On the possibility of an \( \eta \)–meson light nucleus bound state formation

S. A. Rakityansky*, S. A. Sofianos,
Physics Department, University of South Africa, P.O.Box 392, Pretoria 0001, South Africa

V. B. Belyaev,
Joint Institute for Nuclear Research, Dubna, 141980, Russia

W. Sandhas
Physikalisches Institut, Universit\"at Bonn, D-53115 Bonn, Germany

(March 31, 2022)

Abstract

The resonance and bound–state poles of the amplitude describing elastic scattering of \( \eta \)-meson off the light nuclei \(^2H\), \(^3H\), \(^3He\), and \(^4He\) are calculated in the framework of a microscopic approach based on few–body equations. For each of the nuclei, the two–body parameters that enhance the \( \eta N \)–attraction which generate quasi–bound states, are also determined.

I. INTRODUCTION

Experiments on production of low-energy \( \eta \)–mesons in nucleus–nucleus [1], \( N \)–nucleus [2,3], and \( \pi \)–nucleus [4] collisions, revealed an energy dependence of the corresponding cross–sections, which does not contradict the expected strong \( \eta \)–nucleus final–state interaction [5]. This is natural, since the low–energy \( \eta N \)– interaction is resonant. The possibility that this energy dependence is related to the existence of an \( \eta \)–nucleus quasi–bound state cannot be excluded. Indeed it was shown in Ref. [6] that at low energies the \( \eta N \) interaction, in the dominant \( S_{11} \) channel where the near–threshold resonance \( N^*(1535) \) is excited, is attractive.

On the other hand estimations obtained in the framework of the first-order optical potential theory [7,8], put a lower bound on the nucleus atomic number \( A \) for which an \( \eta \)–nucleus bound state could exist, namely, \( A \geq 12 \).

*Permanent address: Joint Institute for Nuclear Research, Dubna, 141980, Russia
However, some speculations on the possibility of a formation of $\eta$–helium bound state still appear [9] despite the discouraging results of the first (and the sole) experiment [10] on direct search for bound states of $\eta$–meson with Lithium, Carbon, Oxygen, and Aluminium. All such speculations are based on the large negative values ($\sim -2$ fm) of the real parts of $\eta$–nucleus scattering lengths calculated within a simplified optical–potential theory [5,9].

To the best of our knowledge, the only microscopic calculations of the scattering lengths was presented in our recent papers [11–13]. In these, it turned out that the $\eta$–helium scattering length could have even larger (negative) real part than earlier estimations. This raises more doubts on the validity of the abovementioned constraint, $A \geq 12$, for the existence of $\eta$–nucleus bound states.

In the present work we examine the possibility of formation of a bound state in the $\eta$–meson $d$, $t$, $^3He$, and $^4He$ systems, in the framework of a microscopic approach, namely, the Finite-Rank Approximation (FRA) of the nuclear Hamiltonian [14,15]. The approximate few–body equations of this approach enable us to calculate the $\eta$–nucleus $T$–matrix $T(\vec{k}',\vec{k};z)$ for any complex total energy $z$, i.e. for any point on the complex plane of the momentum $p = \sqrt{2\mu z}$. In this way we have numerically located the resonance state poles and (for each of the nuclei) we found the parameters that enhance the $\eta N$–attraction which generate quasi–bound state.

II. THE METHOD

The total Hamiltonian of the quantum system consisting of an $\eta$–meson and a nucleus of atomic number $A$, can be written as

$$ H = H_0 + V + H_A, $$

where $H_0$ is the $\eta$–nucleus kinetic energy operator (free Hamiltonian), $V = V_1 + V_2 + \cdots + V_A$ is the sum of $\eta N$–potentials, and $H_A$ is the total Hamiltonian of the nucleus. Elastic scattering process is the transition from the initial $|\vec{k},\psi_0 \rangle$ to the final $|\vec{k}',\psi_0 \rangle$ asymptotic state which differ only in the direction of the relative $\eta$–nucleus momenta $\vec{k}$ and $\vec{k}'$. During this process, the nucleus remains in the ground eigenstate $|\psi_0 \rangle$, of $H_A$

$$ H_A |\psi_0 \rangle = E_0 |\psi_0 \rangle. $$

The $\eta$–nucleus scattering amplitude $f(\vec{k}',\vec{k};z)$, is expressed in terms of the asymptotic states

$$ f(\vec{k}',\vec{k};z) = -\frac{\mu}{2\pi} \langle \vec{k}',\psi_0 | T(z) | \vec{k},\psi_0 \rangle, $$

i.e in terms of the matrix elements of the operator $T$ obeying the Lippmann-Schwinger equation

$$ T(z) = V + V \frac{1}{z - H_0 - H_A} T(z). $$
Here $\mu$ is the $\eta$–nucleus reduced mass. For further developments, we introduce the (auxiliary) operator $T^0$ via

$$T^0(z) = V + V \frac{1}{z - H_0} T^0(z)$$

(5)

and rewrite Eq. (4) in the form

$$T(z) = T^0(z) + T^0(z) \frac{1}{z - H_0} H_A \frac{1}{z - H_0 - H_A} T(z).$$

(6)

Then, using the approximation

$$H_A \approx E_0 |\psi_0> <\psi_0|,$$

(7)

we obtain for the matrix elements $T(\vec{k}', \vec{k}; z) \equiv< \vec{k}', \psi_0 |T(z)| \vec{k}, \psi_0 >$ the following integral equation

$$T(\vec{k}', \vec{k}; z) =< \vec{k}', \psi_0 |T^0(z) |\vec{k}, \psi_0 > + E_0 \int \frac{d^3k''}{(2\pi)^3} \frac{< \vec{k}', \psi_0 |T^0(z) |\vec{k}'', \psi_0 >}{(z - \frac{k''^2}{2\mu})(z - E_0 - \frac{k''^2}{2\mu})} T(\vec{k}'', \vec{k}; z),$$

(8)

which after the partial–wave decomposition becomes one-dimensional and can be easily solved (numerically) if we know the auxiliary matrix

$$< \vec{k}', \psi_0 |T^0(z) |\vec{k}, \psi_0 >.$$

(9)

It is easy to see that the $T^0$–operator describes the scattering of an $\eta$–meson from nucleons fixed at their spatial positions inside the nucleus, because the equation (5), which defines $T^0$, does not contain any operator which acts on the internal nuclear Jacobi coordinates $\{\vec{r}\}$. Therefore all operators in Eq. (4), are diagonal in these variables and thus its momentum representation reads

$$T^0(\vec{k}', \vec{k}; \vec{r}; z) = V(\vec{k}', \vec{k}; \vec{r}) + \int \frac{d^3k''}{(2\pi)^3} \frac{V(\vec{k}', \vec{k}'', \vec{r})}{z - \frac{k''^2}{2\mu}} T^0(\vec{k}'', \vec{k}; \vec{r}; z).$$

(10)

In other words it depends only parametrically on the coordinates $\{\vec{r}\}$. Solving this integral equation and employing the ground state wave function $\psi_0(\vec{r})$ for the nucleus, the input $< \vec{k}', \psi_0 |T^0(z) |\vec{k}, \psi_0 >$ to Eq. (3) is obtained by integrating over all nuclear Jacobi coordinates $\{\vec{r}\}$

$$< \vec{k}', \psi_0 |T^0(z) |\vec{k}, \psi_0 > = \int d^{3(A-1)} r |\psi_0(\vec{r})|^2 T^0(\vec{k}', \vec{k}; \vec{r}; z).$$

(11)

For practical calculations, Eq. (3) is written in terms of the Faddeev components $T^0_i(z)$

$$T^0(z) = \sum_{i=1}^A T^0_i(z),$$

(12)

Introducing the operators
we finally get for $T^0_i$ the following system of integral equations

$$T^0_i(\vec{k}', \vec{k}; \vec{r}; z) = t_i(\vec{k}', \vec{k}; \vec{r}; z) + \int \frac{d^3k''}{(2\pi)^3} \frac{t_i(\vec{k}', \vec{k}''; \vec{r}; z)}{z - \frac{k''^2}{2\mu}} \sum_{j\neq i} T^0_j(\vec{k}'', \vec{k}; \vec{r}; z).$$

The $t_i$ describes the scattering of the $\eta$-meson off the $i$-th nucleon. It is expressed in terms of the corresponding two-body $\eta N$-matrix via

$$t_i(\vec{k}', \vec{k}; \vec{r}; z) = t_{\eta N}(\vec{k}', \vec{k}; z) \exp \left[ i(\vec{k} - \vec{k}') \cdot \vec{r}_i \right]$$

where $\vec{r}_i$ is the vector from the nuclear center of mass to the $i$-th nucleon and can be expressed in terms of the Jacobi vectors $\{\vec{r}\}$.

We emphasize, that the use of $T^0$ in the above scheme is not an additional (fixed-scatterer) approximation. The coupled equations (13) and (14) are exact. The only approximation used here is the one defined by Eq. (7) and corresponds to a truncation of the spectral expansion of the nuclear Hamiltonian. Physically, it means that during the multiple scattering of $\eta$-meson, the nucleus remains in its ground state. This approximation is widely used in nuclear physics and is known as the coherent approximation [16].

### III. RESULTS AND DISCUSSION

As an input information we need the ground-state wave functions $\psi_0$ of the nuclei involved and the two-body $t$–matrix $t_{\eta N}$.

For the bound states we employed simple Gaussian–type functions

$$\psi_d(\vec{x}) = \left( \frac{3}{8\pi <r^2_d>} \right)^{3/4} \exp \left( \frac{-3x^2}{8 <r^2_d>} \right),$$

$$\psi_t(\vec{x}, \vec{y}) = (\sqrt{3}\pi <r^2_t>)^{-3/2} \exp \left[ -\left( \frac{x^2}{2} + \frac{2y^2}{3} \right) / (2 <r^2_t>) \right],$$

$$\psi_\alpha(\vec{x}, \vec{y}, \vec{z}) = \left[ 2 \left( \frac{9}{16\pi <r^2_\alpha>} \right)^3 \right]^{3/4} \exp \left[ -\frac{9}{16 <r^2_\alpha>} \left( \frac{x^2}{2} + y^2 + \frac{z^2}{2} \right) \right],$$

where $\vec{x}, \vec{y},$ and $\vec{z}$ are the Jacobi vectors. These functions were constructed to be symmetric with respect to nucleon permutations, and to reproduce the experimental mean square radii: $\sqrt{<r^2_d>} = 1.956$ fm [17], $\sqrt{<r^2_{3H}>} = 1.755$ fm [18], $\sqrt{<r^2_{3He}>} = 1.959$ fm [18], and $\sqrt{<r^2_\alpha>} = 1.671$ fm [19]. For masses and binding energies of the nuclei we used the experimental values [20].

Since at low energies the $\eta N$ interaction is dominated by the $N^*(1535) S_{11}$ - resonance, we used the following separable form for the $\eta N$ - amplitude
\[ t_{\eta N}(k', k; z) = \frac{\lambda}{(k'^2 + \alpha^2)(z - E_0 + i\Gamma/2)(k^2 + \alpha^2)} \]  

(18)

with \( E_0 = 1535 \text{ MeV} - (m_N + m_\eta) \) and \( \Gamma = 150 \text{ MeV} \)\(^\text{[21]}\). In order to fix the parameter \( \alpha \), we make use of the results of Refs.\(^\text{[6,22]}\), where the same \( \eta N \rightarrow N^* \) vertex function \((k^2 + \alpha^2)^{-1}\) was employed with \( \alpha \) being determined via a two-channel fit to the \( \pi N \rightarrow \pi N \) and \( \pi N \rightarrow \eta N \) experimental data.

Due to experimental uncertainties and differences between the models for the physical processes, one can use three different values for the range parameter \( \alpha \), namely, \( \alpha = 2.357 \text{ fm}^{-1} \)\(^\text{[6]}\), \( \alpha = 3.316 \text{ fm}^{-1} \)\(^\text{[22]}\), and \( \alpha = 7.617 \text{ fm}^{-1} \)\(^\text{[6]}\). Since there is no criterion for singling out one of them, we use all three in our calculation.

The remaining parameter \( \lambda \) is chosen to provide the correct zero-energy on-shell limit, i.e., to reproduce the known \( \eta N \) scattering length \( a_{\eta N} \),

\[ t_{\eta N}(0, 0, 0) = -\frac{2\pi}{\mu_{\eta N}} a_{\eta N}. \]  

(19)

Like the range parameter \( \alpha \), the scattering length \( a_{\eta N} \) is not well known, the estimated values being within the range \( \text{Re} a_{\eta N} \in [0.27, 0.98] \text{ fm} \) and \( \text{Im} a_{\eta N} \in [0.19, 0.37] \text{ fm} \)\(^\text{[23]}\). Our intention is to vary the \( \text{Re} a_{\eta N} \) until the corresponding \( \eta N \) attraction generates a bound state in the \( \eta \)–nucleus system. As the starting value we chose \((0.55 + i0.30) \text{ fm}\) proposed by Wilkin\(^\text{[5]}\). Thus, we take

\[ a_{\eta N} = (g0.55 + i0.30) \text{ fm}, \]  

(20)

where \( g \) is an enhancing parameter.

Since \( a_{\eta N} \) is complex, the \( \eta \)–nucleus Hamiltonian is non–Hermitian and its eigenenergies are generally complex. Hence, we do not expect to find a pole of \( T(z) \) on the positive imaginary axis of the complex \( k \)-plane with any choice of the enhancing factor \( g \). As was shown in Ref.\(^\text{[24]}\), when the interaction becomes complex the bound-state poles move into the second quadrant of the complex \( k \)-plane. Therefore we search in this quadrant in order to locate possible poles of \( T(z) \). However, not all poles in the second quadrant stem from bound states. Indeed, the energy \( E_0 = p_0^2/2\mu \) corresponding to a pole,

\[ E_0 = \frac{1}{2\mu} \left[ (\text{Re} p_0)^2 - (\text{Im} p_0)^2 + 2i(\text{Re} p_0)(\text{Im} p_0) \right], \]  

(21)

has a negative real part only if \( p_0 \) is above the diagonal of this quadrant. Below the diagonal \( \text{Re} E_0 > 0 \) the pole is attributed to a resonance. Therefore this diagonal is the critical border, and when crossing from below, a pole becomes a quasi–bound state.

Fixing the enhancing factor \( g \) of Eq.\(^\text{[20]}\) to the value \( g = 1 \) and making variations of the complex parameter \( p = \sqrt{2\mu z} \) within the second quadrant, we located the poles close to the origin, \( p = 0 \), which are given in the Table 1.
It is seen, that for the $\eta d$, $\eta t$, and $\eta^3He$ systems, these poles lie below the diagonal, i.e. in the resonance region, while the $\eta^4He$ system has a quasi-bound state. On the other hand, all such poles are not far from the border separating the resonance and bound state domains, since the difference between $|\text{Re} p_0|$ and $|\text{Im} p_0|$ is not very large and the $|\text{Re} E_0|$ is rather small for all poles found. Hence, one can expect that small changes of the factor $g$ could place the poles on this border. Following this idea, we varied $g$ until we found the factors which generates poles on the diagonal. They are given in the Table 2.

Table 1. Positions $p_0 = \sqrt{2\mu E_0}$ of the poles of the $\eta$–nucleus amplitudes with $g = 1$. For each of the nuclei the calculations were done with three values of the range parameter $\alpha$.

|          | $p_0$ (fm$^{-1}$)     | $E_0$ (MeV)     | $\alpha$ (fm$^{-1}$) |
|----------|----------------------|-----------------|----------------------|
| $\eta d$ | $-0.90254 + i0.35880$ | 31.448 – i29.698| 2.357                |
|          | $-0.84562 + i0.32422$ | 27.969 – i25.143| 3.316                |
|          | $-0.82460 + i0.30855$ | 26.813 – i23.333| 7.617                |
| $\eta t$ | $-0.56125 + i0.24475$ | 10.818 – i11.650| 2.357                |
|          | $-0.55747 + i0.27050$ | 10.076 – i12.789| 3.316                |
|          | $-0.52717 + i0.28349$ | 8.3770 – i12.675| 7.617                |
| $\eta^3He$ | $-0.54791 + i0.25111$ | 10.056 – i11.669| 2.357                |
|          | $-0.51111 + i0.30709$ | 7.0788 – i13.312| 3.316                |
|          | $-0.47578 + i0.34354$ | 4.5944 – i13.863| 7.617                |
| $\eta^4He$ | $-0.15056 + i0.18278$ | -0.43713 – i2.2399| 2.357                |
|          | $-0.17940 + i0.24300$ | -1.0933 – i3.5484| 3.316                |
|          | $-0.23100 + i0.30850$ | -1.7016 – i5.8006| 7.617                |

Table 2. The enhancing factors $g$ moving the $\eta$–nucleus amplitude poles to the points $p_0 = \sqrt{2\mu E_0}$ on the diagonal. For each of the nuclei the calculations were done with three values of the range parameter $\alpha$.

|          | $g$   | $p_0$ (fm$^{-1}$)     | $E_0$ (MeV)     | $\alpha$ (fm$^{-1}$) |
|----------|-------|----------------------|-----------------|----------------------|
| $\eta d$ | 1.654 | $-0.32545 + i0.32545$ | -i9.7134       | 2.357                |
|          | 1.566 | $-0.33741 + i0.33741$ | -i10.440       | 3.316                |
|          | 1.535 | $-0.33938 + i0.33938$ | -i10.566       | 7.617                |
| $\eta t$ | 1.361 | $-0.33900 + i0.33900$ | -i9.7467       | 2.357                |
|          | 1.310 | $-0.35424 + i0.35424$ | -i10.643       | 3.316                |
|          | 1.260 | $-0.35378 + i0.35378$ | -i10.615       | 7.617                |
| $\eta^3He$ | 1.330 | $-0.34375 + i0.34375$ | -i10.022       | 2.357                |
|          | 1.221 | $-0.36640 + i0.36640$ | -i11.386       | 3.316                |
|          | 1.144 | $-0.38004 + i0.38004$ | -i12.247       | 7.617                |
| $\eta^4He$ | 0.955 | $-0.16164 + i0.16164$ | -i2.1267       | 2.357                |
|          | 0.911 | $-0.19940 + i0.19940$ | -i3.2363       | 3.316                |
|          | 0.899 | $-0.25130 + i0.25130$ | -i5.1403       | 7.617                |
These factors correspond to an $\eta N$ attraction, which just generates an $\eta$–nucleus binding with $Re E_0 = 0$. Further increase of $g$ moves the poles up and to the right, enhancing the binding and reducing the widths of the states.

It is seen, that the real part of the $a_{\eta N}$, providing the critical binding of $\eta$–meson to a light nucleus, lies within the existing uncertainties, $[0.27, 0.98]$ \cite{23}, for this value. Therefore, in reality the $\eta$–nucleus quasi–bound states can exist with $A \geq 2$. If this is not the case, then at least the near–threshold resonances (poles just below the diagonal) must exist. However, as one sees from both tables, the widths of such quasi–bound and resonance states are small only for the $\eta^4He$ system while for the other systems considered, they are rather large $\sim 20$ MeV, which means that such states is difficult to be detected in experiments.
REFERENCES

[1] R. Frascaria et al., Phys. Rev. C 50, R537(1994).
[2] J. Berger et al., Phys. Rev. Lett., 61, 919(1988).
[3] E. Chiavassa et al., Nucl. Phys., A 519, 413(1990).
[4] C. Peng et al., Phys. Rev. Lett., 63, 2353(1989).
[5] C. Wilkin, Phys. Rev., C 47, R938(1993).
[6] R. S. Bhalerao and L. C. Liu, Phys. Rev. Lett., 54, 865(1985).
[7] Q. Haider and L. C. Liu, Phys. Lett., 172 B, 257(1986).
[8] L. C. Liu and Q. Haider, Phys. Rev., C 34, 1845(1986).
[9] S. Wycech, A. M. Green, and J. A. Niskanen, Los-Alamos e-print archive: nucl-th/9502022, (1995)
[10] R.E. Chrien et al., Phys. Rev. Lett., 60, 2595(1988).
[11] S. A. Rakityansky, S. A. Sofianos, and V. B. Belyaev, in Symposium on Effective Inter- actions in Quantum Systems, Ed. S. A. Sofianos, December 1994, UNISA, Pretoria.
[12] S. A. Rakityansky, S. A. Sofianos, W. Sandhas, and V. B. Belyaev, Los-Alamos e-print archive: nucl-th/9504020, (1995), submitted to Phys. Lett. B
[13] V. B. Belyaev, S. A. Rakityansky, S. A. Sofianos, W. Sandhas, and M. Braun, in: European conference on few–body problems in physics, Peniscola, Spain, 5-9 June 1995.
[14] V. B. Belyaev and J. Wrzecionko, Sov. Journal of Nucl. Phys. 28, 78(1978).
[15] V. B. Belyaev, in: Lectures on the theory of few-body systems, Springer-Verlag, Heidelberg, 1990.
[16] A. K. Kerman, H. Mc Manus, and R. Thaler, Ann. Phys. (NY) 8, 551(1959).
[17] S. Klarfsfeld, J. Martorell, and D. W. L. Sprung, J. Phys. G: Nucl.Phys. 10, 165(1984).
[18] A. Amroun et al., Nucl. Phys., A579, 596(1994).
[19] D. R. Tilley, H. R. Weller, and G.M. Hale, Nucl Phys. A 541, 1(1992).
[20] A. H. Wapstra and G. Audi, Nucl. Phys. 432, 1(1985).
[21] Particle Data Group, Phys. Rev. D50(3), 1319(1994).
[22] C. Bennhold and H. Tanabe, Nucl. Phys., A 530, 625(1991).
[23] M. Batinic, A. Svarc, Los-Alamos e-print archive: nucl-th/9503020, (1995),
[24] W. Cassing, M. Stingl, Phys. Rev., C 26, 22(1982).