Tunneling Current–Voltage Characteristics of Graphene Field-Effect Transistor

Victor Ryzhii1,3, Maxim Ryzhii1,3, and Taiichi Otsuji2,3

1 Computer Solid State Physics Laboratory, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8580, Japan
2 Research Institute for Electrical Communication, Tohoku University, Sendai 980-8577, Japan
3 CREST, Japan Science and Technology Agency, Tokyo 107-0075, Japan

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We develop an analytical device model for a graphene field-effect transistor. Using this model, we calculate its current–voltage characteristics at sufficiently high gate voltages when a n–p–n (p–n–p) lateral junction is formed in the transistor channel and the source–drain current is associated with the interband tunneling through this junction. © 2008 The Japan Society of Applied Physics

Progress in the fabrication of graphene, a monolayer of carbon atoms forming a dense honeycomb two-dimensional (2D) crystal structure and the demonstration of its nontrivial properties have caused an extensive experimental and theoretical studies.1–4) The gapless energy spectrum in graphene results in specific features of its transport, plasma and optical properties. The exceptional properties of graphene and graphene-based heterostructures open up prospects in creating novel electronic and optoelectronic devices, for example, prospective devices utilizing the quantum Hall effect,5–7) terahertz plasma phenomena,8–12) graphene nanoribbon field-effect transistors,13) new devices based on the electrically induced p–n junctions.14–16) Recently,16) a graphene-based field-effect transistor (GFET) with an n+1-Si substrate serving as a back gate and a metal top controlling gate was fabricated and characterized. At sufficiently strong negative bias of the top gate, a p-type region under this gate appears in contrast to the conventional FETs in which the depletion region is formed in this section. In this case, the GFET channel is partitioned into three sections: the source n-section, the p-section under the top gate, and the drain n-section, so that a lateral n–p–n junction in the channel is formed. The device structure and its band diagrams are shown in Fig. 1. In such a situation, the source–drain current in a GFET at low temperatures can primarily be associated with the sequential tunneling (which surpasses the thermionic processes and the recombination processes accompanied by some scattering mechanisms) through the n–p and then p–n junctions.5,6) As a result, the GFET characteristics can be determined by the effect of the gate voltage on the tunneling conductance of the n–p and p–n junctions. A phenomenological device model for GFETs operating in the tunneling regime was used in ref. 17 (see also refs. 16 and 18). In this model, the value, $F$, characterizing the slope of the dependence of the potential energy spatial distribution in the lateral n–p and p–n junctions in the channel serves as a parameter of the model. However, this quantity is determined by the GFET geometrical parameters and, more importantly, by the gate voltages. Hence, to obtain the GFET characteristics, one needs to find the dependences of $F$ on the device structure parameters and the gate voltages in a self-consistent manner.

In this paper, we propose a device model for GFETs with a highly conducting substrate serving as the back gate and the top controlling gate, as well as with two contacts (source and drain) to the channel, and calculate the GFET dc characteristics (in particular, the dependence of the tunneling source–drain current on the top-gate voltage). The dc voltages of $V_{th} > 0$ and $V_{b} + V_{d} > 0$ are applied between the substrate (back gate) and the source and drain contacts, where $V_{d}$ is the drain voltage. The top gate voltage $V_{g}$, which controls the source–drain current, is assumed to be negative and sufficiently large so that $V_{b} + V_{g} + V_{th} < 0$, where $W_{b}$ and $W_{g}$ are the gate layer thicknesses, and the GFET channel constitutes a lateral n–p–n structure.

For the calculations of the tunneling current $I_{t}$ and the tunneling conductance $G_{t}$ of the lateral n–p–n structure under consideration with relatively smooth n–p and p–n junctions at low drain voltages, the following equation can be used:14,16,18)

$$I_{t} = \frac{e}{2\pi \hbar} \int_{0}^{\pi} \frac{d\theta}{\sqrt{\omega_{c}^{2} - \omega^{2}}}$$

$$G_{t} = \frac{1}{e^{2}/2\pi \hbar} \int_{0}^{\pi} \frac{d\theta}{\sqrt{\omega_{c}^{2} - \omega^{2}}}$$

Fig. 1. Schematic view of a GFET structure (a) and its band diagrams (spatial profiles of the band’s edge) at different voltages: in equilibrium (b) and (c) and under applied drain voltage (d). The arrow in enlarged section indicates the direction of electron tunneling.
Here, $g_0 = 2e^2/\pi\hbar$, $e$ is the electron charge, $h$ is the reduced Planck constant, $v \approx 10^6$ cm/s is the characteristic velocity, $H$ is the width of the GFET structure in the direction perpendicular to the current flow, $F = e|\phi'|/dx|_{x=x_1}$ is the slope of the potential energy curve at the points $x = \pm x_1$ (tunneling points), at which the electric potential $\phi = V_g$, where $eV_F$ is the Fermi energy of two-dimensional electron gas (2DEG) near the source and drain contacts (where $\phi = 0$ and $\phi = V_g$, respectively). In eq. (1), we considered that the degeneracy factor in graphene is equal to 4 and took into account the presence of two junctions in series that gave a factor of 1/2.

Generally, numerical calculations are required to find this distribution. However, in the case when the electron density in the side sections of the channel markedly exceeds the hole density in the central region, whereby one can assume that $x_I < L_g/2$ (i.e., the tunneling points are under the top gate), an analytical approach becomes possible, as demonstrated below.

The potential distribution in the central section of the channel, i.e., under the top gate ($|x| < L_g/2$, where $L_g$ is the gate length and the axis $x$ is directed along the graphene layer), is governed by the following equation:

$$
\frac{3}{W_g} \frac{d^2 \phi}{dx^2} - \frac{\phi - V_g}{W_g} = \frac{4\pi e}{ae}(\Sigma_e - \Sigma_h).
$$

(2)

Here $\Sigma_e = \Sigma_e(\phi)$ and $\Sigma_h = \Sigma_h(\phi)$ are the electron and hole densities in the channel. Equation (2) is a consequence of the 2D Poisson equation for the region $-L_g/2 \leq x \leq L_g/2$ and $-W_b \leq z \leq W_g$ in the so-called weak nonlocality approximation.19,20 It is valid if the scale of the lateral inhomogeneities $\lambda \leq W_b, W_g$.

There are three regions in the central section of the channel: two the n-regions ($x_I < |x| < L_g/2$), which contact the source and drain n-regions, and the p-region ($|x| < x_I$). Taking into account the linear dispersion relation for electrons and holes, for the degenerate 2DEG and 2D hole gas (2DHG) in the pertinent n- and p-regions of the central section of the channel, one obtains, respectively, the following expressions for the electron and hole densities:5,6

$$
\Sigma_e = \frac{e^2(V_F + \phi)^2}{\pi\hbar^2v^2} \theta(V_F + \phi),
$$

(3)

$$
\Sigma_h = \frac{e^2(V_F + \phi)^2}{\pi\hbar^2v^2} \theta(-V_F - \phi),
$$

(4)

where $V_F = \sqrt{\pi\hbar^2v^2V_b/4e^2\hbar}$ and $\theta(\phi)$ is the unity step function.

As follows from eqs. (2)–(4), the potential distributions in the n-regions ($x_I < |x| < L_g/2$) and in the p-region ($|x| < x_I$) are, respectively, governed by the following equations:

$$
\frac{d^2 \psi}{dx^2} = \frac{3}{W_b W_g} \left[ \psi + \frac{(1 + \psi)^2}{\Theta} + u \right],
$$

(5)

$$
\frac{d^2 \psi}{dx^2} = \frac{3}{W_b W_g} \left[ \psi - \frac{(1 - \psi)^2}{\Theta} + u \right].
$$

(6)

Here, the normalized potential $\psi = \phi/V_F$ as well as the parameters $\Theta = V_0/V_F$ and $u = V/V_F \propto V_b^{-1/2}$ have been introduced, where $V_0 = \pm (W_b + W_g)/4W_b W_g (\pi\hbar^2v^2/4e^2)$ is the characteristic voltage (determined by the geometrical parameters) and $V = -(V_b W_g + V_g W_b)/(W_b + W_g)$ is the effective gate voltage. The right-hand sides of eqs. (5) and (6) are quadratic functions of the potential. This differs from the cases of 2DEG and 2DHG in the degenerate electron–hole systems with quadratic dispersion relations for electrons and holes, where the functions in question are linear functions of $\psi$.

For $ae = 4$, $W_b = 2.8 \times 10^{-5}$ cm (280 nm), and $W_g = 4 \times 10^{-6}$ cm (40 nm), $V_0 \simeq 7, 75$ mV. If $V_0 = 1–10$ V, $V_b \simeq 31–98$ mV, hence, $\Theta \simeq 0.08–0.25$. These back gate voltages correspond to the electron densities $\Sigma\simeq 7.89 \times (10^{10}–10^{11})$ cm$^{-2}$. Hence, one can assume that $\Theta < 1$. This inequality corresponds to relatively high electron density in the source and drain sections of the channel.

Integration of eqs. (5) and (6) results in

$$
\left( \frac{d\psi}{dx} \right)^2 = \frac{3}{W_b W_g} \left[ \psi^2 + \frac{2(1 + \psi)^2}{3\Theta} + 2u\psi + A \right],
$$

(7)

$$
\left( \frac{d\psi}{dx} \right)^2 = \frac{3}{W_b W_g} \left[ \psi^2 - \frac{2(1 - \psi)^2}{3\Theta} + 2u\psi + B \right],
$$

(8)

where $A$ and $B$ are constants. Focusing on the case of $u > 1$, which corresponds to the formation of the p-region in the center of the channel, and considering that in the center of the channel ($x = 0$, i.e., relatively far from the tunneling points, the right-hand side of eq. (6) should be small and $(d\psi/dx)|_{x=0} = 0$. Taking into account the relative smallness of $\Theta$, for $\psi_0 = \psi|_{x=0}$ one can obtain $\psi_0 \sim 1 - \sqrt{\Theta(u - 1)}$. As a result, taking into account that $\psi|_{x=+h} = -1$, for $F = eV_F |d\psi/dx|_{x=+h}$ from eq. (8) we find

$$
F = \frac{2eV_F}{\sqrt{W_b W_g}} \left( u - 1 \right)^{1/2}.
$$

(9)

Using eqs. (1) and (9), we arrive at the following equation for the source–drain tunneling current:

$$
J_t = G_t V_d \left( \frac{V_b - V_g}{V_0} \right)^{3/8}.
$$

(10)

Here

$$
G_t = \frac{g_0 H}{4\pi} \sqrt{\frac{x h \hbar}{2 e^2}}.
$$

(11)

where $W = W_b^{3/8}W_g^{1/8}/(W_b + W_g)^{1/8}$, and the channel threshold voltage is given by

$$
V_{th} = \left[ \frac{V_b W_g + W_b}{W_g} + \sqrt{\frac{V_b W_g + W_b}{W_g}} \right] V_b
$$

< 0.

(12)

Figure 2 shows the dependence of the threshold voltage $V_{th}$ on the back-gate voltage $V_b$ calculated using eq. (12) for GFETs with different thicknesses, $W_b$, of the top gate layer. The tunneling source–drain current as a function of the top-gate voltage at different back-gate voltages, described by eqs. (10) and (12), is demonstrated in Fig. 3. The same parameters of the GFET structure as in the above estimate
were assumed.

Using eqs. (11) and (12), for the GFET transconductance $g_t = (\partial J_t/\partial V_g)|_{V_d, V_b=\text{const}}$ associated with the tunneling, one obtains

$$g_t = - \frac{G_t^* V_d}{V_0^{5/8}(V_{th} - V_g)^{5/8}},$$

(13)

where $G_t^* = G_0 [3W_b/8(W_b + W_g)]$. The quantities $G_t$ and $G_t^*$ are determined by the geometrical parameters of the GFET structure: $W_b$, $W_g$, and $H$. In particular, at $W_g \ll W_b$, $G_t^* \simeq 0.02 \frac{g_0 (H/W_b^{1/4} W_g^{3/4})}{\sqrt{4 \hbar \epsilon F / e}} \propto W_g^{-3/4}$. One can see that $|g_t|$ increases when $V_g$ tends to $V_{th}$. However, at small $V_{th} - V_g$, the activation energy of the barrier formed by the p-region becomes low, so that the thermionic current over this barrier might become essential. Taking into account that according to eq. (9), the activation energy of the barrier formed by the p-region is $e[\sqrt{V_0(V_{th} - V_g) - V_d}/2]$, one might conclude that the decrease in the tunneling current $J_t$ when $V_g$ tends to $V_{th}$ becomes compensated by an increase in the thermionic current $J_{th}$, at $V_0(V_{th} - V_g) \sim (k_B T/e)^2$, where $T$ is the temperature and $k_B$ is the Boltzmann constant. Considering this, from eq. (13), one can arrive at the following estimate:

$$\max |g_t| = G_{t}^* \left( \frac{V_d}{V_0} \right) \left( \frac{eV_0}{k_B T} \right)^{5/4}. \quad (14)$$

In conclusion, we developed an analytical device model for a GFET at sufficiently strong negative bias gate voltages when it operates in the tunneling regime. The GFET tunneling current–voltage characteristics were calculated using this model. It was shown that the source–drain tunneling current is proportional to $(V_{th} - V_g)^{3/8}$. The maximum of the GFET transconductance in the tunneling regime was estimated.

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