Mott Dissociation and Kaon to Pion Ratio in the EPNJL Model

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Abstract—The behaviour of pseudoscalar mesons within the SU(3) PNJL-like models is considered for finite $T$ and $\mu_B$. We compare the pole approximation (Breit–Wigner) with the Beth–Uhlenbeck approach. We evaluate the $K/\pi$ ratios along the phase transition line in the $T-\mu_B$ plane with constant and $T/\mu_B$-dependent pion and strange quark chemical potentials. Using the model, we can show that the splitting of kaon and anti-kaon masses appears as a result of introduction of density and this explains the difference in the $\pi^+/K^+$ ratio and $K^−/\pi^−$ ratio at low $\sqrt{s_{NN}}$ and their tendency to the same value at high $\sqrt{s_{NN}}$. A sharp “horn” effect in the $K^+/\pi^+$ ratio is explained by the enhanced pion production which can be described by occurrence of a non-equilibrium pion chemical potential of the order of the pion mass. We elucidate that the horn effect is not related to the existence of a critical endpoint in the QCD phase diagram.

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INTRODUCTION

Our interest in the "horn" in the $K^+/\pi^+$ ratio (the peak-like structure at the collision energy $\sqrt{s_{NN}} \sim 8$ GeV) appears due to the fact that modern attempts to explain its existence give a good intuitive explanation of the $K/\pi$ bebehaviour in the high-energy region. The smooth, almost constant behaviour at high energies (RHIC, LHC) can be explained by the fact that at high energies, during the heavy-ion collision, the quark-gluon-plasma is created (it is confirmed by other signals). The creation of the QGP is accompanied by a constant yield of strange particles, independent on the temperature. The jump from the maximum value to the almost constant line is connected with the increase of the pion yield in comparison with the strangeness yield when deconfinement appears and it could be a signal of "onset of deconfinement" (the idea was originally predicted by Gazdzicki and Gorenstein) [1].

The description of the $K^+/\pi^+$ at energies <8 GeV has not such a clear explanation. On the one hand, it was shown that the partial chiral symmetry restoration could be responsible for the quick increase in the $K^+/\pi^+$ ratio at low energies and its decrease with increasing energy (as a result of chiral condensate destruction) [2, 3]. On the other hand, it is not clear still if the deconfinement appeared during heavy-ion collision at such energies and if it coincided with the chiral phase transition. And at last, the recent data from the NA61 experiment have shown a strong dependence on the system size with no horn effect in Ar + Sc collisions [1] for which the reason is not yet understood.

The main idea of this work is to find a model able to reproduce both chiral phase transition and deconfinement transition at finite $T$ and $\mu_B$. From this point, the NJL-like models are promising models, as they are capable of describing both the chiral phase transition and the deconfinement transition. The chiral symmetry breaking is referred to the quarks developing quasiparticle masses by propagating in the chiral condensate and the confinement of coloured quark states is effectively taken into account by coupling the chiral quark dynamics to the Polyakov loop and its effective potential. However, in the model the absolute value of the pseudocritical temperature of the chiral crossover transition, above 200 MeV, is too large when
compared to the lattice QCD result for 2 + 1 flavors of $T_c = 156.5 \pm 1.5$ MeV [4].

The so-called entanglement PNJL (EPNJL) model, with the modified scalar four-quark interaction $g_s$, could be one of possible solutions of this problem [5, 6]. Such entanglement leads to a change of the phase diagram: the (pseudo-)critical temperature of the chiral and deconfinement crossover transition at low chemical potentials is lowered towards the value of the lattice QCD prediction. A more physical solution is to go beyond the mean field approximation and to consider the role of hadronic excitations in the medium in melting the chiral condensate [7]. First steps in this direction have been explored, e.g. within a generalized Beth—Uhlenbeck (BU) approach [8, 9]. We also considered the (E)PNJL model with vector interaction to evaluate the effect of changing the chiral phase transition of the first order to the soft crossover in the region of high $\mu_B$. Including the vector interaction in the model gives such possibility to vary the position of the critical end point (CEP) in the phase diagram depending on the value of the coupling $g_v$. When $g_v$ reaches the critical value, the chiral phase transition turns into soft crossover overall [10, 11].

In this work we show that the energy-dependent behaviour of the $K/\pi$ ratio could be rescaled by a new variable, $T/\mu_B$, where both $T$ and $\mu_B$ are chosen along the line of the chiral phase transition, which is supposed to correspond to the chemical freeze-out line in the QCD phase diagram. In this region, it can be important to introduce the nonequilibrium pion chemical potential. The reason to consider the nonequilibrium pion distribution function lies in the behaviour of pions during a heavy-ion collision: their number is quasi conserved over the time scale of the HIC until freeze-out. The resulting nonequilibrium pion distribution function with pion chemical potential has been successfully used in the phenomenological description of the transverse momentum spectra of pions produced in heavy-ion collision experiments, see [12].

MASS SPECTRUM FOR MESONS AT FINITE TEMPERATURE AND DENSITY

We start from the SU(3) PNJL model with the Kobayashi—Maskawa—T’Hooft (KMT) interaction [13]

$$\mathcal{L} = \bar{q}(i\gamma^\mu D_\mu - m - \gamma_5 \mu)q + \frac{1}{2}g_s \sum_{a=0}^8 \left( (\bar{q} \gamma^\mu q) \right)^2 + \left( (\bar{q} \gamma^\mu \gamma^5 q) \right)^2 + g_D \left[ \det[\bar{\eta}(1 + \gamma_5)] + \det[\bar{\eta}(1 - \gamma_5)] - \delta(\Phi, \bar{\Phi}; T) \right]. \tag{1}$$

The model describes the dynamical chiral symmetry breaking as a coupling of quarks to the chiral condensate. The quarks develop a quasiparticle mass even for vanishing current quark masses $m_{0i}$ (chiral limit) by propagating in the chiral condensate. In this model with the Lagrangian (1), the quarks are coupled both to the chiral condensate and to the homogeneous gluon background fields represented by the Polyakov loop dynamics. The confinement properties are described by the effective potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$, which depends on the complex traced Polyakov loop $\Phi = Tr(exp[i(\lambda_4 A^4_0 + \lambda_8 A^8_0)])$ and its conjugate $\bar{\Phi}$. The potential is constructed on the basis of the center symmetry $Z_3$ of the color SU(3) gauge group. The possible temperature dependence of its parameters is fitted to lattice QCD results for the pressure in the pure gauge sector (for details see [14]).

The pseudocritical temperature of the chiral crossover transition at $\mu_B = 0$ GeV in the mean field PNJL model is $T_c = 0.218$ GeV and this value is higher than the $T_c = 156.5 \pm 1.5$ GeV obtained by the ab-initio simulations of Lattice QCD [4].

The PNJL model with rescaled $T_0$ has a problem with correspondence between critical temperature for the chiral condensate and for the Polyakov loop. To improve this disagreement with Lattice data, the so-called EPNJL model was proposed [6]. According to this extension of the PNJL model, a phenomenological dependence of the scalar meson coupling $g_s$ on the Polyakov loop is introduced that does obey the $Z_3$-symmetry

$$\tilde{g}_s(\Phi) = g_s(1 - \alpha_s(\Phi \bar{\Phi} - \alpha_s(\Phi^3 + \bar{\Phi}^3))) \tag{2},$$

where the parameters $\alpha_s = \alpha_2 = 0.2$ were chosen in [5] to reproduce the two-flavor LQCD data. Such rescaling of $g_s$ leads to a rescaling of the pseudocritical temperature in the low-density region [5, 6] to $T_c \sim 0.179$ GeV and now being synchronous with the Polyakov-loop transition.

We also considered the PNJL model with vector interaction to discuss the critical end point position on the phase diagram. The position of the CEP is under debate [10] and can not be settled yet by Lattice QCD simulations. It was shown in the NJL and in the PNJL models with vector interaction, that the CEP can disappear when the vector coupling exceeds a critical value. The Lagrangian of the PNJL model with vector interaction is obtained from the Lagrangian (1) by adding the vector interaction contribution $-\frac{1}{2}g_v \sum_{a=0}^8 (\bar{q} \gamma^\mu \gamma^5 \gamma^a q)^2$; the vector coupling $g_v$ can also be rescaled in the spirit of the EPNJL model, similar to the scalar coupling (2).

The meson spectral properties are encoded in the meson propagator ($M = P, S$)

$$\mathcal{M}_{ij}^M(\omega, \mathbf{q}) = \frac{2P_\parallel}{1 - 4P_\parallel \Pi_\parallel^M(\omega, \mathbf{q})}, \tag{3}$$
where for the non-diagonal pseudo-scalar mesons π, K:

\[ P^\pi_{ij} = g_S + g_D \langle \tilde{q}_i g_q \rangle, \quad P^K_{ij} = g_S + g_D \langle \tilde{q}_i g_q \rangle \]  

and \( \Pi^M_{ij}(P_i) \) is the polarization function. The (complex) mass pole solutions \( z_M = m_M - i\Gamma_M/2 \) in the Breit–Wigner (BW) approximation of the Bethe–Salpeter equation at vanishing meson momentum \( q = 0 \), are particularly interesting:

\[ 1 - 2 P^M_{ij}(z = z_M, q = 0) = 0, \]  

where for the width of the quark-antiquark resonant state \( \Gamma_M \ll m_M \) should hold. When the mass parameter is below the two-quark threshold \( (m_M < m_i + m_j) \), the polarization function is real and Eq. (5) corresponds to the homogeneous Bethe–Salpeter equation in the rest frame of the meson which defines a true bound state with mass \( m_M \) and infinite lifetime \( \Gamma_M = 0 \). \( \Gamma_M \) becomes finite at the Mott transition, when the meson becomes unbound and its mass is larger than that of its constituents \( m_M > m_i + m_j \). The temperatures of the Mott transition for pion and kaon are \( T^\pi_{\text{Mott}} = 0.232 \text{ GeV} \) and \( T^K_{\text{Mott}} = 0.230 \text{ GeV} \), respectively.

An adequate account for both bound and continuum states can be made within the BU approach that is generalized for the case of dense matter and the Mott dissociation of mesons across the chiral/deconfinement transition in [15, 16]. For this, the mesonic propagator can be rewritten in the "polar" representation:

\[ \Pi^M_{ij}(\omega, q) = \Pi^M_{ij}(\omega, q) e^{\delta_{ij}(\omega, \pi)} \]  

with a meson phase shift \( \delta_{ij}(\omega, q) \). In the phase shift representation, the bound state appears at the energy where the phase shift has a jump to the value \( \pi \) [15]. In the rest frame of the meson this energy corresponds to its mass. For energies above the continuum threshold, the phase shift drops and asymptotically vanishes, in accordance with the Levinson theorem.

At zero chemical potential both the BW and the BU approaches show the degeneracy of positive and negative charged meson masses, as for light quarks, \( m_\pi = m_\ell \) was chosen. At nonzero chemical potential and low \( T \), the splitting of mass in charged multiplets appears due to excitation of the Dirac sea modified by the presence of the medium (see [17–19]). In the BU approach an anomalous mode appears after \( T_{\text{Mott}} \) for positively charged kaons. This is due to the fact that with increasing density the scattering mode of the polarization function \( \Pi^M_{ij}(P) \) becomes dominant and induces a bound state pole in the meson propagator (3) [20].

### Kaon to Pion Ratio

In this Section we discuss the application of the PNJL model to the description of the kaon to pion ratio. There are some remarks to our calculations in the frame of the PNJL model.

The first point is that experimental data are shown as a function of the collision energy \( \sqrt{s_{NN}} \) which never appears as a parameter in effective models. We used the fact that in the statistical model for each collision energy the temperature and the baryon chemical potential of freeze-out can be found using some parametrization as suggested, e.g., by Cleymans et al. [21]. Using this parametrization, the \( K/\pi \) ratio can be considered as a function of a new variable \( T/\mu_B \) instead of \( s_{NN} \), where \( (T, \mu_B) \) are taken along the freeze-out line.

The second point is that according to the nonequilibrium statistical models the ratio of the yields of mesons, such as the \( K/\pi \) ratios which were obtained in the midrapidity range, can be calculated in terms of the ratio of the number densities of mesons \( (K^\pm/\pi^\pm = n_{K^\pm}/n_{\pi^\pm}) \) with

\[ n_M = d_M \int_0^\infty \frac{d^3 q}{(2\pi)^3} g_M(E_M), \]

where \( g_M(E) = (e^{(E-M)/T} - 1)^{-1} \) is the Bose function for a meson with energy \( E \) and chemical potential \( \mu_M \). In the generalized BU approach, however, an off-shell generalization of the partial number density of the mesonic species \( M \) holds

\[ n_M(T) = d_M \int \frac{d^3 q}{(2\pi)^3} \frac{d\omega}{2\pi} g_M(\omega) \frac{d\delta_M(\omega, q)}{d\omega}, \]

where \( \delta_M(\omega) = \delta_M(\omega, 0) \) is the meson phase shift that was calculated for mesons at rest, but for which Lorentz-boost invariance \( \omega = \sqrt{q^2 + m^2} \) was assumed (see [16]).

The chemical potential for pions, as a parameter characterizing the nonequilibrium state, has been chosen as a constant close to the pion mass, e.g. \( \mu_\pi = 0.135 \text{ GeV} \), following the works [12, 22]. The chemical potential for kaons is defined here by their chemical composition, i.e. by their quark content, so that \( \mu_{K^\pm} = \mu_u - \mu_s \) and \( \mu_{K^\mp} = \mu_s - \mu_u = -\mu_{K^\pm} \).

For the BU approach, lowering the pseudocritical temperature of the chiral restoration in the EPNJL model leads to a more rapid drop above the horn, than in the BW approximation, when the chemical potential of pion is constant. We introduced the nonequilibrium pion chemical potential and strange quark chemical potential. Now they are assumed to depend on the
specific entropy variable \( x = T / \mu_B \) as functions of the Woods–Saxon form

\[
\mu_\pi(x) = \mu_\pi^{\text{min}} + \frac{\mu_\pi^{\text{max}} - \mu_\pi^{\text{min}}}{1 + \exp(- (x - x_\pi^{\text{th}})/\Delta x_\pi))},
\]

\[
\mu_s(x) = \frac{\mu_s^{\text{max}}}{1 + \exp((x - x_s^{\text{th}})/\Delta x_s)).
\]

The best values of the parameters for the PNJL (EPNJL) model are \( \mu_\pi^{\text{max}} = 147.6 \pm 10(107 \pm 10) \text{ MeV}, \)
\( \mu_\pi^{\text{min}} = 120(92) \text{ MeV}, \)
\( x_\pi^{\text{th}} = 0.370(0.409), \)
\( \Delta x_\pi = 0.015(0.00685). \)

The parameter values \( \mu_s^{\text{max}} / \mu_s^{\text{crit}} = 0.205, \)
\( x_s^{\text{th}} = 0.223, \)
\( \Delta x_s = 0.06 \) hold for both models.

In Fig. 1 we show the \( K^+ / \pi^+ \) and \( K^- / \pi^- \) for the EPNJL model as a function of the specific entropy variable \( x = T / \mu_B \) when the BU approach is applied and the pion and strange quark chemical potentials are either constant (thin lines) or given as functions of \( x \) (right panel) so that the experimental data can be fitted (thick lines). To show the behaviour of \( K / \pi \) ratios on \( T - \mu_B \), we choose the BU approach with nonequilibrium pion chemical potential in the EPNJL model with \( g_\pi = 0.6g_8 \) (when there is no first order chiral phase transition in the system). The dot on the phase diagram (Fig. 2) corresponds to the maximum of the \( K^+ / \pi^+ \) ratio on the line of pseudocritical temperatures for the chiral transition (our proxy for chemical freeze out).

**Fig. 1.** Left panel: The ratios \( K^+ / \pi^+ \) (black lines) and \( K^- / \pi^- \) (red lines) are shown as function of \( T / \mu_B \) along the chemical freeze-out line for the EPNJL model with \( g_\pi = 0 \) within the BU approach. Thin lines correspond to the case with fixed \( \mu_\pi = 147.6 \text{ MeV}. \) The thick lines are obtained when \( \mu_s \) and \( \mu_\pi \) vary with \( T / \mu_B \) as shown in right panel.

**Fig. 2.** \( K^+ / \pi^+ \) (left) and \( K^- / \pi^- \) (right) on the \( T - \mu_B \) plane for the EPNJL model with \( g_\pi = 0 \) (no CEP). The black dot indicates the maximum of the \( K^+ / \pi^+ \) ratio on the line of pseudocritical temperatures for the chiral transition (our proxy for chemical freeze out).
RESULT AND DISCUSSION

In this work, we used the SU(3) PNJL model to describe the QCD matter at finite temperature and density. In the frame of the model, we paid attention to the $K/\pi$ ratio as a function of $T/\mu_B$, where $T$ and $\mu_B$ are chosen along the phase transition line on the phase diagram. We replaced the $\sqrt{s_{NN}}$ by the variable $T/\mu_B$ choosing $T/\mu_B$ along the line of the phase transition motivating that by the observation that along the straight lines of constant $x$ in the phase diagram the value of the specific entropy $S/N_B = s/n$ is constant and belongs to a conserved quantity in the hydrodynamic evolution of the hadronizing fireball created in central nucleus-nucleus collisions.

In the mean field PNJL model, we considered the cases with different values of $g_v$ which move the CEP to lower $T$ till it disappears and shifts the crossover pattern to higher chemical potentials. According to our analysis, the key to understanding the horn lies in the position and slope of the almost straight lines of constant $K^+/\pi^+$ and $K^-/\pi^-$ ratios relative to the curved freeze out line. Both ratios are almost unaffected by the appearance of a change from crossover to first order transition.

In the present work, we have suggested that the sharpness of the horn effect in the $K^+/\pi^+$ ratio is well explained by a Bose-enhanced pion production for $x > x_{norm}$, i.e. for heavy-ion collisions with $\sqrt{s_{NN}} > 8$ GeV, in the region of meson dominance, where a low-momentum enhancement of pion production has also been observed. Such an effect is best described by a nonequilibrium pion distribution function, which according to Zubarev’s concept of the nonequilibrium statistical operator requires an additional Lagrange multiplier, the pion chemical potential, for a consistent description. Within the Beth–Uhlenbeck approach we have provided fit functions for the $x$-dependence of the pion and strange quark chemical potentials that lead to a simultaneous description of the $x$-dependence of both kaon-to-pion ratios in accordance with the experimental data, with a strong horn effect for the $K^+/\pi^+$ ratio.

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