COULD GAUGE GRAVITATIONAL DEGREES OF FREEDOM
PLAY THE ROLE OF ENVIRONMENT
IN “EXTENDED PHASE SPACE” VERSION
OF QUANTUM GEOMETRODYNAMICS

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Abstract

In the context of the recently proposed formulation of quantum geometrodynamics in
extended phase space we discuss the problem how the behavior of the Universe, initially
managed by quantum laws, has become classical. In this version of quantum geometro-
dynamics we quantize gauge gravitational degrees of freedom on an equal basis with
physical degrees of freedom. As a consequence of this approach, a wave function of the
Universe depends not only on physical fields but also on gauge degrees of freedom. From
this viewpoint, one should regard the physical Universe as a subsystem whose proper-
ties are formed in interaction with the subsystem of gauge degrees of freedom. We argue
that the subsystem of gauge degrees of freedom may play the role of environment, which,
being taken into account, causes the density matrix to be diagonal. We show that under
physically reasonable fixing of gauge condition the density matrix describing the physi-
cal subsystem of the Universe may have a Gaussian peak in some variable, but it could
take the Gaussian form only within a spacetime region where a certain gauge condition
is imposed. If spacetime manifold consists of regions covered by different coordinate
charts the Universe cannot behave in a classical manner nearby borders of these regions.
Moreover, in this case the Universe could not stay in the same quantum state, but its
state would change in some irreversible way.

1. Introduction

Quantum geometrodynamics claims to give a description of quantum stage of the Universe
evolution, including an explanation of the fact that the behavior of the Universe, initially
managed by quantum laws, has become classical. According to Halliwell [1], two requirements must be satisfied for a system to be regarded as classical. The first requirement is that its evolution should be described by classical laws in a very good approximation, and the second requirement involves the notion of decoherence—a transition from a pure state to some mixed state described by diagonal density matrix. There has been a number of works where the notion of decoherence was discussed in the context of quantum cosmology based on the Wheeler–DeWitt quantum geometrodynamics. As well known, the destruction of the off-diagonal terms in the density matrix cannot be regarded in the limits of unitary evolution. A possible way to solve this problem is to consider this destruction as a result of interaction with some environment [2, 3]. The application of this idea to quantum cosmology implies splitting the Universe into two subsystems, one of which is a system under investigation and the other plays the role of environment. It has been suggested to consider some modes of scalar, gravitational and other fields as an environment (see, for example, [1, 4]). However, there is no natural way to split the Universe into two subsystems.

The aim of the present work is to discuss these questions in the limits of recently proposed formulation of quantum geometrodynamics in extended phase space [5] – [11]. In this version of quantum geometrodynamics we quantize gauge gravitational degrees of freedom on an equal basis with physical degrees of freedom. The motivation for it was that it is impossible to separate gauge, or “non-physical” degrees of freedom from physical ones if the system under consideration does not possess asymptotic states, and it is indeed the case for a closed universe as well as for a universe with rather nontrivial topology. As a consequence of this approach, a wave function of the Universe depends not only on physical fields but also on gauge degrees of freedom. From this viewpoint one should regard the physical Universe as a subsystem whose properties are formed in interaction with the subsystem of gauge degrees of freedom.

The plan of the work is as follows. We remind basic equations of the “extended phase space” version of quantum geometrodynamics, consider a general structure of a wave function of the Universe and construct a density matrix describing the physical subsystem of the Universe. We then show that under physically reasonable fixing of gauge condition the density matrix may have a Gaussian peak in some variable, say, in a scale factor, while other degrees of freedom (e.g., gravitational waves and matter fields) should be treated quantum mechanically. The important point in this consideration is that the density matrix takes the Gaussian form only within a spacetime region where a certain gauge condition is imposed. In simple cosmological models one can introduce a single gauge condition in the whole spacetime, and the behavior of the Universe can be regarded as classical almost over the whole history of the Universe. Meanwhile, in the case of nontrivial topology spacetime manifold may consist of regions covered
by different coordinate charts, so that one should impose different gauge conditions in these regions. Then, the Universe cannot behave in a classical manner near by borders of these regions. We shall argue that in this case the Universe could not stay in the same quantum state; as a consequence of interaction with the subsystem of gauge degrees of freedom its state would change in some irreversible way.

2. The “extended phase space” version of quantum geometrodynamics: basic formulas

To investigate a system without asymptotic states we make use of the path integral approach \[9, 10\]. It is easy to illustrate the crux of the matter for a simple minisuperspace model with a gauged action

\[
S = \int dt \left\{ \frac{1}{2} v(\mu, Q)^2 \dot{Q}^{a} \dot{Q}^{b} - \frac{1}{v(\mu, Q)} U(Q) + \pi_{\theta} \left( \mu - f_{a} \dot{Q}^{a} \right) - iw(\mu, Q) \dot{\bar{\theta}} \right\}. \tag{2.1}
\]

Here \( Q = \{Q^{a}\} \) stands for physical variables such as a scale factor or gravitational-wave degrees of freedom and material fields, and we use an arbitrary parametrization of a gauge variable \( \mu \) determined by the function \( v(\mu, Q) \). For example, in the case of isotropic universe or the Bianchi IX model \( \mu \) is bound to the scale factor \( a \) and the lapse function \( N \) by the relation

\[
\frac{a^{3}}{N} = v(\mu, Q). \tag{2.2}
\]

\( \theta, \bar{\theta} \) are the Faddeev – Popov ghosts after replacement \( \bar{\theta} \to -i\bar{\theta} \). Further,

\[
w(\mu, Q) = \frac{v(\mu, Q)}{v_{,\mu}}; \quad v_{,\mu} \overset{def}{=} \frac{\partial v}{\partial \mu}. \tag{2.3}
\]

We confine attention to the special class of gauges not depending on time

\[
\mu = f(Q) + k; \quad k = \text{const}, \tag{2.4}
\]

which can be presented in a differential form,

\[
\dot{\mu} = f_{a} \dot{Q}^{a}; \quad f_{a} \overset{def}{=} \frac{\partial f}{\partial Q^{a}}. \tag{2.5}
\]

The Schrödinger equation for this model reads

\[
i \frac{\partial \Psi(\mu, Q, \theta, \bar{\theta}; t)}{\partial t} = H \Psi(\mu, Q, \theta, \bar{\theta}; t), \tag{2.6}
\]

where

\[
H = -\frac{i}{w} \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} - \frac{1}{2M Q^{a}} \frac{\partial}{\partial Q^{a}} MG^{\alpha \beta} \frac{\partial}{\partial Q^{\beta}} + \frac{1}{v} (U - V); \tag{2.7}
\]
$M$ is the measure in the path integral,

$$M(\mu, Q) = v^K(\mu, Q) w^{-1}(\mu, Q);$$

(2.8)

$$G^{\alpha \beta} = \frac{1}{v(\mu, Q)} \left( \begin{array}{cc} f_a f^a & f^a \\ f^a & \gamma^a_{ab} \end{array} \right); \quad \alpha, \beta = (0, a); \quad Q^0 = \mu,$$

(2.9)

$K$ is a number of physical degrees of freedom; the wave function is defined on extended configurational space with the coordinates $\mu, Q, \theta, \bar{\theta}$. $V$ is a quantum correction to the potential $U$, that depends on the chosen parametrization (2.2) and gauge (2.4):

$$V = \frac{5}{12 w^2} (w_{\mu} f_a f^a + 2 w_{\mu} f_a w^a + w_\alpha w^a) + \frac{1}{3 w} (w_{\mu, \mu} f_a f^a + 2 w_{\mu, a} f^a + w_{\mu} f_a + w^a_a) +$$

$$+ \frac{K-2}{6 v w} (v_{\mu} w_{\mu} f_a f^a + v_{\mu} f_a w^a + w_{\mu} f_a v^a + v_\alpha w^a) -$$

$$- \frac{K^2 - 7 K + 6}{24 v^2} (v_{\mu} f_a f^a + 2 v_{\mu} f_a v^a + v_\alpha v^a) +$$

$$+ \frac{1-K}{6 v} (v_{\mu, \mu} f_a f^a + 2 v_{\mu, a} f^a + v_{\mu} f_a + v_\alpha).$$

(2.10)

The Schrödinger equation (2.6) – (2.10) is derived from a path integral with the effective action (2.1) without asymptotic boundary conditions by the standard well-defined Feynman procedure, thus it is a direct mathematical consequence of the path integral. Once we agreed that imposing asymptotic boundary conditions is not correct in the case of a closed universe, we are doomed to come to a gauge-dependent description of the Universe.

3. The general solution to the Schrödinger equation and the density matrix

The general solution to the Schrödinger equation (2.6) has the following structure [10]:

$$\Psi(\mu, Q, \theta, \bar{\theta}; t) = \int \Psi_k(Q, t) \delta(\mu - f(Q) - k) (\bar{\theta} + i \theta) dk.$$  

(3.1)

The dependence of the wave function (3.1) on ghosts is determined by the demand of norm positivity.

Note that the general solution (3.1) is a superposition of eigenstates of a gauge operator,

$$\langle \mu - f(Q) | k \rangle = k | k \rangle; \quad | k \rangle = \delta(\mu - f(Q) - k).$$

(3.2)

It can be interpreted in the spirit of Everett’s “relative state” formulation. In fact, each element of the superposition (3.1) describes a state in which the only gauge degree of freedom $\mu$ is definite, so that time scale is determined by processes in the physical subsystem through
functions $v(\mu, Q)$, $f(Q)$ (see (2.2), (2.4)), while $k$ being determined by initial clock setting. Indeed, according to (2.4), the parameter $k$ gives an initial condition for the variable $\mu$. The function $\Psi_k(Q, t)$ describes a state of the physical subsystem for a reference frame fixed by the condition (2.4). It is a solution to the equation

$$i \frac{\partial \Psi_k(Q; t)}{\partial t} = H_{(phys)} \Psi_k(Q; t),$$

(3.3)

$$H_{(phys)} = \left[ -\frac{1}{2M} \frac{\partial}{\partial Q^a} \frac{1}{v} M \gamma^{ab} \frac{\partial}{\partial Q^b} + \frac{1}{v} (U - V) \right] \bigg|_{\mu = f(Q) + k}. \quad (3.4)$$

In general, one can seek a solution to Eq. (3.3) in the form of superposition of stationary state eigenfunctions:

$$\Psi_k(Q, t) = \sum_n c_n \Psi_{kn}(Q) \exp(-iE_n t); \quad (3.5)$$

$$H_{(phys)} \Psi_{kn}(Q) = E_n \Psi_{kn}(Q). \quad (3.6)$$

The eigenvalue $E$ corresponds to a new integral of motion that emerges in the proposed formulation as a result of fixing a gauge condition and characterizes the gauge subsystem (see below Eq. (4.2)).

In this paper we shall be interested under what conditions the behavior of the Universe can be regarded as classical. As well known, one of necessary requirements is that a wave function could be represented in the WKB form: $\Psi_{kn}(Q) = C(Q) \exp[iS(Q)]$. However, in our formulation there is an additional requirement. In the classical limit the Universe is described by gauge-invariant Einstein equations, so that all vestiges of gauge fixing should be eliminated. In particular, $E$ must take the zero eigenvalue. Thus, in the classical limit the Universe appears to be in the unique eigenstate with $E = 0$.

Strictly speaking, we need some mechanism of the ”reduction” of the wave function (3.5) to the state with $E = 0$. We suppose that such a mechanism involves a specific interaction between gauge and physical subsystems, but we have not been able to give explanation of the mechanism. In this paper we shall assume that, in any region where the quasiclassical approximation exists, the Universe is described by a quasiclassical wave function of the special state with $E = 0$, $\Psi_k(Q, t) = \Psi_{kn}(Q)$, and concentrate on the density matrix of the physical subsystem.

The normalization condition for the wave function (3.1) reads

$$\int \Psi^{*}(\mu, Q, \theta, \bar{\theta}; t) \Psi(\mu, Q, \theta, \bar{\theta}; t) M(\mu, Q) d\mu d\theta d\bar{\theta} \prod_a dQ^a =$$

$$\int \Psi_{k'}^{*}(Q, t) \Psi_{k'}(Q, t) \delta(\mu - f(Q) - k) \delta(\mu - f(Q) - k') M(\mu, Q) dk dk' d\mu \prod_a dQ^a =$$
The solution (3.1) is normalizable under the condition that $\Psi_k(Q, t)$ is a sufficiently narrow packet over $k$. Let us emphasize that the dependence of $\Psi_k(Q, t)$ on $k$ is not fixed by the equation (3.3) in the sense that $\Psi_k(Q, t)$ can be multiplied by an arbitrary function of $k$. One cannot choose the function $\Psi_k(Q, t)$ to be not depending on $k$, since in this case one would obtain a non-normalizable, non-physical state. It would imply that the gauge condition is fixed absolutely precisely ($\delta$-shaped packet), and such a situation is unrealistic from the physical point of view. We should rather consider a narrow enough packet over $k$ to fit a certain classical $\bar{k}$ value:

$$\Psi(\mu, Q, \theta, \bar{\theta}; t) = \frac{1}{\sqrt{2i\alpha\sqrt{\pi}}} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2\alpha^2} \left( k - \bar{k} \right)^2 \right] \Psi_k(Q, t) \delta(\mu - f(Q) - k) \left( \bar{\theta} + i\theta \right) dk =$$

$$= \frac{1}{\sqrt{2i\alpha\sqrt{\pi}}} \exp \left[ -\frac{1}{2\alpha^2} \left( \mu - f(Q) - \bar{k} \right)^2 \right] \Psi_k(Q, t) \left( \bar{\theta} + i\theta \right). \quad (3.8)$$

Since our investigation aims in giving description of a physical Universe, we can introduce a density matrix

$$\rho(Q, Q', t) = \int \Psi^*(\mu, Q, \theta, \bar{\theta}; t) \Psi(\mu, Q', \theta, \bar{\theta}; t) M(\mu, Q) d\mu d\theta d\bar{\theta}. \quad (3.9)$$

For the wave function (3.8) the expression for density matrix reads

$$\rho(Q, Q', t) = \frac{1}{\alpha \sqrt{\pi}} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2\alpha^2} \left( \mu - f(Q) - \bar{k} \right)^2 - \frac{1}{2\alpha^2} \left( \mu - f(Q') - \bar{k} \right)^2 \right] \times$$

$$\times \Psi_k^*(Q, t) \Psi_k(Q', t) M(\mu, Q) d\mu =$$

$$= \frac{1}{\alpha \sqrt{\pi}} \exp \left( -\frac{1}{4\alpha^2} [f(Q) - f(Q')]^2 \right) \Psi_k^*(Q, t) \Psi_k(Q', t) \times$$

$$\times \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{\alpha^2} \left( \mu - \frac{1}{2} [f(Q) - f(Q')] - \bar{k} \right)^2 \right] M(\mu, Q) d\mu =$$

$$= \exp \left( -\frac{1}{4\alpha^2} [f(Q) - f(Q')]^2 \right) \Psi_k^*(Q, t) \Psi_k(Q', t) \times$$

$$\times M \left( \frac{1}{2} [f(Q) + f(Q')] + \bar{k}, Q \right). \quad (3.10)$$

We have taken into account that only the vicinity of the “point” $\mu = \frac{1}{2} [f(Q) + f(Q')] + \bar{k}$ gives a significant contribution to the integral over $\mu$ and replaced $\mu$ by its approximate value in the measure $M(\mu, Q)$. The normalization condition for the density matrix is

$$\int \rho(Q, Q, t) \prod_a dQ^a = \int \Psi_k^*(Q, t) \Psi_k(Q, t) M(f(Q) + \bar{k}, Q) \prod_a dQ^a = 1, \quad (3.11)$$
it corresponds to the condition (3.7).

Thus, we can see that the density matrix contains the factor

$$\exp\left(-\frac{1}{4\alpha^2} [f(Q) - f(Q')]^2\right)$$

(3.12)

so, if we choose $f(Q)$ to be equal to one of physical variables which we shall denote as $q$ (it may be, for example, a scale factor $a$), $f(Q) = q$, the expression (3.12) will take a Gaussian form

$$\rho \sim \exp\left[-\frac{(q - q')^2}{4\alpha^2}\right].$$

(3.13)

its width $\sqrt{2}\alpha$ being determined by a precision with which we can fix the gauge condition. We shall not discuss here what values $\alpha$ could take; it is a subject of quantum theory of measurements.

At first glance, it may seem that the density matrix has a Gaussian peak only in case of rather specific choice of a gauge condition $\mu = q$. In fact, however, when the gauge condition (2.4) is fixed, it does not mean that some reference frame has been already chosen. By the choice of a reference frame we imply, following to Landau and Lifshitz, imposing some conditions on the metric components $g_{0\mu}$ or, in our simplified model, on the lapse function $N$. A reference frame is completely fixed only if the choice of the parameterization function (2.2), as well as the gauge (2.4), is made. Let us emphasize again that the choice of gauge variable parameterization and that of gauge condition have an inseparable interpretation, – they are both determined by the construction of a clock, so without loss of generality any gauge condition can be turned to $\mu = q$ by choosing the function $v(\mu, Q)$. It is confirmed mathematically by the fact that the Hamiltonian operator in physical subspace (3.4) after substitution $\mu = f(Q) + k$ depends on the function $v(f(Q) + k, Q)$, but not on the functions $v(\mu, Q), f(Q)$ separately. Indeed, in (3.4) the quantum correction $V$ can be presented in the form

$$V|_{\mu=f(Q)+k} = \frac{5}{12w^2} \frac{\delta w}{\delta Q^a} \frac{\delta w}{\delta Q^a} + \frac{1}{3w} \frac{\delta^2 w}{\delta Q^a\delta Q^a} + \frac{K - 2}{6vw} \frac{\delta w}{\delta Q^a} \frac{\delta Q^a}{\delta Q^a} - \frac{K^2 - 7K + 6}{24v^2} \frac{\delta^2 v}{\delta Q^a\delta Q^a} + \frac{1}{6v} \frac{\delta^2 v}{\delta Q^a\delta Q^a},$$

(3.14)

where $\frac{\delta v}{\delta Q^a} = (v_{a,f,a} + v_{a})|_{\mu=f(Q)+k}$ is a total derivative with respect to $Q^a$ of the function $v(f(Q) + k, Q)$, etc. Splitting the procedure of fixing a reference frame into choosing the parameterization and imposing a gauge condition is quite conventional, we shall give some examples in the next section. However, the physical part of the wave function $\Psi_k(Q, t)$, satisfying the equation (3.3), does not depend on this splitting, but on a chosen reference frame as a whole.
The condition $\mu = q$ implies that the gauge subsystem interacts with the physical subsystem through the variable $q$. As a result of this interaction, the density matrix becomes about diagonal in $q$. So, the Universe can be regarded as a classical system in any region where the physical part of the wave function $\Psi_k(Q,t)$ can be represented in the WKB form with respect to the variable $q$ and where the gauge condition $\mu = q$ can be imposed. At the same time, the density matrix does not diagonalize in other physical variables (such as gravitational waves and matter fields, with $q$ representing the scale factor), which do not interact with the gauge subsystem and should be treated quantum mechanically. One can assume that these degrees of freedom describe small perturbations on the background of an isotropic universe. A similar approach is adopted, for example, in [1] where the long-wavelength modes of a scalar field play the role of environment while the short-wavelength modes remain quantum mechanical. The authors of the works [1, 4] and others seek for a model in which a density matrix would contain a Gaussian factor. However, it is always questionable what part of the physical Universe should be considered as an environment. On the other side, taking into account the gauge subsystem is thought to be inevitable when constructing mathematically consistent quantum geometrodynamics. Gauge degrees of freedom are not observable directly, but only by its influence on the physical subsystem. The result (3.13), namely, that the density matrix may have a Gaussian peak in some variable under physically reasonable fixing of gauge condition, is quite general. It seems, therefore, to be natural to regard gauge degrees of freedom as the environment for the physical Universe. One could say, in a certain sense, that the physical Universe is a subsystem of itself.

4. The gauge subsystem as a factor of cosmological evolution

When deriving the Schrödinger equation from the path integral with the effective action (2.1), we approximate the path integral on extended gauged set of equations obtained by varying this effective action. The extended set of equations includes ghosts equations and a gauge condition, and equations for physical degrees of freedom also contain gauge-noninvariant terms. So, the gauged Einstein equations look like

$$R^\nu_{\mu} - \frac{1}{2} \delta^\nu_{\mu} R = \kappa \left( T^\nu_{\mu(mat)} + T^\nu_{\mu(obs)} + T^\nu_{\mu(ghost)} \right),$$

(4.1)

where $T^\nu_{\mu(mat)}$ is the energy-momentum tensor of matter fields, $T^\nu_{\mu(obs)}$ and $T^\nu_{\mu(ghost)}$ are obtained by varying the gauge-fixing and ghost action, respectively. $T^\nu_{\mu(obs)}$ describes the observer (the gauge subsystem) in the extended set of equations.

In particular, the $(0,0)$-Einstein equation (Hamiltonian constraint) is transformed to the form
$H = E$, where $H$ is a Hamiltonian in extended phase space and

$$E = - \int \sqrt{-g} T^0_{\text{obs}} d^3x.$$  \hspace{1cm} (4.2)

In quantum theory the modified Hamiltonian constraint leads to a stationary Schrödinger equation (see [360]).

To give a simple example, in this section we shall bear in mind an isotropic universe, then the parameterization function $v(\mu, Q)$, as well as the gauge-fixing function $f(Q)$, will depend only on a scale factor, i.e.

$$\frac{a^3}{N} = v(\mu, a), \quad \mu = f(a) + k.$$  \hspace{1cm} (4.3)

The quasi-energy-momentum tensor of the gauge subsystem reads:

$$T^\nu_{\mu(\text{obs})} = \text{diag} \left( \varepsilon(\text{obs}), -p(\text{obs}), -p(\text{obs}), -p(\text{obs}) \right);$$  \hspace{1cm} (4.4)

$$\varepsilon(\text{obs}) = - \frac{\hat{\pi}_0}{2\pi^2} \frac{v^2(\mu, a)}{a^6 v,\mu} \bigg|_{\mu = f(a) + k};$$  \hspace{1cm} (4.5)

$$p(\text{obs}) = \varepsilon(\text{obs}) \left[ 1 - \frac{a}{3v(\mu, a)} \left( v,\mu f,a + v,a \right) \right] \bigg|_{\mu = f(a) + k}.$$  \hspace{1cm} (4.6)

The last formula gives the equation of state for the gauge subsystem depending on parameterization and gauge condition. Note that, again, the equation of state after substitution $\mu = f(a) + k$ depends on the function $v(f(a) + k, a)$, but not on the functions $v(\mu, a)$, $f(a)$ separately.

Let us choose

$$v(\mu, a) = \frac{a^2}{\mu}, \quad \mu = 1 + \frac{1}{a^4}. \hspace{1cm} (4.7)$$

(Here and below we shall assume that the classical value $\bar{k} = 0$). As follows from (4.3), it corresponds to the condition for the lapse function $N$:

$$N = a + \frac{1}{a^3}. \hspace{1cm} (4.8)$$

This gauge condition is rather interesting in some respects. For large $a$ we have $N = a$ (conformal time gauge), while in the limit of small $a$ the condition (4.8) can be rewritten as $Na^3 = 1$. The latter corresponds to the constraint on metric components $\sqrt{-g} = \text{const}$. This constraint is known to lead to the appearance of $\Lambda$-term in the Einstein equations [12, 8]. For $v(\mu, a)$ and $f(a)$ determined by (4.7) the equation of state (4.6) reduces to

$$p(\text{obs}) = \varepsilon(\text{obs}) \left[ 1 - \frac{1}{3} \left( \frac{4}{1 + a^4} + 2 \right) \right].$$  \hspace{1cm} (4.9)
In the course of cosmological evolution the equation of state changes from \( p_{\text{obs}} = -\varepsilon_{\text{obs}} \) in the limit of small \( a \) to \( p_{\text{obs}} = \frac{1}{3} \varepsilon_{\text{obs}} \) in the limit of large \( a \). The former corresponds a medium with negative pressure typical for an exponentially expanded early universe with \( \Lambda \)-term, the latter is an ultrarelativistic equation of state, and the Einstein equations in the limit of large \( a \) have a solution describing a Friedmann universe in the conformal time gauge \( N = a \). Therefore, we can see that the gauge subsystem appears to be a factor of cosmological evolution; its state changing over the history of the Universe determining a cosmological scenario.

5. Discussion

If we adopt the ADM parameterization, namely, regard the lapse function \( N \) as a gauge variable and impose the condition \( (4.8) \), according the above consideration, we shall come to the conclusion that at large \( a \), when \( N = a \) in a good approximation, the density matrix will have a Gaussian peak \( (3.13) \) with \( q \) representing \( a \). In other words, this simple model confirms that the scale factor becomes a classical variable in the region of large \( a \).

On the other hand, we are free to choose another gauge variable, \( \mu \), giving the function \( v(\mu, Q) \). Then, we shall change properties of environment and the character of its interaction with the physical subsystem. Remind that the density matrix \( (3.9) \) is obtained by integrating out a gauge variable defined by \( (2.2) \). In particular, as was said above, for a given reference frame one can choose \( \mu \) to satisfy the condition \( \mu = a \). So, instead of \( (4.7) \) one can put

\[
v(\mu, a) = \frac{a^2}{1 + \mu^4}, \quad \mu = a. \tag{5.1}\]

It leads to the same condition \( (4.8) \) for \( N \) and the same equation of state \( (4.9) \). The density matrix will have a Gaussian peak \( (3.13) \) in the whole region where the condition \( \mu = a \) can be imposed, or, where the reference frame determined by \( (4.8) \) can be chosen. (One could make the simplest choice \( N = a \), but in this case in the quasiclassical limit one would get a Friedmann universe without \( \Lambda \)-term and, correspondingly, without a stage of inflation.) The choice \( (5.1) \) means that we choose in \( (3.1) \) the basis, which is the set of eigenstates of the gauge operator \( (\mu - a) \):

\[
(\mu - a) |k\rangle = k |k\rangle; \quad |k\rangle = \delta (\mu - a - k). \tag{5.2}\]

The basis \( (5.2) \) plays the role of a preferred basis in the sense that the density matrix is about diagonal. In this case we choose an environment causing the density matrix to diagonalize. However, the variable \( \mu \), which represents the environment, may not have a clear physical meaning like the lapse function \( N \).
The gauge subsystem has also other features, which are usually implied for environment. In particular, in any region where some gauge condition is fixed, a state of the Universe is not disturbed by interaction with the gauge system. Indeed, since one of canonical equations in extended phase space is a gauge condition in a differential form (2.5), a gauge operator commutes with the Hamiltonian (2.7) [10]. For example, when the state of the Universe is a specific state discussed above with \( E = 0 \), described by a quasiclassical wave function \( \Psi_{k0}(Q) \), it will not be disturbed by interaction with the environment.

To summarize, under a suitable definition of the parameterization function \( v(\mu, a) \), one can get that the density matrix would have a Gaussian peak in some variable in any region where a certain reference frame is chosen. An appropriate gauge condition may be rather complicated, as the condition (4.8), which includes different regimes in limiting cases. (In fact, Eq. (4.8) describes a transition from the condition \( Na^3 = 1 \) to \( N = a \).) In simple cosmological models it is possible to introduce a single reference frame in the whole spacetime, and the behavior of the Universe can be regarded as classical almost over the whole its history. The situation is different if one admits nontrivial topology of spacetime. In general, spacetime manifold may consist of regions covered by different coordinate charts, so that one should introduce different reference frames in these regions. A transition from one reference frame to another cannot be described by a single gauge condition like (4.8), and the Universe cannot behave in a classical manner nearby borders of regions where different reference frames are introduced. It especially concerns spacetime manifolds with horizons. Let us note that the quasiclassical approximation is not valid nearby the borders of these regions as well, so that the two requirements – the quasiclassical character of a wave function and fixing a certain reference frame – completely coincide.

In [11] we have considered a small variation of the gauge-fixing function \( f(Q) \) while the parameterization function \( v(\mu, Q) \) being fixed. This variation corresponds to a transition to another reference frame and another basis

\[
(\mu - f(Q) - \delta f(Q)) |k\rangle = k |k\rangle; \quad |k\rangle = \delta (\mu - f(Q) - \delta f(Q) - k). \tag{5.3}
\]

Then the Hamiltonian in physical subspace (3.4) acquires additional terms, which contain anti-Hermitian part in the original subspace defined by the basis (3.2). Accordingly, a measure in the physical subspace \( M (f(Q) + k, Q) \) (see (3.7)) changes to \( M (f(Q) + \delta f(Q) + k, Q) \). The change of the measure and the appearance of an anti-Hermitian part of the Hamiltonian (3.4) show that a transition to another reference frame has an irreversible character. As a consequence of interaction with the subsystem of gauge degrees of freedom, the Universe may not stay in one of states of the superposition (3.5), in particular, in a special state with \( E = 0 \) for which a classical limit could be obtain. Its state would change in some irreversible way.
References

[1] J. J. Halliwell, Phys. Rev. D39 (1989) P. 2912.
[2] W. H. Zurek, Phys. Rev. D24 (1981) P. 1516.
[3] W. G. Unruh, W. H. Zurek, Phys. Rev. D40 (1989) P. 1071.
[4] T. Fukuyama, M. Morikawa, Phys. Rev. D39 (1989) P. 462.
[5] V. A. Savchenko, T. P. Shestakova and G. M. Vereshkov, Int. J. Mod. Phys. A14 (1999) P. 4473.
[6] V. A. Savchenko, T. P. Shestakova and G. M. Vereshkov, Int. J. Mod. Phys. A15 (2000) P. 3207.
[7] T. P. Shestakova, Gravitation & Cosmology 5 (1999) P. 297.
[8] T. P. Shestakova, in Proceedings of the IV International Conference ”COSMION-99” (Moscow, 1999), Gravitation & Cosmology 6, Supplement (2000) P. 47.
[9] V. A. Savchenko, T. P. Shestakova and G. M. Vereshkov, Gravitation & Cosmology 7 (2001) P. 18.
[10] V. A. Savchenko, T. P. Shestakova and G. M. Vereshkov, Gravitation & Cosmology 7 (2001) P. 102.
[11] T. P. Shestakova, in Proceedings of the V International Conference on Gravitation and Astrophysics of Asian-Pacific countries, Gravitation & Cosmology 8, Supplement II (2002) P. 140.
[12] S. Weinberg, Rev. Mod. Phys. 61 (1989) P. 1.