Shaping Effects on Zonal Flow in Tokamaks

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Abstract. Zonal flows are \( m = n = 0 \) electrostatic potential fluctuations with a finite radial wave-number. Plasma shaping effects on zonal flow are studied with gyrokinetic theory. We focus on the effect of the elongation \( E \) particularly. A new coordinates which are related to elongation are rigorously developed in the analysis of gyrokinetic equation. The GAM real frequency and damping rates are inversely proportional to elongation \( E \) which seems to be consistent with that measured in ASDEX-Upgrade. The lower the GAM frequency the more important it is expected to become in moderating the turbulence via shear decorrelation. Reduction of GAM damping rates could help to generate the zonal flow.

1. Introduction
The current studies on zonal flows (ZF) and geodesic acoustic modes (GAM) in the context of tokamak plasmas are motivated by their association with drift wave turbulence and transport. Theoretical works and numerical simulations [1, 2] predicted the formation of static and oscillating poloidal plasma shear flows driven exclusively by nonlinear interactions in gradient driven plasma turbulence [3, 4]. These turbulence-driven flows may in-turn moderate the turbulence and hence affect the plasma transport, via either shear decorrelation or by acting as an energy sink [5, 6].

For circular plasma, the GAM has a natural frequency. However, many experimental measurements show substantial deviations in non-circular plasmas. For instance, initial measurements using beam emission spectroscopy on the DIII-D tokamak indicate the GAM frequency decreases with increasing elongation \( E \) [7]. A similar scaling inversely with the plasma boundary elongation \( E \) was observed on the ASDEX Upgrade tokamak using Doppler reflectometry [8]. It is important to discuss the elongation on the GAM frequency detailly since the GAM may impact on the effective shearing rates and thus turbulence radial correlation length reduction, providing its frequency is lower than the inverse turbulence decorrelation time [5, 9]. Watari is the first one to discuss the shaping effects on GAM systematically [10]. With some simplifications to the dispersion relation the GAM frequency can be shown to scale inversely with \( E \). Damping rates are also important since reduction of damping rates helps to generate zonal flow [11].
In this paper new coordinates which are related to elongation are rigorously developed in the analysis of gyrokinetic equation. It is found that the real frequency and damping rates are inversely proportional to elongation $E$ which seems to be consistent with that measured in ASDEX-Upgrade.

2. Solov’ev Coordinates
We proceed from the Grad-Shafranov equation

$$\nabla \cdot \left( \frac{\nabla \Psi}{R^2} \right) = \frac{\mu_0}{R} j_\phi (R, \Psi)$$

where $\Psi$ is the poloidal magnetic flux, $j_\phi$ is the plasma current density in the toroidal direction. When the plasma current has a quasi-uniform distribution equation (1) has following Solov’ev solution, we get equation (2),

$$\Psi = \Psi_0 \left\{ \frac{1}{E} \left[ \delta R_0^2 + (1- \delta) R^2 \right] Z^2 + \frac{E}{4} \left( R^2 - R_0^2 \right)^2 \right\}$$

where $\Psi_0 = \frac{j_\phi \mu_0 E}{2R_0 \left( 1 + E^2 \right)}$, $E$ is elongation, $\delta$ is parameters of triangularity. Solov’ev coordinates are developed as equation (3),

$$\Psi = \Psi_0 E \left\{ \frac{1}{E^2} \left[ R_0^2 - (1- \delta) R^2 + (1- \delta) R^2 \right] Z^2 + \frac{1}{4} \left( R^2 - R_0^2 \right)^2 \right\}$$

where

$$\Psi_{00} = \frac{\mu_0 j_\phi R_0 E^2}{2 \left( 1 + E^2 \right)}, Y = \frac{R^2}{R_0^2} - 1$$

We set up new coordinates $(x, \theta, \phi)$ which we call Solov’ev coordinates. $(x, \theta, \phi)$ are related to the cylindrical coordinates $(R, \phi, Z)$ by the relations $R = R_0 \sqrt{1 + 2\varepsilon \cos \theta}$, $Z = -\frac{Ex \sin \theta}{\sqrt{1 + (1- \delta)2\varepsilon \cos \theta}}$ where $\varepsilon = \frac{x}{R_0}$, $Y = 2\varepsilon \cos \theta$ and $\theta$ is in clockwise direction.

The Jacobian is

$$J = \frac{1}{\nabla x \cdot \nabla \phi \cdot \nabla \theta} = \frac{xER_0}{\sqrt{1 + 2\varepsilon(1- \delta) \cos \theta}}$$

and the volume $d\tau = J dx d\phi d\theta$. 
Similar coordinates \((\Psi, \chi, \phi)\) are given by M. N. Rosenbluth [12] in which 
\[\text{d}l_q = R_B d\Psi, \quad \text{d}l_x = J_r B_x d\chi, \quad \text{d}l_\phi = R d\phi \quad \text{and} \quad \text{d}\tau = J_k d\Psi d\chi d\phi = J dxd\theta d\phi.\]

Comparing the volume, we obtain the result written as equation (4),
\[J d\theta = J_r d\chi \frac{d\Psi}{dx} = 2x\Psi \omega J_r d\chi\] (4)

3. The Gyro-kinetic Equation in Solov’ev Coordinates

The gyro-kinetic equation is given in Ref. [13] which is simplest developed so far,
\[\frac{\partial}{\partial t} f_i + \nu_q \cdot \nabla f_i - i(\omega - n\omega_E - i\Omega_a) f_i = -\frac{ie}{T}(\omega - \omega^*) J_i(k, \rho) \hat{F}_m \hat{\Phi}_k\] (5)

Zonal flow is axisymmetrical and low frequency mode, that is \(n = 0\). For \(\nu_k = 0\) and \(k_r >> k_\theta\), equation (5) in Rosenbluth’s coordinates [12] is
\[\frac{\partial}{\partial t} f_i + \nu_q \cdot \nabla f_i - i\omega f_i = \frac{\nu_q}{B} \frac{\partial}{\partial \chi} f_i + \nu_q \cdot \nabla f_i - i\omega f_i\]
\[= -\frac{ie}{T}(\omega - \omega^*) J_i(k, \rho) \hat{F}_m \hat{\Phi}_k\] (6)

There is no an express for \(J_r\). Using Solov’ev coordinates and comparing the volume, we can obtain equation (7),
\[(E\omega + \omega_d \sqrt{1 + 2\varepsilon(1 - \delta)} \cos \theta \sin \theta + i\omega_0 \sqrt{1 + 2\varepsilon(1 - \delta)} \cos \theta \frac{\partial}{\partial \theta}) f_i = \frac{e}{T} E\omega F_m \hat{\Phi}_k J_i(k, \rho)\]
(7)

Where \(f_i\) is the non-adiabatic part of the distribution function, and \(\nu_q\) can be written as equation (8),
\[\nu_q = \frac{B_q}{B_x} \nu_d = \frac{\nu_d}{\nabla \Psi / \nabla \chi} = \frac{\nu_d}{2x\Psi_{oo}} \frac{[1 + 2\varepsilon(1 - \delta) \cos \theta]}{E^2 Z}\]
\[= -\frac{\nu_d}{E} \sqrt{1 + 2\varepsilon(1 - \delta)} \cos \theta \sin \theta\]

\[\omega_i = \frac{\nu_\phi}{q R_0}, \quad q = \frac{B_\phi}{2\Psi_{oo}}, \quad \nu_d = \frac{\nu_\phi^2 + \nu_d^2}{\Omega_0 R_0}, \quad \omega_d = k_r \nu_d\]

For large aspect ratio equation (6) is turned to be equation (9),
\[(E\omega + \omega_d \sin \theta + i\omega_0 \frac{\partial}{\partial \theta}) f_i = \frac{e}{T} E\omega F_m \hat{\Phi}_k J_i(k, \rho)\] (9)

which is the same as that in Ref. [11] except \(\omega\) is replaced by \(E\omega\) equation (7) is a new gyrokinetic equation since it includes elongation, \(E\). There is no way including elongation, \(E\), in Ref. [11] which is valid only for circular configuration.

4. Numerical Solution

We solve the eigenequation which comes from quasineutrality condition. The GAM real frequency and damping rates are obtained using the same procedures as in Ref. [11] where plasma has two components, that is, one is hot and another is cold, \(n_h / n_0\) is fraction of the hot component. The GAM
real frequency and damping rates are exactly inversely proportional to elongation E (seen in figure 1) which seems to be consistent with that measured in ASDEX-Upgrade tokamak.

**Figure 1.** The real frequency and damping rates vs. elongation E, where \( q=3 \), \( n_h/n_0 \) is fraction of the hot component.

### 5. Discussion

Shaping effects on zonal flow are studied in this paper. A new coordinates which are related to elongation are rigorously developed in the analysis of gyrokinetic equation. It is found that the GAM real frequency and damping rates are inversely proportional to elongation E which seems to be consistent with that measured in ASDEX-Upgrade. Reduction of GAM damping rate could help to generate the zonal flow [11]. Hopefully, coming experimental data in ASDEX-Upgrade [8] and HL-2A [14] could confirm our statements.

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