Spectral analysis of signals by time-domain statistical characterization and neural network processing: Application to correction of spectral amplitude alterations in pulse-like waveforms

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Abstract—We present a time-domain method to detect and correct spectral alterations of signals by employing statistical characterization of waveforms and a pattern-recognition procedure using simple Artificial Neural Networks. The proposed strategy implements very-fast routines with a computational cost proportional to the number of signal samples, being convenient for applications in embedded environments with limited computational capabilities or fast real-time control tasks. We use the proposed algorithms to correct spectral amplitude attenuations in a pulse-like waveform with a sinc profile as an application example.

Index Terms—signal processing, artificial neural networks, statistical characterization, real-time systems

I. INTRODUCTION

A spectral characterization by Fourier transform is a ubiquitous and powerful tool used to extract behaviors and features of waveforms [1]–[4], particularly of the response of signals affected by physical systems, as it is well known. The response spectrum gives accurate information about how each frequency component of the signal is altered by the system. The computational cost of the Fourier transform is proportional to \(N_s \log_2(N_s)\) (for FFT) [5], where \(N_s\) is the number of signal samples. Nevertheless, in some cases where it is necessary to extract general properties of the signals, the Fourier analysis could have a high computational cost compared with other techniques applied directly to the time domain. In particular, due to this cost, a complete characterization using frequency-domain methods could be inefficient in an embedded environment with limited computational capability or when a very-fast procedure is required. Accordingly, the general properties of the signals can be measured using statistical methods [6]–[15], where moments and cumulants are calculated with a computational cost proportional to \(N_s\).

In this work, we use statistical techniques to characterize alterations or deformations in the signals and their spectra due, for instance, to non-idealities in the communication channels or amplifiers. We aim to identify such alterations and compensate them by applying modifications in the input signal; then, the response or system output signal would be the desired original signal without deformations. We want to apply these proceedings in a real-time control strategy, which can be supported in an embedded processor; thus, for this purpose, we choose a time-domain statistical method as a fast feature extraction procedure. In particular, we use the Extended SSC technique (ESSC) developed by the authors [16] as an extension of the SSC (Statistical Signal Characterization) method created by H.L. Hirsch [6], [7] to characterize the signal. Such an extended method uses the 4-parameters strategy proposed by Hirsch and adds some moments and cumulants of the signal, derivative and integral, conforming to a 30-parameters characterization procedure. The signals we want to describe have pulse-like forms; they are used in applications such as NMR (Nuclear Magnetic Resonance) or MRI (Magnetic Resonance Imaging), radar or communication techniques, etc. In such applications, shaped-pulse methods accomplish different actions, and the efficiency of these methods depends on the precision of the signal form. Thus, the correction strategy is performed by recognizing which waveform is more similar to the response signal between a set of waveforms with known spectral alterations; then, we can modify the input and compensate for such an alteration. This recognition task can be achieved using simple Artificial Neural Networks (ANNs), as we will describe in the following sections.

The remainder of this paper is organized as follows: Section II introduces the method used in this work to characterize spectral alterations by employing a time-domain procedure. Section III presents an application of such a method proposing an algorithm to correct spectral amplitude attenuations affecting a signal or waveform. Accordingly, Section III-A explains the spectral-correction strategy in detail, which implements statistical characterization of the altered waveforms and a pattern-recognition procedure using ANNs; then, Section III-B applies the concepts to correct alterations in a sinc-pulse.
waveform as an example. Finally, Section IV discusses the results and presents the conclusions.

II. TIME-DOMAIN METHOD TO CHARACTERIZE SPECTRAL BANDS OF A SIGNAL

The strategy chosen to detect alterations in the signal spectrum is to split such a spectrum into bands; then, we inspect each frequency band looking for deviations or deformations by comparing it with an unaltered or non-deformed spectrum or signal. This kind of strategy, splitting the spectrum into bands and observing the resulted signals on the time domain, is similar to obtaining a wavelet development for the signal and is common practice in the literature [17], [18]. We want to correct possible amplitude spectrum deformations of the signals; thus, we present the method and results showing amplitude-frequency graphs, but the concepts can also apply to the phase spectrum. Accordingly, in the case of phase corrections, we can use a similar technique that in the amplitude case (or for real and imaginary spectrum parts).

Therefore, we can develop a signal \( f(t) \) using filters to extract each frequency band, this is

\[
f(t) = \mathcal{F}_{\nu \rightarrow t}^{-1} \left\{ \tilde{F}(\nu) \sum_n \tilde{F}_C^{(n)}(\nu) \right\} = \sum_n \mathcal{F}_{\nu \rightarrow t}^{-1} \left\{ F(\nu) \tilde{F}_C^{(n)}(\nu) \right\} = \sum_n g_C^{(n)}(t),
\]

with

\[
\sum_n \tilde{F}_C^{(n)}(\nu) = 1,
\]

and

\[
g_C^{(n)}(t) \equiv \mathcal{F}_{\nu \rightarrow t}^{-1} \left\{ F(\nu) \tilde{F}_C^{(n)}(\nu) \right\},
\]

where \( F(\nu) \equiv F_{\nu \rightarrow \nu} \left\{ f(t) \right\} \) is the Fourier transform of \( f(t) \) from the variable \( t \) to \( \nu \equiv \omega/(2\pi) \), and \( \mathcal{F}_{\nu \rightarrow t}^{-1} \{ \} \) represents the inverse transform. In Eq. (1), the functions \( F_C^{(n)}(\nu) \) are the spectral behaviors of the filters used to decompose the signal spectrum, and \( g_C^{(n)}(t) \) are the functions extracted by such filters from the original signal. Such spectral decomposition is shown in Fig. 1 for a sinc-pulse waveform (a.1) with its amplitude spectrum (a.2). In the graphs, we use a normalized time variable \( \nu \) ranging between 0 and 1; thus, the frequency \( \nu \) scale is reciprocal and arbitrary (e.g., if we have ms in \( t \), then we obtain kHz in \( \nu \), and so on). We can observe the spectral splitting process in different bands by applying the filters with spectrum \( F_C^{(n)}(\nu) \) (dotted light green lines in (b.2), (c.2), (d.2), (e.2), and (f.2)), where we obtain the wavelets \( g_C^{(n)}(t) \) (solid blue lines in (b.1,b.2), (c.1,c.2), (d.1,d.2), (e.1,e.2), and (f.1,f.2)) by means of the inverse Fourier transform over the spectra \( F(\nu) \tilde{F}_C^{(n)}(\nu) \) (Eq. 3).

Such wavelets develop \( f(t) \) (Eq. 1), as is represented in (g.1).

The sum of the spectra of the filters \( F_C^{(n)}(\nu) \) must be 1 (see Eq. 2) to develop the signal adequately, with non-introducing changes in it. However, if the amplitude signal spectrum is bounded in frequencies; this is, it is null or with negligible

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values for frequencies higher than a frequency $\nu_M$; then, we can approximate the development (1) by a finite number $n_M$ of filters that extract the exact spectrum in the range $|\nu| < \nu_M$. Such a bounded development is represented by Fig. 1 wherein (g.2) is shown the sum of the filter spectra (dotted light green line) and the frequency region that accomplishes (2): besides, it is shown the total filtered signal spectrum $\sum_n F(n) \tilde{F}^{(n)}(\nu)$ (solid blue line), whose inverse Fourier transform is shown in (g.1). There is no other theoretical restriction for the spectral profile $\tilde{F}^{(n)}(\nu)$ of filters that extract the exact spectrum in the range $|\nu| < \nu_M$ $(\nu = \nu_k)$, where $\nu_k$ is the period or maximum width of the filters, and $\nu$ is the maximum frequency where we want to accomplish the following band-rejection filter over the signal $F^{(n)}(\nu) = 1 - \Delta_n \tilde{F}^{(n)}(\nu)$. Therefore, for each $n = 0, 1, 2, \ldots, n_M$, we have

$$F^{(n)}(\nu) = \left\{ \begin{array}{ll}
\frac{1}{2} & |\nu| \leq \Delta_n/2, \\
0 & |\nu| > \Delta_n/2.
\end{array} \right. \tag{4}$$

In Eq. (4), $\Delta_n$ is the maximum or period width of the filters, and $\nu_n$ is the central frequency of the $n$th filter. In Fig. 1 we used the filter profile (2); thus, we have

$$\Delta_n = 2 \nu_M/n_M, \quad \nu_n = n \Delta_n/2, \quad |\nu| < \nu_M \tag{5}$$

where $\nu_M$ is the maximum frequency we want to accomplish strictly Eq. (2), and $n_M > 0$ is the chosen value for the maximum filter number in the spectral development of the signal.

Finally, we conclude that we can obtain a time-domain characterization of different spectral bands of a signal by analyzing their associated wavelets $g^{(n)}(\nu)$. In the next section, we use this approach to correct amplitude alterations in the signal spectrum.

III. APPLICATION TO SPECTRAL AMPLITUDE CORRECTION OF A SIGNAL

We aim to correct the spectral deformations of a signal. In order of that, we build a set of waveforms with known alterations (or without changes) by taking the original signal and applying an amplitude attenuation in a frequency band of the development (1); we obtain

$$f_n(t, \Delta_n) = F^{-1}\left\{ F(\nu) \left[ \sum_{m \neq n} \tilde{F}^{(n)}(\nu) + (1-\Delta_n) \tilde{F}^{(n)}(\nu) \right] \right\} \tag{6}$$

$$= F^{-1}\left\{ F(\nu) \left[ 1-\Delta_n \tilde{F}^{(n)}(\nu) \right] \right\} = f(t) - \Delta_n \tilde{g}^{(n)}(t),$$

where $f_n(t, \Delta_n)$ is the altered waveform obtained by attenuating the $n$th band of $f(t)$ in a factor $\Delta_n$ $(0 \leq \Delta_n \leq 1)$. We can see from Eq. (6) that such an alteration is equivalent to applying the following band-rejection filter over the signal

$$F^{(n)}(\nu, \Delta_n) = 1 - \Delta_n \tilde{F}^{(n)}(\nu). \tag{7}$$

The altered-waveform set $\{f_n(t, \Delta_n)\}$, with

$$n = \{0, 1, 2, \ldots, n_M\},$$

and different values of $\Delta_n$, will be called aw-set. In Fig. 2 we see the sets $\{f_n(t, \Delta_n)\}$ (dotted red lines in b.1, c.1, e.1, and f.1), where $n = \{0, 1, 2, 3, 4\}$; which corresponds to the aw-set with 100% $(\Delta_n = 1)$ of attenuation in the bands.

Therefore, we might identify the type of amplitude spectrum alteration by comparing the aw-set and the system response signal in the time domain; then, we would obtain the $\Delta_n$ factors to be correct in the bands. In the following sections, we propose a correction method using these concepts.

A. Description of the spectral-correction method

The spectral-correction method that we propose consists of identifying the type of amplitude spectral alteration by using an Artificial Neural Network (ANN) to extract the probability of similarity between the system response signal and the members of the aw-set in the time domain. Then, because we know the spectral bands and values of the attenuations of the aw-set, we can obtain a series of correction factors employing such probabilities. As was commented, the aw-set is obtained by applying over the original signal (without alterations) the band-rejection filters of Eq. (7) in the range $|\nu| < \nu_M$ and for different attenuations $(\Delta_n)$, where each band is configured with a width and central frequency as in Eq. (5). Then, we train and test the ANN using the algorithm represented in Fig. 2.

In Fig. 2 the schematic diagram describes the following processes. First, we generate an ideal pulse input signal with a time resolution of 10000 points. Such an ideal signal passes through a downsampling stage where the samples are reduced by a factor of $M = 10$ (decimation) and is applied a uniform-random time jitter noise (between 0 and 9 samples) aimed at emulating the effects of an acquisition process. After that, we apply a filter $F^{(n)}(\nu, \Delta_n)$ (Eq. 7) where we choose the band $n$ and the attenuation factor $\Delta_n$ to obtain a waveform member of the aw-set. Following the filter stage, we perform a uniform-random amplitude scaling (up to 75% of the maximum value), and an offset noise introduces a uniform-random continuous level (up to 5% of the maximum amplitude value). Then, the real input signal is finally obtained by adding a Gaussian white noise (with zero mean and the standard deviation set as 5% of the maximum signal amplitude). Therefore, we have a simulation of a real acquired signal for the altered waveform (band $n$ and attenuation $\Delta_n$) to feed the pre-processing algorithm of the ANN.

The pre-processing stage is an algorithm that aims to extract several relevant features of the input signal. Such a stage is
a crucial part of the ANN input, where a waveform with $N_s$ samples is represented by several characteristic features or parameters, reducing the complexity of the processing of the ANN. We use a very-fast statistical method as a pre-processing stage; that is called ESSC (Extended Statistical Signal Characterization) by the authors [16] and is proposed as an extension of the SSC one developed by H.L. Hirsch [6], [7]. This method consists of the calculation of the SSC parameters (4 for each waveform) plus several moments and cumulants of the signal, its derivative and integral, in a total of 30 parameters conforming a ‘fingerprint’ of the waveform. Before the pre-processing stage, the input signal is affected by a median and a mean filter, an offset correction, and a pulse detection routine. Besides, the signal is normalized in amplitude and time, and it is adequate for entry in the ESSC algorithm. That filters+pre-processing block is called 30-parameters ESSC in Fig. 2, and its computational cost is proportional to $N_s$.

We implement a feed-forward back-propagation ANN to solve the pattern-recognition task between the different waveform members of the aw-set. The ANN structure (see the ANN block in Fig. 2) is formed by an input layer with 30 input nodes, where this amount corresponds to the ESSC parameter entries. Then, a hidden layer has between 5 and 40 neurons with a tansig activation function. At the end of the network, an output layer includes neurons and terminals as the number of output classes to be predicted ($n_c$). Such output neurons use the softmax activation function and implement the one-hot encoding method. The softmax’s output is an $n_c$-elements array with the prediction level for each class; this is a range of values between 0 and 1. Such a level is the probability of association of the input parameters (or waveform) with such a class; thus, we use a weighted random selection employing these probabilities to choose the predicted class ($c$) for that input (i.e., the output result of the ANN). To define the number of neurons in the hidden layer, we trained the network with a different quantity of hidden neurons and we evaluated the average cross-entropy of the softmax’s output; then, we used the ANN configuration and the number of hidden neurons with the lowest cross-entropy value.

The process depicted in Fig. 2 is used to train an ANN per frequency band (i.e., for a filter $F_C^{(n)}(\nu, \Delta_a)$ with a fixed value of $n$). We note that an equal input signal with the same filter choice produces different real input signals in the pre-processing stage due to the randomness introduced by several noise sources. Therefore, the training of the ANN is performed by exhaustively running the algorithm shown in Fig. 2.

We used the following five attenuation values

\[ \Delta_a = \{ \Delta_a^{(c)} \} = \{ 1 (100\%), 0.75 (75\%), \]
\[ 0.5 (50\%), 0.25 (25\%), 0 (0\%) \} \tag{8} \]

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\]

to obtain the aw-set of each band:

\[
\left\{ f_n( t, \Delta_a^{(c)} ) = f(t) - \Delta_a^{(c)} g_C^{(n)}(t) \right\}, \tag{9}
\]

with $c = \{ 1, 2, 3, 4, 5 \}$. The 5th class is the non-attenuated waveform, $\Delta_a^{(5)} = 0 (0\%)$. A network is trained using a dataset with arrays of 30-parameters (ESSC) as inputs. Such a dataset corresponds to the parameters of all the waveforms in the aw-set (8), and it is provided the class to which each waveform belongs or true class. Besides, we used a 6th class to identify that the alteration is in another band. In order of that, we train the same network using a dataset with altered waveforms from other bands or filters, where $\Delta_a = \{ 1, 0.75, 0.5, 0.25 \} (\Delta_a = 0$ is yet included in the 5th class). Accordingly, the outputs

\[
\text{Fig. 2. Time-domain algorithm used to train and test the Artificial Neural Networks for the spectral-correction method.}
\]

\[
\text{An ANN is trained using a dataset with arrays of 30-parameters (ESSC) as inputs. Such a dataset corresponds to the parameters of all the waveforms in the aw-set (9), and it is provided the class to which each waveform belongs or true class. Besides, we used a 6th class to identify that the alteration is in another band. In order of that, we train the same network using a dataset with altered waveforms from other bands or filters, where $\Delta_a = \{ 1, 0.75, 0.5, 0.25 \} (\Delta_a = 0$ is yet included in the 5th class). Accordingly, the outputs}
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In our approach, we suppose that general spectral attenuations non-idealities, for instance, in the transmission physical system.

The algorithm is described by the schematic diagram in Fig. 3. Such a comparison is commonly shown in the form of a confusion matrix, where the rows (columns) represent the true classes and the columns (rows) represent the predicted classes; then, the matrix element values are the number of times that a predicted class is obtained as an output of the ANN for a fixed true class. We could obtain a different predicted class value for the same waveform under different algorithm runs; due to the weighted random selection process. Therefore, we calculate an average confusion matrix from several runs with the same waveform to represent the network performance adequately.

After the building, training, and testing of the ANNs for all the bands, we may use the networks to implement the correction algorithm to the input (or original) signal. Such an algorithm is described by the schematic diagram in Fig. 3. The real input signal is the original waveform affected by an arbitrary spectral amplitude attenuation (or alteration) due to non-idealities, for instance, in the transmission physical system. In our approach, we suppose that general spectral attenuations due to the system can be expressed or very well approximated by

$$F_S(\nu) = \prod_n \left(1 - \Xi_{a,n} \tilde{F}_C(n)(\nu)\right); \quad (10)$$

this is, the system spectral response can be developed by a product of band-rejection filters of type (7).

The 30-parameters ESSC block extracts the waveform features using very-fast filters and time-domain statistical preprocessing of the signal; then, a 30-parameters array is sent to each ANN to be analyzed. We have as many ANNs as the number of bands that we want to split the frequency range of the signal, where the ANN\(n\) \((n = 0, 1, \ldots)\) is trained using the aw-set obtained by applying a band-rejection filter \(F_C(n)(\nu, \Delta_{a})\) with a width and central frequency defined in Eq. (5) and with the different attenuations of Eq. (8).

We obtain a mean correction factor for the band \(n\) using the probability of each class of the output softmax function of ANN\(n\), this is

$$\Delta_{a,n} = \sum_{c=1}^{n_c} \Delta_{a,c} p_C(\Delta_{a,c}) = \sum_{c=1}^{n_c} \Delta_{a,c} p_C(n)(\Delta_{a,c}) \quad (11)$$

with \(n_c = 6\), where for simplicity we define \(p_C(n)(\Delta_{a,c}) = \tilde{p}_C\), and \(\Delta_{a,c}^{(6)} = \Delta_{a,c}^{(5)} = 0\). In Eq. (11), \(p_C(n)\) represents the probability associated with the network or band \(n\).

Finally, we use the correction factors (11) to obtain the corrected signal

$$\hat{f}(t) = F_{\nu ightarrow t}^{-1} \left\{ F(\nu) \prod_n \left(1 - \Xi_{a,n} \tilde{F}_C(n)(\nu)\right)^{-1} \right\}$$

$$= F_{\nu ightarrow t}^{-1} \left\{ F(\nu) \sum_{n} \sum_{q_n=0}^{\infty} \Xi_{a,n}^{(n)} \left(\tilde{F}_C(n)(\nu)\right)^{q_n} \right\}$$

$$= \sum_n \left(1 + \Xi_{a,n}\right) g_C(n)(t)$$

$$+ \sum_{q_n, q_1, \ldots} \left(\prod_n \Xi_{a,n}^{(n)}\right) g_{C_{q_n,q_1,\ldots}}(t), \quad (12)$$

where are defined the following wavelets

$$g_{\{q_0, q_1, \ldots\}}(t) = F_{\nu ightarrow t}^{-1} \left\{ F(\nu) \prod_n \left(\tilde{F}_C(n)(\nu)\right)^{q_n} \right\}, \quad (13)$$

and \(\sum_{q_0, q_1, \ldots} \) represents a sum excluding the indices

$$\{q_0, q_1, \ldots, q_n, \ldots\} = \{0, 0, 0, \ldots, 0, \ldots\}; \{1, 0, 0, 0, \ldots\}; \ldots; \{0, 0, 0, 1, \ldots\}; \ldots.$$

The spectrum of the corrected waveform (12) is defined to cancel the system spectral response (10); thus, we obtain the spectrum \(F(\nu)\) of the original waveform by multiplying the Fourier transform of the corrected signal (12) times the system spectral function (10). Therefore, the arbitrary signal spectral attenuation is corrected by using the time-domain development (12) as the input signal, where the compensation factors regulate the amplitude of the wavelet components (3) and (13). Such a development can be approximated by keeping the terms with lower power of \(\Delta_{a,n}\), whose accuracy will depend on the values of these factors. Consequently, if we introduce the waveform (12) into the analyzed physical system, the system output will be the original signal.

In summarizing, the proposed spectral amplitude correction method consists of the following steps

a) Define the maximum frequency \(\nu_M\) and the number of filters or bands \(n_M\) to resolve the signal spectrum.

b) Train and test the ANNs generating the aw-set (9) of each band by the algorithm of Fig. 2.

c) Apply the algorithm of Fig. 3 to an arbitrary altered input signal calculating the mean correction factor of each frequency band (Eq. (11)).

d) Compensate the original signal with the correction factors obtaining the corrected signal (Eq. (12)).

e) Use the corrected signal as system input, and the system output would be the original signal.

As an example, we use the developed algorithms to correct spectral attenuations in a sinc-pulse waveform in the following section.
**B. Example for a sinc-pulse waveform**

In this section, we used the method proposed in Section III-A to detect and correct alterations in a sinc-pulse waveform (Fig. 1 (a.1), and (a.2)), as an application example of the developed algorithm. We chose to split the signal spectrum into bands as in Fig. 1 with that wavelet decomposition \( \{ g_n(t) \} \) (b.1, c.1, d.1, e.1, and f.1), where we selected the maximum frequency \( \nu_M = 7.5 \), and band number \( n_M = 4 \). Thus, we obtain from Eq. (5): \( \Delta \nu = 3.75 \), \( \nu_n = 1.875 \times n \).

The aw-sets \( \{ f_n(t, \Delta a^{(c)}) \} \) obtained for the band numbers \( n = \{ 0, 1, 2, 3, 4 \} \) are shown in Figs. 4-9 and 10. In those figures, we can observe in (a.1, b.1, c.1, and d.1) the waveforms for the following classes and attenuation factors: \( c = \{ 1, 2, 3, 4 \} \), \( \{ \Delta a^{(c)} \} = \{ 100\%, 75\%, 50\%, 25\% \} \). The original waveform, shown in Fig. 1 (a.1), corresponds to \( c = 5 \) and \( \Delta a^{(5)} = 0\% \), and it is shared between all the bands. We show in solid red lines in (a.2, b.2, c.2, and d.2) the normalized amplitude spectra of the waveforms. Such spectra are obtained by multiplying the original waveform spectrum \( F(\nu) \), shown in Fig. 1 (a.2), times the filter profiles \( F_C^{(n)} (\nu, \Delta a^{(c)}) \), shown in dotted light green lines in (a.2, b.2, c.2, and d.2). The waveforms are generated by the inverse Fourier transform of such spectra.
We averaged the results of 50 matrices to obtain the mean matrices. The off-diagonal elements are averaged, and the diagonal ones are obtained resting 1000 minus the sum of the averaged off-diagonal in the same row. The labels indicate the attenuation factors $\Delta_c(a)$ for the classes $c = \{1, 2, 3, 4, 5\}$, and NB stands for Not Is This Band representing that the attenuation is in another band for the class $c = 6$. The results are shown for different frequency bands: $n = 0$ (a), $n = 1$ (b), $n = 2$ (c), $n = 3$ (d), and $n = 4$ (e).

An application example of the spectral-correction algorithm is shown in Fig. 10 for a sinc-pulse waveform $f(t)$ ((b.1), dotted red line). The waveform $\tilde{f}(t)$ ((a.1), solid red line) is a sinc pulse deformed by a particular spectral response $F_S(\nu)$ of a physical system ((a.2), dotted light green line); thus, $\tilde{f}(t)$ is the output signal of the system with $f(t)$ as the input one. The spectrum of $\tilde{f}(t)$ is $\tilde{F}(\nu)$ ((a.2), solid red line), which is obtained by multiplying the spectrum $F(\nu)$ of the sinc pulse ((b.2), dotted red line) times the response $F_S(\nu)$. The corrected signal obtained by applying the algorithm in Fig. 3 is $\hat{f}(t)$ ((a.1), dashed cyan line), whose spectrum is $\hat{F}(\nu)$ ((a.2), dashed cyan line). The outcome of the correction factors is

\[
\Delta_{0},0 = 0.5023, \quad \Delta_{1,0} = 0.1259, \quad \Delta_{2,0} = 0, \\
\Delta_{3,0} = 0, \quad \Delta_{4,0} = 0.0001.
\]

The corrected-signal spectrum $\hat{F}(\nu)$ can be obtained by multiplying $F(\nu)$ times the spectral compensation $F_C(\nu)$ ((a.2), dash-dot blue line), where

\[
F_C(\nu) \equiv \prod_n \left(1 - \Delta_{n,n}\hat{F}_C^{(n)}(\nu)\right)^{-1}.
\]

The signal $f^*(t)$ ((b.1), solid light green line) is the output of the system by applying $\hat{f}(t)$ as the input signal, whose spectrum is $F^*(\nu)$ ((b.2), solid light green line). We can
obtained the maximum absolute value of the signal. In our example, it is taken in the time $t_i$.

Fig. 9. Average confusion matrices (50 matrices averaged) for aw-sets in the sinc-pulse example (1000 elements per output class). The labels denote $\Delta_{\alpha,n}$ for the true and predicted class ($c = \{1, 2, 3, 4, 5\}$); the label NB (Not in this Band) represents $c = 6$. (a,b,c,d, and e) results for different bands.

Consequently, the precision of the correction algorithm is associated with: the quality of the known aw-set to describe the different spectral deformations produced by the physical system, the capacity of the ESSC routine, and the trained ANNs, to identify the altered waveforms.

We observe from Fig. 9, comparing the predicted class output with the known true class, that in the lower bands ($n = \{0, 1, 2\}$), (a), (b), and (c), which hold the most relevant part of the signal spectrum, there is very high effectiveness in recognizing between different waveforms from the 4-factors attenuation group: 100%, 75%, 50%, and 25% ($c = \{1, 2, 3, 4\}$), that is reflected by the high values of the principal diagonal. It exists for a particular factor a very-subtle influence of the immediately lower one, i.e., there are elements in the first diagonal superior with low values. Besides, such a very-high performance is also obtained by discriminating between the 4-factors group and the class 0% ($c = 5$, the original waveform) and NB ($c = 5$). We can conclude that the excellent performance in distinguishing the waveforms is product of generating an aw-set with subtle differences between its members with nearby classes, as it is observed from Figs. 4, 5, and 6, but distinguishable by the ESSC method and the training of the ANNs.

In the following section, we discuss the obtained results and arrive at conclusions about the proposed spectral-correction methodology.

IV. DISCUSSION AND CONCLUSION

The proposed time-domain method to correct spectral amplitude attenuations is based on discriminating between a set of different known altered waveforms (aw-set) by using ESSC to extract signal features and simple ANNs to realize the pattern recognition task. Then, a corrected signal is generated as the system input by employing a wavelet decomposition weighted with a series of predicted mean correction factors. Therefore, the physical system alters the corrected signal, and the output will be the desired waveform.
efficiently recognizing the 4-factors group waveforms in the aw-set, where the signal spectrum has a significant amplitude (see Fig. 1(b.2), (c.2), and (d.2)).

In the case of the band \( n = 3 \), the attenuation factors are strongly influenced by the immediately lower one; this is observed by the same-order values of the principal diagonal and the first diagonal superior elements in (d). That is because an altered-waveform class has a very similar form to the waveform of its nearby lower class for this band, as can be seen in Fig. 1. Therefore, we conclude that the mean correction factor of this band would have an intermediate value between the classes associated with such elements. However, this factor has a poor influence on the corrected signal because of the small-signal spectrum amplitude in the band (see Fig. 1(e.2)).

Finally, the band with \( n = 4 \) holds the higher frequency region, where the signal spectrum has a very low amplitude. Thus, some attenuation in this band does not significantly change the form of the original signal, as we see from Fig. 1 and the predicted class results are attenuations of 0%, as can be seen in the matrix (e). Therefore, the conclusion is that the mean correction factor of this band would have a value close to 0. Nevertheless, the contribution of this factor to the correction algorithm is despicable because the signal spectrum has a negligible amplitude in this band (see Fig. 1(f.2)).

We obtained from the application example shown in Fig. 10 that the spectrum-corrected algorithm outcome is an output-system signal with an RMS error lower than 5% (see Eq. (14)). That performance is achieved with a significantly spectral deformation supposed to the physical system response (see \( F_S (\nu) \) in Fig. 10(a.2), dotted light green line). Therefore, we conclude that the algorithm can adequately compensate these spectral-alteration types on the signal.

There are pending issues to be analyzed, which are out of the scope of this work, between then: i) test the performance of the spectral-correction method by changing the noise level or signal-to-noise ratio, ii) study the precision of recognizing spectral profiles by changing the width of the band-rejection filters and using different band numbers, iii) use a unique global ANN to detect all possible spectral alterations by combining attenuations in different frequency bands.

The approach used in this work is to detect a general spectral profile by splitting the spectrum into frequency bands and using one ANN per band. We suppose that combinations of attenuations in different bands can be detected by the join working of all the ANNs. Therefore, each ANN must discriminate between \( N_p = n_1 + (n - 1) n_2 \) different patterns of altered waveforms, where \( n_1 \) is the number of attenuation levels; then, \( N_p = 21 \) in our case \( (n_1 = 5, \text{ and } n_2 = 4) \). If we use a unique ANN to detect different spectral attenuation, such an ANN is trained using all the possible combinations of attenuations in the bands, but there are many combinations with subtle spectral altered waveforms similar to the original one; thus, there would be more confusion in the detecting procedure. Besides, the number of patterns to discriminate will be \( N_p = n_1^{n_2} + 1 \); then, \( N_p = 3125 \) in our case.

As a final remark, the results obtained from the confusion matrices and the application example of Section III-B permit us to conclude that a spectral-correction procedure can be satisfactorily performed by the proposed time-domain algorithms (using ESSC and simple ANNs). Thus, this method is convenient for embedded or limited computer architectures.

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