A model of recovering the parameters of fast nonlocal heat transport in magnetic fusion plasmas

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Abstract. A model is elaborated for interpreting the initial stage of the fast nonlocal transport events, which exhibit immediate response, in the diffusion time scale, of the spatial profile of electron temperature to its local perturbation, while the net heat flux is directed opposite to ordinary diffusion (i.e. along the temperature gradient). We solve the inverse problem of recovering the kernel of the integral equation, which describes nonlocal (superdiffusive) transport of energy due to emission and absorption of electromagnetic (EM) waves with long free path and strong reflection from the vacuum vessel’s wall. To allow for the errors of experimental data, we use the method based on the regularized (in the framework of an ill-posed problem, using the parametric models) approximation of available experimental data. The model is applied to interpreting the data from stellarator LHD and tokamak TFTR. The EM wave transport is considered here in the single-group approximation, however the limitations of the physics model enable us to identify the spectral range of the EM waves which might be responsible for the observed phenomenon.

1. Introduction

To describe the anomalous heat transport in the magnetized thermonuclear plasma the superdiffusion formalism was suggested (namely, the equation, integral in space coordinates, with a long-tailed kernel and, respectively, dominance of carriers with a long free path). It includes steady-state transport of heat by electron/ion Bernstein [1] and electron cyclotron waves [2], transport of temperature perturbation by electron Bernstein waves [3] which was possible to describe by Biberman-Holstein equation known from the theory of atomic excitation’s transport in spectral lines of atoms/ions; model of fractional derivatives for heat perturbation transport (see references in reviews [4-6]). One of the most interesting examples of fast nonlocal transport is the unusual direction of a heat flux, namely, the instantaneous (in a scale of diffusion time) temperature increase in the plasma center in case of fast cooling the plasma periphery (so called "cold pulse" experiments in tokamaks and stellarators, [4-6]) or an inverse process — the instantaneous temperature fall in the plasma center in case of fast heating the plasma periphery [7].

In this work an inverse problem of recovering the integral equation kernel is formulated, extending/modifying the works [8-10], and solved. The equation describes the nonlocal
(superdiffusive) energy transfer caused by emission and absorption of electromagnetic (EM) waves with a long free path and strong reflection of waves from the walls of vacuum chamber. The developed model is used for interpretation of the "cold pulse" experiments in the stellarator LHD [11, 12] and tokamak TFTR [4, 13].

2. The fast nonlocal transport model

The model is aimed at recovering the main parameters of the unknown fast heat carriers by solving an inverse problem. The model is based on the following assumptions.

— At the initial stage of the event, unusual behavior of electron temperature $T_e$ (e.g., the instantaneous, within the diffusion time scale, electron temperature increase in the plasma center in case of the instantaneous cooling of the plasma periphery) is completely defined by EM waves. Their group velocity is so high that it is possible to neglect retardation.

— Energy density and total energy of carriers are small in comparison with those of the main plasma, and the total power of energy losses (owing to incomplete reflection of waves from walls) does not exceed a small fraction $f$ of the total power of energy losses in plasma.

— Free path length of waves is large, it is not less than the minor radius of toroidal plasma (this condition is satisfied for EM-waves with a frequency significantly higher fundamental cyclotron and plasma frequencies). Waves are strongly reflected from the walls of the vacuum chamber. Then intensity of EM-waves is spatially homogeneous and isotropic in group velocity’s angles.

— EM-waves transport is considered in the one-group (monochromatic) approximation.

— Source and sink functions, i.e. the power density of waves’ emission and the absorption coefficient of waves, are proportional to electron density and the unknown function depending only on temperature. There is a link between these functions corresponding to the Kirchhoff’s law.

— To exclude possible contribution of diffusion processes into the observed evolution of $T_e$ at initial stage, the inverse problem is solved for an energy balance equation only in the central part of plasma.

The first condition allows to obtain the energy balance equation which after simplification takes a form:

$$\left[ K(T(\rho, t))I_{EM}(T(\rho, t)) - K(T_0(\rho))I_{EM}(T_0(\rho)) \right] = \Pi(\rho, t),$$

where $\rho$ is a radial coordinate normalized on the small radius of a toroidal plasma; $K(T)$ is a temperature-dependent factor of electrons in the absorption coefficient of EM-waves (measured here in the inverse seconds); $T_0(\rho)$ and $T(\rho, t)$ are the spatial electron temperature distributions, respectively, at the initial moment (i.e. at quasi-stationary stage before disturbance of temperature via pellet injection into peripheral plasma) and after that; $I_{EM}$ is the waves’ intensity (energy flux density, measured here in temperature measure units, keV; the point in the argument means that it is a mute variable under the integral),

$$I_{EM}(T(\rho, t)) = \frac{\int_0^1 n_e(\rho_i, t)Q(T(\rho, t))\rho_i d\rho_i}{F_w + \int_0^1 n_e(\rho_j, t)K(T(\rho, t))\rho_2 d\rho_2},$$

where the power density of EM-waves source (expression under integral in numerator in (2)) and the absorption coefficient (expression under integral in denominator in (2)) are linked by Kirchhoff’s law (for the long-wavelength radiation, $\hbar \omega \ll T$, it is the Rayleigh–Jeans’s law), and right part of equation (1) is a difference (before and after injection) in balance of plasma thermal energy per one particle (electron density is omitted in the left and right parts in (1) due to its approximate invariance in the central part of plasma),

$$K(T) = Q(T)T, \quad \Pi(\rho, t) = A \frac{d}{dt} \left[ T(\rho, t) - T_0(\rho) \right],$$

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where the coefficient $A$ allows also for the contribution of ions to plasma thermal energy; the constant $F_w$ in (2) is bound to $R_w$, the reflectivity of waves from walls, averaged on the vacuum chamber’s geometry, on the wavelength and the width of spectral range in which the energy transfer takes place (actually, width of the emission and absorption spectrum in central plasma)

$$F_w = (1 - R_w) F_0, \quad \frac{1}{F_0} = \frac{n'_e a}{\pi} \left( \frac{c_w}{\omega} \right) ^2 \frac{\pi^2}{\Delta \omega} \left[ 1 + k^2_{\text{elong}} \right]$$

(4)

here $a$ — effective minor radius of toroidal plasma, $n'_e$ — characteristic electron density ($10^{13}$ cm$^{-3}$), $c_w$ — speed of waves equal to velocity of the light, $\omega$ — the wave frequency in monochromatic transport model, $\Delta \omega$ — the above-mentioned spectral width, $k_{\text{elong}}$ — vertical elongation of poloidal cross-section of toroidal plasma.

The inverse problem is formulated as a recovery of the function $K(T)$ (which is assumed to be monotonic) and the constant $F_w$ for the known functions $T_0(\rho)$, $T(\rho, t)$ and $n_e(\rho, t)$. It corresponds to recovery of the integral operator kernel in the transport equation (1), (2) for the known dependence of perturbation of electron temperature and density on the time and magnetic surface.

3. Method of solving the inverse problem

Numerical calculations are carried out by the method of sequential correction and identification of mathematical model parameters in Eqs. (1) — (4). A method based on a sequential, regularized approximation of the input data by parametric models is used. This approach is ideologically close to the methods of Tikhonov’s regularization [14] (in this paper there is also regularization by smoothness). The regularization criterion is the smoothness of the approximating functions. Smoothness is treated as one of the important characteristics of the solution quality. The general scheme for estimating the modeling error is the cross-validation. Similar approaches to data processing can be found in [15]. The general scheme of the method proposed by A.V. Sokolov has already been successfully applied to various problems, including the processing of geophysical data [16].

To estimate the quality of the parametric models and to choose the direction of their improvement, a large number of auxiliary subtasks has to be solved. Each step of the proposed method is a two-level finite-dimensional non-linear problem of mathematical programming. At the "lower" level, it is necessary to solve a large set of independent subtasks, and this phase of calculations can be significantly accelerated when computing is transferred to a distributed computing environment. To do this, the system of services for solving the non-linear problems of mathematical programming on the Everest platform was used [17], where the package pool (Ipopt, https://projects.coin-or.org/Ipopt) was deployed to solve such problems. For the software implementation of the method, including the description of tasks and the data exchange with packages, the Python language and the Pyomo optimization modeling package are used [18, 19]. The general scheme of the Pyomo application in the Everest system of services coincides with that already used for integration of the optimization modeling language AMPL [20] into the Everest. More detailed information on the proposed numerical method can be found in [21].

4. Calculation results

In this section we show two typical solutions of the inverse task — recovery the $K(T)$ function in the framework of the model (1) — (4).

Space-time dynamics of the electron temperature and density in shot #48708 in stellarator LHD with auxiliary plasma heating (2 MW power from injection of a neutral beam and 1 MW from injection of electron cyclotron waves) is presented in figure 1 in [11] and figure 1 in [12]. Possible solution of the inverse problem for function $K(T)$ by consideration of plasma dynamics after injection in a time interval from 2.8 s to 2.815 s is presented in figure 1.
Figure 1. $K(T)$ is a function of temperature recovered by solving the inverse problem in the model of energy balance (1) — (4), for experimental conditions on stellarator LHD in the discharge #49708 [11, 12] (solid curve) and tokamak TFTR in the discharge #88076.

Pellet injection at 2.797 s led to an instantaneous increase of the density in the plasma periphery and to a sharp temperature drop to a minimum at 2.8 s, then that was replaced by a slowly growth. The chosen time interval is characterized by the almost linear increase of temperature and constant density in the central part of plasma (the normalized radius $\rho \leq 0.45$), whereas in the periphery ($0.7 \leq \rho \leq 1$) the temperature grows more slowly after sharp decrease, than in the centre, and dynamics of the density is such that the increase in density, caused by pellet, slowly moves towards plasma center.

Space-time dynamics of electron temperature in tokamak TFTR in the Ohmic discharge #88076 is presented in figures 2 and 3 in [4]. Possible solution of the inverse problem for the $K(T)$ function if considering the plasma dynamics after injection in a time interval from 3.72 s to 3.74 s, is also presented in figure 1. Pellet injection at 3.71 s produced the same effect, as in case of experiment in stellarator LHD.

5. Limitations of physics model

The physics model imposes restrictions for free parameters of the problem: wavelength $\lambda$ of EM waves, relative spectral width $\Delta \omega/\omega$, reflectivity $R_w$ of EM waves from the wall of the vacuum chamber, and the fraction $f$ of power losses, caused by the outgoing EM waves, in the total power losses $P_{\text{loss}}$.

The limitation on the power losses, caused by the outgoing EM waves, takes a form

$$\frac{I_{\text{EM}}(t)}{\text{keV}} \left(\frac{\Delta \omega}{\omega}\right) \left(\frac{10^{-2} \text{cm}}{\lambda}\right)^3 (1-R_w) \leq f \frac{P_{\text{loss}}}{100 \text{ MW}}.$$  \hspace{1cm} (5)

The condition that free path length is not less than plasma minor radius is as follows:

$$\frac{I_{\text{EM}}(t)}{\text{keV}} \left(\frac{\Delta \omega}{\omega}\right) \left(\frac{10^{-2} \text{cm}}{\lambda}\right)^3 (1-R_w) \leq f \frac{P_{\text{loss}}}{100 \text{ MW}}.$$  \hspace{1cm} (6)

The total energy of waves has to be much less than the total thermal energy of plasma:

$$\frac{I_{\text{EM}}(t)}{\text{keV}} \left(\frac{\Delta \omega}{\omega}\right) \left(\frac{10^{-2} \text{cm}}{\lambda}\right)^3 \ll 10^6 \int_{0}^{10^{13} \text{ cm}^{-3}} n_e(\rho, t) T_e(\rho, t) \rho d \rho.$$  \hspace{1cm} (7)

And at last, for existence of quasi-stationary intensity of EM-waves and notable power exchange between the central and peripheral plasma (i.e. a condition of existence of the studied effect in the considered energy transport model) the contribution to intensity of waves, determined by reflection from the wall and expressed by the first term in denominator of (2), should not exceed the contribution of absorption:
\[
\left( \frac{\Delta \omega}{\omega} \right) \left( \frac{10^{-2} \text{cm}}{\lambda} \right)^3 \left( 1 - R_w \right) < 2.5 \times 10^{-4} \int_0^{10^3 \text{cm}^{-3}} \frac{n_e(\rho, t) K(T_e(\rho, t))}{s^{-1}} \rho d \rho.
\]

(8)

Restrictions (5) — (8) for applicability of solutions of the inverse problem, shown in figure 1, can be presented in the form of the following system of inequalities:

\[
a \left( \frac{\Delta \omega}{\omega} \right) \left( \frac{10^{-2} \text{cm}}{\lambda} \right)^3 \leq b \frac{P_{\text{loss}}}{100 \text{ MW}} \frac{f}{(1 - R_w)},
\]

\[
a \left( \frac{\Delta \omega}{\omega} \right) \left( \frac{10^{-2} \text{cm}}{\lambda} \right)^3 \leq \frac{d}{1 - R_w}, \quad \left( \frac{\Delta \omega}{\omega} \right) \left( \frac{10^{-2} \text{cm}}{\lambda} \right)^3 \ll g.
\]

(9)

The following values of parameters correspond to stellarator LHD: \( a = 0.05 \), \( b = 0.36 \), \( d = 0.19 \), \( g = 10^7 \), \( P_{\text{loss}} = 3 \text{ MW} \), \( f < 0.1 \). For tokamak TFTR we have parameters \( a = 0.0296 \), \( b = 0.419 \), \( d = 0.116 \), \( g = 10^6 \), \( P_{\text{loss}} = 1 \text{ MW} \), \( f < 0.1 \). Apparently, the weak and obviously feasible is the condition (7), and the hardest is the first condition in (9).

An important consequence of physical restrictions (5) — (8) for the model (1) — (4) is that the unified parameter in the center of inequalities (9), which depends on the relative spectral width and the wavelength, gives the values of wavelength in the submillimeter range which is studied substantially less than the ambient spectral ranges.

The most important direction of developing the model (1) — (4) seems to be a transition from one-group transport of EM-waves to multi-group ones.

6. Conclusions

The model is developed to interpret the initial stage of the phenomenon of fast nonlocal transport in which the response of electron temperature space profile to its local perturbation is almost instantaneous in scales of diffusion time, and the total flux of heat is directed opposite ordinary diffusion (i.e. along temperature gradient). The model is based on dominance of nonlocal (superdiffusive) heat transport by EM-waves with a long free path length and the strong reflection of waves from walls of the vacuum chamber.

The method of regularized optimizing identification is used for solving the inverse problem of recovering the kernel of integral operator in the integral-differential equation. The developed model is used for interpretation of the experiments on a fast cooling of peripheral plasma (so-called "cold pulse" experiments) in stellarator LHD and tokamak TFTR. It is shown that for these experiments, close results of recovering the EM-waves source and coefficient of their absorption are obtained. Despite the transport of EM-waves is considered in the one-group approximation, the restrictions of physics model allow to allocate the spectral range of EM-waves which might be responsible for the observed phenomenon. The found solution of the inverse problem will allow us to look for the concrete physical mechanisms which would be compatible with such dependence of the source and sink of EM-waves on the electron temperature and density.

In general, modeling results indicate possible reality of the hypothesis [8] that the studied phenomenon develops according to the following scenario: fast cooling of the plasma periphery leads to a sharp decrease of absorption of EM-waves in this area; it does possible more free circulation of EM-waves and, under condition of their strong reflection from the wall of vacuum chamber, an increase of the intensity of waves; for monotonic profile of temperature dependence of absorption coefficient, this leads, in turn, to temperature growth in the plasma center at initial stage of perturbation evolution. Note that such scenario is compatible also with an inverse process when on the plasma periphery there is a sharp heating, and in the center the temperature falls. Ability of the model to give possible explanation not only to a unexpected temperature response in the center to perturbations of temperature on the periphery, but also to give a qualitative explanation of the
reversibility with respect to the sign of temperature perturbation is, in our opinion, the important advantage of the developed approach.

The realization of the developed physical model was possible only due to carrying out massive numerical calculations in the distributed computing environment.

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