Transformation of vortex structures in the wake of a sphere moving in the stratified fluid with decreasing of internal Froude number

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Abstract. The 3D separated, density stratified viscous fluid flows around a sphere are investigated by means of the direct numerical simulation (DNS) on the basis of the Navier-Stokes equations in the Boussinesq approximation on the supercomputers at the wide range of internal Froude ($Fr$) and Reynolds ($Re$) numbers. For DNS the Splitting on physical factors Method for Incompressible Fluid flows (SMIF) with the hybrid explicit finite difference scheme (second-order accuracy in space, minimum scheme viscosity and dispersion, monotonous) has been used. At $Fr > 10$ with increasing of $Re$ we observed the flow regimes of the homogeneous viscous fluid (including the laminar-turbulent transition in the boundary layer on the sphere). With decreasing of $Fr$ at $Re < 500$ the strong transformation of vortex structures in the sphere wake is demonstrated by means of the $\beta$– visualization. Thus the refined classification of the flow regimes around a sphere moving in the viscous stratified fluid is presented.

1. Introduction
Unsteady 3D separated and undulatory fluid flows around a moving blunt body are very wide spread phenomena in the nature. The understanding of such flows is very important both from theoretical and from practical points of view. In the experiments (Lin et al., 1992; Chomaz et al., 1993) the 2D internal waves structure in the vertical plane and the 3D vortex structure of the wake are observed. Using DNS the full 3D vortex structures of the flow (the 3D internal waves and the 3D wake) can be observed. Besides the numerical studies of the non-homogeneous (stratified) fluids are very rare. In this connection at the present paper the stratified viscous fluid flows around a sphere are investigated by means of DNS on the basis of the Navier-Stokes equations in the Boussinesq approximation on the supercomputers at the wide ranges of the main flow parameters ($Fr$ and $Re$).

2. Numerical method SMIF
2.1. Equations and boundary conditions
Let $\rho(x, y, z) = \rho_0(1 - x/(2C) + S(x, y, z))$ is the density of the linearly stratified fluid where $x, y, z$ are the Cartesian coordinates; $z, x, y$ are the streamwise, lift and lateral directions ($x, y, z$ have been non-dimensionalized by $d/2$); $C = \Delta/d$ is the scale ratio, $\Lambda$ is the buoyancy scale, which is related to the buoyancy frequency $N$ and period $T_b$ ($N = 2\pi/T_b$, $N^2 = g/\Lambda$); $g$ is the scalar of the gravitational acceleration; $S$ is a dimensionless perturbation of salinity. The density stratified viscous fluid flows have been simulated on the basis of the Navier-Stokes equations in the Boussinesq approximation.
(1) – (3) (including the diffusion equation (1) for the stratified component (salt)) with four dimensionless parameters: $Fr, Re, C_o = 1, Sc = v/\kappa = 709.22$, where $\kappa$ is the salt diffusion coefficient.

$$\frac{\partial S}{\partial t} + (v \cdot \nabla) S = \frac{2}{Sc \cdot Re} \Delta S + \frac{v_x}{2C}$$  \hspace{1cm} (1)

$$\frac{\partial v}{\partial t} + (v \cdot \nabla) v = -\nabla p + \frac{2}{Re} \Delta v + \frac{C}{2Fr^2} S \frac{g}{g}$$  \hspace{1cm} (2)

$$\nabla \cdot v = 0$$  \hspace{1cm} (3)

In (1) – (3) $v = (v_x, v_y, v_z)$ is the velocity vector (non-dimensionalized by $U$), $p$ is a perturbation of pressure (non-dimensionalized by $\rho_0 U^2$).

The spherical coordinate system $R, \theta, \phi$ ($x = R \sin \theta \cos \phi, y = R \sin \theta \sin \phi, z = R \cos \theta, v = (v_R, v_\theta, v_\phi)$) and O-type grid are used. On the sphere surface the following boundary conditions have been used:

$\frac{\partial v}{\partial R} = v_\theta = v_\phi = 0,$  \hspace{1cm} (step I)

$\Delta p = -\nabla \cdot \vec{v}$  \hspace{1cm} (step III)

The Poisson equation for the pressure (step III) has been solved by the diagonal Preconditioned Conjugate Gradients Method.

2.2. Splitting scheme of SMIF

Let the velocity, the perturbation of pressure and the perturbation of salinity are known at some moment $t_n = n \tau$, where $\tau$ is time step, and $n$ is the number of time-steps. Then the calculation of the unknown functions at the next time level $t_{n+1} = (n+1) \tau$ for equations (1) – (3) can be presented in the following four-step form:

$$\frac{S^{n+1} - S^n}{\tau} = -(v \cdot \nabla) S^n + \frac{2}{Sc \cdot Re} \Delta S^n + \frac{v_x^n}{2C}$$  \hspace{1cm} (step I)

$$\frac{\vec{v} - v^n}{\tau} = -(v \cdot \nabla) v^n + \frac{2}{Re} \Delta v^n + \frac{C}{2Fr^2} S^{n+1} \frac{g}{g}$$  \hspace{1cm} (step II)

$$\tau \Delta p = \nabla \cdot \vec{v}$$  \hspace{1cm} (step III)

$$\frac{v^{n+1} - \vec{v}}{\tau} = -\nabla p$$  \hspace{1cm} (step IV)

The Poisson equation for the pressure (step III) has been solved by the diagonal Preconditioned Conjugate Gradients Method.

2.3. Finite-difference scheme for the convective terms of the equations (1)-(3) (1D example)

Let us consider the linear model equation:

$$f_t + u f_x = 0, \quad u = \text{const.}$$  \hspace{1cm} (4)

Let

$$\frac{f_{i+1}^{n+1} - f_{i}^{n}}{\tau} + u \frac{f_{i+1}^{n} - f_{i-1}^{n}}{h} = 0$$  \hspace{1cm} (5)

be a finite-difference approximation of equation (4).

Let us investigate the class of the difference scheme which can be written in the form of the two-parameter family which depends on the parameters $\alpha$ and $\beta$ in the following manner:

$$f_i^{n+1} = \begin{cases} \alpha f_{i-1}^{n+1} + (1 - \alpha - \beta) f_i^{n+1} + \beta f_{i+1}^{n+1}, & u \geq 0 \\ \alpha f_{i+1}^{n+1} + (1 - \alpha - \beta) f_i^{n+1} + \beta f_{i+1}^{n+1}, & u < 0. \end{cases}$$  \hspace{1cm} (6)
In this case the first differential approximation for equation (5) has the form

\[ f_i + u f_x = \left[ \frac{h}{2} \nu (1 + 2\alpha - 2\beta) - \frac{\alpha u^2}{2} \right] f_{xx}. \] (7)

If we put \( \alpha = \beta = 0 \) in (6) we'll obtain usual first order monotonic scheme which is stable when

\[ 0 < C = \frac{\tau \nu}{h} \leq 1, \] where \( C \) is the Courant number. (8)

If \( \alpha = 0, \beta = 0.5 \) we'll obtain the usual central difference scheme, and for \( \alpha = -0.5, \beta = 0 \) – the usual upwind scheme. Both last two scheme have second order of accuracy in space variable and are non-monotonic.

It is known that it is impossible to construct a homogeneous monotonic difference scheme of higher order than the first order of the approximation for equation (4). A monotonic scheme of higher order can therefore only be constructed either on the basis of second-order homogeneous scheme using smoothing operators, or on the basis of the hybrid schemes using different switch conditions from one scheme to another (depending on the nature of the solution), possibly with the use of smoothing. Here we are going to consider a hybrid monotonic difference scheme.

Let us investigate schemes with upwind differences, i.e. \( \beta = 0 \). The requirement that the scheme viscosity should be a minimum, as can readily be seen from equation (7), impose the following condition on \( \alpha \):

\[ \alpha = -0.5 \left( 1 - C \right). \] (9)

For schemes with \( \alpha = 0 \), the analogous condition is

\[ \beta = 0.5 \left( 1 - C \right). \] (10)

Since an explicit finite difference scheme considered, we shall restrict the subsequent analysis to the necessary condition for stability in the case of the explicit schemes (8).

Let us assume that there is a monotonic net function \( f_i^n \), for example, \( \Delta f_{i+\frac{1}{2}}^n = f_{i+1}^n - f_i^n \geq 0 \) at any \( i \).

The function \( f_{i+1}^{n+1} \) will also be monotonic when the following conditions are satisfied:

(a) for a scheme with \( \beta = 0 \) and \( \alpha \) from relationship (9), under the condition

\[ \Delta f_{i+\frac{1}{2}}^n \leq \zeta(C) \Delta f_{i-\frac{1}{2}}^n, \] where \( \zeta(C) = 0.5 \left( 1 - C \right) / (2 - C) \);

(b) for a scheme with \( \alpha = 0 \) and \( \beta \) from relationship (10), under the condition

\[ \Delta f_{i+\frac{1}{2}}^n \leq \sigma(C) \Delta f_{i-\frac{1}{2}}^n, \] where \( \sigma(C) = 2 \left( 1 + C \right) / C \).

It can be seen from this that the domains of monotonicity of the homogeneous scheme being considered have a non-empty intersection. Hence, a whole class of hybrid schemes is distinguished by the condition of switching from one homogeneous scheme to another. The general form of this condition is as follows:

\[ \Delta f_{i+\frac{1}{2}}^n = \delta \Delta f_{i-\frac{1}{2}}^n, \] where \( \zeta(C) \leq \delta \leq \sigma(C) \).

The choice of \( \delta = 1 \) corresponds to the points of the interchange of the sign of the second difference \( f_i^n \) and makes it possible to obtain the estimation \( f_{xx} = O(h) \) for the required function \( f \) at the intersection points, by means of which a second-order approximation is retained with respect to the spatial variables of smooth solutions. We used the following switching condition:

if \((u \Delta f)^{i,\frac{1}{2}} \geq 0 \), then the scheme with \( \beta = 0 \) (MUDS) is used;

if \((u \Delta f)^{i,\frac{1}{2}} < 0 \), then the scheme with \( \alpha = 0 \) (MCDS) is used;

where \( \Delta f^{i,\frac{1}{2}} = f_{i+1}^n - f_i^n \).

On smooth solutions this scheme has a second order of approximation with respect to the time and spatial variables. It is stable when the Courant criterion (8) is satisfied and monotonic. More over it was shown that this hybrid scheme comes nearest to the third order schemes.

The generalization of the considered finite-difference scheme for 2D and 3D problems is easily performed for convective terms in (1-2). For the approximation of other space derivatives in equations (1) – (3) the central differences are used.
The efficiency of the method SMIF and the greater power of supercomputers make it possible adequately to model the 3D separated incompressible homogeneous viscous flows past a sphere and a circular cylinder at moderate Reynolds numbers (Gushchin et al., 2002, 2004, 2006) and the air, heat and mass transfer in the clean rooms.

3. The visualization techniques
For the visualization of the 3D vortex structures in the sphere wake the isosurfaces of $\beta$ and $\lambda_2$ have been drawing, where $\beta$ is the imaginary part of the complex-conjugate eigen-values of the velocity gradient tensor $\mathbf{G}$ (Chong et al., 1990), $\lambda_2$ is the second eigen-value of the $\mathbf{S}^2 + \mathbf{\Omega}^2$ tensor, where $\mathbf{S}$ and $\mathbf{\Omega}$ are the symmetric and antisymmetric parts of $\mathbf{G}$ (Jeong et al., 1995). The function $\beta$ has a real physical meaning. Let us consider a local stream lines pattern around any point in a flow (where $\beta > 0$) in a reference frame $\mathbf{x}$ moving with the velocity of this point ($\mathbf{v} = d\mathbf{x}/dt \approx \mathbf{G} \mathbf{x}$, where $\mathbf{v}$ is a velocity of a fluid particle in the considered reference frame $\mathbf{x}$ and $t$ is time). It’s easy to demonstrate (see the theory of the ordinary differential equations) that the local stream lines pattern in the considered reference frame $\mathbf{x}$ is closed or spiral, and $\beta$ is the angular velocity of this spiral motion. The good efficiency of this $\beta$-visualization technique has been demonstrated in (Gushchin et al., 2006).

4. The classification of the flow regimes around a sphere moving in the viscous stratified fluid

4.1. The flow regime I ($Fr > 10$ — “the homogeneous case”).

4.1.1. The laminar-turbulent transition in the sphere wake ($Re < 4 \cdot 10^4$).
For $Fr > 10$ the homogeneous viscous fluid flow regimes are observed. The following classification of flows (Gushchin et al., 2002, 2004, 2006) has been obtained by SMIF at $Re < 4 \cdot 10^4$ (figure 1): 1) $Re \leq 20.5$ — without separation; 2) $20.5 < Re \leq 200$ — a steady axisymmetrical wake with a vortex ring in the recirculation zone (RZ) and vortex sheet surrounding RZ (figure 2a); 3) $200 < Re \leq 270$ — a steady double-thread wake with a deformed vortex ring in RZ; 4) $270 < Re \leq 400$ — a periodical formation of the vortex loops (facing upwards), the periodical separation of the one edge of the vortex sheet (figure 2b); 5) $400 < Re \leq 3000$ — the periodical separation of the opposite edges of the irregularly rotating vortex sheet; 6) $3000 < Re \leq 4 \cdot 10^4$ — a turbulent wake (subcritical regime). The comparison of the time-average sphere drag coefficients $\overline{C_d}$ obtained by SMIF with experimental
\(\bar{C}_d\) (Schlichting, 1979) is presented at figure 1. For \(Re \leq 500\) we have a good comparison with experiment. For \(500 < Re < 5 \cdot 10^5\) we can say about a qualitative agreement with experiment because with increasing of \(Re\) we can observe consequentially the turbulization of the wake, the turbulization of the vortex sheet and the turbulization of the boundary layer (BL).

\[\text{Figure 3. The skin friction patterns on the sphere lee side at } x > 0 \ (Fr > 10): \text{ a-c – } Re = 10^4; 5 \cdot 10^4; 4.1 \cdot 10^5.\]

\[\text{Figure 4. The vortex structures in wake: a-c – } Re = 10^4; 5 \cdot 10^4; 4.1 \cdot 10^5 \ (Fr > 10); \text{ a-c – isosurfaces of } \beta = 1, 2, 2.\]

\[\text{Figure 5. The mean velocity vectors at vertical plane } x-z \ (a) \text{ and the mean pressure distribution on the sphere surface (in comparison with experiment (Bakić, 2002)) (b) at } Re = 5 \cdot 10^4 \ (Fr > 10).\]

4.1.2. The laminar-turbulent transition in the boundary layer on the sphere \((4 \cdot 10^4 < Re < 5 \cdot 10^5)\). At \(4 \cdot 10^4 < Re < 5 \cdot 10^5\) a laminar-turbulent transition in BL on the sphere surface (critical regime) is observed (Matyushin et al., 2007). This critical regime is characterized by the monotonous reduction of \(\bar{C}_d\) from 0.452 to 0.155 due to the reattachment of the separated laminar BL with formation of the separated bubbles within the boundary layer (figure 3-4b). With increasing of \(Re\) the azimuthal length of these separated bubbles is decreased (figure 3-4c). In other words at \(Re = 5 \cdot 10^4\) the streamwise turbulization of BL is realized (figure 3-4b). At \(Re = 4.1 \cdot 10^5\) BL become turbulent both in streamwise...
and azimuthal directions. The process of the generation of a separated bubble within the boundary layer (near the primary separation line) has been demonstrated in (Matyushin et al., 2007). The growing of these separated bubbles and their drift downstream led the vortex loops of the wake to form (figure 4b-c). At $Re = 5 \times 10^4$ the time-averaged angle $\theta$, of the laminar boundary layer separation is equal to 88° (see figure 5b). In the experiment (Bakic, 2002) $\theta$, was equal to 97-100°.

Figure 6. The vortex structures in the stratified fluid around a moving sphere at $Re = 100$: a-e – $Fr = 2, 0.8, 0.5, 0.2, 0.08$; a-e – the isosurfaces of $\beta = 0.005; 0.005; 0.02; 0.02; 0.005$.

4.2. The flow regimes at $0.4 \leq Fr \leq 10$ (figure 6a-c).

For flow regime II ($1.5 \leq Fr \leq 10$ – “the quasi-homogeneous case”) the “homogeneous” vortex structures of the wake (observed at $Fr > 10$) are flattened in the vertical direction and the four additional threads connected with the vortex sheet surrounding the sphere dominate in the wake (figure 6a).

For example for $Fr = 2$ at $200 < Re \leq 270$ the six-threads wake is observed (figure 7-8b). Unlike $Fr = 100$ at $Fr = 2$ (at $200 < Re \leq 270$) the unsteadiness in the form of the periodical fluctuation of the rear stagnation point around axis $z$ is observed. Thus with decreasing of $Fr$ (from 10 to 1.5) the vortex ring is deformed in an oval (figure 7b). In the vertical plane the part of fluid is supplied in RZ (figure 9a). Then this fluid goes through the core of the vortex oval and is emitted downstream in the horizontal plane (figure 10a). The 3D instantaneous stream lines which are going near the sphere surface go around this vortex oval and form the four vortex threads.

At $Fr < 1.5$ ($200 < Re \leq 500$) the big initial vertical flattening of the flow (figure 7-8c) prevent the vortex formation mechanisms typical for the homogeneous fluid. At $Fr < 1.5$ the new vortex formation mechanisms (which are typical for the stratified fluid) are realized with increasing of $Re$. With decreasing of $Fr$ the fluid structures around the sphere are slowly flattened both along the vertical axis $x$ and along the line of the sphere motion (along axis $z$) (the length of the internal waves in the vertical plane is $\lambda/d \approx 2\pi Fr$ (figure 6)). The length of four threads (connected with the vortex sheet surrounding the sphere) is also diminished with reducing of $Fr$ (figure 6a-b) and at $Fr \leq 0.1$ these threads are transformed in the high gradient sheets of density before the sphere (fig. 6e).

During the flow regime III ($0.9 < Fr < 1.5$ – “the non-axisymmetric attached vortex in RZ”) the vortex oval in RZ (figure. 10-11a) has been transformed into the quasi-rectangle with two shot vertical
vortex tubes (figure 10b). With decreasing of \( Fr \) the thickness of this quasi-rectangle in the vertical plane \( x-z \) is diminishing up to zero and this quasi-rectangle is transformed into the system of the two symmetric vortex loops (figure 10c). It means that the wave processes destroy the rectangular vortex in RZ. As you can see from figure 12d-e the top and bottom parts of each wave crest along the axis \( z \) are visualized as two symmetrical V-shaped vortex structures with the sloping ends. The head of this V-shaped vortex structure is connected with the four auxiliary vortex threads induced near the axis \( z \) by the four main vortex threads (connected with the vortex sheet surrounding the sphere). In one's turn these four induced vortex threads are connected with the horizontal arc-shaped vortex structures represented the internal waves. The clear relationship between the isolines of the perturbation of the salinity \( S \) in the vertical plane (the traditional way of the representation of the internal waves (figure 12a)) and the horizontal arc-shaped vortex structures (the new 3D way of the representation of the internal waves) is shown at figure 12.

**Figure 7.** The skin friction patterns on the sphere lee side at \( Re = 250 \): a-c – \( Fr = 100, 2, 1 \).

**Figure 8.** The vortex structure of the sphere wake at \( Re = 250 \): a-c – \( Fr = 100, 2, 1 \); a-c – the isosurfaces of \( \beta = 0.04, 0.04, 0.08 \).

**Figure 9.** The stream lines in the vertical plane (in the reference frame connected with sphere): a-f – \( Fr = 2, 0.8, 0.5, 0.3, 0.08 (Re = 100), 0.05 (Re = 500) \).
During the **flow regime IV** ($0.6 < Fr \leq 0.9$ – “the two symmetric vortex loops in the recirculation zone (RZ)”) the legs of these two symmetric vortex loops are combined with four induced vortex threads (mentioned above) (figure 10c-d) and the primary separation line (and RZ) vanish. Thus at $0.4 \leq Fr \leq 0.6$ the **flow regime V** (“the absence of RZ”) is observed (figures 9c, 10-11d).

4.3. **The flow regimes at Fr < 0.4 (“a new recirculation zone”, figure 6 d-e)**.
With following decreasing of Fr at $Fr = 0.4$ a new recirculation zone (RZ) is generated from the nearest wave crest (figures 9d, 10-11e). During the **flow regime VI** ($0.25 < Fr < 0.4$ – “a new RZ”) the V-shaped vortex structures (mentioned above) are transformed into “dorsals”. At $Fr \leq 0.25$ (with decreasing of Fr) these “dorsales” are shifted more close to the sphere and form a strong vortex envelope (“skeleton”) around the new RZ (figure 6d-e).

Thus at $Fr \leq 0.25$ the **flow regime VII** (“the two vertical vortices in the new RZ (bounded by the internal waves)”) is observed. At $0.03 \leq Fr \leq 0.25$ (at $Re < 120$) the ring-like primary separation line
has been simulated (figure 11f). At $Fr < 0.03$ (at $Re < 120$) the cusp-like primary separation line with four singular points (two nodes (in $x$-$z$ plane) and two saddles (in $y$-$z$ plane)) has been observed (figure 11h).

At $Fr \leq 0.3$, $Re > 120$ the edges of two vertical vortex sheets (on each side of the new quasi-2D RZ) are detached alternatively (figures 9-10f, 11g). The corresponding Strouhal numbers $0.19 < St = f \cdot d/U < 0.24$ (where $f$ is the frequency of shedding) are in a good agreement with the experiment (Lin et al., 1992).

The obtained characteristics of the simulated flows (such as horizontal and vertical separation angles) for $Fr \leq 10$, $Re \leq 500$ are in a good agreement with the experiment (Lin et al., 1992). The drag coefficients also correspond to experimental values.

![Figure 12. Fr = 1, Re = 100: a) the isolines of the perturbation of salinity $S$ ($\delta S = 5 \cdot 10^{-6}$, the darker (blue) isolines correspond to negative $S$), b) the isolines of $\beta > 0$ ($\delta \beta = 0.002$); c-d) the stream lines in the vertical plane (in the reference frame connected with fluid (c) and with sphere (d)); the isosurface of $\beta = 0.02$.](image)

4.4. The diffusion-induced flow around a resting sphere

Let us consider a resting sphere in a continuously stratified fluid. Owing to the numerical method SMIF it was shown (for the first time) that the interruption of the molecular flow (by the resting sphere) not only generates the axisymmetrical flow on the sphere surface (from the equator to the poles) but also creates the short unsteady internal waves (Baydulov et al., 2005). At first a number of these waves is equal to $t/T_b$. For example at $t = 2 \cdot T_b$ four convective cells with the opposite directions of the vorticity (two waves) are presented in the stream lines distribution. A base cell, whose size is determined by the radius of the sphere, is located near the sphere surface. At time more than $37 \cdot T_b$ the sizes and arrangement of cells are stabilized, and only the base cell and two thin adjacent cells with a thickness 2.2 mm are observed both in the salinity perturbation field $S$ and in the stream lines pattern. In other words the high gradient sheets of density with a thickness 2.2 mm are observed near the poles of the resting sphere. The maximum velocity of this quasi-steady flow near the sphere surface is equal to 0.006 mm/s ($Fr = 0.0003$, $Re = 0.12$).

5. Conclusions

The continuous changing of the complex 3D sphere wake vortex structure of the stratified viscous fluid with decreasing of $Fr$ from 100 to 0.02 has been investigated at $Re < 500$ owing to the mathematical modeling on the supercomputers and the $\beta$-visualization of the 3D vortex structures (the internal waves and the sphere wake). The numerical method SMIF and the $\beta$-visualization technique have been briefly described. At $0.6 < Fr < 1.5$ the gradual disappearance of the recirculation zone (RZ) is observed. It is the most interesting and complex transformation of the wake vortex structure. At $Fr = 0.4$ a new RZ is formed from the nearest “wave crest” which is appeared very close to the sphere.
With increasing of $Re$ (from 100 to 500) the number of the degrees of freedom of the flow is increased too. At the same time the stratification stabilizes the flow in the sphere wake. As a result of our DNS at $0.02 \leq Fr \leq 100$ and $Re \leq 500$ the following classification of the flow regimes around a sphere horizontally moving in the viscous stratified fluid has been observed:

I) $Fr > 10$ – the homogeneous case;
II) $1.5 \leq Fr \leq 10$ – the quasi-homogeneous case (with four additional threads connected with the vortex sheet surrounding the sphere);
III) $0.9 < Fr < 1.5$ – the non-axisymmetric attached vortex in RZ;
IV) $0.6 < Fr \leq 0.9$ – the two symmetric vortex loops in RZ;
V) $0.4 \leq Fr \leq 0.6$ – the absence of RZ;
VI) $0.25 < Fr < 0.4$ – a new recirculation zone;
VII) $Fr \leq 0.25$ – the two vertical vortices in the new RZ (bounded by the internal waves).

This classification is more close to the classification from (Chomaz et al., 1993).

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