A model of polymer gravitational waves: theory and some possible observational consequences

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We propose a polymer quantization scheme to derive the effective propagation of gravitational waves on a classical Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime. These waves, which may originate from a high energy source, are a consequence of the dynamics of the gravitational field in a linearized low-energy regime. A novel method of deriving the effective Hamiltonian of the system is applied to overcome the challenge of polymer quantizing a time-dependent Hamiltonian. Using such a Hamiltonian, we derive the effective equations of motion and show that (i) the form of the waves is modified, (ii) the speed of the waves depends on their frequencies, and (iii) quantum effects become more apparent as waves traverse longer distances.

Keywords: Quantum gravity, gravitational waves

1. Introduction

Recent discovery of gravitational waves (GWs) and the rapid increase in the sensitivity of GWs observatories has opened up a great opportunity in connecting
theory and phenomenology with experiment in precision cosmology, black hole physics and quantum gravity among other fields. In particular we are now becoming more hopeful about the observation of signatures of quantum gravity in GWs emitted from black hole mergers and high redshift regions of the cosmos.

There have been numerous studies connecting theories of quantum gravity with potential observations regarding the structure of quantum spacetime. In particular, in Loop Quantum Gravity (LQG) [1], there have been studies to understand the consequence of nonpertubative quantization in propagation of Gamma Ray Bursts (GRBs), other matter fields, and GWs on cosmological or black holes spacetimes (for some examples see, Refs. [2–31] and references within).

In this work we consider GWs as effective perturbations propagating on a classical FLRW cosmological spacetime. The effective form of such waves is derived by applying the techniques of polymer quantization [32–36] to the classical perturbations. Such a quantization is a representation of the classical algebra on a Hilbert space that is unitarily inequivalent to the usual Schrödinger representation. In it, operators are regularized and written in a certain exponential form. In such theories, the infinitesimal generators corresponding to some of the operators do not exist on the Hilbert space. As a consequence, the conjugate variables to those operators only admit finite transformations. Thus, the dynamics of the theory leads to the discretization of the spectrum of the conjugate operators (for more details and some examples of polymer quantization applied to particles and path integral formulation of black holes, see Refs. [34, 35, 37–39]).

In the model we present in this paper, the Hamiltonian is time-dependent and directly polymer quantizing it proves to be quite challenging. Hence, we apply a novel method based on the use of extended phase space, to overcome this issue (see Ref. [40]). The Hamiltonian in the extended phase space is rendered time independent by applying a certain canonical transformation and then polymer quantized using some of techniques developed in the literature [35, 41, 42]. After that, the effective version of this quantum Hamiltonian is made time-dependent again by applying the inverse of the above-mentioned canonical transformation. Finally the system is re-expressed in the usual non-extended phase space. Using this modified Hamiltonian, we derive the effective equations of motion of polymerized GWs and show that i) the form of the waves is modified, ii) the speed of the waves depends on their frequencies, and iii) the quantum effects are amplified by the distance/time the waves travel.

This paper is organized as follows: in Sec. 2, we derive the classical Hamiltonian of perturbations on an FLRW classical background. In Sec. 3, this time-dependent Hamiltonian is turned into a polymer effective time-dependent Hamiltonian by applying a certain method that is inspired by an approach used to deal with time-dependent harmonic oscillators. Using this Hamiltonian, we then derive the effective equations of motion. In Sec. 4, we study the behavior of the solutions in both nonperturbative and perturbative regimes and show that quantum gravitational
effects induce certain imprints on the waveform, frequency, and speed of GWs. Finally, in Sec. 5 we present our concluding remarks.

2. Hamiltonian formalism for GWs

We start with a spacetime manifold $M = T^3 \times \mathbb{R}$ with a spatial 3-torus topology\(^a\), equipped with coordinates $x^j \in (0, \ell)$ and a temporal coordinate $x^0 \in \mathbb{R}$. The background metric $\breve{g}_{\mu\nu}$ is then perturbed by a small perturbation $h_{\mu\nu}$ such that the full metric $g_{\mu\nu}$ can be written as

$$g_{\mu\nu} = \breve{g}_{\mu\nu} + h_{\mu\nu}. \quad (1)$$

GWs are the result of the weak-field approximation to the Einstein field equations for the above metric. As is well-known, a wave traveling along, say, the $x^3$ direction, can be separated into two polarization scalar modes $h_+(x)$ and $h_\times(x)$ as

$$h_{ij}(x) = h_+(x)e^+_{ij} + h_\times(x)e^\times_{ij}, \quad (2)$$

where

$$e^+ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad e^\times = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (3)$$

We would like to study the dynamics of the perturbations on a homogeneous, isotropic universe described by the FLRW metric

$$\breve{g}_{\mu\nu} dx^\mu dx^\nu = -N^2(x^0) d(x^0)^2 + a^2(x^0) dx^2, \quad (4)$$

where $x^0$ is an arbitrary time coordinate, $N(x^0)$ is the lapse function which depends on the choice of $x^0$, and $dx^2 = \sum^3 d(x^i)^2$ is a unit 3-sphere. To obtain a Hamiltonian of a harmonic oscillator, we introduce a new field

$$h_\sigma(x^0, x) = \frac{a^2 \sqrt{\kappa}}{\ell^{3/2}} \sum_k A_{\sigma, k}(x^0) e^{ik \cdot x} \quad (5)$$

which together with its conjugate momentum $E_{\sigma, k}$ constitute a canonically conjugate pair

$$\{ A_{\lambda, k}, E_{\lambda', k'} \} = \delta_{kk'} \delta_{\lambda\lambda'}. \quad (6)$$

Here $\ell$ is the result of our quantization on a lattice which corresponds to an upper limit on the momenta involved. The above canonical pair allow us to write the Hamiltonian of the system as

$$H = \frac{N}{2a^3} \sum_{\sigma = +, \times} \sum_k \left[ E_{\sigma, k}^2 + k^2 a^4 A_{\sigma, k}^2 \right] =: \sum_{\sigma = +, \times} \sum_k h_{\sigma, k}(x^0), \quad (7)$$

\(^a\)To avoid a discussion of boundary conditions on fields (generated by perturbations), we will assume that the spatial 3-manifold is $T^3$.\n
where $N$ is the lapse. It is clear that this last equation represents the Hamiltonian of a set of decoupled harmonic oscillators with a time-dependent frequency $\omega^2 = k^2 a^4$, due to time dependence of $a(t)$.

At this point, we choose the harmonic time gauge where $N(x^0 = \tau) = a^3(\tau)$ to get rid of the factor $a^{-3}$ in front of Eq. (7). Hence, the Hamiltonian of the perturbations (for a fixed mode $k$ and a fixed polarization $\sigma$) over the FLRW background in harmonic time becomes

$$H_{\sigma,k}(\tau) = \frac{1}{2} \left[ \epsilon_{\sigma,k}^2 + k^2 a^4 A_{\sigma,k}^2 \right]. \tag{8}$$

3. Polymer quantization and the effective Hamiltonian

The time-dependence of this Hamiltonian (8) makes deriving its effective polymer corrections quite complicated. This is because its polymer quantization will yield a time-dependent quantum pendulum-type system whose solutions are mathematically difficult to treat. In order to circumvent this issue, we will apply a procedure based on the extended phase space formalism (more details in Ref. [40]).

The idea is to first lift the (action of the) system

$$S = \int \left\{ p \frac{dq}{dt} - H(t) \right\} dt, \tag{9}$$

with time-dependent Hamiltonian of the form

$$H(t) = \frac{1}{2m} p^2 + \frac{1}{2} m \omega(t)^2 q^2, \tag{10}$$

to the extended phase space (EPS). In accordance with Dirac’s formalism, the system is now described by the extended action

$$S = \int \left\{ p \frac{dq}{d\tau} + p_t \frac{dt}{d\tau} - \lambda \phi \right\} d\tau, \tag{11}$$

where

$$\phi = p_t + H(t) \approx 0, \tag{12}$$

is a first class constraint ensuring the compatibility of the two actions (9) and (11) on the constrained surface $\phi = 0$, $\lambda$ is a Lagrange multiplier, and $p_t$ is the momentum conjugate to $t$. This is step (1) in Fig. 1. In step (2) in Fig. 1, we apply a canonical transformation

$$Q = \frac{1}{\rho(t)} q, \tag{13}$$

$$T = \int \frac{1}{\rho^2(t)} dt, \tag{14}$$

$$P = \rho(t) p - m \dot{\rho}(t) q, \tag{15}$$

$$P_T = \rho^2(t) p_t + \rho(t) \dot{\rho}(t) q p - \frac{m}{2} q^2 \left[ \dot{\rho}^2(t) + \frac{W^2}{\rho^2(t)} - \omega^2(t) \rho^2(t) \right], \tag{16}$$
in the extended phase space which removes the time dependency of the Hamiltonian $H(t)$ in $\phi$. Here, $W$ is the time-independent frequency of the time-independent system and $\rho$ is an auxiliary variable to be determined by the specific properties of the system, more precisely by $\omega$ and $W$. After this step the action becomes

$$S = \int \left\{ P \frac{dQ}{d\tau} + P_T \frac{dT}{d\tau} - \lambda \tilde{\phi} \right\} d\tau,$$

where, the first class constraint now reads

$$\tilde{\phi} = \rho^2(T) [P_T + K] \approx 0,$$

and the corresponding Hamiltonian $K$ appearing in it is

$$K = \frac{1}{2m} P^2 + \frac{1}{2} mW^2 Q^2.$$

Moreover, the auxiliary equation used to fix $\rho(t)$ becomes

$$\ddot{\rho}(t) + \omega^2(t) \rho(t) = \frac{W^2}{\rho^3(t)}.$$

This time-independent harmonic oscillator can now be polymer quantized [35, 41, 42] as in step (3) of Fig. 1. We then perform the inverse canonical transformations above in step (4) of Fig. 1. Finally, in step (5) of Fig. 1, we apply the inverse of the canonical transformations above and solve the constraint. This yields the polymer effective Hamiltonian on the usual phase space, where now the Hamiltonian is not just effective but also time-dependent.

By applying this method to our Hamiltonian (7), we obtain an effective polymer Hamiltonian with polymer $\mathcal{E}_{\sigma,k}$ and discrete spectra for $A_{\sigma,k}$ as

$$H^{(E)}_{\text{eff}} = \sum_{\sigma = \pm} \sum_{k \in \mathcal{L}} \left\{ \frac{2}{\mu^2 \rho^2} \sin^2 \left( \frac{\mu (\rho E_{\sigma,k} - \dot{\rho} A_{\sigma,k})}{2} \right) + \frac{\dot{\rho} A_{\sigma,k} E_{\sigma,k}}{\rho} + \frac{A_{\sigma,k}^2}{2} \left[ \omega^2 - \frac{\dot{\rho}^2}{\rho^2} \right] \right\},$$

(21)
where $\mu$ is called the polymer parameter that sets the scale for which the quantum gravity effects become important, and we have set $\hbar = 1$. The corresponding equations of motion read

$$\frac{dA_{\sigma,k}}{dt} = \frac{1}{\rho} \sin \left( \mu \left( \rho E_{\sigma,k} - \dot{\rho} A_{\sigma,k} \right) \right) + \frac{\dot{\rho}}{\rho} A_{\sigma,k}, \quad (22)$$

$$\frac{dE_{\sigma,k}}{dt} = \frac{\dot{\rho}}{\rho^2} \sin \left( \mu \left( \rho E_{\sigma,k} - \dot{\rho} A_{\sigma,k} \right) \right) + \left( \frac{\ddot{\rho}}{\rho} \right)^2 A_{\sigma,k} - \omega^2 A_{\sigma,k} - \frac{\dot{\rho}}{\rho} E_{\sigma,k}. \quad (23)$$

These equations are nonlinear in both $A_{\sigma,k}$ and $E_{\sigma,k}$, and their $\mu \to 0$ limit matches the classical equations of motion as expected. Also notice that in our case, $\rho$ controls the background geometry such that a time-dependent $\rho$ corresponds to a time-dependent background geometry and a constant $\rho$ corresponds to the flat spacetime.

4. Perturbative and nonperturbative numerical solutions

We will now solve Eqs. (22)–(23) for specific field-space configurations, both perturbatively, and numerically and nonperturbatively for both time-dependent and time-independent backgrounds.

4.1. Time-independent background

For a time-independent background, for which $\rho = 1$ and $\dot{\rho} = 0$, we can obtain full nonpertubative numerical solutions for $A(t)$ as seen in Fig. 2. We can also obtain a perturbative solution

$$A(t) \simeq E_I \sin \left[ (1 - (E_I k\mu)^2/16)kt \right]$$

$$- \frac{E_I^3 k^2 \mu^2}{16} \sin^2 \left[ (1 - (E_I k\mu)^2/16)kt \right] \cos \left[ (1 - (E_I k\mu)^2/16)kt \right]. \quad (24)$$

This solution can exhibits a frequency shift of order $\mu^2$, and a cubic correction term. The cubic term can also be rewritten, and thought of, as an introduction of higher harmonics using angle identities. In observations, the frequency shift may be more important to account for than the excited harmonics. This is because the frequency shift can manifest as a phase shift that has considerable time to develop as the wave traverses cosmological distances. In Fig. 2 we demonstrate this, comparing the perturbative solution to the exact and classical ones for the time-independent case.

We can also analyze the above perturbative solutions and obtain some insight into the speed of propagation of the waves. For that, we note that the dominant contributions to Eq. (24) can be written as

$$A(t) \simeq E_I \sin \left[ \left( 1 - \left( E_I k\mu/4 \right)^2 \right) kt \right]. \quad (25)$$

Comparing with the classical solution where we identify $ka^2 = \omega_c$, with $\omega_c$ being the classical angular speed, we notice that up to first order the polymer angular
speed is
\[ \omega_{\mu}^{(E)} \simeq \omega_c \left[ 1 - k^2 \left( \frac{E_\mu}{4} \right)^2 \right]. \quad (26) \]

Although this is a perturbative and approximate result and even though we have neglected higher harmonics in Eq. (24), the above equation reveal a curious phenomenon. Noting that \( \omega_c = ka^2 \) and with the group velocity being
\[ v = \frac{d\omega_{poly}}{d(ka^2)} \quad (27) \]
with \( \omega_{poly} \) being either \( \omega_{\mu}^{(A)} \) or \( \omega_{\mu}^{(E)} \), we obtain
\[ v_{\mu}^{(E)} \simeq 1 - k^2 \left( \frac{E_\mu}{4} \right)^2. \quad (28) \]

where \( v_{\mu}^{(E)} \) is the velocity of the effective waves. One can see from Eq. (28) that the group velocity of the waves is slower than the speed of light by a factor of \( k^2 \left( \frac{E_\mu}{4} \right)^2 \), and it also depends on the initial momentum \( E_I \) of the waves and the polymer parameter \( \mu \) due to the factor \( k^2 \left( \frac{E_\mu}{4} \right)^2 \). More importantly, the deviation from the speed of light also depends on the modes \( k \). Hence, waves with larger \( k \) (i.e., larger energies) have a lower speed compared to the ones with smaller \( k \) and are more affected by the quantum structure of spacetime. Also, notice that this case leads to the violation of Lorentz symmetry as can be seen by squaring both sides of Eq. (26). Of course, due to the sheer smallness of the expected value of \( \mu \), and the appearance of their squares in the above expressions, these effects are very small, but a highly energetic phenomenon with a large \( E_I \) may help to amplify it to an extent that future observatories can detect it. We should emphasize that the presence of the violation of the Lorentz symmetry in this case, as seen from the above results, is a consequence of the polymer quantization and, in particular, this model, and is not a direct consequence of LQG.

4.2. Time-independent background

For the case of a time-dependent background, we can obtain a solution in one of two ways: directly integrating the EOMs, or using the canonical transformation in Eqs. (13)–(16). In either case, we will need to obtain a solution for \( \rho \) by solving Eq. (20). In general, this choice determines whether the mode amplitude will be purely decaying or will contain oscillatory behavior. Here we will seek purely growing solutions for \( \rho \), choosing initial conditions such that oscillatory behavior is minimized; in our case, simply choosing \( \rho = 1 \) and \( \dot{\rho} = 0 \) is sufficient. Choosing a different initial amplitude for \( \rho \) is in any case equivalent to rescaling of the scale factor \( a \), polymer scale, momentum, and time coordinate. The full nonperturbative
solutions are plotted in Fig. 4. We can also obtain a perturbative solution

\[
A(t) \simeq \mathcal{E}_I \rho \sin \left[ (1 - (\mathcal{E}_I k \mu)^2/16)kT(t) \right] \\
- \frac{\mathcal{E}_I^2 k^2 \mu^2}{16} \rho \sin^2 \left[ (1 - (\mathcal{E}_I k \mu)^2/16)kT(t) \right] \cos \left[ (1 - (\mathcal{E}_I k \mu)^2/16)kT(t) \right],
\]

where

\[
T(t) = \int_{t_I}^{t} \frac{dt'}{\rho(t')^2}
\]

For GWs emitted at a time much greater than the characteristic wave time scale, i.e., \( t_I \gg k^{-1} \), where \( T_I \) is the initial time, and for nonoscillatory solutions of \( \rho \), the second-derivative term is small, and solutions to the auxiliary equations are well approximated by a simple power law, \( \rho = 1/a \). In Fig. 3 we show the behavior of \( \rho \) for several sets of initial conditions, and for a universe with a cosmological constant with \( w = -1, a \propto t^{1/3} \), and \( t_I = 10^3 \) (in units of \( k^{-1} \)). In subsequent plots we will use initial conditions that do not result in oscillatory behavior.

From the canonical transformation (13)–(15) (or, rather, its inverse), we see that the time-dependent waveform amplitude will pick up an overall factor of \( \rho \) relative to the time-independent one, the time coordinate will be altered, and the
momentum will be similarly rescaled but will also pick up an additional factor proportional to the wave amplitude. Due to the monotonically decreasing nature of $\rho$ and the smallness of its derivative, this additional factor will be a strongly subdominant contribution. In Fig. 4 we show the final solution for the field $A(t)$ for this time-dependent background. Somewhat counterintuitively, the frequency is seen to increase at later times; more commonly the frequency is considered to decrease (redshift) with cosmological expansion. This is due to the choice of harmonic slicing we have made, with $N = a^3$ instead of the more commonly used $N = 1$ (synchronous) or $N = a$ (comoving) time coordinate.

5. Discussion and Conclusion

In this work we have studied a certain effective form of GWs, considered as quantized perturbations propagating over a classical FLRW spacetime, in order to derive observational signatures to be compared with the results of experiments conducted by GW observatories. We have considered the Hamiltonian of classical gravitational perturbations, a time-dependent Hamiltonian, and have applied the techniques of polymer quantization to it. This polymer quantization was applied to each of the Fourier modes of the GW. A feature of this quantization is that the one-particle Hilbert space is modified and the Lorentz symmetry is no longer present [38]. This modification is “encoded” on the polymer scale $\mu$, which is usually considered to be very small (of the order of the Planck scale). However, our intuition in the present case is that the propagation of the GWs may capture some insights about these modifications despite the small values of the polymer scales.

After deriving a time-dependent effective polymer Hamiltonian using a novel approach, we derived both nonperturbative and perturbative analytical expression for the solutions and analyzed them to obtain further insight into the behavior of such waves. As a result, we found the following:
Fig. 4. Time evolution of $A$ (as in Fig. 2) for two different choices of $\mu$, for the case of a time-dependent background, i.e., $\rho(t)$ as described in the text. The axis is broken to show the behavior at a later time. Again, $\nu$ refers to another representation of the polymer quantization in which the momentum $E$ is discrete, which can be found in our original paper [31].

i) The form of the waves is modified. More precisely, there is a phase shift with respect to the classical case. Furthermore, small-amplitude harmonics are excited.

ii) The speed of the waves turns out to be smaller than the speed of light by a factor $k^2 \left( \frac{E_I \mu}{4} \right)^2$. This factor not only depends on the polymer scale $\mu$ and the initial momentum of the perturbations $E_I$, but also on the wave vector $k$ or, equivalently, the frequency of the waves. Thus, the higher-energy waves show a greater deviation from the classical behavior compared to the low-energy waves.

iii) The modifications to the classical behavior due to quantum effects become increasingly visible as the waves travel: the corrections result in an effective phase shift, which can become of order unity when $E_I \mu^2 k^3 D_s$ is of order unity for a distance $D_s$ traveled.

In a future work, we will proceed to apply our method to the case where both the background spacetime and the perturbations are effective.

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