Principal component analysis of measured data for ultrasound transmission tomography

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Abstract. In this paper, the new version of imaging algorithm for Ultrasonic Transmission Tomography was presented. This algorithm was comprehensively tested with both synthetic and real measurement data. Different configuration of an internal objects was considered. In order to improve the quality of imaging the input data were treated by Principal Component Analysis. The proposed algorithm proved its usefulness and superiority over classical imaging algorithms.

1. Introduction

The quality of the measurement signal in tomographic systems [1-7] has always been a problem. Proper and effective noise suppression is a key problem, which is why so many researchers have devoted themselves to this topic [8,9]. A very successful attempt to reduce the dimensionality of the three-dimensional problem, by eliminating almost linearly dependent measurements. Indirectly thanks to dimensionality reduction the noise was suppressed (see for example [9]).

This article attempts to directly [10] suppress noise using the PCA (Principal Component Analysis) technique.

We focus our attention on PCA, which has three main applications [10]:
1. Preliminary data processing for the purposes of model development;
2. Data compression;
3. Noise reduction.

In this article, we will deal with noise reduction, based on synthetic and measured data and on an optimization approach to imaging [11-26].

2. Optimization approach to imaging in transmission ultrasonic tomography

The new proposed ultrasonic imaging algorithm does not pixelate the area (see papers [11,27,28]) for imaging its interior, but is based on the idea of uniformly distributed many test objects inside the considered area. During the optimization process, some of the test objects will disappear and others will adjust their size and location to real objects. This idea is presented in figure 1(a) and figure 1(b).
The objective function was defined as a measure of the distance of the calculated signal in each iterative step from the measured signal. The objective function (equation (1)), will be minimized by a gradient free optimization method [29].

The size of the task is defined as the number of unknowns which define the location and size of sample objects. We do not know the real image of the interior, so the test objects are arranged on two rays of eight objects on each radius so that they cover the entire study area as accurately as possible (see figure 1).

Each of the test objects is characterized by three parameters: the radius of a circular object, the position vector module (in the polar coordinate system) and the angle of inclination of this vector to the positive direction of the x axis. This gives a total of 8 * 2 * 3 = 48 optimization parameters.

3. Definition of the objective function

In order to match the signal calculated in each iteration step to the measured signal, the following objective function has been defined. This objective function will be subject to minimization with a certain constrains:

\[
\Phi = \sum_{j=1}^{p} \Phi_j = \frac{1}{2} \sum_{j=1}^{p} (f_j - v_{0j})^T (f_j - v_{0j}) = \frac{1}{2} (F - V_0)^T (F - V_0)
\]  

(1)

where: \( \Phi \) – global objective function calculated for all \( p = 16 \) positions of the voltage source (so-called projection angles), \( j=1,2,...,p \), \( \Phi_j \) – objective function for the \( j \)-th position of the voltage source, \( f_j \) – vector of electrodes voltages obtained from calculations in the current iterative step for the assumed distribution of internal objects and for the \( j \)-th position of the voltage source (projection angles), \( v_{0j} \) – vector of measured voltages for \( j \)-th position of the voltage source. The matrices \( F \) and \( V_0 \) are equal respectively:

\[
F = [f_1, f_2, \ldots, f_p] \quad \text{and} \quad V_0 = [v_{01}, v_{02}, \ldots, v_{0p}].
\]

The objective function \( \Phi_j \) is the average square error of image formation for the \( j \)-th position of the sound source. The value of the global target function with determined measurement data depends on the matrix \( F \). In turn, the matrix \( F \) is not only a function of the parameters of internal objects such as the radius of the object, the radius of the location of the object and its angle of inclination relative to the positive direction of the x axis, but also a function of the location of the sound source (so-called projection angle). The matrix \( F \) is determined at each iterative step by analysing the forward problem [11].
4. Inequality constrains

For the case of generalized starting point, 16 internal objects were adopted, each of which has three parameters defining their location. Thus, the total number of decision variables will be $16 \times 3 = 48$. The constraints should be set for these 48 decision variables, in this case preferably inequality constraints. It was assumed that the cross-sectional radius of the trial object and its position radius must be a positive number. This resulted in 32 inequality constrains. In addition, it can be assumed, though not necessary, that the angle of the position vector is also positive, which will increase the number of inequality constraints up to 48.

The starting point shown in Fig. 1 is not the only possible one, but for this kind of starting point, it is necessary to protect against collision of internal objects during the iterative process. This type of restriction can be written as follows:

$$R_i - r_i - (R_{i+1} + r_{i+1}) > R_w \times 0.01$$

where $R_i$ is the position vector module of the $i$-th object, $r_i$ is its internal radius and $R_w$ is the radius of the region.

We do not impose any constraints on the angle between the position vector and positive direction of the $x$ axis. Ultimately, the inequality constraints matrix will have 64 rows and 48 columns.

5. Noise Suppression by the Principal Component Analysis

Experience so far has shown that the parameter responsible for the angle of the position vector is relatively reluctant to change during the iterative process. Therefore, in order to facilitate reaching the optimal solution, the starting point was laid out slightly differently. The way it was changed is shown in figure 1b, and it consists of the fact that the inner layer has been shifted by an angle that the objects of the inner and outer layer would not collide.

This section of the article examines the use of PCA in the aspect of noise reduction. Admittedly, using such a tool as STATISTICS [10] with its toolboxes, functions such as `princomp` and `zscore` are useful and easily available. However, the basic computing platform for ultrasonic imaging in this work is MATLAB [29]. What's more, only the basic MATLAB functions were used to present the application of Principal Component Analysis (PCA) [10,30].

We will carry out PCA on the example of a data set consisting of "measuring" data for the configuration of four internal objects shown in figure 7. To this signal, noise was added at the level of 20% of the maximum value of the synthetic signal. In this way, an artificially noisy matrix of the "measuring" signal is presented as shown in figure 2.

The data set consists of 32 rows corresponding to projection angles and 31 columns containing measurements for a given projection angle (31 because the source sensor is not taken into account). Thus, the data size is a matrix with 32 rows and 31 columns (figure 2).

![Figure 2. "Measuring" matrix with 20% noise.](image_url)
In accordance with the requirements of the imaging algorithm, the measurement matrix in figure 2 was transformed into a vector (single-row matrix) with $32 \times 31 = 992$ columns.

![Figure 3. "Measuring" signal without noise obtained from transformation of a synthetic measuring matrix.](image)

By comparing the noisy signal presented in figure 4 with the synthetic signal in figure 3, we can see how much influence of 20% noise has on the final shape of the signal. It is clear that the signal has lost its characteristic four peaks occurring due to the presence of four internal objects represented by a blue dashed line in figure 7(a) or in figure 7(b).

Starting the PCA, the author of [10] recommends data standardization. Thus, based on the signal, we calculate an exemplary mean vector and an exemplary standard deviation vector. In this case, "standardization" means subtracting the sample mean from each observation, then dividing it by the sample standard deviation. It centres and scales the data. These calculations were carried out using the built-in MATLAB functions:

```matlab
AMean = mean(A)
AStd = std(A)
B = (A - repmat(AMean, [nA, 1]))./repmat(AStd, [nA 1]);
```

where $A$ is the measurement signal matrix and $nA$ is the number of rows of the measurement matrix and matrix $B$ is the standardized measurement matrix.

To calculate the main components coefficients and their respective variances, this can be done by determining the eigenfunctions of the sample covariance matrix:

```matlab
[V D] = eig(cov(B));
```
Figure 4. Noisy "measuring" signal with noise at 20% level of the maximum value of the synthetic signal in figure 3.

Matrix $\mathbf{V}$ contains the coefficients of the main components. The elements of the main diagonal of the matrix $\mathbf{D}$ store variances of the respective main components, which can be seen in figure 5.

Figure 5. Distribution of main components.

Interesting in this case is that the first 22 principal components contain almost 99.9% of the variance of the original data table.
The noise before and after the analysis of the main components is shown in figure 6. The range of the vertical axis for figures 6(a) and 6(b) is the same to make noise reduction more visible.

Figure 6. Noise (a) before PCA (b) after PCA.

Figure 7. Imaging of four small internal objects (a) without PCA (b) with PCA
Multiple noise reduction is clearly visible, which results in improved imaging, which is presented in figure 7. This is not a very significant improvement, but it is clearly visible. The internal objects are marked in yellow as the point from which the optimization process starts. The final result is marked in green and the thick blue dotted line indicates the actual location of the objects inside the area. In figure 7(b) you can observe results of the optimization process. At the beginning optimization process there were sixteen testing objects (yellow) and at the end only four testing objects marked in green remained, whose radius matched the radius of real objects.

It is worth noting that before the analysis of the main components, the objective function decreased by 2.5 times and after PCA by over 2.7 times. So, the difference is small, but as you can see in figure 8 the iterative process with PCA is much shorter and faster.

The measured signal after optimization is close to the synthetic signal without noise, which can be seen in figure 9(a). The difference between these signals is insignificant and visible in figure 9(b). As one can see, the difference is less than 10% of the maximum signal value.

6. Conclusions
In this paper new algorithm for Ultrasound Transmission Tomography was presented. Based on experiments with numerical and measurements one can state that the algorithm for one internal object provides stable and precise images. The more difficult task of distributed internal object algorithm after PCA provides stable results as it was proved based on synthetic data.

As a final thought, one can say that the PCA is a powerful tool, and is quickly computed on current computers, even for large data. While there are limits to what it can do, it is a handy tool which is inexpensive in terms of computation time.
The authors would like to state that all figures were drawn with the aid of MATLAB [29].

7. References

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