Cluster correlation in light nuclei

Y Kanada-En’yo¹, F Kobayashi¹ and T Suhara²

¹Department of Physics, Kyoto University, Kyoto 606-8502, Japan
²Institute of Physics, University of Tsukuba, Tsukuba 305-8571, Japan

Abstract. Cluster aspects in light unstable nuclei as well as light stable nuclei are discussed. The cluster structures of \(^{13}\)Be and \(^{9}\)Li were investigated. The present calculation suggests that the breaking of the neutron magicity in \(^{13}\)Be may occur due to the developed cluster structure as in the cases of \(^{11}\)Be and \(^{12}\)Be. Clustering in highly excited states of \(^{9}\)Li was discussed in conjunction with those of \(^{10}\)Be.

1. Introduction
The existence of a variety of cluster states has been suggested in neutron-rich Be, where the \(2\alpha\) core and surrounding excess neutrons play important roles (for example, Ref. [1] and references therein). In addition to such two-centre cluster structures conjectured for Be and Ne isotopes, three-centre cluster structures have been attracting great interest. For instance, a \(3\alpha\) cluster gas state identified with an excited \(^{12}\)C state is one of the recent hot topics and it is often discussed in conjunction with alpha condensation [2]. In neutron-rich C, geometric \(3\alpha\) configurations, such as a linear-chain structure, were proposed to be found in some excited states [3].

Such rich cluster phenomena tell us that clustering is one of the essential features in unstable nuclei just as in stable nuclei. Various kinds of cluster structure emerge depending on the excitation energy, on the number and kinds of core clusters as well as on the number of excess neutrons. In low-lying states, clusters are in general tightly bound and clustering is characterized by the correlation between clusters and the actual way of cluster formation. On the other hand, in highly excited states, one may often see remarkable cluster structures in which clusters weakly couple to each other. Another cluster aspect peculiar to neutron-rich nuclei is the molecular orbital structure in which clusters are bonded by excess neutrons in molecular orbitals.

In the present work, we investigate the structure of low-lying states of \(^{13}\)Be and discuss the breaking of neutron magicity. We also study the \(^{6}\)He+\(t\) clustering of excited states of \(^{9}\)Li and their analogy with and difference from \(^{6}\)He+\(\alpha\) cluster states in \(^{10}\)Be. We also discuss how clustering properties change from low-lying states to highly excited states as a function of the inter-cluster distance.

2. Formulation
In the framework of antisymmetrized molecular dynamics (AMD) [4], an \(A\)-nucleon wave function is given by a Slater determinant of Gaussian wave packets:

\[
\Phi_{\text{AMD}}(Z) = \frac{1}{\sqrt{A!}} A\{\varphi_1, \varphi_2, ..., \varphi_A\},
\]  

(1)
where the $i$th single-particle wave function is written as
\begin{equation}
\varphi_i = \phi_{X_i} \chi_i \tau_i, \tag{2}
\end{equation}
\begin{equation}
\phi_{X_i}(r_j) \propto \exp\left\{-\nu \left(\frac{r_j}{\nu} - X_i\right)^2\right\}, \tag{3}
\end{equation}
\begin{equation}
\chi_i = \left(\frac{1}{2} + \xi_i\right) \chi^\uparrow + \left(\frac{1}{2} - \xi_i\right) \chi^\downarrow. \tag{4}
\end{equation}
Here $\phi_{X_i}$ and $\chi_i$ are spatial and spin functions, and $\tau_i$ is the isospin function which is fixed to be up (proton) or down (neutron). We take an optimized value of the width parameter $\nu$ for each nucleus. Accordingly, an AMD wave function is expressed by a set of variational parameters, $Z = \{X_1, X_2, \ldots, X_A, \xi_1, \xi_2, \ldots, \xi_A\}$, which indicate the Gaussian centres and the spin orientations for all nucleons. These parameters are determined by energy variation.

To study nuclear structure, spin-parity variation and spin-parity projection are applied to the AMD wave functions. For the study of Be isotopes, the variation after spin-parity projection was performed in the AMD framework (VAP). By choosing specific configurations of $Z$ and superposing several spin-parity projected AMD wave functions $P_{Z,\pm}^{t,s} K \Phi_{\text{AMD}}(Z)$, one is able to perform GCM-type (GCM: generator coordinate method) cluster-model calculations in the AMD framework. To study the excited states of $^9\text{Li}$ and $^{10}\text{Be}$, $^6\text{He}+t$-type and $^6\text{He}+^4\text{He}$-type GCM calculations are performed. We also do $^{12}\text{Be}+n$-type calculations for $^{13}\text{Be}$.

For more details of the present calculations, the reader is referred to Refs. [5, 6]. The general formulation of the AMD method is expounded in Refs. [7, 8].

The effective nuclear interactions used in the present calculations are the Volkov force and the MV1 force [9, 10] with a spin-orbit force supplemented by the Coulomb force.

3. Results
3.1. Structure of $^{13}\text{Be}$

The structure of Be isotopes has been intensively studied by many groups. In Be, the formation of the $2\alpha$-cluster core and the surrounding neutrons play important roles. In $^{11}\text{Be}$, the vanishing of the neutron magic number is known from the abnormal parity of the $1/2^+$ ground state. In recent years, the breaking of the neutron magicity was also found in $^{12}\text{Be}$ as well as $^{11}\text{Be}$. The $^{12}\text{Be}$ ground state is a largely deformed state dominated by the intruder $2h\omega$ configuration though $^{12}\text{Be}$ is an $N = 8$ nucleus. The lowering mechanism of these intruder states in $^{11}\text{Be}$ and $^{12}\text{Be}$ is successfully described by the molecular orbital picture as discussed by von Oertzen and many groups [1]. In the molecular orbital model, a $2\alpha$ core is formed and the valence neutrons occupy the molecular orbitals around the $2\alpha$. In the intruder states, the molecular $\sigma$ orbital is occupied by two valence neutrons. The $\sigma$ orbital has two nodes along the $\alpha-\alpha$ direction, and it gains kinetic energy as the $2\alpha$ cluster develops. The $\sigma$ orbital corresponds to the intruder $sd$-orbit which comes down into the $p$-shell in a developed cluster system. This means that the origins of the breaking of the neutron magicity in neutron-rich Be are the formation of the $2\alpha$-cluster core and the lowering of the $\sigma$ orbital in the developed cluster state. Then a question to be answered is whether magicity is also broken in further neutron-rich Be isotopes. To answer this problem we applied the AMD+VAP method to calculate the structure of $^{13}\text{Be}$.

Recently, the $^{13}\text{Be}$ spectrum has been measured. In the experimental work by Kondo et al [11], the low-lying $1/2^-$ state was reported at the energy much below the $5/2^+$ state. Since $^{13}\text{Be}$ is an $N = 9$ nucleus, the lowest state could be the $5/2^+$ if single-particle levels were ordered normally as would be naively expected from a simple shell model. The observation of the low-lying $1/2^-$ state in $^{13}\text{Be}$ is inconsistent with the normal single-particle level ordering and it suggests that $N = 8$ is not a magic number here since the $sd$-orbit intrudes in the $p$-shell.
We applied the AMD+VAP method to $^{13}$Be and calculated the states up to $J = 5/2$. The calculated energy levels of $^{13}$Be are shown in Fig. 1 compared with the experimental energy levels reported in Ref. [11]. The interaction parameters are those adopted in Ref. [12] for $^{11}$Be. In the present work, we used the set 1 interaction, which reproduces the abnormal parity ground state $1/2^+$, i.e., the parity inversion in $^{11}$Be.

In the present result, the energy levels of $^{13}$Be are found to be out of the normal order. The $1/2^-$ and $3/2^+$ states, almost degenerate, lie lowest, with a $5/2^+$ state above them. The $1/2^-$ and $3/2^+$ states are intruder states dominated by $1\hbar\omega$ and $2\hbar\omega$ excited configurations, respectively, while the $5/2^+$ state is described by a normal $0\hbar\omega$ configuration. The appearance of these intruder states at such low energies reveals that the $N = 8$ magicity is broken in $^{13}$Be, just as in $^{11}$Be.

As is discussed later, the structure of the low-lying $^{13}$Be states obtained with AMD+VAP can be interpreted in a $^{12}$Be+$n$ picture, in which the $^{12}$Be core is in the intrinsic state $^{12}$Be$(0^+_1)$, having an intruder $2\hbar\omega$ configuration, or in the state $^{12}$Be$(0^+_2)$, with a normal $0\hbar\omega$ configuration. In fact, $^{13}$Be is an unbound nucleus and all its states are resonances above the $^{12}$Be+$n$ threshold. In such a case, the asymptotic behaviour of the valence neutron wave function in the outer region can be important, but the AMD method is not suitable to take that into account. To see the effects of the spatial extension of the last neutron on the excitation energies, we also calculated the energy levels in a $^{12}$Be+$n$ model [5]. In the $^{12}$Be+$n$ model the $1/2^-$ state is the lowest.

By analysing the intrinsic structures of the calculated $^{13}$Be states, it is found that a $2\alpha$ core is formed in most of the states just as in $^{10}$Be, $^{11}$Be and $^{12}$Be. In particular, developed $2\alpha$ cluster structures with large deformations are found in the negative-parity states $^{13}$Be$(1/2^-)$, $^{13}$Be$(3/2^-)$ and $^{13}$Be$(5/2^-)$. In positive-parity states, $^{13}$Be$(1/2^+)$ and $^{13}$Be$(5/2^+)$ show relatively weaker cluster structures with smaller deformations while $^{13}$Be$(3/2^+)$ shows a remarkable $2\alpha$ structure similar to the negative-parity states. The intrinsic density distributions of the intruder states of $^{11}$Be, $^{12}$Be and $^{13}$Be are shown in Fig. 2. We can say that the breaking of $N = 8$ magicity in $^{13}$Be can be understood again as resulting from the lowering of the $\sigma$ orbital in the developed cluster system just as in $^{11}$Be and $^{12}$Be.

It is also interesting how the $^{13}$Be states can be interpreted as $^{12}$Be+$n$ states by adding a neutron on top of the $^{12}$Be cores. The $N = 8$ magicity is broken in $^{12}$Be. That is to say, the ground state of $^{12}$Be is well deformed and it is considered to be dominated by the $2\hbar\omega$ excited configuration, while the second $0^+$ state in the low-energy region is described dominantly by the normal $0\hbar\omega$ configuration. In the $^{12}$Be+$n$ picture, $^{13}$Be$(1/2^-)$, $^{13}$Be$(3/2^-)$ and $^{13}$Be$(5/2^-)$ contain the $^{12}$Be$(2\hbar\omega)$ core with one neutron in a $p$-orbit. $^{13}$Be$(3/2^+)$ can be regarded as combined from the $^{12}$Be$(2\hbar\omega)$ core and an $sd$-orbit neutron, while $^{13}$Be$(5/2^+)$ is interpreted as the $^{12}$Be$(0\hbar\omega)$ core and an $sd$-orbit neutron.

![Figure 1. Energy levels of $^{13}$Be calculated with the AMD+VAP method and the $^{12}$Be+$n$ model by using the set 1 interaction of Ref. [12]. The experimental data from Ref. [11] are also shown.](image-url)
3.2. Cluster structure of $^9$Li

As described by the Ikeda diagram [13], cluster structures may appear in excited states near the threshold energy even if the ground state shows no prominent cluster structure. This threshold rule may be valid in unstable nuclei as well as in stable nuclei as is exhibited by various neutron-rich nuclei. Our aim here is to search for developed cluster states in excited states of $^9$Li.

Let us first review the cluster features of $^{10}$Be and then consider the analogy with $^9$Li. The $K^\pi = 0^+_2$ band in $^{10}$Be has been extensively studied in experimental and theoretical works. The $0^+_2$ state at 6.18 MeV just below the $^6$He+$\alpha$ threshold energy is considered to be the band-head state of the $K^\pi = 0^+_2$ band. The $2^+$ state at 7.54 MeV and the $4^+$ state at 10.2 MeV have been suggested by experiments to have developed $^6$He+$\alpha$-cluster structures and may belong to the $K^\pi = 0^+_2$ band [14, 15, 16, 17]. This $0^+_2$ band of $^{10}$Be is regarded as a rotational band constructed from the strong-coupling $^6$He+$^4$He cluster state having an alpha cluster at a head-on position with respect to the deformed $^6$He cluster. This state is also interpreted as a molecular orbital state with two neutrons in the $\sigma$ orbital around the developed $2\alpha$ core. In addition to the $0^+_2$ state, the existence of a higher $0^+$ $^6$He+$\alpha$ resonance state has been recently proposed in theoretical and experimental works [18, 19].

An analogy with the $^6$He+$^4$He cluster states in $^{10}$Be suggests that the Ikeda threshold rule causes $^4$He+$t$ cluster states to appear in $^9$Li near the $^6$He+$t$ threshold. In our previous study of $^9$Li with the ($\beta$-$\gamma$)-constrained AMD [20], we have found a developed $^6$He+$t$ structure at a shallow local minimum in the energy surface on the $\beta$-$\gamma$ plane. According to that calculation, the excitation energy of the $^6$He+$t$ cluster state in $^9$Li is expected to be 2 MeV higher than that of the $^{10}$Be($0^+_2$), and hence it may be a resonance above the $^6$He+$t$ threshold.

To investigate the $^6$He+$t$ cluster states near the $^6$He+$t$ threshold in $^9$Li, we perform GCM calculations in a $^6$He+$t$ cluster model. We superpose the $^6$He+$t$ cluster wave functions with distances $d = 1, 2, \cdots, 8$ fm and obtain energy levels for $^9$Li. The energy levels measured from the $^6$He+$t$ threshold energy are shown in Fig. 3. The developed $^6$He+$t$ cluster states are expected to be a few MeV above the $^6$He+$t$ threshold energy. The cluster states $J^\pi = 1/2^+, 3/2^+, 5/2^-$ and $7/2^-$ form a $K^\pi = 1/2^-$ rotational band, and show strong in-band $E2$ transitions. They show spatially well developed $^6$He+$t$ cluster structure. Compared with the $K^\pi = 0^+_2$ band of $^{10}$Be, in which the $^6$He cluster strongly couples to the neighbouring $^4$He cluster, in the $K^\pi = 1/2^-$ band of $^9$Li the $^6$He cluster couples more weakly to the $t$ cluster.

Unfortunately, there is no experimental information on the $^6$He+$t$-type excited states of $^9$Li. It is a challenging problem to search for such cluster states since they consist of two exotic clusters, $^6$He and $t$.

4. Discussion

Recently, it was revealed that various kinds of cluster structures appear in light stable and unstable nuclei depending on proton and neutron numbers and on the excitation energy. Cluster states near the threshold energy are often characterized as weakly coupled clusters since the
inter-cluster motion decouples from the cluster intrinsic degrees of freedom. On the other hand, in low-lying states we often see strongly coupled cluster structures in which clusters are tightly bound to form relatively compact states. Another type of cluster structure peculiar to neutron-rich nuclei is molecular orbital structure in which clusters are bonded by excess neutrons.

In $^{10}$Be one may see such a change from a deeply bound cluster structure to a weakly coupled structure passing through the molecular orbital structure. Let us discuss the change of the two-cluster structure in $^{10}$Be and $^9$Li as a function of the inter-cluster distance (see the schematic picture in Fig. 4). How do the cluster structures of $^{10}$Be and $^9$Li compare?

The case of the small inter-cluster distance corresponds to the deeply bound cluster states. The ground state is obtained to be such a tightly bound compact cluster structure. With the increase of the inter-cluster distance, the structure of $^{10}$Be changes into a molecular orbital structure, in which excess neutrons are moving around both clusters in the longitudinal $\sigma$ orbital. With a further increase of the inter-cluster distance, the excess neutrons get localized around one of the $\alpha$ clusters to form a $^6$He cluster. There, the $^6$He cluster is still deformed and it may align toward the neighboring $\alpha$ cluster to gain potential energy. Finally, the $^6$He cluster takes an energy eigenstate, and the system tends to a state with a weakly coupled pair of clusters, which is connected to the asymptotic $^6$He+$\alpha$ cluster channel. In the realistic $^{10}$Be system, the $0^+_2$ state is considered to be intermediate between the molecular orbital structure and the aligned $^6$He+$\alpha$-cluster one, while a $0^+_1$ state with a weakly coupled $^6$He+$\alpha$ cluster pair was suggested to be a few MeV higher than the $0^+_2$ state [18, 19].

In $^9$Li one may also see a similar change of cluster structure from deeply bound clusters to the asymptotic $^6$He+$t$ channel. However, the cluster aspects of $^9$Li may be somewhat different. One of the interesting properties of $^9$Li different from $^{10}$Be is that the $t$ cluster in $^9$Li easily decouples from the $^6$He cluster because a $t$ cluster gives a weaker attraction (or potential) than an $\alpha$ cluster. Therefore, the cluster states in $^9$Li may show a trend to the weak coupling feature. Indeed, in the realistic $^9$Li system, the $^6$He+$t$ cluster state in the $K^\pi = 1/2^-$ band, proposed to be near the threshold energy in the present work, is understood as an intermediate structure between the aligned $^6$He+$t$ and the weakly coupled $^6$He+$t$ cluster structures. We could not find
molecular orbital structure in excited states of $^9$Li. That is natural because the excess neutrons feel weaker attraction by the $t$ cluster than by an $\alpha$ and they do not move around the whole $\alpha+t$ core but tend to localize around the $\alpha$ core. This is different from the case of $^{10}$Be, in which a stable molecular structure is formed by $\sigma$-orbital neutrons around the developed $2\alpha$-cluster core and that is an essential property of the $K^\pi = 0^+_2$ band.

Figure 4. A schematic figure for the structure change of $^{10}$Be and $^9$Li toward the $^6$He$^+$$^4$He and $^6$He+$t$ channels with the increase of the inter-cluster distance.

5. Summary
Cluster structures in $^{13}$Be were investigated with AMD+VAP calculations. Low-lying states having intruder $1h\omega$ and $2h\omega$ configurations were obtained, namely, a $1/2^-$ state with a dominant $1h\omega$ excitation, and a $3/2^+$ state with $2h\omega$. The result suggests that the neutron magicity in $^{13}$Be has been broken because of the developed cluster structure, just as in the cases of $^{11}$Be and $^{12}$Be.

Highly excited cluster states of $^9$Li were studied with cluster GCM calculations. The analogy and difference between the clustering properties of $^9$Li and $^{10}$Be were discussed.

It was found that various kinds of cluster structures appear depending on the neutron and proton numbers as well as on the excitation energy. In the neutron-rich nuclei $^{11}$Be, $^{12}$Be and $^{13}$Be cluster structure develops even in the low-lying states. Their clusters are tightly bound or bonded by excess neutrons. On the other hand, there are also weakly coupled cluster states near the cluster thresholds in the same manner as in stable nuclei. The appearance of low-lying cluster states much below the cluster-decay threshold energy seems inconsistent with the Ikeda threshold rule. The weakly coupled cluster states do of course bear out the threshold rule.

The calculations reported in the present work suggest that there exist $^6$He$+t$-type cluster resonance states in $^9$Li. These states are peculiar because they consist of exotic clusters that are themselves unstable nuclei. It is a challenge to experimentalists to find them.

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References

1. Oertzen W, Freer M and Kanada-En’yo Y 2006 Phys. Rep. 432 43
2. Tohsaki A, Horiuchi H, Schuck P and Röpke G 2011 Phys. Rev. Lett. 87 192501
3. Suhara T and Kanada-En’yo Y 2010 Phys. Rev. C 82 044301
4. Kanada-En’yo Y, Horiuchi H and Ono A 1995 Phys. Rev. C 52 628; Kanada-En’yo Y and Horiuchi H 1995 Phys. Rev. C 52 647
5. Kanada-En’yo Y 2012 Phys. Rev. C 85 044320
6. Kanada-En’yo Y and Suhara T 2012 Phys. Rev. C 85 024303
7. Kanada-En’yo Y and Horiuchi H 2011 Prog. Theor. Phys. Suppl. 142 205
8. Kanada-En’yo Y, Kimura M and Horiuchi H 2003 Comptes Rendus Physique Vol. 4 497
9. Volkov A B 1965 Nucl. Phys. 74 33
10. Ando T, Ikeda K and Tohsaki A 1980 Prog. Theory. Phys. 64 1608
11. Kondo Y et al 2010 Phys. Lett. B 690 245
12. Kanada-En’yo Y and Horiuchi H 2002 Phys. Rev. C 66 024305
13. Ikeda K, Tagikawa N and Horiuchi H 1968 Prog. Theor. Phys. Suppl. Extra Number 464
14. Soić N et al 1996 Europhys. Lett. 34 7
15. Liendo J A, Curtis N, Caussyn D D, Fletcher N R and Kurtukian-Nieto T 2002 Phys. Rev. C 65 034317
16. Milin M et al 2005 Nucl. Phys. A 753 263
17. Freer M et al 2006 Phys. Rev. Lett. 96 042501
18. Kobayashi F and Kanada-En’yo Y 2012 arXiv:1208.0387 [nucl-th]
19. Kuchera A N et al 2011 Phys. Rev. C 84 054615
20. Suhara T and Kanada-En’yo Y 2010 Prog. Theor. Phys. 123 303