Modular Inflation and the Curvaton

George Lazarides

Physics Division, School of Technology, Aristotle University of Thessaloniki,
GR-54124 Thessaloniki, Greece

Abstract. Supersymmetric Peccei-Quinn models which provide a suitable candidate for the curvaton field are studied. These models also solve the $\mu$ problem, while generating the Peccei-Quinn scale dynamically. The curvaton is a pseudo Nambu-Goldstone boson corresponding to an angular degree of freedom orthogonal to the axion. Its order parameter increases substantially following a phase transition during inflation. This results in a drastic amplification of the curvaton perturbations. Consequently, these models are able to accommodate low-scale inflation with Hubble parameter at the TeV scale such as modular inflation. We find that modular inflation with the orthogonal axion as curvaton can indeed account for the observations for natural values of the parameters. In particular, the spectral index can easily be made adequately lower than unity in accord with the recent data.

Keywords: Inflation, curvaton

PACS: 98.80.Cq

INTRODUCTION

The precise cosmological observations of the last decade have established inflation (for a review on inflation, see e.g. [1]) as an essential extension of the hot big bang model. However, the case of slow-roll inflation (i.e. the case with inflaton mass $\ll H^*$, the Hubble parameter at the time when the cosmological scales exit the horizon during inflation) suffers from the fact that, typically, supergravity (SUGRA) introduces [2, 3] corrections to the inflaton mass of order $H^*$. One way to keep the inflaton mass under control is to use as inflaton a pseudo Nambu-Goldstone boson (PNGB) field, since the flatness of the potential of such a field is protected by a global $U(1)$ symmetry. Such candidates are [4] the string axions, which are the imaginary parts of string moduli fields with the flatness of their potential lifted only by (soft) supersymmetry (SUSY) breaking. This results in inflaton masses $\sim H_*$. The resulting modular inflation can be of the fast-roll type [5]. Fast-roll inflation lasts only a limited number of e-foldings, which, however, can be enough to solve the horizon and flatness problems.

In modular inflation, $H_* \sim 1$ TeV and, thus, the inflationary energy scale is much lower than the grand unified theory (GUT) scale. As a result, the perturbations of the inflaton are not sufficiently large to account for the required density perturbations for explaining the large scale structure in the universe and the temperature perturbations in the cosmic microwave background radiation (CMBR). (For a low-scale inflation model where the inflaton perturbations are adequate, see [6].) Thus, a curvaton [7] (see also [8]), i.e. another “light” field during inflation, is necessary to provide the observed curvature perturbation. However, even the curvaton cannot [9] generically help us to reduce the inflationary scale to energies low enough for modular inflation. This is possible only in certain curvaton models which amplify [10, 11] additionally the curvaton perturbations.
We will construct [12] a class of SUSY Peccei-Quinn (PQ) models [13] which possess such an amplification mechanism and also solve naturally the strong $CP$ and $\mu$ problems. We use as curvaton an angular degree of freedom orthogonal to the QCD axion, which we will call orthogonal axion (the radial PQ field was used as curvaton in [14]). We study the characteristics of the scalar potential in this class of models. We then focus on curvaton physics and derive a number of important constraints necessary for the model to be a successful curvaton model. Finally, we concentrate on a concrete example of this class of models and show that it can indeed work for natural values of parameters.

**MODULAR INFLATION**

After gravity mediated soft SUSY breaking, the potential of the inflaton $s$, which is a canonically normalized string axion, is [4]

$$V(s) = V_m - \frac{1}{2} m_s^2 s^2 + \cdots,$$

where $V_m \sim (m_{3/2} M_P)^2$ and $m_s \sim m_{3/2}$ with $m_{3/2} \sim 1$ TeV and $M_P \approx 2.44 \times 10^{18}$ GeV being the gravitino mass and the reduced Planck mass respectively. The ellipsis in (1) denotes terms which are expected to stabilize $V(s)$ at $s \sim M_P$. The inflationary potential $V_*$ at the time when the cosmological scales exit the horizon is of intermediate scale:

$$V_*^{1/4} \sim \sqrt{m_{3/2} M_P} \sim 10^{10.5} \text{ GeV}$$

for which $H_* \sim m_{3/2}$.

In this model, inflation can be of the fast-roll type, where

$$s = s_i \exp(F_s \Delta N) \quad \text{with} \quad F_s \equiv \frac{3}{2} \left( \sqrt{1 + \frac{4c}{9}} - 1 \right), \quad c \equiv \left( \frac{m_s}{H_*} \right)^2 \sim 1.$$ 

Here, $\Delta N$ is the number of the elapsed e-foldings and $s_i$ the initial value of the inflaton field $s$. From the above, one can obtain the inflation potential $N$ e-foldings before the end of inflation as

$$V(N) \simeq V_m \left( 1 - e^{-2F_s N} \right).$$

Even in the fast-roll case, modular inflation keeps the Hubble parameter $H$ rather rigid. Indeed, the slow-roll parameter

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2} c^2 \left( \frac{s}{M_P} \right)^2 \ll 1,$$

because $c \sim 1$ and $s \ll M_P$ during inflation (the overdot denotes derivative with respect to the cosmic time $t$).

The initial conditions for the inflaton field are determined by the quantum fluctuations which send the field off the top of the potential hill. Hence, we expect that the initial
value for the inflaton is

\[ s_i \simeq \frac{H_m}{2\pi}, \quad \text{where} \quad H_m \simeq \frac{\sqrt{V_m}}{\sqrt{3}M_P}. \]  

(6)

Assuming that the final value of \( s \) is close to its vacuum expectation value (VEV) \( \langle s \rangle \sim M_P \), we find, from (3), that the total number of e-foldings is

\[ N_{\text{tot}} \simeq \frac{1}{F_s} \ln \left( \frac{M_P}{m_{3/2}} \right), \]  

(7)

where we took into account that \( H_m \sim m_{3/2} \).

### AMPLIFICATION OF THE CURVATON PERTURBATIONS

We consider a PNGB curvaton \( \sigma \) whose order parameter \( v = v(t) \) (determined by the values of the radial fields in the model) takes [10, 11] a different (larger) expectation value in the vacuum than during inflation and, in particular, when the cosmological scales exit the horizon. The potential for the real canonically normalized field \( \sigma \) is

\[ V(\sigma) = (v\tilde{m}_\sigma)^2 \left[ 1 - \cos \left( \frac{\sigma}{v} \right) \right] \Rightarrow V(|\sigma| \ll v) \simeq \frac{1}{2}\tilde{m}_\sigma^2\sigma^2, \]  

(8)

where \( \tilde{m}_\sigma = \tilde{m}_\sigma(v) \) is the variable mass of \( \sigma \). In the vacuum, \( v = v_0 \) and \( \tilde{m}_\sigma = m_\sigma \).

In the curvaton scenario, the curvature perturbation observed by the cosmic microwave background explorer (COBE) [15], \( \zeta \simeq 2 \times 10^{-5} \), is given by

\[ \zeta \sim \Omega_{\text{dec}} \zeta_\sigma, \]  

(9)

where \( \zeta_\sigma \) is the partial curvature perturbation of the curvaton and \( \Omega_{\text{dec}} \) is the ratio of the curvaton energy density \( \rho_\sigma \) to the total energy density of the universe \( \rho \) at the time of the decay of the curvaton:

\[ 10^{-2} \lesssim \Omega_{\text{dec}} \equiv \left. \frac{\rho_\sigma}{\rho} \right|_{\text{dec}} \leq 1. \]  

(10)

The lower bound originates (see [16]) from the 95\% confidence level bound on the possible non-Gaussian component of the curvature perturbation from the CMBR data obtained by the Wilkinson microwave anisotropy probe (WMAP) satellite [17]. The partial curvature perturbation of the curvaton when the latter oscillates in a quadratic potential is given [18] by

\[ \zeta_\sigma \sim \left. \frac{\delta \sigma}{\sigma} \right|_{\text{dec}} \sim \left. \frac{\delta \sigma}{\sigma} \right|_{\text{osc}}, \]  

(11)

where “osc” denotes the onset of curvaton oscillations at a time given by \( H_{\text{osc}} \sim m_\sigma \) (we assume that \( \tilde{m}_\sigma \) had reached its vacuum value before the onset of oscillations).
The phase $\theta \equiv \sigma / v$ corresponding to $\sigma$ remains frozen until the oscillations begin:

$$\theta_{\text{osc}} \simeq \theta_*, \quad \delta \theta_{\text{osc}} \simeq \delta \theta_*,$$

where star denotes the values of quantities at the time when the cosmological scales exit the inflationary horizon and $\delta \theta$ is the perturbation in $\theta$. This implies that

$$\delta \sigma \bigg|_{\text{osc}} \simeq \sigma \bigg|_{\text{osc}} \simeq \delta \theta \bigg|_{\text{osc}} \simeq \delta \sigma \bigg|_*, \tag{13}$$

From the curvaton perturbation during inflation $\delta \sigma_*=H_* / 2\pi$, we then find [12] that

$$\delta \sigma_{\text{osc}} \simeq H_* \left( \frac{1}{2\pi \epsilon} \right)^{1/n+1}, \quad \text{where} \quad \epsilon \equiv \frac{v_*}{v_0} \ll 1. \tag{14}$$

So, after the end of inflation when $v$ assumes its vacuum value, the curvaton perturbation is amplified by a factor $\epsilon^{-1}$. Finally, one can show [12] that

$$\sigma_{\text{osc}} \sim \frac{H_* \Omega_{\text{dec}}}{\pi \epsilon \zeta}, \quad \text{and} \quad \epsilon \geq \epsilon_{\text{min}} \equiv \frac{H_*}{2\pi v_0}, \tag{15}$$

where we used the relations $\delta \sigma_*/\sigma_* \leq 1$ and $\sigma_{\text{osc}} \lesssim v_0$.

**CAN WE CONSTRUCT PQ MODELS WITH A PNGB CURVATON?**

In the SUGRA extension of the minimal supersymmetric standard model (MSSM), there exist D- and F-flat directions in field space which can generate intermediate scales

$$M_1 \sim (m_3/2 M_P^{1/3})^{1/n}, \tag{16}$$

where $n$ is a positive integer. It is natural to identify $M_1$ with the symmetry breaking scale $f_a$ of the PQ symmetry $U(1)_{PQ}$, such that a $\mu$ term is generated with $\mu \sim f_a^{n+1}/M_P^n \sim m_{3/2}$ [19]. This would simultaneously resolve the strong CP and $\mu$ problems of MSSM.

For this, we need a non-renormalizable superpotential term

$$\lambda P^{n+1} h_1 h_2 / M_P^n, \tag{17}$$

where $\lambda$ is a dimensionless parameter, $P$ is a standard model (SM) singlet superfield with $\langle P \rangle \sim M_1$ and $h_1, h_2$ are the electroweak Higgs doublets. One can show [12] that $P$ must necessarily carry a non-zero PQ charge. As a consequence, $P$ has a flat potential.

To lift the flatness of its potential and generate an intermediate VEV for $P \sim M_1$, we must introduce [20, 21, 22] a second SM singlet $Q$ with non-zero PQ charge having a coupling of the type

$$\xi P^{n+3-k} Q^k / M_P^n, \tag{18}$$

where $\xi$ is a dimensionless parameter and $k$ is a positive integer smaller than $n+3$. 
After soft SUSY breaking, the scalar potential possesses \([12]\) non-trivial minima at

\[
|P|, |Q| \sim (m_{3/2}M_{Pl}^\frac{1}{1+\alpha}),
\]

where \(U(1)_{PQ}\) is spontaneously broken and \(f_a\) and \(\mu\) are generated dynamically. The soft masses-squared \(m_P^2, m_Q^2 \sim m_{3/2}^2\) of \(P, Q\) can have either sign, while the coefficient \(A\) of the soft \(A\)-term corresponding to the coupling in (18), which is generally complex with \(|A| \sim m_{3/2}\), must be large enough for the non-trivial minima to exist if \(m_P^2, m_Q^2 > 0\).

To implement our scenario, we need a valley of local minima of the potential which has negative inclination and \(|P| < |P_0|\) and \(|Q| < |Q_0|\), where \(|P_0|\) and \(|Q_0|\) are the vacuum values of \(|P|\) and \(|Q|\) respectively. If the system slowly rolls down this valley during inflation, the order parameter \(v < v_0\) and our amplification mechanism for the curvaton perturbations may work. This can be achieved only if one of the mass-squared \(m_P^2, m_Q^2\) is negative. Let us take \(m_P^2 < 0\) and \(m_Q^2 > 0\). In this case, the scalar potential is \([22]\) unbounded below unless \(k = 1\). So, we restrict ourselves to the case \(k = 1\). One can show \([12]\) that the orthogonal axion in this case acquires a mass of order \(m_{3/2}\) during inflation and, thus, does not qualify as a PNGB curvaton.

The addition of a third SM singlet superfield \(S\), however, with a coupling

\[
\xi_\eta P^{p+3-p-q} Q^p S^q / M_{Pl}^n,
\]

where \(p, q\) are non-negative integers with \(p + q \leq n + 3\) and \(q \geq 3\), can drastically change \([12]\) the situation allowing the implementation of our mechanism. In the next section, we will present a concrete class of models of this category.

**PQ MODELS WITH AN AXION-LIKE CURVATON**

We consider a class of extensions of MSSM which are based on the SM gauge group, but also possess a global anomalous PQ symmetry \(U(1)_{PQ}\), a global non-anomalous \(R\) symmetry \(U(1)_R\), and a discrete \(Z_2^F\) symmetry. Note, in passing, that global continuous symmetries can effectively arise \([23]\) from the discrete symmetry groups of many compactified string theories (see e.g. \([24]\)). In addition to the usual MSSM superfields \(h_1, h_2\) (Higgs SU(2)_L doublets), \(l_i\) (SU(2)_L doublet leptons), \(e_i^c\) (SU(2)_L singlet charged leptons), \(q_i\) (SU(2)_L doublet quarks), and \(u_i^c, d_i^c\) (SU(2)_L singlet anti-quarks) with \(i = 1, 2, 3\) being the family index, the models contain the SM singlet superfields \(P, Q, S\). The charges of the superfields under \(U(1)_{PQ}\) and \(U(1)_R\) are

\[
PQ : P(-2), Q(2), S(0), h_1, h_2(n+1),
\]

\[
R : P(\frac{n+3}{2}), Q(\frac{n-1}{2}), S(\frac{n+1}{2}), h_1, h_2(0)
\]

with the “matter” (quark and lepton) superfields having \(PQ = -(n+1)/2\) and \(R = (n+1)(n+3)/4\). The integer \(n\) is taken to be of the form

\[
n = 4l + 1, \quad \text{where} \quad l = 0, 1, 2, \ldots,
\]
for reasons to be explained below. Finally, under $Z^p_2$, $P$ changes sign. Baryon (and lepton) number is [25] automatically conserved to all orders in perturbation theory as a consequence of $U(1)_R$ (and $U(1)_{PQ}$). The $Z_2^P$ subgroup of $U(1)_{PQ}$ coincides with the matter parity symmetry $Z_{mp}$, which changes the sign of all matter superfields.

The most general superpotential compatible with these symmetries is

$$W = y_{eij} h_1 e_j^c + y_{uij} q_i u_j^c + y_{dij} q_i d_j^c + \lambda P^{n+1} h_2/M_p^n + \sum_{k=0}^{(n+3)/4} \lambda_k S^{n+3-4k}(PQ)^{2k}/M_p^n,$$

where $y_{eij}, y_{uij}, y_{dij}$ are the usual Yukawa coupling constants, $\lambda, \lambda_k$ are complex dimensionless parameters, and summation over the family indices is implied.

### The scalar potential

The resulting scalar potential for PQ breaking after soft SUSY breaking is

$$V = |F_P|^2 + |F_Q|^2 + |F_S|^2 + V_{\text{soft}},$$

where

$$F_P = \sum_{k=1}^{(n+3)/4} 2k\lambda_k S^{n+3-4k}(PQ)^{2k-1}Q/M_p^n,$$

$$F_Q = \sum_{k=1}^{(n+3)/4} 2k\lambda_k S^{n+3-4k}(PQ)^{2k-1}P/M_p^n,$$

and

$$F_S = \sum_{k=0}^{(n-1)/4} (n+3-4k)\lambda_k S^{n+2-4k}(PQ)^{2k}/M_p^n,$$

are the F-terms, and

$$V_{\text{soft}} = m_P^2 |P|^2 + m_Q^2 |Q|^2 + m_S^2 |S|^2 + \left[ A \sum_{k=0}^{(n+3)/4} \lambda_k S^{n+3-4k}(PQ)^{2k}/M_p^n + \text{h.c.} \right]$$

the soft SUSY-breaking terms. Here, the soft SUSY-breaking masses-squared $m_P^2, m_Q^2,$ and $m_S^2$ are of the order of the $m_{3/2}^2$ and can, in principle, have either sign. However, the potential $V$ is [12] bounded below only if $m_P^2$ and $m_Q^2$ are positive. For definiteness, we will take these two soft masses-squared to be equal, i.e. we will put $m_P^2 = m_Q^2 \equiv m^2$. Also, for simplicity, we assumed universal soft SUSY-breaking $A$-terms with $|A| \sim m_{3/2}$.

For reasons which will become clear later, we take $m_S^2 < 0$. Therefore, the origin in field space ($P = Q = S = 0$) is a saddle point of the potential with positive curvature in the $P$ and $Q$ directions and negative in the $S$ direction. We will call it trivial saddle point.
FIGURE 1. Plot of $V$ defined in (24)-(28) in units of $M_1^{14}/M_2^{10}$ with respect to $|S|$ and $|P|$, which are in units of $M_1$. We took $n = 5$, $M_1 \equiv (mM_5^3)^{1/6}$, $m_P^2 = m_Q^2 = m_S^2 \equiv m^2$, $A = -9m$, and $\lambda_0 = \lambda_1 = \lambda_2 = 1$. Also, $|P| = |Q|$ and $\theta_S = \theta_P = \theta_Q = 0$ ($\theta_S$, $\theta_P$, $\theta_Q$ are the phases of $S$, $P$, $Q$) so that the potential is minimized. The trivial and shifted valleys as well as the trivial and non-trivial minima are clearly visible.

One can show [12] that the potential $V$ has a “trivial” valley of local minima which lies on the $S$ axis (i.e. at $P = Q = 0$) and is clearly visible in Figure 1. Moreover, there exists a “trivial” minimum on this valley at $|S| \sim (m_{3/2}M_p^{n})^{1/(n+1)}$, where $U(1)_{PQ}$ is unbroken and no $\mu$ term is generated. So, we should avoid ending up at this trivial minimum.

The potential $V$ possesses [12] non-trivial minima too with

$$|P| = |Q|, \quad |S| \sim (m_{3/2}M_p^{n})^{1/(n+1)},$$

where $U(1)_{PQ}$ is broken and a $\mu$ term is generated. Actually, after soft SUSY breaking (included in $V$), the only global symmetry surviving in a non-trivial minimum is $Z_2^{mp}$.

The shifted valley of minima. We expand $V$ for $|S| \ll |P| \sim |Q|$. The leading term is

$$V(0) = \frac{(n+3)^2}{4}|\lambda_{n+3}|^2|PQ|^{n+1}(|P|^2 + |Q|^2)\frac{m_P^2|P|^2}{M_p^{2n}} + m_P^2|P|^2 + m_Q^2|Q|^2$$

$$-2|A||\lambda_{n+3}|\frac{(|P||Q|)^{n+1}}{M_p^2} \cos \left[\frac{n+3}{2}(\theta_P + \theta_Q)\right],$$

where $\theta_P$ and $\theta_Q$ are the phases of $P$ and $Q$ respectively and $A\lambda_{(n+3)/4}$ is taken real and negative by field rephasing.

The potential $V(0)$ is minimized with respect to the phases $\theta_P$ and $\theta_Q$ for

$$\frac{n+3}{2}(\theta_P + \theta_Q) = 0 \pmod{2\pi}, \quad |P| = |Q|,$$

$$\left(\frac{|P|^{n+1}}{M_p^2}\right)_+ \equiv x_+ = \frac{|A| + \sqrt{|A|^2 - 4(n+2)m^2}}{(n+2)(n+3)|\lambda_{n+3}|}$$

for $|A|^2 > 4(n+2)m^2$. (31)
The presence of the term $\lambda_{n+3/4}(PQ)^{(n+3)/2}/M_\text{Pl}^n$ in the superpotential is vital to the existence of this “shifted” minimum. In view of $Z_2^P$, however, this term can only exist if $(n+3)/2$ is an even positive integer, which implies the restriction in (22).

For $|S| \ll |P| \sim |Q|$, the shifted minimum of $V(0)$ is also a minimum of $V$ with respect to $|P|$ and $|Q|$ at a practically $S$-independent position. Thus, for small values of $|S|$, we obtain a “shifted” valley of minima of $V$ at almost constant values of $|P|$ and $|Q|$. This valley, which is clearly visible in Figure 1, has negative inclination for non-zero and small values of $|S|$, due to the negative mass term of $S$. It starts from the “shifted” saddle point which lies at $|S| = 0$ and $|P|, |Q|$ equal to their values at the shifted minimum of $V(0)$. Note, in passing, that a shifted valley of minima was first used in [26] as an inflationary path in order to avoid the overproduction of doubly charged magnetic monopoles at the end of SUSY hybrid inflation [2, 28] in a Pati-Salam [29] GUT model.

**The PNGB curvaton.** The dominant $S$-dependent part of $V$ can be expressed as

$$V_{(1)} = m_3^2|S|^2 - 2A|\lambda_{n+1}||S|^4(|P||Q|)^{n-1}M_\text{Pl}^n \cos \left(4\theta_S + \frac{n-1}{2}(\theta_P + \theta_Q)\right)$$

$$+ \frac{(n-1)(n+3)}{2} |\lambda_{n+1}|^2 |S|^4(|P|^2 + |Q|^2)^{n-1}M_\text{Pl}^2 \cos \left(4\theta_S - 2(\theta_P + \theta_Q)\right),$$

(32)

where $\theta_S$ is the phase of $S$ and $A\lambda_{(n-1)/4}$ is made real and negative. The potential $V_{(1)}$ is minimized with respect to the phases $\theta_P, \theta_Q, \theta_S$ for $4\theta_S + (n-1)(\theta_P + \theta_Q)/2 = 0$ modulo $2\pi$. Defining the real canonically normalized fields

$$\phi_P \equiv \sqrt{2}|P|\theta_P, \quad \phi_Q \equiv \sqrt{2}|Q|\theta_Q, \quad \phi_S \equiv \sqrt{2}|S|\theta_S,$$

(33)

we find [12] three angular mass eigenstates on the shifted valley for $|S| \ll |P| = |Q|$:  

1. The axion field $a = (\phi_P - \phi_Q)/\sqrt{2}$, which remains massless to all orders.

2. $\phi_{PQ} \equiv (\phi_P + \phi_Q)/\sqrt{2}$ with mass-squared $m_{PQ}^2 = |A|(n+3)|\lambda_{(n+3)/4}|x_+^2/2 \sim m_3^{1/2}$.

3. $\phi_S$, our PNGB curvaton (orthogonal axion) $\sigma$ with suppressed mass-squared

$$\hat{m}_\sigma^2 = \frac{8}{n+2}|\lambda_{n+1}|^4 \frac{|S|^2}{|P|^2} \left[ (n+5)|A| - (n-1)\sqrt{|A|^2 - 4(n+2)m_3^2} \right] \sim \frac{|S|^2}{|P|^2}m_3^{1/2}.$$

(34)

Note that the $Z_2^P$ symmetry is [12] very important for the PNGB nature of $\phi_S$.

SUGRA corrections [2, 3, 30] during inflation can be introduced by simply replacing $A, m_3^2$, and $m_S^2$ by their effective values:

$$\tilde{A} = A + c_A H, \quad \tilde{m}_3^2 = m_3^2 + c_{PQ}H^2, \quad \tilde{m}_S^2 = m_3^2 + c_SH^2 = c_SH^2 - |m_3^2|$$

(35)

respectively. Here, $c_A$ is a complex parameter of order unity, while $c_{PQ}$ and $c_S$ are real and positive parameters again of order unity. We can arrange the parameters so that
$m^2_S > 0$ during the initial stages of inflation. In this case, the shifted saddle point of $V$ becomes a local minimum and the system may be initially trapped in it. $H$ decreases during inflation and, thus, at some moment of time, this minimum turns into a saddle point and the system starts slowly rolling down the shifted valley. During this slow roll-over, $\phi_S \equiv \sigma$ is an effectively massless PNGB field which can act as curvaton.

**CURVATON PHYSICS**

*The required $\varepsilon$. The phase $\theta = \theta_S \sim 1$ corresponding to the curvaton degree of freedom remains frozen until the onset of the curvaton oscillations. Hence, we expect to have $\sigma_{osc} \sim \theta v_0$. From (15), we then find

$$\varepsilon \sim \frac{\Omega_{dec}}{\pi \zeta \theta} \left( \frac{m_{3/2}}{M_P} \right)^{\frac{5}{34}} \gtrsim \varepsilon_{\min} \sim \left( \frac{m_{3/2}}{M_P} \right)^{\frac{5}{34}},$$

(36)

where we have also used that $H_* \sim m_{3/2}$ and $v_0 \sim (m_{3/2} M_P^n)^{1/(n+1)}$. Assuming that $\sigma$ decays before big bang nucleosynthesis (BBN), i.e. its decay width $\Gamma_\sigma \sim m_\sigma^3/v_0^2$ is greater than the Hubble parameter $H_{BBN}$ at BBN, one shows [10, 12] that

$$\varepsilon < \frac{\Omega_{dec}^{\frac{1}{2}}}{\pi \zeta} \left( \frac{M_P}{T_{BBN}} \right)^{\frac{1}{2}} \left( \frac{m_{3/2}}{M_P} \right)^{\frac{5}{4}} \sim 10^{-4} \Omega_{dec}^{-\frac{1}{2}},$$

(37)

where $T_{BBN} \approx 1$ MeV is the cosmic temperature at BBN. From (36) and (37), we find [12] that

$$n > \frac{8 + \log(\Omega_{dec}^{\frac{1}{2}}/\theta)}{7 - \log(\Omega_{dec}^{\frac{1}{2}}/\theta)},$$

(38)

which implies that $n \geq 1$ for $\theta \sim 1$. The requirement that $\Gamma_\sigma > H_{BBN}$ yields [12]

$$m_\sigma \gtrsim 10^{\frac{8-9}{34}} \text{ TeV} \quad \Rightarrow \quad n \leq \frac{9 + \log(m_\sigma/\text{TeV})}{1 - \log(m_\sigma/\text{TeV})}$$

(39)

for $m_\sigma < 10$ TeV. This inequality demands that $n \leq 9$ for $m_\sigma \lesssim 1$ TeV. So, we get $1 \leq n \leq 9$.

*Reheating the universe. We will assume that the curvaton decays after dominating the energy density of the universe, i.e. $\Omega_{dec} \sim 1$, which is [12] crucial for avoiding the overclosure of the universe by axions (see below). The curvaton dominates [12] when $H = H_{dom}$, where

$$H_{dom} \sim \left( \frac{\sigma_{osc}}{M_P} \right)^4 \min\{m_\sigma, \Gamma_{inf}\}$$

(40)

with $\Gamma_{inf} \sim g^2 m_{3/2}$ being the inflaton decay width ($g$ is the coupling constant of the inflaton to its decay products). It can be shown [12] that the requirement that $\Gamma_\sigma < H_{dom}$.
(i.e. \( \sigma \) decays after dominating the energy density of the universe) results in the bound

\[
g > \frac{1}{\theta^2} \left( \frac{m_{3/2}}{M_p} \right)^{\frac{n-2}{n+1}} \Rightarrow n > \frac{30 - \log g - 2 \log \theta}{15 + \log g + 2 \log \theta} \Rightarrow n \geq 2 \quad \text{for } \theta \sim 1. \quad (41)
\]

Hot big bang begins after the decay of the curvaton at a reheat temperature

\[
T_{\text{REH}} \sim \sqrt{\frac{\Gamma_\sigma M_p}{\sigma}} \sim m_{3/2} \left( \frac{m_{3/2}}{M_p} \right)^{\frac{1}{2}(\frac{n-1}{n+1})} \geq T_{\text{BBN}} \quad \text{for } n \leq 9. \quad (42)
\]

**Diluting the axions.** For \( n > 1 \), \( f_a \approx v_0 \sim (m_{3/2} M_p^n)^{1/(n+1)} \gg 10^{12} \) GeV. This normally leads to axion overproduction overclosing the universe. However, if the curvaton dominates the universe before decaying, the entropy generated [31] during its decay

\[
\frac{S_{\text{after}}}{S_{\text{before}}} \sim \left( \frac{H_{\text{dom}}}{\Gamma_\sigma} \right)^{\frac{1}{2}} \sim g \theta^2 \frac{v_0^3}{m_{3/2} M_p^2} \sim g \theta^2 \left( \frac{M_p}{m_{3/2}} \right)^{\frac{n-2}{n+1}} \quad (43)
\]

can adequately dilute the axions (see [32]).

**The evolution of \( v \).** The order parameter \( v \propto |S| \) must be slowly rolling during inflation to preserve the approximate scale invariance of the perturbations (see below). So, it should follow the equation

\[
3H|\dot{S}| + m_S^2 |S| \simeq 0 \quad \Rightarrow \quad \frac{\dot{v}}{v} = \frac{\dot{|S|}}{|S|} = \frac{1}{3} c_S \left( \frac{|m_S^2|}{c_S H^2} - 1 \right) H. \quad (44)
\]

Using (4) and the fact that \( |m_S^2| \equiv c_S H_K^2 \simeq c_S H_m^2 (1 - e^{-2F_0 N_s}) \), this equation becomes

\[
\frac{3}{c_S} \frac{d \ln |S|}{dN} = \frac{e^{-2F_0 N_s} - e^{-2F_0 N}}{1 - e^{-2F_0 N}} \quad (45)
\]

where \( H_K \) and \( N_s \) correspond to the phase transition which changes the sign of \( m_S^2 \) during inflation. The solution of (45) is

\[
\frac{6}{c_S} \ln \left( \frac{|S|^*}{|S|^x} \right) = (1 - e^{-2F_0 N_s}) F_s^{-1} \ln \left( \frac{e^{2F_0 N_s} - 1}{e^{2F_0 N_s} - 1} \right) - 2(N_x - N_s), \quad (46)
\]

where \( |S|^* \equiv |S|(N_s) \sim (\epsilon/\epsilon_{\text{min}}) H_s \) and \( |S|^x \equiv |S|(N_x) \sim H_K/2\pi \) from quantum fluctuations at the phase transition. The contribution to the spectral index \( n_s \) from the evolution of \( v \) during inflation is [12] \(-H_s^{-1} (\dot{v}/v)_s \leq 0 \). The WMAP bound [17] on \( n_s \) then implies

\[
\frac{c_S}{3} e^{-2F_0 N_s} \left( \frac{1 - e^{-2F_0 (N_x - N_s)}}{1 - e^{-2F_0 N_s}} \right) \leq 0.04. \quad (47)
\]

It is important to note that, in the present case, the negative contribution to \( n_s \) from the variation of \( v \) during inflation leads naturally to spectral indices which can be adequately smaller than unity in accordance with the recent WMAP results [17].
A CONCRETE EXAMPLE

From (39), and (41) and in view of (22), we see that not many choices for \( n \) are allowed. In fact, we can only accept the models with \( n = 5, 9 \) (i.e. \( l = 1, 2 \)) with the latter case being marginal. Hence, to illustrate the above, we take an example with \( n = 5 \) (i.e. \( l = 1 \)) and the curvaton assuming a random value after the phase transition, i.e. \( \theta \sim 1 \).

The bound in (39) suggests that this case is acceptable provided that \( m_\sigma \gtrsim 220 \) GeV. Using (36), we find that \( \epsilon \sim 10^{-8.5} \) (recall that \( \Omega_{\text{dec}} \simeq 1 \) and \( \epsilon_{\text{min}} \sim 10^{-12.5} \)), which yields \( |S|_r \sim 10^4 H_* \). We also estimate [12] \( N_* \) to be about 38 for \( H_* \sim m_3/2 \). The reheat temperature turns out to be \( T_{\text{REH}} \simeq 10 \) MeV, while the entropy production at curvaton decay is given by \( S_{\text{after}}/S_{\text{before}} \sim 10^{7.5} g \). From (41), we then conclude that \( g > 10^{-4.5} \).

The dilution of axions by the entropy produced when the curvaton decays after dominating the universe may lead [20] to a cosmological disaster. A sizable fraction of the curvaton’s decay products consists of sparticles, which eventually turn into stable lightest sparticles (LSPs) in models (such as ours) with an unbroken matter parity. The freeze-out temperature of the LSPs is much higher than \( T_{\text{REH}} \). Thus, the LSPs freeze out right after their production and can, subsequently, overclose the universe. This problem can be solved [12] by suppressing the Higgsino components of the lighter neutralinos and charginos below 1% and taking the curvaton adequately light.

One can show [12] that all the requirements mentioned above can be satisfied for

\[
|c_s, c| \lesssim O(10^{-4}) \quad \Rightarrow \quad |m_S|, m_s \lesssim O(10^{-2}) H_*.
\]

The smallness of \( |m_S| \) is due to the requirement that \( |S| \) is slowly rolling during the relevant part of inflation so that the approximate scale invariance of the density perturbations is preserved, whereas the smallness of \( m_s \) to the required value of \( \epsilon (\gg \epsilon_{\text{min}}) \), which demands substantial variation of \( |S| \) from the phase transition during inflation until the time when the cosmological scales exit the inflationary horizon. Such a variation can be achieved with a large number of e-foldings \( (N_{\text{tot}} \lesssim O(10^5 - 10^6)) \), which means that, in our case, modular inflation is not of the fast-roll type.

CONCLUSIONS

We constructed SUSY PQ models generating the PQ scale and the \( \mu \) term dynamically. They contain a successful PNGB curvaton whose perturbations are suitably amplified to account for the observed curvature perturbations even in low-scale inflationary models such as modular inflation where the inflaton is unable to generate these perturbations. The spectral index of density perturbations can easily satisfy the recent WMAP bound in contrast to other inflationary models. However, due to the very low value of the reheat temperature, baryogenesis may be achieved only via some exotic mechanism (see [12]).

ACKNOWLEDGMENTS

This work was supported by the European Union under the contract MRTN-CT-2004-503369.
REFERENCES

1. G. Lazarides, *Lect. Notes Phys.* **592**, 351 (2002) (hep-ph/0111328); hep-ph/0607032.
2. E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, *Phys. Rev. D* **49**, 6410 (1994).
3. M. Dine, L. Randall, and S. Thomas, *Phys. Rev. Lett.* **75**, 398 (1995); *Nucl. Phys. B* **458**, 291 (1996).
4. P. Binétruy, and M. K. Gaillard, *Phys. Rev. D* **34**, 3069 (1986); F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, *ibid.* **47**, 426 (1993); T. Banks, M. Berkooz, S. H. Shenker, G. W. Moore, and P. J. Steinhardt, *ibid.* **52**, 3548 (1995); R. Brustein, S. P. De Alwis, and E. G. Novak, *ibid.* **68**, 023517 (2003).
5. A. D. Linde, *J. High Energy Phys.* **11**, 052 (2001).
6. R. Allahverdi, K. Enqvist, J. Garcia-Bellido, and A. Mazumdar, hep-ph/0605035.
7. K. Enqvist, and M. S. Sloth, *Nucl. Phys. B* **626**, 395 (2002); D. H. Lyth, and D. Wands, *Phys. Lett. B* **524**, 5 (2002); T. Moroi, and T. Takahashi, *ibid.* **522**, 215 (2001), (E) **539**, 303 (2002).
8. S. Mollerach, *Phys. Rev. D* **42**, 313 (1990); A. D. Linde, and V. Mukhanov, *ibid.* **56**, 535 (1997).
9. D. H. Lyth, *Phys. Lett. B* **579**, 239 (2004).
10. K. Dimopoulos, D. H. Lyth, and Y. Rodríguez, *J. High Energy Phys.* **02**, 055 (2005).
11. K. Dimopoulos, *Phys. Lett. B* **634**, 331 (2006).
12. K. Dimopoulos, and G. Lazarides, *Phys. Rev. D* **73**, 023525 (2006).
13. R. Peccei, and H. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977); S. Weinberg, *ibid.* **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).
14. K. Dimopoulos, G. Lazarides, D. H. Lyth, and R. Ruiz de Austri, *J. High Energy Phys.* **05**, 057 (2003).
15. C. L. Bennett *et al.*, *Astrophys. J.* **464**, L1 (1996).
16. D. H. Lyth, C. Ungarelli, and D. Wands, *Phys. Rev. D* **67**, 023503 (2003).
17. D. N. Spergel *et al.*, astro-ph/0603449.
18. K. Dimopoulos, G. Lazarides, D. H. Lyth, and R. Ruiz de Austri, *Phys. Rev. D* **68**, 123513 (2003).
19. J. E. Kim, and H. P. Nilles, *Phys. Lett. B* **138**, 150 (1984).
20. K. Choi, E. J. Chun, and J. E. Kim, *Phys. Lett. B* **403**, 209 (1997).
21. G. Lazarides, and Q. Shafi, *Phys. Rev. D* **58**, 071702 (1998).
22. G. Lazarides, and Q. Shafi, *Phys. Lett. B* **489**, 194 (2000).
23. G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, *Phys. Rev. Lett.* **56**, 432 (1986).
24. N. Ganoulis, G. Lazarides, and Q. Shafi, *Nucl. Phys. B* **323**, 374 (1989).
25. G. Lazarides, and N. D. Vlachos, *Phys. Lett. B* **459**, 482 (1999); G. Lazarides, *PoS(trieste99)008*, 1999 (hep-ph/9905450).
26. R. Jeannerot, S. Khalil, G. Lazarides, and Q. Shafi, *J. High Energy Phys.* **10**, 012 (2000); G. Lazarides, “Supersymmetric Hybrid Inflation,” in *Recent Developments in Particle Physics and Cosmology*, edited by G. C. Branco et al., Kluwer Acad. Pub., Dordrecht, 2001, pp. 399–419 (hep-ph/0011130); R. Jeannerot, S. Khalil, and G. Lazarides, *J. High Energy Phys.* **07**, 069 (2002).
27. G. Lazarides, M. Magg, and Q. Shafi, *Phys. Lett. B* **97**, 87 (1980).
28. G. R. Dvali, Q. Shafi, and R. K. Schaefer, *Phys. Rev. Lett.* **73**, 1886 (1994); G. Lazarides, R. K. Schaefer, and Q. Shafi, *Phys. Rev. D* **56**, 1324 (1997).
29. J. C. Pati, and A. Salam, *Phys. Rev. D* **10**, 275 (1974).
30. M. Dine, W. Fischler, and D. Nemeschansky, *Phys. Lett. B* **136**, 169 (1984); G. D. Coughlan, R. Holman, P. Ramond, and G. G. Ross, *ibid.* **140**, 44 (1984).
31. R. J. Scherrer, and M. S. Turner, *Phys. Rev. D* **31**, 681 (1985).
32. P. J. Steinhardt, and M. S. Turner, *Phys. Lett. B* **129**, 51 (1983); G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, *ibid.* **192**, 323 (1987); G. Lazarides, R. K. Schaefer, D. Seckel, and Q. Shafi, *Nucl. Phys. B* **346**, 193 (1990).