Forecasting Malaysia Under-5 Mortality Using State Space Model

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Abstract: Under-five mortality is a key point for child prosperity and most countries discuss under-five mortality. However, early childhood mortality is still high and turns into a huge problem in some developing countries. The aim of this study is to analyze the trend pattern, develop forecasting models and forecast future trends of under-five mortality in Malaysia by gender. The yearly under-five mortality rates (U5MR) of a 37-year period (1980 – 2016) in Malaysia had been modelled using State Space model. It was found that the U5MR in Malaysia had fluctuated from year to year with a slowly decreasing trend pattern for both genders with males having a higher rate compared to females. Based on Local Linear Trend model, the future trend is increasing slightly and forecast trend for the male population is higher than the female. This study could become a reference for developed and developing countries. It could also become a guideline for human resource management and health care allocation planning.

Keywords: Forecasting, Local Linear Trend Model, State Space Model, Trend, Under-five Mortality

1. Introduction
Mortality of the population, especially at a young age, is one of the key measures of population health. Besides, mortality rates among young children are the best single indicator of child health in low-income and middle-income countries like Malaysia and they are also often used as indicators of social and economic development. The most widely used measure of child mortality in recent years has been the under-five mortality rate (U5MR). The U5MR is defined as the probability of dying between live birth and age five years which include the neonatal, post-neonatal and childhood mortality which are mortality in children younger than 5 years old [1]. A reduction in the U5MR of the population becomes the central development agenda for improving population health in most countries in the world. Therefore, the mortality reduction and the avoidance of premature mortality from any cause become a goal for every health system in a country [2]. The U5MR is a key pointer for child prosperity including wellbeing status and was additionally a broad marker for social and financial advances [3].
Globally, the world as a whole, has been stepping up the effort to reduce U5MR. The U5MR reduction was a global target of the Millennium Development Goals number four (MDG4 – reduce under-five mortality by two thirds between 1990 and 2015) and it was also included in the current Sustainable Development Goals number three (SDG3 – reduce under-five mortality to 25/1000 live births or less by 2030) [4]. Along with the global reduction in U5MR, Malaysia also has a remarkable experience in reducing U5MR. Malaysia is one of the few countries that has already achieved child health goals for MDG4 and SDG3. This has been the result of a synergy of a wide range of policies, strategies, and programs that addressed accesses to services through socio-economic, cultural, educational, gender, and poverty dimensions. In addition, medical advances, the availability of child health services that include control of communicable diseases and immunisation including vaccines and oral rehydration for children have been made widely accessible in Malaysia through the country’s primary health care system. Therefore, with this current scenario, it is worthwhile to report the health conditions of U5MR during this period and predict future U5MRs. This prediction is important as the government and practitioners need to make good planning and decisions in various planning areas such as health policy, education program, population projections, pharmaceutical research, life insurance and social security planning. Forecasting of Malaysia’s U5MR is also beneficial to the Department of Statistics Malaysia (DOSM) and other bodies in identifying the most appropriate approach in producing better U5MR forecasts. Therefore, in line with Malaysia's Eleventh Plan, it is important to have an accurate forecast of U5MR because such knowledge may become a guide to policy makers and practitioners in allocating scarce resources into areas and populations where they are mostly needed. In addition, such knowledge that informs about public health needs in different population groups should be distributed across sub-national regions. At the same time, it is important to explore patterns relating to child death and place of death. Understanding these patterns is necessary to identify the remaining issues in child mortality and to formulate better intervention strategies. The results of this study may serve as a principal guideline in planning intervention for children especially in health care and education, thus, revealing the need for improvement of under-five mortality data collection in health facilities and their corresponding systems.

According to Kamaruddin and Ismail [5], in their study, they found that the mortality rate for infants is higher compared to children, teenagers and adults under the age of 40. However, across the calendar year from 1984 to 2012, child mortality rate from age 0 to age 5 has sharply decreased for these twenty-nine years’ observations. At this stage, it is important to explore the trend and determinants of U5MR in order to reduce the vulnerability of child survival. In addition, forecasting of the under-five mortality is important in tracking under-five mortality reduction progress and evaluating the countries’ performances relating to MDGs and SDGs. In addition, the under-five mortality is crucial to many application areas, especially in government planning. Most countries need to forecast under-five mortality for solvency assessment of life insurers [6].

Globally, it is fortunate that mortality had made strides over the last few decades for most of the countries, but the issue of concern is whether the change was similar according to age. In the case of Malaysia, mortality trends also indicate a decline for all ethnic groups and in all regions of Malaysia. However, these declines are not similar for all specific ages of the population in all periods [7]. In addition, most of the under-five deaths occurred within the first year of life. Moreover, the death incidence rate was higher in males compared to females [8]. Previously, several researchers conducted studies to forecast U5MR using data of their countries such as England and Wales [9], the Kermanshah province in Iran [10] and Zambia [11]. In the case of Malaysia, there were numerous efforts in forecasting its total mortality such as studies done by Husin, Zainol and Ramli [12,13] and Kamaruddin [14]. However, to the researchers’ knowledge, there is limited literature found on forecasting Malaysia’s U5MR to date where the attention is more on infant mortality only. Therefore, this study was conducted to study the trend pattern, develop forecasting models and forecast future trends of U5MR in Malaysia by gender.
2. State Space Methodology

Univariate state space model for time series is also known as structural time series model. Structural time series model is a helpful methodology for time series analysis. A structural time series model has a direct interpretation and is formulated straightforwardly as far as various imperceptibly or inactive components such as trend, cycle or seasonal [15]. Professionals chose the structures and the models for parts that are pertinent in clarifying the elements watched within the information. In order to remove the imperceptible components, the Kalman filter (KF) is kept running upon the state space type of the model. That is the reason why, in some cases, these models were alluded to as state space models. Box-Jenkins Autoregressive Integrated Moving Average (ARIMA) procedure is generally used for analyzing time-series data. However, this study used state space modelling approach using KF because this technique can take into account the time dependency of the underlying parameters [16].

The type of structural time series models are local level model and local linear trend model where the trend in the local level model is just a random walk [17]. Thus

\[ y_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim \text{NID}(0, \sigma^2_{\varepsilon}), \]
\[ \mu_t = \mu_{t-1} + \eta_t, \quad \eta_t \sim \text{NID}(0, \sigma^2_{\eta}) \quad t = 1, \ldots, 37 \]

(1)

where the unpredictable and level disturbance, \( \varepsilon_t \) and \( \eta_t \), are commonly autonomous and the documentation \( \text{NID}(0, \sigma^2_{\varepsilon}) \) signifies regularly and is freely dispersed with mean zero and variance \( \sigma^2_{\varepsilon} \).

At the point when \( \sigma^2_{\eta} \) is zero, the level is consistent. The signal-noise proportion, \( q = \sigma^2_{\eta} / \sigma^2_{\varepsilon} \), assumes the key part in deciding how perceptions ought to be weighted for forecast and signal extraction. When \( q \) was higher, past observations were constrained in estimating the long haul. Essentially, a better \( q \) implies that the nearest perceptions got a better weight when signal extraction was completed. It is noted that despite the fact that the forecast work is even, the model is considered to have a trend, except if \( \sigma^2_{\eta} \) was zero, as the levels changed after a few times. The local linear trend model is progressively broad in the trend component that had a stochastic slope, \( \beta_t \), which it pursues, is a random walk. Hence,

\[ \mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \quad \eta_t \sim \text{NID}(0, \sigma^2_{\eta}), \]
\[ \beta_t = \beta_{t-1} + \zeta_t, \quad \zeta_t \sim \text{N}(0, \sigma^2_{\zeta}). \]

(2)

where the unpredictable, level and slope disturbance, \( \varepsilon_t, \eta_t \) and \( \zeta_t \), separately, are commonly independent. If both variances \( \sigma^2_{\eta} \) and \( \sigma^2_{\zeta} \) are zero, the pattern is deterministic, that is

\[ \mu_t = \mu_0 + \beta_t, \quad t = 1, \ldots, 37 \]

(3)

At that point when just \( \sigma^2_{\zeta} \) is zero, the slope is constant and the trend diminished to a random walk with drift

\[ \mu_t = \mu_{t-1} + \beta + \eta_t. \]

(4)

Permitting \( \sigma^2_{\zeta} \) to be positive, however, setting \( \sigma^2_{\eta} \) to zero gives the coordinate a random walk trend, which, when evaluated, would in general be moderately smooth. The signal-noise proportion is currently given by \( q_{\zeta} = \sigma^2_{\zeta} / \sigma^2_{\varepsilon} \). The model is frequently alluded to as the ‘smooth trend’ model. All else being equal, it is alluring to have a smooth trend since it is less demanding on the eye and more engaging to arrangement procedures. Consequently, in the case of integrated random walk and random walk plus drift trends gave a comparable fit, the integrated random walk might be favoured. Due to that, the factual treatment of imperceptibly component models depends on the state space form.

The parameters of a state space model are fundamentally assessed by the maximum likelihood that is maximized by utilizing the KF and Kalman smoothing (KS) [18]. The KF utilized the observed data to find out about the unobservable state factors, which portray the state of the model. The KF is, for the most part, distribution-free and gives the most excellent straight indicators within the sense of minimizing the mean squared error [19] while KS solves the anticipated esteem of the covered up state.
condition on all the data [20]. The KF is a recursive method that includes steps such as initialization, expectation, adjustment and probability development. This study used $X_{t|t}$ to signify the expectation of the variable X at time t, conditional upon data accessible at times. The KF was initialized by inferring the finest indicator of the beginning state, $z_{0|0}$, and an appraise of its covariance matrix, $\Sigma_{0|0} = E[(z_0 - z_{0|0})(z_0 - z_{0|0})']$. If the procedure was stationary, this is direct, since this study can expand on the unaltering state of the framework. More absolutely, this study can set $z_{0|0} = z^*$ and $\Sigma_{0|0} = \Sigma^*$ such that

1) $z^* = Bz^*$
2) $\Sigma^* = B\Sigma^* B' + \Sigma_w = [I - B \otimes B]^{-1} \text{vec}(\Sigma_w)$. Before moving to the subsequent step, we tend to set $t = 1$ such, consequently, $z_{t-1|t-1} = z_{0|0}$ and $\Sigma_{t-1|t-1} = \Sigma^*$. When time was equal to $t$, $z_{t-1|t-1}$ and $\Sigma_{t-1|t-1}$ were used along with the transition equation, for instance

$$
\begin{align*}
\dot{z}_{t|t-1} &= Bz_{t-1|t-1} \\
\dot{\Sigma}_{t|t-1} &= B \Sigma_{t-1|t-1} B' + \Sigma_w 
\end{align*}
$$


Then, $z_{t|t-1}$ were used in developing the forecast $y_{t|t-1}$ which is equal to $H_{z_{t|t-1}}$. The observed $y_t$ was able to develop the forecast error

$$
\begin{align*}
\dot{u}_t &= y_t - y_{t|t-1} = y_t - H_{z_{t|t-1}} = v_t + H(z_t - z_{t|t-1}).
\end{align*}
$$

Due to Gaussian errors, it sought after that $u_t \sim N(0, \Sigma_u = H \Sigma_{t-1|t-1} H')$. Furthermore, since $y_t = u_t + y_{t|t-1}$, it follows that $f(y_t \mid y_{t|t-1}, \delta) = f(u_t \mid \delta)$. The forecast $z_{t|t-1}$ and $\Sigma_{t|t-1}$ has been overhauled (adjusted) according to Kalman [21] and the formula can be written as

$$
\begin{align*}
\dot{z}_{t|t} &= z_{t|t-1} + K_t(y_t - y_{t|t-1}) = z_{t|t-1} + K_t(y_t - H_{z_{t|t-1}}),
\\
\dot{\Sigma}_{t|t} &= \Sigma_{t|t-1} - K_tH \Sigma_{t|t-1} H' K_t',
\end{align*}
$$

where $K_t = \Sigma_{t|t-1}H' (H' \Sigma_{t|t-1} H' + \Sigma_u)^{-1}$

The past segment had given a way to calculate the chance of an information sample conditional on parameters $\delta$. This likelihood is, commonly, a complex non-linear function of the parameters, specified typically increasing the likelihood that the function would be conducted numerically. A few strategies were accessible to lead to this numerical maximization. In the event that the likelihood function is smooth and nonstop, Gradient-based strategies (e.g. Newton’s method) would permit to determine the Maximum Likelihood methods in an exceedingly easy approach [21].

3. Data Description

The study used secondary data obtained from the Department of Statistics (DOSM). The data available covered the period from 1980 to the year 2016 which includes 37 observations for males and females, respectively and this study covered U5MR in Peninsular Malaysia only. Figure 1 represents the total number of under-five mortality by gender in Peninsular Malaysia for year 2010 to 2016. For the same period, the total number of under-five mortality in Peninsular Malaysia shows that death among males was consistently higher than females. The total mortality for males in 2014 is higher than other years while mortality for females in 2015 was higher than in other years. The least total of mortality for
females was 1732 which was in 2013. Meanwhile, in 2010, the total mortality for males was 2250 which was the lowest compared to other years.

As shown in figure 2, the U5MR for male population was higher than female population since 1980 till now. Moreover, a decreasing trend of all groups in Peninsular Malaysia for U5MR is obvious to the male population and female population from 7.5 in 1980 to 2.5 in 1996. There exists a random shock of irregular components in U5MR data where the rates drastically increased in year 1997 and 1998. This is because Malaysia was having an economic crisis during those years. During the economic crisis, food costs had increased and had a widespread impact on nutritional and health status of the population, particularly among children [22]. This situation has prompted the increase in the U5MR for years 1997 and 1998. However, starting from year 2000 until 2011, the figure shows that the trend of U5MR is a uniformed pattern. In year 2012 to 2015, the U5MR trend slightly increased and dropped back in year 2016.
4. Results

Through this section, the state space methodology was used to analyse U5MR of Malaysian population. In this study, the analysis of state space model had chosen the local linear trend as the suitable model since the data used had inconsistent trend. The parameters of a state space model and fitting a structural model for a time series are fundamentally estimated by maximum likelihood. Firstly, the parameters of the model were estimated by constructing a fit of the local linear trend model for U5MR of the male and female populations and the result is shown in table 1.

Table 1. Estimation parameter by Local Linear Trend Model

|       | Level  | Slope  | Observational |
|-------|--------|--------|---------------|
| Male  | 1.681e-08 | 2.446e-09 | 4.628e-08     |
| Female| 2.878e-09 | 1.967e-09 | 4.086e-08     |

Table 1 presents the maximum likelihood estimates (MLEs) for the variance of level disturbance of male population, $\sigma_\eta^2$ was 1.681e-08 while 4.628e-08 for the variance of the observation disturbances, $\sigma_\epsilon^2$ and the variance of the slope disturbances $\sigma_\xi^2$ was 2.446e-09. Furthermore, 2.878e-09, 1.967e-09, and 4.086e-08 were MLEs for variance of level disturbance, the variance of the slope disturbance and variance of the observation disturbance for the female population. The state variance of both genders for the slope component was almost equal to zero, meaning that the value of the slope hardly changes over time. Furthermore, the parameters within the model were calculated employing a KF rule that tackles the anticipated value of the covered up state at time conditioned on the discovered data up to time. The KF provides the ideal state such as the lowest mean square error that estimates the unobserved data up to time, whereas the KS tackles the expected value of the covered up state conditioned on all the data. Estimators from the KF and KS are the maximum-likelihood estimates. Figure 2 and Figure 3 show that the outcome have been summarized by displaying the fitted values and smoothed from both models. Both figures present time plots for the filtered and smoothed state for male and female populations with point wise 95% confidence intervals. As shown in figure 3 and figure 4, obviously, the smoothed state variables were smoother than the filtered state variables. The confidence intervals for the smoothed state variables were also smaller than those of the filtered state variables.

Figure 3. Kalman filter and smoothing for male population
Figure 4. Kalman filter and smoothing for female population

Figure 5 and figure 6 compared the predicted and actual U5MR for the male and female populations. The predicted values for local linear model for female population was very close and almost overlapping to the actual value compared to the male population. This means that the error prediction for female population was smaller than male population and indicated that the local linear trend model is good enough for both male and female populations since the predicted value fit the model well.

Figure 5. Predicted U5MR of Local Linear Trend Model for Male Population

Figure 6. Predicted U5MR of Local Linear Trend Model for Female Population

Since this study is interested to forecast the U5MR for both populations from year 2017 until year 2030, the fourteen-step ahead forecast were generated. The forecast values for U5MR of Malaysian male and female populations from year 2017 until 2030 are shown in Table 2 and there is a slightly increasing trend from year 2017 until year 2030 which are 0.001891 for year 2017 and 0.001907 for year 2030. Moreover, the female population also shows an increasing trend of forecasts from year 2017 until year 2030 which are 0.001667 to 0.001878. The trend of the forecast U5MR can be clearly seen in Figure 7 which shows the trend of the actual U5MR data from year 1980 to 2016 for both male and female populations and the future trend of U5MR from year 2017 until year 2030. Moreover, the future trend shows a slightly increasing pattern for both male and female populations. However, the future trend for male population is a bit higher than the female population. The forecast pattern shows an increment since the previous series had an unstable pattern.
Table 2. Forecast Values for U5MR of Male and Female Populations

| Year | Male     | Female   |
|------|----------|----------|
| 2017 | 0.001891 | 0.001667 |
| 2018 | 0.001892 | 0.001684 |
| 2019 | 0.001893 | 0.001700 |
| 2020 | 0.001894 | 0.001716 |
| 2021 | 0.001896 | 0.001732 |
| 2022 | 0.001897 | 0.001748 |
| 2023 | 0.001898 | 0.001765 |
| 2024 | 0.001900 | 0.001781 |
| 2025 | 0.001901 | 0.001797 |
| 2026 | 0.001902 | 0.001813 |
| 2027 | 0.001904 | 0.001829 |
| 2028 | 0.001905 | 0.001846 |
| 2029 | 0.001906 | 0.001862 |
| 2030 | 0.001907 | 0.001878 |

Figure 7. Trend of actual and forecast values for U5MR of male and female populations

5. Conclusion
This paper studied the trend pattern of under-five mortality in Malaysia by gender. The trend analysis over the data of U5MR by gender from year 1980 to 2016 found that the series of U5MR for male and female populations were impacted by trend and irregular components. This situation shows that the U5MR series is an unstable pattern. State Space Model was utilized to develop forecasting models for under-five mortality in Malaysia by gender. According to the pattern of the U5MR, it was found that Local Linear Trend model was the most suitable model to forecast Malaysia U5MR for male and female populations. Malaysia has made considerable progress in reducing U5MRs from 1980 to 2016. However, based on forecast values, U5MRs could show a slight upward trend from 2017 to 2030 for both male and female populations. Future considerations for child healthcare include the management of birth asphyxia, congenital heart disease, preterm/low birth weight and other congenital abnormalities. To further reduce the U5MR in Beijing, specific prevention measures should be adopted for children of various age groups [23]. In addition, future researchers can also consider other techniques and
conducting comparison study to identify the most accurate model in predicting Malaysia U5MRs. It is possible, if any, to consider other models, which are able to predict U5MRs more accurately and capture its patterns more specifically, need to be explored. In addition, an intervention analysis in modelling U5MR should also be taken into consideration in order to investigate the possibility of random shock of the data affecting the trend of U5MR in Malaysia.

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