Homoclinic bifurcations in low-Prandtl-number Rayleigh-Bénard convection with uniform rotation

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Abstract – We present results of direct numerical simulations on homoclinic gluing and ungluing bifurcations in a low-Prandtl-number \((0 \leq Pr \leq 0.025)\) Rayleigh-Bénard system rotating slowly and uniformly about a vertical axis. We have performed simulations with stress-free top and bottom boundaries for several values of the Taylor number \((5 \leq Ta \leq 50)\) near the instability onset. We observe a single homoclinic ungluing bifurcation, marked by the spontaneous breaking of a larger limit cycle into two limit cycles with the variation of the reduced Rayleigh number \(r\) for smaller values of \(Ta\) \((< 25)\). A pair of homoclinic bifurcations, instead of one bifurcation, is observed with the variation of \(r\) for slightly higher values of \(Ta\) \((25 \leq Ta \leq 50)\) in the same fluid dynamical system. The variation of the bifurcation threshold with \(Ta\) is also investigated. We have also constructed a low-dimensional model which qualitatively captures the dynamics of the system near the homoclinic bifurcations for low rotation rates. The model is used to study the unfolding of bifurcations and the variation of the homoclinic bifurcation threshold with \(Pr\).

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Nonlinear extended dissipative systems, driven away from equilibrium, reveal a wide range of bifurcations and pattern dynamics [1]. A homoclinic gluing [2] occurs when two limit cycles simultaneously become homoclinic orbits to a single saddle point. This leads to spontaneous merging of two limit cycles to a single limit cycle in the appropriate phase space, as the bifurcation parameter is raised above a critical value. The gluing bifurcation is observed in liquid crystals [3], in fluids [4], and in electronics circuits [5]. Paul et al. [6] and Pal et al. [7] showed recently the possibility of a homoclinic bifurcation in very-low-Prandtl-number thermal convection and investigated the pattern dynamics near the bifurcation point.

The Rayleigh-Bénard convection (RBC), where a horizontal layer of a fluid is subjected to an adverse temperature gradient, has been widely studied to investigate a plethora of patterns [8], bifurcations [9], route to chaos [10] and turbulence [11]. The low-Prandtl-number convection [12–14], which is relevant in the geophysical [15] and the astrophysical [16] context, shows interesting pattern dynamics [17] and a variety of bifurcations. The rotation about a vertical axis introduces the Coriolis force [18,19] as well as the centrifugal force [20] in the flow. They showed interesting effects of the centrifugal force, even if the ratio of the centrifugal force to the force of buoyancy is small \((\sim 10^{-2})\). The Coriolis force breaks the mirror symmetry of the patterns and exhibits several interesting patterns (e.g., [21]). However, the effect of the Coriolis force on homoclinic bifurcations in the RBC is yet to be explored.

We present in this article the results of direct numerical simulations based on the pseudo-spectral method on homoclinic bifurcations and pattern dynamics near the onset of the convection in the low-Prandtl-number rotating Rayleigh-Bénard convection. We observed a nonlocal periodic solution at the primary instability for all values of \(Ta\) \((\leq 50)\), which is different from the chaotic solution observed in the absence of rotation \((Ta = 0)\). The corresponding limit cycle breaks spontaneously into two limit cycles near a homoclinic point with an increasing value of the reduced Rayleigh number \(r\) for \(Ta < 25\). For \(25 \leq Ta \leq 50\), we observe homoclinic gluing as well as ungluing, which is qualitatively new for a fluid dynamical
system. The time period of oscillation diverges in the vicinity of the homoclinic bifurcation thresholds. We also present a simple model that captures the qualitative behaviour for small values of $\Delta r$. We then use the model to obtain the bifurcation diagram and study the Prandtl number dependence of the homoclinic threshold.

We consider a thin horizontal layer of Boussinesq fluid of thickness $d$, kinematic viscosity $\nu$, thermal expansion coefficient $\alpha$, thermal diffusivity $\kappa$, rotating with a uniform angular velocity $\Omega$ about a vertical axis, and subjected to an adverse temperature gradient $\beta$ in the vertical direction. The maximum value of the centrifugal force to the force of buoyancy, is $10^{-4}$ in our simulations. The centrifugal force is less than 0.5% of the Coriolis force. Therefore, we have ignored the effects of the centrifugal force. The hydrodynamics of the rotating Rayleigh-Bénard convection is governed by

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \text{Ro} \theta \mathbf{\lambda} + \nabla^2 \mathbf{v} + \sqrt{\text{Fr}} (\mathbf{v} \times \mathbf{\lambda}),$$

$$\text{Pr} [\partial_t \theta + (\mathbf{v} \cdot \nabla) \theta] = \nabla^2 \theta + v_3,$$

$$\nabla \cdot \mathbf{v} = 0,$$

where $\mathbf{v}(x, y, z, t) \equiv (v_1, v_2, v_3)$, $\theta(x, y, z, t)$ and $p(x, y, z, t)$ are, respectively, the flow velocity, the convective temperature field and the convective pressure field. The unit vector $\mathbf{\lambda}$ is directed against the direction of gravity $g$. All the length scales are made dimensionless by the fluid thickness $d$, the time scale by the viscous diffusive time $d^2/\nu$, and the temperature field by $\nu \beta d/\alpha$ in the above equations. The dimensionless numbers are: i) the Rayleigh number $\text{Ra} = (\alpha \beta d^4) / (\nu \kappa)$, ii) the Taylor number $\text{Ta} = 4 \Omega^2 d^4 / \nu^2$, and iii) the Prandtl number $\text{Pr} = \nu / \kappa$.

We consider thermally conducting and stress-free bounding surfaces. This leads to the boundary conditions $\theta = \partial_x v_1 = \partial_x v_2 = v_3 = 0$ on the boundaries located at $z = 0$ and 1. All the convective fields are assumed to be periodic in the horizontal plane. The direct numerical simulations (DNS) of the hydrodynamic system (eqs. (1)–(3)) with the boundary conditions mentioned above are carried out using an open-source code TARANG [22] based on pseudo-spectral method. All the convective fields are expanded as

$$\Psi(x, y, z, t) = \sum_{l, m, n} \Psi_{lmn}(t) e^{ik(lx + my)} f_n^{(1)}(z),$$

$$\Phi(x, y, z, t) = \sum_{l, m, n} \Phi_{lmn}(t) e^{ik(lx + my)} f_n^{(2)}(z),$$

where $\Psi(x, y, z, t) \equiv (v_3, \theta)^T$, $f_n^{(1)}(z) = \sin(n \pi z)$ and $\Phi(x, y, z, t) \equiv (v_1, v_2)^T$, $f_n^{(2)}(z) = \cos(n \pi z)$. We consider a periodic simulation box of size $2 \pi / k : 2 \pi / k : 1$. The confinement effects due to the side walls are neglected here. The value of $k$ is taken as the critical wave number $k_c(Ta)$ predicted by the linear theory [18]. The spatial grid resolution of $64 \times 64 \times 64$ is used for DNS, which is quite good to resolve the flow structure near onset. The time advancement is done using the standard fourth-order Runge-Kutta (RK4) integration scheme. The time step $\Delta t$ used for the integration varies in the range $0.0005 \leq \Delta t \leq 0.001$. We started our simulations with random initial conditions by fixing the values of the Prandtl number $\text{Pr}$ and the Taylor number $\text{Ta}$. The reduced Rayleigh number $r = \text{Ra}/\text{Ra}_c(Ta)$, where $\text{Ra}_c(Ta)$ is the critical Rayleigh number predicted by the linear theory of Chandrasekhar [18], is increased in small steps $\Delta r (0.001 \leq \Delta r \leq 0.01)$. The reduced Rayleigh number $r$ has been varied in the range $1 \leq r \leq 1.25$ for all values of $\text{Ta}$ considered here. The values of all the fields at the final time step have also been used as the initial condition for the next run at a slightly higher value of $r$. We have performed our runs for different values of Taylor number ($2 \leq \text{Ta} \leq 50$) for various values of the Prandtl number ($0 \leq \text{Pr} \leq 0.04$). We have also performed runs by starting with a higher value of $r$ and then by decreasing its value in small steps. We have not observed any hysteresis in the range of $r$ considered here. We present here extensive results for $\text{Pr} = 0.025$ and $\text{Pr} = 0$. In the limit $\text{Pr} \to 0$, eq. (2) is slaved to the vertical velocity, i.e., $\nabla^2 \theta = -v_3$.

The various flow patterns observed in DNS for different values of the Prandtl number $\text{Pr}$, the Taylor number $\text{Ta}$ and the reduced Rayleigh number $r$ are listed in table 1. The onset of the convection is always a self-tuned oscillation (STO) [14] of two mutually perpendicular sets of rolls. Figure 1(a) displays the temporal variation of the leading Fourier modes ($W_{101}$ and $W_{011}$) at low rotation rate ($\text{Ta} = 10$) for $\text{Pr} = 0.025$ just at the onset. A new set of parallel rolls is self-tuned when the amplitude of the older set of rolls grows close to its maximum value. The older set of rolls disappears with the growth of the new set of rolls. The competition of mutually perpendicular self-tuned rolls is periodic, as opposed to the chaotic dynamics in the absence of the Coriolis force ($\text{Ta} = 0$). This represents a global limit cycle. As the reduced Rayleigh number $r$ is increased, the temporal variation of the leading Fourier modes becomes chaotic. Two sets of mutually perpendicular rolls are self-tuned chaotically (STC).

With further increase in $r$, the minima of the leading modes become non-zero. The corresponding convective patterns are then oscillating cross-rolls (OCR). Depending upon the values of $\text{Pr}$ and $\text{Ta}$, two types of OCR solutions are observed. In one case, we observe the identical temporal variation of the modes $W_{101}$ and $W_{011}$ but with a constant phase difference (see fig. 1(b)). The constant phase difference makes $W_{101} \neq W_{011}$ for most of the time. The corresponding patterns are called here as OCR-I. The other types of solutions with dissimilar temporal variation of the modes $W_{101}$ and $W_{011}$ (see fig 1(c)) are called OCR-II and OCR-III. The lower row of fig. 1 shows the global limit cycle at the onset (fig. 1(d)), the same just before the homoclinic bifurcation (fig. 1(e)) and
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Table 1: Convective flow patterns observed in DNS: self-tuned oscillation (STO) of two sets of mutually perpendicular rolls, self-tuned chaotic patterns (STC), oscillating cross-rolls (OCR-I) with \( |W_{101}| = |W_{011}| \), oscillating cross-rolls (OCR-II) with \( |W_{101}|_{\text{max}} \neq |W_{011}|_{\text{max}} \), stationary cross-rolls (CR) with \( W_{101} \neq W_{011} \) and stationary squares (SQ) with \( W_{101} = W_{011} \).

| Convective patterns | \( Pr = 0 \) | \( Pr = 0.025 \) |
|---------------------|---------------|-----------------|
|                     | \( r (Ta = 10) \) | \( r (Ta = 40) \) | \( r (Ta = 10) \) | \( r (Ta = 25) \) |
| STO                 | 1.001–1.018   | 1.001–1.071     | 1.001–1.02   | 1.001–1.021 |
| STC                 | 1.019–1.088   | 1.072–1.116     | 1.021–1.034  | 1.022–1.037 |
| OCR-II              | 1.019–1.088   | 1.117–1.127     | 1.038–1.139  | 1.140–1.175 |
| OCR-I               | 1.089–1.120   | 1.128–1.154     | 1.035–1.148  | 1.140–1.175 |
| OCR-II              | 1.121–1.137   | 1.155–1.163     | 1.149–1.168  | 1.176–1.186 |
| CR                  | 1.138–1.187   | 1.164–1.183     | 1.170–1.210  | 1.187–1.209 |
| SQ                  | \( \leq 1.189 \) | \( \leq 1.184 \) | \( \leq 1.211 \) | \( \leq 1.210 \) |

Fig. 1: (Color online) The temporal variation of Fourier modes \( W_{101} \) (pink (gray) solid lines) and \( W_{011} \) (blue (black) dotted lines) at (a) \( r = 1.001 \), (b) \( r = 1.147 \), and (c) \( r = 1.149 \) for \( Ta = 10 \) at \( Pr = 0.025 \) computed from DNS. The onset is a non-local solution with a periodic competition between two mutually perpendicular set of self-tuned oscillatory rolls (STO). The corresponding phase portraits in the \( W_{101}-W_{011} \) plane are shown in (d), (e), and (f), respectively.

Fig. 2: (Color online) Homoclinic gluing and ungluing bifurcations, as computed from DNS. The first row shows the phase portraits for \( Pr = 0 \) and \( Ta = 40 \) for (a) \( r = 1.001 \), (b) \( r = 1.127 \), (c) \( r = 1.128 \) and (d) \( r = 1.155 \). The second row shows the phase portraits for \( Pr = 0.025 \) and \( Ta = 25 \) for (e) \( r = 1.001 \), (f) \( r = 1.138 \), (g) \( r = 1.139 \) and (h) \( r = 1.176 \). Single glued limit cycle is in blue (black) and two unglued limit cycles are in blue (black) and pink (gray).

The primary instability is in the form of a global limit cycle describing a periodic competition between two sets of self-tuned rolls (fig. 2(e)). This changes smoothly to a periodic competition of two sets of cross-rolls (OCR-I, fig. 2(f)), as \( r \) is raised. A further increase in \( r \) leads to homoclinic gluing (fig. 2(g)). The glued solution again breaks into two smaller limit cycles, as \( r \) is raised further (fig. 2(h)). The possibility of both homoclinic ungluing and gluing in the same fluid dynamical system near the primary instability is a new phenomenon.

The variation of the homoclinic thresholds \( r_{ug} \) for ungluing and for glue \( r_g \) with the Taylor number \( Ta \) for different values of \( Pr \) is plotted in fig. 3. The homoclinic ungluing threshold \( r_{ug} \), where a global limit cycle (OCR-I) breaks into two smaller limit cycles (OCR-II), increases with \( Ta \). The best fit shows a power law dependence on \( Ta \) (fig. 3(a)). The ungluing threshold \( r_{ug} \) scales with \( Ta \) as \( Ta^{\gamma} \). The exponent \( \gamma \) depends on the Prandtl number \( Pr \).
Fig. 3: (Color online) Variation of homoclinic bifurcation thresholds with Taylor number $Ta$, as obtained from DNS. The variation of homoclinic ungluing threshold $r_{ug}$ with $Ta$ for three values of the Prandtl number ($Pr = 0.01$, $0.025$) are plotted in (a). The points marked by “$\circ$”, “$\Delta$” and “$\ast$” represent the data for $Pr = 0$, $0.01$ and $0.025$, respectively. Variations of homoclinic gluing threshold $r_g$ with $Ta$ for $Pr = 0$ and $0.025$ are plotted in (b). Both $r_{ug}$ and $r_g$ show power law dependence on $Ta$.

The exponent $\gamma = 0.022$ for $Pr = 0$, $0.025$ for $Pr = 0.01$, and $0.029$ for $Pr = 0.025$. The gluing threshold $r_g$ also scales with $Ta$ as $Ta^{\beta}$. The value of $\beta$ also depends on $Pr$. Its value is $0.067$ for $Pr = 0$ and is $0.091$ for $Pr = 0.025$, respectively. Two exponents are quite different. The range of $Ta$ for which homoclinic gluing and/or ungluing can be observed also depends on $Pr$.

Direct numerical simulations are enormously time consuming. We therefore construct a model to capture all the essential features observed in DNS. We construct a smaller model using more energetic modes observed in the simulations. We expand the convective fields compatible with the boundary conditions as

$$\Phi = \sum_{l,m,n} \Phi_{lmn}(t) \cos(lk_c x) \cos(mk_y y)$$

$$+ \Phi_{lmn}(t) \sin(lk_c x) \sin(mk_y y) \tilde{f}_n(z),$$

where $f_n(z) = f_n^{(1)}(z) = \sin(n \pi z)$ in the expansion of the vertical velocity $v_3$ and the convective temperature field $\theta$, while $f(z) = f_n^{(2)}(z) = \cos(n \pi z)$ in the expansion of the vertical vorticity $\omega_3 = (\nabla \times \nu) \cdot \lambda$. We choose the following eight real modes for the vertical velocity ($v_3$): $W_{101}, W_{011}, W_{112}, W_{111}, W_{121}, W_{121};$ twelve real modes for the vertical vorticity ($\omega_3$): $\tilde{Z}_{101}, \tilde{Z}_{011}, \tilde{Z}_{112}, \tilde{Z}_{111}, \tilde{Z}_{121}, \tilde{Z}_{121}, \tilde{Z}_{121}, \tilde{Z}_{110}, \tilde{Z}_{200},$ and $\tilde{Z}_{220};$ and nine real modes for the temperature field $\theta$: $\tilde{\theta}_{101}, \tilde{\theta}_{011}, \tilde{\theta}_{112}, \tilde{\theta}_{112}, \tilde{\theta}_{211}, \tilde{\theta}_{211}, \tilde{\theta}_{121}, \tilde{\theta}_{121}, \tilde{\theta}_{002}$. We then project the hydrodynamic equations (eqs. (1) and (2)) on these modes to obtain a twenty-nine–mode model.

We now compare results of the model with those obtained from DNS. Figure 4 compares the temporal behaviour in the vicinity of the homoclinic bifurcations in a fluid of $Pr = 0.025$ at $Ta = 25$. The temporal variations of Fourier modes $W_{101}$ (pink (gray) solid lines) and $W_{011}$ (blue (black) dotted lines) computed from DNS just before and after the first homoclinic bifurcation (gluing) and just after the second homoclinic bifurcation (ungluing) are displayed in the upper row. The lower row shows results similar to those obtained from the model. They are in good qualitative agreement. Figure 5 compares the variation of the time period of oscillatory solutions with $r$ for $Pr = 0.025$ obtained from DNS (upper row) and the model (the lower row). The time period of the limit cycle solution obtained from DNS ($a$) and the model ($c$) shows a divergence near the homoclinic ungluing for $Ta = 10$. For a slightly larger value of $Ta$ a pair of homoclinic bifurcations is observed. Figure 5(b) and (d) compares the variation of the time period of oscillatory solutions near homoclinic bifurcations for $Ta = 25$. In this case we have a pair of homoclinic points in the same fluid dynamical system. This is possible due to the presence of rotation. The results obtained from DNS (fig. 5(b)) and that computed from the model (5(d)) compare well.

We now use the model to study the sequence of bifurcations in a rotating Rayleigh-Bénard system. The bifurcation diagram for $Pr = 0.025$ at $Ta = 10$ obtained from the model is plotted in fig. 6(a). The model captures the whole sequence of bifurcations including homoclinic gluing observed in DNS (see table 1) quite accurately. For $Ta = 25$, the bifurcation diagram obtained from the model is displayed in fig. 6(b). The onset of the convection appears as self-tuned oscillations of rolls parallel to either the $x$- or the $y$-axis in this case. Two limit cycles representing two oscillations merge into one as $r$ is raised above a critical value $r_g$. For this case we find the spontaneous breaking of the merged (glued) limit cycle again into two
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Fig. 5: (Color online) Comparison of the variation of the dimensionless time period ($\tau$) of limit cycle(s) with $r$ as computed from DNS (upper row) and from the model (lower row) near homoclinic bifurcations for $Pr = 0.025$: $Ta = 10$ (the first column) and $Ta = 25$ (the second column). The data for the first set of two unglued limit cycles (OCR-II'), glued limit cycles (OCR-I), and the second set of unglued limit cycles (OCR-II) are denoted by points marked as “x”, “+”, and “v”, respectively. The data from DNS and model are in good qualitative agreement.

Fig. 6: (Color online) Bifurcation diagrams for $Pr = 0.025$ for (a) $Ta = 10$ and (b) $Ta = 25$, as obtained from the model. The bifurcation diagram for $Ta = 10$ shows a series of patterns: self-tuned oscillations (STO), glued oscillation (OCR-I), unglued oscillations (OCR-II), cross-roll (CR) fixed points, and square (SQ) fixed points. For $Ta = 25$, the bifurcation diagram captures two sets of unglued oscillatory solutions (OCR-II' and OCR-II) instead of one set of unglued solutions as obtained in case (a). The homoclinic thresholds $r_g$ (black square) and $r_{ug}$ (black circle) represent transition points from OCR-II' $\rightarrow$ OCR-I and OCR-I $\rightarrow$ OCR-II, respectively.

Fig. 7: (Color online) Variation of the threshold (a) $r_{ug}$ for homoclinic ungluing and (b) $r_g$ for homoclinic gluing with Prandtl number $Pr$ for three different values of the Taylor number, as obtained from the model.

limit cycles, as $r$ is raised above another threshold $r_{ug}$ for homoclinic ungluing. The sequence of bifurcations is in good qualitative agreement with the results obtained from DNS. The model captures well both homoclinic ungluing and gluing, as found in DNS. The model, however, does not show self-tuned chaos in a very narrow range above the primary instability, as observed in DNS.

The model also predicts two types of fixed-point solutions at relatively higher values of $r$: stationary squares (SQ, with $|W_{101}| = |W_{011}|$) and stationary cross-rolls (CR, $|W_{101}| \neq |W_{011}|$) in the bifurcation diagram. The possibility of SQ patterns is displayed by solid black lines, while CR patterns are marked by the green (gray) curves in the bifurcation diagrams. The stable and unstable fixed-point solutions are represented by the solid and dashed lines, respectively. The eigenvalues obtained from the linear stability analysis for low-Prandtl-number fluids ($Pr \leq 0.025$) at small rotation rates show that all unstable fixed points are saddle fixed points near the instability onset. SQ saddle points have fourfold symmetry and CR are symmetrically placed about the diagonal in the $W_{101}$-$W_{110}$ plane. A glued limit cycle oscillates about all the saddle points in each quadrant of the $W_{101}$-$W_{110}$ plane. There is one such limit cycle in each quadrant. They never glue together in the presence of finite rotation, unlike in the case of without rotation [6]. We never observe chaotic solutions at the instability onset for small rotation rates ($Ta \leq 50$). The amplitude equations [7] in the normal form in the absence of rotation are also derived recently. The symmetry under $W_{101} \leftrightarrow W_{011}$ leads to the same limit cycle, which is a degenerate solution. This limit cycle breaks into two smaller limit cycles only when it comes asymptotically close to a SQ saddle point. The symmetry under $W_{101} \leftrightarrow W_{011}$ now leads to two different limit cycles, each of which oscillates locally around a CR saddle point. The degeneracy is lifted. The time period of the limit cycle solutions diverges when the limit cycle almost touches the SQ saddle point (fig. 6). Thus, the model confirms the cascade of homoclinic gluing and ungluing in the low-Prandtl-number Rayleigh-Bénard convection with uniform rotation and captures the essential behaviour at low rotation rates. A cascade of homoclinic gluing is known to occur in the Taylor-Couette flow [24].

The variations of the threshold $r_g$ for homoclinic gluing and $r_{ug}$ for ungluing as a function of the Prandtl number $Pr$, as computed from the model for different rotation rates are plotted in fig. 7. The threshold $r_{ug}$ for the homoclinic ungluing varies linearly with $Pr$ (fig. 7(a)). The homoclinic ungluing of a larger limit cycle into two smaller
ones is observed in low-Prandtl-number fluids at small rotation rates. The threshold $r_{\text{th}}$ increases with increasing rotation rates. However, the possibility of homoclinic gluing once again occurs at slightly higher rotation rates ($25 \leq \alpha \leq 50$). The threshold $r_g$ for homoclinic gluing (fig. 7(b)) also varies linearly with $Pr$.

We have investigated the possibility of homoclinic bifurcations in the rotating Rayleigh-Bénard convection in low-Prandtl-number fluids with stress-free and thermally conducting boundary conditions. The convection appears as a periodic competition of two sets of mutually perpendicular self-tuned rolls at the instability onset in the presence of even small rotation rates, as opposed to the chaotic behaviour observed in the absence of rotation. The periodic competition representing a larger limit cycle undergoes homoclinic bifurcation and breaks into two smaller limit cycles. The smaller limit cycles may merge together depending on the Prandtl number $Pr$ and the Taylor number $Ta$. The homoclinic gluing and ungluing are associated with the divergence of the time period of oscillatory solutions. The divergence of the time period shows power law behaviour near homoclinic bifurcations. The scaling is asymmetric on two sides of a homoclinic bifurcation point. The periodic solutions undergo inverse Hopf bifurcation to stationary cross-rolls and then to stationary squares via inverse pitchfork, as the reduced Rayleigh number $r$ is raised slowly at a fixed value of $Ta$. We have also constructed a simple model that captures quite well the effect of the Coriolis force on homoclinic bifurcations.

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