We present a methodology to change the state of the Weather Research Forecasting (WRF) model coupled with the fire spread code SFIRE, based on Rothermel’s formula and the level set method, and with a fuel moisture model. The fire perimeter in the model changes in response to data while the model is running. However, the atmosphere state takes time to develop in response to the forcing by the heat flux from the fire. Therefore, an artificial fire history is created from an earlier fire perimeter to the new perimeter, and replayed with the proper heat fluxes to allow the atmosphere state to adjust. The method is an extension of an earlier method to start the coupled fire model from a developed fire perimeter rather than an ignition point. The level set method can be also used to identify parameters of the simulation, such as the fire spread rate. The coupled model is available from openwfm.org and it extends the WRF-Fire code in WRF release.

Keywords: DDDAS, Data assimilation, Wildland fire, Wildfire, Weather, Filtering, Level set method, Parameter estimation, Fuel moisture

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1. Introduction

This article reports on recent developments in building a Dynamic Data Driven Application System (DDDAS) for wildland fire simulations [1][2][3]. A DDDAS is based on the ability to incorporate data into an executing simulation [4]. See Fig. [1] for the overall scheme of the DDDAS.

The paper is organized as follows. In Sec. [2] we review some existing approaches to data assimilation in simulations of wildland fires. In Sec. [3] and [4] we briefly formulate the model and the principal idea of creating and replaying artificial fire history from point ignition to a given perimeter, for reference. Sec. [5] considers several methods for the construction of a level set function, needed for the replay, two from our previous work and a new method, which is an extension of the reinitialization equation approach known in level set methods. In Sec. [6] we present a new method how the level set functions constructed for two perimeters can be used to create and replay...
Large scale weather data from NOAA Rapid Update Cycle runs:
• 12km-resolution initial conditions
• 12km-resolution boundary conditions
Static data:
• High-resolution topography
• Land Use and Soil Data

Static fire data:
• High resolution fuel data:
  • 30m-resolution fuel description
  • 30m-resolution elevation data

Dynamic fire data:
• Satellite & aircraft fire detection
• NDVI-derived fuel moisture, NDVI-corrected fuel data
• MISR plume heights and fire intensity
• Incident fire perimeters
• Ground sensors

High-resolution forecast:
• wind speed and direction
• air temperature
• air humidity
• precipitation
• cloudiness etc...

High-resolution fire forecast:
• fire area
• fire risk
• fire heat flux
• fire intensity
• fire rate of spread
• plume height

Figure 1: Scheme of wildland fire DDDAS.

an artificial fire history between the two perimeters, and a new method that uses the two level set functions for an automatic adjustment of the fire spread rate between the two perimeters. Sec. 7 describes a new moisture model coupled with the fire and atmosphere model, and the possibilities for the assimilation of moisture data. Finally, Sec. 8 is the conclusion.

2. Data assimilation for wildland fires

One way to incorporate data into an executing simulation is by sequential statistical estimation, which takes all available data to date into account, and is known in geosciences as data assimilation. Data assimilation is a standard technique in numerical weather prediction, and the ability to assimilate large amounts of real-time data is behind much of the recent improvement of weather forecast skill [5]. However, data assimilation for wildland fires poses unique challenges, and classical data assimilation methods simply will not work [6, 7, 8]. One of the reasons is that in many other physical systems where standard methods work well, such as pollution transport or atmospheric dynamics, unwanted perturbations tend to dissipate over time; but, in a fire model, once a perturbation ignites an unwanted fire, the fire will keep growing, and after few assimilation cycles, everything burns. Another reason is that a fire as a coherent structure, needs to be moved, started, or extinguished in response to the data, which requires positional, Lagrangean correction; additive corrections of the values of the physical fields are not very useful.

Data assimilation methods by sequential Monte-Carlo methods (SMC), also known as particle filters, were developed in the literature for cell-based fire models [9, 10]. They can handle non-Gaussian distributions, but they are computationally very expensive, because they require very large ensembles to cover a region of the state space by random perturbations. A suitable perturbation algorithm is the key to a successful application. The perturbation methods used in wildland fire modeling range from random modifications of the burn area [9] to genetic algorithms, which evolve the shape of the fire by simulated evolution, where the states with fire regions closer the the data are more likely to survive [11]. While SMC methods with tens of thousands of particles may be feasible for 2D cell models, with relatively small state vectors, they are definitely out of question for a coupled atmosphere-fire model. Methods based on the optimal statistical interpolation and the Kalman filter (KF), such as the ensemble Kalman filter (EnKF),
Figure 2: Visualization in Google Earth client, 2007 Witch Fire. False color shows fire heat flux, superimposed on the Earth surface. Patches of slower fuels keep burning behind the fireline. Reproduced from [18].

assume that the state distribution is at least approximately Gaussian and they modify the state in response to data [5, p. 180] rather than rely on hitting the right answer with random perturbations. Thus, KF-based methods require much smaller ensembles than SMC methods, but still in the range of 20-100 members and easily many hundreds [12]. However, because of the fine resolution of the atmospheric model needed over large areas, and the associated need for small time steps, the simulations are computationally very demanding, and such ensembles are still out of question. FFT-based data assimilation methods, which reduce data assimilation to efficient operations with diagonal matrices [13] and can drastically reduce the required ensemble size, from hundreds to often just 5 or 10 members. However, using the Fourier basis is tantamount to the assumption that the state covariance does not vary spatially [14]. Wavelet estimation can combine the effectiveness of spectral methods with an automatic treatment of spatial locality [15]. Wavelet diagonal approximations of the covariance matrix [16] are of particular interest, as they allow efficient evaluation of the EnKF formulas [17].

Position correction methods, such as morphing [6], can overcome the limitations of changing the state of the simulation by additive corrections only. These methods extend the state by a new variable containing a deformation field, similarly as in optical flow methods [19] and extraction of the wind field from a sequence of radar images [20]. For other related position correction methods, see, e.g., [21]. Our morphing technique is distinguished by replacing linear combinations of member states, which are at the heart of, e.g., the EnKF, by intermediate states, which interpolate both the magnitude and the position of coherent features, such as fires. Time series of station observations could be handled by considering composite states over several time steps. However, while morphing works successfully for fire models [6, 8], it changes the delicate physical balance of the atmospheric equations and limits the possibility of the treatment of the model as a black box. Even a simple linear transformation to move and reshape the vortex in hurricane forecasting needs rebalancing of the atmospheric variables from conservation equations [22, p. 11].

Therefore, an important problem in data assimilation for a coupled atmosphere-fire model is how to adjust the atmosphere state when the state of the fire model changes in response to data. The heat output of the fire is concentrated in a narrow area with active combustion, therefore the fire forcing on the atmosphere is highly localized. If the fire
is just shifted, a position correction alone can be successful to some extent [6,8] because the relationship between the changes in the atmosphere and in the fire is captured in the covariance of their deformation fields. However, in general, the covariance does not contain sufficient information and a spin-up is required to develop proper circulation patterns for the changed fire forcing.

3. The coupled atmosphere – fire model

Over time, the wildland fire DDDAS has evolved from a simple convection-reaction-diffusion equation exploratory model to test data assimilation methodologies [23] and the CAWFE model [24,25], which couples the Clark-Hall atmospheric model with fire spread implemented by tracers (Lagrangean particles), to the currently used Weather Research Forecasting (WRF) mesoscale atmospheric code [26] coupled with a spread model implemented by the level set method [8,27]. The Clark-Hall model has many favorable properties, such as the ability to handle refinement, but WRF is a supported community model, it can execute in parallel, and has built-in export and import of state, which is essential for data assimilation. Also, WRF supports data formats standard in geosciences. The implementation by the level set method was chosen because the level set function can be manipulated much more easily than tracers. The coupled code is available from the Open Wildland Fire Modeling Environment (OpenWFM) [18] at openwm.org, which contains also diagnostic and data processing utilities, including visualization in Google Earth (Fig. 2), which we first proposed in [2]. A subset of the SFIRE code was released with WRF as WRF-Fire. The model is capable of running on a cluster faster than real time with atmospheric resolution in tens of m, needed to resolve the atmosphere-fire interaction, for a fire of size over 10 km [28]. See [8,27] for further details and references.

The state variables of the fire model are the level set function, $\Phi$, the time of ignition $T_i$, and the fuel fraction remaining $F$, given by their values on the nodes of the fire model mesh. At a given simulation time $t$, the fire area is represented by the level set function $\Phi$ as the set of all points $(x,y)$ where $\Phi(x,y) \leq 0$. Since the level set function is interpolated linearly between nodes, this allows a submesh representation of the fire area. In every time step of the simulation, the level set function is advanced by one step of a Runge-Kutta scheme for the level set equation

$$\frac{d\Phi}{dt} = -R|\nabla \Phi|, \quad (1)$$

where $R = R(t,x,y)$ is the fire rate of spread and $|\cdot|$ is the Euclidean norm. The ignition time $T_i = T_i(x,y)$ is then computed for all newly ignited nodes, and it satisfies the consistency condition

$$\Phi(t,x,y) \leq 0 \iff T_i(x,y) \leq t, \quad (2)$$

where both inequalities express the condition that the location $(x,y)$ is burning at the time $t$.

The fire rate of spread $R$ is given by the Rothermel’s formula [29] as a function of the wind speed (at a height dependent on the fuel) and the slope in the direction normal to the fireline. From the level-set representation of the fireline at the time $t$ as $\Phi(t,x,y) = 0$, it follows by an easy calculus that the normal direction is $\nabla \Phi / |\nabla \Phi|$, where $|\cdot|$ is the Euclidean norm. Thus,

$$R = R\left(\frac{u}{|\nabla \Phi|}\right), \quad (3)$$

where $u$ is the wind field.

Once the fuel starts burning, the remaining mass fraction $F = F(t,x,y)$ is approximated by exponential decay,

$$F(t,x,y) = \begin{cases} 
\exp\left(-\frac{t-T_i(x,y)}{\tau_i(x,y)}\right), & t > T_i(x,y), \\
1, & t \leq T_i(x,y),
\end{cases} \quad (4)$$

where $\tau_i$ is the fuel burn time, i.e., the number of seconds for the fuel to burn down to $1/e \approx 0.3689$ of the starting fuel fraction $F = 1$. The heat fluxes from the fire to the atmosphere are taken proportional to the fuel burning rate, $\partial F / \partial t$. The proportionality constants are fuel coefficients. The heat fluxes from the fire are inserted into the atmospheric model as forcing terms in differential equations of the atmospheric model in a layer above the surface, with exponential decay with altitude. This scheme is required because atmospheric models with explicit time-stepping, such as WRF, do not support flux boundary conditions. The sensible heat flux is added to the time derivative of the temperature, while the latent heat flux is added to the derivative of water vapor concentration.
4. Replaying artificial fire history

The SFIRE code as presented in [8, 27] starts from one or more ignition points. The release of the heat from the fire then gradually establishes atmospheric circulation patterns and the fire evolves in an interaction with the atmosphere. There is, however, a practical need to start the simulation from an observed fire perimeter, and to modify the fire perimeter in a running simulation, which presents a particular problem in a coupled model. The atmospheric circulation due to the fire takes time to develop and the heat release from the fire model needs to be gradual, or the model will crash due to excessive vertical wind component.

Therefore, we have proposed creating and replaying an approximate fire history, leading to the desired fire perimeter [30]. Replaying the fire history allows for graduate release of the combustion heat and allows the atmospheric circulation patterns due to the fire to develop. The fire history is encoded as an array of ignition times $T_i(x, y)$, prescribed at all fire mesh nodes. To replay the fire, the numerical scheme for advancing $\Phi$ is suspended, and instead the level set function is set to

$$\Phi(t, x, y) = T_i(x, y) - t.$$  \hspace{1cm} (5)

The fuel decay (4) is then computed from $T_i$, and the resulting heat fluxes are inserted into the atmosphere. After the end of the replay period is reached, the numerical scheme of the level set method takes over.

5. Creating a level set function from a given fire perimeter

A fire perimeter is considered as a closed curve $\Gamma$, composed of linear segments, and given as a sequence of the coordinates of the endpoints of the segments. In practice, such geospatial data are often provided as a GIS shapefile [31], or encoded in a KML file, e.g., from LANDFIRE.

In [30], we have proposed a simple scheme for creating an artificial fire history to be used in the fire replay scheme (5), given an ignition point and ignition time at that point, approximate ignition times $T_i$ at the mesh points are established by linear interpolation between the ignition point and the perimeter (Fig. 3). This simple method was already shown to be successful in starting the model from the given perimeter in a simple idealized case (Fig. 4), with the error in the wind speed of only few %. Extensions of the artificial history scheme will be needed for domains which are not star-shaped with respect to the ignition point. Running the fire propagation backwards in time to find an ignition point is also a possibility, with an intriguing forensic potential [30].

The ignition times $T_i$ at locations outside of the given fire perimeter are perhaps best thought of as what the ignition times at those locations might be in future as the fire keeps burning.

Constructing a level set function from a perimeter is one of the basic tasks in level set methods. Given a closed curve $\Gamma$, one wishes to construct a function $L = L(x, y)$, such that

$$L > 0 \text{ outside of } \Gamma, \quad L < 0 \text{ inside of } \Gamma, \quad L = 0 \text{ on } \Gamma.$$  \hspace{1cm} (6)
(a) The difference in the horizontal wind vector (m/s).  
(b) The relative difference in the wind speed.

Figure 4: The difference in the horizontal wind field at 6.1m between a simulation propagated naturally from a point and another one advanced artificially. The first simulation was ignited from a point in the northeast corner of the domain, and the fire perimeter was recorded after 40 minutes. This perimeter and ignition location were used to generate an artificial history for the first 40 minutes, which was replayed in the second simulation. Both simulations were then allowed to advance another 28 minutes. Reproduced from [30].

In the application to perimeter ignition, one can then set at a fixed instant $t$,

$$T_i (x, y) = c L(x, y) + (t - T), \quad \Phi (t, x, y) = L(x, y)$$

where $c$ is a scaling factor, and proceed with the replay as described in Section 4.

One possibility is simply define the values of $L$ next to $\Gamma$ by the signed distance, and forget about the exact representation of $\Gamma$ as $L = 0$. Instead, in [33], we have proposed to find the values of $L$ next to $\Gamma$ by least squares. Denoting by $u$ the vector of the values $L$ next to $\Gamma$, it is easy to see that $u$ satisfies a homogeneous system of linear equations of the form $Bu = 0$ with at most two nonzeros per rows, and each row corresponding to an edge on the mesh that is intersected by $\Gamma$, as the edge $XY$. We can then find a suitable $u$ minimizing $||u - d||^2$ subject to $Bu = 0$, where $d$ are the signed distances [7]. Once the values of $L$ near $\Gamma$ are found, one can extend $L$ to the whole domain as the distance function by the Fast Marching Method (FMM) [34], or by a simpler and less accurate approximate method suggested in [33].

A better method can be obtained by taking the spread rate into account. The level set function $L$ is a solution of the Hamilton-Jacobi equation

$$R |\nabla L| = 1, \quad L = 0 \text{ on } \Gamma.$$
Figure 5: A level set function linear on the line segments connecting the nodes of the fire mesh cannot be defined at the nodes X and Y consistently as the signed distance from the interface $\Gamma$. The distance of the point X from $\Gamma$ does not depend on the location of the point Z, while the distance of Y does; yet the values of the level set function at X and Y are linear along the segment XY and so fixed by the ratio of their distances from W.

which can be found by solving the reinitialization equation [32, Eq. (7.4)]

$$\frac{\partial L}{\partial t} = \pm (1 - R |\nabla L|)$$

where the sign is taken positive outside of $\Gamma$ and negative inside. Equation (8) is solved by upwinding formulas moving away from $\Gamma$ and starting from the values of $L$ on the other side of $\Gamma$. Alternating the solution process between the outside and the inside of $\Gamma$, the values of $L$ on the two sides of $\Gamma$ “balance out and a steady-state signed distance function is obtained” [32, p. 66].

The situation here is more complicated, because the spread rate $R$ depends on the level set function $L$ following (3). Hence, we freeze $L$ inside $R$ and use successive approximations of the form

$$\frac{\partial L_{k+1}}{\partial t} = \pm \left( 1 - R \left( u \cdot \frac{\nabla L_k}{|\nabla L_k|} \right) \right) \frac{|\nabla L_{k+1}|}{|\nabla L_k|}.$$

6. Data assimilation for the level set fire spread model

Creating a level set function as in [30] and in Sec. 5 allows for starting the coupled model from a given fire perimeter instead of an ignition point. However, a more general approach is needed for data assimilation. Suppose the fire perimeter in the simulation is $\Gamma_1$ at time $t_1$. Then at time $t_2 > t_1$, the fire evolves to fire perimeter $\Gamma_2$. However, data is assimilated, changing the state of the fire model and resulting in a different fire model state with perimeter $\Gamma_a$. First we construct level set functions $L_1$, $L_2$, and $L_a$ for the perimeters $\Gamma_1$, $\Gamma_2$, and $\Gamma_a$, respectively, satisfying (6). We assume that all three level set functions are created using the same method. The resulting approximate formulas will be exact in the case of 1D propagation with the level set functions linear and having the same slope. They will be used point-wise as an approximation otherwise. To emphasize the point-wise application, we write out the arguments $(x, y)$ when present.

6.1. Modifying the fire perimeter dynamically

The state of the atmosphere will no longer match the state of the fire model with the perimeter $\Gamma_a$, and we need to make up the evolution of the atmosphere as the fire progresses from the perimeter $\Gamma_1$ to the perimeter $\Gamma_a$. Since $\Gamma_1$ is
the spread rate after the data assimilation are, respectively, We now use a simple argument of proportions. Assume for the moment 1D fire propagation in one direction and that then satisfies

\[ f_{1,a} = 0 \text{ on } \Gamma_1, \quad 0 < f_{1,a} < 1 \text{ between } \Gamma_1 \text{ and } \Gamma_a, \quad f_{1,a} = 1 \text{ on } \Gamma_2. \]

We can then use the function \( f_{1,a} \) to create artificial ignition times by

\[ T_i (x,y) = t_1 + \frac{L_1 (x,y)}{L_1 (x,y) - L_2 (x,y)} (t_2 - t_1) \]

which interpolates between the perimeters \( \Gamma_1 \) and \( \Gamma_a \), and replay the fire history to release the heat into the atmosphere gradually, as in Sec. 6.2. Dynamic estimate of fire spread rate

A common source of errors in fire modeling is incorrect spread rate. The level set function construction here can be used to adjust the spread rate as well. Define \( f_{1,2} \) similarly to (9),

\[ f_{1,2} (x,y) = \frac{L_1 (x,y)}{L_1 (x,y) - L_2 (x,y)}. \]

We now use a simple argument of proportions. Assume for the moment 1D fire propagation in one direction and that \( f_{1,a} \) and \( f_{1,2} \) are linear. Then \( \Gamma_1, \Gamma_2, \) and \( \Gamma_a \) are points on the real line. and the spread rates of the simulated fire and the spread rate after the data assimilation are, respectively,

\[ R = \frac{\Gamma_2 - \Gamma_1}{t_2 - t_1}, \quad R_a = \frac{\Gamma_a - \Gamma_1}{t_2 - t_1}. \]

However, since \( f_{1,2} \) and \( f_{1,a} \) are linear,

\[ f_{1,2} (x) = \frac{x - \Gamma_1}{\Gamma_2 - \Gamma_1}, \quad f_{1,a} (x) = \frac{x - \Gamma_1}{\Gamma_a - \Gamma_1}, \]

which gives

\[ \frac{R_a (x)}{R (x)} = \frac{\Gamma_a - \Gamma_1}{\Gamma_2 - \Gamma_1} = \frac{f_{1,2} (x)}{f_{1,a} (x)}, \quad x \neq \Gamma_1. \]

Thus, using (9) and (10), (11) suggests to modify the given spread rate \( R \) at a point \((x,y)\) to become the spread rate \( R_a \) after the data assimilation, given by

\[ R_a (x,y) = \frac{L_1 (x,y) - L_a (x,y)}{L_1 (x,y) - L_2 (x,y)} R (x,y). \]

7. Moisture model

Fire spread rate depends strongly on the moisture contents of the fuel. In fact, the spread rate drops to zero when the moisture reaches the so-called extinction value. For this reason, we have coupled the fire spread model with a simple fuel moisture model integrated in SFIRE and run independently at every point of the mesh. The temperature and the relative humidity of the air (from the WRF atmosphere model) determine the fuel equilibrium moisture contents \( E [35] \), and the actual moisture contents \( m = m (t) \) is then modeled by the standard time-lag equation

\[ \frac{dm}{dt} = \frac{E - m}{T_d}, \]

where \( T_d \) is the drying lag time. We use the standard model with the fuel consisting of components with 1, 10, and 100 hour lag time, with the proportions given by the fuel category [36], and the moisture is tracked in each component.
separately. During rain, the equilibrium moisture $E$ is replaced by the saturation moisture contents $S$, and the equation is modified to achieve the rain-wetting lag time $T_r$ only asymptotically for heavy rain,

$$\frac{dm}{dt} = \frac{S - m}{T_r} \left(1 - \exp\left(-\frac{r - r_0}{r_s}\right)\right), \text{ if } r > r_0,$$

where $r$ is the rain intensity, $r_0$ is the threshold rain intensity below which no perceptible wetting occurs, and $r_s$ is the saturation rain intensity, at which $1 - 1/e \approx 63\%$ of the maximal rain-wetting rate is achieved. The coefficients can be calibrated to achieve a similar behavior as accepted empirical models \[13\] \[14\]. See \[39\] \[40\] for other, much more sophisticated models. If moisture measurements are available, they can be ingested in the model \[13\] \[14\] by a fast and cheap Kalman filter in one variable, run at each point independently.

8. Conclusion

We have presented new techniques to assimilate the perimeter data at two different times into a coupled atmosphere-fire model, a new method to estimate the adjustment of the model spread rate between the perimeters towards the data, and a new coupling of the atmosphere-fire model with a third model, a simple time-lag model of fuel moisture. Implementation of the data assimilation is in progress. The moisture model is currently included in the code download and will be treated in more detail elsewhere.

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$S$ is the rain intensity, $T_r$ is the rain-wetting lag time, $r_0$ is the threshold rain intensity below which no perceptible wetting occurs, and $r_s$ is the saturation rain intensity, at which $1 - 1/e \approx 63\%$ of the maximal rain-wetting rate is achieved. The coefficients can be calibrated to achieve a similar behavior as accepted empirical models \[13\] \[14\]. See \[39\] \[40\] for other, much more sophisticated models. If moisture measurements are available, they can be ingested in the model \[13\] \[14\] by a fast and cheap Kalman filter in one variable, run at each point independently.
