Quark mass effects in the thermodynamical properties of an extended (P)NJL model

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We analyze the thermodynamical properties of a system of strongly interacting particles at vanishing quark chemical potential in the framework of a recently developed extension of the Polyakov-Nambu-Jona-Lasinio Model. In addition to eight quark interactions terms, non-canonical terms which explicitly break chiral symmetry up to the same order in a $1/\Nc$ expansion ($\Nc$ number of colors) are included. A recently proposed Polyakov potential is considered and the results are compared to lattice QCD data resulting in a favorable scenario for the recent model variants.

**KEYWORDS:** spontaneous chiral symmetry breaking, explicit chiral symmetry breaking, general spin 0 eight-quark interactions, finite temperature

1. Introduction

Simple effective models are useful tools in the study of systems of strongly interacting particles. Among these, models of the Nambu-Jona-Lasinio type \cite{1}, drawing its power from the fact that they share with QCD its global symmetries and include both spontaneous and explicit chiral symmetry breaking, have been quite successful. The fact that they are not plagued by the sign problem of the \textit{ab initio} approach of lattice QCD (lQCD) at finite quark chemical potential ($\mu_q$) has a twofold consequence: on the one hand this means that their degree of success can be checked by comparison to the result of lQCD simulations at $\mu_q = 0$ and on the other hand they can be used to venture into the region of the phase diagram which is problematic to lQCD.

1.1 The model

Restricting our study to the light quark sector ($u$, $d$ and $s$), a recently developed model lagrangian (NJLH8qm) \cite{2} includes terms with explicit chiral symmetry breaking up to the same order in a $1/\Nc$ expansion as the 't Hooft determinant term ($6q$ interaction term) \cite{3–6} which one must add to the NJL interaction (the $4q$ interaction term which drives the spontaneous breaking of chiral symmetry) to break the unwanted $U_A(1)$ axial symmetry. This is an extension of a previously developed version with $8q$ terms (NJLH8q) \cite{7, 8} and consistently contains all non-derivative spin 0 multiquark interactions up to the the relevant order in $1/\Nc$.

Polyakov loop extension of these models \cite{9–12} has become a very popular way to mimic the inclusion of gluonic degrees of freedom. In these models an additional Polyakov potential must be introduced to drive the deconfinement transition (the Polyakov loop goes from $\phi = 0 \rightarrow 1$). Several forms of this potential have been considered trying to reproduce lQCD results at vanishing $\mu_q$. Here we will consider the recently proposed from \cite{13, 14} and compare lQCD results reported in \cite{15–18}.

A more detailed description of the model can be found in \cite{19} where the results without the Polyakov loop extension and the results considering the Polyakov potential form from \cite{10} are also considered.
2. Results and discussion

Here we will analyze some thermodynamical quantities resulting from the Polyakov extended version of the model using four different parameter sets: two in the NJLH8q model and two in the NJLH8qm model. The sets are labeled with suffix $A/B$ referring to weaker/stronger OZI-violating $8q$ interactions. It should be noted that, while in the NJLH8q model a wide range of fits differing in strengths of the OZI-violating $8q$ interactions is possible (an increase in the $8q$ interaction strength accompanied by a suitable decrease in the NJL $4q$ interaction leaves the meson spectra unchanged apart from a decrease in the $\sigma$ meson mass) [8], in the case of NJLH8qm a much more realistic fit to the meson spectra is achieved while at the same time locking us in a moderately strong $8q$ interaction regime when compared to the NJLH8q case thus reducing the arbitrariness in the $4q/8q$ interactions interplay.

In Fig. 1 we present the $T$ dependence at $\mu_q = 0$ of several thermodynamical quantities: squared speed of sound ($C_s^2$), the energy-momentum tensor trace anomaly ($\Theta$), pressure ($-\Omega$), energy density ($\epsilon$), entropy density ($s$) and specific heat ($C_V$). All these quantities display a marked behavior showing the crossover transition which according to recent lQCD data is expected to occur at a temperature $T \approx 155$ MeV over a range of $\Delta T \approx 20$MeV [17, 18].

In Fig. 2 the fluctuations and correlations of conserved charges (baryon number, electric charge and strangeness) can be observed. These are thought to be good probes for the experimental detection
Fig. 2. Top row, left-to-right displays the fluctuations of conserved charges: $\chi^B_2$, $\chi^Q_2$ and $\chi^S_2$. Bottom row, left-to-right, correlations: $\chi^{BO}_{11}$, $\chi^{BS}_{11}$ and $\chi^{QS}_{11}$. Same model parameters and notation as in Fig. 1. The markers, labeled as WBEoS and HotQCDEoS, correspond to continuum extrapolated lQCD results taken respectively from [15] and [16].

of the transition behavior in relativistic heavy-ion collisions. Note that for that application one should consider finite $\mu_q$ but here we are looking for the validation of the model against lQCD data at $\mu_q = 0$.

As can be seen in Figs. 1 and 2 there is a good general agreement with lQCD simulations. Several points are worthy of note:

- the stronger eight quark interactions appear to be necessary for the reproduction of the local minimum of $C^2_2$;
- $\Theta^\mu_\mu$, $-\Omega$, $\epsilon$ and $s$ are all reasonably reproduced;
- $C_V$ displays a peak (except for the NJLH8qA case) which is absent from the lQCD data which reflects the overall faster transition behavior induced by the OZI-violating 8q interactions;
- baryon number and strangeness fluctuations ($\chi^B_2$ and $\chi^S_2$) as well as the baryon number-electric charge, baryon number-strangeness and electric charge-strangeness correlations ($\chi^{BO}_{11}$, $\chi^{BS}_{11}$ and $\chi^{QS}_{11}$) are reasonably reproduced and have their slope improved by the stronger 8q interaction regime and the inclusion of non-canonical explicit chiral symmetry breaking interactions;
- the electric charge fluctuation $\chi^B_2$ is not accurately reproduced in our model.

3. Conclusions

Building upon the success of the recently proposed NJLH8qm model in reproducing the low-lying spectra of scalar and pseudoscalar mesons, it is shown that its Polyakov loop extended version
is quite successful in reproducing IQCD results for several relevant thermodynamical quantities. This strengthens the importance of this model as a simple tool for the study of the thermodynamics of systems of strongly interacting particles at finite $T$ and $\mu$ for instance in the study of the fluctuations and correlations of conserved charges in regions of the phase diagram relevant to relativistic heavy ion collisions.

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