Study on Residual Displacement of FPB bearing in Long-span Continuous Girder Bridge

Shuai Song¹, Xili Qi¹, Yousheng Qian²*, Liangliang Tan², Lvyang Zhou¹, Yujie Peng¹

¹College of Architectural Engineering, Guizhou Construction Vocational and Technical College, Gui zhou Gui yang 551400
²Civil Engineering College of Guizhou University, Gui zhou Gui yang 550025
*Corresponding author’s e-mail: 414371452@qq.com

Abstract: Based on the principle of mechanical equilibrium, the theory of FPB with residual displacement is analyzed. This analysis derives the stiffness and equivalent damping ratio of FPB with residual displacement, and constructs the hysteretic model with residual displacement according to the existing hysteretic model of FPB. Through midas/civil modeling and analysis, the influence of different friction coefficients on the residual displacement of FPB is studied. When the friction coefficient is small, the distribution of residual displacement of FPB bearings is directional; the positive residual displacement is positively correlated with the friction coefficient, and the negative residual displacement gradually changes to the direction of seismic wave input with the increase of friction coefficient, first decreases and then increases. The positive and negative residual displacements after aftershocks are the same as those after main shocks, and their variation with friction coefficient is related to aftershocks intensity. By analyzing the hysteretic curves of different residual displacement bearings, the correctness of theoretical analysis is proved. At the same time, it is concluded that the larger the residual displacement, the more vulnerable the bearings are to instability, which leads to the failure of the main beam. The study of residual displacement can provide theoretical basis for selecting friction coefficient and controlling residual displacement in FPB design.

1. Introduction
The Friction Pendulum Bearing (FPB) utilizes its unique design to make the friction slider automatically return to the center of the curved surface under its dead load after the earthquake, thus increasing the reliability of the FPB²⁴. However, the self-reset function of FPB is limited, there will be some residual displacements after the earthquake⁵.

Major-aftershock type earthquake accounts for about 30% of earthquakes in China, and aftershocks are often the main factor determining the failure of bridges⁶. After each main shock, a certain amount of residual displacement will be generated for the FPB. When subsequent aftershocks occur, the FPB will be in the working state with residual displacement, which may lead to the destruction of non-structural components passing through the isolation layer, and greatly reduce the seismic isolation performance of the FPB under subsequent aftershocks. The Curved Surface Friction Coefficient of the FPB is the main parameter affecting the residual displacement of the FPB, but its influence law needs to be studied. Taking a long-span continuous girder bridge as an example, the relationship between residual displacement of the FPB and friction coefficient is studied by means of midas/civil finite element software.

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd
2. The opetical analysis of the FPB

2.1. Self-recovery Characteristics

After the earthquake, the FPB is no longer subjected to seismic force, that is, \( F = 0 \). According to the principle of force equilibrium, there is

\[
\frac{W D}{R \cos \theta} = f \cos \theta
\]  

(1)

Usually the “\( \theta \)” is smaller, so there is

\( D = \mu R \)  

(2)

Formula (2) shows that when \( D=\mu R \), the restoring force is balanced with the friction force, the bearing will no longer return to the center, and the bearing will have the largest residual displacement: \( D_r=\mu R \); when \( D>\mu R \), the FPB overcomes friction force and return to its center under the action of its bearing capacity. when \( D<\mu R \), the FPB returns to its center by inertia, so \( D_r<\mu R \). The above analysis shows that the FPB can not be completely reset, and its residual displacement is related to the sliding surface radius and friction coefficient.

2.2. Mechanical Model with Residual Displacement

The FPB with residual displacement can be simplified as a slider moving along the circular sliding surface\[^3,4\]. The radius of the sliding surface is \( R \), the rotation angle of the residual displacement is \( \alpha \), and the corresponding residual displacement is \( d \), as shown in Figure 1.

![Fig.1 Schematic diagram of FPB bearing model](image1)

![Fig.2 hysteresis model of FPB bearing](image2)

In the light of the Force Balance of Slider, when \( \theta + \alpha \) is small, According to formula \( \sum M_c = 0 \), there is:

\[
F = \frac{W}{R} (D - d) + \mu R \text{sgn}(\theta + \alpha)
\]  

(3)

The \( \text{sgn}(\theta + \alpha) \) is Sign function, which can be known as follows:

\[
\text{sgn}(\theta + \alpha) = \begin{cases} 
1, & \theta + \alpha > 0 \\
-1, & \theta + \alpha < 0 
\end{cases}
\]

According to formula (3), the isolation stiffness of the FPB is:

\[
K_{fpb} = \frac{W}{R}
\]  

(4)

Expressions of effective stiffness and effective damping ratio with residual displacement:

\[
K_{eff} = \frac{W}{R} + \frac{\mu W}{D_d - d}
\]  

(5)

\[
\xi_{eff} = \frac{2\mu R}{\pi(D_d - d)}
\]  

(6)
In the formula, W is the dead load of the bearing, R is the radius of the sliding surface, d is the residual displacement, \( \mu \) is the friction coefficient, \( \alpha \) is the rotation angle corresponding to d, and \( D_d \) is the design displacement.

Formulas (4), (5) and (6) show that the existence of residual displacement does not affect the isolation stiffness of the FPB, but has an effect on the effective stiffness and effective damping ratio. In the absence of residual displacement, the hysteretic center of the FPB is at the lowest point of sliding surface; in the presence of residual displacement d, the hysteretic center of FPB bearing is at the lowest point of sliding surface and at the circle of radius of residual displacement D. Compared with the FPB hysteresis model without residual displacement, the FPB hysteresis model with residual displacement moves the distance of residual displacement d to the direction of residual displacement as a whole. As shown in Figure 2, the real line and the dashed line in the figure are respectively the FPB hysteresis model without residual displacement and that with residual displacement.

3. Models and working conditions

3.1 Engineering Example

A bridge is a 3-span prestressed concrete continuous box girder bridge with a span of 70+120+70m. The main beam is a single-box single-chamber box section, the mid-span beam is 3m high, and the pier top beam is 7.5m high. The height of the bottom beam is changed by the secondary parabola. The pier adopts rectangular pier and pile foundation. The seismic fortification intensity is 8 degrees and the site type is II. When building the three-dimensional model of the bridge, the elastic beam element is used in the main girder, considering the plastic hinge effect of the pier, the pile-soil interaction of the elastic support model is used. The bridge model is shown in Fig 3. FPB bearings are arranged according to the bearing arrangement scheme in Figure 4.

There are equivalent friction coefficients, friction coefficients at slow speed and friction coefficients at fast speed in the FPB. An exponential function can be used to describe the relationship between them\(^{[7,8]}\), as follows:

\[
\mu = \mu_f - (\mu_f - \mu_s) \exp^{-\lambda v}
\]  

In the formula, \( \mu \) is the equivalent friction coefficient; \( \mu_f \) is the friction coefficient when the speed is fast; \( \mu_s \) is the friction coefficient when the speed is slow; and \( \lambda \) is the material parameter related to the friction material.

In order to study the influence of the friction coefficient of the FPB on the residual displacement of bearing, assuming that the ratio of \( \mu_f \) to \( \mu_s \) is fixed, \( \mu \) is indirectly changed by changing the value of \( \mu_f \). According to the range of friction coefficient of friction pendulum bearing, the values of \( \mu_f \) are 0.02, 0.04, 0.06, 0.08, 0.1, 0.12 and 0.14, respectively.

3.2 Main and Aftershock Seismic Waves

According to the statistical data of strong aftershocks in 15 years of 115 earthquake sequences from 1800 to 1995 in mainland China, the probability of aftershocks occurring after major earthquakes is
more than 30%, and the relationship between the magnitude of major earthquakes and the magnitude of aftershocks is as follows\textsuperscript{(9)}:

\[
M_{a1} = 0.50M_z + 2.02 \quad (8)
\]
\[
M_{a2} = 0.32M_z + 2.98 \quad (9)
\]

Among them, the unbiased variance estimates of Formula (8) and Formula (9) are respectively \(\sigma_1^2 = 0.24\), \(\sigma_2^2 = 0.21\); \(M_z\) is the main earthquake magnitude; \(M_{a1}\) is the magnitude of strong aftershock 1; \(M_{a2}\) is the magnitude of strong aftershock 2.

The magnitude of an earthquake is related to its epicenter intensity \(I\) as follows\textsuperscript{(10)}:

\[
M = \frac{2}{3}I + 1 \quad (10)
\]

This paper assumes that the intensity of the main shock is the basic fortification intensity of degree VIII, and assumes that strong aftershocks 1 and 2 occur after the main shock. The intensity is determined according to the above-mentioned relationship. On the basis of Elcentro, Taft and Holly waves, three main shock waves and six aftershocks are synthesized artificially according to the three factors of spectrum characteristics, peak value and duration of ground motion, and the frequency characteristics of the main shock and aftershocks are assumed as the same in duration, only considering the variation of the peak value of ground motion. In order to restore the static state of the bridge before the aftershock after the main shock, the free vibration time of 50s is added after the main shock. The synthetic seismic wave is shown in Figure 5-7.

![Synthetic seismic wave](image)

(a) The main earthquake wave  
(b) Aftershock earthquake wave

**Fig. 5 The main and aftershock earthquake wave**

### 4. Research on analysis results

#### 4.1 Effect of Friction Coefficient on Residual Displacement

According to the restoring force characteristics of the FPB, the friction coefficient has a great influence on the residual displacement. After the main shock, the residual displacements corresponding to different friction coefficients are shown in Table 1. The relationship between residual displacements and friction coefficients is shown in Figure 6.

| Friction Coefficient | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 |
|----------------------|------|------|------|------|------|------|------|
| Bearing Number       |      |      |      |      |      |      |      |
| 1#                   | 9.29 | 4.05 | 8.09 | 10.88| 21.96| 24.71| 28.67|
| 3#                   | 14.46| 8.74 | 13.37| 16.08| 27.53| 31.32| 34.30|
| 5#                   | -15.19| -20.40| -16.48| -13.68| -2.725| -0.44| 3.94 |
| 7#                   | -7.87 | -13.05| -9.06 | -6.21| 4.82 | 7.61 | 11.48|

**Tab. 1 Residual displacement**
Tables 1 and Figure 6 show that when the friction coefficient is less than 0.1 and the bearing is symmetrical to the middle of the bridge span, the residual displacement along the direction of seismic wave input is opposite to the sign in the symmetrical bearing. The residual displacement is positive first, then negative, indicating that the residual displacement is directional. The positive residual displacement is positively correlated with the friction coefficient, and increases with the increase of friction coefficient; the negative residual displacement is inversely correlated with the friction coefficient, and changes along the direction of seismic wave with the increase of friction coefficient; when the friction coefficient is less than 0.04, the positive residual displacement decreases with the increase of friction coefficient, while the negative residual displacement increases along the reverse direction of seismic wave.

4.2 Residual Displacement After Aftershocks

After the main shock, the FPB can not be completely restored, leaving residual displacement. On this basis, the variation of residual displacement of the FPB after different aftershocks is further studied. In order to save space, only the residual displacement of 3# bearing and 5# bearing under different aftershocks is listed in Table 2. The variation of residual displacement of other bearings is shown in Figure 7.

| Friction Coefficient | Bearing 3# (4#) | Bearing 5# (6#) |
|----------------------|----------------|----------------|
|                      | Aftershock 1   | Aftershock 2   | Aftershock 1 | Aftershock 2 |
| 0.02                 | 12.52          | 15.89          | -17.54       | -13.73       |
| 0.04                 | 15.86          | 16.90          | -13.75       | -12.44       |
| 0.06                 | 19.35          | 14.72          | -10.47       | -14.56       |
| 0.08                 | 21.01          | 13.79          | -8.96        | -15.32       |
| 0.1                  | 19.91          | 19.02          | -9.82        | -11.1        |
| 0.12                 | 10.40          | 25.35          | -18.8        | -4.96        |
| 0.14                 | 5.12           | 32.69          | -23.7        | 1.93         |

In Table 2 and Figure 7, when the friction coefficient is less than 0.08, the variation of residual displacement with friction coefficient after aftershock 1 is similar to that after main shock, and the residual displacements of 1# and 3# bearings are larger than that after main shock, and the residual displacements of 5# and 7# bearings are smaller than that after main shock. The variation law of
residual displacement with friction coefficient after aftershock 2 is opposite to that of aftershock 1, but the residual displacements of 1# and 3# bearings are still larger than that after main shock, and the residual displacements of 5# and 7# bearings are still smaller than that after main shock; when friction coefficient is greater than 0.08, the variation law of residual displacement after aftershock is opposite to that when friction coefficient is less than 0.08.

4.3 Research on Hysteretic Model

In order to verify the correctness of the hysteretic model with residual displacement, the hysteretic curves of each bearing under aftershock 1 are drawn with friction coefficient of 0.06 as an example, as shown in Figure 8.

According to 3.1 analysis, when the friction coefficient is 0.06, the residual displacement of each bearing is different after the main shock, and the hysteretic curve under the action of aftershock 1 is shown in Figure 8. In Figure 8, the hysteretic curves corresponding to different residual displacements are different. The larger the residual displacement, the larger the moving distance of the hysteretic curves. However, the hysteretic stiffness of each hysteretic curve is close to the absolute value of the positive maximum displacement and the negative maximum displacement. This shows that the hysteretic curves after the residual displacement is in good agreement with the theoretical analysis. Therefore, the larger the residual displacement is after the main shock, the more likely the bearing is to lose stability and cause beam falling damage. When designing the FPB, the influence of the residual displacement of the bearings on the FPB should be considered, and corresponding measures should be taken to reduce the residual displacement.

5. Conclusion

In this paper, a long-span continuous beam bridge with 8 FPBs is taken as an example to study the variation of residual displacement of the FPB with friction coefficient by means of finite element software. The results shows that:

- After the main shock, when the friction coefficient of the FPB is less than 0.1, the residual displacement has directionality, and whether it’s positive or negative is related to the direction of seismic wave. The sign of residual displacement along the direction of seismic wave input is opposite in the symmetrical bearings. Bearings that are subjected to seismic action first have positive residual displacement, and bearings that are subjected to seismic wave action later have negative residual displacement.

- After the main shock, the positive residual displacement is positively correlated with the friction coefficient, and increases with the increase of friction coefficient; the negative residual displacement is inversely correlated with the friction coefficient, and changes along the direction of seismic wave with the increase of friction coefficient, from negative residual displacement to positive residual displacement.
Positive and negative residual displacements after aftershocks are the same as those of main shocks, and their variation law with friction coefficient is related to aftershocks intensity. When the aftershocks intensity is large and the friction coefficient of the FPB is small, the residual displacement of the FPB is larger than that after main shock, and the variation law is similar to that after main shock. With the increase of friction coefficient, the variation law is opposite to that after main shocks. When the intensity of aftershock is small and the friction coefficient is small, the residual displacement value of aftershock is larger than that after the main shock, and the variation law is opposite to that of aftershock when the intensity of aftershock is large.

When the friction coefficient is small, the positive residual displacement is inversely correlated with the negative residual displacement. When the friction coefficient is large, the negative displacement will decrease. When the friction coefficient is large, it will no longer be the same. In order to reduce the residual displacement, the smaller friction coefficient should be chosen in the design and application of the FPB.

The hysteretic curve with residual displacement deduced by the theory is in good agreement with the model analysis, which shows the correctness of the theoretical deduction. At the same time, the hysteretic curve with residual displacement shows that the residual displacement has a great influence on the hysteretic curve of the FPB. The larger the residual displacement is, the more easily the FPB will lose stability, which will lead to the failure of the main beam.

References

[1] Liu Feng, Tao Shijun et al. Seismic isolation design method of continuous beam bridge based on friction pendulum bearing[J]. Highway, 2015, 111(6): 111-116
[2] WANG Kehai, LI Chong. Seismic design method for medium and small span bridges considering bearing friction slip[J]. Engineering mechanics, 2014, 31(6): 85-92
[3] XIA Xiushen, WANG Xihe et al. Effect of Input Ground Motion Dimension on Seismic Response of FPB Isolation Curved Bridge[J]. Journal of Highway and Transportation Research and Development, 2012, 29(11): 69-74
[4] LIU Jun, WANG He-xi. Application of double spherical seismic isolation bearing in a rigid frame—continuous girder bridge[J]. Journal of railway science and engineering, 2012, 9(3): 117-123.
[5] Gong jian, Deng xuesong. Study on theoretical analysis numerical simulation of friction pendulum bearing[J]. Journal of disaster prevention mitigation engineering, 2011, 31(1): 56-62
[6] Chen Yan-jiang, Hao Chao-wei. Seismic Vulnerability Analysis of Continual Rigid-bridge with High Piers Considering Mainshock-aftershock Earthquake Sequences[J]. Earthquake Resistant Engineering and Retrofitting, 2015, 37(4): 132-146
[7] JT/T 852-2013, Friction pendulum seismic isolation bearing for highway bridges[S]. 2013
[8] Zhuang Junsheng. Bridge vibration isolation, base isolation bearing and device[M]. China Railway Publishing House, 2012
[9] WANG Jianqiang, YOU Qingyan, DING Yonggang, LI Dawang. Influence of friction coefficient of bearing on seismic response of base-isolated structure with friction pendulum system[J]. World Earthquake Engineering, 2012, 28(2): 98-102.
[10] LIU Ai rong, XIONG Ren. Deformation and energy seismic damage model considering the impact of strong after shock[J]. Journal of shenzhen university science and engineering, 2011, 18(3): 189-194