Electrodes erosion influence on the interior ballistics of the electrothermal accelerator of macrobodies

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Abstract. The previously developed mathematical model for macrobody ballistics inside an electrothermal accelerator was modified. The model can be used to predict the projectile velocity for given values of the mass and size of the projectile, the electrodynamic parameters of the capacitive energy storage, and accelerator parameters. The problem of determining the projectile velocity was solved in a hydrodynamic formulation by numerical integration of the equations of motion, the energy conservation equation, and the caloric equation of state. In the case of neglecting the dynamically changing friction coefficient, the calculation results differ considerably from the experimental data, but the model qualitatively describes the physical processes of interior ballistics of the electrothermal accelerator. Electrodes erosion influence was shown.

1. Introduction
This paper presents the modification of the previously developed mathematical model [1] of the interior ballistics of an electrothermal accelerator of macrobodies. The electrothermal accelerator (ETA) is used to model the impact of micrometeorites and space debris particles on the structural elements and structural materials of spacecrafts in the laboratory. Some previously obtained results are given in [2, 3]. Use of the ETA allows an increase in the average velocity range (2–4 km/s) [4–6], which is difficult to achieve for traditional gas-dynamic (powder) accelerators and is economically inexpedient when using light-gas guns. These studies have been motivated by the need for adequate modeling of high-velocity impact of micrometeorites on the structural materials of spacecrafts and investigation of their strength, and development of effective means and methods of protecting spacecrafts from impact with micrometeorites and space debris.

The investigated ETA includes elements of both an electric discharge accelerator and a light-gas gun. In the ETA, the working body is formed by decomposition of a hydrogen-containing substance under the action of the plasma generated by the pulsed flow of electric current in the discharge gap during discharge of a capacitive electric energy storage device whose inductance and discharge-circuit resistance have small values that ensure high discharge current.

The physical processes occurring in the high-voltage high-current discharge in the discharge chamber of the ETA are similar to the explosion of an explosive due to the passage of a high electric current through a circuit for a small period of time. A description of these phenomena can be obtained by solving a system of nonlinear differential equations in partial derivatives corresponding to the laws of conservation of mass, momentum, and energy.
The purpose of this study is to develop a mathematical model of internal ballistic processes for predicting the velocity of a projectile for given projectile mass and size, electrodynamic parameters of a capacitive energy storage device, calculated ETA parameters, including the type of electrode material, etc.

2. Experimental research

For experimental tests of the ETA, a setup was developed (figure 1) which includes a diagnostic chamber, a vacuum chamber, and the ETA, in which a projectile was placed immediately ahead of the discharge chamber. Observation of the target was carried out through diagnostic windows located on both sides of the diagnostic chamber. The chambers were evacuated to a residual pressure of 1–13 Pa. A diagram of the ETA is shown in figure 2.

The electrothermal accelerator was connected to a 300 μF capacitive storage device charged to a voltage of up to 20 kV with a maximum energy of 60 kJ. Switching was carried out by an RVU-43 controlled vacuum gap with an operating voltage of up to 30 kV and an operating current of up to 350 kA. In the ETA casing, there are a cathode and a hollow anode partially filled with the actuating medium made of hydrocarbon compound. Directly behind the anode...
are a channel with an inner diameter of 4.15 mm and a length of 100–240 mm and a projectile with a mass of 100–300 mg. In the space between the cathode and the anode, there is a discharge initiator in the form of a carbon filament. Although the anode is pressed into the channel and is in electrical contact with it, there is no discharge current in the channel. This is confirmed by experimental data [2], according to which the erosion of the channel is negligible compared to the erosion of the electrodes of the discharge chamber.

The electric current intensity was measured using a Rogowski loop, the voltage drop in the discharge gap using a resistive divider, and the channel pressure using a piezoelectric sensor. The total inductance of the electrical circuit was determined experimentally and was 0.7 µH. The initial electrical resistance in the discharge gap was 7–14 Ohm. The following physical model is proposed to describe the acceleration of the projectile in the ETA. Pulsed discharge of the capacitive storage device results in the formation of a carbon plasma in the gap between the cathode and the anode, which is used as an energy source. Then the energy is absorbed by the actuating medium consisting of the hydrocarbon compound, which in turn causes a sharp increase in internal energy, pulsed heating of the actuating medium, and its transition to the gaseous state. The subsequent acceleration of the projectile is similar to the processes occurring in light-gas guns. It should be noted that the larger adiabatic index of the hydrogen–carbon mixture for the ETA is compensated by the high temperature of the gas.

In an experiment with a projectile of mass 193 mg, a channel length of 100 mm, a capacitive storage voltage of 15 kV, and stored energy of 33.75 kJ, the maximum current in the discharge chamber reached a value equal to 156 kA. During the experiment, the voltage drop in the discharge gap and the current intensity in the discharge circuit were measured. Using the time dependences of the current intensity and the voltage drop in the discharge gap, the released energy is determined from $W = \int U(t)I(t)dt$. The results of calculations of the released energy and the rate of energy release (instantaneous power) are shown in figure 3.

A T10000 piezoelectric pressure sensor was placed at a distance of 50 mm from the beginning of discharge chamber. The results of measuring the pressure P are shown in figure 4. The pressure increase was fixed in 120 µs after the start of the high-voltage discharge, and in 200 µs, the pressure reached 470 MPa.

Figure 3. Time dependences of the energy release rate and the energy, released in the discharge gap: 1—energy release rate $B$; 2—energy $W$. 
The results of recording the high-speed action of aluminum projectile on a semi-infinite target made of aluminum alloy when illuminated by laser radiation are shown in figure 5. The recording speed was 120 000 fps. Figure 5(a, b) shows an area ahead of the target with calibration marks (5 × 5 mm² squares) and the time of the start of high-voltage discharge at which the accelerator apparently begins to accelerate. The time of the start of the high-voltage discharge was determined using a light-emitting diode (LED), with delay time 3 µs, fed from an additional Rogowski loop. Figure 5(c) shows the target illuminated by the plasma source after the projectile left the accelerating channel. The motion of the projectile is shown in figure 5(d–g), the process of high-velocity interaction in figure 5(h) and figure 5(i), and the process of scattering of the target and projectile fragments in figure 5(j–l). Transverse dimension of the projectile was changed considerably [figure 5(k)] due to deformation during high-velocity impact. The velocity of the projectile determined by the video frames was 2.1 km/s before the impact.

3. Mathematical formulation of the problem
To describe the interior ballistics of the ETA, it is advisable to use the approach proposed in [3]. The equations of motion in a one-dimensional formulation are written in Lagrangian form (the independent variables are the time $t$ and the mass coordinate $m$):

$$ V(x,t) = \frac{1}{\rho(x,t)} = \frac{\partial X(x,t)}{\partial m}, $$

$$ \frac{\partial u}{\partial t} = -\frac{\partial}{\partial m}(P + Q), $$

$$ \frac{\partial E}{\partial t} = -(P + Q) \frac{\partial V}{\partial t} + B(t). $$
Figure 5. Video frames of high-speed recording acceleration of the projectile: \( t = 46.088 \) (a), 46.462 (b), 46.512 (c), 47.036 (d), 47.045 (e), 47.053 (f), 47.061 (g), 47.070 (h), 47.078 (i), 47.236 (j), 47.552 (k), and 47.819 ms (l); 1—light signal from the LED; 2—calibration mark; 3—target; 4—projectile; 5—place of impact; 6—projectile after collision.

Here \( u(x, t) = \frac{\partial X(x, t)}{\partial t} \) is the gas velocity, \( X(x, t) \) is the Euler coordinate defining the position of a gas element at time \( t \), \( x \) is the Lagrangian coordinate defining the position of the gas element at the initial time, \( m(x, t) = \int \rho(x, t)dx \) is the mass of the gas passing through unit area, \( \rho(x, t) \) is the gas density, \( P \) is the pressure, and \( Q \) is a parameter characterizing the artificial viscosity [7, 8] and introduced to eliminate mathematical discontinuities in the solution that arise in the cases where compression leads to the formation of shock waves in the gas:

\[
Q(x, t) = \begin{cases} 
C_1 \frac{(\Delta m)^2}{V} \left| \frac{\partial V}{\partial t} \right|^2 & \text{for } \frac{\partial V}{\partial t} < 0, \\
0 & \text{for } \frac{\partial V}{\partial t} \geq 0.
\end{cases}
\] (4)

Here \( \Delta m \) is a mass element, \( V(x, t) \) is the specific volume of the gas, \( C_1 \) is a parameter whose value for this problem is chosen equal to 6.0, \( E(x, t) \) is the gas energy density, and \( B(t) \) is the intensity of the heat source (the rate of energy transfer to the gas referred to unit mass of the gas).

In the case of the ETA, the intensity of the heat source is the energy released in the discharge gap during discharge of the capacitive electric energy storage. The quantity \( B(t) \) can be obtained experimentally (by measuring the current intensity and the voltage drop in the discharge gap) or analytically (if the electrodynamic parameters of the electrical circuit and storage device...
such as the capacitance of the storage device, the active and reactive circuit resistances, and the charge voltage of the storage are known). The equation of motion of the projectile in the channel includes the velocity and the pressure of the gas on the base of the projectile:

\[ M_{pr} \frac{du(x_{pr}, t)}{dt} = P(x_{pr}, t) A - F_{fr}. \]

Here \( A \) is the cross-sectional area of the channel, \( M_{pr} \) is the mass of the projectile and \( F_{fr} \) is the resistance force, which is introduced under the assumption that all energy losses occurring during projectile motion are due to the presence of a frictional force between the projectile and the channel are directly proportional to the pressure on the projectile:

\[ F_{fr} = f P(x_{pr}, t) A, \]

where \( f \) is the friction coefficient which in the calculations was assumed to be 0.6.

Thus, the formula for the projectile velocity is written as

\[ \frac{du(x_{pr}, t)}{dt} = \frac{P(x_{pr}, t) A}{M_{pr}} (1 - f). \]  

(5)

The caloric equation of state has the same form as in the description of the detonation of explosives:

\[ P = E\rho^{\gamma} (\rho) - 1 \approx E\rho^{0.09 + \rho^2/0.3 + \rho^4}. \]  

(6)

System (1)–(6) is closed by the equation of state

\[ T = \frac{PV}{R}, \]

where \( R = 145 \text{ J/(kg K)} \) is the gas constant for the mixture of sublimated hydrocarbons and carbon in the ETA discharge chamber.

4. Results analysis

The dependence of the pressure, projectile velocity, and the density and temperature of the gas on time and the length of the channel were obtained by numerical solution [1] of system (1)–(6). The following assumptions were used: the friction coefficient is 0.6 (the sliding friction coefficient for aluminum and steel), the pressure at which the projectile begins to move is 0.4 MPa, the initial density of the actuating medium consisting of sublimed hydrocarbon is 0.7 kg/m\(^3\). In the calculations, the energy required to transfer the actuating medium to the gaseous state was subtracted from the total energy released in the discharge gap. Figure 6 shows the pressure versus time in different sections of the channel. Curve 3 corresponds to the section the closest to the projectile.

The time dependence of the projectile velocity is shown in figure 7, the dependences of the pressure on the base of the projectile and the projectile velocity of the channel length in figure 8 and the dependences of the projectile velocity, pressure, and the energy released on time and the position of the projectile in figure 9.

According to the developed mathematical model, for a channel 100 mm long, the velocity of the projectile is 2.7 km/s. In the experiment, the projectile velocity was found to be 2.1 km/s. According to the results of the experiment, the projectile travels a distance of 50 mm in 120 \( \mu \)s, and, according to the results of calculations for the model, in 80 \( \mu \)s (see figures 5 and 9). This difference is apparently due to the fact that the dependence of the friction coefficient on the projectile velocity is not reliably known. Also, model [1] does not consider the erosion of the electrodes of the discharge chamber, resulting in an increase in the total mass and the average molar weight of the actuating medium and hence a reduction in the limiting speed of sound in
Figure 6. Time dependence of the pressure in the channel: 1—section adjacent to the discharge chamber; 2—intermediate section; 3—section the closest to the projectile; 4—energy release rate $B$.

Figure 7. Time dependence of the projectile velocity and energy release rate: 1—velocity $u$; 2—energy release rate $B$.

the actuating medium and the launching velocity in general. This research is aimed to prove such a suggestion by inputting in the model an electrodes erosion coefficient. It is 60 mg/C for
Figure 8. The pressure on the base of the projectile and projectile velocity versus the length of the channel $L$: 1—pressure $P$ (calculation); 2—pressure $P$ (experiment); 3—velocity $u$ (calculation); 4—velocity $u$ (experiment).

Figure 9. Calculated results and experimental data: 1—projectile velocity; 2—the position of the projectile; 3—the energy release rate; 4—the pressure on the base of the projectile versus time; 5, 6—the position of the projectile at the times $t = 80$ and $120 \mu s$; 7—the pressure in the channel $P = 470$ MPa at the time $t = 200 \mu s$ (1, 2, 4, 5—the calculation results; 3, 6, 7—the experimental data).

Cu-electrodes (figure 10) [9] and this correspondences with own experimental results. Figure 11 shows the dependences of the pressure on the base of the projectile and the projectile velocity of the channel length without erosion of electrodes (curves 1 and 2) and with this parameter (curves 3 and 4).
Figure 10. Specific erosion for different materials of anode and cathode.

Figure 11. The pressure on the base of the projectile and projectile velocity versus the length of the channel $L$: 1, 2—calculated results for pressure $P$ and velocity $u$ without erosion of the electrodes; 3, 4—calculated results for pressure $P$ and velocity $u$ with considering the erosion; 5, 6—experimental data for pressure $P$ and velocity $u$.

5. Conclusions

As a result, one can see the sufficient influence of the erosion of the electrodes of the discharge camber of ETA on the results of the mathematical model that was developed to estimate the projectile velocity using electrothermal launching technology. The such initial data as the mass and size characteristics of the projectile (mass, diameter), the electrodynamic parameters of the capacitive energy storage device (capacitance, inductance, resistance), ETA parameters (size of the discharge chamber, channel length), and the parameters of the actuating medium (density and energy of sublimation) were added with erosion coefficient and/or type of material of the electrodes. Since the model does not include the dynamically changing friction coefficient, the
results of calculations of the velocity and the distance traveled by the projectile differ significantly from experimental data. Nevertheless, the developed mathematical model qualitatively describes the physical processes occurring in the discharge chamber and in the ETA channel.

Acknowledgments
The work is supported by the Ministry of Defense of the Russian Federation under the Program of Development of the Peter the Great Military Academy of Strategic Missile Troops.

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