Neural networks application to determine the types and magnitude of aberrations from the pattern of the point spread function out of the focal plane

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Abstract. Recognition of the types of aberrations corresponding to individual Zernike functions were carried out from the pattern of the intensity of the point spread function (PSF) outside the focal plane using convolutional neural networks. The PSF intensity patterns outside the focal plane are more informative in comparison with the focal plane even for small values/magnitudes of aberrations. The mean prediction errors of the neural network for each type of aberration were obtained for a set of 8 Zernike functions from a dataset of 2 thousand pictures of out-of-focus PSFs. As a result of training, for the considered types of aberrations, the obtained averaged absolute errors do not exceed 0.0053, which corresponds to an almost threefold decrease in the error in comparison with the same result for focal PSFs.

1. Introduction

The task of detecting, recognizing and compensating for wavefront aberrations [1, 2] is closely related to the problem of improving the resolution of optical systems [3-8]. The solution to this problem is relevant in many applications, including microscopy [9-15], astronomy [16-20], and ophthalmology [21-23], therefore it has a long history and many different approaches and methods.

One of the tools for describing wavefronts, in addition to the deviation patterns, is the decomposition of aberrations over various bases. The most famous decomposition bases include Zernike polynomials [24-28], as well as Seidel aberrations. However, direct measurement of aberration coefficients is possible only for low levels of aberrations [28].

Recently, to solve the problem of recognition and compensation of optical system aberrations from the PSF intensity pattern in a certain plane, the means of intellectual analysis and machine learning are increasingly being used [29-35]. Some works [33, 34] showed the problems of using focal PSF at low levels of aberrations, when the intensity pattern is almost indistinguishable from the focal spot.

The purpose of this work is to investigate the possibility to improve the results of using neural networks to solve the problem of recognizing the type and magnitude of wavefront aberration from the PSF pattern outside the focal plane. The architecture of the neural network corresponded to the Xception [36] adapted for the regression problem.
Thus, in this work, we use a neural network to solve the regression problem [36]. In a regression problem, we aim to predict the outcome of a continuous value such as probability or price. This problem differs from the classification problem, where we seek to select a discrete quantity from a list of classes. A numerical experiment showed that, trained on 8 separate types of aberrations, matched with the Zernike functions, the neural network confidently copes with the classification task, i.e. detection in a superposition of two different types of aberrations of their presence. The obtained averaged absolute errors do not exceed 0.0053, which corresponds to an almost threefold decrease in the error as compared to the analogous result for focal PSFs.

2. Theoretical foundations

Wavefront (WF) aberrations are usually described in terms of Zernike functions $Z_{nm}(r, \phi)$:

$$W(r, \phi) = \exp[i\psi(r, \phi)], \quad \psi(r, \phi) = 2\pi\alpha \sum_{m=0}^{\infty} \sum_{n=0}^{m} C_{nm} Z_{nm}(r, \phi).$$

(1)

where,

$$Z_n^m(r, \phi) = \frac{A_n R_n^m(r)}{n!} \begin{pmatrix} \cos (m\phi) \\ \sin (m\phi) \end{pmatrix},$$

(2)

$A_n = \sqrt{(n+1)/\pi}$ is the normalization factor, $R_n^m(r)$ are the radial Zernike polynomials [1], and $\alpha$ is the wave aberration level.

PSF outside the focal plane $z$ can be calculated using the Fourier transform with the addition of the defocusing function to the wavefront (1):

$$G(u, v, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x, y) \exp \left[ \frac{i\pi}{\lambda z} (x^2 + y^2) \right] \exp \left[ -2\pi i (ux + vy) \right] dx dy.$$  

(3)

Table 1. Fragment of a dataset depicting extrafocal PSFs of various types of individual wave aberrations for $\lambda = 532$ nm, $f=100$ mm is the focal plane.

| $(n.m)$ | $\alpha$ | $z=0$ | $z=f$ | $z=105$ mm | $(n.m)$ | $\alpha$ | $z=0$ | $z=f$ | $z=105$ mm |
|---------|-----------|-------|-------|-------------|---------|-----------|-------|-------|-------------|
| 0.10    | (2.0) 0.25| 0.10  | 0.10  | 0.10        | (3.1) 0.25| 0.50      | 0.50  | 0.50  | 0.50        |
| 0.50    | 0.10      | 0.10  | 0.10  | 0.10        | (2.2) 0.25| 0.50      | 0.50  | 0.50  | 0.50        |
| 0.10    | (3.3) 0.25| 0.10  | 0.10  | 0.10        | 0.50    | 0.50      | 0.50  | 0.50  | 0.50        |
Table 1 shows a fragment of the dataset and the stages of its modeling. Similar intensity distributions were calculated for aberrations up to 4 orders in [37, 38]. A phase consisting of one Zernike polynomial (2) with a weight coefficient $C_{nm} = 1$ and a variable level of wave aberration $\alpha$ was applied to the input to the mathematical model of the aberrated wavefront (1). Then, using the Fourier transform with the addition of the defocusing function (3) to the wavefront (1), the PSF outside the focal plane was calculated. In a number of numerical experiments, the most optimal value of the parameter $z$ turned out to be equal to 105 mm at a wavelength of $\lambda = 532$ nm.

It can be seen from the intensity distribution that outside the focal plane the "character" of the distribution is preserved, but it becomes more pronounced, including at small aberrations.

### 3. Results and discussion

A dataset with 2 thousand images of out-of-focal PSFs (Table 1) was used to train the neural network. It was divided in a 4:1 ratio into training and validation sets.

To train the neural network, it was decided to use the Google Colab service. The Keras library was used as a machine-learning library, which is an add-on over the Deep learning4j, TensorFlow and Theano libraries. Single-channel images with a size of $128 \times 128$ pixels were fed to the input of the neural network, examples of which are presented in Table 1. The complete architecture of Xception is shown in Figure 1.

![Complete Xception architecture](image)

**Figure 1.** Complete Xception architecture.

To solve the problem of recognizing aberrations, it was decided to use the ready-made Xception architecture, presented in 2015 [36]. A distinctive feature of this model is the use of the Depthwise separable convolution. The essence of this technique is to split the transformations over the input tensor on each layer into two stages:

- at the first stage, the tensor is convoluted with a $1 \times 1$ kernel. This operation is called "pointwise convolution";
- at the second stage, standard convolution is performed. After that, the results of the convolution of the tensor obtained as a result of applying the "pointwise convolution" operation to the input tensor are summed up with all the filters of the layer and form the output tensor of the layer. This operation is called "depthwise convolution".

This technique allows you to significantly reduce the number of weights in the neural network, which significantly affects the learning rate required for training resources and the weight of the
pretrained model. Thus, the Xception architecture is the best choice, providing sufficient training accuracy, while requiring significantly less resources.

Adam was chosen as the optimization algorithm, the initial learning factor of which was $10^{-4}$. The loss function was the mean absolute error. The learning process was carried out in stages with a stepwise decrease in the learning coefficient of the Adam algorithm. The number of epochs was 250. Data augmentation using noise and random image rotation in the range from 0 to 45 degrees did not bring any tangible improvement in the results. The achieved indicators of prediction errors for all aberrations of a certain type - mean absolute error (MAE) and mean square error (MSE), are given in Table. 2.

| $(n,m)$ | MAE    | MSE    |
|--------|--------|--------|
| (1,1)  | 0.0053 | 0.0020 |
| (2,0)  | 0.0047 | 0.0018 |
| (2,2)  | 0.0042 | 0.0016 |
| (3,1)  | 0.0045 | 0.0017 |
| (3,3)  | 0.0042 | 0.0016 |
| (4,0)  | 0.0046 | 0.0017 |
| (4,2)  | 0.0043 | 0.0016 |
| (4,4)  | 0.0041 | 0.0015 |

The computational experiment showed that the trained neural network confidently copes with the classification task, i.e. identifying two non-zero types of aberrations for each image. The main contribution to the error is made by the error in determining the value of the coefficient of two nonzero aberrations. In addition, the dataset calculated in the out-of-focal plane made it possible to determine the level of aberration about 3 times better than the same result for focal PSF [33].

4. Conclusion

In this work, PSF patterns were simulated outside the focal plane for a coherent case, and a neural network training process was carried out to recognize the type and magnitude of different types of aberrations. The use of out-of-focal PSFs made it possible to almost 3 times reduce the recognition error in comparison with the same result for focal PSFs.

The most obvious way to reduce the error in recognizing a single wave aberration is to increase the dataset. In addition, it is planned to consider a dataset calculated in the out-of-focus plane for a superposition of two different types of aberrations. In the future, a full-fledged regression problem will be solved to determine the superposition of aberrations from the PSF pattern.

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