GRAVITATIONAL WAVES FROM COMPACT SOURCES *

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We review sources of high-frequency gravitational waves, summarizing our current understanding of emission mechanisms, expected amplitudes and event rates. The most promising sources are gravitational collapse (formation of black holes or neutron stars) and subsequent ringing of the compact star, secular or dynamical rotational instabilities and high-mass compact objects formed through the merger of binary neutron stars. Significant and unique information for the various stages of the collapse, the structure of protoneutron stars and the high density equation of state of compact objects can be drawn from careful study of gravitational wave signals.

1. Introduction

The new generation of gravitational wave (GW) detectors is already collecting data, improving the achieved sensitivity by at least one order of magnitude compared to operating resonant bar detectors. Broadband GW detectors are sensitive to frequencies between 50 and a few hundred Hz. In their advanced stage, the current GW detectors will have a broader bandwidth but will still not be sensitive enough to frequencies above 500 to 600Hz. Nevertheless, improved sensitivity can be achieved at high frequencies through narrow-band operation 1,2. In addition, there are proposals for constructing wide band resonant detectors in the kHz band 3.

In this short review we will discuss some of the sources that are in the high frequency band (≥ 500 – 600Hz), where the currently operating interferometers are sensitive enough only if they are narrowbanded. Since there exist a variety of GW sources with very interesting physics to be

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explored, this high-frequency window deserves special attention. If either resonant or narrow-band interferometers achieve the required sensitivity, a plethora of unique information can be deduced from detected signals.

2. Gravitational collapse

*Core collapse.* One of the most spectacular astrophysical events is the core collapse of massive stars, leading to the formation of a neutron star (NS) or a black hole (BH). The outcome of core collapse depends sensitively on several factors: mass, angular momentum and metallicity of progenitor, existence of a binary companion, high-density equation of state, neutrino emission, magnetic fields, etc. Partial understanding of each of the above factors is emerging, but a complete and consistent theory for core collapse is still years away.

Roughly speaking, isolated stars more massive than \( \sim 8 - 10 M_\odot \) end in core collapse and \( \sim 90\% \) of them are stars with masses \( \sim 8 - 20 M_\odot \). After core bounce, most of the material is ejected and if the progenitor star has a mass \( M \lesssim 20 M_\odot \) a neutron star is left behind. On the other hand, if \( M \gtrsim 20 M_\odot \) fall-back accretion increases the mass of the formed proton-neutron star (PNS), pushing it above the maximum mass limit, which results in the formation of a black hole. Furthermore, if the progenitor star has a mass of roughly \( M \gtrsim 45 M_\odot \), no supernova explosion is launched and the star collapses directly to a BH\(^4\).

The above picture is, of course, greatly simplified. In reality, the metallicity of the progenitor, the angular momentum of the pre-collapse core and the presence of a binary companion will decisively influence the outcome of core collapse\(^5\). Very massive stars lose mass through strong stellar winds. The mass loss rate is sensitive to the metallicity of the star and can be very high, allowing a 60 \( M_\odot \) star to leave behind a typical 1.4 \( M_\odot \) neutron star, instead of a much more massive black hole. Since mass-loss is a complex phenomenon, current observations are still insufficient to constrain the different possible outcomes of core collapse for stars with \( M \gtrsim 45 M_\odot \). Through mass transfer or common envelope episodes, a binary companion can cause a massive star to enter the rapid mass-losing (Wolf-Rayet) phase earlier, making wind mass loss more effective. Roughly half of all stars are in sufficiently close binaries that binary interactions must be taken into account when trying to predict to outcome of core collapse. Rotation influences the collapse by changing dramatically the properties of the convective region above the proto-neutron star core. Centrifugal forces slow down in-
falling material in the equatorial region compared to material in the polar axis, yielding a weaker bounce. This asymmetry between equator and poles also strongly influences the neutrino emission and the revival of the stalled shock by neutrinos.

The supernova event rate is 1-2 per century per galaxy and about 5-40% of them produce BHs in delayed collapse (through fall-back accretion), or direct collapse.

Initial rotation rates. Of considerable importance is the initial rotation rate of proto-neutron stars, since (as will be detailed in the next sections) most mechanisms for emission of detectable gravitational waves from compact objects require very rapid rotation at birth (rotational periods of the order of a few milliseconds or less). Since most massive stars have non-negligible rotation rates (some even rotate near their break-up limit), simple conservation of angular momentum would suggest a proto-neutron star to be strongly differentially rotating with very high rotation rates and this picture is supported by numerical simulations of rotating core collapse.

On the other hand, observationally, we know of pulsars (that have been associated with a supernova remnant) rotating only as fast as with a 16ms period, which suggest a period of at least several milliseconds at birth. A possible explanation for this discrepancy is that the pre-supernova core has been slowed down by e.g. magnetic torques. The most recent evolutionary models of rotating core-collapse progenitors (including the effect of magnetic torques) suggest that progenitors of neutron stars with typical masses of $\sim 1.4M_\odot$ are indeed slowly rotating, producing remnants with periods at birth of the order of 10 ms. Nevertheless, the same study finds that very massive progenitors evolve so rapidly that the angular momentum transfer out of the core, by magnetic torques, is diminished, yielding heavy proto-neutron stars with rotational periods of the order of a few milliseconds (provided mass-loss takes place to prevent black hole formation). Binary interactions can also accelerate some phases of progenitor evolution, allowing for fast initial spins. If the magnetic torques operate efficiently, it is clear that rapid rotation at birth will have an event rate much smaller than the usual galactic supernova rate. Still, strong emission mechanisms (e.g. bar-mode instabilities) could yield detectable signals at acceptable event rates.

Other ways to form a rapidly rotating proto-neutron star would be through fall-back accretion, through the accretion-induced collapse of a white dwarf, or through the merger of binary white dwarfs in glob-
ular clusters. It is also relevant to take into account current gamma-ray-burst models. The collapsar model requires high rotation rates of a proto-black hole. In addition, a possible formation scenario for magnetars involves a rapidly rotating protoneutron star formed through the collapse of a very massive progenitor and some observational evidence is already emerging.

Gravitational wave emission. Gravitational waves from core collapse have a rich spectrum, reflecting the various stages of this event. The initial signal is emitted due to the changing axisymmetric quadrupole moment during collapse. In the case of neutron star formation, the quadrupole moment typically becomes larger, as the core spins up during contraction. In contrast, when a rapidly rotating neutron star collapses to form a Kerr black hole, the axisymmetric quadrupole moment first increases but is finally reduced by a large factor when the black hole is formed.

A second part of the gravitational wave signal is produced when gravitational collapse is halted by the stiffening of the equation of state above nuclear densities and the core bounces, driving an outwards moving shock. The dense fluid undergoes motions with relativistic speeds \((v/c \sim 0.2\text{-}0.4)\) and a rapidly rotating proto-neutron star thus oscillates in several of its axisymmetric normal modes of oscillation. This quasi-periodic part of the signal could last for hundreds of oscillation periods, before being effectively damped. If, instead, a black hole is directly formed, then black hole quasi-normal modes are excited, lasting for only a few oscillation periods. A combination of neutron star and black hole oscillations will appear if the proton-neutron star is not stable but collapses to a black hole.

In a rotating proto-neutron star, nonaxisymmetric processes can yield additional types of gravitational wave signals. Such processes are dynamical instabilities, secular gravitational-wave driven instabilities or convection inside the proto-neutron star and in its surrounding hot envelope. Anisotropic neutrino emission is accompanied by a gravitational wave signal. Nonaxisymmetries could already be present in the pre-collapse core and become amplified during collapse. Furthermore, if there is persistent fall-back accretion onto a proto-neutron star or black hole, these can be brought into ringing.

Below, we discuss in more detail those processes which result in high frequency gravitational radiation.
2.1. Neutron star formation

Core collapse as a potential source of GWs has been studied for more than three decades (some of the most recent simulations can be found in \cite{26,27,20,11,28,29,30,31,32,33}). The main differences between the various studies are the progenitor models (slowly or rapidly rotating), equation of state (polytropic or realistic), gravity (Newtonian or relativistic) and neutrino emission (simple, sophisticated or no treatment). In general, the gravitational wave signal from neutron star formation is divided into a core bounce signal, a signal due to convective motions and a signal due to anisotropic neutrino emission.

**Core bounce signal.** The core bounce signal is produced due to rotational flattening and excitation of normal modes of oscillations, the main contributions coming from the axisymmetric quadrupole (\(l = 2\)) and quasi-radial (\(l = 0\)) modes (the latter radiating through its rotationally acquired \(l = 2\) piece). If detected, such signals will be a unique probe for the high-density EOS of neutron stars\cite{34,35}. The strength of this signal is sensitive to the available angular momentum in the progenitor core. If the progenitor core is rapidly rotating, then core bounce signals from Galactic supernovae (\(d \sim 10\) kpc) are detectable even with the initial LIGO/Virgo sensitivity at frequencies \(< 1\) kHz. In the best-case scenario, advanced LIGO could detect signals from distances of 1 Mpc, but not from the Virgo cluster (\(\sim 15\) Mpc), where the event rate would be high. The typical GW amplitude from 2D numerical simulations \cite{11,28} for an observer located in the equatorial plane of the source is

\[
h \approx 9 \times 10^{-21} \varepsilon \left(\frac{10\text{kpc}}{d}\right)
\]

where \(\varepsilon \sim 1\) is the normalized GW amplitude. For such rapidly rotating initial models, the total energy radiated in GWs during the collapse is \(< 10^{-6} - 10^{-8} M_\odot c^2\). If, on the other hand, progenitor cores are slowly rotating (due to e.g. magnetic torques\cite{13}), then the signal strength is significantly reduced, but, in the best case, is still within reach of advanced LIGO for galactic sources.

Normal mode oscillations, if excited in an equilibrium star at a small to moderate amplitude, would last for hundreds to thousands of oscillation periods, being damped only slowly by gravitational wave emission or viscosity. However, the proto-neutron star immediately after core bounce has a very different structure than a cold equilibrium star. It has a high internal
temperature and is surrounded but an extended, hot envelope. Nonlinear oscillations excited in the core after bounce can penetrate into the hot envelope. Through this damping mechanism, the normal mode oscillations are damped on a much shorter timescale (on the order of ten oscillation periods), which is typically seen in the core collapse simulations mentioned above.

*Convection signal.* The post-shock region surrounding a proto-neutron star is convectively unstable to both low-mode and high-mode convection. Neutrino emission also drives convection in this region. The most realistic 2D simulations of core collapse to date\(^\text{30}\) have shown that the gravitational wave signal from convection significantly exceeds the core bounce signal for slowly rotating progenitors, being detectable with advanced LIGO for galactic sources, and is detectable even for nonrotating collapse. For slowly rotating collapse, there is a detectable part of the signal in the high-frequency range of 700Hz-1kHz, originating from convective motions that dominate around 200ms after core bounce. Thus, if both core bounce signal and a convection signal would be detected in the same frequency range, these would be well separated in time.

*Neutrino signal.* In many simulations the gravitational wave signature of anisotropic neutrino emission has also been considered\(^\text{36,37,38}\). This type of signal can be detectable by advanced LIGO for galactic sources, but the main contribution is at low frequencies for a slowly rotating progenitor\(^\text{30}\). For rapidly rotating progenitors, stronger contributions at high frequencies could be present, but would probably be buried within the high-frequency convection signal.

Numerical simulations of neutron star formation have gone a long way, but a fully consistent 3D simulation including relativistic gravity, neutrino emission and magnetic fields is still missing. The combined treatment of these effects might not change the above estimations by orders of magnitude but it will provide more conclusive answers. There are also issues that need to be understood such as pulsar kicks (velocities exceeding 1000 km/s) which suggest that in a fraction of newly-born NSs (and probably BHs) the formation process may be strongly asymmetric\(^\text{39}\). Better treatment of the microphysics and construction of accurate progenitor models for the angular momentum distributions are needed. All these issues are under investigation by many groups.
2.2. Neutron star ringing through fall-back accretion

A possible mechanism for the excitation of oscillations in a proto-neutron star after core bounce is the fall-back accretion of material that has not been expelled by the revived supernova shock. The isotropy of this material is expected to be broken due to e.g. rotation or nonaxisymmetric convective motions, thus a large number of oscillation modes will be excited as this material falls back onto the neutron star. This process is, of course, complex and the detectability of gravitational waves from these oscillations will depend on several factors, such as the fall-back accretion rate, the degree of asymmetry of the fall-back material the structure of the proto-neutron star envelope, the presence of magnetic fields etc.

Recently, the ringing of a neutron star through fall-back accretion has been modeled through relativistic 2D nonlinear hydrodynamical simulations\textsuperscript{40}. Quadrupolar shells of matter were accreted on a static neutron star (in the approximation that the background spacetime remains unchanged). Gravitational waves were then extracted through the Zerilli-Moncrief formalism. The gravitational wave signal from such a process comprises a narrow peak at the $l = 2$ normal mode frequency of the neutron star and a very broad peak, featuring interference fringes, centered at a much higher frequency. Since the frequency of the broad peak is still too low to be identified with a $w$-mode of the star, the interpretation for this part of the signal is that it is related to the motion of the fluid shell and the reflection of the gravitational-wave pulse from this motion in the external Zerilli potential, which also creates the interference fringes. The accretion of a quadrupolar shell containing 1\% of the mass of the star releases gravitational waves with a total energy similar to the energy emitted immediately after core bounce. It is thus interesting to consider this mechanism in more detail, since the excitation of the normal modes in the neutron star happens when the star has already cooled somewhat (compared to the proto-neutron star immediately after core bounce) which simplifies the identification of observed oscillations with normal modes of cold neutron star models. The formation of a dense torus as as a result of stellar gravitational collapse, binary neutron star merger or disruption. Such a system either becomes unstable to the runaway instability or exhibit a regular oscillatory behavior, resulting in a quasi-periodic variation of the accretion rate as well as of the mass quadrupole suggesting a new sources of potentially detectable gravitational waves\textsuperscript{41}. 
2.3. **Black hole formation**

The gravitational-wave emission from the formation of a Kerr BH is a sum of two signals: the *collapse signal* and the *BH ringing*. The collapse signal is produced due to the changing multipole moments of the spacetime during the transition from a rotating iron core or proto-neutron star to a Kerr BH. A uniformly rotating neutron star has an axisymmetric quadrupole moment given by\(^\text{42}\)

\[ Q = -\frac{a J^2}{M} \]  

(2)

where \(a\) depends on the equation of state and is in the range of \(2 - 8\) for \(1.4M_\odot\) models. This is several times larger in magnitude than the corresponding quadrupole moment of a Kerr black hole \((a = 1)\). Thus, the *reduction* of the axisymmetric quadrupole moment is the main source of the collapse signal. Once the BH is formed, it continues to oscillate in its axisymmetric \(l = 2\) QNM, until all oscillation energy is radiated away and the stationary Kerr limit is approached.

The numerical study of rotating collapse to BHs was pioneered by Nakamura\(^\text{43}\) but first waveforms and gravitational-wave estimates were obtained by Stark and Piran\(^\text{44}\). These simulations were performed in 2D, using approximate initial data (essentially a spherical star to which angular momentum was artificially added). A new 3D computation of the gravitational wave emission from the collapse of unstable uniformly rotating relativistic polytropes to Kerr BHs\(^\text{45}\) finds that the energy emitted is

\[ \Delta E \sim 1.5 \times 10^{-6}(M/M_\odot), \]  

(3)

significantly less than the result of Stark and Piran. Still, the collapse of an unstable \(2M_\odot\) rapidly rotating neutron star leads to a characteristic gravitational-wave amplitude \(h_c \sim 3 \times 10^{-21}\), at a frequency of \(\sim 5.5\text{kHz}\), for an event at 10kpc. Emission is mainly through the "\(+\)" polarization, with the "\(\times\)" polarization being an order of magnitude weaker.

Whether a BH forms promptly after collapse or a delayed collapse takes place depends sensitively on a number of factors, such as the progenitor mass and angular momentum and the high-density EOS. The most detailed investigation of the influence of these factors on the outcome of collapse has been presented recently in\(^\text{33}\), where it was found that shock formation increases the threshold for black hole formation by \(\sim 20 - 40\%\), while rotation results in an increase of at most 25\%.
2.4. Black hole ringing through fall-back or hyper-accretion

Single events. A black hole can form after core collapse, if fall-back accretion increases the mass of the proto-neutron star above the maximum mass allowed by axisymmetric stability. Material falling back after the black hole is formed excites the black hole quasi-normal modes of oscillation. If, on the other hand, the black hole is formed directly through core collapse (without a core bounce taking place) then most of the material of the progenitor star is accreted at very high rates ($\sim 1 - 2M_\odot/s$) into the hole. In such hyper-accretion the black hole’s quasi-normal modes (QNM) can be excited for as long as the process lasts and until the black hole becomes stationary. Typical frequencies of the emitted GWs are in the range 1-3kHz for $\sim 3 - 10M_\odot$ BHs.

The frequency and the damping time of the oscillations for the $l = m = 2$ mode can be estimated via the relations

$$\sigma \approx 3.2\text{kHz} \, M_{10}^{-1} \left[ 1 - 0.63(1 - a/M)^{3/10} \right]$$

$$Q = \pi \sigma \tau \approx 2 (1 - a)^{-9/20}$$

These relations together with similar ones either for the 2nd QNM or the $l = 2, \ m = 0$ mode can uniquely determine the mass $M$ and angular momentum parameter $a$ of the BH if the frequency and the damping time of the signal have been accurately extracted. The amplitude of the ring-down waves depends on the BH’s initial distortion, i.e. on the nonaxisymmetry of the blobs or shells of matter falling into the BH. If matter of mass $\mu$ falls into a BH of mass $M$, then the gravitational wave energy is roughly

$$\Delta E \gtrsim \epsilon \mu c^2 (\mu/M)$$

where $\epsilon$ is related to the degree of asymmetry and could be $\epsilon \gtrsim 0.01$. This leads to an effective GW amplitude

$$h_{\text{eff}} \approx 2 \times 10^{-21} \left( \frac{\epsilon}{0.01} \right) \left( \frac{10\text{Mpc}}{d} \right) \left( \frac{\mu}{M_\odot} \right)$$

Resonant driving. If hyper-accretion proceeds through an accretion disk around a rapidly spinning Kerr BH, then the matter near the marginally bound orbit radius can become unstable to the magnetorotational (MRI) instability, leading to the formation of large-scale asymmetries. Under certain conditions, resonant driving of the BH QNMs could take place. Such a continuous signal could be integrated, yielding a much larger signal
to noise ratio than a single event. For a $15M_\odot$ nearly maximal Kerr BH created at 27Mpc the integrated signal becomes detectable by LIGO II at a frequency of $\sim 1600\text{Hz}$, especially if narrow-banding is used$^{51}$.

3. Rotational instabilities

If proto-neutron stars rotate rapidly, nonaxisymmetric dynamical instabilities can develop. These arise from non-axisymmetric perturbations having angular dependence $e^{im\phi}$ and are of two different types: the classical bar-mode instability and the more recently discovered low-$T/|W|$ bar-mode and one-armed spiral instabilities, which appear to be associated to the presence of corotation points. Another class of nonaxisymmetric instabilities are secular instabilities, driven by dissipative effects, such as fluid viscosity or gravitational radiation.

3.1. Dynamical instabilities

Classical bar-mode instability. The classical $m = 2$ bar-mode instability is excited in Newtonian stars when the ratio $\beta = T/|W|$ of the rotational kinetic energy $T$ to the gravitational binding energy $|W|$ is larger than $\beta_{\text{dyn}} = 0.27$. The instability grows on a dynamical time scale (the time that a sound wave needs to travel across the star) which is about one rotational period and may last from 1 to 100 rotations depending on the degree of differential rotation in the PNS.

The bar-mode instability can be excited in a hot PNS, a few milliseconds after core bounce, or, alternatively, it could also be excited a few tenths of seconds later, when the PNS cools due to neutrino emission and contracts further, with $\beta$ becoming larger than the threshold $\beta_{\text{dyn}}$ ($\beta$ increases roughly as $\sim 1/R$ during contraction). The amplitude of the emitted gravitational waves can be estimated as $h \sim MR^2\Omega^2/d$, where $M$ is the mass of the body, $R$ its size, $\Omega$ the rotation rate and $d$ the distance of the source. This leads to an estimation of the GW amplitude

$$h \approx 9 \times 10^{-23} \left( \frac{\epsilon}{0.2} \right) \left( \frac{f}{3\text{kHz}} \right)^2 \left( \frac{15\text{Mpc}}{d} \right) M_{1.4}R_{10^3}^2. \tag{8}$$

where $\epsilon$ measures the ellipticity of the bar, $M$ is measured in units of $1.4M_\odot$ and $R$ is measured in units of 10km. Notice that, in uniformly rotation Maclaurin spheroids, the GW frequency $f$ is twice the rotational frequency $\Omega$. Such a signal is detectable only from sources in our galaxy or the nearby ones (our Local Group). If the sensitivity of the detectors
is improved in the kHz region, signals from the Virgo cluster could be detectable. If the bar persists for many (~ 10-100) rotation periods, then even signals from distances considerably larger than the Virgo cluster will be detectable. Due to the requirement of rapid rotation, the event rate of the classical dynamical instability is considerably lower than the SN event rate.

The above estimates rely on Newtonian calculations; GR enhances the onset of the instability, $\beta_{\text{dyn}} \sim 0.24^{52,53}$ and somewhat lower than that for large compactness (large $M/R$). Fully relativistic dynamical simulations of this instability have been obtained, including detailed waveforms of the associated gravitational wave emission. A detailed investigation of the required initial conditions of the progenitor core, which can lead to the onset of the dynamical bar-mode instability in the formed PNS, was presented in\textsuperscript{31}. The amplitude of gravitational waves was due to the bar-mode instability was found to be larger by an order of magnitude, compared to the axisymmetric core collapse signal.

**Low-$T/|W|$ instabilities.** The bar-mode instability may be excited for significantly smaller $\beta$, if centrifugal forces produce a peak in the density off the source’s rotational center\textsuperscript{54}. Rotating stars with a high degree of differential rotation are also dynamically unstable for significantly lower $\beta_{\text{dyn}} \gtrsim 0.01^{55,56}$. According to this scenario the unstable neutron star settles down to a non-axisymmetric quasi-stationary state which is a strong emitter of quasi-periodic gravitational waves

$$h_{\text{eff}} \approx 3 \times 10^{-22} \left( \frac{R_{\text{eq}}}{30\text{km}} \right) \left( \frac{f}{800\text{Hz}} \right)^{1/2} \left( \frac{100\text{Mpc}}{d} \right) M_{1.4}^{1/2}. \quad (9)$$

The bar-mode instability of differentially rotating neutron stars is an excellent source of gravitational waves, provided the high degree of differential rotation that is required can be realized. One should also consider the effects of viscosity and magnetic fields. If magnetic fields enforce uniform rotation on a short timescale, this could have strong consequences regarding the appearance and duration of the dynamical nonaxisymmetric instabilities.

An $m = 1$ one-armed spiral instability has also been shown to become unstable in proto-neutron stars, provided that the differential rotation is sufficiently strong\textsuperscript{54,57}. Although it is dominated by a “dipole” mode, the instability has a spiral character, conserving the center of mass. The onset of the instability appears to be linked to the presence of corotation points\textsuperscript{58}.
(a similar link to corotation points has been proposed for the low-$T/|W|$ bar mode instability and requires a very high degree of differential rotation (with matter on the axis rotating at least 10 times faster than matter on the equator). The $m = 1$ spiral instability was recently observed in simulations of rotating core collapse, which started with the core of an evolved $20M_{\odot}$ progenitor star to which differential rotation was added. Growing from noise level ($\sim 10^{-6}$) on a timescale of $5$ms, the $m = 1$ mode reached its maximum amplitude after $\sim 100$ms. Gravitational waves were emitted through the excitation of an $m = 2$ nonlinear harmonic at a frequency of $\sim 800$Hz with an amplitude comparable to the core-bounce axisymmetric signal.

3.2. Secular gravitational-wave-driven instabilities

In a nonrotating star, the forward and backward moving modes of same ($l, |m|$) (corresponding to ($l, +m$) and ($l, -m$)) have eigenfrequencies $\pm |\sigma|$. Rotation splits this degeneracy by an amount $\delta \sigma \sim m\Omega$ and both the prograde and retrograde modes are dragged forward by the stellar rotation. If the star spins sufficiently rapidly, a mode which is retrograde (in the frame rotating with the star) will appear as prograde in the inertial frame (a non-rotating observer at infinity). Thus, an inertial observer sees GWs with positive angular momentum emitted by the retrograde mode, but since the perturbed fluid rotates slower than it would in the absence of the perturbation, the angular momentum of the mode in the rotating frame is negative. The emission of GWs consequently makes the angular momentum of the mode increasingly negative, leading to the instability. A mode is unstable when $\sigma(\sigma - m\Omega) < 0$. This class of frame-dragging instabilities is usually referred to as Chandrasekhar-Friedman-Schutz (CFS) instabilities.

$f$-mode instability. In the Newtonian limit, the $l = m = 2$ $f$-mode (which has the shortest growth time of all polar fluid modes) becomes unstable when $T/|W| > 0.14$, which is near or even above the mass-shedding limit for typical polytropic EOSs used to model uniformly rotating neutron stars. Dissipative effects (e.g. shear and bulk viscosity or mutual friction in superfluids) leave only a small instability window near mass-shedding, at temperatures of $\sim 10^9$K. However, relativistic effects strengthen the instability considerably, lowering the required $\beta$ to $\approx 0.06 - 0.08$ for most realistic EOSs and masses of $\sim 1.4M_{\odot}$ (for higher masses, such as hypermassive stars created in a binary NS merger, the required rotation rates are even lower).
Since PNSs rotate differentially, the above limits derived under the assumption of uniform rotation are too strict. Unless uniform rotation is enforced on a short timescale, due to e.g. magnetic braking\textsuperscript{69}, the $f$-mode instability will develop in a differentially rotating background, in which the required $T/|W|$ is only somewhat larger than the corresponding value for uniform rotation\textsuperscript{70}, but the mass-shedding limit is dramatically relaxed. Thus, in a differentially rotating PNS, the $f$-mode instability window is huge, compared to the case of uniform rotation and the instability can develop provided there is sufficient $T/|W|$ to begin with.

The $f$-mode instability is an excellent source of GWs. Simulations of its nonlinear development in the ellipsoidal approximation\textsuperscript{71} have shown that the mode can grow to a large nonlinear amplitude, modifying the background star from an axisymmetric shape to a differentially rotating ellipsoid. In this modified background the $f$-mode amplitude saturates and the ellipsoid becomes a strong emitter of gravitational waves, radiating away angular momentum until the star is slowed-down towards a stationary state. In the case of uniform density ellipsoids, this stationary state is the Dedekind ellipsoid, i.e. a nonaxisymmetric ellipsoid with internal flows but with a stationary (nonradiating) shape in the inertial frame. In the ellipsoidal approximation, the nonaxisymmetric pattern radiates gravitational waves sweeping through the LIGO II sensitivity window (from 1kHz down to about 100Hz) which could become detectable out to a distance of more than 100Mpc.

Two recent hydrodynamical simulations\textsuperscript{72,73} (in the Newtonian limit and using a post-Newtonian radiation-reaction potential) essentially confirm this picture. In \textsuperscript{72} a differentially rotating, $N = 1$ polytropic model with a large $T/|W| \sim 0.2 - 0.26$ is chosen as the initial equilibrium state. The main difference of this simulation compared to the ellipsoidal approximation comes from the choice of EOS. For $N = 1$ Newtonian polytropes it is argued that the secular evolution cannot lead to a stationary Dedekind-like state does not exist. Instead, the $f$-mode instability will continue to be active until all nonaxisymmetries are radiated away and an axisymmetric shape is reached. This conclusion should be checked when relativistic effects are taken into account, since, contrary to the Newtonian case, relativistic $N = 1$ uniformly rotating polytropes are unstable to the $l = m = 2$ $f$-mode\textsuperscript{67} – however it has not become possible, to date, to construct relativistic analogs of Dedekind ellipsoids.

In the other recent simulation \textsuperscript{73}, the initial state was chosen to be a uniformly rotating, $N = 0.5$ polytropic model with $T/|W| \sim 0.18$. Again,
the main conclusions reached in $^7$ are confirmed, however, the assumption of uniform initial rotation limits the available angular momentum that can be radiated away, leading to a detectable signal only out to about $\sim 40\text{Mpc}$. The star appears to be driven towards a Dedekind-like state, but after about $10$ dynamical periods, the shape is disrupted by growing short-wavelength motions, which are suggested to arise because of a shearing type instability, such as the elliptic flow instability $^7$.

\textit{r-mode instability}. Rotation does not only shift the spectra of polar modes; it also lifts the degeneracy of axial modes, give rise to a new family of \textit{inertial} modes, of which the $l = m = 2$ \textit{r}-mode is a special member. The restoring force, for these oscillations is the Coriolis force. Inertial modes are primarily velocity perturbations. The frequency of the \textit{r}-mode in the rotating frame of reference is $\sigma = 2\Omega/3$. According to the criterion for the onset of the CFS instability, the \textit{r}-mode is unstable for any rotation rate of the star $^7$. For temperatures between $10^7 - 10^9\text{K}$ and rotation rates larger than $5\text{-}10\%$ of the Kepler limit, the growth time of the unstable mode is smaller than the damping times of the bulk and shear viscosity$^7$. The existence of a solid crust or of hyperons in the core $^8$ and magnetic fields $^9$ can also significantly affect the onset of the instability (for extended reviews see $^9,10$). The suppression of the \textit{r}-mode instability by the presence of hyperons in the core is not expected to operate efficiently in rapidly rotating stars, since the central density is probably too low to allow for hyperon formation. Moreover, a recent calculation$^{11}$ finds the contribution of hyperons to the bulk viscosity to be two orders of magnitude smaller than previously estimated. If accreting neutron stars in Low Mass X-Ray Binaries (LMXB, considered to be the progenitors of millisecond pulsars) are shown to reach high masses of $\sim 1.8M_\odot$, then the EOS could be too soft to allow for hyperons in the core (for recent observations that support a high mass for some millisecond pulsars see $^{12}$).

The unstable \textit{r}-mode grows exponentially until it saturates due to nonlinear effects at some maximum amplitude $\alpha_{\text{max}}$. The first computation of nonlinear mode couplings using second-order perturbation theory suggested that the \textit{r}-mode is limited to very small amplitudes (of order $10^{-3} - 10^{-4}$) due to transfer of energy to a large number of other inertial modes, in the form of a cascade, leading to an equilibrium distribution of mode amplitudes$^{13}$. The small saturation values for the amplitude are supported by recent nonlinear estimations $^{14,15}$ based on the drift, induced by the \textit{r}-modes, causing differential rotation. On the other hand, hydrodynamical
simulations of limited resolution showed that an initially large-amplitude \( r \)-mode does not decay appreciably over several dynamical timescales\(^9\), but on a somewhat longer timescale a catastrophic decay was observed\(^{90}\) indicating a transfer of energy to other modes, due to nonlinear mode couplings and suggesting that a hydrodynamical instability may be operating. A specific resonant 3-mode coupling was identified\(^{91}\) as the cause of the instability and a perturbative analysis of the decay rate suggests a maximum saturation amplitude \( \alpha_{\text{max}} < 10^{-2} \). A new computation using second-order perturbation theory finds that the catastrophic decay seen in the hydrodynamical simulations\(^{90,91}\) can indeed be explained by a parametric instability operating in 3-mode couplings between the \( r \)-mode and two other inertial modes\(^{92,93,94}\). Whether the maximum saturation amplitude is set by a network of 3-mode couplings or a cascade is reached, is, however, still unclear.

A neutron star spinning down due to the \( r \)-mode instability will emit gravitational waves of amplitude

\[
h(t) \approx 10^{-21} \alpha \left( \frac{\Omega}{1\text{kHz}} \right) \left( \frac{100\text{kpc}}{d} \right)
\]

(10)

Since \( \alpha \) is small, even with LIGO II the signal is undetectable at large distances (VIRGO cluster) where the SN event rate is appreciable, but could be detectable after long-time integration from a galactic event. However, if the compact object is a strange star, then the instability may not reach high amplitudes \( (\alpha \sim 10^{-3} - 10^{-4}) \) but it will persist for a few hundred years (due to the different temperature dependence of viscosity in strange quark matter) and in this case there might be up to ten unstable stars in our galaxy at any time\(^{95}\). Integrating data for a few weeks could lead to an effective amplitude \( h_{\text{eff}} \sim 10^{-21} \) for galactic signals at frequencies \( \sim 700 - 1000\text{Hz} \). The frequency of the signal changes only slightly on a timescale of a few months, so that the radiation is practically monochromatic.

**Other unstable modes.** The CFS instability can also operate for core g-mode oscillations\(^{96}\) but also for \( w \)-mode oscillations, which are basically spacetime modes\(^{97}\). In addition, the CFS instability can operate through other dissipative effects. Instead of the gravitational radiation, any radiative mechanism (such as electromagnetic radiation) can in principle lead to an instability.
3.3. Secular viscosity-driven instability

A different type of nonaxisymmetric instability in rotating stars is the instability driven by viscosity, which breaks the circulation of the fluid \(^{98,99}\). The instability is suppressed by gravitational radiation, so it cannot act in the temperature window in which the CFS-instability is active. The instability sets in when the frequency of a prograde \(l = -m\) mode goes through zero in the rotating frame. In contrast to the CFS-instability, the viscosity-driven instability is not generic in rotating stars. The \(m = 2\) mode becomes unstable at a high rotation rate for very stiff stars and higher \(m\)-modes become unstable at larger rotation rates.

In Newtonian polytropes, the instability occurs only for stiff polytropes of index \(N < 0.808\) \(^{99,100}\). For relativistic models, the situation for the instability becomes worse, since relativistic effects tend to suppress the viscosity-driven instability (while the CFS-instability becomes stronger). For the most relativistic stars, the viscosity-driven bar mode can become unstable only if \(N < 0.55\) \(^{101}\). For 1.4\(M_\odot\) stars, the instability is present for \(N < 0.67\).

An investigation of the viscosity-driven bar mode instability, using incompressible, uniformly rotating triaxial ellipsoids in the post-Newtonian approximation \(^{102}\) finds that the relativistic effects increase the critical \(T/|W|\) ratio for the onset of the instability significantly. More recently, new post-Newtonian \(^{103}\) and fully relativistic calculations for uniform-density stars \(^{104}\) show that the viscosity-driven instability is not as strongly suppressed by relativistic effects as suggested in \(^{102}\). The most promising case for the onset of the viscosity-driven instability (in terms of the critical rotation rate) would be rapidly rotating strange stars \(^{105}\), but the instability can only appear if its growth rate is larger than the damping rate due to the emission of gravitational radiation - a corresponding detailed comparison is still missing.

4. Accreting neutron stars in LMXBs

Spinning neutron stars with even tiny deformations are interesting sources of gravitational waves. The deformations might results from various factors but it seems that the most interesting cases are the ones in which the deformations are caused by accreting material. A class of objects called Low-Mass X-Ray Binaries (LMXB) consist of a fast rotating neutron star (spin \(\approx 270 - 650\)Hz) torqued by accreting material from a companion star which has filled up its Roche lobe. The material adds both mass and an-
gular momentum to the star, which, on timescales of the order of tenths of Megayears could, in principle, spin up the neutron star to its breakup limit. One viable scenario suggests that the accreted material (mainly hydrogen and helium) after an initial phase of thermonuclear burning undergoes a non-uniform crystallization, forming a crust at densities $\sim 10^8 - 10^9 \text{g/cm}^3$. The quadrupole moment of the deformed crust is the source of the emitted gravitational radiation which slows-down the star, or halts the spin-up by accretion.

An alternative scenario has been proposed by Wagoner as a follow up of an earlier idea by Papaloizou-Pringle. The suggestion was that the spin-up due to accretion might excite the $f$-mode instability, before the rotation reaches the breakup spin. The emission of gravitational waves will torque down the star’s spin at the same rate as the accretion will torque it up, however, it is questionable whether the $f$-mode instability will ever be excited for old, accreting neutron stars. Following the discovery that the $r$-modes are unstable at any rotation rate, this scenario has been revived independently by Bildsten and Andersson, Kokkotas and Stergioulas. The amplitude of the emitted gravitational waves from such a process is quite small, even for high accretion rates, but the sources are persistent and in our galactic neighborhood the expected amplitude is

$$h \approx 10^{-27} \left( \frac{1.6\text{ms}}{P} \right)^5 \frac{1.5\text{kpc}}{D}.$$  \hfill (11)

This signal is within reach of advanced LIGO with signal recycling tuned at the appropriate frequency and integrating for a few months. This picture is in practice more complicated, since the growth rate of the $r$-modes (and consequently the rate of gravitational wave emission) is a function of the core temperature of the star. This leads to a thermal runaway due to the heat released as viscous damping mechanisms counteract the $r$-mode growth. Thus, the system executes a limit cycle, spinning up for several million years and spinning down in a much shorter period. The duration of the unstable part of the cycle depends critically on the saturation amplitude $\alpha_{\text{max}}$ of the $r$-modes. Since current computations suggest an $\alpha_{\text{max}} \sim 10^{-3} - 10^{-4}$, this leads to a quite long duration for the unstable part of the cycle of the order of $\sim 1\text{Myear}$.

The instability window depends critically on the effect of the shear and bulk viscosity and various alternative scenarios might be considered. The existence of hyperons in the core of neutron stars induces much stronger bulk viscosity which suggests a much narrower instability window for the
$r$-modes and the bulk viscosity prevails over the instability even in temperatures as low as $10^8$K. A similar picture can be drawn if the star is composed of “deconfined” $u$, $d$ and $s$ quarks - a strange star. In this case, there is a possibility that the strange stars in LMXBs evolve into a quasi-steady state with nearly constant rotation rate, temperature and mode amplitude emitting gravitational waves for as long as the accretion lasts. This result has also been found later for stars with hyperon cores. It is interesting that the stalling of the spin up in millisecond pulsars (MSPs) due to $r$-modes is in good agreement with the minimum observed period and the clustering of the frequencies of MSPs.

5. Binary mergers

Depending on the high-density EOS and their initial masses, the outcome of the merger of two neutron stars may not always be a black hole, but a hypermassive, differentially rotating compact star (even if it is only temporarily supported against collapse by differential rotation). A recent detailed simulation in full GR has shown that the hypermassive object created in a binary NS merger is nonaxisymmetric. The nonaxisymmetry lasts for a large number of rotational periods, leading to the emission of gravitational waves with a frequency of $3$kHz and an effective amplitude of $\sim 6 \times 10^{-21}$ at a large distance of $50$Mpc. Such large effective amplitude may be detectable even by LIGO II at this high frequency.

The tidal disruption of a NS by a BH or the merging of two NSs may give valuable information for the radius and the EOS if we can recover the signal at frequencies higher than $1$ kHz.

6. Gravitational-wave asteroseismology

If various types of oscillation modes are excited during the formation of a compact star and become detectable by gravitational wave emission, one could try to identify observed frequencies with frequencies obtained by mode-calculations for a wide parameter range of masses, angular momenta and EOSs. Thus, gravitational wave asteroseismology could enable us to estimate the mass, radius and rotation rate of compact stars, leading to the determination of the ”best-candidate” high-density EOS, which is still very uncertain. For this to happen, accurate frequencies for different mode-sequences of rapidly rotating compact objects have to be computed.

For slowly rotating stars, the frequencies of $f$, $p$- and $w$- modes are
still unaffected by rotation, and one can construct approximate formulae in order to relate observed frequencies and damping times of the various stellar modes to stellar parameters. For example, for the fundamental oscillation \((l = 2)\) mode \((f\text{-mode})\) of non-rotating stars one obtains \(^{35}\)

\[
\sigma (\text{kHz}) \approx 0.8 + 1.6 M_{1.4}^{1/2} R_{10}^{-3/2} + \delta_1 m \bar{\Omega} \\
\tau^{-1} (\text{sec}^{-1}) \approx M_{1.4}^3 R_{10}^{-4} (22.9 - 14.7 M_{1.4} R_{10}^{-1}) + \delta_2 m \bar{\Omega}
\]

(12) \hspace{1cm} (13)

where \(\bar{\Omega}\) is the normalized rotation frequency of the star, and \(\delta_1\) and \(\delta_2\) are constants estimated by sampling data from various EOSs. The typical frequencies of NS oscillation modes are larger than 1kHz. Since each type of mode is sensitive to the physical conditions where the amplitude of the mode is largest, the more oscillations modes can be identified through gravitational waves, the better we will understand the detailed internal structure of compact objects, such as the existence of a possible superfluid state of matter\(^{125}\).

If, on the other hand, some compact stars are born rapidly rotating with moderate differential rotation, then their central densities will be much smaller than the central density of a nonrotating star or same baryonic mass. Correspondingly, the typical axisymmetric oscillation frequencies will be smaller than 1kHz, which is more favorable for the sensitivity window of current interferometric detectors\(^{126}\). Indeed, axisymmetric simulations of rotating core-collapse have shown that if a rapidly rotating NS is created, then the dominant frequency of the core-bounce signal (originating from the fundamental \(l = 2\) mode or the \(l = 2\) piece of the fundamental quasi-radial mode) is in the range 600Hz-1kHz\(^{11}\).

If different type of signals are observed after core collapse, such as both an axisymmetric core-bounce signal and a nonaxisymmetric one-armed instability signal, with a time separation of the order of 100ms, this would yield invaluable information about the angular momentum distribution in the proto-neutron stars.

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