Optical phase conjugation reflectivity via stimulated Brillouin scattering in acousto-optic diffusive semiconductor plasma crystal

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Abstract. Analytical investigation has been made for optical phase conjugation (OPC) reflectivity via stimulated Brillouin scattering (SBS) in acousto-optic diffusive semiconductor plasma crystal. Our analysis is based on hydrodynamic model and coupled mode approach of interacting waves. The numerical estimations are made for n-type InSb semiconductor plasma crystal duly irradiated by CO2 laser. The magnitude of the third-order nonlinear optical susceptibility for III-V semiconductor obtained from our analysis is found to agree well with the experimental values. Maximum OPC reflectivity is obtained when cyclotron frequency is in resonance with applied pump frequency.

1. Introduction
SBS has been extensively investigated due to its important property of optical phase conjugation (OPC), which can be exploited to recover the phase front of a beam [1-4]. Nimje et al. [5] investigated the effects of material parameters on interaction length to occur OPC-SBS in semiconductors and found that when cyclotron frequency is tuned with pump frequency interaction length becomes $10^{-4}$ time smaller than that obtained in absence of magnetic field. Sen et al. [6] studied phase conjugation in magnetised semiconductors via SBS and reported that when pump intensity is well above the threshold; reflectivity approaches its absolute limit $R = 1$ at interaction length $800 \mu m$. The OPC-SBS reflectivity being directly dependent on the third order Brillouin susceptibility, an enhancement in reflectivity is possible even at lower excitation intensity if one can achieve larger third-order susceptibility in the medium. Recently, Nimje et al. [7] have reported that magnetic field enhances the steady-state Brillouin gain through Brillouin susceptibility of acousto-optic semiconductors. In present paper, we aim to study the OPC reflectivity via SBS using hydrodynamic model and coupled mode theory of interacting waves under thermal equilibrium.
2. Brillouin Susceptibility

Here the electric field associated with pump wave is described by \( \tilde{E}_0 = \tilde{x}E_0 \exp \{i(\omega_0 t - k_0 x) \} \). A magnetostatic field \( \tilde{B}_0 = \tilde{z}B_0 \) is applied across the direction of wave propagation \( \tilde{k} = \tilde{x}k \).

We start with the calculation of the total current density for resonant Stokes component arises due to nonlinear interactions of the waves, followed by derivation of the effective Brillouin susceptibility through third-order nonlinear polarisation arising due to nonlinear induced current and acousto-optical strain. Nimje et al. [7] have already derived expression for Brillouin susceptibility in acousto-optic crystal for the above field geometry.

Thus, by following equation (19) of [7] in dispersionless acoustic wave regime, one may obtain the effective Brillouin susceptibility as

\[
\chi_s = -\frac{i\varepsilon (\eta^2 - 1) k^2 \omega_0^*}{4\rho \Gamma \omega_\alpha \omega_\beta (\omega^*_0 - \omega)} \left[ 1 + \frac{\omega_0^2}{\omega_\alpha \omega_\beta} |G| \right],
\]

where the symbols have their usual meaning and well defined in reference [7].

3. Optical phase conjugation reflectivity

In this section, an analytical expression for optical phase conjugation (OPC) reflectivity via stimulated Brillouin scattering (SBS) is derived. For this, we consider the irradiation of the semiconductor crystal by a slightly off-resonant laser of frequency \( 1.78 \times 10^{14} \text{ s}^{-1} \) with excitation intensity well above the threshold value required for the onset of SBS in semiconductor crystal. The pump laser mode undergoes stimulated scattering processes in the medium. We have considered the transverse component of electric field that propagates along x-direction. The propagation of these modes through the transparent optical material can be represented by the general electromagnetic wave equation

\[
\nabla^2 E - \left( \frac{\omega_\text{p}^2}{c^2} \right) E = -\mu_0 \omega_\text{p}^2 P
\]

(2)

The field variation is given by the plane wave approximation \( \exp \{i(\omega t - kx) \} \). \( P \) represents the total induced polarisation in semiconductor crystal comprising of linear as well as the nonlinear components and is given by

\[
\hat{P}(x,t) = \varepsilon_0 [\chi^{(1)} + \chi^{(3)} |E|^2 + \ldots \ldots \ldots \ldots ] \tilde{E}
\]

(3)

Here complex susceptibility \( \chi^{(1)} \) corresponds to linear optical effects such as linear refraction and absorption of wave within the medium, while third-order nonlinear optical susceptibility \( \chi^{(3)} \) is responsible for the nonlinear phenomena that gives rise to the phenomenon of OPC.

We assume that the pump wave \( (\omega_0, \tilde{k}_0) \) is propagating along the negative x direction while the Stokes mode of scattered electromagnetic wave is assumed to be propagating along positive x direction.

On applying slowly varying envelope approximation in equations (2) and (3), we can obtain the corresponding electromagnetic wave equation for \( \tilde{E}_0(\omega_0, \tilde{k}_0) \) (pump wave) and \( \tilde{E}_1(\omega_0, \tilde{k}_1) \) (Stoke mode) as

\[
(\partial / \partial \tilde{x}) \tilde{E}_0 = \alpha_0 \tilde{E}_0 - i\gamma_0 \tilde{E}_0 - (\omega_0^2 / 2k_0^2) \chi^{(3)}_0 |E_1|^2 \tilde{E}_0
\]

(4a)
and

\[
(\partial / \partial x) E_1 = \alpha_i E_1 - i \alpha_f E_1 - (\omega_0^2 / 2k_f^2) \chi^{(3)}_f | E_0 |^2 E_1
\]  

(4b)

where \( \alpha_{0,i} = (\omega_{0,i} / 2c) \chi^{(1)}_0 \) and \( \alpha_{r,0,i} = (\omega_{0,i} / 2c) \chi^{(1)}_r \) stand for the absorption and dispersive properties of the crystal at frequency \( \omega_{0,i} \). \( \chi^{(3)}_0 \) is the third-order optical susceptibility at frequency \( \omega_{0,1} \).

For phase conjugated wave, the phase matching conditions enable one to take \( k_0 = -k_1 \), such that \( k_0 = 2k_0 \); also we take \( |k_0| = |k_1| = k \) with \( |k_0| = 2k \). We assume that \( \omega_1 = \omega_0 = \omega \) and \( \alpha_1 = \alpha_0 = \alpha \).

The phase-conjugate scattered radiations is a function of pump amplitude and is given by

\[
E_1(x) = \beta(x) E_0^*(x),
\]

(5)

where \( \beta(x) \) is a measure of conjugacy with \( |\beta(x)|^2 \) being defined as the OPC reflectivity \( R \) via SBS. On simplifying equation (4a), one can obtained the pump electric field as

\[
E_0(x) = [E_0(L) \exp\{-\alpha_E(L-x)\}] \exp[i\alpha_f(L-x)]
\]

(6)

where \( \alpha_E = \alpha_1 - (i\omega^2 / 2k_c^2) \chi^{(3)}(\omega) |E_1|^2 \) and \( E_0(L) \) is the pump electric field at the entrance window \( L = x \). Equation (6) reveals the x dependence of the pump amplitude as well as the nature of phase variation of the electric field. From equations (6) and (4b), we find the electric field associated with the Stokes mode of the backscattered electromagnetic wave as

\[
E_1(x) = E_1(0) \exp[-\alpha_E x + (\kappa / 2\alpha_E) \{1 - \exp(2\alpha_E x)\}] \exp(i\alpha_E x)
\]

(7)

where \( \kappa = (\omega_0^2 / 2k_c^2) \chi_B E_0^2(L) \exp(-2\alpha_E L) \) and \( E_1(0) \) is the spontaneous noise of Stokes field amplitude at \( x = 0 \). The backscattered Brillouin mode \( E_1(x) \) possesses a gain constant given by

\[
-\{\alpha_E x + (\kappa / 2\alpha_E) [1 - \exp(2\alpha_E x)]\}
\]

For finite gain, the condition should be satisfied

\[
\alpha_E x + (\kappa / 2\alpha_E) [1 - \exp(2\alpha_E x)] < 0
\]

(8)

It is clear from equation (8) that the gain constant depends upon \( \alpha_E \) and \( \kappa \). For bulk semiconducting material of mm dimensions shined by a slightly off-resonant laser with phonon frequency less than the crystal band-gap energy, one may take \( 2\alpha_E x < 1 \). Consequently, the threshold condition for onset of Brillouin gain is \( \kappa \geq \alpha_E \).

Therefore, we can find the threshold value of the excitation intensity that is responsible for Brillouin gain as

\[
I_{\text{th}} \geq \eta \epsilon_0 c^3 k\alpha_E / \omega^2 |\chi_B|
\]

(9)

For pump intensity well above the threshold, on can obtain finite OPC reflectivity \( R \) at the entrance window as

\[
R = |\beta(x = L)|^2 = |E_1(0)/E_0(L)|^2 \exp[2(\kappa - \alpha_E) L]
\]

(10)
Figure. Dependence of OPC reflectivity $R$ with applied magnetic field $B_0$ (in term of $\omega_c$) at the interaction length $L = 10^{-7} m$.

where $L$ is the length of semiconductor crystal, $|E_i(0)|^2$ being the noise intensity for the SBS process [8] and its magnitude is generally taken to be about $10^{-12}$ to $10^{-13}$ times of $|E_0(L)|^2$. Hence, significantly high OPC reflectivity ($R \approx 1$) can be achieved in the crystal only if $(\kappa - \alpha_E) L \approx 15$ i.e. when the gain is nearly equal to $e^{10}$.

4. Results and discussion:
The set of parameters used for numerical appreciation is taken from ref. [8]. The characteristic dependence of OPC reflectivity on magnetic field (in term of $\omega_c$) is plotted in Figure. Figure shows the variation of OPC reflectivity $R$ with cyclotron frequency $\omega_c$. It may be inferred that the finite reflectivity is obtained only when magnetic field is in range from 13.99T to 14.35T. The corresponding cyclotron frequency range is from $1.75 \times 10^{14} s^{-1}$ to $1.81 \times 10^{14} s^{-1}$, respectively. Beyond this limit on both the ends the wave is completely absorbed by the semiconductor i.e. no OPC reflectivity is observed. This behaviour may be used for construction of magnetic switches. Thus 72% OPC reflectivity is obtained only when $\omega_c$ resonant with $\omega_0$.

5. References
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