A Numerical Investigation of the Geometric Characteristics of Floating Wind Turbine Wakes under Axial and Yawed Rotor Conditions

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Abstract. The physics of wind turbine wakes on fixed foundations have been extensively studied through numerical modelling and measurements over the recent decades. While research initiatives in the field of floating wind turbines have increased significantly, various aspects characterising floating turbine wakes are still not well understood. This paper applies a free-wake vortex model to derive the geometric characteristics of the wake of a surging rotor under axial and yawed rotor conditions. The study examines the influence of platform motion on the wake boundaries and centreline up to four rotor diameters downstream. It is shown that floating turbine wakes experience a higher degree of perturbation as compared to those of fixed turbines. This is highly dependent on the sea state and suggests the need to develop new analytical wake models for the wind industry that are able to account the effect of platform motion on wake behaviour.

1. Introduction
Significant progress has been achieved by the wind energy industry in the development of floating offshore wind turbine (FOWT) technologies to exploit the vast global wind resources available in deep seas. Costs of floating farms are predicted to decline, with FOWTs expected to become competitive with water offshore wind turbines in shallow water before 2030 [1]. Understanding the aerodynamics of floating turbine wakes will remain essential role ensure the most economical solutions are brought to the deep offshore wind market. The aerodynamics of wind turbine wakes on fixed foundations has been investigated in depth for both axial and misaligned (yawed) rotors [2, 3]. While the underlying physics of fixed and floating rotors are very similar, the low stiffness of floating platforms causes rotors to exhibit differences in the aerodynamics behavior that would impact farm performance. For example, studies have shown that the cyclic component of the rotor thrust coefficient $C_T$ of a FOWT may reach a substantial proportion of the time-averaged value (as much as 20%) under specific sea states [4, 5, 6]. This impacts the evolution of the wake downstream of the rotor and suggests the need for further research to evaluate the suitability of existing analytical wake models, that have been based on the assumption of a steady $C_T$, for array design [7, 8] or wake steer control [9, 10] applications of floating wind farms. Having better understanding of the geometric characteristics of FOWT wakes is one of the most essential research needs to develop improved wake models specifically for floating farms.
2. Objectives
The main objective of this paper is to compare, through numerical simulations, the geometric characteristics of the wake of a surging FOWT with that of a fixed turbine under axial and yawed rotor operating states. The study focuses on the effect of the FOWT platform surge motion on the outer boundaries and centreline of the helical wake up to 4D downstream of the rotor.

3. Numerical Model
The numerical simulation code used in this study is the open-source Wake Induced Dynamic Simulator (WInDS) developed by Sebastian [11]. The code is a free-wake vortex model (FWVM) designed specifically to model floating rotor aerodynamics, with platform motion prescribed to the code as a function of time. The blades are modelled as lifting lines with the application of the Kutta-Joukowski theorem, thus requiring the input of airfoil lift and drag coefficient data to be able to compute the aerodynamic loads. A cosine distribution of radial elements is applied at the blade tip and root regions to model the steep radial variation of bound circulation in these regions with a higher degree of numerical accuracy. The free-wake is represented through helical vorticity sheets consisting of a mesh of straight-line vortex filaments to model trailing and shed circulation, with one vorticity sheet emerging from the trailing edge of each blade. Viscous effects in the wake are implemented through the addition of a viscous parameter to the Biot Savart equation as documented in [11, 12]. The viscous parameter is based on the Lamb-Oseen vortex core model to model the velocity distribution around the vortex filaments and the Ramasamy-Leishman model to account for the growth of each individual wake filament core with time as applied by Farrugia et al [13].

A time marching algorithm is used to generate the free-wake as a function of time, amending the position of the nodes defining the wake mesh nodes after each rotor time step. Different numerical integration techniques are embedded in WInDS. A Predictor-Corrector with Central-difference (PCC) time integration approach was selected due to its favourable numerical stability [11]. The approach entails three steps for updating the position vector \( x \) of the wake filament nodes at time \( t \) for a resultant velocity vector \( u \):

1. Apply the forward Euler method as a predictor:
\[
x_{t+\Delta t} = x_t + u_t \Delta t
\]

2. Compute the resultant velocities at wake nodes following application of Biot-Savart law:
\[
u_{t+\Delta t} = f(x_{t+\Delta t})
\]

3. Apply correction to prediction:
\[
x_{t+\Delta t} = x_t + \frac{\Delta t}{2} (u_{t+\Delta t} + u_t)
\]

4. Methodology
The NREL\(^1\) 5MW baseline wind turbine rotor model defined by Jonkman et al [14] was implemented in the FWVM code. The wind speed and rotor tip speed ratio were fixed at 11.4 m/s and 7, the latter resulting in optimal efficiency conditions. The presence of wind shear and turbulence were ignored. 2D static airfoil data corrected for 3D effects documented by Jonkman [14] were adopted. The FWVM code was used to generate the free wakes similar to those shown in Figure 1 for four rotor yaw angles (\( \gamma = 0, 15, 30, 45 \) degrees) and three regular wave sea states. The three modelled sea states are summarized in Table 1 below, where \( H \) and \( T_s \) denote the wave height and period, respectively. \( S3 \) represents the most extreme sea condition. \( S0 \) denotes the non-floating condition in which the rotor support structure is assumed to be rigidly fixed. In total 16 different free wake computations were carried out. The surge amplitude (\( A \)) at each sea state was estimated using the NREL code FAST for a 5 MW turbine installed

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on a semi-sub platform [15]. Parameters $A$ and $T_w$ were prescribed to the FWVM code as a sinusoidal function explained in equation below:

$$X = A \sin \left(2\pi \frac{1}{T_w}\right)$$

| Sea State | $H$ (m) | $A$ (m) | $T_w$ (s) | $f$ (Hz) |
|-----------|---------|---------|---------|---------|
| S0        | 0       | 0       | 0       | 0       |
| S1        | 3.66    | 0.75    | 9.50    | 0.10526 |
| S2        | 6.40    | 1.85    | 11.65   | 0.08584 |
| S3        | 9.14    | 3.31    | 13.60   | 0.07353 |

The remaining platform degrees of freedom were fixed. In view of the fact that the present study was specifically intended to study the impact of surge motion on the evolution of the aerodynamic near wake, the same surge motion corresponding to each sea state was applied to each of the four rotor yaw angle settings. The simulation time was selected to ensure that the free wake extended by not less than 8D downstream. In case of floating conditions, this implied at least 6 surge oscillations of the rotor. This was sufficient for the numerical time marching solution to stabilize with time.

A Python code was developed to post process the free wake geometric data to derive the (1) external wake boundary, (2) wake center and (3) wake skew angle. The following procedure was applied for each of the generated free wakes:

- **Step 1:** an algorithm was implemented to track the coordinates of the outermost six trailing vortex filament paths up to 4D downstream (refer to Figure 2). The wake boundary was taken to be the average trajectory of such filament paths.

- **Step 2:** The coordinates of the wake boundaries along a horizontal plane were then selected. Horizontal lines passing through the rotor axis and perpendicular to the rotor plane were plotted and the coordinates of the two points of intersection at the wake boundaries were determined, as explained in Figure 3. The midpoint location of each horizontal lines was identified. This was taken to be the wake centre for a given position downstream of the rotor plane.

- **Step 3:** The midpoint locations identified in Step 2 representing the wake axis were plotted as shown in Figure 4. A linear regression line based on the least squares method was then plotted. Although various studies involving CFD and measurements on fixed (non-floating) rotors have established that the wake centreline (or trajectory) for yawed turbines is non-linear [16, 17, 18], the centreline may be reasonably assumed to be linear up to 4D downstream from the rotor.

![Figure 1: Free-wake generated by FWVM code for $\gamma=30^\circ$ and sea state S0](image-url)
Using the fitted regression line, the two following parameters were computed:

a. the standard error of the linear regression, which was taken to represent the degree of perturbation experienced by the wake

b. the wake skew angle ($\chi$), defined as the angle between the regression line and the rotor axis (refer to Figure 3)

Figure 2: Vortex filament trail paths for $\gamma = 15^0$ and $\lambda = 7$, sea state $S0$

Figure 3: Derivation of wake centerline and skew wake angle ($\chi$)
Figure 4: Linear regression to derive best line fit for wake centerline ($\gamma = 45^0$ and sea state S3). The individual data points (+) represent the wake centre at a given position downstream from the rotor.

5. Results and Discussion

Figures 5 and 6 present the wake boundaries along the horizontal plane passing through the rotor axis for axial and yawed ($\gamma = 30^0$) rotor conditions. The centerline of the wake is also shown. Similar plots for $\gamma = 15^0$ and $45^0$ were also generated. It is observed that the FWVM code predicts perturbations in the wake boundaries for all modelled conditions, including those involving a fixed (non-floating) rotor that is aligned with the wind. No wake perturbation is noted in the close vicinity of the rotor (< 1D) where the wake is known to be dominated by the strong and stable helical tip vortices. The wake expansion within this region for a given yaw angle was also noted to be uninfluenced by the sea state. As the wake convects downstream, the vortex filaments come close to one another. The vortex-to-vortex interaction causes the filaments to become unstable, triggering wake perturbations. In the present FWVM code, the interaction is governed by the viscous core models being coupled to the Biot-Savart equation, as mentioned earlier in Section 3. In reality the flow phenomena involving vortex-to-vortex interaction, which contributes to tip vortex instability and eventual breakdown of the helical wake structure as the latter convects downstream, involves complex flow processes that demand the use of more comprehensive models (such as direct numerical simulation (DNS) using the Navier–Stokes equations). Such processes have been studied in detail for a fixed rotor by Lignarolo et al [19], but similar studies targeting FOWTs are lacking in open literature. Yet the present free-wake vortex model provides important insight about the possible influence of the sea state on the degree of the perturbation experienced by a FOWT wake. It is observed how wake boundaries of surging turbines encounter perturbations at an early stage, closer to the rotor disc, when compared to those of fixed turbines. Furthermore, the degree of perturbation is significantly influenced by the sea state under both axial and yawed conditions, as is evident from Figures 5 and 6.

Figure 7 shows the centerline trajectory for fixed conditions and different yaw angles, as derived using the approach explained in section 2 and Figure 3. The lateral centerline position is expressed in terms of rotor diameter $D$ and with respect to an axis that is aligned with the undisturbed wind direction and passing through the rotor vertical yaw axis. The non-linearity in the centerline is a result of the wake perturbations as simulated by the FWVM code. Two important observations can be noted for the rotor and test conditions being modelled: firstly, while rotor yawing is well-known to induce unsteady aerodynamics in the near wake as a result from a time-varying angle of attack at the blade elements, the amplitude of perturbation in the wake centerline is observed to decrease with yaw angle. Secondly, yawed conditions delay the onset of the wake perturbation to a position further downstream of the rotor.
Figure 5: Wake boundaries and centerline for $\Psi = 0$ deg and $\lambda = 7$

disc. This may be observed through a comparison of the plots for $\gamma = 0^0$ and $45^0$ shown in Figure 6. Such behaviour may be the result of lower values for the thrust coefficient $C_T$ at larger yaw angles ($C_T = 0.74$ at $\gamma = 0^0$ and $C_T = 0.52$ for $\gamma = 45^0$). Similar observations could be noted for the FWVM results for the floating conditions (Figure 8), however the perturbations in the wake centreline are significantly increased as a result of the sinusoidal surge motion of the platform. Furthermore, platform surge causes the wake to encounter perturbation at an earlier stage in a floating rotor as compared to a fixed rotor. For example, while the wake for an unyawed rotor ($\gamma = 0^0$) on a fixed base experiences the first perturbations at 2D (Figure 7), in the case of floating conditions, these are being predicted to occur from 1 D (Figure 8(a)).

Figure 9 shows the standard error for the wake centreline derived from results plotted in Figure 7 and 8 using the linear regression analysis explained in Section 3 (refer to Figures 2 and 3). As noted in Section 3, only the centerline data up to 4D was considered. Higher values for the standard error imply a higher degree of perturbation encountered by the wake. As may be noted from Figure 8, the standard error for fixed conditions is minimal, in the range of $0.016 – 0.021$D ($2.1 – 2.6$ m). Platform motion leads to larger values for the standard error, with the largest value being encountered in sea state $S_3$, correspond-
Figure 6: Wake boundaries and centerline for $\Psi = 30$ deg and $\lambda = 7$

Figure 7: Wake centerline for fixed (non-floating) conditions.
Figure 8: Wake centerline for floating conditions

- ing to the most extreme sea state (Table 1). However increased yaw angles result in lower standard error values, despite higher levels of unsteady aerodynamics induced in the wake as a result of misalignment of the rotor with the wind. As in the case for fixed rotors, this is likely due to the smaller time-average $C_T$ values encountered at the large yaw angles, as may be noted in Figure 10(a). Even though the ratio of the amplitude in $C_T$ to the corresponding time average value increases with yaw angle for each sea state (Figure 10(b)), the maximum amplitude in $C_T$ reached over the 16 simulations (equal to 0.043 at $\gamma = 45^0$) was only 5.9% of the maximum time-averaged $C_T$ (obtained at $\gamma = 0^0$).

Figure 11 plots the variation of the skewed wake angle ($\chi$) with the rotor yaw angle ($\gamma$). As explained earlier in Section 3, the skewed wake angle was derived based on the best line fit for a given yaw angle and sea state. $\chi$ is shown to be larger than $\gamma$ under yawed flow, similar to what is experienced in rotors on fixed foundations. Early studies for fixed rotors [20, 21] have shown that the wake skew angle at a given yaw angle exhibits a quasi linear variation with $C_T$. In the present study, the variation of $\chi$ with $\gamma$ as estimated by the FWVM code was found to be nearly identical at all sea states. This is due to the
Figure 9: Standard error in wake centreline (vertical axis denotes standard error in terms of rotor diameter, $\lambda = 7$)

(a) Time-averaged $C_T$  
(b) Ratio of amplitude to time-averaged $C_T$

Figure 10: Variation of the time-average (fig. a) and amplitude-to-mean thrust coefficient ratio (fig. b) with yaw angle and sea state ($\lambda = 7$)

Figure 11: Variation of wake skew angle ($\chi$) with rotor yaw angle ($\psi$) for different sea states
fact that the time-average $C_T$ at each yaw angle remained nearly unchanged for all sea states, as shown in Figure 10(a).

6. Conclusion
This study has demonstrated, through numerical modelling with a free-wake vortex model, how a FOWT wake experiences a higher degree of perturbation than that of a fixed wind turbine. Such a behavior is significantly dependent on the sea state and is encountered under both axial and yawed rotor conditions. Yet it could be concluded that the turbine yawing may in effect reduce the wake centerline perturbation induced by platform motion. This is possibly a result of the lower time averaged axial thrust coefficient exerted on the rotor as a result of rotor yaw. Although this study has only been limited to the near wake up to 4D and one rotor tip speed ratio, it provides important insight about the impact of platform motion on the wake behavior. The study suggests the need for further research to develop better analytical wake models specifically for floating wind farms that are able to cater for the effect of sea state on the wake behavior. Furthermore, more analysis is required using the more advanced CFD models and wind tunnel measurements on model turbines to capture the viscous effects in the wake evolution process of a surging rotor in a more comprehensive manner.

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