Signal Formation in Various Detectors

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August 18, 2014

Abstract

In this write up we present the general theory of the signal formation in various detectors. We follow a pedagogical analysis and presentation such that the results could easily understood and applied by the interested reader to the different detector configurations. We include few applications to gaseous detectors, namely, Monitored Drift Tubes (MDT) and microstrip pattern detector of the micromegas type.

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1 Signal formation in a small-size detector

Let us consider a detector with small dimensions such that wave propagation effects are not important. This means that for the larger dimension, $d_{\text{max}}$, of the detector we have $d_{\text{max}} \ll \lambda_{\text{min}} = c'/f_{\text{max}}$, where $f_{\text{max}}$ is the maximum frequency needed to describe the detector signal, $c'$ is the electromagnetic wave velocity in the detector.

The situation is semi-static. This means that even though there is time dependence, the time rates of change at any time are so small that the relations of electrostatics hold.

We will prove a generalization of the Shockley-Ramo theorem [1, 2]. This generalization is similar to the one described in reference [3]. The Shockley-Ramo theorem refers to signals induced on grounded conductors due to moving charges in the space between the conductors. In the generalization, the conductors is not needed to be grounded, they may have different potentials and they are connected to external circuit (network).

Let us consider the situation shown in Figure 1, there are $N$, internal, ideal conductors. The conductors are of finite extent. In addition, there is an external conductor $0$ surrounding all the internal conductors. This external conductor is held at a known reference potential which we assume to be zero. The external conductor can be considered to be extended, toalyl or partially, to infinity.

We examine the case when the space between the conductors contains a linear dielectric material with permittivity (dielectric function) $\varepsilon(x) = \varepsilon_r(x)\varepsilon_0$, where $\varepsilon_r$ is the relative permittivity and $\varepsilon_0$ is the vacuum permittivity or electric constant. Thus, the dielectric material might not be homogeneous, on the other hand the dielectric function does not depend on frequency.
Figure 1: System of \( N + 1 \) conductors and an external network to which they are connected. There is a time-variant charge distribution, \( \rho(x,t) \), in the space between the conductors that it moves with velocity \( u(x,t) \).

The shaded part of Figure 1 denotes the space inside the conductors. The space between the conductors is considered as the external space of the conductors and it is denoted by \( \Omega \). This space is multiply connected, so the total surface \( S \) surrounding this volume is the union of all the surfaces of the individual conductors

\[
S = S_0 \cup S_1 \cup S_2 \ldots \cup S_N = \bigcup_{i=0}^{N} S_i
\]

The charge distribution \( \rho \) is, in general, a function of space \( x \) and time \( t \), so \( \rho = \rho(x,t) \). The velocity of the charge distribution is also a function of space and time, \( u = u(x,t) \). In this analysis, these two functions, \( \rho(x,t) \), \( u(x,t) \), are supposed to be known functions.

The signals are taken from the conductors \( 1, 2, \ldots, N \) of the detector and are then fed to the external network. Usually, the detector is characterized as an \( N \) conductor (signal) detector, ignoring the reference conductor \( 0 \). Let us denote \( u(x,t) \) as the drift velocity of the moving charges that are created inside the detector by ionization. The drift velocity, \( u(x,t) \), is a function of the electric field, \( E = E(x,t) \). This electric field depends on the external potentials, the bias potentials of the detector and on the space charges created from the ionization. In addition, if there is an external magnetic field \( B \), the drift velocity will be affected. Usually, the bias voltages are constant in time and furthermore the (external) field they produce is much stronger compared to the (internal) field produced by the space charges. In this case the space charges create a negligible “internal” electric field. In some cases, we might have a large concentration of charges; as a result a strong “internal” electric field is created, and must be taken into account, appropriately. For simplifying our calculations, we will consider that the biasing potentials and space charges produce fields independent of time, thus we can denote the velocity as a function of space, \( u = u(x) \).

Inside the detectors, along with the creation of electrons, positive charges are also created. In the case of gaseous detectors this is due to the ionization process. At each ionization point the negative charges (primarily electrons) and the positive charges (ions) move in opposite directions. The absolute value of the drift velocity of the electrons is much larger than the drift velocity of the ions, \( u_e \gg u_{ion} \) (for Ar gas at normal conditions, \( u_e \approx 200u_{ion} \)). Evidently, the corresponding currents of the positive and negative charges have the same direction. In order to find the total result, we need to apply the superposition principle, taking into account all the contributing charge types. We will examine the case when we consider only one type of charges. We should mention here that one could argue that in this approach, conservation of charge is not taken into account. In reality when an amount of negative charge is created, an equal amount of positive charge is produced at the same time. One could imagine that an equal amount of charge is produced and remains static at the point of ionization. Another argument is that because we treat the problem as a semi-electrostatic problem, there is not use of the full Maxwell equations which contain the conservation of charge.
Let us assume that there is an electrostatic condition at any given moment (Figure 1). We apply the divergence theorem [3, 4, 6] which leads to Green’s reciprocity theorem. This can be proven for a linear dielectric, even when it is not homogeneous, in which case permittivity \( \varepsilon = \varepsilon(x) \) depends on the position \( x \).

The reciprocity theorem can be expressed as follows

\[
\int_{\Omega} \Phi' \rho d^3x + \int_{S} \Phi' \sigma d\alpha = \int_{\Omega} \Phi \rho' d^3x + \int_{S} \Phi \sigma' d\alpha
\]

Eq. (1) states that volume distribution of free charge, \( \rho \), in three dimensional space \( \Omega \) and surface charge distribution, \( \sigma \), on the surface \( S \) surrounding the above space, create a potential \( \Phi \) inside \( \Omega \). Similarly for the primed quantities.

Green’s reciprocity theorem for point charges takes the form

\[
\sum_{l} q_{l} \Phi_{l}' = \sum_{l} q_{l}' \Phi_{l}
\]

In this case, free charges \( q \) create potentials \( \Phi \) in the positions of the \( q' \) and charges \( q' \) create potentials \( \Phi' \) in the positions of the \( q \). Instead of a continuous distribution, one can use a free point charge. This is a mathematically simpler case to treat. Starting from the point charge analysis one could superimpose the appropriate partial results of many point charges to get the result for the continuous charge distribution. We can use the continuous charge distribution result to obtain the moving discrete point charge result, by introducing the appropriate delta function distribution.

We will proceed with the generalization of the Shockley-Ramo theorem, for a continuous charge distribution which changes with time.

The charge distribution shown in Figure 1, in general, depends on time. The rate of change is sufficiently small. There is an external circuit (network) connected to the conductors of the detector and to the surrounding reference conductor, 0. The actual (instantaneous) semi-electrostatic state of Figure 1 is described at time \( t \) by the charge and potential quantities by

\[
[rho(x, t), q_0(t), q_1(t), q_2(t), \ldots, q_N(t)], \quad [phi(x, t), 0, v_1(t), v_2(t), \ldots, v_N(t)]
\]

(2)

The running indices refer to quantities related to the corresponding conductors, and \( x \) is the position vector for points inside \( \Omega \), the space between the conductors.

Let us now imagine \( N \) electrostatic states of the \( N \) conductors of the detector. We use two indices, index \( j = 1, 2, \ldots, N \) describes the state, while index \( l = 1, 2, \ldots, N \) describes the conductor. Here, the potentials and fields are constant and there is no charge inside the space \( \Omega \), i.e. \( \rho(x, t) = 0 \). These states are shown by

\[
[0, Q_{jl}, \ldots, Q_{jl}, \ldots, Q_{jl}], \quad [\Phi_j(x), 0, 0, \ldots, V_{l(=j)}, \ldots, 0] \quad j = 1, 2, \ldots, N
\]

(3)

In Eq. (3), \( Q_{jl} \) is the charge of conductor \( l \) in state \( j \), where only conductor \( l = j \) has a non-zero potential, \( V_{l(=j)} \neq 0 \), while all the others have a zero potential, \( V_{l(\neq j)} = 0 \). \( \Phi_{j}(x) \) is the potential in space \( \Omega \) corresponding to state \( j \).

We apply Eq. (1) for the actual state described by Eqs. (2), and state \( j \), described by Eq. (3), and we get

\[
\int_{\Omega} \Phi_{j}(x) \rho(x, t) d^3x + V_{j} \int_{S_{j}} \sigma_{j}(x, t) d\alpha = \sum_{l=1}^{N} v_{l}(t) Q_{jl}, \quad j = 1, \ldots, N
\]

Evidently, the charge of conductor \( j \) is \( q_{j}(t) = \int_{\Omega} \sigma_{j}(x, t) d\alpha \). We define \( c_{jl} \) as

\[
c_{jl} = \frac{Q_{jl}}{V_{j}}
\]

(4)

therefore the following equations stand at any given moment

\[
\Phi_{j}(t) = \frac{1}{V_{j}} \int_{\Omega} \Phi_{j}(x) \rho(x, t) d^3x = -q_{j}(t) + \sum_{l=1}^{N} c_{jl} v_{l}(t), \quad j = 1, \ldots, N
\]

(5)

The physical meaning of the various quantities is understood from the definitions of states of Eqs. (2), (3), (4). Specifically, \( \Phi_{j}(x) \) is the potential in the space between conductors, when there is no free charge in this space and
only conductor \( j \) has a constant potential \( V_j \), while all the other conductors have zero potential. \( c_{jl} \) is the quotient of the free charge induced in conductor \( l \) when conductor \( j \) alone has potential \( V_j \) while all the rest have zero potential and there is no charge in the space between them, by \( V_j \). For coefficients \( c_{jl} \), we have \( c_{ii} > 0 \) and \( c_{ij} \leq 0, \ i \neq j \). These quantities are often referred to with different names, i.e. electrostatic inductance coefficients. They have the dimension of a capacity but they don’t have an electrotechnical representation; as a matter of fact, some of them are negative.

We will now see that we can introduce capacitances that have the usual physical meaning as the capacitances in electrotechnology. This allows us to use the usual methods of electrical circuit theory for solving the problems of detector signals.

Evidently, from Eq. (5), when \( \rho(x,t) = 0 \), we arrive at Eq. (6), for the charges and the potentials of the conductors. Since we have electrostatic states, we replaced \( q \) by \( Q \) and \( v's \) by \( V \)

\[
Q_j = \sum_{l=1}^{N} c_{jl} V_l, \quad j = 1, \ldots, N
\]

\[
U = \frac{1}{2} \sum_{i,j=1}^{N} c_{ij} V_i V_j
\]

where \( U \) is the electrostatic energy of the system, \( Q_j \) is the charge induced in conductor \( j \) by all the conductors when they have potentials \( V_l, l = 1, 2, \ldots, N \). In order to introduce the "usual" capacitances we need to define the potential differences between the various nodes. For this purpose we successively add to and subtract from the second members of Eqs. (6) the sums \((\sum_{l=1}^{N} c_{jl}) V_j \). This yields to

\[
Q_j = C_{jj} V_j + \sum_{l=1}^{N} C_{jl} (V_j - V_l), \quad j = 1, \ldots, N
\]

where

\[
c_{jl} = C_{lj} = -c_{ij} = -c_{jl} \quad \forall j \neq l, \quad C_{ll} = \sum_{j=1}^{N} c_{lj}
\]

\[
c_{ij} = C_{ij} \quad \forall i \neq j, \quad c_{ii} = \sum_{k=1}^{N} C_{ik}
\]

The coefficients (capital symbols) \( C_{jl} \) are usually referred to as capacity coefficients and are all non-negative, \( C_{jl} \geq 0 \). Coefficients \( C_{jj} \) is the capacity between point (node) \( j \) and reference conductor 0. Coefficients \( C_{ij} \) are the usual capacities describing electrostatic coupling (interaction) between the corresponding conductors of the detector. These capacities are also called two terminal capacitances. In this case, it is noteworthy that, according to Eq. eqrefeq:mmr-1-9, for each coefficient \( C_{jl} \) we have

\[
C_{jl} = -\frac{Q_j}{V_l}, \quad j \neq l
\]

Remember that \( Q_j \) is the induced charge on conductor \( j \), when all other conductors have zero potential, \( V_k = 0 \ \forall k \neq l \), and only conductor \( l \) has a non-zero potential, \( V_l \neq 0 \). For the remaining coefficients we have

\[
C_{jj} = \frac{Q_j}{V_j}
\]

Notice that \( Q_j \) is the charge induced in conductor \( j \) when all other conductors have the same potential as \( j \), namely \( V_l = V_j, \ \forall l = 1, 2, \ldots, N \). Again, we follow the procedure above, i.e. we add to and subtract from the second member of Eq. (5) the sum \((\sum_{l=1}^{N} c_{jl}) v_j \) and, by combining this with the last term \((\sum_{l=1}^{N}) \), we get

\[
\phi_j(t) = \frac{1}{V_j} \int_{V} \phi_j(x) \rho(x,t) d^3 x = -q_j(t) + C_{jj} v_j(t) + \sum_{l=1}^{N} C_{jl} (v_j(t) - v_l(t)) \quad j = 1, \ldots, N
\]
We consider that the conductor currents, charges and the potentials in general change with time, so if we take the time derivatives of these equations, we get

\[
\frac{d\phi_j(t)}{dt} = \frac{d}{dt} \left\{ \frac{1}{V_j} \int_{\Omega} \phi_j(x) \rho(x,t) d^3x \right\} = -\frac{dq_j(t)}{dt} + \sum_{i=1}^{N} C_{ji} \frac{dv_i(t)}{dt} \\
= -\frac{dq_j(t)}{dt} + C_{jj} \frac{dv_j(t)}{dt} + \sum_{i=1}^{N} C_{ji} \frac{v_i(t)}{dt} - \frac{v_j(t)}{dt}
\]  

To determine the instantaneous currents and potentials, we will apply Eq. (5) or Eq. (7) at two nearby times. The external circuit and current sources. We will see more details later.

This leads us to the equivalent electrotechnical circuit of the detector, which is shown in Figure 2, together with the formation. We could say that the charge contributes to the small or large modification of the biasing static electric field. In this case, one can note that only the part of the charge distribution which is moving contributes to the signal formation. We can say that the charge contributes to the small or large modification of the biasing static electric field while the current density is only due to the moving charges. Using Eq. (9), we get

\[
\mathbf{J}(x,t) = \rho(x,t) \mathbf{u}(x) \\
\nabla \cdot \mathbf{J}(x,t) + \frac{\partial \rho(x,t)}{\partial t} = 0
\]

Using the vector identity \( \nabla \cdot (\phi \mathbf{A}) = \mathbf{A} \cdot (\nabla \phi) + \phi (\nabla \cdot \mathbf{A}) \) we get

\[
\frac{d}{dt} \left\{ \frac{1}{V_j} \int_{\Omega} \phi_j(x) \rho(x,t) d^3x \right\} = \frac{1}{V_j} \int_{\Omega} \mathbf{J}(x,t) \cdot \nabla \phi_j(x) d^3x - \frac{1}{V_j} \int_{\Omega} \nabla \cdot [\mathbf{J}(x,t) \phi_j(x)] d^3x \\
+ \frac{1}{V_j} \int_{S_j} \phi_j(x) \rho(x,t) [\mathbf{u}(x) \cdot \mathbf{n}(x)] da
\]

Applying the divergence theorem yields

\[
\frac{d}{dt} \left\{ \frac{1}{V_j} \int_{\Omega} \phi_j(x) \rho(x,t) d^3x \right\} = \frac{1}{V_j} \int_{\Omega} \mathbf{J}(x,t) \cdot \nabla \phi_j(x) d^3x - \frac{1}{V_j} \int_{\Omega} \phi_j(x) \mathbf{J}(x,t) \cdot \mathbf{n}(x) da \\
+ \frac{1}{V_j} \int_{S_j} \phi_j(x) \rho(x,t) [\mathbf{u}(x) \cdot \mathbf{n}(x)] da
\]

Finally, we get

\[
\frac{\nabla \phi_j(x)}{V_j} = -\frac{\mathbf{E}_j(x)}{V_j}
\]

\[
\frac{d}{dt} \left\{ \frac{1}{V_j} \int_{\Omega} \phi_j(x) \rho(x,t) d^3x \right\} = \frac{1}{V_j} \int_{\Omega} \mathbf{J}(x,t) \cdot \nabla \phi_j(x) d^3x = -\frac{1}{V_j} \int_{\Omega} \mathbf{J}(x,t) \cdot \mathbf{E}_j(x) d^3x
\]

We consider that the conductor currents, \( i_j(t) \), are positive when they move away from the conductor. Then,

\[
i_j(t) = -\frac{dq_j(t)}{dt}
\]
The (induced) auxiliary current for conductor \( j \) is defined as follows

\[
I_j(t) = \frac{1}{V_j} \int_{\Omega} J(x, t) \cdot \nabla \Phi_j(x) d^3x = -\frac{1}{V_j} \int_{\Omega} J(x, t) \cdot E_j(x) d^3x
\]  

(10)

If we consider a moving point charge (one type only, positive or negative), whose position is defined by the kinematic equation \( \mathbf{x}_q = \mathbf{x}_q(t) \), then its velocity will be \( \mathbf{v}_q = \dot{\mathbf{x}}_q \). In this case, the charge density and the corresponding current density can be expressed as follows

\[
\rho = q \delta(x - \mathbf{x}_q) \\
\mathbf{J} = \rho \mathbf{u} = q \mathbf{u} \delta(x - \mathbf{x}_q)
\]

Eq. (10) yields

\[
I_j(t) = -\frac{\mathbf{E}_j(x_q(t)) \cdot \mathbf{u}(\mathbf{x}_q(t))}{V_j}
\]  

(11)

The value of \( \mathbf{E}_j \) and velocity \( \mathbf{u} \) are refereed at time \( t \) and space point \( \mathbf{x}_q \) of charge \( q \).

Eq. (8) becomes

\[
I_j(t) = i_j(t) + \sum_{l=1}^{N} c_{jl} \frac{dv_l(t)}{dt}
\]

\[
I_j(t) = i_j(t) + C_{jj} \frac{dv_j(t)}{dt} + \sum_{l=1}^{N} C_{jl} \frac{d(v_j(t) - v_l(t))}{dt}, \quad j = 1, \ldots, N
\]  

(12)

This technique is called the method of weighting field \( \mathbf{E}(\mathbf{x})/V_j = -\nabla \Phi_j(\mathbf{x})/V_j \). It is noteworthy that \( \mathbf{E}_j(\mathbf{x}) \) is an auxiliary, purely electrostatic field depending solely on the conductor geometry and the dielectric between the conductors. It is calculated with no free charges in the space between the conductors, and for the respective auxiliary potentials we have \( V_i \neq 0, V_j \neq 0 \ \forall j \neq l \). These potentials are not related to the biasing voltages of the detector during normal operation.

The solution to the problem, that is the determination of the various signal voltages and signal currents, is obtained by solving the system of (differential) equations shown in Eqs. (12), combined with the equations that hold for the external circuit. The equations for the external circuit relate the currents \( i_k(t) \) and voltages \( v_l(t) \). Eqs. (12) show that, when charges move within the detector space, current sources \( I_j(t) \) correspond to the respective detector electrodes.

According to Eqs. (12), we get the equivalent electrotechnical circuit of Figure 2, where these current sources are part of the total network, that consists of the “internal” network, i.e. capacities \( C_{ij} \), sources \( I_j \) and of the external network. One could see that indeed for the equivalent circuit of the detector, the second set of the Eqs. (12) holds.

![Figure 2: The equivalent detector circuit with three conductors and an external circuit.](image-url)
Included in the external circuit is the circuit providing bias for the detector. Usually, the bias circuit is designed in such a way that it does not affect much the signal form in the detector output. For this reason it is often not taken into account when making calculations.

Thus, instead of solving the differential Eqs. (12) in combination with the external circuit equations, we can solve the electrotechnical problem depicted in Figure 2, where we have current sources $I_j(t)$ and the total (internal and external) network. We can solve the equations by means of different circuit solving methods, such as the method of analyzing in complex space, where the Laplace transformation is used.

The charges move between the electrodes for finite time. In general, after all charges have been collected (at which point the current density is $J(x,t) = 0$). Signal voltages and currents are still present on the detector outputs. Simple cases will be presented in the following sections. We have to remember that (ideal) current sources have infinite resistance, so when their current is zero they represent an open part of the circuit.

We comment on a useful result about total charge through the current source of each conductor, see book by W. Blum, W. Riegler, L. Rolandi [6]. If a point charge $q$, is moving along a trajectory $x(t)$, from position $x_0(t_0)$ to position $x_1(t_1)$, the total charge that flows through current source number $n$, connected to conductor $n$, is given by Eq. (13) below

$$Q_n = \int_{t_0}^{t_1} I_n dt = -\frac{q}{V_n} \int_{t_0}^{t_1} (E_n(x(t))) \cdot u(x(t)) dt = \frac{q}{V_n} (\Phi_n(x_1) - \Phi_n(x_0))$$

We simplified our path description by not including the index $q$. The charge depends only on the end points of the trajectory, it does not depend on the specific path. If a pair of charges $q$, $-q$ are at a point $x_0$, where they were produced, and after some time, charge $q$ moves and arrives at position $x_1$ while $-q$ moves and reaches position $x_2$, the total charge through the current source is given by the following Eq. (14)

$$Q_n = \frac{q}{V_n} (\Phi_n(x_1) - \Phi_n(x_2))$$

If charge $q$ moves to the surface of conductor (electrode of the detector) $n$ while charge $-q$ moves to the surface of some other electrode, the total charge through the source of the $n$ electrode is equal to $q$. When both charges move to other electrodes, the total charge through the $n$ source is zero. The conclusion is that, after all charges have arrived at the different electrodes, the total charge through the source of electrode $n$ is equal to the charge that has arrived at electrode $n$. From this one also concludes that the above currents on electrodes that do not receive any charge are bipolar.

At this point we make some comments related to resistive Micromegas detectors under study to be used among other experiments in the ATLAS experiment. The reading strips (conductors) are under resistive strips, separated by an insulating layer. The various widths are comparable among themselves. This case is similar to the one treated by several people among them Werner Riegler, reference [5]. It is easy to examine the induced current signals in two extreme cases, a) when the resistivity of resistive strips is extremely high so they behave as pure dielectric materials. Then the situation is simple and can be treated easily. The negative charges do not run all the way to the reading strips and the weighting field is appropriately changed. There are “induced” currents on the reading strips. The other extreme case b) is when resistivity is so small that resistive strips are like ideal (grounded) conductors. In this case there are not induced currents on the reading strips, they are shielded, and the signals on the reading strips are due to the couplings between resistive and reading strips. The general case is more complicated and we believe that leads to both, induced signals on the reading strips and signals due to couplings with the resistive strips. Probably a simpler case to be analyzed involves a resistive plane instead of resistive strips.

2 Examples

2.1 Point charge moving within a detector of two conductors

We consider a detector of two conductors, surrounded by a grounded conductor, as shown in Figure 3. The signal is formed on conductor 2, while conductor 1 is connected to the bias source. The equivalent electrotechnical circuit is shown in Figure 4.
Using the first equations from Eq. (12), we find for each conductor

\[ I_1(t) = i_1(t) + c_{11} \frac{dv_1(t)}{dt} + c_{12} \frac{dv_2(t)}{dt} \]

\[ I_1(t) = -\frac{1}{V_1} \int_{\Omega} \mathbf{J}(x,t) \cdot \mathbf{E}_1(x) \, d^3x \]

\[ I_2(t) = i_2(t) + c_{12} \frac{dv_1(t)}{dt} + c_{22} \frac{dv_2(t)}{dt} \]

\[ I_2(t) = -\frac{1}{V_2} \int_{\Omega} \mathbf{J}(x,t) \cdot \mathbf{E}_2(x) \, d^3x \]

(15)

We have \( v_1 = V_a \), so \( \frac{dv_1}{dt} = 0 \). For a point charge at time \( t \) located in position \( x(t) \) with velocity \( u(x(t)) \), the following equations hold for \( I_1, I_2 \)

\[ I_1(t) = -q \frac{\mathbf{E}_1(x(t)) \cdot \mathbf{u}(x(t))}{V_1}, \quad V_2 = 0 \]

\[ I_2(t) = -q \frac{\mathbf{E}_2(x(t)) \cdot \mathbf{u}(x(t))}{V_2}, \quad V_1 = 0 \]

\( i_1(t) \) is of no interest, so out of Eqs. (15) we only keep the equation of conductor 2, namely

\[ I_2(t) = i_2(t) + c_{22} \frac{dv_2(t)}{dt} \]

(16)
where \( v_2(t) \) and \( i_2(t) \) are the two signal forms on the detector output. For the resistor connected to conductor 2, we have \( i_2 = v_2/R \), therefore Eq. (16) yields

\[
I_2(t) = \frac{v_2(t)}{R} + c_{22} \frac{dv_2(t)}{dt} \tag{17}
\]

The solution to this (differential) equation is given by [7]

\[
v_2(t) = c_2(0) + e^{-t/(RC_{22})} \int_0^t e^{t'/(RC_{22})} I_2(t')dt'
\]

If the two conductors of the setup in Figure 3 constitute an ideal capacitor, then the following equations hold

\[-c_{12} = -c_{21} = c_{11} = c_{22} \\
C_{11} = C_{22} = 0 \\
C_{12} = C_{21} = -c_{12} = -c_{21} = c_{22} = C_d
\]

This means that there is only one (independent) coefficient, the capacitance of the detecting conductor (electrode), with respect to the other electrode. The characteristic of this setup is that, when these conductors have opposite charges of the same magnitude, all the field lines beginning from one conductor end on the other. It should be noted that this is not true in the general case, of a pair of conductors.

We are going to examine two extreme cases that could approximately show up in practice. We will be assuming that \( v = v_2, I = I_2 \).

In the first case \( C_dR \gg \tau \), where \( \tau \) is the time needed for \( v \) to change significantly, i.e. \( C_dR|\text{d}v/\text{d}t| \gg |v| \); then we can omit term \( v \) and we have, approximately, from Eq. (17)

\[
I(t)R = C_dR \frac{dv(t)}{dt}, \quad I(t) = C_d \frac{dv(t)}{dt} \\
\frac{dv(t)}{dt} = \frac{I(t)}{C_d}, \quad v(t) = v(0) + \frac{1}{C_d} \int_0^t I(t')dt'
\]

This is equivalent to the case where the resistor (\( R = \infty \)) does not exist and \( I \) is charging capacitance \( C_d \). This is called the "voltage mode of the detector operation".

In the other case \( C_dR \ll \tau \), then \( C_dR|\text{d}v/\text{d}t| \ll |v| \); therefore, we have approximately

\[IR = v\]

This is equivalent to a circuit, were there is no capacitor, \( C_d = 0 \) and the whole current \( I \) is passing through the resistor. This is called the "current mode of the detector operations".

We can solve the problem by considering the electrotechnical circuit of Figure 4. Indeed, by applying Kirchhoff’s laws we finally end up with (differential) Eq. (17). Remember that it can be shown that \( c_{22} = C_{22} + C_{12} \).

2.2 Cylindrical detector with circular cross-section and a circular wire conductor on its axis, MDT like

For this analysis one could use references [6] and [7]. A cross-section of a singe MDT detector of the ATLAS experiment [8] is illustrated in Figure 5. The equivalent electrotechnical circuit can be seen in Figure 6.

We assume that the detector is a cylinder of much bigger length than its cross sectional diameter. We can thus disregard the fringe field effects at the two ends of the detector, i.e. practically, the field is radial everywhere inside the detector. Within the detector a point charge \( q \) is moving radially from the center outwards. In Figures 5 & 6 the bias voltage, \( V_a \), applied between the central detector electrode, conductor 1 and enclosure 0 (at potential zero) is not shown. We assume that the circuit is such that the bias circuit does not affect the detector signal.
According to the above analysis, we examine the detector as an one-conductor device with \( N = 1 \). It can be easily understood that the only capacitance coefficient involved is the detector capacitance, \( C_d \), is the capacitance formed by the two conductors, given by the following formula

\[
C_d = C_l l = l \frac{2\pi \varepsilon}{\ln(b/a)}
\]

where \( l \) is the detector length and \( C_l \) is its capacitance per unit length.

The bias potential biases conductor 1 positively with respect to the enclosure, 0. The (internal) radius of the problem is \( b \) and the radius of the central conductor is \( a < b \). The auxiliary electric field in the space of the cylindrical detector for potential \( V \) in electrode 1 (i.e. the central electrode of the cylindrical detector) is radial with an outwards direction, and is given by

\[
\begin{align*}
E(x) \hat{e}_r &= \frac{C_l V}{2\pi r} \frac{1}{\ln(b/a)} \frac{1}{r} \hat{e}_r \\
&= V \frac{1}{\ln(b/a)} \frac{1}{r} \hat{e}_r 
\end{align*}
\]  

(18)

We use cylindrical coordinates as usual. It has been taken into account that the capacitance per unit length is given by

\[ C_l = \frac{2\pi \varepsilon}{\ln(b/a)} \]

For gaseous detectors, it holds that \( \varepsilon \tau \approx 1 \), \( \varepsilon = \varepsilon_r \varepsilon_0 \approx \varepsilon_0 \).

From Eq. (11) we get

\[
I(t) = -q \frac{1}{\ln(b/a)} \frac{1}{r(t)} u(x(t)) \hat{e}_r
\]

The motion of the point charge is radial, so we have

\[
I(t) = -q \frac{1}{\ln(b/a)} \frac{1}{r(t)} u(r(t))
\]

\( u(r(t)) \) is positive only if the motion is from the center outwards and negative in the opposite case.
We accept that a point charge, \( q \), started at time \( t = 0 \) from position \( r = r_0 > a \), where \( a \) is the wire radius and is moving with instantaneous velocity \( u(r(t)) \). This start position is, in practice, found to be very close to the wire surface. This is because ionization occurs very close to the wire. Positive ions move inside the detector towards the cathode, and negative particles (electrons) move towards the anode. Since electrons are collected in a very short time one could ignore their contribution to the detector signal. Thus, we examine only the positive ions, assuming that there is only one kind. For ions, the drift velocity (for a wide range of fields) is given by

\[
u = \mu E_a \tag{19}\]

where \( \mu \) (mobility) is constant. It must be noted that field \( E_a \) is the real field in the detector due only to the high-voltage detector bias, ignoring the effect of space charges and assuming that the voltage drop in the detector connected to the high-voltage supply is negligible compared to the high voltage. Using Eq. (18) and (19), we get

\[
u = \mu \frac{V_a}{\ln(b/a) r} \tag{20}\]

It is evident that positive point charge \( q \) will be moving from the anode (central electrode, signal electrode) towards the cathode (external electrode) and we will have

\[
I = -\frac{q \mu V_a}{\ln^2(b/a) r^2} \tag{21}
\]

The equivalent circuit contains a current source that is connected to the external network shown in Figure 6. For the point charge of ions we need its position as a function of time. From the equations above we get

\[
\frac{dr}{dt} = \mu \frac{V_a}{\ln(b/a)} \frac{1}{r}, \quad r dr = \mu \frac{V_a}{\ln(b/a)} dt,
\]

\[
r^2 = r_0^2 + 2 \mu \frac{V_a}{\ln(b/a)} t, \quad r^2 = r_0^2 \left(1 + \frac{t}{t_0}\right)
\]

\[
t_0 = \frac{r_0^2}{2 \mu V_a} \ln(b/a),
\]

\[
T = \frac{(b^2 - r_0^2) \ln(b/a)}{2 \mu V_a}
\]

\[
I(t) = -\frac{q \mu V_a}{r_0^2 \ln^2(b/a)} \frac{1}{(1 + t/t_0)}
\]

The \( t_0 \) parameter defines the time scale for the charge carrier motion and the induced signal and \( T \) is the arrival time of point charge on the external enclosure. From Figure 6 and with the help of Eqs. (12) we get

\[
I(t) = \frac{v(t)}{R} + C_d \frac{dv(t)}{dt}
\]

\[
I(t)R = v(t) + RC_d \frac{dv(t)}{dt} \tag{21}
\]

We will analyze two cases:

(a) Assume \( R C_d \ll \tau \).

Here, the characteristic time is \( \tau = t_0 \). Characteristic time is, in general, different for the various carrier types, ions and electrons. We omit the term containing \( R C_d \) and we therefore have approximately

\[
v(t) = I_q R \tag{22}
\]
We might say that in practice with this approximation, the signal lasts for as long as the duration of the charge motion in the detector space (denote this time as $T$), since capacitive effects have been omitted. In a better approximation we might say that after time $T$, the capacitor-detector (with initial voltage $v(T)$) discharges very fast through resistor $R$, with a very small time constant $RC_d$.

Using Eqs. (20) and (22), we get

$$v(t) = I(t)R = -\frac{q\mu V_a R}{r_0^2 \ln^2(b/a)} \left(1 + \frac{t}{t_0}\right) \quad 0 \leq t \leq T$$

$$v(t) = 0 \quad t > T \quad \text{and} \quad t < 0$$

Even better, we could say that when $t > T$ we have

$$v(t) \approx v(T)e^{-t-T/RC_d}, \quad t > T$$

We will determine the charge induced on the detector during the motion of positive charge $q$. We define this charge as positive if we have positive charge moving towards the examined electrode. To find the charge induced from time $t = 0$ to time $t \leq T$, we integrate the current with respect to time. Therefore, we get

$$Q(t) = \int_0^t I(t')dt' = \frac{q}{\ln(b/a)} \int_0^t \frac{u}{r}dr' = \frac{q}{\ln(b/a)} \int_{r_0}^r \frac{dr}{u} = \frac{q}{\ln(b/a)} \ln\left(\frac{r}{r_0}\right)$$

$$Q_t = \frac{q}{\ln(b/a)} \ln\left(\frac{b}{r_0}\right) > 0$$

(b) The other extreme case is for $RC_d \gg \tau = t_0$. In this case, it is as if there were no resistor, $R = \infty$, in the equivalent circuit. In Eqs. (21) we omit the $v(t)$ term, therefore

$$I(t) = C_d \frac{dv(t)}{dt}$$

The solution of the above equation, after using Eq. (20), yields

$$v(t) = v(0) + \frac{1}{C_d} \int_0^t I(t')dt' = -\frac{1}{C_d} \int_0^t I(t')dt' = -\frac{q}{2C_d \ln(b/a)} \ln\left(1 + \frac{t}{t_0}\right), \quad 0 \leq t \leq T$$

and

$$v(t) = \begin{cases} 0 & \text{for } t < 0, \\ v(T) & \text{for } t > T \end{cases}$$

In reality, after time $T$, the capacitor-detector discharges very slowly, with a large time constant $RC_d$, through the resistor $R$

$$v(t) \approx v(t)e^{-(t-T)/RC_d}, \quad t > T$$

In the general case, with no approximations, we give the solution of Eq. (21) for the above cylindrical detector case. The result, under the initial condition $v(0) = 0$ and $RC_d = \tau_d$, yields

$$v(t) = \begin{cases} -\frac{q\mu V_a R}{r_0^2 \ln^2(b/a)} \frac{t_0}{\tau_d} e^{-(t+t_0)/\tau_d} \left[E_1\left(-\frac{t_0}{\tau_d}\right) - E_1\left(-\frac{t+t_0}{\tau_d}\right)\right] & \text{for } 0 \leq t \leq T, \\
\left(E_1\left(-\frac{t}{\tau_d}\right) - E_1\left(-\frac{t+\tau_0}{\tau_d}\right)\right) & \text{for } t > T \\
v(T)e^{-(t-T)/\tau_d} \end{cases}$$

where $E_1(x) = \int_x^\infty e^{-t}/t \, dt$ is the exponential integral. In Figure 7 we see an indicative signal (potential) waveform of a cylindrical detector, $v(t)$, corresponds to a positive point charge (ions) of one type moving radially, before being collected.

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2.3 Signals in a micromegas detector

2.3.1 Overview

The geometry of the detector is a plane geometry, see Figure 8 (see [9, 10]). We imagine two cartesian coordinates, one, \( z \), is normal to the mesh and strip plane, and the other, \( x \), is across the above planes normal to the strip length. We ignore the end effects that involve a very small area of the whole detector. Furthermore, the detector has small dimensions so electromagnetic wave propagation is ignored. We assume there is not longitudinal or transverse diffusion of charges. This means that a point charge remains a point charge as it moves. Let a point charge \( q \) at a point \( A \) inside the gas multiplication region moving with a known drift velocity \( u \). The biasing potentials determine the motion of charges in the detector space. We consider that the biasing potentials are dc (no time dependence) so the drift velocity depends only on the position of the moving point charge. To calculate the signal on a readout strip electrode, we use the generalized Shockley-Ramo theorem, proved above. We have to calculate the current sources corresponding to each one readout strip for each type of moving charge. To do so, we put one at a time readout strip (electrode), \( a \), at potential \( V_a \) and all other electrodes are at zero potential. The electric field at point \( A \), position vector \( \mathbf{x} \), is \( \mathbf{E}_A \). The point \( A \) is the position of the moving point charge. This is an auxiliary field that has no relation to the actual biasing (dc) potentials of the electrodes of the “working” detector. We stress that the biasing potentials are much bigger than the signal potentials. The ratio (weighting field) \( \frac{\mathbf{E}_A}{V_a} \) depends only on the geometry and the electric properties of the materials in the space inside the detector. It is clear that this ratio depends on the position \( \mathbf{x} \) of the point \( A \). It has dimension of inverse length, so in the SI is measured in m\(^{-1}\).

The source current which is equal to the auxiliary (induced) current on electrode \( a \) due to the motion of point charge \( q \) at \( A \), is given by

\[
I_a = -q \frac{\mathbf{E}_A \cdot \mathbf{u}}{V_a} \tag{23}
\]

In a micromegas detector we have both electrons and positive ions drifting in opposite directions, each time starting from the same point in space. The electrons move with much higher speeds than the (positive) ions and we treat each sign of charge separately. The biasing is dc and such that the electrons move from the mesh towards the resistive strips, where they are absorbed and stop.

For a varying continuous charge distribution with current density \( \mathbf{J}(\mathbf{x}, t) \), the auxiliary current is

\[
I_a(t) = -\frac{1}{V_a} \int_{\Omega} \mathbf{J}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}) \, d^3x
\]

The integral is over the volume where the charge is distributed.

Figure 2 shows the equivalent circuit of a multi-electrode (three electrodes) detector connected to an external circuit. To determine the weighting field, one has to solve the Laplace equation assuming that the particular strip has a potential \( V_a \) and all other strips and the mesh are grounded. One determines the potential \( \Phi(\mathbf{x}, z) \) of such a plane geometry and then from that the weighting field is determined. The potential and the field is a complicated function.
of position. There are simplified geometries for which the field can be easily calculated. Summarizing the problem is to calculate the signals induced on the strips of the detector, when a specific point charge moves in a known way inside the amplification region of the detector. For all readout strips we calculate the equivalent current sources. At the end we use the equivalent circuit shown in Figure 2 and the detector signals are determined.

### 2.3.2 Signal formation in resistive micromegas

The geometry of the detector is shown in Figure 8 (see [10]). When electrons enter the space between MESH and resistive strips (gas amplification region), they produce pairs of ions and electrons forming showers, they multiply with increasing multiplicity, as they move from the MESH towards to the resistive strips. The multiplication of charge depends on the (first) Townsend coefficient, which in general, among other things, is a function of the electric field. If \( n \) is the number of one type charged particles at a certain point then at a nearby space point along the electric field the increase of the charges will be \( dn = n \, dr \), where \( dr \) is the (infinitesimal) distance between the two space points. To find the increase of charge one has to integrate the above formula. This is difficult in the general case. We will simplify our calculations by assuming that the Townsend coefficient is constant independent of the electric field. We assume that the resistive strips do not influence much the induced currents on the readout strips. This could be justified if the resistive strips have very high resistance so they behave like insulating material and has the same effect as the shaded insulator between resistive and readout strips, shown in Figure 8. If the resistive strips were pure grounded conductors, it is obvious that no signals could be induced on the readout strips, the latter being shielded by the fully conducting strips. The reality is probably in between these two extreme situations.

![Micromegas geometry of the amplification region.](image)

We assume that the resistive strips have the same relative permittivity, \( \varepsilon_{rm} \) with the material between the two types of strips.

Let us consider a point charge \( q \) at a point A inside the gas multiplication region moving with velocity \( u \). The auxiliary (induced) current on electrode \( a \) due to the motion of point charge \( q \) at A, is given by the Eq. (23).

A simple case is the calculation of the signal on the mesh which is one very large electrode and the weighting field for the mesh is the field formed on a parallel plate capacitor. One may ignores that the field near the strips is not homogeneous. Let us consider that the point charge motion is normal to the strip planes. In this case, of the almost ideal plane capacitor geometry, it is easy to show that (see Figure 8)

\[
\frac{E_a}{V_a} = \frac{1}{d} \left( \frac{d}{d_g} \frac{1}{\varepsilon} \right) \left( 1 - \frac{\varepsilon_r}{\varepsilon_{rm}} \right) + \frac{\varepsilon_r}{\varepsilon_{rm}}
\]

(24)

This quantity, as a vector, is directed from the mesh to the strips. Usually, we define the effective gap \( d_{eff} \) as follows

\[
d_{eff} = d \left[ \frac{d_g}{d} \left( 1 - \frac{\varepsilon_r}{\varepsilon_{rm}} \right) + \frac{\varepsilon_r}{\varepsilon_{rm}} \right]
\]

so relation Eq. (24) is reduced to

\[
\frac{E_a}{V_a} = \frac{1}{d_{eff}}
\]

in the case of the mesh, for a moving point charge \( q \) with velocity \( u \), which has direction opposite to the direction of the auxiliary field, the auxiliary current is

\[
I_{mesh} = \frac{qu}{d_{eff}}
\]
For the strips, things are more complicated because the weighting field for each strip is not so homogeneous, it varies a lot from point to point.

The next step is to consider separately the motion of the electrons and the positive ions and at the same time and find how the moving charges change with position due to the gas amplification phenomenon. The (constant) Townsend coefficient is $\alpha$. Consider that a point charge consisting of electrons enters the gas amplification region, from the mesh. A shower of electrons and positive ions is developing. The numbers of electrons and ions is increasing as the electrons move towards the resistive strips. It is obvious that the positive ions move towards the mesh.

Let the initial absolute value of the charge of entering electrons be $q_0 (>0)$. After time $t$ they drift with drift velocity $u_n$ a distance $z = u_n t$ measured on the normal from the mesh towards the resistive strips. This holds for $0 < t < d / u_n$, where $d / u_n$ is the time it takes for the negative point charge to reach the resistive strip plane. Since this point charge moves, it multiplies and we get the corresponding negative ($q_n$) and positive ($q_p$) charges. At each point, obviously, the number of ions are a bit less than the number of electrons, $q_p (t) = q_n (t) - q_0$. We are referring to the absolute values of charges. We have

$$q_n (t) = q_0 e^{\alpha z} = q_0 e^{\alpha u_n t}, \quad 0 < t < d / u_n$$

$$q_p (t) = q_0 e^{\alpha z} - q_0 = q_0 (e^{\alpha u_n t} - 1), \quad 0 < t < d / u_n$$

The auxiliary current on strip $\alpha$, due to the moving electrons, is

$$I_{n\alpha} (t) = q_0 e^{\alpha u_n t} E_A u_n = q_0 e^{\alpha u_n t} \frac{E_A (x, d - u_n t)}{V_a} u_n$$

For the mesh we have

$$I_{\text{mesh}} (t) = q_0 e^{\alpha u_n t} \frac{u_n}{d_{\text{eff}}}$$

It is clear that

$$I_n = 0, \quad t > d / u_n$$

To find the total induced charge, one has to integrate those formulas from $t = 0$ to $t = d / u_n$. This is easily done for the mesh, for which we get

$$Q_{\text{mesh}} = \frac{q_0}{\alpha d_{\text{eff}}} (e^{ad} - 1)$$

This is the charge that is “moved” by the current source which has a current equal to the corresponding auxiliary current. It has a very clear meaning if one ignores all the (internal) capacitances involved and assumes only one ohmic (external) resistor is connected to each such current source (on each readout strip).

For the positive ions things are a little bit more complicated. The ionization is due to the motion of the electrons and it lasts for time equal to $d / u_n$. The positive ions move with drift speed $u_p$, much smaller than the electron speed $u_n$, $u_p$ is approximately $u_n / 100$ to $u_n / 1000$. The positive ions move towards the mesh.

Figure 9 shows the charge distribution of formed ions in the amplification region. The same is true for the electron distribution. The distribution is "static" in the sense that the produced charges are shown at each point ignoring their drifting!

![Figure 9](image_url)

Figure 9: "Static" charge distribution due to shower development in the gas amplification gap.
The ion auxiliary current on the strip $\alpha$ and on the mesh is

\[ I_{p\alpha}(t) = q_0 u_p \frac{E_A}{V_a} (e^{\alpha u_p t} - e^{\alpha u_n t} - 1) = q_0 u_p \frac{E_x(x,d-u_n t)}{V_a} (e^{\alpha u_n t} - e^{\alpha u_p t} - 1), \quad 0 < t < d/u_n \]

\[ I_{p\text{mesh}}(t) = \frac{q_0 u_p}{d_{\text{eff}}} (e^{\alpha u_n t} - e^{\alpha u_p t} - 1), \quad 0 < t < d/u_n \]

After this time all electrons are collected by the resistive strips while ions continue to move towards the mesh till time $d/u_p$. We have

\[ I_{p\alpha}(t) = q_0 u_p \frac{E_A}{V_a} (e^{\alpha u_n t} - e^{\alpha u_p t} - 1) = q_0 u_p \frac{E_x(x,u_p t)}{V_a} (e^{\alpha d} - e^{\alpha u_p t} - 1), \quad d/u_n < t < d/u_p \]

\[ I_{p\text{mesh}}(t) = \frac{q_0 u_p}{d_{\text{eff}}} (e^{\alpha d} - e^{\alpha u_p t} - 1), \quad d/u_n < t < d/u_p \]

\[ I_p = 0, \quad t > d/u_p \]

One can calculate the signals of the strips using the above auxiliary currents and the respective Figure 2 of the detector. One can calculate charges as we did for the electron currents.

In the simple case of the mesh we get after integration

\[ Q'_{\text{mesh}} = q_0 \frac{e^{\alpha d} - e^{\alpha u_n t}}{d_{\text{eff}}} \left[ \frac{u_p}{u_n} - \frac{u_p}{u_n} - e^{\alpha u_p t} + 1 \right], \quad \text{collected from } t = 0 \text{ to } t = d/u_n \]

\[ Q''_{\text{mesh}} = q_0 \frac{e^{\alpha u_p t}}{d_{\text{eff}}} \left[ e^{\alpha u_p t} - e^{\alpha d} - \alpha d u_p e^{\alpha d} + \alpha d e^{\alpha d} \right], \quad \text{collected from } t = d/u_n \text{ to } t = d/u_p \]

So the total ion charge $Q_{\text{pmesh}}$ is given by

\[ Q_{\text{pmesh}} = \frac{q_0}{\alpha d_{\text{eff}}} \left[ 1 - \frac{u_p}{u_n} \right] \left[ e^{\alpha d} (\alpha d - 1) + 1 \right], \quad \text{collected from } t = 0 \text{ to } t = d/u_p \]

One could get a feeling about the order of magnitude for the currents by examining the simpler case for the mesh. Because $d_{\text{eff}} < d$ the currents are bigger than they are in the case when $d = d_e$ (when there are no resistive strips). At the same time the charges are moving only a distance $d_e < d$.

The total charge due to the ions is bigger than the one due to the electrons, because more ions travel larger distances than electrons do, $Q_{\text{pmesh}} > Q_{\text{emesh}}$.

We mention that the auxiliary currents are defined with different formulae for various time intervals and in the general case of capacitors, inductors and so on, one should remember that the signals on the readout strips exist even when the auxiliary currents become zero.

**Acknowledgments**

We would like to thank our graduate students G. Iakovidis, S. Leontsinis, and K. Ntekas for carefully reading the manuscript and making valuable comments.

The present work was co-funded by the European Union (European Social Fund ESF) and Greek national funds through the Operational Program ”Education and Lifelong Learning” of the National Strategic Reference Framework (NSRF) 2007-1013. ARISTEIA-1893-ATLAS MICROMEGAS.
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