Dispersed Phase of Non-Isothermal Particles in Rotating Turbulent Flows

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We suggest certain effects, caused by interaction between rotation and gravity with turbulence structure, for the cooling/heating of dispersed phase of non-isothermal particles in rotating turbulent fluid flows. These effects are obtained through the derivation of kinetic or probability density function based macroscopic equations for the particles. In doing so, for one-way temperature coupling, we also show that homogeneous, isotropic non-isothermal fluid turbulence does not influence the mean temperature (though it influences mean velocity) of the dispersed phase of particles settling due to gravitational force in the isotropic turbulence.

I. INTRODUCTION

Different phenomena of non-isothermal two-phase flows existing in situations, such as, dispersion of aerosols, dust devils, dust storms and their effects on atmosphere, protoplanetary nebula and disks, birth and growth of cloud droplets, all involve rotation, gravitational field (e.g. see [1–5]). Researchers continue to face challenges in accurately explaining the observed and discovering the new phenomena [6–11] due to the involved turbulence closure problem with complexity further enhanced by the presence of dispersed phase of particles and droplets. In recent years, combination of kinetic approach [12] and theories for classical fluid turbulence closure problem [13] has evolved in a mathematical rigorous approach of probability density function (PDF) capable of capturing certain phenomena for the dispersed phase of particles/droplets in turbulent flows (e.g. see reference [11] and references cited therein). In our ongoing efforts toward unification of various aspects and explanation of the phenomena of two-phase turbulence [10, 11, 14], in this letter we apply PDF approach [15–
to important situation of two-phase rotating, non-isothermal, turbulent flows yielding new effects for collective behavior of particles’ temperature.

II. ANALYSIS

Consider particles dispersed in non-isothermal rotating flows moving due to forces of fluid drag, rotation (Coriolis and centrifugal) and gravity. Particles also exchange heat with the surrounding fluid and are considered here as point particles for analysis purpose. The trajectory and temperature of each particle is governed by the Lagrangian equations for its position $X$, velocity $V$ and temperature $T_p$, written in a coordinate system having constant angular velocity $\Omega$ as

$$\frac{dX_i}{dt} = V_i, \quad \frac{dV_i}{dt} = \beta_v(U_i - V_i) + 2\epsilon_{iab}V_a\Omega_b + f_i, \quad (1)$$

$$\frac{dT_p}{dt} = \beta_\theta(T - T_p) + Q(T_p). \quad (2)$$

Here, $\beta_v = 1/\tau_p$, $\tau_p$ is particle velocity time constant, $U_i$ and $T$ are carrier fluid velocity and temperature in the vicinity of the particle, $\epsilon_{iab}$ is the Levi-Civita’s alternating tensor, and $f_i = f^c_i + f^g_i$ account for components of centrifugal $f^c = -\Omega \times (\Omega \times r)$ and gravitational $f^g = f^g r$ accelerations with $f^g = -(GM/r^3)$ and position vector of particle $r$ and $r = |r|$. Also, $G$ is the gravitational constant, $M$ is the mass responsible for $f^g$, $\beta_\theta$ is inverse of the particle temperature time constant, $Q(T_p)$ is a function of $T_p$ accounting for particle temperature source and/or heating and cooling of particles due to radiation, e.g. see cases of optically thick and thin limits in situation protoplanetary disk [4]. The collective behavior of particles can be represented by the Eulerian instantaneous equations in physical space $x$ and time $t$ for particles number density $n(x, t)$, velocity $V_i(x, t)$ (obtained from Eq. (1)) and for temperature $\Theta$ which can be formed from Eq. (2) in an usual manner, written as

$$\frac{\partial \Theta}{\partial t} + V_j \frac{\partial \Theta}{\partial x_j} = \beta_\theta(T - \Theta) + Q(\Theta). \quad (3)$$

To extract statistical information for collective behavior of particles from the instantaneous equations, the density-weighted average (denoted by overbar) of the equations is preferable instead of non-weighted average due to the established reason [11]. The mean number density $N = \langle n \rangle$ then satisfies $\partial N/\partial t + \partial(NV_j)/\partial x_j = 0$, density-weighted averages $\bar{V}_i$ and
\( \Theta \) satisfy

\[
D_t \nabla_j + \frac{\partial v'_n v'_j}{\partial x_n} = \beta_v (\langle U_j \rangle - \nabla_j) + R_j + \beta_v u''_j, \tag{4}
\]

\[
D_t \Theta + \frac{\partial v'_n \Theta}{\partial x_n} = \beta_\theta (\langle T \rangle - \Theta) + \bar{Q} - \frac{v'_n \theta'}{\partial x_n} \partial \ln N + \beta_\theta \bar{\tau}, \tag{5}
\]

with \( R_j = 2 \epsilon_{jabc} \bar{v}_a \Omega_b + f_j - v'_n v'_j \partial \ln N / \partial x_n \) and operator \( D_t = \partial / \partial t + \bar{V}_i \partial / \partial x_i \). Also for any instantaneous variable \( A \), \( \bar{A} = \langle nA \rangle / N \) with ensemble average denoted by \( \langle \rangle \), \( a' \) and \( a'' \) are fluctuations in \( A = \bar{A} + a' = \langle A \rangle + a'' \) over \( \bar{A} \) and \( \langle A \rangle \), respectively. In Eqs. (4) and (5), \( u''_j \) and \( t'' \) represent respectively, fluctuations in fluid velocity and temperature, in the vicinity of particle, over the fluid mean velocity \( \langle U_i \rangle \) and temperature \( \langle T \rangle \). The appeared unknown terms \( v'_n v'_j, v'_n \theta' \) and, in particular, \( u''_j, t'' \) are difficult to model from the instantaneous equations but can be obtained with ease in PDF approach \[11, 14, 17\].

### III. NON-ISOTHERMAL, ISOTROPIC TURBULENCE CASE

Before obtaining expressions for the unknown terms, consider Eqs. (4) and (5) for an ideal situation homogeneous, isotropic, non-isothermal fluid turbulence with uniform \( \langle T \rangle \), \( \Omega_b = 0 \) and having dispersed particles settling under the gravity with \( Q = 0, f_i = -\delta_{i3} g \) where \( g \) is acceleration due to gravity. The equations simplify to \( \bar{V}_3 = -g / \beta_v + \bar{u}_3' \) and \( \Theta = \langle T \rangle + \bar{\tau} \) having additional drift velocity \( \bar{u}_3' \) and temperature \( \bar{\tau} \), for the particles, caused by the turbulence velocity and temperature structures. For the isotropic case, though \( \bar{u}_3' \) is nonzero \[22\], we show now that \( \bar{\tau} = 0 \) when the effects of particle temperature on the fluid temperature are neglected (i.e. one way coupling of temperature). For the one-way temperature coupling and diffusivity constant \( \alpha \), \( t'' \) is governed by \( \frac{\partial t''}{\partial t} + u''_j \frac{\partial t''}{\partial x_j} = \alpha \frac{\partial^2 t''}{\partial x_j \partial x_j} \), and which suggests that for each realization of \( u''_j \), at any time \( t \), \( t'' \) and \( -t'' \) are equally probable at any location of the dispersed particle. This suggests that the density-weighted average of \( t'' \) vanishes i.e. \( \bar{\tau} = 0 \). All these imply that particles’ mean temperature \( \Theta = \langle T \rangle \) is identical in both the cases of isotropic non-isothermal turbulent and stationary fluids, both having identical uniform mean fluid temperature \( \langle T \rangle \).
IV. PDF APPROACH

We use PDF approach to obtain expressions for unknown terms and, in particular, $\bar{u}_i''$ and $\bar{t}''$ in general situation of turbulent flows. The equation for ensemble average of phase space density $W(x, v, \theta, t)$ of the particles, representing PDF equation, is

$$\frac{\partial \langle W \rangle}{\partial t} + \frac{\partial}{\partial x_i} v_i \langle W \rangle + \frac{\partial}{\partial v_i} \beta_v \langle (U_i) - v_i \rangle \langle W \rangle + \frac{\partial}{\partial v_i} (2\epsilon_{iab} v_a \Omega_b + f_i) \langle W \rangle$$

$$+ \frac{\partial}{\partial \theta} [\beta_\theta (\langle T \rangle - \theta) + Q] \langle W \rangle = - \frac{\partial}{\partial v_i} [\beta_v \langle u_i'' W \rangle] - \frac{\partial}{\partial \theta} [\beta_\theta \langle t'' W \rangle],$$

which can be obtained from Eqs. (1) and (2) and Liouville’s theorem [14]. Here $x, v,$ and $\theta$ are phase space variables corresponding to $X, V,$ and $T_p,$ respectively. The expressions for unknown terms $\langle u''_i W \rangle$ and $\langle t'' W \rangle$ appearing in Eq. (6) can be obtained by employing Furutsu-Donsker-Novikov functional formula [18, 20, 23, 24], which are exact expressions when $u''_i$ and $t''$ along the particle path have Gaussian distribution. The expressions are

$$\beta_v \langle u_i'' W \rangle = - \left[ \frac{\partial}{\partial x_k} \lambda_{ki} + \frac{\partial}{\partial v_k} \mu_{ki} + \frac{\partial}{\partial \theta} \omega_i - \gamma_i \right] \langle W \rangle,$$

$$\beta_\theta \langle t'' W \rangle = - \left[ \frac{\partial}{\partial x_k} \Lambda_k + \frac{\partial}{\partial v_k} \Pi_k + \frac{\partial}{\partial \theta} \Omega - \Gamma \right] \langle W \rangle,$$

where various tensors are

$$\Lambda_k = \beta_v \beta_\theta \int_0^t dt_2 \langle t'' u_i''(t|t_2) \rangle G_{jk}(t_2|t),$$

$$\Pi_k = \beta_v \beta_\theta \int_0^t dt_2 \langle t'' u_i''(t|t_2) \rangle \frac{d}{dt} G_{jk}(t_2|t),$$

$$\Gamma = \beta_v \beta_\theta \int_0^t dt_2 \langle u_i''(t|t_2) \partial t'' / \partial x_k \rangle G_{jk}(t_2|t),$$

$$\Omega = \beta_v \beta_\theta \int_0^t dt_2 \langle t'' u_i''(t|t_2) \rangle G_{jk}(t_2|t) + \Omega_2,$$

with $\Omega_2 = \beta_\theta^2 \int_0^t dt_2 \langle t'' t''(t|t_2) \rangle G_{jk}(t_2|t)$ and expressions for remaining tensors can be obtained by writing $\beta_v u_i''$ instead of $\beta_\theta t''$ on the right-hand side (rhs) of Eqs. (9)-(12) and in $\Omega_2$ and changing $\Lambda_k \rightarrow \lambda_{ki}, \Pi_k \rightarrow \mu_{ki}, \Omega \rightarrow \omega_i$ and $\Gamma \rightarrow \gamma_i.$ In these expressions, shorthand notations $u_i''(t|t_2), t''(t|t_2)$ represent $u_i''(x, v, \theta, t|t_2), t''(x, v, \theta, t|t_2),$ the argument $(x, v, \theta, t|t_2)$ represents the value of $u_i''$ and $t''$ at time $t_2$ in the vicinity of particle that passes through $x$ at time $t$ with velocity $v$ and temperature $\theta.$ Also, $u_i''$ and $t''$ represent $u_i''(x, t)$ and $t''(x, t),$ respectively and now onwards too. The equations for $G_{jk}(t_2|t), G_j(t_2|t), G^\theta(t_2|t)$ $\forall t \geq t_2$ are

$$DG_{jk}(t_2|t) - \beta_v G_{ji} \frac{\partial \langle U_k \rangle}{\partial x_i} - 2\epsilon_{kab} \Omega_b \frac{dG_{ja}}{dt} - \frac{\partial f_k}{\partial x_i} G_{ji} = \delta_{jk} \delta(t - t_2).$$
\[
\frac{d}{dt}G_j(t_2|t) - \beta_0 G_{jk}\frac{\partial(T)}{\partial x_k} - \frac{\partial Q(\theta)}{\partial \theta}G_j + \beta_0 G_j = 0
\]  
\[
\frac{d}{dt}G^\theta(t_2|t) + \beta_0 G^\theta - \frac{\partial Q(\theta)}{\partial \theta}G^\theta = \delta(t - t_2)
\]

where \(D = d^2/dt^2 + \beta_v d/dt\). For later convenience, \(\Lambda_i\) and \(\Gamma\) are written in different forms as

\[
\frac{\Lambda_i}{\beta_\theta} = \int_0^t ds(t''\Delta v_i), \quad \frac{\Gamma}{\beta_\theta} = \int_0^t ds(\Delta v_i \partial t''/\partial x_i)
\]

where \(\Delta v_i = \int_0^s dt_2 \beta_v u''_k(t|t_2) \frac{dG_k(t_2|s)}{ds}\) represents change in particle velocity due to \(\beta_v u''_k(t|t_2)\), during time 0 to \(s\), along the trajectory that passes through \(x, v\) at \(t\).

The various tensors contain statistical properties related to \(u''_i\) and \(t''\) along the particle path and which can be obtained from the modelled equation for \(u''_i\) along the particle path [11], exact equation for \(t''\)

\[
\frac{dt''}{dt} = \alpha \nabla^2 t'' - u''_i \frac{\partial (T)}{\partial x_i} + \langle u''_i \partial t''/\partial x_i \rangle - (U_i - V_i) \frac{\partial t''}{\partial x_i}
\]

with model \(\alpha \nabla^2 t'' = -t''/\tilde{T}_\theta\) based on the simple IEM model [25] suggested for single-phase turbulence. It should be noted that integral time scale \(\tilde{T}_\theta\) is different than the time scale of IEM model. To obtain \(\langle t'' u''_j(t|t_2)\rangle\), multiply Eq. (17) by \(u''_j(t|t_2)\) and take ensemble average to yield

\[
d\langle t'' u''_j(t|t_2)\rangle/dt = -\langle t'' u''_j(t|t_2)\rangle/\tilde{T}_\theta + S_j\]

with \(S_j = -\langle u''_j(t|t_2)\rangle [u''_i \partial (T)/\partial x_i + (U_i - V_i) \partial t''/\partial x_i]\).

Further, the first order solution for \(\langle t'' u''_j(t|t_2)\rangle\) can be obtained by neglecting \(S_j\) and the result is an exponential form \(\langle t'' u''_j(t|t_2)\rangle = \langle t''(t|t_2) u''_j(t|t_2)\rangle e^{-(t - t_2)/\tilde{T}_\theta}\) which is considered here for further analysis.

The continuum equations for the dispersed phase can be obtained by taking various moments of Eq. (6) after substituting for \(\beta_v \langle u_i'W \rangle\) and \(\beta_\theta \langle t'W \rangle\) from Eqs. (7) and (8). The equations for \(\nabla_i = \frac{1}{N} \int v_i(W) d\mathbf{v} d\theta\) and \(\overline{\theta} = \frac{1}{N} \int \theta(W) d\mathbf{v} d\theta\) are identical to Eqs. (4) and (5), respectively, with

\[
\beta_v \overline{u''_j} = -\overline{X_{ij}} \frac{\partial N}{\partial x_i} - \frac{\partial}{\partial x_i} \overline{X_{ij}} + \overline{\nabla}_j,
\]

\[
\beta_\theta \overline{t''} = -\overline{X_k} \frac{\partial N}{\partial x_k} - \frac{\partial \overline{X_k}}{\partial x_k} + \overline{\Gamma}.
\]

Though the equations for higher order correlations \(v'_nv'_j, v'_n\overline{\theta}\) etc. can also be derived from the PDF equation, we focus our discussion on different phenomena contained in Eq. (19) for particles’ temperature.
V. NEW EFFECTS

The phenomena contained in various terms of Eq. (18) for additional drift of particles are described earlier [11]. Here we discuss phenomena related to temperature of collective particles as described by Eq. (19). For the discussion, we consider density weighted average of various tensors to be equal to their respective instantaneous values, e.g. \( \lambda_{ij} = \lambda_{ij} \). It should be noted that the form of Eqs. (5) and (19) remains unchanged irrespective of whether the rotation and gravitation exist or not. The rotation and gravitation effects are felt by these equations through \( G_{ij} \) and \( G_j \) appearing in various tensors. The first term on the rhs of Eq. (19) represents phenomenon of additional increase or decrease in mean temperature of particles caused by combination of statistical nature of the turbulence structure represented by \( \Lambda_k \) and gradient of particles mean number density in general situation of non-isothermal two-phase turbulent flows. The rhs of Eq. (19) vanishes when fluid variables along the particle path are correlated by delta function in time as suggested by expressions for \( \Lambda_k \) and \( \Gamma \) along with \( G_{jk}(t|t) = 0 \) [11]. When the correlation time is finite, the last two terms of Eq. (19) account for certain phenomena which we discuss now for cases of (1) slow rotation i.e. \( \Omega_b \) is small and (2) fast rotation without gravity i.e. \( f_{gi} = 0 \).

VI. SLOW ROTATION CASE

For slow rotation, we simplify the last two terms in Eq. (19) by expanding the solution of Eq. (13) as

\[
G_{jk}^1 - \beta_v G_{ij}^0 \frac{\partial \langle U_k \rangle}{\partial x_i} - 2 \varepsilon_{kab} \Omega_b \frac{dG_{ja}^0}{dt} - \frac{\partial f_k}{\partial x_i} G_{ji}^0 = 0,
\]

using exponential form for \( \langle t'' u_j''(t|t_2) \rangle \) and \( \langle t''(t|t_2) u_j''(t|t_2) \rangle \equiv \langle t'' u_j'' \rangle \). The simplification yields

\[
- \frac{\partial}{\partial x_i} \Lambda_i + \Gamma = -\beta_v \beta_0 \int_0^t dt_1 \langle t'' u_k''(t|t_1) \rangle \frac{\partial G_{ki}^1(t_1)}{\partial x_i} - \beta_v \beta_0 \int_0^t dt_1 G_{ki}^1(t_1|t) e^{-(t-t_1)/\tilde{T}_0} \langle t'' \partial u_k''/\partial x_i \rangle - \langle t'' \partial u_k''/\partial x_k \rangle \beta_0 A \]

where \( A = \{ \tilde{T}_0 [1 - e^{-t/\tilde{T}_0}] + \frac{\tilde{T}_0}{\tilde{T}_0 + 1} [e^{-t(\beta_v + 1)/\tilde{T}_0} - 1] \} \). Using the solution for \( G_{ki}^1 \) from Eq. (20), the second term on the rhs of Eq. (21) can be further simplified as

\[
- \left[ \beta_v^2 \beta_0 \frac{\partial \langle U_i \rangle}{\partial x_k} A'_1(t) - 2 \beta_v \beta_0 \varepsilon_{ikb} \Omega_b A'_2(t) - \beta_v \beta_0 \frac{\partial (f_i^g + f_j^g)}{\partial x_k} A'_1(t) \right],
\]
where \( A'_1 = \int_0^t dt_1 \left[ e^{-\left(\frac{t-t_1}{\tilde{T}_\theta}\right)} f_{t_1}^t dt_2 G^0(t_2|t)G^0(t_1|t_2) \right] \) and \( A'_2 = \int_0^t dt_1 \left[ e^{-\left(\frac{t-t_1}{\tilde{T}_\theta}\right)} f_{t_1}^t dt_2 G^0(t_2|t) \frac{d}{dt_2}G^0(t_1|t_2) \right] \). The first term in (22) represents contribution to heating/cooling of particles due to mean shear rate. The second term in (22) suggests that correlation between \( t'' \) and the component of fluid vorticity at particle location \( (\epsilon_{bik} \frac{\partial u_i}{\partial x_k}) \) in the direction of rotation \( \Omega_b \) produces additional heating/cooling for particle phase and whose origin is the Coriolis force. The last term in (22) represents heating/cooling of the dispersed phase due to the interaction of centrifugal and gravitational forces with turbulence fluctuations \( u''_i \) and \( t'' \). For further discussion, we consider a finite value for rotation only about \( x_3 \) axis i.e. \( \Omega_1 = \Omega_2 = 0 \) and \( \Omega_3 \neq 0 \). Then, the centrifugal part of \( f_k \) is \( f^c_k = x_k \Omega_2^2 - \delta_{k3}x_k \Omega_3^2 \) and its contribution to the last term in (22) becomes

\[
- \beta_v \Omega_3^2 (t'' \left\{ \partial u''_k/\partial x_k - \partial u''_3/\partial x_3 \right\}),
\]

which describes the phenomenon of heating/cooling of the dispersed phase in the presence of a centrifugal force. The effects of gravitational force contained in the last term in (22) can be simplified as

\[
- \beta_v \Omega_3^2 (t'' \left\{ \partial u''_k/\partial x_k - \partial u''_3/\partial x_3 \right\}) \quad \text{fast rotation without gravity} \]

VII. FAST ROTATION WITHOUT GRAVITY

For large value of \( t \), Eqs. (16) suggest that most contributions to \( \Lambda_i \) and \( \Gamma \) come from the correlations of \( t'' \) and \( \partial t''/\partial x_i \) with velocity of particles, between \( s(< t) \) and \( t \), present in the region near to \( x \) and \( t \) and passing through \( x \) at \( t \). Assuming exponential form for these correlations with integral time scale \( \tilde{T}_\theta \), the last two terms in Eq. (19) become approximately equal to \( -\beta_v \Omega_3^2 (t'' \partial u''_k/\partial x_k) f^g + (t'' \partial u''_3/\partial x_3) x_i \frac{\partial f^g}{\partial x_k} \)

where the last term arises due to variation of gravitational acceleration \( f^g \) in space.
VIII. EFFECT OF FLUID COMPRESSIBILITY

Fluid compressibility gives rise to interesting effects, of turbulent thermal diffusion and barodiffusion of particles, which are contained in Eq. (18) [10]. For particles dispersed in a compressible ideal gas, compressibility effects on the temperature of the particles can be obtained by using the following relation [14]

$$\langle t'' \partial u_k'' / \partial x_k \rangle \approx \langle t'' u_k'' \rangle [\partial \ln \langle T \rangle / \partial x_k - \partial \ln \langle P \rangle / \partial x_k].$$ (25)

Substituting it into the last term in Eq. (21) suggests turbulent thermal and pressure effects on the heating and cooling of the dispersed phase analogous to the two phenomena of turbulent thermal diffusion and barodiffusion [10]. Substituting Eq. (25) into Eqs. (23) and (24) suggests that the first term on the rhs of these equations further adds to these thermal and pressure effects in the presence of rotation and gravitation field.

IX. CONCLUDING REMARKS

It has been our goal to accurately describe and discover the phenomena related to important situation of dispersed phase of non-isothermal particles in rotating turbulent flows under the influence of gravitational field. The mean temperature $\overline{\Theta}$ of the particle phase is found to be affected by the term $\overline{t''}$ in Eq. (5) containing new effects caused by mutual interactions of turbulent structure of fluid temperature and velocity with the rotation and gravitation field. These effects and their accurate description have been surfaced through the application of mathematical rigorous PDF approach having established capability in capturing certain phenomena (see [11] and references cited therein).

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