A Collection of Polarized Parton Densities

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Abstract

A significant number of parameterizations for the polarized parton densities have appeared in the literature. Using the CTEQ evolution package, these distributions have been evolved consistently preparatory to compilation into an integrated package in the spirit of PDFLIB by Plohow-Besch. Here, a comparison of a few of the more recent distributions are made.
Table 1: This is a partial listing of parameterizations for the polarized parton distribution functions.

| ∆PDFs | AUTHORS | REFERENCE |
|-------|---------|-----------|
| BT-95 | Bartelski & Tatur | preprint, CAMK 95-288 |
| GRSV-95 | Gluck, Reya, Stratmann & Vogelsang | preprint, DO-TH 95/13 |
| GRV-95 | Gluck, Reya & Vogelsang | preprint, DO-TH 95/11 |
| CLW-95 | Cheng, Liu and Wu | preprint, IP-ASTP-17-95 |
| BS-95 | Bourrely and Soffer | Nucl. Phys. B445 (1995) 341 |
| F-95 | de Florian, et al. | Phys. Rev. D51 (1995) 37 |
| GS-95 | Gehrmann and Stirling | Z. Phys. C65 (1995) 461 |
| BBS-95 | Brodsky, Burkardt & Schmidt | Nucl. Phys. B441 (1995) 197 |
| N-94 | Nadolsky | Z. Phys. C63 (1994) 601 |
| CCGN-93 | Chiappetta, et al. | Z. Phys. C59 (1993) 629 |
| F-93 | de Florian, et al. | Phys. Lett. B319 (1993) 285 |
| CW-92 | Cheng and Wai | Phys. Rev. D46 (1992) 125 |
| SL-92 | Sridhar and Leader | Phys. Lett. B295 (1992) 283 |
| CN-91 | Chiappetta and Nardulli | Z. Phys. C51 (1991) 435 |
| GRV-90 | Gluck, Reya and Vogelsang | Nucl. Phys. B329 (1990) 347 |
| G-90 | Gupta, et al. | Z. Phys. C46 (1990) 111 |
| CL-90 | Cheng and Lai | Phys. Rev. D41 (1990) 91 |

1 Introduction

The ideas for computing polarized parton densities have been around for a long time[1]. What has been lacking is sufficient experimental data to define those densities. Good progress has been made from deep inelastic scattering experiments. At present, the data ranges are from $0.003 < x < 0.8$ and $1 \text{GeV}^2 < Q^2 < 60 \text{GeV}^2$, and this has provided reasonable fits describing the up quark and down quark distributions. Data intended to constrain the gluon and sea quark densities tightly has yet to be obtained.

Recent theoretical progress has given us the next to leading order Altarelli-Parisi splitting kernels for polarized partons[2]. The first helicity distributions based on this higher order evolution have already appeared[3].

A partial listing of polarized parton densities is given in Table 1. As we can see, 1995 has been a good year. Soon, we should have about as many distributions as there are data points to fit. This table mainly focuses on those distributions from the 1990’s, but there are other distributions that people have found useful.
Table 2: This is a listing of input parameters for the polarized parton distribution functions in Table 1.

| ∆PDFs | Mode | ∆PDF | Order | $Q_0^2$ (GeV$^2$) | $\Lambda_{QCD}^{(4)}$ (MeV$^2$) | Unpolarized PDF |
|-------|------|-------|-------|-------------------|---------------------|----------------|
| 270   | BT-95| LO    | 4.0   | 230               | U-MRSA (set A)      |
| 260   | CLW-95| LO    | 10.0  | 231               | U-MRSA' (set A')    |
| 250   | GRSV-95| NLO   | 0.34  | 200               | U-GRVt-95           |
| 240   | GRV-95| LO    | 0.23  | 200               | U-GRVt-95           |
| 230   | BS-95| LO    | 3.0   | 200               | BS-95               |
| 220   | F-95 | LO    | 10.0  | 230               | U-MRS' (set D')     |
| 210   | GS-95| LO    | 4.0   | 177               | U-O                 |
| 200   | BBS-95| LO    | 4.0   | 230               | U-MRS' (set D'_0)   |
| 190   | N-94 | LO    | 11.0  | 200               | U-GRVt-92           |
| 180   | CCGN-93| LO    | 1.0   | 260               | U-DFLM (avg. set)   |
| 170   | F-93 | LO    | 4.0   | 168               | U-CTEQ1             |
| 160   | CW-92| LO    | 10.0  | 260               | U-DFLM (avg. set)   |
| 150   | SL-92| LO    | 4.0   | 177               | U-O                 |
| 140   | CN-91| LO    | 1.0   | 260               | U-DFLM (avg. set)   |
| 130   | GRV-90| LO    | 10.0  | 360               | U-GHR               |
| 120   | GPS-90| LO    | 5.0(15.0) | 200(90)    | U-EHLQ(U-EMC)      |
| 110   | CL-90| LO    | 10.7  | 260               | U-DFLM (avg. set)   |

that appeared before 1990\[4\].

Using the CTEQ evolution package\[5\], these distributions have been evolved consistently to allow for a comparison of a few of the more recent distributions as well as to facilitate future research. A library in the spirit of PDFLIB by Plothow-Besch\[6\] is under development and will soon be available for distribution. In the remainder of this report, details of the evolution are presented and some comparisons between a few distributions from the 1990’s are made.

2 On the Parameters of the $Q^2$ Evolution

To have a properly defined parton distribution function (PDF) requires that a number of parameters be defined from the outset (something about which many papers are not very explicit). In Table 2 the values of the parameters used in this evolution are presented. Generally, the evolution for each parton distribution starts with an initial distribution at a given energy scale ($Q_0$) as provided in

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Table 3: This is a listing of references for the unpolarized parton distribution functions associated with the $\Delta$PDFs in Table 1.

| Unpolarized PDFs | AUTHORS | REFERENCE |
|------------------|---------|-----------|
| U-GRVt-95        | Gluck, Reya and Vogt | Z. Phys. C67 (1995) 433 |
| U-BS             | Bourrely and Soffer  | Nucl. Phys. B445 (1995) 341 |
| U-BBS            | Brodsky, Burkardt & Schmidt | Nucl. Phys. B441 (1995) 197 |
| U-MRSA'          | Martin, Stirling & Roberts | Phys. Lett. B354 (1995) 155 |
| U-MRSA           | Martin, Roberts & Stirling | Phys. Rev. D56 (1994) 6734 |
| U-MRS'           | Martin, Roberts & Stirling | Phys. Lett. B306 (1993) 145 |
| U-CTEQ1          | CTEQ Collaboration | Phys. Lett. B304 (1993) 159 |
| U-GRVt-92        | Gluck, Reya & Vogt  | Z. Phys. C53 (1992) 127 |
| U-O              | Owens             | Phys. Lett. B266 (1991) 126 |
| U-DFLM           | Diemoz, et al.    | Z. Phys. C39 (1988) 21 |
| U-EMC            | Sloan, Smajda and Voss | Phys. Rep. 162 (1988) 45 |
| U-DO             | Owens             | Phys. Rev. D30 (1984) 49 |
| U-EHLQ           | Eichten, et al.   | Rev. Mod. Phys. 56 (1984) 579 |
| U-GHR            | Gluck, Hoffman & Reya | Z. Phys. C13 (1982) 119 |

The quark masses are not mentioned in the tables. A few of the distributions begin their evolution at scales of $Q_0$ around 1 GeV, which can be below the charm and bottom quark thresholds. This has its effect on the evolution when these quark thresholds are crossed. Some authors indicate what quark masses were used, like in GRV-95 where $m_c = 1.5\text{ GeV}$ and $m_b = 4.5\text{ GeV}$ were given, but for most of the cases values of $m_c = 1.6\text{ GeV}$ and $m_b = 5.0\text{ GeV}$ have been adopted. In all the evolution performed here, the top quark mass has been set to $m_t = 180\text{ GeV}$.

Since the data has been at low $Q^2$, quite a number of papers perform their fits based on evolutions with three quark flavors. For the evolutions here, however, the full six flavors are used with the understanding that some people want to do computations at higher energy scales.

Each set of helicity distributions has been associated in some manner with a specific set of unpolarized PDFs. In most cases, this appears through the appli-
cation of a dilution model, whereby the $\Delta$PDF is written as a linear combination of unpolarized PDFs weighted by a phenomenological function in $x$. This association between the $\Delta$PDFs and the unpolarized PDFs is tabulated in Table 3. In performing the evolutions, these associations have been maintained.

The $\Delta$PDFs that have been chosen for presentation are N-94 (sets 1 and 2), BBS-95, GS-95 (sets A, B, and C), and GRV-95 ("standard" and "valence" scenarios). In Figs. 1-3, these distributions are shown at the scale $Q = 15 \text{ GeV}$. To evolve the BBS-95 helicity densities, it was necessary to assume a form for the sea quark distributions. Since the BBS-95 distributions are close to the MRSD0′ form, those sea quark densities were used to provide the helicity densities for the sea; namely, at the initial energy scale it was assumed that $\Delta \bar{u}(x) = \frac{\bar{u}(x)}{2}$ and $\Delta \bar{d}(x) = \frac{\bar{d}(x)}{2}$.

3 The Up and Down Quarks

The two distributions best defined by the present experimental data are the up and down quark densities. Looking at Fig. 1, there is nice agreement between the different fits for $\Delta u$; it isn’t until the larger $x$ are reached (where the error bars on the data increase) that significant deviations occur. The down quark densities appear to have a few atypical contenders, but it should be noted that in the plot of $x \Delta d(x)$, the BBS-95 and N-94 (set 2) distributions both allow $\Delta d(x)$ to cross over into positive values at some $x$. Since the $\Delta$PDFs are constrained by the moments, where $\Delta d = \int \Delta d(x) dx < 0$, the BBS-95 and N-94 (set 2) distributions compensate for their positive contribution to the integral with a more negative $\Delta d(x)$ below the crossover point.

4 The Gluon and Sea Quark Densities

Since the gluon and sea quark densities are, to a large degree, unconstrained, it is here where the parameterizations are most distinguishable. Some models have negligible or large $\Delta G(x)$ and $\Delta s(x)$ while others may carry more moderate $\Delta G(x)$ and $\Delta s(x)$. Hopefully, this issue will be settled with results from hadron-hadron collisions at laboratories like the RHIC.

5 Large $x$ Limits

As discussed by Farrar and Jackson, our expectation is that as the momentum fraction of the parton approaches unity that the helicity of the parton should coincide with that of its parent hadron. In other words, we have the limit

$$\frac{\Delta q(x)}{q(x)} \rightarrow 1 \quad \text{as} \quad x \rightarrow 1.$$ (1)
The plots in Fig. 2 illustrate the polarization of some of the ∆PDFs in the large $x$ limit. The BBS-95 distributions carry a smooth transition of the polarization towards unity with large $x$ while other distributions have polarizations that sharply rise at very large $x$, plateau around $x \gtrsim 0.6$, or ignore this large $x$ behavior. Numerically, the PDFs are small as $x$ nears unity, which minimizes the effect such variations in the ∆PDF may have on physics. Nevertheless, it is important to note that it is in the large $x$ region that the polarization distinguishes between the different models and fits[9]. In particular, different parameterizations of the ∆d distribution cross over from negative to positive values at different $x$.

6 Small $x$ Extrapolations

With the larger energy colliders like HERA, the RHIC or the LHC comes the possibility of investigating polarization physics at higher energy scales and smaller momentum fraction than fixed target experiments. In Fig. 3, the small-$x$ extrapolation for a sample of ∆PDFs is displayed. These results indicate that, as usual, care must be exercised when extending the use of the ∆PDFs beyond the range of the data with which they were fit. The view generally taken is that the polarization of the helicity distributions should vanish as $x \to 0$[10]. Using the distributions beyond their range of validity can produce unreasonable results.

7 $Q^2$ Evolution

In Fig. 4 the $Q^2$ evolution is performed for four of the ∆PDFs we have been examining. What is specifically shown is the charge weighted sum over the quark helicity distributions,

$$\frac{1}{2} \sum_i e_i^2 \Delta f_i(x, Q^2),$$

for $Q = 0.015, 0.1, 1, 10$ TeV and where $i$ runs over the quark types $u, d, s, \bar{u}, \bar{d}, \bar{s}$. (I do not call this $g_1$ because some papers, such as GS-95, define this structure function differently by including the anomalous gluon contribution.) The thing to note in Fig. 4 is that the $Q^2$ evolution reveals distinguishing features of the different helicity distributions, indicating that the evolution properties themselves will be useful to consider when establishing the ∆PDFs[11].

8 Caveats

This work has mainly been a presentation of polarized parton densities available in the literature. Though some features of these distributions have been discussed, no attempt has been made here to determine the quality or correctness of the fits.
or the procedures used. In some cases the comparison would be pointless because of the improvements in data and theory over the past decade.

There are a few caveats for the user. The many distributions have been fit with different data at different times and may eventually become outdated. Furthermore, not all the distributions have been determined from the same perspective within QCD. An example of such variations can be found in the different definitions used for the structure function $g_1$. Inconsistencies, many of which are irrelevant due to the lack of constraining data on the gluon and sea quark densities, may also appear in the fits (e.g., higher order unpolarized PDFs sometimes have been used as the input distribution for helicity densities evolved in leading order). Nonetheless, with the variety available among all the $\Delta$PDFs, a wide range of possible physics can be investigated.

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Figure 1: The helicity distributions for up, down and strange quarks and for gluons are shown at $Q = 15$ GeV: N-94 sets 1 and 2 (solid lines), BBS-95 (dash-dot), GS-95 sets A, B, and C (dotted), and the GRV-95 “standard” and “valence” scenarios (dashed).
Figure 2: The polarization of the partons in the proton as described by the ratios $\Delta u(x)/u(x)$, $\Delta d(x)/d(x)$, $\Delta g(x)/g(x)$ and $\Delta s(x)/s(x)$. Curves are as in Fig. 1.
Figure 3: The small-$x$ extrapolations for the helicity distributions of the up, down and strange quarks and for the gluons are shown. Curves are as in Fig. 1.
Figure 4: The $Q^2$ evolution for four of the $\Delta$PDFs are displayed.