Analysis and Optimization of a Pulse Repetition Frequency for Along-Track Distributed Multiple-Input Multiple-Output Synthetic Aperture Radar

Li Pengcheng, Sun Zaoyu*, He Feng, Zhang Yongsheng

School of Electronic Science, National University of Defense Technology, Changsha, Hunan 410073, China
lipengcheng18@nudt.edu.cn

Abstract. A distributed multiple-input multiple-output synthetic aperture radar (MIMO-SAR) system combines azimuth multi-channel and digital beamforming technologies to produce data with a high-resolution azimuth and wide swath. The MIMO-SAR system uses a tight along-track formation, and the spacing between its satellites is typically 100 m. The space sampling characteristic is sensitive to changes in pulse repetition frequency (PRF). This paper presents a method for optimizing the PRF of a distributed MIMO-SAR system based on the performance of the latter. The optimized PRF adapts well to variations in satellite spacing. These results show that PRF optimization can effectively improve the performance of the distributed MIMO-SAR system.

1. Introduction
Distributed satellites are arranged in a compact formation along the track [1], a number of receiving channels are placed on different satellite platforms [2], and several small satellites simultaneously transmit signals with a low pulse repetition frequency (PRF). Further, several channels receive echo signals, and when combined with digital beamforming technology [3], they process data received on multiple channels, thereby eliminating azimuth ambiguity and obtaining high-resolution wide mapping band imaging. A distributed multiple-input multiple-output synthetic aperture radar (MIMO-SAR) system can receive and send signals separately; the channels are typically spaced 100 m apart. The PRF offset directly affects the spatial sampling characteristics of the SAR signals. To process SAR signals, the 100 m intervals at which the channels are spaced must be equivalent to the distance between sampling points, which are intervals of several meters, and the PRF deviation must be magnified tens of times during the equivalent processing. There is no equivalent of the latter for single-satellite multi-channel SAR as their channels are spaced at a distance of only more than a few meters.

Some studies [4] have established models for the influence of PRF error from a distributed SAR system on along-track interferometry phases and proposed the precision required to optimize the PRF; for example, the German Aerospace Center has proposed the concept of variable PRF SAR [5] [6]. Other studies [7] have focused on selecting PRFs that are variable and can be used in azimuthal multi-carrier MIMO-SAR systems. Several researchers [8] have analyzed the antenna length and PRF selection in azimuth multi-beam SAR systems, while others [9-10] have analyzed the influence of PRF selection on imaging performance under azimuth multi-channel high-resolution SAR systems; some of the latter have proposed PRF optimization methods for azimuth and width measurements [10]. Still other works [11-12] have presented the constraints on PRF selection in traditional SAR systems. Current PRF...
optimization methods for SAR systems are mainly used in single-satellite multi-channel SAR systems and are not suitable for use in distributed MIMO-SAR systems, where the characteristics of spatial sampling are sensitive to the PRF. To resolve this, PRF selection must be optimized as an optimized PRF can adapt to changes in spacing and improve system performance.

2. Distributed multiple-input multiple-output synthetic aperture radar (MIMO-SAR) system model

2.1. Formation configuration

Small satellites follow a compact formation configuration along a given heading and have the exact same sub-astral point trajectory. By controlling the difference in right ascension of the ascending node to overcome the influence of the Earth’s rotation, and there is the same ground track[13], and controlling the difference in the angle of the flat approach results in control of the distance between small satellites along a heading; together, these processes can be used to retain a compact formation along the given heading. All the orbital elements are the same except for slight differences between the right ascension of the ascending node and angle of the flat approach. To avoid collisions between adjacent small satellites, the critical distance between any two small satellites is maintained at 100 m [14].

This is the solution of the Hill equation for the formation configuration along a course [13].

\[
\begin{align*}
\dot{x}(t) &= 0 \\
y(t) &= a(\Delta M + \Delta \Omega \cos i) \\
z(t) &= \frac{n_e}{n} \sin i \cdot a \Delta M \cdot \cos nt. \\
\dot{\tilde{x}}(t) &= 0 \\
\dot{\tilde{y}}(t) &= 0 \\
\dot{\tilde{z}}(t) &= -n_e \sin i \cdot a \Delta M \cdot \sin nt.
\end{align*}
\]

where \( n_e \) is the angular velocity of the rotation of the Earth, \( n \) is the average angular velocity of the reference satellite, \( a \) is the semimajor axis, \( \Delta M \) is the difference in mean anomaly at epoch, \( \Delta \Omega \) is the difference in longitude of the ascending node, and \( i \) is the orbital inclination.

2.2. Signal model

A distributed MIMO-SAR system uses a low PRF to transmit a wide beam to multiple satellites to receive the echo signal simultaneously. This study uses the “two transmissions and four receivers” mode as an example, as shown in Figure 1. T 1 and T 2 represent the satellites launched, with T 1 denoted the reference satellite. Four small satellites receive the echo signal simultaneously and separate the echo at the receiving end. The \( m^{th} \) receiving satellite is denoted by \( R_m \) and the \( m^{th} \) receiving satellite is denoted \( x_m \) relative to the heading distance of the reference satellite.

Assuming that the signal for the launch of a small satellite is \( a(t) \exp(j2\pi f_c t) \) and \( W_m(t) \) represents the two-way antenna pattern. Therefore, assuming that the antenna patterns of each small satellite are identical, \( (x, y, z) \) can represent the position of the ground scattering point \( P \), \( \tau \) represents the fast time variable, and \( t \) represents the slow time variable. The signal received by the \( m^{th} \) satellite can then be expressed as follows:

\[
Z_m(t, \tau) = \iiint_{(x,y,z)} W_m(t) \cdot a \left( \tau - \frac{2}{c} R_m(t) \right) \exp \left( j2\pi f_c (\tau - \frac{2}{c} R_m(t)) \right) dx dy dz, \quad (3)
\]

where

\[
R_m(t) = \sqrt{\frac{(vt - y)^2 + x_m^2 + z_m^2}{2} + \frac{(vt + x_m - y)^2 + x^2 + z^2}{2}}. \quad (4)
\]

When equation (4) is expanded at \( t = x/v \), a second-order approximation similar to the following is obtained:

\[
R_m(t) \approx \sqrt{\left( vt + \frac{x_m}{2} - y \right)^2 + x^2 + z^2 + \frac{x_m^2}{8R_0}}. \quad (5)
\]
Figure 1. Space geometry of the distributed satellite MIMO-SAR system along the course.

After demodulating the echo signal, the following is obtained:

\[ z_m(t, \tau) = \iiint W_m(t) a \left( \tau - \frac{R_m(t)}{c} \right) \exp \left( -j \frac{4\pi}{\lambda} R_m(t) \right) \exp \left( -j \frac{\pi^2}{2\lambda R_0} \right) dx dy dz \]

\[ = A(t) S(t) + N(t). \]  

(6)

The least squares method is then used to construct and obtain filter groups as follows:

\[ W(t) = A(t) \left( A(t)^H A(t) \right)^{-1}. \]  

(7)

As the spacing between small satellites is in the order of 100 m, it is necessary to multiply the echo signal by the constant phase term \( \exp \left( j \frac{\pi^2}{2\lambda R_0} \right) \) to compensate for the additional phase term before the signal is reconstructed.

3. Analysis of pulse repetition frequency (PRF) parameters for the distributed MIMO-SAR system

The spatial sampling characteristics of the distributed MIMO-SAR system are shown in Figure 2. Assuming an adjacent satellite interval \( D \), uniform sampling can be achieved when the PRF and spacing between the satellites satisfy the following relationships:

\[ k \frac{v}{PRF} = m \frac{D}{2}, \quad \frac{v}{PRF} = \frac{2k\pi}{m} \]

where the uniform sampling PRF is

\[ PRF_{uni} = \frac{2k\pi}{m}, \quad \frac{v}{PRF} = \frac{2k\pi}{m} \]

and \( m \) is the number of small satellites.

When the sampled point coincides with the second equivalent phase center, the equivalent sampled point must coincide. The relationship between the coincidence of the sampled point and \( \tau \) is given below.

\[ \frac{x\nu}{PRF} = \frac{D}{2}, \quad (\alpha \in \mathbb{Z}^+) \]  

(10)

\[ \text{PRF} = \frac{2x\nu}{D}, (\alpha \in \mathbb{Z}^+) \]  

(11)

Assuming that the coordinates of adjacent small satellites \( Ri \) and \( Ri+1 \) in the orbital plane determined by the Hill equation are \((x_i, y_i, z_i)\) and \((x_{i+1}, y_{i+1}, z_{i+1})\), respectively, the spacing of adjacent small satellites along their heading is expressed as follows:

\[ D_{i,i+1} = \sqrt{(y_{i+1} - y_i)^2}. \]  

(12)

Substituting (1) into (12) results in

\[ D_{i,i+1}(t) = \sqrt{(a(\Delta M + \Delta \Omega))^2}. \]  

(13)
To realize the coincidence of the sub-satellite trajectories of adjacent satellites, the following relationships should be satisfied [14]:

\[
\Delta t = \frac{-\Delta M}{n} = \frac{\Delta \Omega}{n_k} \quad (14)
\]

Substituting (13) and (14) into (8) results in the following:

\[
PRF = \frac{2 \times k}{m} \frac{\nu}{\sqrt{(a \Delta t(n_k-n))^2}}, \quad m \in \{1, 2, \ldots, (2 \times M - 1)\}, \quad M \geq 2, \quad k \in \mathbb{Z}^+ \quad (15)
\]

Thus, the relationship of the transformation between PRF and the distance of the formation satellite is established using equation (15) above.

The spacing between small satellites can be equivalent to \(D - n \cdot \nu / PRF\), where \(\nu\) is the velocity of the small satellites. When \(n\) is high, the equivalent sampling interval is several meters. As shown in Figure 3, increasing the PRF error of 0.1 Hz to 20 times results in a 4.7 m deviation in the sampling interval. Therefore, the spatial sampling characteristic of a distributed MIMO-SAR system is sensitive to the PRF, and the error in the latter deteriorates easily.

![Figure 2. Spatial sampling characteristics of the distributed multiple-input multiple-output synthetic aperture radar satellites.](image)

The 1st PRT
The 2nd PRT
The 3rd PRT
Corresponding equivalent phase center
Equivalent sampling point

![Figure 2. Spatial sampling characteristics of the distributed multiple-input multiple-output synthetic aperture radar satellites.](image)

The 1st PRT
The 2nd PRT
The 3rd PRT
Corresponding equivalent phase center
Equivalent sampling point

Figure 3. Effect of a pulse repetition frequency (PRF) on the formation satellites.

This study uses a PRF that results in an efficient azimuth ambiguity ratio (AASR) and signal-to-noise scaling factor, as per the performance requirement of the system. The initial selection of PRF allows the system to maintain a good performance in a certain range. When the primary PRF is optimized, the PRF that adapts well to changes in satellite spacing is selected; this ensures that the optimized PRF can maintain the good performance required and also improve the anti-jamming ability of the system.

4. Analysis of simulation and optimization
4.1. Reconstruction performance indicators

In a distributed MIMO-SAR, the reconstruction of the azimuth signal affects mainly the azimuth ambiguity ratio (AASR) and SNR. Xi-Le [2] detailed the derivation of the equation for the AASR for a single base azimuth multiple-phase centered SAR system. The AASR formula for a distributed MIMO-SAR system is given as follows:

\[
AASR = \frac{\sum_{m=-N}^{N-1} \int_{-B_P}^{B_P} W^2(f_m + m(2M-1) - PRF) df}{\int_{-B_P}^{B_P} W^2(df)}.
\]  

(16)

In terms of the SNR, more attention is given to the modulation of reconstructed signal. The signal-to-noise scaling factor is defined as the ratio of the input to the output signal to noise ratio as follows:

\[
\phi = \frac{SNR_{in}}{SNR_{out}} = E\left[\|W^H\|^2\right].
\]

(17)

where \(SNR_{in}\) denotes the input SNR and \(SNR_{out}\) denotes the output SNR.

4.2. Analysis of performance simulation

This study simulates and compares the performance of single platform and distributed MIMO-SAR systems; the simulation parameters are shown in Table 1 and the results are shown in Figures 4 and 5. The latter show that the AASR and signal-to-noise scaling factor of MIMO-SAR are stationary, the signal-to-noise scaling factor decreases as the PRF increases and improves gradually and that de-ambiguity is performed well. The AASR of the distributed MIMO-SAR system fluctuates markedly with the change in PRF. The deviation in the PRF often leads to a failure in system reconstruction; However, for some PRFs, the distributed MIMO-SAR system performs better than the single-platform MIMO-SAR system.

Table 1. Simulation parameters of the multiple-input multiple-output synthetic aperture radar (MIMO-SAR) system.

| Parameters             | Single-platform MIMO-SAR | Distributed MIMO-SAR |
|------------------------|--------------------------|----------------------|
| Velocity (m/s)         | 7482                     | 7482                 |
| Receiving channel distance (m) | 1.80                 | 100                  |
| Carrier frequency (GHz) | 9.6                      | 9.6                  |
| Wavelength (m)         | 0.031                    | 0.031                |
| Downcast angle (°)     | 35                       | 35                   |
| Number of sub-pulses   | 2                        | 2                    |
| Reconstruction bandwidth (Hz) | 7600                  | 7600                 |

Figure 4. Azimuth ambiguity ratio results from the simulation.

Figure 5. Signal-to-noise scaling factor results from the simulation.
4.3. PRF optimization

Assuming that four small satellites are formed along the course with a half-length axis of 7121.03 km, an eccentricity of $1.02 \times 10^{-5}$, an orbital inclination angle of 98.36671 degrees, a perigee angle of 0 degrees, and an interval of 100 m between two small satellites, the difference between the angles of the mean proximity points and right ascension points are $1.40 \times 10^{-5}$ degrees and $1.94930 \times 10^{-6}$ degrees, respectively, to overcome the influence of the rotation of the Earth. For the two-body motion model, the approximate cosine variation of the spacing of the satellite in one period is obtained without considering the perturbation effect, as shown in Figure 6.

![Figure 6. Variation in the distance of adjacent satellites within one orbital period.](image)

The PRF with better adaptability to the variation in the satellite spacing is selected based on an analysis of system performance. PRFs are first separated based on the reconstruction performance of the system, after which the PRF that has a signal-to-noise scaling factor of less than 0 dB and an AASR of less than -30 dB is selected.

Taking PRF = 1606 Hz as an example, this study simulated the performance of the optimized SAR system. As shown in Figure 7, the distributed MIMO-SAR AASR fluctuates around -27.2 dB in one orbital period, and the system maintains good reconstruction performance. The signal-to-noise scaling factor varies consistently around -3.0 dB, as shown in Figure 8. The results show that the optimization of PRF can effectively improve the system performance and the anti-jamming ability of the system by changing the spacing between the small satellites.

![Figure 7. Fluctuation of the azimuth ambiguity ratio in the simulation.](image)  ![Figure 8. Fluctuation in the signal-to-noise scaling factor in the simulation.](image)
5. Conclusion
In this study, a PRF selection optimization method aimed for use with a distributed MIMO-SAR is proposed. The method is based on using system performance to improve the system reconstruction performance under a special formation structure. The spatial sampling characteristics of a distributed MIMO-SAR system are sensitive to the PRF. Therefore, the PRF bias is weighted to be amplified when the small satellites are spaced at equal intervals. This sensitivity is verified by comparing the reconstruction performance of both a single-platform MIMO SAR and a distributed MIMO SAR system. A compact satellite formation is designed without perturbation based on the two-body motion model, the orbit parameters are set, and variation in the spacing between adjacent small satellites is then simulated. Under this configuration, the PRF can be optimized effectively using the variation of satellite spacing to improve system performance.

References
[1] J. Ender 2002 Spacebased SAR/MTI using multistatic satellite configurations J.
[2] Wang J, Ding C B and Liang X D 2018 Research outline of airborne MIMO-SAR system with same time-frequency coverage. J. Radars 7220–34
[3] Xi-le, M 2014 Research on high-resolution wide-swath imaging technologies of azimuth multiple phase center SAR. PhD. School of Electronic Science, National University of Defense Technology, Changsha
[4] Zhou W, Liu Y X and Li X 2014 Brief analysis on the development and application of multi-input multi-output synthetic aperture radar J. Radars 3 10–18
[5] Zeng B, Zhang X L and Shi J 2007 The influence of PRF error to velocity measurement in distributed SAR. J. Electron. Inf. Technol., 029 831–35
[6] Luo X, Wang R, Xu W et al. 2014 Modification of multichannel reconstruction algorithm on the SAR with linear variation of PRI. IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens. 7 3050–59
[7] Villano M, Krieger G and Moreira A 2014 Staggered SAR: high-resolution wide-swath imaging by continuous PRI variation. IEEE Trans. Geosci. Remote Sens. 52 4462–79
[8] Yu Y J 2016 Study of imaging method on the SAR with continuous PRF variation. Dissertation. University of Electronic Science and Technology of China, Chengdu
[9] Fan Q, Li G C and Lv X D 2004 Determination of antenna length and pulse repetition frequency in multiple azimuth beam synthetic aperture radar system. Mod. Radar, 26 66–7
[10] Gebert N and Krieger G, 2009 Azimuth phase center adaptation on transmit for high-resolution wide-swath SAR imaging. IEEE Geosci. Remote Sens. Lett., 6 782–86
[11] Gebert N, Krieger G and Moreira A. 2009 Errata: digital beamforming on receive: techniques and optimization strategies for high-resolution wide-swath SAR imaging. IEEE Trans. Aerosp. Electron. Sys., 45 564–92
[12] Cantafio L J 1989 Space-Based radar handbook. (Boston: Artech House)
[13] Cumming, L G and Wong F H 2012 Digital processing of synthetic aperture radar data: algorithms and implementation. ed. Hong Wen, Hu Donghui, Han Bing. (Beijing: Publishing House of Electronics Industry)
[14] Zheng X N, Wang W and Gao Y D D 2003 Fundamentals of near-earth spacecraft orbit. (Changsha: National University of Defense Technology Press)
[15] Wang T and Bao Z 2006 Improving the image quality of spaceborne multiple-aperture SAR under minimization of sidelobe clutter and noise. IEEE Geosci. Remote Sens. Lett., 3 297–301