Neutrino masses, dark energy and the gravitational lensing of pre-galactic H I

R. Benton Metcalf

Max Planck Institut für Astrophysics, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany

ABSTRACT

We study the constraints which the next generation of radio telescopes could place on the mass and number of neutrino species by studying the gravitational lensing of high-redshift 21-cm emission in combination with wide-angle surveys of galaxy lensing. We use simple characterizations of reionization history and of proposed telescope designs to forecast the constraints and detectability threshold for neutrinos. It is found that the degeneracy between neutrino parameters and dark energy parameters is significantly reduced by incorporating 21-cm lensing. The combination of galaxy and 21-cm lensing could constrain the sum of the neutrino masses to within \( \sim 0.04 \) eV and the number of species to within \( \sim 0.1 \). This is an improvement of a factor of 2.6 in mass and 1.4 in number over a galaxy lensing survey alone. This includes marginalizing over an 11-parameter cosmological model with a two-parameter model for the dark energy equation of state. If the dark energy equation of state is held fixed at \( w = -1 \), the constraints improve to \( \sim 0.025 \) eV and 0.04. These forecasted errors depend critically on the fraction of sky that can be surveyed in redshifted 21-cm emission (25 per cent is assumed here) and the redshift of reionization (\( z = 7 \) is assumed here). It is also found that neutrinos with masses too small to be detected in the data could none the less cause a significant bias in the measured dark energy equation of state.

Key words: gravitational lensing – intergalactic medium – cosmological parameters – dark matter – large-scale structure of Universe.

1 INTRODUCTION

One of the most tantalizing questions in experimental particle physics – the nature of neutrino mass – is connected to one of the most tantalizing questions in observational cosmology – the nature of dark energy. Cosmological observations presently put the most stringent upper limits on the absolute mass of neutrinos (Seljak et al. 2005; Spergel et al. 2007) and might continue to do so for some time to come. However, a partial degeneracy between neutrino masses and the dark energy equation of state limits how well either one can be determined from cosmological measurements based on the evolution of structure formation.

Atmospheric and solar neutrino oscillation experiments strongly indicate that neutrinos have mass and that the sum of their masses is larger than the measured mass splittings \( \sum m_\nu \geq \sqrt{\Delta m_{\text{atm}}^2} + \sqrt{\Delta m_{\text{sol}}^2} \simeq 0.05 \) eV. Direct measurements from \( \beta \)-decay experiments give an upper bound on the electron neutrino mass of 2.5 eV; there are no competing measurements of the other flavours (see Fogli et al. 2006 for a review of experimental results). Combinations of cosmological data give a constraint of \( \sum m_\nu < 0.66 \) eV assuming a flat cosmology with a cosmological constant, but this limit is considerably looser if the density of dark energy is allowed to evolve with time (Hannestad 2005; Seljak et al. 2005; Spergel et al. 2007).

Massive neutrinos in the appropriate mass range are relativistic when they decouple from the photons, electrons and baryons when the temperature of the Universe is a few MeV. They have the effect of suppressing the power spectrum of density fluctuations on small scales (scales smaller than the horizon when the neutrinos become non-relativistic, \( k > k_{\text{eff}} \simeq 0.018 \sqrt{m_\nu \Omega_\nu h} \) \( \text{Mpc}^{-1} \) \( \text{eV}^{-1/2} \), where \( k \) is the Fourier wave number, \( m_\nu \) is the neutrino’s mass and \( h \) is the Hubble parameter in units of 100 km s\(^{-1}\) Mpc\(^{-1}\)) due to free streaming. This suppression begins at the time of decoupling and continues until today because the thermal velocity of the neutrinos is still significant [\( v_{\text{therm}} \simeq 150 (1 + z) m_\nu \text{ km s}^{-1} \text{eV}^{-1} \) when non-relativistic]. The free-streaming scale continues to decrease from its maximum of \( k_{\text{eff}} \) as \( k_{\text{eff}} \simeq 0.82 \frac{\sqrt{\Delta m_{\text{atm}}^2} \sqrt{\Delta m_{\text{sol}}^2}}{(1+z)^2} \left( \frac{m_\nu}{\text{eV}} \right) h \) \( \text{Mpc}^{-1} \).
Neutrinos will resist falling into dark matter haloes with velocity dispersions \( \lesssim v_{\text{esc}} \). For a review of neutrinos in cosmology, see Lesgourgues & Pastor (2006).

It has been proposed that neutrino masses could be measured by the next generation of weak gravitational lensing surveys (Euclid\(^1\), Large Synoptic Survey Telescope\(^2\), PanSTARRS\(^3\), Dark Energy Survey\(^4\), Hannestad, Tu & Wong 2006; Kitching et al. 2008) and that the existence of massive neutrinos may limit the ability of these surveys to measure properties of the dark energy (Hannestad 2005; Kiakotou, Elgaroy & Lahav 2007). Dark energy causes structure formation to evolve differently than it would otherwise at low redshift (\( z \lesssim 1 \)). A degeneracy arises between this and the late-time free streaming of massive neutrinos since at scales significantly smaller than \( k_H \), the suppression is scale independent and the lensing surveys will be sensitive to a limited range in \( k \). However, at \( z \gg 1 \) dark energy is expected to have negligible effects on structure formation, so if higher redshifts can be probed the degeneracy can be removed.

The cosmic microwave background (CMB) provides information on the power spectrum at \( z \sim 1000 \), but Silk damping limits its sensitivity to scales below \( k_H \). Gravitational lensing of pre-galactic 21-cm radiation can provide a probe of structure formation on the scales needed, at the redshifts needed.

The prospects for measuring gravitational lensing of the pre-galactic 21-cm radiation have been studied by a number of authors (Zahn & Zaldarriaga 2006; Hilbert, Metcalf & White 2007; Lu & Pen 2007; Metcalf & White 2007, 2009). This paper is an extension to the forecasts given in Metcalf & White (2009) and the methods used here are described there in more detail.

## 2 Formalism and Modelling

### 2.1 Pre-galactic 21-cm radiation

A thorough review of pre-galactic 21-cm radiation can be found in Furlanetto, Oh & Briggs (2006). For this paper, we will mention only the barest essentials needed to specify our model for 21-cm emission. The fluctuations in the 21-cm brightness temperature depend on the spin temperature, \( T_s \), the ionization fraction, \( x_{HI} \), and the density of \( H_1 \) through

\[
\delta T_b \simeq 24(1 + \delta_b)x_{HI} \left( \frac{T_s - T_{\text{CMB}}}{T_s} \right) \left( \frac{\Omega_m h^2}{0.02} \right) \left( \frac{1.15}{\Omega_m h^2} \right)^{1/2} \mathrm{mK}
\]

(\( T_{\text{CMB}} \) = 2.73 K). As is commonly done, we will assume that the spin temperature is much greater than the CMB temperature. This leaves fluctuations in \( x_{HI} \) and the baryon density \( \delta_b = (\rho_b - \bar{\rho}_b)/\bar{\rho}_b \) as the sources of brightness fluctuations. We will make the simplifying assumption that \( x_{HI} = 1 \) until the Universe is very rapidly and uniformly reionized at a redshift of \( z_{\text{reion}} \). We will take \( \delta_b \) to be distributed in the same way as dark matter according to the cold dark matter model. Non-linear structure formation (Peacock & Dodds 1996) and linear redshift distortion (Kaiser 1987) are included. Realistically, the reionization process will be inhomogeneous and may extend over a significant redshift range. This will increase \( C_r(z) \) by perhaps a factor of 10 on scales larger than the characteristic size of the ionized bubbles (Zaldarriaga, Furlanetto & Hernquist 2004) and make the distribution non-Gaussian.

### 2.2 Gravitational lensing

It is convenient to express the lensing formalism in terms of the convergence, \( \kappa(\theta, z) \), at a position \( \theta \) on the sky which to an excellent approximation is related directly to the distribution of matter through which the light passes

\[
\kappa(\theta, z) = \frac{3}{4} H_0 \Omega_m \int_0^\infty \frac{dz}{E(z)} \left( \frac{1 + z}{1 + z_s} \right) g(z, z_s) \delta(\theta, z)
\]

\[
\approx \frac{3}{4} H_0 \Omega_m \int_0^\infty \left( \frac{\delta(\theta, z)}{\delta(\theta, z_s)} \right) \frac{dz}{E(z)} \left( \frac{1 + z}{1 + z_s} \right) g(z, z_s)
\]

\[
= \sum_i G(z_i, z_s) \delta(\theta, z_i)
\]

with

\[
g(z, z_s) = \frac{\int_z^{z_s} dz' \eta(z', z_s) D(z', 0) D(z', z_s)}{D(z, 0)}
\]

\( H_o \) is the Hubble parameter. The convergence can be thought of as a projected dimensionless surface density. The weighting function for the source distance distribution, \( \eta(z) \), is normalized to unity. \( D(z', z) \) is the angular size distance between the two redshifts and \( \delta(x, z) \) is the fractional density fluctuation at redshift \( z \) and perpendicular position \( x \). The function

\[
E(z) = \sqrt{\Omega_m (1 + z)^3 + \Omega_\Lambda (1 + z)^2}^{-1/2},
\]

where \( \Omega_m \) and \( \Omega_\Lambda \) are the present day densities of matter and dark energy measured in units of the critical density. It is assumed that the Universe is flat (\( \Omega_m + \Omega_\Lambda = 1 \)). The function describing the evolution of dark energy with redshift can be written as

\[
f(z) = \frac{1}{\ln(1 + z)} \int_{\ln(1+z)}^{0} \left[ 1 + w(a) \right] \frac{\mathrm{d} \ln a}{a}
\]

where \( w(a) \) is the equation of state parameter for the dark energy – the ratio of the of its pressure to its density – and \( a = (1 + z)^{-1} \) is the scale parameter.

For our purposes, it is convenient to express equation (3) as a matrix equation,

\[
K = \delta \Phi,
\]

where the components of \( K \) are the convergences running over all position angles \( \theta \) and source redshifts, \( z_s \). The components of the vector \( \delta \) run over all position angles and foreground redshifts \( z_i \).

1. http://www.dune-mission.net
2. http://www.lsst.org
3. http://pan-starrs.ifa.hawaii.edu
4. https://www.darkenergysurvey.org/
where $C^I_\ell(k)$ is the power spectrum of the actual brightness temperature, while $C_\ell(k) = C^I_\ell(k) + C^N_\ell(k)$ is the observed power spectrum which includes noise and $\ell$ is the angular Fourier mode number. Because the estimator is a sum over all the observed pairs of visibilities, it will (by the central limit theorem) be close to Gaussian distributed even though it is quadratic in the visibilities.

The $\kappa(\ell, z)$ can be grouped into a data vector

$$D = \hat{K} - K$$

$$= \hat{K} - G\delta,$$

where the components run over all the combinations of $z_i$ and $\ell$ that are measured. To a good approximation, modes from different bands and $\ell$ separated by more than the resolution of the telescope are statistically independent, so the covariance matrix for $D$ is diagonal

$$N_{ij} \simeq \delta_{ij}N^2(\ell_i, v_i),$$

with $|\ell_i - \ell_j| > \delta$ larger than the resolution. The likelihood function for the correlations in $\kappa(\ell, z)$ is given by

$$\ln \mathcal{L} = -\frac{1}{2} \hat{\kappa}^T C^{-1} \hat{\kappa} - \frac{1}{2} |C|,$$

where

$$C = N + C_k$$

(Metcalfe & White 2009). $C_k$ here is the (cross-)power spectrum of the convergence for two different source redshifts,

$$[C_k]_{ij} = \langle \kappa(z_i, \ell) \kappa(z_j, \ell) \rangle.$$

This can be calculated using expression (2) and a model for the power matter spectrum.

2.3 Structure formation with massive neutrinos

To calculate the linear matter power spectrum, we use the analytic formulae of Eisenstein & Hu (1999) with the modification of Kiakotou et al. (2007) that improve accuracy when the neutrino masses are small. We use the method of Peacock & Dodds (1996) to transform this linear power spectrum into a power spectrum with non-linear structure formation. This method has not been tested thoroughly against simulations of non-linear structure formation with massive neutrinos, but we have compared our resulting non-linear power spectra with those from the simulations of Brandbyge et al. (2008). We find that the maximum suppression caused by neutrinos (at $k \sim 1 h \text{Mpc}^{-1}$) is reproduced very well ($<1$ per cent accuracy) while the region around the baryon oscillation peaks are not produced as well (peaks of $\sim 10$ per cent inaccuracy) mostly because the Eisenstein & Hu (1999) fitting formula does not reproduce these oscillations in the linear power spectrum. A number of authors have attempted to describe structure formation with neutrinos analytically (see, for example, Bond, Efstathiou & Silk 1980; Singh & Ma 2003; Ringwald & Wong 2004; Lesgourgues et al. 2009), but these results are either not applicable to sufficiently small scales or difficult to implement numerically over the large range in scale and redshift that we require. Our method reproduces the non-linear power spectrum accurately enough for the simple forecasting methods used here.

We assume that there are $N_\nu$ species of neutrinos with the same mass. The total density of neutrinos is fixed by the physics at the time of decoupling to be $\Omega_\nu h^2 = \sum m_\nu/93.14 \text{eV}$. If the masses are not degenerate, the measured $N_\nu$ will not be an integer. These are the parameters we will try to constrain. Roughly speaking, $N_\nu$ controls the average mass of the neutrino species and through this the free-streaming scale while $\Omega_\nu h^2$ affects the degree of suppression to the power spectrum.

The measurements we consider are actually sensitive to any light particle species with significant cosmological densities not just neutrinos. The interpretation of the parameters would be different for relic particles that were produced in a different way, for example axions, but the late-time physics would be the same.

2.4 Model observations

2.4.1 21-cm observations

We will concentrate on the planned the Square Kilometre Array (SKA) because it is the only planned telescope that will have a large enough size to be relevant to neutrino constraints. It is only the core of the telescope that will be used for observing pre-galactic 21-cm radiation. Plans for the SKA core have not been finalized, but it is expected that we will have to design the core to be able to have a diameter of $D_\text{tel} \sim 6 \text{ km}$ ($\ell_{\text{max}} \sim 10^8$), a aperture covering fraction of $f_\text{cover} \sim 0.02$ [the total collecting area of the telescopes divided by $\pi(D_\text{tel}/2)^2$] and a frequency range extending down to $\sim 100 \text{ MHz}$ which corresponds to $z \sim 13$. It is expected that the SKA will be able to map the 21-cm emission with a resolution of $\Delta \theta \sim 1 \text{ arcmin}$. For reference, one arcminute (full width at half-maximum) corresponds to baselines of 5.8 km at $z = 7$ and 11 km at $z = 15$. We will take $\ell_{\text{min}} = 10$ to be the lowest mode to be measured.

The noise in each visibility measurement will have a thermal component and a component resulting from imperfect foreground subtraction. Here, we model only the thermal component. If the telescopes in the array are uniformly distributed on the ground, the average integration time for each baseline will be the same and the power spectrum of the noise will be

$$C^N_\ell = \frac{2\pi}{\Delta\nu T_{\text{sys}}} \left( \frac{T_{\text{sys},\lambda}}{f_{\text{cover}} D_\text{tel}} \right)^2 = \frac{(2\pi)^2 T^2_{\text{sys}}}{\Delta\nu T_{\text{sys}} f_{\text{cover}}^2 \ell_{\text{max}}(\nu)^2},$$

(Zaldarriaga et al. 2004; Morales 2005; McQuinn et al. 2006) where $T_{\text{sys}}$ is the system temperature, $\Delta\nu$ is the bandwidth, $\ell$ is the total observation time and $\ell_{\text{max}}(\lambda) = 2\pi D_\text{tel}/\lambda$ is the highest multipole that can be measured by the array, as set by the largest baselines. At the relevant frequencies, the overall system temperature is expected to be dominated by galactic synchrotron radiation. We will approximate the brightness temperature of this foreground as $T_{\text{sky}} = 180 \text{K}(v/180 \text{ MHz})^{-2.6}$, as appropriate for regions well away from the Galactic plane (Furlanetto et al. 2006). This results in larger effective noise for higher redshift measurements of the 21-cm emission. We will consider an observation time of 90 d which might be achievable within three seasons of observation and we will assume that the survey covers 25 per cent of the sky.

2.4.2 Galaxy weak lensing survey

For comparison and for combining with the 21-cm lensing, we will consider a model galaxy lensing survey. The noise in power spectrum estimates from such a survey can be written as $N_\ell(\ell) = \sigma^2/\ell^g$, where $\sigma_\ell$ is the angular number density of background galaxies and $\sigma\ell$ is the rms intrinsic ellipticity of those galaxies. This neglects all systematic errors as well as photometric redshift uncertainties.

$^5$ www.skatelescope.org
Following standard assumptions, we model the redshift distribution of usable galaxies as $\eta(z) \propto z^2 e^{-l(z)_{0.3}^{1.5}}$, where $z_0$ is set by the desired median redshift, and we adopt $\sigma_v = 0.25$. The Euclid satellite (the imaging part of which was previously known as DUNE$^6$) proposes to survey 20,000 deg$^2$ on the sky to a usable galaxy density of $n_g \simeq 35$ arcmin$^{-2}$ with a median redshift of $z \sim 0.9$. Several planned ground-based surveys – LSST, PanSTARRS – will cover comparable areas to Euclid at a similar depth. In order to use tomographic information, we divide the galaxies into 10 redshift bins each containing the same number of galaxies.

### 2.4.3 CMB observations

The Planck Surveyor$^7$ will do a full sky survey of the CMB radiation with higher resolution and more sensitivity to polarization than is now available. We incorporate these future measurements into our forecasts by calculating the expected Fisher matrix. To do this, we use the same technique as described in the appendix of Rassat et al. (2008). This includes the contributions from both the scalar and tensor perturbations.

### 2.5 The cosmology

We use an 11-parameter cosmological model. The energy densities in dark energy and baryons are $\Omega_m$ and $\Omega_\Lambda$ in units of the critical density. The density of matter is fixed to $\Omega_m = 1 - \Omega_\Lambda$ to make the geometry flat. The primordial power spectrum is

$$P_s(k) = A_s \left( \frac{k}{H_0} \right)^n_{s} \Delta n_{s} \ln k.$$  

The $\Delta n_{s}$ parameter is included because if the primordial power spectrum is not a pure power law, it might partially mock the effect of early neutrino free streaming on the power spectrum. The time-dependent dark energy equation of state has two parameters $w(a) = w_0 + w_a(1 - a)$. There are two neutrino parameter $\nu_m$ and $\sum m_\nu$. Including $\Delta n_{s}$ and a non-constant $\nu_m$ makes this a more general set of parameters with more potential degeneracies than has usually been used when predicting constraints on massive neutrinos. The optical depth to CMB last scattering surface is $\tau$. The fiducial model is set to $\{\tau, h, \Omega_\Lambda h^2, \Omega_m h^2, n_s, \Delta n_{s}, w_0, w_a, N_e\} = \{0.09, 0.705, 0.361, 0.0227, 1, 0, -1, 0, 3\}$. The normalization, $A_s$, is set so that the fluctuations within a sphere of radius 8 Mpc is $\sigma_8 = 0.812$ in the fiducial model. These are the Wilkinson Microwave Anisotropy Probe 5 yr concordance model values (Komatsu et al. 2009). Two values for $\Sigma, m_\nu$ are used, 0.66 and 0.09 eV. All calculated parameter constraints are marginalized over the other parameters.

### 3 RESULTS

To assess how well the proposed observations could measure neutrino properties, we adopt two statistical methods that are popular in the literature – Fisher matrix forecasts and the Bayesian evidence method.

$^6$ www.dune-mission.net

$^7$ www.rssd.esa.int/index.php?project=Planck

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Figure 1. The forecasted constraints on the combined mass and number of neutrinos with the fiducial model $\{\sum m_\nu, N_e\} = \{0.66 \text{eV}, 3\}$ marginalized over all the other cosmological parameters. The dotted curves are for a Euclid-like galaxy lensing survey, the dashed curves are for a 21-cm lensing survey, the solid curves are for the tomographic combination of the two surveys and the dot-dashed curve is for Planck by itself.

### 3.1 Fisher matrix forecast

The maximum likelihood estimate for any parameter can be found by maximizing (12) with respect to that parameter. The error in this estimator is often forecast using the Fisher matrix defined as

$$F_{ij} = - \left( \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right).$$

The expected error in the parameter $p_i$ marginalized over all other parameters, is $\sigma^2_i = (F^{-1})_{ii}$. The un marginalized error estimate (the error when all other parameters are held fixed) is $(F_{ii})^{-1}$.

Since $\ell$ modes separated by more than the resolution of the telescope will not be correlated, we can break the likelihood function up into factors representing each resolved region in $\ell$ space (Metcalfe & White 2007). The result is that there are $\sim (2\ell + 1)f_{\text{sky}}$ independent measured modes for each value of $\ell$, where $f_{\text{sky}}$ is the fraction of sky surveyed. The Fisher matrix can then be further simplified to the widely used form,

$$F_{ab} = \frac{1}{2} \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1) f_{\text{sky}} \text{tr} \left[ C^{-1} C^{-1} C^{-1} \right].$$

The error estimates on the neutrino parameters are shown in Figs 1 through 3 for combinations of different data and model assumptions. Fig. 1 shows the constraints using the three kinds of data by themselves and the combination of galaxy and 21-cm lensing. These lensing surveys are combined tomographically not just by adding their Fisher matrices as would be the case if they were independent measurements. 10 redshift bins with equal numbers of galaxies are used and 10 redshift bins of fixed width are used for the 21-cm lensing. The CMB does a relatively good job of constraining the $N_e$, but not as good a job of constraining the sum of the masses.

In Fig. 2, the CMB constraints are combined with the lensing constraints. There is a drastic improvement because the parameter degeneracies are significantly reduced (note the change in scale from Fig. 1). The forecasted errors are 0.04 eV and 0.1 for the combination of lensing surveys with $\sum m_\nu = 0.66$. This is an improvement of 2.6 in mass and 1.4 in $N_e$ over either of the lensing surveys by themselves. In Fig. 3, the dark energy equation of state is kept fixed ($w = -1$). The improvement in the constraints illustrates the degeneracy between neutrinos and dark energy parameters. The
that neutrinos are massless, the maximum likelihood would not give
the correct value for \( w_o \).

These calculations are in agreement with previous calculations of
the constraints from future galaxy lensing surveys (Hannestad et al.
2006; Kitching et al. 2008) where more restricted cosmological
models were used and thus stronger constraints were found.

### 3.2 Bayesian evidence

Another statistical question that could be asked is whether the data
require massive neutrinos. One way to answer this question is us-
ing the average ratio of the Bayesian evidence (Jeffreys 1961) for
models with and without massive neutrinos. This technique has
been used extensively in cosmological parameter estimation (Saini,
Weller & Bridle 2004; Liddle et al. 2006; Heavens, Kitching &
Verde 2007 for example) and galaxy lensing surveys specifically
(Kitching et al. 2008).

The Bayesian evidence is the probability of the data, \( D \), given
the model, \( M \), marginalized over the parameters of that model, \( \{\theta\} \),

\[
E(D|M) = \int d\theta p(D|\theta, M)p(\theta|M).
\]

(19)

The integral is over all of the parameter space. Bayes factor is the
ratio of the evidences for two competing models,

\[
B = \frac{E(D|M_o)}{E(D|M_1)}.
\]

(20)

Here, model \( M_o \) is the simpler model with \( n_o \) parameters and model
\( M_1 \) is more complex and has more parameters, \( n_1 > n_o \).

To make a forecast, Bayes factor must be averaged over expected
data sets. It will be assumed that model \( M_o \) is the real case so
that the averaging is according to this model. We are asking how
well we can expect to rule out model \( M_1 \). In calculating this,
we use the approximation to \( \langle B \rangle \) derived by Heavens et al. (2007) the
Savage-Dickey ratio

\[
\langle B \rangle \simeq (2\pi)^{-(n_1 - n_o)/2} \left| \frac{F^{(1)}}{F^{(0)}} \right| \exp \left\{ -\frac{1}{2} \delta^2 \theta_o F^{(1)}_{\alpha\alpha} \delta \theta_o \right\},
\]

(21)

where

\[
\delta \theta_i = \left[ (F^{(0)})^{-1} \right]_{ij} F^{(1)}_{\alpha\mu} \delta \theta_{\alpha}
\]

(22)

and the range of the indexes are \( i = 1, \ldots, n_o, \alpha = n_o + 1, \ldots, n_1 \)
and \( \mu, \nu = 1, \ldots, n_1 \). We have assumed that there are no a priori
limits on the range of the neutrino parameters.

Fig. 5 shows evidence ratio as a function of the deviation from
the fiducial model, \( \{\sum_{\alpha}m_\alpha, N_\alpha\} = \{0, 3\} \). Again, we see that
the constraints from the lensing of galaxies and 21 cm put similar
constraints on the neutrino parameters. By combining these data
sets, the constraints are improved by more than a factor of 2 in
\( \sum_{\alpha}m_\alpha \). A combined mass as small as 0.2 eV should be detected, 0.19
eV if \( N_\alpha \) is fixed to three. Fig. 6 shows how these constraints change
if the dark energy equation of state is fixed to the cosmological
constant value, \( \{w_o, w_1\} = \{-1, 0\} \). The constraints drastically
improve which again demonstrates the degeneracy between dark
energy and massive neutrinos. In this case, a combined mass of
0.09 eV would be detectable.

### 4 DISCUSSION

If it is assumed that neutrinos are massless, or have too small a
mass to be of significance, the dark energy equation of state, \( w = p/\rho \), could be underestimated from cosmological constraints. This
would be the case even if the data does not allow for a detection of neutrino mass. It is also true that any independent constraint on the neutrino mass would improve future cosmological constraints on dark energy. The Karlsruhe Tritium Neutrino Experiment (KATRIN)\(^8\) \(\beta\)-decay experiment expects to reach a level of 0.2 eV for the electron neutrino mass and thus might have an impact on dark energy constraints. In Section 3, it was shown that a neutrino mass as small as \(\sum m_{\nu}/\eta_B = 0.03\) eV could bias \(w_o\) high by \(\sim 1\sigma\) in future galaxy lensing surveys if it is not accounted for and if it is accounted for the error bars increase by a factor of 3.4 for \(w_o\) and 2 for \(w_a\). Lensing of high-redshift 21 cm is a way to reduce this degeneracy by adding constraints on the growth of structure at higher redshifts where dark energy is presumed to contribute very little. Type Ia Supernovae surveys and surveys aimed at measuring the baryon acoustic oscillations at \(z \sim 1\) will put constraints on dark energy that are based on the luminosity distance redshift relation and so are independent of the growth of structure. It is important that methods based both on structure formation and on luminosity distance are fully realized to avoid systematic errors. The two probes are also necessary to test alternative theories of gravity on large scales which might also provide an explanation for the apparent acceleration of the cosmological expansion.

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