I. INTRODUCTION

Mixing of different baryonic states in nuclear systems remains a topic rather exciting but being far from sophisticated understanding. A long time ago, it was speculated that a nucleus is not generally a pure nucleonic system. Due to $NN \leftrightarrow N\Delta$ and $NN \leftrightarrow \Delta\Delta$ transitions, a nuclear wave function incorporates some admixture of states with one (or even more) $\Delta$ baryon(s) \cite{1}. Strictly speaking, also heavier baryons should be taken into account.

Admixtures of $\Sigma$ states in $\Lambda$ hypernuclei probably are more important for hypernuclear dynamics. The $\Lambda\Sigma$ mass difference (80 MeV) is rather less than the $\Delta N$ difference (300 MeV). Moreover, pionic exchange between $\Lambda$ and $N$ necessarily gives rise to virtual $\Sigma$ because of zero $\Lambda$ isospin. It was suggested a long ago \cite{2} that the $\Lambda N - \Sigma N$ coupling is responsible for the so-called $\frac{3}{2}^+ \Lambda$He overbinding problem, which was confirmed recently in a consistent study of $s$-shell hypernuclei \cite{6, 7}. Probably, the $\Lambda N - \Sigma N$ coupling plays the crucial role in binding of hypertriton \cite{2}. The coupling contributes greatly to the $\Lambda$ binding in nuclear matter \cite{2}.

However, there are no direct unambiguous measurements of the baryonic admixtures so far. Various processes are considered to detect $\Delta$ admixtures in ordinary nuclei, but discrimination between $\Delta$ components pre-existing in a nucleus and $\Delta$ baryons produced in a reaction is a difficult problem (for some recent attempts, see \cite{2, 8}). For contributions of $\Lambda N - \Sigma N$ mixing to $\Lambda$ hypernuclear binding energies, alternative dynamical pictures without explicit $\Sigma$ admixtures usually exist. For instance, when a single channel $\Lambda N$ interaction is described in terms of correlated two-pion exchange, it is probable that the dynamics of virtual intermediate $\Sigma$'s is included implicitly. Also effective $\Lambda NN$ force can mimic effects of the $\Lambda N - \Sigma N$ coupling without explicit $\Sigma$ degrees of freedom.

Several other implications of the $\Lambda N - \Sigma N$ coupling providing in principle ways to identify the $\Sigma$ admixtures, have been discussed. It was suggested that measurement of hypernuclear magnetic moments is promising for this aim \cite{3}. The probability of the rare $\pi^+\Lambda$ channel of hypernuclear weak decay is sensitive to $\Sigma^+$ admixture \cite{11}. The coupling can lead in some cases to the reverse order of spin doublet levels \cite{12}. Also production of neutron-rich $\Lambda$ hypernuclei from $(K^-, \pi^+)$ and $(\pi^-, K^+)$ reactions can proceed via $\Sigma^-$ admixture as a doorway state \cite{13}. Relevant data are not available so far. Moreover, in all of these cases, some background effects [as meson charge exchange in the $\pi^+\Lambda$ decay as well as in the $(K^-, \pi^+)$ and $(\pi^-, K^+)$ reactions] occur, which can hinder detection of $\Sigma$ admixture.

The $\Lambda N - \Sigma N$ coupling in double-strangeness hypernuclei is of particular interest, since the relevant mass difference (about 25 MeV) is lowest among all possible known baryonic couplings. However, experimental information on double-strangeness systems is rather scarce so far, and no definite knowledge of the coupling exists yet.

Theoretically, the $\Lambda\Lambda - \Xi N$ coupling in $\Lambda\Lambda$ hypernuclei\footnote{Strictly speaking, the term “$\Lambda\Lambda$ hypernucleus” in this context means a state of an $S = -2$ hypernucleus with dominant $\Lambda\Lambda$ component.} have been considered by several groups \cite{4, 12, 14, 15, 16, 17, 18, 19, 20, 21}. Mostly, hypernuclei observed experimentally have been studied, namely, $^6\Lambda\Lambda$He \cite{12, 16, 17, 18}, $^{10}\Lambda\Lambda$Be, $^{13}\Lambda\Lambda$B \cite{16, 17}. With meson-exchange coupling potentials, probabilities of $\Xi$ admixtures less than 1% were
obtained. Contributions of the coupling to the binding energies are as small as several tenths of MeV except the case of extremely strong ΛΛ attraction providing ΛΛ bound state \[16\,18\]. When the contribution can reach several MeV. Much larger coupling has been obtained \[17\] within a quark model predicting free bound \(H\) dibaryon. In this case, not only \(ΣN\), but also \(ΣΣ\) component is of a great weight (more than 10%).

Myint and Akaishi \[14\] argued that the \(ΛΛ - ΣN\) coupling is considerably enhanced in five-baryon hypernucleus \(\Lambda_ΛH\). A proton, appearing from the \(ΛΛ \rightarrow Σ^-p\) transition, can be bound rather strongly in the \(α\) particle. Thus, the difference between the thresholds of channels \(3H + Λ + Λ\) and \(4He + Σ^-\) is reduced to 8 MeV from 29 MeV for free \(ΛΛ\) and \(Σ^-p\) pairs. Myint and Akaishi \[14\] obtained 1% for the \(Σ^-\) admixture probability and 0.5 MeV for the binding excess appearing due to the coupling. These values are larger than those typically obtained by other authors for other hypernuclei, but still small to provide more or less unambiguous signature of the coupling.

In the studies performed in the 1990s, Nijmegen hard-core model D (NHC-D) \[20\] has been used popularly as a standard meson-theoretical model for \(S = -2\) interactions. The reason was that this model is compatible with strong \(ΛΛ\) attraction (\(ΔB_{ΛΛ} \approx 4–5\) MeV) supported by earlier data on \(ΛΛ\) hypernucleus \[21\,22\]. This strong \(ΛΛ\) attraction of NHC-D is due to its specific feature that only the scalar singlet is taken into account. In the cases of the other Nijmegen models incorporating the whole scalar nonet, the meson-exchange parts of the \(ΛΛ\) interactions are much weaker than those of NHC-D. In the case of the hard-core model F (NHC-F) \[23\], for instance, the strength of that part is about a half of NHC-D. Of course, in these models the hard-core radii can be treated as adjustable parameters to reproduce any strength of \(ΛΛ\) interactions. Then, it is difficult to discriminate between strong and weak meson-exchange attraction compensated by small and large hard-core radii, respectively, in \(ΛΛ\) single-channel treatment.

On the other hand, the \(ΛΛ - ΣN\) coupling of NHC-D is relatively weak because of small contributions of strange mesons, which leads to small \(Σ\) admixtures appeared in structure calculations of double-\(Λ\) hypernuclei \[6\]. In the case of NHC-F, for instance, the \(ΛΛ - ΣN\) coupling has been known to be stronger by about two times than that of NHC-D. As shown later, the strength of the \(ΛΛ - ΣN\) coupling of NHC-D is the weakest among the various Nijmegen models due to the above-mentioned specific feature.

Since the discovery of Nagata event identified uniquely as \(6ΛHe\) \[24\] in 2001, \(ΛΛ\) attraction is believed to be rather less (\(ΔB_{ΛΛ} \approx 1\) MeV). On the basis of this new datum, Hiyama et al. have performed the systematical analysis for light double-\(Λ\) hypernuclei \[27\]. The strength of obtained \(ΛΛ\) interaction is very similar to the meson-exchange part of NHC-F. Even if NHC-D is used, it is possible to reproduce \(ΔB_{ΛΛ}(6ΛHe) \approx 1\) MeV by taking a larger value of its hard-core radius appropriately. However, weak \(ΛΛ\) attraction consistent with \(ΔB_{ΛΛ} \approx 1\) MeV can be obtained more plausibly by the other Nijmegen models. Therefore, it is likely that their stronger \(ΛΛ - ΣN\) coupling interactions are more realistic and the mixing in \(ΛΛ\) hypernuclei is more significant than it was speculated earlier from NHC-D.

Very recently, Myint et al. \[19\] performed a new study of the five-baryon \(ΛΛ\) hypernuclei \(5ΛHe\) and \(5ΛHe\) with diagonal \(ΛΛ\) and coupling \(ΛΛ - ΣN\) \(G\)-matrix interactions deduced from various meson-exchange models. While Myint et al. \[19\] employed a single-channel approach, in which the coupling is involved effectively in the diagonal potential, we solve the relevant two-channel problem explicitly. Particularly, we incorporate the diagonal Ξ potential into the calculation and emphasize that effect of this potential is rather important. We investigate also the role of the other inputs (\(Λ - ΣZ\) and \(ΛΛ\) potentials).

In Sec. \[II\] we describe our model, choice of phenomenological hyperon-core potentials, and also \(G\)-matrix derivation of the diagonal \(ΛΛ\) and coupling \(ΛΛ - ΣN\) effective interaction. Section \[III\] contains presentation and discussion of our results. Also comparison of our approach with that of Myint et al. \[19\] is outlined. Section \[IV\] is devoted to concluding remarks and some outlook.

## II. MODEL AND INTERACTIONS

### A. Model

Accurate five-body calculations for hypernuclei \(5ΛHe\) and \(5ΛHe\) are available so far only in the single-channel approach without baryonic mixings \[27\]. In \[14\,19\], three-body models were utilized with free two-baryon \(S = -2\) potentials (more precisely, their phase-equivalents). Since we also do not solve the five-body problem directly, we use effective \((G\)-matrix\) two-body \(ΛΛ - ΛΛ\) and \(ΛΛ - ΣN\) interactions instead of free-space ones. Here, we adopt the Hartree-Fock (HF) description, which is not only simpler, but also is suitable for the \(G\)-matrix formalism defined in a single-particle model space.

Because density distributions of the lightest nuclei are known to be described poorly in the HF approximation, we do not apply the HF description for all the five baryons. Instead, we treat the hypernuclei as \(5Σ + Λ + Λ\) and \(4He + Σ\) in the \(ΛΛ\) and \(Σ\) channels, respectively. The
nuclear cores are treated as inert, and hyperon-nucleus interactions are described fully by $\Lambda - ^3Z$ and $\Xi - ^4He$ potentials. This HF approach is similar to that in [28], but it is extended to incorporate the second channel.

We adopt the wave function of the five-baryon hypernucleus as $\Psi = \psi_3 \psi_\Lambda(r_1) \psi_\Lambda(r_2) + \psi_3 \psi_N(r_1) \psi_\Xi(r_2)$, where $\psi_3$, $\psi_\Lambda$, $\psi_N$, and $\psi_\Xi$ are the three-nucleon core wave function and single-particle wave functions of $\Lambda$, nucleon ($N$), and $\Xi$, respectively. The $N$ and $\Xi$ components appear from the $\Lambda\Lambda$ nucleon ($\Lambda\Lambda$) and $\Xi\Xi$ potentials. This HF approach is similar to that in [28], whereas $\psi_\Lambda$ and $\psi_N$ depend on $\Lambda\Lambda$- and $\Xi\Xi$-core interactions, respectively. If $\psi_3$ and $\psi_N$ are normalized to unity, the normalizing condition for $\varphi$ and $\chi$ is $\int_0^\infty \rho_\varphi^2 dr + \int_0^\infty \chi^2 dr = 1$. Since $U_\Lambda$ and $\sigma$ depend on $\varphi$, the system must be solved self-consistently as usually in HF approaches.

In [14], only one state ($\alpha + \Xi$) is kept. Generally, it is an important problem how to take account of other states beyond the $1s$ shell. It was shown that the $\Lambda\Lambda - \Xi\Xi$ coupling contributes to the $^6\Lambda\Lambda$He binding energy sizably (though not so much for realistic interactions) [14, 17, 18], despite that all the nucleonic $1s$ states are occupied. In our approach, highly-excited components are renormalized into the effective $\Lambda\Lambda$ interactions in some approximate way, as commented in the end of Subsec. II C.

### B. $\Lambda$- and $\Xi$-core interactions

Our main purpose in this work is to calculate the quantity $\Delta B_{\Lambda\Lambda}$ defined as

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_\Lambda,$$

where $B_\Lambda$ and $B_{\Lambda\Lambda}$ denote binding energies of $\Lambda$ and a pair of $\Lambda$’s in a nuclear core, respectively. In order to obtain $\Delta B_{\Lambda\Lambda}$ values of $^5\Lambda\Lambda$H, $^5\Lambda\Lambda$He, and $^6\Lambda\Lambda$He, we need $\Lambda - ^3Z$, $\Lambda - ^3Z$, and $\Lambda - ^4He$ potentials.

For $\Lambda$-core interactions, we use phenomenological prescriptions. For $\Lambda - ^3Z$ potential $U_{\Lambda\Lambda}$ we use the Isle-type form [29]

$$U_{\Lambda\Lambda} = U_{\Lambda\Lambda}^{\text{Isle}} (1.106 \exp(-r_1^2/r_0^2) - \exp(-r_2^2/r_0^2))$$

with $r_1 = 1.25$ fm, $r_2 = 1.41$ fm. We fit the strength to binding energies of $^4\Lambda\Lambda$H and $^4\Lambda\Lambda$He in the ground and the first excited states, and then average the potential over the singlet and triplet states. The obtained values are $U_{\Lambda\Lambda}^{\text{Isle}} = 322.8$ MeV ($^4\Lambda\Lambda$H) and $U_{\Lambda\Lambda}^{\text{Isle}} = 338.3$ MeV ($^4\Lambda\Lambda$He). Quantity $B_{\Lambda\Lambda} (1.25$ MeV in $^4\Lambda\Lambda$H and $1.53$ MeV in $^4\Lambda\Lambda$He) in [30] is calculated in this averaged potential. For the $\Lambda - ^4He$ potential, fitting is performed to $B_{\Lambda\Lambda} (^4\Lambda\Lambda$He), which gives $U_{\Lambda\Lambda}^{\text{Isle}} = 394.9$ MeV.

To address sensitivity of the results to the shape of the hyperon-nucleus potentials, we examine another force. Namely, $U_{\Lambda\Lambda}$ is taken as one-range Gaussian (ORG):

$$U_{\Lambda\Lambda} = U_{\Lambda\Lambda}^{\text{ORG}} \exp(-r_1^2/r_0^2$$

with $r_0 = 1.5656$ fm and $U_{\Lambda\Lambda}^{\text{ORG}} = -43.93$ MeV for $^4\Lambda\Lambda$He [30]. The fitting procedure for the potential strengths is the same as described above. We obtain $U_{\Lambda\Lambda}^{\text{ORG}} = -38.38$ ($-39.78$) MeV for $^4\Lambda\Lambda$H ($^4\Lambda\Lambda$He).

It is well known that the effect of the $\Lambda N - \Sigma N$ coupling is especially important just in the $^4\Lambda\Lambda$H and $^4\Lambda\Lambda$He hypernuclei. Thus, this effect may be expected to be important also in the five-body $\Lambda\Lambda$ hypernuclei. In our model, the $\Lambda N - \Sigma N$ coupling is incorporated effectively at the mean-field level in the $\Lambda$-nucleus potentials, since they are determined so as to reproduce the experimental $\Lambda$ binding energies in $^4\Lambda\Lambda$H, $^4\Lambda\Lambda$He, and $^4\Lambda\Lambda$He. It should be noted that some direct interplay between two types of baryonic mixing is also possible, which is beyond the mean-field treatment. For instance, a $\Sigma$ hyperon appeared from the $\Lambda N - \Sigma N$ coupling can participate in further couplings (like $\Lambda\Sigma - \Sigma N$), however, since the $\Sigma$ admixtures are not larger than 2% [32, 31], we omit such effects.

In our two-channel model, important roles are played by $\Xi$-core interactions. Particularly, we study effects of the $\Xi\varphi$ potential $U_{\Xi\varphi}$ for $\Xi$ admixtures in $^5\Lambda\Lambda$H and $^5\Lambda\Lambda$He. Unfortunately, too little is known about $\Xi$-nucleus interaction. The $\Xi$-core attractions comparable to $\Lambda$-core ones were deduced by analyzing the compilation of the emission events of $^\Xi$ hypernuclei” in the past [32, 33], although none of individual events was identified unambiguously. The recent data on $^{12}\text{C}(K^-, K^+)\text{ reaction}$ [32, 34] give some evidence that the $\Xi$-nucleus interaction, being attractive, is weaker by about half than the corresponding $\Lambda$-nucleus interaction. We consider that this information is most reliable in the present stage. We adopt here a simple folding model to obtain the strength of $\Xi\varphi$ interaction compatible with $\Xi$ well depth [35, 36] of 14 MeV.
in $^{11}$B. The $\Xi\alpha$ potential is deduced by folding of the obtained $\Xi N$ interaction with the density of $\alpha$, and then $\Xi$ binding energies in the $\Xi\alpha$ systems are calculated. We find no bound state of $\Xi(\Lambda\Lambda\alpha)\nu$, while $\Xi^-$ hyperon is bound by $\alpha$ particle with $B_{\Xi^-}(\frac{5}{2}^-\Lambda\Lambda\alpha)=0.4$–0.5 MeV (depending on details of the folding procedure). This value is much greater than the corresponding Coulomb energy (about 0.1 MeV). The rms radius (about 6 fm) also confirms that it is a nuclear (or, maybe, “hybrid Coulomb-assisted”) state. We consider also the $\Xi$ well depth in $^{11}$B of 24 MeV according to the earlier analysis [32]. In this case, bound $\Xi^-(\Lambda\Lambda\alpha)\nu$ appears with $B_{\Xi^-}=0.9$–1.1 MeV. The much stronger $\Xi\alpha$ potential suggested by Filikhin and Gal [30] gives $B_{\Xi^-}=2.1$ MeV and thus it is probably incompatible with the data [33,34].

For radial dependence of $\Xi\alpha$ potential $U_{\Xi\alpha}$, we adopt the Isle-type form (4) as well. We fit strength $U_{\Xi\alpha}^{\text{Isle}}$ to $B_{\Xi^-}(\frac{5}{2}^-\Lambda\Lambda\alpha)=0.5$ MeV (potential Xa1). For comparison, we examine the Filikhin and Gal [30] choice ($B_{\Xi^-}(\frac{5}{2}^-\Lambda\Lambda\alpha)=2.1$ MeV, potential Xa2). At last, we test also zero $\Xi\alpha$ potential Xa0 (to be consistent with calculations [19], we switched off also Coulomb $\Xi^-$ interaction in the last case). For consistency with the $\Lambda$-core potentials, we prepare also the $\Xi\alpha$ potentials in the ORG form with $r_0=2.145$ fm, which corresponds to folding of a $\Xi N$ ORG-type potential with Gaussian $\alpha$ density with reasonable range parameters. Potential Xa3 is fitted to the same value $B_{\Xi^-}(\frac{5}{2}^-\Lambda\Lambda\alpha)=0.5$ MeV as potential Xa1. The strongest ORG-type potential Xa4 is fitted to $B_{\Xi^-}(\frac{5}{2}^-\Lambda\Lambda\alpha)=1.06$ MeV, which is compatible with $\Xi$ well depth of 24 MeV [32] in $^{11}$B, being rather less than the potential by Filikhin and Gal [30] predicts. The $\Xi\alpha$ potentials used are presented in Table I.

C. Diagonal $\Lambda\Lambda$ and coupling $\Lambda\Lambda-\Xi N$ interactions

Let us derive the effective $\Lambda\Lambda-\Xi N$ interactions suitable for our HF model space, starting from the underlying free-space interactions. We adopt here the $\Lambda\Lambda-\Xi N$ sectors of the SU(3)-invariant OBE models by Nijmegen group; not only the hard-core models NHC-D and -F, but also the soft-core models NSC89 [37] and NSC97 [38]. There are six versions (a–f) of the NSC97 model. In this work, we choose the e and f versions as typical examples.

The $G$-matrix theory is most convenient to derive an effective interaction in some model space: Our effective interactions in the HF single-particle space are given by the $G$-matrix interactions $G_{\Lambda\Lambda,\alpha\alpha}$ into which high-momentum transfer components beyond our model space are renormalized. On the other hand, the effective $\Lambda\Lambda-\Xi N$ coupling interaction $G_{\Lambda\Lambda,\Xi N}$ controls the mixing of $\Xi$ components with $\Lambda\Lambda$ states within our HF model space.

We note here the basic feature of our model for $^6\Lambda\Lambda\alpha\nu$ and $^6\Lambda\Lambda\alpha\nu$ (of $^6\Lambda\Lambda\alpha\nu$): In the case of $^6\Lambda\Lambda\alpha\nu$, where the mixing of $\Xi N$ component is out of our model space due to

The Pauli effect, the $\Lambda\Lambda$ state is described by $G_{\Lambda\Lambda,\Lambda\Lambda}$ in the single-channel treatment. In the case of $^5\Lambda\Lambda\alpha\nu$ (of $^5\Lambda\Lambda\alpha\nu$), for which the $\Xi N$ mixing is treated explicitly within our model space, the $\Lambda\Lambda$ and $\Xi N$ mixed states are described by $G_{\Lambda\Lambda,\Lambda\Lambda}$ and $G_{\Lambda\Lambda,\Xi N}$ in the two-channel treatment.

In this work we construct simply the $G$-matrix interactions in nuclear matter [7], which can be considered approximately as the effective interactions in our finite model space. This nuclear-matter approach is sufficient for our purpose to explore $\Lambda\Lambda-\Xi N$ coupling effects, considering uncertainties of underlying free-space interactions. The coupled-channel $G$-matrix equation for $\Lambda\Lambda$ and $\Xi N$ pairs in symmetric nuclear matter with a Fermi momentum $k_F$ is written as

$$G_{\text{cc}} = v_{\text{cc}} + \sum_{\epsilon'} v_{\epsilon\epsilon'} Q_{\epsilon\epsilon'} G_{\epsilon'\epsilon'} c_0,$$

where $c$ denotes a relative state for a pair $y=(\Lambda\Lambda)$ or $(\Xi N)$, and $v_{\epsilon\epsilon'}$ are free-space interactions. The starting channel $c_0$ correspond to $y=(\Lambda\Lambda)$. For $y=(\Xi N)$ the Pauli operator $Q_{\Xi N}$ acts on intermediate nucleon states. The energy denominator for $y \rightarrow y'$ transition is given by $e_{yy'}$. For the intermediate spectrum we adopt the so-called gap choice in which no potential term is taken into account.

This coupled-channel treatment can be extended straightforwardly to the $\Lambda\Lambda-\Xi N-\Sigma\Sigma$ three-channel case. In the cases of using NHC-D, -F and NSC89, for simplicity, we derive the $G$-matrix interactions in the $\Lambda\Lambda-\Xi N$ two-channel treatment. For the NSC97 models, however, we perform the $\Lambda\Lambda-\Xi N-\Sigma\Sigma$ three-channel calculations because of the following reason: The coupling features of NSC97e and NSC97f are fairly different. The diagonal potential derived from NSC97e in the two-channel treatment is considerably more attractive than the corresponding one from NSC97f, but they become similar to each other in the three-channel treatment. Namely, the effect of $\Sigma\Sigma$ channel is (not) substantial in the case of NSC97e (NSC97f). On the other hand, the effects of $\Sigma\Sigma$ channels for the $\Lambda\Lambda-\Xi N$ effective coupling potentials are not so important both in the cases of NSC97f and NSC97f.

The $G$-matrix interactions in coordinate space are represented by Gaussian functions as follows: First we calculate the momentum-space matrix elements $\langle k|G_{\Lambda\Lambda,\Lambda\Lambda}|k\rangle$ and $\langle k|G_{\Lambda\Lambda,\Xi N}|k\rangle$ by solving Eq. (6). Next, we assume effective local potentials, which simulate the calculated $G$-matrix elements, in three-range Gaussian forms:

$$V(r) = a \exp(-r^2/r_1^2) + b \exp(-r^2/r_2^2) + c \exp(-r^2/r_3^2).$$

with $r_1=1.5$ fm, $r_2=0.9$ fm, $r_3=0.5$ fm. This Gaussian form is used both for the diagonal $\Lambda\Lambda$ interaction $V_{\Lambda\Lambda}$ and the $\Lambda\Lambda-\Xi N$ coupling interaction $V_{\Lambda\Lambda,\Xi N}$. Parameters $a$, $b$, and $c$ are determined so that $\langle k|V_{\Lambda\Lambda}|k\rangle$ and $\langle k|V_{\Lambda\Lambda,\Xi N}|k\rangle$ simulate the corresponding $G$-matrix elements $\langle k|G_{\Lambda\Lambda,\Lambda\Lambda}|k\rangle$ and $\langle k|G_{\Lambda\Lambda,\Xi N}|k\rangle$, respectively. The $G$-matrices are calculated at a low den-
sity \( (k_F = 1.0 \text{ fm}^{-1}) \), because our concern in this work is light double-\( \Lambda \) hypernuclei. Here the uncertainty for choosing the \( k_F \) value is not so problematic in our analyses due to the following reason: The diagonal parts of our \( G \)-matrix interactions are adjusted so as to reproduce \( \Delta B_{\Lambda \Lambda}(^6 \Lambda \Lambda \text{He}) = 1.0 \text{ MeV} \), and the \( k_F \)-dependences of their coupling parts are not so strong.

The old models NHC-D and NHC-F incorporate hard cores, which radii \( r_c \) can be treated as free parameters. We choose \( r_c = 0.535 \text{ fm} \) (NHC-D) and \( r_c = 0.475 \text{ fm} \) (NHC-F) which give (for the Isle-type \( \Lambda \Lambda \)-He potential) \( \Delta B_{\Lambda \Lambda}(^6 \Lambda \Lambda \text{He}) = 0.90 \) and \( 1.02 \text{ MeV} \), respectively, rather close to \( \Delta B_{\Lambda \Lambda}(^6 \Lambda \Lambda \text{He}) = 1.0 \text{ MeV} \). These hard cores are entirely phenomenological and the results are quite sensitive to their values. Here, some comment should be given concerning the parametrization of the coordinate-space interaction. With the hard-core models, the above procedure to derive the three-range potential \( \xi \) gives rise to the strongly repulsive core in the \( \Lambda \Lambda \) diagonal channel, which is too singular to treat it in our HF model space. So, we derived the version whose repulsive cores are comparable to those for the cases of the soft-core models NSC89/97 with a sacrifice of reproducing accurately the \( k \)-dependence of \( \langle k|G_{\Lambda \Lambda,\Lambda \Lambda}|k \rangle \). The resulting core heights are similar to those of the \( \Lambda \Lambda \) \( G \)-matrix interactions derived from NHC-D and -F according to another method \( \xi \).

On the other hand, the core parts of the Nijmegen soft-core models are modeled more sophisticatedly on the basis of the \( SU(3) \) symmetry. In the cases of NSC97 models, the potentials for \( S = -2 \) sector are determined with no additional free parameters. In the case of NSC89, it is necessary to choose the \( S \)-wave form-factor mass in the \( S = -2 \) channel, though the resulting potential is not so sensitive to this quantity: We take 1050.0 MeV rather tentatively. Then, the NSC89 model somewhat overestimates \( \Lambda \Lambda \) attraction \( (\Delta B_{\Lambda \Lambda} = 1.43 \text{ MeV in } ^6 \Lambda \Lambda \text{He}) \) whereas the NSC97 models give a slight \( \Lambda \Lambda \) repulsion: \( \Delta B_{\Lambda \Lambda} = -0.32 \text{ MeV} \) \( (e) \) and \( \Delta B_{\Lambda \Lambda} = -0.04 \text{ MeV} \) \( (f) \). In all the cases, we change the repulsive short-range part \( c \) in the diagonal \( \Lambda \Lambda \) interaction given by \( \xi \) so as to reproduce the Nagara datum. Since the existing models as well as data are far from being certain, we do not consider the agreement/disagreement too seriously. Instead, we compare the different potential models considering them as examples of possible effective interactions motivated by microscopic pictures, but phenomenological to some extent.

With the ORG-type \( \Lambda \) core potentials, we adopt the same procedure. Modifications required in the \( \Lambda \Lambda \) diagonal potentials with respect to the Isle-type case are typically small. Parameters of the diagonal \( \Lambda \Lambda \) and coupling \( \Lambda \Lambda - \Xi N \) effective interactions are presented in Table II.

The last column in Table II shows the volume integrals for the coupling potentials \( \int V_{\Lambda \Lambda,\Xi N}(r) \, dr \). This quantity reflects nicely the net strengths of the coupling potentials. The strongest (weakest) coupling is seen to be NHC-F(D). One should be careful in this case, however, that not only the diagonal part, but also the coupling part depends on the hard-core radius. As found in Table II for instance, the volume integral for NHC-F is 582.3 MeV fm\(^3\). This strength is obtained by taking \( r_c = 0.475 \text{ fm} \), which leads to the reasonable diagonal \( \Lambda \Lambda \) attraction. When we take \( r_c = 0.53 \text{ fm} \) (the same value as that in the \( ^1S_0 \) NN channel), the value of the volume integral is reduced to 338.1 MeV fm\(^3\).

Myint et al. \([19]\) used the simple Gaussian potentials which are phase-shift equivalents to NSC97e, NHC-D and NHC-F. In order to compare our method with theirs, let us derive the \( G \)-matrix interactions from their phase-equivalent potential to NSC97e. The value of the volume integral of their coupling potential in the isospin-singlet state is 525.2 MeV fm\(^3\). The corresponding value for the \( G \)-matrix version derived from their potential is 401.8 MeV fm\(^3\), which is not far from our value 370.2 MeV fm\(^3\) for NSC97e in Table II. It is notable here that the \( G \)-matrix coupling strength is demonstrated to be less than the free-space one.

All \( \Lambda \Lambda \) diagonal potentials used here are fitted so as to reproduce the experimental value \( \Delta B_{\Lambda \Lambda}(^6 \Lambda \Lambda \text{He}) = 1.0 \text{ MeV} \) \( \xi \) using the single-channel approximation [equation II without the coupling term]. From a general point of view, it may be more reasonable to take into account the \( \Lambda \Lambda \) \( - \Xi N \) mixing explicitly not only in \( ^3 \Lambda \Lambda \text{He} \) \( (\Lambda \Lambda \text{He}) \), but also in \( ^1 \Lambda \Lambda \text{He} \). Previously, some estimations for the \( \Lambda \Lambda \) \( - \Xi N \) mixing effect in \( ^1 \Lambda \Lambda \text{He} \) were reported: For instance, Carr et al. \([16]\) obtained the contribution to the binding energy about \( 0.2 \text{ MeV} \) unless \( \Lambda \Lambda \) attraction is uniformly strong, while Yamada and Nakamoto \([17]\) presented the contribution as large as \( 0.4 \text{ MeV} \). Both the results were obtained using the NHC-D model (or its simplified version) specified by a strong \( \Lambda \Lambda \) attraction and a weak \( \Lambda \Lambda \) \( - \Xi N \) coupling. The difference between those results exhibits some uncertainties in current ap-

| No. | \( u^{\text{Isle}}_0 \) | \( B_{\Lambda \Lambda}(^6 \Lambda \Lambda \text{He}) \) | \( B_{\Xi N}(^5 \Lambda \Lambda \text{He}) \) | No. | \( u^{\text{ORG}}_0 \), \( r_0 = 2.145 \text{ fm} \) | \( B_{\Lambda \Lambda}(^6 \Lambda \Lambda \text{He}) \) | \( B_{\Xi N}(^5 \Lambda \Lambda \text{He}) \) |
|-----|----------------|------------------|----------------|-----|----------------|------------------|----------------|
| Xa1  | 158.8          | unbound          | 0.50           | Xa3 | -1.816         | unbound          | 0.50           |
| Xa2  | 319.9          | 2.09             | 3.25           | Xa4 | -3.108         | 1.06             | 2.14           |
| Xa0  | 0              | -                | -              | Xa0 | 0              | -                | -              |

TABLE I: Parameters of the Isle-type \( (u^{\text{Isle}}_0) \) and ORG \( (u^{\text{ORG}}_0) \) \( \Xi \alpha \) potentials. Corresponding \( \Xi \) binding energies are also shown. All the quantities are in MeV.
TABLE II: Parameters of the diagonal ($V_{\Lambda\Lambda}$) and coupling ($V_{\Lambda\Lambda,\Xi N}$) potentials and volume integrals $\int V_{\Lambda\Lambda,\Xi N}(r) \, d^3r$ for various potential models. All the entries are in MeV except the rightmost column, which is in MeV fm$^3$.

| Model  | $a$  | $b$  | $c$(Isle) | $c$(ORG) | $a$  | $b$  | $c$  | Vol. Int. |
|--------|------|------|-----------|----------|------|------|------|----------|
| NHC-D  | -5.659 | -177.8 | 925.0 | 916.0 | 0.1841 | 102.8 | -244.1 | 250.9 |
| NSC97f | -5.380 | -157.3 | 810.0 | 808.0 | 1.361  | 109.2 | -193.5 | 334.2 |
| NSC97e | -5.227 | -168.7 | 867.0 | 863.0 | 1.146  | 96.07 | -59.37 | 370.2 |
| NSC89  | -2.447 | -98.60 | 436.0 | 463.0 | -0.5035 | 128.7 | -68.23 | 465.5 |
| NHC-F  | -1.768 | -105.9 | 462.0 | 488.0 | -0.9449 | 199.3 | -300.5 | 582.3 |

approaches (generally, more elaborated than our one). It can be deduced from [16, 18] that the coupling contribution decreases if $\Lambda\Lambda$ attraction weakens, so probably Nagara event [24] implies a less value than previous estimations. On the other hand, models with stronger $\Lambda\Lambda - \Xi N$ coupling strengths may give more substantial contributions.

Our approach is different from theirs on the basic point. Namely, we renormalize contributions from high-lying $\Xi N$ states into the diagonal $\Lambda\Lambda$ interaction through the $G$-matrix procedure, and only low-lying $\Xi N$ states are treated explicitly in our model space. In the cases of $^5\Lambda\Lambda$H and $^5\Lambda\Lambda$He, there appears the particular low-lying $\alpha + \Xi$ configuration. Then, the intermediate nucleon is strongly bound in the $\alpha$ particle. Such an intermediate state is forbidden by the Pauli principle for a nucleon in the case of $^6\Lambda\Lambda$He: Low-lying $\Xi N$ states coupled to the $\Lambda\Lambda$ state are $\alpha + \Xi + N$ configurations, in which $N$ (and, probably, also $\Xi$) lies in continuum. Energy differences of these intermediate $\Xi N$ states from the $\Lambda\Lambda$ ground state are substantially greater than that for the $\alpha + \Xi$ configuration. Similar contributions from $^3$H(He) + $N + \Xi$ continuum configurations occur also in $^5\Lambda\Lambda$H ($^5\Lambda\Lambda$He). In principle, it would be reasonable to take into account the continuum configurations in $^6\Lambda\Lambda$He as well as in $^5\Lambda\Lambda$H ($^5\Lambda\Lambda$He).

In our single-channel approximation for $^6\Lambda\Lambda$He, however, the “diagonal” $\Lambda\Lambda$ potential fitted to the experimental binding energy incorporates effectively contributions from $\Xi N$ intermediate states in which the nucleon is outside from the 1s shell. The reasons to justify our procedure are as follows: First, such contributions are expected to be small enough due to large energy differences and small overlaps of wave functions of the $\Lambda\Lambda$ bound state and $\Xi N$ continuum states. Secondly, these contributions in the cases of $^6\Lambda\Lambda$He and $^5\Lambda\Lambda$H ($^5\Lambda\Lambda$He) are supposed to be roughly equal to each other and, therefore, to be simulated well by the diagonal $\Lambda\Lambda$ interaction. Being simplified, our approach enables us to avoid uncertainties arising from a treatment of the coupling in $^6\Lambda\Lambda$He.

If the coupling contribution to the binding energy of $^6\Lambda\Lambda$He is comparable to that from the diagonal $\Lambda\Lambda$ potential, our results may be less reliable quantitatively. However, there is no reason to expect that the main effect (difference of the couplings in $^5\Lambda\Lambda$H and $^5\Lambda\Lambda$He) can disappear even in this unfavorable case. It should be emphasized that the coupling in $^6\Lambda\Lambda$He anyway deserves further careful study by itself.

III. RESULTS AND DISCUSSION

First, we discuss the results obtained with Isle-type hyperon-nucleus potentials.

In Table III $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$H) and $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$He) calculated in the single-channel approximation without the coupling are presented. Since all the diagonal $\Lambda\Lambda$ potentials are fitted to $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$H) = 1.0 MeV, they give also values close to each other for $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$H) = 0.58–0.63 MeV and $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$He) = 0.65–0.69 MeV. It is seen that even without the coupling, $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$H)–$\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$He)>0. This nonzero difference was first obtained in the five-body calculation [27] and then confirmed and explained in [40]. The origin of this difference is charge symmetry breaking $\Lambda\Lambda$N interaction. Since $B_{\Lambda\Lambda}$($^5\Lambda\Lambda$H)–$B_{\Lambda\Lambda}$($^5\Lambda\Lambda$He)>0, $\Lambda$ hyperons in $^5\Lambda\Lambda$H move closer to the center and, therefore, closer to each other than in $^5\Lambda\Lambda$H. So they attract each other somewhat stronger in $^5\Lambda\Lambda$H than in $^5\Lambda\Lambda$He. This difference is less than 0.1 MeV (whereas the difference in the $B_{\Lambda\Lambda}$ values in the corresponding single-$\Lambda$ hypernuclei is about 0.3 MeV) if the $\Lambda\Lambda$ attraction is compatible with Nagara event and can be greater for stronger $\Lambda\Lambda$ attraction, but not greater than several tenths of MeV [40]. It is seen that the coupling effect increases the difference considerably (columns labeled cc in Table III corresponding to the Xa1 potential.).

In Fig. III a), $\Delta B_{\Lambda\Lambda}$ values obtained from the full calculation with various Isle-type $\Xi\alpha$ potentials are shown as functions of volume integral $\int V_{\Lambda\Lambda,\Xi N}(r) \, d^3r$.

It is seen that the coupling effect is anyway meaningful and may be rather high. Even with moderate $\Xi\alpha$ potential Xa1, full $\Delta B_{\Lambda\Lambda}$ is more than twice as large as the single-channel value for strong coupling interactions. For the NHC-F model, the difference $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$H)–$\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$He) is about 0.4 MeV. For the strongest $\Xi\alpha$ potential Xa2, $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$H) can reach 2.3 MeV (remind that $\Delta B_{\Lambda\Lambda}$($^6\Lambda\Lambda$He)= 1.0 MeV). Even for zero $\Xi\alpha$ potential Xa0, $\Delta B_{\Lambda\Lambda}$($^5\Lambda\Lambda$He) can exceed $\Delta B_{\Lambda\Lambda}$($^6\Lambda\Lambda$He) as has been pointed out in [40].

The $\Xi$ admixture probability $p_\Xi = \int_0^\infty \chi^2 \, dr$ is pre-
sent in Fig. 1(b). Its dependence on the volume integral follows closely the corresponding curves for $\Delta B_{\Lambda \Lambda}$. Most of the values in Fig. 1(b) exceed considerably $p_\Xi$ obtained in $\Lambda\Lambda$ hypernuclei with meson-exchange models earlier. In the extreme case of the strong coupling and strong $\Xi\alpha$ potential, $p_\Xi$ reaches 14%. The same value has been obtained in $^{10}\Lambda\Lambda(Be)$ in a quark model with strong dibaryonic mixing.

It is seen that $\Delta B_{\Lambda \Lambda}$ and $p_\Xi$ are smooth functions of the volume integral. At relatively weak couplings, $\Delta B_{\Lambda \Lambda}$ and $p_\Xi$ are nearly quadratic in the volume integral according to the lowest order of perturbation theory. Generally the coupling effect depends on the coupling potential strength (volume integral) rather stronger than on its shape.

The coupling contributions to $\Delta B_{\Lambda \Lambda}$’s in our calculations with no $\Xi\alpha$ potential are systematically smaller than those obtained by Myint et al. The numerical comparison between the two approaches can be exemplified for the NSC97e model. The $G$-matrix interaction derived from their simplified version (97e) is similar qualitatively to that from the original NSC97e, as mentioned before. The obtained values of $\Delta B_{\Lambda \Lambda}$ for $^5\Lambda\Lambda$ H and $^5\Lambda\Lambda$ He are 0.96 MeV and 1.28 MeV, respectively. These values are considerably larger than our obtained values not only with Xa0 (0.79 and 0.94 MeV), but also with Xa1 (see Table III).

Certainly, our approach differs from theirs by many features. Here, it is worthwhile to comment critically the method applied by Myint et al. to reduce the two-channel problem to the single-channel one. For this aim, the effective $\Lambda\Lambda$ diagonal potential

$$V_{\text{eff}}^{\Lambda\Lambda} = V_{\Lambda\Lambda} - V_{\Lambda\Lambda,\Xi N} \frac{1}{\Delta E} V_{\Xi N,\Lambda\Lambda}$$

is used incorporating the coupling due to the second term (for brevity, we do not write down separately the terms corresponding to the $\Xi^{-}p$ and $\Xi^{0}n$ channels). Essential approximation of Myint et al. is replacement of the full energetical denominator by average constant $\Delta E$, which is adjusted to $\Lambda\Lambda$ scattering properties calculated in a meson-exchange model.

The contribution of the second term of (8) to a $\Lambda\Lambda$ hypernuclear energy can be easily expressed as

$$\epsilon = -\frac{\sum_i |\langle \phi | V_{\Lambda\Lambda,\Xi N} | \chi_i \rangle|^2}{\Delta E},$$

where $\phi$ is the wave function of the first ($\Lambda\Lambda$) channel solved with $V_{\text{eff}}^{\Lambda\Lambda}$, and $|\chi_i\rangle$ represents the complete set of states of the second ($\Xi N$) channel.
On the other hand, the accurate expression for the second-order perturbative contribution, considering the Pauli suppression effect on nucleon occupied states, is given as

$$\epsilon = -\sum_{i \notin P} \frac{|(\phi_0 | V_{NN,X} | \chi_i)|^2}{E_i - E_0}$$

(10a)

$$= -\sum_{i} \frac{|(\phi_0 | V_{NN,X} | \chi_i)|^2}{E_i - E_0} + \sum_{i \notin P} \frac{|(\phi_0 | V_{NN,X} | \chi_i)|^2}{E_i - E_0}$$

(10b)

where $\phi_0$ and $E_0$ correspond to the unperturbed (uncoupled) state and $P$ denotes Pauli forbidden $\Xi N$ states. The summation in the first term in (10a) runs over all the $\Xi N$ states and corresponds to (9). The second term comes from the Pauli correction, corresponding to $\Delta V_{Pauli}$ in (14). Since the $\Xi$ admixtures are typically not so high, the lowest order of perturbation is appropriate at least qualitatively; the difference between $\phi$ and $\phi_0$ is probably small. To reduce the first term of (10a) to (9), one should assume some averaged value for the denominator. Clearly, this averaged value is defined by the $\Xi$ hypernuclear spectrum and bears no relation to the two-baryon c.m. averaged energy $\Delta E$ adjusted so as to simulate the $\Lambda\Lambda$ scattering parameters. On the other hand, Myint et al. use the energy denominators conforming to (10) in the calculation of many-body corrections $(\Delta V_{Pauli}$ and $\Delta V_{alpha}$ in their notations). Conceptually, in (10a) $\Delta E$ is used in the first term, but not in the second term. It leads to the contradiction pointed out by Filikhin et al. The correction introduced to exclude the Pauli forbidden state, if calculated properly, is greater than the whole coupling effect. This contradiction is considered to be originated from the inconsistent treatments for the two terms in Eq. (10a), which are obtained only by reforming the single term (10a). Namely, their treatment does not accord to the clear-cut expression (10) and the obtained results are considered to be questionable.

We stress also the role of the $\Xi\alpha$ potential omitted in (10). Naturally, the stronger is $\Xi\alpha$ attraction, the greater is the coupling. Substitution of zero $\Xi\alpha$ potential Xa0 by even relatively weak attractive Xa1 gives an energy gain up to 0.2 MeV in $^5\Lambda\Lambda$H and 0.3 MeV in $^5\Lambda\Lambda$He. For the Xa2 potential, the gain can reach 0.5 MeV ($^5\Lambda\Lambda$H) and 1.1 MeV ($^5\Lambda\Lambda$He). The $\Xi\alpha$ potential is found to amplify the effect, making it observable more simply. On the other hand, it is clear that reliable quantitative extraction of the coupling strength from future data is improbable until one deduces reliable $\Xi\alpha$ potential.

Qualitatively, the effect of the $\Xi\alpha$ potential can be described as follows. Introducing $\Xi\alpha$ attraction, one essentially moves the unmixed $^5\frac{Z}{2}$ state closer to the unmixed $^5\Lambda\Lambda Z$ state. Naturally, the smaller is the energy difference, the greater is the mixing. Seemingly, similar effect can be provided by changing of the $\Lambda\Lambda$ diagonal potential. But for the $\Lambda\Lambda$ potential, this is not the case. If the diagonal $\Lambda\Lambda$ interaction increases (the $^5\Lambda\Lambda Z$ level moves down and away from the $^5\frac{Z}{2}$ one), the coupling can even grow. The reason is that the weaker is $\Lambda\Lambda$ attraction, the more extended is $\Lambda$ spatial distribution. Therefore, overlap of the $\Lambda$ wave function with the nucleonic one becomes poorer (note that the nucleon is bound in the $\alpha$ particle rather strongly) and the coupling strength is reduced. This factor can overcome the decrease of the initial energy difference. Some schematic numerical illustrations have been presented in [20].

Here we demonstrate another example of the importance of the hyperonic spatial distributions. We repeat the calculations using ORG-type $U_{\alpha\lambda}$ and $U_\Xi$. The $\Delta B_{\Lambda\Lambda}$ values are shown in Fig. (a) and $\Xi$ probabilities are presented in Fig. (b). It is seen that the coupling effect increases considerably with respect to the Isle-type potential form. In the extreme cases, $\Delta B_{\Lambda\Lambda}(\Lambda\Lambda\Lambda\Lambda \text{He})$ reaches huge values, almost five times as $\Delta B_{\Lambda\Lambda}(\Lambda\Lambda\Lambda\Lambda \text{He})$ from the single-channel calculation (remind that “extreme” ORG-type $\Xi\alpha$ potential Xa4 is weaker than Xa2 of the Isle-type). The $\Xi$ admixture probabilities are larger too, though to a less extent.

The reason for this enhancement is similar to that discussed above. The ORG-type $\Lambda$-core potential gives much more concentrated hyperon spatial distributions than the Isle-type potential does. It is seen from Fig. 3, where the hyperonic radial wave functions in $^5\Lambda\Lambda$H are shown in comparison with the proton wave function. Therefore, the overlap becomes larger, and the mixing matrix element $\sigma$ in (11) and (2) increases. It is interesting that replacement of the strongest ORG-type Xa4 potential by still stronger Isle-type potential Xa2 does not enhance further the coupling since the shapes of the $\Lambda - ^3\frac{Z}{2}$ and $\Xi\alpha$ potentials become inconsistent.

Usually, the Isle-type potentials are treated as more realistic for light hypernuclei. However, it is instructive that there exists such a high sensitivity of the coupling effect to the shape of the $\Lambda$-core and $\Xi\alpha$ potentials. Though the coupling effect in the calculation with the ORG-type potentials is probably overestimated, it is still possible that the true effect is somewhat larger than that obtained with the Isle-type potentials.

IV. CONCLUSION

We studied properties of mirror hypernuclei $^5\Lambda\Lambda$H and $^5\Lambda\Lambda$He in the two-channel approach with G-matrix effective interactions. The $\Lambda\Lambda - \Xi N$ coupling is particularly efficient in these hypernuclei owing to the small energy differences between the $\Lambda\Lambda$ and $\Xi N$ states. Most importantly, the coupling in $^5\Lambda\Lambda$He is substantially larger than in $^5\Lambda\Lambda$H. The $\Lambda\Lambda - \Xi N$ coupling leads to considerable difference of the binding energies of $^5\frac{Z}{2}$He and $^5\Lambda\Lambda Z$. This difference hardly can be explained by any other reasons and can be a clear signature of the baryonic mixing. Possibly, it is the most unambiguous signature of baryonic mixing among various suggestions considered so far for ordinary nuclei as well as hypernuclei.
FIG. 2: The same as in Fig. 1 for the ORG-type \( \Xi \alpha \) potentials \( Xa3 \) (crosses), \( Xa4 \) (triangles), and \( Xa0 \) (diamonds) and the ORG-type \( \Lambda \)-core potentials.

Essentially, the effect analyzed here is a charge symmetry breaking effect of an unusual (many-body) nature. Namely, the \( \Lambda \Lambda \) interaction in the mirror \( \Lambda \Lambda \) hypernuclei appears to be different due to electromagnetic mass difference of \( \Xi^- \) and \( \Xi^0 \), since different nucleonic states in the mirror cores are occupied. Another charge symmetry breaking mechanism for free \( \Lambda N \) interaction, originating from the mass difference of \( \Sigma^+ \) and \( \Sigma^- \), has been suggested by Dalitz and von Hippel [42] (for further studies, see [43, 44]). By contrast, our mechanism is essentially many-body, existing only due to the nuclear environment.

Our analysis demonstrates that detailed knowledge of the \( \Xi \alpha \) and \( \Lambda - \frac{1}{2} \Sigma \) potentials is needed in order to extract the \( \Lambda \Lambda - \Xi N \) coupling strength quantitatively from future data on \( {}^5_{\Lambda \Lambda} \)H and \( {}^5_{\Lambda \Lambda} \)He. Whereas the \( \Lambda \)-nucleus interaction, though being far from complete understanding, is recognized to a large extent, \( \Xi \)-nucleus interaction is known quite poorly at present. Our consideration exhibits that generally there are no separate branches of \( \Lambda \Lambda \) and \( \Xi \) hypernuclear dynamics, but rather the unified field of \( S = -2 \) hypernuclei. We showed the example when not only the \( \Lambda \Lambda - \Xi N \) coupling interaction, but also the \( \Xi \)-nucleus diagonal potential, is essential for \( \Lambda \Lambda \) hypernuclear properties. On the other hand, the same \( \Lambda \Lambda - \Xi N \) potential is responsible for conversion widths of \( \Xi \) hypernuclei whereas branching ratios of conversion channels depend substantially on the \( \Lambda \Lambda \) potential [45].

Due to the small mass difference of the \( \Xi N \) and \( \Lambda \Lambda \) pairs, the \( \Xi N \) admixtures in \( \Lambda \Lambda \) hypernuclei are not only important, but rather its importance is drastically different in different hypernuclei (and, probably, in different states). Evidently, the \( \Xi N \) mixing in the five-baryon \( \Lambda \Lambda \) hypernuclei is considerably larger than in \( {}^5_{\Lambda \Lambda} \)He. It was suggested that the \( \Lambda \Lambda - \Xi N \) coupling effect in \( {}^4_{\Lambda \Lambda} \)H is also large [12]. It should be noted, however, that baryonic spatial distributions in \( {}^4_{\Lambda \Lambda} \)H are expectedly rather extended. We showed here that it is an unfavorable factor for the coupling. This problem is very interesting in view of the question whether \( {}^4_{\Lambda \Lambda} \)H is bound or not, which was answered oppositely in two recent studies [46, 47] basing on comprehensive four-body (however,
single-channel) calculations.

Lastly, some comments on the possible observation of the five-body $\Lambda\Lambda$ hypernuclei are in order. The $^{3}_{\Lambda\Lambda}H$ hypernucleus can be produced both from $\Xi^{-}$ capture reactions \cite{47,48} or the $^{7}\text{Li}(K^{-}, K^{+})$ reaction \cite{49} and detected by characteristic $\pi^{-}$ emission \cite{50}. However, detection by the $\Lambda\Lambda \rightarrow \Sigma^{-} p$ weak decay suggested in \cite{49} is problematic in view of recent calculations \cite{50,51,52} predicting very small branching ratios for the $\Sigma$ emission channel. On the other hand, production rates of $^{\Lambda\Lambda}5\text{He}$ from $\Xi^{-}$ capture by various nuclei are much smaller \cite{47,48}. Possibly, $^{5}_{\Lambda\Lambda}\text{He}$ can be produced with a detectable probability from $\Xi^{-}$ capture by lithium. However, the detection of $^{5}_{\Lambda\Lambda}\text{He}$ is rather difficult since its $\pi^{-}$ decay is expectedly rather improbable. The technique to detect $^{5}_{\Lambda\Lambda}\text{He}$ is, therefore, an open problem.

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