ON FREQUENCY-DEPENDENT ROCK EXPERIMENTS: A COMPARATIVE REVIEW

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August 7, 2022

ABSTRACT

Rock properties are environment- and condition-dependent which render field-laboratory comparisons ambiguous for a number of known and unknown reasons that constitute the upscaling problem. Unknowns are first transformed into knowns in a controlled environment (laboratory) and second in a volatile environment (field). Causality-bound dispersion and attenuation are respectively defined as rock properties that are frequency- and distance-dependent: dispersion implies non-zero attenuation and vice versa. Forced-Oscillation (FO), Resonant Bar (RB), and Pulse-Transmission (PT) are the customary techniques to measure rock properties at Hz, kHz, and MHz frequencies. Notably FO has emerged as the current champion in bridging the field-laboratory void in recent years. Not only is FO probing seismic (Hz) frequencies but with \( \sim 10^{-6} \) strain amplitudes it is also similar to field seismic. RB and PT are concisely however FO is verbosely elaborated by chronologically compiling most (if not all) FO studies on sedimentary rocks and comparing all available FO measurements on reference materials such as lucite, aluminium, and PEEK. First of its kind, this inter-laboratory comparison may serve as a reference for others who seek to verify their own results. Differences between FO are discussed with alternative strain and stress sensors being the focal points. Other techniques such as Resonant Ultrasound Spectroscopy (RUS), Laser Ultrasounds (LUS), and Differential Acoustic Resonance Spectroscopy (DARS) that are similar to FO, RB and PT are also described. Only time will tell what the future holds for FO but plausible improvements for the future are ultimately given which may elevate it even further. Experimental combined with numerical novelty will extend the probeable frequencies beyond their current limits.

Key words: Dispersion; Attenuation; Forced-Oscillation; Resonant Bar; Pulse-Transmission
1 INTRODUCTION

Laboratory and field measurements are not easily compared for a number of known and unknown reasons that constitute the upscaling problem. These measurements are based on wave reflection and refraction (field) or transmission and to some extent echo (laboratory). Due to the dispersive (frequency-dependent) nature of fluid-saturated rocks, traditional laboratory (MHz) and field (Hz and kHz) measurements are incompatible. Dispersion is both a laboratory and field phenomenon that occurs whenever properties measured at different frequencies are compared. For example, White et al. [1] inadvertently described dispersion from seismic to sonic field data while studying anisotropy. Caution must thus be exercised when comparing data or basing models (measured or valid at certain frequencies) on dispersive parameters (measured at other frequencies). Attenuation effects are included in many algorithms for waveform modeling, imaging, and full-waveform inversion among other field-relevant applications. Experimental studies in controlled conditions are paramount to universally understand this fluid-related phenomenon and its mechanisms.

Forced-Oscillation (FO), Resonant Bar (RB), and Pulse-Transmission (PT) are three common techniques to measure rock properties at seismic (Hz), sonic (kHz), and ultrasonic (MHz) frequencies, respectively. Particularly FO and to some extent also RB are increasingly recognized as means to cover seismic and sonic frequencies (from Hz to kHz) in the laboratory. These techniques share the common denominator that all were first used to study metals by physicists but were later adopted by rock physicists to also study rocks. Progress is inevitable as the number of laboratory studies at similar frequencies as in the field are rapidly increasing despite measuring dissimilar parameters. In fact, also the measured parameters are environment-dependent. For example, FO in its basic (longitudinal) form measures Young’s modulus and Poisson’s ratio instead of P- and S-wave velocities at seismic frequencies, while its auxiliary (uniaxial) form measures uniaxial modulus equivalent to P-wave velocity. Transformation between moduli and velocities inevitably introduces errors despite being trivial for isotropic and non-trivial for anisotropic rocks. FO novelty is driven by extending the frequencies at which it operates, what properties it measures and how in terms of sensor types, improving its boundary conditions (dead volume), and recently also its concurrent imaging via CT.

This study is in its entirety based on Rørheim [4] but with minor modifications for formatting purposes. Rørheim [5] was its first iteration and Rørheim and Holt [6] its second but it has since been expanded into greater detail while being continuously updated to keep up with time as novel experiments are becoming customary. In fact, unknown to Rørheim [5] at the time, Subramaniyan et al. [7] is an excellent review of apparatuses with the ability to measure seismic attenuation in reservoir rocks. Similarities between this study and Subramaniyan et al. [7] are evident as both are inter-laboratory comparisons however the focus of Subramaniyan et al. [7] was on the standardization need of FO apparatuses whereas the present is on comparing FO measured moduli and attenuations on lucite, aluminium, and PEEK reference specimens. Subramaniyan et al. [7] did not cover all apparatuses and only considered reservoir rocks such as sandstones and carbonates unlike unconventional ones like shale. Validating attenuation measurements by literature comparison was the basis of Rørheim and Holt [6]. Like Subramaniyan et al. [7], the three most common techniques (FO, RB, and PT) are the focal points (FO more so than RB and PT), but unlike Subramaniyan et al. [7], other novel techniques such as Resonant Ultrasound Spectroscopy (RUS), Differential Acoustic Resonance Spectroscopy (DARS), and Laser UltraSonics (LUS) are also elaborated. The primary three techniques (FO, RB, and PT) are also theoretically defined whereas the secondary ones (RUS, DARS, and LUS) are not. Recent numerical advances are also included as the value and significance of Digital Rock Physics (DRP) surely will improve in the future when elevated from its current infancy.
2 THEORY

RB, PT, and FO are related due to all three exploiting mechanical disturbances in a material to deduce its properties. Here the theory of FO is elaborated while that of RB and PT are mostly unelaborated due to FO being the primary focus from the outset and throughout. Relevant models commonly used to simulate the dispersive (or frequency-dependent) behaviour of rocks across frequencies are also briefly explained.

2.1 MECHANICAL EQUATIONS

RB exploits harmonic waves in the axial direction with radial $u_r$, circumferential $u_\theta$, and axial $u_z$ components of displacement. $z$-dependent but $\theta$-independent motions are separated into torsional waves with $u_\theta$ and longitudinal waves with $u_r$ and $u_z$. Flexural waves are $z$- and $\theta$-dependent motions. Phase velocities $V$ at resonating frequencies $f$ relate to bar length $L$ as

$$V = \frac{2L}{n} f, \quad \lambda$$

where $n \in \{1, 2, 3, \ldots\}$ denotes different modes or harmonics and $\lambda$ is the wavelength. $n = 1$ is the fundamental frequency. Young’s modulus $E$ from longitudinal and flexural modes is $E = \rho V_E^2$ whereas shear modulus $G$ from the torsional mode is $G = \rho V_G^2$, with $V_E$, $V_G$, and $\rho$ being longitudinal or flexural velocity, torsional velocity, and density [8]. PT exploits the time of flight principle to determine P- and S-wave velocities

$$V = \frac{L}{\Delta t}, \quad (2)$$

where $V$ is the velocity of either body wave type, $L$ is the specimen length, and $\Delta t$ is the travel time (Figure 1a). FO measures different moduli and ratios depending on excitation modes: (i) longitudinal, (ii) torsional, (iii) flexural, (iv) volumetric, and (i) uniaxial. Higher harmonics are also explorable for FO like for RB. Elastic moduli are generally defined as stress over strain $M = \sigma/\epsilon$. (i) yields Young’s modulus and Poisson’s ratio $\nu$ respectively as

$$E = \frac{\sigma_{ax}}{\epsilon_{ax}}, \quad (3)$$

$$\nu = -\frac{\epsilon_{rad}}{\epsilon_{ax}}, \quad (4)$$

with $\sigma$ and $\epsilon$ being stress and strain either in ax(ial) or rad(ial) direction (Figure 1b). (ii) provides shear modulus $G$ as the shear stress-strain ratio

$$G = \frac{\sigma_s}{\epsilon_s}, \quad (5)$$

in which $\sigma_s$ and $\epsilon_s$ are shear stress and shear strain, respectively. Kelvin [9] regarded $K$ and $G$ as principal elasticities with special significance. (iii) adds flexural modulus $E_F$ but remains undefined due to its condition-dependent definitions. Flexural and Young’s moduli are theoretically equivalent $E = E_F$ but practically inequivalent $E \neq E_F$. (iv) grants bulk modulus $K$ as the three-dimensional extension of Young’s modulus $E$. It is defined as an object’s proclivity to deform in all directions under hydrostatic stress or pressure regimes

$$K = \frac{\sigma_p}{\epsilon_{vol}}, \quad (6)$$
where $\sigma_p$ (or $P_p$) is hydrostatic stress (or pressure) and $\epsilon_{\text{vol}}$ is volumetric strain. (v) gives uniaxial (or P-wave) modulus $H$ which is akin to Young’s modulus $E$ but for $\epsilon_{\text{rad}} = 0$ enforcing uniaxial-strain instead of uniaxial-stress conditions (Figure 1c).

$$H = \frac{\sigma_{\text{ax}}}{\epsilon_{\text{ax}}} \bigg|_{\epsilon_{\text{rad}} = 0}.$$  

Lamé’s $\lambda_L$ and $\mu_L$ parameterize $K = \lambda_L + 2/3\mu_L$ where $\mu_L = G$ and $\lambda_L$ is physically equivocal. P- and S-wave velocities $V_P$ and $V_S$ are then

$$V_P = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} = \sqrt{\frac{\lambda_L + 2\mu_L}{\rho}},$$
$$V_S = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{\mu_L}{\rho}},$$

whereas $V_P/V_S = \sqrt{K/G + 4/3} = \sqrt{\lambda_L/\mu_L + 2}$ is density-independent.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Amplitudes $B$, $\epsilon$, and $\sigma$ versus time $t$ exemplified for three different techniques: (a) PT and two FO versions: uniaxial-stress (b) and uniaxial-strain (c). Arrows in (a) indicate first maxima.}
\end{figure}
2.1.1 ISOTROPY VERSUS ANISOTROPY

Anisotropic or isotropic is the medium whose elastic properties change or unchange with direction. Hookean theory \[11, 12\] first formulated as “ut tensio, sic vis” \[11\] relates stress $\sigma_{ij}$ and strain $\epsilon_{kl}$ as

$$
\sigma_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} C_{ijkl} \epsilon_{kl}
$$

where $C_{ijkl}$ is stiffness and $i, j = 1, 2, 3$ using Einstein notation \[13\].

Transforming between $E$, $\nu$, $G$, $K$, and $H$ is isotropically trivial (Table 1) yet anisotropically non-trivial as the number of specimens and stiffnesses required for full description increases from one (Figure 2) to three (Figure 3) and from two (Equation 8) to five (Equation 9) using Voigt notation \[14\] for Transverse Isotropy (TI), respectively. Stress $\vec{\sigma} = (\sigma_{11} \sigma_{22} \sigma_{33} \sigma_{23} \sigma_{13} \sigma_{12})$ and strain $\vec{\epsilon} = (\epsilon_{11} \epsilon_{22} \epsilon_{33} \epsilon_{23} \epsilon_{13} \epsilon_{12})$ combined with $\vec{C} = C_{ij}$ denoting either two stiffnesses $C_{11}$ and $C_{44}$ describe isotropic rocks as

$$
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{13} \\
\sigma_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{11} - 2C_{44} & C_{11} - 2C_{44} & 0 & 0 & 0 \\
C_{11} - 2C_{44} & C_{11} & C_{11} - 2C_{44} & 0 & 0 & 0 \\
C_{11} - 2C_{44} & C_{11} - 2C_{44} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{44}
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{33} \\
\epsilon_{23} \\
\epsilon_{13} \\
\epsilon_{12}
\end{bmatrix},
$$

(8)

or three additional stiffnesses $C_{13}$, $C_{33}$, and $C_{66}$ describe anisotropic rocks as

$$
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{13} \\
\sigma_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{11} - 2C_{66} & C_{13} & 0 & 0 & 0 \\
C_{11} - 2C_{66} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{33} \\
\epsilon_{23} \\
\epsilon_{13} \\
\epsilon_{12}
\end{bmatrix},
$$

(9)

where $C_{11}$, $C_{13}$, $C_{33}$, $C_{44}$, and $C_{66}$ combinations yield the anisotropic versions of $E$, $\nu$, $G$, $K$, and $H$. For example, without further elaboration, isotropic $E$ and $\nu$ expand to anisotropic $E_V$, $E_{45}$, and $E_H$ as well as $\nu_{VH}$, $\nu_{HV}$, and $\nu_{HH}$ for the longitudinal mode, while $H = C_{33}$ in Equation 7 for the uniaxial mode of a $0^\circ$ specimen.

---

\[1\] As the extension, so the force" is Hooke’s 1678 solution \[11\] to his 1675 anagram “ceiioonssssttuu” \[12\].

---

Figure 2 Geometry of an isotropic specimen. Triangles indicate measurement directions of biaxial strain gauges.
Figure 3 Geometries of (a) 0°, (b) 45°, and (c) 90° specimens assuming TI symmetry. Triangles indicate measurement directions of biaxial strain gauges.

2.2 ATTENUATION EQUATIONS

Frequency-dependent rock behaviour caused by wave-induced fluid motions at different scales within the porous network is related to the causal-consistent dispersion and attenuation phenomena. The quality factor $Q$ is a measure of a material’s dissipativity [15]. “The greater the value of $Q$, the smaller the internal friction” [18] which is intuitive since, vice versa, low $Q$-values imply large dissipations. $Q$ or $Q^{-1}$ customarily describes attenuation and comes in many forms including as the imaginary $M_\text{Im}$ and real $M_\text{Re}$ parts of complex modulus $M$ [19]

$$Q^{-1} = \frac{M_\text{Im}}{M_\text{Re}}$$

(10)

as the ratio of energy stored $W_m$ to energy loss per cycle $\Delta W$ [20]

$$Q^{-1} = \frac{\Delta W}{2\pi W_m},$$

(11)

or as the tangent of the phase angle $\Delta \theta$ between the applied stress $\theta_\sigma$ and the resulting strain $\theta_\epsilon$ [21]

$$Q^{-1} = \tan(\theta_\epsilon - \theta_\sigma),$$

(12)

which in practice corresponds to the phase angle $\Delta \theta$ between the standard $\theta_\text{STD}$ and the specimen $\theta_\text{SPE}$ exemplified by Figure 4. Equation 12 is fundamental for FO because oscillating stress $\sigma(t) = \sigma e^{i\omega t}$ and strain $\epsilon(t) = \epsilon e^{i\omega t - \Delta \theta}$ are causality-coupled, whereas RB and PT respectively rely on the width of the resonant peak or the time constant of the resonant decay and the Spectral Ratio (SR) method to determine $Q$. Morozov [22] questioned the usefulness of $Q$ as a measure and if its many definitions are “true”, “assumed”, or “apparent”.

To evaluate mechanisms, Winkler and Nur [23] assumed an isotropic solid described by complex moduli with small imaginary parts to transform between two moduli

$$\frac{(1 - \nu)(1 - 2\nu)}{Q_P} = \frac{1 + \nu}{Q_E} - \frac{2\nu(2 - \nu)}{Q_G},$$

(13a)
Table 1 Tabulated relationships among elastic constants in an isotropic material modified from Mavko et al. [15] who based it on Birch [16]. Similar tabulated versions of these relations are common in literature (even Gassmann [17] included one). Given any combination of two parameters, any other elastic properties are uniquely determined from these formulae. The last row features the relationship as functions of velocities $V_p$ and $V_S$ as well as density $\rho$. The last column features $V_p/V_S$ described by different pairs of elastic constants.

| Parameters       | $K$                    | $E$                    | $\lambda_L$               | $\nu$ | $H$                    | $\mu_L$               | $V_p/V_S$               |
|------------------|------------------------|------------------------|---------------------------|-------|------------------------|------------------------|--------------------------|
| ($\lambda_L, \mu_L$) | $\lambda_L + \frac{2}{3} \mu_L$ | $\mu_L \left( \frac{3\lambda_L + 2\mu_L}{\lambda_L + \mu_L} \right)$ | $\frac{\lambda_L}{2(\lambda_L + \mu_L)}$ | $\lambda_L + 2\mu_L$ | ($\frac{\lambda_L + 2\mu_L}{\mu_L} \right)^{\frac{1}{2}}$ |
| ($K, \lambda_L$)   | $9K \left( \frac{K - \lambda_L}{3K - \lambda_L} \right)$ | $\frac{\lambda_L}{3K - \lambda_L}$ | $3K - 2\lambda_L$ | $\frac{3(3K - \lambda_L)}{2(3K + \mu_L)}$ | ($\frac{3\lambda_L - 2K}{\lambda_L - K} \right)^{\frac{1}{2}}$ |
| ($K, \mu_L$)         | $\frac{9K\mu_L}{3K + \mu_L}$ | $K - \frac{2}{3} \mu_L$ | $\frac{3K - 2\mu_L}{2(3K + \mu_L)}$ | $K + \frac{4}{3} \mu_L$ | ($\frac{K + 4\mu_L}{\mu_L} \right)^{\frac{1}{2}}$ |
| ($E, \mu_L$)         | $\frac{E_{in}}{3(4\mu_L - K)}$ | $\mu_L \left( \frac{E_{2\mu_L}}{4\mu_L - 1} \right)$ | $\frac{E}{4\mu_L - 1}$ | $\mu_L \left( \frac{4\mu_L - E}{4\mu_L - 1} \right)$ | ($\frac{E - 4\mu_L}{E - 3\mu_L} \right)^{\frac{1}{2}}$ |
| ($K, E$)            | $3K \left( \frac{3K - E}{9K - E} \right)$ | $\frac{3K - E}{6K}$ | $3K \left( \frac{3K + E}{9K - E} \right)$ | $\frac{3KE}{9K - E}$ | ($\frac{E + 3K}{E} \right)^{\frac{1}{2}}$ |
| ($\lambda_L, \nu$)  | $\lambda_L \left( \frac{1 + \nu}{\nu} \right)$ | $\lambda_L \left( \frac{1 + \nu}{1 - 2\nu} \right)$ | $\lambda_L \left( \frac{1 - 2\nu}{1 - 2\nu} \right)$ | $\lambda_L \left( \frac{1 - 2\nu}{1 - 2\nu} \right)$ | $\lambda_L \left( \frac{1 - 2\nu}{1 - 2\nu} \right)$ |
| ($\mu_L, \nu$)       | $\nu \left[ \frac{2(1 + \nu)}{3(1 - 2\nu)} \right]$ | $2\mu_L \left( 1 + \nu \right)$ | $\mu_L \left( \frac{2\nu}{1 - 2\nu} \right)$ | $\mu_L \left( \frac{2\nu}{1 - 2\nu} \right)$ | $\mu_L \left( \frac{2\nu}{1 - 2\nu} \right)$ |
| ($K, \nu$)           | $3K \left( 1 - 2\nu \right)$ | $3K \left( \frac{\nu}{1 + \nu} \right)$ | $3K \left( \frac{1 - 2\nu}{2 + 2\nu} \right)$ | $3K \left( \frac{1 - 2\nu}{2 + 2\nu} \right)$ | $3K \left( \frac{1 - 2\nu}{2 + 2\nu} \right)$ |
| ($E, \nu$)           | $\frac{E}{3(1 - 2\nu)}$ | $\frac{E}{(1 + \nu)(1 - 2\nu)}$ | $\frac{E}{(1 + \nu)(1 - 2\nu)}$ | $\frac{E}{(1 + \nu)(1 - 2\nu)}$ | $\frac{E}{(1 + \nu)(1 - 2\nu)}$ |
| ($H, \nu$)           | $H - \frac{4}{3} \mu_L$ | $\mu_L \left( \frac{3H - 4\mu_L}{H - \mu_L} \right)$ | $\frac{H - 2\mu_L}{2(M - \mu_L)}$ | $\frac{H - 2\mu_L}{2(M - \mu_L)}$ | $\frac{H - 2\mu_L}{2(M - \mu_L)}$ |
| ($V_p, V_S$)         | $\rho \left( V_p^2 - \frac{4}{3} V_S^2 \right)$ | $\rho V_S^2 \left( \frac{3V_p^2 + 3V_S^2}{V_p^2 + V_S^2} \right)$ | $\rho \left( V_p^2 - 2V_S^2 \right)$ | $\rho \left( V_p^2 - 2V_S^2 \right)$ | $\rho \left( V_p^2 - 2V_S^2 \right)$ |
\[
\frac{1 - 2\nu}{Q_K} = \frac{3}{Q_E} - \frac{2(\nu + 1)}{Q_G},
\]
(13b)

\[
\frac{1 + \nu}{Q_K} = \frac{3(1 - \nu)}{Q_P} - \frac{2(1 - 2\nu)}{Q_G},
\]
(13c)

with \(Q_E = E_R/E_\ell\), \(Q_K = K_R/K_\ell\), and \(Q_G = G_R/G_\ell\) while simultaneously proving that one of these attenuation relations must be true:

\[
Q_K > Q_P > Q_E > Q_G, \quad (14a)
\]
\[
Q_K < Q_P < Q_E < Q_G, \quad (14b)
\]
\[
Q_K = Q_P = Q_E = Q_G. \quad (14c)
\]

Johnston et al. [24] summarized a series of \(Q\)-dependencies based on the accumulation of individual attenuation measurements (frequency, strain amplitude, fluid saturation, pressure and stress, and temperature) and elaborated on attenuation mechanisms: (i) matrix anelasticity, (ii) viscosity and flow of saturating fluids, and (iii) scattering from inclusions. Mechanism (i) is related to solid friction between grains in dry rocks. Mechanism (ii) includes local Biot flow within the pore space often termed “squirt-flow” [25], flow in heterogeneous media [26], and flow related to the wavelength-scale equilibration [27] at the respective microscopic, mesoscopic, and macroscopic scales. Mechanism (iii) tends to be significant for wavelengths close to the size of any heterogeneities. Mechanism (ii) is typically caused by Wave-Induced Fluid Flow (WIFF) [28] but evidence for Wave-Induced Gas Exsolution Dissolution (WIGED) [29] also exists. Fundamental to poroelasticity [17, 25, 30, 31], are the opposing concepts of (i) drained and (ii) undrained conditions defined as the fluid’s ability (i) or inability (ii) to flow in or out of the porous medium. Biot [27] predicted the existence of two regimes of fluid motion governed by (i) viscous effects at the low frequency limit [25] and by (ii) inertia effect at the high frequency limit [30]. Equal viscous and inertial forces of the fluid occur at Biot’s characteristic frequency \(f_b\), while viscous flow and inertial drag respectively dominate below and above. These concepts define characteristic frequencies for the drained-undrained and quasistatic-dynamic transitions. Low- and high-frequency behaviours (“relaxed” versus “unrelaxed” states) are distinguished by characteristic frequencies \(f_c\) at different scales: microscopic \(f_c \rightarrow f_s\), mesoscopic \(f_c \rightarrow f_p\), and macroscopic \(f_c \rightarrow f_d, f_b\) are respectively defined as

\[
f_s = \frac{\xi^3 K_d}{\eta},
\]
(15)

\[
f_p = \frac{\kappa K_S}{\pi L^2 \eta},
\]
(16)

\[
f_d = \frac{\kappa K_u}{\eta L^2},
\]
(17)

\[
f_b = \frac{\eta \phi}{2 \pi \rho_f K},
\]
(18)

where \(\xi\) is aspect ratio, \(\eta\) is viscosity, \(\kappa\) is permeability, \(\phi\) is porosity, \(\rho_f\) is pore fluid density, while \(K_d, K_u,\) and \(K_S\) are drained, undrained, and solid bulk moduli [15, 32].
Figure 4 Equation 12 graphically displayed in which the phase shift $\Delta \theta$ between $\theta_{\text{STD}}$ and $\theta_{\text{SPE}}$ yields $Q^{-1}$. Both strain amplitudes are equal $B_{\text{STD}} = B_{\text{SPE}}$ for the sake of simplicity.

Bulk and shear viscosities are dissimilar but the controlling molecular motion are probably similar [33]. The former is not only poorly defined but also rarely measured compared to the latter. Viscosity discernibility is crucial since bulk dominates shear in dispersivity and attenuativity of rocks saturated with highly viscous fluids. Moreover, stronger bulk losses characterize saturated sediments comprising rounded sand grains rather than silt-sized quartz particles [34].

2.3 MODELS

“All models are wrong but some are useful” in simulating experimental behaviour and isolating the dominant mechanisms. “Numerous theories on wave propagation exist but these concepts remain unencumbered by measured data needed to prove, delimit or extend them” [36]. An increasing number of phenomenological and non-phenomenological (predictive) models that combine flow at different scales are continuously being proposed. Since different experimental techniques cover different frequencies, models that simulate dispersive behaviour across frequencies are imperative to unite results from different experiments. Cole-Cole (CC) [37] and Standard Linear Solid (SLS) (or Zener) [38] models are among the most common ones alongside the analytical Kramers-Kronig relations (KKR) [39, 40]. CC is based on KKR and reduces to the fundamental Debye model [41] if $\alpha \in [0, 1] \rightarrow \alpha = 0$, while SLS is based on the Maxwell [42] and Kelvin-Voigt [43, 44] models but with different combinations of springs and dashpots to represent elastic and viscous components, respectively. Burgers viscoelastic model [45] is a combination of the other models (Maxwell, Kelvin-Voigt, and Zener). KKR are the bidirectional mathematical solutions to the problem of connecting the real and imaginary parts of any complex function that is constrained by causality and analytic in the upper half-plane. Spencer [21] extended the frequency-dependent CC model from the realm of the dielectric constant of liquids to the realm of the real and imaginary parts of porous media moduli impacted by relaxation processes. Mikhailsevitch et al. [46] considered an isotropic viscoelastic material regarded as a dynamic system for which the stress-strain relationship is linear with given mechanical properties as a numerical solution of KKR. In general, KKR is superior to CC due to its analytical nature but necessitates mechanical and attenuation measurements at all frequencies unlike CC that can be fitted using only mechanical measurements. Rørheim [4] describes a joint-fit-based procedure based on CC.

2\(^{\text{nd}}\) Common aphorism generally attributed to Box [35].
that uses both mechanical and attenuation measurements as input. CC is however limited to describing one relaxation time, transition, or mechanism albeit multiple may concurrently exist.

3 METHODS

Experimental determination of dynamic rock properties chronologically began with Resonant Bar (RB), Pulse-Transmission (PT), and Forced-Oscillation (FO) at the respective sonic, ultrasonic, and seismic frequencies (Figure 5). All three techniques are elaborated but FO in greatest detail with (to our knowledge) all accessible studies described and summarized.

![Figure 5](image)

**Figure 5** Probeable frequencies subdivided into seismic, sonic, and ultrasonic for the three primary techniques: RB, PT, and FO

3.1 RESONANT BAR (RB)

Ide [47] invented a technique based on Cady [48], Quimby [49], and Boyle and Sproule [50] for dynamic determination of rock properties that produced longitudinal vibrations of rods by electrostatic traction. Inspired by the novelty of their colleague’s approach, Birch and Bancroft [18] studied the longitudinal, flexural, and torsional modes of a long granite column forced into resonant vibrations excited by an alternating magnetic field. This ultimately became known as the widely acknowledged Resonant Bar (RB) technique whose function is to probe sonic frequencies. However rare measurements in the sonic frequency range may be, experiments using long and slender samples are less so than experiments involving conventional core specimens. The longer the resonator, the lower the characteristic frequency. Since frequency is lowered as a function of bar length, it is common practice to cement short-length bars together due to the difficulty to directly core sufficiently long bars [51]. A sinusoidal force is traditionally submitted to one end of a rock bar fixed at its center of mass (which also coincides with the location of minimum displacement) while the resultant vibrations are measured at the other. Piezoelectric or electromagnetic transducers excite either the extensional or the torsional mode of vibration. Moduli are calculated from the resonance frequencies that depend on the material’s velocity, dimensions, and density. Attenuation is determined by the width of the resonant peak or the time constant of the resonant decay.

Despite the obvious advantage of probing sonic frequencies, Wang [52] identified (i) extensive system calibration and data corrections as well as difficulty with (ii) obtaining high-pressure, high-temperature data and (iii) pure shear mode as disadvantages. Disadvantageous is also that the characteristic frequency is a function of the bar length. Although Jones and Nur [53] extended the investigations of the effects of confining pressure, pore pressure, and degree of saturation by Winkler and Nur [54] with elevated temperatures, RB measurements at *in-situ* conditions belong to the rarities. RB suffered dimensional limitations until Tittmann [55] proposed the possibility to reduce resonance frequencies by the use of additional mass that changes the moment of inertia without altering the rigidity. These words resonated with Nakagawa [56] who realized that
small specimens could be explored using a modified version of RB. Nakagawa [56] designed an apparatus geometrically similar to the Split-Hopkinson Pressure Bar (SHPB) described by Kolsky [58] and named after Hopkinson [57]. Nakagawa [56] added mass to his Split-Hopkinson Resonance Bar (SHRB) by a pair of 40.6 cm steel extension bars with attached PZT source and accelerometer receivers. Extensive corrections for the jacket and the interfaces are however applied in a numerical inversion model to correct for the specimen-rod interface effects. Table 3 lists authors and techniques that have successfully probed the sonic frequency range as functions of frequency and specimen dimensions categorized into measured parameters. Featured is not only RB and FO but also the Pulse-Tube (PTU), Gas Hydrate Resonant Column (GHRC), and Resonant Ultrasound Spectroscopy (RUS) techniques.

3.2 PULSE-TRANSMISSION (PT)

Pulse-Transmission (PT) was extended from metals [59] to rocks [60–62] and further disseminated [63–65] as a technique. PT determines the travel-time of an elastic wave propagating between two PZC (normally quartz or PZT) transducer elements in which one generates and the other records the ultrasonic pulse. The piezoceramic size determines its resonant (or center) frequency. Scattering effects are avoided if specimen heterogeneities are eclipsed in size by the wavelength. Since distance travelled (equals specimen length) and travel-time are known, the basic equation of motion yields velocity based on the time of flight principle. Piezoceramic (PZC) elements deform mechanically and generate electricity respectively as external voltages and forces are applied. P- and S-wave transducers are sensitive to their respective wave types. Polarization-alignment is paramount for the S-wave transducers due to the physical nature of S-waves. In practice, the quality of the signal depends on the transducer-specimen coupling being adequate or inadequate while the accuracy of the measurements is related to the adequacy of the velocity picks. Normal P-waves are trivial but oblique P-waves and any S-waves are non-trivial to pick. Practice and theory are advanced by transducer and algorithm novelty. Eternal is the group versus phase velocity conundrum for oblique P-waves that depend on transducer size. PT attenuation is determined by the Spectral Ratio (SR) method which compares spectral amplitudes at different distances from the slope of the logarithmic decrement [15]. Otherwise attenuation is also determined by comparing two geometrical identical specimens: a non-dispersive reference specimen and the studied specimen [8]. This is the same principle as for FO in comparing non-dispersive reference specimens with dispersive rock specimens. Pulse-Echo (PE) is physically different from PT (echo versus transmission) but principally similar in determining attenuation by standard-specimen comparisons.

3.3 FORCED-OSCILLATION (FO)

Bruckshaw and Mahanta [66], Peselnick and Outerbridge [67], and Usher [68] performed the first Forced-Oscillation (FO) measurements that determined the intrinsic attenuation and elastic moduli of rocks. The two extremes investigated the longitudinal mode (Young’s modulus $E$ and attenuation $Q^{-1}$), whereas the middle developed a pendulum able to generate harmonic torques (shear modulus $G$ and attenuation $Q^{-1}$). Truth be told, internal friction of metals [69, 70] and polymers [71, 73] were measured by FO before extended to rocks [66, 68]. Strain amplitude was not controlled in the first generation of FO experiments but was later constrained to $10^{-6}$ to remain within the linear elastic domain [71]. Spencer [21] measured the stress-strain behaviour of rocks via transducers recording the axially applied sinusoidal force and the resulting displacement. Adelinet et al. [75] applied hydrostatic oscillations to measure bulk modulus $K$ of isotropic rocks via confining pressure pulsations. Jackson et al. [76] modified their torsional capabilities to also accommodate for measurements of the flexural mode which Woirgard et al. [77] and Woirgard and
Guéguen [78] also measured for metals and rocks in the past. Unlike Adelinet et al. [75], Lozovyi and Bauer [79] enforced uniaxial instead of hydrostatic conditions by adding axial oscillations to investigate anisotropic rocks. Suarez-Rivera et al. [80] measured uniaxial modulus decades before Lozovyi and Bauer [79] without disclosing any details about the apparatus. FO requires (i) a force generator with alterable frequency, (ii) a force sensor to estimate the applied stress, and (iii) strain sensors for the specimen. Modulus and attenuation are given by the stress-strain ratio and the phase angle between the stress and the strain (or the area of the hysteresis loop), respectively. If an elastic standard is chosen over a transducer to measure the stress, modulus and attenuation are given by the relative strain amplitude of the standard versus the specimen and the phase angles between the standard and the specimen. Table 4 tabulates apparatuses for (i) longitudinal, (ii) torsional, (iii) flexural, (iv) volumetric, and (v) uniaxial stress-strain oscillations. Type (ii), (iii), and (iv), and (v) apparatuses exist but type (i) is still predominant.

FO is theoretically trivial but practically non-trivial. Misalignments related to manufacturing tolerances primarily affecting attenuation measurements (inconsistent phase angles) are particularly challenging [81,83]. In the case of Rørheim and Holt [6], this problem originated from misaligned interfaces. Uneven stress distribution may be observed on the semiconductor resistivities during axial loading. Numerous approaches have been applied to alleviate systematic errors caused by misalignment if all interfaces are not flawlessly parallel and smooth when manufactured. Paffenholz and Burkhardt [84], Takei et al. [85], and Ikeda et al. [86] neutralized potential misalignments with three axially symmetrical transducers. Resonances enforce a natural upper limit for FO measurements in the low-frequency regime. Due to the nature of resonances (nodes and antinodes as well as their distribution), Batzle et al. [36] extended this limit by identifying the antinodal frequencies and adjust the measurements accordingly. Fluid-flow related dispersion occurring at the intermediate sonic frequency range may be inferred by using a very viscous fluid as described by Fortin et al. [87] and Pimienta et al. [88]. Since the time-scale of diffusion processes are related to the dynamic viscosity of the fluid, the frequency of the measurement can be scaled by the viscosity of the fluid [89].

Despite being less resonance-prone than other transducers, strain gauges are largely limited to investigating pore-scale processes [90]. Averaging multiple strain measurements at different positions on the specimen surface may however approximate the bulk mechanical properties of a rock [91]. The smaller the specimen, the better the averaging result (proportional to the covered area). Chapman and Quintal [90] favoured measuring bulk strain (cantilevers) in addition to or instead of local strains (strain gauges) due to the locality of heterogeneities. Bulk-strain approaches are superior for isotropic rather than anisotropic rocks due to Poisson’s ratios being problematic to measure. Tisato [92] explained the double-interface problem related to radial cantilevers on jacketed (sealed) specimens. Spencer [21] however solved this problem with non-contact capacitive sensors and Tisato [92] explored effect sensors. Fibre optic DAS is another possibility [90,94,96] that faces similar problems. Simultaneous bulk and local strain measurements could perhaps also distinguish between microscopic and mesoscopic dispersion and attenuation mechanisms [90]. Unlike Adam and Batzle [97] but like Chapman and Quintal [90], Sun et al. [98] analyzed individual strains before averaging to distinguish local from global flow. Stress-strain signal magnitudes and phases are computed by FFT after being digitized and averaged by physical or virtual lock-in amplifiers.

FO is applied to measure properties of all sedimentary rock types: sandstones [21,36,46,84,94,95,98,149], carbonates [21,36,67,84,97,99,123,124,148,150,163], and shales [32,80,99,108,123,164,177]. FO is also applied to igneous rocks and other polycrystalline materials [21,75,76,78,178,187] that are beyond the scope of this study (due to their non-sedimentary nature) but are nonetheless represented in Table 4. Suarez-Rivera et al. [80] is the only inter-frequency study comparing FO, RB and PT. McCann et al. [188] used FO to measure fluid properties (bulk modulus $K$ and attenuation $Q^{-1}$) at high pressures. Be advised that Table 4
may list the same apparatus multiple times because they are continuously being improved or adjusted with time. Subramaniyan et al. [7] summarized attenuation studies on conventional reservoir rocks (i.e. sandstones and carbonates), while recent studies on conventional rocks as well as unconventional shales are added herein to update and complete the list:

**Sandstones** Spencer [21] initially observed negligible $Q^{-1}$ of a vacuum-dried Navajo specimen that eventually became significant once saturated with water, ethanol, or n-decane. Paffenholz and Burkhardt [84] evaluated four different sandstones at different saturation levels and observed that $E$ and $G$ decrease with increasing saturation at $< 50\%$ saturation but is independent of water content at $> 50\%$ saturation. In comparison, $Q^{-1}_E$ and $Q^{-1}_G$ are less pronounced with decreasing saturation and their peaks shift towards higher frequencies. Yin et al. [128] studied Berea specimens using FO and RB; $Q^{-1}_E$ increases with frequency and brine-saturation. Cherry et al. [137] saw $E$ stiffening and $Q^{-1}_E$ weakening of a Lyon’s specimen with decreasing saturation. Chelidze et al. [146] also studied Lyon specimens but instead the effect of acetone and water-surfactant on $E$ and $Q^{-1}_E$. $Q^{-1}_G$ of dry or fluid-saturated ($< 50\%$) Berea specimens are low and approximately frequency-independent [100, 101, 103, 105, 107, 109, 112, 117, 189] or high and frequency-dependent [101, 103, 104, 107, 109, 112, 190]. Batzle et al. [36] attributed such behaviour to fluid mobility $M_F$ defined as $M_F = k/\eta$ where $k$ is permeability and $\eta$ is viscosity without presenting $Q^{-1}_P$, $Q^{-1}_K$, and $Q^{-1}_G$ of actual rocks: deflating $M$ is coupled with inflated relaxation time required to equilibrate the pore pressure. Fluid-saturated rocks will consequently appear stiffer under conditions in which the period of elastic perturbation is shorter than the fluid relaxation time than at lower frequencies. David [102] attempted to measure $E$, $Q^{-1}_E$, and $\nu$ of dry, water and glycerine-saturated Fontainebleau specimens but ultimately questioned the reliability of the results due to inconsistencies related to FO eclipsing PT in $E$ and $\nu$ magnitudes. Tisato and Quintal [105] explained their high and frequency-dependent $Q_E$ measurements for a nearly fully water-saturated Berea specimen by WIFF at the mesoscopic scale. Mikhailstevich et al. [109] observed decreasing and frequency-dependent $Q^{-1}_E$ with increasing confining pressure culminating at 1 Hz for a fully water-saturated Donnybrook specimen. Spencer [106] measured $E$ and $Q^{-1}_E$ as well as $\nu$ and $Q^{-1}_v$ in McMurray bitumen sand specimens containing residual air where calculated $Q^{-1}_P$, $Q^{-1}_K$, and $Q^{-1}_G$ peaks shifted towards lower frequencies as viscosity increased (by decreasing temperature) demonstrating strongly viscosity-dependent attenuation mechanisms. Tisato et al. [29] provided the first experimental evidence of WIGED in gas (air, $N_2$, or $CO_2$) and water equilibrated Berea specimens which was also numerically supported. Subramaniyan et al. [191] disclosed that “a decrease in viscosity of the saturating fluid shifted the attenuation curve to higher frequencies” while “an increase in confining pressure caused a decrease in the overall magnitude of attenuation” in Fontainebleau specimens. “Squirt-flow” is implied as the dominant mechanism for their glycerine data. Pimienta et al. [192, 193] studied $E$ and $K$ frequency dependence and attenuation in water and glycerine-saturated Fontainebleau specimens. Two frequency-dependent phenomena interpreted as drained-undrained and undrained-unrelaxed $E$-transitions [192] and underestimation of the drained-undrained transition implying a direct drained-unrelaxed $K$-transition [193] are observed. Spencer and Shine [110] concluded that “the modulus dispersion and attenuations ($Q^{-1}_P$ and $Q^{-1}_G$) in saturated sandstones are caused by a pore-scale, local-flow mechanism operating near grain contacts.” Higher harmonics allowed Rivière et al. [111] to study frequency, pressure, and strain dependence of nonlin-
ear elasticity in dry Berea specimens. Chapman et al. [112] measured $E$ and $Q_E^{-1}$ of a Berea specimen at various saturation levels and confining pressure: attenuation negligible at $< 80\%$ and significant at $> 91\%$ saturation also reduced and shifted towards lower frequencies by increasing confining pressure. Consistent with WIFF in response to heterogeneous water distribution in the pore space (patchy saturation), high enough fluid pressure to ensure full saturation also renders attenuation negligible. Ma et al. [113] attempted to validate their apparatus by aluminium and lucite measurements: $E$ and $\nu$ but no $Q_E^{-1}$. $V_P$ and $V_S$ converted from $E$ and $\nu$ also increased with pressure and saturation. Mikhaltsevitch et al. [46] demonstrated the causality-consistency of dry and water-saturated Donnybrook and Harvey as well as glycerol-saturated Berea specimens using KKR. Massaad [114] and Agoffack et al. [120] researched the effect of CO$_2$ on $E$ and $\nu$ transformed into $V_P$ and $V_S$ in a Berea specimen at Hz and MHz. Tisato et al. [115] proved the feasibility of combining FO with CT [194] to study a dry and partially saturated Berea specimen from 0.1 to 25 Hz in a CT transparent cell. Insignificant changes between these different conditions aside, this was the first time elastic measurements and imaging was combined for FO. Explained by a transition from WIFF to WIGED and inspired by Tisato et al. [29], Chapman et al. [195] observed a significant steepening of the high-frequency asymptote of the measured attenuation in Berea specimens caused by a minor change in water saturation (but a significant modification in the pore fluid distribution). Pimienta et al. [116] reported drained-undrained transitions in Wilkenson, Berea, and Bentheim specimens saturated by fluids of varying viscosities. Berea features $E$ and $\nu$ dispersion as well as $Q_E^{-1}$ and $Q_\nu^{-1}$ attenuation towards higher frequencies. Bentheim and Wilkenson only feature $E$ dispersion and $Q_E^{-1}$ attenuation. Pimienta et al. [116] interpreted Wilkenson’s consistency and Bentheims’s inconsistency with Zener’s model [38] as “squirt-flow” or measurement inaccuracy (possibly also another physical effect). Sun et al. [118] measured two attenuation peaks probably caused by two different mechanisms for a tight sandstone saturated between 45 and 85% but no peaks beyond these limits. Sun et al. [119] focused on presenting an enhanced FO version void of subsonic resonances based on numerical modelling but also included inconclusive $E$ and $\nu$ measurements. Chapman et al. [121] observed “frequency-dependent attenuation and the associated moduli dispersion in response to the drained–undrained transition (0.1 Hz) and ‘squirt-flow’ (> 3 Hz)” in Berea specimens. Yin et al. [122] sought to elucidate Gassmann’s fluid substitution theory by studying a clay-bearing Thüringen specimen by combining FO and PT. Dry specimens are non-dispersive (and thus non-attenuative) whereas water-saturated specimens are dispersive (and thus attenuative) which is attributed to the drained-undrained transition. Gassmann’s theory is consistent with their undrained $K$ but not with their water-softened $G$ which is significant at seismic but masked by “squirt-flow” stiffening at ultrasonic frequencies. Yin et al. [122] ascribed this reduction in surface free energy to chemical interaction between pore fluid and rock frame which eludes Gassmann’s theory. Li et al. [124] observed a broad distribution of $E$ and $Q_E^{-1}$ across 1-1000 Hz with peak attenuation at 60% saturation in a tight sandstone. Borgomano et al. [125] enforced drained or undrained conditions on glycerine-saturated or -unsaturated Bleurswiller specimens for $K$ and $Q_K^{-1}$ determination with microvalves (combined dead volume less than 40 $\mu$l). Gallagher et al. [126] focused on dry, brine-, and glycerine-saturated sandstones. Sun et al. [127] compacted Bleurswiller specimens beyond “the critical pressure which characterizes the onset of pore collapse and...
grain crushing” to the point that the critical frequency of “squirt-flow” dispersion (relaxed-unrelaxed transition) was shifted towards higher frequencies beyond the seismic band “allowing Biot-Gassmann to fully apply”. This result was interpreted as a consequence of increased crack aspect ratio after compaction. Li et al. [129] complimented the tight rocks of Li et al. [124] with a weakly consolidated sandstone. Also a study about saturation effects, peak attenuation occurs at 60 Hz and at 79% saturation. Öğünsamı et al. [130] performed the first systematic inter-laboratory study applying three different FO devices to cross-validate the elastic properties of a reservoir sandstone. Inter-laboratory frequency and fluid-saturation effects are consistently proven pressure-dependent at dry and decane-saturated conditions. Sun et al. [98] investigated the impact of microstructure heterogeneity and local measurements on dispersion and attenuation of dry plus brine- and oil-saturated Shahejie specimens from 1 to 300 Hz. Local diverges from global flow by being influenced by the position of the strain gauges which Sun et al. [98] attributed to crack-aspect-ratio heterogeneity since porosity and crack density are homogeneous. Sun et al. [98] measured not only $E$ and $Q_{1}^{-1}$ but also $\nu$ and $Q_{\nu}^{-1}$ owing to biaxial semiconductor strain gauges. Zhao et al. [131] discovered one or two attenuation peaks related to micro- and mesoscopic flow between 1 and 2000 Hz affected by saturation degree, oil viscosity, and confining pressure. A dual-scale fluid flow model suggested “that the attenuation mechanisms at different scales interplay with each other and jointly dominate the attenuation behavior of the partially saturated specimen.” Yurikov et al. [94] explored fibre optic [DAS] instead of strain gauges on a dry Bentheimer specimen. Chapman et al. [132] observed significant $E$ and $K$ but insignificant $G$ dispersion and attenuation caused by fluid pressure diffusion (FPD) in partially saturated Berea specimens featuring CO$_2$-exsolution by depressurisation. Tisato et al. [135] measured dry and partially saturated Berea specimens as function of confining pressure in which the former are frequency-independent and the latter are frequency-dependent at pressures below 14 MPa. Tisato et al. [135] also used KKR consistent models to determine the mechanism [WIFF] but was unable to distinguish between “squirt-flow” and patchy-saturation as sub-mechanisms. Wei et al. [196] assumed isotropy to converted $K$ and $Q_{K}^{-1}$ from measured $E$ and $Q_{E}^{-1}$ plus $\nu$ and $Q_{\nu}^{-1}$ for two sandstones at different saturation states. Bentheimer and Bandera are opposites: $K$ dispersion is entirely absent in the former but present and increasing with saturation in the latter. Peculiar is however that only converted and no measured parameters are disclosed. Han et al. [134] measured significant Young’s modulus $E$ and Poisson’s ratio $\nu$ transformed to P- and S-wave velocities $V_{P}$ and $V_{S}$ dispersion at seismic and insignificant dispersion from seismic to ultrasonic frequencies for oil- and glycerine-saturated tight sandstones. He et al. [135] extended on previous studies by introducing a squirt-flow extended patchy saturation model: $V_{P}$ is both measured and modelled but $Q_{V_{P}}^{-1}$ is only modelled. Ma [136] studied $V_{P}$ of water- and glycerine-saturated specimens. Yin et al. [138] appears to be a reiteration of Yin et al. [122]. Riabokon et al. [139] studied the non-linearity of 286 dry specimens at different strain amplitudes where Eddy Current Probe (ECP) and Laser Vibrometer (LV) measured axial and radial strains. Lu [140] measured axial and shear strains of artificial specimens (3D printed and otherwise) at different stress and loading conditions from 0.01 to 20 Hz using a pair of Laser Displacement Sensors (LDS). Chapman et al. [141][142] questioned the assumed adiabatic (thermodynamically unrelaxed) and instead argued for isothermal (thermodynam-
ically relaxed) interaction between multiple fluid phases by studying microscopic gas bubbles. Evident $E$, $K$, and $G$ dispersion and $Q_E^{-1}$, $Q_K^{-1}$, and $Q_G^{-1}$ attenuation peaks at $\sim$ 100 Hz are interpreted as thermodynamic relaxed-unrelaxed transitions. $G$ and $Q_G^{-1}$ are possibly affected by pore-scale heterogeneities as experiments and numerics coincide. Yurikov et al. [96] is a reiteration of Yurikov et al. [94, 95]. He et al. [143] combined FO and PT to calculate and measure $V_P$ for three dry, brine-, and glycerine-saturated sandstones as a function of effective pressure. Pore microstructure significance is implied by simultaneously increasing effective pressure and decreasing dispersion as well as simulated by a triple porosity model with combined “squirt-flow” effects. Triple signifies equant, intermediate, and compliant pores with inhomogeneous aspect ratio distributions. Mews et al. [144] studied the impact of strain amplitude on $E$ for a water-saturated Bentheimer specimen as a function of effective stress. Increasing order of harmonics were also briefly analyzed in an attempt to understand non-linearities and non-elastic mechanisms coupled to strain amplitude that are fundamental for static-dynamic property bridging [197]. Han et al. [145] described measured pressure and frequency dependencies of an oil-saturated sandstone with a modified “squirt-flow” model assuming a triple porosity model similar to He et al. [143].

### Carbonates

Spencer [21] measured $E$ and $Q_E^{-1}$ of Spergen specimens at different saturation and temperature conditions. Spencer’s vacuum-dried specimen is the stiffest and least dispersive and attenuative but his water-saturated specimens are increasingly attenuative and dispersive yet softer from 2°C to 25°C. Paffenholz and Burkhardt [84] also observed $E$ and $G$ stiffening with corresponding $Q_E^{-1}$ and $Q_G^{-1}$ weakening as a function of decreasing saturation. Batzle et al. [36] studied P- and S-wave velocities as a function of frequency and temperature for heavy oil-saturated Uvalde specimens that decreased in velocities with increasing temperature. Adam et al. [162] explored Gassmann’s theory applicability to $G$ and $K$ dispersion in what was claimed the first controlled laboratory experiments on carbonates at seismic frequencies. Behura et al. [163] saw significant $Q_G^{-1}$ temperature dependence (and thus viscosity dependence) in bitumen-saturated Uvalde specimens similar to the observations of Spencer [106] for McMurray bitumen sand. Adam et al. [150] calculated $Q_K^{-1}$, $Q_P^{-1}$, and $Q_G^{-1}$ from measured $Q_E^{-1}$ and the complex $\nu$. In contrast to sandstone observations, $Q_E^{-1}$ is higher when dry rather than fully water-saturated: $Q_E^{-1}$ is frequency-dependent in both dry and saturated scenarios. Zhao et al. [151, 152] are the same study presented twice: FO, DARS, and PT are combined to measure $V_P$ and $V_S$ in partially saturated specimens across seismic, sonic, and ultrasonic frequencies. Mikhailsevitch et al. [153] also studied Gassmann’s theory applicability for elastic moduli prediction and the influence of partial water saturation on elastic and anelastic properties of Savoinnière specimens. Borgomano et al. [154] found dispersion at around 200 Hz affecting all moduli but $G$ in water-saturated Lavoux specimens. Mikhailsevitch et al. [155] demonstrated that non-ideal boundary conditions (enforced by varying dead volumes) caused significant dispersion in fully decane-saturated Savoinnière specimens if the dead volume exceeds the pore space. Borgomano et al. [157] interpreted the observed dispersion and attenuation of Coquina, Rustrel, and Indiana (intact and thermally cracked) specimens in terms of transitions between drained, undrained and unrelaxed fluid-flow regimes at water and glycerine-saturated conditions. Biot-Gassmann theory consistency is proven at seismic frequencies. Pore type is correlated to “squirt-flow” dispersion absent in rocks featuring intragranu-
lar microporosity and present in rocks featuring cracked intergranular cement and uncemented grain contacts at seismic and sonic frequencies. Li et al. [124] also studied a tight carbonate at various degrees of saturation devoid of any noteworthy attenuation and dispersion features beyond an accretion of $Q_F^{-1}$ towards 1000 Hz. Tan et al. [198] complemented Mikhaltsevitch et al. [156] with a 1D poroelastic model based on Müller and Sahay [199] that quantified the dead volume dependence predicted by Pinienda et al. [88] and Sun et al. [200]. Ikeda et al. [201] measured increasing $E$ and decreasing $Q_F^{-1}$ with increasing water-saturation for a polymineralic carbonate. Riabokon et al. [158] also studied the non-linearity of dry carbonates. Mikhaltsevitch et al. [155, 156] whose results are also modelled by modified Gassmann theory. Sun et al. [160] studied the impact of partial saturation by (i) drying and (ii) imbibition: P-wave dispersion (and attenuation) is significant for (i) but not for (ii) whereas S-wave dispersion (and attenuation) is insignificant for both (i) and (ii) at $> 80\%$ RH Mesoscopic WIFF controlled by geometry and pore fluid distribution is the main mechanism causing P-wave dispersion. Gallagher et al. [161] hydrostatically investigated unfractured and fractured Rustrel specimens [157] at different effective pressures in triaxial and undrained conditions to better understand the effect of fractures and validate computational fracture models. Aside from a local negative phase shift for the fractured specimen at saturated conditions, no attenuation is observed at dry conditions.

**Shales**

Batzle et al. [108, 173] considered clay particle interactions with bound water responsible for shale attenuation and dispersion. Duranti et al. [174] discovered that dispersion in shales occurs between sonic and ultrasonic frequencies in addition to the seismic band being characterized by nearly constant attenuation. Hofmann [99] and Sarker and Batzle [175] studied the frequency dependence of water and glycerine saturated Mancos specimens. Despite proven able to measure $Q_F^{-1}$ [36], Hofmann [99] and Sarker and Batzle [175] solely focused on $E$ and $\nu$ converted into $V_P$, $V_P$, $V_P$, $V_S$, and $V_S$ via $C_{ij}$. Delle Piane et al. [32, 176] made a two-fold discovery: (i) attenuation is greater normal to bedding, and (ii) partial saturation increases the attenuation (likely due to micro and mesoscopic flow). Failure to disclose $E$ and $\nu$ was due to deficient calibration of the absolute displacement signals related to different amplification factors. Mikhaltsevitch et al. [177] linked seismic and ultrasonic measurements of dry and wet 0° Eagle Ford specimens found to be non-dispersive and non-attenuative by assuming isotropy. Both measurement types were concluded to be in the high-frequency regime due to partial saturation. Huang et al. [164] measured Mancos dispersion and attenuation: $E_V > E_H$ and $Q_{E_V}^{-1} > Q_{E_H}^{-1}$. Low $Q_E$ was also measured for Xianjing specimens. Mikhaltsevitch [166] studied dispersion and attenuation of fully saturated Wellington (0, 45, and 90°) and a partially saturated Mancos (0 and 90°) specimens. $Q_E^{-1} > Q_H^{-1} > Q_V^{-1}$ appears to be a clandestine trend within the scattered Wellington data $Q_E^{-1} < 0.0075$ in which $x$ refers to subscripts $V$, $H$, and 45. Trend or no trend, Mikhaltsevitch [166] measured Wellington at non-dispersive frequencies due to the low $Q_{E_H}^{-1}$ devoid of any noteworthy peaks. Like Wellington $Q_E^{-1}$, $Q_{E_H}^{-1}$, and $Q_{E_V}^{-1}$, Mancos $Q_{E_V}^{-1}$, and $Q_{E_H}^{-1}$ are also insignificant and unnoteworthy at $\text{RH}$s below 97.5%. Unlike at other $\text{RH}$s, 97.5% features $Q_{E'}^{-1}$ and $Q_{E_H}^{-1}$ peaks. $Q_{E_V} > Q_{E_H}$ and $E_H < E_V$ universally applies to Mikhaltsevitch [166] who also observed gradual softening accompanied by monotonically decreasing $E_V$ and $E_H$ with increasing saturation. Mikhaltsevitch et al. [202, 203] are based on
the Wellington experiments summarized in Mikhaltsevitch [166]. Mikhaltsevitch et al. [204] is the recent extension of the Mancos experiments [166, 205, 206]. Anisotropic permeability is their explanation to peak $Q_{E_V}^{-1}$ and $Q_{E_H}^{-1}$ occurring at different frequencies. Global flow ensured by the drained-undrained transition explains the presence or absence of dispersion at the probed frequencies. Wellington is also studied by Mikhailstevitch et al. [169] but at different saturation states with similar dispersion features (and explanation) as Mancos [204]: peak $Q_{E_V}^{-1}$ and $Q_{E_H}^{-1}$ at 2 and 6 Hz. Mikhailstevitch et al. [169] also includes the Eagle Ford experiments by Chavez et al. [123]. Xiao et al. [170] compared different attenuation mechanisms based on characteristic frequencies for 0 and 90° specimens of an unnamed shale.

Rørheim [4] observed non-simultaneous uniaxial-stress $E_V$ and uniaxial-strain $C_{33}$ behaviours as well as decreasing $Q_{E_V}^{-1}$ and $Q_{C_{33}}^{-1}$ with decreasing saturation for five differently oriented and saturated Pierre specimens. $E_V$ is doubled once and $\nu_{VH}$ is halved twice with decreasing saturation. $E_V < E_{15} < E_H$ is continuous while initially $Q_{E_H}^{-1} < Q_{E_V}^{-1} < Q_{E_{15}}^{-1}$ ultimately becomes $Q_{E_V}^{-1} < Q_{E_{15}}^{-1} < Q_{E_{45}}^{-1}$ with increasing frequency. FO-measured $C_{33}$ and PT-measured $V_{P}$, decrease simultaneously with decreasing saturation despite the frequency gap. No $Q_{E_V}^{-1}$ attenuation peaks but $E_V$ dispersion were noticed. Li et al. [171] examined the impact of maturity on attenuation of four different oven-dried 0° specimens: Barnett, Eagle Ford, Mancos, and Jungar. Constant dispersion and insignificant attenuation are features of the three former however the latter feature strong dispersion and significant dispersion which decreases with increasing pressure. The controlling factor is not thermal maturity or clay content but instead the geochemical index attributed to viscous friction between inorganic grains and organic matter. Long et al. [172] excluded capillary effects and WIFF but included viscoelastic bulk and shear moduli as plausible bound water attenuation mechanisms while researching seven unnamed shale specimens.

Although unable to record $Q_{E_E}^{-1}$ measurements, Szewczyk et al. [207] observed strong softening at seismic and hardening counteracting this softening at ultrasonic frequencies (due to increasing dispersion) with increasing water saturation in Mancos specimens. $\nu$ also strongly increased but appeared nearly frequency-independent. Types of shales also differ in stress sensitivity during hydrostatic loading which also differs at seismic and ultrasonic frequencies. Li et al. [167] referred to Huang et al. [164] when measuring $E$ and $\nu$ dispersion of an unnamed field shale. Szewczyk et al. [208] assumed variable and linearly decreasing $C_{ij}^{\text{dry}}$ with increasing RH for anisotropic Gassmann’s theory to be applicable to Mancos. Lozovyi [168] succeeded Szewczyk [208] by comparing the static and dynamic stiffness of Opalinus Clay and three other anonymous specimens [210, 212]. Chavez et al. [123] conducted FO creep tests on three dry sedimentary rocks at constant 2 Hz for 120 hours. Creep effects significantly impacted the moduli for all three specimens but the shale was most affected. Mikhailstevitch et al. [204] questioned the applicability of anisotropic Gassmann theory to Mancos proposed by Szewczyk et al. [208] on the basis that $C_{ij}^{\text{sat}} > C_{ij}^{\text{dry}}$ is repudiated by the non-linearity of the decreasing $C_{44}$ and $C_{66}$ with saturation. Be the fact that $C_{ij}^{\text{sat}} > C_{ij}^{\text{dry}}$ and $C_{ij}^{\text{sat}} < C_{ij}^{\text{dry}}$, as it may, the non-linearity is questioned (only applies to $C_{44}$ and $C_{66}$ at 9% RH). Rørheim et al. [213] performed FO measurements as functions of frequency as well as time on a CO$_2$-exposed Draupne specimen where $E_V$ and $\nu_{VH}$ insignificantly changed over 575 hours. Neither PT-measured $V_{P}$ nor $V_{S}$ significantly changed perhaps due to Draupne’s low calcite content. Rørheim et al. [214]
combined FO and PT at elevated temperatures for 0 and 90° Pierre specimens. FO measured $E_V$ and $E_H$ dispersion shifted towards higher frequencies while FO converted and PT measured $V_{P0}$, $V_{P90}$, $V_{S0}$, and $V_{S90}$ oppositely decreased and increased with temperature. Two simple frequency-dependent bound water models were also proposed.

Numerical studies supplementing experimental ones are becoming customary. Experimentally proven by Tisato and Madonna [101], Quintal et al. [215] and Quintal et al. [216] were the numerical impetus of Chapman and Quintal [90], Tsato and Quintal [105], Chapman et al. [132, 141], Gallagher et al. [161], Tisato and Quintal [189], Kuteynikova et al. [190], Lissa et al. [217, 219], Alkhimenkov and Quintal [220, 221], Quintal et al. [222], Hunziker et al. [223], Quintal et al. [224], Alkhimenkov et al. [225], and Lissa et al. [226] in terms of attenuation modelling. Zhang and Toksöz [227], Das et al. [228], and Jänicke et al. [229] used similar Digital Rock Physics (DRP) approaches to study attenuation mechanisms and fluid-solid interactions. Balcewicz et al. [230] used DRP to study other properties. Focusing on enhancing an existing FO approach, Sun et al. [119] tried to alleviate the experimental destructive subsonic resonances via numerical modelling to redefine their apparatus design. Sun et al. [119] achieved this by inverting Tittmann’s principle manifested by Nakagawa’s loaded resonator: the shorter the apparatus, the higher the characteristic frequency. Sun et al. [119] inspired Liu et al. [231] to investigate the cause and effect of subsonic resonances for the apparatus described by Mikhailsevitch et al. [109] in a similar manner. Sun et al. [200] 3D modelled the drained-undrained transition for the frequency-dependent elastic moduli and attenuation. In order to inaugurate an upper frequency-limit, Borgomano et al. [125] and Li et al. [232] adopted the COMSOL based numerical approach by Sun et al. [119] to deduce at which frequencies their apparatus resonates. Unlike Sun et al. [119] and Borgomano et al. [125], Li et al. [232] included jointing conditions in their model because the stress-field is non-homogeneous. As a result, elastic properties depend on strain gauge position. Be aware however that numerical need not equal experimental due to the ideal versus non-ideal states predication. Ikeda et al. [201] compared experimental and numerical results: 10% separates DRP derived and FO measured $E$.

FO apparatuses are proven by their ability to measure attenuation of standard materials often used for calibration purposes: lucite, aluminium, and PEEK. Other calibration materials exist (e.g. Borgomano et al. [125] calibrated for glass and gypsum in addition to lucite, while Brunner et al. [233] used a unspecified “viscoelastic structure”) but are excluded due to their rarity. To the best of their knowledge, Rørheim and Holt [6] compiled and studied all published FO measurements involving these three materials (Table 2) to validate their findings by literature comparison (Figure 1). Lucite is dispersive opposed to non-dispersive aluminium and PEEK. Questioned is the calibration applicability of high modulus aluminium dissimilar to rocks but not that of low moduli lucite and PEEK similar to rocks. Batzle et al. [36] recognized that the composition of lucite can be variable from batch to batch. In fact, Bonner [239] claimed that the hardener-resin ratio determines the quality of the material. Unlike the inexpensive resin, the hardener is expensive. It is thus common for resin to be the dominant component of the hardener-resin ratio. This primarily affects the absolute mechanical properties but not the overall frequency dispersion characteristics [178]. Figure 6 possibly features this effect as Sun et al. [119] is an evident outlier compared to all others.

*Please be advised that all external measurements from other authors are digitized with variable resolution during which resonance-affected data are omitted. Measurements at non-ambient temperatures [234–238] are also omitted.
Table 2: All known $Q_E$ measurements of lucite, aluminium, and PEEK alphabetically sorted and categorized by author(s) and specimen material, respectively. Color-bars based on Author # and the (cividis) color-map are used instead of legends. *Rørheim and Holt* is in bold because this is based on that study in which all known FO studies were compared.

| Author(s)     | Author # | Specimen(s) | Lucite | Aluminium | PEEK  |
|---------------|----------|-------------|--------|-----------|-------|
| Batzle et al. | 1        |             | ✓      | ✓         | ✓     |
| Borgomano et al. | 2       |             | ✓      | ✓         | ✓     |
| Cao et al.    | 3        |             | ✓      | ✓         | ✓     |
| Cherry et al. | 4        |             | ✓      | ✓         | ✓     |
| Fliedner and French | 5       |             | ✓      | ✓         | ✓     |
| Huang et al.  | 6        |             | ✓      | ✓         | ✓     |
| Koppelmann    | 7        |             | ✓      | ✓         | ✓     |
| Koppelmann via Lakes | 8       |             | ✓      | ✓         | ✓     |
| Li et al.     | 9        |             | ✓      | ✓         | ✓     |
| Lienert and Manghani | 10    |             | ✓      | ✓         | ✓     |
| Liu and Peselnick | 11    |             | ✓      | ✓         | ✓     |
| Lu            | 12       |             | ✓      | ✓         | ✓     |
| Madonna et al. | 13      |             | ✓      | ✓         | ✓     |
| Madonna and Tisato | 14    |             | ✓      | ✓         | ✓     |
| McCann        | 15       |             | ✓      | ✓         | ✓     |
| Mikhailsevitch et al. | 16   |             | ✓      | ✓         | ✓     |
| Nakagawa      | 17       |             | ✓      | ✓         | ✓     |
| Paffenholz and Burkhardt | 18  |             | ✓      | ✓         | ✓     |
| Pimienta et al. | 19    |             | ✓      | ✓         | ✓     |
| **Rørheim and Holt** | 20 |             | ✓      | ✓         | ✓     |
| Spencer       | 21       |             | ✓      | ✓         | ✓     |
| Sun et al.    | 22       |             | ✓      | ✓         | ✓     |
| Takei et al.  | 23       |             | ✓      | ✓         | ✓     |
| Tisato and Madonna | 24  |             | ✓      | ✓         | ✓     |
| Tisato et al. | 25       |             | ✓      | ✓         | ✓     |
| Yao           | 26       |             | ✓      | ✓         | ✓     |
| Yee and Takemori | 27  |             | ✓      | ✓         | ✓     |
| Yin et al.    | 28       |             | ✓      | ✓         | ✓     |
Figure 6 Young’s modulus $E$ and attenuation $Q^{-1}$ versus frequency $f$. Color-bars denote authors according to Author # listed in Table 2. Enhanced distinguishability is ensured by solid symbols for the bolded Rørheim and Holt [6] and open symbols for all other authors.

3.4 OTHERS

Beyond the traditional RB, PT, and FO techniques, Resonant Ultrasound Spectroscopy (RUS), Differential Acoustic Resonance Spectroscopy (DARS), and Laser UltraSonics (LUS) are other novel techniques exploited in rock physical research. Maynard [274] not only introduced the term RUS “to encompass all techniques in which ultrasonic resonance frequencies are used to determine elastic moduli” but also traced the history of RUS back to Fraser and LeCraw [275], Schreiber and Anderson [276], and Demarest [277]. In fact, RUS is in many aspects a continuation of
Table 3  Sonic frequency probing techniques with primary focus on Resonant Bat (RB) but secondary also on Pulse-Tube (PTU), Gas Hydrate Resonant Column (GHRC), Resonant Ultrasound Spectroscopy (RUS), and Differential Acoustic Resonance Spectroscopy (DARS).

| Author(s) | Technique(s) | Frequency (Hz) | Length (cm) | Diameter (cm) | Parameter(s) |
|-----------|--------------|----------------|-------------|---------------|---------------|
| Batzle et al. [36] | FO | 5 E0 – 2.5 E3 | 23 | | $E, V_p, V_S, Q_e$ |
| Birch and Bancroft [18] | RB | 1.4 E2 – 4.5 E3 | 244 | 32 | $Q$ |
| Born [51] | RB | 9.3 E2 – 1.28 E4 | 14 – 124 | 12 | $N = \pi Q^{-1}$ |
| Bourbié and Zinszner [8] | RB | 3 E3 – 5 E3 | | | $V_e, Q$ |
| Cadoret et al. [247] | RB | 1 E3 | 110 | 8 | $V_e, V_S, V_P$ |
| Cadoret et al. [248] | RB | 1 E3 | 110 | 8 | $Q_G, Q_E$ |
| Gardner et al. [249] | RB | 2 E3 – 3 E3 | 5 – 30.0 | 5 | $\delta_E, \delta_S$ |
| Goldberg and Zinszner [250] | RB | 5 E3 – 25 E3 | 25 | 2.5 | $Q_P$ |
| Ide [47] | RB | 4 E3 – 12 E3 | 25 | 5.1 | $E$ |
| Harris et al. [251] | DARS | 1 E3 – 2 E3 | | | $K, Q_K$ |
| Jones and Nur [53] | RB | 1.7 E2 – 3.4 E3 | | | $V_S, Q_G$ |
| Lucet et al. [252] | RB | 5 E3 – 2 E4 | 25 – 30 | 2.5 | $V_e, V_P, Q_E, Q_G$ |
| Lucet and Zinszner [253] | RB | 3 E3 – 1 E4 | 30 | 2.5 | $Q_E$ |
| Lucet and Zinszner [254] | RB | 2 E3 – 2 E4 | | | $Q_E, V_S/V_{US}, V_P/V_{PFB}$ |
| McCann et al. [255] | RB | 1 E3 – 1 E4 | 60 | 6.9 | $V_P, Q_P, T$ |
| Murphy [256] | RB | 3 E2 – 1.4 E4 | 20 – 100 | | $V_e, Q_e, Q_S, Q_G$ |
| Murphy [257] | RB | 1 E3 | 20 – 25 | 19 | $V_S, V_E, Q_E, Q_G$ |
| Nakagawa and Kneafsey [258] | SHR | 3.5 E2 – 2.35 E3 | 6.2 | 3.75 | $E, G, \nu, V_P, V_S, Q$ |
| Nakagawa [56] | SHR | 4 E2 – 2.3 E3 | 6.22 | 3.81 | $E, G, \nu, V_P, V_S, Q$ |
| Nakagawa and Kneafsey [259] | SHR | 3 E2 – 1.5 E3 | 7.62 | 3.75 | $E, G, \nu, V_P, V_S, Q$ |
| O’Hara [260] | RB | 3 E2 – 3 E3 | 38 | 2.22 | $V_e, V_S, \delta$ |
| Priest et al. [261] | GHRC | 4 E2 | 14 | 7 | $Q_E, Q_G$ |
| Tittmann [55] | RB | 2.2 E4 – 2.3 E4 | | | $Q_E, Q_G$ |
| Tittmann et al. [262] | RB | 7 E3 – 9 E3 | 12 | 1.5 | $Q_E$ |
| Waite et al. [263] | SHR | 3.6 E2 – 1.6 E3 | 7.62 | 3.81 | $V_p, V_S$ |
| Wegel and Walther [264] | RB | 1 E2 – 1 E5 | 30 | 1 | $Q_E, Q_G$ |
| Winkler and Nur [25] | RB | 5 E2 – 1.7 E3 | 100 | | $V_e, V_S, V_P, Q_E, Q_G, Q_K, Q_P$ |
| Winkler and Nur [54] | RB | 5 E2 – 9 E3 | 100 | | $V_e, V_S, V_P, Q_E, Q_G, Q_K, Q_P$ |
| Wyllie et al. [265] | RB | 2 E3 | 1.9 – 2.50 | | $V_e, V_S, \nu, \delta_E, \delta_S$ |
| Yin et al. [266] | RB | 1.6 E2 – 1.8 E3 | 39 – 53 | 5 | $E, Q_E$ |
| Zadler et al. [267] | RUS | 1.4 E2 – 8.8 E4 | 7.1 | 2.5 | $Q_E, Q_G, V_P, V_S$ |
| Mode | Author(s) | Specimen(s) | Force generator | Force sensor | Displacement sensor | Frequency (Hz) | Parameter(s) | Institution(s) |
|------|-----------|-------------|----------------|-------------|--------------------|---------------|--------------|----------------|
| (i) Longitudinal | Letto et al. [189] | Sandstone | PVDF actuator | Aluminium STD | Fibre optics | 5E0 – 2 E1 | E, Q_s, ν | | |
| | Lecce et al. [200] | Sandstone | PVDF actuator | Aluminium STD | Fibre optics | 5E0 – 2 E1 | E, Q_s, ν | | |
| | Boccaletti et al. [201] | Limestone | Lead cell | Aluminium STD | Inductive transducer | 4E1 – 3 E1 | E, Q_s, ν | | |
| | | | | | | | | | |
| (ii) Torsional | Berchichan et al. [202] | Limestone | PVDF actuator | Aluminium STD | Inductive transducer | 5E0 – 3 E1 | E, Q_s, ν | | |
| | Groll and Landwehr [203] | Limestone | Lead cell | Aluminium STD | Inductive transducer | 5E0 – 3 E1 | E, Q_s, ν | | |
| | Le et al. [204] | Limestone | Lead cell | Aluminium STD | Interferometer | 4E0 – 1 E1 | E, Q_s, ν | | |
| | | | | | | | | | |
| (iii) Flexural | Jacquet et al. [205] | Glass ceramic | Lead cell | Aluminium STD | Inductive transducer | 1E0 – 1 E1 | E, Q_s, ν | | |
| | | | | | | | | | |
| (iv) Volumetric | Adel et al. [206] | Lead cell | PVDF actuator | Lead cell | Inductive transducer | 5E0 – 3 E1 | E, Q_s, ν | | |
| | | | | | | | | | |
| (v) Uniaxial | Li et al. [207] | Sandstone | PVDF actuator | Lead cell | Inductive transducer | 1E0 – 1 E1 | E, Q_s, ν | | |

* is for Semiconductor strain gauges. ** is for Porcelain strain gauges.
RB but differs due to its ability to capture numerous resonance peaks across wider frequencies irrespective of material shape and dimensions. Be it extensional, torsional, and flexural modes, Zadler et al. \[266\] and Ulrich et al. \[278\] describe RUS as a technique able to ascertain the suite of elastic moduli for anisotropic rocks during a single frequency sweep (Figure 7). For orthorhombic symmetry, eight independent groups of free-vibration modes containing all possible combinations of these symmetries (including isotropic, cubic, hexagonal, and tetragonal) exist: one extensional and torsional, plus three shear and flexural \[277, 279\]. RUS is as numerical as it is experimental in the sense that (i) forward and (ii) inverse problems need be solved to validate the experiment: (i) compute the frequencies and shapes of the normal modes, and (ii) apply a non-linear inversion algorithm to find the elastic constants from these normal-mode frequencies. Since RUS detects numerous resonance peaks from all eight groups, exact correspondence between resonance peaks and vibration modes is paramount as any dissonance makes for erroneous measurements. Batzle et al. \[36\] remarked on RB versus RUS that the narrow bandwidth of RB limits the probed frequencies to the primary resonance and a few overtones whereas the extended frequencies of RUS renders it susceptible to jacketing and suspension procedures plus inhomogeneous strain conditions for the torsional and flexural modes. Indeed, Zadler et al. \[266\] acknowledged that in-situ measurements involving strong coupling to a pressurized medium violate the stress-free boundary conditions RUS is based upon.

![Figure 7](image-url) Exaggerated surface particle displacement for extensional, torsional, and flexural modes for RUS (but also applies to FO) measurements. Modified from Zadler et al. \[266\].

Harris \[280\] envisioned DARS to measure the change in resonance frequency of a submerged cavity caused by the absence or presence of a foreign object inside the cavity which perturbs the resonance properties of the cavity. DARS is based on perturbation theory which relates the frequency shift between a cavity with or without a specimen to the acoustic properties of said specimen \[251, 281, 282\]. In other words, DARS is restricted to the determination of bulk modulus as the change between the normal modes of two volumetric effects. Keys to this technique are (i) the cavity immersed in a fluid containing vessel, (ii) piezoceramic sources used to excite fluid resonances, and (iii) a hypersensitive hydrophone embedded on the cavity surface for detection of acoustic pressure signal changes. Added to these three key elements are also (iv) a computer-controlled step motor for accurate and repeatable specimen positioning, and (v) a phase-sensitive lock-in amplifier connected to source and receiver that uses a predefined frequency sweep (around the natural resonance of the cavity) to select the resonance curve of the fundamental mode in order to recognize the received signal at a specific reference frequency and phase. Wang et al. \[281\] implemented a FEM-based simulation to better understand the DARS system and improve its accuracy in estimating acoustic properties.

Scruby et al. \[283\] first described LUS as a technique to “study thermoelastic generation of elastic waves in a metal by unfocused laser radiation.” As such, LUS is able to characterize waves in terms of velocity and attenuation. Instead of relying on mechanical coupling between transducer and specimen like PT and FO, LUS is strictly non-contact due to two lasers generating
and recording elastic waves at the specimen surface from afar. A short pulse of electromagnetic radiation delivered by the first laser causes thermoelastic-expansion-induced ultrasonic waves recorded by the second laser (interferometer) at an arbitrary point on the specimen surface [284]. Since the surface area beamed by the laser is significantly smaller than the area coupled to PZT transducers, LUS enables multidirectional characterisation of waves from a single specimen while also guaranteeing that group velocity is unambiguously measured. The eternal group versus phase velocity conundrum is thus avoided as the former is easily converted to the latter using established algorithms. It is also claimed that the non-contact generation and detection is not frequency-limited by the physical dimensions of the transducer elements [285–287], yet it is curious that this possibility remains unexplored as most studies are limited to ultrasonic frequencies (thus the name) considering that seismic frequencies are alpha and omega in practice. If LUS is indeed universal for all frequencies, RB, PT, and FO would all be redundant. However, even if LUS is theoretically able but practically unable to probe all frequencies, it is still an improvement compared to PT because LUS leaves less ultrasonic frequencies unprobed. In the realm of rocks, measured parameters at ultrasonic frequencies are often regarded to coincide with the high-frequency limit but this need not be true and could be further explored by LUS. For example, there is evidence for ultrasonic attenuation in rocks depending on the enforced conditions (e.g. Johnston and Toksöz [288]). LUS is extended from metals to rocks in the absence and (recently) in the presence of pseudo in-situ conditions [287, 289–300]. Pseudo is emphasized since it is still restricted to isotropic stress conditions but remain unrestricted in terms of temperature [287, 290, 292].

4 DISCUSSION

Technique-bias currently exists because several different techniques need be exploited to access Hz, kHz, and MHz frequencies. These techniques also measure different parameters that need be transformed for comparability which is in itself is an error source. Only if theoretically understood in a controlled environment (laboratory) may dispersion also be practically understood in an uncontrolled environment (field). To the end of understanding dispersion as a phenomenon, all frequencies need be accessible for the transitions between regimes to be studied. These transitions are mechanism-dependent and observed as changes in moduli or attenuation peaks with frequency. Dispersion and attenuation are indeed causality-bound. They are also key to identify and understand the physical mechanisms causing dispersion. Multiple transitions probably occur in any given rock but singular transitions are mostly studied due to frequency-limitations of the investigation techniques. The limiting factors for FO, RB, and PT are resonances (upper boundary), specimen size, and piezoceramic size, respectively. Resonances are constructive for RB and destructive for FO but the same principle applies to both: The smaller (and lighter) the resonator, the higher the characteristic frequency, and vice versa. In fact, this principle also applies to PT where larger piezoceramics equal lower frequency. PT piezoceramics for this purpose seldom recede below ~50 kHz. No universal technique probing all frequencies of interest alas exists at this time. As a result, existing techniques are enhanced or manipulated to extend their operating frequencies. For example, FO apparatuses are becoming smaller and lighter to increase the upper boundary at which they resonates. Numerical investigations during the design process accelerated this evolution. Batzle et al. [36] identified nodes and antinodes to extend this upper boundary by distributing the measurements accordingly. Since dispersion is a fluid-dependent phenomenon, viscosity-manipulation either by fluids such as glycerine [36, 102, 126, 134, 136, 157] or temperature [36, 53, 106, 214] may shift the transition from unprobed to probed frequencies due to altered fluid mobility. Viscosity-manipulation by temperature [301] is explored at high but unexplored at low temperatures for FO. Among RUS, LUS and DARS,
particularly LUS is enticing due to its non-contact nature and its supposed ability to “generate acoustic energy” and “detect surface displacements over a broad frequency band (typically from continuous to GHz)” [285]. Even if LUS is unable to probe seismic frequencies it is proven to probe a wider range of ultrasonic frequencies than PT. Thus far only isotropic (hydrostatic) conditions are possible [287] but applying a load along the axis of two synchronized stepper motors above and below the specimen to achieve biaxial conditions while still preserving its rotation ability should be fairly simple. Since most of the sensors are outside the pressure vessel, it is also an ideal candidate to fit inside a CT scanner provided that the laser signals are not somehow distorted by x-rays. “A respectable frequency gap remains to be bridged” [18] are still true words that are becoming less so with time and technological advancements.

Models compensate for the unprobed frequencies and also connect different measurements at different frequencies. Commonest amongst a multitude of models are CC, SLS, and KKR. KKR is superior to CC and SLS due to its analytical nature but requires parameters (real part) and their corresponding attenuations (imaginary part) at all frequencies to be valid. Many WIFF models based on Biot [25] exist because “squirt-flow” is the dominant dispersion mechanisms at microscopic scale in sandstones. Models simulating dispersive behaviour related to mechanisms valid at mesoscopic and macroscopic scales are rarities in comparison. Since multiple transitions may occur in the same specimen, it is important to simulate mesoscopic and macroscopic flows in addition to microscopic flow. As described in Section 3, not only analytical but also numerical models are developed to describe these mechanisms. The paucity of bound water models is concerning because it is perhaps the least known yet also the most critical mechanism due to its experimentally [302–305] and numerically [306] proven non-zero shear modulus and enhanced viscosity. Simultaneous bound water conversion to free water and layer thickness reduction are also proven at elevated temperatures [307]. Bound water is also proven to convert to free water and DRP is progressing and may be a valuable tool in the future. Calibrating DRP with 3D printed or otherwise synthetic specimens with known geometry and properties would surely be a great leap forward if it becomes a possibility. More knowns than unknowns is key.

Subramaniyan et al. [7] is also a precautionary tale that discusses potential problems and possible solutions with implementing FO. As such, advantages and disadvantages with different components as well as boundary effects and strain amplitudes are discussed. Rørheim and Holt [6] discovered that modulus is unaffected but attenuation is affected by minor misalignments as foreseen by Liu and Peselnick [82]: smooth and parallel surfaces are paramount. For longitudinal excitation, deviatoric stress $\sigma_{\text{dev}} = \sigma_{\text{ax}} - P_c$ (where $P_c$ is the previously undefined confining pressure) can be a mitigation measure to ensure adequate coupling and uniform stress distribution. Other mitigation measures include adhesive [21] and aluminium foil [167] at the specimen-endcap interfaces. FO apparatuses differ in primarily strain and secondary stress measurements (Table 4). Most measure local strains using a set of specimen-attached strain gauges while few measure bulk strain using capacitors, cantilevers, LVDTs, lasers, or recently even fibre optics DAS sensors. Pro et contra of bulk versus local measurements are elaborated (Section 3) where bulk is considered superior to local (especially for heterogeneous rocks) but more problematic for radial measurements due to the double-interface problem. Strain gauges are more directional-versed and require less calibration than the others but must be mounted to each individual specimen where the wires cause sealing-related problems. DAS is particularly interesting as it offers the opportunity to bridge the laboratory-field void because equivalent sensors are used in both environments. Mechanisms can thus be studied independent of potential sensor-bias. DAS is also more strain-sensitive (as low as $10^{-11}$ strain amplitudes) and less temperature-sensitive than semiconductors among several advantages [95]. Elastic standards are customary among numerous stress sensors. These standards are commonly non-dispersive metals such as aluminium or titanium attached with foil or semiconductor strain gauges. Due to the high stiffnesses of both metals, high-sensitive semiconductors are preferred over low-sensitive foil strain gauges. Stress

26
\( \sigma_{ax} \) and strain \( \epsilon_{ax} \) proportionally increase while Young’s modulus \( E \) is constant (Equation 3) if the area \( A \) is lowered since \( \sigma_{ax} = F/A \) where \( F \) is force. Increasing the strain amplitudes of the standard improves the accuracy of foil strain gauges in regard to amplitude and phase sensitivities. For example, Rørheim [4] designed a temperature-compensated “hollow dog bone” aluminium standard equipped with four biaxial foil gauges where \( A \) was decreased by a factor of five. Note that electronic-induced errors are a concern if different circuitries are used for stress and strain measurements [36, 103, 207].

FO as a measure to probe seismic frequencies is a force to be reckoned with manifested by the growing number of apparatuses. Most of which are based on the design of Batzle et al. [36] with specimen-attached strain gauges for local strain measurements opposed to specimen-unattached transducers for bulk strain measurements like Spencer [21]. Simultaneous local and bulk strain measurements combined in one apparatus could possibly distinguish between microscopic, mesoscopic, and macroscopic mechanisms as dispersion and attenuation sources [90]. Other possible FO improvements for the future include:

- Numerical-based designs using, for example, COMputer SOLution (COMSOL) or Multi-physics Object Oriented Simulation Environment (MOOSE) framework are crucial in understanding the nature of resonance and predicting their nodal and antinodal distributions with frequency. Be it DRP or otherwise, numerical studies complementing experimental ones are also advancing the universal understanding of dispersion and attenuation mechanisms.
- Tisato et al. [115] was the first of its kind but others will surely follow because the CT-FO combination may elucidate the spatial distribution of fluids and thus also the fluid-solid interactions that are the dispersion and attenuation causing mechanisms. Most phenomena and their respective mechanisms are explained if porosity is adequately described as it is the critical factor separating solid and porous media. Measuring physical properties while simultaneous imaging phase distributions at pore-scale is common for other techniques except FO 3D printed or otherwise synthetic rock specimens with known geometry and properties subject to CT plus FO could also contribute to this end. Fractals may also be experimentally investigated if specimen geometries are scale-independent and fractal-consistent.
- Imaging fluid-solid interactions with time is especially enticing for Carbon Capture and Storage (CCS) purposes (as a natural extension to Rørheim et al. [213]).
- Multi-element PZT transducers could also excite torsional and flexural modes as well as mitigate any potential misalignments by redistributing the stress field. Nakagawa [104] partially converted longitudinal into torsional excitations using a novel “compression-torsion coupler”. Misalignment mitigation and stress field redistribution is already achieved by employing three PZT transducers [84–86].
- True triaxial conditions (\( \sigma_x \neq \sigma_y \neq \sigma_z \) [309, 310]) are achieved if there is independent control of two radial strains in which \( H = C_{11} \) in Equation 7. Pistons in addition to confining pressure are required to achieve these conditions which render specimen-attached strain gauges inadequate due to the fact that they would be destroyed upon impact of the piston. A cantilever solution in both axial and radial directions could however be a possibility.
- Virtual instead of physical lock-in amplifiers [107, 244] would offer greater data control as well as being an inexpensive option to split strain gauge elements to focus on local pore-scale processes by analyzing individual strains before averaging to distinguish local from global flow.
- Temperature control by heating circulators would render experimental temperature independent from laboratory temperature which is sensitive. Improved temperature control would not only allow for unexplored (sub- as well as superambient) temperatures to be ex-
explored but also keep the temperature constant for the duration of the experiment. Subambient temperatures are an unexplored curiosity shifting the mechanism-dependent transitions towards lower frequencies due to decreased fluid mobility.

- Experiments dedicated to elucidate bound water as an attenuation mechanism are crucial to understand attenuation as a phenomenon. FO is exploited to this end as well albeit still in its infancy [172, 214]. Other novel techniques or combinations of techniques may however be more applicable.

- Machine Learning (ML) is bound to somehow be exploited in the laboratory as it is in the field [311] to ease data acquisition as well as analysis. PT is perhaps favoured over FO though as automated arrival time picking ML is an immediate extension of Zhu and Beroza’s PhaseNet. Especially S-waves could benefit from automated picking due to their ambiguity compared to P-waves, ML could also possibly be combined with DRP.

- Exploring higher harmonics is not only possible for RB at sonic but also for FO at seismic frequencies. Harmonics are key to understand non-linearities and non-elastic mechanics and their dependency on strain amplitude, frequency, pressure, and fluid saturation [111, 144, 312]. Nonlinear attenuation is also temperature-dependent: mesoscopic \( f_s \) increases and macroscopic \( f_b \) decreases with temperature [313]. The static-dynamic property discrepancy is caused by mechanisms such as drainage conditions, dispersion, heterogeneities, and strain amplitude [197]. Strain amplitude dependency on non-linearities and non-elastic mechanisms are crucial to resolve this discrepancy.
  - Pressure- and temperature-induced non-linear elastic behaviour in rocks is also explored by LUS [292, 293].

5 CONCLUSION

Novel experimental techniques need be developed or enhanced to improve the conditions at which rocks are experimented for the field-laboratory gap to be bridged. Among the usual techniques, FO is perhaps the current champion due to its similarities with field seismic. Despite its promise, caution need be exercised when comparing laboratory with field data due to the omnipresent upscaling problem that transcends frequencies. Novel are not only the experimental techniques but also the analytical and numerical models that are continuously being developed. These models primarily describe WIFF at microscopic scale but secondary models to describe flows at mesoscopic and macroscopic are required for full understanding since multiple transitions are measured in rocks. Bound water should also be further explored as a mechanism due to its experimentally and numerically proven non-zero shear modulus and enhanced viscosity. DRP is gaining momentum and may possibly be integral in understanding rock phenomena and mechanisms in the future.

ACKNOWLEDGEMENTS

Rune M. Holt and Andrew J. Carter at NTNU are acknowledged for their encouragements. Current and former SINTEF employees are also acknowledged: Andreas Bauer, Jørn F. Stenebråten, Serhii Lozovyi, Dawid Szewczyk, Lars Erik Walle, Andreas N. Berntsen, Eyvind F. Sonstebø, M. Hossain Bhuiyan, Anna M. Stroisz, Pierre R. Cerasi, and Sigurd Bakheim. Lara Blazevic is finally acknowledged for proofreading.
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**ACRONYMS**

**ANU** Australian National University.

**CC** Cole-Cole.

**CCS** Carbon Capture and Storage.

**CLSC** Closed-Loop Servo Controlled.

**CNRS** Centre national de la recherche scientifique.

**CO₂** Carbon dioxide.

**COMSOL** COMputer SOLution.

**CSM** Colorado School of Mines.

**CT** Computed Tomography.

**CU** Curtin University.

**CUP** China University of Petroleum.

**DARS** Differential Acoustic Resonance Spectroscopy.
| Abbreviation | Full Form |
|--------------|-----------|
| DAS          | Distributed Acoustic Sensing. |
| DRP          | Digital Rock Physics. |
| ECP          | Eddy Current Probe. |
| ENS          | École Normale Supérieure (Paris). |
| ETH          | Eidgenössische Technische Hochschule (Zürich). |
| FEM          | Finite Element Method. |
| FFT          | Fast Fourier Transform. |
| FO           | Forced-Oscillation. |
| FU           | Frankfurt University. |
| GE           | General Electric. |
| GHRC         | Gas Hydrate Resonant Column. |
| ICL          | Imperial College London. |
| INGV         | Istituto Nazionale de Geofisica e Vulcanologia. |
| IPGS         | Institut de Physique du Globe de Strasbourg. |
| KKR          | Kramers-Kronig relations. |
| LBL          | Lawrence Berkeley National Laboratory. |
| LDS          | Laser Displacement Sensors. |
| LUS          | Laser UltraSonics. |
| LV           | Laser Vibrometer. |
| LVDT         | Linear Variable Displacement Transducer. |
| ML           | Machine Learning. |
| MOOSE        | Multiphysics Object Oriented Simulation Environment. |
| N₂           | Dinitrogen. |
| NTNU         | Norwegian University of Science and Technology. |
| PE           | Pulse-Echo. |
| PEEK         | Polyether Ether Ketone. |
| PT           | Pulse-Transmission. |
| PTU          | Pulse-Tube. |
| PZC          | Piezoceramic. |
| PZT          | Lead Zirconate Titanate (piezoceramic material). |
| RB           | Resonant Bar. |
RH  Relative Humidity.
RU  Rice University.
RUS  Resonant Ultrasound Spectroscopy.
SHPB  Split-Hopkinson Pressure Bar.
SHRB  Split-Hopkinson Resonance Bar.
SLS  Standard Linear Solid.
SR  Spectral Ratio.
TI  Transverse Isotropy.
TUB  Technische Universität Berlin.
UA  University of Aberdeen.
UCB  University of Colorado Boulder.
UH  University of Houston.
UO  University of Oxford.
UofA  University of Alberta.
UofT  University of Toronto.
UoH  University of Hawaii.
US  University of Stuttgart.
USGS  United States Geological Survey.
UT  University of Tokyo.
UT(A)  University of Texas (at Austin).
UW  University of Wisconsin-Madison.
WIFF  Wave-Induced Fluid Flow.
WIGED  Wave-Induced Gas Exsolution Dissolution.

NOMENCLATURE

\( \alpha \)  Cole-Cole\'s spreading factor.  \( \in [0, 1] \)
\( \vec{C} \)  Stiffness tensor.
\( \vec{\epsilon} \)  Strain tensor.
\( \vec{\sigma} \)  Stress tensor.
\( \Delta \theta \)  Phase angle.
\( \Delta t \)  Travel time.
\(\Delta W\) Energy loss per cycle.
\(\epsilon\) Strain.
\(\epsilon_{\text{ax}}\) Axial strain.
\(\epsilon_{\text{rad}}\) Radial strain.
\(\epsilon_s\) Shear strain.
\(\epsilon_{\text{vol}}\) Volumetric strain.
\(\epsilon_{11}\) One of the strains \(\epsilon_{ij}\).
\(\epsilon_{12}\) One of the strains \(\epsilon_{ij}\).
\(\epsilon_{13}\) One of the strains \(\epsilon_{ij}\).
\(\epsilon_{22}\) One of the strains \(\epsilon_{ij}\).
\(\epsilon_{23}\) One of the strains \(\epsilon_{ij}\).
\(\epsilon_{33}\) One of the strains \(\epsilon_{ij}\).
\(\epsilon_{ij}\) Einstein notation for strain.
\(\eta\) Viscosity.
\(\kappa\) Permeability.
\(\lambda\) Wavelength.
\(\lambda_L\) Lamé’s first parameter.
\(\mu_L\) Lamé’s second parameter.
\(\nu\) Poisson’s ratio.
\(\nu_{\text{HH}}\) One of three Poisson’s ratios assuming [TT] symmetry.
\(\nu_{\text{HV}}\) One of three Poisson’s ratios assuming [TT] symmetry.
\(\nu_{\text{VH}}\) One of three Poisson’s ratios assuming [TT] symmetry.
\(\omega\) Angular frequency defined as \(\omega = 2\pi f\).
\(\phi\) Porosity.
\(\pi\) Pi. \(3.141592653\ldots\)
\(\rho\) Density.
\(\rho_f\) Pore fluid density.
\(\sigma\) Stress.
\(\sigma_{\text{ax}}\) Axial stress.
\(\sigma_{\text{dev}}\) Deviatoric stress.
\(\sigma_p\) Hydrostatic stress (or pressure).
\( \sigma_s \)  
Shear stress.

\( \sigma_x \)  
One of the principal stresses in \( z \)-direction.

\( \sigma_y \)  
One of the principal stresses in \( y \)-direction.

\( \sigma_z \)  
One of the principal stresses in \( z \)-direction.

\( \sigma_{11} \)  
One of the stresses \( \sigma_{ij} \).

\( \sigma_{12} \)  
One of the stresses \( \sigma_{ij} \).

\( \sigma_{13} \)  
One of the stresses \( \sigma_{ij} \).

\( \sigma_{22} \)  
One of the stresses \( \sigma_{ij} \).

\( \sigma_{23} \)  
One of the stresses \( \sigma_{ij} \).

\( \sigma_{33} \)  
One of the stresses \( \sigma_{ij} \).

\( \sigma_{ij} \)  
Einstein notation for stress.

\( \theta_{\text{SPE}} \)  
Specimen phase.

\( \theta_{\text{STD}} \)  
Standard phase.

\( \theta_\varepsilon \)  
Strain phase.

\( \theta_\sigma \)  
Stress phase.

\( \xi \)  
Aspect ratio.

\( A \)  
Area.

\( B_{\text{SPE}} \)  
Strain amplitude of specimen.

\( B_{\text{STD}} \)  
Strain amplitude of standard.

\( C_{11} \)  
One of five stiffnesses assuming \( T\Gamma \) symmetry.

\( C_{13} \)  
One of five stiffnesses assuming \( T\Gamma \) symmetry.

\( C_{33} \)  
One of five stiffnesses assuming \( T\Gamma \) symmetry.

\( C_{33} \)  
One of five stiffnesses assuming \( T\Gamma \) symmetry.

\( C_{44} \)  
One of five stiffnesses assuming \( T\Gamma \) symmetry.

\( C_{66} \)  
One of five stiffnesses assuming \( T\Gamma \) symmetry.

\( C_{ijkl} \)  
Einstein notation for stiffnesses.

\( C_{ij} \)  
Voigt notation for stiffnesses.

\( E \)  
Young’s modulus.

\( E_H \)  
One of three Young’s moduli assuming \( T\Gamma \) symmetry.

\( E_V \)  
One of three Young’s moduli assuming \( T\Gamma \) symmetry.

\( E_{45} \)  
One of three Young’s moduli assuming \( T\Gamma \) symmetry.
\( F \quad \text{Force.} \\
\( f \quad \text{Resonant frequency.} \\
\( f_b \quad \text{Biot’s characteristic frequency.} \\
\( f_c \quad \text{Characteristic frequency.} \\
\( f_d \quad \text{Drained-undrained characteristic frequency.} \\
\( f_p \quad \text{Patchy-saturation characteristic frequency.} \\
\( f_s \quad \text{“Squirt-flow” characteristic frequency.} \\
\( G \quad \text{Shear modulus.} \\
\( H \quad \text{Uniaxial (or P-wave) modulus.} \\
\( K \quad \text{Bulk modulus.} \\
\( K_d \quad \text{Drained bulk modulus.} \\
\( K_S \quad \text{Solid bulk modulus.} \\
\( K_u \quad \text{Undrained bulk modulus.} \\
\( L \quad \text{Specimen length.} \\
\( M \quad \text{Complex modulus.} \\
\( M_{\mathcal{I}} \quad \text{Imaginary part of complex modulus} \ M. \\
\( M_{\mathcal{R}} \quad \text{Real part of complex modulus} \ M. \\
\( n \quad \text{Positive integer in which} \ n \in \{1, 2, 3, \ldots\} \text{representing nodes or harmonics.} \\
\( P_c \quad \text{Confining pressure.} \\
\( Q \quad \text{Quality factor.} \\
\( Q^{-1} \quad \text{Inverse quality factor.} \\
\( Q_{E_H}^{-1} \quad \text{Inverse quality factor of Young’s modulus} \ E_H \text{ assuming TI symmetry.} \\
\( Q_{E_V}^{-1} \quad \text{Inverse quality factor of Young’s modulus} \ E_V \text{ assuming TI symmetry.} \\
\( Q_{E_{45}}^{-1} \quad \text{Inverse quality factor of Young’s modulus} \ E_{45} \text{ assuming TI symmetry.} \\
\( Q_E^{-1} \quad \text{Inverse quality factor of Young’s modulus} \ E. \\
\( Q_G^{-1} \quad \text{Inverse quality factor of shear modulus} \ G. \\
\( Q_K^{-1} \quad \text{Inverse quality factor of bulk modulus} \ K. \\
\( Q_\nu^{-1} \quad \text{Inverse quality factor of Poisson’s ratio} \ \nu. \\
\( u_r \quad \text{Radial displacement.} \)
| Symbol | Description |
|--------|-------------|
| $u_x$  | Axial displacement. |
| $u_\theta$ | Circumferential displacement. |
| $V$  | Velocity. |
| $V_{E}$ | Longitudinal or flexural velocity. |
| $V_{G}$ | Torsional velocity. |
| $V_{P_0}$ | P-wave velocity normal to bedding. |
| $V_{P_{45}}$ | P-wave velocity oblique ($45^\circ$) to bedding. |
| $V_{P_{90}}$ | P-wave velocity perpendicular to bedding. |
| $V_P$ | P-wave velocity. |
| $V_P/V_S$ | Ratio between compressional (P) and shear (S) wave velocities. |
| $V_{S_0}$ | S-wave velocity normal to bedding. |
| $V_{S_{90}}$ | S-wave velocity perpendicular to bedding. |
| $V_S$ | S-wave velocity. |
| $W_m$ | Energy stored. |
| $E_\Im$ | Imaginary part of Young’s modulus $E$. |
| $E_\Re$ | Real part Young’s modulus $E$. |
| $G_\Im$ | Imaginary part of shear modulus $G$. |
| $G_\Re$ | Real part of shear modulus $G$. |
| $K_\Im$ | Imaginary part of bulk modulus $K$. |
| $K_\Re$ | Real part of bulk modulus $K$. |
| $Q_E$ | Quality-factor of Young’s modulus $E$. |
| $Q_G$ | Quality-factor of shear modulus $G$. |
| $Q_K$ | Quality-factor of bulk modulus $K$. |