On quantum plasma: A plea for a common sense

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Abstract \textendash The quantum plasma theory has flourished in the past few years without much regard to the physical validity of the formulation or its connection to any real physical system. It is argued here that there is a very limited physical ground for the application of such a theory.

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In the past few years one could observe a lot of activity in the literature regarding the quantum plasma. The amount of publications is already measured by three-digit numbers. Practically without exceptions those works deal with collective interactions in plasmas and with consequent waves, instabilities, soliton formations, vortices, etc., and also in almost all cases such studies are performed within the framework of fluid theory. It is a common wisdom that the fluid approach is valid only if the spatial scales of the physical problem are considerably larger than the mean distance between the plasma particles $\lambda_m$, and also, above the mean free path of particles $\lambda_f = \nu_j/\nu_j$, $\nu_j^2 = \kappa T_j/m_j$. The latter applies to the kinetic theory too, which operates with distribution functions (Maxwellian or some of its modifications), and those are mainly achieved in the processes of thermalization (i.e., collisions) or fluctuations. At such scales the quantum effects are simply out of scope.

Normally, the quantum effects would appear primarily for lighter particles (electrons). In reality, however, the quantum effects in plasmas are typically of very limited importance. This may be seen from basic books on plasma theory, see, e.g., in ref. [1]. The electron temperatures above which the classical plasma description is valid are determined by the following limit:

$$T_e(n_e) = \left[ \left( \frac{\hbar}{2} \right)^3 \left( \frac{1}{3m_e\kappa} \right)^{3/2} \cdot n_e \right]^{2/3} = 7.37 \cdot 10^{-17} n_e^{2/3}.$$  \hspace{0.5cm} (1)

Here, the electron number density is per cubic meter, and the electron temperature is in Kelvin degrees.

![Fig. 1: The locus of pairs $n_e, T_e$ determining the boundary between the classical and quantum domains.](image)

The temperature-density boundary separating the classic and quantum domains, for some typical plasma densities, is presented in fig. 1. It is seen that extremely low temperatures (on the order of $10^{-5}$ K) are needed in order to observe quantum effects.

This is presented also in table 1 for a much larger span of the electron number density, see the first two rows. For electrons, some quantum effects may become of interest only at the number density of the order of $10^{30}$ m$^{-3}$ and small temperatures of a few thousand K. In this limit only, the uncertainty in the position becomes comparable to the distance between electrons. However, for ions even the number densities of around $10^{30}$ m$^{-3}$ are not high enough. This is shown by several values in brackets in

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ties of the order of \(n \sim 10^{35} - 10^{41} \text{ m}^{-3}\) cold plasma [2].

Because the mean distance between particles is \(\lambda_m \simeq 1/n^{1/3}\), for those extremely large number densities of the order of \(n_\ell \simeq 10^{30} - 10^{40} \text{ m}^{-3}\) one finds \(\lambda_m \simeq 10^{-10} - 10^{-13} \text{ m}\). Hence, the quantum effects are expected at atomic and subatomic scales, i.e., those on the order of the Bohr radius or below it [3,4]. Studying quantum corrections therefore may have sense for elementary processes like the ionization, attachment, detachment etc.

We stress also that the kinetic and fluid theories, used in studying collective interactions in plasmas and therefore the phenomena mentioned above (waves, instabilities, etc.), imply weakly non-ideal plasmas. Those are plasmas close to thermodynamic equilibrium, and in the same time comprising particles whose energy of thermal motion is considerably above the energy of the electrostatic interaction of the charged particles. In practical terms this implies the presence of a relatively high number of particles within the Debye sphere, or in other words the mean distance \(\lambda_m\) between two charged particles should be well below the Debye radius \(r_d = v_\ell/\omega_\ell\), where \(\omega_\ell = q^2 n_\ell/(\varepsilon_0 m_\ell)\) is the plasma frequency. The total internal energy of such a plasma can be written as \(U = U_{th} + U_{int}\), where the first and second terms are, respectively, the thermal energy and the energy of electrostatic interaction. This can further be written as \(U = (3n\kappa T/2)[1 - \lambda_m^3/\langle 12\pi r_d^3 \rangle]\), where \(n = \sum n_j\). Here, the ratio \(\lambda_m^3/r_d^3\) must be well below unity (the standard condition of a weakly non-ideal plasma). However, the third row in table 1 shows just the opposite in the domain of temperatures and densities where the quantum effects do play a role: the mean distance between the particles is far above the Debye radius. The plasma is thus far from total or partial thermodynamic equilibrium. This thermodynamic equilibrium is in fact an underlying condition for the collective behavior of a standard plasma description used in numerous papers. Since this condition is often violated, the application of such a description is not valid.

### Table 1: The electron number density and the corresponding temperature, eq. (1), below which the quantum effects should be included (the values in brackets are for protons), and the corresponding ratio of the mean distance between particles and the Debye length.

| \(n_e\) (m\(^{-3}\)) | \(10^6\) | \(10^{12}\) | \(10^{18}\) | \(10^{24}\) | \(10^{26}\) | \(10^{28}\) | \(10^{30}\) |
|---------------------|--------|--------|--------|--------|--------|--------|--------|
| \(T_e\) (K)         | 7. \(10^{-13}\) | 7. \(10^{-9}\) | 7. \(10^{-5}\) | 0.7    | 16     | 342(0.18) | 1587(0.85) | 7369(3.9) |
| \(\lambda_m/r_{de}\)| 173    | 78     | 36     | 25     | 17     |

It is evident that the quantum domain is never of any importance in most plasmas, except in the core of neutron stars which can be characterized as a dense \((n \sim 10^{35} - 10^{41} \text{ m}^{-3})\) cold plasma [2].

The degeneracy of electrons, as an additional manifestation of quantum effects, appears at temperatures (in K) below

\[
T_{deq} = |\hbar^2/(2m_e\kappa)|[10^{-6}n_e/(2.76\pi)]^{3/2}.
\]

It is seen that at \(n_e = 10^{30} \text{ m}^{-3}\) this value for electrons becomes very low, viz. \(T_{deq} = 41\text{ K}\). Such temperatures can indeed be found in astrophysical molecular clouds, however, the accompanying plasma density is many \((e.g., 15-20)\) orders of magnitude below the necessary value. Hence, the degeneracy is of no importance.

To summarize, our criticism is aimed at the huge number of studies on waves and instabilities in plasmas, particularly within the fluid domain, that are so frequently seen in the literature in the past few years. Typically, in these studies some “quantum” correction terms are included in the momentum equations for electrons (and ions as well), and this all for parameters where they cannot possibly play any role. Particularly meaningless are such examples of works dealing with wave dynamics in dusty plasmas, where, first, spatial scales at which dust grain dynamics takes place are many orders of magnitude above any quantum scale correction, and, second, it is clear that finding an appropriate physical system (extremely dense and simultaneously extremely cold) to apply these quantum corrections becomes practically impossible. In our view, the arguments given in the text above should be kept in mind and the criteria for the applicability of the theory should be checked before any practical step.

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