Heavy Quark Production from Relativistic Heavy Ion Collisions

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Abstract.
We study the production of heavy quarks, charm at BNL-RHIC ($\sqrt{s} = 200$ GeV/nucleon) and CERN-LHC ($\sqrt{s} = 5.5$ TeV/nucleon) and bottom at CERN-LHC from heavy ions colliding at relativistic energies. We consider initial fusion of gluons (and quark-anti-quark annihilation), pre-thermal parton interactions and interactions in thermalized quark gluon plasma. We also consider free-streaming partons as another extreme and compare the results with those from a thermalized plasma of partons. The pre-thermal contribution is calculated by considering interaction among partons having large transverse momenta (jet-partons) after the initial interaction, and from passage of these partons through a thermalized quark gluon plasma. Charm production from pre-thermal processes is found to be comparable to that from prompt (initial) interactions at LHC. It is suggested that this may have important implications for the study of nuclear modification factor, $R_{AA}$ as well as for back-to-back correlation of heavy quarks and production of dileptons having a large mass.

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1. Introduction

Investigation of the properties of quark gluon plasma, a deconfined strongly interacting matter, remains a major activity of present day high energy nuclear physics. This holds out a promise of a deeper understanding of the laws of Quantum Chromo-Dynamics (QCD) and of early universe which was in the form of quark-gluon plasma. Relativistic heavy ion collisions studied at the Brookhaven National Laboratory have produced extremely valuable results which confirm elliptic flow of hadrons [1], suppression of hadrons having large transverse momenta or jet-quenching [2], recombination of partons as a mechanism of hadronization [3], radiation of thermal photons [4], and suppression [5] and regeneration [6] of J/ψ (see, Ref. [7]). These findings clearly confirm formation of quark gluon plasma, which is strongly interacting and has almost a vanishing viscosity [8].

We are now ready to explore the next and more involved questions about the quark-gluon plasma in greater detail and precision. These questions, in no particular order, could be the following. When and how quickly does the plasma thermalize? Does the time of thermalization depends on the flavors of the partons? What is the flavor dependence of energy loss of partons? How does the plasma hadronize? How does it evolve in space and time? How does the restoration of the chiral symmetry upon formation of the plasma and its breaking upon hadronization affect the evolution of the system? How does it break-up? What is the order of phase transition? Is there a tricritical point? Where does it lie? Shall we witness some of the more exotic signatures of the formation of the plasma like the local CP violation and chiral condensates?

A related set of questions concern the production and propagation of the heavy quarks in such collisions. Heavy quarks offer some very distinct advantages. They are mainly produced from prompt fusion of gluons (\(gg \rightarrow Q\overline{Q}\)) and quark-antiquark annihilation (\(q\overline{q} \rightarrow Q\overline{Q}\)). These processes can be accurately described up to next to leading order using perturbative QCD. Their large mass and the necessity to produce them as a \(Q\overline{Q}\) pair, ensures that their production at later times could be limited. Again, due to their very large mass they move slowly, some-what like Gulliver in the Land of Lilliput, through the partonic wind of light quarks and gluons. They will lose energy as they are buffeted by the partons; through collisions and radiation of gluons. Several studies (see, e.g. [9] and references therein) have made detailed evaluations for this energy loss.

Consider b-quarks moving through the plasma. Due to their large mass it is expected that while they will lose energy and momentum, the direction of their motion may not change substantially due to these interactions which may involve small momentum transfers, individually. Their momentum does not undergo a large change as they fragment or coalesce with a light quark to form B-mesons. The B-mesons may also not change their direction drastically due to their interaction with pions. If true, this should make them a valuable probe for the reaction plain dependence of the properties of the plasma, especially for non-central collisions. Similar considerations may apply to
charm quarks as well, to a great extent. It has also been suggested that the partonic flow may sensitively affect the back to back correlations of heavy-quarks [10, 11].

The modification of the back-to-back correlations reported earlier considers production at lowest order [10] and also at NLO [11], the later contributing substantially to the production of charm quarks at LHC. It is quite clear that the heavy-quarks produced from splitting of gluons, for example would not move back-to-back.

If other mechanisms, e.g., a thermal production of heavy-quarks, or a pre-equilibrium production of heavy-quarks due to interaction between partons having large transverse momenta, or a production due to passage of a parton having a large transverse momentum (jet) through the thermalized QGP makes a substantial contribution, then these back-to-back correlations will be affected strongly. Of course the so called nuclear modification factor $R_{AA}$ will have to be accounted for. It is also expected that the relative importance of these contributions will depend on the transverse momentum distribution of the heavy quarks, which will introduce further richness in these studies. The correlated decay of charm and bottom mesons is also known to lead to a substantial contribution to dileptons [12], which have long been considered a reliable signature of quark gluon plasma [13].

In order to put these possibilities to a rigorous test, as a first step we consider production of charm quarks at RHIC and LHC energies and of bottom quarks at LHC energies due to prompt interactions, thermal productions, and pre-equilibrium productions due to interaction of two partonic jets and due to passage of a partonic jet through the quark gluon plasma. Our results suggests that there is a substantial production of charm quarks at LHC following the initial (prompt) interaction.

The paper is organized as follows. In the next section we give the formulations of prompt, jet-jet, jet-thermal, and thermal interactions leading to production of heavy quarks. We shall also consider a scenario, where the initially produced partons undergo free-streaming, as an alternative to fully thermalize expansion. The results are discussed in Sect. 4. Finally in Sec. 5, we give our conclusions.

2. Initial Fusion (Prompt Interaction)

Heavy ion collisions at RHIC (gold on gold) or at LHC (lead on lead) lead to heavy quark productions primarily through gluon fusion ($gg \rightarrow Q\overline{Q}$) and also from light quark annihilation ($q\overline{q} \rightarrow Q\overline{Q}$). The flavor excitations for intrinsic heavy quarks ($qQ \rightarrow qQ$ or $gQ \rightarrow gQ$) is suppressed when next-to leading order processes are considered (see, Ref. [14]).

The differential cross-section for $gg \rightarrow Q\overline{Q}$ and $q\overline{q} \rightarrow Q\overline{Q}$ can be written as:

$$\frac{d\sigma}{d\hat{t}} = \frac{|M|^2}{16\pi\hat{s}^2} \tag{1}$$

where the invariant amplitude $|M|^2$ is given by [15]:

$$|M|^2_{q\overline{q} \rightarrow Q\overline{Q}} = \frac{64\pi^2a_s^2}{9} \left[ \frac{(M^2 - \hat{t})^2 + (M^2 - \hat{u})^2 + 2M^2\hat{s}}{\hat{s}^2} \right], \tag{2}$$
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Figure 1. The $p_T$ distribution for charm production from initial fusion (solid curve), jet-jet (dashed curve), jet-thermal (dash-dotted curve), thermal (stars), and free-streaming (solid circles) processes with initial time 0.15 fm/c, in central collision of gold nuclei at RHIC at $\sqrt{s} = 200$ AGeV.

and

$$|M|_{gg \to Q\bar{Q}}^2 = \pi^2 \alpha_s^2 \left[ \frac{12}{s^2} (M^2 - \hat{t})(M^2 - \hat{u}) + \frac{8}{3} \frac{(M^2 - \hat{t})(M^2 - \hat{u}) - 2M^2(M^2 + \hat{t})}{(M^2 - \hat{t})^2} + \frac{8}{3} \frac{(M^2 - \hat{t})(M^2 - \hat{u}) - 2M^2(M^2 + \hat{u})}{(M^2 - \hat{u})^2} - \frac{2}{3} \frac{M^2(\hat{s} - 4M^2)}{(M^2 - \hat{t})(M^2 - \hat{u})} - 6\frac{(M^2 - \hat{t})(M^2 - \hat{u}) + M^2(\hat{u} - \hat{t})}{\hat{s}(M^2 - \hat{t})} - 6\frac{(M^2 - \hat{t})(M^2 - \hat{u}) + M^2(\hat{t} - \hat{u})}{\hat{s}(M^2 - \hat{u})} \right]. \quad (3)$$

The running coupling constant $\alpha_s$ for lowest order is given by

$$\alpha_s = \frac{12\pi}{(33 - 2N_f) \ln(Q^2/\Lambda^2)}. \quad (4)$$

where $N_f=3$ is the number of flavors and $\Lambda$ is the QCD scale parameter used in the running coupling constant.
The cross-section for the production of heavy quarks from proton-proton collisions at leading order \cite{16} is given by

\[
\frac{d\sigma}{dy_1dy_2d^2p_T} = 2x_1x_2 \sum_{ij} \left[ f^{(1)}_i(x_1,Q^2)f^{(2)}_j(x_2,Q^2) \frac{d\hat{\sigma}_{ij}(\hat{s},\hat{t},\hat{u})}{d\hat{t}} + f^{(1)}_j(x_1,Q^2)f^{(2)}_i(x_2,Q^2) \frac{d\hat{\sigma}_{ji}(\hat{s},\hat{u},\hat{t})}{d\hat{t}} \right] / (1 + \delta_{ij}),
\]

where i and j are the interacting partons and \( f_i \) and \( f_j \) are the partonic structure functions, and \( x_1 \) and \( x_2 \) are the momentum fractions of the parent nucleons carried by the interacting partons. The effect of nuclear shadowing which becomes more pronounced at small \( x \) has been included using the EKS98 \cite{17} parametrization. We use CTEQ5L structure function set for nucleons \cite{18}. The intrinsic transverse momentum of the interacting partons is neglected.

The light quark (u, d, s) and gluon production is similarly calculated with \( d\sigma/d\hat{t} \) taken from Ref. \cite{19} with vanishing quark masses.

For heavy ion collisions the \( p_T \) spectrum for heavy quark production is given by

\[
\frac{dN}{d^2p_Tdy} = T_{AA} \frac{d\sigma}{d^2p_Tdy}
\]
Figure 3. The $p_T$ distribution for charm production from initial fusion (solid curve), jet-jet (dashed curve), jet-thermal (dash-dotted curve), thermal (stars), and free-streaming (solid circles) processes with initial time 0.07 fm/$c$, in central collision of lead nuclei at LHC at $\sqrt{s} = 5500$ AGeV.

where for central collisions, $T_{AA}=286$ fm$^{-2}$ for Au+Au at RHIC and $T_{AA}=292$ fm$^{-2}$ for Pb+Pb at LHC. We account for higher order corrections by taking a constant K-factor $\approx 2.5$ (see, Ref. [9]).

3. Interaction among Partons

The initial hard scattering between the partons of the two nuclei will lead to gluons and light quarks having large transverse momenta. These will ultimately fragment and lead to a stream of hadrons into a narrow cone or jets. We shall continue to call gluons and light quarks having large transverse momenta as jet particles. Being copious in number, they would interact again and may even approach thermalization. The first collisions among two gluonic jets or a quark and anti-quark jet are likely to have sufficient energy to produce a pair of heavy quarks. Thus it is felt that jet-jet interaction in relativistic heavy ion collision can be an interesting source of production of heavy quarks [20].

Several authors have predicted that very large initial temperatures could be attained at RHIC [4] and LHC energies [21]. This suggests that even a thermal production of heavy quarks through the interaction of thermalized quarks and gluons can take place.

An interesting new source of high momentum photons [22] and dileptons [23] has recently been proposed, which arises from the passage of high momentum quark or gluon jets through the thermalized quark gluon plasma. It is of interest to see it whether the
same process could lead to production of heavy quarks. We shall call this process as jet conversion or jet-thermal interaction [24].

The general expression for the production of a heavy quark at central rapidity is given by [20, 27]:

\[
E \frac{d^3 N}{d^3 p} \bigg|_{y=0} = \int d^4x \int \frac{1}{16(2\pi)^8} \frac{d^3p_1d^3p_2d^3p'}{\omega_1\omega_2} \times \delta^4(\sum p^\mu) / E' \times |M|^2 F(\vec{x}, \vec{p}_1, t) F(\vec{x}_2, \vec{p}_2, t)
\]

(7)

where \( \sum p^\mu = p_1 + p_2 - p - p' \), \( p_1 \) and \( p_2 \) are the four-momenta of the incoming partons and \( p \) and \( p' \) are the same for outgoing heavy quark and anti-quark. \( F(\vec{x}, \vec{p}, t) \) gives the phase space distribution function for the incoming partons.

Before deriving results for different processes by choosing appropriate phase space distributions etc., we can perform the following simplifications:

Writing \( d^3p_i/\omega_i = p_{T_i}dp_{T_i}d\phi_i dy_i \) and integrating over \( d^3p' \), we get for \( y=0 \) (\( p_z=0 \),...
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Figure 5. The $p_T$ distribution for bottom production from initial fusion (solid curve), jet-jet (dashed curve), jet-thermal (dash-dotted curve), thermal (stars), and free-streaming (solid circles) processes with initial time $0.07\text{ fm}/c$, in central collision of lead nuclei at LHC at $\sqrt{s} = 5500\text{ AGeV}$.

and $p_T=p$:

$$
\int \frac{d^3p_1 d^3p_2 d^3p'}{\omega_1 \omega_2 E'} \delta^4 \left( \sum p' \right) |M|^2 F(\vec{x}, \vec{p}_1, t) F(\vec{x}, \vec{p}_2, t)
= \int dy_1 dy_2 d\phi_1 d\phi_2 p_{T2}dp_{T2} p_{T1}dp_{T1} \frac{\delta(\sum E)}{E'} |M|^2
\times F(\vec{x}, p_{T1}, \phi_1, y_1, t) F(\vec{x}, p_{T2}, \phi_2, y_2, t),
$$

(8)

where

$$
\frac{\delta(\sum E)}{E'} = \frac{\delta(p_{T1} - p_{T1,0})}{[p_{T2}(\cosh(y_1 - y_2) - \cos(\phi_1 - \phi_2)) - (E \cosh y_1 - p \cos \phi_1)]},
$$

(9)

and we have

$$
p_{T1,0} = \frac{p_{T2}(E \cosh y_2 - p \cos \phi_2)}{[p_{T2}(\cosh(y_1 - y_2) - \cos(\phi_1 - \phi_2)) - (E \cosh y_1 - p \cos \phi_1)]}.\)

(10)

Now we proceed to evaluate individual contributions.

3.1. Jet-Jet interaction of partons

In order to estimate the contribution of the jet-jet interaction to the production of heavy quarks, we approximate [20] the phase-space distribution of the gluon, quark, or anti-quark jets produced in initial (prompt) scattering of the partons in a central collision as:

$$
F(\vec{x}, \vec{p}, t) = f_{\text{jet}}(\vec{x}, \vec{p}, t),
$$

(11)
Figure 6. The $p_T$ distribution for bottom production from initial fusion (solid curve), jet-jet (dashed curve), jet-thermal (dash-dotted curve), thermal (stars), and free-streaming (solid circles) processes with initial time 0.5 fm/c, in central collision of lead nuclei at LHC at $\sqrt{s} = 5500$ AGeV. The jet-jet contribution in this is thus truly pre-thermal.

where [20]

$$f^i_{\text{jet}}(\vec{x}, \vec{p}, t) = \frac{(2\pi)^3}{g_i \tau \pi R_T^2 p_T} \frac{dN_i}{dy d^2p_T} \delta(y - \eta) \Theta(\tau_f - \tau) \Theta(\tau - \tau_i),$$

with $p_T > 2$ GeV. Here $\delta(y - \eta)$ denotes the Bjorken correlation for space-time and energy-momentum rapidities and $'i'$ stands for quarks, anti-quarks, or gluons. The degeneracy of quarks and gluons is given by $g_{g/q}$ such that $g_g = 8 \times 2$ and $g_q = 3 \times 2$ [25]. Further $R_T$ is the transverse radius of the nucleus and $dN_i/d^2p_Tdy$ is the transverse momentum distribution of partons for $p_T > 2$ GeV. We neglect the dependence of this distribution on the momentum rapidity [20] as we are calculating the results for heavy quarks at $y=0$, when only very small values of $y_1$ and $y_2$ contribute, and the rapidity dependence is marginal. For results at $y \neq 0$, appropriate distributions will need to be used.

The momentum space distribution of the jets at RHIC and LHC are taken from parametrization [23] given earlier, where the jet distributions were calculated in LO-pQCD with a K-factor ($\approx 2.5$), CTEQ5L structure functions and EKS98 shadowing functions, which we have used here for calculation of initial production of heavy quarks. Thus we have,

$$\frac{dN}{dy d^2p_T} = T_{AA} \frac{d\sigma^{\text{jet}}}{d^2p_Tdy} \bigg|_{y=0} = K \frac{C}{(1 + p_T/B)^\beta},$$

where $[20]$
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and

\[ h^i_{\text{jet}}(p_T) = \frac{1}{g_i} \frac{dN}{d^2p_Tdy} \bigg|_{y=0}, \quad (14) \]

where K, C, B, and \( \beta \) are taken from reference [23].

We neglect transverse expansion of the system, which should be valid at early times when most of the heavy quarks are produced. Now taking

\[ d^4x = \tau d\tau r dr d\eta d\phi_r, \quad (15) \]

we can perform the integration over \( r, \phi_r \), and \( \tau \).

Thus the \( p_T \) distribution of open heavy quark production from jet-jet interaction can be written as

\[
E \frac{d^3N}{d^3p} \bigg|_{y=0} = \frac{\ln(\tau_f/\tau_i)}{16(2\pi)^2(\pi R_T^2)} \int d\eta \ dp_T \ d\phi_1 \ d\phi_2 \\
\times \left[ \frac{1}{p_{T2}(1 - \cos(\phi_1 - \phi_2)) - (E \cosh \eta - p \cos \phi_1)} \right] \\
\times \left[ \frac{1}{2} g_1^2 h_{\text{jet}}^g(p_{T1,0}) h_{\text{jet}}^g(p_{T2}) |M|^2_{gg \rightarrow Q\bar{Q}} \right. \\
\left. + g_q^2 N_f h_{\text{jet}}^q(p_{T1,0}) h_{\text{jet}}^q(p_{T2}) |M|^2_{qq \rightarrow Q\bar{Q}} \right], \quad (16)
\]

where \( N_f = 3 \) is the number of quark flavors [26].

While writing the above, we have used the Bjorken correlations \( \delta(y_1 - \eta) \) and \( \delta(y_2 - \eta) \) (see Eq.12). With this, the Eqs.(9) and (10) reduce to,

\[
\delta(\sum E) \quad \frac{E}{E'} = \frac{\delta(p_{T1} - p_{T1,0})}{p_{T2}(1 - \cos(\phi_1 - \phi_2)) - (E \cosh \eta - p \cos \phi_1)}, \quad (17)
\]

and

\[
p_{T1,0} = \frac{p_{T2}(E \cosh \eta - p \cos \phi_2)}{p_{T2}(1 - \cos(\phi_1 - \phi_2)) - (E \cosh \eta - p \cos \phi_1)}, \quad (18)
\]

and thus Eq.(8) reduces to Eq.(16), above. Numerical integration of the Eq.16 gives pre-thermal heavy quark production. We shall see that the major contribution to heavy quark production comes from gluon fusion.

We add that Levai et al. [27] calculated the so-called pre-equilibrium (jet-jet) contribution by assuming that this mechanism operates during \( \tau \in [0.1 - 0.5] \) fm/c. We shall explore some other options as well.

We re-iterate that the above expressions do not account for energy loss of the energetic gluons and quarks as they traverse the plasma and thus they provide the upper limit of the heavy quark productions.

3.2. Interaction among thermally and chemically equilibrated partons

If multiple scatterings occur in a rapid succession, QGP may reach thermal equilibrium quite early and follow hydrodynamics evolution till it hadronizes. During this phase of evolution, heavy quarks may be produced provided the initial temperature is high [28].
We can estimate the initial temperature by assuming Bjorken hydrodynamics \cite{29} which relates it to the particle rapidity density by

\[ \frac{2\pi^4}{45\zeta(3)\pi R_T^4} \frac{dN}{dy} = 4aT_0^3 \tau_0 \]  

where \( dN/dy \) is the particle rapidity density, \( a=42.25\pi^2/90 \) for massless light quarks and gluons and \( R_T = 1.2A^{1/3} \). We take particle rapidity density as 1260 estimated experimentally \cite{30} at RHIC and assumed \cite{21} that the particle rapidity density at LHC is about 5625. Some recent works suggest a smaller value for \( dN/dy \approx 3000-3500 \) at LHC \cite{31} from considerations of parton saturation. However, larger values have also been suggested \cite{32}. The initial experimental results from \( pp \) collisions at LHC at 0.9 TeV, 2.36, and 7 TeV already show an increase in the particle rapidity densities, which is steeper than expected \cite{33}. In any case, results for any other rapidity density can be easily obtained.

The time evolution of the temperature of thermalized QGP for a boost-invariant longitudinal expansion is governed by \cite{29}:

\[ T_0^3 \tau_0 = T^3 \tau = \text{const.} \]  

Assuming a rapid thermalization and chemical equilibration, we get the lowest estimate of \( \tau_0 \) from the above by taking \( \tau_0 \approx 1/3T_0 \) \cite{21}. This provides that \( \tau_0 \approx 0.15 \text{ fm}/c \) at RHIC. Such a small value for \( \tau_0 \) is supported by the single photon data \cite{4}. As an alternative, we also consider a much larger time of thermalization, \( \tau_0 \approx 0.5 \text{ fm}/c \), with \( T_0 \) calculated from Eq. 19. We add that several studies, especially the ones related to the flow of hadrons tend to use a larger value of \( \tau_0 \approx 0.6 \text{ fm}/c \) \cite{1}, even though more recent studies find only a weak dependence of \( \tau_0 \) on the flow for hadrons \cite{34}. For LHC, we have similarly assumed \( \tau_0 \approx 0.07 \text{ fm}/c \) and 0.5 \text{ fm}/c.

Taking the critical temperature is taken to be 0.170 GeV, we see that the end of the QGP phase occurs at:

\[ \tau_f = (\text{const.}/0.170^3) \]  

and then the thermal production mechanism would operate during \( \tau_0 \) to \( \tau_f \).

We take the phase space distribution for the thermalized quarks and gluons as,

\[ f_{th}(p_T, y, \eta) = \exp \left[ -p_T \cosh(y - \eta)/T \right]. \]  

Thus the transverse momentum distribution of thermally produced charm given by

\[
E \frac{d^3N}{d^3p_{\perp}} \bigg|_{y=0} = \frac{\pi R_T^2}{16(2\pi)^3} \int \tau \, d\tau \, d\eta \, dp_{T2} \, d\phi_1 \, d\phi_2 \, dy_1 \, dy_2 \\
\times \frac{p_{T2}(\cosh(y_1 - y_2) - \cos(\phi_1 - \phi_2)) - (E \cosh y_1 - p \cos \phi_1)}{[p_{T2}(\cosh(y_1 - y_2) - \cos(\phi_1 - \phi_2)) - (E \cosh y_1 - p \cos \phi_1)]} \\
\times [f_{th}(p_{T1,0}, y_1, \eta)f_{th}(p_{T2}, y_2, \eta)] \times \left[ \frac{1}{2}g_5^2 \left| M_{gc-\bar{Q}Q} \right|^2 + g_5^2N_f \left| M_{gq-\bar{Q}Q} \right|^2 \right]
\]  

(23)
Just as in pre-thermal case, we calculate the final charm production at \( y = 0 \) or central rapidity and thus \( p_z = 0 \) and \( p_T = p \). We now have the kinematical constraint:

\[
\frac{\delta(\sum E)}{E'} = \frac{\delta(p_{T1} - p_{T1,0})}{[p_{T2}(\cosh(y_1 - y_2) - \cos(\phi_1 - \phi_2)) - (E \cosh y_1 - p \cos \phi_1)]} \tag{24}
\]

and

\[
p_{T1,0} = \frac{(p_{T2}(E \cosh y_2 - p \cos \phi_2))}{[p_{T2}(\cosh(y_1 - y_2) - \cos(\phi_1 - \phi_2)) - (E \cosh y_1 - p \cos \phi_1)]} \tag{25}
\]

Numerical integration of the Eq. 23 for different initial conditions gives us the contribution from thermalized QGP.

### 3.3. Interactions of free-streaming partons

As an extreme, we consider free-streaming partons, as a model of evolution of the system of deconfined quarks and gluons, which completely relaxes the condition of thermalization. The initial distribution at \( t = \tau_0 \) and \( z = 0 \) is obtained by assuming maximum entropy \[21\], so that

\[
f(p, x) = \frac{dN}{d^4pd^3x} = \exp\left(-\frac{E}{T_0}\right), \tag{26}
\]

and the condition that needs to be satisfied is

\[
p^\mu \frac{\partial f(x, p)}{\partial x^\mu} = 0. \tag{27}
\]

We assume boost invariance along the z-axis with

\[
f(p, x) = f(p_T, p_z t - Ez). \tag{28}
\]

The solution which satisfies the differential Eq. [27] is

\[
f(p, x) = \exp\left[-\frac{\sqrt{p_T^2 + (p_z t - Ez)^2 / \tau_0^2}}{T_0}\right]. \tag{29}
\]

Now using

\[
p_z = p_t \sinh y, \ E = p_t \cosh y, \]
\[
z = \tau \sinh \eta, \ t = \tau \cosh \eta, \tag{30}
\]

Eq. 29 becomes

\[
f(p_T, \eta, y) = \exp\left[-\frac{p_T \sqrt{1 + \tau^2 \sinh^2(y - \eta) / \tau_0^2}}{T_0}\right]. \tag{31}
\]

Thus the final integration to calculate \( p_T \) distribution for heavy quark production from free streaming partons is given by
\[
E \frac{d^3 N}{d^3 p} \bigg|_{y=0} = \frac{\pi R_T^2}{16(2\pi)^3} \int \tau d\tau \ d\eta \ dp_{T1} \ d\phi_1 \ dp_{T2} \ d\phi_2 \ dy_1 \ dy_2 \\
\times \frac{(p_{T1} p_{T1,0})}{[p_{T2}(\cosh(y_1 - y_2) - \cos(\phi_1 - \phi_2)) - (E \cosh y_1 - p \cos \phi_1)]} \\
\times \left[ f(p_{T1,0}, \eta_1, y_1) f(p_{T2}, \eta_2, y_2) \times \left[ \frac{1}{2} g_g^2 \left| M_{gg\rightarrow Q\bar{Q}} \right|^2 + g_q^2 N_f \left| M_{q\bar{q}\rightarrow Q\bar{Q}} \right|^2 \right] \right]
\]
(32)

The initial conditions for the free-streaming case are taken to be the same as that for the thermal production, whereas the final time is taken as \( R_T/c \), the transverse radius of the nuclei, after which the system would surely expand rapidly along the transverse direction as well and disintegrate.

### 3.4. Charm production from the passage of jets through thermal medium

Now we discuss the production of heavy quarks by passage of light quark and gluonic jets through thermalized QGP. In order to proceed, we can differentiate the phase space distribution into jet partons and thermalized partons. The phase space distribution for equilibrated medium is given by

\[
f_{th} = \exp \left[ -p_T \cosh(y - \eta)/T \right].
\]
(33)

The jet distribution is given by Eq. 11 and 12 for \( p_T > 2 \text{ GeV} \). We have already discussed \( \tau_i \) or \( \tau_0 \), which gives the start of the time from when we consider the system to be in the form of QGP. We define \( \tau_d \) as the time which a jet takes to reach the surface of the quark gluon plasma. Consider a jet formed at \( \vec{r} \) with velocity \( \vec{v} \) which travels to the surface of plasma. The distance, \( d \), covered in this process is given by,

\[
d = -r \cos \phi + \sqrt{R_T^2 - r^2 \sin^2 \phi},
\]
(34)

where \( \phi = \cos^{-1}(\hat{v} \cdot \hat{r}) \), and \( R_T \) is the radius of the system. A massless quark or a gluon would take a time

\[
\tau_d = d/c
\]
(35)

for this journey. Considering that QGP would cool down to the critical temperature by \( \tau = \tau_f \), the time spent by the jet in the plasma would be \( \tau_{\text{max}} = \min \left[ \tau_f, \tau_d \right] \) (see Ref. [23]), and the jet-thermal production mechanism would be in operation during \( \tau \in [\tau_i, \tau_{\text{max}}] \).

The thermalization time is taken as either 0.15 fm/c or 0.5 fm/c at RHIC and as either 0.07 fm/c or 0.5 fm/c at LHC as before.

The final result for jet-thermal interaction is given by
\[ E \frac{d^3 N}{d^3 p} \bigg|_{y=0} = \frac{1}{16(2\pi)^4 \pi R_T^2} \int d\tau \, rdr \, d\eta \, d\phi_1 \, d\phi_2 \, dy_1 \, dp_{T2} \]

\[
\times \left( p_{T2}(\cosh(y_1 - \eta) - \cos(\phi_1 - \phi_2)) - E \cosh y_1 + p_z \sinh y_1 + p_T \cos \phi_1 \right) - Hp_{T1,0} \]

\[
\times f_{\text{in}}(p_{T1,0}, y_1, \eta) \left[ g_q^2 N_f h^q_{\text{jet}}(p_{T2}) |M^2_{qg-Q\bar{Q}}| + \frac{1}{2} g_g^2 h^g_{\text{jet}}(p_{T2}) |M^2_{gg-Q\bar{Q}}| \right],
\]

which we evaluate numerically.

We add that these results do not account for the energy loss suffered by the high energy quark of gluon as it traverses the plasma, and during which it may even change its flavour. This would necessitate a treatment along the lines of Ref. [35] for production of photons from a similar process.

4. Results

Now we discuss our results. As a first step we show the results for charm production at RHIC and LHC.

4.1. Charm at RHIC

In Fig. 1 we plot the results at RHIC for charm production from prompt interactions (initial fusion), jet-jet interactions of partons, thermal production and from the passage of high momentum jets through thermalized quark-gluon plasma. We also give the free-streaming results to compare with thermal production. We consider central collision of gold nuclei at \( \sqrt{s} = 200 \) AGeV.

We see that the contribution of the initial fusion dominates at all \( p_T \). Considering initial formation time \( \tau_0 \) as 0.15 fm/c, we can consider two extremes for the jet-jet interactions as indicated earlier. For one extreme we consider that jet-jet interaction continues till jets reach the surface of the system (\( \tau_f \approx R_T/c \)). The other extreme could be to assume that it operates only till the time of thermalization, if it is large (\( \tau_f \approx 0.5 \) fm/c), so that it is considered as the pre-equilibrium contribution [27]. In any case the time integration for this contribution reduces to ln(\( \tau_f/\tau_i \)), (see Eq. 16) and thus one can easily obtain this results for any choice of initial conditions.

Our results for this contribution at RHIC are similar in magnitude and form to those reported by Lin and Gyulassy [20] for RHIC energies. Recall that we consider only partons having \( p_T > 2 \) GeV as constituting the jets. The jet-thermal contribution is seen to be comparable to the contribution of the jet-jet interaction.

We find that the contribution of thermal production at large \( p_T \) is rather small. However it is larger by a factor of about 3 at lower \( p_T \), compared to the jet-jet and jet-thermal contributions. This has its origin in large initial temperature for low \( \tau_0 \) value assumed here and exclusion of partons having \( p_T < 2 \) GeV in the jet distribution.
The contribution obtained by assuming free-streaming partons having same initial $\tau_0$ and operating till $\tau_f = R_T/c$ is quite similar to thermal contribution. We also note that both thermal and free-streaming contributions drop a lot more rapidly for larger $p_T$ compared to contributions initiated by prompt and jet interactions. The latter contribute at a level of about 4-5 % for large $p_T$ as compared to the initial fusion for $\tau_0 = 0.15$ fm/$c$ and $T_0 = 447$ MeV. We add that a much larger initial temperature of 700 MeV and $K=4$ has been assumed in the calculations by Liu and Fries [24] for the jet conversion in thermalized medium.

In Fig.2, we give results obtained by using the formation time of the plasma as 0.5 fm/$c$, and further assuming that the jet-jet interaction is in operation during $\tau \approx 0.15$ fm/$c$ to 0.5 fm/$c$, so that it can be considered as a pre-equilibrium contribution [27]. In any case, we find that initial fusion gives the largest contribution to charm production, followed by jet-jet and jet-thermal interactions. At high $p_T$, jet-jet as well as jet-thermal interactions give a contribution of about 1% of the initial fusion. Now the thermal and free-streaming contribution though remaining similar in magnitude, are much smaller and also fall more rapidly with $p_T$. They give a contribution at a level of 1% of the initial fusion contribution at $p_T < 3$ GeV.

We realize that even though small in magnitude, the jet-jet and jet-thermal contributions may still give a discernible feature to the back-to-back correlation of charm-quarks at RHIC. The correlation of charm quarks from these should be distinct from initial fusion (back-to-back for LO contribution and forward peaked for NLO contribution). This aspect is under study.

We have not given our results for bottom production at RHIC energies here as we have found that bottom production from the jet-jet mechanism is less than 3 orders of magnitude, the thermal mechanism is less than 6 orders of magnitude and the jet-thermal mechanism is less than 4 orders of magnitude of that obtained from initial (prompt) interaction [9]. We do add, however, that the decay of bottom into charm quarks adds an interesting richness to the study of charm production as well as charm-correlation.

4.2. Charm and bottom at LHC

Next we discuss our results for charm and bottom production at LHC.

We consider central collision of lead nuclei at $\sqrt{s} = 5500$ AGeV. Other initial conditions have already been discussed. We find that (see Fig.3) the charm production from initial fusion is about a factor of 10 or more than that at RHIC, and of-course its fall with $p_T$ is considerably slower, as one would expect.

We plot our results at LHC, taking time of formation of QGP to be 0.07 fm/$c$. We have assumed the initial time of jet-jet interaction to be from $\tau_i < 0.1$ fm/$c$ until the jets reach the surface of the system, $\tau_f \approx R_T/c$ as one of the extremes. We find jet-jet contribution even exceeds the initial fusion for $p_T > 2$ GeV. Of-course since we include jets having $p_T > 2$ GeV, this contribution decreases for lower $p_T$. We shall return to
this. The thermal production of the charm, taking advantage of the initial temperature when $\tau_i \approx 0.07 \text{ fm}/c$, is almost 40% of the initial fusion contribution at low $p_T$, and drops to about 5–10% of the same at $p_T \approx 5 \text{ GeV}$. The jet-thermal contribution is also seen to be considerable at large $p_T$, though smaller than jet-jet contribution. It is seen to be comparable to the thermal contribution at lower $p_T$. Of-course this has its origin in the large initial temperature.

Next in Fig.4, we plot our results at LHC for another set of initial conditions. Taking time of equilibration to be 0.5 fm/c, we have pre-thermal charm production resulting from jet-jet interactions starting at $\tau_i = 0.07 \text{ fm}/c$ and operating till the time of thermalization. Jet-jet interaction gives a contribution which is comparable to that from initial fusion. Once again the decrease at lower $p_T$ occurs due to the exclusion of jets having $p_T < 2 \text{ GeV}$. We also note that for $p_T \sim 8 \text{ GeV}$, the jet-jet interaction contributes at a level of 10% of the initial fusion. The jet-thermal contribution on the other hand starts at about 20% of the initial fusion contribution at the lowest $p_T$ and rises to about one-third of initial contribution for larger $p_T$. The thermal and the free-streaming contributions are seen to be quite small for larger $p_T$, though around $p_T \approx 8 \text{ GeV}$, they contribute at a level of 10% of the initial fusion.

The thermal and the free-streaming contributions shown in Figs. 3 and 4 deserve more attention. We see that the two contributions for larger initial temperatures at LHC differ by a factor of about 2 or more, while for the smaller initial temperature at RHIC they are of similar magnitude. This we feel has its origin in the large initial temperature which enhances the phase-space contribution to the thermal production of charm quarks. In fact in Figs. 5 and 6, where we plot the contributions for the bottom quarks production at LHC, this difference again shows up.

Coming back to our earlier observation about the contribution of these processes to the back-to-back correlation of charm quarks, we realize that the large contribution from processes like jet-jet, jet-thermal and even thermal production of charm quarks at LHC is likely to drastically alter the conclusions about this which were arrived at from studies [10, 11] which invoked only the initial fusion processes.

As indicated earlier, we give our results (see Figs. 5 and 6) for the production of bottom quarks at LHC for two initial conditions discussed earlier. Of-course we find that the thermal production of bottom quarks is quite negligible at LHC. The jet-jet contribution is seen to be at a level of 2–6% of the initial fusion. The jet-thermal contribution is also negligible.

Thus we feel that a high statistics data may still be able to discern the contribution of processes other than initial fusion to the back-to-back correlation. We shall report this in our future publications.

5. Summary and Conclusions

We have reported results on charm production at RHIC and LHC and bottom production at LHC from initial fusion and multiple scattering processes treated as jet-
jet interaction and thermal production and passage of jets through QGP. Two different initial conditions, one with early thermalization and other with thermalization at $\tau_i \approx 0.5 \text{ fm}/c$ have been used. Substantial production of charm, specially at LHC, is seen in addition to the production due to initial fusion.

The jet-jet contribution to charm quarks at LHC even exceeds the initial fusion contribution for intermediate $p_T$ for suitable initial conditions. We have argued that since back-to-back correlation for charm quarks from the processes under consideration could be different from those from initial fusion, the results for that, specially at LHC would be considerably affected. This may even be discernible for back-to-back correlation of bottom quarks at LHC. As indicated earlier, these results will be published shortly.

One could think of several improvements. The obvious one would be to include energy loss suffered by the interacting gluons and quarks before fusion and those by the heavy quarks after production. We feel that the inclusion of energy loss before fusion may have smaller effect on over-all production of the heavy quarks as these would be limited to the earliest times when the momenta and the temperature are still very large. The energy loss suffered by the heavy quarks after the production can not be neglected (see e.g., Ref. [9]). This will alter the $p_T$ distribution of the heavy quarks. However, as it will affect the heavy quarks produced by all the processes, their relative contributions will remain largely unaffected. Relaxing the condition of chemically equilibrated plasma [27] will also be useful. It should be of interest to, alternatively, estimate the prompt charm production using the colour glass condensate model which predicts different scalings for different rapidities [36].

A more complete calculation using parton cascade model [37] will be reported shortly (see also, Ref. [38]).

We conclude that production of charm quarks at LHC and even at RHIC from processes other than initial fusion can be large and can play a significant role in our study of back-to-back correlation. This will have important implications for the study of the nuclear modification factor $R_{AA}$ as well as large mass dileptons having their origin in the correlated charm decay [12, 28].

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