The Gribov problem in presence of background field for $SU(2)$ Yang-Mills theory

Fabrizio Canfora$^1$, Diego Hidalgo$^{1,2}$, Pablo Pais$^{1,3}$

$^1$ Centro de Estudios Científicos (CECS), Casilla 1469, Valdivia, Chile.
$^2$ Departamento de Física, Universidad de Concepción, Casilla 160, Concepción, Chile.
$^3$ Physique Théorique et Mathématique, Université de Bruxelles and International Solvay Institutes, Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium.

E-mail: $^1$canfora@cecs.cl, $^2$dhidalgo@cecs.cl, $^3$pais@cecs.cl

Abstract. The Gribov problem in the presence of a background field is analyzed: in particular, we study the Gribov copies equation in the Landau-De Witt gauge as well as the semi-classical Gribov gap equation. As background field, we choose the simplest non-trivial one which corresponds to a constant gauge potential with non-vanishing component along the Euclidean time direction. We show that the Gribov copies equation is affected directly by the presence of the background field, constructing an explicit example. The analysis of the Gribov gap equation shows that the larger the background field, the smaller the Gribov mass parameter. These results strongly suggest that the relevance of the Gribov copies (from the path integral point of view) decreases as the size of the background field increases.

1. Introduction

The main tool to compute observable quantities in QFT is perturbation theory. In gauge theories, and in Yang-Mills (YM) theory in particular, a fundamental problem to solve in order to compute physical quantities is the over-counting of degrees of freedom related to gauge invariance (for a detailed analysis see [1]). The Faddeev-Popov (FP) gauge fixing procedure is the cornerstone which allows using the Feynman rules and Feynman diagrams in all applications of the standard model. The obvious fundamental hypothesis is that the gauge-fixing condition must intersect once and only once every gauge orbit. Locally, in the space of gauge fields, this hypothesis requires that the FP operator should not have zero modes so that the FP determinant is different from zero. The reason is that the existence of a proper gauge transformation preserving the gauge-fixing would spoil the whole quantization procedure since it would imply that the FP recipe does not completely eliminate the over-counting of degrees of freedom.

However, in [2], Gribov showed that in non-Abelian gauge theories (in flat, topologically trivial space-times) the FP procedure fails at non-perturbative level. The reason is that a proper gauge fixing is not possible due to the appearance of Gribov copies: namely, gauge equivalent configurations satisfying the Coulomb gauge. Later, Singer [3] showed that if Gribov ambiguities occur for the Coulomb gauge, they occur for all gauge fixing conditions involving derivatives of the gauge field.

Naively, one could expect to completely avoid the Gribov problem by simply choosing algebraic gauge fixings like the axial gauge or the temporal gauge, which are free of Gribov
copies. However, these choices have their own, and even worse, problems (for a detailed reviews see [4] [5]).

On the other hand, the existence of Gribov copies is not just a problem since, as Gribov himself argued, the natural way to solve such a problem is able to shed considerable light on the infrared (IR) region of YM theory. Such solution is to restrict the path-integral only to a region \( \Omega \), which is called Gribov region, where FP operator is definite-positive [2, 6, 7, 8, 9] (detailed reviews are [10] and [5]) so that there are no Gribov copies connected to the identity. It is worth to mentioning that some Gribov copies are still left within the Gribov region [6]. In order to restrict the path integral to the Gribov region, one can use the Gribov-Zwanziger (GZ) approach [11, 12]. When the space-time geometry is flat and the topology trivial this method is able to reproduce the usual perturbation theory encoding, at the same time, the effects related to the elimination of the Gribov copies. For instance, it allows the computation of the glueball masses in excellent agreement with the lattice data [19, 20]. Within the same framework, it is also possible to solve the sign problem for the Casimir energy and force in the MIT-bag model [21].

Thus, it is natural to wonder whether or not this approach works so well also in the presence of a background gauge field. From the theoretical point of view, this analysis is very important as it discloses how strongly the presence of a background field can affect the Gribov region and the whole issue of Gribov copies. One of the most relevant applications of the background field method is the computation of the vacuum expectation value of the Polyakov loop [27] in which the presence of the Polyakov loop manifests itself as a constant background field with component along the Euclidean time. Formally, a constant background gauge field with only the timelike component non-vanishing is related to a bosonic chemical potential [29]. On the other hand, the physical interpretation of such Bosonic chemical potential is rather obscure in the case of non-perturbative gluons and so it will not be discussed in the present case. Another very important non-perturbative phenomenon in which the presence of a background gauge field plays a key role is the (both Abelian and non-Abelian) Schwinger effect [30, 31, 32]. Also in the case of the non-Abelian Schwinger effect, the relevant background gauge fields are constant \( A_\mu \) which have components both along time and space directions. From the point of view of applications, such an analysis can also be quite relevant in relation with quark-gluon plasma [33, 34], color superconductivity in QCD [35], astrophysics [36, 34], and cosmology [37, 38].

The Background Field Method (BFM) [39, 40, 41] together with the techniques developed in [13, 14, 15, 16, 17] suggest that background fields can play a prominent role within the GZ approach to YM theory. Here we show that the Gribov copies equation and the gap equation are affected directly by the presence of a background field.

The paper is organized as follows. In the second section, the Gribov problem in the Landau-De Witt gauge is introduced. In the third section, explicit examples of Gribov copies in the Landau-De Witt gauge are studied. In the fourth section, the Gribov gap equation within a background field is analyzed. Some conclusions and discussions are drawn at the end.

2. A brief review of the Gribov-Zwanziger action

In this section we present an outlook of the GZ-approach without background consider background fields, which is the aim of Section 3. The Euclidean Yang-Mills action

\[
S_{YM} = \frac{1}{4} \int d^4x F^a_{\mu \nu} F^{a\mu\nu}, \quad F^a_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,
\]

is invariant under the gauge transformation

\[
A_\mu \rightarrow A_\mu = U^{-1} A_\mu U + U^{-1} \partial_\mu U,
\]
with $U \in SU(N)$. In order to take into account the existence of Gribov copies due to the
this gauge transformation, Gribov proposed [2] to restrict the domain of integration in the path
integral to a region in functional space where the eigenvalues of the FP operator $\mathcal{M}^{ab}$ are strictly
positive. This region is known as the Gribov region $\Omega$, and is defined as

$$
\Omega = \{ A^a_\mu \mid \delta_\mu A^a_\mu = 0; \quad \mathcal{M}^{ab} = -\partial_\mu (\partial^\mu \delta^{ab} - g f^{abc} A^c_\mu) = -\partial_\mu D^{ab}_\mu > 0 \},
$$

(3)

where $D^{ab}_\mu = \partial_\mu \delta^{ab} - g f^{abc} A^c_\mu$ is the usual covariant derivative, which depends on $A_\mu$. The
boundary of this region is called the first Gribov horizon. Later on, Zwanziger [11] implemented
the Gribov region $\Omega$ in Euclidean Yang-Mills theories resulting

$$
S_{GZ} = S_{FP} + S_\gamma + S_0,
$$

(4)

$$
S_{FP} = \int d^4 x \left( \frac{1}{4} F^a_{\rho\sigma} F^{a\rho\sigma} + i b^a \partial_\rho A^a_\rho + \bar{c}^a \mathcal{M}^{ad} c^d \right),
$$

(5)

$$
S_\gamma = \int d^4 x \left( \gamma^2 g f^{abc} A^a_\rho (\varphi^b_\rho - \varphi^c_\rho) + \gamma^4 \int d^4 x h(x) \right),
$$

(6)

$$
S_0 = \int d^4 x \left( -\bar{\varphi}^{ac} \mathcal{M}^{ab} \varphi^b_\rho + \bar{\omega}^{ac} \mathcal{M}^{ab} \omega^b_\rho + g f^{amb} (\partial_\rho \bar{\varphi}^{ac}) (D^m_\rho c^f) \varphi^b_\sigma \right),
$$

(7)

where Greek indexes run from $\mu = 1 \ldots d$ and latin indexes from $a = 1 \ldots N^2 - 1$. The fields
$(\varphi^a_\mu, \varphi^a_\mu)$ are a pair of complex conjugate bosonic fields, while $(\bar{\omega}^{ac} \omega^{ac})$ are anti-commuting
fields, and the Gribov parameter $\gamma$ is determined by the following gap equation

$$
\frac{\partial \tilde{\kappa}_{\text{vac}}}{\partial \gamma^2} = 0.
$$

(8)

Therefore, equation (8) represents the horizon condition formula which will allow us to determine
the Gribov parameter later on.

3. Gribov-Zwanziger action in a background field

As we present a brief introduction in Section 2 of the GZ-approach, now we analyze what
happens if we take into account a background field. We consider the $SU(N)$ Yang-Mills theory
in $d = 4$ Euclidean dimensions defined in Eq.(4). In the BFM (see [4, 42] for more details), one
introduces a fixed background gauge field configuration $B_\mu$ through the splitting

$$
A_\mu \rightarrow a_\mu \equiv A_\mu + B_\mu,
$$

(9)

where $A_\mu$ and $B_\mu$ play completely different roles. On the one hand, $A_\mu$ represents the quantum
fluctuations of the gauge field. On the other hand, the background field $B_\mu$ plays the role
of a classical background. (this approach is quite relevant in the case of the Polyakov loop
computation [27]). The gauge symmetry (2) changes with this background field as

$$
A_\mu + B_\mu \rightarrow A'_\mu + B'_\mu = U^{-1} \partial_\mu U + U^{-1} (A_\mu + B_\mu) U.
$$

(10)

Although it is not mandatory, in many applications (such as the already mentioned case of the
Polyakov loop) it is convenient to demand that the background gauge field is fixed (namely,
it does not transform under gauge transformations (2)). Consequently, it follows the natural
requirement

$$
\delta B^a_\mu = 0.
$$

(11)

In this case, the symmetry transformation in Eq. (10) can be written as

$$
A_\mu \rightarrow A'_\mu = U^{-1} \partial_\mu U + U^{-1} A_\mu U + \left( U^{-1} B_\mu U - B_\mu \right),
$$

(11)
where it has been explicitly taken into account that $B_\mu$ is not affected by the gauge transformation. At the infinitesimal level, $U \approx I + \omega^a \tau_a$, $\omega \ll 1$, one recovers the usual infinitesimal gauge transformations with a background gauge field [4, 42]

$$\delta A^a_\mu = f^{abc} \omega^b (A^c_\mu + B^c_\mu) + \frac{1}{g} \partial_\mu \omega^a,$$

$$\delta B^a_\mu = 0. \quad (13)$$

Correspondingly, the Landau gauge-fixing condition is also modified. In the presence of a background field, the most convenient gauge-fixing condition takes the form

$$\tilde{G}^a_\mu[B] \equiv \tilde{D}^{ab}_\mu A^b_\mu = 0, \quad \tilde{D}^{ab}_\mu := \partial_\mu \delta^{ab} + g f^{acb} B^c_\mu,$$

known as the Landau-DeWitt (LDW) gauge fixing condition. The GZ method can be applied to this situation by means a suitable choose of the background field $B^a_\mu$ in order to the new FP operator $M^a_{bc} \equiv -\tilde{D}^{ab}_\mu (B^b_\mu) D^c_{\mu} (a)$ is invertible inside the Gribov region $\Omega$. Following the lines of [43] and [27], the GZ action under the LDW gauge acquires the form

$$S_{GZ} = \int d^4 x \left( \frac{1}{4} F^{a}_{\mu \nu} F^{\mu \nu a} + c^a \tilde{D}_\mu (B) D_\mu (a) c^a - \frac{(\tilde{D}_\mu (B) A_\mu)^2}{2 \xi} + \tilde{\varphi}^{0 a} \tilde{D}_\sigma (B) D^b_\sigma (a) \varphi^d c^d - \omega^{ac} \tilde{D}^{ab}_\nu (B) D^d_\nu (a) \omega^{dc} - g \gamma^2 f^{abc} A^a_\mu (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) - \gamma^4 d (N^2 - 1) \right). \quad (15)$$

There are two key mathematical requirements (described in the references [2, 11]) necessary in order to write down a local GZ action in the presence of a background field. The first one corresponds to the condition that the FP operator at tree-level must be invertible. The second key requirement is the validity of the Dell’Antonio-Zwanziger theorem [9], which also could change in the presence of a background, but in the case of the LDW gauge provided the background field is constant and commutes with itself, and then the theorem remains valid.

4. The simplest non-trivial background field

In order to describe the effects of a background field, we will consider the simplest non-trivial background gauge field

$$B^a_\mu = \frac{r_0}{g} \delta^{a 3} \delta_{\mu 0}. \quad (16)$$

Here we have chosen the above background gauge field with only Euclidean time component since it allows to construct explicitly analytic examples of Gribov copies as well as to solve the semi-classical Gribov gap equations (which, quite consistently, shows that the Gribov mass decreases with the increase of $r_0$).

In this case, the LDW gauge fixing reads

$$\tilde{G}^a_\mu[B] = 0, \quad (17)$$

so that the Gribov copies equation becomes

$$\partial^\mu A^{U}_\mu + g [B^\mu, A^{U}_\mu] = 0, \quad (18)$$

where $A^{U}_\mu$ is defined in Eq. (11). It is worth emphasizing that the background gauge field $B^\mu$ identically satisfies the LDW gauge-fixing: $\tilde{D}^{ab}_\mu \tilde{B}^b_\mu = 0$. 


The following standard parametrization of the $SU(2)$-valued functions $U(x^i)$ is useful
\[ U = Y^0 \mathbf{1} + Y^a \tau_a, \quad (Y^0)^2 + Y^a Y_a = 1, \]
where $Y^0$ and $Y^a$ are functions on the coordinates $x^i$, and the sum over repeated indices is understood also in the case of the group indices (in which case the indices are raised and lowered with the flat metric $\delta_{ab}$). The $SU(2)$ generators $\tau^a$ satisfy $\tau_a \tau_b = -\delta_{ab} \mathbf{1} - \epsilon_{abc} \tau^c$, where $\mathbf{1}$ is the identity $2 \times 2$ matrix and $\epsilon_{abc}$ are the components of the totally antisymmetric Levi-Civita tensor with $\epsilon^{123} = 1$.

4.1. Gribov copies of the vacuum
In the present case, the gauge transformations of the vacuum have the expression (see Eq. (11))
\[ 0 \rightarrow U^{-1} \partial_{\mu} U + (U^{-1} B_{\mu} U - B_\mu) . \]
Correspondingly, the equation for the Gribov copies of the vacuum in the presence of a background field reads
\[ \partial^\mu \left( U^{-1} \partial_\mu U + (U^{-1} B_\mu U - B_\mu) \right) + g [ B^{\mu}, U^{-1} \partial_\mu U + (U^{-1} B_\mu U - B_\mu) ] = 0 . \]
This, actually, is a system of coupled non-linear partial differential equations. In order to reduce it consistently to a single differential equation a particular hedgehog ansatz can be used [16] (see Appendix A for the details on the vacuum case). This corresponds to the following ansatz for the gauge copy
\[ U = Y^0(x^i) \mathbf{1} + Y^a(x^i) \tau_a, \quad Y^0(x^i) = \cos \alpha(x^i), \quad Y^a(x^i) = \hat{n}^a \sin \alpha(x^i) \]
being $\hat{n}_a$ normalized with respect to the internal metric $\delta_{ab}$ as
\[ \delta_{ab} \hat{n}^a \hat{n}^b = 1 . \]

4.2. Vacuum Gribov copies with $T^3$ topology
Let us analyze the Gribov copies equation in a flat spatial space with $T^3$-topology. Such choice of topology can be very useful in relation with lattice studies [44, 45]. We take the metric
\[ ds^2 = \sum_{i=1}^{i=3} \lambda_i^2 d^2 \phi_i, \]
where the $\lambda_i \in \mathbb{R}$ represents the length of the torus along the $i$-axis and the coordinates $\phi_i \in [0, 2\pi)$ corresponds to the $i$–th factor $S^1$ in $T^3$. In the $T^3$ case, the gauge transformation $U$ is independent of the Euclidean temporal coordinate $x^0$ and is proper when [16]
\[ U(\phi_i + 2m_i \pi) = U(\phi_i), \quad m_i \in \mathbb{Z}, \quad i = 1, 2, 3. \]
The generalized hedgehog ansatz adapted to this topology reads
\[ \alpha = \alpha(\phi_1), \quad \hat{n}^1 = \cos(p\phi_2 + q\phi_3), \quad \hat{n}^2 = \sin(p\phi_2 + q\phi_3), \quad \hat{n}^3 = 0, \]
with $p, q$ arbitrary integers. From this ansatz, the equation (A.1) is reduce to the following single scalar non-linear differential equation (see Appendix A for details),
\[ \frac{d^2 \alpha}{d\phi_1^2} = \xi \sin(2\alpha), \quad \xi = \frac{\lambda_1^2}{2} \left( \frac{p^2}{\lambda_2^2} + \frac{q^2}{\lambda_3^2} + 4 \frac{r_1^2}{g} \right), \]
The norm of the copies $\frac{N[U]}{(2\pi)^2}$, according to (29), in the case $p = q = \lambda_i = 1$ versus the background $r_0$ for $k = 1$ (in red) and $k = 2$ (in green). The solutions of $\alpha(\phi_1)$ fulfil the condition (26).

and, according to (23), the condition

$$\alpha(\phi_1 + 2\pi) = \alpha(\phi_1) + 2\pi k, \quad k \in \mathbb{Z}$$

must be fulfilled. The equation (25) can be reduced to a first order conservation law

$$V = \frac{1}{2} \left[ \left( \frac{d \alpha}{d \phi_1} \right)^2 + \xi \cos(2\alpha) \right] \Rightarrow \phi_1 - \phi_0 = \pm \int_{\alpha(\phi_0)}^{\alpha(\phi_1)} \frac{dy}{\sqrt{2V - \xi \cos(2y)}}, \quad (27)$$

where $\phi_0$ and $V$ are integration constants.

The most enlightening way to see the background’s effect on the Gribov copies is by means of its norm. In fact, as it is well known [46], the weight of a given copy $U$ is related to its norm

$$N[U] = \int_{T^3} d^3x \sqrt{g} Tr \left[ (U^{-1}\partial_\mu U + U^{-1}B_\mu U - B_\mu)^2 \right], \quad (28)$$

where in this case $g$ refers to the determinant of the metric associated to the line element (22), setting the coupling constant to be zero. In particular, the bigger is $N[U]$, the less relevant the copy is from the path integral point of view. As in this case there is a background potential, the integral (28) can be written as

$$N[U] = \frac{(2\pi)^2 \lambda_2 \lambda_3}{\lambda_1} \int_0^{2\pi} d\phi_1 \left( 2V + 3\xi \sin^2\alpha(\phi_1) - \xi \cos^2\alpha(\phi_1) \right), \quad (29)$$

where in the last equality we used the definition (27) of the constant $V$. In Figure 1, we show the norm $N[U]$ for $p = q = \lambda_i = 1$ increases when $r_0$ grows both for $k = 1$ and $k = 2$, at least in the range $r_0 \in (0, 1.0)$, for solutions $\alpha(\phi_1)$ such that fulfil the condition (26) and $\alpha(0) = 0$. Consequently, in this region, the bigger $r_0$ the smaller the importance of Gribov copies of the form considered here. It is necessary more computational power to see how is the behavior of the norm outside the region studied here (for instance, $|p| > 1$ and $|q| > 1$).
5. **Solving the GZ gap equation for SU(2) with constant background field**

In order to determine the gap equation, we will proceed first to show the effective potential to GZ action at one-loop approximation for the SU(2) internal gauge group in the presence of a background potential discussed in Section 3. We will work at a small enough but non-zero temperature, taking into account the background field as

\[ B^\alpha_p = \frac{T}{\rho} r \delta^{\alpha\beta} \delta_{\rho \phi}, \]

(30)

where \( r \) is a dimensionless parameter related to the background field \( r_0 \) defined in (16) as \( r = r_0/T \), with \( T \) the temperature. At a first glance, the above background field should have no physical at all as it is pure gauge. However, the gauge transformation which would remove it is not periodic in Euclidean time. Thus, such a gauge transformation is improper and it is not allowed. The present framework, despite its simplicity, it is still able to disclose in a very clean way the effects of the background gauge potential on the Gribov parameter. We will focus only on the dependence of the Gribov parameter with respect to the background parameter, keeping the temperature constant.

In order to obtain the vacuum energy at one loop, we consider only from (15) the quadratic terms in the fields which are functionally integrated (see (31) for higher order corrections). We find [27]

\[ \varepsilon(r, \lambda^2) = -\frac{d(N^2 - 1)}{2Ng^2} \lambda^4 + \frac{T}{2V} (d - 1) \text{Tr} \ln \frac{D^4 + \lambda^4}{\Lambda^4} - d \frac{T}{2V} \text{Tr} \ln \frac{-D^2}{\Lambda^2}, \]

(31)

where \( V \) is the Euclidean space volume, \( \lambda^4 = 2Ng^2 \gamma^4 \), being \( \gamma \) is the Gribov parameter, \( D \) is the covariant background derivative in the adjoint representation defined in (14), and \( \Lambda^2 \) is a scale parameter in order to regularize the result. We can rewrite (31), taking into account the Cartan subalgebra of SU(2) is one-dimensional and the thermodynamic limit \( V \rightarrow \infty \), as

\[ \varepsilon(r, \lambda^2) = -\frac{d(N^2 - 1)}{2Ng^2} \lambda^4 + \frac{1}{2} (d - 1) \sum_{s = -1}^{s = 1} [I(sr, i\lambda^2) + I(sr, -i\lambda^2)] - d \frac{s}{2} \sum_{s = -1}^{s = 1} I(sr, 0), \]

(32)

where \( s \) is the isospin SU(2), and we defined the function

\[ I(u, m^2) = \frac{T}{V} \text{Tr} \ln \left(-\frac{D^2 + m^2}{\Lambda^2}\right) = T \sum_{n = -\infty}^{+\infty} \int \frac{d^3q}{(2\pi)^3} \ln \left[(2\pi nT + u\tilde{q})^2 + q^2 + m^2\right]. \]

(33)

In the last definition, we expanded in the Fourier space the zero-component momentum in the Matsubara bosonic frequencies \( 2\pi nT \) [47, 48], and \( \tilde{q} \) denotes the spatial momentum vector. We will compute first (33) using similar techniques which were already applied in GZ approach [27, 49], which lead us the result (see details in [27])

\[ I(u, m^2) = \frac{m^4}{32\pi^2} \left[ \ln \left(\frac{m^2}{\Lambda^2}\right) - \frac{3}{2}\right] - \frac{T^2m^2}{\pi^2} \sum_{n = 1}^{+\infty} K_2 \left(\frac{\sqrt{m^2}}{T}\right) \frac{\cos(nu)}{n^2}, \]

(34)

where \( K_2 \) is the modified Bessel function of the second kind extended to the complex plane [50]. Inserting (34) into (32), we have

\[ \varepsilon(r, \lambda^2) = -\frac{d(N^2 - 1)}{2Ng^2} \lambda^4 - \frac{3}{2} \frac{(d - 1)\lambda^4}{32\pi^2} \left[ \ln \left(\frac{\lambda^2}{\Lambda^2}\right) - \frac{3}{2}\right] \]

\[ - \frac{i\lambda^2T^2}{2} (d - 1) \sum_{s = -1}^{s = 1} \sum_{n = 1}^{+\infty} \left[ K_2 \left(\frac{\sqrt{m^2}}{T}\right) - K_2 \left(\frac{\sqrt{m^2}}{T}\right) \right] \frac{\cos(nrs)}{n^2} - d \frac{s}{2} \sum_{s = -1}^{s = 1} I(rs, 0). \]

(35)
The gap equation (36) can be solved using numerical techniques. In Figure 2 (a), it is plotted the left hand side of the gap equation (36) as a function of $\lambda^2$ for different values of $r$. We see the intersection value of the curve (which is the solution for a given value of $r$) decrease when the background $r$ grows, as it is shown more clearly in Figure 2 (b), where it is shown the parameters $\lambda$ which are solution of gap equation at $T/\lambda_0 = 1$ versus $r$. We could interpret this as the theory becomes less confined as the Gribov parameter reduces (see Section 6) at least in the range $r \in [0, 1]$. The present expression of the Gribov parameter is only valid as long as $\partial \lambda^2 / \partial r \neq 0$. The points where the latter derivative vanishes could signal a change on the phase diagram. Thus, when $\partial \lambda^2 / \partial r \neq 0$, the present semi-classical approximation is not valid anymore. Therefore, we have included the plots in Figure 2 only the region in which our approximation can be trusted.

6. Conclusions and perspectives

In the present paper, it has been shown that the Gribov copies equation is affected directly by the presence of a background gauge field. In particular, explicit examples have been constructed...
in which the norm of the Gribov copies satisfying the usual boundary conditions increases when the size of the background field is very large.

The analysis of the semi-classical Gribov gap equation in the chosen background gauge potential and of the dependence of the Gribov mass on the background potential itself, quite consistently, confirms the above results. Namely, we have shown that the larger is the size of the background gauge potential, the smaller is the corresponding Gribov mass.

Moreover, constant background gauge potentials are very important also in relation with the non-Abelian Schwinger effect [31, 32]. Although the constant gauge potentials considered in that references allow a complete analysis of the semi-classical Gribov gap equation, they make extremely difficult to construct explicit examples of Gribov copies.

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Appendix A. Gribov copies with a constant background field
In this appendix we consider the derivations and properties of the equation of gauge-equivalent fields satisfying the LDW gauge in the presence of background field. Our aim is to calculate the following ordinary differential equation

\[ U^{-1} \partial_\mu U + U^{-1} B_\mu U - B_\mu = (Y^0 \partial_\mu Y^c - Y^c \partial_\mu Y^0 + \epsilon_{abc} Y_a \partial_\mu Y_b) \tau_c - 2\gamma_0 \epsilon_{\alpha\beta\gamma} \left( Y_\alpha Y^{\beta} + (Y^0)^2 \right) \tau_c + \frac{2\gamma_0}{g} \delta_{\mu\alpha} Y^\alpha \tau_3. \]

The next step is to apply to this last expression the covariant background derivative and set it to be zero according to (18). This results in the following expression

\begin{align*}
(−Y^0 □ Y^c − Y^c □ Y^0 + \epsilon_{\alpha\beta\gamma} Y_\alpha □ Y^\beta) \tau_c − \frac{2\gamma_0}{g} \left( Y^\gamma Y^3 + Y^\gamma Y^3 + \epsilon_{\alpha\beta\gamma} Y_\alpha Y^\beta \right) \tau_c &
\end{align*}

\[ = -2\gamma_0 \epsilon_{\alpha\beta\gamma} \left( Y_0 Y^b - Y_0 Y^b - Y^c Y^3 + Y^c Y^3 + Y^c Y^3 \right) - \frac{2\gamma_0^2}{g} \left( Y_0 Y^c + \epsilon_{\alpha\beta\gamma} Y_\alpha Y^\beta \right) \tau_c + \frac{4\gamma_0}{g} \psi Y_3 = 0 \]  

where \( □(\ldots) = \partial_\mu \partial^\mu (\ldots) \), and de dot represents the derivative with respect to the component which the background field belongs. In the particular case of flat spatial space \( T^3 \) (22) for the \( Y^\mu \) prescription (20), and for the hedgehog ansatz (24) the set of equations (A.1) reduces to the following ordinary differential equation

\[ \frac{d^2 \alpha}{d\phi^2} - \beta(p, q) \sin(2\alpha) - \frac{2\gamma_0^2}{g} \sin(2\alpha) = 0, \quad \beta(p, q) = \lambda_2^2 \left( \frac{p^2}{\lambda_2^2} + \frac{q^2}{\lambda_3^2} \right). \]

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