Magnetic Component of Quark-Gluon Plasma is also a Liquid!

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The magnetic scenario recently suggested in [1] emphasizes the role of monopoles in strongly coupled quark-gluon plasma (sQGP) near/above the deconfinement temperature, and specifically predicts that they help reduce its viscosity by the “magnetic bottle” effect. Arguments for “magnetic liquid” in 1-2Tc based on lattice results of monopole density were provided in [2]. Here we present results for monopole-(anti)monopole correlation functions from the same classical molecular dynamics simulations, which are found to be in good agreement with lattice results in [3]. We show that the magnetic Coulomb coupling runs in the direction opposite to the electric one, as expected, and it is roughly inverse of the asymptotic freedom formula for the electric one. However, as T decreases to Tc, the magnetic coupling never gets too weak, with the plasma parameter always large enough (Γ > 1). This nicely agrees with empirical evidences from RHIC experiments, implying that magnetic objects cannot have large mean free path and should also form a good liquid with low viscosity.

I. RUNNING COUPLINGS

Discussions of electric-magnetic duality appear in theoretical literature regularly since Maxwell’s time. Especially important for what follows are: (i) the celebrated Dirac quantization condition [4], (ii) ’t Hooft-Polyakov monopole solutions [7], (iii) “dual superconductor” idea of confinement by ’t Hooft and Mandelstam and (iv) Seiberg-Witten solution of the N = 2 SUSY gauge theory [6], identifying properties and dynamical role of magnetically charged objects in this setting.

Before we focus on our main subject – monopole mutual interactions – let us first comment on other theoretical issues related with situations in which both “electric” and “magnetic” particles are present at the same time.

For pure gauge fields electric/magnetic duality simply means rewriting magnetic field B as gradient and electric field E as a curl of a “dual” potential: however there are nontrivial questions about the sources (and boundary conditions). The electric objects – quarks – are traditionally present in the Lagrangian as Noether charges, while monopoles are solitonic solutions carrying topological charges. Can “magnetic” formulation be consistently defined, interchanging their roles and putting monopoles in the Lagrangian instead? Can even a situation be found in which both formulations are similar? These ideas were discussed starting from the famous paper [8].

Since both description should describe the same theory serious issues of consistency appear. At the quantum mechanics level the famous Dirac [4] condition must be held, demanding basically that the product of two couplings is fixed to an integer (put here to 1)

\[ \alpha_E \ast \alpha_M = 1 \] (1)

So while one of them may be small, the other must necessarily be large.

At the level of quantum field theory the Dirac condition elevates into a requirement that two couplings must run into the opposite directions

\[ \tilde{\beta}(\alpha)_E + \tilde{\beta}(\alpha)_M = 0 \] (2)

where the beta functions are \( \tilde{\beta}(\alpha) = \frac{\mu}{\alpha} \frac{d\alpha}{d\mu} = 2\beta(g)/g \) for the electric and magnetic couplings respectively, with \( \beta(g) = \mu g \frac{\beta g}{g} \) being the usual beta function. This indeed is what happens in Seiberg-Witten solution, in which electric coupling is weak at large momenta due to asymptotic freedom, and magnetic is weak at small ones due to U(1) “Landau pole”.

In our previous paper [1] we discussed for sQGP the important role of this generic feature. As it is known for 30 years, QGP at high T can be described perturbatively, with e.g. small quark and gluon effective masses \( M/T \sim \sqrt{\rho_{\text{electric}}} << 1 \). The monopoles in this case are heavy composites which play a minor role, although they are strongly interacting and form an interesting sub-sector in which perturbative analysis is impossible. However as T goes down and one approaches the deconfinement transition \( T \rightarrow T_c \), the inverse is expected to happen: electrically charged particles – quarks and gluons – are getting heavier and more strongly coupled. There are strong evidences that both phenomena do happen: lattice data show that quark (baryon) masses seem to be large in sQGP near \( T_c \) [9], while RHIC’s “perfect liquid” [10,11] supports the idea of strong coupling.

However at this point one may ask what happens with monopoles: as \( T \rightarrow T_c \) the same logic suggests that they must become lighter and more important. At some point their masses (and roles) get comparable to that of the electric objects, after which the tables are turned and their fortune reversed. Electric objects gets strongly coupled and complicated while monopoles gets lighter, proliferate and eventually take over the bulk, expelling electric fields into the flux tube. As shown in [12], this may...
happen in the plasma phase, before confinement transition is reached. The corresponding phase diagram was discussed in [1]. Here are two questions, on which will we be focused below: (i) Are there evidences that the magnetic coupling does run in the opposite direction? (ii) How small does the magnetic coupling become at \( T \rightarrow T_c \), and is a perturbative description of magnetic plasma possible? As the reader will see, we will answer “yes” to the first and “no” to the second question.

(A digression about the most symmetric \( N=4 \) SYM theory which is conformal. How do we know that its coupling does not run? One may calculate the first coefficient of the beta function, and will indeed see that negative gauge contribution is nicely cancelled by fermions and scalars. But there are infinitely many coefficients, and one has to check them all! An elegant way to prove the case is based on another outstanding feature of the \( N=4 \) SYM: this theory is (nearly) self-dual under electromagnetic duality. As we discussed above, the Dirac condition requires the product of electric and magnetic couplings to be constant. But, as shown by Osborn [13], in Higgs case the multiplet of (lowest) magnetic objects of the \( N=4 \) SYM theory include 5 scalars, 4 fermions (monopoles plus one gluino zero mode occupied), plus 1 spin-1 particle (3 polarizations), exactly the same set of states as in the original electric multiplet (gluon-gluinos-Higgses). Thus effective magnetic theory has the same Lagrangian as the original electric formulation and the same beta function. That would conflict with requirement that both couplings run in the opposite direction, unless they do not run at all!)

II. MOLECULAR DYNAMICS WITH MONOPOLES

It has become apparent around 2003 [10,11] that Quark-Gluon Plasma discovered at Relativistic Heavy Ion Collision (RHIC) is the most “perfect liquid” known. Theorists have since been taking the big challenge to explain the remarkable properties of sQGP with diverse approaches, ranging from those borrowed from classical plasmas to AdS/CFT duality: for recent reviews see [14].

Molecular Dynamics (MD) simulations are proved to be powerful tools widely used for studying the conventional EM plasma, especially in the strongly coupled regime when the analytic approaches are difficult. It provides detailed microscopic real time information about correlation functions and transport properties. It has recently been employed to simulate sQGP in [15,1]. The key classical Coulomb plasma parameter \( \Gamma \) is defined by

\[
\Gamma \equiv \frac{\alpha_C / \left( \frac{4\pi}{3} \right)^{1/3}}{T}
\]

with \( \alpha_C \) the Coulomb coupling between charges, \( Q = E \) or \( M \), and \( n \) the density of the corresponding charges. It means the ratio of the potential energy (interaction) to the thermal kinetic energy; thus a weakly coupled plasma has \( \Gamma \ll 1 \). Plasmas with \( \Gamma > 1 \) are known as strongly coupled, and for \( \Gamma = 1 \), \( \Gamma_c \sim 100 \) it is in the liquid regime, becoming solid for \( \Gamma > \Gamma_c \). Less precisely defined “glassy” regime is at \( \Gamma \approx 10 \ldots \Gamma_c \) in between. The electric \( \Gamma \) parameter in sQGP is believed to be \( \Gamma_E \sim 3 \) at \( T = (1 - 2)T_c \) [15], so it corresponds to a liquid. The value of the magnetic parameter \( \Gamma_M \) is the subject of this Letter.

In our previous paper [1] we explored (to our knowledge, for the first time) a novel plasmas made of a mixture of both electric and magnetic charges. Using MD and standard Kubo formulae we calculated its transport properties, i.e. the shear viscosity and diffusion constant. We found that an equal 50%-50% mixture has smallest viscosity (the shortest mean free path, if one want to use such language) because of the “magnetic bottle’ effect.

Here we present an important property that was not discussed in [1], namely the particle-particle equal-time density-density correlator depending on the distance \( r \):

\[
G_{ab}(r) = \frac{1}{V} \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \frac{\delta(|r_i^a - r_j^b| - r)}{\rho_a \rho_b 4\pi r^2}
\]

where \( a, b \) denotes two species of particles with total numbers \( N_a, N_b \) in a volume \( V \) and \( \rho_a, \rho_b \) their density. In our MD simulation the monopole and anti-monopole numbers are both set to be 250 inside a sphere with radius 10 \( a_{MD} \). \( a_{MD} \) is the length unit in MD and corresponds to about 0.2\( fm \) after mapping to sQGP system: see [1] for more details. The correlator of the monopole-anti-monopole pair is particularly instructive because it tells how strongly the monopoles are correlated. The result is shown in Fig.1, for the 50%-50% electric-magnetic symmetric plasma with \( \Gamma = 2.3 \). It features a considerable nearest-neighbor peak in the monopole-anti-monopole correlator, reaching around 1.25, with hints to small secondary correlations: a typical liquid behavior, see various

![Figure 1](image-url)
examples in e.g. [16]. The same correlator for a weakly coupled gas will have a very weak peak, barely above 1, while a solid will have strong multiple peaks. The monopole-monopole correlator shows strong suppression at short distance while stays very close to 1 at larger $r$: also typical of Coulomb plasma. The reader shall recall that we do not consider here the usual Coulomb plasma due to presence of equal number of electric particles in our mixture – with which monopoles interacts with a strong Lorentz force, leading to very complicated motion – so the result is far from trivial.

To get a quantitative measure we have fitted them at large $r$ (where deviation from 1 is small and linearized screening is correct) by the Debye formula

$$G_{ab}(r) \sim e^{\pm \frac{\alpha_M e^{-r/R_d}}{rT}}$$

FIG. 2. (color online) Monopole-antimonopole (the upper two curves) and monopole-monopole (the lower two curves) correlators at $1.1 T_c$ (red long dashed) and $3.8 T_c$ (blue dot dashed): points with error bars are lattice data [3], the dashed lines are our fits (see text).

III. LATTICE DATA AND MONOPOLE CORRELATIONS

Lattice studies of monopoles have a long history which we would not even attempt to summarize here (see e.g. [17]). We will not discuss here the properties of these monopoles – such as masses or their interaction with quarks/ghons or “Higgs field” $A_0$ - to be addressed in [18] ). We will focus entirely on one aspect of these data, related to the correlators (4) and $\Gamma$ parameter just discussed. Those have been obtained recently in [3], for technical reasons for the simplest SU(2) pure gauge theory. Fig.2 shows two sets of their data for $T = 1.1 T_c$ and $T = 3.8 T_c$ for monopole-antimonopole and for monopole-monopole correlations.

Note that the curves are very similar to the ones in our MD studies shown in previous section, especially the shape and magnitude of the opposite-sign peak. The most important feature apparent from these (and other) plots is that the correlation gets stronger at the higher $T$, confirming our expectation that the magnetic component of sQGP gets stronger coupled at high $T$, oppositely to the electric component.

To get a quantitative measure we have fitted them at large $r$ (where deviation from 1 is small and linearized screening is correct) by the Debye formula

$$G_{ab}(r) \sim e^{\pm \frac{\alpha_M e^{-r/R_d}}{rT}}$$

FIG. 3. (color online) (a) The magnetic coupling $\alpha_M$ (on Log$_{10}$ scale) versus $T/T_c$ fitted from the monopole-antimonopole (boxes with solid blue curve) and monopole-monopole (triangles with dashed blue curve) correlators. Their inverse, the corresponding $\alpha_E$ from the Dirac condition, are shown as stars with solid red curve and diamonds with dashed red curve respectively, together with an asymptotic freedom (green dotted) curve (see text); (b) Effective magnetic plasma coupling $\Gamma_M$ versus $T/T_c$, with the gray band schematically showing high-T limit $\Gamma_M^*$ (see text).

In (5) the positive sign in the exponent is for monopole-antimonopole ($ab = + -$) while the negative for monopole-monopole ($ab = + +$), with $\alpha_M$ is the magnetic coupling, and the $R_d$ magnetic screening radius. See the dashed curves in Fig.2. Some details about the fitting for each data set: we exclude the last few
points that are rather flat and marginally lower/higher than 1 for $G_{++}/G_{++}$, which the fitting formulæ can not reach: the rule to determine how many points from large $r$ toward the peaks to be included is to make $\chi^2/d.o.f$ as close to 1 as possible: the $\chi^2/d.o.f$’s for the eight $T’$s are 1.16,1.96,1.17,0.46,0.82,0.16,1.16,1.23 for monopole-antimonopole correlators and 3.65,2.87,2.16,2.48,2.02,1.06,0.89,1.36 for monopole-monopole correlators. The values of $\alpha_{\chi^2} = \frac{M}{\chi^2}$ obtained are all in 2–3 region with slight tendency to decrease toward higher $T$.

The resulting values of magnetic coupling from fitting both monopole-antimonopole and monopole-monopole correlators for all available temperatures are shown in Fig.3(a): as expected $\alpha_M$ is getting weaker at low $T$ and stronger at high $T$. Inverting these values (because of the Dirac condition (1)) one gets the respective electric couplings, which we compare to the dashed green curve corresponding to the pure gauge SU(2) one-loop asymptotic freedom expression $\alpha_E \approx \frac{2}{\ln(2(\ln C + T/T_c))}$ with the coefficient $C = 54$ determined from the last point at $T_c = 3.8T_c$. The one-loop running becomes much slower as $T$ decreases, which is normal and shall be cured by higher order loops and more importantly by non-perturbative corrections such as instanton, see [19].

Finally, in Fig.3(b) we combine the data on the magnetic coupling and the monopole density from the same work to get the dimensionless magnetic plasma parameter (3) in the studied range $T \approx 1–4T_c$. We use a formula $n(T)/T^3 = 0.557/[\ln(2.69 + T/T_c)]^2$ from [3] which was found to nicely fit the monopole density data. The $\alpha_M$ used is the average of the fitted values from monopole-antimonopole and monopole-monopole correlator, with half of their difference as the error bar. As one can see, magnetic component of QGP never gets to be a weakly coupled gas, as $\Gamma_M > 1$ at all $T$ even close to $T_c$. On the other hand, even at the highest $T$ the value of $\Gamma_M$ does not reach large values $> 10$ at which liquids are known to become glass-like and viscosity starts growing.

In the high $T$ limit we expect “magnetic scaling” $n_M \sim (\alpha_E T)^{3}$, thus plasma parameter approach a fixed point $\Gamma_M \sim \alpha_M n_M^{1/3}/T = \Gamma^*_M$, due to Dirac condition. The curve in Fig.3(b) indicate $\Gamma^*_M \sim 5$.

**IV. SUMMARY**

In this Letter we have shown that gauge theory monopoles in a deconfined phase behave as charges in a Coulomb plasmas. Furthermore, we show that the temperature dependence (running) of the magnetic couplings in gauge theories is indeed the inverse of the electric one, electric-magnetic duality arguments [1]. Good agreement was found (in shape and magnitude) of the correlators we calculated in MD simulation [1] for novel electric-magnetic plasmas and recent lattice results [3]. More specifically, we concluded that the magnetic part of QGP at $T = (1 - 4)T_c$ has an effective plasma parameter in the “good liquid” domain [20] $\Gamma = 1 - 4.5$, not spoiling the “perfect liquid” observed at RHIC. We predict existence of a fixed point for $\Gamma$ at high $T$.

**Acknowledgments.**

We are grateful to A. D’Alessandro and M. D’Elia, who provided us with extensive set of their data in tabular form, going beyond the content of their paper [3]. We thank Claudia Ratti for helpful discussions. This work was supported in parts by the US-DOE grant DE-FG-88ER40388.

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Based on the monopole density they obtained on lattice, the authors of [2] suggested the magnetic component becomes gaseous above $2T_c$, see also separate argument in [17]. However the correlators from [3] clearly showed stronger correlation at higher $T$ (see Fig.2), and our results for $\Gamma$ in Fig.3(b) also showed that at $T > 2T_c$ the magnetic component is still a liquid with even larger $\Gamma_M$. 