Nonlinear absorption of Alfvén waves: model taking into account photorecombination radiation

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Abstract. Damping of the Alfvén wave falling onto the boundary of the dissipative plasma is considered. In this model, the following dissipation mechanisms are included: the hydrodynamic viscosities and thermal conductivities of electrons and ions, magnetic viscosity, thermal relaxation, losses due to bremsstrahlung as well as the synchrotron and photorecombination radiations. These studies are based on the equations of the two-fluid electromagnetic hydrodynamics that fully account for the electron inertia. Numerical analysis based on the implicit finite-difference scheme indicates the existence of a quasi-stationary regime of the Alfvén wave damping in dissipative plasma and shows that the Alfvén wave can be imprisoned in plasma.

1. Introduction

The temperatures observed in the solar corona (up to $\sim 6 \cdot 10^6$ K) are about $10^3$ times higher than those of the Sun photosphere and chromosphere; these layers are located below the Sun corona closer to its center. A possible reason for this anomaly was formulated in [1] and analyzed in the present study, as well as in some previous authors’ publications [2–4]. Namely, such abnormal temperature distribution may form due to the heat released in the corona as a result of the nonlinear absorption of the Alfvén waves generated in the photosphere. In this case, bremsstrahlung of electrons and ions in the corona considerably affects these processes [3].

The Alfvén waves are the transverse sinusoidal vibrations of plasma propagating along the magnetic field. During their propagation, only the transverse components of the plasma vector parameters vary, while their longitudinal components and thermodynamic parameters remain constant.

Within the classical MHD theory, the waves of this kind were described by H. Alfvén in 1942 [5]. The distinctive feature of the Alfvén waves is the fact that they are the exact solutions of both the equations of plasma dynamics linearized in the vicinity of a homogeneous state and the nonlinear plasma dynamics equations (classical MHD [6]; two-fluid EMHD [7], and other models). This fact makes it possible to study such waves in detail and compare the predictions of the linear and the nonlinear theories.

The nonlinear absorption of the Alfvén waves considered in the present and our previous papers occurs due to dissipative factors such as the magnetic and hydrodynamic viscosities, relaxation of ion...
and electron temperatures, as well as radiation. In [3, 4], we examined the contribution of electron bremsstrahlung, and in the present paper, for the Alfvén waves with large amplitudes, we will additionally take into account the synchrotron and the photorecombination radiations.

Since the dissipative processes under consideration are the small-scale phenomena, for the cosmic plasma, their consideration in the framework of the classical MHD encounters an obvious obstacle: in the classical MHD problems, the characteristic length scale $L$ must satisfy the condition (which is obviously true for cosmic plasma) $L \gg \ell_s = c/\omega_p$ (here, $\ell_s$ is the skin length and $\omega_p$ is the plasma frequency), whereas, for the lengths $\gg \ell_s$, the dissipative effects are vanishingly small. There are two ways to get over this difficulty: either use the turbulent MHD theories of plasma [8, 9] or use the two-fluid description based on the electromagnetic hydrodynamics (EMHD) of plasma [7], which does not require the restriction $L \gg \ell_s$ to be fulfilled. If we follow the first approach, then, in order to obtain the closed set of equations of turbulent motion, it will be necessary to calculate the semiempirical turbulence coefficients (viscosity, et al. [8]) that is quite problematic for astrophysical problems. Therefore, we prefer the second approach.

The hypothesis on plasma incompressibility is true (see [10]), if the relations $v^2 \bar{\pm} c^2 \bar{\pm}$ hold, where $v_\pm$ are the hydrodynamic velocities of electrons and ions, respectively; $c_\pm$ are their speeds of sound; $\rho_\pm = (k_B T / m_\pm)^{1/2}$, and $k_B$ is the Boltzmann constant. Calculations show that, when considering the nonlinear absorption of the Alfvén waves in the solar corona, the incompressibility conditions are satisfied, despite the low plasma density: according to [11], typical plasma densities in the solar corona are in the range of $\rho = 10^{-12} - 10^{-15}$ g/cm$^3$. This conclusion is confirmed by the results of [2], where the corona plasma was assumed to be compressible.

The studies presented show that the synchrotron radiation has almost no effect on the nonlinear absorption process of the Alfvén waves, while the effect of the photorecombination radiation on the absorption is only quantitative. The basic qualitative conclusions of [3] still remain valid: (a) the absorbed Alfvén wave penetrates the solar corona to a finite depth, which is considerably smaller than the penetration depth calculated without allowance for the photorecombination radiation; (b) the parameters of the absorbed Alfvén wave stabilize to the quasi-stationary parameters, which can be found from the solution of the boundary value problem described by a set of ordinary differential equations defined on the half-line; (c) in calculations performed with allowance for photorecombination radiation, the electron and ion temperatures are much lower than those obtained with allowance for only the bremsstrahlung.

2. Equations of electromagnetic hydrodynamics of plasma (EMHD)

It is assumed that the electron and ion dynamics in plasma can be described by the following equations of incompressible EMHD [7,12,13]:

$$\rho = \text{const}, \quad \text{div} \mathbf{U} = 0, \quad \frac{\partial \rho \mathbf{U}}{\partial t} + \text{Div} \mathbf{P} = \text{Div} \mathbf{P}$$

$$\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \text{rot} \mathbf{E} = 0, \quad \text{div} \mathbf{E} = 0, \quad \text{div} \mathbf{H} = 0, \quad j = \frac{c}{4\pi} \text{rot} \mathbf{H}$$

$$\mathbf{E} + \frac{c^2 \lambda \lambda}{4\pi \rho} \text{rot} \mathbf{E} = \frac{1}{\sigma c} [\mathbf{U}, \mathbf{H}] + \frac{1}{\rho} \text{Div} \mathbf{W},$$

where $\rho = \rho_e + \rho_i$ is the total density of electrons and ions, and $\mathbf{U} = (\rho_e \mathbf{v}_e + \rho_i \mathbf{v}_i)/\rho$ is their summarized mass velocity. Hereinafter, the indices $\pm$ mark the parameters of electrons and ions; $\lambda_\pm = m_\pm / e_\pm$, and $\rho_\pm, \mathbf{v}_\pm, m_\pm,$ and $e_\pm$ are the densities, hydrodynamic velocities, masses, and absolute values of electron and ion charges, respectively; electrons and ions are assumed to be
polytropic gases with the same adiabatic index $\gamma$. For incompressible plasma, the momentum flux tensor $\Pi = \Pi^{(h)} + \Pi^{(p)} + \Pi^{(c)}$, the viscous stress tensor $P = \Pi^{(h)} + \Pi^{(i)}$, and the Hall stress tensor $W$ have the following forms:

$$
\Pi^{(h)} = \rho U U + \rho_{2} I_{3}, \ \Pi^{(p)} = \frac{H_{2}^{2}}{8\pi} I_{3} - \frac{HH}{4\pi}, \ \Pi^{(c)} = \lambda_{4} \frac{jj}{\rho}, \ p_{x} = p_{a} + p_{s},
$$

$$
W = (\lambda_{6} - \lambda_{8})(\Pi^{(h)} + \Pi^{(c)}) + (\lambda_{p} - \lambda_{s}, p) I_{3} + \lambda_{6} \lambda_{8}(jU + Uj) - U^{(h)} - \Pi^{(c)},
$$

$$
\Pi^{(h)} = 2\mu_{s} D^{(h)}, \ \Pi^{(p)} = 2\mu^{(h)} D^{(p)}, \ \Pi^{(c)} = 2\mu^{(c)} D^{(c)}, \ \Pi^{(i)} = 2\mu_{s} D^{(i)}.
$$

Here, $D^{(h)} = \text{def}U$, $D^{(p)} = \text{def}(j/\rho)$, and $D^{(i)} = \text{def}v_{s}$ are the strain tensors; $\mu_{s}$ are the hydrodynamic viscosities of electrons and ions; $\mu_{e} = \mu_{e} + \mu_{e}$, $\mu_{i} = \lambda_{6} \mu_{e} - \lambda_{8} \mu_{i}$, and $\mu^{(i)} = \lambda_{6} \mu_{e} + \lambda_{8} \mu_{i}$. Taking into account the fact that the transport coefficients depend on the electron and ion temperatures $T_{\pm}$, we should add the following equations for the temperatures to set of equations (1) [10]:

$$
\rho_{c} c_{p}^{3} \left[ \frac{\partial T_{\pm}}{\partial t} + v_{s} \cdot \text{grad}T_{\pm} \right] = \text{div} \left( \chi_{\pm} \nabla T_{\pm} \right) + \text{tr} \left( \Pi_{\pm} D_{\pm} \right) + \frac{m_{e}^{2}}{m_{s}^{2}} j_{\pm}^{2} \pm b(T_{\pm} - T_{s}) - p_{\pm}^{\pm}, \quad (2)
$$

where $p_{\pm}^{\pm} + p_{\pm}^{\pm} + p_{\pm}^{\pm}$, $c_{p}^{3} = k_{0}/((\gamma - 1)m_{e})$ are the thermal capacities at constant pressure; $\pm b(T_{\pm} - T_{s})$ is the quantity of heat acquired by the plasma components in elastic collisions; $b$ and $\chi_{s}$ are the thermal relaxation rate and heat conductivity coefficients for electrons and ions, respectively; $p_{\pm}^{\pm}$, $p_{\pm}^{\pm}$, and $p_{\pm}^{\pm}$ are the losses due to bremsstrahlung, photorecombination radiation, and synchrotron radiation of electrons and ions, respectively. Equations (1)–(2) form the closed determined set of equations for finding the unknown functions $\rho$, $U$, $T_{\pm}$, $H$, and $E$.

The $\mu_{s}, \chi_{s}, \sigma$, and $b$ transport coefficients and the $p_{\pm}^{\pm}, p_{\pm}^{\pm}$, and $p_{\pm}^{\pm}$ losses can be obtained from approximate solutions of the kinetic equations [14], and, for $Z = 1$, they are assumed to be the following [14–21]:

$$
\mu_{s} = 3.44 \cdot 10^{-18} \left( \frac{m_{s}}{m_{e}} \right)^{1/2} T_{s}^{3/2}, \ \mu_{e} = 1.857 \cdot 10^{-18} T_{s}^{5/2}, \ \sigma = 0.906 \cdot 10^{-7} T_{s}^{3/2}, \ b = 1.353 \cdot 10^{-16} \left( \frac{m_{e}}{m_{s}} \right)^{3} \frac{p_{3}^{4}}{3^{2}},
$$

$$
\chi_{s} = 0.244 \cdot 10^{-5} T_{s}^{5/2}, \ \chi_{e} = 0.429 \cdot 10^{-5} \left( \frac{m_{e}}{m_{e}} \right)^{1/2} T_{s}^{3/2}, \ p_{T}^{h} = 6.777 \cdot 10^{-21} n_{e} n_{e} T_{s}^{1/2}, \ p_{T}^{s} = \frac{4}{3} \frac{k_{B} e^{4}}{m^{3} c^{2}} H^{2} n_{e} \Phi,
$$

where $p_{T}^{h} = p_{T}^{h} = p_{T}^{h} = 0$; the $T_{s}$ temperatures are in units of Kelvin; $p_{T}^{h}$, and $p_{T}^{s}$ are calculated in units of erg/(s·cm$^{3}$) and, for $Z = 1$, $p_{T}^{h}$ / $p_{T}^{h}$ is calculated as follows: $p_{T}^{h} = p_{T}^{h}$, $p_{T}^{h} = p_{T}^{h}$,

$$
\frac{p_{T}^{h}}{p_{T}^{h}} = \frac{10^{3} k_{0}}{T_{s} (K)}.
$$

(3)

For the temperatures of the order of $\sim 10^{3} K$ and $\sim 10^{4} \times 10^{6} K$, we set $k_{s} = 3.33$ [22] and $k_{t} = 371.2$ [20], respectively; for the temperatures higher than $\geq 10^{7} K$, the photorecombination radiation can be neglected [23]. The synchrotron radiation can also be neglected, as shown in [23].
3. The Alfvén wave consideration using the EMHD equations

For the dissipativeless plasma with the planar geometry, the EMHD equations have the following exact solutions [2]:

\[ U_\perp = u(t)e^{i\kappa x}, \quad H_\perp = h(t)e^{i\kappa x}, \quad E_\perp = e(t)e^{i\kappa x}, \quad T_\parallel = \text{const}, \quad \rho = \text{const}, \quad U_x = 0, \]

which are called the plane Alfvén waves. Here, \( \kappa > 0 \) is an arbitrary coefficient, \( U_\perp = U_\perp^x + iU_\perp^y \), \( H_\perp = H_\perp^x + iH_\perp^y \), and \( E_\perp = E_\perp^x + iE_\perp^y \); besides, \( H_\perp \) is \( \text{const} \) and \( e(t) \) can be explicitly expressed in terms of \( u(t) \) and \( h(t) \) [2]. The functions \( u(t) \), \( h(t) \) have the following form:

\[ u(t) = C_1 e^{i\omega_0 t} + C_2 e^{i\omega_1 t}, \quad h(t) = (4\pi\rho)^{1/2} (kv_\perp)^{-1} \left( C_1 \omega_0 e^{i\omega_1 t} + C_2 \omega_1 e^{i\omega_1 t} \right). \] (5)

Here, \( r = \kappa c / \omega_p \), \( \omega_p = (4\pi\rho)^{1/2} (\vec{\lambda}, \vec{\lambda})^{-1/2} \) is the plasma frequency; \( C_1 \) and \( C_2 \) are arbitrary complex constants. The transverse component of the current density \( j_\parallel = j_\parallel^x + i j_\parallel^y \) has the form of \( j_\parallel = j(t)e^{i\kappa x} \), and \( j_x = 0 \), where \( j(t) = -\frac{\kappa c}{4\pi} h(t) \). Finally, we obtain:

\[ \omega_0 = \omega_0(\kappa) = \frac{k v_\perp}{2} \left( \frac{r \lambda^2}{1 + r^2} \pm \frac{4}{(1 + r^2)^2} \right)^{1/2}, \] (6)

where for \( H_\perp > 0 \), \( -\omega^-_\perp < \omega < \omega^+_\perp \), and for \( H_\perp < 0 \), \( -\omega^-_\perp < \omega < \omega^+_\perp \).

In the long-wave limit, for \( r \ll 1 \), we have \( \omega_0(\kappa) \sim \pm \kappa v_\perp \) and the solution obtained above turns into the classical Alfvén wave.

4. Statement of the spatial absorption problem

We consider the plane Alfvén wave propagating from left to right in the domain \( x < 0 \) and falling onto the boundary \( x = 0 \) of the dissipative plasma occupying the half-space \( x > 0 \). Further propagation of this wave in the domain \( x > 0 \) is attended with its absorption, which is the subject of our studies. Further, we consider the case of \( H_\perp < 0 \). Plasma in the domain \( x > 0 \) is assumed to be magnetized, motionless, homogeneous, and isothermal. Thus, at the initial time, we have the following initial conditions in the domain \( x > 0 \):

\[ U_\perp |_{x=0} = 0, \quad U_\parallel |_{x=0} = 0, \quad T_\parallel |_{x=0} = T_0, \quad \rho |_{x=0} = \text{const}, \quad H_\perp |_{x=0} = 0, \]

where the constant \( \rho \) and the longitudinal magnetic field \( H_\perp \) are the same as in the domain \( x < 0 \), whence the Alfvén wave comes.

The dissipative plasma parameters at the boundary \( x = 0 \) coincide with those of the Alfvén wave for \( x = 0 \). At infinity, the parameters of the dissipative plasma coincide with those of the unperturbed plasma.

In the case of plane symmetry (\( \partial / \partial y = \partial / \partial z = 0 \)), we write set of equations (1)–(3) in dimensionless form and set \( U = U_\perp \), \( H = H_\perp \), \( E = E_\perp \), and \( j = j_\perp \). Thus, we obtain the following equations:

\[ \rho \frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left( \frac{H_\perp H - \mu \varepsilon \partial U}{\sigma} - \xi \mu \frac{\partial j}{\partial x} \rho \right) = 0, \quad \frac{\partial H_\perp}{\partial t} + \frac{\partial j}{\partial x} E = 0, \quad j = \frac{\partial H_\perp}{\partial x}, \]

\[ E - \frac{\varepsilon^2}{\rho} \frac{\partial^2 E}{\partial x^2} = \frac{\xi}{\sigma} \frac{j}{\sigma} + iH_\perp U + \frac{\xi}{\rho} \frac{\partial}{\partial x} \left[ \Lambda H_\perp H - \mu \frac{\partial U}{\partial x} - \xi \mu \frac{\partial j}{\partial x} \rho \right], \]
is the Coulomb logarithm, and \( \chi_{\pm} \) can be found in [3, 4]. For \( Z = 1 \), they are given in Section 2 and, in dimensionless form, they are as follows: \( \mu_a = \mu + \mu_c \), \( \mu_c = (\lambda_c / \lambda) \mu \), \( \mu_a = (\lambda_c / \lambda) \mu \), \( \mu_c = (\lambda_c / \lambda) \mu \), \( \mu_a = T^{3/2} / R_s \), \( \sigma = \sigma_o T^{3/2} \), and \( \chi_{\pm} = C_s (\xi / \zeta) T^{3/2} \), where \( \sigma_o = 2.59 \), \( C_s = (m_e / m_p) \cdot 0.563 \), \( R_s = (\zeta / \xi) \cdot (m_p / m_e) \cdot 0.277 \), and \( C_s = (m_e / m_p) \cdot 2.11 \).

At last, \( \xi, \zeta, \) and \( \xi_1 \) are the similarity numbers, which have the following form:

\[
\xi = \frac{\xi_c}{L_0} = \frac{c \sqrt{\lambda_c \lambda}}{4 \pi \rho_o L_0}, \quad \zeta = \frac{(4 \pi \rho_p)^{1/2} c e^b}{2 \pi \rho_s L_0 \left(1 + Z \frac{m_p}{m_e}\right)}, \quad \xi_1 = \frac{\rho_e^{3/2}}{H_0^2} \frac{10^3 \rho e^b}{c^2 h m_e} 64 \pi^{3/2} \left(1 + Z \frac{m_p}{m_e}\right)^{-3/2}
\]

where \( \xi_c = c / \alpha_p \) is the skin length, \( L = 15 \) is the Coulomb logarithm, \( h \) is the Planck constant; and \( L_0 \), \( \rho_o \), \( H_0 \), etc., are the characteristic scales of length, density, magnetic field strength, etc., respectively. When going to the dimensionless parameters, we have assumed that \( t_0 = L_0 \cdot v_0 \), \( v_0 = v_\lambda = H_0 / (4 \pi \rho_p)^{1/2} \), \( E_0 = v_0 H_0 / c \), \( j_0 = c H_0 / (4 \pi L_0) \), and \( T_0 = v_\lambda^2 \lambda_c e / (2 k_o) \).

Thus, we have to find a solution of the initial boundary-value problem, described by set of equations (7), on the half-line \( x \geq 0 \) with the following dimensionless initial conditions:

\[
U |_{x=0} = 0, \quad H |_{x=0} = 0, \quad E |_{x=0} = 0, \quad T_s |_{x=0} = T^0, \quad x \geq 0,
\]  

and, for \( x = 0 \) and \( x = +\infty \), the dimensionless boundary conditions are as follows:

\[
U |_{x=0} = U_0 e^{i\omega t}, \quad H |_{x=0} = \frac{\rho U_0^{3/2}}{H_0} e^{i\omega t}, \quad E |_{x=0} = -\frac{\rho U_0^{3/2}}{H_0} e^{i\omega t}, \quad E |_{x=-\infty} = \frac{i U_0^{3/2}}{H_0} \left( H_0 + H_s \Lambda \xi \omega - \xi^2 \omega^2 \right) e^{i\omega t},
\]

\[
U |_{x=\infty} = 0, \quad H |_{x=\infty} = 0, \quad E |_{x=\infty} = 0, \quad j |_{x=\infty} = 0, \quad T_s |_{x=\infty} = T^0.
\]  

For the amplitudes \( U_0 < 1 \) and \( U_0 \geq 1 \), we assumed \( k_i = 3.33 \) and \( k_i = 371.2 \), respectively.

5. Numerical analysis

The numerical method used here for finding an approximate solution of problem (7)–(9) is described in [23].

Below, we present some results obtained for \( \rho_o = 10^{-12} g / cm^3 \), \( H_0 = 1 G \), and \( Z = 1 \). In this case, \( \zeta = 3 \cdot 10^{-4} \), \( \xi_1 = 3 \), and the skin length is \( \xi_c = c / \alpha_p \approx 1 cm \). With allowance for the small-scale absorption processes, we set \( L_0 = \xi_c \), and then we obtain \( \xi = 1 \). For the above parameters of the background dissipative plasma, the Alfvén velocity is equal to \( v_\lambda = H_0 / (4 \pi \rho_p)^{1/2} = 2.8 \cdot 10^6 cm / s \), \( t_0 = L_0 / v_\lambda = 3.45 \cdot 10^{-6} s \), \( T_0 \approx 3 \times 10^3 K \), \( \omega = -30 \), and \( H_s = -1 \).

For the small transverse velocity amplitude \( U_0 \left(|U_0| = 0.1\right) \) of the incident Alfvén wave (for which \( k_i = 3.33 \)), the calculation results can be found in [23]. Below, we present the results for the large
amplitude $|U_0|=1$ and $k_r=371.2$ (figures 1–4). These graphs show that the stabilization of the electron temperature occurs much faster than that of the ion temperature. The maximal values of the electron and ion temperatures are 16% lower than those obtained with allowance for merely bremsstrahlung. The penetration depth also becomes smaller: for electrons and ions, it is $\sim 6.5$ and $\sim 5.5$ times smaller, respectively. When both bremsstrahlung and recombination radiation are taken into account, the stabilization rate of the Alfven wave parameters is 6 times higher than that in the case of taking into account only bremsstrahlung. We note that it takes 4 days for the PC with Intel Core i5-7400 processor to perform calculations in both cases: for $U_0=1$ [23] (in the case of taking into account only bremsstrahlung) and for $U_0=1$ and $k_r=371.2$ (the case of additional taking into account the photorecombination radiation).

**Figure 1.** Electron temperature stabilization for $U_0=1$.

**Figure 2.** Ion temperature stabilization for $U_0=1$.

**Figure 3.** Steady state electron temperature profiles with allowance for radiation for $U_0=1$.

**Figure 4.** Steady state ion temperature profiles with allowance for radiation for $U_0=1$.

**Acknowledgments**

This work was supported by the Russian Science Foundation (project no. 16-11-10278).

**References**

[1] McIntosh S W, Pontien B P, Carlsson M, Hansteen V, Boerner P and Goossens M 2011 *Nature* 475 478

[2] Gavrikov M B and Taiurskii A A 2013 *Matematiceskoe modelirovanie* 25 8 65
[3] Gavrikov M B and Taiurskii A A 2017 *Herald of the Bauman Moscow State Tech. Univ. Nat. Sci.* 2 40
[4] Gavrikov M B and Taiurskii A A 2018 *Herald of the Bauman Moscow State Tech. Univ. Nat. Sci.* 3 82
[5] Alfvén H 1950 *Cosmical electrodynamics* (Oxford University Press)
[6] Landau L D, Lifshits E M and Pitaevskii L P 1984 *Electrodynamics of Continuous Media* vol 8 (2nd ed Butterworth–Heinemann)
[7] Gavrikov M B 2018 *Two-fluid electromagnetic hydrodynamics* (Moscow: KRASAND)
[8] Gorbatskiy V G 1977 *Space gasdynamics* (Moscow: Nauka) 360 p
[9] Pai S I 1962 *Magnetogasdynamics and Plasma Dynamics* (Springer) 197 p
[10] Landau L D and Lifshitz E M 1987 *Fluid Mechanics* vol 6 (2nd ed Butterworth-Heinemann)
[11] Allen K U *Astrophysical quantities* 1973 (University of London Athlone Press)
[12] Gavrikov M B 2006 The basic equations for two–fluid electromagnetic hydrodynamics Part I *Preprint* 59 (Inst. Appl. Math., the Russian Academy of Sciences) 28 p
[13] Gavrikov M B and Taiurskii A A 2015 Electron inertia effect on incompressible plasma flow in a planar channel *J. Plasma Phys.* 81 495810506
[14] Braginskii S I 1965 Transport processes in Plasma In *Reviews of Plasma Physics* vol 1 ed M A Leontovich (New York: Consultants Bureau) p 205
[15] Spitzer L *Physics of Fully Ionized Gases* 1962 (New York: 2nd ed Interscience)
[16] Chapman S and Cowling T G 1952 *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press)
[17] Imshennik V S 1961 *Astronomy Reports* 38 652
[18] Landau L D 1937 *JETP* 7 203
[19] Chukbar K V 2008 *Lectures on transport phenomena in plasmas* (Dolgoprudnyi: Izdatelskii dom “Intellekt”) 256 p
[20] Grim G 1983 Radiation Processes in Plasma, in *Basics of Plasma Physics* vol 1 (Moscow: Energoatomizdat)
[21] Trubnikov B A 1973 in *Voprosy teorii plazmy* vol 7 ed B B Kadomtsev (Moscow: Energoatomizdat) 274
[22] Morozov A I 2006 *Introduction to plasma dynamics* (Moscow: FIZMATLIT) 576 p
[23] Gavrikov M B and Taiurskii A A 2019 *Matematicheskoe modelirovanie* 31 12 71