A Simplified Algorithm for Detecting Power Recirculation within 1-Dof Multi-Entity Planetary Gear Trains

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Abstract: Because there is power recirculation around internal loops in many transmissions, an undesirable increase in tooth meshing loss can arise in the system which results in lower efficiency. A single graph is used to illustrate the power flow magnitudes and directions. A simple formula for detecting power circulation in one degree of freedom multi-entity parallel-connected planetary gear trains is developed. In the absence of friction, power recirculation mainly depends on the speed ratios of the individual entities that constitute the PGT. The main novelty of the current method is in its extreme simplicity.

1. Introduction
A planetary gear train (PGT) that contains a single carrier is called a planetary gear train entity (PGTE), whereas two or more PGTEs linked together form a compound PGT or simply a PGT. Figure (1) shows a four-entity planetary gear train.

![Figure 1. A four-entity planetary gear train.](image)

Any 1-dof multi-entity planetary gear train can be reduced to two parallel-connected PGTs that have two interconnected and one fixed link as shown in Figure (2). Ignoring the losses, only the $p_y$ portion of the input power is passed through the second gear train entity. The $p_y$ and $p_x$ portions that pass through the interconnected links determine whether the input power is either divided or circulated in internal loops. Planetary gear trains that circulate power bring the disadvantages of significant friction tooth loss,
increased power and deterioration of bearing life. In some cases, the efficiency between 60 and 70 % is justified as the increase in power supplied can be compensated by reduced volume or cost. However, there is a need to improve the efficiency of the planetary gear trains in which the circulating power is inherent. Therefore, it is necessary to study the topic of power recirculation because it results in harmful work [1].

Figure 2. (a) Input-split compound PGT (b) Output-split compound PGT.

Figure (2) (b) illustrates the same PGT shown in Figure (2) (a) with the flow of powers in the opposite direction.

White [2] developed a method for examining planetary gear trains arrangements that split input power to get a certain velocity ratio. A similar approach for multi-path arrangements is discussed by Ciobotaru, et al. [3]. To determine the conditions under which power re-circulation occurs in two-Dof PGTs, Gupta et al. [4] made, in their analysis, some basic errors. Ismael [5] identified the correct conditions. In the absence of friction, a nomograph-based methodology was developed to analyze power re-circulation in PGTs [6].

The key step in analyzing mechanical efficiency is to check the direction of power flow [7] and verify power recirculation. Several studies have addressed the power recirculation through PGTs [8-11]. The direct method for power flow analysis depends on a complete static force analysis [12-14]. Speed ratio sensitivity is used in some studies [15–18]. In other studies, gear ratios are used to calculate the power flow [19 and 20].

The aim of the present work is to develop a single formula to diagnose internal power recirculation. What distinguishes the current method from other methods is its extreme simplicity.

2. Kinematic Analysis

For any GPE, the “planet gear ratio” can be written as:

\[ R_{i,p}^c = \frac{Z_p}{Z_i} = \frac{\omega_i - \omega_c}{\omega_p - \omega_c} \]

where \( \omega_i \), \( \omega_p \), and \( \omega_c \) are the speeds of links \( i \), \( p \), and \( c \). \( Z_i \) and \( Z_p \) are the teeth numbers on gears \( i \) and \( p \). \( R_{E,p}^c \) and \( R_{p,p}^c \) are obtained from Eq. (1) as \( R_{E,p}^c = 0 \) and \( R_{p,p}^c = 1 \).

For any PGTE and / or PGT, the \( R_{m,n}^u \) can be written as Esmail [21] from the two shared links \( s_1 \) and \( s_2 \) as:

\[ R_{m,n}^u = \frac{R_{m,s_2}^1 - R_{n,s_2}^1}{R_{m,s_2}^1 - R_{n,s_2}^1} = \frac{\omega_m - \omega_u}{\omega_n - \omega_u} \]

Figure (3) also illustrates the interconnected links with detailed labelling.
3. Power Recirculation Analysis
In the present work, both of the split power configurations shown in Figure (2) (a) and (b) will be considered. Input and output powers are as shown by the arrows. Any PGTE and/or PGT must be under power balance, therefore, we can write for torque balance:

\[ T_m + T_n + T_u = 0 \]  

(3)

for power balance:

\[ T_m \omega_m + T_n \omega_n + T_u \omega_u = 0 \]  

(4)

For the PGT shown in Fig. (3) and from Eqs. (3) and (4), we get:

\[ \frac{\tau_y}{R_{z',x}} = -T_{z'} = \frac{T_y'}{(1-R_{z',x}^y)} \]  

(5)

From Eq. (5)

\[ P_{z'} = T_{z'} \omega_{z'} = -\frac{T_y \omega_y}{R_{z',x}^y \omega_{z'}} \]  

(6)

and

\[ P_{y'} = T_{y'} \omega_{y'} = \frac{T_y \omega_y}{R_{z',x}^y \omega_{z'}} \left(1 - R_{z',x}^y\right) \]  

(7)

Dividing Eq. (7) by Eq. (6), we get:

\[ \frac{P_{y'}}{P_{z'}} = \left(R_{z',x}^y - 1\right) R_{y'',z''}^f \]  

(8)

Simply, when \( \left(R_{z',x}^y - 1\right) R_{y'',z''}^f < 0 \), there is power circulation. There is no need to go deeper into studying torques and speeds as they are unnecessary. Also, \( R_{x,z'}^y \) and \( R_{y'',z''}^f \) depend on the teeth numbers of the gears constituting the planetary gear trains.

4. Algorithm for power recirculation

Step 1: It can be shown that any planetary gear train can be reduced to two parallel connected PGTs that have two interconnected and one fixed link as shown in Figure (5) (a) and (b).

Step 2: Enter the labels of the interconnected links as shown in Figure (3).

Step 3: After entering the corresponding numbers for \( x, y', y'', z', z'' \), and \( f \) in Eq. (8), the speed ratios \( R_{z',x}^y \) and \( R_{y'',z''}^f \) are calculated.

Step 4: When \( \left(R_{z',x}^y - 1\right) R_{y'',z''}^f < 0 \), there is power circulation
Example
By choosing different connections for different planetary gear train entities, many possible systems can be obtained. Figure (4) shows a three-entity planetary gear train as an illustrative example to show how the power flows into parallel-connected planetary gear train.

![Figure 4. A three-entity PGT.](image)

Figure (4) shows a PGT consisting of three PGTEs; PGTE₁ containing links 1, 2, 3 and 4, PGTE₂ containing links 5, 6, 7 and 8, and PGTE₃ containing links 9, 10, 11 and the fixed carrier 12 (f).

The interconnection of three or more planetary gear train entities can always be reduced to the diagrams shown in Figure (2) (a) or (b). Figure (5) shows two cases where the three-PGTEs shown in Figure (4) can be reduced to two interconnected PGTs. In Figures (5)(a) and (c), PGT₁ contains only PGTE₁ while PGT₂ contains PGTE₂ and PGTE₃. In Figure (5)(b) and (d), PGT₁ contains PGTE₁ and PGTE₂ while PGT₂ contains only PGTE₃.

![Figure 5. The reduction of three-entity PGT shown in Figure (4) to only two interconnected gear trains.](image)

For the case shown in Figure (5) (a) and (c), \( x = 1, y' = 3, y'' = 7 \), and \( z' = 4, z'' = 8 \) and \( f = 12 \). Therefore, we can write Eq. (8) as:
\[
\frac{P_5}{P_6} = (R_{4,1}^3 - 1)R_{7,8}^{12}
\]  
(9)

From Eqs. (1) and (2)

\[
R_{4,1}^3 = \frac{Z_1}{Z_1 + Z_3}
\]  
(10)

And

\[
R_{7,8}^{12} = \frac{Z_{11} - Z_9}{Z_{11} - Z_9}
\]  
(11)

Since \(Z_1\) and \(Z_3\) are both positive, therefore, from Eq. (10) we can conclude that \(0 < R_{4,1}^3 < 1\).

Also, since \(Z_{11} < Z_9\), then \(Z_{11} - Z_9 < 0\) and \(Z_{11}/(Z_{11} - Z_9)\) is negative. Also \([Z_{11}/(Z_{11} - Z_9)] - (Z_9/Z_7)\) is negative which makes \(R_{7,8}^{12}\) positive and greater than one.

Returning to Eq. (9), and since \(0 < R_{4,1}^3 < 1\) and \(R_{7,8}^{12} > 1\), therefore \((R_{4,1}^3 - 1)R_{7,8}^{12} < 0\) and there is power circulation in this internal loop.

For the case in Figure (5) (b), \(x \equiv 1, y' \equiv 5, y'' \equiv 11, and z' \equiv 8, z'' \equiv 9 and f \equiv 12\). Therefore, we can write Eq. (8) as :

\[
\frac{P_5}{P_6} = (R_{6,1}^5 - 1)R_{11,9}^{12}
\]  
(12)

From Eqs. (1) and (2)

\[
R_{11,9}^{12} = \frac{-Z_9}{Z_{11}}
\]  
(13)

and

\[
R_{6,1}^5 = \frac{Z_7Z_9}{Z_7Z_9 - Z_9Z_5}
\]  
(14)

Since \(Z_9\) and \(Z_{11}\), in Eq. (13), are both positive, therefore, \(R_{11,9}^{12} < 0\). In Eq. (14), \(Z_7Z_1 - Z_3Z_5\) may be larger or smaller than zero. When \(Z_7Z_1 - Z_3Z_5 > 0\), i.e. \(Z_7Z_1 > Z_3Z_5\), then \(R_{6,1}^5 > 1\), and \((R_{6,1}^5 - 1)R_{11,9}^{12} < 0\) and there is power recirculation in this loop. When \(Z_7Z_1 - Z_3Z_5 < 0\), i.e. \(Z_7Z_1 < Z_3Z_5\), then \(R_{6,1}^5\) is negative and \((R_{6,1}^5 - 1)R_{11,9}^{12} > 0\) and there is no power circulation in this internal loop.

5. Conclusions

A simplified algorithm has been introduced to detect power recirculation within 1-dof multi-entity parallel-connected PGTs. A single graph is used to illustrate power flow magnitudes and directions. A simple formula for detecting power recirculation is obtained in multi-entity PGTs. In the absence of friction, which is an acceptable hypothesis in the study of power recirculation, power recirculation mainly depends on the speed ratios of the individual entities that constitute the PGT. The speed ratios are functions of the type of the PGTE and teeth numbers. The main benefit of the current method is in its extreme simplicity.

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