We present a calculation of the strong decays of the exotic states $Z_b(10610)$ and $Z_b'(10650)$ using a covariant quark model. We use a molecular-type four-quark current for the coupling of the $Z_b(10610)$ and $Z_b'(10650)$ to the constituent heavy and light quarks.

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I Introduction

A few years ago the Belle Collaboration [1] reported on the observation of two charged bottomoniumlike resonances in the mass spectra of $\pi^\pm \Upsilon(nS)$ ($n = 1, 2, 3$) and $\pi^\pm h_b(mP)$ ($m = 1, 2$) in the decays $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-, h_b(mP)\pi^+\pi^-$. The measured masses and widths were given by

$$M_{Z_b} = (10607.2 \pm 2.0) \text{ MeV}, \quad \Gamma_{Z_b} = (18.4 \pm 2.4) \text{ MeV},$$
$$M_{Z_b'} = (10652.2 \pm 1.5) \text{ MeV}, \quad \Gamma_{Z_b'} = (11.5 \pm 2.2) \text{ MeV}. \quad (1)$$

The existence of these two states was later confirmed by the same collaboration [2,3] in differing decay channels.

In [2] the Belle Collaboration rediscovered the two states in $e^+e^-$ annihilation into $\Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$). It was found that the favored quantum numbers for both $Z_b$ states are $I^G(J^P) = 1^+(1^+)$. In the paper [3] the Belle Collaboration reported on the results of an analysis of the three-body processes $e^+e^- \to BB^\pi^\pm, BB^*\pi^\pm,$ and $B^*B^*\pi^\pm$. It was found that the transitions $Z_b^+(10610) \to [BB^* + c.c.]^\pm$ and $Z_b'(10650) \to [B^*B^*]^\pm$ dominate among the corresponding final states. The fit to the $BB^*\pi$ and $B^*B^*\pi$ data gave the values of $Z_b$ masses and widths

$$M_{Z_b} = (10605 \pm 6) \text{ MeV}, \quad \Gamma_{Z_b} = (25 \pm 7) \text{ MeV},$$
$$M_{Z_b'} = (10648 \pm 13) \text{ MeV}, \quad \Gamma_{Z_b'} = (23 \pm 8) \text{ MeV}. \quad (2)$$

The relative decay fractions for the $Z_b$-decays were determined assuming that they are saturated by the $\Upsilon(nS)\pi$ ($n = 1, 2, 3$), $h_b(mP)\pi$ ($m = 1, 2$) and $B^*(\pi^+)B^*$ channels.

The theoretical structure assignments for the two hidden-bottom meson resonances proposed immediately after their observation [4–22] were based on a molecular [4, 7, 9, 11, 16] and a tetra-quark interpretation [12, 13] using the analogy to the corresponding states in the charmonium sector. In [8] the new resonances were identified as a hadro-quarkonium system based on the coupling of light and heavy quarkonia to intermediate open-flavor heavy-light mesons. Subsequently these states have been studied extensively using various assignments within different approaches: chiral quark model [14], using phenomenological Lagrangians [15, 17, 19], effective field theory [18, 22], QCD sum rules [20], meson exchange model [21], effective range theory [23], and holographic QCD [24].

In Ref. [16] some of us presented a detailed analysis of the strong decays $Z_b^+ \to \Upsilon(nS) + \pi^+$ and $Z_b'^+ \to \Upsilon(nS) + \pi^+$, $n = 1, 2, 3$ using a phenomenological Lagrangians formulated in terms of hadronic degrees of freedom. The hadronic molecular approach was developed in Refs. [25] and is based on the compositeness condition formulated in [26, 27]. The compositeness condition implies that the renormalization constant of the hadron wave function is set equal to zero or that the hadron is a bound state of its constituents.

In this paper we study the strong decays of the $Z_b$ and $Z_b'$ states using the covariant confined quark model (CCQM) proposed in Refs. [28, 29]. The CCQM has been successfully applied to the description of the properties of the exotic $X(3872)$, $Z_c(3900)$, $Z(4430)$, and $X(5568)$ states [28–31]. The present study complements the analysis performed in
II  \( Z_b(10610) \) and \( Z_b(10560) \) as molecular-type four-quark states

Since the masses of the \( Z_b^+(10610) \) and \( Z_b'^+(10560) \) resonances are very close to the respective \( B^+ \bar{B} \) (10604 MeV) and \( B^* B^* \) (10649 MeV) thresholds, in Ref. [4] it was suggested that they have molecular-type binding structures. Their observed quantum numbers are \( I^G(J^{PC}) = 1^+(1^+) \), so that the neutral isotopic states with \( I_s = 0 \) have the quantum numbers \( J^{PC} = 1^+ \). As a result the allowed interpolating four-quark currents have the form:

\[
J_{Z_b^+}^\mu = \frac{1}{\sqrt{2}} \left[ (d\gamma_5 b)(\bar{b}\gamma^\mu u) + (\bar{d}\gamma_5 b)(b\gamma^\mu u) \right] ,
\]

\[
J_{Z_b'^+}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}(\bar{d}\gamma_5 b)(b\gamma_\alpha u) .
\]

Such a choice guarantees that the \( Z_b \)-state can only decay to the \([\bar{B}^* B + c.c.]\) pair whereas the \( Z_b'^+ \)-state can decay only to a \( B^* B^* \) pair. Decays into the \( B B \)-channels are forbidden.

The exotic states can also decay into a bottomonium state plus a charged light meson. We start with the classification of such two-body decays. In Table I we show a list of the orbital excitations of the \( b\bar{b} \) states with spin 0 and 1 by analogy with the charmonium excitations as given in Ref. [32].

| quantum number \( I^G(J^{PC}) \) | name | quark current | mass (MeV) |
|----------------------------------|------|---------------|------------|
| \( 0^+(0^-) \) (\( S = 0, L = 0 \)) | \( ^1S_0 = \eta_b(1S) \) | \( \bar{b} \gamma^5 b \) | 9399.00 ± 2.30 |
| \( 0^- (1^-) \) (\( S = 1, L = 0 \)) | \( \gamma \) | \( \bar{b} \gamma^\mu b \) | 9460.30 ± 0.26 |
| \( 0^+(0^+) \) (\( S = 1, L = 1 \)) | \( P_0 = \chi_{60} \) | \( \bar{b} b \) | 9859.44 ± 0.52 |
| \( 0^+(1^+) \) (\( S = 1, L = 1 \)) | \( P_1 = \chi_{01} \) | \( \bar{b} \gamma^\mu \gamma^5 b \) | 9892.72 ± 0.40 |
| \( 0^- (1^-) \) (\( S = 0, L = 1 \)) | \( P_1 = h_b(1P) \) | \( \bar{b} \gamma^\mu \gamma^5 b \) | 9899.30 ± 0.80 |

\( G \)-parity is a multiplicatively quantum number conserved in strong interactions. Keeping in mind that \( G(\pi^+) = -1 \) and \( G(\rho^+) = +1 \), the decays \( Z_b \to \bar{Y} \rho, \eta_b \pi, \chi_{61} \pi, h_b \rho \) are forbidden. The decay \( Z_b \to \chi_{61} \rho \) is not allowed kinematically. There are therefore only the three allowed decays: \( Z_b^+ \to \bar{Y} + \pi^+ \), \( Z_b^+ \to h_b + \pi^+ \) and \( Z_b^+ \to \eta_b + \rho^+ \).

Let us discuss the spin kinematics for the three decays \( 1^+ \to 1^- + 0^- \) (\( Z_b^+ \to \bar{Y} + \pi^+ \), \( Z_b^+ \to [\bar{B}^* B^* + c.c.] \)), \( Z_b^+ \to \eta_b + \rho^+ \), \( 1^- \to 1^- + 0^- \) (\( Z_b^+ \to h_b + \pi^+ \)) and \( 1^+ \to 1^- + 1^- \) (\( Z_b^+ \to \bar{B}^* B^* \)).

- The decay \( 1^+ \to 1^- + 0^- \).

The momenta and the Lorentz indices of the polarization four-vectors in the decay are labelled according to the transition matrix element

\[
M = \langle 1^- (q_1; \delta), 0^- (q_2) | T | 1^+(p; \mu) \rangle .
\]

The product of the parities of the two final state mesons is \(+1\) which matches the parity of the initial state. Thus the two final state mesons must have even relative orbital momenta. In the present case these are \( L = 0, 2 \). The
spins $s_1$ and $s_2$ of the two final state mesons couple to the total spin $S = 1$. Thus one has the two $(LS)$ amplitudes ($L = 0, S = 1$) and ($L = 2, S = 1$). The covariant expansion of the transition matrix is given by

$$M = \left( A \rho^{\mu\delta} + B q_1^\mu q_2^\delta \right) \varepsilon_\mu \varepsilon_\rho^{\varepsilon}.$$

(6)

Alternatively one may describe the transition amplitude by the helicity amplitudes $H_{\lambda\lambda'\lambda''}$. The helicity amplitudes may be expressed as a linear superposition of the invariant amplitudes $A$ and $B$. One has

$$H_{00} = - \frac{E_1}{M_1} A - \frac{M}{M_1} |q_1|^2 B, \quad H_{+1+1} = H_{-1-1} = - A.$$

(7)

Since the particles of the initial and final states are on their mass-shells one has $p^2 = M^2$, $q_1^2 = M_1^2$, $q_2^2 = M_2^2$ and $p^\mu \varepsilon_\mu = 0$, $q_1^\mu \varepsilon_\mu = 0$. The magnitudes of the final state three-momentum and energy in the rest frame of the initial particle is given by $|q_1| = \sqrt{M^2 M_1^2 M_2^2}/2M$ and $E_1 = (M^2 + M_1^2 - M_2^2)/2M$, respectively.

The rate of the decay $1^+(p) \to 1^-(q_1) + 0^-(q_2)$ finally reads

$$\Gamma = \frac{|q_1|}{24\pi M^2} \left\{ \left( 3 + \frac{|q_1|^2}{M_1^2} \right) A^2 + (M^2 + M_1^2 - M_2^2) \left[ \frac{|q_1|^2}{M_1^2} AB + \frac{M^2}{M_1^2} |q_1|^4 B^2 \right] \right\},$$

(8)

\begin{itemize}
  \item The decay $1^+ \to 1^+ + 0^-$. \end{itemize}

The matrix element is described by

$$M = \langle 1^+(q_1; \delta), 0^-(q_2) | T | 1^+(p; \mu) \rangle.$$

(9)

In this decay the product of the parities of the two final state mesons is $(-1)$ which does not match the parity of the initial state. Thus the two final state mesons must have odd relative orbital momenta. In the present case this is $L = 1$. The spins $s_1$ and $s_2$ of the two final state mesons couple to the total spin $S = 1$. Thus one has only one $(LS)$ amplitude with $L = 1, S = 1$. This implies that there is only one invariant amplitude in this transition. Accordingly there is only one term in the covariant expansion of the matrix given by

$$M = C \varepsilon_{q_1q_2\mu\delta} \varepsilon_\mu \varepsilon_\rho^{\varepsilon}.$$

(10)

where $\varepsilon^{q_1q_2\mu\delta} = q_1^{\alpha} q_2^{\beta} \varepsilon^{\alpha\beta\mu\delta}$.

The rate of the decay $1^+(p) \to 1^+(q_1) + 0^-(q_2)$ can be seen to be given by

$$\Gamma = \frac{|q_1|^3}{12\pi M^2} C^2.$$

(11)

This decay is suppressed kinematically due to the p-wave suppression factor $|q_1|^2$.

\begin{itemize}
  \item The decay $1^+ \to 1^- + 1^-$. \end{itemize}

By naive counting the covariant expansion of the matrix element

$$M = \langle 1^-(q_1; \delta), 1^- (q_2; \rho) | T | 1^+(p; \mu) \rangle.$$

(12)

involves five invariant amplitudes whereas there are only three independent $(LS)$ amplitudes. This can be seen as follows. The product of the parities of the two final state mesons is $(+1)$ which matches the parity of the initial state. Thus the two final state mesons must have even relative orbital momenta. In the present case these are $L = 0, 2$. The spins $s_1$ and $s_2$ of the two final state mesons couple to the total spins $S = 0, 1, 2$. Thus one has three $(LS)$ amplitudes with $L = 0, S = 1$, $L = 2, S = 1$ and $L = 2, S = 2$. Returning to the covariant form, the naive expansion of the matrix element reads

$$M = (A_1 \varepsilon^{q_1q_2\mu\delta} + A_2 \varepsilon^{q_2q_3\mu\delta} + A_3 \varepsilon^{q_3q_4\mu\rho} q_2^\delta + A_4 \varepsilon^{q_4q_5\mu\rho} q_3^\delta + A_5 \varepsilon^{q_5q_6\mu\rho} q_4^\delta) \varepsilon_\mu \varepsilon_\rho^{\varepsilon} \varepsilon_\sigma^\varepsilon.$$

(13)

taking into account the transversality conditions $p^\mu \varepsilon_\mu = 0$, $q_1^\mu \varepsilon_\mu = 0$ and $q_2^\mu \varepsilon_\mu = 0$. However, in four dimensions there are two linear relations between the five covariants which can be calculated using the Schouten identity. In fact, one has

$$\varepsilon^{q_1q_2\mu\rho} q_1^\rho = \varepsilon^{q_1q_2\rho\delta} q_1^\mu - \varepsilon^{q_1\mu\rho\delta} q_1 q_2 + \varepsilon^{q_2\mu\rho\delta} q_1^2,$$

$$\varepsilon^{q_1q_2\mu\rho} q_2^\rho = \varepsilon^{q_1q_2\rho\delta} q_1^\mu + \varepsilon^{q_1\mu\rho\delta} q_2^2 - \varepsilon^{q_2\mu\rho\delta} q_1 q_2.$$

(14)
The matrix element can therefore be written as

\[ M = \left( B_1 \varepsilon^{q_1 q_2 \rho \delta} q_1^\mu + B_2 \varepsilon^{q_1 \mu \rho \delta} + B_3 \varepsilon^{q_2 \mu \rho \delta} \right) \varepsilon_{\mu \delta \rho}. \]  

(15)

where

\[ B_1 = -(A_3 + A_4 + A_5), \]
\[ B_2 = -(A_1 + q_2^2 A_3 - q_1 q_2 A_4), \]
\[ B_3 = -(A_2 - q_1 q_2 A_3 + q_1^2 A_4). \]  

(16)

The relation between the helicity amplitudes \( H_{\lambda_1 \lambda_2} (\lambda = \lambda_1 - \lambda_2) \) and the invariant amplitudes can be calculated to be

\[ H_{0;+1+1} = -H_{0;-1-1} = -E_1 A_1 - E_2 A_2 - M|q_1|^2 A_5, \]
\[ H_{+1;+10} = -H_{-1;-10} = \left( \frac{E_1 M - M_1^2}{M_2} \right) A_1 + M_2 A_2 - \frac{M^2}{M_2}|q_1|^2 A_4, \]
\[ H_{-1;0+1} = -H_{+1;0-1} = M_1 A_1 + \left( \frac{E_1 M - M_1^2}{M_1} \right) A_2 - \frac{M^2}{M_1}|q_1|^2 A_3. \]  

(17)

The rate of the decay \( 1^+(p) \to 1^- (q_1) + 1^- (q_2) \), finally, reads

\[ \Gamma = \frac{|q_1|}{24\pi M^2} \cdot 2 \left\{ M^2 |q_1|^4 B_1^2 + \left[ (1 + \frac{M^2}{M_2})|q_1|^2 + 3M_1^2 \right] B_2^2 + \left[ (1 + \frac{M^2}{M_1})|q_1|^2 + 3M_2^2 \right] B_3^2 \right. \]
\[ + \left. (M^2 + M_1^2 - M_2^2)|q_1|^2 B_1 B_2 + (M^2 - M_1^2 + M_2^2)|q_1|^2 B_2 B_3 + \left[ 3(M^2 - M_1^2 - M_2^2) - 2|q_1|^2 \right] B_3 B_4 \right\} \]
\[ = \frac{|q_1|}{24\pi M^2} \cdot 2 \left\{ |H_{0;+1+1}|^2 + |H_{+1;+10}|^2 + |H_{-1;0+1}|^2 \right\}. \]  

(18)

III Two-body decays of \( Z_b(10610) \) and \( Z'_b(10650) \) in a covariant quark model

The nonlocal renditions of the local four-quark currents written down in Eqs. (3) and (4) are given by

\[ J_{Z_b}^{\mu}(x) = \int dx_1 \ldots \int dx_4 \, \Phi_{Z_b} \left( \sum_{i < j} (x_i - x_j)^2 \right) \pi^{\mu}(x_1, \ldots, x_4), \]  

(19)

\[ J_{Z'_b}(x) = \frac{1}{\sqrt{2}} \left\{ (\bar{d}(x_3) \gamma_5 b(x_1))(\bar{b}(x_2) \gamma^\mu u(x_4)) + (\bar{d}(x_3) \gamma^\mu b(x_1))(\bar{b}(x_2) \gamma_5 u(x_4)) \right\} \]  

(20)

and

\[ J_{Z'_b}^{\mu\nu}(x) = \varepsilon^{\mu\nu\alpha\beta} (\bar{d}(x_3) \gamma_\alpha b(x_1))(\bar{b}(x_2) \gamma_\beta u(x_4)) \]  

(21)

The effective interaction Lagrangians describing the coupling of the \( Z_b \) and \( Z'_b \) states to its constituent quarks is written in the form

\[ \mathcal{L}_{\text{int.,}Z_b} = g_{Z_b} \, Z_b(x) \cdot J_{Z_b}^{\mu}(x) + \text{H.c.} \]  

(22)

\[ \mathcal{L}_{\text{int.,}Z'_b} = \frac{g_{Z'_b}}{2M_{Z'_b}} \, Z'_b(x) \cdot J_{Z'_b}^{\mu\nu}(x) + \text{H.c.}, \]  

(23)

where \( Z_{b,\mu} = \partial_\mu Z_{b,\nu} - \partial_\nu Z_{b,\mu} \) is the stress tensor of the \( Z'_b \) field. We have included a factor \( 1/M_{Z'_b} \) in the interaction Lagrangian for the \( Z_b, \mu \) state to have the same dimension for the \( g_{Z_b} \) and \( g_{Z'_b} \) couplings.

The coupling constants \( g_H = Z_b, Z'_b \) in Eqs. (22) and (23) are determined by the normalization condition called the compositeness condition (see, Refs. [27] and [32] for details).

\[ Z_H = 1 - g_H^2 \bar{\Pi}_H(M_{Z_H}^2) = 0, \]  

(24)
where $\Pi_H(p^2)$ is the scalar part of the vector-meson mass operator

$$\bar{\Pi}_{H}^{\mu\nu}(p) = g^{\mu\nu} \bar{\Pi}_H(p^2) + p^\mu p^\nu \bar{\Pi}_H^{(1)}(p^2),$$

$$\bar{\Pi}_H(p^2) = \frac{1}{3} \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Pi_{H}^{\mu\nu}(p).$$

(24)

The Fourier-transforms of the $Z_b$ and $Z_b'$ mass operators are given by

$$\bar{\Pi}_{Z_b}^{\mu\nu}(p) = \frac{9}{2} \prod_{i=1}^{3} \int \frac{d^4 k_i}{(2\pi)^4 i} \bar{\Phi}_{Z_b} (\vec{\omega}^2)$$

$$\times \left\{ \tr \left[ \gamma_5 S_1(\hat{k}_1) \gamma_5 S_1(\hat{k}_3) \right] \tr \left[ \gamma_\mu S_4(\hat{k}_4) \gamma_\nu S_2(\hat{k}_2) \right] + \tr \left[ \gamma_\mu S_1(\hat{k}_1) \gamma_\nu S_3(\hat{k}_3) \right] \tr \left[ \gamma_5 S_4(\hat{k}_4) \gamma_5 S_2(\hat{k}_2) \right] \right\}$$

(25)

and

$$\bar{\Pi}_{Z_b'}^{\mu\nu}(p) = -9 \prod_{i=1}^{3} \int \frac{d^4 k_i}{(2\pi)^4 i} \bar{\Phi}_{Z_b'} (\vec{\omega}^2) \frac{\epsilon^{\mu\nu\rho\delta} \epsilon^{\nu\rho\sigma}}{M_{Z_b'}^2}$$

$$\times \tr \left[ \gamma_\rho S_1(\hat{k}_1) \gamma_\sigma S_3(\hat{k}_3) \right] \tr \left[ \gamma_\delta S_4(\hat{k}_4) \gamma_\sigma S_2(\hat{k}_2) \right],$$

(26)

where

$$\hat{k}_1 = k_1 - w_1 p, \hat{k}_2 = k_2 - w_2 p, \hat{k}_3 = k_3 + w_3 p, k_4 = k_1 + k_2 - k_3 + w_4 p,$$

$$\vec{\omega}^2 = 1/2 (k_1^2 + k_2^2 + k_3^2 + k_1 k_2 - k_1 k_3 - k_2 k_3)$$

(27)

and $\epsilon^{\mu\nu\rho\delta} = p_\mu \epsilon^{\nu\rho\sigma}$. 

The matrix elements of the two-body decays are given by

$$M^{\mu\delta} (Z_b(p, \mu) \to \Upsilon(q_1, \delta) + \pi^+(q_2)) = \frac{3}{\sqrt{2}} g_{Z_b} g_{\Upsilon} g_{\pi}$$

$$\times \int \frac{d^4 k_1}{(2\pi)^4 i} \int \frac{d^4 k_2}{(2\pi)^4 i} \bar{\Phi}_{Z_b} (\vec{\eta}^2) \bar{\Phi} (\vec{k}_1 + v_1 q_1 (2) \bar{\Phi} (\vec{k}_2 + u_1 q_2 (2))$$

$$\times \left\{ \tr \left[ \gamma_5 S_1(\hat{k}_1) \gamma_\delta S_2(\hat{k}_1 + q_1) \gamma_\mu S_4(\hat{k}_2) \gamma_5 S_3(\hat{k}_2 + q_2) \right] + \tr \left[ \gamma_\mu S_1(\hat{k}_1) \gamma_\delta S_2(\hat{k}_1 + q_1) \gamma_5 S_4(\hat{k}_2) \gamma_5 S_3(\hat{k}_2 + q_2) \right] \right\}$$

$$= A_{Z_b} \tau_\pi g^{\mu\delta} + B_{Z_b} \tau_\pi \eta_1 \eta_2,$$

(28)

$$M^{\mu\delta} (Z_b'(p, \mu) \to \Upsilon(q_1, \delta) + \pi^+(q_2)) = 3 g_{Z_b'} g_{\Upsilon} g_{\pi} \frac{i\epsilon^{\mu\nu\rho\delta}}{M_{Z_b'}}$$

$$\times \int \frac{d^4 k_1}{(2\pi)^4 i} \int \frac{d^4 k_2}{(2\pi)^4 i} \bar{\Phi}_{Z_b'} (\vec{\eta}^2) \bar{\Phi} (\vec{k}_1 + v_1 q_1 (2) \bar{\Phi} (\vec{k}_2 + u_1 q_2 (2))$$

$$\times \tr \left[ \gamma_\rho S_1(\hat{k}_1) \gamma_\delta S_2(\hat{k}_1 + q_1) \gamma_\mu S_4(\hat{k}_2) \gamma_5 S_3(\hat{k}_2 + q_2) \right]$$

$$= A_{Z_b'} \tau_\pi g^{\mu\delta} + B_{Z_b'} \tau_\pi \eta_1 \eta_2,$$

(29)

$$M^{\mu\rho} (Z_b(p, \mu) \to \eta_0(q_1) + \rho(q_2, \rho)) = \frac{3}{\sqrt{2}} g_{Z_b} g_{\eta_0} g_{\rho}$$

$$\times \int \frac{d^4 k_1}{(2\pi)^4 i} \int \frac{d^4 k_2}{(2\pi)^4 i} \bar{\Phi}_{Z_b} (\vec{\eta}^2) \bar{\Phi}_{n_0} (\vec{k}_1 + v_1 q_1 (2) \bar{\Phi}_{\rho} (\vec{k}_2 + u_1 q_2 (2))$$

$$\times \left\{ \tr \left[ \gamma_5 S_1(\hat{k}_1) \gamma_5 S_2(\hat{k}_1 + q_1) \gamma_\mu S_4(\hat{k}_2) \gamma_\rho S_3(\hat{k}_2 + q_2) \right] + \tr \left[ \gamma_\mu S_1(\hat{k}_1) \gamma_5 S_2(\hat{k}_1 + q_1) \gamma_5 S_4(\hat{k}_2) \gamma_\rho S_3(\hat{k}_2 + q_2) \right] \right\}$$

$$= A_{Z_b n_0} g^{\mu\rho} - B_{Z_b n_0} q_2 g_1,$$

(30)
The argument $w$ and the two-body reduced masses as

\[ M^{\mu \rho} (Z_{b}^{+}(p, \mu) \rightarrow \eta_{b}(q_{1}) + \rho(q_{2}, \rho)) = 3 g_{Z_{b}^{+}g_{b}\bar{g}_{B}} \frac{i \varepsilon^{\mu \rho \nu \delta}}{M_{Z_{b}^{+}}} \]

\[ \times \int \frac{d^{4}k_{1}}{(2\pi)^{4}i} \int \frac{d^{4}k_{2}}{(2\pi)^{4}i} \Phi_{Z_{b}^{+}}(-\vec{\delta}^{2}) \bar{\Phi}_{b} \left( (k_{1} + v_{1}q_{1})^{2} \right) \bar{\Phi}_{\rho} \left( (k_{2} + u_{4}q_{2})^{2} \right) \]

\[ \times \text{tr} [\gamma_{a} S_{1}(k_{1})\gamma_{5} S_{2}(k_{1} + q_{1})\gamma_{\beta} S_{4}(k_{2})\gamma^{\rho} S_{3}(k_{2} + q_{2})] \]

\[ = A_{Z_{b}^{+}g_{b}\bar{g}_{B}} g^{\mu \rho} - B_{Z_{b}^{+}BB} q_{2}^{\mu} q_{1}^{\rho}. \]  

(31)

\[ M^{\mu \delta} (Z_{b}^{+}(p, \mu) \rightarrow h_{b}(q_{1}, \delta) + \pi^{+}(q_{2})) = \frac{3}{\sqrt{2}} g_{Z_{b}^{+}g_{b}\pi} \frac{i \varepsilon^{\mu \rho \nu \delta}}{M_{Z_{b}^{+}}} \]

\[ \times \int \frac{d^{4}k_{1}}{(2\pi)^{4}i} \int \frac{d^{4}k_{2}}{(2\pi)^{4}i} \Phi_{Z_{b}^{+}}(-\vec{\delta}^{2}) \bar{\Phi}_{h_{b}} \left( (k_{1} + v_{1}q_{1})^{2} \right) \bar{\Phi}_{\pi} \left( (k_{2} + u_{4}q_{2})^{2} \right) \]

\[ \times \left\{ \text{tr} [\gamma_{a} S_{1}(k_{1})\gamma_{5} \cdot (2k_{1}^{i}) S_{2}(k_{1} + q_{1})\gamma^{\mu} S_{4}(k_{2})\gamma_{\delta} S_{3}(k_{2} + q_{2})] \right. \]

\[ + \text{tr} [\gamma^{\mu} S_{1}(k_{1})\gamma_{5} \cdot (2k_{1}^{i}) S_{2}(k_{1} + q_{1})\gamma_{\beta} S_{4}(k_{2})\gamma_{\delta} S_{3}(k_{2} + q_{2})] \}

\[ = e^{\rho \mu q_{1} q_{2}} A_{Z_{b}^{+}h_{b}\pi}. \]  

(32)

\[ M^{\mu \delta} (Z_{b}^{+}(p, \mu) \rightarrow h_{b}(q_{1}, \delta) + \pi^{+}(q_{2})) = 3 g_{Z_{b}^{+}g_{b}\pi} \frac{i \varepsilon^{\mu \rho \nu \delta}}{M_{Z_{b}^{+}}} \]

\[ \times \int \frac{d^{4}k_{1}}{(2\pi)^{4}i} \int \frac{d^{4}k_{2}}{(2\pi)^{4}i} \Phi_{Z_{b}^{+}}(-\vec{\delta}^{2}) \bar{\Phi}_{h_{b}} \left( (k_{1} + v_{1}q_{1})^{2} \right) \bar{\Phi}_{\pi} \left( (k_{2} + u_{4}q_{2})^{2} \right) \]

\[ \times \text{tr} [\gamma_{a} S_{1}(k_{1})\gamma_{5} \cdot (2k_{1}^{i}) S_{2}(k_{1} + q_{1})\gamma_{\beta} S_{4}(k_{2})\gamma_{\delta} S_{3}(k_{2} + q_{2})] \]

\[ = e^{\rho \mu q_{1} q_{2}} A_{Z_{b}^{+}h_{b}\pi}. \]  

(33)

The argument $\vec{\delta}^{2}$ of the $Z_{b}$ and $Z_{b}^{+}$ vertex functions is given by

\[ \vec{\delta}^{2} = \vec{\eta}^{2} + \vec{\eta}^{2} + \eta_{3}^{2}, \]

\[ \eta_{1} = \frac{1}{2\sqrt{2}} (2k_{1} + (1 + w_{1} - w_{2})q_{1} + (w_{1} - w_{2})q_{2}), \]

\[ \eta_{2} = \frac{1}{2\sqrt{2}} (2k_{2} - (w_{3} - w_{4})q_{1} + (1 - w_{3} + w_{4})q_{2}), \]

\[ \eta_{3} = \frac{1}{2} ((1 - w_{1} - w_{2})q_{1} - (w_{1} + w_{2})q_{2}), \]  

(34)

where $w_{i} = m_{i}/(m_{1} + m_{2} + m_{3} + m_{4})$. The quark masses $m_{i}$ are specified as $m_{1} = m_{2} = m_{b}$, $m_{3} = m_{4} = m_{d} = m_{u}$, and the two-body reduced masses as $v_{i} = m_{i}/(m_{1} + m_{2}) (i = 1, 2)$ and $w_{j} = m_{j}/(m_{3} + m_{4}) (j = 3, 4)$.

The matrix elements of the decays $Z_{b}^{0} \rightarrow B^{0} + B^{+}$ and $Z_{b}^{*} \rightarrow B^{*0} + B^{+}$ read

\[ M^{\mu \rho} (Z_{b}^{0}(p, \mu) \rightarrow \bar{B}^{0}(q_{1}) + B^{+}(q_{2}, \rho)) = \frac{9}{\sqrt{2}} g_{Z_{b}^{0}g_{B}\bar{g}_{B}} \]

\[ \times \int \frac{d^{4}k_{1}}{(2\pi)^{4}i} \int \frac{d^{4}k_{2}}{(2\pi)^{4}i} \bar{\Phi}_{Z_{b}^{0}} (-\vec{\delta}^{2}) \bar{\Phi}_{B} \left( (k_{2} + v_{2}q_{1})^{2} \right) \bar{\Phi}_{B^{+}} \left( (k_{1} + u_{1}q_{2})^{2} \right) \]

\[ \times \text{tr} [\gamma^{\mu} S_{1}(k_{1})\gamma^{\rho} S_{3}(k_{1} + q_{1})] \text{tr} [\gamma_{5} S_{4}(k_{2})\gamma_{5} S_{2}(k_{2} + q_{2})] \]

\[ = A_{Z_{b}^{0}BB^{+}} g^{\mu \rho} - B_{Z_{b}^{0}BB^{+}} q_{2}^{\mu} q_{1}^{\rho}. \]  

(35)
\[
M^{\mu \alpha} (Z^+_b(p, \mu) \rightarrow B^{*0}(q_1, \delta) + B^+(q_2)) = \frac{9}{\sqrt{2}} g Z_b g_B g_B \\
\times \left( \frac{d^4 k_1}{(2\pi)^4 i} \right) \left( \frac{d^4 k_2}{(2\pi)^4 i} \right) \Phi_{Z_b} \left( -\delta^2 \right) \Phi_{B^*} \left( -(k_1 + \hat{v}_1 q_1)^2 \right) \Phi_{B} \left( -(k_2 + \hat{u}_4 q_2)^2 \right) \\
\times \text{tr} \left[ \gamma_5 S_1(k_1) \gamma_5 S_3(k_1 + q_2) \right] \text{tr} \left[ \gamma_\mu S_4(k_2) \gamma_\delta S_2(k_2 + q_1) \right] \\
= A_{Z_b\alpha\beta} g^{\mu \delta} + B_{Z_b\alpha\beta} q_1^\mu q_2^\delta. \tag{36}
\]

The argument of the vertex function is given by
\[
\delta^2 = \delta_1^2 + \delta_2^2 + \delta_3^2, \\
\delta_1 = -\frac{1}{2\sqrt{2}} (k_1 + k_2 + (w_1 - w_2)q_1 + (1 + w_1 - w_2)q_2), \\
\delta_2 = -\frac{1}{2\sqrt{2}} (k_1 + k_2 + (1 - w_3 + w_4)q_1 - (w_3 - w_4)q_2), \\
\delta_3 = \frac{1}{2} (k_1 - k_2 + (w_1 + w_2)q_1 - (1 - w_1 - w_2)q_2). \tag{37}
\]

The quark masses are specified as \(m_1 = m_2 = m_b, m_3 = m_4 = m_d = m_u\), and the two-body reduced masses as \(\hat{v}_2 = m_2/(m_2 + m_4), \hat{v}_4 = m_4/(m_2 + m_4)\) and \(\hat{u}_1 = m_1/(m_1 + m_3), \hat{u}_3 = m_3/(m_1 + m_3)\).

Finally, we consider the \(Z^+_b \rightarrow B^{*+} + B^{*0}\) decay. This process is described by the invariant matrix element, which is expressed in terms of three relativistic amplitudes \(B_i, (i = 1, 2, 3)\) as
\[
M^{\mu \beta} (Z^+_b(p, \mu) \rightarrow B^{*0}(q_1, \delta) + B^{*+}(q_2, \rho)) = 9 g Z_b g_B g_B \frac{g_{\mu \rho \delta}}{M_{Z_b}^2} \\
\times \left( \frac{d^4 k_1}{(2\pi)^4 i} \right) \left( \frac{d^4 k_2}{(2\pi)^4 i} \right) \Phi_{Z_b} \left( -\delta^2 \right) \Phi_{B^{*+}} \left( -(k_1 + \hat{v}_1 q_1)^2 \right) \Phi_{B^{*0}} \left( -(k_2 + \hat{u}_4 q_2)^2 \right) \\
\times \text{tr} \left[ \gamma_\alpha S_1(k_1) \gamma_\delta S_3(k_1 + q_1) \right] \text{tr} \left[ \gamma_\beta S_4(k_2) \gamma_\rho S_2(k_2 + q_2) \right] \\
= B_1 q_1^\mu \epsilon_{\alpha \nu \rho \delta} + B_2 \epsilon_{\alpha \mu \nu \rho} + B_3 \epsilon_{\alpha \mu \rho \delta}. \tag{38}
\]

### IV Numerical results

First of all, we would like to note that all adjustable parameters of our model (constituent quark masses, infrared cutoff, and size parameters) have been fixed in our previous studies by a global fit to a multitude of experimental data [3]. The only two new parameters are the size parameters of the two exotic \(Z_0^b\) states. As a guide to adjust them we take the experimental values of the largest branching fractions presented in Ref. 3:
\[
\mathcal{B}(Z^+_b \rightarrow [B^+ B^{*0} + B^{*0} B^{*+}]) = 85.6^{+1.5+1.5}_{-2.0-2.1}\% ,
\]
\[
\mathcal{B}(Z^+_b \rightarrow B^{*+} B^{*0}) = 73.7^{+3.4+3.2}_{-4.4-3.5}\% . \tag{39}
\]

By using the central values of these branching rates and total decay widths given in Eq. 2 we find the central values of our size parameters \(\Lambda_{Z_b} = 3.45\text{ GeV}\) and \(\Lambda_{Z_b'} = 3.00\text{ GeV}\). Allowing them to vary in the interval
\[
\Lambda_{Z_b} = 3.45 \pm 0.05\text{ GeV} \quad \Lambda_{Z_b'} = 3.00 \pm 0.05\text{ GeV}, \tag{40}
\]
we obtain the values of various decay widths shown in Table III

The total widths are equal to \(\sum_i \Gamma_i(Z_b) = 30.9^{+2.3+2.3}_{-2.1-2.1}\text{ MeV}\) and \(\sum_i \Gamma_i(Z_b') = 34.1^{+2.8+2.8}_{-2.5-2.5}\text{ MeV}\) which should be compared with the experimental values \(\Gamma(Z_b) = 25 \pm 7\text{ MeV}\) and \(\Gamma(Z_b') = 23 \pm 8\text{ MeV}\), respectively. The Belle observations indicate that the decays involving bottomonium states are significantly suppressed compared with the \(B\)-meson modes. In our calculation we find that the modes with \(\Upsilon(1S)\pi^+\) and \(\eta_b \rho^+\) are suppressed but not as much.
TABLE II: Particle decay widths for the $Z_b(10610)$ and $Z_b^\prime(10650)$.

| Channel | $Z_b(10610)$ | $Z_b(10650)$ |
|---------|--------------|--------------|
| $\Upsilon(1S)\pi^+$ | $5.9\pm0.4$ | $9.5^{+0.7}_{-0.6}$ |
| $h_b(1P)\pi^+$ | $(0.14\pm0.01)\cdot10^{-1}$ | $0.74^{+0.05}_{-0.04}\cdot10^{-3}$ |
| $h_b\rho^+$ | $4.4\pm0.3$ | $7.5^{+0.6}_{-0.5}$ |
| $B^B B^{*0} + B^{0} B^{*+}$ | $20.7^{+1.6}_{-1.5}$ | $-17.1^{+1.5}_{-1.4}$ |

as in the data. As one can see from Table II the ratios of decay rates are

$$\frac{\Gamma(Z_b \to \Upsilon(1S)\pi)}{\Gamma(Z_b \to BB^* + \text{c.c.})} \approx 0.29,$$

$$\frac{\Gamma(Z_b \to h_b\rho^+)}{\Gamma(Z_b \to BB^* + \text{c.c.})} \approx 0.21,$$

$$\frac{\Gamma(Z_b^\prime \to \Upsilon(1S)\pi)}{\Gamma(Z_b^\prime \to BB^* + \text{c.c.})} \approx 0.56,$$

$$\frac{\Gamma(Z_b^\prime \to h_b\rho^+)}{\Gamma(Z_b^\prime \to BB^* + \text{c.c.})} \approx 0.44 .$$

The decays into the $h_b(1P)\pi^+$ mode are suppressed by the $p$-wave suppression factor in the rate expression (41).

V Summary

By using molecular-type four-quark currents for the recently observed resonances $Z_b(10610)$ and $Z_b^\prime(10650)$, we have calculated their two-body decay rates into a bottomonium state plus a light meson as well as into $B$-meson pairs.

We have fixed the model size parameters by adjusting the theoretical values of the largest branching fractions of the modes with the $B$-mesons in the final states to their experimental values.

We found that the modes with $\Upsilon(1S)\pi^+$ and $h_b\rho^+$ in the final states are suppressed but not as much as the Belle Collaboration reported.

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[1] A. Bondar et al. (Belle Collaboration), Phys. Rev. Lett. 108, 122001 (2012).
[2] A. Garmash et al. (Belle Collaboration), Phys. Rev. D 91, 072003 (2015).
[3] A. Garmash et al. (Belle Collaboration), Phys. Rev. Lett. 116, no. 21, 212001 (2016).
[4] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk and M. B. Voloshin, Phys. Rev. D 84, 054010 (2011).
[5] M. B. Voloshin, Phys. Rev. D 84, 031502 (2011).
[6] J. R. Zhang, M. Zhong and M. Q. Huang, Phys. Lett. B 704, 312 (2011). C. Y. Cui, Y. L. Liu, M. Q. Huang, Phys. Rev. D 85, 074014 (2012).
[7] Y. Yang, J. Ping, C. Deng and H. S. Zong, J. Phys. G 39, 105001 (2012).
[8] I. V. Danilkin, V. D. Orlovsky and Yu. A. Simonov, Phys. Rev. D 85, 034012 (2012).
[9] Z. F. Sun, J. He, X. Liu, Z. G. Luo and S. L. Zhu, Phys. Rev. D 84, 054002 (2011).
[10] D. Y. Chen, X. Liu and S. L. Zhu, Phys. Rev. D 84, 074016 (2011); D. Y. Chen and X. Liu, Phys. Rev. D 84, 094003 (2011); X. Liu and D. Y. Chen, Few Body Syst. 54, 165 (2013).

[11] M. Cleven, F. K. Guo, C. Hanhart and U. G. Meissner, Eur. Phys. J. A 47, 120 (2011).

[12] T. Guo, L. Cao, M. Z. Zhou and H. Chen, arXiv:1106.2284 [hep-ph].

[13] F. S. Navarra, M. Nielsen, J.-M. Richard, J. Phys. Conf. Ser. 348, 012007 (2012).

[14] M. T. Li, W. L. Wang, Y. B. Dong and Z. Y. Zhang, J. Phys. G 40, 015003 (2013).

[15] D. Y. Chen, X. Liu and T. Matsuki, Chin. Phys. C 38, 053102 (2014).

[16] Y. Dong, A. Faessler, T. Gutsche and V. E. Lyubovitskij, J. Phys. G 40, 015002 (2013); Few Body Syst. 54, 1011 (2013).

[17] G. Li, F. I. Shao, C. W. Zhao and Q. Zhao, Phys. Rev. D 87, 034020 (2013).

[18] M. Cleven, Q. Wang, F. K. Guo, C. Hanhart, U. G. Meissner and Q. Zhao, Phys. Rev. D 87, 074006 (2013).

[19] S. Ohkoda, S. Yasui and A. Hosaka, Phys. Rev. D 89, 074029 (2014).

[20] Z. G. Wang and T. Huang, Nucl. Phys. A 930, 63 (2014). Z. G. Wang, Eur. Phys. J. C 74, 2963 (2014).

[21] J. M. Dias, F. Aceti and E. Oset, Phys. Rev. D 91, 076001 (2015).

[22] W. S. Huo and G. Y. Chen, Eur. Phys. J. C 76, 172 (2016).

[23] X. W. Kang, Z. H. Guo and J. A. Oller, Phys. Rev. D 94, 014012 (2016).

[24] T. Gutsche, V. E. Lyubovitskij and I. Schmidt, arXiv:1706.07716 [hep-ph].

[25] A. Faessler, T. Gutsche, V. E. Lyubovitskij and Y. L. Ma, Phys. Rev. D 76, 014005 (2007); Phys. Rev. D 76, 114008 (2007); Phys. Rev. D 77, 114013 (2008); Phys. Rev. D 76, 014003 (2007); Phys. Rev. D 77, 094013 (2008); J. Phys. G 38, 015001 (2011); J. Phys. G 40, 015002 (2013); Phys. Rev. D 88, 014030 (2013); T. Branz, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D 79, 014035 (2009); Phys. Rev. D 80, 054019 (2009); Y. Dong, A. Faessler, T. Gutsche, Sergey Kovalenko, and V. E. Lyubovitskij, Phys. Rev. D 79, 094013 (2009); T. Branz, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D 82, 054010 (2010); Phys. Rev. D 82, 054025 (2010); Y. Dong, A. Faessler, T. Gutsche, S. Kumano and V. E. Lyubovitskij, Phys. Rev. D 82, 034035 (2010); T. Gutsche, M. Kesenheimer and V. E. Lyubovitskij, Phys. Rev. D 90, 094013 (2014); Y. Dong, A. Faessler, T. Gutsche, Q. Liu and V. E. Lyubovitskij, arXiv:1705.09631 [hep-ph].

[26] S. Weinberg, Phys. Rev. 130, 776 (1963); A. Salam, Nuovo Cim. 25, 224 (1962); K. Hayashi, M. Hirayama, T. Muta, N. Seto and T. Shirafuji, Fortsch. Phys. 15, 625 (1967).

[27] G. V. Efimov and M. A. Ivanov, The Quark Confinement Model of Hadrons, (IOP Publishing, Bristol & Philadelphia, 1993).

[28] S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov and J. G. Körner, Phys. Rev. D 81, 114007 (2010).

[29] S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, J. G. Körner, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D 84, 014006 (2011).

[30] T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, Phys. Rev. D 94, 094012 (2016).

[31] F. Goerke, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli, Phys. Rev. D 94, 094017 (2016).

[32] M. A. Ivanov, J. G. Körner and P. Santorelli, Phys. Rev. D 71, 094006 (2005) [Phys. Rev. D 75, 019901(E) (2007)].

[33] G. V. Efimov and M. A. Ivanov, Int. J. Mod. Phys. A 4, 2031 (1989); I. V. Anikin, M. A. Ivanov, N. B. Kulimanova and V. E. Lyubovitskij, Z. Phys. C 65, 681 (1995); M. A. Ivanov, M. P. Locher and V. E. Lyubovitskij, Few Body Syst. 21, 131 (1996); M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner and P. Kroll, Phys. Rev. D 56, 348 (1997); M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D 60, 094002 (1999); A. Faessler, T. Gutsche, M. A. Ivanov, V. E. Lyubovitskij and P. Wang, Phys. Rev. D 68, 014011 (2003); M. A. Ivanov, J. G. Körner and P. Santorelli, Phys. Rev. D 73, 054024 (2006); A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, D. Nicmoruss and K. Pumsa-ard, Phys. Rev. D 73, 094013 (2006); A. Faessler, T. Gutsche, B. R. Holstein, V. E. Lyubovitskij, D. Nicmoruss and K. Pumsa-ard, Phys. Rev. D 74, 074010 (2006); A. Faessler, T. Gutsche, B. R. Holstein, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, Phys. Rev. D 78, 094005 (2008); A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, Phys. Rev. D 80, 034025 (2009); T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, Phys. Rev. D 81, 034010 (2010); M. A. Ivanov, J. G. Körner, S. G. Kovalenko, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D 85, 034004 (2012); T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli, Phys. Rev. D 86, 074013 (2012); Phys. Rev. D 87, 074031 (2013); Phys. Rev. D 88, 114018 (2013); Phys. Rev. D 90, 114033 (2014); Phys. Rev. D 92, 114008 (2015); Phys. Rev. D 93, 034008 (2016); T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli and N. Habyl, Phys. Rev. D 91, 074001 (2015) [Phys. Rev. D 91, 119907(E) (2015)]; T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, V. V. Lyubushkin and P. Santorelli, arXiv:1705.07290 [hep-ph].

[34] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, 100001 (2016).