Enhanced performance of joint cooling and energy production

O. Entin-Wohlman, 1, 2 Y. Imry, 3 and A. Aharony 1, 2

1 Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel
2 Physics Department, Ben Gurion University, Beer Sheva 84105, Israel
3 Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100 Israel

(Dated: October 21, 2014)

The efficiencies/coefficients of performance of three-terminal devices, comprising two electronic terminals and a thermal one (e.g., a boson bath) are discussed. In particular, two procedures are analyzed. (a) One of the electronic terminals is cooled by investing thermal power (from the thermal bath) and electric power (from voltage applied across the electronic junction); (b) The invested thermal power from the boson bath is exploited to cool one electronic terminal and produce electric power. Rather surprisingly, the coefficient of performance of (b) can be enhanced as compared to that of (a).

PACS numbers: 73.50.Lw, 72.20.Pa, 84.60.Rb

I. INTRODUCTION

Achieving high efficiencies and coefficients of performance in thermoelectric nanodevices is one of the main goals of contemporary research on these effects. It is well-known that a strong energy dependence of the electronic transport is a prerequisite for efficient thermoelectric phenomena involving charge carriers, e.g. the Seebeck effect. In a seminal paper Mahan and Sofo proposed (Kedem and Caplan advanced related ideas in 1965) that high values of the thermoelectric efficiency of a two-terminal electronic device are obtained when the energy-dependent conductance has a sharp structure away from the nearly-common chemical potentials of the leads. Following this proposal there were quite a few attempts to achieve effectively narrow electronic bands, especially in nanostructures with transmission resonances and where the enhanced scattering of phonons at interfaces also reduces the phononic heat conductance. Examples are quantum-well superlattices and quantum wires and crystalline arrays of quantum dots. Likewise, the possibility to approach in nanostructures the limit of reversible processes, for which the efficiency is the highest, has been pursued as well as the maximal efficiency at finite output power (see e.g. Ref. [7] and references therein).

A different way to achieve strongly energy-dependent transport is to consider inelastic processes, in which the electrons interchange energy with a boson bath, e.g., photons or electron-hole excitations (see Fig. 1). The boson bath coupled to the electronic system represents the third terminal, making the setup a three-terminal one. In such devices, in addition to the normal thermoelectric effects in the two electronic terminals, there can arise thermoelectric phenomena due to the energy transfer between the thermal terminal and the electronic ones. Physically, this is because the energy exchange between the electronic and bosonic systems induces electronic (charge and heat) currents.

Various three-terminal thermoelectric devices and processes have been proposed: setups designed for cooling-by-heating (CBH) processes, for which the bias voltage is kept zero and one of the electronic terminals is cooled by the thermal bath three-terminal junctions based on molecular bridges, where the charge carriers interchange energy with the vibrations of the molecule forming the bridge; rectification of thermal fluctuations in chaotic cavities two-sites nanostructures based on the inelastic phonon-assisted hopping of cooling a two-dimensional electron gas (which plays the role of the thermal bath) at low temperatures by elastic electron transitions to and from the leads; quantum ratchet converting the nonequilibrium noise of a nearby quantum point contact to dc current carbon nanotubes designed to extract energy from a discrete local oscillator at ultralow temperatures junctions connected to several electronic terminals and cooling the vibrational motion by charge current.

For a two-terminal system converting thermal energy into work or conversely cooling one of the terminals by investing work, the maximal efficiency is achieved in the Carnot thermodynamic cycle, where entropy production vanishes. When there are more than two terminals, one may define various efficiencies (or equivalently, coefficients of performance) and explore their limits in a reversible process. In Sec. II we examine, on general thermodynamic grounds, two cooling scenarios feasible in a three-terminal setup and analyze their efficiencies for zero entropy production. Section III defines the currents and the thermodynamic forces driving them, and uses those to re-express the coefficients of performance introduced in Sec. II. In these two sections we compare the coefficients of performance achieved in the corresponding reversible processes, and show that the one belonging to the three-terminal setup can be higher than the related coefficient of the pure CBH process, but is still lower than the usual Carnot efficiency for cooling by investing work (in a two-terminal junction). Things become more exciting once a specific model is introduced (in Sec. IV), and the efficiency is analyzed allowing for ‘parasitic’ processes in the junction, represented in our case by (unavoidable)
phonon heat conductances (see Sec. [V]). We find that the three-terminal setup has interesting features. (a) When used to cool one of the electronic terminals by investing work (in the form of electric power) and heat from the boson bath, the working range of the device increases as the latter increases; (b) Depending on phonon conductances, the coefficient of performance for joint cooling and power production may be enhanced compared to the situation where work is invested. We summarize our results in Sec. [VI].

II. GENERAL THERMODYNAMIC CONSIDERATIONS

The ubiquitous electronic thermoelectric nanodevice consists of a junction bridging two electronic terminals held at different temperatures and chemical potentials. We denote those by \( \mu_L \) and \( T_L \) for the “left” electronic terminal, and \( \mu_R \) and \( T_R \) for the “right” one. Our three-terminal setup includes in addition a thermal terminal supplying bosons (e.g., phonons, photons, electron-hole excitations), thus allowing for inelastic processes of the charge carriers moving in-between the electronic terminals. The boson bath is kept at yet another temperature, \( T \).

Quite generally, a “double-driving” can be exploited, comprising both work \( W \) (in an important special case where \( I \) is the electric current and \( V \) is the bias voltage) and heat \( Q_T \) (produced by the thermal reservoir at temperature \( T_T \)), to take heat \( Q_L \), from the \( L \) terminal and dump heat \(-Q_R\) into the \( R \) one.\(^{[2]}\) In the usual two-terminal thermoelectric cooling scenario \( Q_T = 0 \) and the cooling is accomplished by the work \( W \). The other particular scenario is when no work is invested, \( W = 0 \), and cooling is achieved by \( Q_T \), i.e., the CBH process. To interpolate between these two limits we introduce the parameter \( \alpha = W/Q_T \), in the first scenario \( \alpha = \infty \) and in the second \( \alpha = 0 \). In the thermoelectric model of Ref. \(^{[1]}\) used below, \( \alpha \) depends on the model parameters and on the ratio of the driving forces, \( Q_T \) and the bias voltage producing the work.

The cooling efficiency for this process, i.e., the COP, is defined by

\[
\eta_a = \frac{Q_L}{W + Q_T} = \frac{Q_L}{-Q_R - Q_L},
\]

where the second equality results from energy conservation

\[
-Q_R = Q_L + W + Q_T.
\]

Since the reservoirs are each in equilibrium, the corresponding entropies are

\[
S_\ell = Q_\ell/T_\ell, \quad (\ell = L, R, T), \quad (4)
\]

and the total entropy production in the process, \( S \), is

\[
S = S_L + S_T + S_R. \quad (5)
\]

In a reversible process \( S = 0 \); the corresponding COP, \( \eta'_a \), is then [using Eqs. (3), (4), and (5)]

\[
\eta'_a = \eta_{cbe} \bigg[ 1 - \frac{Q_R}{Q_T(1 + \alpha)} \bigg]. \quad (6)
\]

Here

\[
\eta_{cbe} = \frac{T_L}{(T_R - T_L)}, \quad (7)
\]

is the efficiency (COP) of the first scenario above, where cooling is achieved by investing work alone. As expected, large values of \( \alpha \) give \( \eta_{cbe} \) while \( \alpha = 0 \) yields the maximal efficiency of the cooling-by-heating process,

\[
\eta_{cbh} = \eta_{cbe} [1 - (T_R/T_T)] = 1 - \frac{(T_R/T_T)}{(T_R/T_L)}. \quad (8)
\]

Any finite positive \( \alpha \) gives \( \eta'_a > \eta_{cbh} \), as follows trivially from Eq. (6). So, we lose efficiency by adding the CBH power. On the other hand, \( \eta'_a > \eta_{cbh} \), so we gain efficiency compared to a pure CBH process. The advantages of the double driving are that we get more cooling power, that the working region of the device increases (see below) and that for realistic purposes, one could use the free sun as the thermal terminal, which reduces the efficiency by a few percent only.

Next we consider the case where part of the invested heat \( Q_T \) from the thermal terminal is exploited to cool the left electronic terminal and part of it is converted to useful work, \( W \). Energy conservation in this case yields

\[
Q_T + Q_L = W - Q_R, \quad \text{while the efficiency is}
\]

\[
\eta_b = \frac{Q_L + W}{Q_T}. \quad (9)
\]
Since the heat from the thermal terminal produces work in addition to cooling, the ratio \( Q_L/Q_T \) in a reversible process deviates from \( \eta_{\text{cbh}} \), Eq. [5]. Indeed, using Eq. [4] in conjunction with energy conservation yields

\[
\frac{Q_L}{Q_T} = \eta_{\text{cbh}} - \frac{W/Q_T}{(T_R/T_L) - 1} .
\]  

(10)

Note that when \( W = 0 \) this result reproduces \( \eta_{\text{ir}} \), Eq. [6] for the \( \alpha = 0 \) scenario there. On the other hand, we now find that

\[
\eta_{\text{ir}} = \eta_{\text{cbh}} + \frac{W}{Q_T} \left( 1 - \frac{1}{(T_R/T_L) - 1} \right) .
\]  

(11)

It is seen that for \( T_R - T_L < T_L \), the efficiency increases with \( |W| \) for \( W < 0 \), the more so with decreasing \( T_R - T_L! \).

For completeness, we recast the above reasoning into the configuration in which the thermal terminal is cooled by both work done (e.g., by electrical current between the configuration in which the thermal terminal is cooled and for a reversible process \([i.e.,|W| = 0]\) scenario there. On the other hand, we now find that

\[
\eta_{\text{ir}} = \eta_{\text{cbh}} - \frac{W}{Q_T} \left( 1 - \frac{1}{(T_R/T_L) - 1} \right) .
\]  

(12)

and for a reversible process \([i.e., S = 0, \text{see Eq. [5]}]\)

\[
\eta_{\text{ir}} = \eta' [1 - \frac{T_R}{T_L (1 + \alpha')}] ,
\]  

(13)

where \( \alpha' = W/Q_L \) and \( \eta' = T_R/(T_R - T_L) \), the usual Carnot efficiency for cooling a terminal at temperature \( T_T \) by moving heat to a terminal having temperature \( T_R \), using pure work.

When \( \alpha' \to \infty \) the reversible-process efficiency becomes simply \( \eta' \), while for \( \alpha' \to 0 \) the expression in the square brackets in Eq. [13] becomes \( 1 - (T_R/T_L) \). This is just the Carnot efficiency for converting the heat moved between the left and the right terminals to work. Equation [13] is then the reversible efficiency for CBH, as found and explained by Ref. 8 [see their Eq. (11)].

The above is very general, beyond linear transport. It is straightforward to introduce the (negative) correction to the efficiency when the total entropy production (waste) is positive. This is discussed in Sec. V.

III. CURRENTS AND FORCES

Having set the thermodynamic basis for defining efficiencies (or coefficients of performance) in a three-terminal device, we proceed to examine those from another point of view, by considering the currents flowing in the system and the thermodynamic forces driving them, within linear response.

The thermodynamic driving forces and the currents conjugate to them can be defined unambiguously by considering the entropy production of the device. In terms of the heat/entropy production in the thermal terminal and in the electronic \( L \) and \( R \) terminals, \( \dot{Q}_L, \dot{Q}_R \) respectively, the entropy production is [see Eqs. [4] and [5]]

\[
\dot{S} = \frac{\dot{Q}_L}{T_L} + \frac{\dot{Q}_R}{T_R} ,
\]  

(14)

where \( S \) is the total entropy of the system. The heat produced in each of the electronic reservoirs is \( \dot{Q}_L = E_L - \mu_L N_L \), \( L = L \) or \( R \) (\( E_L \) is the total energy of the \( t \)-th electronic reservoir, and \( N_L \) is the number of charge carriers there). Energy conservation implies that

\[
\dot{Q}_L + \dot{Q}_R = -\mu_L \dot{N}_L - \mu_R \dot{N}_R ,
\]  

(15)

while charge conservation gives \( \dot{N}_L + \dot{N}_R = 0 \). Since the particle current emerging from the left electronic terminal is

\[
J_L = -\dot{N}_L ,
\]  

(16)

it is seen that the right-hand side of Eq. (15) is just the Joule heating in the system, \( J_L (\mu_L - \mu_R) \).

Exploiting the conservation laws, the entropy production relation Eq. (14) becomes

\[
T_R \dot{S} = J_L (\mu_L - \mu_R) + J_L^Q \left( 1 - \frac{T_R}{T_L} \right) + J_R^Q \left( 1 - \frac{T_L}{T_R} \right) ,
\]  

(17)

Here we have chosen the temperature of the right electronic bath as the reference temperature, and introduced the heat currents leaving the left terminal

\[
J_L^Q = -\dot{Q}_L ,
\]  

(18)

and the one leaving the thermal one

\[
J_R^Q = -\dot{Q}_R .
\]  

(19)

As usual, the entropy production Eq. (17) appears as a (scalar) product of a vector consisting of the three currents, \( J_L, J_L^Q, \) and \( J_R^Q \), with a vector comprising the driving forces, the electric one,

\[
\mu_L - \mu_R = |e| V ,
\]  

(20)

and the two thermal ones, given by the temperature difference across the electronic junction and between the electronic system and the boson bath. We shall confine ourselves to the configuration where the left electronic terminal is to be cooled and will also use the thermal terminal as the chief power supplier. Then the lowest temperature of the three is \( T_L \) and the highest one is \( T_R \). Accordingly, the thermal driving forces are \( (T_R/T_L) - 1 \) and \( 1 - (T_R/T_L) \).

One can imagine various scenarios for cooling the \( L \)-electronic terminal in the three-terminal junction depicted in Fig. 1. Here, as in Sec. III, we focus on two specific situations. We present for each of them the relevant
The COP coefficient of performance (COP) and then examine the best value it can have, which is obtained when the cooling and the accompanying processes are reversible, i.e., the entropy production vanishes. The power(s) obtained in such a process is (are) unfortunately vanishingly small. Nonetheless, it is instructive to study these upper limits that are the target of many studies in thermoelectricity [an example is the proposal of Ref. 1]. More realistic configurations and the corresponding coefficients of performance will be considered in Sec. V.

(a) As in Sec. [I] the L–electronic terminal can be cooled by investing power extracted from the thermal terminal and an electric power. The COP of this process, denoted \( \eta_a \), is the ratio of the thermal power gained and the two invested powers:

\[
\eta_a = \frac{J_L^Q}{|e| J_L V + J_T^2}. \tag{21}
\]

The COP attains its highest possible value in a reversible process, for which \( \dot{S} = 0 \). Indeed, upon using Eq. (17) for \( \dot{S} = 0 \) we find that in a reversible process

\[
\eta_a^{\text{rev}} = \frac{1}{(T_R/T_L) - 1} \times \frac{|e| J_L V + |1 - (T_R/T_L)| J_L^Q}{|e| J_L V + J_T^2}. \tag{22}
\]

When the electric power vanishes Eq. (22) reproduces the COP of the ‘cooling-by-heating’ process in the reversible limit given in Eq. [3], while in the more mundane scenario of cooling by investing only electric power, the COP \( \eta_{\text{cbe}} \) is given by Eq. (7).

Obviously, when the entropy production vanishes (or is rather small) the cooling-by-heating process is less effective than cooling by electric power; the joint three-terminal COP lies in-between these two limits,

\[
\eta_{\text{cbe}} < \eta_a^{\text{rev}} < \eta_{\text{cbe}}. \tag{23}
\]

The best performance in a reversible process is thus reached upon cooling by investing electric power. However, as compared to the reversible cooling-by-heating option proposed by Cleuren et al., for which the electric voltage vanishes, investing electric power in addition to the thermal one improves the COP. As mentioned, the three-terminal arrangement also extends the range of the ‘working condition’ of the device as compared to a two-terminal setup, see Sec. [V].

(b) The left electronic terminal is cooled and at the same time electric power is being produced. Accordingly, the ratio between the powers gained to that invested, i.e., the COP \( \eta_b \), is

\[
\eta_b = \frac{J_L^Q + |eVJ_L|}{J_T^2}. \tag{24}
\]

When the process is reversible the COP of this process is

\[
\eta_b^{\text{rev}} = \left(1 - \frac{T_R}{T_T}\right) \frac{J_L^Q + |eVJ_L|}{J_T^2/[\theta(R/T_L) - 1] + |eVJ_L|}. \tag{25}
\]

We see that the reversible value of the COP when the voltage vanishes is given by Eq. (8). On the other hand, if the electronic left bath is not cooled at all, then the device works as a ‘solar cell’ (in the case where the thermal terminal is the sun), yielding electric power in response to the temperature difference between the electronic system and the thermal terminal. The best value of the COP for this configuration is the textbook Carnot efficiency \( \eta_C \)

\[
\eta_C = 1 - \frac{T_R}{T_T}. \tag{26}
\]

As in case (a) above, \( \eta_{\text{rev}}^{\text{b}} \) is bounded in-between its two limiting values. But in contrast to case (a), which of the two is the bigger and which is the smaller depends on the temperature difference across the electronic reservoirs. When that temperature difference is significant, \( (T_R/T_L) - 1 > 1 \), i.e., \( \eta_{\text{cbe}} < 1 \) [Eq. (7)] we find

\[
\eta_C < \eta_{\text{rev}}^{\text{b}} < \eta_{\text{cbe}}, \quad \eta_{\text{cbe}} < 1. \tag{27}
\]

When the temperature difference across the electrons is small then the inequality is reversed, i.e.,

\[
\eta_{\text{cbe}} < \eta_{\text{rev}}^{\text{b}} < \eta_C, \quad \eta_{\text{cbe}} > 1. \tag{28}
\]

IV. AN EXAMPLE OF A THREE-TERMINAL THERMOELECTRIC DEVICE

In order to consider the coefficients of performance away from reversibility one needs to invoke a specific system and find for it the currents flowing in the setup in response to the driving forces. In the linear-response regime the relation between two vectors, the one of the currents and the one of the driving forces [cf. Eq. (17) and the discussion around it] can be written in a matrix form

\[
\begin{bmatrix}
|e|J_L \\
J_L^Q \\
J_T^Q
\end{bmatrix} = \frac{1}{T_R} M \begin{bmatrix}
V \\
-(T_R/T_L) - 1 \\
1 - (T_R/T_T)
\end{bmatrix}, \tag{29}
\]

where the \((3 \times 3)\) matrix \( M \) comprises the transport coefficients. Within the linear-response regime \( M \) does not depend on the driving forces and is determined by the thermal-equilibrium properties of the setup. It then obeys the Onsager relations; for instance, for a system invariant to time-reversal—the case studied here—it is symmetric. One notes that the matrix \( M \) is nonnegative definite, to ensure the positiveness, or vanishing, of the entropy production \( \dot{S} \), Eq. (17). The matrix \( M \) may be singular, for example, when the first two rows are proportional to one another and then the two corresponding currents (for example, the electronic charge and thermal currents) are proportional to each other. This special situation, which was termed by Kedem and Caplan‘strong coupling’, is also behind the mechanism of Mahan and Sofo involving transport in a very narrow energy band.
This yields high values of the COP. In the case of Ref. [4], each transferred electron carries the same energy and heat. We will come back to this point below. It can also happen that the third row is also proportional to the first one and then all the three currents, in the three-terminal case, are proportional to each other, which may be termed 'full strong coupling'.

We obtain the transport coefficients, i.e., the matrix $\mathcal{M}$, for the two-level model of Ref. [11], depicted in Fig. 2. The model of Ref. [8] has two such two-level pairs, whose effects add in the cooling and subtract in the electrical current, causing the latter to vanish. We assume that the boson-assisted hopping is the strongly-dominant electronic transport channel. The specific conditions for neglecting elastic (so-called co-tunneling) transmission are analyzed in detail in Ref. [11]. Any relevant inelastic transmission will be exponentially small when the temperature approaches zero. As in previous publications on this issue [8], we discuss noninteracting quasiparticles.

Under these circumstances, the matrix $\mathcal{M}$ gives the relations among the currents and the driving forces [see Eq. (29)], is

$$
\frac{1}{V_T} \mathcal{M} = G_{in} \begin{bmatrix}
\frac{1}{E/|e|} \left\{ T_T/|e| \right\} & \frac{1}{E/|e|} \left\{ E/|e| \right\} & \frac{1}{E/|e|} \left\{ E/|e| \right\} \\
\frac{1}{|e|} & \frac{1}{E/|e|} & \frac{1}{E/|e|} \\
0 & 0 & 0
\end{bmatrix}
$$

Here $G_{in}$ is the hopping conductance of the electronic junction. The first term on the right-hand side of Eq. (32) contains the strongly-coupled part of the transport matrix. The second term there, describing the parasitic phonon conductances $K_p$ and $K_{PP}$, spoils this 'strong coupling' property.

The working condition [23] for cooling the left electronic terminal requires that the heat current emerging from it be positive. In our model, this amounts to

$$
\frac{|e|V}{E} - \left( \frac{T_R}{T_L} - 1 \right) + \frac{\omega}{1 - \frac{T_R}{T_T}} \geq \frac{e^2 K_p}{E} G_{in} \left( \frac{T_R}{T_L} - 1 \right).
$$

Note that the phonon thermal conductance on the right-hand side, $K_p$, is scaled by the "bare" electronic thermal conductance of the junction, $G_{in} E^2/e^2$ [the transport coefficient contained in the 22- element of the matrix $\mathcal{M}$, see Eq. (32)].

It is illuminating to re-examine the entropy production Eq. (17) in the framework of our model in conjunction with the working condition for cooling the left electronic terminal. Using Eq. (32) in the relation (29), and inserting the resulting currents into Eq. (17) yields

$$
T_R \dot{S} = \frac{G_{in}}{e^2} \left[ \frac{|e|V - E/|e| \left( \frac{T_R}{T_L} - 1 \right) + \omega \left( 1 - \frac{T_R}{T_T} \right)}{e^2} \right]^2
+ K_p \left( \frac{T_R}{T_L} - 1 \right)^2 + K_{PP} \left( 1 - \frac{T_R}{T_T} \right)^2.
$$
As might have been expected, the “parasitic” heat flows carried by the phonons [the last two terms in Eq. (34)] make the entropy production positive and the thermoelectric transport irreversible. When these are ignored (or are very small) and our setup approaches the strong-coupling limit then the cooling process will be reversible provided that \(|e|V/\mathcal{E} - [(T_R/T_L) - 1] + (\omega/\mathcal{E})[1 - (T_R/T_T)] = 0\), i.e., when the heat flowing out of the left electronic terminal vanishes. In other words, when the cooling process is reversible it yields, as is always the result, zero output power.

V. IRREVERSIBLE COEFFICIENTS OF PERFORMANCE

Exploiting the transport coefficients of the specific device described in Sec. IV the electric current leaving the left terminal is

\[
|e|J_L = \frac{G_{\text{in}}}{|e|} \frac{\mathcal{E}}{E} \left[ |e|V + T_R T_L \right] + \frac{\omega}{\mathcal{E}} \left( 1 - \frac{T_R}{T_T} \right). \tag{35}
\]

Likewise, the electronic heat current from that terminal is

\[
J^Q_L = \frac{\mathcal{E}}{|e|} (|e|J_L - K_P \left(T_L^2 - 1\right)) \tag{36}
\]

The working condition for the left terminal to be cooled is the positiveness of \(J^Q_L\); therefore \(J_L\) is necessarily positive. This means that as long as the bias voltage is positive, electric power is being invested in the system, while when \(V\) is negative (i.e., the electric current flows against the voltage) the device produces electric power. Thus, the COP is given by \(\eta_a\), Eq. (21), when the voltage is positive and by \(\eta_b\), Eq. (24), when it is negative.

Adding the expression for the heat current leaving the boson bath

\[
J^Q_T = \frac{\omega}{|e|} (|e|J_L + K_{PP} \left(1 - \frac{T_R}{T_T}\right)), \tag{37}
\]

we find that the COP’s of our system are

\[
\eta_a = \frac{B - \kappa_P \left(T_L^2 - 1\right)}{\frac{|e|V + \omega}{\mathcal{E}} B + \kappa_{PP} \left(1 - \frac{T_R}{T_T}\right)} \tag{38}
\]

for \(V \geq 0\), and

\[
\eta_b = \frac{(1 - \frac{|e|V}{\mathcal{E}}) B - \kappa_P \left(T_L^2 - 1\right)}{\frac{\omega}{\mathcal{E}} B + \kappa_{PP} \left(1 - \frac{T_R}{T_T}\right)} \tag{39}
\]

for \(V \leq 0\). Here we have introduced for brevity the notation

\[
B = \frac{|e|V}{\mathcal{E}} + \frac{T_R}{T_L} + \frac{\omega}{\mathcal{E}} \left(1 - \frac{T_R}{T_T}\right), \tag{40}
\]

and measured the phonon conductances in terms of the bare electronic heat conductance,

\[
\kappa_p = e^2 K_p / (\mathcal{E}^2 G_{\text{in}}), \quad \kappa_{PP} = e^2 K_{PP} / (\mathcal{E}^2 G_{\text{in}}). \tag{41}
\]

Note that the bias should exceed a certain threshold, \(V_c\), dictated by the working condition Eq. (33),

\[
\frac{|e|V_c}{\mathcal{E}} = \frac{(T_R T_L - 1)}{(1 + \kappa_p) - \frac{\omega}{\mathcal{E}} \left(1 - \frac{T_R}{T_T}\right)}. \tag{42}
\]

As discussed in Sec. III joint cooling and energy harvesting requires negative values of the bias voltage, and hence negative values of the threshold \(V_c\); a large enough phonon conductance \(K_P\) (the actual value depends on the model parameters) will therefore prevent such a combined action.

The figures below display the COP’s \(\eta_a\) and \(\eta_b\) as functions of the bias voltage, the former for positive values of the bias voltage and the latter for its negative values, as explained in Sec. III; see Eqs. (21) and (24). In all figures the temperature difference between the electronic \(L\) and \(R\) terminals is kept constant, chosen to be \(T_R/T_L = 3/2\) and yielding \(T_{\text{Carnot}} = 2\) for the Carnot coefficient of performance for refrigeration by investing work in a two-terminal setup [see Eq. (7) and the discussion in Sec. III]. The various curves in each figure are for different values of \(1 - T_R/T_T\) (explicit values are given in the captions). The parasitic phonon conductances in dimensionless units, see Eqs. (41) of the electronic junction \((\kappa_P)\) and between the electronic system and the thermal terminal \((\kappa_{PP})\) are taken to be equal for definiteness and vary from 4 to 0.2, whereupon the COP increases. In all three figures we see that the ‘working regime’, i.e., the voltage range over which cooling is possible, increases with the driving force of the thermal terminal \(1 - T_R/T_T\), namely the threshold \(V_c\) decreases.

Figure 3 is for \(\kappa_P = \kappa_{PP} = 4\); cooling exists only beyond a positive threshold of the bias voltage [see Eq.
so no joint cooling and energy harvesting is possible. However, that is already possible for $\kappa_P = \kappa_{PP} = 2$, as shown in Fig. 4. Very interestingly, the COP increases with decreasing $V < 0$ and has a small peak, which is yet smaller than the one for $V > 0$. What is, seemingly, most surprising is that for even smaller parasitics, e.g., $\kappa_P = \kappa_{PP} = .2$, still, away from the reversible limit, as seen in Fig. 5 the values of the COP at $V < 0$ can be substantially larger than those for $V > 0$. This means that harvesting energy may actually *increase the COP compared to investing energy*, a truly remarkable property of the three-terminal setup!

Note also that, again for small negative voltages, the COP increases with the absolute value of the voltage. This is understandable, considering the reversible limit, Eq. (11). There $W$ represents the useful work the thermal terminal is providing, in addition to cooling the left electronic terminal. From the discussion around that equation in Sec. 11 it is seen that for $T_R - T_L < T_L$, the efficiency increases with $|W|$ for $W < 0$, the more so with decreasing the temperature difference across the electronic system, $T_R - T_L$. This tendency is exemplified by Figs. 6 which display $\eta_b$, Eq. (39), as a function of that temperature difference for negative values of the voltage. It is seen that indeed $\eta_b$ increases as the absolute value of $V$ increases. However, because of the condition Eq. (42) necessary for the device to operate, the range of temperature differences between the thermal terminal and the electrons is reduced as well. The various curves in each of the panels in Figs. 6 correspond to different values of $1 - (T_R/T_L)$. As $|V|$ is increased, the working condition Eq. (42) allows only for $T_R$–values which are closer and closer to $T_T$. 

![FIG. 4: (Color online) The efficiency of a three-terminal junction as a function of $|eV/E|$ for various values of $1 - T_R/T_T$, for $\kappa_P = \kappa_{PP} = 2$, $(T_R/T_L) - 1 = 0.5$ and $\omega/E = 2$. The various curves in the upward direction are for $1 - T_R/T_T = 0$ [the lowest solid (purple) line], 0.25, 0.5, 0.75 and 1 [the upper dotted (magenta) curve].](image1)

![FIG. 5: (Color online) The efficiency of a three-terminal junction as a function of $|eV/E|$ for various values of $1 - T_R/T_T$, for $\kappa_P = \kappa_{PP} = 0.2$, $(T_R/T_L) - 1 = 0.5$ and $\omega/E = 2$. The various curves in the upward direction are for $1 - T_R/T_T = 0$ [the lowest solid (purple) line], 0.25, 0.5, 0.75 and 1 [the upper dotted (magenta) curve].](image2)

![FIG. 6: (Color online) The efficiency $\eta_b$, Eq. (39) as a function of the temperature difference across the electronic system, for various values of the temperature difference between the thermal reservoir and the electrons, $1 - (T_R/T_L) = 0.25$, the dashed (brown) curve, $= 0.5$, the dotted (magenta) curve, $= 0.75$, the solid (green) line and $= 1$, the dashed (blue) curve. The upper panel is for $|eV/E| = -0.1$, the mid one is for $|eV/E| = -0.5$, and the lower panel is for $|eV/E| = -1$. The working condition Eq. (42) is not fulfilled for all values of $1 - (T_R/T_L) = 0.25$ and therefore some of the curves are missing in the two lower panels. Here $\kappa_P = \kappa_{PP} = 0.05$, and $\omega/E = 2$.](image3)
VI. SUMMARY AND CONCLUSIONS

We have analyzed the efficiency of two cooling processes possible in a mixed three-terminal thermoelectric junction, comprising two electronic terminals that interchange energy with a third, thermal contact. For concreteness, we have concentrated mainly on the possibility to cool one of the electronic terminals, either by investing thermal energy extracted from the thermal terminal and electric power supplied by a bias voltage on the electrons, or by exploiting the thermal energy of the boson bath to cool the electronic terminal and to produce electric power. (Cooling of the thermal contact has been mentioned in Sec. II.) We have found that one advantage of the three-terminal setup compared with the two-terminal one is the increase in the working regime of the device. In our case, this is manifested by the extended range of bias-voltage values for which cooling is possible. This increase is dominated by the parasitic phonon conductance; in our case it is the phonon conductance, $K_p$, of the electronic junction, which should not be too large. With a suitable choice of $K_p$ the threshold of the voltage becomes negative, and then the three-terminal setup cools and at the same time, produces electric power. In our model system, we find that the COP for this dual action can be enhanced compared with its value (for the same parameters) obtained when the device works just as a refrigerator. This enhancement, necessitating not-too-large parasitic thermal conductances, should become huge when the cooling is for a small cooling temperature increment, $T_R – T_L << T_L$.

Acknowledgments

This work was supported by the Israeli Science Foundation (ISF) and the US-Israel Binational Science Foundation (BSF). We thank J-H Jiang, J-L Pichard, D. Shalhar, G. C. Tewar and G. Zeltzer for discussions on related questions.