Modelling magnetic oscillators

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Abstract. The effect of magnetic fields complicates the analysis of nonradial pulsations of stars. The angular dependence of the eigenfunction of a pulsation mode cannot be represented by a single spherical harmonic, and another type of oscillations enters into the equations. We discuss different methods employed to analyse nonradial pulsations in the presence of a magnetic field and compare the results from the different methods. Then, we discuss some important properties caused by the presence of a strong magnetic field, and discuss some asteroseismological works.

1. Introduction
Stellar oscillations (or pulsations) under the influence of a strong magnetic field occur in relatively cooler magnetic Ap stars which have global magnetic fields in the range of kG to a few 10 kG. The pulsation frequencies of these stars are much higher than the fundamental pulsation frequency so that they are called rapidly oscillating Ap (roAp) stars. The first roAp star, Przybylski’s star (HD 101065) was discovered by Don Kurtz in 1978 (Kurtz 1978). Since then more than 30 roAp stars were discovered [see Kurtz and Martinez (2000) for a review of observational, mainly photometric, results; the list of roAp stars is updated in Kurtz et al. (2006)]. On the HR diagram, roAp stars lie in the δ Scuti instability region. The pulsation periods range from 6 min to 21 min, being much shorter than those of δ Scuti variables, the pulsations correspond to low-degree high radial order p-modes. The pulsations seem to be axisymmetric with respect to the magnetic axis which is inclined to the rotation axis (oblique pulsator; Kurtz 1982). Recently, accurate frequencies immune to the 1-day alias became available from coordinated multi-site observations (e.g., Kurtz et al. 2005; Handler et al. 2006) and from space photometry (e.g., Gruberbauer et al. 2007; Huber et al. 2007). In addition, recent time-resolved spectroscopic studies have revealed many intriguing phenomena in the atmospheres of roAp stars (see Kochukhov 2007 for a recent review).

To model the pulsations of roAp stars, we have to include the coupling between pulsation and magnetic fields, which causes various special properties in eigenfunctions and frequency spectra of pulsations (see Cunha 2007 for a recent review on theories). In this paper, we first discuss the methods to calculate eigenfrequencies for pulsations of magnetic stars and compare results obtained by different methods.
2. Pulsation equations with a magnetic field

A linearized momentum equation for nonradial pulsations under the influence of a magnetic field may be written as
\[
\frac{dv}{dt} = \frac{\rho'}{\rho} \frac{dp}{dr} e_r - \frac{1}{\rho} \nabla p' + \frac{1}{4\pi \rho} (\nabla \times B') \times \mathbf{B}_0,
\]  

(1)

where \( v \) is pulsation velocity, \( \rho \) matter density, \( p \) pressure, \( \mathbf{B} \) magnetic field, and \( e_r \) is the unit vector in the radial direction. The prime (') indicates the Eulerian perturbation of the quantity, and the subscript 0 to \( \mathbf{B} \) means its equilibrium value. The equilibrium magnetic field is assumed to be force free (\( \nabla \times \mathbf{B}_0 = 0 \); usually a dipole field is assumed). In the above equation and in the following discussion, we neglect the effect of rotation (cf. Bigot and Dziembowski 2002). A linearized form of the continuity equation may be written as
\[
\frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho v) = 0.
\]

(2)

If magnetic fields were absent, equations (1) and (2), with the adiabatic relation between \( p' \) and \( \rho' \), would close the set of equations for adiabatic nonradial pulsations, in which the angular dependence is separated using the spherical harmonic \( Y_{m}^{\ell} (\theta, \phi) \), and the eigenvalues (the square of the frequencies) are all real [see e.g. Unno et al. (1989) for basic properties of nonradial pulsations]. In the presence of a magnetic field, the Lorentz force appears in the momentum equation (the last term in eq. 1) and hence we need to include equations describing the magnetic field perturbations. Assuming the ideal MHD condition, a linearized induction equation may be written as
\[
\frac{\partial \mathbf{B}'}{\partial t} = \nabla \times (v \times \mathbf{B}_0).
\]

(3)

Because of the Lorentz force, the angular dependence of nonradial pulsations cannot be expressed by a single spherical harmonic, and nonradial pulsations are coupled with magnetic oscillations. Therefore, obtaining eigenfrequencies and eigenfunctions for nonradial pulsations of a magnetized star are considerably complicated.

3. Methods to obtain eigenfrequencies and eigenfunctions

In most part of a magnetic star, the Alfvén velocity is much slower than the sound velocity so that the magnetic effect on acoustic waves is small. In an outer boundary layer, however, the Alfvén speed is comparable to or faster than sound speed, so that the full equations including Lorentz force must be solved. Roberts and Soward (1983) investigated the magnetic field effect on the pulsation by getting analytical solutions in the boundary layer to fit interior solutions. They have found that magnetic slow waves, generated in outer layers, propagate inward but are dissipated before they reach the stellar center because the wavelength of these waves decreases rapidly with decreasing ratio of Alfvén to sound speed. Roberts and Sowards argued that slow waves provide an important source of damping for the pulsations. Their finding forms a foundation in later works on the pulsations of magnetized stars.

Campbell and Papaloizou (1986) obtained numerically eigenfunctions in the outer boundary layer for which a plane parallel approximation was employed with a local Cartesian coordinate. They have found that for high-order p-modes horizontal motion can reach about 10% of the radial motion in agreement with the result of Roberts and Soward (1983), and that the loss of pulsation energy via slow waves is significant.

Magnetic effects on pulsation frequencies for axisymmetric (\( m = 0 \)) modes were first computed by Dziembowski and Goode (1996) taking into account the boundary layer. They divided the stellar interior into two parts; the outer thin boundary layer at \( r > r_{\text{fit}} \) and the remaining inner part, where \( r_{\text{fit}} \) took a value between 0.98 and 0.99. In the outer boundary layer,
using the plane-parallel approximation, as Campbell and Papa loizou (1986), they integrated the pulsation equations including the Lorentz force, from the surface to the bottom of the layer

\[ r = r_{\text{fit}} \]

with a trial eigenvalue. They obtained, at various co-latitudes \( \theta \), a solution in which magnetic slow waves propagated inward at \( r = r_{\text{fit}} \), in accordance with the suggestion of Robert and Soward (1983). Then, the boundary layer solution, at \( r = r_{\text{fit}} \), say \( y'(r_{\text{fit}}, \theta) \) for a trial eigenvalue was used as the boundary condition for the interior calculation \( (r \leq r_{\text{fit}}) \), where the Lorentz force is neglected; i.e., the usual nonradial pulsation equations without magnetic fields were solved. Since the angular dependence of \( y'(r_{\text{fit}}, \theta) \) cannot be described by a single Legendre polynomial, it was expanded as

\[ y'(r_{\text{fit}}, \theta) = \sum_{\ell} y_{\ell}^{i}(r_{\text{fit}}) P_{\ell}(\cos \theta), \tag{4} \]

where \( y_{\ell}^{i} \) are solutions for degree \( \ell \) of the equations of nonradial pulsations satisfying the inner boundary condition at \( r = 0 \). In other words, for \( r \leq r_{\text{fit}} \) pulsation equations were solved without magnetic fields using \( y_{\ell}^{i}(r_{\text{fit}}) \) as the outer boundary condition. Bigot et al. (2000; BPPDG hereafter) extended Dziembowski and Goode’s method for non-axisymmetric \( (m \neq 0) \) pulsations.

Cunha and Gough (2000; CG00 hereafter) computed solutions for the outer boundary layer similarly as in the above mentioned works; i.e., they integrated the full equations including magnetic field perturbations from the surface to the base of the boundary layer using the plane-parallel approximation. They took, however, a different approach in connecting the boundary-layer solutions to the interior solutions. Below the boundary layer, they represented eigenfunctions in an asymptotic form as

\[ \delta p(r, \theta, \phi) \propto \sin \left( \int_{r_{\text{fit}}}^{R^*} \kappa dr + \delta(R^*, \theta) \right) Y_{l}^{m}(\theta, \phi), \tag{5} \]

where \( \delta p \) is the Lagrangian perturbation of pressure, \( R^* \) is the distance from the center at the base of the boundary layer \( (= r_{\text{fit}} \) in eq. 4), and \( \kappa \) is the vertical wavenumber of the pulsation. The phase \( \delta(R^*, \theta) \) at the base of the boundary layer is obtained from integration through the boundary layer. CG00 constructed variational principle to obtain the frequency shift \( \Delta \nu = \nu(B_p) - \nu_0 \) from the phase shift \( \Delta \delta = \delta(R^*, \theta) - \delta_0 \) as

\[ \Delta \nu \propto \int_{\theta_0}^{\pi} \Delta \delta(R^*, \theta) Y_{l,m}^{m} Y_{l,m}^{m} \sin \theta d\theta. \tag{6} \]

One of the advantages of this method is the fact that the magnetic configuration can be changed easily. Cunha (2006) obtained frequency shifts caused by quadrupolar magnetic fields.

Saio and Gauthschy (2004; SG04 hereafter) analyzed axisymmetric \( (m = 0) \) adiabatic pulsations using a method somewhat different from those discussed above. They did not compute the boundary layer separately but included the whole star to solve for complex eigenfrequencies and eigenfunctions. They expressed the angular dependence of a variable as a sum of terms proportional to the spherical harmonics;

\[ p' = \sum_{\ell} p_{\ell}^{j} Y_{l}^{0}, \quad \xi = \sum_{\ell} \left( \xi_{r} Y_{l}^{0} e_r + \xi_{\theta} \frac{dY_{l}^{0}}{d\theta} e_{\theta} \right), \quad B' = \sum_{\ell} \left( b_{r} Y_{l}^{0} e_r + b_{\theta} \frac{dY_{l}^{0}}{d\theta} e_{\theta} \right). \tag{7} \]

where \( \ell = 2j - 1 \) for odd modes and \( \ell = 2(j - 1) \) for even modes with \( j = 1, 2, \ldots, j_{\text{max}} \). \( Y_{l}^{0} \) means a spherical harmonic \( Y_{l}^{m}(\theta, \phi) \) for \( m = 0 \). The expansion was truncated usually at \( j_{\text{max}} = 10 \sim 12 \); they solved for all variables simultaneously but set running-wave conditions for
Figure 1. Schematic description of the computational method adopted by Saio and Gautschy (2004). For the perturbations of the magnetic field, the vacuum condition $\nabla \times \mathbf{B}' = 0$ was imposed at the outer boundary, and a running wave condition was adopted at $r = 0.95R_\ast$, where the wavelength of magnetic slow waves are very short because the Alfvén speed is much higher than the sound speed. For mechanical variables (pressure perturbations and radial displacement), usual outer and central boundary conditions were imposed. The magnetic and mechanical variables were solved for simultaneously to obtain eigenfrequencies.

the magnetic variables at $r = 0.95R_\ast$, below this layer the effect of magnetic field was neglected (see Fig. 1). Saio (2005; S05 hereafter) extended this method to nonadiabatic pulsations.

In the presence of a magnetic field, the eigenfunction of a pulsation mode is expressed by a superposition of components associated with various $\ell$ values. To identify the latitudinal dependence of a mode we sometimes use $l_m$ to indicate the $\ell$ value of that component with the largest kinetic energy.

We note that in all the methods discussed above, the coupling between nonradial pulsation and magnetic field is taken into account by solving both pulsational and magnetic variables in the outer boundary layer where the Alfvén speed is comparable to or exceeds the sound speed. At some early stage of research on the pulsations of magnetic stars, however, approaches were pursued that treated the effect of magnetic field as a perturbation to nonradial pulsations without taking into account the outer boundary layer (e.g., Shibahashi and Takata 1993). We do not discuss those approaches in this paper.

4. Comparison of the results from different methods
In this section, we compare frequency shifts obtained by different methods as discussed in the previous section. The frequency shift $\Delta \nu (\equiv \nu(B_p) - \nu_0)$ varies as a function of $\nu_0$ and $B_p$; for a polytrope with index $\alpha$, $\Delta \nu$ is a function of $\nu_0 B_p^{1/(1+\alpha)}$ (Roberts and Soward 1983; Campbell and Papaloizou 1986), where $B_p$ is the polar strength of a dipole magnetic field.

Figure 2 compares frequency shifts obtained for a polytrope of index 3 at $B_p = 1$ kG by CG00
Figure 2. Frequency shifts of high-order $p$-modes for a polytropic model (index = 3) with $(M, R) = (2M_\odot, 2R_\odot)$ (the asymptotic large separation of frequencies is $62\mu\text{Hz}$) obtained by CG00 (blue solid lines for $l = 1$ and red dashed lines for $l = 3$) are compared with the results of SG04 (blue and red symbols). The top and bottom panels contain the real part and the imaginary part of frequency, respectively.

$\nu_0(m\text{Hz})[B_p(k\text{G})]^{0.25}$

In the analysis of SG04, the truncated expansion [eq.(7)] fails to converge around a jump in the real part of $\Delta \nu$ because a considerable fraction of kinetic energy is distributed into high $\ell$ components. Therefore, sequences of triangles and pentagons in Fig. 2 have gaps around those jumps. Despite the large difference in the method of calculation, SG04 results agree qualitatively with those of CG00. In particular, the real part of the shift for $l_m = 3$ agrees well. Before the first jump, $\Delta \nu$ of $l_m = 1$ agrees very well, but the amplitudes of the jumps are considerably larger in the results of CG00 than of SG04. The difference might come from the significant
Figure 3. Frequency shifts of high-order $p$-modes at $B_p = 1$ kG for a zero-age main-sequence model of $1.8M_\odot$ obtained by BPBDG (filled symbols) are compared with the results obtained by the method of SG04 (open symbols). The top and bottom panels contain the real part and the imaginary part of frequency, respectively. Cunha (2006) showed comparisons between the results from the methods of CG00 and SG04 for a $1.8M_\odot$ main-sequence model. Dziembowski and Goode (1996) and BPBDG did not obtain jumps in $\Delta \nu$. This can be understood as that their analyses did not extend to high enough $\nu_0$ or $B_p$. Figure 3 compares results of BPBDG (filled symbols) for axisymmetric modes of $l_m = 0$ and $l_m = 1$ with the results obtained by the method of SG04 (open symbols) for a $1.8M_\odot$ ZAMS model similar to the model used by BPBDG. Dipole modes (triangles) agree well for both real and imaginary parts. (The agreement is much better than the comparison given in Fig. 27 of SG04. The reason for the difference is that the $T - \tau$ relation for optically thin layers used in SG04 was different from that used in BPBDG.) The $l = 0$ modes (real parts in particular) of BPBDG are closer to $l_m = 2$ modes of SG04 (blue squares) rather than $l_m = 1$ for $\nu_0 > 1.5$ mHz. This is probably due to the ambiguity in mode identification. The kinetic energy associated with the $l = 0$ component...
of a $l_m = 2$ mode with $\nu_0 > 1.6$ mHz is larger than 60% of that of $l = 0$ component so that these modes have properties intermediate between $l = 0$ and $l = 2$ modes. Also, as seen in Fig. 3, frequencies of $l_m = 0$ modes are very close to those of $l_m = 2$ when the magnetic effect is small (see also Fig. 8 in SG04). Therefore, the $l = 0$ modes of BPBDG with $\nu_0 > 1.5$ mHz are probably the same modes as $l_m = 2$ of SG04 in Fig. 3. From this figure, we can conclude that the method of BPBDG yields results that are very similar to those from the method of SG04.

5. Magnetic effects on nonradial p-mode pulsations

In addition to the pulsation frequency shifts by the effect of a magnetic field, the pulsation tends to be damped by the leakage of pulsation energy due to slow waves. S05 claimed that the slow-wave damping is enough to stabilize low-order ($\delta$ Scuti type) pulsations which are not observed in roAp stars (e.g., Kurtz 2000). We should note, however, that helium depletion by diffusion might play a more important role in the stability of low-order modes of roAp stars (e.g., Gautschy et al. 1998; Théado and Cunha 2006).

Latitudinal distribution of pulsation amplitude is considerably modified in the presence of a strong magnetic field as discussed in SG04 and S05. Generally, the amplitude is suppressed considerably around the magnetic equator and tends to be strongly confined to the polar regions. This effect was confirmed observationally by Kochukhov (2004) who obtained velocity amplitude distribution of the roAp star HR 3831 and found that the amplitude distribution can be described by a superposition of $\ell = 1$ and $\ell = 3$ Legendre functions. We also note that Handler et al. (2006) found a notable $\ell = 3$ contribution in the photometric amplitude modulation due to stellar rotation of the roAp star HD 99563. The theoretical prediction of the amplitude confinement might be also supported by the observational fact that pulsational radial velocity variations are largest for spectral lines of rare earth elements which tend to be concentrated toward the magnetic poles (Kochukhov et al. 2002).

The modification of amplitude distribution also affects line-profile variations caused by pulsations. Including the magnetic effects yields narrower features in line-profile variations (Saio and Gautschy 2004b). Therefore, it seems to be necessary to include the magnetic effects on the amplitude distribution to analyze line profile variations observed in many roAp stars (Kochukhov et al. 2007).

6. Some asteroseismological results

Figure 4 shows frequencies of a few high-order p-modes with $0 \leq l_m \leq 3$ for a $2M_\odot$ main-sequence star model as a function of the polar strength, $B_p$, of a dipole magnetic field. The large separation, the frequency difference between two adjacent radial order for a given $l_m$ $[\nu(l_m, n) - \nu(l_m, n-1); n = \text{radial order}]$, is not affected very much by the magnetic field strength, because frequency shift is not very sensitive to radial order. However, small separations such as $\nu(l_m, n) - \nu(l_m + 2, n - 1)$ are affected very much by the presence of magnetic fields as is apparent in Fig. 4 even showing mode crossings. This opens possibilities of an asteroseismological determination of the strength of magnetic fields of roAp stars, or of testing the validity of the theory by comparisons with observed frequencies.

Recently, the Canadian photometric satellite MOST obtained accurate pulsation frequencies for the two roAp stars $\gamma$ Equ (Gruberbauer et al. 2007) and 10 Aql (Huber et al. 2007). Comparing the observed frequencies with theoretical ones, Gruberbauer et al. obtained $B_p \approx 8kG$ for $\gamma$ Equ, while Huber et al. concluded that the number of observed frequencies of 10 Aql is too small to make an asteroseismic estimate of the magnetic field strength. On the other hand, with ground-based high-precision time-resolved spectroscopic observations, Mkrtichian et al. (2007) found 15 frequencies in Przybylski’s star and obtained a good fit with observed frequencies at $B_p \approx 9kG$. 

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Figure 4. An example of high-order ($19 \leq n \leq 22$ for $l_m = 1$) p-mode frequencies versus $B_p$ (polar strength of dipole magnetic fields) for a $2M_\odot$ main-sequence model. Filled and open symbols are excited and damped modes, respectively. Sometimes, $l_m$ shifts between 0 and 2, and 1 and 3, because the distribution of kinetic energy among expanded components changes with $B_p$. Each sequence is terminated when $l_m$ shifts to values larger than 3 or when the expansion of eigenfunctions (Eq. 7) stops converging (the expansion truncated at $j_{\text{max}} = 10$ for these calculations). Frequencies of $l_m = 0$ modes (and $l_m = 2$ modes with a large contribution from $\ell = 0$ component) increase most rapidly as $B_p$ increases.

7. Concluding remarks
We have discussed different methods to model stellar pulsations under the influence of a strong magnetic field. It is found that frequency shifts obtained by different methods at various pulsation frequencies and $B_p$ agree at least qualitatively with each other. Also, observed frequencies of some roAp stars seem to fit reasonably well with theoretical frequencies calculated nonadiabatically by the method of S05. These are encouraging results for asteroseismology of roAp stars which has just started recently. Further observational results in the future should provide further opportunity to test models and to obtain information on the outer layers of these stars.

Despite the early success of reproducing frequencies of some observed roAp stars, there are many things to be improved in our modelling. For the three stars for which detailed comparisons
of frequencies were made, theoretical predictions of the pulsational instability did not work; i.e., all the modes fitted to the observed ones were damped modes (Gruberbauer et al. 2007; Huber et al. 2007; Mkrtichian et al. 2007). The excitation of high-order \( p \) modes is due to the kappa-mechanism in the hydrogen ionization zone (Dziembowski and Goode 1996; Gautschy et al. 1998; Balmforth et al. 2001; Cunha 2002). Since the excitation occurs far out in the envelope compared with the case of low-order pulsations, the stability seems to be sensitive to the position of the outer boundary. If the outer boundary is shifted from \( \tau = 10^{-3} \) to \( 10^{-4} \), for example, the excited frequency range shifts considerably (S05, Elkin et al. 2005). This sensitivity might partially come from the diffusion approximation used even above the photosphere to calculate radiation flux variation. More sophisticated treatment for radiation in the optically thin layer must be used. Such improvement might be important in comparing theoretical results with the intriguing phenomena recently discovered by accurate spectroscopies (e.g., Elkin, Kurtz, and Mathys 2005; Kurtz, Elkin, and Mathys 2006; Ryabchikova et al. 2007ab).

Observationally, no roAp star seems to significantly deviate from the oblique pulsator model invented by Kurtz (1982); that is, all roAp pulsation modes seem mainly to be axisymmetric \( (m = 0) \) with respect to the magnetic axis (Kochukhov 2004; Kochukhov et al 2007). Theoretically, however, the results by BPBDG and CG00 do not indicate special properties for non-axisymmetric \( (m \neq 0) \) modes. (The non-axisymmetric extension of the method of SG04 has not been successful yet.) Bigot and Dziembowski (2002) suggested that a coupling with rotation forces the pulsation axis to deviate from the magnetic axis giving rise to considerable non-axisymmetric components. [Their model for HR 3831, however, seems to contradict the result of Kochukhov (2004)]. Non-axisymmetric pulsations in the presence of a magnetic field couple with Alfven waves which generate toroidal components in pulsations. Probably, we have to better understand these properties before we understand the non-axisymmetric pulsations of magnetic stars and their coupling with rotation.

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References
Balmforth N J, Cunha M S, Dolez N, Gough D O and Vauclair S 2001 MNRAS 323 362
Bigot L and Dziembowski W A 2002 A\&A 391 235
Bigot L, Provost J, Berthomieu G, Dziembowski W A and Goode P R 2000 A\&A 356 218 (BPBDG)
Campbell C G and Papaloizou J C B 1986 MNRAS 220 577
Cunha M S 2002 MNRAS 333 47
Cunha M S 2006 MNRAS 365 153
Cunha M S 2007 Comm. Asteroseismology 150, 48
Cunha M S and Gough D 2000 MNRAS 319 1020 (CG00)
Dziembowski W A and Goode P R 1996 ApJ 458 338
Elkin V G, Kurtz D W and Mathys G 2005 MNRAS 364 864
Elkin V G, Riley J D, Cunha M S, Kurtz D W and Mathys G 2005 MNRAS 358 665
Gautschy A, Saio H and Harzenmoser H 1998 MNRAS 301 31
Gruberbauer M, Saio H, Huber D, Kallinger T, Weiss W W, Guenther D B, Kuschnig R, Matthews J M, Moffat A F J, Rucinski S, Sasselov D and Walker G A H 2007 submitted to A\&A
Handler G, Weiss W W, Shobbrook R R, Paunzen E, Hempel A, Anguina S K, Kalebwe P C, Kilkenny D, Martinez P, Moalusi M B, Garrido R and Medupe R 2006 MNRAS 366 257
Huber D, Saio H, Gruberbauer M, Weiss W W, Rowe J, Hareter M, Kallinger T, Reegen P, Guenther D B, Kuschnig R, Matthews J M, Moffat A F J, Rucinski S, Sasselov D and Walker G A H 2007 submitted to A\&A
Kochukhov O 2004 ApJ 615 L149
Kochukhov O 2007 Comm. Asteroseismology 150, 39
Kochukhov O, Piskunov N, Ilyin I., Ilyina S and Tuominen I. 2002 A\&A 389 420
Kochukhov O, Ryabchikova T, Weiss W W, Landstreet J D and Lyashko D 2007 MNRAS 376, 651
Kurtz D W 1978 Inf. Bull. Var. Stars No.1436
Kurtz D W 1982 MNRAS 200 807
Kurtz D W 2000 *ASP Conf. Ser.* 210 287
Kurtz D W and Martinez P 2000 *Baltic Astr.* 9 253
Kurtz D W, Cameron C, Cunha M S, Dolez N, Vauclair G, Pallier E, Ulla A, Kepler S O, da Costa A, Kanaan A et al. 2005 *MNRAS* 358 651
Kurtz D W, Elkin V G, Cunha M S, Mathys G, Hubrig S, Wolff B, and Savanov I 2006 *MNRAS* 372 286
Kurtz D W, Elkin V G and Mathys G 2006 *MNRAS* 370 1274
Mkrtichian D E, Hatzes A P, Saio H and Shobbrook R 2007 submitted to *A&A*
Roberts P H and Soward A M 1983 *MNRAS* 205 1171
Ryabchikova T, Sachkov M, Kochukhov O and Lyashko D 2007b *A&A* 473 907
Ryabchikova T, Sachkov M, Weiss W W, Kallinger T, Kochukhov O, Bagnulo S, Ilyin I, Landstreet J D, Leone F, Lo Curto G, Lüftinger T, Lyashko D and Magazzù 2007a *A&A* 462 1103
Saio H 2005 *MNRAS* 360 1022 (S05)
Saio H and Gautschy A 2004a *MNRAS* 350 485 (SG04)
Saio H and Gautschy A 2004b *ASP Conf. Ser.* 310 478
Shibahashi H and Takata M 1993 *PASJ* 45 617
Théado S and Cunha M S 2006 Comm. Asteroseismology 147 101
Unno W, Osaki Y, Ando H, Saio H and Shibahashi H 1989 *Nonradial Oscillations of Stars*, (Tokyo: University of Tokyo Press)