The evolution of information entropy components in relativistic heavy-ion collisions

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Abstract The time evolution process of thermodynamic entropy $S_{\text{thermal}}$, multiplicity entropy $S_{\text{mul}}$, and configuration entropy $S_{\text{conf}}$ at the relativistic heavy-ion collisions is studied using the AMPT model to generate central Au + Au collision events. By superimposing the three kinds of information entropy, we can get a complete information entropy of the system to describe the physical process of the relativistic heavy-ion collisions. The results show that the four stages of the time evolution process of the system entropy $S$ seem to correspond to the four physical processes in the relativistic heavy-ion collision, indicating that the total entropy of the system can reflect the physical information more accurately in the relativistic heavy-ion collision. This also shows that Shannon information entropy does provide an effective tool to study the evolution process in the relativistic heavy-ion collisions.

1 Introduction

The process of phase transition in Relativistic heavy-ion collisions (RHIC) includes the transformation of quark–gluon plasma (QGP) into hadron gas. Physicists believe that this process reproduces the situation of the first 10 $\mu$s after the Big Bang [1]. Further study of the heavy-ion collision process can bring a deeper understanding of the properties of the nuclear matter produced in the interaction of heavy nuclei and the mechanism of the origin of the universe. Generally, The reaction process of RHIC is divided into several stages in different theories: The initial interaction produced many partons. If the colliding nucleus is heavy enough and the incident energy is high enough, the partonic system will reach equilibrium or local equilibrium and forming quark–gluon plasma (QGP). After that, the system expands and the temperature decreases into the hadron gas (HG) stage by phase transition. Finally, Hadrons continue to scatter until the system reaches equilibrium, forming final state particles. Due to the complexity of the dynamic properties of heavy-ion collisions, there is no unified theory to describe the whole reaction process at present. In this dilemma, researchers use different theoretical approaches to describe different stages in nuclear collisions [2].

Although the evolution process of RHIC is so complex, if we take a view from the perspective of change types in RHIC, we will find that there are always accompanied with three types of change in the evolution of RHIC: the scattering between particles, the production and annihilation of particles, and the variation of the internal structure of particles. If we can make a quantitative description of the different types of changes in the process of RHIC, a unified description of this process can be obtained from a new perspective. This is the main motivation of introducing information entropy into the study of heavy-ion collisions.

The concept of entropy originated from the development of thermodynamics and statistical mechanics. Based on the works [3–6] of Boltzmann and Gibbs in the last 20 years of the nineteenth century, today we understand that the entropy of the system can be determined by its specific probability distribution $p_i$, for a system in state $i$, which can be expressed as follows

$$S = -k_B \sum_i p_i \ln p_i,$$  \hspace{1cm} (1)

where $p_i$ (i=1, 2, ..., n) is the independent probabilities of events in a system and $k_B$ is Boltzmann’s constant. For a given set of constraints, when $p_i$ is the most probable state of the system, the entropy has a maximum value. In terminology of physics, when the system is in the most probable distribution, its entropy is the maximum, which corresponds to its equilibrium state. However, in 1948, Shannon found a theorem similar to the definition of “entropy” in physics.
to measure the amount of information [7, 8], which can also be expressed as Eq. (1). It’s important to note that shannon’s entropy is more general than boltzmann’s entropy, because shannon’s entropy does not require that the probabilities of the microscopic states in the system must be uniform. But this assumption is made for many systems also in boltzmann’s approach. This enables shannon entropy to study not only equilibrium systems but also non-equilibrium systems. This advantage makes it a good tool to measure the chaotic evolution and information loss in dynamical evolution of system.

In the 1980s, some researchers simultaneously devoted themselves to the study of entropy generated in collision process by using different dynamic models [9]. The adoption and development of shannon information entropy in heavy-ion collisions are frequently by reserachers in recently [10]. Cao and Hwa first applied Shannon information entropy to the study of chaotic behavior caused by particle production in branching process [11]. In 1996, Ma adopted the idea of “event entropy” to obtain a novel signature of liquid gas phase transition under the references of transition temperature in 1999 [12]. In the study of intermediate mass fragments, Ma et al. adopted the event information entropy found the isobaric scaling phenomenon in neutron-rich projectile fragmentation reactions [13], as well as, in the fragments differing of different neutron-excesses [14]. Recently, Xu and Ko investigated the chemical freeze-out conditions in RHIC by specific entropy of hadrons [15].

The concept of entropy has an internal relationship with the complexity of the system, so generally speaking, it can not be simply divided into different entropy. In order to observe the changes of different types of information in the process of heavy ion collision evolution, the entropy of the system is divided into thermodynamic entropy, multiplicity entropy and configuration entropy. We assume that the total entropy of the system is exactly the algebraic sum of the entropy of each part. Here, the distribution \( p_i \) of thermodynamic entropy is defined by the distribution of 6-dimensional phase space of particles, which measures the disorder degree of particles in phase space, and the distribution \( \{ p_i \} \) of multiplicity entropy is defined by the event probability of having \( i \) particles produced, which measures the disorder of particles in event phase. The adoption of configuration entropy in this work is inspired by Csernai et al. [16] and Lichtenberg [17], makes us consider the information variables generated by the different composition of quarks inside hadrons, because hadrons are not a point particle. The introduction of these three different entropies will be detailed in the following section. After obtained various types of information entropy, a unified quantitative description to the process of heavy-ion collision will be obtained in the end.

The organization of the present paper is as follows. In Sect. 2, The A Multiphase Transport Model (AMPT), which we used in this work, will be briefly introduced. In Sects. 3, 4 and 5, the thermodynamic entropy, multiplicity entropy and configuration entropy in the evolution of heavy-ion collisions is investigated. In Sect. 6, the total information entropy in the evolution of heavy-ion collisions is investigated as well. A conclusion of this paper is presented in Sect. 7.

2 The AMPT model

A multiphase transport model [18] is used to analyze the evolution of information entropy, which is a widely used theoretical tool for relativistic heavy-ion collisions. The AMPT model is based on nonequilibrium transport dynamics, which consists of four parts: the Heavy-Ion Jet I Nteraction Generator (HIJING) model [19] for generating the initial-state information, Zhangs parton cascade (ZPC) model [20] for modeling partonic scatterings, the Lund string fragmentation model or a quark coalescence model for hadrons formation, and a relativistic transport (ART) model [21] for treating the resulting hadron scatterings. These are combined to give a coherent description of the dynamics of relativistic heavy ion collisions.

Here we choose the string melting version of the AMPT model (v2.269b;isopt=5), in which partons freeze-out according to local energy density. The hadronization process is realized by a quark cascade model, which combines two nearest partons into a meson and three nearest quarks (anti-quarks) into a baryon (anti-baryon). The method of determining hadron species is done by the flavor and invariant mass of coalescing partons. The impact parameter is in the range \( b \leq 3 \text{ fm} \) and the parton cross section is taken to be 10 mb. To give a general picture of the evolution of particle production after collision, we obtained the time evolution of the number of partons and hadrons in the central Au+Au collision at the \( \sqrt{s_{NN}} = 200 \text{ GeV} \) by using AMPT. As shown in Fig. 1, a large number of partons (quark, anti-quark, and gluons) are produced in the early stage of collision (\( t < 5 \text{ fm/c} \), while
later only a few hadrons are produced. During these times, partons are dominant, and the whole system is in a deconfined phase, which is in the perturbative QCD vacuum, with only a few of hadrons [22]. After that, the number of hadrons increases with the decrease of the number of partons, and the system goes through a phase transition into hadron gas phase.

3 The thermodynamic entropy

The thermodynamic entropy contains the information about the momentum and position of particles in heavy-ion collisions, which can be calculated by the 6-dimensional phase space distribution $p_i(p_x, p_y, p_z, x, y, z)$. Under the definition of shannon entropy, the $p_i$ is the ratio of the particle number in the local 6-dimensional phase-space i.e. the $i$-th bin in the global 6-dimensional phase space. Here, the number and size of 6-dimensional phase space units are set to sufficiently contain all the mechanical information after the average of all events. It should be noted that the distribution of the system is based on the number of certain particles versus the total number of particles. The thermodynamic entropies of the partons, hadrons, and system are calculated by Eq. (1) in Au+Au collisions at $\sqrt{s_{NN}} = 64.2$ GeV, 200 GeV, and 800 GeV.

We can be seen from Fig. 2a, that the thermodynamic entropy of partons at different center of mass of energies (c.m. energy) have similar evolution curves. The trend of the thermodynamic entropy decreases first, then goes up and reaches a vertex, and then decreases. The reason for the decrease of parton thermodynamic entropy in the first stage is that strings are not included in our statistics, and then the energy released by the string melting is used as the external energy input to the system. The increase of the thermodynamic entropy of partons in the second stage is due to the scattering and generation of partons. The decrease of thermodynamic entropy of partons in the third stage is due to the disappearance of cooling of partons. It can be observed that the thermodynamic entropy of the final parton disappears earlier in low-energy collisions because the central local temperature is lower in low-energy collisions, so the partons freeze out faster.

From Fig. 2b, the thermodynamic entropy of all hadrons increase rapidly in the early stage at three different c.m. energies, due to the rapid generation of hadrons. Then, the thermodynamic entropy of hadrons gradually tends to saturation and reaches a maximum, which means that the system tends to equilibrium.

Figure 3 shows the time evolution of thermodynamic entropy for the whole collision system, which goes through three stages. The first stage is the initial state of parton, its total thermodynamic entropy decreases and increases due to the energy released and gained by string formation and melting. It also can be interpreted in physical images as entropy reduction caused by compression and expansion after collision. At the same time, the quark gluon plasma (QGP) is formed when the entropy decreases to the minimum. In the second stage, the thermodynamic entropy linearly increases at $t = 5$ to 10 fm/c, which corresponds to the phase transition process of the system from QGP to hadronic gas. The third stage is the process of thermodynamic entropy approaching saturation, which corresponds to the final equilibrium state of hadron scattering. The deep valley of “thermodynamic entropy” shown in early stage may be also related to the fact that, as hadrons are formed, there is a rapid decrease in
the total number of particles in the system, because now the scale at which the particles are identified has changed from partons to hadrons. In addition, the higher the c.m. energy in the high-energy collision, the faster the thermodynamic entropy of the system reaches the equilibrium state. This is because the particles produced by collisions with higher energies have higher momentum, which makes the system approach the maximum disorder faster.

4 The multiplicity entropy

The multiplicity entropy contains the information of the type and number of particles, which introduced by Ma in 1999 [12] to diagnose a nuclear liquid gas phase transition by finding the maximum value of multiplicity entropy in a certain state of the system to determines the critical point. Here, in the context of multiplicity entropy, the probability distribution $p_i$ is related to the ratio of the particle numbers $N_i$ produced in the $i$-th bin to the total particle number $N$, i.e. $p_i = N_i / N$. $p_i$ is the normalized probability distribution, where $\sum_i p_i = 1$. The time evolution of the multiplicity entropy of partons and hadrons at different c.m. energies is calculated by Eq. (1), as shown in Fig. 4.

From Fig. 4a, we can see that the time distribution of multiplicity entropy of partons are all similar under different c.m. energies of Au–Au collisions. The multiplicity entropy of partons decreases slowly before 10 fm/c of the evolution process, which is due to the fact that the distribution of various kinds of partons remains almost unchanged at this stage. Under the conditions of $\sqrt{s_{NN}} = 200$ GeV and 800 GeV, the multiplicity entropy of partons decreases slightly after 10 fm/c, which is due to the faster cooling of energetic quarks at the later stage of the collision system. Under the condition of $\sqrt{s_{NN}} = 800$ GeV, the change of multiplicity entropy is smaller than that of $\sqrt{s_{NN}} = 200$ GeV, because the energetic quarks can exist longer under higher energy collisions, which has little effect on the overall distribution of partons. At $\sqrt{s_{NN}} = 64.2$ GeV collisions, the multiplicity entropy drops sharply near 30 fm/c because the partons almost completely freeze out.

In Fig. 4b, a significant inflection point appears about 6–8 fm/c of hadronic multiplicity entropy, which is the maximum of multiplicity entropy at different c.m. energies. This seems to imply the critical point of the system from QGP to hadronic gas. This inflection point gives a good sign of phase transition because the maximum of the multiplicity entropy reflects the largest fluctuation of the multiplicity probability distribution at the critical point. From the perspective of information theory, the prediction of which hadron will appear in the system at this moment is the most difficult. Fig. 5a shows that the maximum value of multiplicity entropy appears at about 7 fm/c, and the corresponding temperature in Fig 5b at the same time is 147 MeV, which is almost the same as the chemical freeze-out temperature of 141 MeV obtained by specific entropy in reference [15]. Both results are slightly lower than those extracted from the experimental data based on the statical model [23–27].

It should be noted that the critical point obtained by the multiplicity entropy is very close to the starting point of the linear increase of the thermodynamic entropy, which indicates that the chaotic degree of the system increases sharply after the system evaporates from QGP state to hadronic gas. The critical point will be reached later at higher c.m. energies, due to the higher temperature in the central region at higher c.m. energies. Thus the cooling time of QGP into hadronic gas will be longer. After the critical point, the multiplicity entropy of hadrons decreases gradually to a stable value, which reflects the decay of unstable excited hadrons into more stable hadrons.

The multiplicity entropy of the whole system is also plotted in Fig. 4c, but this result is unsatisfactory for the prediction of critical point. The reason for this defect is that the particles in the global region can not correspond to those particles in the region where the phase transition actually occurs, while the region where the hadron is produced corresponds to the region where the phase transition occurs. Therefore, it
5 The configuration entropy

In the previous studies, we regard both partons and hadrons as point particles without internal structure, but in fact mesons and baryons are composed of quarks, in which the mesons are composed of two quarks and most baryons are composed of three quarks. For mesons consisting of two quarks, there is only one space configuration, but for baryons consisting of three quarks, the internal space configuration is not clear. It should be noted here that after nuclear collision, the system will inevitably be accompanied by changes in the amount of information in the process from point particles without internal structure to nucleons with internal structure. Here we attempt to quantify the information variable caused by the change of particle internal configuration.

Inspired by the work of Csernai et al. [16], we consider that particles consisting of three nucleons have four possible configurations, each of which has a different probability of formation, which is related to: direction dependence of the links, different constituents, different (energetic) weights of the links, dynamical freedom of the length or angle of the link, etc. In this way, we can get the corresponding topological Shannon entropy through the probability distribution of different binding modes of quarks in nucleon.

It is interesting that the quark–diquark model mentioned by Lichtenberg [17,28,29] in 1982 coincides with the above viewpoint. This model described baryon as a bound state of one quark with one diquark, in which the diquark is formed by the attraction of two quarks with color and spin anti-symmetry, when both quarks are correlated in this way they tend to form a very low energy configuration. In this model, there are four possible configurations inside baryons (we only consider those particles with quark numbers less than or equal to 3). For example, the four internal configurations of protons are shown in Fig. 6. Here we assume equal probability for each topological configuration in baryon, so that the probability of any configuration in baryons is 1/4. However, we also need to consider that there are other particles with only one internal configuration in the collision process, so there are:

\[ p_{\text{baryon}} = \frac{N_{\text{baryon}}}{N}, \quad p_{\text{other}} = 1 - p_{\text{baryon}}. \]  

Then the configuration entropy of system is:

\[ S_{\text{conf}} = \langle p_{\text{other}} \rangle \ln(p_{\text{other}}) + 4 \times \frac{1}{4} \langle p_{\text{baryon}} \rangle \ln(p_{\text{baryon}}). \]  

The configuration entropy of system in Au + Au collisions at different c.m. energies is plotted in Fig. 7. One can see that the time evolution of configuration entropy at different c.m. energies are similar in Fig. 7. The configuration entropy of a collision system with a higher c.m. energy is lower than that of a collision system with a lower c.m. energy, because...
are calculated under this idea. The results of the total entropy
The cumulative values of the above three kinds of entropy
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Hativistic heavy ion collision system should include as much
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tion of Au+Au collisions. This enlightens us that a complete
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energy are showing by these different kinds of entropy.

In Ma’s review [10], “the basic scientific meaning of Shan-
on information entropy applied in heavy-ion collisions is
to indicate the chaoticity of nuclear matter in the colliding
nuclear system” is mentioned. This reminds us that all three
types of chaotic phase space, particle type and internal configuration of particles, reflect the
chaotic nature of the system in the evolution of relativistic
heavy ion collisions. This enlightens us that a complete
description of the degree of chaos in the evolution of relativistic
heavy ion collision system should include as much
information entropy changes as possible in different aspects.
The cumulative values of the above three kinds of entropy
are calculated under this idea. The results of the total entropy
$S (= S_{\text{thermal}} + S_{\text{mul}} + S_{\text{conf}})$ of the system during the evolu-
tion of Au+Au collisions are shown in Fig. 8, where the time
development appear smooth.

From Fig. 8, we can see that the time evolution of the total
entropy $S$ in the Au + Au collisions system goes through
four stages: The first stage, about $0 < t < 3$ fm, entropy
decreases due to the energy released by string formation and
melting and compression after collision; Second, entropy is
at the minimum of the system about $3 < t < 5$ fm, corre-
ponding to the parton rescattering phase, which the system
is in the quark gluon plasma (QGP) state; Then the entropy
of the system increases rapidly from $t = 5$ fm to $9$ fm, which
may correspond to the transformation process from QGP to
hadron phase; After that, the entropy distribution tends to be
saturated, which would correspond to state of hadron scatter-
ing. In addition, the higher the c.m. energy in the Au + Au col-
ision, the faster the entropy of the system reaches the equi-
librium state. This is because the particles produced by col-
lisions with higher energies have higher momentum, which
makes the system approach the maximum disorder faster.

7 Conclusion and discussion

Entropy (thermodynamic entropy) in physics is usually
defined as a measure of the disorder degree of the system.
Our total entropy gives a more complete description of the
disorder degree of the system, which evolves smoothly with
the system. This is more in line with our intuition.

It is important to remind readers that in this study, our sim-
ulated data was generated from the Monte Carlo simulation,
but the way we analyzed it avoided the ergodic assumption,
which probably does not hold up while the system is evolving
over time. Our analysis is done by considering the instantan-
eous distribution and performing the analysis on the system
configuration at these moments separately. For this reason,
the “entropies” studied in this paper is more similar to ana-
lyzing the phase space distribution.

At the same time, the fact that ergodic assumption does
not hold usually means the deviation from boltzmann statis-
tics to Tsallis statistics. In Sect. 3, we refer to the one of the
explanations of the deep valley of thermodynamic entropy
cauased by the scale change of the particles in the system. It is
very interesting to obtain from the considerations about scale
invariance in field theory for we know that scale invariance
is a fundamental aspect of Yang-Mills field (YMF) theory.
Recent studies [30,31] have shown that, by associating Yang-
Mills fields with complex fractal-type structures, it is possible
to associate a thermodynamic description of the hot and dense
medium formed in high energy collisions. The formation of
fractal structures by system can be described by Yang-Mills
fields theory, and the fractal structures can cause the emer-
gence of nonextensivity in system, which can be described
by Tsallis statistics. Therefore, the Tsallis statistical method
may present a better description of the deep valley of ther-
modynamic entropy. Several studies [32–37] have shown that
the non-extensive approach can more satisfactorily describe
any aspect of high-energy collisions in the case of violation
of the ergodic assumption, and some of those work present
a detailed analysis of thermodynamical quantities (including
entropy) that show results consistent with LQCD calcula-
tions well. We look forward to improving the accuracy of
our work with Tsallis statistics in the future.

In addition, our quantitative description of heavy-ion col-
ision system also conforms to the points of view mentioned

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**Fig. 8** Time evolution of total information entropy for system in Au–
Au collisions at different c.m. energies

higher c.m. energies collide to produce a larger proportion of
mesons without internal structures.

6 The total entropy

Above all, we have calculated the thermodynamic entropy, multiplicity entropy, and configuration entropy of a Au +
Au collision system at different c.m. energies of 64.2 GeV,
200 GeV, and 800 GeV, respectively. Now we need to rethink
that what characteristics of a Au + Au collisions at the RHIC
energy are showing by these different kinds of entropy.

In Ma’s review [10], “the basic scientific meaning of Shan-
on information entropy applied in heavy-ion collisions is
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in the the reference [16]: (1) all degrees of freedom should be taken into account, (2) these should be independent degrees of freedom (orthogonal), (3) the degrees of freedom must be quantized, also for continuous degrees of freedom. This last point was recognized after the development of quantum mechanics, which led to the quantization of phase-space volume cells. Ponit (1) is violated in this AMPT evaluation on the string degrees of freedom are not taken into account and this led to a temporary decrease of the total entropy. Such decrease should not happen in a closed system. This missing entropy of string is shown clearly in Fig. 8 at \( t < 4 \) fm.

In this paper, the time evolution process of thermodynamic entropy \( S_{\text{thermal}} \), multiple entropy \( S_{\text{mul}} \), and configuration entropy \( S_{\text{conf}} \) in relativistic heavy-ion collision is studied carefully using AMPT model to generate central Au + Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV with \( |y| < 1 \) and \( p_T < 5 \) GeV. In this way, we obtain the time evolution distribution image of the total entropy \( S \) in the relativistic heavy ion collision. The results show that the four stages of the time evolution process of the system entropy \( S \) have obvious correspondence with the four physical processes experienced in the relativistic heavy ion collision, indicating that the total entropy of the system can more accurately reflect the physical information in the relativistic heavy ion collision. This further indicates that shannon information entropy, as an effective tool for analyzing the dynamic process of a system, does provide a model-independent method for studying the evolution of a collision nuclear system.

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