Modelling the propagation of an ultrasonic ray through a heterogeneous medium

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Abstract

The estimation of mechanical properties of concrete is one of the most important applications of the ultrasonic method. This technique often involves the measuring of the ultrasonic pulse propagation velocity. However, there exist some factors influencing the effective ultrasonic pulse propagation velocity in concrete. This paper is devoted to a discussion of the behavior of the effective ultrasonic pulse propagation velocity as a function of the free water contained in concrete. The travelling of an ultrasonic ray through a heterogeneous medium composed by cement and aggregates (solid phase), water (liquid phase) and air (gaseous phase) was empirically modelled. A relationship between the transition time of the ultrasonic ray in the testing piece and the free water quantity was considered. We assumed an empirical intermediate relationship between the ultrasonic path length in porous space in concrete filled with water and the free water quantity in the testing piece. From experimental data points we evaluated the parameters of the model and obtained

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a fitting function for describing the dependence between the effective ultrasonic pulse propagation velocity and the free water quantity in concrete. We also analyzed the error propagation through the algorithm used. It allowed to decide under which conditions the linear approach between the effective ultrasonic pulse propagation velocity and the free water quantity in concrete can be applied.
I. INTRODUCTION

The estimation of mechanical properties of concrete, in particular, that of compressive strength, is one of the most important applications of the ultrasonic method [1–6]. However, the effective application of this method is subject to the influence of several factors [5,7–10]. One of these factors is the water which penetrates in concrete through the pores [11–14]. Under the influence of this factor, the compressive strength and the effective ultrasonic pulse propagation velocity differently behave as the free water content changes. In fact, the latter increases as the free water content in the specimen increases. On the other hand, the compressive strength practically behaves constant when the free water content is modified [2,15].

Under these circumstances, some considerations will be made. The free water in concrete is one of the most important building problems widely considered today. The concrete strength is generally considered its most important property among other characteristics for establishing its quality. This property of the concrete may be influenced by its free water content. Several authors consider that the durability of a building structure is determined by the rate at which water and chemical components that it contains infiltrate and move through the porous building materials [24].

On the other hand, the ultrasonic transmission method is often used for estimating, among other properties, the concrete strength. The application of the ultrasonic transmission technique to measuring the ultrasound propagation velocity through concrete allows to estimate the value of concrete strength [5]. The relationship between these two quantities is often established when the concrete in service is dried. When free water infiltrates and moves through the pores the measuring of the effective ultrasonic pulse propagation velocity can not be directly used for estimating the value of the concrete strength. It is still an open problem. However, the application of the ultrasonic transmission method for determining
the ultrasound pulse velocity can allow to estimate the free water content in the porous structure but it can not allow to decide about the damage of the building. The free water content affects the ultrasonic pulse propagation velocity but this measure says nothing about the current strength of concrete. If the dried state of concrete can be obtained back again, then the measuring of the ultrasonic pulse propagation velocity made in that condition could indicate a possible change in the value of concrete strength. This problem has a high attracting social attention in a tropical country where several types of meteorological events often produce a high level of moisture in building during several days. M. Rodriguez [5] has studied the influence of moisture in building in tropical condition finding that this phenomenon produces a significatively high damage in building structure.

M. Kuntz and P. Lavallée [24] have studied the infiltration of liquid and the propagation of the moisture front in non-saturated porous media using an one-dimensional diffusion equation for describing the phenomenon. They considered a generalized scaling law of the type $x t^{-\alpha}$ with $\alpha = 1/(n + 1)$ and being $n \in \mathbb{R}$. The water transfer in partially saturated materials was assumed to follow a general nonlinear diffusion equation being water content the variable to determine. They found the values $\alpha = 0.58$ in brick and $\alpha = 0.61$ in the limestone diverging both from the value $\alpha = 0.5$. The results mean that the volume of absorbed water in this materials may be underestimated, having a particular importance for evaluating the effect of water content in building structures. This result expresses that there exists a great theoretical interest about the complexity of this problem along with the practical approach.

E. Ohdaira and N. Masuzawa [16] have examined the possibility of the NDE of concrete from a knowledge of the behavior of the ultrasound propagation velocity with water content in concrete. They measured the ultrasound velocity and the frequency component on ultrasonic propagation as a function of water content in concrete. They made testing pieces of concrete being 10 cm in diameter and 20 cm in length. These testing pieces were embedded
in water for 50 days. After this, the water in each test piece was gradually extracted and the ultrasound propagation velocity was conveniently measured for having 15 points in the relationship between water content and the ultrasound propagation velocity. They found that the ultrasound propagation velocity decreases approximately linearly in proportion to the decrease in water content. Similarly the same occurs for the frequency of the maximum transmission.

M. Rodriguez and R. Bonal [23] made a comment on the paper referred above. They considered that the relationship between the ultrasound propagation velocity and water content can better be described by using a statistical model. They fitted the experimental data points obtained by Ohdaira and Masuzawa [16] by applying this method and obtaining an empirical fitting equation. They concluded that the ultrasound propagation velocity exponentially depends on the free water quantity in concrete.

In this work an empirical nonlinear model for the relationship between the effective ultrasonic pulse propagation velocity \( v \) and the quantity of free water in concrete \( (m_w) \) was considered by us to better describe that behavior. The experimental data given in [16] were considered again. It was also discussed under which conditions the linear approach can be used.

II. MODELLING THE TRAVELLING OF AN ULTRASONIC RAY THROUGH A HETEROGENEOUS MEDIUM

We consider the experimental disposition scheme for applying the transmission technique in ultrasonic method to determine the ultrasonic pulse propagation velocity. An ultrasonic ray travelling through a concrete testing piece containing free water can be modelled by considering an equivalent scheme consisting of a heterogeneous medium composed by three phases of the matter: gaseous phase (air), liquid phase (free water), and solid phase (fine
and coarse aggregate, cement). The total transmission time $t$ of the ultrasonic ray in the acoustical system may be decomposed as

$$t = t_a + t_w + t_s + \delta$$  \hspace{1cm} (1)

where we have separately considered the time interval spent by the ultrasonic ray for travelling through each phase in the heterogeneous medium: $t_a$ corresponds to the air, $t_w$ corresponds to the free water, and $t_s$ corresponds to the solid phase of concrete. Finally, $\delta$ represents the time interval spent by the ultrasonic ray for travelling outside the testing piece, and it will be considered a constant value during the measuring procedure on a given testing piece. Its numerical value in all these experiments was taken to be $\delta = 0.6\mu s$ \[16\]. The equation (1) may be written as

$$t - \delta = \frac{h_a}{v_a} + \frac{h_w}{v_w} + \frac{h_s}{v_s} = \frac{h}{v},$$  \hspace{1cm} (2)

where $h_a$, $h_w$, $h_s$ are the ultrasonic ray path lengths in air, in free water, and in solid phase, respectively, $h$ is the testing piece length (200 mm), $v_a = 330$ m/s, $v_w = 1500$ m/s, $v_s$ is the ultrasonic propagation velocity in the concrete solid phase, and finally, $v$ represents the effective ultrasonic propagation velocity in the heterogeneous medium (concrete).

When the testing piece does not exhibit moisture at all it can be considered that the term $h_w/v_w$ in the equation (2) is approximately equal to zero and the transition time $t$ of the ultrasonic ray through the testing piece takes its maximum value $t_{max}$. It corresponds to a minimum value of the effective ultrasonic propagation velocity $v$ in the heterogeneous medium. Besides, the variable $h_a$ takes its maximum value $h_p$ which corresponds to the ultrasonic ray path length in porous space when only the air is present. In this case, the equation (2) can be written as

$$t_{max} - \delta = \frac{h_p}{v_a} + \frac{h_s}{v_s} = \frac{h}{v_{min}},$$  \hspace{1cm} (3)

On the other hand, if the testing piece is quite saturated with the free water, we can consider that the term $h_a/v_a$ in the equation (2) is approximately equal to zero, the variable
$h_w$ takes its maximum value $h_p$, the variable $t$ takes its minimum value $t_{min}$ corresponding to the maximum value of the variable $v$. Then, the equation (2) will become to

$$t_{min} - \delta = \frac{h_p}{v_w} + \frac{h_s}{v_s} = \frac{h}{v_{max}},$$

(4)

We have assumed in the analysis above that the quantities $h_s, v_s$, and $h$ are fixed. It is also considered that

$$h_w + h_a = h_p = constant,$$

(5)

$$h_p + h_s = h = constant$$

(6)

From the equations (3) and (4) we can find that

$$h_p = \frac{(t_{max} - t_{min})}{k_1},$$

(7)

where the constant $k_1 = 2364\mu s/m$. By using the equation above, we can estimate the ultrasonic ray path length in porous space, where $t_{max}$ corresponds to the transition time of the ultrasonic ray in the testing piece when it is saturated with free water, and $t_{min}$ is the value of the same variable when the testing piece is quite dried.

On the other hand, we can derive a relationship for the dependence of the variable $t$ (transition time of the ultrasonic ray in the testing piece) with the variable $h_w$ (ultrasonic ray path length in the free water in the porous space in concrete). In fact, from the equations (2), (3), and (5) it can be written that

$$t_{max} - t = h_w \times k_1$$

(8)

It can be observed that the correction term $\delta$ in equation (2) cancels itself and it does not appear in equation (8). Variables $t$ and $h_w$ in the equation (8) are well defined in the intervals $t \in [t_{max}, t_{min}]$ and $h_w \in [0, h_p]$. The variable $t$ can be directly measured, but the variable $h_w$ can not. Here we considered that the variable $h_w$ depends on the quantity of free water ($m_w$) in the testing piece, but such a dependence will not be a simple one.
The way in which \( h_w \) depends on \( m_w \) is not a known matter for theoretical explaining, but we suppose that such a dependence is influenced by the morphology of pores and by the statistical distribution law of pores in the testing piece volume, among other factors. These aspects will be considered in further investigations. We know that the variable \( h_w \) and the variable \( m_w \) are well defined in the intervals \( h_w \in [0, h_p] \) and \( m_w \in [0, m_{w,max}] \). We assume the following empirical relationship

\[
h_w = Q \cdot m_w^\gamma, \quad Q > 0, \quad 1 > \gamma > 0,
\]

being \( Q \) a positive dimensional constant and the exponent \( \gamma \) should be in the interval \( \gamma \in (0, 1) \). The particular values which the proportional constant \( Q \) and the exponent \( \gamma \) take in a particular experiment will be determined by the factors indicated above. Introducing the equation (9) in the equation (8) the following relationship is obtained

\[
t_{\text{max}} - t = m_w^\gamma \cdot Q \cdot k_1,
\]

where now both variables \( t \) and \( m_w \) can be directly measured. It is convenient to linearizing the equation (10) by applying natural log function to both sides of the equation,

\[
\log(t_{\text{max}} - t) = \gamma \cdot \log(m_w) + \log(Q \cdot k_1)
\]

Applying a linear regression method to a log-log plotting of the variables \( (t_{\text{max}} - t) \) and \( (m_w) \) the best straight line is fitted to the experimental data point \( (m_w, t_{\text{max}} - t) \), and the model parameters \( \gamma \) and \( Q \) can be determined. In this plot, the point \( (0,0) \) can not be taken into account.

**III. EXPERIMENTAL DATA POINTS AND FITTING FUNCTION**

In order to evaluate the proposed empirical model, given by the equation (11), we considered the experimental data points obtained by Ohdaira and Masuzawa [16]. They measured the transition time \( t \) of an ultrasonic ray in a cylindrical testing piece of concrete being 200 mm in length by applying the ultrasonic transmission technique. The corresponding value of
the quantity of free water $m_w$ in the testing concrete piece was measured by using a special device. The cotas of error for the variables $t$ and $m_w$, and the parameter $h$, all of them directly measured, are given. They applied the procedure to three types of testing piece named A-1, B-1, and C-1. From the experimental data points obtained by these authors we only considered those whose were subjects of our analysis. Besides, the experimental data points considered by us were organized as data vectors because it made easier data processing when certain software was used. Ohdaira and Masuzawa [16] measured the transition time $t$ five times for each value of the free water $m_w$. Statistical validation allows us to average these five values by taking the mean value as one element (in $\mu s$) of the data vector called $\langle time \rangle$. The vector $\langle water \rangle$ contains the values of the variable $m_w$ in kilogram. These vectors are separately given below for each testing piece.

Testing piece A-1:

\[
water = [0.2321 0.2109 0.1969 0.1824 0.1694 0.1571 0.1404... \nonumber \\
0.1227 0.1060 0.0787 0.0615 0.0419 0.0285 0.0089 0.0000]; \quad (12) \nonumber \\
\]

\[
time = [44.48 45.16 46.08 46.64 46.48 46.44 46.60 46.48... \nonumber \\
47.24 47.80 48.24 49.16 49.76 51.08 51.32]; \quad (13) \nonumber \\
\]

Testing piece B-1:

\[
water = [0.2500 0.2165 0.2001 0.1850 0.1724 0.1614 0.1449... \nonumber \\
0.1278 0.1093 0.0786 0.0613 0.0406 0.0240 0.0084 0.0000]; \quad (14) \nonumber \\
\]

\[
time = [44.88 46.32 46.12 46.32 46.80 46.44 47.48 47.00... \nonumber \\
47.24 47.76 48.28 49.20 50.84 51.60 51.80]; \quad (15) \nonumber \\
\]

Testing piece C-1:

\[
water = [0.2479 0.2257 0.2021 0.1871 0.1693 0.1555 0.1436... \nonumber \\
0.1259 0.1082 0.0896 0.0656 0.0531 0.0379 0.0241 0.0088 0.0000]; \quad (16) \nonumber \\
\]
\[
time = [46.64 47.12 48.52 48.52 48.76 49.20 49.16 49.40... \\
49.64 50.08 50.28 50.52 51.64 52.40 52.64 53.08]; \quad (17)
\]

We estimated the random cota of error for the vector \(\langle \time \rangle\) and composed it together with the instrumental cota of error whose value is equal to 0.2 \(\mu s\) [16] for having the vector \(\langle \cota - \time \rangle\) in \(\mu s\),

Testing piece A-1:
\[
\cota - \time = [0.31 0.22 0.25 0.34 0.36 0.22 0.28 ... \\
0.22 0.23 0.23 0.28 0.38 0.30 0.22 0.24]; \quad (18)
\]

Testing piece B-1:
\[
\cota - \time = [0.21 0.22 0.21 0.21 0.23 0.23 0.24 ... \\
0.26 0.22 0.26 0.26 0.22 0.28 0.29 0.22]; \quad (19)
\]

Testing piece C-1:
\[
\cota - \time = [0.20 0.22 0.24 0.21 0.24 0.22 0.23 ... \\
0.22 0.23 0.27 0.23 0.24 0.34 0.32 0.54 0.28]; \quad (20)
\]

The vector \(\langle \time \rangle\) was corrected by the constant value \(\delta = 0.6 \mu s\) (equation (1)) for obtaining the vector \(\langle \timec \rangle\) in \(\mu s\),

Testing piece A-1:
\[
\timec = [43.88 44.56 45.48 46.04 45.88 45.84 46.00 ... \\
45.88 46.64 47.20 47.64 48.56 49.16 50.48 50.72]; \quad (21)
\]

Testing piece B-1:
\[ timec = [44.28 \ 45.72 \ 45.52 \ 45.72 \ 46.20 \ 45.84 \ 46.88 \ldots \ 46.40 \ 46.64 \ 47.16 \ 47.68 \ 48.60 \ 50.24 \ 51.00 \ 51.20]; \]  \hspace{1cm} (22)

Testing piece C-1:

\[ timec = [46.04 \ 46.52 \ 47.92 \ 47.92 \ 48.16 \ 48.60 \ 48.56 \ldots \ 48.80 \ 49.04 \ 49.48 \ 49.68 \ 49.92 \ 51.04 \ 51.80 \ 52.04 \ 52.48]; \]  \hspace{1cm} (23)

By using the vector \( \langle timec \rangle \) the vector \( \langle velocity \rangle \) was constructed by applying the relationship

\[ v = h/(t - \delta) \]  \hspace{1cm} (24)

where \( h = 200 \ mm \) is the length of each testing piece and \( t - \delta \) are the elements of the vector \( \langle timec \rangle \). The elements of the vector \( \langle velocity \rangle \) are given in \( m/s \),

Testing piece A-1:

\[ velocity = [4557.9 \ 4488.3 \ 4397.5 \ 4344.0 \ 4359.2 \ 4363.0 \ 4347.8 \ldots \ 4359.2 \ 4288.2 \ 4237.3 \ 4198.2 \ 4118.6 \ 4068.3 \ 3962.0 \ 3943.2]; \]  \hspace{1cm} (25)

Testing piece B-1:

\[ velocity = [4516.7 \ 4374.5 \ 4393.7 \ 4374.5 \ 4329.0 \ 4363.0 \ 4266.2 \ldots \ 4310.3 \ 4288.2 \ 4240.9 \ 4194.6 \ 4115.2 \ 3980.9 \ 3921.6 \ 3906.3]; \]  \hspace{1cm} (26)

Testing piece C-1:

\[ velocity = [4344.0 \ 4299.2 \ 4173.6 \ 4173.6 \ 4152.8 \ 4115.2 \ 4118.6 \ldots \ 4098.4 \ 4078.3 \ 4042.0 \ 4025.8 \ 4006.4 \ 3918.5 \ 3861.0 \ 3843.2 \ 3811.0]; \]  \hspace{1cm} (27)

Finally, the cotas of error for the transition time and for the length of the testing piece \( \delta h = 0.0005 \ m \) were propagated through the formula (24) written in the form of finite increment,
$$\delta v = \frac{h}{(t-\delta)^2} \delta t + \frac{1}{(t-\delta)} \delta h$$  (28)

The vector $\langle \delta velocity \rangle$ in m/s represents the cota of error vector of the vector $\langle velocity \rangle$ propagated by the equation above. For each type of testing piece we obtained,

Testing piece A-1:

$$\delta velocity = [43.31 \ 33.65 \ 35.15 \ 42.66 \ 45.21 \ 32.10 \ 36.93... \ 31.36 \ 32.01 \ 31.06 \ 35.04 \ 42.21 \ 34.94 \ 27.52 \ 28.65];$$  (29)

Testing piece B-1:

$$\delta velocity = [32.30 \ 31.55 \ 30.86 \ 30.64 \ 32.19 \ 32.94 \ 32.66... \ 35.00 \ 31.20 \ 34.33 \ 33.16 \ 28.84 \ 32.03 \ 32.09 \ 26.48];$$  (30)

Testing piece C-1:

$$\delta velocity = [30.10 \ 30.66 \ 31.48 \ 28.37 \ 31.08 \ 28.84 \ 29.93... \ 28.65 \ 29.45 \ 31.78 \ 28.96 \ 29.41 \ 35.67 \ 33.69 \ 49.35 \ 29.86];$$  (31)

By considering the equation (11) the log-log plotting was applied to each of the three types of testing pieces. Figures 1, 2 and 3 show the results. The best straight line fitting the experimental data points produced the following values for the model parameters $\gamma$ and $Q$,

$$\gamma_{A-1} = 0.8707, \ \gamma_{B-1} = 0.9205, \ \gamma_{C-1} = 0.7923.$$  (32)

$$Q_{A-1} = 0.0111, \ Q_{B-1} = 0.0124, \ Q_{C-1} = 0.0080.$$  (33)

Figure 4 depicts the plotting of the equation (9) using the values of the model parameters indicated above.

On the other hand, by applying the equation (7), the parameter $h_p$ in meter was estimated
\[ h_{p,A-1} = 0.0029, \quad h_{p,B-1} = 0.0029, \quad h_{p,C-1} = 0.0027 \]  

(34)

From the equation (2) and (10), the effective ultrasonic pulse propagation velocity \( v \) as a function of \( m_w \) is given by

\[
v = \frac{h/k_2}{1 - (Qk_1/k_2)m_w^\gamma}, \quad k_2 = t_{\text{max}} - \delta
\]

(35)

This equation is the fitting function for the experimental data points \((m_w, v)\). Using the numerical values of the parameters \( h, k_2, Q, k_1, \) and \( \gamma \), the equation (35) for each of the testing pieces are respectively,

\[
v_{A-1} = \frac{3943.22}{1 - (0.52524)m_w^{0.8707}}, \quad (36)
\]

\[
v_{B-1} = \frac{3906.25}{1 - (0.58125)m_w^{0.9205}}, \quad (37)
\]

\[
v_{C-1} = \frac{3810.98}{1 - (0.365854)m_w^{0.7923}} \quad (38)
\]

Figures 5, 6, and 7 separately depict the experimental data points \((m_w, v)\) and the fitting function given in each case by the equations (36), (37), and (38).

**IV. DISCUSSION OF THE RESULTS**

The influence of moisture in the testing piece of concrete on the effective ultrasonic pulse propagation velocity is highly significant. Experimental results confirm this fact. Though the ultrasonic transmission technique for measuring the effective ultrasonic pulse propagation velocity does not exhibit high accuracy, it does allow to determine the level of such an influence and the behavior of the effective ultrasonic pulse propagation velocity with water content can be well defined. The functional form by which such an influence is determined depends, among other factors, on the morphology of pores and on the statistical distribution law of pores in the testing piece volume. However, we have considered an empirical relationship between the variable \( h_w \) and the variable \( m_w \) that indirectly very well matches to experimental data points through the relationship (35) between the effective ultrasonic
pulse propagation velocity \( v \) and the quantity of free water \( m_w \) in the testing piece. The finding of this fitting function has been the aim of this work. Figure 4 depicts the empirical relationship between \( h_w \) and \( m_w \) for each type of testing pieces. It is interesting to observe that the values of the parameters \( \gamma \) and \( Q \) increases with the increase in cement content in the testing piece in the considered range. It is just a conjecture. This aspect will be considered in further investigations.

We observe that the relationship between \( v \) and \( m_w \) is not a linearly proportional function. Figures 5, 6 and 7 for testing pieces A-1, B-1, and C-1, show a suitable fitting of the experimental data points by the equation (35).

It is important to check whether the proposed model correctly describes the behavior of the experimental data \( (v = f(m_w)) \). The equation (35) can not be linearized, but if \( m_w = 0 \), then the velocity coincides with the minimum velocity \( v_0 \) (velocity in dry state) and, consequently, equation (35) can be written in the form

\[
y = \frac{a}{1 - bx^c},
\]

where \( y = v \) is the effective ultrasonic pulse propagation velocity, \( a = v_0 \) represents the minimum velocity (in dry state), \( x = m_w \) is the free water content, \( b = Q(k_1/k_2) \) and \( c = \gamma \) are parameters related with the parameters indicated in the equation (35). When \( x = 0 \), from equation (39) is obtained that \( y = a \).

The results of the statistical analysis are presented in table no.1, for each one of the samples \( A_1, B_1, \) and \( C_1 \).

The model very well fits the experimental data, and the values of parameter \( c \) do not significatively differ from those of \( \gamma \) obtained in the equation (32). In all the cases they are within the deviation range proposed by the model. One can conclude that the model given
in equations (36, 37, 38), appropriately describes the experimental data set.

Next, we analyze under which conditions the behavior of the variable $v$ with the variable $m_w$ can be considered a linearly proportional relationship. From the equation (35) we can obtain the following relationship by applying the Newtonian binomial formula. The quadratic approximation is

$$v^{(2)} \cong (h/k_2) * (1 + (Qk_1/k_2)m_w^\gamma + (Qk_1/k_2)^2 * m_w^{2\gamma}), \quad (Qk_1/k_2)m_w^\gamma << 1 \quad (40)$$

The linear approximation of the equation above is

$$v^{(1)} \cong (h/k_2) * (1 + (Qk_1/k_2)m_w^\gamma), \quad (Qk_1/k_2)m_w^\gamma << 1 \quad (41)$$

and the contribution to the effective ultrasonic propagation velocity of the quadratic term is,

$$v^{(2)} - v^{(1)} \cong (h/k_2) * (Qk_1/k_2) * m_w^\gamma \quad (42)$$

The vector $\langle \text{error}_v \rangle$ in m/s contains the contributions of the quadratic term in equation (42). For each testing piece we can write,

Testing piece A-1:

$$\langle \text{error}_v \rangle = [82.28 69.64 61.79 54.08 47.55 41.70 34.29... 27.12 21.02 12.51 8.15 4.18 1.77 0.28 0.00]$$

Testing piece B-1:

$$\langle \text{error}_v \rangle = [99.72 76.52 66.19 57.29 50.31 44.56 36.53... 28.99 21.74 11.85 7.50 3.51 1.33 0.19 0.00]$$

Testing piece C-1:
\[
\langle \text{errorv} \rangle = [53.77 \ 46.34 \ 38.90 \ 34.43 \ 29.38 \ 25.68 \ 22.63 \ldots \\
18.38 \ 14.45 \ 10.72 \ 6.54 \ 4.68 \ 2.74 \ 1.34 \ 0.27 \ 0.00]; \quad (45)
\]

We can compare the vectors \(\langle \delta \text{velocity} \rangle\) and \(\langle \text{errorv} \rangle\) for deciding when the equation (41) is suitable to apply. When \(m_w \geq (0.16)\) in kilogram it can be observed that the contribution of the quadratic term (equation 42) is greater than the propagated cota of error for the experimental value of the effective ultrasonic propagation velocity \(v\) given by the equation (28). The equation (41), considering that the exponent \(\gamma \cong 1\), approximately well describes the linear behavior considered in [16], but this approach implies a greater error in velocity when \(m_w\) is approximately greater than 0.16 kg.

On the other hand, the equation (35) very well quantitatively describes the effect of moisture in concrete on \(v\). In fact, the formula (35) can be conveniently written as

\[
\frac{\delta v}{v} = \frac{k_1 * h}{(t_{\text{max}} - h_w * k_1)^2} (100 * h_p/h) \quad (46)
\]

where \(100 * h_p/h\) represents the ratio in percent between the ultrasonic ray path length in porous space and the testing piece length. Applying this equation to experimental data points for each testing piece of concrete it may be found that such a small value of the ratio \(100 * h_p/h\) as \(\cong 1.45\) percent can yield a highly significative value of the fraction \(\delta v/v\), approximately 14 – 16 percent. This result is well confirmed by the experimental data points \((v, m_w)\).

V. CONCLUSIONS

The result of this analysis confirms that the empirical fitting function very well quantitatively describes the behavior of effective ultrasonic pulse propagation velocity with the free water content in concrete testing piece and that this empirical model well explains under which condition the linear approximation can be applied. It also allows to estimate the
propagation of error for the indirectly-measured involved variables.

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VI. TABLES
### Table no.1: Test Piece A-1

| R       | R.squared | R.squared adj | Typical error of the estimate | Durbin-Watson |
|---------|-----------|---------------|-------------------------------|---------------|
| .955    | .912      | .905          | .26315                        | 1.187         |

|          | Non standardized | Standardized Coefficients | t  | Sig. | Confidence Interval 95 % |                          |                |
|----------|-------------------|---------------------------|----|------|----------------------------|--------------------------|------------------|
|          | Coefficients      |                           |    |      |                            |                          |                  |
| B        | Typ. Error        | Beta                      |    |      | Lower bound                | Upper bound              |                  |
| Ln b     | -8.944            | .197                      | -45.314 | .000 | -9.374                     | -8.514                  |                  |
| C        | .870              | .078                      | .955 | 11.177 | .000                       | .700                     | 1.040            |
Table no.1: Test Piece B-1

| R    | R.squared | R.squared adj | Typical error of the estimate | Durbin-Watson |
|------|-----------|---------------|-------------------------------|---------------|
| .948 | .898      | .89           | .31323                        | 1.667         |

| Coefficients | Non standardized | Standardized Coefficients | t     | Sig. | Confidence Interval 95 % |
|--------------|------------------|---------------------------|-------|------|--------------------------|
| B            | Typ. Error       | Beta                      |       |      | Lower bound | Upper bound |
| Ln b         | -8.828           | .227                      | -38.867 | .000 | -9.323 | -8.333     |
| C            | .921             | .089                      | .948  | 10.294 | .000 | .726 | 1.116     |
Table no.1: Test Piece C-1

| R    | R.squared | R.squared adj | Typical error of the estimate | Durbin-Watson |
|------|-----------|---------------|--------------------------------|---------------|
| .977 | .955      | .951          | .16848                         | 1.265         |

|         | Non standardized | Standardized Coefficients | t   | Sig. | Confidence Interval 95 % |
|---------|-------------------|----------------------------|-----|------|--------------------------|
|         | B                 | Typ. Error                | Beta |      | Lower bound | Upper bound |
| Ln b    | -9.271            | .122                      | -75.767 | .000 | 9.535       | 9.006       |
| C       | .793              | .048                      | .977 | 16.557 | .000       | .689        | .896        |
VII. FIGURES
FIG. 1. Linear regression, testing piece A-1

FIG. 2. Linear regression, testing piece B-1
FIG. 3. Linear regression, testing piece C-1

FIG. 4. Curves of the empirical model: ultrasonic ray path length in the free water in the porous space in concrete versus free water content
FIG. 5. Fitting function, testing piece A-1

FIG. 6. Fitting function, testing piece B-1
Velocity versus water content: testing piece C-1

FIG. 7. Fitting function, testing piece C-1
log-log plotting, testing piece A–1

Fitting line

Experimental data

\[ \log(t_{\text{max}} - t), \ t_{\text{max}} \text{ and } t \text{ in s} \]

\[ \log(mw), mw \text{ in kg} \]
log–log plotting, testing piece B–1

Experimental data

log(t_{max} - t), t_{max} and t in s

Fitting line

log(mw), mw in kg
log–log plotting, testing piece C–1

\[ \log(mw), mw \text{ in kg} \]

\[ \log(t_{\text{max}} - t), t_{\text{max}} \text{ and } t \text{ in s} \]

Fitting line

Experimental data
Water content, in kg

Ultrasonic ray path length in water, in mm

- * Testing piece A-1
- o Testing piece B-1
- + Testing piece C-1
Velocity versus water content: testing piece A–1

- **Velocity, in m/s**
- **Water content, in kg**

- ○: Experimental value
- ★: Fitted value
Velocity versus water content: testing piece B−1

Velocity, in m/s

Water content, in kg

- ○ Experimental value
- ● Fitted value
