STABILITY OF YANG-MILLS FIELDS SYSTEM IN THE HOMOGENEOUS (ANTI-)SELF-DUAL BACKGROUND FIELD

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Abstract

Stability of Yang-Mills fields system in the background field is investigated basing on Toda criterion, Poincare sections and the values of the maximal Lyapunov exponents. The existence of the region of regular motion at low densities of energy is demonstrated. Critical energy density of the order-chaos transition is analyzed for the different values of the model parameter.

Introduction

In contrast to electrodynamics, the dynamics of Yang-Mills fields is inherently non-linear and chaotic at any density of energy. This assumption was confirmed analytically and numerically\textsuperscript{[1, 2, 3]}. Further analysis of spatially homogeneous field configurations\textsuperscript{[4]} showed that inclusion of Higgs field leads to order-chaos transition at some density of energy of classical gauge fields\textsuperscript{[5, 6, 7]}. Classical Higgs field regularizes chaotic dynamics of classical gauge fields below critical energy density and leads to the emergence of order-chaos transition.

Chaos in Yang-Mills fields\textsuperscript{[8]} and vacuum state instability in nonperturbative QCD models\textsuperscript{[9, 10, 11]} are also considered in connection with confinement. It has been also shown recently that interaction of the constant chromo-magnetic field with axial field could generate confinement\textsuperscript{[12]}. These results indicate the importance of nonperturbative background fields.

In our previous paper\textsuperscript{[13]} we have investigated the stability of Yang-Mills-Higgs fields and described analytically the regions of chaotic and stable motion. In this work Yang-Mills fields are considered on the background of the homogeneous (anti-)self-dual field\textsuperscript{[14]}. As the dynamics of arbitrary field configurations is too complicated, we follow\textsuperscript{[15]} and reduce our model to spatially homogeneous fields which depend on time. After that we are left with only finite number of degrees of freedom (two in our case) which allows us to investigate the dynamics of the system using conventional methods developed for mechanical systems.

In this work one more mechanism of the chaos suppression in the Yang-Mills fields models is proposed. Homogeneous (anti-)self-dual field eliminates chaoticity of Yang-Mills dynamics below critical energy density.

1 Homogeneous (anti-)self-dual field

In this paper classical dynamics of SU(2) model gauge fields system is considered on the background of the homogeneous (anti-)self-dual field. Various properties of this

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solution of the Yang-Mills equations in SU(2) theory were investigated originally by other authors [9, 16, 17, 18]. It was demonstrated that self-dual homogeneous field provides the Wilson confinement criterion [19]. Therefore this field is at least a possible source of confinement in QCD if it is a dominant configuration in the QCD functional integral.

Homogeneous self-dual field is defined by the following expressions [14]:

\[ B_\mu^a = B n^a b_{\mu \nu} x_\nu, \]
\[ F_{\mu \nu}^a = -2 B n^a b_{\mu \nu}, \]

where \( B \) - value of the field strength, vector \( n^a \) and tensor \( b_{\mu \nu} \) characterize the direction of the field, respectively, in color space and in space-time. The latter has the following properties

\[ b_{\mu \nu} = -b_{\nu \mu}, \quad b_{\mu \nu} b_{\mu \rho} = \delta_{\nu \rho}, \]
\[ \tilde{b}_{\mu \nu} = \frac{1}{2} \varepsilon_{\mu \nu \alpha \beta} b_{\alpha \beta} = \pm b_{\mu \nu}, \]

where positive and negative signs in last expression correspond, respectively, to self-dual and anti-self-dual cases.

As the directions of the background field in color space and in space-time can be chosen arbitrarily, we will assume that the gauge field has color components \((n^1, n^2, n^3) = (0, 0, 1)\) and space-time components \(B = (B_1, B_2, B_3) = (0, 0, B)\).

2 Model potential of the system

The Lagrangian of SU(2) gauge theory in Euclidean metrics is

\[ L = -\frac{1}{4} G_{\mu \nu}^a G_{\mu \nu}^a, \]

where \( G_{\mu \nu}^a \) is a field tensor which has the following form:

\[ G_{\mu \nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \varepsilon^{abc} A_\mu^b A_\nu^c. \]

In last expression \( A_\mu^a, a = 1, 2, 3 \) are the three non-abelian Yang-Mills fields and \( g \) denotes the coupling constant of these fields.

We consider the fluctuations around background homogeneous self-dual field. Self-dual field is regarded as external one and it is taken into account by substituting modified vector potential in the Yang-Mills Lagrangian

\[ A_\mu^a \rightarrow A_\mu^a + B_\mu^a \]

where \( A_\mu^a \) is the fluctuation to the background field \( B_\mu^a \).

We use the gauge:

\[ A_4^a = 0, \]

and consider spatially homogeneous field configurations [15]

\[ \partial_i A_\mu^a = 0, \quad i = 1..3. \]
Our model of Yang-Mills fields in (anti-)self-dual field is constructed in Euclidean space. In order to analyze the model by using analytical and numerical methods we should switch to Minkowski space. We consider chromo-magnetic model. Thus we put chromo-electric field is equal to zero. If $A_1 = q_1, A_2 = q_2$ and the other components of the perturbative Yang-Mills fields are equal to zero, the potential of the model is:

$$V = \frac{1}{2} g^2 q_1^2 q_2^2 + \frac{1}{2} H^2 - g H q_1 q_2 + \frac{1}{8} g^2 H^2 (x^2 q_1^2 + y^2 q_2^2), \quad (1)$$

where $H$ - chromo-magnetic background field strength, $x$ and $y$ - coordinates which play the role of the parameters, $q_1$ and $q_2$ - field variables.

### 3 Stability of the model

#### 3.1 Toda criterion

At first, stability of the model is investigated using well known technique based on the Toda criterion of local instability [20, 21] which allows us to obtain the value of the critical energy density of order-chaos transition in the system. This energy and minimum of the energy as the functions of the model parameter $s = g H xy$ are shown on the fig.1.

![Figure 1: Critical energy density of order-chaos transition (thin line) and minimum of the energy (thick line) as a functions of the model parameter $s = g H xy$.](image)

Critical and minimal energies are close to each other for $s \in (-4, 4)$. This behavior indicates the absence of the region of regular motion in the system. In other case ($s \in (-\infty, -4)$ or $s \in (4, \infty)$), the critical energy density is much larger than minimal one and the system is regular up to this energy. These results will be checked using numerical methods in next subsection.

#### 3.2 Numerical calculations

The system is investigated using Poincare sections and Lyapunov exponents for wide range of model parameter values. These numerical methods could indicate global regular regimes of motion whereas Toda criterion reveals only the local chaotic...
properties of the trajectories [22]. Thus numerical methods are more precise for stability analysis.

![Figure 2: Poincare sections for two dimensional Yang-Mills system in the background field for \( s = 0, H = 1 \).](image1)

Results of the numerical calculations for the system with model parameter \( s = 0 \) are shown on the fig.2 and fig.3. Contrary to Toda criterion, system is regular at small energies below the energy of the background field \( E_c = E_{vac} = \frac{1}{2} H^2 \). There are only regular regimes of motion for small energies (e.g. fig.2a and fig.3a) and two types of motion for energies above \( E_c \) (fig.2b and fig.3b).

![Figure 3: Maximal Lyapunov exponents for two dimensional Yang-Mills system in the background field for \( s = 0, H = 1 \), (a)\( E = 0.15 \) and (b)\( E = 0.68 \).](image2)

We have the following Poincare sections (fig.4) for large model parameter values. It is seen that system is fully regular (fig.4a) for high values of energy \( (E_{vac} < E < E_c) \) as it was revealed by Toda criterion. All trajectories have zero maximal Lyapunov exponents for this case. Two types of trajectories (chaotic and regular) are present in the system with energies above the critical one (fig.4b).

It is seen that Toda criterion rather good describes the region of the large values of model parameter \( s \) and fails for \( s \in (-4, 4) \).

Numerical calculations have shown that there is a region of regular motion at low densities of energy in our system at any value of the model parameter. Therefore, homogeneous (anti-)self-dual field regularizes chaotic dynamics of Yang-Mills fields system.
Figure 4: Poincare sections for two dimensional Yang-Mills system in the background field for $s = 25.5, g = 0.1, H = 1, x = 15, y = 17, (a)E = 21$ and (b)$E = 100$. Thin line - border of the phase space.

Conclusions

The existence of the nonperturbative component of the Yang-Mills field is crucial for the confinement phenomenon. On the other hand, classical dynamics of Yang-Mills field is chaotic at any density of energy in the absence of background fields and can be regularized only if some other fields, for example, the Higgs field, are included in the model.

In this work we have demonstrated that homogeneous (anti-)self-dual background field has similar properties. Yang-Mills field on such background has the region of regular motion at low densities of energy. There is order-chaos transition in the system at any values of model parameters. The critical density of energy of this transition is equal to the background field energy for small parameters and is much larger for large parameter values.

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