OFDM based FFT compared with OFDM based wavelet transform

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Abstract. In recent year the orthogonal frequency division multiplexing widely used in wireless communication system, the conventional structure of OFDM used Inverse Fast Fourier Transform for modulation and multiplexing the data at the transmitter side and used Fast Fourier Transform at the receiver to demodulation and demultiplexing the data. In this paper, Inverse Wavelet Transform IWT used in transmitter instead of IFFT and Wavelet Transform WT in receiver side instead of FFT and compared between two methods with different noise in the AWGN channel. The performance found by bit error rate BER. Also different modulation scheme have been used (16-QAM, 32-QAM, 64-QAM and 128-QAM) and different types of wavelet filters Daubechies (Db-3, Db-5, Db-8, Db-10) and Haar filter to compare between them and find which achieved better performance.

Keywords: FFT, DWT, OFDM, BER.

1. Introduction
Increasing demand for high performance wireless communication systems find key interest in a multicarrier transmission technique called as orthogonal frequency division multiplexing (OFDM). The requirement of high data rate is achieved by means of OFDM technique [1]. It is employed to improve the spectral efficiency and to reduce the channel distortion. The core concept of OFDM is splitting the entire bandwidth to number of subcarriers on which modulation and multiplexing is performed.

The general OFDM system incorporates IFFT (Inverse Fourier transform) and FFT (Fourier transform) to multiplex the signals together at transmitter and de-multiplex the signal at the receiver respectively. It adds the usage of cyclic prefix (CP) to the transformed signal and transmits to the channel. The purpose of combine cyclic prefix is to reduce inter symbol interference (ISI). It also has the disadvantage of reducing the power efficiency, data throughput and spectral containment of the channel.

In this paper, wavelet transform WT is used in place of FFT block of OFDM system. WT finds it primary application in the field of signal processing which is the base of any wireless communication system. It involves the mathematical properties and benefits of both fractional Fourier and wavelet transform. Due to this, the ISI which results in loss of information in communication system can be reduced. In this work an investigation into the performance of discrete wavelet transform based multicarrier system using zero forcing equalization in time domain is presented. The results produced were then compared to that of conventional FFT-based OFDM system. The proposed system shows superior BER performance.
2. OFDM basics

During modulation, OFDM symbols are typically divided into frames, so that the data will be modulated frame by frame in order to the received signal be in sync with the receiver. Long symbol periods diminish the probability of having inter-symbol interference, but could not remove it. To make ISI nearly eliminated, a cyclic extension (or cyclic prefix) is added to each symbol period. An exact copy of a fraction of the cycle, typically 26% of the cycle, taken from the end is added to the front. As shown in figure 1, a guard period, another name for the cyclic extension, is the amount of uncertainty allowed for the receiver to capture the starting point of a symbol period, such that the result of FFT still has the correct information [2].

![Figure 1. Cyclic Extension Tolerance.](image)

We will separate some of the basics of OFDM used widely in the communication system where we can consider it a kind of modulation technique and also a kind of technique multiplexing. If we talk about the basic of OFDM is a splitting or separation high data rate stream for low data rate stream. They are all sent at the same time on more than one carrier. The frequency band is divided into more than one sub-channel and is called a multi carrier modulation and is transmitted via a sub-channel and each two subcarriers is perpendicular and each is separated separately figure 2.

![Figure 2. OFDM Basic.](image)

2.1. What is Multipath Transmission?

Through the above figure we note that if we had a user trying to use my mobile while moving from one place to another, we will notice there are some obstacles that hinder it. For example, the buildings and cars are all obstacles to the transmission of the signal from transmission to receiver, we find that the signal collides with these obstacles and change the path, so access to the receiver at different times depending on the type of obstacle that hit him.

One of the causes of inter symbol interference is multipath propagation in which a wireless signal from a transmitter reaches the receiver via multiple paths. The causes of this include reflection (for instance, the signal may bounce off buildings), refraction (such as through the foliage of a tree) and atmospheric effects such as atmospheric ducting and ionosphere reflection [3]. Since the various paths can be of different lengths, this results in the different versions of the signal arriving at the receiver at different times. Another cause of inter symbol interference is the transmission of a signal through a band limited channel, i.e., one where the frequency response is zero above a certain frequency (the cutoff frequency). Passing a signal through such a channel results in the removal of frequency
components above this cutoff frequency. In addition, components of the frequency below the cutoff frequency may also be attenuated by the channel.

2.2 Inter Symbol Interference (ISI) Problem

As mentioned above, the signal that transmission from the transmission to the receiver will take different paths and thus reach the distortion at different times leads to the overlap of the symbol with each other so the attenuation is very large.

This phenomenon occurs when the transmitter is a high data rate because the time of each symbol is relatively small [4]. If the symbol time is relatively less than the delay time between the arrivals of the symbol with each other, this problem appears very clearly, especially in the systems which use one carrier only one frequency which is the way data transfer is used but there are solutions to overcome this phenomenon:

1. Equalizers channel: An extra circuit is placed before the transmission process so that it operates a balance for frequency component power so that each component is the same power. In other words, if we have high frequencies and often have very low power or frequencies low and often have a very high power here in both cases, power will be different. We came to this way to work balance for power.

2. Reduce data rate: Reduce the proportion of data for the system, but this method is relatively unsuccessful, a more efficient approach was achieved by increasing the time of the symbol and at the same time was used more than a carrier orthogonal.

In other words, each symbol is divided by more than one frequency, so we can overcome this phenomenon (ISI). The duration of each session is divided by the symbol on the sub-channel and is greater than the delay time between the arrival of the symbol and some of them.

Thus, this method was used to maintain the ratio of the system data because more than one carrier was used. This method is called serial to parallel conversion. You can reduce ISI using a number of methods. Obviously making the symbols very long in comparison to the channel impulse response will reduce ISI relative to signal power, but this severely limits your signaling rate. Better approaches are to use a matched filter (filter whose impulse response is the time-reversed complex conjugate of the convolution of the transmitter pulse shape and the channel impulse response). You can also employ a multitude of channel equalization schemes either in time or frequency domain, with or without decision feedback.

2.3 Orthogonality

The key to OFDM is maintaining orthogonality of the carriers. If the integral of the product of two signals is zero over a time period, then these two signals are said to be orthogonal to each other. Two sinusoids with frequencies that are integer multiples of a common frequency can satisfy this criterion. Therefore, orthogonality is defined by [5]:

\[
\int_{0}^{T} \cos(2\pi nfot)\cos(2\pi m福特)dt = 0(n \neq m)
\]

where \( n \) and \( m \) are two unequal integers;
\( f_0 \): is the fundamental frequency;
\( T \): is the period over which the integration is taken for OFDM;
\( t \): is one symbol period and \( f_0 \) set to \( 1/T \).

So orthogonality of sub-carriers:

Encode: frequency-domain samples to time-domain sample (IFFT)

In time - domain it will be

\[
x(t) = \sum_{k=-N/2}^{N/2-1} X[k]e^{j2\pi kt/N}
\]

In frequency - domain it will be

\[
X[K]=\frac{1}{N} \sum_{t=-N/2}^{N-1} x(t)e^{-j2\pi kt/N}
\]

Decode: time-domain samples to frequency-domain sample (FFT)

Orthogonality of any two bins:
\[
\sum_{t=N/2}^{N-1} e^{-j2\pi k t} = 0, \forall P \neq K
\]  

2.4 Structure of OFDM By Using IFFT and FFT

As shown in figure 3 we have to implement the OFDM System. Simulation flowchart is MATLAB code for 256 bits processing is given below.

![OFDM block diagram](image)

Figure 3. OFDM block diagram.

And then modulation process until the data modulation is converted from serial to parallel, where we can send each set of bits at the same moment and then insert them into (IFFT) until the guard band is done for each symbol. We can overcome the delay spread in each channel. It is then converted into a radio wave format until it is transmitted via a channel. In the receiver, the process is almost the opposite. It is then inserted into the (FFT) to separate each carrier from the other and data is then converted from parallel to series, by bit and then the process of demodulation is done until the signal reaches the receiver in each symbol is drawn on the other and then the process channel encoder down to the source decoder until the data is restored to its original format.

1. **Source Encoder:** There is a transmission that which data sent by image or voice or text and converted to binary data bit.

2. **Source Decoder:** The original bit is extracted again, and the data format is restored to its original form, whether image, voice or text.

3. **Channel Encoder:** be at transmission and used in the work of encrypting data. There are many types of code used for example (RCPC) rate compatible punctured convolution and this type is used to protect data during transmission in the channel.

4. **Channel Decoder:** When the receiver is used to restore the original data again.

5. **Mapping of Symbol:** When transmission for single carrier transmission group or collect a bit depending on the type of inclusion used for the carrier transmission multi zeros is added to bit to be a symbol.

6. **Recovery of Symbol:** When a receiver is retrieved bit of the symbol and restore the bit of its natural form.

7. **Modulation:** We also note in the diagram below in figure 4 that each symbol consists of a set of serial bits are entered in serial to parallel is a register with one entry and has more than the exit outlets are dealt with through the range of the carrier.
8. Fast Fourier Transform Inverse (IFFT)
The main purpose of the transmission is to transfer data from the frequency domain to the time domain and then send it via channel.

9. Fast Fourier Transform (FFT)
(FFT) is the conversion of data from time domain to frequency domain and retrieval of data in its original form.

10. Guard Band: After the IFFT operation occurs in the transmitter, the guard band is added to the beginning of each symbol and is required to be greater than the maximum delay of the channel so that the (ISI) phenomenon arising from the multipath does not occur. The simulation result in figure 5.
Figure 5. Simulation Result using FFT.

3. Wavelet Transform
A wavelet is a wave-like oscillation with an amplitude that begins at zero, increases, and then decreases back to zero. It can typically be visualized as a "brief oscillation" like one recorded by a seismograph or heart monitor. Generally, wavelets are intentionally crafted to have specific features that make them useful for data processing. Using a "reverse, shift, multiply and integrate" technique called convolution, wavelets can be combined with known portions of a damaged signal to extract information from the unknown portions [6].

An accurate bandwidth estimation scheme based on wavelet transform (WT) is proposed for OFDM, which estimates OFDM bandwidth using wavelet decomposition, signal reconstruction, moving covariance calculation and statistic histogram method in turn. Without requiring information
about incoming signal and SNR level, the proposed scheme can estimate bandwidth parameter automatically and accurately even at low SNRs. Performance of the proposed scheme is shown by the theoretical arguments and simulations.

Used WT. Generally, an approximation to WT is used for:

1. data compression if a signal is already sampled, and the CWT for signal analysis Thus, WT approximation is commonly used in engineering and computer science, and the CWT in scientific research. Like some other transforms, wavelet transforms can be used to transform data, then encode the transformed data, resulting in effective compression. For example, JPEG 5000 is an image pressure standard that uses by orthogonal wavelets. This means that although the frame is over complete, it is a tight frame (see types of frames of a vector space), and the same frame functions (except for conjugation in the case of complex wavelets) are used for both analysis and synthesis, i.e., in both the forward and inverse transform. For details see wavelet compression.

2. A related use is for smoothing/denoising data based on wavelet coefficient thresholding, also called wavelet shrinkage. By adaptively thresholding the wavelet coefficients that correspond to undesired frequency components smoothing and/or denoising works can be performed. Denoising in digital signal by soft threshold rule gets its acme at level (2), where much noise can be removed in it as possible while signal information is affect least [7]. In figure 6 some example of denoising signal by using wavelet transform with different level.

3. Wavelet transforms are also starting to be used for communication applications. Wavelet OFDM is the basic modulation scheme used in HD-PLC (a power line communications technology developed by Panasonic), and in one of the optional modes included in the IEEE 1901 standard. Wavelet OFDM can achieve deeper notches than traditional FFT OFDM, and wavelet OFDM does not require a guard interval (which usually represents significant overhead in FFT OFDM systems).
3.1. Structure of OFDM by using IWT and WT

As shown in Figure 7, the block is similar to the block diagram of FFT but used WT at the receiver and the IWT at the transmitter.

3.1.1. Inverse Wavelet Transform Block (IWT)

Is exactly like (IFFT) the main purpose of the transmission is to convert data from the frequency domain to the time domain and then send it by channel each OFDM symbol is transformed into time domain by WT. The cyclic prefix is added to the beginning of the symbol and is removed at the receiver. This causes each subcarrier to be affected by flat fading.

After the IWT operation occurs in the transmitter, the guard band is added to the beginning of each symbol and is required to be greater than the maximum delay of the channel so that the (ISI)phenomenon arising from the multipath does not occur so when we want to send data without
guard band as in the figure 8 that we have three different symbols. Each symbol has a different carrier modulation. As we mentioned previously, the carrier must be orthogonal with some of it. Here we have three paths of the symbol from which it is sent.

1. path zero: It is the most difficult types of paths and it will arrive in same time and thus does not occur any overlap with the symbol and some.

2. path one: It is relatively late for path zero and thus delay shifting in path will occur, so that it will overlap, it will be simple in the symbol and some.

3. path two: It is relatively late for path one and therefore there will be a very large overlap.

![Figure 8. Send without guard band.](image)

3.1.2. Wavelet Transform Block (WT)

Is exactly like (FFT) the main purpose of the transmission is to convert data from the time domain to the frequency domain and then send it by channel.

After the WT operation occurs in the receiver, the guard band is removed to the beginning of each symbol and is required to be greater than the maximum delay of the channel so that the inter symbol interference phenomenon arising from the multipath does not occur so when we want to send data with zero guard bands as in the figure 9 that there is a time period free of any data left between each symbol and the other. Here too we have three path to the symbol through which we send

1. path zero: will reach the symbol without any overlap or without any distortion.

2. path one: There will be some delay and thus will be part of one symbol.

3. path two: which suffers from greater delay will be extracted the largest part of the symbol one. Thus, will not occur interference or distortion in the symbol and some. The output signal is shown in figure 10.

![Figure 9. Send with zero guard band.](image)
4. Compared between FFT and Wavelet Transform

The Fast Fourier Transform (FFT) and the Discrete Wavelet Transform (DWT) are both linear operations that generate a data structure that contains log2(N) segments of various lengths, usually filling and transforming it into a different data vector of length 2N.

The mathematical properties of the matrices involved in the transforms are similar as well. The inverse transform matrix for both the FFT and the DWT is the transpose of the original. As a result, both transforms can be viewed as a rotation in function space to a different domain. For the FFT, this new domain contains basis functions that are Sines and Cosines. For the Wavelet Transform, this new domain contains more complicated basis functions called Wavelets, Mother Wavelets, or analysing wavelets. Both transforms have another similarity. The basic functions are localized in the frequency, making mathematical tools such as power spectra (how much power is contained in a frequency interval) and scalograms (to be defined later) useful at picking out frequencies and calculating power distributions. The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space. Fourier sine and cosine functions are not. This localization feature, along with wavelets’ localization of frequency, makes many functions and operators using wavelets “sparse” when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detection feature in images, and removing noise from time series [7]. The square wave window truncates the sine or cosine function to fit a window of a particular width. Because a single window is used for all frequencies in the Windows Fourier Transform, the resolution of the analysis is the same at all locations in the time-frequency plane. An advantage of Wavelet Transforms is that the windows vary. In order to isolate signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basis functions. A way to achieve this is to have short high-frequency ones. One thing to remember is that Wavelet Transforms do not have a single set of basic functions like the Fourier Transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. Thus, wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis [7].

5. Simulation And Results

The performance of used FFT and WT in OFDM is evaluated in this section where used (16-QAM, 32-QAM, 64-QAM and 128-QAM) for mapping the data and (AWGN) noise in the channel since it is
the noise in OFDM. In wavelet transform used different types of filters (Db-3, Db-4, Db-8, Db-10 and Haar) in order to compare between them and with AWGN noise 20db the results of BER of each case in table 1, which conclude that WT (Haar filter) better than other types of wavelet filter and FFT. This mean that Haar filter more suitable for OFDM signal. The suitability come from the shape of the Haar filter near to the shape of the OFDM signal. In table 2 increase the AWGN noise to 30db and found the BER.

**Table 1.** The result of BER for FFT and WT and AWGN noise 20db.

| Modulation scheme | WT With different Filter | FFT |
|-------------------|--------------------------|-----|
|                   | Db-3                     | Db-5| Db-8| Db-10| Haar |
| 16-QAM            | 8.76123*10⁻¹            | 8.9354*10⁻¹| 3.72122*10⁻⁴| 5.57611*10⁻⁴| 13.34325*10⁻⁶| 6.45632*10⁻⁵|
| 32-QAM            | 8.8726*10⁻¹            | 9.01215*10⁻¹| 5.74321*10⁻³| 6.83254*10⁻³| 4.18747*10⁻³| 8.26788*10⁻⁴|
| 64-QAM            | 9.47103*10⁻¹           | 9.6548*10⁻¹| 8.21984*10⁻²| 7.92533*10⁻²| 7.43276*10⁻²| 5.43287*10⁻²|
| 128-QAM           | 9.74823*10⁻¹          | 9.94651*10⁻¹| 6.47258*10⁻¹| 8.03627*10⁻¹| 3.04345*10⁻¹| 3.89321*10⁻¹|

**Table 2.** The result of BER for FFT and WT and AWGN noise 30db.

| Modulation scheme | WT With different Filter | FFT |
|-------------------|--------------------------|-----|
|                   | Db-3                     | Db-5| Db-8| Db-10| Haar |
| 16-QAM            | 14.454237*10⁻¹          | 15.44271*10⁻¹| 9.23991*10⁻⁴| 8.14439*10⁻⁴| 21.3425*10⁻⁶| 11.12462*10⁻⁵|
| 32-QAM            | 16.04652*10⁻¹          | 15.81231*10⁻¹| 7.82713*10⁻³| 10.36278*10⁻³| 11.51219*10⁻³| 9.61451*10⁻³|
| 64-QAM            | 17.49237*10⁻¹          | 17.72334*10⁻¹| 6.40247*10⁻²| 13.35612*10⁻²| 10.42523*10⁻²| 7.27647*10⁻²|
| 128-QAM           | 17.3764*10⁻¹          | 19.15731*10⁻¹| 9.33926*10⁻¹| 18.92912*10⁻¹| 12.3519*10⁻¹| 20.88352*10⁻¹|

6. Conclusions
The conventional structure of OFDM system utilizes FFT as a multiplexer, in this paper proved wavelet transform more suitable and contrary for noise without denoising in the system since WT is mapping the signal respect to the frequency and location. Where different types of wavelet filters managed in order to find the more performance in OFDM signal which proved the Haar filter is the better one using BER evaluation. Also, FFT need to cyclic prefix but in the WT do not need to the cyclic prefix because the overlapping properties of WT thus the bandwidth efficiency increased. Also, the modulation scheme 128-QAM is more suitable than other types of modulation scheme.

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