Magnetically induced signatures in a thin film superconductor

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Abstract

We study the interaction between a one dimensional magnetic nanostructure and a thin film superconductor. It is shown that different magnetic distributions produce characteristic magnetic field signatures. Moreover, the magnetic structure can induce a weak link in the superconducting film, or be positioned directly above a predefined nonsuperconducting weak link. We estimate the magnetic flux associated with such a structure, and discuss a general expression for the energy calculated within the London model.
Weak links in superconductors have generated a lot of interest over the past decades, both in the study of conventional and high $T_c$ superconductors. More recently, research has also been focused towards junctions in thin films, due to their potential in future technology (see e.g. Ref. [1] and references therein). For such junctions it has been found that the field is a superposition of fields from Pearl vortices along the junction with a certain line density [1]. An interesting special case is the generation of weak links with spatially localized magnetic fields. Creation of a weak link in a bulk (or thick film) superconductor using a magnetic domain wall was first proposed by Sonin [2]. In that paper expressions for the magnetic fields where found, and it was argued that domain walls can induce movable weak links. In the current paper we try to extend the idea of Ref. [2] to thin film superconductors, and estimate the magnetic field and flux distribution associated with an one dimensional magnetic nanostructure. The weak link could be a domain wall, generated and controlled by a stress pattern or external field, or it could be a stationary prefabricated nanomagnetic stripe. A particular advantage of using magnetic domain walls as weak links is that they can be moved at high speeds, and may therefore have potential applications in future fluxtronics devices. On the other hand, in a prefabricated magnetic stripe (e.g. permalloy) the polarity of the magnetic vector could easily be switched by an external field [3]. This could also be of interest for creation and annihilation of vortices. Since the vortex pinning energy often can be regarded as proportional to the thickness of the superconductor, it is reasonable to assume that pinning by nonmagnetic sources are negligible in a thin film. Therefore, it should in principle be possible to produce film systems in which the magnetic texture determines the junction properties.

Consider a thin superconducting film located at $z = 0$ with thickness much smaller than the penetration depth of the superconductor. The surface is covered by a one dimensional magnetic structure centered at $x = 0$, with thickness much smaller than that of the superconducting film. It is assumed that the magnetic structure is sufficiently long that end effects are not a problem, and that it consists of surface charges separated from the superconductor by a very thin (negligible) oxide layer to avoid the exchange of electrons or spin (see e.g. Ref. [4]). We will here analyze the case where the magnetization is perpendicular to the plane of the superconducting film, since we expect this geometry to give a stronger coupling to the superconductor. In order to gain some insight into the behavior of the weak link, let us first compare the magnetic fields from the two following models for the magnetization
distribution:

\[
M^G = M_0 \exp(-\alpha x^2) \delta(z) \hat{e}_z ,
\]

and

\[
M^S = \begin{cases} 
M_0 \delta(z) \hat{e}_z & \text{if } -W \leq x \leq W, \\
0 & \text{if } |x| \geq W,
\end{cases}
\]

where \( M_0 \) and \( \alpha \) are constants. Here we assume that \( \alpha = 2W^{-2} \).

Using the method Ref. [5, 6], we find that the gaussian magnetization distribution results in the following field components (per unit length):

\[
H^G_z(x, z) = \lambda e M_0 \sqrt{\pi \alpha} \int_0^\infty \frac{k_x^2 \exp(-k_x^2/4\alpha) \cos(k_x x)}{1 + 2\lambda_e k_x} \exp(-k_x |z|) dk_x ,
\]

\[
H^G_x(x, z) = \lambda e M_0 \sqrt{\pi \alpha} \int_0^\infty \frac{k_x^2 \exp(-k_x^2/4\alpha) \sin(k_x x)}{1 + 2\lambda_e k_x} \exp(-k_x |z|) dk_x .
\]

On the other hand, the step magnetization distribution gives

\[
H^S_z(x, z) = \frac{2\lambda_e M_0}{\pi} \int_0^\infty \frac{k_x \sin(k_x W) \cos(k_x x)}{1 + 2\lambda_e k_x} \exp(-k_x |z|) dk_x ,
\]

\[
H^S_x(x, z) = \frac{2\lambda_e M_0}{\pi} \int_0^\infty \frac{k_x \sin(k_x W) \sin(k_x x)}{1 + 2\lambda_e k_x} \exp(-k_x |z|) dk_x .
\]

Figure 1 shows \( H^S_z \) when \( z = \lambda_e/200 \) (solid line) and \( z = 0 \) (dash-dotted line). Note that when \( z = 0 \) the magnetic field oscillates due to the steep magnetization gradient. This could therefore be interpreted as Gibbs oscillations, well known in Fourier analysis. At a certain height above the surface these oscillations are smoothed out due to the exponential decay factor. Figure 2 shows \( H^G_z \) (solid line) and \( H^G_x \) (dash-dotted line) when \( W = \lambda_e/40 \) and \( z = 0 \). Note that the peak of the \( z \) component is located at the origin. This is in contrast to Fig. 1, where the maximum field is located near the edges. Moreover, the two negative peaks of the field are much less pronounced for a gaussian distribution.

It is clear that if the magnetic field exceed the critical field of the superconductor, then a weak link is generated at which vortices may exists[2]. The current across the link is, in absence of any external currents, given by

\[
J^s_x = dJ_c \sin \left( \phi_2 - \phi_1 + 2\pi \frac{\Phi}{\Phi_0} \right) ,
\]

\[
J^s_y = dJ_c \cos \left( \phi_2 - \phi_1 + 2\pi \frac{\Phi}{\Phi_0} \right) ,
\]

\[
J^s_z = dJ_c \cos \left( \phi_2 - \phi_1 + 2\pi \frac{\Phi}{\Phi_0} \right) .
\]
where \( \phi = \phi_2 - \phi_1 + 2\pi \Phi/\Phi_0 \) is the phase difference across the junction, \( \Phi \) the magnetic flux through the junction and \( \Phi_0 \) the flux quantum. The magnetic flux for a gaussian magnetization distribution is estimated to be

\[
\Phi \approx \mu_0 \int_{y_1}^{y_2} \int_{-W}^{W} H_z dydz = \frac{\mu_0 \Delta y \lambda_e M_0}{\sqrt{\pi \alpha}} \int_0^\infty k_x \frac{\exp(-k_x^2/4\alpha) \sin(k_x W)}{1 + 2\lambda_e k_x} dk_x ,
\]

(7)

where \( \Delta y = y_2 - y_1 \) is the length of the weak link. It is seen that the flux, and therefore also the phase, is dependent on the magnetization, the wall width and the penetration depth.

Let us now consider a normal weak link (i.e. not magnetic), in which the phase can be found by using the approach of Ref. [1]. If we position a very thin magnetic structure (as discussed above) directly over the predefined weak link, a flux will flow through this junction. Here we may set \( \lambda_e \to \infty \), since the junction is entirely nonsuperconducting, and this results in

\[
\Phi \approx \frac{\mu_0 \Delta y M_0}{2\sqrt{\pi \alpha}} \int_0^\infty \exp(-k_x^2/4\alpha) \sin(k_x W) dk_x = C \mu_0 M_0 \Delta y ,
\]

(8)

where \( C \) is an unimportant constant \( (C = [\exp(-2)/\sqrt{\pi}] \int_0^{2\sqrt{2}} \exp(k_x^2) dk_x) \), and we have assumed that \( \alpha = 2W^{-2} \). This approach also gives a reasonable description of the case where the strength of the magnetic field from the magnetic structure breaks down superconductivity. However, the simple treatment given here only accounts for the direct magnetic flux through the junction, and neglects the vortices distributed around the weak link. In general, the full expression for \( \phi \) can not be found explicitly, but can be obtained by first determining the vortex interspacing distance by evaluating the expression for the energy (i.e. find the zero energy). From the given distribution of vortices one may obtain the magnetic field \( H_{vy} \), and finally take advantage of the expression \( J_z = dJ_x \sin \phi = -2H_{vy} \) to obtain an expression for the phase \( \phi \). Such a numerical analysis is outside the scope of this Brief Communication. However, let us for completeness discuss some properties of the energy of the system, which is in general found by evaluating

\[
E = \int_V \left( \frac{1}{2} \mu_0 H^2 + \frac{1}{2} \mu_0 \lambda^2 J_z^2 - \mu_0 M \cdot H \right) dV .
\]

(9)

Here \( \mu_0 \) is the permeability, and the current due to supercurrents and magnetization gradients can be written as

\[
J = J_s + J_m = J_s + \nabla \times M
\]

(10)
Note that the integration over the energy density is taken over the whole space, although the current can only flow in the volume of the thin film superconductor. I.e., we do not adopt the usual approach of dividing the space into superconducting and nonsuperconducting regions. Thus, we believe that the approach shown below can be applied to a more general class of systems, as long as there is no exchange of electrons and spin between the magnetic and superconducting structures. In order to obtain a more useful expression for the energy, we first transform the part associated with kinetic energy of the superconducting electrons

\[
E_{\text{kinetic}} = \frac{1}{2} \mu_0 \int_V \lambda^2 J_s^2 dV = \frac{1}{2} \mu_0 \int_V \lambda^2 J_s \cdot [\nabla \times (\mathbf{H} - \mathbf{M})] dV .
\]

(11)

Here we have used that \( J_s = \nabla \times (\mathbf{H} - \mathbf{M}) \). To further transform this integral, we note that a surface integral over the kernel \( J_s \times (\mathbf{H} - \mathbf{M}) \) vanishes when the surface is located far from the system. This means that we can write

\[
E_{\text{kinetic}} = \frac{1}{2} \mu_0 \int_V \lambda^2 J_s^2 dV = \frac{1}{2} \mu_0 \int_V \lambda^2 \mathbf{H} \cdot \nabla \times J_s dV .
\]

(12)

But now it should be remembered that the London equation gives

\[
\nabla \times J_s = -\frac{1}{\lambda^2} \mathbf{H} + \frac{1}{\lambda^2} \mathbf{V}
\]

(13)

where \( \mathbf{V} \) is the vortex source function, which represents all the vortices in the system (In the case of Pearl vortices it is simply a sum of delta functions). We obtain

\[
E_{\text{kinetic}} = \frac{1}{2} \mu_0 \int_V \lambda^2 J_s^2 dV = \frac{1}{2} \mu_0 \int_V [\mathbf{H}^2 + \mathbf{M} \cdot \mathbf{H} + (\mathbf{H} - \mathbf{M}) \cdot \mathbf{V}] dV .
\]

(14)

In total, the energy becomes

\[
E = \frac{1}{2} \mu_0 \int_V [\mathbf{M} \cdot \mathbf{H} + (\mathbf{H} - \mathbf{M}) \cdot \mathbf{V}] dV .
\]

(15)

Equation (15) can be applied to systems which are sufficiently local that the surface corrections can be neglected. The usefulness of Eq. (15) relies on the fact that for a thin film superconductor it reduces to a two dimensional integral, since the vortex source function and the magnetization distribution both are assumed to be located at \( z=0 \). Then the Fourier analysis of Refs. [4, 5] can be applied to obtain simple integrals for the energy. In their interesting paper Erdin and coworkers demonstrated a different method for calculating the energy within the London approximation [4]. The technique presented here gives an alternative and perhaps more intuitive route to evaluate the energy. It is instructive to divide the
energy terms in three different parts. The self-energy of the magnetic structure is

\[
E_m = -\frac{1}{2} \mu_0 \int_V M \cdot H_m dV ,
\]

the energy associated with the vortex is

\[
E_v = \frac{1}{2} \mu_0 \int_V H_v \cdot V dV ,
\]

and the interaction energy

\[
E_{vm} = -\frac{1}{2} \mu_0 \int_V [M \cdot H_v - H_m \cdot V + M \cdot V] dV .
\]

It is seen that the \( E_{vm} \) consists of the interaction between the magnetization and the vortex field, the magnetically generated field and the vortex source function and also the interaction between the magnetization and the vortex source function. Note that the two first contributions have opposite sign, and their sum is equal to \(-\mu_0 \int_V M \cdot H_v dV\) only in the case when they are equal. Although this is a reasonable approximation in some particular cases, the above analysis shows that it is not correct in general\[5, \ 7, \ 8\]. Moreover, one should take into account the finite distribution of the vortex source function.

In conclusion, we have discussed some properties of thin film superconductors in the close vicinity of one dimensional magnetic structures. It was found that the magnetic field depends strongly on the magnetic distribution function, and simple estimates for the flux through a weak link were derived. Finally, we derived an expression for the energy associated with a more general class of systems.
[1] V.G. Kogan, V.V. Dobrovitsky, Y. Mawatari, R.G. Mints and J.R. Clem, Phys. Rev. B 63, 144501 (2001).

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[3] G. Yi, P.R. Aitchison, W.D. Doyle, J.N. Chapman and C.D.W. Wilkinson, J. Appl. Phys. 92, 6087 (2002).

[4] S. Erdin, A.F. Kayali, I.F. Lyuksytov, and V.L. Pokrovsky, Phys. Rev. B 65, 014414 (2002).

[5] L.E. Helseth, Phys. Rev. B 66, 104508 (2002).

[6] On p. 1, Eq. 1 in Ref. [5] the term containing the vortex source function should be multiplied with $\delta(z)$, and also on p. 4 (line 7) the magnetization distribution should be multiplied with $\delta(z)$. These errors did not influence the final result.

[7] Even though the expression for the interaction energy is not complete, it should be pointed out that Eqs. 30, 38 and 44 in Ref. [5] may serve as a reasonable starting point for computing the energy when the magnetization is directed perpendicular to the film plane.

[8] In Figs. 2 and 4 in Ref. [5] an attractive force is defined to be pointing towards the magnetic bubble. Thus, an attractive force for negative $\rho$ is defined to be positive and points to the right, whereas an repulsive force points to the left. In the case of $\rho > 0$, an attractive force is negative and points to the left, whereas an repulsive force points to the right. In sum, the force is allways attractive or repulsive, depending on the polarities of the magnetic bubble as well as the vortex. Although this sign convention is not mathematically pleasing (because negative $\rho$ is allowed), it is nonetheless allowed physically as long as one clearly points out the definition. Unfortunately, this was not done in Ref. [5]. The author thanks F.M. Peeters for a discussion on this matter.
FIG. 1: The z component of the magnetic field generated by a step-like magnetization distribution when $z = 0$ (dash-dotted line) and $z = \lambda_e/200$ (solid line). Here $W = \lambda/40$, and the curves have been normalized with respect to the maximum peak of the dash-dotted line.
FIG. 2: The $x$ (dash-dotted line) and $z$ (solid line) component of the magnetic field generated by a gaussian magnetization distribution. Here $z=0$, $W = \lambda/40$ and $\alpha = 2W^{-2}$. The curves are normalized with respect to the maximum peak of the $z$ component.