Positive Cross Correlations in a Normal- Conducting Fermionic Beam Splitter

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We investigate a beam splitter experiment implemented in a normal conducting fermionic electron gas in the quantum Hall regime. The cross-correlations between the current fluctuations in the two exit leads of the three terminal device are found to be negative, zero or even positive depending on the scattering mechanism within the device. Reversal of the cross-correlations sign occurs due to interaction between different edge-states and does not reflect the statistics of the fermionic particles which ‘antibunch’.

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In mesoscopic physics, the investigation of the conductance of small electronic devices is widely used to obtain their transport properties. Additionally to such time-averaged measurements, the temporal fluctuations (noise) in the current caused by the granularity of charge and diffraction of the wave-function provide us with important supplementary information about electronic transport [1]. Nonequilibrium noise has been widely explored to determine, for example, the effective charge of carriers [2, 3, 4, 5] or to study the transmission properties of quantum coherent devices such as quantum point contacts [6, 7], diffusive wires [8] or chaotic cavities [9]. Universally, the statistical correlations due to the Pauli exclusion principle are responsible for the negative current-current correlations between different leads in multi-terminal devices [10, 11]. Such negative correlations have been observed in distinct experiments [12, 13, 14, 15]. It has also been shown that for a diluted electronic stream obeying classical statistics the negative correlations vanish [14].

In contrast, a positive cross-correlation (i.e. ‘bunching’) is predicted to occur in devices with ‘non-normalconducting’ contacts [16] like hybrid-structures which use a superconductor as current-injector: two entangled electrons, forming a Cooper in the superconductor, are simultaneously emitted into different exit leads giving rise to a positive correlation [17, 18, 19, 20, 21, 22, 23]. Alternatively, devices with ferromagnetic contacts can show positive cross-correlations due to ‘opposite spin-bunching’ [24] or dynamical spin-blockade [25].

In this article we are interested in a discussion by Tétié and Büttiker [26] about the occurrence of positive cross-correlations in a purely normal-conducting fermionic device. As the authors show, the effect is due to current redistribution among different conducting states. We consider this idea here experimentally in a beam-splitter configuration (Fig. 1(a)) where a current $I$ injected at contact 1 is split into two equal parts exiting into contacts 2 and 3. Our main result is the observation of positive cross-correlations between the two exit contacts for a particular implementation of the beam-splitter according to Ref. [26]. Although predicted by various theoretical works, such positive cross-correlations have not been seen before in mesoscopic devices. We further conclude from our experimental observation that a positive correlation in fermionic systems can be interpreted as a sign of entanglement only if effects as the one shown here can be ruled out.

Fig. 1(b) gives an ‘inside view’ of the physical implementation of the beam splitter configuration used to study the cross-correlations sign reversal from negative to positive: A two-dimensional electron gas (2DEG) is exposed to a perpendicular magnetic field so that the current flows in edge-states along the border of the device [27, 28]. Edge-states provide natural fermionic ‘beams’ which are, thanks to their chirality, easily split by a quantum point contact (QPC) into a transmitted and a reflected part [10]. The two (tunable) QPC’s in series play different roles: the first one ($A$) introduces noise within the edge-state(s) while the second one ($B$) acts as a beam splitter that exits the edge-states or parts of them into different contacts. In former experiments, only one single spin-degenerated edge-state was populated [14]. It was shown that the correlations are always negative as expected for a single stream of fermionic particles showing ‘antibunching’ behavior [14, 26].

In the following, we consider the case that exactly the last two spin-degenerated Landau levels are fully occupied. Partitioning at $A$ with the transmission probability $T_A^{ii}$ gives rise to current fluctuations $\Delta I^{ii}$ in the second edge-state ($ii$). Their power spectral density is $\langle (\Delta I^{ii})^2 \rangle = 2 G_0 T_A^{ii} (1 - T_A^{ii}) \mu_I$ with $\mu_I$ the electrochemical potential of contact 1 and $G_0 = 2 e^2/h$ [29, 30, 31]. The first edge-state remains noiseless because it is transmitted at $A$ with unit probability ($T_A^{ii} = 1$). Inter-edge-state equilibration introduced via an extra voltage probe 4 redistributes the current fluctuations $\Delta I^{ii}_{\text{in}}$ in the current $I^{ii}_{\text{in}}$ incident to the mixing contact 4 between the two outgoing edge-states $I^{ii}_{\text{out}}$ and $I^{ii}_{\text{out}}$: $\Delta I^{ii}_{\text{out}} = \Delta I^{ii}_{\text{in}} = \Delta I^{ii}/2$. Finally, the ‘beam splitter’ $B$ separates the two edge-states into two different contacts 2 and 3. Since the current fluctuations in both edge-states originate from the same scattering process at $A$ the cross-correlations are expected to be positive. Their power spectral density $\langle \Delta I_2 \Delta I_3 \rangle = \langle \Delta I^{ii}_{\text{out}} \Delta I^{ii}_{\text{out}} \rangle$ divided by the Poissonian value
between contact 2 and 3 the time dependent currents are given by:

\[
\frac{\langle \Delta I_2 \Delta I_3 \rangle}{2e|I|} = \frac{\langle (\Delta I_2^{ii})^2 \rangle}{8e|I|} = + \frac{1}{4} \frac{T_{\alpha A}^{\mu} (1 - T_{\alpha A}^{\mu})}{1 + T_{\alpha A}^{\mu}}.
\]

Here, \( I = G_0 (1 + T_{\alpha A}^{\mu}) \mu_1/e \) describes the total current injected at contact 1.

Experimentally, the device illustrated in Fig. 1(b) is implemented in a standard GaAs/Al\(_{0.3}\)Ga\(_{0.7}\)As-heterostructure. The QPC’s A and B are defined by metallic split-gates on top of the 2DEG, which forms 60 nm below the surface (Fig. 1(c)). Two samples with different path lengths L (200 and 14 \( \mu \)m) between the two QPC’s have been measured. The solid curve in Fig. 2(a) shows the normalized reflected current \( I_3/I \) as function of the voltage applied to gate B with gate A open. It is given by \( I_3/I = 1 - (T_{ji}^B + T_{ji}^B)/2 \). For \( I_3/I < 0.5 \) we obtain the transmission \( T_{ji}^B \) by measuring \( I_3 \) (\( T_{ji}^B \equiv 1 \)). The transmission \( T_{ji}^B \) is determined similarly.

In order to detect the current-current cross-correlations between contact 2 and 3 the time dependent currents \( I_\alpha(t) \) (\( \alpha = 2, 3 \)) are converted to voltage signals \( V_\alpha(t) \) by two series resistors \( R_{\alpha A} = h/4e^2 + R_{0,\alpha} \) implemented by means of additional ohmic contacts 2 and 3. \( R_{0,\alpha} \) denotes the contact resistances of the ohmic contacts, which is of the order 0.5 - 3 k\( \Omega \). The voltage fluctuations \( \Delta V_\alpha(t) = \Delta I_\alpha(t) R_{\alpha A} \) are measured by two low-noise amplifiers and fed into a spectrum analyzer which calculates the power spectral density. The RC-damping of the voltage noise due to the finite capacitance of the measurement lines (Fig. 2(b)) and the offset-noise \( S_0 \) due to the amplifiers are obtained from a calibration measurement of the Nyquist noise \( 4k_B T R \) as function of the bath temperature \( T \) for a given resistance \( R \). The noise measurements are performed in a frequency range of 20 to 70 kHz with typical bandwidths of 5 kHz. The measurement frequencies as well as the current bias are chosen such that contributions from \( 1/f \)-noise are negligible. All measurements were performed in a \( ^3 \)He-cryostat with a base temperature of 290 mK.

**FIG. 1:** (a,b) Implementation of the beam splitter in the quantum Hall regime with two edge-states \((i, ii) (B = 1.6 \) Tesla\). Equilibration between them occurs along the path from A to B when the current flows into an additional voltage probe 4. (c) SEM image: QPC’s are formed by split gates on top of a two-dimensional electron gas.

**FIG. 2:** Current fluctuations due to scattering at A are redistributed among the two edge-states so that positive cross-correlations are observed. Partial scattering at the second point contact B reveals the fermionic nature of the edge-states and yields a negative correlation. The correlations are zero in case that no partial scattering occurs at the QPC’s. The data (\( \bigcirc \)) are measured on sample 2. (a) Reflected current at 3 with gate C closed. (b) RC-damping of the voltage noise.

Fig. 2 gives the cross-correlations \( S_{1,23} = \langle \Delta I_2 \Delta I_3 \rangle_\omega \) measured on sample 1 for different configurations of gate A and B and with gate C open. In a first measurement \( T_{\alpha A}^B \) equals \( 0.5 \) and the beam splitter B is adjusted such that the second edge-state is totally reflected \( (T_{ji}^B = 0) \), which corresponds to the configuration shown in Fig. 1(b). For these parameters we indeed observe a positive cross-correlation (red full-circles). The solid line is the maximal positive cross-correlation given by Eq. 1. For comparison the Poissonian-noise \( S_0 = 2e|I| \) is given as dotted line. The total offset \( S_\ell^0 \) equals \( 3.13 \cdot 10^{-27} \) A\( ^2 \)/Hz. The current-noise of the amplifiers gives an offset \( S_{1,\ell,=0}^0 \) of \( 3.91 \cdot 10^{-27} \) A\( ^2 \)/Hz, which we obtain from several temperature calibrations. From these two values the thermal correlations between contact 2 and 3 can be calculated \( S_{1,23}(T = 0) = S_{1,\ell,=0}^0 - S_{1,\ell,=0}^0 = -7.9 \cdot 10^{-28} \) A\( ^2 \)/s, which turn out to be negative. Thermal correlations are always negative [11, 12]. They are not related to the statistics of the charge carriers but occur due to charge conservation. The measured value is in reasonable agreement with the theoretical prediction of \( -k_B \theta T G_0(3 - T^2) = -8.8 \cdot 10^{-28} \) A\( ^2 \)/s for \( T_{ji}^B = 0, T_{ji}^B = T = 0.5 \) and \( \theta = 290 \) mK [26].

For transparencies \( T_{ji}^B > 0 \) the second edge-state is only partially reflected at the ‘beam splitter’. Consequently, the statistical properties of the electrons in the fermionic ‘beam’ become apparent and the cross-correlations change sign from positive to negative. The solid line indicates the ‘full anti-
The curves are shifted in $V$ for clarity. (a) The edge-states equilibrate between $A$ and $B$ within a floating voltage probe. (b) Gate $C$ closed: no equilibration occurs and the currents carried by the two edge-states are unequal if the second edge-state is only partially transmitted at $A$. (c) $I_{23}/I$ vs. gate $C$ with gate $B$ closed and $T_{ii}^A \simeq 0.4$. Contact 4 is on the ground. (d) A positive cross-correlation is observed with gate $C$ open which disappears for gate $C$ closed.
(black circles). For sample 2 with a smaller path length \( L \) between the QPC’s the data points (open circles) are rather close to the expected value. Although the QPC \( B \) is adjusted to a plateau with high precision the second edge-state (ii) might not be reflected perfectly. Already a tiny transmission \( T_B^{ii} \) of 2% reduces the maximal positive cross-correlation by 23\%, illustrated by one of the dashed curves in Fig.4(a). The positive correlations completely disappear for \( T_B^{ii} > 9\% \). The high sensitivity to any changes from \( T_B^{ii} = 0 \) thus might explain the deviations from the solid curve. The open squares in Fig.4(a) are the results from sample 2 where gate \( C \) is closed so that the state-mixing voltage probe 4 is disconnected. Cross-correlations larger than 0.043 - \( S_0 \) could theoretically occur due to additional scattering of the first edge-state at \( A \). The dashed-dotted curve gives an example for \( T_A^{ii} = 0.96 \) instead of 1 that would yield a maximal positive correlation of 0.051 - \( S_0 \).

Fig 4(b, c) summerize the negative correlations obtained for \( T_A^{ii} = 1 \) and 0, respectively. The data points do not exactly agree with the expected values according Eq. 2 (solid curves). The dashed curves denote the changes that would occur due to additional scattering at the first QPC \( A \), yielding a small positive contribution to the negative correlations [14]. However, the transmission at \( A \) equals 0 or 1 (open gate) with quite high precision (\( |\Delta T_A^{ii}| \leq 0.03 \)) and we think that the deviations observed here are related to non-equal transmissions of the two spin-polarized parts in the second edge-state. The dotted lines in Fig.4(b, c) are the negative correlations for 20, 40 and 100 % unequal transmission (from bottom to top). For one spin-polarized edge-state totally transmitted and the other totally reflected the correlations would be zero for \( (T_B^{ii}) = 0.5 \).

From the data we estimate that the differences between the two transmissions are in the order of 20 - 40% of \( T_B^{ii} \).

In conclusion we have observed positive cross-correlations in a multi-terminal electronic device. This positive correlations occur due to interactions between different current carrying states inside the device and can be switched on and off by means of an external gate voltage, which controls the interaction inside the device.

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[1] For a review, see: Y. M. Blanter and M. Böttiker, Phys. Rep. 336, 1 (2000).
[2] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 79, 2526 (1997).
[3] R. de-Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahalu, Nature 389, 162 (1997).
[4] A. A. Kozhevnikov, R. J. Schoelkopf, and D. E. Prober, Phys. Rev. Lett. 84, 3398 (1990).
[5] X. Jelj, P. Payet-Burin, C. Baraduc, R. Calemczuk, and M. Sanquer, Phys. Rev. Lett. 83, 1660 (1999).
[6] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, Phys. Rev. Lett. 75, 3340 (1995).
[7] A. Kumar, L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 76 2778 (1996).
[8] M. Henny, S. Oberholzer, C. Strunk, and C. Schönnerberger, Phys. Rev. B 59, 2871 (1999).
[9] S. Oberholzer, E. Sukhorukov, and C. Schönnerberger, Nature 415, 765 (2002).
[10] M. Böttiker, Phys. Rev. B 46, 12485 (1992).
[11] Th. Martin and R. Landauer, Phys. Rev. B 45, 1742 (1992).
[12] M. Henny, S. Oberholzer, C. Strunk, T. Heinzel, K. Ensslin, M. Holland, and C. Schönnerberger, Science 284, 296 (1999).
[13] W. D. Oliver, J. Kim, R. C. Liu, Y. Yamamoto, Science 284, 299 (1999).
[14] S. Oberholzer, M. Henny, C. Strunk, C. Schönnerberger, T. Heinzel, K. Ensslin, and M. Holland, Physica E 6, 314 (2000).
[15] H. Kiesel, A. Renz, and F. Hasselbach, Nature 418, 392 (2002).
[16] for a review, see: M. Böttiker, "Reversing the sign of current-current correlations" in "Quantum Noise", edited by Yu. V. Nazarov and Ya. M. Blanter, Kluwer, p. 3 - 31 (2003).
[17] M. P. Anantram and S. Datta, Phys. Rev. B 53, 16390 (1996).
[18] Th. Martin, Phys. Lett. A 220, 137 (1996).
[19] J. Torres and Th. Martin, Eur. Phys. J. B 12, 319 (1999).
[20] T. Gramnespacher and M. Böttiker, Phys. Rev. B 61, 8125 (2000).
[21] P. Recher, E. V. Sukhorukov, and D. Loss, Phys. Rev. B 63, 165314 (2001).
[22] J. Börlin, W. Belzig, and C. Bruder, Phys. Rev. Lett. 88, 197001 (2002).
[23] P. Samuelsson and M. Böttiker, Phys. Rev. Lett. 89, 046601 (2002).
[24] O. Sauret and D. Feinberg, Phys. Rev. Lett. 92, 106601 (2004).
[25] A. Cottet, W. Belzig, and C. Bruder, Phys. Rev. Lett. 92, 206801 (2004).
[26] C. Teixier and M. Böttiker, Phys. Rev. B 62, 7454 (2000).
[27] B. I. Halperin, Phys. Rev. B 25, 2185 (1982).
[28] M. Böttiker, Phys. Rev. B 38, 9375 (1988).
[29] V. A. Khvesh, Sov. Phys. JETP 66, 1243 (1987).
[30] G. B. Lesovik, JETP Lett. 49, 592 (1989).
[31] M. Böttiker, Phys. Rev. Lett. 65, 2901 (1990).
[32] B. W. Alphenaar, P. L. McEuen, R. G. Wheeler, and R. N. Sacks, Phys. Rev. Lett. 64, 677 (1990).
[33] B. J. van Wees, E. M. M. Willems, C. J. P. M. Harmans, C. W. J. Beenakker, H. van Houten, J. G. Williamson, C. T. Foxon, and J. J. Harris, Phys. Rev. Lett. 62, 1181 (1989).