Chiral Phonon Mediated High-Temperature Superconductivity

Yi Gao,† Yang Pan,† Jun Zhou,‡ and Lifa Zhang

Phonon Engineering Research Center of Jiangsu Province, Center for Quantum Transport and Thermal Energy Science, Institute of Physics Frontiers and Interdisciplinary Sciences, School of Physics and Technology, Nanjing Normal University, Nanjing 210023, China

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Breaking down the traditional perception on phonons which are achiral, the recent discovery of chiral phonon carrying angular momentum provides possible ways to couple electron, photon, spin, magnon and exciton, etc. We theoretically proposed an electron-chiral phonon interaction with two-phonon process, in contrast to conventional electron-phonon interaction, and a kind of effective Hubbard U through exchanging two chiral phonons is proposed. Taking two-dimensional diatomic honeycomb lattice as an example, we found this repulsive Hubbard U mediated by chiral phonons induces unconventional and high-temperature superconductivity. Moreover, the numerical calculations show an inverse isotope effect which is consistent with experimental observations in high-Tc superconductors. Our finding on electron-chiral phonon and the associated Cooper pair provides a path to understand the high-Tc superconductivity.

The electron-phonon (E-P) interaction is not only the key to conventional superconductivity according to the Bardeen-Cooper-Schrieffer (BCS) theory, but also an important factor of the mechanism of high transition temperature (Tc) superconductivity [1]. Pairs of bound electrons, also called “Cooper pairs”, can be formed though exchanging intermediate phonons which leads to an effective attraction between electrons. It was still under debate that whether the E-P interaction is adequate to explain the high-Tc superconductivity in cuprates because of the peculiar phenomena including: the weak or inverse isotope effect, the linear electrical resistivity, and the kink in the electronic dispersion relations [2,3], etc. Other E-P interaction effects such as the unconventional E-P interaction [4], the polaronic effect [1], the non-adiabatic effect [5], spatial charge inhomogeneity [6], and the rapid increase of E-P coupling strength below the critical point in optimally doped strange metal [7] further complicate the underlying mechanisms. Beside phonons, other bosons like magnons and plasmons were proposed to be responsible for the formation of Cooper pairs in doped Mott insulator [8].

Recently, a nontrivial chiral phonon effect, which characterizes the phonon angular momentum (PAM) [9], has been theoretically proposed by Zhang et al. [10,11] and has been experimentally observed in both two-dimensional materials [12] and simple ferromagnets [13]. Interestingly, the chiral phonon effect was also observed through the measurement of thermal Hall conductivity in the pseudogap phase of cuprates [14] which are typical high-Tc superconductors. This finding could be attributed to the spin-phonon interaction [13,15]. It is straightforward to speculate that the PAM would strongly affect the interplay of spin, charge and phonon in high-Tc superconductors and consequently modify the superconducting (SC) phase transition.

This chiral phonon effect cannot be simply incorporated into the BCS theory. Because the key quantities of conventional E-P interaction in BCS theory are phonon frequency ω, phonon density-of-states (DOS), and E-P coupling constant. The PAM is characterized by $L^p_{ls} = u_{ls} \times p_{ls}$ which describes the rotation of ions around their equilibrium positions [10]. $u_{ls}$ and $p_{ls}$ are the displacement and momentum of s-th atom in l-th unit cell, respectively. Usually, the overall PAM $L^p = \sum_{l,s} L^p_{ls}$ vanishes due to the time-reversal and inversion symmetries. A nonzero $L^p$ and the corresponding phonon magnetization ($M^p$) can be obtained in nonequilibrium chiral system where both time-reversal and inversion symmetries are broken [16,17].

In this Letter, we consider the electron-chiral phonon (E-CP) interaction in the framework of strong-coupling theory in the Hubbard model [18]. This interaction is non-adiabatic and is beyond the Born-Oppenheimer approximation because $L^p_{ls}$ is a function of both $u_{ls}$ and $p_{ls}$. In other words, the electronic motion does not only depend on ion coordinations but also on their momenta. Our numerical calculations reveal the significant contribution of E-CP interaction to superconductivity.

Method.– The Hamiltonian is consisted of four parts. $H = H_0 + H_{ee} + H_{ep} + H_{e-cp}$ where $H_0$ is the non-interacting part, $H_{ee}$ is the electron-electron interaction, $H_{ep}$ is the conventional E-P interaction, and $H_{e-cp}$ describes the E-CP interaction. We choose the Planck constant $\hbar$, Boltzmann constant $k_B$ and lattice constant to be 1 for simplicity. We adopt a tight-binding model for a two-dimensional (2D) diatomic honeycomb lattice, then $H_0 = \sum_{k\sigma} \psi_{k\sigma}^\dagger M_k \psi_{k\sigma}$, where $\psi_{k\sigma}^\dagger = (c_{k1\sigma}^\dagger, c_{k2\sigma}^\dagger)^T$ and $\psi_{k\sigma} = (c_{k1\sigma}, c_{k2\sigma})^T$, $c_{k\sigma}$ and $c_{k\sigma}^\dagger$ are creation and annihilation operators of electrons with momentum $k$ and spin $\sigma$ ($\sigma = \uparrow, \downarrow$) on sublattice $s$ ($s = 1, 2$).

$$M_k = \begin{pmatrix} -\mu & \varepsilon_k \\ \varepsilon_k & -\mu \end{pmatrix},$$

(1)
where \( \varepsilon_k = t [e^{i(\frac{1}{2}k_x + \frac{\sqrt{3}}{2}k_y)} + e^{i\sqrt{3}k_y} + e^{i(\frac{1}{2}k_x + \frac{\sqrt{3}}{2}k_y)}] \), \( t \) is the nearest-neighbor hopping integral, and the electron wave vector is \( k = (k_x, k_y) \).

The electron-electron interaction is considered as the Hubbard type, which is \( H_{ee} = \frac{1}{N} \sum_{k,k', q, \sigma} c_{k+q, \sigma}^\dagger c_{k, \sigma} c_{k', q+\sigma} c_{k', q+\sigma} \), where \( N \) is the number of unit cells and \( U \) is the Coulomb interaction. The conventional E-P interaction is written as \( H_{ep} = \frac{g_{ep} m^{-1/4}}{\sqrt{N}} \sum_{q, k, s, \sigma} \xi^\dagger_{k+q, s, \sigma} c_{k+q, s, \sigma} A^\dagger_q \), where \( g_{ep} = g_{ep} m^{-1/4} \) is the E-P coupling strength and \( m \) is the mass of atom. \( A_q = a_q^+ + a_q^\dagger \) where \( a_q^\dagger \) (\( a_q^+ \)) creates (annihilates) a phonon with wave vector \( q \) and mode \( \nu \).

We assume that electron spin \( (S_{l,s}) \) feels a magnetic field induced by the local atomic rotation where the magnetic field is proportional to the PAM. Then the E-CP interaction is analogous to the form of spin-orbit coupling \( \eta L_{l,s} S_{l,s} \) [19] where \( \eta \) is the coupling strength parameter. In order to simplify our calculations, we assume that \( \eta \) is a constant and \( L_{l,s} S_{l,s} \) and \( L_{l,s} S_{l,s} \) are negligible. The Hamiltonian of E-CP interaction is written as

\[
H_{e-ep} = 2\eta \sum_{l,s} L_{l,s}^+ S_{l,s}^\dagger
= -\frac{\sqrt{2}\eta}{N} \sum_{\nu, \nu', q, a, q', s} g^{\nu', \nu, a}_{q, q', s} A^\dagger_q B^\nu_{q', a} - (q' - q)^\nu q^\nu q',
\]

(2)

where \( B^\nu_{q} = a^\nu_{q} - a_{q}^\dagger \) and \( S_{l,s}^\nu = \frac{1}{\sqrt{2}} \sum_{k, \sigma} \sigma \xi^\dagger_{k+q, s, \sigma} c_{k+q, s, \sigma}. \)

Detailed derivation of \( H_{e-ep} \) is given in Supplemental Material. The matrix element is defined as

\[
g^{\nu', \nu, a}_{q, q', s} = \sqrt{\frac{\omega^\nu_{q', \sigma}}{\omega^\nu_{q, \sigma}}} \xi^\dagger_{q, \nu}(s) \left( \begin{array}{cc}
0 & -i \\
i & 0
\end{array} \right) \xi_{q', \nu}(s),
\]

(3)

where \( \xi_{q, \nu}(s) \) is the polarization vector and \( \omega^\nu_{q} \) is the phonon frequency. It satisfies \( g^{\nu', \nu, a}_{q, -q', s} = g^{\nu', \nu, a}_{q, q', s} \).

The Feynman diagrams of E-CP interaction are shown in Fig. 1. This interaction is non-adiabatic, in which electrons are affected by both the displacement and momentum of the atoms. The phonon operators in the E-CP interaction are \( A^\dagger_q B^\nu_{q', a} \) which can be expanded as

\[
-\delta^\nu_{q', q} a^\nu_{q, a} + \delta^\nu_{q', q} a^\dagger_{q, a} - \delta^\nu_{q', q} a^\dagger_{q, a} + \delta^\nu_{q', q} a^\nu_{q, a}.
\]

(4)

So there are three kinds of two-phonon processes: emitting one phonon and absorbing one phonon; emitting two phonons; absorbing two phonons, which are shown in Fig. 1 (a). Consequently, the second-order of the aforementioned two-phonon processes give rise to two kinds of effective electron-electron interactions as shown in Fig. 1 (b). Unlike the BCS mechanism in which Cooper pairs are formed through exchanging one phonon, the E-CP interaction gives another mechanism of the formation of Cooper pairs via an effective Hubbard U induced through exchanging two chiral phonons. The Hamiltonian of the effective electron-electron interaction is given in Supplemental Material. We must emphasize that the E-CP interaction is fundamentally different from the conventional anharmonic E-P interaction with two-phonon absorption and emission. The chiral phonons are able to affect the spin fluctuation and the magnetic order which cannot be affected by anharmonic E-P interaction.

We adopt the widely used Green’s function method to investigate the effect of E-CP interaction on high-\( T_c \) superconductivity. First we calculate the electron’s normal Green’s function matrix \( G(k) \) according to Eq. 1. Then the irreducible susceptibility matrix \( \chi_0(q) \) can be calculated based on \( G(k) \). Then within the random phase approximation (RPA), we can calculate the longitudinal and transverse spin susceptibility matrices, \( \chi_{zz}^\nu(q) \) and \( \chi_{x}^\nu(q) \), respectively, as well as the charge susceptibility matrix, \( \chi_{c}(q) \).

When the temperature is close to \( T_c \), the linearized Eliashberg equation can be expressed as [15]

\[
\lambda^\nu \delta_{\alpha \alpha'}(k) = -\frac{T}{N} \sum_{q} \sum_{a_1, a_2, a_3, a_4} G^{a_1 a_2}(k-q) G^{a_3 a_4}(q-k) \delta_{\alpha a_3}(k-q) V^{a_1 a_2, a_4 a_6}(q) ,
\]

(5)

where \( \delta(k) \) is the electron’s anomalous self-energy. \( T_c \) is reached when the largest eigenvalue \( \lambda \) in Eq. 5 reaches 1. The singlet pairing interaction in Eq. 3 is [13]

\[
V(q) = \frac{1}{2} \tilde{S}(q) \chi_{zz}(q) \tilde{S}(q) + U_s \chi_{z}^\dagger_{-}(q) U_s
- \frac{1}{2} \tilde{C}(q) \chi_{c}(q) \tilde{C}(q) + \frac{1}{2} [\tilde{S}(q) + \tilde{C}(q)],
\]

(6)

where
Here \(\omega_1 = \omega_{1q_1}^{\alpha}, \omega_2 = \omega_{1q_1+q}^{\alpha}\) are the phonon dispersion relations and \(N_1 = \omega_{1q_1}^{\alpha}, N_2 = \omega_{1q_1+q}^{\alpha}\) are the Bose distribution functions. \(\omega_n = 2n\pi T\) is the Matsubara frequency where \(n\) is integer. The derivations and expressions of \(G(k), \chi_0(q), \chi_s^{zz}(q), \chi_s^+(q), \chi_c(q), U_s\) and \(C(q)\) can all be found in the Supplemental Material.

In the following, we set \(U = 2.3t\) and adjust the chemical potential \(\mu\) to ensure the electron filling \(n_f = 0.95\) where \(n_f = 1 + TN^{-1} \sum_{\alpha} \sum_k G^{a_\alpha a_\alpha}(k)\). The band width of Eq. (1) is about \(6t\), therefore this choice of \(U\) corresponds to an intermediate strength of electron-electron interaction. Furthermore since the band structure has particle-hole symmetry, we consider only the hole-doped case and the choice of \(n_f\) corresponds to 5\% hole doping. The number of unit cells is set to be \(N = 32 \times 32\) and we use 16384 Matsubara frequencies. The summation over momentum and frequency are both done by fast Fourier transformation. The E-CP coupling matrix element \(\tilde{g}_{q, q', s}\) is calculated by given values of \(m_1\) and \(m_2\) which are the masses of two atoms, respectively. When \(m_1 = m_2 = 1\), we rescale the phonon’s dispersion so the maximal \(\omega_q^{\alpha} \approx 0.097t\). The exact calculation of E-CP coupling strength \(\eta\) is not an easy task. We speculate that the E-CP interaction is similar to the spin-orbit coupling by simply considering an ion rotating around an electron [19]. Then the value of \(\eta\) must decrease with increasing temperature because of the enlarged distance between electron and ion. Therefore, we assume a simple exponential decay of \(\eta\) as follows

\[
\eta = \eta_0 e^{-T/T^*}.
\] (8)

Here we set \(\eta_0 = 4.6574 \times 10^{-4}t\) and \(T^* = 0.02t\).

**Results and discussions.** – Starting from the electron filling \(n_f = 0.95\), we calculate the largest eigenvalue of \(\chi_0(q, i\omega_n = 0)\tilde{S}(q, i\omega_n = 0)\) over all \(q\) and denote it as \(\alpha_s^{zz}\). Its value has to be less than 1 to prevent the formation of static magnetic order. We further denote \(\alpha_s^{+-}\) and \(\alpha_c\) as the largest eigenvalues of \(\chi_0(q, i\omega_n = 0)\tilde{U}_s\) and \(-\chi_0(q, i\omega_n = 0)\tilde{C}(q, i\omega_n = 0)\), respectively. Similarly, both of them have to be less than 1 for the system to stay away from static magnetic and charge ordering.

In the absence of E-CP and E-P couplings, i.e., \(\eta = \tilde{g}_{ep} = 0\), we show the evolution of \(\alpha_s^{zz}, \alpha_s^{+-}\) and \(\alpha_c\) with temperature in solid curves in Fig. 2(a). It is obvious that \(\alpha_s^{zz} = \alpha_s^{+-}\). Through the temperature range we investigated, \(\alpha_s^{zz}\) and \(\alpha_s^{+-}\) are always less than 1. On the other hand, in Fig. 2(b) we show the largest eigenvalue of Eq. (3), \(\lambda\), as a function of \(T\). We can see that \(\lambda\) increases with decreasing \(T\) and reaches 1 at \(T \approx 0.0025t\), suggesting that \(T_c \approx 0.0025t\) when there is neither E-CP nor E-P interaction. If the E-P coupling is present, for example, \(\eta = 0\) and \(\tilde{g}_{ep} = 0.15t\), dotted curves in Figs. 2(a) and 2(b) show that \(\alpha_c\) and \(T_c\) increase to 0.5 and 0.0035t, respectively. The spin susceptibility related \(\alpha_s^{zz}\) and \(\alpha_s^{+-}\) are unaffected by the E-P interaction.

In the presence of the E-CP interaction (but no E-P interaction), the evolution of \(\alpha_s^{zz}\) and \(\lambda\) with temperature are shown in Figs. 2(c) and 2(d), respectively. As we can see, both \(\alpha_s^{zz}\) and \(\lambda\) increase with decreasing temperature, suggesting the trend to have magnetic phase
transition and superconductivity phase transition. However, since when $T$ decreases, $\lambda$ reaches 1 before $\alpha^{zz}_s$ does, thus the superconductivity phase transition actually occurs and the corresponding $T_c \approx 0.0155t$ as indicated by the vertical dashed line in Fig. 2(d). Therefore, the E-CP coupling can greatly enhance $T_c$ compared to its value when $\eta = 0$ as shown in Fig. 2(b). Moreover, $\alpha^{+-}_s$ and $\alpha_c$ are not shown because only the 2-component is considered in Eq. 2, then both of them stay unchanged as shown in Fig. 2(a). Our numerical calculations further confirm that the E-P interaction, by setting $\tilde{g}_{ep} = 0.15t$ in this case, hardly change the value of $T_c$. This proves that the great enhancement of $T_c$ is indeed resulted from the E-CP interaction.

This finding reveals that the E-CP interaction leads to an unambiguous increase of $\alpha^{zz}_s$. The reason for this increase is, the phonon contributions, i.e., the last two terms in Eq. (7), are positive when $a_1 = a_2$ and negligibly small when $a_1 \neq a_2$. Therefore the phonon can effectively increase the value of $U$ for the longitudinal spin susceptibility, leading to an increased $\alpha^{zz}_s$, which means an increased longitudinal spin fluctuation. It then enhances the term $\frac{1}{2} \tilde{S}(q)\chi^{zz}_s(q)\tilde{S}(q)$ in Eq. 6, leading to an enlarged pairing interaction, which then results in an enhanced $T_c$.

We now turn to investigate the superconductivity pairing symmetry. We project $\phi(k)$ in Eq. (5) onto each band and since in our hole-doped case, only the lower band crosses the Fermi level, therefore we consider only the pairing function on this band and denote it as $\Delta^{11}(k)$. When $\eta = \tilde{g}_{ep} = 0$, the superconductivity pairing is resulted solely from the electron-electron interaction. In this case, at $T = 0.002t$ slightly below $T_c = 0.0025t$, we show the real and imaginary parts of $\Delta^{11}(k, i\pi T)$ in Figs. 3(a) and 3(b), respectively. One can see that $\text{Re}\Delta^{11}(k, i\pi T)$ and $\text{Im}\Delta^{11}(k, i\pi T)$ differ only in magnitude, but have the same structure in momentum space. Therefore the phase factor between the real and imaginary parts is global and $\Delta^{11}(k, i\pi T)$ can be taken as real, i.e., no time-reversal symmetry breaking. Furthermore, there exist sign changes in $\Delta^{11}(k, i\pi T)$, indicating that the pairing symmetry is unconventional (not isotropic s-wave). When $\eta \neq 0$ and $\tilde{g}_{ep} = 0$, although $T_c$ is greatly enhanced, the pairing function at $T = 0.015t$ slightly below $T_c = 0.0155t$ shown in Figs. 3(c) and 3(d) remains qualitatively the same as that for $\eta = 0$. The pairing still does not break the time-reversal symmetry and is unconventional as well. By examining the phonon contribution to $\tilde{S}(q)$ we found that, the last two terms in Eq. (7) are slightly $q$-dependent, therefore the E-CP coupling will not vary the pairing symmetry significantly as compared to the $\eta = 0$ case. We have also verified that, in the above two cases, including the E-P interaction at $\tilde{g}_{ep} = 0.15t$ does not change the pairing function qualitatively.

Thus, by introducing the E-CP interaction, high-temperature and unconventional superconductivity can be induced in systems with relatively weak electron-electron interaction. For the present system, taking $t = 1$ eV as the energy unit, the calculated superconductivity transition temperature $T_c$ can be boosted from 2.5 meV (29K) to 15.5 meV (180K). Furthermore, if the phonon couples to the $S^+$ and $S^-$ components of the electrons, $T_c$ may be further enhanced since the factor is 1 in the $U_s\chi^{+-}_s(q)U_s$ term in Eq. 6, instead of $\frac{1}{2}$ in the $\tilde{S}(q)\chi^{zz}_s(q)\tilde{S}(q)$ term. This will be studied in our future.
work.

The anomalous isotope effect has been found in many kinds of superconductors [21, 22]. The E-CP interaction provides an alternative explanation of such isotope effect. First we consider a case where the masses of the two sublattices are equal \((m_1 = m_2 = m)\). Fig. 4 shows that \(T_c\) increases with increasing \(m\). Such inverse isotope effect is opposite to the conventional E-P coupling in BCS theory where \(T_c\) decreases with increasing \(m\). Because the most significant term, \(\omega_n = 0\) component in Eq. (7), can be written as

\[
S^{a_1a_1, a_2a_2}(\mathbf{q}, i\omega_n = 0, m, T) = \sqrt{m} S^{a_1a_1, a_2a_2}(\mathbf{q}, i\omega_n = 0, 1, \sqrt{m}T),
\]

when the phonon dispersion relation is proportional to \(1/\sqrt{m}\) and \(U = 0\). This means that the matrix elements of \(\tilde{S}(\mathbf{q})\) are enhanced as \(m\) increases, leading to an inverse isotope effect. Detailed discussion of isotope effect is given in Supplemental Material. In addition, we also investigate the isotope substitution effect in Fig. 4 where \(m_1\) is fixed to be 1 and \(m_2\) changes from 1 to 1.6. We can see that \(T_c\) also increases with increasing \(m_2\). However, the increasing rate is smaller than that of Fig. 4. Our calculation results provide a possible explanation of weak and inverse isotope effect in cuprate superconductors with oxygen isotope substitution (\(\text{O}^{16} \rightarrow \text{O}^{18}\)) [21, 22].

In summary, we introduce the electron-chiral phonon interaction into the current theory of high \(T_c\) superconductivity. This interaction leads to an interplay of electron, spin, and charge in superconductors. The numerical calculation results show a remarkable enhancement of \(T_c\) induced by this interaction. Moreover, both unconventional paring and peculiar weak and inverse isotope effect are found which are able to explain the experimental observations.

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* These authors contributed equally to this work.
† zhujunzhou@njnu.edu.cn
‡ phyzlf@njnu.edu.cn

[1] V. Z. Kresin and S. A. Wolf, Rev. Mod. Phys. 81, 481 (2009).
[2] A. Lanzara, P. V. Bogdanov, X. J. Zhou, S. A. Kellar, D. L. Feng, E. D. Lu, T. Yoshida, H. Eisaki, A. Fujimori, K. Kishio, J. I. Shimoyama, T. Noda, S. Uchida, Z. Hussain, and Z. X. Shen, Nature 412, 510 (2001).
[3] F. Giustino, M. L. Cohen, and S. G. Louie, Nature 452, 975 (2008).
[4] T. Egami, in Symmetry and Heterogeneity in High Temperature Superconductors, edited by A. Bianconi (Springer, Berlin, 2006) Chap. II.1, pp. 79–86.
[5] P. Bañacký, in Symmetry and Heterogeneity in High Temperature Superconductors, edited by A. Bianconi (Springer, Berlin, 2006) Chap. II.2, pp. 87–101.
[6] D. Reznik, L. Pintschovius, M. Ito, S. Ikubo, M. Satō, H. Goka, M. Fujita, K. Yamada, G. D. Gu, and J. M. Tranquada, Nature 440, 1170 (2006).
[7] Y. He, M. Hashimoto, D. Song, S. D. Chen, J. He, I. M. Vishik, B. Moritz, D. H. Lee, N. Nagaosa, J. Zaanaen, T. P. Devereaux, Y. Yoshida, H. Eisaki, D. H. Lu, and Z. X. Shen, Nature 412, 510 (2001).
[8] P. A. Lee, N. Nagaosa, and X. G. Wen, Rev. Mod. Phys. 78, 17 (2006).
[9] A. G. McLellan, J. Phys. C: Solid State Phys. 21, 1177 (1988).
[10] L. Zhang and Q. Niu, Phys. Rev. Lett. 112, 085503 (2014).
[11] L. Zhang and Q. Niu, Phys. Rev. Lett. 115, 115502 (2015).
[12] H. Zhu, J. Yi, M. Li, J. Xiao, L. Zhang, C. Yang, R. A. Kaindl, L. Li, Y. Wang, and X. Zhang, Science 359, 579 (2018).
[13] S. R. Tauchert, M. Volkov, D. Ehberger, D. Kazenwadel, M. Evers, H. Langer, A. Donger, A. Book, W. Kreupelpaintner, U. Nowak, and P. Baum, Nature 602, 73 (2022).
[14] G. Grissonnanche, S. Thériault, A. Gourgout, M. E. Boulanger, E. Lefrançois, A. Ataei, F. Laliberté, M. Dion, J. S. Zhou, S. Pyon, T. Takayama, H. Takagi, N. Doiron-Leyraud, and L. Taillefer, Nat. Phys. 16, 1108 (2020).
[15] C. Stamm, T. Kachel, N. Pontius, R. Mitzner, T. Quast, K. Hölldack, S. Khan, C. Lupulescu, E. F. Azia, M. Wietstruk, H. A. Dür, and W. Eberhardt, Nat. Mater. 6, 740 (2007).
[16] M. Hamada, E. Minamitani, M. Hirayama, and S. Murakami, Phys. Rev. Lett. 121, 144306 (2018).
[17] M. Hamada and S. Murakami, Phys. Rev. Research 2, 023275 (2020).
[18] T. Takimoto, T. Hotta, and K. Ueda, Phys. Rev. B 69, 104504 (2004).
[19] R. Shankar, Principles of Quantum Mechanics, 2nd ed. (Plenum Press, New York, 1994) p. 468.
[20] B. Stritzker and W. Buckel, Z. Phys. 257, 1 (1972).
[21] H. J. Bornemann, D. E. Morris, and H. B. Liu, Physica C 182, 132 (1991).
[22] A. M. Schaefier, S. R. Temple, J. K. Bishop, and S. Dee-myad, Proc. Natl. Acad. Sci. USA 112, 60 (2015).
[23] A. Stucky, G. W. Scheerer, Z. Ren, D. Jaccard, J.-M. Pounirol, C. Barreteau, E. Giannini, and D. Van Der Marel, Sci. Rep. 6, 37582 (2016).
[24] D. J. Pringle, G. Williams, and J. L. Tallon, Phys. Rev. B 62, 12527 (2000).