Synchronize accelerated clock in a multipartite relativistic quantum system

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We perform a protocol for multipartite quantum clock synchronization in a relativistic setting. The dynamics of the multipartite quantum system consisting of Unruh-DeWitt detectors when one of the detectors under accelerated motion is obtained. To estimate the time difference between the clocks, we calculate the time probability and analyze how the probability is influenced by the Unruh thermal noise and other factors. It is shown that both relativistic motion and the interaction between atom and the external scalar field affect the choice of optimal number of excited atoms in the initial state, which leads to a higher clock adjustment accuracy. The time probabilities for different types of initial states approach to the same value in the limit of infinite acceleration, while they tend to different minimums with increasing number of atoms. In addition, the accuracy of clock synchronization using bipartite initial state is found always higher than multipartite systems, while the Z-type multipartite initial states always perform better than the W-type states.

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I. INTRODUCTION

High-accuracy synchronization of separated clocks is of fundamental interest in fundamental physics and a wide range of applications such as the global positioning system (GPS), navigation, gravitational wave observation, airport traffic control, as well as the laser interferometer gravitational wave observation (LIGO) [1, 2]. There are two approaches to synchronize a pair of clocks. Einstein’s synchronization proposal [3] is implemented by exchanging pulses between two spatially separated clocks and measuring their arrival time. An alternative approach is the Eddington clock transport proposal [4], which is based on sending a locally synchronized clock from one part to the other part. However, both of them are limited by the unstable intervening media, and some quantum strategies of clock synchronization [5–22] have been performed to achieve an enhancement of accuracy. The main advancement of the quantum clock synchronization (QCS) proposals are employing quantum entanglement and state squeezing to improve the precision of synchronization.

On the other hand, the feasibility of QCS under the influence of clocks’ relativistic motion has to be concerned because time dilation induced by relativistic effects is experimentally observed due to velocities of several meters per second by comparing a moving clock with a stationary clock [23]. We studied the influence of relativistic gravitational frequency shift effect on satellite-based QCS and found that the precision of QCS is remarkably affected by the relativistic effect of the Earth [24]. When it comes to the accelerated clock, the Unruh effect [25–26], a significant prediction in quantum field theory is necessary to be considered. The Unruh effect shows that a uniformly accelerated observer would see a thermal bath, whose temperature is proportional to the proper acceleration possessed by observers. The influence of Unruh effect on quantum systems [27–34] is a focus of study recently because such studies provide insights into some key physical questions such as nonlocality, causality, and the information paradox of black holes. Since it’s hard to observe the Unruh effect in laboratory, people approaches to the Unruh’s original derivation with the semiclassical Unruh-DeWitt detector model [35]. Recently, the dynamics of entanglement [36], quantum discord [37], quantum nonlocality [38], and quantum coherence [39] of two entangled detectors under the influence of Unruh thermal noise have been discussed. Although some efforts have been devoted to the influence of Unruh effect on quantum information, these studies are confined to two-detector systems, while the behavior of many-detector system in an acclerated frame is an ongoing topic of research.

In this paper, we perform a multipartite protocol for QCS in a relativistic frame, where each clock corresponds to a two-level atom modeling by the Unruh-DeWitt detector. We assume that \( n \) detectors are prepared in an entangled initial state and timing information is determined by the observable time probability of the final state. Unlike previous multipartite QCS protocols [40–41], we concern about the problem of QCS for an accelerated clock and the influence of Unruh thermal noise is involved. Without loss of generality, we assume that the first static clock in the \( n \)-partite system has been synchronized and regarded as the standard clock. To analyze how the number of atoms and quantum entanglement of the initial state influence the accuracy of QCS, we distinguish the QCS in the two-party system from two different initial states in the multipartite system.

The outline of the paper is organized as follows. In Section III we study the evolution of the multiparty system when one clock is carried by the accelerated detector. In Sec. IV we study the QCS for a accelerated clock in the multipartite system and analyze how the Unruh thermal noise and other
factors impact the accuracy of our scheme. In Sec. IV we make a conclusion.

II. EVOLUTION OF THE MANY-DETECTOR SYSTEM WHEN AN ATOM ACCELERATED

To synchronize \( n \) spatially separated clocks, Krco and Paul [40] promoted a multiparty synchronization protocol, in which any one of the clocks can be taken as the standard clock. In their proposal, the initial shared among these clocks is a W-type state [42]

\[
|\psi_W\rangle = \frac{1}{\sqrt{n}}(|10\ldots 00\rangle + |01\ldots 00\rangle + \ldots + |00\ldots 01\rangle),
\]

which involves \( n \) subsystems and each of them only contains a single qubit in the excited state. A few years later, Ben-Av and Exman employed the Z-type state as a generalization of \( W \) state and found that the clock adjustment accuracy was improved via Z-state properties [41]. To study the dynamics of the many-detector system under relativistic motion, we are going to work out the evolution of \( Z \)-type initial state in a relativistic setting firstly. The dynamics of the \( W \) state can be obtained in a similar way. The \( Z \) state is an entangled state with \( n \) two-level noninteracting atoms and fully symmetric under the operation of particle exchange, which owns the form

\[
|\psi_{z}\rangle = \frac{1}{\sqrt{n!}}\sum_{k=0}^{n} \frac{(-k)!k!(n-k)!}{n!} |010\ldots 00\rangle
+ |001\ldots 00\rangle + \ldots + |000\ldots 11\rangle,
\]

where \( k \) atoms are involved in the excited energy eigenstate \(|1\rangle\), while the rest lie in the ground state \(|0\rangle\).

The total Hamiltonian of the whole system is

\[
H_{n,\phi} = \sum_{i=1}^{n} H_{i} + H_{KG} + H_{int}^{R,\phi},
\]

where \( H_{KG} \) stands for the Hamiltonian of the massless scalar field satisfying the K-G equation \( \Box\phi = 0 \). In Eq. 3, \( H_{i} = \Omega D_{i}^2 + \frac{\Omega_{\phi}}{2\kappa} D_{i}^2 \) are the Hamiltonian of each atom, where \( D_{i} \) and \( D_{i}^\dagger \) represent the creation and annihilation operators of the \( i \)-th atom, respectively. And \( \Omega \) is the energy gap between \(|0\rangle\) and \(|1\rangle\). In our system, the timing information reads the proper time of each atom. We assume that the second atom of the multiparticle system, carried by an observer Bob, is uniformly accelerated for a duration time \( \Delta \), while other atoms keep static and are switched off all the time. The running of Bob’s clock is affected by the relativistic motion and needs to be synchronized. The world line of Bob’s detector is described by

\[
t(\tau) = a^{-1} \sinh a\tau, z(\tau) = a^{-1} \cosh a\tau, y(\tau) = z(\tau) = 0,
\]

where \( a \) is the proper acceleration of Bob. The interaction Hamiltonian \( H_{int}^{R,\phi} \) between Bob’s detector and the field is

\[
H_{int}^{R,\phi}(t) = \epsilon(t) \int_{\Sigma_t} d^3x \sqrt{-g(x)} [\chi(x) D^2 + \nabla(x) D^2],
\]

where \( \epsilon(t) \) is the coupling constant, \( g_{ab} \) is the Minkowski metric and \( \eta \equiv \det(g_{ab}) \). In Eq. 5, \( \Sigma_t \) represents that the integration takes place over the global spacelike Cauchy surface and \( \phi(x) \) is the scalar field operator, while \( \chi(x) \) ensuring the interaction only occurs in the neighbourhood of Bob’s detector.

By introducing a compact support complex function \( f(x) = \epsilon(t) e^{-i\Omega t} \chi(x) \), we have \( \phi(x) f = R f - A f \), where \( A \) and \( R \) are the advanced and retarded Green functions. Then we obtain

\[
\phi(f) = \int d^3x \sqrt{-g} \phi(x) f = i[a_{RI}(\Gamma^-) - a_{RI}(\Gamma^+)],
\]

where \( \Gamma^- \) and \( \Gamma^+ \) represent the negative and positive frequency parts of \( \phi(f) \) respectively, and \( a_{RI} \) and \( a_{RI}^\dagger \) are Rindler creation and annihilation operators in Rindler region \( I \). Since \( \epsilon(t) \) is a roughly constant for \( \Delta \gg \Omega^{-1} \), the test function \( f \) approximately owns the positive-frequency part, which means \( \Gamma^- \approx 0 \). And if we define \( \lambda = -\Gamma^+ \), Eq. 2 could be rewritten as \( \phi(f) \approx i\lambda^t(\lambda) \).

Here we only consider the first order under the weak-coupling limit. With Eq. 5 and Eq. 6, the final state of the whole system in the interaction picture at time \( t > t_0 + \Delta \) is found to be

\[
|\Psi_t^{n,\phi}\rangle = |\Psi_{t_0}^{n,\phi}\rangle + \frac{1}{(1-q)^{1/2}}\sum_{k=0}^{n} C_n^k \left\{ (|11\ldots 00\rangle + \ldots + |01\ldots 11\rangle) \otimes q^{k/2}|1F_{2\Omega t}\rangle \right\}_{k}\cdots\left\{ (|10\ldots 00\rangle + \ldots + |00\ldots 11\rangle) \otimes |1F_{\Omega t}\rangle \right\}_{k}.
\]

III. QCS FOR THE ACCELERATED CLOCK IN A MULTIPARTITE QUANTUM SYSTEM

In this section, we study the QCS in the many-detector quantum system. To synchronize Bob’s accelerated clock, we
select the first atom carried by a static observer Alice as the standard clock. In the beginning, Alice and Bob make an appointment to the starting time $\tau$ to measure their local state. However, since Bob's clock has been accelerated, they do not have a synchronized clock to start with together. Therefore, they can only start their measurements at their own proper time $\tau_i = \tau (i = A, B)$, which are relative to the time readings of their own clock and are different due to relativistic effects. After the accelerated motion of Bob, Alice measures her state when her clock points at $\tau_A = \tau$, which reduces the collapse of wave packet of the state shared between Alice and Bob. Then Bob also measures his state at time $\tau_B = \tau$, which lags behind than Alice's measuring time $\tau_A = \tau$. In this way we obtain the time probability which is used to estimate the difference between Alice's and Bob's proper time.

Since we only need to synchronize Bob's clock according to Alice's standard clock, the degree of freedom for all the rest atoms should be traced out. Indeed, any accelerated clock in the $n$-party system can be synchronized with Alice's clock, whereas any static clock can be selected as the standard clock. Tracing out the the rest sub-systems in Eq. (9), we obtain the reduced density matrix for Alice and Bob

$$\rho^{AB} = \frac{k(n-k)}{|c|^2 n(n-1)} \begin{pmatrix} S_1 & 0 & 0 & 0 \\ 0 & S_2 & 1 & 0 \\ 0 & 1 & S_3 & 0 \\ 0 & 0 & 0 & S_4 \end{pmatrix},$$  \hspace{1cm} (10)

in their own basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, where $S_1 = \frac{n-k-1}{n-k} + \frac{\alpha^2}{1-q}$, $S_2 = 1 + \frac{q\alpha^2(n-k-1)}{(1-q)k}$, $S_3 = 1 + \frac{\alpha^2(k-1)}{(1-q)(n-k)}$ and $S_4 = \frac{k-1}{n-k} + \frac{\alpha^2}{1-q}$.

To synchronize Bob's clock, we adopt the dual basis $|\text{pos}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|\text{neg}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ as the measurement basis, which are obtained from $|0\rangle$ and $|1\rangle$ through the Hadamard transformation. In the measurement basis, Eq. (10) is rewritten into

$$\rho^{AB} = \frac{1}{8+4\alpha^2} \begin{pmatrix} 4 + \alpha_+ & \beta_+ & \alpha_- & -4 + \beta_- \\ \beta_+ & \alpha_+ & -\alpha_- & \beta_- \\ -\alpha_- & -\beta_+ & \alpha_+ & 4 + \alpha_- \\ -4 + \beta_- & \alpha_- & \beta_+ & 4 + \alpha_+ \end{pmatrix},$$  \hspace{1cm} (11)

where $\alpha_{\pm} = \frac{(1-q-q^2\nu^2)(n-k-1)+\nu^2 k}{(1-q)k} \pm \frac{(1-q+\nu^2)(k-1)}{(1-q)(n-k)} + \frac{q\nu^2}{1-q}$ and $\beta_{\pm} = \frac{(1-q-q^2\nu^2)(n-k-1)+\nu^2 k}{(1-q)k} \pm \frac{(1-q+\nu^2)(k-1)}{(1-q)(n-k)} + \frac{q\nu^2}{1-q}$.

If Alice measures her atom with $|\text{pos}\rangle$ at the upon arranged time $\tau_A = \tau$, the state of Bob's atom immediately collapses to the following form

$$\rho^{B}_{\tau_A=\tau} = \gamma \begin{pmatrix} 4 + \alpha_+ & \beta_+ \\ \beta_+ & 4 + \alpha_- \end{pmatrix},$$  \hspace{1cm} (12)

where $\gamma = \frac{1}{4+2\alpha^2}$. When Alice's clock reads $\tau_A = \tau$, Bob's clock does not point at $\tau$. In other words, their local clocks have a time difference $\delta = \tau_A - \tau_B$, which leads Bob's state evolves to the following form at $\tau_B = \tau$

$$\rho^{B}_{\tau_B=\tau} = \gamma \begin{pmatrix} 2 + \alpha_+ + 2\cos\Omega\delta & \beta_+ + 2i\sin\Omega\delta \\ \beta_+ - 2i\sin\Omega\delta & 2 + \alpha_- - 2\cos\Omega\delta \end{pmatrix},$$  \hspace{1cm} (13)

in the measurement basis $|\text{pos}\rangle, |\text{neg}\rangle$.

Therefore, the probability for Bob measuring $|\text{pos}\rangle$ at time $\tau_B = \tau$ is

$$P(|\text{pos}\rangle) = \frac{1}{2} + \frac{k(n-k)(1-q)\cos(\Omega\delta)}{(n-1)((1-q)n+\nu^2 k + q\nu^2(n-k))},$$  \hspace{1cm} (14)

from which the measuring results obtained by Bob allow him to estimate the time difference $\delta$. In other words, the information of time difference is exposed by the observable probability $P(|\text{pos}\rangle)$ and Bob can adjust his clock accordingly if the energy gap and time difference satisfy the condition $|\Omega\delta| < 2\pi$. Also, we obtain the probability $P(|\text{neg}\rangle)$ with $|\text{neg}\rangle$ to estimate the time difference $\delta$ in the same way.

To obtain a higher clock adjustment accuracy, we firstly discuss the role of $k$, the number of excited atoms in the initial state. Eq. (2). After some calculations, the optimal $k$ is found to be

$$k_{\text{opt}} = |Y|$$  \hspace{1cm} (15)

for $Y = \frac{1}{(1+q\nu^2)(n-1)}[(1-q)(1+q\nu^2)q^2(1-q)^2 + \sqrt{n^2(1-q)(1-q)\nu^2 - 4\nu^2 - 4} - 4\nu^2 - 4]$.

Similarly, the time probability $P_W(|\text{pos}\rangle)$ for $W$ state is found to be

$$P_W(|\text{pos}\rangle) = \frac{1}{2} + \frac{(1-q)\cos(\Omega\delta)}{(1-q)n+\nu^2 k + q\nu^2(n-k))},$$  \hspace{1cm} (16)

For $W$-type initial states, the parameter $k$ is not required to be optimized since $k = 1$ in this case.

In Fig. 1, we analyze the time probability $P$ as a function of the number of atoms $n$ both for the $Z$-type and $W$-type initial states. Such time probabilities are obtained when Bob measures the final states of the system by $|\text{pos}\rangle$. The energy gap $\Omega$ and the time difference $\delta$ are fixed as $\Omega\delta = 2\pi$. In this case the value of time probability equals to the amplitude of the probability, which is an indicator of the clock adjustment accuracy. It is showed that the time probability $P$, i.e., the clock at $\tau_B = \tau$.
adjustment accuracy of the synchronization for \(W\)-type initial state equals to the accuracy of \(Z\)-type initial state for \(n = 2\). In addition, the time probability \(P\) decreases with the growth of the atoms’ number \(n\). This attributes to the decrease of entanglement since comparing to the bipartite quantum system, the initial entanglement have been distributed to other subsystems in multipartite systems. That is, Alice and Bob in a bipartite system share more bipartite entanglement than in a multiparty system. What’s more, we find that the time probability \(P\) of \(Z\) state with \(k = \{Y\}\) is always more than that of \(W\) state for the same \(n\), which means the \(Z\) states have higher clock adjustment accuracy for the synchronization. In addition, it is found that the time probability \(P\) approaches to different final values for different initial states in the limit of the infinite number of atoms.

To get a better understanding on how the entanglement shared between Alice and Bob influences the adjustment accuracy of the synchronization, we analyze the clock synchronization for a general bipartite entangled initial state, which has the form

\[
|\psi^{AB}\rangle = \sin \theta |01\rangle + \cos \theta |10\rangle. \tag{17}
\]

Similarly, we let Alice own the static standard clock while Bob’s accelerated clock needs to be synchronized. We calculate the probability and quantum entanglement for the same state. To quantify entanglement, we employ the concurrence \([43, 44]\), which is

\[
C(\rho) = 2 \max \left\{0, \tilde{C}_1(\rho), \tilde{C}_2(\rho) \right\}
\]

for a \(X\)-type quantum state. Here \(\tilde{C}_1(\rho)\) and \(\tilde{C}_1(\rho) = \tilde{C}_2(\rho) = \sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{23}\rho_{32}}\) and \(\rho_{ij}\) are matrix elements of the state density matrix.

The probability for the general bipartite initial state is

\[
P_2(|\text{pos}\rangle) = \frac{1}{2} + \frac{(1 - q) \sin 2\theta \cos(\Omega\delta)}{2(1 - q) + 2\nu^2(\cos^2 \theta + q \sin^2 \theta)}. \tag{18}
\]

If we assume \(\theta = \pi/4\) and \(q\) vanishes, which means the initial state is maximally entangled and the relativistic motion is not considered, Eq. (18) can be casted as \(P_2(|\text{pos}\rangle) = \frac{1}{2} + \frac{1}{2} \cos(\Omega\delta)\), identifying with the results in a non-relativistic system \([5, 40, 41]\).

In Fig. 2, we compare the time probability \(P\) and entanglement as a function of the initial state parameter \(\theta\) for some fixed effective coupling parameters \(\nu\). It is shown that the amplitude of the time probability \(P\) has similar variation trend with the entanglement as the initial state parameter \(\theta\) changes. Specifically, the time probability \(P\) gets the maximum when we adopt the maximal entangled initial state. Then we conclude that the entanglement shared between the observers enhances the accuracy of clock synchronization in the relativistic setting.

We are interested in how the accelerated motion and the coupling between the atom and field influence the accuracy of quantum clock synchronization. In Fig. 3, we study the influence of the acceleration parameter \(q\) on time probability \(P\) and compare the results of bipartite quantum clock synchronization with multipartite quantum clock synchronization (take \(n = 20\) as an example). We find that the time probability \(P\) sharply decreases with increasing acceleration of Bob, which reveals that the accuracy of the clock synchronization is sharply reduced by the Unruh thermal noise. It is worthy note that the time probabilities for all the three different initial states approach to 0.5 when the acceleration parameter \(q \rightarrow 1\), i.e., in the limit of infinite acceleration. Such behavior is quite different from the dynamics of probabilities with increasing number of atoms. Moreover, the accuracy of clock synchronization using bipartite initial state is always better than the accuracy of multipartite systems and we find again the \(Z\) state perform better than the \(W\) state.

In Fig. 4, we show the time probability \(P\) as a function of the effective coupling parameter \(\nu\). We find that with increases of the effective coupling parameter \(\nu\), the time probability \(P\) decreases, which means that the interaction between the accelerated atom and scalar field would reduce the clock adjustment accuracy. In addition, we find that whatever is the value of the effective coupling parameter \(\nu\), the time probability \(P\) for a pair of entangled atoms is always greater than for twenty entangled atoms. Unlike the effect of acceleration, the time probability is more robust on the coupling parameter \(\nu\).
FIG. 4: (Color online) The measurement probability as a function of the effective coupling parameter $\nu$ for two-party state (solid line), $W$ state (dashed line) and optimal $Z$ state (dot line), independently. The acceleration parameter $q$ is fixed as $q = 0.8$. The product of the energy gap parameter $\Omega$ and the time difference parameter $\delta$ are fixed as $\Omega\delta = 2\pi$.

IV. CONCLUSIONS

In conclusion, we have studied the dynamics of a many Unruh-DeWitt-detector system and performed a proposal for multipartite QCS in a relativistic setting. We work out the final state of the many-detector system both for a general $Z$-type initial state. The information of time difference is expose by the observable time probability and the clock can be adjusted accordingly. We distinguish the QCS in the two-party system from two different initial states in the multipartite system. It is shown that both the coupling parameter and accelerating parameter affect the choice of optimal number of excited atoms $k$ in the initial $Z$ state, which leads to a higher clock adjustment accuracy. The clock adjustment accuracy would decrease with the increased number of the atom, which attributes to the weakness of the bipartite entanglement between Alice and Bob. By comparing the behavior of time probability with that of entanglement, we find the entanglement of the initial state would enhance the clock adjustment accuracy. It is worthy note that the time probabilities for all the three different initial states approach to the same in the limit of infinite acceleration, while they tend to different minimums with increasing number of atoms. In addition, the accuracy of clock synchronization using bipartite initial state is found always better than that of multipartite states, while the $Z$ state performs better than the $W$ state. Like the Unruh thermal noise, the interaction between the accelerated clock and the external scalar field also reduces the accuracy of clock synchronization.

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