Universality of short-range correlations in light nuclei

W. Horiuchi$^{1,2}$, H. Feldmeier$^1$, T. Neff$^3$, and Y. Suzuki$^{3,4}$

$^1$GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany
$^3$RIKEN Nishina Center, RIKEN, Wako 351-0198, Japan
$^4$Department of Physics, Niigata University, Niigata 950-2181, Japan

E-mail: whoriuchi@riken.jp (W. Horiuchi)

Abstract. In this contribution, we investigate the structure of short-range correlations in many-body states. We obtain the highly correlated many-body states with an explicitly correlated basis which enables us to get a precise solution of a many-body Schrödinger equation for a realistic interaction. We show two-body density distributions calculated from three- and four-body states to investigate the short-range correlations between nucleon pairs. At distances below 1 fm a universal behavior is found which does not depend on the many-body states. The universality is also seen in the high momentum components of the two-body momentum distributions.

1. Introduction

In nuclei, the nuclear interaction induces strong short-range correlations among the nucleons. Realistic nucleon-nucleon interactions, which reproduce the nucleon-nucleon scattering phase-shifts and deuteron properties, contain short-range repulsive and tensor components. Due to the short-range repulsion, nucleon pairs will not be found at distances below 0.5 fm. This is reflected by a high momentum component in the momentum distribution. The tensor correlations induce further momenta above the Fermi momentum. Though these correlations can only be measured indirectly, some physical observables may reflect the high momentum component. These days short-range correlations attract increasing interest. Recent measurements which try to extract information on the short-range correlations have been carried out at JLab [1]. These correlations also provide important information on the saturation property in nuclear matter.

We investigate the structure of short-range correlations in many-body states. The highly correlated many-body states are represented with an explicitly correlated basis which enables us to get a precise solution of a many-body Schrödinger equation for a realistic interaction [2, 3]. We show two-body density distributions calculated from three- and four-body states to investigate the short-range correlations between nucleon pairs. At distances below 1 fm a universal behavior is found which does not depend on the many-body states. Two-body momentum distributions reflecting the high momentum transfers of the short-range repulsive and tensor correlations are also discussed.

$^2$ Present address: RIKEN Nishina Center, RIKEN, Wako 351-0198, Japan, supported by the Special Postdoctoral Researchers Program of RIKEN.
2. Numerical Calculations
We assume that an $A$-nucleon state can be expanded in terms of a combination of basis states, each of which is a product of space, spin and isospin parts,

$$|\Phi; JM\rangle = \sum_{i=1}^{K} C_{i} A\left\{ \left| \psi_{i}^{(\text{space})} \psi_{i}^{(\text{spin})} \psi_{i}^{(\text{isospin})} \right\rangle_{JM} \right\}.$$

(1)

Here $A$ is the antisymmetrizer and the square bracket $[\cdots]$ stands for the angular momentum coupling. The spin and isospin parts are expanded using the basis of successive coupling, e.g.,

$$|\psi_{i}^{(\text{spin})}\rangle = \left[ \cdots \left[ \left| \frac{1}{2} \right| s_{123} \right| s_{123} \cdots \right] S_{i} M_{S} \right\rangle,$$

(2)

where the set of intermediate spins $(S_{12}, S_{123}, \ldots)$ takes all possible values compatible with the total spin $S_{i}$ of the $i$th basis. The isospin mixing is ignored in this contribution, so that the total isospin $T_{i}$ is kept fixed to $T$. The orbital part $\psi_{i}^{(\text{space})}$ is expressed in terms of the explicitly correlated Gaussian basis [2, 3]. The variational parameters in the basis are determined by a stochastic variational method [2, 4] and the basis dimension is increased until good convergence is reached. We investigate the wave functions of $^3\text{H}$, $^3\text{He}$ and $^4\text{He}$ using the Argonne V8’ interaction [5]. We also calculate the first excited state of $^4\text{He}$ that exhibits dilute 3+1 cluster structures [6].

The antisymmetrized many-body state $|\Phi; JM\rangle$ contains all information about the nuclear system. Its $A$-body density is a function of $A$ position or momentum vectors and $4*A$ spin-isospin possibilities and hence can not be visualized easily. Therefore we integrate and sum over $A-2$ single-particle degrees of freedom and are left with the two-body density. This represents an average over all particle pairs in the many-body state. In addition we integrate over the center of mass position of the pair and obtain the two-body densities for the four spin-isospin channels which are possible for a nucleon pair. The two-body density for the relative motion is given by

$$\rho_{SM_{S}, TM_{T}}^{\text{rel}}(r) = \frac{1}{2J+1} \sum_{M} \langle \Phi; JM\rangle \sum_{i<j} P_{ij}^{SM_{S}} \hat{P}_{ij}^{TM_{T}} \delta^{3}(\vec{r}_{i} - \vec{r}_{j} - \vec{r}) |\Phi; JM\rangle.$$

(3)

This is the probability density to find a nucleon pair at the relative position $r = r_{1} - r_{2}$ in the spin $S$, $M_{S}$ and isospin $T$, $M_{T}$ channel. The operators $\hat{P}_{ij}^{SM_{S}}$ and $\hat{P}_{ij}^{TM_{T}}$ project on spin and isospin of the pair, respectively. The corresponding distribution of the relative momentum $k = (k_{1} - k_{2})/2$ of the particle pair with total spin $S$, $M_{S}$ and isospin $T$, $M_{T}$ is defined as

$$n_{SM_{S}, TM_{T}}^{\text{rel}}(k) = \frac{1}{2J+1} \sum_{M} \langle \Phi; JM\rangle \sum_{i<j} P_{ij}^{SM_{S}} \hat{P}_{ij}^{TM_{T}} \delta^{3}\left(\frac{1}{2} (\vec{k}_{i} - \vec{k}_{j}) - \vec{k}\right) |\Phi; JM\rangle.$$

(4)

3. Results
Here we focus on the two-body density distributions with the $S=1$, $T=0$ channel. In Fig. 1 the spatial two-body densities $\rho_{11,00}^{\text{rel}}(r)$ of four different states are displayed. The striking observation is that at short distances they look very similar independently of the many-body state. That means that the correlation felt by a particle pair in the $S=1$, $T=0$ channel is the same at short distances independently of the remaining particles in the system. In regions where the potential is attractive, $r \approx (0, 0, \pm 1 \text{ fm})$, the densities are large and in regions where the interaction is repulsive or close to zero the probability of finding the particle pair is small. At
Figure 1. (Color online) From left to right: Two-body densities in coordinate space for a pair of nucleons with $S=1$, $M_S=1$ and $T=0$ in the ground states of $^2$H, $^3$H and $^4$He and the 20.21 MeV excited state of $^4$He denoted by d, t, $\alpha$ and $\alpha^*$, respectively. The densities have rotational symmetry around the $z$-axes and range from black = 0 to bright (yellow) = maximum. Maxima assume values of $0.008\,\text{fm}^{-3}$ for d, $0.015\,\text{fm}^{-3}$ for t, $0.035\,\text{fm}^{-3}$ for $\alpha$ and $0.015\,\text{fm}^{-3}$ for $\alpha^*$.

Figure 2. Cuts of $\rho_{11,00}^\text{rel}(r)/\rho_{11,00}^\text{rel}(r_n)$ at $r_n=(0,0,1\,\text{fm})$ for deuteron (d), triton (t), $^3$He (h), $^4$He ($\alpha$) and the first excited state of $^4$He ($\alpha^*$). The result with the unitary correlation operator method (UCOM) for $^4$He is also displayed.

short distances below 0.5 fm the AV8' potential is so strongly repulsive that the pair densities in all many-body states are pushed down towards zero. One should bear in mind that in a simple shell model many-body state these correlations can not be represented and the two-body densities have actually their maximum at relative distance $r=0$.

Fig. 2 shows cuts of the normalized two-body density $\rho_{11,00}^\text{rel}(r)/\rho_{11,00}^\text{rel}(r_n)$ along the $z$-direction and the $x$-direction. We normalize the quantities at $r_n=(0,0,1\,\text{fm})$, where the densities are close to their maximum value. It is surprising to see in Fig. 2 that for small distances all five densities practically coincide along the $z$-axis. The same holds true when going along the $x$-axis, although the normalization was done on the $z$-axis. This means that not only the central correlations but also the angular dependence of the tensor correlations are almost identical at short distances. The short-range central and tensor correlations exhibit universal behavior at short distances below about 1 fm. They do not depend on the nuclear many-body states for which they have been calculated.
Figure 3. Two-body densities as a function of relative momentum \( k \) for the \( S=1, T=0 \) channel multiplied by the same normalization factors for the coordinate space. Ground state densities of \(^2\text{H}, \(^3\text{H}, \(^3\text{H}, \) and \(^4\text{He} \) are denoted by \( d, t, h, \) and \( \alpha \), respectively. The excited state of \(^4\text{He} \) is labeled with \( \alpha^* \).

This universality confirms the basic assumption of the unitary correlation operator method (UCOM) [7]. In the UCOM short-range correlations are described explicitly with central and tensor correlation operators. In Fig. 2, we also display the two-body density of \(^4\text{He} \) calculated with the UCOM. The uncorrelated wave function is assumed to be a \((0s)^4 \) harmonic oscillator shell model wave function. The UCOM result is in very good agreement with the accurate calculation obtained by our correlated basis approach.

Fig. 3 shows the normalized two-body density in the momentum space. The same normalization factors as for the two-body densities in coordinate space are used. As seen in the figure, we also find the universality at high momenta beyond 3 fm\(^{-1} \) for all many-body states.

4. Summary
We have investigated the structure of short-range correlations in many-body states with the explicitly correlated basis using a realistic nucleon-nucleon interaction. We have shown two-body density distributions for the \( S=1, T=0 \) channel calculated from three- and four-body states to investigate the short-range correlations between nucleon pairs. At distances below 1 fm a universal behavior is found which does not depend on the many-body states. Two-body momentum distributions also exhibit the universal behavior at high momenta beyond 3 fm\(^{-1} \). A study of the other spin-isospin channels are interesting and will be reported soon.

References
[1] Subedi R et al. 2008 Science \textbf{320} 1476
[2] Varga K and Suzuki Y 1995 \textit{Phys. Rev. C} \textbf{52} 2885
[3] Suzuki Y, Horiuchi W, Orabi M, and Arai K 2008 \textit{Few Body Syst.} \textbf{42} 33
[4] Suzuki Y and Varga K 1998 “Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems”, Lecture Notes in Physics, Vol. 54, Springer, Berlin Heidelberg
[5] Wiringa R B, Stoks B G J and Schiavilla R 1995 \textit{Phys. Rev. C} \textbf{51} 38
[6] Horiuchi W and Suzuki Y 2008 \textit{Phys. Rev. C} \textbf{78} 034305
[7] Roth R, Neff T, and Feldmeier H 2010 \textit{Prog. Part. Nucl. Phys.} \textbf{65} 50