Effect of peculiar velocities on the gravitational potential in cosmological models with perfect fluids

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Abstract

We consider a universe filled with perfect fluid with the constant equation of state parameter $\omega$. In the theory of scalar perturbations, we study the effect of peculiar velocities on the gravitational potential. For radiation with $\omega = 1/3$, we obtain the expression for the gravitational potential in the integral form. Numerical calculation clearly demonstrates the modulation of the gravitational potential due to the presence of peculiar velocities. We also show that peculiar velocities affect the gravitational potential in the case of the frustrated network of cosmic strings with $\omega = -1/3$.

Keywords: cosmology, scalar perturbations, peculiar velocities, gravitational potential

1. Introduction

Recently\textsuperscript{[1]}, it was demonstrated that peculiar velocities of dark matter inhomogeneities in the $\Lambda$CDM model play an essential role in structure formation. They affect the contrast density and collective gravitational potential of inhomogeneities. For example, it was shown in\textsuperscript{[1]} that peculiar velocities contribute into the effective length of exponential screening of the gravitational potential at large cosmological scales. In turn, the shape of the gravitational potential affects the formation of structures in the Universe. It is of interest to investigate this problem for models where the matter source is different from dark matter. Obviously, because the Universe during its evolution passes through the radiation dominated stage with the shape of the gravitational potential at large cosmological scales. In turn, the shape of the gravitational potential affects the formation of structures in the Universe. It is of interest to investigate this problem for models where the matter source is different from dark matter. Obviously, because the Universe during its evolution passes through the radiation dominated stage with the equation of state $\omega = 1/3$, radiation is the most compelling case. The literature also addresses models with various equations of state. For this reason, we start, for generality, with a model for an arbitrary $\omega = \text{const.} \neq -1/3$. Then we concentrate on radiation and demonstrate that acoustic oscillations modulate gravitational potential due to the presence of peculiar velocities. We also discuss briefly the exceptional case $\omega = -1/3$ (e.g., the frustrated network of cosmic strings) and demonstrate here the effect of peculiar velocities on the gravitational potential.

2. Setup of the model

We consider a universe filled with perfect fluid having the equation of state

$$p = \omega \rho, \quad \omega = \text{const.}$$

(1)

For this model, the background Friedmann equation is

$$\frac{3H^2}{a^2} = \frac{3 H^2}{c^2} = \kappa \rho,$$

(2)

where the Hubble parameter is $H = (da/dt)/a = (c/a)(a' / a) \equiv (c/a)H$. $c$ is the speed of light and synchronous and conformal times are related as follows: $a d\eta = c d t$. Hereafter, the prime denotes the derivative with respect to the conformal time ($\eta$) and the overbar indicates the background values. The constant $\kappa \equiv 8\pi G_N / c^4$ is introduced as well ($G_N$ is the Newtonian gravitational constant). Since the equation of state parameter $\omega$ is the constant, background equation of state is

$$\rho = \omega \rho,$$

(3)

Therefore, from the energy conservation equation we get

$$\dot{\rho} = \frac{\dot{A}}{a^{3(1+\omega)}},$$

(4)

where $\dot{A}$ is the constant of integration. Then, integration of the Friedmann equation (2) for $\omega \neq -1/3$ results in $a \sim \rho^{2/(1+3\omega)} \sim t^{2/[3(1+\omega)]}$ and $\eta \sim t^{1+3\omega/[3(1+\omega)]}$. For example, in the case of radiation $\omega = 1/3$ where Eq. (4) takes the form

$$\dot{\rho} = \frac{\dot{A}}{a^4},$$

(5)

we get $a(\eta) = A_1 \eta^{3/2}$, $(0 \leq \eta < +\infty)$, $a(t) = \sqrt{2\dot{A}c^2}$ and $ct = (1/2)A_1 t^2$ where $A_1 \equiv (\kappa A c^2 / 3)^{1/2}$. In the exceptional case $\omega = -1/3$, where Eq. (4) is transformed into

$$\dot{\rho} = \frac{\dot{A}}{a^2},$$

(6)
integration of Eq. 2 gives \( a(\eta) = Ce^{\eta \omega} \), \((-\infty < \eta < +\infty)\), where \( C \) is the constant of integration, \( a(t) = Bct \) and \( ct = (C/B)e^{\eta \omega} \). Positive dimensionless constant \( B \equiv (\kappa\tilde{A}_x/3)^{1/2} > 0 \).

Background matter is perturbed by inhomogeneities of perfect fluid. We consider only scalar perturbations. Then, in conformal Newtonian gauge perturbed metrics is \([2][3]\)

\[
ds^2 = a^2(\eta)[(1 + 2\Phi)dt^2 - (1 - 2\Phi)dr^2],
\]

and the perturbed Einstein equations read

\[
\Delta \Phi - \frac{3\alpha^2}{a^2} \left( \Phi' + \frac{\alpha'}{a'} \Phi \right) = \frac{1}{2}\kappa a^2 \delta \varepsilon, \tag{8}
\]

\[
\Phi' + \frac{\alpha'}{a'} \Phi = -\frac{1}{2}\kappa a^2 \delta \rho, \tag{9}
\]

\[
\Phi'' + \frac{3\alpha''}{a'} + \frac{\alpha'}{a} \Phi' + \frac{3\alpha^2}{a^2} \Phi = -\frac{1}{2}\kappa a^2 \delta \rho, \tag{10}
\]

where \( \delta \rho = \omega \delta \varepsilon \). \( \tag{11} \)

Therefore, the squared speed of sound is equal to the parameter of equation of state:

\[
u_s^2 = \frac{\delta \rho}{\delta \varepsilon} = \frac{\delta \rho}{\tilde{\varepsilon}} = \omega. \tag{12} \]

The energy density fluctuation can be expressed as follows \([4][5][6][7]\)

\[
\delta \varepsilon = \frac{\delta A}{A^{1+\omega}} + 3(1 + \omega) \tilde{\varepsilon} \Phi, \tag{13} \]

where \( \Phi \) is singled out.

We rewrite the perturbed Einstein equation in momentum space with the help of the Fourier transform:

\[
F(r) = (2\pi)^{-3/2} \int_{\mathbb{R}^3} d\mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} \tilde{F}(\mathbf{k}). \tag{14} \]

Hereafter the tilde denotes the Fourier transformed quantities. For example, Eq. 5 reads:

\[
k^2\tilde{\Phi} + \frac{3\alpha'}{a} \tilde{\Phi}' + \frac{\alpha'}{a^2} \tilde{\Phi} = -\frac{1}{2}\kappa a^2 \delta \varepsilon. \tag{15} \]

Then, the sum of Eq. 15 times \( \omega \) and transformed Eq. 10 results in

\[
\Phi'' + \frac{3\alpha''}{a'} + \left( 1 + nu_s^2 \right) \Phi' + k^2 u_s^2 \Phi = 0, \tag{16} \]

where we took into account the relation

\[
2\frac{\alpha''}{a} - \frac{\alpha^2}{a^2} (1 - 3\omega) = -\kappa a^2 (\tilde{\rho} - \omega \tilde{\varepsilon}) = 0, \tag{17} \]

following from the Friedmann equations. The case of dust \( u_s = 0 \) was investigated in \([\underline{1}]\). Therefore, we will not consider it in our paper. The physical wavelength corresponding to the momentum \( k \) is \( \lambda_x = a/k \) and the sound horizon is \( \lambda_x \equiv u_s H^{-1}c \).

Since in the case \( \omega \neq -1/3 \) the scale factor behaves as \( a(\eta) \sim \eta^{2/(1+3\omega)} \), then Eq. 16 reads

\[
\tilde{\Phi}'' + \frac{6(1 + \omega)}{1 + 3\omega} \tilde{\Phi}' + u_s^2 k^2 \tilde{\Phi} = 0 \tag{18} \]

with the general solution (see, e.g., 53:3.7 in \([8]\))

\[
\tilde{\Phi}(\eta) = C_1 \eta^3 J_{-3/2}(u_s \eta k) + C_2 \eta^{3/2} J_{3/2}(u_s \eta k), \tag{19} \]

where \( J_{\nu} \) are Bessel functions and

\[
u = -\frac{5 + 3\omega}{2(1 + 3\omega)}. \tag{20} \]

3. Relativistic perfect fluid

In this section, we consider only the case of radiation in detail. Since for radiation \( \omega = 1/3 \), the general solution \([19]\) reads

\[
\tilde{\Phi}(\eta) = C_1 \eta^{3/2} J_{3/2}(u_s \eta k) + C_2 \eta^{-3/2} J_{-3/2}(u_s \eta k). \tag{21} \]

In the considered case, \( \lambda_x = a/k = A_1 \eta k \) and \( \lambda_x = u_s A_1 \eta^2 \)

and for the modes outside the horizon \( \lambda_x = A_1 \eta k \gg \lambda_x = u_s A_1 \eta^2 \implies u_s \eta k << 1 \). Taking into account the properties of the Bessel functions \([8][9]\)

\[
\sqrt{\frac{\pi}{2x}} J_{3/2}(x) = j_{1,2}(x) = -\frac{\cos x}{x} + \frac{\sin x}{x^2}, \tag{22} \]

\[
\sqrt{\frac{\pi}{2x}} J_{-3/2}(x) = j_{1,2}(x) = -\frac{\cos x}{x^2} - \frac{\sin x}{x}, \tag{23} \]

(where \( j_{1,2} \) are the spherical Bessel functions) and asymptotes for small \( x \)

\[
J_{3/2}(x) \rightarrow \frac{4}{3\pi} x^{3/2} \rightarrow 0, \tag{24} \]

\[
J_{-3/2}(x) \rightarrow \frac{1}{2} \pi x^{-1/2} \rightarrow -\infty, \tag{25} \]

we can easily see that the only non-falling mode is

\[
\tilde{\Phi}(\eta) = \Phi_0(\eta) \sqrt{\frac{\pi}{2(u_s \eta k)^{3/2}}} J_{3/2}(u_s \eta k) \tag{26} \]

\[
= -3\Phi_0(\eta) \sqrt{\frac{\pi}{2(u_s \eta k)^{3/2}}} \left[ \cos(u_s \eta k) - \frac{\sin(u_s \eta k)}{u_s \eta k} \right] \]

in full agreement with \([2]\). This solution tends to the constant: \( \Phi(\eta) \rightarrow \Phi_0(\eta) \) outside the sound horizon \( u_s \eta k \rightarrow 0 \). The r.h.s. of \([26]\) demonstrates that acoustic oscillations modulate the gravitational potential which we will show explicitly below.

Therefore, the Fourier transformed l.h.s. of Eq. 9 is

\[
\tilde{\Phi}' + \frac{\alpha'}{a^2} \tilde{\Phi} = 3\Phi_0(\eta) \sqrt{\frac{\pi}{2(u_s \eta k)^{3/2}}} \left[ \cos(u_s \eta k) - \frac{\sin(u_s \eta k)}{u_s \eta k} \right] + \sin(u_s \eta k) \]

\[
= -\tilde{\Phi}(\eta) \sqrt{\frac{\pi}{2(u_s \eta k)^{3/2}}} \left[ 2 + \frac{(u_s \eta k)^2 \sin(u_s \eta k) - \sin(u_s \eta k)}{u_s \eta k} \right]. \tag{27} \]
We can use this formula to determine the peculiar velocity via Eq. (2). It shows that the peculiar velocity undergoes the acoustic oscillations. On the other hand, substitution of this expression into Eq. (13) eliminates the time derivative allowing us to define the Fourier transform of the gravitational potential

\[
\Phi = \frac{-\kappa \frac{\delta \lambda_r}{a}}{\frac{k^2 - \frac{3}{\eta^2} \left[ \frac{(u_k k)^2}{u_k k \cos(u_k k) - \sin(u_k k)} + \frac{a^2}{\lambda_r^2} \right]}{a^3}},
\]

(28)

where we used the Fourier transform of the energy density fluctuation (13) for radiation

\[
\delta \tilde{\epsilon} = \frac{\delta A_r}{a^4} + 4 \frac{\lambda_r}{a^2} \tilde{\Phi}.
\]

(29)

We also introduced the screening length

\[
\lambda_r^2 \equiv \left[ \frac{2\kappa A_r}{\lambda_r^2} \right]^{-1}.
\]

(30)

It can be easily seen that

\[
a^2 \lambda_r^2 = \frac{6}{\eta^2},
\]

(31)

and \( \lambda_r = \lambda_s / \sqrt{2} \).

Since the combination \( \tilde{\Phi}' + (a'/a) \tilde{\Phi} \) is proportional to the peculiar velocity, it describes in (15) the effect of the peculiar velocity on the gravitational potential. If we neglect the contribution of peculiar velocity, then the matter density fluctuation is

\[
\frac{\delta \lambda_r}{a^4} = -\frac{2}{\kappa a^2} \left[ k^2 + \frac{a^2}{\lambda_r^2} \right] \tilde{\Phi} = -\frac{2}{\kappa a^2} f_1(k) \tilde{\Phi}.
\]

(32)

On the other hand, the peculiar velocity contribution leads to the appearance of an additional k-dependent term:

\[
\frac{\delta \lambda_r}{a^4} = -\frac{2}{\kappa a^2} \left[ \frac{k^2 - \frac{3}{\eta^2} \left( \frac{(u_k k)^2}{u_k k \cos(u_k k) - \sin(u_k k)} \right) + \frac{a^2}{\lambda_r^2} \right] \tilde{\Phi} = -\frac{2}{\kappa a^2} f_2(k) \tilde{\Phi},
\]

(33)

where we took into account the relation (31).

To obtain an explicit form of the gravitational potential, it is necessary to determine the expression for the matter density fluctuations \( \delta \lambda_r / a^4 \). Now we consider the model where this fluctuation is a localized inhomogeneity in the form of the delta function

\[
\frac{\delta \lambda_r}{a^4} \rightarrow \frac{M c^2 \delta(r)}{a^3},
\]

(34)

where \( M \) is an effective mass of this inhomogeneity. With the help of the inverse Fourier transform we get

\[
\frac{\delta \lambda_r}{a^4} = \frac{M c^2 (2\pi r)^{-3/2}}{a^3}.
\]

(35)

Now, we can describe preliminary some properties of the gravitational potentials in the presence or absence of the peculiar velocities. First, both (32) and (33) deep inside the horizon \( \Lambda_k = a/k \ll \Lambda_r \) behaves as

\[
\frac{\delta \lambda_r}{a^4} \approx -\frac{2}{\kappa a^2} \frac{k^2}{\lambda_r^2} \tilde{\Phi}.
\]

(36)

Such a large \( k \) limit corresponds to the short distances from an inhomogeneity. Therefore, this formula demonstrates that at short distances the gravitational potential has the Newtonian behavior for both cases.

Eq. (32) shows that the gravitational potential satisfies the Helmholtz equation. Therefore, if we neglect the peculiar velocity, the gravitational potential created by individual inhomogeneity has the Yukawa potential form with characteristic length of interaction \( \lambda_r / a \). That is, at large distances (i.e., small \( k \)), for example, outside of the horizon \( a/k \ll \lambda_r \ll \kappa \eta \ll 1 \), the gravitational potential decreases exponentially with the exponent \( a/\lambda_r \). In the case of Eq. (33) (i.e., in the presence of the peculiar velocities), the Helmholtz equation is corrupted because of the additional \( k \)-dependent term. Hence, the gravitational potential is not the Yukawa one. As we show below, the potential is approximately described by the Yukawa potential modulated by acoustic oscillations. Moreover, outside the horizon \( a/k \gg \lambda_r \ll \kappa \eta \ll 1 \) Eq. (33) accepts the following form

\[
\frac{\delta \lambda_r}{a^4} \approx -\frac{2}{\kappa a^2} \left( k^2 + \frac{3a^2}{2\lambda_r^2} \right) \tilde{\Phi}.
\]

(37)

Therefore, the effect of the peculiar velocity results in the prefactor \( 3/2 \) in comparison with Eq. (32), and at large distances the gravitational potential decreases exponentially with the exponent \( \sqrt{3/2} a/\lambda_r \).

Coming back to Eqs. (32) and (33), we can express the gravitational potentials in the position space as

\[
\Phi_i(r) = \frac{1}{(2\pi)^{3/2} 2a^3} \int_{k > 0} d\kappa e^{ikr} \frac{\delta \lambda_r}{f(k)}
\]

(38)

\[
= \frac{G \kappa M}{c^2 \ar \eta} \int_0^{\infty} dk \frac{3 \sin(kr)}{f(k)}, \quad i = 1, 2,
\]

where we used the relation (35). Obviously, in the case of the Helmholtz Eq. (32), the gravitational potential has the Yukawa form

\[
\Phi_i(r) = -\frac{G \kappa M}{c^2 \ar} e^{-ar/\lambda_r}.
\]

(39)

However, in the case of Eq. (33), the integral (38) can be calculated only numerically. Since \( a^2 / \lambda_r^2 = 6/\eta^2 \), the gravitational potential depends parametrically on the conformal time \( \eta \).

\[\text{We do the numerical calculations in Python by discretizing the integral in (38). Since we know the analytic formula for } \Phi_i, \text{ to determine the efficiency of the code we check the relative error } e = |x_n - x_d|/|x_n| \text{ where } x_n \text{ and } x_d \text{ are the numerical and the analytical solutions, respectively. We have found that the error is in the order of } 10^{-8} \text{ at most in the steep region of the potential and it rapidly decreases in the flat region as expected. Regarding the pure numerical case } \Phi_i, \text{ by using the same code, we only can compare two successive numerical solutions, say } x_n \text{ and } x_{n+1}, \text{ as } e = |x_n - x_{n+1}|/|x_n| \text{ to estimate the relative error. In this case, the error is in the order of } 10^{-8} \text{ at most.}\]
Figure 1: Gravitational potentials in the case of radiation for different values of the parameter $\eta$. The dashed blue line corresponds to pure Yukawa potential and the solid orange line takes into account the effect of peculiar velocity.

In Figure 1, we represent pictures for three different values of $\eta$ where the dimensionless potentials are defined as

$$\hat{\Phi}_i(r) = -\frac{c^2a_i}{G_NM} \Phi_i, \quad i = 1, 2.$$  \hfill (40)

These pictures demonstrate the effect of peculiar velocities on the form of the gravitational potential (see the difference between $\Phi_1$ and $\Phi_2$), in particular, the modulation of the gravitational potential by acoustic oscillations in $\Phi_2$.

It is also convenient to introduce new designations as $k \eta \equiv l$ and $r/\eta \equiv \xi$. Then, for dimensionless gravitational potentials we have

$$\phi_i(\xi) = -\frac{c^2a_i}{G_NM} \Phi_i = \frac{1}{\xi \pi} \int^\xi_0 dl \frac{l \sin(l \xi)}{f_i(l)}, \quad i = 1, 2,$$  \hfill (41)

where

$$f_1(l) \equiv l^2 + 6,$$  \hfill (42)

$$f_2(l) \equiv l^2 - 3 \frac{(u_\xi l)^2 \sin(u_\xi l)}{u_\xi l \cos(u_\xi l) - \sin(u_\xi l)}.$$  \hfill (43)

Thus, for all values of $\eta$ we have only one figure depicted in Figure 2.

4. The frustrated network of cosmic strings: $\omega = -1/3$

In this section, we briefly discuss the exceptional case $\omega = -1/3$. Such a model can describe the frustrated network of cosmic strings. Despite the negative sign of the speed of sound squared, it was shown \[10, 11\] that such a component could be stable if sufficiently rigid.

In the considered case, Eq. (15) is

$$\ddot{\Phi} + 2B\dot{\Phi} - \frac{k^2}{3} \Phi = 0$$  \hfill (44)

with the general solution

$$\Phi = C_1 e^{-B \sqrt{\frac{k^2}{3} + 1} \eta} + C_2 e^{B \sqrt{\frac{k^2}{3} + 1} \eta}.$$  \hfill (45)

Obviously, the first solution is the decreasing one and we neglect it. Hence,

$$\Phi = \Phi_1 e^{-B \sqrt{\frac{k^2}{3} + 1} \eta} = \Phi_1 \left( \frac{B}{C} \right)^{-1 + \sqrt{1 + \frac{k^2}{3B^2}}}.$$  \hfill (46)

where $\Phi_1$ is the value of $\Phi$ at $\eta = 0$. Therefore, the l.h.s. of Eq. (9) is

$$\ddot{\Phi} + a^\prime \ddot{\phi} = \sqrt{B^2 + \frac{k^2}{3}} \ddot{\phi}$$  \hfill (47)
and defines the peculiar velocity.

If we neglect the contribution of the peculiar velocity into equation (8) and consequently drop off the expression (47), then we get

$$\frac{\delta \Lambda_{st}}{a^2} = -\frac{2}{\kappa a^2} \left( k^2 + \frac{\alpha^2}{\lambda_{st}^2} \right) \Phi \equiv -\frac{2}{\kappa a^2} f_1(k) \Phi, \quad (48)$$

where we took into account that the energy density fluctuations (13) in the considered case is

$$\ddot{\varepsilon} = \frac{\delta \Lambda_{st}}{a^2} + 2 \frac{\dot{\Lambda}_{st}}{a^2} \Phi. \quad (49)$$

We introduced the screening length

$$\lambda_{st}^2 \equiv \left[ \frac{\kappa \dot{\Lambda}_{st}}{a^2} \right]^{-1}. \quad (50)$$

It can be easily seen that

$$\frac{a^2}{\lambda_{st}^2} = 3B^2 = \text{const} \quad (51)$$

and $\lambda_s \equiv | \rho_s / H^2 c = \lambda_{st}.$

Obviously, Eq. (48) is pure Helmholtz equation. However, if we preserve the velocity depended expression (47) in Eq. (8), then an additional $k$-depended term comes into play:

$$\frac{\delta \Lambda_{st}}{a^2} = \frac{2}{\kappa a^2} \left( k^2 + 3B^2 + \frac{k^2}{3} + \frac{a^2}{\lambda_{st}^2} \right) \Phi \equiv \frac{2}{\kappa a^2} f_2(k) \Phi, \quad (52)$$

corrupting the Helmholtz equation.

Let us consider now the model where the matter fluctuation is a localized inhomogeneity:

$$\frac{\delta \Lambda_{st}}{a^2} = \frac{M c^2 \delta(r)}{a^3} \quad (53)$$

It can be easily seen from (48) and (52), that, similar to the case of radiation, at short distances from the inhomogeneity (i.e., large $k$ limit), the gravitational potentials have Newtonian behavior. On the other hand, at large distances (e.g., outside the horizon $a/k \gg \lambda_{st}$)

$$f_2(k) \approx k^2 + 2 \frac{a^2}{\lambda_{st}^2}. \quad (54)$$

Therefore, both gravitational potentials exponentially decrease at large distances. However, the consideration of peculiar velocities leads to a change in the screening length.

The gravitational potentials in the position space are given by Eq. (38) where the functions $f_i(k)$ are defined in Eqs. (48) and (52). These integrals depend on the parameter $B$.

In Figure (3), we present dimensionless potentials for a particular case $B = 0.1$. The difference between the two lines shows the effect of peculiar velocities.

We can also construct the $B$-independent dimensionless potentials as

$$\phi_i(\xi) \equiv -\frac{c^2 a}{G_N MB} \Phi_i = \frac{1}{2} \frac{\xi}{\pi} \int_0^\infty dl \frac{\sin(l \xi)}{f_i(l)}, \quad i = 1, 2. \quad (55)$$

where $k/B \equiv l$, $Br \equiv \xi$ and

$$\bar{f}_1(l) \equiv l^2 + 3, \quad (56)$$

$$\bar{f}_2(l) \equiv l^2 + 3 \sqrt{1 + \frac{2}{3}} + 3. \quad (57)$$

The corresponding potentials are depicted in Figure (4).

Figure 3: Gravitational potentials in the case of the frustrated network of cosmic strings for parameter $B = 0.1$. The dashed blue line corresponds to pure Yukawa potential, and the solid orange line takes into account the effect of peculiar velocity.

Figure 4: Gravitational potentials \( \phi(\xi) \) where $\xi = Br$. The dashed blue and the solid orange lines have the same meaning as in Figure (3).

5. Conclusion

In the present paper, we have investigated the effect of the peculiar velocities on the form of the gravitational potential in cosmological models with perfect fluids. We have considered...
perfect fluids with the constant parameter $\omega$ of the equation of state. Starting from an arbitrary value of $\omega$, we then concentrated on relativistic fluid with $\omega = 1/3$. Here, peculiar velocities undergo acoustic oscillations [2]. In the momentum space, we have obtained the formulas for the gravitational potentials both in the presence and absence of peculiar velocities. To get the exact form of potentials in the position space, we have assumed that the matter fluctuation is a localized inhomogeneity in the form of the delta function. If we neglect peculiar velocities, then the gravitational potential has the form of the Yukawa potential. Since the Fourier integral for the velocity-dependent potential can be calculated only numerically, we have depicted the results graphically in Figures (1) and (2). These figures clearly demonstrate the modulation of the gravitational potential by acoustic oscillations due to the presence of peculiar velocities.

To illustrate the effect of the peculiar velocities on the gravitational potential, we also considered the case of the frustrated network of cosmic strings with $\omega = -1/3$. The result is depicted in Figures (3) and (4). In this exceptional case, acoustic oscillations are absent. Nevertheless, the difference between the figures demonstrates the effect of peculiar velocity.

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