Quark distributions in polarized $\rho$ meson and its comparison with those in pion

A.G. Oganesian*
Institute of Theoretical and Experimental Physics,
B.Cheremushkinskaya 25, Moscow,117218,Russia

Abstract

Valence quark distributions in transversally and longitudinally polarized $\rho$ mesons in the region of intermediate $x$ are obtained by generalizied QCD sum rules. Power corrections up to $d=6$ are taken into account. Comparison of the results for $\pi$ and $\rho$ mesons shows, that polarization effects are very significant and SU(6) symmetry of distribution functions is absent. The strong suppression of quark and gluon sea distributions in longitudinally polarized $\rho$ mesons is found.

*e-mail address: armen@vitep5.itep.ru
1 Introduction

Structure functions are one of the most significant characteristics of the inner structure of hadrons. For nucleon and pion there are experimental results (see, e.g. [1]-[4] for the nucleon, and [5]-[7], for the pion). For other hadrons, one should use some models based on additional suppositions about inner structure. A very significant question about quark distribution dependence on hadron polarization has no model independent answer (moreover in various models it is usually simply supposed that there is no significant influence).

That is why determination of quark distribution functions in a model-independent way in QCD sum rules based only on QCD and the operator product expansion (OPE) seems to be very important task especially for polarized hadron (in this talk we will discuss polarized $\rho$ meson case).

QCD sum rules for valence quark distribution at intermediate $x$ was suggested in [8] and developed in [9]-[11]. The method was based on the fact, that the imaginary part (in $s$-channel) of 4-point correlator, corresponding to the forward scattering of a virtual photon on the current with the quantum numbers of hadron of interest is dominated by contribution of small distances (at intermediate $x$) if the virtualities of the photon and hadronic current ($q^2$ and $p^2$, respectively) are large and negative $|q^2| \gg |p^2| \gg R_c^2$, where $R_c$ is the confinement radius. So the operator product expansion in this $x$ region is applicable. Then, comparing dispersion representation of the forward amplitude in terms of physical states with that, calculated in OPE and using Borel transformation to suppress higher resonance contributions, one can find quark distribution functions at intermediate $x$. Unfortunately, the accuracy of the sum rules for nucleon, obtained in [9, 11], are bad, (especially for $d$-quark), moreover it was found to be impossible to calculate quark distributions in the $\pi$ and $\rho$ mesons in this way. The reason is that the sum rules in the form used in [9, 11] have a serious drawback.

This drawback comes from the fact that contribution of nondiagonal transitions wasn’t suppressed in sum rules by borelization, and special additional procedure, which was used in [9, 11] to supress them is incorrect for such sum rules. There is no time to discuss this in details, the full analisys can be found in our papers [12] and [13]. In this papers the modified method of calculating of the hadron structure functions (quark distributions in hadrons) was suggested. The problem of supressing the nondiagonal terms is eliminated and valence quark distributions in pion and polarized $\rho$ meson were calculated.

2 The method

Let me briefly present the method. Consider the non-forward 4-point correlator

$$\Pi(p_1, p_2; q, q') = -i \int d^4x d^4y d^4z e^{ip_1x+iqy-ip_2z} \times \langle 0 | T \{ j^h(x), j^{el}(y), j^{el}(0), j^h(z) \} | 0 \rangle$$

(1)

Here $p_1$ and $p_2$ are the initial and final momenta carried by hadronic the current $j^h$, $q$ and $q' = q + p_1 - p_2$ are the initial and final momenta carried by virtual photons and Lorentz indices are omitted. It will be very essential for us to consider unequal $p_1$ and $p_2$ and treat $p_1^2$ and $p_2^2$ as two independent variables. However, we may put $q^2 = q'^2$ and $t = (p_1 - p_2)^2 = 0$. The general
form of the double dispersion relation (in $p_1^2, p_2^2$) of the imaginary part of $\Pi(p_1^2, p_2^2, q^2, s)$ in the $s$ channel has the form

$$Im\Pi(p_1^2, p_2^2, q^2, s) = a(q^2, s) + \int_0^\infty \frac{\varphi(q^2, s, u)}{u - p_1^2} du + \int_0^\infty \frac{\varphi(q^2, s, u)}{u - p_2^2} du$$

$$+ \int_0^\infty du_1 \int_0^\infty du_2 \frac{\rho(q^2, s, u_1, u_2)}{(u_1 - p_1^2)(u_2 - p_2^2)}$$

(2)

Double Borel transformation in $p_1^2$ and $p_2^2$ to (2) eliminates three first terms and we have

$$\mathcal{B}_{M_1^2} \mathcal{B}_{M_2^2} Im\Pi(p_1^2, p_2^2, q^2, s) = \int_0^\infty du_1 \int_0^\infty du_2 \rho(q^2, s, u_1, u_2) \exp \left[ - \frac{u_1}{M_1^2} - \frac{u_2}{M_2^2} \right]$$

(3)

where $M_1^2$ and $M_2^2$ are the squared Borel mass. The integration region with respect to $u_1, u_2$ may be divided into four areas:

I. $u_1 < s_0, \quad u_2 < s_0$;
II. $u_1 < s_0, \quad u_2 > s_0$;
III. $u_1 > s_0, \quad u_2 < s_0$;
IV. $u_1, u_2 > s_0$.

Here $s_0$ is the continuum threshold in the standard QCD sum rule model of the hadronic spectrum with one lowest resonance plus continuum. Area I obviously corresponds to the resonance contribution and the spectral density in this area can be written as

$$\rho(u_1, u_2, x, Q^2) = g_h^2 \cdot 2\pi F_2(x, Q^2) \delta(u_1 - m_h^2) \delta(u_2 - m_h^2),$$

(4)

where $g_h$ is defined as

$$\langle 0 | j_h | h \rangle = g_h$$

(5)

(For simplicity we consider the case of the Lorentz scalar hadronic current. The necessary modifications for the $\pi$ and $\rho$ mesons will be presented below). If the structure, proportional to $P_\mu P_\nu [P_\mu = (p_1 + p_2)_\mu/2]$ is considered in $Im\Pi(p_1, p_2, q, q')$, then in the lowest twist approximation $F_2(x, Q^2)$ is the structure function of interest.

In area IV, where both variables $u_{1,2}$ are far from the resonance region, the nonperturbative effects may be neglected and, as usual in the sum rules, the spectral function of a hadron state is described by the bare loop spectral function $\rho^0$ in the same region:

$$\rho(u_1, u_2, x) = \rho^0(u_1, u_2, x)$$

(6)

In areas II and III one of the variables is far from the resonance region, but another is in the resonance region, and the spectral function in this region is some unknown function $\rho = \psi(u_1, u_2, x)$, which corresponds to the transitions like $h \to \text{continuum}$.

Taking all these facts into account, the physical side of the sum rule (3) can be rewritten as

$$\hat{B}_1 \hat{B}_2 [Im\Pi] = 2\pi F_2(x, Q^2) \cdot g_h^2 e^{-m_h^2(M_1^2 + M_2^2)} + \int_0^{s_0} du_1 \int_0^\infty du_2 \psi(u_1, u_2, x) e^{-(u_1^2 + u_2^2)/(M_1^2 + M_2^2)}$$

3
\[
+ \int_{s_0}^{\infty} du_1 \int_{s_0}^{\infty} du_2 \psi(u_1, u_2, x) e^{-\frac{u_1^2 + u_2^2}{M_1^2}} + \int_{s_0}^{\infty} du_1 \int_{s_0}^{\infty} du_2 \rho^0(u_1, u_2, x) e^{-\frac{u_1^2 + u_2^2}{M_2^2}}
\] (7)

In what follows, we put \( M_1^2 = M_2^2 \equiv 2M^2 \). (As was shown in [4], the values of the Borel parameters \( M_1^2, M_2^2 \) in the double Borel transformation are about twice as large as those in the ordinary ones).

One of the advantages of this method is that after the double Borel transformation the unknown contributions of II and III areas [the second and the third terms in (7)] are exponentially suppressed. So, and we would like to emphasise this, we do not need in any additional artificial procedure such as the differentiation with respect to the Borel mass. Using standard duality arguments, we estimate the contribution of all nonresonance region (i.e., areas II, III, IV) as the contribution of the bare loop in the same region and demand their value to be small (less than 30%). Finally, equating the physical and QCD representations of \( \Pi \) one can write the following sum rules:

\[
Im \Pi_0^{QCD} + \text{Power correction} = 2\pi F_2(x, Q^2) g_h^2 e^{-m_h^2 \left( \frac{1}{M_1^2} + \frac{1}{M_2^2} \right)}
\]

\[
Im \Pi_0^{QCD} = \int_{s_0}^{\infty} \int_{s_0}^{\infty} \rho^0(u_1, u_2, x) e^{-\frac{u_1 + u_2}{2M^2}}
\] (8)

It is worth mentioning that if we consider the forward scattering amplitude from the very beginning, (i.e. put \( p_1 = p_2 = p \) as in [8]-[11]) and perform the Borel transformation in \( p^2 \), then the contributions of the second and third terms in (2), in contrast to (7), would not be suppressed comparing with the lowest resonance contribution we are interesting in. They correspond to the nondiagonal transition matrix elements discussed in the Introduction.

3 Quark distributions in pion

Let me briefly discuss the main points of the calculation and the results for the pion structure function, which can be treated as a check of the accuracy of the method due to fact that for pion the experimental results are available. To find the pion structure function by the method, described in the previous section, one should consider the imaginary part of 4-point correlator (1) with two axial and two electromagnetic currents. Since \( \bar{d}(x) = u(x) \), it is enough to find the distribution of the valence \( u \)-quark in \( \pi^+ \). The most suitable choice of the axial current is

\[
\bar{u}_\mu = \bar{u} \gamma_\mu \gamma_5 d
\] (9)

The electromagnetic current is chosen as \( u \)-quark current with the unit charge

\[
\bar{u}_\mu = \bar{u} \gamma_\mu u
\] (10)

The most convenient tensor structure, which is chosen to construct the sum rule, is a structure proportional to \( P_\mu P_\nu P_3 P_\sigma / \nu \).

The sum rule for the valence \( u \)-quark distribution in the pion in the bare loop approximation is [12]:

\[
Im \Pi_0^{QCD} + \text{Power correction} = 2\pi F_2(x, Q^2) g_h^2 e^{-m_h^2 \left( \frac{1}{M_1^2} + \frac{1}{M_2^2} \right)}
\]

\[
Im \Pi_0^{QCD} = \int_{s_0}^{\infty} \int_{s_0}^{\infty} \rho^0(u_1, u_2, x) e^{-\frac{u_1 + u_2}{2M^2}}
\] (8)

It is worth mentioning that if we consider the forward scattering amplitude from the very beginning, (i.e. put \( p_1 = p_2 = p \) as in [8]-[11]) and perform the Borel transformation in \( p^2 \), then the contributions of the second and third terms in (2), in contrast to (7), would not be suppressed comparing with the lowest resonance contribution we are interesting in. They correspond to the nondiagonal transition matrix elements discussed in the Introduction.
\[ u_\pi(x) = \frac{3}{2\pi^2} \frac{M^2}{f_\pi^2} x (1 - x) (1 - e^{-s_0/M^2}) e^{m_0^2/M^2}, \]  

where \( s_0 \) is the continuum threshold. In [12] the following corrections to Eq. (11) were taken into account:

1. Leading order (LO) perturbative corrections proportional to \( \ln(Q^2/\mu^2) \), where \( \mu^2 \) is the normalization point. In what follows, the normalization point will be chosen to be equal to the Borel parameter \( \mu^2 = M^2 \).

2. Power corrections – higher order terms of OPE. Among them, the dimension-4 correction, proportional to the gluon condensate \( \langle 0 | \alpha_s \pi G_{\mu\nu}^a G_{\mu\nu}^b | 0 \rangle \) was first taken into account, but it was found that the gluon condensate contribution to the sum rule vanishes after double borelization. There are two types of vacuum expectation values of dimension 6. One, involving only gluonic fields:

\[ \frac{g_s}{\pi} \alpha_s f^{abc} \langle 0 | G_{\mu\nu}^a G_{\nu\lambda}^b G_{\lambda\mu}^c | 0 \rangle \]  

and the other, proportional to the four-quark operators

\[ \langle 0 | \bar{\psi} \Gamma \psi \cdot \bar{\psi} \Gamma \psi | 0 \rangle \]  

It was shown in [12] that terms of the first type cancel in the sum rule and only terms of the second type survive. For the latter, one may use the factorization hypothesis which reduces all the terms of this type to the square of the quark condensate.

A remark is in order here. As was mentioned in the Introduction, the present approach is invalid at small and large \( x \). No-loop 4-quark condensate contributions are proportional to \( \delta(1 - x) \) and, being outside of the applicability domain of the approach, cannot be taken into account. In the same way, the diagrams, which can be considered as radiative corrections to those, proportional to \( \delta(1 - x) \) must be also omitted.

All dimension-6 power corrections to the sum rule were calculated in [12] and the final result is given by (the pion mass is neglected):

\[ x u_\pi(x) = \frac{3}{2\pi^2} \frac{M^2}{f_\pi^2} x^2 (1 - x) \left[ 1 + \left( \frac{a_s(M^2) \cdot \ln(Q_0^2/M^2)}{3\pi} \right) \times \left( \frac{1 + 4x \ln(1 - x)}{x} - \frac{2(1 - 2x) \ln x}{1 - x} \right) \right] (1 - e^{-s_0/M^2}) \]

\[ - \frac{4\pi \alpha_s(M^2) \cdot 4\pi \alpha_s a^2}{(2\pi)^4 \cdot 3^2 \cdot 2^6 \cdot M^6} \cdot \frac{\omega(x)}{x^3(1 - x)^3}, \]  

where \( \omega(x) \) is the fourth degree polynomial in \( x \),

\[ a = -(2\pi)^2 \langle 0 | \bar{\psi} \psi | 0 \rangle \]

\[ \omega(x) = -5784x^4 - 1140x^3 - 20196x^2 \]

\[ + 20628x - 8292 \ln(2) + 4740x^4 + 8847x^3 \]

\[ + 2066x^2 - 2553x + 1416 \]
The function \( u_\pi(x) \) may be used as an initial condition at \( Q^2 = Q^2_0 \) for solution of QCD evolution equations (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations).

In the numerical calculations we choose: the effective \( \Lambda_{QCD}^{LO} = 200 \text{ MeV}, \ Q^2_0 = 2 \text{ GeV}^2, \ a_s(a(1\text{GeV}^2) = 0.13 \text{ GeV}^6 \). The continuum threshold was varied in the interval \( 0.8 < s_0 < 1.2 \text{ GeV}^2 \) and it was found, that the results depend only slightly on its variation. The analysis of the sum rule (14) shows, that it is fulfilled in the region \( 0.15 < x < 0.7; \) the power corrections are less than 30\% and the continuum contribution is small (\(< 25\%\)). The stability in the Borel mass parameter \( M^2 \) dependence in the region \( 0.4 \text{GeV}^2 < M^2 < 0.6 \text{GeV}^2 \) is good. The result of our calculation of the valence quark distribution \( xu_\pi(x,Q^2_0) \) in the pion is shown in Fig. 1. In Fig. 1, we plot also the valence \( u \)-quark distribution found in [6] by fitting the data on the production of \( \mu^+\mu^- \) and \( e^+e^- \) pairs in pion-nucleon collisions (Drell-Yan process). When comparing with the distribution found here it should be remembered, that the accuracy of our curve is of order of 10-20\%, (see discussion in Section 5). The \( u \)-quark distribution found from the experiment is also not free from uncertainties (at least 10-20\%, see [5] - [7]). For all these reasons, we consider the agreement of two curves as being good.

Assume, that \( u_\pi(x) \sim 1/\sqrt{x} \) at small \( x < \sim 0.15 \) according to the Regge behaviour and \( u_\pi(x) \sim (1-x)^2 \) at large \( x > \sim 0.7 \) according to quark counting rules. Then, matching these functions with (14), one may find the numerical values of the first and the second moments of the \( u \)-quark distribution:

\[
\mathcal{M}_1 = \int_0^1 u_\pi(x)dx \approx 0.84 \ (0.85) \tag{17}
\]

\[
\mathcal{M}_2 = \int_0^1 xu_\pi(x)dx \approx 0.21 \ (0.23) \tag{18}
\]

where the values in the parentheses correspond to behavior \( u_\pi(x) \sim (1-x) \) at large \( x \). The results depend only slightly on the matching points (not more than 5\%, when the lower matching point is varied in the region 0.15 - 0.2 and the upper one in the region 0.65 - 0.75). The moment \( \mathcal{M}_1 \) has the meaning of the number of \( u \) quarks in \( \pi^+ \) and it should be \( \mathcal{M}_1 = 1 \). The deviation of (17) from 1 characterizes the accuracy of our calculation. The moment \( \mathcal{M}_2 \) has the meaning of the pion momentum fraction carried by the valence \( u \) quark. Therefore, the valence \( u \) and \( \bar{d} \) quarks carry about 40\% of the total momentum.

The valence \( u \)-quark distribution in the pion was calculated recently in the instanton model [15]. At intermediate \( x \), the values of \( xu_\pi(x) \) found in [15] are not more than 20\% higher that our results. Recently, the pions valence quark momentum distribution using a model, based on the Dyson-Schwinger equation was also calculated [16]. Our results are in a reasonable agreement with the results of [16]. Our estimation (18) of the second moment of the valence quark distribution can be also compared with the calculation of the second moment of the total (valence plus sea) quark distribution in the pion [17]. The value, obtained in [17], is 0.6 for the total momentum carried by all quarks (valence plus sea) with the accuracy about 10\%. Taking into account, that sea quarks usually supposed to carry 15\% of the total momentum, one can estimate from the result of [17], that the second moment of one valence quark distribution should be about 20-22\%, which is in a good agreement with our result (18). The quark distribution in the pion was calculated also in [18] by using sum
rules with nonlocal condensates. Unfortunately it is impossible to perform the comparison directly, because the quark distribution is calculated in [18] only at very low normalization point. But, comparing his result with different models, the author of this paper arrived at the conclusion, that the result is in agreement with experimental data at $Q^2=20 \text{ GeV}^2$ within the accuracy about 20%. Our result is close also to experimental fit, so we can believe that our results are in an agreement with those of [18].

4 Quark distributions in $\rho$ meson

Let us calculate valence $u$-quark distribution in the $\rho^+$ meson. The choice of hadronic current is evident

$$j_{\rho}^\mu = \bar{u} \gamma_\mu d$$

(19)

The matrix element $\langle \rho^+ | j_{\rho}^\mu | 0 \rangle$ is given by

$$\langle \rho^+ | j_{\rho}^\mu | 0 \rangle = \frac{m_{\rho}^2}{g_\rho} e_\mu$$

(20)

where $m_\rho$ is the $\rho$-meson mass, $g_\rho$ is the $\rho - \gamma$ coupling constant, $g_\rho^2/4\pi = 1.27$ and $e_\mu$ is the $\rho$ meson polarization vector. Consider the coordinate system, where the collision of the $\rho$-meson with momentum $p$ and the virtual photon with momentum $q$ proceeds along $z$-axes. Averaging over $\rho$ polarizations is given by the formulas:

$$e^L_\mu e^L_\nu = \left( q_\mu - \frac{\nu p_\mu}{m^2_\rho} \right) \left( q_\nu - \frac{\nu p_\nu}{m^2_\rho} \right) \frac{m^2_\rho}{\nu^2 - q^2 m^2_\rho}$$

(21)

for longitudinally polarized $\rho$ and

$$\sum_T e^T_\mu e^T_\nu = -\left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{m^2_\rho} \right) - \frac{m^2_\rho}{\nu^2 - q^2 m^2_\rho} \left( q_\mu - \frac{\nu p_\mu}{m^2_\rho} \right) \left( q_\nu - \frac{\nu p_\nu}{m^2_\rho} \right)$$

(22)

for transversally polarized $\rho$.

The imaginary part of the forward $\rho - \gamma$ scattering amplitude $W_{\mu\nu\lambda\sigma}$ (before multiplication by $\rho$ polarizations) satisfies the equations $W_{\mu\nu\lambda\sigma} q_\mu = W_{\mu\nu\lambda\sigma} q_\nu = W_{\mu\nu\lambda\sigma} p_\lambda = W_{\mu\nu\lambda\sigma} p_\sigma = 0$, which follow from current conservation. The indices $\mu, \nu$ refer to the initial and final photon; $\lambda, \sigma$ – to the initial and final $\rho$. The general form of $W_{\mu\nu\lambda\sigma}$ is:

$$W_{\mu\nu\lambda\sigma} = \left[ \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \left( \delta_{\lambda\sigma} - \frac{p_\lambda p_\sigma}{m^2_\rho} \right) A - \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \left( q_\lambda - \frac{\nu p_\lambda}{m^2_\rho} \right) \left( q_\sigma - \frac{\nu p_\sigma}{m^2_\rho} \right) B \right. \right.$$

$$- \left( p_\mu - \frac{\nu q_\mu}{q^2} \right) \left( p_\nu - \frac{\nu q_\nu}{q^2} \right) \left( \delta_{\lambda\sigma} - \frac{p_\lambda p_\sigma}{m^2_\rho} \right) C + \left( p_\mu - \frac{\nu q_\mu}{q^2} \right) \left( p_\nu - \frac{\nu q_\nu}{q^2} \right)$$

$$\left. \times \left( q_\lambda - \frac{\nu p_\lambda}{m^2_\rho} \right) \left( q_\sigma - \frac{\nu p_\sigma}{m^2_\rho} \right) D \right]$$

(23)
where \( A, B, C \) and \( D \) are the invariant functions. By averaging Eq. \((23)\) over polarizations for longitudinal and transverse \( \rho \) mesons, one can find that the structure function \( F_2(x) \) proportional to \( p_\mu p_\nu \) is given in the scaling limit \((\nu^2 \gg |q^2| < m_\pi^2)\) by the contribution of invariants \( C + (\nu^2/m_\pi^2)D \) and \( C \) in the cases of the longitudinal and the transversal \( \rho \) meson, respectively. This means that, in the forward scattering amplitude \( W_{\mu \nu \lambda \sigma} \) \((23)\), one must separate the structure proportional to \( p_\mu p_\nu p_\lambda p_\sigma \) in the first case and the structure \( \sim p_\mu p_\nu \delta_{\lambda \sigma} \) in the second case.

Consider now the non-forward 4-point correlator

\[
\Pi_{\mu \nu \lambda \sigma}(p_1, p_2; q, q') = -i \int d^4 x d^4 y d^4 z e^{i p_1 x + i q y - i p_2 z} \times \langle 0 | T \{ j_\mu^e(x), j_\nu^e(y), j_\nu^i(0), j_\sigma^i(z) \} | 0 \rangle,
\]

where the currents \( j_\mu^e(x) \) and \( j_\lambda^i(x) \) are given by Eq.\((11)\) and \((20)\). It is evident from the consideration, that in the non-forward amplitude the most suitable tensor structure for determination of \( u \)-quark distribution in the longitudinal \( \rho \) meson is that proportional to \( P_\mu P_\nu P_\lambda \), while \( u \)-quark distribution in the transverse \( \rho \) can be found by considering the invariant function at the structure \((-P_\mu P_\nu \delta_{\lambda \sigma})\).

In the case of longitudinal \( \rho \) meson the tensor structure, that is separated is the same as in the case of the pion. Since bare loop contributions for vector and axial hadronic currents are equal at \( m_q = 0 \), the only difference from the pion case is the normalization. It can be shown, that \( u \)-quark distribution in the longitudinal \( \rho \) meson can be found from Eq.\((14)\) by substituting \( m_\pi \rightarrow m_\rho \), \( f_\pi \rightarrow m_\rho/g_\rho \) and, therefore, one can easily write down sum rules for this distribution:

\[
x_u^L(x) = \frac{3}{2\pi^2} \frac{M^2 g_\rho^2}{m_\rho^2} e^{m_\rho^2/M^2} x^2 (1 - x) \left[ 1 + \left( \frac{a_s(M^2) \cdot \ln(Q_0^2/M^2)}{3\pi} \right) + \left( 1 + 4x\ln(1-x) - 2(1-2x)\ln x \right) \left( 1 - e^{-s_0/M^2} \right) - \frac{a_s(M^2) \cdot \alpha_s a^2}{\pi^2 \cdot 3^7 \cdot 2^6 \cdot M^6} \cdot \frac{\omega(x)}{x^3(1-x)^3} \right],
\]

where \( a \) and \( \omega(x) \) are given by Eqs.\((15)\) and \((16)\), respectively. Sum rules for \( u_\rho^L(x) \) are satisfied in the wide \( x \) region: \( 0.1 < x < 0.85 \). The Borel mass \( M^2 \) dependence is weak in the whole range of \( x \), except \( x \leq 0.15 \) and \( x \geq 0.7 \). Figure. 2 presents \( x_u^L(x) \) as a function of \( x \). The values \( M^2 = 1 \text{ GeV}^2 \) and \( s_0 = 1.5 \text{ GeV}^2 \), \( Q_0^2 = 4 \text{ GeV}^2 \) were chosen, the parameters \( \Lambda^{LO}_{QCD} \) and \( \alpha_s a^2 \) are the same as in the calculation of \( x_u(\pi) \).

Let us now consider the case of transversally polarized \( \rho \)-meson, i.e., the term proportional to the structure \( P_\mu P_\nu \delta_{\lambda \sigma} \). The bare loop contribution (with leading order perturbative corrections) is

\[
u^T(x) = \frac{3}{8\pi^2} \frac{g_\rho^2}{m_\rho^2} e^{m_\rho^2/M^2} \cdot M^4 \cdot E_1 \left( \frac{s_0}{M^2} \right) \cdot \varphi_0(x)
\]

\[
\left[ 1 + \frac{\ln(Q_0^2/\mu^2) \cdot \alpha_s(\mu^2)}{3\pi} \cdot \left( 4x - 1/\varphi_0(x) + 4\ln(1-x) - \frac{2(1 - 2x + 4x^2)\ln x}{\varphi_0(x)} \right) \right]
\]

\[(26)\]
where

\[ \varphi_0(x) = 1 - 2x(1 - x) \]  \hspace{1cm} (27)

We consider now the power correction contribution to the sum rules. The power correction of the lowest dimension is proportional to the gluon condensate \( \langle G_{\mu\nu} G_{\rho\sigma}^a \rangle \) with \( d = 4 \). The \( \langle G_{\mu\nu} G_{\rho\sigma}^a \rangle \) correction was calculated in the standard way in the Fock-Schwinger gauge \( x_\mu A_\mu = 0 \).

The quark propagator \( iS(x, y) = \langle \psi(x) \psi(y) \rangle \) in the external field \( A_\mu \) has the well-known form [14, 15]. In contrast to the pion case, the \( \langle G_{\mu\nu} G_{\rho\sigma}^a \rangle \) correction for transversally polarized \( \rho(\rho_T) \), does not vanish.

\[ Im \Gamma^{(d=4)} = -\frac{\pi}{8x} \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^2 | 0 \rangle \]  \hspace{1cm} (28)

There are a great number of loop diagrams for \( d = 6 \) correction. It is convenient to divide them into two types and discuss these types separately. Type I diagrams are those, in which only the interaction with the external gluon field is taken into account and type II diagrams are those in which expansion of the quark field is also taken into account.

Discuss briefly the special features of calculating diagrams of these two types. The type-I diagrams are obviously proportional to \( \langle 0 | g^2 f_{abc} G_{\mu\nu}^a G_{\rho\sigma}^b G_{\rho\sigma}^c | 0 \rangle \), \( \langle 0 | D_{\mu} G_{\nu\mu}^a D_{\nu} G_{\alpha\beta}^a | 0 \rangle \), \( \langle 0 | G_{\mu\nu}^a D_{\rho} D_{\sigma} G_{\alpha\beta}^a | 0 \rangle \).

One may demonstrate [21] that these tensor structures are proportional to two vacuum averages

\[ \langle 0 | g^2 j_{\mu}^2 | 0 \rangle \] and \( \langle 0 | g^3 G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c f_{abc} | 0 \rangle \).

By using the factorization hypothesis the first of them, \( \langle 0 | g^2 j_{\mu}^2 | 0 \rangle \) easily reduces to \( \langle g\bar{\psi}\psi \rangle^2 \), which is well known:

\[ \langle 0 | g^2 j_{\mu}^2 | 0 \rangle = -(4/3)[\langle 0 | g\bar{\psi}\psi | 0 \rangle]^2. \]  \hspace{1cm} (29)

But \( \langle 0 | g^3 G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c f_{abc} | 0 \rangle \) is not well known; there are only some estimates based on the instanton model [22, 23]. In contrast to \( \pi \) and \( \rho_1 \)-meson cases, the terms proportional to \( \langle 0 | g^3 f_{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c | 0 \rangle \) are not cancelled for \( \rho_T \) and one should estimate it. The estimation based on the instanton model [22] gives

\[ -\langle g^3 f_{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle = \frac{48\pi^2}{5} \frac{1}{\rho_c^2} \langle 0 | (\alpha_s/\pi) G_{\mu\nu}^2 | 0 \rangle, \]  \hspace{1cm} (30)

where \( \rho_c \) is the effective instanton radius.

Among type-II diagrams only those, in which the interaction with the vacuum takes place inside the loop are considered. Such diagrams cannot be treated as the evolution of any non-loop diagrams and are pure power corrections of dimension 6.

The total number of \( d = 6 \) diagrams is enormous – about 500. Collecting the results we get finally the following sum rules for the valence \( u \)-quark distribution in the transversally polarized \( \rho \) meson.

\[ x\rho^T(x) = \frac{3}{8\pi^2} \rho_c^2 m_\rho^2 M^2 \frac{M^4}{m_\rho^4} \left\{ \varphi_0(x) E_1 \left( \frac{s_0}{M^2} \right) \left[ 1 + \frac{1}{3\pi} \ln \left( \frac{Q_0^2}{M^2} \right) \alpha_s(M^2) \left( \frac{4x - 1}{\varphi_0(x)} + \right) \right. \right. \]
The values of moments, obtained by assuming that $u$ parentheses.

for the longitudinally polarized $\rho$ chosen as $x$ in numerical calculations. Equation (30) was used and the effective instanton radius $x$ where

\[
\langle \frac{\alpha_s(M^2)}{2^5 \cdot 3^5 \pi^2 M^6 x^3 (1-x)^3} \rangle = 1
\]

where

\[
\begin{align*}
\xi(x) &= -1639 + 8039x - 15233x^2 + 10055x^3 - 624x^4 - 974x^5 \\
\chi(x) &= 8513 - 41692x + 64589x^2 - 60154x^3 + 99948x^4 \\
&\quad -112516x^5 + 45792x^6 + (-180 - 8604x + 53532x^2 \\
&\quad - 75492x^3 - 28872x^4 + 109296x^5 - 55440x^6)ln2
\end{align*}
\]

The standard value of the gluonic condensate $\langle 0 | (\alpha_s / \pi) G^2 | 0 \rangle = 0.012 GeV^4$ was taken in numerical calculations. Equation (30) was used and the effective instanton radius $\rho_c$ was chosen as $0.5 fm$. This value is between the estimations of $[22]$, $\rho_c = 1 fm$ and $[23]$, $\rho_c = 0.33 fm$. (In the recent paper $[24]$ it was argued that the liquid gas instanton model overestimates higher order gluonic condensates and, in order to correct this effect, larger values of $\rho_c$ comparing with $[23]$ should be used). The Borel mass dependence of $xu_T(x)$ in the interval $0.2 < x < 0.65$ is weak at $0.8 < M^2 < 1.2 GeV^2$. Figure 3 shows $xu_T(x)$ at $M^2 = 1 GeV^2$ and $Q_0^2 = 4 GeV^2$. Dashed and thin solid lines demonstrate the influence of the variation in $\rho_c$ on the final result: the lower line corresponds to $\rho_c = 0.6 fm$ and the upper $\rho_c = 0.4 fm$. Our results are reliable at $0.2 < x < 0.65$, where $d = 4$ and $d = 6$ power corrections each comprise less than 30% of the bare loop contribution. (The contributions $\langle 0 | (\alpha_s / \pi) G^2 | 0 \rangle$ and $\langle 0 | g^3 f^{abc} G_{\mu\nu}^{a} G_{\nu\lambda}^{b} G_{\lambda\mu}^{c} | 0 \rangle$ are of the opposite sign and compensate one another, $\alpha_s(M^2)\alpha_s a^2$ contribution is negligible.) At $\rho_c = 0.4 fm$ the applicability domain shrinks to $0.25 < x < 0.6$.

The moments of quark distributions in the longitudinal $\rho$ meson are calculated in the same way, as it was done in for the case of pion: by matching with Regge behavior $u(x) \sim 1/\sqrt{x}$ at low $x$ and with quark counting rule $u(x) \sim (1 - x)^2$ at large $x$. The matching points were chosen as $x = 0.10$ at low $x$ and $x = 0.80$ at large $x$. The numerical values of the moments for the longitudinally polarized $\rho$ are

\[
\begin{align*}
\mathcal{M}_1^L &= \int_0^1 dxu_\rho^L(x) = 1.06 \quad (1.05) \\
\mathcal{M}_2^L &= \int_0^1 xdxu_\rho^L(x) = 0.39 \quad (0.37)
\end{align*}
\]

The values of moments, obtained by assuming that $u(x) \sim (1 - x)$ at large $x$ are given in the parentheses.

Reliable calculation of moments for the case of transversally polarized $\rho$ meson is impossible, because of a narrow applicability domain in $x$ and expected double humps shape of the
Now let us discuss the nonpolarized $\rho$-meson case. The quark distribution function $u(x)$ in this case is equal to

$$u_\rho(x) = \frac{u^L_\rho(x) + 2u^T_\rho(x)}{3}$$

and we can determine $u(x)$ only in the region, where sum rules for $u^L_\rho(x)$ and $u^T_\rho$ are satisfied, i.e. in $0.2 \lesssim x \leq 0.65$. In this region, $u_\rho(x)$ is found to be very close to $u_\pi(x)$ (the difference in whole range of $x$ is no more than 10-15%).

### 5 Summary and discussion

Figure 4 gives the comparison of the valence $u$-quark distributions in the pion and longitudinally and transversally polarized $\rho$ mesons. The shapes of the curves are quite different, especially of $xu^T_\rho(x)$ in comparison with $xu^L_\rho(x)$ and $xu_\pi(x)$. Strongly different are also the second moments in the pion and longitudinal $\rho$-meson: the momentum fraction, carried by valence quarks and antiquarks $-(u + \bar{d})$ in the longitudinal $\rho$ meson is about 0.8, while in the pion it is much less – about 0.4-0.5. All these differences are very large and many times larger than estimated uncertainties of our results. In the case of $u$-quark distribution in the pion the main source of them is the value of the renorminvariant quantity $(2\pi)^4\alpha_s\langle|\bar{\psi}\psi|0\rangle^2$. In our calculations, we took it to be equal to 0.13 GeV$^6$. In fact, however, it is uncertain by a factor of 2. (Recent determination \cite{25} of this quantity from $\tau$-decay data indicates that it may be two times larger). The perturbative corrections also introduce some uncertainties, especially at large $x(x > 0.6)$ where the LO correction, which is taken into account, is large. The estimation of both effects shows that they may result in 10-20% variation (increase at $x < 0.3$ and decrease at $x > 0.3$) of $xu_\pi(x)$.

For the $u$-quark distribution in the longitudinally polarized $\rho$ meson, the uncertainties in $\alpha_s\langle|\bar{\psi}\psi|0\rangle^2$ do not play any role, because of higher $M^2$ values, so the expected accuracy is even better. One should note, that the accuracy of prediction of moments for longitudinally polarized $\rho$ meson are very high, because, as one can see from Fig.4, sum rules predict quark distribution almost in whole region of $x$ (from 0.1 up to 0.9). The fact that the first moment is so close to 1, also confirm that accuracy is high. Large value of the second moment mean, that valence quarks in longitudinally polarized $\rho$ meson carried about 0.8 of the total momentum, so one can conclude that the total (gluon and quark) sea is strongly suppressed. Moreover, this prediction for the second moment is close to those obtained in \cite{26} from quite another sum rules, and also agreed with the lattice calculation results \cite{27}, so the sea suppression in longitudinally polarized $\rho$ meson can be treated as a theoretically well established fact.

The accuracy of our results for the $u$-quark distribution in the transversally polarized $\rho$ meson is lower, because of a large role of $d = 4$ and $d = 6$ gluonic condensate contributions. For the latter, as was discussed before, there are only estimations, based on the instanton model, and the gluon condensate $\langle0 | (\alpha_s/\pi)G^2 | 0\rangle$ is also uncertain by a factor 1.5. Estimations show that they result in not more than 30-40% variation in $xu^T_\rho$ at $x \approx 0.3 - 0.4$, but in much less variation at $x \approx 0.5 - 0.6$. (The LO perturbative corrections are not more than 20% at small $x$ and negligible at large $x$). So one can see, that the difference, obtained
in the quark distribution in the pion, longitudinally and transversally polarized $\rho$-meson in fact are much more larger than any possible uncertainties of the results.

In summary the main physical conclusion are the following:

(1) The sea is strongly suppressed in the longitudinally polarized $\rho$-meson.

(2) The quark distribution in the polarized $\rho$-meson are significantly dependent on polarization.

(3) The quark distributions in the pion and $\rho$-meson have a not too much in common. The specific properties of the pion, as a Goldstone boson, manifest themselves in the quark distributions different from those in the $\rho$ meson. $SU(6)$ symmetry may, probably, take place for the static properties of $\pi$ and $\rho$, but not for their inner structure. We have no explanation, why $u$-quark distributions in the pion and unpolarized $\rho$-meson at $0.2 < x < 0.65$ are close to one another – whether it is a pure accident or there are some deep reasons for it.

This study was supported in part by Award no. RP2-2247 of U.S. Civilian Research and Development Foundation for Independent States of Former Soviet Union (CRDF), by the Russian Foundation of Basic Research, project no. 00-02-17808 and by INTAS Call 2000, project 587.
References

[1] A.M. Cooper-Sarkar, R.C.E. Davenish, and A.De Roeck, Int. J. Mod. Phys. A 13, 3385 (1998).

[2] H.L.Lai et al. (CTEQ Collaboration), Eur. Phys. J. C 12, 375 (2000).

[3] A.D. Martin, R.G. Roberts, W.J. Stirling, and R.S. Thorne, Eur. Phys. J. C 4, 463 (1998).

[4] M. Glück, E. Reya, and A. Vogt, Eur. Phys. J. C 5, 461 (1998).

[5] P. Aurenche et al., Phys. Lett. B 233, 517 (1989).

[6] M. Glück, E. Reya, and A. Vogt, Z. Phys. C 53, 651 (1992).

[7] P.J. Sutton, A.D. Martin, B.G. Roberts, and W.J. Stirling, Phys. Rev. D 45 2349 (1992).

[8] B.L. Ioffe, JETP Lett. 42, 327 (1985).

[9] V.M. Belyaev and B.L. Ioffe, Nucl. Phys. B 310, 548 (1988).

[10] A.S. Gorsky, B.L. Ioffe, A.Yu. Khodjamirian, and A.G. Oganesian, Z. Phys. C 44, 523 (1989).

[11] B.L. Ioffe and A.Yu. Khodjamirian, Phys. Rev. D 51 3373 (1995).

[12] B.L. Ioffe and A.G. Oganesian, Eur. Phys. J. C 13, 485 (2000).

[13] B.L. Ioffe and A.G. Oganesian, Phys. Rev. D 63 096006 (2001).

[14] B.L. Ioffe and A.V. Smilga, Nucl. Phys. B 216, 373 (1983).

[15] A.E. Dorokhov and L. Tomio, Phys. Rev. D 62, 014016 (2000).

[16] M.B. Hecht, C.D. Roberts, and S.M. Schmidt, Phys. Lett. B 495, 136 (2000).

[17] V.M. Belyaev and B.Yu. Block, Yad.Fiz. 43, 706 (1986) Sov. J. Nucl. Phys 43, 450, (1986)

[18] A. Belitsky, Phys. Lett. B 386, 359 (1996).

[19] A.V. Smilga, Sov. J. Nucl. Phys. 35, 271 (1982).

[20] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Fortschr. Phys. 32, 585 (1984).

[21] S.N. Nikolaev and A.V. Radyushkin, Nucl. Phys. B 213, 189 (1983).

[22] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Phys. Lett. B 86, 347 (1979).

[23] T. Schäfer and E.V. Shuryak, Rev. Mod. Phys. 70, 323 (1998).
[24] B.L. Ioffe and A.V. Samsonov, Phys. At. Nucl. 63, 1448 (2000).

[25] B.L. Ioffe and K.N. Zyablyuk, [hep-ph/0010089].

[26] A. Oganesian and A. Samsonov, JHEP 0109: 002, (2001).

[27] C. Best et al Phys. Rev. D 56, 2743 (1997).
Figure 1: Quark distribution function in the pion (thick line) and the fit from [6] (thin line).

Figure 2: Quark distribution function for longitudinally polarized $\rho$ meson

Figure 3: Quark distribution function for transversally polarized $\rho$-meson at three choices of instanton radius $\rho_c = 0.4, 0.5, 0.6$ fm., (curves from top to bottom, respectively)
Figure 4: Valence $u$ quark distributions for $\rho^T$ (thick curve), $\rho^L$ (curve with squares) and $\pi$-meson (thin curve)