A Robust Approach to Chance Constrained Optimal Power Flow with Renewable Generation

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Abstract—Optimal Power Flow (OPF) dispatches controllable generation at minimum cost subject to operational constraints on generation and transmission assets. The uncertainty and variability of intermittent renewable generation is challenging current deterministic OPF approaches. Recent formulations of OPF use chance constraints to limit the risk from renewable generation uncertainty, however, these new approaches typically assume the probability distributions which characterize the uncertainty and variability are known exactly. We formulate a Robust Chance Constrained (RCC) OPF that accounts for uncertainty in the parameters of these probability distributions by allowing them to be within an uncertainty set. The RCC OPF is solved using a cutting-plane algorithm that scales to large power systems. We demonstrate the RCC OPF on a modified model of the Bonneville Power Administration network, which includes 2209 buses and 176 controllable generators. Deterministic, chance constrained (CC), and RCC OPF formulations are compared using several metrics including cost of generation, area control error, ramping of controllable generators, and occurrence of transmission line overloads as well as the respective computational performance.

I. INTRODUCTION

A. Motivation

The continually growing penetration of intermittent renewable energy resources, e.g., wind and solar photovoltaic, is revealing a number of drawbacks in existing power system operational procedures that may limit the integration of these new resources. Wind generation is an intermittent and not fully dispatchable generation technology that imposes challenges to least-cost, risk-averse management of generation and transmission assets. One approach to address these challenges is strategic investments in more transmission and controllable generation capacity to enhance the system flexibility [1]. [2].

These investments are costly and subject to a variety of regulatory and policy limitations. On the other hand, improving operating protocols may create additional flexibility in the existing system by replacing ad hoc deterministic policies for limiting system risk from intermittent generation with probabilistic formulations that account for intermittency in a principled manner.

Historically, these deterministic policies were designed to account for less challenging deviations of load from its forecast value. They have performed very well for their design conditions where fluctuations in load are a small fraction of the total load, however, they are not expected to cost effectively manage risk when net-load fluctuations are large. Recent regulatory initiatives, such as Federal Energy Regulatory Commission (FERC) Orders 764 [3] and 890 [4] have identified the need for a new generation of operating protocols and decision-making tools for the successful integration of renewable generation.

In this manuscript, we implement a distributionally robust chance constraint (RCC) optimal power flow (OPF) model and compare it with the deterministic OPF and chance constrained (CC) OPF models. The deterministic OPF is a typical short-term decision-making tool used by a number of utilities and its implementation in this work aims to give a reasonable benchmark for comparison. The chance constrained (CC) OPF limits the probability of violating transmission or generation constraints using a statistical model of wind deviations from forecast values that is zero mean and Gaussian distributed. As compared to the CC OPF, the RCC OPF is a generalization that allows for uncertainty in the mean and variance of the wind forecast error. To demonstrate the effect of using probabilistic methods in an OPF, we compare these different OPF formulations in the setting of vertically-integrated grid operations, specifically, on a modification of the Bonneville Power Administration (BPA) system.

B. Literature review

Traditional deterministic OPF models [5] dispatch controllable generation using the central (most likely) wind forecast, i.e., they do not endogenously account for the variability and uncertainty of wind generation [6]. Exogenously calculated reserve margins and heuristic policies [7], [8] are often used to enforce additional security requirements to ensure system reliability. However, these heuristic approaches are limited in their ability to produce cost-efficient solutions [9], [10].

Recently, a number of transmission-constrained OPF and unit commitment (UC) models based on stochastic programming [11]– [13], interval programming [14], [15], chance constrained optimization [18] and robust optimization [16], [17] have been proposed for endogenous risk-averse decision making. A common drawback of stochastic programming and chance constrained optimization is a requirement for accurate statistical models of wind generation uncertainty and variability. In practice, wind generation is modelled using the uni- or multivariate Gaussian distribution, which unavoidably results in solution inaccuracy [19]. In addition, stochastic programming typically requires the generation of a relatively large number of scenarios leading to impractically large computing times, even for relatively large values of the duality gap [20]. If the number of scenarios is reduced by means of scenario reduction techniques [21], the monetary benefits attained with stochastic programming may reduce accordingly [13], [20].

In contrast, interval programming and robust optimization models allow wind generation fluctuations within a certain
range around a central forecast. By disregarding the likelihood of individual scenarios within this range, these methods result in an overly conservative solution as compared to stochastic programming [23]. Although the conservatism of robust optimization can be reduced by adjusting the budget of uncertainty [23] or using dynamic uncertainty sets [24], there is no systematic methodology to choose the value of budget of uncertainty a priori.

Here, we leverage recent work by Bienstock et al. [18] that formulated and implemented a CC OPF using a cutting-plane approach and demonstrated its scalability to large systems. Based on the CC OPF, Bienstock et al. [18] also envisioned an RCC OPF that allowed for uncertainties in the mean and variance of wind forecast errors, but did not implement or test the RCC-OPF. In this manuscript, we build upon the RCC OPF formulation from [18] to reduce the inaccuracy of assuming Gaussian distributions for wind forecast errors. The RCC OPF is implemented in JuMPChance [25], an open source optimization package developed to model ordinary source optimization package developed to model ordinary OPF, CC OPF, and RCC OPF models on this test system.

Finally, Sec. IV presents conclusions and possible directions for future work.

II. RCCOPF FORMULATION

In Sec. I of [18] we review the CC OPF formulation from [18] and restate it with a few modifications. Section I-C discusses a model for distributionally robust chance constraints which are incorporated into a RCC-OPF formulation in Section I-D.

A. Notations

- \( B \) — set of buses
- \( r \) — index of reference bus (\( \in B \))
- \( L \) — set of lines
- \( G \) — set of controllable generators
- \( G_b \) — subset (\( \subset G \)) of generators located at bus \( b \)
- \( \mathcal{W} \) — subset of buses with wind farms
- \( \beta_{mn} \) — susceptance of line \((m, n)\)
- \( f_{mn} \) — real power flow over line \((m, n), \text{MW}\)
- \( \theta_b \) — phase angle at bus \( b \)
- \( p_i \) — output of controllable generator \( i \), MW
- \( d_b \) — demand at bus \( b \), MW
- \( p_{imi} \) — minimum output of generator \( i \), MW
- \( p_{imax} \) — maximum output of generator \( i \), MW
- \( f_{max} \) — capacity of line \((m, n), \text{MW}\)
- \( R_{ui} \) — max ramp-up of generator \( i \) in the OPF period, MW/h
- \( R_{di} \) — max ramp-down of generator \( i \) in the OPF period, MW/h
- \( w_f^b \) — forecast output of wind farm at bus \( b \), MW
- \( \omega_i(t) \) — actual deviation from forecast \( w_f^b \) at time \( t \), MW
- \( c_{i1} \) — linear coefficient of cost for generator \( i \), $/MW
- \( c_{i2} \) — quadratic coefficient of cost for generator \( i \), $/MW²

In the rest of this manuscript, bold symbols denote random variables. In particular, \( \omega \) models deviations \( \omega_i(t) \) within the OPF period, which drive random fluctuations in controllable generator injections \( p_i \), bus phases \( \theta_b \), and power line flows \( f_{mn} \) (described below). We denote the total deviation from the forecast as \( \Omega = \sum_{b \in W} \omega_b \). In the CC-OPF, the deviations \( \omega_b \) are assumed independent and normally distributed with zero mean and known variance \( \sigma_b² \). In the RCC-OPF formulation discussed in Sec. II-D, this assumption is relaxed by introducing symmetric intervals \([-\bar{\mu}_b, \bar{\mu}_b]\) and \([\sigma_b², \bar{\sigma}_b² + \bar{\sigma}_b²]\) for the mean and variance of \( \omega_b \).

B. CCOPF Formulation

The CC-OPF formulation is derived from the following single-stage deterministic OPF:

\[
\min_{\mathcal{G}, \theta} \sum_{i \in \mathcal{G}} (c_{i2}p_i^2 + c_{i1}p_i) \tag{1}
\]

subject to

\[
\sum_{n \in B} B_{bn}\theta_n = \sum_{i \in \mathcal{G}_b} p_i + w_f^b - d_b, \quad \forall b \in B, \tag{2}
\]

\[
p_{i1}^\text{min} \leq p_i \leq p_{i1}^\text{max}, \quad \forall i \in \mathcal{G}, \tag{3}
\]

\[
f_{mn} = \beta_{mn}(\theta_m - \theta_n), \quad \forall \{m, n\} \in L, \tag{4}
\]

\[
-f_{mn}^\text{max} \leq f_{mn} \leq f_{mn}^\text{max}, \quad \forall \{m, n\} \in L, \tag{5}
\]

where \( B \) is the \(|B| \times |B|\) bus admittance matrix defined by:

\[
B_{mn} = \begin{cases} \sum_{k: \{k,n\} \in \mathcal{E}} \beta_{kn}, & m = n \in \mathcal{L} \\ -\beta_{mn}, & m \neq n \in \mathcal{L} \\ 0, & \text{otherwise} \end{cases} \tag{6}
\]

In the deterministic OPF formulation in (1)-(5), the controllable generation set points \( p_i \) are optimized to minimizing the total cost of generation for the forecast wind generation \( w_f \) and demand \( d_b \), subject to operating constraints on generators and transmission lines. In the presence of deviations \( \omega_b \) from \( w_f^* \), we model the proportional response of generators:

\[
p_i = p_i - \alpha_i \Omega, \tag{7}
\]

Here, \( \alpha_i \geq 0 \) is the participation factor for controllable generator \( i \). When \( \sum_i \alpha_i = 1 \), the response rule (7) guarantees that generation and load remain balanced, but does not limit the magnitude of the response of the generators or the resulting flow on the power lines.

As shown in [18], the deterministic OPF in (1) can be reformulated as a CC-OPF by introducing probabilistic constraints
on $f_{mn}$ and $p_i$ and modelling the participation factors $\alpha_i$ as decision variables. We modify the CC OPF formulation in [18] as follows:

$$\min \sum_{i \in \mathcal{G}} \left( c_{i2}(p_i^2 + \var(\Omega)\alpha_i^2) + c_{i1}p_i \right)$$

subject to:

$$\sum_{n \in \mathcal{B}} B_{bn}\theta_n = \sum_{i \in \mathcal{G}_h} p_i + w'_b - d_b, \quad \forall b \in \mathcal{B}$$

$$p_i^{\min} \leq p_i \leq p_i^{\max}, \quad \forall i \in \mathcal{G}$$

$$f_{mn} = \beta_{mn}(\theta_m - \theta_n), \quad \forall \{m, n\} \in \mathcal{L}$$

$$|f_{mn}| \leq f_{mn}^{\max}, \quad \forall \{m, n\} \in \mathcal{L}$$

$$P(p_i - \Omega\alpha_i > p_i^{\max}) < \epsilon_i, \quad \forall i \in \mathcal{G}$$

$$P(p_i - \Omega\alpha_i < p_i^{\min}) < \epsilon_i, \quad \forall i \in \mathcal{G}$$

$$P(\Omega\alpha_i > RD_i) < \epsilon_i, \quad \forall i \in \mathcal{G}$$

$$P(f_{mn} + \beta_{mn}(\Omega\delta_m - \Omega\delta_n) + \beta_{mn}\omega^T(\pi_m - \pi_n) > f_{mn}^{\max}) < \epsilon_{mn}, \quad \forall \{m, n\} \in \mathcal{L}$$

$$P(f_{mn} + \beta_{mn}(\Omega\delta_m - \Omega\delta_n) + \beta_{mn}\omega^T(\pi_m - \pi_n) < -f_{mn}^{\max}) < \epsilon_{mn}, \quad \forall \{m, n\} \in \mathcal{L}$$

$$\sum_{i \in \mathcal{G}} \alpha_i = 1, \quad \alpha \geq 0$$

$$\sum_{i \in \mathcal{G}_r} \alpha_i = \delta_r, \quad \theta_r = 0.$$ 

$$\sum_{n \in \mathcal{B}, n \neq r} B_{bn}\delta_n = \alpha_b, \quad \forall b \in \mathcal{B} \setminus \{r\}$$

Here the decision variables are $p, \theta, \delta, \alpha$ and $f$, where $\delta$ is a nonphysical auxiliary variable. For the quadratic cost of production (assumed to be convex), [18] shows that

$$\mathbb{E}[\sum_{i \in \mathcal{G}} c_{i2}p_i^2 + c_{i1}p_i] = \sum_{i \in \mathcal{G}} \left( c_{i1}(p_i^2 + \var(\Omega)\alpha_i^2) + c_{i2}p_i \right).$$

Therefore, in (8), the CC-OPF seeks to minimize the convex quadratic expected cost of production. Under these assumptions, the CC-OPF is tractable and representable using second-order conic programming (SOCP).

Constraints (9)-(12) are deterministic and enforce power flow feasibility, generator limits, and power line flow limits for $\omega^T$, similar to those in the deterministic OPF of [10]. In chance constraints (13)-(14), the controllable generator outputs are now random and given by Eq. (7). As in [18], these chance constraints bounded by $\epsilon_i$, the probability of the fluctuating generator outputs exceeding their upper limits $p_i^{\max}$ or lower limits $p_i^{\min}$.

Relative to [18], we add constraints (15) and (16) that limit the probability of the real-time response $\alpha_i\Omega$ to wind deviations from the forecast from exceeding $RU_i$ for positive changes and $RD_i$ for negative changes. Here, $RU_i$ and $RD_i$ are the continuous ramping constraints on the generators over the OPF time step.

Chance constraints (13)-(16) on the controllable generator injections are expressed explicitly in terms of $\alpha$ and the random wind fluctuations $\omega$. Chance constraints (17)-(18), which bound the line flows $f_{mn}$, are more subtle. The flows $f_{mn}$ also change with the fluctuating wind injections and the controllable generator response, however, the $f_{mn}$ depend on the wind deviations in an implicit manner, i.e. $f_{mn} = \beta_{mn}(\theta_m - \theta_n)$, where

$$\sum_{n \in \mathcal{B}} B_{bn}\theta_n = \sum_{i \in \mathcal{G}_h} (p_i - \alpha_i\Omega) + w'_b - d_b + \omega_b, \quad \forall b \in \mathcal{B}.$$ 

Bienstock et al. [18] derive explicit equations for $f_{mn}$ by observing that once a reference bus $r \in \mathcal{B}$ is chosen and $\theta_r$ and $\alpha_r$ fixed to zero, the system of equations (23) is invertible and the adjusted phase angles $\theta$ (and hence $f$) can be expressed as a linear function of $\theta$, $\Omega\delta$, and $\omega$:

$$\theta_b = \theta_b - \Omega\delta_b + \pi^T_b\omega$$

where $\theta$ satisfies (9), $\delta$ satisfies (21), and $\pi_b$ is the $b$th row of $\hat{B}^{-1}$ (oriented as a column vector), $B$ is the $(|\mathcal{B}| - 1 \times |\mathcal{B}| - 1)$ submatrix of $\hat{B}$ with the row and column corresponding to the reference bus removed. The variable $\delta$ is introduced solely for computational convenience. The chance constraints (17)-(18) use (24) to express $f_{mn}$ explicitly in terms of the random variables and decision variables.

While chance constraints like (13)-(18) are often nonconvex and difficult to treat in general [26], under the assumption of normality, they are both convex and computationally tractable [27]. In particular, a chance constraint of the form

$$\mathbb{P}(\xi^T x \leq b) \geq 1 - \epsilon$$

is equivalent to

$$\mu^T x + \Phi^{-1}(1 - \epsilon)\sqrt{x^T\Sigma x} \leq b,$$

when $\xi \sim N(\mu, \Sigma)$ where $\Phi^{-1}$ is the inverse cumulative distribution function of the standard normal distribution. In the following, we assume $\epsilon < \frac{1}{2}$ so that $\Phi^{-1}(1 - \epsilon) > 0$ and constraint (26) is convex in $(x, b)$. Note that in this model, we treat each chance constraint independently. Although it would also be natural to pose a model which attempts to enforce that multiple linear constraints hold jointly with high probability, convexity in this case remains an open question, even under the assumption of normality [26, 28].

Constraint (26) is not only convex; it can be represented as a second-order conic constraint handled by many off-the-shelf optimization packages like CPLEX [29] and Gurobi [30]. Indeed, we see that (26) is satisfied iff

$$||\Sigma^{1/2} x||_2 \leq (b - \mu^T x)/\Phi^{-1}(1 - \epsilon).$$
While the representation of the CC-OPF in [8]–[20] with the reformulation of the chance constraints per [26] is quite useful, [18] observed that off-the-shelf solvers were not capable of solving large scale CC-OPF instances. Instead, [18] implemented a specialized algorithm based on sequential linearizations of [26] where they derived an explicit SOCP representation of the above CC-OPF formulation. In this manuscript, the formulation is provided as stated in [8]–[20] to the modeling tool JuMPChance [25], which automatically transformations chance constraints into a form that is recognizable by existing commercial solvers, eliminating a potential source of error in model transcription.

C. Distributionally Robust Chance Constraints

In the analytic reformulation of the chance constraints in [26], the wind deviations $\omega_b$ are assumed to be normally distributed with known (zero) mean and variance, i.e. $\omega \sim N(\mu, \Sigma)$. This approach is computationally tractable but has drawbacks. The assumption of normality is often an approximation, and even when valid, $\mu$ and $\Sigma$ are typically estimated from data and not known exactly. Often, we can only say with confidence that $(\mu, \Sigma)$ fall in some uncertainty set $U$. By reformulating the chance constraints of Sec. II-B to so-called distributionally robust chance constraints, the constraint $\xi^T x \leq b$ is required to hold with high probability under all possible distributions within $U$, i.e.,

$$P_{\xi \sim N(\mu, \Sigma)}(\xi^T x \leq b) \geq 1 - \epsilon, \quad \forall (\mu, \Sigma) \in U. \quad (28)$$

For each $(\mu, \Sigma) \in U$, we have a single convex constraint of the form [25], therefore (28) is a potentially infinite set of convex constraints and is convex itself. Ben-Tal et al. [31] derive semidefinite programming representations of (28) for special cases of $U$ while which theoretically tractable, may be computationally challenging. In contrast, Bienstock et al. [18] describe a cutting-plane algorithm which we adopt and demonstrate is capable of handling large-scale instances.

The cutting-plane approach in [18] iteratively solves a sequence of relaxations of (28). At each iteration, we must verify if (28) is satisfied. In this manuscript, we consider uncertainty sets $U$ that can be partitioned into a product $U = U_\mu \times U_\Sigma$ where $(\mu, \Sigma) \in U$ iff $\mu \in U_\mu$ and $\Sigma \in U_\Sigma$. In this case, (28) holds iff

$$\max_{\mu \in U_\mu} x^T \mu + \Phi^{-1}(1-\epsilon) \sqrt{\max_{\Sigma \in U_\Sigma} x^T \Sigma x} \leq b. \quad (29)$$

For fixed $x$, both of the inner maximization problems in (29) have a linear objective, and so detecting if (28) is satisfied for a given $x$ can be computed by optimizing a linear function over the sets $U_\mu$ and $U_\Sigma$. If the solution to (29) shows that (28) is satisfied, then the algorithm terminates. Otherwise, (29) is used to find the corresponding $(\mu, \Sigma)$ that violates (28), and we add a linearization of the corresponding constraint (26) to the relaxation which cuts off the current solution. Therefore, at any iteration, the relaxation we solve is a linear program, similar to the linearization scheme for CC-OPF. This process repeats until (28) is satisfied within numerical tolerances. Although this algorithm does not have polynomial convergence guarantees in general, it is an immensely powerful approach that also underlies standard techniques such as Benders decomposition. For further discussion, see [18].

D. Formulation for RCC-OPF

We adapt the discussion in Sec. II-C to formulate a RCC-OPF. First, we adopt the assumptions in [18] that the fluctuations at the different wind sites are independent within an OPF time step, i.e. $\Sigma = \text{diag}(\sigma^2)$. The RCC-OPF models uncertainty in the $\omega$ distribution parameters at each bus $b$ as intervals $[\mu_b, \bar{\mu}_b]$ and $[\sigma^2_b, \bar{\sigma}^2_b, \sigma^2_b + \bar{\sigma}^2_b]$ for $\mu$ and $\sigma$, respectively. To represent the aggregate uncertainty, we construct polyhedral uncertainty sets

$$U_\mu = \left\{ \mu \in \mathbb{R}^{|W|} : |\mu_b| \leq \bar{\mu}_b, \sum_{b \in W} |\mu_b| / \mu_b \leq \Gamma_\mu |W| \right\} \quad (30)$$

and

$$U_{\sigma_2} = \left\{ s \in \mathbb{R}^{|W|} : \exists t : s = \sigma^2 + t, |t| \leq \bar{\sigma}^2_b, \sum_{b \in W} |t_b| / \sigma_b^2 \leq \Gamma_\sigma |W| \right\} \quad (31)$$

for the mean, and variance, similar to those proposed by Bertsimas and Sim [32]. The parameters $\Gamma_\mu$ and $\Gamma_\sigma$ are the uncertainty budgets used to adjust the level of conservatism of the resulting RCC-OPF algorithm. The $\Gamma$’s may be interpreted as a bound on the proportion of wind farms that may take their worst case distribution. The least conservative limit $\Gamma = 0$ recovers the standard CC-OPF model. The most conservative limit $\Gamma = 1$ ensures feasibility when each wind farm can take on its worst-case production distribution. For the uncertainty sets in (30) and (31), the optimization (29) required to check feasibility of each robust chance constraint (28) reduces to a cheap sorting operation [32]. Note that, similar to the CC-OPF, the constraints (13)–(18) are treated separately, i.e. when they are “robustified” with respect to the distribution parameters, the worst-case distribution may be different for each constraint. For the objective (22), we simply take the nominal value for $\var(\Omega)$. In the remainder of this manuscript, we set $\Gamma = \Gamma_\mu = \Gamma_\sigma$ and investigate the results as a function of $\Gamma$.

III. Case Study

We investigate the performance and benefits of a RCC-OPF approach relative to CC-OPF and deterministic OPF approaches by implementing all three in the setting of vertically-integrated grid operations, specifically, on a modification of the Bonneville Power Administration (BPA) system. This setting allows us to evaluate the effect of using probabilistic methods in an OPF. We use $\Gamma$ as a parameter to study the impact of robust conservatism on the cost of generation and the statistics of Area Control Error (ACE), generator ramping, and power line loading.

A. Test System and Data

We use a modification of the BPA system with 2866 transmission lines and 2209 buses including 676 load buses,
176 controllable generators, and 24 wind farms. The total installed capacity of controllable generators is 40.6 GW, composed of 133 hydro generators (28 GW), 41 gas-fired generators (9.6 GW), 1 coal-fired generator (1.2 GW), and 1 nuclear generator (1.8 GW). The total installed capacity of 24 wind farms is 4.6 GW. The technical characteristics of the controllable generators and the network configuration are adapted from PowerWorld [33]. Tie-line power flows to neighboring interconnections are modeled as loads at the ends of the tie lines.

The case study spans the Winter season from December 2012 to March 2013 so that the factors affecting wind, load, and hydro generation are relatively stationary. Hour-ahead, hour-resolution load forecasts and their actual 5-minute realizations are taken from [34]. As provided, these data are aggregated across the entire BPA system. We disaggregate the load forecasts and realizations among 676 load buses based on their population density. For each wind farm location, archived NOAA forecast data [35] are used to provide hour-ahead, hour-resolution wind speed forecasts. Actual, 5-minute realizations of wind power using an equivalent wind turbine for each wind farm location [36]. We use the methodology from [37] to compute the intervals $[\hat{\mu}_b, \hat{\mu}_b]$, $[\hat{\sigma}_b^2 - \hat{\sigma}_b^2, \hat{\sigma}_b^2 + \hat{\sigma}_b^2]$ for the uncertainty sets in Eq. (29) and (31).

In this study, we seek to realistically emulate the short-term operational planning of the BPA system operator; however, not all required data are available. To fill in the missing data, we make the following operational assumptions:

a) **Nuclear Generators**: The single nuclear unit in the BPA system is assumed to be a “must run” unit, and its hourly power output $p_i$ is set to the 95% of its nameplate capacity. Its participation factor $\alpha_i$ in real-time balancing is set to zero [38].

b) **Gas and Coal Generators**: The power outputs $p_i$ and participation factors $\alpha_i$ of all gas-fired and one coal-fired generators are decision variables.

c) **Hydro Generators**: Dispatch decisions for hydro generators often depend more on water flow considerations rather than on power system operations [39]. Instead of being co-optimized with thermal and nuclear generators, hydro dispatch levels $p_i$ are fixed in all OPF formulations considered here as exogenous parameters [40]. System aggregated, hour-resolution hydro generation is taken from historical BPA data [34] and disaggregated to individual hydro generators based on their installed capacity. The assignment of participation factors is also affected by water flow conditions beyond the scope of this work. Lacking operational data, we set the participation factors $\alpha_i$ of the hydro generators to a common value, which itself is a decision variable.

We refer interested readers to [41] for the input data and the code used in this case study.

**B. Evaluation Procedure**

The evaluation procedure includes two steps, which emulate the hour-ahead scheduling and real-time dispatch, respectively, and are organized as follows:

1) **Step 1**: The RCC OPF, CC OPF (i.e. RCC OPF with $\Gamma = 0$), and a deterministic OPF are solved for hour $t$ using hour-ahead, hour-resolution wind power forecasts $\omega_{t,b}^\ell$ and load forecasts $d_{t,b}^\ell$ at each bus $b$ and the generator commitment decisions $u_{t,i}$ described in III-A. The result is an optimal hourly dispatch $p_{t,i}^\star$ and hourly participation factors $\alpha_{t,i}^\star$.

2) **Step 2**: Next, these optimal decisions $p_{t,i}^\star$ and $\alpha_{t,i}^\star$ are evaluated in a quasi-static power flow simulation of the system behavior using the actual, 5-minute realizations of wind power $\omega_{t,b}(\tau)$ and demand $d_{t,b}(\tau)$ where $\tau$ refers to the twelve 5-minute intervals of hour $t$. For every $\tau$, we compute the actual power output of each controllable generator $p_{t,i}(\tau)$ as $p_{t,i}(\tau) = \max[p_{t,i}^{\min}, \min[p_{t,i}(\tau), p_{t,i}^{\max}]]$ where

$$p_{t,i}(\tau) = p_{t,i}^\star - \alpha_{t,i}^\star \sum_{j \in W} \omega_{t,b}(\tau). \quad (32)$$

Using $p_{t,i}(\tau)$, a power flow calculation yields the $f_{t,mn}(\tau)$. The actual power output $p_{t,i}(\tau)$ is then used to calculate the actual hourly operating cost $C_i$ by summing the cost of $p_{t,i}(\tau)$ over all 5-minute intervals $\tau$. This emulation process reflects the vertically-integrated utility setting; i.e. costs are allocated according to energy delivered to the system with no markup cost for providing regulation.

Steps 1 and 2 are repeated for every operating hour in the period from December 2012 to March 2013. The results for each $\tau$ are analyzed for power system area control error (ACE) statistics, generator ramping statistics, and power line flow statistics as described below.

**C. Cost performance**

The RCC-OPF includes three user-defined parameters related to its probabilistic nature. The first of these is $\Gamma$ which determines the budget of uncertainty in wind forecast probability distributions defined by the uncertainty sets in [30] and [31]. Wind conditions change frequently, and we expect that this uncertainty parameter will be determined by short-term policy decisions of the system operators. In contrast, the parameters $\epsilon_1$ and $\epsilon_{mn}$ limit the probability that equipment constraints are violated, i.e. generation output or ramping limits described in (13)-(16) and power flow limits on lines described in (17)-(18). These parameters are directly related to the impact on power system assets, and we expect these are determined by long-term, e.g. seasonal policies.

In our case study, these parameters are set in a sequential process with the results shown in Tables I and II. Fixing the parameters $\epsilon_1$ and $\epsilon_{mn}$ at 1/6 and 0.0025, respectively, Steps 1 and 2 from subsection III-B are executed to determine the actual cost of generation $C^a$ over the entire study period for $\Gamma$ between 0.0 and 1.0 and for a deterministic OPF. The participation factors are not decision variables in the deterministic OPF. Instead, a fixed participation factor $\alpha = 0.05$ is used for all thermal and hydro generators. Table II shows the lowest $C^a$ is found at $\Gamma = 0.6$.

Fixing $\Gamma = \Gamma^* = 0.6$, Table II displays the sensitivity of the actual cost $C^a$ of the RCC-OPF solution to the parameters $\epsilon_i$.

1Recall that the CC OPF is the RCC OPF with $\Gamma = 0$. 
and $\epsilon_{\text{nn}}$. We begin the discussion with the results in the row for $\epsilon_{\text{nn}}=0.01$. For this larger $\epsilon_{\text{nn}}$, potential network congestion plays less of a role, i.e. the selection of the generators’ $p_{t,i}^*$ and $\alpha_{t,i}^*$ is less dependent on their location in the network. Instead, their selection is more sensitive to generator costs and constraints. Therefore, as $\epsilon_i$ decreases (moving left to right) in the $\epsilon_{\text{nn}}=0.01$ row, the main impact is to spread the $p_{t,i}^*$ and $\alpha_{t,i}^*$ more uniformly across the fleet to reduce the ramping duty of any particular generator. As $\epsilon_i \rightarrow 1/48$, more duty is placed on higher cost generators driving up the operating cost via the $\text{var}(\Omega)$ term in (8). However for our case study, the high percentage of very flexible hydro generators suppresses this cost increase—a result that is not expected to carry over to other power systems with different controllable generation fleets.

Next, we consider the small $\epsilon_{\text{nn}}$ limit shown in the $\epsilon_{\text{nn}}=0.0001$ row in Table II. Here, avoiding potential network congestion plays a larger role in the selection of the $p_{t,i}^*$ and $\alpha_{t,i}^*$ with the results becoming less sensitive to the generator cost and risk parameter $\epsilon_i$. This is reflected in the elevated and relatively flat cost even as $\epsilon_i \rightarrow 1/48$. In between the two extremes of $\epsilon_{\text{nn}}$, there is a relatively strong trade off in $C^a$ between $\epsilon_{\text{nn}}$ (network risk) and $\epsilon_i$ (generator risk).

The results in Tables I and II suggest that, for the test system used in our case study, the RCC-OPF model achieves the best cost performance with $\Gamma^* = 0.6$, $\epsilon_{\text{nn}} = 0.0025$, $\epsilon_i^* = 1/6$. In the remainder of this manuscript, we assume that the long-term policy parameters are fixed at $\epsilon_{\text{nn}}^* = 0.0025$ and $\epsilon_i^* = 1/6$, and present the technical analysis for variable $\Gamma$.

### Table I

| CC OPF | RCC OPF | OPF |
|--------|---------|-----|
| $\Gamma$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | – |
| $C^a$, MS | 112.2 | 111.6 | 110.9 | **108.7** | 112.7 | 114.9 | 115.6 |
| $\Delta C$, MS | – | -0.589 | -1.296 | **-3.471** | 0.459 | 2.716 | 3.401 |
| $\Delta \%$ | – | -0.524 | -1.155 | **-3.093** | 0.409 | 2.420 | 3.032 |

The actual realized generation cost $C^a$ for the period from December 2012 to March 2013 is computed in Step 2 using the dispatches $p_{t,i}^*$ and participation factors $\alpha_{t,i}^*$ computed in Step 1 of subsection III-B. The least cost solution is found for $\Gamma=0.6$ and is marked in bold. Also displayed are the changes in cost and fractional changes in cost relative to the $\Gamma=0$ case. In all of these cases, $\epsilon_{\text{nn}}=0.0025$ and $\epsilon_i^*=1/6$.

#### D. Technical analysis

1) **ACE performance:** The Area Control Error (ACE) is computed for each 5-minute interval $\tau$ in hour $t$ as:

$$ACE_i(\tau) = \sum_{b \in B} (d_{t,b}(\tau) - w_{t,b}(\tau)) - \sum_{i \in G} p_{t,i}(\tau). \quad (33)$$

Figure 1a displays the cumulative distribution function (CDF) of all the $ACE_i(\tau)$ in the study period, and Fig. 1b displays the average of $ACE_i(\tau)$. Starting from the most conservative $\Gamma = 1.0$, the average of $ACE_i(\tau)$ displays a slow but monotonic increase showing that $\Gamma$ effectively controls the system's technical performance. At $\Gamma < 0.6$, the average $ACE_i(\tau)$ increases more rapidly to the CC-OPF at $\Gamma=0$ and displays a significant jump from the CC-OPF to the deterministic OPF. By accounting for fluctuations in wind, the CC-OPF outperforms the deterministic OPF in controlling the ACE, and by progressively accounting for uncertainty in the parameters of the distribution describing the fluctuations, the RCC-OPF outperforms the CC-OPF.

In Fig. 1b, the difference between the CDFs for $\Gamma = 1.0, 0.8$, and 0.6 is not very significant. As our measure of conservatism is relaxed further (i.e. $\Gamma < 0.6$), the CDFs show a general increase in the frequency of ACE events of all

### Table II

| $\epsilon_{\text{nn}}$ | $\epsilon_i$ | $\epsilon_{\text{nn}}$ | $\epsilon_i$ | $\epsilon_{\text{nn}}$ | $\epsilon_i$ |
|----------------------|--------------|----------------------|--------------|----------------------|--------------|
| 0.01                 | 0.0025       | 0.0025               | 0.0025       | 0.0025               | 0.0025       |
| 0.0025               | 0.001       | 0.001                | 0.001       | 0.001                | 0.001       |
| 0.0001               | 0.0001      | 0.0001               | 0.0001      | 0.0001               | 0.0001      |

Percentage changes in actual generation cost $C^a$ relative to the $\epsilon_i = 1/6$, $\epsilon_{\text{nn}} = 0.0025$ case. All cases use $\Gamma = \Gamma^*=0.6$.
sizes and the emergence of a longer tail of large $ACE_t(\tau)$ values. We also note that $\Gamma \sim 0.6$ is the value where the ACE statistics first begin to significantly deteriorate and where RCC-OPF cost takes on its minimum value. Above $\Gamma = 0.6$, little additional ACE control performance is gained for the additional cost. This analysis suggests that, in the setting of vertically-integrated grid operations, the RCC-OPF with an appropriately chosen $\Gamma$ will result in better compliance with the control performance standards (CPS) [42] at a lower operating cost.

2) Ramping performance: The RCC-OPF also reduces generator ramp rate (RR) violations as compared to the CC-OPF and deterministic OPF, potentially avoiding generator wear-and-tear effects [43]. After the $\alpha_{t,i}^*$ are chosen, the generator ramp rates are simply $\alpha_{t,i}^* \sum_{b \in V_t} \omega_{t,b}(\tau)$. Figure 2(a) and b) display the number of RR violations for individual generators for the CC-OPF and for the RCC-OPF for different $\Gamma$, respectively.

The impact of the RCC-OPF on RR violations is twofold. First, as the robustness of the RCC-OPF increases, the number of generators affected by RR violations is reduced from 11 (for the CC-OPF, i.e. $\Gamma = 0$), to 4 with $\Gamma = 0.2$, and to 2 with $\Gamma = 1.0$. Second, the number RR violations per generator is also greatly reduced as $\Gamma$ increases. It is noteworthy that both effects can be observed even for a relatively small level of robustness, e.g. $\Gamma = 0.2$. Combined with the results from Table I this analysis suggests that the RCC-OPF with an appropriately chosen $\Gamma$ achieves lower operating cost and avoids indirect costs related to wear-and-tear effects on controllable generators. In contrast, RR violations for the deterministic OPF (not shown in Fig. 2) are observed on 39 generators—24 of which experience RR violations in more than one 5-minute interval.

3) Transmission overload performance: From the $p_{t,i}(\tau)$, $\omega_{t,b}(\tau)$, and $d_{t,b}(\tau)$, a power flow solution yields $f_{t,mn}(\tau)$ from which power line overloads are computed. Figure 3 displays a histogram of the number of overloads per power line for the four most frequently overloaded lines in the RCC-OPF for $\Gamma = 0.0, 0.2, 0.6$, and 1.0. Several other lines are overloaded during the study period, but these overloads only occur during one 5-minute period. Interestingly, $\Gamma$ does not have a significant impact on the frequency of overloads for the lines in Fig. 3.

Fig. 2. a) Histogram of the number of generator ramp rate (RR) violations per generator over the entire study period for the CC-OPF (i.e. the RCC-OPF with $\Gamma = 0$). b) Same as a) but for the RCC-OPF with $\Gamma = 0.2, 0.6$, and 1.0. All results are computed with $\epsilon_{min} = 0.0025$ and $\epsilon_t^* = 1/6$.

Fig. 3. Comparison of overloads that are observed with the CC and RCC OPF models during more than one 5-minute interval. All results are computed with $\epsilon_{min} = 0.0025$ and $\epsilon_t^* = 1/6$.

E. Computational performance

The computations were carried out with CPLEX v. 12.1 [29] as an LP solver on a Intel Xenon 2.55 GHz processor with at least 32 GB RAM on the Hyak supercomputer system at the University of Washington [44]. All RCC-OPF modeling was done using JuMPChance [25], a freely available extension for the JuMP [43] modeling language. The average wall-clock time for a RCC-OPF instance was approximately 8 seconds with an increase to ~ 20 seconds for instances with $\Gamma = 1.0$. Such an increase is expected because of the larger number of cutting plane iterations required for constrained problems. However, the computational times show that the RCC-OPF formulation is very tractable on the large-scale power system model used here.

IV. Conclusions

Based on [18], we have developed and implemented algorithms to compute a distributionally-robust chance constrained optimal power flow (RCC-OPF) that accounts for uncertainty in the parameters of statistical models that describe the deviations of wind (or other intermittent) generation from its forecast. We have demonstrated the scalability of the RCC-OPF by performing a seasonal case study on a modification of the BPA system. In this setting of vertically integrated grid operations, the case study shows that, compared to deterministic or even chance constrained OPF (CC-OPF), the RCC-OPF distributes both generation and regulation in a manner that can result in both cost savings and better technical performance; including fewer violations of transmission line limits, generator ramping limits, and smaller Area Control Error values.

The work in this manuscript points to several areas for potential future work, including:

- similar case studies should be performed on power systems that are dominated by fossil generation instead of hydro generation
the RCC-OPF formulation should be extended to include the effects of reactive power on nodal voltage magnitudes, transmission line limits, and generator limits

to better model generator ramping constraints, the RCC-OPF should be modified to a time-extended or look-ahead formulation consistent with the operation of modernized power systems

current formulation should be extended to market-based operations to incorporate the cost of procuring frequency regulation capacity.

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