Hydrophone’s sensitivity calibration based on its complex transfer function

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Abstract. A novel approach to calibrate ultrasonic hydrophone’s sensitivity magnitude has been developed. The primary calibration method known as self-reciprocity with an auxiliary transducer was improved with a measurement technique based on the assessment of complex transfer function of the system. The protocol was experimented from 1.0 MHz to 7.0 MHz, and thereafter compared with a previous calibration realized at the National Physical Laboratory (NPL, Teddington, UK). Within the frequency range of interest, the novel calibration results were statistically identical to the previous calibration data. The 95% confidence level uncertainty varied from 6.6% to 7.5%.

1. Introduction

For ultrasound hydrophones, calibration means to determine their sensitivity, that is, its voltage response (output) to a dynamic pressure that reaches its active element. The ratio between the amplitude of the voltage measured at the output of the hydrophone and the amplitude of the acoustic pressure wave incident on the active element of the hydrophone defines the reception sensitivity (V/Pa). Self-reciprocity method is a traditional, long time validated and well known on ultrasonics [1]. Reciprocity refers to a reciprocal transducer that acts as a projector and receptor. The reciprocity coefficient ($J$) is defined as the ratio of the free-field voltage sensitivity of the transducer, $M$, to its transmitting response to current, $S$, [1], [2]. Typically, measurements are carried out at discrete frequencies. A relatively simple measurement setup is used, as far as reciprocity procedure demands electrical measurements and mechanical quantities in the reciprocal coefficient. The self-reciprocity method requires a measurement of an open circuit voltage appearing at the terminals of the transducer when it is operating as a receiver in a pulse-echo mode.

A novel technique to calibrate the magnitude of hydrophone’s sensitivity is introduced, using the transducer transfer function. The method employed in the present work is based on auxiliary transducer acting in a pulse-echo setup and on electrical measurements such as, voltage and current.

An advantage with the proposed system and method, it is possible to calibrate hydrophones from common equipment in a laboratory of ultrasound. The obtained hydrophone sensitivity was compared
with their sensitivity as calibrated at NPL (UK’s National Physical Laboratory). To establish a metrological base of comparison, an uncertainty model was developed, and the results are the kernel of the present document.

2. Theoretical approach

2.1. Self-reciprocity

The theoretical principles of self-reciprocity calibration are described in [2] and [3], and standardized by [1]. As it is an absolute calibration method, the pressure on the transducer face is obtained directly from the electrical voltage and current, and the current obtained indirectly by measuring the impedance of the probe, and mechanical quantities present in the parameter plane wave reciprocity \( J = \frac{2A}{\rho c} \), where \( A \) is the effective area of the auxiliary transducer, \( \rho \) is the density of the propagation medium and \( c \) is the propagation velocity propagation in medium. The echo is generated by a reflector (flat face and the reflection coefficient calibrated in advance) placed in a specific field position, as shown in Figure 1. Knowing the transducer transfer function, as a function of frequency \( \omega \), it is possible to determine the pressure field at the point where the target reflector was positioned.

![Figure 1. Self-reciprocity pulse-echo scheme.](image)

2.2. Determination of the transfer function

Traditional transfer function assessment is based in the complex frequency spectrum analyses, and it has been published in the literature [4, 5]. The novel approach used in the present work deals with the transfer function in the time domain, instead, as presented by [6]. Despite the technique uses the waveforms in the time domain, as they are used single frequency tone burst as excitation, it is straightforward to compute all parameters as function of \( \omega \), i.e, the angular frequency.

2.3. Magnitude of the sensitivity

Joining both methods, i.e., self-reciprocity and determination of the transfer function, the hydrophone sensitivity (M) can be expressed as function of \( \omega \) by:

\[
M_h(\omega) = \frac{V_h(\omega)\sqrt{2A \cdot R(\omega)Z(\omega)G_1(\omega)}}{\sqrt{V_{open}(\omega)\frac{I_1(\omega)}{I_0(\omega)}V(\omega)\rho_0c_0 \cdot G_2(\omega)}}
\]

where \( V_h(\omega) \) is the output signal from the hydrophone placed at the same position that the transducer previously calibrated by self-reciprocity; \( G_1(\omega) \) and \( G_2(\omega) \) and are the correction due diffraction in either way from the transducer to the target and vice-versa, respectively; \( V_{open}(\omega) \) is the apparent open circuit voltage measured in the reception activity of the transducer when loaded by a finite impedance; \( \frac{I_1(\omega)}{I_0(\omega)} \) is a correction factor to compute correctly the open circuit voltage, being \( I_0(\omega) \) the measured current and \( I_1(\omega) \) is computed to correct the measurement to the open circuit situation;
$V(\omega)$ is the driven voltage to the transducer in the output mode; $\rho_0$ and $c_0$ are the density and the speed of sound in the medium, respectively; $A$ is the effective radiation area of the transducer; $R(\omega)$ is the reflection coefficient of the reflecting target; and $Z(\omega)$ is the complex impedance of the transducer. The complete derivation of (1) is omitted for simplification in this text.

2.4. Measurement uncertainty
The approach depicted in [7] was applied in (1), deriving the Type B uncertainties for the hydrophone sensitivity. Type A source of uncertainty was calculated as the standard deviation of the mean after a repeatability experiment.

3. Material and method
A schematic drawing of the experimental setup of the system to perform calibration of a hydrophone is presented in Figures 2.

![Figure 2. Experimental setup](image)

The following parameter are recorded in the during a hydrophone calibration (see (1)): frequency (MHz), $V(\omega)$, $V_{\text{open}}(\omega)$, $V_h(\omega)$, water temperature (°C), pulse echo distance $d_1$, $R(\omega)$, $Z(\omega)$, $R_k(\omega)$, $G_1(\omega)$. The correction factor $G_2(\omega)$ should be applied to the pressure reaching the surface of the sensor element hydrophone as a function of normalized distance and the ratio of the effective radius of the probe emission and hydrophone. The short-circuit voltage $V_k(\omega)$ and short-circuit resistance $R_k(\omega)$ both used to assess $I_k(\omega)$. The transducer and the target reflector should be fully submerged in water, at least 100 mm below the water level in the tank. The transducer should be aligned with the target reflector. The hydrophone under test used must be aligned with the transducer. The water temperature should be measured with the measuring end of the probe located at least 100 mm below the water level in the tank. The central region of the face of the hydrophone’s active element must be located approximately 180 mm from the geometric center of the face of the target reflector toward the horizontal axis (x-axis). The amplitude measured generator to 20 V peak to peak with a "burst" of 20 cycles. The transducer was moved in the Z axis to position $d_1$ away from the target reflector. The distance $d_1$ is determined using the formula $Z_0 = d^2/4\lambda$, where $d$ is the diameter of the transducer, and $\lambda$ is given by $\lambda = c/f$, where $c$ is the propagation speed of ultrasound in water and $f$ is operating frequency of the transducer. The two way pulse echo distance $2d_1$ should be within the following
range: \( 1.5Z_e \leq 2d \leq 3.0Z_e \) [1]. Type A uncertainty is assessed as the precision under repeatability conditions after 4 repetitions. The high frequency signal was generated using an arbitrary wave generator model AWF 33250 (Agilent Technologies, USA), and voltage measurements are undertaken with an oscilloscope model TDS 3012 (Tektronix Inc., USA). \( R_s \) (in \( \Omega \)) and the 50 \( \Omega \) shunt are in series and parallel to the circuit, respectively. The reflecting target is a stainless steel cylinder (\( \varnothing = 80 \text{ mm diam} \times 80 \text{ mm long} \)).

4. Results

Table 1 disclose the results, comprising the calibrated values using the present method \( M_h (\omega) \) and its expanded uncertainty (with 95% confidence level), the calibration values from the NPL with their uncertainty, and the normalized error (\( E_{\text{norm}} \)).

| \( F \text{[MHz]} \) | \( M_h \text{[V/Pa]} \) | \( U_u \text{[V/Pa]} \) | \( M_h \text{[V/Pa]} \) | \( U_u \text{[V/Pa]} \) | \( E_{\text{norm}} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1               | 66.7            | 4.9             | 71              | 4.3             | 0.66            |
| 2               | 67.7            | 4.6             | 74              | 4.4             | 0.99            |
| 3               | 70.5            | 4.7             | 75              | 4.5             | 0.69            |
| 5               | 78.4            | 5.2             | 76              | 4.6             | 0.35            |
| 7               | 80.4            | 5.3             | 76              | 4.6             | 0.63            |

5. Conclusion

Table 1 summarize the results of this work. For all frequencies tested, the difference between the calibration from NPL and the results with the proposed method are statistically equal, using both uncertainties as parameter. It can be affirmed as the differences in all cases are lower than the addition of the respective uncertainties. From that point of view, the present method could be considered validated. Further, the uncertainties obtained are numerically not much greater than those provided by NPL. Some improvements particularly in the voltage measurements should be applied, the uncertainty would be even lower.

Acknowledgments

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6. References

[1] IEC International Electrotechnical Commission 2013 62127-2:2013 Ultrasonics – Hydrophones – Part 2: Calibrations for Ultrasonic Fields up to 40 MHz, ed IEC Central Office (Geneva)
[2] Foldy and Primakoff H 1945 The Journal of the Acoustical Society of America 17 109-120
[3] Carstensen E L 1947 The Journal of the Acoustical Society of America 19(4) 961-965
[4] Ludwig G and Brendel K 1988 Transactions on Ultrasonics Ferroelectrics Frequency Control 35(2) 168-174
[5] Radusescu E G, Lewin P A, Wojcik J and et al 2003 *Ultrasonics* 41 247-254
[6] Neer P L M J V, Vos H J and Jong N D 2011 *Ultrasonics* 51 1-6
[7] BIPM Bureau International des Poids et Measures 2008 *Evaluation of Measurement – Guide to the Expression of Uncertainty in Measurement*, (First Edition September 2008), available in [www.bipm.org](http://www.bipm.org) accessed in February 2014.