Scale Invariance in Heavy Hadron Molecules

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(Dated: January 18, 2018)

We discuss a scenario in which the \( P_c(4450)^+ \) heavy pentaquark is a \( \Sigma_c D^- \Lambda_c(2595) \bar{D} \) molecule. The \( \Lambda_c D \rightarrow \Sigma_c D^- \Lambda_c(2595) \bar{D} \) transition is mediated by the exchange of a pion almost on the mass shell that generates a long-range \( 1/r^2 \) potential. This is analogous to the effective force that is responsible for the Efimov spectrum in three-boson systems interacting through short-range forces. The equations describing this molecule exhibit approximate scale invariance, which is anomalous and broken by the solutions. If the \( 1/r^2 \) potential is strong enough this symmetry survives in the form of discrete scale invariance, opening the prospect of an Efimov-like geometrical spectrum in two-hadron systems. For a molecular pentaquark with quantum numbers \( \frac{3}{2}^- \) the attraction is not enough to exhibit discrete scale invariance, but this prospect might very well be realized in a \( \frac{3}{2}^+ \) pentaquark or in other hadron molecules involving transitions between particle channels with opposite intrinsic parity and a pion near the mass shell. A very good candidate is the \( \Lambda_c(2595) \bar{D}_s - \Sigma_c \Xi_c \) molecule. Independently of this, the \( 1/r^2 \) force is expected to play a very important role in the formation of this type of hadron molecule.

The onset of scale invariance in two-body systems is a remarkable property. It connects a series of seemingly disparate low-energy phenomena in atomic, nuclear and particle physics under the same theoretical description [1]. When the scattering length \( a_0 \) of a two-body system is much larger than any other scale, i.e. \( a_0 \rightarrow \infty \), the system is invariant under the scale transformation \( r \rightarrow \lambda r \) with arbitrary \( \lambda \). The low energy properties of this two-body system can be fully explained independently of the underlying short-range dynamics. That is, few-body systems with a large scattering length admit a universal description. Efimov discovered that three-boson systems exhibit a characteristic three-body spectrum for \( a_0 \rightarrow \infty \), where the binding energy of the states is arranged in a geometric series [2]. The continuous scale invariance of the three-body equations is anomalous and the spectrum only shows discrete scale invariance under the transformation \( r \rightarrow \lambda_0 r \) where the value of \( \lambda_0 \) is now fixed. Conversely if \( E_n \) is the binding energy of a three-body state there is another state with binding \( E_{n+1} = E_n/\lambda_0^2 \), a prediction that was confirmed experimentally with Cs atoms a decade ago [3]. This type of discrete geometrical spectrum also happens in three-body systems containing at least two-identical particles [4], or when the scattering is resonant in higher partial waves [2, 6]. This mechanism might be responsible for the binding of the triton [7], a series of halo nuclei [8, 12] and the Hoyle state [13, 14].

There is a two-body system that is intimately related to the Efimov effect, which is the \( 1/r^2 \) potential. At zero energy the reduced Schrödinger equation for the s-wave becomes

\[
-u''(r) + \frac{g}{r^2} u(r) = 0,
\]

which is obviously scale invariant. The connection with the three-body system is apparent when one realizes that it also contains a similar equation with an effective \( 1/r^2 \) potential in the hyperradius \( \rho \) [15]. For \( g > -1/4 \) the equation above admits power-law solutions of the type

\[
u(r) = c_+ r^{5/2+\nu} + c_- r^{3/2-\nu},
\]

where \( c_\pm \) constants and \( \nu = \sqrt{1/4 + g} \), where scale invariance is lost. For \( g < -1/4 \) we have instead solutions of the type

\[
u(r) = c r^{1/2} \sin(\nu \log \lambda_2 r),
\]

with \( c \) a constant, \( \lambda_2 \) an energy scale that depends on the short-range physics and \( \nu = \sqrt{1/4 - g} \). Now the solutions display discrete scale invariance with \( r \rightarrow \lambda_0 r \), where \( \lambda_0 = e^{\nu/\nu} \). In turn there is a geometric bound state spectrum where \( E_{n+1} = E_n/\lambda_0^2 \), with \( E_n \) and \( E_{n+1} \) the energy of two consecutive states. Here we make the observation that the \( 1/r^2 \) potential can appear in heavy hadron molecules, for instance the \( P_c(4450)^+ \) heavy pentaquark if it happens to be molecular. There might be hadronic systems where the potential might be attractive enough to exhibit discrete scale invariance.

The heavy pentaquarks \( P_c(4380)^+ \) and \( P_c(4450)^+ \), \( P_c^* \) from now on, were discovered by the LHCb [16] and are a recent and interesting addition to a growing family of exotic hidden charm (and bottom) hadrons that began with the \( X(3872) \) more than ten years ago [17]. There is still a lot of discussion regarding the nature of the \( P_c^* \) and \( P_c^* \), from the role of threshold effects [18–21], to baryocharmonia [22], a compact pentaquark [23–26], a heavy baryon-antimeson molecule [27–31] or other

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more exotic possibilities [32, 33]. The \( P^c_c \) is an interesting molecular candidate because its width is not particularly big \( \Gamma = 35 \pm 19 \) MeV and its closeness to the \( \Sigma_c \bar{D}^* \) threshold, see Fig. [11]. As a matter of fact a series of works predicted the possibility of a heavy baryon-antimeson molecule before the discovery of the \( P^c_c \) [34–37]. The probable quantum numbers of the \( P_c \) and \( P^c_c \) are \( \frac{3}{2}^- \) and \( \frac{1}{2}^+ \) respectively, followed by \( \frac{3}{2}^+ \) and \( \frac{3}{2}^- \). The standard molecular explanation for the \( P^c_c \) heavy pentaquark is that of a \( \Sigma_c \bar{D}^* \) bound state, which prefers the quantum number \( \frac{3}{2}^- \) for the \( P^c_c \). Here we discuss the scenario in which the molecular \( P^c_c \) also contains a \( \Lambda_c \bar{D} \) component in addition to \( \Sigma_c \bar{D}^* \), where \( \Lambda_c \) denotes the \( \Lambda_c \) (2595). Burns [38] proposed this idea on the analogy between the \( D \bar{D}^* + D^* \bar{D} \) and the \( Y_c \bar{D} + Y_c \bar{D} \) systems, i.e. the X(3872) and the \( P^c_c \), where \( Y_c, Y^*_c \) are charmed baryons. He argued that the most natural analog to the \( D \bar{D}^* + D^* \bar{D} \) system is \( \Sigma_c \bar{D}^* – \Lambda_c \bar{D} \) on the basis that the mass difference of the \( \Lambda_c \) and \( \Sigma_c \) is very close to the \( D^* \) and \( D \) splitting. Here we will explore this possibility.

The low-energy dynamics of \( \Sigma_c \bar{D}^* – \Lambda_c \bar{D} \) is driven by one pion exchange (OPE) and is fascinating for two reasons. First, if the \( \Lambda_c \bar{D} \) pair exchanges a pion to become a \( \Sigma_c \bar{D}^* \) pair the pion will be almost on the mass shell, leading to a unusual long-range potential for strong interactions. Second, the intrinsic parities of \( \Lambda_c (\frac{1}{2}^-) \) and \( \Sigma_c (\frac{1}{2}^+) \) are different while the ones for the \( D \) and \( D^* \) are the same. As a consequence OPE will switch odd (even) waves in the \( \Lambda_c \bar{D} \) channel to even (odd) waves in the \( \Sigma_c \bar{D}^* \) one. That is, there is a vector force: the analogous \( \Lambda \bar{D} \) potential dominates the long-range dynamics of the system for \( 1/m_c < r < 1/|\mu_c| \). We stress that scale invariance is only approximate and broken by three factors: (i) the pion is not exactly on the mass shell, (ii) the \( \Sigma_c \bar{D}^* \) and \( \Lambda_c \bar{D} \) thresholds are a pair of MeV away from each other and (iii) the widths of the \( \Sigma_c \) and \( \Lambda_c \) baryons. Owing to the exploratory nature of this work we will assume \( \mu_c = 0 \) from now on, which implies overlapping thresholds, and we will ignore the widths. The potential in the \( I = 1/2 \) channel reads

\[
\langle \Lambda_c \bar{D} | V_{\text{OPE}}(\vec{r}) | \Sigma_c \bar{D}^* \rangle \approx \frac{g \mu_c}{4\sqrt{2}} \frac{m_c}{\mu_c} \sqrt{\frac{4}{3} \vec{\epsilon} \cdot \vec{r}}.
\]

If we consider \( J^P = \frac{3}{2}^- \) (the standard quantum numbers for a molecular pentaquark), the partial waves contributing are \( \Sigma_c D^*(2D_3/2), \Sigma_c D^*(4S_3/2), \Sigma_c D^*(4D_3/2) \) and \( \Lambda_c \bar{D}(2P_3/2) \). In this partial wave basis the reduced Schrödinger equation at zero energy reads

\[
-u'' + \left[ 2\mu_{P_c} V_{\text{OPE}} + \frac{L^2}{r^2} \right] u = 0,
\]

where \( u \) is the wave function in vector notation and \( \mu_{P_c} \) the reduced mass of the molecule (actually there is one reduced mass for each particle channel, but here we can take their geometric mean). The combination of the vector OPE potential and the centrifugal barrier reads

\[
2\mu_{P_c} V_{\text{OPE}} + \frac{L^2}{r^2} = \frac{g R \rho}{r^2} = \frac{1}{r^2} \left( \begin{array}{ccc} 6 & 0 & g \\ 0 & 0 & 0 \\ g & -g & 2 \end{array} \right).
\]

That is, we have a four channel version of Eq. [11]. We can diagonalize the matrix \( g \left( \frac{3}{2}^- \right) \), in which case we end up with four equations of the type

\[
-u'' + \frac{g_1}{r^2} u_i = 0,
\]

where the \( g_i \)'s \( (i = 1, 2, 3, 4) \) are the eigenvalues of \( g \left( \frac{3}{2}^- \right) \). There are three positive and one negative eigenvalue

\[
g_i = \{ 6, 2, 3 + \sqrt{9 + 3g^2}, 3 - \sqrt{9 + 3g^2} \},
\]

where the negative one can trigger discrete scale invariance. This will happen if \( |g| > 5 / (4 \sqrt{3}) \approx 0.7217 \). However the value of \( g \) for the \( P^c_c \) molecule is \( g = 0.60 \pm 0.10 \) h2, where we have used \( g_1 = 0.59 \pm 0.01 \pm 0.07 \) from \( D^* \rightarrow D \pi \) and \( D^* \rightarrow D \gamma \) decays [40, 41]. This requires \( h_2 > 1.21^{+0.25}_{-0.19} \), which is well above \( h_2 = 0.60 \pm 0.07 \) from CDF [42] or \( h_2 = 0.63 \pm 0.07 \) from the analysis of Ref. [43], where in both cases \( h_2 \) is extracted from \( \Gamma (\Lambda_c \rightarrow \Sigma_c \pi) \). That is, there is not enough attraction to achieve discrete scale invariance. For the \( \frac{3}{2}^- \) molecule the matrix is different but the attractive eigenvalue is still \( g - \left( \frac{3}{2}^- \right) = 3 - \sqrt{9 + 3g^2} \), requiring \( |g| > 5 / (4 \sqrt{3}) \). The most interesting pentaquark-like molecule is the \( \frac{1}{2}^- \), with partial waves \( \Sigma_c \bar{D}^*(2P_{1/2}), \Sigma_c \bar{D}^*(4P_{1/2}) \) and \( \Lambda_c \bar{D}(2S_{1/2}) \), where

\[
g \left( \frac{1}{2}^- \right) = \left( \begin{array}{ccc} 2 & 0 & g \\ 0 & 2 & -\sqrt{2}g \\ g & -\sqrt{2}g & 0 \end{array} \right).
\]
The attractive eigenvalue is \( g_\pm (k_+^2) = 1 - \sqrt{1 + 3g^2} \), which requires \( |g| > \sqrt{3}/4 \approx 0.4330 \) and \( |h_2| > 0.73^{+0.11-0.06} \), i.e. overlapping with current estimations of \( h_2 \). Finally the \( \frac{3}{2}^+, \frac{5}{2}^+ \) and \( \frac{5}{2}^- \) cases require \( |g| > 7\sqrt{3}/4 \), \( |g| > 7\sqrt{3}/4 \) and \( |g| > 15\sqrt{3}/4 \). That is, the strength of the vector force is in general weak in the pentaquark-like molecules to achieve discrete scale invariance, with the notable exception of \( \frac{1}{2}^+ \) which lies on the limit. Though the \( \frac{1}{2}^+ \) is not the ideal candidate for the \( P_c^* \), it is nonetheless a probable hadronic molecule with interesting properties.

Yet the \( P_c^* \) is not the only system where this can happen. The general conditions for a \( H_1 H_2 \rightarrow H_1' H_2' \) hadronic molecule to have scale invariance are: (i) the hadrons are particularly long-lived, (ii) the mass difference of the hadrons in each vertex is similar to that of a pseudo-Goldstone boson \( m(H_1)' - m(H_1) \approx m(H_2) - m(H_2)' \approx m_P \) (iii) the intrinsic parity of \( H_2 \) and \( H_2' \) is the same, while that of \( H_1 \) and \( H_1' \) is different and (iv) \( H_1 \) and \( H_1' \) have the same spin for the pseudo-Goldstone boson to be emitted in s-wave. Notice that it is not strictly necessary to exchange a pion near the mass-shell to have a long-range \( 1/r^2 \) force. A kaon near the mass-shell will also generate this type of force.

If we have the \( \Lambda_c\Sigma_c \) on the one side, besides the \( D^* \), the \( \Xi_b\Xi_b' \) bottom baryon combination also fulfills the previous conditions, see Fig. 1 for the threshold location. In this regard the \( \Lambda_c\Xi_b\Sigma_c\Xi_b' \) system seems to be the best candidate for a scale invariant molecule in the heavy sector. The \( \Lambda_c\Xi_b \rightarrow \Sigma_c\Xi_b' \) potential for \( I = 1/2 \) reads

\[
\langle \Sigma_c\Xi_b' | \sigma_{\text{OPE}}(\vec{r}) | \Lambda_c\Xi_b \rangle \approx \frac{g_3 h_2 m_\Xi}{8\pi} \frac{2g_\tau}{r^2},
\]

where \( g_3 \) is the axial coupling for the \( \Xi_b\Xi_b\pi \) vertex and \( g_\tau \) is the Pauli matrix for that vertex. If we consider states in which the \( \Sigma_c\Xi_b' \) is in s-wave or alternatively in p-wave where the tensor force is attractive, we have

\[
0^+ = \Sigma_c\Xi_b' (3P_0) - \Lambda_c\Xi_b (1S_0),
\]

\[
0^- = \Sigma_c\Xi_b' (1S_0) - \Lambda_c\Xi_b (3P_0),
\]

\[
1^- = \Sigma_c\Xi_b' (3S_1 - 3D_1) - \Lambda_c\Xi_b (1P_1 - 3P_1).
\]

In these partial wave bases the \( g \) matrices read

\[
g(0^+) = \begin{pmatrix} 2 & g \\ g & 0 \end{pmatrix},
\]

\[
g(0^-) = \begin{pmatrix} 0 & g \\ g & 2 \end{pmatrix},
\]

\[
g(1^-) = \begin{pmatrix} 0 & \frac{1}{\sqrt{3}} g & \frac{1}{\sqrt{3}} g & \frac{1}{\sqrt{3}} g \\ \frac{1}{\sqrt{3}} g & 0 & \sqrt{2} g & -\sqrt{2} g \\ \frac{1}{\sqrt{3}} g & -\sqrt{2} g & 0 & 2 g \\ \frac{1}{\sqrt{3}} g & 2 g & -\sqrt{2} g & 0 \end{pmatrix}.
\]

For \( |g| > 3/4 \) the attractive eigenvalue of the matrices above will trigger discrete scale invariance. The evaluation of \( g \) depends on the axial coupling \( g_3 \), which can be extracted from the \( \Sigma_c \rightarrow \Lambda_c \pi \) decay, yielding \( g_3 = 0.973^{+0.019}_{-0.042} \) \cite{43}. This translates into \( g = 1.12^{+0.03}_{-0.03} h_2 \) requiring \( |h_2| > 0.67^{+0.03}_{-0.02} \), which is within the error of the \( h_2 = 0.63 \pm 0.07 \) \cite{43}.

Even if the vector force is not enough for the molecule to display discrete scale invariance, it can still play an important role in binding owing to the long-range attraction that the \( 1/r^2 \) potential provides. If the binding mechanism is s-wave short-range attraction, a way to test this hypothesis is the following: for \( r \leq R_s \) we will assume that OPE is not valid and model the short-range interaction with a delta-shell instead:

\[
V(r) = V_{\text{OPE}}(r) \theta(r - R_s) + \frac{C_0(R_s)}{4\pi R_s^3} \delta(r - R_s),
\]

where \( R_s \) is the radius at which we expect short-range physics to dominate. We have chosen the delta-shell because it is a convenient regulator for the short-range physics (the conclusions will be similar for other regulators).

Then we calculate the relative strength of the coupling \( C_0 \) required to have a bound state at zero energy in the presence/absence of a vector force. In the one-channel problem of Eq. (11) for \( g > -1/4 \) and in the absence of OPE, the relative strength of \( C_0 \) is \( (1/2 + \nu) \) of that required to bind if \( g = 0 \), while for \( g < -1/4 \) it always binds. (independently of the choice of \( R_s \).) Thus if \( \nu \rightarrow 0 \) \( (g \rightarrow -1/4) \) the short-range potential only has to be half the normal strength to be able to bind the system. If there is OPE there will be a dependence on \( R_s \), as we are assuming a short-range binding mechanism, we have to choose \( R_s \geq R_c \) with \( R_c \) the critical radius at which OPE alone will bind the molecules, which is about 0.8 fm.

\[\text{FIG. 1. Location of the thresholds for the two scale invariant molecule candidates considered in this work, the } \Lambda_c D^* \Sigma_c D^* \text{ and the } \Lambda_c \Xi_b \Sigma_c \Xi_b. \text{ We also show the location of the } P_c^* \text{ for comparison.}\]
for both the $P_c^+$ and $\Lambda_{c1} \bar{\Xi}_b - \Sigma_c \bar{\Xi}_b'$. We take $R_c = 1$ fm, $\mu_c = 0$ and $h_2 = 0.63$ (notice that for $h_2 > 0.73 / 0.67$ the $P_c^+(1^{+-}) / \Lambda_{c1} \bar{\Xi}_b - \Sigma_c \bar{\Xi}_b'$ will always bind for $\mu_c = 0$).

For the $P_c^+(1^{+-})$ we obtain that with the $\Lambda_c D$ the attraction required to bind is 75% of that of a standard $\Sigma_c D^*$ molecule, representing a moderate improvement (where for the $\Sigma_c \Sigma_c \pi$ axial coupling we use $g_3 = -1.38$ [43]).

For the heavy baryonium the numbers are 46% ($0^-$) and 53% ($1^-$) respectively, where the $0^+$ case. Here we have not considered the $P_c^+(1^{+-}) / \Sigma_c \bar{\Xi}_b(0^+)$ as they bind in p-wave. That is, the existence of the $\Lambda_c D$ and $\Lambda_{c1} \bar{\Xi}_b$ channels reduce the s-wave attraction required to bind by one fourth and a half respectively. The conclusion is that the vector force can be a really important mechanism in the formation of this type of hadronic molecules.

Scale invariant hadron molecules are an intriguing theoretical possibility. They are the two-body realization of a type of universality that is usually only found in three-body atomic and nuclear systems. There are clear theoretical requirements for a hadron molecule to show scale invariance at long distances, where the most natural mechanism is the exchange of a pion almost on the mass shell between initial and final two-hadron states with opposite intrinsic parity. If we consider heavy hadrons, the candidates include $\Lambda_{c1} D^* - \Sigma_c D$, i.e. the molecular interpretation of the recently discovered $P_c^+$ pentaquark state, while the most likely scale invariant molecule is probably the $\Lambda_{c1} \bar{\Xi}_b - \Sigma_c \bar{\Xi}_b'$ baryonium. Discrete scale invariance requires that the couplings have a minimal strength, a condition that a $1/2^+$ heavy pentaquark and a $\Lambda_{c1} \bar{\Xi}_b - \Sigma_c \bar{\Xi}_b'$ molecule can meet. Even if there is no geometrical spectrum in these molecules, the long-range attraction provided by the vector force might very well play an important role as a binding mechanism, which is indeed an analogous situation to that of the triton, a few halo nuclei and a series of cold atoms systems, to name a few examples.

\section*{ACKNOWLEDGMENTS}

This work is partly supported by the National Natural Science Foundation of China under Grants No. 11375024, No.11522539 and the Fundamental Research Funds for the Central Universities.

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