UNIFIED DESCRIPTION OF THE NON – SINGLET SPIN DEPENDENT STRUCTURE FUNCTION $g_1$
INCORPORATING ALTARELLI-PARISI EVOLUTION AND THE DOUBLE LOGARITHMIC $\ln^2(1/x)$ EFFECTS AT LOW $x$

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Abstract

A unified equation for the non-singlet spin dependent structure function $g_1^{ns}(x,Q^2) = g_1(x,Q^2) - g_n(x,Q^2)$ which incorporates the complete leading order Altarelli-Parisi evolution at finite $x$ and double logarithmic $\ln^2(1/x)$ effects at $x \to 0$ is formulated. This equation is solved assuming simple phenomenological parametrisation of the non-perturbative part of the structure function. Reasonable agreement is obtained with the recent data obtained by the Spin Muon Collaboration. Predictions for the small $x$ behaviour of $g_1^{ns}(x,Q^2)$ in the kinematical range which may be relevant for the possible HERA measurements are given. The contribution to the Bjorken sum coming from the region of very small values of $x$ is quantified. Extrapolation of $g_1(x,Q^2)$ to the region of small $x$ and small $Q^2$ is also discussed.

1. Introduction

Inelastic scattering of polarised leptons on polarised nucleons is a powerful tool to study the internal spin structure of the nucleon. Measurements on proton, deuteron and neutron targets allow verification of sum rules like e.g. the Bjorken sum rule, \cite{1}, which is a fundamental relation of QCD and which refers to the first moment of the non-singlet spin dependent structure function, $g_1^{ns}(x,Q^2)$. Here the variables $x$ and $Q^2$ are conventionally defined as $x = Q^2/(2pq)$ and $Q^2 = -q^2$ where $q$ and $p$ denote the four momentum transfer between the leptons and the four momentum of the proton respectively. Evaluation of the sum rules thus requires knowledge of spin dependent structure functions over the entire region of $x$. Since the experimentally accessible $x$ range is limited (presently $x > 0.003$, \cite{2}), extrapolations to $x = 0$ and $x = 1$ are unavoidable. The latter is not critical, giving only a small contribution to the moments. However, the small $x$ behaviour of $g_1(x)$ is theoretically not well established and results on the moments depend drastically on the assumptions made for this extrapolation, cf. \cite{2,3}. Theoretical studies of the problem are thus awaited for. This is even more important in view of a possible future polarising the proton beam at HERA, \cite{4}, and thus investigating the polarised electron-proton collisions
It has recently been pointed out that the small $x$ behaviour of both singlet and non-singlet spin dependent structure function $g_1(x, Q^2)$ is controlled by the double logarithmic terms i.e. by those terms which correspond to powers of $\alpha_s \ln^2(1/x)$ \cite{1,4}. The double logarithmic terms also appear in the non-singlet structure function $F_2^{ns}(x, Q^2) = F_2^g(x, Q^2) - F_2^u(x, Q^2)$ \cite{5} but the leading small $x$ behaviour of the $F_2^{ns}$ which they generate is overridden by the (non-perturbative) contribution of the $A_2$ Regge pole \cite{6}. They can however generate the leading small $x$ behaviour of the spin dependent structure functions $g_i$ where the relevant Regge poles are those which correspond to axial vector mesons \cite{7,8}, expected to have low ($\sim 0$) intercept. Let us recall that the Regge pole model gives the following small $x$ behaviour of the structure functions $F_2^{ns}(x, Q^2)$ and $g_i^1(x, Q^2)$ where $g_i^1(x, Q^2), i = s, ns$ denote either singlet ($g_i^1(x, Q^2) = g_i^p(x, Q^2) + g_i^n(x, Q^2)$) or non-singlet ($g_i^{ns}(x, Q^2) = g_i^p(x, Q^2) - g_i^n(x, Q^2)$) combination of structure functions:

\begin{equation}
F_2^{ns}(x, Q^2) \sim x^{1-\alpha_{A_2}(0)}
\end{equation}

\begin{equation}
g_i^1(x, Q^2) \sim x^{-\alpha_i(0)}
\end{equation}

In eqs (1) and (2) $\alpha_{A_2(0)} \sim 1/2$ is the intercept of the $A_2$ Regge pole trajectory and $\alpha_{s,ns}(0)$ denote the intercept of the Regge pole trajectory corresponding to the axial vector mesons with $I = 0$ or $I = 1$ respectively for which we expect $\alpha_{s,ns}(0) \leq 0$. For $\alpha_{A_2}(0) \sim 1/2$ the resummation of the double $\ln^2(1/x)$ terms does not affect substantially the small $x$ behaviour of $F_2(x, Q^2)$ and its leading behaviour at low $x$ remains to be given by equation (1). On the contrary, the Regge behaviour of the spin dependent structure functions $g_i^1(x, Q^2)$ is unstable against the resummation of the $\ln^2(1/x)$ terms which generate more singular $x$ dependence than that implied by eq.(2) for $\alpha_{s,ns}(0) \leq 0$.

The double logarithmic $\ln^2(1/x)$ effects go beyond the standard LO (and NLO) QCD evolution of spin dependent parton densities \cite{11,17}. They can be accomodated for in the QCD evolution formalism based upon the renormalisation group equations after including the complete "small $x$" anomalous dimensions with the $\ln^2(1/x)$ terms resummed to all orders \cite{18}. The purpose of this paper is to develop alternative formalism based on unintegrated distributions in analogy to the unpolarised case \cite{13,20} but for simplicity limited to the non-singlet structure function $g_i^{ns}(x, Q^2)$. The content of the paper is as follows: in the next section we recall the basic integral equation for the non-singlet combination of the unintegrated quark distributions which resums the double logarithmic $\ln^2(1/x)$ terms. In Sec.3 we formulate the unified equation which incorporates the effects of the small $x$ resummation and the LO Altarelli-Parisi evolution. Numerical analysis of this equation is presented in Sec.4 starting from the simple parametrisation of the nonperturbative part of the spin structure function $g_i^{ns}$. Possible extrapolation of the spin dependent structure function to the region of low $Q^2$ is also discussed in this section. Finally in Section 5 we give a summary of our main results.

2. Double logarithmic $\ln^2(1/x)$ resummation

The double logarithmic terms in the non-singlet part of the $g_1(x, Q^2)$ (or of the $F_2(x, Q^2)$) are generated by ladder diagrams with quark (antiquark) exchange, Fig.1 \cite{21,23}. 


To be precise for the spin dependent structure function $g_1(x, Q^2)$ one has also to include certain class of non-ladder diagrams. In the non-singlet case however their contribution is non-leading in the large $N_c$ limit ($N_c$ is a number of colours). It turns out to be numerically small for $N_c = 3$ and will be neglected. The contribution of non-ladder diagrams is however non-negligible for the singlet spin dependent structure function. In what follows we shall consider only the non-singlet structure function $g_{1ns}(x, Q^2) = g_{1p}(x, Q^2) - g_{1n}(x, Q^2)$. For notational simplicity we shall drop the superscript ‘$ns$’ i.e. we will denote $g_{1ns}$ as $g_1$ etc.

When discussing the leading (double) logarithmic $\ln^2(1/x)$ resummation effects in the spin dependent structure function $g_1(x, Q^2)$ it is convenient to work with the (combination) of the unintegrated spin dependent quark distributions, $f(x', k^2)$:

$$f(x', k^2) = \frac{1}{6}(f_u(x', k^2) - f_d(x', k^2))$$  \hspace{1cm} (3)

where $x'$ is the longitudinal momentum fraction of the parent proton carried by a quark and $k^2$ denotes the quark transverse momentum squared. The sum of double logarithmic $\ln^2(1/x)$ terms corresponding to ladder diagrams is generated by the following integral equation for the (unintegrated) structure function $f(x', k^2)$:

$$f(x', k^2) = f^{(0)}(x', k^2) + \bar{\alpha}_s(k^2) \int_{x' \to x} \frac{dz}{z} \int_{k_0^2}^{k^2/z} \frac{dk'^2}{k'^2} f \left( \frac{x'}{z}, k'^2 \right)$$  \hspace{1cm} (4)

where

$$\bar{\alpha}_s(k^2) = \frac{2\alpha_s(k^2)}{3\pi}$$  \hspace{1cm} (5)

In this equation only the leading part of the splitting function $P_{qq}(z)$ in the $z = 0$ limit is retained. The variables $k^2(k'^2)$ denote the transverse momenta squared of the quarks, $k_0^2$ is the infrared cut-off and the inhomogeneous term $f^{(0)}(x', k^2)$ will be specified later. The integration limit $k'^2/z$ follows from the requirement that the quark virtuality is controlled by the transverse momentum squared. The structure function $g_1(x, Q^2)$ is related in the following way to $f(x', k^2)$:

$$g_1(x, Q^2) = g_1^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f(x' = x(1 + k^2/Q^2), k^2)$$  \hspace{1cm} (6)

where $g_1^{(0)}(x)$ is the nonperturbative part of the structure function and

$$W^2 = Q^2(\frac{1}{x} - 1)$$  \hspace{1cm} (7)

To be precise, the kinematic limit for the integration over $dk^2$ should be set equal to $W^2/4$ but we have checked that the results are relatively insensitive to this change.
The relation \( x' = x(1 + k^2/Q^2) \) follows from the on-shell condition imposed upon the struck quark i.e. \( (q + k)^2 = 0 \) where \( k \) is now the four momentum of the probed quark. This can be seen after we decompose this four momentum into basic light-like vectors \( p \) and \( q' = q + xp \):

\[
k = x'p - \beta q' + k_t
\]  

(8)

and assume that integration over \( d\beta \) which is implicit in the definition of the distribution \( f(x', k^2) \) is restricted to the region \( \beta << 1 \). In the limit \( k^2 << Q^2 \) we do, of course, get \( x' = x \).

The origin of the nonperturbative part \( g_1^{(0)}(x) \) can be viewed upon as originating from the non-perturbative region \( k^2 < k_0^2 \), i.e.

\[
g_1^{(0)}(x) = \int_0^{k_0^2} \frac{dk^2}{k^2} f(x, k^2)
\]  

(9)

The origin of the double logarithmic \( \ln^2(1/x) \) terms in \( g_1(x, Q^2) \) can be traced to the fact that the conventional single logarithmic terms coming from the logarithmic integration over the longitudinal momentum fraction \( z \) are enhanced by the logarithmic integration over the transverse momentum up to the \( z \) dependent limit \( k^2/z \) in eq.(10) and up to the \( x \) dependent limit \( W^2 = Q^2(1/x - 1) \) in eq. (11). Those double logarithmic \( \ln(1/x) \) terms will generate the leading small \( x \) behaviour of the structure function \( g_1(x, Q^2) \) provided that the input structure functions \( g_1^{(0)}(x, Q^2) \) and \( f^{(0)}(x, k^2) \) are non-singular for \( x \to 0 \).

For fixed (i.e. non-running) coupling \( \bar{\alpha}_s(k^2) \to \tilde{\alpha}_s \), the small \( x \) behaviour is then found to be given by

\[
g_1(x, Q^2) \sim x^{-\lambda}
\]  

(10)

where

\[
\lambda = 2\sqrt{\tilde{\alpha}_s}
\]  

(11)

and where we have neglected in eq.(10) the slowly varying logarithmic factors.

3. Unified equation incorporating Altarelli-Parisi evolution and the double \( \ln^2(1/x) \) resummation.

In order to derive the unified equation which will include complete LO Altarelli-Parisi evolution and the \( \ln^2(1/x) \) terms resummation let us at first confront eqs (11) and (12) with the LO Altarelli-Parisi evolution equation for the non-singlet structure function \( g_1(x, Q^2) \) [11, 12]. This equation can be written in the following form in terms of the unintegrated distribution \( f(x, k^2) \):

\[
f(x, k^2) = \frac{\alpha_s(k^2)}{2\pi} \left[ \frac{4}{3} \int_x^1 \frac{dz}{z} \frac{(1 + z^2)g_1^{(0)}(z) - 2zg_1^{(0)}(x)}{1 - z} + \left( \frac{1}{2} + \frac{8}{3} \ln(1 - x) \right) g_1^{(0)}(x) \right] + \frac{\alpha_s(k^2)}{2\pi} \int_{k_0^2}^{k^2} \frac{dk^2}{k^2} \left[ \frac{4}{3} \int_x^1 \frac{dz}{z} \frac{(1 + z^2)f(z, k^2) - 2zf(x, k^2)}{1 - z} + \left( \frac{1}{2} + \frac{8}{3} \ln(1 - x) \right) f(x, k^2) \right]
\]  

(12)

and

\[
g_1(x, Q^2) = g_1^{(0)}(x) + \int_{k_0^2}^{Q^2} \frac{dk^2}{k^2} f(x, k^2)
\]  

(13)
Equivalence of the eqs (12, 13) to the Altarelli-Parisi equation can be easily seen by observing that the left hand side of eq. (12) is equal to

\[ k^2 \frac{\partial g_1(x, k^2)}{\partial k^2} \]

and its right hand side is:

\[ \frac{\alpha_s(k^2)}{2\pi} \left[ \frac{4}{3} \int_x^1 \frac{dz}{z} \left( 1 + z^2 \right) g_1(z, k^2) - 2zg_1(x, k^2) \right] + \left( \frac{1}{2} + \frac{8}{3} \ln(1 - x) \right) g_1(x, k^2) \]

As usual the small \( x \) behaviour of the solution of the Altarelli-Parisi equation depends upon the small \( x \) behaviour of the input structure function. If the input structure function \( g_1^{(0)}(x) \) is not singular at \( x \to 0 \) then the leading small \( x \) behaviour of the structure function \( g_1(x, Q^2) \) obtained from the solution of the Altarelli-Parisi equation is given by:

\[ g_1(x, Q^2) \sim \exp\left[ 2\sqrt{\xi(Q^2)\ln(1/x)} \right] \]

where

\[ \xi(Q^2) = \frac{4}{3} \int_{k_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \]

This leading small \( x \) behaviour is generated by the approximate form of the Altarelli-Parisi equation in which at \( z \to 0 \) we keep only the leading term of the splitting function \( P_{qq}(z) \). This approximate equation has the following form

\[ f(x, k^2) = f^{(0)}(x, k^2) + \bar{\alpha}_s(k^2) \int_x^1 \frac{dz}{z} \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} f\left( \frac{x'}{z}, k'^2 \right) \]

with the structure function \( g_1(x, Q^2) \) given as usual by eq. (13). This equation has the same structure as eq. (4) except for the upper limit of integration over \( dk'^2 \). It is therefore evident from eq. (4) that the LO Altarelli-Parisi evolution is incomplete at low \( x \) since it does not generate the double but only the single \( \ln(1/x) \) terms.

Equation (4) generates correctly the leading small \( x \) behaviour but it is inaccurate in describing the evolution in \( Q^2 \) both for very small and, of course, for large values of \( x \). In order to have the correct \( Q^2 \) evolution one should thus include in the formalism the complete splitting function \( P_{qq}(z) \) and not only its leading component at \( z \to 0 \). In order to have at the same time the possibility to generate the double logarithmic \( \ln(1/x) \) terms at low \( x \) one has to modify eq. (12) and change appropriately the upper limit of integration over \( k'^2 \) to \( k^2/z \) (as in eq. (4)) in that part of the QCD evolution which is controlled by the leading part of the splitting function \( P_{qq}(z) \) at \( z \to 0 \). The corresponding equation then reads:

\[ f(x', k^2) = f^{(0)}(x', k^2) + \bar{\alpha}_s(k^2) \int_{x'}^1 \frac{dz}{z} \int_{k_0^2}^{k^2/z} \frac{dk'^2}{k'^2} f\left( \frac{x'}{z}, k'^2 \right) + \]

\[ \frac{\alpha_s(k^2)}{2\pi} \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} \left[ \frac{4}{3} \int_{x'}^1 \frac{dz}{z} \frac{(z + z^2)f(x', k^2) - 2zf(x', k^2)}{1 - z} + \left( \frac{1}{2} + \frac{8}{3} \ln(1 - x') \right) f(x', k^2) \right] \]

where

\[ f^{(0)}(x', k^2) = \frac{\alpha_s(k^2)}{2\pi} \left[ \frac{4}{3} \int_{x'}^1 \frac{dz}{z} \frac{(1 + z^2)g_1^{(0)}(x') - 2zg_1^{(0)}(x')}{1 - z} + \left( \frac{1}{2} + \frac{8}{3} \ln(1 - x') \right) g_1^{(0)}(x') \right] \]
and the function $\bar{\alpha}_s(k^2)$ is given by eq.(5). The structure function $g_1(x, Q^2)$ is related to $f(x', k^2)$ by eq.(19). Equations (17),(18) and (3) which treat both potentially large logarithms $\ln(1/x)$ and $\ln(Q^2)$ on equal footing will be the basis of our analysis. Similar treatment of the unpolarised (singlet) structure function and of gluon distributions which combine the Altarelli-Parisi and BFKL evolution(s) has been proposed in refs [19, 20]. Finally let us observe that we will need the unintegrated distribution only in the ”perturbative” domain, $k^2 > k^2_0$, while the non-perturbative contribution is parametrised in terms of the input distribution $g_1^{(0)}(x)$, cf. eqs (4), (9) and (18).

4. Numerical results

We solve equation (17) assuming the following simple parametrisation of $g_1^{(0)}(x)$:

$$g_1^{(0)}(x) = \frac{2}{3} g_A (1 - x)^3$$

where $g_A$ is the axial vector coupling constant (we set $g_A = 1.257$). In the small $x$ limit we have $g_1^{(0)}(x) \rightarrow \text{const}$ that corresponds to the assumption that $\alpha_{A1}(0) = 0$ where $\alpha_{A1}(0)$ is the intercept of the $A_1$ Regge pole trajectory (cf. eq. (2)). The function $g_1^{(0)}(x)$ given by eq. (13) does also satisfy the Bjorken sum rule, i.e.

$$\int_0^1 dx g_1^{(0)}(x) = \frac{g_A}{6}$$

The parameter $k^2_0$ is set $k^2_0 = 1 \text{ GeV}^2$.

In Fig.2 we compare the calculated $g_1(x, Q^2)$ with recent measurements of the SMC which extend to presently lowest values of $x$. In view of the simplicity of the parametrisation (19) with no free parameters, agreement with the data can be regarded as quite satisfactory. Parametrisation of $g_1^{(0)}(x)$ can certainly be made more flexible, with phenomenological parameters to be fitted to the data but large experimental errors make such refinements irrelevant at present.

In Fig.3 we show the structure function $g_1(x, Q^2)$ as the function of $x$ for $Q^2 = 10 \text{ GeV}^2$ obtained from eq.(17) with $f(x', k^2)$ obtained from solving eq.(17). For comparison we also show $g_1(x, Q^2)$ obtained from the standard LO QCD evolution (cf. eqs(12,13)) as well as the input structure function $g_1^{(0)}(x)$ given by eq.(19). We can see that the $\ln^2(1/x)$ resummation gives steeper structure function than that generated from the LO evolution and this effect is already visible for $x \lesssim 10^{-2}$.

The structure function $g_1(x, Q^2)$ obtained from equations (3) and (17) still satisfies the Bjorken sum rule despite the fact that the double logarithmic terms might in principle lead to its violation. This can be seen as follows. One can show from eq.(3) that the first moment of the function $g_1$ is related to the first moment of $f$:

$$\int_0^1 dx g(x, Q^2) = \int_0^1 dx g_1^{(0)}(x) + \int_{k^2_0}^\infty \frac{dk^2}{k^2(1 + k^2/Q^2)} \int_0^1 df(u, k^2)$$

It can easily be shown that the first moment of the function $f$ vanishes and so the first moment of $g_1(x, Q^2)$ remains to be given by the first moment of $g_1^{(0)}$ which is equal to $g_A/6$, etc.
cf. eq. (20). Vanishing of the first moment of $f$ follows from the fact that eq. (17) would lead to the corresponding inhomogeneous integral equation for the moment function of $f$ with the vanishing inhomogeneous term for the first moment. This inhomogeneous term is just the first moment of $f^{(0)}$ defined by eq. (18). The solution of such an equation then also vanishes. Vanishing of the first moment of $f^{(0)}$ on the other hand follows from the fact that $f^{(0)}$ is proportional to the convolution of the splitting function $P_{qq}$ with $g^{(0)}$, cf. eq. (18), and the first moment of $P_{qq}$ is equal to zero.

Equations (6) and (17) generate dynamically the small $x$ extrapolation of $g_1(x, Q^2)$ and so one may in particular determine the contribution to the Bjorken integral coming from the region of very small values of $x$ which is not being experimentally probed at present. We have thus estimated a contribution

$$
\Delta I(x_a, x_b, Q^2) = \int_{x_a}^{x_b} dx g_1(x, Q^2)
$$

(22)
to the Bjorken integral and found that $\Delta I(0, 0.003, 10 \text{ GeV}^2) = 0.0057$ for the pure Altarelli-Parisi evolution calculations and 0.0080 for these with double logarithmic terms included. These numbers have to be compared with 0.004 obtained when a $g_1 = \text{const}$ behaviour, consistent with Regge prediction has been assumed and fitted to the lowest $x$ data points for proton and deuteron targets (cf. [2, 3]). An analogous number in the present case, $\Delta I(0, 0.003) = 0.0026$, where we have used $g_1^{(0)}$ instead of $g_1$ in the integrand of $\Delta I$, eq. (22).

![Figure 2](image.png)

Figure 2. Non-singlet part of the proton spin structure function $g_1(x, Q^2)$. Points show the recent SMC results, [2, 3], where $g_1^{ns} \equiv g_1^p - g_1^n = 2(g_1^p - g_1^n/(1 - 3 \omega_D/2))$ was obtained from the proton and deuteron data ($\omega_D$ denotes a probability of the $D$ state of the deuteron; $\omega_D = 0.05$). Error bars are statistical; systematic errors are of the order of 20% of the statistical ones (in the lowest $x$ bin). Values of $Q^2$ change with $x$ as marked in the figure. Continuous line corresponds to the model calculations, eqs (6), (17) and (19), performed at the measured values of $(x, Q^2)$; broken line is a pure leading order Altarelli–Parisi prediction, eqs (12), (13) and (19), at the same points.

Let us finally comment upon the possible low $Q^2$ extrapolation of the structure function $g_1(x, Q^2)$. For $Q^2 \to 0$ (for fixed $pq$) the structure function $g_1(x, Q^2)$ should be a finite
function of $pq$, free from any kinematical singularities or zeros at $Q^2 = 0$. It may be seen that the structure function $g_1(x, Q^2)$ defined by eqs (6,17) and with $g_1^{(0)}(x)$ given by parametrisation (19) fulfills those criteria. This may not always hold for arbitrary parametrisation of $g_1^{(0)}(x, Q^2)$ which may include kinematical singularities at $x = 0$. In such a case one may just use the same prescription as that which was adopted in ref. [24] i.e. to replace in eq.(6) the function $g_1^{(0)}(x)$ by $g_1^{(0)}(\bar{x})$ where
\[ \bar{x} = x \left(1 + \frac{k_0^2}{Q^2}\right) \] (23)
and to leave remaining parts of the calculation unchanged. After this simple rearrangement the structure function $g_1(x, Q^2)$ can be extrapolated to the low $Q^2$ region (for fixed $2pq = Q^2/x$) including the point $Q^2 = 0$. It has however to be remembered that the (extrapolated) partonic contribution to the low $Q^2$ region may not be the only one there.

5. Summary and conclusions

In this paper we have formulated the unified equation for the spin dependent non-singlet structure function $g_{1s}(x, Q^2)$ which incorporated the LO Altarelli-Parisi evolution and the double logarithmic $\ln^2(1/x)$ effects at small $x$. The equation treats on equal footing two potentially "large" logarithms $\ln(Q^2)$ and $\ln(1/x)$. It also allows to extrapolate $g_{1s}^{(0)}$ dynamically to the region of small $x$. 

Figure 3. Non-singlet part of the proton spin structure function $g_1(x, Q^2)$ as a function of $x$ for $Q^2 = 10$ GeV$^2$. Continuous line corresponds to the model calculations, eqs (6), (17) and (19), broken line is a pure leading order Altarelli–Parisi prediction, eqs (12), (13) and (19), and a dotted one corresponds to $g_1^{(0)}$, eq.(19).
The non-singlet case has been chosen for its simplicity; dominant part of the double logarithmic \( \ln^2(1/x) \) terms and the QCD evolution here are generated by ladder diagrams with quark and antiquark exchange. It has also been chosen in order to meet the experimental need to know the low \( x \) behaviour of \( g_1^{ns} \), necessary when evaluating the Bjorken sum rule from the data. Elaboration of similar formalism for the singlet case is in progress.

We solved that equation starting from the simple parametrisation of the non-perturbative part of \( g_1^{ns}(x,Q^2) \). We found that the \( \ln^2(1/x) \) effects are very significant already for \( x \lesssim 10^{-2} \) which may be probed in the possible future HERA measurements. We have also estimated the contribution from the small \( x \) region \( (x < 0.003) \) to the Bjorken sum rule and found it to be around 4% of the value of the sum. We have pointed out that the structure function \( g_1^{ns}(x,Q^2) \) obtained from our formalism can easily be extrapolated to the region of low \( Q^2 \) although it may not be the only contribution in this region.

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