BPS states and supersymmetric index

in $N = 2$ type I string vacua

Jose F. Morales$^a$ and Marco Serone$^{a,b}$

$^a$International School for Advanced Studies, ISAS-SISSA
Via Beirut n. 2-4, 34013 Trieste, Italy

$^b$Istituto Nazionale di Fisica Nucleare, sez. di Trieste, Italy

e-mail: morales, serone@sissa.it

Abstract

We study the moduli dependence of a class of couplings in $K3 \times T^2$ compactifications of type I string theory, for which one-loop amplitudes can be written in terms of an $N = 2$ supersymmetric index. This index is determined for generic models as a function of the BPS spectrum. As an application we compute the one-loop moduli dependence of the $F_g W^{2g}$ couplings, where $W$ is the $N = 2$ gravitational superfield, for type I compactifications based on the Gimon-Johnson $K3$ orientifolds, showing explicitly their dependence on the aforementioned index.

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1. Introduction

During the recent developments achieved in string and quantum field theories with extended supersymmetry, it has become evident that a prominent role in governing the dynamics is played by BPS states. These special states provide important clues in the exploration of the strong coupling regimes of the corresponding theories. The appearance of solitonic massless BPS states in $N = 2$ supersymmetric Yang-Mills theories \cite{1}, the prediction of U-duality in type II string theories of the existence of BPS states carrying Ramond-Ramond charges \cite{2}, subsequently identified with D-branes \cite{3}, and the resolution of the conifold singularity via a solitonic charged BPS state \cite{4} are only few examples of the vast number of results supporting their importance.

An interesting connection between BPS states and generalized Kac-Moody currents has been found in \cite{5}. In this reference the authors study threshold and gravitational corrections in $N = 2$ four-dimensional compactifications of the heterotic string. These corrections can be written in terms of the $N = 2$ supersymmetric index defined in \cite{6} and are determined purely in terms of BPS states. In particular the index is shown to count the difference between the number of hyper and vectormultiplets in the effective four-dimensional theory.

The scope of this paper is to show that an analogous result holds in $N = 2$ type I string compactifications. The $N = 2$ supersymmetric index is realized in type I compactifications and then related to the corresponding threshold and gravitational corrections. The connection between type I amplitudes and this index was first pointed out by Antoniadis et al. in ref.\cite{7}. For a generic compactification we compute this index as a function of the BPS spectrum, finding again that BPS contributions enter only through the difference between hyper and vectormultiplets \cite{1}. As an application of these results we will explicitly compute one-loop amplitudes of higher derivative F-terms $F_2(X)W^2$, with $W$ the $N = 2$ gravitational superfield and $X$ a generic chiral vector superfield, for a class of type I string

\footnote{Strictly speaking this is not the case for the $F_1$-coupling, see below.}
compactifications obtained as an orientifold of type IIB theory [8, 9, 10, 11], generalizing
the computation performed in [12] for the $\mathbb{Z}_2$ model [13, 14] and pointing out explicitly
the BPS dependence of these amplitudes. This tower of gravitational couplings has been
extensively studied in the context of string dualities. In particular for type II strings they
are given by topological partition functions in twisted Calabi-Yau sigma models [15], while
in heterotic compactifications they can be extracted from a simple one-loop amplitude
[16, 17]. The type I computation of these terms, like in the heterotic case, is given by
a one-loop amplitude which can be written in terms of the mentioned supersymmetric
index. This index encodes the compactification model-dependence of these couplings, while
an universal part coming from the spacetime correlations completes its structure.

The paper is organized as follows. In the next section we realize the supersymmetric
index in $N = 2$ type I compactifications and compute it for a generic model as a function
of its four dimensional BPS spectrum. In section three we briefly review the construction
of $K3$ orientifolds given by [4] and then we compute the $F_g$ couplings in these vacua. In
the two appendices we report some technical details needed to compute the correlation
function and the values of the indices for the various models. Some conclusions are given
in the last section.

2. BPS states and the $N = 2$ supersymmetric index

In this paper we study the moduli dependence of one-loop amplitudes of couplings in
$N = 2$ supersymmetric four-dimensional lagrangians arising as $K3 \times T^2$ compactifications
of type I string theory. We will consider in detail the tower of $F_g$ gravitational couplings,
although in this section we will restrict our analysis to the main features of the involved
amplitudes in order to allow a straightforward generalization to a wider class of terms.
In particular we discuss some examples of couplings in which one-loop corrections can
be written in terms of the “new supersymmetric index” [6] realized in $N = 2$ type I
compactifications. The important point in the considered amplitudes is that the internal theory, entering simply through an index, receive contributions only from its ground states. As it will be explicitly shown in our computations, the space-time part of the correlation function can be reduced to a supersymmetric partition function in the odd-spin structure, where a cancellation between fermion and boson determinants holds and again only ground states give a non-vanishing result. In this way we reduce the amplitude to a sum of BPS states contributions.

One loop amplitudes in string computations involve in general a sum over the spin structures carried by the involved surfaces. Using the Riemann identity we can relate the sum over spin structures to an odd spin structure correlation of new operators obtained by a triality rotation of the original ones [18]. In the last section we use this procedure to reduce the relevant amplitude for the $F_g$ couplings (a bunch of gravitons and graviphotons) to an odd spin structure correlation. Other physical one-loop corrections as thresholds to gauge couplings are also determined from odd spin structure computations [19]. In this section we will see how these odd correlations are related to a realization of the mentioned supersymmetric index.

2.1. One-loop corrections in type I string compactifications

In order to understand better the general structure of the kind of amplitudes involved in our discussion, let us briefly work it out two simple examples of relevant odd spin structure correlations in $K3 \times T^2$ type I compactifications.

a) $F_1$ gravitational coupling

In this first example we discuss the moduli dependence of the $R^2$ coupling in the effective four-dimensional action, with $R_{\mu\nu\rho\lambda}$ the Riemann tensor. This can be extracted [19] from the on-shell three-point function of two gravitons and a T-modulus computed in the odd
spin structure. Type I computations involve also a sum over the surfaces that we will denote respectively by \( T, K, A, M \) for the torus, Klein bottle, annulus and Möbius strip. The torus contribution can be written as

\[
\epsilon^{\mu\nu\lambda\rho} p_1 \lambda p_2 \rho p_2^\beta \Theta_T^{(T)} = \int_{\Gamma} \frac{d^2 \tau}{\tau_2} \int \prod_{i=1}^{3} d^2 z_i \langle V^\mu_1(p_1, z_1) V^\nu_2(p_2, z_2) \times V^{(-1,-1)}_T(p_3, z_3) T_F(z) \tilde{T}_F(\bar{z}) \rangle_{\text{odd}}
\]

where \( \tau = \tau_1 + i \tau_2 \) is the modular parameter of the world-sheet torus, \( \Gamma \) its fundamental domain and \( V_T^{(-1,-1)} \) is the \( T \)-vertex operator in the \((-1,-1)\) left-right ghost picture

\[
V_T^{(-1,-1)}(p) = e^{-\phi(z)} \Psi_+(z, \bar{z}) e^{ip \cdot X}
\]

\( \Psi_+(z, \bar{z}) \) being a primary field of dimension \((1/2, 1/2)\). The \((-1,-1)\) ghost picture takes into account the left-right Killing spinors on the world-sheet torus while the supercurrent insertions \( (T_F = T^+_F + T^-_F + \text{space-time part}, \text{and the same for the left one} \tilde{T}_F) \) soak up the gravitino zero modes. The \( V_h \)'s represent the gravitons, whose vertices are given by:

\[
V^\mu_\nu_\lambda(p) = (\partial X^\mu + ip \cdot \bar{\psi} \psi^\mu)(\bar{\partial} X^\nu \cdot i p \cdot \bar{\tilde{\psi}} \tilde{\psi}^\nu) e^{ip \cdot X}
\]

The analysis of this amplitude is just a left-right symmetric version of the one performed in [19]. The OPE of the \( N = 2 \) internal superconformal algebras are given by

\[
\tilde{T}^+_F(\bar{w})T^+_F(w) \Psi_\pm(z, \bar{z}) = \mp \tilde{J}(\bar{w})J(w) \Phi_\pm(z, \bar{z}) + ...
\]

where \( J, \tilde{J} \) are the \( U(1) \) currents associated to the \( N = 2 \) superconformal algebras, \( \Phi_\pm \) is the dimension \((1,1)\) upper component of \( \Psi_\pm \) and ... are contour integrals that give vanishing contribution to this amplitude. We can then relate \( \Theta_T \) to a \( T \)-derivative of a quantity \( \Delta \) written as

\[
\Theta_T^{(T)} = i \partial_T \Delta^{(T)}
\]

with

\[
\Delta^{(T)} = -i \int_{\Gamma} \frac{d^2 \tau}{\tau_2} C_T
\]
and

$$C_T \equiv \text{Tr}_{RR}(-1)^{F_L + F_R} F_L F_R q^\Delta \bar{q}^\bar{\Delta}$$

(2.5)

where \( q = e^{2 \pi i \tau} \), the trace is restricted to the Ramond-Ramond sector and contains the momenta lattice sum in the torus direction, and \( \Delta \) is the conformal dimension of the state propagating around the loop. \( F_L, F_R \) are the fermionic numbers, i.e. the zero modes of the \( U(1) \) currents \( J_L, J_R \), which soak up the four zero modes of the free fermions associated to the torus direction and then are necessary to get a non-vanishing result. The space-time part contributes only through the eight zero modes needed in the torus for the odd-spin structure. In the last section we will study the whole tower of \( F_g \) couplings, which includes, besides the two gravitons, an additional bunch of \( (2g - 2) \) spacetime operators, obtained from the graviphotons through a triality rotation induced by the spin-structure sum. The internal structure is therefore untouched and only the space-time part of the amplitude will be modified. In this section we will concentrate in this internal part \( C_T \) (and similar quantities for the rest of the surfaces) which realizes the \( N = 2 \) supersymmetric index. The contributions given by the other surfaces can be analyzed in an analogous way. Proceeding along the same lines followed for the torus, we relate the amplitudes to an integration in the corresponding worldsheet moduli of a spacetime correlation and an internal contribution through an index written as:

$$C_K \equiv \text{Tr}_{RR} \left( \frac{F_L + F_R}{2} \right) (-1)^{F_L + F_R} \Omega q^\Delta \bar{q}^\bar{\Delta}$$

$$C_A \equiv \text{Tr}_R (-1)^F F q^\Delta \bar{q}^\bar{\Delta}$$

$$C_M \equiv \text{Tr}_R (-1)^F F \Omega q^\Delta \bar{q}^\bar{\Delta}$$

(2.6)

Again the \( F \) insertions provide the correct number of zero modes (two in this case) that we need to soak up in order to get a non-vanishing result.

b) Threshold corrections to gauge couplings

The second example refers to moduli dependence of one-loop threshold corrections to
gauge couplings. More general corrections of this kind have been studied in \cite{20}. We restrict ourselves to show the connection of these amplitude with the considered index. As before, we can extract the moduli dependence from a three-point function where now, instead of gravitons, we insert the gauge field vertex operators:

$$V^\mu,a_A(p, z) = (\partial_\tau X^\mu + ip_\tau \cdot \psi \psi^\mu) e^{ip X} \lambda^a$$  \hspace{1cm} (2.7)

with “a” an index in the adjoint of the gauge group and $\lambda^a$ the corresponding Chan-Paton matrix. The relevant amplitude is then given by

$$\epsilon^{\mu\nu\lambda\rho} p_1 \lambda \rho \delta^{ab} \Theta_{T}^{A,M} = \int \frac{dt}{t} \prod_{i=1}^{2} dt_i d^2 z \langle V_{A}^{\mu,a}(p_1, t_1) V_{A}^{\nu,b}(p_2, t_2) V_{T}^{(-1)}(p_3, z) T_F(z_0) \rangle_{odd}^{A,M}$$  \hspace{1cm} (2.8)

where $V_T^{(-1)}$ is the closed modulus

$$V_T^{(-1)}(p) = (e^{-\phi} \Psi(z, \bar{z}) + e^{-\tilde{\phi}} \tilde{\Psi}(\bar{z}, z)) e^{ip X}$$  \hspace{1cm} (2.9)

where $\Psi$ and $\tilde{\Psi}$ are respectively the components of dimensions $(1/2,1)$ and $(1,1/2)$ of an $N=2$ superfield and $T_F \equiv T_F + \tilde{T}_F$ is the left-right symmetric picture changing operator.

The four spacetime zero modes required in the odd spin structure come from the fermion part of the gauge field vertices reproducing the correct kinematic factor in (2.8). As before the $N=2$ OPE allows us to write the internal contribution as a $T$-derivative of a trace in the Ramond sector

$$\Theta_{T}^{A} = i \partial_T \Delta_{a}^{A}$$

with

$$\Delta_{a}^{A} \sim \int \frac{dt}{t} C_{a}^{A}$$  \hspace{1cm} (2.10)

and

$$C_{a}^{A} \equiv T_R(-1)^F F \left(Q^2 a q^A \bar{q} \bar{\Delta} \right)$$  \hspace{1cm} (2.11)

where $Q^2_a$ is the charge of the state propagating around the loop. The Möbius strip contribution on the other hand is just given by an $\Omega$ insertion in this trace. We recognize again
the indices found in the previous example. In this way we have expressed some one-loop corrections to four-dimensional effective actions arising from compactifications of type I strings, in terms of a realization of the \( N = 2 \) supersymmetric index in these theories.

### 2.2. BPS states and \( N = 2 \) Supersymmetric indices

In the beginning of this section we argued that the quantities (2.5,2.6) can be written as a sum over BPS contributions. This subsection is devoted to determine them in terms of this spectrum for a generic \( K3 \times T^2 \) type I compactifications. Threshold and gravitational one-loop corrections for analogous compactifications of the heterotic string are also written in terms of this \( N = 2 \) supersymmetric index. Exploiting the representation properties of the internal superconformal algebra for these compactifications, Harvey and Moore \[5\] found that the index counts the difference between the number of BPS hyper and vectormultiplets at each level of mass. We will follow the lines of this reference to find similar results for the relevant indices involved in type I compactifications. The analysis is performed for the case in which no Wilson lines are turned on. Our considerations are however general, and the modifications brought by their inclusions will be pointed out.

The internal superconformal theory associated to \( K3 \times T^2 \) compactifications of type I theory is a sum of two pieces, corresponding to the open and closed string sectors. The open sector is realized with a \( (c = 3, N = 2) \oplus (c = 6, N = 4) \) SCFT while the closed sector is associated to the conformal theory that arises after an \( \Omega \)- projection of the \([(c = 3, N = 2) \oplus (c = 6, N = 4)]_L \otimes [(\tilde{c} = 3, N = 2) \oplus (\tilde{c} = 6, N = 4)]_R \) SCFT, where \( \Omega \) is the world-sheet parity operator. In order to relate (2.5,2.6) to a counting of four-dimensional BPS states let us review the structure and superconformal content of these states in type I compactifications.
String states in the open sector satisfy the mass condition (in the Neveu-Schwarz sector):

\[ \frac{1}{2} M^2 = \frac{1}{2} p^2 + (N - \frac{1}{2}) + h_{int} \]  

(2.12)

with \( N \) the oscillator number associated to the space-time and torus directions, \( h_{int} \) the conformal weight in the \( N = 4 \) SCFT and \( p \) the Kaluza-Klein momentum coming from the torus. The BPS bound \( M^2 = p^2 \) is then satisfied only by the six-dimensional massless Neveu-Schwarz states \( (N = 1/2, h_{int} = 0) \) and \( (N = 0, h_{int} = 1/2) \), which after a further torus compactification generate all the four dimensional vector and hyper BPS multiplets. Note that this is not what happens in \( N = 2 \) heterotic models where the BPS condition (in the Neveu-Schwarz sector) reads

\[ \frac{1}{8} M^2 = \frac{1}{2} p_R^2 = \frac{1}{2} p_L^2 + (h - 1) \]  

(2.13)

with \( p_L, p_R \), the torus momenta including winding and Kaluza-Klein modes, and \( h \) the conformal weight of the states in the \( c = 26 \) CFT. In this case, the number of BPS states depends on the level of mass, since for a fixed \( p_R^2 \) each point in the lattice \( p_R^2 - p_L^2 \in 2\mathbb{Z} > 0 \) defines additional BPS states with \( h > 1 \) besides the six-dimensional massless one \( h = 1 \).

The structure of the type I open Ramond sector is simply obtained by spectral flow of the two massless Neveu-Schwarz representations seen before. The conformal content is displayed in the following table:

| Sector   | Vectormultiplets          | Hypermultiplets            |
|----------|---------------------------|----------------------------|
| Ramond   | \((1/8, \pm 1/2) \otimes (1/4, 1/2)\) | \(2 \times (1/8, \pm 1/2) \otimes (1/4, 0)\) |

where, following the notation of [3], we denote with \((h, q) \otimes (h', I)\) a state with conformal weight \( h \) and U(1) charge \( q \) of the \( c = 3, N = 2 \) theory and weight \( h' \) and representation \( I \) of the SU(2) current of the \( c = 6, N = 4 \) theory.

We are now ready to compute the indices (2.6) associated to the open string sector. The fermionic numbers decompose as \( F = F^{(1)} + F^{(2)} \), \( F^{(1)} \) and \( F^{(2)} \) being the zero modes of the U(1) currents \( J^{(1)} \) and \( J^{(2)} \) associated to the \( N = 2 \) and \( N = 4 \) superconformal
algebras respectively \((J^{(2)} = 2J^3, \text{ where } J^3 \text{ is the Cartan element of the } N = 4 \text{ SU}(2) \text{ current})\). Since in \(SU(2)\) representations the eigenvalues of \(J^3\) come always in pairs, the only non-vanishing contribution to the indices come from the \(F^{(1)}\) insertion. Equivalently, only the \(F^{(1)}\) insertion, which soaks up the zero modes of the free fermions associated to the \(T^2\) torus, give a non-vanishing result. We are then left with the trace

\[
C_A + C_M = 2 \text{Tr}_{N=2}F^{(1)}(-)^{F^{(1)}}\text{Tr}_{N=4}(-)^{F^{(2)}}q^\Delta,
\]

where the trace now runs over the \(\Omega\)-invariant states \(\Gamma\) including the Chan-Paton degrees of freedom. The \(N = 2\) part is common to both multiplets, while the \(N = 4\) SCA enters only through the Witten indices \([21]\):

\[
\begin{align*}
\text{Tr}_{(1/4,0)}(-)^{F^{(2)}} &= 1 \\
\text{Tr}_{(1/4,1/2)}(-)^{F^{(2)}} &= -2
\end{align*}
\]

Finally we find

\[
C_A + C_M = 2 \sum_{\Gamma \in \Gamma} \left( \frac{1}{2} e^{i\pi t} - \frac{1}{2} e^{-i\pi t}\right) (\text{Tr}_{I=0}(-)^{F^{(2)}} + \text{Tr}_{I=1/2}(-)^{F^{(2)}}) e^{-\pi t|p|^2} = 4i \left( n_H^{\text{open}} - n_V^{\text{open}} \right) \sum_{\Gamma \in \Gamma} e^{-\pi t|p|^2}
\]

where \(\Gamma\) represents the lattice momenta sum and \(n_V^{\text{open}}, n_H^{\text{open}}\) are respectively the number of massless four dimensional vector and hypermultiplets in the open string sector. We have extracted the overall factor \((n_H^{\text{open}} - n_V^{\text{open}})\), corresponding to the common degeneration to all levels of mass of the BPS number, as was already discussed before. More general backgrounds including Wilson lines on the torus can be analyzed. In this case, the \(T^2\) momenta lattice will be shifted by the included gauge field expectation values, and a given number of massless vector and hypermultiplets will get masses. Notice, however, that their difference will be left invariant, since this Higgs mechanism will always give mass to an hyper-vector pair.

\(^2\)Note that the \(\Omega\)-projection in the open sector gives constraints only on the Chan-Paton degrees of freedom.
Let us now turn to the closed string spectrum. The bosonic BPS content in this sector arises from a tensor product of the aforementioned NS-NS (R-R) massless representations symmetrized (antisymmetrized) under $\Omega$. Let us first notice that for each R-R ground state in the $N = 4$ SCFT we can construct four states, taking into account the multiplicities coming from the $N = 2$ space-time and torus algebra representations. Although the trace runs only in the R-R sector, using spectral flow, we can see that all the bosonic content of BPS multiplets is taken into account, since $\Omega$-even contributions to this trace count the NS-NS states kept by the $\Omega$-projection. The counting of R-R ground states in the $N = 4$ theory is simply given by the geometrical structure of $K3$: 20 states $(1/4, 0) \otimes (1/4, 0)$, corresponding to the $h^{1,1} = 20$ cohomologically distinct $(1, 1)$ differential forms on $K3$ and one multiplet $(1/4, 1/2) \otimes (1/4, 1/2)$ counting the four forms given by $h^{0,0}, h^{2,0}, h^{0,2}, h^{2,2}$. As is clear, there are no states $(1/4, 1/2) \otimes (1/4, 0)$ or vice versa because $h^{1,0} = h^{0,1} = h^{2,1} = h^{1,2} = 0$. We can finally write the torus index as

$$C_T = \text{Tr}_{N=4}^{N=2} \left[ F_L (-)^{F_L + F_R} q^\Delta \bar{q}^{\bar{\Delta}} \right] = - (n_H^{\text{closed}} + n_V^{\text{closed}}) \sum_{p \in \Gamma} e^{-\pi \tau_2 |p|^2}$$

(2.17)

where $n_H^{\text{closed}} + n_V^{\text{closed}}$ are the 24 massless four dimensional hyper and vectormultiplets, corresponding to the Witten index $\text{Tr}_{N=4}^{N=2} \left[ (-)^{F_L + F_R} \right] = \chi(K3)$. The $N = 2$ part enters only through the lattice sum and the overall $(-1)$ factor.

The last involved quantity is the index related to the Klein Bottle. If we call $\Omega_{int}$ the worldsheet parity operator restricted to the $K3$ part, we can observe that vectors, being constructed from left-right symmetric spacetime+torus combinations of states, are counted by $\Omega_{int}$ with a minus sign in the $RR$ sector. This observation allows us to write finally:

$$C_K = \text{Tr}_{N=4}^{N=2} \left[ \frac{F_L + F_R}{2} (-)^{F_L + F_R} \Omega q^{\Delta + \bar{\Delta}} \right] = i(n_H^{\text{closed}} - n_V^{\text{closed}}) \sum_{p \in \Gamma} e^{-\pi t |p|^2}$$

(2.18)

where again the overall factor $(i)$ comes from the $N = 2$ part.

We achieved the final goal of this section. The contributions associated to the Klein bottle, annulus and Möbius strip depend only on the difference between the number of hyper
and vector BPS states of a given compactification. As noted in [5], this dependence ensures
the smoothness of amplitudes in the moduli space, since BPS states in $N = 2$ theories
always appear and disappear in hypermultiplet-vectormultiplet pairs. Notice, however, that
$C_T$ enters in a different way, but being associated to corrections in $N = 4$ supersymmetric
theories, it does not contribute in general. Among the couplings we analyzed, it gives a
non-trivial contribution only to the gravitational coupling $F_1$, as we have seen before and
we will see in greater detail in the next section.

3. $F_g$ terms on K3 orientifolds

As an application of the results found in the last section, we want explicitly compute in
the following the gravitational couplings $F_g$ at 1-loop for K3 orientifold models, showing
their BPS dependence.

3.1. Review of K3 orientifolds

In this subsection we briefly review the construction of orientifolds of Type IIB string
theory [8, 9, 10, 11], following in particular the construction given by [9]. An orientifold [23]
is a generalization of an orbifold where the discrete group by which we mod out contains in
general also the world-sheet parity operator $\Omega$. We will consider in particular orientifolds of
K3, in its orbifold limits $T^4/Z_N$ with $N = 2, 3, 4, 6$. As discussed in [7], there is a freedom
in choosing the orientifold group; closure under group multiplication allows two choices:

$$Z^A_N = \{1, \Omega, \alpha_N^k, \Omega_j\} \quad k, j = 1, 2, ..., N - 1; \quad (N = 2, 3, 4, 6)$$ \hspace{1cm} (3.1)

and

$$Z^B_N = \{1, \alpha_N^{2k-2}, \Omega_{2j-1}\} \quad k, j = 1, 2, ..., N/2; \quad (N = 4, 6)$$ \hspace{1cm} (3.2)

where, following the same notation of [4], $\alpha_N$ is the generator of the discrete group $Z_N$ and
$\Omega_j \equiv \Omega \cdot \alpha_N^j$. The cancellation of tadpole divergencies will require in general the addition of
sources of R-R charges, i.e. of D-branes, with the corresponding open string sectors. It has been found in [9] that for the A-models we always need 32 D-9 branes and 32 D-5 branes, excluding the case of $\mathbb{Z}_3^A$ where we do not have D-5 branes at all. On the other hand the $\mathbb{Z}_4^B$ model does not have open sectors at all, while the $\mathbb{Z}_6^B$ has only 32 D-5 branes. Since, as we will show in the next subsection, the amplitudes we want to compute are invariant for small perturbations of the models, like moving the D-5 branes or varying vacuum expectation values continuously, we only consider here the massless spectrum of these models, in the case of maximum gauge group, when all the D-5 branes are overlapped on a fixed point of the $K3$ orbifold.

As was argued in section two only the six-dimensional massless states are relevant to the indices computation. This spectrum is reported in [9] for the different orientifold models which we are considering. It always contains a universal part from the closed string sector formed by the gravitational multiplet and one tensor-multiplet. On the other hand the open sector provides the gauge group and charged matter in a given representation. It has been shown in detail in [24] for the $\mathbb{Z}_2^A$ model that potentially dangerous U(1) anomalies, in general presented in these models, are simply removed by a Higgs mechanism that gives mass to the corresponding U(1) gauge field. Our amplitudes, however, being proportional to the supersymmetric index, do not depend on this Higgs phenomenon that give mass to an hyper-vector multiplet pair. In the following we will then simply ignore it. The four dimensional models we want to consider are obtained by a further compactification on a $T^2$ torus, giving the same hypermultiplet massless spectrum and $n_T + 3$ further vectormultiplets coming from the closed string sector, where $n_T$ is the number of tensormultiplets in six dimensions.

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3 Strictly speaking we are not considering here the number of dynamical D-branes in the model, but the number of values the Chan-Paton factors take.

4 Note that in [11] it has been found, among others, a model without tensormultiplets.
3.2. Computation of the $F_g$ couplings

The relevant amplitude we consider here to compute the $F_g$ couplings involves two gravitons and $2g - 2$ graviphotons whose vertex operators are:

$$V_g^{\mu\nu}(p) = (\partial X^\mu + ip \cdot [\psi, \psi^\mu])(\bar{\partial} X^\nu + i\bar{p} \cdot \bar{\psi}, \bar{\psi}^\nu)e^{ip \cdot X}$$

$$V_\gamma(p) = (Q_1^{(L)} + Q_1^{(R)})(Q_2^{(L)} + Q_2^{(R)})V_g(p) \quad (3.3)$$

where

$$Q_{\alpha,1}^{(L)} + Q_1^{(R)} = \oint dze^{-\frac{H_T}{2}}S_\alpha \Sigma e^{\frac{H_T}{2}}(z) + \oint d\bar{z}e^{-\frac{\bar{H}}{2}}\bar{S}_\alpha \bar{\Sigma} e^{\frac{\bar{H}}{2}}(\bar{z})$$

$$Q_{\alpha,2}^{(L)} + Q_2^{(R)} = \oint dze^{-\frac{\bar{H}}{2}}S_\alpha \bar{\Sigma} e^{\frac{\bar{H}}{2}}(z) + \oint d\bar{z}e^{-\frac{H_T}{2}}\bar{S}_\alpha \Sigma e^{\frac{H_T}{2}}(\bar{z}) \quad (3.4)$$

are the supersymmetric charges and $e^{-\frac{H}{2}}, e^{\frac{\bar{H}}{2}}$ are the bosonization of the superghosts and of the complex fermion associated to the internal torus respectively, $S_\alpha$ is the space-time spin field operator and $\Sigma$ and its complex conjugate are the $K3$ internal spin field operators; bosonizing the $U(1)$ Cartan current in the $SU(2)$ algebra of the internal $N = (4, 4)$ SCFT as $J_3 = i\sqrt{2}H$, $\Sigma$ can be written as $\Sigma = e^{i\frac{\sqrt{2}}{2}H}$. The same thing applies of course for the right-moving sector.

The 1-loop amplitude involves a sum over the torus, Klein bottle, annulus and Möbius strip surfaces. Since the $2g - 2$ graviphoton vertex operators are inserted in the (-1) ghost picture, we need to include in the correlation function $2g - 2$ picture changing vertex operators. Moreover, due to the +1 charge carried by $V_\gamma$ in the torus direction, the non-vanishing contribution of the picture changing operators will come only from the part $(e^\phi e^{-iH_T} \partial Z_5^+ \text{+ right-moving})^5$ where $Z_5^+$ is the complex scalar associated to the torus direction. The $2g - 2 \partial Z_5^+$ and $\bar{\partial} Z_5^+$ cannot contract with anything in the correlation function, so that only their zero modes part give a non-vanishing contribution. We can use the method of

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5The discussion done here and in what follows also applies to the torus contribution to the amplitude, reminding that in this case left and right sectors are unrelated, of course.
images, as described in [25], in order to compute the boson and fermion propagators on all the surfaces starting from those on the torus. Bosonizing fermions and superghosts, we can then compute each term using the results of [26]. The important point to note is that we have always a cancellation between the contributions of the superghosts and the fermions of the torus direction. We are left in this way with a sum over the spin structures for the remaining four directions, that can be related to a triality rotation in the corresponding SO(8) lattice [18]; for each term, given the structure of the graviton and graviphoton vertex operators, the spin-structure dependent part of the amplitude, for generic arguments \(a_1, a_2\), is always of the form

\[
\sum_{g_1, g_2} \sum_{\alpha, \beta} (-)^{4\alpha\beta} \theta^2 \left[ \frac{\alpha}{\beta} \right] (a_1) \theta \left[ \frac{\alpha + g_1}{\beta + g_2} \right] (a_2) \theta \left[ \frac{\alpha - g_1}{\beta - g_2} \right] (a_2) \tag{3.5}
\]

where the first sum is over the twisted sectors of the orbifold (for the open string sector \(g_1 = 0\)), the second is over the spin structures, the first two theta functions refer to the space-time coordinates and the remaining two to the internal \(K3\) directions. We can now perform the spin structure sum using the Riemann identity:

\[
\sum_{g_1, g_2} \sum_{\alpha, \beta} (-)^{4\alpha\beta} \theta^2 \left[ \frac{\alpha}{\beta} \right] (a_1) \theta \left[ \frac{\alpha + g_1}{\beta + g_2} \right] (a_2) \theta \left[ \frac{\alpha - g_1}{\beta - g_2} \right] (a_2) = \\
\sum_{g_1, g_2} \theta^2 \left[ \frac{1/2}{1/2} \right] (\tilde{a}_1) \theta^2 \left[ \frac{1/2}{1/2} \right] (\tilde{a}_2) \theta^2 \left[ \frac{1/2 + g_1}{1/2 + g_2} \right] (0) \theta^2 \left[ \frac{1/2 - g_1}{1/2 - g_2} \right] (0) \tag{3.6}
\]

where \(\tilde{a}_{1,2} = a_1 \pm a_2\). We can reinterpret this result as an amplitude in the odd spin structure of new vertex operators obtained from the original one through the SO(8) triality rotation. As shown in [18], the graviton vertices are left invariant by the map while the graviphoton operators are transformed to:

\[
V_\gamma(p^\pm) \rightarrow \left[ (\partial + \bar{\partial}) Z^\pm_2 + i p_1^\pm (\psi^+_1 - \bar{\psi}^+_1)(\psi^+_2 - \bar{\psi}^+_2) e^{ip_1^\pm Z_1^\pm} \right] \tag{3.7}
\]

where we did a convenient choice of the kinematical structure. The remarkable fact is that the correlation functions now depend only on the space-time coordinates, the internal \(K3\) part entering through its partition function in the odd spin structure. After having
performed the aforementioned steps and having extracted appropriately the kinematical structure, we can define a generating function for the $F_g$ couplings, whose expression is given in terms of a simple correlation in a $\lambda$-perturbed action:

$$F(\lambda) \equiv \sum_{g=1}^{\infty} g^2 \lambda^2 F_g = \frac{\lambda^2}{\pi^2} \sum_{\alpha,M,K} \int [dM]_{\alpha} \sum_{p \in \Gamma} e^{-\frac{\pi|p|^2}{t^2 \alpha^2}} C_{\alpha}^{(0)}([t]) \langle V_g^+ V_g^- e^{-S_0 + \lambda S} \rangle_{\alpha}$$  \hspace{1cm} (3.8)

where $C_{\alpha}^{(0)}$ represent the $K3$ part of the studied indices and the rest of the notation follows [12]. In all the surfaces, because of the four zero modes of the new action,

$$\langle V_g^+ V_g^- e^{-S_0 + \lambda S} \rangle_{\alpha} = \frac{t^2}{2 d^2} \langle e^{-S_0 + \tilde{\lambda} S} \rangle_{\alpha}$$  \hspace{1cm} (3.9)

For $\tilde{\lambda} = 0$, i.e. for $F_1$, there is an additional contribution coming from the the torus, since in this case the action has eight zero modes. This further contribution is simply:

$$F_{\text{torus}}^1 = 4 \int_{\Gamma} \frac{d^2 \tau}{\tau_2} C_T$$  \hspace{1cm} (3.10)

where $\Gamma$ is the fundamental domain of the torus. We are then left to evaluate determinants of space-time bosons and fermions and the indices $C_{\alpha}([t])$. Eq. (3.10) represents the only non-vanishing contribution given by the torus, since the corresponding determinant is $\lambda$-independent. For the other surfaces the determinants can be simply computed using the corresponding modes expansion of the fields reported in Appendix A. For $n \neq 0$, boson and fermion determinants always cancel. For $n = 0$, in the annulus and Möbius strip the $\lambda$-dependent term in the fermion part of the action drops out, while the bosonic contribution reduces to:

$$\langle e^{-S_0 + \tilde{\lambda} S} \rangle_{A} = \langle e^{-S_0 + \tilde{\lambda} S} \rangle_{M} = \frac{1}{t^3} \prod_{m=1}^{\infty} \left( 1 - \frac{\tilde{\lambda}^2}{m^2} \right)^{-2} = \frac{1}{t^3} \left( \frac{\tilde{\lambda} \pi}{\sin \lambda \pi} \right)^2$$  \hspace{1cm} (3.11)

For the Klein bottle, on the other hand, besides the bosonic contribution there is also, for $n = 0$, a fermionic contribution for $m = \text{odd}$ leading to:

$$\langle e^{-S_0 + \tilde{\lambda} S} \rangle_{K} = \frac{4}{t^3} \left[ \prod_{m=1}^{\infty} \left( 1 - \frac{\tilde{\lambda}^2}{m^2} \right)^{-2} \right] \left[ \prod_{k=0}^{\infty} \left( 1 - \frac{4 \tilde{\lambda}^2}{(2k+1)^2} \right)^2 \right] = \frac{4}{t^3} \left( \frac{\tilde{\lambda} \pi}{\sin \lambda \pi} \right)^2 \cos^2 \tilde{\lambda} \pi$$  \hspace{1cm} (3.12)

\footnote{Strictly speaking $F_1$ should be evaluated by a three-point function, as we have done in subsection 2.1., but the computation reported here reproduces the same result.}
where the factor four between the contribution of the Klein bottle with that of the annulus and Möbius strip is due to the different modular parameter of the covering tori (see Appendix A). Note that the space-time contribution to the amplitude for all the surfaces is given by the string states with $n = 0$, i.e. with oscillation number zero. This observation, together with the analysis performed in the last section for the internal part, allows us to conclude that only BPS states are contributing to the considered correlation functions. Putting all the results together, for large torus compactification\footnote{We take this limit simply to perform the $\tau_1$ modular integration in the torus contribution.}, we have:

$$F(\lambda) = 2\lambda^2 \int_0^\infty \frac{dt}{t} \sum_{p \in \Gamma} e^{-\pi t |p|^2} \left[ \left( n_{\text{H}}^\text{total} - n_{\text{V}}^\text{total} \right) \frac{d^2}{d\lambda^2} \left( \frac{\bar{\lambda}}{\sin \lambda} \right)^2 + 4n_{\text{closed}}^V \right]$$

(3.13)

with $\bar{\lambda} = \lambda t p/2\sqrt{2U_2}$ and $U_2$ the Kähler class of the $T^2$ torus and where we have used the results of section two:

$$\frac{C_A^{(0)} + C_M^{(0)}}{4} + C_K^{(0)} = n_{\text{H}}^\text{total} - n_{\text{V}}^\text{total}$$

$$C_T^{(0)} - C_K^{(0)} = 2n_{\text{closed}}^V$$

Comparing the spectrum in \cite{9} with the values of $C_\alpha^{(0)}$ given in Appendix B (table 1), we can explicitly check that this index reproduces separately for all the sectors the difference between the number of four-dimensional hyper and vectormultiplets. These are, of course, particular free models with no background fields; in general we will have a mixing between the sectors, but such that the total contribution $(C_A + C_M)/4 + C_K$ will always count the number of BPS (hypers-vectors), as showed in general in section 2.

It is worth while to point out that the results obtained here for the $C_\alpha^{(0)}$ are not in contradiction with what found in \cite{12} for the $\mathbb{Z}_2^A$ model. The values obtained in \cite{12}, that is

$$C_T^{(0)} = 8, C_K^{(0)} = 0, C_A^{(0)} + C_M^{(0)} = 4 \cdot 240$$

take into account the $U(1)$ anomalies that give masses to 16 hypermultiplets in the closed string spectrum and 16 vectormultiplets to the open one.
4. Conclusions

In this paper we have studied one-loop amplitudes of $N = 2$ supersymmetric effective actions arising from $K3 \times T^2$ compactifications of type I string theory. The important point is the realization of an $N = 2$ supersymmetric index which provides an important information about the BPS spectrum of the different type I compactifications. Many of the corrections to chiral terms in these effective actions can be written in terms of this supersymmetric index, encoding the compactification model dependence. Using the superconformal algebras underlying these compactifications the index is reduced to the counting of BPS states similar to the one found for the heterotic string $\text{[5]}$. As in that case, we proved that this index counts simply the difference between the number of four-dimensional hyper and vectormultiplets of the corresponding model.

Among the one-loop amplitudes related to this index we studied in detail those corresponding to gravitational higher derivative couplings $F_\sigma W^{2\sigma}$ for type I compactifications continuously connected to $K3$ orientifolds. These couplings are completely determined by the BPS spectrum of the corresponding model, encoded in the aforementioned index.

These results are independent of any string duality statement. In particular all the considered orientifold models besides the $Z_2$ case cannot have a weak heterotic dual in the limit of large torus, (where the heterotic winding modes decouple and a weak-weak duality makes sense) since they contain more than one tensormultiplet in six dimensions. The structure of the one loop amplitudes we considered here for type I string compactifications are, however, the same than what previously found for the heterotic case. This illustrates once more how BPS states contributions in correlation functions of different string theories are organized in an unified way and point out again the importance of these states for a better understanding of the string dynamics.

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Appendix A

Modes expansions

We present here the modes expansion of the bosonic and fermionic fields in the annulus, Möbius strip and Klein bottle surfaces, needed to evaluate the corresponding determinants. We take as fundamental region of each surface $0 \leq \tau \leq t, 0 \leq \sigma \leq 1$, where $t$ is the corresponding modulus. Following Burgess and Morris [25], we can consider each surface as a torus modded out by a given projection, whose action on the bosonic and fermionic fields defines the corresponding boundary and crosscap conditions. The modes expansion for the fields in the annulus is then given by:

$$x_A(\tau, \sigma) = \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{\infty} \alpha_{m,n} e^{2i\pi m\tau} \cos \pi n\sigma$$

$$\psi_A(\tau, \sigma) = \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{2i\pi m\tau} e^{i\pi n\sigma}$$

$$\tilde{\psi}_A(\tau, \sigma) = \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{2i\pi m\tau} e^{-i\pi n\sigma}$$

(A.1)

after having extended the fields to the torus $0 \leq \tau \leq 2t, 0 \leq \sigma \leq 2$ and identified $x(\tau, \sigma) = x(\tau, 2-\sigma), \psi(\tau, \sigma) = \tilde{\psi}(\tau, 2-\sigma)$. For the Möbius strip we extend the field to the torus $0 \leq \tau \leq 2t, 0 \leq \sigma \leq 2$ by identifying $x(\tau, \sigma) = x(1+\tau, 1-\sigma), x(\tau, \sigma) = x(\tau, 2-\sigma), x(\tau, \sigma) = x(2t-\tau, \sigma)$.

Note that since we need to compute determinants in the odd spin structure, we will consider in the following fermionic fields on this spin structure only.
ψ(τ, σ) = ˜ψ(1 + τ, 1 − σ), ψ(τ, σ) = ˜ψ(τ, 2 − σ). Then the mode expansion is:

\[ x_M(τ, σ) = \sum_{m=-\infty}^{+\infty} \sum_{n \geq 0} \alpha_{m,n} e^{i\pi m \tau} \cos \pi n \sigma \quad m + n = \text{even} \]

\[ \psi_M(τ, σ) = \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{i\pi m \tau} e^{i\pi n \sigma} \quad m + n = \text{even} \quad (A.2) \]

\[ \tilde{\psi}_M(τ, σ) = \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{i\pi m \tau} e^{-i\pi n \sigma} \quad m + n = \text{even} \]

The bosonic and fermionic mode expansion in the Klein bottle are:

\[ x_K(τ, σ) = \frac{1}{2} \sum_{m=-\infty}^{+\infty} \alpha_{m,n} e^{i\pi m \tau} (e^{2i\pi n \sigma} + (-)^m e^{-2i\pi n \sigma}) \]

\[ \psi_K(τ, σ) = \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{i\pi m \tau} e^{2i\pi n \sigma} \quad (A.3) \]

\[ \tilde{\psi}_K(τ, σ) = \sum_{m,n=-\infty}^{+\infty} d_{m,n} (-)^m e^{i\pi m \tau} e^{-2i\pi n \sigma} \]

resulting from the identification \( x(τ, σ) = x(1 + τ, 1 − σ), \psi(τ, σ) = \tilde{\psi}(1 + τ, 1 − σ) \) on the torus \( 0 \leq τ \leq 2t, 0 \leq σ \leq 1 \). Note that the modular parameter of the covering tori for the three surfaces with the aforementioned projections is respectively \( τ = it/2, it, 2it \). The factor of two between the annulus and Möbius strip parameters is due to the fact that the corresponding tori cover two times the annulus and four the Möbius surface.

Appendix B

\( C_\alpha \) for \( K3 \) orientifolds

In this appendix we report the value of the indices \( C_{\alpha}^{(0)} \), i.e. of the indices (2.5,2.6) restricted to the \( K3 \) part, for the orientifold models considered in section three. In order to avoid an heavy notation, we will omit in the following the superscript \((0)\) in \( C_{\alpha}^{(0)} \).

\( C_T = \text{Tr}_{RR}(-)^{F_L+F_R} \) is the Witten index [22], whose value gives the Euler characteristic of
that is +24, independently of the model\[^9\]. The value of \( C_K = \text{Tr}_{RR} \Omega (-)^{F_L+F_R} \) can be written for any A-model as

\[
C_K(Z^A_N) = -\frac{1}{N} \sum_{k=0}^{N-1} 4 \sin^2 \frac{2\pi k}{N} + n_{(\frac{N}{2}, \frac{N}{2})} \tag{B.1}
\]

where the first factor is due to the untwisted sector while \( n_{(\frac{N}{2}, \frac{N}{2})} \) is the contribution of the sector twisted by \( \alpha_N^{N/2} \) (when it exists, i.e. for \( N \neq 3 \)) and equals the number of fixed points invariant under \( \alpha_N \) present in that sector. The other twisted sectors give a vanishing contribution because the world-sheet parity operator \( \Omega \) interchanges sectors twisted by \( g \) (a generic group element) with the ones twisted by \( g^{-1} \) and then eigenvalues of \( \Omega \) come always in pairs. For the B-models \( C_K = \text{Tr}_{RR} \Omega \alpha_N (-)^{F_L+F_R} \) and its value is

\[
C_K(Z^B_N) = -\frac{2}{N} \sum_{k=1}^{N/2} 4 \sin^2 \frac{2\pi (2k-1)}{N} + n_{(\frac{N}{2}, \frac{N}{2})} \tag{B.2}
\]

where \( n_{(\frac{N}{2}, \frac{N}{2})} \) is again the contribution of the \( \alpha_N^{N/2} \)-twisted sector, but it now counts the number of fixed points invariant under \( \alpha_N^2 \), weighted by their eigenvalues under \( \alpha_N \)\[^11\].

Let us now turn our attention to the open string indices \( C_A = \text{Tr}_{R} (-)^F \) and \( C_M = \text{Tr}_{R} \Omega (-)^F \), considering separately the 99, 55 and 95+59 sectors. Taking into account the results of \[^9\] for the open massless spectrum, we can write for all the A-models in the 99 sector:

\[
C_{99}^A(Z^A_N) = -\frac{1}{N} \sum_{k=0}^{N-1} 4 \sin^2 \frac{\pi k}{N} (\text{Tr} \gamma_{k,9})^2 \\
C_{99}^M(Z^A_N) = +\frac{1}{N} \sum_{k=0}^{N-1} 4 \sin^2 \frac{\pi k}{N} \text{Tr} (\gamma_{-1}^{-1} \Omega_{k,9} \gamma_{k,9}) \tag{B.3}
\]

following the same notation of \[^9\], where the minus sign in \( C_A \) is due to the fermionic charges of the two spin fields in the Ramond sector. The B-models do not have D9 branes

\[^9\]From now on it will be understood that the trace is performed on the \( \alpha_N \)-invariant states on all the sectors, twisted and untwisted.

\[^10\]Note that due to the world-sheet parity operator \( \Omega \), \( F_L = F_R \) so that \( (-)^{F_L+F_R} \) is completely irrelevant.

\[^11\]Remember that in these models there are only the sectors twisted by an even number of \( \alpha_N \)'s, so that \( n_{(\frac{N}{2}, \frac{N}{2})} = 0 \) for the \( Z_6^B \) model.
| Model | \((C_A + C_M)/4\) | \(C_K\) |
|-------|-----------------|--------|
| \(Z_2^A\) | 99: -16 | +16 |
| \(Z_2^A\) | 55: -16 | 95+59: +256 |
| \(Z_3^A\) | 99: -28 | -2 |
| \(Z_4^A\) | 99: -8 | 55: -8 +8 |
| \(Z_4^A\) | 59+95: +128 |  |
| \(Z_6^A\) | 99: -20 | 55: -20 +4 |
| \(Z_6^A\) | 59+95: +96 |  |
| \(Z_4^B\) | - | 0 |
| \(Z_6^B\) | 55: -28 | -2 |

Table 1: Values of the indices \(C_\alpha\) for the various surfaces in the open and closed string sectors.

at all. In the 55 sector

\[ C_A^{55}(Z_N^A) = -\frac{1}{N} \sum_{k=0}^{N-1} 4 \sin^2 \frac{\pi k}{N} (\text{Tr} \gamma_{k,5})^2 \]

\[ C_M^{55}(Z_N^A) = +\frac{1}{N} \sum_{k=0}^{N-1} 4 \cos^2 \frac{\pi k}{N} \text{Tr}(\gamma^{-1}_{\Omega_{5,5}} \gamma^t_{\Omega_{5,5}}) \]  

(B.4)

where, since \(\Omega_5^{3,4} \Omega^{-1} = -\psi_0^{3,4}\) in the 5-sector, we have \(\sin^2 \rightarrow \cos^2\) in \(C_M\). For the \(Z_6^B\) model

\[ C_A^{55}(Z_6^B) = -\frac{2}{6} \sum_{k=0}^{2} 4 \sin^2 \frac{2\pi k}{6} (\text{Tr} \gamma_{2k,5})^2 \]

\[ C_M^{55}(Z_6^B) = +\frac{2}{6} \sum_{k=1}^{3} 4 \cos^2 \frac{\pi (2k-1)}{6} \text{Tr}(\gamma^{-1}_{\Omega_{2k-1,5}} \gamma^t_{\Omega_{2k-1,5}}) \]  

(B.5)
Finally, in the 95+59 sector:

\[ C^A_{95+59}(Z_N^A) = + \frac{2}{N} \sum_{k=0}^{N-1} (\text{Tr}\gamma_{k,9}) (\text{Tr}\gamma_{k,5}) \]  \hspace{1cm} (B.6)  

Given the solution for the matrices \( \gamma \)'s representing the orientifold group \( [9] \), we can explicitly compute the values of these indices for all the models (see table 1).

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