MHD NATURAL CONVECTION BOUNDARY-LAYER FLOW
OVER A SEMI-INFINITE HEATED PLATE WITH
ARBITRARY INCLINATION

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Abstract. The aim of this article is to study the natural convection boundary-layer flow over a semi-infinite heated plate with arbitrary inclination. Existing solutions of similar models can be recovered as the limiting cases of horizontal and vertical plates from our generalized problem. Moreover, porous effects and the influence of transverse magnetic field; fixed to the fluid or the plate are accounted. The dimensionless velocity, in conjunction with the corresponding skin friction, have been presented as the sum of mechanical, thermal and concentration components. Furthermore, the contribution of the system parameters to the fluid motion in question has been depicted graphically. The novelty of the present study is to analyze the effect of angle of inclination of the plate and the case when the magnetic field is fixed relative to the fluid or to the plate on the fluid motion.

1. Introduction. For the past few years, the study of magnetohydrodynamic (MHD) natural convection flow of electrically conducting fluids, with heat and mass transfer, has gained special attention due to its multiple applications in the field of meteorology, electrical power generation, solar physics, geophysics, and chemical engineering. Furthermore, the study of such model for the case of a moving inclined plate has numerous useful applications. Hence, it has attracted the attention of several researchers. In 1990, Umemura and Law [23] noted that the flow characteristics of the natural convection boundary layer flows over a flat plate with an arbitrary inclination, also depends on the extent of inclination and the distance from the leading edge. Moreover, Palani [12] had studied the convection effects on the flow past an inclined plate with variable surface temperatures in water at 4°C. Regarding to viscous dissipative heat, Uddin and Kumar [22] examined the unstable free convection in a fluid flow past an infinite inclined plate, which is immersed in a porous medium. Additionally, they observed that, the value of the friction factor and the heat transfer coefficient decreases with the increasing angle of the plate from the vertical direction. Subsequently, this observation attracted the interest of chemical engineers. Furthermore, Singh and Makinde [19] investigated MHD free convection flow along an inclined plane with Newtonian heating in the presence of

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an exponentially decaying volumetric heat source. In addition, the effect of thermal radiation on an unsteady MHD flow past an inclined plate in the presence of a chemical reaction and viscous dissipation was investigated by Barik et al. [2].

\[\text{Nomenclature}\]

\begin{align*}
D & \quad \text{chemical molecular diffusivity} \\
C & \quad \text{dimensional concentration in the fluid} \\
C_{nw} & \quad \text{concentration of the fluid near the plate} \\
C_{\infty} & \quad \text{concentration of the fluid far away from the plate} \\
T & \quad \text{dimensionless temperature of the fluid} \\
T_{w} & \quad \text{constant temperature of the plate} \\
T_{\infty} & \quad \text{free stream temperature} \\
K & \quad \text{permeability of porous medium} \\
u & \quad \text{velocity of the fluid} \\
u_{0} & \quad \text{characteristic velocity of the plate} \\
S_{c} & \quad \text{Schmidt number} \\
M & \quad \text{ratio of the buoyancy forces} \\
cp & \quad \text{specific heat at constant pressure} \\
C & \quad \text{dimensionless fluid concentration} \\
G_{c} & \quad \text{mass Grashof number} \\
Gr & \quad \text{thermal Grashof number} \\
g & \quad \text{acceleration due to gravity} \\
Pr & \quad \text{Prandtl number} \\
R & \quad \text{Chemical reaction parameter} \\
r & \quad \text{radiative heat flux} \\
B & \quad \text{magnetic field strength} \\
Pr_{eff} & \quad \text{effective Prandtl number} \\
Nr & \quad \text{radiation conduction} \\
k_{r} & \quad \text{Rosseland mean attenuation coefficient} \\
\mu & \quad \text{coefficient of viscosity} \\
\nu & \quad \text{kinematic coefficient of viscosity} \\
\beta_{C} & \quad \text{volumetric coefficient of expansion with concentration} \\
\rho & \quad \text{density} \\
\sigma & \quad \text{electric conductivity} \\
\tau & \quad \text{skin friction} \\
\gamma & \quad \text{inclination angle from the vertical direction} \\
\beta_{T} & \quad \text{volumetric coefficient of thermal expansion} \\
k & \quad \text{thermal conductivity of the fluid} \\
\end{align*}

Chen [3] investigated the natural flow over a spongy inclined surface with a variable wall temperature and concentration. He observed that the effect of the buoyancy force decreases when the angle of inclination is increased. Vieru et al. [24] analytically solved the MHD natural convection flow along with mass diffusion and Newtonian heating when the plate applies an arbitrary time-dependent shear stress on the viscous fluid. Fetecau et al. [6] examined the slip effects on the radiative MHD free convection flow over a moving plate with a heat source and mass diffusion. Later, Fetecau et al. [5] discussed the MHD flow with a heat source, radiative effects, and shear stress on the boundary to obtain a general solution. On the other hand, all these reviews and many earlier published papers consider the scenario of magnetic lines of force being fixed relative to the fluid. In recent years, Narahari and Debnath [9] have considered both scenarios, i.e., when the magnetic lines of force are fixed relative to the fluid or the plate for an unsteady MHD free convection flow with a constant heat source and heat flux. These exact solutions are obtained for the fluid motions, since there exists an exponentially accelerated or constantly accelerating plate. The first exact solutions of this kind appear to be those obtained by Tokis [21]. His results are related to the motions produced by uniform, constantly accelerating or decaying oscillatory translations of the plate. Moreover, the mass transfer that is fundamental to several biological and chemical processes and, additionally, for the permeable effects of various engineering and geophysical applications has not been taken into consideration in the aforementioned papers. Furthermore, there are numerous interesting papers, such as those by Reddy [13], Abid et al. [1], Khan et al. [7], Sreekala et al. [20], Narahari et al. [11], Zhang et al. [25], and Seth et al. [16] in which exact solutions are obtained for the hydromagnetic free convection...
flows through permeable media with heat and mass transfer, but in a scenario where the magnetic field lines of force are fixed relative to the fluid.

In this article, our aim is to present a general investigation of MHD natural convection flow over a moving infinite inclined plate embedded in a permeable medium with constant concentration and chemical reaction. However, our motivation is to not only to generalize the previous results by considering concentration effects and mass transfer but also produce the new results regarding the general and oscillating motions of the inclined plate. It is noteworthy that the fluid velocity does not remain zero at infinity if the magnetic field is fixed to the plate [18]. Furthermore, in order to highlight the contribution of mechanical, thermal, and concentration on the fluid velocity, we will express the velocity as a sum of mechanical, thermal, and concentration components. In addition, their contribution concerning the fluid motion will be graphically illustrated and discussed with respect to slowly accelerating motions of the plate.

2. Statement of the problem. Consider the unsteady free convection flow of an electrically conducting incompressible viscous fluid over a non-conducting semi infinite inclined plate in the presence of a uniform transverse magnetic field of strength \( B_0 \) and our investigation comprises of the two cases namely when magnetic lines of force are fixed to the fluid or to the plate. Initially, at \( t = 0 \) the plate and the fluid are at rest at the constant temperature and constant species concentration. After the time \( t = 0 \), the plate begins to slide in its plane against the gravitational field with certain velocity \( u_0 \sin(wt) \) and its temperature \( T' \) is maintained at some fixed value \( T'_w \). Here, \( u_0 \) is a constant velocity. The plate is also maintained at a constant concentration \( C'_w \). Following Narahari and Debnath [9], we also assume that all physical properties are constant except the density variation with temperature in the body force and the induced magnetic field is negligible in comparison with the applied magnetic field. Furthermore, porous and radiative effects and the chemical reaction between the fluid and species concentration are taken into consideration while the viscous dissipation and Joule heating are neglected. With these conditions, by choosing a suitable cartesian coordinate system and using the usual Boussinesq approximation our problem reduces to the next set of partial differential equations Narahari and Debnath [9], Shah et al. [17]

\[
\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta_T(T' - T'_w)\cos\gamma + g\beta_C(C' - C'_w)\cos\gamma, \tag{1}
\]

\[
\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - R'(C' - C'_w); \quad y', t' > 0, \tag{2}
\]

with the initial and boundary conditions

\[
u'(y', 0) = 0, \quad T'(y', 0) = T'_w, \quad C'(y', 0) = C'_w; \quad y' \geq 0, \tag{3}
\]

\[
u'(0, t') = u_0 \sin(\omega t'), \quad T'(0, t') = T'_w, \quad C'(0, t') = C'_w; \quad t' > 0, \tag{4}
\]

\[
u'(y', t') < 0, \quad T'(y', t') \to T'_w, \quad C'(y', t') \to C'_w \quad \text{as} \quad y' \to \infty, \tag{5}
\]

where \( u'(y', t') \), \( T'(y', t') \) and \( C'(y', t') \) for the velocity, the temperature and the species concentration while \( \nu, g, \beta_T, \beta_C, K, \sigma, \rho, c_p, k, D, R \) and \( q_r \) are defined in the nomenclature. The parameter \( \ell = 0 \) when the magnetic field is fixed relative to the fluid (MFFRF) and \( 1 \) when the magnetic field is fixed relative to the
plate (MFFRP). Adapting the Rosseland diffusion approximation for an optically thick fluid (see Seth et al. [15] or Narahari and Dutta [10]),
\[
q_r = -\frac{4\sigma}{3kR} \frac{\partial T^4}{\partial y^s},
\]
assuming the difference between the fluid temperature and the free stream temperature to be small enough, the energy equation (2) can be rewritten in the form [4]
\[
Pr_{eff} \frac{\partial T(y', t)}{\partial t'} = \nu \frac{\partial^2 T(y', t)}{\partial y'^2}; \quad y', t' > 0,
\]
where \( Pr_{eff} = \frac{Pr}{1 + N_r} \) is the effective Prandtl number [8] and \( Pr = \frac{\nu c_p}{\kappa}, N_r = \frac{\sigma}{\kappa kR} T_\infty^3 \) are Prandtl number and the radiation-conduction parameter respectively.

Now, introducing the dimensionless variables, functions and parameters
\[
y = \frac{u_0}{\nu} y', t = \frac{u_0^2}{\nu} t', u = \frac{u'}{u_0}, T = \frac{T' - T_\infty}{T_w - T_\infty},
\]
and choosing the characteristic velocity \( u_0 \) to be equal with \( \sqrt{\nu g \beta_T (T_w - T_\infty)} \), our problem reduce to the following dimensionless partial differential equations
\[
\frac{\partial u(y, t)}{\partial t} = \frac{\partial^2 u(y, t)}{\partial y^2} + T(y, t) \cos \gamma + NC(y, t) \cos \gamma
\]
\[
- K u(y, t) - M \left( u(y, t) - \ell \sin t \right); \quad y, t > 0,
\]
\[
Pr_{eff} \frac{\partial T(y, t)}{\partial t} = \frac{\partial^2 T(y, t)}{\partial y^2} \frac{\partial C(y, t)}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C(y, t)}{\partial y^2} - RC(y, t); \quad y, t > 0
\]
with initial and boundary conditions
\[
u(y, 0) = 0, \quad T(y, 0) = 0, \quad C(y, 0) = 0; \quad y \geq 0,
\]
\[
u(0, t) = \nu_0 \sin t, \quad T(0, t) = 1, \quad C(0, t) = 1; \quad t > 0,
\]
\[
u(y, t) < \infty, \quad T(y, t) \to 0, \quad C(y, t) \to 0 \text{ as } y \to \infty.
\]
The ratio of the buoyancy forces \( N \), the magnetic parameter \( M \) and Schmidt number \( S_c \) are defined by
\[
N = \frac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}, \quad M = \frac{\sigma B_0^2}{\rho u_0^2}, \quad S_c = \frac{v}{D}.
\]
It is important to mention that \( Pr_{eff} \) and \( S_c \) are transport parameters regarding the thermal and mass diffusivity while \( N \) represents the relative contribution of the mass transport rate on the free convection flow Narahari and Dutta [10]. Since \( \beta_C \) can be positive (or negative) and \( \beta_T \geq 0 \) always non-negative. We have three possibilities for \( N \), either \( N < 0 \) or \( N > 0 \) or \( N = 0 \).

3. Solution of the problem. Solving Eqs. (10) and Eqs. (10) along with the initial condition (12) and Eqs. (12) boundary condition Eqs. (13), we get
\[
T(y, t) = \frac{y \sqrt{Pr_{eff}}}{2 \sqrt{\pi t}} \exp(-\frac{y^2 Pr_{eff}}{4t}), \quad C(y, t) = \psi(y \sqrt{S_c}, t, R, 0).
\]
Next, we find the fluid velocity. For this purpose, we use the Laplace transform technique. Now, we need the Laplace transforms of $T(y, t)$ and $C(y, t)$, i.e.,

$$
\bar{T}(y, q) = \exp^{-y\sqrt{q\text{Pr}_{eff}}}, \quad \bar{C}(y, q) = \frac{1}{q}\exp^{-y\sqrt{S_c(q+R)}},
$$

(15)

Here, $q$ is the transform parameter. Applying the Laplace transform to Eqs. (9) and using the corresponding initial conditions, we find

$$
q\bar{u}(y, q) = \frac{\partial^2 \bar{u}(y, q)}{\partial y^2} + \bar{T}(y, q)\cos \gamma + N\bar{C}(y, q)\cos \gamma - K\bar{u}(y, q)
$$

(16)

with the boundary conditions

$$
\bar{u}(0, q) = \frac{1}{1 + q^2}, \quad \bar{u}(y, q) < \infty \quad \text{as} \quad y \to \infty.
$$

(17)

Here, $\bar{u}(y, q)$ denote the Laplace transforms of $u(y, t)$. Substituting Eqs. (15)_1 and Eqs. (15)_2 into (16), we get

$$
\left( \frac{\partial^2}{\partial y^2} - (q + H) \right) \bar{u}(y, q) = -M\frac{1}{q^2 + 1} - \exp^{-y\sqrt{q\text{Pr}_{eff}}} \cos \gamma
$$

(18)

$$
- N\frac{1}{q}\exp^{-y\sqrt{S_c(q+R)}} \cos \gamma,
$$

where $H = M + K$. Solving the ordinary differential equation (18) with the boundary conditions (17), we get

$$
\bar{u}(y, q) = \frac{\exp^{-y\sqrt{q+H}}}{1 + q^2} + \frac{M\ell}{(q^2 + 1)(q + H)} \left( 1 - \exp^{-y\sqrt{q+H}} \right) + \frac{1}{1 - \text{Pr}_{eff}} \frac{1}{q + C}
$$

$$
\left( \exp^{-y\sqrt{q\text{Pr}_{eff}}} - \exp^{-y\sqrt{q+H}} \right) \cos \gamma + \frac{N}{(S_cR - H)} \left( \frac{1}{q} - \frac{1}{q + D} \right)
$$

(19)

Finally, applying the inverse Laplace transform and using the convolution theorem and Eqs. (A1) and (A2) from Appendix, we can present the velocity field as the sum of mechanical, thermal and concentration contributions as follows

$$
u(y, t) = u_m(y, t) + u_T(y, t) + u_C(y, t),
$$

(20)

where

$$
u_m(y, t) = \frac{\ell M}{2\sqrt{\pi}} \int_0^t \frac{\sin(t - s)}{s\sqrt{s}} \exp(-\frac{y^2}{4s} - Hs)ds + \int_0^t \sin(t - s) \exp^{-Hs} \text{erf}(\frac{y}{2\sqrt{s}})ds.
$$

(21)

Mechanical component of velocity,

$$
u_T(y, t) = \frac{1}{1 - \text{Pr}_{eff}} \left( \psi(y\sqrt{\text{Pr}_{eff}}, t; 0, -C) - \psi(y, t; H, -C) \right) \cos \gamma.
$$

(22)
Thermal component of velocity

\[ u_c(y, t) = \frac{N}{(H - S_c R)} \left( \psi(y \sqrt{S_c}; t; R, 0) - \psi(y, t; H, 0) \right) \]

\[ - \psi(y \sqrt{S_c}; t; R, -D) + \psi(y, t; H, -D) \right) \cos \gamma. \]

Concentration component of velocity.

It is not difficult to show that \( u(y, t) \), given by Eqs. (20)-(23), satisfies the imposed initial and boundary conditions. To verify the boundary condition (12), \( u_m(y, t) \) can be rewritten in an equivalent form as

\[ u_m(y, t) = \frac{y}{2 \sqrt{\pi}} \int_0^t \sin(t - s) \exp\left(-\frac{y^2}{4s} - Hs\right) \mathrm{d}s + \ell M \int_0^t \sin(t - s) \exp^{-Hs} \text{erf}\left(\frac{y}{2 \sqrt{s}}\right) \mathrm{d}s. \]

As regards the limit of velocity at infinity, it results that

\[ \lim_{y \to \infty} u(y, t) = \begin{cases} 0, & \text{if } \ell = 0 \\ M \int_0^t \sin(t - s) \exp^{-Hs} \text{erf}\left(\frac{y}{2 \sqrt{s}}\right) \mathrm{d}s. & \text{if } \ell = 1. \end{cases} \]

As evident for Eqs. (25), in the case when the magnetic field is fixed relative to the plate, the fluid does not remain at rest far away of the plate.

Next, to determine the skin friction or shear on the plate, i.e.

\[ \tau = -\frac{\partial u(y, t)}{\partial y} \bigg|_{y=0} . \]

As Previous, we find the skin friction in terms of three components.

\[ \tau = \tau_m + \tau_T + \tau_c, \]

where

\[ \tau_m = \frac{1}{2 \sqrt{\pi}} \int_0^t \sin(t - s) \exp^{-Hs} \mathrm{d}s - \ell M \int_0^t \sin(t - s) \exp^{-Hs} \sqrt{s} \mathrm{d}s. \]

Mechanical part of skin friction.

\[ \tau_T = \frac{1}{1 - Pr_{eff}} \left( \phi(t; H, -C) - \phi(t; 0, -C) \sqrt{Pr_{eff}} \right) \cos \gamma, \]

Thermal part of skin friction.

\[ \tau_c = \frac{N}{S_c R - H} \left[ \phi(t; H, 0) - \phi(t; H, -D) \right] \]

\[ - \sqrt{S_c} \left( \phi(t; R, 0) - \phi(t; R, -D) \right) \right) \cos \gamma, \]

Concentration part of skin friction.

Moreover, the function \( \phi(t; a, b) \) is defined in the Appendix. For \( k = 0 \) and \( \gamma = 0 \) into Eqs. (20) and (27) we recover the corresponding results of ([18], Eqs. (20) and (27)). As the concentration and thermal parts of velocity and skin friction are independent of \( sint \), so in the following section we will discuss the special cases regarding to the mechanical part of velocity and skin friction.
4. **Numerical results and discussion.** The natural-convection boundary-layer flow over a semi-infinite heated plate of arbitrary inclination is studied by first identifying a set of combined boundary-layer variables. Porous effects are taken into consideration and the transverse magnetic field is fixed to the fluid or to the plate. The dimensionless velocity, as well as the corresponding skin friction, is presented as sum of mechanical, thermal and concentration components. Moreover, the contribution of the system parameters to the fluid motion is graphically brought to light. The novelty of the present study is to analyze the effect of angle of inclination of the plate and the case when the magnetic field is fixed relative to the fluid or to the plate.

In Figs. 1 to 4, we plotted the concentration part of velocity versus $y$ for different values of the inclination of the plane $\gamma$, ratio of buoyant forces $N$, chemical reaction parameter $R$, and Schmidt number $Sc$. Moreover, Fig. 5 is prepared for comparison of concentration part of velocity for different values of dimensionless time $t$. We have noticed that for increasing values of $\gamma$ the inclination of the the plate with vertical wall, the concentration part of velocity decreases. For increasing values of $N$ ratio of buoyancy forces, concentration part of velocity increases. For increasing values of $R$ the chemical reaction parameter, absolute values of concentration part of velocity also increases. For increasing values of $Sc$, Schmidt number, the concentration part of velocity decreases. Finally, for fixed value of $y$ concentration part of velocity decreases with increasing values of $t$. Figs. 6 and 7 shows the effect of angle of inclination of the plate with the vertical wall and Prandtl number on the thermal part of velocity with respect to $y$. We have observed that the increase in the value of $\gamma$ decreases the absolute value of velocity and for different value of Prandtl Number absolute value of thermal part of velocity increases. The effects of porous medium parameter $k$ on mechanical part of velocity are shown in Fig. 8, it is observed that the absolute value of velocity decreases with increasing values of $k$. In the Fig. 9, we consider both the cases when magnetic lines of forces are fixed to plate, i.e., for $\ell = 1$ and for magnetic field is fixed relative to the fluid, i.e., for $\ell = 0$. Fig. 9 clearly shows that absolute value of velocity corresponding to $\ell = 1$ are appreciably increased as compared with the case for the same value of $k$. On the other hand, the effects of magnetic field parameter $M$ are in the Fig.10. It is observed that absolute value of velocity decreases with the increasing value of the magnetic field parameter $M$ for $\ell = 1$. In the Fig. 11, we have make comparison of magnetic field parameter $M$ for both the cases when $\ell = 1$ and $\ell = 0$. It is noticed that for the same value of $M$ absolute value of velocity is higher for $\ell = 1$ as compared to the case when $\ell = 0$. Fig. 12 shows the combine effects of velocity and its components.

For the same sum of velocity components such as concentration, mechanical and thermal velocity, the combined velocity $u(y, t)$ is greater than to sum of all of its components. Moreover, the velocity $u(y, t)$ is higher for $\ell = 1$ as compared to the case for $\ell = 0$.

**Appendix.**

\[
L^{-1}[\exp(-y\sqrt{q})] = \frac{y}{2t\sqrt{\pi t}} \exp\left(-\frac{y^2}{4t}\right), \quad L^{-1}\left[\frac{\exp(-y\sqrt{q})}{q}\right] = \text{erfc}\left(\frac{y}{2\sqrt{t}}\right), \\
L^{-1}\left[\frac{\exp(-y\sqrt{q} + a)}{q - b}\right] = \psi(y, t; a, b).
\]  

(A1)
ψ(y, t; a, b) = \frac{e^{bt}}{2} \left[ e^{-y\sqrt{a+1}} e^{-\frac{y}{2\sqrt{t}}} e^{-\sqrt{(a+b)t}} \right] + e^{y\sqrt{a+1}} e^{-\frac{y}{2\sqrt{t}}} e^{\sqrt{(a+b)t}} \right]. \quad (A2)

L^{-1}[qF(q)] = f'(t) + \delta(t)f(0),
\text{if } L^{-1}[F(q)] = f(t), \delta \text{ is the Dirac delta function.} \quad (A3)

L^{-1} \left[ \frac{1}{(q+b)\sqrt{q+a}} \right] = \frac{e^{-bt}}{\sqrt{a-b}} \text{erf}(\sqrt{(a-b)t}), \quad L^{-1} \left[ \frac{1}{\sqrt{q}} \right] = \frac{1}{\sqrt{\pi} t}. \quad (A4)

L^{-1} \left[ \frac{\sqrt{q+a} + a}{q + b} \right] = \frac{e^{-bt}}{\sqrt{\pi} t} + \frac{1}{\sqrt{a-b}} e^{-bt} \text{erf}(\sqrt{(a-b)t}) = \phi(t; a, b). \quad (A5)

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