Control of crack pattern using memory effect of paste

Akio Nakahara, Yuu Shinohara and Yousuke Matsuo
Laboratory of Physics, College of Science and Technology, Nihon University, Funabashi 274-8501, Japan
E-mail: nakahara@phys.ge.cst.nihon-u.ac.jp

Abstract. A densely packed colloidal suspension, called as a paste, remembers the direction of external mechanical fields, such as flow and vibration. When the pastes are dried, memories in pastes are visualized as macroscopically anisotropic crack patterns, such as lamellar, radial, ring and spiral. Here, we experimentally investigate how pastes remember such experiences by using paste with different size distribution of colloidal particles. We find that a paste with smaller particles have a better memory, in the sense it remembers external mechanical fields at smaller solid volume fraction, which implies that interparticle forces between colloidal particles play an important role in memory effects, causing a quantitative change in the phase diagram for the same material. This result supports the hypothesis that memories in pastes are maintained as microscopically anisotropic network structure of colloidal particles, connected via interparticle forces between colloidal particles, such as van der Waals interaction.

1. Introduction
When cracks are formed, they emerge suddenly and unavoidably, and it is often difficult to control the formation of these cracks [1, 2]. For example, if you drop a glass cup to the ground and the cup is broken, it is difficult to predict the precise shape of each fragments. In these situations, there appear statistical laws instead, and a mass distribution of fragments obeys a kind of scaling laws [3, 4]. Only few successes were reported where the morphology of 1-dimensional cracks is controlled in a case of quenched glass plate. A stability of a propagating straight crack is studied experimentally and a transition from straight to oscillatory and branched cracks is obtained by controlling cracking condition like a cooling rate [5, 6, 7, 8].

Recently, it is reported that the morphology of 2-dimensional crack pattern can also be controlled in the case of desiccation cracks [9]. Usually when a paste made of colloidal particles and water dries, isotropic and cellular desiccation crack patterns appear, with the characteristic spacing between cracks almost same as the depth of the paste [10, 11, 12, 13]. When a paste is vibrated or fluidized before drying, however, pastes remember the direction of flow and vibration, and memories in pastes are visualized as morphology of desiccation crack patterns [9, 14, 15, 16]. That is, the plasticity of paste enables itself to remember external mechanical fields [17, 18, 19, 20]. This result opens a new way to control crack formation, because, by imprinting memory into paste beforehand, the future desiccation crack patterns can be controlled as we want [15, 16, 21, 22]. We would like to know the mechanism of memory effects for better control of crack formation.

This paper is constructed as follow; In Sec. 2, we review the experimental findings of memory effects in paste and distinguish memory of flow from memory of vibration. In Sec. 3, we present morphological phase diagrams of desiccation crack patterns, and show experimental conditions...
in which pastes remember external mechanical fields, such as vibration and flow. Section 4 is the main part of this paper. We perform experiments using pastes with different size distribution of colloidal particles to investigate the effects of interparticle forces on the formation of memories in pastes. Finally, Sec. 5 is devoted to conclusions with further applications.

2. Memory effects of paste

As was introduced above, it was found that a densely packed colloidal suspension, called as a paste, has memories of external mechanical fields, such as flow and vibration. Here, the size of colloidal particles are distributed from sub micron to several ten micron. For example, a mixture of calcium carbonate (CaCO$_3$) and water remembers the direction of vibration, when it is vibrated in the case of a water-poor condition. When we vibrate a water-poor CaCO$_3$ paste horizontally in one direction for a short period, stop the vibration, and dry the paste at 25°C and 30% humidity, a lamellar desiccation crack pattern appears with the directions of crack propagation all perpendicular to the direction of the initial vibration [9, 14]. We imagine that, when the water-poor CaCO$_3$ paste was vibrated, a longitudinal density fluctuation was formed inside paste along the direction of the initial vibration, and that deformed structure remains due to the presence of the plasticity. Note that paste of CaCO$_3$ remembers only the direction of vibration and cannot remember flow direction.

On the other hand, it is reported that some pastes can remember not only the vibration but also a flow. Such pastes can remember the direction of flow when they are fluidized in a water-rich condition. Paste of magnesium carbonate hydroxide, carbon paste (mixture of activated charcoal with water), and clay paste such as kaolin are examples of pastes having memory of flow. When a water-rich paste is vibrated, it is easily fluidized, and the resultant desiccation crack pattern has a structure which is similar to the flow pattern that we observed at the initial

![Figure 1](image-url)

**Figure 1.** Transition in the direction of lamellar crack patterns as the value of the solid volume fraction at the initial vibration is decreased. The paste used in this figure is magnesium carbonate hydroxide (Kanto Chemical, Tokyo). The arrow indicates the direction of the initial vibration, where the amplitude $r$ and the frequency $f$ of the vibration are 15mm and 2Hz, respectively, i.e., the maximum strength of the vibration $4\pi^2rf^2$ is 2.4m/s$^2$. We vibrate the container horizontally in one direction for 1 min., stop the vibration, and dry the paste until desiccation cracks are formed. The lengths of the sides of both square acrylic containers are 200mm, and the mass of powder in each container is set to be 100g so that the final spacing between cracks will be almost the same. (a) Memory of vibration. The value of the solid volume fraction $\rho$ is 12.5%, so the paste is in a water-poor condition. Such a water-poor paste remembers a direction of the initial vibration, and the direction of lamellar cracks is perpendicular to the direction of the initial vibration. (b) Memory of flow. The value of the solid volume fraction $\rho$ is 6.7%, so the paste is in a water-rich condition. Such a water-rich paste remembers a flow direction, and the direction of lamellar cracks is parallel to the direction of the flow induced by the initial vibration [15].
vibration. That is, when a paste has a memory of flow, the directions of desiccation cracks become parallel to the flow direction [15]. In Table 1, we summarize which kind of memory effects each paste has. Figure 1 shows that there exists a transition in the memory of paste from a memory of vibration to a memory of flow, which can be visualized as a transition in the direction of crack propagation.

To distinguish memory of flow from memory of vibration, we perform experiments by using containers with various shapes. As for the memory of vibration shown in Fig. 2(a), the directions of lamellar cracks are anywhere perpendicular to the direction of the initial vibration. On the other hand, as for the memory of flow shown in Fig. 2(b), the direction of lamellar cracks in square container is parallel to the direction of the initial vibration, while in parallelogrammatic containers, the direction of lamellar cracks is parallel to the direction of the flow along the oblique direction and not to the direction of the initial vibration. That is, we can distinguish memory of flow from memory of vibration.

**Table 1. Memory effects**

|                          | Memory of vibration | Memory of flow |
|--------------------------|---------------------|----------------|
| calcium carbonate (CaCO₃)| YES                 | NO             |
| magnesium carbonate hydroxide | YES             | YES            |
| activated charcoal (carbon) | YES               | YES            |
| clay (kaolin)            | YES                 | YES            |

**Figure 2.** Comparison between two memory effects. The length of each side of the square container is 200mm and the lengths of the neighboring sides of the two parallelogrammatic containers are 200mm and 200√2mm. The arrows indicate the direction of the initial vibration, where the amplitude $r$ and the frequency $f$ of the vibration are 15mm and 1Hz, respectively, i.e., the maximum strength of the vibration $4\pi^2rf^2$ is 0.6m/s². (a) Memory of vibration. Here, we use a water-poor CaCO₃ paste with a solid volume fraction of $\rho = 44\%$. The direction of crack propagation is anywhere perpendicular to the direction of the initial vibration. As for the memory of vibration, the boundary condition for the movement of paste is the bottom of the container. (b) Memory of flow induced by the initial vibration [15]. Here, we use a water-rich paste of magnesium carbonate hydroxide with a solid volume fraction of $\rho = 7.7\%$. The flow patterns caused by the initial external vibration are memorized and are exhibited by the crack patterns. As for the memory of flow, the boundary condition for flow motion is the side walls of containers.
3. Morphological phase diagram of desiccation crack patterns

To investigate the mechanism of memory effects, systematic experiments are performed to obtain a morphological phase diagram of desiccation crack patterns of pastes. The pastes we use here are magnesium carbonate hydroxide and activated charcoal (carbon) (Kanto Chemical, Tokyo). Both pastes have memories of both vibration and flow. First, we observe sizes and shapes of dry powder particles by Scanning Electron Microscope (SEM) (Fujitsu, Tokyo). Figure 3 shows that particles of magnesium carbonate hydroxide are much more smaller than those of activated charcoal (carbon). Figure 3 also shows that the shape of magnesium carbonate hydroxide is disk-like, while the shape of activated charcoal (carbon) is crashed rod-like.

To make morphological phase diagrams, the solid volume fraction, i.e., the volume fraction of powder in the mixture, plays an important role in the memory effect. When the solid volume fraction is small, colloidal particles are distributed sparsely, so that the mixture is regarded as a Newtonian viscous fluid. On the other hand, when the solid volume fraction exceeds a threshold value (Liquid-Limit), a network of colloidal particles is formed via van der Waals attractive interaction, and the rheology of the mixture changes drastically. This very concentrated suspension of colloidal particles is regarded as a paste. Also, a strength of the initial external vibration should take an appropriate value to imprint a memory into a paste. Here, the maximum strength of the vibration is given by $4\pi^2 r f^2 [\text{m/s}^2]$, where $r [\text{m}]$ is the amplitude and $f [1/\text{s}]$ is the frequency of the vibration.

Morphological phase diagrams of desiccation crack patterns are shown in Fig. 4, as a function of both the solid volume fraction and the strength of the initial vibration. Here, Fig. 4(a) shows a case where we use magnesium carbonate hydroxide, and Fig. 4(b) shows the one where we use activated charcoal (carbon). The size of the square container is 200mm with each side. The mass of colloidal particles in the mixture in each container is fixed as 100g, so that the final thickness of paste with different solid volume fractions is equalized when they dry and thus the characteristic sizes of final crack patterns are also equalized. We vibrate the container

Figure 3. Images of dry particles of magnesium carbonate hydroxide and active charcoal (carbon) taken by Scanning Electron Microscope (SEM): (a) magnesium carbonate hydroxide. The scale in the figure indicates that the distance between neighboring small white dots just above parameters is 0.20µm, so the horizontal size of the figure is 8.0µm. The shape of magnesium carbonate hydroxide is disk-like. (b) active charcoal (carbon). The scale in the figure indicates that the distance between neighboring small white dots just above parameters is 5.00µm, so the horizontal size of the figure is 200.0µm. The shape of activated charcoal (carbon) is crashed rod-like. From (a) and (b), we see that particles of magnesium carbonate hydroxide are much more smaller than those of activated charcoal (carbon).
Figure 4. Morphological phase diagram of desiccation crack patterns, shown as a function of the solid volume fraction $\rho$ and the maximum strength of the initial vibration $4\pi rf^2$. The length of each side of the square container is 200 mm, and mass of powder in each container is fixed as 100g. The vertical dotted lines represent the Liquid-Limit and the vertical dashed-and dotted lines represent Plastic-Limit. The solid lines represent the yield stress lines, on which the maximum value of the shear stress induced by the vibration becomes same as that of the yield stress of paste. The broken lines represent boundaries between regions. Open circles denote isotropic cellular crack patterns. Solid squares denote lamellar crack patterns, the direction of which is perpendicular to the direction of the initial vibration, and is the visualization of the memory of vibration. Open squares with a plus inside denote lamellar crack patterns, the direction of which is parallel to the direction of the flow induced by the initial vibration, and is the visualization of the memory of flow. (a) Paste made of magnesium carbonate hydroxide and water [15]. (b) Paste made of activated charcoal (carbon) and water. For both pastes, in regions A below yield stress line, pastes do not move at all, so they have no memory and only isotropic cellular crack patterns appear. Pastes remember vibration in regions B, and remember flow in regions C, but in regions D where pastes were in turbulent states we only get cellular cracks again. We see that both pastes have an ability to remember the directions of vibration and flow. horizontallly in one direction for 1 min., stop the vibration, and dry the paste until desiccation cracks are formed.

The paste of magnesium carbonate hydroxide, shown in Fig. 4(a), has memories of both flow and vibration. The region between two vertical lines, i.e., Liquid-Limit at 4% and Plastic-Limit at 18%, is divided by the solid and the dashed curves into four regions, A, B, C, and D. The solid curve corresponds to the yield stress line, on which the maximum strength of the initial vibration is equal to the strength of the yield stress of the paste. Isotropic cellular crack patterns appear in regions A and D. In region A, the maximum value of the strength of the initial vibration is smaller than that of the yield stress of paste, so paste did not experience any motion at the initial vibration. On the other hand, in region D, the paste were in a turbulent state at the initial vibration. We consider that the paste remembers the direction of the turbulent flow, but it is difficult to confirm the relation between the direction of turbulent flow and the morphology of random cellular cracks. In regions B just above the yield stress line, a water-poor paste is vibrated just like the east part of Japan was hit by an huge earthquake at 11 March 2011, and the paste remembers the direction of the vibration even after the vibration stops. In region C where a water-rich paste is fluidized in one direction like a lamellar flow (i.e., not in a turbulent state), the paste memorizes the direction of the flow induced by the initial vibration. We also get a similar result when we perform experiments using paste of activated charcoal (carbon),
and its result is shown in Fig. 4(b).

On the other hand, the paste of calcium carbonate cannot remember flow direction. That is, even when water-rich CaCO$_3$ paste is fluidized just like in regions C in Fig. 4, CaCO$_3$ paste cannot remember flow direction, and only isotropic cellular crack patterns appear. Recent experiments[23] have clarified the reason why CaCO$_3$ paste cannot remember flow direction: it is due to the existence of long-range Coulombic repulsive interaction between charged colloidal particles in water. In fact, like pastes which remember flow direction, such as magnesium carbonate hydroxide and activated charcoal (carbon), colloidal particles are not charged in water. If the Coulombic repulsion is screened by adding sodium chloride into CaCO$_3$ paste, CaCO$_3$ paste gets an ability to remember flow direction.

4. Size dependence on memory effect

Then, why can pastes remember the external mechanical fields, such as vibration and flow? In this section, we perform experiments where we use pastes with different size distribution of colloidal particles. If the memory effect is just a result of jamming phenomena of inelastic particles at high solid volume fraction, we will see no change in memory effects as we keep the solid volume fraction constant and decrease the mean size of colloidal particles. Here, we speculate that the memory of paste is maintained in a network structure of colloidal particles, which are connected each other via interparticle forces between colloidal particles. When the mean size of colloidal particles decreases but the value of the solid volume fraction is still kept constant, the ratio of the surface of colloidal particles to their volume increases, and the effect of interparticle forces increases. We will check whether we can observe any changes in memory effects as we change the size distribution of colloidal particles in paste.

Here, we use Japan Industrial Standards (JIS) test powders of CaCO$_3$ particles with different size distribution. They are called test powders, because they are used to examine whether sizes of pores in filters fit to JIS. We use JIS Z 8901 test powder 1-16 (CaCO$_3$) and JIS Z 8901 test powder 1-17 (CaCO$_3$). First, we observe sizes and shapes of dry powder particles by Scanning Electron Microscope (SEM). From Fig. 5, we see that particles of JIS Z 8901 test powder 1-16

(a) JIS Z 8901 test powder 1-16 (CaCO3)  (b) JIS Z 8901 test powder 1-17 (CaCO3)

Figure 5. Images of dry particles of calcium carbonate (CaCO$_3$) taken by Scanning Electron Microscope (SEM). The scale in each figure indicates that the distance between neighboring small white dots just above parameters is 0.50μm, so the horizontal size of each figure is 20.0 μm. (a) Japan Industrial Standards (JIS) Z 8901 test powder 1-16 (CaCO$_3$). (b) JIS Z 8901 test powder 1-17 (CaCO$_3$). The shapes of both CaCO$_3$ test powder particles in (a) and (b) are similar to each other and are rough just like rocks and sand particles.
Figure 6. Cumulative mass distribution of colloidal particles, i.e., the fraction of the total mass of colloidal particles with diameters larger than $d \mu m$, expressed as $F(d)\%$, is represented as a function of $d$. Solid circles and its solid line denote $F(d)\%$ of JIS Z 8901 test powder 1-16 (CaCO$_3$), solid squares and its solid line denote $F(d)\%$ of JIS Z 8901 test powder 1-17 (CaCO$_3$), open squares and its dotted line denote $F(d)\%$ of magnesium carbonate hydroxide, and open triangles and its dotted line denote $F(d)\%$ of activated charcoal (carbon). The dashed line denotes the value of $F(d) = 50\%$, which corresponds to the characteristic size of colloidal particles. Except particles of activated charcoal which are much larger than other powder particles, the characteristic sizes of each colloidal particles can be estimated to be 4.5$\mu m$ (JIS Z 8901 test powder 1-16 (CaCO$_3$)), 2.0$\mu m$ (JIS Z 8901 test powder 1-17 (CaCO$_3$)), and 3.0$\mu m$ (magnesium carbonate hydroxide), respectively.

(CaCO$_3$) are larger than that of JIS Z 8901 test powder 1-17 (CaCO$_3$). Shapes of both CaCO$_3$ test powder particles in (a) and (b) are similar to each other and are rough just like rocks and sand particles. That is, we do not see any big differences in shapes between two JIS test powders.

Next, we measure the size distribution of colloidal particles in each paste using sedimentation method based on Stokesian approximation. Figure 6 shows cumulative mass distribution of colloidal particles, $F(d)\%$, i.e., the fraction of total mass of colloidal particles with diameters larger than $d \mu m$. Here, let us define a characteristic size of colloidal particles with a diameter where the value of $F(d) = 50\%$. From Fig. 6, we see that the characteristic size of JIS Z 8901 test powder 1-16 (CaCO$_3$) is 4.5$\mu m$, while that of JIS Z 8901 test powder 1-17 (CaCO$_3$) is 2.0$\mu m$. That is, the particles of JIS Z 8901 test powder 1-16 (CaCO$_3$) are almost two times larger than that of JIS Z 8901 test powder 1-17 (CaCO$_3$). Since the characteristic size of magnesium carbonate hydroxide is 3.0$\mu m$ and is just between those of two CaCO$_3$ JIS test powders, the ability to remember flow direction is not determined solely by the value of the characteristic size of colloidal particles, as was confirmed in [23].

Morphological phase diagrams of desiccation crack patterns of CaCO$_3$ pastes are shown in Fig. 7, as a function of both the solid volume fraction and the maximum strength of the initial vibration. Here, Fig. 7(a) shows experimental results where we use JIS Z 8901 test powder 1-16 (CaCO$_3$), and Fig. 7(b) shows the one where we use JIS Z 8901 test powder 1-17 (CaCO$_3$). As was discussed above, CaCO$_3$ pastes cannot remember flow directions. The size of the square container is 200mm with each side, and the mass of CaCO$_3$ particles in each container is fixed as 360g in both (a) and (b).

Figure 7 shows that, by decreasing the characteristic size of colloidal particles, the values of Liquid-Limit and Plastic-Limit decrease, and the value of the yield stress of paste increases.

This
Figure 7. Morphological phase diagram of desiccation crack patterns, shown as a function of the solid volume fraction $\rho$ and the maximum strength of the initial vibration $4\pi^2rf^2$. The length of each side of the square container is 200 mm, and mass of powder in each container is fixed as 360g. The vertical dotted lines represent the Liquid-Limit and the vertical dashed-and dotted lines represent Plastic-Limit. The solid lines represent the yield stress lines, on which the maximum value of the shear stress induced by the vibration becomes same as that of the yield stress of paste. The broken lines represent boundaries between regions. Open circles denote isotropic cellular crack patterns. Solid squares denote lamellar crack patterns, the direction of which is perpendicular to the direction of the initial vibration, and is the visualization of the memory of vibration. (a) Paste made of JIS Z 8901 test powder 1-16 (CaCO$_3$). The characteristic size of colloidal particles is given by 4.5\,\mu m from Fig. 6. (b) Paste made of JIS Z 8901 test powder 1-17 (CaCO$_3$). The characteristic size of colloidal particles is given by 2.0\,\mu m from Fig. 6. For both pastes, in regions A below yield stress line, pastes do not move at all, so they have no memory and only isotropic cellular crack patterns appear. Pastes remember vibration in regions B, but in regions D where pastes were fluidized we only get cellular cracks again. That is, CaCO$_3$ pastes cannot remember flow directions. Comparing (a) and (b) we see that, by decreasing the characteristic size of colloidal particles, the values of Liquid-Limit and Plastic-Limit decrease, the value of the yield stress increases, the strength of viscosity and plasticity increase, and pastes come to remember vibration even at smaller solid volume fraction. Since decreasing the characteristic size of colloidal particles corresponds to increasing the ratio of the surface of colloidal particles to their volume, all these results indicate that interparticle forces play important roles in memory effects, as they connect colloidal particles to form intelligent network structure.
effects. Of course, for memory effects to hold, the size of colloidal particles must not exceed several hundred micron. When we performed same experiments using carbon paste made of huge carbon particles with a characteristic size of particles about 300\(\mu m\), we could not get desiccation crack patterns. That is, all these huge carbon particles are separated into individual particles as they dry, just like drying an assembly of sand particles.

We speculate that the origin of the memory effect is the deformation of network structure of colloidal particles, which are connected with each other via interparticle forces [24, 25, 26, 27]. When a water-poor paste is vibrated just like an earthquake, the longitudinal density fluctuation emerges inside the network structure. On the other hand, when a water-rich paste is fluidized, the dilute network structure is elongated along flow direction, unless particles are charged and the network is easily broken. These structural changes under external mechanical fields survive due to its plasticity, and are visualized as the morphology of desiccation crack patterns in situations where weaker points must be broken.

5. Conclusions

Pastes remember the directions of vibration and flow that they have suffered. We find that a paste with smaller particles have a better memory, in the sense it remembers external mechanical fields at smaller solid volume fraction. This result implies that interparticle forces between colloidal particles play an important role in memory effects, causing a quantitative change in the phase diagram for the same material. We speculate that memories in pastes are maintained as microscopically anisotropic network structure of colloidal particles, connected via interparticle forces between colloidal particles, such as van der Waals interaction. Memories in pastes can be visualized as crack patterns which appear when pastes dry. By using the memory effect of paste, we can control the morphology of desiccation crack patterns and make any geometrical crack patterns that streamlines can make. Recently there appear new challenges to control the formation of desiccation cracks by using electrical and magnetic fields [28, 29, 30, 31]. We would like to combine all these methods for the better control of crack formation.

Acknowledgments

We would like to acknowledge K. Hoshino and H. Nakayama for performing experiments with us and Ooshida Takeshi, M. Otsuki, S. Kitsunezaki, S. Goto, T. Matsumoto, T. Hatano, N. Ito, Ferenc Kun, T. Dutta and S. Tarafdar for valuable discussions. This work was supported by Grant-in-Aid for Scientific Research (KAKENHI) (B) 22340112 and (C) 21540388 and 23540452 of Japan Society for the Promotion of Science (JSPS). This project was also supported by JSPS and HAS under the Japan - Hungary Research Cooperative Program and by JSPS and DST under the Japan - India Science Cooperative Program.

References

[1] Herrmann H J and Roux S 1990, Statistical Models for the Fracture in Disordered Media, (Amsterdam: North-Holland)
[2] Lawn B 1993, Fracture of Brittle Solids, 2nd ed., (Cambridge: Cambridge)
[3] Oddershede L, Dimon P, and Bohr J 1993, Phys. Rev. Lett. 71 3107
[4] Herrmann H J, Wittel F K, and Kun F 2006, Physica A 371 59
[5] Yuse A and Sano M 1993, Nature 362 329
[6] Mandler M 1994, Phys. Rev. E 49 R51
[7] Hayakawa Y 1994, Phys. Rev. E 49 R1804
[8] Sasai S-I, Sekimoto K, and Nakaniishi H 1994, Phys. Rev. E 50 R1733
[9] Nakahara A and Matsuo Y 2005, J. Phys. Soc. Jpn. 74 1362
[10] Groisman A and Kaplan E 1994, Europhys. Lett. 25 415
[11] Allain C and Limat L 1995, Phys. Rev. Lett. 74 2981
[12] Komatsu T S and Sasa S 1997, Jpn. J. Appl. Phys. 36 391
[13] Kitsunezaki S 1999, Phys. Rev. E 60 6449
[14] Nakahara A and Matsuo Y 2006, *J. Stat. Mech.: Theory Exp.* P07016
[15] Nakahara A and Matsuo Y 2006, *Phys. Rev. E* **74** 045102(R)
[16] *Physics Today* 2007, vol 60, no 9, p 116
[17] Ooshida Takeshi and Sekimoto K 2005, *Phys. Rev. Lett.* **95** 108301
[18] Otsuki M 2005, *Phys. Rev. E* **72** 046115
[19] Ooshida Takeshi 2008, *Phys. Rev. E* **77** 061501
[20] Ooshida Takeshi 2009, *J. Phys. Soc. Jpn.* **78** 104801
[21] Kitsunezaki S 2009, *J. Phys. Soc. Jpn.* **78** 064801
[22] Kitsunezaki S 2010, *J. Phys. Soc. Jpn.* **79** 124802
[23] Matsuo Y and Nakahara A, arXive:1101.0953v1 [cond-mat.soft].
[24] Tanaka H and Araki T 2000, *Phys. Rev. Lett.* **85** 1338
[25] Butter K, Bomans P H H, Frederik P M, Vroege G J and Philipse A P: *Nature Materials* **2** 88
[26] Butt H-J, Kappl M 2010, *Surface and Interfacial Forces*, (Weinheim: WILEY-VCH)
[27] Israelachvili J N 2011, *Intermolecular and Surface Forces*, 3rd ed., (San Diego: Elsevier)
[28] Mal D, Sinha S, Middya T R, and Tarafdar S 2007, *Physica A* **384** 182
[29] Pauchard L, Elias F, Boltenhagen P, Cebers A and Bacri J C 2008, *Phys. Rev. E* **77** 021402
[30] Ngo A T, Ricardi J and Pileni M P 2008, *Nano Letters* **8** 2485
[31] Ngo A T, Ricardi J and Pileni M P 2008, *J. Phys. Chem. B* **112** 14409