Form-Finding with and without Cable Reinforced Thomsen Surface in Tensioned Fabric Structure

Yee HM*, Abdul Hadi MN
Faculty of Civil Engineering, Universiti Teknologi MARA, 13500 Permatang Pauh, Pulau Pinang, Malaysia

E-mail: nasayz@yahoo.com, minyh@ppinang.uitm.edu.my*corresponding author

Abstract. Tensioned Fabric Structure (TFS) is structure that is composed of tensioned fabric as structural members. The form of Thomsen surface applied in TFS has been developed in this study because the form of Thomsen surface has not been studied by others researcher. The objective of this study is to determine initial equilibrium shape in Thomsen form and cable reinforced of Thomsen surface in TFS with variable \( u=v=0.8 \) using nonlinear analysis method. The surface of Thomsen surface, \( u=v=0.8 \) shows the least square error of total warp and fill stress deviation is less than 0.01. From the result, initial equilibrium shape in Thomsen form and cable reinforced of Thomsen surface, \( u=v=0.8 \) is corresponding to equal tension surface. In conclusion, Thomsen form and cable reinforced of Thomsen surface with variable \( u=v=0.8 \) will provide an alternative shape for designer to be considered for adopted in tensioned fabric structure.

1. Introduction

Fabric structure is known as tensile structure or membrane structure. Generally, the structural in fabric structure has applied in roof and tent. Fabric structure contained fabric surface as main component in fabric structure. Figure 1 shows the example of tensioned fabric structure. TFS with different surface form could be realized. Their variations as possible choice form of minimal surface for TFS have been studied. In the analysis, the form of Thomsen surface has been developed. The shape of Thomsen surface has not been studied by other researchers. Besides, no other work on Thomsen surface applied in TFS have been found. The process of this study began with structural analysis in TFS which is called form-finding. Form-Finding is the process to determine the initial equilibrium shape under prescribed pre-stress system and boundary condition.

The form-finding computational analysis used in this study is nonlinear analysis method proposed by [1]. Previous studies have shown that form-finding using nonlinear analysis method in Handkerchief, Thomsen, Bour’s, Catenoid, Helicoid, Scherk, Enneper, Oval, Costa, Möbius strip, Monkey Saddle and Chen-Gackstatter TFS models [1-16]. [17] has proposed the algorithm for warp direction in TFS. Before form-finding, mesh generation of TFS is needed.

Understanding of the possible form of Thomsen surface in TFS is important because will provide alternative shape for structural designers to be considered. In this paper, Thomsen surface in tensioned fabric structure with variable \( u=v=0.8 \) have been carried out.
2. Generation of Thomsen Surface
Figure 2 shows the form of Thomsen surface. The form of Thomsen surface can be obtained from [18] and [19]. Equation (1) shows the equation for Thomsen Surface.

\[
X = \frac{\alpha \beta u + \sqrt{1 + \beta^2 \sinh \alpha u \cos \alpha v}}{\alpha^2}, \quad Y = \frac{\alpha \beta v + \sqrt{1 + \beta^2 \sinh \alpha u \cos \alpha v}}{\alpha^2}, \quad Z = \sinh \alpha u \sin \alpha v
\]

\[\alpha = 1, \quad \beta = 0\]

Where \(u = v = \text{constant}\), \(u\) and \(v\) represented as the point \((u, v)\) mapped to \((X(u, v), Y(u, v), Z(u, v))\). While, \(\alpha\) and \(\beta\) represented as the unit circle centered at the origin.

3. Computational Method using Nonlinear Analysis Method
The principle of nonlinear analysis method is based on [1]. The large displacement finite element formulation used for analysis of structural behaviour under external loads. Since the method can be
used for both the converged shape problem and load analysis, the approach using nonlinear analysis is quite common. The basic equation used is expressed as follows:

\[
(\frac{\partial^2 K}{\partial \Delta t^2} + \frac{\partial K}{\partial \Delta f})u = \frac{\partial F}{\partial \Delta f}
\]  

(2)

Where \(\frac{\partial K}{\partial \Delta t}\) is linear strain incremental stiffness matrix, \(\frac{\partial K}{\partial \Delta f}\) is nonlinear strain incremental stiffness matrix, \(\frac{\partial f}{\partial \Delta f}\) is vector internal forces, \(\frac{\partial F}{\partial \Delta f}\) is load vector and \(u\) is vector of increment in displacement.

A nonlinear finite element analysis program by [1] for the analysis of tensioned fabric structures has been used in this study. The procedure adopted is based on the work as specified in [1]. 3-node plane stress element has been used as element to model the surface of TFS. All x, y and z translation of nodes lying along the boundary edge of the Thomsen surface have been restrained. The member pretension in warp and fill direction, is 2000N/m, respectively. The shear stress is zero.

Two stages of analysis were involved in the procedures of form-finding in one cycle proposed by [1]. First stage (denoted as SF1) is analysis which starts with an initial guess shape in order to obtain an updated shape for converged shape. The initial guess shape can be obtained from any pre-processing software and reference [1] is chosen for this study. This is then followed by the second stage of analysis (SS1) aiming at checking the convergence of updated shape obtained at the end of stage (SF1). During stage (SF1), artificial tensioned fabric properties, E with very small values are used. Both warp and fill tensioned fabric stresses are kept constant. In the second stage of (SS1), the actual values of tensioned fabric properties are used. Resulting warp and fill tensioned fabric stresses are checked at the end of the analysis against prescribed tensioned fabric stresses. Then, iterative calculation has to be carried out in order to achieve convergence where the criteria adopted is that the average of warp and fill stress deviation should be < 0.01. The resultant shape at the end of iterative step n (SSn) is considered to be in the state of converged shape under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed membrane stresses relative to the prescribed stress is negligibly small. Such checking of difference in the obtained and prescribed stresses has been presented in the form of total stress deviation in warp and fill direction versus analysis step. As a first shape for the start of form-finding procedure adopted in this study, initial guess shape is needed. For the generation of such initial guess shape, knowledge of the requirement of anti-clastic nature of TFS is used. The incorporation of anti-clastic feature into the model will help to produce a better initial guess shape.

4. Computational Result

The program of Adina System [20] represents as finite element (FE) software. The program are developed computational form-finding of TFS using nonlinear analysis method.

![Reinforcing Cables](image)

**Figure 3.** Location of reinforcing cables of Thomsen Surface
Form-finding using nonlinear analysis method of converged shape and cable reinforced for Thomsen surface has been carried out. Area of cable and pretension of cable used for Thomsen surface are 0.005m³ and 15000N/m, respectively. Warp and fill fabric surface is 2000N/m. In this study, the variable has been used in this study is Thomsen surface, \( u=v=0.8 \). Figure 3 shows the location of one reinforcing cables for Thomsen surface, \( u=v=0.8 \). The number of nodes and triangular elements for Thomsen surface, \( u=v=0.8 \) is 175 and 288, respectively.

4.1 Thomsen Surface, \( u=v=0.8 \)

Figure 4 shows initial guess shape for Thomsen surface model, \( u=v=0.8 \). Figure 5 shows converged shape for Thomsen TFS model, \( u=v=0.8 \). Figure 6 shows convergent curve of Thomsen surface model, \( u=v=0.8 \). From the graph, the warp and fill direction for Thomsen surface model, \( u=v=0.8 \) is 0.000758 and 0.000060, respectively. The surface of Thomsen TFS model, \( u=v=0.8 \) shows the total warp and fill stress deviation is less than 0.01. From the result, the converged shape of Thomsen surface model, \( u=v=0.8 \) is corresponding to equal tension surface.

![3D view](image1)

![Front View](image2)

![Side View](image3)

![Top View](image4)

**Figure 4.** Initial guess shape for Thomsen TFS model, \( u=v=0.8 \)
4.2 Cable Reinforced Thomsen Surface, \( u=v=0.8 \)

Figure 7 shows converged shape for cable reinforced Thomsen surface model, \( u=v=0.8 \). The location of cable reinforced Thomsen TFS model as shown in Figure 3. The dimension of cable reinforced Thomsen surface model, \( u=v=0.8 \) is 1.776×1.6×1.274 units. The cable pretension for cable reinforced Thomsen surface model, \( u=v=0.8 \) with one cable is 13258.18N. Figure 8 shows convergent curve of cable reinforced Thomsen surface model, \( u=v=0.8 \). Based on the graph, warp and fill direction for cable reinforced Thomsen surface model, \( u=v=0.8 \) is 0.001638 and 0.001462, respectively.
5. Conclusion
The initial equilibrium shape in Thomsen form and cable reinforced of Thomsen surface, \( u=v=0.8 \) have been carried out successfully using the procedure adopted which is based on nonlinear analysis method [1]. The Thomsen form and cable reinforced of Thomsen surface with variable \( u=v=0.8 \) to be obtained in this study will provide an alternative shape for designer to be considered for adopted in tensioned fabric structure.
References
[1] Yee H M 2011 A computational strategy for form-finding of tensioned fabric structure using nonlinear analysis method PhD dissertation (School of Civil Engineering, Universiti Sains Malaysia, Pulau Pinang, Malaysia)
[2] Wan Ibrahim M H, Abdul Hadi M N and Yee H M 2018 Form-Finding Using Nonlinear Analysis Method in Tensioned Fabric Structure in The Form of Handkerchief Surface, Journal of Physics 995 1-9
[3] Wan Ibrahim M H, Abdul Hadi M N, and Yee H M 2018 Form-Finding of Thomsen Surface Using Nonlinear Analysis Method Journal of Physics 995 1-8
[4] Yee H M, Abd Malek N A and Aziz N S A 2017 Form-finding of Sustainable Cable Reinforced Tensioned Fabric Structure in Different Pre-Stress, 35th Conference of ASEAN Federation of Engineering Organisations Bangkok Thailand 16-18 November
[5] Yee H M and Choong K K 2016 A Computational Mechanics using Nonlinear Analysis Method in Tensioned Fabric Structure Int. J. Mech 10 261-265
[6] Yee H M and Abdul Hadi M N 2016 Tensioned Fabric Structures with Surface in the Form of Chen-Gackstatter MATEC Web Conf. 64 7001
[7] Yee H M, Choong K K and Abdul Hadi M N 2015 Sustainable Development of Tensioned Fabric Green Structure in the Form of Enneper Int. J. Mater. Mech. Manuf. 3(2) 125-128
[8] Yee H M and Abdul Hadi M N 2015 Enneper in Tensioned Fabric Structures Engineering Development Conference on Mathematical and Computational Methods in Science and Engineering Malaysia, 23-25
[9] Yee H M and Abdul Hadi M N 2015 Tensioned Fabric Structures with Surface in the Form of Chen-Gackstatter and Monkey Saddle Int. J. Struct. Civ. Eng. Res. 4(4) 331-335
[10] Yee H M, Abdul Hamid H, and Abdul Hadi M N 2015 Computer Investigation of Tensioned Fabric Structure in the Form of Enneper Minimal Surface Appl. Mech. Mater. 754-755 743-746
[11] Mohd Noor M S, Yee H M, Choong K K and Haslinda A H 2013 Tensioned Membrane Structures in the Form of Egg Shape Appl. Mech. Mater. 405-408 989-992
[12] Yee H M and Abdul Hadi M N 2015 Soap film Enneper model in structure engineering Proceedings of the International Conference on Advanced Materials, Structures and Mechanical Engineering, South Korea 29-31 pp. 1
[13] Yee H M, Abdul Hadi M N, Ghani K A and Hamid N H A 2015 Tensioned Fabric Structures with surface in the form of Monkey Saddle surface Proceedings of the 2nd International Conference of Advanced Materials, Mechanical and Structural Engineering South Korea 18-20 pp. 191
[14] Yee H M and Samsudin A 2014 Mathematical and Computational Analysis of Moebius Strip Int. J. Math. Comput. Simul. 8 197-201
[15] Yee H M, Kim J Y, and Mohd Noor M S 2013 Tensioned Fabric Structures in Oval Form Appl. Mech. Mater. 405-408 1008-1011
[16] Yee H M, Choong K K and Kim J Y 2011 Form-Finding Analysis of Tensioned Fabric Structures Using Nonlinear Analysis Method Adv. Mater. Res. 243-249 1429-1435
[17] Yee H M and Choong K K 2013 Proposed Algorithm for Warp Direction Checking in Tensioned Fabric Structures Int. J. Sci. Res. Knowl. 13-19
[18] Dierkes U, Hildebrandt D and Sauvigny F 2010 Minimal Surface Springer Berlin, Heidelberg 339 pp. 53-90
[19] MathWorld W. 1999 Thomsen Surface, Wolfram Research, Inc. [Online]. Available: http://demonstrations.wolfram.com/ThomsenSurfaces/. [Accessed: 15-Feb-2018]
[20] ADINA System 8.1 2003 R&D Inc