On the Symmetry Constraints of CP Violations in QCD

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(June 18, 2018)

Abstract

Three symmetry constraints on the CP violations in QCD are discussed in this paper. In order to generate CP violating observables from QCD, these constraints require: (1) spontaneous chiral symmetry breaking, (2) explicit chiral symmetry breaking, e.g., finite quark masses, (3) $U_A(1)$ anomaly, in addition to a nonzero $\theta$ parameter. A pictorial illustration is used to unify and elucidate these constraints and indicate a dual relation between quark mass and the quark condensate. Based on the symmetry constraints, a dynamical suppression scenario to solve the strong CP problem within QCD is examined. We conclude that a solution of the strong CP problem has to involve physics beyond the standard model.

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I. MOTIVATIONS

In spite of many research efforts, CP violation in the subnuclear world remains one of the most important challenges yet to be solved by physicists [1]. While CP violations in the underlying dynamics of elementary particles are required to explain the observed baryon asymmetry in our universe [2], we have only one system ( $K_L, K_S$ mesons, in their decay modes into two or three pions ) which demonstrates that CP is not a good symmetry of the weak interactions [3]. The experimental evidence we have acquired is insufficient to pin down a unique theory.

On the other hand, it is believed that we have a good understanding of parity violation in the weak interactions, and the existing measurements indicate that strong and electromagnetic interactions conserve C, P and T to a high accuracy, e.g., the measurements of a neutron electric dipole moment [4]. While the above interactions are successfully described by the standard model, it is possible to incorporate CP violation in the $K$ meson system and thus provide predictions for other phenomena, e.g., CP violation in the $B$ meson system, and/or explanations of the baryon asymmetry. In so doing, one is naturally led to the following question: Why are the existing discrete symmetry breakings only observed in the weak sector and not in the strong one? Such an innocent puzzle may have profound ramifications; for example, if one insists that the strong interaction is governed by a non-Abelian gauge theory like QCD, the nontrivial topological structure ( à la instantons ) implies a $\theta$ vacuum which violates CP [5]. The situation is further complicated by the presence of a quark mass matrix, which connects weak CP violations to the strong ones under chiral transformations. From a theoretical perspective, it is not clear why QCD should conserve CP and this is generally referred to as a strong CP problem [6].

One can answer this question by asserting that the parameter ( which will be specified later ) associated with CP violations in QCD is small, so that the observed effects are tiny. We shall consider such a solution undesirable, as a vanishing parameter without increasing the symmetry of QCD should be considered "unnatural", according to t’Hooft [7]. Further-
more, there is no way to calculate the CP violating parameter from QCD itself. However, if we treat QCD as a low-energy effective interaction of some high-energy theory (e.g., some grand unified theory), then such a CP violating parameter can be calculated and can be shown to be naturally small in certain models \[8\]. Another solution to the strong CP problem is given by introducing an extra pseudoscalar particle, namely, the axion \[9\]. In either case, an explanation of the small CP violating parameter necessarily involves physics beyond the standard model. Nevertheless, these scenarios seem to receive little experimental support at this moment.

In between these alternative solutions to the strong CP problem, one can imagine that there exists the possibility that CP violations in a non-Abelian gauge theory are dynamically suppressed. That is, the intrinsic symmetry of the theory leads to small coefficients of CP violating observables, e.g., neutron electric dipole moment, in company with a function of the CP violating parameter which can be of order \(O(1)\). If this were the case, a natural solution of CP violations in QCD can be obtained without invoking extra particles or modifications of the gauge structure. It is to this point that this paper is addressed, and we shall examine a dynamical suppression mechanism in terms of symmetry constraints on CP violations in QCD, which is the main subject of this paper.

II. INTRODUCTION

Even though we shall focus on the CP violations of QCD in this paper, it is helpful to begin our discussion with CP violations in the standard model, which includes the electroweak theory and QCD as two ingredients. As the definition of the strong CP problem is a subtle issue, it is important to specify our domain in order to avoid extra complications.

First of all, by standard model we mean the minimal theory with a \(SU_C(3) \times SU_L(2) \times U(1)\) gauge structure and one Higgs doublet together with fundamental fermions (quarks and leptons) as matter fields. Second, there are three places in the standard model Lagrangian where one can naturally incorporate CP violation without modifying the gauge
structure and/or changing the particle content of the theory. These are: (1) charged current vertices; (2) Yukawa couplings between fermions and Higgs particle; (3) gauge field anomalies. Third, it is important to notice that the three sources of CP violation are not independent, due to the reparametrization invariance of the generating functional. In particular, under a general chiral rotation in the fermion flavor space, it is possible to shift the CP violating parameters among the three terms. Consequently, there are only two independent CP violating parameters in the three-generation standard model with massless neutrinos.

Fourth, if we are only concerned with low energy physics, it is useful to look at the effective theory, where we integrate out all heavy particles (e.g., Higgs particle, $W$ and $Z$ bosons, plus heavy fermions) of the standard model. In the low energy effective theory, the first two terms mentioned above reduce to: (1') flavor changing 4-fermion (weak) interactions and (2') fermion mass matrix, and the CP violation can be characterized by a Cabibbo-Kobayashi-Maskawa (CKM) phase\textsuperscript{1}. If we neglect the electroweak interactions in our discussion, as we shall do later, the combined effects of (2') and (3) can be characterized by a single parameter, which is referred to as the $\tilde{\theta}$ parameter. Finally, while it is complicated to give reparametrization and renormalization group invariant definitions of these two CP violating parameters in the three-generation standard model, it is worthwhile to notice that there exists a particular representation of the standard model Lagrangian in which (2') is CP even but (1') and (3) violate CP. Henceforth, we shall refer to the effects derived from the CKM phase as weak CP violations and those related to the $\tilde{\theta}$ parameter as strong CP violations\textsuperscript{2}.

\textsuperscript{1}The existence of such a CP violating phase requires at least three generations of fermion doublets.

\textsuperscript{2}The $U(1)$ gauge anomaly is physically irrelevant because of the trivial topological structure of the Abelian gauge theory. The $SU_L(2)$ gauge anomaly, due to the $V-A$ structure of weak interaction, is also unimportant in our discussion.
III. STRONG CP VIOLATIONS IN QCD

Having defined the strong CP problem within the framework of the standard model, we shall write down the explicit definitions to set up proper notations for further reference. For the sake of simplicity, we shall discuss a theory with one quark flavor. The generalization to a multi-flavor case can be found elsewhere [10].

Normally, the (CP conserving) QCD Lagrangian is given by

\[
\mathcal{L}_{QCD} \equiv \bar{\psi}i \not{D}\psi + m_q \bar{\psi}\psi + \frac{1}{4} G^2 \tag{1}
\]

where

\[
\not{D} \equiv (\partial_\mu + ig_s B_\mu^a \frac{\lambda^a}{2} \cdot \gamma^\mu) \tag{2}
\]

The meanings of various symbols are:

\[
\begin{align*}
\psi &: \text{quark field} \\
\bar{\psi} &: \text{Dirac adjoint of the quark field, } \bar{\psi} \equiv \psi^\dagger \gamma_0 \\
B^a_\mu &: \text{gluon field, } a = 1, \ldots, 8 \\
\frac{\lambda^a}{2} &: \text{generators of the color SU}(3) \text{ gauge group, } a = 1, \ldots, 8 \\
G_{\mu\nu} &: \text{gluonic tensor field, } G_{\mu\nu} \equiv [\partial_\mu + ig_s B_\mu \cdot \partial_\nu + ig_s B_\nu], \\
B_\mu &\equiv B^a_\mu \frac{\lambda^a}{2}, \quad G^2 \equiv G_{\mu\nu}G^{\mu\nu} \\
g_s &: \text{strong coupling constant in QCD}
\end{align*}
\]

To calculate various correlation functions in the quantum theory, it is useful to define the QCD generating functional (denoted by \( Z \)):

\[
Z[\zeta, \bar{\zeta}, J_\mu] \equiv \frac{1}{N} \int [D\psi][D\bar{\psi}][DB_\mu] e^{iS_{QCD} + \bar{\psi}\gamma^\mu\zeta + \bar{\zeta}\gamma^\mu J_\mu} \tag{3}
\]

where the QCD action

\[
S_{QCD} \equiv \int d^4x \, \mathcal{L}_{QCD} \tag{4}
\]

\(^3\text{The simplification is due to the fact that strong CP violation is a flavor-singlet problem.}\)
The normalization constant for the generating functional is

\[ N \equiv \int [D\psi][D\bar{\psi}][DB_\mu]e^{iS_{QCD}} \] (5)

such that

\[ Z[\zeta = 0, \bar{\zeta} = 0, J_\mu = 0] = 1 \] (6)

Notice that in the generating functional (Eq. 3), the fermion field \(\psi, \bar{\psi}\) and the gluon field \(B_\mu^a\) are dummy variables; only the external source fields \(\zeta, \bar{\zeta}, J_\mu\) specify the physical ground state (QCD vacuum) of the theory. Therefore, we can redefine these dummy variables freely without changing the physical contents of the theory. In particular, we can perform a \(U_A(1)\) chiral rotation on the fermion field:

\[ \psi \to \psi ' \equiv e^{i\theta \gamma_5} \psi \] (7)

or

\[ \psi _i' = [\cos \theta (I)_{ij} + i \sin \theta (\gamma_5)_{ij}] \psi_j \] (8)

While it is clear that the quark mass term \(m_q \bar{\psi} \psi\) transforms into \(m_q \bar{\psi} e^{2i\theta \gamma_5} \psi\) under a \(U_A(1)\) chiral rotation, it is not a trivial task to show that a \(U_A(1)\) chiral rotation is not an unitary transformation (\(U_A(1)\) anomaly) and a Jacobian associated with this change of variables in the functional space has to be implemented. As shown by Fujikawa [11], the fermionic measure in the functional integral \([D\psi][D\bar{\psi}]\) transforms, under a \(U_A(1)\) chiral rotation,

\[ [D\psi][D\bar{\psi}] \to [D\psi][D\bar{\psi}] e^{i\frac{g^2}{12\pi^2} \int d^4x G\tilde{G}(x)} \] (9)

Hence, we generate a new (but equivalent) QCD Lagrangian with two extra terms (together with a change of the chiral phases of the external fermion source fields \(\zeta, \bar{\zeta}\)):

\[ m_q \bar{\psi} (e^{i2\theta \gamma_5} - 1) \psi \approx im_q \sin 2\theta \bar{\psi} \gamma_5 \psi, \quad \frac{g^2}{32\pi^2} G\tilde{G} \] (10)

Several comments are in order:

(1) These two terms carry the same quantum numbers and both are odd under parity (P) and time reversal (T) transformations. We shall refer to the former as a quark
pseudomass term, and the latter as a gluon anomaly term. We emphasize that these are the lowest dimensional CP violating operators that one can write down in the QCD Lagrangian, consistent with basic requirements of a relativistic quantum field theory.

(2) We can generalize the discussion to a (generally CP violating) QCD Lagrangian with an arbitrary quark pseudomass term \( m_q \bar{\psi} e^{i \theta_q} \gamma_5 \psi \) and a gluon anomaly term \( \frac{g^2 \theta_G}{32 \pi^2} \tilde{G} G \).

\[
\mathcal{L}_{QCD; \theta_G, \theta_q} \equiv \bar{\psi} i\not{D} \psi + m_q \bar{\psi} e^{i \theta_q} \gamma_5 \psi + \frac{1}{4} G^2 + \frac{g^2 \theta_G}{32 \pi^2} \tilde{G} G \tag{11}
\]

It can be shown that, under a \( U_A(1) \) chiral rotation (Eq. 7), both \( \theta_q \) and \( \theta_G \) change by \( 2\theta \). Therefore, the difference

\[
\bar{\theta} \equiv \theta_G - \theta_q \tag{12}
\]

is an invariant of the \( U_A(1) \) chiral rotation, which can be used to label the classes of equivalent QCD Lagrangians. Since the physical observables are independent of the representations of the generating functional, we conclude that any CP violating observable has to be proportional to the \( U_A(1) \) invariant chiral phase \( \bar{\theta} \).

(3) The \( \bar{\theta} \) parameter, being a difference between two chiral phases, is an angular variable with period \( 2\pi \) (in the case of one quark flavor). Consequently, any physical observable has to be a periodic function of the \( \bar{\theta} \) parameter.

(4) In the multi-flavor case of QCD, the number of quark chiral phases is equal to the number of quark flavors. Nevertheless, since we can perform a \( U_A(1) \) chiral rotation independently on each flavor, there is still only one physical parameter which characterizes the strength of strong CP violations. In that case, the \( \bar{\theta} \) parameter is defined as

\[
\bar{\theta} \equiv \theta_G - \sum_j \theta_{q_j} \tag{13}
\]

where \( j \) is the flavor index for light quarks.

\[\text{These requirements include: (1) Hermiticity, (2) Lorentz invariance, and (3) gauge invariance.}\]
IV. SYMMETRY CONSTRAINTS OF CP VIOLATIONS IN QCD

The previous discussions seem to suggest that there are close relationships between the $U_A(1)$ chiral symmetry and the CP violations in QCD. Indeed, it is necessary that the chiral symmetry is broken both spontaneously and explicitly so that strong CP violations are possible. We shall refer to these relations as symmetry constraints and discuss their meanings and implications in this section.

(1) Non-Perturbative Nature of CP Violations in QCD

Given the fact that the gluon anomaly term $G\tilde{G}$ can be written as a total divergence of the Chern-Simon current $K_\mu$,

$$K_\mu = \frac{g_s^2}{16\pi^2} \sum \epsilon_{\mu\nu\rho\sigma} B^{a\nu} \{ \partial^\rho B^{a\sigma} + \frac{1}{3} f_{abc} B^{b\rho} B^{c\sigma} \}$$

(14)

$$\partial^\mu K_\mu = \frac{g_s^2}{32\pi^2} G\tilde{G}$$

(15)

it is not too surprising that this term has no effect on a perturbative calculation of any CP violating observable. Since a potential modification of the perturbative expansion caused by the gluon anomaly term can only be related to the gluonic propagator in the Feynman rules, it turns out such a contribution to the gluonic propagator is zero, as can be verified by a direct calculation.

Since we can always perform chiral rotations to shift the strong CP violating phases into the gluon anomaly term (i.e., $\theta_q = 0$, $\bar{\theta} = \theta_G$), and any physical observable should not depend on the particular representation we choose to do a calculation, we conclude that the strong CP violation has to be a purely nonperturbative effect. However, it remains to see how a calculation of CP violating observables based on the quark pseudomass term leads to the same conclusion, if we insist on the reparametrization invariance of the CP violating observables. This is a problem, because it seems that the presence of a quark pseudomass term will lead to a modification of the quark propagator, hence generate contributions to the CP violating observables in perturbative calculations. Apparently, such an effect is in contradiction to our previous observation.
Indeed, if we perform a calculation of a quark electric dipole moment (denoted as qEDM) using a perturbative expansion on both fine structure constant $e^2/\hbar c$ and strong coupling constant $g_s$, we do find a finite contribution to a tensor structure which corresponds to a qEDM, with the strength proportional to the quark chiral phase $\sin \theta_q$, see Fig. 2. Nevertheless, we should be careful not to interpret this result as a physical observable. As we discussed before, $\theta_q$, by itself, is a representation-dependent parameter and CP violating physical observables should depend only on $\bar{\theta}$. What goes wrong here?

The answer is that there are other representation-dependent chiral phases we should include in the extraction of a physical observable associated with a chirally covariant tensor. For example, in the case of a particle EDM, the relevant tensor $i\sigma_{\mu\nu}\gamma_5$ is mixed with the tensor associated with the anomalous magnetic moment (denoted as AMM) $\sigma_{\mu\nu}$ under a $U_A(1)$ chiral rotation on the particle field. Besides that, the same $U_A(1)$ chiral rotation also causes the mass of the particle to develop a chiral phase, $M \rightarrow Me^{i\alpha\gamma_5}$, which can be viewed as a mixing between $I$ and $i\gamma_5$ tensors, see Fig. 1. We need to subtract the relative phases, $\arctan(\frac{\text{EDM}}{\text{AMM}})$ and $\alpha$, in order to define a representation-independent answer for the physical observables. It can be verified that in the calculation of the qEDM, both $\arctan(\frac{\text{EDM}}{\text{AMM}})$ and $\alpha$ are equal to $\theta_q$; hence there is no quark EDM from the perturbative calculation in any representation of the QCD generating functional.

With a suitable generalization, one can convince oneself that this conclusion holds true to all orders in QCD, i.e., the inclusions of any higher order loops does not affect the conclusion. One can also apply the same argument to the bound states of quarks, i.e., hadrons, and show that any CP violating observables have to come from the nonperturbative contributions of QCD.

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5This point has been emphasized by E.P. Shabalin [12]. However, the argument in support of the conclusion in these works seems unclear to the present author.

6By perturbative contributions, we mean those arising from an expansion of any physical observ-
Since the nonperturbative property of QCD can be characterized in terms of the spontaneous chiral symmetry breaking, which is manifested by the presence of vacuum condensates, we can rephrase the conclusion in the following way:

If there is no spontaneous chiral symmetry breaking in QCD, there is no strong CP violation, even with a nonzero $\bar{\theta}$.

(2) Chiral Limit and CP Violations in QCD

Another important constraint on the strong CP violations has to do with a nonvanishing quark mass term in the QCD Lagrangian. It was first pointed out by Peccei and Quinn [9] that there is no strong CP violation if there exists a massless quark in the QCD Lagrangian.

This constraint can be understood in terms of the functional integral formalism. Since a quark chiral phase $\theta_q$ is ill-defined if the quark mass is zero, we can take advantage of this fact to rotate away (via an $U_A(1)$ chiral transformation) the gluon chiral phase $\theta_G$, so that any QCD Lagrangian with a massless quark is equivalent to a CP conserving one. Therefore, it is necessary to have all quark masses finite to generate a CP violating observable from QCD.

The point of this argument is that we need to have a well-defined chirally covariant phase, in addition to the gluon chiral phase $\theta_G$, such that, after the reduction of the irrelevant degrees of freedom by reparametrization invariance, we are left with a chirally invariant CP violating parameter, e.g., $\bar{\theta}$. For instance, we can replace the quark mass term by a higher dimensional operator which violates the chiral symmetry explicitly, e.g., $\bar{q} \sigma_{\mu\nu} e^{i\bar{\gamma} \gamma_5} q G^{\mu\nu}$ [13]. In this case, even though the quark is massless, we still can have a CP violating observable; proportional to the chirally invariant phase $\theta_G - \beta$. Consequently, the second constraint can be phrased as follows:

If there is no explicit chiral symmetry breaking, there is no strong CP violation in QCD.

able in a power series of the strong coupling constant $g_s$. A purely nonperturbative observable has zero coefficients to all order in its power series expansion. For example, $\langle \bar{q}q \rangle \propto e^{-1/g_s^2}$.
(3) $U_A(1)$ Anomaly Constraint of CP Violations in QCD

The third constraint was first discussed by M.A. Shifman, A.I. Vainshtein and V.I. Zakharov [14], and then rediscovered by S. Aoki, A. Gocksch, A.V. Manohar and S.R. Sharpe [15]. They pointed out that, in a diagrammatical language, the strong CP violations only contribute to physical observables through the internal fermion loops with a pseudo–mass insertion. Such a diagram is a manifestation of the anomalous Ward identity associated with the flavor singlet axial current and has a close relationship with the $U_A(1)$ anomaly in QCD. This connection has been examined in the context of chiral perturbation theory by S. Aoki and T. Hatsuda [16], and H-Y Cheng [17]. Essentially, this constraint requires that the chiral anomaly provides a solution to the $U_A(1)$ problem [14] [18]. If this is not the case, then there is no strong CP violation. In a hadronic calculation, this constraint implies that CP violating observables should be proportional to the difference between $m_\pi^2$ and $m_\eta'$ [16] [17]. A quantitative realization of this constraint in QCD implies that CP violating observables should be proportional to the anomalous gluon condensate $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$ [19].

It should come as no surprise that these three constraints are not independent since ultimately both the anomalous gluon condensate $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$ and the quark chiral radius [10]

$$R_q^2 \equiv \left[ \langle q\bar{q}\rangle_{\theta_q,\theta_G} \right]^2 + \left[ i\langle q\gamma_5\bar{q}\rangle_{\theta_q,\theta_G} \right]^2$$

(16)
can be related to the QCD scale $\Lambda_{QCD}$. Indeed, through the use of a generalized anomalous Ward identity, we can prove that $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$ is proportional to the product of $m_q$, $R_q$ and $\sin \bar{\theta}$ [10].

To summarize, the bottom line of the study of symmetry constraints is that we need three finite QCD parameters: $m_q$, $R_q$ and $\sin \bar{\theta}$ to generate a CP violating observable from QCD.

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7Therefore, it is impossible to calculate the effect of strong CP violations in a quenched lattice calculation [15].
V. A GRAPHIC ILLUSTRATION OF THE SYMMETRY CONSTRAINTS OF CP VIOLATIONS IN QCD

From the previous discussions, it is clear that to generate a CP violating observable from QCD, we need three nonzero parameters, \( m_q, R_q \) and \( \bar{\theta} \). This conclusion was derived through the use of a functional integral formalism. While these elegant derivations are exact and nonperturbative in nature, they lack the intuition and simplicity to help us grasp the basic ideas. In view of this, we would like to show a different approach to understand how these symmetry constraints work in QCD.

In fact, there is a simple way to visualize the symmetry constraints without relying on the functional integral formalism. Specifically, we shall use a graphic illustration to show that there is no strong CP violation if there is no spontaneous chiral symmetry breaking and/or the quark mass vanishes.

To begin with, the set of CP violating QCD Lagrangians with two CP violating parameters, as defined in Sec. III, can be represented in a two dimensional plane (phase space), where a given pair of CP violating parameters corresponds to a point on the plane. It is useful to choose the polar representations for the CP violating parameters (i.e., \( \theta_q, \theta_G \)) so that the periodic structures of these parameters implies an identification of the boundaries of the squares (e.g., \( \theta_q = 0 \equiv \theta_q = 2\pi \)) and the phase space of the CP violating QCD Lagrangians becomes a torus, see Fig. 3.

On the two dimensional plane, we can identify the equivalent classes of the CP violating QCD Lagrangians (those with the same \( \bar{\theta} \)) by the straight lines \( \theta_G - \theta_q = constant \). After the identification of the boundaries of the square, these straight lines map onto a family of nonintersecting closed loops winding over the torus. Any two points on the same curve, which correspond to equivalent CP violating QCD Lagrangians with the same \( \bar{\theta} \), describe

\[ ^8 \text{As we have explained before, the } U_A(1) \text{ anomaly constraint is not really independent from the first two constraints. Hence, we shall ignore this constraint in this section.} \]
the same physics (reparametrization invariance), see Fig. 4.

The connection with the symmetry constraints is established once we specify the length scales of the torus: the large radius, conjugate to the $\theta_G$ angular variable, is related to some function of $R_q$, $f(R_q)$; and the small radius, conjugate to the $\theta_q$ angular variable, is some function of $m_q$, $g(m_q)$. Since we are only interested in a qualitative description of the symmetry constraints in this section, the actual form of the function is not important, except that the function has to vanish when its argument is zero. With these specifications, we can study the change of geometries of the torus in two special limits:

(1) chiral limit ($m_q \to 0$):

In this limit, as the small radius $g(m_q)$ shrinks to zero, the torus degenerates into a circle with radius $f(R_q)$ and all equivalent loops collapse onto the same circle. If we insist on the single-valuedness of the physics as all the equivalent classes collapse onto the CP conserving one, it is natural to conclude that all strong CP violations vanish, see Fig. 5.

(2) no spontaneous chiral symmetry breaking ($R_q \to 0$):

In this limit, as the large radius $f(R_q)$ shrinks to zero, the torus degenerates into a sphere with radius $g(m_q)$. The equivalent loops become eight–shaped curves and they all intersect with the CP conserving loop at two points. Again, using the single-valuedness argument, we conclude that if there is no spontaneous chiral symmetry breaking ($R_q = 0$), the QCD Lagrangians conserve P and CP even with a nonzero $\bar{\theta}$, see Fig. 6.

It is worth mentioning that such a graphic illustration indicates a dual relationship between $m_q$ and $R_q$. In addition, we hope that the geometrical pictures can be used quantitatively. For example, by choosing some suitable functions $g(m_q)$ and $f(R_q)$ of both radii of the torus, we might be able to relate the EM moments of particles to certain geometrical measures (e.g., surface area enclosed by certain contour on the torus) or fluxes through the loops.
VI. WHY STRONG CP VIOLATIONS ARE SMALL

As we mentioned in the motivation, the discussions of the (chiral) symmetry constraints on the strong CP violations in QCD is not just of theoretical interest. One practical implication we hope to infer from these general constraints is whether QCD can cure the strong CP problem by itself without resorting to an unnaturally small \( \bar{\theta} \) parameter. This suggestion might seem too ambitious in view of the amazing experimental data on the CP violation observables, e.g., the search for a neutron electric dipole moment at a level of \( 10^{-26} e \cdot cm \). To achieve such a tiny effect (the ratio of the EDM to the AMM of a neutron is less than \( 10^{-12} \)) requires a delicate cancellation in any calculation if \( \bar{\theta} \) is of order unity. However, we feel that it is worthwhile to consider this problem in a quantitative way, as the result involves only nonperturbative contributions, which can be related to other observables in the physics of the strong interactions. Such a study has been done in the framework of the low-energy effective theory of QCD \[16\] \[17\]. A more direct approach, based on the QCD parameters, appeared only recently \[19\].

If we formulate the calculation of the CP violating observables based on the QCD Lagrangian, one useful technique is provided by the method of operator product expansion (OPE), as exemplified in the practice of QCD sum rule calculations \[20\] and heavy quark expansion \[21\]. This is a systematic expansion of physical observables in the dimensions of QCD operators (over some suitable energy scale, e.g., \( \frac{\Lambda_{QCD}^2}{Q^2} \), \( \frac{\Lambda_{QCD}^2}{M^2} \)). The physics associated with short-distance and long-distance fluctuations are factorized into Wilson coefficients and matrix elements of the QCD operators, respectively. Such a scheme is particularly useful in QCD because the nonperturbative aspects of the theory are parametrized in terms of various condensates, for instance: quark condensate \( \langle \bar{q}q \rangle \) and anomalous gluon condensate \( \langle G\tilde{G} \rangle_{q,\theta_G} \) \[14\], and the nonperturbative contributions to the Wilson coefficients become important only in higher dimensional operators \[20\]. If we assume that low energy hadronic observables can be approximately saturated by the first few lower dimensional operators and the Wilson coefficients are dominated by perturbative contributions, then OPE series
of a given correlation function can give us quantative information of low energy hadronic observables in terms of QCD parameters.

For a given physical quantity, the numerical value is a function of the Wilson coefficients and various matrix elements (condensates), together with other parameter, e.g., $\bar{\theta}$. If we can show that the Wilson coefficients are small (e.g., due to small quark masses and/or other cancellations) and the observable only receives contributions from higher dimensional operator because of the symmetry constraints, then the smallness of that observable can be considered as natural, because such a small number comes from a dynamical suppression instead of an unexplained tiny input parameter. In the case of CP violating observables, we can establish a relation between hadronic observables and QCD parameters and thus use it to answer the question of why the strong CP violations are small.

We have performed a calculation of the nucleon electric dipole moments, based on the method of QCD sum rule [19], to study the above question. While we are able to demonstrate that the three chiral symmetry constraints hold explicitly in our calculation without assuming a small $\bar{\theta}$, the numerical result indicates that the dynamical suppression is not sufficient to achieve the experimental upper bound for the neutron EDM. Thus, the answer to the question whether QCD can cure the strong CP problem by itself, from the perspective of the symmetry constraints, seems to be negative.

VII. SUMMARY AND CONCLUSION

In this paper, we study the symmetry constraints on the strong CP violations in QCD. Previous findings on the special features of the strong CP problem, including: (1) its non-perturbative nature; (2) the importance of nonzero quark masses; (3) the chiral anomaly

\footnote{Our calculation, which generates a functional dependence of the nucleon EDM on the $\bar{\theta}$ parameter, satisfies the current upper limit from cold neutron experiments [1], with a $\bar{\theta}$ parameter of the order $10^{-9}$.}
constraint, are unified and elucidated in a dual relation between quark mass \(( m_q )\) and the quark condensate \(( R_q )\), which can be visualized in a pictorial way without much mathematical complication.

We examine the possibility of a dynamical suppression mechanism of CP violating observables, e.g., neutron electric dipole moment, due to the symmetry constraints in QCD. In a sum rule calculation of the nucleon EDMs [19], we obtain a negative answer to the question of the existence of a natural solution to the strong CP problem within QCD. Hence, to explain a small \( \bar{\theta} \) parameter necessarily involves physics beyond the minimal standard model.

ACKNOWLEDGMENTS

The author would like to thank Prof. E.M. Henley, Prof. T. Hatsuda, Prof. A. Nelson, Prof. P. Arnold and Dr. T. Meissner for many useful discussions. This work was supported by Nuclear Theory Group of Department of Physics at University of Washington, under the grant DE-FG-03-97ER41014.
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FIG. 1. Quark propagator in the presence of pseudo-mass term

FIG. 2. Quark EM moments in the presence of pseudo-mass term
FIG. 3. The phase plane and CP torus of QCD

FIG. 4. The equivalent classes of QCD Lagrangian
FIG. 5. The chiral limit of the CP torus of QCD

FIG. 6. The chiral symmetric CP torus of QCD
On the Symmetry Constraints of CP violations in QCD

1. Motivations

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4. Symmetry Constraints of CP Violations in QCD
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