Theoretical Status of Muon (g-2)

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Abstract. The theoretical status of the muon anomaly is reviewed including the recent change in the light by light hadronic correction. Specific attention is given to the implications of the shift in the difference between the BNL experimental result and the standard model prediction for sparticle mass limits. The implication of the BNL data for Yukawa unification is discussed and the role of gaugino mass nonuniversalities in the satisfaction of Yukawa unification is explored. An analysis of the BNL constraint for the satisfaction of the relic density constraint and for the search for dark matter is also given.

INTRODUCTION

In this talk we discuss the current status of theory vs experiment for \( a_\mu = (g_\mu - 2)/2 \) and the implications for new physics. Recently a reevaluation of the light by light hadronic contribution to \( a_\mu \) has resulted in a change in the sign of this contribution reducing the difference between the BNL experimental result and the standard model prediction from 2.6\( \sigma \) to 1.6\( \sigma \). In view of this change we reconsider the implications for supersymmetry. We carry out the analysis using a 1\( \sigma \) and a 1.5\( \sigma \) error corridor around the central value of the difference between experiment and theory. For the 1\( \sigma \) analysis we find that the upper limits on sparticle masses remain unchanged from those predicted with the 2.6\( \sigma \) difference between experiment and the standard model result with a 2\( \sigma \) error corridor. For the 1.5\( \sigma \) analysis we find that the upper limits are substantially increased from the old analysis and the upper limits of the sparticle masses may lie on the borderline or beyond of what is accessible at the Large Hadron Collider. An important result that arises from the Brookhaven experiment is that the sign of the \( \mu \) parameter is determined to be positive for a broad class of supersymmetric models. However, it is known that Yukawa coupling unification typically prefers a negative \( \mu \). We discuss a possible way out of this problem using nonuniversality of gaugino masses. Finally we consider the implications of the Brookhaven result for neutralino relic density and for the direct detection of supersymmetric dark matter in dark matter detectors.

\( G_\mu - 2 \): EXPERIMENT VS STANDARD MODEL

Over the last three months the theoretical prediction of \( a_\mu \) in the standard model has undergone a significant revision because of the change in sign of the light by light (LbL) hadronic correction to \( a_\mu \). Thus the previous average for \( a_\mu^{\text{had}} (\text{LbL}) \) was [1, 2]
\[ a_{\mu}^{\text{had}}(LbL) = -8.5(2.5) \times 10^{-10} \]. However, recent reevaluations\[3, 4, 5, 6, 7\] give a \( a_{\mu}^{\text{had}}(LbL) \) opposite in sign to the previous evaluations. The reevaluations are summarized in Table 1.

| authors                        | \( a_{\mu}^{\text{had}}(LbL) \)   |
|-------------------------------|----------------------------------|
| Knecht et. al.[3]             | \( 8.3(1.2) \times 10^{-10} \)   |
| Hayakawa & Kinoshita[4]       | \( 8.9(1.5) \times 10^{-10} \)   |
| Bijnens et. al.[6]            | \( (8.3 \pm 3.2) \times 10^{-10} \) |
| Blockland et. al.[5] \((\pi^0)\) | \( 5.6 \times 10^{-10} \) |

Now with the old value of the LbL hadronic correction the total standard model prediction of \( a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{had}} \) was \( a_{\mu}^{\text{SM}} = 11659159.7(6.7) \times 10^{-10} \). Using the BNL experimental result[8] of \( a_{\mu}^{\text{exp}} = 11659203(15) \times 10^{-10} \) one finds \( a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 43(16) \times 10^{-10} \) which gives the old 2.6\( \sigma \) deviation between experiment and the standard model. However, taking an average of the top three entries in Table 1 for \( a_{\mu}^{\text{had}}(LbL) \) the revised difference between experiment and the standard model is

\[ a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 26(16) \times 10^{-10} \]

which is now only a 1.6\( \sigma \) deviation between experiment and the standard model prediction. [More recently another evaluation of \( a_{\mu}^{\text{had}}(LbL) \) based on chiral perturbation theory has been given in Ref.[7] which gives \( a_{\mu}^{\text{had}}(LbL) = (5.5^{+5}_{-3} + 3.1\hat{C}) \times 10^{-10} \) where \( \hat{C} \) stands for unknown low energy constants arising from subleading contributions. The authors of Ref.[7] view a \( \hat{C} \) range of \(-3 \) to \( 3 \) or even larger as not unreasonable. The result of Eq.(1) corresponds essentially to a \( \hat{C} = 1 \) and a much larger value will significantly affect Eq.(1) and the conclusions resulting from it.] Aside from the issue of LbL hadronic correction, the remaining part of the hadronic correction contains \( \alpha^2 \) and \( \alpha^3 \) vacuum polarization corrections. In the deduction of Eq.(1) we used the evaluation of Ref.[8] for the \( \alpha^2 \) correction. However, there is considerable amount of controversy regarding these corrections and this issue is still under scrutiny[10].

**SUPERSYMMETRIC CORRECTION TO \( G_{\mu} - 2 \)**

If indeed there is discrepancy between experiment and the standard model prediction of \( a_{\mu} \) then it would have important implications for new physics. Such new physics could be supersymmetry, compact extra dimensions, muon compositeness, techni-color, anomalous W couplings, new gauge bosons, leptoh-quarks, or radiative muon masses[11]. We focus here on supersymmetric models and specifically on supergravity models[12] which arise from gravity mediated breaking of supersymmetry. The soft SUSY breaking sector of the minimal supergravity model (mSUGRA) is defined by four parameters: these consist of the universal scalar mass \( m_0 \), the universal gaugino mass \( m_{1/2} \), the universal trilinear coupling \( A_0 \) and \( \tan\beta = \langle H_2 \rangle / \langle H_1 \rangle \) where \( H_2 \) gives mass
to the up quark and $H_1$ gives mass to the down quarks and the leptons. We will use SUGRA models as a benchmark and similar analyses can be carried out in other models such as those based on gauge mediation and anomaly mediation breaking mechanisms of supersymmetry. We begin by discussing the basic one loop contribution to $g_{\mu} - 2$ in supersymmetry\[13\]. Here the basic contributions are from the chargino $\tilde{W}$ and neutralino $\chi_i$ ($i=1,\ldots,4$) exchange. For the CP conserving case the chargino contribution is the largest and here one has

$$a_{\mu} = \frac{m_\mu^2}{48\pi^2} \frac{A_{R}^{(a)} A_{R}^{(a)}}{m_{\tilde{W}_a}^2} F_1 \left( \frac{m_\tilde{\nu}}{m_{\tilde{W}_a}} \right)^2 + \frac{m_\mu}{8\pi^2} \frac{A_{L}^{(a)} A_{L}^{(a)}}{m_{\tilde{W}_a}} F_2 \left( \frac{m_\tilde{\nu}}{m_{\tilde{W}_a}} \right)^2$$

(2)

where $A_L(A_R)$ are the left(right) chiral amplitudes and are defined by

$$A_R^{(1)} = -\frac{e}{\sqrt{2}\sin\theta_W} \cos\gamma_1; \quad A_L^{(1)} = (-1)^\theta \frac{em_\mu \cos\gamma_2}{2M_W \sin\theta_W \cos\beta}$$

$$A_R^{(2)} = -\frac{e}{\sqrt{2}\sin\theta_W} \sin\gamma_1; \quad A_L^{(2)} = -\frac{em_\mu \sin\gamma_2}{2M_W \sin\theta_W \cos\beta}$$

(3)

and where $\gamma_i$ are the mixing angles and $F_1, F_2$ are form factors. Recently, the absolute signs of the supersymmetric contribution was checked by taking the supersymmetric limit\[14\]. There are some interesting features of the SUSY contribution. One finds that since $A_L \sim 1/\cos\beta$ one has\[15, 16\] $a_{\mu}^{SUSY} \sim \tan\beta$. Further, it is easy to show that the sign of $a_{\mu}^{SUSY}$ is correlated with the sign of $\mu$\[15, 16\]. Thus one finds that $a_{\mu}^{SUSY} > 0$ for $\mu > 0$ and $a_{\mu}^{SUSY} < 0$ for $\mu < 0$ where we use the sign convention of Ref.\[17\].

**IMPLICATIONS OF DATA**

*Upper limits on sparticle masses.* Soon after the BNL result became available\[8\] a large number of analyses appeared in the literatures exploring the implications of the results for new physics\[18\]. These analyses were based on the result $a_{\mu}^{exp} - a_{\mu}^{SM} = 43(16) \times 10^{-10}$ which as is now realized is based on using the wrong sign of the light by light hadronic correction. Using the above result and a the 2\(\sigma\) error corridor so that $10.6 \times 10^{-10} < a_{\mu}^{SUSY} < 76.2 \times 10^{-10}$ the BNL data leads to the following sparticle mass limits in mSUGRA\[19\]: $m_{\tilde{W}} \leq 650$ GeV, $m_{\tilde{\nu}} \leq 1.5$ TeV ($\tan\beta \leq 55$) and $m_{1/2} \leq 800$ GeV, $m_0 \leq 1.5$ TeV ($\tan\beta \leq 55$). Since the LHC can explore squarks/gluinos up to 2 TeV the BNL result implies that sparticles should become visible at the LHC\[20\].

Next we assess the situation as a consequence of the change in the sign of the light by light hadronic correction which results in Eq.(1). In this case a 2\(\sigma\) error corridor would not lead to upper limits for the sparticle masses. However, one can get interesting constraints if one imposes a 1\(\sigma\) or a 1.5\(\sigma\) constraint. A 1\(\sigma\) constraint actually yields exactly the same upper limits as before so in this case the analysis of Ref.\[19\] remains valid as far as the upper limits are concerned. The case of 1.5\(\sigma\) was analyzed in Ref.\[21\] and it was found, as expected, that the upper limits go up considerably. In Fig.(1) results
are presented for the case of \( \tan \beta = 45 \). Here one finds that the upper limits on \( m_0 \) and \( m_{1/2} \) are considerably larger than for the analysis of Ref.\[19\]. Specifically one finds from Fig.(1) that the upper limit on \( m_0 \) in the range of the parameter space exhibited already exceeds 2.5 TeV which is on the borderline of the reach of the LHC. The upper limits for other values of \( \tan \beta \) are sharply dependent on the value of \( \tan \beta \). A more complete analysis of the constraint for the 1.5\( \sigma \) case can be found in Ref.\[21\].

Another interesting implication of the BNL result is that under the assumption of CP conservation and setting \( a_{\mu}^{\text{SUSY}} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} \) the BNL data determines \( \text{sign}(\mu) = +1 \) (see, e.g., Ref.\[19\]). It known that \( \mu > 0 \) is favored by the \( b \to s + \gamma \) constraint\[22, 23\] and also favored for dark matter searches. The implications of \( \mu > 0 \) for dark matter will be discussed in further detail below. One issue of concern relates to the possibility that the supersymmetric effects may be masked by effects arising from low lying extra dimensions. This possibility was examined in Ref.\[24\] in a model with one large extra dimension compactified on \( S^1/Z_2 \) with radius \( R \) \((M_R = 1/R = O(\text{TeV}))\). The extra dimension contributes to the Fermi constant and by a comparison of the standard model prediction with the experimental value of the Fermi constant\[24\] one can place a limit on the extra dimension of about \( M_R > 3 \) TeV. With this size value of \( M_R \) one finds that the contribution of the extra dimension to \( a_{\mu} \) is negligible\[24\] compared to the supersymmetric contribution. For the case of strong gravity the effect on \( a_{\mu} \) from the Kaluza-Klein excitations of the graviton in the case \( d=2 \) is also small\[25\] since here the fundamental Planck scale \( M_* \) is found to have a lower limit of \( M_* > 3.5 \) TeV from the recent gravity experiment\[26\]. The above exhibits the fact that \( g_{\mu} - 2 \) is not an efficient probe of extra dimensions. Other techniques such as energetic dileptonic signals at LHC would be more efficient signals for the exploration of extra dimensions\[27\].
important effect that can modify the supersymmetric contribution is the effect of CP violating phases. This topic has been analyzed in several works [14]. Specifically it is found that the BNL data can be used to constrain the CP phases and strong constraint on the phases are found to exist [28].

**Positivity of $\mu$ and Yukawa Unification.** We discuss now another aspect of the $g_\mu - 2$ constraint and this concerns Yukawa unification in supersymmetric models. It has been known for some time that $b - \tau$ Yukawa coupling unification typically prefers a negative $\mu$ [29, 30]. Thus the positively of the $\mu$ sign implied by the BNL data appears a priori to pose a problem for Yukawa unification. Now the reason why Yukawa unification typically prefers a negative $\mu$ is easily understood from the fact that such unification requires a negative supersymmetric correction to the $b$ quark mass and a negative correction to the $b$ quark mass is easily obtained when $\mu$ is negative. To illustrate this phenomenon we consider the gluino and chargino exchanges which contribute the largest supersymmetric correction to the $b$ quark mass. Thus one has [31]

$$\Delta_\tilde{g} = \frac{2\alpha_3 \mu M_{\tilde{g}}}{3\pi} \tan \beta I(m^2_{\tilde{g}}, m^2_{\tilde{g}}, M^2)$$

and

$$\Delta_\tilde{\chi}^+ = \frac{Y_t \mu A_t}{4\pi} \tan \beta I(m^2_{\tilde{t}}, m^2_{\tilde{t}}, \mu^2)$$

where $I > 0$. Generally the gluino exchange contribution tends to be the larger one and here one finds that for a positive $M_{\tilde{g}}$ which is typically the case a negative $\mu$ indeed leads to a negative correction to the $b$ quark mass which in turn leads to the usual result that $b - \tau$ unification prefers a negative $\mu$. There are several solutions discussed recently to overcome this problem [32, 33, 34, 35]. One simple possibility discussed in Refs. [34, 35] is that of nonuniversal gaugino masses where the sign of the gluino mass is negative relative to the mass of the SU(2) gaugino mass. In this case one can obtain a negative contribution to the $b$ quark mass while maintaining a positive $\mu$. The basic idea is that with nonuniversalsities one can have the sign of $SU(2)$ and $SU(3)$
gauginos to be opposite. A positive positive $\mu$ and a positive $\tilde{m}_2$ are consistent with the BNL data, while a positive $\mu$ and a negative $\tilde{m}_3$ gives a negative correction to the $b$ quark mass and leads to $b - \tau$ unification. We considered two classes of models, one based on SU(5) and the other on SO(10). For the SU(5) case one has that the the gaugino mass matrix can transform like the symmetric product of $(24 \times 24)_{\text{sym}}$ which has the expansion of $1 + 24 + 75 + 200$. In this case the gaugino masses arising from the 24 plet has the opposite sign between the SU(2) and SU(3) gauginos\[^{[36]}\]. Thus one can choose $\mu$ positive and the gluino mass to be negative which gives a negative contribution to the $b$ quark mass and allows for the satisfaction of the $b - \tau$ unification constraints. The degree of unification defined by $\delta_{b\tau} = (|\lambda_b - \lambda_{\tau}|)/\lambda_{\tau}$ vs the correction to the $b$ quark mass is exhibited in Fig.(2) for the 24 plet case with a positive $\mu$ sign. One finds that $b - \tau$ unification can be satisfied to a high degree of accuracy with an appropriate negative correction $\Delta_b$ to the $b$ quark mass. A similar analysis holds for the SO(10) case. Here the gaugino mass matrix can transform like symmetric product of $(45 \times 45)_{\text{sym}}$ which has the expansion of $1 + 54 + 210 + 770$. Here for the case when the symmetry breaking occurs pattern is of the form $SO(10) \rightarrow SU(4) \times SU(2) \times SU(2) \rightarrow SU(3) \times SU(2) \times U(1)$ one finds that the $SU(3), SU(2), U(1)$ gaugino masses arising from the 54 plet are in the ratio\[^{[57]}\] $M_3 : M_2 : M_1 = 1 : -3/2 : -1$. However, for the symmetry pattern $SO(10) \rightarrow SU(2) \times SO(7) \rightarrow SU(3) \times SU(2) \times U(1)$ one finds that the $SU(3), SU(2), U(1)$ gaugino masses arising from the 54 plet are in the ratio\[^{[57]}\] $M_3 : M_2 : M_1 = 1 : -7/3 : 1$. We will call this case $54'$. An analysis similar to that for the 24 plet case can be carried out for the 54 and $54'$ cases and one finds that $b - \tau$ unification occurs once again for these cases.

**Implications for Relic Density and Dark Matter Search.** As noted earlier the BNL data indicates a positive value of $\mu$ for mSUGRA. This result has important implications for dark matter. As already indicated a positive $\mu$ is preferred by the constraint imposed by the flavor changing neutral current process $b \rightarrow s + \gamma$\[^{[12]}\] in that the experimental value for this branching ratio imposes severe constraints on the SUSY parameter space for the negative sign of $\mu$ but imposes much less stringent constraints on the parameter space for a positive value of $\mu$. Thus a positive $\mu$ is more favorable for supersymmetric dark matter analysis in that it allows for a large amount of the parameter space of the model where relic density constraints can be satisfied along with satisfying the $b \rightarrow s + \gamma$ constraint. Furthermore, it also turns out that detection rates for a positive $\mu$ are generally larger than for a negative $\mu$. Thus after the BNL data became available it was immediately realized that the positive $\mu$ sign indicated by the BNL data was favorable for dark matter searches\[^{[13], [14]}\]. More detailed analyses were done in several subsequent works and the parameter space of mSUGRA was further constrained from the relic density constraints. Now another way that the BNL data constrains dark matter is through the Yukawa unification conditions. Here we discuss the implications of this constraint on dark matter. As discussed in the section on $b - \tau$ unification above, one finds that this unification can be achieved with a positive $\mu$ in a variety of ways. One possibility discussed above arose from nonuniversal gaugino masses. We discussed two main scenarios for nonuniversalities corresponding to the SU(5) and SO(10) cases. For SU(5) the gaugino mass nonuniversalities arising from the 24 plet case allows a negative contribution to the $b$ quark mass with a positive $\mu$ and leads to regions of the parameter
space where \( b - \tau \) unification occurs. Analysis in this region exhibits that all of the spectrum lies within the usual naturalness limits\[38\]. It is interesting to investigate if this region of the parameter space also leads to a satisfaction of the relic density constraint. We consider a rather liberal corridor here corresponding to the range \( 0.02 \leq \Omega h^2 \leq 0.3 \). The analysis shows that significant regions of the parameter space exist where these constraints are satisfied. An analysis of the detection rates in the direct detection of dark matter was also given\[21\]. One finds\[21\] that the detection rates lie in a range that can be fully explored in the new generation of experiments currently underway and those which are planned in the future (see, e.g., Ref.\[39\]). A similar analysis can be carried out for the SO(10) case. The sparticle masses consistent with the BNL 1\( \sigma \) constraint as given by Eq.\( (1) \), consistent with the \( b \rightarrow s + \gamma \) constraint and consistent with the relic constraint are given in Table 2 for the 24 plet case of SU(5) and for the 54 and 54' cases of SO(10).

Table 2: Sparticle mass ranges for 24, 54, and 54' cases from Ref.\[21\]

| Particle | 24 (GeV) | 54 (GeV) | 54' (GeV) |
|----------|---------|---------|---------|
| \( \chi_1^0 \) | 32.3 - 75.2 | 32.3 - 81.0 | 32.3 - 33.4 |
| \( \chi_1^\pm \) | 86.9 - 422.6 | 94.6 - 240.8 | 145.8 - 153.9 |
| \( \tilde{g} \) | 479.5 - 1077.2 | 232.5 - 580.3 | 229.8 - 237.4 |
| \( \tilde{\mu}_1 \) | 299.7 - 1295.9 | 480.5 - 1536.8 | 813.1 - 1196.3 |
| \( \tilde{\tau}_1 \) | 203.5 - 1045.1 | 294.2 - 1172.6 | 579.4 - 863.7 |
| \( \tilde{u}_1 \) | 533.6 - 1407.2 | 566.7 - 1506.4 | 822.9 - 1199.8 |
| \( \tilde{d}_1 \) | 535.1 - 1407.5 | 580.3 - 1546.2 | 845.1 - 1232.5 |
| \( \tilde{t}_1 \) | 369.9 - 975.2 | 271.5 - 999.6 | 513.7 - 819.9 |
| \( \tilde{b}_1 \) | 488.2 - 1152.8 | 158.1 - 1042.0 | 453.2 - 749.9 |
| \( h \) | 104.3 - 114.3 | 103.8 - 113.3 | 108.1 - 110.9 |

Here one finds some interesting features in the spectrum. Thus in these scenarios the neutralino mass lies below 81 GeV and the higgs boson mass lies below 115 GeV in the three scenarios considered in Table 2. The Higgs mass ranges of Table 2 are consistent with the current Higgs mass limits from LEP\[40\] taking into account the tan\( \beta \) dependence\[41\]. Further these mass ranges can be fully explored in RUNII of the Tevatron. Similarly the mass ranges of the other sparticle masses of Table 2 can be explored in RUNII of the Tevatron via the trileptonic signal\[42\] and other techniques\[17\] while the full range for most of the spectrum of Table 2 can be explored at the LHC\[20\].

**CONCLUSION**

There is a significant amount of more data from the 2000 runs and BNL eventually hopes to measure \( a_\mu \) to an accuracy of \( 4 \times 10^{-10} \). On the other hand, reanalyses of the hadronic correction are still underway to pin down further the size of these corrections. If the deviation between the central value of experiment and the standard model pre-
diction persists at the current level but the error is significantly reduced one could still see a possibility of approaching the discovery limit. Needless to say the implications of a sizable deviation between experiment and theory are enormous as foreseen in early works[13] and elucidated further in several subsequent works[15, 16, 18, 19]. Specifically, the light Higgs boson should show up in RUNII of the Tevatron and most of the sparticles ($\tilde{g}, \tilde{q}, \tilde{W}$, etc) should become visible at the LHC. Further, a positive $\mu$ sign implied by the BNL data along with a low lying sparticle spectrum is very encouraging for the search for supersymmetric dark matter.

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