U-duality and Network Configurations of Branes

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ABSTRACT

We explicitly write down the invariant supersymmetry conditions for branes with generic values of moduli and U-duality charges in various space-time dimensions $D \leq 10$. We then use these results to obtain new BPS states, corresponding to network type structure of such branes.
1 Introduction

String [1-15] and Brane [16-18] Networks have recently attracted a great deal of attention in various contexts. In the realm of string theory, the fact that they come out as stable non-perturbative BPS states implies that a complete knowledge of these states and their dynamics will make our understanding of non-perturbative regime more transparent. More optimistically, it has been suggested, that it may be possible to construct a non-perturbative formulation of string theory by treating these configuration as fundamental objects. In fact, a non-perturbative, background independent formulation for the dynamics of these networks has already been proposed, albeit in an utterly different context [13-15]. Among other aspects, these networks have also made their appearances in connection with certain non-trivial compactifications [18]. More precisely, when M-theory is considered on a toric geometry, such configurations appear along the locus of the vanishing cycle. Moreover these networks have also been used for understanding the symmetry enhancements in F-Theory from type IIB string theory point of view [4]. In this context, it has been pointed out that the states corresponding to string junctions and nets are responsible for the symmetry enhancement to the exceptional groups in F-Theory.

The networks are also intimately connected to gauge theories due to the fact that the branes, in the weak coupling limit, get decoupled from the bulk supergravity theory and corresponding low energy effective dynamics is described by SYM gauge theories. These connections predict non-perturbative states in gauge theories. In particular, a finite network with transverse branes at the ends gives rise to non-perturbative states in the world-volume gauge theories of the boundary branes. Conversely, the positions of the branes at the boundary arise as moduli in the gauge theory of the network. By varying these and other moduli, including those for internal branes, one can study the non-perturbative aspects of the theory like the coulomb and the Higgs branch and phase transitions between them. Similar kinds of networks have also been found to play crucial roles in various other contexts like quantum gravity and branched polymer [14].

Most of the discussions of the networks, however, involve strings and branes in ten dimensions. In particular, in type IIB string theory, there exists $SL(2, Z)$ S-duality group and various supersymmetric networks with S-duality charges have been constructed by making use of it. However, it is well known that compactified theories in lower dimensions contain various branes which are not present in the ten dimensional theories. In addition to that, in lower dimensions, the $U$-duality group is much bigger and therefore can accommodate various new supersymmetric configurations. In view of this, and keeping in mind the varied applications of networks, it is interesting and useful to consider networks in lower dimensions which will provide deeper insights in the domain of non-perturbative physics.

In this paper, we give a detailed analysis of brane configurations with U-duality charges. More precisely, we write down the U-duality invariant conditions on supersymmetry transformation parameters for various branes with U-duality charges, referred
as U-branes, in type II theories in space-time dimensions $D \leq 10$. Although some of these results are implicit in the supersymmetry properties of branes discussed in other contexts [19,21], the network construction requires an explicit expression for the supersymmetry charges or, alternatively, the corresponding parameters. We write down these expressions for a number of examples. To argue that such conditions are indeed true, we point out that brane solutions, with generic U-duality charges, can be obtained from some specific configuration by U-duality transformations. The condition on the supersymmetry parameters are then also generated by the same transformation. The consistency of such conditions on supersymmetry parameters is also seen from the fact that by setting all the moduli to zero, they reproduce the supersymmetry condition for an appropriate brane configuration in ten dimensional type IIA/IIB theory and eleven dimensional M-theory. Moreover, our results, in several examples, are also consistent with a classification of U-branes in a different context [21].

We then use the above expressions of the preserved supersymmetry charges to construct several new non-perturbative BPS configurations. It is observed that the supersymmetry condition for a U-brane involves a map from the physical space to the internal space of $U$-duality. This suggests that one can construct a supersymmetric configuration of arbitrary number of branes, corresponding to a network structure, provided they are oriented in the physical space appropriately [9]. These configurations preserve one-quarter or less supersymmetries.

The plan of the paper is as follows. We start by reviewing the supersymmetries of the brane configurations in ten dimensions in the next section. In section-3 we obtain the explicit form of the supersymmetries of U-branes using the transformations of fields and supercharges. We will consider all lower dimensions down to four and will restrict ourselves to the toroidal compactification of type IIA/B theories. In section-4 we will present various brane networks that preserve a certain amount of supersymmetry. We will conclude with the discussions of the results and comments about further work. A sketch for the derivation of the supersymmetry conditions is presented in an appendix.

## 2 Branes in ten dimensions

To start with, we review the situation in ten dimensional type IIB theory and present the results for the supersymmetry of various brane configurations. As is well known, the massless sector contains a complex scalar $\tau$, two 2-form fields $B^{(1)}$ and $B^{(2)}$ and a self-dual 4-form field $A$. The $U$ or $S$-duality group is $SL(2)$ with the maximal compact subgroup being $U(1)$. The antisymmetric tensor fields transform under $SL(2)$, the supercharges transform under $U(1)$ and the complex scalar $\tau$ which parametrizes the coset $SL(2)/U(1)$ transform under both.

In order to give the explicit transformation let us introduce the $2 \times 2$ upper triangular vielbein matrix $\frac{1}{\tau_2} \begin{pmatrix} \tau_1 & \tau_2 \\ 0 & 1 \end{pmatrix} = V^{a}_i(\tau)$ where $i$ and $a$ are the $SL(2)$ and $SO(2)$ indices.
respectively. Under a generic $SL(2)$ element $\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ it transform as

$$V(\tau) \longrightarrow V(\tau') = \Lambda V(\tau)\mathcal{O}^{-1},$$

(2.1)

where $\mathcal{O}$ is the corresponding $SO(2)$ element with angle $\text{arg}(c\tau + d)$. One can then easily check that under this transformation,

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}.$$

The self-dual 4-form field is a singlet while the 2-form fields transform as

$$\begin{pmatrix} B^{(1)} \\ B^{(2)} \end{pmatrix} \longrightarrow \begin{pmatrix} a & -b \\ c & -d \end{pmatrix} \begin{pmatrix} B^{(1)} \\ B^{(2)} \end{pmatrix},$$

(2.2)

Finally, the parameters of supersymmetry transform as

$$(\epsilon_L - i\epsilon_R) \rightarrow \exp\left(\frac{i}{2}\text{arg}(c\tau + d)\right)(\epsilon_L - i\epsilon_R).$$

(2.3)

Now we consider branes with generic $SL(2)$ charges. Type IIB theory allows only odd $p$-branes and every $p$-brane couples electrically (or magnetically) to $(p+1)$-form (or $(7-p)$-form) field. So there are instantons (7-branes) coupled to scalars, strings (5-branes) coupled to $B$ and dyonic 3-brane coupled to $A$. The condition on the supersymmetry parameter can now be written down from the symmetry and charges.

2.1.1 Strings and five branes

Let us start with the string. The supersymmetric configurations involving $(p, q)$ strings and 5-branes have already been studied in some detail in literature [3, 17]. However we review them for completeness. For a $(1, 0)$ or a D-string, the supersymmetry parameters mentioned above satisfy the condition:

$$\epsilon_L = \Gamma_{18}\epsilon_R.$$

(2.4)

This can be complexified into a form:

$$\epsilon_L + i\epsilon_R = i\Gamma_{18}(\epsilon_L - i\epsilon_R).$$

(2.5)

The $SL(2)$ invariant form of the above equation is given by [3]

$$\epsilon_L + i\epsilon_R = e^{i\alpha}\Gamma_{18}(\epsilon_L - i\epsilon_R),$$

(2.6)

where $\alpha$ is a phase defined in terms of the $SL(2)/U(1)$ moduli $\tau$ and the $SL(2)$ charges $(p, q)$ as: $p + q\tau = |p + q\tau|e^{i\alpha}$.

This $SL(2)$ invariant relation can be obtained by starting from the condition for $(1, 0)$ string with trivial modulus $\tau = i$ and making an $SO(2)$ transformation $\frac{1}{\sqrt{p^2 + q^2}} \begin{pmatrix} p & -q \\ q & p \end{pmatrix}$.
which takes \((1, 0)\) to \((p, q)\) but keeps \(\tau\) invariant. Then to get the nontrivial moduli one has to make another \(SL(2)\) transformation by the element \(\frac{1}{\tau_2} \begin{pmatrix} \tau_1 & \tau_2 \\ 0 & 1 \end{pmatrix}\) which leads to the above condition \([23]\).

A similar construction is also possible for 5-branes. One can start with the supersymmetric condition for the D-5 brane along \((5 \cdots 9)\) hyperplane with trivial moduli:

\[
\epsilon_L + i\epsilon_R = i\Gamma_{1..4}(\epsilon_L - i\epsilon_R),
\]

and then make the transformations to get the \((p, q)\) 5-brane with general moduli. An individual \((p, q)\) 5-brane preserves a supersymmetry which is similar to the one in \((2.6)\) and has a form:

\[
\epsilon_L + i\epsilon_R = e^{i\alpha}\Gamma_{1..4}(\epsilon_L - i\epsilon_R).
\]

These supersymmetry conditions have the interesting implication that one can construct networks of strings or 5-branes which break one-quarter supersymmetry and thus lead to stable BPS states. As has been shown in literature, one can consider any number of planar \((p, q)\) strings provided their orientations are parallel to the orientations of the respective charge vectors in a charge plane. Similarly, the web or net of \((p, q)\) 5-branes can be constructed by having four common directions for each of the 5-brane and by aligning one of the edges of each of them in a plane, similar to the case of \((p, q)\) string.

### 2.1.2 Three Brane

The D3-brane couples to the self-dual 4-form field which is invariant under the \(S\)-duality. The supersymmetry parameters satisfy the condition:

\[
\epsilon_L + i\epsilon_R = -i\Gamma_{1..6}(\epsilon_L + i\epsilon_R).
\]

The supersymmetry properties of configurations of 3-branes therefore depend only on their orientations in space. Though the 3-branes by themselves can not be used to construct non-trivial junctions, they do play an important role in the analysis of junctions and webs.

### 2.1.3 Seven Brane and Instanton

Finally we consider the 7-brane which is magnetically charged with respect to the scalar fields \([23]\). The supersymmetry condition for a D-7 brane along \((3, 4, \cdots 9)\) is given by \(\epsilon_L = \Gamma_{12}\epsilon_R\). By writing this relation as

\[
\epsilon_L + i\epsilon_R = -i\Gamma_{12}(\epsilon_L + i\epsilon_R),
\]

we observe that it is manifestly \(S\)-duality invariant and therefore represents the supersymmetry condition for a general \((p, q)\) 7-brane as well. It is noted that a generic \((p, q)\) brane preserves a supersymmetry which is independent of both, charges and moduli.
The results of the above paragraph shows that any number of parallel 7-branes of arbitrary \((p, q)\) charges can be introduced without breaking further supersymmetry. This is consistent with the analysis of BPS states in F-Theory using string junctions. It has been pointed out \cite{4} that the exceptional groups observed in F-Theory are reproduced from the type IIB point of view by introducing \((p, q)\) branes for \(p, q \neq 1, 0\), and by analysing the states constructed by connecting these branes. We notice that all such branes are parallel to each other, irrespective of the values of \(p\) and \(q\).

The case of \((p, q)\) instanton \cite{25} in type IIB theory is similar, as they have similar \(SL(2)\) properties as that of a 7-brane. One has to take euclidean signature and replace \(\Gamma_{12}\) by unity.

3 Supersymmetry conditions for \(U\) branes in lower dimensions

3.1 Nine dimension

Now we consider the type II theory compactified on an \(S^1\). The massless spectrum contains 3 scalar fields parametrizing the coset \(GL(2, R)/SO(2)\) \cite{28}. The other antisymmetric fields are three 1-form, two 2-form and one 3-form fields. So in this case we will have all the branes starting from instanton to 6-branes.

The \(U\)-duality group is \(SL(2, Z) \times Z_2\) which is the discrete subgroup of the supergravity duality group \(SL(2, R) \times O(1, 1)\). The \(SL(2)\) is identical to the S-duality in ten dimensions. Under \(SL(2)\), two 1-form fields transform as a doublet and one as a singlet. The 2-forms transform as a doublet and the 3-form is a singlet. Two of the three scalars form the coset \(SL(2)/SO(2)\). Their transformations, like in ten dimensions, can be written in terms of \(SL(2)/SO(2)\) vielbein. However there is an extra scalar coming from the volume of the \(T^2\) which remains singlet under \(SL(2)\). The supersymmetry parameters \(\epsilon_\pm\) transform as a spinor of \(SO(2)\). In the following analysis we concentrate on the \(SL(2)\) part of the duality symmetry only.

3.1.1 Strings and Four branes

The strings and the 4-branes couple electrically and magnetically respectively to the 2-form field. From the M-theory point of view they are precisely the M2-branes and M5-branes wrapped on 1-cycles of \(T^2\) and form a doublet. On the other hand, from type IIB side the strings are the same \((p, q)\) strings of ten dimension while 4-branes are the \((p, q)\) 5-branes wrapped on \(S^1\).

Since the 2-form fields transform as a doublet under \(SL(2)\), we will have a doublet of strings and 4-branes. Following the discussion in ten dimensions, we can write the
$SL(2)$ invariant supersymmetric condition for a string along $X^8$ as:

$$
\begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix}
= i
\begin{pmatrix}
0 & \epsilon^{i\alpha} \\
\epsilon^{-i\alpha} & 0
\end{pmatrix}
\Gamma_{1..7}
\begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix},
$$

(3.1)

where $\epsilon_+$ and $\epsilon_-$ are the spinors in nine dimensions and $\alpha = \text{arg}(p + q\tau)$. Similarly, for the 4-branes, one can write the supersymmetry condition in the same manner. One has to just replace the $i\Gamma_{1..7}$ by $\Gamma_{1..4}$ for a 4-brane along $X^{5..8}$ hyperplane.

The relative as well as overall signs in eqn. (3.1) as well as others below can be fixed by comparing with 10-dimensional results. We also like to mention that all these relations are ambiguous upto the choice of spinor basis and do not affect the counting of the number of supersymmetries in network constructions.

Since the supersymmetry conditions for strings and 4-branes define a map from physical space to the internal space, one can form networks with these objects having arbitrary charges in a similar way as in ten dimensions.

### 3.1.2 Membranes and Three branes

Membranes and U3-branes couple electrically and magnetically to the 3-form field in nine dimensions which is a singlet under this $SL(2)$. As a result, these branes are also invariant under $SL(2)$. In fact, they can be obtained from the ten dimensional IIB theory by considering the D3-brane either along $S^1$ or perpendicular to it. From M-theory side, the membrane is the M2-brane and U3-branes can be obtained by wrapping M5-branes on $T^2$. The invariant supersymmetry for membranes along $(78)$ plane can be written as:

$$
\epsilon_\pm = -i\Gamma_{1..6}\epsilon_\pm.
$$

(3.2)

The condition for the U3-brane is also similar. For a 3-brane along $(678)$ direction the condition on the supersymmetry parameter is

$$
\epsilon_\pm = \Gamma_{1..5}(\epsilon_\pm).
$$

(3.3)

As an application, these branes can be attached at the ends of the external branes of a network and will lead to BPS states in the corresponding gauge theory.

### 3.1.3 Particles and Five branes

The particles and the U5-branes couple to 1-form fields. As mentioned earlier, there are three of them which form a doublet as well as a singlet. One obtains $SL(2)$ doublets of branes by wrapping $(p, q)$-strings on $S^1$ or reducing $(p, q)$ 5-branes in a direction orthogonal to $S^1$. The singlet particle is the KK momentum mode and the U5-brane is its magnetic dual. In M-theory side, doublet of particles are the KK momentum modes arising due to compactification on $T^2$ while the singlet is the wrapped M2-brane. Also, doublet of U5-branes are the KK magnetic monopoles while the singlet is simply the M5-brane when reduced to nine dimensions.
We can now write down the conditions on supersymmetry parameters for the doublet. These are given by,

\[
\begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix} = i \begin{pmatrix}
0 & e^{i\alpha} \\
e^{-i\alpha} & 0
\end{pmatrix} \Gamma_{1..8} \begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix} = \begin{pmatrix}
0 & e^{i\alpha} \\
e^{-i\alpha} & 0
\end{pmatrix} \Gamma_{1..3} \begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix}
\]

for U-particles and U5-branes respectively. Similarly, the singlets satisfy the supersymmetry conditions:

\[
\epsilon_\pm = \Gamma_{1..8} \epsilon_\pm \quad \text{and} \quad \epsilon_\pm = -i \Gamma_{1..3} \epsilon_\pm.
\]

These conditions individually lead to one-half BPS states. However a particle (or 5-brane) charged with respect to both, doublet and singlet fields, corresponds to one-quarter BPS states.

We have therefore discussed different U-branes in nine dimensions and their supersymmetries. As we now go down to lower dimensions, we encounter larger U-duality groups and networks with more generalized structure.

### 3.2 Eight Dimension

We now discuss the situation in eight-dimensional space-time. The U-duality symmetry in the present case is given by \(SL(3) \times SL(2)\). The massless spectrum contains a set of 5 scalars which form the \(SL(3)/SO(3)\) moduli and a set of two scalars corresponding to \(SL(2)/U(1)\) moduli. Apart from these, one has 1-form and 2-form fields in (3,2) and (3,1) representations respectively. They couple to particles (or U4-branes) and strings (or U3-branes) respectively.

The scalars can be arranged in a vielbein form. The \(SL(2)/U(1)\) part of the vielbein is same as in ten dimensions, while the \(SL(3)\) part can be written as an upper triangular matrix \(\lambda_i^a\) with unit determinant and \((i, a)\) are \(SL(3)\) and \(SO(3)\) indices respectively. This is given by

\[
\lambda_i^a = \begin{pmatrix}
e^{-\frac{(\phi+\alpha)}{2}} & \chi e^{\frac{\phi}{2}} & e^{\frac{\phi}{2}} \eta_1 \\
0 & e^{\frac{\alpha}{2}} & e^{\frac{\alpha}{2}} \eta_2 \\
0 & 0 & e^{\frac{\chi}{2}}
\end{pmatrix},
\]

where \(\phi, \alpha, \chi, \eta_1, \eta_2\) are the five scalar fields parametrizing the moduli. Two \(SL(2)\) subgroups correspond to setting \(\phi = \eta_1 = \eta_2 = 0\) and \(\phi = -\alpha, \chi = \eta_1 = 0\). Under a generic \(SL(3)\) transformation \(\Lambda\) the transformation of the scalar fields can be given by

\[
\lambda_i^a(\mu) \rightarrow \lambda_i^a(\mu') = \Lambda_i^j \lambda_j^b \tilde{O}_b^a
\]

where, \(\mu\) are the scalar parameters and \(O\) represents the \(SO(3)\) matrix corresponding to the \(SL(3)\) element \(\Lambda\).
In eight dimensions, the $N = 2$ supergravity is parameterized by four pseudo-Majorana spinor parameters transforming in $(2_+, 2_-)$ representation of the maximal compact subgroup of $U$-duality group which is $SU(2) \times U(1)$. Explicitly, they are given by: $\epsilon^1_{\pm}$ and $\epsilon^2_{\pm}$ where $\pm$ are the $U(1)$ charges. Under a generic $SL(3)$ transformation $\Lambda$, they transform as: $\epsilon_+ \rightarrow U \epsilon_+$ and $\epsilon_- \rightarrow U \epsilon_-$, where the spinors $\epsilon_{\pm}$ are defined as:

$$\epsilon_{\pm} \equiv \left(\begin{array}{c} \epsilon^1_{\pm} \\ \epsilon^2_{\pm} \end{array}\right),$$

and the transformation $U \in SU(2)$ can be obtained from the vielbein. These supersymmetry parameters in eight dimensions are obtained by the compactification of the 10-dimensional spinors $\epsilon_{L,R}$. We now present the supersymmetry conditions for various U-branes in 8-dimensions.

### 3.2.1 Strings and Three Branes

In eight-dimensions, U-strings couple electrically to three antisymmetric tensor fields which transform in a $(3, 1)$ representation under $SL(3) \times SL(2)$. Each string is therefore denoted by three $SL(3)$ charges $(p, q, r)$. From the M-theory point of view, this can be thought of as the M2-brane wrapped on the $(p, q, r)$ one-cycle of $T^3$. The type IIB $SL(2)$ is contained in $SL(3)$ and the extra $SL(2)$ corresponds to the constant deformations of $T^2$ on which the type IIB is compactified. So from the type IIB point of view these strings correspond to the ten dimensional $(p, q)$ strings and the wrapped 3-branes.

A detailed analysis of the supersymmetry property of the string case has been done in a recent paper by the present authors \cite{9}. The supersymmetry parameters for a string along $X^7$ satisfy:

$$\begin{pmatrix} \epsilon_+ \\ \epsilon_- \end{pmatrix} = -i \begin{pmatrix} \chi \\ 0 \end{pmatrix} \Gamma_{1..6} \begin{pmatrix} \epsilon_+ \\ \epsilon_- \end{pmatrix},$$

where $\chi$ is a $2 \times 2$ matrix defined in terms of an $SO(3)$ vector as: $\chi = X_i \sigma_i$. The components of the charge vector $\tilde{X}$ are given in terms of $SL(3)$ charges $p_i$’s and $SL(3)/SO(3)$ moduli $\lambda^a_i$ as $X_i = \lambda^a_i p_a$. $|X|$ is the magnitude of this vector. Above condition is then manifestly $SL(3)$ invariant.

One can also obtain this relation in a similar way as the $(p, q)$ string. This is done by starting from the supersymmetry condition for $(1, 0, 0)$ with trivial moduli and then considering an $SO(3) \in SL(3)$ transformation that will generate $(p, q, r)$ charges but keep the moduli fixed. Finally, to have non-trivial moduli, one makes another $SL(3)$ transformation given by the vielbein $\lambda^a_i$.

The objects hodge dual to strings in 8-dimensions are the U3-branes. From the M-theory point of view, they can be seen as M5-branes wrapped on $(p, q, r)$ 2-cycles. From type IIB side these are the combinations of the wrapped $(p, q)$ 5-branes on $T^2$ and the 3-brane. The U-duality invariant supersymmetry condition for a 3-brane is similar
to the one in eqn. (3.9). The only modification is the number of \( \Gamma \) projections. For a 3-brane along \( X^{567} \) hyperplane we therefore have:

\[
\left( \begin{array}{c}
\epsilon_+ \\
\epsilon_-
\end{array} \right) = \left( \begin{array}{cc}
\frac{\chi}{|\beta|} & 0 \\
0 & \frac{\chi}{|\beta|}
\end{array} \right) \Gamma_{1..4} \left( \begin{array}{c}
\epsilon_+ \\
\epsilon_-
\end{array} \right).
\]

These invariant relations lead to the fact that one can place many strings or 3-branes with arbitrary charges in a manner keeping some supersymmetry unbroken, provided they are properly oriented. All these states preserve one-eighth or more supersymmetry.

### 3.2.2 Four Brane and Zero Brane

The objects that couple to the 1-forms are the 0-branes and 4-branes. From M-Theory point of view they are the M2-branes (M5-branes) wrapped on different two-cycles (one-cycles) together with the KK momentum modes (its magnetic dual) along three \( S^1 \). From type IIB side they are \( (p,q) \) strings (and 5-branes) with various wrapping and the KK momentum modes. They transform in the \( (3,2) \) representation of the \( U \)-duality symmetry. So the charges can be written as \( Q_{ia} \) where \( i \) and \( a \) are the \( SL(3) \) and \( SL(2) \) vector indices respectively.

The invariant supersymmetry can now be written by observing the form of equations (3.9) and (2.6). In terms of \( SL(3) \times SL(2) \) charges and moduli, the supersymmetry condition for a 0-brane is:

\[
\left( \begin{array}{c}
\epsilon_+ \\
\epsilon_-
\end{array} \right) = \frac{1}{\Delta} \left( \begin{array}{cc}
0 & -Q \\
Q & 0
\end{array} \right) \Gamma_{1..7} \left( \begin{array}{c}
\epsilon_+ \\
\epsilon_-
\end{array} \right),
\]

where we have written the \( SL(3) \times SL(2) \) charges in the form of complex \( 2 \times 2 \) matrices \( Q = Q_{i} \sigma_i = (Q_{i1} + iQ_{i2}) \sigma_i \). For the special case \( n_i = i\epsilon_{ijk} Q_j Q_k = 0 \) the phase factors of the \( Q_i \)’s are equal. Eqn. (3.11) then implies the breaking of one-half supersymmetry. However, for \( n_i \neq 0 \) one has to impose an additional condition namely, \( n_i \sigma_i \epsilon_\pm = \pm |n| \epsilon_\pm \) and therefore only one-quarter supersymmetry is preserved. In eqn. (3.11) \( \Delta \) is a normalization factor given by \( \Delta^2 = Q_i \tilde{Q}_i + |n| \).

For one-half BPS states the supersymmetry condition can be written in a simpler form as

\[
\left( \begin{array}{c}
\epsilon_+ \\
\epsilon_-
\end{array} \right) = \left( \begin{array}{cc}
\frac{\chi}{|\beta|} & 0 \\
0 & \frac{\chi}{|\beta|}
\end{array} \right) \left( \begin{array}{cc}
0 & -e^{i\alpha} \\
e^{-i\alpha} & 0
\end{array} \right) \Gamma_{1..7} \left( \begin{array}{c}
\epsilon_+ \\
\epsilon_-
\end{array} \right),
\]

where the phase \( \alpha \) is defined in terms of \( SL(2) \) charges and moduli as before and \( \chi = Q e^{-i\alpha} \).

For the 4-branes, the invariant supersymmetry condition can be written just by replacing \( \Gamma_{1..7} \) by \( i \Gamma_{1..3} \) and \( -Q \) by \( Q \) keeping \( \tilde{Q} \) unchanged in eqn. (3.11). Since these are again extended objects, one can construct BPS webs using properly charged 4-branes.
3.2.3 Membranes

Membranes or U2-branes are the dyonic objects in eight dimensions and couple to the 3-form fields. In type IIB side these are 3-branes wrapped on a \((p,q)\) cycle of \(T^2\). Since the 3-form field in eight dimensions comes from the self-dual 4-form in ten dimensions, membranes form an \(SL(2,Z)\) multiplet. These membranes can also be thought of as coming from the M2-brane as well as wrapped M5-brane and is invariant under the \(SL(3)\) part of the \(U\)-duality group. The supersymmetry condition can then be written as

\[
\begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix} = \begin{pmatrix}
0 & e^{i\alpha} \\
e^{-i\alpha} & 0
\end{pmatrix} \Gamma_{1..5} \begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix},
\]

(3.13)

where the parameter \(\alpha\) is defined as \(\alpha = \text{arg}(p + q\tau)\) like the case of string in ten dimension. Since these membranes form an \(SL(2)\) multiplet, one can construct the network of membranes preserving one-quarter supersymmetry.

We have therefore given the supersymmetry conditions for various U-branes in eight dimensions. As mentioned, one can reproduce the 10-dimensional results for both type IIA and IIB string theory. In other words the branes in lower dimensions combine the results for the type IIA as well as IIB theory. Moreover, the 8-dimensional branes in the presence of moduli and arbitrary charges have non-perturbative information about string theory, which is not contained in the original 10-dimensional branes.

3.3 Seven dimension

After discussing the 8-dimensional case, we now extend these results to other lower dimensional cases. In 7-dimensions, the \(U\)-duality symmetry is \(SL(5)\). The massless spectrum contains 14 scalars, ten 1-form fields and five 2-form fields. So we have all the branes starting from (-1)-brane (instanton) to 4-brane. Under \(SL(5)\) the ten 1-form fields transform in an antisymmetric representation of \(SL(5)\) and the five 2-form fields transform as vector. The 14 scalars, transform under both the \(SL(5)\) and its maximal compact subgroup \(SO(5)\) as the \(SL(5)/SO(5)\) coset. To get their explicit transformation we write down the \(5 \times 5\) upper triangular matrix with unit determinant: \(V^a_i\) is parametrized by the 14 scalars which represent the vielbein for \(SL(5)/SO(5)\). The transformation is given by:

\[
V^a_i(\mu) \rightarrow V^a_i(\mu') = g^{-1}_{ij}V^b_j(\mu)\tilde{O}_{ba},
\]

(3.14)

where as before \(\mu\) and \(\mu'\) are the moduli parameter and indices \(i\) and \(a\) correspond to \(SL(5)\) and \(SO(5)\) respectively.

The supersymmetry parameters correspond to 4 spinors which transform among themselves in the 4-dimensional spinor representation of \(SO(5)\).

\[
\Lambda : \epsilon^a \rightarrow U^{a}_{b} \epsilon^b,
\]

(3.15)

Where \(U\) is an \(SO(5)\) element in the spinor representation. We now analyse the supersymmetry properties of 7-dimensional branes.
3.3.1 Particle and Three Brane

The 0-brane (and 3-brane) couple to ten vector fields which transform in a 10-dimensional representation of the U-duality symmetry $SL(5)$. These can be thought of as the M2-brane (and M5-branes) wrapped on different two-cycles of $T^4$ together with the KK momentum modes arising due to the compactification. In type IIB side they are wrapped strings (5-branes) on one-cycles (two cycles) and the corresponding KK momentum modes.

The charge of the 0-brane is given by a $5 \times 5$ antisymmetric tensor $Q_{ab}$ which transforms as 10 of $SL(5)$. To write down the supersymmetry condition in the present case we define a tensor $\chi_{ij}$ as

$$\chi_{ij} = \lambda_{ia}^{-1} \lambda_{jb}^{-1} Q_{ab} \quad (3.16)$$

where $\lambda_{ia}$ are the vielbein representation of the $SL(5)/SO(5)$ moduli.

Now, to write down the supersymmetry condition, we define a matrix $\hat{\chi}$ in terms of $SO(5)$ Gamma matrices $\gamma^i$ as:

$$\hat{\chi} = \chi_{ij} \gamma^{ij} \quad (3.17)$$

These constructions are the generalizations of similar ones for $SL(3)$ vectors in eight dimensions. The $SL(5)$ invariant supersymmetry condition is then written as:

$$\epsilon = \frac{\hat{\chi}}{\Delta} \Gamma_{1..3} \epsilon, \quad (3.18)$$

where $\Delta$ is a normalization factor obtained the same manner as in the case of 0-branes in eight dimensions. In other words, the tensorial nature of $\gamma^{ij}$ is compensated by that of $\chi_{ij}$ to give an $SL(5)$ invariant. The amount of the supersymmetry broken will depend on the charge. If $n_m \equiv \epsilon_{ijklm} \hat{\chi}_{ij} \hat{\chi}_{kl} = 0$ then it breaks one-half supersymmetry whereas $n_m \neq 0$ will preserve only one-quarter. This condition is derived from the consistency of eqn.(3.18) under multiple operations and is similar to the one in [21], where it was obtained from a general classification of branes using properties of central charges.

The condition for one-half BPS states (3.18) can be generated from that for a 3-brane in ten dimensions by applying $SL(5)$ transformations as in the earlier cases. The U3-branes magnetically couple to 1-form fields in seven dimensions and satisfy the same supersymmetry condition (3.18) with $\Gamma_{1..6}$ replaced by $\Gamma_{1..3}$.

3.3.2 String and Membrane

In seven dimensions one also has U-strings and membrane which are dual to each other and have charges which transform as a vector under $SL(5)$. The M2-brane reduced to seven dimensions and the M5-branes wrapped on four 3-cycles constitute the multiplet of U2-brane. In type IIB side these are 3-branes on three 1-cycles and $(p, q)$ 5-brane on a 3-cycle. Similarly the U-string is coming from M2-brane on four 1-cycles and M5-brane on one 4-cycles while in type IIB it is a $(p, q)$ string alongwith the D3-brane on
three 2-cycles. U2 and U1-branes in seven dimensions satisfy similar conditions. Their supersymmetry property is similar to that of a U-string in 8-dimensions since both have charges which transform as a vector.

Since the supersymmetry parameters transform in a spinorial representation while the charges are in vector one, to define the invariant supersymmetry condition we introduce $SO(5)$ charges in a spinor representation as a matrix: $\phi \equiv Y_i \gamma_i$, where $Y_i = V^i_a Y^a_i$. $V^a_i$s are the $SL(5)/SO(5)$ vielbeins and $Y^a_i$s are the $SL(5)$ charges. Now one can write down the invariant supersymmetry relations for strings and U2-branes as:

$$\epsilon = \frac{\phi}{|\phi|} \Gamma_{1..5} \epsilon \quad \text{and} \quad \epsilon = \frac{\phi}{|\phi|} \Gamma_{1..4} \epsilon$$

repectively. These supersymmetic conditions are also a simple generalizations of the result in [4] and in eqn. (3.9). In addition, in seven dimensions, there are 14 scalars parameterizing the $SL(5)/SO(5)$ which couple to the instantons and 4-branes. Their explicit solution and supersymmetry condition will also be of interest to analyse.

We have therefore discussed different U-branes in seven dimensions. In this case, we have also seen the application of tensorial charges in obtaining the invariant supersymmetry condition. The charges span a five dimensional internal space and such branes can therefore give rise to five dimensional webs with 1/32 or more supersymmetry.

### 3.4 Six Dimension

We now discuss the case in six dimensions. The U-duality group is $SO(5,5)$. The massless spectrum contains 25 scalars parametrizing the coset $SO(5,5)/SO(5) \times SO(5)$, sixteen 1-form fields ($A_\mu$) in a 16-dimensional spinorial representation of $SO(5,5)$ and five 2-form fields ($B_{\mu\nu}$) forming a 10-dimensional representation of $SO(5,5)$ by decomposing the $B_{\mu\nu}$’s into self-dual and anti-self-dual forms. The six dimensional theory therefore has the brane configurations: U3-brane and U-instantons which couple to the scalars, U2 and U0-branes which couple to vectors and finally U1-brane coupling to the ten 2-form fields.

Now consider the representations of different fields. The maximal compact subgroup of $SO(5,5)$ is $SO(5) \times SO(5)$. The scalars, as in earlier cases, transform under both $SO(5,5)$ and $SO(5) \times SO(5)$. They can be written down in the form of a vielbein given by [26]:

$$U = \begin{pmatrix} a^a_m & b^a_{\hat{m}} \\ c^a_m & d^a_{\hat{m}} \end{pmatrix},$$

where $a^a_m$, $b^a_{\hat{m}}$, $c^a_m$, $d^a_{\hat{m}}$ are $5 \times 5$ matrices, $\hat{m}, m (= 1..5)$ together are the $SO(5,5)$ vector indices and $a$ and $\hat{a}$ are the vector indices for the two $SO(5)$’s respectively. The ‘dot’ is to distinguish the second $SO(5)$ from the first. The vielbeins transform as:

$$U(\mu) \rightarrow U(\mu') = gU(\mu)h^{-1}, \quad \text{where} \quad g \in SO(5,5), \quad h \in SO(5) \times SO(5)$$
and $\mu$ is the scalar parameters. As in other cases, using these moduli one can associate an $SO(5,5)$ element with an $SO(5) \times SO(5)$ one.

Since the vector fields transform in the Majorana-Weyl spinor representation of $SO(5,5)$ we present it explicitly. To define those representation let us consider the Clifford algebra for $SO(5,5)$. The $SO(5,5)$ gamma matrices are given by tensor product of two $SO(5)$ gamma matrices and pauli matrix as

$$
\Gamma^\hat{m} = \gamma^\hat{m} \times 1 \times \sigma_1, \quad \hat{m} = \hat{1}, \ldots, \hat{5}, \quad \text{and}
$$

$$
\Gamma^\dot{m} = 1 \times \gamma^\dot{m} \times -i\sigma_2, \quad \dot{m} = \dot{1}, \ldots, \dot{5}.
$$

We define $\Omega$ as $\gamma^T = \Omega \gamma \Omega^{-1}$, where $\gamma$ is a $SO(5)$ Gamma matrix. The $\Omega$ serves as the raising and lowering operator for $SO(5)$ spinor index: $\psi^{\alpha} = \Omega^{\alpha\beta} \psi_\beta$. The $SO(5,5)$ Gamma matrices satisfy

$$
\Gamma^{m*} = B\Gamma^m B^{-1}, \quad \text{where} \quad B = \Omega \times \Omega \times 1, \quad \text{and},
$$

$$
\Gamma^{mT} = C\Gamma^m C^{-1}, \quad \text{where} \quad C = \Omega \times \Omega \times \sigma_1.
$$

A 32 component spinor $\psi$ of $SO(5,5)$ consists of two $SO(5) \times SO(5)$ spinors $\xi^{\mu\dot{\mu}}$ and $\chi^{\mu\dot{\mu}}$ which are of opposite chirality. If we impose the Majorana-Weyl condition:

$$
\bar{\Gamma} \psi \equiv -\psi, \quad \psi^* = B\psi, \quad \text{where} \quad \bar{\Gamma} = \Gamma^1 \ldots \Gamma^5 \Gamma^1 \ldots \Gamma^5
$$

we get $\psi = \begin{pmatrix} 0 \\ \chi \end{pmatrix}$. Under $U$-duality group it transforms as

$$
\chi^{\mu\dot{\mu}} \rightarrow (S^T)_{\mu\dot{\mu}}^{\nu\dot{\nu}} \chi^{\nu\dot{\nu}}
$$

where $S^{\nu\dot{\nu}}$ is the spinor transformation matrix of $SO(5) \times SO(5)$.

In order to write down the invariant supersymmetric relation we have to transform the $SO(5,5)$ charges into $SO(5) \times SO(5)$ charges. This can be achieved by using an alternative form of moduli which is a $16 \times 16$ matrix denoted by $V^{\mu\dot{\mu}}$, where $\alpha$ and $\dot{\alpha}$ are the $SO(5) \times SO(5)$ spinor indices. These moduli are related to the vector moduli in the following way [26]:

$$
a^{\alpha}_{\hat{m}} = \frac{1}{16} V^{\mu\dot{\mu}\alpha\dot{\alpha}\nu} (\gamma^\mu)_{\mu}^{\nu} (\gamma^\alpha)_{\alpha}^{\beta} V_{\nu\dot{\mu}\beta\dot{\alpha}} \quad \text{etc.}
$$

The supersymmetry parameters transform only under the maximal compact subgroup $SO(5) \times SO(5)$. One can write them in two sets of four Majorana-Weyl spinors in six dimension $\epsilon^1$ and $\epsilon^2$ which transform in a representation $(4,1)$ and $(1,4)$. Using the vielbeins one can associate a unique $SO(5) \times SO(5)$ element with a generic $SO(5,5)$ which determines the transformations like that in other dimensions. We now discuss the supersymmetry conditions for U-branes in six dimensions.
3.4.1 Particle and Membrane

Let us consider the particles and membranes which are dual to each other in six dimensions. They have the 16 charges in the spinor representation of $SO(5,5)$. Sixteen U2-branes from M theory point of view are coming from the M2-brane and wrapped M5-brane on ten 3-cycles of $T^5$ and five KK momentum modes. From type IIB side they are $(p,q)$ 5-branes wrapped on four 3-cycles of $T^4$, 3-branes on four 1-cycles and four KK momentum modes. The U0-branes come from dual configurations. In order to write down a supersymmetry configuration we need to transform the $SO(5,5)$ spinor charges of the vector fields into the spinors of $SO(5) \times SO(5)$.

Making use the moduli $V^\alpha_{\hat{\alpha}}$, one can define $SO(5) \times SO(5)$ spinor charges $Q^\alpha_{\hat{\alpha}}$ from the original $SO(5,5)$ charges $Q^\mu_{\dot{\mu}}$. One can then write a U-duality invariant supersymmetry condition for U2-branes in six dimensions as:

$$\epsilon_\alpha = \frac{1}{\Delta} Q^\alpha_{\hat{\alpha}} \Gamma_{1..3} \epsilon_{\hat{\alpha}} \quad \text{and} \quad \epsilon^{\hat{\alpha}} = -\frac{1}{\Delta} \tilde{Q}^{\dot{\alpha}\alpha} \Gamma_{1..3} \epsilon_\alpha.$$  \hspace{1cm} (3.28)

where $\tilde{Q}^{\dot{\alpha}\alpha}$ is simply the contravariant tensor corresponding to $Q^\alpha_{\hat{\alpha}}$ and indices are raised and lowered using $\Omega$’s defined earlier. $\Delta$ is again a normalization factor. Once again a similar condition will be satisfied by a U0-brane, with Gamma matrix projection being: $\Gamma_{1..5}$ and $-\tilde{Q}$ is replaced by $\tilde{Q}$.

The consistency between two equations in (3.28) requires $\tilde{Q} \tilde{Q} = \tilde{Q} \tilde{Q} = 1$. One can argue that when charges are restricted in this manner, the supersymmetry condition corresponds to one-half BPS state. Otherwise, supersymmetry will be broken further. Once again similar results appear in other contexts as well [21].

3.4.2 String

Finally, we discuss the situation with strings (U1-branes) in six dimensions. This case is similar to the one in four dimensions, where one has both the electric and magnetic charged objects. Here, one can combine the "electric" and "magnetic" field strengths and form a ten dimensional vector under $SO(5,5)$.

The charges, which transform as vectors $(5,1) + (1,5)$ under the $SO(5) \times SO(5)$ symmetry group are constructed by using the vielbeins $a^a_{\dot{m}}$, $b^\dot{a}$ etc. in eqn. (3.20). We denote these $SO(5)$ vectors as $X^1_a$ and $X^2_{\dot{a}}$ respectively. The invariant supersymmetry conditions are then similar to the ones in higher dimension for a string and can be written as:

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \begin{pmatrix} \chi^1 \\ 0 \\ 0 \\ \chi^2 \end{pmatrix} \Gamma_{1..4} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}.$$  \hspace{1cm} (3.29)

where, $\chi^1 = X^a \gamma_a$ and $\chi^2 = X^{\dot{a}} \gamma_{\dot{a}}$. If the $SO(5,5)$ charge vector $(X^\dot{m}, X^{\hat{m}})$ is null, $\chi^1$ and $\chi^2$ can be simultaneously normalized to unity. Then this condition breaks one-half supersymmetry. Otherwise supersymmetry breaks to one-quarter, as the consistency of eqn. (3.29) then requires setting all the components of $\epsilon_1$ or $\epsilon_2$ to zero.
3.5 U-branes in Four and Five Dimensions

In five dimensions, the full U-duality symmetry is a non-compact version of $E_6$. The massless $p$-form fields consist of 42 scalars and 27 vector fields. So it has the U(-1)-branes and strings as well as their duals namely, U2 and U0-branes. They couple to the scalar and vector fields respectively.

In this case the maximal compact subgroup is $USp(8)$. The 42 scalars can be written as a vielbein $V_{\alpha\beta}^{ab}$ for the coset space $E_6/USp(8)$, where $\alpha$, $\beta$ are the $E_6$ indices and $a$, $b$ are $USp(8)$ indices. The transformation is once again like

$$V(\mu) \rightarrow V(\mu') = gV(\mu)h^{-1}, \quad g \in E_6, \quad h \in USp(8).$$

(3.30)

The 27 vector fields $A_{\mu}^{\alpha\beta}$ transform in the fundamental representation of $E_6$ and are singlets of the $USp(8)$. The supersymmetry parameters are given by 8 spinors in five dimensions. Under $USp(8)$ they transform in the fundamental representation.

The U-string and the U0-brane in this case couple to the 1-form field and so the charges transform in the fundamental of $E_6$. To find out the invariant supersymmetry, one uses the moduli $V_{\alpha\beta}^{ab}$, to transform the charges into a 27-dimensional representation of $USp(8)$. We denote these charges as: $X_{ab}$, with raising and lowering defined by the symplectic metric $\omega^{ab}$. The simplest case now preserves one-half supersymmetry and the invariant condition for U1-brane has a form:

$$\epsilon^a = X^{ab} \Gamma_{1..3} \epsilon_b,$$

(3.31)

where $X_{ab}$ are the $8 \times 8$ antisymmetric matrices satisfying $\omega^{ab} X_{ab} = 0$ and are properly normalized. The U0-brane can be found by replacing $\Gamma_{1..3}$ by $i\Gamma_{1..4}$ in (3.31). In the 1/2 supersymmetry case it simply corresponds to $X^2 = -1$. It is expected that there are other charges as well which breaks supersymmetry to one quarter and one eighth. However we postpone these discussions for future.

By continuing to 4-dimensions, the U-duality group is $E_7$ with maximal compact subgroup: $U(8)$. Here we have 28 vector fields and 70 scalar fields. One can form the $E_7/U(8)$ vielbeins using these scalar fields. The vielbeins are 56 dimensional matrices which transform under $E_7$ as 56 while under $U(8)$ it transforms as $28 + \bar{28}$. The vector fields are the self-dual fields in 4-dimensions. To get the transformations one has to split the 28 vector field strengths in self-dual and anti-self-dual parts. These electric and magnetic fields then combine to give a 56-dimensional representation of $E_7$. The sixteen supercharges $\epsilon^L$ and $\epsilon^R$ transform as a $8_2$ and $\bar{8}_2$ representation of $U(8)$. One also has 70 scalar fields which parameterize a coset $E_7/SU(8)$. These scalars can be parameterized by a $56 \times 56$ matrix, which plays the role of a vielbein in the present situation. In particular, it transforms as a 56-representation of $E_7$ and $28 + 28$ representaion of $SU(8)$. Using these vielbein’s one can construct the charges with tensor structure 28 and 28 of $SU(8)$. We represent these charges as $\chi_{ij}$ and $\chi^{ij}$. It is then straightforward to write down the supersymmetry conditions for the dyonic 0-brane in four dimensions. For a U0-brane, we have:

$$\epsilon^L_i = \chi_{ij} \Gamma_{1..3} \epsilon^R j, \quad \epsilon^R i = -\chi^{ij} \Gamma_{1..3} \epsilon^L_j,$$

(3.32)
and a similar condition for $\epsilon^R$. This condition represents the one half BPS states when $\chi^{ij} \chi_{jk} = -\delta^i_k$. However generalizations to cases preserving less amount of supersymmetries should also be possible.

We now apply the results derived in this section for the construction of network and web of U-branes.

4 Applications

So far we have mainly discussed supersymmetries of individual $U$-branes with generic charges in various dimensions. We have seen that each of them preserves some amount of supersymmetry. It will be interesting to obtain such U-brane solutions directly from supergravity and verify our results. More interesting objects are the intersecting $U$-branes, as well as the network or web type configurations. By using the supersymmetry conditions one can construct various intersecting brane configurations and webs which represent BPS states. It should also be possible to construct the corresponding supergravity solutions.

We now discuss the applications of the results in the previous section to construct general network like configurations in various dimensions. The requirements of such constructions is the presence of a certain amount of supersymmetry when an arbitrary number of such branes are put together. In this connection, interesting objects are the branes with $U$-duality charges. Then one can form junction where three branes meet. In general they can meet along any hypersurface with dimension smaller than the particular branes involved. One can then make use of these junctions to build web structures. Apart from the supersymmetry condition, each brane-junction has to satisfy the charge balance and the tension balance conditions as well. One can also construct junctions of more than three branes at a time, but these junctions can be thought of as a degeneration of a net due to the vanishing of a polygon face. Another set of interesting objects in this case are the nets with branes ending on other orthogonal branes. In this context, whether a brane can end on another one will be fixed by the $U$-duality group. For example, in 8-dimension $(1,0,0)$ string can end on $(0,1,0)$ as they come from $F$ and $D$ strings of ten dimension. Other cases, related to this one through $U$-duality are therefore also compatible.

For the construction of webs, the ten dimensional cases have been well studied. Since the strings and five branes have $(p,q)$ charges, one can construct the planar webs of strings and five branes. Note that it is not possible to consider a non-planar BPS configuration as the charge space is two dimensional. In this case one also has D3-branes which are invariant under S-duality group. So one can consider the configurations involving the 3-branes and 5-branes or strings. These configurations gives rise to interesting gauge theories. Also, one can consider the three string junction with strings ending on three D3-branes. These objects have been identified as the BPS states in $SU(3)$ SYM. A gauge theory structure may be possible to write down directly for the webs, although they have not yet been constructed explicitly. However zero mode analysis have been
done in this context to obtain an S-Matrix structure for these theories.

We now consider the eight-dimensional case. $U$-strings and $U3$-branes now have charges which are vector under $SL(3)$. They are neutral with respect to the remaining part of the $U$-duality, namely $SL(2)$. The $U4$-brane charges are vector under $SL(3)$ as well as $SL(2)$. The $U2$-brane is a singlet of $SL(3)$ and a doublet of $SL(2)$. The essential idea for constructing webs then is to present an identification between the coordinates of the ‘internal space’ and that of the physical space.

For the eight-dimensional case, the non-planar network of strings with 1/8 supersymmetry has been discussed previously. The internal space is three dimensional and the direction of one of the charge vector is given by $\vec{X}$. The corresponding string is then aligned along the direction $\hat{n}$ in the $(123)$ space. The supersymmetry condition for this general orientation of string can be written in a covariant form as,

$$i\hat{n} \cdot \sigma \Gamma_{jk} \frac{\epsilon_{\pm}}{2} = -\frac{i}{2} (\hat{n} \cdot \sigma) e_{ijk} \Gamma_{4..7} \frac{\epsilon_{\pm}}{2}, \quad (4.1)$$

where $\hat{n} = \frac{\vec{X}}{|X|}$ and $e_{ijk}$ is the Levi-Civita tensor. Now, any rotation of the string in the 3-dimensional space, spanned by $(1, 2, 3)$ directions, is associated with a rotation in the charge space so as to preserve the above condition. Then, any number of strings can be put together with appropriate orientation and charges. All of them together break one-eighth supersymmetry with the condition:

$$i\sigma_i \epsilon_{\pm} = \frac{1}{2} e_{ijk} \Gamma_{jk} \epsilon_{\pm}, \quad \text{and} \quad \Gamma_{4..7} \epsilon_{\pm} = \epsilon_{\pm}. \quad (4.2)$$

These networks can also be obtained from M-theory compactification on a non-trivial geometry along the line of [15].

Similarly, following an earlier work on the web structure of 5-branes in ten dimensions, we can also construct webs of eight-dimensional $U3$-branes. For this, one can take two directions of each of the 3-branes as common. The remaining direction can be aligned in exactly the same manner as the strings. This will again give a configuration with 1/8 supersymmetry. As the $U3$-branes intersect along a two dimensional surface, one should be able to write a three dimensional worldvolume theory describing such configurations. We however do not understand the full structure of this gauge theory at the moment.

We next discuss a web constructed of $U4$-branes in eight dimensions. The basic difference in this case with respect to the webs of strings and 3-brane is due to the fact that $U4$-branes carry both $SL(3)$ and $SL(2)$ charges. In this case, to construct a class of supersymmetric configuration, we write the supersymmetry condition for a 4-brane, when one of its edges is aligned in a three dimensional space, making an angle $\theta$ from $x^{3}$-axis and an angle $\phi$ from $x^{1}$-axis in the $x^{1} - x^{2}$ plane. Similarly, we choose another plane spanned by coordinates $x^{4} - x^{5}$. One more edge of the 4-brane lies in this plane making an angle $\alpha$ from the $x^{4}$ axis. The 4-branes are then left only with two common
directions. The supersymmetry condition of eqn. (3.12) then has a generalized form:

\[
\begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix} =
\begin{pmatrix}
\hat{n} \cdot \sigma & 0 \\
0 & \hat{n} \cdot \sigma
\end{pmatrix}
\begin{pmatrix}
0 & e^{i\alpha} \\
e^{-i\alpha} & 0
\end{pmatrix}
(G_1 \Gamma_2 \cos \theta
+
\Gamma_2 \Gamma_3 \sin \theta \cos \phi + \Gamma_3 \Gamma_1 \sin \theta \sin \phi)(\Gamma_5 \cos \alpha + \Gamma_4 \sin \alpha)
\begin{pmatrix}
\epsilon_+ \\
\epsilon_-
\end{pmatrix}.
\] (4.3)

To obtain projections conditions on the supersymmetry parameters for arbitrary angles, we parameterize the matrix \(\chi\) as in [9]. Then the invariant supersymmetry parameters can be shown to satisfy the independent conditions:

\[
\Gamma_{23} \epsilon_+ = i \sigma_3 \epsilon_+, \quad \Gamma_{23} \epsilon_+ = i \sigma_1 \epsilon_+, \quad (1 + i \Gamma_{45}) \epsilon_- = 0, \quad \Gamma_5 \epsilon_- = \epsilon_+.
\] (4.4)

Together they give rise to solutions preserving 1/16 supersymmetry. The geometric realization of similar configurations has been discussed earlier [18]. Once again, it will be interesting to construct the low energy gauge theory for these webs in the line of \((p,q)\) 5-branes [17].

The remaining U-brane in eight dimensions, namely \(U2\)-brane, is a singlet under \(SL(3)\) part of the \(U\)-duality group. One can therefore consider the networks of other \(U1(U3)\)-branes where the external branes end on parallel or orthogonal \(U2\)-branes. This configuration breaks the supersymmetry by further one-half and should correspond to non-trivial BPS states in the low energy world volume theory of the \(U2\)-branes. In eight dimensions we also have \(U0\)-branes. They can also possibly be used for obtaining more exotic constructions of webs. For example, they can form the vertices in a web consisting of strings and membranes, as they saturate the \(SL(3)\) charges of a string and \(SL(2)\) charges of membranes.

We have therefore shown the existence of non-perturbative configurations consisting of webs of 1, 2, 3 and 4-branes. We again emphasize that these are novel objects which do not exist in higher dimensions. For example, a \(U2\)-brane carries charges with respect to both \(A_{\mu\nu\rho}^+\) and \(A_{\mu\nu\rho}^-\). Such charged objects do not exist in the ten dimensional theory.

Let us now consider the situation in seven dimensions. In this case, we have \(U2\) and \(U3\)-branes with charges transforming as vectors and tensors of \(SL(5)\). The internal space now is five dimensional. By utilizing the full five dimensional structure of the charge space one can obtain configurations where supersymmetry is broken to 1/32. However one can restrict oneself to a subspace of the full five dimensional space and obtain BPS states with more supersymmetry. The \(U2\)-branes in seven dimensions can be dealt with in the same manner as the strings and 3-branes in eight dimensions, since their transformation properties are similar.

The case of \(U3\)-branes in seven dimensions is more interesting since they transform as a higher dimensional representation of the \(U\)-duality symmetry. We restrict ourselves to the one-half BPS \(U3\)-branes only. To obtain a web like structure out of them, we write down the supersymmetry condition when a 2-plane of the brane is oriented in a five
dimensional subspace of the seven dimensional space-time in special way. By choosing such an orientation the supersymmetry condition can be written in a form:

\[ \epsilon = \hat{\chi} \frac{\chi_{ij}}{|\chi| |\chi|} \Gamma_{ij} \epsilon. \]  

(4.5)

where \( \hat{\chi} \equiv \chi_{ij} \gamma^{ij} \) as in eqn.(3.17). Equation (4.5) is simply a generalization of eqn.(3.18) to the case when 3-branes have a special orientation in the five dimensional space. The first factor \( \frac{\chi_{ij}}{|\chi|} \) gives the orientation of the 3-brane in the internal space, whereas a factor \( \chi_{ij} \Gamma_{ij} \) gives its orientation in the spatial directions. The presence of the same parameters \( \chi_{ij} \) in both places implies the alignment of the 3-branes in these directions. In other words, the parameters \( \chi_{ij} \) specify the position of the 3-branes in the same manner that the angles \( \theta \) and \( \phi \) parameterized the position of a string in a three dimensional space.

Now, to construct a network structure consisting of these branes, we solve the conditions (4.5) by applying projections:

\[ \Gamma_{ij} \epsilon = \gamma^{ij} \epsilon. \]  

(4.6)

For a generic situation these are ten projection conditions on \( \epsilon \) and therefore break supersymmetry completely. However one can restrict to a subset when some of the moduli are set to zero and the 2-plane lies in a smaller subspace. One can then have supersymmetric solutions as well.

In six dimensions, to construct a string network, the simplest possibility is to choose the \( SO(5,5) \) moduli such that matrices \( \chi_1 \) and \( \chi_2 \) are identified in equation (3.29). Then the situation becomes identical to the one for the string network solution in seven dimensions with 1/32 or more supersymmetry. However the supersymmetry condition (3.29) in six dimensions may allow a more general possibility. This is because the left and right-moving modes of a string represent independent degrees of freedom of a Narain lattice and can point in different directions. The Gamma matrices in two eqns. (3.29) can then be different and may be used for having a web structure on a compact space. It is however not clear whether such BPS states are also possible in non-compact spaces. Other possibility in six dimensions is to consider U2-branes. The charges are now in spinor representation whereas the membrane orientation is represented by a vectorial direction. It is again not clear how to use an arbitrary number of these objects for constructing BPS states. However it should be possible to construct them by switching off certain moduli.

In five dimensions, the only possible object to be used are the strings. The charges now describe a 27-dimensional vector. A simple way to construct the BPS configuration once again will be to switch off a set of moduli, so that the 27 of \( USp(8) \) breaks into a representation of orthogonal groups such as \( SO(6) \times SO(2) \). One can then use the standard ways, outlined in several examples above, to construct the BPS configurations.
5 Discussion

To conclude, we have analyzed the supersymmetry properties of U-branes in various dimensions. We have shown that there exist supersymmetric configurations of $U$-branes, their intersections, as well as web structures carrying different $U$-duality charges in various dimensions. Each of them represent BPS states. In particular, we have presented new BPS configurations of string theory preserving $1/32$ or more supersymmetries.

It should be possible to construct some of these solutions explicitly from supergravity point of view [30]. This is certainly true for the U-branes and their orthogonal intersections. It seems to be technically non-trivial to obtain net or web type solutions from supergravity point of view. However, the geometry of the nets is obtainable explicitly from the consideration of the membranes and five branes of M theory [6].

It has been suggested that certain stable non-BPS states of string theory [12, 29] can be represented as web-like structures through certain moduli deformations. It will be of interest to generalize these for other $U$-branes. It will also be interesting to obtain the low energy effective theory of these $U$-branes and nets. In ten dimensions, when $\alpha' \to 0$ the branes decouple from the bulk supergravity and one can write down a low energy effective theory which is the SYM (DBI) in respective dimension. It is likely that a similar limit will exist in other cases as well. An approach in this direction may be the zero mode analysis of the branes and nets. This may lead to new phenomena in the realm of QFT’s which can not be obtained otherwise.

Finally, these $U$-branes and nets can possibly be related to compactifications on certain non-trivial manifolds. $D5$-branes of 8-dimensions has already been shown to play an essential role in compactification on $T^3$ fibered Calabi-Yau manifold [23]. Moreover, these $U$-brane junctions were also discussed in connection with M-theories on toric geometries. There, the network structures are expected to be associated with the locus of the vanishing cycles of the toric hypersurface [18]. In the ten dimensional case such a relation has already been argued for the 5-brane webs [17]. The extension of these explicit mappings to the cases of other networks should be interesting to study as well.

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A Appendix

To obtain the $U$-duality invariant conditions on supersymmetry parameters for different $U$-branes, we will start from the conditions for wrapped M2/M5 branes in eleven dimensions.

The condition on supersymmetry parameters for M2-branes (along $X^9 X^{10}$) and M-
branes (along $X^6...X^{10}$) in eleven dimensions are:

$$
\epsilon^{(11)} = \Gamma_{1,8}\epsilon^{(11)}, \quad \epsilon^{(11)} = \Gamma_{1,5}\epsilon^{(11)},
$$

(A.1)

where $\Gamma$’s and $\epsilon^{(11)}$ are the eleven dimensional gamma matrices and supersymmetry parameters respectively. The supersymmetry conditions in lower dimensions can then be obtained in the following way.

- **U-string in eight dimensions**:

  The eleven dimensional Gamma matrices can be written as

  \[
  \Gamma_\mu = \gamma_\mu \times 1, \quad \mu = 0, 1, \ldots, 7 \\
  \tilde{\gamma} = i\gamma_0\ldots\gamma_7
  \]

  (A.2)

  Where $\gamma_\mu$ are eight dimensional Gamma matrices and $\sigma_i$’s are Pauli spin matrices. A U-string along $X^7$ is an M2 brane along $X_7$ and in some other direction $X_7^+$. Using the above splitting, the condition (A.2) becomes

  \[
  \epsilon^{(8)}_\pm = i\gamma_{1,6}\sigma_i\epsilon^{(8)}_\pm
  \]

  (A.3)

  where $\epsilon^{(8)}$ are eight dimensional spinors and $\pm$ are the chirality indices. The condition (3.9) can then be obtained from (A.2) by making $SL(3)$ rotations mentioned earlier.

- **U0-brane in eight dimensions**

  A U0-brane can be thought of as a wrapped M2-brane on $X^{7+i}X^{7+j}$ plane. The corresponding supersymmetry condition will be

  \[
  \epsilon^{(8)}_+ = \gamma_{1,7}\sigma_i\epsilon^{(8)}_-
  \]

  (A.4)

  $SL(3)$ rotations will then generalize the $\sigma_i$ in (A.4) to $\chi$ in (3.12). Finally one also has to make $SL(2)$ transformations which lead to the $SL(3) \times SL(2)$ invariant relations in (3.11).

- **U0-branes in seven dimensions**

  In this case the eleven dimensional Gamma matrices can be written as

  \[
  \Gamma_\mu = \gamma_\mu \times \rho_5, \quad \mu = 0, 1, \ldots, 7 \\
  \rho_5 = \rho_1 \cdots \rho_4 \\
  \Gamma_{6+i} = 1 \times \rho_i, \quad i = 1, 2, 3, 4.
  \]

  (A.5)

  Where $\gamma_\mu$ and $\rho_i$ are Gamma matrices in seven and four dimensions respectively.
U0-brane in seven dimensions can be obtained by wrapping M2-brane on some plane $X^{6+i}X^{6+j}$. The supersymmetry condition (A.5) then becomes

$$\epsilon^{(7)} = \rho_{ij}\gamma_{1..6}\epsilon^{(7)},$$

(A.6)

where $\epsilon^{(7)}$ are seven dimensional spinors. Moreover, by making $SL(5)$ rotations, $\rho_{ij}$ can be generalized to $\chi_{ij}$ in condition (3.18), where $\chi^2 = -1$ and $\epsilon_{ijklm}X_{ij}X_{kl} = 0$. This represents $U$-duality invariant supersymmetry condition for one-half BPS U0-branes in seven dimensions. It can be generalized further to one-quarter BPS states.
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