The Trojan Horse Method has been developed and applied by the Catania Nuclear Astrophysics group over the last three decades. In this work, the basic features of the method together with a review of validity checks and recent applications will be reviewed. In particular examples of studies of reactions induced by charged particles and neutrons in presence of radioactive species will be shown.

KEYWORDS: Nuclear Astrophysics, Indirect Methods, Trojan Horse

1. Introduction

It is widely known that cross sections of nuclear reactions between charged particles involved in astrophysical scenarios, that are the main subject of experimental Nuclear Astrophysics, are often very small (order of nano- or picobarns or even less) [1]. On the other hand, neutron induced nuclear reactions, also of huge importance in this field, are quite difficult to measure owing to the technical difficulties of producing well collimated beams with also a well defined energy.

To overcome these and other difficulties, a series of indirect methods aiming at measuring the cross sections at hand were developed. The common feature of these methods is that of trying to derive pieces of information on the cross section of a reaction of interest for astrophysics from a that of a different process that is easier to measure from the experimental point of view. Among these methods, very popular ones are the Coulomb Dissociation Method (CD) and the Asymptotic Normalization Constant Method (ANC). A detailed discussion of these methods goes beyond the scope of this paper and here only their gross features are outlined.

The CD was developed since the '80 of last century (see e.g. [2]) and it allows for the measurement of radiative capture reactions at low energies exploiting the photo-dissociation of a suitably chosen nucleus passing through a high electric field generated by virtual photons in the vicinity of a target heavy, high Z nucleus [3]. It is assumed that in the photo-dissociation process the heavy target nucleus remains in its ground state and the process is purely electric. Both these assumptions have been criticized and the excitation of the heavy nucleus as well as the contribution of magnetic multipoles and nuclear potentials of the interacting particles have been shown to have a role in the reaction mechanism at hand. Nonetheless, using more complicated theoretical approaches CD has proven to be an important tool in Nuclear Astrophysics [4].

The ANC also deals with the measurements of radiative captures reactions exploiting direct transfer reactions. The explanation of the method and references to applications of it to particular cases can be found on the review paper on indirect methods [5].

The remaining part of this paper is devoted to an introduction of the Trojan Horse Method (THM) that was developed by the Catania Nuclear Astrophysics group along the last three decades.
2. Trojan Horse Method

The THM has been proposed by G. Baur [6]. In Baur’s approach, the method was based on generic direct reactions while its implementation focusing on a specific reaction mechanism, the quasi free one, was first suggested by C. Spitaleri [7] and widely applied by the Catania Nuclear Astrophysics group since mid 90s of last century [8]. THM is presently widely recognized by the scientific community as a powerful tool to study nuclear processes of astrophysical relevance. A review can be found in the already cited paper [5]. In the present work the method is presented in its simplest, though effective, form. It is also worth noting that THM was developed as an application and an extension of studies done during the ’70s and ’80s of last century by the nuclear physics group in Catania in the field of direct, and in particular quasi-free, reaction mechanisms (see e.g. [9], [10], [11]). THM has also been recently applied to reactions induced by neutrons [12] and also involving radioactive species, by using radioactive ion beams (RIB) [13].

While CD and ANC deal with radiative capture reactions, hence typically with one particles and one photon in the final state, THM allows for the measurement of cross section of thermonuclear reactions with two massive particles in the final state (say, process I)

\[ A + x \rightarrow c + d \] (1)

at astrophysical energies by measuring that of a reaction with three body in the final state (process II)

\[ A + B \rightarrow c + d + s. \] (2)

This is possible provided a quasi-free (QF) reaction mechanism is dominant in process II in a specific kinematical region. In such a case the reaction can be thought to proceed via a polar mechanism that is schematically represented by the Feynman diagram shown in figure 1. In this picture, particle B has a strong x \(\oplus\) s cluster structure. The lower pole in figure 1 represents the virtual decay of B into x and s. Then, particle x is brought inside the nuclear field of nucleus A (while being hidden inside the "trojan horse" nucleus B) and interacts with A giving rise to the reaction of astrophysical interest (I) without suffering the exponential decrease with energy of the tunneling probability that would take place in a direct measurement. Particle s, on the other hand, remains spectator to the process. If one assumes that the virtual decay occurs when B has already overcome the Coulomb barrier due to the presence of A, then x can interact with A without feeling the strong cross section suppression that

\[
\text{Fig. 1. The diagram sketches the pole approximation for the process II. Particle B virtually decays into particle x and s: while s behaves as a spectator to the process, x interacts with A in the (virtual) } \\
A + x \rightarrow c + d \quad \text{reaction.}
\]
it would experience in a direct measurement, as pictorially described in figure 2. This "cross section amplification" effect is actually the reason for introducing any indirect method. The description underlying figure 1 is usually addressed to as the impulse approximation (IA) [14]. This approximation to be valid, the following assumption must be fulfilled: (a) the momentum transfer is sufficiently high, i.e. the associated wavelength is sufficiently small (less than 1 fm); (b) the incident energy in the center of mass system of the three body process is higher than the binding energy of the clusters x-s inside B.

These conditions are necessary but not sufficient ones, so the actual dominance of the QF mechanism has to be checked on a case-by-case base. A powerful, though yet not sufficient, test of the QF mechanism dominance is the Treyman-Yang criterion. This criterion has been presented for high energy reaction by S. Treiman and C. Yang [15] and extended to low energy nuclear physics reaction by I.S. Shapiro [16]. Recently, the Treyman-Yang criterion as been applied by the Catania group to the d(10B; 7Be,α)n reaction used to study the p(10B; 7Be,α) two body process [18, 19].

If the QF mechanism is dominant, the cross section for the three body process can be written e.g. in plane-wave impulse approximation (PWIA) as

\[
\frac{d^3\sigma}{d\Omega\,dE_c\,d\Omega_d} \propto (KF) \left( \frac{d\sigma}{d\Omega} \right)_N |\Phi(p_s)|^2
\]  

where (KF) is a kinematical factor, \(\Phi(p_s)\) is the Fourier transform of the radial wave function of the x-s intercluster motion and \(\left( \frac{d\sigma}{d\Omega} \right)_N\) is the cross section of the virtual two body process, by construction sensitive exclusively to the nuclear part of the interaction between a and x particles as the Coulomb field between the colliding particles has been overcome thanks to the TH mechanism. According to the post collision prescription, this process occurs at a center of mass energy that is given by \(E_{cm} = E_{c-d} - Q_{2b}\), where \(E_{c-d}\) is the relative energy between the c and d outgoing particles and \(Q_{2b}\) is the two-body reaction (process I) Q-value. It has to be stressed that, choosing proper kinematical conditions, \(E_{cm}\) can assume values that can be higher as well as lower than the Coulomb barrier for the x-a system. From Eq. 3 it is clear that one could extract \(d\sigma/d\Omega\) for process I from a measurement of the cross-section of process II. In the case of interest for astrophysics, i.e. when the energy \(E_{cm}\) is lower than the Coulomb barrier, the cross section obtained using the THM cannot be compared immediately with that of direct experiments, as the former misses the suppression factor originating from the presence of the Coulomb barrier. This effect has to be calculated and put back by hand into the THM result in order to compare it with direct data:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{THM} \propto G_i^C \frac{d^3\sigma}{d\Omega\,dE_c\,d\Omega_d} / \left[ (KF) |\Phi(p_s)|^2 \right],
\]
where $G_C$ is the probability for tunneling under the Coulomb barrier for the $x-a$ system that can be calculated theoretically [8]. This $(d\sigma/d\Omega)_{\text{THM}}$ cross section is directly comparable with direct measurement data, once it has been normalized to these data in an energy region higher than that of astrophysical interest (even above the Coulomb barrier).

It is also important to note that the THM cross section $(d\sigma/d\Omega)_{\text{THM}}$ obtained in this way is completely insensitive to the electron screening effects. Actually, the projectile b enters the electron cloud with such a high energy (condition (b) in IA, i.e. order of MeV) that any influence of the electrons on its motion can be disregarded. In this sense, THM not only provides a powerful tool to measure cross sections of astrophysical interest but it also allows for studying the electron screening problem in a decoupled way with respect to direct measurement.

Without entering into details, it is clear that also the centrifugal barrier existing between the particles when the interaction proceeds via waves other then $l=0$ can be accounted for in this framework. As results, the THM mechanism described above can be applied also to reaction involving non-charged particles (i.e. neutrons) as in this case the barrier between the interacting nuclei will be just the centrifugal one. This possibility is of the utmost interest when dealing with process of astrophysical interest induced by neutrons on short lived species (mean life of the order of 1 minute or less), as this kind of reactions cannot, at the present status of the art of experimental facilities, be studied in direct way.

3. Recent THM results: reactions involving RIB and neutrons

![Fig. 3](image)

Fig. 3. Preliminary results for the $^{18}\text{F}(n,\alpha)^{15}\text{N}$ excitation function obtained from $^{18}\text{F}(d,\alpha)^{15}\text{N})p$ via THM . Note the correspondence of the obtained spectrum with the known $\alpha$-decaying levels in $^{19}\text{F}$. The dotted purple ellipse around 1-1.5 MeV roughly indicates a region on $^{19}\text{F}$ excitation energies where no $\alpha$-decaying levels are expected.

Recent results of the THM applied to neutron induced reactions and to reaction involving ra-
Fig. 4. The $^{18}\text{F}(p,\alpha)^{15}\text{O}$ astrophysical S factor from [13]. The full dots are THM experimental data with the assumption of $J^\pi = 3/2^+$ for the resonance at $E = 6460$ keV in $^{19}\text{Ne}$, the open ones correspond to the assumption of $J^\pi = 5/2^-$ (the difference from this last assumption to the other possible value $1/2^-$ and $3/2^-$ being negligible within the errors). The solid and dashed lines shown in figure are calculations presented and discussed in [20] smeared to the present experimental resolution.

dioactive ion beams (RIB) are shown in figure 3 and 4 respectively. Namely, the processes of figure 3 shows preliminary results for the study of the $^{18}\text{F}(n,\alpha)^{15}\text{N}$ process where a radioactive ion interacts with a neutron. Figure 4 shows the results of a recent study on the process of interest for the Nova phenomenon $^{18}\text{F}(p,\alpha)^{15}\text{O}$ reported in [13]. Both results are obtained from experimental runs performed at the CRIB apparatus [17] of the University of Tokyo CNS, based at RIKEN campus in Wako, Japan. In both cases the three body reactions used in the THM analysis were initiated by bombarding a deuterated target (thickness of roughly 150 $\mu$g/cm$^2$) with a 50 MeV $^{18}\text{F}$ beam.

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References
[1] C. E. Rolfs and W. S. Rodney, *Cauldrons in the cosmos: Nuclear Astrophysics*, (University of Chicago Press, Chicago, 1988).
[2] C.A. Bertulani, G. Baur and H. Rebel, Nucl. Phys. A 458, 188 (1986).
[3] E. Fermi, Z. Phys. 29, 315 (1924).
[4] Motobayashi T., and Sakurai H., Prog. Theor. Exp. Phys. 2012, 03C001 (2012).
[5] Tribble R.E., Bertulani C.A., La Cognata M., Mukhamedzhanov A.M., Spitaleri C., Rep. Prog. Phys. 77 106901 (2014).
[6] G. Baur, Phys. Lett. B 178, 135 (1986).
[7] C. Spitaleri, *Proceedings of the Fifth Hadronic Physics Winter Seminar*, Folgaria - TN, Italy, Ed. World Scientific, Singapore, (1990).
[8] S. Cherubini et al., Astrophys. J. 457 (1996) 855.
[9] M. Lattuada, F. Riggi, C. Spitaleri, D. Vinciguerra, Nuovo Cimento A 83, 151 (1984).
[10] M. Zadro et al., Phys. Rev. C 40, 181 (1989).
[11] G. Calvi et al., Phys. Rev. C 41, 1848 (1990).
[12] M. Gulino et al., Phys. Rev. C 87, 012801(R) (2013).
[13] S. Cherubini et al., Phys. Rev. C 92, 015805 (2015).
[14] G. F. Chew and G. C. Wick, Phys. Rev. 85, 636 (1952).
[15] S. Treiman and C. Yang, Phys. Rev. Lett. 8, 140 (1962).
[16] I. S. Shapiro et al., Nuclear Physics 61, 353 (1965).
[17] T. Teranishi, S. Kubono, S. Shimoura et al., Nucl. Phys. A 719, 253c (2003).
[18] S.S. Perrotta, MSc Thesis, Treiman-Yang Criterion as a test for the dominance of the Quasi-Free reaction mechanism: Application to the d(10B; 7Be,α)n process, DFA-Università di Catania, (2018), http://www.infn.it/thesis/thesis_dettaglio.php?id=12675
[19] S.S. Perrotta et al., EPJ Web Conf., Volume 184, 2018, 9th European Summer School on Experimental Nuclear Astrophysics, https://doi.org/10.1051/epjconf/201818402012
[20] C.E. Beer et al., Phys. Rev. C 83, 042801, (2011).