Reflective scattering, color conductivity, and centrality in hadron reactions

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Abstract

The reflective scattering mode is supposed to play a significant role in hadron interactions at the LHC energy region and beyond. We discuss connection of this mode to the color conducting medium formation in hadron collisions and its role in centrality determination. The issues of centrality in view of the measurements at the LHC are relevant for enlightening the asymptotic dynamics.
1 Introduction

Hadrons are composite, extended objects and their formfactors are described by nontrivial functions. So, a seemingly natural expectation is an increase with energy of the relative weight of all the inelastic interactions. Otherwise, a significantly increasing relative contribution to $pp$–interactions from elastic scattering events was observed, i.e. the ratio of elastic to total cross-sections $\sigma_{el}(s)/\sigma_{tot}(s)$ rises with energy, while the ratio $\sigma_{inel}(s)/\sigma_{tot}(s)$ decreases accordingly. Such trends in behavior of elastic and inelastic interactions have been confirmed by the recent measurements at $\sqrt{s} = 13$ TeV [1]. Thus, the experimental results are divergent from the above expectation which looked as a natural one for the time being. However, such divergence between experimental results and theoretical expectation is not surprising due to neglect of the color confinement effects under formulation of such naive suggestion. Evidently, the role of confinement becomes more significant as the hadron collision energy increases.

An important role of the elastic component of strong interactions dynamics was underlined long time ago by Chew and Frautchi [2] in their study based on the Mandelstam representation and principle of maximal strength of strong interaction. The term “strongest possible” interaction means maximality of the total interaction rate $N_{tot}$ which is a product of the collider luminosity by the effective total cross–section, i.e. $N_{tot} = \mathcal{L}\sigma_{tot}$. It corresponds to maximal possible values of imaginary parts of the partial amplitudes since sum of those results in an effective total cross-section according to optical theorem. Chew and Frautchi noted that a “characteristic of strong interactions is a capacity to “saturate” the unitarity condition at high energies”. It results in a hint on the dominating contribution of the elastic scattering (decoupled from the multiparticle production) due to unitarity saturation at $s \to \infty$. Unitarity saturation corresponds to zeroing of the real part of partial amplitudes\footnote{It is evident from the Argand plot for a partial amplitude.}.

It should also be remarked here (enhancing what was said above), that the upper bound for the inelastic cross–section obtained recently [3] excludes $\ln^2 s$–dependence for $\sigma_{inel}(s)$ at $s \to \infty$ when the ratio $\sigma_{tot}(s)/\sigma_{tot}^{max}(s)$ follows its limiting behavior, i.e. it tends to unity at $s \to \infty$. Here $\sigma_{tot}^{max}(s) (\equiv (4\pi/t_0) \ln^2(s/s_0))$, represents the Froissart–Martin boundary dependence for the total cross-sections and results from saturation of unitarity and the elastic scattering amplitude analyticity in the Lehmann–Martin ellipse.

Extrapolation of the observed experimental dependencies to higher energies can be performed in a twofold way: one can assume equipartition of the elastic and inelastic contributions at $s \to \infty$ (black disc) or saturation of the unitarity bound leading to the asymptotic behavior of the ratios $\sigma_{el}(s)/\sigma_{tot}(s) \to 1$ and
Some intermediate dependencies of these ratios at $s \to \infty$ can also be envisaged (cf. [4]). Both of the above mentioned modes are in agreement with unitarity, but rejection of the reflective scattering by assuming $f(s, b) \leq 1/2$, where $b$ is an impact parameter of the colliding hadrons (note that $l = b\sqrt{s}/2$ and the real part of the amplitude is neglected) means ad hoc limitation imposed by assumption that the only possible scattering mode it is “the shadow mode” and restricts the wealth of the possible dynamics provided by the unitarity. Moreover, it has been known for a long time that absorption is not a consequence of unitarity [5].

The reflective scattering mode corresponds to unitarity saturation at $s \to \infty$, decoupling of elastic scattering from multiparticle production in this limit and, hence, is consistent with “strip approximation” of [2] which suppose dominance of double spectral functions in the areas determined by elastic unitarity condition. The main feature of the mode is a negative value of the elastic scattering matrix element $S(s, b)$, it leads to asymptotic dominance of the elastic scattering and peripheral character of the inelastic scattering overlap function in the impact parameter space. Decoupling of the elastic scattering from the multiparticle production starts to occur at small values of the impact parameter $b$ first and expands to larger values of $b$ while collision energy increases. Such a behavior corresponds to increasing self–dumping of inelastic contributions to unitarity equation [6]. The knowledge of the decoupling dynamics is essential for the hadron interaction’s studies, e.g. for the development of QCD in its nonperturbative sector where the color confinement plays an essential role.

The aforementioned properties of the reflective scattering underlines importance of this mode in the context of the hadron dynamics study. The interest is actualized also by the results of the recent measurements at the LHC [7,8]. The essential features of the reflective scattering mode are discussed in the first part of this note.

The $b$–dependence of the scattering amplitude as well as of the inelastic overlap function are associated with a collision geometry. The quantitative description of these functions is important for the reflective scattering mode detection occurring initially at small impact parameter values. This can be performed using centrality variable for the events classification. We consider suggestion for centrality definition in the case of small systems in the second part of this note.

2 Reflective scattering in hadron interaction

The main features of the reflective scattering mode are listed below. Partial wave matrix element of the elastic scattering is related to the corresponding amplitude
function $f_l(s)$ by the relation

$$S_l(s) = 1 + 2if_l(s),$$

where the amplitude $f_l(s)$ obey the unitarity equation:

$$\text{Im}f_l(s) = |f_l(s)|^2 + h_{l,\text{inel}}(s).$$ \hspace{1cm} (1)

It is convenient to use an impact parameter representation, which provides a simple semiclassical picture of hadron scattering, recall that $l = b\sqrt{s}/2$. For simplicity, we use a common assumption on the smallness of the real part of the elastic scattering amplitude in the impact parameter representation $f(s, b)$ and perform replacement $f \rightarrow if$. It should be noted that this assumption correlates with unitarity saturation. Indeed, unitarity saturation means that $\text{Im}f(s, b) \rightarrow 1$ at $s \rightarrow \infty$ and fixed $b$. It can easily be seen that this limiting behavior implies that $\text{Re}f(s, b) \rightarrow 0$ at $s \rightarrow \infty$ and fixed $b$ and this leads to inconsistency of Maximal Odderon with unitarity saturation. Neglect of the real part contribution is justified at least qualitatively since the recent analysis is consistent with this conclusion. However, the problem of the real part cannot be considered as solved nowadays due to insufficient experimental data. Its value is sensitive to the possible violation of dispersion relations. It should be noted that there were considered different amplitude parameterizations in which the ones with negative imaginary part at large values of $-t$ and all are consistent, nonetheless, with peripheral form of the inelastic overlap function. For discussion of the imaginary part of the scattering amplitude sign, see [13].

Unitarity equation provides the evident relation for the dimensionless differential distribution of the inelastic collisions over $b h_{\text{inel}}(s, b)$ in case of proton–proton scattering

$$h_{\text{inel}}(s, b) = f(s, b)(1 - f(s, b)).$$ \hspace{1cm} (2)

It constraints variation of the amplitude $f(s, b)$ by the values from the interval $0 \leq f \leq 1$. The value of $f = 1/2$ corresponds to the complete absorption of the initial state and means that the elastic scattering matrix element is zero, $S = 0$ (note that $S = 1 - 2f$). If the amplitude $f(s, b)$ at $b = 0$ (beyond some threshold value of energy) becomes greater than $1/2$, then the maximal value of differential distribution of inelastic collisions is $1/4$ at $b > 0$ (cf. Eq. (2)). Thus, approaching unitarity saturation limit in the region where $f > 1/2$ leads to a peripheral nature of the inelastic hadron collisions’ differential distribution over impact parameter (inelastic overlap function). Such peripheral character is a straightforward result of a probability conservation, i.e. unitarity.

Reflective scattering mode appears in the region of $s$ and $b$ where the amplitude $f$ variates in the range $1/2 < f \leq 1$. It means that $S$ is negative and lies in the region $-1 \geq S < 0$. A negative $S$ is the reason for the term “reflective scattering”. Its interpretation will be discussed in the next section.
The value of the collision energy corresponding to the complete absorption of the initial state under the central collisions \( S(s, b) |_{b=0} = 0 \) is denoted as \( s_r \) and the estimates for the value of \( s_r \) are of order of few TeV \([14]\). At the energies \( s \leq s_r \) the scattering in the whole range of impact parameter variation has a shadow nature (Fig. 1), it means that solution of the unitarity equation for the elastic amplitude has the form:

\[
f(s, b) = \frac{1}{2} \left[ 1 - \sqrt{1 - 4h_{\text{inel}}(s, b)} \right],
\]

which assumes a direct coupling of elastic scattering to multiparticle production often called shadow scattering. When the energy value becomes larger than \( s_r \), the scattering picture at small values of impact parameter \( (b \leq r(s)) \), where \( S(s, b = r(s)) = 0 \) starts to acquire a reflective contribution. At such energy and impact parameter values unitarity gives for the elastic amplitude another form:

\[
f(s, b) = \frac{1}{2} \left[ 1 + \sqrt{1 - 4h_{\text{inel}}(s, b)} \right].
\]

Eq. (4) corresponds to growing decoupling of the elastic scattering from multiparticle production. It can exist in the limited range of the impact parameter values only, namely, at \( b \leq r(s) \) since at larger values of \( b \): \( f \sim h_{\text{inel}} \). At \( s > s_r \) the function \( h_{\text{inel}}(s, b) \) has a peripheral \( b \)-dependence. Note, that

\[
\frac{\partial h_{\text{inel}}(s, b)}{\partial b} = S(s, b) \frac{\partial f(s, b)}{\partial b},
\]

Figure 1: Schematic representation of the regions in \( s \) and \( b \) plane corresponding to absorptive \((S > 0)\) and reflective \((S < 0)\) scattering modes.
where $S(s, b)$ is negative at $s > s_r$ and $b < r(s)$.

### 3 Reflection and color conductivity

The elastic scattering matrix element can be written in the form

$$S(s, b) = \kappa(s, b) \exp[2i\delta(s, b)].$$  \hspace{1cm} (6)

Here $\kappa$ and $\delta$ are the real functions and $\kappa$ can vary in the interval $0 \leq \kappa \leq 1$. This function is called an absorption factor and its value $\kappa = 0$ corresponds to complete absorption of the initial state,

$$\kappa^2(s, b) = 1 - 4h_{\text{inel}}(s, b).$$  \hspace{1cm} (7)

The function $S(s, b)$ can be nonnegative in the whole region of the impact parameter variation or have negative values in the region $b < r(s)$ when the energy is high enough, i.e. at $s > s_r$. Under the reflective scattering, $f > 1/2$, an increase of elastic scattering amplitude $f$ corresponds to decrease of $h_{\text{inel}}$ according to

$$(f - 1/2)^2 = 1/4 - h_{\text{inel}}$$

and, therefore, the term antishadowing has been initially used for description of such scattering mode emphasizing that the reflective scattering is correlated with the self-damping of the inelastic channels contribution [6] and increasing decoupling of the elastic scattering from multiparticle production dynamics.

Transition to the negative values of $S(s, b)$ means that the phase $\delta$ changes its value from $0$ to $\pi/2$. The term reflective is taken from optics since the phases of incoming and outgoing waves differ by $\pi$. It happens when the reflecting medium is optically denser (i.e. it has a higher refractive index than the medium where incoming wave travels before encounter the scatterer). Thus, there is an analogy with the sign change under reflection of the electromagnetic wave by surface of a conductor. This occurs because of the electromagnetic field generates a current in the medium. The energy evolution of the scatterer leads to appearance of the reflective scattering mode if one admits the unitarity saturation in the limit of $s \to \infty$.

Reflective scattering mode does not assume any kind of hadron transparency during the head-on collisions. Contrary, it is about the geometrical elasticity (cf. [15]). The term transparency is relevant for the energy and impact parameter region responsible for the shadow scattering regime only, i.e. where $f < 1/2$ (cf. Fig 1). The interpretation of the reflective scattering mode based on the consideration of inelastic overlap function alone is, therefore, a deficient one. It could lead to an incorrect rendering based on idea of the formation of the hollow
fireball (e.g. filled by the disoriented chiral condensate) in the intermediate state of hadron–hadron interaction (cf. [16]) and consideration of the central region as the transparent one.

The emerging physical picture of high energy hadron interaction region in transverse plane can be visualized then in a form of a reflecting disk (with its albedo approaching to complete reflection at the center) which is surrounded by a black ring (with complete absorption, $h_{inel} = 1/4$) since the inelastic overlap function $h_{inel}$ has a prominent peripheral form at $s \to \infty$ in this scattering mode. The reflection mode implies that the following limiting behavior $S(s, b)|_{b=0} \to -1$ will take place at $s \to \infty$. Of course, it is supposed a monotonic increase of the amplitude $f$ with energy to its unitarity limit $f = 1$, and an artificial option of its nonmonotonic energy dependence at fixed values of $b$ is excluded.

QCD is a theory of hadron interactions with colored objects confined inside those entities. Thus, one can imagine that the color conducting medium is being formed instead of color insulating one when the energy of the interacting hadrons increases beyond some threshold value. Properties of such medium are under active studies in nuclear collisions, but color conducting phase can be generated in hadron interactions too. Therefore, one can try to associate appearance of the reflective scattering mode with formation of the color conducting medium in the intermediate state of hadron interaction (cf. Fig. 2). Such idea was briefly mentioned in ref. [16]. The analogy is based on replacement of an electromagnetic field by a chromomagnetic field of QCD. As another example of a source for analogy the phenomenon of Andreev reflection at the boundary of the normal and superconducting phase applied to quark scattering at the interface between the cold quark-gluon phase and the color-superconductor can be pointed out [19, 20].

Formation of the color conducting medium in hadron collisions might also be responsible for a number of collective effects such as correlations, anisotropic flows and others observed in small systems. Such effects can arise due to a ring-like shape of the impact–parameter region responsible for the multiparticle production processes. Such ring-like shape is a result of the reflective scattering mode appearance. It implies an important role of a coherent behavior of the deconfined matter formed under hadron interactions, resulting finally in the explicit collective effects [21].

We do not specify the nature of color conducting medium, namely, there is no need to discuss what kind of constituents form it — light point-like current colored quarks or massive quasi-particles — colored constituent quarks. But, in any case one can expect appearance of color conductivity in this medium under

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2Despite the limiting behavior of $S(s, b)$ corresponds to $S \to -1$ at $s \to \infty$ and fixed $b$, the gap survival probability, contrary to conclusion of [17], tends to zero at $s \to \infty$ (cf. [15]).

3Such nonmonotonic behavior might result (after integration over $b$) in peculiar distortions superimposed onto the rising energy dependence of the total cross-section.
Indeed, the LHC experiments lead to an unexpected discovery of collective effects in small systems [22] (for comprehensive list of references to the experimental results of ALICE, ATLAS, CMS and LHCb Collaborations cf. [23] and for a brief review — [24]). A prominent example is an observation of a "ridge" effect in two–particle correlations in $pp$-collisions in the events with high multiplicities [25].

Interest to the collective effects is supported by their relation to dynamics of quark-gluon plasma formation [26]. And confinement of color is associated with collective, coherent interactions of quarks and gluons. It results in the formation of the asymptotic colorless states–hadrons.

The reflective scattering mode formation is consistent with the results of the impact parameter analysis at the LHC energy $\sqrt{s} = 13$ [11]. This mode can be interpreted as a manifestation of the color conducting phase formation. The observation of the reflective scattering mode makes important the events classification depending on the impact parameter of the collision. This mode most significantly affects collisions with small impact parameters and hence it is important to classify an impact parameter values of the particular hadron collision events.

It should be emphasized that the collision geometry describes the hadron interaction region entirely but not the spacial properties of the each participating hadron.

In what follows we consider definition of centrality in small systems and the
role of the reflective mode at the LHC energy range.

4 Centrality in nucleus–nucleus collisions

The centrality is commonly accepted variable for description and classification of the collision events in nuclei interactions. This variable is related to the collision geometry– degree of the collision peripherality. Thus, the centrality is given by the impact parameter value associated with the general geometrical characteristics of a particular collision event.

As it was noted in [27] and [28], in experiments with nuclei (including hadron-nucleus reactions) the knowledge of centrality $c_N^b$ is extracted from either number of charged particles registered in the respective detector or the transverse energy measured in the calorimeter. Those quantities are both denoted by $n$ and are relevant to experimentally measurable quantity $c_N^N$ also called centrality. Superscript $N$ means nuclear collisions while superscript $h$ denotes pure hadron collisions. The definitions of $c_N^b$ and $c_N^N$ [27] are as follows

$$c_N^b \equiv \frac{\sigma_{inel}^b}{\sigma_{inel}^b},$$

where

$$\sigma_{inel}^b = \int_0^b P_{inel}^N(b') 2\pi b' db$$

and $P_{inel}^N(b)$ is probability distribution of the inelastic collisions over the impact parameter $b$, while the experimentally measurable quantity

$$c_N^N \equiv \int_{n}^{\infty} P^N(n')dn'.$$

includes distribution over the multiplicity or the total transverse energy in the final state.

It should be noted that the energy dependence of the above quantities is tacitly implied and not indicated explicitly. The energy dependence, however, can be a nontrivial one in the collisions of nuclei as well as of hadrons, since size of interaction region, probabilities of interactions, multiplicities and transverse energies are the energy–dependent quantities in both cases. Evidently, the effects related to the energy dependence of all these quantities should be taken into account under analysis of the experimental data at the same value of centrality but at different energies.

Under assumption that the probability $P^N(n)$ has a Gaussian at a fixed value of the impact parameter $b$ [29], the relation between $c_N^N$ and $c_N^b$ has been obtained and discussed in [27]. The improvement of this reconstruction procedure
with gamma distribution was performed in [28]. This prescription allows one to extract a knowledge on the impact parameter value from the experimental data independently of collision dynamics. Assumptions on the Gaussian or gamma distributions are general ones and do not depend on the structure of the object under consideration, i.e. it could be applied equally for nuclei and hadrons as well. Gamma distribution is preferable in the regime where application of the central limit theorem is not justified [28]. It seems that this approach to the impact parameter reconstruction can be used for hadron reactions also.

It is important to emphasize again that the proposed reconstruction of the impact parameter is not based on a particular nuclear interaction model and/or concept of participating nucleons.

5 Reflective mode and centrality in hadron reactions

In view of the prominent collective effects observed in small systems, such as \( pp \)-collisions together with indications on the reflective scattering mode observation at the LHC, an introduction of centrality valid for any values of energy is useful to classify the collision events.

The hadron scattering has similarities as well as differences with the scattering of nuclei. Geometrically, hadrons are the extended objects too, but a significant contribution to \( pp \)-interactions is provided by the elastic scattering with the ratio of elastic to total cross-sections \( \sigma_{el}(s)/\sigma_{tot}(s) \) rising with energy. The elastic scattering of nuclei is not significant at high energies, nucleons are not confined in nucleus. Hence, the geometrical characteristics of hadron collisions associated with the elastic scattering are essential for the hadron dynamics, i.e. for the development of QCD in its nonperturbative sector where the confinement plays a crucial role.

It will be argued further that definition of centrality based on the use of a straightforward analogy with nuclei interactions is not appropriate for the hadron interactions at the energies where the reflective scattering gives a significant contribution. To obtain a relevant universal definition, we propose to use a full probability distribution \( P_{tot}^{h}(s,b) \) in order to take into account the elastic channel events. The neglect of the elastic scattering events would lead to wrong estimation of centrality for the particular hadron collision. Thus, for the centrality \( c_{b}^{h}(s,b) \) the following definition is suggested

\[
c_{b}^{h}(s,b) \equiv \frac{\sigma_{b}^{b}(s)}{\sigma_{tot}(s)}, \tag{10}
\]

where

\[
\sigma_{tot}(s) = 8\pi \int_{0}^{b} \text{Im} f(s,b)b'/db'
\]
is the impact–parameter dependent cumulative contribution into the total cross–section, $\sigma_b^{b\text{tot}}(s) \rightarrow \sigma_{\text{tot}}(s)$ at $b \rightarrow \infty$. In Eq. (10) the total (elastic plus inelastic) contribution replacing the inelastic cross–section only were used.

It should be noted, that there is nothing wrong with definition of centrality in the form of Eq. (8) in case of hadron scattering at the energies where the reflecting scattering mode is not presented, but its presence at higher energies changes form of an inelastic overlap function $h_{\text{inel}}(s, b)$ from a central to peripheral one with maximum at $b \neq 0$. Therefore, Eq. (8) for centrality in this case ceases to be valid. The use of centrality in this form would lead to a distorted dependence when its value would not reflect real collision geometry.

Contrary to the inelastic overlap function $h_{\text{inel}}(s, b)$, the function $\text{Im}f(s, b)$ at the LHC energies has a central impact parameter profile with a maximum located at $b = 0$ [11]. The amplitude $f(s, b)$ is the Fourier–Bessel transform of the scattering amplitude $F(s, t)$:

$$F(s, t) = \frac{s}{\pi^2} \int_0^\infty db f(s, b) J_0(b \sqrt{-t}).$$ \hspace{1cm} (11)

The definition Eq. (10) can be inverted, namely, one can consider centrality as an observable measured in hadron collisions. Eq. (10) then can be used for restoration of the elastic scattering amplitude, more specifically the function $\text{Im}f(s, b)$ can be calculated, if the impact parameter dependence of $c_b^{h\text{tot}}(s, b)$ is experimentally known. The inverted relation corresponding to Eq. (10) written in the differential form gives:

$$\text{Im}f(s, b) = \frac{\sigma_{\text{tot}}(s)}{8\pi b} \frac{\partial c_b^{h\text{tot}}(s, b)}{\partial b}.$$ \hspace{1cm} (12)

The impact parameter representation provides a simple semiclassical picture of hadron scattering, e.g. head–on or central collisions correspond to small impact parameter values. From Eq. (12) one can easily get the inequality

$$0 \leq \frac{\partial c_b^{h\text{tot}}(s, b)}{\partial b} \leq \frac{8\pi b}{\sigma_{\text{tot}}(s)}$$ \hspace{1cm} (13)

or in the integral form

$$0 \leq c_b^{h\text{tot}}(s, b) \leq \frac{4\pi b^2}{\sigma_{\text{tot}}(s)}$$ \hspace{1cm} (14)

for $b \leq R(s)$, $R(s) \sim \frac{1}{\mu} \ln s$, where $\mu$ is determined by the value of a pion mass.

To demonstrate a transition to the reflective scattering mode explicitly we use the unitarization scheme which represents the scattering amplitude $f(s, b)$ in the rational form of one-to-one transform and allows its variation in the whole interval allowed by unitarity [30]. Respective form for the function $S(s, b)$ is written in
this case as a known Cayley transform mapping nonnegative real numbers to the interval $[-1, 1]$:

$$S(s, b) = \frac{1 - U(s, b)}{1 + U(s, b)}. \quad (15)$$

It should be repeated here that we neglect by the real part of the scattering amplitude $f(s, b)$. The real, nonnegative function $U(s, b)$ can be considered as an input or bare amplitude which is subject to the unitarization procedure. The models of a different kind can be used for construction of a particular functional form of $U(s, b)$. The most of the models provide monotonically increasing dependence of the function $U(s, b)$ on energy (e.g. power-like one) and its exponential decrease with the impact parameter (due to analyticity in the Lehmann-Martin ellipse). The value of the energy corresponding to the complete absorption of the initial state at the central collisions $S(s, b)|_{b=0} = 0$ is denoted as $s_r$ and determined by the equation $U(s_r, b)|_{b=0} = 1$. In the energy region $s \leq s_r$ the scattering in the whole range of impact parameter variation has a shadow nature and high multiplicities are associated with central collisions in the geometrical models.

6 Centrality and geometrical models

A wide class of the geometrical models (relevant for the centrality discussion) allows one to assume that $U(s, b)$ has a factorized form (cf. [31] and references therein):

$$U(s, b) = g(s) \omega(b), \quad (16)$$

where $g(s) \sim s^\lambda$ at the large values of $s$, and the power dependence guarantees asymptotic growth of the total cross-section $\sigma_{tot} \sim \ln^2 s$. Such factorized form corresponds to a common source for the increase with energy of the total cross-sections and the slope of the diffraction cone in elastic scattering. The particular simple form of the function $\omega(b) \sim \exp(-\mu b)$ has been chosen to meet the analytical properties of the scattering amplitude. This form of $\omega(b)$ also assumed by the physical picture grounded on its representation as a convolution of the two energy-independent hadron pionic-type matter distributions (cf. Fig. 3, it illustrates notion of centrality in hadron scattering) in transverse plane:

$$\omega(b) \sim D_1 \otimes D_2 \equiv \int db_1 D_1(b_1)D_2(b - b_1). \quad (17)$$

Parameter $\mu$ should be then equal to the doubled value of a pion mass.

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4This one-to-one transform maps upper half-plane into a unit circle in case when $U$ and $S$ both are complex functions and the value of the function $S = 0$ is reached at finite values of energy and impact parameter.

5The old (pre-LHC) numerical estimates of $s_r$ have given for its value $\sqrt{s_r} = 2 - 3 TeV$ [14].

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Figure 3: Schematic view of hadron scattering with the impact parameter $b$ in the geometric models (cf. e. g. [32–35]).

Of course, a weak energy dependence of centrality can allow one to use it as a parameter for the data analysis at different energies in hadron reactions. Indeed, asymptotically, the centrality $c_b^b(s, b)$, defined according to Eq. (10), decreases with energy slowly, like $1/\ln^2(s)$ at fixed impact parameter values. Unitarization generate its dependence $\sim b^2/\ln^2(s)$ (since $f(s, b)$ saturates unitarity limit, i.e. $f(s, b) \rightarrow 1$, at $s \rightarrow \infty$). Such slow energy decrease of centrality allows one to compare the data at different energies and approximately the same value of centrality provided the energy values are high enough and are not too much different. Contrary, a strong energy dependence of centrality would bring problems under comparison of the data obtained at the same values of centrality and different energies.

To justify further use of the suggested form of Eq. (10) for centrality instead of Eq. (8), we examine Eq. (8) aiming to construct a counterexample, i.e. to demonstrate that such option is leading to the strong energy dependence of centrality defined that way, and therefore Eq. (8) is not an appropriate definition for small systems.

Thus, inelastic overlap function $h_{inel}(s, b)$ in Eq. (??) when the function $U(s, b)$ is chosen in the form of Eq. (16) allows one to calculate explicitly the centrality given by Eq. (8). In this case:

$$c_b^b(s, b) = \frac{1}{\ln(1 + g(s))} \left[ \ln \frac{1 + g(s)}{1 + g(s) \exp(-\mu b)} - \mu b f(s, b) \right], \quad (18)$$
where
\[ f(s, b) = \frac{g(s) \exp(-\mu b)}{1 + g(s) \exp(-\mu b)}. \] (19)

It leads to conclusion that at \( s \to \infty \) and fixed value of \( b \) one would expect strong energy decrease of the function \( c^b_h(s, b) \) under such option, i.e.
\[ c^b_h(s, b) \sim \frac{1}{s^\lambda \ln(s)}. \] (20)

Eq. 20 corresponds to the statement made in \([36]\), where it has been shown that centrality defined in a straightforward analogy with the case of nuclei collisions cannot serve as a measure of the impact parameter but this quantity is to be associated with the dynamics of the multiparticle production when the value of the impact parameter can variate in the narrow region around \( b = r(s) \). In this region the absorption is maximal in case of the reflective scattering domination at \( s \to \infty \) and centrality constructed in that way is just a cumulative contribution of the edge (cf. \([37]\)). Contrary, it is proposed to define centrality as an impact–parameter dependent cumulative contribution of all the interactions, elastic and inelastic ones, and it has a central impact parameter profile corresponding to an intuitive expectation.

An important problem is how to estimate the impact parameter value in a given \( pp \)-collision event from the data. It is evident that the event classification by multiplicity of the final state is not relevant for that purpose since a contribution of the elastic channel is then almost neglected. Moreover, centrality defined in that way has a strong energy dependence as it has been shown due to increasing peripherality of the probability distribution over impact parameter \( P^h_{\text{inel}}(s, b) \) with rising energy.

The most relevant observable seems to be a sum of the transverse energies of the final state particles. We can assume, following \([27]\) and \([28]\), the Gaussian or gamma distributions of the transverse energy for the fixed value of impact parameter \( b \) and extend the conclusions of \([27]\) and \([28]\) for nucleus-nucleus or proton-nucleus collisions to proton-proton collisions. Namely, fitting the experimental data and using Bayes’ theorem for conditional probability one can effectively reconstruct the distribution of the impact parameter for a given value of centrality determined by the total transverse energy of all final particles. Further details of such reconstruction can be found in the papers \([27]\) and \([28]\).

It should also be noted that observation of the non–Gaussian elliptic flow fluctuations in PbPb collisions \([38]\) makes use of gamma distribution preferable at the LHC energies since the degree of the initial–state spatial anisotropy resulting in the elliptic flow is correlated with the collision impact parameter value of the particular event.
7 Conclusion

The features associated with a character of $b$–dependence of hadron interactions are considered in this note. The $b$-dependent differential quantities are much more sensitive to the interaction dynamics that the overintegrated over $b$ ones. They are responsible for a number of conclusions such as the peripheral form of the inelastic overlap function and, correspondingly, the central distribution of the elastic overlap function over the impact parameter.

As it was mentioned in the beginning of this note, the problem of the scattering amplitude real part account has no final solution nowadays. Its account could, in principle, change the form of the inelastic overlap function since $h_{\text{inel}}$ transforms in this case into the sum $h_{\text{inel}} + (\text{Re} f)^2$ in the unitarity equation [9]. Despite such hypothetical possibility exists, it is not consistent with the experiment. Namely, the results of the quantitative impact parameter analysis performed in [11] with account of the real part are not in favor of such option. The resulting form of inelastic overlap function remains to be a peripheral one and a relevant form of the elastic overlap functions is not changed, it remains to be a central one. This is in favor of the real part neglect at least under qualitative considerations and justifies the approximation by an imaginary scattering amplitude. The vanishing role of the real part of the scattering amplitude account has been emphasized in [13], too. It also provides a hint on the unitarity saturation at the asymptotic energies.

There were proposed a rendering for the reflective scattering mode based on formation of color–conducting medium in the intermediate state and definition of centrality has been given with account of this mode existence in small systems like $pp$–collisions. The use of the transverse energy measurements in a calorimeter seems to be a more relevant method for centrality estimations than the method based on multiplicity measurements. The use of transverse energy measurements is a more universal method since it includes the case of unitarity saturation.

Nowadays, ATLAS and CMS experiments at the LHC are indeed using transverse energy measurements for centrality determination, but in the collisions of nuclei only. Experimental feasibility of centrality measurements in case of hadron reactions needs to be considered also.

Centrality extracted from the experimental data can be used for the elastic scattering amplitude reconstruction in the impact parameter space according to Eq. [12]. The magnitude of this amplitude at small values of $b$ is essential for reflective scattering mode detection. It is due to the fact that dynamics of elastic $pp$–scattering is described by complex function $F(s, t)$ of the two Mandelstam variables $s$ and $t$. Any quantities integrated over $b$ (i.e. those taken at $-t = 0$) are not sensitive to the details of their dependencies on $b$ and/or $t$, respectively, and therefore they cannot provide required information relevant for the available accelerator energies. Much higher energies are needed for the possibility of making
definite conclusions on the new scattering mode appearance if one proceeds from the overintegrated quantities only.

The relation of centrality and \( b \)-space elastic amplitude is similar to the optical theorem (it relates the elastic scattering amplitude with properties of all the collisions including elastic and inelastic ones).

Evidently, the important issue is further search for the unambiguous experimental manifestations of the reflective mode in elastic and inelastic hadron interactions. The centrality measurements in \( pp \)-collisions seems to be a promising way for that purpose since it would help to extract information on the typical impact parameter values of a particular collision event.

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