INTRODUCTION

Unconventional reservoirs are characterized by low permeability, low porosity, and tight structure, which bring great difficulties in gas and oil production. Multistage hydraulic fracturing and horizontal drilling technology have been used to solve these problems successfully.1-3 Drilling horizontal wells on shale reservoirs causes hydraulic fractures and many secondary fractures, forming fracturing effect zones, as shown in Figure 1, which consist of the horizontal wells with fracture zones.
injected fracturing fluid, the hydraulic fracture zone formed by perforation and the reservoir reconstruction zone.

The SEM images of shale gas reservoir cores are shown in Figure 1C. The reservoirs are mainly composed of organic and inorganic materials under microscopic mechanisms, and there are still some natural fractures in the matrix, but they are not developed and rarely connected. The gas in shale gas reservoir exists in three forms: (a) adsorbed gas in organic matter, and (b) free gas in pores and fractures, (c) dissolved gas in liquid gas. The gas flow pattern is also different at different pore sizes. This research is going to increase the area of fracture and drive out the oil and gas in them, so the reservoir can be divided into two parts: matrix and fracture. In this study, a dual-porosity dual-permeability model for shale rocks was established, which considers the interactions between matrix and fracture and the flow performances separately.

The permeability of tight rock is closely related to stress. The reservoir pressure will decline with shale gas extraction, and this decline disrupts the pressure balance between the fracture and the matrix, causing gas to flow from the matrix to the fracture and ultimately to the bottom hole. In addition, the mechanical properties of shale gas reservoirs are affected by effective stress and strain, such as the porosity and the apparent permeability. In order to accurately evaluate gas production in shale reservoirs, many scholars use continuum mechanics models to study gas flow in shale reservoirs with different flow mechanisms in consideration of dynamic reservoir properties and different scales, such as the single-porosity, dual-porosity model, and triple-porosity model. Among them, the influence of surface diffusion on gas production cannot be ignored, which is an important gas migration mechanism in shale matrix. The dual-porosity model provides a good explanation for the gas transfer mechanism in kerogen and inorganic matrix. The mass transfer mechanism of absorption, Knudsen diffusion, surface diffusion, and Darcy flow coexist in the numerical simulation. Some researches considered the effective stress elastic and gas adsorption strain, and established the apparent permeability model of the reservoir. Shabro et al. and Civan et al. made introduced and analyzed for the important parameters of the apparent permeability model. Peng et al. and Cao et al. considered that the shale gas reservoir, especially the nanometers porous reservoir, is composed of inorganic minerals and kerogen, and modeled the influence of the flow pattern and diffusion on the apparent permeability. Xin et al. established a full coupling model including the shale matrix and discrete fractures, considering the influence of fracture seepage loss and reservoir temperature. However, all these models only cover the influence of various factors on reservoir permeability and do not consider the influence of effective stress on natural fractures. Some researches based on the dual medium model discussed the "soft hard" compaction effect of fractures on gas production, but they did not analyze the relationship between hydraulic fracture effective permeability and gas pressure. In previous research, some scholars did not consider the change of pore structure of the shale reservoir, mostly focused on the gas flow principles and the matrix deformation. However, recent studies have shown that the effect of pore structure on permeability cannot be ignored. This is because the complex pore structure leads to complex gas flow mechanism. Fractal theory can well describe its structural properties, such as pore size distribution, channel curvature, and surface roughness. In this study, the coupling relationship among solid deformation, gas adsorption strain, gas flow in the matrix, gas surface diffusion, fractal properties, and gas flow in hydraulic fractures were investigated.

Fracture parameters have direct impacts on the shale gas production, and the degree of impacts varies with parameters. A discrete fracture network model was established to study the effect of fracture-related properties on the reservoir productivity. Zhang et al. established a triple-continuum model to analyze the relationship between kerogens and production. And a fully coupled flow deformation model was used to analyze the influence of fracture diversion loss on
cumulative production. Besides, Wang et al.\textsuperscript{18} established a two-phase flow-back model to analyze the impact of fracture parameters on production rate. These researches all analyzed the relationship between fracture features and production, but they did not go further to the impact of each fracture feature on the gas yield. Therefore, this research analyzes the fracture features and quantitatively compares their effects on the gas production.

The purpose of this paper is to establish a comprehensive model based on the flow mechanism of single-porosity medium and dual-porosity medium in different zones to analyze the influence of key fracture parameters on shale gas production. Firstly, the deformation stress mechanism of shale reservoir is analyzed. The apparent permeability model of matrix and fracture system is established by considering the adsorption, diffusion and flow mechanism of the gas in the pore system and fractal properties. Secondly, according to the law of mass conservation of fluid and the principle of effective stress, the governing equations of gas flow for fracture and matrix systems are established. After that, the multiphase simulators are used for numerical simulation to verify the mathematical model. Finally, the influence of key fracture parameters on shale gas recovery is analyzed.

## 2 | FORMULATION OF THE CONCEPTUAL MODEL

The derivations in this paper are based on the following assumptions: (a) Shale reservoirs are double porous isotropic linear elastic media; (b) Reservoirs are isothermal and the fluid is saturated; (c) The plane strain condition with small deformation is adopted; (d) There is compressible fluid single-phase flow (methane in this study) in the fracture system following the Darcy’s law; (e) The Langmuir equation is used to describe the adsorption/desorption of gas.

### 2.1 | Mechanical equilibrium equation

#### 2.1.1 | Deformation in single-porosity medium

For the gas-saturated homogeneous porous elastic media, the coupling between shale rocks deformation and gas flow is described by elastic theory. The deformation-constitutive equation can be written as

\[
\varepsilon_{ij} = \frac{1}{2G} \sigma_{ij} - \left( \frac{1}{6G} - \frac{1}{9K} \right) \sigma_{kk} \delta_{ij} + \frac{\beta_k}{3K} p_k \delta_{ij} + \frac{\varepsilon_s}{3} \delta_{ij} \quad (1)
\]

where \( i \) and \( j \) are two orthogonal directions in the x and y directions, \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are the components of the total stress and total strain respectively. \( \sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33} \) is the volume stress. \( G = E/(2 + 2\nu) \) is the shear modulus, with \( E \) denoting the Young’s modulus of reservoir and \( \nu \) denoting the Poisson’s ratio. \( \varepsilon_s = p_k \varepsilon_k / (p_L + p_L) \) is the expansion strain of the shale matrix caused by adsorption,\textsuperscript{36} with \( \varepsilon_k \) and \( p_L \) denoting the Langmuir volumetric strain constant and Langmuir pressure constant, respectively. \( p_k \) is the gas pressure.

The deformation equilibrium equation of shale gas reservoir is as follows

\[
\sigma_{ij} + F_i = 0 \quad (2)
\]

where \( \sigma_{ij} \) is the total stress component and \( F_i \) is the volume force component. The geometric equation of deformation can be expressed as

\[
\varepsilon_{ij} = \frac{1}{2} (u_{ij} + u_{ji}) \quad (3)
\]

Combination of Equations (1)–(3) yields the Navier equation as

\[
G \varepsilon_{ij} + \frac{1}{1 - 2\nu} u_{ij,j} - \beta_k p_{k,i} = \frac{K \varepsilon_k p_{L,i}}{(p_m + p_L)^2} (p_m + p_L) + F_i = 0 \quad (4)
\]

#### 2.1.2 | Deformation in dual-porosity medium

Dual-porosity medium has different stiffness and deformation of fractures and matrix. Therefore, the deformation-constitutive equation is slightly modified as

\[
\varepsilon_{ij} = \frac{1}{2G} \sigma_{ij} - \left( \frac{1}{6G} - \frac{1}{9K} \right) \sigma_{kk} \delta_{ij} + \frac{\beta_m}{3K} p_m \delta_{ij} + \frac{\beta_f}{3K} p_f \delta_{ij} + \frac{\varepsilon_s}{3} \delta_{ij} \quad (5)
\]

where \( p_m \) and \( p_f \) represent the gas pressure of matrix and fracture, respectively. \( \beta_m \) and \( \beta_f \) are the Biot coefficients for the matrix and fracture, respectively, and can be expressed as\textsuperscript{37,38}:

\[
\begin{align*}
\beta_f &= 1 - K/K_s \\
\beta_m &= K/K_s - K/K_m
\end{align*} \quad (6)
\]

where \( K = E/(3(1 - 2\nu)) \) is the volume modulus of shale rock. \( K_s = E_s/(3(1 - 2\nu)) \) is the shale grain. \( E_s \) is the Young’s modulus of the shale matrix. \( K_m \) is the bulk modulus of the shale skeleton, which can be calculated by \( K_m = K_s / [ 1 - 3\phi(1 - \nu) / [2(1 - 2\nu)] ] \textsuperscript{39} \). The obtained governing equation of the shale stress and deformation is the modified Navier equation, as follows:

\[
G \varepsilon_{ij} + \frac{1}{1 - 2\nu} u_{ij,j} - \beta_m p_{m,i} - \beta_f p_{f,i} - \frac{K \varepsilon_k p_{L,i}}{(p_m + p_L)^2} (p_m + p_L) + F_i = 0
\]

\[
(7)
\]
According to Equation (7), the volume stress forming the shale gas reservoirs is from gravity, fluid flow in pores and fractures, and gas desorption/adsorption.

2.2 | The apparent permeability of Single-Porosity medium

2.2.1 | Knudsen Number

Shale gas reservoirs have micro- and nano-scale pores filled with free gas. The adsorbed gas is mainly concentrated near the substrate surface, gas flow patterns also vary depending on pore size. As Figure 2 shows, the Knudsen number changes with the pore pressure as the pore radius changes from 1 nm to 50 um, at the temperature of 350 K. According to the Knudsen number, the flow of gas molecules can be divided into four forms: (a) \( K_n < 0.01 \), the viscous flow, which is applicable to conventional hydrodynamics; (b) \( 0.01 < K_n < 0.1 \), the slip flow, which is formed by the gas molecules sliding at the solid interface; (c) \( 0.1 < K_n < 10 \), the transition flow, which is generated when the collision probability between molecules is similar; (d) \( K_n > 10 \), the free molecular flow (Knudsen flow), which controls most gas transportation and in which the collision between molecules and pore wall is the most intensive.

Gas transport in shale gas reservoirs is mainly controlled by Knudsen diffusion. The Knudsen number defined by Langmuir is used to determine the gas flow pattern, which is expressed as

\[
K_n = \frac{\lambda}{d_p}
\]  

where \( d_p \) is the pore radius, \( \lambda \) is the average free path of gas, which can be expressed as

\[
\lambda = \frac{k_BT}{\sqrt{2\pi P_k d_m^2}}
\]

where \( k_B \) is the Boltzmann constant, \( T \) is the absolute temperature and \( d_m \) is the collision diameter of gas molecules. Therefore, Equation (8) can be rewritten as

\[
K_n = \frac{k_BT}{d_p\sqrt{2\pi P_k d_m^2}}
\]

2.2.2 | Formulation of gas diffusion in shale matrix

For the shale gas reservoirs, a large number of nanopores exist in two systems of organic (Kerogen) and inorganic matrix, which can form a variety of gas transmission forms, as shown in Figure 3. There are three main gas transport mechanisms for shale matrices, and the total gas flux is the sum of viscous flow, Knudsen diffusion, and surface diffusion, as described below.

(1) Knudsen diffusion

The gas flux of Knudsen diffusion is defined as:

\[
J_k = -\frac{\phi_k}{\tau} \frac{\sigma P_k d_m^2 D_k M_g}{RT} \nabla P_k
\]

where \( \delta \) is the ratio of gas molecules diameter to pore diameter, \( \delta = d_m/d_p \) \( D_k \) is the Knudsen diffusion constant, which is
where $\tau$ is the tortuosity of pore channels. $D_f$ is the fractal coefficient of pore surface. $M_g$ is the molar mass of gas, $R$ is the general gas constant.

(2) Viscous flow

The viscous flow was obtained using tortuosity's modified Hagen-Poiseuille equation with slip boundary conditions:\textsuperscript{5,14}

$$J_v = -\frac{\phi_k}{\tau} \frac{D_f d_m^2 M_g p_k}{8\pi RT} \nabla p_k$$  \hspace{1cm} (13)

The diffusion coefficient induced by viscous flow is as follows:

$$D_v = \left[ 1 + \frac{\mu}{d_m p_k} \left( \frac{2}{\alpha_r} - 1 \right) \right] \sqrt{\frac{8\pi RT}{M_g}}$$  \hspace{1cm} (14)

where $\alpha_r$ is the tangential momentum accommodation coefficient.

(3) Surface diffusion

In addition to free gas flow, surface diffusion of adsorbed gas also occurs. When the influence of capillary pressure between the bulk gas phase and the adsorbed gas phase on mass transport can be ignored, the surface diffusion flux is expressed as:\textsuperscript{15}

$$J_s = -\frac{\xi_{ms}}{\kappa} \frac{D_s A\theta M_g}{k_d^3 N_A \rho_g} \nabla p_k$$  \hspace{1cm} (15)

where $\xi_{ms}$ is the correction factor for surface diffusion for adsorbed gas, which can be expressed as:

$$\xi_{ms} = \frac{\phi_k}{\tau} \left( 1 - \frac{d_m}{d_p} \right)^2 \left[ \left( 1 - \frac{d_m}{d_p} \right)^{-2} - 1 \right]$$  \hspace{1cm} (16)

The surface diffusion coefficient is derived using the kinetic method:\textsuperscript{16}

$$D_s = \frac{D_0 s}{1 - \theta} + \frac{\xi^2 (2 - \theta) + H(1 - \kappa) (1 - \kappa) \frac{\xi^2 \theta^2}{2}}{\left( 1 - \theta + \frac{\xi^2}{2} \right)^2}$$  \hspace{1cm} (17)

where $D_s$ is the surface diffusion coefficient. $D_0^s$ is the surface diffusion coefficient at zero gas coverage. $\theta$ is the surface coverage. $\kappa$ is the blockage parameter to express the degree of pore blocking. $H$ is the isosteric-adsorption heat.

(4) Apparent permeability

The total gas diffusion of matrix pores originates from the three sources mentioned above:\textsuperscript{17,18}

$$J = \frac{K_n}{1 + K_n} J_k + \frac{1}{1 + K_n} J_v + J_s$$  \hspace{1cm} (18)

The apparent permeability is then obtained as

$$k_k = -\frac{J \mu}{\rho_g} \nabla p$$  \hspace{1cm} (19)

2.3 | The apparent permeability of dual-porosity medium

2.3.1 | The apparent permeability of the shale matrix

We analyzed the shale gas reservoir with obvious diffusion and flows. The apparent permeability of shale matrix can be expressed by the Knudsen formula:\textsuperscript{24,42}
where \( h \) denotes the slip coefficient, which is usually set as \(-1\) and \( \alpha \) is the sparse coefficient of the Knudsen function, which can be obtained by:

\[
\alpha = \alpha_0 \frac{2}{\pi} \tan^{-1} (\alpha_1 K_n^\phi) \tag{21}
\]

where \( \alpha_0 \) is the sparse coefficient of \( K_n \to \infty \), \( \alpha_1 \) and \( \beta \) are the dimensionless fitting constants, which can be obtained by numerical simulation and experimental data. Figure 4 shows the relationship between \( \alpha \) and the Knudsen number.\(^{18}\) When the correlation coefficients in Table 1 are used, the analytic solution fits the experimental data from Tison\(^{43}\) well.

The intrinsic permeability of shale matrix can be determined according to its relationship with porosity\(^{44}\) as follows

\[
k_\infty = k_{\infty 0} \left( \frac{\phi_m}{\phi_{m0}} \right)^3 \tag{22}
\]

where \( k_{\infty 0} \) is the initial intrinsic permeability.\(^{7}\) The matrix porosity can also be expressed by the relationship between the porosity and effective stress.\(^{26}\)

\[
\frac{\phi_m}{\phi_{m0}} = \exp \left\{ \left( 1 - \frac{1}{K_P} + \frac{1}{K_n} \right) \left[ \bar{\sigma} - \bar{\sigma}_0 - (P - P_0) \right] \right\} = \exp \left\{ -C_m \left( \bar{\sigma} - \bar{\sigma}_0 - (P - P_0) \right) \right\} \tag{23}
\]

where subscript 0 refers to the initial value for shale rocks, \( K_P \) denotes the bulk modulus of the shale pore system and \( \bar{\sigma} = -\sigma_{kk}/3 \) is the average compressive stress. Because the bulk modulus \( K \) is several orders larger than \( K_P \), \( 1/K - 1/K_P \approx -1/K_P \) can be approximated. Define the compressibility \( C_m = 1/K_P \), which represents the mechanical properties of porous media. Assuming that the calculation model of the shale gas reservoir is a plane problem,\(^{26}\) we get

\[
\varepsilon_{zz} = \frac{1}{2G} \sigma_{zz} - \left( \frac{1}{6G} - \frac{1}{9K} \right) \sigma_{kk} + \frac{1}{3K} \left( \beta_m p_m + \beta_j p_j \right) + \frac{\varepsilon_s}{3} = 0 \tag{24}
\]

With

\[
\sigma_{zz} = \nu (\sigma_{xx} + \sigma_{yy}) - (1 - 2\nu) \left( \beta_m p_m + \beta_j p_j \right) - \frac{E\varepsilon_s}{3} \tag{25}
\]

The mean stress \( \bar{\sigma} \) is

\[
\bar{\sigma} = -\frac{1}{3} \sigma_{kk} = -\frac{1}{3} + \frac{1}{3} (\sigma_{xx} + \sigma_{yy}) + \frac{1 - 2\nu}{3} (\beta_m p_m + \beta_j p_j) + \frac{E\varepsilon_s}{9} \tag{26}
\]

Finally, the formula for the ultimate change of the matrix porosity can be obtained by substituting Equation (23) into Equation (26):

\[
\phi_m = \exp \left\{ -c_m \left[ \left( \frac{1 - 2\nu}{3} \right) \left( \beta_m (p_m - p_0) - \beta_j (p_j - p_0) \right) \right] \right\}
\]

\[
= \exp \left\{ -c_m \left[ -(P_m - P_0) + \frac{E\varepsilon_s}{9} \left( p_m + p_L - \frac{p_0}{p_L} \right) \right] \right\} \tag{27}
\]

Combining Equation (27) with Equation (20), we get the formula for calculating the variation of the shale matrix apparent permeability:

**TABLE 1** Parameters used in the apparent permeability model

| Physical                  | Value  | Unit     |
|---------------------------|--------|----------|
| Gas type                  | CH₄    |          |
| Universal gas constant, \( R \) | 8.314  | J/(mol K) |
| Temperature, \( T \)      | 350    | K        |
| Poisson's ratio of shale, \( \nu \) | 0.33   |          |
| Tortuosity, \( \tau \)   | 3      |          |
| Boltzmann constant, \( k_B \) | 1.3805 \times 10^{-23} | J/K |
| Molecular diameter of methane, \( a_m \) | 3.8 \times 10^{-10} | m   |
| Residual aperture, \( d_p \) | 1 \times 10^{-7} | m   |
| Rarefraction coefficient at \( K_n \to \infty, a_0 \) | 64/15\pi |          |
| Fitting constant, \( a_1 \) | 4.0    |          |
| Fitting constant, \( \beta \) | 0.4    |          |
| Gas slip constant, \( h \) | -1     |          |
2.3.2 | The apparent permeability of the fracture network

Figure 5 is a model showing the porosity and permeability of the fractured system in the shale gas reservoir, in which the shale matrix block is a cube.\(^{45,46}\) We selected a unit (outlined by the red dotted line in Figure 5A) from the model to analyze the stress between one fracture and one matrix. The unit is composed of a shale matrix block (outlined by the black solid line) and the surrounding fractured cube. The side length of the matrix block is \(s\), and the fracture opening between two matrix blocks is \(b\). Due to the uniaxial stress \(\sigma\) on the matrix block, the unit has some deformation as shown in Figure 5B. For the cube geometry models, the initial porosity and permeability of the fractures are:

\[
k_f = \frac{b^3}{12s}, \quad \phi_f = \frac{3b}{s} \tag{29}
\]

The effective strain \(\Delta \sigma_{el}\) can cause fracture opening change \(\Delta b\), which can be expressed as the difference between the whole reservoir deformation and matrix deformation:\(^{47}\)

\[
\Delta b = (b + s) \frac{\Delta \sigma_{el}}{E} - s \frac{\Delta \sigma_{el}}{E_S} = s \left(1 - \frac{E}{E_S}\right) \frac{\Delta \sigma_{el}}{E} + b \frac{\Delta \sigma_{el}}{E_S} \tag{30}
\]

The constitutive relation of the known shale rocks is

\[
\Delta \sigma_{el} = \frac{\Delta \varepsilon_v - \Delta \varepsilon_s}{3} \tag{31}
\]

After getting the value of \(\Delta b\), the porosity change of the fracture system can be obtained by

\[
\Delta \phi_f = \phi_f - \phi_{f0} = \frac{3\Delta b}{s} = \left(1 - \frac{E}{E_S}\right) \left(\Delta \varepsilon_v - \Delta \varepsilon_s\right) \tag{32}
\]

where \(\Delta \varepsilon_v\) is the volume strain increment in the shale rocks and \(\Delta \varepsilon_s\) is the adsorption-induced expansion strain increment.\(^{48}\)

\[
\Delta \varepsilon_v = \frac{1-2v}{E} - \Delta \sigma_{kk} = \frac{(1-2v)(1+v)}{E(1-v)} \left[\beta_m(p_m-p_0) + \beta_f(p_f-p_0)\right]
\]

\[
+ \frac{v}{3(1-v)} \Delta \varepsilon_s \tag{33}
\]

\[
\Delta \varepsilon_s = \frac{\varepsilon_L p_m}{(p_m + p_L)} - \frac{\varepsilon_L p_0}{(p_L + p_0)} = \frac{\varepsilon_L p_f(p_m-p_0)}{(p_L + p_0)(p_m + p_L)} \tag{34}
\]

Therefore, the porosity of the fracture system can be obtained by:

\[
\phi_f = \phi_{f0} + \left(1 - \frac{E}{E_S}\right) \left[\left(1-\frac{2v}{1-v}\right) - \frac{2(v-2)}{3(1-v)} \frac{\varepsilon_L p_f p_m}{(p_L + p_0)(p_m + p_L)} \right] \tag{35}
\]

Meanwhile, based on the fractal distribution of fracture length,\(^{10}\) the permeability of the fracture system is finally obtained by:

\[
k_f = \frac{\zeta^2 r_{max}^2}{12} \left(2 - D_f \frac{1 - \phi_f^{4/\phi_f}}{1 - \phi_f^2/\phi_f}\right) \tag{36}
\]

where \(D_f\) is the Fracture length fractal dimension. \(l_{max}\) is the Maximum length of FN, \(\zeta\) is the Proportionality coefficient.
3 | THE GAS FLOW EQUATION

The shale gas reservoirs were divided into two regions, each with different gas flow and solid deformation mechanism in shale matrix. Figure 6 shows details of diffusion and deformation in each zone. Their governing equations are described below.

3.1 | Governing equation of single-porosity medium

The mass conservation equation of shale gas is expressed as:

$$\frac{\partial (m_k)}{\partial t} + \nabla \cdot \left( - \frac{k_g}{\mu} \rho_{gk} \nabla p_k \right) = 0$$  \hspace{1cm} (37)

where $m_k$ is the gas mass including free gas and adsorbed gas and is expressed as:

$$m_k = \rho_{gk} \phi_k + \rho_{gr} \phi_r \frac{V_L p_k}{p_k + p_L}$$  \hspace{1cm} (38)

In the above equation, $\rho_{gk}$ is the gas density, $\rho_{gr}$ is the gas density in the standard condition, $\rho_r$ is the density of shale matrix, and $V_L$ is the Langmuir volume constant. According to the ideal gas law, the gas density is defined as

$$\rho = \frac{pM_g}{ZRT}$$  \hspace{1cm} (39)

where $Z$ is the compression coefficient, which can be estimated by the development correlation between the pseudo-reduced pressure ($p_r$) and pseudo-reduced temperature ($T_r$),

$$Z = 0.702e^{-2.37p_r^2} - 5.524e^{-2.37p_r} + (0.0447T_r^2 - 0.1647T_r + 1.15)$$  \hspace{1cm} (40)

Here $p_r = p/p_c$ and $T_r = T/T_c$, with $p_c$ and $T_c$ denoting the critical pressure and temperature of methane, respectively. Gas viscosity can also be estimated using the following correlation:

$$\mu = (9.379 + 0.01607M_g)(1.8T)^{1.5}$$  \hspace{1cm} (41)

$$\begin{align*}
\xi &= \frac{986.4 + 0.01009M_g}{1.8T} \\
X &= 3.448 + \frac{986.4 + 0.01009M_g}{1.8T} \\
Y &= 2.447 - 0.2224X \\
\end{align*}$$

where $\phi_k$ is the porosity of the single-porosity zone. Considering the significant influence of effective stress and gas flow state on the apparent permeability, the dynamic porosity can be expressed as

$$\phi_k = \phi_{10} \exp \left[ - C_m (\bar{\sigma} - \bar{\sigma}_0 - (P_k - P_0)) \right]$$  \hspace{1cm} (42)

Substituting Equation (38) into Equation (37) obtains the governing equation of gas flow in shale matrix:

$$\begin{align*}
\phi_k + p_r \rho_r \frac{V_L p_k}{(p_k + p_L)^2} \frac{\partial p_k}{\partial t} + p_k \frac{\partial \phi_k}{\partial t} + \nabla \cdot \left( - \frac{k_g}{\mu} \rho_{gk} \nabla p_k \right) &= 0 \\
\end{align*}$$  \hspace{1cm} (43)

3.2 | Governing equation of dual-porosity medium

The mass conservation equations of the gas flow in matrix and fracture system are:

$$\begin{align*}
\frac{\partial m_m}{\partial t} + \nabla \cdot \left( - \rho_m \frac{k_m}{\mu} \nabla p_m \right) &= q_m \\
\frac{\partial m_f}{\partial t} + \nabla \cdot \left( - \rho_f \frac{k_f}{\mu} \nabla p_f \right) &= -q_m \\
\end{align*}$$  \hspace{1cm} (44)
where $\mu$ is the gas dynamic viscosity, $\rho_m$ and $\rho_f$ are the density of the shale gas reservoir matrix and fracture, respectively. $q_m$ is the gas source or sink, $t$ is time. Generally, the storage space and flow channel of the shale gas are the inorganic matrix and fracture, respectively.$m_m$ is the gas mass of the shale matrix with both the adsorbed gas and the free gas, and $m_f$ is the mass of the free gas in the fractures, as the mass of other gases is negligible.$^5^1$

$$
\begin{align*}
\begin{cases}
  m_m &= \rho_m \phi_m + \rho_g \rho_f, \\
  m_f &= \rho_f \phi_f
\end{cases}
\end{align*}
$$

The right term of (44) is the source or sink of gas, indicating that the gas diffusion process is transferred from the pore system to the matrix system$^5^2$: 

$$
q_m = -\frac{\pi^2 \rho_m k_m}{\mu} \left( \frac{1}{L_x} + \frac{1}{L_y} \right) (p_f - p_m) 
$$

where $L_x$ and $L_y$ are the fracture diversion capacity in the x and y directions, respectively. Therefore, the partial differential governing equations of the matrix and fracture system of Equation (44) are:

$$
\begin{align*}
\begin{cases}
  \rho_m &\frac{\partial \rho_m}{\partial t} - \nabla \cdot (\rho_m \frac{k_m}{\mu} \nabla p_m) = \frac{\pi^2 \rho_m k_m}{\mu} \left( \frac{1}{L_x} + \frac{1}{L_y} \right) (p_f - p_m) \\
  \phi_f + p_f \beta_f (1 - \frac{E}{E_s} (1 - 2\nu)(1 + \nu) \frac{E(1 - \nu)}{2}) &\frac{\partial p_f}{\partial t} - \nabla \cdot (p_f \frac{k_f}{\mu} \nabla p_f) = \frac{\pi^2 \rho_f k_f}{\mu} \left( \frac{1}{L_x} + \frac{1}{L_y} \right) (p_m - p_f)
\end{cases}
\end{align*}
$$

3.3 Governing equations of the gas flow in hydraulic fractures

With the depletion of the bottom-hole pressure, the shale gas flows to the inner boundary of the reservoir, also known as the hydraulic fracture system, and then flows into the bottom-hole along the tangential direction of the fracture. The gas flows in the hydraulic fracture following the Darcy’s law.$^5^3$ In the fracture system, the mass conservation equation of the compressible fluid is

$$
d_f \frac{\partial (\rho_f \phi_f)}{\partial t} + \nabla \cdot (\rho_f \phi_f) = 0 
$$

where $d_f$ is the width of the hydraulic fracture and $\phi_f$ is the porosity. The boundary velocity along the fracture can be obtained by the Darcy’s law:

$$
q_{hf} = -d_hf \frac{k_{hf}}{\mu} \nabla T
$$

where $d_{hf}$ is the hydraulic fracture width, $p_{hf0}$ is the initial shale reservoir pressure. With the continuous fracturing, the reservoir gas pressure gradually decreases, resulting in the effective permeability reduction in the supporting fractures.$^2^2,^5^4$ The effective permeability of the hydraulic fracture is related to gas pressure as follows

$$
k_{hf} = k_{hf0} \exp \left( -c_f (p_{hf0} - p_{hf}) \right)
$$

where $k_{hf0}$ is the initial permeability of the hydraulic fracture, $c_f$ is the pressure sensitive coefficient. The gas permeability of the fracture is relatively high, which has slight influence on the flowing pattern, this indicates that effective stress is an important factor in gas production.
MODEL VERIFICATIONS

We verified the model by comparing the simulation results with field data of gas production of the Marcellus shale and Barnett shale. The 300 days production data of the Marcellus shale were adopted to verify the reliability of the model in short-term production and the 4 years’ production data of the Barnett shale were used to verify the model in long-term production. The governing equations were implemented and solved by the COMSOL Multiphysics. The mechanical equilibrium equation of shale gas reservoir is solved by solid mechanics module and the gas flow equation is solved by PDE module. As Figures 7 and 8 shows, the numerical simulation results are basically consistent with the field data. In the early stages of shale gas production, the free gas from the reservoir fractures moves rapidly toward horizontal Wells, so gas production drops sharply from the highest to the low. In the later stage, the adsorbed gas from the matrix accounted for most of the production, while the desorption and diffusion process of the gas were slow and lasted for a long time, leading to the slow decline in the production in this stage.

RESULTS AND DISCUSSION

We established a fully coupled of gas flow mechanism and effective stress model of shale gas to analyze the effects of some key fracture parameters on the long-term gas exploration in multi-stage hydraulic fracturing, which the base geometric model is shown in Figure 9 (referred to as the first geometric model here after). These parameters are geometric parameters of hydraulic fractures, the fracture half-length, width and fracture diversion capacity of the hydraulic fracture and the initial permeability and fracture opening of the natural fracture.

In the first geometric model (Figure 9), the simulated area is 600 m in length and 300 m in width, which is divided into three zones: a single-porosity medium zone (100 m in high), the dual-porosity medium zone (200 m in high), and the hydraulic fracture zone (shown by the red line at the bottom of the Figure 9). The distance between the adjacent fractures is...
70 m, and there are a total of 8 hydraulic fractures with the same length of 120 m. There is no gas flow around the model boundary. The bottom-hole pressure is $p_w$. The maximum pressure at the upper boundary is 40 MPa, and the minimum pressure at the right boundary is 35.2 MPa. The axis support is set at the lower boundary and the left boundary. The specific parameters of this simulation are shown in Table 2.

Figure 10 shows the reservoir pressure distribution of the first geometric model at different exploration stages. The pressure scope continues expanding with the production time. After 1 year’s production, the pressure in the fracture becomes the same as the bottom-hole pressure, and the daily gas production reaches the peak. The gas in the reservoir is absorbed into the hydraulic fracture because of the diffusion mechanism. In the following years, the pressure diffusion did not change significantly compared with the previous years, indicating that the gas exploitation has reached the peak. Figure 11 shows the reservoir pressure profile along the cutting line A-B of the first geometric model in different exploration periods. The gas pressure is very high at the initial stage, but it gradually decreases and finally becomes equivalent to the bottom-hole pressure. In addition, the pressure at the hydraulic fracture drops rapidly, indicating that the hydraulic fracture can improve the reservoir performance very well.

5.1 Influence of the geometric parameters of hydraulic fractures

To analyze the influence of geometric parameters of hydraulic fractures on the gas production, we established two more geometric models (referred to as the second model and the third model). As Figure 12A shows, the fractures of the second model have the same geometric shape but different lengths (between 80 and 120 m) with the first model (Figure 9). The length of primary hydraulic fracture in the third model (Figure 12B) is the same as that in the first model, both of which are 120 meters. However, there are many secondary fracture networks, which are consistent with the complex fracture network generated in the actual situation. The total length of eight main hydraulic fractures in the three geometric models is the same.

The increase percentage of three geometric models on shale gas production is presented in Figure 13. The third model has the highest gas production. The gas production...
of the second mode was higher than that of the first mode. Although the total length of hydraulic fractures is the same, the longer hydraulic fractures have the larger the contact surface in the three-dimensional domain. This makes the longer hydraulic fractures have higher gas production rate. Because the hydraulic fractures are longer than the second model, the third models have higher fracture diversion capacity. The total length of the primary fractures is the same, while many secondary hydraulic fractures make fractures diversion capacity relatively stronger, which make the greater treatment area between two adjacent fractures. Therefore, under the same hydraulic conductivity, the wider fracture diversion capacity caused by the longer hydraulic fractures contains larger treatment area, which is the effective way to improve gas production efficiency.

5.2 Effect of fracture half-length

We simulated the dynamic changes of the horizontal well production, using the fracture half-length of 40, 80, 120, and 160 m, as shown in Figure 14. Gas production grows with the fracture half-length, indicating that the production is positively correlated with the fracture half-length. After 1200 days, the cumulative gas production of the half-lengths of 40, 80, 120, and 160 m is $6.49 \times 10^7$ m$^3$, $6.69 \times 10^7$ m$^3$, $7.01 \times 10^7$ m$^3$, and $7.56 \times 10^7$ m$^3$, respectively. Therefore, one of the effective ways to improve shale gas reservoir production can be increasing the half-length of hydraulic fractures.

5.3 Effect of fracture width

Another important parameter affecting the shale gas production is the hydraulic fracture width. Figure 15 simulates the relationship between the shale gas production and hydraulic fracture width. The daily gas production decreases rapidly in the initial stage and becomes stable gradually. The main reason is that large aperture fracture has high conductivity in a short time, which improves the production. However, the width of the hydraulic fracture should be no larger than 0.003 m for stable gas production.
Effect of fracture diversion capacity

The fluid supply capacity provided by the supporting fracture is called the fracture diversion capacity. We simulated the gas production of horizontal wells with the hydraulic fracture diversion capacity of 0.3 D cm, 0.6 D cm, 0.9 D cm, and 1.2 D cm as shown in Figure 16. After 1200 days, the cumulative gas production is $5.88 \times 10^7$ m³, $6.65 \times 10^7$ m³, $7.31 \times 10^7$ m³, and $7.53 \times 10^7$ m³ for the four hydraulic fracture diversion capacity. It shows that improving fracture conductivity can increase shale gas production, but the production growth tends to be slow, indicating that there is an optimal fracture diversion capacity. This is because the total amount of gas in the reservoir is constant under current conditions and will not change due to the different diversion capacity of fractures. Therefore, when the development enters the later stage, its production is gradually the same.

To find out which of the above factors has a more obvious effect on production, we also compared the contribution of the above hydraulic fractures three parameters (half-length, width, fracture diversion capacity) on the increment of the
cumulative gas production. The increase percentage of gas production (CIP) is defined as:

\[
CIP = \frac{\text{production@single change parameter} - \text{production@Basic parameters}}{\text{production@Basic parameters}} \times 100\% \quad (51)
\]

As Figure 17 shows, when the characteristic change value is 1, the half-length, fracture diversion capacity, and width of hydraulic fractures in 1200 days are 12.79%, 10.21%, and 4.33%, respectively. It is also found that the influence of fracture diversion capacity has a faster growth trend. Therefore, the best way to improve the shale gas production is to increase the half-length of hydraulic fractures, then the fracture diversion capacity, and finally the fracture width.

5.5  Effect of natural fracture

We also analyzed the influence of some properties of the natural fractures on gas production, including the initial natural permeability and natural fracture opening. As Figure 18 shows, the shale gas production grows with the increase of natural fracture permeability. When the permeability value reaches \(1 \times 10^{-16}\) m², the shale gas production tends to be stable, indicating that a proper initial permeability value brings high gas production, and too high-permeability does not improve the production significantly.

We also analyzed the relationship between the natural fracture opening and the gas production as shown in Figure 19. The natural fracture opening is positively correlated with production, which has obvious influence on the production. For example, after 600 days, the fracture opening of \(1 \times 10^{-6}\) m has the cumulative production of \(5.58 \times 10^7\) m³, while the fracture opening of \(1 \times 10^{-7}\) m has the cumulative production of \(6.20 \times 10^7\) m³.

The above analysis only considers the influence of a single factor on the output, and there are certain limitations in
this study. Therefore, according to the changes of initial permeability and opening of natural fractures, we adopted the orthogonal experimental design method to discuss the cumulative production of horizontal Wells under different parameter combinations, analyze the importance of each factor on the production, and obtain the optimal fracture parameter group. The factors affecting recovery test are shown in Table 3.

We compared the contributions of the three geometric models to gas production. The three models have natural fractures with different opening and the initial permeability. Figure 20A shows the production of 1200 days brought by the first and second models. The production of the second model is 7.21% higher than that of the first model, as it significantly improves the reservoir storage. It shows that the second model is applicable to the poor quality of the shale gas reservoir, such as the reservoir with the natural fracture opening of $1 \times 10^{-7}$ m and the initial permeability of $1 \times 10^{-18}$ m$^2$. As Figure 20B shows, the production of 1200 days of the third model is larger than that of the first model. Therefore, for shale gas reservoirs with high-permeability and fractures development well, increasing the length of hydraulic fracture can greatly improve the production.

6 | CONCLUSIONS

In this work, a fully coupled model of gas flow and effective stress in shale gas reservoirs composed of single-porosity medium zone, dual-porosity medium zone, and hydraulic fracture zone was established. The mechanisms of reservoir deformation, the gas multiscale adsorption, surface diffusion, and flow mechanisms in each zone were studied. Based on the theory of porous elastic medium, Fractal dimensions evolution, Darcy’s law, and the law of conservation of mass, a new apparent permeability model of matrix and fracture system was established which could well simulate the actual situation of shale gas reservoir. The influence of key fracture parameters on short- and long-term recovery of shale gas was discussed by using the three zones model. Based on these preliminary studies, the following conclusions can be drawn:

First, the new dual-porosity model of three zones after the surface diffusion and fractal theory is added to describe the multiscale mechanism of gas flow and deformation in different zones. The gas production rate at different stages reflects the different effective stress and flow properties in each zone.

Secondly, the related parameters of hydraulic fractures are the key factors affecting the production of shale gas reservoirs. By comparing the cumulative production growth rates of the parameters, it is found that increasing the hydraulic fracture half-length brings the highest growth of shale gas production, followed by increasing the fracture diversion capacity and width. In addition, natural fractures also have a great impact on shale gas production. The higher fracture permeability and wider opening are favorable for high production.

Finally, to further improve the production of shale gas reservoir, we can also change the geometry of hydraulic fracture. For reservoirs with large-fractured and relatively high-permeability, increasing the length of hydraulic fractures can bring higher production, while for reservoirs with micro-fractured and relatively low-permeability, more complex shapes hydraulic fractures can contribute to increase production.

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NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| $a$    | shape factor, m$^{-2}$ |
| $b$    | fracture opening between the two matrix blocks, m |
| $\Delta b$ | increment of fracture aperture, m |
| $c_m$ | Concentration of gas in the matrix blocks, kg/m$^{-3}$ |
| $c_f$ | Concentration of gas in the fractures, kg/m$^{-3}$ |
| $d_{hf}$ | width of the fracturing fracture, m |
| $d_p$ | pore diameter, nm |
| $d_m$ | collision diameter of gas molecules, nm |
| $D_f$ | a fractal coefficient of pore surface |
| $D_s$ | surface diffusion coefficient, m$^2$/s |
| $D_s^0$ | surface diffusion coefficient at zero gas coverage, m$^2$/s |
| $E$ | Young’s modulus of shale, MPa |
| $E_s$ | Young’s modulus of shale grains, MPa |
| $F_i$ | component of the body force in the i-direction, N |
| $G$ | shear modulus of shale, $G = E / (2 + 2v)$, MPa |
| $h$ | slip coefficient |
| $H$ | isosteric-adsorption heat, J/mol |
| $J_k$ | gas flux of Knudsen diffusion, kg/(m$^2$ s) |
| $J_s$ | flux of surface diffusion, kg/(m$^2$ s) |
| $J_v$ | modified gas flux of viscous flow, kg/(m$^2$ s) |
| $J$ | total flow flux of matrix, kg/(m$^2$ s) |
| $k_f$ | permeability of the fractures, m$^2$ |
| $k_m$ | permeability of the matrix, m$^2$ |
| $k_{hf}$ | permeability of hydraulic fractures, m$^2$ |
| $k_{s0}$ | permeability of single-porosity medium zone, m$^2$ |
| $k_{so}$ | intrinsic permeability of the matrix, md |
| $k_B$ | Boltzmann constant, J/K |
| $K$ | volume modulus of shale, $K = E / 3 (1 - 2v)$, MPa |
| $K_n$ | Knudsen number |
| $K_p$ | bulk modulus of pores, MPa |
| $K_s$ | bulk modulus of shale grains, $K_s = E_s / 3 (1 - 2v)$, MPa |
| $K_m$ | bulk modulus of shale skeleton, MPa |
| $m_g$ | gas quality of the matrix, kg/m$^3$ |
| $m_l$ | gas quality of the fracture, kg/m$^3$ |
| $M_g$ | molecular mass of methane, kg/mol |
| $N_A$ | the Avogadro constant, mol$^{-1}$ |
\( p_{0} \) initial pore pressure, Pa
\( p_{f} \) pressure in the fractures, Pa
\( p_{b} \) gas pressure in the single-porosity medium zone, Pa
\( p_{m} \) Pressure in the matrix blocks, Pa
\( p_{w} \) bottom-hole pressure, Pa
\( p_{L} \) Langmuir pressure constant, Pa
\( p_{r} \) pseudo-reduced pressure, Pa
\( q_{m} \) gas source, kg/(m².s)
\( R \) universal gas constant, J/(mol·K)
\( s \) length of the matrix block, m
\( T \) temperature, K
\( T_{r} \) pseudo-reduced temperature, K
\( V_{L} \) Langmuir volume constant, m³/kg
\( t \) time, s
\( Z \) gas compressibility factor

**Greek symbols**
- \( \Delta \varepsilon \) increment of volumetric strain
- \( \alpha \) sparse coefficient of the Knudsen function
- \( \alpha_{0} \) rarefaction coefficient at \( K_{n} \rightarrow \infty \)
- \( \alpha_{j} \) fitting constant
- \( \alpha_{s} \) a tangential momentum accommodation coefficient
- \( \beta \) dimensionless fitting constants
- \( \beta_{m} \) Effective stress coefficient for pore
- \( \beta_{f} \) Effective stress coefficient for fracture
- \( \delta \) ratio of gas molecules diameter to pore diameter, \( \delta = \frac{d_{m}}{d_{p}} \)
- \( u_{ij} \) derivative of displacement \( u_{i} \) with respect to \( j \) direction
- \( \varepsilon_{L} \) Langmuir volumetric strain constant
- \( \varepsilon_{s} \) sorption-induced swelling strain of the shales
- \( \varepsilon_{x} \) matrix\( \varepsilon_{x} = \frac{p_{m}V_{L}}{f_{L}} \left( \frac{p_{m} + p_{L}}{p_{m} + p_{L}} \right) \)
- \( \theta \) surface coverage, \( \theta = \frac{p_{L}}{p_{L} + p_{L}} \)
- \( \kappa \) a blockage parameter to express the degree of pore blocking
- \( \lambda \) molecular mean free path, nm
- \( \mu \) dynamic viscosity, Pa·s
- \( \nu \) the Poisson’s ratio of shale
- \( \rho_{m} \) gas density in the matrix, kg/m³
- \( \rho_{g0} \) density of gas under the standard conditions, \( \rho_{g0} = \frac{p_{0}M_{f}}{ZRT} \), kg/m³
- \( \xi_{m} \) a correction factor for surface diffusion for adsorbed gas
- \( \overline{\sigma} \) mean stress, \( \overline{\sigma} = -\sigma_{kk}/3 \), MPa
- \( \sigma_{kk} \) volumetric stress of shale matrix, \( \sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33} \), MPa
- \( \sigma_{ij} \) total stress tensor, MPa
- \( \tau \) tortuosity of pore channels
- \( \phi_{m} \) porosity of the matrix
- \( \phi_{f} \) porosity of the fractures
- \( \phi_{hf} \) porosity of the hydraulic fracture
- \( \phi_{p} \) porosity of single-porosity medium zone

**Subscripts**
- \( 0 \) Initial value of the variable
- \( m \) Matrix
- \( f \) fractures
- \( i, j \) index of tensor, equals to 1, 2 for 2D domain, equals to 1, 2, and 3 for 3D domain

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