A new duplex Stirling engine concept for solar powered cooling

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Abstract: The global cooling demand is one of the fastest growing energy demands and is putting a strain on the electricity infrastructure. Solar powered cooling could provide most of the cooling demand due to the coincidence of the cooling demand and the solar irradiance. In particular, the solar powered Stirling-cycle cooler has low maintenance requirement, high theoretical efficiency and use of environmentally friendly gases. However, Stirling-cycle coolers are expensive due to high driving temperatures, complex heat exchangers and expensive solar tracking so that they have so far only been successful at high temperature difference applications. This study introduces a novel directly-coupled Solar Stirling cooler for which the hot engine cylinders are deployed inside evacuated tube collectors. The machine uses air as working fluid and its driving mechanism is based on the free-piston, balanced compound technology that was patented by Finkelstein. A second-order mathematical model is used to investigate the performance of the machine for different cylinder arrangements, gas leakage rates, chilling temperatures and solar irradiance. In addition, the regenerators are optimised to maximise the cold production. It is shown that mechanical friction can be reduced to 20% by selecting an appropriate cylinder arrangement. The solar cooler achieves a maximum cold production rate of 367.5 W/m² without using external heat exchangers at load temperature of 7°C, which is comparable to photovoltaic powered coolers. In addition, the machine is relatively simple, has safe and quiet operation, uses ambient air as working gas and is able to produce a wide range of chilling including sub-zero temperatures without changing the working gas. The direct thermal coupling of the Stirling cooler to evacuated tube collectors significantly reduces the complexity of the machine and removes intermediate heat transfer steps which reduce the performance. Thus, the suggested cooling technology has great potential for solar refrigeration, especially for low power and near ambient cooling.

Keywords: Solar cooling, Stirling cycle, Franchot engine, evacuated tubes, free-piston, balanced compounding

1 Introduction

Near ambient solar refrigeration and cooling has gained attention in recent years due to increasing cooling demand and the coincidence of the cooling demand to the peak electricity demand for grid powered vapour compression cycle (VCC) refrigerators. Among the different technologies for solar cooling, photovoltaic (PV) powered VCC and sorption technologies have been commercialised. Although PV panel prices are declining fast, PV powered refrigeration is still more expensive than cheap grid-connected coolers. Even though, solar PV powered refrigerators are the most economical option for low power applications and in remote areas where the electricity grid is not reliable. However, PV modules are still expensive and the VCC refrigerators often use non-environmentally-friendly refrigerants, are prone to refrigerant leaking and failure of its mechanical parts. Thus, there is demand for an alternative, reliable and environmentally-friendly solar powered cooling technology.

The Stirling-cycle refrigerator is a cooling technology that has safe and quiet operation, low maintenance requirement, high theoretical efficiency and uses environmentally friendly gases. The Stirling refrigerator is already competitive in high-lift refrigeration where it achieves better performance and is cheaper than some cryocooling technologies. The one stage Stirling refrigerator reaches its performance peak for an absolute temperature ratio of around two, for which the performance of the VCC deteriorates. Although, the ideal Stirling cycle has a higher COP than the inverse Rankine cycle the Stirling-cycle refrigerator has not been commercially successful at low lift refrigeration due to the lower power density. In contrast to VCC refrigerators, only a small number of solar Stirling refrigerators are available due to their initial costs. Moreover, coupling and hermetic
sealing of both the engine and the compressor parts is another obstacle for Stirling-cycle refrigerators. Ferreira and Kim suggested using a solar parabolic dish Stirling engine powered VCC refrigerator for which the power is transferred electrically from the dish focal point to a remote VCC refrigerator. They showed it can be cheaper than the absorption refrigerator powered by a parabolic trough collector but is not competitive with the PV powered VCC. In this device, the engine and refrigerator work at high efficiency and are hermetically sealed. However, using a commercial electric generator and motor that each have 90% efficiency reduces the total system efficiency to 81% and increases the costs compared to a system with direct mechanical coupling. Berchowitz and co-workers have developed a free-piston refrigerator for domestic refrigeration and portable coolers. The refrigerator achieves 35% of Carnot efficiency, which is comparable to a VCC refrigerator. The portable coolers are powered by a PV collector and have a COP of three while the Carnot COP is 9.1. Oguz and Ozkadi tested the free-piston Stirling refrigerator for domestic refrigeration and showed that the main challenge is due to the complicated heat exchangers. Heat exchangers especially the hot ones are still the major cost of Stirling engines beside the cost due to using high pressures and temperatures. Thus, the Stirling cooler could be an alternative to the VCC if the cost and complexity of the heat exchangers are reduced while maintaining the performance.

Mahkamov et al. studied a liquid piston converter fed by evacuated tube collectors (ETC). The machine is most suitable for near ambient refrigeration due to the low temperature reduction of 11°C. C. Langdon-Arms et al. developed a liquid piston double acting Stirling machine coupled to a moderate range solar collector with temperatures up to 200°C. The machine is able to generate cooling at 5°C below ambient but still requires additional components for the thermal coupling to the solar collector. ThermoLift presented a novel idea to overcome this challenge by installing the machine in the parabolic dish base. In their design, two small mirrors are added in the focal point to reflect the solar irradiance to the dish base where the stationary machine is located. The new design is based on their 3.5-ton gas-fired model that has a calculated COP of 0.8-1.2 for air conditioning and a maximum experimental heating COP of 1.65 at 8°C. Dai et al. showed in their finite time thermodynamic model that the cooling rate of a dish Stirling refrigerator can be maximised for a specific collector optical efficiency. On the other hand, increasing the heat transfer coefficient of the Stirling machine slightly improves the cooling rate. To the best of the authors knowledge and in contrast to the ETC and parabolic dish, no Stirling engine has been directly coupled to the compound parabolic collector (CPC). The CPC can achieve moderate temperatures up to 200°C and presents a good compromise between simplicity and thermal performance. Grosu et al. showed by exergy analysis that compound parabolic collector (CPC) is a high-performance collector and can technically and economically achieve temperatures as high as 180°C. However, to the best of the authors knowledge, no Stirling engine has been directly coupled to the CPC.

In the authors previous work, the cylinder heated and cooled Franchot engine and a novel isothermaliser design that improves the power density were developed. The cylinder heated and cooled engine benefits from the whole working cylinder area for heat exchanging and can achieve high thermal efficiencies. Although the heat exchanging mechanism is simple, the engine uses long cylinders to enhance the heat transfer and power generation. Due to the long cylinders, the engine requires long cranks that result in a long engine and large side forces. By increasing the number of cylinders the mechanical vibrations are reduced and the Franchot engine becomes self-starting. A new cranking mechanism based on the balanced compounding method for the multi-cylinder engine removes the need for rotational cranks and reduces the engine size. This mechanism is suitable for driving linear alternators, fluid pumps and heat pumps. In addition, the long and narrow cylinders of
the multi-cylinder engine open new possibilities for thermal coupling to solar collectors. However, many other free piston configurations with different geometry and applications exist 27.

In this study, the cylinder heated and cooled Franchot engine with the balanced compounding cranking mechanisms is directly coupled to a line focus collector. This system has the advantage of using a simple solar tracking system and also reduces the complexity of the thermal coupling between the Stirling engine and the solar collector. This has the potential to produce a more efficient and cheaper solar cooling machine. The characteristics and performance of this integrated system are evaluated with a validated mathematical model.

2 Direct thermal coupling of the Franchot engine with CPC

Compound parabolic collectors (CPC) consist of parabolic reflectors that focus the incoming sunlight onto a line collector (usually an evacuated tube collector) 28. The line absorbers are particularly suited to directly heat the multi-cylinder Franchot engine due to the similarity in geometry. The multi-cylinder Franchot engine is a multiple engine configuration for which each engine has a distinct regenerator (see Figure 1). Further details about the working principle of the multi-cylinder Franchot engine can be found in 25 and 26. These engines are either kinematically coupled to a common crankshaft or used the balanced compounding method to drive a free-piston Stirling cooler. The balanced compound Franchot engine needs at least three Franchot engines to make a complete thermodynamic cycle.

A novel direct thermal coupling is achieved by deploying the hot cylinders of the multi-cylinder Franchot engine inside the evacuated tubes. The heat is transferred to the hot cylinder by closely attaching it to the inner surface of the absorber pipe as depicted in Figure 1A). A thermally conductive fitting or wire mesh can be added to the annulus between the glass tube and hot cylinders to reduce the effect of the cylinder thermal expansion and vibration on the glass tube. In the future, it might be possible to directly use the absorber glass tube as engine cylinder if a strong and vibration resistant glass is manufactured especially for engines with low maximum pressure. The direct thermal coupling removes the need for extra components such as heat transfer fluid, heat exchangers, piping, auxiliary power supply and fluid pump. Thus, the direct thermal coupling simplifies the design and can potentially lead to systems with higher efficiencies and lower costs.

The large surface area of the cold cylinders of the engine enables the heat exchange with the ambient air by either natural or forced convection. Additional fins can be attached to the cold pipes, which increase the heat transfer area and thus improve the heat transfer. It is also important to acknowledge the potential of using the CPC reflector as an external fin by attaching it to the cold cylinders to benefit from its large surface area. An annular heat exchanger filled with heat transfer fluid such as water or water-glycol mixture might also be used if even higher heat transfer rates are required.

For solar refrigeration, the Stirling refrigerator is mechanically coupled to the engine using the balanced compounding mechanism 26. The compression cylinder in the cooling machine can reject heat to the ambient similarly to the cold cylinder of the engine. For transferring cold energy to the chilled space, the expansion or chilling cylinders can be directly deployed in the chilled space. Additionally, fins or an annual heat exchanger can be added to improve the heat transfer.

The remainder of this study evaluates the theoretical performance of this novel concept. Further technical and commercial issues related to the collector performance, engine vibrations, tube strength and tube bore smoothness go beyond the scope of this study but should be addressed in future studies.
3 Methodology

The mathematical model used in this study is an extension of a previous contribution derived for the Franchot engine\(^{23}\). In this study, the model assumes a non-ideal Franchot engine and refrigerator with direct coupling to a solar collector. The friction forces due to piston weight, gas friction in the machine cylinders and non-ideal regenerators are included. The effect of piston weight on the thermodynamic forces is ignored due to the horizontal installation of the proposed machine. By applying Newton’s second law of motion to each reciprocator, the force balance equation \(^{26}\) is given by

\[
\begin{bmatrix}
F_{1}

F_{2}

\vdots

F_{n}
\end{bmatrix}
= 
\begin{bmatrix}
m_{1}

m_{2}

\vdots

m_{n}
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_{1}

\ddot{x}_{2}

\vdots

\ddot{x}_{n}
\end{bmatrix}
+ 
\begin{bmatrix}
F_{el1}

F_{el2}

\vdots

F_{eln}
\end{bmatrix}
\]
where $F_i$ is the thermal driving force, $m_i$ is the total mass of reciprocating elements, $F_{il}$ is the load force, $x_i$ is the reciprocator displacement. Unless otherwise stated, the parameter $i$ goes from 1 to the number of reciprocators.

The power is calculated for the $n - p h$ Franchot engine, which is a multi-cylinder Franchot engine comprised of $n$ of distinct Franchot engines, as follows

$$P = F_1 x_1 + F_2 x_2 + \cdots + F_n x_n$$

The thermal forces applied to each connecting rod are calculated from

$$\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} = \begin{bmatrix} \Delta p_{E1} \\ \Delta p_{E2} \\ \vdots \\ \Delta p_{En} \end{bmatrix} \begin{bmatrix} A_{h1} & A_{h2} & \cdots & A_{hn} \\ A_{k1} & A_{k2} & \cdots & A_{kn-1} \end{bmatrix}$$

where $A_{hi}$ and $A_{ki}$ are the cross-sectional area of the hot and cold pistons respectively, $\Delta p_{Ei}$ is the pressure difference across the pistons of the Franchot engine, $F_i$ is the force due to the load and the subscript $E$ denotes the Franchot engine.

If all reciprocators have the same cross-sectional area and mass then the acceleration of the pistons can be calculated by combining Equations 1 and 2 to get

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_n \end{bmatrix} = \frac{1}{m} \begin{bmatrix} \Delta p_{E1} + \Delta p_{En} \\ \Delta p_{E2} + \Delta p_{E1} \\ \vdots \\ \Delta p_{En} + \Delta p_{En-1} \end{bmatrix} - \begin{bmatrix} F_{il} \\ F_{i2} \\ \vdots \\ F_{in} \end{bmatrix}$$

The speed and displacement of the pistons are calculated by calculating the integral and double integral of the acceleration matrix, respectively.

The friction is a load that reduces the Stirling engine performance and that needs to be minimised. There are two mechanical friction sources in this configuration: friction due to the weight of the reciprocating masses and friction due to the side forces created by the cylinder offset. The side load due to piston friction can be calculated as

$$F = \frac{y}{L} \Delta F$$

where $y$ is the crank length and $L$ is the distance between engine and cooler, respectively.
The load force due to the mechanical friction can be written as

$$\Delta F = [\Delta F_1, \Delta F_2, \ldots, \Delta F_n]^T = A \begin{bmatrix} \Delta p_{E_1} - \Delta p_{E_1} \\ \Delta p_{E_2} - \Delta p_{E_2} \\ \vdots \\ \Delta p_{E_{n-1}} - \Delta p_{E_{n-1}} \end{bmatrix}$$

where $g$ and 0.2 are the gravitational acceleration and dry coefficient of friction given by Hirata.

The pressure variation on one side of the heated and cooled cylinder Franchot engine is calculated from

$$\dot{p} = -p \left( \frac{v_e}{T_{re}} + \frac{v_c}{T_{cr}} \right) + \frac{R}{C_p} \left( \frac{\dot{Q}_e}{T_{re}} + \frac{\dot{Q}_c}{T_{cr}} \right)$$

where $v, T,$ and $\dot{Q}$ denote the volume, temperature and heat flow rate in the working spaces, respectively, and subscripts $e, r$ and $c$ indicate the expansion, regeneration and compression space, respectively. The volume on one side of the Franchot machine is calculated from

$$v = \frac{\pi D^2}{4} \chi$$

The volume on the other side of the Franchot engine is calculated from

$$v = \frac{\pi D^2}{4} (L_e - x)$$

where $D$ and $L_e$ are the piston diameter and cylinder length, respectively.

External irreversibility is considered through the heat addition and removal which are calculated from Newton’s law of cooling

$$\dot{Q} = h A \Delta T$$
where \( h \) is the convective heat transfer coefficient, which holds for Reynolds’ numbers between 1000 and 100,000 and is calculated as \(^{31}\)

\[
\begin{align*}
h_e &= 0.042D_h^{-0.42}x^{0.58}p^{0.58}T^{-0.19} \\
h_c &= 0.0236D_h^{-0.47}x^{0.53}p^{0.53}T^{-0.11}
\end{align*}
\]

where \( \Delta T, D_h, h_e \) and \( h_c \) are the temperature difference between the working gas and cylinder wall, hydraulic diameter, convective heat transfer during the expansion and compression, respectively.

Regenerator end temperatures are calculated as \(^{23}\)

\[
T_{rh} = \frac{-\phi i \dot{m}_e T_e}{\phi(1 - i) \dot{m}_e} \quad 13
\]

\[
T_{rk} = \frac{-\phi j \dot{m}_c T_c}{\phi(1 - j) \dot{m}_c} \quad 14
\]

where the parameters \( i \) and \( j \) are given by

\[
i = \begin{cases} 1, & \dot{m}_e < 0 \\ 0, & \dot{m}_e \geq 0 \end{cases} \quad 15
\]

\[
j = \begin{cases} 1, & \dot{m}_c < 0 \\ 0, & \dot{m}_c \geq 0 \end{cases} \quad 16
\]

Hence, the average regenerator temperature is

\[
T_r = \frac{T_{rh} - T_{rk}}{\ln \frac{T_{rh}}{T_{rk}}} \quad 17
\]

To increase the accuracy of the model the reheat and pressure losses of the regenerator are considered. The effect of having imperfect regeneration is considered by modifying the regenerator gas stream temperatures as \(^{32}\)

\[
T_{rho} = T_{rk} + \varepsilon(T_{rh} - T_{rk}) \quad 18
\]

\[
T_{rk0} = T_{rh} - \varepsilon(T_{rh} - T_{rk}) \quad 19
\]
where $T_{rh0}, T_{rk0}$ and $\varepsilon$ are the hot outlet gas temperature, cold outlet gas temperature and regenerator effectiveness, respectively. The effectiveness is calculated according to Tanaka \textsuperscript{33} by

$$
\varepsilon = \frac{Ntu}{Ntu + 2}
$$

where $Ntu$ is the number of transfer units and calculated from

$$
Ntu = \frac{4\bar{Nu}L_r}{P_rR_e d_h}
$$

where $\bar{Nu}, P_r, R_e$ and $d_h$ are the average Nusselt number, Prandtl number, average Reynolds number and regenerator hydraulic diameter, respectively.

The Nusselt number is correlated according to Tanaka as follows

$$
\bar{Nu} = 0.33 R_e^{-0.67}
$$

The pressure loss due to the gas friction with the regenerator material is calculated from

$$
\Delta p_{loss} = \frac{0.5f_h \rho L_r U_{max}^2}{d_h}
$$

where $\Delta p_{loss}$ is the pressure loss and $f_h$ is the friction factor calculated according to Tanaka from

$$
f_h = 1.6 + \frac{175}{R_{e_{max}}}
$$

where $R_{e_{max}}$ is the maximum Reynolds’ number obtained from

$$
R_{e_{max}} = \frac{D_h U_{max} \rho}{\mu}
$$

where $U_{max}$ is the maximum gas speed which can be calculated based on average gas speed from \textsuperscript{34}

$$
U_{max} = \pi \frac{X_{m1}}{60} = \frac{\pi}{2} U_{av}
$$

The pressure loss due to the connecting pipe is calculated as

$$
\Delta p_{loss} = -\frac{2f_{Re} \mu L_r U_{av}}{d_h}
$$

where $f_{Re}, \mu, L_r$ and $d_h$ are the friction factor, dynamic viscosity, connecting pipe length and pipe hydraulic diameter, respectively. The friction factor $f_{Re}$ is calculated by \textsuperscript{35}
\[ f_{Re} = \begin{cases} 
16 & \text{Re} < 2000 \\
7.343 \times 10^{-4} \text{Re}^{1.3142} & 2000 < \text{Re} < 4000 \\
0.0791 \text{Re}^{0.75} & \text{Re} > 4000 
\end{cases} \]

The solar energy intercepted by the collector is calculated from

\[ \dot{Q}_{\text{collector}} = AG \]

Where \( A \) and \( G \) are solar collector area and solar radiation intensity per unit of collector area in [W/m\(^2\)], respectively. The energy absorbed by the collector is calculated using

\[ \dot{Q}_i = \zeta \dot{Q}_{\text{collector}} \]

where \( \zeta \) is the solar collector efficiency calculated by \(^{22,36}\)

\[ \zeta = \zeta_0 - \frac{a_1 \Delta T}{G} - \frac{a_2 \Delta T^2}{G} \]

where the constants \( \zeta_0, a_1 \) and \( a_2 \) are given by the CPC manufacturer. \( \Delta T \) is the temperature difference between the hot engine cylinder and ambient temperature. Hence, the hot temperature is calculated from

\[ T_h = T_k + \Delta T \]

The multi-cylinder model is validated against the Der Minassians \(^{37}\) double acting \(3\) – \(ph\) FPSE prototype shown in Figure 2 A). The system has three cylinders, which each have two diaphragm pistons. One diaphragm motion is reversed using a novel mechanical reverser. In addition to the working gas spring, each engine uses springs due to the diaphragm pistons, flexure and magnetic coupling.

\[ \text{(A)} \]

\[ \text{(B)} \]

*Figure 2: Der Minassians engine A) picture from \(^{38}\) B) schematic diagram with reverser.*
According to Figure 2, Equation 4 is rewritten to consider the reversal of one piston and the use of external springs as

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_n \end{bmatrix} = \frac{1}{m} \left( A \begin{bmatrix} p_{E1} + p_{En} - 2p_o \\ p_{E2} - p_{E1} \\ \vdots \\ p_{En} - p_{En-1} \end{bmatrix} - \begin{bmatrix} F_{I1} \\ F_{I2} \\ \vdots \\ F_{In} \end{bmatrix} - k_p \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \right)$$

where $x_n$, $p_o$, and $k_p$ are the hot side piston motion, ambient pressure and piston system stiffness, respectively. The engine expansion volume is calculated from

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} V + Ax_1 \\ V + Ax_2 \\ \vdots \\ V + Ax_n \end{bmatrix}$$

The compression swept volume, which is reversed for the last piston, is calculated from

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} V - Ax_2 \\ V - Ax_3 \\ \vdots \\ V + Ax_1 \end{bmatrix}$$

To apply the three-control volume model, the regenerator end temperatures were set to the source temperature and the heat transfer to the swept volume is ignored. The Der Minassians engine parameters are given in Table 1.

Table 1: The parameters of the 3-ph Der Minassians engine

| Name                     | Symbol | Value/unit |
|--------------------------|--------|------------|
| Swept volume             | $V$    | $93.2 \times 10^{-6} \ m^3$ |
| Piston area              | $A$    | $45.6 \times 10^{-4} \ m^2$ |
| Reciprocating mass       | $m$    | $0.64 \ kg$ |
| Total dead volume        | $V_r$  | $163.2 \times 10^{-6} \ m^3$ |
| Hot, cold temperatures   | $T_h, T_k$ | $420 \ K, 300 \ K$ |
| Working gas pressure     | $p$    | $100 \ kPa$ |
| Ambient air constant     | $MR$   | $0.119 \ J/K$ |
| Reciprocator mass        | $m$    | $0.1 \ kg$ |
| Number of phases         | $n$    | $3$ |
| Piston system stiffness  | $k_p$  | $3.58 \ kN/m$ |
| Damping factor           | $c$    | $11.2 \ N.s/m$ |

Table 2 shows a comparison between the experimental data, Der Minassians calculation and this work. The phase shift and phase angles match exactly. The frequency is slightly higher than that calculated by Der Minassians because this work assumes adiabatic swept volumes which increase the gas stiffness and hence the frequency. The frequency discrepancy with experimental data is attributed to
the effective piston area that reduces the working gas stiffness. The swept volume does not increase linearly with the stroke as the flat shape of the diaphragm piston deforms with the stroke. The model in this work anticipates that the system starts-up at a temperature of 400 K with the damping load considered. It is higher than the calculated temperature by Der Minassians as the isothermal model assumes that the real gas temperature differences are larger than in the adiabatic model. In contrast to the experimental engine, the discrepancy in estimating the stroke and frequency is due to the finite heat transfer and effective piston diameter in the test engine, respectively. Thus, the start-up temperature is overestimated because the damping losses are increased due to the larger anticipated frequency. However, both the phase shift and phase angle for the multiphase engine were accurately obtained. The model, which considered the exact speed, piston diameter and heat transfer, is validated in [25] by applying it to the Karabulut engine [39]. Thus, the model can be extended for the balanced compound engine.

| Variable                              | Experimental | Der Minassians isothermal model | This work   |
|---------------------------------------|--------------|---------------------------------|-------------|
| Phase shift                           | 60°          | 60°                             | 60°         |
| Phase angle                           | 120°         | 120°                            | 120°        |
| Frequency                             | 16 Hz        | 19 Hz                           | 20 Hz       |
| Start-up temperature at cold side temperature of 313K | 373 K       | 367 K                           | 400 K       |
| Stroke                                | 1.4 \times 10^{-2} m |                                | 1.7 \times 10^{-2} m |

4 Results and discussion

All the results in this chapter are theoretical and based on the mathematical model presented in section 3. The model has been implemented in Matlab/Simulink with a time step of 10^{-4} s and using the fourth order Runge-Kutta solver.

4.1 Cylinder arrangement

Each machine reciprocator is composed of four pistons (see Figure 1 C) and hence, four force signals are contributing in the total reciprocator driving force as shown in Figure 3. These forces are different in amplitude and phase shift but share the same frequency. Due to these differences, the reciprocator generates unwanted moment forces, which increase the friction between the pistons and cylinders. The side forces can be reduced by reducing the crank length y, increasing the length of the piston rod and decreasing the coefficient of friction according to Equations 5 and 7.
Figure 3. The four piston forces acting on one reciprocator over one crank rotation. Conditions: $T_l=T_k = 300 \text{ K}$, $T_h = 450 \text{ K}$, $L_e=L_c=0.5 \text{ m}$, $D_e=D_c = 0.025 \text{ m}$ and ideal regenerator.

In addition, the expansion and compression cylinders of the refrigerator can be easily interchanged by changing the regenerator connections. If the refrigerator has the same regenerator connecting order as the engine, the expansion cylinders of the refrigerator will be inline with the expansion cylinders in the engine and similarly for the compression cylinders. This is the forward regenerator connection. If instead, the expansion cylinders are inline with the compression cylinders of the refrigerator (reversed regenerator connection), a more balanced force combination leads to reduced side forces. However, no other change can be made to the arrangement of the refrigerator cylinders. Figure 4 shows the effect of interchanging the cooler cylinders on the side forces. The side force in the case of reversed regenerator connection is reduced to around 20% of the side force for the forward regenerator connection. Thus, according to Equation 7, the mechanical friction associated with the side forces will also be reduced to around 20% of the mechanical friction for the forward regenerator connection.
Figure 4. Total side forces of the duplex engine for the forward regenerator connection (expansion cylinders of the engine are in line with the cooler expansion cylinders) and reversed regenerator connection. Conditions: $T_{l}=T_{k}=300\, K$, $T_{h}=450\, K$, $L_e=L_c=0.5\, m$, $D_e=D_c=0.025\, m$, ideal regenerator and crank length of 0.04 m.

However, the side forces are not completely cancelled out by interchanging the refrigerator cylinders because of the phase shift between the piston forces shown in Figure 3. The moment force would only be removed if the refrigerator works as an engine with the same temperature levels as the engine. In his case, the two engines generate identical force signals.

4.2 Base design of the coupled solar cooler

The CPC collector XL15/26P made by Ritter-XL is chosen as a reference in this study. The collector is commercially available for the domestic sector and has high performance. It can achieve temperatures of 185°C above ambient at a solar to thermal efficiency of 50%. In addition, it can achieve high stagnation temperatures up to 338°C at solar irradiance of 1kW/m². However, the useful energy at the stagnation temperature is zero. The high temperatures of this collector positively affect the engine performance at high loads and enable the engine to self-start.

The machine dimensions are governed by the dimensions of the XL15/26P collector and the $3 - ph$ engine. Each ETC has a collector sheet of $11.6 \times 10^{-2} \times 1.5\, m$. However, shorter lengths of 0.5 and 1.0 m are investigated as well. The chiller and engine are chosen to have the same geometry. The engine diameter is fixed to $2.5 \times 10^{-2} \, m$ so that it fits inside the evacuated tube. The machine is assumed horizontal and the effect of gravity on the piston dynamics is eliminated. The crank length is fixed at $4\times 10^{-2} \, m$ due to the ETC diameter. A piston length of $5\times 10^{-2} \, m$ is chosen to reduce gas leakage. The reciprocating mass including the link, piston rods and pistons mass was calculated to
be 0.31 kg, 0.6 kg and 0.9 kg for the cylinder lengths of 0.5 m, 1 m and 1.5 m, respectively. The regenerator length is kept constant at 0.1 m to fit between the collector cylinders. The regenerator wire mesh diameter is fixed at 90 microns. For simplicity and friction reduction, the $3 - \phi h$ balanced compound Franchot machine is used with the reversed regenerator connection (see Figures 3 and 4). As the refrigeration cycle has monotonic performance and is directly driven by a Stirling cycle engine, the cooler is assumed to have the same dimensions and operating conditions of the driving engine.

Cooling machines are usually manufactured based on maximum loads \textsuperscript{41}42 and optimised for constant irradiance \textsuperscript{43}. For example, the irradiance is often fixed at a midday peak of 1 kW/m\textsuperscript{2} which coincides with the maximum cooling need. The cold cylinders are assumed to have a constant and consistent temperature of 27°C and the chilling cylinders are assumed to have a constant temperature of 7°C.

The regenerator diameter and porosity were optimised for the maximum cold production using the ‘Particle Swarm Optimisation (PSO)’ function in Matlab while the other parameters are not changed. For example, the phase angle is defined by the number of cylinders, the piston diameter is limited to the ETC diameter and speed is governed by the moving masses and engine geometry. In addition to the PSO, the regenerator diameter and porosity are manually rounded to the nearest centimetre and integer number, respectively.

### 4.3 Optimised solar cooler

The performance of the optimised solar powered machine for different cylinder lengths are summarised in Table 3. It is assumed that there is no gas leakage through tight piston rod seals. The regenerator diameter and porosity were optimised for maximum cold production. For example, the optimised regenerator diameter of 0.1 m and porosity of 89% are obtained for a cylinder length of 0.5 m and charging pressure of one atm. The maximum cooling power in this case is 55.7 W. This gives a solar COP (COPs) of 0.334 for a collector temperature of about 135°C at an efficiency of 58.6%.

| Parameters                | $L_e=0.5$ m | $L_e=1$ m | $L_e=1.5$ m |
|---------------------------|-------------|-----------|-------------|
| Reciprocator mass         | 0.31 kg     | 0.6 kg    | 0.9 kg      |
| Regenerator diameter      | 0.1 m       | 0.11 m    | 0.11 m      |
| Regenerator porosity      | 89%         | 90%       | 89%         |
| Average stroke            | 0.44 m      | 0.94 m    | 1.41 m      |
| Frequency                 | 3.22 Hz     | 1.95 Hz   | 1.48 Hz     |
| Collector temperature     | 135°C       | 134.5°C   | 142°C       |
| Collector efficiency      | 58.6%       | 58.6%     | 57.3%       |
| Engine solar efficiency   | 7.1%        | 7.1%      | 8.95%       |
| Cooling power             | 55.7 W      | 114.6 W   | 168.4 W     |
| Maximum solar COP         | 0.334       | 0.344     | 0.337       |
| Peak cooling power per m\textsuperscript{2} | 334.2 W/m\textsuperscript{2} | 343.8 W/m\textsuperscript{2} | 336.8 W/m\textsuperscript{2} |

The prime mover works at an efficiency smaller than Curzon and Ahlborn efficiency and at low collector temperature. Based on Curzon and Ahlborn efficiency and the efficiency of the XL15/26P...
collector for $T_K = 27^\circ C$, the maximum solar engine efficiency is 10.1%. This means that the performance of this machine can be further enhanced if more parameters are optimised for the maximum cold generation. For example, when the engine and refrigerator regenerators were optimised separately, a maximum cooling power of 61 W was generated for a cylinder length of 0.5 m. The engine regenerator diameter of 0.1 m with a porosity of 90% and the refrigerator regenerator diameter of $7.35 \times 10^{-2}$ m with a porosity of 89% were obtained. The cold production improved to around 10% which is very close to the Curzon and Ahlborn efficiency.

By considering gas leakage through the piston rod seals, the average working pressure of the machine is assumed to be 1 atm. Interestingly, the optimised regenerator diameter of $8.7 \times 10^{-2}$ m at a porosity of 88% for the engine and refrigerator is smaller than for the machine without gas leakage. It also gives a cooling power of 61 W making a specific solar cooling power of 367.5 W/m². However, the engine efficiency is 9% and can only be enhanced to 10% by separately optimising the regenerator of the prime mover and refrigerator.

The solar powered machine could also be optimised based on the phase angle $23^\circ$ in order to decrease the size of the regenerator by increasing the number of cylinders, which in addition, increase the total cold production. The machine performance can be enhanced by choosing different dimensions for the engine and refrigerator from each other. Moreover, the refrigerator can be isothermalised to improve cold production $24^\circ$. On the other hand, the engine power maximum is limited by the solar irradiance and isothermalising the engine does not add any benefit.

In terms of the solar coefficient of performance (COPs), this technology is comparable to the basic PV powered VCC and absorption coolers in terms of the solar COP as reported by Lazzarin $36$. For example, the PV powered VCC approaches a solar COP of 0.5 for air conditioning. The single stage absorption chiller has a solar COP of 0.45 and 0.3 at 90 °C for ETC and flat plate collector, respectively. The double effect absorption cooler using ETC collectors at a driving temperature of 160 °C achieves a maximum solar COP of 55% and single effect absorption give 45% and 30%. Also it reported that Rankine engine driven compressor can achieve a solar COP of 38% at 160 °C $44$. However, the Stirling cycle technology developed in this study has the advantages of using stationary thermal collectors, fewer components and devises, no toxic materials, the same technology for the engine and refrigerator and benign refrigerants. In addition, it enjoys quiet operation, low maintenance requirements and wide range of temperatures for refrigeration and heat pumping without changing the refrigerant (air).

4.4 Influence of irradiance

Cooling requirements vary based on solar irradiance and ambient temperature. Hence, the machine is investigated for different load conditions and different irradiances as shown in Figure 5. It shows that the cold production improves for higher solar irradiance, which adds more energy to the engine, and for higher load temperature, which reduces the load on the engine. The solar COP and machine stroke are increased although the solar collector temperature decreased. However, the solar collector efficiency increases for decreasing hot cylinders temperature, consequently more energy is absorbed by the engine.
Figure 5. Effect of increasing the load temperature on cooling power, solar COP, stroke and CPC temperature at solar irradiance of 1 kW/m² and 700 W/m².

For the range of load conditions investigated, no change was observed for the machine frequency, which is almost constant at 3.4 Hz. At the load temperature of 300 K and solar irradiance of 1 kW/m² the pistons hit the cylinder heads. Therefore, the engine must be kept loaded at high solar irradiances unless hitting the cylinder heads is prevented by an additional mechanism such as using cushioning springs. However, using mechanical springs is impractical due to the number of springs and due to the high cylinder temperature. Gas cushioning can easily be implemented by decreasing the regenerator dead volume but it decreases the cold production.

5 Conclusion

A novel solar powered cooler based on direct thermal coupling of a Stirling engine with a solar collector has been introduced and evaluated with a validated mathematical model. The hot cylinders of the 3-ph duplex Franchot engine are deployed directly inside evacuated tube collectors. This eliminates the need for additional heat transfer components, such as heat transfer fluid, pumps, complex heat exchangers and an external power supply. Moreover, the cold and chilling cylinders can directly exchange heat with the cooling or chilled spaces due to their large surface area. Fins, fans or cylindrical exchangers could be used to improve the heat transfer if higher heat transfer rates are required. This novel design can produce to more efficient and cheaper solar coolers because of the simplified design and removal of intermediate heat transfer steps.

A validated mathematical model of the novel machine has been implemented in Matlab/Simulink. The model has been used to evaluate the performance for different machine configurations and to optimise the regenerator configuration. The results show a promising machine concept for near ambient refrigeration. The friction losses can be reduced to 20% of the maximum friction by selecting the reversed regenerator connection. The COPs can be up to 0.367 for the basic Stirling-cycle which is
comparable to a commercial PV powered VCC that has a PV efficiency of 10% and COP of 3.5. The solar powered cooler can be further optimised for higher power densities by using distinct regenerator configurations for the engine and refrigerator parts.

The novel design has potential to compete with the PV driven VCC in terms of total efficiency, cost, quietness, reliability and safety. Moreover, the machine can achieve the optimised response without hitting cylinder heads and hence no additional stroke control mechanism is needed. The machine showed a capability for sub-zero refrigeration using ambient air at atmospheric pressure, which is considered a big advantage over many other cooling technologies. Thus, the Stirling technology is highly recommended for near ambient solar cooling.

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