Lattice PUF: A Strong Physical Unclonable Function Provably Secure against Machine Learning Attacks

Ye Wang, Xiaodan Xi, and Michael Orshansky
Department of Electrical and Computer Engineering
The University of Texas at Austin
Austin, TX, USA
{lhywang, paul.xiaodan, orshansky}@utexas.edu

Abstract—We propose a strong physical unclonable function (PUF) that is provably secure against machine learning (ML) attacks with both classical and quantum computers. The security is derived from cryptographic hardness of learning decryption functions of semantically secure public-key cryptosystems within the probably approximately correct (PAC) framework. The proposed PUF compactly realizes the decryption function of the learning-with-errors (LWE) public-key cryptosystem as the core block. Due to the fundamental connection of LWE to lattice problems, we call the construction the lattice PUF.

The lattice PUF is lightweight and fully digital. It is constructed using a weak PUF, as a physically obfuscated key (POK), an LWE decryption function block, a pseudo-random number generator in the form of a linear-feedback shift register (LFSR), a self-incrementing counter, and a control block. The POK provides the secret key of the LWE decryption function. A fuzzy extractor is utilized to ensure stability of the POK. The proposed lattice PUF significantly improves upon a direct implementation of LWE decryption function in terms of challenge transfer cost by exploiting distributional relaxations allowed by recent work in space-efficient LWEs. Specifically, only a small challenge-seed is submitted, the value of a self-incrementing counter is embedded into the challenge seed.

We develop a measure of ML resistance in terms of the total number of operations needed to learn the model of a PUF: a PUF has $k$-bit ML resistance if a successful ML attack requires $2^k$ operations. We construct a lattice PUF that realizes a challenge-response pair space of size $2^{136}$, requires 1160 POK bits, and guarantees 128-bit ML resistance. Assuming a bit error rate of 5% for SRAM-based POK, 6.5K SRAM cells are needed. The PUF shows excellent uniformity, uniqueness, and reliability. The mean (standard deviation) of uniformity is 49.92% (1.58%), of inter-class Hamming distance (HD) is 50.00% (1.58%), and of intra-class HD is 4.43% (6.96%). We implement the PUF on a Spartan 6 FPGA. It requires only 45 slices for the lattice PUF proper, and 233 slices for the fuzzy extractor.

Index Terms—Strong PUF, PAC Learning, Lattice Cryptography, ML Resistance.

I. INTRODUCTION

Silicon physical unclonable functions (PUFs) are security primitives widely used in device identification, authentication, and cryptographic key generation [38]. Given an input challenge, a PUF exploits the randomness inherent in CMOS technology to generate an output response. In contrast to weak PUFs, also called physically obfuscated keys (POKs) using the taxonomy of [17], which supply limited amount of challenge-response pairs (CRPs), strong PUFs have an exponentially large CRP space. Strong PUFs are superior to POKs because the large size of the CRP space permits direct challenge-response authentication without other cryptographic primitives.

In order for a strong PUF to be an effective security primitive, the associated CRPs need to be unpredictable: given a certain set of known CRPs, it should be hard to predict the unobserved CRPs with high probability. In other words, strong PUFs are required to be resilient to modeling attacks. However, the effectiveness of modeling attacks via machine learning (ML) on many strong PUFs has been widely demonstrated [18], [37]. A list of common ML techniques includes support vector machines (SVM), logistic regression (LR), neural networks (NN), and evolutionary strategies (ES). Most proposed modifications of the original arbiter PUF aimed to strengthen ML resistance, including the XOR arbiter PUF and the feed-forward PUF [28], [33], [38], [42], have also been broken via various ML attacks [4], [13], [14], [37]. By exploiting higher intrinsic nonlinearity, some strong PUFs [27], [40] exhibit empirically-demonstrated resistance to some ML algorithms. Empirical demonstrations of ML resistance are not fully satisfactory since they can never rule out the possibility of other more effective ML algorithms. The so-called controlled PUF setting [15] attempts to ensure the ML resistance via cryptographic primitives such as hash functions. However, the use of hash functions inside a PUF endangers the promise of a strong PUF as a lightweight structure. Strong PUF constructions using established cryptographic ciphers, such as AES [6], have similar challenges. An additional challenge our paper addresses is that an ideal ML resistant PUF should exhibit security against both classical as well as quantum algorithms. In summary, the question of whether it is possible to engineer a provably ML secure lightweight strong PUF has been a long-lasting challenge [39].

In this paper, we propose a strong PUF that is secure
against ML attacks with both classical and quantum computers. The security is guaranteed by engineering a PUF for which modeling is, provably, a computationally hard problem. The main insight is the mapping of ML attack resistance in a PUF to hardness of learning a decryption function of a cryptosystem. As a formal framework, we adopt the probably approximately correct (PAC) theory of learning [35]. The specific insight, which allows us to build a novel strong PUF, is our reliance on the earlier proof that PAC-learning a decryption function of a semantically secure public-key cryptosystem entails breaking that cryptosystem [24], [25], [26]. The PAC non-learnability of a decryption function implies that with a polynomial number of samples, with high probability, it is not possible to learn a function accurately by any means. Specifically, we develop a PUF for which the task of modeling is equivalent to PAC-learning the decryption function of a learning-with-errors (LWE) public-key cryptosystem. LWE cryptosystems are based on the hardness of LWE problem that ultimately is reduced to the hardness of several problems on lattices [36]. The input-output mapping between the PUF and the underlying LWE cryptosystem can be briefly summarized as follows: challenge ⇐⇒ ciphertext and response ⇐⇒ decrypted plaintext. Notably, LWE is believed to be secure against both classical and quantum computers. Because of the intrinsic relation between the proposed PUF and the security of lattice cryptography we call our construction the lattice PUF.

The lattice PUF is constructed using a POK, an LWE decryption function block, a linear-feedback shift register (LFSR), a self-incrementing counter, and a control block. The entire implementation is lightweight and fully digital. The LWE decryption function block is the core module of the lattice PUF: generating response (plaintext) to each submitted challenge (ciphertext). Design parameters of the LWE decryption function in the lattice PUF are chosen by balancing the implementation costs, statistical performance, and the concrete hardness of ML resistance. We develop a measure of ML security in terms of the total number of operations needed to learn the model of the PUF. Such concrete hardness is established by the analysis of state-of-the-art attacks on the LWE cryptosystem [29], [34] and evaluated by the estimator developed by Albrecht et al. [2]. Using this estimator, we say that a PUF has $k$-bit ML resistance if a successful ML attack requires $2^k$ operations. We implement the LWE decryption function that takes 1168-bit input challenges while guaranteeing 128-bit ML resistance.

The likely deployment model for strong PUFs presumes a clear distinction between a device being authenticated (a PUF proper) and an authenticating server sending challenges to the PUF and analyzing its responses. It is natural to assume that the server is less resource-constrained than the PUF. It is thus desirable to place the computationally-costly part of challenge generation, that utilizes a relatively more expensive encryption function of LWE, on the server. The theoretical security guarantee, cited above, assumes that the inputs to the LWE decryption function are generated by an encryption function operating on the uniformly random plaintexts: we call such allowed queries “challenges generated by a ciphertext distribution”. However, we found that a direct implementation of the lattice PUF, in which the server fully generates ciphertext, is inefficient due to the well-known high ratio of ciphertext to plaintext: a PUF with a 128-bit concrete ML hardness requires transmitting $116.8K$ challenge bits in order to produce a 100-bit response string.

We solve this problem by exploiting distributional relaxations allowed by recent work in space-efficient LWEs. First, our approach replaces the component of ciphertext which dominates transmission cost, by a uniformly sampled random vector, such that the resulting distribution is statistically close to the original ciphertext distribution [1]. The advantage of the above replacement is that, as shown in [12], multiple simple pseudo-random number generators (PRNGs), including those based on a linear-feedback shift register (LFSR), are capable of producing it. Specifically, [12] shows that input challenges generated by PRNGs provide similar concrete security guarantees against standard attacks on LWE. The proposed strategy allows introducing a low-cost PRNG based on an LFSR and transmitting only a small seed. This results in a dramatic reduction of the effective challenge size. In the improved design with the same parameters chosen above, only 928 bits are needed to produce a 100-bit response. This is a $100X$ reduction of communication cost in authentication, in contrast to the direct PUF implementation as LWE decryption function.

The focus of the paper is a PUF that is secure against passive attacks in which the observed challenges can be used to derive an internal model of the PUF. However, the LWE decryption function is vulnerable to an active attack that supplies arbitrary input challenges. (As we show, this risk also carries into an LFSR-based variant). We overcome the risk of such an attack by adopting the technique in [43]: we introduce a self-incrementing counter to embed the counter value into a challenge seed. This makes the attack impossible as the counter restricts the attacker’s ability to completely control input challenges to the LWE decryption function.

Our reliance on LWE to formally guarantee resistance to ML attacks is novel. We highlight a critical difference between our work and recent work [11], [17], [20], [11], [17], [20] have also utilized lattice-based problems, including learning-parity-with-noise (LPN) and LWE, to realize computationally-secure fuzzy extractors (FEs) and, as a byproduct, to construct strong PUFs. Here, CRP generation is based on generating multiple private keys (playing the role of PUF responses) and multiple public keys (playing the role of PUF challenges). Although both constructions utilize lattice-based cryptosystems, they differ fundamentally in the root of their security guarantees. The fundamental security property that [11], [17], [20] rely upon is the computational hardness of recovering a private key from a public key in a public-key cryptosystem.

1A computational FE guarantees absence of information leakage from publicly shared helper data via computational hardness in contrast to conventional FEs that need to limit their information-theoretic entropy leakage.
It turns out that, in a strong PUF context, this security is insufficient. The vulnerability is due to the fact that the challenges are by definition publicly known [11], [17], [20], and an attacker can use multiple challenges (public keys) to recover the private key. This is only possible because multiple public keys are derived using a fixed (same) source of secret POK bits, embedded in the error term of LPN or LWE. As was shown in [3], the fact that multiple CRPs have shared error terms can be easily exploited allows a computationally-inexpensive algorithm for solving an LPN or LWE instance, thus compromising the hardness of LPN or LWE problems. Thus, by itself [11], [17], [20], the resulting PUF does not have resistance against ML modeling attacks. This vulnerability is fixed in [17], [20] by introducing a cryptographic hash function to hide the original CRPs.

In stark contrast, the proposed lattice PUF derives its security by directly exploiting a distinctly different property of public-key cryptosystems: the theoretically-proven guarantee that their decryption functions are not PAC-learnable. (As shown later, this property stems from semantic security of a cryptosystem coupled with the ease of generating multiple ciphertexts.) In the lattice PUF, the above-discussed vulnerability is absent since the publicly known challenges are ciphertexts and the security of the cryptosystem ensures that a fixed private key (the POK, in our case) cannot be recovered from ciphertexts.

We implement the PUF on an FPGA which requires a 1160-bit secret key. The secret key is generated from POK bits. We use an FE with concatenated codes to reconstruct stable POK bits. The random source of the POK can be any weak PUF. The SRAM PUF power-up states are used in our paper. Assuming an average bit error rate (BER) of 5% for raw SRAM cells, the total number of raw SRAM bits needed is 6.5K, in order to achieve a key reconstruction failure rate of $10^{-6}$. The LFSR utilizes a 256-bit seed. The self-incrementing counter produces a 128-bit output. Additional 128 bits are concatenated with the counter output to form the input seed to the LFSR. Thus, the resulting lattice PUF is able to achieve a CRP space of size $2^{136}$. The mean BER (intra-class Hamming distance (HD)) is 4.43%. The lattice PUF also shows excellent uniformity and uniqueness. The mean (standard deviation) of uniformity is 49.92% (1.58%) and of inter-class HD is 50.00% (1.85%). The hardware implementation on a Xilinx Spartan 6 FPGA utilizes only 45 slices for the lattice PUF logic and 233 slices for the concatenation-code-based FE. Compared to several known strong PUFs, the proposed PUF is significantly more resource-efficient.

II. LWE Decryption Functions Are Hard to Learn

This section formally defines ML resistance of strong PUFs via the notion of PAC learning and shows why LWE decryption functions are attractive for constructing post-quantum ML-resistant PUFs. In this section, we focus on passive attacks in which the attacker can observe the challenges sent to the verifier but is unable to generate challenges of his or her choice.

A. ML Resistance as Hardness of PAC Learning

A strong PUF can be modeled as a function $f : C \rightarrow \mathcal{R}$ mapping from the challenge space $C$ (usually $\{0, 1\}^n$) to the response space $\mathcal{R}$ (usually $\{0, 1\}$). We call $f$ the true model of a strong PUF since it captures the exact challenge-response behavior.

ML attacks are usually performed by relying on a functional class of candidate models, collecting CRPs as the training data, and running a learning algorithm to obtain a model from the candidate class which best approximates the true model. In addition to the approximation quality, the criteria of evaluating the effectiveness and efficiency of the learning algorithm also include the sample and time complexity. To claim that a strong PUF is easy to learn, one can propose a learning algorithm which finds a CRP model with good approximation quality using a small number of sample CRPs and terminates in a short time. The converse is difficult: to claim that a PUF is hard to learn, one must show that all possible learning algorithms fail to provide models with good approximation quality, or they require a large number of CRPs or a long running time.

Almost all previous work on strong PUFs attempts to demonstrate their ML resistance by using a set of well-known ML attacks, such as LR, SVM, NN and ES. The problem is that even if these attacks fail with CRPs of moderate size under moderate running time, the ML resistance is still questionable since one cannot enumerate all possible learning algorithms. In particular, subtle differences in learning algorithms, e.g. the number of hidden layers and the number of neurons per layer in an NN, can result in significant difference in the learning results as well as different sample and time complexity [16].

We argue that the only known framework for seeking a provable notion of ML resistance with a formal analysis of approximation quality, sample size, and time complexity is the PAC learning model [35]. We now formalize the passive modeling attack scenario in the context of PAC learning. A PAC-term for a true model $f$ of a strong PUF is a concept. Denote as $\mathcal{F}$ the set of all possible PUF-realized functions (every instance of a PUF creates its unique functional mapping $f$). The set of candidate models used in the learning algorithm is the hypothesis set $\mathcal{H}$. The goal of a learning algorithm is to select a candidate model that matches the true model well. Importantly, as shown later, the proof of PAC-hardness guarantees that $\mathcal{H}$ does not have to be restricted to be the same as $\mathcal{F}$ of true models. This generalization permits a stronger representation-independent PAC-hardness proof. While not always possible, representation-independent hardness can be proven for PAC-learning of decryption functions ensuring that no matter how powerful and expressive the chosen $\mathcal{H}$ is, PAC learning decryption function requires exponential time.

2Further, the construction of [11], [17], [20] is expensive since (1) recovering secrets from helper data requires solving a system of linear equations, and (2) implementing a hash function is costly. In contrast, our PUF is lightweight since it implements only the LWE decryption function rather than the expensive encryption function.
Within the PAC model, CRPs in a training set are assumed to be independent and identically distributed (i.i.d.) under a certain distribution \( \mathcal{D} \).

We say a set \( \mathcal{F} \) of strong PUFs is PAC-learnable using \( \mathcal{H} \), if there exists a polynomial-time algorithm \( \mathcal{A} \) such that \( \forall \epsilon > 0, \forall \delta > 0 \), for any fixed CRP distribution \( \mathcal{D} \), and \( \forall f \in \mathcal{F} \), given a training set of size \( m \), \( \mathcal{A} \) produces a candidate model \( h \in \mathcal{H} \) with probability of, at least, \( 1 - \delta \) such that

\[
\Pr_{{(c,r) \sim \mathcal{D}}} [f(c) \neq h(c)] < \epsilon.
\]

In conclusion, our strategy is to say that a strong PUF is ML-resistant if it is not PAC-learnable (i.e., that it is PACHARD). PAC-hardness implies that any successful ML attack requires at least an exponential running time.

### B. Decryption Functions Are not PAC Learnable

What is critically important is that there exist functions that are known to be not PAC-learnable. Specifically, a class of decryption functions of secure public-key cryptosystems is not PAC-learnable, as established by [24], [26]. We outline their proof below.

A public-key cryptosystem is a triple of probabilistic polynomial-time algorithms \( (\text{Gen}, \text{Enc}, \text{Dec}) \) such that: (1) \( \text{Gen} \) takes \( n \) as a security parameter and outputs a pair of keys \((pk, sk)\), the public and private keys respectively; (2) \( \text{Enc} \) takes as input the public key \( pk \) and encrypts a message \( (\text{plaintext}) \) \( r \) to return a ciphertext \( c = \text{Enc}(pk, r) \); (3) \( \text{Dec} \) takes as input the private key \( sk \) and a ciphertext \( c \) to decrypt a message \( r = \text{Dec}(sk, c) \).

We only need to discuss public-key cryptosystems encrypting 1-bit messages. One of the security requirements of a public-key cryptosystem is that it is computationally infeasible for an adversary, knowing the public key \( pk \) and a ciphertext \( c \), to recover the original message, \( r \). This requirement can also be interpreted as the need for indistinguishability under the chosen plaintext attack (also often referred to as semantic security requirement) [23].

Given the encryption function \( \text{Enc} \) and the public key \( pk \), the goal of an attacker is to devise a distinguisher \( \mathcal{A} \) to distinguish between encryption \( \text{Enc}(pk, r) \) of \( r = 0 \) and \( r = 1 \) with non-negligible probability:

\[
\Pr[\mathcal{A}(pk, \text{Enc}(pk, 0)) = 1] - \Pr[\mathcal{A}(pk, \text{Enc}(pk, 1)) = 1] \geq \epsilon.
\]

A cryptosystem is semantically secure if no polynomial-time attacker can correctly predict the message bit with non-negligible probability.

The connection between the above-stated security of a public-key cryptosystem and the hardness of learning a concept class associated with its decryption function was established in [24], [26]. The insight of [24], [26] is that PAC-learning is a natural result of the ease of encrypting messages with a public key. Since the encryption function \( \text{Enc} \) and the public-key \( pk \) is known, the distinguishing algorithm can sample independent training examples in the following way: (1) picking a plaintext bit \( r \) uniformly randomly from \( \{0, 1\} \), (2) encrypting \( r \) to get the ciphertext \( c = \text{Enc}(pk, r) \). (We later refer to the resulting distribution of ciphertext as the "ciphertext distribution".) Next, the distinguishing algorithm passes the set of training examples \((c, r)\)'s into an algorithm for learning the decryption function \( \text{Dec}(sk, \cdot) \). The PAC learning algorithm returns a model \( h(\cdot) \) that aims to approximate \( \text{Dec}(sk, \cdot) \). Using \( h(\cdot) \), one could distinguish between ciphertexts stemming from \( r = 0 \) and \( r = 1 \) with non-negligible probability. This would entail violating the semantic security of the cryptosystem. Technically, this can be summarized as follows [24], [26].

**Theorem 1:** If a public-key cryptosystem is secure against chosen plaintext attacks, then its decryption functions are not PAC-learnable (under the ciphertext input distribution).

### C. LWE Is Post-Quantum Secure

According to the cryptographic hardness above, decryption functions of any secure public-key cryptosystem, such as Rivest–Shamir–Adleman (RSA) and elliptic-curve cryptography (ECC), can be used to construct ML-resistant PUFs. However, integer-factoring-based cryptosystems, including RSA and ECC above, become insecure with the development of quantum computers. Among all post-quantum schemes [5], the LWE cryptosystem based on hard lattice problems appears to be most promising due to its implementation efficiency and stubborn intractability since 1980s.

A lattice \( \mathcal{L}(V) \) in \( n \) dimensions is the set of all integral linear combinations of a given basis \( V = \{v_1, v_2, \ldots, v_n\} \) with \( v_i \in \mathbb{R}^n \):

\[
\mathcal{L}(V) = \{a_1 v_1 + a_2 v_2 + \cdots + a_n v_n : \forall a_i \in \mathbb{Z}\}.
\]

The LWE problem is defined on the integer lattice \( \mathcal{L}(V) = \{(a, (a, s))\} \) with a basis \( V = (I, s) \), in which \( I \) is an \( n \)-dimensional identity matrix and \( s \) is a fixed row vector (also called the secret) in \( \mathbb{Z}_q^n \). Throughout this paper, vectors and matrices are denoted with bold symbols with dimension on superscript, which can be dropped for convenience in case of no confusion. Unless otherwise specified, all arithmetic operations in the following discussion including additions and multiplications are performed in \( \mathbb{Z}_q \), i.e. by modulo \( q \).

For the lattice \( \mathcal{L}(V) = \{(a, (a, s))\} \) with dimension \( n \), integer modulus \( q \) and a discrete Gaussian distribution \( \Psi_n \), for noise, the LWE problem is defined as follows. The secret vector \( s \) is fixed by choosing its coordinates uniformly randomly from \( \mathbb{Z}_q \). Next \( a_i \)'s are generated uniformly from \( \mathbb{Z}_q \). Together with the error terms \( e_i \), we can compute \( b_i = (a, s) + e_i \). Distribution of \( (a_i, b_i) \)'s over \( \mathbb{Z}_q^n \times \mathbb{Z}_q \) is called the LWE distribution \( A_{s, \Psi_n} \). The most important property of \( A_{s, \Psi_n} \) is captured in the following lemma:

**Lemma 1:** Based on hardness assumptions of several lattice problems, the LWE distribution \( A_{s, \Psi_n} \) of \( (a, b) \)'s is indistinguishable from a uniform distribution in \( \mathbb{Z}_q^n \times \mathbb{Z}_q \).

Solving the decision version of LWE problem is to distinguish with a non-negligible advantage between samples from \( A_{s, \Psi_n} \) and those generated uniformly from \( \mathbb{Z}_q^n \times \mathbb{Z}_q \). This LWE problem is shown to be intractable to solve, without knowing the secret \( s \), based on the worst-case hardness of...
several lattice problems [36]. Errors $e$ are generated from a discrete Gaussian distribution $\Psi_{\alpha}$ on $\mathbb{Z}_q$ parameterized by $\alpha > 0$: sampling a continuous Gaussian random variable with mean 0 and standard deviation $\alpha q/\sqrt{2\pi}$ and rounding it to the nearest integer in modulo $q$. Notice that error terms are also essential for guaranteeing the indistinguishability: without noise $(a, b)$ becomes deterministic and the secret $s$ can be solved efficiently via Gaussian elimination methods.

We now describe a public-key cryptosystem based on the LWE problem above in [8]:

**Definition 1:** (LWE cryptosystem)
- **Private key:** $s$ is uniformly random in $\mathbb{Z}_q^n$.
- **Public key:** $A \in \mathbb{Z}_q^{m \times n}$ is uniformly random, and $e$ $\in \mathbb{Z}_q^n$ with each entry from $\Psi_{\alpha}$. Public key is $(A, b = As + e)$.
- **Encryption:** $x \in \{0, 1\}^m$ is uniformly random. To encrypt a one-bit plaintext $r$, output ciphertext $c = (a, b) = (A^T x, b^T x + r[q/2])$.
- **Decryption:** Decrypt the ciphertext $(a, b)$ to 0 if $b - \langle a, s \rangle$ is closer to 0 than to $[q/2]$ modulo $q$, and to 1 otherwise. Notice that each row in the public-key $(A, b)$ is an instance from the LWE distribution $A, \Psi_{\alpha}$.

Correctness of the LWE cryptosystem can be easily verified: without the error terms, $b - \langle a, s \rangle$ is either 0 or $[q/2]$, depending on the encrypted bit. Semantic security of the LWE cryptosystem follows directly from the indistinguishability of the LWE distribution from the uniform distribution in $\mathbb{Z}_q^n \times \mathbb{Z}_q$. Ciphertexts $(a, b)$ are either linear combinations or shifted linear combination of LWE samples, both of which are indistinguishable from the uniform distribution. This is true because shifting by any fixed length preserves the shape of a distribution. Therefore, an efficient algorithm that can correctly guess the encrypted bit would be able to distinguish LWE samples from uniformly distributed samples. This allows [36] to prove that:

**Theorem 2:** Based on the hardness assumptions of several lattice problems, the LWE cryptosystem is secure against the chosen-plaintext attacks using both classical and quantum computers.

When the error terms $e_i$’s are introduced:

$$b - \langle a, s \rangle = \sum_{i \in S} b_i + \left[\frac{q}{2}\right]r - \sum_{i \in S} a_i s_i$$

$$= \sum_{i \in S} \langle a_i, s_i \rangle + e_i - \left[\frac{q}{2}\right]r - \sum_{i \in S} a_i s_i$$

$$= \left[\frac{q}{2}\right]r - \sum_{i \in S} e_i,$$

in which $S$ is the set of non-zero coordinates in $x$. For a decryption error to occur, the accumulated error $\sum_{i \in S} e_i$ must be greater than the decision threshold $[q/4]$. The probability of the error is given by [34]:

$$\text{Error}_{\text{LWE}} \approx 2(1 - \Phi\left(\frac{q/4}{\alpha q \sqrt{m}/2 \sqrt{2\pi}}\right))$$

$$= 2(1 - \Phi\left(\frac{\pi}{2\alpha \sqrt{m}}\right)),$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard Gaussian variable. We later use this expression to find the practical parameters for the lattice PUF.

### III. Design of Lattice PUF

A strong PUF is a function $f$ that maps input challenges $c$ to output responses $r$ in such a way that its physical instantiations are unique, robust, and show good randomness. We now show how to realize a PUF whose challenge-response behavior is defined by the decryption function of the LWE cryptosystem so that its ML resistance is guaranteed. Such a PUF is achieved by implementing the LWE decryption function using a POK-derived secret as the private key. In such a PUF, ciphertexts and decrypted plaintexts are treated as PUF challenges and responses respectively. However, such a direct implementation results in a very large challenge word, making challenge-transfer costs prohibitive. We overcome this problem by exploiting distributional relaxations allowed by recent work in space-efficient LWEs.

The top-level architecture of the proposed lattice PUF is shown in Figure 1.

#### A. LWE Decryption Function

Figure 2 shows the core component of the proposed lattice PUF: the LWE decryption function. It takes a binary challenge vector $c = \{c_0, c_1, \ldots, c_{N-1}\}$ of size $N = (n+1)\log q$ which maps to a ciphertext $(a, b)$ in the following way:

$$a_i = \sum_{j=0}^{\log q-1} c_{(i-1)\log q+j}2^j, \forall i \in \{1, 2, \ldots, n\},$$

$$b = \sum_{j=0}^{\log q-1} c_{n\log q+j}2^j.$$

Here $a_i$ denotes the $i$-th element of the integer vector $a \in \mathbb{Z}_q^n$. In this paper, without specification, $\log_q(x)$ refers to $\log_2(x)$. Similarly, the private key $s$ for the corresponding LWE decryption function is realized by a binary secret key $W = \{W_0, W_1, \ldots, W_n \log q - 1\}$ of size $n \log q$:

$$s_i = \sum_{j=0}^{\log q-1} W_{(i-1)\log q+j}2^j, \forall i \in \{1, 2, \ldots, n\}.$$

A modulo-dot-product $b - \langle a, s \rangle$ is computed using the modulo-multiply-accumulate unit. It can be implemented in a serial way using $n$ stages. Recall that all additions and multiplications are performed in modulo $q$. Since $q$ is a power of 2 in our construction, modulo addition and multiplication can be naturally implemented by integer addition and multiplication that keep only the last $\log q$-bit result. Finally the response $r$ is produced by a quantization operation $r = Q(b - \langle a, s \rangle)$:

$$Q(x) = \begin{cases} 0 & x \in [0, \frac{q}{4}] \cup (\frac{3q}{4}, q - 1], \\ 1 & x \in (\frac{q}{4}, \frac{3q}{4}] \cup [\frac{3q}{4}, q - 1]. \end{cases}$$

The computation above can be directly implemented as a strong PUF with $2^N$ CRPs since it maps a challenge vector
\( c \in \{0, 1\}^N \) into a binary response \( r \in \{0, 1\} \). We now discuss parameter selection for the LWE decryption function. In general, we seek to find design parameters such that (1) the resulting PUF has excellent statistical properties, such as uniformity, uniqueness, and reliability, (2) successful ML attacks against it require an un-affordably high time complexity in practice, and (3) its hardware implementation costs are minimized.

Prior theoretical arguments establish the impossibility of a polynomial-time attacker. To guarantee practical security, we need to estimate the number of samples and the actual running time (or a number of CPU operations) required for a successful ML attack. [36] shows that a small number of samples are enough to solve an LWE problem, but in an exponential time. Thus, we refer to runtime as concrete ML resistance (or ML hardness) and say that a PUF has \( k \)-bit ML resistance if any successful ML attack requires at least \( 2^k \) operations. We adopt the estimator developed by Albrecht et al. [2] to estimate concrete ML hardness. The concrete hardness of an LWE problem increases with the increase of LWE parameters \( n, q, \alpha \) for all types of attacks. Recall that \( n \) represents the lattice dimension, \( q \) represents the range of integer for each dimension, and \( \alpha \) reflects the noise level in CRP (ciphertext) generation. For a given set of parameters, the estimator compares the complexity of several most effective attacks, including decoding, basis reduction, and meet-in-the-middle attacks [9], [19], [29]. We utilize the estimator in a black-box fashion to find the set of parameters with the target of 128-bit concrete ML resistance.

We consider two metrics of implementation cost, both of which scale with \( n \): the number of challenge and secret bits needed \((n \log q)\), and the number of multiply-accumulate (MAC) operations \((n)\). This motivates the need to decrease \( n \).

For conventional PUFs, such as arbiter PUF and SRAM PUF, an output error is due to environmental noise, e.g. delay changes in arbiter PUF and FET strength changes in SRAM PUF with both voltage and temperature. In contrast, output errors of the lattice PUF come from two sources: (1) environmental errors of secret bits, and (2) errors of decryption during response generation. The former can be thought as the failure of key reconstruction in POKs. Since a single bit-flip completely changes the challenge-response behavior of LWE decryption function, the failure rate of key reconstruction needs to be low, e.g. \( 10^{-6} \) (as widely adopted in other PUF applications [31]). Section IV describes how the target failure rate can be achieved via a conventional FE based on the error-correcting codes. The latter corresponds to the decryption error and is orthogonal to errors in the secret key \( s \). Recall that in CRP generation of the lattice PUF, a bit of plaintext \( r \) is sampled and the ciphertext \( c \) is produced by a noisy encryption function \( c = Enc(r) \). Given ciphertext \( c \) as input challenge, the decryption function can output a wrong response \( r' \neq r \) when the accumulated error \( \sum_{i \in S} e_i \) in the encryption function exceeds the decision boundary.

The model for evaluating the decryption error rate is shown in Section II. In order for a strong PUF to be used in direct authentication, its decryption error rate should be small enough for reliable distinguishability of long strings. Following [43], we set the target at 5%. Figure 3 explores the trade-off between the number of secret bits and the decryption error rate needed for 128-bit concrete ML hardness. It shows that, at fixed concrete ML hardness, the decryption error rate decreases super exponentially with the number of secret bits.

Considering the design metrics above, a feasible set of parameters is found using the estimator in [2]. By setting \( n = 145, q = 256, m = 256 \) and \( \alpha = 2.77% \), we achieve a lattice PUF with 128-bit concrete hardness and a decryption error rate of 4.55%.

In order to get a 1-bit response, \((n + 1) \log q = 1168\) bits need to be sent to the lattice PUF as a challenge. For direct authentication applications, usually around 100 bits of responses are required. Therefore, the direct implementation described so far would require \( C = 116.8K \) challenge bits. This high ratio of challenge length to response length limits its practical use in many scenarios when communication is expensive. We next describe an improved design that overcomes this limitation.

**B. Challenge Compression through Distributional Relaxation**

The basic design described in the previous section requires a challenge \( c \) in the form \( c = (a, b) \) to be sent from the
server to the PUF. To represent vector $a \in \mathbb{Z}_q^n$ requires $n \log q$ bits while to represent scalar $b \in \mathbb{Z}_q$ requires only $\log q$ bits. Thus, the major cost of transmission is in sending $a$. We wish to avoid sending $a$ directly and, instead, to send a compressed (shorter) version of $a$ and re-generate its full-size version on the PUF. Our approach is enabled by the recent results on the distributional behavior of $a = A^T x$ [1] and the concept of space-efficient LWE [12].

Recall that $b$ is given by:
\[
\begin{align*}
b &= b^T x + r \lfloor q/2 \rfloor \\
&= (A s + e)^T x + r \lfloor q/2 \rfloor \\
&= (A^T x)^T s + e^T x + r \lfloor q/2 \rfloor.
\end{align*}
\]

First, we replace the component $a = A^T x$ by $a^*$ uniformly randomly sampled from $\mathbb{Z}_q^n$. That allows us to represent challenge $c = (a, b)$:
\[
\begin{align*}
a &= A^T x \\
b &= (A^T x)^T s + e^T x + r \lfloor q/2 \rfloor
\end{align*}
\]
as $c^* = (a^*, b^*)$:
\[
\begin{align*}
a^* &= a^T s + e^T x + r \lfloor q/2 \rfloor \\
b^* &= a^T s + e^T x + r \lfloor q/2 \rfloor.
\end{align*}
\]

In [1], it is proven that distribution of $c^* = (a^*, b^*)$ is statistically close to the original ciphertext distribution, therefore the required security properties are preserved.

The advantage of the above approximation is that, as shown by [12], several low-complexity PRNGs are capable of producing an output string $\alpha'$ suitably close to $a^* \in \mathbb{Z}_q^n$ within the context of LWE cryptosystem. In particular, an LFSR is an especially simple PRNG having the right properties. Specifically, a vector $\alpha'$ generated by an LFSR provides similar concrete security guarantees against standard attacks on LWE, such as CVP reduction, decoding, and basis reduction [12]. This is because LFSR-generated $\alpha'$ maintains good properties including:

- it is hard to find “nice” bases for a lattice with basis from LFSR-generated $\alpha'$;
- given an arbitrary vector in $\mathbb{Z}_q^n$, it is hard to represent it as a binary linear combination of LFSR-generated $\alpha$’s;
- it is hard to find a short vector $w$ that is orthogonal to LFSR-generated $\alpha$’s.

The ability to rely on a simple PRNG to produce $\alpha'$ allows a dramatic reduction in challenge transfer cost. Now, the challenge $c'$ contains only a small seed $\alpha'$ into the PRNG and the corresponding $b'$ as
\[
b' = (\alpha')^T s + e^T x + r \lfloor q/2 \rfloor = \text{LFSR}(\text{seed}_\alpha)^T s + e^T x + r \lfloor q/2 \rfloor.
\]

Here LFSR$(\cdot)$ denotes the output generated by an LFSR.

With LWE parameters chosen as Section III-A, using a seed of length $l = 256$ is able to reduce the challenge length from 1108 to 256 + 8 = 264 per one bit of response. The improvement of efficiency becomes more pronounced for generating multiple responses: This is because $\alpha'_1 \ldots \alpha'_l$ can be generated sequentially from the $l$-bit seed, so that only the seed and $\alpha'_1 \ldots \alpha'_l \in \mathbb{Z}_q$ are required to be sent to the PUF side. 100 bits of responses now require only transmitting $128 + 100 \times \log 256 = 928$ bits for challenges.

C. Countermeasure for Active Attack

In this section, we introduce a simple defense to protect our PUF against a standard attack on the LWE decryption function. The attack is premised on the ability to supply arbitrary challenges (ciphertexts) as inputs to the decryption function. The attack proceeds as follows. The attacker fixes $a$ and enumerates all possible $b \in \mathbb{Z}_q$ for challenge $c = (a, b)$. As $b$ increases from $0$ to $q - 1$, the response $r = Q(b - \langle a, b \rangle)$ changes from $Q(b - \langle a, s \rangle) = 0$ to $Q(b + 1 - \langle a, s \rangle) = 1$ exactly when $b$ satisfies
\[
b - \langle a, s \rangle = q/4.
\]

We denote this specific value of $b$ as $b$. The exact value of $\langle a, s \rangle$ can then be extracted by $\langle a, s \rangle = b - q/4$. By repeating this procedure $n$ times, the attacker is able to set up $n$ linear equations (without errors):
\[
\begin{align*}
\langle a_0, s \rangle &= \hat{b}_0 - q/4, \\
\langle a_1, s \rangle &= \hat{b}_1 - q/4, \\
&\vdots \\
\langle a_{n-1}, s \rangle &= \hat{b}_{n-1} - q/4.
\end{align*}
\]

Gaussian elimination can then be used to solve for $s$. The reason the attack succeeds is that attackers are able to fix $a$ and use it for multiple values of $b$.

We propose a modification of the lattice PUF that eliminates the vulnerability to the above attack. This can be achieved by embedding a self-incrementing counter into the system design, in a way similar to [43]. As shown in Figure 1, the
concatenation of the challenger-provided seed and the counter value \( t \) (i.e. \( \text{seed}_a || t \)) is used as the seed for generating \( a \). The self-incrementing counter restricts the attacker’s ability to fix \( a \) since the attacker does not control the counter value. As a result, the attacker cannot enumerate all values of \( b \) while keeping \( a \) unchanged. The counter value is public and is incremented by 1 on each response generation.

### IV. Experimental Results

In this section we evaluate statistical properties of the lattice PUF, including uniformity, uniqueness, and reliability with parameters chosen in Section III. We also present the implementation cost on the FPGA platform and compare it with prior work.

#### A. Statistical Analysis

**Uniformity** of a PUF characterizes unbiasedness, namely, the proportion of ‘0’s and ‘1’s in the output responses. For an ideal PUF \( f \), the proportion needs to be 50%. We adopt the definition of uniformity in [32] based on the average Hamming weight \(\text{HW}(f)\) of responses \( r \) to randomly sampled challenges \( c \)’s:

\[
\text{HW}(f) = E_r[\text{HW}(r)] = E_c[\text{HW}(f(c))].
\]

Here \( E_X \) represents expectation over random variable \( X \). Note that \( c \) follows the ciphertext distribution rather than the usual uniform distribution [32]. Figure 4 shows uniformity obtained using 1000 randomly selected challenges. The distribution is centered at 49.92%, the standard deviation is 1.58%.

**Uniqueness** measures the ability of a PUF to be uniquely distinguished among a set of PUFs. Based on [32], we define this metric to be the average inter-class HD of responses \( r_i, r_j \) under the same challenges \( c \) for a randomly picked PUF pair \( (f_i, f_j) \):

\[
\text{HD}(f_i, f_j) = E_{(r_i, r_j)}[\text{HD}(r_i, r_j)] = E_c[\text{HD}(f_i(c), f_j(c))].
\]

For ideal PUFs, responses under the same challenges are orthogonal, namely, \(\text{HD}(f_i, f_j)'s \) are close to 50%. Uniqueness is also evaluated under the ciphertext distribution.

Uniqueness is shown in Figure 5, evaluated for 1000 PUF instances. The lattice PUF achieves near-optimal uniqueness: inter-class HD is centered at 50.00%, its standard deviation is 1.58%.

**Reliability** of a PUF \( f \) is characterized by the average BER of outputs with respect to their enrollment values:

\[
\text{BER} = E_{f'}[\text{HD}(f, f')] = E_{f', c}[\text{HD}(f(c), f'(c))].
\]

As discussed in Section III, the overall BER of the lattice PUF is due to two components: the failure rate of key reconstruction and LWE decryption error rate. Intra-class HD in Figure 5 reflects the result of decryption errors by assuming a perfect key reconstruction.

| Raw BER (%) | Error-Correcting Code | Raw POKs |
|-------------|-----------------------|----------|
| 1           | [236, 128, 14]        | N/A      | 2,360   |
| 5           | [218, 128, 11]        | [3, 1, 1]| 6,540   |
| 10          | [220, 128, 12]        | [5, 1, 2]| 11,000  |
| 15          | [244, 128, 15]        | [7, 1, 3]| 17,080  |

**Fig. 4:** Uniformity of lattice PUF output.

**Fig. 5:** Uniqueness and reliability of lattice PUF output.

**Fig. 6:** POK uses an FE to ensure stability of the secret seed.


TABLE II: (a) Area consumption and (b) runtime of our reference lattice PUF implementation on Xilinx Spartan-6 FPGA.

(a)

| Module      | Size [slices] |
|-------------|---------------|
| LFSR        | 27            |
| LWEDec      | 2             |
| Controller  | 16            |
| Total       | 45            |

(b)

| Step                                  | Time [µs] |
|---------------------------------------|-----------|
| Seed seed $x_k||t$ load for LFSR      | 8         |
| 1-bit decryption from LWEDec          | 39        |
| Total @ 33 MHz                        | 47        |

TABLE III: Hardware implementation costs of strong PUFs.

| Design                | Platform      | PUF Logic [Slices] |
|-----------------------|---------------|--------------------|
| POK+AES [6]           | Spartan 6     | 80                 |
| Controlled PUF [15]   | Spartan 6     | 127                |
| CFE-based PUF [17], [20] | Zynq-7000      | 9,825              |
| Lattice PUF           | Spartan 6     | 45                 |

B. Hardware Implementation Results

We now present the details of implementing the lattice PUF, as shown in Figure 1. The design was synthesized, configured, and tested on a Xilinx Spartan-6 FPGA (XC6SLX45), a low-end FPGA in 45nm technology. The core block implementing the LWE decryption function (LWEDec) includes an 8-bit MAC and a quantization block, as shown in Figure 2). The 256-bit LFSR is implemented using RAM-based shift registers.

We use an FE to generate a key of 1160 bits with the failure rate of reconstruction targeted at $10^{-6}$. We adopt the homogeneous error assumption, i.e., all cells have the same BER [7]. Prior work shows that intrinsic BERs of the various POKs range from 0.1% [22] to 15% [30]. We study the POK designs under four levels of raw BER: 1%, 5%, 10%, and 15% to explore design costs. We use concatenated error-correcting codes, with a repetition code as the inner code, and a shortened BCH code as the outer code. Concatenated codes are typically more efficient than single codes in terms of code length and hardware cost [7]. Table I lists the configuration of error-correcting codes used at different BER levels. At the raw BER of 5%, 6.5 K cells are needed to construct the secret $s$ of length 1160 bits at the target failure rate $10^{-6}$.

The total size of the lattice PUF (without FE) for the Spartan-6 platform is 45 slices, most of which is taken up by the LFSR and the controller. Table IIa shows the breakdown of resources needed to realize the various modules. The total latency (at 33.3MHz clock) to generate a 1-bit PUF response is 47µs, and the total time to generate a 100-bit PUF response is, approximately, $8µs + 100 \times 39µs \approx 3.9ms$ since seed loading is only executed once. Table IIb lists the latency of each step of response generation.

We compare the implementation cost of the lattice PUF against established strong PUF designs [6], [15], [20] in Table III. The original strong PUF based on AES [6] is implemented as an ASIC. Here, we adopt [10] as an FPGA alternative to estimate the implementation cost of AES. Notice that [6] uses no error correction since it guarantees reliability via dark bit masking. Similarly, the FPGA implementation of SHA-3 [21] is adopted to estimate the cost of a hash function for the controlled PUF [15]. The FPGA utilization result of the strong PUF based on the computational FE (CFE) is presented via the number of LUTs in [20]. We estimate the corresponding slice count using [41]. We find that the implementation cost of the lattice PUF (without FE) is cheaper than that of AES on POK, controlled PUF, and CFE-based PUF.

Detailed costs of error correction in POKs for the lattice PUF with different raw BERs are presented in Table IV. Assuming raw POK BER of 5%, the FE design of the lattice PUF requires 233 slices. This is cheaper than linear solver block used in the CFE-based strong PUF [17], [20] which requires 65, 700 LUTs and 16, 425 slices.

V. Conclusion

In this paper, we described a new strong physical unclonable function (PUF) that is provably secure against machine learning (ML) attacks with both classical and quantum computers. The security is derived from cryptographic hardness of learning decryption functions of semantically secure public-key cryptosystems within the probably approximately correct framework. The proposed PUF compactly realizes the decryption function of the learning-with-errors (LWE) public-key cryptosystem as the core block. We implemented a lattice PUF on a Spartan 6 FPGA. The design realizes a challenge-response pair space of size $2^{136}$, requires 1160 physically obfuscated key bits, and guarantees 128-bit ML resistance. The PUF shows excellent uniformity, uniqueness, and reliability.

REFERENCES

[1] A. Akavia, S. Goldwasser, and V. Vaikuntanathan. Simultaneous hardcore bits and cryptography against memory attacks. In Theory of Cryptography Conference, pages 474–495. Springer, 2009.
[2] M. R. Albrecht, R. Player, and S. Scott. On the concrete hardness of learning with errors. Journal of Mathematical Cryptology, 9(3):169–203, 2015.
[3] D. Apon, C. Cho, K. Eldefrawy, and J. Katz. Efficient, reusable fuzzy extractor from lwe. In International Conference on Cyber Security Cryptography and Machine Learning, pages 1–18. Springer, 2017.
[4] G. T. Becker. The gap between promise and reality: On the insecurity of xor arbiter pufs. In International Workshop on Cryptographic Hardware and Embedded Systems, pages 535–553. Springer, 2015.
[5] D. J. Bernstein. Introduction to post-quantum cryptography. In Post-quantum cryptography, pages 1–14. Springer, 2009.
[6] M. Bhargava and K. Mai. An efficient reliable puf-based cryptographic key generator in 65nm cmos. In Proceedings of the conference on Design, Automation & Test in Europe, page 70. European Design and Automation Association, 2014.
[7] C. Bosch, J. Guajardo, A.-R. Sadeghi, J. Shokrollahi, and P. Tuyls. Efficient helper data key extractor on fpas. In International Workshop on Cryptographic Hardware and Embedded Systems, pages 181–197. Springer, 2008.
TABLE IV: Hardware utilization in FE design on Spartan 6 FPGA.

| Raw BER (%) | Outer Code | Inner Code | Total |
|-------------|------------|------------|-------|
|             | Reg LUT Slice | Reg LUT Slice | Reg LUT Slice |
| 1           | 905 895 276   | 0 0 0      | 905 895 276   |
| 5           | 730 688 232   | 0 0 0      | 730 688 232   |
| 10          | 785 740 243   | 0 0 0      | 785 740 243   |
| 15          | 973 913 326   | 0 0 0      | 973 913 326   |

[8] Z. Brakerski, A. Langlois, C. Peikert, O. Regev, and D. Stehlé. Classical hardness of learning with errors. In Proceedings of the forty-fifth annual ACM symposium on Theory of computing, pages 575–584. ACM, 2013.

[9] Y. Chen and P. Q. Nguyen. Bkz 2.0: Better lattice security estimates. In International Conference on the Theory and Application of Cryptology and Information Security, pages 1–20. Springer, 2011.

[10] J. Chu and M. Benaisa. Low area memory-free fpga implementation of the aes algorithm. In Field Programmable Logic and Applications (FPL), 2012 22nd International Conference on, pages 623–626. IEEE, 2012.

[11] B. Fuller, X. Meng, and L. Reyzin. Computational fuzzy extractors. In International Conference on the Theory and Application of Cryptology and Information Security, pages 174–193. Springer, 2013.

[12] S. D. Galbraith. Space-efficient variants of cryptosystems based on learning with errors. url: https://www.math.auckland.ac.nz/~sgal018/compact-LWE.pdf, 2013.

[13] F. Ganji, S. Tajik, F. Füllér, and J.-P. Seifert. Strong machine learning attack against pufs with no mathematical model. In International Conference on Cryptographic Hardware and Embedded Systems, pages 1–411. Springer, 2016.

[14] F. Ganji, S. Tajik, and J.-P. Seifert. Pac learning of arbiter pufs. Journal of Cryptographic Engineering, 6(3):249–258, 2016.

[15] B. Gassert, M. V. Dijk, D. Clarke, E. Torlak, S. Devadas, and P. Tuyls. Controlled physical random functions and applications. ACM Transactions on Information and System Security (TISSEC), 10(4):3, 2008.

[16] I. Goodfellow, Y. Bengio, A. Courville, and Y. Bengio. Deep learning, volume 1. MIT press Cambridge, 2016.

[17] C. Herder, L. Ren, M. van Dijk, M.-D. Yu, and S. Devadas. Trapdoor computational fuzzy extractors and stateless cryptographically-secure physical unclonable functions. IEEE Transactions on Dependable and Secure Computing, 14(1):65–82, 2017.

[18] G. Hospodar, R. Maes, and I. Verbauwhede. Machine learning attacks on 65nm arbiter pufs: Accurate modeling poses strict bounds on usability. In Information Forensics and Security (WIFS), 2012 IEEE International Workshop on, pages 37–42. IEEE, 2012.

[19] N. Howgrave-Graham. A hybrid lattice-reduction and meet-in-the-middle attack against stm. In Annual International Cryptology Conference, pages 159–169. Springer, 2007.

[20] C. Jin, C. Herder, L. Ren, P. H. Nguyen, B. Fuller, S. Devadas, and M. van Dijk. Fpga implementation of a cryptographically-secure puf based on learning parity with noise. Cryptography, 1(3):23, 2017.

[21] J.-P. Kaps, P. Yalla, K. K. Surapathi, B. Habib, S. Vadlamudi, S. Gurung, and J. Pham. Lightweight implementations of sha-3 candidates on fpgas. In International Conference on Cryptology in India, pages 270–289. Springer, 2011.

[22] J. W. Lee, D. Lim, B. Gassend, G. E. Suh, M. Van Dijk, and S. Devadas. A technique to build a secret key in integrated circuits for identification and authentication applications. In VLSI Circuits, 2004. Digest of Technical Papers. 2004 Symposium on, pages 176–179. IEEE, 2004.

[23] R. Lindner and C. Peikert. Better key sizes (and attacks) for lwe-based encryption. In Cryptographers’ Track at the RSA Conference, pages 319–339. Springer, 2011.

[24] R. Maes, P. Tuyls, and I. Verbauwhede. A soft decision helper data algorithm for sram pufs. In Information Theory, 2009. ISIT 2009. IEEE International Symposium on, pages 2101–2105. IEEE, 2009.

[25] R. Maes, A. Van Herewege, and I. Verbauwhede. Pulky: A fully functional puf-based cryptographic key generator. In International Workshop on Cryptographic Hardware and Embedded Systems, pages 302–319. Springer, 2012.

[26] A. Maiti, V. Gunreddy, and P. Schaumont. A systematic method to evaluate and compare the performance of physical unclonable functions. In Embedded systems design with FPGAs, pages 245–267. Springer, 2013.

[27] R. Kumar and W. Burleson. On design of a highly secure puf based on non-linear current mirrors. In Hardware-Oriented Security and Trust (HOST), 2014 IEEE International Symposium on, pages 38–43. IEEE, 2014.

[28] J. W. Lee, D. Lim, B. Gassend, G. E. Suh, M. Van Dijk, and S. Devadas. A technique to build a secret key in integrated circuits for identification and authentication applications. In VLSI Circuits, 2004. Digest of Technical Papers. 2004 Symposium on, pages 176–179. IEEE, 2004.

[29] R. Lindner and C. Peikert. Better key sizes (and attacks) for lwe-based encryption. In Cryptographers’ Track at the RSA Conference, pages 319–339. Springer, 2011.

[30] R. Maes, P. Tuyls, and I. Verbauwhede. A soft decision helper data algorithm for sram pufs. In Information Theory, 2009. ISIT 2009. IEEE International Symposium on, pages 2101–2105. IEEE, 2009.

[31] R. Maes, A. Van Herewege, and I. Verbauwhede. Pulky: A fully functional puf-based cryptographic key generator. In International Workshop on Cryptographic Hardware and Embedded Systems, pages 302–319. Springer, 2012.

[32] A. Maiti, V. Gunreddy, and P. Schaumont. A systematic method to evaluate and compare the performance of physical unclonable functions. In Embedded systems design with FPGAs, pages 245–267. Springer, 2013.

[33] M. Majzoobi, F. Koushanfar, and M. Potkonjak. Lightweight secure pufs. In Computer-Aided Design, 2008. ICCAD 2008. IEEE/ACM International Conference on, pages 670–673. IEEE, 2008.

[34] D. Micciancio and O. Regev. Lattice-based cryptography. In Post-quantum cryptography, pages 147–191. Springer, 2009.

[35] M. Mohri, A. Rostamizadeh, and A. Talwalkar. Foundations of machine learning. MIT press, 2012.

[36] O. Regev. On lattices, learning with errors, random linear codes, and cryptography. Journal of the ACM (JACM), 56(6):34, 2009.

[37] U. Rührmair, F. Sehnke, J. Söltner, G. Dror, S. Devadas, and J. Schmidhuber. Modeling attacks on physical unclonable functions. In Proceedings of the 17th ACM conference on Computer and communications security, pages 237–249. ACM, 2010.

[38] G. E. Suh and S. Devadas. Physical unclonable functions for device authentication and secret key generation. In Proceedings of the 44th annual Design Automation Conference, pages 9–14. ACM, 2007.

[39] A. Vijayakumar, V. C. Patil, C. B. Prado, and S. Kundu. Machine learning resistant strong puf: Possible or a pipe dream? In Hardware Oriented Security and Trust (HOST), 2016 IEEE International Symposium on, pages 19–24. IEEE, 2016.

[40] X. Xi, H. Zhuang, N. Sun, and M. Orshansky. Strong subthreshold current array puf with 2 65 challenge-response pairs resilient to machine learning attacks in 130nm cmos. In VLSI Circuits, 2017 Symposium on, pages C268–C269. IEEE, 2017.

[41] Xilinx. Zynq-7000 SoC Data Sheet: Overview, 7 2018. v1.11.1.

[42] X. Xu, U. Rührmair, D. E. Holcomb, and W. Burleson. Security evaluation and enhancement of bistable ring pufs. In International Workshop on Radio Frequency Identification: Security and Privacy Issues, pages 3–16. Springer, 2015.

[43] M.-D. Yu, M. Hiller, J. Delvaux, R. Sowell, S. Devadas, and I. Verbauwhede. A lockdown technique to prevent machine learning on pufs for lightweight authentication. IEEE Transactions on Multi-Scale Computing Systems, 2(3):146–159, 2016.