The Inner Forces Redistribution of Continuous Beams

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Abstract. The technical plastic theory is applied for steel structures design in actual standards. This theory and standard calculate procedures assumed the plastic redistribution of internal forces by arising and development of plastic hinges and plastic mechanism failure of the structure. The plastic redistribution of internal forces enables the loading of structure to increase. The plastic redistribution and increasing of the loading cause also changes in the support reactions that are generally needed to consider as the actions on the structural supports. The redistribution of internal forces and the changes of support reactions depend on the elastic-plastic properties of constructive steels that are variable. Therefore, plastic redistribution of internal forces may not be absolute, or may not arise at all. In the paper, the effect of elastic-plastic redistribution of internal forces on the load-carrying capacity and the changes of support reactions for selected continuous beams are analyzed.

1. Introduction
The actual international and national technical standards equalize the elastic and plastic calculation methods of steel structures. The elastic calculation can be used in any case. For the plastic calculation, it is necessary to meet certain load, material and structural-strength assumptions and requirements according to individual standards. These standard assumptions and requirements, in line with the development of scientific and technical knowledge, are gradually being refined. This usually expands the possibilities of applying plasticity calculation. In this context, it can be stated that the new international and national standards (EN 1993-1-1, CSN EN 1993-1-1 and other) allow a wider application of plasticity calculation of steel structures than the previous standards.

Plasticity calculation of steel structures by actual standards is based on the technical theory of plasticity. This theory assumes an idealized elastic-plastic or strength-plastic working diagram of applied steel and the gradual formation and arising of plastic hinges in the most stressed cross-sections of the structure. In the final stage, it is assumed that a complete plastic mechanism of failure will occur without previous stability effects. These assumptions and the idealization of plastic hinges and plastic mechanism make it generally possible to simplify the calculation of steel structures. This simplification is particularly evident for continuous beams. Assumption of plastic hinges in the most stressed cross-sections with theoretical plastic bending resistance \( M_{pl} \), resp. \( M_{pl,Rd} \) leads to simplification of the calculation down to the calculation level of simple statically certain beams. Moreover, in the case of continuous beams, the use of plasticity seems to be very effective due to the redistribution of internal forces.
The plastic redistribution of internal forces generally increases the load-bearing capacity of steel structures. This increase depends on the structural arrangement and the load on the structure. By increasing the load, support reactions also change as a result of the redistribution of internal forces. However, the redistribution of internal forces and changes in support reactions depend on the actual material properties of the steels used. Due to a certain random variability in the material properties of the structural steels used, the assumed plastic redistribution of the internal forces of the steel structures may not be complete but only partial, or in extreme cases may not be manifested at all. In this context, the paper analyses the effect of the assumed plastic redistribution of internal forces on the load-bearing capacity and changes in support reactions of selected groups of two-span and three-span continuous beams [1-7].

2. Analysis of inner forces redistribution effect – experimental knowledge

In accordance with demonstrable experimental results, it can be stated that the real working diagram of structural steels at bending or bending-shear loading of beams does not correspond to the assumed idealized working diagrams. Under bending or bending-shear loading of beams occur after reaching the limit of proportionality, resp. limit elasticity and yield strength to fluently strengthening of structural steels. Simplified identification of proportional, elastic and yield strength limits for conventional structural steels does not have a significant negative effect. The significantly positive effect has its subsequent continuous strengthening in the whole region of plastic loading and deformation. Consequently, the actual load-carrying capacity of the cross-sections at the assumed plastic hinges as well as the total load-carrying capacity of the continuous beams is, therefore, greater than their theoretical load-carrying capacity as determined by the assumptions considered [2-4].

The differences between the predicted and the actual working diagram of structural steels result in the absence of complete plastic hinges and plastic failure mechanisms. Therefore, the course and values of internal forces, as well as the proportional values of the support reactions of continuous beams, are not entirely theoretical. The differences in internal forces and reactions ultimately depend on the geometrical-static arrangement and load of the beams.

The yield strengths of structural steels as well as the dimensions of the cross-sections and hence the cross-sectional characteristics of the beams are random variables. The required reliability of their design is therefore ensured by design strength values of applied structural steels. The differences between the actual yield strength and the design value of the steel strength do not result in the assumed plastic hinges and plastic failure mechanism. Thus, there is no complete redistribution of internal forces. Taking into account the random variability and the probability of occurrence of the design strength of the steel, it can be assumed that practically always only a partial redistribution will occur. In the extreme case, if the difference between the actual yield strength and the design value of the steel strength is greater, it can be assumed that there will be no redistribution of internal forces at all. As a result, there are again differences between the actual and design values of internal forces and responses. As a result, there are again differences between the actual and design values of internal forces and responses [2-4].

Favorable differences in the actual and assumed material properties of structural steels and the bending resistance of cross-sections do not negatively affect the overall load-carrying capacity of the beams. Therefore, in view of the reliability of their plasticity design, this need not be specifically taken into account. However, the actual elastic-plastic action also affects the overall and relative values of the support reactions of continuous beams. These reactions throughout the construction have to be considered in many cases as actions, resp. loading on other support parts. For example, if we consider the roof construction and the purlins form continuous beams, then their reactions are considered as a load on the trusses. In such cases, it is obviously necessary to consider the greatest possible reactions of the purlins. This fact was taken into account in the previous standards (CSN 73 1401). But in the
actual standards (EN 1993-1-1, CSN EN 1993-1-1 and other) this phenomenon is not specifically mentioned. However, this does not mean that the largest - the most unfavorable values of support reactions should not be taken into account when applying the plasticity calculation. From a practical point of view, it is therefore very useful and necessary to know what the possible differences and consequences are when considering only the values of the support reactions that result from the plasticity calculation. Therefore, the following numerical study was performed [2-4].

3. Numerical study of inner forces redistribution effect

Two-span and three-span continuous beams were considered in accordance with the intention. The spans of the individual fields were assumed to be the same, denoted uniformly with l. The load on the beams was assumed differently. It was considered to take into account, as far as possible, the practical possibilities and, in particular, the possible differences in the support reactions that result from the assumed plastic or possible elastic action. The load of the individual beams is obviously from the static schemes shown in Tables 1 and 2.

For the continuous beams thus defined, the following have been determined by:
- elastic calculation of the limit loads $q_{el}$ or $F_{el}$;
- plasticity calculation of the limit loads $q_{pl}$ or $F_{pl}$ and the reactions of $A_{pl}$ and $B_{pl}$
- elastic calculation of the reactions $A_{el,pl}$ and $B_{el,pl}$ corresponding to the load $q_{pl}$ or $F_{pl}$.

The determined limit loads and reactions are for all continues beams listed in Tables 1 and 2. Here, a comparison of the respective limit loads $q_{pl}$ and $q_{el}$ or $F_{pl}$ and $F_{el}$ is also made, as well as a comparison of the limit reactions $A_{pl} - A_{el,pl}$ and $B_{pl} - B_{el,pl}$.

The relative values of $q_{pl} / q_{el}$ and $F_{pl} / F_{el}$ express the rate of increase in the load-carrying capacity of beams using the plasticity of structural steels. They point out the efficiency of using plasticity calculation when designing continuous steel beams. The relative values of $q_{pl} / q_{el}$ and $F_{pl} / F_{el}$ indicated for each case of beams in Tables 1 and 2 only characterize the contribution of stress redistribution. Accordingly, it is appropriate to define the contribution of plastic redistribution of stress separately by means of the so-called stress redistribution rates:

$$ r = \frac{q_{pl}}{q_{el}} \frac{W_{el}}{W_{pl}}, \text{ resp. } r = \frac{F_{pl}}{F_{el}} \frac{W_{el}}{W_{pl}}. $$

Thus, the values of the $q_{pl} / q_{el}$ and $F_{pl} / F_{el}$ ratios indicated for each case of beams in Tables 1 and Table 2 are the values of the respective stress redistribution rate $r$. The $r$ values corresponding to the individual beams are different. They depend on the static diagram and the load on the beams. For the contemplated two-span continuous beams, $r = 1.111$ to $1.457$ and for the three-span continuous beams, $r = 1.050$ to $1.275$. The overall higher values of the redistribution rate $r$ belong to two-span continuous beams, which generally appear to be more advantageous in terms of production and assembly. The highest value $r$ of the considered beams belongs to a two-span uniformly loaded continuous beam, suitable especially for roof and ceiling constructions.

From the point of view of the intention of the paper, the relative values of ultimate limit and internal support reactions of continuous beams A and B, which were determined by plasticity calculation and by elastic calculation with respective limit plastic load - ratios $A_{pl} / A_{el,pl}$ and $B_{pl} / B_{el,pl}$. The values of these ratios characterize the differences resulting from the assumed plastic and possible elastic action of the continuous beams. It should be noted that these are maximum values. These values belong to extreme situations where the positive differences between the assumed design and the actual strength of the steel used are so large that the beams act flexibly until the theoretical load is reached. It is probably more likely that these strength differences are not so great and the beams act to some extent also plastically. This is not a definite indefinite elastic-plastic action of the beams.
### Table 1. Two-span continuous beams

| Static scheme of the beams | qel, Fel | Ael,pl | Bpl | qpl, Fpl | Apl | Bpl | Ael,Apl | Bel,Bpl |
|---------------------------|----------|--------|-----|----------|-----|-----|----------|----------|
| N210                      | 10,449 f_y W_{el} / l^2 | 0.414 q_{pl} l | 0.672 q_{pl} l | 11,656 f_y W_{pl} / l^2 | 0.437 q_{pl} l | 0.625 q_{pl} l | 1,116 W_{pl} / W_{el} | 0.947 | 1,075 |
| N220                      | 8,000 f_y W_{el} / l^2 | 0.414 q_{pl} l | 1,172 q_{pl} l | 11,656 f_y W_{pl} / l^2 | 0.375 q_{pl} l | 1,250 q_{pl} l | 1,457 W_{pl} / W_{el} | 1,104 | 0.938 |
| N211                      | 4,924 f_y W_{el} / l | 0.333 F_{pl} | 0.833 F_{pl} | 6,000 f_y W_{pl} / l | 0.406 F_{pl} | 0.688 F_{pl} | 1,218 W_{pl} / W_{el} | 0.820 | 1,211 |
| N221                      | 5,333 f_y W_{el} / l | 0.333 F_{pl} | 1,334 F_{pl} | 6,000 f_y W_{pl} / l | 0.312 F_{pl} | 1,375 F_{pl} | 1,125 W_{pl} / W_{el} | 1,067 | 0.970 |
| N212                      | 3,600 f_y W_{el} / l | 0.750 F_{pl} | 1,500 F_{pl} | 4,000 f_y W_{pl} / l | 0.833 F_{pl} | 1,333 F_{pl} | 1,111 W_{pl} / W_{el} | 0.900 | 1,125 |
| N222                      | 3,000 f_y W_{el} / l | 0.750 F_{pl} | 2,500 F_{pl} | 4,000 f_y W_{pl} / l | 0.667 F_{pl} | 2,667 F_{pl} | 1,333 W_{pl} / W_{el} | 1,125 | 0.938 |
| N213                      | 2,612 f_y W_{el} / l | 1,166 F_{pl} | 2,167 F_{pl} | 3,000 f_y W_{pl} / l | 1,266 F_{pl} | 1,969 F_{pl} | 1,148 W_{pl} / W_{el} | 0.921 | 1,101 |
| N223                      | 2,134 f_y W_{el} / l | 1,166 F_{pl} | 3,668 F_{pl} | 3,000 f_y W_{pl} / l | 1,031 F_{pl} | 3,938 F_{pl} | 1,405 W_{pl} / W_{el} | 1,131 | 0.931 |

From the viewpoint of the required design reliability, it seems reasonable to consider the maximum possible differences in support reactions of beams. If these differences are then more significant, they should be considered in the design of the support members and the structure.

The question is therefore: what values can take on the differences between the support reactions of beams determined by their plastic or elastic action. The answer to this question is the results of the numerical study performed, represented by the ratios of A_{pl} / A_{el,pl} and B_{pl} / B_{el,pl} as it is shown in Tables 1 and 2.
In the case of contemplated two-span continuous beams, the ratios of $A_{pl}/A_{el,pl} = 0.820$ to $1.125$ and $B_{pl}/B_{el,pl} = 0.938$ to $1.211$. In the case of contemplated three-span continuous beams, the ratios of $A_{pl}/A_{el,pl} = 0.784$ to $1.036$ and $B_{pl}/B_{el,pl} = 0.987$ to $1.160$.

Table 2. Three-pole continuous beams

| Static scheme of the beams | $q_{pl}$, $F_{pl}$ | $A_{pl}$ | $B_{pl}$ |
|----------------------------|------------------|---------|---------|
|                            | $q_{el}, F_{el}$ | $A_{el,pl}$ | $B_{el,pl}$ |
| $N_{320}$                  | $9,881 \ t_y W_{el} / l^2$ | $0.414 \ q_{pl}$ | $0.586 \ q_{pl}$ |
|                            | $11,656 \ t_y W_{pl} / l^2$ | $0.450 \ q_{pl}$ | $0.550 \ q_{pl}$ |
|                            | $1,180 \ W_{pl} / W_{el}$ | $0.920$ | $1,065$ |
| $N_{330}$                  | $10,000 \ t_y W_{el} / l^2$ | $0.414 \ q_{pl}$ | $1,086 \ q_{pl}$ |
|                            | $11,656 \ t_y W_{pl} / l^2$ | $0.400 \ q_{pl}$ | $1,100 \ q_{pl}$ |
|                            | $1,166 \ W_{pl} / W_{el}$ | $1,035$ | $0.987$ |

The analysis carried out shows that the values of the ratios of the $A_{pl}/A_{el,pl}$ and $B_{pl}/B_{el,pl}$ support reactions do not depend unambiguously on the degree of redistribution of internal forces $r$, but depend on the specific static arrangement and loading of the beams. Practical calculation and design should be therefore investigated and considered.

In the context of the presented considerations and the results of the performed analysis, it should be recalled that the preliminary standard ENV 1993-1-1 and consequently the standards CSN and STN...
enable to use some redistribution of internal forces of continuous beams even at their elastic
calculation, by allowing the distribution of up to 15% of the extremely elastic moments of each
member, provided that the internal forces remain in equilibrium with the external load (Article 6.5.2)
or the plastic bending resistance of the most stressed cross-section can be considered (Article 6.5.3).
This allows better utilization of the load-carrying capacity of continuous beams and statically
indeterminate structures at all if they have class 1 or 2 cross-sections. From this point of view, this
procedure is well-founded and very simple. However, determining the equilibrium state of the internal
and external forces (and hence the support reactions) applied to the redistribution of extreme elastic
moments also requires a certain elastic-plastic action, which is certainly more problematic than the
assumed plastic action and corresponding plasticity calculation. Therefore, the finally EN 1993-1-1
does not seem to allow for the redistribution of extreme elastic moments (up to 15%) but leaves the
possibility of using the plastic bending resistance of the most stressed cross-section. Finally, both the
analysis and the results of the numerical study show that they cannot be of more important meaning
when using the plastic bending resistance only at the most stressed cross-section, but even with
a limited redistribution of extreme elastic moments without increasing the load. They can only have
a minor effect on support reactions. However, the requirement to determine and consider the
equilibrium state of internal and external forces also indirectly draws attention to the need to
determine and consider the maximum possible reactions in plasticity calculation of continuous beams.

4. Conclusions

The redistribution of the internal forces of the continuous beams and the statically indeterminate
structures at all assumed in their plasticity calculation depends on the actual material properties of the
steels used. This should be considered in particular when determining support responses. The support
reactions should be determined to assume the plastic and eventual elastic action of the continuous
beams and the maximum possible effects should be considered on the support structures.

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