Flavor diagonal tensor charges of the nucleon from $2+1+1$ flavor lattice QCD

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We present state-of-the-art results for the matrix elements of flavor diagonal tensor operators within the nucleon state. The calculation of the dominant connected contribution is done using eleven ensembles of gauge configurations generated by the MILC collaboration using the HISQ action with $2+1+1$ dynamical flavors. The calculation of the disconnected contributions is done using seven (six) ensembles for the strange (light) quarks. These high statistics simulations allowed us to address various systematic uncertainties. A simultaneous fit in the lattice spacing and the light-quark mass is used to extract the tensor charges for the proton in the continuum limit and at $M_\pi = 135$ MeV: $g_T^p = 0.784(28)$, $g_7^p = -0.204(11)$ and $g_T^s = -0.0027(16)$. Implications of these results for constraining the quark electric dipole moments and their contributions to the neutron electric dipole moment are discussed.

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I. INTRODUCTION

High precision calculations of the matrix elements of flavor diagonal quark bilinear operators, $\bar{q}\Gamma q$ where $\Gamma$ is one of the sixteen Dirac matrices, within the nucleon state provide a quantitative understanding of a number of properties of nucleons and their interactions with electroweak probes. In this paper, we present results for the tensor charges, $g_T^d$, $g_T^s$ and $g_T^u$, that give the contribution of the electric dipole moment (EDM) of these quark flavors to the EDM of the nucleon. They are defined as the nucleon matrix elements of the renormalized tensor operator, $Z_T \bar{q} \sigma^{\mu\nu} q$ with $\sigma^{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$, $Z_T$ the renormalization constant and $q$ the bare quark field:

$$\langle N(p,s)|Z_T \bar{q} \sigma^{\mu\nu} q|N(p,s)\rangle = g_T^d \bar{u}_N(p,s)\sigma_{\mu\nu}u_N(p,s).$$

(1)

Experimentally, they can be extracted from semi-inclusive deep-inelastic scattering (SIDIS) data [1-3]. These tensor charges also provide the hadronic input to the WIMP-nucleus cross section in dark matter models that generate tensor quark-WIMP operators [4].

Our first calculations of these tensor charges were reported in Refs. [5,6], and here we provide an update using significantly more data that has allowed us to control all systematic uncertainties in both the connected and disconnected contributions. We show that even though the disconnected contributions are small, $O(0.01)$, the data are precise enough to extrapolate them to the continuum limit and evaluate them at the physical pion mass $M_\pi = 135$ MeV. In particular, we report a signal in $g_T^s$, whose contribution to the neutron EDM can be enhanced versus $g_T^p$ by $m_s/m_u \approx 40$ in models in which the chirality flip is provided by the Standard Model Yukawa couplings.

II. LATTICE METHODOLOGY

All the calculations were done on ensembles with $2+1+1$-flavors of HISQ fermions [7] generated by the MILC Collaboration [8]. In order to calculate the matrix elements of flavor diagonal operators, one needs to evaluate the contribution of both the “connected” and “disconnected” diagrams. The lattice methodology and our strategy for the calculation and analysis of the two-point and connected three-point functions using Wilson clover fermions on the HISQ ensembles has been described in Refs. [5,9-11] and for the disconnected contribution in Refs. [5,12].

The details of the calculation and analysis of the connected contributions on eleven ensembles covering the range 0.15-0.06 fm in the lattice spacing, $M_\pi = 135$–320 MeV in the pion mass, and $M_\pi L = 3.3$–5.5 in the lattice size have been presented in Ref. [13] and readers are referred to it. These high-statistics calculations allowed us to analyze the three systematic uncertainties...
due to lattice discretization, dependence on the quark mass and finite lattice size, by making a simultaneous fit in the three variables $a, M_x^2$ and $M_x L$. The final results for the connected contribution to the proton with this chiral-continuum-finite-volume extrapolation, reproduced from Ref. [12], are

$$
g^{u-d}_T = 0.989(32) \quad g_T^{u-d}|_{\text{conn}} = 0.590(25),$$
$$
g^{u+d}_T|_{\text{conn}} = 0.790(27) \quad g_T^{u+d}|_{\text{conn}} = -0.198(10).$$

In this paper, we focus on the analysis of the disconnected contributions and extracting the final results for the flavor diagonal tensor charges by combining them with the connected contributions. The lattice parameters of the seven ensembles used in the analysis of the disconnected contributions for the light and strange quarks are the same as in Ref. [12], as are the number of configurations analyzed and the number of random sources used to stochastically evaluate the disconnected quark loop on each configuration.

### III. CONTROLLING EXCITED-STATE CONTAMINATION (ESC)

The first step in the analysis is to understand and remove the excited state contamination (ESC) in the disconnected contribution. A number of features stand out in the data shown in Fig. 1. First, for a given value of the source-sink separation $\tau$, the data are much more noisy compared to the corresponding connected contribution analyzed in Ref. [13]. Second, within statistical uncertainties, there is no clear distinction between results at different $\tau$ values. In fact, the data at the various source-sink separations $\tau$ overlap for both $g^l_T$ and $g^r_T$, and no ESC is apparent. Lastly, the magnitude, in most cases, is smaller than 0.01, which is smaller than the statistical uncertainty in the connected contribution. Thus, the ESC is expected to be even smaller, so for the estimate on each ensemble, we take a simple average over the multiple $\tau$ data. The results for the bare charges, $g^l_T$ and $g^r_T$, obtained from the average shown in Fig. 1 are given in Table I.

Note that, since there is no evidence for ESC we average the data over the various $\tau$ values, and the disconnected contribution is very small, the uncertainty due to analyzing the connected and disconnected contributions separately using the QCD spectral decomposition, as discussed in Refs. [12] [13], is expected to be even smaller.

### IV. RENORMALIZATION OF THE OPERATORS

Flavor diagonal light quark operators, $\bar{q} \Gamma q$, can be written as a sum over isovector $(u-d)$ and isoscalar $(u+d)$ combinations which renormalize differently—isovector with $Z^{\text{isovector}}$ and isoscalar with $Z^{\text{isoscalar}}$. The isovector renormalization constants are given in Ref. [13]. The difference between $Z^{\text{isovector}}$ and $Z^{\text{isoscalar}}$ starts at two-loops in perturbation theory and is expected to be small. For the tensor operators, this expectation has been checked for the twisted mass action; explicit non-perturbative calculations have shown that $Z^{\text{isovector}} = Z^{\text{isoscalar}}$ to within a percent [14] [15]. Since the errors in our data are larger, we take $Z^{\text{isoscalar}} = Z^{\text{isovector}}$ and renormalize all three charges, $g^{u}_T$, $g^{d}_T$ and $g^{s}_T$, using $Z^{\text{isovector}}$. Using these factors, both the connected and disconnected contributions are renormalized in two ways:

$$
g^{l,s}_T|_{R1} = g_T \times Z^{\text{isovector}},$$
$$
g^{l,s}_T|_{R2} = \frac{g_T}{g_V} \times Z^{\text{isovector}}_{V}. \quad (3)$$

The second definition uses the conserved vector charge condition $g^{u-d}_V \times Z^{V}_V = 1$. These two results for the renormalized disconnected contributions on each ensemble are also given in Table I. They are extrapolated separately to the continuum limit and $M_x = 135$ MeV, and the extrapolated results are given in Table II.

### V. THE CONTINUUM-CHIRAL EXTRAPOLATION

The last step in the analysis is to evaluate the results at $M_{x_0} = 135$ MeV and in the continuum and infinite volume limits, $a \rightarrow 0$ and $M_x L \rightarrow \infty$. Since the range of $M_x L$ spanned by our disconnected data is small, $3.9 < M_x L < 4.8$, and the connected contributions showed no significant finite volume corrections, we neglect these in the analysis of the disconnected contributions. Thus, we fit the renormalized data given in Table II keeping just the leading correction terms in $a$ and $M_x$:

$$
g^{l,s}_T(a, M_x, L) = c_1 + c_2 a + c_3 M_x^2 + \ldots, \quad (4)$$

The data with the renormalization method $R2$ and the results of the fits are shown in Fig. 2. The dependence of both $g_T$ and $g_V$ on $M_x$ and $a$ is small and the extrapolated value is consistent with what we obtain by just averaging the six (seven) points. We consider the final errors from the fit reasonable as they are larger than those in most individual points and cover the total range of variation between the points. The extrapolated results for the two ways of doing the renormalization are given in Table II along with the $\chi^2$/DOF of the fit using Eq. (4). The final value is taken to be the average of the two and summarized in both Tables II and III.

### VI. COMPARISON WITH PREVIOUS WORK

In Table III, we also give results obtained by the ETMC collaboration [14] using a single physical mass...
TABLE II. Results for the renormalized disconnected contributions to the proton obtained in the limit $a = 0$ and $M_{q}\pi = 135$ MeV using Eq. (3). To perform the continuum-chiral extrapolation, separate extrapolations (labeled $g_{1}$ and $g_{2}$) have been carried out for the two ways of constructing the renormalized charges defined in Eq. (3). We also give the fits and the final results, which are obtained by averaging $Z_{\text{const}}$ columns four–seven. They are obtained using a simple average over the data shown in Fig. 1 since no significant ESC is evident. In columns four–seven, we give the renormalized charges $g_{1}$ for $\text{disc}$ and $\text{isc}$.

| $\tau$   | $g_{1}^{\text{disc}}$ | $g_{1}^{\text{isc}}$ | $\chi^{2}/\text{DOF}$ |
|----------|------------------------|------------------------|------------------------|
| $\tau = \infty$ | 0.0022(10)             | 0.0022(10)             | 0.31                   |
| $\tau = 9$   | 0.0016(21)             | 0.0016(21)             | 0.31                   |
| $\tau = 10$  | 0.0016(9)              | 0.0016(9)              | 0.31                   |
| $\tau = 11$  | 0.0021(8)              | 0.0021(8)              | 0.31                   |
| $\tau = 12$  | 0.0022(9)              | 0.0022(9)              | 0.31                   |

FIG. 1. The data for the disconnected contribution of the light (top two rows) and strange (bottom two rows) quarks to the bare tensor charge $g_{\text{T}}^{\text{asc}}$ are given in columns two and three. They are obtained using a simple average over the data shown in Fig. 1 since no significant ESC is evident. The boxcar renormalization constant $Z_{\text{box}}$ is used in all cases as discussed in the text.
FIG. 2. The data for the disconnected contribution of the light (left two panels) and strange (right two panels) quark to the tensor charges, $g_{T,disc}^{u}$ and $g_{T,disc}^{s}$. The results, in the continuum limit and at the physical pion mass, using the fit ansatz given in Eq. (4), are shown by the red star. In each panels, the result of the fit is shown versus $a$ ($M_{T}$), with the other variable set to its physical value. For comparison, the grey band shows the fit versus only $M_{T}^{2}$, i.e., ignoring the dependence on $a$.

FIG. 3. (Left) Constraints on the BSM couplings of the CP-violating quark EDM operator using the current experimental bound on the nEDM $(2.9 \times 10^{-26} \text{ ecm})$ and assuming that only these couplings contribute. (Right) Regions in $M_{T}$-$\mu$ plane corresponding to various values of $d_{n}/d_{e}$ in split SUSY, obtained by varying $g_{T,disc}^{u,d,s}$ within our estimated uncertainties. The solid black line correspond to $d_{e} = 8.7 \times 10^{-29} \text{ cm}$ with $\sin \phi = 1$. The allowed region lies above it, in which, in a band of constant $d_{n}/d_{e}$, the allowed values of both $d_{n}$ and $d_{e}$ decrease.
TABLE III. Final results for the individual connected and disconnected contributions to the flavor diagonal charges. Our new results, labeled PNDME’18, are the sum of the two contributions and given in the third row. In the fourth row we give the results from the ETMC collaboration [14] for comparison. These were obtained from a single physical mass ensemble at $a=0.0938(4)$ fm, $M_{\pi}=130.5(4)$ MeV, i.e., without a continuum extrapolation. In the last row we reproduce our PNDME’15 results [5] to highlight the improvement achieved with higher statistics and more ensembles.

|          | $g_n^d$ | $g_s^d$ | $g_L^d$ |
|----------|---------|---------|---------|
| Connected| 0.790(27) | -0.195(10) |         |
| Disconnected| -0.0064(33) | -0.0064(33) | -0.0027(16) |
| PDNME’18 (Sum) | 0.784(28) | -0.204(11) | -0.0027(16) |
| ETMC’17 [14] | 0.782(21) | -0.219(17) | -0.00319(72) |
| PNDME’15 [5] | 0.774(66) | -0.233(28) | 0.008(9) |

VII. IMPLICATIONS FOR NEUTRON ELECTRIC DIPOLE MOMENT

The tensor charges for the neutron, in the isospin symmetric limit, are obtained by interchanging the light quark labels, $u \leftrightarrow d$, in the results for the proton given in Table III. Using these and the experimental bound on the nEDM ($d_n \leq 2.9 \times 10^{-26} \text{ e cm}$ [19]), the relation

$$d_n = d_n^{u T} + d_n^{d T} + d_n^{s T},$$

provides constraints on the CP violating quark EDMs, $d_n^{q T}$, arising in BSM theories, assuming that the quark EDM is the only CP-violating BSM operator. The bounds on $d_n^{q T}$ are shown in the left panel of Fig. 3. Of particular importance is the reduction of the error in $g_{T}^{d}$ compared to our previous result in Ref. [5]. The new results let us bound $d_n^{q T}$. Conversely, the overall error in $d_n$ is reduced even if $d_n^{q T}$ is enhanced versus $d_n^{q}$ by $m_s/m_u \approx 40$ as occurs in models in which the chirality flip is provided by the Standard Model Yukawa couplings.

In general, BSM theories generate a variety of CP-violating operators that all contribute to $d_n$ with relations analogous to Eq. [5]. As discussed in Ref. [6], in the “split SUSY” model [17,19], the fermion EDM operators provide the dominant BSM source of CP violation. In Fig. 3 (right), we update the contour plots for $d_n / d_L$ in the gaugino ($M_{\tilde{g}}$) and Higgsino ($\mu$) mass parameter plane with the range 500 GeV to 10 TeV. For this analysis, we have followed Ref. [20] and set $\tan \beta = 1$.

Thanks to the greatly reduced uncertainty in the tensor charges (factor of $\approx 6$ for $g_L^d$ and $\approx 2-3$ for $g_T^d$), the ratio $d_n/d_c$ is much more precisely known in terms of SUSY mass parameters. This allows for stringent tests of the split SUSY scenario with gaugino mass unification [17,19]. In particular, our results and the experimental bound $d_c < 8.7 \times 10^{-29} \text{ e cm}$ [21], imply the split-SUSY upper bounds $d_n < 3 \times 10^{-28} \text{ e cm}$. This limit is falsifiable by the next generation experiments.

VIII. CONCLUSIONS

The data in Table III show that the PNDME’18 results (this work) with much higher statistics are a significant improvement over the PDNME’15 [5] values, in which we had neglected the disconnected contribution to $g_{T}^{u}$ and $g_{T}^{d}$ because the value on each ensemble was consistent with zero and the quality of the data was insufficient to perform a chiral-continuum fit. The reduced uncertainty has tightened the constraints on the quark EDM couplings and on the ratio $d_n / d_c$ in the split SUSY scenario with gaugino mass unification [17,19] as shown in Fig. 3.

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