AN AXISYMMETRIC, HYDRODYNAMICAL MODEL FOR THE TORUS WIND IN ACTIVE GALACTIC NUCLEI. II. X-RAY–EXCITED FUNNEL FLOW

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ABSTRACT

We have calculated a series of models of outflows from the obscuring torus in active galactic nuclei (AGNs). Our modeling assumes that the inner face of a rotationally supported torus is illuminated and heated by the intense X-rays from the inner accretion disk and black hole. As a result of such heating, a strong biconical outflow is observed in our simulations. We calculate three-dimensional hydrodynamical models, assuming axial symmetry and including the effects of X-ray heating, ionization, and radiation pressure. We discuss the behavior of a large family of these models, their velocity fields, mass fluxes, and temperature, as functions of the torus properties and X-ray flux. Synthetic warm-absorber spectra are calculated, assuming pure absorption, for sample models at various inclination angles and observing times. We show that these models have mass fluxes and flow speeds comparable to those inferred from observations of Seyfert 1 warm absorbers, and that they can produce rich absorption-line spectra.

Subject headings: acceleration of particles — galaxies: active — hydrodynamics — methods: numerical — quasars: absorption lines — X-rays: galaxies

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1. INTRODUCTION

One of the insights provided by observations of Seyfert galaxies and some quasars is the prevalence in their X-ray spectra of spectral lines and bound-free continua from ions of intermediate-Z elements. Early observations of Seyfert 1 galaxies using proportional counters and solid-state detectors revealed spectra with strong absorption features in the 0.1–10 keV range (Halpern 1984). These features were attributed mostly to the edges of hydrogen and helium-like oxygen. The term “warm absorber” was proposed, owing to the fact that the observed X-ray–absorbing gas has an electron temperature lower than it would be if a similar level of ionization were produced by collisional ionization. However, more detailed spectroscopic studies were hampered by the limited X-ray resolution of the ASCA and ROSAT satellites. The grating spectographs on the X-ray telescopes Chandra and XMM-Newton provide unprecedented spectral resolution up to ~10 keV. These show that X-ray spectra obtained from approximately half of low-redshift active galactic nuclei (AGNs) contain many lines from ions of Fe, Si, S, O, Mg, and Ne, and that these are generally broadened and blueshifted by 100–500 km s$^{-1}$ (Kaspi et al. 2002; Steenbrugge 2005). The presence of X-ray–absorbing gas has been confirmed in the majority of AGNs that are bright enough to allow detections (Reynolds 1997; McKernan et al. 2007). There is also a partial correspondence between UV and X-ray absorbers (Crenshaw et al. 1999).

X-ray observations of warm absorbers are consistent with the Seyfert 1/Seyfert 2 dichotomy. For example, the properties of the X-ray emission in the Seyfert 2 galaxy NGC 1068 correspond to the scattered emission expected from warm absorbers in Seyfert 1 galaxies (Kinkhabwala et al. 2002).

Constraints on the position and dynamics of the X-ray–absorbing gas can be deduced from the observed widths and virial arguments, and also from the variability studies of these spectra (Behar et al. 2003; Netzer et al. 2003). These show an absence of any correlated response of the warm-absorber gas to rapid changes (on the order of days) in the continuum. This implies that the ionization timescale in the warm-absorber gas is long (on the order of a month or more). Taken together, the line blueshifts, widths, and time variability analysis favor an origin of the warm-absorber gas at $R \gtrsim 1$ pc away from the black hole (BH). This estimate coincides with the likely location of absorbing matter responsible for obscuration in Seyfert 2 galaxies (Krolik & Begelman 1988). The existence of an outflow from the torus has been suggested by Krolik & Begelman (1986, 1988), and as the source of warm-absorber flows by Krolik & Kriis (1995, 2001).

It is believed that this matter is in the form of a molecular torus that is responsible for obscuring the broad-line region in Seyfert 2 galaxies, and that is thought to exist in most low- and intermediate-luminosity AGNs (Antonucci & Miller 1985). A growing body of direct observational evidence advocates for the existence of the obscuring torus. Mid-infrared high-spatial-resolution studies of the nucleus of NGC 1068 using the Very Large Telescope Interferometer have resolved a dusty structure 2.1 pc thick and 3.4 pc in diameter (Jaffe et al. 2004). Observations support a multitemperature model: the temperature of the warm component was established to be 300 K, and inside of it a second, compact and hot (>800 K) component has been found. Further studies of NGC 1068 systematically reduced estimates of the temperatures of different components (Poncet & Perrin 2006). Observations of the Circinus galaxy, which is among the closest prototype Seyfert 2 galaxies, also revealed a dense and warm $T \gtrsim 300$ K component at about 0.2 pc from the BH, and a cooler $T < 300$ K component at 1 pc (Tristram et al. 2007). If the hotter component is located closer to the X-ray source, it could be attributed to the inner part of the torus, heated by the radiation of the compact nucleus. Although the evidence is strongest for nearby active galaxies, there is also a strong motivation to think that within the same obscuring torus paradigm exist quasars whose central regions are heavily obscured by gas and dust (Type II quasars). Evidence for this comes...
from spectropolarimetric observations by Zakamska et al. (2006).

This paper is part of a series whose main goal is to test the hypothesis that the torus is the origin of the warm-absorber flow. Preliminary results of this work have been reported in Dorodnitsyn et al. (2008, hereafter Paper I), in which we presented the results from a sample model and showed that the adopted model is promising in explaining the warm-absorber phenomenon. In this paper, we provide more details of our methods, and display the results of models that span the space of the input parameters. We present and discuss the hydrodynamic quantities that characterize our models: mass fluxes, velocity fields, and temperature structure. We also show sample X-ray spectra, which we will discuss extensively in a later paper.

Our approach has three basic parts. (1) First, we set up initial conditions, which requires defining an initial torus configuration and making assumptions about the external source of radiation. (2) Then, we implement the wind driving force (local heating/cooling rates and radiation pressure force) and actual two-dimensional (2D) hydrodynamical calculations. The latter include the numerical solution of the time-dependent 2D system of equations, which takes into account centrifugal forces, radiation pressure, and heating terms. (3) Finally, we calculate the X-ray line spectra using a code that adopts Sobolev radiation transfer and ionization calculations for plasma in the intense X-ray field. Each of these steps is described below.

2. GOVERNING EQUATIONS

We solve the following system of equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho \nabla \Phi + \rho g_{\text{rad}}, \quad (2)
\]

\[
\frac{\partial T}{\partial t} + \nabla \cdot [\mathbf{v}(\epsilon + p + \rho \Phi)] = H, \quad (3)
\]

which are the conservation equations for mass, momentum, and energy. Heating and cooling processes are described by the function \( H \) (erg cm\(^{-3}\) s\(^{-1}\)), where \( \epsilon \) is the sum of the kinetic and internal energy densities: \( \epsilon = \rho \nu^2/2 + \epsilon \). These equations should be supplemented by the equation of state, which we assume to be polytropic: \( H = K \rho^\gamma \), where \( \gamma = 1 + 1/n \) and \( n \) is the polytropy index, and \( K \) = \( \gamma - 1 \) e. A one-component, one-temperature \( T = P \mu/R \) (where \( \mu \) is the mean molecular weight per particle and \( R = 8.31 \times 10^5 \) erg K\(^{-1}\) g\(^{-1}\) is the universal gas constant), and a plasma with \( \gamma = 5/3 \) are assumed to constitute the flow. All three components of the flow velocity \( \mathbf{v} = (v_x, v_y, v_z) \) are calculated, assuming azimuthal (\( \partial / \partial \phi \equiv 0 \)) symmetry. Equations (1)–(3) are cast in a nondimensional form, with the characteristic scales set by the properties of the plasma orbiting at a characteristic distance \( R_0 \) from a black hole of mass \( M_6 \) (in units of \( 10^6 \) M\(_\odot\)). The characteristic scales are respectively \( t_0 = R_0^{3/2}(GM_6)^{-1/2} \approx 4.5 \times 10^{11} R_0^{3/2} M_6^{-1/2} \) (s) for the time, where \( R_0 \) is the distance in pc, and \( V_0 = (GM_6/R_0)^{1/2} \approx 6.6 \times 10^3 M_6^{1/2} R_0^{1/2} \) (cm s\(^{-1}\)) for the velocity.

3. FORCES DRIVING THE FLOW

3.1. Heating and Cooling of the Gas

The forces accelerating the wind in our model result from the gradient of gas pressure and from radiation pressure. The thermodynamic properties of X-ray–heated gas depend on the spectrum of the incident radiation, as well as on the local atomic physics. Under the assumption of photoionization equilibrium, the thermodynamic state of photoionized gas can be parameterized in terms of the ratio of radiation energy density to baryon density (Tarter et al. 1969):

\[
\xi = 4\pi F_X/n, \quad (4)
\]

where \( F_X = L_X e^{-\tau}/(4\pi r^2) \) is the local X-ray flux, \( L_X \) is the X-ray luminosity of the nucleus, \( \tau = \int_0^\infty \kappa \rho \, dr \) is the optical depth, and \( n \) is the number density. We assume that the attenuation is dominated by Thomson scattering: \( \kappa = 1.8 \times 10^5 R_0^{-1} \) cm\(^{-2}\), where \( X_0 \) is the mass fraction of hydrogen, and the factor \( e^{-\tau} \) approximately accounts for the attenuation of the radiation flux on the way from the source toward a fiducial point. The methods adopted in this paper for treating the effects of radiation are essentially the same as those described in Proga et al. (2000, 2007) and have been applied to various problems in the study of AGNs and X-ray binaries.

Assuming that the there is a fraction \( f_X \) of the total accretion luminosity \( L_{\text{BH}} \) available in X-rays, and that the disk radiates a fraction \( \Gamma \) of its Eddington luminosity \( L_{\text{Edd}} = 1.25 \times 10^{34} M_6 \), we estimate \( \zeta \approx 3 \times 10^5 f_X L_{\text{BH}}/(N_250 \, \text{pc}) \), where \( N_250 \) is the column density in \( 10^{23} \) cm\(^{-2}\). If the dynamical time within the flow is much larger than the characteristic time of the photoionization and recombination, then the ionization balance is determined by the condition of photoionization equilibrium. The rates of Compton and photoionization heating, and Compton, radiative recombination, bremsstrahlung, and line cooling are then given by approximate formulae, modified from those of Blondin (1994) for these processes:

\[
\Gamma_{\text{IC}} \quad \text{(erg cm}^{-3} \text{ s}^{-1}) = 8.9 \times 10^{-36} \xi (T_X - 4T)^3 \quad (5)
\]

for the Compton heating/cooling,

\[
\Gamma_X \quad \text{(erg cm}^{-3} \text{ s}^{-1}) = 1.5 \times 10^{-21} \zeta^{1/4} T^{-1/2} (T_X - T) T_X^{-1} \quad (6)
\]

for the photoionization heating and recombination cooling, and

\[
\Lambda \quad \text{(erg cm}^{-3} \text{ s}^{-1}) = 3.3 \times 10^{-27} T^{1/2}
\]

\[
+ [4.6 \times 10^{-17} \exp (-1 \times 10^5 / T) \times (\xi^{0.8-0.98n})^2 - 1/2] \times 10^{-24} \quad (7)
\]

for the bremsstrahlung and line cooling.

These formulae were originally derived for a 10 keV bremsstrahlung spectrum \( T_X = 2.6 \times 10^7 \) K and were found to be in reasonable (~25%) agreement with numerical simulations (Blondin 1994). Equations (3)–(7) are slightly modified versions of those of Blondin (1994), accommodating new atomic data. Using the XSTAR code (Kallman & Bautista 2001), we recalculated heating/cooling rates for the incident spectrum, which is a power law with energy index \( \alpha \), and found results essentially equivalent to those given by equations (3)–(7). Note that in the case of a bremsstrahlung spectrum, a formal value of \( \alpha = 0 \) should be used in equation (7). For a power law with energy index \( \alpha = 1.1 \), the results differ by \( <30\% \) (see Fig. 1). Given these rates of energy deposition from the radiation to the flow, we write the total radiative heating/cooling function as \( H = \Gamma_{\text{IC}} + \Gamma_X - \Lambda \). We have also performed several runs of our hydrodynamical models with different assumptions about heating/cooling and found no important difference between the flow dynamics calculated using equations (3)–(7) and those calculated using the original formulae of Blondin (1994), nor between bremsstrahlung and power-law spectra for several values of \( \alpha \). It appears
that, for example, the effects of the optical depth are much more important. That is, the difference between curves for the power-law and the bremsstrahlung spectra at small $\xi$ (correspondingly high density) in Figure 1 becomes unimportant.

3.1.1. Radiation Pressure

The radiation pressure force consists of the force due to continuum absorption $g_{\text{cont}} = F_{\text{UV}} \kappa/c$ and to lines

$$ g_{\text{rad}} = \left( F_{\text{UV}} \kappa / c \right) M(t), $$

where $M(t)$ is the force multiplier (Castor et al. 1975), and $F_{\text{UV}}$ is the local UV flux. We make use of the particular form (Owocki et al. 1988)

$$ M(t) = k t^{-\alpha} \left[ (1 + \tau_{\text{max}})^{-\alpha} - 1 \right] / \tau_{\text{max}}^{-\alpha}, $$

where $t = \tau / \eta$ is the optical depth parameter, $\eta = \kappa / \sigma_e$ is the line strength parameter, $\sigma_e$ is the Thomson cross section, and $\tau_{\text{max}} = \eta_{\text{max}}$. A parameter $\eta_{\text{max}}$ was introduced by Owocki et al. (1988) and Stevens & Kallman (1990) in order to limit the effect of very strong lines. That is, they assume a line number distribution that satisfies $dN/d(\eta, \nu) \sim \eta^{\alpha-2} \exp(-\eta/\eta_{\text{max}})$, where $N(\eta, \nu)$ is the line number distribution. If $\eta_{\text{max}} \to \infty$, such that lines are distributed as a power law, one recovers the result of Castor et al. (1975): $M(t) \sim k t^{-\alpha}$. In the opposite case of $\tau_{\text{max}} \to 0$, the force multiplier is independent of $t$, and $M_{\text{max}} \sim k \eta_{\text{max}}^{\alpha}$. As a result of this maximum line strength cutoff, a correction factor appears in the relation for $M(t)$ (eq. [9]). The dependence of $k$ and $\eta_{\text{max}}$ on $\xi$ has been numerically calculated and then fitted by analytical formulae (Stevens & Kallman 1990):

$$ k = 0.03 + 0.385 \exp(-1.4 \xi^{0.6}), $$

$$ \log \eta_{\text{max}} = \begin{cases} 
6.9 \exp(0.16 \xi^{0.4}), & \log \xi \leq 0.5 \\
9.1 \exp(-7.96 \times 10^{-3} \xi), & \log \xi > 0.5 
\end{cases} $$

From these, one can see that $M(t)$ can depend significantly on the ionization parameter. Taking a fiducial $\alpha = 0.5$ (the value adopted in all our calculations), one finds that $M_{\text{max}} = 585$ at $\xi = 0$, and then has two local maxima, $M_{\text{max}} = 724$ at $\xi = 0.3$, and $M_{\text{max}} = 743$ at $\xi = 3.1$. $M_{\text{max}}$ then drops to 1.7 at $\xi = 100$ and decreases gradually to $M_{\text{max}} = 0.01$ at $\xi = 1000$.

3.2. Initial Configuration: Rotating Torus with Arbitrary Compton Optical Depth

We begin from a rotating toroidal configuration in equilibrium in the external gravitational field of the BH. The equation of state of the torus interior is described by the polytrope $P = K \rho^{1+1/n}$. The distribution of the density (or pressure) in the torus interior was given by Papaloizou & Pringle (1984; “PP-torus” hereafter), who assumed that the distribution of the specific angular momentum inside the torus is constant. In our case, such a torus would not be in equilibrium because of the radiation pressure from the central object. Therefore, we modify the equilibrium equations of Papaloizou & Pringle (1984) to include the radiation pressure term. Since this cannot be done in a closed analytical form, we can write an approximate equation,

$$ \frac{P}{\rho} \approx \frac{1}{n+1} \left( \frac{1 - \Gamma e^{-\tau(t)}}{r} - \frac{1}{2r^2 \sin^2 \theta} - C \right). $$

Note that equation (11) must be understood as a bridging formula between two limiting cases: optically thin, when $e^{-\tau(t)} \sim 1$ (in which case it represents a PP-torus with $1 - \Gamma$ reduced gravity), and optically thick, when $g_{\text{rad}} \sim 0$ (the PP-torus case). The constant $C$ in equation (11) parameterizes the distribution of the torus models and is connected with the distortion of the torus (this is described in more detail below). Including the radiation pressure reduces the effective gravity, and thus the torus gas needs less entropy to sustain it against vertical collapse. In both of these limiting cases, this equation is exact.

Note that the problem of toroidal equilibrium in the presence of heating (or other radiation transfer effects) introduces a
characteristic length scale through the optical depth $\tau$, leading to non-self-similarity of the model. Equation (11) was derived by assuming that the distribution of the specific angular momentum inside the torus is constant. Choosing nondimensional units and working in terms of $\omega$ (the cylindrical radius in units of $R_0$), if we define the nondimensional density $\rho$ such that $\rho(\omega = 1) = 1$, and the nondimensional pressure $P$ and internal energy $e$ such that $P = (\gamma - 1)e$ and $e(\omega = 1) = \rho_0$, then $\rho_0 = [n/(n + 1)](1 - (\Gamma)(0.5 - C)\theta)(1/(x) - (1/2(\pi^2) - C)]^{-1/n}$. The inner and outer edges of the torus are located at $\omega = \omega^+$ and $\omega^*$, respectively. Bounded configurations exist only for $0 < C < 0.5$, and the distortion of the torus is described by the parameter $d = (\omega^- + \omega^*)/2 = 1/(2C)$. The boundary of the torus is matched to the exterior by the condition $P = 10^{-6}$. The PP-torus is unstable to non-axisymmetric perturbations (Papaloizou & Pringle 1984). However, this effect cannot be numerically investigated in the azimuthal symmetry that we adopt, since no signals can propagate in the $\phi$ direction. At $\omega > 1$, matter that constitutes the torus has an excess of angular momentum with respect to the local “equilibrium” Keplerian value, i.e., $l(\omega, z) > l(r)$, while the opposite is true in the inner parts of the torus; i.e., at $\omega < 1, l(\omega, z) < l(r)$. It is the internal pressure of the torus (eq. [11]) that inhibits matter from settling to smaller (or larger, depending on angular momentum) orbits. The gas first evaporates from the part of the torus that is closer to the source of radiation and tends to settle at larger $\omega$, as soon as the back-pressure supporting it drops.

We begin our simulations from the stationary configuration determined from equation (11). We follow the torus evolution as it is being heated by X-rays. No replenishing of the gas that constitutes the initial torus is provided. Therefore, the torus will eventually lose almost all its mass and will completely evaporate. However, in the regime we are investigating, the evaporation is not dramatic and does not significantly deplete the torus during the characteristic dynamical time.

In the following sections, we will show that the existence and character of the flow from the heated torus depends critically on the geometry. That is, it depends on the divergence of the flow streamlines, the strength and incident angle of the X-ray illumination, and the direction of the effective gravity in the rotating frame of the torus. The flow is intrinsically two-dimensional, and therefore cannot be adequately described a priori by 1D models, such as those preformed by Chelouche & Netzer (2005). Furthermore, the shape of the torus, and thus the launching surface for the flow, is affected by the flow. Therefore, the torus interior cannot be considered as a boundary condition (e.g., as in Balsara & Krolik 1993); we need to include it in the computational domain.

4. METHODS

For our computations, we adopt a spherical-polar coordinate system $(r, \theta)$, extending the computational domain $\{r_i, \theta_i\}$ from $r_{in} = 0.01$ to $r_{out} = 50$ in radius, and from 0 to $\pi$ in the polar domain, making no assumption about equatorial symmetry. The number of points in the radial, $N_r$, and polar, $N_\theta$, directions are taken to be equal: $N = 140$ in low-resolution grids, and $N = 300$ in high-resolution grids. The $\{r_i\}$ grid is nonuniformly spaced, i.e., $r_{i+1} = r_i + (r_{out} - r_{in})(k_{i+1}^{(N_r - 1)} - 1)/(k_{N_r}^{(N_r - 1)} - 1)$, and $r_{i-1} = r_i + (r_{out} - r_{in})k_{i-1}^{(N_r - 1)}$ for $i = 2, N_r - 1$, and a refinement factor of $k_r = 4$. In order to achieve better resolution of the flow rather than the torus interior, we also adopt a polar grid with nonuniform spacing $\theta_{i+1} = \theta_i - \Delta \theta$, so that the maximum refinement is approached at $\theta = \pi/4$: $\delta_{i+1} = \delta_i/k_{\theta}^{(N_\theta - 1)}$ at $0 < \theta < \pi/4$, and $\delta_{i+1} = \delta_i k_{\theta}^{(N_\theta - 1)}$ at $\pi/4 < \theta < \pi/2$ (with analogous spacing in the southern hemisphere). Boundary conditions are axially symmetric at $\theta = 0$ and $\pi$, and outflowing at $r_{in}$ and $r_{out}$.

To numerically solve the system of hydrodynamical equations (1)–(3), we use the code ZEUS2D (Stone & Norman 1992). Note that the characteristic time of X-ray heating/cooling can be much shorter than the dynamical time, which in such a case introduces strong stiffness to the system of equations (1)–(3). To overcome this difficulty, some modifications have been made to the code. The most important one is the implementation of a fully implicit update of the energy in equation (3) just prior to the transport step in ZEUS2D. In addition, we account for the radiation pressure (eq. [8] term) as an initial test, we evolved a toroidal distribution of matter for two rotational periods and found the configuration to be stable. The gas is illuminated by the incident X-ray radiation, which has a power-law spectrum with an energy index $\alpha = 1$. The heating/cooling rates are described by the approximate analytical formulae given in equations (3)–(7).

4.1. Warm Absorbers

We test the output of our hydrodynamical models against the ability to predict warm-absorber spectra. To do this, we use the output from the hydrodynamical code of $\rho, v$, and $T$ as an input to the calculation of X-ray line and photoelectric absorption spectra. The numerical code has been specifically developed for calculation of spectra in the X-ray domain and makes use of procedures developed for the XSTAR code (Kallman & Bautista 2001), while calculating the ionization structure and distribution of opacities and treating the radiation transfer in the Sobolev approximation (Rybicki & Hummer 1983). Although the goal of this paper is to show that purely hydrodynamic 2D models can produce warm-absorber spectra, we present here only sample spectra, assuming pure absorption. We postpone a more detailed discussion, including a full 3D transfer calculation, to a separate publication.

5. RESULTS

The most important parameters that determine the properties of the warm-absorber flow are the initial Compton optical depth of the torus $\tau_X^i = \tau(\theta = 90^\circ)$ (or equivalently, the maximum initial torus density $n_{max}$) and the distance from the BH, $R_0$. We also explore the dependence on $\Gamma$ and $d$. Typical values are chosen for some other parameters: the mass of the black hole $M_{\text{BH}} = 10^6 M_\odot$, the Compton temperature of the X-ray radiation $T_X = 10$ keV, and the fraction of X-rays and UV radiation $f_X = 0.5$. (For rotating flows exposed to multitemperature radiation, see, e.g., Proga et al. 2008.) This last parameter is consistent with typical energy distributions of radiation close to the BH (Laor et al. 1997). We neglect any changes in the BH luminosity. The important thermal timescales within the flow, namely the Compton heating and cooling time $t_{X}$ and the dynamical time $t_{dyn}$, may be of the same order: $t_X \sim t_{dyn} \sim 10^{10}$ s. This is discussed in more detail later in this section. Thus, the outflowing gas may not be in thermal equilibrium, and adiabatic losses are likely to be important. Note that a nearly hydrostatic Compton-heated corona can exist only at $r \approx R_{IC} = GM_{\text{BH}} \mu m_{\text{p}}/RT_X \approx 8 \times 10^{16}(M_\odot/T_X)$ cm, where $T_X$ is the Compton temperature in terms of $10^7$ K. In all of our models, the major flow is located at $r \gg R_{IC}$.

We have calculated 20 models, including combinations for $\Gamma = 1.3$ (models A) and (models B); $R_0 = 0.5, 1, 1.5, \Gamma = 0.1, 0.3, 0.5, 0.7, 0.9$, all with $d = 2.5$; and two models with $d = 5$ (models C). These are summarized in Table 1, where some of the characteristic results from the computed models are presented. In what follows, we describe in detail the cases that best illustrate...
the most important results. We also discuss the dependence of our results on parameters, based on the behavior of the ensemble of models.

The model $A_6$ is similar to the model described in Paper I, although the initial torus in the model described here has a different distribution of $\rho$ and $e$ (see eq. [11] and the discussion thereafter), and a smaller $\tau_c^\iota$. In Paper I, this model is described in detail. Calculations presented here reveal more details and confirm the conclusions of Paper I. We begin here by describing results from model $B_6$ and later discuss how this model differs from model $A_6$.

Model $B_6$ has $\tau_c^\iota = 40$, $R_0 = 1$, and $\Gamma = 0.5$, and corresponds to a Compton-thick ($\tau_c^\iota \approx 40$) torus with a large $n_{\text{max}} = 10^7$ $\text{cm}^{-3}$ and a mass $M_{\text{torus}} = 9.3 \times 10^3 M_\odot$. Results are displayed in Figure 2, where the evolution of the distribution of density is shown as a function of time (the density scale is such that 0 corresponds to $10^7$ $\text{cm}^{-3}$), in Figure 3, where the distribution of pressure is shown at $t = 3$ (the pressure scale is such that 0 corresponds to $4.7 \times 10^{-4}$ $\text{dyn cm}^{-2}$), in Figure 4 for various quantities as a function of the inclination $\theta$, in Figure 5 (left), where the effect of the distortion parameter $d$ is demonstrated, and in Figure 6, which shows horizontal “slices” of the velocity and temperature at constant height, $z$. In the case of this model, the torus column is high enough to effectively screen the torus interior from penetrating X-rays. This leads to the formation of a nearly pure funnel flow, i.e., the torus interior, and hence the shape of the surface responsible for launching and collimating the flow is essentially unaffected by X-ray heating on timescales $\lesssim t_{\text{phot}}$.

Here and below, we discuss the time evolution of our models in terms of $t$, measured in units of the characteristic time of rotation, $t_0$. After $t = 1$, a high-pressure region created by X-ray heating expands to $r \approx 4.5$–5 pc throughout the area not shadowed by the high-density torus. At this time, the torus is located at $\theta < 50^\circ$. The distortion parameter has a value $d \approx 2.5$; i.e., the torus shape is almost unchanged from its initial value. This is shown in the top left panel of Figure 2. Within the part of the flow that is not shadowed by the torus, high-temperature gas expands in a spherical bubble of radius $r \approx 5.2$ pc, in which the temperature is $T \sim (3 - 10) T_\text{vir}(r)$, where $T_\text{vir} = 2.6 \times 10^5 M_\odot/r_{\text{pc}}$ is the local virial temperature. An axisymmetric region exists between $\varpi < 0.75$ pc and $z < 2$ pc, where the temperature $T \sim 10 T_\text{vir}(r)$. That is, high-temperature ($T \sim 3 \times 10^6$ K) but low-density gas fills the torus funnel. The ionization parameter (eq. [4]) in this region is $Z \approx 10^4$–$10^5$. The outer edge of the torus extends to $\approx 4.25$ pc in temperature and to $\approx 4.5$ pc in density contours.

Figure 2 (bottom left) shows density and velocity fields for model $B_6$ at $t = 3$. Figure 3 shows that a high-pressure region expands to a height $z \approx 6$ pc from the equatorial plane. The torus inner edge is inferred from the temperature and density maps to be $\varpi \approx 0.83$ pc. Inside this radius, a region that we refer to as the “torus throat,” the temperature is $T \approx 10^6$–$10^7$ K. A wide nozzle with $(\varpi_{\text{max}} - \varpi_{\text{min}})/\varpi_{\text{max}} \approx 2.12$, where $\varpi_{\text{max}} \approx 0.4$ pc, is formed, with an inner radius of $\varpi \approx 0.85$ pc. The torus outer edge is slightly shifted to $\varpi \approx 4.5$ pc. The values of $\xi_{\text{min}}$ (the minimum ionization parameter along a radial line) and the column density vary significantly with the inclination angle. Figure 4 shows the distribution of radial and poloidal velocity, $\xi$, density, and the rate of growth of number density with radius as a function of $\theta$ at $t = 3$ for model $B_6$. Near the axis, $\xi_{\text{min}}(\theta \approx 20^\circ) = 10^7$, and the column density is $N_{\text{23}} = 10^{-3}$. Note that if $\xi_2 \approx \xi_{\text{dyn}}$, i.e., if the gas is not in thermal equilibrium, then $\xi_2$ is not meaningful as when $\xi_2 < \xi_{\text{dyn}}$. When $\xi_2 \approx \xi_{\text{dyn}}$, adiabatic losses strongly affect the temperature of the gas. At larger heights, the ionization parameter decreases: $\xi_{\text{min}}(\theta \approx 25^\circ) = 3 \times 10^3$, and at higher inclination, $\xi$ gradually reduces from $\xi_{\text{min}}(\theta = 45^\circ) = 12$, eventually becoming $\xi_{\text{min}}(\theta = 60^\circ) = 2.5$. At a critical angle ($\theta \approx 40^\circ$), a strong rise in the column density reflects the fact that the line of sight penetrates the dense torus body rather than the wind (see Fig. 4, bottom right). The column density increases from $N_{\text{23}} = 0.3$ at $\theta = 45^\circ$ to $N_{\text{23}} \approx 100$ at $\theta = 60^\circ$, providing total obscuration. Figure 3 also shows the position of the sonic surface determined by the relation $v_p/c_s = 1$, where $v_p = (v_r^2 + v_\theta^2)^{1/2}$ is the poloidal velocity, and $c_s = (R T / \mu)^{1/2}$ is the speed of sound. Behind the torus, a low-entropy region exists that is bounded

| Model | $\tau_c^\iota$ | $R_0$ | $\Gamma$ | $d$ | $v_{\text{max},r=3}^{\text{los}}$ | $v_{\text{max},r=5}^{\text{los}}$ | $v_{\text{max},r=3}^{\text{los}}$ | $v_{\text{max},r=5}^{\text{los}}$ | $M_{\text{los}}$ | $M_{\text{los}}$ |
|-------|----------------|-------|---------|-----|-----------------|-----------------|-----------------|-----------------|-------------|-------------|
| A1    | 1.3            | 0.5   | 0.1     | 2.5 | 516             | 317             | 624             | 332             | 4.09 \times 10^{-4} | 6.54 \times 10^{-3} |
| A2    | 1.3            | 0.5   | 0.3     | 2.5 | 710             | 317             | 847             | 330             | 1.48 \times 10^{-3} | 4.31 \times 10^{-3} |
| A3    | 1.3            | 0.5   | 0.5     | 2.5 | 707             | 267             | 760             | 291             | 2.34 \times 10^{-3} | 2.14 \times 10^{-2} |
| A4    | 1.3            | 1     | 0.1     | 2.5 | 547             | 189             | 514             | 217             | 1.76 \times 10^{-3} | 1.66 \times 10^{-2} |
| A5    | 1.3            | 1     | 0.3     | 2.5 | 526             | 179             | 605             | 343             | 9.65 \times 10^{-3} | 6.34 \times 10^{-3} |
| A6    | 1.3            | 1     | 0.5     | 2.5 | 570             | 235             | 670             | 293             | 2.02 \times 10^{-2} | 1.8 \times 10^{-2} |
| A7    | 1.3            | 1.5   | 0.1     | 2.5 | 360             | 197             | 413             | 230             | 6.21 \times 10^{-3} | 1.64 \times 10^{-2} |
| A8    | 1.3            | 1.5   | 0.3     | 2.5 | 388             | 169             | 540             | 310             | 1.40 \times 10^{-2} | 5.68 \times 10^{-2} |
| B1    | 1.3            | 1.5   | 0.5     | 2.5 | 317             | 207             | 663             | 370             | 2.66 \times 10^{-2} | 1.23 \times 10^{-3} |
| B2    | 1.3            | 1.5   | 0.5     | 5   | 789             | 788             | 772             | 770             | 1.35 \times 10^{-2} | 8.01 \times 10^{-3} |

* In $v_{\text{max},r=3}^{\text{los}}$, $\theta$ is the inclination angle, and $T$ is the observing time.
from the sides by a quasi-stationary shock. The existence of this structure can be understood from the following considerations. If the flow were perfectly symmetric in both hemispheres, then it should have \( v_z = 0 \) at \( z = 0 \), and the \( z = 0 \) plane would be the equivalent of a rigid wall (i.e., a reflecting boundary). Thus, if \( v_z < 0 \) behind the torus, the formation of a shock structure is anticipated. Generally, this is the kind of picture one expects to observe from a supersonic wind flowing over a rigid obstacle.

At \( t = 5 \) in model B6 (Fig. 2, bottom right), the density maximum is located at \( \varpi \approx 2 \) pc. The inner edge of the torus does not shift significantly from its position at \( t = 3 \) (\( \varpi^- \approx 0.75 \) pc in density maps and \( \sim 1 \) pc in temperature maps), and the outer edge is at \( \varpi^+ \approx 4.3 \) pc. The temperature of the torus interior is in the range \( 1 \times 10^5 \) to \( 6 \times 10^5 \) K. A hot flow is located near the axis, bounded from the sides by the torus throat and with high temperature \( \sim 1 \) few \( \times 10^6 \) K. A significant drop of the ionization parameter \( \xi \) from \( \sim 6 \times 10^3 \) to \( \sim 6 \) occurs again at \( \theta \gtrsim 45^\circ \) to \( 50^\circ \) (see Fig. 4), where the column density also rises from \( N_{23} \approx 0.04 \) to 30 at \( \theta \gtrsim 60^\circ \). The aspect ratio of the torus is \( \Delta = R_0/H \sim 1 \), in accordance with what is inferred from observations (Krolik & Begelman 1986; Jaffe et al. 2004). At low inclinations, \( \theta \lesssim 10^\circ \), and everywhere in the wind, the poloidal component of the velocity is determined by \( v_r \). However, at \( \theta > 50^\circ \) inside the torus throat, the \( v_\theta \) component is important; i.e., \( v_\theta \sim v_r \) at \( \varpi < 1 \) pc.

Model A6 has \( \tau_{\xi} = 1.3, R_0 = 1, \) and \( \Gamma = 0.5 \), and is very similar to the model described in Paper I. It differs from model B6 in that the smaller optical depth of the torus interior cannot shield the gas from extensive X-ray heating, and the torus loses mass from large parts of its surface. The initial maximum density of the torus is \( n_{\text{max}} = 10^6 \) cm\(^{-3} \), which corresponds to an initial torus mass \( M_{\text{tor}} = 9 \times 10^4 M_\odot \). Figure 7 shows the distributions of poloidal velocity, \( \xi \), density, and the rate of growth of number density with radius as a function of \( \theta \) at \( t = 3 \) for model A6 (in the same format as Fig. 4). During the evolution, a region of high pressure extends from \( r \approx 4.5 \) to \( 5 \) pc at \( t = 1 \), to \( r \approx 12 \) pc at \( t = 3 \), and to \( r \approx 20 \) pc at \( t = 5 \). The inner edge of the nozzle shifts slightly from \( \varpi^- \approx 0.8 \) pc at \( t = 1 \) to \( \varpi^- \approx 0.83 \) pc at \( t = 3 \), and \( \varpi^- \approx 0.75 \) pc at \( t = 5 \). At later times, the behavior of the model A6 is similar to that of models A1 and A5, as can be inferred from Figure 8.

It has been mentioned that in model B6, much of the torus interior is opaque to penetrating X-rays. Remarkably, the minimum nozzle cross section does not change much at late times, implying...
that the mass-loss rate becomes quasi-saturated. Note that in the case of a 1D flow, $\dot{M}$ is roughly set by the position of the sonic point, which in turn is set by gravity. In the case of a 2D nozzle, the mass-loss rate is determined by X-ray heating, gravity, and the minimum nozzle cross section. In the case of model B6, the latter remains almost unchanged in time. We believe that this model is probably most representative in showing the key features of X-ray–excited flow. However, models A1 may generally have broader angular patterns, in which a warm-absorber spectrum is observed, as will be discussed below. Only comparing synthetic spectra with observations can answer the question of what model is more adequate in describing the phenomenon of warm absorbers.

5.1. Mass Loss within the Funnel Flow

It is instructive to consider the distribution of variables within a horizontal cross section at a certain height above the equatorial plane. In so doing, we interpolate the solution from an $(r, \theta)$ spherical grid to a $(z, \varpi; 100 \times 100)$ Cartesian grid. Figure 6 shows the distribution of temperature, and the z-component of velocity in terms of the escape velocity, $U_{esc} = (2GM_{BH}/r)^{1/2}$, at different heights for model B6.

A hot region extends to $\varpi \approx 1$ pc at $z = 0.2$ and to $\varpi \approx 2$ pc at $z = 1$. The “funnel” can be seen in distributions of both temperature and velocity. At the X-ray–heated boundary of this nozzle, gas is being heated so that its temperature suddenly increases to $\sim 10^6 - 10^7$ K. This fact reveals an analogy between the torus flow and X-ray–excited winds in X-ray binaries (Basko et al. 1977; McCray & Hatchett 1975); we discuss this further later in this section. Note that in our case, the inner surface of the torus serves both as a copious source of gas and as a collimating funnel.

Figure 5 shows models B6 and C2 at $t = 4$, and Figure 8 shows density and velocity streamlines for models A1 and A3 at $t = 4$. Note that there is little difference between Figure 8 (left, model A1) and Figure 8 (right, model A3); the effect of a smaller $R_0$ is partially compensated for by the fact that $\Gamma$ is also smaller, thus reducing the effective gravity. If $\Gamma \approx 0$, then $\varpi^{-} \approx 0.5$ (for $C = 0.2$ in eq. [11]). However, when $\Gamma = 0.5$, as in model B6 (Fig. 5, left), the effective gravity at the innermost optically thin edge of the torus is reduced by half. Figure 5 (right) shows a model (C2) with an initially large distortion parameter $d = 5$ ($C = 0.1$; cf. Table 1).

In model A6, a well-developed wind is observed in the vicinity of the high-density torus, following the equal-pressure contours; the maximum radial velocity is observed close to the axis at $v_{\max}(\theta \approx 3\degree) = 700$ km s$^{-1}$. As a general trend, at $t = 3$ the maximum velocity has a plateau at $20\degree < \theta < 50\degree$, $v_{\max} = 220$ km s$^{-1}$, and at lower values closer to the equatorial plane (Fig. 7). The flow is approximately symmetric in both hemispheres. At later times, $t = 4$ and $t = 5$, the behavior of the model is similar to $t = 3$; namely, $v_{\max}(\theta \approx 4\degree, T = 5) = 900$ km s$^{-1}$, and $v_{\max} \approx 380$ km s$^{-1}$ on the plate. The torus is losing mass in all directions, although with very different speeds at different inclinations. Because we are solving equations of ideal hydrodynamics (with only a small numerical viscosity), accretion through the inner boundary (at $r = 0$) is negligible: $M_{\dot{m}}(M_{\odot} \text{ yr}^{-1}) < 10^{-5}$. The maximum mass flux per unit solid angle $M_{\dot{m}}(M_{\odot} \text{ yr}^{-1} \text{ sr}^{-1})$ peaks at $\theta \approx 13\degree$ at $t = 3$, i.e., at much higher inclinations than $v_{\max}$; $M_{\dot{m}}(M_{\odot} \text{ yr}^{-1} \text{ sr}^{-1})$ at $t = 3, M_{\dot{m}}(M_{\odot} \text{ yr}^{-1} \text{ sr}^{-1}) = 0.02$ at $t = 5$. The total mass-loss rate at $t = 3$ is $M(\dot{M}_{\odot} \text{ yr}^{-1}) \approx 7 \times 10^{-3}$.

The mass-loss rate is $M(\dot{M}_{\odot} \text{ yr}^{-1}) \approx 2.4 \times 10^{-2}$ at $t = 4$ and $M(\dot{M}_{\odot} \text{ yr}^{-1}) \approx 4 \times 10^{-2}$ at $t = 5$, and the change of the mass-loss rate with time is $dM/dt (\dot{M}_{\odot} \text{ yr}^{-1}) \approx 10^{-6}$. Comparing distributions of $v$ and $n$, we conclude, for example, that the apparent minima of $v_{\varpi} \approx v_{\varpi}$ correlate (with a certain lag) with maxima of $n$ and vice versa, reflecting conservation of mass flux.

As in model A6, the model B6 funnel wind carries mass flux that does not change much during its evolution. The maximum velocity is as high as $\sim 1000$ km s$^{-1}$ near the axis, and typically $200$ km s$^{-1} \leq v_{\max} \leq 600$ km s$^{-1}$ at $15\degree \leq \theta \leq 50\degree$. The bulk of the gas, which may potentially produce warm-absorber features, moves with comparable speed. However, the largest observed velocity in model B6 is $v_{\max}(\theta \approx 3\degree) = 1200$ km s$^{-1}$ at $t = 5$. The mass-loss rate is $M(\dot{M}_{\odot} \text{ yr}^{-1}) \approx 3.4 \times 10^{-2}$ at $t = 3$, and $M(\dot{M}_{\odot} \text{ yr}^{-1}) \approx 7 \times 10^{-2}$ at $t = 5$.

5.1.1. Spectra

Computing absorption spectra is a key test for the warm-absorber flow model. Several sample spectra are shown here, although a detailed discussion of methods and results of calculations of such spectra are postponed to a later paper.

Figure 9 shows the model A6 spectrum observed at different inclinations. The figure shows the warm-absorber spectrum at $t = 3$ and $t = 4$. At $t = 4$, a rich X-ray line-absorption spectrum exists in the range $43\degree < \theta \leq 52\degree$, and in the range $47\degree < \theta \leq 55\degree$ at $t = 5$.

At $t = 3$, model B6 predicts a rich spectrum for $42\degree < \theta \leq 47\degree$. At later times, a similar spectrum appears at lower inclinations. Figure 10 shows the model B6 spectrum observed at different inclinations at $t = 4$. At $t = 5$, the spectrum exists between $45\degree < \theta \leq 50\degree$. Note that the region of the funnel wind in this model is bounded by the area unshadowed by the torus: $0\degree \leq \theta \leq 40\degree$. At $\theta \geq 30\degree$, the column density becomes $N_{\odot} \approx 0.45$, and the ionization parameter $\xi \approx 20$. At higher inclinations, the X-ray flux in the 1 keV $< E < 2$ keV range becomes severely absorbed.

Figure 11 shows the evolution of the observed properties of the warm-absorber flow with time (in the same time units) for model A3. It can be seen that warm-absorber spectra are changing.
Fig. 4.—Distributions of \( v_r \) and the poloidal velocity \( v_p \) (top left), \( \xi \) (top right), \( n \) (bottom left), and the rate of growth of number density with radius (bottom right) at time \( t = 3 \) for model B6. Curves are marked by an inclination angle \( \theta \). The horizontal axis gives the distance in pc. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 5.—Effect of distortion parameter \( d \), with velocity streamlines superimposed on contours of the number density at time \( t = 4 \). Left: Model B6, Right: Model C2. The axes give the distance in pc. [See the electronic edition of the Journal for a color version of this figure.]
slowly on a timescale $\Delta t \sim 1$. This is typical for most of our models and shows the range of times over which our solution can be considered as a representation of a steady state warm-absorber flow. A quantitative analysis of our synthetic spectra and comparison with observations will be done in a later paper. This is due in part to the need for a full three-dimensional treatment of the transfer and scattering of line photons, which we do not present here. Rather, the spectra in Figures 9, 10, and 11 are calculated assuming pure absorption. We can crudely calculate some of the properties of individual lines, and show that these are generally consistent with observations. A convenient way to do this is to discuss the profile of what is likely to be the strongest line in any synthetic spectrum, the $\text{Ly}$ line of O viii. In model A6 at $t = 3$, the full width at half-maximum (FWHM) of this line is $\sim 200$ km s$^{-1}$. Closer to the BH, the maximum observed velocity is greater; i.e., models A3 and B3 give FWHMs of $\sim 400$ km s$^{-1}$ at $\theta \sim 43^\circ$ and $40^\circ$, respectively. The centroid energy of the line is at a blueshifted velocity $(50 \sim 200)$ km s$^{-1}$ with respect to line center. These velocities are less than those observed from, e.g., NGC 3783, but are comparable to those observed from other objects (McKernan et al. 2007). Such comparisons should also include the effects of scattered emission, which may skew the line centroid and red edge, and which we have not considered here.

5.1.2. Analytical Estimates of the Mass-Loss Rate

The mass-loss rate found from numerical calculations is in approximate agreement with theoretical expectations. The value of the mass-loss rate $\dot{M}$ can be estimated by integrating the average mass flux $\dot{m}$ over the surface area of the torus exposed to X-ray radiation, $\Sigma \sim 2\pi R_0^2/\Delta$, where $\Delta = R_0/H \sim 1$. Here, $\dot{m}$ may be estimated using the same arguments as those of Basko et al. (1977) and McCray & Hatchett (1975). Namely, that heating from a BH creates a narrow transition layer—a “skin” on the surface of the torus. Within this layer, the temperature rises almost discontinuously from an inner “cold” ($T \sim 10^4$, $T \leq T_{\text{vir}}$) to an outer “hot” ($T \geq T_{\text{vir}}$) value. This transition can be seen in Figure 6.

Matching momentum $p + \rho v^2$ and mass flux $\dot{\rho} = \rho v$ below and above this discontinuity, we obtain the well-known relation
\[ j^2 = (P_h - P_0)/(\rho_0^{-1} - \rho_h^{-1}) \]

where the subscripts 0 and \( h \) refer to values below and above the discontinuity, respectively. Being heated, the gas expands, and its specific volume \( V = 1/\rho \) increases. Above the discontinuity, the flow is assumed to be isothermal, so that \( P \sim 1/V \). In the \( P-V \) plane, the transition between points \( P_0, V_0 \) and \( P_h, V_h \) goes through the straight line with an inclination \((P_h - P_0)/(V_h - V_0) > (dP/dT)_T\), and it follows that \( j^2 < -(dP/dV)_T = \rho_h c_{s,h}^2 \), where \( c_s = (RT/\mu)^{1/2} \) is the velocity of sound. Since \( j = \rho_h v_h \), it follows that \( v_h < c_{s,h} \) and that the flow immediately above the discontinuity is subsonic (Basko et al. 1977; McCray & Hatchett 1975). From momentum conservation, \( P_0 \approx P_m = P_h + \rho_h v_h^2 \), and the mass flux associated with such heating can be estimated as \( \langle j \rangle = P_m/[\rho_h M_h(1 + M_h^2)] \), where \( M_h \) is the Mach number above the discontinuity.
Fig. 10.—Model B₆ spectrum, observed at time $t = 4$.

Fig. 11.—Model A₃ X-ray spectra, observed at $\theta = 45^\circ$, as a function of time.
discontinuity, }P_m\text{ is the pressure below the discontinuity, and for simplicity we assume that }v_b \simeq c_s\text{.}

McCray & Hatchett (1975) have calculated the state of the gas in the optically thin layer of a stellar atmosphere heated by X-rays. From their results, it follows that the relation between }P_m\text{ and }F_X\text{ can be cast in the form }P_m = 10^{-12} \alpha_\ldots F_X, \text{ where }\alpha_\ldots \sim 1, \text{ reflecting the shape and the effective temperature of the incident spectrum (Basko et al. 1977). Although it is essential (in order to obtain stationary transonic flow, correctly matching boundary conditions at infinity) that the flow above the discontinuity be subsonic, we assume that the sonic surface is located not far from the discontinuity, estimating }v_2 = c_s\text{, and }M_{b_1} = 1. \text{ Next, we write }F_X = \left[ L_X/(4\pi R^2)\right]/(1 + \varphi), \text{ where }A\text{ is the effective X-ray albedo of the X-ray–heated skin (which we simply assume to be optically thin); we take }A = 0.4\text{ and assume }\mu = 0.5. \text{ Calculating }M = \langle j \rangle \Sigma, \text{ we finally obtain}

\[ \dot{M} (\text{M}_\odot \text{ yr}^{-1}) \simeq 0.16 \frac{F_X \Gamma}{T_{b_6}} \frac{M_6}{\Delta}, \] (12)

where }T_{b_6}\text{ is the temperature above the discontinuity in units of }10^6 \text{ K. Inserting relevant parameters such as }\Gamma = 0.5, F_X = 0.5, \text{ }R = 1 \text{ pc, }M_6 = 1, \text{ and }\Delta = 1, \text{ and adopting the value of }T_b\text{ taken from our numerical model }A_b, \text{ we estimate the mass-loss rate as }\dot{M} = (M_6 \text{ yr}^{-1}) \simeq 4 \times 10^{-2}. \text{ Comparing results from this approximate formula with those summarized in Table 1, we conclude that they are in good accord. Given the torus mass in model }A_6, \text{ we conclude that it may sustain such mass loss for }\sim 1 \times 10^6 \text{ yr. The upper limit may be inferred from Table 1, and is found to be }\sim 10^5 \text{ yr.}

5.1.3. Adiabatic Losses

The characteristic timescale at which energy is deposited into the flow via Compton processes, }t_X\text{, can be cast in the form

\[ t_X (\text{s}) \simeq 9.4 \times 10^{10} \frac{n_{pc}^2}{k_F} \frac{T}{\Gamma X} \frac{T}{T - 1}, \] (13)

where }T = T/T_X\text{ and }T_X = 2.9 \times 10^7 \text{ K. This should be compared with the dynamical time of the flow,

\[ t_{\text{dyn}} (\text{s}) \simeq 4.3 \times 10^{10} r_{pc} \sqrt{\frac{T}{T}}. \] (14)

When }t_{\text{dyn}} \leq t_X\text{, the outflowing gas departs from thermal equilibrium, and one must account for adiabatic losses }\dot{\Lambda}_{ad}\text{ when calculating the temperature of the gas. Note that the properties of the two-phase (or multi-phase) gas are conventionally described by the S-curve on the }T-Z\text{ diagram (Krolik et al. 1981), where }Z = F_X/(nkT)\text{ is the other form of the ionization parameter. That is, on the }T-Z\text{ plot, those places where }dT/dZ > 0\text{ are stable to isobaric perturbations. Places where }dT/dZ < 0\text{ are unstable. Including }\dot{\Lambda}_{ad}\text{ may significantly lower the temperature of the hot phase (Chelouche & Netzer 2005). This temperature can be estimated by equating the Compton heating rate }\dot{\Lambda}_{IC} \simeq 4kF_X\langle \sigma_v n/m_c^2 \rangle F_X \text{ with the adiabatic losses rate }\dot{\Lambda}_{ad} \sim \langle v/r \rangle p c^2. \text{ The flow near the funnel walls is less divergent than it would be in the case of a spherically symmetric wind, in which case the latter expression is a factor of 2 larger. Assuming that the }\text{above the discontinuity }v \sim c_s, \text{ we obtain

\[ T_b (\text{K}) \simeq 5.7 \times 10^6 \left( \frac{k\Gamma}{r_{pc}} \right)^{2/3}. \] (15)

which gives }T_b \approx 2 \times 10^8 \text{ K for parameters adopted in this paper. This value is in good agreement with the value of }T_b\text{, which is found from }T(z, x, z)\text{ distributions shown in Figure 6. Three major regions within the funnel flow may be emphasized: (1) a "discontinuity," where temperature is rising from the inner torus value to }T_b \approx 10^6 \text{ K; (2) a "plateau," where }T \approx T_b, \text{ and the thermodynamic characteristics of the flow result from the interplay between }\dot{\Lambda}_{IC} \text{ and }\dot{\Lambda}_{ad}, \text{ and (3) a region of hot, overionized flow, where }T \rightarrow T_X.\n
5.1.4. Returning Current

From Figures 5 and 8, we see that there exists a region behind the dense torus where outflow switches to inflow. This gas rejoins the torus in the shadowed region. For example, taking the model }A_6\text{ and integrating the mass flow over the region where }v_p < 0\text{, we obtain }\dot{M} = 4 \times 10^{-6} M_\odot \text{ yr}^{-1} \text{ at }t = 3. \text{ That makes }\sim 6\%\text{ of the total accretion rate }\dot{M}_{\text{accr}}\text{ required to maintain the }0.5 L_{\text{edd}}\text{ luminosity of the BH, given the efficiency of accretion }\eta = 0.06. \text{ At the same time, much more mass, }\sim 2 \times 10^{-3} M_\odot \text{ yr}^{-1}, \text{ is lost within the funnel (}\theta \leq 50^\circ\text{) in the X-ray–excited wind. Matter that is removed from the funnel is replaced by gas from the torus interior. Thus, a weak large-scale convection flow is observed in the simulations. This effect is most clearly seen in models with large }d\text{, such as model }C_2\text{, shown in Figure 5, and is due to a strong drop of }v_p\text{ as the outflowing gas passes the shock wave front behind the torus (cf. Fig. 3) and is unable to escape from the potential well.}

5.1.5. Radiation Force

The dependence of the radiation pressure on the ionization parameter }\xi\text{ is determined by equations (8) and (10). In the region of the fast flow, the wind is too overionized for the radiation force to be important. This resembles the case of low-mass X-ray binaries (LMXBs), in which the radiation pressure is also found to be insufficient to drive a significant outflow (Proga & Kallman 2002). The ionization parameter drops below }\sim 100\text{ at }\theta > \sim 40^\circ, \text{ the value determined by the torus aspect ratio }\Delta. \text{ Thus, the radiation pressure may be of importance at higher }\theta\text{, and at these inclinations its relative strength is determined by the attenuation of the X-ray and UV fluxes. For model }A_6, \text{ we have }\gamma \approx 6 \text{ at }\theta > \sim 90^\circ\text{ and }\gamma \approx 1 \text{ at }\theta > 60^\circ; \text{i.e., the torus becomes Compton thin at }\theta > 60^\circ. \text{ The radiation pressure exhibits complicated behavior with varying }\theta\text{, having multiple maxima and minima. The force multiplier }M(t, \xi(\theta))\text{ peaks at }\sim 44^\circ\text{ at }\tau \sim 2, \text{ where }\gamma_{\text{ad}}/\gamma_{\text{grav}} \sim 5. \text{ Generally, two maxima of }\gamma_{\text{ad}}\text{ are observed at a given }\theta\text{ along a radial line. The second peak becomes smaller at higher inclinations; i.e., in models }A_6\text{, the radiation pressure is determined mainly by the properties of X-ray heating [i.e., }\xi(\theta)]\text{ rather than by the attenuation of the UV flux. At higher }\theta, \text{ smaller maxima occur at smaller }r\text{; the inner skin of the torus exerts considerable radiation pressure, although at large }\theta\text{ it is opposed by the back-pressure of the torus interior. We calculated a model with the same parameter values as model }A_6\text{, but with }\gamma_{\text{ad}} = 0. \text{ At }t = 3, \text{ this model gives }v_{\text{max}}(\theta = 10^\circ) = 564 \text{ km s}^{-1}\text{ and }v_{\text{max}}(\theta = 45^\circ) = 194 \text{ km s}^{-1}. \text{ Comparing with Table 1, we see that for the range of angles where warm-absorber flow is observed, the radiation pressure does not play a major role in the flow acceleration. In models }B_6\text{, the attenuation is much stronger than in models }A_6, \text{ and consequently the secondary maxima of }\gamma_{\text{ad}}\text{ observed in models }A_6\text{ are suppressed by the }e^-\text{ attenuation. The radiation pressure is important only on the skin of the torus, but almost everywhere points in the wrong direction, opposing the back-pressure of the torus interior. Only at }\theta > 45^\circ\text{ does it point in the}
direction tangential with the torus surface, but as \( r \approx 3 \) pc, the density drops and \( \zeta \) rises so that \( M(t, \zeta) \) becomes small.

5.1.6. Dependence on \( \Gamma, R_g,0, \tau_v^\pm \), and \( d \)

If the interior of the torus is optically thick to X-rays, then the torus loses mass mostly from the surface, much as in the "self-excited wind" scenario for X-ray binaries (Basko et al. 1977). As shown above, in such a case the torus throat serves as a funnel, and the gas is injected into the flow from the funnel walls.

Note that the location of the narrowest part of this funnel determines the characteristic terminal speed of the wind. In order to explore this, we have made a set of runs similar to models A3 and A6, but with reduced \( \tau_v \). For model A3, which has \( \tau_v = 2 \) and \( R_g = 0.5 \), we find that for \( \theta = 10^\circ \) and \( t = 4.5 \) the maximum velocity \( v_p^\text{max} \) is 378 km s\(^{-1}\). For model A6, which has \( \tau_v = 2 \) and \( R_g = 1 \), we find \( v_p^\text{max} = 432 \) km s\(^{-1}\) for the same \( \theta \). If in the latter model we make the optical depth smaller (\( \tau_v = 1 \)), we obtain \( v_p^\text{max}(t = 4.5, \theta = 10^\circ) = 400 \) km s\(^{-1}\). This shows, in accordance with our expectations of the mass flux conservation, that the torus is losing mass from deeper inside. As shown in Table 1, reducing \( R_g \) has the effect of increasing the maximum velocity. An increase of \( \Gamma \) has the same effect. However, this maximum velocity may be observed at a different inclination. Increasing the distortion parameter \( d \) somewhat increases the maximum velocity, redistributing \( v_p^\text{max}(\theta) \) to higher inclinations.

From the numerical solution, we note that the torus aspect ratio \( \Delta = r/H \approx 1 \) does not strongly influence the evolution. This is because it is the innermost part of the throat that determines the dynamics of an evaporative flow. This inner throat is located at high \( \theta \), so that it remains optically thick most of the time. Numerical experiments confirm that the geometry of this inner throat remains approximately unchanged in time.

Figure 2 shows that the geometry of the innermost part of the torus, i.e., the densest part (roughly located between 0.5 and 2 pc) shrinks considerably in the vertical direction during the process of evolution. This is the result of the joint action of the radiation pressure and the back-pressure of the hot evaporative flow. This is particularly interesting, as it resembles the geometrically thick outskirts of AGN accretion disks, which are known to be unstable to self-gravity (Kolyzkhalov & Syunyaev 1980; Shlosman & Begelman 1989). The physics of such systems is complicated, and is subject to various possible competing effects. The self-gravitating instability may also operate in the torus body, perhaps leading to a dynamical system of molecular-dusty self-gravitating clouds (as in Krolik & Begelman 1988). If this is the case, the optical depth of the torus \( \tau_v \) is crucial, as in the optically thin case the torus will effectively cool and collapse to a thin disk with subsequent star formation (Toomre 1964). At the other extreme (\( \tau_v \gg 1 \)), the released energy can go to increase the velocity dispersion of the clouds, effectively supporting the torus thickness (Pacynska 1978). Strong IR radiation pressure exerted on these clouds, which can come from internal reprocessing of X-rays, can produce significant vertical force (Thompson et al. 2005; Hönig & Beckert 2007) and may suppress the self-gravity instability while at the same time providing pressure support against vertical collapse (Krolik 2007). Vertical support and partial suppression of gravitational collapse may also be provided by radiation pressure from star formation within the torus or the obscuring flow (Wada & Norman 2002). Further heating and mass loss induces a torus to expand and change shape.

We have calculated models B6 and A6 with \( 100 \times 100 \) resolution further in time to investigate the late stages of their evolution. At \( T = 17 \) in model B6, the torus has two extended lobes in both hemispheres with an opening angle of 45°. The lobes have a certain degree of asymmetry with respect to the equatorial plane. The shape of the obscuring structure no longer resembles the initial torus, and the column densities are in the range \( N_{23} = 10-1000 \) at 30°. The radial velocities in this structure are in the range \( 200-400 \) km s\(^{-1}\). Our model does not allow for replenishing of the torus; obviously, the torus will evaporate completely given enough time. Thus, in model A6, the torus evaporates completely by time \( T = 15 \). These results imply that in order to get a quasi-stationary warm-absorber flow, the replenishing time should be on the order of the mass-loss time. The whole torus configuration may be unstable in a secular sense; the instability is driven by the long characteristic time of the global torus heating/cooling (due to expansion, winds, and radiative losses), advection of heat in the torus body by internal flows, etc. For example, the mass-loss rate \( M \sim \Sigma \), and the surface area \( \Sigma \) increases during the torus expansion. If after some time of extensive heating the torus separates into several parts, further mass loss will increase due to the larger total surface area of the fragments.

6. CONCLUSIONS

We have studied X-ray–excited winds from the putative gas-dusty torus in AGNs. We approached this problem using numerical methods combining detailed hydrodynamical modeling with calculation of the warm-absorber spectra. Our hydrodynamical calculations included two-dimensional, axially symmetric rotating flow, driven primarily by X-ray heating. Compton, bremsstrahlung, and photoionization heating/cooling processes were taken into account, as well as the radiation pressure force, which was calculated in the Sobolev approximation. A code combining XSTAR for photoionization calculations with the Sobolev radiation transfer was developed for the calculation of the spectra.

We find that a rotationally supported torus heated by radiation from the inner accretion disk and black hole can indeed be a source of the material that we observe in the warm-absorber flow. We find that the inner throat of the torus is not only important as a source of gas but also because it creates a funnel for the outflowing wind. This leads generally to larger velocities within the funnel and a different velocity distribution within the warm-absorber flow from those derived from models based on spherically symmetric winds. The wind mass-loss rate within the funnel is not very sensitive to the details of the initial torus distribution and approaches \( \sim 0.02-0.09 \) \( M_g \) yr\(^{-1}\). Strong X-rays heat the gas within the funnel, producing a fast \( \sim 1000 \) km s\(^{-1}\) ionized flow near the axis and a slower \( \leq 500 \) km s\(^{-1}\) flow closer to the funnel walls. This is where optical depth effects become important and a warm-absorber spectrum is produced. Using methods developed in studies of X-ray binaries, we were able to estimate the mass-loss rate from such a funnel flow, finding it to be in good agreement with our numerical solution.

The funnel flow is found to be promising with respect to obtaining high-velocity warm-absorber flows. What is beyond the scope of our models is the possibility of having multiple phases in such a high-velocity flow on spatial scales smaller than our grid resolution. Our treatment of the thermal properties of the gas will produce two-phase behavior at our grid resolution; we do not find this behavior, owing to the fact that the cooling timescales are generally too long. The answer to the question of whether there can be high-velocity "bullets" or "embedded clouds" on length scales smaller than the resolution of the grid is related to the problem of the origin of broad and narrow UV/optical line-emitting clouds, and requires different computational methods from those employed here.
Our models, which have initial Compton depths $\tau_{18}^C \geq 1$, aspect ratio $R_0/H \sim 1$, and are located at $0.5 \text{ pc} \leq r \leq 1.5 \text{ pc}$, predict warm-absorber spectra, thus confirming the main conclusion made in Paper I. The existence of such spectra depends on the fact that the flow is intrinsically two-dimensional, meaning both that the dynamics of the funnel flow are different from 1D models, and that optical depth effects are important, as they strongly depend on inclination. The latter point requires that we include the entire torus in the computational domain rather than considering it as a boundary condition. The distribution of the ionization parameter $\xi$ depends strongly on $\theta$, further confining the range of angles where conditions are right for the warm-absorber flow to be observed. In most of our models, warm-absorber-like spectra are produced in a $10^\circ$ range, at $\theta \approx 40^\circ \pm 5^\circ$. This range is set both by the initial aspect ratio of the torus, which we take to be $\sim 1$, and by the thickness of the X-ray–heated skin of the torus. More optically thin models produce warm-absorber-like spectra for $\theta \approx 40^\circ \pm 10^\circ$, as they potentially provide more partially optically thin gas for evaporation.

The bulk of the gas in this scenario has a terminal velocity on the order of the escape velocity at the inner torus edge. Because of the funnel mechanism, part of the gas is redistributed to lower inclinations and acquires a higher terminal speed of $\sim 1000 \text{ km s}^{-1}$. In a real AGN environment, such flow may contain clumps, irregularities, and even dust, features that are not captured in our studies because of the intrinsic limitations our methods. Accounting for the multiple phases of a gas (on a subcellular level) may reveal this in more detail and may also broaden the range of angles where the warm absorbers appear.

The part of the flow that is shielded by the optically thick part of the torus body can also flow out as part of a global torus expansion. Thus, it strongly depends on the deposition of energy directly to its interior. This problem is related to one of the infrared support of the AGN torus’ vertical structure against gravitational collapse (Krolik 2007) and also requires additional investigation.

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REFERENCES

Antonucci, R. R. J., & Miller, J. S. 1985, ApJ, 297, 621
Balsara, D. S., & Krolik, J. 1993, ApJ, 402, 109
Basko, M. M., Hatchett, S., McCray, R., & Sunyaev, R. A. 1977, ApJ, 215, 276
Behar, E., Rasmussen, A. P., Blustin, A. J., Sako, M., Kahn, S. M., Kaastra, J. S., Branduardi-Raymont, G., & Steenbrugge, K. C. 2003, ApJ, 598, 232
Blondin, J. M. 1994, ApJ, 435, 756
Castor, J. I., Abbott, D. C., & Klein, R. I. 1975, ApJ, 195, 157
Chelouche, D., & Netzer, H. 2005, ApJ, 625, 95
Crenshaw, D. M., et al. 1999, ApJ, 516, 750
Dorodnitsyn, A., Kallman, T., & Proga, D. 2008, ApJ, 675, L5 (Paper I)
Halpern, J. P. 1984, ApJ, 281, 90
Hönig, S. F., & Beckert, T. 2007, MNRAS, 380, 1172
Jaffe, W., et al. 2004, Nature, 429, 47
Kallman, T., & Bautista, M. 2001, ApJS, 133, 221
Kaspi, S., et al. 2002, ApJ, 574, 643
Kinkhabwala, A., et al. 2002, ApJ, 575, 732
Kolykhalov, P. I., & Syunyaev, R. A. 1980, Soviet Astron. Lett., 6, 357
Krolik, J. H. 2007, ApJ, 661, 52
Krolik, J. H., & Begelman, M. C. 1986, ApJ, 308, L55
———. 1988, ApJ, 329, 702
Krolik, J. H., & Kriss, G. A. 1995, ApJ, 447, 512
———. 2001, ApJ, 561, 684
Krolik, J. H., McKee, C. F., & Tarter, C. B. 1981, ApJ, 249, 422
Laor, A., Fiore, F., Elvis, E., Wilkes, B. J., & McDowell, J. C. 1997, ApJ, 477, 93
McCray, R., & Hatchett, S. 1975, ApJ, 199, 196
McKerrn, B., Yuqooob, T., & Reynolds, C. S. 2007, MNRAS, 379, 1359
Netzer, H. et al. 2003, ApJ, 599, 933
Owocki, S. P., Castor, J. I., & Rybicki, G. B. 1988, ApJ, 335, 914
Paczyński, B. 1978, Acta Astron., 28, 91
Papaloizou, J. C. B., & Pringle, J. E. 1984, MNRAS, 208, 721
Poneclet, A., Perrin, G., & Sol, H. 2006, A&A, 450, 483
Proga, D. 2007, ApJ, 661, 693
Proga, D., & Kallman, T. R. 2002, ApJ, 565, 455
Proga, D., Ostriker, J. P., & Kurosawa, R. 2008, ApJ, 676, 101
Proga, D., Stone, J. M., & Kallman, T. R. 2000, ApJ, 543, 686
Reynolds, C. S. 1997, MNRAS, 286, 513
Rybicki, G. B., & Hummer, D. G. 1983, ApJ, 274, 380
Shlosman, I., & Begelman, M. C. 1989, ApJ, 341, 685
Steenbrugge, K. C. 2005, A&A, 432, 453
Steenbrugge, K. C. 2005, A&A, 432, 453
Stone, J. M., & Norman, M. L. 1992, ApJS, 80, 753
Tarter, C. B., Tucker, W., & Salpeter, E. E. 1969, ApJ, 156, 943
Thompson, T. A., Quataert, E., & Murray, N. 2005, ApJ, 630, 167
Toomre, A. 1964, ApJ, 139, 1217
Tristram, K. R. V., et al. 2007, A&A, 474, 837
Wada, K., & Norman, C. A. 2002, ApJ, 566, L21
Zakamska, N., et al. 2006, AJ, 132, 1496