Isoscalar dipole excitations in $^{16}$O

Yoshiko Kanada-En’yo and Yuki Shikata

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

Isoscalar (IS) monopole and dipole excitations in $^{16}$O were investigated by the method of shifted basis antisymmetrized molecular dynamics combined with the generator coordinate method. Significant strengths of the IS monopole and dipole transitions were obtained in the low-energy region below the giant resonances. In addition to the compressive mode, which mainly contributes to the high-energy strengths for the IS dipole giant resonance, we obtained a variety of low-energy dipole modes such as the vortical dipole mode in the $1^{-}$ state of the vibrating tetrahedral $4\alpha$ and the $^{12}$C+α cluster structure in the $1^{-}$ state. The $1^{-}$ state contributes to the significant low-energy strength of the IS dipole transition as 5% of the energy-weighted sum rule, which describes well the experimental data observed by the α inelastic scattering.

I. INTRODUCTION

In the past decades, low-energy monopole and dipole excitations have been attracting great interests (see, for example, reviews in Refs. [1–5] and references therein). A central issue is possible appearance of new excitation modes decoupled from collective vibration modes corresponding the giant resonances (GR). In experiments with α inelastic scattering extensively performed for study of isoscalar (IS) monopole and dipole excitations, significant low-energy strengths with the fraction of several percentages of the energy weighted sum rule have been observed in various stable nuclei such as $^{16}$O, $^{40}$Ca, and $^{208}$Pb [6–8]. The questions to be answered are what is the origin of these IS low-energy dipole (LED) strengths and how the dipole modes come down to the energy much lower than the IS giant dipole resonances (GDR).

In order to understand the IS LED strengths, the vortical dipole (VD) mode (called also the torus or toroidal mode) has been studied firstly with hydrodynamical models [9, 10], and later with microscopic approaches [2, 11–13]. The VD mode is characterized by the nuclear vorticity and has a unique feature different from the standard IS dipole mode so-called compressive dipole (CD) in the IS GDR. Since the nuclear density is conserved in the VD mode, its energy can be lower than the IS GDR involving compression of nuclear density. As a measure of the nuclear vorticity in the dipole excitations, the toroidal dipole (TD) operator has been introduced [9, 14]. The TD operator is given by the rotational component (a curl term) of the transition current density and the counter part of the compressive dipole (CD) operator with the irrotational component (a divergence term) of the transition current density, and has been proved to be a good probe for the low-energy VD mode [14].

In light nuclei, also cluster states may contribute to the low-energy IS monopole (IS0) and dipole (IS1) transition strengths because the IS0 and IS1 operators contain higher order $r^{\lambda+1}$ terms and can excite not only the compressive vibration modes but also the inter-cluster motion in the cluster states as pointed out by Yamada et al. [20] and Chiba et al. [21]. Indeed, the low-energy IS monopole strengths in $^{16}$O have been described well by cluster states with a semi-microscopic $4\alpha$-cluster model [20]. It is an important issue to clarify the IS dipole excitations in $^{16}$O, in particular, the cluster and vortical aspects of the low-energy modes.

Theoretical calculations with cluster models have been performed for $^{16}$O and suggested a variety of cluster structures such as the tetrahedral $4\alpha$ and $^{12}$C+α structures [20, 22–27]. However, there have been no microscopic calculation that successfully describes the energy spectra of $^{16}$O. Recently, we applied a microscopic model of the antisymmetrized molecular dynamics (AMD) [38–41] to $^{16}$O, and obtained reasonable reproduction of the energy spectra of $^{16}$O such as $0^{+}_2$, $2^{+}_1$, $4^{+}_1$, $1^{+}_{2}$, and $3^{-}_2$ states in the positive- and negative-parity bands with the $^{12}$C+α structure and $3^{+}_1$ and $4^{+}_2$ states in the ground band with the tetrahedral $4\alpha$ structure [36, 42].

Our aim is to investigate the IS dipole excitations in $^{16}$O. Main interest are properties of the IS LED modes such as the cluster and vortical aspects. For this aim, we apply the method of the shifted basis AMD (sAMD) [43–45] combined with the cluster generator coordinate method (GCM). The sAMD+GCM has been recently constructed to describe both the single-particle excitation and large amplitude cluster mode. This method has been applied to $^{12}$C to discuss the cluster, vortical, and compressive IS dipole modes, and proved to be a powerful approach for the IS monopole and dipole excitations in a wide energy range including the low-energy states and high-energy GRs. [44, 46].

In our previous work of $^{16}$O[42], we have investigated the cluster states with variation after spin-parity projections (VAP) [47] combined with the $^{12}$C+α-cluster GCM, which we called the VAP+GCM, but not the IS GDR because the sAMD bases have not been adopted in the previous work. The great advantages of the present sAMD+GCM are that it describes both the low-energy cluster state and the GDR in a unified framework owing to inclusion of one-particle and one-hole (1p-1h) excitations in the sAMD bases, and is suitable to discuss details of the IS dipole excitations. In this paper, we show the IS monopole and dipole strength functions in $^{16}$O in a wide energy range covering the low-lying vortical and cluster modes, and also the high-energy compressive vibration...
modes of the GRs. For detailed analysis of the monopole and dipole transitions, we calculate the form factors and transition densities and compare them with experimental data measured by the electron scattering. We discuss the vortical and cluster aspects of the IS LED states and clarify properties of the IS dipole excitations.

The paper is organized as follows. The formulation of the sAMD+GCM for $^{16}$O is explained in Sec. II. Section III shows the calculated results and discusses the properties of the IS monopole and dipole modes. Finally, the paper is summarized in section IV. In appendix sections, the definitions of the transition operators, densities, and strengths are given.

II. FORMULATION

In order to calculate the IS monopole and dipole excitations in $^{16}$O, we combine the sAMD with the previous VAP+GCM model [12]. Namely, we prepare the sAMD wave functions and combine them with the basis wave functions adopted in the previous VAP+GCM calculation. We call the present calculation “sAMD+GCM”. In this section, we explain the framework and procedure of the present calculations of $^{16}$O. For details of the VAP+GCM and the AMD, the reader is referred to Refs. [12] [14] [16] [18] and references therein.

A. VAP+GCM with AMD wave functions

An AMD wave function is given by a Slater determinant of single-particle Gaussian wave functions,

$$\Phi_{AMD}(Z) = \frac{1}{\sqrt{A!}} A\{\varphi_1, \varphi_2, \ldots, \varphi_A\}, \quad (1)$$

$$\varphi_i = \phi_{X_i}(\chi_i, \tau_i), \quad (2)$$

$$\phi_{X_i}(r_j) = \left(\frac{2\nu}{\pi}\right)^{4/3} \exp\left\{-\nu(r_j - X_i)^2\right\}, \quad (3)$$

$$\chi_i = \left(\frac{1}{2} + \xi_i\right)\chi^+ + \left(\frac{1}{2} - \xi_i\right)\chi^-. \quad (4)$$

where $A$ is the antisymmetrizer, $\phi_{X_i}$, $\chi_i$, and $\tau_i$ are the spatial, spin, and isospin functions of the $i$th single-particle wave function, respectively. The isospin part is fixed to be up (proton) or down (neutron). $\nu$ is the width parameter, which is fixed to be $\nu = 0.19$ fm$^{-2}$ used in the previous calculation. The condition $\sum_{i=1}^{Z/A} X_i/A = 0$ is always kept and the contribution of the center of mass motion is exactly removed from the total system. The AMD wave function is specified by the set of variational parameters $Z = \{X_1, \ldots, X_A, \xi_1, \ldots, \xi_A\}$ for the centroids of single-nucleon Gaussian wave packets and nucleon-spin orientations, which are determined by the energy variation.

It should be stressed that, in the AMD model, the existence of any clusters is not a priori assumed because Gaussian centroids, $X_1, \ldots, X_A$, of all single-nucleon wave packets are independently treated as variational parameters. Nevertheless, the model wave function can describe various cluster wave functions, and also shell-model wave functions because of the antisymmetrization of Gaussian wave packets.

To obtain the AMD wave function optimized for the $J^\pi$ state, the VAP is performed with respect to the variation of $Z$ by

$$\delta \langle \Phi | H | \Phi \rangle = 0, \quad (5)$$

for the $J^\pi$-projected AMD wave function $\Phi = P_{MK}^J \Phi_{AMD}(Z)$, where $P_{MK}^J$ is the spin-parity projection operator. For the AMD wave function $\Phi_{AMD}^{16}_O(Z)$ of $^{16}$O, we perform the VAP with $J^\pi_c = 0^+_2, 2^+_1, 4^+_2, 1^+_1, 2^+_1, 3^+_1$, and $5^+_1$, and obtain nine configurations of $\Phi_{AMD}^{16}_O(Z)$ with the parameters $Z^\text{opt}_c$ optimized for each $\beta = J^\pi_c$ state. In the simple VAP calculation, we superpose the nine configurations.

In the GCM calculation, we adopt the $^{12}$C+$\alpha$ cluster wave functions, where the angular momentum projection and internal excitations of the sub system $^{12}$C-cluster are considered. We first perform the VAP calculation of the subsystem $^{12}$C for three states $^{12}$C$(0^+_1)$, $^{12}$C$(0^+_2)$, and $^{12}$C$(1^-_1)$. Using the obtained $^{12}$C-cluster wave functions, the $^{12}$C+$\alpha$ wave function is constructed as done in Ref. [12]. The relative distance $d$ between $^{12}$C and $\alpha$ clusters is treated as a generator coordinate. The angular-momentum projection of the subsystem $^{12}$C is also practically performed by taking into account rotation of the $^{12}$C-cluster.

B. sAMD+GCM: combination of sAMD with VAP+GCM

In addition to the VAP and $^{12}$C+$\alpha$ wave functions, the sAMD wave functions are also superposed to describe 1p-1h excitations on the ground state. Starting from the ground state wave function $\Phi_{AMD}^{16}_O(Z^\text{opt}_0)$ obtained by the VAP, we consider small variations of single-particle wave functions by shifting the Gaussian centroid of each single-particle wave function, $X_i \rightarrow X_i + \epsilon \sigma$, (the spatial position parameters), of $Z^\text{opt}_c$ in the AMD wave function. Here $\epsilon$ is an enough small constant, $\epsilon \sigma$ ($\sigma = 1, \ldots, 8$) are unit vectors for 8 directions. Spin non-flip and flip states and recoil effects are taken into account as explained in Ref. [14]. Consequently, totally $16A = 256$ bases of the shifted AMD wave functions are superposed in addition to the VAP and $^{12}$C+$\alpha$ wave functions in the sAMD+GCM calculation of $0^+$ and $1^-$ states.

In the present sAMD+GCM calculation, we use the $\Phi_{AMD}^{12}_C(Z^\text{opt}_{\beta=0^+_1})+\alpha$ configuration with the inter-cluster distances of $d = \{1.2, 2.4, \ldots, 7.2$ fm$\}$ and $\Phi_{AMD}^{12}_C(Z^\text{opt}_{\beta=0^+_2, 1^-_1})+\alpha$ configurations with $d = \{1.2, 2.4, \ldots, 7.2$ fm$\}$.  


give the opposite ordering of the \(0^+\)+\(\text{AMD+GCM}\) and \(V\text{AP+GCM}\) calculations eventually be assigned to the experimental \(0^-\) MeV energy gain of the \(0^+\) state. For the \(2^+\) state, the \(s\text{AMD+GCM}\) obtains 2 bases but not by the \(s\text{AMD}\) bases. It is not the case two calculations for these states, because the developed \(V\text{AP+GCM}\), there is no essential difference between the levels in \(E\) obtained in the final model calculations. We note that, the calculated fourth is much better than the preceding microscopic cluster \(s\text{AMD+GCM}\) is not as good as the \(V\text{AP+GCM}\), but it by about 2 MeV in the \(s\text{AMD+GCM}\). As a result, the energy position of the excited \(0^+\) improves the ground state correlations. Because of this \(V\text{AP+GCM}\) meaning that the \(s\text{AMD}\) bases efficiently presented in the previous paper \[42\].

The IS0 and IS1 transition strengths are calculated with the \(0^+\) and \(1^-\) states obtained by the \(s\text{AMD+GCM}\). The form factors and transition densities are also calculated. The definitions of the operators, matrix elements, strengths, form factors, and transition densities are given in appendixes.

III. RESULTS

A. Structure properties of low-energy levels of \(0^+\) and \(1^-\) states

The \(s\text{AMD+GCM}\) result of the binding energy, root-mean-square (rms) matter radii, and excitation energies of low-lying \(0^+\) states are listed in Table I, and those of the \(1^-\) state are shown in Table II. For comparison, values calculated with the \(V\text{AP}\) (without the \(12\text{C}+\alpha\) nor \(s\text{AMD}\) bases) and those of the \(V\text{AP+GCM}\) (without the \(s\text{AMD}\) bases) are also shown in the tables. These correspond to the \(V\text{AP}\) and \(V\text{AP+GCM}\) calculations presented in the previous paper \[42\].

Various cluster states are obtained in the excited \(0^+\) levels in \(E \lesssim 20\) MeV. Compared the \(s\text{AMD+GCM}\) and \(V\text{AP+GCM}\), there is no essential difference between the two calculations for these states, because the developed cluster states are dominantly contributed by the GCM bases but not by the \(s\text{AMD}\) bases. It is not the case for the ground state, but the \(s\text{AMD+GCM}\) obtains 2 MeV energy gain of the \(0_1^+\) state compared with the \(V\text{AP+GCM}\) meaning that the \(s\text{AMD}\) bases sufficiently improve the ground state correlations. Because of this additional energy gain of the ground state, the relative energy position of the excited \(0^+\) states are raised up by about 2 MeV in the \(s\text{AMD+GCM}\). As a result, the agreement with the experimental energy spectra in the \(s\text{AMD+GCM}\) is not as good as the \(V\text{AP+GCM}\), but it is much better than the preceding microscopic cluster model calculations. We note that, the calculated fourth \(0^+\) state with the \(12\text{C}(2_1^-)\)+\(\alpha\) cluster structure should be assigned to the experimental \(0_1^+\) state, because the \(s\text{AMD+GCM}\) and \(V\text{AP+GCM}\) calculations eventually give the opposite ordering of the \(0_1^+\) and \(0_2^+\) states as discussed in the previous paper.

In the calculated \(1^-\) levels, the \(1_1^-\) and \(1_2^-\) states are obtained in \(E \approx 15\) MeV. The higher state (\(1_2^-\)) is the well developed cluster state and regarded as the band-head state of the \(K^\pi = 0^-\) \(12\text{C}+\alpha\) band, which is the parity doublet of the \(K^\pi = 0_2^-\) \(12\text{C}+\alpha\) band built on the band-head \(0_2^-\) state. The lower state (\(1_1^-\)) has the small rms radii comparable to that of the ground state and shows less prominent cluster structure than the \(1_2^-\) and \(0_2^+, 0_3^+, 1_1^-\) states. Comparing with the \(V\text{AP+GCM}\), the \(s\text{AMD+GCM}\) gives the smaller radius of the \(1_1^-\) state. Moreover, the excitation energy of the \(1_1^-\) state is almost same between the \(s\text{AMD+GCM}\) and \(V\text{AP+GCM}\) calculations indicating that the \(s\text{AMD}\) bases describe additional correlations contributing the size shrinkage and the 2 MeV energy gain comparable to that of the ground state.

B. Cluster structures of low-lying states

Cluster aspects of the low-lying states have been investigated in Ref. \[12\]. We here briefly review the cluster structures of the \(0_1^+, 0_2^+, 1_1^-,\) and \(1_2^-\) states following the discussions in the previous paper based on the analysis of the intrinsic wave functions, \(\Phi_{\text{AMD}}^{\text{ss}}(Z_{\beta=J^m})\), obtained by the \(V\text{AP}\) calculation.

Figure II shows the intrinsic density distribution of the \(0_1^+, 0_2^+,\) and \(1_2^-\) states. The \(0_1^+\) state shows the tetrahedral \(\alpha\) cluster structure, in which three \(\alpha\) form the
is sitting on the Z(vertical) axis (Fig. 1(a)). Its cluster development is not so remarkable as seen in the compact density distribution. The 1\(^{-}\) state also has a tetrahedral 4\(\alpha\) clustering with a compact density distribution similar to the 0\(^{+}\) state, but the orientation of the triangle 3\(\alpha\) part is somewhat tilted from the 0\(^{+}\). This tilting motion of the triangle 3\(\alpha\) produces the dipole excitation with K\(^{\pi}\) = 1\(^{-}\) in the 1\(^{-}\). This mode is similar to the vibration mode of the tetrahedral 4\(\alpha\) discussed by the algebraic 4\(\alpha\) cluster model [51, 52]. However, the 0\(^{+}\) and 1\(^{-}\) states obtained in the present calculation are not the equilateral tetrahedral states but the prolately deformed one with the 3\(\alpha\)+\(\alpha\) configuration and contain the \(\alpha\) breaking component.

The 0\(^{2}\) state has the developed 12\(^{\alpha}\)C+\(\alpha\) cluster structure, in which 4\(\alpha\) clusters are arranged in a planar-like configuration. Because of the remarkably developed 12\(^{\alpha}\)C+\(\alpha\) clustering, the 0\(^{2}\) state shows a largely deformed intrinsic density compared with the 0\(^{+}\). The developed 12\(^{\alpha}\)C+\(\alpha\) clustering constructs the K\(^{\pi}\) = 0\(^{+}\) band and the parity-partner K\(^{\pi}\) = 0\(^{-}\) band starting from the the bandhead 1\(^{-}\) state.

We should note that, even though the 0\(^{1}\), 0\(^{2}\), 1\(^{-}\), and 1\(^{2}\) show the formation of four \(\alpha\) clusters, the clusters are not necessarily the ideal \(\alpha\) clusters with the (0s)\(^{3}\) configuration but contain the \(\alpha\)-cluster breaking because of the spin-orbit interaction. We can evaluate the \(\alpha\)-cluster breaking component from the expectation value of the squared proton spin \((S_{p}^{2})\) because it measures the S = 1 mixing induced by the \(\alpha\) breaking. The values calculated with the VAP are \((S_{p}^{2}) = 0.07, 0.71, 0.35, \) and 0.79 for the 0\(^{1}\), 0\(^{2}\), 1\(^{-}\), and 1\(^{2}\), respectively, indicating the slight breaking in the 0\(^{1}\) and the significant \(\alpha\) breaking in the 0\(^{2}\), 1\(^{-}\), and 1\(^{2}\).

It should be also commented that, these VAP configurations couple with other configurations such as the 12\(^{\alpha}\)C-cluster rotation and 1p-1h excitations in the sAMD+GCM calculation, but they still give significant contributions and roughly describe main properties of the 0\(^{1}\), 0\(^{2}\), 1\(^{-}\), and 1\(^{2}\) states.

### C. Transition strengths

The calculated IS0 and IS1 transition strengths to the 0\(^{2}\), 3\(^{+}\), 4\(^{+}\), and 1\(^{2}\) states are listed in Tables III and IV. Here the strengths \(B(E\pi) = B(IS1)/4\) are compared with the experimental data. The observed \(B(E\pi)\) of the 0\(^{2}\), 0\(^{3}\), and 0\(^{5}\) state are reproduced well by the sAMD+GCM calculation. In the dipole excitations, the remarkably large \(B(IS1)\) is obtained for the 1\(^{-}\) with the energy weighted sum rule ratio of 5\%, where as the much weaker IS1 transition is obtained for the 1\(^{-}\) state in the 12\(^{\alpha}\)C+\(\alpha\) band. The relatively weak IS1 transition to the cluster state seems to contradict the naive expectation that the compressive operator could excite cluster states, but it is not true the case of the 1\(^{-}\) state. As mentioned previously, the 1\(^{-}\) state in the 12\(^{\alpha}\)C+\(\alpha\) band has the planar-like configuration and shows the different orientation of the

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
& VAP & VAP & sAMD & exp \\
& +GCM & +GCM & & \\
\hline
\(E_{\pi}(1^{+})\) (MeV) & 10.3 & 9.4 & 9.6 & 7.12 \\
\(E_{\pi}(1^{-})\) (MeV) & 17.0 & 12.1 & 14.4 & 9.59 \\
\(R(1^{+})\) (fm) & 2.76 & 2.87 & 2.80 & \\
\(R(1^{-})\) (fm) & 2.96 & 3.58 & 3.37 & \\
\(B(IS1;0^{+}_{1} \rightarrow 1^{+}_{1})\) & 124.5 & 165.5 & 169.8 & \\
\(P_{IS1}(1^{+})\) & 0.042 & 0.048 & 0.051 & 0.42\(^{a}\) \\
\(B(IS1;0^{+}_{1} \rightarrow 1^{-}_{1})\) & 7.9 & 2.9 & 10.2 & \\
\(P_{IS1}(1^{-}_{1})\) & 0.0044 & 0.0011 & 0.0045 & \\
\hline
\end{tabular}
\caption{Properties of the 1\(^{-}\) and 1\(^{+}\) states, excitation energies, rms radii, the IS1 strengths, and the EWSR ratio \(P_{IS1}\) of the energy-weighted IS1 strengths. The present result of the sAMD+GCM and those of the VAP and VAP+GCM calculations from Ref. [42] are shown compared with the experimental data [49]. The experimental data of the EWSR calculations from Ref. [42] are shown compared with the experimental data of EWSR.}
\end{table}

\begin{figure}[h]
\begin{subfigure}[b]{0.43\textwidth}
\centering
\includegraphics[width=\textwidth]{fig1a.png}
\caption{(a) 0\(^{+}_{1}\)}
\end{subfigure}
\begin{subfigure}[b]{0.43\textwidth}
\centering
\includegraphics[width=\textwidth]{fig1b.png}
\caption{(b) 0\(^{+}_{2}\) (K=0\(^{+}\))}
\end{subfigure}
\begin{subfigure}[b]{0.43\textwidth}
\centering
\includegraphics[width=\textwidth]{fig1c.png}
\caption{(c) 1\(^{-}_{1}\) (K=1\(^{-}\))}
\end{subfigure}
\caption{(color online) Density distributions in the intrinsic states obtained by the VAP for the 0\(^{+}_{1}\), 0\(^{+}_{2}\), and 1\(^{-}\) states. The densities integrated along the Y, X, and Z axes are plotted on the (left) X-Z, (middle) Y-Z, and (right) X-Y planes, respectively. Figures corresponds to those of Ref. [42], but are reconstructed from the wave functions.}
\end{figure}
triangle $^{12}\text{C}$-cluster from the initial $0_1^+$. Therefore, the $0_1^+$ to $1_2^+$ excitation involves not only the inter-cluster excitation but also the $^{12}\text{C}$-cluster rotation, which can not be directly excited by the IS1 operator.

The IS0 and IS1 strength functions up to up to $E = 60$ MeV are shown in Fig. 2. The energy weighted sum rule ratios calculated with the sAMD+GCM are plotted. In the IS0 strength function, a large fraction of the strengths are distributed in $E \leq 40$ MeV. The cluster states significantly contribute to the lower part of the strengths in $E \leq 20$ MeV, which are not clearly separated from the GMR strengths. On the other hand, in the IS1 strength function, the $1_1^{-}$ state contributes to the significant low-energy strength separated from the IS GDR peak around 40 MeV.

FIG. 2: The EWSR ratio of the IS0 and IS1 transition strengths calculated with the sAMD+GCM.

D. Form factors and transition densities

Figure 3 shows the calculated elastic and inelastic form factors of the IS0 and IS1 transitions from the ground state to the $0_{1,2,3}^+$ and $1_{1,2}^{-}$ states in comparison with experimental data observed by electron scattering [53].

The calculated form factors of the $0_{1,2,3}^+$ states are in good agreement with the experimental data in the low-momentum region. In the shape of the observed inelastic form factors, a difference can be seen between the $0_2^+$ and $0_3^+$ states. The form factor of the $0_2^+$ drops off at the smaller transfer momentum $q$ than the $0_3^+$ reflecting the broader radius dependence of the transition density of the $0_2^+$. This trend is qualitatively described in the present calculation and understood by the difference in the cluster structures between the $0_2^+$ and $0_3^+$ states: the dominant $^{12}\text{C}(0_3^+)+\alpha$ component in the $0_2^+$ state and the $^{12}\text{C}(2_3^+)+\alpha$ component in the $0_3^+$ state.

For the dipole transition to the $1_2^{-}$, the magnitude and shape of the experimental form factor are nicely reproduced by the present calculation. Compared to the $1_2^{-}$, the calculated IS1 transition to the $1_2^{-}$ is quite weak. At the maximum peak, the form factor of the $0_1^+ \rightarrow 1_2^{-}$ transition is about two orders less than that of the $0_1^+ \rightarrow 1_1^{-}$ transition. Moreover, the shape of the form factor is different between the $1_1^{-}$ and $1_2^{-}$ states because of the structure difference. The form factor of the $1_2^{-}$ in the $^{12}\text{C}+\alpha$ band shows two peak structure with a dip at $q \sim 2 \text{ fm}^{-2}$, which can not be seen in the form factor of the $1_1^{-}$ state with the compact tetrahedral $4\alpha$.

For further discussions of the IS0 and IS1 transitions, we show the transition densities for the $0^+$ and $1^-$ states with $B(IS0) > 10 \text{ fm}^2$ and $B(IS1) > 10 \text{ fm}^2$ in Fig. 4. The transition density of the $0^+$ states in Fig. 4(a) shows qualitatively similar behavior with one node around $r = 2.5 - 3.0$ fm, but one can see a quantitative difference between the $0_2^+$ and high-energy $0^+$ states. The transition density in the $0_2^+$ state is expanded outward and its node is located at the largest position $r \sim 3$ fm due to the developed $^{12}\text{C}(0_3^+)+\alpha$ cluster structure. Conversely, the transition density of higher states in $E > 22$ MeV is contracted inward. This trend can be understood by the character of small amplitude vibration in the high-energy monopole excitations. The transition density for other $0^+$ states in $15 < E < 22$ MeV shows the intermediate feature.

Compared with the monopole transitions, the IS1 transition density sensitively reflects different characters of dipole excitations. In particular, one can see clear differences in the transition density between the $1_1^{-}$, $1_2^{-}$, and high-energy GDR. The transition density in the $1_1^{-}$ with the compact $4\alpha$ structure shows the most contracted distribution with a node at $r < 3$ fm and the surface peak at $r \sim 4$ fm. On the other hand, in the $1_2^{-}$ state assigned to the $^{12}\text{C}+\alpha$ band, the transition density has two nodes and shows the broadly stretched distribution with the surface peak at $r \sim 5$ fm. In the high-energy GDR transition, which are contributed by the $1^-$ states in $40 < E < 44$ MeV, the transition density shows the intermediate feature with one node at $r \sim 3.5$ fm and the surface peak at $4 \lesssim r \lesssim 4.5$ fm. In $15 < E < 30$ MeV, most of the $1^-$ states have the GDR-like transition density but a few states show the $1_1^-$-like contracted behavior.

E. Vortical nature of dipole excitations

In order to clarify properties of the LED and GDR states, we calculate the transition strengths with the CD and TD operators. Note that the CD strength, which is
in principle equivalent to the IS1 strength, is sensitive to the compression dipole mode, whereas the TD strength can probe the nuclear vorticity in the dipole excitation.

The calculated CD and TD strength functions are shown in Fig. 5. In the CD transitions, we obtain the significant strength below 10 MeV for the $1^-_1$ state and the huge peak around $E = 40$ MeV for the IS GDR. In contract to the CD strength, there is no remarkable TD strength in the high-energy region for the IS GDR. From this result, it is concluded that the IS GDR do not have the vortical feature but is the normal compressive mode. Instead, the TD strength is concentrated on the $1^-_1$ state probing the vortical nature. The $1^-_1$ in the $^{12}$C+$\alpha$ band has the weak CD and TD transitions because this state is the inter-cluster excitation involving the $^{12}$C-cluster.
rotation and is weakly excited by the CD and TD operators.

The present result indicates quite different characters of the dipole excitations between the $1^{-}_1, 1^{-}_2$, and IS GDR states: the strong CD and TD transitions in the $1^{-}_1$, weak CD and TD transitions in the $1^{-}_2$, and strong CD but weak TD transitions in the IS GDR. In particular, one of the prominent features of the $1^{-}_1$ is the strong TD strength. In the analysis of the intrinsic wave functions, we find that the TD strength in the $0^+_1 \rightarrow 1^{-}_1$ is contributed by the dominant $K = 1$ component of the prolately deformed $3\alpha+\alpha$ structure of the $1^{-}_1$. On the other hand, the CD strength in the $0^+_1 \rightarrow 1^{-}_1$ is mainly contributed by the $K = 0$ component. In the $4\alpha$ structure, the $K = 1$ and $K = 0$ components have large overlap and mixes to each other because of the bosonic symmetry of $\alpha$ clusters. This is a unique feature of the dipole excitation in $^{16}\text{O}$, in which the $1^{-}_1$ state has the strong TD and CD strengths.

To illustrate the vortical and compressive natures of the $1^{-}_1$, we show in Fig. 6 the transition current density before the parity projection, and (e) (f) that after the projection. Here, the transition current density before the parity projections at the $Y = 0$ and $X = 0$ planes, (c) (d) that after the $K$ projection before the parity projection, and (e) (f) that after the $K$ and parity projections are shown. The nucleon number density of the $0^+_1$ and $1^{-}_1$ states are also shown by solid and dashed lines, respectively. Note that, the parity (axial) symmetry is broken in the intrinsic states before the parity projection ($K$ projection) but it is restored after the projection.

In the transition current density before the $K$ and parity projections, a vortex is created at the lower part by the tilting motion of the triangle $3\alpha$ configuration as seen in Fig. 6(a) and (b). After the $K = 1$ projection, where the nuclear current is averaged around the $Z$-axis, a $K = 1$ vortex appears clearly at the lower part of Fig. 6(c). Then, after the parity projection, the vortical current is duplicated and two vortexes appear in the lower and upper parts. The $K = 1$ vortexes aligned along the $\gamma$-ray is the feature of the $K = 1$ VD mode in the prolately deformed system. This mode differs from the torus-shape vortex, which has been originally proposed in the $K = 0$ dipole excitation (obviously, the torus current is allowed only in the $K = 0$ dipole excitation because of the mathematical condition.) The geometrical shape of the current in the $K = 1$ VD mode is described in detail in our previous paper [54].

Let us turn to the nuclear current in the $K = 0$ component shown in Fig. 6(d) and (f) before and after the parity projection, respectively. The $0^+_1 \rightarrow 1^{-}_1$ excitation also contains the relative motion between the last $\alpha$ cluster and the $3\alpha$. In the $K = 0$ component, this corresponds to the $L = 1$ excitation of the $3\alpha-\alpha$ relative distance. The relative oscillation of the last $\alpha$ cluster against the $3\alpha$ induces the compressive nuclear current as seen in Fig. 6(d) and (f) and contributes to the significant CD strength in the $0^+_1 \rightarrow 1^{-}_1$ transition.

Strictly speaking, it is not be able to uniquely define the intrinsic frame for physical states with eigenvalues of angular momentum, but in the present case that the system has the prolative deformation because of the tetrahedral $3\alpha+\alpha$ configuration, the discussion in the “intrinsic” frame can be useful to get the intuitive understanding.

FIG. 5: (color online) The energy weighted strength functions of the CD and TD transitions calculated with the sAMD+GCM. The scaled strengths $B(D)$ of discrete states are smeared by Gaussian with the range $\gamma = 1/\sqrt{\pi}$ MeV.

IV. SUMMARY AND OUTLOOK

The IS monopole and dipole excitations in $^{16}\text{O}$ were investigated with the sAMD+GCM. The significant IS0 and IS1 transition strengths were obtained in the low-energy region in addition to the GRs. The $1^{-}_1$ state contributes to the significant low-energy strength of the IS1 transition with 5% of the energy-weighted sum rule, which describes well the experimental data observed by $\gamma$ inelastic scattering. The calculated form factors of the inelastic transitions to the $0^+_2$, $0^+_3$, and $1^{-}_1$ states reproduce the experimental electron scattering form factors. The transition densities were also analyzed.

The different characters of the dipole excitations were found in the $1^{-}_1, 1^{-}_2$, and IS GDR: the strong CD and TD transitions in the $1^{-}_1$, the weak CD and TD transitions in the $1^{-}_2$, and the strong CD but weak TD transitions in the IS GDR. Cluster and vortical aspects of the low-energy dipole states were investigated. In conclusion, we regard the $1^{-}_1$ as the vortical vibration mode with the tetrahedral $4\alpha$ structure, the $1^{-}_2$ as the $^{12}\text{C}+\alpha$ cluster mode, and the IS GDR as the collective vibration of the compressive dipole mode.
Appendix A: Transition densities

The density and current density operators for the nuclear matter are defined as
\[ \rho(r) = \sum_k \delta(r - r_k), \quad \text{and} \quad j(r) = -\frac{i\hbar}{2m} \sum_k \nabla_k \delta(r - r_k) + \delta(r - r_k) \nabla_k. \]

Here, \( j(r) \) includes only the convection term of the nuclear current but not the spin term of magnetization. The transition density and current density for the \( |0^+\rangle \rightarrow |f\rangle \) transition are given as
\[ \rho^{(tr)}_{0\rightarrow f}(r) = \langle f | \rho(r) | 0 \rangle, \]
\[ \delta j^{(tr)}(r) = \langle f | j(r) | 0 \rangle. \]

The \( \lambda \)th transition density is obtained from the multipole decomposition of the transition density,
\[ \rho^{(tr)}_{0\rightarrow f}(r) = \frac{1}{\sqrt{2J_f + 1}} \sum_{\lambda} \rho^{(tr)}_{\lambda,0\rightarrow f}(r) \]
\[ \times \sum_{\mu} Y_{\lambda,\mu}^*(\hat{r}) (J_i M_i \lambda \mu | J_f M_f), \]
where \( J_i \) and \( M_i \) \((J_f \text{ and } M_f)\) are the spin quantum numbers of the initial \(|0\rangle \) (final \(|f\rangle \) state. The \( \lambda \)th multipole component of the so-called longitudinal form factor is related to the Fourier-Bessel transform of the transition charge density \( \rho^{ch}_{\lambda,0\rightarrow f}(r) \) by
\[ F(q) = \frac{\sqrt{4\pi}}{Z} \frac{1}{\sqrt{2J_i + 1}} \int dr r^2 j_{\lambda}(qr) \rho^{ch}_{\lambda,0\rightarrow f}(r), \]
where \( \rho^{ch}_{\lambda,0\rightarrow f}(r) \) is calculated by taking into account the proton charge radius and assuming the mirror symmetry.

Appendix B: IS monopole and dipole operators and transition strengths

The standard compressive-type ISA operators of the IS monopole and dipole excitations are defined as
\[ M_{IS0} \equiv \int dr \rho(r) r^2, \]
\[ M_{IS1}(\mu) \equiv \int dr r^2 \rho(r) Y_{1\mu}(\hat{r}). \]

The IS0 and IS1 transition strengths for \( |0^+_\lambda\rangle \rightarrow |J^+_f\rangle \) are given by the reduced matrix elements as
\[ B(IS\lambda) = \frac{1}{2J_i + 1} |\langle J^+_f | M_{IS\lambda} | 0^+_\lambda \rangle|^2, \]
where the angular momentum of the initial state is \( J_i \) and that of the final state is \( J^+_f \). For \( \lambda = 0 \) and

Acknowledgments

The authors would like to thank the Dr. Nesterenko and Dr. Chiba for fruitful discussions. The computational calculations of this work were performed by using the supercomputer in the Yukawa Institute for theoretical physics, Kyoto University. This work was supported by JSPS KAKENHI Grant Nos. 18K03617 and 18J20926.
discussed in Ref. [14]. They are defined as sure of the nuclear vorticity in the dipole excitations as the latter (TD) has been proved to be as a good mea-
and sensitive to the compressive dipole excitations, and former (CD) corresponds to the standard IS1 operator and TD operators are used as done in Refs. [43, 46]. The energies, for which the empirical values of
\[ \langle J_k^\pm | M_{IS\lambda} | 0^\pm \rangle = \sqrt{4\pi} \int dr r^2 \rho^{(tr)}_{\lambda;0 \to f}(r) \] (B4)
for the IS0 transition and
\[ \langle J_k^\pm | M_{IS\lambda} | 0^\pm \rangle = \int dr r^2 \rho^{(tr)}_{\lambda;0 \to f}(r) \] (B5)
for the IS1 transition.
The energy-weighted sum rule of the IS0 operator is
\[ \sum_k (E_k - E_0) B(IS0; 0^+_1 \to 0^+_k) = \frac{2\hbar^2 A}{m} \langle r^2 \rangle \] (B6)
with the mean square radius \( \langle r^2 \rangle = \langle 0^+_1 | \sum_i r_i^2 | 0^+_1 \rangle / A \) of the ground state. For the IS1 operator, we use the following energy-weighted sum rule from Ref. [9],
\[ \sum_k (E_k - E_0) B(IS1; 0^+_1 \to 1^-_k) = \frac{3\hbar^2 A}{32m\pi} \left( 11\langle r^4 \rangle - \frac{25}{3} \langle r^2 \rangle^2 - 10\epsilon \langle r^2 \rangle \right) , \] (B7)
where \( \langle r^4 \rangle = \langle 0^+_1 | \sum_i r_i^4 | 0^+_1 \rangle / A \) and \( \epsilon = (4/E_2 + 5/E_0)\hbar^2 / 3mA \). Here \( E_2 \) and \( E_0 \) are the IS GQR and GMR energies, for which the empirical values of \( E_2 = 63A^{-1/3} \) MeV and \( E_0 = 80A^{-1/3} \) MeV are used, respectively.

Appendix C: CD and TD strengths

In the analysis of isoscalar dipole excitations, the CD and TD operators are used as done in Refs. [14, 49]. The former (CD) corresponds to the standard IS1 operator and sensitive to the compressive dipole excitations, and the latter (TD) has been proved to be as a good measure of the nuclear vorticity in the dipole excitations as discussed in Ref. [14]. They are defined as
\[ M_{CD}(\mu) = \frac{-i}{2\sqrt{3}c} \int dr \delta j(r) \cdot \left[ \frac{2\sqrt{2}}{5} r^2 Y_{12\mu}(\hat{r}) - r^2 Y_{10\mu}(\hat{r}) \right] , \] (C1)
\[ M_{TD}(\mu) = \frac{-i}{2\sqrt{3}c} \int dr \delta j(r) \cdot \left[ \frac{\sqrt{2}}{5} r^2 Y_{12\mu}(\hat{r}) + r^2 Y_{10\mu}(\hat{r}) \right] , \] (C2)
where \( Y_{\lambda L \mu} \) is the vector spherical harmonics.

The matrix elements of these IS dipole operators for the \( |0^+_1 \rangle \to |1^-_k \rangle \) transitions are given as
\[ \langle 1^-_k | M_{CD}(\mu) | 0^+_1 \rangle = \frac{-i}{2\sqrt{3}c} \int dr \delta j(r) \cdot \left[ \frac{2\sqrt{2}}{5} r^2 Y_{12\mu}(\hat{r}) - r^2 Y_{10\mu}(\hat{r}) \right] , \] (C3)
\[ \langle 1^-_k | M_{TD}(\mu) | 0^+_1 \rangle = \frac{-i}{2\sqrt{3}c} \int dr \delta j(r) \cdot \left[ \frac{\sqrt{2}}{5} r^2 Y_{12\mu}(\hat{r}) + r^2 Y_{10\mu}(\hat{r}) \right] . \] (C4)

Using the continuity equation, the CD matrix element is related to the matrix element of the standard IS1 operator \( M_{IS1} \) as
\[ \langle 1^-_k | M_{CD}(\mu) | 0^+_1 \rangle = -\frac{E}{10h^2c} \langle 1^-_k | M_{IS1}(\mu) | 0^+_1 \rangle . \] (C5)

The CD and TD strengths, which are scaled with the factor \( \left( \frac{10h^2c}{E} \right)^2 \), are defined as
\[ \hat{B}(CD,TD; 0^+_1 \to 1^-_k) \equiv \left( \frac{10h^2c}{E} \right)^2 |\langle 1^-_k || M_{CD,TD} || 0^+_1 \rangle|^2 . \] (C6)

[1] M.N. Harakeh, A. van der Woude, Giant Resonances, Oxford University Press, 2001.
[2] N. Paar, D. Vretenar, E. Khan and G. Colo, Rept. Prog. Phys. 70, 691 (2007).
[3] T. Aumann and T. Nakamura, Phys. Scr. T152, 014012 (2013).
[4] D. Savran, T. Aumann and A. Zilges, Prog. Part. Nucl. Phys. 70, 210 (2013).
[5] A. Bracco, F. C. L. Crespi and E. G. Lanza, Eur. Phys. J. A 51, 99 (2015).
[6] M. N. Harakeh and A. E. L. Dieperink, Phys. Rev. C 23, 2329 (1981).
[7] P. Decowski, H. P. Morsch and W. Benenson, Phys. Lett. 101B, 147 (1981).
[8] T. D. Poelhekken, S. K. B. Hesmondhalgh, H. J. Hofmann, A. van der Woude and M. N. Harakeh, Phys. Lett. B 278, 423 (1992).
[9] S. F. Semenko, Sov. J. Nucl. Phys. 34, 356 (1981).
[10] D. G. Ravenhall and J. Wambach, Nucl. Phys. A 475, 468 (1987).
[11] D. Vretenar, N. Paar, P. Ring, and T. Nikšić Phys. Rev. C 65, 021301 (2002).
[12] N. Ryezayeva et al., Phys. Rev. Lett. 89, 272502 (2002).
[13] P. Papakonstantinou, V. Y. Ponomarev, R. Roth and J. Wambach, Eur. Phys. J. A 47, 14 (2011).
[14] J. Kvasil, V. O. Nesterenko, W. Kleinin, P.-G. Reinhard and P. Vesely, Phys. Rev. C 84, 034303 (2011).
[15] A. Repko, P.-G. Reinhard, V. O. Nesterenko and J. Kvasil, Phys. Rev. C 87, 024305 (2013).
[16] J. Kvasil, V. O. Nesterenko, W. Kleinig and P.-G. Reinhard, Phys. Scripta 89, 054023 (2014).
[17] V. O. Nesterenko, J. Kvasil, A. Repko, W. Kleinig and P.-G. Reinhard, Phys. Atom. Nucl. 79, 842 (2016).
[18] V. O. Nesterenko, A. Repko, J. Kvasil and P. G. Reinhard, Phys. Rev. Lett. 120, no. 18, 182501 (2018).
[19] V. M. Dubovik and A. A. Cheshkov, Sov. J. Part. Nucl. 5, 318 (1975).
[20] T. Yamada, Y. Funaki, T. Myo, H. Horiuchi, K. Ikeda, G. Röpke, P. Schuck and A. Tohsaki, Phys. Rev. C 85, 034315 (2012).
[21] Y. Chiba, M. Kimura and Y. Taniguchi, Phys. Rev. C 93, 034319 (2016).
[22] J. A. Wheeler, Phys. Rev. 52, 1083 (1937); ibid. 52, 1107 (1937).
[23] D. M. Dennison, Phys. Rev. 96, 378 (1954).
[24] D. M. Brink, H. Friedrich, A. Weiguny and C. W. Wong, Phys. Lett. B33, 143 (1970).
[25] Y. Suzuki, Prog. Theor. Phys. 55, 1751 (1976).
[26] Y. Suzuki, Prog. Theor. Phys. 56, 111 (1976).
[27] Y. Fujiwara et al., Prog. Theor. Phys. Suppl. 68, 29 (1980).
[28] M. Libert-Heinemann, D. Baye and P.-H. Heenen, Nucl. Phys. A 339, 429 (1980).
[29] W. Bauhoff, H. Schultheis, R. Schultheis Phys. Rev. C 29, 1046 (1984).
[30] P. Descouvemont, Nucl. Phys. A 470, 309 (1987).
[31] P. Descouvemont, Phys. Rev. C 44, 306 (1991).
[32] P. Descouvemont, Phys. Rev. C 47, 210 (1993).
[33] K. Fukatsu and K. Katô, Prog. Theor. Phys. 87, 151 (1992).
[34] Y. Funaki, T. Yamada, H. Horiuchi, G. Röpke, P. Schuck and A. Tohsaki, Phys. Rev. Lett. 101, 082502 (2008).
[35] Y. Funaki, T. Yamada, A. Tohsaki, H. Horiuchi, G. Röpke and P. Schuck, Phys. Rev. C 82, 024312 (2010).
[36] Y. Kanada-En’yo, Phys. Rev. C 89, 024302 (2014).
[37] W. Horiuchi and Y. Suzuki, Phys. Rev. C 89, 011304 (2014).
[38] Y. Kanada-En’yo, H. Horiuchi and A. Ono, Phys. Rev. C 52, 628 (1995).
[39] Y. Kanada-En’yo and H. Horiuchi, Phys. Rev. C 52, 647 (1995).
[40] Y. Kanada-En’yo and H. Horiuchi, Prog. Theor. Phys. Suppl. 142, 205 (2001).
[41] Y. Kanada-En’yo, M. Kimura and A. Ono, PTEP 2012, 01A202 (2012).
[42] Y. Kanada-En’yo, Phys. Rev. C 96, no. 3, 034306 (2017).
[43] Y. Kanada-En’yo, Phys. Rev. C 93, 024322 (2016).
[44] Y. Kanada-En’yo, Phys. Rev. C 93, 054307 (2016).
[45] M. Kimura, Phys. Rev. C 95, no. 3, 034331 (2017).
[46] Y. Kanada-En’yo, Y. Shikata and H. Morita, Phys. Rev. C 97, no. 1, 014303 (2018).
[47] Y. Kanada-En’yo, Phys. Rev. Lett. 81, 5291 (1998).
[48] Y. Kanada-En’yo and Y. Shikata, Phys. Rev. C 95, no. 6, 064319 (2017).
[49] D. R. Tilley, H. R. Weller and C. M. Cheves, Nucl. Phys. A 564, 1 (1993).
[50] I. Angeli and K. P. Marinova, At. Data Nucl. Data Tables 99, 69 (2013).
[51] R. Bijker and F. Iachello, Phys. Rev. Lett. 112, no. 15, 152501 (2014).
[52] R. Bijker and F. Iachello, Nucl. Phys. A 957, 154 (2017).
[53] T. N. Buti et al., Phys. Rev. C 33, 755 (1986).
[54] Y. Shikata, Y. Kanada-En’yo and H. Morita, arXiv:1902.10962 [nucl-th].