Criterion of lifting force influence on the motion of buoyancy vortex ring

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Abstract. Criterion characterizing the lifting force influence on buoyancy vortex ring motion is worked out. If this criterion is small compared unity then lifting force influence on buoyancy vortex ring motion is weak, if criterion is large compared unity then such influence is strong. To check criterion experiments with buoyancy vortices moving in the direction of lifting force are carried out. Comparison of the experimental results with theoretical predictions shows their agreement.

1. Introduction

Several studies [1, 2] are devoted to investigations of the motion of homogeneous and inhomogeneous buoyant vortices in the gravity field. At the same time, the motion of buoyancy vortices having initial velocity has practically not been investigated. Such a situation is associated with the complexity of formation of the buoyancy vortex having initial velocity and containing a set amount of substance, which is more lightweight than the environmental one. In particular it is unknown the manner in which the lifting force impact on the vortex ring motion is varied with the initial velocity or corresponding Reynolds number. However, an understanding of it represents scientific and practical interest with respect to the problem of the dynamics of vorticity and the problems related to the processes of transfer and mixing. Experimental data are necessary for constructing theoretical models, which are adequate for observations.

One of the bright properties of vortex rings is the linearity of increasing their sizes in dependence on the distance traversed, at least at the initial stage of motion. This effect is characterized by the ring expansion coefficient $\alpha$, which is equal to $dR/dz$, $R$ is the ring radius, and $z$ is the coordinate along which the motion proceeds. At the present time, it is established that the expansion coefficient of buoyant vortices in gravity field are an order of magnitude larger than that of vortices in a homogeneous liquid [2, 3]. So the buoyancy or lifting force has a great influence on the vortex ring dynamics. In this connection it is useful to work out criterion characterizing the lifting force influence on vortex ring motion.

2. Lifting force influence criterion

A vortex ring moving along $z$-axis parallel to the gravitational acceleration $g$ in an unbounded viscous fluid of density $\rho_0$ at infinity is considered. From experiments it is known that a vortex ring during the motion retains its structure. Approximately it is composed of a toroidal core with a circular cross-section, containing a fluid of constant vorticity, and the atmosphere surrounding the core with zero vorticity [5]. For buoyancy vortex it is further assumed that all inhomogeneity is contained within the
core. Then as the characteristic parameters of the flow are taken: radius of the vortex ring \( R \) (equal to the radius of the center line of the torus), radius of the cross section of the core \( a \), velocity of the ring \( u \), density \( \rho_0 \), kinematic viscosity \( \nu \) and integral buoyancy force \( F \). Other vortex parameters such as circulation \( \Gamma \), momentum \( P \) can be obtained from characteristic one.

Criterion characterizing the buoyancy force influence on vortex ring motion must be constructed as a dimensionless complex containing characteristic parameters. So determination of the vortex characteristic parameters from its formation conditions leads to the criterion calculation and prediction of the buoyancy force influence on vortex ring motion.

To estimate buoyancy force influence we will compare expansion coefficients of homogeneous and buoyancy vortices. If these coefficients are distinguished significant that influence is significant.

In [4] it is presented analytical model describing the motion of homogeneous and buoyancy heterogeneous vortex rings. In particular in this work it is obtained formulae for calculation of initial expansion coefficient \( \alpha_i \), which may be written as:

\[
\alpha_1 = \alpha_* + \frac{\beta F}{2\pi \rho_0 \nu^2 \text{Re}_1^2} \tag{1}
\]

\( \beta = \text{const} \approx 0.3 \), \( \text{Re}_1 = u_1 R_1 / \nu \), \( R_1 \) and \( u_1 \) are initial radius and velocity of the vortex. Parameter \( \alpha_* \) can be considered as an expansion coefficient of ring in homogeneous liquid because the case \( F = 0 \) is consistent with the motion in homogeneous liquid. If the motion occurs in the direction of lifting force then \( F > 0 \), if in opposite one then \( F < 0 \). From this equation it follows that lifting force increases the expansion coefficient if the motion occurs in its direction and decreases if the motion is the opposite.

Let

\[
k = \frac{\beta F}{2\pi \rho_0 \nu^2 \text{Re}_1^2} \tag{2}
\]

Then equation (1) may be written as

\[
\alpha_1 = \alpha_*(1 + k) \tag{3}
\]

From (3) it follows that distinction \( \alpha_1 \) from \( \alpha_* \) depends on the magnitude of \( k \). So \( k \) may be adopted as criterion characterizing the buoyancy force influence on vortex ring motion. To final calculate \( k \) it must be given parameter \( \alpha_* \). Because \( \alpha_* \) may be adopted as expansion coefficient of ring in homogeneous liquid then for it calculation we use empirical equation obtained in [3] for \( \text{Re}_1 > 3.5 \times 10^3 \). This equation has the next form

\[
\alpha_* = 3 \cdot 10^{-3} + \frac{50}{\text{Re}_1 - 10^3} \tag{4}
\]

Substituting (4) in (2) we obtain final expression for \( k \).

\[
k = \frac{\beta F}{2\pi \rho_0 \nu^2 (3 \cdot 10^{-3} + 50/(\text{Re}_1 - 10^3))} \tag{5}
\]

From (5) it follows that \( k \) is monotonically decreasing function of \( \text{Re}_1 \) if \( F > 0 \) and in contrast if \( F < 0 \). So the lifting forces influence on vortex ring motion decreases with increasing of Reynolds number and the peak distinguish between \( \alpha_1 \) and \( \alpha_* \) will be attained at minimum Reynolds number.

To establish the boundary value of \( k \) between strong and weak influence of lifting force we must prescribe corresponding value to \( k \). Put that this boundary corresponds to \( |k| = 0.5 \). Note that from (3) it follows that for buoyancy vortex moving in the direction of lifting force \( F > 0 \) this boundary
corresponds to the case $\alpha_1 = 1.5 \alpha_*$. After substitution of $k = 0.5$ in (5) we obtain equation for minimum lifting force at which their influence will be appreciable

$$ F = \frac{\pi \rho_0 \nu^2 \text{Re}_1^2}{\beta \left( 3 \cdot 10^{-3} + \frac{50}{\text{Re}_1 - 10^4} \right) } $$

(6)

3. Experimental results

Experiments with buoyancy vortex rings moving in water in the direction of lifting force have been carried out to check criterion characterizing the lifting force influence on vortex motion. In the experiments for creating the buoyancy, air bubbles are put in the vortex. In this case evidently $F = \rho g V$, $g$ is gravitational acceleration, $V$ is volume of the air captured in vortex. The air bubbles are captured by a formed vortex, and, then, fall in its torus shaped core under the action of centrifugal force, where they are accumulated on the axial line of the torus displaced together with the vortex. The motion of the vortex ring is recorded by a high speed velocity video camera. The shadow transmission shooting is made perpendicularly to the direction of motion. The position of the vortex is visualized because of the presence air bubbles in it. We measure the external transverse size $d$ of the region occupied with bubbles in dependence on the distance $z$ travelled by the vortex.

In figure 1 the photograph of the air bubbles captured by the vortex is shown. In figure 2 the typical dependences $d(z)$ are shown by the points. The solid lines in figure 2 are the linear regressions drawn through the experimental points. From the half angle of the inclination of the linear regressions graphs, we find the expansion angle of the vortex ring (because the expansion angle is related to the radius instead of the diameter).

![Figure 1. Air bubbles captured in the vortex_ring core.](image1)

![Figure 2. The external transverse size $d$ of the region occupied with bubbles in dependence on the distance $z$ travelled by the vortex. $V = 7.5$ cm$^3$, Re$_1$ = 5180 (1), 6480 (2) and 7520 (3).](image2)

Using equation (6) we can estimate a minimum volume $V_m$ of air bubbles captured by the vortex which is needed for influence of lifting force will be essential.

The right side of (6) is monotonically increasing function on $\text{Re}_1$. Then the minimum volume will be achieved at minimum Reynolds number. A minimum Reynolds number at which bubbles are held in the vortex core according to experiments is approximately equal to 3500. Substituting in (6) $F = \rho g V$, $\text{Re}_1 = 3500$ we obtain $V_m \approx 0.3$ cm$^3$. During the calculation we put $\nu = 10^{-2}$ cm$^2$/s, $g = 980$ cm/s$^2$, $\beta = 0.3$. 


In the experiments both kinds of vortices are investigated, with small buoyancy, \( V = 0.3 \text{cm}^3 \) and large buoyancy \(- V = 7.5 \text{cm}^3\); Reynolds numbers are varied from 4200 to 28500. For vortices with small buoyancy parameter \( k \) is varied from 0.4 to 0.04, with large buoyancy \( k \) is varied from 140 to 70. In figure 3 by the points it is shown expansion coefficients of vortices with small buoyancy, in figure 4 – expansion coefficients of vortices with small and large buoyancy. The solid line in the figures is empirical dependence (4) of expansion coefficient of vortices in homogeneous liquid, taken from [3].

![Figure 3](image1.png)

**Figure 3.** 1 – expansion coefficients of buoyancy vortex ring containing in the core air of volume \( V = 0.3 \text{cm}^3 \) and 2 – empirical dependence (4).

![Figure 4](image2.png)

**Figure 4.** 1 – expansion coefficients of buoyancy vortex ring containing in the core air of volume \( V = 0.3 \text{cm}^3 \), 2 – of volume \( V = 7.5 \text{cm}^3 \) and 3 – empirical dependence (4).

### 4. Analysis of the results and conclusions.

From figures 3 and 4 it follows that for buoyancy vortex ring for which parameter \( k \gg 1 \), also \( \alpha_1 \gg \alpha_* \). For buoyancy vortex ring for which parameter \( k \) is small compared 1 expansion coefficient is distinguished weakly from \( \alpha_* \) – expansion coefficient of vortex in homogeneous liquid (figure 3). From this we conclude that parameter \( k \) can be considered as criterion characterizing lifting force influence on the motion of buoyancy vortex ring.

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