Measuring $\beta$ in $B \rightarrow D^{(*)+} D^{(*)-} K_s$ Decays

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Abstract

We consider the possibility of measuring both $\sin(2\beta)$ and $\cos(2\beta)$ in the KM unitarity triangle using the process $B^0 \rightarrow D^{*+} D^{*-} K_s$. This decay mode has a higher branching fraction (O(1%)) than the mode $B^0 \rightarrow D^{*+} D^{*-}$. We use the factorization assumption and heavy hadron chiral perturbation theory to estimate the branching fraction and polarization. The time dependent rate for $B^0(t) \rightarrow D^{*+} D^{*-} K_s$ can be used to measure $\sin(2\beta)$ and $\cos(2\beta)$. Furthermore, examination of the $D^{*+} K_s$ mass spectrum may be the best way to experimentally find the broad $1^+$ p-wave $D_s$ meson.


1 Introduction

The decay $B^0 \rightarrow J/\psi K_s$ is expected to provide a clean measurement of the angle $\sin(2\beta)$ in the unitarity triangle $\Gamma$. However, other modes can also provide relevant information on the angle $\beta$. An example of such a mode is the decay $B^0 \rightarrow D^{(*)}\bar{D}^{(*)}$. In this mode $B^0 \rightarrow D^{*+}D^{-}$, the vector-vector final state in general is an admixture of CP odd and even eigenstates, because s, p and d partial waves with different CP-parities can contribute. Since the CP asymmetry has opposite sign for the two CP states they tend to cancel or dilute the overall asymmetry. The amount of dilution of the CP asymmetry is represented by the dilution factor, $D$, which depends on the CP composition of the final state. The presence of two CP components in the final state of $B^0 \rightarrow D^{*+}D^{-}$ makes the dilution factor, $D < 1$ for this decay. This is unlike the case for a mode such as $B^0 \rightarrow D^+D^-$ where the final state is a CP eigenstate and $D=1$ as there is no dilution of the CP asymmetry. An angular analysis can extract the contribution of the different CP eigenstates leading to a measurement of $D$ and hence of $\sin(2\beta)$ $\Gamma$. However, in the factorization approximation and using Heavy Quark Effective Theory (HQET) it can be shown that the final state in $B^0 \rightarrow D^{*+}D^{-}$ is dominated by a single CP eigenstate $\Gamma$. To the extent that this is valid, the angle $\sin(2\beta)$ can be determined without the need for an angular analysis. The decay $B^0 \rightarrow D^{*+}D^{*-}$ may be preferred to $B^0 \rightarrow D^+D^-$ because contamination from penguin contributions and final state interactions (FSI) are expected to be smaller in the former decay $\Gamma$.

In this work we consider the possibility of extracting $\beta$ from the decay $B^0 \rightarrow D^{(*)}\bar{D}^{(*)}K_s$. These modes are enhanced relative to $B^0 \rightarrow D^{(*)}\bar{D}^{(*)}$ by the factor $|V_{cs}/V_{cd}|^2 \sim 20$. As in the case of $B^0 \rightarrow J/\psi K_s$ decay, the penguin contamination is expected to be small in these decays. Moreover these decays can be used to probe both $\sin 2\beta$ and $\cos 2\beta$ which can resolve $\beta \rightarrow \pi/2 - \beta$ ambiguity $\Gamma$.

The possibility that a large portion of the $b \rightarrow c\bar{c}s$ decays materialize as $B \rightarrow D\bar{D}K$ was first suggested by Buchalla et al. $\Gamma$. Using wrong sign $D$-lepton correlations, experimental evidence for this possibility was found by CLEO, who observed $B(B \rightarrow DX) = (7.9 \pm 2.2\%)$ $\Gamma$. Later, CLEO $\Gamma$, ALEPH $\Gamma$ and DELPHI $\Gamma$ reported full reconstruction of exclusive $D\bar{D}K$ final states with branching fractions that are
consistent with the result from $D$-lepton correlations.

CLEO obtained $\mathcal{B}(\overline{B}^0 \rightarrow D^{*+} \overline{D}^{*0} K^-) = (1.30^{+0.61}_{-0.47} \pm 0.27)\%$ and $\mathcal{B}(B^- \rightarrow D^{*0} \overline{D}^{*0} K^-) = (1.45^{+0.78}_{-0.58} \pm 0.36)\%$. These values should be approximately equal to the branching fraction for $\mathcal{B}(B^0 \rightarrow D^{*+} D^{*-} K^0)$.

We use the latter value for the purpose of a sensitivity estimate. Taking into account $\mathcal{B}(K_0 \rightarrow K_s) = 0.5$, $\mathcal{B}(K_s \rightarrow \pi^+ \pi^-) = 0.667$, and assuming that the $K_s$ reconstruction efficiency is $\sim 0.5$, we can estimate the ratio of the tagged $B^0 \rightarrow D^{*+} D^{*-} K_s$ events to the tagged $D^{*+} D^{*-}$ events. Assuming $\mathcal{B}(B^0 \rightarrow D^{*+} D^{*-}) = 6 \times 10^{-4}$, which is the central value of the recent CLEO measurement[11], we find that the ratio of the number of events is $\sim 4.0$. Therefore, this mode could be more sensitive to the CP violation angle $\sin(2\beta)$ than $B^0 \rightarrow D^{*+} D^{*-}$. However, if the final state contains a resonance then $B^0$ and $\overline{B}^0$ can be distinguished and there is additional dilution of the CP asymmetry. For the decay $B \rightarrow f$ and $\overline{B} \rightarrow \bar{f}$ the dilution factor, $D$, measures the ratio of the overlap of the amplitudes for $B \rightarrow f$ and $\overline{B} \rightarrow \bar{f}$ to the average of the decay rate for $B \rightarrow f$ and $\overline{B} \rightarrow \bar{f}$.

Clearly $D = 1$ when the amplitudes for $B \rightarrow f$ and $\overline{B} \rightarrow \bar{f}$ decays are equal. When the final state in the decay $B \rightarrow D^{*+} D^{*-} K_s$ contains a resonance the amplitude for $B$ and $\overline{B}$ decays are different because the resonance in the $B$ and $\overline{B}$ final state occur at different kinematical points. This causes additional mismatch of the $B$ and $\overline{B}$ amplitudes which results in the further dilution of the CP asymmetry. A similar conclusion is obtained in the comparison of $B^0 \rightarrow D^{+} D^{-} K_s$ to $B^0 \rightarrow D^{+} D^-$. The above conclusions are detector dependent; a somewhat pessimistic estimate of the $K_s$ reconstruction efficiency is used here while the detection efficiency for the $D^{*+} D^{*-}$ final state is assumed to be similar for both cases. Better determination of the CP sensitivities will require more precise measurements of the branching fractions for the $D^* \overline{D}^* K$ decay modes and will also depend on details of the experimental apparatus and reconstruction programs.

The amplitude for the decay $B^0 \rightarrow D^* \overline{D}^* K_s$ can have a resonant contribution and a non-resonant contribution. For the resonant contribution the $D^* K_s$ in the final state comes dominantly from an excited $D_s(1^+)$ state. In the approximation of treating $D^* \overline{D}^* K_s$ as $D^* D_s(excited)$, there are four possible excited p-wave $D_s$ states which might contribute. These are the two states with the light degrees of freedom in a
\[ j^P = 3/2^+ \] state and the two states with light degrees of freedom in a \([j^P = 1/2^+]\) state. Since the states with \([j^P = 3/2^+]\) decay via d-wave to \(D^*K_s\), they are suppressed. Of the states with light degrees of freedom in \([j^P = 1/2^+]\) states, only the \(1^+\) state contributes. The \(0^+\) state is forbidden to decay to the final state \(D^*K_s\).

To estimate the above contribution and to calculate the non-resonant amplitude, we use heavy hadron chiral perturbation theory (HHCHPT)\(^{[12]}\). The momentum \(p_k\) of \(K_s\) can have a maximum value of about 1 GeV for \(B^0 \rightarrow D^{*+}\bar{D}^{*-}K_s\). This is of the same order as \(\Lambda_\chi\) which sets the scale below which we expect HHCHPT to be valid. It follows that in the present case it is reasonable to apply HHCHPT to calculate the three body decays.

In the lowest order in the HHCHPT expansion, contributions to the decay amplitude come from the contact interaction terms and the pole diagrams which give rise to the non-resonant and resonant contributions respectively. The pole diagrams get contributions from the various multiplets involving \(D_s\) type resonances as mentioned above. In the framework of HHCHPT, the ground state heavy meson has the light degrees of freedom in a spin-parity state \([j^P = 1^-]\), corresponding to the usual pseudoscalar-vector meson doublet with \([J^P = (0^-,1^-)]\). The first excited state involves a p-wave excitation, in which the light degrees of freedom have \([j^P = 1^+\) or \(3^+\). In the latter case we have a heavy doublet with \([J^P = (1^+,2^+)]\). These states can probably be identified with \(D_{s1}(2536)\) and \(D_{sJ}(2573)\)\(^{[13]}\). Heavy quark symmetry rules out any pseudoscalar coupling of this doublet to the ground state at lowest order in the chiral expansion \(^{[14]}\); hence the effects of these states will be suppressed and we will ignore them in our analysis. In fact there is an experimental upper limit on inclusive \(B \rightarrow D_{s1}(2536)X < 0.95\%\) at 90\% C.L \(^{[15]}\). Since the total \(D\bar{D}K\) rate is about 8\%, this confirms that the narrow p-wave states do not account for a significant fraction of the total \(D\bar{D}K\) rate.

The other excited doublet has \([J^P = (0^+,1^+)]\). These states are expected to decay rapidly through s-wave pion emission and have large widths \(^{[16]}\). Observation of the \(1^+\) state in the \(D\) system was recently reported by CLEO \(^{[17]}\). Only the \(1^+\) can contribute in this case. For later reference, we denote this state
by $D_{s1}'$. However, quark model estimates suggest \[18\] that these states should have masses near $m + \delta m$ with $\delta m = 500$ MeV, where $m$ is the mass of the lowest multiplet.

We will assume that the leading order terms in HHCHPT give the dominant contribution to the decay amplitude and so we will neglect all sub-leading effects suppressed by $1/\Lambda$ and $1/m$, where $m$ is the heavy quark mass. We show that from the time dependent analysis of $B^0(t) \rightarrow D^{*+}D^{*-}K_s$ one can extract $\sin(2\beta)$ and $\cos(2\beta)$. Measurement of both $\sin(2\beta)$ and $\cos(2\beta)$ can resolve the $\beta \rightarrow \pi/2 - \beta$ ambiguity \[3, 18, 20\]. The measurement of $\sin(2\beta)$ can be made from the time dependent partial rate asymmetry while a fit to the time dependent rate for $\Gamma[B^0(t) \rightarrow D^{*+}D^{*-}K_s] + \Gamma[\bar{B}^0(t) \rightarrow D^{*+}D^{*-}K_s]$ may be used for the extraction of $\cos(2\beta)$. Note that the $\cos(2\beta)$ term measures the overlap of the imaginary part of the amplitudes for $B \rightarrow D^{*+}D^{*-}K_s$ and $\bar{B} \rightarrow D^{*+}D^{*-}K_s$ decays and is non zero only if there is a resonance contribution.

As in the case for $B \rightarrow D^{*+}D^{*-}$ the asymmetry in $B \rightarrow D^{*+}D^{*-}K_s$ is also diluted. For the non resonant contribution to $B \rightarrow D^{*+}D^{*-}K_s$ the final state is an admixture of CP states with different CP parities. This leads to the dilution of the asymmetry and this is the same dilution of the asymmetry as in the case for $B \rightarrow D^{*+}D^{*-}$. As already mentioned above, when the resonant contribution is included there is further dilution of the asymmetry from the additional mismatch of the amplitudes for $B$ and $\bar{B}$ decays. One can reduce the additional dilution of the CP asymmetry by imposing cuts to remove the resonance. A narrow resonance is preferable as it can be more effectively removed from the signal region than a broad resonance. In this work we examine several cuts that can be used to remove the resonance and lessen the dilution of the CP asymmetry. When we include the resonance contribution we find that a broader resonance leads to a larger value of $D$ and is a more useful probe of $\cos(2\beta)$ because of the larger overlap of the amplitudes for $B \rightarrow D^{*+}D^{*-}K_s$ and $\bar{B} \rightarrow D^{*+}D^{*-}K_s$ decays.

We also point out that from the differential decay distribution of the time independent process $B^0 \rightarrow D^{*+}D^{*-}K_s$ one can discover the $1^+$ resonance $D_{s1}'$. We show that the differential decay distribution for small values of $E_k$, the kaon energy, shows a clear resonant structure which comes from the pole contribution
to the amplitude with the excited $J^P = 1^+$ intermediate state. Therefore, examination of the $D^*K_s$ mass spectrum may be the best experimental way to find the broad $1^+$ p-wave $D_s$ meson and a fit to the decay distribution will measure its mass and the coupling.

A similar analysis can be performed for $B^0 \rightarrow D^+D^-K_s$ [5, 21]. However the predictions of HHCHPT for this mode may be less reliable because of the larger energy of the $K_s$. The effects of penguin contributions, though small, may also be more important in $B^0 \rightarrow D^+D^-K_s$ than in $B^0 \rightarrow D^*D^*K_s$ as in the two body case [3].

In the next section we describe the extraction of of $\sin 2\beta$ and $\cos 2\beta$ from the time dependent rate for $B(t) \rightarrow D^{*+}D^{*-}K_s$. In the next section we present the the amplitude for $B \rightarrow D^*\bar{D}^*K_s$ in the factorization approximation and using HHCHPT. In the last section we discuss and present our results.

### 2 Extraction of $\sin 2\beta$ and $\cos 2\beta$

In this section we discuss the extraction of $\sin 2\beta$ and $\cos 2\beta$ from the time dependent rate for $B(t) \rightarrow D^{*+}D^{*-}K_s$. We define the following amplitudes

$$a^{\lambda_1, \lambda_2} = A(B^0(p) \rightarrow D_{\lambda_1}^{*+}(p_+)D_{\lambda_2}^{*-}(p_-)K_s(p_k)), \quad \bar{a}^{\lambda_1, \lambda_2} = A(\bar{B}^0(p) \rightarrow D_{\lambda_1}^{*+}(p_+)D_{\lambda_2}^{*-}(p_-)K_s(p_k)), \quad (1)$$

where $B^0$ and $\bar{B}^0$ represent unmixed neutral $B$ and $\lambda_1$ and $\lambda_2$ are the polarization indices of the $D^{*+}$ and $D^{*-}$ respectively.

The time-dependent amplitudes for an oscillating state $B^0(t)$ which has been tagged as a $B^0$ meson at time $t = 0$ is given by,

$$A^{\lambda_1, \lambda_2}(t) = a^{\lambda_1, \lambda_2} \cos \left( \frac{m t}{2} \right) + i e^{-2i\beta} \bar{a}^{\lambda_1, \lambda_2} \sin \left( \frac{m t}{2} \right), \quad (2)$$

and the time-dependent amplitude squared summed over polarizations and integrated over the phase space angles is:

$$|A(s^+, s^-; t)|^2 = \frac{1}{2} \left[ G_0(s^+, s^-) + G_c(s^+, s^-) \cos \Delta m t - G_s(s^+, s^-) \sin \Delta m t \right] \quad (3)$$
with

\[ G_0(s^+, s^-) = |a(s^+, s^-)|^2 + |\bar{a}(s^+, s^-)|^2, \]
\[ G_c(s^+, s^-) = |a(s^+, s^-)|^2 - |\bar{a}(s^+, s^-)|^2, \]
\[ G_s(s^+, s^-) = 2 \Re \left( e^{-2i\beta} \bar{a}(s^+, s^-) a^*(s^+, s^-) \right) \]
\[ = 2\sin(2\beta) \Re (\bar{a}a^*) + 2\cos(2\beta) \Im (\bar{a}a^*). \]

(4)

(5)

(6)

The variables \( s^+ \) and \( s^- \) are the Dalitz plot variable

\[ s^+ = (p_+ + p_k)^2, \quad s^- = (p_- + p_k)^2 \]

The transformation defining the CP-conjugate channel \( B^0(t) \to D^{*-}D^{*+}K_s \) is \( s^+ \leftrightarrow s^- \), \( a \leftrightarrow \bar{a} \) and \( \beta \to -\beta \). Then:

\[ |\bar{A}(s^-, s^+; t)|^2 = \frac{1}{2} [G_0(s^-, s^+) - G_c(s^-, s^+) \cos \Delta m t + G_s(s^-, s^+) \sin \Delta m t]. \]

(7)

Note that for simplicity the \( e^{-\Gamma t} \) and constant phase space factors have been omitted in the above equations.

It is convenient in our case to replace the variables \( s^+ \) and \( s^- \) by the variables \( y \) and \( E_k \) where \( E_k \) is the \( K_s \) energy in the rest frame of the \( B \) and \( y = \cos \theta \) with \( \theta \) being the angle between the momentum of \( K_s \) and \( D^{*+} \) in a frame where the two \( D^* \) are moving back to back along the \( z \)-axis. This frame is boosted with respect to the rest frame of the \( B \) with \( \vec{\beta} = -(\vec{p}_k/m_B)(1/(1 - E_k/m_B)) \). Note \( s^+ \leftrightarrow s^- \) corresponds to \( y \leftrightarrow -y \). The variable \( y \) can be expressed in terms of variables in the rest frame of \( B \). For instance

\[ E_+ = \frac{E'_B E'_+ - p'_B p'_+ y}{m_B} \]

where \( E_+ \) and \( E'_+ \) are the energy of the \( D^{*+} \) in the rest frame of the \( B \) and in the boosted frame while \( E'_B \) is the energy of the \( B \) in the boosted frame. The magnitudes of the momentum of the \( B \) and the \( D^{*+} \) in the boosted frame are given by \( p'_B \) and \( p'_+ \) respectively.

In the approximation of neglecting the penguin contributions to the amplitude there is no direct CP violation. This leads to the relation

\[ a^{\lambda_1, \lambda_2}(\vec{p}_{k1}, E_k) = \bar{a}^{-\lambda_1, -\lambda_2}(-\vec{p}_{k1}, E_k) \]

(8)
where $\vec{p}_{k1}$ is the momentum of the of the $K_s$ in the boosted frame. The above relations then leads to

\[
G_0(-y,E_k) = G_0(y,E_k) \quad (9)
\]
\[
G_c(-y,E_k) = -G_c(y,E_k) \quad (10)
\]
\[
G_{s1}(-y,E_k) = G_{s1}(y,E_k) \quad (11)
\]
\[
G_{s2}(-y,E_k) = -G_{s2}(y,E_k) \quad (12)
\]

where we have defined

\[
G_{s1}(y,E_k) = \Re(\bar{a}a^*) \quad (13)
\]
\[
G_{s2}(-y,E_k) = \Im(\bar{a}a^*) \quad (14)
\]

Carrying out the integration over the phase space variables $y$ and $E_k$ one gets the following expressions for the time-dependent total rates for $B^0(t) \rightarrow D^{*+} D^{-} K_s$ and the CP conjugate process

\[
\Gamma(t) = \frac{1}{2}[I_0 + 2 \sin(2\beta) \sin(\Delta mt) I_{s1}] \quad (15)
\]
\[
\Gamma(t) = \frac{1}{2}[I_0 - 2 \sin(2\beta) \sin(\Delta mt) I_{s1}] \quad (16)
\]

where $I_0$ and $I_{s1}$ are the integrated $G_0(y,E_k)$ and $G_{s1}(y,E_k)$ functions. One can then extract $\sin(2\beta)$ from the rate asymmetry

\[
\frac{\Gamma(t) - \Gamma(t)}{\Gamma(t) + \Gamma(t)} = D \sin(2\beta) \sin(\Delta mt) \quad (17)
\]

where

\[
D = \frac{2I_{s1}}{I_0} \quad (18)
\]

is the dilution factor.

The $\cos(2\beta)$ term can be probed by by integrating over half the range of the variable $y$ which can be taken for instance to be $y \geq 0$. In this case we have

\[
\Gamma(t) = \frac{1}{2}[J_0 + J_c \cos(\Delta mt) + 2 \sin(2\beta) \sin(\Delta mt) J_{s1} - 2 \cos(2\beta) \sin(\Delta mt) J_{s2}] \quad (19)
\]
\[
\Gamma(t) = \frac{1}{2}[J_0 + J_c \cos(\Delta mt) - 2 \sin(2\beta) \sin(\Delta mt) J_{s1} - 2 \cos(2\beta) \sin(\Delta mt) J_{s2}] \quad (20)
\]
where \( J_0, J_c, J_{s1} \) and \( J_{s2} \), are the integrated \( G_0(y, E_k), G_c(y, E_k), G_{s1}(y, E_k) \) and \( G_{s2}(y, E_k) \) functions integrated over the range \( y \geq 0 \). One can measure \( \cos(2 \beta) \) by fitting to the time distribution of \( \Gamma(t) + \bar{\Gamma}(t) \). Measurement of the \( \cos(2 \beta) \) can resolve the \( \beta \rightarrow \frac{\pi}{2} - \beta \) ambiguity.

3 Amplitude and Decay Distribution

In this section we present the amplitude and decay distribution for the decay \( B \to D^{*+} D^{*-} K_s \). Details of the calculation of the amplitudes using the factorization assumption and HHCHPT are given in Appendix A.

The non-resonant amplitude for the three body decay \( \bar{B}^0(v, m) \to D^{*+}(\epsilon_1, v_+), m_1)D^{*-}(\epsilon_2, v_-, m_2)K_s(p_k) \), after setting \( m_2 = m_1 \), is given by

\[
\pi_{\text{non-res}} = K \sqrt{\frac{m}{m_1}} m_1 \xi(v \cdot v_+) \frac{f_{D^*}}{f_K} \left[ i \varepsilon^{\mu \nu \alpha \beta} \epsilon_1^* \epsilon_2^* \epsilon_1 v_{+\beta} + \epsilon_1^* \cdot v_2^* \cdot v_+ - \epsilon_1^* \cdot \epsilon_2^* (v \cdot v_+ + 1) \right]
\]

where

\[
K = \frac{G_F}{\sqrt{2}} V_c \left( \frac{\alpha_c}{\sqrt{2}} + c_2 \right).
\]

Note that the amplitude above is the same as the amplitude for \( B^0 \to D^{*-} D^{*+} \) except for a constant multiplicative factor \( \sim 1/f_K \).

To a good approximation one can use \( \bar{v} \sim 0 \) where \( \bar{v} \) is the velocity of the \( \bar{B}^0 \) in the boosted frame where the two \( D^* \) are moving back to back. The \( K_s \), in this limit, is emitted in a s-wave configuration as the amplitude is independent of the angles that specify the \( K_s \) momentum in the boosted frame. Then, as in the \( \bar{B}^0 \to D^{*+} D^{*-} \) case there are three helicity states allowed, \((+,+), (-,-) \) and \((0,0)\) with the corresponding helicity amplitudes \( H_{++}, H_{--} \) and \( H_{00} \). The helicity states are not CP eigenstates but one can go to the partial wave basis or the transverse basis where the states are CP eigenstates. The transverse basis amplitudes are related to the helicity amplitudes as

\[
A_\parallel = \frac{H_{++} + H_{--}}{\sqrt{2}}
\]

\[
A_\perp = \frac{H_{++} - H_{--}}{\sqrt{2}}
\]

\[
A_0 = H_{00}
\]
The three partial waves that are allowed in this case, $s$, $p$ and $d$ are then given by

$$
 s = \frac{\sqrt{2}A_{\parallel} - A_0}{\sqrt{3}} \\
p = A_{\perp} \\
d = \frac{\sqrt{2}A_0 + A_{\parallel}}{\sqrt{3}}
$$

(23)

The CP of the final state is given by $\eta(-)^L$ where $\eta$ is the intrinsic parity of the final states and $L$ is the relative angular momentum between $D^{*+}$ and $D^{*-}$.

In the approximation $\vec{v} \sim 0$ one can write the non-resonant amplitude for $B^0(v, m) \to D^{*+}(\epsilon_1, v_+, m_1)D^{*-}(\epsilon_2, v_-, m_1)K_s(p_k)$

$$
a_{non-res} = K\sqrt{m}m_1\xi(v \cdot v_-)f_{Ds}f_K
$$

$$
\left[ -i\varepsilon^{\mu\nu\alpha\beta}\epsilon_{2\mu}^*\epsilon_{1\nu}^*v_\alpha v_-\beta + \epsilon_2^* \cdot v_\epsilon_1^* \cdot v_- - \epsilon_1^* \cdot \epsilon_2^*(v \cdot v_- + 1) \right]
$$

(24)

There can also be pole contributions of the type shown in Figure 1. 

Figure 1: The pole contribution to the process $B \to D^{*}D^{*}K_S$. The intermediate state $I$ can be $D_{s1}^{**}$ or $D_{s}^{*}$. The solid square represents the weak vertex while the solid circle represents the strong vertex.

These give the decay sequences

$$
\bar{B}^0 \to D^{*+}D_{s1}^{**}\to D^{*+}D^{*-}K^0
$$

and

$$
\bar{B}^0 \to D^{*+}D_{s}^{*}\to D^{*+}D^{*-}K^0
$$
The propagator for the vector resonance is given by

\[
S_{\mu\nu} = \frac{i(V_\mu V_\nu - g_{\mu\nu})}{2V \cdot k}
\]  

(25)

where the momentum of the propagating particle \( P = m_I V + k \) where \( m_I \) is the mass of the intermediate particle in Figure. 1.

The contribution from the pole diagrams are given by \( \pi_{1\text{res}} \) and \( \pi_{2\text{res}} \), where \( \pi_{1\text{res}} \) is, with \( m_I = m^{\prime}\),

\[
\pi_{1\text{res}} = K \sqrt{m} \sqrt{m_1} \sqrt{m_1} \sqrt{m^{\prime}} \xi(v.v_+) \frac{f_{D^{\prime}11}^{*}}{f_K} \frac{h_{p_k \cdot v_-}}{(p_k \cdot v_- + m_1 - m^{\prime} + \frac{i\Gamma_{D^{\prime}11}}{2})}
\]

(26)

Note that the above amplitude can be rewritten as

\[
\pi_{1\text{res}} = -\pi_{\text{non-res}} \frac{f_{D^{\prime}11}^{*}}{f_{D^{\prime}}} \sqrt{\frac{m^{\prime}}{m_1}} \frac{h_{p_k \cdot v_-}}{(p_k \cdot v_- + m_1 - m^{\prime} + \frac{i\Gamma_{D^{\prime}11}}{2})}
\]

(27)

\( \pi_{2\text{res}} \) is given by, with \( m_I = m^{\ast} \) where \( m^{\ast} \) is the \( 1^\text{-} D^{\ast}_{s} \) mass,

\[
\pi_{2\text{res}} = K \sqrt{m_1} \sqrt{m_1} \sqrt{m^{\ast}} \xi(v.v_+) \frac{f_{D^{\ast}11}^{*}}{f_K} \frac{g}{(p_k \cdot v_- + (m_1 - m^{\ast}) + \frac{i\Gamma_{D^{\ast}11}}{2})} X
\]

(28)

\[
X = -i\varepsilon^{\mu\nu\alpha\beta} \epsilon_{2uP_k v_+} v_+ v_\mu v_\nu v_\alpha v_\beta + i\varepsilon^{\mu\nu\alpha\beta} \epsilon^{*}_{1u} \epsilon^{*}_{2v} \epsilon^{*}_{2u} \epsilon^{*}_{2v} \epsilon^{*}_{2v} \epsilon^{*}_{2v} \epsilon^{*}_{2v} \epsilon^{*}_{2v} (v_+ v_+ + 1) + (\epsilon^{*}_{2u} v_+ v_+ v_\mu v_\nu v_\alpha v_\beta - \epsilon^{*}_{2u} v_+ v_+ v_\mu v_\nu v_\alpha v_\beta) + (\epsilon^{*}_{2u} v_+ v_+ v_+ v_+ v_\mu v_\nu v_\alpha v_\beta - \epsilon^{*}_{2u} v_+ v_+ v_+ v_+ v_\mu v_\nu v_\alpha v_\beta)
\]

The amplitude \( \pi_{2\text{res}} \) gives a tiny contribution to the total amplitude and can be neglected. In fact, this amplitude vanishes in the small velocity limit where the \( D^{\ast} \) are almost at rest \([22]\). We note that the process with the \( 0^+ \) intermediate state

\[
\bar{B}^0 \rightarrow D^{\ast+} D_{s0} \rightarrow D^{\ast+} D^{\ast-} K^0
\]

is not allowed due to parity conservation while the amplitude with the \( 0^- \) intermediate state

\[
\bar{B}^0 \rightarrow D^{\ast+} D_{s}^{-} \rightarrow D^{\ast+} D^{\ast-} K^0
\]

is expected to be small compared to \( \pi_{1\text{res}} \). The propagator term in the above amplitude goes as approximately \( 1/(E_K + (m_{D^{\ast}} - m_{D_s})) \) which does not have a pole as in \( \pi_{1\text{res}} \). Moreover, the amplitude is further
suppressed with respect to $\alpha_{\text{res}}$ by a factor $\sim p_k/E_k$ or $|\vec{v}|/v_0$, where $\vec{v}$ and $v_0$ are the three velocity and the time component of the velocity four vector of the $D^*$, from the $D_s^+D^{*+}K^0$ vertex.

The total amplitude for $\bar{B}^0(v,m) \rightarrow D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$ can be written as

$$\alpha = \alpha_{\text{non-res}}[1 - P_1]$$

and the total amplitude for $B^0(v,m) \rightarrow D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$ can be written as

$$a = a_{\text{non-res}}[1 - P_2]$$

with

$$P_1 = \frac{f_{D_{s1}^{*}}}{f_{D^*}} \sqrt{\frac{\epsilon}{m_1}} \frac{h p_k \cdot v_-}{(p_k \cdot v_+ + m_1 - m_1' + \frac{i\Gamma_{D_{s1}^{*}}}{2})}$$

$$P_2 = \frac{f_{D_{s1}^{*}}}{f_{D^*}} \sqrt{\frac{\epsilon}{m_1}} \frac{h p_k \cdot v_+}{(p_k \cdot v_+ + m_1 - m_1' + \frac{i\Gamma_{D_{s1}^{*}}}{2})}$$

Note $P_1$ and $P_2$ can be expressed in terms of $E_k$ and $y$ and $P_1(y,E_k) = P_2(-y,E_k)$. The relation between quantities in the boosted frame and the rest frame of the $B$ along with the calculation of the squared amplitude are given in Appendix B.

The double differential decay distribution for the time independent process

$$\bar{B}^0(v,m) \rightarrow D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$$

can be written as

$$\frac{1}{\Gamma} \frac{d\Gamma}{dydE_k} = \frac{f(y,E_k)}{\int f(y,E_k) \frac{p_k p_{s1}^{*}}{m} dydE_k}$$

where $p_k'$ and $p_s'$ are the magnitudes of the three momentum of the $K_s$ and $D^{*+}$ in the boosted frame and the expression for $f(y,E_k)$ can be found in Appendix B. The differential distribution depends only on $f_{D_{s1}^{*}}$ and the mass $m_{s1}^{*}$ and the coupling $h$ of the $D_{s1}^{*}$ state. It is expected that $f_{D_{s1}^{*}} \approx f_{D_s^*}$ and in the $SU(3)$ limit $f_{D_s^*} = f_{D^*}$. So in the $SU(3)$ limit a two parameter fit to the differential decay distribution can determine the mass and the coupling of the $D_{s1}^{*}$ state.
The widths of the positive parity excited states are expected to be saturated by single kaon transitions \[4\]. In our calculation we require the width of the \(D_{s1}^{*+} \) state. Assuming

\[
\Gamma_{D_{s1}^{*+}} \approx \Gamma(D_{s1}^{*+} \to D^{*+} K^0) + \Gamma(D_{s1}^{*+} \to D^{*0} K^+) \tag{34}
\]

one can write

\[
\Gamma_{D_{s1}^{*+}} = \frac{\hbar^2}{\pi f_K} \frac{m_1}{m^{*'}} (m^{*'} - m_1)^2 p \tag{35}
\]

where \(p\) is the magnitude of the three momentum of the decay products in the rest frame of \(D_{s1}^{*+}\) and \(m_1\) and \(m^{*'}\) are the masses of the \(D_{s}^{*}\) and \(D_{s1}^{*+}\) state.

It is clear that if \(a = \bar{a}\) then the dilution factor \(D = 1\). However that is not the case here. For the non resonant contribution, in the approximation of small velocity of the \(B\), the final state is an admixture of CP states with different CP parities. This leads to \(D < 1\). This is the same dilution of the asymmetry as in the case for \(B \to D^{*+} D^{*-}\). When the resonant contribution is included the amplitude \(a\) and \(\bar{a}\) have an asymmetric dependence on the variable \(y\). This reflects the fact that in the process \(\bar{B}^0 \to D^{*+} D_{s1}^{*-} \to D^{*+} D^{*-} K^0\) the kaon emerges most of the time closer to \(D^{*-}\) than the \(D^{*+}\). The situation is reversed for \(B^0\) decays. Consequently there is additional mismatch between the amplitudes \(a\) and \(\bar{a}\) which leads to further dilution of the asymmetry. One can reduce the dilution of the asymmetry, \(i.e.,\) increase \(D\), by imposing cuts so as to reduce the resonant contribution. We consider several cases where cuts may be employed to decrease the dilution of the asymmetry. From Eq. (31-32) it is clear that resonance occurs when the following condition is met

\[
p_k \cdot v_+ = m^{*'} - m_1 \tag{36}
\]

\[
p_k \cdot v_- = m^{*'} - m_1 \tag{37}
\]

If, in the allowed region of \(E_k\), we can find a value \(E_{k0}\) such that for values of \(E_k \geq E_{k0}\) the above conditions are not satisfied for \(-1 \leq y \leq 1\) then we can remove the resonance by using the cut \(E_k \geq E_{k0}\). The value of \(E_{k0} \sim 0.76\) GeV in our case. We will call this case cut 1 for future reference.
Another possible cut is to include the whole range of \( E_k \) but in the region \( E_k \leq E_{k0} \) we remove the resonance by cutting on the variable \( y \). We can use the region \(-0.5 \leq y \leq 0.5\) since for most values of \( E_k \) the resonance condition is satisfied in the range \(-1 \leq y \leq -0.5\) and \(0.5 \leq y \leq 1\). We will call this case cut 2 for future reference. In any event, the cuts can be optimized after the resonance has been seen experimentally. However as we try to increase the value of \( D \) by reducing the resonance through cuts we also lessen the branching ratio from loss of signal.

4 Results and Discussions

![Graph](image)

Figure 2: The branching fraction for \( \bar{B}^0 \to D^{*+}D^{*-}K_s \) as a function of the \( h \) with and without cuts.

As inputs to the calculation, we use \( f_{D^*} \approx f_{D^*_s} = 200 \) MeV and take the mass of the \( D^*_s \) state to be 2.6 GeV. For the Isgur-Wise function we use the form

\[
\xi(\omega) = \left(\frac{2}{1 + \omega}\right)^2.
\]

QCD sum rules have been used to compute the strong coupling constants \( g \) and \( h \) \cite{Ref}. We will use \( g = 0.3 \) as obtained in Ref \cite{Ref} but keep \( h \) as a free parameter because this coupling plays a more important role.
Figure 3: The Dilution factor $D$ as a function of the $h$ with and without cuts.

in the decay widths.

Fig. 2 shows the branching fraction for $\bar{B}^0 \to D^{*+} D^{*-} K_s$ as a function of the coupling $h$. A QCD sum rule calculation gives $h \sim -0.5$ \cite{23}. We use the same sign of $h$ as obtained in QCD sum rule calculation but vary $h$ from $-0.6$ to $-0.1$. For this range of $h$ the branching fraction can vary in the range $0.45 - 0.93\%$ when we employ no cuts. For $h = -0.4$ which corresponds to a $D_s^{*+}$ state with a width of about 150 MeV the branching fraction is $0.83\%$. In our calculation this corresponds to a branching ratio $\mathcal{B}(B^0 \to D^{*-} D^{*+} K^0)\approx\mathcal{B}(B^0 \to D^{*-} D^{*0} K^+)\approx\mathcal{B}(B^+ \to \bar{D}^{*0} D^{*+} K^+)\approx\mathcal{B}(B^+ \to \bar{D}^{*0} D^{*+} K^0)\approx 0.9 - 1.86\%$. This is consistent with the CLEO measurements mentioned above. Fig. 2 also shows the branching fractions after cuts have been applied to reduce the dilution of the CP asymmetry.

Fig. 3 shows a plot of the dilution factor $D$ versus the coupling $h$. In the absence of any cuts we find that larger values of $|h|$ gives a larger value of $D$ and hence less dilution in the asymmetry because for a broad $D_s^{*+}$ state there is more overlap between the amplitudes for $B^0 \to D^{*+} D^{*-} K_s$ and $\bar{B}^0 \to D^{*+} D^{*-} K_s$. For $h = -0.4$ the dilution factor is about 0.75 with no cuts. For the case of cut 1, where we use the cut $E_k > E_{k0}$ to effectively remove the resonance, the dilution factor increases with smaller $|h|$. This is because
Figure 4: The squared amplitude for $B^0 \rightarrow D^{*+} D^{*-} K_s$ $\bar{B}^0 \rightarrow D^{**} D^{*-} K_s$ as a function of the variable $y$ for $h = -0.4$ which corresponds to a $D_{s1}^{*'}$ state with a width of about 150 MeV.

for smaller $|h|$ and $E_k > E_{k0}$ the resonant amplitude is small and the total amplitude is dominated by the non-resonant amplitude which give a larger value for $D$. For the case of cut 2, as in the case with no cuts, the dilution factor $D$ decreases with smaller $|h|$. This is because we are using the entire region of $E_k$ and not removing the resonance by the cut $E_k > E_{k0}$ as in the case of cut 1. Consequently a broader resonance and hence a larger value of $|h|$ gives a larger value of $D$ and vice versa.

Figure 4 shows the squared amplitude for $B^0 \rightarrow D^{*+} D^{*-} K_s$ and $\bar{B}^0 \rightarrow D^{**} D^{*-} K_s$ as a function of the variable $y$ for $h = -0.4$. As mentioned above the nature of the two curves reflects the fact that in the process $\bar{B}^0 \rightarrow D^{**} D_{s1}^{*'} \rightarrow D^{*+} D^{*-} K^0$ the kaon emerges most of the time closer to $D^{*-}$ than the $D^{*+}$ while the situation is reversed for $B^0$ decays.

Fig. 5 shows the plot of the function $G_0, G_c, G_{s1}$ and $G_{s2}$ as a function of $y$ for $E_k = 0.6$ GeV and for $h = -0.4$. From the figure we see that the functions $G_0$ and $G_{s1}$ are symmetric in $y$ while $G_c$ and $G_{s2}$ are antisymmetric in $y$. This follows from the absence of direct CP violation as shown in Eq. (19-22).

In Fig. 6 we show the the decay distribution $d\Gamma/dE_k$ versus the kaon energy $E_k$. For small values of
$E_k$ the decay distribution shows a clear resonant structure which comes from the pole contribution to $a^{1\text{res}}$ with the excited $J^P = 1^+$ intermediate state. Therefore, examination of the $D^*K_s$ mass spectrum may be the best experimental way to find the broad $1^+$ p-wave $D_s$ meson and as mentioned in the previous section a fit to the decay distribution will measure its mass and the coupling.

In Fig. 7 we show the functions $G_0$, $G_c$, $G_{s1}$ and $G_{s2}$ integrated over the $y \geq 0$ as a function of $h$. $J_0$ and $J_c$ refer to the integrated $G_0$ and $G_c$ functions while $J_{s1}$ and $J_{s2}$ refer to the integrated $G_{s1}$ and $G_{s2}$ functions. As already mentioned, restricting the integration range to $y \geq 0$ allows a probe of the $\cos(2\beta)$ term in the time dependent rate for $B^0(t) \to D^{(*)}D^{(*)}K_s$ decays. It is clear from the figure that a broader resonance is more favorable to probe $G_{s2}$ which is the coefficient of the $\cos(2\beta)$ term.

In summary, we have studied the possibility of extracting $\sin(2\beta)$ and $\cos(2\beta)$ from time dependent $B^0 \to D^{(*)}\overline{D}^{(*)}K_s$ decays. These decays are expected to have less penguin contamination and much larger branching fractions than the two body modes $B^0 \to D^{(*)}\overline{D}^{(*)}$. Using HHCHPT we have calculated the branching fractions and the various coefficient functions that appear in the time dependent rate for $B^0 \to D^{(*)}D^{(*)}K_s$. We also showed that a examination of the $D^*K_s$ mass spectrum may be the best
Figure 6: The decay distribution $d\Gamma/dE_k$ versus the kaon energy $E_k$.

experimental way to find the broad $1^+$ p-wave $D_s$ meson and measure its mass and coupling.

5 Acknowledgements

We thank Yuval Grossman for an important observation. This work was supported in part by the United States Department of Energy (T. Browder and S. Pakvasa), and by the Natural Sciences and Engineering Research Council of Canada (A. Datta and P. J. O’Donnell).
Figure 7: The functions \( G_0, G_c, G_{s1} \) and \( G_{s2} \) integrated over the \( y \geq 0 \) as a function of \( h \). \( J_0 \) and \( J_c \) refer to the integrated \( G_0 \) and \( G_c \) functions while \( J_{s1} \) and \( J_{s2} \) refer to the integrated \( G_{s1} \) and \( G_{s2} \) functions. The values of the integral can be obtained by multiplying by \( \Gamma_B \) where \( \Gamma_B \) is the width of the \( B \).

**Appendix A**

In the Standard Model (SM) the amplitudes for \( B \to D(\bar{D}^{(*)})K_s \) are generated by the following effective Hamiltonian \cite{24, 25}:

\[
H_{\text{eff}}^q = \frac{G_F}{\sqrt{2}} [V_{tb}V_{tq}^*(c_1O_{1f}^q + c_2O_{2f}^q) - \sum_{i=3}^{10} (V_{ub}V_{uq}^* c_i^u + V_{cb}V_{cq}^* c_i^c + V_{tb}V_{tq}^* c_i^t) O_{i}^q] + H.C. ,
\]  

(A. 1)

where the superscript \( u, c, t \) indicates the internal quark, \( f \) can be \( u \) or \( c \) quark, \( q \) can be either a \( d \) or a \( s \) quark depending on whether the decay is a \( \Delta S = 0 \) or \( \Delta S = -1 \) process. The operators \( O_i^q \) are defined as

\[
O_{1f}^q = \bar{q}_\alpha \gamma_\mu L f_\beta \bar{f}_\gamma \gamma_\mu L b_\alpha, \quad O_{2f}^q = \bar{q}_\gamma \mu L f_\gamma \gamma_\mu L b ,
\]

\[
O_{3,5}^q = \bar{q}_\gamma \mu L b q'_\gamma \gamma_\mu L(R) q', \quad O_{4,6}^q = \bar{q}_\alpha \gamma_\mu L b_\beta \bar{q}_\rho \gamma_\mu L(R) q'_\alpha,
\]

\[
O_{7,9}^q = \frac{3}{2} \bar{q}_\gamma \mu L b e_{q'} q'^{\gamma_\mu} R(L) q', \quad O_{8,10}^q = \frac{3}{2} \bar{q}_\alpha \gamma_\mu L b_\beta e_{q'} q'_\beta \gamma_\mu R(L) q'_\alpha,
\]

where \( R(L) = 1 \pm \gamma_5 \), and \( q' \) is summed over all flavors except \( t \). \( O_{1f,2f} \) are the current-current operators that represent tree level processes. \( O_{3-6} \) are the strong gluon induced penguin operators, and operators
$O_{7-10}$ are due to $\gamma$ and $Z$ exchange (electroweak penguins), and “box” diagrams at loop level. The Wilson coefficients $c_i^f$ are defined at the scale $\mu \approx m_b$ and have been evaluated to next-to-leading order in QCD. The $c_i^f$ are the regularization scheme independent values obtained in Ref. $[26]$. We give the non-zero $c_i^f$ below for $m_t = 176$ GeV, $\alpha_s(m_Z) = 0.117$, and $\mu = m_b = 5$ GeV,

\[
\begin{align*}
c_1 &= -0.307, \quad c_2 = 1.147, \quad c_3 = 0.017, \quad c_4 = -0.037, \quad c_5 = 0.010, \quad c_6 = -0.045, \\
c_7^t &= -1.24 \times 10^{-5}, \quad c_8^t = 3.77 \times 10^{-4}, \quad c_9^t = -0.010, \quad c_{10}^t = 2.06 \times 10^{-3}, \\
c_{5,6}^{u,e} &= -c_{4,6}^{u,e}/N_c = P_5^{u,e}/N_c, \quad c_{7,9}^{u,e} = P_7^{u,e}, \quad c_{8,10}^{u,e} = 0
\end{align*}
\]

where $N_c$ is the number of color. The leading contributions to $P_i^{u,e}$ are given by: $P_s^i = (\frac{n_c}{5\pi})c_2(\frac{10}{9} + G(m_i, \mu, q^2))$ and $P_e^i = (\frac{n_c}{9\pi})(N_c c_1 + c_2)(\frac{10}{9} + G(m_i, \mu, q^2))$. The function $G(m, \mu, q^2)$ is given by

\[
G(m, \mu, q^2) = 4 \int_0^1 x(1-x)\ln\frac{m^2 - x(1-x)q^2}{\mu^2} \, dx .
\]

All the above coefficients are obtained up to one loop order in electroweak interactions. The momentum $q$ is the momentum carried by the virtual gluon in the penguin diagram. When $q^2 > 4m^2$, $G(m, \mu, q^2)$ becomes imaginary. In our calculation, we use $m_u = 5$ MeV, $m_d = 7$ MeV, $m_s = 200$ MeV, $m_c = 1.35$ GeV $[27, 13]$.

In the factorization assumption the amplitude for $B \to D^{(*)} \bar{D}^{(*)} K_s$ can now be written as

\[
M = M_1 + M_2 + M_3 + M_4
\]

where

\[
\begin{align*}
M_1 &= \frac{G_F}{\sqrt{2}} X_1 < \bar{D}^{(*)} K_s | \bar{s} \gamma^\mu (1 - \gamma^5) c | 0 > < D^{(*)} | \bar{c} \gamma_\mu (1 - \gamma^5) b | B > \\
M_2 &= \frac{G_F}{\sqrt{2}} X_2 < \bar{D}^{(*)} D^{(*)} | \bar{c} \gamma^\mu (1 - \gamma^5) c | 0 > < K_s | \bar{s} \gamma_\mu (1 - \gamma^5) b | B > \\
M_3 &= \frac{G_F}{\sqrt{2}} X_3 < \bar{D}^{(*)} D^{(*)} | \bar{c} \gamma^\mu (1 + \gamma^5) c | 0 > < K_s | \bar{s} \gamma_\mu (1 - \gamma^5) b | B > \\
M_4 &= \frac{G_F}{\sqrt{2}} X_4 < \bar{D}^{(*)} K_s | \bar{s} (1 + \gamma^5) c | 0 > < D^{(*)} | \bar{c} (1 - \gamma^5) b | B > \end{align*}
\]
where

\[ X_1 = V_c \left( \frac{c_1}{N_c} + c_2 \right) + B_3 + \frac{B_4}{N_c} + \frac{B_9}{N_c} + B_{10} \]

\[ X_2 = V_c \left( c_1 + \frac{c_2}{N_c} \right) + B_3 + \frac{1}{N_c} B_4 + B_9 + \frac{1}{N_c} B_{10} \]

\[ X_3 = B_5 + \frac{1}{N_c} B_6 + B_7 + \frac{1}{N_c} B_8 \]

\[ X_4 = -2 \left( \frac{1}{N_c} B_5 + B_6 + \frac{1}{N_c} B_7 + B_8 \right) \]  \hspace{1cm} (A. 8)

We have defined

\[ B_i = - \sum_{q=u,c,t} c_i^q V_q \]  \hspace{1cm} (A. 9)

with

\[ V_q = V_{qs}^* V_{qb} \]  \hspace{1cm} (A. 10)

In the above equations \( N_c \) represents the number of colors. It is usually the practice in the study of two body non-leptonic decays to include non-factorizable effects by the replacement \( N_c \to N_{eff} \). Since it is not obvious that \( N_{eff} \) for two body non-leptonic decays is the same for non-leptonic three body decays we will use \( N_c = 3 \) in our calculation.

As already mentioned, we expect the contribution from penguin diagrams to be small and so as a first approximation we will neglect \( M_3 \) and \( M_4 \). Furthermore, from the values of the Wilson coefficients \( c_{1,2} \) given above in the previous section it is clear that the amplitude \( M_2 \) is suppressed with respect to \( M_1 \) with the Wilson coefficients associated with \( M_2 \) being about 7 \% of the Wilson coefficients associated with \( M_1 \). We also note that the currents \( < \overline{D}^{(*)} K_s | \bar{s} \gamma^\mu (1 - \gamma^5) c | 0 > \) and \( < K_S | \bar{s} \gamma_\mu (1 - \gamma^5) b | B > \), which appear in \( M_1 \) and \( M_2 \) respectively, receive contributions from both the contact terms and the pole terms. For the former current the pole terms are proportional to \( 1/(E_K - \delta m) \) while for the latter the pole term goes as \( 1/(E_K + \delta m) \). This also leads to a further suppression of \( M_2 \) relative to \( M_1 \). We therefore neglect \( M_2 \) and only retain \( M_1 \) in our calculation. We will also neglect CP violation in the \( K^0 - \bar{K}^0 \) system and so (with an appropriate choice of phase convention) we can write

\[ K_s = \frac{K^0 - \bar{K}^0}{\sqrt{2}} \]

21
To calculate the various matrix elements in $M_1$ above we use Heavy Hadron Chiral Perturbation Theory (HHCHPT). In HHCHPT, the ground state ($j^P = \frac{1}{2}^-$) heavy mesons are described by the $4 \times 4$ Dirac matrix

$$ H_a = \frac{(1 + \not{v})}{2} \left[ P_{a\mu}^* \gamma^{\mu} - P_a \gamma_5 \right] $$  \hspace{1cm} (A. 11)

where $v$ is the heavy meson velocity, $P_{a\mu}^*$ and $P_a$ are annihilation operators of the $1^-$ and $0^-$ $Q \bar{q}_a$ mesons ($a = 1, 2, 3$ for $u, d$ and $s$): for charm, they are $D^*$ and $D$ respectively. The field $\Pi_a$ is defined by

$$ \Pi_a = \gamma^0 H^\dagger \gamma^0 $$

Similarly, the positive parity $1^+$ and $0^+$ states ($j^P = \frac{1}{2}^+$) are described by

$$ S_a = \frac{(1 + \not{v})}{2} \left[ D_{1\mu}^* \gamma^{\mu} \gamma_5 - D_0 \right] . $$  \hspace{1cm} (A. 12)

In the above equations $v$ generically represents the heavy meson four-velocity and $D^*\mu$ and $D$ are annihilation operators normalized as follows:

$$ \langle 0 | D | c \bar{q}(0^-) \rangle = \sqrt{M_H} $$

$$ \langle 0 | D^*\mu | c \bar{q}(1^-) \rangle = \epsilon^\mu \sqrt{M_H} . $$  \hspace{1cm} (A. 13)

Similar equations hold for the positive parity states $D_{1\mu}^*$ and $D_0$. The vector states in the multiplet satisfy the transversality conditions

$$ v^\mu D^*_\mu = v^\mu D^*_{\mu} = 0. $$

For the octet of the pseudo Goldstone bosons, one uses the exponential form:

$$ \xi = \exp \frac{iM}{f_\pi} $$  \hspace{1cm} (A. 14)

where

$$ M = \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta \\
\pi^- \\
K^- \\
\sqrt{\frac{1}{2}} \pi^0 - \sqrt{\frac{1}{6}} \eta \\
\pi^+ \\
K^0 \\
\sqrt{\frac{1}{2}} \pi^0 - \sqrt{\frac{1}{6}} \eta \\
K^+ \\
\end{pmatrix} $$  \hspace{1cm} (A. 15)

and $f_\pi = 132 \text{ MeV}$. 

22
The lagrangian describing the fields $H$, $S$ and $\xi$ and their interactions, under the hypothesis of chiral and spin-flavor symmetry and at the lowest order in light mesons derivatives, is\cite{14}:

$$
\mathcal{L} = \frac{f^2}{8} \mathrm{Tr} \left[ \partial^{\mu} \Sigma \partial_\mu \Sigma^\dagger + iH_b v^\mu D_{\mu ba} \bar{H}_a \right] 
+ \mathrm{Tr} \left[ S_b \left( i v^\mu D_{\mu ba} - \delta_{ba} \Delta \right) \bar{S}_a \right] + ig \mathrm{Tr} \left[ H_b \gamma_\mu A_{ba}^\mu \bar{H}_a \right] 
+ ig' \mathrm{Tr} \left[ S_b \gamma_\mu \gamma_5 A_{ba}^\mu \bar{S}_a \right] + i g \mathrm{Tr} \left[ S_b \gamma_\mu \gamma_5 A_{ba}^\mu \bar{H}_a \right] + \text{h.c.}
$$

(A. 16)

where $\mathrm{Tr}$ means the trace, and

$$
D_{\mu ba} = \delta_{ba} \partial_\mu + V_{\mu ba} = \delta_{ba} \partial_\mu + \frac{1}{2} \left( \xi^\mu \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right)_{ba}
$$

(A. 17)

$$
A_{\mu ba} = \frac{1}{2} \left( \xi^\mu \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right)_{ba}
$$

(A. 18)

$$
\Sigma = \xi^2 \text{ and } \Delta \text{ is the mass splitting of the } S_a \text{ states from the ground state } H_a.
$$

The currents involving the heavy $b$ and $c$ quarks, $J^\mu_V = \langle D^*(\epsilon_1, p_1) | \bar{c} \gamma^\mu (1 - \gamma_5) b | B(p) \rangle$ can be expressed in general in terms of form factors \cite{28}

$$
J^\mu_V = -\frac{2iV(q^2)}{m + m_1} \epsilon^{\mu\nu\alpha\beta} \epsilon^*_{1\nu} \epsilon_{P_1\beta} - (m + m_1) A_1(q^2) \epsilon^*_{1\mu} + \frac{A_2(q^2)}{m + m_1} \epsilon^*_{1\mu} \cdot q (p + p_1)^\mu 
+ 2m_1 A_3(q^2) \frac{\epsilon^*_{1\mu}}{q^2} q^\mu - 2m_1 A_0(q^2) \frac{\epsilon^*_{1\mu}}{q^2} q^\mu
$$

(A. 20)

with

$$
A_3(q^2) = \frac{m + m_1}{2m_1} A_1(q^2) - \frac{m - m_1}{2m_1} A_2(q^2)
$$

$$
A_3(0) = A_0(0)
$$

(A. 21)

where $q = p - p_1$ is the momentum transfer and $m$ and $m_1$ are the masses of $B$ and $D^*$. In the heavy quark limit the various form factors are related to a universal Isgur-Wise function $\xi(v \cdot v_1)$ where $v$ and $v_1$ are the four velocities of the $B$ and the $D^*$ meson. One can write

$$
J^\mu_V = \sqrt{m} \sqrt{m_1} \xi(v \cdot v_1) \left[ -i \epsilon^{\mu\nu\alpha\beta} \epsilon^*_{1\nu} v_{1\beta} + \epsilon^*_{1\mu} v_1 - \epsilon^*_{1\mu} (v \cdot v_1 + 1) \right]
$$

(A. 22)
The weak current $L_a^\mu = \overline{q} \gamma^\mu (1 - \gamma_5) Q$ can be written in the effective theory as

$$L_a^\mu = \frac{if_H \sqrt{m_H}}{2} Tr \left[ \gamma^\mu (1 - \gamma_5) H \beta_{ba}^+ \right]$$

(A. 23)

where $f_Q$ is the heavy meson decay constant. One can therefore write

$$< D^* (\epsilon_2, v_2) K_0 | s \gamma^\mu (1 - \gamma_5) c | 0 > = \frac{i m_2 f_{D^*} \epsilon_{2\mu}^s}{f_K}$$

(A. 24)

Appendix B

The total amplitude for $\bar{B}^0(v, m) \to D^{*-}(\epsilon_1, v_+, m_1) D^{*-}(\epsilon_2, v_-, m_1) K_s(p_k)$ can be written as

$$a = a_{non-res} [1 - P_1]$$

(B. 1)

and the total amplitude for $B^0(v, m) \to D^{*-}(\epsilon_1, v_+, m_1) D^{*-}(\epsilon_2, v_-, m_1) K_s(p_k)$ can be written as

$$a = a_{non-res} [1 - P_2]$$

(B. 2)

with

$$P_1 = \frac{f_{D^{*-}_{1\ell}}}{f_{D^{*-}}} \sqrt{\frac{m_{1\ell}}{m_1}} \frac{h p_k \cdot v_-}{(p_k \cdot v_- + m_1 - m_{1\ell} + \frac{i \Gamma_{D^{*-}_{1\ell}}}{2})}$$

(B. 3)

$$P_2 = \frac{f_{D^{*-}_{1\ell}}}{f_{D^{*-}}} \sqrt{\frac{m_{1\ell}}{m_1}} \frac{h p_k \cdot v_+}{(p_k \cdot v_+ + m_1 - m_{1\ell} + \frac{i \Gamma_{D^{*-}_{1\ell}}}{2})}$$

(B. 4)

In the boosted frame we can write

$$p_k \cdot v_- = \frac{E'_k E'_- + p'_k p'_- y}{m_1}$$

(B. 5)

$$p_k \cdot v_+ = \frac{E'_k E'_+ - p'_k p'_+ y}{m_1}$$

(B. 6)

where $E'_k$ and $p'_k$ are the energy and the magnitude of the momentum of the kaon in the boosted frame, $E'_\pm$ and $p'_\pm$ are the energies and the magnitude of the momenta of the $D^{*\pm}$ in the boosted frame and $m_1$ is the $D^*$ mass. In the boosted frame we have the following relations

$$E'_k = \gamma (E_k - \vec{\beta} \cdot \vec{p}_k)$$

(B. 7)
\[
E_k = \frac{1}{\sqrt{1 - \frac{E_{k}^2 - m_{k}^2}{m^2(1 - \frac{E_{k}}{m})^2}}} \left[ E_k + \frac{E_{k}^2 - m_{k}^2}{m(1 - \frac{E_{k}}{m})} \right]
\]  
(B. 8)

\[
p'_k = p'_{B} = \sqrt{E_{k}^2 - m_{k}^2}
\]  
(B. 9)

\[
p'_+ = p'_- = \sqrt{E_+^2 - m_1^2}
\]  
(B. 10)

\[
E'_+ = E'_- = \frac{E'_B - E'_k}{2}
\]  
(B. 11)

where \(E_k\) and \(p_k\) are the energy and magnitude of the momentum of the \(K_s\) is the \(B\) rest frame, \(E'_B\) and \(p'_B\) are the energy and magnitude of the momentum of the \(B\) in the boosted frame and \(m, m_1\) and \(m_k\) are the \(B, D^*\) and \(K_s\) masses.

Note from the above relations that \(P_1\) and \(P_2\) can be expressed in terms of \(E_k\) and \(y\) and \(P_1(y, E_k) = P_2(-y, E_k)\).

Squaring the amplitudes and summing over polarizations one can write

\[
|\overline{a}|^2 = |a_{\text{non-res}}|^2 |1 - P_1|^2
\]  
(B. 12)

\[
|a|^2 = |a_{\text{non-res}}|^2 |1 - P_2|^2
\]  
(B. 13)

\[
a^*\overline{a} = a^*_{\text{non-res}} a_{\text{non-res}} (1 - P_2)^* (1 - P_1)\]  
(B. 14)

where

\[
|\overline{a}_{\text{non-res}}|^2 = \kappa^2 [-x^2 + 2(2x_1x_2 + x_2)x + 2x_1^2 - x_2^2 + 4x_1 + 2]
\]  
(B. 15)

\[
|a_{\text{non-res}}|^2 = \kappa^2 [-x^2 + 2(2x_1x_2 + x_1)x + 2x_2^2 - x_1^2 + 4x_2 + 2]
\]  
(B. 16)

\[
a^*_{\text{non-res}} \overline{a}_{\text{non-res}} = \kappa^2 [x^2 + (x_1 + x_2 - 2)x + 2x_1 + 5x_1x_2 + 2x_2 + 2 + O(p_k^2/m^2)]
\]  
(B. 17)

where

\[
\kappa = \frac{G_F}{\sqrt{2}} V_c \left( \frac{c_1}{N_c} + c_2 \right) \sqrt{m \sqrt{m_1 m_1}}
\]

\[
x_1 = v \cdot v_+ = \frac{E'_B E'_+ - p'_B p'_+ y}{m m_1}
\]

\[
x_2 = v \cdot v_- = \frac{E'_B E'_- + p'_B p'_- y}{m m_1}
\]
\[ x = v_+ \cdot v_- = \frac{E'_+ E'_- + p'_+ p'_-}{m_1^2} \]

The double differential decay distribution for the time independent process

\[ \bar{B}^0(v, m) \rightarrow D^{*+}(\epsilon_1, v_+, m_1)D^{*-}(\epsilon_2, v_-, m_1)K_s(p_k) \]

can be written as

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dydE_k} = \frac{f(y, E_k)}{\int f(y, E_k) \frac{p'_k p'_-}{m} dydE_k} \tag{B.18}
\]

\[
f(y, E_k) = \left[ -x^2 + 2(2x_1x_2 + x_2) x + 2x_1^2 - x_2^2 + 4x_1 + 2 \right] |1 - P_1|^2 \tag{B.19}
\]

where \( p'_k \) and \( p'_- \) are the magnitudes of the three momentum of the \( K_s \) and \( D^{*+} \) in the boosted frame.

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