INTRODUCTION TO LEPTOGENESIS

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The discovery of neutrino masses makes leptogenesis a very attractive scenario for explaining the puzzle of the baryon asymmetry of the Universe. We present the basic ingredients of leptogenesis, explain the predictive power of this scenario (and its limitations), and describe recent theoretical developments.

1 The puzzle of the baryon asymmetry

The baryon asymmetry, that is the difference between the number densities of baryons \( n_B \) and of antibaryons \( n_{ar{B}} \) normalized to the entropy density \( s \), is extracted from observations of light element abundances and of the cosmic microwave background radiation:

\[
Y_B^{\text{obs}} \equiv \frac{n_B - n_{ar{B}}}{s} = (8.7 \pm 0.3) \times 10^{-11}. \tag{1}
\]

There are three conditions that have to be met in order that a dynamical generation of the baryon asymmetry ("baryogenesis") becomes possible:

1. Baryon number violation;
2. C and CP violation;
3. Departure from thermal equilibrium.

In principle, the Standard Model (SM) of particle physics could satisfy all three conditions and lead to successful baryogenesis:

1. Sphaleron interactions violate baryon-number \( (B) \) and lepton number \( (L) \), though they conserve \( B - L \). These interactions are related to quantum anomalies. They are faster than the expansion rate of the Universe in the temperature range \( 10^2 \, \text{GeV} \lesssim T \lesssim 10^{12} \, \text{GeV} \).

2. Weak interactions violate charge-conjugation \( (C) \) in a maximal way. For example, the weak-force-carriers couple to the left-handed down and up quarks, but not to the left-handed down and up antiquarks. They also violate CP via the Kobayashi-Maskawa phase \( \delta_{\text{KM}} \).

3. The electroweak phase transition (EWPT), that occurred around \( T \sim 100 \, \text{GeV} \), could be a first order phase transition and therefore depart from thermal equilibrium. (The EWPT is the transition from an \( SU(2) \times U(1) \) symmetric Universe, with massless weak force carriers and fermions, to a Universe with a broken electroweak symmetry, massive \( W \) and \( Z \) vector-bosons and massive quarks and leptons.)

In reality, however, only the first ingredient is fulfilled in a satisfactory way. As concerns CP violation, the contribution from \( \delta_{\text{KM}} \) to baryogenesis is suppressed by a tiny factor,

\[
\frac{(m_t^2 - m_c^2)(m_d^2 - m_u^2)(m_s^2 - m_u^2)(m_t^2 - m_c^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)}{T_{\text{eq}}^{12}} s_{12} s_{13} s_{23} \sin \delta_{\text{KM}} \sim 10^{-18}, \tag{2}
\]

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where $T_c \sim 100 \text{ GeV}$ is the temperature of the EWPT, $s_{ij} \equiv \sin \theta_{ij}$ and $\theta_{ij}$ are the three CKM mixing angles. Thus, the CP violation of the SM is much too small to explain (1). Furthermore, the EWPT would be first order only if the Higgs particle were light, $m_H \lesssim 70 \text{ GeV}$. The experimental limit, $m_H \gtrsim 115 \text{ GeV}$, implies, however, that the transition from $\langle H \rangle = 0$ to $\langle H \rangle \neq 0$ was smooth.

These failures of the SM constitute the problem that baryogenesis poses to particle physics. New physics, beyond the SM, is required to explain it, with the following ingredients:

1. $B - L$ must be violated. In this statement, we refer to two aspects of the sphaleron interactions. First, if the new physics violates $B + L$ but not $B - L$, the sphaleron interactions will erase the asymmetry. Second, if the new physics violates $L$ but not $B$, the sphaleron interactions will generate $B \neq 0$.

2. There must be new sources of CP violation, with suppression factor that is $\gg 10^{-10}$.

3. Either the Higgs sector is extended in such a way that the EWPT does provide the necessary departure from thermal equilibrium, or new out-of-equilibrium situations appear (such as the out-of-equilibrium decays of heavy new particles).

## 2 Neutrino masses and the see-saw mechanism

Measurements of fluxes of atmospheric and solar (and later also reactor and accelerator) neutrinos have established that neutrinos are massive and mix. In particular, two mass-squared differences ($\Delta m^2_{ij} \equiv m_i^2 - m_j^2$) among the three Standard Model neutrinos have been measured:

$$|\Delta m^2_{32}| \sim 2.5 \times 10^{-3} \text{ eV}^2, \quad \Delta m^2_{21} \sim 8 \times 10^{-4} \text{ eV}^2.$$  \hspace{1cm} (3)

The first measurement implies that at least one of the neutrinos is heavier than 0.05 eV. Cosmological considerations and direct searches imply that the neutrinos are lighter than $\sim 1 \text{ eV}$.

Within the SM, the neutrinos are massless. The reason is that the model has an accidental $B - L$ symmetry that forbids Majorana masses for the neutrinos. Dirac masses are impossible in the absence of singlet neutrinos (i.e. neutrinos that, unlike the SM ones, have not even weak interactions). It is clear, however, that the SM is not a full theory of Nature (it certainly cannot be valid above the Planck scale, and there are good reasons to think that it fails at a much lower scale) but only a low energy effective theory. In that case, we must add non-renormalizable terms to the Lagrangian. Already at dimension five, we find a set of terms that involve the lepton doublets $L_i$ and the Higgs field $\phi$,

$$\mathcal{L}_{d=5} = \frac{Z_{ij}}{\Lambda} L_i L_j \phi \phi,$$  \hspace{1cm} (4)

where $Z_{ij}$ is a symmetric matrix of complex, dimensionless couplings and $\Lambda$ is the scale where the Standard Model description breaks. These terms lead to light neutrino masses:

$$m_\nu = \frac{Z \langle \phi \rangle^2}{\Lambda}.$$  \hspace{1cm} (5)

Thus, simply taking into account that the SM is an effective theory that is valid only up to some high scale $\Lambda \gg \langle \phi \rangle$, we not only accommodate neutrino masses but also gain an understanding why they are much lighter than the charged fermions. The mass scale of the latter is set by $\langle \phi \rangle$, while that of neutrinos – arising only from non-renormalizable terms – is further suppressed by the ratio $\langle \phi \rangle / \Lambda \ll 1$.

What could be the full high energy theory that leads to the non-renormalizable terms of Eq. (4)? The simplest realization is to add heavy singlet neutrinos $N_\alpha$. These are new fermions
that are neutral under the SM gauge group. Consequently, they have none of the SM gauge interactions (strong, electromagnetic and weak). Still, there are two types of terms that are added to the Lagrangian when we add \( N_\alpha \)'s to the list of elementary particles:

\[
\mathcal{L}_N = M_\alpha N_\alpha N_\alpha + \lambda_{\alpha i} N_\alpha L_i \phi,
\]

where \( M \) is a Majorana mass matrix for the singlet neutrinos, and \( \lambda \) is a Yukawa matrix that couples them to the light lepton doublets. At scales well below the masses \( M_\alpha \), the leading effect of these new interactions is to generate the dimension five terms of Eq. (4), with \( \Lambda = \lambda^T M^{-1} \lambda \).

The scale \( \Lambda \) acquires a concrete interpretation: It is the mass scale of the heavy singlet neutrinos. The heavier these neutrinos are, the lighter the active (that is, the SM) neutrinos become, hence the name “see-saw mechanism” for this way of generating light neutrino masses.

Beyond the generation of light neutrino masses, the Lagrangian terms of Eq. (6) have three features that are important for our purposes:

1. It is impossible to assign a lepton number to the \( N_\alpha \)'s in such a way that \( \mathcal{L}_N \) is \( L \)-conserving: The \( M \)-terms require \( L(N) = 0 \) while the \( \lambda \)-terms require \( L(N) = -1 \). Thus, \( \mathcal{L}_N \) breaks \( L \) and (since it does not break \( B \)) \( B - L \).

2. We can choose the phases of the \( N_\alpha \) fields in a way that makes \( M \) real, but then \( \lambda \) will have physical, irremovable phases. Thus \( \mathcal{L}_N \) violates CP.

3. The Lagrangian \( \mathcal{L}_N \) allows for \( N \) decays via \( N \to L \phi \). If, however, the Yukawa couplings are small enough, the \( N \)-decays occur out of equilibrium.

We learn that the singlet neutrinos, which were introduced to explain the light neutrino masses via the see-saw mechanism, fulfill all three requirements that were specified in Section 1 in order that the baryon asymmetry might be explained.

3 Leptogenesis

Leptogenesis is a term for a scenario where new physics generates a lepton asymmetry in the Universe which is partially converted to a baryon asymmetry via sphaleron interactions.\(^2\)\(^3\) In the previous section we learned that the introduction of singlet neutrinos with Majorana masses and Yukawa couplings to the doublet leptons fulfills Sakharov conditions. This means that, if the see-saw mechanism is indeed the source of the light neutrino masses, then qualitatively leptogenesis is unavoidable. The question of whether it solves the puzzle of the baryon asymmetry is a quantitative one. To answer that, we must be more specific about the details of how leptogenesis works.

The Majorana nature of the singlet neutrino masses implies that any single heavy mass eigenstates can decay to both \( L \phi \) and \( \bar{L} \phi \). If we assign the \( N \) mass eigenstates a lepton number zero, the first mode is \( \Delta L = +1 \) while the second is \( \Delta L = -1 \). Thus, lepton number is violated in these decays.

The decay is dominated by the single tree diagram of Fig. 1. There are, however, corrections coming from the one loop diagrams. If there is more than a single \( N_\alpha \), then there is a relative CP-violating phase between the tree and the loop diagram. For example, for \( N_1 \) decay, the relative phase between the tree diagram and the loop diagram with an intermediate \( N_2 \) will be the phase of \( (\lambda \lambda^\dagger)_{12} \). Thus, CP is violated in these decays. Indeed, one can define the following CP asymmetry:

\[
\epsilon_{N_\alpha} = \frac{\Gamma(N_\alpha \to \ell \phi) - \Gamma(N_\alpha \to \bar{\ell} \phi^\dagger)}{\Gamma(N_\alpha \to \ell \phi) + \Gamma(N_\alpha \to \bar{\ell} \phi^\dagger)}.
\]
In a model with two singlet neutrinos, we have \( x_{12} \equiv M_1/M_2 \)

\[
\epsilon_{N_\alpha} = g_\alpha(x_{12}) \frac{\Im[(\lambda\lambda^\dagger)_2^2]}{(\lambda\lambda^\dagger)_{\alpha\alpha}},
\]

where \( g_{1,2}(x_{12}) \) can be found in the literature. 4

Finally, the decay is out of equilibrium if the decay rate is slower than the expansion rate of the Universe when the temperature is of the order of the mass of the decaying singlet neutrino, \( \Gamma _\alpha \lesssim H(T \sim M_\alpha) \). This can be translated into the following condition on the Lagrangian parameters:

\[
\bar{m}_\alpha \equiv \frac{(\lambda\lambda^\dagger)_{\alpha\alpha}\langle \phi \rangle^2}{M_\alpha} \lesssim m_\alpha \sim 10^{-3} \, eV.
\]

For \( M_1 \ll 10^{14} \, GeV \), the final baryon asymmetry is given, to a good approximation, by the following expression:

\[
Y_B = -1.4 \times 10^{-3} \sum_{\alpha, \beta} \epsilon_{N_\alpha} \eta_{\alpha\beta},
\]

where \( \eta_{\alpha\beta} \) parametrizes the washout of the \( \epsilon_{N_\alpha} \) asymmetry due to \( N_\beta \) interactions.

In the case that (a) the lepton asymmetry is dominated by the contribution from \( \epsilon_{N_1} \), that is, the contribution from the lightest singlet neutrino decays, (b) the masses of the singlet neutrinos are strongly hierarchical, and (c) \( N_1 \) decays at \( T \gtrsim 10^{12} \, GeV \), this mechanism of leptogenesis becomes very predictive (see e.g. 5). Among the interesting features of this scenario are the following:

(i) For \( x_{12} \ll 1 \), there is an upper bound on \( \epsilon_{N_1} \):

\[
|\epsilon_{N_1}| \leq \epsilon^{DI} \equiv \frac{3}{16\pi} \frac{M_1(m_3 - m_2)}{v^2}.
\]

Given that \( m_3 - m_2 \leq (\Delta m_{32})^{1/2} \sim 0.05 \, eV \), Eqs. (10) and (11) provide a lower bound on \( M_1 \) which, for initial zero abundance of \( N_1 \), reads:

\[
M_1 \geq 2 \times 10^9 \, GeV.
\]

This, in turn, implies a lower bound on the reheat temperature after inflation, \( T_{RH} \), that is in possible conflict with an upper bound that applies in the supersymmetric framework (to avoid the gravitino problem).

(ii) The washout parameter \( \bar{m}_1 \) cannot be too large, or else \( Y_B \) becomes too small. Roughly speaking, \( \bar{m}_1 \lesssim 0.1 - 0.2 \, eV \) is required. Since

\[
\bar{m}_1 \geq m_1,
\]

this implies an upper bound on \( m_1 \). Furthermore, requiring that \( \Delta L = 2 \) washout effects are also consistent with successful leptogenesis puts a bound of the same order, \( \bar{m} \lesssim 0.1 - 0.2 \, eV \), where

\[
\bar{m} = (m_1^2 + m_2^2 + m_3^2)^{1/2}.
\]
We learn that, if $N_1$-leptogenesis is indeed the source of the observed baryon asymmetry, then the absolute scale of neutrino masses is known to within a factor $\sim 3$, that is $0.05 \leq m_3 \lesssim 0.15 \text{eV}$.

(iii) If the initial abundance of $N_1$ is zero, then $\tilde{m}_1$ cannot be too small, or else the $N_1$ abundance was never large enough to generate $Y_B$. The situation is optimal for $\tilde{m}_1 \sim 10^{-3} - 10^{-1} \text{eV}$, where, on one hand, $Y_B$ is independent of the initial conditions and, on the other, the washout effects are mild. From the theoretical point of view, one expects $\tilde{m}_i$ to be at a scale similar to $(\Delta m^2_{21})^{1/2} \sim 10^{-2} \text{eV}$. This fact makes leptogenesis a very plausible scenario.

4 Recent developments

In the previous section, we described the predictive power of the standard leptogenesis scenario. The analysis of this scenario has been refined in recent years, including $O(0.1)$ effects such as finite temperature effects and spectator processes.

It is important, however, to realize that if any of the conditions that we specified for this scenario is violated, then some or much of the predictive power is lost. In particular, this would happen if any of the following applied:

- No strong hierarchy among the $M_\alpha$;
- $T_{\text{leptogenesis}} \lesssim 10^{12} \text{GeV}$;
- $\epsilon_{\nu_\alpha>1}$ contributes significantly.

We now briefly describe the consequences of each of these ingredients.

4.1 The role of hierarchy

Much of the constraining power of the standard scenario relies on the Davidson-Ibarra bound. In particular, the leading term in an expansion in $M_1/M_{2,3}$ of $\epsilon_{N_1}$ vanishes in the limit of degenerate light neutrinos. It has been realized, however, that the sub-leading terms do not vanish in this limit. Instead, one has

$$|\epsilon_{N_1}| \lesssim \max \left( \epsilon_{\text{DI}}, \frac{M^3_1}{M_3 M_2^2} \right).$$

The situation is even more extreme if the heavy neutrino masses are quasi-degenerate. If the mass splitting is of the order of the width, then a resonant enhancement of the CP asymmetry is possible, with $|\epsilon_{N_{1+2}}|$ coming close to its maximal value of one.

4.2 The role of flavor

If leptogenesis took place at temperatures higher than about $10^{12} \text{ GeV}$, then the flavor composition (i.e. the $\tau, \mu, e$ mixture) of the doublet state $\ell_1$ to which $N_1$ decays is unimportant. Essentially, $\ell_1$ propagates as a coherent state, and would further undergo either gauge interactions, which leave its flavor composition unchanged, or $\lambda_{\alpha 1}$-related processes – inverse decays and scatterings – which determine the washout factor.

The situation is, however, quite different if the temperature that is relevant to leptogenesis is below $10^{12} \text{ GeV}$. In that case, the tau Yukawa interactions are faster than the expansion rate of the Universe, and the $\ell_1$ state is quickly projected onto either $\ell_\tau$ or the orthogonal direction $\ell_\alpha$ (a combination of $\ell_\mu$ and $\ell_e$). If the temperature is even lower, $T \lesssim 10^9 \text{ GeV}$, when the muon Yukawa interactions become faster than the expansion rate, then $\epsilon_{\nu_\ell}$ is projected onto the three flavor directions. Each of the flavored asymmetries $\epsilon_{N_1}^i$ is subject to its own washout factor,

$$\eta_{11}^i = \min(\eta_{11} / K_i, 1),$$

as if you were reading it naturally.
\begin{equation}
K_i = |\langle \ell_i | \ell_1 \rangle|^2.
\end{equation}

The time evolution of the flavor asymmetries can then be quite different from the case that flavor effects are absent. In particular, if leptogenesis occurs at $10^9 \lesssim T \lesssim 10^{12}$ GeV ($T \lesssim 10^9$ GeV), and if $K_\tau \sim K_\alpha$ ($K_\tau \sim K_\mu \sim K_e$), the flavor effects enhance the final baryon asymmetry by a factor $\sim 2$ ($\sim 3$).

Another interesting flavor-related effect is the possibility that the decay products of $N_1$ decays, $\ell_1$ and $\bar{\ell}_1$, are not CP-conjugate of each other. Such a mismatch, when accompanied by $K_i \ll 1$ for one of the relevant flavors, can enhance the final asymmetry by an order of magnitude.

4.3 The role the heavier singlet neutrinos

The contribution of the CP asymmetries induced by the heavier singlet neutrinos, $\epsilon_{N_2,3}$, is often ignored in analyses of leptogenesis. The common wisdom is that, since $N_1$ becomes abundant after (or is abundant when) $N_{2,3}$ decay, and since it induces lepton number changing processes, it erases any pre-existing asymmetry and, consequently, only $\epsilon_{N_1}$ is important for the final outcome.

Obviously, this line of reasoning does not hold when $N_1$ is very weakly coupled, that is $\tilde{m}_1 \ll m_\star$. But, more surprisingly, the argument is also false in the case that $N_1$ is strongly coupled, that is $\tilde{m}_1 \gg m_\star$. If, at the time of $N_2$ decays, the $N_1$-related interactions are very fast then, somewhat similarly to the flavor effects, $\epsilon_{N_2}$ will be projected onto the directions that are aligned with and orthogonal to $\ell_1$. While $\epsilon_{||\ell_1}$ can be washed out, $\epsilon_{\perp\ell_1}$ is protected against the $N_1$-related washout, and therefore conserved.

Since it is impossible to have all three of $\ell_{1,2,3}$ aligned (that would lead to two massless light neutrinos), it is always the case that there is a component in either or both of $\epsilon_{N_{2,3}}$ that cannot be washed-out by the interactions of the lighter singlet neutrinos.

The conclusion is that, in general, $N_{2,3}$ leptogenesis cannot be ignored. It is irrelevant only if $\epsilon_{N_{2,3}} \to 0$, or $T_{RH} \ll M_2$, or $\tilde{m}_{2,3} \gg m_\star$, or $T \lesssim 10^9$ GeV.

5 Conclusions

The interested reader can find a comprehensive study, a pedagogical introduction, and a clear overview of recent developments in several excellent reviews.

Leptogenesis provides an attractive and plausible solution to the puzzle of the baryon asymmetry. Qualitatively, the power of this idea stems from the fact that it arises automatically when the see-saw mechanism is invoked to explain why neutrinos are massive and why they are so light. Quantitatively, the range of parameters that makes the simplest leptogenesis scenario successful and independent of initial conditions is precisely the range preferred by the measured light neutrino parameters.

Yet, it is difficult if not impossible to test leptogenesis in a stringent way. The number of parameters that play a role in leptogenesis is much larger than the number of measurable parameters. The predictive power applies only in the simplest scenario, but some of the necessary conditions that lead to the simplifications are unjustified under general circumstances.

Furthermore, it is impossible to directly observe the CP and lepton number violating processes that are relevant to leptogenesis. The reason is that they involve new particles – singlet neutrinos – that are, very likely, much heavier than the energies accessible in experiments. Furthermore, these particles have none of the Standard Model gauge interactions, and therefore will not be produced even if they are light enough. They only have Yukawa interactions, but the lighter they are, the weaker their Yukawa couplings are likely to be.
It is, however, possible – at least in principle – to establish that CP is violated and that lepton number is violated in neutrino interactions. If the first assumption is confirmed by observing CP violation in long baseline neutrino experiments, and the second by observing neutrinoless double beta decay, then the plausibility of leptogenesis as the source of the observed baryon asymmetry will be even stronger than it is today.

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