Glassy phase in quenched disordered crystalline membranes

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We investigate the flat phase of $D$-dimensional crystalline membranes embedded in a $d$-dimensional space and submitted to both metric and curvature quenched disorders using a nonperturbative renormalization group approach. We identify a second order phase transition controlled by a finite-temperature, finite-disorder fixed point unreachable within the leading order of $\epsilon = 4 - D$ and $1/d$ expansions. This critical point divides the flow diagram into two basins of attraction: that associated with the finite-temperature fixed point controlling the long-distance behaviour of disorder-free membranes and that associated with the zero-temperature, finite-disorder fixed point. Our work thus strongly suggests the existence of a whole low-temperature glassy phase for quenched disordered crystalline membranes and, possibly, for graphene and graphene-like compounds.

PACS numbers: 87.16.D-, 11.10.Hi, 11.15.Tk

Introduction. Graphene [1] is now well recognized as a unique material due to its outstanding mechanical, optical, thermal, chemical and electronic properties [2–5]: high mechanical strength, optical transmittance, thermal conductivity and carrier mobility. These properties make it one of the most studied compounds both from fundamental and practical viewpoints. At the fundamental level, graphene has launched very challenging problems, notably that of unraveling the physics of Dirac massless fermions propagating in a fluctuating curved space that mimics quantum gravity [6]. At the practical level, graphene has been regarded as a very promising candidate for a broad range of technological applications going from energy storage, ultrafiltration, gas/electrochemical sensor to drug/gene delivery, bioimaging, chemo/bio sensing and so on (see for instance [7]). This situation has stimulated the design and study of other two dimensional (2D) materials such as silicene, germanene, phosphorene, hexagonal boron nitride or transition metal dichalcogenides – such as MoS$_2$ – endowed with properties analogous to those of graphene except for the presence of a nonvanishing or tunable band gap making them, notably, good candidates for the design of semiconductor devices [5, 8].

It is worth recalling that the outstanding properties of graphene and, more generally, graphene-like compounds mainly rely on the extreme purity and regularity of their periodic lattice structure. However, as in all materials, lattice imperfections such as defects (dislocations, grain boundaries, etc.), impurities or vacancies are either naturally present or are generated during the manufacturing process. These defects or impurities can dramatically deteriorate the performances of pristine graphene, such as its carrier mobility or thermal conductivity (see, for instance, [9, 10]). Conversely, defective graphene has been shown to display enhanced sensing properties and a paradoxical increase of its elastic modulus for moderate density of vacancies [11]. Moreover, and of utmost importance, the production of vacancies, introduction of impurities, or attachment of a chemical functional group have been proposed as one of the possible mechanisms to open a tunable band gap. This defect engineering of graphene is, however, still at an early stage as even the role of defects on its mechanical properties is still not fully understood. Also, many properties of defective graphene seem to lack universality as they strongly rely on the nature and mobility of the defects involved, on the chemical functional group possibly attached to vacancy defects, etc. [9, 10].

We propose here a first step toward the understanding of quenched disorder in graphene-like materials by investigating the long-distance effective action of quenched disordered crystalline membranes (see [12, 13] for reviews) that describes the elastic and curvature degrees of freedom of these systems. The relevance of our predictions to genuine graphene-like materials obviously relies on the robustness of our results with respect to the introduction of electronic degrees of freedom, that are neglected here. We, however, emphasize the remarkable success of this kind of approach that has already successfully explained the existence and stability of pristine graphene (see, for instance, [14]), which is a priori forbidden by the Mermin-Wagner theorem, by the emergence of a nontrivial scaling of the physical parameters in the deep infrared (IR), notably by the infinitely growing bending rigidity constant, $\kappa(q) \sim q^{-\eta}$. Accurate computations of the exponent $\eta$ have been realized by various field-theoretical techniques – perturbation theory [15–17], self consistent screening approximation (SCSA) [18–23], nonperturbative renormalization group (NPRG) approaches [24–27] – that compare very well to results obtained by Monte-Carlo and molecular dynamics simulations of graphene [28]. Following up on this success we consider here the influence of quenched disorder on the
long-distance behaviour of generic crystalline membranes and, hopefully, on graphene-like materials.

Largely motivated by the observation of a remarkable wrinkling transition to a glassy phase upon cooling of partially polymerized phospholipid vesicles [29–31], quenched disorder has been thoroughly studied along these lines (for a review see [32]). Nelson and Radzihovsky [33, 34], using self-consistent techniques and \( \epsilon = 4 - D \) expansion, have found that purely metric disorder was irrelevant at any finite temperature \( T \), the renormalization group (RG) trajectories being attracted toward the finite-\( T \), vanishing-disorder fixed point. At vanishing temperatures, disorder would lead to a destabilization of the flat phase through a disorder-induced softening of the effective bending rigidity, which has led these authors to speculate about the existence of a spin-glass-like phase. Soon thereafter Morse et al. [35, 36], extending the work done in [33, 34] by adding curvature disorder, have demonstrated the irrelevance of this kind of disorder at any finite temperature. Moreover, they have shown that the interplay between metric and curvature quenched disorders gives rise to a \( T = 0 \) fixed point unstable with respect to temperature. Associated with this fixed point, one should observe, at sufficiently low temperatures and high values of disorder, an anomalous disorder-induced scaling regime [35, 36]. These works have been pursued by an explicit search for either flat-glassy or crumpled-glassy phases by means of mean-field approximations in the case of short-range [37–41] or long-range [23, 42, 43] disorders.

We have revisited the model considered in [35, 36] within a NPRG framework in order to go beyond the early \( \epsilon, 1/d \) expansions and SCSA approaches and have obtained a surprising result: in the presence of both metric and curvature disorders, there exists a finite-\( T \), unstable fixed point missed within these approaches. This fixed point which is unstable with respect to temperature and, thus, associated with a second-order phase transition, divides the space of coupling constants into two basins of attraction: one controlled by the finite-\( T \), vanishing-disorder fixed point associated with disorder-free membranes and another one controlled by the \( T = 0 \), finite-disorder fixed point identified in [35, 36] that we now find to be stable within our approach. We thus predict a whole “glassy” [72] phase controlled by this fixed point.

Effective action. Let us consider a \( D \)-dimensional membrane embedded in a \( d \)-dimensional space. Each point of the membrane is identified, within the membrane, by \( D \) internal coordinates \( x \equiv x_i, \ i = 1 \ldots D \) and, in the embedding space, by the \( d \)-dimensional vector field \( R(x) \). The long-distance, effective, action which we consider is given by [73]:

\[
S[R] = \int d^Dx \left\{ \frac{K}{2} (\partial^2_i R(x))^2 + \frac{\lambda}{2} u_{ij}(x)^2 + \mu u_{ij}(x)^2 \right. \\
- c(x) \partial^2_i R(x) - \sigma_{ij}(x) u_{ij}(x) \right\}
\]

(1)

with summation over repeated indices. In Eq.(1) the first term represents curvature energy with bending rigidity \( K \), and the second and third terms the elastic energies with Lamé coefficients \( \lambda \) and \( \mu \); stability considerations require \( K, \mu \), and \( \lambda/(2/D) \) to be positive. The fourth and fifth terms represent couplings of disorder fields \( c(x) \) and \( \sigma_{ij}(x) \) to the curvature \( \partial^2_i R(x) \) and strain tensor \( u_{ij}(x) \), respectively. These fields are chosen to be short-ranged quenched Gaussian ones with zero-mean value and variances given by [35, 36]:

\[
[c_i(x) \ c_j(x')] = \Delta_\kappa \delta_{ij} \ \delta^{(D)}(x - x')
\]

\[
[\sigma_{ij}(x) \ \sigma_{kl}(x')] = (\Delta_\lambda \delta_{ijkl} + 2\Delta\mu I_{ijkl} \delta^{(D)}(x - x'))
\]

(2)

where \( I_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \), with \( i, j, k, l = 1 \ldots D \), \(...\) denotes an average over Gaussian disorder and where \( \Delta_\kappa, \Delta_\mu \) and \( \Delta_\lambda + (2/D)\Delta\mu \) are positive. For disorder-free membranes, the strain tensor \( u_{ij} \) in Eq.(1) is given by: \( u_{ij} = \frac{1}{2}(g_{ij} - g^0_{ij}) = \frac{1}{2}(\partial_i R_{,j} - \partial_j R_{,i} - g^{0}_{ij} \partial_i \partial_j R) \) where \( g_{ij} \) represents a metric on the membrane and

\[
R^0(x) = [[[R(x)]]] = \zeta \epsilon_i e_i
\]

(3)
in which \( ... \) denotes a thermal average. In Eq.(3) the \( \epsilon_i \) form an orthonormal set of \( D \) vectors so that \( R^0(x) \) represents a flat configuration of reference. In this configuration the tangent vectors \( \partial_i R^0 \) take nonvanishing values, \( \partial_i R^0 = \zeta \epsilon_i \), where \( \zeta \) is the stretching factor, so that \( g^0_{ij} \) is just the flat metric: \( g^0_{ij} = \zeta^2 \delta_{ij} \). In the presence of disorder, the reference configuration is no longer the flat one. Quenched random fields induce [35, 36] \( i) \) a local deformation of \( g^0_{ij} \), noted \( \delta g_{ij}(x) \), given by: \( \sigma_{ij}(x) = 1/2(\lambda \delta_{ij} - 2\mu I_{ijkl}\delta \delta_{kl}(x)) \) and \( ii) \) a random spontaneous curvature: \( \partial^2_i R(x) = c(x) / \kappa \).

The action Eq.(1) is very close to that considered in [35, 36]. However ours differs from the latter one by the fact that it is entirely written in terms of the field \( R \) instead of the fields \( u \) and \( h \) that generally parametrize fluctuations around the flat phase. This allows one to keep a fully rotationally invariant formalism – where, notably, the full strain tensor \( u_{ij} \) is considered without any approximation – in which the crumpling-to-flat transition can be investigated as well [24–27, 44]. The flat phase is obtained through the limit where the running stretching factor \( \zeta_k \) – see below – that signals the appearance of a spontaneous symmetry breaking, takes a finite value or, equivalently, where the dimensionless stretching factor \( \zeta_k \) goes to infinity (see for instance [45]).

NPRG approach. To derive the RG equations for the coupling constants entering in action Eq.(1) we first av-
verage over quenched disorder by means of replica formalism. Then we employ a method based on the concept of effective average action $\Gamma_k$ – where $k$ is a running scale – analog to a Gibbs free energy where only fluctuations of momenta $q \geq k$ have been integrated out (see [46] and [47–54] for reviews and, for recent applications to frustrated magnets [55–57], turbulence [58], Kardar–Parisi–Zhang equation [59, 60] or random fields models [61–66]). At the microscopic lattice scale $\Lambda \propto a^{-1}$ where $a$ is the typical length of a chemical bond, $\Gamma_k = \Lambda$ coincides with the microscopic action $S[R]$ while, at long distance, $k = 0$, it coincides with the usual Gibbs free energy $\Gamma[R]$. The effective average action $\Gamma_k$ follows an exact equation [67]:

$$\partial_t \Gamma_k[R] = \frac{1}{2} \text{Tr} \left\{ \partial_i R_k (\Gamma_k^{(2)}[R] + R_k)^{-1} \right\}$$

(4)

where $t = \ln k / \Lambda$ and where the trace must be understood as a space (or momentum) $D$-dimensional integral as well as a summation over implicit vectorial and replica indices. In Eq.(4), $\Gamma_k^{(2)}[R]$ is the inverse propagator, the second derivative of $\Gamma_k$ with respect to the field $R$. Finally, $R_k(q)$ is a cut-off function that suppresses the propagation of modes with momenta $q < k$ and makes that $\Gamma_k$ encodes only modes with momenta $q > k$ [47]. We consider a cut-off function diagonal both in vector and replica space – $R_k^{(2)}[q](q) = R_k(q) \delta_{q, q}$ and – and we present our analytical results for the ”$\Theta$” cut-off [68], $R_k(q) \propto (k^2 - q^2) \Theta(k^2 - q^2)$. Equation (4) is a functional partial differential equation that cannot be solved exactly so that approximations are required. We use here the field, field-derivative expansion [47] where $\Gamma_k[R]$ is expanded in powers of the order parameter $\partial_i R$ and its derivatives around the nontrivial minimum, Eq.(3), while preserving the nonperturbative content of Eq.(4). We are thus led to the following effective action:

$$\Gamma_k[R] = \int d^D x \left\{ \frac{Z_k}{2} \left( \partial_i^2 R^\alpha \right)^2 - \frac{\Delta_k}{2} \partial_i^2 R^\alpha \partial_j^2 R^\beta J^{\alpha \beta} \right\} + \frac{\Delta_k}{8} \left( \partial_i R^\alpha \partial_j R^\alpha - D\xi_k^2 \right)^2 + \frac{\mu_k}{4} \left( \partial_i R^\alpha \partial_j R^\alpha - \delta_{ij} \xi_k^2 \right)^2 - \frac{\Delta_k}{8} \left( \partial_i R^\alpha \partial_j R^\alpha - D\xi_k^2 \right)^2 \right\}$$

(5)

where $J^{\alpha \beta}$ is the unit matrix in replica space. Note that although limited to fourth order in powers of the order parameter $\partial_i R$ and to first order in powers of its derivatives, this truncation has led to very well converged results for pure membranes, [see [24–27]], leading in particular to the value of $\eta \approx 0.85$ perfectly confirmed numerically [28]. In Eq.(5) we have done the rescaling $R \rightarrow T^{1/2} Z_k^{1/2} \eta^{-1/2} R$, where $Z_k$ is a running field renormalization and introduced the running coupling constants: $\lambda_k = \lambda T Z_k^2 k^{-2}$, $\mu_k = \mu T Z_k^2 k^{-2}$, $\Delta_k = \Delta_k T^{-1} Z_k k^{-1}$ and $\tilde{\xi}_k = \xi_k T^{-1/2} Z_k^{1/2} k^{1/2}$. Note that $\tilde{\mu}_k$ and $\tilde{\lambda}_k$ can be used as a measure of the temperature $T$ where $\Delta_k$ diverges at vanishing temperatures. In order to study this latter regime we introduce, as in [35, 36], $\tilde{\mu}_k = \mu_k \Delta_k$ and $\tilde{\lambda}_k = \lambda_k \Delta_k$ that stay finite at any temperature. The field renormalization $Z_k$ and rescaled curvature disorder variance $\Delta_k$ allow to define the running anomalous dimensions $\eta_k = -\partial_t \ln Z_k$ and $\eta'_k = \eta_k + \partial_t \ln \Delta_k$ that, at a fixed point, characterize the scaling behaviour of the thermal – $\chi(q)$ – and disorder – $C(q)$ – correlation functions, respectively, that are defined from the two-point correlation function $G_{RR}(q) = \langle [R(q) R(-q)] \rangle$ by:

$$G_{RR}(q) = \langle \delta R(q) \delta R(-q) \rangle + \langle [R(q) R(-q)] \rangle$$

(6)

with $\delta R(q) = R(q) - \langle R(q) \rangle$, and behave, at low momenta, as:

$$\chi(q) \sim q^{-(4-\eta)} , \ C(q) \sim q^{-(4-\eta')} .$$

(7)

Finally, we define from $\eta$ and $\eta'$ the exponent $\phi$ by [35, 36]: $\phi = \eta' - \eta$ that describes, in particular, the flow of the temperature near $T = 0$.

**RG equations.** We consider the RG equations obtained from Eq.(5) in the flat phase. We restrict our study to the attractive hypersurface defined by $\Delta_k = -2/D \tilde{\mu}_k$ and $\tilde{\lambda}_k = -2/D \tilde{\mu}_k - 2/D \tilde{\mu}_k$ – where we have introduced the notation $D_n = D + n$ for $n > 0$ – that generalizes to any dimension $D$ the one considered near $D = 4$ in [33–36] (see also [74]). The RG equations, in terms of dimensionless coupling constants, $\tilde{\pi}_k = Z_k^{-2} k^{D-4} \tilde{\mu}_k$, $\tilde{\lambda}_k = Z_k^{-2} k^{D-4} \tilde{\lambda}_k$, $\tilde{\xi}_k = Z_k^{-1} \tilde{\xi}_k$ and $\tilde{\mu}_k = Z_k^{-3} k^{D-4} \tilde{\mu}_k$ read:

$$\partial_t \tilde{\pi}_k = (D - 4) \tilde{\pi}_k + 4d \tilde{\pi}_k \tilde{\pi}_k \tilde{\pi}_k + 3 \tilde{\mu}_k \tilde{\pi}_k$$

$$\partial_t \tilde{\lambda}_k = (D - 4) \tilde{\lambda}_k +$$

$$+ 8d (\tilde{\lambda}_k \tilde{\pi}_k \tilde{\pi}_k + 3 \tilde{\pi}_k \tilde{\pi}_k \tilde{\pi}_k)$$

$$\tilde{\xi}_k = \tilde{\xi}_k (\eta_k - 4 D (D - 1) \tilde{\pi}_k)$$

$$\partial_t \tilde{\pi}_k = (D - 4 + 3 \eta_k) \tilde{\pi}_k$$

$$- 4 \tilde{\mu}_k (D (D - 1) \tilde{\pi}_k + 4d (\tilde{\pi}_k \tilde{\pi}_k \tilde{\pi}_k + 3 \tilde{\mu}_k \tilde{\pi}_k)$$

(8)

where $\eta_k$ is given by:

$$\eta_k = \frac{32 (D - 1) D_k (2 D \tilde{\mu}_k + D \tilde{\pi}_k - \tilde{\pi}_k) A_D}{D D_2 D_8 + 32 (D - 1) (2 D \tilde{\mu}_k + D_8 (\tilde{\mu}_k - \tilde{\pi}_k)) A_D}$$

(9)

and

$$\eta'_k = 2 \eta_k - 4 D (D - 1) \tilde{\pi}_k$$

(10)
where \( \Delta_D = 16 A_D (D_8 - \eta_k )/(D D_2 D_4 D_8 ) \), \( \bar{A}_D = 16 A_D (D_{12} - \eta_k )/(D D_2 D_4 D_{12} ) \), \( \bar{A}_D = 16 A_D (D_{16} - \eta_k )/(D D_2 D_4 D_{16} ) \), \( A_{D} = 2 ^ {D+1} \pi D ^ {2/2} \Gamma (D/2) \), \( \Gamma (\ldots ) \) being Euler’s gamma function and \( d_c = d - D \). The RG Eqs.(8)-(10) generalize those derived perturbatively in [33-36] with the major difference that our expressions of \( \eta_k \) and \( \eta_k ^ \prime \) involve nonpolynomial, thus nonperturbative, contributions of the disorder that play a crucial role, see below.

\( \epsilon \)-expansion. Close to \( D = 4 \), expanding our equations in powers of the coupling constants and in \( \epsilon = 4 - D \) we recover [75] those obtained in [35, 36]. Let us recall their content in \( D < 4 \). There exists one fully stable fixed point, called \( P_4 \), lying at finite temperature – \( \bar{\eta}_4 = 96 \pi ^ 2 \epsilon / (24+d_c) \) – characterized by vanishing disorder coupling constants \( \Delta_\mu^4, \bar{\eta}_4 \) and by \( \eta_4 = \eta_4 ^ \prime /2 = 12 \epsilon / (24+d_c) \); it is thus associated with disorder-free membranes. There exists another fixed point, named \( P_5 \), lying at vanishing temperature – \( \bar{\eta}_5 = 0 \) – characterized by nonvanishing disorder coupling constants \( \Delta_\mu^5, \bar{\eta}_5 \) and by \( \eta_5 = \eta_5 ^ \prime = 3 \epsilon / (6+d_c) \), i.e. \( \phi_5 = 0 \) at order \( \epsilon \). A further stability analysis [35, 36] shows that \( P_5 \) is marginally unstable with respect to temperature so that \( P_5 \) should be relevant to the physics of membranes only up to a typical lengthscale \( L_c \sim \epsilon ^ {1/\alpha T} \) where \( \alpha \) depends on the membrane parameters [35, 36].

Nonperturbative analysis. Studying now directly the nonperturbative equations (8)-(10) we are led to a substantively different conclusion. First, we well identify, as in [35, 36], the fully stable finite-\( T \), vanishing-disorder fixed point \( P_4 \) and the \( T = 0 \), finite-disorder fixed point \( P_5 \). However, we find that \( P_5 \) is, contrary to what has been found in [35, 36], stable with respect to temperature, see Fig.(1). This situation relies on the fact that there exists a supplementary – critical – fixed point, that we call \( P_c \), lying between \( P_4 \) and \( P_5 \), unstable with respect to temperature, and which governs a second order phase transition between the disorder-free and the “glassy” phases associated with \( P_4 \) and \( P_5 \), respectively, see Fig.(1). It is very instructive to derive from Eqs.(8)-(10) the coordinates of \( P_c \), the critical exponent \( \eta_c \) and the eigenvalue \( y_c \) associated with the relevant direction near \( D = 4 \). At leading nontrivial order in \( \epsilon \) one has [76]: \( \bar{\eta}_c = 4 \pi ^ 2 \epsilon ^ 2 (27 + 5 d_c )/15 (6 + d_c ^ 2) \), \( \bar{\Delta}_c = 24 \pi ^ 2 \epsilon / (6 + d_c ) \), \( \bar{\eta}_c = 4 \pi ^ 2 \epsilon / (6 + d_c ) \), \( \eta_c = 3 \epsilon / (6 + d_c ) \) equal to \( \eta_c ^ \prime \) and \( \eta_c ^ \prime \) vanishes at \( P_4 \) and, finally, \( y_c = (27 + 5 d_c ) \epsilon ^ 2 / 20 \epsilon (6 + d_c ^ 2) \). These quantities are, at first order in \( \epsilon \), strictly equal to those associated with \( P_4 \). Therefore \( P_4 \) cannot be distinguished, at this order, from \( P_3 \); this is the reason why it has been missed in [35, 36] while it is clearly identified within our approach, already at order \( \epsilon ^ 2 \) – via \( \bar{\eta}_c \) and \( y_c \) – and, even more clearly, in the physical \( D = 2 \) and \( d = 3 \) – case, see Fig.(1). To complete our results we provide in Table I the critical exponents \( \eta, \eta \prime \) and the lower critical dimensions \( D_{lc} \) associated with \( P_5 \) and \( P_c \) in the physical case both for the NPRG and SCSA [77]. While the quantitative agreement at \( P_5 \) is remarkable as for \( \eta_5 \) and \( D_{lc5} \), what firmly validates our approach, a decisive discrepancy between the NPRG and SCSA is that the fixed point \( P_c \) is present within the former and absent within the latter. This also results in a disagreement concerning the stability of \( P_5 \), already observed at order \( \epsilon ^ 2 \) where we find \( \phi_5 = - y_c \) while the SCSA leads to \( \phi_5 = 0 \). The origin of this discrepancy is less trivial to identify than that occurring between the perturbative (including leading order of \( 1/d \) expansion) and nonperturbative approaches because of the complexity of the set of approximations – notably partial resummation of infinite amount of diagrams – performed within the SCSA [20]. This is under investigations [69].

| \( \eta_5 \) | \( \eta_5 ^ \prime \) | \( \phi_5 \) | \( D_{lc5} \) | \( \eta_5 ^ \prime \) | \( y_c \) | \( D_{lc} \) |
|---|---|---|---|---|---|---|
| SCAS [23] | 0.449 | 0.449 | 0 | 1.70 | \( \times \) | \( \times \) | \( \times \) |
| NPRG | 0.449 | 0.277 | -0.172 | 1.71 | 0.492 | 0.131 | 1.67 |

TABLE I: The critical exponent \( \eta, \eta \prime \) and \( D_{lc} \) at the fixed points \( P_5 \) and \( P_c \).

FIG. 1: The RG flow in \( D = 2 \) and \( d = 3 \) within the space \( \bar{\eta}_k, \bar{\eta}_k, \eta_k, \Delta_{ck}, \Delta_{ck} = - \frac{1}{2} \Delta_{ck}, \eta_k = - \frac{1}{2} \eta_k \). \( P_1 \) is the Gaussian, unstable, fixed point. \( P_4 \) is fully attractive and associated with disorder-free membranes. \( P_5 \) is also fully attractive and controls the low-temperature phase of disordered membranes. Finally, \( P_c \), unstable with respect to temperature governs the phase transition between the two phases.

Conclusion. Investigations of the nonperturbative regime of quenched disordered crystalline membranes have revealed the existence of a second order phase transition between a disorder-free phase and a glassy phase controlled by the \( T = 0 \) fixed point. Several issues should be settled in the near future. First, the question of the numerical and experimental probe for this phase should be addressed. Thanks to its potentially wide range of applicability, our prediction could \( a \ priori \) be tested on
a large variety of systems. In the context of biological physics, cell membranes with imperfectly polymerized cytoskeleton – inducing metric disorder – and with inclusion of asymmetrical proteins – generating curvature disorder – are possible candidates. In the context of 2D electronic crystalline membranes, an obvious candidate, among others, is graphene in which inclusion of lattice defects can induce, in addition to metric alterations, a rearrangement of sp²-hybridized carbon atoms into non-hexagonal and, thus, nonvanishing curvature structures. Second, at the formal level, the very nature of the putative glassy phase should be clarified [69]. Also, more specifically in the context of graphene-like membranes, the remarkable coupling between electronic and elastic degrees of freedom raises both theoretically and experimentally very challenging issues of understanding how electronic, transport, thermal, and optical properties are altered within the expected glassy phase.

ACKNOWLEDGEMENTS

We thank B. Delamotte, J.-N. Fuchs, P. Le Doussal, L. Radzihovsky, G. Tarjus and M. Tissier for helpful discussions.

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[72] We call – loosely speaking – this phase “glassy” in reference to the fact that it is purely controlled by disorder fluctuations.
[73] We closely follow the notations of Refs.[35, 36] in order to make comparisons easier.
[74] We indicate that only in the case $D < 4$ we find it attractive in the IR.
[75] Up to a redefinition of the coupling constants by a factor $1/(8\pi)^2$.
[76] All the quantities of order $\epsilon^2$ given here are approximative since finite-order derivative approximations of the NPRG approach are not exact at order $\epsilon^2$ [70].
[77] For numerical results we have employed two families of cut-off functions $\tilde{R}_k(q) = C Z_k q^4 / (\exp(q^4/k^4) - 1)$ and $\tilde{R}_k(q) = C Z_k k^4 \exp(-q^4/k^4)$ where $C$ is a free parameter used to investigate the cut-off dependence of physical quantities. Varying $C$ allows one to optimize each cut-off function inside its family, i.e. to (try to) find stationary values of these quantities, see for instance [71].