An aggregation decomposition for the analysis and synthesis of spacecraft control systems

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Abstract. This paper focuses on the interconnections between the variants of assembling the elements of spacecraft control systems within the considered alternative graphic formalization approach. Under these circumstances, the assembly variants of the system elements or their sets are depicted as peaks of an alternative graph, with arcs representing interconnections. The aggregation decomposition method in the analysis and synthesis of the structure of complex systems includes two interrelated stages: sequential decomposition of objectives, functions, tasks, etc., performed by the system, and their aggregation at appropriate detail levels.

1. Introduction

Complex systems are characterized by the complexity of interaction between their structural elements, which may be dispersed over a large territory and interconnected through complex dynamic interconnections. These may occur in the process of the systems’ functioning. A typical example of such a system is the multifunctional distributed automated information and management framework (e.g.: a unified national transportation network). Several important issues arise in the process of analyzing and synthesizing such systems, the most critical of which concerns the systems’ structure, which, largely, determines the properties of the system and characteristics of its functioning [1–3].

The structure is understood to be the organization of the system from individual elements with their interconnections, which are determined by the distribution of functions and executed tasks. Thereby the structure is a way of organizing a whole from separate component parts. The structure reflects the structure and internal organizational form and the existing relatively stable interconnections between separate elements. Complex systems have a large number of elements, properties, and interconnections between elements, making, therefore, a single description of the structural aspects of the system a complicated theoretical and practical task. In practice, these processes are analyzed and studied by examining separate aspects of the system’s internal structure. For instance, in the design of distributed
automated information and management systems, we can distinguish the following structural types: organizational, topological (spatial), functional, the data structure of the computer network, etc.

The system’s structure determines the composition and interconnections of two types of elements – control and production (or information) transport elements, as well as the distribution of management functions by elements of the organizational hierarchy and preplanned tasks of the control system through the elements of information transport system.

The problem of synthesizing the control system’s structure includes: (1) selecting a number of control levels and subsystems (the control hierarchy); (2) coordinating the tasks of the subsystems at different levels; (3) developing decision-making contours; (4) distributing executive functions (tasks, information arrays, and procedures) to levels and units of the system; (5) selecting the technical means for information transfer and processing [4–6].

2. The aggregation decomposition method

Within the framework of aggregation decomposition method, the structure of the projected system can be described at different detail levels by a set of alternative graphs. The objectives and functions of the system are formalized as an interconnected set of alternative graphs GF and GI.

In some cases, the objectives and functions can be detailed to separate control tasks, arrays and information processing procedures (figure 1, where 1 represents functions, goals; 2 represents tasks, arrays, procedures; 3 represents control units; 4 is the onboard software; 5 is the spacecraft; 6 are the information elements).

![Figure 1. Interconnected set of alternative columns.](image)

The control nodes are formalized as an interconnected set of alternative graphs G_f and G_C. Likewise, the controlled system is formalized at the level of received information from the spacecraft through separate payload blocks (graphs G_r and G_p (figure 1)).

At the system’s multilevel model, obtained in the form of an interconnected set of alternative graphs varying in elements detail, the tasks of synthesizing the structure of subsystems and the system as a whole are set. In this case, two types of problems can be classified:

- Optimization of the system’s structure at the same level of detail (formally, it is reduced to searching for optimal subgraphs on the generated models of systems of this level).
- The optimal structure of the system (formally reduced to constructing and selecting the optimal display of alternative graphs of different levels).
In order to formalize the interconnections between different variants of the system’s structure, by means of the considered approach, the alternative-graphic formalization with various variants for arranging the system’s elements (or their sets) is established in the form of alternative graph peaks. The arcs, in their turn, reflect the interconnections between the peaks [7–9].

The structure synthesis task by means of such a formalization lies in determining the optimal display of a set of interrelated functions (tasks, arrays, and procedures) and variants of their execution (I) through a set of interrelated nodes and variants for their layout (J). These two variables are described by the alternative graphs $G_1$ and $G_l$. The resulting alternative graph $G_0(H_0,D_0)$ formalizes the existing options for the structure. A characteristic feature is the multi-domain individual reflection of the distribution of the functions of each node and the arcs of the graph, which characterize the interconnection between the nodes.

A wide array of tasks for the synthesis of the system’s structure through this formalization is formulated as: an alternative graph of possible $G_0$ implementation options is shown; it is necessary to choose the subheading $G^+ \subset G_0$ optimizing the given characteristics.

$$F_0(G^+) \rightarrow \text{opt},$$

and satisfying the limitations set in the subheading as a whole:

$$F_\alpha(G^+)RP_\alpha, \alpha = 1, \alpha_0$$

(2)

on separate parts of graph $G^+_\mu \subset G_\mu, G_\mu \subset G_0$ (including the elements of the sets (I, J, H_0))

$$F_\beta(G^+_\mu)RP_\beta, \beta = 1, \beta_0; \mu = 1, \mu_0; \; G^+ \subset G_0.$$ (3)

The emerging problem statements are classified by the presence and method of option formalization and the interconnections between the elements of the graph, the properties of the graph, and the type of target functions and their constraints.

The main characteristics of the structure of the systems are expressed in additive form if they depend only on the composition of the selected elements in the graph, or in a recursive-computable form. This is true if the dependency is both on the corresponding characteristics of the elements of the selected options and the structure of their interconnection.

The main qualitative characteristics of the synthesized variants of the structure include the cost of system’s development and operation, the time characteristics of task execution, and the loading of nodes. Depending on the type of characteristics used for a criterion and its constraints, selecting the structure of the system represents a linear or nonlinear integer programming task. Non-linear characteristics, such as system performance, can sometimes be reduced to linear form by introducing additional variables and constraints. This allows using standard methods of linear integer programming for solving the considered tasks. For more complex problems, it is necessary to develop special solving methods such as ‘branches and boundaries’.

Consider the formalization of tasks for synthesizing the structure of a spacecraft control system (it is necessary to define a set of control units within the system at each level of the hierarchy and connections between them, a set of control tasks and their distribution by levels and nodes of the system). In this task, we will consider the system at two description levels: the level of functional control tasks and the level of organizational structure elements of the system (control nodes) (figure 2). $G_1$ shows the composition and interconnections (logical, temporal, volumetric) between the control tasks performed in the system, along with options for their execution; $G_2$ shows the composition, interconnections, and options for building control nodes in the system. Various characteristics are assigned to the elements of graphs $G_1$ and $G_2$, such as the cost of developing or operating the element, the time characteristics of its operation, etc.
The task of structural synthesis comes down to determining the optimal display \((m)\) of a set of interconnected variants of functions, tasks, and stages on a set of interconnected variants of system elements assembly.

Note that the set of \(m\) displays is equivalent to the set of subgraphs of a given type of graph \(G_O\), which formally can be obtained by applying the composition operation to graphs \(G_I\) and \(G_J\): 
\[
G_O = G_J \circ G_I
\]

The composition of graphs \(G=G_1 \circ G_2\) is a binary operation on graphs, the result of which is graph \(G(V,X)\), where \(V=V_1 \times V_2\) and the vertex \((U_1,U_2)\) is adjacent to \((\nu_1, \nu_2)\) if and only if \((U_1\text{ is adjacent to } \nu_1 \text{ or } U_1=\nu_1)\) and \(U_2\text{ is adjacent to } u_2\). The same result is given by the application of the graph conjunctions operation \((G_O=G_1 \land G_T\), where graph \(G_T\) contains loops in each vertex \((10, 11)\)).

In this case, the task can be set either in graphical terms, or as a task of mathematical programming.

The considered task of structural synthesis, which takes into account the interconnections between the tasks of volumetric type control, consists in determining system nodes and variants of task solution considering the expense of data exchange between the tasks, which reduces the exploitation costs of the developed system:

\[
\min \left\{ \sum_{i,j,k} b_{ijkl} x_{ijk}x_{ij'k'} + \sum_{i,j} c_{ij}x_{ij} \right\}
\]

where \(b_{ijkl}x_{ij}x_{ij'}x_{ij'}\) is \(a_{ijk}\), if the indexes of the variables \(x_{ij}\) and \(x_{ij'}\) coincide, and \(b_{ijkl}x_{ij}x_{ij'}\) otherwise; \(x_{ijk} = 1\), if the \(i\)-th task is solved in the \(j\)-th node in the \(k\)-th variant, and \(x_{ijk} = 0\) otherwise; \(a_{ij}\) are the operational costs of solving the \(i\)-th task with the \(k\)-th variant in the \(j\)-th node; \(a_{ik}x_{ij'}\) is the average volume of transmitted data between the tasks \(ik\) and \(ij'\); \(y_{ij'}\) are the costs of transmitting a unit of data between the nodes \(j\) and \(j'\); \(c_{ij}\) are the costs of operating the \(l\)-th technical means in the \(j\)-th node.

The following are subject to restrictions:

- Each of the tasks is performed in only one of the nodes
\[
\sum_{i,j} x_{ijk} = 1, \quad i = 1, l
\]

- The development resources do not exceed the set value
\[
\sum_{i,j} R_{ij}x_{ij} + \sum_{i,j} R_{ik}x_{ijk} \leq R
\]

here, \(R_l\) are the capital expenditures for equipment; \(R_{ik}\) are the development and implementation costs of the \(i\)-th task in the \(k\)-th variant.

- Other restrictions are recorded as linear inequalities from the introduced variables.
In [12], the proposed algorithms for solving the problem (4) – (6) are based on local optimization schemes and ‘branches and boundaries’. The solution of a special linear problem is used as estimates in branching.

The difficulties of the practical implementation of mathematical programming models are connected, firstly, with ensuring the completeness, accuracy, and reliability of the initial necessary data. This includes the dynamics of the system’s functioning and the multi-criteria nature of the selection of the variant structure.

The parameters of the synthesized systems, in some cases, might only be known approximately, with some distribution of the probabilities of their value or with a degree of parameter values belonging to given intervals. This leads to the ambiguity of determining the optimal variant of the synthesized system structure. Depending on the degree of awareness and type of uncertainty at the stage of structure synthesis, as well as on the objectives and value of the acceptable risk, different sets of objectives for the structural synthesis are possible. From a mathematical point perspective, the considered statements lead to the maximum problems of mathematical programming (if a guaranteed result is necessary), problems of stochastic programming, and problems formulated on fuzzy sets [13, 14].

A designated place is occupied by the approach during the process of formalization and solving problems for synthesis of system structure, which allows to take into account the uncertainty associated with dynamic and stochastic characteristics of system functioning.

In some cases it is necessary to consider various types of the uncertainties. For example, if the values of a number of structural parameters Ω are set vaguely, or it is known that they change in the process of system functioning in a certain range of values [Ω, Ω], then it is necessary to study the stability of the selected variant; for instance, selecting acceptable qualitative characteristics with unstable system parameters.

Considering the dynamics of the system’s functioning at the structural synthesis stage makes it mandatory to use optimization and simulation models. For the majority of practical tasks only simulation methods are viable for formalizing the functioning dynamics.

Consequently, there are issues considering the rational combination of such models for the reception of optimum (rational) variants for each system structure. This approach triggers certain iterative procedures for determining rational variants of the system’s structure of system. Using optimization and simulation models, it is possible to construct, estimate, and select rational options for the system assembly during the synthesis process.

3. Conclusion
The proposed approach includes a number of stages for a human-machine procedure: (1) the analysis of the initial problem; (2) the decomposition of the problem in order to separate the dynamic and stochastic characteristics; (3) the identification and study of interconnections between the stochastic and dynamic characteristics; (4) the construction of parametric optimization models; (5) the construction of appropriate simulation models; (6) the development of formalized procedures for analyzing the optimization and simulation results and algorithms for adjusting the optimization models parameters. From a mathematical perspective, the discussed objectives belong to a class of mathematical programming tasks, in which the number of limitations (and in some cases the target function) are set not in the explicit form (analytical expressions), but algorithmically by means of simulation models.

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