ PRESS } = 3.67\]

Residual plots for these two models do not indicate any serious model inadequacy. Two-dimensional contour plots for these models are also analyzed and corresponding characterization based on all the three models developed are summarized in Table 6.

| Response | Requirement | Plot of       | Nature of stationary point | Level of driers to meet the requirements |
|----------|-------------|---------------|----------------------------|----------------------------------------|
|          |             | Drier 1 vs. 2 | Max. Response              |                          Drier 1 ➔ High               |
| Gloss    | Minimum 80  | Drier 1 vs. 3 | Saddle Point               | Drier 2 ➔ High               |
|          |             | Drier 2 vs. 3 | Saddle Point               | None                          |
| Clarity  | Minimum 80  | Drier 1 vs. 2 | Saddle Point               | None                          |
|          |             | Drier 1 vs. 3 | Saddle Point               | None                          |
|          |             | Drier 2 vs. 3 | Min. Response              | Drier 2 ➔ Low                |
| Drying quality | At least ‘Good’ (*) | Drier 1 vs. 2 | Max. Response              |                          Drier 1 ➔ Low               |
|          |             | Drier 1 vs. 3 | Min. Response              |                          Drier 2 ➔ High               |
|          |             | Drier 2 vs. 3 | Min. Response              |                          Drier 2 ➔ High               |

Note (*): For being at least ‘Good’, pseudo-observation for drying quality should be at least 0.4197.

5. Determination of Optimal Values of the Input Variables

It was observed from the above summarization and corresponding contour plots that some of the stationary points are saddle points whereas some points of maximum or minimum response are outside the experimental region. Charnes et al. (1955) formulated the multi-objective optimization problem as a goal programming problem. Goal programming is a multicriteria decision making process where the decision maker have more than one objective (or goal or target) and these objectives are conflicting in nature and he/she must somehow reach a decision taking all of them into account. In weights method of goal programming algorithm, the experimenter tries to construct a single objective function of minimizing the weighted sum of the unwanted deviations from the set of target values, where the weights reflect the decision maker’s preference regarding the relative importance of each goal. Thus, goal programming gives a compromise solution based on the relative importance of each goal.

In the current problem, since the goals were conflicting in nature it was decided to use Charnes et al. (1955) proposed goal programming approach for arriving at the optimum levels of input variables, i.e. three types of driers. In this problem, the goals were made more optimistic and taken as

i) Measured value of gloss \(y_1\) should be at least 90,

ii) Measured value of clarity \(y_2\) should be at least 90, and

iii) Drying quality \(y_3\) should be at least ‘Good’, i.e. pseudo-observation \(y_3\) should be at least 0.4197.

and it was assumed that all goals were of equal importance, consequently equal weights were assigned to each goal.

The resulting formulation was solved by ‘Solver’ add-in of MS Excel (2007). MS Excel spreadsheet for implementing the goal programming formulation is given in Figure 6. The resulting optimal values of the input variables (i.e. quantities of the three types of driers needed to be added) in terms of coded as well as actual values are given in Table 7. Taking into account the measurement capability of the weighting instrument in use, the actual values of the input variables are rounded off and recommended for implementation. Table 7 also contains the recommended rounded off values of the input variables.
Figure 6. MS Excel spreadsheet for goal programming

| Input variables       | Optimal solution                  |
|-----------------------|-----------------------------------|
|                       | Coded value | Actual value | Rounded value |
| Quantity of Drier 1   | -0.94341    | 4.173 gm.   | 4.2 gm.       |
| Quantity of Drier 2   | 1.682       | 2.341 gm.   | 2.3 gm.       |
| Quantity of Drier 3   | 1.44077     | 2.720 gm.   | 2.7 gm.       |

Expected values of gloss, clarity and drying quality at the suggested factor level combination of the input variables are given in Table 8.

| Response    | Expected value | Implication    |
|-------------|----------------|----------------|
| Gloss       | 83.6           | Acceptable     |
| Clarity     | 89.0           | Acceptable     |
| Drying quality | 0.91        | Outcome is at least ‘Good’ |

It may be noted that due to rounding off of the required addition quantities of the three types of driers, the expected value of gloss is obtained to be less than the corresponding ‘optimistic’ goal, but still this value is much higher than the corresponding manufacturing goal of 80.

6. Implementation

The management of the organization agreed to implement the recommended solution. Accordingly, 5 batches of the paint formulation were manufactured and the responses were measured. The responses for the different batches are given in Table 9. The responses for all these batches were observed to fulfill the respective requirements. Encouraged by the result, management has since implemented the solution.
Table 9. Result of implementation

| Batch Number | Drying quality | Observed responses | Clarity |
|--------------|----------------|--------------------|---------|
| 1            | Good           | 82.3               | 87      |
| 2            | Excellent      | 84.6               | 90      |
| 3            | Good           | 82.8               | 89      |
| 4            | Good           | 81.8               | 91      |
| 5            | Excellent      | 84.7               | 92      |

7. Conclusion

Every manufacturing industry often faces several quality problems which demand for quick resolution for sustaining in today’s highly competitive market. It is found that application of properly planned statistical methodologies can be an effective approach for solving quality problems in paint manufacturing industry, which is essentially a chemical industry. Presence of interactions is very much likely in case of chemical processes. Also there is high possibility of curvature in such manufacturing systems. This paper demonstrates that system’s curvature and interaction effects can be well modeled by carrying out experimentation using rotatable central composite design (RCCD) and fitting response surfaces. Multiple responses can then be effectively optimized using goal programming approach. The problem solving approach demonstrated in this paper can be extremely useful in a wide variety of manufacturing processes.

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Appendix 1

A Brief Description of Nair’s Method for Detecting Location and Dispersion Effects in Categorical Data

Let 

- \( n \) = number of experimental trials,
- \( K \) = number of categories, and
- \( y_{ik} \) = observed frequency of category \( k \) in \( i \)-th experimental trial.

\[ i = 1, 2, \ldots, n \quad \text{and} \quad k = 1, 2, \ldots, K \]

Further, let \( q_k \) be the overall proportion of observations that fall into category \( k \). Then

\[
q_k = \frac{\sum_{i=1}^{n} y_{ik}}{\sum_{i=1}^{n} \sum_{j=1}^{K} y_{ij}}
\]

Let, \( \tau_k = \sum_{j=1}^{K-1} q_j + \frac{q_k}{2} \), so that \( \tau_k \) is proportional to the midrank for category \( k \). Then the location scores are defined as:

\[
l_k = \frac{\tilde{\tau}_k}{\sqrt{\sum_{j=1}^{K} q_j \tilde{\tau}_j^2}}, \quad (k = 1, 2, \ldots, K), \quad \text{where} \quad \tilde{\tau}_k = \tau_k - \sum_{j=1}^{K} q_j \tau_j = \tau_k - 0.5.
\]
The dispersion scores are defined as:

\[
d_k = \frac{e_k}{\sqrt{\sum_{j=1}^{K} q_j e_{j}^2}}, \quad (k = 1, 2, \ldots, K),
\]

where 

\[
e_k = l_k \left( l_k - \sum_{j=1}^{K} q_j l_j^3 \right) - 1.
\]

Then the location pseudo-observation for the \( i \)-th experimental trial is given by:

\[
L_i = \sum_{k=1}^{K} y_{i,k} l_k, \quad i = 1, 2, \ldots, n;
\]

and the dispersion pseudo-observation for the \( i \)-th experimental trial is given by:

\[
D_i = \sum_{k=1}^{K} y_{i,k} d_k, \quad i = 1, 2, \ldots, n.
\]