Modeling and Simulating on the Bandwidth and Capacity of Underwater Acoustic Channel

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Abstract. For underwater acoustic communication, the acoustic signals produce great attenuation in the propagation, and the complex environmental noise leads to the extremely limited bandwidth and capacity of the underwater acoustic channel. In this paper, the bandwidth and capacity of the channel are modeled and simulated under the condition that the transmitted power spectral density is constant. The theoretical analysis and simulation results show that the lower limit of the bandwidth of the underwater acoustic channel is determined by the noise and the upper limit is determined by the attenuation of the channel. Therefore, it is concluded that the narrow bandwidth of underwater acoustic channel is determined by attenuation and noise.

1. Introduction
Underwater acoustic channel is one of the most complex communication channels at present. In addition to the characteristics of multipath time extension and obvious Doppler effect [1], it also has the features of narrow bandwidth and small capacity. Due to the rapid attenuation of electromagnetic wave in seawater, sound wave with lower frequency can only be used as the information carrier of underwater communication at present. Furthermore, factors such as the diffusion and attenuation of sound wave during its propagation in seawater not only cause the distortion of received signals, but also ultimately lead to the extremely limited bandwidth of underwater acoustic channel [2]. Different from radio wireless electromagnetic channels, acoustic wave propagation loss increases with frequency and distance in underwater, and the main reason of propagation loss is the absorption and diffusion of acoustic wave. Marine ambient noise is the main noise of underwater acoustic channel, which will affect the detection and recognition of sonar signal. In recent years, many achievements have been made in the modeling of multipath effect and Doppler effect of underwater acoustic channels [3][4], but relatively few achievements have been acquired in terms of bandwidth and capacity of underwater acoustic channels. Based on the combination of acoustic theory and the actual marine environment, this paper analyzes and studies the law of underwater acoustic channel. In view of the unique characteristics of the bandwidth and capacity of the channel, the relevant mathematical model is established in line with the actual situation, and the research is carried out through modeling and simulating.

The remainder of this paper is organized as follows. In Section II, attenuation and noise of underwater acoustic channel are introduced respectively. Section III briefly discusses the theoretical model about channel bandwidth and capacity. Section IV contains the results of simulation. Conclusions are summarized in Section V.
2. Attenuation and Noise of Underwater Acoustic Channel

2.1. Attenuation of underwater acoustic channels
Different from radio wireless electromagnetic channels, acoustic wave propagation loss increases with frequency and distance in underwater, and the main cause of propagation loss is the absorption and diffusion of acoustic wave. Attenuation loss and spread loss are the main forms of acoustic propagation loss in media. Attenuation loss mainly refers to the effects of scattering, absorption and leakage of sound energy from the sound channel. Spread loss, sometimes refers to the geometrically attenuated effect of the sound signal from the sound source as it propagates outward. When specific propagation conditions do not need to be taken into account, for acoustic channels with transmitting signal frequency $f$ and path length $l$, the following formula can be used to roughly estimate propagation loss assuming the attenuation is represented by dB

$$10 \log \left( \frac{A(l, f)}{A_0} \right) = n \cdot 10 \log l + 10 \log \alpha(f)$$

where, $A_0$ represents the unit normalized constant; $n$ represents the coefficient of different propagation conditions, usually its value range is 0~4; $l$ represents the distance from the sound source, and its unit is m; $\alpha$ represents the absorption coefficient, and its unit is dB/m.

The absorption capacity of sea water to sound wave is related to the frequency of sound wave. Schulkin and Marsh concluded the semi-empirical formula of absorption coefficient based on the 30000 measurements of frequency 2~25 kHz and distance within 22 km, which can be expressed as follows [5]

$$\alpha = ASf \cdot f^2 / (f^2 + f_r^2) + BF^2 / f_r$$

where, $A = 1.89 \times 10^{-5}$; $B = 2.72 \times 10^{-5}$; $S$ denotes salinity (‰); $f$ denotes acoustic frequency (kHz); $f_r$ is the relaxation frequency (kHz), which is equal to the reciprocal of the relaxation time and related to temperature

$$f_r = 2.19 \times 10^7 \left( \frac{1520}{T} \right)$$

where, $T$ is absolute temperature (K). For high frequency signals (i.e., when $f \gg f_r$), or for low frequency signals (i.e., when $f \ll f_r$), the attenuation coefficient is proportional to the square of the frequency of the sound wave. In short, the bandwidth of the system with long communication distance is only a few kHz due to absorption loss and spread loss. The bandwidth of the medium distance communication system is about 10 kHz, while the short distance communication can obtain a wider bandwidth of more than 100 kHz. In summary, the bandwidth of most underwater acoustic channels is limited.

2.2. Noise of underwater acoustic channel
The marine environmental noise can be divided into marine dynamic noise, traffic and industrial noise, biological noise, under-ice noise and seismic noise according to its causes of production. In addition, marine environmental noise is also related to different sea areas, meteorological conditions, hydrophone location and signal frequency, which is complex and changeable. In 1948, Knudson et al. studied the marine environmental noise spectrum level [6]. The so-called environmental noise spectrum level refers to the sound pressure level within the 1Hz bandwidth received by the non-directional hydrophone. Their research results show that the environmental noise level increases with the sea state or wind power, and decreases with the increase of frequency in a certain law. The environmental noise of the ocean can be simulated with four kinds of noise sources: turbulence noise, ship noise, wave noise and thermal noise. The empirical formula for the power spectral density of the four kinds of noises in ocean noise is given below [7]

$$10 \log N_t(f) = 17 - 30 \log f$$

$$10 \log N_s(f) = 40 + 20(s - 0.5) + 26 \log f - 60 \log(f + 0.03)$$
\[ 10 \log N_w(f) = 50 + 7.5 \log f + 20 \log f - 40 \log (f + 0.4) \] (6)
\[ 10 \log N_{th}(f) = -15 + 20 \log f \] (7)

Where, \( f \) is frequency, in kHz; \( N_t, N_s, N_w, N_{th} \) represents turbulence noise, ship noise, sea wave noise and thermal noise respectively; \( s \) denotes the activity level of the vessel, and \( s \in [0,1] \); \( w \) represents the wind speed over the sea.

3. Bandwidth and Capacity of Underwater Acoustic Channel

According to the characteristics of the underwater acoustic channel attenuation and noise, the noise at low frequency is larger, this determines the lower limit of the bandwidth about the channel; and the values of the channel attenuation in high frequency region is larger, this determines the upper limit of the bandwidth to the channel. As a result, the underwater acoustic channel is a narrow bandwidth which is determined by the attenuation and noise. In Literature [8], only the main path in the multipath of the channel is selected for studying and analyzing through simulation. The frequency band is divided into several small narrow bands, which are expressed as \( \Delta f \), and the Gaussian noise power spectral density is used to approximate the noise power spectral density within \( \Delta f \). By using the power spectral density \( N(f) \) and attenuation of noise \( A(l, f) \), the receiver can obtain the signal-to-noise ratio (SNR) formula within \( \Delta f \)

\[ \text{SNR}(l, f) = \frac{P/A(l, f)}{N(f) \cdot \Delta f} = \frac{S_l(f)}{A(l, f) \cdot N(f)} \] (8)

Where, \( f \) is the carrier frequency; \( l \) is the transmission distance; \( S_l(f) \) is the power spectral density function, and \( P \) is the transmitting power. Then the SNR within the entire bandwidth can be expressed as follows

\[ \text{SNR}(l, B(l)) = \frac{\int_{B(l)} S_l(f) A^{-1}(l, f) df}{\int_{B(l)} N(f) df} \] (9)

where \( B \) denotes the bandwidth. Afterwards the transmitted power is given by

\[ P = \int_{B(l)} S_l(f) df \] (10)

It can be known from Equation (8) that the SNR of different frequencies is determined by \( A(l, f) \cdot N(f) \), and the optimal carrier frequency \( f_0(l) \) of different transmission distances is the frequency point corresponding to the highest SNR. The channel capacity is calculated according to the Shannon formula, and the results are as follows

\[ C(l) = \sum \Delta f \log_2 \left[ 1 + \frac{S_l(f_0 A^{-1}(l, f))}{N(f)} \right] = \int_{\Delta f} \log_2 \left[ 1 + \frac{S_l(f) A^{-1}(l, f)}{N(f)} \right] df \] (11)

4. Simulation Analysis

By applying the empirical formula expressed in Equations (4)-(7) to compute and simulate, the change curve of power spectral density of underwater noise can be obtained, as shown in Figure 1. Meanwhile, the law of interaction between attenuation and noise can also be acquired, as shown in Figure 2. As can be seen from Figure 2, the simulation is carried out at different transmission distances, and there is an optimal frequency that minimizes attenuation, and the frequency value is different for different transmission distances, and is lower when the signal is transmitted in a short distance.
The channel capacity of underwater acoustic is discussed below when the transmitted power spectral density \( S_l(f) \) of the signal is constant. First, 3dB bandwidth is defined, which is denoted by \( B_3 \), and it is expressed as \( A(l,f) \cdot N(f) \)

\[
B_3 = f_h - f_i \quad (12)
\]

where, \( f_h \) and \( f_i \) represent the highest frequency and the lowest frequency respectively. When \( f \in [f_i, f_h] \), the following formula holds

\[
\text{SNR}(l,f) > \text{SNR}(l,f_0(l))/2 \quad (13)
\]

where, \( f_0(l) \) represents the optimal carrier frequency when the transmission distance is \( l \). Several typical values about \( f_0(l) \) and \( B_3 \) can be obtained from Equation (8), and specific data are shown in Table 1[7].

When \( S_l(f)=S \) and \( B(l)=B_3(l) \), namely, the power spectral density is constant and the bandwidth is 3dB, the channel capacity can be calculated by the following formula

\[
C_3(l) = \int_{f_i}^{f_h} \log_2 \left[ 1 + \frac{P_3(l)/B_3(l)}{A(l,f) \cdot N(f)} \right] df \quad (14)
\]

### Table 1. Optimal carrier frequency and bandwidth under different transmission distance

| Distance \( l \) [km] | The optimal \( f_0 \) [kHz] | 3dB bandwidth \( B_3 \) [kHz] | \([f_i, f_h]\) [kHz] |
|------------------------|-----------------------------|-------------------------------|------------------|
| 5                      | 8.626                       | 10.884                        | [3.783, 14.667]  |
| 10                     | 5.923                       | 7.652                         | [2.475, 10.127]  |
| 20                     | 4.009                       | 5.489                         | [1.502, 6.991]   |
| 50                     | 2.096                       | 3.490                         | [0.650, 4.140]   |
| 100                    | 0.888                       | 1.809                         | [0.432, 2.241]   |
| 150                    | 0.680                       | 1.025                         | [0.357, 1.382]   |
| 200                    | 0.581                       | 0.739                         | [0.314, 1.053]   |

In the case of 3dB bandwidth, the simulation can obtain the channel capacity, bandwidth and bandwidth utilization ratio at different multipath number and different transmission distance, as shown in Figure 3 and Figure 4. With the increase of the number of channel multipath, the channel space will increase, but the increase is not significant. In addition, it can be seen from Figure 4 that the bandwidth utilization ratio of the channel almost does not change with the transmission distance, but
with the increase of the number of channel multipath; the bandwidth utilization of the channel significantly increases, but the extent of the increase gradually decreases.

Figure 3. Channel capacity and bandwidth via different path and distance

Figure 4. Bandwidth utilization ratio via different path and distance

Furthermore, Figure 5 is a simulation analysis of the relationship between the channel capacity and the SNR. It can be seen from Figure 5 that, with the increase of the SNR, the channel capacity and bandwidth utilization ratio will significantly increase, while under the same conditions, with the increase of the propagation distance, the capacity of the channel will gradually decrease.

Figure 5. Channel capacity via different SNR and transmission distance

5. Conclusions
Due to the complexity of underwater acoustic channel, most of the current research on the channel is some approximate statistical models obtained through simulation research or measurement. This paper focuses on the bandwidth and capacity of underwater acoustic channel, using the mathematical model about attenuation of acoustic signal and the complex environmental noise in the channel, through theoretical analysis and computer simulation experiments, the physical quantity that affects the bandwidth and capacity of the channel is modeled and simulated. The results show that the bandwidth and capacity of underwater acoustic channel are affected by many factors, such as signal-to-noise ratio, propagation distance and number of multipath. Channel bandwidth and capacity is finally determined by the sound attenuation and the environmental noise, namely the low frequency channel noise determines the lower limit of the bandwidth, the attenuation of high frequency area determines the upper limit of the bandwidth.
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