Compressive X-ray Tomosynthesis based on Exposure Sequence Optimization

Wang Dong¹², Ma Xu¹*, Zhao Qile¹, Xu Hao¹ and Xu Tingfa¹²,*

¹ Key Laboratory of Photoelectronic Imaging Technology and System of Ministry of Education of China, School of Optics and Photonics, Beijing Institute of Technology, Beijing, 100081, China
² Beijing Institute of Technology Chongqing Innovation Center, Chongqing, 401120, China
³Corresponding author’s e-mail: maxu@bit.edu.cn, ciom_xtf1@bit.edu.cn

Abstract. X-ray tomosynthesis technique can reconstruct three-dimensional object of interest from two-dimensional projection measurements. This paper proposes a compressive X-ray tomosynthesis imaging method to inspect the moving object on the conveyor. Coding masks are used to modulate the intensity distribution of X-ray sources and reduce the radiation dose. Low-cost uniform linear array detector is used to collect measurement data. Considering the rigid motion of the object, the proposed method uses a dynamic imaging model and a reconstruction framework based on the variation of the sensing geometry. Subsequently, an optimization method is proposed to design the exposure sequence of X-ray sources based on the genetic algorithm. Optimizing the exposure sequence can ameliorate the condition of the ill-posed inverse reconstruction problem. Simulations show that the optimized exposure sequence can effectively improve the reconstruction performance of the compressive X-ray tomosynthesis.

1. Introduction

X-ray tomosynthesis (XT) is an inspection technology that interrogates three-dimensional (3D) object at numerous angles to obtain two-dimensional (2D) projection measurements, from which the cross-section information of object can be reconstructed with computed synthesis algorithms. XT has various applications in industrial manufacturing, military and medical diagnosis [1-2].

In the applications of medical imaging, reduction of radiation dose is a core issue of concern. In the past, this is mainly achieved by reducing the number of projection angles [1]. However, if the number of projections is quite small, the reconstruction quality of object will be degraded. For example, the detailed features may be contaminated by the artifacts [3]. In 2006, Candès et al. proposed the theory of compressed sensing (CS), which paves a new way to solve the ill-posed XT reconstruction problem with less projection measurements [4]. In 2009, Choi and Brady et al. proposed a compressive XT method by using coding masks to modulate the structured illumination of X-ray sources [5]. Kaganovsky et al. compared four different kinds of tomography subsampling methods, and proved that random subsampling on the detector was a more efficient strategy than the others [6]. Recently, a set of optimization method of coding mask patterns were proposed to further improve the reconstructed images quality [7-10].

This paper uses a dynamic imaging model to depict the inspection process of moving object in the compressive XT system. As shown in Figure 1, a group of X-ray sources are uniformly distributed on
the source plane to illuminate the moving object on the conveyor. The coding masks with random patterns are placed under each X-ray source to block a part of X-rays and modulate the intensity distributions. As the object moves forward, the X-ray sources are turned on one by one. The measurements during different exposures are collected by the uniform linear array (ULA) detector, which is cost-efficient compared to the area detector. The 3D object can be reconstructed from the measurements using CS algorithm aided by the sparsity assumption on the original object [11].

Hereafter, the paper develops an optimization method for the exposure sequence of X-ray sources based on genetic algorithm (GA). The attenuation effect of the object on X-rays can be represented by a sensing matrix, which is affected by the position relationship among the X-ray source, object and detector. It is desired to wisely design the exposure sequence according to the moving trail of object to improve the condition of sensing matrix, thus further improving the reconstruction performance. A certain X-ray source is allowed to be turned on several times or always turned off. The optimization cost function is constructed based on the restricted isometric property (RIP) in the CS theory.

Figure 1. Sketch of the compressive XT system to inspect moving object.

2. Dynamic imaging model of XT system

Consider an X-ray with incident intensity $I_0$ passes through the object with an attenuation coefficient $x(r)$ and reaches a position at a distance $r$ from the source. According to the Beer-Lambert’s law, the intensity after attenuation is $I = I_0 \exp\left(-\int_0^r x(r) dr\right)$. The continuous representation of the projection measurement $y(r)$ is formulated as

$$y(r) = -\ln(I/I_0) = \int_0^r x(r) dr.$$  \hspace{1cm} (1)

The discretize form of equation (1) is $y = Ax$, where $y \in \mathbb{R}^{D_1}$ is the raster-scanned vector representing the discretized projection measurements collected by the ULA detector, and $x \in \mathbb{R}^{F_1}$ represents the attenuation coefficients in the space.

Define a object space $\Omega_0$ with $P = P_x \times P_y \times P_z$ 3D mesh, which is located below the coding masks and directly above the conveyor. The object is gridded into $Q = Q_x \times Q_y \times Q_z$. The object moves from one side of conveyor to the other side within the space $\Omega_0$. The detection plane is expressed as a $D = D_x \times D_y$ mesh. The subscripts associated with these constants indicate the $x$, $y$, and $z$ axes. Assume the total number of X-ray sources is $S$, and the number of exposures is $R$. Suppose the ULA detector consist of $E$ parallel equidistant strip detectors along the $y$ axis, so the ULA detector includes $H = D_x \times E$ pixels.

Notation $A \in \mathbb{R}^{D_1 \times F_1}$ represents the system matrix generated by the ASTRA tomography toolbox [12]. As the object moves, the subspace occupied by the object changes constantly, but the object is always expressed as the same vector $f \in \mathbb{R}^{D_1}$. That means the system matrix $A$ continues to change, since the sensing geometry is changed along with the movement of object. According to the positions of the object in $\Omega_0$ and the strip detectors, a compressed version of system matrix $A' \in \mathbb{R}^{H \times D_1}$ can be extracted from the original system matrix $A$ by deleting some rows and columns. It is noted that $A'$ means the compressed system matrix for the $s^{th}$ ($s=1,2,\cdots,S$) source at the $r^{th}$ ($r=1,2,\cdots,R$) exposure. The projection
vector $y_i'$ is formulated as $y_i' = A_i' f$.

We first stack matrices $A_i'$ and vector $y_i'$ for different sources at different exposures to facilitate the optimization of the exposure sequence, the imaging model can be extended to

$$\hat{y} = \hat{A} f,$$  \hspace{1cm} (2)

where $\hat{y} = [y_1^T, y_2^T, \ldots, y_s^T]^T$ and $\hat{A} = [A_1^T, A_2^T, \ldots, A_s^T]^T$. Equation (2) includes all of the information of the sensing geometries for all sources at every exposure.

Next, define a diagonal matrix $N' \in \mathbb{R}^{s \times s}$ for the $r^{th}$ exposure to indicate which source is turned on. Since only one source is allowed to be turned on, there is only one diagonal element in $N'$ to be 1, and the rest are 0. For the $r^{th}$ exposure, the indicator matrix $M'$ is defined as the Kronecker product of $N'$ and an identity matrix $I \in \mathbb{R}^{m \times m}$, written as $M' = N' \otimes I$. Considering all exposures, the combination indicator matrix $M$ can be expressed as $M = N \otimes I$, where $N = \text{diag}(N^1, N^2, \ldots, N^S)$ and $M = \text{diag}(M^1, M^2, \ldots, M^S)$.

The pixels on coding masks have one-to-one correspondence to the pixels on the detector. Numerically, the unblocked and blocked pixels on the coding masks are presented by 1 and 0, respectively. The only pixels associated with the ULA detector are considered, which are referred to as variable pixels. The coding matrix $C_i \in \mathbb{R}^{m \times h}$ is defined as a diagonal matrix whose diagonal elements include all variable pixels on the $i^{th}$ coding mask. The coding masks are kept the same in every exposure. Place all $C_i$, $s=1,2,\ldots,S$ on the diagonal blocks in turn and loop $R$ times, we can obtain a combination coding matrix $C$, written as $C = \text{diag}(C_1, C_2, \ldots, C_S, C_1, C_2, \ldots, C_S \ldots)$.

Multiplying $M$ and $C$ on the right side of equation (2), the imaging model can be formulated as

$$\hat{y} = MC \hat{A} f.$$  \hspace{1cm} (3)

Assume $f$ can be expressed as the product of a sparse basis $\Psi$ and a coefficient vector $z$, equation (3) can be rewritten as $\hat{y} = MC \hat{A} \Psi z = Bz$, where the sensing matrix of the compressive XT is given by

$$B = MC \hat{A} \Psi.$$  \hspace{1cm} (4)

3. Optimization of Exposure Sequence

3.1. Cost Function

The object $f$ can be reconstructed by solving the following inverse problem based on CS theory:

$$\hat{z} = \text{argmin}_z \| y - Bz \|_2^2 + \alpha \| z \|_1,$$  \hspace{1cm} (5)

where $\alpha$ is the weight of sparsity prior term, and $\| \cdot \|$ is the symbol of norm. Define $\hat{B}$ as the normalized version of $B$ in each column. According to RIP criterion, to successfully solve $z$, $\hat{B}$ should satisfy $(1-\epsilon) \| z \|_2 \leq \| \hat{B} z \|_2 \leq (1+\epsilon) \| z \|_2$, where $\epsilon$ is a constant that is small enough. In addition, $\epsilon$ is restricted by the inequality $\epsilon_0 \leq \| \hat{B} \hat{B} - I \|_F$ [13]. Thus, the cost function $F$ to optimize the exposure sequence can be set as

$$F = \| \hat{B} \hat{B} - I \|_F.$$  \hspace{1cm} (6)

3.2. Optimization Algorithm

According to equations (4) and (6), the $F$ is a function of the combination indicator matrix $M$, which is determined by the exposure sequence. The goal is searching for an exposure sequence to minimize the cost function, while the coding masks adopt random binary patterns. This paper uses GA to implement the global search, since it convergences quickly and does not depend on the gradient. In general, set $W$ chromosomes in the population $U$. Each chromosome consists of $R$ genes, and each gene represents the order number of the source used in a certain exposure. Calculate the cost function value of all chromosomes. Search for the smallest value $F_{\text{min}}$ and the corresponding chromosome $u_{\text{min}}$ in the first generation, and record them as the temporary optimized solution $F_o, u_o$. As illustrated in Figure 2, in the
following iterations, the population $U^n$ is continuously updated by the genetic manipulations such as selection, crossover, mutation, and reinsertion. Accordingly, the optimized solution $F_o$, $U_o$ are updated.

Figure 2. Schematic diagram of genetic manipulations.

4. Simulations

Based on the proposed compressive XT method, simulation and analysis are performed. Nine X-ray sources are distributed on a $3\times3$ array in the source plane. The simulation parameters are summarized as follows: $P = P_x \times P_y \times P_z = 64 \times 128 \times 6$, $D = D_x \times D_y = 128 \times 256$, $H = D_x \times E = 128 \times 31$, $Q = Q_x \times Q_y \times Q_z = 32 \times 32 \times 4$, and $R=13$. The transmittance of coding masks is 50%. The GA algorithm is run for 200 iterations. The sparse basis $\Psi$ in equation (4) is determined by the Kroneck product of the 2D Haar wavelet basis and 1D discrete cosine transform basis. We add white Gaussian noise to the measurement intensity $I$ to simulate the actual XT measurements. The level of the additive noise is quantified by the signal-to-noise ratio (SNR), that is, the ratio of the signal power to the noise power.

Figure 3 depicts the reconstruction images for the 4 layers in 3D Shepp-Logan phantom object with 50dB measurement noise. The peak signal-noise ratio (PSNR) of the reconstructed images are provided, where the original images are used as the benchmark. It is shown that the optimization of exposure sequence will increase the average reconstruction PSNRs of the 4 layers from 30.34dB to 40.26dB. The initial exposure sequence is set as '3389499167625', and the optimized exposure sequence is '3638885663999', where the digital number means the order number of X-ray sources. As shown in Figure 4, the value of $F$ convergences rapidly from 70.39 to 57.87, then the optimal value $F_o=57.87$ keeps almost constant. Figure 5 illustrates the impact of measurement noise level on the reconstruction quality. Within 60dB, the reconstruction performance is improved as the noise level reduces.

Figure 3. Simulations of the proposed optimization method for the Shepp-Logan phantom object.
5. Conclusion
This paper developed a compressive XT approach to inspect moving object with optimized exposure sequence. This paper described the dynamic imaging model and inverse reconstruction framework to consider the variation of sensing geometry. Subsequently, an optimization method of exposure sequence was applied to improve the reconstruction performance in low radiation dose. In the future, fast co-optimization method for exposure sequence and coding masks may be studied.

Acknowledgments
National Natural Science Foundation of China (NSFC) (61527802); Fundamental Research Funds for the Central Universities (2018CX01025).

References
[1] Grant, D. G. (1972) Tomosynthesis: a three-dimensional radiographic imaging technique. IEEE Trans. Bio. Eng., 19:20-28.
[2] Reiser, I., Glick, S. (2014) Tomosynthesis imaging. Taylor and Francis.
[3] Hou, W., Zhang, C. (2014) Analysis of compressed sensing based CT reconstruction with low radiation. In: Proc. ISPACS. Kuching, pp.291–296.
[4] Candès, E. J., Romberg, J., Tao T. (2006) Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. IEEE Trans. Inf. Th., 52:489-509.
[5] Choi, K., Brady, D. J. (2009) Coded aperture computed tomography. In Proc. SPIE 7468, 74680B.
[6] Kaganovsky, Y., Li, D., Holmgren, A., et al. (2014) Compressed sampling strategies for tomography. J. Opt. Soc. A. A, 31: 1369-1394.
[7] Ma, X., Zhao, Q., Cuadros, A., et al. (2019) Source and coded aperture joint optimization for compressive X-ray tomosynthesis. Opt. Exp, 27: 6640-6659.
[8] Cuadros, A. P., Peitsch, C., Arguello, H., et al. (2015) Coded aperture optimization for compressive X-ray tomosynthesis. Opt. Exp., 23:32788-32802.
[9] Cuadros, A. P., Arce, G. R. (2017) Coded aperture optimization in compressive X-ray tomography: a gradient descent approach. Opt. Exp., 25:23833-23849.
[10] Mao, T., Cuadros, A. P., Ma, X., et al. (2018) Fast optimization of coded apertures in X-ray computed tomography. Opt. Exp., 26: 24461-24478.
[11] Figueiredo, M. A. T., Nowak, R. D., Wright, S. J. (2007) Gradient projection for sparse reconstruction: application to compressed sensing and other inverse problems. IEEE J.Sel.Top.Sig. 4:586-597.
[12] Van Aarle, W., Palenstijn, W. J., Beenhouwer, J., et al. (2015) The ASTRA toolbox: a platform for advanced algorithm development in electron tomography. Ultramicroscopy, 157: 35-47.
[13] Rauhut, H. (2010) Compressive sensing and structured random matrices. Radon Series Comp. Appl. Math, 9:1-92.