Impulsive Pressure Activity on MHD Flux Generalized Burgers Fluid

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Abstract. The main purpose of the work is to analyse studies of the magnetohydrodynamic "MHD" flow for a fluid of generalized Burgers’ "GB" within an annular pipe submitted under impulsive pressure "IP" gradient. Closed form expressions for the velocity profile, impulsive pressure gradient have been taken by performing the finite Hankel transform "FHT" and Laplace transform "LT" of the successive fraction derivatives. As a result, many figures are planned to exhibit the effects of (different fractional parameters "DFP", relaxation and retardation times, material parameter for the Burger’s fluid) on the profile of velocity of flows. Furthermore, these figures are compared the flow without MHD.

Keywords: Generalized Burgers’ fluid, impulsive pressure gradient, magnetohydrodynamic flow.

1. Introduction

Due to the increasing interest in the flow of different types of Newtonian and non-Newtonian fluids, Kashif and others [1] discussed general solutions for six types of liquids and described various fluid flow standards. Muhammad [2] also compared the effect of parameters on the velocity of Newtonian and non-Newtonian fluids. A burger model is a rapidly developing model and applied to describe various viscous materials in many areas like asphalt in geology mechanics, blood in medical sciences, cheese in food products and much else, see [3], [4], [5], [6].

MHD flow problems have been attracted considerable attention due to extensive engineering and medical applications. Moreover, MHD principles are used in plasma flows, nuclear reactor dynamics, pumps, radar systems…etc. Recently MHD flow is extensively studied by the group of researchers, see [7], [8], [9], [10], [11].

Many problems began to appear by adding different types of pressure. In this regard, Liancun and others [12] added the constant pressure on the flow of three-dimensional. Hanan F. and Ghada H. [13] discussed the results of a generalized fluid flow of an unstable viscous burger in the annular tube of a fractional model under impulsive pressure.
Our interest in this paper is to study the flow of unstable viscous liquid in a conical tube with the generalized fractional liquid model under impulsive pressure exerted by magnetic field MHD. We compare it with flow under impulsive pressure without magnetic field[13].

2. Dominant Equations

The constitutive equations of a GB fluid are compressible and given by

\[ T = -p + \mathbf{S} , \quad \left( 1 + \lambda_2^a \mathbf{D}_t^a + \lambda_2^b \mathbf{D}_t^b \right) \mathbf{S} = \mu \left( 1 + \lambda_3^b \mathbf{D}_t^b \right) \mathbf{A}_1 \]

(1)

Where \( T \) refers to Cauchy stress, \(-p\) is a non-specific spherical stress, \( \mathbf{S} \) indicates the extra stress tensor, \( \mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T \) is the first Rivlin- Ericksen tensor with gradient the velocity progression where \( \mathbf{L} = \mathbf{g} \), \( \mu \) means viscous fluid dynamic , \( \lambda_1 \) and \( \lambda_3 \) (\(< \lambda_4 \)) are the relaxation and retardation times, respectively, \( \lambda_4 \) is a new material parameter for the Burger’s fluid, \( \alpha \) and \( \beta \) are the parameters of the fractional calculus such as \( 0 \leq \alpha \leq \beta \leq 1 \) and \( \mathbf{D}_t^\alpha \) is the upper converted fractional derivative attributive and defined by

\[ \mathbf{D}_t^\alpha \mathbf{S} = \mathbf{L}_t^\alpha \mathbf{S} + (\nabla \cdot \nabla) \mathbf{S} - \mathbf{L} - \mathbf{S} \mathbf{L}^T, \]

\[ \mathbf{D}_t^\beta \mathbf{A}_1 = \mathbf{L}_t^\beta \mathbf{A}_1 + (\nabla \cdot \nabla) \mathbf{A}_1 - \mathbf{L} \mathbf{A}_1 - \mathbf{A}_1 \mathbf{L}^T \]

(2)

where \( \mathbf{D}_t^\alpha \) and \( \mathbf{D}_t^\beta \) are the fractional differentiation operators of order \( \alpha \) and \( \beta \) depend on the Riemann-Liouville definition, which are described as follows:

\[ \mathbf{L}_t^\alpha [f(t)] = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} f(\tau) \, d\tau , \quad 0 \leq \alpha \leq 1 \]

\[ \mathbf{L}_t^\beta [\mathbf{S}] = \mathbf{L}_t^\beta (\mathbf{L}_t^\beta \mathbf{S}) \]

(3)

here \( \Gamma(\cdot) \) is the Gamma function.

We assume that the generalized normal Oldroyd-B model has an one-way flow for both the velocity field and shear stress as shown

\[ \mathbf{V}(r, t) = \omega(r, t) \mathbf{e}_y \]

\[ \mathbf{S} = \mathbf{S}(r, t) \]

(4)

where \( \mathbf{e}_y \) is the unit vector along \( y \)-direction. Replacing equation (4) into (1) and taking consideration of the initial situation

\[ \mathbf{S}(r, 0) = 0 \]

(5)

We obtain

\[ (1 + \lambda_2^a \mathbf{D}_t^a + \lambda_2^b \mathbf{D}_t^b) \mathbf{S}_r = \mu \left( 1 + \lambda_3^b \mathbf{D}_t^b \right) \partial_x \omega(r, t) \]

\[ (1 + \lambda_2^a \mathbf{D}_t^a + \lambda_2^b \mathbf{D}_t^b) \mathbf{S}_y = 2\mathbf{S}_r \left( \lambda_2^a + \lambda_2^b \mathbf{D}_t^b \right) \partial_x \omega(r, t) = -2\mu \lambda_3^b (\omega(r, t))^2 \]

\[ \mathbf{S}_r = \mathbf{S}_y = \mathbf{S}_x = \mathbf{S}_{xx} = 0 \]

(6)

Moreover, By adding a magnetic field \( \mathbf{H} = \left[ 0, H_y, 0 \right] \) which acts in the \( y \)-direction, the magnetic force on the body is expressed by \( \sigma \mathbf{H}_y^2 \omega \), where \( \sigma \) Refers to electrical accessibility of fluid. Then, we will get the following motion equation

\[ \rho \frac{d}{dt} + \frac{\partial}{\partial r} \left( r \mathbf{S}_r \right) + \sigma \mathbf{H}_y^2 \omega \]

(7)

By removing amidst equation (6) and equation (7), we obtain the differential fractional equation that is given by
\begin{equation}
(1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu) \frac{\partial}{\partial t} \left( \frac{\partial}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \omega - \frac{1}{\mu} \left( 1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu \right) \frac{\partial}{\partial t} (1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu) \frac{\partial}{\partial r^2} \omega
\end{equation}

\begin{equation}
\frac{\sigma n H_0}{\mu} \left( 1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu \right) \frac{\partial}{\partial t} \omega
\end{equation}

3. Flow of Plane Poiseuille

The problem of an incompressible generalized Burgers’ fluid flow is firstly at rest between two long coaxial infinitely cylinders of radii \( R_0 \) and \( R_1 \). As a result, the fluid is generated from an impulsive pressure gradient at time \( t = 0 \) which acts on liquid in \( y \)-direction. Pointing to Eq. (8), the coinciding differential fractional partial equation which describe such flow has the following form

\begin{equation}
\frac{\partial}{\partial t} \left( \frac{\partial}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \omega - K(1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu)\delta(t) - M(1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu) \omega
\end{equation}

(9)

where \( \nu = \frac{\mu}{\rho} \) indicate the kinematic viscosity, \( K(t) = \frac{1}{\mu} \frac{d}{dt} \) denotes the fixed pressure gradient and \( M = \frac{\sigma n H_0}{\mu} \) designates the dimensionless magnetic number.

The related beginning and ending states are as follows

\begin{align}
\omega(r, 0) &= \delta_1 \omega, \omega(r, 0) = \delta_1 \omega, \omega(r, 0) = 0, R_0 \leq r \leq R_1 \nonumber \\
\omega(R_1, 0) &= \omega(r_1, 0) = 0, t > 0 
\end{align}

(10)

To earn the accurate analytical resolution of the previous problem (9)- (10), we apply the principle of Laplace transform[14] with regard to \( t \). Then, we have

\begin{equation}
S(1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu) \Omega = \nu \left( 1 + \lambda_4^a L_t^a \right) \left( \frac{\partial}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \Omega - K(1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu) - M(1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu) \Omega
\end{equation}

\begin{equation}
\Omega(R_0, 0) = \delta_1 \omega, \Omega(R_1, 0) = \delta_1 \omega, \Omega(r, 0) = 0, t > 0
\end{equation}

(11)

when \( \Omega(r, S) \) designate to the image function of \( \Omega(r, t) \) and \( S \) indicates a converting parameter. We use the restricted Hankel transform [14] and describe it as follows

\begin{equation}
\frac{\partial}{\partial r} = \frac{\pi}{2} \sum_{i=1}^{\infty} k_i^2 \frac{\partial}{\partial r} \Omega(k_i R_0) \frac{\partial}{\partial \Omega(k_i R_1)}
\end{equation}

(12)

So that \( k_i \) are the positive roots of equation \( B_0(K_1 k_i) = 0 \) and \( B_0(k_i) = J_0(k_i)Y_0(K_1 k_i) = 0 \)

where \( J_0(\cdot) \) and \( Y_0(\cdot) \) are the functions of Bessel of both the first and second types of order zero.

By using the restricted Hankel transform to (11) Eqs. with respect to \( r \), we obtain

\begin{equation}
\Omega = \frac{-K(1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu)}{(S \Omega)(1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu) + \nu k_i^2 (1 + \lambda_4^a L_t^a + \lambda_5^a L_t^a\mu)}
\end{equation}

(13)

Now, writing Eq. (13) in series form as follows
\[ \omega_H = -K(1 + \lambda_1^2 S^2 + \lambda_2^2 S^2\alpha^2) \sum_{k=0}^{m} (-1)^k \sum_{a,b,c,d,n \geq 0} \frac{a!b!c!d!n!}{a!b!c!d!n!} \sum_{k=0}^{m} (-1)^k \sum_{a,b,c,d,n \geq 0} \frac{a!b!c!d!n!}{a!b!c!d!n!} \]

(14)

where \( \alpha = \beta - 2\alpha(k + a + n) - (b - a) - \alpha \).

We apply discrete inverse Laplace transform [14] will take the following form

\[ \omega_H = \left\{ \begin{aligned}
&-K \sum_{k=0}^{m} (-1)^k \sum_{a,b,c,d,n \geq 0} \frac{a!b!c!d!n!}{a!b!c!d!n!} \sum_{k=0}^{m} (-1)^k \sum_{a,b,c,d,n \geq 0} \frac{a!b!c!d!n!}{a!b!c!d!n!} \\
&\left\{ L_k^{\alpha+1} \alpha^k + L_k^{\beta+1} \beta^k \right\} \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right) + L_k^{\alpha+1} \alpha^k \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right) + L_k^{\beta+1} \beta^k \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right)
\end{aligned} \right\}
\]

(15)

where \( L_k^{\alpha+1} (z) = \sum_{j=0}^{m} \frac{(j+m)!z^j}{j! (\alpha + \alpha + \beta)} \) indicates the generalized Mittag-Leffler function [14] and to earn Eq. (15), the following feature of reverse Laplace transform is applied [14]

\[ L^{-1} \left[ \frac{\Gamma(\alpha \alpha + \beta)}{\Gamma(\alpha \alpha + \beta)} \right] = T^\alpha + \mu^{-1} E_{\alpha,\mu} \left( \pm \xi T^\alpha \right) , \quad (R \in \mathbb{R}) \]

(16)

eventually, using the inverse Hankel transform to obtain the analytic resolution of velocity distribution \( \omega(r, t) = \)

\[ \left\{ \begin{aligned}
&-K \sum_{k=0}^{m} (-1)^k \sum_{a,b,c,d,n \geq 0} \frac{a!b!c!d!n!}{a!b!c!d!n!} \sum_{k=0}^{m} (-1)^k \sum_{a,b,c,d,n \geq 0} \frac{a!b!c!d!n!}{a!b!c!d!n!} \\
&\left\{ L_k^{\alpha+1} \alpha^k + L_k^{\beta+1} \beta^k \right\} \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right) + L_k^{\alpha+1} \alpha^k \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right) + L_k^{\beta+1} \beta^k \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right)
\end{aligned} \right\}
\]

(17)

3.1 The limiting status

Making the limits of Eq. number (17), when \( \alpha \rightarrow 0 \), \( \lambda_1 \rightarrow 0 \) (b=0), we can obtain the distribution of velocity for a generalized Oldroyd-B fluid. Thus, the field of velocity reduces to

\[ \omega(r, t) = \left\{ \begin{aligned}
&-K \sum_{k=0}^{m} (-1)^k \sum_{a,b,c,d,n \geq 0} \frac{a!b!c!d!n!}{a!b!c!d!n!} \sum_{k=0}^{m} (-1)^k \sum_{a,b,c,d,n \geq 0} \frac{a!b!c!d!n!}{a!b!c!d!n!} \\
&\left\{ L_k^{\alpha+1} \alpha^k + L_k^{\beta+1} \beta^k \right\} \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right) + L_k^{\alpha+1} \alpha^k \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right) + L_k^{\beta+1} \beta^k \left( -\frac{\eta}{\sigma^2} L_k^{\alpha+1} \right)
\end{aligned} \right\}
\]

(18)

where \( \sigma = k + l(a - 1) - \alpha \).

4. Discussion and Numerical results:

In this study, the flow in an annular tube is discussed due to the fluid flow gradient of a generalized Burger's fluid with a magnetic field. The precise accuracy of the speed\( \dot{u} \) field is obtained through the separate Laplace application and restricted Hankel transformations. Additionally, few numbers are plotted to detect the behavior of the various parameters involved in speed\( \dot{u} \) expressions. The flow from the burst pressure gradient (plate a) and the flow from the continuous pressure gradient (plate b) are also approximated graphically in Figures 1-6.

Figs. 1 and 3, are provided the graphical illustrations for the effect of the non-integer fractional parameter\( \alpha \) and the graphical explanation of the effect of repose parameter\( \lambda_1 \) on the fields of velocity. Velocity is decreasing with the increased of \( \alpha \) and \( \lambda_1 \) in (a) without MHD but increased in (b) when \( \alpha \) and \( \lambda_1 \) are increased with MHD.
Fig. 2 shows that the field of velocity is increased with the increasing of both cases with MHD.

Fig. 4 is prepared to show the effect of the material parameter $\lambda_2$ on the field of velocity. The velocity is increased with the increase of $\lambda_2$ in both cases, and decreased with MHD.

Fig. 5 is prepared to show the tardiness parameter $\lambda_3$ on the field of velocity. The field of velocity is decreased with the increase of $\lambda_3$ with MHD, and vice versa in (a).

Fig. 6 is prepared to show the time parameter $t$ on the field of velocity. The field of velocity is decreased with the increase of $t$ without MHD, and increased greatly with the increase of $t$ with MHD.

![Figure 1. Velocity of different value of $\alpha$ while maintaining another parameters constant a) flow due to impulsive pre. grad. b) flow due to impulsive pre. Grad with MHD.](image1)

![Figure 2. Velocity of different value of $\beta$ while maintaining another parameters constant a) flow due to impulsive pre. grad. b) flow due to impulsive pre. Grad with MHD.](image2)
Figure 3. Velocity of different value of $\lambda_1$ while maintaining another parameters constant a) flow due to impulsive pre. grad. b) flow due to impulsive pre. Grad with MHD.

Figure 4. Velocity of different value of $\lambda_2$ while maintaining another parameters constant a) flow due to impulsive pre. grad. b) flow due to impulsive pre. Grad with MHD.

Figure 5. Velocity of different value of $\lambda_3$ while maintaining another parameters constant a) flow due to impulsive pre. grad. b) flow due to impulsive pre. Grad with MHD.
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