Abstract

We examine the potential for gamma-ray conversion to electron-positron pairs, either in the field of a nucleus or of an electron of a detector, to measure the fraction \( P \) of linear polarization of cosmic gamma sources. For this purpose we implement, validate and use an event generator based on the HELAS amplitude calculator and on the SPRING event generator.

We characterize several ways to measure \( P \). Past proposals to increase the polarization sensitivity by the selection of a fraction of the events in a subset of the available phase space are found to be inefficient, due to the loss in statistics. The use of an optimal variable that includes the full 5D probability density function is found to improve the precision of the measurement of \( P \) of a factor of approximately 2.

We then study the dilution of the asymmetry that parametrize the degradation of the precision due to experimental effects such as multiple scattering. In a detector made with a succession of converter slabs and tracker foils, the dependence of the dilution is found to be different from that predicted assuming a given (the most probable) value of the pair opening angle. The limitations of a slab detector are avoided by the use of an active target, in which conversion and tracking are performed by the same device, in which case the dilution of the measurement of \( P \) is found to be manageable. Based on a realistic sizing of the detector, and for an effective exposure of 1 year, we estimate the precision for a Crab-like source on the full energy range to be approximately 1.4 %.

Key words: polarimetry, gamma rays, TPC, pair conversion, triplet, event generator

In section 1 we present a physics case for \( \gamma \) polarimetry in the MeV – GeV photon energy range. Section 2 is an introduction to the measurement technique, focused on pair conversion. In section 3 various aspects of the measurement are studied at generator level, that is in absence of any experimental effect. The generator is validated by comparison with published results based on analytical calculations. The distributions of several kinematic variables of interest, such as the opening angle and the transferred momentum, are studied. Some differences between nuclear and triplet conversion are pointed out. We examine several past proposals to improve the \( P \) sensitivity, after which we apply the technique of “optimal variables” to make use of all the information present in the 5D probability density function (pdf). We then study the effects of multiple scattering and of experimental cuts in section 4. Finally we describe the performance of a realistic detector, a 1 m³ 5 bar argon gas TPC.

1. A science case for \( \gamma \) Polarimetry

In many sources of gamma rays, the models proposed to explain the emission have very different polarization signatures. Depending on the orientation of the magnetic field at the source and on the primary emission mechanism, different degrees and directions of polarization are anticipated. \( \gamma \)-ray polarimetry can therefore be used to probe the nature and geometry of many objects including pulsars, binary systems, gamma-ray bursts and active galactic nuclei.

In rotation powered pulsars (RPPs), the pulsed, non-thermal radiation from relativistic particles in the magnetosphere of the neutron star is highly beamed along its magnetic field lines. This means that the emitted radiation should be highly polarized either parallel or perpendicular to the field lines. Although the different high-energy emission models share common emission mechanisms, these processes occur at different locations in the pulsar system depending on the geometry of the particular emission model. Thus, the polarization signatures of these models are expected to differ greatly from each other. Reference 2 shows the different polarization signatures expected at optical wavelengths. Similar degrees of polarization are expected in the keV - MeV energy range but with a different polarization angle. This energy-dependent rotation of the polarization direction could be used to locate the sites of emission and to probe the emitting particle population and their energetics [3].

Polarized high-energy emission has also been detected from the Crab pulsar wind nebula (PWN) between 200 keV and 800 keV and between 100 keV and 1 MeV. In both cases the polarization was aligned with the spin axis of the neutron star.

Other galactic systems are also predicted to emit polarized gamma rays. Many models for the X-ray emission from accretion powered pulsars (APPs) predict high degrees of linear polarization which varies both with pulse phase and with energy [3]. Using the calculations of Ref. 6, Ref. 7 discusses
how the polarization of the emission from these objects up to \( \approx 85 \text{ keV} \) can be used to decipher the long-standing problem of the geometry of the emitting regions and also to search for the signature of vacuum resonance (photon propagation through the birefringent strongly magnetized plasma leads to polarized emission, but for a critical value of the field, the birefringence of the vacuum cancels that of the plasma, leading to resonant emission, but for a critical value of the field, the birefringence of the geometry of the emitting regions and also to search for how the polarization of the emission from these objects up to high energies in the X-ray regime can be used to test strong gravity in galactic black hole binaries.

GRB fall broadly into two categories that are postulated to be those created by the explosion of a hypernova or as a result of the coalescence of two compact objects (e.g., neutron stars, white dwarfs, black holes). In the relativistic jets that are thus produced, synchrotron emission is thought to be the dominant emission process. Electrons are accelerated to near light speed by the relativistic shocks and, given the presence of a strong magnetic field, the degree of polarization of the photons emitted by these jets is expected to be very high. Polarization measurements of the emission should therefore provide information critical to distinguishing between the many emission models that exist for GRBs (see, for example, Refs. [10, 11, 12, 13]). Further Compton scattering of synchrotron photons on cold electrons is expected to decrease the polarization fraction and to rotate its angle by 90° at high energy, so polarimetry in energy bins is needed [13].

Another source class whose study should benefit from polarimetry studies is AGN and, in particular, the blazar subclass, which are strong emitters in the \( \gamma \) energy range. The models proposed to explain their emission fall into two broad categories, namely, hadronic and leptonic. One study of the polarization properties of relativistic jets [13] has shown that the observed degree of X-ray polarization should be sufficiently different for two of the most commonly postulated leptonic emission mechanisms, external-Compton (EC) and synchrotron-self-Compton (SSC), to determine which (if either) is the dominant process in blazars. Analytical calculations and simulations show that in contrast with EC emission, which is expected to be polarized at a level of \( \ll 1 \% \), SSC emission of a blazar can be polarized to more than 50% and that the degree of polarization rises above the MeV range, while it plateaus at X-rays energies [16]. If it can be shown that hadrons are accelerated to high energies in the jets of AGN, this source class would be a strong candidate for the accelerators of the high-energy cosmic rays, a long-standing mystery in astrophysics.

In the course of the efforts to build a quantized theory of gravitation, the possibility of Lorentz and CPT invariance violation has been considered. \( \gamma \) polarimetry turns out to be the most sensitive tool to test such an effect. In the framework of effective field theories, a birefringence effect of the vacuum is predicted [17]. The linear polarization direction would be rotated through an energy-dependent angle, due to different phase velocities for opposite helicities. This vacuum birefringence would rotate the polarization direction of monochromatic radiation, or could depolarize linearly polarized radiation composed of a spread of energies. The rotation angle is expressed as \( \theta \approx \xi E^2 t/(2M_P) \) where \( E, t, M_P \) are the photon energy, the propagation time, and the Planck mass, and \( \xi \) is a dimensionless Lorentz-violating effective field theory (EFT) parameter. The present limit is of \( |\xi| < 3.4 \times 10^{-16} \) [13] based on the observation of the polarization of the X-soft-\( \gamma \) emission of GRB 061122 in the 250 – 800 keV energy range with IBIS on INTEGRAL. Due to the squared dependence \( \theta \propto E^2 \), extending the polarization measurements to higher energies would lead to an improved sensitivity to a possible Lorentz invariance violation.

Last but not least, GRB polarimetry allows us to search for hints of the presence of the axion, the pseudo-scalar field associated with the \( U(1) \) symmetry devised to solve the QCD CP problem. The dichroism induced by the coupling of the propagating photon with the axion in the presence of the magnetic field generated by the GRB would lead to a rotation of the polarization direction which is here proportional to the photon energy: again, due to the width of the energy spectrum the polarization would be blurred. The actual observation of a non-zero polarization fraction would allow us to obtain an upper limit on the axion-to-two-photon coupling \( g_{\gamma \gamma} \), which is GRB-model dependent, but it is presently the best limit for an axion mass close to 1 meV [19]. As the limit is proportional to \( 1/\sqrt{E} \), extending the polarization measurement to higher energies would lead to an improved value, or even to a detection.

2. \( \gamma \)-ray polarimetry

The polarization fraction of a gamma-ray beam is measured by analyzing the distribution of the azimuthal angle, \( \phi \), of its conversion in a detector, which is given by the differential interaction rate

\[
\frac{d\Gamma}{d\phi} \propto (1 + A\cos(2(\phi - \phi_0))),
\]

where \( P \) is the fraction of the linear polarization of the photon beam and \( A \) is the polarization asymmetry of the conversion process. There are several ways to define the azimuthal angle, \( \phi \), as we shall see later. The angle origin, \( \phi_0 \), determines the orientation of polarization of the photon flux from a given cosmic source, with respect to a given fixed direction.

2.1. Compton scattering

In the case of Compton scattering, \( A \approx 2m/E \) at high photon energy, \( E \), so the sensitivity to polarization strongly decreases above a few MeV [20].

2.2. Pair conversion

A photon with energy above the pair creation threshold can convert to an \( e^+ e^- \) pair in the electric field of a charged particle of the detector. The process is described in the Born approximation by the diagrams shown in Fig. [1].

2.2.1. Nuclear pair conversion

“Nuclear” conversion, in the field of a nucleus, was first considered for polarimetry in 1950 [21]. This process dominates the cross-section at high energy where \( A \) goes to a constant. In that case, \( \phi \) is measured from the \( e^+ e^- \) pair.
It was soon realized, however, that multiple scattering of the tracks blurs the measurement of $\phi$, which induces a damping of the modulation in eq. (1). The asymmetry is reduced and an effective asymmetry $A_{\text{eff}}$ can be written as $A_{\text{eff}} = D \times A$, where $D$ is a dilution factor, $D = e^{-2\sigma_{x}}$ \cite{22, 23, 24}, and $\sigma_{x}$ is the $x$ angle resolution, which after the propagation over a length $L$ of material with radiation length $X_{0}$, is $\sigma_{x} \approx 14 \sqrt{L/X_{0}}$ \cite{22, 23}. A dilution of $D = 1/2$ is obtained for a resolution of $\sigma_{x} = \sigma_{c} \equiv \sqrt{\ln 2/2} \approx 0.59$ rad. This limitation is energy independent because, despite the fact that the electrons from high-energy photon conversions have larger momenta and therefore suffer less multiple scattering, the pair is emitted in the forward direction with a typical opening angle that decreases with increasing photon energy as $E^{-x}$, where $m$ denotes the electron mass. The direction of recoil of the nucleus also keeps track of the polarization of the photon, but the recoil momentum is of the order MeV/c and the path length of the nucleus is too short to be detected and therefore to enable a measurement of $\phi$.

Using this approach, a $\gamma$-ray polarimeter using nuclear conversion with a detector comprised of slabs would require a huge number of extremely thin converters, which was considered to be infeasible.

### 2.2.2. Triplet conversion

When the incoming photon converts in the field of an electron, $\gamma e^{-} \rightarrow e^{-} e^{+} e^{-}$, the recoiling electron is emitted at a large angle with respect to the photon direction, and therefore the measurement of $\phi$ is easier. Three tracks are observed in the final state, and thus the process is named “triplet” conversion \cite{25}. Since two electrons are present in the final state, four additional “exchange” diagrams are present, which are not shown in Fig. 1. Votruba first derived an expression, albeit a very tedious one, for the total cross-section taking into account all eight of the diagrams \cite{26}. Borsellino subsequently showed that diagrams (a) and (b) dominate the triplet cross-section at high energy and that the $\gamma - e$ diagrams and the exchange diagrams could therefore be neglected. Most of the subsequent works (e.g. Refs. \cite{27}, \cite{28}) studied the energy range above 50 MeV, which is unfortunate as for cosmics sources most of the signal is below that value.

#### 2.2.3. Sign of the asymmetry

It has been shown \cite{29} that in the case of nuclear conversion, the plane of emitted $e^{+} e^{-}$ pair correlates with the direction of the polarization of the photon in such a way that the pair is preferably emitted in the plane of polarization of the photon ($\mathcal{A} > 0$) \cite{30} while for triplet conversion the recoil electron is preferably emitted in the plane orthogonal to the direction of the photon polarization ($\mathcal{A} < 0$) \cite{30, 31}. The simulation developed here confirms the theoretical calculations discussed in Ref. \cite{29}.

Obviously the point here is not the charge of the recoiling particle, as changing its sign would be simply equivalent to a flip of the photon electric field (e.g. from $x$ to $-x$) that does not affect the polarization. Simply, an azimuthal angle determined from the plane of the pair yields a positive $\mathcal{A}$, while an azimuthal angle determined from an (approximately) normal to that plane (such as the pair momentum or the recoiling particle direction) yields a negative $\mathcal{A}$. Needless to say, exchanging by mistake the electron and the positron of the pair does not affect the measurement.

Asymmetries have been measured in two experiments, both with nuclear conversion and at high energy ($\approx$ GeV); strict acoplanarity was not required \cite{32, 33}. The sign of the asymmetry is not discussed explicitly in either of these works. Reference \cite{32} seems to obtain a positive $\mathcal{A}$, and so is Ref. \cite{33} (See their Fig. 6) when an offset of $\phi_{0} = \pi/2$ is taken into account. In the following we do not consider this issue explicitly, and assume implicitly a positive value for $\mathcal{A}$, for convenience.

#### 2.3. Energy range of interest

The probability of the conversion of a photon in a detector is determined by the interaction length $\Lambda = 1/(\rho H)$, where $H$ is the mass attenuation coefficient in the detector material and $\rho$ is the material density. The value of $H$ is tabulated by NIST \cite{34} as a function of photon energy. It rises rapidly above threshold, which is $2mc^{2}$ for nuclear conversion and $4mc^{2}$ for triplet. Most cosmic $\gamma$-ray sources have a power-law spectrum that decreases with energy, such that the flux, $F \propto E^{-\Gamma}$, where $\Gamma$ is the spectral index, which typically has a value of $\approx 2$. The energy range of interest is obtained by examining the variation with $E$ of the product $F \times H$ (Fig. 2). Most of the conversions and therefore most of the sensitivity to polarization clearly lie below 50 MeV.

The high-energy asymptotic differential cross-section, which is valid both for triplet and nuclear conversion (after $Z^{2}$ scaling) is \cite{30}:

$$
\frac{d\sigma}{d\phi} \propto \sigma_{t} \left[ \frac{28}{9} \ln 2\omega - \frac{218}{27} \right] - P \cos(2\phi - \phi_{0}) \left[ \frac{4}{9} \ln 2\omega - \frac{20}{27} \right],
$$

\footnote{Except for the events for which the pair is almost coplanar to the photon direction \cite{29}, for which the asymmetry changes sign, which is confirmed by our simulation.}

\footnote{Bogdan Wojtsekhowski, private communication, May 2013.}

Figure 1: Feynman diagrams describing photon conversion in the Born approximation; (a) and (b) are called Borsellino diagrams, and (c) and (d) are called $\gamma - e$ diagrams. Particles are noted + (positron), − (electron) and r (recoil), and labeled by their momentum, $p$. 

\[\]
with $\omega \equiv E/m$. The fine-structure constant is denoted $\alpha$ and the classical radius of the electron, $r_0$. This expression leads to an asymptotic value for $A$ of $1/7 \approx 14\%$ and undergoes an unphysical change of sign for $\ln 2\omega = 5/3$, that is for $E = 1.35\text{ MeV}$. Corrections of order $m/E$ to $A$ have been computed in Ref. [35], but the same behavior remains, with $A$ decreasing and changing sign at low energy.

3. The event generator

This work builds on the previous opus by Endo and Kobayashi[29]. In Ref. [28] the differential cross-section for triplet conversion was computed exactly, including the eight diagrams, within the Born approximation and without screening. After the amplitudes were computed using the HELAS software [36], the differential cross-section was integrated using the BASES integrator[37]. Triplet production was studied in the event generator SPRING[38]. We extend the study to nuclear conversion, and we compare the HELAS computation to the Bethe-Heitler (BH) differential cross section [39], an analytical expression based on the two Borsellino or BH diagrams. In both cases, the energy that is carried away by the recoiling particle is also taken into account. The screening of the electric field of the recoiling particle is also taken into account.

The incoming photon is assumed to have a momentum $\vec{k}$ parallel to $z$. We denote $\vec{p}$, $E$, $\theta$, $\phi$ the momentum, energy, polar and azimuthal angle of the outgoing particles, with subscripts $+$, $-$, and $r$ for the positron, electron, and recoiling particle, respectively (Fig. 5). The fraction of the photon energy carried away by a lepton is denoted $x$, e.g. $x_+ \equiv E_+/E$, and $\vec{q} \equiv \vec{k} - \vec{p}_+ - \vec{p}_-$. is the momentum transferred to the recoiling particle.

With three particles in the final state, and taking energy-momentum conservation into account, the final state is described by five variables. These can be the azimuthal and polar angles describing the two track of the pair plus the already mentioned energy fraction that is $\phi_+$, $\theta_+$, $\phi_-$, $\theta_-$ and $x$, all in the laboratory frame, as in the expression of the differential rate established by Bethe and Heitler [39]. Instead, as in Ref. [28], we first generate the azimuthal and polar angles of the recoiling particle and of one of the track, the positron, all in the center-of-mass frame, and the invariant mass of the pair.

3.1. Validation

As the 5D differential cross-section is rather involved, with strongly peaked variables such as the polar angle of the electrons of the pair, we validate the behavior of the generator against known properties of the pdf available in the literature.

3.1.1. cross-section

We first compare the triplet cross-section obtained by the integrator of the event simulation to the computation by Mork [40], as was done in Ref. [28] (no screening). This is shown in Fig. 28 (Supplementary data).

In practice the charged particle on which the photon converts is embedded into an atom of the detector. When the differential cross-section is computed in the Born approximation, the screening of the electric field of that particle by the (other, in the case of triplet conversion) electrons of the atom can be described by a form factor. In the case of nuclear conversion, the recoiling nucleus is slow enough that the collision as regarded by the atom can be considered as elastic, while in the case of triplet conversion the electron is ejected and the atom, ionized.

Screening in nuclear conversion is taken into account by multiplying the differential cross-section by $[1 - F(q)]^2$ where $F(q)$ is the Mott atomic form factor, $F(q) = 1/(1 + (111(q/mc)^2Z^{-1/3})^2)$. We note that screening affects the $q$ spectrum below a typical value $q_Z = mcZ^2/111 \approx 12\text{ keV/c}$ for argon. The $q$ distribution is narrow and centered around $1\text{ MeV/c}$ at low energy. It becomes extended at high energy (see Fig. 19) with asymptotically a lower kinematic limit of $q_m = 2m^2/E$ (e.g. [25]). We see that screening affects the lower part of the $q$ spectrum for photon energies larger than $\approx 2m^2/q_Z$ (43 MeV for argon). For triplet conversion, screening is taken
into account by multiplying the differential cross-section by the incoherent scattering factor, $S(\nu)$, where $\nu \propto q^{2}\gamma \frac{\gamma}{\gamma-1}$. At low $q$ ($\nu < 0.01$) we use $S(\nu) = 1.38 \sqrt{\nu}$ [42]. At a higher value, $1 > \nu > 0.01$, we use $S(\nu) = \sqrt{1 - (1 - \nu)^{2}}$, which we found to be a good representation of the data tabulated in table I of Ref. [52]. For $\nu > 1$ we use $S(\nu) = 1$.

The mass attenuation coefficients computed with the simple Anz"{a}tze used here are compared to the full computation from the NIST server [34] in Fig. 29 (Supplementary data, left). A more precise comparison is presented in Fig. 29 (Supplementary data, right), where the ratio of the cross-sections with/without screening is compared to the values tabulated in Table 1 (nuclear) and Table 5 (triplet) of Ref. [43], for aluminium. The fair agreement meets the needs of the present study.

At low energy, just above threshold, the interference between the diagrams in triplet conversion leads to a decrease of the cross-section, from 1.4 to 1.11 $\mu$b at $\omega = 4.4$ ($E \approx 2.25$ MeV) that is compatible with the analytical computation by Mork (1.1 $\mu$b [40]). The systematic comparison of the distributions of a series of kinematic variables, $q, \theta_{\perp}, \theta_{\parallel}, x_{i} \equiv E_{i}/E$ for the BH and H generators do not show any difference within statistical fluctuations for nuclear conversion (plots not shown).

3.1.2. Opening angle

The distribution of the opening angle $\theta_{\perp}$ of the pair was computed by Olsen from a high energy approximation of the differential cross-section, with a most probable value, $\hat{\theta}_{\perp}$, that decreases with photon energy as $E_{0}/E$ with $E_{0} \approx 1.6$ MeV [44]. Experimentally, some differences between the distribution of nuclear and triplet conversions have been noted [45]. We fit the peak of the $\theta_{\perp}$ distribution with a third degree polynomial, we compute $\hat{\theta}_{\perp}$ and we present the variation of $\hat{\theta}_{\perp} \times E$ as a function of $E$ in Fig. 4 (left).

- For nuclear conversion, the values for the Bethe-Heitler (BH) and the full (H) amplitude show compatible results.
- When no screening is applied, triplet and nuclear conversion both show a value compatible with that of Ref. [44] at high energy.
- When screening, which is active only at high energy, is taken into account, $\hat{\theta}_{\perp}$ increases and increases with energy.
- For triplet conversion, $\hat{\theta}_{\perp}$ decreases strongly at low energy, which has been observed experimentally [46], with a reduction of almost a factor of two just above threshold.

Since there are two negative electrons in the final state for triplet conversion, we choose by convention that the one with the smallest polar angle ($\theta$) wrt the direction of the incoming photon in the laboratory frame be the member of the pair. The other one is considered to be the recoiling particle. Neglecting the exchange diagrams for triplet conversion, or even restricting the amplitude to the Borsellino diagrams, does not affect the value of $\hat{\theta}_{\perp}$ (Fig. 4 right).

3.1.3. Recoil momentum distribution

The ratio of the distributions of the recoil momentum for triplet conversion to nuclear conversion is shown in Fig. 5. When screening is not taken into account, a pattern similar to the analytical computation by Mork [40] is observed (Fig. 5 left). The difference between the effect of screening for triplet and nuclear conversion is visible at low $q$ (Fig. 5 right).

In a given experiment, the analysis of triplet conversion events can be performed only for recoil momentum $q$ larger than some threshold $q_{0}$. The magnitude of the cross-section for these events, $\sigma(q > q_{0})$ is obviously a concern. In the high-energy approximation, it was shown that $\sigma(q > q_{0})$ is energy independent [51,47]. The way that the full calculation performed here tends to the high-energy approximation [47] is shown in Fig. 6.

4. Precision of the measurement of $P$

When the measurement of the polarization is performed with a fit of the $\phi$ distribution with the pdf of eq. (1), the RMS resolution of the measurement of $\mathcal{AP}$ is given by $\sigma_{\mathcal{AP}} \approx \sqrt{\frac{2}{N}}$, that is:

$$\sigma_{\mathcal{P}} \approx \frac{1}{\mathcal{A}} \sqrt{\frac{2}{N}}, \quad (3)$$

where $N$ is the number of events in the sample.

The value of $\mathcal{AP}$ can also be computed from the moments of appropriate weight $w_{i}$, for event $i$, $i = 1 \cdots N$. In general, the

\[3\text{With this approach, the direction of the polarization of the source, that is}\]
Figure 6: Variation of the triplet cross-section $\sigma(q > q_0)$ above a recoil momentum threshold, $q_0$, as a function of $q_0$ in units of $mc$, compared with the high-energy asymptotic expression of EPJC (2011) 71,1778 (thick line). With (dashed line) and without (solid line) form factor applied.

Figure 7: Distribution of $2\cos(2\phi)$ for the triplet conversion of 40 MeV photons.

The expectation value $E(w)$ of $w$ is a function of the parameter(s) of the distribution (here of $P$). We can then obtain an estimator of the parameter(s) from the measured mean value $\langle w \rangle$. The expression for $E(w)$ reads:

$$E(w) = \int w(\phi) \frac{dT}{d\phi} d\phi$$

For $w = 2 \cos 2\phi$ (Fig. 7), we obtain $E(w) = A_P$. The uncertainty is obtained from the expression for the variance of $w$

$$\sigma_P = \frac{1}{A \sqrt{N}} \sigma_w,$$

with the RMS $\sigma_w$ of $w$, $\sigma_w = \sqrt{E(w^2) - E(w)^2}$. Here, $E(w^2) = 2$, so that

$$\sigma_P = \frac{1}{A \sqrt{N}} \sqrt{2 - (A_P)^2},$$

which is smaller than the value obtained by the fit (eq. (5)) and that tends to it in the approximation of a small asymmetry and/or polarization. The variation of $A$ obtained from

the angle $\phi_0$ (see eq. 1), can be measured by a combined analysis of weights $2\cos 2\phi$ and $2\sin 2\phi$.

E(2 cos(2$\phi$)) with photon energy is shown in Fig. 8 (left). In the above, the angle $\phi$ can be either the azimuthal angle of the recoiling particle, or of the pair, that are back-to-back. In the case of nuclear conversion though, since the recoil of the nucleus goes undetected, $\phi$ cannot be measured directly[4]. The azimuthal angle of one of the tracks, e.g. that of the positron $\phi_+$, may be used, in which case the effective polarization asymmetry decreases [48, 49]. The decrease is extremely strong at low energy (Fig. 9 right). The angle that can be measured is the azimuthal angle that connects the positron to the electron, $\omega_{+-}$ (Fig. 9), as the direction of the incoming photon is not known, in general. The use of the angle $\omega_{+-}$ provides a partial recovery of the sensitivity (49) and Fig. 9 right).

The variation of $A$, obtained from $\phi$, $\phi_+$ and $\omega_{+-}$, with photon energy for nuclear conversion (no screening).

4.1. Attempts to increase the polarization asymmetry

There have been many attempts to make use of the variation of the polarization asymmetry, $A$, with the kinetic variables that describe an event, so as to increase the effective value of $A$ by a judicious event selection. The asymmetry was found to be larger when the two electrons of the pair share the energy equally [51, 50], it varies with the (azimuthal) acoplanarity, $\phi_+ - \phi_-$, between the two electrons of the pair [29], and it is larger for small pair opening angles [28]. This has lead to proposals of strategies that augment the mean value of $A$ by an event selection.

Unless the source would be alone in the sky, or some extra information is used to determine the true direction of the incoming photon, such as time windowing for a GRB, or such as phase information for a pulsar.
Such a kinetic variable is generically noted \( \chi \) in the following. We find the polarization asymmetry \( \mathcal{A} \) to be larger at low (logarithm of the) recoil momentum \( \log_{10}(q/\text{MeV}/c) \), at low opening angle \( \theta_{-} \) (here rescaled by the photon energy) and when the energy is balanced between the tracks of the pair \( (x_{s} \approx 0.5) \) (Fig. 11).

Selecting events so as to increase the value of \( \mathcal{A} \) for the selected sample has a cost in terms of statistics and therefore in terms of analyzing power. The typical range of interest is where the figure of merit \( F = \frac{\mathcal{A}(\chi)}{\sigma_{\chi}(\chi)} \) is maximum (e.g. [28]). The variation of \( F(\chi) \) for the three variables \( q, \theta_{-} \), and \( x_{s} \) is shown in Fig. 12.

We study such cuts here, making an event selection taking as an example an upper side cut \( \chi < \chi_{c} \) as is the case for variables \( q \) and \( \theta_{-} \). Optimizing such cuts involves minimizing the precision of the measurement of \( P \), eq. (5), in which the factors \( \mathcal{A}, N \) and \( \sigma_{\chi} \) are estimated with the cut applied. In practice we use here its inverse, normalized to an initial total sample of one event, and we note this new figure of merit \( G \):

\[
G(\chi_{c}) = \frac{\mathcal{A}(\chi_{c})}{\sigma_{\chi}(\chi_{c})} \sqrt{\frac{N(\chi_{c})}{N}}.
\]

The variation of the figure of merit \( G \) with \( \chi_{c} \) (Fig. 13) shows that this selection strategy brings very little, if any, improvement. For example, for triplet conversion at photon energy of 40 MeV, a \( \theta_{-} \times E < 2 \text{ rad MeV} \) cut would result in an increase of the effective asymmetry from 21.3 to 40.6 % at the cost of a cut efficiency of 38%, with a small \( G \) improvement from 0.153 to 0.185.

4.2. The use of an optimal variable

A way to improve the polarization sensitivity, beyond the simple cuts used above, is to make an optimal use of the information contained in the multi-dimensional probability density function (pdf) of the signal, through the use of an optimal variable (see e.g. [52][53][54][55]). We note \( \Omega \), the set of kinematic variables that fully describes an event, \( \rho(\Omega) \) its pdf, that depends on the parameter (here \( P \)) that we want to measure.

We are looking for a weight \( w(\Omega) \), so that the \( P \) dependence of \( E(w) \) allows a measurement of \( P \), and that the variance of such a measurement is minimal. The solution is

\[
w_{\text{opt}} = \frac{\partial \ln \rho(\Omega)}{\partial P}.
\]

[28] is due to their further selecting kinematic parameters according to a dedicated experiment — in particular the opening angle peaks at a larger value of \( \gamma_{m}/E \) in Ref. [28].
In the particular case of a polarization measurement
\[ p(\Omega) = f(\Omega) + P \times g(\Omega), \]  
with \( \int f(\Omega) d\Omega = 1 \) and \( \int g(\Omega) d\Omega = 0 \), we obtain
\[ w_{\text{opt}} = \frac{g(\Omega)}{f(\Omega) + P \times g(\Omega)}. \]

In this study, we rescale the weight by a factor of 2, so as to provide an asymmetry that is commensurate with \( \mathcal{A} \). Also, in the approximation that the polarization is small \(^{[52]}\), we can neglect the \( P \times g(\Omega) \) term in the denominator, and obtain
\[ w_0 = 2 \frac{g(\Omega)}{f(\Omega)}. \]

In the case of the reduced equation (eq. \([1]\)) we find:
\[ w_1 = 2 \cos 2\phi, \]  
(a weight that therefore makes optimal use of the reduced information present in the 1D distribution.

We can now estimate the loss of information when using the 1D reduced distribution by comparing the performance of the 1D weight \( w_1 \) with that of the 5D weights \( w_0 \). In the case of

the Bethe-Heitler (BH) pdf, \( f(\Omega) \) and \( g(\Omega) \) are simply obtained from eq. \([9]\). In the case of the HELAS computation of the pdf (H), the unpolarized part \( f(\Omega) \) is obtained as the average of an \( x \)-polarized and \( y \)-polarized differential cross-section. Both calculations of the nuclear 5D weights yield results that agree perfectly with each other (Fig. \([14]\), while a slight discrepancy between BH and H is visible at low energy for triplet conversion. We can see that at a given photon energy, the three weights provide similar values of the asymmetry, while the uncertainty is improved by a factor larger than 2 for a 5D optimal weight, resulting in an increase of \( G \) by the same factor. If it were possible to measure the kinematic variables that describe the final state of an event with sufficient precision, using a 5D optimal weight would therefore improve the precision of the measurement of \( P \) by up to a factor of about 2.

We note that \( E(w_0) \) goes to an asymptote at high \( q \) and \( \theta_{-+} \) (Fig. \([15]\)), in contrast with \( E(w_1) \) for which it keeps on decreasing to zero (Fig. \([17]\).

After having characterized various ways to measure \( P \) at generator level, we now consider the consequences of the experimental effects on the measurement of \( P \).

5. Experimental effects

5.1. Nuclear conversion

We first re-examine the configuration that has been studied in the literature, i.e. a detector composed of a series of converter slabs in which photons convert, interleaved with tracking detectors in which electrons are tracked. We then turn to an active target, i.e. a single homogeneous detector which at the same time converts photons and tracks electrons. For this study of experimental effects in nuclear conversion, the 1D weight is computed with angle \( \omega_{-+} \), which is the quantity that is available in practice.
5.1.1. Slab converter

For the first case (a slab detector), tracking is performed after the tracks exit the converter. The angular resolution for these tracks is therefore at least equal to the average deflection angle over their path length \( x \) inside the slab\footnote{We consider here that the tracks traverse the same full thickness \( x \), i.e. we don’t consider the fluctuation of the conversion point.}. The order of magnitude of the effect on the measurement of \( \phi \) is first estimated following Refs. [22, 23], updating the numerical values they used to parametrize the multiple scattering. In the small angle approximation, the \( \phi \) angle resolution can be expressed as

\[
\sigma_\phi = \frac{\theta_{0\phi} \oplus \theta_{0\phi}}{\theta_{\phi-}},
\]  

where \( \theta_{0\phi} \) and \( \theta_{0\phi} \) are the RMS width of the projected deflection distribution of the positron and of the electron, respectively, and \( \theta_{\phi-} \) is the opening angle of the pair. \( \theta_0 \) is obtained from the Gaussian approximation of multiple scattering [50]:

\[
\theta_0 = \frac{p_0}{B_0} \sqrt{\frac{x}{X_0}},
\]

where \( p_0 = 13.6 \text{ MeV}/c \) and the small logarithmic correcting factor has been neglected.

Analytical approximate expression, using the most probable opening angle. We first use the approximation made in Refs. [22, 23] of an opening angle \( \theta_{\phi-} \) equal to its most probable value \( \theta_{\phi-} = E_0/E \). At equipartition, \( p_\phi \approx p_\perp \), we obtain:

\[
\sigma_\phi \approx \frac{2 \sqrt{2} p_0}{E_0} \sqrt{\frac{x}{X_0}} \approx \sigma_0 \sqrt{\frac{x}{X_0}},
\]

where \( \sigma_0 \equiv 2 \sqrt{2} p_0/E_0 \approx 24 \text{ rad} \). We note that the expression for \( \sigma_\phi \) is independent of \( E \): at higher energy the track suffers less multiple scattering but the pair opening angle gets smaller. Taking the small logarithmic correcting factor into account in the expression for \( \theta_0 \) for that particular value of the thickness leads to a 28% decrease of \( \sigma_0 \) (to about 17 rad) and of \( \sigma_\phi \). The critical thickness \( x_c \), which induces a dilution of the polarization asymmetry of a factor of 2 is therefore such that the normalized thickness \( t_c \equiv x_c/X_0 \equiv (\sigma_\phi/\sigma_\perp)^2 \approx 1.2 \times 10^{-3} \), which amounts, for example, to 110 \( \mu \text{m} \) of silicon.

Simulation. We now use the full 5D simulation. We first cross-validate the above analytical results and the simulation, parametrizing the multiple scattering with the approximation of eq. (14), and selecting events with a pair opening angle close to the maximum, \(|\theta_{\phi-} \times E - E_0| < 0.2 \text{ MeV} \), and energy sharing close to equipartition, \( 0.8 < E_\perp/E < 1.2 \): we obtain an RMS width for \( \sigma_{\phi-} \) of 0.581 \( \pm \) 0.005 rad, which is compatible with that of \( \sigma_\phi \). Releasing the selection produces a more peaked \( \omega_{\phi-} \) residual distribution, with a similar RMS, but that is smaller by about 8%. Using the full expression for the multiple scattering [56] further decreases the RMS. Finally an RMS equal to \( \sigma_\phi \) is obtained for a normalized thickness of \( t_c \approx 1.7 \times 10^{-3} \), that is for

\[
\sigma_\phi = \sigma_c / \sqrt{E} \approx 14.6 \text{ rad}.
\]

The dilution of the polarization asymmetry so induced is visible in Fig. [16] center and right. Since the distribution of the azimuthal angle residual is non Gaussian in the full simulation (Fig. [16] left), the dilution factor can be different from \( 1/2 \) for an RMS equal to \( \sigma_c \).

The variation of the dilution factor, \( D \), with the normalized thickness, \( t \), is shown in Fig. [17] The dilution factor is smaller for the 5D weight than for the 1D weight, which indicates that the track angular resolution affects not only the azimuthal angle but also the detail of the 5D differential cross-section. Note that in the case of the 5D weight, the dilution is also affected by effects that are not taken into account in the present study, such as the resolution of the measurement of the energy-momentum of each track. Also the single-track angular resolution and the non-observation of the recoil in the case of nuclear conversion induce an uncertainty in the definition of the \( z \) axis. The simpler estimator based on the weight computed from angle \( \omega_{\phi-} \) is immune from these effects, and therefore we will use the 1D weight as a conservative benchmark of the precision on \( P \).

The variation of \( D(t) \) obtained with the simulation, which includes the full \( \theta_{\phi-} \) distribution, turns out to be very different from the expression \( D(t) = e^{-2t/\lambda_{\phi-}} \) based on the particular value \( \theta_{\phi-} \) of \( \theta_{\phi-} \).

One might be tempted to select events with large opening angle \( \theta_{\phi-} \) in the hope of getting a better dilution, but for \( E(w_0) \), the asymmetry is not improved, and we merely get a loss in statistics. For \( E(w_1) \) it is even worse (plots not shown), as the asymmetry itself decreases at high \( \theta_{\phi-} \) (Fig. [11]).

5.1.2. Thin detectors

In a thin detector, the optimal, minimal value of the single-track angular resolution in the low-momentum, multiple-scattering-dominated regime is given by [57]:

\[
\sigma_{\phi 1 L} \approx (2\sigma)^{1/4}l^{1/8}X_0^{-3/8}(p/p_\perp)^{-3/4} = (p/p_\perp)^{-3/4},
\]

with:

\[
p_1 = p_0 \left( \frac{4\sigma^2 l}{X_0^2} \right)^{1/6},
\]

Figure 17: Variation of the dilution factor \( D \) with the slab normalized thickness \( t \), the polarized fraction \( P \) being estimated with the expectation value of either the 1D weight or of the 5D weight, for photon energies of 10 and 100 MeV (nuclear conversion). The thin line shows the approximation using the most probable opening angle (\( \theta_0 = 14 \text{ rad} \)).
The azimuthal angle of the pair is:

\[ \theta \sim \frac{1}{E L} - \frac{1}{E p} \]

Again, we first consider the most probable opening angle. Analytical approximate expression, using the most probable for optimal fits in the presence of multiple scattering [58]. Such Equations (16), (17) were obtained [57] from the expressions for argon, \( \sigma \).

The variation of \( E_L \) with the detector density, \( \rho \), is shown in Fig. 19. In a dense TPC such as a liquid or solid [7] polarimetry using nuclear conversion is hopeless, since the value is smaller than 1 MeV. But, in contrast with conventional belief, polarimetry using nuclear conversion should be possible in the energy range of interest with a high-precision gas TPC using an optimal track fitting.

**Simulation.** We use the simulation to determine the value of the dilution, \( D \), as a function of photon energy and of the detector parameter \( p_1 \). Again, the dilution is stronger (\( D \) smaller) when \( E \) is measured with the 5D weight (Fig. 20, right) than for the 1D weight (Fig. 20, left). The data are well fitted by a function

\[ D(E, p_1) = \exp[-2(a p_1^b E)^c], \]

with coefficients \( a, b \) and \( c \) given in Table [1].

Liquid neon does not allow electrons to drift, but solid neon does [63].
5.1.3. Precision of the measurement of $P$

In the general case, the precision of the measurement of $P$ is given by eq. (5). The number of events is

$$N = \int T \eta \epsilon A_{\text{eff}}(E) \frac{dN}{dE} dE$$

(21)

where $A_{\text{eff}}(E)$ is the effective area, $T$ and $\epsilon$ the exposure duration and the efficiency. The exposure fraction, $\eta$, is the “equivalent on-axis exposure duration” divided by the effective exposure duration $T_{\text{FM}}$. For the Fermi LAT, for example, with an equivalent on-axis exposure duration of $3.2 \pm 1.0$ Ms over an effective exposure duration of 29 Ms, $\eta = 0.11 \pm 0.03$, depending on the source location and on the telescope configuration (see Fig. 3 of Ref. [62]). In the case of a stand-alone detector such as a TPC, the exposure fraction is expected to be larger than that of a two-component telescope like the LAT (a tracker and a calorimeter), and therefore we can assume that $0.5 > \eta > 0.1$ for steady sources. The gamma-ray flux for a Crab-like source in the MeV-GeV energy range is

$$N \approx \frac{F}{E^2}$$

(22)

with $F = F_0 = 10^{-3}$ MeV/(cm$^2$s) [60].

- We obtain a naive estimate of the precision from eq. (5), with $N$ from eq. (21) and $A$ equal to a constant value of 0.25. The results are displayed in Table 2 as $N$ and $\sigma_{p0}$. As expected, the number of conversion events increases with $Z$ for nuclear conversion for a given detector mass, while the number of triplet conversions is nearly independent of $Z$, the number of electrons per unit detector mass being (almost) a constant of Nature.

For other configurations, the precision is readily obtained from $\sigma_{p0}$ by

$$\sigma_P = \sigma_{p0} \sqrt{\frac{T_0 F_0 M_0}{\eta \epsilon TF M}}$$

(23)

- A more precise estimate of the asymmetry and of the precision is obtained from the mean value and the variance of the weights over the full spectrum. This calculation involves the average value and RMS of the weight at a given photon energy $E$, taken from fits of the data presented in Fig. 14 with a heuristic function $f(E) = \alpha + \beta/E$, which are presented in Fig. 21. The results are displayed in Table 2 ($A$ 1D, $A$ 5D, $\sigma_P$ 1D, $\sigma_P$ 5D).

5.1.4. Dilution

In practice, for operation in space, the overall size of the detector might be a more limiting factor than the detector mass, and one might want to increase the gas pressure to increase the sensitive mass. Several factors limit high-pressure operation; micro-pattern gas detectors show an operational gain that decreases at high density; even though the mass of the container is a pressure-independent fraction of the mass of the contained gas, building a low-mass high-pressure container for a large volume detector will doubtlessly be an issue. Here we examine the relation between detector density and polarization dilution. For a given sensitive volume, $V = 1$ m$^3$, we compute the number and spectrum of converted photons (Fig. 2). The dilution factor is taken from eq. (20), in which $p_1$ depends on the pressure $P$ through $X_0$ (see eq. (17)).

The average asymmetry and the precision for the whole spectrum are plotted as a function of detector density, normalized to that of gas at 1 bar, in Fig. 22. For the density range available to gas detectors, the increase in statistics overrides the degradation of the dilution, and the performance of the detector improves with pressure. It’s only at an even larger density, close to that of the dense (liquid or solid) phase, that the two effects compensate.

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\*36% for a spherical titanium container of argon gas, at the limit of elasticity.
Performing polarimetry with a liquid TPC is most likely extremely difficult in practice. Not only does the sensitivity rely on the events with a large opening angle (Fig. [19]) but also on the lowest part of the photon spectrum (Fig. [20]). At these energies, triggering in the presence of background may be difficult and extracting the signal of a given source from the irreducible photon background could also be a problem.

A configuration that is easier to handle is a 5 bar argon TPC with the nominal parameters used in this study (T = 1 year, V = 1 m³, σ = l = 0.1 cm, η = ε = 1). The conversion of a 10 MeV photon in 5 bar argon is shown in Fig. [27]. The precision of the (1D) measurement of P for this setup would be about 1 %, with an effective polarization asymmetry of about 15 %.

5.1.5. Background

γ-ray telescopes are affected by a huge background. The dominating contributions are from (charged) cosmic rays that traverse the detector and from the albedo photons that enter the detector from below and pair convert into it. These should be easily rejected, thanks to the excellent pattern recognition capability of a TPC. γ-ray Compton interactions in the detector leave single tracks that are easily rejected too. The production of irreducible photons by the interaction of a cosmic ray into the vessel material, outside of the detector and of the coverage of a cosmic ray veto, yields photons that enter into the detector from below, which are rejected. In the following, we consider that the background is dominated by the galactic gamma emission.

In the presence of background, the expression for the precision of the measurement of P eq. [5] becomes

\[ \sigma'_{p} = \sigma_{w}/(A' \sqrt{S + B}), \]

where \( S \) and \( B \) are the number of signal and background events, respectively. The measured polarization asymmetry, \( A' \), computed using a weight is \( A' = A \times S/(S + B) \), assuming a non-polarized background with \( \sigma_{w} \) unchanged, so that

\[ \sigma'_{p} = \sigma_{p} \sqrt{\frac{S + B}{S}}. \] (24)

The degradation of the performance of the polarimeter is sizable when \( B \geq S \). In the energy range of interest here, the background in the galactic plane amounts to \( f \approx 10^{-2} \text{MeV/(cm}^2\text{sr)} \) [64], so that \( B = S \) when summing over a solid angle \( \Delta \Omega = F/f \approx 0.1 \text{ sr} \), corresponding to an apex angle \( \theta = \sqrt{\Delta \Omega/\pi} \approx 10^\circ \). The background is not expected to be too much of an issue, except at the lowest energies at which, for a gaseous TPC, the photon-resolution angle of nuclear conversion is dominated by the non-observation of the momentum transferred to the recoiling nucleus [57]. In that context, a 10° 68%-containment angle is reached for \( E = 5.6 \text{MeV} \).

5.2. Towards an actual experiment: Full simulation

Finally we simulate a full spectrum for a 5 bar argon detector. We first cross validate (not shown) the computation of the asymmetry, dilution and effective asymmetry that were used in Fig. [22]. We then implement the following experimental cuts:

- A cut on the opening angle, \( \theta_{\text{pair}} > 0.1 \text{ rad} \), to ensure that the tracks are sufficiently separated for pattern recognition can be performed;
- A cut on the reconstructed direction of the photon, to ensure background rejection, \( \theta_{\text{pair}} < 10^\circ \), as determined above;
- A cut on the (kinetic) energy of each of the exiting leptons, to ensure proper track reconstruction (\( E_{\text{pair}} > 0.5 \text{ MeV} \), \( E_{\text{kin}} > 0.5 \text{ MeV} \), for which the path length in 5 bar argon is \( \approx 30 \text{ cm} \)).

Figure [23] shows the variation of the cut efficiency \( \epsilon_{c} \), of the polarization asymmetry \( A \) and of the precision \( \sigma_{p} \) with these cuts. Applying all cuts results in an efficiency of 45 %, an (1D) asymmetry of 16.6 % and a precision of \( \sigma_{p} \approx 1.4\% \).
5.2.1. GRB

For a bright GRB such as GRB 041219A \cite{65} the expected number of nuclear conversions is \( \approx 900 \). After the \( \theta_{\text{cut}} \) and \( E_{\text{kin}} \) cuts, 564 events would remain with an (1D) asymmetry of 18 \% and a precision of \( \sigma_{\text{P}} \approx 33 \% \), and an (5D) asymmetry of 16 \% and a precision of \( \sigma_{\text{P}} \approx 18 \% \).

5.3. Triplet conversion

Only a recoiling electron that has a sufficiently long path length inside the detector can be tracked correctly. Also, multiple scattering affects the measurement of its azimuthal angle. In Fig. 24, we compare both effects, namely we compare the path length to the distance \( x_\epsilon \) after which the average deflection angle is equal to the critical angle \( \theta_c \) (from eq. \ref{eq:14}). In the kinetic-energy range of interest, the limiting effect is multiple scattering. This validates a posteriori the constant-momentum, that is the small energy loss, approximation under which \( x_\epsilon \) is obtained.

We can then obtain the fraction of triplet events for which the recoil electron is measurable, that is for which \( x_\epsilon \) is larger than a given value, say 2 cm (Fig. 25). For example in 1 bar Argon, \( x_\epsilon = 2 \text{ cm} \) is reached for \( E_{\text{kin}} \approx 0.17 \text{MeV} \), corresponding to \( p_\epsilon \approx 0.42 \text{MeV}/c \), that is, for 4 MeV photons, \( \approx 70 \% \) of the events. Note that the resolution \( \sigma_{\text{cut}} \) obtained from the naive application of eq. \ref{eq:16} yields a value of \( \sigma_{\text{cut}} \approx 0.29 \text{ rad} \), smaller than \( \sigma_{\text{eff}} \) : after the \( x_\epsilon \) cut has been applied, we can expect the dilution to degrade the measurement only slightly. For triplet conversion, increasing the pressure augments the statistics, but degrades the efficiency of the recoil momentum cut.

The interplay of these effects is estimated with the simulation. The resolution of the measurement of the azimuthal angle of the recoiling electron \( \phi_\epsilon \) is parametrized using eqs. \ref{eq:16} and \ref{eq:17} and presented in Fig. 25. For 5 bar argon, we expect a precision of \( \approx 8 \% \) with an effective asymmetry of 21 \%.

\footnote{The spectrum of GRB 041219A was kindly provided to us by D. Götz up to 100 MeV, above which we extrapolated it with a \( \Gamma = 2.06 \) powerlaw, for a duration of 120 s.}
that attempts to increase the statistics and therefore the precision by using a liquid TPC would provide a small improvement only.

6. Conclusion

We have implemented an event generation, building on the work of Ref. [28]. We have interfaced the amplitude calculator HELAS with the event generator SPRING. For nuclear conversion, the validation showed no difference between the HELAS pdf and the Bethe-Heitler analytical pdf. Differences between the triplet and nuclear distributions that had been mentioned previously, such as the strong decrease of the most probable value of the opening angle at low energy for triplet conversion, or the strong difference between the triplet and nuclear q distributions above a couple of MeV/c, are observed. Past proposals to increase the polarization sensitivity by a judicious selection of the events turn out to be inefficient because the possible improvement is actually counter balanced by the the loss in statistics. Only the use of an optimal variable that makes use of all of the information present in the 5D pdf allows an improvement of the precision by a factor of about 2.

The dilution of the polarization asymmetry for nuclear conversion due to multiple scattering in a slab detector is studied with the full 5D pdf; its variation with slab thickness is found to be quite different from that which had been computed under the fixed-opening-angle assumption. The limitations of the converter/trackers remain though, and we turn to the use of an active target. In that case, in the multiple-scattering dominated regime that is relevant in the photon energy range considered here, the momentum dependence of the single-track angular resolution is parametrized with a single parameter, a critical momentum, $p_1$, that depends on the characteristics of the detector: the dilution is then parametrized as a function $D(E, p_1)$. We found that with a low density active target, such as a gas TPC, the sensitivity to polarization of nuclear conversion events is recovered over most of the photon energy range of interest.

Finally, the simulation of a realistic case, with a 1 m$^3$ 5 bar argon TPC, a one year exposure to a Crab-like source (with an exposure fraction of $\eta = 1$) and with experimental cuts applied yields a $P$ precision of 1.4\% (that is, $<4.4\%$ for $\eta > 0.1$). The precision for GRBs is lower due to the smaller statistics, and we may only be able to perform GRB polarimetry above 1 MeV for the brightest bursts.

**Note added in proof:** A recent publication [67] extends previous calculations of the polarization fraction of the emission of blazars in leptonic and hadronic jet models up to 500 MeV, that is the gamma energy range relevant to the present study. It includes an estimate of the dilution due to the external Compton radiation (assumed unpolarized) and presents predictions for a sample of sources.

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A. Supplementary data

Figure 28: Comparison of the total cross section for triplet conversion (bullets, no screening) with the computation by Mork (solid line) [40].

Figure 29: Left: Comparison of the total mass attenuation coefficients (argon: with (squares) and without (bullets) screening) with the data from NIST [59] (curves). Right: ratio of the cross sections with/without screening (symbols) with the results from Ref. [43] (curves).