A Theoretical Study on Nozzle Design for Gas–Particle Mixture Flow

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(Received on October 3, 1988; accepted in the final form on December 20, 1988)

The theoretical equations governing the nozzle flows of a gas–particle mixture was extended to the complicated case where a continuous distribution of particle sizes is present and some calculation examples were demonstrated in the previous paper. But, the governing equations were applicable only to the case where the gas-phase velocity is subsonic. Then, we have here, rewritten an equation to obtain the gas-phase velocity in the form different from the previous case so that the governing equations are valid to the whole gas velocity region including the supersonic flow. Thereby, a few sample calculations have been illustrated for supersonic mist flows composed of air and water–particles, which are commonly applied to the secondary cooling zone of continuously cast slabs. The results so obtained have been discussed from a numerical point of view. Also, it has been shown that the nozzle geometry can be designed in accord with the pressure profile given along a nozzle axis in advance.

KEY WORDS: gas–particle mixture; mist cooling; nozzle geometry; constant mass; slip ratio; mass flow rate; pressure profile; mist flow; equilibrium; nonequilibrium; supersonic flow.

1. Introduction

The so called two-phase flows are observed everywhere in nature. In reality, we can cite many familiar instances such as rain, snow, clouds, etc. Also, a number of processes in iron- and steelmaking industry positively introduce the utilization of two-phase or multiphase flows. In the blast furnace ironmaking, the pulverized coal injection method, the coal-oil mixture injection method and the coal-tar mixture injection method have been developed and now put into practical use. In addition, the steelmaking process in converter is basically multiphase reaction. In order to enhance the reaction, stirring of the molten steel bath is regarded as most effective, so that mechanical stirring, electromagnetic stirring and gas injection stirring have been adopted. All of the foregoing processes are concerned with two-phase or multiphase flows.

This paper is concerned with the nozzle flows of gas–particle mixture. The analysis of two-phase flow in a nozzle has some attractive aspects. First, the motion of particles aerothermodynamically interacts with that of gas-phase. Second, the particle-to-gas mass flow rate ratio takes a considerable influence on flow properties such as particle velocity, particle temperature, gas-phase velocity and pressure. Third, the difference in particle size has an appreciable effect on the ratio of particle velocity to gas velocity (slip ratio), when a continuous distribution of particle size is taken into consideration.

According to Zucrow and Hoffman,10 the system of equations governing the steady quasi-one-dimensional flow of a gas–particle mixture consists of particle continuity equation, particle momentum equation, particle energy equation, particle equation of state, gas continuity equation, gas momentum equation, gas energy equation, and gas equation of state. In the previous paper,5) we have modified the above system of equations in such a way as to give a fit to the case where particles have a continuous distribution of size. Then the nozzle flow of mist consisting of gas and liquid particle, which is commonly applied to the secondary cooling zone of continuously cast slabs, has been analyzed from a numerical point of view and it has been demonstrated how the phase nonequilibrium phenomena occur in the subsonic nozzle flow. To do so, the case has been premised where a gas containing suspended liquid particles is initially stored in a considerably large reservoir and the gas–particle mixture directly flows through a nozzle. However, the reservoir pressure is not permitted to be so high that the gas velocity reaches the sonic state in the throat region of a De Laval nozzle. That is, the system of equations described in Ref. 2) is singular in the transonic region.

In the present paper, we wish to extend the governing equations so that they enable us to determine the flow properties from the subsonic regions to the supersonic regions through the throat of a converging–diverging nozzle. This is not tedious in the calculation. The governing equations for the determination of all of the flow properties except the gas velocity are the same with the previous case (see Ref. 2)). Only the equation to obtain the gas velocity is different from the previous one in the form. That is, the equation is rewritten into the form including the term of the pressure gradient along the nozzle axis. Therefore, it is necessary to give the pressure distribution previously as the known parameter. Thereby, the
International steady flow. The no-interact material drag at a particle 

\[ p \equiv p(\xi), \quad \rho \equiv \rho(x), \]
\[ \tilde{p} \equiv \tilde{p}(\xi), \quad \tilde{\rho} \equiv \tilde{\rho}(x), \]
\[ \tilde{V} \equiv \tilde{V}(\xi), \quad \tilde{V} \equiv \tilde{V}(x), \]
\[ \tilde{V}_p \equiv \tilde{V}_p(\xi, r_p), \quad \tilde{V}_p \equiv \tilde{V}_p(x, r_p), \]
\[ \tilde{T} \equiv \tilde{T}(\xi), \quad \tilde{T}_p \equiv \tilde{T}_p(\xi, r_p), \quad \tilde{T}_p \equiv \tilde{T}_p(x, r_p). \]

We note that in the later the independent variable \( x \) will be omitted with minimum of inconvenience to descriptions such as \( V(x) \to V \).

\[ A_{\rho} = \frac{9}{2} \frac{\tilde{p} \tilde{L}_0}{\tilde{\rho} \tilde{p} \tilde{L}_0 \alpha} \quad \text{and} \quad \tilde{f}_p = \tilde{C}_D \tilde{C}_D \tilde{B}_\text{Stokes} \hspace{2cm} (7) \]

where, \( \tilde{C}_D \): the drag coefficients based upon the Henderson correlating equations \( \tilde{C}_D \tilde{B}_\text{Stokes} \to 2 \tilde{s}/\tilde{p} \): the Stokes law of drag coefficient for spheres.

\[ Re_p = \frac{Re_{p0} \tilde{r}_p |V - V_p(\tilde{r}_p)|}{T_p} \hspace{2cm} (8) \]
\[ \lambda = \frac{2\tilde{C}_D}{3Pr\tilde{C}_D}, \quad \tilde{g}_p = \frac{Na}{Na_{\text{Stokes}}}, \quad \theta = \tilde{C}_p/\tilde{C}_{\text{Stokes}} \hspace{2cm} (9) \]

where, \( Na \): the particle Nusselt number based on the empirical expression by Carlson and Hoglund

\[ Na_{\text{Stokes}} = 2 \]: the Nusselt number in the Stokes flow regime.

\[ V_p(r_p) \frac{d}{dx} V_p(r_p) = A_{\rho0} f_p \frac{T_p}{T_p} |V - V_p(r_p)| \hspace{2cm} (10) \]
\[ V_p(r_p) \frac{d}{dx} T_p = \lambda A_{\rho0} \tilde{g}_p \frac{T_p}{r_p} |T - T_p(r_p)| \hspace{2cm} (11) \]
\[ \tilde{m}_p(r_p) = \rho_p(r_p) A V_p(r_p), \]
\[ \tilde{M}_p = \tilde{\rho}_p(r_p) d \tilde{r}_p \hspace{2cm} (12) \]
\[ \rho V \frac{dV}{dx} + \nu_0 \int \rho_p(r_p) V_p(r_p) \frac{dV_p(r_p)}{dx} dr_p + \frac{1}{\tilde{g}} \left[ 0 T_p(r_p) \right] \hspace{2cm} (13) \]
\[ T = 1 + \frac{M_p}{\tilde{M}_p} \hspace{2cm} \text{and} \hspace{2cm} \tilde{M}_p \hspace{2cm} (14) \]
\[ \rho_p = \rho / T \hspace{2cm} (15) \]
\[ \tilde{M}_p = \rho AV \hspace{2cm} (16) \]

Besides these equations, \( \tilde{p} \) is given in the form of function of nozzle axis \( x \), that is,

\[ \tilde{p} = f(x) \hspace{2cm} (17) \]
To complete the system of Eqs. (10) to (17), another equation should be introduced for the determination of gas velocity \( V \). In the previous paper,\(^5\) the next equation,

\[
\frac{dV}{dx} = -\frac{M^2}{1-M^2} \left\{ \frac{T}{V} \frac{dA}{dx} - \frac{\nu_0 \bar{A}_{\rho} T^2}{\rho} \int_{\rho} \frac{p_\rho(r_p)}{r_p^2} \right. \\
\times \left( \frac{V-V_p(r_p)}{V(r_p)} \right) + \int_{\rho} \frac{\phi(\bar{g}_p)}{r_p^2} \right\} V \int_{\rho} \frac{p_\rho(r_p)}{r_p^2} dr_p
\]

has been used. The above equation is clearly singular at \( M=1 \). Therefore, it is not applicable to the case where \( M \neq 1 \) in a nozzle.

Then, combining Eq. (10) and Eq. (13), we have

\[
\frac{dV}{dx} = -\frac{\nu_0}{\rho V} \int_{\rho} \frac{p_\rho(r_p)}{r_p^2} \int_{\rho} \frac{T^2}{T^2} \left[ V-V_p(r_p) \right] dr_p \\
- \frac{1}{\bar{r}_p V} \frac{dp}{dx}
\]

Here, the gas velocity \( V \) is calculated by Eq. (19) instead of Eq. (18).

2.4. Numerical Treatment

Among the equations mentioned above, Eqs. (10) to (16) comprise a set of 7 equations for determining 7 flow properties \( \rho, T, V, \rho_\rho(r_p), V_p(r_p), T_p(r_p), \) and \( A \). These equations should be combined in a form appropriate for seeking a numerical solution. However, the initialization is very tedious at the start of numerical calculation.

Let us suppose that a gas containing suspended liquid-particles is initially stored in an appreciably large reservoir, and that the gas-particle mixture directly flows through a nozzle. In this case, it is far more common to consider that the gas velocity as well as particle velocity is zero at the reservoir. Hence, the initial situation corresponds to the assumption that the sectional area of nozzle is infinite at its entrance. So, the problem remains how the numerical treatment should be made at the first computational step.

When \( V_p(r_p) = V \) and \( T_p(r_p) = T \) for all \( r_p \), the particles are commonly said to be in velocity and thermal equilibrium with the gas. Also, such a mixture flow is called the equilibrium flow. While the flow obeying the Eqs. (10) to (16) is termed the nonequilibrium flow.

Then, it is assumed that all of the particles are in velocity and thermal equilibrium with the gas only near the reservoir so that the gas as well as all of the particles may take a non-zero velocity at the entrance of the nonequilibrium region. In short, one should bear in mind that the initial point of nonequilibrium region, in a numerical sense, corresponds to the end point of equilibrium flow, and that the aforementioned system of equations governing the nonequilibrium flow is solved as a perturbation from an equilibrium reference flow.

Hence, we wish to describe the problem of the constant lag approximation. Let us now define the following two lag factors,

\[
K_p(r_p) = \frac{V_p(r_p)}{V}
\]

and

\[
L_p(r_p) = \frac{1-T_p(r_p)}{1-T}
\]

As can be understood from the numerical procedure mentioned above, it holds true in the equilibrium region that \( K_p(r_p) = 1 \) and \( L_p(r_p) = 1 \) for all \( r_p \). While \( K_p(r_p) \) as well as \( L_p(r_p) \) is not constant in the non-equilibrium region, but variable along the nozzle axis \( x \). In other words, \( K_p(r_p) = \bar{K}_p(x, r_p) \) and \( L_p(r_p) = \bar{L}_p(x, r_p) \). In fact, the results of an equilibrium analysis for the case where \( 1-K_p = 1-L_p = 0 \) is an upper limit for an actual flow, and the frozen analysis for the case where \( 1-K_p = 1-L_p = 1 \) is a lower limit. One should keep in mind that the actual values of the flow properties must lie between the values for such two limiting situations.

For the general case of constant lags including the equilibrium flow, we have

\[
\bar{p}p = \frac{1}{\rho_0 \bar{p} \bar{t}} = 1
\]

Here, \( \bar{t} \) is derived as follows:

\[
\bar{t} = \frac{\gamma(1+\Gamma_1)(1+\Gamma_2)}{\gamma(1+\Gamma_1)(1+\Gamma_2) - (\gamma-1)(1+\Gamma_2)}
\]

where,

\[
\Gamma_1 = \frac{\nu_0}{\rho_0} \int_0^\infty \left[ \phi(\bar{g}_p) \right] \frac{e^{2r_2}K_\rho^2(r_p)dr_p}{r_p^2}
\]

\[
\Gamma_2 = \frac{\nu_0}{\rho_0} \int_0^\infty \left[ \phi(\bar{g}_p) \right] \frac{e^{2r_2}K_\rho^2(r_p)dr_p}{r_p^2}
\]

\[
\Gamma_3 = \frac{\nu_0}{\rho_0} \int_0^\infty \left[ \phi(\bar{g}_p) \right] \frac{e^{2r_2}K_\rho^2(r_p)dr_p}{r_p^2}
\]

Again, in the region from the reservoir up to the end point of the equilibrium flow both the particle velocity lag and the thermal lag are here taken to be zero (i.e., \( K_p(r_p) = 1 \) and \( L_p(r_p) = 1 \) for all \( r_p \)). All of the flow properties in the equilibrium region can be calculated by

\[
\rho = \bar{p} \bar{t} \bar{\rho}
\]

\[
T = \bar{p} \bar{t} \bar{\rho}
\]

\[
V = \left( \frac{2(1-T_1)}{1-T_1} \right)^{1/2}
\]

\[
M_\rho = \rho AV
\]

Finally, we should like to note that in our calculation, the position where \( \bar{p} \) is somewhat arbitrarily 0.998 is selected as the initial point of nonequilibrium region and corresponds to \( x = -110 \). In passing, the Runge-Kutta method is used to find the numerical solutions.

3. Numerical Experiments

A few sample calculations will be demonstrated on the basis of the theoretical prescription mentioned above. In the present numerical calculation, a gas-
particle mixture composed of air and water-particles is treated in relation to the nozzle flow of mist applied to the secondary cooling zone of continuously cast slabs.

The physical constants as well as the reference conditions adopted in this numerical experiment are listed in Table 1. These parameters are fixed throughout the present paper.

In the numerical experiments, let it be assumed that the continuous size distribution function of water-particles mixing in the mist is the same with the case of the previous paper:

\[ \phi_d(r_p) = \frac{J r_p}{250} \exp \left( - \frac{r_p}{250} \right) \]

in which the normalizing factor \( J \) has been found to be 1/12,580 (\( \mu \)m\(^{-2} \)). Also, the minimum and the maximum radii of the particle contained in the mixture are specified as

\[ r_{p, \min} = 1.0 \, (\mu \text{m}) \quad \text{and} \quad r_{p, \max} = 50.0 \, (\mu \text{m}) \]

The average radius \( L_{\rho 0} = 17.396 \, (\mu \text{m}) \). For the convenience of the numerical treatment, the continuous distribution of the particle size from 1.0 to 50.0 (\( \mu \)m) is represented by 11 discrete sizes so that the size interval is constant (4.9 (\( \mu \)m)). Each size is made dimensionless on the basis of the definition by

\[ r_p(i) = [r_{p, \min} + 4.9(i-1)]/L_{\rho 0} \quad (i = 1 \sim 11) \] (30)

According to Eq. (1), the dimensionless distribution function of particle size is easily expressed as

\[ \phi_d[r_p(i)] = 2.4288r_p(i) \exp [-1.2103r_p(i)] \quad (i = 1 \sim 11) \] (31)

Again, the above function is, of course, normalized so that the value of definite integral in \([r_{p, \min}, r_{p, \max}]\) takes unity.

Next, we must give a pressure profile, as expressed in Eq. (17), in the form of a function of \( x \). In principle, the pressure profile is permitted to be arbitrarily determined as one desires. First, we wish to treat the case where \( p_0 = 1.0 \times 10^5 \) (Pa) at the reservoir and \( p_e = 1.0 \times 10^5 \) (Pa) at the nozzle exit. Second, we wish to adopt a pressure profile such that \( p \) takes 0.528 (= \( p_e/p_0 = [(r+1)/2]^{1/2} \)) at \( x = 0 \) for gas-only flow and \( p \) at the nozzle exit (at \( x = X_e \)) becomes equal to the ambient gas pressure \( (p_e = 1.0 \times 10^5 \) (Pa) or \( p_e = 0.1 \), and also \( p \) reaches unity as \( x \to x_0 \). Hence, let us adopt the following pressure profile,

\[ p = -\frac{1}{2} \left( \frac{g(x)}{2} \right)^{2/3} + 0.5; \]

\[ g(x) = k_1 x + k_2 \] (32)

in which \( k_1 = 0.13894 \) and \( k_2 = -0.05609 \). We note that the position of \( x = 0 \) corresponds to that of the throat for gas-only flow and that the position of \( x = X_e \) corresponds to that of the nozzle exit. Fig. 1 indicates the relationship of \( p \) with \( x \).

Fig. 2 shows the nozzle geometry along the nozzle axis, that is, the variation of \( L \) with \( x \), with \( \nu \) as a parameter. Here, it should be remarked that \( \nu = 0 \) corresponds to single-phase flow (gas-only flow), and that the position of the throat \( (A_x = 1) \) does not correspond to \( x = \nu \) in the case of \( \nu \). The throat tends to be located at the negative side of \( x \) with the increase in \( \nu \) (at \( x = -0.223 \) for the case of \( \nu = 1 \), at \( x = -0.331 \) for \( \nu = 2 \) and at \( x = -0.397 \) for \( \nu = 3 \)). It can be understood from this figure that there seems to be no significant difference in the nozzle size of the convergent part regardless of the magnitude of \( \nu \), but the nozzle diameter expands in the divergent part with the increase in \( \nu \).

Figs. 3(a) to 3(c) indicate the variation of the

![Fig. 1. Pressure profile along nozzle axis to be introduced to numerical experiments. Note that this is in accord with Eq. (32).](image)

![Fig. 2. Nozzle geometry along nozzle axis. The indication of \( \nu = 0 \) corresponds to gas-only flow. Note that there seems to be no significant difference in the nozzle size of the convergent part regardless of the magnitude of \( \nu \), but \( L \) is extended in the divergent part as \( \nu \) is increased.](image)
density, temperature and velocity, respectively, of gas-phase along the nozzle axis $x$ with $\nu$ as a parameter. It is noted that the indication of these flow properties is in exact accord with that of the nozzle geometry shown in Fig. 2. It can be seen from these figures that the decreasing behavior in $\rho$ and $T$ with $x$ is smooth. In case where $\nu=0$, $T=0.833$ and $\rho=0.634$ at $x=0$. The values agree with these calculated from $T=2/(\gamma+1)$ and $\rho=\gamma\nu^\nu$. $\rho$ of the case where $\nu\neq0$ is small in comparison with the case of gas-only flow (see Fig. 3(a)), and $T$ indicates the opposite (Fig. 3(b)). Again, the gas velocity is seen to increase monotonously from the reservoir to the nozzle exit. But, with increasing $\nu$ the gas velocity becomes small due to the drag force exerted by all of the particles on the gas. Fig. 4 gives the variation of the local Mach number $M(V/V\sqrt{T})$ of the gas-phase along the nozzle axis with $\nu$ as a parameter. The case where $\nu=0$ exhibits $M=1$ at the throat. But, for the case where $\nu\neq0$, the position of $M=1$ is observed to be located downstream of the throat, and with the increase in $\nu$ such a tendency presents itself in a more explicit way.

Figs. 5(a) to 5(c) give the variation of the particle velocity $V_p(i_{\nu}(i))$ ($i$: size number, see Eq. (30)) along the nozzle axis with $i$ as a parameter. It can be observed from these figures that the particle velocity is increased smoothly with $x$, and particles with smaller size tend to travel at the higher velocity. In addition, the increase in $\nu$ leads to decreasing the particle velocity, although it is not so remarkable in the present range of $\nu$.

Figs. 6(a) to 6(c) exhibit the variation of the slip ratio, that is, the ratio of the particle velocity to the gas velocity, defined by Eq. (20), along the nozzle axis with the particle size as a parameter. The slip ratio is higher as the particle size becomes smaller. Again, the ratio is slightly variable along the nozzle axis: It can be seen that the ratio is decreased in the region of $M<1$ and the opposite occurs in the region of $M>1$. Furthermore, it should be emphasized that the difference in $\nu$ gives not so significant change to the slip ratio.

Figs. 7(a) to 7(c) present the variation of the thermal lag factor $L_\theta(i_{\nu}(i))$, defined by Eq. (20), with $x$. It can be said that the difference in $\nu$ brings about not so appreciable change in the thermal lag factor. This is the same with the case of the slip ratio.

Fig. 8 gives the variation of the total number of particles per unit volume $N_p$ with $x$. We find that $N_p$ increases towards the nozzle throat, takes the peak value near the end of convergent part of nozzle, and thereafter decreases towards the nozzle exit.

4. Discussion

According to the theoretical procedure described in the second section the sample calculations have been demonstrated for the nozzle flow of mist con-
sisting of air and water-particles. Here, the gas velocity \( V \) has been calculated from Eq. (19) instead of Eq. (18). Thereby, it has been illustrated that all of the flow properties from the subsonic region to the supersonic region through the transonic region can be calculated according to the pressure profile given previously along the nozzle axis.

First, we wish to discuss the comparison between Eqs. (18) and (19). The former involves the term concerning the nozzle geometry. Therefore, the determination of \( V \) from Eq. (18) requires to give the nozzle configuration in advance. The latter involves the term concerning the pressure profile along the nozzle axis. So, in order to obtain \( V \) by Eq. (19), \( p \) must previously be given over the whole length of nozzle axis. The sectional area of nozzle is uniquely determinable in the form corresponding to the given pressure profile. Eq. (18) is essentially applicable to the subsonic nozzle flow regime, as been demonstrated in the previous paper.\(^3\) Or, if the region from the reservoir up to a certain position of \( M>1 \) (say, \( M=1.05 \)) is allowed to be regarded as the equilibrium region, Eq. (18) is applicable to the determination of the supersonic velocity in the subsequent region.

Fig. 5. Variation of particle velocity along nozzle axis with size number, \( i \), as a parameter for \( \nu=1.0 \) (a), \( \nu=2.0 \) (b) and \( \nu=3.0 \) (c).

Fig. 6. Ratio of particle velocity to gas velocity (slip ratio) along nozzle axis for \( \nu=1.0 \) (a), \( \nu=2.0 \) (b) and \( \nu=3.0 \) (c) (\( i \); size number). Note that the difference in \( \nu \) exhibits almost no influence on the slip ratio, but the difference in particle size has a considerable influence on it.
However, errors to an appreciable degree seem to be unavoidable in the above numerical procedure. This is because the velocity lag factor as well as the thermal lag factor is significantly smaller even in the region of Mach number less than unity, so far as the particle size is not very small (see Figs. 6 and 7). Although Eq. (18) as well as Eq. (19) is consistent with each other from an aerodynamic point of view, we should like to emphasize that Eq. (19) has no singularity over all Mach number ranges.

In this sense, the use of Eq. (19) is convenient in that the nozzle configuration can be obtained in obedience to the desired pressure profile, for example, as given in the form of Eq. (32). Fig. 9 shows the nozzle geometry calculated by the pressure profile different from the case of Fig. 1. The other computational conditions are the same with the previous case. Comparing the pressure profile shown in Fig. 9 with that of Fig. 1, there is observed the appreciable difference between the two. It should be noted that the nozzle configurations so obtained, therefore, are different from each other (see Figs. 2 and 9).

Next, we wish to examine the relation between $\dot{M}_p$ and $\nu$. It should be noted that $\dot{M}_p$ decreases with the increase in $\nu$. In the present numerical experiment $\dot{M}_p=0.4918$ at $\nu=1$, $\dot{M}_p=0.4360$ at $\nu=2$ and $\dot{M}_p=0.3948$ at $\nu=3$. From Eq. (4), we have

$$\nu = \frac{\dot{M}_p}{\dot{M}_q} = \frac{\dot{M}_p}{\dot{M}_p} = \frac{R_p \rho A}{\dot{M}_p} = \frac{M_p}{R_p \rho_0}$$

in the equilibrium region ($K_p(r_p)=1$). Therefore, it has been proved that $\dot{M}_p=M_p$ in the dimensionless space. But, it is, of course, not true that $\dot{M}_p=M_p$ in

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**Fig. 7.** Variation of thermal lag factor along nozzle axis with particle size as a parameter for $\nu=1.0$ (a), $\nu=2.0$ (b) and $\nu=3.0$ (c).

**Fig. 8.** Distribution of total number of particles per unit volume along $x$-axis with $\nu$ as a parameter.

**Fig. 9.** Nozzle configurations calculated by the pressure profile different from the case of Fig. 1 with $\nu$ as a parameter. Note that the difference in pressure profile gives an appreciably significant change to the nozzle geometry (compare this with that of Fig. 2).
the dimensioned space. From Eq. (4), we have
\[ \dot{M}_g = \dot{M}_p \rho_g A_\phi (\gamma R T_p)^{\nu/\alpha} \]
and \[ \dot{M}_p = \nu \dot{M}_g \]
So, \[ \dot{M}_p = 0.001501 \text{ (kg/s)} \] at \( \nu = 1 \),
\[ \dot{M}_p = 0.002662 \text{ (kg/s)} \] at \( \nu = 2 \),
\[ \dot{M}_p = 0.003615 \text{ (kg/s)} \] at \( \nu = 3 \).

Finally, we wish to discuss the problem of phase-transformation from liquid particles to solid ones. According to Fig. 3(b), the gas temperature is decreased to a considerable degree as \( x \to X_g \) (=10). That is, \( T \) falls below the freezing point: In case where \( \nu = 1 \), \( T_p \approx 0.6 \) (\( T_p = 194 \text{ K} \)) at \( x = X_g \). So, the problem remains whether or not the phase-transformation of particles occurs. Figs. 10(a) to 10(c) show the variations of \( T_p \) against \( x \), with \( i \) as a parameter. It can be seen from the figure that water–particles with a smaller size attain a lower temperature state at the nozzle exit, and that such a tendency becomes slightly conspicuous with the decrease in \( \nu \). In case where \( \nu = 1 \), the particles of the size number \( i = 1 \) are markedly cooled at \( T_p \approx 0.87 \) (\( T_p = 281 \text{ K} \)) at \( x = X_g \) (=10). No phase-transformation narrowly occurs in the present numerical experiments.

If the water–particles are transformed into the icy ones, various phenomena must occur: the generation of latent heat of solidification, the expansion of particle volume, the change in particle-phase and gas-phase velocities due to the phase-transformation, etc. Presumably, such phenomena may be unfavorable to controlling the cooling intensity for the solidified shell from a point of view of the nozzle applied to the secondary cooling zone of continuously cast slabs. For example, the volume expansion of particles due to the phase-transformation can cause the choking of nozzle outlet.

5. Conclusion

The theoretical equations governing the nozzle flows of a gas–particle mixture have been extended to the case where they enable us to determine the flow properties in the whole region from the subsonic to the supersonic region through the throat of a converging–diverging nozzle. That is, the equation to determine the gas-phase velocity has been rewritten into the form including the term of the pressure profile along the nozzle axis instead the term concerning the cross sectional area of nozzle. Therby the sample calculations have been demonstrated for the nozzle flow of mist composed of air and water–particles. We have suggested that the gas velocity from the subsonic to the supersonic flows through the transonic region as well as the configuration of a converging–diverging nozzle can be obtained in the theoretically clear form. In the present numerical experiment, it has been understood that there seems to be no significant difference in the nozzle geometry of the convergent part regardless of the magnitude of loading ratio, but the nozzle diameter expands in the divergent part with the increase in the loading ratio.

In conclusion, we note that the theoretical procedure mentioned here has an attractive feature in that the nozzle configuration can be obtained in accord with the desired pressure profile along the nozzle axis.

Nomenclature

- \( a \): local sonic speed
- \( A \): dimensionless sectional area of nozzle
- \( A_{\phi} \): dimensionless friction factor defined in Eq. (7)
- \( C_D \): drag coefficient
- \( C_{pp} \): gas specific heat at constant pressure
- \( C_{pp} \): specific heat of particle material
\( f_p \): momentum transfer parameter defined in Eq. (7)

\( g_p \): heat transfer parameter defined in Eq. (9)

\( K_p \): velocity lag factor defined in Eq. (20)

\( l_p \): average particle radius

\( I_p \): thermal lag factor defined in Eq. (20)

\( L \): radius of nozzle

\( \dot{m}_p \): dimensionless mass flow rate function of particle \((\dot{m}_p = \dot{m}_p(r_p))\)

\( M \): gas-phase Mach number

\( \dot{M}_p, M_p \): dimensionless total mass flow rate of gas and particle, respectively, defined in Eq. (4)

\( n_p \): dimensionless number density function of particle \((n_p = n_p(r_p))\)

\( N_p = \int n_p(r_p)\,dr_p = \dot{N}_p/\dot{N}_p^0 \) (see Eq. (3))

\( N_p^0 \): particle Nusselt number

\( P \): dimensionless pressure

\( Pr \): gas-phase Prandtl number

\( \dot{R}_p \): particle Reynolds number defined in Eq. (8)

\( r_p \): dimensionless particle radius \((=\rho_p/\rho_p^0)\)

\( R_p \): density of condensed particle per unit volume of flowing medium

\( r_{p, \text{max}} \): maximum radius of particle

\( r_{p, \text{min}} \): minimum radius of particle

\( T, T_p \): dimensionless gas- and particle-phase temperatures \((T_p = T_p(r_p))\), respectively

\( V, V_p \): dimensionless gas- and particle-phase velocities \((V_p = V_p(r_p))\), respectively

\( x \): dimensionless coordinate along nozzle axis

\( \gamma \): gas specific heat ratio

\( \dot{\gamma} \): modified gas specific ratio defined in Eq. (22)

\( \Gamma_1, \Gamma_2, \Gamma_3 \): specified parameters to determine \( \dot{\gamma} \) defined in Eqs. (23), (24) and (25), respectively

\( \delta \): exponent in the viscosity-temperature equation (8)

\( \theta = C_{pp}/C_{pp}^0 \)

\( \lambda = 2C_{pp}^0/(3\rho C_{pp}^0) \)

\( \overline{\rho}_p \): gas viscosity

\( \nu = (M_p/M_p^0\rho) \)

\( \nu_p = R_p/\rho_p^0 \) defined in Eq. (5)

\( \rho \): dimensionless gas density

\( \rho_{sp} \): particle material density

\( \rho_p \): dimensionless particle density function \((\rho_p = \rho_p(r_p))\)

\( \phi \): dimensionless continuous distribution function of particle sizes

Superscript

\(^\sim \): dimensioned quantity

Subscripts

0: quantity in reservoir or stagnation state

E: quantity at nozzle exit

* : quantity at nozzle throat

g: gas-phase

p: particle-phase

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