Theory of magnetic domains in uniaxial thin films

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Abstract

For uniaxial thin films, properties of magnetic domains are usually described within the Kittel model, which assumes that domain walls are much thinner than the domains. In this work we present a simple model that includes a proper description of the magnetostatic energy of domains and domain walls and also takes into account the interaction between both surfaces of the film. Our model describes the behaviour of domain and wall widths as a function of film thickness, and is especially well suited for the strong-stripe phase. We prove the existence of a critical value of magneto-crystalline anisotropy above which stripe domains exist for any film thickness and justify our model by comparison with exact results. The model is in good agreement with experimental data for hcp cobalt.

(Some figures may appear in colour only in the online journal)

1. Introduction

Thin magnetic films with stripe domains and perpendicular magneto-crystalline anisotropy present a high scientific interest. They are model systems to understand the domain structure of ferromagnetic, as well as ferroelectric, thin films. Such materials are used for the fabrication of memories and spin injection devices. Considering a thin film with its bulk easy axis perpendicular to the surface, the magnetization direction in the domains can depend on the film thickness. Thick films behave as bulk material but for thin films, the competition between exchange, magnetostatic and magneto-crystalline energy can tip up the magnetization into the plane. A very well-known theory of magnetic domain structure has been developed by Kittel [1]. In this model, the film magnetization is described by a square profile. Domain wall energy is allowed for the energy balance, but not for the magnetization profile, which determines the magnetostatic energy. Such an approximation is only valid when the domain width is much larger than the width of domain walls. Therefore, this model is bound to fail for very thin films, where the domain width vanishes. Furthermore, the magnetostatic interaction between the top and bottom surfaces is disregarded, which leads to wrong results unless the domain width is much less than the film thickness. The latter does not hold for very thin strongly anisotropic films.

Within the Kittel model, the width of stripe domains is proportional to the square root of the film thickness. The model also predicts the existence of a critical film thickness below which the magnetization direction flips from the out-of-plane to in-plane direction. Later on, it was shown that for materials with strong magneto-crystalline anisotropy \( K \) (in comparison with the square of the saturation magnetization \( M_0 \)) the magnetostatic interaction energy between top and bottom interface cannot be neglected. The ratio \( 2K/(\mu_0 M_0^2) \) (in SI units) is also known as the quality factor \( Q \) and the magnetization direction remains perpendicular to the surface for all values of film thickness for \( Q > 1 \). Numerous theoretical and experimental works have been devoted to such materials with strong perpendicular magnetic anisotropy [2–4]. It has been shown that in such materials the domain width has a minimum as a function of film thickness, increasing towards smaller or larger thicknesses. Materials with strong perpendicular anisotropy are also of technological importance in the field of magnetic multilayers (see [5] and references therein).

Just above the critical film thickness, the magnetization still lies predominantly in the plane of the film, deviations from the homogeneous in-plane orientation being very small. This permits an exact mathematical solution of the problem of stripe domain nucleation at the critical point [6]. Near the critical point the out-of-plane component of the magnetization remains much smaller than the saturation value \( M_0 \) and there is a strong variation of the magnetization direction across the film thickness. The corresponding phase is usually called the weak-stripe phase. As the film thickness increases, the...
magnetization direction in the domains tends more and more
towards the bulk easy axis, which is perpendicular to the film.
Gradually, the weak-stripe phase evolves towards the strong-
stripe phase, where the magnetization is predominantly parallel
(or antiparallel) to the bulk easy axis.

For a detailed analysis of the weak-stripe phase a
micromagnetic numerical analysis is indispensable (see [7–11]
and many other works). However, simple analytical models
are helpful for getting a quick, albeit crude understanding of
the stripe phases. These models restrict the magnetization
direction to the vertical plane parallel to the stripes (i.e. they
consider domain walls of Bloch type) but improve the Kittel
model. One can distinguish between models with a linear
domain wall profile [12] or others which use the Jacobi sine
function to parametrize the magnetization profile [13,14].

Also a sinewave magnetization profile was proposed by
Saito [15] to treat the weak-stripe phase in an approximate
manner. In the following we present a thorough analytical and
numerical analysis of a simple but quite complete model for
the strong-stripe phase. We adopt a sinewave magnetization
profile in the wall and take into account the magnetostatic
interaction between the top and bottom surfaces of the film.
Our model applies to hexagonal Co, for which a critical
thickness between 25 nm [16, 17] and 40 nm [18] has been
reported, as well as to materials with high magneto-crystalline
anisotropy, such as FePd(001) or garnet films [4,18,19].

Another possible application concerns ferromagnetic thin films of
Mn5Ge3 that were recently synthesized [20, 21]. Our model
unifies previous results for materials with strong anisotropy
(\(O \gtrsim 1\)) [2–4] and those with medium anisotropy (\(\frac{1}{4} \lesssim Q < 1\))
[12], containing them as special cases. The explicit treatment
of the domain wall is an important improvement with respect
to the Kittel model since it allows the determination of the
in-plane remanent magnetization, which is important for the
analysis of experimental data. We give numerical results for
the dependence of domain and wall widths on the film thickness
and calculate the critical thickness at which the magnetization
switches from the out-of-plane to in-plane direction if \(Q < 1\).
We show that the critical thickness tends to zero as \(Q \rightarrow 1\).
This behaviour is identical in our model and in the exact
solution [6]. In the case of strong anisotropy (\(Q \gtrsim 1\)) stripe
magnetic domains exist at any film thickness.

Our theory considers a non-magnetostrictive film: the
magneto-elastic energy is equal to zero. But our theory
contains the case of ultra-thin magnetic films as one possible
limit. If the thickness is only a few monolayers, the surface
anisotropy has to be taken into account. In that case, Yafet
and Gyorgy [22] predicted a critical perpendicular anisotropy at
which the magnetic domains appear. Their domain wall profile
is identical to ours but they analyse the magnetostatic energy
only for a thickness that is much smaller than the domain width.
Our model is not limited by that restriction.

We concentrate on the strong-stripe phase since the details
of the weak stripe phase are hard to capture in a simple model.
In materials with high anisotropy the weak-stripe phase does
not appear at all and it will be shown below that even in
materials with medium anisotropy (\(\frac{1}{4} \lesssim Q < 1\)) it is of
rather limited importance. In the latter case the weak-stripe

phase occurs only in a narrow interval of thicknesses around
the critical point. This region of the weak-stripe phase is not
correctly described in our model. On the other hand, even in the
case of weak anisotropy (\(Q \ll 1\)), our model provides a correct
qualitative description of the strong-stripe phase, although it
neglects such important phenomena as closure domains.

Our paper is organized as follows. After a short summary
of known results (Kittel’s theory, sawtooth magnetization
model, exact results for stripe phase nucleation) we present
our model in section 3. In section 4 we show its analytical
and numerical solution. The usefulness of our theory is
demonstrated in section 5 by way of theoretical analysis of
published experimental data for the well-known hcp cobalt
system. In section 6 we compare our model with other
approaches and expose its strengths and limitations.

2. Known results

2.1. Kittel’s model

This model was developed for ferromagnetic films with
uniaxial anisotropy perpendicular to the film. It determines the
width of magnetic stripe domains \(d\) as a function of the film
thickness \(h\), and the critical thickness where the magnetization
direction flips from out-of-plane to in-plane (see figure 1). The
total energy density (per unit volume) is given by

\[
E_{\text{total}} = 0.136\mu_0 M_0^2 \frac{d}{h} + \frac{\sigma}{\bar{d}},
\]

where \(M_0\) denotes the saturation magnetization and \(\sigma\) the
surface wall energy (SI units). The latter is obtained in a
separate variational calculation for an isolated Bloch wall [23],
\(\sigma = 4\sqrt{AK}\), where \(A\) is the exchange stiffness constant.
The same calculation yields the wall width, \(\delta = \pi \sqrt{A/K}\).
It is convenient to express all spatial dimensions in the units
of \(\sqrt{A/K}\):

\[
\tilde{h} = \frac{h}{\sqrt{A/K}}, \quad \tilde{d} = \frac{d}{\sqrt{A/K}}, \quad \tilde{\delta} = \frac{\delta}{\sqrt{A/K}}.
\]

Thus, Kittel’s reduced wall width is a constant,

\[
\tilde{\delta} = \pi.
\]
The only essential material parameter in the model is the dimensionless quality ratio, $Q = 2K/(\mu_0 M_0^2)$. The first term of equation (1) refers to the magnetostatic energy of a rectangular domain profile, neglecting the influences of finite wall width and film thickness. Therefore, strictly speaking, the Kittel model is only applicable when the following strong inequalities hold:

$$\delta \ll d \ll h,$$

or alternatively, $\delta \ll d \ll \tilde{h}$. The right-hand condition ensures the negligibility of the magnetostatic interaction between the top and the bottom surfaces of the film. By minimizing the energy with respect to $d$, one obtains the equilibrium domain width:

$$\tilde{d} = 3.84\sqrt{Q\tilde{h}},$$

and the minimum energy density:

$$E_{\text{total}}^{\text{min}} = 2\sqrt{0.136\mu_0 M_0^2\sigma}/h.$$ (6)

As one can see, the minimum energy density $E_{\text{total}}^{\text{min}}$ tends to infinity as $h \to 0$. Therefore, the multi-domain structure cannot be stable in that limit within the Kittel model. In fact, for a uniform in-plane magnetization (perpendicular to the bulk easy axis), the magnetostatic and the exchange energies are equal to zero, and the anisotropy energy is maximal and equal to $K$. The critical point is obtained by equating equation (6) to $K$. Therefore, we can express the reduced critical thickness as follows:

$$\tilde{h}_c'' = 4.35Q^{-1}.$$ (7)

Within the Kittel model, the domain width is identical to the half-period $\tilde{d}$ (see figure 1). Its value at the critical point is obtained by setting equation (7) into (5), which results in

$$\tilde{d}'' = 8.$$ (8)

These equations permit to obtain a qualitative evaluation of the critical quantities. But they fail in the neighbourhood of the critical thickness, where domain and wall widths become close, see equations (3) and (8). Furthermore, by equations (7) and (8) in the critical region the ratio $h/\tilde{d}$ equals 0.544/4 = 0.272$\mu_0 M_0^2/K$, i.e. also the second precondition of the Kittel model (4) is violated in the case of large anisotropy or small magnetization, $Q \gtrsim 1$. As a result, Kittel’s model makes a wrong prediction for large $Q$: according to equation (7), $\tilde{h}_c''$ decreases asymptotically, remaining nonzero at any finite $Q$. In actual fact, the critical thickness should vanish at $Q = 1$, as shown by rigorous calculations [6].

For that reason we decided to develop a model that would not break down in the strong-anisotropy case, by allowing for the electrostatic interaction between the top and bottom surfaces of the film and taking into account the wall width explicitly. Within that improved model we have obtained a good description of domain and wall width variation as a function of the film thickness (see section 3).

2.2. Sawtooth magnetization model

The main goal of Kittel’s model is the description of the strong-stripe phase. Yet there is no direct transition between strong stripes and planar magnetization. Rather, it proceeds via an intermediate weak-stripe configuration. The latter can be described by sawtooth magnetization model [15]. In this model the canting angle of the magnetization out of the homogeneous in-plane direction, $\Theta(x)$, is assumed to be a sawtooth curve with a maximum value of $\Theta_0$ and a minimum value of $-\Theta_0$, $\Theta_0 < \pi/2$. Between the extrema the variation is linear and the half period is denoted by $d$. The sawtooth magnetization model is an approximate one, since it neglects the variation of the magnetization direction across the film thickness. The magnetostatic interaction between the top and bottom surfaces is not taken into account either. From the equation for the total energy below one can obtain the expression for the critical thickness where the magnetization starts to flip out-of-plane by a small deflection angle. The first part of the total energy corresponds to the magnetostatic energy; $C_0$ is the leading coefficient of the Fourier series obtained from periodic profile of the deflection angle. The maximum angle $\Theta_0$ is a variational parameter, $0 < \Theta_0 < \pi/2$.

$$E_{\text{total}} = \mu_0 M_0^2 C_0 \tilde{d} + 4\Theta_0^2 h + K \left[ \frac{1}{2} \sin^2 \Theta_0 \right] \frac{\Theta_0}{\Theta_0^2}$$

with $C_0 = 2\Theta_0^2 \pi^2 / 4 - \Theta_0^2$. (9)

The critical thickness is expressed as

$$\tilde{h}_c''' = 27(2/\pi)^5 Q^{-1} = 2.82Q^{-1}.$$ (10)

The expression above has the same form as Kittel’s critical thickness $\tilde{h}_c''$, equation (7), but with a smaller pre-factor. This opens up a possibility to interpret $\tilde{h}_c''$ and $\tilde{h}_c'''$ as the upper and lower bounds of an interval where the weak-stripe phase exists. The latter is a transitional state between the homogeneously magnetized in-plane configuration, stable below $\tilde{h}_c''$, and Kittel’s strong-stripe domain structure, taking place above $\tilde{h}_c''$. One observes the ‘wrong’ behaviour of $\tilde{h}_c'''(Q)$ at $Q > 1$, similar to that of $\tilde{h}_c''(Q)$. Equation (10) also misbehaves at $Q \ll 1$. Rigorous calculations show (see section 2.3) that the true lower bound of the weak-stripe phase does not diverge as $Q \to 0$.

2.3. Exact description of stripe domain nucleation

The exact solution of the nucleation problem was given in 1961 by Muller [6]. A modern presentation, summarized here, can be found in the book of Hubert and Schäfer [7]. The theory was formulated for a thin magnetic film (thickness $h$, magnetization $M_0$) with uniaxial anisotropy perpendicular to the film (anisotropy constant $K$). The energy expression contains the magnetostatic energy, the exchange (stiffness $A$) and anisotropy contributions; the theory imposes no restrictions on the magnetization direction. Below the critical thickness $h_c$ the magnetostatic energy forces the magnetization into the plane of the film. At $h = h_c$ there...
is an instability and the weak-stripe phase emerges. In the neighbourhood of the critical point, deviations from a homogeneous in-plane orientation of magnetization are small, which allows one to linearize the system of micromagnetic equations and find the exact analytical form of the instability mode. The reduced critical thickness $\tilde{h}_c$ is a universal function of the quality ratio $Q$, see figure 4 (solid line). (The quantity plotted in the original drawing, figure 3.109a of [7], is a factor $2\pi$ smaller.) At the same time, the critical half-period $d_c$ can be found, as well as the distribution of magnetization directions in the critical mode. In the weak-anisotropy limit, $Q \to 0$, both $\tilde{h}_c$ and $d_c$ tend to the same finite value,

$$\tilde{h}_c \to d_c \to 2\pi.$$  

As $Q$ increases, the critical thickness $\tilde{h}_c$ decreases and vanishes at $Q = 1$, whereas $d_c$ diverges at that point. The exact results will be used later on to judge the validity of the sinewave wall model (SWM).

### 3. Sinewave wall model

We propose a model stripe domain structure as shown in figures 2 and 3. Like in Kittel’s model, the half-period is denoted by $d$, but it now includes a domain wall of width $\delta$ (see figure 2). The inner domains, with constant magnetization equal to $\pm M_0$, have a width of $d - \delta$. The walls separating different domains are assumed to be of Bloch type, with linear variation of the angle $\Theta(x)$ between the magnetization direction and the $y$-axis. This results in a magnetization profile of sinewave form, which is a good approximation of the profile obtained by the variational method. The schematic representation in figure 2 shows that we consider parallel stripe domains infinite in the $y$-direction. In the $x$-direction, the periodic magnetization profile is as shown in figure 3. The $z$ dimension is restricted to the film thickness $h$.

To calculate the magnetostatic energy, we use an analogy with the electrostatic field calculation for alternating, positively and negatively, charged stripes (see Landau–Lifschitz [24]). We consider the realistic magnetization profile of figure 3 (as opposed to a simple rectangular meander) as well as the magnetostatic interaction between the top and bottom surfaces of the film.

The magnetostatic energy density (per volume) can be written as

$$E_{mag} = \frac{\mu_0 d}{\pi h} \sum_{k=1}^{\infty} |C_k|^2 k \left[ 1 - \exp \left( -\frac{\pi k \tilde{h}}{d} \right) \right],$$  

with

$$C_k = \frac{2M_0}{k\pi} \frac{1}{\left[ 1 - k^2 (\delta/d)^2 \right]} \cos \left( \frac{k\pi \delta}{2d} \right).$$

where $k$ is an odd number, $C_k$ is the Fourier coefficient of $M_z(x)$ (see figure 3). The domain structure is determined by the interplay between magnetostatic, exchange and anisotropy energies. The latter two contributions are as follows:

$$E_{exch} = \frac{1}{2d} \int_0^{2d} A \left( \frac{d\Theta}{dx} \right)^2 dx = \frac{\pi^2}{d\delta} A$$

$$E_{aniso} = \frac{1}{2d} \int_0^{2d} K \cos^2[\Theta] dx = \frac{\delta}{2d} K.$$  

The half-period $d$ and the wall width $\delta$ are then calculated by minimizing the total energy,

$$E_{total} = E_{mag} + E_{exch} + E_{aniso}.$$  

Since it is not possible to solve analytically the two variational equations $\partial_d E_{total} = 0, \partial_\delta E_{total} = 0$ in the general case, we use a Nelder–Mead numerical method. The results are presented in the next section.

### 4. Analytical and numerical results

#### 4.1. Large film thickness

Before presenting the numerical solution in the general case let us discuss some specific situations. For sufficiently thick films the strong inequalities (4) are fulfilled and the expression for the Fourier coefficients, equation (13), becomes

$$C_k = \frac{2M_0}{k\pi}.$$  

Figure 2. Sketch of periodic stripe domains in a magnetic thin film with domain walls of finite thickness. In the domain walls, the magnetization points towards the same direction $-\gamma$ which explains the in-plane remanent magnetization (see section 4.4.)

Figure 3. One full period of the domain structure of figure 2. $M_z(x)$ (solid line) denotes the $z$ component of the film magnetization, and $\Theta(x)$ is the angle between the magnetization direction and the film surface (dashed line). The domain wall width is denoted by $\delta$. 

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Then Kittel’s form is recovered for the magnetostatic energy density:

\[ E_{\text{mag}} = \frac{4\mu_0 M_0^2 d}{\pi^3 h} \sum_{k=1}^{\infty} k^{-3} = 0.136\mu_0 M_0^2 d h. \] (18)

The wall energy associated with the above magnetostatic expression is a minimum for the following reduced wall width:

\[ \delta_\infty = \pi \sqrt{\frac{\tilde{d}}{2}}. \] (19)

The corresponding surface wall energy, \( \sigma = \pi \sqrt{\frac{\tilde{d}}{2}} \Delta K \), is slightly (11%) larger than the variational result, \( 4\sqrt{\Delta K} \), because of the imposed sinewave wall profile of \( M_z(x) \). In the thick-film limit the dependence of the domain width on film thickness can be expressed analytically,

\[ \tilde{d} = 4.05\sqrt{Q h}, \] (20)

and is very similar to the result of the Kittel model, equation (5).

### 4.2. Critical thickness and critical anisotropy

As the film thickness is reduced, the domain width \( d \) decreases and the wall width \( \delta \) increases. Just above the critical thickness, the magnetization profile is purely sinewave (\( \delta \to d \)), forming an unstable spiral magnetic configuration. In this limit equation (12) can be simplified: the sum disappears since only the leading term of the Fourier series, with \( k = 1 \), survives. The magnetostatic energy becomes

\[ E_{\text{mag}} = \frac{\mu_0 M_0^2 d}{4\pi h} \left[ 1 - \exp \left( -\frac{\pi}{d} \right) \right]. \] (21)

and anisotropy plus exchange terms (14) and (15), can be written as

\[ E_{\text{wall}} = \frac{K}{2} + \frac{\pi^2 A}{d^2}. \] (22)

Minimizing the total energy, \( E_{\text{mag}} + E_{\text{wall}} \), with respect to \( d \) and equating the result to \( K \) yields the critical thickness \( \tilde{h}_c \): for this film thickness the total energies of the stripe domain structure and of a mono-domain state with in-plane magnetization are equal. The corresponding value of the half-period is \( \tilde{d}_c \). Both \( \tilde{h}_c \) and \( \tilde{d}_c \) depend on the quality ratio, \( Q = 2K/(\mu_0 M_0^2) \). This dependence can be presented in parametric form by introducing an auxiliary quantity, \( t = \pi h_c/d_c \). One then finds

\[ Q = \frac{3}{2} \left[ 1 - \left( \frac{1}{3} + \frac{1}{t} \right) e^{-t} \right], \] (23)

\[ \tilde{h}_c = \sqrt{6} t \sqrt{\frac{1 - \left( 1 + t/3 \right) e^{-t}}{1 - \left( 1 + t \right) e^{-t}}}, \] (24)

and

\[ \tilde{d}_c = \pi t^{-1} \tilde{h}_c. \] (25)

The parameter \( t \) runs from zero to infinity. The resulting \( \tilde{h}_c \)-versus-\( Q \) dependence is displayed in figure 4 (dashed curve). Within the SWM, the strong-stripe phase should go over directly to the phase with homogeneous in-plane magnetization. However, in reality there is an intermediate region where a weak-stripe structure exists. The latter is characterized by a significant inhomogeneity of magnetization across the thickness of the film and cannot be described within the SWM. The existence of the weak-stripe phase has been confirmed by rigorous numerical calculations of Vukadinovic et al [11], whose point P lies at (0.35, 9.3) in figure 4, i.e. in the weak-stripe region. Weak stripes have also been observed experimentally in thin films with \( Q < 1 \) [16]. The solid line separating the weak-stripe region from the area of homogeneous magnetization is an exact result due to Muller [6, 7]. Thus, the SWM predicts the limits of the strong-stripe phase but fails to describe correctly the magnetic structure beyond the boundary, i.e. left of the dashed line in figure 4. Furthermore, in the SWM the transition at the boundary should proceed abruptly, as a first-order transition. In reality, however, a gradual evolution takes place near the dashed line, as if the line were blurred.

For \( Q \) small, \( Q \to 0, t \to \infty \), the critical thickness \( \tilde{h}_c \) diverges,

\[ \tilde{h}_c \approx \frac{3}{2} \sqrt[3]{\frac{3}{2} Q^{-1}} \approx 3.67 Q^{-1}, \] (26)

whereas

\[ \tilde{d}_c \to \pi \sqrt{6}. \] (27)

The opposite limiting case is \( t \to 0, Q \to 1 \). In that limit \( \tilde{h}_c \) tends to zero,

\[ \tilde{h}_c \approx 4\sqrt{2(1 - Q)} , \] (28)

while \( \tilde{d}_c \) diverges,

\[ \tilde{d}_c = \pi \sqrt{\frac{2}{1 - Q}}. \] (29)
A simple and accurate interpolation formula for $\tilde{h}'_c$ is readily constructed by combining both asymptotic expressions, equations (26) and (28):

$$\tilde{h}'_c = \frac{1}{Q} \sqrt{\frac{(27 + 37Q)(1 - Q)}{2}}.$$  

(30)

For strong anisotropy, $Q > 1$ or $K > \frac{1}{2} \mu_0 M_0^2$, there is no physically meaningful $h'_c$ or $d'_c$ and, without a magnetic field, the mono-domain structure with in-plane magnetization is unstable for any $h$. We find it remarkable that our model reproduces the exact value for the critical quality ratio, $Q = 1$ [6, 7]. The Kittel model is limited to systems with small quality ratios. However, in many strongly anisotropic materials $Q$ is large and no critical thickness is observed [4, 19, 25].

4.3. Numerical results

Away from the critical region, the total energy (16) has to be minimized with respect to $d$ and $\delta$. In general, for arbitrary $M_0$, $K$, $A$ and $h$, the result cannot be expressed analytically and we have to resort to a numerical procedure. The dependence on $A$ and $K$ can still be taken into account by changing over to the dimensionless variables (2). Then we are left with two quantities $\tilde{d}$ and $\tilde{d} - \tilde{\delta}$ versus $\tilde{h}$ with only one parameter $Q$. In figure 5 we prefer to plot the difference $\tilde{d} - \tilde{\delta}$ rather than $\tilde{d}$ since $\tilde{d} - \tilde{\delta}$ can be regarded as an order parameter. The numerical curves confirm all analytical results, in particular, those describing the asymptotic behavior at $\tilde{h} \gg 1$, equations (19) and (20). Thus, $\tilde{\delta}(\tilde{h})$ can be seen to tend to a universal limit.

One observes in figure 5 two distinct regimes, for $Q < 1$ and for $Q > 1$. If $Q > 1$, the stripe domain structure is stable at any $\tilde{h}$. The half-period of the structure, $\tilde{d}(\tilde{h})$ or $\tilde{d}(\tilde{h})$, has a minimum at a certain finite thickness, increasing towards smaller and larger $\tilde{h}$. A prominent feature of the curves with $Q < 1$ is the presence of a critical thickness $\tilde{h}'_c$ where the width of the inner domains, $\tilde{d} - \tilde{\delta}$, vanishes. At that point the sample contains nothing but domain walls, the magnetic structure being a spiral of period $2d_c = 2\delta_c$. Obviously, the wall width cannot be neglected, especially at $Q \approx 1$.

4.4. In-plane remanent magnetization

In a magnetic field directed in the plane of the film the stripe domains arrange themselves parallel to the external field (see [16, 17]). Each domain wall has a magnetic moment component parallel to the in-plane field direction, all of them being directed in the same sense for a small but finite external field. These contributions sum up to a finite in-plane remanent magnetization with a corresponding hysteresis, as observed in thin hcp Co or Mn$_5$Ge$_3$ films [17, 21]. This remanent in-plane magnetization (when the external field tends to zero) is easy to calculate in our model. The sinewave magnetization profile in the domain wall leads to the following expression:

$$\frac{M_r}{M_0} = \frac{1}{d} \int_0^\delta \sin \frac{\pi x}{\delta} \, dx = \frac{2\delta}{\pi d}.$$  

(31)

At the critical thickness, the remanent magnetization is equal to $2/\pi \approx 63.7$ % of the saturation magnetization. Just below this point we would expect the weak-stripe phase [15] in a narrow interval of thicknesses before the transition into the planar mono-domain state. In the weak-stripe phase the maximum $z$-component of the magnetization is less than $M_0$, like in sawtooth magnetization model [15]. However, the weak-stripe phase is beyond the scope of our model.

5. Hexagonal Co

Cobalt hexagonal thin films have been intensively studied. Below a critical thickness, such films exhibit planar magnetization. Recent experimental publications [17, 18, 26] report critical thicknesses between 25 and 50 nanometers. Since the study of Brandenburg et al [18] is the most complete one, SWM has been tested using those data, especially the half-period as a function of film thickness, shown in figure 6. The application of SWM to other thin film materials with different $Q$ values can be found in [27].

SWM has three parameters to characterize different materials: the magneto-crystalline anisotropy constant $K$, the saturation magnetization $M_r$ and the exchange stiffness constant $A$. The saturation magnetization was fixed to
Table 1. Summary of important physical constants obtained for cobalt. The asterisks indicate fixed parameters.

| Material | Kittel | SWM | Measurements |
|----------|--------|-----|--------------|
| $K$ (kJ m$^{-3}$) | 1200 | 820 | 820$^a$, 920$^b$ |
| $A$ (pJ m$^{-1}$) | 76 | 45 | 28$^c$ |
| $M_0$ (MA m$^{-1}$) | 1.43$^*$ | 1.43$^*$ | 1.43$^d$ |
| $h_\text{c}$ (nm) | 36 | 36 | 36 ± 3 |
| $d_\text{c}$ (nm) | 63 | 67 | 63 ± 5 |

Note: Experimental data are extracted: $^a$ from [30], $^b$ from [16], $^c$ from [29] and $^d$ from [18, 28].

the experimental value of $M_0 = 1.43$ MA m$^{-1}$, while the anisotropy and the exchange constant were chosen as fitting parameters. The best fit was obtained with $K = 820$ kJ m$^{-3}$, which is rather close to the bulk anisotropy constant (see table 1), and $A = 45$ pJ m$^{-1}$, which exceeds the value deduced from inelastic neutron scattering experiments [29]. However, it should be taken into account that $A$ is not known accurately, since relating neutron scattering data with our continuum model involve several approximations.

For comparison, we also fitted the experimental data to the Kittel model (see figure 6), leading to the parameter values presented in table 1. The saturation magnetization was fixed to the same value, but the anisotropy and exchange constants deviate more strongly from the experiment than those obtained in the SWM fit. This suggests that the Kittel model is less realistic than SWM. We also observe important differences between both models near the critical thickness. The curvature of $d(h)$ near $h_\text{c}$ is different in the two models. In SWM, for $h$ between 40 and 80 nm the wall is wider than the inner domain, whereas in the Kittel model the wall width remains constant and always inferior to the inner domain width.

The quality factor for Co is $Q = 0.64$, which is less than one. We expect that for materials with stronger anisotropy, i.e. with larger $Q$, the difference between SWM and the Kittel model will be even more significant.

6. Comparison

Our model is constructed in such a way that it applies at any $Q$, including the strong-anisotropy case, $Q > 1$, where there is no critical thickness. In that limit, our model goes over into the model proposed by Malek and Kambersky (MK) [2] (see also [3]) as is demonstrated in figure 7. The MK model treats the domain walls in the same way as in the Kittel model (equation (1)) but takes into account the magneto-crystalline interaction energy between top and bottom surface. Since it lacks an explicit treatment of the domain walls, it misses the critical point at $Q = 1$ which is improved in the SWM. For $Q > 1$, the period $d$ as a function of thickness $h$ is very close in both models (MK and SWM). The small, systematic difference at large $Q$ arises due to the different treatments of domain walls.

The SWM can also be applied to ultra-thin films of only a few monolayers. In that case it goes over into the Yafet–Gyorgy (YG) model [22]. For ultra-thin films, the thickness $h$ is much smaller than $d$ and $\delta$ and varies only slightly. Due to the surface anisotropy $K_{\perp}$, a small variation of $h$ may induce an important change in the total anisotropy. All anisotropy contributions may be incorporated into the model parameter $K^\prime = K_a + K_{\perp}/h$, where the magneto-crystalline anisotropy of the bulk is now denoted by $K_a$. To apply SWM to that case we fix $h$ and vary $K^\prime$, i.e. the quality factor $Q$. As can be seen from figures 5 and 7, for $h \ll 1$, a small variation of $Q$ increases the period $d$ in a dramatic way which was first found by Yafet and Gyorgy [22] and which is not shown here. Also, since in the YG model the domain wall is explicitly taken into account, the transition at $Q = 1$ occurs in the same way as in the SWM, it means that $d = \delta$ at $Q = 1$ (see figure 5). However, there is no weak-stripe phase for ultra-thin films. That is easily recognized in figure 4 if one considers a crossing of the critical lines for $h \ll 1$ by varying $Q$. In that region of the phase diagram, the width of the weak stripe phase goes to zero.

The most interesting region is the one of medium anisotropy, $1/2 \lesssim Q < 1$. To illustrate that region, we chose hexagonal cobalt as an example. With the parameters of table 1 (SWM) we evaluated the critical thickness in SWM, $h^\prime_c = 36$ nm. This agrees with the experiment of Brandenburg et al [18], who find a critical thickness of about 40 nm. However, in Muller’s exact theory [6] the critical thickness comes out much smaller, $h_c = 23$ nm. One has to take into account, though, that stripe domains were observed by other groups in Co films as thin as 25 nm and that weak stripes are certainly hard to see. Our interpretation is that the interval between Muller’s $h_c = 23$ nm and the SWM $h^\prime_c = 36$ nm is a region of the weak-stripe phase.

In the case of cobalt, such an interpretation is supported by a calculation of the weak-stripe phase within the sawtooth magnetization model. The two variational parameters are the maximum canting angle $\Theta_0$ and the half-period $d$. For better precision, we included the complete Fourier series as well as the magnetostatic interaction between the top and bottom surfaces. The numerical results are presented in figure 8. The weak-stripe phase sets in at $h = 28$ nm and has a lower energy than the strong-stripe phase up to 44 nm. Between 28 and 44 nm the maximum canting angle increases from $0^\circ$ to $76^\circ$. Hence, in the case of hexagonal cobalt, we would expect the existence of the weak-stripe phase between 23 and about
44 nm. At larger thicknesses the strong-stripe phase prevails and SWM is more appropriate than the sawtooth magnetization model.

Let us now briefly discuss the region of very small anisotropy, \( Q \ll 1 \). It allows mathematical simplifications, as already discussed in section 4. There, it was shown that the range of thicknesses where the weak-stripe phase could be expected becomes wider accordingly as \( Q \) decreases, i.e. as the magnetic anisotropy weakens. For very weak anisotropy we also expect other complications, such as the presence of closure domains. We believe our model can still be applied if the layer of the closure domains is much thinner than the film as a whole.

7. Conclusion

We present a model (SWM) for the strong-stripe phase in magnetic thin films with perpendicular anisotropy, which improves the well-known Kittel model in two respects. Firstly, our model includes the magnetostatic interaction between the top and bottom surfaces of the film. This is important for materials with strong magneto-crystalline anisotropy. Secondly, the domain walls are treated explicitly, which improves the numerical accuracy and allows the calculation of the in-plane remanent magnetization. The model has the same magnetization profile as the Yafet–Gyorgy model [22] which was, however, constructed for ultra-thin films. We show that a consequent analysis without any thickness restriction leads to reasonable description of the strong-stripe phase. Our model is simple, but general enough to contain many other models that were proposed before as limiting cases. It is especially suited to interpret experimental data concerning the thickness dependence of the domain width or the remanent magnetization. We demonstrate the existence of a critical anisotropy above which stripe domains exist at any film thickness. SWM reproduces the exact threshold value, \( Q = 1 \). We derive a number of analytical results facilitating the estimation of important parameters. The numerical results obtained for different quality factors \( Q \) show the evolution of domain and wall widths as functions of film thickness.

With the new model we are able to correctly describe the behaviour of magnetic domains in cobalt thin films. It allows one to explain the experimental observations of Brandenburg et al [18] with more realistic fitting parameters than using the Kittel model. By comparing the exact critical thickness for stripe nucleation \( h_s \) with our critical thickness \( h_c' \), which corresponds to the on-set of strong stripes, we are able to estimate the range of thicknesses where weak stripes are expected. We show that for Co this interval is only about 15 nm wide, which validates our model. We should remark that the currently available experimental data for Co do not suffice for locating the thickness range of the weak-stripe phase accurately. We find it important that our model takes the wall width into account in an adequate way. In the neighbourhood of the critical thickness, domain and wall width are of comparable size. Thanks to its universality, our model can be applied to other types of ferromagnetic films, e.g. FePd or Mn3Ge3.

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