A phenomenological study of 5d supersymmetry

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Abstract

Supersymmetry and extra dimension need not be mutually exclusive options of physics for the TeV scale and beyond. Intrinsically higher dimensional top-down scenarios, e.g. string models, often contain supersymmetry at the weak scale. In this paper, we envisage a more phenomenological scenario by embedding the 4d constrained minimal supersymmetric standard model in a flat 5d $S_1/Z_2$ orbifold, with the inverse radius of compactification at the TeV scale. The gauge and Higgs supermultiplets are placed in the bulk. We assume that only the third generation matter multiplet accesses the bulk, while the first two generations are confined to a brane on an orbifold fixed point. From a 4d perspective, the bulk has $N = 2$ supersymmetry which entails a special non-renormalization theorem giving rise to a significant numerical impact on the renormalization group running of various parameters. The brane supersymmetry corresponds to $N = 1$ which we assume to be broken in an unspecified but phenomenologically acceptable way. Given this set-up, we study how the gauge and Yukawa couplings and the $N = 1$ brane supersymmetry breaking soft masses run through the energy scale exciting the Kaluza-Klein states at regular interval. In the process, we ensure that electroweak symmetry does break radiatively. We confront our low energy parameters with the experimental measurements or limits of different observables, e.g. LEP lower limits on the lightest Higgs boson and the chargino, the $(g-2)$ of muon, the branching ratio of $b \rightarrow s\gamma$, and the WMAP probe of relative dark matter abundance. We present our results by showing the allowed/disallowed zone in the plane of the common scalar mass ($m_0$) and common gaugino mass ($M_{1/2}$) for both positive and negative $\mu$ parameter. Our plots are the first 5d versions of the often displayed 4d $m_0 - M_{1/2}$ plots, and we provide reasons behind the differences between the 4d and 5d plots.

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1 Introduction

The yet elusive Higgs boson is the primary target of the CERN Large Hadron Collider (LHC). But, LHC is also expected to reveal a new ruler of the tera-electron-volt (TeV) territories. The standard model (SM) has so far been remarkably successful in explaining physics up to a few hundred GeV energy scale. But theoretical inconsistencies of the SM (like, gauge hierarchy problem) and experimental requirements (like, a candidate to account for the dark matter of the universe) suggest that there are good reasons to believe that new physics beyond the SM is just around the corner crying out for verification. Among the different possibilities, supersymmetry and extra dimension stand out as the two leading candidates for dictating terms in the TeV regime. These two apparently distinct classes of scenarios cover a wide variety of more specific models. The usual practice from a bottom-up approach is to attach an ‘either/or’ tag on supersymmetry and extra dimension, as if the presence of one excludes the other. A more careful thought would reveal that the relationship between
these two is *not necessarily* mutually exclusive. In fact, the presence of higher dimensions is a common feature of any fundamental theory valid at high scale. We will get back to this issue a little later. For the moment, to put things into perspective, we recapitulate the chronological evolution of the extra dimensional scenarios without invoking supersymmetry *a priori*. We restrict our discussion to the flat space scenarios, as we are not pursuing the warped path in this paper besides mentioning it as an aside.

Flat extra dimensions were first studied \[1\] in a scenario where gravity propagates in a millimeter (mm) size compact space dimension, with the SM particles confined to a 4d brane. The motive was to bring down the fundamental Planck scale to about a TeV. Subsequently, it was conceived that the brane where the SM particles live may actually have a very small size, like \[10^{-16}\] cm \(\sim\) TeV\(^{-1}\), leading to the concept of a ‘fat brane’ \[2\]. In the context of the present paper, we stick to the fat brane scenario. What are the experimental bounds on the fatness of such a brane, more precisely, on the radius of compactification \(R\)? For universal extra dimension (UED) models \[3\], in which all the SM particles access the extra dimensional bulk, a safe estimate is \(R^{-1} \gtrsim 500\) GeV. More specifically, the \(g - 2\) of the muon \[4\], flavor changing neutral currents \[5-7\], \(Z \rightarrow b \bar{b}\) decay \[8\], the \(\rho\) parameter \[9\], and hadron collider studies \[10\] reveal that \(R^{-1} \gtrsim 300\) GeV. Consideration of \(b \rightarrow s\gamma\), however, implies a somewhat tighter bound \((R^{-1} \gtrsim 600\) GeV \[12\]). Methods to decipher its signals from the LHC data have recently been discussed too \[13\]. On the other hand, in the non-universal scenario where both the SM gauge bosons and the Higgs boson propagate in the bulk but the fermions are confined to a 4d brane \[14\], \(R^{-1}\) cannot be below \((1-2)\) TeV \[15\]. The reason behind the difference in constraints is the following. The KK parity, defined by \((-1)^n\) for the \(n\)th KK label, is conserved in UED, while it is not a good symmetry in the non-universal scenario. As a result, while in the non-universal models KK states can mediate many processes at tree level yielding strong constraints, in the UED model, thanks to the KK parity, KK states appear only inside a loop leading to milder constraints. In any case, in the presence of supersymmetry, all those analyses need to be modified with more parameters, which would expectedly lead to a set of more relaxed bounds on \(R^{-1}\).

Now, what is the motivation of studying a TeV scale (or, a fat brane) extra dimension scenario? The implications of such models have been investigated from the perspective of string theory, phenomenology, cosmology/astrophysics and high energy experiments. Such models provide a cosmologically stable dark matter candidate \[16\], trigger electroweak symmetry breaking successfully through a composite Higgs \[17\], address the fermion mass hierarchy problem from a different point of view \[18\], and stimulate power law renormalization group (RG) running yielding a lower (few tens of a TeV) gauge coupling (near-)unification scale \[20-22\]. Besides, the running of neutrino mixing angles generated from effective Majorana mass operator in a 5d set-up has been studied both in non-supersymmetric \[24\] and supersymmetric \[25\] contexts.

But, what is the advantage of supersymmetrizing it? We argue that supersymmetry and extra dimension need not always be seen as two new physics considered simultaneously. In fact, they may nicely complement each other in *some situations* through mutual requirements. This can be seen as follows:

1. From a top-down approach, string theory provides a rationale behind linking supersymmetry and extra dimension. The string models are intrinsically extra dimensional, and more often than not contain supersymmetry as an integral part. That said, we must also admit that establishing a rigorous connection between a realistic low energy supersymmetric model with string theory is still a long shot, though a lot of efforts have already been put in that direction \[26\].

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1. Generation non-universality in fermion localization imposes \(R^{-1} > 5000\) TeV due to large flavor-changing neutral currents and CP violation \[19\].
2. The power law loop corrections are admittedly ultraviolet (UV) cutoff dominated. It has been argued that if the higher dimensional theory contains a larger gauge symmetry which is perturbatively broken then the difference of gauge couplings of the unbroken subgroups is a calculable quantity independent of UV completion \[23\].
2. The higher dimensional field theories are non-renormalizable, and computation of quantum corrections in such scenarios require the existence of a UV completion. Superstring theories, which contain supersymmetry, provide the main hope in this context. In fact, a joint rôle of ‘partial supersymmetry’ (broken at very high scale) and warped extra dimension/compositeness has been envisaged from AdS/CFT point of view to account for both the ‘little’ and the ‘big’ hierarchies [27].

3. Even after embedding the SM in an extra dimensional set-up, the scalar potential remains unstable under quantum correction. Supersymmetrization stabilizes it and ameliorates the hierarchy problem. It is interesting to note that by admitting chiral fermions and their scalar partners in the same multiplet tacitly provides a rationale behind treating the Higgs boson as an elementary object. An elementary Higgs can be perfectly accommodated in a flat extra dimensional set-up. As a corollary, the upper limit on the lightest supersymmetric neutral Higgs is relaxed beyond the 4d upper limit of 135 GeV due to the presence of the KK towers of top/stop chiral multiplets, and the hitherto disfavored low tan \( \beta \) region can be resurrected [28].

4. Each 4d supersymmetric scenario has its own supersymmetry breaking mechanism. The origin of this mechanism may be linked to the existence of extra dimension. In fact, one of the earliest motivations of a TeV scale fat brane was to relate the scale of 4d supersymmetry breaking with \( R^{-1} \) [2, 29].

Keeping these in mind, we outline the formalism of a 5d supersymmetric model in an \( S^1/Z_2 \) orbifold which contains the 4d supersymmetric states as zero modes. In section 2 we state our assumptions leading to the construction of the 5d model and comment on supersymmetry breaking. Furthermore, we explicitly write down the particle content and their 5d Lagrangian and illustrate the KK decompositions of the different 5d fields. In section 3 we derive the beta functions of the gauge and Yukawa couplings as well as those of the different soft supersymmetry breaking parameters diagram by diagram, pointing out how they are all modified from their 4d values due to the presence of KK states. In section 4 we discuss the numerical effects of RG running and highlight the reason behind the differences between the 4d and 5d scenarios. We also point out under what conditions we can ensure electroweak symmetry breaking. In section 5 we exhibit the numerical impact of RG running through plots showing constraints in the \( m_0-M_{1/2} \) plane. We standardize our numerical codes by reproducing the known 4d plots before encoding the necessary alterations for producing the new plots pertaining to 5d supersymmetry. This also enables us to compare and contrast the 4d and 5d allowed/disallowed zones. Finally, in section 6 we showcase the essential features we have learnt from this analysis.

2 5d supersymmetry

2.1 A snapshot of our model

We highlight the salient features of supersymmetry in higher dimension and outline below the various assumptions that lead to a calculable phenomenological framework.

1. We consider a 5d flat space time metric. The 5th dimension is compactified on a \( S_1/Z_2 \) orbifold. Orbifolding is necessary to reproduce chiral zero mode fermions as a 5d theory is vector-like.

2. We embed the minimal supersymmetric standard model (MSSM) in this higher dimensional set-up (several consequences of such embedding, mainly the effects on gauge and Yukawa couplings’ evolution,
have been studied in [20]). From a 4d point of view this leads to a tower of KK states. The massless sector corresponds to the 4d MSSM states.

3. Since in 5d bulk the fermion representation is vectorial, the two-component spinor $Q$ that generates 4d supersymmetry will in 5d be accompanied by its chiral conjugate mirror $Q^c$. Thus a $N = 1$ supersymmetry in 5d corresponds to two different $N = 1$ supersymmetry, or equivalently, a $N = 2$ supersymmetry from a 4d perspective. In fact, all members of a given KK mode must fall into a valid representation of $N = 2$ supersymmetry. In fact, each 4d supermultiplet is augmented by new chiral conjugate states and together they form a hypermultiplet.

4. Here we are talking about a massive representation of supersymmetry, where the supersymmetry preserving Dirac mass plays the role of central charge for $N = 2$ supersymmetry. This charge is not renormalized, as a consequence of which the bulk hypermultiplets do not receive any wave-function renormalization [20,30]. We observe that this $N = 2$ non-renormalization has serious numerical consequences in RG evolution of parameters. The most notable effect is the blowing up of the Yukawa couplings into the non-perturbative regime around 18 TeV, which we will take to be the cutoff of our theory. This is below the scale of perturbative gauge coupling unification, which is around 30 TeV. Recall that in 5d we encounter power law running which results in early (compared to 4d) unification.

5. We allow the gauge and the Higgs multiplets access the 5d bulk. Thus far what we said is nothing but a supersymmetrization of UED. Only the matter multiplets make the difference. In the UED framework, all SM particles access the bulk, and thus even though there are two fixed points, there is no brane. One could as well have kept some or all of the fermion generations in a brane at a fixed point; the difference would be that the scenario would cease to be universal. In the present supersymmetric context too we have the freedom of keeping some or all of the matter multiplets at an orbifold fixed point. We note that unless we confine at least two generations of matter multiplets on a brane, the requirement of perturbative gauge coupling unification leads to a constraint $R^{-1} > 10^{10}$ GeV [22], spoiling its relevance for LHC. On the other hand, unless we keep the third family of matter multiplet in the bulk we cannot ensure electroweak breaking. In view of the above, we let the third generation matter multiplet access the bulk, but fix the first two generations at the $y = 0$ brane.

6. $N = 2$ supersymmetry forbids Yukawa interaction in the 5d bulk as this interaction involves odd (three) number of chiral multiplets. Therefore, we localize Yukawa interaction at the orbifold fixed point where the supersymmetry corresponds to $N = 1$.

7. Now we come to the important question as how we break the residual $N = 1$ supersymmetry. Different ideas have been advanced for its realization. One way is to break it by the Scherk-Schwarz mechanism in which fermions and bosons satisfy different periodic conditions over the compactified dimension.

Explicit realizations towards this using a TeV-scale orbifold can be found in [32]. Another interesting approach was to break the residual supersymmetry by a second compactification on an orbifold with two reflection symmetries, viz. $S^1/(Z_2 \times Z_2')$ [33]. This can be viewed as a discrete version of the Scherk-Schwarz mechanism. Both these scenarios yield soft masses which are UV insensitive due to the non-local nature of supersymmetry breaking. From a completely different viewpoint, supersymmetry

\[ \text{In other words, for } N = 2 \text{ supersymmetry it turns out that } m_R = m_B, \text{ which is analogous to } q_R = q_B \text{ for } N = 4 \text{ supersymmetry. Here } m \text{ is the Dirac mass (central charge) and } g \text{ is gauge coupling, while } R \text{ and } B \text{ are labels for renormalized and bare quantities. Since the Dirac mass of } N = 2 \text{ hypermultiplets appears on the right-hand side of the anti-commutation relation of the conserved supersymmetry charges, this mass cannot be renormalized. This is intertwined with the observation that only those terms are renormalized which can be written as integrals over all superspace volume. The kinetic term of } N = 2 \text{ hypermultiplets cannot be written as any such integral (see discussions and related earlier references in [20]).} \]
breaking may be infused from the brane-bulk interface \[34\], or transmitted from a distant brane \[35\], or arisen from a gaugino mediation set-up \[36\] (possibly with a much lower cutoff than \(10^{16} \text{ GeV}\)), or triggered by some completely unknown brane dynamics, for example, by a spurion \(F\)-term vacuum expectation value (vev).

In the context of the present analysis, we keep the exact mechanism of the \(N = 1\) brane supersymmetry breaking \textit{unspecified}. We assume that the supersymmetry breaking scale is of the order of the inverse of the compactification radius, for example \(c/R\), where \(c\) is an \(O(1)\) dimensionless parameter.

8. \textit{Our main goal is the following}: Just like in the conventional but constrained version of 4d supersymmetry one starts with a common scalar and a common gaugino mass at high scale (e.g. the GUT scale) and then run them down using the MSSM beta functions to find the weak scale spectrum, we do exactly the same here by assuming a common scalar mass \((m_0)\) and a common gaugino mass \((M_{1/2})\) at low cutoff scale \((18 \text{ TeV})\) and follow the running using the KK beta functions through successive KK thresholds to obtain the weak scale parameters. By adopting a phenomenological approach, we scan \(m_0\) and \(M_{1/2}\) over a set of values \(c/R\), with \(c\) varying in the range 0.1 to 1 and \(R^{-1}\) fixed at 1 TeV.

2.2 \textbf{Multiplet Structures}

As mentioned in the Introduction, from a 4d perspective the KK towers of matter and gauge fields rearrange in the form of \(N = 2\) hypermultiplets. A judicious choice of \(Z_2\) parity of the 5d fields allows us to break the \(N = 2\) supersymmetry to \(N = 1\) supersymmetry. We briefly review below the multiplet structures of the fields following the prescription suggested in \[37\].

2.2.1 \textbf{Vector hypermultiplet}

The 5d super Yang-Mills theory contains a 5-vector gauge field, a 4-component Dirac gaugino and a real scalar. When dimensionally reduced to 4d, the gauge field splits into a 4-vector and a scalar, the gaugino splits into 2 Majorana gauginos, and we still have the real scalar previously mentioned. All these fit into a vector multiplet and a chiral multiplet in \(N = 1\) language. If we represent the \(N = 2\) vector supermultiplet by \(V\), the 4-vector gauge field by \(A_\mu\), the gauginos by \(\lambda\) and \(\psi\), and define a complex scalar field \(\phi \equiv \frac{1}{\sqrt{2}} (\Sigma + i A_5)\), where \(\Sigma\) is the 5d real scalar and \(A_5\) is the 5th component of the 5-vector field, then one can schematically represent the 5d vector supermultiplet as

\[
V \equiv \begin{pmatrix} A_\mu & \phi \\ \lambda & \psi \end{pmatrix}.
\]

From a 4d perspective (where the compactified 5th coordinate \(y\) is just a label), and in the \(N = 1\) language, one can visualize the vector hypermultiplet by a vector multiplet \(\mathcal{V}\) (first column) and a chiral multiplet in the adjoint representation by \(\Phi\) (second column):

\[
\mathcal{V}(x, y) = -\theta \sigma^\mu \theta A_\mu(x, y) + i \theta^2 \theta \lambda(x, y) - i \theta^2 \theta \bar{\lambda}(x, y) + \frac{1}{2} \theta^2 \theta^2 D_{\mathcal{V}}(x, y),
\]

\[
\Phi(x, y) = \phi(x, y) + \sqrt{2} \theta \psi(x, y) + \theta^2 F_\Phi(x, y).
\]

The \(Z_2\) parity of \(V\) is so chosen that the \(\mathcal{V}\) contains a zero mode, but \(\Phi\) does not have any zero mode.
The gauge invariant action may be written as (\( \int d^5 x \equiv \int d^4 x \int dy \))

\[
S_{\text{gauge}}^5 = \int d^5 x \left[ \frac{1}{4g^2} \int d^2 \theta \ W^\alpha W_\alpha + \text{h.c.} + \int d^4 \theta \ \frac{1}{g^2} \left( \partial_5 \mathcal{V} - \frac{1}{\sqrt{2}} (\Phi + \Phi^\dagger) \right)^2 \right],
\]

where the \( W^\alpha (x, y) \) is the field strength superfield corresponding to \( \mathcal{V}(x, y) \).

### 2.2.2 Higgs hypermultiplets

From the \( N = 1 \) perspective, the \( N = 2 \) hypermultiplet splits into two chiral multiplets. Thus have a \( H_u \) hypermultiplet and a \( H_d \) hypermultiplet. We can represent them as (the tilde symbol represents Higgsino)

\[
H_{(u,d)} \equiv \begin{pmatrix} H_L^{(u,d)} \\ H_R^{(u,d)} \end{pmatrix}.
\]

If we denote the two chiral multiplets inside the hypermultiplet \( H(x, y) \) as \( h(x, y) \) in left column and \( h^c(x, y) \) in right column, then one can expand the chiral superfield as

\[
h/\bar{h}^c = H_{L/R} + \sqrt{2} \theta \bar{H}_{L,R} + \theta^2 F_{h/\bar{h}^c}.
\]

We assign even \( Z_2 \) parity to \( h \) so that it has a zero mode, and odd \( Z_2 \) parity to \( h^c \) which does not have zero mode. The free action of the hypermultiplets can be written as

\[
S_{\text{Higgs}}^5 = \int d^5 x \left[ \int d^4 \theta \left( \bar{h} h^c + \bar{h} h \right) + \left( \int d^2 \theta \ h^c (\partial_5 + m) h + \text{h.c.} \right) \right].
\]

### 2.2.3 Matter hypermultiplets

Matters have similar hypermultiplet structures similar to Higgs:

\[
\Psi \equiv \begin{pmatrix} \phi_L & \phi_R \\ \psi_L & \psi_R \end{pmatrix},
\]

where, \( \mathcal{F}_L \equiv (\phi_L, \psi_L) \ (Z_2 \text{ even}) \) and \( \mathcal{F}_R \equiv (\phi_R, \psi_R) \ (Z_2 \text{ odd}) \) represent the two \( N = 1 \) chiral multiplets. The free matter hypermultiplet action will be similar to Eq. (6). There are five matter representations, two SU(2) doublets \( Q \) and \( L \) and three singlets \( u, d, e \), where the symbols have their standard meaning.

### 2.2.4 Gauge interactions

When the hypermultiplets are charged under gauge symmetry, their free action can be promoted to take care of the interaction in the following way:

\[
S_{\text{int}}^5 = \int d^5 x \left[ \int d^4 \theta \left( \mathcal{F}_L e^\gamma \mathcal{F}_L^\dagger + \mathcal{F}_R e^{-\gamma} \mathcal{F}_R^\dagger \right) + \left( \int d^2 \theta \mathcal{F}_L \left( m + \partial_5 - \frac{1}{\sqrt{2}} \Phi \right) \mathcal{F}_R + \text{h.c.} \right) \right],
\]

where, \( \gamma = \gamma^a T^a \) and \( \Phi = \Phi^a T^a \) are Lie-algebra-valued gauge and matter superfields.
2.2.5 Yukawa Interactions

We denote the Yukawa part of the superpotential by \( W_Y \), which contains the usual chiral superfield combinations \( QH_u, QH_d \) and \( LH_d e \). Since an Yukawa interaction involves three (i.e. odd number) chiral superfields, it is not possible to write a bulk Yukawa interaction maintaining \( N = 2 \) supersymmetry. For this reason, we confine Yukawa interaction at the branes even though the associated superfields are in the bulk. A generic action (e.g. involving the third generation bulk matter) can be written as

\[
S^5_{\text{Yuk}} = \int d^5x \left( \int d^2\theta W_Y \right) \left[ \delta(y) + \delta(y - \pi R) \right].
\]  

(9)

As the \( Z_2 \) odd fields vanish at the fixed points, they do not contribute to Yukawa interactions. We note that the first two matter generations can be kept either at either of the two fixed points. Also, the above Yukawa action could have been confined at either of the two branes instead of symmetrically on both.

2.3 KK decomposition of fields

In order to obtain the action in terms of 4d component fields, we need to write down the KK decomposition of the 5d fields in terms of zero modes and higher KK modes [3]. Each 5d field is either \( Z_2 \) even or \( Z_2 \) odd. Only the even fields have zero modes. The decomposition of the Higgs fields will be exactly like the matter fields.

\[
\begin{align*}
V(x, y) &= \frac{\sqrt{2}}{\sqrt{2\pi R}} V^{(0)}(x) + \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} V^{(n)}(x) \cos \frac{ny}{R}, \\
\Phi(x, y) &= \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \Phi^{(n)}(x) \sin \frac{ny}{R}, \\
F_L(x, y) &= \frac{\sqrt{2}}{\sqrt{2\pi R}} F^{(0)}_L(x) + \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} F^{(n)}_L(x) \cos \frac{ny}{R}, \\
F_R(x, y) &= \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} F^{(n)}_R(x) \sin \frac{ny}{R}.
\end{align*}
\]

(10)

3 RG evolution and derivation of the beta functions

The technical meaning of RG evolution in a higher dimensional context has been amply clarified in [20], and we merely reiterate it in the present context. The multiplicity of KK states renders any such higher dimensional scenario non-renormalizable. So ‘running’ of couplings or parameters with the energy scale does not make much of a sense. Rather, one can estimate the finite quantum corrections that these couplings/parameters receive whose size depends on some explicit cutoff \( \Lambda \). The contribution comes from \( \Lambda R \) number of KK states which lie between the scale of the first KK state, which is \( 1/R \), and the cutoff \( \Lambda \). With this interpretation of RG running, we compute the one loop beta functions of the gauge and Yukawa couplings and various soft supersymmetry breaking masses. We make the following observations:

1. The contribution to the beta function from a given KK mode does not depend on its KK label.
2. When we consider different KK thresholds we neglect their zero mode masses, i.e. we assume that the $n$th level KK state is kicked into life when we cross the energy scale $n/R$.

3. As we cross different KK thresholds, the beta functions also change. The beta function of the quantity $X$ at an energy scale $Q$, where $n \lesssim QR < (n + 1)$, can be written as ($t = \ln(Q/Q_0)$, where $Q_0$ is a reference scale, e.g. the electroweak scale)

$$\frac{\partial X}{\partial t} = \beta_X, \text{ where } \beta_X = \beta_{0X} + n\tilde{\beta}_X.$$ \hspace{1cm} (11)

Here $\beta_{0X}$ is the contribution induced by the zero mode (i.e. ordinary 4d MSSM) states (which may be found, for example, in the review [38]) and $\tilde{\beta}_X$ arises from a single KK mode. Eq. (11) is our master equation using which we perform a diagram by diagram calculation for the estimation of $\tilde{\beta}_X$ for various couplings and parameters.

3.1 Gauge couplings and gaugino masses

The running of the gauge couplings ($g_i$) and gaugino masses ($M_i$) are controlled by

$$\beta_{g_i} = \frac{g_i^3}{16\pi^2} \left[ b_i^0 + n\tilde{b}_i \right], \quad \beta_{M_i} = \frac{g_i^2 M_i}{16\pi^2} \left[ b_i^0 + n\tilde{b}_i \right].$$ \hspace{1cm} (12)

For the gauge groups $U(1)$ (which corresponds to $g_1 = \sqrt{5/3}g'$, which unifies), $SU(2)$ and $SU(3)$, $b_i^0 = (33/5, 1, -3)$ [38], and $\tilde{b}_i = (26/5, 2, -2)$ [22], respectively.

3.2 Yukawa and scalar trilinear couplings

We recall that $N = 1$ non-renormalization relates the beta functions of the Yukawa couplings ($y_{ijl}$) to the anomalous dimension matrices ($\gamma_{ij}^l$) of the superfields. This theorem implies that logarithmically divergent contributions can always be written in terms of wave-function renormalizations. Generically, $y_{ijl}$ may be written as

$$\beta_{y_{ijl}} = \gamma_{ij}^l y_{ijl} + \gamma_{ij}^k y_{ijnk} + \gamma_{ij}^n y_{ijn}.$$ \hspace{1cm} (13)

The Feynman diagrams showing the KK contributions to the wave-function renormalizations of the scalars and fermions are displayed in Fig. 1. The contribution from the gauge sector cancels exactly as a consequence of the $N = 2$ non-renormalization theorem mentioned in section 2. Diagrammatically, the origin of this cancellation may be traced to a relative sign between the $A_{\mu\nu}$- and $\phi$-propagators - see Eq. (1). Only the brane localized Yukawa interactions contribute to the Yukawa evolution. We also keep track of the fact that the $Z_2$ odd fields have vanishing wave-functions at the two branes, leaving the even fields alone to contribute to the diagrams in Fig. 1. Here we have made a tacit assumption that although the Yukawa interaction is brane localized, only one KK level ($n$) states float inside the loop at a time. This is a technical assumption to ensure calculability by avoiding KK divergence which would have arisen while summing more than one KK index in a loop calculation.

To appreciate the numerical impact of the bulk $N = 2$ non-renormalization, we first write down the conventional 4d MSSM beta functions (i.e. those coming from zero mode states in the 5d context) which contribute to the evolution of the third generation Yukawa couplings [38]:

$$\beta_{0t}^0 = \frac{y_t}{16\pi^2} \left[ 6g_1^2 y_t + g_0^2 y_0 - \frac{16}{3} g_2^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right].$$
contributions (proportional to Eq. (16)) would still remain while computing the KK contribution. All in all, contributions to the beta functions from the zero mode (i.e. 4d MSSM) states [38]:

\[ \beta_{t,b,\tau}^0 = \frac{y_t}{\alpha_t^2} \left[ 18 y_t^* y_t + y_t^* y_b - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2 \right], \]

(14)

The corresponding KK contributions are given by

\[ \tilde{\beta}_f = \beta_f^0 (g_i \to 0) \quad (f \equiv t, b, \tau), \]

(15)

where the vanishing gauge contributions is a direct consequence of the bulk \( N = 2 \) non-renormalization.

The effects of the above non-renormalization can also be felt in the evolution of the trilinear scalar couplings. The relevant Feynman diagrams are displayed in Fig. 2. Again, for illustration, we first write down the contributions to the beta functions from the zero mode (i.e. 4d MSSM) states [38]:

\[ \beta_{u_t}^0 = \frac{1}{\alpha_t^2} \left[ a_t \left( 18 y_t^* y_t + y_t^* y_b - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 \right) + 2 a_t y_b^* y_t \
+ y_t \left( \frac{32}{3} g_3^2 M_3 + 6 g_2^2 M_2 + \frac{26}{15} g_1^2 M_1 \right) \right], \]

(16)

\[ \beta_{a_t}^0 = \frac{1}{\alpha_t^2} \left[ a_t \left( 18 y_t^* y_b + y_t^* y_t + y_t^* y_t - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2 \right) + 2 a_t y_b^* y_b + 2 a_t y_t^* y_t \
+ y_b \left( \frac{32}{3} g_3^2 M_3 + 6 g_2^2 M_2 + \frac{14}{15} g_1^2 M_1 \right) \right], \]

As expected, the beta functions of the soft supersymmetry breaking parameters are proportional not only to those parameters but to others as well, since any non-renormalization theorem ceases to work when supersymmetry is broken. For the computation of \( \tilde{\beta}_{a_f} \), we need to keep in mind the essence of Eq. (15), i.e. the absence of gauge contributions in \( \tilde{\beta}_f \), while solving the coupled differential equations. However, that part of the gauge contributions (proportional to \( g_i^2 \)) to the trilinear scalar couplings which multiply the gaugino masses (\( M_i \)) in Eq. (16) would still remain while computing the KK contribution. All in all,

\[ \tilde{\beta}_{a_f} = \beta_{a_f}^0 (a_f g_i \to 0). \]

(17)
3.3 Scalar masses

We make three observations regarding the KK contributions to the evolution of scalar masses (see Fig. 3):

1. The two diagrams in the lower row of Fig. 3 depend on the Yukawa couplings. Hence, they are important only for the third generation matter fields.

2. Recall that in the evolution of the Yukawa couplings the KK contributions from the gauge field $A_{\mu}$ and the complex scalar $\phi$ exactly cancelled thanks to the bulk $N = 2$ non-renormalization. However, their fermionic superpartners contribute to the scalar mass evolution and those contributions add up instead of canceling out. This happens because these contributions yield gaugino masses which are $N = 1$ supersymmetry breaking parameters and hence the non-renormalization theorem ceases to be applicable.

3. Each KK state in the two diagrams in the top row of Fig. 3 contributes twice that of the SM because of the doubling of the fermions (this factor of 2 is highlighted in bold-face in Eqs. (18) and (19) below, and also marked on the two diagrams in top row in Fig. 3). However, each KK state at the lower row diagrams contributes the same as in the SM because the odd fermion modes vanish at the brane where Yukawa interaction is confined.

The zero mode contributions to the beta functions can be found in [38]. Below we display the KK contributions to the beta functions only for the third generation scalars in external lines ($S \equiv \text{Tr}[Y_i m_{\phi_i}^2]$, where the index $i$ runs over the third generation matter states which are in the bulk):

$$
\tilde{\beta}_{u_3} = \frac{1}{16\pi^2} \left[ 2 \left( 2\tilde{y}_t^2 \left( m_{H_{u}}^2 + m_{Q_3}^2 + m_{u_3}^2 \right) + 2a_t^2 \right) - 2 \left( \frac{32}{3} g_3^2 |M_3|^2 + \frac{32}{15} g_1^2 |M_1|^2 + \frac{4}{5} g_1^2 S \right) \right],
$$
\[ \beta_{\tilde{d}_3} = \frac{1}{16\pi^2} \left[ 2 \left( 2y_6 \left( m_{Hd}^2 + m_{Q_3}^2 + m_{\tilde{d}_3}^2 \right) + 2a_6^2 \right) - 2 \left( \frac{32}{3} g_3^2 |M_3|^2 + \frac{8}{15} g_1^2 |M_1|^2 - \frac{2}{5} g_1^2 S \right) \right], \]

\[ \beta_{\tilde{Q}_3} = \frac{1}{16\pi^2} \left[ \left( 2y_7 \left( m_{Hd}^2 + m_{Q_3}^2 + m_{\tilde{e}_3}^2 \right) + 2a_7^2 \right) + \left( 2y_7 \left( m_{Hd}^2 + m_{Q_3}^2 + m_{\tilde{d}_3}^2 \right) + 2a_7^2 \right) \right. \]

\[ \left. - 2 \left( \frac{32}{3} g_3^2 |M_3|^2 + 6g_2^2 |M_2|^2 + \frac{2}{15} g_1^2 |M_1|^2 - \frac{1}{5} g_1^2 S \right) \right], \]

\[ \beta_{\tilde{L}_3} = \frac{1}{16\pi^2} \left[ \left( 2y_7 \left( m_{Hd}^2 + m_{L_3}^2 + m_{\tilde{e}_3}^2 \right) + 2a_7^2 \right) \right. \]

\[ \left. - 2 \left( \frac{6}{5} g_1^2 |M_1|^2 + \frac{3}{5} g_1^2 S \right) \right], \]

\[ \beta_{\tilde{e}_3} = \frac{1}{16\pi^2} \left[ 2 \left( 2y_7 \left( m_{Hd}^2 + m_{L_3}^2 + m_{\tilde{e}_3}^2 \right) + 2a_7^2 \right) - 2 \left( \frac{24}{5} g_1^2 |M_1|^2 - \frac{6}{5} g_1^2 S \right) \right]. \]

The beta functions for the Higgs scalars are given by

\[ \beta_{H_u} = \frac{1}{16\pi^2} \left[ 3 \left( 2y_6 \left( m_{H_u}^2 + m_{Q_3}^2 + m_{\tilde{u}_3}^2 \right) + 2a_6^2 \right) - 2 \left( 6g_2^2 |M_2|^2 + \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 S \right) \right], \]

\[ \beta_{H_d} = \frac{1}{16\pi^2} \left[ 3 \left( 2y_6 \left( m_{H_d}^2 + m_{Q_3}^2 + m_{\tilde{d}_3}^2 \right) + 2a_6^2 \right) + \left( 2y_7 \left( m_{Hd}^2 + m_{L_3}^2 + m_{\tilde{e}_3}^2 \right) + 2a_7^2 \right) \right. \]

\[ \left. - 2 \left( 6g_2^2 |M_2|^2 + \frac{6}{5} g_1^2 |M_1|^2 + \frac{3}{5} g_1^2 S \right) \right]. \]
Note that for the first and second generation soft scalar masses only the second diagram in the top row of Fig. 3 would contribute.

4 Special numerical features of RG running in 5d scenario

In this section, we highlight the special features of RG evolution in the 5d scenario. We also compare and contrast them with the 4d features. For all our numerical estimates we have fixed $1/R = 1 \text{ TeV}$.

4.1 The Gauge and Yukawa couplings

The power law running of the gauge and Yukawa couplings has been discussed in [20, 22] for the non-supersymmetric scenario and in [20] for the supersymmetric case. As far as the Higgs multiplets are concerned, there is a crucial difference between our model and that considered in [20]. In our scenario there are separate up- and down-type Higgs hypermultiplets - see Eq. (4). Inside each hypermultiplet only the left column with label $(L)$ is $Z_2$ even and its scalar zero mode receives a vev, whereas the right column with label $(R)$ is projected out. In other words, the hypermultiplet $H_u$ contains the vev $v_u$ and, similarly, $H_d$ contains $v_d$. On the other hand, [20] contains a single hypermultiplet, each column of which has a zero mode, one to be identified with the up-type chiral multiplet which contains the vev $v_u$, and the other to be identified with the down-type containing $v_d$. While our approach constitutes a straightforward generalization of $H_u$ and $H_d$ from chiral multiplets to hypermultiplets, the choice made in [20] requires non-trivial boundary conditions. These two different assumptions lead to significant numerical differences. In our approach, the gauge couplings converge to one another but actually do not meet at a single point, while in [20] the gauge couplings do meet at a point. The difference in the number of KK scalar excitations makes the difference between the two approaches.

Indeed, both gauge and Yukawa couplings exhibit power law running due to summation over the KK states as one crosses the energy thresholds. As we have mentioned in section 2, keeping the first two matter generations confined at the brane ensures that the couplings remain perturbative even with $R^{-1}$ as low as 1 TeV. Starting from their LEP-measured values at the weak scale, as we extrapolate the gauge couplings using the KK beta functions we observe that the three couplings approach very close to one another near 32 TeV, but they do not actually meet at any point, as mentioned in the previous paragraph.

A crucial point of immense numerical significance is that on account of the special $N = 2$ bulk non-renormalization, the third generation matter hypermultiplet kept in the bulk does not receive any wave-function renormalization from the gauge hypermultiplet, which we have illustrated below Eq. (13). As an important consequence of this the Yukawa coupling blows up to large (non-perturbative) values around $\Lambda \sim 18$ TeV, which we therefore take to be the effective cutoff of our theory.

4.2 The gaugino and scalar masses

We assume that at the highest scale $\Lambda = 18$ TeV of our theory, i.e. just before the Yukawa couplings blow up, all scalar masses unify to $m_0$ and all gaugino masses to $M_{1/2}$. Our high scale parameters are then $m_0, M_{1/2}, \text{sgn}(\mu)$ and $\tan \beta \equiv v_u/v_d$. 

12
The gaugino mass running is governed by the evolution of gauge couplings. Since gauge couplings nearly meet around 32 TeV, the gaugino masses tend to converge also at that scale. But in the present context, as mentioned before, we forced the gaugino masses to unify at 18 TeV. Recall that in 5d the running is short but fast (power law), but in 4d it is long and slow (logarithmic). This leads to a general expectation that starting from a given high scale value, the low scale predictions would be similar in 4d and 5d. But since we forcibly unified the gaugino masses in our set-up, earlier than otherwise expected, we obtain a somewhat different set of low scale values. The gaugino mass scaling in 5d is shown in Fig. 4, while in the inset is displayed the 4d running. A rough comparison of the weak scale ratios of the three gaugino masses is the following:

\[
M_1, M_2, M_3 \sim (0.4, 0.8, 3.0) \times M_{1/2} \text{ (in 4d)},
\]

\[
M_1, M_2, M_3 \sim (0.7, 0.8, 2.0) \times M_{1/2} \text{ (in 5d)}.
\] (20)

If \(R\)-parity remains conserved, the lightest neutralino remains the lightest supersymmetric particle (LSP), only that its mass is heavier than what is expected in the standard 4d scenario - see Eq. (20).

Fig. 5 shows the running of the soft scalar masses. The large top quark Yukawa coupling continues to play a crucial rôle as in 4d. A rough comparison of the weak scale predictions in 4d and 5d is:

\[
m_{Q_3}^2 \sim m_0^2 + 5.5 M_{1/2}^2 \text{ (in 4d)},
\]

\[
m_{Q_3}^2 \sim m_0^2 + 3.5 M_{1/2}^2 \text{ (in 5d)}.
\] (21)

Even for the brane localized scalars, the 5d model predicts slightly higher weak scale masses compared to 4d\(^4\). The relatively compressed spectrum in 5d, as expressed through Eqs. (20) and (21), is indicative of a lesser fine-tuning compared to the conventional MSSM.

During power law running we ensure that radiative breaking of electroweak symmetry does happen at the desired scale\(^5\). Just like in 4d, only \(m_{H_u}^2\) turns negative while all other scalars remain positive. Again, the large top quark Yukawa coupling drives this phase transition. A point to note is that unless we keep the third generation matter in the bulk, electroweak symmetry would never break radiatively in our class of models. Furthermore, we had to take in 5d a factor 1 to 3 larger (than 4d) value of \(\mu\) at the cutoff scale.

5 The \(m_0 - M_{1/2}\) parameter space

5.1 Numerical procedure

For our numerical estimates we go through the following steps:

1. We scan \(m_0\) and \(M_{1/2}\) through the range \([0.1 - 1.0]/R\). We fix \(\tan \beta = 10\) and take both positive and negative values of \(\mu\). Our cutoff is 18 TeV, which is the scale where the Yukawa couplings blow up to non-perturbative values. We use one loop RG equations as displayed in section 3. To obtain the weak scale parameters we follow the logarithmic evolution like in the 4d desert, but now using different

---

\(^4\)When the \(N = 1\) supersymmetry breaking is induced by brane-localized spurion vev, the structure of the third generation soft masses may, in principle, be different from what we assumed due to KK mixing. For calculational simplicity we have ignored KK mixing which, we believe, would not affect our main qualitative features.

\(^5\)Radiative electroweak symmetry breaking has been discussed in the context of some specific realization of supersymmetry breaking in an orbifold \([33, 39, 40]\).
beta functions for different energy intervals separated by successive KK thresholds - see Eq. (11). So, operationally, power law running is treated by taking many logs.

2. For each input combination, we perform a consistency check to ensure correct electroweak symmetry breaking, and accept only those inputs which admit this phenomenon.

3. We then feed the weak scale spectrum into the code micrOMEGAS [41], and using this software package calculate the dark matter density ($\Omega_{DM}$), $\text{Br} (b \to s \gamma)$, $\Delta a_\mu = (g - 2)\mu/2$, and $\Delta \rho$. Since we consider $1/R = 1$ TeV, even the lightest KK states are somewhat heavier than the lighter section of the zero mode spectra. As a result, without any significant numerical compromise we neglect the direct loop contributions of the virtual KK particle in other words, the KK effects feed into the calculation of low energy spectra via power law running, but after that we rely on the standard 4d computations encoded in micrOMEGAS. This approximation is good enough for our purpose.

4. We compare the predictions of the above observables with their experimental values/constraints, and translate the information into the inclusion/exclusion plots (Figs. 6 and 7) in the $m_0 - M_{1/2}$ plane. The 4d plots have been reproduced to serve as a guide to the eyes for capturing the 5d subtleties. We note that our 4d plots are in agreement with the ones in the existing literature, e.g. with [43].

5.2 Comparison between 4d and 5d models

We highlight only the major differences between the 4d and 5d models that appear on the $m_0-M_{1/2}$ plane.

1. We assume that $R$-parity is conserved. In the 4d scenario the lightest neutralino is the most likely candidate for an LSP. In the 5d model the situation is somewhat tricky. Indeed, the 4d LSP is still an LSP here which is the zero mode neutralino. Besides, if the KK parity remains conserved, then the $n = 1$ mode of photon tower, namely $\gamma_1$, and its supersymmetric partner $\tilde{\gamma}_1$ are also stable dark matter candidates. However, the KK parity is unlikely to respected by the brane-bulk interaction. In our numerical analysis, we have treated the zero mode LSP as the dark matter candidate.

2. We have taken a $3\sigma$ range of the five year average of WMAP dark matter density ($0.087 < \Omega_{DM}h^2 < 0.138$) [44]. We raise a caution here that if KK parity remains conserved and we have two more dark matter candidates, as mentioned above, then the edge of the allowed band arising from the lower limit of $\Omega_{DM}$ would be further stretched. Note further that in the 5d case there is a slight broadening of the WMAP allowed strip compared to 4d. This happens because of a combined effect of Eqs. (20) and (21) leading to a reduced sensitivity to $M_{1/2}$ variation.

3. The region where the lightest neutralino satisfies the dark matter constraints extends to a higher value of $M_{1/2}$ in 5d compared to 4d. What matters most in this context is the mass difference between the chargino and the lightest neutralino, i.e. $(M_2 - M_1)$. It is clear from Eq. (20) that starting from a given $M_{1/2}$ this difference is smaller in 5d than in 4d. This explains the shift of the band to the right side.

4. We have taken $2.65 \times 10^{-4} \lesssim \text{Br} (b \to s \gamma) \lesssim 4.45 \times 10^{-4}$ [45], and $10.6 \times 10^{-10} \lesssim \Delta a_\mu = (g - 2)\mu/2 \lesssim 43.6 \times 10^{-10}$ [46]. There is nothing much to distinguish between 4d and 5d from these two observables.

Although $b \to s \gamma$, $Z \to bb$, $\Delta \rho$, $B_q-\bar{B}_q$ are all finite up to one loop in UED or its supersymmetric version thanks to the KK number conservation at the tree level Lagrangian, in scenarios containing brane-bound fermions one encounters a UV sensitivity even at one loop [42]. A sensible UV completion is needed to counter this. See also [23] and footnote 2 of this paper.
5. We have not included the direct loop effects of the virtual KK states for any of the weak scale observables. For $R^{-1} = 1$ TeV or more, for processes like muon anomalous magnetic moment or $b \to s\gamma$, such effects are numerically negligible, but only for the Higgs mass it makes a difference. In Figs. 6 and 7, the entire region to the left of the line marked with $m_h = 114$ GeV which looks otherwise disfavored will be resurrected if we include the KK loop correction to the Higgs mass [28].

6 Conclusions and Outlook

We reiterate once again that supersymmetry and extra dimension are interestingly intertwined both from theoretical and phenomenological perspectives. The presence of extra dimensions is an essential part of any high scale fundamental theory, and supersymmetry is quite often an integral component of such theories. Supersymmetry, on the other hand, provides a sensible UV completion of the extra dimensional models. The connection between supersymmetry and extra dimension is further strengthened by the consideration that the origin of supersymmetry breaking may reside in extra dimension. In a general class of scenarios in which TeV-size soft masses are generated at a relatively low cutoff scale (like 20-30 TeV) with intermediate resonances, one can find a common ground where our analysis will be applicable.

The logarithmic running in 4d from 100 GeV to $10^{16}$ GeV is replaced in 5d by fast power law running on a shorter interval from 100 GeV to about 30 TeV thanks to the KK states. This is a feature of extra dimension. What is technically/operationally special about 5d supersymmetry is that it provides a special $N = 2$ non-renormalization that forces us to consider an early cutoff ($\sim 18$ TeV).

The constraints in the $m_0$–$M_{1/2}$ plane have been placed for the first time in this paper. The ratio $M_1/M_{1/2}$ is higher in 5d compared to 4d. For this reason the allowed region in the 5d plot extends to larger values of $M_{1/2}$ compared to the 4d plot.

It should be noted that $R^{-1}$ could in principle be much smaller, e.g. a few hundred GeV, than our reference value of 1 TeV. In fact, the first KK excitations of SM particles could very well be lighter than zero mode superpartners. It is neither our intention, nor it is possible within the scope of the present paper, to distinguish the two different options.

Two issues require further studies beyond the scope of the present paper: (i) Besides the lightest neutralino (the usual 4d LSP), if KK parity remains conserved, there may be two other candidates of dark matter in this model. One is $\gamma_1$, the $n = 1$ level photon, and the other is its superpartner $\tilde{\gamma}_1$. Although brane-bulk interface is likely to frustrate KK parity conservation, yet for the sake of completeness one should study the combined effects of all three dark matter candidates. (ii) It will also be interesting to revisit the lower limit on $R^{-1}$ in a supersymmetric scenario, which we suspect would be relaxed.

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Figure 4: RG running of the gaugino masses.

Figure 5: RG running of the scalar masses and radiative electroweak symmetry breaking.
Figure 6: The $m_0 - M_{1/2}$ parameter space for $\mu > 0$. The 4d plot is also shown to guide the eye. We keep $\tan\beta = 10$ for all the plots. The region ruled out by $\text{Br}(b \rightarrow s\gamma)$ is shaded in light green (lightest shade), the $\tilde{\tau}$ LSP region is shaded in red (darker shade) and the region favored by $(g-2)_\mu$ is the region between the two blue (darkest shade) lines. The WMAP allowed region where $0.087 < \Omega_{DM} h^2 < 0.138$ is shaded in black. We also show the contours for $m_h = 114 \text{GeV}$ and $m_{\tilde{\chi}^\pm} = 103.3 \text{GeV}$, the region to the left of these lines are ruled out by LEP exclusion limits. For the 5d models, the Higgs contour shown does not include the virtual KK contribution.
Figure 7: Same as in Fig. 6 but for $\mu < 0$. The entire region is now disfavored by $(g - 2)_{\mu}$. 

22