We investigate collision of a point particle and an infinitely thin planar domain wall interacting gravitationally within the linearized gravity in Minkowski space-time of arbitrary dimension. In this setting we are able to describe analytically the perforation of the wall by an impinging particle, showing that it is accompanied by excitation of the spherical shock branon wave propagating outwards with the speed of light. Formally, the shock wave is a free solution of the branon wave equation which has to be added to ensure the validity of the retarded solution at the perforation point. Physically, the domain wall gets excited due to the shake caused by an instantaneous change of sign of the repulsive gravitational force. This effect is shown to hold, in particular, in four space-time dimensions, being applicable to the problem of cosmological domain walls.

1. INTRODUCTION

Gravitational interaction of relativistic extended objects has some unusual features. It is an essentially relativistic problem even if their relative velocity is small, since the brane tension, causing gravitational repulsion, contributes to interaction on equal footing with the energy density. The net effect of gravitational interaction of two branes therefore varies with dimensionality of the world-volumes and codimension of their embedding into space-time. It is repulsive for codimension one (domain walls), locally vanishes for codimension two (strings) and attractive in other cases.

Another new feature due to the extended nature of branes is possibility of their free oscillations which may accompany the generic collision process. While two point particles under collision just change their momenta, but remain in the same intrinsic state, the branes will get excited and will not remain in the initial state even asymptotically. Physically, the most interesting case is interaction of domain walls with particles in four dimensions. This was investigated long ago in connection with topological defects in cosmology. The solution of linearized Einstein equations for gravitational field of thin planar domain wall was found by Vilenkin [1, 2]. Gravity of domain walls is repulsive, so a particle impinging on the wall may be reflected at some finite distance, transferring the momentum to the wall. This gives rise to the friction force acting upon the wall moving in cosmic plasma [3]. Here we will be interested in more subtle effect of excitation of the wall during the collision, especially in the situation when the particle energy is enough to reach the wall and to pierce it.

Fortunately, such piercing collision may be treated analytically with the linearized gravity theory. In fact, one important feature associated with codimension of embedding of an extended object into the bulk is the degree of singularity of the linearized gravitational field at its location. Gravitational field diverges as a negative power of the distance for codimension greater than two, it diverges logarithmically for codimension two, but it remains finite in the case of the domain wall. Therefore, though generically gravitational collision of two infinitely thin branes is a singular problem, the collision particle – domain wall turns out to be tractable within the linearized gravity.

Since gravity of the domain wall remains finite when the particle pierces it, the energy-momentum is not exchanged at perforation, so such particles will not contribute to the friction force. We will see, however, that the wall does not remain insensitive to piercing by a point particle, but gets excited. The excitation has a form of the spherical branon wave arising at the moment of perforation and propagating outwards with the velocity of light. This effect was found in five dimensions in the Randall-Sundrum (RS) type of setting [4] and here it will be shown to exist in arbitrary space-time dimensions, including the case of domain walls in the four-dimensional cosmology.

Our treatment of perforation is essentially local, so in order to establish its validity region we should invoke some results obtained in the full non-linear theory. It is well-known that the linear approximation for gravity of the domain wall breaks down at large distances. It is also known [5, 6] that no static solution of full non-linear Einstein equations exists which could fit to the linearized solution of [1, 2], the consistent non-linear ansatz being time-dependent. In particular, an exact solution found by Vilenkin [5] turns out to be a segment of an accelerated spherical domain wall [7, 8] which comes in from infinity, turns around, and heads back out to infinity. Subsequently, Linet have shown that the static solution still does exist [9], but at the expense of introducing a cosmological constant in the bulk. More recently domain walls attracted much attention in the brane-world scenarios [10–14] and especially in the Randall-Sundrum models [15–17]. In fact, the RSII model with one brane is based on the exact static solution of Einstein equations with negative cosmological constant. As it could be expected, this solution reduces to static solution of five-dimensional linearized Einstein equations in the vicinity of the brane (after a suitable coordinate transformation [4]), while corrections due to...
the cosmological constant enter only in the second post-linear order. The static domain wall solutions (thick walls) exist also in field-theoretical models with scalar fields whose vacuum manifold contains disconnected components, in particular, supergravities/effective string theories [18, 19].

Apart from calculation of the friction force, gravitational interaction of domain walls and more general $p$–branes with massive bodies was studied previously in other physical contexts. Some important applications were related to black holes in the brane-world scenarios (for a recent review and further references see [20]). In the Rundall-Sundrum [15–17] setup the particles which are allowed to live in the bulk are expelled from the brane (domain wall) and move along the geodesics into the AdS bulk [21–23]. Their gravity and the corresponding perturbation of the induced metric on the brane was studied in [24]. Black holes, which in such scenarios can be created in high energy particle collisions, also may escape from the brane into the bulk [25–27]. The brane – black-hole system, including the process of the merging, was investigated in detail using the model of a test brane in the black hole background [28–32]. Perforation of domain walls by black holes was qualitatively studied within the field-theoretical model of hybrid defects (axion) [33, 34] suggesting this mechanism as relevant to the cosmological domain wall problem. Numerical studies of interaction of black holes with field-theoretical domain walls are also available [35].

Our approach in this paper is much simpler and it can be considered complementary to the above studies. We treat both the domain wall and the bulk particle as test bodies propagating in $D$–dimensional Minkowski space-time and interacting via linearized gravity. Such a setting seems adequate to give a local description of the process of perforation with possibility to treat the domain wall dynamically.

The paper is organized as follows. In Sec. 2 we derive the brane metric in the linearized gravity, consider its relation to some exact solutions and explore interaction of static branes of different dimensionality dependent on their codimension. In Sec. 3 we consider motion of a point particle in the gravitational field of the domain wall. The next Sec. 4 is devoted to general description of the deformation of the domain wall under gravitational collision and the derivation of the branon wave equation with a source. In Sec. 5 we construct the retarded solution for the branon wave equation with the source generated by gravity of the perforating particle in even and odd dimensions $D > 4$. The Sec. 6 is devoted to the limiting case of the light–light perforation. Then in Sec. 7 we consider the case of the domain wall in the four-dimensional bulk which requires special treatment, and in the last Sec. 8 we discuss some tentative applications. In the Appendix the evaluation of typical integrals involved in the calculations is presented.

2. GRAVITY OF INFINITELY THIN PLANAR WALLS IN $D$ DIMENSIONS

A. Exact solutions

For more generality we start with considering an arbitrary $p$-brane propagating in $D$–dimensional curved space-time (the bulk). We denote the bulk metric as $g_{MN}$, $M, N = 0, 1, 2, ..., D – 1$, and define the brane world-volume $V_{p+1}$ by the embedding equations $x^M = X^M(\sigma^\mu)$, parameterized by arbitrary coordinates $\sigma^\mu$, $(\mu = 0, ..., p)$ on $V_{p+1}$. The corresponding action in the Polyakov form is a functional of $X^M(\sigma^\mu)$ and the metric $\gamma_{\mu\nu}$ on $V_{p+1}$:

$$S_p = -\frac{\mu}{2} \int \left[ X^M_X^N g_{MN} \gamma^{\mu\nu} - (p - 1) \right] \sqrt{|\gamma|} d^{p+1}\sigma. \quad (2.1)$$

Here $\mu$ is the brane tension, $X^M_\mu = \partial X^M / \partial \sigma^\mu$ are the tangent vectors and $\gamma^{\mu\nu}$ is the inverse metric on $V_{p+1}$, $\gamma = \det \gamma_{\mu\nu}$. Also, $\kappa_D^2 \equiv 16\pi G_D$, the metric signature is $+ - - ...$ and our convention for the Riemann tensor is $R^B_{NP,M} \equiv \Gamma^B_{NP,R} - ...$. Variation of (2.1) with respect to $X^M$ gives the brane equation of motion

$$\partial_\mu \left( X^N g_{MN} \gamma^{\mu\nu} \sqrt{|\gamma|} \right) = \frac{1}{2} g_{NP,M} X^N_\mu X^P_\nu \gamma^{\mu\nu} \sqrt{|\gamma|}, \quad (2.2)$$

which is covariant with respect to both the space-time and the world-volume diffeomorphisms. Variation over $\gamma^{\mu\nu}$ gives the constraint equation

$$\left( \gamma^{\mu\nu} \gamma^{\lambda\tau} X^M_\lambda X^N_\tau \right) g_{MN} + \frac{p - 1}{2} \gamma_{\mu\nu} = 0, \quad (2.3)$$

whose solution defines $\gamma_{\mu\nu}$ as the induced metric on $V_{p+1}$:

$$\gamma_{\mu\nu} = X^M_\mu X^N_\nu g_{MN} \big|_{x=X}. \quad (2.4)$$

Adding to (2.1) the Einstein action with the cosmological constant

$$S_E = -\frac{1}{\kappa_D^2} \int (R_D + 2\Lambda) \sqrt{|g|} d^Dx, \quad (2.5)$$

and varying $S_p + S_E$ with respect to the space-time metric $g_{MN}$ we obtain Einstein equations

$$R_{MN} = \frac{1}{2} g_{MN} R = \kappa_D^2 T_{MN} + \Lambda g_{MN}. \quad (2.6)$$
with the source term
\[ T^{MN} = \mu \int X^M \nu^N \gamma^{\mu} \frac{\delta^D(x - X(\sigma))}{\sqrt{|g|}} \sqrt{|\gamma|} d^{D-1}\sigma. \] (2.7)

We will be interested in static solutions of the system (2.2, 2.3, 2.6) for planar branes described by the linear embedding functions
\[ X^M = \Sigma^M_{\mu} \sigma^\mu \] (2.8)
with constant \( \Sigma^M_{\mu} \). In what follows we will mostly use the coordinates \( \sigma^\mu \) coinciding with \( x^\mu \), such that \( \Sigma^M_{\mu} = \delta^M_{\mu} \), but in some cases \( \sigma^\mu \) will be also invoked to avoid confusion.

Consistency of the above coupled system involving singular delta sources depends on codimension \( \tilde{d} = D - p - 1 \) of the embedding of the brane world-volume into the bulk. Strictly speaking, for \( \tilde{d} \geq 3 \) the use of distributions in the full non-linear gravity is not legitimate, though the presence of delta-sources in classical \( p \)-brane solutions in supergravities sometimes still can be detected [18]. The case \( \tilde{d} = 2 \) and \( \Lambda = 0 \), as it is well-known from an example of the cosmic string in four-dimensional space-time [3], is exceptional: in this case the cylindrically symmetric field configurations exist for which Einstein equations reduce to two-dimensional Laplace equation with the delta-source leading to static locally flat conical transverse space. The case \( \tilde{d} = 1 \) (domain wall) is legitimate too, but has a peculiar feature: for \( \Lambda = 0 \) exact solutions of Einstein equations are non-static [1, 2]. However, static solutions in this case do exist for some special value of the cosmological constant \( \Lambda < 0 \), the notorious example being the Randall-Sundrum metric in \( D = 5 \) [15–17]. Indeed, with an ansatz
\[ ds^2_{D} = e^{-2F(\bar{z})}\eta_{\mu\nu} dx^\mu dx^\nu - d\bar{z}^2, \] (2.9)
the Einstein equations reduce to:
\[ (D - 2)F'' = \frac{\mu \kappa^2}{2} \delta(\bar{z}), \quad (D - 1)(D - 2)(F')^2 + 2 \Lambda = 0, \] (2.10)
where a prime denotes the differentiation over \( \bar{z} \). This system has an exact solution provided
\[ \Lambda = -\frac{\mu^2 \kappa^2}{32} \frac{(D - 1)}{(D - 2)}. \] (2.11)
Imposing the additional \( \mathbb{Z}_2 \)-symmetry \( F(-\bar{z}) = F(\bar{z}) \), one obtains
\[ F = k|\bar{z}|, \] (2.12)
where
\[ k = \frac{\mu \kappa^2}{4(D - 2)}, \] (2.13)
so an exact solution of Einstein equations with the cosmological constant (2.6) reads:
\[ ds^2 = e^{-2k|\bar{z}|}\eta_{\mu\nu} dx^\mu dx^\nu - d\bar{z}^2. \] (2.14)
For comparison, we also give here the time-dependent solution found by Vilenkin [1, 3] and Ipser and Sikivie [7, 8] in four dimensions which exists in absence of the cosmological constant:
\[ ds^2 = \left(1 - k|z|\right)^2 dt^2 - e^{2kt} \left(1 - k|z|\right)^2 (dx^2 + dy^2) - dz^2. \] (2.15)
Here \( k \) is a free parameter.

B. Linearized gravity

Now we pass to linearized gravity in Minkowski space-time assuming \( \Lambda = 0 \) and expanding the metric as
\[ g_{MN} = \eta_{MN} + \kappa_D h_{MN}. \] (2.16)
All subsequent operations with indices of \( h_{MN} \) will be performed with respect to the Minkowski metric, e.g.,
\[ g^{MN} \approx \eta^{MN} - \kappa_D h^{MN}. \] In the Lorentz gauge
\[ \partial_N h^{MN} = \frac{1}{2} \partial^M h, \quad h = h^M_M, \] (2.17)
the linearized Einstein equations reduce to
\[ \Box h_{MN} = -\kappa_D \left( T_{MN} - \frac{1}{D-2} T \eta_{MN} \right), \quad T = T_M^M, \] (2.18)

with \( \Box \equiv \partial_M \partial^M \). Consider again an arbitrary plane unexcited \( p \)-brane described by the embedding functions (2.8), choose the coordinates on \( V_{p+1} \) as \( x^0 = \tilde{x}^0 \equiv t \), \( x^i = x^i, i = 1, \ldots, p \) and denote the coordinate transverse to the brane as \( y^k, k = 1, \ldots, \tilde{d} \). Then the brane stress-tensor \( T_{MN} \) will have non-zero only the components \( \mu, \nu = 0, i \) given by
\[ T_{\mu\nu} = \mu \eta_{\mu\nu} \delta^\tilde{d}(\tilde{z}), \] (2.19)
where \( \eta_{\mu\nu} \) is Minkowski metric on the brane (and unity in the case \( p = 0 \)), leading to
\[ ds^2 = \left( 1 + 4k(\tilde{d} - 2)\Phi_{\tilde{d}} \right) \eta_{\mu\nu} dx^\mu dx^\nu - \left( 1 - 4k(p + 1) \Phi_{\tilde{d}} \right) dz_k^2. \] (2.20)

Here \( \Phi_{\tilde{d}} \) is the solution of the transverse Poisson equation
\[ \Delta_{\tilde{d}} \Phi_{\tilde{d}}(\tilde{z}) = \delta^\tilde{d}(\tilde{z}), \] (2.21)
which reads explicitly
\[ \Phi_{\tilde{d}}(\tilde{z}) = \begin{cases} \frac{|\tilde{z}|^2}{2}, & \tilde{d} = 1 \\ (2\pi)^{-\frac{1}{2}} \ln |\tilde{z}|, & \tilde{d} = 2 \\ - (\tilde{d} - 2)^{-1} \Omega_{\tilde{d} - 1}^{-1} |\tilde{z}|^{2 - \tilde{d}}, & \tilde{d} \geqslant 3 \end{cases}, \] (2.22)
where \( \Omega_{\tilde{d} - 1} \) is the volume of the \( \tilde{d} - 1 \)-dimensional unit sphere in \( \tilde{d} \)-dimensional euclidean space: \( \Omega_{\tilde{d} - 1} = \frac{2\pi^{\tilde{d}}}{\Gamma(\tilde{d})} \).

C. Domain walls

In the case \( \tilde{d} = 1 \) the metric (2.20) reads
\[ ds^2 = \left( 1 - 2k|\tilde{z}| \right) \eta_{\mu\nu} dx^\mu dx^\nu - \left( 1 - 2k(D - 1)|\tilde{z}| \right) dz^2. \] (2.23)

This solution generalizes to arbitrary \( D \) the Vilenkin solution [1, 2] of the four-dimensional vacuum linearized gravity, and, similarly, it can not be regarded as linearization of any static metric satisfying \( D \)-dimensional vacuum Einstein equations. However it can be viewed as linearization of the Randall-Sundrum type exact solution (2.14) of the Einstein equations with the negative cosmological constant (2.11). Indeed, the linearization of (2.14) for small \( k|\tilde{z}| \) reads
\[ ds^2 = \left( 1 - 2k|\tilde{z}| \right) \eta_{\mu\nu} dx^\mu dx^\nu - dz^2. \] (2.24)

It looks different from our solution (2.23), but in fact (2.24) does not satisfy the Lorentz gauge condition (2.17) contrary to (2.23). Therefore, within the validity of the linear approximation, the solutions (2.24) and (2.20) must be related by some coordinate transformation. It is easy to check that
\[ \tilde{z} = z - \frac{D - 1}{2} k z^2 \text{sgn}(z), \] (2.25)
does this job in the linear order in \( k|\tilde{z}| \). Note that the right hand side of this equation is continuous at \( z = 0 \), so in the vicinity of this point \( |\tilde{z}| = |z| \). Therefore, the linearized domain wall can be viewed as the small distances limit \( k|\tilde{z}| \ll 1 \) of the exact solution (2.14). Note in passing that the required cosmological constant \( \Lambda \) in (2.6) is quadratic in \( k \), so it can be neglected in the linear order in \( k \).

D. Interaction between plane parallel branes

It is instructive to explore linearized gravitational interaction between two plane parallel branes \( p \) and \( \tilde{p} \leqslant p \), sitting at some finite distance. We split the space-time coordinates as \( x^M = (t, x, y, z) \), where \( x \in \mathbb{R}^p, y \in \mathbb{R}^{p - \tilde{p}}, z \in \mathbb{R}^{\tilde{d}} \). Let the first \( p \)-brane occupy the sector \( x^a = (t, x) \) at the position \( z = 0 \) in the overall transverse space, while the second extends in the sector \( x^a = (t, x) \) at the position \( z = \tilde{z} \). To extract the effective interaction potential we start with the action
\[ S_{\text{int}} = -\frac{\kappa_D}{2} \int h_{MN} T^{MN} d^D x, \] (2.26)
where \( h_{MN} \) is the linearized metric of the \( p \)-brane and \( \hat{T}^{MN} \) is the stress-tensor of the \( \hat{p} \) brane (or vice-versa) and insert as \( h_{MN} \) the solution of the corresponding d’Alembert equation. Using the scalar Green’s function of the d’Alembert equation

\[
\Box_D G(x, x') = \delta^D(x - x'),
\]

we obtain the bilinear form of the stress-energy tensors

\[
S_{\text{int}} = -\frac{x^2}{2} \int G(x, x') \left( T^{MN}(x) \hat{T}_{MN}(x') - \frac{1}{D-2} T(x) \hat{T}(x') \right) d^Dx d^Dx'.
\]

Substituting here the corresponding quantities for both branes at rest, we find that the integral (2.28) reduces to that over time and the spatial coordinates \( x \) of the \( \hat{p} \)-brane, allowing for introduction of the effective potential \( U_{\text{eff}} \) per unit volume of the smaller brane:

\[
S_{\text{int}} = -\int U_{\text{eff}}(\vec{z}) dt d\mathbf{x},
\]

which explicitly reads

\[
U_{\text{eff}} = \frac{x^2 \mu \bar{\mu} (\bar{p} + 1) (\bar{d} - 2)}{2(D - 2)} \Phi_d(\vec{z}).
\]

Inserting here the transverse potential (2.22) we finally obtain

\[
U_{\text{eff}} = -\frac{x^2 \mu \bar{\mu} (\bar{p} + 1)}{2(D - 2)} \begin{cases} \bar{z}/2, & \bar{d} = 1 \\ 0, & \bar{d} = 2 \\ \Omega_d^{-1} |\vec{z}|(2-\bar{d}), & \bar{d} \geq 3 \end{cases}.
\]

Thus, the character of interaction depends on codimension of the embedding of the bigger \( p \)-brane into the bulk: the potential is repulsive for \( \bar{d} = 1 \), there is no force for \( \bar{d} = 2 \) and it is attractive for \( \bar{d} > 2 \). This simple picture, however, holds only in the static case. As we will see, situation becomes more sophisticated when branes are in motion.

### 3. Interaction of Domain Wall with Moving Point Particle

Now we wish to consider the system of the gravitationally interacting domain wall and a moving point particle (\( p = 0 \) brane). This can be done adding to the sum \( S_p + S_E \) the particle action

\[
S_0 = -\frac{1}{2} \int \left( e g_{MN} \dot{z}^M \dot{z}^N + \frac{m^2}{e} \right) d\tau,
\]

where \( e(\tau) \) is the ein-bein of the particle world-line and dots denote derivatives with respect to \( \tau \). Varying \( S_0 \) with respect to \( \dot{z}^M(\tau) \) and \( e(\tau) \) one obtains the geodesic equation in arbitrary parametrization

\[
\frac{d}{d\tau} \left( e \dot{z}^N g_{MN} \right) = \frac{e}{2} g_{NP,M} \dot{z}^N \dot{z}^P,
\]

and the constraint equation

\[
e^2 g_{MN} \dot{z}^M \dot{z}^N = m^2.
\]

The corresponding energy-momentum tensor reads

\[
\hat{T}^{MN} = \int e \dot{z}^M \dot{z}^N \delta^D(x - z(\tau)) \frac{d\tau}{\sqrt{|g|}}.
\]

Both the domain wall and the point particle will be treated on an equal footing in the framework of the linearized gravity on Minkowski background. So we expand the total metric similarly to (2.16) adding to the metric perturbation \( h_{MN} \), which will be still associated with the brane, the metric perturbation \( \hat{h}_{MN} \) due to the particle (preserving the notation of sec. 2D):

\[
g_{MN} = \eta_{MN} + \kappa_D \left( h_{MN} + \hat{h}_{MN} \right).
\]

The Lorentz gauge condition (2.17) will be assumed for both components independently.

To treat the interaction problem in terms of formal expansions in the gravitational coupling we have now to expand the embedding functions \( X^M(\sigma) \) and \( z^M(\tau) \) as well as the Lagrange multipliers \( \gamma_{\mu \nu} \) and \( e(\tau) \) in powers of \( \kappa_D \) to the first order, which amounts to replacing these quantities by \( X^M + \delta X^M, z^M + \delta z^M, \gamma_{\mu \nu} + \delta \gamma_{\mu \nu} \) and \( e + \delta e \). Here \( X^M, z^M, \gamma_{\mu \nu}, e \) will now correspond to free motion in Minkowski space-time, while \( \delta X^M, \delta z^M, \delta \gamma_{\mu \nu} \) are the perturbations of the brane variables due to the gravitational field of the particle \( h_{MN} \), and \( \delta e \) are the perturbations of the particle variables due to the gravitational field of the brane \( h_{MN} \) (and we omit singular self-interaction terms).
A. Perturbation of the particle world-line

The unperturbed domain wall is described by the embedding function (2.8) and the corresponding induced metric is

$$\gamma_{\mu\nu} = \eta_{\mu\nu}. \quad (3.6)$$

Its gravitational field can be read off from Eq. (2.23), or explicitly

$$h_{MN} = \frac{\kappa D\mu}{2} \left( \Xi_{MN} - \frac{D-1}{D-2} \eta_{MN} \right) |z| = \frac{\kappa D\mu}{2(D-2)} \text{diag}(-1, 1, ..., 1, D-1), \quad (3.7)$$

where $\Xi_{MN} \equiv \Sigma^\mu_M \Sigma^\nu_N \eta_{\mu\nu}$. Assuming the particle to move orthogonally to the wall, we parameterize the unperturbed world-line as

$$z^M(\tau) = u^M \tau, \quad u^M = \gamma (1, 0, ..., 0, v), \quad \gamma = 1/\sqrt{1 - v^2}. \quad (3.8)$$

This trajectory intersects the domain wall at the moment of proper time $\tau = 0$, the corresponding coordinate time also being zero, $t = 0$. Using (3.7) and (3.8) in the Eqs. (3.3) and (3.2) one obtains for $\delta e$ and $\delta z^M$ the system of equations

$$\delta e = -\frac{m}{2} \left( \kappa_D h_{MN} u^M u^N + 2 \eta_{MN} u^M \delta \dot{z}^N \right) \quad (3.9)$$

and

$$\frac{d}{d\tau} (\delta e u_M + m \delta \ddot{z}_M) = -\kappa_D m \left( h_{PM,Q} - \frac{1}{2} h_{PQ,M} \right) u^P u^Q, \quad (3.10)$$

which upon the elimination of $\delta e$ gives for $\delta z^M$:

$$\Pi^{MN} \delta \ddot{z}_N = -\kappa_D \Pi^{MN} \left( h_{PN,Q} - \frac{1}{2} h_{PQ,N} \right) u^P u^Q, \quad (3.11)$$

where

$$\Pi^{MN} = \eta^{MN} - u^M u^N \quad (3.12)$$

is a projector onto the subspace orthogonal to $u^M$. Let us now choose the overall gauge condition

$$g_{MN} z^M z^N = 1, \quad (3.13)$$

with $z^M$ including the perturbation. In view of the zero order parametrization assumed (3.8), this amounts to the condition $\delta e = 0$, i.e.,

$$\frac{m}{2} \left( \kappa_D h_{MN} u^M u^N + 2 \eta_{MN} u^M \delta \dot{z}^N \right) = 0. \quad (3.14)$$

Going back to eq. (3.10) one has thereby

$$\delta \ddot{z}_M = -\kappa_D \left( h_{PM,Q} - \frac{1}{2} h_{PQ,M} \right) u^P u^Q, \quad (3.15)$$

or, in components,

$$\delta \ddot{z}^0 = 2kv \gamma^2 \text{sgn}(\tau), \quad \delta \ddot{z}^1 = k(D\gamma^2 v^2 + 1) \text{sgn}(\tau), \quad (3.16)$$

so, the force is repulsive as expected.

Integrating (3.16) twice with initial conditions $\delta z^M(0) = 0$, $\delta \dot{z}^M(0) = 0$, one has

$$\delta z^0 = kvt^2 \gamma^2 \text{sgn}(\tau), \quad \delta z^1 = \frac{1}{2} k\tau^2 (D\gamma^2 v^2 + 1) \text{sgn}(\tau). \quad (3.17)$$

Substituting (3.17) into (3.9) one can check that the gauge condition (3.14) holds.

Since we are using the perturbation theory, corrections to the uniform particle motion must be small (assuming appropriate initial conditions), so the particle always hits the brane and perforates it, reappearing on the other side. The reflection is of course physically possible, if the velocity is not high enough, but this is beyond the validity of our approximation. Our aim is to consider in detail what happens when the particle pierces the brane, so the case of reflection is outside the scope of the present treatment.
B. Acceleration discontinuity at the moment of perforation

According to (3.17), the perturbation of the particle energy $\delta p^0 = m\delta \dot{z}^0$ and the momentum $\delta p^z = m\delta \dot{z}$ have no discontinuity at the location of the brane $z = 0$, but their derivative have. The point-like particle therefore perforates the domain wall without loss of the energy-momentum, but its acceleration is finite and instantaneously changes sign. The sign rule in (3.16) corresponds to gravitational repulsion, as could be expected in the case of co-dimension one. Therefore the discontinuity of acceleration has a simple physical meaning: the repulsive force changes its sign at the moment of perforation. For consistency of our perturbative approach we have to ensure that the first order correction to the particle momentum remains small in the vicinity of the wall $k|z| \ll 1$ where the linearized approximation for wall’s gravity is valid. Since in the zero order $z = \nu \gamma \tau$, from (3.17) we find that this is true indeed:

$$|\delta \dot{z}^0| \sim k|z|\gamma \ll \gamma = u^0.$$  \hfill (3.18)

4. DEFORMATION OF DOMAIN WALL

Now we explore perturbations of the domain wall due to gravitational interaction with the perforating particle. For this we need to know the metric perturbation due to the particle. In accordance with the iterative approach adopted here, we must neglect the particle acceleration in the wall’s gravity when we calculate its proper time. For this we need to know the metric perturbation due to the perforating particle. In accordance with the iterative approach

$$\delta \gamma_{\mu\nu} = 2 \delta_{(\mu} \delta X_{\nu)}^N \eta_{MN} + \kappa_D \delta h_{MN} \Sigma^M \Sigma^N,$$  \hfill (4.6)

where brackets denote symmetrization over indices with the factor 1/2. Then linearizing the rest of the Eq. (2.2), after some rearrangements one obtains the following equation for deformation of the wall:

$$\Pi_{MN} \Box^{D-1} \delta X^N = \Pi_{MN} J^N, \quad \Pi^{MN} = \eta^{MN} - \Sigma^M \Sigma^N \eta_{\mu\nu},$$  \hfill (4.7)

A. Particle gravity

In what follows, the stress-tensor and the gravitational field of the particle will be denoted by bar. The metric perturbation moving along the straight line in the Minkowski space satisfies the equation

$$\Box^D \bar{h}_{MN} = -\kappa_D \left( \bar{T}_{MN} - \frac{1}{D-2} \bar{T} \eta_{MN} \right),$$  \hfill (4.1)

with the source term

$$\bar{T}^{MN}(x) = m \int u^M u^N \delta^D(x - ut) \, d\tau,$$  \hfill (4.2)

which has only $t, z-$ components non-zero. Passing to the $D$-dimensional Fourier-transforms

$$\bar{h}_{MN}(x) = \frac{1}{(2\pi)^D} \int e^{-iqx} \bar{h}_{MN}(q) \, d^D q,$$

$$\bar{T}^{MN}(x) = \frac{1}{(2\pi)^D} \int e^{-iqx} \bar{T}^{MN}(q) \, d^D q,$$  \hfill (4.3)

we obtain from (4.1) the retarded solution in the momentum representation

$$\bar{h}_{MN}(q) = \frac{2\pi \kappa_D m^2}{q^2 + i\varepsilon q^0} \left( u^M u^N - \frac{1}{D-2} \eta_{MN} \right).$$  \hfill (4.4)

In the coordinate representation we find (for $D \geq 4$):

$$\bar{h}_{MN}(x) = -\kappa_D m^2 \frac{\Gamma \left( \frac{D-4}{2} \right)}{4\pi^{D-2}} \left( u^M u^N - \frac{1}{D-2} \eta_{MN} \right) \frac{1}{\left[ \gamma^2 (z - \nu \tau)^2 + r^2 \right]^{D-3}},$$  \hfill (4.5)

where $r = \sqrt{\delta_{ij} x^i x^j}$ is the radial distance on the wall from the perforation point. This is just the Lorentz-contracted $D$-dimensional Newton field of the uniformly moving particle.

B. Perturbation of the wall

Perturbations of the Nambu-Goto branes in external gravitational field were expensively studied in the literature, see e.g. [36, 37]. On the Minkowski background the derivation is particularly simple. First, from Eq. (2.4) we find the perturbation of the induced metric

$$\delta \gamma_{\mu\nu} = 2 \delta_{(\mu} \delta X_{\nu)}^N \eta_{MN} + \kappa_D \delta h_{MN} \Sigma^M \Sigma^N,$$  \hfill (4.6)
where \( \Box_{D-1} \equiv \partial_\mu \partial^\mu \) and \( \Pi^{MN} \) is the projector onto the (one-dimensional) subspace orthogonal to \( V_{D-1} \). The source term in (4.7) reads:

\[
J^N = \kappa_D \Sigma^\mu Q^\nu \eta_{\mu\nu} \left( \frac{1}{2} \hat{h}^{PQ,N} - \hat{h}_N^{NP,Q} \right) \bigg|_{z=0} .
\]

Using the aligned coordinates on the brane \( \sigma^\mu = (t, \mathbf{r}) \), we will have \( \delta^M = \Sigma^M \), so the projector \( \Pi^{MN} \) reduces the system (4.7) to a single equation for \( M = 0 \) component. Thus only the \( z \)-components of \( \delta X^M \) and \( J^M \) are physical. Generically, the transverse coordinates of the branes can be viewed as Nambu-Goldstone bosons (branons) which appear as a result of spontaneous breaking of the translational symmetry [38]. These are coupled to gravity and matter on the brane in the brane-world models via the induced metric (for a recent discussion see [39, 40]). In our case of co-dimension one there is only one such branon. The remaining components of the perturbation \( \delta X^M \) can be removed by suitable transformation of the coordinates on the world-volume, so \( \delta X^n = 0 \) is nothing but the choice of gauge. Note that in this gauge the perturbation of the induced metric \( \delta \gamma_{\mu\nu} \) does not vanish, as it was for the perturbation of the particle ein-bein.

Denoting the physical component as \( \Phi(\sigma^\mu) \equiv \delta X^z \) we obtain the branon \((D-1)\)-dimensional wave equation:

\[
\Box_{D-1} \Phi(\sigma^\mu) = J(\sigma^\mu) , \tag{4.9}
\]

with the source term \( J \equiv J^z \). Substituting (4.5) into the eq. (4.8) we obtain the source term for the branon:

\[
J(\sigma) = -\kappa_D \left[ \frac{1}{2} \eta_{\mu\nu} \tilde{h}^{\mu\nu,z} - \tilde{h} z_{0,0} \right] = -\frac{\lambda vt}{[\gamma^2 v^2 t^2 + r^2]^{1/2}} , \tag{4.10}
\]

where

\[
\lambda = \frac{\kappa^2 D m \gamma^2 \Gamma \left( \frac{D-1}{2} \right)}{4 \pi^{D-1} \left[ r^2 + \alpha^2 \right]^{1/2}} \left( \gamma^2 v^2 + \frac{1}{D-2} \right) . \tag{4.11}
\]

C. Nature of singularity of the source

The source-current has some peculiar features. It is a smooth function of \( r, t \) except for the point \( r = 0 \) where it has singularity at the moment \( t = 0 \) of perforation. This singularity is due to singular nature of the Coulomb field of the point-like particle. Of course the linearized gravity theory can not be trusted as description of the gravitational field near the singularity. Nevertheless, we know that the Fierz-Pauli theory can be used to consistently describe gravitational interaction of point masses treated in terms of distributions. Since the above singularity is essentially due to the point-like nature of the particle stress-tensor, one can hope to be able to describe the whole situation in terms of distributions too.

Consider the branon wave equation with the source treated as distribution. It is easy to see, that in the limit \( t \to 0 \) or \( v \to 0 \) \((\gamma \to 1)\), the source exhibits properties of the \( n = (D-2) \)-dimensional delta-function. Denoting \( \alpha = \gamma vt \), one has:

\[
\lim_{\alpha \to \pm 0} \frac{\alpha}{(r^2 + \alpha^2)^{1/2}} = \begin{cases} 0, & r \neq 0, \\ \pm \infty, & r = 0. \end{cases} \tag{4.12}
\]

The integral of \( J(x) \) over the \( n \)-dimensional space is \( \alpha \)-independent (up to the sign) and finite:

\[
\int J(x) \, d^{D-2}x = -\lambda \Omega_{n-1} \int_0^{\infty} \frac{\alpha \, r^{n-1}}{(r^2 + \alpha^2)^{1/2}} \, dr = -\frac{\pi^{n+1} \lambda}{\Gamma \left( \frac{n+1}{2} \right)} \text{sgn}(\alpha) ,
\]

so the integrand is proportional to the \( n \)-dimensional delta-function. Since \( \text{sgn}(\alpha) = \text{sgn}(t) \), we get therefore:

\[
\lim_{vt \to \pm 0} J(x) = -\frac{\pi^{n+1} \lambda}{\Gamma \left( \frac{n+1}{2} \right)} \text{sgn}(t) \delta^n(r) .
\]

It is worth noting that this limiting distribution will be the same either we consider \( \text{time} \) in the close vicinity of the perforation moment \( t \to 0 \) for any velocity \( v \) of the mass \( m \), or if we consider the limit of the \( \text{small velocity} \) \( v \to 0 \). In the latter case (the quasi-static perforation) this limit holds for sufficiently large \( t \), and since the coefficient \( \lambda \) remains finite as \( v \to 0 \), the point-like source at the right hand side of the branon field equation (4.9) may be attributed to an effective branon “charge”, or the perforation charge.

This notion allows us to better understand the difference between two cases. The first is the static point mass sitting on the brane eternally. Then, coming back to the Eq. (4.7) for branon perturbations, we find that the source term at the right hand side will be zero (i.e. there is no perforation charge in absence of perforation). On the contrary, if the perforation takes place even adiabatically slowly, the branon charge is non-zero. Indeed, in the limit \( v \to 0 \) we will have a point-like source of the branon field:

\[
\lim_{v \to 0} J(x) = Q_B \delta^n(r) ,
\]

where \( Q_B \) is the branon charge.
where an effective branon charge is given by
\[ Q_B = -k \mathrm{sgn}(t). \tag{4.16} \]
This “charge” is a manifestly non-conserved quantity, changing sign at the moment of perforation. For an observer on the brane the perforation therefore looks like a sudden shake, and, as we will see in the next section, the corresponding branon field will be not a static Coulomb field, but an expanding wave.

\section{Constructing the Retarded Solution}

In view of causality, it is reasonable to construct the retarded solution of the branon wave equation (4.9) generated by the source. This can be done using the standard retarded Green’s function on a \((D-1)\)-dimensional flat manifold:
\[ G_{\text{ret}}(x - x') = \frac{1}{(2\pi)^{D-1}} \int \frac{e^{-ik(x-x')}}{k_\mu k^\mu + 2i\epsilon k^\theta} d^{D-1}k, \tag{5.1} \]
satisfying
\[ \Box_{D-1} G_{\text{ret}}(x - x') = \delta^{D-1}(x - x'). \tag{5.2} \]
Parameterizing the \((D-1)\)-dimensional wave-vector as \(k^\mu = (\omega, \mathbf{k})\) and denoting \(k = |\mathbf{k}|\) (not to be confused with \(k\) in the domain wall metric) we present the retarded solution of (4.9) as
\[ \Phi(x^\mu) = -\frac{1}{(2\pi)^{D-1}} \int \frac{e^{-ikx}}{\omega^2 - k^2 + 2i\epsilon \omega} J(k^\mu) d^{D-1}k, \tag{5.3} \]
where \(J(k^\mu)\) is the Fourier-transform of the source\(^1\) (4.10):
\[ J(k^\mu) = -\frac{2\pi^{\frac{D-1}{2}}}{\Gamma(D-\frac{1}{2})} \frac{i\omega}{\gamma^2 \nu^2 k^2 + \omega^2}. \tag{5.4} \]

Integration in (5.3) is straightforward, but it is worth giving here some details in order to show the origin of two physically different components of the solution. Substituting (5.4) into (5.3) we pass to spherical coordinates in the spatial sector of the momentum space \(d^{D-1}k = k^{D-3} dk d\omega d\Omega_{D-3}\). Using \(kr = kr \cos \theta\), we integrate over the angles:
\[ \int e^{\pm i\epsilon \cos \theta} d\Omega_n = \frac{(2\pi)^{\frac{D-1}{2}}}{z^{\frac{D-1}{2}}} J_{\frac{D-1}{2}}(z), \tag{5.5} \]
(derivation is given in (A.4)), and split the integrand into the sum of three terms:
\[ \Phi = \frac{2^{D/2} k \lambda}{\sqrt{\pi} \gamma \Gamma \left(D-\frac{1}{2}\right)} \int_0^\infty dk J_{D-4}(kr) k^\frac{D-4}{2} \int_{-\infty}^\infty d\omega e^{-i\omega t} \left( \frac{1}{\omega - k + i\epsilon} + \frac{1}{\omega + k + i\epsilon} - \frac{2\omega}{\omega^2 + \gamma^2 \nu^2 k^2} \right). \tag{5.6} \]
Note that the first two terms in the bracket correspond to solution of the homogeneous equation, while the last is related to the source. All the three integrals over \(\omega\) can be evaluated by the contour integration:
\[ \int_{-\infty}^\infty e^{-i\omega t} \frac{\omega}{\omega^2 + \gamma^2 \nu^2 k^2} d\omega = -i\pi \mathrm{sgn}(t) e^{-k\gamma \nu |t|}, \tag{5.7} \]
where \(\theta(t)\) is the Heaviside function and \(\mathrm{sgn}(t)\) is the sign function. So we obtain:
\[ \Phi = \Phi_a + \Phi_b, \quad \Phi_a = -\Lambda \mathrm{sgn}(t) I_a, \quad \Phi_b = 2\Lambda \theta(t) I_b, \quad \Lambda = \frac{\sqrt{\pi} \lambda}{2^{D/2} \gamma \Gamma \left(D-\frac{1}{2}\right)}, \tag{5.8} \]
\(^1\) Computation is presented in the Appendix A, the Eq.(A.8).
where

\[ I_a(t, r) = \frac{1}{r} \int_0^\infty dk \frac{J_{D-4}(kr)}{\sqrt{kr}} e^{-k\gamma v|t|}, \]  

\[ I_b(t, r) = \frac{1}{r^{D+4}} \int_0^\infty dk (t,r)J_{D-4}(kr) k^{D-6} \cos kt. \]  

Being averaged over the time of collision, the first term \( \Phi_a \) ("antisymmetric") in (5.8) vanishes; furthermore, it is suppressed by the factor \( \gamma \) in the exponent for an ultrarelativistic collision. On the contrary, the second term \( \Phi_b \) ("branon") starts to be active at the moment of perforation and remains non-zero afterwards. Note that the quantities \( I_a \) and \( I_b \) are defined as time-symmetric. Thus we have two type of integrals over \( k \) involving Bessel functions of an argument \( kr \). The first, \( \Phi_a \), can be integrated for general \( D \) in terms of hypergeometric function:

\[ \Phi_a = \frac{\lambda}{2\gamma^3 \Gamma\left(\frac{D-1}{2}\right) r^{D-3}} \left[ \gamma v t \Gamma\left(\frac{D-3}{2}\right) \left[ 2F1\left(1, \frac{D-3}{2}; \frac{3}{2}; -\frac{\gamma^2 v^2 t^2}{r^2}\right) - \frac{\sqrt{r} \gamma v t}{2} \text{sgn}(t) \Gamma\left(D-\frac{4}{2}\right) \right] \right], \]  

while for \( \Phi_b \) there is no universal formula. Taking into account the recurrence relation for Bessel functions

\[ \frac{1}{z^{\nu+1}} \frac{\partial}{\partial z} J_{\nu}(z) = \frac{J_{\nu+1}(z)}{z}, \]  

the integrals of both types can be obtained by consequent differentiation over \( r \), taking the lowest dimensions \( D = 5 \) and \( D = 6 \) as generating.

### A. Odd \( D \)

Consider first the case \( D = 5 \). For arbitrary \((t, r)\) we start directly with (5.9) substituting \( J_{1/2}(z) = \sqrt{2/\pi} z \sin z \):

\[ I_a = \frac{1}{\sqrt{kr}} \int_0^\infty \frac{J_{1/2}(kr)}{\sqrt{kr}} e^{-k\gamma v|t|} dk = \frac{1}{\sqrt{2\pi} r} \left( \pi - 2 \arctan \frac{\gamma v |t|}{r} \right) = \frac{2}{\sqrt{2\pi} r} \arctan \frac{r}{\gamma v |t|}, \]  

which covers all values of \( t \) and \( r \). Then, using the recurrence relation (5.12) one obtains for general odd \( D \geq 5 \):

\[ I_a = \frac{2}{\sqrt{2\pi}} \left( -\frac{1}{r} \frac{\partial}{\partial r} \right) \frac{\gamma v |t|}{r} \arctan \frac{r}{\gamma v |t|}, \]

\[ \Phi_a = -\frac{2\Lambda}{\sqrt{2\pi}} \left( -\frac{1}{r} \frac{\partial}{\partial r} \right) \frac{\gamma v |t|}{r} \arctan \frac{r}{\gamma v |t|}. \]  

The second type integral (5.10) in five dimensions is

\[ I_b = \frac{2}{\sqrt{2\pi} r} \int_0^\infty \frac{\sin kr}{k} \cos kt dk = \frac{\sqrt{2\pi}}{4r} \left[ \text{sgn}(r+t) + \text{sgn}(r-t) \right]. \]  

Multiplying by the identity \( 1 = \theta(t) + \theta(-t) \), for \( t > 0 \) one has \( \theta(t) \text{sgn}(r+t) \equiv 1 \) and furthermore \( \text{sgn}(r+t) + \text{sgn}(r-t) = \theta(r-t) \), while for \( t < 0 \) one has \( \theta(t) \text{sgn}(r-t) \equiv 1 \) and \( \text{sgn}(r+t) + \text{sgn}(r-t) = \theta(t+r) \). Thus we deduce

\[ I_b = \frac{\sqrt{2\pi}}{2r} \theta(r-|t|), \]

\[ \Phi_b = \frac{\sqrt{2\pi} \Lambda}{r} \theta(t) \theta(r-t), \]  

which is the spherical shock wave starting at the moment of perforation moving outward. Applying again the recurrent relation (5.12) one obtains for general odd \( D \geq 5 \):

\[ I_b = \frac{\sqrt{2\pi}}{2} \left( -\frac{1}{r} \frac{\partial}{\partial r} \right) \frac{\gamma v |t|}{r} \theta(r-|t|), \]

\[ \Phi_b = \sqrt{2\pi} \Lambda \theta(t) \left( -\frac{1}{r} \frac{\partial}{\partial r} \right) \frac{\gamma v |t|}{r} \theta(r-t). \]  

At the moment of perforation \( t = 0 \) the \( \Phi_b \) part of the solution has a jump at any point on the wall outside the perforation point. Its value is obtained by differentiation of \( 1/r \), keeping \( \theta(r-t) \) unchanged:

\[ \delta \Phi_b = \sqrt{2\pi} \Lambda \left( -\frac{1}{r} \frac{\partial}{\partial r} \right) \frac{\gamma v |t|}{r} \theta(r-t). \]
B. Even $D$

Now we start with $D = 6$. The integral (5.9) gives:

$$I_a = \frac{1}{r} \int_0^\infty J_1(kr) e^{-k\gamma v|t|} \, dk = \frac{1}{r^2} \left( 1 - \frac{\gamma v|t|}{\sqrt{\gamma^2 v^2 + r^2}} \right),$$

(5.19)

while the second integral (5.10) is

$$I_b = \frac{1}{r} \int_0^\infty J_1(kr) \cos(kt) \, dk = \frac{\theta(r - |t|)}{r^2} - \frac{\theta(|t| - r)}{\sqrt{r^2 - |t|^2 + \sqrt{r^2 - r^2}}}. $$

(5.20)

Applying the recurrence relation (5.12) we obtain for generic even $D \geq 6$:

$$I_a = \left( - \frac{1}{r} \frac{\partial}{\partial r} \right)^{D-6} \left[ \frac{1}{r^2} \left( 1 - \frac{\gamma v|t|}{\sqrt{\gamma^2 v^2 + r^2}} \right) \right],$$

(5.21)

and

$$I_b = \left( - \frac{1}{r} \frac{\partial}{\partial r} \right)^{D-6} \left( \frac{\theta(r - |t|)}{r^2} - \frac{\theta(|t| - r)}{\sqrt{r^2 - |t|^2 + \sqrt{r^2 - r^2}}} \right). $$

(5.22)

The value of the jump at $t = 0$ is given by

$$\delta \Phi_b = 2\Lambda \left( - \frac{1}{r} \frac{\partial}{\partial r} \right)^{D-6} \frac{1}{r^2} = \frac{2\Lambda(D - 6)!}{r^{D-4}}, \quad D = \text{even}. $$

(5.23)

C. Properties of the solution

The solution obtained has some unexpected features. One could think that the particle approaching the wall will continuously deform it outward through the gravitational repulsion. After perforation similar deformation could be expected in the opposite direction. This kind of deformation is present in our solution indeed as the antisymmetric in time component $\Phi_a$. However, this component alone does not satisfy the branon wave equation (4.9) at the moment of perforation in the sense of distributions. Indeed, at the moment of perforation $t = 0$ the function $\Phi_a$ has a discontinuity for all $r > 0$ equal to

$$\delta \Phi_a = - \frac{\lambda \sqrt{\pi^2}}{2 \gamma^3} \frac{(D-2)}{r^{D-4}}, $$

(5.24)

that immediately follows from the second term in the parenthesis of (5.11). Physical origin of this jump is obvious: the force of interaction remains constant while particle approaches the wall, but it instantaneously changes sign at the moment of perforation. To see the details we first act by the $(D-1)$—dimensional box on the function $I_a$. Since (5.9) does not contain stepwise functions, the second time derivative will be continuous:

$$\tilde{I}_a = \frac{\gamma^2 v^2}{r^{D-4}} \int_0^\infty dk J_{D-2}(kr) k^{\frac{D-2}{2}} e^{-k\gamma v|t|}. $$

(5.25)

The $n$-dimensional laplacian of $I_a$ is acting only on the radial variable, so we can replace it by

$$\Delta_{D-2} = \frac{1}{r^{D-3}} \frac{\partial}{\partial r} \left( r^{D-3} \frac{\partial}{\partial r} \right).$$

Using (5.12) and the identity

$$\left( \frac{1}{z} \frac{\partial}{\partial z} \right) \left( z^\nu J_\nu(z) \right) = z^{\nu-1} J_{\nu-1}(z),$$

(5.26)

one obtains

$$\Delta_{D-2} I_a = - \frac{1}{r^{D-2}} \int_0^\infty dk J_{D-2}(kr) k^{\frac{D-2}{2}} e^{-k\gamma v|t|}. $$

(5.27)

---

2 Hereafter dots over $\Phi(r,t)$ and its constituents denote the derivative over $t$. 

Combining (5.25) and (5.27) and using the table integral from [41] we obtain:

\[
\Box_D^{-1} I_a = \frac{\gamma^2}{r} \int_0^\infty dk J_{D-4}(kr) k^{\frac{D-6}{2}} e^{-k \gamma vr} = \frac{\Gamma\left(\frac{D-1}{2}\right)}{2^{\frac{D-2}{2}} \sqrt{\pi} \gamma^2 v^2 t^2 + r^2} \frac{\gamma^3 v |t|}{kr}.
\]  

(5.28)

Now apply the box to the antisymmetric part. Since \( I_a \) is time-symmetric, then \( \tilde{I}_a \) is time-antisymmetric, so in the sense of distributions \( \tilde{I}_a(t=0) = 0 \), thus \( \delta(t) \tilde{I}_a = 0 \). Hence

\[
\Box_D^{-1} \Phi_a = -\Lambda \text{sgn}(t) \Box_D^{-1} I_a - 2 \Lambda \delta'(t) I_a |_{t=0}.
\]

The first term comes from (5.28), while the second peaks up the jump of \( \Phi_a \):

\[
I_a |_{t=0} = -\frac{\delta \Phi_a}{2},
\]

(5.29)

with \( \delta \Phi_a \) given by (5.24). Therefore acting on \( \Phi_a \) by the box operator we will get the extra term at the right hand side, namely

\[
\Box_D^{-1} \Phi_a = -\frac{\lambda vt}{[\gamma^2 v^2 t^2 + r^2]^\frac{D-4}{2}} + \delta \Phi_a \delta'(t),
\]

(5.30)

Since the delta-derivative term is not present in the source of the branon equation (4.9), this function is not the full solution yet.

The second part of the solution \( \Phi_b \) just compensates the delta-derivative term. In fact its presence in the full deformation was discovered by careful evaluation of the retarded solution. It is instructive first to act by the box operator on the function \( I_b \). Differentiating separately (in the integral form (5.10)) on time and spatial coordinates one finds, using the Fourier-transforms (5.6), (5.8) and (5.10) in any dimension

\[
\tilde{I}_b = \Delta_D^{-2} I_b = -\frac{1}{r^2} \int_0^\infty dk J_{D-4}(kr) k^{\frac{D-6}{2}} \cos kt = -\frac{2^{\frac{D-2}{2}} \sqrt{\pi} |t| t^2 - |r|^2 \frac{2}{\Gamma\left(\frac{D-2}{2}\right)}}{\gamma^2 v^2 t^2 + r^2},
\]

(5.31)

so it is a solution of the homogeneous wave equation

\[
\Box_D^{-1} I_b = 0.
\]

(5.32)

Note that in odd dimensions the function (5.31) can be expressed by derivatives of the delta-function by virtue of the distributional property [43]

\[
\lim_{\lambda \to n} \frac{|x|^\lambda}{\Gamma(\lambda + 1)} = \delta^{(n-1)}(x), \quad n \in \mathbb{N}.
\]

Using this we obtain for the second time derivative

\[
\tilde{I}_b = -2^{\frac{D-2}{2}} \sqrt{\pi} t \delta\left(\frac{D-2}{2}\right)(t^2 - r^2).
\]

(5.33)

However, \( I_b \) represents the solution part only for \( t > 0 \), so acting by box on \( \Phi_b \) we obtain (again using the \( t = 0 \) limit in (5.11)):

\[
\Box_D^{-1} \Phi_b = 2 \Lambda \delta'(t) I_b |_{t=0} = \frac{\lambda \sqrt{\pi} \Gamma\left(\frac{D-4}{2}\right)}{2 \gamma^2 v^2 t^2} \frac{D-4}{r^{D-4}} \delta'(t).
\]

(5.34)

The right hand side exactly compensates the undesirable delta-derivative term in (5.30). According to definitions (5.9,5.10), the \( t = 0 \) limits \( I_b \) and \( I_a \) coincide:

\[
I_b |_{t=0} = I_a |_{t=0} = \frac{1}{r^2} \int_0^\infty dk J_{D-4}(kr) k^{\frac{D-6}{2}},
\]

(5.35)

so \( I_b |_{t=0} \) is also given by (5.29). So we conclude that \( \delta \Phi_b = -\delta \Phi_a \) and the total \( \Phi(r,t) \) is continuous at \( t = 0 \) for any \( r > 0 \).

Another split is also useful: both expressions (5.21) and (5.13) for \( I_a \) (with the corresponding differentiation for higher odd dimensions) contain two terms – one is reminiscent of the static contribution computed in (5.23):

\[
I_a = I_a^{\text{stat}} + I_a^{\text{dyn}}, \quad I_a^{\text{stat}} = \frac{2^{\frac{D-6}{2}} \Gamma\left(\frac{D-4}{2}\right)}{r^{D-4}},
\]

(5.36)

and another (“dynamical”) vanishing at \( t = 0 \) represents the hypergeometric part in (5.11):

\[
I_a^{\text{dyn}} = -2^{\frac{D-2}{2}} \Gamma\left(\frac{D-3}{2}\right) \frac{\gamma \gamma v |t|}{\sqrt{\pi} r^{D-3}} 2F_1\left(\frac{1}{2}, \frac{D-3}{2}; \frac{3}{2}; \frac{\gamma^2 v^2 t^2}{r^2}\right).
\]

(5.37)

In this split the component \( I_a^{\text{stat}} \) satisfies the static Poisson equation:

\[
\Delta_D^{-2} I_a^{\text{stat}} = -(2\pi)^\frac{D-2}{2} \delta^{D-2}(r),
\]

(5.38)

where \( r \) is a spatial coordinate on the wall. Such a decomposition will be useful in constructing solution in four dimensions.
FIG. 1: Behavior of $\Phi_\alpha$ in five (a) and six (b) dimensions for fixed value of $t > 0$ in $t = 1$ scale and units $\kappa_D = m = 1$, for $\gamma = 100$ (black, dashdotted), 500 (red, solid) and 1000 (green, dashed).

1. Other limits

Some other limiting cases will be useful:

$r = 0$, $t$ fixed. First consider the values of $\Phi$ at the perforation point $r = 0$ for any time except the perforation moment. In the Eqs.(5.9, 5.10) one makes use

$$x^{-n}J_n(x) = \frac{1}{2^n \Gamma(n+1)} \left(1 + O(x^2)\right)$$

(5.39)

to obtain

$$I_a(t, 0) = \frac{1}{2^{\frac{D-5}{2}} \Gamma(\frac{D-2}{2})} \int_0^\infty dk k^{D-5} e^{-k\gamma v|t|} \frac{\Gamma(D-4)}{2^{\frac{D-5}{2}} \Gamma(\frac{D-2}{2})} \frac{1}{(\gamma v|t|)^{D-4}},$$

(5.40)

and

$$I_b(t, 0) = \frac{1}{2^{\frac{D-5}{2}} \Gamma(\frac{D-2}{2})} \int_0^\infty dk k^{D-5} \cos(kt) \frac{1}{2^{\frac{D-4}{2}} \Gamma(\frac{D-2}{2})} \times \begin{cases} \frac{\sqrt{2}}{\pi} \gamma v |t|^{-\frac{D-4}{2}} \delta(\frac{D-5}{2}) \left(t\right), & D = \text{odd} \\ \frac{\sqrt{2}}{\pi} \gamma v |t|^{-\frac{D-4}{2}} \Gamma(\frac{D-4}{2}) \left(t\right), & D = \text{even} \end{cases}.$$  

(5.41)

As a function of $\gamma$, $I_a(t, 0)$ scales as $(\gamma v)^{-(D-4)}$. In the ultrarelativistic case $v \simeq 1$, $\gamma \gg 1$ and combined with $\Lambda \Phi_\alpha$ scales as

$$\Phi_\alpha(t, 0) \simeq -\frac{\kappa_\alpha^2 m \Gamma(D-4)}{2^{D-1} \pi^{\frac{D-2}{2}} \Gamma(\frac{D-2}{2})} \frac{1}{(\gamma v|t|)^{D-4}} \sgn(t).$$

(5.42)

Plots of $\Phi_\alpha$ as function of $r$ with fixed $t > 0$ and ultrarelativistic $\gamma$ are given in the Fig. 1.

The behavior of curves on these plots near $r = 0$ confirms the $\gamma$-scaling (5.42).

$r \gg t$, $t$ fixed. For $D = 5$ one takes directly (5.13) and (5.16) to deduce:

$$I_a \simeq \sqrt{\frac{\pi}{2^6 r}} \frac{1}{r \rightarrow 0}, \quad I_b \simeq \frac{\sqrt{2\pi}}{2 r} \frac{1}{r \rightarrow 0}.$$  

(5.43)

For $D > 5$ we use an asymptotic expansion $J(x) \simeq \sqrt{\frac{2}{\pi x}} \cos(x + \pi/4)$ to estimate

$$|I_a| \leq \sqrt{\frac{2}{\pi}} \frac{1}{2^{\frac{D-7}{2}}} \int_0^\infty dk k^{\frac{D-7}{2}} e^{-k\gamma v|t|} = \sqrt{\frac{2}{\pi}} \frac{\Gamma(\frac{D-5}{2})}{\gamma v|t|^D} \frac{1}{2^{\frac{D-4}{2}}} \frac{1}{r \rightarrow 0},$$

(5.44)

$$|I_b| \leq \frac{Q(t)}{2^{\frac{D-7}{2}}} \frac{1}{r \rightarrow 0}, \quad Q(t) = \sqrt{\frac{2}{\pi}} \int_0^\infty dk k^{\frac{D-7}{2}} \cos(kt).$$

(5.45)

Thus both $I_b$ and $I_a$ decay at large distances.

The characteristic behavior of $I$ and $\Phi$ at fixed $t = 1$ is given on Fig. 2 ($D = 5$) and Fig. 3 ($D = 6$).
**FIG. 2:** Dependence of brane perturbations on \( r \) for fixed value of \( t \) for \( D = 5, \gamma = 2 \).

**FIG. 3:** \( D = 6, \gamma = 2 \).

**t \gg r, r \text{ fixed.}** First consider \( D = 5 \) case: from (5.13) and (5.16) we deduce directly

\[
I_a \big|_{t \gg r} \approx \frac{2}{\sqrt{2\pi}} \frac{1}{\gamma \nu |t|} \quad \text{as} \quad t \to \infty \to 0, \quad \lim_{r \to \infty} I_b = \lim_{r \to \infty} \frac{\sqrt{2\pi}}{2r} \theta(r - |t|) = 0. \tag{5.46}
\]

According to [42], the function \(|x^{-\lambda}J_\lambda(x)|\) reaches its absolute maximum at \( x = 0 \) for all \( \lambda \geq -1/2 \), thus in (5.9) the Bessel function is restricted by its near zero value (5.39), implying

\[
|I_a| \leq \frac{\Gamma(D - 4)}{2^{D-2} \Gamma(D-2)} \frac{1}{(\gamma \nu |t|)^{D-4}} \quad t \to \infty \to 0 \tag{5.47}
\]
by virtue of (5.40).

Now pass to $I_b$ given by (5.10): in (5.35) we have shown that the integral
\[
\frac{1}{r^{D-2}} \int_0^\infty dk J_{D-4}(kr) k^{D-6} \to 0
\]
converges for all $D > 4$ with the boundary value (5.29). Thus, according to the Riemann–Lebesgue lemma,
\[
\lim_{t \to \infty} I_b = 0.
\]

The characteristic behavior of $I$ and $\Phi$ at fixed $r = 1$ is given on Fig. 4 ($D = 5$) and Fig. 5 ($D = 6$).

Note that after the perforation $t > 0$ the shock wave $\Phi_b$ satisfies the homogeneous wave equation, describing the free branon wave propagating independently of further motion of the particle. One can notice certain analogy with the effect of propagation of the string boundary of the hole, punctured by a body of finite size (such as black hole) in the domain wall in the field-theoretical models admitting hybrid topological defects. In that case the string boundary may expand outwards eating the wall as was argued in [33].

6. LIGHT-LIKE PERFORATION

It is interesting to consider separately an ultrarelativistic limit $\gamma \to \infty$, covering also the case of the massless particle.

A. Shock-wave in arbitrary dimensions

First we derive the metric generated by a light-like point particle with the energy $\mathcal{E}$ moving in the $D$–dimensional bulk along the trajectory $z = t$. It was found that the infinitely-boosted Schwarzschild metric by Aichelburg and Sexl [44] in four dimensions, being exact, coincides with the solution of linearized theory with point-like massive source, moving with a speed of light. For completeness we rederive the linearized solution in our notation. The field (4.5), being expressed via the energy $\mathcal{E}$ and the momentum $p^M$, reads
\[
\bar{h}_{MN}(x) = -\frac{\mathcal{E} \Gamma(D-3)}{4\pi^{D-1} \mathcal{E}^2} \left( \frac{\mathcal{E}^2}{\mathcal{E}^2} p_M p_N - \frac{1}{D-2} \eta_{MN} \right) \left[ \frac{1}{[\mathcal{E}^2 (z-t)^2 + r^2]^{1/2}} \right].
\]
Reducing for large $\gamma$ to
\[
\bar{h}_{MN}(x) = -\frac{\mathcal{E} \Gamma(D-3)}{4\pi^{D-1} \mathcal{E}^2} \lim_{\gamma \to \infty} \frac{\gamma}{[\mathcal{E}^2 (z-t)^2 + r^2]^{1/2}}.
\]
Consider first the case $D = 5$:
\[
\bar{h}_{MN}(x) = -\frac{\mathcal{E} \Gamma(2)}{4\pi^2} c_M c_N \lim_{\gamma \to \infty} \frac{\gamma}{[\mathcal{E}^2 (z-t)^2 + r^2]^{1/2}}
\]
with $c_M \equiv p_M/\mathcal{E} = (1,0,0,0,1)$ being the null ”unit” vector. Introducing $\alpha = r/\gamma$, one ends up with
\[
\bar{h}_{MN}(x) = -\frac{\mathcal{E} \Gamma(2)}{4\pi^2} c_M c_N \lim_{\alpha \to 1} \frac{\alpha}{(z-t)^2 + \alpha^2} = -\frac{\mathcal{E} \Gamma(2)}{4\pi^2} \delta(z-t) c_M c_N,
\]
which is just the five-dimensional shock-wave metric.

In higher dimensions we start with (4.4):

\[ h_{MN}(q) = \frac{2\pi \kappa_D \delta(q^0 - q^0)}{q^2 + i\varepsilon q^0} \left( \reality{MN} - \frac{1}{D - 2} \frac{\varepsilon^2}{\gamma^2} \right), \]  

(6.5)

reducing in the limit \( v = 1, \gamma \rightarrow \infty \) to:

\[ h_{MN}(q) = \frac{2\pi \kappa_D \delta(q^0 - q^0)}{q^2 + i\varepsilon q^0} c_{MN}. \]  

(6.6)

Inverting the Fourier-transform according to the definition (4.3) and integrating over \( q^0 \) we obtain

\[ h_{MN}(x) = -\frac{\kappa_D \varepsilon}{(2\pi)^{D-1}} c_{MN} \int \frac{e^{iqr}}{q^1} d^{D-2}q \int e^{iq(t - t')} dq^1. \]  

(6.7)

This gives the product of \( \delta(z - t) \) with the Coulomb potential in \( \mathbb{R}^{D-2} \), so in \( D > 4 \) one has:

\[ h_{MN}(x) = -\frac{\kappa_D \varepsilon}{4\pi^{D-2} r^{D-4}} \delta(z - t) c_{MN} \]  

(6.8)

and in four dimensions

\[ h_{MN}(x) = \frac{\kappa_D \varepsilon \delta(z - t)}{2\pi} \ln r c_{MN}. \]  

(6.9)

For \( D = 5 \) (6.8) coincides with (6.4).

Going back from (6.8) to (6.2) one finds the following identity for the delta-like sequence:

\[ \lim_{\alpha \rightarrow 0+} \frac{\alpha^{2n-1}}{(x^2 + \alpha^2)^{\gamma}} = \frac{\sqrt{\pi \Gamma(n - \frac{1}{2})}}{\Gamma(n)} \delta(x). \]  

(6.10)

\[ B. \text{ Branon solution} \]

The source term in the branon equation in the light-like limit reads:

\[ \lim_{\gamma \rightarrow \infty} J(x) = -\frac{\kappa_D^2 \varepsilon \Gamma \left( \frac{D-1}{2} \right)}{4\pi^{D-1}} \frac{\gamma^3 t}{\left( \gamma^2 t^2 + r^2 \right)^{\frac{D-1}{2}}} = \frac{1}{2} \frac{\kappa_D^2 \varepsilon \Gamma \left( \frac{D-1}{2} \right)}{4\pi^{D-1}} \frac{\partial}{\partial \gamma} \left( \gamma^2 t^2 + r^2 \right)^{\frac{D-1}{2}} \delta'(t). \]  

(6.11)

Thus the source flashes only at the moment of perforation \( t = 0 \), and the domain wall gets excited only after the collision. The surviving term in (6.11) should serve as a source for both the antisymmetric part \( \Phi_a \) and the branon component \( \Phi_b \) of the full deformation. However, one can see that the antisymmetric part is absent for the light-like perforation. Indeed, for \( D = 6 \) the limit of (5.19) is zero

\[ I_a = \frac{1}{r^2} \lim_{\gamma \rightarrow \infty} \left[ 1 - \frac{\gamma v|t|}{\sqrt{\gamma v^2 t^2 + r^2}} \right] = \lim_{\gamma \rightarrow \infty} \frac{1}{2(\gamma vt)^2 + O(\gamma^{-4})} = 0. \]  

(6.12)

In higher even dimensions \( I_a \) can be obtained by consecutive application of the operator \( -2\partial/\partial r^2 \). In accord with the Leibnitz rule, differentiation does not affect the \( \gamma \rightarrow \infty \)-limit, the resulting expression

\[ \left( \frac{-1}{r \partial \partial r} \right)^m \left[ 1 - \frac{\gamma v|t|}{\sqrt{\gamma v^2 t^2 + r^2}} \right] = \frac{(2m - 1)! v|t|}{\left( \gamma^2 v^2 t^2 + r^2 \right)^{m+1/2}} \arctan \frac{r}{\gamma v|t|}, \quad m \geq 1 \]

going to zero as \( \gamma \rightarrow \infty \) too.

In odd dimensions \( I_a \) the leading in \( \gamma \) contribution to \( I_a \) is given by

\[ I_a = \frac{2}{\sqrt{2\pi}} \left( \frac{-1}{r \partial \partial r} \right)^m \frac{1}{r} \arctan \frac{r}{\gamma v|t|} \simeq \frac{2}{\sqrt{2\pi}} \frac{(2m - 1)!! |v| |t|^{2m}}{r^{2m+1} \left( \gamma^2 v^2 t^2 + r^2 \right)^m} \arctan \frac{r}{\gamma v|t|}, \quad m \geq 0 \]  

(6.13)

and it is also vanishing in the limit \( \gamma \rightarrow \infty \).

Next, coming to \( \Phi_b \)-part, we see that, taking into account (6.12), the Eq. (5.34) tends to

\[ \lim_{\gamma \rightarrow \infty} \Box_{D-1} \Phi_b = \frac{\kappa_D^2 \varepsilon \Gamma \left( \frac{D-1}{2} \right)}{8\pi^{D-4}} \delta'(t) = \lim_{\gamma \rightarrow \infty} \Box_{D-1} \Phi, \]  

(6.14)

in agreement with (6.11). We conclude that in the light-like limit the full deformation is \( \Phi = \Phi_b \), where

\[ \Phi_b = \frac{\kappa_D^2 \varepsilon}{(2\pi)^{D/2}} \theta(t) I_b, \]  

(6.15)

with \( I_b \) given by (5.17) and (5.22) for odd and even bulk dimensions, respectively.
7. PERFORATION OF THE DOMAIN WALL IN FOUR DIMENSIONS

The case $D = 4$ requires special consideration since some intermediate formulas fail. It is also physically most interesting allowing for applications to the cosmological domain wall problem. In fact, the primordial domain walls moving in the ultrarelativistic gas should be excited via the mechanism described in the previous section.

The linearized fields generated by the particle and the domain wall will read in $D = 4$:

\[
\tilde{h}_{MN}(x) = - \frac{\kappa_4 m}{4\pi} \frac{u_M u_N - \frac{1}{2} \eta_{MN}}{[\gamma^2 (z - vt)^2 + r^2]^{1/2}},
\]

\[
h_{MN}(x) = \frac{\kappa_4 \mu |z|}{4} \text{diag}(-1, 1, 1, 3).
\]

The branon equation (4.9) takes the form

\[
\Box_3 \Phi = - \frac{\lambda vt}{[\gamma^2 v^2 t^2 + r^2]^{3/2}}, \quad \lambda = \frac{\kappa_4^2 m \gamma^2}{8\pi} \left( \gamma^2 v^2 + \frac{1}{2} \right).
\]

Trying naively to adapt for $D = 4$ the integral (5.8)

\[
\Phi = \frac{\lambda}{\gamma^3} \int_0^\infty dk \frac{J_0(kr)}{k} \left( -\text{sgn}(t) e^{-k \gamma v |t|} + 2 \theta(t) \cos kt \right),
\]

one finds that it diverges at $k = 0$, so we need some regularization. To achieve this goal we notice that the solution in the next even dimension $D = 6$ must follow from the $D = 4$ solution by differentiation rules described in Sec. 5. Also taking into account that splitting of the full $\Phi$ into two parts for $D > 4$ is related to causality which is preserved by the differentiation over $r$, we can set following requirements on the $D = 4$ solution:

- the action of $-\frac{1}{r} \frac{\partial}{\partial r}$ must generate separately the six-dimensional solutions (5.20) and (5.19):
- the higher-dimensional expressions remaining regular in the limit $D \to 4$ should be trusted in the $D = 4$ case, namely
  - the static part $I_a^{\text{stat}}$ should satisfy the condition (5.38), while the dynamical part $I_a^{\text{dyn}}$ have to coincide with (5.37);
  - the matching conditions should hold: $I_a|_{t=0} = I_b|_{t=0}$;
  - $\Box \Phi_a$ has to agree with (5.28) and $\Box \Phi_b$ has to agree with (5.30);
- both $I_a$ and $I_b$ have to be the even functions of $t$ while $\Phi_a$ and $\Phi_b$ should be defined via $I_a$ and $I_b$ in the same way as in (5.9,5.10).

Starting with $I_a^{\text{stat}}$, we see that the Eq. (5.38) is regular at $D = 4$, so using the relevant Green’s function we get

\[
\Delta_3 I_a^{\text{stat}} = -2\pi \delta^3(r), \quad I_a^{\text{stat}} = -\ln r.
\]

Next, the dynamical component $I_a^{\text{dyn}}$ is given by the hypergeometric function in (5.37):

\[
I_a^{\text{dyn}} = -\frac{\gamma v |t|}{r} 2F_1\left( \frac{1}{2}, 1; \frac{3}{2}; -\frac{\gamma^2 v^2 t^2}{r^2} \right) = -\text{Arsh} \frac{\gamma v |t|}{r},
\]

which is analytical for any values of $t$ and $r$. In order to check the differentiation rule it is convenient to modify the inverse trigonometric functions as \(^3\):

\[
I_a = -\ln r - \text{Arsh} \frac{\gamma v |t|}{\sqrt{\gamma^2 v^2 t^2 + r^2}}.
\]

Now one can easily see that (7.7) does coincide with (5.19) upon action of $2 \partial/\partial r^2$, with obvious correspondence to static and dynamical parts of (5.19). Thus $\Phi_a$ becomes

\[
\Phi_a = \frac{\lambda}{\gamma^3} \left( \text{Arsh} \frac{\gamma v t}{r} + \text{sgn}(t) \ln r \right),
\]

\(^3 I_a\) presented here, does not satisfy the condition $I_a \to 0$ for $\gamma \to \infty$ only with logarithmic precision: $\ln \left( \frac{\gamma v |t|}{r} \right) + \text{Arsh} \frac{\gamma v |t|}{r} \to 0$. Notice that this condition is not mandatory.
with the jump at \( t = 0 \) equal to
\[
\delta \Phi_a = \frac{2\lambda}{\gamma^3} \ln r. \tag{7.9}
\]
One can check that
\[
\square_3 I_a = \frac{\gamma^3 v|t|}{[\gamma^2 v^2 t^2 + r^2]^{3/2}}, \quad \square_3 \Phi_a = \frac{2\lambda \ln r}{\gamma^3} \delta(t) - \frac{\lambda vr}{[\gamma^2 v^2 t^2 + r^2]^{3/2}}, \tag{7.10}
\]
in agreement with (5.28) and (5.30).

Finally, turning to the shock-wave part \( I_b \), we see that the action of \( 2\partial/\partial r^2 \) must give (5.20). The expression (5.20) contains two step functions \( \theta(r - t) \) and \( \theta(t - r) \), projecting the continuous functions onto the non-overlapping domains before and after the wave-front \( t = r \). Therefore, both components of (5.20) should be obtainable by the action of \( 2\partial/\partial r^2 \) on the corresponding terms separately, without generating terms localized on the front \( t = r \). In other words, \( I_b \) should be continuous. Thus, multiplying (5.20) by \( r \) and integrating, one obtains:
\[
I_b = -\ln r \theta (r - |t|) - \ln \left(t + \sqrt{t^2 - r^2}\right) \theta(|t| - r). \tag{7.11}
\]
Note that the derivatives of \( I_b \) over \( r \) do not contain \( \delta(|t| - r) \) (canceled between two terms), so the continuity requirement is preserved and (5.20) is reproduced under the action of \( 2\partial/\partial r^2 \) indeed.

It remains to check that the expression (7.11) does satisfy the homogeneous d’Alembert equation. Acting on it by the box we obtain
\[
\square \left[\ln \left(t + \sqrt{t^2 - r^2}\right)\right] = \frac{2 + \ln |t|}{r} \delta(|t| - r) = -\square \left[\ln r \theta (r - |t|)\right], \tag{7.12}
\]
so \( \square I_b = 0 \) indeed.

Hence, according to (5.34), \( \square \Phi_b \) is determined by the value \( I_b|_{t=0} = -\ln r \). It is equal to \( I_a|_{t=0} \) and satisfies the condition (7.7). The required jump condition \( \delta \Phi_b = -\delta \Phi_a \) is satisfied too by virtue of relations between \( \Phi_a \) and \( \Phi_b \) with \( I_a \) and \( I_b \), respectively.

Thereby \( \Phi_b \) satisfies
\[
\Phi_b = -2\lambda \theta(t) \left[\ln r \theta (r - t) + \ln \left(t + \sqrt{t^2 - r^2}\right) \theta(t - r)\right], \tag{7.13}
\]
\[
\square_3 \Phi_b = -\frac{2\lambda \ln r}{\gamma^3} \delta(t), \tag{7.14}
\]
in agreement with (5.34).

8. SUMMARY AND CONCLUDING REMARKS

In this paper we have considered the piercing collision of a point particle with the Nambu-Goto domain wall within the linearized gravity. We have shown that, in spite of the singular nature of the particle’s gravitational field at the perforation point in space-time, this process can be safely described in terms of distributions. From our analysis it follows that the domain wall gets excited after the perforation via creation of the spherical branon shock wave propagating outwards with the speed of light. This wave is the reaction of the wall on the instantaneous change of particle’s acceleration on a finite amount due to homogeneous nature of the domain wall gravitational field.

This effect completes the standard picture of interaction of domain walls with particles developed earlier to describe propagation of the domain walls through matter in the early universe [3]. At low temperature most of the surrounding particles are reflected from the wall transferring to it some momentum. This results in the friction force acting on the domain wall. We have shown that those particles, which are not reflected, do not transfer momentum to the wall, but cause an excitation effect. Since we have restricted our consideration here by the linearized approximation, the resulting energy dissipation could not be seen, but in fact it can be expected in the form of gravitational radiation once higher-order effects are included. For this one has to extend the theory to the next post-linear order similarly to the case of colliding particles [45–48].

One could notice certain analogy between our effect and an essentially non-linear phenomenon which may accompany perforation of the field-theoretical domain wall in the models admitting hybrid wall/cosmic string

\[\text{Making use of the identity } Arsh x = \ln \left(x + \sqrt{x^2 + 1}\right), \quad I_a \text{ (7.7) can be equivalently presented as} \]
\[I_a = -\ln \left(\gamma v|t| + \sqrt{\gamma^2 v^2 t^2 + r^2}\right).\]

Acting by the box on \( I_a \) we do not get the \( \delta^2(\tau) \)-term.
defects. Namely, the hole in the wall may be surrounded by a cosmic string which either collapses restoring the initial shape of the domain wall or expands to infinity completely “eating” the wall [33]. This was suggested as mechanism of destruction of domain walls in the early universe. Further investigation have shown that the issue of the above dilemma depends on the number of the created holes: for destruction to occur it is necessary that at least four such holes were produced in the domain wall [34]. Of course these effects can not be touched within our approach, but their nature is still similar: both are related to an intrinsic tension of the relativistic extended objects. Moreover, it is likely that branon excitations in the non-linear field-theoretical domain wall – black hole system were already observed in numerical experiments [35].

As another line of applications, we could mention the Rundall-Sundrum type scenarios [15–17]. Bulk matter perforating the brane representing our universe could generate “unmotivated” explosive events [4] in the observed Universe. Indeed, the branon field universally interacts with matter on the brane via the induced metric [38–40], and, in the non-linear regime the branon explosion will transform into the matter explosion. Speculating further, one could mention that in the brane-world scenarios the branons appear as new relevant low-energy particles [49, 50] whose couplings are suppressed by the brane tension [51]. They are therefore generically stable and weakly interacting, the features making them natural dark matter candidates [52] (for a recent review see [53]). Our mechanism of branon excitation by bulk matter should be explored in this context as well.

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Appendix A: Computation of the Fourier-transforms

Here we give a typical calculation of the Fourier-transforms used in this paper. Consider the branon source in the coordinate space $J(\sigma)$ as given by (4.10):

$$J(\sigma) = -\frac{\lambda v t}{[\gamma^2 t^2 + \sigma^2]^\frac{D-4}{2}}.$$

Substituting this to (4.3) one introduces spherical angles such that $d^{D-2}r = \Omega_{D-4} r^{D-3} \sin^{D-4}\theta dr d\theta$ with $\theta$ being the angle to the polar axis chosen along $k$:

$$J(k) = -\lambda v \Omega_{D-4} \int_0^\pi \int_0^{2\pi} e^{i(z \cos \theta - kr \sin \theta)} \sin^{2k} \theta d\theta d\phi = \frac{2^k \Gamma(k + 1/2) \sqrt{z}}{k^k} J_k(z)$$

(A.2)

Combining the integral [41]

$$\int_0^\pi e^{\pm i z \cos \theta} \sin^{2k} \theta d\theta = \frac{2^k \Gamma(k + 1/2) \sqrt{z}}{k^k} J_k(z)$$

(A.3)

with the volume measure $\Omega_{n-1}$, the full angular integral of $e^{\pm i z \cos \theta}$ can be presented as

$$\int e^{\pm i z \cos \theta} d\Omega_n = \frac{(2\pi)^{\frac{D-2}{2}}}{z^{\frac{D-2}{2}}} J_{\frac{D-2}{2}}(z).$$

(A.4)

Thereby one gets

$$J(k) = -\frac{(2\pi)^{\frac{D-2}{2}} \lambda v}{k^{\frac{D-2}{2}}} \int_0^\pi e^{i\omega t \frac{D-2}{2}} \frac{r^{D-2}}{[\gamma^2 t^2 + r^2]^{\frac{D-4}{2}}} J_{\frac{D-4}{2}}(kr) dt dr.$$

(A.5)

Integration over $r$ is done using the integral:[54]

$$\int_0^\infty dy \frac{y^{n+1} J_n(by)}{y^2 + a^2} = \frac{b^{m-1} a^{n-m+1}}{2^{m-1} \Gamma(m)} K_{n-m+1}(b|a)),$$

(A.6)

and then

$$J(k) = -\frac{\pi^{\frac{D-1}{2}} \lambda}{\Gamma \left(\frac{D-2}{2}\right)} \int_{-\infty}^{\infty} e^{i\omega t} e^{-\gamma \nu |t|} \text{sgn} t dt,$$

(A.7)
where only the imaginary part of the exponent survives by parity. Finally, one ends up with

$$J(k) = -rac{2\pi^{D-2}}{\Gamma(D-1)} \int_0^\infty e^{-\gamma \omega t} \sin\omega t \, dt = \frac{-2\pi^{D-2}}{\Gamma(D-1)} \frac{i\omega}{\gamma} \frac{\omega^2 + \gamma^2 v^2 k^2}{\omega^2}.$$  \hspace{1cm} (A.8)

In the context of the RSII models we need to require that the dominant contribution in the \(r\)-integration will come from the region of validity of linearized approximation, which means

$$r < 1/k, \quad \gamma vt < 1/k.$$  

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