On the gravitational redshift

Klaus Wilhelm
Max-Planck-Institut für Sonnensystemforschung (MPS),
37077 Göttingen, Germany
wilhelm@mps.mpg.de

Bhola N. Dwivedi
Department of Physics,
Indian Institute of Technology (Banaras Hindu University),
Varanasi-221005, India
bholadwivedi@gmail.com

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Abstract

The study of the gravitational redshift—a relative wavelength increase of $\approx 2 \times 10^{-6}$ was predicted for solar radiation by Einstein in 1908—is still an important subject in modern physics. In a dispute whether or not atom interferometry experiments can be employed for gravitational redshift measurements, two research teams have recently disagreed on the physical cause of the shift. Regardless of any discussion on the interferometer aspect—we find that both groups of authors miss the important point that the ratio of gravitational to the electrostatic forces is generally very small. For instance, the gravitational force acting on an electron in a hydrogen atom situated in the Sun’s photosphere to the electrostatic force between the proton and the electron is approximately $3 \times 10^{-21}$. A comparison of this ratio with the predicted and observed solar redshift indicates a discrepancy of many orders of magnitude. Here we show, with Einstein’s early assumption of the frequency of spectral lines depending only on the generating ion itself as starting point, that a solution can be formulated based on a two-step process in analogy with Fermi’s treatment of the Doppler effect. It provides a sequence of physical processes in line with the conservation of energy and momentum resulting in the observed shift and does not employ a geometric description. The gravitational field affects the release of the photon and not the atomic transition. The control parameter is the speed of light. The atomic emission is then contrasted with the gravitational redshift of matter-antimatter annihilation events.
I. INTRODUCTION

The study of the gravitational redshift, a relative wavelength increase of $\Delta \lambda/\lambda \approx 2 \times 10^{-6}$ was predicted for solar radiation by Einstein\textsuperscript{15} in 1908, is still an important subject in modern physics\textsuperscript{8,31,34,63}. Jewell\textsuperscript{30} had found in electric arc spectra:

“[...] that the metallic lines were almost invariably displaced toward the violet, when compared with the corresponding solar lines.”

At that time—in 1896—a high pressure in the solar atmosphere was erroneously considered as causing the shift\textsuperscript{41}. Measurements of the gravitational redshift of solar spectral lines are inherently difficult, because high speeds of the emitting plasmas in the atmosphere of the Sun lead to line shifts due to the classical Doppler effect. Improved observational techniques\textsuperscript{7,40,62}, have nevertheless established a shift of

$$c_0 \frac{\Delta \lambda}{\lambda} \approx 600 \text{ m s}^{-1},$$

where $c_0 = 299 792 458$ m s\textsuperscript{-1} is the speed of light in the vacuum\textsuperscript{6} remote from any masses. This shift is consistent with Einstein’s General Theory of Relativity (GTR)\textsuperscript{18}. Together with various other aspects of GTR—from the deflection of light by a gravitational centre\textsuperscript{13,16,18,42,56} to Mercury’s perihelion precession\textsuperscript{17,46,64,66}, the current attempts to measure the Lense-Thirring effect\textsuperscript{38} on the planets’ motions caused by the solar rotation\textsuperscript{28,29}, and the Shapiro delay\textsuperscript{32,54,55}—the gravitational redshift is one of the experimental tests of GTR\textsuperscript{66}.

Atom interferometry experiments can be used to measure the acceleration of free fall, see, for instance, Müller et al.\textsuperscript{44}, Peters et al.\textsuperscript{49}. The same research team has in the meantime argued that atom interferometry can also perform gravitational redshift measurements at the Compton frequency. This claim was criticized as incorrect by Wolf et al.\textsuperscript{68} leading to a response in support of the original result\textsuperscript{45}. This controversy has continued until recently\textsuperscript{25–27,69,70}.

II. IS THERE A PHYSICAL PROCESS CAUSING THE REDSHIFT?

One aspect of the dispute between Müller et al.\textsuperscript{44} and Wolf et al.\textsuperscript{68} is particularly disturbing and will be analysed here in some detail: Even after the prediction of the gravitational
redshift by Einstein\textsuperscript{15} for over a century and the many observational confirmations mentioned in Section \[ \text{I} \] there appears to be no consensus on the physical process(es) causing the shift. This can be exemplified by two conflicting statements. The first made by Wolf et al.\textsuperscript{68} reads:

“The situation is completely different for instruments used for testing the universality of clock rates (UCR). An atomic clock delivers a periodic electromagnetic signal the frequency of which is actively controlled to remain tuned to an atomic transition. The clock frequency is sensitive to the gravitational potential \( U \) and not to the local gravity field \( g = \nabla U \). UCR tests are then performed by comparing clocks through the exchange of electromagnetic signals; if the clocks are at different gravitational potentials, this contributes to the relative frequency difference by \( \Delta \nu / \nu = \Delta U / c^2 \).”

Whereas in the second statement it is claimed by Müller et al.\textsuperscript{45}:?

“We first note that no experiment is sensitive to the absolute potential \( U \). When two similar clocks at rest in the laboratory frame are compared in a classical red-shift test, their frequency difference \( \Delta \nu / \nu = \Delta U / c^2 \) is given by \( \Delta U = gh + \mathcal{O}(h^2) \), where \( g = \nabla U \) is the gravitational acceleration in the laboratory frame, \( h \) is the clock’s separation, \( c \) is the velocity of light, and \( \mathcal{O}(h^2) \) indicates terms of order \( h^2 \) and higher. Therefore, classical red-shift tests are sensitive to \( g \), not to the absolute value of \( U \), just like interferometry red-shift tests.”

The potential at a distance \( r \) from a gravitational centre with mass \( M \) is constraint in the weak-field approximation for non-relativistic cases\textsuperscript{25} by

\[- c_0^2 \ll U = - \frac{G_N M}{r} \leq 0 , \tag{2}\]

where \( G_N \) is Newton’s constant of gravity. The authors of Ref.\textsuperscript{68} could refer to many publications in their support\textsuperscript{15,37,48,53,57,65}. However, it would be required to define explicitly a reference potential \( U_0 \). A definition in line with Eq. \( \text{(2)} \) would give \( U_0 = 0 \) for \( r = \infty \). Experiments on Earth\textsuperscript{11,23,33,50,51}, in space\textsuperscript{2} and in the Sun-Earth system\textsuperscript{3,5,7,41,59,60,62} have quantitatively confirmed in this approximation a relative frequency shift of

\[
\frac{\nu' - \nu_0}{\nu_0} = \frac{\Delta \nu}{\nu_0} \approx \frac{\Delta U}{c_0^2} = \frac{U - U_0}{c_0^2} , \tag{3}
\]
where $\nu_0$ is the frequency of a certain transition at $U_0$ and $\nu'$ the observed frequency there, if the emission caused by the same transition had occurred at a potential $U$. The question whether the shift happens during the emission process or is a result of a propagation effect is left open by Dicke in the final section of Ref.12:

“To return briefly to the question of the gravitational red shift, it is concluded that there could be two different red-shift effects. One would be interpreted in the usual way as a light propagation effect. The other, if it exists, would be interpreted as resulting from an intrinsic change in an atom with gravitational potential. The experiment employing an atomic clock in space would be one way of observing this effect directly, if it exists.”

There appears to be agreement, however, that the energy of a photon, $E_\nu = h \nu$, with Planck’s constant $h$, does not vary during the propagation in a static gravitational field—excluding a variation of $\nu$ with changing $U$, if $\nu$ is measured against the coordinate or world time. This is consistent with the time dilation of atomic clocks derived from the GTR and, consequently, the matter would be settled, if geometric effects were considered to be an adequate cause of the gravitational redshift. Straumann discussed the modification of the electric potential by gravity in this context.

Wolf et al. and Müller et al. have tried, however, to explore physical processes that cause the shift; yet both attempts are problematic in view of the fact that the gravitational force acting on the electron in transition is extremely small relative to the internal forces. This can easily be verified by a comparison of the weak solar gravitational force $K_G$ acting on the electron in a hydrogen atom in the photosphere of the Sun with the electrostatic force $K_E$:

\[
\frac{||K_G||}{||K_E||} = \frac{G N M_\odot m_e}{R_\odot^2} \left(\frac{e^2}{4 \pi \varepsilon_0 a_0^2}\right)^{-1} = \frac{r_S^\odot}{2 R_\odot^2} m_e c_0^2 \left(\frac{e^2}{4 \pi \varepsilon_0 a_0^2}\right)^{-1} = 3.031 \times 10^{-21}
\] (4)

with $G_N = 6.674 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$; $M_\odot = 1.989 \times 10^{30}$ kg, the mass and $R_\odot = 6.960 \times 10^8$ m, the radius of the Sun; $m_e = 9.109 \times 10^{-31}$ kg, the mass of an electron; $e = 1.602 \times 10^{-19}$ C, the elementary charge; $\varepsilon_0 = 8.854 \times 10^{-12}$ F m$^{-2}$, the permittivity of the vacuum; $a_0 = 5.292 \times 10^{-11}$ m, the Bohr radius; and $r_S^\odot = 2 G N M_\odot/c_0^2 = 2950$ m, the Schwarzschild radius of the Sun.

The early attempts to measure the gravitational redshift of solar spectral lines as well as those of the white dwarf star Sirius B have been reviewed by Hetherington. In particular,
the wrong value of 21 km s\(^{-1}\) published by Adams\(^1\) has been contrasted with the result of (89 ± 16) km s\(^{-1}\) obtained by Greenstein et al.\(^2\) for the companion of Sirius with \(R/R_\odot = 0.0078 \pm 0.0002\) and \(M/M_\odot = 1.20 \pm 0.25\). These radius and mass data inserted into Eq. (4) instead of the solar values give \(5.9 \times 10^{-17}\). Mean gravitational redshifts of (53 ± 6) km s\(^{-1}\) for six white dwarfs in the Hyades have been measured by Greenstein and Trimble.\(^2\)

Even for the very strong gravitational field of the neutron star EXO 0748-676, for which Cottam et al.\(^1\) found a redshift of \(z = 0.35\) in Fe\(\text{xxvi}\) and Fe\(\text{xxv}\) as well as in O\(\text{viii}\) lines, a calculation similar to Eq. (4) yields

\[
\frac{|\mathbf{K}_{\text{NS}}|}{|\mathbf{K}_{\text{E}}|} = \frac{r_S}{2R^2} m_e c_0^2 \left( \frac{e^2}{4\pi\varepsilon_0 a_0^2} \right)^{-1} \approx 2.5 \times 10^{-11}
\]

with \(R = 9.15\) km and \(r_S = 4130\) m for \(M = 1.4\) \(M_\odot\). These values of \(R\) and \(r_S\) lead to the observed redshift of

\[
z = \left(1 - \frac{r_S}{R} \right)^{-\frac{1}{2}} - 1 = 0.35.
\]

Leventhal et al.\(^3\) and Bowers\(^4\) discuss whether a spectral feature at \(\approx 400\) keV observed in the Crab Nebula might be the gravitationally redshifted 511 keV electron-positron annihilation line from the surface of the pulsar. Ramaty\(^5\) concluded that the relatively narrow widths of annihilation lines from gamma-ray bursts indicates emitting material close to the surface of a neutron star.

A gravitational redshift from galaxies in clusters has also been reported.\(^6\)

### III. TOWARDS A SOLUTION

#### A. Emission of spectral lines

The ratios obtained in Eqs. (4) and (5) support Einstein’s early assumption:\(^15\):

„Da der einer Spektrallinie entsprechende Schwingungsvorgang wohl als ein intraatomischer Vorgang zu betrachten ist, dessen Frequenz durch das Ion allein bestimmt ist, so können wir ein solches Ion als eine Uhr von bestimmter Frequenz \(\nu_0\) ansehen.”

(Since the oscillation process corresponding to a spectral line can probably be seen as an intra-atomic process, the frequency of which is determined by the ion alone, we can consider such an ion as a clock with a certain frequency \(\nu_0\)).
We feel that this view merits to be fully appraised, although Einstein later concluded that (atomic) “clocks” would slow down near gravitational centres.

Einstein also emphasized the importance of the momentum transfer during the absorption or emission of radiation:

„Bewirkt ein Strahlenbündel, daß ein von ihm getroffenes Molekül die Energiemenge $h \nu$ in Form von Strahlung durch einen Elementarprozeß aufnimmt oder abgibt (Einstrahlung), so wird stets der Impuls $\frac{h \nu}{c}$ auf das Molekül übertragen, und zwar bei der Energieaufnahme in der Fortpflanzungsrichtung des Bündels, bei der Energieabgabe in der entgegengesetzten Richtung. [...]“

(A beam of light that induces a molecule to absorb or deliver the energy $h \nu$ as radiation by an elementary process (irradiation) will always transfer the momentum $\frac{h \nu}{c}$ to the molecule, directed in the propagation direction of the beam for energy absorption, and in the opposite direction for energy emission.)

„Aber im allgemeinen begnügt man sich mit der Betrachtung des Energie-Austausches, ohne den Impuls-Austausch zu berücksichtigen.“

(However, in general one is satisfied with the consideration of the energy exchange, without taking the momentum exchange into account.)

Let us first assume an atom $A$ with mass $m$ in the ground state located at the gravitational potential $U_0 = 0$ and, therefore, with an energy of $E_0 = mc^2_0$. With an energy difference $\Delta E_0$ from the ground state to the excited atom $A^*$, the mass in this state is $14,36,37$

$$m + \Delta m = \frac{1}{c^2_0} (E_0 + \Delta E_0) .$$  \(7\)

The masses $M$, $m$, and $\Delta m$ constituting the total system considered here are assumed to comply with the inequality $M \gg m \gg \Delta m$, so that higher orders can be neglected in some of the equations. The “rest energy” with respect to the centre of gravity of $M$ and $m$ of the ground state at $U$ will then be $37$

$$E = E_0 + U m .$$  \(8\)

The definition of the rest energy in this context calls for some further explanations. If a particle with mass $m$ is lowered from $U_0 = 0$ to $U$, the potential energy will be converted, for instance, into kinetic energy of the particle, $E_{\text{kin}} = -U m$. The total energy of the particle
at \( U \) will thus be \( E_0 + E_{\text{kin}} \). Provided the kinetic energy is subsequently absorbed as thermal energy at \( U \), the remaining energy \( E_0 \) of the particle—at rest with respect to the centre of gravity—is obviously different from the rest energy in Eq. (8). The energy \( E_0 \) will, however, not be available for any photon emission at \( U \), because a lifting of the mass \( m \) to \( U_0 \) would require the potential energy \( U m \), whereas a photon would not change its energy during the transit from \( U \) to \( U_0 \), and could then be converted to mass there. This accounts for the difference between \( E_0 \) and the rest energy.

As will be shown later, see, e.g., Eq. (27), momentum considerations also lead to the requirement that only the rest energy of Eq. (8) can be emitted as photon.

We now consider the rest energy \( E^* \) of the excited atom \( A^* \) at \( U \) and find

\[
E^* = E_0 + \Delta E_0 + U m + U \Delta m ,
\]

where the remarks above apply as well. In view of these energy equations, the transition of \( A^* \) to the ground state at \( U \) can provide an energy of

\[
\Delta E = E^* - E = \Delta E_0 + U \Delta m ,
\]

which is in principle available for the photon emission. Whether the emitted photon has the expected energy and frequency, can be determined by observations; and the gravitational redshift measurements mentioned in Sect. II confirm indeed the right energy

\[
\Delta E = h \nu' ,
\]

where \( \nu' \) is measured with respect to the world time.

Nevertheless, the question remains how the atom can sense the potential \( U \) at the emission site and react accordingly. We will argue that—in line with Einstein’s remarks quoted—the momentum exchange must be taken into account, in addition to the interaction of the radiation energy with the potential energy of the emitting system. In preparation for this task, we list some relevant relations.

The momentum of a photon emitted at \( U_0 \) with frequency \( \nu_0 \) is

\[
p_0 = \frac{h \nu_0}{c_0} = \frac{\Delta E_0}{c_0},
\]

where \( \Delta E_0 = h \nu_0 \) is its energy. At \( U < 0 \), the energy of the photon can be written as

\[
\Delta E_0 = pc
\]
with a speed of light\(^4\)

\[
c \approx c_0 \left(1 + \frac{2U}{c_0^2}\right).
\] (14)

This speed is in agreement with an evaluation by Schiff for radial propagation in a central gravitational field\(^5\). A decrease of the speed of light near the Sun of this amount is not only supported by the predicted and subsequently observed Shapiro delay\(^3,5\), but also indirectly by the deflection of light\(^1,8\).

The problem can then be illustrated by different scenarios for the emission process:

(a) Under the assumption that the atom can somehow locally sense the gravitational potential \(U\), but not the speed \(c\), the energy given by Eq. (10) would lead to a momentum

\[
p = \frac{\Delta E}{c_0} = \frac{\Delta E_0 + U \Delta m}{c_0}
\] (15)
of the photon after the emission. We could then estimate its energy by applying Eqs. (13) and (14)

\[
p c \approx \frac{\Delta E_0 + U \Delta m}{c_0} \cdot c_0 \left(1 + \frac{2U}{c_0^2}\right) \approx \Delta E_0 + 3U \Delta m, \quad (16)
\]

with \(\Delta E_0/c_0^2 = \Delta m\) according to Eq. (7), and neglecting higher orders of \(U/c_0^2\). The energy thus obtained is in conflict with Eq. (10).

(b) If the atom can, however, sense the local speed of light \(c\), but not the potential \(U\), the photon emission energy will be \(\Delta E_0\), which is also in conflict with Eq. (10).

(c) If the atom can sense both the speed \(c\) and the potential \(U\), it then has to reduce the photon emission energy by a factor of \((1 + U/c_0^2)\) and, at the same time, increase the photon momentum by a factor of \((1 - U/c_0^2)\). Although this scenario is formally correct, it involves very unlikely processes.

(d) If Einstein’s assumption that only intra-atomic processes are of importance is valid, this is equivalent to the statement that the atom can sense neither \(U\) nor \(c\). The internal transition of \(A^*\) to the ground state of atom \(A\) then proceeds in the same way at \(U_0\) and \(U\); in both cases, accompanied by an energy release of \(\Delta E_0\) and a momentum of \(\Delta E_0/c_0\). The adjustment of the energy and momentum transfers to the rest system of the centre of gravity will be achieved during the actual photon emission at the speed \(c\), as will be detailed below.
The intra-atomic processes are indicated in rows 2 to 4 of Table 1. Starting from an excited atom $A^*$ at $U$, the transition energy and momentum are given according to Eqs. (7) and (12). We argue that only the propagation speed $c$ of photons in the environment of the emission location provides the necessary information for the energy and momentum adjustments in line with the corresponding conservation laws.

The sequence of events will be modelled according to an explanation of the Doppler effect based on energy and momentum conservations by Fermi\textsuperscript{20}, which has some resemblance to the Compton effect\textsuperscript{9}. Fermi discussed the interaction of the liberated energy during an atomic transition with the kinetic energy of the emitter and its momentum in a non-relativistic approximation.

In our case, the interactions of the potential energy and momentum during the emission of a photon can be formulated by the introduction of an arbitrary differential momentum vector $x$ parallel to $p_0$, which has to be determined by solving the momentum and energy equations of the atom-photon system in rows 6 and 7 of Table 1. Row 6 is clearly consistent with momentum conservation and row 7 leads to

$$\Delta E_0 - ||x|| c_0 = ||p_0 - x|| c_0 = p c = ||p_0 + x|| c$$

(17)

for the energy relationship. The kinetic energy $E_{\text{kin}}$, the recoil energy, can be neglected, because it is already very small with our assumption $m >> \Delta m$, but has been further reduced in the Pound–Rebka-experiment\textsuperscript{50} with the help of the Mößbauer effect\textsuperscript{43}. From Eq. (17), it follows with Eq. (14)

$$\frac{p_0 - x}{p_0 + x} = \frac{c}{c_0} \approx 1 + \frac{2 U}{c_0^2} ,$$

(18)

where $p_0 = ||p_0||$ and $x = ||x||$. The evaluation yields in our approximation

$$x \approx -p_0 \frac{U}{c_0^2} .$$

(19)

Hence, we get for the momentum of the photon

$$p \approx p_0 \left(1 - \frac{U}{c_0^2} \right) .$$

(20)

The result is that $p$ will be larger than $p_0$. This can be understood by considering that the energy transfer of $||x|| c_0$ in Eq. (17) back to the atom in the gravitational field of the mass $M$ must be accompanied by a momentum transfer of $p_0 U/c_0^2$ and a corresponding reaction on
the photon in line with Eq. (20). Note that the energy transfer \( x c_0 = -p_0 u / c_0 = -U \Delta m \) is of the same amount as the difference of potential energy gains by lowering \( m + \Delta m \) and \( m \) in the field. Taking the remarks related to Eqs. (8) and (9) into account, the energy levels before the emission of the photon are \( \Delta E_0 = \Delta m c_0^2 \) at \( U_0 = 0 \) and

\[
\Delta E = \Delta m c_0^2 - U \Delta m
\]  

(21)

at \( U \), where \(-U \Delta m\) is the potential energy at \( U_0 \) relative to \( U \) converted, for instance, into kinetic energy of the atom. Assuming it is brought to a halt by constraining forces, an energy \( \Delta E' = \Delta E_0 = \Delta m c_0^2 \) remains. As we have seen, it cannot directly be converted into energy, because of momentum considerations, but

\[
h \nu = \Delta E_0 \left( 1 + \frac{U}{c_0^2} \right) = \Delta m c_0^2 + U \Delta m
\]  

(22)

can be emitted and can propagate to \( U_0 \). The conversion of \( \Delta m \) into energy entails a loss of the potential energy gain of \(-U \Delta m\) mentioned above. It will be replenished by the energy transfer \( x c_0 \). The energy budget after the photon emission then is \( \Delta m c_0^2 + U \Delta m \) at \( U_0 \) plus \(-2U \Delta m \) at \( U \) giving a total of \( \Delta m c_0^2 - U \Delta m \) in agreement with Eq. (21). The gravitational redshift in Eq. (22) is consistent with Eq. (3) and observations.

B. The Compton frequency controversy of Wolf et al. and Müller et al.

In a formal way, we can also compare \( E^* \) of Eq. (9) with

\[
E_1 = E_0 + U_1 m ,
\]  

(23)

the rest energy of the ground state at a different potential \( U_1 = U + \delta U \) at a position close to that of the potential \( U \). If \( U_1 \) is chosen such that

\[
U_1 m = U (m + \Delta m) ,
\]  

(24)

subtraction of Eq. (23) from Eq. (9) gives

\[
E^* - E_1 = \Delta E_0 ,
\]  

(25)

which suggests that the energy \( \Delta E_0 \) would be available assuming a more or less instantaneous shift of the atom from \( U \) to \( U_1 \). This is, however, not possible. The selection of \( U_1 \) in Eq. (24), nonetheless, leads to the interesting relation

\[
U \Delta m = m \delta U ,
\]  

(26)
which shows that the energy difference will be determined by the gravitational potential, if a mass variation $\Delta m$ is involved. On the other hand, the potential difference $\delta U$ is of importance, if the emitter with mass $m$ changes its position. In this sense, both statements cited above contain some truth. It would, however, be required to formulate the corresponding premises in great detail.

C. Pair annihilation

We first formulate the rest energy of both particles involved—here an electron and a positron—at the gravitational potential $U$ as

$$2 E^\pm = 2 E^\pm_0 + 2 U m_e$$

with rest energies of $E^\pm_0 = m_e c_0^2$ at $U_0 = 0$. We will neglect any transitions from its excited states and assume a final state that eventually disintegrates into two $\gamma$-ray photons of equal energy $E$, but in opposite directions. In a formal way, in analogy to Sect. III A, each photon can only get half the energy given by Eq. (27) in the rest system of the centre of gravity.

As for the photon emission of an atomic particle, the question arises which parameter controls this emission energy. The answer again is that the speed of light $c$ at $U$ is the decisive factor. In Table 2 are summarized the momentum and energy terms—written under the assumption that the initial annihilation is not dependent on the gravitational potential $U$, but the emission process of the photons is affected by the speed of light in accordance with the results in Sect. III A. The momentum conservation follows from the symmetry of the emissions. The energy equations for each of the photons in line with energy conservation can be written as

$$E^\pm_0 - X c_0 = (P_0 - X) c_0 = (P_0 + X) c = h \nu ,$$

where $P_0 = || \pm P_0 ||$, $X = || \pm X ||$, and $\pm X$ are arbitrary differential momentum vectors parallel to $\pm P_0$, which have to be determined by solving Eq. (28) related to row 8 of Table 2. With Eq. (14) it follows

$$\frac{P_0 - X}{P_0 + X} = \frac{c}{c_0} \approx 1 + \frac{2 U}{c_0^2}$$

and

$$X = -P_0 \frac{U}{c_0^2} .$$
Notice, in this case, that the energy $2 X c_0 = -2 P_0 U/c_0$ corresponds to the potential energy $-2 U m_e$ of the electron and positron at $U_0$ with respect to $U$.

The same arguments as those for spectral lines in Sect. III A then result in a relative gravitational redshift consistent with Eq. (3).

IV. CONCLUSION

In summary, it can be concluded that the internal processes of an atom or ion during transitions between different energy states will not be significantly influenced by a moderate gravitational field, but the conversion of the liberated energy into a photon will be affected by the local gravitational potential via the speed of light and gives the observed redshift. Matter-antimatter pair annihilation leads to the same relative redshift, albeit with a slightly different interaction process in the near-field radiation region.

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1 Adams, W.S., The relativity displacement of the spectral lines in the companion of Sirius, Proc. Nat. Acad. Sci. USA, 11, 382–387 (1925).
2 Bauch, A., and Weyers, S., New experimental limit on the validity of local position invariance, Phys. Rev. D, 65, 081101-1–4 (2002).
3 Blamont, J.E., and Roddier, F., Precise observation of the profile of the Fraunhofer strontium resonance line. Evidence for the gravitational red shift on the Sun, Phys. Rev. Lett., 7, 437–439 (1961).
4 Bowers, R.L., Gravitationally redshifted gamma rays and neutron star masses, Astrophys. J., 216, L63–L65 (1977).
5 Brault, J., Gravitational redshift of solar lines, Bull. Am. Phys. Soc., 8, 28 (1963).
6 Bureau International des Poids et Mesures, Le Système International d’Unités (SI), 8e édition, BIPM, Sèvres, p. 37 (2006).
7 Cacciani, A., Briguglio, R., Massa, F., and Rapex, P., Precise measurement of the solar gravi-
tational red shift, Celest. Mech. Dyn. Astron., 95, 425–437 (2006).
8 Chou, C.W., Hume, D.B., Rosenband, T., and Wineland, D.J., Optical clocks and relativity, Science, 329, 1630–1633 (2010).
9 Compton, A.H., A quantum theory of the scattering of X-rays by light elements, Phys. Rev., 21, 483–502 (1923).
10 Cottam, J., Paerels, F., and Mendez, M., Gravitationally redshifted absorption lines in the X-ray burst spectra of a neutron star, Nature, 420, 51–54 (2002).
11 Cranshaw, T.E., Schiffer, J.P., and Whitehead, A.B., Measurement of the gravitational red shift using the Mössbauer effect in Fe$^{57}$, Phys. Rev. Lett., 4, 163–164 (1960).
12 Dicke, R.H., Eötvös experiment and the gravitational red shift, Am. J. Phys., 28, 344–347 (1960).
13 Dyson, F.W., Eddington, A.S., and Davidson, C., A determination of the deflection of light by the Sun’s gravitational field, from observations made at the total eclipse of May 29, 1919, Phil. Trans. Roy. Soc. Lond. A, 220, 291–333 (1920).
14 Einstein, A., Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig? Ann. Phys. (Leipzig), 323, 639–641 (1905).
15 Einstein, A., Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen, Jahrbuch der Radioaktivität und Elektronik 1907, 4, 411–462 (1908).
16 Einstein, A., Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes, Ann. Phys. (Leipzig), 340, 898–908 (1911).
17 Einstein, A., Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie, Sitzungsberichte Königl. Preus. Akad. Wiss., XLVII, 831–839 (1915).
18 Einstein, A., Die Grundlage der allgemeinen Relativitätstheorie, Ann. Phys. (Leipzig), 354, 769–822 (1916).
19 Einstein, A., Zur Quantentheorie der Strahlung, Phys. Z., XVIII, 121–128 (1917).
20 Fermi, E., Quantum theory of radiation, Rev. Mod. Phys., 4, 87–132 (1932).
21 Greenstein, J.L., and Trimble, V.L., The Einstein redshift in white dwarfs, Astrophys. J. 149, 283–298 (1967).
22 Greenstein, J.L., Oke, J.B., and Shipman, H.L., Effective temperature, radius, and gravitational redshift of Sirius B, Astrophys. J., 169, 563–566 (1971).
23 Hay, H.J., Schiffer, J.P., Cranshaw, T.E., and Egelstaff, P.A., Measurement of the red shift in
an accelerated system using the Mössbauer effect in Fe$^{57}$, Phys. Rev. Lett., 4, 165–166 (1960).

24 Hetherington, N.S., Sirius B and the gravitational redshift: An historical review, Q. J. R. astr. Soc., 21, 246–252 (1980).

25 Hohensee, M.A., Chu, S., Peters, A., and Müller, H., Equivalence principle and gravitational redshift, Phys. Rev. Lett., 106, 151102 (2011).

26 Hohensee, M.A., Chu, S., Peters, A., and Müller, H., Comment on “Does an atom interferometer test the gravitational redshift at the Compton frequency?” Class. Quantum Grav., 29, 048001 (2012).

27 Hohensee, M., and Müller, H., Terrestrial vs. spaceborne, quantum vs. classical tests of the equivalence principle, arXiv:1307.5987v1 [gr-qc] (2013).

28 Iorio, L., Is it possible to measure the Lense-Thirring effect on the orbits of the planets in the gravitational field of the Sun? Astron. Astrophys., 431, 385–389 (2005).

29 Iorio, L., Constraining the angular momentum of the Sun with planetary orbital motions and general relativity, Sol. Phys., 281, 815–826 (2012).

30 Jewell, L.E., The coincidence of solar and metallic lines. A study of the appearance of lines in the spectra of the electric arc and the Sun, Astrophys. J., 3, 89–113 (1896).

31 Kollatschny, W., AGN black hole mass derived from the gravitational redshift in optical lines, Proc. IAU Symp., 222, 105–106 (2004).

32 Kramer, M., Stairs, I.H., Manchester, R.N., and 12 coauthors, Tests of general relativity from timing the double pulsar, Science, 314, 97–102 (2006).

33 Krause, I.Y., and Lüders, G., Experimentelle Prüfung der Relativitätstheorie mit Kernresonanzabsorption, Naturwiss., 48, 34–36 (1961).

34 Lämmerzahl, C., What determines the nature of gravity? A phenomenological approach, Space Sci. Rev., 148, 501–522 (2009).

35 Landau, L.D., and Lifshitz, E.M., Course of theoretical physics 1, Mechanics, 3rd edition, Pergamon Press, Oxford, New York, Toronto, Sydney, Paris, Frankfurt (1976).

36 von Laue, M., Das Relativitätsprinzip, Friedr. Vieweg und Sohn, Braunschweig (1911).

37 von Laue, M., Zur Theorie der Rotverschiebung der Spektrallinien an der Sonne, Z. Phys., 3, 389–395 (1920).

38 Lense, J., and Thirring, H., Über den Einfluß der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, Phys. Z., 19
156–163 (1918).

39 Leventhal, M., MacCallum, C.J., and Watts, A.C., Possible gamma-ray line from the Crab Nebula, Nature, 266, 696–698 (1977).

40 LoPresto, J.C., Chapman, R.D., and Sturgis, E.A., Solar gravitational redshift, Sol. Phys., 66, 245–249 (1980).

41 LoPresto, J.C., Schrader, C., and Pierce, A.K., Solar gravitational redshift from the infrared oxygen triplet, Astrophys. J., 376, 757–760 (1991).

42 Mikhailov, A.A., The deflection of light by the gravitational field of the Sun, Mon. Not. R. astr. Soc., 119, 593–608 (1959).

43 Mössbauer, R.L., Kernresonanzfluoreszenz von Gammastrahlung in Ir$^{191}$, Z. Physik., 151, 124–143 (1958).

44 Müller, H., Peters, A., and Chu, S., A precision measurement of the gravitational redshift by the interference of matter waves, Nature, 463, 926–929 (2010a).

45 Müller, H., Peters, A., and Chu, S., Müller, Peters and Chu reply, Nature, 467, E2 (2010b).

46 Nobili, A.M., and Will, C.M., The real value of Mercury’s perihelion advance, Nature, 320, 39–41 (1986).

47 Okun, L.B., Photons and static gravity, Mod. Phys. Lett. A, 15, 1941–1947 (2000).

48 Okun, L.B., Selivanov, K.G., and Telegdi, V.L., On the interpretation of the redshift in a static gravitational field, Am. J. Phys., 68, 115–119 (2000).

49 Peters, A., Chung, K.-Y., and Chu, S., A measurement of gravitational acceleration by dropping atoms, Nature, 400, 849–852 (1999).

50 Pound, R.V., and Rebka, G.A., Gravitational red-shift in nuclear resonance, Phys. Rev. Lett., 3, 439–441 (1959).

51 Pound, R.V., and Snider, J.L., Effect of gravity on gamma radiation, Phys. Rev., 140, B 788–B 803, (1965).

52 Ramaty, R., Gamma ray line astronomy, Proc. 4th Moriond Astrophys. Meeting, p. 243–254, La Plagne, France (1984).

53 Schiff, L.I., On experimental tests of the general theory of relativity, Am. J. Phys., 28, 340–343 (1960).

54 Shapiro, I.I., Fourth test of general relativity, Phys. Rev. Lett., 13, 789–791 (1964).

55 Shapiro, I.I., Ash, M.E., Ingalls, R.P., and five coauthors, Fourth test of general relativity: New
radar result, Phys. Rev. Lett., 26, 1132–1135 (1971).

56 Shapiro, S.S., Davis, J.L., Lebach, D.E., and Gregory, J.S., Measurement of the solar gravitational deflection of radio waves using geodetic Very-Long-Baseline Interferometry data, 1979–1999, Phys. Rev. Lett., 92, 121101-1–4 (2004).

57 Sinha, S., and Samuel, J., Atom interferometry and the gravitational redshift, Class. Quantum Grav., 28, 145018–1–8 (2011).

58 Smith, C., Bound state description in quantum electrodynamics and chromodynamics. Binding energy effects on annihilation rates and spectra, Dissertation, Louvain-la-Neuve (2002).

59 Snider, J.L., New measurement of the solar gravitational red shift, Phys. Rev. Lett., 28, 853–856 (1972).

60 St. John, C.E., Evidence for the gravitational displacement of lines in the solar spectrum predicted by Einstein’s theory, Astrophys. J., 67, 195–239 (1928).

61 Straumann, N., Reflections on gravity, ESA-CERN Workshop, CERN, 5–7 April 2000, arXiv:astro-ph/0006423v1 (2000).

62 Takeda, Y., and Ueno, S., Detection of gravitational redshift on the solar disk by using iodine-cell technique, Sol. Phys., 281, 551–575 (2012).

63 Turyshev, S.G., Testing fundamental gravitation in space, Nucl. Phys. B (Proc. Suppl.) 243–244, 197–202 (2013).

64 Le Verrier, U., Lettre de M. Le Verrier á M. Faye sur la théorie de Mercure et sur le mouvement du périhélie de cette planète, Compt. rend. hebdomad. sanc. Acad. sci., Paris, 49, 379–383 (1859).

65 Will, C.M., Gravitational red-shift measurements as tests of nonmetric theories of gravity, Phys. Rev. D, 10, 2330–2337 (1974).

66 Will, C.M., The confrontation between general relativity and experiment, Living Rev. Relativity, 9 (2006), 3. URL (accessed 15 Dec. 2013).

67 Wojtak, R., Hansen, S.H., and Hjorth, J., Gravitational redshift of galaxies in clusters as predicted by general relativity, Nature, 477, 567–569 (2011).

68 Wolf, P., Blanchet, L., Bordé, C.J., and three coauthors, Atom gravimeters and gravitational redshift, Nature, 467, E1 (2010).

69 Wolf, P., Blanchet, L., Bordé, C.J., and three coauthors, Does an atom interferometer test the gravitational redshift at the Compton frequency? Class. Quantum Grav., 28, 145017 (2011).
Wolf, P., Blanchet, L., Bordé, C.J., and three coauthors, Reply to the comment on, “Does an atom interferometer test the gravitational redshift at the Compton frequency?” Class. Quantum Grav., 29, 048002 (2012).

| 1 | Transition of Atom A* at U |
|---|--------------------------|
| 2 | Energy                   |
| 3 | Momentum: $\Delta m c^2 = \Delta E_0 = ||p_0||c_0$ |
| 4 | direction $\leftarrow$  $\rightarrow$ |
| 5 | $\leftarrow$ $\leftarrow$ $\leftarrow$ $\Rightarrow$ $\Rightarrow$ |
| 6 | $-p_0 - x$ $-x$ $p_0 + x$ |
| 7 | $-U \Delta m$ $E_{\text{kin}} \ll \Delta E_0$ $||p_0 - x||c_0$ $||p_0 + x||c$ |
| 8 | Atom A | Interaction region | Photon |

Table 1: Transition of an excited atom to the ground state at a gravitational potential $U$. In rows 2 to 4 of the central column—called “Interaction region”—the left-hand side is related to the atom and the right-hand side refers to the near-field radiation during the emission process, which, according to Einstein’s early assumption quoted above, is controlled by the atom alone and therefore does not dependent on $U$. In rows 5 to 7, the photon emission and the reaction onto the emitter are indicated in line with momentum and energy conservation, cf., Eqs. (19) and (17). The Mößbauer effect can be employed to increase the mass of the emitter and allow us to neglect $E_{\text{kin}}$. The momentum vectors are drawn by solid arrows, whereas the propagating photon is characterized by open momentum arrows.
Table 2: Pair annihilation of an electron and a positron at a gravitational potential $U$. The table is structured similar to Table 1, but the near-field interaction region now concerns the momentum and energy relationships during the emissions of the photons 1 and 2. In rows 7 and 8, the momentum and energy relationships are indicated, cf., Eqs. (30) and (28). As in Table 1, the momentum vectors are drawn by solid arrows, whereas the propagating photons are characterized by open momentum arrows.

| 1 | Electron | $E_0^- = m_e c_0^2$ | $E_0^+ = m_e c_0^2$ | Positron |
|---|----------|----------------------|----------------------|----------|
| 2 | Energy   | $|| - P_0 || c_0$ | $|| + P_0 || c_0$ | cf., Eq. (12) |
| 3 | Momentum | $-P_0$ | $+P_0$ | |
| 4 |          | $\leftarrow$ | $\rightarrow$ | |
| 5 |          | $\rightarrow$ | $\leftarrow$ | |
| 6 |          | $\leftrightarrow$ | $\implies \Rightarrow$ | |
| 7 | $-P_0 - X$ | $+X$ | $-X$ | $+P_0 + X$ |
| 8 | $|| - P_0 - X || c$ | $|| - P_0 + X || c_0$ | $|| + P_0 - X || c_0$ | $|| + P_0 + X || c$ |
| 9 | Photon 1 | Interaction | region | Photon 2 |