Multiple Inertial Measurement Units—An Empirical Study

ARIEL LAREY, ELIEL AKNIN, AND ITZIK KLEIN, (Member, IEEE)

Faculty of Electrical Engineering, Technion – Israel Institute of Technology, Haifa 3200003, Israel
Department of Marine Technology, University of Haifa, Haifa 3498838, Israel
Corresponding author: Itzik Klein (kitzik@univ.haifa.ac.il)

ABSTRACT An inertial navigation system is commonly used in various marine platforms above and below the sea surface to calculate the position, velocity and orientation of its carrier platform. Such systems contain an inertial measurement unit (IMU) to measure the specific force and angular velocity which in turn are integrated to obtain the navigation state. Due to sensor noises and other error terms, the navigation solution drifts in time. In situations of pure inertial navigation (no external aiding), multiple IMUs (MIMU) can be used to improve the performance of a single unit. In this paper, we explore the benefits of using a MIMU system for common navigation operations. To that end, a 32 MIMU architecture (192 inertial sensors) was designed and constructed for the experimental evaluations. Utilizing this system we examined the effect of the number of sensors in the architecture versus position accuracy, stationary calibration, coarse alignment and gyro free design. We derive closed form empirical expressions enabling insight to the connection between number of IMUs to the expected performance.

INDEX TERMS Inertial measurement unit, multiple sensors, calibration, gyro-free, coarse alignment.

I. INTRODUCTION

Inertial Navigation System (INS) consist of an Inertial Measurements Unit (IMU) and algorithms to calculate the navigation solution: position, velocity, and orientation of a platform. The IMU has two types of sensors, accelerometers to measure the specific force and gyroscopes to measure the angular velocity. A typical low-cost IMU is based on Micro-Electro-Mechanical-System (MEMS) technology and contains 3-axis orthogonal gyroscopes and accelerometers on the same silicon die [1]. IMUs are used in a variety of applications such as medical [2], sports [3] but are particularly used in navigation of various platforms such as autonomous underwater vehicles [4], unmanned ground [5] and aerial [6] vehicles and smartphones [7].

The IMU measurements are not accurate and have significant measurement errors that cause the navigation solution to drift over time. To circumvent the drift, INS are fused with external sensors [8]. However, in some situations the external sensor is not available (for example, GPS in an indoor environment) the navigation solution will rely only on the INS - a pure inertial navigation solution.

One of the approaches to mitigate the drift while maintaining a low-cost system, is to use a multiple IMU (MIMU) architecture [9], [10].

The motivation for using MIMU were recently summarized in a comprehensive and thorough paper [11]. Those include, higher accuracy, reliability and dynamic measurement range compared to a single IMU. Also, MIMU architecture has the ability to estimate angular motion from accelerometer data and direct estimation of angular acceleration. Several experimental architectures were developed for the MIMU system, e.g. an architecture using eight sensors with an optimized design [12], or an 18 IMUs design including a novel communication interface to receive sensory data simultaneously [13]. Another advantage of using MIMU, is the option to continue collecting data from the system even when one of the sensors malfunctions for some reason, which is impossible to do when using only one IMU [14].

A private case of MIMU is a gyro-free INS (GFINS) architecture which consists of multiple accelerometers but has no gyroscopes [15]. A GFINS consists of at least six distributed accelerometers capable of obtaining linear and angular acceleration and thereby capable of functioning as a conventional INS. Most of the research in the gyro-free is focused on seeking optimal configurations [16] since the
number of accelerometers and their location has a significant influence on the overall system performance. A practical GF kinematic equation of motion and corresponding error-state models, fitting any accelerometer configuration was derived [17]. Recently, a state-of-the-art literature review of GFINS and a control theoretic point of view was provided in [18]. As in MIMU, GFINS utilizes the same benefits for the calculation of the specific force and angular acceleration vectors used to propagate the gyro-free navigation equations.

In this research, we greatly elaborate our initial work on experimental MIMU [19], where a feasibility study of MIMU consisting of 32 IMUs (96 accelerometers and gyroscopes) was conducted. To further examine the influence of the number of IMUs in the MIMU architecture (and their location) relative to a single IMU, four different topics related to basic INS operations were addressed:

1) The improvement of the navigation solution (position, velocity and orientation) accuracy using a MIMU architecture.
2) Stationary calibration of a MIMU architecture.
3) Coarse alignment of MIMU architecture improvements relative to a single IMU in terms of accuracy and time duration of the alignment.
4) Influence of different sensor locations in the MIMU architecture on the specific force estimation.

To that end, a 32 MIMU architecture (192 inertial sensors) was designed and constructed for experimental evaluations. Utilizing this system we examined the affect of the number of sensors in the architecture (6 to 192) in each of the topics. We further derive closed form empirical expressions enabling insight to the connection between number of sensors to the expected performance.

This rest of the paper is organized as follows: Section II gives the problem formulation, including the navigation equations and a brief description of GFIMU theory. Section III presents the methodology while Section IV explains the MIMU structure, hardware and software. Section V presents the experiments and results. Section VI gives the conclusions.

II. PROBLEM FORMULATION

This section provides the equations of motion for a single IMU, GFIMU formulation and coarse alignment equations for a single IMU. In the following subsections, those equations will be elaborated for a MIMU system.

A. INS EQUATIONS OF MOTION

Given initial conditions, specific force and angular velocity measurements, the navigation solution can be calculated. Low-cost gyroscopes cannot measure the Earth angular velocity, thus it may be neglected in the equations of motion. Also, when considering short time navigation, the position vector can be obtained by direct integration on the velocity vector. We define a local coordinate frame, \( l \), where \( x \)-axis points forward, the \( z \)-axis points down in the gravity direction and the \( y \)-axis completes a right-handed orthogonal frame.

The rate of change of the position vector is given by

\[
\dot{p}^l = v^l
\]

(1)

where the position vector is expressed in the local coordinate frame such that \( p^l = [x \ y \ z]^T \), the velocity state vector is expressed in the same coordinates \( v^l = [v_x \ v_y \ v_z]^T \). The rate of change of the velocity vector is

\[
v^l = T_b^l \dot{p}^b + g^l
\]

(2)

where \( \dot{p}^b \) is the specific force vector as measured by the accelerometers, \( T_b^l \) is the transformation matrix from body to the local frame, \( g^l \) is the gravity vector expressed in the local frame. The rate of change of the transformation matrix is

\[
\dot{T}_b^l = T_b^l \Omega_{ib}^b
\]

(3)

where \( \Omega_{ib}^b \) is skew-symmetric form of the angular velocity vector as measured by the gyro

\[
\Omega_{ib}^b = \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}
\]

(4)

where \( \omega_i \) is the angular velocity component around the \( i \)-axis. From the transform matrix, assuming a \( 3 \rightarrow 2 \rightarrow 1 \) rotation sequence, the Euler’s angles can be extracted [1]

\[
\phi = \arctan2 \left( T_{b}^{l}[3, 2], T_{b}^{l}[3, 3] \right)
\]

(5)

\[
\theta = -\arcsin \left( T_{b}^{l}[3, 1] \right)
\]

(6)

\[
\psi = \arctan2 \left( T_{b}^{l}[1, 2], T_{b}^{l}[1, 1] \right)
\]

(7)

B. GYRO-FREE INS

The GFIMU calculates the system specific force and angular acceleration vectors through the linear accelerometers measurements. To that end, GFIMU configuration matrix, \( H \), is employed. It is a function of the number of accelerometers \( N \) in the GFIMU architecture, accelerometers location and direction given by

\[
H = \begin{bmatrix}
(r_1 \times d_1)^T & \vdots & d_1 \\
: & \vdots & : \\
(r_N \times d_N)^T & \vdots & d_N
\end{bmatrix}_{N \times 6}
\]

(8)

where \( d_i \) is the direction vector of the \( i \)-th accelerometer and \( r_i \) is the position vector of the \( i \)-th accelerometer relative to the origin (the point in which the navigation solution is calculated). Using, the configuration matrix (8), the GF-IMU outputs the specific force and the angular acceleration vectors [17]

\[
\begin{bmatrix}
\dot{\Omega}_{ib}^b \\
\dot{f}^b
\end{bmatrix} = \begin{bmatrix}
H_{\dot{\Omega}} \\
H_{\dot{f}}
\end{bmatrix} (Y - M)
\]

(9)

where \( \dot{\Omega}_{ib}^b \) is the angular acceleration vector, \( Y \) is the vector of consisting of the \( N \) accelerometers measurements

\[
Y = [f_1, \cdots, f_N]^T,
\]

(10)
$M$ is the matrix relating between the body angular velocity and the GFIMU architecture and

$$
\begin{bmatrix}
H_{\Omega} \\
H_{\dot{r}}
\end{bmatrix} = (H^T H)^{-1} H^T
$$

(11)

In this paper, we consider only stationary conditions for the GFINS system, thus $M$ is neglected.

### C. COARSE ALIGNMENT

In order to solve the navigation equations of motion (1)-(3), initial conditions are required. While the position and velocity are taken from external sources, the INS initial attitude [1] as well as the GFINS initial roll and pitch [20] are determined by the inertial sensors. In stationary conditions, the pitch and roll angles can be obtained directly from the accelerometers measurements [1]

$$
\phi_{CA} = \arctan 2 (-f_y, -f_z)
$$

(12)

$$
\theta_{CA} = -\arctan \left( \frac{-f_x}{f_y^2 + f_z^2} \right)
$$

(13)

where $f_i$ is the specific force component in the $i$-axis direction, expressed in the body frame.

### D. MULTIPLE-IMU

In theory, a system with a large amount of the same sensors, has a smaller statistical error than a single sensor system. According to the central limit theorem, when the number of sensors in the system is $N$, and $N$ approaches infinity, the expected value (average) of the error is 0 (a system without white noise). In practice, when $N$ is finite, the resulting standard deviation is

$$
\bar{\sigma}_{\text{acc}} = \sigma_{\text{acc}} / \sqrt{N}
$$

(14)

$$
\bar{\sigma}_{\text{gyr}} = \sigma_{\text{gyr}} / \sqrt{N}
$$

(15)

where $\bar{\sigma}_{\text{acc}}$ and $\bar{\sigma}_{\text{gyr}}$ are the resulting standard deviation of the accelerometers and gyroscopes, respectively. In both sensors, the standard deviation was reduced by a factor of $\sqrt{N}$. Thus, for the MIMU, average sensor readings can be plugged into the equations of motion (2)-(3) and mitigate the solution drift.

### III. METHODOLOGY

Both research and MIMU architecture methodologies are considered. First, several research directions were selected to highlight the benefits of using a low-cost MIMU architecture instead of a single IMU. Second, to address those research directions, an experimental MIMU architecture was designed and constructed. It contains 32 IMUs with a total of 192 inertial sensors. Further details on the architecture are given in Section IV. Using this experimental MIMU architecture, dedicated field experiments were made to answer the selected research directions. After analyzing the data and reaching some conclusions we further derive closed form empirical expressions to enable insight to the relationship between number of sensors in the architecture to the expected performance. In that manner, we provide a simple tool to MIMU architecture designers to evaluate the quality of their architectures prior to construction.

Returning to the research methodology, we aim to examine the influence of the number of IMUs in the MIMU architecture relative to a single IMU. To that end, four different topics related to basic INS operations were considered. Before addressing those subjects, two preliminary topics were addressed

- **Noise Reduction.** What is the best strategy for noise reduction in the MIMU architecture? Is it the same as in a signal IMU? We shall examine several approaches and compare the results between a single IMU and a MIMU architecture.

- **Averaging Strategy.** When to perform the averaging? In the input to the equation of motion (IMU raw data) or on their output (navigation solution)? In linear equations it dose not matter and the result will be the same, however the navigation equations of motion are nonlinear and therefore should be examined.

After finding an appropriate noise reduction and averaging strategies, we address four research questions in MIMU theory using our experimental MIMU architecture:

1) **Navigation Solution Accuracy.** How does the navigation solution (position, velocity and orientation) accuracy improves as a function of the number of IMUs in the MIMU architecture? Is there a certain number of IMUs where the improvement is marginal? To answer those questions the INS equations of motion (1)-(3) are solved for a stationary condition scenario.

2) **Stationary Calibration.** Considering zero order stationary calibration (mean), can the MIMU architecture reduce the required time to meet the calibration level of a single IMU? Or given an a priori time limit for the calibration process, can the MIMU improve a single IMU performance?

3) **Coarse Alignment.** Given the time for stationary coarse alignment, what is the improvement in the initial roll and pitch angles as a function of the number of IMUs in the MIMU architecture? To that end, the initial roll and pitch were calculated using (12)-(13).

4) **GFIMU Architectures.** Given that the number of IMUs in the MIMU architecture is fixed, how does their location influence the accuracy of the specific force vector calculation in a GFIMU configuration (9)? To that end, we considered several GFIMU configurations; each had a different configuration matrix as defined in (8).

A flowchart summarizing our research directions as explained above is presented in Figure 1.

### IV. MIMU ARCHITECTURE

The MIMU architecture designed and constructed for the research is based on the 32 MPU-6050 IMU [21] and 16 Arduino controllers. A schematic diagram of the architecture containing eight (out of 32) IMUs is presented in Figure 2.
Each pair of MPU6050 IMU was connected to a single Arduino Nano controller by an I2C protocol. The Arduino controller redirects the output (accelerometers and gyroscopes readings), from the IMUs to a PC at a sampling rate of 100Hz.

Each of the two IMUs pairs was connected to the ground (GND), clock (SCL) and data (SDA) on IMU side and respectively to the Arduino controller side. Additionally, the first IMU voltage source is on the VCC port, and the second IMU was connected to the AD0 port. In that way, each IMU is sending its data to two different addresses, so the Arduino can read both data simultaneously. The proper setup can be seen in Figure 3. All components (IMUs and Arduinos) were placed on a single plane with four different breadboards to facilitate the connections and information analysis. All four Arduino Nano components were connected to a USB Hub with four ports which were connected to a computer. An Arduino controller program code was implemented to receive the MIMU components data. Sensor readings were exported to a CSV file that included the measurements of the accelerometers and gyroscopes. Afterward, we used a dedicated tool to analyze the data and calculate the position, velocity and orientation of the MIMU system. The main functions of the tool include:

1) **Samples Synchronization.** Usually the beginning and the end of the recording time has some difference between the Arduinos in the MIMU system. Therefore, the tool has a synchronization algorithm to match all sensor readings.

2) **Noise Reduction.** A dedicated tool to analyze the data of all IMUs and averaging their results. This tool uses a low pass filter (LPF) as part of the analysis implemented with the a fast Fourier transform (FFT) algorithm. Moreover, the tool also uses Tukey’s Fences (TF) algorithm [22] to remove outliers.

3) **Main Algorithm.** Used to evaluate all necessary code for the experiments required to answer the research topics as addressed in Section III.

V. EXPERIMENTAL RESULTS

To evaluate and compare the performance as a function of the number of IMUs in the system, the root mean square (RMS) error is employed as the performance measure. In all experiments the MIMU system was at stationary conditions. The nominal values for the position, velocity and attitude are zeros. Thus, at each time step the position, velocity and attitude RMS errors are defined as follows:

\[
\delta p_{RMS}(t_k) = \sqrt{\frac{1}{3} \sum_{j=x,y,z} [p_j(t_k)]^2} \\
\delta v_{RMS}(t_k) = \sqrt{\frac{1}{3} \sum_{j=x,y,z} [v_j(t_k)]^2} \\
\delta a_{RMS}(t_k) = \sqrt{\frac{1}{3} \sum_{j=\phi,\theta,\psi} [a_j(t_k)]^2}
\]

where \( t_k \) is the time at epoch \( k \), \( p_j \) is the position vector component \( j = x, y, z \), \( v_j \) is the velocity vector component \( j = x, y, z \) and \( a_j \) is the Euler angle \( j = \phi, \theta, \psi \).

A. NOISE REDUCTION

In the sensor measurement noise reduction approach, we consider LPF and TF algorithms with different parameters:

- **LPF** – no frequency limit, maximum frequency: 2Hz, 3Hz, 4Hz, 5Hz, 6Hz.
- **TF** - no use of TF, K values: 0.75, 1.5, 3.

The MIMU system was in stationary conditions and recordings were made for a time duration of five minutes. The
raw data was calibrated before noise reduction techniques were applied. This experiment was repeated three times to give three different datasets. Next, the IMU measurements were averaged prior to their substitution into the navigation equations (1)-(3). This process was repeated for all LPF and TF parameter combinations as described above. The RMS errors at the end of the scenario, for the position, velocity, and attitude errors are presented in Figure 4 for a single IMU compared to the 32 MIMU architectures. As expected, regardless of the noise filter parameters, the 32 MIMU architecture outperforms the single IMU in all navigation states. For example, when not using any noise reduction filters, the position error of a single IMU is almost 6 times bigger from the 32 MIMU system. The noise reduction filter parameters examined had no significant influence on the single IMU performance. For the MIMU, a 3% difference in the RMS error was obtained between using or not the noise filters. The configuration that was chosen for the next experiments was the configuration that uses TF algorithm when $K = 3$, and LPF with maximum frequency of 3Hz. It is important to note that the system is noisy; that is why the maximum frequency is low. Also, if the system will be in motion, probably a higher maximum frequency will be chosen.

Using those parameters, the position error growth for the single IMU and 32 MIMU system is plotted in Figure 5. As expected, the error increases more rapidly for the single IMU. In this example, the position error of the MIMU after 5 minutes was about 15 meters, while the same error was obtained in the single IMU after about 125 seconds, resulting in a 175 seconds difference. For a time duration less than 30 seconds, no significant difference was found between the single and MIMU systems.

**B. AVERAGING STRATEGY**

There are two options for averaging in a MIMU system:

1) **Navigation solution.** Each IMU measurements, in the MIMU system, is plugged into a single set of equations of motion (1)-(3). The resulting $N$ navigation solutions are then averaged to give a single navigation solution.

2) **IMU raw data.** Average the $N$ IMU measurements (all IMUs in the MIMU system) and substitute the result into a single set of equations of motion (1)-(3) to yield a single navigation solution.

Both approaches are illustrated in Figure 6. If the equations of motion were linear, that theoretically there will be no difference between the two options. However, since they are not, both options should be compared. For that evaluation, the 32 MIMU system was in stationary conditions. The system sensors were measured for 5 minutes. This experiment was repeated three times. Before examining the two averaging approaches, measurement noise reduction was made using the parameters found in Section V-A. The RMS results of the position, velocity, and attitude errors for both approaches are presented in Figure 7. As can be seen, averaging on the navigation solution obtained better performance than on the IMU raw data. The position RMS error was improved by 27%, velocity RMS error by 23% and attitude RMS error by 20%. Therefore, we consider averaging on the
navigation solution for the rest of the paper. Notice however, the trade-off between the two options. Although the error is lower, the computational load increases since the navigation equations of motion needs to be solved $N$ times.

C. STATIONARY CALIBRATION
Calibration performance in terms of time and accuracy of the process are compared between a single IMU and MIMU system. To that end, zero-order calibration in stationary conditions is considered. In that approach, after a predefined time duration, the sensors (both accelerometers and gyroscopes) measurements in each axis were averaged. The resulting values were treated as the sensors biases and were then removed in each measurement epoch. Data was recorded for a time duration of five minutes, plugged into the equation of motion (1)-(3) to calculate the position vector. Without calibration, both single and MIMU system obtained RMS position error of tens kilometers. Although the former improved the error by a factor of five the results are still unacceptable for navigation. When considering 10 seconds for the calibration process, the error reduces by three orders of magnitudes both for the single IMU and 32 MIMU architecture. The position error of the MIMU architecture was 13 meters, while for the single IMU 105 meters, that is eight time worse.

Furthermore, we also addressed the following scenario: for stationary conditions, using the experimental MIMU system and calibration time of 10 seconds, how much time is needed to calibrate a single IMU to receive the same position error as in the MIMU system? To that end, the single IMU was examined in time windows iterations. For each iteration, 5 seconds were added to the calibration time, until the single IMU had almost the same position error as the 32 components MIMU system after 10 seconds of calibration. The results as a function of the number of IMUs in the system. The solid curve is an empirical approximation of the error.

The RMS position error at the end of the trajectory is presented in Figure (9). As the number of IMUs increases in the architecture, the position error decreases. The level of improvement, however, reduces as the number of IMUs increases. For this example, after 20 IMUs, no significant position error improvement was achieved. For a single IMU, the position error after five minutes is 88m while for 20 IMUs it was reduced to 20m, a 78% improvement. The position error from 32 IMUs was about 15m, 83% improvement relative to a single IMU and 25% relative to 20 IMUs. This means that it is not necessary to use as much as possible IMUs (require more space and power consumption) but rather check how much IMUs are required to obtain the desired improvement relative to a single IMU.

A closed form solution representing the position RMS error, $\delta_{\text{RMS}}^t$, behavior at the end of the trajectory as a function of the number of IMUs was derived. To that end, several functions were regressed and the one with the minimum mean square error was chosen. Details on this procedure are provided in the Appendix. The empirical closed form expression for the position RMS is

$$\delta_{\text{RMS}}^t(t = 300) = \frac{84.46}{\sqrt{N}} + 1.11$$  \hspace{1cm} (19)

The RMS position error of (19) is also presented in Figure (9). Based on our 32 IMU experimental MIMU system, if the number of IMUs was increased to infinity the resulting RMS position error would be 1.11m.

The same procedure is repeated for the velocity, $\delta_{\text{RMS}}^a$, and attitude, $\delta_{\text{RMS}}^t$ RMS errors as presented in Figures (10)-(11). As expected, increase in the number of IMUs in the MIMU architecture results in improved performance. Closed form solutions representing the velocity and attitude RMS errors.

FIGURE 8. Position error of a single IMU as a function of the calibration. The constant line presents the position error of the MIMU architecture with 10 seconds of calibration.

FIGURE 9. RMS position errors at the end of the scenario as a function of the number of IMUs in the MIMU architecture. The solid curve is an empirical approximation of the error.
at the end of the trajectory as a function of the number of IMUs were derived giving

\[ \delta v_{RMS}(t = 300) = \frac{0.57}{\sqrt{N}} + 0.02 \]  
\[ \delta a_{RMS}(t = 300) = 183.66 \times e^{-0.6\sqrt{N}} + 12.45 \]  

(20)

(21)

Next, we focus on the position error and aim to find an empirical closed form solution as a function of time (not only at the end of the five minutes trajectory as in (19)). The required structure of the solution should be as (19), thus the numerator and offset should be determined as a function of time. To that end, empirical closed form solutions were derived for the position error after 60, 120, 180 and 240 seconds (see Appendix). Each of those expressions had a different value for the numerator and offset. By regression, those values were found as a function of time. The resulting closed form solution for the position error as a function of time and number of IMUs in the system is

\[ \delta p_{RMS}(t) = \frac{94 \times 10^{-3}t^2}{\sqrt{N}} + 108 \times 10^{-4}t^2 \]  

(22)

To validate (22), the position errors after 60, 120, 180 and 240 seconds as a function of the number of IMUs in the system was calculated using (22) and compared to those from the empirical derivation (see Appendix). The error is presented in Figure 12. The largest error occurs for a single IMU and rapidly decreases as the number of IMUs increases for all time instances examined. For example, the maximum approximation error was for a single IMU after 240s. Meaning, that using (22) at \( t = 240s \), the empirical solution will have an error of 1.74m in approximating the single IMU position error. The actual error of the single IMU at \( t = 240s \) is 56.5m, thus using (22) 2% from the true value was achieved. Therefore, the closed form empirical expression (22) can be used to approximate the position error in the MIMU system as a function of time and the number of IMUs.

**E. COARSE ALIGNMENT**

The influence of the number of IMUs in the system used for stationary coarse alignment process is examined. There, the low-cost gyro sensors cannot measure the earth turn rate and thus cannot calculate the initial heading angle. Therefore, only the initial roll and pitch angles, as measured by the accelerometers, are considered.

The system accelerometers were recorded for a time duration of 30 seconds, three times. For each number of IMUs ranging from a single IMU to the maximum of 32 IMUs, the accelerometer measurements were averaged per sensitive direction. The resulting specific force vector components were plugged into (12)-(13) to calculate the initial pitch and roll angles. The nominal roll and pitch angles are zero. The RMS of the initial pitch and roll error as a function of the numbers of IMUs is presented in Figure 13. Using a single IMU and RMS error of 1.2deg was obtained while using 32 IMUs the error was reduced to 0.15deg. When the number of components is greater than 28, the change in the error is minor. Using curve fitting techniques a closed form empirical
expression is derived based on the results to predict the coarse alignment RMS, $\delta \Theta$, error based on the number of IMUs

$$\delta \Theta = \frac{1.51}{\sqrt{N}} - 0.34$$  (23)

Using (23) an approximation of the error as a function of the IMUs in the architecture can be found.

F. GFIMU ARCHITECTURES

When addressing the experimental 32 MIMU architecture as a gyro-free system, only the 96 accelerometers are considered. Their location and direction influence the accuracy of the estimated specific force and angular acceleration vectors (9). In the baseline configuration, the breadboards (each containing 8 IMUs) are in the shape of square and close to each other. In this experiment four different configurations, as presented in Figure 14 for the accelerometers locations are considered:

1) **Baseline configuration.** Square configuration where the breadboards are stacked to each other to form a square, as shown in Figure 14(a).
2) **Square configuration.** The baseline configuration is spited in a manner that each breadboard is located at 297mm (A4 page length) from the other, as shown in Figure 14(b).
3) **Rectangle configuration.** The breadboards are placed one to each other to form a rectangle shape configuration, as presented in Figure 14(c).
4) **Extended rectangle configuration.** The distance between two neighboring breadboards is set to 297mm while maintaining a rectangle form, as presented in Figure 14(d).

To compare between the different configurations, they are placed in stationary conditions. Raw data was collected for a time duration of 5 minutes, three times. A calibration time of 10 seconds was applied to remove the sensors biases. The accelerometers location and direction were defined with respect to the center of the configuration and then the configuration matrix (8) was calculated for each architecture. The accelerometers raw data and the configuration matrix are substituted into the GF IMU (9) to calculate the specific force and angular acceleration vectors. Since we are addressing stationary conditions, the expected angular acceleration is zero and cannot be measured by the system. Therefore, only the specific force vector estimation from (9) is considered.

The RMS errors of the specific force vector of The GFINS configurations are presented in Figure 15. In all four configurations, as expected, the calibration stage helped reducing the error in the specific force calculation. When the breadboards are placed in a rectangle shape configuration, the errors are smaller (more than 50% reduction) compared with the square shape configurations. Moreover, in both rectangle and square configuration, when the breadboards are far from each other, the error is smaller. Recall, that the estimated GFINS specific force (9) is a function of the accelerometer measurements and the configuration matrix. Both elements contribute to the accuracy of the calculation of the specific force. As pointed out in [15], [16] as the distance between the accelerometers increases, the projection of the accelerometer measurement (geometric factor) is more accurate leading to a reduced error in the GFINS calculations. it is to this reason, the rectangular configuration, particularly the extended one, obtained better performance.

VI. CONCLUSION

An experimental 32 IMU system was designed and constructed to perform an empirical study in stationary conditions. The MIMU architecture can be used to improve the performance of a single IMU in situations when pure inertial navigation is required. In this research, noise reduction analysis and averaging strategy were first examined. Optimal parameters of LPF were found and sensor measurement outlier rejection parameters using TF were determined. It was shown, that averaging on the navigation solution is preferred than averaging on the raw IMU data. Although the former improved the position error by 27%, the computational load increases since the navigation equations of motion need to be solved $N$ times. Thus, the architecture designer should consider the trade-off between accuracy improvement (average on navigation equations) and computational load (average on the raw data).

After addressing those tasks, we examined the benefits of MIMU in stationary calibration, navigation solution...
accuracy, coarse alignment and GFIMU architecture. Using the experimental IMU, it was shown that the position error with a single IMU is four times bigger. In addition, it was shown that after taking 20 IMUs in the architecture, no significant position error improvement was achieved relative to 32 IMUs. This conclusion highlights the fact that the number of IMUs in the MIMU architecture should be examined carefully to meet the accuracy goal since unnecessary IMUs still require space and power consumption.

When applying zero-order stationary calibration, results show that a single IMU system requires about 215 seconds of calibration to obtain the same performance (navigation solution error) as a 32 IMU architecture calibrated only for 10 seconds. Thus, until 215 seconds of calibration, the MIMU architecture obtains better performance for the same calibration time.

In the coarse alignment procedure, as the number of IMUs was increased in the architecture, the accuracy of the initial roll and pitch angles was improved. RMS error of 0.15deg for the MIMU compared to 1.2deg for a single IMU. However, as in the position error, above 28 IMUs, in the architecture has no significant improvement relative to 32 IMUs architecture.

Next, a GF architecture using the 96 MIMU accelerometers was examined in terms of the influence of the accelerometer’s geometry in the architecture on the specific force calculations. To that end, four configurations were examined two in a square shape and two in a rectangle shape. In a rectangle shape configuration, the errors are smaller (more than 50%) compared to the square shape configurations.

Finally, closed form empirical expressions to evaluate performance given the number of IMUs in the architecture were derived. Those, enable insight and give an analytical tool to the MIMU architecture designer to understand the expected behavior of the system performance as a function of the number of IMUs. Particularly, empirical expressions were derived for the position, velocity, attitude and coarse alignment errors. Using those relations a connection between the expected error at the end of the examined five minutes trajectory to the number of IMUs in the architecture is provided. Furthermore, the same analysis was repeated for different time duration (1, 2, 3, 4 and 5 minutes) on the position error. For each time, a closed form empirical solution was derived to show the position error as a function of the number of IMUs. Using all those expressions, we derived a single closed form empirical expression for the position error as a function of time and number of IMUs in the system.

APPENDIX I. DERIVATION OF THE POSITION EMPIRICAL EXPRESSION

To derive the closed form empirical expressions representing the RMS errors the following families of functions were examined:

\[
y = \frac{a}{\sqrt{N}} + b, \quad j = 2, 3 \tag{24}
\]

\[
y = ae^{-bN^j} + c, \quad j = 1/2, 1, 2 \tag{25}
\]

The parameters of each one of the seven functions were regressed. Then, the mean square error (MSE) between the empirical function and the actual quantity, as a function of the number of IMUs in the system at a specific time was calculated. The function with the minimum MSE was chosen. At the end of the trajectory \( t = 300s \), the resulting position RMS is given in (19), velocity RMS (20) and attitude RMS (21).

To derive the position RMS as a function of time, empirical functions for \( t = 60, 120, 180 \) and \( 240s \) are derived in the same manner yielding:

\[
\delta p_{RMS}(t = 60) = \frac{2.38}{\sqrt{N}} + 0.27 \tag{28}
\]

\[
\delta p_{RMS}(t = 120) = \frac{12.67}{\sqrt{N}} + 0.27 \tag{29}
\]

\[
\delta p_{RMS}(t = 180) = \frac{30.18}{\sqrt{N}} + 0.41 \tag{30}
\]

\[
\delta p_{RMS}(t = 240) = \frac{56.17}{\sqrt{N}} + 0.31 \tag{31}
\]

Each of those empirical expressions (28)-(31) has a different value for the numerator and offset. By regression, those values were found as a function of time. To that end, the following families of function were examined:

\[
y = at^j, \quad j = 1, 2, 3, 4 \tag{32}
\]

\[
y = e^{at} \tag{33}
\]

After regression, the numerator value as a function of time was found to be \( 94 \times 10^{-3}t^2 \) and the offset \( 108 \times 10^{-4}t^2 \). The resulting closed form solution for the position error as a function of time and number of IMUs in the system is given by (22).

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ARIEL LAREY was born in Jerusalem, Israel, in 1992. He is currently pursuing the M.Sc. degree in electrical and computer engineering with the Technion – Israel Institute of Technology, Haifa, Israel. His research interests are in image processing, machine learning, and control systems.

ELIEL AKNIN was born in Paris, France, in 1993. He is currently pursuing the M.Sc. degree in electrical and computer engineering with the Technion – Israel Institute of Technology, Haifa, Israel. His research interests are in control systems, signal processing, and machine learning.

ITZIK KLEIN (Member, IEEE) received the B.Sc. and M.Sc. degrees in aerospace engineering and the Ph.D. degree in geo-information engineering from the Technion – Israel Institute of Technology, Haifa, Israel, in 2004, 2007, and 2011, respectively. He is currently with the Department of Marine Technologies, University of Haifa. His research interests include inertial navigation, autonomous underwater vehicles, sensor fusion, and estimation theory.