A shallow layer laboratory model of large-scale atmospheric circulation

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ABSTRACT

A new shallow layer laboratory model of global atmospheric circulation is realised and studied by experiments and numerical simulations. A shallow rotating cylindrical fluid layer of 30 mm thickness and 690 mm diameter, with a localised heater at the bottom periphery and localised cooler in the central part of the upper boundary is considered. The rim heater imitates the equator heating and disc cooler – the North pole cooling. The flow transforms from the Hadley-like regime to the baroclinic wave regime through transitional states. The decrease in the thermal Rossby number for the fixed value of Taylor number results in the regularisation of the baroclinic waves. All wave regimes, even with regular wave structures, are characterised by strong non-periodic fluctuations. The observed baroclinic wave structures are a combination of temporarily evolving different baroclinic modes. The important outcome of the shallow layer model is a realisation of the Earth-like meridional three-cell structure. It is shown that the three-cell structure with analogs of polar, Ferrel and Hadley cells exist only in a limited range of parameters. A comparison of the results for the water and silicon oil demonstrated that the physical properties of the fluid can have a strong impact on the baroclinic wave structure.

1. Introduction

The global atmospheric circulation plays a crucial role in weather and climate processes providing the transfer of heat and angular momentum. The structure and dynamics of the global atmospheric circulation are very complex and depend on multiple factors such as rotation, solar heating, surface topography and so on. Remarkable progress in the numerical modelling of global circulation over the last decades has been made. However, this should not be misleading, because the atmosphere is a highly non-linear system and we are far from full understanding of complex interactions and links between its different elements. The global atmospheric circulation has a convective nature and its driver is a meridional temperature difference, which can variate due to different processes. For
example, substantial warming amplification over the Arctic Pole results in a decrease of the temperature contrast between the pole and the equator (You et al. 2021).

Decades of studies have proven that relatively simple laboratory models can help to understand the nature of complex atmospheric processes (Von Larcher and Williams 2014). Laboratory modelling makes it possible to isolate some key processes and study them with a reproducible and controlled series of experiments.

The main laboratory model for mid-latitude circulation was developed by Hide (1953) and consists of a rotating annular vessel with three concentric cylinders, where narrow gap of the outer annulus is filled with warm fluid, the inner cylinder is filled with cold fluid and the central part is the experimental chamber filled with water. The well-controlled boundary conditions and similarity with Eady model (Eady 1949) help to analyse the experimental results. This configuration has been successfully used for many years to study various aspects related to baroclinic wave formation in mid-latitudes (Read et al. 2014). Recent studies are focused on the influence of the mechanical and thermal inhomogeneity on the baroclinic wave formation (Marshall and Read 2018, 2020), the generation of different waves in the barostrat instability experiment (Rodda et al. 2018). First laboratory studies of mid-latitude circulation in a frame of a polar amplification scenario (Vincze et al. 2017, Rodda et al. 2022) showed that a decrease in the meridional temperature difference slows down the eastward propagation of the jet stream and complexifies its structure. A good agreement between the probability distribution of extreme events in a laboratory experiment and in the atmospheric case proves the usefulness of laboratory modelling for studying the dependence of the probability of extreme events on climate change (Harlander et al. 2022).

However, the real atmosphere is rather characterised by effective heating near the ground in the tropics and cooling in the upper layer in the polar region, which limits the applicability of classical annular configuration with vertical isothermal boundaries and motivates the realisation of a new laboratory configuration for the modelling of atmospheric circulation (Scolan and Read 2017). The horizontal and vertical displacements of heat and cold sources may have a strong influence on the flow structure and dynamics. Indeed, the large-scale flows in the alternative configuration, so-called dishpan experiments, with a cylindrical vessel with the rim heating at the bottom periphery and cooling at the centre (Fultz et al. 1959) were very similar to the typical atmospheric flows but less steady and regular than in the classical annular configuration. The complexity and irregularity of flows in a dishpan configuration was the main reason why a rotating annular vessel with isothermal sidewalls was chosen as a basic model of large-scale atmospheric circulation.

Only recently, first step toward a more complex atmospheric-like model (Scolan and Read 2017) was done. In a new proposed model, the convection in a rotating annular vessel is driven by local heating at the bottom and cooling at the centre of the upper boundary. It is shown that the flows in this configuration exhibit more spatio-temporal complexity, including coexistence and interaction between free convection and baroclinic wave modes.

Another important parameter of global atmospheric circulation modelling is the aspect ratio. The ratio of the characteristic vertical scale (about 20 km) to the horizontal one (more than 1000 km) for large-scale atmospheric flows is very small. For the classical annulus configuration, with isothermal vertical walls the aspect ratio is usually larger than 1 (Read et al. 2014), except recent experiments (Rodda et al. 2020) with aspect ratio close to 0.17.
In the model with a more realistic representation of heating and cooling sources (Scolan and Read 2017), the aspect ratio is about 0.65.

It can be concluded that laboratory experiment is an efficient tool for the investigation of the atmosphere-like dynamics but the problem of the optimal realistic framework is still open. Here, we present a first series of experiments using a shallow layer model of large-scale atmospheric circulation with horizontally and vertically displaced heater and cooler.

The structure of this paper is as follows. The experimental set-up is described in section 2. The main results are presented in section 3, including the description of the flow regimes (section 3.1), mode analysis (section 3.2) and results of numerical simulations (section 3.3). A summary and conclusion is given in section 4.

2. Experimental set-up

The principal scheme of the laboratory model of the general atmosphere circulation is shown in figure 1(a). The shallow rotating cylindrical layer of fluid with the localised heater at the bottom periphery and localised cooler in the central part of the upper boundary is considered. The rim heater imitates the equator heating and disc cooler – the North pole cooling. The rim heater is intentionally shifted from the sidewall to decrease the influence of the no-slip vertical boundaries and provide anticyclonic–cyclonic motion in the upper layer, imitating the formation of large-scale zonal flows (easterlies and westerlies) in the low latitudes.

The experimental model is a rectangular tank of a square cross-section with a side $L = 700$ mm and height $H = 200$ mm (figure 1). The sidewall and bottom are made of Plexiglas with a thickness 20 mm. For the realisation of the cylindrical layer the Plexiglas cylinder with a 3-mm wall and diameter $D = 690$ mm is inserted into the tank. There is no cover or additional heat insulation at the sidewall.

The way of realisation of the thermal boundary conditions may have a strong influence on the flow structure. We chose boundary conditions of the second kind (constant heat flux) because they are more realistic for the atmosphere. The cylindrical insert is installed on a false bottom, the upper part of which is a foil-coated textolite. By cutting out the tracks in the foil, it is possible to make heaters of different configurations. At present case, the heater is a circular stripe of width $l = 25$ mm, heated by electrical current. The distance from the cylindrical sidewall to the outer border of the heater is 40 mm. The heater is a very thin copper foil (of about 50 $\mu$m), so the temperature of its surface depends strongly on the flow structure. The heating power is controlled and kept constant during the experiment. The room temperature is kept constant by an air-conditioning system, and cooling of the fluid is provided by the heat exchange with the surrounding air on the free surface, the central cooling system and some heat losses through the sidewall. The cooling system includes a thick (10 mm) copper disc with diameter $d = 54$ mm partially inserted into the upper layer of the fluid (about 2 mm). We chose the cooler of relatively small area to minimise the impact of the solid lid, because the friction on solid boundaries plays an important role in an angular momentum balance in rotating convection (Evgrafova and Sukhanovskii 2022). The upper surface of the copper disc is cooled by a thermoelectric (Peltier) cooler. To remove heat from the hot side of the thermoelectric cooler a radiator with a forced air circulation is used. For minimisation of the impact of the air circulation,
Figure 1. (a) Scheme of the laboratory model, 1 – rotating table, 2 – rim heater, 3 – cooler, 4 – LED illumination, 5 – CCD camera; (b) Photo of the experimental set-up.

The cooling system is surrounded by an additional open box. It has been checked that air circulation does not result in fluid motion at the open surface. The temperature of the cooler was measured by a copper–constantan thermocouple installed into the copper disc, and the cooling power is estimated using the known relation between the Nusselt number and Rayleigh number.

The experimental model is placed on a rotating horizontal table. The rotating table provides a uniform rotation in the angular velocity range $0.02 \leq \Omega \leq 0.30$ rad s$^{-1}$ (with accuracy of $\pm 0.001$ rad s$^{-1}$). The choice of working fluid is an important issue. An open water
**Table 1.** The main fluid properties and parameters of the experimental model.

| Fluid properties          | Symbol | Value | Units  |
|--------------------------|--------|-------|--------|
| Density                  | $\rho$ | 911   | kg m\(^{-3}\) |
| Kinematic viscosity      | $\nu$  | $5.2 \times 10^{-6}$ | m\(^2\) s\(^{-1}\) |
| Thermal diffusivity      | $\kappa$ | $8.3 \times 10^{-8}$ | m\(^2\) s\(^{-1}\) |
| Thermal expansion coeff. | $\alpha$ | $9 \times 10^{-4}$ | K\(^{-1}\) |
| Prandtl number           | $Pr = \nu \kappa^{-1}$ | 62.7  |

**Experimental set-up**

| Parameter               | Value | Units |
|-------------------------|-------|-------|
| Layer radius            | 345   | mm    |
| Layer depth             | 30    | mm    |
| Heater width            | 25    | mm    |
| Heater radius           | 293   | mm    |
| Cooler radius           | 28    | mm    |
| Heating power           | 123   | wt    |
| Cooling power           | $\approx 3$ | wt |

Surface is practically always characterised by appearance of the so-called surface active agents (surfactants), which can lead to noticeable changes in surface properties. Often the open water surface behave more like a deformable film than a free surface (Manikantan and Squires 2020). Similar effects were found with the transformer oil (Sukhanovsky et al. 2012). To avoid this “rigid film” effect the silicon oil PMS-5 (see table 1) is used as the working fluid, but the appearance of Marangoni flows due to surface tension gradients cannot be excluded (Sukhanovskii et al. 2016). It should be mentioned that PMS-5 has a substantially higher value of the Prandtl number (about 63) in comparison with water, which can result in a less regular flow (Fein and Pfeffer 1976). In all presented experiments, the depth of the fluid layer $h$ was 30 mm and the surface of the fluid was open. The temperature inside the fluid layer was measured at mid-height ($z = 15$ mm) and $R = 180$ mm by the copper–constantan thermocouple and used for the estimation of the mean temperature of the fluid. The main fluid properties and parameters of the experimental set-up are provided in table 1. The direction of rotation in all presented experiments is clockwise.

Aluminium flakes are used to visualise the flow structure in the upper layer. The illumination of the tracers is provided by LED (light-emitting diode) strip placed on the perimeter of the experimental model above the fluid layer. The aluminium flakes are oriented along the flow, so they are bright when the flow is horizontal and dark when vertical motions are dominant. The recording was provided by 4 Mpx CCD camera Bobcat 2020. Most of the images were recorded with 1 fps.

As non-dimensional governing parameters, following Scolan and Read (2017) we use the thermal Rossby number $Ro_T$, the Taylor number $Ta$, and the Ekman number $E$:

$$ Ro_T = \frac{g \alpha h \Delta T}{\Omega^2 R^2}, \quad (1) $$

$$ Ta = \frac{4 \Omega^2 R^5}{h \nu^2}, \quad (2) $$

$$ E = \frac{\nu}{\Omega h^2}, \quad (3) $$

where $g$ is the gravitational acceleration, $\alpha$ thermal expansion coefficient, $\Delta T$ temperature difference between heater and cooler, $R$ radius of the layer, $\nu$ kinematic viscosity. Other important non-dimensional parameters, which describe convection and the heat transfer
are the Rayleigh number $Ra$ and the Nusselt number $Nu$ (ratio of the full heat flux to the conductive one):

$$Ra = \frac{g \alpha \Delta T \theta^3}{\nu \kappa},$$

(4)

$$Nu = \frac{f h}{\lambda \Delta T},$$

(5)

where $\kappa$ is the thermal diffusivity, $\lambda$ is the thermal conductivity and $f$ is the heat flux. For the calculation of the thermal Rossby number, the temperature of the heater is required. The measurement of the temperature of the thin foil is a technically complex problem. The known relation between Nusselt number and Rayleigh number $Nu = CRa^a$ can be used for the estimation of the mean heater temperature. In case of the developed convective regime, $a$ is equal to 1/3 and values of $C$ vary from 0.1 to 0.2 (Golitsyn 1979, Evgrafova and Sukhanovskii 2019), on the base of our numerical simulations (described in section 3.3) we chose $C = 0.2$. After simple calculations for the fixed heat flux ($f_h = 2.45 \times 10^3$ Wt m$^{-2}$), we obtain the mean value of the temperature difference between the fluid and the heater about 17 K. Using the same approach, we estimate the heat power of the cooler (the temperature of the cooler is measured). For the substantial temperature difference between the cooler and the fluid (about 8 K), the cooling power in the presented experiments is approximately 3 wt and the mean heat flux $f_c \approx 1.2 \times 10^3$ wt m$^{-2}$. This remarkable difference in the heating and cooling power is a consequence of the large difference in the heating and cooling areas. The cooler is substantially less than the heater (almost 19 times) and so is the total cooling power. This means that most of the cooling is provided by the heat exchange between the fluid and air on the open surface.

### 3. Main results

#### 3.1. Flow regimes

For the first series of experiments (see table 2), we fix the heating and cooling power and change only the rotation rate. The measurements are realised in a quasi-stationary state, when the net heat flux is approximately zero, which requires about 2 h after the heating and cooling initiation.

| Exp. | $\Omega$, rad s$^{-1}$ | $\Delta T$ | $Ro_T$ | $Ta$    | $E$          |
|------|----------------|---------|-------|--------|-------------|
| 1    | 0.08           | 25.1    | 8.4   | $1.7 \times 10^8$ | 0.068       |
| 2    | 0.09           | 25.7    | 6.5   | $2.3 \times 10^8$ | 0.061       |
| 3    | 0.11           | 26.9    | 4.9   | $3.2 \times 10^8$ | 0.05        |
| 4    | 0.13           | 24.5    | 3.1   | $4.7 \times 10^8$ | 0.042       |
| 5    | 0.17           | 25.1    | 1.9   | $7.5 \times 10^8$ | 0.033       |
| 6    | 0.23           | 23.9    | 1.0   | $1.4 \times 10^9$ | 0.024       |
| 7    | 0.37           | 23.9    | 0.4   | $3.6 \times 10^9$ | 0.015       |
| 8a   | 0.48           | 24.5    | 0.2   | $6 \times 10^9$ | 0.012       |
| 8b   | 0.48           | 10.2    | 0.1   | $6 \times 10^9$ | 0.012       |
| 8c   | 0.48           | 7.2     | 0.06  | $6 \times 10^9$ | 0.012       |
The flow structure for the relatively small angular velocity (exp. 1) is shown in figure 2. Please note that the size of the cooler, which is in direct contact with a fluid, is substantially less than the visible part of the cooling system, including the radiator and open box. The circular stripe (figure 2(a)) visible in the periphery is a heater, which produces intensive convective flow, consisting of multiple plumes. In agreement with a scheme presented in figure 1(a), an ascending convective flow over the heater results in formation of the convergent and divergent radial flows in the upper layer. Radial transport of angular momentum provides an anticyclonic circulation (easterlies) near the sidewall and cyclonic circulation (westerlies) at radii smaller than the radius of the heater. The observed cyclonic part of the flow is nearly axisymmetric and corresponds to the Hadley-like regime (Fultz et al. 1959, Batalov et al. 2010, Scolan and Read 2017). The complex pattern formed by aluminium flakes can be used for the reconstruction of velocity field by PIV (Particle Image Velocimetry) technique (Raffel et al. 1998). The mean velocity field is presented in figure 2(b). There is an anticyclonic belt near the sidewall and substantially more intensive cyclonic circulation in the central part.

The increase in rotation rate results in an increase in the Taylor number and a decrease in the thermal Rossby number. In our experiments in agreement with Fultz et al. (1959), Read et al. (2014) and Scolan and Read (2017) it leads to the instability of the axisymmetric flow. The instantaneous images in figure 3 illustrate transition from Hadley-like regime to the regime with baroclinic waves. There is an important difference between the flow structure and dynamics in the presented experiments and in a classical annulus configuration (Read et al. 2014) and a new one proposed in Scolan and Read (2017). In a classic configuration, there is an area of parameters on a map $Ro_T - Ta$ with steady regular waves. Vertical separation of the heating and cooling (Scolan and Read 2017) decreases this area. Here, in regimes with evident baroclinic waves, as in figure 3(c), the flow is constantly evolving, which leads to the changes in the number, amplitude and shape of the waves.
The movies, which illustrate this temporal flow evolution, are presented in Supplementary materials. The time of living of the regular structures strongly variates from several to tens of rotation periods. Regime with regular but unsteady waves is sensitive to the control parameters, and relatively small variation in $Ro_T$ and $Ta$ (exp. 8a, figure 4a) may lead to the irregular flow structure. Decreasing of $Ro_T$ by decreasing the heating power ($P_h = 24.5$ wt, for exp. 8b) results in the regularisation of the flow (figure 4b). The baroclinic waves with the main mode $m = 3$ are formed. Further decrease of $Ro_T$ ($P_h = 8$ wt, exp. 8c) leads to the weakening of baroclinic waves (figure 4c) and we can expect the achievement of the lower symmetrical regime (Mason 1975) with a subsequent decrease in $Ro_T$.

### 3.2. Mode analysis

In the previous section, we provide a qualitative description of the flow structure and its evolution with the variation of the control parameters. Fourier decomposition of the brightness $I$ field of instantaneous grey-scale images can provide valuable information about the modes of baroclinic waves and their temporal behaviour. The circular area near the mid-radius ($R_c = 160$ mm) of 11 mm (40 pixels) width is used for Fourier decomposition (figure 5). The Fourier components $A_n$, $B_n$ and energy of Fourier modes $E_n$ are given.
Figure 5. The thick black circle shows the area used for the Fourier decomposition of the brightness $I$ field. (a) corresponds to $t = 665$ s in figure 7(a,b) to $t = 155$ s in figure 7(b).

By

$$A_n(t) = \frac{1}{L} \sum_k I_k(t) \cos(2\pi nk/L),$$

$$B_n(t) = \frac{1}{L} \sum_k I_k(t) \sin(2\pi nk/L),$$

$$E_n(t) = A_n(t)^2 + B_n(t)^2,$$

where $t$ is a time, $k$ is an angle, $I_k(t)$ is a brightness value averaged over $40 \times 40$ pixels area, $L = 360$.

The energy of the main modes of baroclinic waves for a series of experiments with increasing rotation rates is presented in figure 6. These results are in good agreement with our qualitative observations. Increasing the rotation rate leads to the instability of Hadley-like regime and the formation of baroclinic waves. For the used dishpan configuration, the flow in a baroclinic regime is a combination of different baroclinic wave modes. There are evident dominant modes for exp. 6 and exp. 7 ($m = 4$ and $m = 3$) but with further increase of rotation rate the baroclinic waves become irregular (exp. 8), hence, the interval of parameters with regular baroclinic waves is very narrow. The time evolution of different wave modes (figure 7) shows that even in exp. 6 and exp. 7 the dominant modes are characterised by strong non-periodic fluctuations (figure 7(a) and (b)). It is interesting that unlike experiments in Hide configuration (Rodda et al. 2020), in presented experiments increasing of rotation rate results in decreasing of the main wave number. A substantial decrease in $Ro_T$ for the fixed $Ta$ results in a regular $m = 3$ wave, but its amplitude also strongly and non-periodically fluctuates (figure 7(d)). The map of regimes in a plane $Ro_T - Ta$ is shown in figure 8, where results from Scolan and Read (2017) are plotted for comparison.
3.3. Numerical simulation

One of the goal of the laboratory modelling is the implementation of the realistic Earth-like mean meridional and zonal circulation. Presented experimental results are focused on the flow structure at the upper layer and do not provide full information about three-dimensional flow structure. To complement experimental results, we provide results of numerical simulation for two different fluids, silicon oil, which is used in presented experiments and water, with significantly lower value of the Prandtl number (about 5). The mathematical modelling in formulation close to the laboratory one is described in detail in Vasiliev et al. (2023) and here we provide only its brief description. A direct numerical simulation of thermal convection in a rotating cylindrical layer is performed using the freely distributed computational fluid dynamics package OpenFOAM v2106. The computational domain is a digital copy of the experimental model in respect of its geometric dimensions, the location of the rim heater and the central disc cooler. The obvious gain in the volume of information occurs due to a remarkable growth of time costs, so to calculate 1 s of physical time (on 112 computing cores), it requires about 300 s of calculations. Here we present results of four simulations (table 3). The simulations 1 and 2 correspond to the experiments 1 and 7 (table 2). The simulations 3 and 4 are done for the same heating (cooling) power and rotation rate as simulations 1 and 2, but for the different working fluid (water).

Numerical simulations for the silicon oil show good qualitative agreement with experiments, including flow structure and characteristic values of temperature and velocity, hence simplified boundary conditions at the open surface (constant heat flux) is not of crucial importance for the studied regimes. The instantaneous radial velocity fields at the free
Figure 7. Time evolution of main wave modes: (a) exp. 6, (b) exp. 7, (c) exp. 8 and (d) exp. 8b. Colour bar in arbitrary units (Experiment) (Colour online).

surface (figure 9) demonstrate the transition from Hadley-like to wave regime. The mean meridional circulation for two different regimes is shown in figure 10. The circulation in the axisymmetric regime corresponds to the general scheme presented in figure 1(a). The Hadley-like cell occupies most of the layer, but because of the shift of the rim heater, an additional cell is formed near the sidewall, producing an anticyclonic belt. In the baroclinic regime, corresponding to the experiment 7 (table 2) one can see the appearance of the extra cell at the middle radii. This is a laboratory analog of the Ferrel cell, which is seen only in the mean (over azimuthal coordinate and time) meridional field, similar to the real Ferrel cell. Decreasing the heating power (Vasiliev et al. 2023) leads to the more regular waves as in
Figure 8. Map of regimes. Squares – experiment, crosses – numerical simulation. Results from Scolan and Read (2017) (triangles) are plotted for comparison. AV – amplitude vacillation, SV – shape vacillation (Colour online).

Table 3. Main parameters of numerical simulations.

| Sim. | $\Omega$, rad s$^{-1}$ | $\Delta T$ | $R_0$ | $T_a$ | $E$ | Pr |
|------|-----------------|---------|------|------|-----|----|
| 1    | 0.08            | 26.1    | 8.72 | $1.7 \times 10^8$ | 0.068 | 62.7 |
| 2    | 0.37            | 25.1    | 0.41 | $3.6 \times 10^5$ | 0.015 | 62.7 |
| 3    | 0.08            | 8.4     | 0.72 | $4.6 \times 10^9$ | 0.013 | 5.4 |
| 4    | 0.37            | 7.8     | 0.04 | $9.7 \times 10^{10}$ | 0.003 | 5.4 |

figure 4(a) and disappearance of the middle (Ferrel-like) cell. Meridional transport of angular momentum determines the structure of mean zonal flows (figure 11). In the Hadley-like regime, cyclonic circulation dominates except for the peripheral area near the sidewall. The structure of zonal flows in the atmospheric-like regime is more complex; here we can see a strong anticyclonic flow near the sidewall, an isolated cyclonic flow formed by analog of Hadley cell, very weak zonal circulation between analogs of Ferrel and polar cells, and cyclonic (in the upper layer) and anticyclonic (in the lower layer) flows in analog of polar cell. We can assume that the described differential rotation has substantial influence on the baroclinic waves and provides their spiral structure. The mean distributions of the temperature also show strong differences between the Hadley-like atmospheric-like regimes. For the latter, we can see distinct area with inclined isotherms; characterised by substantial horizontal gradients. Intensive convective flows determine the temperature distribution in the heating and cooling areas.

In contrast to classical Hyde model, baroclinic waves in the dishpan configuration are unstable. Experiments have shown that the observed baroclinic waves are a superposition of different wave modes, which drift in the rotating system in the cyclonic direction. The amplitude of these wave modes varies significantly with time and without a dedicated frequency. It is useful to consider the distributions of pulsations, the main source of which is wave motion. In the axisymmetric mode (figure 13a), radial velocity pulsations are created by small-scale convective structures (thermals and plumes) concentrated in the areas of rising and sinking currents above the heater and under the cooler. In the wave regime, the
Figure 9. Transition from the axisymmetric flow to the baroclinic waves: (a) $\text{Ro}_T = 8.7$, $\text{T}_a = 1.7 \times 10^8$ (corresponds to Exp.1, table 2), (b) $\text{Ro}_T = 0.4$, $\text{T}_a = 3.6 \times 10^9$ (corresponds to exp. 7, table 2). Instantaneous fields of radial velocity (in mm/s) at the free surface are shown. Numerical simulation.

Figure 10. Mean meridional circulation for different regimes (averaged over azimuthal coordinate and time): (a) axisymmetric regime (Hadley), $\text{Ro}_T = 8.7$, $\text{T}_a = 1.7 \times 10^8$, (b) atmospheric-like regime $\text{Ro}_T = 0.4$, $\text{T}_a = 3.6 \times 10^9$. Colour bar shows the magnitude of velocity (in mm/s). Numerical simulation.

Pulsations are localised in the upper part of the layer, in the area of baroclinic wave formation (figure 13b). This localisation of the baroclinic waves in the upper layer is consistent with Rodda et al. (2020), where a strong dependence of the wave formation on the layer depth was found and with analysis of planetary upper-level frontal zones (Durneva and Chkhetiani 2021).
Figure 11. Mean zonal circulation for different regimes (averaged over azimuthal coordinate and time): (a) axisymmetric regime (Hadley), $Ro_T = 8.7$, $Ta = 1.7 \times 10^8$, (b) atmospheric-like regime $Ro_T = 0.4$, $Ta = 3.6 \times 10^9$. Colour bar shows the magnitude of velocity (in mm/s). Numerical simulation.

Figure 12. Mean temperature distribution for different regimes (averaged over azimuthal coordinate and time): (a) axisymmetric regime (Hadley), $Ro_T = 8.7$, $Ta = 1.7 \times 10^8$, (b) atmospheric-like regime $Ro_T = 0.4$, $Ta = 3.6 \times 10^9$. Colour bar shows the magnitude of temperature (in K). Numerical simulation.
The physical properties of the fluid, such as thermal diffusivity and kinematic viscosity, defined by the Prandtl number, can have a substantial impact on the convective flow. In Fein and Pfeffer (1976), it was shown that increasing the Prandtl number leads to less regular flows. Using water as a working fluid requires a significant change in the experimental setup (the heater design), so to understand the role of the physical properties of the fluid, we run numerical simulations for the same conditions (rotation rate, heating and cooling power) as in experiments 1 and 7 (table 2) but with water instead of silicon oil. The control parameters of these numerical simulations are presented in table 3. It can be seen that substantial decrease in Prandtl number leads to a much smaller temperature difference at the same heating (cooling) power. The control parameters of simulations 2 (silicon oil) and 3 (water) are comparable, so one would expect a similar flow structure. Indeed, the baroclinic waves (figures 9b and 14a), mean meridional circulations (figures 10b and 15a), mean zonal flows (figures 11b and 16a) and mean temperature fields (figures 12b and 17a) are similar, but with some important differences. The analog of the Ferrel cell in the case of water is larger and the analogs of polar and Hadley cells are smaller than in the case of silicon oil. The baroclinic waves in water occupy a larger area than in silicon oil. This is clearly seen in the distributions of the energy of pulsations (figures 13b and 18a). Comparing mean temperature fields, we can see that in case of water distinct area with inclined isotherms is larger and maybe it is the main factor, which is responsible for the size of the baroclinic waves. The described variation in the spatial structure of baroclinic waves with Prandtl number is an interesting feature and needs further study. Decreasing of $Ro_T$ and increasing in $Ta$ (simulation 4) leads to an irregular wave regime (figures 14–18) and destruction of the three-cell meridional structure. The temperature field in the irregular wave regime is characterised by significantly larger area with inclined isotherms, which is favourable for the wave formation.
Figure 14. Typical structure of the radial flow in water simulations: (a) \( Ro_T = 0.72, Ta = 4.6 \times 10^9 \), (b) \( Ro_T = 0.04, Ta = 9.7 \times 10^{10} \). Instantaneous radial velocity fields (in mm/s) at the free surface are shown. Numerical simulation.

Figure 15. Mean meridional circulation for different regimes (averaged over azimuthal coordinate and time): (a) atmospheric-like regime, \( Ro_T = 0.72, Ta = 4.6 \times 10^9 \), (b) irregular wave regime, \( Ro_T = 0.04, Ta = 9.7 \times 10^{10} \). Colour bar shows magnitude of velocity (in mm/s). Numerical simulation.

A comparison of the results for different working fluids and control parameter values shows that a three-cell structure consisting of Hadley cell, Ferrell cell and polar cell analogs exists within a certain range of control parameters.
Figure 16. Mean zonal circulation for different regimes (averaged over azimuthal coordinate and time): (a) atmospheric-like regime, $Ro_T = 0.72$, $Ta = 4.6 \times 10^9$, (b) irregular wave regime, $Ro_T = 0.04$, $Ta = 9.7 \times 10^{10}$. Colour bar shows magnitude of velocity (in mm/s). Numerical simulation.

Figure 17. Mean temperature distribution for different regimes (averaged over azimuthal coordinate and time): (a) atmospheric-like regime, $Ro_T = 0.72$, $Ta = 4.6 \times 10^9$, (b) irregular wave regime, $Ro_T = 0.04$, $Ta = 9.7 \times 10^{10}$. Colour bar shows the magnitude of temperature (in K). Numerical simulation.
Figure 18. RMS of radial velocity (averaged over azimuthal coordinate): (a) atmospheric-like regime, $R_{OT} = 0.72$, $Ta = 4.6 \times 10^9$, (b) irregular wave regime, $R_{OT} = 0.04$, $Ta = 9.7 \times 10^{10}$. Colour bar shows the magnitude of rms (in mm$^2$/s$^2$). Numerical simulation (Colour online).

4. Summary and conclusion

A first series of experiments using a shallow layer model of large-scale atmospheric circulation with horizontally and vertically displaced heater and cooler is realised. The configuration of the model is similar to the dishpan configuration in Fultz et al. (1959) but with some important differences. The heater in the shape of a rim is shifted from the sidewall. It provides an anticyclonic belt in the upper layer near the sidewall, modelling easterly winds and decreasing the influence of the sidewall on the flow formation. Low-viscous silicon oil instead of water is used to avoid complex effects provided by the formation of a thin film of a surface-active substance on the open surface. The transformation of the flow structure with the increase in the rotation rate generally agrees with previous results (see regime diagram in Scolan and Read 2017). The flow transforms from the Hadley-like regime to the baroclinic wave regime through transitional states. The decrease in the thermal Rossby number for the fixed value of Taylor number results in the regularisation of the baroclinic waves.

Since all laboratory models have some limitations, it is necessary to understand their main differences. This will help to choose the most appropriate model for the studying of the specific problems related to atmospheric baroclinic waves. The important difference between the presented and classical annulus configurations is the absence of the steady waves. All wave regimes, even with regular wave structures, are characterised by strong non-periodic fluctuations. The observed baroclinic wave structures are a combination of temporarily evolving different baroclinic modes. We assume that more stochastic and irregular flows in the presented configuration can be explained by the specific realisation of the heating. Rim heating results in the formation of multiple plumes and non-homogeneous temperature distribution in the azimuthal direction. In a rotating layer,
the structure of convective flow in the periphery becomes more non-uniform due to the formation of vortices at the border between anticyclonic and cyclonic flows. This strongly non-homogeneous azimuthal distribution of warm fluid results in the excitation of baroclinic waves of different modes, unlike the case when the heating is provided by isothermal sidewall. A similar result (the absence of steady waves) was obtained in Scolan and Read (2017) with the heater and cooler at horizontal boundaries. This proves that the spatial distribution of heating and cooling, their location and the type of boundary condition are important for the baroclinic wave stability.

In a shallow layer of viscous fluid \( (Pr \gg 1) \) the role of viscous boundary layers cannot be neglected. The thickness of the Ekman layer \( \delta_E = (v/\Omega)^{1/2} \) in our experiments and numerical simulations varies from 2 to 8 mm, so the processes in the Ekman layer can have a substantial influence on the flow structure. In the absence of thermal convection, the Ekman layer is essentially important for a spin-up in homogeneous and stably stratified fluids (see references in Duck and Foster 2001). The shear of azimuthal velocity at the solid boundary provides horizontal flow in the boundary layer (so-called Ekman pumping), which draws the fluid from the upper layers and transports it into corner regions, providing efficient injection of the angular momentum into the bulk of the fluid. Theoretical analysis and experiments (Walin 1969, Buzyna and Veronis 1971, Flór et al. 2002) show that the depth of corner flow penetration can be much deeper than the thickness of the Ekman layer and depends on the stratification. For the rotating linearly stratified bounded layers, the depth of penetration is proportional to the ratio \( R\Omega/N \), where \( N \) is the buoyancy frequency. This estimation for linearly stratified fluids was obtained for a rapidly rotating layer and should be used for our system with caution; because we consider moderate rotation rates and nonlinear stratification. The characteristic value of \( N \) in our experiments (in area of baroclinic waves) is about 1 rad/s, and if we use the estimation of Ekman flow penetration as \( \delta_p \simeq R\Omega/N \), it would strongly exceed the whole layer depth. This means that circulation provided by the processes in the Ekman layer would occupy the whole fluid layer. We can estimate the influence of Ekman pumping by our mean fields for azimuthal and meridional flows (figures 11a and 10a). There is a cyclonic flow near the bottom and the Ekman pumping should provide inward radial flow in the Ekman layer, but as we can see, there is outward radial flow. We can conclude that in our case, thermal convection is a main driver of the meridional circulation, and radial flows induced in the viscous boundary layer are not of primary importance. The distribution of velocity pulsations show that except the heating area, they mostly concentrated in the upper layer. The velocity pulsations are damped in the Ekman layer and as a result baroclinic waves in our model are localised in the upper layer, which is consistent with Rodda et al. (2020) and Durneva and Chkhetiani (2021). We note, however, that the interaction between viscous fluid and solid boundaries plays a crucial role in the formation of differential rotation in rotating convection (Williams 1968, Read 1986, Batalov et al. 2010, Evgrafova and Sukhanovskii 2022). The exchange of angular momentum occurs inside viscous boundary layers, the Ekman layer at the bottom and the Stewartson layer at the sidewall, and the meridional convective circulation provides transport of angular momentum in the fluid layer. The strong differential rotation in baroclinic wave regime may be responsible for their spiral shape.

Another important parameter is the aspect ratio of the model. Its variation leads to a strong shift of the areas with different regimes in the diagram \( \text{Ro}_T - Ta \). The important outcome of the shallow layer model is a realisation of the Earth-like meridional three-cell
structure. Numerical simulation with different working fluids showed that the three-cell structure with analogs of polar, Ferrel and Hadley cells exist only in a limited range of parameters. A comparison of the results for the water and silicon oil also proved that the physical properties of the fluid can have a strong impact on the baroclinic wave structure. Finally, we can conclude that the presented model provides the formation of atmospheric-like flows, characterised by complex temporal behaviour and can serve as an efficient tool for studying different aspects of global atmospheric circulation.

Authors’ contributions

A.S. did the conceptualisation of the study, the experimental data curation and wrote the original draft. E.P. prepared experimental setup, did experimental measurements and reviewed and edited the manuscript, A.V. prepared mathematical model, did numerical simulations and reviewed and edited the manuscript.

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Data availability

The data acquired during this study are available from the corresponding author upon reasonable request. Supplementary materials are available at https://www.icmm.ru/en/nauka/programmigranti/77-rnf/1167-development-of-laboratory-model-of-general-atmospheric-circulation.

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