Dynamical properties of a large-scale many-body system: visualized quantum walk, out-of-time ordered correlators and the butterfly velocity

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Quantum many-body systems are never short of novel and fascinating dynamics, yet its simulation by classical computers requires exponentially-scaled computation resources, which renders the research on large-scale many-body dynamics fiendishly difficult. In this letter, we explore the dynamic behavior of 2D large-scale ferromagnetic $J_1$-$J_2$ Heisenberg model both theoretically and experimentally. First, the analytical solution of magnon dynamics is obtained to show an obvious ballistic propagation of magnon, which is typical for quantum walk. Then, we verify the dynamic behavior of the system through numerical approach of exact diagonalization. We also calculate out-of-time ordered correlators and butterfly velocities among different lattice points, finding that they can well depict the competition between different couplings. Finally, a quantum walk experiment is designed and conducted on the basis of IBM programmable quantum processors, and the experimental results are in consistence with our theoretical predictions. Since the analytical results can be used, in principle, to predict the behavior of large-scale quantum many-body systems and even those infinitely large, this work will help facilitate further research on quantum walk and quantum many-body dynamics in large-scale lattice systems, guide future design of quantum computers, as well as popularize quantum computers until they are known and available to every household in the world.

Introduction.—Dynamical properties of the quantum many-body system are intriguing and important in condensed matter physics [1–3]. Quantum walk (QW), as one of the few controllable dynamical phenomena in quantum many-body systems, promises to bring a multitude of technological applications in quantum computing and other related fields [4–17]. In 2009, Prof. A. M. Childs et al first proved that continuous QW can be used as the computational framework of programmable quantum computers [6, 7], and the following years have witnessed the rise of QW as a hot research topic. By now, QW has been experimentally realized in a variety of platforms such as optical systems [18–39], trapped ions [40–44], cold atomic gases [45–49], superconducting circuits [50–53] and so on.

On the other hand, since transport properties of quantum many-body system hinge on how fast quantum information spreads, out-of-time ordered correlators (OTOCs), which are commonly used to describe thermalization and information scrambling, have aroused extensive attention in recent years [54–58]. As a quantum version of the Classical Poisson Bracket, OTOCs can accurately reflect the growth rate of operators’ noncommutativity over time. Recently, OTOCs have been used to depict information propagation in more and more systems, inclusive of quantum many-body dynamic systems [59–71], localization systems [72–79], quantum chaotic systems [82, 89] and black hole systems [90–93]. Note that, Prof. J. Maldacena, S. H. Shenker and D. Stanford theoretically proved, for the first time, the existence of propagation limit (exponential upper boundary) in black hole system and quantum SYK model [91]. This study also verified the duality between Anti-de Sitter spacetime and conformal field theory [92–93]. Meanwhile, the measurement of OTOCs has been realized in multiple table-top platforms [59, 62, 69–72, 79, 94, 95]. Today, OTOC is among the most desirable observables in describing the properties of information propagation in dynamic systems.

Though a lot of research has been done on the stationary state problem of many-body systems, dynamics of large-scale lattice systems remains a territory insufficiently charted. This letter will be devoted to the study of dynamic properties in large-scale many-body systems.

Visualized QW: A Spin Wave Theory.—The model adopted in this letter is a square lattice ferromagnetic $J_1$-$J_2$ Heisenberg model, and the related Hamiltonian can be written as

$$H = J_1 \sum_{m,n} (\hat{S}_{m,n} \hat{S}_{m+1,n} + \hat{S}_{m,n} \hat{S}_{m,n+1}) + J_2 \sum_{m,n} \hat{S}_{m,n} \hat{S}_{m+1,n+1},$$

(1)

where $J_1$ and $J_2$ correspond to the nearest-neighboring (nn) and the next-nearest-neighboring (nnn) spin exchange coupling, respectively. For simplicity, we consider only the ferromagnetic case, i.e., $J_1 < 0$ and $J_2 < 0$.

Based on the linear spin wave (SW) theory, one can exactly diagonalize the Hamiltonian of the system (see supplementary materials [96] for details). By performing Holstein-Primakoff transformation and Fourier transformation for Eq. (1), Hamiltonian of the magnon in diagonal form can be obtained as

$$\hat{A}_k = \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k,$$

(2)
where $\omega_k$ stands for the energy of magnons, which has the following expressions,

$$\omega_k = 2[J_1(1 - \gamma_{1k}) + J_2(1 - \gamma_{2k})],$$  \hspace{1cm} (3)

$$\gamma_{1k} = \frac{1}{2}(\cos k_x + \cos k_y),$$  \hspace{1cm} (4)

$$\gamma_{2k} = \frac{1}{2}[\cos (k_x + k_y) + \cos (k_x - k_y)].$$  \hspace{1cm} (5)

**B. Spin-Flip Dynamics and Magnon’s QW**—Then, one can readily study the propagation behavior of magnons with the diagonalized Hamiltonian, and figure out how the single spin flip information diffuses in the background of the ferromagnetic ground state. Let’s start with calculating the density distribution versus time, which is a key observable easy to measure experimentally in studying QW, with its expression as [45][47].

$$\xi(r, t) = \langle \phi | \hat{S}^+_{r_{nn}}(t) \hat{S}^-_{r_0} | \phi \rangle,$$  \hspace{1cm} (6)

with

$$\hat{S}^\pm_{r_{nn}}(t) = e^{iHt/h} \hat{S}^\pm_{r_{nn}} e^{-iHt/h},$$  \hspace{1cm} (7)

where $|\phi\rangle$ is the initial state, i.e., the ferromagnetic ground state and $r = r_{nn} - r_0$. In the expression, $\hat{S}^z_{r_{nn}}$ represents the spin operator at $r_{nn}$ in $z$-direction, while $\hat{S}^+_{r_0}$ and $\hat{S}^-_{r_0}$ denote the spin flip up and down operators at $r_0$, respectively. The diffusion behavior of the magnon can be revealed by measuring the evolution of density distribution with time.

By simple calculations, we obtain the analytical expression of magnon’s evolution (see supplementary material [96] for details), i.e.,

$$\xi(r, t) = \left| \sum_k \frac{e^{-i(kr + \omega_k t)/\hbar}}{N^2} \right|^2.$$  \hspace{1cm} (8)

The process of magnon’s dynamic evolution under different parameters $J_2$ is visualized in Fig. 1(a-f). As shown in the figure, when $J_2 = 0$, the system degenerates to the standard Heisenberg model. The density distribution then features ballistic propagation, typical for QW. When $J_2 \neq 0$, the nn coupling comes into play. As shown in Fig. 1 the influence of the nn coupling on dynamics of the system can be distilled into three points:

I. $J_2$ causes the magnon to travel faster in all directions. As we know, the computation speed of quantum computers is closely related to the speed of information propagation between qubits. In other words, the acceleration of information propagation is an effective way to speed up quantum computation.

II. The acceleration is anisotropic, maximum in the $x$- and $y$-axis direction and minimum in the diagonal direction. Therefore, new quantum computers can be designed on the basis of the anisotropic propagation, which will play a pivotal role in conducting anisotropic computation and quantum simulation effectively.

III. The competition between nn and nnn coupling cannot
destroy the ballistic propagation behavior. However, the geometric structure of ballistic propagation can be changed by manipulating parameters $J_1$ and $J_2$. As shown in Fig. 1(a-f), the system transforms from a square structure to a rhombic QW.

C. OTOCs and Butterfly Velocities—Furthermore, we calculate OTOCs of the system and the corresponding butterfly velocity to reveal the propagation characteristics [52][71]. Note that, butterfly velocity and the Lieb-Robinson bounds (the upper limit of the information propagation speed in quantum systems) are highly interdependent, which reflects information propagation behavior in many-body systems [1][2][54][77][105]. Detailed derivation of OTOCs and butterfly velocity in $J_1$-$J_2$ model is included in Supplementary Material [96].

By simple operator gymnastics, one can obtain the analytic expression of the system’s OTOCs, which reads

$$\tilde{F}(t) = \langle \phi| \hat{S}_{r_2}^z \hat{S}_{r_1}^z (t) \hat{S}_{r_2}^x (0) \hat{S}_{r_2}^z (0) \hat{S}_{r_1}^z |\phi\rangle. \quad (9)$$

Then, we get OTOCs’ analytical expression as

$$\tilde{F}(t) = 1 - 8/N^2 \Omega_1 \Omega_2 + 8/N^4 \Omega_1 \Omega_2 \Omega_1 \Omega_2, \quad (10)$$

where,

$$\Omega_1 = \sum_k e^{-i k \cdot (r_2 - r_1)} e^{i \omega k \cdot t / \hbar},$$

$$\Omega_2 = \sum_k e^{i k \cdot (r_1 - r_2)} e^{-i \omega k \cdot t / \hbar}. \quad (11)$$

Based on the SW theory, the propagation properties of magnetic information (a spin-flip signal) in the large-scale $J_1$-$J_2$ Heisenberg model can be revealed at a lower computational cost. In addition, through Baker-Campbell-Hausdorff formula, we obtain

$$F(t) = c_1 - c_2 e^{\lambda (t - \left| y_0 \right| / \nu_b)}, \quad (12)$$

where $c_1$ and $c_2$ are constants with $\lambda$ being the Lyapunov index of the system, while $\nu_b$ refers to the butterfly velocity. The results of OTOCs and the butterfly velocity are plotted in Fig. 1(g-i). We define the time when OTOC begins to decline as the critical time, denoted as $t_d$. The illustration reflects the relationship between $t_d$ and $r$. As shown in Fig. 1(g) [6b], the OTOCs along the $x$, $y$-axis [diagonal] direction oscillate versus time, where the amplitude of OTOCs decreases and $t_d$ increases with the increasing $r$. Since the magnon wave packet itself is conservative in the process of evolution, the probability of magnon distribution at a distant point is relatively small, hence the relatively small amplitude [54][56][96]. Besides, the propagation mode of the system is determined by the growth of the local Heisenberg operator, therefore, the longer the propagation time of the distant lattice magnon, the larger the corresponding $t_d$. The above two points well reflect that the magnon information diffuses in a way of QW. Note that, the slope of $t_d$ in the illustration is the butterfly velocity $[82][84]$. Further, we calculate how the butterfly velocity of the system changes with $J_2$ [see Fig. 1(i)]. As shown in the figure, when $J_2$ increases, the butterfly velocity in the diagonal direction will slowly surpass that in the $x$-direction, which indicates that the butterfly velocity can very well reflect the competitive relationship between nn coupling and nnn coupling in the system.

In short, based on OTOCs and butterfly velocity, one can understand the features of magnon QW in $J_1$-$J_2$ model from the following three perspectives:

I. The superposition of another QW on top of the previous one will lead to the acceleration of information propagation;

II. The competition between the two results in the change from diagonal dominance to $x$- and $y$-axis dominance of QW, and correspondingly, the geometric structure changes from square to rhombus;

III. The introduction of nn coupling will not destroy the ballistic propagation mode, therefore, the system still displays QW.

Digital quantum simulation—In principle, SW theory can very effectively describe the diffusion process of information in large-scale or even infinite lattice ferromagnetic system, thus makes it an ideal way to test the computational power of quantum computers. Although at present the several largest superconducting circuits quantum computers in the world total less than 70 quantum bits [52][53], we believe that the coming decades will see a great increase in the number of qubits, as well as the realization of error-correcting qubits. With the unstoppable progress of technological infrastructure, more phenomena in many-body dynamics, such as QW in large-scale systems and controllable magnetic transport, will be verified by more programmable quantum processors in the future.

As a principle testing, here we conduct experiments on a 5-qubit quantum processor and compare the experimental results with the theoretical ones. In order to conduct an experiment of scrambling dynamics on the IBM quantum processor, one must translate the time evolution into the language of quantum circuits, i.e., a series of standard quantum gates. First, the Heisenberg model described in boson language can be written as

$$H = -\frac{1}{2} N + 2 \sum_l (\hat{a}_l^\dagger \hat{a}_l^\dagger - \hat{a}_l^\dagger \hat{a}_l^\dagger + \hat{a}_l^\dagger \hat{a}_{l+1}^\dagger + \hat{a}_l^\dagger \hat{a}_{l+1}^\dagger). \quad (13)$$

Here we take $J_1 = -1$ and $S = 1/2$. By ignoring the constant term, which only brings in a global phase factor, the time evolution operator can be written as

$$\hat{U}(\delta t) = e^{-i H \delta t} = e^{-i \sum_l (\hat{a}_l^\dagger \hat{a}_l^\dagger - \hat{a}_l^\dagger \hat{a}_l^\dagger + \hat{a}_l^\dagger \hat{a}_{l+1}^\dagger + \hat{a}_l^\dagger \hat{a}_{l+1}^\dagger) \delta t}. \quad (14)$$

For tiny $\delta t$, the Trotter decomposition can be used to break the evolutionary operator into the following steps,

$$\hat{U}(\delta t) \approx e^{-i \sum_l (\hat{a}_l^\dagger \hat{a}_l^\dagger) \delta t} e^{-i \sum_l (\hat{a}_l^\dagger \hat{a}_{l+1}^\dagger + \hat{a}_l^\dagger \hat{a}_{l+1}^\dagger) \delta t} e^{-i \sum_l (\hat{a}_l^\dagger \hat{a}_l^\dagger) \delta t} = \hat{U}_1(\delta t) \hat{U}_2(\delta t) \hat{U}_1(\delta t). \quad (15)$$

where $\hat{U}_1$ and $\hat{U}_2$ represent the contribution of the on-site term and nn term. Next, we need to implement the $\hat{U}_1$ and $\hat{U}_2$ operators with a series of single and double qubit logic gates. $\hat{U}_1$
FIG. 2: (Color online). The illustration of quantum circuit. (a) The quantum circuit of two-qubit time-evolution operation, where $Z^x = e^{i\delta t/2}\sigma_3$, and $Y^z = e^{i\theta t/2}\sigma_3 = e^{i\theta t/2}\sigma_3$. (b) The quantum circuit which realizes the complete time-evolution operation, where $P(\theta)$ is the standard phase transition gate and $\theta = \delta t$. (c) The single spin-flip dynamics of theoretical and experimental approaches when $t = 0$, $t = 0.25$, $t = 0.5$, and $t = 0.75$, respectively.

**Note added.**—After the completion of this work, we notice that Google has recently conducted a relevant research on OTOCs and information scrambling via 53-qubits [108]. Besides, IBM has increased the number of programmable qubits to 127 [109]. The rapid progress in supporting technology will help verify the theory in this letter and facilitate further exploration in large-scale many-body quantum systems.

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[96] Supplementary Materials I: Detailed linear spin wave method, magnon dynamics, otoc and butterfly velocities are provided in S1-S3. S4 provide the comparison of OTOCs between theoretical predictions and the results of numerical Exact diagonalization. Supplementary Materials II: Movie of visualized quantum walks. https://youtu.be/xX1M-N0CM9o

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Supplemental Material

Contents

S1. Details of linear spin wave theory  
S2. Details of magnon dynamics  
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S1. Details of linear spin wave theory

Standard $J_1$-$J_2$ Heisenberg model Hamiltonian reads

$$\hat{H} = J_1 \sum_{m,n} (\hat{S}_{m,n}^x \hat{S}_{m\pm 1,n}^x + \hat{S}_{m,n}^y \hat{S}_{m\pm 1,n}^y) + J_2 \sum_{m,n} \hat{S}_{m,n}^z \hat{S}_{m\pm 1,n\pm 1}^z,$$

(S1)

First, we conduct second quantization via standard Holstein-Primakoff transformation (HP transformation), which can be defined as

$$\hat{S}_{m,n}^x = (2S - \hat{a}_{m,n}^\dagger \hat{a}_{m,n})\hat{a}_{m,n},$$
$$\hat{S}_{m,n}^y = \hat{a}_{m,n}^\dagger (2S - \hat{a}_{m,n}^\dagger \hat{a}_{m,n}),$$
$$\hat{S}_{m,n}^z = 1/2 - \hat{a}_{m,n}^\dagger \hat{a}_{m,n},$$

(S2)

where $m, n$ are the grid labels. For spin-1/2 ferromagnetic (FM) systems, the transformation is equivalent to

$$\hat{S}_{m,n}^x = \hat{a}_{m,n},$$
$$\hat{S}_{m,n}^y = \hat{a}_{m,n}^\dagger,$$
$$\hat{S}_{m,n}^z = 1/2 - \hat{a}_{m,n}^\dagger \hat{a}_{m,n},$$

(S3)

Then a fourier transformation is applied by plugging the following expression

$$\hat{a}_{m,n} = N^{-1} \sum_{k_x,k_y} e^{ik_x m a + ik_y n a} \hat{a}_{k_x,k_y},$$

(S4)

where $a = 1$ is lattice constant. Then one can obtain diagonal Hamiltonian as,

$$\hat{H}_k = \sum_k \omega_k \hat{a}_{k}^\dagger \hat{a}_{k},$$

(S5)

where $\omega_k$ stands for the energy of magnons and we have

$$\omega_k = 2[J_1 (1 - \gamma_{1k}) + J_2 (1 - \gamma_{2k})],$$

(S6)

$$\gamma_{1k} = \frac{1}{2} (\cos k_x + \cos k_y),$$

(S7)

$$\gamma_{2k} = \frac{1}{2} (\cos (k_x + k_y) + \cos (k_x - k_y)).$$

(S8)

S2. Details of magnon dynamics

The time-dependent density $\xi$ is defined as

$$\xi(r, t) = \langle \hat{\Phi}^\dagger (\hat{\Phi}) \hat{\phi} \rangle,$$
$$\hat{\phi} = e^{iHt/\hbar} \hat{\phi},$$

(S9)

Using the same Paradigm of spin wave (SW) theory, we apply HP transformation to get the expression in the bosonic language, i.e.,

$$\xi(r, t) = \langle \hat{\Phi}^\dagger (\hat{\Phi}) \hat{\phi} \rangle,$$
$$\hat{\Phi} = e^{iHt/\hbar} \hat{\Phi},$$

(S10)

where $\hat{\Phi}$ is the particle number operator. We can then naturally rewrite the expression of the density distribution as

$$\xi(r, t) = \langle \hat{\Phi}^\dagger (\hat{\Phi}) \hat{\phi} \rangle,$$

(S11)

with

$$\hat{\Phi}^\dagger (\hat{\Phi}) \hat{\phi}.$$

(S12)

By using Fourier transformation, we have

$$\hat{\Phi}^\dagger (\hat{\Phi}) \hat{\phi}.$$

(S13)

Considering the operation rules of creation and annihilation operators and substituting the diagonal hamiltonian, one can obtain the analytical expression of wavefunction, i.e.,

$$\hat{\Phi}^\dagger (\hat{\Phi}) \hat{\phi}.$$

(S14)

where $r = r_{m,n} - r_0$. By substituting it into Eq. (S11), the density distribution $\xi(r, t)$ can be obtained. The analytical expression reads

$$\xi(r, t) = \frac{1}{N^2} \sum_k e^{-i(kr_0 k)} \hat{a}_k e^{-iHt/\hbar} \hat{a}_k^\dagger \hat{\phi},$$

(S15)

S3. Out-of-time ordered correlators and butterfly velocity

The general definition of four-point correlation function out-of-time ordered correlators (OTOCs) is

$$\hat{F}(t) = \langle \hat{W}^\dagger (t) \hat{W}^\dagger (0) \hat{W}(t) \hat{W}(0) \rangle,$$

(S16)

where,

$$\hat{W}(t) = e^{iHt/\hbar} \hat{W}(0) e^{-iHt/\hbar}.$$

(S17)

In the initial state, $\hat{W}$ and $\hat{V}$ are commutative since they are operators of different positions. With time evolution, however,
the evolution process. So OTOCs show a decline versus time, where the high order terms in the above expression lose commutability as time goes, and gradually dominate OTOCs in the evolution process. So OTOCs show a decline versus time, i.e.,

\[
\hat{W}(t) = \sum_{k=0}^{\infty} \frac{(ip)^k}{k!} [\hat{H}, ..., [\hat{H}, \hat{W}], ...],
\]

(S18)

where the high order terms in the above expression lose commutability as time goes, and gradually dominate OTOCs in the evolution process. So OTOCs show a decline versus time, i.e.,

\[
F(t) = c_1 - c_2 e^{i\mathcal{W}(t - |v_b|/v_b)},
\]

(S19)

Here, $v_b$ is the so-called butterfly velocity which describes the growth rate of a local operator versus time. The butterfly velocity in a chaotic system can define the chaotic boundary closely related to the Lieb-Robinson bound, i.e., light cone of the chaotic system. Here, we choose $\hat{W} = \hat{S}^z_{r_{nm}}$ and $\hat{V} = \hat{S}^z_{r_{n'}}$, where $|r|$ denotes the distance between two sites, while $c_1$ and $c_2$ are constants. Then, the OTOCs become

\[
F(t) = \langle \phi | \hat{S}^z_{r_{n}}(t) \hat{S}^z_{r_{n'}}(0) \hat{S}^z_{r_{n}}(t) \hat{S}^z_{r_{n'}}(0) | \phi \rangle.
\]

(S20)

which can help effectively understand the information spread behavior in quantum systems.

S4. The mechanics of quantum walk

One can understand the quantum walk (QW) and information spread process of the $J_1$-$J_2$ Heisenberg system from the following three perspectives.

First, QW behavior of the system can be explained based on the results of OTOCs and butterfly velocity. The propagation velocity of a single spin-flip information in the $J_1$-$J_2$ model will be determined by the competition of direction-dependent spin exchange coupling. When $|J_2|$ is increasing, the corresponding propagation velocity in the axial direction accelerates much faster compared with that in the diagonal direction, leading to the change of the information propagation velocity.

Second, the $J_1$-$J_2$ QW can be explained in a more intuitive way. For an FM system only with nearest-neighboring(nn) coupling, the real space Hamiltonian of the system can be described by bosonic language as

\[
\hat{H} = \sum_{m,n,m',n'} P_{mm'nn'} \hat{d}^\dagger_{m,n} \hat{d}_{m',n'},
\]

(S21)

where the probability of spin-exchange reads

\[
P_{mm'nn'} = J_1 \left( 2 \delta_{m=m',n=n'} + \frac{1}{2} \delta_{m=m'\pm 1,n=n'} + \frac{1}{2} \delta_{m=m',n=n'\pm 1} \right) + J_2 \left( 2 \delta_{m=m',n=n'} + \frac{1}{2} \delta_{m=m'\pm 1,n=n'} \right).
\]

(S22)

Fixing the nn term, we plot the magnons’ probability distributions $P_{mm'nn'}$ under different next-nearest-neighbor(nnn) interaction in Fig. S1. As for $J_2 = 0$, the spin-exchange coupling can only occur between the four nearest sites [see Fig. S1(a)], which is in consistence with the standard QW in Heisenberg model. This is why the QW exhibits a square structure and the transport front appears at the square’s corners after a period of evolution. Furthermore, the introduction of nn coupling will increase the probability in the diagonal directions, finally resulting in a hollow square $P_{mm'nn'}$ structure [see Fig. S1(b-f)]. That leads to the rhombus QW.

![Fig. S1: (Color online). The spin-exchange probability $P_{mm'nn'}$ of a single magnon in FM system with different $J_2$. The corresponding parameters are marked.](image)

Third, we can also provide a mathematical structural explanation for the QW phenomenon. When we only consider the spin-exchange coupling between nn sites($J_2 = 0$), the expression of $\omega_{\mathbf{k}}$ will degenerate into $\omega_{\mathbf{k}} = J_1 \left( 2 - \cos(kx) - \cos(ky) \right)$, resulting in a square QW structure. The nn term, however, equals to adding an operation of $J_2 \left( 2 - \cos(kx + ky) - \cos(kx - ky) \right)$ on the previous basis. To be specific, it can be regarded as an operation of $45^\circ$ rotation plus a spacing stretching on the lattice. While it also keeps the QW behavior, the propagation structure here changes from a square to rhombus. For clearness, we plot Fig. S2 to show the evolution characteristics versus time considering only the contribution of nn [Fig. S2(a)] and nnn [Fig. S2(b)] interaction. The fact that the highest density is distributed at the four corners denotes that it is actually a form of ballistic diffusion, with the highest density distributed at the spread front, instead of the classical wave mode (denser inside rather than outside).

In a word, one can easily understand the QW dynamics by using OTOCs and butterfly velocities, i.e.,

I. Another type of QW (induced by nnn term) will be superimposed on the previous one (induced by nn term), leading to a nontrivial acceleration.

II. The competition between two QW leads to structure changing from square to a rhombus.

III. The ballistic propagation (QW mechanism) keeps.
S5. Comparison of spin wave theory with numerical exact diagonalization

In this section, we verify the correctness and accuracy of SW theory. Size, dimension and changes in $\hat{W}$ and $\hat{V}$ operators are discussed respectively in how they are affecting OTOCs of the system. As verification, we compare the results with the exact diagonalization. First, we prove the accuracy of SW method in describing information spread of FM system. We compared the above analytical results with the results of exact diagonalization(ED), which are shown in Fig. S3.

Next, we turn to the two-dimensional case. Fig. S3(d-f) corresponds to $r_{m,n} = r_0$ (on-site), $r_0 + x$ (nn) and $r_0 + x + y$ (nnn), respectively. By comparing with the exact diagonalization results, we prove that SW theory is highly effective and accurate not only in describing high dimensional system, but also in depicting OTOCs of the coordinate axis and the diagonal direction.

S6. Details about the digital quantum simulation

A. Quantum circuit for spin-flip dynamics

As shown in the main text, the quantum circuit for realizing digital quantum simulation(DQS) of 1D spin-flip propagation is constituted by the time evolution quantum gate $\hat{U}(\delta t)$, which is decided by the system hamiltonian

$$\hat{H} = -J \sum_l \hat{S}_l \cdot \hat{S}_{l+1}. \quad (S23)$$

It can also be represented in bosonic language through the standard Holstein-Primakoff transformation, which reads

$$\hat{H} = -0.5N + 2 \sum_i \hat{a}_i^\dagger \hat{a}_i - \sum_i (\hat{a}_i^\dagger \hat{a}_{i+1}^\dagger \hat{a}_i \hat{a}_{i+1}). \quad (S24)$$

Then, we can obtain the explicit form of time evolution operator

$$\hat{U}(\delta t) = e^{-i\delta t \hat{H}} = e^{-i\sum_l \hat{a}_l^\dagger \hat{a}_l \delta t} e^{-i\sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \hat{a}_i^\dagger \hat{a}_i) \delta t} \quad (S25)$$

where, $\delta t$ is a tiny period and we neglect the global phase caused by the constant term, which will not influence the probability density distribution. To demonstrate the effect of this operation via quantum gates, a Trotter decomposition is needed and the operator reads

$$\hat{U}(\delta t) = e^{-i \sum_l \hat{a}_l^\dagger \hat{a}_l \delta t} e^{-i \sum_i (\hat{a}_i^\dagger \hat{a}_{i+1}^\dagger \hat{a}_i \hat{a}_{i+1}) \delta t} e^{-i \sum_l \hat{a}_l^\dagger \hat{a}_l \delta t} = \hat{U}_1(\delta t) \hat{U}_2(\delta t) \hat{U}_1(\delta t) \quad (S26)$$

In order to demonstrate the process of operation more intuitively, we first introduce the explicit representation of state...
FIG. S3: (Color online). The comparison between the results of SW and ED (lines: SW results, dots: ED results). (a)∼(c): Comparison of OTOC result when $r_{m,n} = r_0$ in one-dimensional system with 3, 7 and 11 sites, respectively. (d)∼(f): Comparison of OTOC result in two-dimensional systems with 9 sites, when $r_{m,n}$ is the on-site, the nn and the nnn site of the spin-flipped position.

vectors and operators. We have

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix},
|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

\begin{equation}
\hat{a}_t^\dagger = |1\rangle_{Q_1}\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},
\end{equation}

\begin{equation}
\hat{a}_t = |0\rangle_{Q_1}\langle 1| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.
\end{equation}

$$\hat{a}_t^\dagger\hat{a}_{t+1} = |0\rangle_{Q_1}\langle 1| \otimes |1\rangle_{Q_{t+1}}\langle 0| = |01\rangle_{Q_1, t+1}\langle 01|$$

\begin{equation}
= |01\rangle_{Q_1, t+1}\langle 01| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\end{equation}

\begin{equation}
\hat{a}_t^\dagger\hat{a}_{t+1} = |1\rangle_{Q_1, t+1}\langle 0| \otimes |0\rangle_{Q_{t+1}}\langle 1| = |10\rangle_{Q_1, t+1}\langle 01|
\end{equation}

\begin{equation}
= |10\rangle_{Q_1, t+1}\langle 01| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\end{equation}

Here, the time evolution operation is divided into two fundamental parts, a single-qubit (on-site term) operation $\hat{U}_1$ and a two-qubit (nn term) operation $\hat{U}_2$. Next, we are going to realize each operation by certain standard quantum gates. Starting from the single-qubit operation, $\hat{U}_1$ can be rewritten as

$$\hat{U}_1(\delta t) = e^{-i\sum_{l}^{\text{on-site}} \hat{a}_t^\dagger \hat{a}}\delta t$$

\begin{equation}
= \prod_l e^{-i[0 0 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}]\delta t}
\end{equation}

\begin{equation}
= \prod_l \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\delta t} \end{pmatrix}
\end{equation}
It is easy to find that $\hat{U}_1$ shares the similar configuration with the matrix representation of phase transition gate $P(\theta)$. Thus, by comparing the matrix representation of them, namely

$$\hat{U}_1(\delta t) = \prod_{i} \left( \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & e^{-i\delta t} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\delta t} \end{array} \right)$$

$$= \prod_{i} \left( \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \prod_{i} P(\theta),$$

one can easily realize $\hat{U}_1$ operation with phase transition gate when the parameter of which $\theta = \delta t$.

In terms of the two-qubit operation, it is no longer so straightforward, but the idea of seizing equivalent effect after operating remains unchanged. We first illustrate the result of operating $\hat{U}_2$ on each two-qubit state. The matrix form of $\hat{U}_2$ reads

$$\hat{U}_2(\delta t) = e^{i\sum_i (\hat{a}_i\hat{a}_i^\dagger + \hat{a}_i^\dagger\hat{a}_i)\delta t}$$

$$= \prod_{i} e^{i\delta t (\hat{a}_i\hat{a}_i^\dagger + \hat{a}_i^\dagger\hat{a}_i)} = \prod_{i} e^{i\delta t (\hat{a}_i^\dagger\hat{a}_i)}$$

$$= \prod_{i} \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) = \prod_{i} e^{i\delta t (\hat{a}_i^\dagger\hat{a}_i)}$$

$$= \prod_{i} \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Applying the above representations inversely, we have

$$\hat{U}_2(\delta t) = \cos \delta t | 01 \rangle_{Q_1, i} \langle 01 | + | 10 \rangle_{Q_1, i} \langle 10 | - i \sin \delta t | 01 \rangle_{Q_1, i} \langle 10 | + | 10 \rangle_{Q_1, i} \langle 01 |)$$

(S35)

$$+ | 00 \rangle_{Q_{1,i}} \langle 00 | + | 11 \rangle_{Q_{1,i}} \langle 11 |.$$
FIG. S4: (Color online). The illustration of digital experiment on IBM cloud platform. (a) The circuit that simulates spin-flip dynamics in 0.5 time period and conducts measurement on the first qubit. (b) The elements of the group 'ut01' in (a), which realizes 0.1 time period of evolution. (c) The example of measurement result from the circuit.

C. Full data of the measured density distribution

In this section, we demonstrate our experiment data of 5 qubits in 4 specific time points. The results are provided in Fig. S5

S7. Visualized acceleration of quantum walk

Please see the attached file of evolution.mp4

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[S1] D.V. Else, F. Machado, C. Nayak, and N.Y. Yao, Improved Lieb-Robinson bound for many-body Hamiltonians with power-law interactions, Phys. Rev. A 101, 022333 (2020).

[S2] T. Kuwahara and K. Saito, Strictly Linear Light Cones in Long-Range Interacting Systems of Arbitrary Dimensions, Phys. Rev. X 10, 031010 (2020).
\begin{table}[h]
\centering
\begin{tabular}{c|ccccccc}
\hline
$t=0$ & 1 & 2 & 3 & 4 & 5 & 6 \\
qubit\times & & & & & & & \\
$q_0$ & 0.00525 & 0.00783 & 0.00498 & 0.00295 & 0.00296 & 0.00504 \\
$q_1$ & 0.00840 & 0.00685 & 0.00498 & 0.01083 & 0.01281 & 0.00504 \\
$q_2$ & 0.97899 & 0.97358 & 0.97413 & 0.96063 & 0.96946 & 0.97782 \\
$q_3$ & 0.00420 & 0.00685 & 0.00398 & 0.00492 & 0.00591 & 0.00504 \\
$q_4$ & 0.00315 & 0.00489 & 0.01194 & 0.02067 & 0.00887 & 0.00504 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{c|ccccccc}
\hline
$t=0.25$ & 1 & 2 & 3 & 4 & 5 & 6 \\
qubit\times & & & & & & & \\
$q_0$ & 0.13477 & 0.12764 & 0.11037 & 0.14517 & 0.15214 & 0.13477 \\
$q_1$ & 0.18838 & 0.21580 & 0.23266 & 0.22692 & 0.22683 & 0.18838 \\
$q_2$ & 0.33805 & 0.29690 & 0.31022 & 0.28823 & 0.27939 & 0.33805 \\
$q_3$ & 0.19881 & 0.21509 & 0.19911 & 0.19662 & 0.19917 & 0.19881 \\
$q_4$ & 0.13999 & 0.14457 & 0.14765 & 0.14306 & 0.14246 & 0.13999 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{c|ccccccc}
\hline
$t=0.50$ & 1 & 2 & 3 & 4 & 5 & 6 \\
qubit\times & & & & & & & \\
$q_0$ & 0.11119 & 0.13988 & 0.16210 & 0.19623 & 0.17516 & 0.18953 \\
$q_1$ & 0.26685 & 0.27034 & 0.27057 & 0.23270 & 0.21982 & 0.22843 \\
$q_2$ & 0.21682 & 0.18023 & 0.22243 & 0.21887 & 0.20447 & 0.21287 \\
$q_3$ & 0.23905 & 0.23806 & 0.23157 & 0.20377 & 0.24913 & 0.25601 \\
$q_4$ & 0.16609 & 0.17149 & 0.11335 & 0.14843 & 0.15143 & 0.11315 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{c|ccccccc}
\hline
$t=0.75$ & 1 & 2 & 3 & 4 & 5 & 6 \\
qubit\times & & & & & & & \\
$q_0$ & 0.22590 & 0.21007 & 0.20073 & 0.19347 & 0.19672 & 0.21727 \\
$q_1$ & 0.19639 & 0.20147 & 0.20133 & 0.21096 & 0.19915 & 0.20800 \\
$q_2$ & 0.17108 & 0.17568 & 0.19406 & 0.19814 & 0.17365 & 0.18540 \\
$q_3$ & 0.21265 & 0.21744 & 0.22074 & 0.21620 & 0.23497 & 0.20510 \\
$q_4$ & 0.19398 & 0.19533 & 0.18314 & 0.18124 & 0.19551 & 0.18424 \\
\hline
\end{tabular}
\end{table}

FIG. S5: Full data of density distribution from IBM quantum processor for $t = 0, 0.25, 0.5, 0.75$. 
