Coherence length of cosmic background radiation enlarges the attenuation length of the ultra-high energy proton

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Abstract

It is pointed out that an agreement of the one particle energy spectrum of the cosmic background radiation (CMBR) with Planck distribution of 2.725 [K] does not give a strong constraint on the coherence length of CMBR if the mean free path of CMBR is very long. The coherence length in this situation is estimated as a few times of \( k_B T \). Due to this finite coherence length, the attenuation length of ultra-high energy cosmic rays (UHECR) is reduced in the \( \Delta \) resonance region, i.e., around \( 10^{20} \) [eV]. The small attenuation length makes the suppression of the flux of cosmic rays in this energy region less prominent than the naive estimation.

1 Introduction

UHECR beyond \( 10^{20} \) [eV] is expected to be suppressed by its pion production collision with CMBR by GZK bound. In this paper we assume that the proton is UHECR and study its scattering with CMBR. Recently observations of UHECR became possible by several experiments [3, 4, 5, 6, 7], and a possible signal of UHECR around this energy region has been found, although experiments are controversial. A new mechanism which modifies the pion production probability of UHECR is proposed in this letter.

CMBR has the temperature at around 2.7 [K] and is regarded as a wave packet in the present work. Coherence is kept within the size of wave packet, and we call it a coherence length. We show that the finite coherence length modifies the pion production probability in the \( \Delta \) resonance region.

When the invariant mass of the initial states composed of UHECR and a CMBR exceeds the pion production threshold, the attenuation length of UHECR becomes short. GZK predicted the reduction of the flux beyond this energy. In the inelastic cross section, the \( \Delta \) resonance contribution is most
important. The attenuation length beyond this energy becomes the order of 20 [Mpc], which is less than the size of the universe. The UHECR can not propagate long distance in the universe, then. So the flux of cosmic ray should be suppressed beyond this energy, known as GZK bound. For the estimation of GZK bound, Lorentz invariance is assumed and the cross section between gamma and nucleon collisions in laboratory frame is used.

Experiments in this energy region are in progress and the source of UHECR will be identified in the future. Then an experimental determination of the attenuation length would become possible.

We propose a new mechanism of correction to the GZK bound in this paper. Most CMBR photons have been produced before the decoupling time in the early universe and may have finite coherence length. High energy charged particle also has a finite coherence length due to the collinear mass singularity. In the collisions of CMBR with UHECR beyond $10^{20}$ [eV] region, the finite coherence length makes the total photon-nucleon energy to spread. If this energy width is larger than or equivalent to the width of the $\Delta$ resonance, the total amplitude in the $\Delta$ resonance region is suppressed. The attenuation length of UHECR is modified, then.

The effect of the finite coherence length is negligible in the ordinary high energy scattering since the coherence length is much larger than the de Broglie wave length. So this effect has not been taken into account in the previous works on GZK bound. However, we show in the present work that the finite coherence length of CMBR gives a sizable effect to UHECR’s attenuation length even though the CMBR spectrum agrees with the Planck distribution.

2 Wave packet and the coherence length

In ordinary scattering experiments, a position where the beam particle is produced is known and the wave packet size is determined from the mean free path of the particle in matters and its size is semi-microscopic with much larger value than de Broglie length of high energy particle. Its effect is negligible, then. Let us call this length as the first coherence length. In a dilute system we study, a position where the particle is produced is unknown and the wave packet size, i.e., the coherence length is determined by the amplitude of many particle states in the production process.

In a system where each particle has a large mean free path, particle states preserve coherence for long time and coherence length due to mean free path is negligible. In this region, particles are described by a many body wave function and coherence length due to many body effects becomes important. Final state of the scattering matrix is a linear combination of the momentum states with the weight of scattering amplitude, unless a measurement of the final state is made. Consequently a correlation for one particle state of the different momenta, which is defined from the product between the scattering amplitude and its complex conjugate, becomes finite. This correlation length due to many body wave functions becomes important in the dilute system.
We estimate this coherence length of photon from the final states of Thomson scattering of Fig. 1. Let us focus on the final scattering of CMBR. CMBR is composed of almost free photon with the Planck distribution as far as single particle energy distribution is concerned. Thomson scattering amplitude is independent from the scattering angle at the low energy. Also the production region is not identified for CMBR. Consequently the photon in the final states of Thomson scattering is a coherent linear combination of momentum states with different orientations. This correlation of photons with different momenta is computed from these amplitudes where in the initial states the photon follows the Planck distribution with arbitrary angle and the electron follows the Fermi-Dirac distribution with arbitrary angle. We found that the correlation of final photons agrees almost perfectly with that of Gaussian wave packet \[10\]. Its width is a few times of \(k_B T\). So even though photon follows the Planck distribution it has a correlation of Gaussian wave packet of the width of a few \(k_B T\).

We study its implications in the collision of CMBR with UHECR. \(\Delta\) resonance lives for short period and its amplitude has a finite energy width. Hence if the time scale for UHECR to overlap with CMBR is shorter than the life time of the \(\Delta\) resonance, the amplitude is reduced by the finite coherence length. Then cross section due to the \(\Delta\) resonance gets a sizable correction from the finite coherence length.

Scattering amplitudes for the wave packets of a finite spatial extension in relativistic field theory have been formulated in Ref. [11]. It is shown that the amplitudes are consistent with the probability interpretation, despite non-orthogonality of the different states, and that the conservation of the energy and momentum is satisfied within an uncertainties given by the finite interaction area of the wave packets, since the states defined by wave packets have finite extensions. Furthermore, the asymptotic condition is satisfied with an finite initial time \(T_i\) and an finite final time \(T_f\), and the scattering probability has a position-dependence in addition to a momentum-dependence. From these properties, the wave packet scattering amplitude is different from a simple linear combination of the plane wave amplitude.

Gaussian wave packets, i.e., coherent states, of spherical symmetry are
defined as
\[ \langle \vec{p}\,|\vec{P}_0,\vec{X}_0 \rangle = N_3(\sigma_x^2)\frac{2}{\pi} e^{-i\vec{p} \cdot \vec{X}_0 - \frac{\sigma_x^2}{2}(\vec{P}_0 - \vec{p})^2} N_4^2 = (\pi \sigma_x^2)^{-\frac{3}{2}} \frac{\vec{P}_0}{\hbar} = \vec{k}_0. \]  
(1)

Generalizations to the asymmetric wave packets and to non-minimum wave packets are straightforward. The set of functions for one value of \( \sigma \) satisfy the completeness condition.

The time evolution of the free wave is determined by the free Hamiltonian, and creation and annihilation operators which satisfy \([a(\vec{p}, t), a^\dagger(\vec{p}', t')]\delta(t - t') = \delta(\vec{p} - \vec{p}')\delta(t - t')\) The operator \( A(\vec{P}_0, \vec{X}_0, T_0, t) \) that annihilates the state described by the wave packet is defined by
\[ A(\vec{P}_0, \vec{X}_0, T_0, t) = \int d\vec{p} a(\vec{p}, t) \langle \vec{p}\,|\vec{P}_0, \vec{X}_0, T_0 \rangle \]  
(2)

and the creation operator is defined by its conjugate. The particle states expressed by the wave packets follow classical trajectories and have finite spatial extensions. Consequently overlap region is finite and the energy-momentum conservation is only approximate.

The wave packet spreads with time and the spreading velocity in the transverse direction, \( v_T \), and the longitudinal direction, \( v_L \), are given by
\[ v_T = \sqrt{\frac{2}{\sigma_x^2} \frac{1}{E(\vec{P}_0)}}, \quad v_L = \sqrt{\frac{2}{\sigma_x^2} \frac{m^2}{(E(\vec{P}_0))^3}}. \]  
(3)

The \( v_T \) depends on the energy and the \( v_L \) depends on the energy and the mass. A massive wave packet spreads in both directions but a massless wave packet spreads only in the transverse direction. After a macroscopic time, any wave packet of the massive particle spreads to huge size. These wave may be treated as a plane wave approximately. However any wave packet of the massless particle does not spread and its size is kept fixed in the longitudinal direction. Thus, the wave packet of massless particle remains for the long period and its effect is important.

### 3 Resonance in the wave packet scattering

In the scattering of high energy proton with CMBR, the square of the center of mass energy, \( S \), is defined by
\[ S = (M_p^2 + 2E_pE_\gamma - 2|\vec{P}_p|\vec{P}_\gamma|\cos \theta) \simeq (M_p^2 + 2E_p \cdot E_\gamma(1 - \cos \theta)) \]  
(4)

where \( (E_\gamma, \vec{P}_\gamma), (E_p, \vec{P}_p) \), and \( \theta \) are four momenta of the photon, of the proton and the collision angle. The mass and width of the \( \Delta \), \( M_\Delta \) and \( \Gamma \), are \( M_\Delta = 1232[\text{MeV}], \Gamma = 120[\text{MeV}] \).

Breit-Wigner partial wave amplitude and the total cross section are,
\[ f_l(\theta) = \frac{\sqrt{2l + 1}}{p} \frac{\Gamma/2}{\sqrt{S - M_\Delta} + i\Gamma/2}, \quad \sigma_l = \frac{4\pi(2l + 1)}{p^2} \frac{\left( \frac{\Gamma}{2} \right)^2}{(\sqrt{S - M_\Delta})^2 + \left( \frac{\Gamma}{2} \right)^2}. \]  
(5)

4
The photon wave function of the momentum $\vec{p}_0$ which we obtained is expressed by a minimum wave packet,

$$|\psi_\gamma(\vec{p} - \vec{p}_0)| = N \exp \left[ -\frac{(\vec{p} - \vec{p}_0)^2}{2\sigma^2} \right].$$

We have studied also non-minimum case by multiplying a polynomial $h_m(\vec{p})$ to the last term but our conclusion of the present work is the same. For an asymmetric wave packet, an asymmetric $\sigma$ is used. Actually a high energy charged particle is combined with coherent soft photons in order to cure infrared divergence\cite{12, 13} caused by massless particle, photon. So a charged particle system is spread in the momentum and energy.

4 Cross section and attenuation length of UHECR

The average cross section for the CMBR of finite coherence length is given by,

$$\sigma_{\text{CMBR}} = \int d\nu U(\nu) \left[ \int d^3p \, \psi_{\gamma+p}(\vec{p} - \vec{p}_0) \frac{4\pi}{P_{\text{CM}} \sqrt{S - M^2/2}} \right]^2,$$

$$U(\nu, T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1}, \quad |\vec{p}_0| = h\nu$$

where $P_{\text{CM}}$ is momentum in the center of mass frame and parameters $h$, $c$, $k_B$, $T$, are Planck constant, speed of light, Boltzmann constant, and temperature of the CMBR. $\psi_{\gamma+p}(\vec{p})$ is the wave function of the photon-proton system. It should be noted that the integration on $\vec{p}$ is taken in the amplitude because the strict energy conservation does not hold for the wave packet scattering.

Using the cross section, we calculate the average attenuation length of UHECR in the parameter range $\sigma \leq 10 k_B T$, where the temperature is regarded as that of the decoupling time until this point. However to compare with the current observation, we use the current value of the temperature, $T = 2.725[K]$. The inelasticity of the UHECR is given by the ratio of energy loss of the UHECR, $E_{p,f} - E_{p,i}$, over the initial energy in the scattering $K_p = \frac{E_{p,i} - E_{p,f}}{E_{p,i}}$. The attenuation length is obtained by integrating the product of the above ratio, $K_p$, with the cross section. The result is given in Fig. 2. The attenuation length becomes longer by factor 10 if the coherent energy spread is wide. Since the $\sigma$ includes both effects of CMBR and UHECR, larger values of the $\sigma$ is included here.

5 Lorentz invariance

By a Lorentz transformation of the frame, one value of the momentum is transformed to another value of the momentum \cite{14, 15, 16}. Amplitude for the plane
wave is covariant and the cross section is invariant under the Lorentz transformation. So the cross section for the plane wave for CMBR is computed easily from the experimental value of the photon-nucleon reactions in the laboratory frame. The amplitude for the wave packets, however, should be treated carefully, since the wave packet is a linear combination of the momentum states.

We calculated the total cross section for the wave packet explicitly and find the sizable difference compared to the plane wave. The finite size effect of the photon is negligible in the laboratory frame of the pion production process in photon-nucleon reactions but is not negligible in the present situation. In the latter system, a small coherent energy spread of CMBR leads the center of mass energy to spread finite amount. A product of the small coherent energy spread with an extremely large energy of UHECR becomes finite. The variance of center of mass energy $S$ of the laboratory frame, $\Delta S_l$, and of the CMBR frame, $\Delta S_{CMBR}$, are given by

$$\Delta S_l = 2m_p\Delta E_l,$$
$$\Delta S_{CMBR} = 2p\Delta E_{CMBR}\delta(1-a),$$

where “a” is of order 1. If the energy variances of the photon are the same in both systems, the ratio of two values of the center of mass energy is given by $\frac{p}{m_p}$. This ratio is of order $10^{11}$ for UHECR. So in this case $\Delta S_{CMBR}$ is much larger than $\Delta S_l$ and the finite coherent length effect in the CMBR frame of UHECR is much larger than that of the laboratory frame. This shows the reason why a naive Lorentz invariance does not hold for the wave packet scattering.
6 Summary

We found that the attenuation length of UHECR in the $10^{20}$ [eV] region varies depending on the coherence length of CMBR. It is clear from the Fig. 2 that the attenuation length for UHECR becomes longer if the CMBR photon has a finite coherence length. The effect becomes important in the pion production threshold energy region, where a new data from Auger collaboration concluded that the UHECR in this energy region comes sources within 75 [Mpc]. Our calculation at the coherence length of $3.5 k_B T$ suggests that the value becomes 150 [Mpc] instead of 75 [Mpc]. When the precise value of the flux of UHECR in wider energy region will be known, better informations will be obtained.

Our study shows the importance of the finite coherence length of CMBR in analyzing the attenuation length of UHECR.

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