VISUALIZING 2D QUANTUM FIELD THEORY:
GEOMETRY AND INFORMATICS OF MOBILE VISION

To the Bicentenary of N.I.Lobachevskii.

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Abstract. This article is devoted to some interesting geometric and informatic interpretations of peculiarities of 2D quantum field theory, which become revealed after its visualization.

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The present research note, being addressed to specialists in the modern quantum field theory as well as in the computer graphics and the scientific visualization, is an attempt (in my best estimations only suggestive or rather discussion) to describe some peculiarities of two-dimensional quantum field theory becoming revealed after its visualization. The problem of visualization is considered from both geometric and informatic points of view, which are in some sense complementary to each other. Our crucial example is Mobile Vision (a certain "anomalous virtual reality" "naturalizing" 2D quantum projective (sl2;C)(invariant) field theory (precise definitions of the concepts of anomalous virtual reality and the naturalization see below); so the theme of the article is geometry and informatics of Mobile Vision.

The text is organized as follows; it includes two paragraphs: in the first one the visualized peculiarities of 2D quantum field theory are described in geometric terms, in the second (in informatic ones); of course, both paragraphs are interrelated by computational constructions (although considered from rather different points of view) in accord with the thought that "Die Dualität der Natur".

1. GEOMETRY OF MOBILE VISION

It is well-known that many geometric discoveries were motivated by preceding exploration of various natural phenomena. It seems that not less interesting geometric constructions may appear after investigations of artificial (in an (made) system; especially, of computer ones, which are partially based on modelling of a class of the most intriguing natural phenomena.
1.1. Interpretative geometry and anomalous virtual realities [1].

Interpretative geometry is a certain geometry related to interactive computer graphical psycho-kinematic systems. Mathematical data in such systems exist in the form of an interrelation of an interior geometric image (figure) in the subjective space of observer and an exterior computer graphical representation; the least includes the visible elements (draws of figure) as well as of the invisible ones (e.g., analytic expressions and algorithms of the constructing of such draws). Process of the corresponding of a geometric image (figure) in the interior space of observer to a computer graphical representation (visible and invisible elements) is called translation; the visible object may be non-identical to the figure, e.g., if a 3-dimensional body is defined by an axonometry, in three projections, cross-sections or cuts, or in the window technique, which allows to scale up a concrete detail of a draw (which is a rather pityy operation for the visualization of fractals), etc., in this case partial visible elements may be regarded as modules, which translation is realized separately; the translation is called by interpretation if the translation of partial modules is realized depending on the result of the translation of preceding ones.

Definition 1. A figure, which is obtained as a result of the interpretation, is called interpretational figure; the draw of an interpretational figure is called symbolic.

The computer (geometric) description of mathematical data in interactive information system is deeply related to the concept of the anomalous virtual reality. It should be mentioned that there exist several approaches to foundations of geometry: in one of them the basic geometric concept is a space (a medium, a field), geometry describes various properties of a space and its states, which are called the draws of figures; it is convenient to follow this approach for the purposes of the describing of geometry of interactive information system; the role of the medium is played by an anomalous virtual reality, the draws of figures are its certain states.

Definition 2.

A. An anomalous virtual reality (AVR) in a narrow sense is a certain system of rules of non-standard descriptive geometry adopted to a realization on videocomputer (or multisensor system of "virtual reality" [2,3]); an anomalous virtual reality in a wide sense contains also an image in the cyberspace made accordingly to such system of rules; we shall use this term in a narrow sense below.

B. Naturalization is the corresponding of an AVR to an abstract geometry or a physical model; we shall say that the AVR naturalizes the model and such model transcends the naturalizing AVR. Visualization in a narrow sense is the corresponding of certain images or visual dynamics in the AVR to objects of the abstract geometry or processes in the physical model; visualization in a wide sense also includes the preceding naturalization.

C. An anomalous virtual reality, whose images depends on an observer, is called intentional anomalous virtual reality (IAVR); generalized perspective laws in IAVR contain the equations of dynamics of observed images besides standard (geometric) perspective laws; a process of observation in IAVR contains a physical process of observation and a virtual process of intention, which directs an evolution of images accordingly to dynamical laws of perspective.

In the intentional anomalous virtual reality objects of observation present themselves being connected with observer, who acting on them in some way, determines their observed states, so an object is thought as a potentiality of a state from the defined spectrum, but its realization depends also on observer; the symbolic draws of interpretational figures are presented by states of a certain IAVR.

It should be mentioned that a difference of descriptive geometry of computer graphical information system from the classical one is the presence of colors as important bearers of visual information; a reduction to shape graphics, which is adopted in standard descriptive geometry, is very inconvenient, since the use of colors is very familiar in the scientific visualization [4(6)]. The approach to the computer graphical interactive information system based on the concept of anomalous virtual reality allows to consider an investigation of structure of a color space as a rather pityy problem of descriptive geometry, because such space may be much larger than the usual one and its structure may be rather complicated.
1.2 Quantum projective field theory and M obile vision [1,7,8].

It seems to be a significant fact that 2D quantum field theory may be expressed in terms of interpretational geometry, so that various objects of this theory are represented by interpretational guises. Our crucial example is M obile vision (M V). M V is an IAVR naturalizing the quantum projective field theory (Q PFT; [8] and refs therein); the process of naturalization is described in [7,1]; its key points will be presented below, here our attention is concentrated on the basic concepts of the Q PFT, which naturalization M obile vision is.

Definition 2D. A set of continuously distributed visual characteristics of an image in an anomalous virtual reality is called anomalous color space; elements of an anomalous color space, which have non-color nature, are called overcolors, and quantities, which transcendize them in the abstract model, are called "latent lights". Color (perspective) system is a fixed set of generalized perspective laws in an anomalous color space; two AVRs are called equivalent if their color (perspective) systems coincide; an AVR, which is equivalent to one realized on the video computer, is called marginal.

Definition 3A. Q PFT (operator algebra of the quantum field theory, vertex operator algebra) is the pair \((H; t^k_{ij}(x); H)\) is a linear space, \(t^k_{ij}(x)\) is the (valued tensor field such that \(t^k_{lm}(x) t^l_{jk}(y) = t^k_{mj}(x y) t^l_{mk}(y)\).

Let us introduce the operators \(L_v (e_i)_j = t^k_{ij}(x) e_k\), then the following relations will hold: \(L_v (e_i)_j = t^k_{ij}(x) L_v (e_k)\) (operator product expansion) and \(L_v (e_i)_j = L_v (e_k)_j\) (duality). A loo an arbitrary Q PFT (operator algebra one can define an operation depending on the parameter \(m\), \(e_i e_j = t^k_{ij}(x) e_k\); for this operation the following identity holds: \(m_x (i; f(\quad j;)) = m_y (m \quad y (\quad;) ;\); the operators \(H(x)\) are the operators of the left multiplication in the obtained algebra.

Definition 3B. Q PFT (operator algebra \((H; t^k_{ij}(u); u C)\) is called (derived) Q PFT (operator algebra) if \((H)\) is the sum of Verma modules \(V\) over \(sl(2; C)\) with the highest vectors \(v\) and the highest weights \(\lambda_k (v)\) is a prim ary field of spin \(h\), i.e. \([L_k; \lambda_k (v)] = (u)^k (u^1 + k + 1)h \lambda_k (v)\), where \(L_k\) are the \(sl(2; C)\) generators \([L_i; L_j] = (i j) L_{i+j}; L_0 = 1; O; 1\). (3) the (derived) rule of descendants generation holds \([L_1 \lambda_1 (f)] = \lambda_1 L_1 (f)\). Derived Q PFT (operator algebra \((H; t^k_{ij}(u))\) is called projective \(G\) (hyper multiset), i.e. the group \(G\) acts \(\text{by automorphism}\) on other sets, the space \(H\) possesses a structure of the representation of the group \(G\), the representation operates with the action of \(sl(2; C)\) and \(L_1 (g f) = T (g) L_1 (f) T (g^{-1})\).

The linear spaces of the highest vectors of the derived weight form subrepresentations of \(G\), which are called multilets of projective \(G\)-hyper multiset.

As it was mentioned above M V is a certain AVR which naturalizes the Q PFT. Possibly, M V is not its unique naturalization in M V. Unless the abstract model (Q PFT) has a quantum character the images in its naturalization (M V) are classical; the transition from the quantum field model to classical one is done by standard rules [16], namely, the classical field with Taylor coefficients \(a_k\) is corresponded to the element \(a_k L_k^{\text{}}\text{\, of the Q PFT (operator algebra). Under the naturalization these classical fields are identified with fields of three basic colors (red, green and blue), other fields with fields of overcolors; there are pictured only the color characteristics for the xed moment of time on the screen of the video computer as well as the perception of the overcolors by an observer is determined by the intentional character of the AVR of M obile vision. Nam ely, during the process of the evolution of the image, produced by the observation, the vacillations of the color fields take place in accordance to the dynamical perspective laws of M V (Euler formulas or Euler (Mold equations). These vacillations depend on the character of an observation (i.e. an eye movement or another dynamical parameters); the vacillation of image depends on the distribution of the overcolors, that allow to interpret the overcolors as certain vacillations of the ordinary colors. So the overcolors of M V are vacillations of the xed type and structure of ordinary colors with the dependence on the parameters of the observation process; the vacillating "latent lights" are the quantized fields of the basic model of the Q PFT.

\[ ~x \]
\[ ~y \]
\[ ~z \]
The presence of the SU(3) symmetry of classical color space allows us to suppose that the QFT (operator algebra) of the initial model is the projective SU(3) (hypers multil et).

1.3. Quantum conform al and \( q_8 \) (conform al eld theories; quantum ( eld analogs of Euler (A mold tops [L,8]).

**Definition 4A [9-11].** The highest vector \( T \) of the weight 2 in the QFT (operator algebra) will be called the conform al stress-energy tensor if \( T(u) = l_2(T) = 1_k(u)^k \), where the operators \( L_k \) form the V irasoro algebra: \( L_l;L_j = (i+j)L_{l+j} + \frac{j}{12}c \). The set of the highest vectors \( J \) of the weight 1 in the QFT (operator algebra) will be called the set of the a ne variables \( J(\ u) = J_k(u)^k \), where the operators \( J_k \) form the a ne Lie algebra: \( [J_l;J_j] = cJ_{l+j+k} \).

If there is de ned a set of the a ne variables in the QFT (operator algebra) then one can construct the conform al stress-energy tensor by use of Sugawara construction or moreover generally by use of the Virasoro master equations. Below we shall be interested in the special de nitions of the quantum conform al eld theories (nam ely, ones with central charge \( c = 4 \)) in class of the quantum projective ones, which will be called quantum \( q_8 \) (conform al eld theories; the crucial role is played by so-called Lobachevskii algebra in their constructions. In the Poincare realization of the Lobachevskii plane (the realization in the unit disk) the Lobachevskii metric may be written as \( a = q_8^{-1} dzd\overline{z} + (q_4(z,j))^2 \); one can construct the C algebra (Lobachevskii algebra), which may be considered as a quantization of such metric, namely, let us consider two variables \( t \) and \( t \), which obey the following commutation relations: \( [t; t] = 0, [t; t] = q_8 (l \ t \ t) \) (or in an equivalent form \( \{s; s\} = 0, [s; s] = (l \ q_8 ss) \), where \( s = (q_8)in \); one may realize such variables by bounded operators in the Vem a module over a sl(2,C) of weight \( h = q_8^{-1}+1 \); if such Vem a module is realized in polynomials one complex variable \( z \) and the action of sl(2,C) has the form \( L_1 = z, L_0 = z\theta_2 + h, L_{-1} = z\theta_2^2 + 2h\theta_2 \), then the variables \( t \) and \( t \) are represented by tensor operators \( D = \theta_2 \) and \( F = z(2\theta_2 + 2h) \). These operators are bounded if \( q_8 > 0 \) and therefore one can construct a Banach algebra generated by them and observe the commuting commutation relations; the structure of C (algebra) is quite obvious: an involution is de ned on generators in a natural way, because the corresponding tensor operators are conjugate to each other.

**Definition 4B.** The highest vector \( T \) of the weight 2 in the QFT (operator algebra) will be called the \( q_8 \) (conformal stress-energy tensor if \( T(u) = l_2(T) = 1_k(u)^k \), where the operators \( L_k \) form the \( \{ \text{Virasoro} \} \) algebra: \( L_l;L_j = (i+j)L_{l+j} + (i+j) \). The set of the highest vectors \( J \) of the weight 1 in the QFT (operator algebra) will be called the set of the \( q_8 \) (a ne variables \( J_k = J_T^k, [J_T;J] = cJ \), where \( J_T = f(t+1)T \), \( [T;J] = f(t)J = 0, f(t+2h) = 0 \), if \( f(t+2h) = 0 \), and \( c = (q_8)^{-1} \), if \( c = (q_8)^{-1} \).

It should be mentioned that \( q_8 \) (a ne variables and \( q_8 \) (conformal stress-energy tensor are just the \( \text{sl}(2;\mathbb{C}) \) (primary elds in the Vem a module \( V_{\theta} = \theta_2^{q_8^{-1}+1} \)) over \( \text{sl}(2;\mathbb{C}) \) of spin 1 and 2, respectively; if such module is realized as before then \( J_k = \theta_2^{q_8^{-1}+1} \), \( J_k = z + (2h + k + 1) \); \( L_2 = (3h)\theta_2, L_1 = (2h)\theta_2, L_0 = h, L_{-2} = z^2 + 2h(2h + 1), \) \( = z\theta_2 \). So the generators \( J_k \) of \( q_8 \) (a ne algebra may be represented via generators of Lobachevskii C algebra: \( J_k = J_T^k \); if \( k = 0 \) and \( J_T = cJ \)). That means that \( q_8 \) (a ne algebra admits a hom omorphism in a tensor product of the universal enveloping algebra \( U \) of the Lie algebra \( q_8 \) and Lobachevskii algebra. The (derived) QFT (operator algebra) is generated by \( q_8 \) (a ne variables are called canonical projective G (hypers multilets). The primary eld \( V_k(\ u) = \exp(kQ + R(V_1(\ u)du)) \) \( R(u) = \text{sgn}(nu)^2, \) i.e. \( R \) is the H ebert transform \( f(\exp(\text{det})) \) \( = \frac{1}{\text{cos}(2\text{det})} \) of \( f(\exp(\text{det})) \) of a charge \( Q \) is the derived as \( Q(z) = \text{const} \exp(\text{det}(u)) \).
vertex operators $e_{\lambda B_k} (u; r_k)$ [16,17], are not mutually local. Nevertheless, their commutation relations may be described as follows: 

$$V (u) V (v) = S (u + v) V (u) V (v) = S (q_i) V (u) = 0$$

If $a$ is a current; it may be easily performed by an perturbation of simple formulas for such objects for a singular part of a current, as it was stated in [7] such perturbation by a regular part does not change the resulting monodromy.

Let $H$ be an arbitrary direct sum of Vem a module over $sl(2; C)$ and $P$ be a trivial bundle over $C$ with bers isomorphic to $H$; it should be mentioned that $P$ is naturally trivialized and possesses a structure of $sl(2; C)$ (homogeneous bundle. $A (u; \theta; u) = (u; \theta; u) = T (u; \theta; u)$, $k$ the coe"cients of such expansion are just $sl(2; C)$ formulas (primary) $t$; the equation $\theta_t(t) = A (u; \theta; u) t$, where $t$ belongs to $H$ and $u = u (t)$ is the function of scanning, is a quantum analog of the Euler formulas; such analog describes an evolution of M V image under the observation. I once rigorously, such evolution is de ned in the dual space $H$ by formulas $\theta_t(t) = A^T (u; \theta; u) t$. Regarding canonical conjunctive $G$ (hypermultiplet we may construct a quantum analog of the Euler (A mol) equations $\theta(t) = f H; A g$, where an angular $e$ $A (u; \theta; u)$ is considered as an element of the canonical conjunctive $G$ (hypermultiplet being expanded by $sl(2; C)$ (primary) $t$ ax $G$ of this hypermultiplet, $H$ is the quadratic element of $S (g)$, $f$ is canonical Poisson brackets in $S (g)$. It is quite natural to dem $H$ be a solution of the V famous master equation. If we consider a conjunctive $G$ (hypermultiplet, which is a semidirect product of the canonical one and a trivial one (i.e. with $l_t(f) = 0$), it will be possible to combine Euler (A mol) equations with Euler formulas to receive the complete dynamical perspective laws of the M V.

1.A. Organizing M V cyberspace.

M V cyberspace consists of a space of images $V_I$ with the fundamental length (a step of the lattice) $\theta$ and a space of observation $V_0$ with the fundamental length $\theta$. The space of images $V_I$ is one where pictures are form ed, whereas the space of observation $V_0$ is used for a detection of motion of eyes; it is natural to claim that $\theta << A_{tr}$, where $A_{tr}$ is an amplitude of the eye trem or, as well as $\theta >> \theta$. The Euler formulas may be written as $\theta_t(t) = A (u; \theta; u) t$, $A (u; \theta; u)$ may be considered approxim ately in the form $M_1 (t) \Omega (u_1 V_1) + M_2 (t) \Omega (u_2 V_2) + M_3 (t) \Omega (u_3 V_3) = \theta_t(t)$, where $t$ is the quadratic element of $S (g)$, $f = g$ are canonical Poisson brackets in $S (g)$. It is quite natural to dem $H$ be a solution of the V famous master equation. If we consider a conjunctive $G$ (hypermultiplet, which is a semidirect product of the canonical one and a trivial one (i.e. with $l_t(f) = 0$), it will be possible to combine Euler (A mol) equations with Euler formulas to receive the complete dynamical perspective laws of the M V.

$\overline{\theta}$

$$\overline{\theta} = \frac{1}{1 - u x^2} (2 h + 1) \times \frac{1}{1 - u x^2} \theta_x + \frac{h (2 h + 1)}{3} \frac{1}{1 - u x^2}$$

The matrices $A^T (u; \theta; u)$ of size $N \times N$ ($N$ is a number of points of $V_I$) should be expanded in a sum of three terms $M_1 (t) \Omega (u_1 V_1) (t)$, where $V_I (t)$ are matrices of size $N \times N$, depending on param $u e$; this param may have M different values (M is number of points of $V_0$). Matrices $\frac{1}{1 - u x^2}$ are easily calculated, one should obtain the complete matrices $V_I (t)$ making a conjugation in the unitary locals $sl(2; C)$ (modules $V_h$). Derivatives should be replaced by differences everywhere in a standard way. Formulas for $M_1 (t)$ may be received from [19].

\[
\begin{align*}
A & = \exp (u), \quad \text{where } A (u) \text{ and } \text{ are regular and singular parts of } R (V_I (u) du), \\
(\lambda) & = \log (1 + u D), \quad (u) = \log (1 + u D) D, \quad \text{the operator } A \text{ is de ned as } \\
A & = (k) F K D = -i k = \text{Const sum } (2 \lambda z + 2 h) = (2 \lambda z + 2 h), Q = \frac{d}{d z} A \quad \text{Const.}
\end{align*}
\]
1.5. Non-Alexandrian geometry of M obilight.

It should be noted that almost all classical geometries use a certain postulate, which we shall call the Alexandrian, but do not include it in their axiomatics explicitly. For our purposes we prefer to give a precise formulation of this postulate.

A non-Alexandrian postulate. Any statement holding for a certain geometric configuration remains true if this configuration is considered as a subconfiguration of any of its extension.

A non-Alexandrian postulate means that an addition of any subsidiary objects to a given geometric configuration does not influence this configuration. It is convenient to describe a well-known example of non-Alexandrian geometry (which may be called Einstein geometry).

Example of non-Alexandrian geometry. Objects of geometry are weighted points and lines. Weighted points are pairs (a standard point on a plane, a real number). They define a (singular) metric on a plane via Einstein-type equations $R(x) = m \cdot (x \cdot x)$, where $(x \cdot m)$ are weighted points and $R(x)$ is a scalar curvature. Lines are geodesics for this metric. The basic relation is a relation of an incidence.

It can be easily shown that a non-Alexandrian postulate does not hold for such geometry, which contains a standard Euclidean one (extracted by the condition that all "masses" $m$ are equal to 0).

Kinematics and process of scattering of quton may be illustrated by another important example of non-Alexandrian geometry | geometry of solitons [20, 21]. The basic objects of $KdV$ (soliton geometry) are moving points on a line; a configuration of such points defines a $n$ (soliton) solution of $KdV$ (equation $u_t = 6u_{xx} - u_{xxx}$) by the formula $u(x; t) = 2(\log(\det(E + C)))_{xx}$, where $C_{nm} = c_n(t)(c_m(t) - 1/n + 1/m)c(t) + c_n(t)(0)\exp(4(t^2/2));$ such solution is asymptotically free, i.e. may be represented as a sum of soliton solutions (solitons) whereas $t \rightarrow 0$. Soliton has the form $u(x; t) = 2(\cosh 2((x - 4t^2''))) \exp(2(2' - 2'' + 2'''))$, phase is an initial position of soliton and $v = 4(2''')$ is its velocity; scattering of solitons is a two-particle, the shift of phases is equal to $1\sqrt{2} log((1 + 2))((1 + 2))$ for the first (quick) soliton and $2(1 \log ((1 + 2))((1 \log ((1 + 2)))$ for the second (slow) one. Analogous scheme may be realized for $\sinh(Gordon equation, which has the form $u = u + \sin u = 0$ in the lightcone variables; SG (soliton) is defined by the expression $u(t; x) = 4 \arctan((1 \cdot v^2)^1/2(0 \cdot v)))$; the shift of phases under the scattering is equal to $2(1 \cdot v^2)^1/2 \log(v)$ for the slow soliton (and by such expression with the opposite sign for the quick one) in the coordinates of the mass centre, where $v$ is a relative velocity of the quick soliton. Another example of soliton geometry is connected with the nonlinear Schrodinger equation [22]. All examples of soliton geometry concern the opinion that a breaking of the ALEXANDRIAN postulate is generated by an interaction of geometric objects, in particular, such interaction may be defined by a non-linear character of their evolution.

Let's consider now an interpretation of scattering. As it was stated below a true in interpretational geometry is described by a pair $(\text{int}; \text{ext})$, where $\text{int}$ is an interior image in the subjective space of observer and $\text{ext}$ is its exterior counterpart; $\text{int}$ is a result of interpretation of $\text{ext}$. It is natural to suppose that $\text{int}$ depends functionally on $\text{ext}$ and as a rule nonlinearly; moreover, if $\text{ext}$ is asymptotically free then $\text{int}$ is also asymptotically free. Thus, a nontrivial scattering of interacting interpretational objects exists (i.e. although we do not know an explicit form of dynamical equations for $\text{int}$, their solutions, nevertheless, in view of our assumptions may be considered as soliton-like (!), so interpretational geometries may be considered as non-ALEXANDRIAN ones; it should be specially marked that the breaking of an ALEXANDRIAN postulate is realized on the level of qutons themselves, but it is not observed on one of their fields.)
2. INFORMATICS OF MOBILE VISUALIZATION

2.1. Information transmission via anomalous virtual realities: AVR (photodosy).

As it was mentioned above informatics may be considered as a point of view on mathematical objects complex entry to geometric one; so it seemed to be useful to reformulate our main definitions in informatics terms. The concepts of AVR (photodosy and its formal matrices) are a natural parallel to ones of anomalous virtual reality and the transcending abstract model.

Definition 5A. The transmission of information via anomalous virtual reality by "latent lights" is called AVR (photodosy; the system of algebraic structures of the initial abstract model), which characterizes a process of AVR (photodosy via naturalization, is called the formal matrices of AVR (photodosy).

It should be mentioned that the concept of AVR (photodosy and its formal matrices) is deeply related to one of anomalous color space, since the usage of such spaces allows to transmit diverse information in different forms, and an investigation of the information transmission via AVRs, which character depends on a structure of color space, is an important mathematical problem (cf. [23]); it should be marked now that the results of such researches may be used for elaboration of computer systems of psychophysiological self-regulation, hypnosis and suggestion. Structure of AVR (photodosy) is determined by its formal matrices; in view of the quantum character of formal algebraic structures of M V, AVR (photodosy via it is quantum [24]); it seems that this fact needs a special attention. A few words more about M V formal matrices: the investigation of a certain system of algebraic structures of the quantum projective field theory, which may be considered as possible candidates on the role of formal algebraic matrices of M V now, was begun by the author and his collaborators in various directions in series of papers devoted to QFT and is intensively continued in present time; such investigations are far from a finish and there are a lot of hopes that many new interesting algebraic objects will be found in the nearest future; it should be marked that the state of the art is very interesting yet, the main directions of researches are (1) the model formalism [25, 15-17, 26], (2) the formalism of projective Fubini-Veneziano fields [13, 14], (3) the formalism of local field algebras [25, 26], (4) a search for hidden symmetries [27-30], (4) the commutative exterior differential calculus [31, 32]; this article is not a suitable place to describe the obtained results, a reader should address to the cited list of references, especially to [25].

2.2. Information transmission via intentional anomalous virtual realities: IAVR (teleaesthesis).

At first, it should be mentioned that any IAVR is polysemic, it means that quantity and structure of an information received by AVR (photodosy via it, depends on observer; so it is a natural problem to describe the informatics of interactive computer graphical psychophysical interaction system s, which contain more than one observer, in particular, a correlation of different observations. Such system may be regarded as realizing an interactive M ISD. Multiple Instruction (Single Data) architecture with parallel interpretation processes for different observers (this fact should be considered in context of a remark on a quantum (logical character of AVR (photodosy below); in this way we encounter with the phenomenon, which is specific for such system s but may have a more general meaning: namely, the processes of observation by different observers induce a certain information exchange between them.

Definition 5B. AVR (photodosy via IAVR from one observer to another is called IAVR (teleaesthesis); if during IAVR (teleaesthesia) AVR (photodosies) from different observers do not satisfy the superposition principle then it will be said that the collective effect in IAVR (teleaesthesia holds.

It should be mentioned that (1) the process of IAVR (teleaesthesia has a bilateral character; the observers entering into a teleaesthetic communication, are simultaneously as inducers (observers sending information) as receptors (ones receiving it), moreover, as it was mentioned above, quantity and structure of received information depends on receptor as well as on inducer; (2) the collective effect in IAVR (teleaesthesia means that the different inducers in IAVR does not perceived as independent; the transmitted information is not a sum of informations sent by separate observers, so the partial stream of information enter into the exchange interaction, which form a specific information received by receptor. A relation of an origin of IAVR (teleaesthesia to the fact that interactive computer graphical psychophysical interaction system s realize an interactive M ISD architecture should be specially marked.
Definition 5C. An observer in a marginal AVR, to which there is no corresponded any observer in the AVR, realized on the videocomputer, is called virtual; a virtual observer, whose process of observation depends on several real observers is called a collective virtual observer.

The presence of a virtual observer means that a mixed part of the received information is interpreted as an information sent by this observer, which does not exist really. The presence of a collective virtual observer is not obligatory but is usual for interactive computer graphs. System states in the multiuser mode; this fact also should be considered in a context of the previous remark that such systems realize an interactive MISD architecture with parallel interpretation processes for different users.

Let us now illustrate our constructions on the example of MV. It should be mentioned that

$q_k (n)$ (confom al eld theory is a certain deformation of $c = 4$ conform al theory; this deformation is infinitesimally defined by the next Poisson bracket on the virasoro algebra with $c = 4$: $\mathcal{L}_i$, $i = 1, 2$, $\mathcal{L}_i \mathcal{L}_j = 0$). The initial quantum conformal eld theory is local (the locality means that $[\mathcal{L}_i, \mathcal{L}_j(Y)] = 0$ if $\mathcal{L}_i$ and $\mathcal{L}_j$ are local). Let us now consider $q_k (n)$ conformal eld theory with non-zero $q_k$; the corresponding partial angular elds are not mutually local (and therefore, a collective effect in IAVR (teleaesthesis is not observed and a collective virtual observer does not exist). Let us now consider $q_k (n)$ conformal eld theory with non-zero $q_k$; the corresponding partial angular elds are not mutually local (and therefore, a collective effect in IAVR (teleaesthesis holds in the corresponding M V anomalous virtual reality)); nevertheless, they may be expanded as $A(\mathcal{L}_i; \mathcal{L}_j; q_k) = A(\mathcal{L}_i; \mathcal{L}_j; q_k)$, where $A(\mathcal{L}_i; \mathcal{L}_j; q_k) = 0$ (so $q_k$ is a measure of the collective effect); the multiobserver Euler formulas being of the form $t = A(\mathcal{L}_i; \mathcal{L}_j; q_k)$ (where $A(\mathcal{L}_i; \mathcal{L}_j; q_k)$) may be written as $t = \frac{1}{2} A(\mathcal{L}_i; \mathcal{L}_j; q_k) + q_k A(\mathcal{L}_i; \mathcal{L}_j; q_k) t$ (where $A(\mathcal{L}_i; \mathcal{L}_j; q_k)$ and $q_k A(\mathcal{L}_i; \mathcal{L}_j; q_k)$). These equations may be considered as dynamical equations in a marginally IAVR, real observers in which are described by conformal (mutually local) angular elds $A(\mathcal{L}_i; \mathcal{L}_j; q_k)$ and a collective virtual observer is described by the angular eld $q_k A(\mathcal{L}_i; \mathcal{L}_j; q_k)$; the process of observation by a collective virtual observer is reduced to a virtual process of intention, the intention of a collective virtual observer is measured by the parameter $q_k$. It should be marked here that $A$ and $A(\mathcal{L}_i; \mathcal{L}_j; q_k)$ are not mutually local.

This example clarifies the notion of a collective virtual observer; it shows that (1) only a part of received information is interpreted as an information sent by a collective virtual observer (i.e., its presence does not diminish in any way the presence of real ones), (2) a process of intention of a collective virtual observer is completely determined by real ones in interaction (i.e., a collective virtual observer is a specific unified state of real ones entering an interactive computer graphics information system in the multiuser mode), (3) a collective virtual observer communicates with real observers being interpreted (at least, formally) as an independent one.

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