QCD sum rule for nucleon in nuclear matter

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Abstract. We consider the two-point function of nucleon current in nuclear matter and write a QCD sum rule to analyse the residue of the nucleon pole as a function of nuclear density. The nucleon self-energy needed for the sum rule is taken as input from calculations using phenomenological NN potential. Our result shows a decrease in the residue with increasing nuclear density, as is known to be the case with similar quantities.

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1 Introduction

An important topic in strong interaction at non-zero temperature and chemical potential is the propagation of hadrons through these media. Their modified couplings and self-energies are useful not only in analysing the experimental data on heavy-ion collisions, but also in extracting indications of an eventual phase transition in the medium.

At low density, such a propagation in the hadronic phase can, in principle, be studied by invoking chiral perturbation theory to evaluate the appropriate two-point function \cite{12}. This effective theory of QCD is of particular advantage at low temperature (and zero chemical potential), when the heat bath is dominated by pions. Their couplings with themselves and with other hadrons are highly constrained by the chiral symmetry of (massless) QCD. Thus not only are the vertices with pions related to those without the pions, but also they are suppressed by powers of pion momenta. Using these vertices, one can get the hadron parameters to a good accuracy by evaluating only a few relevant Feynman graphs. It has been applied with much success in calculating the proper-ties of the pion and the nucleon at finite temperature \cite{3,4,6,7,8}.

In this work we are interested in finding the effect of strong interaction on the propagation of nucleon at non-zero nucleon chemical potential (\(\mu\)) i.e. in nuclear matter, at zero temperature (\(\beta^{-1} = 0\)). Quite generally, this propagation is studied by considering the two-point function,

\[ I(E,p) = i \int dt d^3x e^{iEt - p \cdot x} \langle T \eta(x) \eta(0) \rangle \]  

of a three-quark current \(\eta(x)\), having the quantum numbers of the nucleon \cite{9,10}. Here \(\langle \cdots \rangle\) denotes ensemble average: For any operator \(O\),

\[ \langle O \rangle = \frac{\text{Tr}[e^{-\beta(H-\mu N)}O]}{Z}, \quad Z = \text{Tr} e^{-\beta(H-\mu N)} \]

where \(H\) and \(N\) are the Hamiltonian and the number operator of the system.

Unfortunately, a straightforward calculation of Feynman graphs in this case, similar to the one at finite temperature, is not possible at present. The difficulty is due to the appearance of new and presumably large couplings. They are shown in Fig. 1, where the complete \(\eta\)-nucleon vertex and the complete nucleon self-energy are analysed in terms of low order perturbative vertices, whose structures may be obtained by the methods of chiral perturbation theory.

Thus, up to terms proportional to a single nucleon field \(\psi(x)\), the current \(\eta(x)\) is given by \cite{11}

\[ \eta(x) = \lambda \left( 1 + \frac{i \phi(x) \cdot \tau}{2 F_\pi} \gamma_5 + \cdots \right) \psi(x), \]  

where \(\phi(x)\) is the pion field and the dots stand for terms with increasing number of pion fields. Here \(F_\pi\), the so-called pion decay constant, is defined by the vacuum-to-pion matrix element of the axial-vector current \(A^\mu_\lambda(x)\),

\[ \langle 0 | A^\mu_\lambda(x) | \pi^j(k) \rangle = i \delta^{ij} k_\mu F_\pi e^{ik \cdot x}, \quad F_\pi = 93 \text{ MeV}, \]  

just as \(\lambda\) is defined by the vacuum-to-nucleon matrix element of the nucleon current \(\eta(x)\)

\[ \langle 0 | \eta(x) | N(p) \rangle = \lambda u(p)e^{ip \cdot x}, \]  

where \(u(p)\) is a positive energy Dirac spinor. The value of \(\lambda\) is obtained from QCD sum rules for nucleon in vacuum \cite{9},

\[ \lambda^2 = (1.2 \pm 0.6) \times 10^{-3} \text{GeV}^6. \]  

Terms in $\eta$ proportional to $\bar{\psi}\psi\psi$ may also be obtained in the same way, bringing in two more new coupling constants \[11\]. We now see in Fig. 1 that unlike the vertex $\eta\bar{\psi}\phi$ in graph (b), which is related to the vertex $\eta\bar{\psi}$ of graph (a) itself, the vertex $\eta\bar{\psi}\psi\psi$ in graph (c) is unknown and unlikely to be small.

For the self-energy graphs, we note the pion-nucleon interaction Lagrangian \[12\],

$$L_{\text{int}} = -\frac{g_A}{2F_\pi} \bar{\psi}(x)\gamma^\mu\gamma_5\tau^i\psi(x)\partial^\mu\phi^i(x) + \cdots,$$

where $g_A$ is the axial-vector coupling constant of the nucleon, $g_A = 1.26$. We again see in Fig. 1 that unlike the self-energy graph (e), which can be calculated with this Lagrangian, the other graph (f) poses difficulty, even though chiral symmetry dictates the form of the four-nucleon effective Lagrangian, whose coefficients can be fixed by the NN scattering lengths \[13\]. Indeed, if one does calculate the graph (f) with this four-nucleon interaction, one gets an unacceptably large value for the nucleon self-energy \[14\]. The problem can be traced to the fact that there are bound and virtual states very close to threshold in the NN system.

In view of these difficulties, we give up calculating the nucleon self-energy and content ourselves with evaluating only the nucleon pole residue, by writing a QCD sum rule for an appropriate amplitude representing the two-point function. (There are several approaches to QCD sum rules in medium \[15\]–\[16\]–\[17\]. The one closest to ours is of Ref.\[17\].) To determine the nucleon pole term in the medium, we still need the nucleon self-energy, which we take from the variational and the Brueckner type calculations of the field theory in medium, where a two-point function assumes the form of a $2 \times 2$ matrix \[21\]. (This formulation is reviewed in \[22\].) But the dynamics is given essentially by a single analytic function, obtained by diagonalising the original matrix. Thus if $\Pi_{11}(E, p)$ is the 11-component of the original matrix amplitude, the corresponding analytic function, to be denoted by the same symbol as in Eq. (1), has the spectral representation \[23\],

$$\Pi(E, p) = \int_{-\infty}^{\infty} \frac{dE'}{2\pi} \frac{\sigma(E', p)}{E' - E - i\eta(E')}$$

where the spectral function is related to the imaginary part of $\Pi_{11}$ by

$$\sigma(E, p) = 2\coth\{\beta(E - \mu)/2\} \text{Im}\Pi_{11}(E, p).$$

For generality we calculate the amplitudes retaining both $\mu$ and $\beta$ and take the limit of zero temperature later.

The two-point function due to the free propagation of the nucleon, namely

$$-\frac{\lambda^2}{\bar{p} - m + i\epsilon}$$
is modified by the vertex and the self-energy corrections of Figs. 2 (a), (b) and (c) to
\[ \Pi(E, \mathbf{p})|_{(a+b+c)} = -\frac{\lambda^2}{\theta - m - \Sigma(p)} \] (9)
where \( \lambda^* \) is the modified coupling parameter in nuclear matter and \( \Sigma(p) \) is the nucleon self-energy acquired in this medium.

In this work we restrict to \( \mathbf{p} = 0 \), when \( \Sigma \) has the simple Dirac decomposition, \( \Sigma = \Sigma_S + \gamma^0 \Sigma_V \). Then Eq. (9) may be rewritten as
\[ \Pi(E)|_{(a+b+c)} = -\lambda^2 \frac{\gamma^0 (E - \Sigma_V) + m^*}{(E - m_1)(E - m_2)}. \] (10)
The scalar part \( \Sigma_S \) of the self-energy changes the mass of the free particle to the effective mass \( m^* \) in the medium, \( m^* = m + \Sigma_S \). Similarly the vector part \( \Sigma_V \) shifts the rest energies, \( \pm m \) of the free nucleon and the antiquinucleon, to \( m_1 = m^* + \Sigma_V \) and \( m_2 = -m^* + \Sigma_V \) respectively of the corresponding quasi-particles. Following our discussion in the Introduction, we work with the subtracted nucleon pole term,
\[ \mathcal{P}(E)|_{(a+b+c)} = \Pi(E)|_{(a+b+c)} + \lambda^2 \frac{\gamma^0 E + m}{E^2 - m^2}. \] (11)
Here and below we use a bar over amplitudes \( \Pi \) and spectral densities \( \sigma \) to denote subtraction of the corresponding vacuum contributions.

We now evaluate the remaining graphs of Fig. (2). The imaginary part of the 11 component of the matrix amplitude for graph (d) is given by [23,24,25]
\[ \text{Im} \Pi(E)|_{11}|_d = -\frac{3 \lambda^2 \pi}{4 F_\pi^2} \int \frac{d^4q}{(2\pi)^3 4\omega_1 \omega_2} \times \]
\[ \left[ (-\gamma^0 \omega_1 + m)(1 + n_+ + n)\delta(E - \omega_1 - \omega_2) + (n_+ + n)\delta(E - \omega_1 + \omega_2) \right] \omega_{1,2} \rightarrow \omega_{1,2}, n_+ \rightarrow n_-, n_+ \rightarrow n_. \] (12)
where \( \omega_1 = \sqrt{m^2 + q^2}, \omega_2 = \sqrt{m_2^2 + q^2} \) and \( n_+ \) and \( n_\pm \) are respectively the distribution functions for nucleons, antinucleons and pions,
\[ n_\pm(\omega_1) = \frac{1}{e^{\theta(\omega_1 \mp m)} + 1}, \quad n(\omega_2) = \frac{1}{e^{\theta(\omega_2 - m)} - 1} \] (13)
Terms without the \( n \)'s are the vacuum contributions, which we subtract out, as already stated above. Further, we restrict to zero temperature so that we have to calculate only the term proportional to \( n_+ \rightarrow \theta(\mu - \omega_1) \): we thus get
\[ \sigma(E)|_{11} = \frac{3 \lambda^2}{16\pi F_\pi^2 E} \sqrt{\omega^2 - m^2}(\gamma^0 \omega - m) \] (14)
on the Landau cut and the negative of the same quantity on the unitary cut. Here \( \omega \) is the nucleon energy expressed in terms of the total energy \( E, \omega = (E^2 + m^2 - m^2_\pi)/(2E) \).

Because of the \( \theta \)-function in the integrand, the two cuts originally over the regions \( 0 \leq E \leq m - m_\pi \) (Landau) and \( m + m_\pi \leq E \leq \infty \) (unitary) shrink respectively to
\[ \mu - \sqrt{\mu^2 - m^2 + m_\pi^2} \leq E \leq m - m_\pi \] (15)
and
\[ m + m_\pi \leq E \leq \mu + \sqrt{\mu^2 - m^2 + m_\pi^2}. \] (16)
Then the desired analytic function in the form of Eq. (7) is given by
\[ \mathcal{P}(E)|_{(d)} = \int_C \frac{dE'}{2\pi} \frac{\sigma(E')|_{(d)}}{E' - E - i\eta(E')} \] (17)
where the subscript \( C \) denotes the difference of two integrals,
\[ \int_C = \int_{\text{Landau}} - \int_{\text{unitary}} \] (18)
Similarly the graphs (e) and (f) give
\[ \mathcal{P}(E)|_{(e+f)} = \frac{2(\gamma^0 E + m)}{E^2 - m^2} + i\epsilon \int_C \frac{dE'}{2\pi} \frac{\sigma(E')|_{(e+f)}}{E' - E - i\eta(E')} \] (19)
where
\[ \sigma(E)|_{(e+f)} = \frac{\lambda^2 g_A}{16\pi F_\pi^2 E} \sqrt{\omega^2 - m^2}(m^2 - E\omega + \gamma^0 m(E - \omega)) \] (20)
Of course, we have yet to subtract out from Eq. (19) its nucleon pole part, as the complete nucleon pole contribution is already represented by Eq. (11).

At this point we adopt an improved choice of the amplitude following the suggestion in Ref. [20]. Of the two quasi-particle poles, the one corresponding to the antinucleon cannot be treated as having narrow width, as we do in Eqs. (10) and (11)), because it can be annihilated strongly in nuclear medium. This ill-represented contribution may be suppressed by working with a modified amplitude. We split \( \mathcal{P} \) into even and odd parts,
\[ \mathcal{P}(E) = \mathcal{P}(E^2) + E\mathcal{P}(o)(E^2), \] (21)
and deal with the combination,
\[ \mathcal{P}(E) = \mathcal{P}(E^2) - m_2\mathcal{P}(o)(E^2) \] (22)
(Recall that \( m_2 \) is the pole position of the quasi-antinucleon.) Indeed, if we separate the unsubtracted pole amplitude \( \Pi(E) \) given by Eq. (10) into even and odd parts and combine them as in Eq. (22), we get the amplitude
\[ -\frac{2\lambda^2 m^*}{E^2 - m^2} \frac{1}{2}(1 + \gamma^0) \] where the pole at \( E = m_2 \) is removed to \( E = m_1 \). Note also that it is proportional to \( \frac{1}{2}(1 + \gamma^0) \). As we are interested in a sum rule for \( \lambda^* \), we shall project all amplitudes on to this combination in Dirac space.\(^{1}\)

\(^{1}\) The amplitudes proportional to \( \frac{1}{2}(1 - \gamma^0) \) are expected to be relatively small and so comparable to those left out in our approximation. Thus the corresponding sum rule may not be reliable, besides the fact that it will not involve \( \lambda^* \).
We thus get the amplitudes corresponding to the different graphs of Fig. (2) as
\[ \Pi(E^2)|(a+b+c) = -2\frac{\lambda^2m^2}{E^2 - m_i^2} + \frac{\lambda^2(m - m_2)}{E^2 - m_2^2} \]
\[ \Pi(E^2)|(d) = \frac{3\lambda^2}{32\pi^2F^2\pi} \int\frac{dE'}{E'} f(E') \]
\[ \Pi(E^2)|(e+f) = -3\frac{\lambda^2 a}{16\pi^2F^2\pi} \int\frac{dE'}{E'} E' + m f(E') \]
Here \( f(E) \) is given by
\[ f(E) = (E - m_2)\sqrt{\omega^2 - m^2}(\omega - m) \]
In the last amplitude we have removed the nucleon pole contribution from graphs (e) and (f). The sum of these amplitudes will give the low energy part of the spectral side of the sum rule.

3 Operator side of sum rule

In obtaining the operator product expansion, we need the explicit form of the quark current \( \eta(x)_{i,j} \) with spin and isospin indices \( D \) and \( i \). Of the two independent possibilities involving three quark fields, we choose here the one, which for proton (\( i = 1 \)) is
\[ \eta(x)_{D,1} = \epsilon^{abc}(u^aT(x)C\gamma^\mu u^b(x))(\gamma_5\gamma_\mu d^c(x))_D, \]
where \( C \) is the charge conjugation matrix and \( a, b, c \) are the colour indices. Because of subtraction of the vacuum amplitude, the unit operator in the operator product does not appear in \( \Pi(E) \). As is well-known, at higher dimensions, there are two sets of contributing operators in the in-medium sum rule: the old set, appearing already to the first order in \( \ln(Q^2/\mu^2) \) in writing the \( \Theta \) functions, and the new set, involving \( u^\mu \), the four-velocity of the medium. From the ensemble average
\[ \langle E, \mu \rangle G^{\alpha \beta}(E, \mu, \nu) = \int\frac{dE'}{E'} G^{\alpha \beta}(E', \mu, \nu), \]
we have
\[ \langle E, \mu \rangle G^{\alpha \beta}(E, \mu, \nu) \sim \frac{6}{\pi^2} \epsilon^{abc}u^aT(x)^\mu u^b(x)C\gamma_\mu d^c(x), \]
with coefficients to zeroth order in \( \alpha_s/\pi \). The renormalization scale \( \mu \) is taken at 1 GeV.

The four-quark condensates in medium encountered above have been factorised in a manner similar to that by saturating the vacuum condensates with the vacuum intermediate state – an approximation that may not be as good as in vacuum. To rectify the error, we adopt the suggestion of Ref. [20] to interpolate the factorised \( \langle iu \rangle \) between its values in vacuum and in medium and in the terms of the above operators, the operator expansion gives
\[ \Pi(E, 0) \xrightarrow{\text{OP}} \frac{1}{4\pi^2} \langle \bar{u}u \rangle + 4\langle u^i u \rangle \gamma^0 E^2 \ln(-E^2/\mu^2) \]
\[ -\frac{1}{6\pi^2} \left( \frac{3}{16} \langle G^2 \rangle + 5\langle \Theta^f \rangle \right) \gamma^0 E \ln(-E^2/\mu^2), \]
\[ -\frac{2E}{3E^2}(\gamma^0 \langle \bar{u}u \rangle)^2 + 2\langle \bar{u}u \rangle \langle u^i u \rangle \]
with coefficients to zeroth order in \( \alpha_s/\pi \). The renormalization scale \( \mu \) is taken at 1 GeV.

Recalling that the combination (22) of amplitudes is a function of \( E^2 \), we can go to the spacelike region by setting \( E^2 = -Q^2, \ Q^2 > 0 \). While \( \mu^2 = 1 \text{ GeV}^2 \) is a natural scale for the expectation values of the operators, it is not convenient for the coefficients, as they contain powers of \( \ln(Q^2/\mu^2) \) in higher orders. As is well-known in the context of deep inelastic scattering, it is possible to use the renormalization group equation to get rid of these \( \ln(Q^2/\mu^2) \) logarithms by shifting the scale for the coefficients from \( \mu^2 \) to \( Q^2 \). The process brings in the (small) anomalous dimensions of the operators; also the operators \( \Theta^f \) and \( \Theta^g \) will mix [25,26]. However, as we shall vary the Borel mass in the neighbourhood of 1 GeV, the renormalization
\[ \langle iu \rangle \xrightarrow{\text{OP}} (1 - f)\langle 0|iu|0 \rangle + f\langle \bar{uu} \rangle \]
where \( f \) is a real parameter in the range \( 0 \leq f \leq 1 \).
group improvement will give small corrections, which we shall ignore in the present analysis.

The nucleon number density $\pi$ is related to the Fermi momentum $p_F$ by $\pi = 2p_F^2/(3\pi^2)$. In normal nuclear matter, it is given by $\pi_0 = (110\text{MeV})^3$ corresponding to $p_F = 270\text{MeV}$. To first order in $\pi$, the change in the expectation value of an operator $O$ in nuclear matter relative to that in vacuum is given by its nucleon matrix element as

\[ \langle O \rangle = \langle 0 | O | 0 \rangle + \frac{\langle p | O | p \rangle}{2m} \pi \]  

(31)

We apply this equation to the different operators. For $\bar{u}u$ and $u^\dagger u$, we get

\[ \langle \bar{u}u \rangle = \langle 0 | \bar{u}u | 0 \rangle + \frac{\sigma}{2m} \pi \]  

(32)

where $\sigma$ is the so-called nucleon $\sigma$-term, $\pi$, obtained by continuing the result for operator expansion to the time-like region. Here $\pi$ stands for the constants.

The quark mass and the vacuum condensate are related by the Gell-Mann, Oakes and Renner formula $\pi$, and

\[ \sigma = \hat{m}(\langle \bar{u}u \rangle - \langle u^\dagger u \rangle) / m = 45 \pm 8\text{MeV}. \]  

(33)

The quark mass and the vacuum condensate are related by the Gell-Mann, Oakes and Renner formula $\pi$.

\[ F^2 m_\pi^2 = -2\hat{m}\langle 0 | \bar{u}u | 0 \rangle. \]  

(34)

Two determinations of these quantities exist in the literature, namely

\[ \hat{m} = 7.2\text{MeV}, \quad \langle 0 | \bar{u}u | 0 \rangle = -\langle 2(225\text{MeV})^3 \]  

(35)

\[ \hat{m} = 5.5\text{MeV}, \quad \langle 0 | \bar{u}u | 0 \rangle = -\langle 2(245\text{MeV})^3 \]  

(36)

obtained respectively in Refs. [32] and [33]. Let us note here that using the formula (34), we may write the condensate (32) in nuclear matter as

\[ \langle \bar{u}u \rangle = \langle 0 | \bar{u}u | 0 \rangle \left( 1 - \frac{\sigma \pi}{m_\pi^2 F^2} \right), \]  

(37)

which vanishes at $\pi = 2.8\pi_0$.

Next, for $\Theta^I$ we can write the nucleon matrix element as

\[ \langle p | \Theta^I_{\mu\nu} | p \rangle = 2A^I(p_{\mu}p_{\nu} - g_{\mu\nu}m^2)/4, \]  

(38)

where the coefficient $A^I$ is determined by the first moment sum rule for the quark distribution function in deep inelastic scattering $\pi$. Evaluated at the momentum scale of 1 GeV, it has the value $A^I = 0.62 \pi$. Then noting the normalization condition, $\langle 0 | \Theta^I_{\mu\nu} | 0 \rangle = 0$, we get from Eqs. (31) and (38),

\[ \langle \Theta^I \rangle = \frac{3}{4}mA^I \pi \]  

(39)

Finally for the operator $C^2$, we use the trace anomaly to relate it to the trace of the full energy momentum tensor $\Theta_{\mu\nu}$,

\[ \Theta^\mu_{\mu} = -\frac{9}{8} G^2 + 2\hat{m}u \bar{u} + c \cdot 1 \]  

(40)

where we add the $c$-number term to fix again its vacuum normalization, $\langle 0 | \Theta^\mu_{\mu} | 0 \rangle = 0$. Taking the vacuum and the ensemble expectation values, we get

\[ \langle G^2 \rangle = \langle 0 | G^2 | 0 \rangle - \frac{8}{9}(m - \sigma) \pi \]  

(41)

With the above results, we can subtract out the vacuum contributions from Eq.(29) for $I(E, 0)$ and write the result for the amplitude combination (22) as

\[ \tilde{I}(Q^2) \overset{\text{OPE}}{=} \left[ -\frac{A}{8\pi^2} Q^2 \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{B m_2}{8\pi^2} \ln \left( \frac{Q^2}{\mu^2} \right) - \frac{2 C m_2}{3 Q^2} \right] \pi \]  

(42)

where $A, B$ and $C$ stand for the constants,

\[ A = \frac{\sigma}{m} + 12, \]  

\[ B = 5mA^I - \frac{2}{9}(m - \sigma), \]  

\[ C = \langle 0 | \bar{u}u | 0 \rangle \left( \frac{f}{m} + 3 \right) \]  

(43)

4 Sum rule

It is now simple to take the Borel transform of the spectral and the operator sides and get the desired sum rule

\[ \chi^2 = \chi^2 e^{m_\pi^2/M^2} \left( \frac{m - m_2}{2m^*} e^{-m_2/M^2} - \frac{3}{64\pi^2 F^2 m^*} \times \right. \]  

\[ \int C \frac{dE}{E} f(E) \left\{ 1 - 2gA \left( \frac{E + m}{E - m} \right) \right\} e^{-E^2/M^2} \]  

\[ - \frac{M^2}{2\chi^2 m^*} \left( \frac{M^2}{8\pi^2} AV_2 + \frac{m_2}{8\pi^2} BV_1 + \frac{2Cm_2}{3M^2} \right) \tilde{\pi} \]  

(44)

where $f(E)$ is given by Eq. (26) and

\[ V_1 = 1 - e^{-W^2/M^2}, \quad V_2 = 1 - (1 + W^2/M^2)e^{-W^2/M^2} \]

The deviation of $V_{1,2}$ from unity represents the contribution from the high energy region on the spectral side, obtained by continuing the result for operator expansion to the time-like region. Here $W$ is a parameter determining the onset of this continuum contribution. We take $W = 2\text{GeV}$, as assumed for the vacuum sum rules $\pi$.

Let us recall here that a special feature of the present QCD sum rule is its sensitivity to the medium dependent quantities, requiring a more careful saturation of its spectral and operator sides. Since we consider nuclear matter at zero temperature, it consists only of nucleons. Then, on the spectral side, the two-particle density dependent contributions can arise from intermediate states consisting only of $N$ with $\pi, \rho, \omega, \phi, \cdots$. As seen from the inequalities (15,16), the range of energy over which these states may contribute is highly restricted. Thus at normal nuclear density, these ranges are only $1080\text{MeV} \leq E \leq 1290\text{MeV}$ on the unitary cut and $670\text{MeV} \leq E \leq 800\text{MeV}$ on the Landau cut. Inclusion of more particles in the intermediate state will, of course, increase these ranges, but their contributions will fall off exponentially in the integrand. So we include only the continuum contribution from the $\pi N$ intermediate state, in addition to the modified nucleon
pole. Although the continuum contributions over the Landau and the unitary cuts are individually rather big, the two largely cancel out in the difference (see Eq. (18)), leaving the nucleon pole contribution to dominate the spectral side.

The operator side is also well saturated with the usual low dimension operators. The leading operators, \( \overline{q}q \) and \( q^i q \) bring in a large contribution, as represented by the term with \( A \) in Eqs. (42, 44), compared to those of dimensions four and six retained in the sum rule. Also the numerous operators of dimension five and higher do not appear to contribute significantly, at least individually [20].

### 5 Evaluation

In presenting numerical results, we observe that among all the parameters, it is the \( \lambda \) which enters most sensitively in the sum rule and also suffers from the largest uncertainty in its value (Eq. (5)). Under these circumstances, we fix \( \lambda \) by requiring maximal stability of the results against variation with respect to the Borel mass in a range similar to that required for the vacuum sum rules [9]. Numerical evaluation shows that bigger the value of \( \lambda \), the more stable is the result. We show this stability in Fig. 3 for different relative densities, taking \( \lambda^2 = .0012 \text{ GeV}^6 \), the central value and \( \lambda^2 = .0018 \text{ GeV}^6 \), the largest in the allowed range. (Here we take the self-energies from Ref. [19], those from Ref. [18] give even better stability of the results.) With the latter value of \( \lambda \), we see that there is a reasonable plateau up to about normal nuclear density, vindicating our saturation scheme. At higher densities, however, the plateau seems to disappear, indicating insufficient saturation.

We next consider the uncertainties in the results from the remaining inputs. First consider the self-energies. As already stated, these have been determined by two groups [18][19], using entirely different methods based on phenomenological NN potentials. Fig. 4 depicts results using these two sets of self-energies. Clearly it is not a source of any significant uncertainty in the results, at least up to normal nuclear density.

Finally we vary the values for the sigma term and for the pair, the quark mass and the quark condensate. As seen from Fig. 5, the uncertainty in these parameters again does not give rise to any significant spread in the values of \( \lambda^* \). Also the term with the parameter \( f \) in Eq. (43) arising from the approximation to the four-quark condensate is relatively too small to change the results appreciably. We thus show unambiguously a decreasing trend for \( \lambda^* \) with the rise of density at least up to normal nuclear density.

### 6 Discussion

We describe here a method to find \( \lambda^* \), the parameter coupling the nucleon current to the nucleon state in nuclear matter. The method consists of writing down a \( QCD \) sum rule in this medium for an appropriate combination of amplitudes representing the ensemble averaged two-point function of nucleon current. With the present scheme of saturating the sum rule, the results are reliable up to normal nuclear density, within which one finds definitely a decrease of \( \lambda^* \) with increase in density. For quantitative results beyond this density, we have to improve upon our saturation scheme. A test of this improvement at a higher density is offered by observing a plateau in \( \lambda^* \) at such density as a function of the Borel mass. If, however, we continue the present calculation beyond its range of validity, it goes to zero at about twice this density.
of these amplitudes in the neighbourhood of the latter to be poorly determined by the sum rules. As self-energies and the nucleon residue. They, however, find plitmates and hence three sum rules to determine the two authors work for a significant contribution.

Also if the many operators of higher dimensions contribute with the same sign, they together may bring in included. Also if the many operators of higher dimensions contribute with the same sign, they together may bring in significant.

In looking for additional sources of significant contributions at higher densities, we note that as the chemical potential increases, the Landau and the unitary cuts extend over wider intervals in $E$, so that contributions from intermediate states with higher thresholds need be included. Also if the many operators of higher dimensions contribute with the same sign, they together may bring in a significant contribution.

It is appropriate here to compare our work with that of Ref. [14], where similar sum rules are discussed. These authors work for $p \neq 0$, getting three independent amplitudes and hence three sum rules to determine the two self-energies and the nucleon residue. They, however, find the latter to be poorly determined by the sum rules. As we point out in footnote 1, there is only one combination of these amplitudes in the neighbourhood of $p = 0$, which has a large nucleon pole term. We thus write a single sum rule for just this amplitude, supplying the (unknown) self-energies from independent theoretical determinations. It is this focusing on the single large amplitude that makes our result for the residue quantitative.

In our present calculation we face a problem in that the coupling parameter $\lambda$ in vacuum itself is not well determined. This, however, is not the case with the couplings of vector and axial-vector currents with the relevant particles, though the problem of determining the self-energies of these particles in nuclear medium still remains. Once the latter problem is treated properly, calculation of such parameters may well be quantitative.

The present work is restricted to nuclear matter at zero temperature, so that the pions appearing in the loop graphs of Figs. 1 and 2 are virtual. It may, however, be easily extended to finite temperature, when there will appear real pions, exciting the nucleons into nucleonic resonances, such as $\Delta(1237)$ with high degeneracy factors. Then contributions from intermediate states like $\pi\Delta$ need also be included in the spectral side of the sum rule.

As we work with nuclear matter, we ignore electromagnetic interaction. However, in the realistic case of nuclear density created, for example, in heavy ion collisions, it may not be negligible [36].

Finally we put together some similar, known results for current-particle couplings and the quark condensate. Consider first the pionic medium at low temperature. The coupling parameter $F_\pi$ of the axial-vector current with pion in vacuum changes to $F^T_\pi$ [38],

$$F^T_\pi = F_\pi \left(1 - \frac{T^2}{12F^2_\pi}\right).$$  

(45)

The coupling of the baryonic current with nucleon, that we are considering here, also changes from the vacuum value $\lambda$ to $\lambda^T$ [8].

$$\lambda^T = \lambda \left(1 - \frac{T^2}{32F^2_\pi}\right),$$  

(46)

For the quark condensate, we have

$$\langle \bar{q}q \rangle^T = \langle 0 | \bar{q}q | 0 \rangle \left(1 - \frac{T^2}{8F^2_\pi}\right),$$  

(47)

where we keep only the leading term, though it has been calculated up to $O(T^6)$ [12].

Considering nuclear matter (at zero temperature), the Lorentz invariance already breaks at leading order for the axial-vector current coupling to pion,

$$k_{\mu} F_\pi \rightarrow k_0 F^T_\pi \delta_{\mu 0} + k_i F^T_\pi \delta_{\mu i}$$  

(48)

giving rise to two decay parameters. They change with nuclear density as [37, 38]

$$F^T_\pi = F_\pi \left(1 - 0.26 \pm 0.04 \frac{F_\pi}{\langle \bar{q}q \rangle_0}\right)$$  

(49)

$$F^T_\pi = F_\pi \left(1 - 1.23 \pm 0.07 \frac{F_\pi}{\langle \bar{q}q \rangle_0}\right)$$  

(50)

Observe that the two parameters have quite different density dependence. But one can argue [37] that it is the temporal component that reflects the spontaneous breaking of chiral symmetry. To these pion decay parameters, we add the result of the present work,

$$\lambda^\ast = \lambda \left(1 - 0.20 \pm 0.04 \frac{F_\pi}{\langle \bar{q}q \rangle_0}\right),$$  

(51)

obtained as a linear fit to the curves of Fig. 5 up to normal nuclear density. Also the quark condensate in this medium is given already by Eq.(37).

All these results are low-density expansions (in pionic and nuclear media) for the current-particle couplings and the quark condensate. What is worth noting is that at such densities all the couplings along with the quark condensate...
definitely decrease with rise of density. The significance of this decrease is clear. In vacuum, $F_\pi (\lambda)$ measures the overlap of the pion (nucleon) state with those obtained by applying the axial (nucleonic) charge on the vacuum. In medium their decrease indicates that the external sources, $A_\mu (x)$ and $\eta (x)$ become gradually weak in exciting pion and nucleon in the respective media. The decrease of the quark condensate implies a trend towards restoration of chiral symmetry.

It is interesting to recall here that although QCD is believed to be the theory of strong interaction amongst quarks and gluons, it is asymptotically free at short distances. This feature leads to the common belief that with the rise of temperature and/or density, hadronic matter will undergo one or more phase transitions, restoring chiral symmetry and liberating colour to form quark-gluon plasma. The corresponding order parameters are the quark condensate and the Polyakov loop, which must be evaluated on the lattice to investigate such transitions. Any density expansion of the parameters, even if carried to arbitrarily high order, would not be valid close to phase transition [39].

It is, however, the case that often the above first order formulae give critical values in qualitative agreement with lattice and exact model calculations. The best example is $(\bar{q}q)^T$, for which the value given by Eq. (47) is of the same order as the one from the lattice [40]. Thus at finite temperature, the same value approximately for the coefficients of $T^2$ in $F_\pi ^T$, $\lambda ^T$ and $(\bar{q}q)^T$ tends to support the expectation that they all go to zero at the same (critical) temperature [5]. In the same way, at finite nuclear chemical potential, we find that the coefficients of $\pi$ are again approximately the same for all the three quantities, allowing us to expect that they all disappear together at the same critical density, which is several times away from the normal nuclear density. Our speculation has an added importance in that quantitative calculation on the lattice proves difficult at finite chemical potential [41].

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