The measurement problem and the completeness of quantum state

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Abstract: Based on the equivalence of quantum-mechanical reduced state and mixed state which we demonstrate in this paper, we give a possible explanation of the quantum measurement problem. We describe quantum measurement process as two-step physical process: one microscopically controllable process which produces entanglement, and one macroscopic process (uncontrollable microscopically) which detects given states (or properties) of some microscopic objects. With the explanation, we conclude that the measurement postulate is just a corollary of the completeness of quantum state. Our explanation is totally rooted in the unitarity of quantum mechanics, and does not need any extension or modification of the formulation of orthodox quantum mechanics. We also point out that quantum randomness comes from entanglement and wavefunction collapse in quantum mechanics is subjective process.

Keywords: measurement problem; quantum state; quantum mechanics;

1. Introduction

The measurement problem is an important issue in researches on the foundation of quantum mechanics [1-15]. It has been studied extensively and seriously, but there are no satisfying answers accepted by most of physicists so far [16, 17]. Most of answers to the measurement problem focus on the following topics.

The measurement postulate in quantum mechanics tells us that if a physical system is in a state \( \sum_i c_i |a_i\rangle \) (where \( \{|a_i\rangle\} \) is a basis of the Hilbert space assigned to the system), then the system will collapse to a state \( |a_i\rangle \) with corresponding probability \( |c_i|^2 \) after being measured in \( \{|a_i\rangle\} \) basis. Measurement causes discontinuous change of quantum state, which conflicts with the continuous evolution ruled by Schrodinger equation. Since proposed, it gives rise to a lot of doubts and controversies and has been regarded as the key point in the measurement problem [2-4, 7, 10, 18].

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Moreover, there is no further explanation in the measurement postulate for what is quantum measurement and how a measurement occurs. According to von Neumann’s interpretation about quantum measurement [1], if a measured system and a measuring device are in the states $\sum_i c_i |a_i\rangle$ and $|m_i\rangle$ respectively before measurement, then the total system will be in the state $\sum_i c_i |a_i\rangle |m_i\rangle$ (where, the state $|m_i\rangle$ is the state of the measuring device corresponding to the state $|a_i\rangle$ of the measured system) after measurement, that is, the total system is in a superposition of different states, but not the one of these states. Some physicists believe that, to explain why the measured system is always in one definite state with some specific probability, but not in the superposition is the key issue in the measurement problem [2, 6, 7, 19, 20].

Quantum mechanics is believed to be the most fundamental theory of physics. Every observation on objective world thus is regarded as a kind of measurement, and the measurement problem becomes a more general problem accordingly. The question of why we have never observed macroscopic system in superposition, and why macroscopic systems emerge always with some specific objective attributes, should be resolved as parts of the measurement problem [8, 19, 20].

There exist a lot of quests for solutions to the measurement problem. Some physicists attempt to resolve the measurement problem in the framework of orthodox quantum mechanics [3, 4, 12, 14, 19-21], and some others endeavor to re-interpret or extend the formulation of quantum mechanics in order to obtain some satisfying answers to the problem [2, 10, 22]. In this paper, we try to give a possibly satisfactory solution to the problem, which is totally analyzed under orthodox quantum mechanics.

2. Two noteworthy points in researches of the measurement problem

In quantum measurement, the measured physical system is a microscopic object (such as a photon or an electron), and the measuring device (such as a photoelectricmeter or a fluoroscope) is a macroscopic instrument, therefore, a quantum measuring process as a whole is a macroscopic action. Quantum mechanics comes from the description of microscopic phenomena, however, when we apply it to macroscopic process like measurement, we have to be extremely careful.
Let \( A \) be a measured physical system and \( M \) a measuring device; \( A \) and \( M \) constitute a closed system. If the system \( A \) is in a superposition state \( \sum_i c_i |a_i\rangle \) and the device \( M \) is in a state \( |m_r\rangle \), then, according to Schrödinger equation which governs the evolution of a closed system, the evolution of the compound system will be determined by a unitary linear operator \( U \). The operator \( U \) will extend the superposition of the measured system \( A \) to the superposition of total system \( A + M \), that is,

\[
\left( \sum_i c_i |a_i\rangle \right) |m_r\rangle \xrightarrow{U} \sum_i c_i |a_i\rangle |m_i\rangle.
\]

This process is the process 2 in the von Neumann’s interpretation to quantum measurement [1]. Since its introduction, it is used explicitly or implicitly in researches on the measurement problem. But this argument is not logical sound as it appears, and needs being analyzed carefully.

A measurement as a macroscopic action contains a lot of uncontrollable microscopic degrees of freedom. Every run of a measurement, these degrees of freedom are different in general. First, the same macroscopic state of a macroscopic device corresponds actually to its many different microscopic states. Every macroscopic state \( |m\rangle \) is in fact a collection of microscopic states \( \{|m_\alpha\rangle\} \) (here, \( \alpha \) denotes the microscopic degrees of freedom uncontrollable by experimenters; for simplicity, suppose that these degrees are discrete.). Second, the linear evolution ruled by Schrödinger equation also depends on the degrees of freedom uncontrollable by experimenters. Even for a closed system of which the energy remains constant, the particles made up the system and interactions between them are different in every run of the measurement from the microscopic view, and consequently its Hamiltonian operator are totally different too. Therefore, every run of a measurement, while same as a macroscopic process, but it experiences microscopically different linear evolution every time. When a measured system and a measuring device is in a state \( (\sum_i c_i |a_i\rangle)|m_{r\alpha}\rangle \), the measuring process causes a linear evolution of the compound system, denoted this evolution operator by \( U' \). According to the linearity of \( U' \),

\[
U' \left( \left( \sum_i c_i |a_i\rangle \right) |m_{r\alpha}\rangle \right) = \sum_i c_i U'(|a_i\rangle |m_{r\alpha}\rangle).
\]

However, the \( U' \) represents the evolution of the measuring process when the total system is in state \( (\sum_i c_i |a_i\rangle)|m_{r\alpha}\rangle \), from the microscopic view, it is
completely different with $U''$ which is another evolution operator which represents the measurement when the total system is in the state $|a_i⟩|m_{ra''}⟩$. Thus, the derivation from $U''(|a_i⟩|m_{ra''}⟩) = |a_i⟩|m_{iar''}⟩$ to $U′(|a_i⟩|m_{ra'}⟩) = |a_i⟩|m_{iar}⟩$, is unreasonable, and accordingly it cannot be deduced out that

$$U′\left(\sum_i c_i|a_i⟩\right)|m_{ra}⟩ = \sum_i c_i|a_i⟩|m_{iar}⟩.$$  

From the above argument, the idea that linear evolution of measurement would extend superposition state of microscopic system into superposition of macroscopic system is problematic.

Another point needed more attention refers to quantum state, especially entangled state. In quantum mechanics, any quantum state $|φ⟩$ of a quantum system, can be resolved as $|φ⟩ = \sum λ_i|b_i⟩$ for any basis $\{|b_i⟩\}$ of the Hilbert space of the system. It is just a mathematical expression and has no physical significance. The system now does not possess any property $|b_i⟩⟨b_i|$ at all (except in the situation with one $λ_i$ such that $|λ_i|^2 = 1$, other $λ_i = 0$), even though according to the measurement postulate, measuring the system in the basis $\{|b_i⟩\}$ would collapse the system to one state $|b_i⟩$ (i.e., it will have the property $|b_i⟩⟨b_i|$). In other words, a microscopic system in a definite state (pure state) has not any other property except the property corresponding the state, and it is measurement making it to possess some properties which it has not before measurement. For compound system, situation becomes more complicated. When a compound system is in a definite state (except product states), the state of any of its subsystems is totally indefinite (that is, it has not any property at all). As an example, consider two electrons of which their spins are in a Bell state

$$Φ = \frac{1}{\sqrt{2}}(|↑⟩⟨↑| + |↓⟩⟨↓|).$$

Here, two electron’s spins are uncertain, i.e., they have not any spin in any direction, and cannot be understood as being in a coexist state in which two electrons all are in the state $|↑⟩$ and all are in the $|↓⟩$. It is not only a consequence of the formulation of quantum mechanics, but a fact verified by many experiments. If two electrons have spins in all direction, then these spins can be regards as “hidden variables” which would make the state $Φ$ satisfying Bell inequality[23, 24], but all experiments carried out so far have confirmed that the state $Φ$ does not meet Bell inequality, but the prediction made by quantum
mechanics[25]. The fact that the states of subsystems in a definite entangled
state are all indefinite is overlooked intentionally or otherwise when discussing
the measurement problem. For example, as Brukner [26] discussed the Wigner’s
friend problem, he thought definitely that, when electrons, measuring devices
and observers is in the state
\[
\frac{1}{\sqrt{2}} (|z+\rangle_1 |z+\rangle_2 |z-\rangle_3 |\text{knows "up"}_4 + |z-\rangle_1 |z-\rangle_2 |z+\rangle_3 |\text{knows "down"}_4).
\]
the observers are in the definite state |\text{knows "up"}_4 or definite state
|\text{knows "down"}_4. Frauchiger et al [27] believed improperly that the
experiment in a supposition state is in a certain state (consequently, they have
definite ideas and others), and therefore, they obtained incorrectly a conclusion
that quantum mechanics cannot describe itself consistently.

The neglection about indefinite state appears also in the decoherence
solution to the measurement problem. In the solution, the measured microscopic
object, the measuring device and the environment are in a superposition state
\[
\sum_i c_i |a_i\rangle |m_i\rangle |e_i\rangle.
\]
According to quantum mechanics, their states are all indefinite,
but this is totally contradiction with our common experiences—the environment
presents itself definitely at all time. Moreover, in the decoherence scheme, the
measured object and the measuring device is in a mixed state
\[
\sum_i |c_i|^2 |a_i\rangle |m_i\rangle \langle a_i| \langle m_i|\text{ after tracing out states of the environment. The mixed
state, however, is from a superposition state in which the states of every system
is indefinite. Thus, the mixed state consists of indefinite states, not the expected
mixed state that would be composed of definite measurement results.}

While the measurement problem involves deeply a lot of aspects of
quantum mechanics, the uncomplete considerations mentioned above make it
more unclear and intricacy, and it is one of reasons why the problem has not
obtained a distinct and rational explanation.

3. How to understand quantum measurement

A solution to the measurement problem means that it gives answers to all
(or parts) of problems in Introduction, that is, removing the measurement
postulate; answering what is quantum measurement and giving the necessary
condition of measurement, if not sufficient and necessary conditions; explaining
why macroscopic objects have not been observed in superposition states and
what is the origin of the quantum randomness, if macroscopic physical processes are regarded as quantum processes.

The splendid success of quantum mechanics is not accident. The seemingly weird measurement postulate has been verified by innumerable applications, and thus there must be profound and unrefusable reasons in it. We believe that quantum mechanics contains all indispensable elements needed by solutions to the measurement problem, and what we need to do is digging into it and find answers out.

3.1 The completeness of quantum state—the equivalence of reduced state and mixed state

In history of quantum mechanics, the quantum description of microscopic objects had been strongly suspected by many physicists. Einstein had thought that quantum states are not complete and they do not include all physical elements in objects which they describe. He believed that some requisite “hidden variables” must be added to quantum states in order to describe microscopic objects completely [28]. By Bell’s researches and many experimental results, it has been confirmed that “hidden variable” theories are invalid, and quantum states are complete. Even though not all “hidden variable” theories of quantum mechanics are eliminated so far, it can be safely said that quantum mechanics is correct about the completeness of quantum state [25, 29].

For a subsystem of a compound system in a definite (pure state), quantum mechanics provides a specific description—reduced state which can be obtained by tracing out states of other subsystems in the compound system. In general, reduced state is regarded as a convenient tool for calculations of the expected value of observables on subsystems, but not the complete description of them [1, 24, 30].

The key point in our solution to the measurement problem is the insistence and the emphasis on the completeness of quantum state, especially on the completeness of reduced state, which we believe, contains all information of subsystems when they are considered as independent and separate objects in physical processes.

The idea that reduced state is complete seems thoughtless at first glance. An intuition is from an entangled state of spins of two electrons, \( \Phi = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \). In terms of quantum mechanics, the reduced states of two electrons are \( \rho_1 = \)
\( \rho_2 = \frac{1}{2}I \), and at this case, their spins are indefinite. But, if an electron (or an ensemble of same electrons) is generated in a state \( |\uparrow\rangle \) or \( |\downarrow\rangle \) each with 50% probability, then it is in a mixed state, the quantum description of which is also \( \rho = \frac{1}{2}I \). If quantum states describe completely microscopic objects, then, due to their same quantum states, electrons in two situations will be totally indistinguishable and would have exact same physical effects in any case consequently. In the latter situation, however, the electron’s spin is in a definite state, \( |\uparrow\rangle \) or \( |\downarrow\rangle \), and in the former situation, the electron’s spin is totally indeterminate, thus, it is taken for granted that they should have different physical effects. Therefore, even though electrons in two situations have same description in quantum mechanics, they are in general regarded to be different. It is one of reasons why reduced state is just considered as a calculational tool, but not the complete description of quantum objects.

We can conceive a communication protocol to show the above intuition is incorrect. Suppose that electrons in two cases mentioned above are different on physical effects, that is, there is at least a physical process \( P \) which can distinguish them and produce distinct result \( r_1 \) and \( r_2 \), respectively. (The process can be implemented for a single electron, or for an ensemble of electrons in a same state.) Suppose that Alice and Bob are far from each other, and they prepared a lot of pairs of electrons in the entangled state \( \Phi = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \) before they separated. Alice possesses the electron 1, and Bob possesses the electron 2. They divide pairs of electrons into \( n \) groups, each of which has same number of pairs. If Alice intends to send a binary bit 0 to Bob through a group of electron pairs, then she does nothing to electrons she possesses in each pair; if she intends to send a binary bit 1 to Bob, then she measures her electrons in the \{\( |\uparrow\rangle \), \( |\downarrow\rangle \}\} basis. Bob, in the other end, carries out physical process \( P \) on his electrons. For pairs of electrons on which Alice does nothing, Bob’s electrons are in the reduced state of \( \Phi \), and he gets the result \( r_1 \) after the process \( P \), and thus he gets the information 0; for pairs of electrons on which Alice measures, according to quantum mechanics electrons collapse into the definite state \( |\uparrow\rangle \) or \( |\downarrow\rangle \) with 1/2 chance respectively after the measurement, and consequently, the Bob’s corresponding electrons are in same state, i.e., these electrons are in a mixed state of \( |\uparrow\rangle \) and \( |\downarrow\rangle \), and thus belong to the latter case mentioned in the last paragraph. After the process \( P \), Bob records the result \( r_2 \) and gets the
information 1. Thus, this protocol would provide a communication on space-like spacetime distance which is directly in contradiction with special relativity. The correctness of quantum mechanics and special relativity means electrons in two cases are indistinguishable physically. This argument shows convincingly the equivalence of reduced state and mixed state.

Note that the communication protocol depends heavily on the measurement postulate. In the next section, based on the equivalence of reduced state and mixed state, we will give a possible explanation of quantum measurement, and take the measurement postulate as a consequence of the equivalence, and remove it from quantum mechanics. But replacing the measurement postulate with the equivalence in quantum mechanics is not our best choice. Even though the equivalence is better than the measurement postulate (at least, the equivalence does not refer to any unexplained concepts, such as measurement, measuring device, observer and so on), it is still wired compared with other postulates in quantum mechanics. Fortunately, we do not need to do so. What we do is to insist the completeness of quantum state, and that reduced state and mixed state as special kinds of quantum state, are complete too. Based on it, a reduced state and the same mixed state would have totally same physical effects and are indistinguishable thoroughly, that is, they are equivalent. We do not change anything in quantum mechanics and just emphasize the universal validity of the completeness of quantum state.

3.2 Measurement process

Based on the equivalence of reduced state and mixed state, we are able to give a possibly accurate explanation of quantum measurement.

We demonstrate measurement process with an example. Suppose that a 1/2-spin particle has the “up” spin in \( a \) direction (denoted the state as \( |a⟩ \)), and an experimenter intends to measure its spin in \( z \) axis direction (denoted the “up” and “down” spin in \( z \) axis as \( |↑⟩ \) and \( |↓⟩ \)). According to quantum mechanics, \( |a⟩ \) can be represented as \( |a⟩ = α|↑⟩ + β|↓⟩ \). In this state, however, the particle has not the spin in \( z \) axis (suppose that \( a \neq z \)). Physically, an indefinite (precisely, nonexistent) state (or the corresponding property) cannot generate physical effects, that is, it cannot play a role in physical processes and produce definite observable results. Therefore, the first step of quantum measurement has to contain a physical process through which the measured
state (or the property) will be marked by a state (or a property) of another particle. In order to make measurement to be possible and precise, the marking would be in one-one correspondence between the original state (or the property) and the marking state (or the property), and the marking state (or property) would be definite and existent. Quantum mechanics provides a very good marking method—entanglement. In our example, we can make the measured particle interacting with another particle, which generates the ready state $|a\rangle_r = \alpha|\uparrow\rangle|0\rangle + \beta|\downarrow\rangle|1\rangle$, in which $|0\rangle$ and $|1\rangle$ are two orthogonal states of the marking particle (for instance, vertical and horizontal polarization, if the marking particle is a photon) which mark $|\uparrow\rangle$ and $|\downarrow\rangle$ states of the measured particle, respectively. Two particles now constitute a compound system. According to quantum mechanics, the reduced state of the marking particle is $|\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$, and actually it is not in any definite state. Now, here is the crux. According to our argument, the reduces state is equivalent physically to a mixed state, in which the marking particle is in a definite state, $|0\rangle$ or $|1\rangle$ with probabilities $|\alpha|^2$ and $|\beta|^2$ respectively. A definite state (or a property) can participate in physical processes, and can produce physical effects. Thus, the next step of measurement is a macroscopic physical process which takes the particle in definite states (or a property) as input, and produces desired observable results. In our example, the second step is to design a macroscopic physical process in terms of some physical effects, in which, when the marking particle is in state $|0\rangle$, then the pointer of the measuring device would point to the scale 1; when the particle is in state $|1\rangle$, the pointer would point to the scale 2. Because the reduced state of the marking particle is $|\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$, after this macroscopic process, the pointer would point to the scale 1 with probability $|\alpha|^2$ and the scale 2 with probability $|\beta|^2$. When the marking particle is in the definite state $|0\rangle$ (or $|1\rangle$), the state of the measured particle must be in the corresponding state $|\uparrow\rangle$ (or $|\downarrow\rangle$), that is, if we measure its spin in $z$ axis once again, we will find the “up” spin (or “down” spin) absolutely. Since the state of two particles before the second macroscopic process is $|a\rangle_r = \alpha|\uparrow\rangle|0\rangle + \beta|\downarrow\rangle|1\rangle$, it would be in contradiction with the quantum description if two particles are not in matching states after the macroscopic process.

In the above example, in order to describe conveniently, we suppose the entanglement exists between two particles. In actually measurements, the
entanglement in most cases occurs between different properties of same particle. For instance, the above 1/2-spin particle in a state $|a⟩ = α|↑⟩ + β|↓⟩$ can be entangled with its locations as $|a⟩_r = α|↑⟩|x_1⟩ + β|↓⟩|x_2⟩$, in which $|x_1⟩$ and $|x_2⟩$ are its two locations in space. In quantum mechanics, the description of compound property of an object is like the description of compound system. The Hilbert space of a compound property is the product of the Hilbert space of its individual properties, provided these properties can be measured simultaneously, more formally, their corresponding Hermitian operators have to be commutable. When considering one property independently, we can obtain its description by tracing out the other properties. Taking as an example, $\text{Tr}_{\text{spin}}(|a⟩_r) = \text{Tr}_{\text{spin}}(α|↑⟩|x_1⟩ + β|↓⟩|x_2⟩) = |α|^2⟨x_1| + |β|^2⟨x_2|$. The argument about the equivalence of reduced state and mixed state can be modified a little to be applied on property, that is, we can design a similarly protocol to finish space-like communications, if reduced property and mixed property are not equivalent.

To sum up, quantum measurement process is a two-step physical process. The first step is a microscope process which entangles the state (or the property) of the measured system with a state (or a property) of a marking system, or with a property of itself. The second step is a macroscopic process which detects a specific state (or a property) of the marking system or itself. It is the reduced state (or the reduced property) that, which is equivalent to a mixed state (or a mixed property), guarantees accordingly that it can take part into physical processes which generate desired results.

3.3 Famous experiments in quantum mechanics

The above interpretation of quantum measurement is universally valid and can be used to explain some famous experiments in quantum mechanics.

In a Stern-Gerlach experiment, an electron (a silver atom in actual experiments) with the “up” spin in $y$ axis flies along $x$ axis into an inhomogeneous magnetic field in the $z$ axis. By this physical process, the “up” spin in $y$ axis which can be resolved into spins in $z$ axis as $|y_+⟩ = \frac{1}{\sqrt{2}}(|z_+⟩ + i|z_−⟩)$ causes the entanglement between its spins in $z$ axis and its paths through the magnetic field, resulting in the entangled state $|y_+⟩_r = \frac{1}{\sqrt{2}}(|z_+⟩|\text{upper}) + i|z_−⟩|\text{lower})$, where $|\text{upper})$ and $|\text{lower})$ represent two paths through which the electron passes. The detector screen is used to check
which path the electron passes through but not the electron’s spin, so what do works is the path information, that is, the reduced property $\text{Tr}_{\text{spin}} |y_+\rangle = \frac{1}{2} |\text{upper}\rangle \langle \text{upper}| + \frac{1}{2} |\text{lower}\rangle \langle \text{lower}|$. Thus, the detector screen finds that half electrons appear in the upper path and the other half in the lower path. In conclusion, when measured the $\textbf{z}$ axis spin of electrons with the “up” spin in $\textbf{y}$ axis, half electrons have “up” spin in $\textbf{z}$ axis, and half electrons have “down” spin in $\textbf{z}$ axis.

In a Mach-Zehnder interference experiment, after a photon goes through the first half-silvered mirror, it propagates along two paths, upper and lower, to the photon receivers. In orthodox quantum mechanics, the photon’s state is described by $|\text{photon}\rangle = \frac{1}{\sqrt{2}} (|\text{upper}\rangle + |\text{lower}\rangle)$, which is a superposition state and is used to explain the interference when the second silvered mirror is inserted. Here, the two photon receivers finish the detection of which path the photon pass through.

However, photon receivers only check the photon’s position where the photon arrives at, but not the paths through which it goes. It is the correspondence between the position and its path that points which path the photon passes. So, more completely, in the course of the photon’s propagating from the first half-silvered mirror to receivers (or the second half-silvered mirror), the photon is in a state $|\text{photon}\rangle = \frac{1}{\sqrt{2}} (|\text{upper}\rangle |x_1(t)\rangle + |\text{lower}\rangle |x_2(t)\rangle)$ at every moment $t$, in which $|x_1(t)\rangle$ and $|x_2(t)\rangle$ are positions of the photon on the corresponding path at the moment $t$. When there is no second half-silvered mirror, photon receivers detect the photon’s position, which can be obtained by tracing out the paths, i.e., $\frac{1}{2} |x_{\text{receiver}_1}\rangle \langle x_{\text{receiver}_1}| + \frac{1}{2} |x_{\text{receiver}_2}\rangle \langle x_{\text{receiver}_2}|$, where $x_{\text{receiver}_1}$ and $x_{\text{receiver}_2}$ are the positions of two photon receivers, respectively. So, the photon has half chance to be at $x_{\text{receiver}_1}$ and half chance at $x_{\text{receiver}_2}$, that is, the photon goes along two paths each with 50% probability.

On the other hand, if the second half-silvered mirror is inserted, the photon will arrives at the same position on the mirror, and its state now is $|\text{photon}\rangle = \frac{1}{\sqrt{2}} (|\text{upper}\rangle + |\text{lower}\rangle) |x_{\text{second-mirror}}\rangle$ in which the photon’s position state $|x_{\text{second-mirror}}\rangle$ is separated out and the photon is in a path superposition state. Now, the second mirror causes the interference.

The Born interpretation of wave function can be explained in similar way. Take the two-slit experiment as an example. When a photon (or an electron)
passes along \( y \) axis through two slits, it is in a superposition state \( |\text{photon}\rangle = \frac{1}{\sqrt{2}} (|\text{slit}_1\rangle + |\text{slit}_2\rangle) \), in which \( |\text{slit}_1\rangle \) and \( |\text{slit}_2\rangle \) represents the state in which the photon takes across the slit 1 or slit 2, respectively. The receiving screen is placed along \( x \) axis. For simplicity, suppose that location states \( \{ |x\rangle \} \) form a complete basis, i.e., \( \int |x\rangle\langle x| dx = 1 \). The photon’s state can be resolved along \( x \) axis as \( |\text{photon}\rangle = \int (w_1(x) + w_2(x)) |x\rangle dx \), in which \( w_1(x) = \frac{1}{\sqrt{2}} \langle \text{slit}_1 | x \rangle \) and \( w_2(x) = \frac{1}{\sqrt{2}} \langle \text{slit}_2 | x \rangle \) are wave functions corresponding to slit 1 and slit 2, respectively. While the photon moves to the receiving screen from slits, experiments can detect the location in \( x \) axis at each moment. Thus, the photon has path property, even though in a superposition of paths, the path is indefinite. Therefore, the total description of the photon in the course of moving from slits to the receiving screen should include the path information, that is, the photon’s state is \( |\text{photon}\rangle = \int (w_1(x) + w_2(x)) |\text{path}_x\rangle |x\rangle dx \), in which \( |\text{path}_x\rangle \) is the path through which the photon passes from slits to the \( x \) position on the screen. It is an entangled state. The screen only detects the photon’s location, but not paths, so tracing out path states, the state of the photon’s location is \( \int [w_1(x) + w_2(x)]^2 |x\rangle\langle x| dx \). So, the probability of receiving the photon at some \( x \) interval on the screen is \( [w_1(x) + w_2(x)]^2 dx \). It is the Born interpretation of wave function.

The explanation of measurement process in fact establishes a necessary condition for quantum measurement. According to the condition, some common devices in quantum experiments are not measuring devices, such as half-silvered mirror in Mach-Zehnder interferometer and magnet devices in Stern-Gerlach experiments, because they just finish the first step of quantum measurement, but lack the second step. It coincides with our practical experiences in quantum measurement.

4. Randomness, definite unitary evolution, collapse of wave function and superposition state of macroscopic object

Randomness is essential in quantum mechanics and is embodied in the measurement postulate, but there is no any explanation for the origin of randomness in quantum mechanics.

However, it is known from the exposition of quantum measurements in the last section that, even though quantum objects evolve according to definite Schrodinger equation and definite states cannot evolve into indefinite states, but
due to the equivalence of reduced state and mixed state, the statistical feature in mixed state leads to the occurrence of randomness in quantum mechanics when states (or corresponding properties) in mixed state take part into physical process. Because entangled states between subsystems generate truly mixed states (for a compound system in a product state, its subsystem is in a pure state, not a mixed state), it is entanglement which causes randomness, and it is the equivalence of reduced state and mixed state which brings randomness into reality.

If the measured system and the measuring devices are regarded as a close system, then quantum measurement process can be seen as a combination of two definite unitary evolutions. The first unitary evolution generates the entangled state of the measured system and the “marking” system or of properties of the measured system. The second unitary evolution is the macroscopic physical process which finishes the interaction between the “marking” system and the measuring device. The first process is controllable microscopically, but the second process is not. As we stressed on in section 2, the latter macroscopic unitary evolutions are different in every run of same macroscopic physical process. This kind of unitary evolutions cannot cause the superposition of macroscopic states. Moreover, even though measurement results are random, but the evolution of the total system is definite and unitary, and thus is not in contradiction with the definite dynamics of quantum mechanics.

If a measured system is in a state $\sum_i c_i |a_i\rangle$, according to our interpretation of quantum measurement, after the first step, it entangles with the “marking” system and its state is $\sum_i |c_i|^2 |a_i\rangle \langle a_i|$ if it is considered independently as a single system. After the second macroscopic process, the “marking” system is determined to be a state $|m_i\rangle$, then the measured system is in the corresponding state $|a_i\rangle$. As an independent single system, it’s state varies in a whole measurement as

$$\sum_i c_i |a_i\rangle \rightarrow \sum_i |c_i|^2 |a_i\rangle \langle a_i| \rightarrow |a_i\rangle.$$

This state’s change is called collapse of wave function in quantum mechanics. From our interpretation of quantum measurement, the collapse occurs in the second step of quantum measurement and it is only a change in observer’s knowledge, that is, if we adopt the ignorance interpretation of mixed
state, then the state’s change is caused by the change of observer’s knowledge about system’s state and does not happen physically, but epistemologically. This explain why the collapse never be observed in experiments. From physical view, the state of the measured system is always $\sum |c_i|^2 |a_i\rangle\langle a_i|$. Only for the observer who knows that the state of the “marking” system is $|m_i\rangle$, the state of the measured system is $|a_i\rangle$. Therefore, there is no collapse of wave function, but variation of observer’s knowledge.

No matter in the interpretation of quantum measurement in orthodox quantum mechanics or in our interpretation, macroscopic physical processes cannot generate superpositions of macroscopic states. In order to expanse a superposition of microscopic states to a superpositions of macroscopic states, a unitary evolution $U$ must satisfy $U(\sum_i c_i |a_i\rangle) |m_{r\alpha}\rangle = \sum_i c_i U(|a_i\rangle |m_{r\alpha}\rangle) = \sum_i c_i |a_i\rangle |m_{i\alpha}\rangle$. It must be the same evolution for all different states $|a_i\rangle$ of the measured system and all initial states $|m_{r\alpha}\rangle$ of the measuring device. Moreover, for the unitarity, it is required essentially to be defined on all states of the measured system and the measuring device. This requirement necessitates $U$ to control all degrees of freedom in the measuring device. Due to tremendous numbers of degrees of freedom in a macroscopic device, designing this kind of unitary operators, if not impossible, is extremely difficulty. The superposition state of macroscopic object exists only in theories or thought experiments so far.

5. Conclusions

We give a possibly promising solution to the quantum measurement problem. It is totally expounded in orthodox quantum mechanics, and does not need any extension or modification of the formulation of quantum mechanics, and also does not depend on any weird interpretation for quantum mechanics.

The most critical starting point in our solution is the emphasis and the insistence on the completeness of quantum state. Especially, the completeness of reduced state of subsystem, when a subsystem is treated as an independent object, leads directly to the equivalence of reduced state and mixed state. We provide a direct argument about the equivalence which is based on correctness of quantum mechanics and special relativity and we think the argument is persuasive.

Based on the equivalence of reduces state and mixed state, we exposit quantum measurement and take the measurement postulate as its corollary. We
describe measurement process as two-step physical process: one microscopically controllable process which produces entanglement, and one macroscopic process (uncontrollable microscopically) which detects given states (or properties) of some microscopic objects. It constitutes the necessary condition of quantum measurement, and thus give partly an answer to what is quantum measurement.

We also point out that quantum randomness comes from reduced state superficially and in fact from entanglement deeply. As a compound system is in a pure state, the reduced states of its subsystems are equivalent to some mixed states which are probabilistic. It is the probabilistic feature of mixed state which embodies quantum randomness. Moreover, the quantum state of a measured system is a reduced state after measurement, and due to the equivalence of reduced state and mixed stat, when an observer knows the measurement result, the measured system is regarded as in a definite state. Thus, the collapse of wave function is not an objective physical process, but a subjective change of observer’s knowledge.

Even though we take the completeness of quantum state as a fundamental principle and regard the equivalence of reduced state and mixed state as its consequence, the equivalence itself is worth researching seriously and deeply. Our argument just shows its inevitability, but does not answer why. In quantum mechanics, quantum state is the most fundamental concept and there are no any deeper concepts under it. Therefore, it seems impossible to explain the equivalence in quantum mechanics. This situation is similar to the equivalence of inertial reference frames and the equivalence of non-inertial reference frame and the gravity field, they as the basic principles in special relativity and general relativity, cannot be explained in the theories based on them. However, as Bell’s saying about quantum nonlocality, the scientific attitude is crying out for explanation. For the equivalence of reduced state and mixed state, we anticipate more researches on it and more understanding about it.

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