QUANTUM MECHANICS VIOLATING EFFECTS TO MASSES OF NAMBU-GOLDSTONE BOSONS : A LESSON FOR MAJORON

Yūichi Chikashige and Tadashi Kon

Faculty of Engineering, Seikei University, Musashino, Tokyo 180, Japan

Abstract

We study gravitational quantum mechanics violating (QMV) effects to masses of Nambu-Goldstone bosons, taking majoron as an example. We show a supersymmetric majoron has either mass of O(keV) for the dimension five potential or smaller mass for effective potentials with higher dimensions. We extend the Dashen’s formula for pseudo Nambu-Goldstone bosons to include possible effects of QMV.
Majoron is a Nambu-Goldstone boson associated with the spontaneous breaking down (SSB) of global U(1)$_{B-L}$ symmetry \cite{1, 2}. It was originally introduced to give mass of right-handed neutrino in the seesaw model \cite{1, 3}. Majoron is massless, unless gravitational interaction is introduced. It has been widely argued that quantum gravitational interaction should not respect any kind of global symmetry, since black holes are pointed out to cause information loss \cite{4}. Therefore one can say that quantum mechanics violating (QMV) effects through creation and successive evaporation of black holes should give majoron nonvanishing mass. Other Nambu-Goldstone bosons like pion and axion should also get through QMV effects such additional masses which would be relatively small values compared to electroweak masses \cite{5}. Majoron is different kind from those pseudo Nambu-Goldstone particles at this point, for this particle has no anomaly to generate electroweak mass and the U(1)$_{B-L}$ symmetry does not have the gravitational anomaly to induce gravitational mass if there would be gravitational instantons.

This paper is concerned with two ways to describe QMV contribution to masses of Nambu-Goldstone bosons, dealing with majoron as a special example. One is the effective potential approach which has been extensively discussed so far \cite{6, 9}. However the other seems to be a bit novel approach which we call the Dashen’s formula with QMV effects. The latter is based on the interesting proposal given by Ellis, Hagelin, Nanopoulos and Srednicki \cite{7} to illustrate Hawking’s idea on QMV effects in which a pure initial state of a system evolves into a final mixed state \cite{8}.

Rothstein, Babu and Seckel in ref.\cite{6} and Akhmedov, Berezhiani, Mohapatra and Senjanović in ref.\cite{9} wrote down such an effective potential for majoron in terms of expansion with inverse powers of the Planck mass, $M_{pl}$, learning lessons of axion case \cite{5}. Akhmedov et al. examine the dimension five potential which includes the perturbative term $V_{pl}(\phi, \sigma)$
with mass-dimension five in addition to the standard Higgs potential $V_0(\phi, \sigma)$ as follows:

$$V = V_0(\phi, \sigma) + V_{pl}(\phi, \sigma), \quad (1)$$

$$V_{pl}(\phi, \sigma) = V_1(\sigma) + V_2(\phi, \sigma). \quad (2)$$

Here $\phi$ denotes the standard isodoublet Higgs field whose vacuum expectation value, $V \simeq 246$ GeV, gives the Dirac mass of neutrinos and $\sigma$ represents the isosinglet one whose vacuum expectation value, $V_{BL}$, gives the Majorana mass of the right-handed neutrinos. The minimum of $V_0(\phi, \sigma)$ determines $V_{BL}$ which is the scale of the violation of $B - L$ number conservation. Then $\sigma$ is written as

$$\sigma = \frac{V_{BL} + \rho}{\sqrt{2}} \exp(i \frac{\chi}{V_{BL}}), \quad (3)$$

where $\chi$ is the majoron, and $\rho$ is an isosinglet Higgs scalar.

$$V_1(\sigma) = \alpha_1 \frac{\sigma^5}{M_{pl}} + \alpha_2 \frac{\sigma^* \sigma^4}{M_{pl}} + \alpha_3 \frac{\sigma^2 \sigma^3}{M_{pl}} + \text{h.c.}, \quad (4)$$

and

$$V_2(\phi, \sigma) = \beta_1 \frac{(\phi^* \phi)^2 \sigma}{M_{pl}} + \beta_2 \frac{(\phi^* \phi) \sigma^2 \sigma^*}{M_{pl}} + \beta_3 \frac{(\phi^* \phi) \sigma^3}{M_{pl}} + \text{h.c.}. \quad (5)$$

are the dimension five forms written in ref. [9]. According to the relative magnitude between $V$ and $V_{BL}$, ref. [9] classifies the two cases (A) and (B). $V < V_{BL}$ corresponds to the case (A) where the mass of majoron, $m_\chi$, becomes approximately

$$m_\chi \simeq \sqrt{\beta_1 \left( \frac{V}{V_{BL}} \right)} \text{keV}, \quad (6)$$

while $V_{BL} < V$ is the case (B) and here

$$m_\chi \simeq \sqrt{\frac{29}{3} \alpha_1 + \frac{9}{2} \alpha_2 + \frac{1}{2} \alpha_3 \left( \frac{V_{BL}}{V} \right)} \text{keV}. \quad (7)$$

Akhmedov et al. mentioned the upper bound for $V_{BL}$ is constrained from the cosmological mass density to be $10 \text{ TeV}$. But no further strong arguments were not given to specify the value of $V_{BL}$ by them.
Now let us turn to see what happens if we look at supersymmetric (SUSY) version of majoron. Shiraishi, Umemura and Yamamoto argued in detail such a model \cite{10} and the identical model was independently discussed by Giudice, Masiero, Pietrini and Riotto around the same time \cite{11}. Following the notation of ref. \cite{11}, we have the potential of Minimal Supersymmetric Standard Model (MSSM), $V_0$, after SUSY breaking as

$$V_0 = V_0(H_1^0, H_2^0) + V_0(N, \Phi) + V_0(\nu, N, \Phi, H_1^0, H_2^0).$$

(8)

In this equation, $V_0(H_1^0, H_2^0)$ is the usual MSSM Higgs potential, and $V_0(N, \Phi)$ and $V_0(\nu, N, \Phi, H_1^0, H_2^0)$ are the terms which are both responsible to break $U(1)_{B-L}$ symmetry and $R$-parity as well. The soft SUSY breaking masses included in $V_0$ are supposed to be an order of 1 TeV as usual. Then as noted by both groups of the authors of refs. \cite{10} and \cite{11}, the consistency requires that $R$-parity and $U(1)_{B-L}$ symmetry should also be broken down spontaneously at the same order, 1 TeV. Thus we admit $V_{BL} \sim O(1\text{TeV})$ for the SUSY majoron. Now the case (B) in ref. \cite{9} should be chosen for our SUSY majoron and its mass is said by the above mentioned effective potential approach to QMV effects of ref. \cite{9} to be an order of keV. Actually Berezinsky and Valle expected that a very weakly interacting keV majoron is considered to be a good candidate for a dark matter particle \cite{12}.

Then a fundamental question is arisen why the dimension five effective potential could be more important than other effective potentials with higher dimensions for our majoron. Let us take an effective potential $V_n$ with an arbitrary dimension $n$ for the case (B) as follows:

$$V_n = \alpha_n \frac{\sigma^n}{M_{pl}^{n-4}}$$

(9)

This gives majoron such mass as

$$m_{\chi} = \frac{(n + 4)(n + 3)}{2^\frac{n}{2}} \alpha_n \frac{V_{BL}}{M_{pl}} \# V_{BL}.$$  

(10)
Our SUSY majoron would obtain such an order of mass for each dimension $n$.

$$m_x \simeq \begin{cases} 
10^{-5}\text{GeV} & \text{for dim 5 (}n=1\text{)} \\
10^{-13}\text{GeV} & \text{for dim 6 (}n=2\text{)} \\
10^{-21}\text{GeV} & \text{for dim 7 (}n=3\text{)} \\
\vdots
\end{cases} \quad (11)$$

At this stage we don’t have any motivations forcing us to choose a special value of mass among the above. What one can say at most is only that the origin of mass of majoron should be the QMV effects.

Now let us turn to an alternative approach. Hawking pointed out the fact that creation and evapolation of black holes let a system loose quantum coherence [13]. He then tried to present axioms suitable to quantum theory of gravity and construct the superscattering operator to represent loss of quantum coherence [8]. Following his idea, Ellis et al. proposed a special form of a differential equation for a density matrix $\rho$ which describes evolution of a system from a pure state to a mixed state [7]. Although Banks, Susskind and Peskin wrote a paper in which this differential equation might cause either breakdown of causality or violation of energy-momentum conservation [14], Unruh and Wald have recently published a paper in which they argue such undesirable features would hardly been seen in our laboratories [15]. We are going to follow this viewpoint of Unruh and Wald.

The equation for $\rho$ written by Ellis et al. [7] is as follows, according to Unruh and Wald [15],

$$\dot{\rho} = -i[H, \rho] - \sum_i \lambda_i (Q_i\rho + \rho Q_i - 2Q_i\rho Q_i). \quad (12)$$

The first term of the right-hand side in eq.(12) is a conventional quantum mechanical one. The second term of the right-hand side in eq.(12) in which $Q_i$ is an hermite projection operator, $Q_i^\dagger = Q_i$, and $Q_i^2 = Q_i$, implies such a peculiar evolution of the system from a pure state to a mixed state, namely, QMV development. Unruh and Wald have written the Heisenberg equation with Hamiltonian $H$ for a Heisenberg operator $A_H$ in the following
\[ \dot{A}_H = i[H, A_H] - \sum_i \lambda_i (Q_i A_H + A_H Q_i - 2Q_i A_H Q_i) \]  
\[ = i[H, A_H] + \sum_i \lambda_i [Q_i, [A_H, Q_i]]. \]

(This was noted by Lindblad [L6] and Gorini, Frigerio, Verri, Kossakowski and Sudarshan [L7].)

Now we recall that mass of pseudo Nambu-Goldstone particle obeys the Dashen’s formula [L8]:

\[ m^2 = -\frac{1}{f^2} \langle 0 | [Q_5, \dot{Q}_5] | 0 \rangle \]

where \( Q_5 \) is a generator of some global symmetry which would be broken down spontaneously with decay constant \( f \). Therefore we can write such a formula, using the evolution equation for \( Q_5 \), as

\[ m^2 = \frac{i}{f^2} \langle 0 | [Q_5, [Q_5, H]] | 0 \rangle - \frac{1}{f^2} \sum_i \lambda_i \langle 0 | [Q_5, [Q_i, [Q_5, Q_i]]] | 0 \rangle. \]

The second term in the right-hand side of the above equation represents QMV contribution to the mass of the Nambu-Goldstone boson. If gravitational interaction would be neglected, this QMV mass should disappear. Thus one could expect that either \( \{\lambda_i\} \) would include suppression factors of \( 1/M_{pl}^k \) or small values of the matrix elements due to the presence of the projection operators \( \{Q_i\} \) which communicate Hilbert space relating to black holes to Hilbert space in our laboratories or both kinds of suppression would be included. As for majoron, \( B - L \) current has no anomaly, so that the first term in the right-hand side of eq.(16) disappears, contrasted with other pseudo Nambu-Goldstone particles like pion, axion and so on. Therefore the generator of \( B - L \) symmetry, \( Q_{B-L} \), and the projection operators \( Q_i \) would give majoron \( \chi \) such mass as

\[ m^2_{\chi} = -\frac{1}{f^2} \sum_i \lambda_i \langle 0 | [Q_{B-L}, [Q_i, [Q_{B-L}, Q_i]]] | 0 \rangle, \]
if we follow the argument in ref. [7]. The parameters \( \lambda_i \) should determine an order of magnitude of \( m_{\chi} \).

Ref. [7] mentions an interesting inequality which is said as an accidental coincidence

\[
\lambda \leq 2 \times 10^{-21} \text{GeV}
\]  

(18)

from long baseline neutron interferometry experiment and \( K^0-\bar{K}^0 \) system, where \( \lambda \) in their paper plays essentially the same rôle as our \( \{ \lambda_i \} \) play. We have again another accidental coincidence with such a value as \( 10^{-21} \) GeV in the previous effective potential with dimension seven in eq.(11). Of course we cannot take it too seriously at this stage. Moreover, there seems to be no reason why we would expect to have a universal contribution of QMV effects. It should be noted here, however, that some physical effects caused by such tiny mass as \( 10^{-21} \) GeV may be feasibly triggered for neutrino oscillations in the case of scalar light particle [19].

Hawking stressed that there shouldn’t be any suppression factors with inverse powers of \( M_{\text{pl}} \) for matrix elements of the scalar particles in contrast with those of vector bosons and spin 1/2 particles [8]. Hawking, Page and Pope once argued furthermore that there may be even a scalar tachyon [20]. If we would follow this opinion, we should think doubtfully that the effective potential in eq.(4) has such an suppression factor as \( 1/M_{\text{pl}}^{n-4} \). We see an advantage of the approach of the Dashen’s formula on that point, since this formula can be written down in any case with suppression factors or without them. Certainly one has \textit{a priori} no reason to expect non-negative contribution from QMV effects in the second term of our Dashen’s formula, eq.(16). That means we need to take definitely much more efforts to examine carefully this vacuum expectation value of commutators with two generators of a global symmetry and a couple of projection operators in that term.

In this note we have given a couple of descriptions for masses of Nambu-Goldstone
particles, namely, the effective potential and the Dashen’s formula. For majoron the effective potential approach needs to have the value of $V_{BL}$ and a specification of dimension as well in order to predict the mass. SUSY majoron can provide an interesting value of $V_{BL}$. The Dashen’s formula needs to analyze deeply the matrix elements of commutation relations in the second term of the right-hand side in eq.(16) and in eq.(17). Otherwise we would never understand what kind of physical process would control masses of Nambu-Goldstone bosons through QMV effects.

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