Dissecting the origin of the submillimetre emission in nearby galaxies with Herschel and LABOCA

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ABSTRACT
We model the infrared to submillimetre spectral energy distribution of 11 nearby galaxies of the Key Insights on Nearby Galaxies: A Far-Infrared Survey with Herschel sample using Spitzer and Herschel data and compare model extrapolations at 870 µm (using different fitting techniques) with Large APEX BOlometer CAmera (LABOCA) 870 µm observations. We investigate how the differences between predictions and observations vary with model assumptions or environment. At global scales, we find that modified blackbody models using realistic cold emissivity indices ($\beta_c = 2$ or 1.5) are able to reproduce the 870 µm observed emission within the uncertainties for most of the sample. Low values ($\beta_c < 1.3$) would be required in NGC 0337, NGC 1512 and NGC 7793. At local scales, we observe a systematic 870 µm excess when using $\beta_c = 2.0$. The $\beta_c = 1.5$ or the Draine & Li (2007) models can reconcile predictions with observations in part of the discs. Some of the remaining ‘excesses’ occur towards the centres and can be partly or fully accounted for by non-dust contributions such as CO(3–2) or, to a lesser extent, free–free or synchrotron emission. In three non-barred galaxies, the remaining excesses rather occur in the disc outskirts. This could be a sign of a flattening of the submm slope (and decrease of the effective emissivity index) with radius in these objects.

Key words: galaxies: ISM.

1 INTRODUCTION

Studying dust emission helps us to probe the star formation activity obscured by dust grains and thus to understand how galaxies evolve through time. Many studies using telescopes such as the Infrared
Astronomical Satellite (IRAS) or the Spitzer Space Telescope (Spitzer) have been carried out to better understand the different grain populations contributing to the near-to-far-infrared (NIR, FIR) emission of nearby galaxies and investigate their hot and warm dust components. Recent observations of the FIR and submillimetre (submm) using the Herschel Space Observatory (Herschel) allow us to probe, with increased resolution compared to Spitzer, the cold dust emission, study its properties (temperature, grain opacity) within galaxies rather than on integrated scales and analyse how they vary with the interstellar medium (ISM) physical conditions (see for instance the analysis on nearby resolved galaxies of Boquien et al. 2011; Aniano et al. 2012; Bendo et al. 2012a; Foyle et al. 2012; Mentuch Cooper et al. 2012; Smith et al. 2012; Draine et al. 2013; Galametz et al. 2013a).

Submm observations of nearby galaxies with Herschel and from the ground have suggested that the cold dust properties could vary with the ISM physical conditions, for instance with metallicity. Deriving the dust masses of low-metallicity galaxies using standard dust properties for instance often leads to unphysical dust-to-gas mass ratios (D/G), compared to those expected from their lack of metals (Galametz et al. 2010; Meixner et al. 2010; Galliano et al. 2011). An excess at submm wavelengths is also often reported in those objects (Galliano et al. 2003; Dumke, Krause & Wielebinski 2004; Galliano et al. 2005; Bendo et al. 2006; Marleau et al. 2006; Galametz et al. 2009; Bot et al. 2010b; O’Halleran et al. 2010; Rémy-Ruyer et al. 2013). The submm ‘excess’ is usually defined as the excess emission above that predicted from FIR data (excluding the submm measurement) using a single $\lambda^{-\beta}$ emissivity law, (but we will also compare our observations with predictions from a more realistic dust model in the resolved analysis, Section 4). The presence of submm excess in a substantial number of galaxies challenges the models standardly used (more particularly how the submm regime is modelled) and raises the issue of how cold grain properties vary with environment (metallicity, temperature, star formation activity, etc.). The origin of the observed excess emission is still an open question. The various hypotheses investigated so far can be classified in two categories.

(1) On the one hand, the excess could result from an additional dust component not accounted for in the current models. Some studies suggest that this excess could be linked with a reservoir of very cold dust ($<15$ K) distributed in dense clumps (Galliano et al. 2005; Galametz et al. 2011) or magnetic dipole emission from magnetic grains (Draine & Hensley 2012). Others propose that the excess characterizes the so-called anomalous microwave emission generally associated with ‘spinning dust’ emission (Draine & Lazarian 1998) with a peak frequency which varies with environment or grain properties (Bot et al. 2010a; Murphy et al. 2010; Ysard & Verstraete 2010; Peel et al. 2011; Planck Collaboration et al. 2011b), although Draine & Hensley (2012) have argued that rotational emission should not significantly contribute to the 870 $\mu$m emission.

(2) On the other hand, the excess could be linked with a variation of dust properties with environment, in particular with temperature or density. Analysis of spatially resolved dust properties in the Galaxy suggest, for instance, that the emissivity of dust grains seems to increase towards the coldest regions of molecular clouds (Juvela et al. 2011), supporting the hypothesis of grain coagulation processes in those environments (Paradis, Bernard & Mény 2009). Moreover, the use of amorphous carbon (AC) instead of standard graphite is sometimes preferred to model carbon dust in the spectral energy distribution (SED) fitting of thermal dust emission in dwarf galaxies (Galametz et al. 2010; Meixner et al. 2010; O’Halleran et al. 2010; Galliano et al. 2011). Indeed, AC grains have a lower emissivity index, resulting in a flatter submm spectrum, and thus require less dust to account for the same emission. This usually leads to lower dust masses and seems in those cases to reconcile the dust-to-gas mass ratios with those expected from the low metallicity of these objects. Many more investigations are necessary to link this excess with the ISM conditions and enable us to disentangle its different possible origins.

Even if mostly observed in low-metallicity objects, a submm (850 or 870 $\mu$m) excess has also been detected on global scales in solar-metallicity objects (Galametz et al. 2011). Submm excess emission is usually not (or barely) detected at 500 $\mu$m in spiral galaxies (Bendo et al. 2010; Dale et al. 2012; Smith et al. 2012; Kirkpatrick et al. 2013), suggesting that observations beyond Herschel are necessary to properly probe the presence of excess in these galaxies. We now investigate the potential excess emission beyond 500 $\mu$m in 11 nearby galaxies chosen from the KINGFISH programme (Key Insights on Nearby Galaxies: A Far-Infrared Survey with Herschel; Kennicutt et al. 2011), adding 870 $\mu$m measurements taken with LABOCA (on APEX) to the NIR-to-FIR data set. These 11 objects were selected because they are large enough to use the LABOCA mapping mode and their diffuse emission is expected to be bright enough to be detected by the instrument.

Our Spitzer+Herschel+LABOCA data set is ideal to resolve the potential 870 $\mu$m excess and understand its origin. This paper complements the work of Galametz et al. (2012) (thereafter [G12]) that studies the spatially resolved cold dust properties (temperature, emissivity index, mass) for the same sample. The paper is structured as follows. In Section 2, we describe the data sets we use. In Section 3, we study how the 870 $\mu$m emission compares with various model extrapolations from the Herschel wavebands on global scales. In Section 4, we simulate 870 $\mu$m maps using various resolved SED modelling techniques and compare them with our LABOCA observations in order to spatially analyse the differences between these maps. We summarize the analysis in Section 5.

2 OBSERVATIONS AND DATA REDUCTION

This work is based on a combination of Spitzer, Herschel and LABOCA observations available for a sample of 11 nearby galaxies. A description of the sample is available in the first part of this study ([G12]) and general properties are summarized in Table 1. We provide some details of the observations and data reduction processes in this section.

2.1 Spitzer data

Warm dust can contribute a non-negligible amount of the 70 $\mu$m emission in galaxies. In order to constrain the mid-infrared (MIR) SED, observations from the Multiband Imaging Photometer for Spitzer (MIPS; Rieke et al. 2004) at 24 and 70 $\mu$m are included in the data set. The 11 galaxies have been observed with the MIPS as part of the Spitzer Infrared Nearby Galaxies Survey (SINGS; Kennicutt et al. 2003). The full-width maxima (FWHM) of the MIPS 24 and 70 $\mu$m PSFs are 6 arcsec and 18 arcsec, respectively. Some galaxies within the sample are also part of the Local Volume Legacy (LVL) survey. All the data were processed with the latest LVL pipeline1 (Dale et al. 2009), whether as part of that sample or to make them consistent with it.

1 http://irsa.ipac.caltech.edu/data/SPITZER/LVL/LVL_DR5_v5.pdf
Table 1. Galaxy data. (1) Galaxy name. (2) Morphological type from Kennicutt et al. (2011). NGC 0337, NGC 1097 and NGC 3627 have also been classified as ‘peculiar’ by Buta et al. (2010). (3) J2000.0 Right ascension. (4) J2000.0 Declination. (5) Distance in megaparsecs from Kennicutt et al. (2011). (6) Mean disc oxygen abundance obtained using the calibration of Pilyugin & Thuan (2005) taken from Moustakas et al. (2010). (7) Stellar masses from Skibba et al. (2011) updated for the quoted distances. (8) Total infrared luminosities in the 3- to 1100 µm from Galametz et al. (2013b). (9) Optical spectral classification of the nucleus as proposed by Moustakas et al. (2010).

| Galaxy       | Optical morphology | \(\alpha_0\) (J2000) | \(\delta_0\) (J2000) | Distance (Mpc) | 12+log(O/H) | \(M_\star\) (log \(M_\odot\)) | \(L_{\text{TIR}}\) (log \(L_\odot\)) | Nuclear classification |
|--------------|--------------------|----------------------|----------------------|---------------|-------------|-------------------------------|-------------------------------|-----------------------|
| NGC 0337     | SBD                | 00^h59^m50.7^s        | -07^d34’44”          | 19.3          | 8.18        | 9.32                          | 10.03                         | SF                    |
| NGC 0628     | SAC                | 01^h36^m41.8^s        | 15^d47’17”           | 7.2           | 8.35        | 9.56                          | 9.84                          | SF                    |
| NGC 1097     | SBB                | 02^h46’18.0”          | -30^d16’42”          | 14.2          | 8.47        | 10.5                          | 10.62                         | AGN                   |
| NGC 1291     | SB0/a              | 03^h17’39.1”          | -41^d06’32”          | 10.4          | 8.52        | 10.8                          | 9.43                          | AGN                   |
| NGC 1316     | SAB0               | 03^h22’41.2”          | -37^d12’10”          | 21.0          | 8.77        | 11.5                          | 9.77                          | AGN                   |
| NGC 1512     | SBab               | 04^h03’55.0”          | -43^d20’44”          | 11.6          | 8.56        | 9.92                          | 9.53                          | AGN                   |
| NGC 3351     | SBb                | 10^h43’57.5”          | 11^d42’19”           | 9.3           | 8.60        | 10.2                          | 9.84                          | SF                    |
| NGC 3621     | SAd                | 11^h18’18.3”          | -32^d48’55”          | 6.5           | 8.27        | 9.38                          | 9.83                          | AGN                   |
| NGC 3627     | SAbb               | 11^h20’13.4”          | 12^d59’27”           | 9.4           | 8.34        | 10.5                          | 10.40                         | AGN                   |
| NGC 4826     | SAb                | 12^h56’42.8”          | 21^d40’50”           | 5.3           | 8.54        | 9.94                          | 9.56                          | AGN                   |
| NGC 7793     | SAd                | 23^h57’50.4”          | -32^d35’30”          | 3.9           | 8.31        | 9.00                          | 9.22                          | SF                    |

2.2 Herschel data

The 11 galaxies of the sample have been observed with Photodetector Array Camera and Spectrometer (PACS) and Spectral and Photometric Imaging Receiver (SPIRE) on-board Herschel as part of the KINGFISH programme. Those images allow us to sample the peak of the thermal dust emission and to probe the cold dust phases. PACS observed at 70, 100 and 160 µm with FWHSs of 5.2 arcsec, 7.7 arcsec and 12 arcsec, respectively (Poglitsch et al. 2010). The data are processed to Level 1 using the Herschel Interactive Processing Environment (HIPE) software. Scanamorphos was then used to produce the final PACS and SPIRE maps (Roussel 2013). The scanamorphos algorithm was preferred over the standard madmap mapmaking technique because it can subtract the brightness drifts caused by low-frequency noise using the redundancy built in the observations. PACS maps are calibrated in Jy pixel\(^{-1}\) and have a final pixel size of 1.4 arcsec, 1.7 arcsec and 2.85 arcsec at 70, 100 and 160 µm, respectively. The PACS calibration uncertainties are ∼10 percent (see the PACS Observer’s Manual\(^2\)). SPIRE produced maps at 250, 350 and 500 µm, with FWHSs of 18 arcsec, 25 arcsec and 36 arcsec, respectively (Griffin et al. 2010)\(^3\). SPIRE maps are calibrated in Jy beam\(^{-1}\). The final pixel size is 6 arcsec, 10 arcsec and 14 arcsec at 250, 350 and 500 µm, respectively, with calibration uncertainties of ∼7 percent for the three wave bands (see the SPIRE Observer’s Manual\(^4\)). We estimate the SPIRE beam sizes using the flux calibration paper of Griffin et al. (2013). We apply their factors to convert from the point source pipeline to the extended source pipeline. Characterizing each band by a solid angle appropriate for a spectrum of the form \(I \propto \nu^{-v}\), we derive beam sizes of 469.1, 827.2 and 1779.6 arcsec\(^2\) for 250, 350 and 500 µm, respectively. Fluxes are then converted in MJy sr\(^{-1}\). More details on the data reduction of Herschel maps are available in the overview paper of the KINGFISH programme (Kennicutt et al. 2011).

2.3 LABOCA 870 µm maps

In order to search for potential excess emission beyond 500 µm, we complement the NIR-to-FIR coverage of the dust emission with LABOCA observations of our galaxies at 870 µm. LABOCA is a multichannel bolometer array for continuum observations located on the Atacama Pathfinder Experiment (APEX) telescope in the Atacama desert, Chile (Siringo et al. 2009). The FWHM of the point spread function (PSF) at 870 µm is ∼19.2 arcsec (Weiß et al. 2009), thus close to that of SPIRE 250 µm.

Data are taken from various projects (C-082.F-0005B, E-080.B-3057A, E-082.B-0679A, E-083.B-0813A, E-086.B-0927A, M-079.F-0129, M-081.F-0012, M-083.F-0052 and M-085.F-0058). We refer to Albrecht et al. (in preparation) for details on the iterative data reduction procedure used to produce the LABOCA maps. A summary is provided here. The mapping is performed in spiral mode (raster pattern). Correction for atmospheric attenuation was enabled through determinations of the zenith opacity via sky-dips (∼ every two hours). The flux calibration (achieved through observations of Mars, Uranus, Neptune and secondary calibrators) is estimated to be accurate within 9 per cent rms. The time-ordered data streams are reduced using the BOA software\(^5\), following steps of calibration, opacity and temperature drifts correction, flat-fielding, conversion to Jy, flagging of dead or bad channels, removal of the correlated noise (on the global array or induced in groups sharing the same electronics), flagging of stationary points or data taken above a given scanning velocity and acceleration threshold, \(^5\)He temperature drift correction, median baseline removal and despiking. A final intensity map is obtained by co-adding the individual subscans (pixel size of 4 arcsec × 4 arcsec) along with a weight map. The steps of correlated noise or median baseline removal can lead to a subtraction of real flux in the maps. This can be solved by using an iterative approach. From the first map produced using the technique previously described, a source model is created by isolating the pixels superior to a given signal-to-noise ratio (areas of apparent source emission defined with polygons) and the data

\(^2\)http://herschel.esac.esa.int/Docs/PACS/html/pacs.om.html
\(^3\)http://herschel.esac.esa.int/twiki/bin/view/Public/SpiRePhotometerBeamProfileAnalysis
\(^4\)http://herschel.esac.esa.int/Docs/SPIRE/html/spire.om.html
\(^5\)BOA was developed at MPIfR (Max-Planck-Institut fr Radio Astronomy, Bonn, Germany), AIfA (Argelander-Institut für Astronomie, Bonn, Germany), AIRUB (Astronomisches Institut der Ruhr-Universität, Bochum, Germany) and IAS (Institut d’Astrophysique Spatiale, Orsay, France).
3 THE 870 µm EMISSION ON GLOBAL SCALES

3.1 Global flux densities

Along with a detailed description of the LABOCA observations and data reduction, Albrecht et al. (in preparation) provide the global photometry of the sample. Their elliptical apertures are defined to cover the extended emission as observed in the Spitzer/MIPS 160 µm maps.

Global radio emission, namely thermal bremsstrahlung emission (free–free) from electrons in the hot ionized gas or synchrotron emission from relativistic electrons moving in a magnetic field, is a possible contribution to the 870 µm continuum emission. Albrecht et al. (in preparation) estimate the free–free contributions from Hz measurements (with which free–free emission directly scales) using integrated Hz + [NII] measurements corrected for [NII] contribution within the Hz filters (Kennicutt et al. 2009). Assuming a thermal electron temperature of 10^4 K and the prescription from Niklas, Klein & Wielebinski (1997) (their equation 2), they convert the Hz fluxes to a free–free emission at 870 µm of

$$ S_{\text{Hz}} \left( \frac{\text{mJy}}{\text{erg cm}^{-2} \text{s}^{-1}} \right) = 5.329 \times 10^{11} \frac{S_{\text{Hz}}}{\text{erg cm}^{-2} \text{s}^{-1}} ,$$

(1)

where K is determined to be ~2 in the sample of Niklas et al. (1997). The free–free contribution is then subtracted from global radio continuum fluxes from the literature to determine the synchrotron emission. They determine the radio spectral index α (defined as $S \propto \nu^{-\alpha}$) and extrapolate towards the LABOCA frequency. They also used a composite model combining a thermal and non-thermal fraction (with 0.1 as the thermal spectral index and a free non-thermal spectral index), model that leads to similar contributions, namely a total free–free+synchrotron contribution at 870 µm of less than 2 per cent on average for the sample. We refer to their study for further details on the technique and results for individual objects.

CO line emission is another possible non-dust contribution to the 870 µm emission since the CO(3–2) transition occurs at 867 µm. When available, Albrecht et al. (in preparation) determine the CO(3–2) line emission from CO(3–2) observations, deriving the flux densities from

$$ S_{\text{CO}(3-2)} = \frac{2k\nu}{c^3\Delta\nu} \int I_{\text{CO}(3-2)} \, d\Omega ,$$

(2)

with k the Boltzmann constant, v the line frequency, c the speed of light, Δν the bolometer bandwidth and $I_{\text{CO}(3-2)}$ the integral of the velocity-integrated intensity over the solid angle of the source Ω (Ω is derived from the main beam-size in the case of single-point CO observations). CO(3–2) estimates are otherwise derived from CO(1–0) or CO(2–1) observations (see Section 4.4). The global CO(3–2) contributions are ranging from less than 3 per cent in NGC 0337 and NGC 0628 to up to 14 per cent in NGC 4826 Albrecht et al. (in preparation). We use the global 870 µm values corrected for free–free + synchrotron and CO line contribution to the 870 µm flux in the following section. Because local contributions from CO, free–free and synchrotron emission can vary across the galaxies, we return to the estimates of the resolved non-dust contributions in the individual galaxies in Section 4.4.

We compare the integrated LABOCA flux densities with 870 µm estimates extrapolated from observations at shorter IR/submm wavelengths. The global Spitzer and Herschel flux densities are taken from Dale et al. (2007) and Dale et al. (2012), respectively. Their photometric elliptical apertures (provided in [G12]) are chosen to encompass the total emission at each Spitzer and Herschel wavelength. Uncertainties on Spitzer and Herschel fluxes were computed as a combination in quadrature of the calibration uncertainty and the measurement uncertainty. The SPIRE beam sizes have been updated since the publication of Dale et al. (2012) and [G12]. The SPIRE global flux densities are thus rescaled to the new beam size estimates (see Section 2.2 for values), leading to a 7–9 per cent correction compared to [G12]. Discrepancies can be observed between MIPS 70 µm and Herschel/PACS 70 µm fluxes (Aniano et al. 2012) because PACS is less sensitive than MIPS to diffuse emission and because the instruments possess different filter profiles. We thus decide to use both observations in our modelling, to be conservative. Colour corrections will be applied during the SED model fitting. The contribution of C+ to the 160 µm emission is considered to be minor (C II/L_{TIR} ~ 0.15 per cent from Brauher, Dale & Helou 2008 or Malhotra et al. 2001 and L_{TIR}/vF_{(160)} ~ 2.5 from Dale et al. 2009 so C II/vF_{(160)} ~ 0.4 per cent). The SPIRE flux densities are also not corrected for line contribution but potential line features (such as [NII]205 µm or CO) are considered to be minor compared to continuum emission.

Comparison with Planck data. We can compare the global Herschel 350 and 500 µm flux densities and the LABOCA 870 µm flux densities with the Planck global flux densities at 350 µm, 550 µm, 850 µm (FWHMs of 4.3 arcmin, 4.7 arcmin, 4.8 arcmin, respectively). The Planck flux densities are provided by the Planck Catalogue of Compact Sources (PCCS Planck Collaboration et al. 2013) and obtained through the IPAC Infrared Science Archive\(^7\). Most of our sources are resolved by Planck. We choose the flux densities estimated using the APERTFLUX technique (fitting of a variable circular aperture centred at the position of the source). The Planck global flux densities at 350 µm, 550 µm, 850 µm and 1.3 mm are overlaid on the global SEDs of the sample (see Section 3.2 for the description of the modelling) in Fig. 2. We observe a very good agreement between the global flux densities in the [350–850] µm range. The mean value of the Planck350/Spire350 µm flux density ratios is 0.96 ± 0.19 and the mean value of the Planck850/LABOCA870 µm flux density ratios is 0.87 ± 0.28. The mean value of the Planck550/Spire550 µm flux density ratios is 0.64 ± 0.15, but becomes 0.94 ± 0.23 when Planck 550 µm flux densities are converted to 500 µm flux densities using a (550/500)\(^6\)

\(^6\) http://www.astro.princeton.edu/~ganiano/Kernels.html

\(^7\) http://irsa.ipac.caltech.edu/applications/planck/

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Figure 1. Herschel/SPIRE 350 µm maps of the sample in MJy sr⁻¹ (colour scales are provided for each object in linear scale), with the signal-to-noise of the LABOCA 870 µm maps overlaid as contours. The signal-to-noise maps have been convolved to a resolution of 25 arcsec (that of SPIRE 350 µm) before display. Contours are, for NGC 0337: 3, 6, 9, 12 and 17σ where σ = 0.17 MJy sr⁻¹, for NGC 0628: 3, 6, 9 and 12σ where σ = 0.19 MJy sr⁻¹, for NGC 1097: 3, 6, 9, 20 and 30σ where σ = 0.13 MJy sr⁻¹, for NGC 1291: 3 and 6σ where σ = 0.13 MJy sr⁻¹, for NGC 1316: 3 and 6σ where σ = 0.20 MJy sr⁻¹, NGC 1316 is the southern object in this map, NGC 1317 the northern object. Contours are, for NGC 1512: 3, 6, 9, 12, 17 and 22σ where σ = 0.09 MJy sr⁻¹, for NGC 3351: 3, 5, 12 and 21σ where σ = 0.16 MJy sr⁻¹, for NGC 3621: 3, 6, 9, 15 and 21σ where σ = 0.16 MJy sr⁻¹, for NGC 3627: 3, 6, 9, 12, 17 and 27σ where σ = 0.22 MJy sr⁻¹, for NGC 4826: 3, 9, 15, 27 and 40σ where σ = 0.22 MJy sr⁻¹, for NGC 7793: 3, 6, 9, 12 and 17σ where σ = 0.19 MJy sr⁻¹.
factor between the two (i.e. assuming dust with a $\nu^2$ opacity and a Rayleigh–Jeans behaviour). Using the GAUFLUX values from the PCCS (fitting of a variable Gaussian model to the source), ratios are equal to $1.20 \pm 0.21$ (when converted as above) at 500 $\mu$m and $1.07 \pm 0.30$ at 870 $\mu$m, which is also in very good agreement as well. In particular for the galaxy NGC 1512, the GAUFLUX estimates are more consistent with the *Herschel* and LABOCA data than the APERFLUX estimates. In some galaxies, the dispersion in the *Planck*-to-SPIRE or *Planck*-to-LABOCA ratios are relatively high compared to the calibration uncertainties, i.e. 1 per cent at

![NGC1512](image1)

![NGC3351](image2)

![NGC3621](image3)

![NGC4826](image4)

![NGC7793](image5)

**Figure 1 — continued**
Figure 2. Global SEDs obtained with a two-temperature fitting technique. The model using an emissivity index of the cold component $\beta_c$ fixed to 2.0 is shown by a dotted blue line, with $\beta_c$ fixed to 1.5 by a long dashed green line and with a free $\beta_c$ by a black solid line. The corresponding warm components (with $\beta_w$ fixed to 2.0 in the three cases) are overlaid in grey. MIPS, PACS and SPIRE data are overlaid with black rectangles. Observed LABOCA flux densities (not included in the fit) are overlaid in red and have been corrected for CO and radio continuum (see Section 3.1). We also indicate, when available, Planck global flux densities at 350 $\mu$m, 550 $\mu$m, 850 $\mu$m and 1.3 mm (grey crosses) but do not include them in the fit. The height of the grey crosses represents the uncertainties of the Planck fluxes.

850 $\mu$m and 10 per cent at 350 and 550 $\mu$m (Planck Collaboration et al. 2013), 7 per cent for SPIRE (see Section 2.2 for references) and 9 per cent for LABOCA. The dispersions in the Planck/SPIRE ratios is however consistent with what has been reported in other studies (see the work of Herranz et al. 2013, for instance). Some contamination of the Planck fluxes could originate from the large beam sizes of the telescope, in particular for NGC 0337 which is smaller than Planck's beam. This could explain the large differences we observe for this particular galaxy. Differences between the spectral responses of the Planck, LABOCA and SPIRE instruments are not taken into account in this comparison either, which could also explain the dispersions.

3.2 Global SED modelling technique

As in [G12], we modelled the global SED of thermal dust emission of our galaxies from 24 to 500 $\mu$m using modified blackbodies (MBB). We refer to this study for a discussion of the model, the choice of parameters, and the possible degeneracies. We remind some of the potential issues of the use of MBBs at the end of this section. Since we want to investigate potential excess emission at 870 $\mu$m, the LABOCA observations are not included in the fitting procedure. In [G12], we show that a non-negligible fraction of the 70$\mu$m fluxes can be attributed to warm dust. We thus choose to account for this contribution by using a two-temperature
(warm + cold dust) fitting technique, each component being modelled as an MBB:

\[ L_\nu(\lambda) = A_\nu \lambda^{-\beta_\nu} B_\nu(\lambda, T_\nu) + A_c \lambda^{-\beta_c} B_c(\lambda, T_c). \]

\( B_\nu \) is the Planck function, \( T_\nu \) and \( T_c \) are the temperature of the warm and cold component, respectively, while \( \beta_\nu \) and \( \beta_c \) are the emissivity index of the warm and cold component, respectively. \( A_\nu \) and \( A_c \) are scaling coefficients, functions of the optical depths contributed by the cold and warm dust components at a given reference wavelength. We determine the colour correction for each band by convolving our 2MBB spectra with the instrumental response function using the different Spitzer and Herschel user’s manual conventions.\(^8\)

The curve fitting procedure applies a Levenberg–Marquardt \( \chi^2 \) minimization and weights flux densities by the inverse squared of the uncertainty. The emissivity index of the warm component is fixed to a value \( \beta_\nu = 2.0 \), an approximation of the opacity in the standard (Li & Draine 2001) dust models. Discussion on the influence of the warm dust component and of the 24-to-100 \( \mu m \) flux variations in the sample can be found in [G12] (their section 3.4). We fix the emissivity index of the cold dust component to two different values: \( \beta_c = 2.0 \) and \( \beta_c = 1.5 \), in order to study how the two models match the observations at 870 \( \mu m \). Degeneracies between the temperature and the emissivity index are observed when the two parameters are allowed to vary at the same time. This can lead to an artificial anticorrelation between the emissivity index and the temperature due to noise effects (Shetty et al. 2009, among others). An ‘apparent’ flattening of the slope of the SED could be created as well from temperature mixing effects. On global scales as well as on local scales (our pixels cover large ISM elements), various dust grain populations with different temperatures could be mixed, leading to biases in the emissivity estimates. For both reasons, [G12] caution the use of a variable emissivity to model SEDs at the local scale of consideration. We will not apply this technique on the resolved analysis (Section 4). We nevertheless model the free \( \beta_c \) assumption at global scales in order to test which values are invoked to explain the SPIRE slopes without a priori assumption the dust emissivity index value and detect potential strong flattening of the submm slope in our objects.

While Klaas et al. (2001) or Dunne & Eales (2001) showed that the SEDs of nearby galaxies could be modelled with either one MBB with a variable \( \beta \) or multiple MBB with \( \beta = 2.0 \), others showed that the bulk of the FIR emission from dust grains could be fitted with dust heated by a single radiation field (Draine et al. 2007). Recently, other studies have shown that dust emitting in the 100-to-500 \( \mu m \) range may be heated by both the star-forming regions and the older stellar populations (Boquien et al. 2011; Bendo, Galliano & Madden 2012b), which questions the single MBB fitting of that full wavelength range. Isothermal or two-temperature fits are not fully able to model the range of grain temperatures observed in each of our resolved ISM elements. We remind the reader that what we call ‘emissivity index’ in the following analysis is thus, in fact, the ‘effective emissivity index’ (i.e. the power-law index resulting from the different grain populations) rather than the ‘intrinsic emissivity index’ (i.e. the power-law index of the MBB modelling the emission of a dust grain at a given temperature). The following analysis thus primarily investigates submm slope trends with environment rather than directly probing the intrinsic emissivity properties of cold grains.

We present the global SEDs obtained using the three different hypotheses: \( \beta_c \) fixed to 2.0 (dotted blue line), fixed to 1.5 (dashed green line) or free (plain black line) in Fig. 2. We overlay the warm dust component for each model in grey, PACS and SPIRE data with black rectangles and the LABOCA 870 \( \mu m \) flux densities corrected for free–free and synchrotron and CO line contribution (taken from Albrecht et al. in preparation) with red rectangles. As mentioned before, we also overlay (when available) the Planck global flux densities at 350 \( \mu m \), 550 \( \mu m \), 850 \( \mu m \) as well as the Planck 1.3 mm flux density but did not use them in the fit.

3.3 870 \( \mu m \) predictions and comparison with observations

We deduce global flux estimates at 870 \( \mu m \) from the different 2MBB models and estimate the uncertainties using a Monte Carlo technique: we generate 100 sets of modified 24-to-500 \( \mu m \) constraints, with fluxes varying randomly within the individual error bars (following a normal distribution around their nominal value). The absolute calibration is the dominating source of uncertainty and is highly correlated across the SPIRE bands (SPIRE’s observers’ manual). SPIRE measurements are thus randomly modified but in a similar correlated way. We then apply our fitting technique to the 100 modified data sets. The global 870 \( \mu m \) estimates we extrapolate are the medians of these distributions; we take the standard deviation of each distribution as the uncertainty.

Table 2 summarizes for each galaxy the global 870 \( \mu m \) estimate and 1\( \sigma \) uncertainty obtained using our 3 different MBB fitting techniques (\( \beta_c = 2.0, 1.5 \) and \( \beta_c \) free). The ratios between the observed 870 \( \mu m \) flux densities of Albrecht et al. (in preparation), corrected for free–free and synchrotron emission and line contribution (called ‘870 obs corrected’ in the table), and our extrapolations from the different models (\( \beta_c = 2.0 \) or 1.5 and \( \beta_c \) free) are also provided. The update of the SPIRE beam sizes discussed in Section 3.1 does not significantly change the \( \beta \) values derived when this parameter is free in the models compared to the results of [G12] (variations by less than 5 per cent on average).

For 7 galaxies (NGC 1097, NGC 1291, NGC 1316, NGC 3351, NGC 3621, NGC 3627 and NGC 4826), the observed 870 \( \mu m \) flux densities match the values estimated using \( \beta_c = 2.0 \), within the uncertainties. The global 870 \( \mu m \) emission in those objects could thus be explained by thermal emission modelled using dust grains with standard properties. However, we note that for NGC 1316 and NGC 1097, using \( \beta_c = 1.5 \) does provide a better prediction of the observed 870 \( \mu m \) (observed-to-modelled ratio closer to unity) than the ‘\( \beta_c = 2.0 \)’ case.

The four remaining galaxies (NGC 0337, NGC 0628, NGC 1512 and NGC 7793) show an 870 \( \mu m \) excess when \( \beta_c = 2.0 \) is used. For NGC 0628 and NGC 1512, using a flatter emissivity index \( \beta_c = 1.5 \) can reconcile the observed and modelled 870 \( \mu m \) global emission within the uncertainties, as shown by their observed-to-modelled ratio close to unity. For the two remaining galaxies NGC 0337 and NGC 7793, the submm slope constrained from Spitzer+Herschel data is already flat (\( \beta_c < 1.7 \) when free to vary) and we observe an excess of the 870 \( \mu m \) emission compared to the extrapolations, even when an emissivity \( \beta_c = 1.5 \) is used (∼50 per cent above the model predictions on average), even if this excess is statistically weak (inferior to a 2\( \sigma \) level). A cold dust component with an even lower emissivity index could be necessary in these galaxies to match the observations. Including the 870 \( \mu m \) data in the fitting procedure leads to \( \beta_c \) values of 1.33 ± 0.15 and 1.19 ± 0.14 for NGC 0337 and

\(^8\) The link to PACS and SPIRE observer manuals in which these conventions can be found are provided earlier in this paper. The reader can access the MIPS Instrument handbook here: [http://irsa.ipac.caltech.edu/data/SPITZER/docs/mipsinstrument handbook/](http://irsa.ipac.caltech.edu/data/SPITZER/docs/mipsinstrument handbook/).
NGC 7793, respectively, thus at the lower end of values typically used in the literature. This questions the origin of the excess detected at 870 µm and the use of a simple isothermal component to fit the cold dust population up to 870 µm in those two objects.

As mentioned before, using $\beta_c = 1.5$ reconciles the observed and modelled 870 µm global emission for NGC 1512, within the 870 µm uncertainty. Nevertheless, when $\beta_c$ is allowed to vary, the model fit to Spitzer and Herschel data favours very low $\beta_c$ values ($\beta_c = 1.16$). Including the 870 µm data in the fitting procedure, we find $\beta_c = 1.22 \pm 0.15$ for NGC 1512. The 870 µm observation thus confirms the shallow slope. The 870 µm observation reveals an extended structure in the south-east of NGC 1512 that is not observed in the SPIRE maps (Fig. 1) and accounts for 30–40 per cent of the global flux. Albrecht et al. (in preparation) could not firmly determine the nature of this extended emission that contaminates the map and therefore the global 870 µm flux estimates. They note that the global flux density is only a factor of 1.3 higher than the Planck value reported for this object in the Early Release Compact Source Catalogue (Planck Collaboration et al. 2011a, GAUFLUX technique). This is coherent with the contamination we estimate.

3.4 Dependence with global properties

We now analyse how the excess (or deficit) of the observed 870 µm emission compared to our extrapolations varies with the global characteristics of our objects. In Fig. 3, we plot the ratios between the observed 870 µm flux densities and the 870 µm estimates extrapolated from the $\beta = 2$ (top panel) and $\beta = 1.5$ (bottom panel) models as a function of the average metallicity expressed as oxygen abundances (left) and as a function of the total stellar mass in $M_\odot$ (middle). Metallicities, listed in Table 1, are taken from Kennicutt et al. (2011) and obtained using the calibration of Pilyugin & Thuan (2005). We use the stellar masses derived by Skibba et al. (2011). The observed 870 µm flux densities are corrected for free–free and synchrotron emission and CO(3–2) line contribution. In this figure, the ratio of the observed-to-modelled flux can be used as a ‘proxy’ for submm slope variations to a certain extent, since it characterizes how the observed slope varies compared to a theoretical slope of $\beta = 2$ or $\beta = 1.5$.

The difference we derive between the observed and the modelled 870 µm flux densities varies from one object to the other but does not seem to show a particular trend with metallicity, at least within
Figure 3. Ratio between the observed 870 µm flux densities and the 870 µm estimates predicted from the β = 2 (top) and β = 1.5 (bottom) model as a function of, from left to right, metallicity (expressed as oxygen abundances), stellar mass in (M⊙) and total infrared luminosity (in L⊙). The observed 870 µm flux densities are corrected for free–free and synchrotron emission and line contribution (see Section 3.1). Labels indicate the galaxies’ NGC numbers. The horizontal line indicates unity.

4 THE 870 µm EMISSION ON LOCAL SCALES

In order to understand the range of emissivity index values we observe on global scales, we now try to characterize variations of the submm slope at 870 µm within our galaxies.

4.1 Extrapolations using 2MBB techniques

We convolve our Spitzer and Herschel images to the lowest resolution of the data set, namely that of SPIRE 500 µm, using the convolution kernels generated by Aniano et al. (2011). Maps are then projected to a common sample grid with a pixel size of 14 arcsec (the standard pixel size of the SPIRE 500 µm images). This size corresponds to ISM elements ranging from ~265 pc for our closest galaxy NGC 7793 to ~1.4 kpc for NGC 1316. We then use our two-temperature fitting technique to model the SED in each pixel.

As previously mentioned in Section 3.2, we choose to fix βc in the following analysis. This will (i) prevent biases (T–β degeneracy, temperature mixing) from affecting our maps and (ii) enable us to study the behaviour of the submm slope on resolved scales compared to models for which dust properties are perfectly constrained. Beyond the 870 µm excess issue, we
thus also probe potential variations of the submm slope in our objects.

We produce 870 μm maps of our 11 galaxies for two different cases, βc, fixed to 2.0 and 1.5. We also convolve the LABOCA images to the SPIRE 500 μm resolution using the convolution kernels developed by Aniano et al. (2011) (see Section 2.2 for web reference) and regrid the maps to our common pixel grid (pixel size: 14 arcsec). For this study, the 870 μm maps have not been corrected for CO or radio contamination. We discuss their potential contribution to the observed 870 μm fluxes in Section 4.4. Fig. 4 gives an example of the observed 870 μm map of NGC 3627 (top line) along with the 870 μm maps extrapolated from the models with βc = 2.0 (second line, first panel) or 1.5 (second line, middle panel). The maps of the full sample derived using the 2MBB techniques are given in appendix in Fig. A1. We use the same colour scale for the observed and modelled 870 μm maps to allow a direct comparison.

4.2 Extrapolations using the Draine & Li (2007) model

We want to compare the 870 μm estimates obtained from the 2MBB resolved techniques with those predicted by a more physical (as far as dust composition and ISM physics are concerned) SED model. We thus also build pixel-by-pixel 870 μm maps using the resolved SED modelling of the KINGFISH sample provided in Aniano et al. (2012) and Aniano et al. (in preparation) where Spitzer + Herschel data up to 500 μm are modelled using the Draine & Li (2007) dust models (hereafter [DL07]) on a local basis. Maps are derived at the resolution of SPIRE 500 μm, for a 14 arcsec × 14 arcsec pixel grid. We refer to the two previously mentioned studies for details on the SED modelling technique. We will remind the reader of a few principles in this section. In addition to the KINGFISH sample, this SED fitting technique has also been used on M31, modelled out to 1.5 optical radius in Draine et al. (2013). They show that deviations from the [DL07] model appear in the centre (galactocentric distances lower than 6 kpc), suggesting steeper submm opacity spectral index but that the [DL07] model can fit the emission out to 500 μm elsewhere, within the uncertainties.

In the [DL07] dust models, the dust size distribution and its composition are considered to be uniform in each resolution element for which the model is run. Sources of the infrared emission are old stars, Polycyclic Aromatic Hydrocarbons, graphite and silicate grains. The stellar emission (>3 μm) is mostly constrained by the two first Spitzer/IRAC bands and modelled by scaling a blackbody function, using a photospheric temperature of 5000 K (Bendo et al. 2006). In each pixel, the dust is assumed to be heated by a distribution of starlight intensities, but with most of the dust heated by a single starlight intensity that is interpreted as the starlight intensity in the diffuse ISM. The spectral shape of the radiation field is assumed to be that of the Galactic diffuse ISM estimated by Mathis, Mezger & Panagia (1983). The modelling procedure uses the bandpasses of the instruments to apply colour-corrections and convergence to a preferred model is reached using a χ2 minimization technique.

We extrapolate 870 μm maps of the sample from the [DL07] resolved SED models of Aniano et al. (in preparation). Fig. 4 shows an example of the modelled 870 μm map for NGC 3627 (second line, third panel). We provide the 870 μm maps of the full sample derived with this SED modelling technique in appendix in Fig. A1 (bottom panels). Here, again, the same colour scale was applied between observed and modelled 870 μm maps to allow a direct comparison.

The modelled 870 μm maps obtained using the [DL07] formalism are quantitatively similar to those obtained using a 2MBB model with βc = 1.5. In the wavelength range covered by SPIRE, the SED extrapolated from the [DL07] models is in fact similar to a single temperature MBB with βc = 2.0 but the model can reach colder temperatures due to the mixture of temperatures it includes. The [DL07] dust models also incorporate small modifications to the amorphous silicate opacity, especially for λ > 250 μm, in order to better match the average high Galactic latitude dust emission spectrum measured by Cosmic Background Explorer-Far Infrared Absolute Spectrophotometer (COBE-FIRAS; Wright et al. 1991; Reach et al. 1995; Finkbeiner, Davis & Schlegel 1999). Thus, [DL07] models already include a ‘Galactic submm excess’ (modifications inferior to 12 per cent for the 250 μm < λ < 1100 μm range). Emission in excess of the [DL07] models thus means an excess compared to what could be explained by Galactic-like dust only.

4.3 Absolute and relative difference maps at 870 μm

In order to compare the modelled luminosity Lmod at the observed luminosity Lobs at 870 μm, we derive maps of the absolute difference defined as

\[ \text{absolute difference} = L_{\text{mod}}(870 \, \mu m) - L_{\text{obs}}(870 \, \mu m) \]

and maps of the relative difference defined as:

\[ \text{relative difference} = \frac{L_{\text{mod}}(870 \, \mu m) - L_{\text{obs}}(870 \, \mu m)}{L_{\text{mod}}(870 \, \mu m)}. \]

Fig. 4 shows an example of the 870 μm absolute difference maps (third line panels) and relative difference maps (bottom line panels) obtained from our three resolved modelling techniques for the galaxy NGC 3627. We provide the absolute and relative difference maps of the full sample in appendix in Fig. A1. We indicate zeroes in the colour scales with a white marker to allow a visual separation between 870 μm excess or deficit compared to the different SED models. The signal-to-noise of the LABOCA 870 μm map is overlaid as contours on the absolute and relative difference maps. The first contour shown corresponds to a 3σ 870 μm detection. As mentioned in the previous section, the modelled 870 μm maps obtained using the [DL07] formalism and those obtained using an MBB with βc = 1.5 are quantitatively similar so our conclusions are the same for these two methods.

When βc is fixed to a standard value of 2, we systematically observe regions with a positive difference between the observed and the modelled 870 μm emission in our maps. This positive difference can be localized in the centre of the galaxy, like in NGC 1097, or extended across the object (NGC 0337 or NGC 7793 for instance). The 870 μm maps extrapolated from the βc = 1.5 model are brighter than those with βc = 2.0 due to the flatter submm slope induced by this SED model that leads to higher extrapolations at 870 μm. As a direct consequence, the absolute difference derived using βc = 1.5 is systematically smaller than that derived using βc = 2.0.

We now qualitatively and quantitatively describe the trends observed for each individual galaxy. Part of the excess we detect can be linked to contamination by free–free, synchrotron, or CO emission. We quantify this contribution in Section 4.4.

NGC 0337. The distributions of the observed and modelled 870 μm emission are very similar. However, quantitatively, the observed 870 μm is significantly higher than our model predictions, regardless of the model we use. The absolute excess above the model prediction at 870 μm is radially decreasing, following the distribution of the star formation activity of the galaxy as well as that of the dust mass surface density mapped in [G12]. The relative excess...
Figure 4. Top panel: flux density at 870 µm observed with LABOCA (in MJy sr$^{-1}$) for the galaxy NGC 3627 not corrected for CO and radio continuum. Second line panels: modelled 870 µm maps (in MJy sr$^{-1}$) derived, from left to right, using our two-MBB procedure with $\beta_c$ fixed to 2, fixed to 1.5 or using the [DL07] formalism. Third line panels: absolute difference between the observed and the modelled 870 µm map in MJy sr$^{-1}$ (defined as observed flux − modelled flux) for the different SED models (see Section 4.3). Bottom line panels: Relative difference defined as (observed flux − modelled flux)/(modelled flux). We provide the colour scales for each line on the right-hand side. Zeroes are indicated with a white marker on the colour bars. The signal-to-noise of the LABOCA 870 µm map is overlaid as contours on the absolute and relative difference maps (same levels than in Fig. 1).
seems to be non-homogeneous across the galaxy. The relative excess in regions detected with LABOCA at a 3σ level is ~70 per cent on average when $\beta_\sigma = 1.5$, with a standard deviation of 25 per cent. This is slightly higher than what is found on global scales for this object (but we remind the reader that we are excluding the lowest surface brightness regions here). Unfortunately, because this is one of most distant galaxies of the sample, our poor resolution does not allow us a very detailed study of the 870 μm residual distribution.

NGC 0628. As one can see from Fig. 1, part of the emission in the south-east side of the galaxy is not detected in the LABOCA map. The modelled 870 μm maps are thus brighter than the emission actually observed with LABOCA, whatever the model, and the absolute difference thus shows a deficit of emission in that south-east region. For the rest of the galaxy, the distributions of the observed and modelled maps are similar, with the 870 μm emission (modelled and observed) following the distribution of the emission traced by SPIRE wavebands. We observe a strong 870 μm excess above the $\beta_\sigma = 2.0$ model across the galaxy. The difference between the observed and modelled 870 μm emission partly but not fully disappears when we use a lower $\beta_\sigma = 1.5$ or the [DL07] dust model. This excess emission is not homogeneously distributed across the disc of the galaxy and does not follow the galaxy structure. Indeed, the absolute and relative difference maps of NGC 0628 do not show a statistically significant excess emission in the centre but peaks in the low-surface brightness regions of the galaxy where cold dust at 870 μm emission is detected by LABOCA at a reasonable 3σ level.

NGC 1097. The observed and modelled 870 μm maps are similar, with a very strong 870 μm emission in the centre of the galaxy (the galaxy possesses a moderate AGN) and along the bar, and two fainter spiral arms. A positive difference between the observed and modelled map is seen across the galaxy when using the $\beta_\sigma = 2.0$ MBB model. The 870 μm emission seems to be better explained when a $\beta_\sigma = 1.5$ MBB model or the [DL07] formalism are used. An excess above the model prediction remains nonetheless in the central region when $\beta_\sigma = 1.5$ (we obtain an average of ~54+35−25 per cent in regions with a 20σ detection) and in the few inter-arm resolved elements that are modelled.

NGC 1291. The three modelled 870 μm maps follow the distribution of the SPIRE maps, namely a bright centre elongated in the north–south direction and a diffuse cold dust ring (prominent longward of 160 μm) with localized bright spots in the ring in the north-west and south-east arcs (see Hinz et al. 2012, for a detailed analysis of this cold dust ring with Herschel). We analyse the excess in resolved elements detected at 870 μm to a 2σ level, which, unfortunately, restricts the study to the centre of the galaxy where a strong excess above the $\beta_\sigma = 1.5$ model prediction is detected (~59 per cent on average in the regions we modelled, with a standard deviation of 30 per cent). The major part of the cold dust ring is not detected with LABOCA (see Fig. 1), which prevents us to conclude about any trend in the ring of NGC 1291. It also implies that the global 870 μm flux density provided for this galaxy in Albrecht et al. (in preparation) should be considered as a lower limit. An emission in excess is systematically detected in the centre of the galaxy, whatever the model, but the relative 870 μm difference is small (<30 per cent for regions with a 3σ LABOCA detection). An 870 μm excess above the $\beta_\sigma = 1.5$ model is also observed and radically increasing to a factor of 2 in the north-east region of the galaxy, i.e. in the only region of the disc detected with LABOCA at a 3σ level.

NGC 3351. The modelled 870 μm maps show a central peak in the emission surrounded by a diffuse disc emission but only part of the emission in the disc is detected with the LABOCA instrument (see Fig. 1). For this galaxy as well, the non-detection of disc structure with LABOCA indicates that the global 870 μm flux density provided for this galaxy in Albrecht et al. (in preparation) should be considered as a lower limit. An emission in excess is systematically detected in the centre of the galaxy, whatever the model, but the relative 870 μm difference is small (<30 per cent for regions with a 3σ LABOCA detection). An 870 μm excess above the $\beta_\sigma = 1.5$ model is also observed and radically increasing to a factor of 2 in the north-east region of the galaxy, i.e. in the only region of the disc detected with LABOCA at a 3σ level.

NGC 3621. Our modelled maps qualitatively match the observed distribution. They show a radial decrease of the 870 μm emission and two peaks in the centre of the galaxy, the southern being the strongest. However, the observed 870 μm emission is brighter than our $\beta_\sigma = 2.0$ model prediction, with a distribution of the absolute difference map following the galaxy structure and the dust mass surface density mapped in [G12]. This trend remains when the $\beta_\sigma = 1.5$ model or the [DL07] model are used. The excess is, however, statistically weak in these two cases, with an 870 μm relative excess of 19+12−6 per cent on average in regions with a 5σ LABOCA detection. Peaks of the relative excess emission (barely visible for the $\beta_\sigma = 1.5$ or [DL07] models) are distributed in low-surface brightness regions rather than in the centre of the galaxy.

NGC 3627. The 870 μm map follows the distribution of the SPIRE observations, namely a strong emission in the centre and peaks at the bar ends (bar in the NW-SE direction) where molecular hydrogen (Wilson et al. 2012) and large dust reservoirs ([G12]) are located. The star formation primarily occurs in these two knots where the SF efficiency is enhanced (Watanabe et al. 2011). For all models, the absolute 870 μm excess follows the spiral structure of the galaxy (and the dust mass surface density), with excess maxima...
corresponding to 870 μm peaks and spiral arms still distinguishable. The relative 870 μm excess above the $\beta_c = 1.5$ or [DL07] models shows moderate peaks in the centre and each end of the bar. We obtain an average relative excess of 30 per cent for these regions (detected by LABOCA at a 20σ level), with a standard deviation of 19 per cent.

NGC 4826. The map of the 870 μm absolute difference between observation and predictions, like that of the 870 μm emission itself, shows a smooth radial decrease of the excess in this object. The relative difference is non-homogeneous and statistically weak on average across the whole object in the $\beta_c = 1.5$ or [DL07] cases. This is consistent with what is found at global scale for NGC 4826.

NGC 7793. The modelled maps derived for the galaxy follow the distribution of the cold dust emission traced by SPIRE and the 870 μm observation. NGC 7793 is a flocculent galaxy with peaks of star formation spread out in the disc (but only partly resolved at the resolution we are working at in this study) and most of the disc is detected with LABOCA at a level $> 3\sigma$. Like for the galaxy NGC 0337 (a galaxy that shows a similar flattening of the submm slope at global scale), we observe a strong 870 μm absolute excess throughout the galaxy. It however does not follow the spiral structure in this object and peaks are observed within the disc rather than in the centre (the relative excess is lower than 30 per cent in the nucleus). The trend is confirmed by the relative difference maps in which the 870 μm difference between the observations and the models seems to radially increase (by more than a factor of 2) towards the south of the galaxy.

For half of the sample, the global fit favours steeper slopes ($\beta_c = 2.0$) compared to the resolved study. This could primarily be linked with the fact that we are restricting our resolved study to pixels with good detections in the LABOCA data, excluding the faint outskirts where the 870 μm emission is weak. These global versus resolved discrepancies could also partly be due to effects of non-linearity in the SED models we are using and reported for instance in Galliano et al. (2011) or [G12]. Comparing estimates obtained on global and local scales, they show that the dust mass obtained from integrated fluxes could be much lower than that obtained on a resolved basis and attribute this effect to a possible dilution of the cold dust regions in hotter regions. Integrated models could consequently be unable to account for the cold dust populations, biasing the SEDs towards warmer dust (or steeper submm slope) in MBB models or higher minimum heating intensities for the [DL07] models. The global versus resolved discrepancies we observe re-inforce the importance of systematically confronting integrated results (obtained for instance for high-redshift sources that are lacking spatial resolution) and resolved properties derived in the nearby Universe.

4.4 Non-dust contribution to the 870 μm emission

4.4.1 Free–free and synchrotron contribution

As mentioned in Section 3, free–free and synchrotron emission can be a source of contamination of our 870 μm observations but does not significantly (~2 per cent) contribute, at global scale at least, to the 870 μm continuum emission in most of our objects. In the galaxy centres, contribution to the 870 μm by synchrotron emission could increase in case the galaxy hosts an AGN. Two galaxies, NGC 1097 and NGC 1316 possess moderate AGNs. Quantifying the contribution of synchrotron to the 1.3 mm emission in a large sample of galaxies (they neglect the free–free emission), Albrecht, Krügel & Chini (2007) showed that the mean contribution of AGN is on average of ~8 per cent at this wavelength while less than 2 per cent in normal galaxies. We thus expect a more significant non-dust contribution from the synchrotron emission in the centre of NGC 1097 and NGC 1316 than for the rest of the sample. Radio contamination could also moderately affect the central resolved elements of other galaxies whose nucleus is classified as ‘AGN’ in the Moustakas et al. (2010) classification. Finally, the properties of the dust populations in the centre of NGC 1097 could also be affected by the presence of a prominent starbursting ring (Hummel, van der Hulst & Keel 1987).

4.4.2 Local CO contribution

The $^{12}$CO(3–2) line emission can also contribute to the 870 μm emission in some galaxies of the sample. The 870 μm maps we use to derive the difference maps are not corrected for the CO(3–2) contribution. Albrecht et al. (in preparation) provide details of the CO observations available for our sample from which they derive the global estimates of the CO line contribution at 870 μm we quote in this section. At present, there are no CO observations available for the galaxies NGC 1512 and NGC 3621.

From single-point measurements. Direct CO(3–2) single-point measurements (~22 arcsec beam) were taken towards the centre of NGC 1097 (Petitpas & Wilson 2003) and NGC 4826 (Mao et al. 2010). For NGC 1097, the contribution of the 3–2 line to the measured 870 μm emission in this beam is $S_{(3-2)} \sim 101$ mJy. This represents 20 per cent of the total LABOCA flux contained in the 56 arcsec central region. Added to the synchrotron contribution to the 870 μm (8 per cent), this could mostly explain the central excess (40 per cent on average) we observe. For NGC 4826, a contributing $S_{CO(3-2)}$ flux density of 122 mJy is estimated, representing 14 per cent of the total flux, here again a non-negligible contribution to the central 870 μm excess emission. A CO(2–1) single-point measurement (22 arcsec beam) was taken towards the centre of NGC 1316 (Horellou et al. 2001). Using a brightness temperature ratio $R_{3-2,1-0} = 0.36$ (mean value of the ratio derived in Wilson et al. 2012), Albrecht et al. (in preparation) estimated a contribution of $S_{CO(3-2)} \sim 12$ mJy, which represents, like in NGC 1097, up to 20 per cent of the central 56 arcsec central region. Here, again, added to the radio contribution to the 870 μm, we can explain half of the emission in excess in the centre. Finally, only CO(1–0) single-point measurements (43 arcsec beam) are available towards the centre of NGC 1291 (Tacconi et al. 1991) and NGC 7793. Using a brightness temperature ratio $R_{3-2,1-0} = 0.18$ (mean value of the ratio derived in Wilson et al. 2012), Albrecht et al. (in preparation) estimate the $S_{CO(3-2)}$ flux density to reach 28 mJy at most in NGC 1291, minor compared to the excess emission we observe, and 240 mJy in NGC 7793. This represents in both cases ~6.5 per cent of the 870 μm emission, not sufficient to explain the excess we observe. We note that in NGC 7793, the single-point observation is, anyhow, not sufficient to probe CO contribution in the low-surface brightness regions where the excess emission is mostly detected.

From CO maps. CO(2–1) maps of NGC 0337, NGC 0628, NGC 3351 and NGC 3627 were obtained as part of the HERA CO-Line Extragalactic Survey (HERACLES; Leroy et al. 2009). We derive maps of the CO(3–2) line contribution to the 870 μm maps, using a brightness temperature ratio $R_{3-2,1-0} = 0.36$ to convert the CO(2–1) to CO(3–2) emission. Fig. 5 (left-hand column) shows the CO(3–2)-to-870 μm flux density ratios for these four galaxies. MIPS 24 μm contours are overlaid to indicate the distribution of the star-forming regions across the galaxies. We also show the absolute difference maps (Fig. 5, middle column) and the relative difference

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Figure 5. Left column: CO(3–2) contribution to the 870 μm emission (in per cent, see Section 4.4.2). Middle column: excess emission on absolute scales (in MJy sr\(^{-1}\)) corrected for CO(3–2). The modelled 870 μm map is obtained using the [DL07] formalism. Right column: corresponding relative excess corrected for CO(3–2). MIPS 24 μm contours are overlaid: for NGC 0337: 0.4, 1.5, 4 and 7 MJy sr\(^{-1}\); for NGC 0628: 0.25, 0.7, 1.5 and 2.1 MJy sr\(^{-1}\); for NGC 3351: 0.1, 0.3, 1.1 and 10 MJy sr\(^{-1}\); for NGC 3627: 0.6, 2.2, 6 and 15 MJy sr\(^{-1}\).
Figure 6. Top: absolute difference between the observed 870 µm flux densities (not corrected for CO and radio continuum) and the 870 µm estimates extrapolated from the [DL07] dust model as a function of galactocentric radius (left, see Section 4.5.1) and 24 µm surface brightness in units of W kpc$^{-2}$ (right, see Section 4.5.2). The different galaxies of the sample are colour coded. The horizontal dashed line indicates when the observation matches the model. Bottom: same with relative differences.

### 4.5 Analysis

#### 4.5.1 Average radial dependence

Resolved analysis of the IR emission of nearby galaxies have showed that dust properties are particularly expected to vary from the nuclei to the outskirts of galaxies (see for instance the radial profiles of various dust properties like the TIR-to-UV ratio, the dust-to-gas mass ratio or the Polycyclic Aromatic Hydrocarbons fraction in Muñoz-Mateos et al. (2009), of dust abundance gradients in Mattsson & Andersen (2012) or the bulge versus disc comparison of Engelbracht et al. (2010), among others). In order to study the radial dependence of our results, we plot in Fig. 6 the absolute (top left) and relative (bottom left) difference between the observed and modelled 870 µm emission (using the [DL07] formalism) as a function of galactocentric radius. For each object, we normalize the radius to the approximate major radius of the galaxy. After correction, a weak relative excess (<0.5 so 50 per cent above the modelled flux) remains in the centre and on each side of the bar of the galaxy. Directly using the CO(3–2) observation of NGC 3627 taken from the James Clerk Maxwell Telescope (JCMT) Nearby Galaxy Legacy Survey (NGLS; Wilson et al. 2012) lowers the relative excess to 40 per cent in the centre and at each end of the bar.

Maps (Fig. 5, right-hand column) now corrected for CO(3–2) emission. The modelled 870 µm maps used in this analysis are those obtained using the [DL07] model. For NGC 0337 and NGC 0628, the CO(3–2) line contamination is estimated to represent less than 3 per cent of the 870 µm emission on average across the galaxy, with a peak at 12 per cent in the centre for NGC 0628. This contribution is not sufficient to explain the 870 µm excesses we observe above our model extrapolations for these two objects. For NGC 3351, an 8 per cent CO(3–2) line contamination to the 870 µm emission is derived in the centre, which can fully explain the weak central excess we detect for this object. Finally, an 11 per cent line contamination is derived on average for NGC 3627, but with a peak of the CO contribution to the 870 µm in the centre of the galaxy. After correction, a weak relative excess (<0.5 so 50 per cent above the modelled flux) remains in the centre and on each side of the bar of the galaxy. Directly using the CO(3–2) observation of NGC 3627 lowered the relative excess to 40 per cent in the centre and at each end of the bar.
the low statistics we have in the first case and the unknown nature of the 870 μm southern structure in the second case. For the remaining objects, we restrict the analysis to resolved elements with a signal-to-noise ratio greater than 5 in the 870 μm band. We bin our resolved elements in an annulus of 0.3 r25 radii are not corrected for inclination and take the medians in these annuli. The optical radius of NGC 0337 is only 1.5 arcmin, which corresponds to six resolved elements of 14 arcsec. Elements are thus binned in 5 annuli rather than 10 for this object. We calculate the median when three resolved elements at least fulfill our criteria to work on better statistics. Uncertainties are the standard deviations in each annulus.

The absolute differences decrease with radius in most cases, with a systematic 870 μm excess (positive difference) in the centre (typically regions located within 0.3 r25). Some galaxies like NGC 0337 show a smooth decrease with radius. Others have a sharp 870 μm excess profile in the very centre such as for NGC 1097, NGC 3627 or NGC 4826, essentially linked with the contribution of CO and radio to the 870 μm emission, as discussed in the previous section. The galaxy NGC 1316 hosts a low-luminosity X-ray AGN (Kim & Fabbiano 2003) but the possible radio contamination of the 870 μm emission does not lead to a similar broken profile. We observe a sharp decrease of the absolute difference in NGC 3351 as well but this could be due to the non-detection of the 870 μm emission previously mentioned, leading to an underestimate of the observed 870 μm flux density across most of the disc of the galaxy. The two non-barred objects NGC 0628 and NGC 7793 have very flat profiles, i.e. almost no dependence of the absolute excess compared to model predictions on radial distance. The results of [G12] also suggest differences in the submm spectra between barred and non-barred galaxies (when βc is allowed to vary). We discuss this further in Section 4.5.3.

The radial profiles of most of the galaxies are confined to the ±50 per cent range, which suggests that the [DL07] model is, on average per radial bin, sufficient to explain the 870 μm emission we observe, within uncertainties. We observe a noticeable relative difference in NGC 0337 for which the 870 μm difference is systematically above the ±50 per cent threshold. Even if the relative 870 μm excess of NGC 0337 and NGC 7793 seems to increase with radius across the disc of the objects before dropping in the outskirts, the trend is, within the uncertainties, consistent with a constant relative (and positive) difference. Some of the decreases obtained in the outskirts regions could be linked with the low statistics (and low 870 μm surface brightnesses) towards r25 radii.

4.5.3 Decrease of the effective emissivity index?

In Fig. 6, we observe from the profiles of resolved elements with a 870 μm detection at 5σ averaged in annuli that the relative excess does not show a particular dependence with radius. However, we can see from Fig. A1 that the relative differences between observed and modelled 870 μm emission preferentially peak in the disc or the outskirts for the galaxies NGC 0628 and NGC 7793 (in NGC 3621 as well to a lesser extent) when probed in low-surface brightness regions. Whatever causes the excess emission in these objects seems to have a predominant effect in these regions rather than in the centre. Modelling the thermal dust emission in a strip of the Large Magellanic Cloud, Galliano et al. (2011) also derived a submm excess map of the region (excess at 500 μm in their study) and found that the excess is inversely correlated with the dust mass surface density, thus increasing towards low-surface brightnesses as well. In [G12], we showed that when the temperature and the emissivity are allowed to vary in the resolved 2MBB fitting process, the ‘effective’ emissivity index seems to radially decrease towards low surface brightness regions. The high 870 μm excess emission observed in the north-east part of the disc in NGC 3351 (unfortunately the only region of the disc detected with LABOCA) also corresponds to a region where low values of the emissivity index βc were derived in [G12]. The two independent results (870 μm excess detected in the disc or outskirts when βc is fixed versus radial flattening of the [250–500μm] slope when βc is a free parameter) could both be explained by a radial flattening of the submm slope in those particular objects.

NGC 0628, NGC 3621 and NGC 7793 are non-barred galaxies, and three of the four late-type objects of the sample (SaCs, SaDs and SaD morphology, respectively). The fourth late-type galaxy is NGC 0337 in which we also detect an excess predominant in the outskirts, even if the compactness of the source does not allow us to properly analyse the excess distribution. The radial decrease of the
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5 CONCLUSIONS

We combine Spitzer, Herschel and LABOCA data to model with MBBs the global IR-to-submm emission of 11 nearby galaxies. We investigate how the potential excess emission at submm wavelengths varies with various model assumptions (MBB with \( \beta_c \) fixed or free) and show the following.

(i) For seven galaxies of the sample (NGC 1097, NGC 1291, NGC 1316, NGC 3351, NGC 3621, NGC 3627 and NGC 4826), the global 870 \( \mu \)m emission can be explained, within the uncertainties, by thermal emission using an emissivity index \( \beta_c = 2.0 \).

(ii) For the four other objects (NGC 0337, NGC 0628, NGC 1512 and NGC 7793), the integrated 870 \( \mu \)m emission is in excess compared to that expected from cold dust with an emissivity index \( \beta_c = 1.5 \). Using an emissivity index \( \beta_c = 1.5 \) reproduces the global excess we observe in NGC 0628, and also lead to better predictions of the 870 \( \mu \)m emission in the previously mentioned two galaxies NGC 1097 and NGC 1316. For NGC 1512, the observed 870 \( \mu \)m flux and the 870 \( \mu \)m modelled using \( \beta_c = 1.5 \) are consistent within the error bars but the model fitted to Spitzer+Herschel data favours very low \( \beta_c \), values when allowed to vary (\( \beta_c = 1.16 \)). Lower values (\( \beta_c < 1.3 \)) would also be required to fit the 870 \( \mu \)m data in NGC 0337 and NGC 7793, which questions the use of an isothermal component to fit the cold dust population up to 870 \( \mu \)m in those objects.

We apply the same methodology on local scales and produce 870 \( \mu \)m maps using 2MBBs models with fixed \( \beta_c \) or the [DL07] dust models in order to compare them to the observed emission.

(i) We observe a systematic 870 \( \mu \)m excess when the emissivity index of \( \beta_c = 2.0 \) is used to model the ISM elements we selected (above a given signal-to-noise criterion). Using \( \beta_c = 1.5 \) can, in many cases, reconcile the observed and the modelled 870 \( \mu \)m emission. The (absolute and relative) difference maps derived using the [DL07] formalism are similar to those obtained using \( \beta_c = 1.5 \).

(ii) Maps of the absolute difference between observed and modelled 870 \( \mu \)m emission show, in many cases, a decrease of this quantity with radius and an increase with star formation (traced using the 24 \( \mu \)m surface brightness). In NGC 1097, NGC 1316, NGC 3627 and NGC 4826, this excess can be partly or fully explained by contributions from CO(3–2) emission and/or free–free and synchrotron radiation.

(iii) The relative excess profiles have larger uncertainties with various behaviours from one galaxy to another and no clear dependence in the case of NGC 0337 and NGC 7793 with either parameter (radius or star formation). This raises the issue of what could be the drivers of the major excesses we detect in these two objects. This study should be extended to closer (and thus more resolved) objects to increase our statistics.

(iv) In the non-barred spirals NGC 0628, NGC 3621 and NGC 7793 however, the relative excess maps show peaks in the disc or towards low surface brightnesses. A radial flattening of the submm slope (so a decrease of the ‘effective’ emissivity index with radius) as suggested by the studies of [G12], could explain the distribution of the 870 \( \mu \)m excess we observe.

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REFERENCES

Albrecht M., Krügel E., Chini R., 2007, A&A, 462, 575
Aniano G., Draine B. T., Gordon K. D., Sandstrom K., 2011, PASP, 123, 1218
Aniano G. et al., 2012, ApJ, 756, 138
Bendo G. J. et al., 2006, ApJ, 652, 283
Bendo G. J. et al., 2010, A&A, 518, L65
Bendo G. J. et al., 2012a, MNRAS, 419, 1833
Bendo G. J., Galliano F., Madden S. C., 2012b, MNRAS, 423, 197
Boquien M. et al., 2011, AJ, 142, 111
Bot C., Ysard N., Paradis D., Bernard J. P., Lagache G., Israel F. P., Wall W. F., 2010a, A&A, 523, A20
Bot C. et al., 2010b, A&A, 524, A52
Brauer J. R., Dale D. A., Helou G., 2008, ApJS, 178, 280
Buta R. J. et al., 2010, ApJS, 190, 147
Calzetti D. et al., 2007, ApJ, 666, 870
Dale D. A. et al., 2007, ApJ, 655, 863
Dale D. A. et al., 2009, ApJ, 703, 517
APPENDIX A: 870 µm MAPS PREDICTED FROM OUR DIFFERENT SED MODELS AND 870 µm ABSOLUTE AND RELATIVE DIFFERENCE MAPS

NGC0337

Figure A1. For each galaxy: top panel: flux density at 870 µm map observed with LABOCA (in MJy sr⁻¹) not corrected for CO and radio continuum. Second line panels: modelled 870 µm maps (in MJy sr⁻¹) derived, from left to right, using our two-MBB procedure with βc fixed to 2, fixed to 1.5 or using the [DL07] formalism. Third line panels: absolute difference between the observed and the modelled 870 µm map in MJy sr⁻¹ (defined as observed flux – modelled flux) for the different SED models. Bottom line panels: relative excess defined as (observed flux – modelled flux)/(modelled flux). We provide the colour scales for each line on the right-hand side. Zeroes are indicated with a white marker on the colour bars. The signal-to-noise of the LABOCA 870 µm map is overlaid as contours on the absolute and relative difference maps (same levels than in Fig. 1).
NGC0628

$\beta_c = 2.0$ model  $\beta_c = 1.5$ model  [DL07] model

Figure A1 – continued
NGC1097

Figure A1 – continued

\( \beta_c = 2.0 \) model

\( \beta_c = 1.5 \) model

[DL07] model
NGC1291

\[ \beta_c = 2.0 \text{ model} \quad \beta_c = 1.5 \text{ model} \quad [DL07] \text{ model} \]

\[ 870 \mu \text{m Modelled} \quad 870 \mu \text{m Observed} \]

\[ \text{Absolute Difference} \quad \text{Relative Difference} \]

Figure A1 – continued
NGC1512

Figure A1 – continued
NGC3621

\[ \beta_c = 2.0 \text{ model} \quad \beta_c = 1.5 \text{ model} \quad \text{[DL07] model} \]

Figure A1 — continued
NGC4826

**Figure A1 – continued**

- **β_c = 2.0 model**
- **β_c = 1.5 model**
- **[DL07] model**

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NGC7793

\[ \beta_c = 2.0 \text{ model} \]

\[ \beta_c = 1.5 \text{ model} \]

\[ \text{[DL07] model} \]

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