On identities in Hom-Malcev algebras

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Abstract
In a Hom-algebra an identity, equivalent to the Hom-Malcev identity, is found.
2000 MSC: 17A20, 17A30.

1 Introduction and statement of results

Hom-Lie algebras were introduced in [3] as a tool in understanding the structure of some q-deformations of the Witt and the Virasoro algebras. Since then, the theory of Hom-type algebras began an intensive development (see, e.g., [2], [4], [6], [7], [8], [12], [13], [14], [15]). Hom-type algebras are defined by twisting the defining identities of some well-known algebras by a linear self-map, and when this twisting map is the identity map, one recovers the original type of considered algebras.

In this setting, a Hom-type generalization of Malcev algebras (called Hom-Malcev algebras) is defined by D. Yau in [15]. Recall that a Malcev algebra is a nonassociative algebra \((A, \cdot)\), where the binary operation \(\cdot\) is anti-commutative, such that the identity

\[ J(x, y, x \cdot z) = J(x, y, z) \cdot x \] (1.1)

holds for all \(x, y, z \in A\) (here \(J(x, y, z)\) denotes the Jacobian, i.e. \(J(x, y, z) = xy \cdot z + yz \cdot x + zx \cdot y\)). The identity (1.1) is known as the Malcev identity. Malcev algebras were introduced by A.I. Mal’tsev [9] (calling them Moufang-Lie algebras) as tangent algebras to local smooth loops, generalizing in this way a result in Lie theory stating that a Lie algebra is a tangent algebra to a local Lie group (in fact, Lie algebras are special case of Malcev algebras). Another approach to Malcev algebras is the one from alternative algebras:
every alternative algebra is Malcev-admissible [9]. So one could say that
the algebraic theory of Malcev algebras started from Malcev-admissibility
of algebras. The foundations of the algebraic theory of Malcev algebras go
back to E. Kleinfeld [5], A.A. Sagle [10] and, as mentioned in [10], to A.A.
Albert and L.J. Paige. Some twisting of the Malcev identity (1.1) along any
algebra self-map $\alpha$ of $A$ gives rise to the notion of a Hom-Malcev algebra
$(A, \cdot, \alpha)$ ([15]; see definitions in section 2). Properties and constructions
of Hom-Malcev algebras, as well as the relationships between these Hom-
algebras and Hom-alternative or Hom-Jordan algebras are investigated in
[15]. In particular, it is shown that a Malcev algebra can be twisted into
a Hom-Malcev algebra and that Hom-alternative algebras are Hom-Malcev
admissible.

In [15], as for Malcev algebras (see [10], [11]), equivalent defining iden-
tities of a Hom-Malcev algebra are given. In this note, we mention another
identity in a Hom-Malcev algebra that is equivalent to the ones found in
[15]. Specifically, we shall prove the following

**Theorem.** Let $(A, \cdot, \alpha)$ be a Hom-Malcev algebra. Then the identity

$$J_{\alpha}(w \cdot x, \alpha(y), \alpha(z)) = J_{\alpha}(w, y, z) \cdot \alpha^2(x) + \alpha^2(w) \cdot J_{\alpha}(x, y, z)$$

$$- 2J_{\alpha}(y \cdot z, \alpha(w), \alpha(x)) \quad (1.2)$$

holds for all $w, x, y, z$ in $A$, where $J_{\alpha}(x, y, z) = xy \cdot \alpha(z) + yz \cdot \alpha(x) + zx \cdot \alpha(y)$. Moreover, in any anti-commutative Hom-algebra $(A, \cdot, \alpha)$, the identity (1.2) is equivalent to the Hom-Malcev identity

$$J_{\alpha}(\alpha(x), \alpha(y), x \cdot z) = J_{\alpha}(x, y, z) \cdot \alpha^2(x) \quad (1.3)$$

for all $x, y, z$ in $A$.

Observe that when $\alpha = Id$ (the identity map) in (1.3), then (1.3) is (1.1)
i.e. the Hom-Malcev algebra $(A, \cdot, \alpha)$ reduces to the Malcev algebra $(A, \cdot)$
(see [15]).

In section 2 some instrumental lemmas are proved. Some results in these
lemmas are a kind of the Hom-version of similar results by E. Kleinfeld [5]
in case of Malcev algebras. The section 3 is devoted to the proof of the
theorem.

Throughout this note we work over a ground field $\mathbb{K}$ of characteristic 0.
2 Definitions. Preliminary results

In this section we recall useful notions on Hom-algebras ([8], [12], [13], [15]), as well as the one of a Hom-Malcev algebra [15]. In [5], using an analogue of the Bruck-Kleinfeld function, an identity (see identity (6) in [5]) characterizing Malcev algebras is found. This identity is used in [10] to derive further identities for Malcev algebras (see [10], Proposition 2.23). The main result of this section (Lemma 2.7) proves that the Hom-version of the identity (6) of [5] holds in any Hom-Malcev algebra.

Definition 2.1. A multiplicative Hom-algebra is a triple \((A, \mu, \alpha)\), in which \(A\) is a \(K\)-module, \(\mu : A \times A \to A\) is a bilinear map (the binary operation), and \(\alpha : A \to A\) is a linear map (the twisting map) such that \(\alpha\) is an endomorphism of \((A, \mu)\). The Hom-algebra \((A, \mu, \alpha)\) is said anticommutative if the operation \(\mu\) is skew-symmetric, i.e. \(\mu(x, y) = -\mu(y, x)\), for all \(x, y \in A\).

In the rest of this paper, we will use the abbreviation \(x \cdot y = \mu(x, y)\) in a Hom-algebra \((A, \mu, \alpha)\).

Remark. The multiplicativity of the twisting map is not necessary in the definition of a Hom-algebra (see, e.g., [6], [8]). The multiplicativity is included here for convenience.

Definition 2.2. Let \((A, \cdot, \alpha)\) be an anticommutative Hom-algebra.

(i) The Hom-Jacobian ([8]) of \((A, \cdot, \alpha)\) is the trilinear map \(J_\alpha(x, y, z)\) on \(A\) defined by \(J_\alpha(x, y, z) = xy \cdot \alpha(z) + yz \cdot \alpha(x) + zx \cdot \alpha(y)\).

(ii) \((A, \cdot, \alpha)\) is called a Hom-Lie algebra ([3]) if the Hom-Jacobi identity \(J_\alpha(x, y, z) = 0\) holds in \((A, \cdot, \alpha)\).

Definition 2.3. ([15]) A Hom-Malcev algebra is an anticommutative algebra \((A, \cdot, \alpha)\) such that the Hom-Malcev identity (see (1.3))

\[ J_\alpha(\alpha(x), \alpha(y), x \cdot z) = J_\alpha(x, y, z) \cdot \alpha^2(x) \]

holds in \((A, \cdot, \alpha)\).

Remark. When \(\alpha = Id\), then the Hom-Jacobi identity reduces to the usual Jacobi identity \(J(x, y, z) := xy \cdot z + yz \cdot x + zx \cdot y = 0\), i.e. the Hom-Lie algebra \((A, \cdot, \alpha)\) reduces to the Lie algebra \((A, \cdot)\). Likewise, when \(\alpha = Id\), the Hom-Malcev identity reduces to the Malcev identity (1.1), i.e. the Hom-
Malcev algebra \((A, \cdot, \alpha)\) reduces to the Malcev algebra \((A, \cdot)\).

The following simple lemma holds in any anticommutative Hom-algebra.

**Lemma 2.4.** In any anticommutative Hom-algebra \((A, \cdot, \alpha)\) the following holds:

(i) \(J_\alpha(x, y, z)\) is skew-symmetric in its three variables.

(ii) \(\alpha^2(w) \cdot J_\alpha(x, y, z) - \alpha^2(x) \cdot J_\alpha(y, z, w) + \alpha^2(y) \cdot J_\alpha(z, w, x)
- \alpha^2(z) \cdot J_\alpha(w, x, y)
= J_\alpha(w, x, z, y, \alpha(z)) + J_\alpha(y, z, \alpha(w), \alpha(x)) + J_\alpha(w, y, \alpha(z), \alpha(x))
+ J_\alpha(z, x, \alpha(w), \alpha(y)) - J_\alpha(z, w, \alpha(x), \alpha(y)) - J_\alpha(x, y, \alpha(z), \alpha(w)),
for all \(w, x, y, z\) in \(A\).

**Proof.** The skew-symmetry of \(J_\alpha(x, y, z)\) in \(w, x, y, z\) follows from the skew-symmetry of the operation \(\cdot\).

Expanding the expression in the left-hand side of (ii) and then rearranging terms, we get (by the skew-symmetry of \(\cdot\))

\[
\alpha^2(w) \cdot J_\alpha(x, y, z) - \alpha^2(x) \cdot J_\alpha(y, z, w) + \alpha^2(y) \cdot J_\alpha(z, w, x)
- \alpha^2(z) \cdot J_\alpha(w, x, y)
= -\alpha^2(z) \cdot (wx \cdot \alpha(y)) + \alpha^2(y) \cdot (wx \cdot \alpha(z))
-\alpha^2(x) \cdot (yz \cdot \alpha(w)) + \alpha^2(w) \cdot (yz \cdot \alpha(x))
-\alpha^2(x) \cdot (wy \cdot \alpha(z)) - \alpha^2(z) \cdot (yw \cdot \alpha(x))
+\alpha^2(w) \cdot (zx \cdot \alpha(y)) + \alpha^2(y) \cdot (xz \cdot \alpha(w))
-\alpha^2(x) \cdot (zw \cdot \alpha(y)) + \alpha^2(y) \cdot (zw \cdot \alpha(x))
+\alpha^2(w) \cdot (xy \cdot \alpha(z)) - \alpha^2(z) \cdot (xy \cdot \alpha(w)).
\]

Next, adding and subtracting \(\alpha(yz) \cdot \alpha(wx)\) (resp. \(\alpha(wx) \cdot \alpha(yz)\),
\(\alpha(zx) \cdot \alpha(wy), \alpha(wy) \cdot \alpha(zx), \alpha(xy) \cdot \alpha(zw)\) and \(\alpha(zw) \cdot \alpha(xy)\)) in the first
(resp. second, third, fourth, fifth, and sixth) line of the right-hand side expression in the last equality above, we come to the equality (ii) of the lemma.

In a Hom-Malcev \((A, \cdot, \alpha)\) we define the multilinear map \(G\) by

\[
G(w, x, y, z) = J_\alpha(w, x, \alpha(y), \alpha(z)) - \alpha^2(x) \cdot J_\alpha(w, y, z)
- J_\alpha(x, y, z) \cdot \alpha^2(w)
\]

\[\text{(2.1)}\]
for all $w, x, y, z$ in $A$.

**Remark.** (1) If $\alpha = Id$ in (2.1), then $G(w, x, y, z)$ reduces to the function $f(w, x, y, z)$ defined in [5] which in turn is a variation of the Bruck-Kleinfeld function defined in [1].

(2) If in (2.1) replace $J_\alpha(t, u, v)$ with the Hom-associator [8] as $(t, u, v)$, then one recovers the Hom-Bruck-Kleinfeld function defined in [15].

**Lemma 2.5.** In a Hom-Malcev algebra $(A, \cdot, \alpha)$ the function $G(w, x, y, z)$ defined by (2.1) is skew-symmetric in its four variables.

**Proof.** From the skew-symmetry of “$\cdot$” and $J_\alpha(t, u, v)$ (see Lemma 2.4(i)) it clearly follows that

\[
G(x, w, y, z) = -G(w, x, y, z)
\]

\[
G(w, x, z, y) = -G(w, x, y, z).
\]

Next, using the skew-symmetry of $J_\alpha(t, u, v)$,

\[
G(y, x, y, z) = J_\alpha(y \cdot x, \alpha(y), \alpha(z)) - J_\alpha(x, y, z) \cdot \alpha^2(y)
\]

\[
= J_\alpha(\alpha(y), \alpha(z), y \cdot x) - J_\alpha(y, z, x) \cdot \alpha^2(y)
\]

\[
= J_\alpha(y, z, x) \cdot \alpha^2(y) - J_\alpha(y, z, x) \cdot \alpha^2(y) \quad \text{(by (1.3))}
\]

\[
= 0.
\]

Likewise, one checks that $G(w, y, y, z) = 0$. This suffices to prove the skew-symmetry of $G(w, x, y, z)$ in its variables. \[\square\]

As we shall see below, the following lemma is a consequence of the definition of $G(w, x, y, z)$ and the skew-symmetry of $J_\alpha(t, u, v)$ and $G(w, x, y, z)$.

**Lemma 2.6.** Let $(A, \cdot, \alpha)$ be a Hom-Malcev. Then

\[
J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(x \cdot y, \alpha(z), \alpha(w)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))
\]

\[
+ J_\alpha(z \cdot w, \alpha(x), \alpha(y)) = 0;
\]

\[\text{(2.2)}\]

\[
2G(w, y, y, z) - \alpha^2(w) \cdot J_\alpha(x, y, z) + \alpha^2(x) \cdot J_\alpha(w, y, z) - \alpha^2(y) \cdot J_\alpha(z, w, x)
\]

\[
+ \alpha^2(z) \cdot J_\alpha(w, x, y) = J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)), \quad \text{(2.3)}
\]

for all $w, x, y, z$ in $A$.

**Proof.** From the definition of $G(w, x, y, z)$ (see (2.1)) we have

\[
J_\alpha(w \cdot x, \alpha(y), \alpha(z)) = G(w, x, y, z) + \alpha^2(x) \cdot J_\alpha(w, y, z) + J_\alpha(x, y, z) \cdot \alpha^2(w),
\]

\[
\text{Proof.} \quad \text{From the definition of } G(w, x, y, z) \text{ (see (2.1)) we have}
\]

\[
J_\alpha(w \cdot x, \alpha(y), \alpha(z)) = G(w, x, y, z) + \alpha^2(x) \cdot J_\alpha(w, y, z) + J_\alpha(x, y, z) \cdot \alpha^2(w),
\]
Therefore, by the skew-symmetry of "\(\cdot\)" , \(J_\alpha(x, y, z)\) and \(G(w, x, y, z)\), we get
\[
J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(x \cdot y, \alpha(z), \alpha(w)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))
\]
\[
= G(w, x, y, z) + G(x, y, z, w) + G(y, z, w, x) + G(z, w, x, y)
\]
\[
= G(w, x, y, z) - G(w, x, y, z) + G(y, z, w, x) - G(y, z, w, x)
\]
\[
= 0,
\]
which proves (2.2).
Next, again from the expression of \(G(w, x, y, z)\),
\[
J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) + J_\alpha(z \cdot w, \alpha(x), \alpha(y))
\]
\[
= [G(w, x, y, z) + \alpha^2(x) \cdot J_\alpha(w, y, z)] + [J_\alpha(x, y, z) \cdot \alpha^2(w)]
\]
\[
+ [G(y, z, w, x) + \alpha^2(z) \cdot J_\alpha(y, w, x)] + [J_\alpha(z, w, x) \cdot \alpha^2(y)]
\]
\[
= 2G(w, x, y, z) - \alpha^2(w) \cdot J_\alpha(x, y, z) + \alpha^2(x) \cdot J_\alpha(y, z, w) - \alpha^2(y) \cdot J_\alpha(z, w, x)
\]
\[
+ \alpha^2(z) \cdot J_\alpha(w, x, y)
\]
so that we get (2.3).

From Lemma 2.5 and Lemma 2.6, we get the following expression of \(G(w, x, y, z)\).

**Lemma 2.7.** Let \((A, \cdot, \alpha)\) be a Hom-Malcev. Then
\[
G(w, x, y, z) = 2[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))] \tag{2.4}
\]
for all \(w, x, y, z\) in \(A\).

**Proof.** Set \(g(w, x, y, z) = J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(x \cdot y, \alpha(z), \alpha(w)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) + J_\alpha(z \cdot w, \alpha(x), \alpha(y))\). Then (2.2) says that \(g(w, x, y, z) = 0\) for all \(w, x, y, z\) in \(A\). Now, by adding \(g(w, x, y, z) - g(x, w, y, z)\) to the right-hand side of Lemma 2.4(ii), we get
\[
\alpha^2(w) \cdot J_\alpha(x, y, z) = \alpha^2(x) \cdot J_\alpha(y, z, w) + \alpha^2(y) \cdot J_\alpha(z, w, x) - \alpha^2(z) \cdot J_\alpha(w, x, y)
\]
\[
= J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) + J_\alpha(z \cdot w, \alpha(x), \alpha(y))
\]
\[
+ J_\alpha(x \cdot y, \alpha(z), \alpha(w)) - J_\alpha(x \cdot y, \alpha(z), \alpha(w))
\]
\[
+ J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(x \cdot y, \alpha(z), \alpha(w))
\]
\[
+ J_\alpha(y \cdot z, \alpha(w), \alpha(x)) + J_\alpha(z \cdot w, \alpha(x), \alpha(y))
\]
\[
= 0
\]
which proves (2.3).
\[ J_\alpha(x \cdot w, \alpha(y), \alpha(z)) - J_\alpha(w \cdot y, \alpha(z), \alpha(x)) \]
\[ - J_\alpha(y \cdot z, \alpha(x), \alpha(w)) - J_\alpha(z \cdot x, \alpha(w), \alpha(y)) \]
\[ = 3J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + 3J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \]

\[ J_\alpha(y \cdot z, \alpha(x), \alpha(w)) - J_\alpha(z \cdot x, \alpha(w), \alpha(y)) \]
\[ = 3J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + 3J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \]

\[ \alpha^2(w) \cdot J_\alpha(x, y, z) = \alpha^2(x) \cdot J_\alpha(y, z, w) \]
\[ + \alpha^2(y) \cdot J_\alpha(z, w, x) - \alpha^2(z) \cdot J_\alpha(w, x, y) \]
\[ = 3[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))]. \quad (2.5) \]

Next, adding (2.3) and (2.5) together, we get

\[ 2G(w, x, y, z) = \alpha^2(x) \cdot J_\alpha(x, y, z) + \alpha^2(x) \cdot J_\alpha(y, z, w) \]
\[ - \alpha^2(y) \cdot J_\alpha(z, w, x) + \alpha^2(z) \cdot J_\alpha(w, x, y) \]
\[ + \alpha^2(y) \cdot J_\alpha(x, y, z) - \alpha^2(x) \cdot J_\alpha(y, z, w) \]
\[ + \alpha^2(y) \cdot J_\alpha(z, w, x) - \alpha^2(z) \cdot J_\alpha(w, x, y) \]
\[ = J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \]
\[ + 3[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))] \]

\[ \alpha^2(w) \cdot J_\alpha(x, y, z) - \alpha^2(x) \cdot J_\alpha(y, z, w) \]
\[ + \alpha^2(y) \cdot J_\alpha(z, w, x) - \alpha^2(z) \cdot J_\alpha(w, x, y) \]
\[ = 3[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))]. \quad (2.5) \]

Next, adding (2.3) and (2.5) together, we get

\[ 2G(w, x, y, z) = 4[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))] \]
and (2.4) follows.

3 Proof

Relaying on the lemmas of section 2, we are now in position to prove the theorem.

**Proof of the theorem.** First we establish the identity (1.2) in a Hom-Malcev algebra. We may write (2.1) in an equivalent form:

\[ J_\alpha(w \cdot x, \alpha(y), \alpha(z)) = \alpha^2(x) \cdot J_\alpha(w, y, z) + J_\alpha(x, y, z) \cdot \alpha^2(w) \]
\[ + G(w, x, y, z). \quad (2.6) \]

Now in (2.6), replace \( G(w, x, y, z) \) with its expression from (2.4) to get

\[ -J_\alpha(w \cdot x, \alpha(y), \alpha(z)) = \alpha^2(x) \cdot J_\alpha(w, y, z) + J_\alpha(x, y, z) \cdot \alpha^2(w) \]
\[ + 2J_\alpha(y \cdot z, \alpha(w), \alpha(x)), \]

which leads to (1.2).
Now, we proceed to prove the equivalence of (1.2) with (1.3) in an anti-commutative Hom-Malcev algebra. First assume (1.3). Then Lemmas 2.4, 2.5, 2.6, and 2.7 imply that (1.2) holds in any Hom-Malcev algebra.

Conversely, assume (1.2). Then, setting $w = y$ in (1.2), we get, by the skew-symmetry of $J_{\alpha}(x, y, z)$,

\[ J_{\alpha}(y \cdot x, \alpha(y), \alpha(z)) = \alpha^2(y) \cdot J_{\alpha}(y, z, x) - 2J_{\alpha}(\alpha(y), \alpha(x), y \cdot z). \]  

(2.7)

Now, the permutation of $z$ with $x$ in (2.7) gives

\[ J_{\alpha}(y \cdot z, \alpha(y), \alpha(x)) = \alpha^2(y) \cdot J_{\alpha}(y, x, z) - 2J_{\alpha}(\alpha(y), \alpha(z), y \cdot x), \]

i.e.

\[ 2J_{\alpha}(y \cdot z, \alpha(y), \alpha(x)) = -2\alpha^2(y) \cdot J_{\alpha}(y, z, x) - 4J_{\alpha}(\alpha(y), \alpha(z), y \cdot x), \]

or

\[ 4J_{\alpha}(\alpha(y), \alpha(z), y \cdot x) = -2\alpha^2(y) \cdot J_{\alpha}(y, z, x) - 2J_{\alpha}(y \cdot z, \alpha(y), \alpha(x)). \]  

(2.8)

Next, the subtraction of (2.8) from (2.7) gives (keeping in mind the skew-symmetry of $J_{\alpha}(x, y, z)$)

\[ -3J_{\alpha}(\alpha(y), \alpha(z), y \cdot x) = 3\alpha^2(y) \cdot J_{\alpha}(y, z, x) \]

i.e.

\[ J_{\alpha}(\alpha(y), \alpha(z), y \cdot x) = J_{\alpha}(y, z, x) \cdot \alpha^2(y) \]

so that we get (1.3).

\[ \square \]

Remark. If set $\alpha = Id$, then the identity (1.2) (resp. (1.3)) reduces to the identity (2.26) (resp. (2.4)) of [10]. The equivalence of (2.4) and (2.26) of [10] could be deduced from the works [10] and [11].

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