Novel Differential Conductance Oscillations in Asymmetric Quantum Point Contacts

Hao Zhang,1 Phillip M. Wu,2 and A. M. Chang1

1Department of Physics, Duke University, Physics Building, Science Drive, Durham, North Carolina 27708, USA
2Department of Applied Physics and Geballe Laboratory for Advanced Materials, Stanford University, Stanford, California 94305

(Dated: September 22, 2014)

Small differential conductance oscillations as a function of source-drain bias were observed and systematically studied in an asymmetric quantum point contact (QPC). These oscillations become significantly suppressed in a small in-plane magnetic field (∼0.7 T) or at higher temperatures (∼800 mK). Qualitatively, their temperature evolution can be simulated numerically based on smearing of the Fermi distribution, whereas features near zero-bias cannot. Single particle scenarios are unsatisfactory in accounting for the oscillations, suggesting that they are likely caused by electron and spin correlation effects.

After years of search, the elusive quantum wigner crystal-like state in 1-dimension and quasi-1-dimension—a state in which an uncertainty in the momentum yields a relevant kinetic energy scale, but Coulomb repulsion wins out—still has not been unequivocally demonstrated. The most promising system, in which a systematic study can be carried out using transport measurements, is the QPC system. Despite the lack of a smoking-gun signature, such as a threshold for conduction analogous to the motion of a pinned charge density wave (or at least a highly nonlinear increase beyond some pinning voltages), to date, several puzzling characteristics have been discovered in ballistic QPCs, when the number of conduction channels (transverse modes) is tuned toward the single channel limit. These include the well-studied 0.7 anomaly, the suppression (or destruction) of the first quantized conductance plateau, as well as higher plateaus, and the recently discovered conductance resonances in geometrically asymmetric QPCs. All of these features point beyond the single particle picture, as none can be explained adequately without introducing Coulomb interaction and spin effects.

In this letter, we report the discovery of a new signature unique to the QPC system. Specifically, we focus on unusual oscillations in the differential conductance (dI/dV) as a function of source-drain bias, in an asymmetric ballistic QPC. The magnetic field behavior shows that these oscillations are readily suppressed under a small in-plane magnetic field (∼0.7T), while the temperature dependence shows that these oscillations are washed out at around T = 800 mK. It is worthwhile to emphasize that this type of oscillations as a function of source-drain bias has not been reported in the published literature in any electronic system; the only possible exception is unpublished work in a quadruple-quantum-dot, which is entirely different from the QPC system. Similar oscillations are reproduced in a second sample at 30 mK and zero magnetic field.

Our QPC samples were fabricated by electron beam lithography, evaporation of the Cr/Au surface gates and lift-off. The two-dimensional electron gas (2DEG) is located at a shallow depth 85 nm below the surface of the GaAs/AlGaAs heterostructure crystal. The carrier density and mobility are 3.8 × 10^11/cm^2 and 9 × 10^5 cm^2/Vs respectively, giving a mean free path ∼9 µm. An excitation voltage V_ac = 5 µV at 17.3 Hz was applied, and the current was measured in a PAR124A lock-in amplifier after conversion to a voltage using a home-made current preamplifier. The measurement was first carried out in a He-3 fridge with a base temperature 300 mK, and then repeated in a dilution refrigerator with a base temperature below 30 mK. All data shown in this paper are qualitatively reproducible with thermal cycling.

Before presenting the main data for the dI/dV oscillations...
tions in the 100 nm channel length, asymmetric QPC, we begin by addressing the issue of donor/impurity-induced disorder in our QPC devices. To establish with confidence that what we observe are intrinsic effects, and not from disorder, we contrast the behavior of the linear conductance in an asymmetric QPC, against symmetric QPCs with smooth and gradual entrances and exits. Fig. 1 shows the linear conductance for both symmetric (left panels) and asymmetric QPCs (right panels). Symmetric QPCs have two geometrically symmetric split gates while the asymmetric ones were fabricated by replacing one split gate with a long gate to introduce asymmetry. This long gate is labeled the wall gate while the other short gate labeled the finger gate. The scanning electron microscope (SEM) images of asymmetric QPCs can be found in Ref. 6, 8. The left two figures in Fig. 1 show the linear conductance for two asymmetric QPCs with different channel lengths (100 nm and 500 nm respectively), while the right two figures show the linear conductance for one asymmetric QPC at 300 mK and 30 mK, respectively. Notably, Fig. 1(b) and (d) are the linear curves of the same asymmetric QPC but with different cool downs. During the measurement, one gate (the wall gate for the asymmetric QPC) voltage (V_{wall}) was held fixed, while the other gate voltage (V_{finger}) was scanned to measure the conductance.

In our previous work \cite{6}, conductance resonances, and also their behavior as a function of the channel length and width, were reported. The dirt effects, caused by dopant impurity and lithographic imperfections, were also discussed in details and were largely ruled out. Here using the symmetric QPCs as a control group, we show more clear-cut evidence to rule out the dirt effects. Especially for Fig. 1(c), in a symmetric QPC with a long channel length, almost no conductance resonances are observed (except for the 0.7 effect \cite{1}), down to 300mK. While for the asymmetric QPC, fabricated by the same method on the same crystal with the same channel length, heavily dense conductance resonances are observed (in Fig. 2 (c) of Ref. 8). If the resonances in asymmetric QPCs were caused by dirty effects, then the symmetric QPCs (Fig. 1(a)(c)) should also show dirt-induced resonances. The symmetric QPCs showing no resonances besides the well established quantized plateaus clearly rule out impurity effects. Thus the linear conductance resonances reported before are indeed intrinsic, which as ascribed, are due to electron correlations. Besides resonances, the modulation of conductance plateaus was also reported for asymmetric QPCs. \cite{8,9} Here the symmetric QPCs (Fig. 1(a)(c)), used as a control group again, show no plateau modulation as the V_{wall} was tuned from the left most to the right most, e.g. the value of plateau remains at the quantized value throughout. While for the asymmetric QPC (Fig. 1(b)(d)), clearly modulation of the first and second quantized plateaus are observed. The dotted lines mark the quantized plateaus positions.

During the measurement, one gate (the wall gate for the asymmetric QPC) voltage (V_{wall}) was swept back and forth at different speeds (bottom) to -0.71 V (top), T = 300 mK. (b): V_{wall} = -0.85 V, V_{finger} = -1.84 V (bottom) to -1.65 V (top), T = 30 mK. Inset: a typical dI/dV curve for left and right sweeping V_{sd}, vertical offset by 0.06 · 2e^2/h for clarity.

Fig. 2 shows the dI/dV as a function of source drain bias (V_{sd}) for fixed V_{wall} and different V_{finger} values (represented by different colors) (a): V_{wall} = -1.35 V, V_{finger} = -0.89 V (bottom) to -0.71 V (top), T = 300 mK. (b): V_{wall} = -0.85 V, V_{finger} = -1.84 V (bottom) to -1.65 V (top), T = 30 mK. Inset: a typical dI/dV curve for left and right sweeping V_{sd}, vertical offset by 0.06 · 2e^2/h for clarity.

The dI/dV measurement was configured in the following way: during the measurement, V_{wall} was held fixed, while V_{finger} was varied for different individual curves (different colors online). V_{wall} in Fig. 2(a) is set to the same value as the rightmost red curve in Fig. 1(b), while Fig. 2(b) has the same V_{wall} value as the leftmost red curve in Fig. 1(d). The dI/dV curves, corresponding with other red linear curves of Fig. 1(b)(d), were also systematically measured. These additional dI/dV curves show very similar behavior with those presented in this paper (including their magnetic field and temperature dependences). The three linear red curves in Fig. 1(b) or (d) represent three linear conductance regions, which alternately exhibit a weak, strong and then weak first quantized plateau. The consistency of the three dI/dV sets corresponding with the red linear curves in Fig. 1(b) or (d) for different cool downs, suggests that the oscillations exist in a wide gate voltage range. For each dI/dV trace, V_{sd} was swept back and forth at different speeds between ~2mV and 2mV. The inset in Fig. 2 contains the back and forth sweeping for one typical curve to demonstrate reproducibility.
FIG. 3. (Color online) dI/dV curves of Fig. 2, after subtracting smoothed background. Vertical offset: 0.016 (a) and 0.04 (b) for clarity. The colored curves correspond to the curves in Fig. 2 of the same color. The curves enclosed by the black dashed rectangles in (a) and (b) correspond to the respective curves enclosed by the dashed rectangular boxes in Fig. 2(a) and (b), respectively.

To highlight these oscillations, a smoothed background was subtracted. The background was generated by smoothing over a relatively wide V_{sd} range to average out the small oscillations (the thin red curve, passing through the thick green curve in Fig. 2(b), represents a typical smoothed background of the thick green curve). Fig. 3 shows the dI/dV curves, after subtractions of the smoothed background, offset for clarity. (Note that the thick green curve in Fig. 3(b) corresponds with the thick green curve in Fig. 2(b) after subtracting the red curve). A typical dI/dV curve has around 5 ~ 7 oscillations. A typical oscillation amplitude at T = 30 mK is roughly 0.035 × 2e^2/h. At T = 300 mK, the oscillation size is roughly halved, as indicated by a comparison of the oscillations in the dashed rectangular boxes in Figs. 3 (a) and (b), where the vertical scale for (b) is twice as for (a). The separation between neighboring oscillations is ~ 330 μV in V_{sd} at T = 30 mK, comparable to their separation 300 mK.

Fig. 4(a) shows the in-plane magnetic field dependence of a typical dI/dV trace. Different magnetic fields are represented with different colors. The curves in Fig. 4(b) correspond with those in (a) with the smoothed background subtracted. Fig. 4(d) shows the in-plane magnetic field dependence for a second set of fixed V_{wall} and V_{finger} values, (also with smoothed background subtracted). Figs. 4(b)(d) indicate that at a low in-plane magnetic field (∼ 0.3 T), the oscillation sizes are already significantly reduced, and are nearly fully suppressed by ∼ 0.7 T. This surprising in-plane magnetic field behavior is systematically characterized and examined below.

The magnetic field dependence of nine dI/dV traces were measured to obtain reasonable statistics. These nine sets were chosen in the following way: for each red curve in Fig. 1(d), the magnetic field dependence of three dI/dV traces, corresponding with three different V_{finger} settings, were measured. Fig. 4(b) and (d) represent two of the nine dI/dV sets. Fig. 4(c) shows a summary for the nine sets, providing a general trend of how the average oscillation size evolves with in-plane field. The average oscillation size was calculated in the following way: for each curve at a specific magnetic field, after subtracting the smoothed background, the average oscillation size was estimated by calculating the average absolute value for all data points of this new curve. Then we normalize the size against the size of each curve at B = 0 T. For each field, there are nine red dots (some not visible) representing the average oscillation amplitude of each of the nine dI/dV curves, while the black dot represents the mean value of the nine red dots. The fact that the oscillation amplitude does not decay fully to zero may be due to non-zero contributions not related to the oscillations, such as noise. Another method, based on the average root-mean-square, instead of the average absolute value for all data points of this new curve. Then we normalize the size against the size of each curve at B = 0 T. For each field, there are nine red dots (some not visible) representing the average oscillation amplitude of each of the nine dI/dV curves, while the black dot represents the mean value of the nine red dots. The fact that the oscillation amplitude does not decay fully to zero may be due to non-zero contributions not related to the oscillations, such as noise. Another method, based on the average root-mean-square, instead of the average absolute value, showed similar trends as Fig. 4(c). At B = 0.7 T, the mean oscillation size (black dot) of these nine dI/dV curves is nearly saturated in its decay trend.

In Fig. 4(a), the orange curve overlapping the black curve (B=0 T), is identical with the upper orange curve (B = 0.7 T), but without offset. As can be seen, the orange curve, for which the oscillations are nearly sup-
pressed, passes through the black curve for the most part. This behavior is found for all the other \(dI/dV\) traces: these curves with reduced oscillations pass through the \(B=0\) \(T\) curves, when plotted without vertical offset. This behavior will be discussed in the supplemental document [9] as one evidence to rule out the weak/anti-weak localization scenario. If we were to assume a typical g-factor of \(-0.44\), the Zeeman energy scale would be \(51 \mu \text{eV/Tesla}\), a factor of three smaller than than the oscillation separation (\(\sim 330 \mu \text{eV}\)) even at \(B = 2 T\). We emphasize that the sensitivity to such a small in-plane B field (\(\sim 0.3T\)) is highly unusual and completely unexpected!

![Figure 5](image_url)

**FIG. 5.** (Color online) Temperature evolution of the small \(dI/dV\) oscillations. \(T = 30 \, \text{mK}, \, 450 \, \text{mK}, \, 800 \, \text{mK}\) for (a)(b)(c) respectively. (d) is the numerical simulation for \(T=450 \, \text{mK}\). Inset in (b)(c): one typical \(dI/dV\) (black) and numerical(red) curve after subtracting smoothed background, for \(T = 450 \, \text{mK}\) (b) and \(800 \, \text{mK}\) (d), respectively. Inset in (d): statistics for the average oscillation amplitude for different gate voltages at different temperatures.

Fig. 5 shows the temperature evolution of these \(dI/dV\) oscillations, for which the \(V_{\text{wall}}\) corresponds to the middle red linear conductance curve in Fig. 1(d), at \(T = 30 \, \text{mK}\) (a), \(450 \, \text{mK}\) (b) and \(800 \, \text{mK}\) (c). The oscillations become smaller at higher temperatures and are almost washed out at \(800 \, \text{mK}\). The temperature evolution of the oscillations, corresponding to the right and left red curves in Fig. 1(d), show consistency with Fig. 5. The \(V_{\text{wall}}\) in Fig. 5(a) is held fixed at \(-1.05 \, \text{V}\), while at higher temperatures(\(450 \, \text{mK}, \, 800 \, \text{mK}\)), due to drift, and by comparing the linear conductance traces, \(V_{\text{wall}}\) was fixed at \(-1.10 \, \text{V}\) in Figs. 5(b) and (c).

Fig. 5(d) shows the numerical simulation of the \(dI/dV\) at \(T = 450 \, \text{mK}\), which qualitatively agrees with the measured data in Fig. 5(b). The simulation, based on the thermal broadening of the Fermi distribution, involves convoluting the transmission probability with the Fermi function to deduce the conductance at higher tempera-

Tures. Details are discussed in the supplemental document [9].

To better compare the measured oscillations with the simulation at each temperature, in the insets in Fig. 5(b)(c), we show the \(dI/dV\) after subtracting the smoothed background, for one representative trace in Fig. 5(b) or (c). By comparing the measured data (black) and the simulated ones (red) at \(T = 450 \, \text{mK}\) and \(800 \, \text{mK}\) in the inset of (b)(c), the oscillation sizes are seen to be semi-quantitatively in agreement, for the oscillations off zero bias. However, the structure at zero bias does not agree with, and is obviously smaller than, the simulation results. This difference in behavior suggests that the two bias regions may have different energy scales and thus are related to different origins, e.g. multi-Kondo scales or a new mechanism.

The inset in Fig. 5(d) shows the statistics for the average oscillation size of every \(dI/dV\) curves at different temperatures. This statistics gives a general trend of how the oscillation sizes decreases as increasing the temperatures. As can be seen, the oscillations are almost washed out at \(T = 800 \, \text{mK}\). The corresponding thermal energy scale, is \(2.6 \, \mu \text{eV}, \, 39 \, \mu \text{eV}, \, \text{and} \, 69 \, \mu \text{eV}\), respectively. The average oscillation size is calculated using the same method discussed in the statistics of magnetic field dependence. At a given temperature, the black vertical line indicates the spread in the average oscillation size for individual curves at a particular gate voltage, while each red dot is the mean value of all curves contained within the black line.

In the above, we have provided a fairly complete characterization of the oscillations. At present, there does not exist any theory, which can be used to interpret the unusual \(dI/dV\) oscillations. Several possibilities that may arise within simple single particle scenario are considered and ruled out [3], due to the fact that the size of the oscillations and its magnetic field dependence behavior do not agree with any of these models: For example, the contribution from the 2DEG reservoir can only give rise to an oscillation with a size much smaller than measured. Thus the oscillations are from the QPC channel. The possible excitation to quasi-bound states, modulation of energy bands due to magnetic field, field misalignment, Aharonov-Bohm effects and weak or anti-weak localizations, are also carefully considered, but are inconsistent with the data (and its magnetic field dependence). Detailed discussion is presented in the supplemental document.

By virtue of having ruled out all sensible single particle pictures, we are led to the conclusion that our observation must be related to electron-electron interactions. Since an in-plane magnetic field mainly interacts with electron spin, the magnetic field dependence suggests that the small oscillations are related to electron spin correlation. Based on the estimated 1D electron density in the single channel limit [10], a rough estimate of the electron
number inside the QPC channel is around $3 \sim 6$, which is of the same order as the number of oscillations for a typical $dI/dV$ curve. In future studies, it will be interesting to directly relate such oscillations to the formation of localized charge state inside the QPC channel due to correlation effects, possible of relevance to the exotic zigzag type of Wigner-crystal-like states 11, 12.

We thank M. Melloch for the GaAs/AlGaAs crystal. This work was supported in part by NSF DMR-0701948, and by the Academia Sinica, Taipei.

[1] K. J. Thomas, J. T. Nicholls, M. Y. Simmons, M. Pepper, D. R. Mace, and D. A. Ritchie, Phys. Rev. Lett. 77, 135 (1996).
[2] A. P. Micolich, Journal of Physics: Condensed Matter 23, 443201 (2011).
[3] W. K. Hew, K. J. Thomas, M. Pepper, I. Farrer, D. Anderson, G. A. C. Jones, and D. A. Ritchie, Phys. Rev. Lett. 101, 036801 (2008).
[4] W. K. Hew, K. J. Thomas, M. Pepper, I. Farrer, D. Anderson, G. A. C. Jones, and D. A. Ritchie, Phys. Rev. Lett., 102, 056804 (2009).
[5] P. M. Wu, Peng Li, Hao Zhang, and A. M. Chang, Phys. Rev. B 85, 085305 (2012).
[6] H. Zhang, P. M. Wu, and A. M. Chang, Phys. Rev. B 88, 075311 (2013).
[7] Runan Shang et al., arXiv:1312.2376
[8] Hao Zhang, PhD Thesis, Duke University (2014).
[9] Hao Zhang, Phillip M. Wu, and A.M. Chang, supplemental document to this manuscript.
[10] Phillip M Wu, PhD Thesis, Duke University (2010).
[11] A. D. Klironomos, J. S. Meyer and K. A. Matveev, Europhysics Letters 74, 679 (2006).
[12] A. C. Mehta, C. J. Umrigar, J. S. Meyer, and H. U. Baranger, Phys. Rev. Lett. 110, 246802 (2013).
Novel Differential Conductance Oscillations in Asymmetric Quantum Point Contacts
(Supplemental)

Hao Zhang,1 Phillip M. Wu,2 and A. M. Chang1

1Department of Physics, Duke University, Physics Building, Science Drive, Durham, North Carolina 27708, USA
2Department of Applied Physics and Geballe Laboratory for Advanced Materials, Stanford University, Stanford, California 94305

(Dated: September 22, 2014)

NUMERICAL SIMULATION FOR THE TEMPERATURE DEPENDENCE OF THE OSCILLATIONS IN FIG. 5 OF THE MAIN TEXT

The simulation is done in the following way: since the short QPC channel (≈ 100nm) is coupled to the two Fermi sea reservoirs, the QPC transport will be dominated by the Fermi statistics of these two reservoirs. Thus the conductance will be the integral of the product of electron transmission probability \((T(E))\), derivative of Fermi-Dirac function \((\partial f/\partial E)\), density of states and velocity as a function of energy, for both left and right Fermi sea reservoirs. The expression for the differential conductance is obtained and shown in the following equation:

\[
\frac{dI}{dV} = \frac{2e^2}{h} \int_{-\infty}^{\infty} \left[ \alpha \frac{\partial f(E-\mu_L,kT)}{\partial E} + \beta \frac{\partial f(E-\mu_R,kT)}{\partial E} \right] T(E) dE \tag{1}
\]

where \(\mu_L\) and \(\mu_R\) are the quasi-Fermi level for the left and right reservoir, \(\alpha\) and \(\beta\) represent the portion of bias distributed between the left and right reservoir respectively. Thus \(\alpha + \beta = 1\). During the measurement, one reservoir is virtually grounded, leading to \(\alpha = 1, \beta = 0\). The simulation shown in Fig.5 is based on this assumption. At \(T = 30mK\), the width of the derivative of Fermi-Dirac function, \(3.5kT \approx 9\mu eV\), is much smaller than the width of the small oscillations (≈ 330\(\mu V)\), thus to the first order approximation, the \(dI/dV\) curves at \(T = 30mK\) can be treated as if it was at \(T = 0K\). At \(T = 0K\), the derivative of Fermi-Dirac function in Equation 1 becomes a delta function, thus \(T(E)\) can be extracted. Using the \(dI/dV\) curves at \(T = 30mK\) as an input to extract \(T(E)\), the high temperature evolution can be simulated by plugging in \(T(E)\) in Equation 1.

One typical simulation for \(T = 450mK\), using Fig.5(a) as an input, is shown in Fig.5(d). As can be seen, the oscillation sizes qualitatively agree with the measured \(dI/dV\) at the same temperature (Fig. 5(b)). The simulation for \(T = 800mK\) (not shown here) also qualitatively agrees with Fig.5(c). Besides this set, the simulation for other sets of \(dI/dV\) curves, with the wall voltage corresponding to the leftmost and rightmost red curves in Fig. 1(d), show consistency with the measured \(dI/dV\) oscillations at \(T = 450mK, 800mK\). Besides the assumption \(\alpha = 1, \beta = 0\), simulations based on \(\alpha = 0.9, \beta = 0.1\) and \(\alpha = 0.7, \beta = 0.3\) are also implemented and show similar results compared with the \(\alpha = 1, \beta = 0\) case.

DISCUSSION OF SIMPLE SINGLE-PARTICLE PICTURES

We first rule out any artifacts from either the indium contacts, or from the two-dimensional electron gas (2DEG) regions leading into the QPC, as we discuss below. Consequently, we can ascribe the observed novel behaviors to the 1d QPC channel with full confidence. Let us begin by examining the issue of contact resistance. Typical good quality indium contacts to the 2DEG are below 1\(\Omega\) at low temperature. This magnitude is far smaller than the observed oscillation amplitude, which is as large as 200\(\Omega\). Therefore, artifacts from the indium contacts can be safely ruled out.

As far as the 2DEG regions, which behave as excellent metallic conductors, the only possibility for any variation in the resistance (or conductance) in this temperature range \((30mK - 1K)\) comes from quantum interference effects, such as Aharonov-Bohm interference, or weak localization/anti-localization. Given the long \(l_o\) typically exceeding 10\(\mu m\) at 30\(mK\), we expect such effects to be of order \(G_0 = 2e^2/h\), compared to the 2DEG conductance of \(G_{2D} \approx 60G_0\) (equivalent to \(\sim 220\Omega\)). A change of order \(G_0\) in the 2DEG will yield a corresponding resistance change of \(\sim 6\%\), or 13\(\Omega\), far below the observed resistance modulation of 200\(\Omega\). Therefore, the 2DEG regions cannot account for our observations. These considerations give substantial confidence that our findings are associated with the 1d channel.

Next, we examine several single particle effects occurring within the quasi-1-d QPC channel proper, including: (1) excitation into higher longitudinal bound states associated with quantization along the length of the channel, (2) misalignment of the in-plane magnetic field (\(\sim 7^\circ\)), giving rise to a vertical component out of the 2DEG plane, and (3) quantum interference, weak-localization/anti-localization, and Aharonov-Bohm interference type effects. We argue that none of these scenarios is consistent with observations.

One possible origin causing these oscillations may be the excitations to the longitudinal quasi-bound states formed in the channel, associated with the channel length. If this were the case, the effective wave number and channel length (100nm) for the Nth quasi-bound states should satisfy \(k_N \cdot L = N\pi\), similar with the relation of a Fabry – Pérot interferometer. Thus even the
smallest energy level spacing, which is between $N=1$ and $N=2$, is estimated to be $\Delta E = \frac{(\hbar k)^2}{2m} = 1.7$ meV. This energy level spacing is 5 times greater than the spacing between the neighbor oscillations, which is around 330μV in $V_{sd}$. Besides this level spacing, a small in-plane magnetic field $\sim 0.3T$ is expected to do little to the electron transport in this quasi-bound state excitation picture. But the $dI/dV$ oscillations were significantly modified (suppressed) under such small in-plane magnetic field. These considerations suggest that the oscillations cannot be due to this excitation into higher single particle quantum levels.

Now we estimate how the magnetic field modifies the potential profile of the QPC in the 2DEG. The profile is assumed to be a parabola shape: $\frac{1}{2}m\omega^2x^2$ along the direction perpendicular to the channel (out-of-plane). The $\omega$ was estimated to be $3.14 \times 10^{12}/s$, using the magnetic depopulation method. Applying a magnetic field perpendicular to the 2DEG will hybridize this parabola with Landau levels to a new parabola with a new $\omega$. Since the in-plane magnetic field was aligned with the 2DEG within $7^\circ$, the modification of $\omega$ due to this out-of-plane magnetic field hybridization at $B = 0.3T$ is estimated to be less than $0.2%\omega$. Such a small modification of the potential profile has little effect on the electron transport, as the effective $\omega$ increases quadratically with $B$. Thus this single particle picture cannot give rise to the significant modulation of the small oscillations either.

Another way to interpret these oscillations is to relate them with the Aharonov-Bohm effect. But the estimated Aharonov-Bohm phase shift at $B = 0.3T$, due to the out-of-plane magnetic field component, is $\Delta \phi = \frac{\Phi_B}{\Phi_0} \approx 0.05$ radians, which is much too small to affect the transport. In this estimation, to calculate $\Phi_B = B_{\perp}A$, where $A$ is the effective area of the QPC channel, the channel length is assumed to be the same with the lithographic length, 100nm, and the effective channel width is assumed to be 10% of the estimated 80 nm channel width in the single channel limit. Within a ballistic channel, the fact that the wave function phase does not sample across the entire physical channel width gives rise to this reduction factor, in contrast to a diffusive case.

In the above, we have already established that Fermi-Dirac smearing is able to account for the reduction of the small oscillations amplitude with increasing temperature. We have also argued, based on the magnetic field dependence (and accounting for a small $7^\circ$ misalignment with the 2DEG plane), that quantum interference/weak localization type mechanism is unable to account for them. Here, we further point out that in the unlikely event that quantum interference or weak localization becomes of relevance, any issue of the quantum phase coherence length, $\xi$, should not lead to discernible effects, while further emphasizing the unlikelihood of the quantum interference scenario.

At low temperatures, at low energies $l_\phi$ typically exceeds $20\mu m$ below $50mK$ for high mobility ($\sim 10^6 cm^2/Vs$) devices. The temperature evolution of $l_\phi$ is typically power law, behaving as $T^{-\frac{1}{4}}$ or $T^{-\frac{1}{2}}$, depending on whether it is limited by small energy transfer or large energy transfer. Even at $450mK$, $l_\phi$ is expected to exceed $5\mu m$, still far longer than the QPC channel length of $\sim 100nm$. Therefore, there should be little influence on transfer in the linear regime (low energies) due to weak/anti-weak localization. Moreover, the $dI/dV$ oscillations occur at a source drain bias voltage of $\sim 330\mu eV$ and higher, far exceeding the thermal energy at $800mK$ and below. Therefore, the relevant $l_\phi$ at such high energies should be completely insensitive to temperature in the range of our study. Besides that, for weak/anti-weak localization, the conductance at a finite magnetic field is either higher or lower than the zero magnetic field case, but as shown in Fig. 4(a) of the main text (the orange curve online), the conductance at a finite magnetic field goes through the zero magnetic field case, suggesting that it is not weak/anti-weak localization.

[1] Phillip M Wu, PhD Thesis, Duke University (2010).
[2] B. J. van Wees, L. P. Kouwenhoven, H. van Houten, C. W. J. Beenakker, J. E. Mooij, C. T. Foxon, and J. J. Harris, Phys. Rev. B 38, 3625 (1987).
[3] G. Timp, A. M. Chang, J. E. Cunningham, T. Y. Chang, P. Mankiewich, R. Behringer, and R. E. Howard, Phys. Rev. Lett. 58, 2814 (1987).
[4] A. M. Chang, H. U. Baranger, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 73, 2111 (1994).