A Nonlocal Wave-Particle Duality

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PACS 03.65.Ud – Entanglement and quantum nonlocality
PACS 03.65.Ta – Foundations of quantum mechanics

Abstract – Analyzing a proposed modified ghost-interference experiment, we show that revealing the particle-nature of a particle passing through a double-slit hides the wave-nature of a spatially separated particle which it is entangled with. We derive a nonlocal duality relation, \(D_1^2 + V_2^2 \leq 1\), which connects the path distinguishability of one particle to the interference visibility of the other. It extends Bohr’s principle of complementarity to a nonlocal scenario. We also propose a ghost quantum eraser in which, erasing the which-path information of one particle brings back the interference fringes of the other.

Introduction. – The two-slit interference experiment has become a cornerstone of the issue of wave-particle duality and Bohr’s complementarity principle. So beautifully and simply does it capture the dual nature of particles and light and the superposition principle that it has become symbolic of the mysterious nature of quantum mechanics. The fact that the wave and particle nature cannot be observed at the same time, appears to be so fundamental that Bohr elevated it to the level of a new principle, the principle of complementarity \(1\). Bohr asserted that if an experiment clearly revealed the particle nature, it would completely hide the wave nature, and vice-versa. This principle has now been made quantitatively precise by a bound on to what extent the two natures could be simultaneously observed \(2,3\). The extent to which one can distinguish which of the two slits a particle passes through, is given by a quantity \(D\), and the visibility of the interference by \(V\). The quantities \(D\) and \(V\) are so defined that they can take values only between 0 and 1. The relation putting a bound on the two is given by the so-called Englert-Greenberger-Yasin (EGY) duality relation \(2,3\)

\[V^2 + D^2 \leq 1.\]  \(1\)

The above relation implies that a full which-path information (\(D = 1\)) would definitely wash out the interference pattern completely (\(V = 0\)).

It is quite obvious that when we talk of which-path distinguishability, we talk of the which-path knowledge of the same particle which contributes to the interference pattern. In this sense the EGY relation is local. In the following we propose and theoretically analyze an experiment involving pairs of entangled particles in which we relate the which-path information of one particle to the fringe visibility of the other.

Ghost interference. – The starting point of our analysis is the well known ghost-interference experiment carried out by Strekalov et al. \(4\). In this experiment, pairs of entangled photons are generated from a spontaneous parametric down conversion (SPDC) source. In the path of photon 1 is kept a double-slit and further down in the path is a \(\text{fixed}\) detector \(D_1\). Photon 2 travels undisturbed and is ultimately detected by the movable detector \(D_2\). The detectors \(D_1\) and \(D_2\) are connected to a coincidence counter. In coincident counts, a two-slit interference pattern is seen by detector \(D_2\) for photon 2. Note that photon 2 does not pass through any double-slit. This interference was appropriately called ghost interference, and has been understood to be a consequence of entanglement. This experiment generated lot of research attention in subsequent years \(5,13\).

Our proposed experiment is shown in Fig.1 Entangled particle pairs emerge from a source \(S\). For the sake of generality, let us assume massive particles, although the ghost interference experiment is done with photons. Particle 1 passes through a double-slit and also interacts with a which-path detector. We do not specify any form of
the which-path detector, but just assume that it is a two-state system with states $|d_1\rangle$ and $|d_2\rangle$ which get entangled with the two paths of particle 1. This entanglement is a must in order that the which-path detector acquires the relevant information about the particle. Particle 1 then travels and reaches a fixed detector D1. Particle 2 travels unhindered to the detector D2. As the two particles have to be counted in coincidence, the paths travelled by both the particles, before reaching their respective detectors, are equal. In the absence of the which-path detector, this is just the original ghost interference experiment where particle 2 displays an interference pattern.\[4\]

**Which-path information.** – We assume $|d_1\rangle, |d_2\rangle$ to be normalized, but not necessarily orthogonal. The ultimate limit to the knowledge we can acquire as to which slit particle 1 went through is set by how distinct the states $|d_1\rangle, |d_2\rangle$ are. If $|d_1\rangle, |d_2\rangle$ are orthogonal, we can in principle know with hundred percent accuracy which slit the particle went through. With this thinking we define which-path distinguishability for particle 1 as

$$D_1 = \sqrt{1 - (|d_1|^2|d_2|^2)}.$$  

In order to quantify the effect of the which-path detector on the ghost interference shown by particle 2, we carry out a quantum mechanical analysis of the dynamics of the entangled particles. We assume that the particles travel in opposite directions along the $x$-axis. The entanglement is in the $z$-direction. The best state to describe momentum-entangled particles is the generalized EPR state \[14\]

$$\Psi(z_1, z_2) = C \int_{-\infty}^{\infty} dp e^{-p^2/4\hbar^2} e^{-ipz_2/\hbar} e^{ipz_1/\hbar} e^{-\left[(z_1 + z_2)^2/4\Delta^2\right]},$$  

where $C$ is a normalization constant, and $\sigma, \Omega$ are certain parameters. In the limit $\sigma, \Omega \to \infty$ the state \[3\] reduces to the EPR introduced by Einstein, Podolsky and Rosen \[15\].

After performing the integration over $p$, \[3\] reduces to

$$\Psi(z_1, z_2) = \frac{\sigma}{\pi \Omega} e^{-(z_1 - z_2)^2/\sigma^2} e^{-(z_1 + z_2)^2/4\Omega^2}.$$  

It is straightforward to show that $\Omega$ and $\sigma$ quantify the position and momentum spread of the particles in the $z$-direction.

We assume that after travelling for a time $t_0$, particle 1 reaches the double slit ($vt_0 = L_2$), and particle 2 travels a distance $L_2$ towards detector D2. Using the strategy outlined in the preceding discussion, we can write the state of the entangled photons after a time $t_0$ as follows:

The state of the entangled system, after this time evolution, can be calculated using the Hamiltonian governing the time evolution, given by $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\hbar}{2m} \Omega^2$. After a time $t_0$, \[4\] assumes the form

$$\Psi(z_1, z_2, t_0) = C_{t_0} \exp \left[ \frac{-(z_1 - z_2)^2}{\sigma^2} \right] \exp \left[ \frac{-(z_1 + z_2)^2}{4\Omega^2} \right].$$

where $C_{t_0} = \left( \pi(\Omega + \frac{\hbar t_0}{2m})(1/\sigma + \frac{\hbar t_0}{2m}) \right)^{-1/2}$.

We take into account the effect of the double-slit on the entangled state as follows. We assume that the double-slit allows the portions of the wavefunction in front of the slit to pass through, and blocks the other portions. We assume that what emerge from the double-slit are localized Gaussian wavepackets, whose width is the width of the slit. The two slits being A and B, the wavepackets which pass through, are denoted by $|\phi_A(z_1)\rangle$ and $|\phi_B(z_1)\rangle$, respectively. The portion of particle 1 which gets blocked is, say, $\chi(z_1)$. These three states are obviously orthogonal, and the entangled two-particle state can be expanded in terms of these. We can thus write:

$$|\Psi(z_1, z_2, t_0)\rangle = |\phi_A\rangle|\phi_A\rangle + |\phi_B\rangle|\phi_B\rangle + |\chi\rangle|\chi\rangle,$$

where the corresponding states of particle 2 are given by

$$\psi_A(z_2) = \langle \phi_A(z_1)|\Psi(z_1, z_2, t_0)\rangle$$

$$\psi_B(z_2) = \langle \phi_B(z_1)|\Psi(z_1, z_2, t_0)\rangle$$

$$\psi_\chi(z_2) = \langle \chi(z_1)|\Psi(z_1, z_2, t_0)\rangle.$$  

In addition, the wavepackets of particle 1 get entangled with the two states of the which-path detector $|d_1\rangle, |d_2\rangle$. So, the state we get after particle 1 crosses the double-slit is:

$$|\Psi(z_1, z_2)\rangle = |d_1\rangle|\phi_A\rangle|\psi_A\rangle + |d_2\rangle|\phi_B\rangle|\psi_B\rangle + |\chi\rangle|\psi_\chi\rangle.$$  

The first two terms represent the amplitudes of particle 1 passing through the double-slit, and the last term represents the amplitude of it getting blocked. Unitarity of the dynamics assures that these two parts of the wavefunction will evolve independently, without affecting each other. Since we are only interested in situations where particle 1 passes through the slit, we will throw away the term which represents particle 1 not passing through the slits. To do that, we just have to renormalize the remaining part of the wavefunction, which looks like

$$|\Psi(z_1, z_2)\rangle = \frac{1}{C} (|d_1\rangle|\phi_A\rangle|\psi_A\rangle + |d_2\rangle|\phi_B\rangle|\psi_B\rangle).$$

$p$
where \( C = \sqrt{\langle \psi_A | \psi_A \rangle + \langle \psi_B | \psi_B \rangle} \).

In the following, we assume that |\( \phi_A \rangle, |\phi_B \rangle \), are Gaussian wave-packets:

\[
\phi_A(z_1) = \frac{1}{\sqrt{\pi/2}} e^{-(z_1-z_0)^2/\epsilon^2}, \\
\phi_B(z_1) = \frac{1}{\sqrt{\pi/2}} e^{-(z_1+z_0)^2/\epsilon^2},
\]

where \( z_0 \) is the z-position of slit A and B, respectively, and \( \epsilon \) their widths. Thus, the distance between the two slits is \( 2z_0 \equiv d \).

Using (7) and (5), wavefunctions |\( \psi_A \rangle, |\psi_B \rangle \) can be calculated, which, after normalization, have the form

\[
\psi_A(z_2) = C_2 e^{-(z_2-z_0)^2/\epsilon^2}, \quad \psi_B(z_2) = C_2 e^{-(z_2+z_0)^2/\epsilon^2},
\]

where \( C_2 = (2/\pi)^{1/4}(\sqrt{1+R} + \sqrt{1/R})^{-1/2} \),

\[
z_0' = \frac{z_0}{4 \sqrt{1+R} \sigma + 1 + \frac{4 \pi}{4 \sqrt{1+R} \sigma}}
\]

and

\[
\Gamma = \frac{c^2 + \frac{4 \pi}{4 \sqrt{1+R} \sigma}}{1 + \frac{4 \pi}{4 \sqrt{1+R} \sigma}} + \frac{2 \hbar t}{m} + \frac{2 \hbar t}{m}.
\]

Thus, the state which emerges from the double slit, has the following form

\[
\Psi(z_1, z_2) = c |d_1 \rangle e^{-(z_1-z_0)^2/\epsilon^2} e^{-(z_2-z_0')^2/\epsilon^2} + c |d_2 \rangle e^{-(z_1+z_0)^2/\epsilon^2} e^{-(z_2+z_0')^2/\epsilon^2},
\]

where \( c = (1/\sqrt{\pi}) (\sqrt{1+R} + \sqrt{1/R})^{-1/2} \). Particles travel for another time \( t \) before reaching their respective detectors. We assume that the wave-packets travel in the \( x \)-direction with a velocity \( v \) such that \( \lambda = h/mv \) is the d’Broglie wavelength. Using this strategy, we can write \( h(t+2t_0)/m = \lambda D/2\pi \), \( \hbar t_0/z = \lambda L/2\pi \). The expression \( \lambda D/2\pi \) will also hold for a photon provided, one uses the wavelength of the photon for \( \lambda \). The state acquires the form

\[
\Psi(z_1, z_2, t) = C_t |d_1 \rangle e^{-(z_1-z_0)^2/\epsilon^2} e^{-(z_2-z_0')^2/\epsilon^2} \exp \left[ \frac{-(z_2-z_0')^2}{\epsilon^2 + \frac{4 \pi I_1}{\Gamma} \lambda} \right] e^{(\Gamma + \frac{4 \pi I_1}{\Gamma} \lambda) t}, \quad + C_t |d_2 \rangle e^{-(z_1+z_0)^2/\epsilon^2} e^{-(z_2+z_0')^2/\epsilon^2} \exp \left[ \frac{-(z_2+z_0')^2}{\epsilon^2 + \frac{4 \pi I_1}{\Gamma} \lambda} \right] e^{(\Gamma + \frac{4 \pi I_1}{\Gamma} \lambda) t},
\]

where

\[
C_t = \frac{1}{\sqrt{\pi} \sqrt{\epsilon + i L_1 \lambda / \pi \sqrt{\sqrt{I_1} + \sqrt{\Gamma + i L_1 \lambda / \pi \sqrt{\sqrt{I_1}}}}}} \cdot \exp \left[ \frac{-(z_2-z_0')^2}{\epsilon^2 + \frac{4 \pi I_1}{\Gamma} \lambda} \right] e^{(\Gamma + \frac{4 \pi I_1}{\Gamma} \lambda) t}.
\]

In order to get simplified results, we consider the limit \( \Omega \gg \epsilon \) and \( \Omega \gg 1/\sigma \). In this limit

\[
\Gamma^2 \approx \gamma^2 + 4 \hbar t_0/m, \quad \gamma^2 = \epsilon^2 + 1/\sigma^2 \text{ and } z_0' \approx z_0.
\]

**Nonlocal wave-particle duality.** – We are now in a position to calculate the probability of particle 2 at a position \( z_2 \), provided that D1, which is fixed at \( z_1 = 0 \), detects particle 1. This probability density is given by \( |\Psi(0, z_2, t)|^2 \), which has the following form

\[
|\Psi(0, z_2, t)|^2 = |C_t|^2 e^{-(z_2-z_0')^2/\epsilon^2} e^{-(z_2+z_0')^2/\epsilon^2} \cosh \left[ \frac{4 \pi t_2}{\gamma^2 + \frac{4 \pi I_1}{\Gamma} \lambda} \right] \times \left\{ 1 + |\langle d_1 | d_2 \rangle|^2 \right\}
\]

Eqn. (17) represents a ghost interference pattern for particle 2, eventhough it has not passed through any double-slit.

We can calculate the fringe visibility of the interference formed by particle 2. Fringe visibility is defined as \( V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \), where \( I_{max}, I_{min} \) is the maximum and minimum intensity in neighboring region of the interference pattern (16). The fringe visibility for particle 2, from (17), is given by

\[
\mathcal{V}_2 = \frac{|\langle d_1 | d_2 \rangle|^2}{\cosh \left[ \frac{4 \pi t_2}{\gamma^2 + \frac{4 \pi I_1}{\Gamma} \lambda} \right]}.
\]

As \( \cosh(\alpha) \geq 1 \), we can write the inequality

\[
\mathcal{V}_2 \leq |\langle d_1 | d_2 \rangle|.
\]

Using (2), the above inequality yields

\[
\mathcal{D}_2^2 + \mathcal{V}_2^2 \leq 1
\]

The inequality (20) is a very interesting one. It puts a bound on how much which-path information for particle 1 and visibility of interference fringes for particle 2 we can get at the same time. Clearly, full which-path information for particle 1 implies that the interference pattern of particle 2 will be completely washed out.

Bohr’s complementarity principle is made quantitatively precise by the EGY inequality (1). However, here we have a curious scenario where complementarity is governing two separated particles which are not even interacting with each other. By virtue of entanglement, their natures are also entwined with each other. Revealing the particle (bsket) of one, hides the wave nature of the other! It appears that in this kind of entangled state, the two particle can either reveal their wave-nature together, or particle nature together.

**Ghost quantum eraser.** – In the preceding section we saw that extracting which-path information in particle 1, leads to disappearance of interference in particle 2. For a conventional two-slit experiment it is well known that if we devise a way to erase the which-path information, it is possible to recover the lost interference. This phenomenon goes by the name of quantum eraser (17,18).
In the following we propose a quantum eraser experiment which can be performed with entangled photons. The setup is shown in Fig. 2, and is motivated by a two-slit quantum eraser demonstrated by Walborn et al. \[19\] \[20\]. Properties of entangled photons have also been used to construct a quantum eraser before \[21\]. Our proposal is radically different from those in that the which-path information and interference is probed, not in the same photon, but in two different photons. The setup consists of a type I SPDC source generating pairs of photons which we call 1 and 2. There is a double-slit in the path of photon 1, followed by a fixed detector D1. Photon 2 travels undisturbed to detector D2 which scans various positions, and acts like a screen. The two detectors are connected to a coincidence counter. Behind the double-slit are kept two quarter-wave plates which convert the passing photons to linear polarizations. All the photons reach detector D2 which scans various positions, and acts like a screen. The two detectors are connected to a coincidence counter. Behind the double-slit are kept two quarter-wave plates which convert the passing photons to linear polarizations. All the photons reach detector D2 which scans various positions, and acts like a screen. The two detectors are connected to a coincidence counter. Behind the double-slit are kept two quarter-wave plates which convert the passing photons to linear polarizations.

These orthogonal polarization states of photon 1 play the role of $|d_1\rangle$ and $|d_1\rangle$, as described in the preceding analysis. However, $|d_1\rangle$ and $|d_1\rangle$ are completely orthogonal in this case. The state of the two photons, when the reach their respective detectors, is given by $|\Psi_1\rangle$. The coincident probability of detecting photon 2 is given by $|\langle q_1|\Psi(z_1, z_2, t)\rangle|^2$ is given by (14).

The coincident probability of detecting photon 2 is now given by

$$|\langle q_1|\Psi(0, z_2, t)\rangle|^2 = \frac{|C_1|^2}{2} e^{-\frac{2z_2}{\lambdaD}} \frac{e^{-\frac{2z_2}{\lambdaD}}} {\cosh \left[ \frac{4z_2 f_0}{c^2 + \lambdaD^2} \right]} \right) (2)$$

The above represents an interference pattern, even in the presence of the quarter-wave plates. The horizontal polarizer has erased the which-path information for photon 1 and the interference for photon 2 has come back. This scenario is depicted in the lower diagram of Fig. 2.

**Conclusions.** – In conclusion, we have theoretically analyzed a modified ghost-interference setup where a which-path detector for particle 1 has been introduced. Unravelling the particle aspect of photon 1 hides the wave aspect of photon 2. This appears to be a nonlocal extension of Bohr’s complementarity principle. We also derive a nonlocal duality relation connecting the which-path distinguishability of particle 1 with the interference fringe visibility of particle 2. Because of entanglement, the wave and particle aspects of the two particles are no longer independent. We propose a realizable *ghost quantum eraser* experiment. Here erasing the which-path information of one photon recovers the interference for the other photon. The aspects discussed in this investigation reveal highly nonclassical and nonlocal features of entangled systems.

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M.A. Siddiqui thanks the University Grants Commission, India for financial support.

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A Nonlocal Wave-Particle Duality

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