Spin-lattice relaxation time in pressure-induced two-leg ladder cuprate superconductors

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Abstract. Spin-lattice relaxation time in pressure-induced two-leg ladder cuprate superconductors is investigated based on the kinetic energy driven superconducting mechanism. It is shown that the spin-lattice relaxation time exhibits a temperature linear dependence at low temperature followed by a peak developed below the superconducting transition temperature, in qualitative agreement with experiments.

The great interest of superconducting (SC) spin ladder cuprates lies in that its ground state may be a spin liquid state with a finite spin gap of the magnetic excitations[1]. This spin liquid state may play a crucial role in superconductivity of doped cuprates as emphasized by Anderson [2]. Experimentally it has been shown that the doped two-leg ladder copper oxide material Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ has a SC phase at high pressure [3]. Moreover, the structure under high pressure remains the same as the case in ambient pressure [4], and the spin background in the SC phase does not drastically alter its spin gap properties [1]. Recently, the dynamical spin response on the doped two-leg ladder cuprate Sr$_2$Ca$_{12}$Cu$_{24}$O$_{41}$ in the SC state under pressure has been detected by virtue of nuclear magnetic resonance and nuclear quadrupole resonance [5]. It is indicated that the spin-lattice relaxation time possesses temperature linear dependence at low temperature followed by a peak appears below the SC transition temperature $T_c$.

Within the charge-spin separation (CSS) fermion-spin theory [6], the dynamical spin response of Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ in the normal state has been studied [7]. Furthermore, the pressure-induced superconductivity in Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ [8] has been discussed based on the kinetic energy driven SC mechanism [9]. In this paper, we apply the kinetic energy driven SC mechanism to discuss the spin-lattice relaxation time in the pressure-induced two-leg ladder cuprate superconductor Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ in the SC state.

We start from the two-leg $t$-$J$ ladder model,

$$H = -t_{\|} \sum_{i\sigma} C_{ia\sigma}^\dagger C_{i+\|a\sigma} - t_{\perp} \sum_{i\sigma} (C_{i1\sigma}^\dagger C_{i2\sigma} + H.c.) - \mu \sum_{i\sigma} C_{ia\sigma}^\dagger C_{ia\sigma} + J_{\|} \sum_{i\sigma} S_{ia} \cdot S_{i+\|a\sigma} + J_{\perp} \sum_{i} S_{i1} \cdot S_{i2},$$

(1)

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supplemented by the single occupancy local constraint \( \sum_{\sigma} C_{i_{\sigma}} C_{i_{\bar{\sigma}}} \leq 1 \), where \( i \) runs over all rungs, \( \eta = \pm \hat{x}, a(= 1, 2) \) and \( \sigma(= \uparrow, \downarrow) \) are leg and spin indices, respectively, \( C_{i_{\sigma}} \) and \( C_{i_{\bar{\sigma}}} \) are the electron creation (annihilation) operators, \( S_{i_{\sigma}} = C_{i_{\sigma}}^{\dagger} C_{i_{\bar{\sigma}}} / 2 \) are the spin operators with \( \bar{\sigma} = (\sigma_{x}, \sigma_{y}, \sigma_{z}) \) as the Pauli matrices, and \( \mu \) is the chemical potential. The local constraint can be treated properly within the CSS fermion-spin theory, \( C_{i_{\uparrow}} = h_{i_{\uparrow}}^{\dagger} S_{i_{\uparrow}}^{-}, C_{i_{\downarrow}} = h_{i_{\downarrow}}^{\dagger} S_{i_{\downarrow}}^{+} \). In this CSS fermion-spin representation, the low-energy behavior of the \( t-\hat{J} \) ladder Hamiltonian (1) can be expressed as,

\[
H = t_{\parallel} \sum_{i\bar{\eta}} (h_{i+\bar{\eta}a_{\bar{\eta}}1}^{\dagger} S_{i_{\bar{\eta}}}^{+} S_{i+\bar{\eta}a_{\bar{\eta}}1} + h_{i+\bar{\eta}a_{\bar{\eta}}1}^{\dagger} h_{i+\bar{\eta}a_{\bar{\eta}}1}^{\dagger} S_{i+\bar{\eta}a_{\bar{\eta}}1}^{+} S_{i_{\bar{\eta}}}^{+}) \\
+ t_{\perp} \sum_{i} (h_{i1}^{\dagger} S_{i1}^{+} S_{i1} + h_{i1}^{\dagger} h_{i2}^{\dagger} S_{i2}^{+} S_{i1} + h_{i1}^{\dagger} h_{i2} h_{i2}^{\dagger} S_{i1}^{+} S_{i2}^{+}) \\
+ \mu \sum_{i\sigma} h_{i\sigma}^{\dagger} h_{i\sigma} + J_{\parallel \text{eff}} \sum_{i\bar{\eta}} S_{i_{\bar{\eta}}} \cdot S_{i+\bar{\eta}a_{\bar{\eta}}1} + J_{\perp \text{eff}} \sum_{i} S_{i_{1}} \cdot S_{i_{2}},
\]

(2)

where \( J_{\parallel \text{eff}} = J_{\parallel} (1-p)^{2}, J_{\perp \text{eff}} = J_{\perp} (1-p)^{2}, \) and \( p = \langle h_{i_{\text{eff}}}^{\dagger} h_{i_{\text{eff}}} \rangle = \langle h_{i_{\text{eff}}}^{\dagger} h_{i_{\text{eff}}} \rangle \) is the charge carrier doping concentration.

Based on the kinetic energy driven SC mechanism, the dynamical spin structure factor of the doped two-leg ladder cuprate superconductors under pressure can be obtained [10] as,

\[
S(k, \omega) = -2[1 + n_{B}(\omega)][\text{Im} D_{L}(k, \omega) + \text{Im} D_{T}(k, \omega)]
\]

\[
= -2[1 + n_{B}(\omega)] \frac{B_{i_{\text{eff}}}^{2} \text{Im} \Sigma_{s}^{(1)}(k, \omega)}{[\omega^{2} - \left( \omega_{1k} \right)^{2} - B_{1} \text{Re} \Sigma_{s}^{(1)}(k, \omega)]^{2} + \left( B_{1} \text{Im} \Sigma_{s}^{(1)}(k, \omega) \right)^{2}},
\]

(3)

where \( n_{B}(\omega) \) is the boson distribution function, \( \text{Im} \Sigma_{s}^{(1)}(k, \omega) = \text{Im} \Sigma_{L}^{(s)}(k, \omega) + \text{Im} \Sigma_{T}^{(s)}(k, \omega), \text{Re} \Sigma_{s}^{(1)}(k, \omega) = \text{Re} \Sigma_{L}^{(s)}(k, \omega) + \text{Re} \Sigma_{T}^{(s)}(k, \omega), \text{Im} \Sigma_{L}^{(s)}(k, \omega)[\text{Re} \Sigma_{L}^{(s)}(k, \omega)] \) and \( \text{Im} \Sigma_{T}^{(s)}(k, \omega)[\text{Re} \Sigma_{T}^{(s)}(k, \omega)] \) are the corresponding imaginary (real) parts of the spin longitudinal and transverse self-energy, respectively, while the spin longitudinal and transverse self-energy, \( \Sigma_{L}^{(s)}(k, \omega) \) and \( \Sigma_{T}^{(s)}(k, \omega) \), \( B_{1} \text{Re} \) and the mean-field spin excitation spectrum \( \omega_{1k} \) have been given in Refs. [10, 11].

In the dynamical spin response of the pressure-induced two-leg ladder cuprate superconductors, one of the characteristic features is the spin-lattice relaxation time \( T_{1} \), which is closely related to the dynamical spin structure factor, and can be expressed as,

\[
\frac{1}{T_{1}} = \frac{2K_{B}T_{c}}{g^{2} \mu_{B}^{2}} \lim_{\omega \to 0} \frac{1}{N} \sum_{k} F_{\alpha}^{2}(k) \chi''(k, \omega) / \omega,
\]

(4)

where \( g \) is the lande-factor, \( \mu_{B} \) is the Bohr magneton, and \( F_{\alpha}(k) \) is the form factors, while the dynamical spin susceptibility \( \chi''(k, \omega) = (1 - e^{-\beta \omega}) S(k, \omega) \). This form factors \( F_{\alpha}(k) \) can be set to constant without loss of generality [7]. The pressure effects are imitated through the variation of the values of \( J_{\perp} / J_{\parallel} \) and \( t_{\perp} / t_{\parallel} \) [8]. \( (t_{\parallel} / t_{\perp})^{2} \) is chosen the same as that of \( J_{\perp} / J_{\parallel} \). In this case, \( 1/T_{1} \) has been evaluated and the result at \( p = 0.20 \) for \( t_{\parallel} / J_{\parallel} = 2.5 \) and \( t_{\perp} / t_{\parallel} = 0.7 \) (underpressure) is plotted in Fig. 1 in comparison with the experimental data [5] taken from \( \text{Sr}_{1-x} \text{Ca}_{x} \text{Cu}_{2+y} \text{O}_{4+x} \) at \( x = 12 \left( p \approx 0.20 \right) \), where we have chosen units \( \hbar = K_{B} = 1 \).

Our theoretical results show that \( 1/T_{1} \) exhibits a linear temperature dependent behavior at low temperatures \( (T > T_{c}) \) followed passes through a minimum and displays a tendency towards an increase with decreasing temperatures, which is dominated by a peak developed below the SC transition temperature \( T_{c} \). Our results are in qualitative agreement with the experiments [5].
Figure 1. The temperature dependence of the spin-lattice relaxation time $1/T_1$ in both logarithmic scales at $p = 0.20$ for $t_\parallel /J_\parallel = 2.5$ and $t_\perp /t_\parallel = 0.7$. Inset: the experimental result on Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ taken from Ref. [5].

Furthermore, this peak can be attributed to a SC coherence peak while the temperature linear dependence of $T_1^{-1}$ at low temperatures to K Corringa-type behavior. In the doped two-leg ladder cuprate superconductors, the charge carrier-spin bound state in the normal state is due to the interaction between charge carriers and spins from the kinetic energy term in the $t$-$J$ ladder (2) [7]. At low temperatures ($T > T_c$), the spin in the charge carrier-spin bound state moves almost freely, which induces the temperature-linear component in $T_1^{-1}$, even if the most of spins in the system form the spin liquid state [7].

In conclusion, we have discussed the spin-lattice relaxation time in the pressure-induced two-leg ladder cuprate superconductor Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ based on the kinetic energy driven SC mechanism. Our results show that the spin-lattice relaxation time exhibits a temperature linear dependence at low temperature followed by a peak developed below the SC transition temperature, in qualitative agreement with experimental measurements on Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ in the pressure-induced SC state.

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