Primordial Magnetic Fields via Spontaneous Breaking of Lorentz Invariance

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Abstract

Spontaneous breaking of Lorentz invariance compatible with observational limits may realistically take place in the context of string theories, possibly endowing the photon with a mass. In this process the conformal symmetry of the electromagnetic action is broken allowing for the possibility of generating large scale ($\sim Mpc$) magnetic fields within inflationary scenarios. We show that for reheating temperatures safe from the point of view of the gravitino and moduli problem, $T_{RH} \lesssim 10^9$ GeV for $m_{3/2} \approx 1$ TeV, the strength of the generated seed fields is, in our mechanism, consistent with amplification by the galactic dynamo processes and can be even as large as to explain the observed galactic magnetic fields through the collapse of protogalactic clouds.

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1. Introduction

Coherent magnetic fields are observed over a wide range of scales in the Universe from the Earth, Solar System and stars to galaxies and clusters of galaxies [1].

Magnetic fields play an important role in a variety of astrophysical processes. For instance, the galactic field affects the dynamics of the galaxy as it confines cosmic rays, influences the dynamics of compact stars and the process of star formation [2]. Large scale magnetic fields are also quite important in quasars, active galactic nuclei and in intercluster gas or rich clusters of galaxies.

The current estimates for the magnetic field of the Milky Way and nearby galaxies is $B \sim 10^{-6}$ G, which are supposed to be coherent over length scales comparable to the size of the galaxies themselves [1]. The origin of the astrophysical mechanisms responsible for these galactic fields is however poorly understood.

Possibly the most plausible explanation for the observed galactic magnetic field involves some sort of dynamo effect [2, 3]. In this mechanism, turbulence generated by the differential rotation of the galaxy enhances exponentially, via non-linear processes, a seed magnetic field up to some saturation value that corresponds to equipartition between kinetic and magnetic energy. A galactic dynamo mechanism along these lines can enhance a seed magnetic field by a factor of several orders of magnitude. Indeed, if one assumes that the galactic dynamo has operated during about 10 Gyr, then a seed magnetic field could be amplified by a factor of $e^{30}$ corresponding to about 30 dynamical timescales or complete revolutions since the galaxy has formed. Hence the observed galactic magnetic fields at present may, via the dynamo amplification, have its origin in a seed magnetic field of about $B \sim 10^{-19}G$.

Naturally, the galactic magnetic fields can emerge directly from the compression of a primordial magnetic field, in the collapse of the protogalactic cloud. In this case, it is required a seed magnetic field of $B \sim 10^{-9}G$ over a comoving scale $\lambda \sim Mpc$, the comoving size of a region which condenses to form a galaxy. Since the Universe through most of its history has behaved as a good conductor it implies that the evolution of any primeval cosmic magnetic field will conserve magnetic flux [4, 5]. Therefore, the ratio denoted by $r$, of the
energy density of a magnetic field \( \rho_B = \frac{B^2}{8\pi} \) relative to the energy density of the cosmic microwave background radiation \( \rho_\gamma = \frac{\pi^2}{15} T^4 \) remains approximately constant and provides an invariant measure of magnetic field strength.

At present, for galaxies \( r \sim 1 \), so pregalactic magnetic fields of about \( r \approx 10^{-34} \) are required if one invokes dynamo amplification processes, and \( r \approx 10^{-8} \) if the observed galactic magnetic fields are created from the compression of the primordial magnetic field in the collapse of the protogalactic cloud without the help of dynamo type processes.

A number of proposals have been put forward to explain the way a primordial magnetic field could be generated (see [6] for a review). In many of these proposals, changes in the nature of the electromagnetic interaction at the period of inflation [4, 7] are involved together with the process of structure formation [4, 8]. Other proposals invoke collision of phase transition bubbles [9], fluctuating Higgs field gradients [10], superconducting cosmic strings [11] and non-minimal coupling between electromagnetism and gravity via the Schuster-Blacket relation [12]. In this work we shall describe a mechanism to produce a seed magnetic field that is based on a possible violation of Lorentz invariance in solutions of string field theory and that uses inflation for amplification of quantum fluctuations of the electromagnetic field. We show that these fluctuations are compatible with both galactic dynamo mechanisms and protogalactic cloud collapse scenario.

Invoking a period of inflation to explain the creation of seed magnetic fields is a quite attractive suggestion as inflation provides the means of generating large-scale phenomena from microphysics that operates on subhorizon scales. More concretely, inflation, through de Sitter-space-produced quantum fluctuations, provides the means of exciting the electromagnetic field allowing for an increase of the magnetic flux before the Universe gets filled with a highly conducting plasma. Furthermore, via a mechanism akin to the superadiabatic amplification, long-wavelength modes, for which \( \lambda \gtrsim H^{-1} \), are during inflation and reheating enhanced. It is certainly quite interesting that inflation can play, for generating primordial magnetic fields, the same crucial role it plays in solving the problems of initial conditions of the cosmological standard model.

However, as pointed out by several authors [4, 7], it is not possible to produce the required
seed magnetic fields from a conformally invariant theory as it happens with the usual U(1) gauge theory. The reason being that, in a conformally invariant theory, the magnetic field decreases as $a(t)^{-2}$, where $a(t)$ is the scale factor of the Robertson-Walker metric, and during inflation, the total energy density in the Universe is constant, so the magnetic field energy density is strongly suppressed, yielding $r = 10^{-104} \lambda_{Mpc}^{-4}$, which is far too low for a seed field candidate. It then follows that conformal invariance of electromagnetism must be broken.

In the context of string theories, conformal invariance may be broken actually due to the possibility of spontaneous breaking of the Lorentz invariance [13] (this breaking can also lead to the breaking of CPT symmetry [14]). This possibility arises explicitly from solutions of string field theory, at least for the open type I bosonic string, as interactions are cubic in the string field and these give origin in the static field theory potential to cubic interaction terms of the type $SSS$, $STT$ and $TTT$, where $S$ and $T$ denote scalar and tensor fields. The way Lorentz invariance may be broken can be seen, for instance, from the static potential involving the tachyon and a generic vector field as can be explicitly computed [13]:

$$V(\varphi, A_\mu, ...) = -\frac{\varphi^2}{2\alpha'} + ag\varphi^3 + bg\varphi V_\mu V^\mu + ..., \quad (1)$$

$a$ and $b$ being order one constants and $g$ the on-shell three-tachyon coupling.

The vacuum of this model is clearly unstable and this instability gives rise to a mass-square term for the vector field that is proportional to $\langle \varphi \rangle$. If $\langle \varphi \rangle$ is negative, then the Lorentz symmetry is spontaneously broken as the vector field can acquire itself a non-vanishing vacuum expectation values. This mechanism can give rise to vacuum expectation values to tensor fields inducing for the fields that do not acquire vacuum expectation values, such as the photon, to mass-square terms proportional to $\langle T \rangle$ (this possibility has been briefly discussed in [13]). Hence, one should expect from this mechanism, terms for the photon such as $\langle T \rangle A_\mu A^\mu$, $\langle T_{\mu\nu} \rangle A_\mu A^\nu$ and so on. Naturally, these terms break explicitly the conformal invariance of the electromagnetic action.

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1 Actually, it is known that negative quadratic mass states corresponding to tachyonic solutions are admitted in the representation space of massive vector particles implying in a violation of the rotational sector of Lorentz invariance [15].
Observational constraints on the breaking of the Lorentz invariance arise from measurements of the quadrupole splitting time dependence of nuclear Zeeman levels along Earth’s orbit, the so-called Hughes-Drever experiment \cite{17, 18}, and have been performed over the years \cite{19, 20}, the most recent one indicating that \( \delta < 3 \times 10^{-21} \) \cite{21}. Bounds on the violation of momentum conservation and existence of a preferred reference frame can be also extracted from limits on the parametrized post-Newtonian parameter \( \alpha_3 \) obtained from the pulse period of pulsars \cite{22} and millisecond pulsars \cite{23}. This parameter vanishes identically in general relativity and the most recent limit \( |\alpha_3| < 2.2 \times 10^{-20} \) obtained from binary pulsar systems \cite{24} implies the Lorentz symmetry is unbroken up to this level. These limits indicate that if the Lorentz invariance is broken then its violation is suppressed by powers of energy over the string scale. Similar conclusions can be drawn for putative violations of the CPT symmetry \cite{14}.

In order to relate the theoretical possibility of spontaneous breaking of Lorentz invariance to the observational limits discussed above we parametrize the vacuum expectation values of the Lorentz tensors in the following way:

\[
<T> = m_L^2 \left( \frac{E}{M_S} \right)^{2l},
\]

where \( m_L \) is a light mass scale when compared to string typical energy scale, \( M_S \), presumably \( M_S \approx M_P \); \( E \) is the temperature of the Universe in a given period and \( 2l \) is a positive integer. Given that the expansion of the Universe is adiabatic, we shall further replace in what follows the temperature of the Universe by the inverse of the scale factor (the proportionality constant is absorbed in the yet unspecified light mass scale, \( m_L \)). Notice that parametrization (2) used here is somewhat different than the ones used in previous work \cite{14, 25, 26}.

2. Generation of Seed Magnetic Fields

We are going to consider spatially flat Friedmann-Robertson-Walker cosmologies, where the stress tensor is described by a perfect fluid with an equation of state \( p = \gamma \rho \). The metric in the conformal time, \( \eta \), is given by:
\[ g_{\mu\nu} = a(\eta)^2 \text{diag}(-1, 1, 1, 1) \tag{3} \]

where \( a(\eta) \) is the scale factor.

The present value of the Hubble parameter is written as \( H_0 = 100 \ h_0 \ \text{km s}^{-1} \ \text{Mpc}^{-1} \) and the present Hubble radius is \( R_0 = 10^{26} \ h_0^{-1} \ \text{m} \), where \( 0.4 \leq h_0 \leq 1 \). We shall assume the Universe has gone through a period of exponential inflation at a scale \( M_{\text{GUT}} \) and whose associated energy density is given by \( \rho_I \equiv M_{\text{GUT}}^4 \). The details of this de Sitter phase are not relevant and will play, as discussed in [4], no role in our mechanism for generating a primordial seed magnetic field. From the Friedmann equation, \( H_{dS} = \left( \frac{8\pi G}{3} \rho_I \right)^{1/2} = \left( \frac{8\pi}{3} \right)^{1/2} \frac{M_{\text{GUT}}^2}{M_P} \), where \( M_P \) is the Planck mass.

From our discussion on the breaking of Lorentz invariance we consider for simplicity only a single term, namely \( \langle T \rangle A_\mu A^\mu \), from which implies the following Lagrangian density for the photon:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu
u} F^{\mu\nu} + M_L^2 a^{-2l} A_\mu A^\mu, \tag{4} \]

where \( M_L^2 \equiv \frac{m_L^2}{M_P^2} \).

The field strength tensor, \( F_{\mu\nu}, \) is given by

\[
F_{\mu\nu} = a(\eta)^2 \begin{pmatrix}
0 & -E_x & -E_y & -E_z \\
E_x & 0 & B_z & -B_y \\
E_y & -B_z & 0 & B_x \\
E_z & B_y & -B_x & 0
\end{pmatrix}
\]

and it satisfies the Bianchi identity

\[ \partial_\mu F_{\lambda\kappa} + \partial_\lambda F_{\kappa\mu} + \partial_\kappa F_{\mu\lambda} = 0 \tag{5} \]

as well as the equation of motion for the photon field

\[ \nabla^\mu F_{\mu\nu} + M_L^2 a^{-2l} A_\nu = 0 \tag{6} \].
Eqs. (5) and (6) can be then explicitly written as:

\[
\frac{1}{a^2} \frac{\partial}{\partial \eta} a^2 \vec{B} + \vec{\nabla} \times \vec{E} = 0 ,
\]

(7)

\[
\frac{1}{a^2} \frac{\partial}{\partial \eta} a^2 \vec{E} - \vec{\nabla} \times \vec{B} - \frac{n}{\eta^2} \vec{A} = 0 ,
\]

(8)

where \( n \equiv -\eta^2 M^2 a^{-2l+2} \) and \( \vec{A} \) is the vector potential.

Taking the curl of Eq. (8) and using Eq. (7) we obtain the wave equation for the magnetic field:

\[
\frac{1}{a^2} \frac{\partial^2}{\partial \eta^2} a^2 \vec{B} - \vec{\nabla}^2 \vec{B} + \frac{n}{\eta^2} \vec{B} = 0 .
\]

(9)

The corresponding equation for the Fourier components of \( \vec{B} \) is given by:

\[
\ddot{\vec{F}}_k + k^2 \vec{F}_k + \frac{n}{\eta^2} \vec{F}_k = 0 ,
\]

(10)

where the dots denote derivatives according to the conformal time and

\[
\vec{F}_k(\eta) \equiv a^2 \int d^3 x e^{ik \cdot \vec{x}} \vec{B}(\vec{x}, \eta) ,
\]

(11)

\( \vec{F}_k \) being a measure of the magnetic flux associated with the comoving scale \( \lambda \sim k^{-1} \). It follows that the energy density of the magnetic field is given by \( \rho_B(k) \propto \frac{|\vec{F}_k|^2}{a^4} \).

For modes well outside the horizon, \( a\lambda >> H^{-1} \) or \( |k\eta| << 1 \), solutions of Eq. (10) are given in terms of the conformal time:

\[
|\vec{F}_k| \propto \eta^{m_{\pm}}
\]

(12)

where

\[
m_{\pm} = \frac{1}{2} \left[ 1 \pm \sqrt{1-4n} \right].
\]

(13)

In order to estimate \( n \) we consider the different phases of evolution of the Universe. By requiring that \( n \) is not a growing function of conformal time, it follows that \( n \) has to be
either a constant or that $2l$ is negative, which is excluded by our assumption (2). Hence for different phases of evolution of the Universe:

(I) Inflationary de Sitter (dS) phase, where $a(\eta) \propto -\frac{1}{\eta H_{dS}}$, it follows that $l = 0$ and

$$n = -\frac{M_{dS}^2}{H_{dS}^2},$$  \hspace{1cm} (14)

where we denote the light mass $M_L$ by the index of the relevant phase of evolution of the Universe.

(II) Phase of Reheating (RH) and Matter Domination (MD), where $a(\eta) \propto \frac{1}{4} H_0^2 R_0^3 \eta^2$, yields from the condition $n$ is a constant that $2l = 3$ and

$$n = -\frac{4M_{MD}^2}{H_0^2 R_0^3}. \hspace{1cm} (15)$$

(III) Phase of Radiation Domination (RD), where $a(\eta) \propto H_0 R_0^2 \eta$, from which follows that $l = 2$ and

$$n = -\frac{M_{RD}^2}{H_0^2 R_0^4}. \hspace{1cm} (16)$$

It is clear that in this case last case $n \ll 1$.

We mention that, we could have obtained the same results comparing Eq. (10) to the corresponding equation of the Fourier modes of a scalar field, coupled non-minimally to gravity through the term $\frac{1}{2} \xi R \phi^2$, where $R = 6 \ddot{a} a^{-3}$ is the Ricci scalar. The relevant Lagrangian density is:

$$\mathcal{L}_\phi = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \xi R \phi^2. \hspace{1cm} (17)$$

In terms of the $k$-th Fourier component of the combination $w = a \phi$, the equation of motion of a non-minimally coupled scalar field can be written as (4):

$$\ddot{w}_k + k^2 w_k + \frac{n_\phi}{\eta^2} w_k = 0, \hspace{1cm} (18)$$

where $n_\phi = \eta^2 (6 \xi - 1) \ddot{a} a^{-3}$ and as before $\rho_\phi(k) \propto |w_k|^2 a^2$. 

Notice that the correspondence between $w_k$ and the components of $\vec{F}_k$ implies, for instance, that if $\xi = \frac{1}{6}$ then the components of $A_\mu$ behave like a scalar field conformally coupled and if $\xi = 0$ then the components of $A_\mu$ behave like a scalar field minimally coupled. Furthermore, a correspondence between $w_k$ and $\vec{F}_k$, requires that $n = n_\phi$ implying in the following condition:

$$\ddot{a} + \frac{M_L^2}{(6\xi - 1)} a^{-2l+3} = 0 .$$ (19)

It is easy to show that for different phases of evolution of the Universe one can obtain, via Eq. (19), essentially the same results for $n$ as the ones we have obtained from requiring that $n$ is constant.

Hence for each phase of evolution of the Universe we find from the corresponding values of $l$ and associated conditions to $M_L$ the following behaviour for $|\vec{F}_k|$:

(I) de Sitter Phase

$$|\vec{F}_k| \propto a^{-m_{\pm}} ,$$ (20)

where

$$m_{\pm dS} = \frac{1}{2} \left[ 1 \pm \sqrt{1 + \left( \frac{2M_{dS}}{H_{dS}} \right)^2} \right]$$ (21)

and

$$\rho_{dS} \propto a^{-2m_{dS}} .$$ (22)

Notice that the most relevant exponent is given by $m_{-dS}$, as it corresponds to the fastest growing solution for $\vec{F}_k$.

(II) Phase of Reheating and Matter Domination

$$|\vec{F}_k| \propto a^{\frac{1}{2}m_{\pm}} ,$$ (23)

where
\[ m_{\pm RH} = \frac{1}{2} \left[ 1 \pm \sqrt{1 + 16 \frac{M_{RH}^2}{H_0^2 R_0^3}} \right], \]  

(24)

and therefore

\[ \rho_B \propto a^{m_{\pm RD} - 4} . \]  

(25)

The relevant exponent here is \( m_{+RD} \), as it corresponds to the fastest growing solution for \( \vec{F}_k \).

(III) Phase of Radiation Domination

\[ |\vec{F}_k| \propto a^{m_{\pm}} , \]  

(26)

where

\[ m_{\pm RD} = 1, 0 \]  

(27)

from which follows that

\[ \rho_B \propto a^{-2} \text{ or } \rho_B \propto a^{-4} . \]  

(28)

As expected, for \( m_{-RD} \) one obtains that \( \rho_B \propto a^{-4} \).

These results enable us to estimate strength of the primordial magnetic field. Assuming the Universe has gone through a period of inflation at scale \( M_{GUT} \) and that fluctuations of the electromagnetic field have come out from the horizon when the Universe had gone through about 55 e-foldings of inflation, then in terms of \( r \) \[ r \approx (7 \times 10^{25})^{-2(p+2)} \times \left( \frac{M_{GUT}}{M_p} \right)^{4(q-p)/3} \times \left( \frac{T_{RH}}{M_p} \right)^{2(2q-p)/3} \times \left( \frac{T_*}{M_p} \right)^{-8q/3} \times \lambda_{Mpc}^{-2(p+2)} , \]  

where \( T_* \) is the temperature at which plasma effects become dominant, that is, the temperature when the Universe first becomes a good conductor and that can be estimated from details of the reheating process \( T_* = \min\{ (T_{RH} M_{GUT})^{\frac{1}{2}} ; (T_{RH}^2 M_p)^{\frac{1}{3}} \} \) \[ r \approx (7 \times 10^{25})^{-2(p+2)} \times \left( \frac{M_{GUT}}{M_p} \right)^{4(q-p)/3} \times \left( \frac{T_{RH}}{M_p} \right)^{2(2q-p)/3} \times \left( \frac{T_*}{M_p} \right)^{-8q/3} \times \lambda_{Mpc}^{-2(p+2)} , \]  

for \( T \leq T_* \), \( \rho_B \) necessarily evolves as \( \propto a^{-4} \). For the reheating temperature we assume either a poor or a quite
efficient reheating, $T_{RH} = \{10^9 \text{ GeV}; M_{GUT}\}$, see however the discussion below. Finally, $p \equiv m_{-dS} = \frac{1}{2} \left[ 1 - \sqrt{1 + \left( \frac{2M_{dS}}{H_{dS}} \right)^2} \right]$ and $q \equiv m_{+RH} = \frac{1}{2} \left[ 1 + \sqrt{1 + 16 \frac{M_P^2}{H_0^2} H_{dS}} \right]$ are the fastest growing solutions for $F_k^i$ in the de Sitter and Reheating phases, respectively.

In order to obtain numerical estimates for $r$ we have to compute $M_L$. At the de Sitter phase we have that $M_{dS} = m_{dS}$. As we have seen $m_L$ is a light energy scale when compared to $M_P$ and $M_{GUT}$, the energy scale of the de Sitter phase. Thus in order to estimate $M_L$ at the de Sitter phase we introduce a parameter, $\chi$, such that $m_{dS} = \chi M_{GUT}$ and $\chi \ll 1$.

At the matter domination phase, we have to impose that the mass term $M_{MD} = m_{MD} \left( \frac{T_\gamma}{M_P} \right)^l$, $T_\gamma$ being the temperature of the cosmic background radiation at about the recombination time, is consistent with the present-day limits of experiments and observations to the photon mass. Thus, at the matter domination phase, we have to satisfy the condition, $M_{MD} \leq m_\gamma$ which implies that $m_{MD} \left( \frac{T_\gamma}{M_P} \right)^{3/2} \leq 3 \times 10^{-36} \text{ GeV}$ [27], following that $m_{MD} \leq 7.8 \times 10^4 \text{ GeV}$. A more stringent bound on $m_{MD}$ could be obtained from the limit $m_\gamma \leq 1.7 \times 10^{-42} h_0 \text{ GeV}$ arising from the absence of rotation in the polarization of light of distant galaxies due to Faraday effect [28].

In tables I, II and III we present our estimates for the ratio $r$ for different values of $M_{GUT}$. One can see that we obtain values that are in the range $10^{-37} < r < 10^{-5}$. For $M_{GUT} = 10^{15}, 10^{16} \text{ GeV}$ one sees that a poor reheating and the lower values for $\chi$ tend to render $r$ too low even for an amplification via dynamo processes. On the other hand, for $M_{GUT} = 10^{17} \text{ GeV}$ a quite efficient reheating leads for $\chi > 5 \times 10^{-2}$ to rather large primordial magnetic fields. Actually, primordial fields greater than $3.4 \times 10^{-9} (\Omega_0 h_{50})^{1/2} \text{ G}$, where $\Omega_0$ is the density parameter at present and $h_{50}$ is the present value of the Hubble parameter in units of $50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, are ruled out as they lead to more anisotropies than the ones observed in the microwave background radiation [29]. Hence, our results indicate that models with $M_{GUT} = 10^{17} \text{ GeV}, \chi > 5 \times 10^{-2}$ and $T_{RH} = M_{GUT}$ should be disregarded.

However, an important issue is that in supersymmetric theories the reheating temperature is severely constrained in order to avoid that gravitinos and moduli are not copiously

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2This constraint is much more stringent than the ones arising from nucleosynthesis that imply, at the end of nucleosynthesis when $T_\gamma \approx 0.01 \text{ MeV}$, that $B_P < 2 \times 10^9 G$ and that $\rho_B \leq 0.28 \rho_\nu$ [30].
regenerated in the post-inflationary epoch. This is indeed a difficulty as as once regenerated beyond a certain density these particles dominate the energy density of the Universe or, if decay, have undesirable effects on nucleosynthesis and lead to distortions of the microwave background. The relevant bounds are the following (see [31] and references therein):

\[ T_{RH} \leq (2 - 6) \times 10^9 \text{ GeV} \quad \text{for} \quad m_{3/2} = (1 - 10) \text{ TeV}. \] (30)

Therefore it follows that for \( M_{GUT} = 10^{15} \ (10^{16}) \text{ GeV} \), only for \( \chi > 5 \times 10^{-4} \ (\chi > 5 \times 10^{-3}) \) the generated seed magnetic fields are large enough. For \( M_{GUT} = 10^{17} \text{ GeV} \), the generated seed fields, although rather small, can be still compatible with observations, for the chosen \( \chi \) values, via the dynamo amplification process.

Finally, we mention that from the comparison with a non-minimally coupled scalar field, our results allow inferring, for example that for \( M_{GUT} = 10^{17} \text{ GeV} \), at the de Sitter phase \( \xi = -0.02 \) and \( \xi = -0.04 \) for \( \chi = 5 \times 10^{-2} \) and \( \chi = 6 \times 10^{-2} \), respectively. This allows us to conclude that the growth in magnetic flux is analogous to the phenomenon of superadiabatic amplification, as the components of \( A_\mu \) behave like the scalar field with a negative mass-square term. For the matter domination phase we find that \( \xi = \frac{1}{6} \), actually the expected result.

3. Conclusions

Galactic dynamo or protogalactic cloud collapse can explain the observed magnetic fields of galaxies at present provided a primordial seed magnetic field is generated prior the Universe turns a good conductor and the magnetic flux turns into a conserved quantity.

A particularly interesting proposal is to invoke inflation for explaining the origin of the primordial magnetic field. This is a quite good idea since inflation naturally relates microphysics with macrophysics as it allows for amplification of quantum fluctuations of fields, and in particular of the electromagnetic field, stretching fluctuations beyond the horizon prior the Universe becomes a good conductor and locks the growth of electromagnetic flux. However, this scenario requires the breaking of the conformal invariance as otherwise the \( r \) will be diluted to quite small values \( r < 10^{-104} \lambda_{Mpc}^{-4} \) at \( Mpc \) scales. In Ref. [4] it was
suggested, in order to break the conformal symmetry of electromagnetism, to introduce in a somewhat *ad hoc* way terms such as $R A_\mu A^\mu$, $R_{\mu\nu} A^\mu A^\nu$, etc, into the electromagnetic action. In this work we have pointed out that the existence of string field theory solutions, in the context of which Lorentz invariance can be spontaneously broken, leads for the photon to terms like $\langle T \rangle A_\mu A^\mu \langle T_{\mu\nu} \rangle A^\mu A^\nu$, hence breaking conformal symmetry. We then showed that this allows inflation to generate rather large seed magnetic fields.

Furthermore, we have demonstrated that the strength of the magnetic field produced by our mechanism is sensitive to the values of a light mass, $m_L$, (cf. Eq. (2)), $M_{\text{GUT}}$ and the reheating temperature $T_{\text{RH}}$. Our results indicate that for rather diverse values of these parameters we can obtain values for $r$ that are consistent with amplification via galactic dynamo or collapse of protogalactic clouds. For $M_{\text{GUT}} = 10^{15}, 10^{16} \text{ GeV}$ we find that poor reheating and $\chi = 5 \times 10^{-4}$ and $\chi = 5 \times 10^{-3}$ tend to give rise to too low $r$ values. For $M_{\text{GUT}} = 10^{17} \text{ GeV}$, very efficient reheating leads for $\chi = 6 \times 10^{-2}$ to rather large primordial magnetic fields, these being actually incompatible with upper limits derived from the study of the microwave background anisotropies. Since in supersymmetric theories the reheating temperature is strongly constrained not to be greater than about $10^9 \text{ GeV}$ for $O(\text{TeV})$ gravitino and moduli masses, our results indicate that for $M_{\text{GUT}} = 10^{15}, 10^{16} \text{ GeV}$ only by choosing $\chi > 5 \times 10^{-4}$ and $\chi > 5 \times 10^{-3}$, respectively, the generated seed fields are large enough for accounting the observations. There is actually no such a limitation for $M_{\text{GUT}} = 10^{17} \text{ GeV}$. 
### Tables

| $\chi$   | $p$   | $q$   | $T_{RH}(GeV)$ | $T_*(GeV)$ | $\log \, r$ |
|----------|-------|-------|---------------|------------|-------------|
| $5 \times 10^{-4}$ | $-1.67$ | $1$   | $10^9$        | $10^{12}$  | $-37$       |
| $5 \times 10^{-4}$ | $-1.67$ | $1$   | $10^{15}$     | $10^{15}$  | $-31$       |
| $6 \times 10^{-4}$ | $-2.08$ | $1$   | $10^9$        | $10^{12}$  | $-21$       |
| $6 \times 10^{-4}$ | $-1.08$ | $1$   | $10^{15}$     | $10^{15}$  | $-13$       |

Table 1: Values of $r = \frac{\rho_B}{\rho_r}$ at 1 Mpc for $M_{GUT} = 10^{15}$ GeV

| $\chi$   | $p$   | $q$   | $T_{RH}(GeV)$ | $T_*(GeV)$ | $\log \, r$ |
|----------|-------|-------|---------------|------------|-------------|
| $5 \times 10^{-3}$ | $-1.67$ | $1$   | $10^9$        | $2.3 \times 10^{12}$ | $-35$       |
| $5 \times 10^{-3}$ | $-1.67$ | $1$   | $10^{16}$     | $10^{16}$  | $-27$       |
| $6 \times 10^{-3}$ | $-2.08$ | $1$   | $10^9$        | $2.3 \times 10^{12}$ | $-18$       |
| $6 \times 10^{-3}$ | $-1.08$ | $1$   | $10^{16}$     | $10^{16}$  | $-9$        |

Table 2: Values of $r = \frac{\rho_B}{\rho_r}$ at 1 Mpc for $M_{GUT} = 10^{16}$ GeV

| $\chi$   | $p$   | $q$   | $T_{RH}(GeV)$ | $T_*(GeV)$ | $\log \, r$ |
|----------|-------|-------|---------------|------------|-------------|
| $5 \times 10^{-2}$ | $-1.67$ | $1$   | $10^9$        | $10^{13}$  | $-33$       |
| $5 \times 10^{-2}$ | $-1.67$ | $1$   | $10^{17}$     | $10^{17}$  | $-24$       |
| $6 \times 10^{-2}$ | $-2.08$ | $1$   | $10^9$        | $10^{13}$  | $-16$       |
| $6 \times 10^{-2}$ | $-1.08$ | $1$   | $10^{17}$     | $10^{17}$  | $-5$        |

Table 3: Values of $r = \frac{\rho_B}{\rho_r}$ at 1 Mpc for $M_{GUT} = 10^{17}$ GeV
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