The Box-Problem in Deformed Special Relativity

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Abstract

We examine the transformation of particle trajectories in models with deformations of Special Relativity that have an energy-dependent and observer-independent speed of light. These transformations necessarily imply that the notion of what constitutes the same space-time event becomes dependent on the observer’s inertial frame. To preserve observer-independence, the such arising nonlocality should not be in conflict with our knowledge of particle interactions. This requirement allows us to derive strong bounds on deformations of Special Relativity and rule out a modification to first order in energy over the Planck mass.

1 Introduction

It is generally believed that the Planck mass $m_{Pl}$ is of special significance. As the scale where effects of the yet-to-be-found theory of quantum gravity are expected to become important, it has been argued the energy associated to the Planck mass should have an observer-independent meaning. Lorentz-transformations however do not leave any finite energy invariant. Thus, the requirement of assigning an observer-independent meaning to the Planck mass seems to necessitate a modification of Special Relativity and a new sort of Lorentz-transformations. This modification of Special Relativity, which does not introduce a preferred frame but instead postulates the Planck mass as an observer-independent invariant, has become known as “Deformed Special Relativity” (DSR) [1, 2, 3, 4].

The deformed Lorentz-transformations that leave the Planck mass invariant under boosts can be explicitly constructed. There are infinitely many of such

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deformations, and they generically result in a modified dispersion relation and an energy-dependent speed of light \[5\]. In the low energy limit, this energy-dependent speed of light coincides with the speed that we have measured. Depending on the sort of deformation, the speed of light can increase, decrease, or remain constant with energy. We will here examine the case where it is not constant.

These deformations of Special Relativity have recently obtained increased attention since measurements of gamma ray bursts observed by the Fermi Space Telescope have now reached a precision high enough to test a modification in the speed of light to first order in the energy over the Planck mass \[6, 7, 8\]. While such modifications could also be caused by an actual breaking of Lorentz-invariance that introduces a preferred frame, models that break Lorentz-invariance are subject to many other constraints already \[9\]. This makes DSR the prime candidate for an energy dependent speed of light. We will here argue that DSR necessitates violations of locality that put much stronger bounds on an energy-dependent speed of light already than the recent measurements of gamma ray bursts.

This paper is organized as follows. In the next section we will study a thought-experiment that lays out the basic problem that an energy-dependent but observer-independent speed of light renders locality a frame-dependent notion. In section \[3\] we will transfer this thought-experiment into a realistic setting. We will show that, with a first order modification of the speed of light, the violations of locality would be within current measurement precision and thus cannot be dismissed on grounds of practical impossibility of detection. In section \[4\] we will consider a variant of the setup that covers the case in which there is an additional enhanced quantum mechanical uncertainty in DSR and show that still the problem is within current measurement precision. This then requires us to put bounds on the energy dependence of the speed of light such that the previously studied effect is not in conflict with already existing measurements. This will be done in section \[5\]. In section \[6\] we will consider some alternative options to prevent these bounds but have to conclude that these are all implausible. We use the convention \(c = \hbar = 1\).

## 2 The Box-Problem, Version 1.0

In the cases of DSR we will examine, the speed of light is a function of energy \(\tilde{c}(E)\), such that this function is the same for all observers. Thus, in a different rest frame where \(E\) was transformed into \(E'\) under the deformed Lorentz-transformation, the speed of light would be \(\tilde{c}'(E') = \tilde{c}(E')\). In ordinary Spe-
cial Relativity it is only one speed, \( \lim_{E \to 0} \tilde{c}(E) = 1 \), that is invariant under the Lorentz-transformations. This is a result of deriving Lorentz-transformations as the symmetry-group of Minkowski space and not an assumption for the derivation. It is thus puzzling how an energy-dependent speed of light that takes different values can also be observer-independent.

The intuitive problem can be seen in the following scenario. Consider the case in which the speed of light was decreasing monotonically and finally reached zero when the energy equaled the Planck mass. Then, a photon with \( E = m_{Pl} \) would be at rest. We put this photon inside a box. The box represents a classical, macroscopic, low-energy object, one for which modifications of Special or General Relativity are absent or at least negligible.

What does an observer moving relative to the box with velocity \( v \) see? He sees the box move with \(-v\) relative to him. The photon’s energy in his restframe is also the Planck mass, since it is an invariant of the deformed Lorentz-transformation. Consequently the photon is also at rest, and cannot remain inside the box. Indeed, if the observer only waits long enough, the photon will be arbitrarily far outside the box.

If we bring another particle into the game, for example an electron, that in the restframe of the box interacts with the photon, then the moving observer will generically see the particles interact outside the box (except for the specifically timed case in which the electron just meets the photon in the moment when the photon is also in the box). The different transformation behavior of the world-lines of the box and the photon thus results in an observer-dependent notion of what constitutes ‘the same’ spacetime event. In contrast to the observer-dependence of ‘the same’ moment in time that one also has in Special Relativity, this concerns the observer dependence of what happens at the same time and the same place. Since two straight, non-parallel lines always meet in one point, an example requires at least three lines, rspt. three objects moving with constant velocity, here the photon, the electron and the box. In one reference frame they all meet in the same space-time point. In another reference frame they do not. This poses significant challenges if one wants to accommodate it in a local theory.

While this setting exemplifies the box-problem, it can be criticized on the grounds that experimentalists do not have many reasons to worry about particles with energies of \( 10^{19} \) GeV. We will thus in the next section study an actually observable situation. This will be a more complicated setup, but the underlying cause of the problem remains the same. It is the requirement that the speed of photons changes with energy but changes in an observer-independent way that forces upon us that the world-lines of particles transform differently depending on the

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particle’s energy. This then has the effect that the question what constitutes ‘the
same’ spacetime event becomes observer-dependent, which can run into conflict
with observations that have confirmed the locality of particle interactions to high
precision.

3 The Box-Problem, Version 2.0

Consider a gamma ray burst (GRB) at distance $L \approx 4$ Gpc that, for simplicity, has
no motion relative to the Earth. This source emits a photon with $E_\gamma \approx 10$ GeV,
such that it arrives in the Earth restframe at $(0,0)$ inside a detector. Together with
the 10 GeV photon there is a low-energetic reference photon emitted. The energy
of that photon can be as low as wanted.

In the DSR scenario we are considering the dispersion relation of photons is
modified to

$$E^2 = p^2 + 2\alpha \frac{E^3}{m_{Pl}} + \text{higher order},$$

and the phase velocity depends on the photons’ energy. To first order

$$\tilde{c}(E) \approx \left(1 + \alpha \frac{E}{m_{Pl}}\right) + O\left(\frac{E^2}{m_{Pl}^2}\right).$$

where we will neglect corrections of order higher than $E_\gamma/m_{Pl}$ in the following,
and set $\alpha = -1$, in which case the speed of light decreases with increasing energy.

The important point is that Eq. (1) and (2) are supposed to be observer inde-
dependent, such that these relations have the same form in every reference frame.
This then requires the non-linear, deformed Lorentz-transformations in momentum space. These transformations depend on the form of the modified dispersion relation. We will however here work in an approximation and only need to know
that the Lorentz-transformations receive to lowest order a correction in $E/m_{Pl}$.

The higher energetic photon is slowed down and arrives later than the lower
energetic one. One has for the difference $\Delta T$ between the arrival times of the high
and low energetic photon

$$\Delta T = L \left(\frac{1}{\tilde{c}(E_\gamma)} - 1\right) = L \frac{E_\gamma}{m_{Pl}} + O\left(\frac{E^2_\gamma}{m_{Pl}^2}\right).$$

4
With 4 Gpc $\approx 10^{26}$ m, $E_{\gamma} \approx 10^{-18} m_{Pl}$, the delay is of the order 1 second, take or give an order of magnitude. Strictly speaking, this equation should take into account the cosmological redshift since the photon propagates in a time-dependent background. However, for our purposes of estimating the effects it will suffice to consider a static background, since using the proper General Relativistic expression does not change the result by more than an order of magnitude [10, 11].

We further consider an electron at $E_e \approx 10$ MeV emitted from a source in the detector’s vicinity such that it arrives together with the high energetic photon at $(0,0)$ inside the detector. The source can be as close as wanted, but to make a realistic setup it should be at least of the order 1 m away from the detection point. The low energetic photon leaves the GRB together with the high energetic photon at $(x_e, t_e) = (-L, -L/c)$. It arrives in the detector box at $(x_a, t_a) = (0, L(1 - 1/c))$, by $-t_a$ earlier than the electron. We have chosen the emission time such that $-t_a = \Delta T$, and the electron arrives with the same delay after the low energetic photon as the high energetic photon.

With an energy of 10 MeV, the electron is relativistic already, but any possible energy-dependent DSR effect is at least 3 orders of magnitude smaller than that of the photon, and due to the electron’s nearby emission the effects cannot accumulate over a long distance. The electron’s velocity is

$$v_e \approx \left(1 - \frac{1}{2} \frac{m_e^2}{E_{e}^2}\right) \approx (1 - 10^{-3}) + O(E_{e}^2/m_{Pl}^2) . \quad (4)$$

Inside the detector at $x = 0$ the photon scatters off the electron. The photon changes the momentum of the electron, which triggers a bomb and the lab blows up. That is of course completely irrelevant. It only matters that the elementary scattering process can cause an irreversible and macroscopic change. This setup is depicted in Fig. 1.

Also in the picture is a satellite moving relative to the Earth restframe (thick grey line in Fig. 1). From that satellite, a team of physicists observes and tries to describe the processes in the lab. The satellite crosses the lab just when the bomb blows off at $(0,0)$. That’s somewhat of a stretch, but let’s not overdo it with the realism. The typical speed of a satellite in Earth orbit is $v_S = -10$ km/s, or, in units of c, $v_S \approx -3 \times 10^{-5}$, and the gamma factor is approximately $\gamma_S \approx 1 + 10^{-9}$ for the relative motion between lab and satellite. Of course the satellite is bound in the gravitational field of the Earth and not on a constant boost, but on the timescales that matter for the following this is not relevant. Alternatively, replace Earth by a space station with negligible gravitational field.
Figure 1: Labframe. The gamma ray burst (thick red line) is in rest with the detector (grey shaded area). It emits at the same time one low energetic photon (thin red line) and one high energetic photon (dotted purple line) that is slowed down due to the energy-dependent speed of light. From a source close to the detector, there is an electron emitted (blue line) that meets the low energetic photon in the detector. The electron scatters on the photon, changes momentum and triggers a bomb. A satellite flies by towards the gamma ray burst and crosses the detector just when the photon also meets the electron. The thin grey lines depict the light-cone in the low energy limit.

Now let us look at the same scenario from the satellite restframe, shown in Fig 2. We will denote the coordinates of that restframe with \((x', t')\). The satellite is moving towards the GRB, thus the electron’s and photons’ energies are blueshifted. We have

\[
E'_\gamma = \sqrt{\frac{1-v_S}{1+v_S}} E_\gamma + O \left( \frac{E_\gamma^2}{m_{Pl}^2} \right),
\]

\[
(5)
\]

and the energy of the very low energetic photon remains very low energetic. The low-energetic photon crosses the satellite at \((x, t) = (L(1/\tilde{c}(E_\gamma) - 1)/(1 - 1/v_S), L(1/\tilde{c}(E_\gamma) - 1)/(v_S - 1))\). In the satellite frame the time passing between
the arrival of the low energetic reference photon and the electron at $x' = 0$ is

$$t'_{a} = \frac{L}{\gamma_{S}} \frac{1/\tilde{c}(E_{\gamma}) - 1}{1 - v_{S}}.$$  

(6)

(Note that this is not the Lorentz-transformation of $t_{a}$, as becomes clear from the figures.) The formulation of DSR in position space has been under debate. It has been argued that the space-time metric should become energy-dependent [14, 15, 16, 17], and in [18] it was shown that keeping the energy-dependent speed of light observer-independent forces one to accept also the transformations in position-space become dependent on an external parameter characterizing the particle (for example its energy), though the interpretation remains unclear. Thus, to keep track of assumptions made, let us point out that we are talking here about an observation made on two low energetic particles from a very macroscopic, non-relativistic satellite. Even if there was a DSR-modification to the above transformation, it could come in here only through corrections of the order $E_{e}/m_{Pl}$, and do so without this tiny contribution being able to add up over a long distance.

With higher energy, the speed of the electron increases. The speed of the photon also changes but, and here is the problem, according to DSR by assumption the function $\tilde{c}$ is observer-independent. In the satellite frame one then has

$$\tilde{c}(E_{\gamma}') = 1 - \frac{E_{\gamma}'}{m_{Pl}} = 1 - \sqrt{\frac{1 - v_{S}}{1 + v_{S}} \frac{E_{\gamma}}{m_{Pl}}} + O \left( \frac{E_{\gamma}^{2}}{m_{Pl}^{2}} \right),$$  

(7)

and the distance the photons travel until they reach the satellite is

$$L' = \gamma_{S} \left( \frac{v_{S}}{\tilde{c}(E_{\gamma}) - 1} \right) L.$$  

(8)

Thus, the time passing between the arrival of the reference photon and the high energetic photon at the satellite is

$$\Delta T' = \frac{E_{\gamma}'}{m_{Pl}} L' = \frac{1 - v_{S}}{1 + v_{S}} \Delta T + O \left( \frac{E_{\gamma}^{2}}{m_{Pl}^{2}} \right).$$  

(9)

Again, the question arises whether there could be some energy dependence in this transformation. Since we are talking about passive transformations here, this creates an interpretational mess, but nevertheless we will discuss this possibility later in section 5. With the above, in the satellite frame the high energetic photon thus arrives later than the electron by

$$\Delta T' - t'_{a} = \left( \frac{1 - v_{S}}{1 + v_{S}} - \frac{1}{\gamma_{S}(1 - v_{S})} \right) \Delta T + O \left( \frac{E_{\gamma}^{2}}{m_{Pl}^{2}} \right).$$  

(10)
Figure 2: The same scenario as in Fig. 1 as seen from the satellite restframe. The gamma ray burst (thick red line) now moves to the right, and emits the low energetic photon (thin red line) and the high energetic photon (dotted purple line) at slightly blueshifted energies. The high energetic photon is slowed down even more, misses the electron and the bomb is not triggered.

Inserting \( \frac{1}{\gamma_S} \approx 1 - \frac{1}{2}v_S^2 \) for \( v_S \ll 1 \), one finds

\[
\Delta T' - t'_a \approx -3 \Delta T \left( v_S - \frac{1}{2}v_S^2 \right) \approx 10^{-5} \Delta T.
\]

In the satellite frame, the low-energetic photon thus misses the electron by \( \approx 10^{-5} \) seconds. Possible additional DSR effects for the electron are negligible because of its low energy and short travel distance and thus cannot save the day.

Now \( 10^{-5} \) seconds might not appear much given the typical time resolution for detection of such particles is at best of the order milliseconds. However, multiplied by the speed of light, the high energetic photon is still lagging behind as much as a kilometer when it arrives in the detector. It only catches up with the electron at

\[
x' = \frac{t'_a - \Delta T'}{1/c(E'_\gamma) - 1/v'_e} \approx (t'_a - \Delta T') \frac{E'_e^2}{m_e^2} \approx 10^5 \text{ m},
\]

\[(12)\]
and thus safely outside the detector. The photon then cannot scatter off the electron in the detector, and the electron cannot trigger the bomb to blow up the lab. The physicists in the satellite are puzzled.

4 The Box-Problem, Version 2.1

An assumption we implicitly made in the previous section was that the quantum mechanical space- and time-uncertainties $\Delta t, \Delta x$ are not modified in DSR, such that the GeV photon can be considered peaked to a $\Delta t$ smaller than the distance to the electron at arrival. For a distance of 1 km, this is about 19 orders of magnitude higher than $1/E_\gamma$ and thus an unproblematic assumption.

Whether or not DSR has a modification of quantum mechanics is hard to say in absence of a formulation of the model in position space, so let us just examine the possibilities. There either is a modification, or there is not. The previous section examined the case in which there is no modification. Here we will consider the case that there was a modification of quantum mechanics. We will show that if the difference in arrival time in the Earth frame $\Delta T$ was of the order seconds, this would either be incompatible with experiment, or with observer independence. Later, we can use the experimental limits to obtain a on bound the possible delay compatible with experiment.

The question whether or not the wave function spreads in DSR depends on how one interprets the modified dispersion relation. It is supposed to describe the propagation of a particle in a background that displays quantum gravitational effects. Yet the question is whether this modification should be understood as one for a plane wave or for a localized superposition of plane waves already. In the first case a wave-packet would experience enhanced dispersion, in the latter case not. In the absence of a derivation, both interpretations seem plausible.

Let us point out that we are here talking about the dispersion during propagation and the position uncertainty resulting from this and not a modification of the maximally possible localization itself. DSR generically does not only have an energy-dependence of the speed of light, but also an energy-dependence of Planck’s constant $\hbar$ [5]. This results in a generalized uncertainty principle which in particular has the effect that particles with momentum approaching the Planck scale have an increasing position uncertainty, as opposed to the limit on position uncertainty monotonically decreasing with the ordinary Heisenberg relation. However, these DSR corrections to $\hbar$ also go with powers of $E/m_{Pl}$. This means that the maximally possible localization of the 10 GeV photon at emission is af-
fected, but to an extent that is negligible. The relevant contribution to the uncer-
tainty would be the one stemming from the dispersion during propagation.

In case there is a modification caused by a dispersion of the wave-packet, then
the uncertainty of the slowed down, high energetic photon at arrival would be
vastly larger than the maximal localization of the Heisenberg limit allows. If one
starts with a Gaussian wave-packet localized to a width of $\sigma_0$ at emission and
tracks its spread with the modified dispersion relation, one finds that to first order
the now time-dependent width is

$$\sigma(t) = \sigma_0 \sqrt{1 + \left(\frac{2t}{m_p \sigma_0^2}\right)^2}$$  \hspace{1cm} (13)

If we start with a width of $\sigma_0 \approx 1/E_\gamma$, then for times $t \gg m_p \sigma_0^2$ (which amounts
for the values we used to $t \gg 10^{-6}$ seconds), one finds that the width is to first
order $\sigma(t) \approx 2t E_\gamma / m_p$. Or, in other words, in the worst case the uncertainty
of the wave-packet at arrival is about the same size as the time delay $\Delta t \approx \Delta T$. In
this case the photon at arrival would be smeared out over some hundred thousand
kilometers. A delay of $\Delta T$ with an uncertainty of $\Delta T$ is hard to detect, but it would
also be impossible to find out whether or not the center of the wave-packet had
been dislocated by a factor five orders of magnitude smaller than the width of the
wave-packet. This is sketched in Fig. 3.

We recall however that the box-problem was caused by the unusual transfor-
mation behavior of $\Delta T$. To entirely hide this behavior, the quantum mechanical
uncertainty $\Delta t$ needs to be much larger than the delay $\Delta T - t_a$ in all restframes,
such that it was practically unfeasible to ever detect a tiny difference in probability
with the photons we can receive, say, in the lifetime of the universe. We run into a
problem when the delay between the electron and the slow photon is about equal
to or even smaller than the uncertainty of the slow photon. The two times $\Delta T$ and
$t_a$ however transform differently, since the one is determined by the requirement
of leaving the energy-dependent speed of light observer-independent, whereas the
other is determined by the crossing of worldlines of particle for which all DSR-
effects are negligible. As a consequence, the delay will in some reference frames
be larger than or of the same order as the uncertainty.

To see this, let us boost into a reference frame with $\nu = 1 - \varepsilon$, such that $\gamma \approx$
Figure 3: Satellite frame, with increased quantum uncertainty. The same scenario as in Fig. 2 with added space and time uncertainty for the high energetic photon (purple area). The photon is smeared out all over the detector. It interacts with the electron and triggers the bomb without that interaction appearing nonlocal for the observer in the satellite.

\[ \frac{1}{\sqrt{2\varepsilon}} \] The inequality that needs to be fulfilled to hide the delay is then

\[ |\Delta T' - t'_a| \ll |\Delta'| \] \hspace{1cm} (14)

\[ \Leftrightarrow |\varepsilon - \sqrt{\frac{2}{\varepsilon}}| \ll \varepsilon , \] \hspace{1cm} (15)

which is clearly violated without even requiring extreme boosts. To put in some numbers, consider an observer in rest with the electron with \( \varepsilon = 10^{-3} \), and \( \gamma \approx 20 \). We then have

\[ |\Delta T' - t'_a| \approx 10^4 \Delta' . \] \hspace{1cm} (16)

Similarly, if we boost into the other direction \( v = -1 + \varepsilon \), the requirement to
hide the delay takes the form
\[
|\Delta T' - t'_a| \ll |\Delta t'|
\]
\[
\Leftrightarrow \left| \frac{2}{\varepsilon} - \sqrt{\frac{\varepsilon}{2}} \right| \ll \frac{2}{\varepsilon},
\]
which is also clearly violated. Though in this case the delay does not actually get much larger than the uncertainty, they both approach the same value. We would then be comparing the probability of interaction at the center of the wave-packet with one at a distance comparable to its width. In this case then the probability of interaction, if we consider a Gaussian wave-packet, had fallen by a factor of order one. Thus, in some reference frames the particles would be able to interact inside the box with some probability (depending on the cross-section), whereas in other frames they would only interact in a fraction of these cases, in conflict with observer independence. This would require several photons to get a proper statistic, but it is a difference in probability that is feasible to measure within the lifetime of the universe, and thus is still in conflict with observer-independence. The advantage of boosting to a velocity in the opposite direction as the photon is that the delay itself does not also decrease.

Let us mention again that we have considered here a photon whose approximate uncertainty in momentum space is at emission comparable to the mean value, which is quite badly localized. If the photon’s momentum had instead an uncertainty of \( \approx 100 \text{ MeV} \) only, then the mismatch in timescales was by two orders of magnitude larger.

We have here assumed that it is appropriate to use the normal Lorentz-boosts to calculate the time span \( t'_a \), but to what precision do we know these? The transformation behavior under Lorentz-boosts has been tested to high precision in particle collisions where boosts from the center of mass system to the laboratory restframe are constantly used. For the time-dilatation in particular, the decay-time of muons is known to transform as \( \Delta t' = \gamma \Delta t \) up to a \( \gamma \)-factor of 30 to a precision of one per mille [12]. Note however that \( \gamma = 30 \) is only marginally larger than in the example we have used. If the arising mismatch thus was a timescale smaller than the scattering process could test, then we would not have a problem. We will exploit this later to obtain a bound on the delay still compatible with experiment.

To further distinguish possible options, let us notice that the latter argument actually referred to an active Lorentz-boost rather than a passive one. An active boost is needed to describe in our coordinate system properties of the same physical system at different relative velocities, such as the muons at different rapidity.
A passive boost on the other hand is used to describe the same physical system as seen from two observers at different velocities, such as the box in the Earth frame and the satellite frame. In Special Relativity, both boosts are identical (rsp. the one is the inverse of the other). Due to the human body commonly being in very slow motion compared to elementary particles, experimental tests for passive boosts are very limited. In the limit of small boosts where we can test both, they agree and confirm Special Relativity. Otherwise we would have to take great care which boost we should be using to describe signals from GPS satellites or read out spectra of atoms in motion [13].

We are thus lead to consider the option that the active boost describing the fast moving muon is not identical with a passive boost that would be needed to describe the muon/electron from a reference frame at such a high boost. That would then mean a muon in rest in our reference frame does not appear to a fast moving observer as the fast moving muon to us. To be concrete, while the muon’s lifetime might be enhanced to \( \Delta T' = \gamma_{\text{active}} \Delta T \) for us when we accelerate it, the alien-observer at high \( \gamma \) might see our muon in rest decaying with \( \Delta T' = \gamma_{\text{passive}} \Delta T \), where \( \gamma_{\text{passive}} \approx 1 - v \), such that the box-problem caused by the different transformation behaviors would be avoided. That however is either in disagreement with observer independence or with experiment, which can be seen as follows.

Consider an ultra-high energetic proton that hits our detector. Are we supposed to describe it by applying an active boost to protons in rest on Earth, or are we supposed to describe it by a passive boost, assuming that we should instead transform our coordinate system to that of the proton? The only way to answer this question is to decide whether or not the proton has been “actively” boosted. But this boost would necessarily be a boost relative to something. We might for example be tempted to call the proton actively boosted because it moves fast relative to us or the cosmic microwave background, but that notion depends on the presence of a preferred frame. In the case of our box-problem the question comes down to which reference frame is the right one to decide whether or not the electron interacts with the slow moving photon inside the box (with some probability), and why that particular frame was the right one to pick.

Alternatively, we could try to find out whether the particle we aim to describe has ever been accelerated after its formation. Since acceleration is an absolute notion, the particle’s initial restframe could then hold as a reference frame to define further active boosts without singling out a globally preferred frame. Leaving aside the problem of defining a restframe for massless particles, this would mean the boost we needed to describe a particle depended on the previous history of the particle. In particular this would mean properties of particles produced at high ra-
pidity in a collision would have to be transformed into the lab frame by a passive boost. This boost would in high energy collisions have to differ by many orders of magnitude from the standard Lorentz-transformation, a modification we would long have seen. But in addition, this would mean that the muon-decay actually does probe passive rather than active boosts and thus provides the constraint we were using.

To summarize this argument, we have seen that an increased quantum mechanical uncertainty $\Delta t$ that scales with the delay between the high- and low-energetic photon $\Delta T$ cannot in all reference frames bridge the distance the photon is lagging behind the electron when we use a normal Lorentz-boost. And that even though we have used an at emission very badly localized photon already. Active boosts have been tested up to the necessary precision such that a delay of $\Delta T$ of the order seconds would result in a conflict with observer-independence. If passive boosts were different from active boosts, this would necessitate the introduction of a preferred frame and thus disagree with our aim to preserve observer independence. Either way we turn it, quantum mechanics does not solve the box-problem. We will thus in the following section draw consequences.

It is worthwhile to note however that adding quantum mechanical uncertainty does solve the box problem, version 1.0, discussed in section[2]. This is because, as previously noted, DSR generically also implies a modification of the maximally possible localization due to an energy dependence of Planck’s constant. Take for example the dispersion relation [4]:

$$\frac{E^2}{(1 + E/m_{Pl})^2} = p^2.$$  \hspace{1cm} (19)

It has the property of setting a maximal possible value for the momentum, $p = m_{Pl}$, which is only reached for $E \to \infty$. In this case the energy-dependent speed of light and Planck’s constant are [18]:

$$\tilde{c}(E) = \frac{1}{1 + E/m_{Pl}} \quad \tilde{\hbar}(E) = 1 + E/m_{Pl}.$$  \hspace{1cm} (20)

Thus, while the speed of light goes to zero, Planck’s constant goes to infinity. For the photon in rest in the box this would result in an infinite position uncertainty, such that neither observer could plausibly say whether the particle is inside the box or not.
5 Bounds

What if we tried to live with the electron scattering off the photon 10 meters outside the detector? This would require the cross-section for Coulomb-scattering in the satellite frame to be dramatically different from what we have measured in the Earth frame. In the Earth frame, this scattering process probes a typical distance inverse to the center of mass energy of the scattering particles. In the satellite frame, the cross-section must be the same for the distance the photon is lagging behind the electron. This cross-section might not indeed have been measured in any satellite, but this is unnecessary because if it was different from that in our Earth frame this would be incompatible with observer-independence.

The logic of the here presented argument is as follows. If there was an energy-dependent speed of light that resulted in the 10 GeV photon arriving about 1 second later than the low-energetic photon, then the requirement of observer-independence implies violations of locality that are incompatible with previously made experiments. Note that it is not necessary to actually perform the experiment as in the setup explained in the previous sections since observer-independence means we can rely on cross-sections previously measured on Earth. In that sense, the experiment has already been done. The setup has only been added to make clear that the effect is not in practice undetectable and thus cannot be discarded as a philosophical speculation. To then resolve the disagreement, we either have to give up observer-independence, which would mean we are not talking about DSR any longer, or, if we want to stick with DSR, the violations of locality should be small enough to not be in conflict with any already made experiment.

This means one can use the excellent knowledge of QED processes to constrain the possibility of there being such a DSR modification by requiring the resulting mismatch in arrival times not to result in any conflict with cross-sections we have measured.

Let us first consider the case where there is no DSR-modification of the quantum mechanical uncertainty. The distance $L = \text{some Gpc}$ is as high as we can plausibly get in our universe, and the 10 GeV photon is as high as we have reliable observational data from particles traveling that far. The center of mass energy of the electron and the high energetic photon is $\sqrt{s} \approx 15 \text{ MeV}$. The process thus probes distances of $\approx 10 \text{ fm}$. If the photon and the electron were in the satellite frame closer already than the distance their scattering process probes, we would not have a problem. Requiring $|\Delta T' - t'_a| < 10 \text{ fm}$ leads to a bound on the delay
between the low and high energetic photon of
\[ \Delta T < 10^{-17} \text{s} \quad , \] (21)
in order for there not to be any conflict with known particle physics. If we reinsert the \( \alpha \) that we set to one from Eq. (2), we can write the bound as \( \alpha < 10^{-18} \). This is what we find from the requirement that there be no problem in the satellite frame in the case without an additional dispersion of the photon’s wave-packet. With such a dispersion, there is no problem in the satellite frame.

However, according to our argumentation in the previous section we can trust Lorentz-boosts up to \( \gamma \approx 30 \). Using this boost increases the mismatch to \( |\Delta T' - t'_a| \approx 80\Delta T \), and the requirement that it be unobservable with presently tested QED precision amounts to
\[ \Delta T < 10^{-23} \text{s} \quad , \] (22)
or \( \alpha < 10^{-24} \). Note that this does take into account a possible DSR-modification of quantum mechanics already, and thus covers both cases, the one with and without spread of the wave-packet. However, since the ratio \( E_\gamma/m_{Pl} \) is approx \( 10^{-18} \), present-day observations do already rule out any first order modification in the speed of light, and come indeed close to testing a second order modification. The here offered analysis however depends on the scaling in Eq. (3) and thus applies only for modifications linear in the energy.

It is quite possible that the energies we have chosen and the setup we have used do not yield the tightest constraints possible. One could for example have used a photon scattering off another photon or more complicated scattering processes involving neutrinos or other light elementary particles, or have the electron be emitted from a different source such that the center of mass energy is higher. We will not examine all of these cases here, but it seems feasible to get the bound another one or two orders of magnitude stronger. Even stronger constraints might arise from considering high energetic scattering processes in the early universe.

6 Discussion

Let us now see whether there are other options to save DSR in face of the box problem. First we notice that the problem evidently stems from the transformation behavior of \( \Delta T \) in Eq. (9). This behavior is a direct consequence of requiring the energy-dependent speed of light \( \tilde{c} \) to be observer-independent, together with
applying a normal, passive, Lorentz-transformation to convert the distance $L$ into the satellite restframe. Now if one would use a modified Lorentz-transformation also on the coordinates, a transformation depending on the energy of the photon, then $\Delta T$ could indeed transform properly and both particles would meet also in the satellite frame. This would require that the transformation on the distance $L$ was modified such that it converted the troublesome transformation behavior of $\Delta T$ back into a normal Lorentz-transformation. Then, all observers would agree on their observation.

The consequence of that would be that the distance between any two objects would depend on the energy of a photon that happened to propagate between them, an idea that is hard to make sense of. But even if one wants to swallow this, the result would just be that the distance between the GRB and the detector was energy-dependent such that it got shortened in the right amount to allow the slower photon to arrive in time together with the electron. That however meant of course the speed of the photon would not depend on its energy. The confusion here stems from having defined a speed from the dispersion relation without that speed a priori having any meaning in position space. Thus, this possibility does indeed solve the box-problem, but just reaffirms that observer-independence requires the speed of light to be constant.

Or, one might want to argue that maybe in the satellite frame the both photons were not emitted at the same time, such that still the electron could arrive together with the high energetic photon. This however just pushes the bump around under the carpet by moving the mismatch in the timescales in the satellite frame away from the detector and towards the source. One could easily construct another example where the mismatch at the source had macroscopic consequences. This therefore does not help solving the problem.

Another option would be to exploit that the problem arises from the same fact that made the time-delay of the photon observable in the first line: the long distance traveled. One could thus demand the cross-section to depend on the history of the photon, such that it was only the long-traveled photons that required strong modifications on QED cross-sections. Basically, this would mean that any particle’s cross-section was dependent on the particle’s history. This is unappealing, but worse, then cross-sections had to be modified for all ultra-high energetic particles that have travelled long distances, and there is so far no indication for that. In particular, since interstellar space is not actually empty, a large increase in the photon-photon cross-section would not allow the high energetic photons to arrive on Earth at all.

Then, finally, one could try to accept that the electron just does not scatter
off the photon. This would mean that the macroscopic history an observer sees depended on his relative velocity. This would certainly have made stays in space stations much more interesting.

Let us point out that the box-problem does not exist in theories that break rather than deform Lorentz-invariance. The reason is that in the case Lorentz-invariance is broken, the speed of the high energetic photon is not an observer-independent function of the energy. Instead, the relations (1) and (2) only hold in one particular frame, and in all other frames they contain the velocity relative to that particular frame. There are however strong constraints on the breaking of Lorentz-invariance already from many other observations, see e.g. [9] and references therein.

We started with the motivation that the requirement of the Planck energy being observer-independent seems to necessitate a modification of Lorentz-invariance that can result in an energy-dependent speed of light. This energy-dependent speed of light has then lead us to violations of locality that are hard to reconcile with experiment. That DSR implies a frame-dependent meaning of what is “near” was mentioned already in [19]. Serious conceptual problems arising from this were pointed out in [20, 18], and here we demonstrated a conflict with experiment to very high precision.

It has however been argued in [21] that the requirement of the Planck scale being observer-independent does not necessitate it to be an invariant of Lorentz-boosts, since the result of such a boost does not itself constitute an observation. It is sufficient that experiments made are in agreement over that scale. In particular if the Planck length plays the role of a fundamentally minimal length no process should be able to resolve shorter distances. This does require a modification of interactions in quantum field theory at very high center-of-mass energies and small impact parameters, but it does not necessitate a modification of Lorentz-boosts for free particles. In this case, the speed of light remains constant and the box is not a problem.

7 Conclusion

We have studied the consequences of requiring an energy-dependent and observer-independent speed of light in Deformed Special Relativity. We have shown it to result in an observer-dependent notion of what constitutes the same space-time event and thus were lead to consider violations of locality arising from such a transformation behavior. Using the concrete example of a highly energetic pho-
ton emitted from a distant gamma ray burst, we have shown that these violations of locality would be in conflict with already measured elementary particle interactions if the energy dependence was of first order in the energy over the Planck mass. This in turn was used to derive a bound on the still possible modifications in the speed of light, which is 22 orders of magnitude stronger than previous bounds that were obtained from direct measurements of delays induced by the energy-dependence. This new bound rules out modification to first order in the energy over the Planck mass.

Acknowledgements

I want to thank Giovanni Amelino-Camelia, Stefan Scherer, and Lee Smolin for helpful comments.

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