Magnetization study on the field-induced quantum critical point in YbRh$_2$Si$_2$

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Abstract. We study the field-induced quantum critical point (QCP) in YbRh$_2$Si$_2$ by low-temperature magnetization, $M(T)$, and magnetic Grüneisen ratio, $\Gamma_{\text{mag}}$, measurements and compare the results with previous thermal expansion, $\beta(T)$, and critical Grüneisen ratio, $\Gamma_{\text{cr}}(T)$, data on YbRh$_2$(Si$_{0.95}$Ge$_{0.05}$)$_2$. In the latter case, a slightly negative chemical pressure has been used to tune the system towards its zero-field QCP. The magnetization derivative $-dM/dT$ is far more singular than thermal expansion, reflecting a strongly temperature dependent pressure derivative of the field at constant entropy, $(dH/dP)_S = \frac{V_m\beta}{(dM/dT)}$ ($V_m$: molar volume), which saturates at $(0.15\pm0.04)$ T/GPa for $T \to 0$. The line $T^*(H)$, previously observed in Hall- and thermodynamic measurements, separates regimes in $T$-$H$ phase space of stronger ($\epsilon > 1$) and weaker ($\epsilon < 1$) divergent $\Gamma_{\text{mag}}(T) \propto T^{-\epsilon}$.

1. Introduction

Quantum criticality in YbRh$_2$Si$_2$ (YRS) has recently attracted considerable interest because of divergences of the quasiparticle (qp) effective mass and qp-qp scattering cross-section [1, 2], as well as a drastic change of the Hall-coefficient [3] upon tuning through the quantum critical point (QCP), which is also accompanied by signatures in thermodynamic and magnetic properties [4]. Undoped YRS at ambient pressure shows very weak antiferromagnetic (AF) order below $T_N = 70$ mK, which could be tuned continuously down to $T = 0$ by either application of a small magnetic field or by a tiny volume expansion (negative chemical pressure) in YbRh$_2$(Si$_{0.95}$Ge$_{0.05}$)$_2$. Important advantages of using magnetic field compared to chemical pressure for tuning the system towards the QCP are that fields could be changed continuously and no disorder is introduced by doping.

Recently, the Grüneisen ratio, $\Gamma \propto \beta/C$, where $\beta$ and $C$ denote the volume thermal expansion and specific heat, respectively, has been identified as the most sensible probe of the nature of quantum criticality for pressure-driven QCPs and correspondingly, the magnetic Grüneisen ratio, $\Gamma_{\text{mag}} = -(dM/dT)/C$ ($M$: magnetization) for the field-induced case [5, 6]. For Ge-doped YRS, the critical contribution to the Grüneisen ratio has been found to diverge with fractional exponent, $\Gamma_{\text{cr}} \propto T^{-0.7}$, below 0.3 K [7]. This temperature dependence is in strong disagreement with the prediction of the traditional scenario for an itinerant AF QCP [5] and has been taken as evidence for local quantum criticality in this system [7]. Very recently, a detailed study of the magnetic Grüneisen ratio in the $T$-$H$ plane of the phase diagram of undoped YRS has been reported [8]. $\Gamma_{\text{mag}}$ diverges when approaching the QCP either at $H = H_c$ as a function of temperature or within the Landau-Fermi liquid regime (as $T \to 0$) at $H > H_c$, when tuning the
field towards $H_c$. For the latter case, $\Gamma_{\text{mag}}(T \to 0) = -0.3/(H - H_c)$ has been found. Such a $(H - H_c)^{-1}$ dependence is expected from a scaling ansatz for a field-driven QCP. Furthermore, the prefactor should equal the exponent of the divergence of the specific heat coefficient, which is indeed fulfilled [8]. In contrast to this rather clear situation in the approach of the QCP at $H > H_c$ by reduction of the field towards $H_c$, more complicated behavior has been found for the temperature dependence at $H = H_c$ [8].

At high temperatures, an unexpectedly strong divergence $\Gamma_{\text{mag}} \propto T^{-2}$ is found with a crossover at about 0.3 K towards a weaker $T^{-0.7}$ divergence. In this paper, we discuss how this complicated behavior may arise as consequence of the additional low-energy scale $T^*(H)$, previously observed in Hall- and thermodynamic measurements [3, 4]. We also investigate the relation between chemical-pressure and field tuning in YRS by comparing the temperature derivative of the magnetization with the volume thermal expansion.

2. Experimental

High-quality single crystals ($\rho_0 = 1 \, \mu\Omega\text{cm}$) were grown from In-flux as described earlier [9]. The DC magnetization was measured utilizing a high-resolution capacitive Faraday magnetometer [10]. In this paper, we analyze magnetic Grüneisen parameter results from [8] and compare them with thermal expansion data from [7].

3. Results and discussion

![Figure 1](image1.png)

**Figure 1.** Temperature derivative of the magnetization $-dM/dT$ of YbRh$_2$Si$_2$ at $H=0.06$ T (left axis) and volume thermal expansion $-\beta$ of YbRh$_2$(Si$_{0.95}$Ge$_{0.05}$)$_2$ [7] at zero field (right axis) as a function of temperature (on a logarithmic scale).

![Figure 2](image2.png)

**Figure 2.** $V_m\beta/(dM/dT)$ as a function of temperature. Grey and black lines indicate a third polynomial and a linear dependence (fitted below 0.3 and 0.2 K), respectively. Inset: Pressure dependence of AF critical field of YbRh$_2$Si$_2$ [11]. Solid line represents $H_N(P) = a + bP + cP^2$ with initial slope $b = 0.11$ T/GPa.

At first, we compare the temperature derivative of the magnetization, which equals the field derivative of the entropy, for undoped YRS at $H = H_c$ with the volume thermal expansion for Ge-doped YRS near the critical chemical pressure. The latter property equals the pressure-derivative of the entropy, so the ratio $V_m\beta/(dM/dT) = (dH/dP)_S$ ($V_m$: molar volume) equals the pressure derivative of the field at constant entropy. A comparison of the two former properties is shown in Figure 1, whereas their ratio is displayed in the main part of Figure 2. This ratio is temperature dependent and decreases with decreasing $T$, extrapolating to a value of $(0.15 \pm 0.04)$ T/GPa where the lower and upper boundaries are given by either a linear or a
third polynomial extrapolation as indicated by the black and gray lines, respectively. This value could be compared with a measurement of the pressure dependence of the critical field of AF ordering in YRS, derived from magnetization measurements under hydrostatic pressure [11]. As shown in the inset of Figure 2, the initial pressure derivative of the Néel field is derived as 0.11 T/GPa which is close the lower bound of $(dH/dP)_S$ derived from the ratio of thermal expansion to $dM/dT$, proving the consistency of our thermodynamic data.

The temperature dependences of $dM/dT$ for undoped YRS at $H = H_c$ and $\beta(T)$ at $H = 0$ for Ge-doped YRS are quite different, in particular at high temperatures. Whereas the former strongly increases upon cooling to below 4 K, reverse behavior is found in the latter property, resulting in a strongly temperature dependent $(dH/dP)_S$. For the analysis of the critical Grüneisen ratio $\Gamma^{cr}$ at low temperatures [7], a substantial constant "background contribution" in thermal expansion divided by temperature has been subtracted whereas for the study of the magnetic Grüneisen ratio no background in $(dM/dT)/T$ needs to be subtracted, since a possible constant contribution is negligibly small [8]. Both, the critical "thermal" Grüneisen ratio $\Gamma^{cr}(T)$, as well as the magnetic Grüneisen ratio, $\Gamma_{mag}(T)$, display a crossover towards a $T^{-0.7}$ divergence below 0.3 K, which for the former property has been taken as evidence for local quantum criticality [7]. The low-$T$ magnetic susceptibility and specific heat coefficient of YRS also display crossovers at 0.3 K [2]. The analysis of the exponent of the magnetic Grüneisen ratio presented in the following, suggests that these crossovers are related to the additional low-energy scale $T^*(H)$.

**Figure 3.** Contour plot in $T$-$H$ phase space for the power-law exponent of the magnetic Grüneisen ratio in YbRh$_2$Si$_2$ ($H \perp c$). Positions of $T^*(H)$ determined by $\Delta M/\Delta T(H)$ [8], susceptibility and magnetostriction [4], as well as Hall effect [3] are plotted for comparison. The black and red lines represent the boundary of the AF state and the cross-over towards Fermi-liquid behavior in the electrical resistivity, respectively [4].
Figure 3 displays the evolution of the magnetic Grüneisen ratio exponent $\epsilon$, derived from $\Gamma_{\text{mag}}(T) \propto T^{-\epsilon}$, in the temperature-field phase diagram of YRS. Within the Fermi-liquid regime, $\epsilon = 0$ is predicted [5 6]. We note, that this regime (cf. the violet colored region in Fig. 3) increases roughly linearly with increasing $H$ and extends to temperatures about twice as large than $T_{\text{FL}}(H)$ as derived from the resistivity measurements [4]. The boundary of the Fermi liquid regime is known to be different for different physical quantities.

The scaling analysis predicts a divergence of the magnetic Grüneisen ratio according to $\Gamma_{\text{mag}} \propto T^{-1/\nu z}$ within the quantum critical regime ($\nu$ and $z$ are correlation-length exponent and dynamical exponent, respectively). However, $\Gamma_{\text{mag}}(T)$ at $H = 0.06 \, \text{T}$ does not show a single power-law exponent, but rather displays a strong temperature dependence of the exponent. Most interestingly, $\epsilon \approx 1$ is found close to $T^*(H)$, whereas larger and smaller values are found at $T > T^*(H)$ and $T < T^*(H)$, respectively. The broad region (cf. the green colored region in Fig. 3) where this change of the exponent takes place suggests that the 0.3 K crossovers in the various physical properties at $H = 0$ and $H = H_c$ [2] may also result as consequence of $T^*(H)$. Recently, a critical Fermi surface model for a local QCP has been proposed, in which the electronic criticality is described by $\nu = 2/3$, $z = 3/2$, and $d = 1$ [12]. In our previous study, from the measured $\Gamma_{\text{mag}}$ we derived the quantity $G_r = \nu(d - z) = -0.3$, which is in good agreement with $G_r = -1/3$ from the critical Fermi surface model. Furthermore, this model predicts $\epsilon = 1/\nu z = 1$. Remarkably, the $T^*(H)$ line traces a constant-exponent region with $\epsilon \sim 1$.

In conclusion, our comparison of the temperature derivative of the magnetization for undoped YRS with the thermal expansion of Ge-doped YRS provides information on the pressure derivative of the field at constant entropy, $(dH/dP)_{S}$. This property is strongly temperature dependent and saturates for $T \rightarrow 0$ at $(0.15 \pm 0.04) \, \text{T/GPa}$. The contour plot of the magnetic Grüneisen ratio suggests that the temperature dependence within the non-Fermi liquid region of the $T$-$H$ phase diagram is strongly affected by $T^*(H)$, which may explain the crossover scales near 0.3 K observed in various magnetic and thermodynamic properties. Our thermodynamic data are compatible with the recently proposed critical Fermi surface model [12].

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4. References

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