Scalar Dark Matter Formation in Electron - Axion Like Collider
Xuan N.N¹, Thang V.T²

¹Department of Physics, Le Qui Don University, Ha noi, Vietnam
Email: xuanmn@lqdu.edu.vn.com
²Faculty of Basic Science, Ngo Quyen University, Binh Duong, Vietnam
Email: vuthang76@gmail.com

Received: 19 Nov 2021,
Received in revised form: 20 Dec 2021,
Accepted: 28 Dec 2021,
Available online: 11 Jan 2022
©2022 The Author(s). Published by AI Publication. This is an open access article under the CC BY license (https://creativecommons.org/licenses/by/4.0/).

Abstract—The aim paper is devoted to study the interaction of axion like (ALPs) with fermions when expanding the standard model. The process of collider between axion-electron, which produce scalar dark matter and release electron has been studied. The scattering amplitude and cross section of this collision in the center of mass system (CMS) have been calculated by using Mandelstam variables and then plotted its dependence on scattering angle and energy.

Keywords—Axion – Axion like, Dark Matter, Standard Model, New Physics

I. INTRODUCTION
In recent years, the search for particles outside the standard model with small mass and weak interaction with the standard model has attracted a lot of attention from the scientific community, especially those who interested in interactions of high-energy elementary particles. The motive of this interest is to answer the question of whether there is new physics; Are the new particles energetic or light? Theoretical studies also want to show that the light particles that interact weakly with the particles in the standard model are spontaneously generated by the extension of the standard model, as well as the dark matter (DM) formation process.

Axion like particles (ALPs) are the scalar (or pseudo – scalar) particles that appear in many different physical models, they can act as a Goldstone Boson of the U(1)PQ group (Peccei Quinn) [1,2] and can also appears as a component of the Chiral super-field in the super-symmetry theory (SUSY) [2]. ALPs with mass around MeV are associated with a wide range of phenomena for cosmology and astrophysics [3], such as affecting the Bigbang Nucleosynthesis (BBN), CMB, and stellar evolution. ALPs are also involved in the formation of cold dark matter and can be used to account for a large number of astronomical singularities such as the extreme cold efficiency of a class of stars, the transparency to the strangeness of the universe with its super-high-energy gamma ray [4] or the hiding of monochromatic X-ray around the energy of 3.5KeV [5]. ALPs also play a key role in breaking electroweak symmetry and in solving hierarchy problems through relaxation mechanisms. ALPs also give us the exciting ability to connect the standard model to potential dark matter particles [6].

The aim of this paper is to consider the interaction of scalar particles as ALPs with fermions when expanding the standard model, it occurs in most of the strong CP problems, in the model with break super-symmetry. The ALPs are pseudo-Goldstone bosons, they are light and very weakly bound together. The process of axion-electron interaction through positron exchange will study. It
produces scalar dark matter and release electrons. We will calculate the scattering amplitude and scattering cross section of the collision in the center of mass system (CMS) with Mandelstam variables and investigate the graphing number.

The paper is constructed as follow: In Sec.2, we find an expression of the scattering amplitude based on the Feynman diagram at the s-channel and the t-channel with Mandelstam variables used. In Sec. 3, the scattering cross section is deduced and numerically calculated, and then plotted its dependence on scattering angle and energy. In Section 4, we give some discussion and conclusion about the obtained results.

II. ALPS – ELECTRON SCATTERING AMPLITUDE

In this section, we will derive an expression for the scattering amplitude of the process

\[ a(k_1)e^-(p_1)\rightarrow\phi(k_2)e^-(p_2), \]

which ALPS collider with electron to create e+ through positron exchange.

The propagator of an electron has the form

\[ D(q) = \frac{1}{q^2 - m^2_e + i\epsilon}, \]

where \( q^2 = \frac{m^2_e}{2} \) is the square of momentum transfer.

The Lagrangian of axion – electron interaction is [7,8]

\[ L_{ae} = -g_s\bar{e}\gamma_s e, \]

where \( g_s \approx \frac{m_s m_e}{m_f} = 4.07 \times 10^{11} m_e \) is the axion – electron coupling [7,9].

From (1) and (2), we use Feynman diagram law to compute the scattering amplitude for above process. In this process, ALPS – electron interaction is carried out simultaneously in both channels, s-channel and t-channel. So the total scattering matrix is

\[ M = M_s + M_t. \]

Where \( M_s \) and \( M_t \) are the scattering matrices of s-channel and t-channel respectively.

2.1. s- Channel scattering amplitude

In this channel (Fig.1), an electron with momentum \( p_1 \) absorbs an axion with momentum \( k_1 \) to produce a positron, which then radiates out scalar dark matter with momentum \( k_2 \) and produces an electron with momentum \( p_2 \).

Based on the rule of calculating amplitude according to Feynman diagram, we have

\[ iM_s = \bar{u}(p_2)\left(-\frac{im_em_f}{m_e}\gamma_5\right)u(p_1)\left(\frac{1}{q_s^2 - m^2_e}\right) \]

\[ = \frac{im_em_f}{m_e} \bar{u}(p_2)\gamma_5u(p_1) \]

\[ = \alpha(k_1) \]

\[ \rightarrow \]

\[ e^-(p_1) \rightarrow e^+(q_2) \]

\[ e^+(p_2) \]

\[ \phi(k_2) \]

\[ \text{Fig. 1: s – Channel} \]

Note that the sum is taken according to the spin states of the particle

\[ u(p)\bar{u}(p) = (p_\mu\gamma^\mu \pm m)_{\text{anomaly}} = (\hat{p} \pm m)_{\text{anomaly}} \]

Here (+) for particles and (-) for anti-particles. Thus,

\[ |M_s|^2 = \frac{m^2_em^2_f}{m^2_e(q^2 - m_e^2)^2} Tr\left[ (\hat{p}_1 + m_e)(\gamma^\mu\gamma_5)(\hat{p}_1 - m_e) \right] \]

We already know the properties of gamma matrices

\[ \gamma_5 = -\gamma_s, \]

\[ (\gamma_5^\mu)^* = -\gamma_s(\gamma_5^\mu)^* = -\gamma_s(k = 1, 2, 3), \]

\[ \gamma_\mu = i\gamma^\mu_5, \gamma_5 = i\gamma_\mu = \gamma_\mu, \gamma_\mu^* = \gamma_\mu^*, \gamma_\mu^* = \gamma_\mu, \gamma_\mu^* = \gamma_\mu^* \]

\[ \gamma^2_5 = 1 \]

and \( \gamma_s \) is anti-commutative with the other gamma matrices, then

\[ (\hat{p}_1 + m_e)\gamma_5 = \gamma_5(\hat{p}_1 - m_e) \]

Putting eqs. (6), (7) into eq.5 we get

\[ |M_s|^2 = \frac{m^2_em^2_f}{m^2_e(q^2 - m_e^2)^2} Tr\left[ p_1^\mu(\gamma_\mu\gamma_5 - m_e^2)(\hat{p}_1 - m_e) \right] \]

Since the product of an odd number of Dirac matrices equals zero, and

\[ Tr\left[ \gamma_\mu(\gamma_5^\mu)(\gamma_\mu^*\gamma_5^\mu - m_e^2) \right] = Tr\left[ p_1^\mu(\gamma_\mu\gamma_5 - m_e^2) \right] = p_1^{\mu_1} p_1^{\mu_2} (4g_{\mu_1\mu_2} - 4m^2_e) = 4(p_1^\mu - m_e^2) \]

Substitute (9) into (8), we obtain:

\[ |M_s|^2 = \frac{m^2_em^2_f}{m^2_e(4E_1^2)^2} 4(p_1^\mu - m_e^2) \]

Considering the center of mass reference system, using the law of conservation of momentum:
\[ \mathbf{p}_1 + \mathbf{k}_1 = 0 \Rightarrow \mathbf{p}_1 = -\mathbf{k}_1 = \mathbf{p}; \]
\[ \mathbf{p}_2 + \mathbf{k}_2 = 0 \Rightarrow \mathbf{p}_2 = -\mathbf{k}_2 = \mathbf{k}; \]

Here Mandelstam variables are used:
\[ p_1 = (E_1, \mathbf{p}); k_1 = (E_2, -\mathbf{p}); \]
\[ p_2 = (E_3, \mathbf{k}); k_2 = (E_4, -\mathbf{k}); \]

\[ p_1, p_2, k_1, k_2 \] are the 4 − component vectors; \( \mathbf{p}, \mathbf{k} \) are the 3 − component momenta of incident and scattered particles.

We have \( \mathbf{p}^2 = E^2 - m^2 \) then
\[ \mathbf{p}^2 = E_1^2 - m_1^2 = E_2^2 - m_2^2; \quad \mathbf{k}^2 = E_3^2 - m_3^2 = E_4^2 - m_4^2 \]

\[ s = q^2 = (p_1 + k_1)^2 \]
\[ = m_1^2 + m_2^2 + 2(E_1 E_2 + \sqrt{E_1^2 - m_1^2} \sqrt{E_2^2 - m_2^2}) \]

\[ s = m_3^2 + m_4^2 + 2(E_3 E_4 + \sqrt{E_3^2 - m_3^2} \sqrt{E_4^2 - m_4^2}) \]

Since the law of conservation of energy:
\[ E_1 + E_2 = E_3 + E_4 \]

Ignore the mass of the particles besides the energy terms
\[ |\mathbf{p}| = E_1 = E_3 = E; \quad |\mathbf{k}| = E_2 = E_4 \]

\[ q_1^2 - m_1^2 \approx 4E_1 E_2 \]

So we have
\[ \begin{align*}
  p_1^2 + p_2^2 &= E_3 - \mathbf{p} \cdot \mathbf{k} = E_1 E_3 - |\mathbf{p}| |\mathbf{k}| \cos \theta \\
  &= E_1 E_3 - \sqrt{E_1^2 - m_1^2} \sqrt{E_3^2 - m_3^2} \cos \theta \\
  &\approx E_1 E_3 - E_1 E_3 \cos \theta = E_1 E_3 (1 - \cos \theta)
\end{align*} \]

Then substitute (19) into (10), we derive
\[ |M_s|^2 = \frac{m_1^2 g_s^2}{m_2^2 (4E_2)} \left[ 4(E_1 E_3 (1 - \cos \theta) - m_1^2) \right] \]
\[ \approx \frac{m_1^2 g_s^2}{4m_2^2 (4E_2)} E_1 E_3 (1 - \cos \theta) \]

From (13), (16), (17) and (18) while ignoring the electron mass \( m_e \)
\[ \sqrt{E_1^2 - m_1^2} + E_1 = 2E \Rightarrow E_1 = E \left( 1 + \frac{m_1^2}{4E} \right) = E \left( 1 + \frac{m_e^2}{4s} \right) \]
\[ E_3 = 2E - E_4 = E - \frac{m_2^2}{4E} = E \left( 1 - \frac{m_2^2}{4s} \right) \]
\[ s = q_1^2 - m_1^2 = q_2^2 = 4E_1 E_2 = 4E^2 \]

So the final expression for the \( s \) - channel scattering amplitude is
\[ |M_s|^2 = \frac{m_1^2 g_s^2}{4m_2^2 E} \left( 1 - \frac{m_2^2}{s} \right) (1 - \cos \theta) \]

\[ = \frac{m_1^2 g_s^2}{4m_2^2 E} \left( 1 - \frac{m_2^2}{4s} \right) (1 - \cos \theta) \]

\[ = \frac{m_1^2 g_s^2}{4m_2^2 E} \left( 1 - \frac{m_2^2}{4s} \right) (1 + \cos \theta)^2 \]

### 2.2. \( t \)-channel scattering amplitude

In this channel, an electron radiates out scalar dark matter and produces positron, which immediately combines with axion to produce electron.

Doing the same as the \( s \)-channel transform, we have
\[ |M_t|^2 = \frac{m_1^2 g_s^2}{m_2^2 (q_1^2 - m_1^2)} 4E_1 (1 - \cos \theta) \]
\[ = \frac{m_1^2 g_s^2}{m_2^2 (q_1^2 - m_1^2)} 4E^2 \left( 1 - \frac{m_2^2}{4E^2} \right) (1 - \cos \theta) \]

\[ \phi(k_2) \quad e^+(p_1) \quad e^-(p_2) \]

\[ \text{Fig. 2: } t \text{- Channel} \]

Further more
\[ q_1^2 = (p_1 - k_2)^2 = p_1^2 + k_2^2 - 2p_1k_2 \]
\[ = m_1^2 + m_2^2 - 2(E_1 E_3 + \sqrt{E_1^2 - m_1^2} \sqrt{E_3^2 - m_2^2} \cos \theta) \]

Ignore the mass of the particles besides the energy term \( E_1, E_3 \) to derive
\[ q_1^2 - m_1^2 = 2E^2 \left( \frac{m_2^2}{4E^2} - 1 \right) (1 + \cos \theta) \]

Thus, the expression of the \( t \)-channel scattering amplitude is
\[ |M_t|^2 = \frac{m_1^2 g_s^2}{m_2^2 E^2 \left( \frac{m_2^2}{4E^2} - 1 \right)} (1 + \cos \theta)^2 \]

\[ = \frac{m_1^2 g_s^2 (1 - \cos \theta)}{m_2^2 E^2 \left( \frac{m_2^2}{4E^2} - 1 \right)} (1 + \cos \theta)^2 \]

\[ = \frac{m_1^2 g_s^2 (1 - \cos \theta)}{m_2^2 E^2 \left( \frac{m_2^2}{4E^2} - 1 \right)} (1 + \cos \theta)^2 \]

### 2.3. Total scattering amplitude

To continue calculating the total scattering amplitude according to (3), we have to calculate two more terms \( M_s M_s^*, M_t M_t^* \).

Firstly,
\[ M, M^* = \frac{m^2_{\gamma} g^2_{\gamma}}{m^2_{\gamma}(q^2_{-} - m^2_{\gamma})(q^2_{+} - m^2_{\gamma})} \text{Tr}[(\hat{p}_{\gamma} + m_{\gamma})\gamma_5(\hat{p}_{\gamma} + m_{\gamma})\gamma_5] \]

\[ = -\frac{m^2_{\gamma} g^2_{\gamma}(1 - \cos \theta)}{8m^2_{\gamma}E^2 (1 + \cos \theta)} \]

and similarly \[ M^*_M = -\frac{m^2_{\gamma} g^2_{\gamma}(1 - \cos \theta)}{8m^2_{\gamma}E^2 (1 + \cos \theta)} \]

Then, the expression of scattering amplitude is

\[ |M|^2 = \frac{m^2_{\gamma} g^2_{\gamma}}{4m^2_{\gamma}E^2} \left( 1 - \frac{m^2_{p}}{s} \right)(1 - \cos \theta) \]

\[ + \frac{m^2_{\gamma} g^2_{\gamma}(1 - \cos \theta)}{m^2_{\gamma}E^2 \left( 1 - \frac{m^2_{p}}{s} \right)(1 + \cos \theta)^2} - \frac{m^2_{\gamma} g^2_{\gamma}(1 - \cos \theta)}{2m^2_{\gamma}E^2 (1 + \cos \theta)} \]

\[ = \frac{m^2_{\gamma} g^2_{\gamma}}{m^2_{s}} (1 - \cos \theta) \times \]

\[ \left[ 1 - \frac{m^2_{p}}{s} \right] + \frac{4}{\left( 1 - \frac{m^2_{p}}{s} \right)(1 + \cos \theta)^2} - \frac{2}{(1 + \cos \theta)} \]

\[ \text{(31)} \]

**III. CROSS SECTION**

In this section we will calculate the differential scattering cross section and the total scattering cross section of the above collision. These are important physical parameters that can be compared with experimental measurements.

By definition, the differential cross-section is equal to

\[ \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2s} \left| \frac{d\hat{\sigma}}{d\hat{\Omega}} \right| |M|^2 \]  \[ \text{(32)} \]

Substitute the expression of the total scattering amplitude in (31) into (32), we have

\[ \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2s} \frac{m^2_{\gamma} g^2_{\gamma}}{m^2_{\gamma} E^2} (1 - \cos \theta) \times \]

\[ \left[ 1 - \frac{m^2_{p}}{s} \right] + \frac{4}{\left( 1 - \frac{m^2_{p}}{s} \right)(1 + \cos \theta)^2} - \frac{2}{(1 + \cos \theta)} \]

\[ = \frac{1}{64\pi^2s^2} \frac{m^2_{\gamma} g^2_{\gamma}}{m^2_{\gamma}} (1 - \cos \theta) \left[ \left( 1 - \frac{m^2_{p}}{s} \right) - \frac{2}{(1 + \cos \theta)} \right] \]

\[ \text{(33)} \]

Integrating this expression with attention \[ d\Omega = 2\pi d(\cos \theta) \] , we get the total scattering cross section

\[ \sigma = \frac{1}{32\pi s^2} \frac{m^2_{\gamma} g^2_{\gamma}}{m^2_{\gamma}} (1 - \cos \theta) \left[ \left( 1 - \frac{m^2_{p}}{s} \right) - \frac{2}{(1 + \cos \theta)} \right]^2 d(\cos \theta) \] \[ \text{(34)} \]

Set \[ A = 1 - \frac{m^2_{p}}{s} ; x = 1 + \cos \theta \] then

\[ \sigma = \frac{1}{32\pi s^2} \frac{m^2_{\gamma} g^2_{\gamma}}{m^2_{\gamma}} \left[ -\frac{1}{2} A^2 x^2 + 2(A^2 + 2A)x - 4(A - 1) \ln \frac{\frac{8}{x} - 1}{8} \right] \] \[ \text{(35)} \]

We choose the input parameters as follows [8]:

\[ m_{\gamma} = 0.511 \text{ MeV} = 5.11 \times 10^{-4} \text{ GeV}; g_{\gamma} = 4.0710^{-11} m_{\gamma} \]

We then plot the dependence of the differential cross section in terms of \( \cos \theta \) when choosing \( s = 14 \text{ TeV} \) with different \( m_{\gamma} \).

According to the Fig. 3, it is clear that the dark matter effect in this scattering process can only be clearly observed at small scattering angles \( (\theta \approx 0) \), while at large angles, the effect is negligible.
We plot the dependence of the total scattering cross section on the collision energy, with the scattering angle $\theta = 0$. We see, the total scattering cross section in this case is inversely proportional to the collision energy, the larger energy collision, the smaller the total scattering cross-section.

![Graph of the dependence of the total cross section on the collision energy](image)

*Fig. 4: Graph of the dependence of the total cross section on the collision energy*

And finally we plot the dependence of the total scattering cross-section on the $m_\phi$ represented as follows

![Graph of the dependence of the total cross section on the mass of dark matter](image)

*Fig. 5: Graph of the dependence of the total cross section on the mass of dark matter*

The total scattering cross section in this process is directly proportional to the mass, as the mass increases; the total scattering cross section also increases. This is the same as in the case of $\gamma\gamma \rightarrow \gamma\gamma$ scattering [12,13] where radions are involved.

IV. CONCLUSION

In this work we have discussed theoretical situation concerning ALPs particles that interact with Standard Model particles via couplings to electron. It should be emphasized that the effects of the radion have been found to be quite strong [11,12,13]. A scenario of particular interest is ALPs coupled electron to a light scalar DM particle. In this case, DM may pair – annihilate into photons DM can couple-annihilate into photons and therefore it is very difficult for us to observe DM-generating effects experimentally. Our results are attractive because of possible connection to radion and dark matter. We hope that future experiments will confirm the existence of radion. Works along these lines are in progress.
ACKNOWLEDGEMENTS

The author (Xuan N.N) would like to thank to Prof. N. S. Han for supporting and making useful remarks to improve this paper. This work is also supported by Le Qui Don University.

REFERENCES

[1] R. D. Peccei and H. R. Quinn (1977). Constraints imposed by CP conservation in the presence of pseudoparticles. Phys. Rev. D 16, 1791.
[2] Joerg Jaeckel and Michael Spannowsky (Sep.2015). Probing MEV to 90 GeV axion-like particles with LEP and LHV. arXiv:1509.00476v [hep-ph]
[3] D. Cadamuro and J. Redondo (2012). Cosmological bounds on pseudo Nambu-Goldstone bosons. JCAP 02, 032 [arXiv:1110.2895] [INSPIRE].
[4] M. Millea, L. Knox and B. Fields (2015). Phases of new physics in the CMB. Phys. Rev. D 92, 023010 [arXiv:1501.04097] [INSPIRE].
[5] P. Arias, D. Cadamuro, M. Goodsell, J. Jaeckel, J. Redondo and A. Ringwald (2012). WISPy Cold Dark Matter. JCAP 06, 013 [arXiv:1201.5902] [INSPIRE].
[6] Matthew J. Dolan, Torben Ferber, Christopher Hearty, Felix Kahlhoefer and Kai Schmidt-Hoberg (2017). Revised constraints and Belle II sensitivity for visible and invisible axion-like particles. JHEP12, 094 [arXiv:1709.00009]
[7] J .E. Kim (1984). A Common Scale for the Invisible Axion, Local SUSY GUTs and SaxinoDecay. Phys. Lett.B136, 378
[8] Giovanni Carugno (2015). Cosmological Axion Search: A Brief and Partial Introduction. SIF ROMA SETTEMBRE
[9] Ricardo Z. Ferreira, Alessio Notari, and Fabrizio Rompineve (2021). Dine-Fischler-Srednicki-Zhitnitsky axion in the CMB. Phys. Rev. D 103, 063524
[10] C. Boehm and P. Uwer (2006). Revisiting Bremsstrahlung emission associated with Light Dark Matter annihilations. arXiv:hep-ph/0606058.
[11] D.V. Soa, D.T.L. Thuy, N.H. Thao and T.D. Tham (2012). Radion production in gamma-electron collisions. Mod. Phys. Lett. A27, 1250126.
[12] S.T.L. Anh, H.H. Bang et al (2018), Unparticle effects on bhabha scattering, Can. J. Phys. 96:3, 268.
[13] S.T.L. Anh, H.H. Bang et al (2018), Unparticles effects on axion like particles production in e+e- collisions, Int. J. Theor. Phys 149.