I. BRANES, SUPERGRAVITY AND SUPERSPACE

I.1 $p$-branes: $D = 11$, $p = 2$

Branes play important roles in string theory and M-theory. They are non-perturbative objects that may be described as solitons of the low-energy effective supergravity theories (see refs. [1] and [2] for extensive reviews). Here, I will concentrate on the dynamics of branes, as described by their actions [3, 4, 5, 6, 7]. There are a number of different branes in string theory and M-theory, most conveniently characterised by their field content when seen as a field theory on the world-volume. The simplest ones, the so-called $p$-branes, have a scalar multiplet on the world-volume. D-branes contain a vector multiplet, coupling to string endpoints [8], and the M5-brane has a self-dual tensor.

As a model for the simplest branes I will treat the membrane in eleven dimensions [4]. The action for a brane typically consists of two parts, a kinetic term proportional to the invariant volume, and a Wess–Zumino term specifying the minimal coupling to a $(p + 1)$-form potential, under which the brane carries charge:

$$S = -T \int d^3 \xi \sqrt{-|g|} + T \int C .$$

(1)

$T$ is the membrane tension. The action (1) looks like an action describing bosonic degrees of freedom contained in its transverse fluctuations. How do we describe a supersymmetric brane, containing an equal number of fermionic degrees of freedom? One simple and very efficient way, in many aspects much simpler than a component approach, is to consider the dynamics to be described by the same formal action, but
where the bosonic world-volume is embedded in superspace\(^1\). The target superspace has coordinates \(Z^M = (X^m, \theta^\alpha)\) (the corresponding inertial indices are \(A = (a, \alpha)\), a Lorentz vector and some spinor), and the background fields entering the action (1) are pullbacks from superspace to the world-volume, \(g_{ij} = E_i^a E_j^b \eta_{ab}\), \(C_{ijk} = E_k^c E_i^B E_j^A C_{ABC}\), with \(E_i^A = \partial_i Z^M E^A_M\), \(E^A_M\) being the target space super-vielbein.

Let us now investigate what this means for the supermembrane. The eight transverse bosonic oscillations must be matched in number by eight fermionic degrees of freedom if the action is to be supersymmetric. A (Majorana) spinor in \(D = 11\) has 32 components. The number is reduced by half by the equations of motion, as usual, but it is clear that an additional local symmetry is required in order to get eight physical spinor degrees of freedom. This is the so called \(\kappa\)-symmetry, parametrised by a half spinor. \(\kappa\)-symmetry is a local (in terms of the location on the brane) translation of the brane in a fermionic direction in superspace. As such, it is generated by a superspace vector field pointing in fermionic directions only:

\[
\kappa_M \partial_M = \kappa_\alpha E^\alpha M \partial_M,
\]

The transformation of the coordinates is

\[
\delta_\kappa Z^M = \kappa^i P^{ij} \, \Theta_j.
\]

Pullbacks of superspace forms are transformed by the Lie derivative,

\[
\delta_\kappa f^* \Omega = f^* L_\kappa \Omega = f^* (i\kappa d + di\kappa) \Omega,
\]

which after a brief calculation implies

\[
\delta_\kappa C = i\kappa H + di\kappa C,
\]

\[
\delta_\kappa E^A = D\kappa^A + i\kappa T^A,
\]

\[
\delta_\kappa g_{ij} = 2E_i^a E_j^b \kappa^\alpha T^{\alpha \beta} \eta_{\alpha \beta}.
\]

where \(H = dC\) is the background tensor superfield strength and \(T^A = DE^A\) the super-space torsion.

To determine how the action transforms (modulo boundary terms), we only need \(i\kappa H\) and \(i\kappa T^a\). In \(D = 11\) supergravity \([9, 10]\) this is particularly simple, the only non-vanishing components of \(H\) and \(T^a\) with at least one spinorial form-index are the dimension 0 ones, \(T^{\alpha \beta} = 2\Gamma^{\alpha \beta}_a\), \(H_{\alpha \beta} = 2(\Gamma_{ab})_{\alpha \beta}\). A short calculation yields

\[
\delta_\kappa S = - \int d^3\xi \sqrt{-g} E_i^\alpha (\Gamma^i - \frac{1}{2\sqrt{-g}} e^{ijk} \Gamma_{jk}) \kappa^\alpha \kappa^\beta.
\]

with the obvious notation for pullbacks of \(\Gamma\)-matrices. The combination of \(\Gamma\)-matrices in the last term may be written as

\[
\Gamma^i - \frac{1}{2\sqrt{-g}} e^{ijk} \Gamma_{jk} = \Gamma^i (\mathbb{I} - \frac{1}{6\sqrt{-g}} e^{ijk} \Gamma_{ijk}) = \Gamma^i (\mathbb{I} - \Gamma),
\]

and is seen to provide a projection on \(\kappa\), since \(\Pi_\pm = \frac{1}{2}(\mathbb{I} \pm \Gamma)\), due to the identities \(\Gamma^2 = \mathbb{I}\) and \(\text{tr}\Gamma = 0\), are projection matrices splitting a 32-component spinor in two halves. The only chance that this variation vanishes is thus that \(\Pi_- \kappa = 0\). This is indeed the half spinor of local fermionic symmetry that was needed for the matching of bosonic and fermionic degrees of freedom. Since setting the dimension 0 torsion to a \(\Gamma\)-matrix

\[\text{There exists a framework, the so called embedding formalism, where both target space and the world-volume are superspaces [6]. I will not consider it here.}\]
puts the background on shell [11], the supermembrane has \( \kappa \)-symmetry in any on-shell background of \( D = 11 \) supergravity.

Analogous calculations hold for other \( p \)-branes in other supergravities, and show that for general on-shell backgrounds, the actions are \( \kappa \)-symmetric. \( \kappa \)-symmetry is related to the fact that the branes are BPS-saturated configurations—the supersymmetry algebra generating the multiplets on the branes (in the present case a scalar multiplet) contains half the number of fermionic generators compared to the target space supersymmetry, and half of the target space supersymmetry is broken (the world-volume fields are Goldstone fields corresponding to broken symmetries of the background). The projection matrices are related to (target space) supersymmetry algebras with “central” tensorial charges, that get projected out by a half-rank projection \( \Pi \).

We may also note that the formalism presented here, with the brane embedded in an arbitrary target superspace background, actually is as simple as in a flat superspace. Working with explicit fermionic coordinates becomes complicated, since the expression for a tensor potential is complicated, while the (gauge invariant) field strength is simple.

As presented here, the branes are viewed as infinitely thin objects moving in superspace. They may also be seen as solitons in the low-energy effective supergravity theories [12]. All fields on branes arise as Goldstone modes corresponding to broken symmetries of the background theory. Scalars and fermions correspond to broken translational symmetries and supersymmetry, while vectors on D-branes and tensors on M5-branes arise as Goldstone modes for large gauge symmetries of target space tensors, i.e., gauge transformations that take different values “on the brane” and in the asymptotic region [13].

### I.2 \( D = 11 \) supergravity

The \( \Gamma \)-matrix constraint on the dimension 0 torsion puts the theory on shell [11]. The tensor field arises naturally from the superspace geometry, and it is not necessary to separately require the existence of a closed 4-form on superspace. The Bianchi identity for \( H \) at dimension 0 becomes \( 0 = (dH)_{\alpha \beta \gamma \delta} = 6T_{\alpha \beta \gamma |\delta} = 24\Gamma_{\alpha \beta \gamma |\delta} \), which is fulfilled due to a Fierz identity in eleven dimensions, and at dimension 1, the non-vanishing torsion is

\[
T_{\alpha \beta} = \frac{1}{36} \Gamma^{bcd}_{\alpha \beta} H_{abcd} + \frac{1}{288} \Gamma^{b c d e}_{\alpha \beta} H_{b c d e} .
\tag{7}
\]

Actually, the superspace Bianchi identities also leave room for a spinor \( \omega_\alpha \) at dimension \( \frac{1}{2} \) and a vector \( \omega_a \) at dimension 1 in \( T \), the Bianchi identities further require that these be integrable to \( \omega_A = D_A \phi \), and the “conformal compensator” \( \phi \) can then be removed by a conventional constraint, or alternatively by the enlargement of the structure group to include Weyl rescalings\(^2\).

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\(^2\) There is a disagreement on this point. The view presented here is that of refs. [11, 14], while the authors of ref. [15] claim that the conformal compensator has to play a rôle in a (yet unknown) supersymmetric off-shell formulation of eleven-dimensional supergravity.
I.3 D-branes, type II supergravity

Type II superstring theories, and their low-energy effective theories, type IIA and IIB supergravity, contain tensor fields in the Ramond-Ramond sector. For type IIA the potentials have odd rank, $C = C(1) \oplus C(3) \oplus C(5) \oplus \ldots$, and for type IIB even, $C = C(0) \oplus C(2) \oplus C(4) \oplus \ldots$. The corresponding field strengths are required to be self-dual, so in principle we have a redundant set of potentials, which is useful when considering brane actions. A five-brane, e.g., couples minimally to a 6-form potential, whose 7-form field strength is dual to the 3-form. In addition there is the the NS-NS 2-form $B$.

D-branes are exactly the non-perturbative objects carrying charge under the RR fields. They act as hypersurfaces where fundamental strings are allowed to end, and contain vector degrees of freedom, coupling minimally to the world-lines of the string ends [8]. This picture resulted in an effective action for D-branes [16, 17]:

$$S = -\int d^{p+1} \xi e^{-\phi} \sqrt{-|g_s + F|} + \int e^F C$$

(8)

There are some things to explain in this expression. The field $\phi$ is the dilaton field, and the factor $\exp(-\phi)$ means that the D-brane tension in the “string frame” is proportional to $g^{-1}$, where $g = \exp(\phi)$ is the string coupling. In the second line, the action has been rewritten in terms of the Einstein metric $g_E = \exp(\phi/2)g_s$, which is sometimes convenient, especially when I later want to consider SL(2; $\mathbb{Z}$) duality symmetry. The second, Wess–Zumino, term in the action is evaluated with wedge products, and the $(p+1)$-form is extracted, so that for the D3-brane, e.g., it reads $\int (C(4) + F \wedge C(2) + \frac{1}{2} F \wedge F \wedge C(0))$. There is a U(1) vector field $A$ on the world-volume, and the field strength $F$ contains the NS-NS 2-form potential $B$ through $F = dA - B$. The gauge transformations of $B$, $\delta_B A = d\lambda$, also act on $A$ as $\delta A = \lambda$, so that $F$ is invariant. This means that an expectation value for $F$ can be traded for a background $B$ field. Apart from the dilaton factor, the first, kinetic, term is of Dirac–Born–Infeld type.

The RR tensors have “modified” field strengths $R = e^B d(e^{-B} C) = dC - H \wedge C$, and their Bianchi identities read $dR + H \wedge R = 0$. Gauge transformations $\delta_C C = e^B d\lambda$ leave the WZ term invariant up to a total derivative.

As was done for the membrane in the previous section, the D-brane actions (8) are promoted to actions for supersymmetric D-branes by letting the the embedding be in a target superspace of type IIA or IIB. Superspace formulations of the type IIA and type IIB supergravities are given in refs. [19] and [20].

The essential check is again $\kappa$-symmetry. The calculations are somewhat more complicated than for a brane with a scalar multiplet, so I refer to ref. [7] for more details. In type IIB, the two spinor coordinates have the same chirality, and instead of introducing

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3 This implies that such configurations do not break supersymmetry, which can be seen in the supergravity solutions corresponding to D-branes with constant $F$, or equivalently, in a background $B$ field [18]. What instead happens is that the part of supersymmetry remaining unbroken is a different projection than for $F = 0$, due to the $F$-dependence of the projector (13).
explicit indices, I include this in a IIB spinor index \( \alpha = 1, \ldots, 32 \), and introduce the basis of \( 2 \times 2 \)-matrices \( \{ \mathbb{1}, I, J, K \} \) with \( I = i\sigma_2, J = \sigma_1, K = \sigma_3 \) (they can be seen as a basis for the split quaternions). The the relevant fields at dimension 0 are

\[
T_{\alpha\beta}^a = 2\Gamma_{\alpha\beta}^a, \tag{9}
\]

\[
H_{\alpha\beta} = -2e^{\frac{\phi}{2}}(\Gamma_{a}K)_{\alpha\beta}, \tag{10}
\]

\[
R_{a_1 \ldots a_{n-2}\alpha\beta} = 2e^{\frac{\phi}{2}}(\Gamma_{a_1 \ldots a_{n-2}}K_{a}l_{\alpha\beta}I). \tag{11}
\]

Since the ten-dimensional supergraviqeties have spinors, \( \lambda_{\alpha} = D_{\alpha}\phi \) of dimension \( \frac{1}{2} \), these will also occur in the dimensions \( \frac{1}{2} \) components of \( H, R \) and \( T \), which I do not list here.

The variation of the action requires that we specify the transformation of the vector \( A \). In order to get something gauge invariant we must take \( \delta_{\kappa}A = i_{\kappa}B \), which implies that \( \delta_{\kappa}F = -i_{\kappa}H \). Then the variation of the WZ term becomes \( \delta_{\kappa}(e^{F}C) = e^{F}i_{\kappa}R \) modulo boundary terms. Going through the procedure of inserting the variations of the fields in the lagrangian yields an expression

\[
\delta_{\kappa}L \propto E_{i}^{\alpha} \Gamma^{i}(\mathbb{1} - \Gamma)\kappa^{\alpha}, \tag{12}
\]

where \( \Gamma \) is a more complicated expression than eq. (6), containing different powers of the field strength \( F \), and thus providing field-dependent half-rank projections of a spinor.

For the D3-brane, it takes the form

\[
\Gamma = \frac{e^{ijkl}}{\sqrt{|g + e^{-\phi/2}F|}}\left( -\frac{1}{2}e^{-\phi/2}F_{ij} \Gamma_{kl}I + \frac{1}{8}e^{-\phi}e^{ij}F_{ij}F_{kl}I \right), \tag{13}
\]

and similar expressions hold for other D-branes. This shows \( \kappa \)-symmetry of the D-branes in an arbitrary on-shell supergravity background.

### I.4 SL(2; \mathbb{Z}), tensor democracy

Type IIB supergravity has an SL(2; \mathbb{R}) symmetry, which at the quantum level is broken to the SL(2; \mathbb{Z}) S-duality group. Since S-duality is non-perturbative, the representations under SL(2; \mathbb{Z}) contain perturbative and non-perturbative states, and can not be manifested in perturbative string theory. Nevertheless, it can be manifested in the effective supergravity, and it is meaningful to ask whether it is possible to treat all branes with NS-NS and RR charges, including the fundamental string, on an equal footing, thus manifesting the S-duality symmetry.

The scalars, the dilaton and axion, belong to the coset SL(2; \mathbb{R})/U(1). This is a combination of NS-NS and RR fields, since the axion is identified with \( C_{(0)} \). The NS-NS and RR 2-forms \( B_{(2)} \) and \( C_{(2)} \) combine into an SL(2) doublet, and the 4-form (with selfdual field strength) is an SL(2) singlet. Higher rank tensors are dual to those already mentioned. The representations of branes reflect those of the tensor fields: The strings and five-branes come with charges that form an SL(2; \mathbb{Z}) doublet \((p,q)\), while the D3-brane forms a singlet. I will not examine higher-dimensional branes here.
The scalars of type IIB supergravity are described as a complex doublet $U^r$, $r = 1, 2$, subject to the constraint $\frac{i}{2} \epsilon_{rs} U^r \bar{U}^s = 1$ (which is the condition that $U$ has unit determinant when seen as a real $2 \times 2$ matrix). The coset is obtained from gauging the U(1) acting as $U \rightarrow e^{i \theta} U$. One forms the left-invariant Maurer–Cartan forms

$$Q = \frac{1}{2} \epsilon_{rs} dU^r \bar{U}^s,$$

$$P = \frac{1}{2} \epsilon_{rs} dU^r U^s,$$

with Bianchi identities (Maurer–Cartan equations)

$$dQ - iP \wedge \bar{P} = 0,$$

$$DP = dP - 2iP \wedge Q = 0.$$

The scalars act as a bridge between objects that are SL(2) doublets and real objects that are SL(2) singlets but carry U(1) charge. If we write the 3-form doublet of the field strengths as $H_{(3)} = dC_{(2)}$, the SL(2) singlet field strength is $\mathcal{H}_{(3)} = U^r H_{(3)r}$. Notice that this is necessary when writing a kinetic term as proportional to $H \cdot \bar{H}$. The Bianchi identity is $D \mathcal{H} + i \mathcal{H} \wedge P = 0$ (recall that $\mathcal{H}$ has U(1) charge 1 while $P$ has charge 2). The singlet 5-form is constructed as $H_{(5)} = dC_{(4)} + \text{Im}(\mathcal{C}_{(2)} \wedge \mathcal{H}_{(3)})$, with Bianchi identity $dH_{(5)} - i \mathcal{H}_{(3)} \wedge \mathcal{H}_{(3)} = 0$.

We now come to the crucial point in describing brane dynamics SL(2)-covariantly. It is not sufficient to introduce one vector field on the brane. Remember that the field strength was $F = dA - B$, where $B$ was the NS-NS 2-form. It is clear that another vector, combining with the RR 2-form is needed, so that they form a doublet. One should thus have $F_r = dA_r - C_{(2)r}$, reflecting the fact that strings of different charges $(p, q)$ can end on a brane. Once this step has been taken, it is equally natural to introduce a form of rank $p$ for each background tensor fields of rank $p + 1$, reflecting the fact that a $p$-brane can end on the brane we describe, and coupling minimally to its boundary. For this reason, such a formulation, with complete “tensor democracy” on the branes, should most naturally encode the coupling of branes to background fields. Gauge invariance (in target space and on the brane) demands that also the tensors on the brane have modified Bianchi identities. The 2-form and 4-form on any brane are

$$\mathcal{F}_{(2)} = U^r dA_{(1)r} - C_{(2)},$$

$$F_{(4)} = dA_{(3)} - C_{(4)} + \text{Im}(\mathcal{A}_{(1)} \wedge \mathcal{H}_{(3)}),$$

with Bianchi identities

$$D \mathcal{F}_{(2)} + i \mathcal{F}_{(2)} \wedge P = -\mathcal{H}_{(3)},$$

$$dF_{(4)} = -H_{(5)} - \text{Im}(\mathcal{F}_{(2)} \wedge \mathcal{H}_{(3)}).$$

In a generic situation, the procedure seems to give too many bosonic fields, and there must be ways to reduce the number in order to recover an SL(2)-covariant description of brane dynamics. The key is selfduality, and I will sketch how it works for different branes. I refer the readers to refs. [21, 22] for details. All cases described may be shown to be $\kappa$-symmetric, along similar lines as in the previous sections. The actions do not divide into Born–Infeld plus Wess–Zumino, since this presumes a division into NS-NS and RR fields.
The \((p,q)\) strings. The vectors \(A_r\) have no local degrees of freedom on the two-dimensional world-sheet, so we do not have to worry about removing degrees of freedom. The only degrees of freedom of vectors is a quantised electric flux (see ref. [23] for one vector), so the description given rises to a pair of integers \((p,q)\), which are the charges of string. In this way, the whole spectrum of \((p,q)\) strings is described within one single action [21]. That the description is correct is checked by \(\kappa\)-symmetry and by the fact that the correct tensions [24] are produced.

The 3-brane. Having two vector potentials gives too many degrees of freedom, and one of them has effectively to be removed. This is obtained by imposing a selfduality relation on the complex field strength \(\mathcal{F}: \mathcal{F} = i \ast \mathcal{F} + \text{higher order terms}\). It turns out that not any non-linear selfduality relation is allowed. Its exact form is dictated by consistency with the coupling to the background fields, and also, independently, by \(\kappa\)-symmetry, and it encodes in a manifest way the earlier observed Poincaré selfduality of the 3-brane. A formulation of the dynamics of the type IIB 3-brane is obtained [22] that naturally encodes in a most symmetric way all couplings to background fields, and thereby the possibilities for the 3-brane to host brane boundaries [22].

The \((p,q)\) 5-branes. This case is not constructed in detail, but the general scheme is described in ref. [22]. There is a duality relation between the 4-form and the 2-forms. The fact that the corresponding supergravity solution could be described analytically [25] makes it reasonable to believe that the dynamics can be described covariantly, in spite of problems with dualisation in six dimensions [26].

The M5-brane and type IIA. The formalism is not restricted to type IIB. It was successfully applied to write down a “quasi-action” (the equations of motion follows, but not the selfduality, which however is uniquely determined by consistency with background couplings and by \(\kappa\)-symmetry) for the M5-brane [27]. It is also applicable to type IIA branes, and will also there encode the background interactions in the most natural way.

I.5 Summary

I have described brane dynamics by embedding in superspace, given a detailed account of the mechanisms behind \(\kappa\)-symmetry and focussed on the couplings of branes to fields in the background effective supergravity.

It is known that the effective supergravity theories following from string theory or M-theory receive corrections to higher order in \(\alpha'\) than the lowest order ones used in this talk. Some \(\alpha'\)-corrections to the brane actions themselves are also known [28, 29]. What happens to the brane dynamics when \(\alpha'\) corrections are turned in in target space? It is clear that a superspace formulation is desirable in order to answer such questions. In the following lecture I will describe some recent progress in the superspace formulations of \(D = 11\) supergravity and \(D = 10\) super-Yang–Mills theory, both relevant for string/M-theory.
II. STRING/M-THEORY CORRECTIONS TO SUPERGRAVITY
AND SUPER-YANG–MILLS

II.1 $D = 11$ supergravity cont’d

We will now continue the discussion of the superspace formulation of eleven-dimensional supergravity in superspace [10, 11, 14]. The vielbeins are $E^A = dZ^M E_M^A$, and the resulting torsion 2-form is $T^A = DE^A = dE^A + E^B \wedge \omega_B^A$, where the structure group is the Lorentz group, i.e., the spin connection satisfies $\omega_{\alpha \beta}^{\gamma} = \frac{1}{4}(\Gamma_{\alpha \beta})^{\gamma}_{\gamma} \omega_{\gamma}^a$. The curvature is $R^A_B = d\omega^A_B + \omega^A_C \wedge \omega^C_B$. The Bianchi identities for torsion and curvature are $DT^A = E^B \wedge R^A_B$ and $D R^A_B = 0$. Of these, one needs only to use the first one.

As long as torsion and curvature are constructed from vielbeins and spin connections, the Bianchi identities are automatically fulfilled. In order to reduce the enormous amount of fields contained in these, one has however to impose “conventional constraints” connecting the different components. Then the Bianchi identities become integrability conditions that have to be checked, and which imply the equations of motion (this is true for the maximally supersymmetric theories I deal with in this lecture).

The conventional constraints are of two types. The first one uses the freedom in the definition of the torsion to shift it into the spin connection when possible. These constraints do not eliminate all of the torsion (as it does in bosonic gravity), but have the effect of determining the spin connection if terms of the vielbein, which is desirable.

The second type uses a redefinition of the tangent bundle, $E^A \rightarrow E^B M^A_B$, while keeping the spin connection, and thus the curvature, invariant (although their components vary due to the change of basis). We want to use this freedom to the extent that it enables us to express all vielbein components in terms of the dimension $-\frac{1}{2}$ one, $E_{\mu}^a$.

Let us now examine the lowest-dimensional torsion components, $T_{\alpha \beta}^{\gamma}$, at dimension 0. I already mentioned that putting it equal to a $\Gamma$-matrix takes the theory on-shell, so in order to incorporate corrections to the ordinary supergravity this constraint (which is not a conventional constraint) has to be modified. A general expansion yields, since the torsion is symmetric in the spinor indices,

$$ T_{\alpha \beta}^{\gamma} = 2(\Gamma_{\alpha \beta}^{\gamma} X_d^c + \frac{1}{2!} \Gamma^{d_1 d_2}_{\alpha \beta} X_{d_1 d_2}^c + \frac{1}{3!} \Gamma^{d_1 ... d_5}_{\alpha \beta} X_{d_1 ... d_5}^c) $$

Decomposing into irreducible representations of the Lorentz group, we find that the three “$X$-tensors” contain $((20000) \oplus (01000) \oplus (00000)) \oplus ((11000) \oplus (00100) \oplus (10000)) \oplus ((10002) \oplus (00002) \oplus (00010))$, where standard Dynkin labels for highest weights are used.

If this representation content is compared to the one in the dimension-0 matrices for redefining the vielbein, $M_a^{\mu}$ and $M_{\alpha}^{\beta}$, which is $((20000) \oplus (01000) \oplus (00000)) \oplus ((00002) \oplus (00010) \oplus (00100) \oplus (01000) \oplus (10000) \oplus (00000))$, we see that the only remaining components are $X_{d_1 d_2}^{c \left| (11000) \right.}$ and $X_{d_1 ... d_5}^{c \left| (10002) \right.}$, i.e., the “irreducible hooks” [30, 14]. These superfields should encode which the corrections to the supergravity are, and the equations of motion for any version of $D = 11$ supergravity should follow from the solution of the Bianchi identities with a suitable choice of these tensors. This will be even clearer when we consider spinorial cohomology in a little while.
Solving the Bianchi identities turns out to be quite complicated, and we have not succeeded in doing it in full generality. In ref. [14], we were able to show that the gravitino equation of motion received a correction, by solving the Bianchi identities up to dimension \(3^2\), encountering on the way some remarkable numerical coincidences. We found no contribution to the Weyl curvatures up to this level, which means that the elimination of the conformal compensator by a conventional constraint is still valid.

II.2 \(D = 10\) super-Yang–Mills

The study of the general superspace formulation of \(D = 10\) super-Yang–Mills is motivated by its connection to string theory and the relevance for finding non-abelian analogues of the Born–Infeld action [31, 32]. An advantage with the system is that it is much easier to analyse than \(D = 11\) supergravity, so we hoped that it would be more manageable.

I now work in flat superspace, with \(T_{\alpha\beta} = 2\Gamma_{\alpha\beta}^a \) and the rest of the torsion vanishing. The gauge potential is a superspace 1-form with components \(A_A = (A_a, A_\alpha)\), and the field strength is \(F = dA + A \wedge A\) with Bianchi identity \(DF = 0\). In components, the Bianchi identity reads:

\[
\text{dim. } 3^2 : \quad D(\alpha F_{\beta\gamma}) + 2\Gamma_{(\alpha\beta} F_{c|\gamma)} = 0 ,
\]

\[
2 : \quad 2D_{(\alpha F_{\beta\gamma})} + D_{c} F_{\alpha\beta} + 2\Gamma_{\alpha\beta} F_{dc} = 0 ,
\]

\[
\frac{5}{2} : \quad D_{\alpha} F_{bc} + 2D_{[b} F_{c]} = 0 ,
\]

\[
3 : \quad D_{[a} F_{bc]} = 0
\]

(23)

(24)

(25)

(26)

Taking \(F_{\alpha\beta} = 0\) puts the theory on-shell [33], and it must be relaxed if we want to incorporate corrections. The general expansion is

\[
F_{\alpha\beta} = \Gamma_{\alpha\beta}^a J_a + \frac{1}{5!} \Gamma_{\alpha\beta}^{a_1...a_5} J_{a_1...a_5} .
\]

(27)

The vector can always be set to zero as a conventional constraint, to eliminate the “extra” vector potential occurring at \(\theta\)-level in \(A_\alpha\). We then have \(A_a = -\frac{1}{2\Gamma} \Gamma_{\alpha}^a D_\alpha A_\beta\) (in the abelian case). The relevant deformation lies in the five-form, which is automatically (anti-)selfdual, due to the chirality of the spinors. In reference [34] we were able to solve the Bianchi identities completely for arbitrary \(J\), whose components act as a super-current multiplet, and obtain the equations of motion,

\[
0 = D^b F_{ab} - \lambda \gamma \lambda - 8D^b K_{ab} + 36w_a - \frac{4}{3} \{\lambda, \tilde{J}_a\} - 2\bar{J}_b \Gamma_{a} F_{b} + \frac{1}{140\cdot 3!} \bar{J}_{bcd} \Gamma_{a} \tilde{J}_{bcd} + \frac{1}{4524} \{D_f J_{fhde}, J_{a}^{bcd}\} + \frac{1}{42} \cdot \frac{1}{4!} D_f J_{fhde}, J_{a}^{bcd}\) ,
\]

(28)

\[
0 = D^b \lambda - 30\psi + \frac{4}{3} D^a \tilde{J}_a + \frac{5}{128\cdot 3!} \Gamma^{abcd} \tilde{J}_a J_{abcd} .
\]

Apart from \(F_{ab}\) and \(\lambda^a\) (which appears in the field strength as \(\lambda^a = \frac{1}{10} \Gamma^{a\alpha\beta} F_{a\beta}\)), the quantities appearing in these equations all arise, as explained in ref. [34], in the \(\theta\)
expansion of \( J_{abcde} \), \( \vec{J} \)'s at first, \( K \)'s at second, \( \psi \) at third, and \( \omega \) at fourth order in \( \theta \). Explicitly, their precise relations to \( J_{abcde} \) are given by

\[
\vec{J}_a = \frac{1}{1680} \Gamma^{bcde} DJ_{bcdea},
\]

\[
\vec{J}_{abc} = -\frac{1}{12} \Gamma^{de} DJ_{deabc} - \frac{1}{224} \Gamma_{[ab}^{fdefg} DJ_{fde[|c]e]},
\]

\[
\vec{J}_{abcde} = DJ_{abcde} + \frac{5}{6} \Gamma_{[ab}^{f} \Gamma^{fghi} DJ_{ghi|cde]} + \frac{1}{24} \Gamma_{[abcd}^{fghi} DJ_{fghi|e]},
\]

\[
K_{ab} = \frac{1}{3376} (D\Gamma^{cde} D) J_{cdeab},
\]

\[
K_{abcd} = \frac{1}{480} (D\Gamma_{a}^{fg} D) J_{fg[|bcd|]},
\]

\[
\psi_\alpha = -\frac{1}{840} \cdot \frac{3! \cdot 5!}{5!} \Gamma_{abc}^{\beta\gamma} \Gamma_{de}^{\delta} D_{[\beta} D_{\gamma} D_{\delta]} J_{abcde},
\]

and finally

\[
w_a = \frac{1}{4032} \cdot \frac{4! \cdot 5!}{4!} \Gamma_{abc}^{[ab} \Gamma_{def}^{\delta} D_{\alpha} D_{\beta} D_{\gamma} D_{\delta]} J_{bcdef}.\]

I will soon show how one may use this formalism to deduce possible forms of \( \alpha' \)-corrections allowed by supersymmetry. The idea is thus to take advantage of the fact (normally considered as a drawback) that the superspace formulation takes the theory on-shell.

II.3 Fields and deformations from spinorial cohomology

Before becoming more specific about string-related corrections to super-Yang–Mills theory, I would like to digress on an amusing mathematical structure that has something to tell about maximally supersymmetric theories.

The basic idea is that the theories we consider are gauge theories, and that, in a superspace formulation, where all potentials and field strengths are forms on superspace, all components except the purely spinorial ones are redundant. Since all physical fields are contained in the objects carrying spinorial form indices only, it is interesting to examine the structure arising from these. Our complexes are of the form

\[
r_0 \xrightarrow{\Delta_0} r_1 \xrightarrow{\Delta_1} r_2 \xrightarrow{\Delta_2} \ldots \xrightarrow{\Delta_{n-1}} r_n \xrightarrow{\Delta_n} \ldots,
\]

where \( r_p \), for some \( p \geq 0 \), is the representation carried by a gauge transformation, \( r_{p+1} \) that of a potential and \( r_{p+2} \) that of a field strength. I will refer to the representations \( r_n \) as \( n \)-forms, a notation not to be confused with that of a tensor antisymmetric in vector indices. The exact definitions are given, both for gauge theory and supergravity, in the following sections, where it will also be clear why \( \Delta \) is a nilpotent operator. The rôle of \( r_{p+3} \) is as a Bianchi identity.

Let me describe in more detail how the complexes work, with the super-Yang–Mills theory as an example. We have already seen that \( A_{a} \) contains the fields of the theory. The relevant part of the field strength, as argued above, lies in \((00020)^4\), and does

\[\text{I use standard Dynkin labels for SO(1,9)}.\]
not contain \( A_\alpha \). We also note [33, 34] that part of the dimension-\( 3 \) Bianchi identity states the vanishing of the (00030) component of \( D_\alpha F_{\beta\gamma} \). These observations make it natural to consider, not the sequence of completely symmetric representations in spinor indices, but a restriction of it, namely the sequence of Spin(1,9) representations \( r_n \equiv (00n0) \). They are the part of the totally symmetric product of \( n \) chiral spinors that has vanishing \( \Gamma \)-trace, and may be represented tensorially as \( C_{\alpha_1...\alpha_n} = C(\alpha_1...\alpha_n) \), \( \Gamma^\alpha_{\alpha_1\alpha_2} C_{\alpha_1\alpha_2\alpha_3...\alpha_n} = 0 \). For \( n = 2 \), \( C \) is an anti-selfdual five-form, for \( n = 3 \) a \( \Gamma \)-traceless anti-selfdual five-form spinor, etc.

The operator \( \Delta_n: r_n \rightarrow r_{n+1} \) can schematically be written as \( \Delta_n C_n = \Pi(r_{n+1}) D C_n \), where \( D \) is the exterior covariant derivative \( D = d\theta^\alpha D_\alpha \) and \( \Pi(r_n) \) is the algebraic projection from \( \otimes^n(0010) \) to \( (00n0) \). It is straightforward to write an explicit tensorial form for \( \Delta \) by subtracting \( \Gamma \)-traces from \( DC \), but it will not be used here. It is also straightforward to see that, for an abelian gauge group and standard flat superspace, the sequence \( (36) \) forms a complex, i.e., \( \Delta^2 = 0 \). This follows simply from the fact that while \( \{ D_\alpha, D_\beta \} = -\Pi T_{\alpha\beta} \), the torsion only has a component \( 2T_{\alpha\beta} C_{\alpha\beta} \) which is projected out by \( \Pi(r_n) \). This means that for non-abelian gauge theory the complex should be considered in a flat background, and the deformations yielded are infinitesimal.

We would now like to calculate the cohomology \( H^n = \text{Ker} \Delta_n / \text{Im} \Delta_{n-1} \) of the complex associated with \( D = 10 \) super-Yang–Mills. This can be done by considering the decomposition into irreducible representations of the representation sitting at level \( \ell \) in \( r_n, r'_\ell \equiv \wedge^\ell S \otimes r_n \). This is easily done, e.g. with the help of the program LiE [35]. One then follows each of the irreducible representations at a given dimension through the subcomplex

\[
\begin{align*}
r'_0 & \rightarrow r'_1 \rightarrow r'_2 \rightarrow \ldots \rightarrow r'_{\ell-1} \rightarrow r'_\ell.
\end{align*}
\]

Let me illustrate the calculation by examining the field content. We then look into the spinor potential of dimension \( \frac{3}{2} \), which contains all fields in the vector multiplet, so we should examine the first cohomology. The vector (dimension 1) sits at \( \ell = \frac{1}{2} \) and the spinor (dimension \( \frac{3}{2} \)) at \( \ell = 1 \). The subcomplexes under consideration are \( r'_0 \rightarrow r'_1 \rightarrow r'_2 \) and \( r'_3 \rightarrow r'_4 \rightarrow r'_5 \). Checking the multiplicities of the relevant representations, (10000) and (00001), in these, we obtain the sequences \( 0 \rightarrow 1 \rightarrow 0 \) and \( 0 \rightarrow 1 \rightarrow 0 \rightarrow 0 \). The components of the cohomology in these representations and dimensions clearly contain the physical fields. This can be understood in a traditional framework as removing degrees of freedom in a superfield gauge transformation (removing the image from the left) and imposing the vanishing of the field strength \( F_{\alpha\beta} \) (removing the complement of the kernel from the right). Analogous considerations tell us that the second cohomology contains a spinor of dimension \( \frac{5}{2} \) and a vector of dimension 3. These are interpreted as belonging to a current supermultiplet, i.e., fields entering the right hand sides of the equations of motion. This goes well together with the observation that modifications of the theory are introduced by deforming the constraint \( F_{\alpha\beta} = 0 \) [33, 36, 34, 37]. The relevance of the cohomology is explained by the facts that deformations introduced by relaxing \( F_{\alpha\beta} = 0 \) have to fulfill the Bianchi identity (removing the complement of the kernel from the right), and that relevant deformations are counted modulo field redefinitions (removing the image from the left). See also the following section for a fuller discussion.
A complete calculation of the cohomology requires that one considers all irreducible representations occurring at arbitrary levels. This quickly becomes untractable to do by hand. The method for calculating cohomologies is by using the program LiE [35]. The method will be presented in detail in a forthcoming publication [38]. The complete cohomology consists of

\[ \mathcal{H}^0 = (00000)_0 \quad \text{(gauge transformations)} \quad (37) \]
\[ \mathcal{H}^1 = (10000)_1 \oplus (00001)_{3/2} \quad \text{(fields)} \quad (38) \]
\[ \mathcal{H}^2 = (00100)_{5/2} \oplus (10000)_3 \quad \text{(deformations)} \quad (39) \]
\[ \mathcal{H}^3 = (00000)_4 \quad (?) \quad (40) \]

where the subscript indicates dimension.

Similar cohomologies may be calculated for the \( D = 11 \) supergravity [38], and they confirm in a nice way the conclusions presented earlier in this lecture. An interesting observation is that one can choose either to consider the vielbein or the 3-form, and in either case are all the fields and deformations of the supergravity contained. It looks as though a superspace 3-form potential automatically contains gravitational degrees of freedom, although it is difficult to envisage how the dynamics should be formulated without reference to geometry.

II.4 \( F^4 \) terms

I would like to sketch how the superspace methods already described are used to derive \( \alpha' \)-corrections to \( D = 10 \) super-Yang–Mills. The method for \( D = 11 \) supergravity is in principle analogous, but much more complicated. So far, the corrections allowed by supersymmetry have been determined up to order \( \alpha'^2 \) [37], and although the level of technical complexity is high, it seems reasonable to continue one or two levels.

We need to specify what \( J_{abcde} \) is in terms of the fundamental superfields \( F \) and \( \lambda \). We first observe that there are no corrections at order \( \alpha' \). For dimensional reasons, \( F_{\alpha\beta} \) has to be proportional to \( \lambda^2 \), which does not contain the representation (00020). Then, starting at order \( \alpha'^2 \), there are two types of possible terms, modulo the lowest order field equations (\( A, B, \ldots \) are adjoint gauge group indices, not to be confused with \( A = (a, \alpha) \) used earlier):

\[
J^A_{abcde} = -\frac{1}{2} \alpha'^2 M^A_{BCD} (\lambda^B \Gamma^f \Gamma_{abcde} \Gamma^g \lambda^C) F^D_{fg} \\
+ \frac{1}{6} \alpha'^2 N^A_{BC} \left( D_{[a} \lambda^B \Gamma_{bcd]} D_{e]} \lambda^C - \text{dual} \right). \quad (41)
\]

These satisfy the (00030) constraint at linear order, which is easily seen by acting with a spinor derivative and perform tensor multiplication of the representations of the fields. Here, \( M \) and \( N \) are some invariant tensors carrying adjoint indices of the gauge group.

Not all deformations in (00020) are relevant, as explained in the previous section. Those that are in the image of \( \Delta_1 \) correspond to field redefinitions of \( A_\alpha \) and are trivial. A careful examination of field redefinitions shows that only the first term in eq. (41)
is relevant, and the other can be discarded. In addition, $M_{ABCD}$ can be taken to be completely symmetric in adjoint indices.

A lengthy calculation gives the deformed equations of motion at order $\alpha'^2$ by acting with spinor derivatives on $J_{abcde}$, and inserting in eq. (28). These may subsequently be integrated to a component action, which reads

$$\mathcal{L} = -\frac{1}{4} G^{Aij} G^A_{ij} + \frac{1}{2} \chi^A D \chi^A$$

$$-6 \alpha'^2 M_{ABCD} \left[ \text{tr} G^A G^B G^C G^D - \frac{1}{4} (\text{tr} G^A G^B) (\text{tr} G^C G^D) \right]$$

$$-2 G^{Aik} G^{Bjk} (\chi^C \Gamma_{ik} D j \chi^D) + \frac{1}{2} G^{Ail} D_l G^{Bjk} (\chi^C \Gamma_{ijk} \chi^D)$$

$$+ \frac{1}{180} (\chi^A \Gamma^{ijk} \chi^B)(D_i \chi^C \Gamma_{ijk} D j \chi^D) + \frac{1}{10} (\chi^A \Gamma^{ijk} \chi^B)(D_i \chi^C \Gamma_{ijk} D j \chi^D)$$

$$+ \frac{7}{60} f^{DEF} G^{Aij} (\chi^B \Gamma_{ijk} \chi^C)(\chi^E \Gamma^k \chi^F)$$

$$- \frac{1}{360} f^{DEF} G^{Aijklm} (\chi^B \Gamma^{klm} \chi^C)(\chi^E \Gamma_{ijklm} \chi^F) + O(\alpha'^3).$$

The spinor $\lambda$ has been replaced by $\chi$ and $F$ by $G$, since there is a field redefinition involved in reaching this final form. It agrees with previous work [39] on previously known terms (up to quadratic in fermions).

With only a minor further restriction on $M$, the action has a second non-linearly realised supersymmetry when the gauge group has a U(1) factor, as is the case when one considers field theory on multiple branes. The “symmetrised trace prescription” of Tseytlin [31] is consistent with our results, but supersymmetry does not completely specify it, even at the $F^4$ level. It will of course be interesting to continue the analysis to higher orders. The (00030) Bianchi identity will necessarily lead to corrections at order $\alpha'^4$ and higher, and a complete action will be non-polynomial. It is not clear whether any closed, Born–Infeld-like form exists. It is even not known if new “invariants” arise that start at higher orders, or if everything follows uniquely once the $\alpha'^2$ correction is determined.

II.5 Branes? Conclusions

The properties of the maximally supersymmetric field theories we have considered have been turned into a tool for studying restrictions imposed by supersymmetry on self-interactions. Much more is to be done, both for super-Yang–Mills and supergravity, but it will be necessary to use computer programs, e.g. LiE [35] and the Mathematica package GAMMA [40], to a higher degree.

A question which so-far remains unaddressed is what happens to branes moving in backgrounds with $\alpha'$-corrections from string/M-theory. To investigate this one will need more informations about $\alpha'$-corrected supergravity. Will the actions still be formally the same, and the dynamics only change through the coupling to background fields? I would tend to answer in the positive, although nothing is known. One difficulty immediately presents itself, namely that the tensor field strengths will take non-zero values even for the components of negative dimension [14]. Since $\kappa$-symmetry relies on cancellations of contributions from the kinetic and WZ terms, the resulting variations would have no
contribution from the torsion to cancel against. One possibility is that also the condition that \( \kappa \) is purely spinorial is modified. One preliminary investigation would consist of checking \( \kappa \)-symmetry for supersymmetric Wilson loops [41] in a deformed super-Yang–Mills background.

ACKNOWLEDGMENTS

The author would like to thank the organisers of the XXXVII Karpacz Winter School for two very pleasant and inspiring weeks.

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