WEAKLY SUPERVISED NEURO-SYMBOLIC MODULE NETWORKS FOR NUMERICAL REASONING

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ABSTRACT

Neural Module Networks (NMNs) have been quite successful in incorporating explicit reasoning as learnable modules in various question answering tasks, including the most generic form of numerical reasoning over text in Machine Reading Comprehension (MRC). However, to achieve this, contemporary NMNs need strong supervision in executing the query as a specialized program over reasoning modules and fail to generalize to more open-ended settings without such supervision. Hence, we propose Weakly-Supervised Neuro-Symbolic Module Network (WNSMN) trained with answers as the sole supervision for numerical reasoning based MRC. It learns to execute a noisy heuristic program obtained from the dependency parsing of the query, as discrete actions over both neural and symbolic reasoning modules and trains it end-to-end in a reinforcement learning framework with discrete reward from answer matching. On the numerical-answer subset of DROP, WNSMN outperforms NMN by 32% and the reasoning-free language model GenBERT by 8% in exact match accuracy when trained under comparable weak supervised settings. This showcases the effectiveness and generalizability of modular networks that can handle explicit discrete reasoning over noisy programs in an end-to-end manner.

1 INTRODUCTION

End-to-end neural models have proven to be powerful tools for an expansive set of language and vision problems by effectively emulating the input-output behavior. However, many real problems like Question Answering (QA) or Dialog need more interpretable models that can incorporate explicit reasoning in the inference. In this work, we focus on the most generic form of numerical reasoning over text, encompassed by the reasoning-based MRC framework. A particularly challenging setting for this task is where the answers are numerical in nature as in the popular MRC dataset, DROP (Dua et al., 2019). Figure 1 shows the intricacies involved in the task, (i) passage and query language understanding, (ii) contextual understanding of the passage date and numbers, and (iii) application of quantitative reasoning (e.g., max, not) over dates and numbers to reach the final numerical answer.

Three broad genres of models have proven successful on the DROP numerical reasoning task. First, large-scale pretrained language models like GenBERT (Geva et al., 2020) uses a monolithic Transformer architecture and decodes numerical answers digit-by-digit. Though they deliver mediocre performance when trained only on the target data, their competency is derived from pretraining on massive synthetic data augmented with explicit supervision of the gold numerical reasoning. Second kind of models are the reasoning-free hybrid models like NumNet (Ran et al., 2019), NAQANet (Dua et al., 2019), NABERT+ (Kinley & Lin, 2019) and MTMSN (Hu et al., 2019), NeRd (Chen et al., 2020). They explicitly incorporate numerical computations in the standard extractive QA pipeline by learning a multi-type answer predictor over different reasoning types (e.g., max/min, diff/sum, count, negate) and directly predicting the corresponding numerical expression, instead of learning to reason. This is facilitated by exhaustively precomputing all possible outcomes of discrete operations and augmenting the training data with the reasoning-type supervision and numerical expressions that lead to the correct answer. Lastly, the most relevant class of models to consider for this work are the modular networks for...
reasoning. Neural Module Networks (NMN) (Gupta et al., 2020) is the first explicit reasoning based QA model which parses the query into a specialized program and executes it step-wise over learnable reasoning modules. However, to do so, apart from the exhaustive precomputation of all discrete operations, it also needs more fine-grained supervision of the gold program and the gold program execution, obtained heuristically, by leveraging the abundance of templatized queries in DROP.

While being more pragmatic and richer at interpretability, both modular and hybrid networks are also tightly coupled with the additional supervision. For instance, the hybrid models cannot learn without it, and while NMN is the first to enable learning from QA pair alone, it still needs more finer-grained supervision for at least a part of the training data. With this, it manages to supercede the SoTA models NABERT and MTMSN on a carefully chosen subset of DROP using the supervision. However, NMN generalizes poorly to more open-ended settings where such supervision is not easy to handcraft.

### Need for symbolic reasoning.

One striking characteristic of the modular methods is to avoid discrete reasoning by employing only learnable modules with an exhaustively precomputed space of outputs. While they perform well on DROP, their modeling complexity grows arbitrarily with more complex non-linear numerical operations (e.g., exp, log, cos). Contrarily, symbolic modular networks that execute the discrete operations are possibly more robust or pragmatic in this respect by remaining unaffected by the operation complexity. Such discrete reasoning has indeed been incorporated for simpler, well-structured tasks like math word problems (Koncel-Kedziorski et al., 2016) or KB/Table-QA (Zhong et al., 2017; Liang et al., 2018; Saha et al., 2019), with Deep Reinforcement Learning (RL) for end-to-end training. MRC however needs a more generalized framework of modular neural networks involving more fuzzy reasoning over noisy entities extracted from open-ended passages.

In view of this, we propose a Weakly-Supervised Neuro-Symbolic Module Network (WNSMN)

- A first attempt at numerical reasoning based MRC, trained with answers as the sole supervision;
- Based on a generalized framework of dependency parsing of queries into noisy heuristic programs;
- End-to-end training of neuro-symbolic reasoning modules in a RL framework with discrete rewards;

To concretely compare WNSMN with contemporary NMN, consider the example in Figure 1. In comparison to our generalized query-parsing, NMN parses the query into a program form (\(\text{MAX}(\text{FILTER}(\text{FIND('Carpenter')}, 'goal'))\)), which is step-wise executed by different learnable modules with exhaustively precomputed output set. To train the network, it employs various forms of strong supervision such as gold program operations and gold query-span attention at each step of the program and gold execution i.e., supervision of the passage numbers \((23, 26, 42)\) to execute \(\text{MAX}\) operation on.

While NMN can only handle the 6 reasoning categories that the supervision was tailored to, WNSMN focuses on the full DROP with numerical answers (called DROP-num) that involves more diverse reasoning on more open-ended questions. We empirically compare WNSMN on DROP-num with the SoTA NMN and GenBERT that allow learning with partial or no strong supervision. Our results showcase that the proposed WNSMN achieves 32% better accuracy than NMN in absence of at least one or more types of supervision and performs 8% better than GenBERT when the latter is fine-tuned only on DROP in a comparable setup, without additional synthetic data having explicit supervision.

## 2 Model: Weakly Supervised Neuro-Symbolic Module Network

We now describe our proposed WNSMN that learns to infer the answer based on weak supervision of the QA pair by generating the program form of the query and executing it through explicit reasoning.
**Parsing Query into Programs** To keep the framework generic, we use a simplified representation of the Stanford dependency parse tree (Chen & Manning, 2014) of the query to get a generalized program (Appendix A.5). First, a node is constructed for the subtree rooted at each child of the root by merging its descendants in the original word order. Next an edge is added from the left-most node (which we call the root clause) to every other node. Then by traversing left to right, each node is organized into a step of a program having a linear flow. For example, the program obtained in Figure 1 is \( X1 = \text{('which is the longest')}, X2 = \text{('goal by Carpenter', X1)} \). Answer = Discrete-Reasoning('which is the longest', X2). Each program step consists of two types of arguments (i) Query Span Argument obtained from the corresponding node, indicates the query segment referred to, in that program step e.g., ‘goal by Carpenter’ in Step 2 (ii) Reference Argument(s) obtained from the incoming edges to that node, refers to the previous steps of the program that the current one depends on e.g., X1 in Step 2. Next, a final step of the program is added, which has the reference argument as the leaf node(s) obtained in the above manner and the query span argument as the root-clause. This step is specifically responsible for handling the discrete operation, enabled by the root-clause which is often indicative of the kind of discrete reasoning involved (e.g., max). However this being a noisy heuristic, the QA model needs to be robust to such noise and additionally rely on the full query representation in order to predict the discrete operation. For simplicity we limit the number of reference arguments to 2.

## 2.1 Program Execution

Our proposed WNSMN learns to execute the program over the passage in three steps. In the preprocessing step, it identifies numbers and dates from the passage, and maintains them as separate canonicalized entity-lists along with their mention locations. Next, it learns an entity-specific cross-attention model to rank the entities w.r.t. their query-relevance (§2.1.1), and then samples relevant entities as discrete arguments (§2.1.2) and executes appropriate discrete operations on them to reach the answer. An RL framework (§2.1.3) trains it end-to-end with the answer as the sole supervision.

### 2.1.1 Entity-Specific Cross Attention for Information Extraction

To rank the query-relevant passage entities, we model the passage, program and entities jointly.

**Modeling interaction between program and passage** This module (Figure 2, left) learns to associate query span arguments of the program with the passage. For this, similar to NMIN, we use a BERT-base pretrained encoder (Devlin et al., 2018) to get contextualized token embeddings of the passage and query span argument of each program step, respectively denoted by \( P_k \) and \( Q_k \) for the \( k \)-th program step. Based on it, we learn a similarity matrix \( S \in \mathbb{R}^{l \times n \times m} \) between the program and passage, where \( l, n, \) and \( m \) respectively are the program length and query span argument and passage length (in tokens). Each \( S_k \in \mathbb{R}^{n \times m} \) represents the affinity over the passage tokens for the \( k \)-th program argument and is defined as \( S_k(i, j) = w^T \cdot [Q_k; P_k; P_k \cdot P_k] \), where \( w \) is a learnable parameter and \( \cdot \) is element-wise multiplication. From this, an attention map \( A_k \) is computed over the passage tokens for the \( k \)-th program argument as \( A_k(i, j) = \text{softmax}_j(S_k(i, j)) = \frac{\exp(S_k(i, j))}{\sum_j \exp(S_k(i, j))} \).

Similarly, for the \( i \)-th token of the \( k \)-th program argument the cumulative attention \( a_{ki} \) is defined as \( a_{ki} = \text{softmax}_i(\sum_j S_k(i, j)) \). A linear combination of the attention map \( A_k(i, \cdot) \) weighted by \( a_{ki} \) gives the expected passage attention for the \( k \)-th step, \( \alpha_k = \sum_i a_{ki} A_k(i, \cdot) \in \mathbb{R}^m \).

**Span-level attention.** To facilitate information spotting and extraction over contiguous spans of text, we regularize the passage attention so that the attention on a passage token is high if the attention over its neighbors is so. We achieve this by adopting a heuristic smoothing technique (Huang et al., 2020), taking a sliding window of different lengths \( \omega = \{1, 2, \ldots, 10\} \) over the passage, and replacing the token-level attention with the attention averaged over the window. This results in 10 different attention maps over the passage for the \( k \)-th step of the program: \( \{\alpha_k^\omega | \omega \in \{1, 2, \ldots, 10\}\} \).

**Soft span prediction.** This network takes a multi-scaled (Gupta et al., 2020) version of \( \alpha_k^\omega \), by multiplying the attention map with \( |s| \) different scaling factors \( \omega = \{1, 2, 3, 10\} \), yielding a \( |s| \)-dimensional representation for each passage token, i.e., \( \bar{\alpha}_k^{\omega} \in \mathbb{R}^{m \times |s|} \). This is then passed through a \( L \)-layered stacked self-attention transformer block (Vaswani et al., 2017), which encodes it to \( m \times d \) dimension, followed by a linear layer of dimension \( d \times 1 \), to obtain the span prediction logits: \( \alpha_k^\omega = \text{Linear}(\text{Transformer}(\text{MultiScaling}(\bar{\alpha}_k^{\omega}))) \in \mathbb{R}^m \). Further the span prediction logits at
A critical requirement for a meaningful attention over entities is to incorporate information extrac-
tion windows is where the elements of \( \alpha_k \) are computed as 
\[
\alpha_k^{\text{num}}(i,j) = \sum_{\omega \in \Omega} \omega \cdot P_k(i,j) 
\]
where the elements of \( S^\text{num}_k \) are computed as 
\[
S^\text{num}_k(i,j) = P_k W_n P_{kn,j} 
\]
with \( W_n \in \mathbb{R}^{d \times d} \) being a learnable projection matrix and \( n_j \) being the passage location of the \( j \)-th number token. These similarity scores are additively aggregated over all mentions of the same number entity in the passage.

The relation between program and entities is then modeled as \( \tau_k^{\text{num}} = \text{softmax}(\sum_{\omega} \alpha_k^{\text{num}}(i,j)) \) in \( R^N \), which gives the expected distribution over the \( N \) number tokens for the \( k \)-th program step and using \( \omega \) as the smoothing window size. The final stacked attention map obtained for the different windows is \( \tau_k^{\text{num}} = \{ \tau_k^{\text{num}}(\omega) : \omega \in \{1, 2, \ldots, 10\} \} \). Similarly, for each program step \( k \), we also compute a separate stacked attention map \( \tau_k^{\text{date}} \) over the unique date tokens, parameterized by a different \( W_d \).

A critical requirement for a meaningful attention over entities is to incorporate information extraction capability in the number and date attention maps \( A_k^{\text{num}} \) and \( A_k^{\text{date}} \), by enabling the model to attend over the neighborhood of the relevant entity mentions. This is achieved by minimizing the unsupervised auxiliary losses \( L_{\text{aux}}^{\text{num}} \) and \( L_{\text{aux}}^{\text{date}} \) in the training objective, which impose an inductive bias over the number and date entities, similar to Gupta et al. (2020). Its purpose is to ensure that the passage attention is densely distributed inside the neighborhood of \( \pm \Omega \) (a hyperparameter, e.g., \( 10 \)) of the passage location of the entity mention, without imposing any bias on the attention distribution outside the neighborhood. Consequently, it maximises the log-form of cumulative likelihood of the attention distribution inside the window and the entropy of the attention distribution outside of it.

\[
L_{\text{aux}}^{\text{num}} = -\frac{1}{l} \sum_{k=1}^l \sum_{i=1}^m \left[ \sum_{j=1}^N \log(N) \sum_{j=1}^N \mathbb{1}_{n_j \in [i \pm \Omega]} a_{kij}^{\text{num}} \right] - \sum_{j=1}^N \sum_{n_j \notin [i \pm \Omega]} a_{kij}^{\text{num}} \log(a_{kij}^{\text{num}}) \]  
(1)

where \( \mathbb{1} \) is indicator function and \( a_{kij}^{\text{num}} = A_k^{\text{num}}(i,j) \). \( L_{\text{aux}}^{\text{date}} \) for date entities is similarly defined.
Apart from the

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(with output dimension $d$)

The model next learns to execute a single step\footnote{This is a reasonable assumption for DROP with a recall of 90\% on the training set. However, it does not limit the generalizability of WNSMN, as with standard beam search it is possible to scale to an $l$-step MDP.} of discrete reasoning (Figure 3) based on the final program step. The final step contains (i) root-clause of the query which often indicates the type of discrete operation (e.g., ‘what is the longest’ indicates max, ‘how many goals’ indicates count), and (ii) reference argument indicating the previous program steps the final step depends on. Each previous step (say $k$) is represented as stacked attention maps $T_k^{num}$ and $T_k^{date}$, obtained from §2.1.1

**Operator Sampling Network** Owing to the noisy nature of the program, the operator network takes as input: (i) BERT’s [CLS] representation for the passage-query pair and LSTM (Hochreiter & Schmidhuber 1997) encoding (randomly initialized) of the BERT contextual representation of (ii) the root-clause from the final program step and (iii) full query (w.r.t. passage), to make two predictions:

- **Entity-Type Predictor Network**, an Exponential Linear Unit (Elu) activated fully-connected layer followed by a $\text{softmax}$ that outputs the probabilities of sampling either date or number types.
- **Operator Predictor Network**, a similar Elu-activated fully connected layer followed by a $\text{softmax}$ which learns a probability distribution over a fixed catalog of 6 numerical and logical operations (count, max, min, sum, diff, negate), each represented with learnable embeddings.

Apart from the $\text{diff}$ operator which acts only on two arguments, all other operations can take any arbitrary number of arguments. Also some of these operations can be applied only on numbers (e.g., sum, negate) while others can be applied on both numbers or date (e.g., max, count).

**Argument Sampling Network** This network learns to sample date/number entities as arguments for the sampled discrete operation, given the entity-specific stacked attentions ($T_k^{num}$ and $T_k^{date}$) for each previous step (say, $k$), that appears in the reference argument of the final program step. In order to allow sampling of fixed or arbitrary number of arguments, the argument sampler learns four types of networks, each modeled with a $L$-layered stacked self attention based $\text{Transformer}$ block (with output dimension $d$) followed by different non-linear layers embodying their functionality and a $\text{softmax}$ normalization to get the corresponding probability of the argument sampling (Figure 3).

- **Sample Arbitrary Argument**: $\text{Multinomial}$ (Entity-Ranked Distribution, Counter Prediction).

Depending on the number of arguments needed by the discrete operation and the number of reference arguments in the final program step, the model invokes one of **Sample 1, 2, Arbitrary** Argument. For instance, if the sampled operator is $\text{diff}$ which needs 2 arguments, and the final step has 1 or 2 reference arguments, then the model respectively invokes either **Sample 2 argument** or **Sample 1 argument** on the stacked attention $T$ corresponding to each reference argument. And, for operations needing arbitrary number of arguments, the model invokes the **Sampling Arbitrary Argument**. For the

**Figure 3: Operator & Argument Sampling Network and RL framework over sampled discrete actions**
Arbitrary Argument case, the model first predicts the number of entities $c \in \{1, \ldots, 10\}$ to sample using the Counter Network, and then samples from the multinomial distribution based on the joint of $c$-combinations of entities constructed from the output distribution of the Entity Ranker module.

2.1.3 Training with Weak Supervision in the Deep RL Framework

We use an RL framework to train the model with only discrete binary feedback from the exact match of the gold and predicted numerical answer. In particular, we use the REINFORCE [Williams, 1992] policy gradient method where a stochastic policy comprising a sequence of actions is learned with the goal of maximizing the expected reward. In our case, the discrete operation along with argument sampling constitute the action. However, because of our assumption that a single step of discrete reasoning suffices for most questions in DROP, we further simplify the RL framework to a contextual multi-arm bandit (MAB) problem with a 1-step MDP, i.e., the agent performs only one step action.

Despite the simplifying assumption of 1-step MDP, the following characteristics of the problem render it highly challenging: (i) the action space $A$ is exponential in the order of number of operations and argument entities in the passage (averaging to 12K actions for DROP-num); (ii) the extreme reward sparsity owing to the binary feedback is further exacerbated by the presence of spurious rewards, as the same answer can be generated by multiple diverse actions. Note that previous approaches like NMN can avoid such spurious supervision because they heuristically obtain additional annotation of the question category, the gold program or gold program execution at least for some training instances.

In our contextual MAB framework, for an input $x = (\text{passage}(p), \text{query}(q))$, the context or environment state $s_\phi(x)$ is modeled by the entity specific cross attention (§2.1.1) parameterized by $\phi$ between the (i) passage (ii) program-form of the query and (iii) extracted passage date/number entities. Given the state $s_\phi(x)$, the layout policy (§2.1.2) parameterized by $\theta$ then learns the query-specific inference layout, i.e., the discrete action sampling policy $P_\theta(a|s_\phi(x))$ for action $a \in A$. The action sampling probability is a product of the probability of sampling entities from the appropriate entity type ($P_\theta^{\text{type}}$), probability of sampling the operator ($P_\theta^{\text{op}}$), and probability of sampling the entity argument(s) ($P_\theta^{\text{arg}}$), normalized by number of arguments to sample. Therefore, with the learnable context representation $s_\phi(x)$ of input $x$, the end-to-end objective is to jointly learn $\{\theta, \phi\}$ that maximises the expected reward $R(x, a) \in \{−1, +1\}$ over the sampled actions ($a$), based on exact match with the gold answer.

To mitigate the learning instability in such sparse confounding reward settings, we initialize with a simpler iterative hard-Expectation Maximization (EM) learning objective, called Iterative Maximal Likelihood (IML) [Liang et al., 2017]. With the assumption that the sampled actions are extensive enough to contain the gold answer, IML greedily searches for the good actions by fixing the policy parameters, and then maximises the likelihood of the best action that led to the highest reward. We define good actions ($A_x^{\text{good}}$) as those that result in the gold answer itself and take a conservative approach of defining best among them as simply the most likely one according to the current policy.

$$J^{\text{IML}}(\theta, \phi) = \max_x \sum_{a \in A_x^{\text{good}}} \log P_{\theta, \phi}(a|x)$$  \hspace{1cm} (2)

After the IML initialization, we switch to REINFORCE as the learning objective after a few epochs, where the goal is to maximise the expected reward ($J^{\text{RL}}(\theta, \phi) = \sum_x \mathbb{E}_{\theta, \phi}(a|x)R(x, a)$) as

$$\nabla_{(\theta, \phi)}J^{\text{RL}} = \sum_x \sum_{a \in A} P_{\theta, \phi}(a|x)(R(x, a) - B(x))\nabla_{\theta, \phi}(\log P_{\theta, \phi}(a|x))$$  \hspace{1cm} (3)

where $B(x)$ is simply the average (baseline) reward obtained by the policy for that instance $x$. Further, in order to mitigate overfitting, in addition to $L_2$-regularization and dropout, we also add entropy based regularization over the argument sampling distribution, in each of the sampling networks.

3 Experiments

We now empirically compare the exact-match performance of WNSMN with SoTA baselines on versions of DROP dataset and also examine how it fares in comparison to strong supervised skylines. The Primary Baselines for WNSMN are the explicit reasoning based NMN (Gupta et al., 2020)
which uses additional strong supervision and the BERT based language model GenBERT (Geva et al., 2020) that does not embody any reasoning and autoregressively generates numeric answer tokens. As the Primary Dataset we use DROP-num, the subset of DROP with numerical answers. This subset contains 45K and 5.8K instances respectively from the standard DROP train and development sets. Originally, NMN was showcased on a very specific subset of DROP, restricted to the 6 reasoning-types it could handle, out of which three (count, date-difference, extract-number) have numeric answers. This subset comprises 20K training and 1.8K development instances, out of which only 10K and 800 instances respectively have numerical answers. We further evaluate on this numerical subset, referred to as DROP-Pruned-num. In both the cases, the training data was randomly split into 70%:30% for train and internal validation and the standard DROP development set was treated as the Test set.

Figure 4 shows the t-SNE plot of pretrained Sentence-BERT (Reimers & Gurevych, 2019) encoding of all questions in DROP-num-Test and also the DROP-Pruned-num-Test subset with different colors (red, green, yellow) representing different types. Not only are the DROP-num questions more diverse than the carefully chosen DROP-Pruned-num subset, the latter also forms well-separated clusters corresponding to the three reasoning types. Additionally, the average perplexity (using nltk) of the DROP-Pruned-num and DROP-num questions was found to be 3.9 and 10.65 respectively, further indicating the comparatively open-ended nature of the former.

For the primary baselines NMN and GenBERT, we report the performance on in-house trained models on the respective datasets, using the code open-sourced by the authors. The remaining results, taken from Geva et al. (2020), Kinley & Lin (2019), and Ran et al. (2019), refer to models trained on the full DROP dataset. All models use the same pretrained BERT-base. Also note that a primary requirement of all models other than GenBERT and WNSMN i.e., for NMN, MTMSN, NABERT, NAQANET, NumNet, is the exhaustive enumeration of the output space of all possible discrete operations. This simplifies the QA task to a classification setting, thus alleviating the need for discrete reasoning in the inference process.

Table 1 presents our primary results on DROP-num, comparing the performance of WNSMN (accuracy of the top-1 sampled action by the RL agent) with various ablations of NMN (provided in the authors’ implementation) by removing atleast one of Program, Execution, and Query Attention supervision (Appendix A.4.1) and GenBERT models with pretrained BERT that are finetuned on DROP or DROP-num (denoted as GenBERT and GenBERT-num). For a fair comparison with our weakly supervised model, we do not treat NMN with all forms of supervision or GenBERT model pretrained with additional synthetic numerical and textual data as comparable baselines. Note that these GenBERT variants indeed enjoy strong reasoning supervision in terms of gold arithmetic expressions provided in these auxiliary datasets.

NMN’s performance is abysmally poor, indeed a drastic degradation in comparison to its performance on the pruned DROP subset reported by Gupta et al. (2020) and in our subsequent experiments in Table 2. This can be attributed to their limitation in handling more diverse classes of reasoning and open-ended queries in DROP-num, further exacerbated by the lack of one or more types of strong supervision. Our earlier analysis on the complexity of the questions in the subset and full DROP-num further quantify the relative difficulty level of the latter. On the other hand, GenBERT delivers a mediocre performance, while GenBERT-num degrades additionally by 4%, as learning from numerical answers alone further curbs the language modeling ability. Our model performs significantly better than both these baselines, surpassing GenBERT by 8% and the NMN baseline by around 32%. This showcases the significance of incorporating explicit reasoning in neural models in comparison to the vanilla large scale LMs like GenBERT. It also establishes the generalizability of such reasoning.

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*Both the results and limitations of NMN in Table 4 and Table 2 were confirmed by the authors of NMN as well.*
based models to more open-ended forms of QA, in comparison to contemporary modular networks like NMN, owing to its ability to handle both learnable and discrete modules in an end-to-end manner.

Next, in Table 2 we compare the performance of the proposed WNSMN with the same NMN variants (as in Table 1) on DROP-Pruned-num. Some of the salient observations are: (i) WNSMN in fact reaches a performance quite close to the strongly supervised NMN variant (first row), and is able to attain at least an improvement margin of 4% over all other variants obtained by removing one or more types of supervision. This is despite all variants of NMN additionally enjoying the exhaustive precomputation of the output space of possible numerical answers; (ii) WNSMN suffers only in the case of extract-number type operations (e.g., max, min) that involve a more complex process of sampling arbitrary number of arguments (iii) Performance drop of NMN is not very large when all or none of the strong supervision is present, possibly because of the limited diversity over reasoning types and query language; and (iv) Query-Attention supervision in fact adversely affects NMN’s performance, in absence of the program and execution supervision or both, possibly owing to an undesirable biasing effect. However when both supervisions are available, query-attention is able to improve the model performance by 5%. Further, we believe the test set of 800 instances is too small to get an unbiased reflection of the model’s performances.

In Table 3, we additionally inspect recall over the top-k actions sampled by WNSMN to estimate how it fares in comparison to the strongly supervised skylines: (i) NMN with all forms of strong supervision; (ii) GenBERT variants +ND, +TD and +ND+TD further pretrained on synthetic Numerical and Textual Data and both; (iii) reasoning-free hybrid models like MTMSN (Hu et al., 2019) and NumNet (Ran et al., 2019), NAQANet (Dua et al., 2019) and NABERT, NABERT+ (Kinley & Lin, 2019). Note that both NumNet and NAQANet do not use pretrained BERT. MTMSN achieves SoTA performance through a supervised framework of training specialized predictors for each reasoning type to predict the numerical expression directly instead of learning to reason. While top-1 performance of WNSMN (in Table 1) is 4% worse than NABERT, Recall@top-2 is equivalent to the strongly supervised NMN, top-5 and top-10 is comparable to NABERT+, NumNet and GenBERT models +ND, +TD and top-20 nearly achieves SoTA. Such promising recall over the top-k actions suggests that more sophisticated RL algorithms with better exploration strategies can possibly bridge this performance gap.

4 ANALYSIS & FUTURE WORK

Performance Analysis Despite the notorious instabilities of RL due to high variance, the training trend, as shown in Figure 5(a) is not afflicted by catastrophic forgetting. The sudden performance jump between epochs 10-15 is because of switching from iterative ML initialization to REINFORCE objective. Figure 5(b) shows the individual module-wise performance evaluated using the noisy pseudo-rewards, that indicate whether the action sampled by this module led to the correct answer or not (details in Appendix A.6). Further, by bucketing the performance by the total number of passage entities in Figure 5(c), we observe that WNSMN remains unimpacted by the increasing number of date/numbers, despite the action space explosion. On the other hand, GenBERT’s performance drops linearly beyond 25 passage entities and NMN-num degrades exponentially from the beginning, owing to its direct dependency on the exponentially growing exhaustively precomputed output space.

| Supervision-Type | Acc. (%) | Count | Date-differ |
|------------------|----------|-------|-------------|
| NMN-num Variants |          |       |             |
| ✓ ✓ ✓             | 68.6     | 50.0  | 88.4        | 72.5 |
| ✓ ✓ ×             | 42.4     | 24.1  | 73.9        | 36.4 |
| ✓ × ×             | 54.3     | 47.9  | 80.7        | 40.9 |
| × ✓ ×             | 48.3     | 38.1  | 72.4        | 41.9 |
| ✓ ✓ ×             | 61.0     | 44.7  | 81.1        | 63.2 |
| ✓ ✓ ✓             | 62.3     | 43.7  | 84.1        | 67.7 |
| ✓ ✓ ✓             | 62.1     | 46.8  | 83.6        | 66.1 |

| WNSMN             |          |       |             |
| ✓ × ×             | 66.5     | 58.8  | 66.8        | 75.2 |

4.1 Table 2: DROP-Pruned-num-Test Performance of NMN variants and WNSMN

| Strongly Supervised Models | Acc. (%) |
|---------------------------|----------|
| NMN-num (all supervision) | 58.10    |
| GenBERT+ND                | 69.20    |
| GenBERT+TD                | 70.50    |
| GenBERT+ND+TD             | 75.20    |
| NAQANet                   | 44.97    |
| NABERT                    | 54.27    |
| NABERT+                 | 66.60    |
| NumNet                   | 69.74    |
| MTMSN                    | 75.00    |

| Recall@top-k actions of WNSMN (%) |
|-----------------------------------|
| k = 2 | k = 3 | k = 4 | k = 5 | k = 10 | k = 20 |
| 58.6 | 63.0 | 65.4 | 67.4 | 72.3 | 74.2 |

4.2 Table 3: Skylines & WNSMN top-k performance on DROP-num-Test
Figure 5: (a) Training trend showing the Recall@top-k and all actions, accuracy of Operator and Entity-type Predictor, estimated based on noisy pseudo rewards (Appendix A.6), (b) Module-wise performance (using pseudo-reward) on DROP-num-Test, (c) Bucketing performance by total number of passage entities for WNSMN, and the best performing NMN and GenBERT model from Table 1.

More Stable RL Framework

The training trend in Figure 5(a) shows early saturation and the module-wise performance indicates overfitting despite the regularization tricks in §2.1.3 and Appendix A.6. While more stable RL algorithms like Actor-Critic, Trust Region Policy Optimization (Schulman et al., 2015) or Memory Augmented Policy Optimization (Liang et al., 2018) can mitigate these issues, we leave them for future exploration. Also, though this work’s objective was to train module networks with weak supervision, the sparse confounding rewards in the exponential action space indeed render the RL training quite challenging. One practical future direction to bridge the performance gap would be to pretrain with strong supervision on at least a subset of reasoning categories or on more constrained forms of synthetic questions, similar to GenBERT. Such a setting would require inspection and evaluation of generalizability of the RL model to unknown reasoning types or more open-ended questions.

5 RELATED WORK

In this section we briefly compare our proposed WNSMN to the two closest genre of models that have proven quite successful on DROP3 i) reasoning free hybrid models NumNet, NAQANet, NABERT, NABERT+, MTMSN, and NeRd ii) modular network for reasoning NMN. Their main distinction with WNSMN is that in order to address the challenges of weak supervision, they obtain program annotation from the QA pairs through i) various heuristic parsing of the templatized queries in DROP to get supervision of the reasoning type (max/min, diff/sum, count, negate). ii) exhaustive search over all possible discrete operations to get supervision of the arguments in the reasoning.

Such heuristic supervision makes the learning problem significantly simpler in the following ways

- These models enjoy supervision of specialized program that have explicit information of the type of reasoning to apply for a question e.g., SUM(10,12)
- A simplistic (contextual BERT-like) reader model to read query related information from the passage trained with direct supervision of the query span arguments at each step of the program
- A programmer model that can be directly trained to decode the specialized programs
- Executing numerical functions (e.g., difference, count, max, min) either by i) training purely neural modules in a strong supervised setting using the annotated programs or by ii) performing the actual discrete operation as a post processing step on the model’s predicted program. For each of these previous works, it is possible to directly apply the learning objective on the space of decoded program, without having to deal with the discrete answer or any non-differentiability.

However, such heuristic techniques of program annotation or exhaustive search is not practical as the language of questions or the space of discrete operations become more complex. Hence WNSMN learns in the challenging weak-supervised setting without any additional annotation through

- A noisy symbolic query decomposition that is oblivious to the reasoning type and simply based on generic text parsing techniques

3A more detailed related work section is presented in the Appendix A.4
• An entity specific cross attention model extracting passage information relevant to each step of the decomposed query and learning an attention distribution over the entities of each type
• Learning to apply discrete reasoning by employing neural modules that learn to sample the operation and the entity arguments
• Leveraging a combination of neural and discrete modules when executing the discrete operation, instead of using only neural modules which need strong supervision of the programs for learning the functionality
• Fundamentally different learning strategy by incorporating inductive bias through auxiliary losses and Iterative Maximal Likelihood for a more conservative initialization followed by REINFORCE

These reasoning-free hybrid models are not comparable with WNSMN because of their inability to learn in absence of any heuristic program annotation. Instead of learning to reason based on only the final answer supervision, they reduce the task to learning to decode the program, based on heuristic program annotation. NMN is the only reasoning based model that employ various auxiliary losses to learn even in absence of any additional supervision, similar to us.

To our knowledge WNSMN is the first work on modular networks for fuzzy reasoning over text in RC framework, to handle the challenging cold start problem of the weak supervised setting without needing any additional specialized supervision of heuristic programs.

6 CONCLUSION

In this work, we presented Weakly Supervised Neuro-Symbolic Module Network for numerical reasoning based MRC based on a generalized framework of query parsing to noisy heuristic programs. It trains both neural and discrete reasoning modules end-to-end in a Deep RL framework with only discrete reward based on exact answer match. Our empirical analysis on the numerical-answer only subset of DROP showcases significant performance improvement of the proposed model over SoTA NMNs and Transformer based language model GenBERT, when trained in comparable weakly supervised settings. While, to our knowledge, this is the first effort towards training modular networks for fuzzy reasoning over RC in a weakly-supervised setting, there is significant scope of improvement, such as employing more sophisticated RL framework or by leveraging the pretraining of reasoning.

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A APPENDIX

A.1 QUALITATIVE ANALYSIS

| Weakly Supervised Neuro-Symbolic Module Network | GenBERT |
|-----------------------------------------------|---------|
| 1. Query: how many times did a game between the patriots versus colts result in the exact same scores?, Ans: 2 | Predicted AnsType: Decoded |
| Num. of Passage Entities: Date(10), Number(9) | Decoder output: 2 |
| D, N = Entity-Attention(‘how many times’) // D, N are the attention distribution over date and number entities | Span extracted: “colts” |
| D1, N1 = Entity-Attention(‘did a game between the patriots versus colts result in the exact same scores’, (D, N)) | Answer = 2 |
| ‘Number’, ‘Count’ = Entity-Operator-Selector(‘how many times’, Query) | |
| Answer = 2 = Count(N1) | |
| 2. Query: how many people in chennai, in terms of percent population, are not hindu?, Ans: 19.3 | Predicted AnsType: Decoded |
| Num. of Passage Entities: Date(2), Number(26) | Decoder output: 19.3 |
| D, N = Entity-Attention(‘how many people in chennai, in terms of percent population’) | Span extracted: “80.7” |
| D1, N1 = Entity-Attention(‘are not hindu’, (D, N)) | Answer = 19.3 |
| ‘Number’, ‘Negate’ = Entity-Operator-Selector(‘are not hindu’, Query) | |
| 1 = Count(N) | |
| {80.7} = Sample-Arbitrary-Arguments(N1, 1) | |
| Answer = 19.3 = Negate(80.7) | |
| 3. Query: how many more percent of the population was male than female?, Ans: 0.4 | Predicted AnsType: Decoded |
| Num. of Passage Entities: Date(4), Number(29) | Decoder output: 3.2 |
| D, N = Entity-Attention(‘how many’) | Span extracted: “49.8” |
| D1, N1 = Entity-Attention(‘more percent of the population was male’, (D, N)) | Answer = 3.2 |
| D2, N2 = Entity-Attention(‘than female’, (D, N)) | |
| ‘Number’, ‘Difference’ = Entity-Operator-Selector(‘how many’, Query) | |
| 50.2 = Sample-1-Argument(N1) | |
| 49.8 = Sample-2-Argument(N2) | |
| Answer = 0.4 = Difference(50.2, 49.8) | |
| 4. Query: how many more, in percent population of aigle were between 0 and 9 years old than are 90 and older?, Ans: 9.8 | Predicted AnsType: Decoded |
| Num. of Passage Entities: Date(0), Number(25) | Decoder output: 1.7 |
| D, N = Entity-Attention(‘how many more’) | Span extracted: “0.9” |
| D1, N1 = Entity-Attention(‘in percent population of aigle were between 0 and 9 years old’, (D, N)) | Answer = 1.7 |
| D2, N2 = Entity-Attention(‘than are 90 and older’, (D, N)) | |
| ‘Number’, ‘Difference’ = Entity-Operator-Selector(‘how many more’, Query) | |
| 10.7 = Sample-1-Argument(N1) | |
| 9.8 = Sample-2-Argument(N2) | |
| Answer = 9.8 = Difference(10.7, 0.9) | |
| 5. Query: going into the 1994 playoffs, how many years had it been since the suns had last reached the playoffs?, Ans: 3 | Predicted AnsType: Decoded |
| Num. of Passage Entities: Date(3), Number(17) | Decoder output: 7 |
| D, N = Entity-Attention(‘going into the 1994 playoffs : how many years’) | Span extracted: “1991” |
| D1, N1 = Entity-Attention(‘had it been since the suns had last reached the playoffs’, (D, N)) | Answer = 7 |
| ‘Date’, ‘Difference’ = Entity-Operator-Selector(‘going into the 1994 playoffs : how many years’, Query) | |
| {1991, 1994} = Sample-2-Argument(D) | |
| Answer = 3 = Difference(1991, 1994) | |
| 6. Query: how many more points did the cats have in the fifth game of the AA championship playoffs compared to st. paul saints?, Ans: 3 | Predicted AnsType: Decoded |
| Num. of Passage Entities: Date(3), Number(12) | Decoder output: 3 |
| D, N = Entity-Attention(‘how many’) | Span extracted: “4 - 1 in the fifth game” |
| D1, N1 = Entity-Attention(‘more points did the cats have in the fifth game of the AA championship playoffs’, (D, N)) | Answer = 3 |
| D2, N2 = Entity-Attention(‘compared to the st. paul saints’, (D, N)) | |
| ‘Number’, ‘Difference’ = Entity-Operator-Selector(‘how many’, Query) | |
| 5.0 = Sample-1-Argument(N1) | |
| 2.0 = Sample-1-Argument(N2) | |
| Answer = 3.0 = Difference(5.0, 2.0) | |
| 7. Query: how many total troops were there in the battle?, Ans: 40000 | Predicted AnsType: Decoded |
| Num. of Passage Entities: Date(1), Number(3) | Decoder output: 100000 |
Attention refers to that learnt entity-specific cross attention described in §2.1.1. It then performs work. A detailed pseudo-code of the WNSMN algorithm is provided below. (ii) On the other hand, the steps of the program generated by WNSMN first parses the dependency structure in the query into a program-form. Next, for each step of the program, it generates an attention distribution over the date and number entities. Entity-Attention refers to that learnt entity-specific cross attention described in §2.1.1. It then performs the discrete reasoning by sampling an operation and specific entity-arguments, in order to reach the answer. Entity-Type-Operator-Selector refers to the Entity-Type and Operator Predictor in Operator Sampling Network and Sample-*-Argument refers to the Argument Sampling Network described in §2.1.2. Sum/Difference/Logical-Not are some of the discrete operations that are executed to get the answer. In some of the cases, (e.g., Query 3.) despite wrong parsing the model was able to predict the correct operation even though the root clause did not have sufficient information. In Query 10., the correct operation is Max, but WNSMN reaches the right answer by sampling only the maximum number entity through the Sample-Arbitrary-Argument network and then applying a spurious Sum operation on it.

(ii) On the other hand, the steps of the program generated by NMN-num first compute or further filter attention distribution over the passage or entities which are then fed into the learnable modules (Passage-Attn-To-Count, Year-Difference) that predict the answer. In order to do so, it needs to precompute all possible outputs of numerical operations that generate new numbers for e.g. year-diffs in Example 9. Because of the relatively poorer performance of NMN-num, its outputs are only reported for the last 3 instances, which were cherrypicked based on NMN-num’s predictions.

(iii) GenBERT first predicts whether the answer should be decoded or extracted from passage span and accordingly uses the Decoder output or extracted span as the answer. By design, the modular networks provide a more interpretable output than the monolithic encoder-decoder model GenBERT.

A.2 IMPLEMENTATION & PSEUDO-CODE

The source-code and models pertaining to this work would be open-sourced on acceptance of this work. A detailed pseudo-code of the WNSMN algorithm is provided below.
Algorithm 1 WNSMN Algorithm

Input: (Query \((q)\), Passage \((p)\)) = \(x\)
Output (or Supervision): Answer\((y)\) ∈ \(\mathbb{R}\)

Preprocessing:
\[
[num_1, num_2, \ldots, num_N] = Num = Extract-Numbers(p) \quad // \text{Number and Date}
\]
\[
[date_1, date_2, \ldots, date_D] = Date = Extract-Dates(p) \quad // \text{Entity and Passage Mentions}
\]

Inference:
\[
[(q_1, ref_1), \ldots, (q_k, ref_k), \ldots, (q_l, ref_l)] = Program = Query-Parsing(q)
\]
\[
\text{for step } (q_k, ref_k) \in \text{Program do}
\]
\[
(A_k^{num}, T_k^{num}), (A_k^{date}, T_k^{date}) = Entity-Attention(q_k, p, ref_k, Num, Date)
\]
\[
\text{end for}
\]
\[
L_{aux}^{num}, L_{aux}^{date} = Entity-Inductive-Bias(A^{num}, A^{date}) \quad \text{Equation (1)}
\]
\[
q_i = \text{Query Span Argument of Last Step} \quad // \text{Program Arguments and Stacked Attention}
\]
\[
ref_i = \text{Reference Argument of Last Step} \quad // \text{Map over Entities for Last Step}
\]
\[
T^{num} = \{T_k^{num} | k \in ref_i\}, T^{date} = \{T_k^{date} | k \in ref_i\}
\]
\[
\text{Operators} = \{op_1, op_2, \ldots, op_{k1}\} = \text{Operator-Predictor}(q_i, q) \quad // \text{Operator and EntityType}
\]
\[
\text{EntTypes} = \{type_1, type_2, \ldots, type_{k1}\} = \text{Entity-Type-Predictor}(q_i, q) \quad // \text{Sampling}
\]
\[
\text{Actions} = \{\}\quad // \text{Action Sampling for each Operator}
\]
\[
\text{for } op, type \in (\text{Operators}, \text{EntTypes}) \text{ do}
\]
\[
\text{if type is Number then}
\]
\[
T = T^{num}
\]
\[
\text{else if type is Date then}
\]
\[
T = T^{date}
\]
\[
\text{end if}
\]
\[
\text{if op is diff then}
\]
\[
\text{if } |ref_i| = 2 \text{ then}
\]
\[
arg_1 = \{arg_{11}, arg_{12}, \ldots, arg_{1k2}\} = \text{Sample-1-Argument}(T_0)
\]
\[
arg_2 = \{arg_{21}, arg_{22}, \ldots, arg_{2k2}\} = \text{Sample-1-Argument}(T_1)
\]
\[
arg = \{(a_1, a_2) | (a_1, a_2) \in (arg_1, arg_2)\}
\]
\[
\text{else if } |ref_i| = 1 \text{ then}
\]
\[
arg = \{arg_1, arg_2, \ldots, arg_{k2}\} = \text{Sample-2-Argument}(T_0)
\]
\[
\text{end if}
\]
\[
\text{else if op is count then}
\]
\[
arg = \{count_1, count_2, \ldots, count_{k2}\} = \text{Count-Network}(\sum_j T_j)
\]
\[
\text{else}
\]
\[
arg = \{arg_1, arg_2, \ldots, arg_{k2}\} = \text{Sample-Arbitrary-Argument}(\sum_j T_j)
\]
\[
\text{end if}
\]
\[
\text{probs} = \{(p^{type} \cdot p^{op} \cdot p) | p \in p^{arg}\} \in \mathbb{R}^{k^2} \quad // \text{p’s refer to the corresponding probabilities}
\]
\[
\text{answers} = \{\text{Execute-Discrete-Operation}(type, op, arg) | arg \in \text{args}\} \in \mathbb{R}^{k^2}
\]
\[
\text{actions} = \{(prob, answer) | \text{prob} \in \text{probs}, \text{answer} \in \text{answers}\}
\]
\[
\text{Actions} = \text{Actions} \cup \text{actions}
\]
\[
\text{end for}
\]

Training:
\[
\text{for } i \in \{1, \ldots, N_{IML} + N_{RL}\} \text{ do}
\]
\[
\text{for } (x, y) \in \mathcal{D} \text{ do}
\]
\[
A(x) \leftarrow \text{Actions sampled for input(x)} \quad // \text{Using above Algorithm}
\]
\[
R(x, a, y) \leftarrow \text{Exact Match Reward for action } a \text{ for instance } x \text{ with gold answer } y
\]
\[
\text{if } i \leq N_{IML} \text{ then}
\]
\[
(\theta, \phi) \leftarrow \max_{\theta, \phi} J_{IML} \text{ over } (A, R) + \min_{\phi} \mathcal{L}_{aux} \quad J_{IML} \text{ from Equation (2)}
\]
\[
\text{else}
\]
\[
(\theta, \phi) \leftarrow \max_{\theta, \phi} J_{RL} \text{ over } (A, R) + \min_{\phi} \mathcal{L}_{aux} \quad J_{RL} \text{ from Equation (3)}
\]
\[
\text{end if}
\]
\[
\text{end for}
\]
\[
\text{end for}
\]
### Qualitative Inspection of WNSMN Predictions

**Good Action:** Action Resulting in exact match with gold answer  
**Correct Action:** Action Manually annotated to be correct

| Category | Value |
|----------|-------|
| Number of test instances (DROP-num Test) | 5800 |
| Number of instances with atleast 1 good action | 4868 |
| Number of instances with more than 1 good action | 2533 |
| Average number of good actions (where there is atleast 1 good action) | 1.5 |
| Average number of good actions (where there is more than 1 good action) | 2.25 |
| Number of instances where the top-1 action is good action | 2956 |
| Number of instances where top-1 is the only good action | 2335 (79% of 2956) |
| Number of instances with possibility of top-1 action being spuriously good | 620 (21% of 2956) |
| Number of instances manually annotated (out of possible cases of spurious top-1 action) | 334 (out of 620) |
| Number of instances where top-1 action is found to be spurious | 28 (8.4% of 334) |
| Avg Ratio of Probability of Top Action and Maximum Probability of all other spuriously good actions (if any) | 4.4e+11 |

Table 5: Analysis of the predictions of WNSMN on DROP-num Test

**Generic Observations/Notes**

- **Note:** When the model selects a single number in the Argument Sampling network and the Operator sampled is not of type count, we forcefully consider the operation as a NO-OP. For example sum/min/max over a single number or date is treated as NO-OP.

- One potential source of spuriously correct answer is the neural ‘counter’ module which can predict numbers in [1, 10]. However, out of the cases where atleast one of the top-50 actions is a good action we observe that the model is able to learn when the answer is directly present as an entity or can be obtained through (non count) operations over other entities and when it cannot be obtained directly from the passage but needs to aggregate (i.e., count) over multiple entities. Table 6 below gives some examples of hard instances where the WNSMN Top-1 prediction was found to be correct.

| True Reasoning | Model Prediction | Count |
|----------------|------------------|-------|
| negate a passage entity i.e., 100 - number | the model was able to select negate of the correct entity as the top action. | 34 |
| min/max of a set of passage entities | the model instead directly sampled the correct minimum/maximum entity as a single argument and then applied NO-OP operation over it. | 11 |
| select one of the passage entities | the model was able to select the right entity and apply NO-OP on it as the top action. | 18 |
| count over passage entities | the model was able to put count as the top action and the spurious actions came much lower with almost epsilon probability | 88 |
| difference over passage entities (the same answer could be spuriously obtained by other non-difference operations over unrelated entities) | the model was able to put difference as the top action and the spurious actions came much lower with almost epsilon probability | 89 |
| difference over passage entities (the same answer could be spuriously obtained by difference over other unrelated entities) | the model was able to put difference over the correct arguments as the top action | 66 |

Table 6: Case Study of the 306 instances manually annotated as Correct out of 334 instances
### True Reasoning

| True Reasoning                                      | Model Prediction                                      | Count |
|-----------------------------------------------------|--------------------------------------------------------|-------|
| difference of dates/months                         | count over years                                       | 4     |
| sum(number1, count([number2]))                      | count over numbers                                     | 1     |
| difference between entities                        | sum over two arguments (both arguments wrong)          | 1     |
| difference between entities                        | difference over two arguments (both arguments wrong)   | 1     |
| difference between entities                        | count over entities                                     | 1     |
| question is vague/incomplete/could not be answered manually | count or difference                                     | 2     |
| counting over text spans (Very rare type of question, only 2 found out of 334) | wrong operator                                         | 2     |
| miscellaneous                                      | wrong operator                                          | 7     |
| miscellaneous                                      | correct operator wrong arguments (one correct)         | 2     |
| miscellaneous                                      | correct operator wrong arguments (all wrong)           | 5     |

Table 7: Case Study of the 28 instances manually annotated as Wrong out of 334 instances.

### Table 8: Manual Analysis of a few hard instances (with Question and Relevant Passage Excerpt) where WNSMN top-1 prediction was found to be correct

| Question                                                                 | Relevant Passage Excerpt                                                                 | Model Prediction Analysis                                                                                                                                                                                                                                                                                                                                 |
|-------------------------------------------------------------------------|----------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| How many printing had Green Mansions gone through by 1919?              | “W. H. Hudson which went through nine printings by 1919 and sold over 20,000 copies....” | Model was able to rank the operation sum([9.0]) highest. the count-number operator had near-epsilon probability, indicating that indeed it did not find any indication of the answer being 9 by counting entities over the passage. This is despite the fact that most of the “how many” type questions need counting.                                                                                   |
| The Steelers finished their 1995 season having lost how many games difference to the number of games they had won? | “In 1995, the Steelers overcame a 3-4 start (including a 20-16 upset loss to the expansion 1995 Jacksonville Jaguars season) to win eight of their final nine games and finished with an record, the second-best in the AFC...” | Model had to avoid distracting numbers (3,4) and (20,16) to understand that the correct operation is difference of (9-8)                                                                                                                                                                                                                                                                               |
| How many more field goals did Longwell boot over Kasay?                  | “26-yard field goal by kicker Ryan Longwell ... Carolina got a field goal with opposing kicker John Kasay. ... Vikings would respond with another Longwell field goal (a 22-yard FG) ... Longwell booted the game-winning 19-yard field goal ” | Question needed counting of certain events and none of these appeared as numbers. Model was able to apply count over number entities correctly                                                                                                                                                                                                                                                                                             |
| How many delegates were women from both the Bolshevik delegates and the Socialist Revolutionary delegates? | “Of these mandatory candidates, only one Bolshevik and seven Socialist Revolutionary delegates were women.” | Model was able to apply sum on the correct numbers, even though many of the “how many” type questions need counting                                                                                                                                                                                                                                                                                                                                       |
| How many years in a row did the GDP growth fall into negatives?         | “Growth dropped to 0.3% in 1997, -2.3% in 1998, and -0.5% in 1999.”                      | Model had to understand which numbers are “negative”. It also needed to understand to count the two events instead of taking difference of the years                                                                                                                                                                                                                                                                                                                                 |
| At it’s lowest average surface temperature in February, how many degrees C warmer is it in May? | “The average surface water temperature is 26-28 C in February and 29 C in May.” | Passage had distractive unrelated numbers in the proximity but the model was able to select the lowest temperature out of (26,28) and then take difference of (29-26)                                                                                                                                                                                                                                                                                                                                 |
| How many years before the blockade was the Uqair conference taken place? | “Ibn Saud imposed a trade blockade against Kuwait for 14 years from 1923 until 1937... At the Uqair conference in 1922...” | Passage had other distractive unrelated numbers in the proximity but the model was able to select the correct difference operation                                                                                                                                                                                                                                                                                                                                   |

### A.4 BACKGROUND: NUMERICAL REASONING OVER TEXT

The most generic form of Numerical reasoning over text (NRoT) is probably encompassed by the machine reading comprehension (MRC) framework (as in Dua et al. (2019)), where given a long passage context, c, the model needs to answer a query q, which can involve generating a numerical or textual answer or selecting a numerical quantity or span of text from the passage or query. The distinguishing factor from general RC is the need to perform some numerical computation using the entities and numbers in the passage to reach the goal.
**Modular Networks for Reasoning**: In the early NRoT datasets [Koncel-Kedziorski et al., 2016], which deal with simpler math word problems with a small context and few number entities, symbolic techniques to apply discrete operations were quite popular. However, as the space of operations grow or the question or the context becomes more open-ended these techniques fail to generalize. Incorporating explicit reasoning in neural models as discrete operations requires handling non-differentiable components in the network which leads to optimization challenges.

**Discrete/Symbolic Reasoning in NRoT**: Recently Deep Reinforcement Learning (DRL) has been employed in various neural symbolic models to handle discrete reasoning, but mostly in simpler tasks like KBQA, Table-QA, or Text-to-SQL [Zhong et al., 2017; Liang et al., 2018; 2017; Safa et al., 2019; ?; ?]. Such tasks can be handled by well-defined components or modules, with well structured function-prototypes (i.e., function arguments can be of specific variable-types e.g., KB entities or relations or Table row/column/cell values), which can be executed entirely as a symbolic process. On the other hand, MRC needs more generalized frameworks of modular networks involving fuzzy forms of reasoning, which can be achieved by learning to execute the query over a sequence of learnable neural modules, as explored in [Gupta et al., 2020]. This was inspired by the Neural Modular Networks which have proved quite promising for tasks requiring similar fuzzy reasoning like Visual QA ?.

**SoTA models on DROP**: While the current leaderboard-topping models already showcase quite superior performance on the reasoning based RC task, it needs closer inspection to understand whether the problem has been indeed fully solved.

**Pre-trained Language Models**: On one hand, the large scale pretrained language models [Geva et al., 2020] use Transformer encoder-decoder (with pretrained BERT) to emulate the input-output behavior, decoding digit-by-digit for numeric and token-by-token for span based answers. However such models perform poorly when only trained on DROP and need additional synthetic dataset of numeric expressions and DROP-like numeric textual problems, each augmented with the *gold numeric expression* form.

**Reasoning-free Hybrid Models**: On the other hand, a class of *hybrid* neural models have also gained SoTA status on DROP by explicitly handling the different types of numerical computations in the standard extractive QA pipeline. Most of the models in this genre, like NumNet [Ran et al., 2019], NAQANet [Dua et al., 2019], NABERT+ [Kinley & Lin, 2019], MTMSN [Hu et al., 2019] and NeRd [Chen et al., 2020] do not actually treat it as a reasoning task; instead they precompute an exhaustive enumeration of all possible outcomes of numerical and logical operations (e.g., sum/diff, negate, count, max/min) and augment the training data with knowledge of the query-type (depending on reasoning-type) and *all* the numerical expression that leads to the correct answer. This reduces the question-answering task to simply learning a multi-type answer predictor to classify into the reasoning-type and directly predict the numerical expression, thus alleviating the need for rationalizing the inference or handling any (non-differentiable) discrete operation in the optimization. Some of the initial models in this genre are NAQANet [Dua et al., 2019] and NumNet [Ran et al., 2019] which are respectively numerically aware enhancements of QANet(?) and the Graph Neural Networks. These were followed by BERT-based models, NABERT and NABERT+ [Kinley & Lin, 2019], i.e. a BERT version of the former, enhanced with *standard numbers and expression templates* for constraining numerical expressions. MTMSN [Hu et al., 2019] models a specialized multi-type answer predictor designed to support specific answer types (e.g., count/negation/add/sub) with supervision of the arithmetic expressions that lead to the gold answer, for each type.

**Modular Networks for Reasoning**: NMN [Gupta et al., 2020] is the first model to address the QA task through explicit reasoning by learning to execute the query as a specialized program over learnable modules tailored to handle different types of numerical and logical operations. However, to do so, it further needs to augment the training data with annotation of the *gold program* and *gold program execution* i.e. the *exact* discrete operation and numerical expression (i.e., the numerical operation and operands) that leads to the correct answer for e.g., the supervision of the gold numerical expression in Figure 1 is $\text{SUM}(23, 26, 42)$. This is usually obtained through manual inspection of the data through regex based pattern matching and heuristics applied on the query language. However, because of the abundance of templated queries in DROP this pattern matching is infact quite effective and noise-free, resulting in the annotations acting as strong supervision.

However such a manual intensive process severely limits the overall model from scaling to more general settings. This is especially true for some of the previous reasoning based models, NABERT+,
NumNet and MTMSN which perform better than NMN (in fact achieve SoTA performance) on
the full DROP dataset. But we do not consider them as our primary baselines, as, unlike NMN,
these models (Hu et al. (2019); Dua et al. (2019); Ran et al. (2019)) do not have any provision to
learn in absence of the additional supervision generated through exhaustive enumeration and manual
inspection. (Gupta et al., 2020) have been the first to train a modular network strong,

Our work takes it further along the direction in two ways

• while NMN baseline can handle only 6 specific kinds of reasoning, for which they tailored
  the program generation and gold reasoning annotation, our model works on the full DROP-
  num, that involves more diverse kinds of reasoning or more open-ended questions, and
  requires evaluating on a subset $\times 7.5$, larger by training on $\times 4.5$ larger training data.

• while NMN generalized poorly on the full DROP-num, especially when only one or more
  types of supervision is removed, our model performs significantly better without any of
  these types of supervision.

Together, NMN and GenBERT are some of the latest works in the two popular directions (reasoning
and language model based) for DROP that allow learning with partial no strong supervision and
hence act as primary baselines for our model.

Since in this work we are investigating how neural models can incorporate explicit reasoning, we
focus on only answering questions having numerical answer (DROP-num), where we believe the
effect of explicit reasoning is more directly observable. This is backed up by the category-wise
performance comparison of reasoning-free language model GenBERT (reported in Gupta et al., 2020)
with other hybrid models (MTMSN and NABERT+) that exploit numerical computation required
in answering DROP questions. While, on DROP-num, there is an accuracy gap of 33% between
the GenBERT model and the hybrid models (when all are trained on DROP only), there is only a
2-3% performance gap on the subset having answers as single span, despite the latter also needing
reasoning. This evinces that the performance gap is indeed due to exploiting explicit reasoning under
such strong supervised settings.

A.4.1 LIMITATIONS OF NMN

The primary motivation behind our work comes from some of the limitations of the contemporary
neural module networks, NMN and the reasoning-free hybrid models MTMSN, NABERT+, NumNet,
NAQANet; specifically their dependence on the availability of various kinds of strong supervision.
For that we first describe the nature of programmatic decompositions of queries used in the modular
architectures in the closest comparable work of NMN.

NMN defined a program structure with modules like ‘find’, ‘filter’, ‘relocate’, ‘find-num’, ‘find-date’,
‘year-difference’, ‘max-num’, ‘min-num’, ‘compose-number’ etc., to handle a carefully chosen subset
of DROP showcasing only 6 types of reasoning, (i.e. Date-Difference, Count, Extract Number,
Number Compare). For e.g. for the query Which is the longest goal by Carpenter? the program
structure would be (MAX(FILTER(FIND('Carpenter'), ‘goal’)), where each of these operations are
learnable networks. However to facilitate learning of such specialized programs and the networks
corresponding to these modules, the model needs precomputation of the exhaustive output space
for different discrete operation and also various kinds of strong supervision signals pertaining to the
program generation and execution.

Precomputation of the Exhaustive Output-Space: For operations that generate a new number as its
output (e.g., sum/diff), the annotation enumerates the set of all possible outputs by computing over
all subsets of number or date entities in the passage. This simplifies the task by allowing the model
to directly learn to optimize the likelihood of the arithmetic expression that lead to the final answer,
without any need for handling discrete operations.

Program Supervision provides supervision of the query category out of the 6 reasoning categories, on
which their program induction grammar is tailored to. With this knowledge they can directly use the
category specific grammar to induce the program ( for e.g. SUM(FILTER(FIND)) in Fig[1]. Further all
these models (NMN, MTMSN, NABERT+, NumNet, NAQANet) use the supervision of the query category to understand whether the discrete operation is of type count or add/sub or max/min. which includes the knowledge of the ‘gold’ discrete operation (i.e. count or max/min or add/sub) to perform.

Query Attention Supervision provides information about the query segment to attend upon in each step of the program, as the program argument for e.g. in Fig[1] ‘Carpenter’ and ‘goal’ in the 1st and 2nd step of the program.

Execution Supervision: For operations that select one or more of the number/date entities in the passage, (for e.g. max/min), rule based techniques provide supervision of the subset of numbers or dates entities from the passage, over which the operation is to be performed.

These annotations are heuristically generated through manual inspection and regular expression based pattern matching of queries, thus limiting their applicability to a small subset of DROP only. Furthermore, using a hand-crafted grammar to cater to the program generation for each of their reasoning categories, hinders their generalizability to more open ended settings. While this kind of annotation is feasible to get in DROP, this is clearly not the case with other futuristic datasets, with more open-ended forms of query, thus calling for the need for other paradigms of learning that do not require such manually intensive annotation effort.

A.4.2 Pretraining Data for GenBERT

While GenBERT (Geva et al. (2020)) greatly benefits from pretraining on synthetic data, there are few notable aspects of how the synthetic textual data was carefully designed to be similar to DROP. The textual data was generated for the same two categories nfl and history as DROP with similar vocabulary and involving the same numerical operations over similar ranges of numbers (2-3 digit numbers for DROP and 2-4 digit numbers for synthetic textual data). The intentional overlap between these two datasets is evident from the t-SNE plots (in Figure 6) of the pretrained Sentence-Transformer embedding of questions from DROP-num (blue) and the Synthetic Textual Data (red). Further, while the generalizability of GenBERT was tested on add/sub operations from math word problems (MWP) datasets ADD-SUB, SOP, SEQ, their synthetic textual data was also generated using the same structure involving world state and entities and verb categories used by ? to generate these MWP datasets. Such bias limits mitigates the real challenges of generalizability, limiting the true test of robustness of such language models for numerical reasoning.

A.5 Query Parsing: Details

The Stanford Dependency parse tree of the query is organized into a program structure as follows

- **Step 1)** A node is constructed out of the subtrees rooted at each immediate child of the root, the left-most node is called the root-clause
- **Step 2)** Traversing the nodes from left to right, an edge is added between the left-most to every other node, and each of these are added as steps of the program with the node as the query span argument of that step and the reference argument as the incoming edges from past program steps
- **Step 3)** The terminal (leaf) nodes obtained in this manner are then further used to add a final step of the program which is responsible for handling the discrete operation. The query-span argument of this step is the root-clause, which often is indicative of the kind of discrete reasoning to perform. The reference arguments of this step are the leaf nodes obtained from Step 2).

Figure 7 provides some example queries similar to those in DROP along with their Dependency Parse Tree and the Simplified Representation obtained by constructing the nodes and edges as in Step 1) and 2) above, and the final program which is used by WNSMN. Note that in this simplified representation of the parse tree the root-word of the original parse tree is absorbed in its immediate
Figure 7: Examples of Programs for WNSMN obtained from the Dependency Parse Tree of the Query

succeeding child. Also we simplify the structure in order to limit the number of reference arguments in any step of the program to 2, which in turn requires the number of terminal nodes (after step 2 of the above process) to be limited to 2. This is done in our left to right traversal by collapsing any additional terminal node into a single node.

A.6 RL Framework: Details

In this section we discuss some additional details of the RL framework and tricks applied in the objective function.

Iterative ML Objective: In absence of supervision of the true discrete action that leads to the correct answer, this iterative procedure fixes the policy parameters to search for the good actions \( \mathcal{A}_{\text{good}} = \{ a : R(x, a) = 1 \} \) and then optimizes the likelihood of the best one out of them. However, the simple, conservative approach of defining the best action as the most likely one according to the current policy can lead to local minima and overfitting issues, especially in our particularly sparse and confounding reward setting. So we take a convex combination of a conservative and a non-conservative selection that respectively pick the most and least likely action according to the current policy out of \( \mathcal{A}_{\text{good}} \) as best. Hyperparameter \( \lambda \) weighs these two parts of the objective and is chosen to be quite low \( (1e^{-3}) \), to serve the purpose of an epsilon-greedy exploration strategy without diverging significantly from the current policy.

\[
J^{\text{ML}}(\theta, \phi) = \sum_x (1 - \lambda) \max_{a \in \mathcal{A}_{\text{good}}} \log P_{\theta, \phi}(a|x) + \lambda \min_{a \in \mathcal{A}_{\text{good}}} \log P_{\theta, \phi}(a|x)
\]

Using Noisy Pseudo-Reward: In addition to using the REINFORCE objective to maximise the likelihood of actions that lead to the correct answer, we can also obtain different noisy pseudo rewards \( \in \{-1, +1\} \) for the different modules that contribute towards the action sampling (i.e. the operator and the entity-type and different argument sampler networks). Towards this end, we...
define pseudo-reward for sampling an operator as the maximum of the reward obtained from all the actions involving that operator. Similarly, we can also define reward for predicting the entity-type (date or number) over which the discrete operation should be executed. Following the same idea, we also obtain pseudo rewards for the different argument sampling modules. For e.g. if the most likely operator (as selected by the Operator Sampler) is of type count and it gets a pseudo-reward of +1, then, in that case, we can use the reward obtained by the different possible outputs of the Counter network as a noisy pseudo-label supervision and subsequently add an explicit loss of negative log-likelihood to the final objective for the Counter module. Similar pseudo-reward can be designed for the Entity-Ranker module when the most likely operator sampled by the Operator Sampler needs arbitrary number of arguments. Treating the pseudo-reward as a noisy label can lead to a negative-log-likelihood based loss on output distribution from the Entity-Ranker, following the idea that the correct entities should atleast be ranked high so as to get selected when sampling any arbitrary number of entities.