Numerical simulation of carpet cloaking device in terahertz frequency range

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Abstract. This work is devoted to the numerical calculation of the effective constitutive parameters of the carpet cloaking device and to the numerical simulation of this cloak using finite element method (FEM) for the terahertz frequency range.

1. Introduction
In recent years methods of design of composite structure called metamaterials are rapidly improving. Thereby there are unprecedented opportunities to manipulate of electromagnetic radiation. Transformation optics is new science of radiation control [1], it is very powerful instrument for emission control. This approach is based on differential geometry and tensor analysis. Transformation optics shows that space can be arbitrarily curved according to predetermined manner [2]. The rays in such space will have special trajectories. The Fermat’s principle is applied to control light by changing of the medium parameters. This is accompanied by a conversion from virtual space to physical space by coordinate transformations.

One of the most interesting devices based on this principle are cloaks of invisibility. Special case of such devices is cover that crushes a cloaked object to a flat ground plane in virtual space [3, 4]. In fact there is an object and cover lying on the surface but it is perceived as the actual ground plane to the observer. This effect is achieved by coordinate transformations. The term "carpet cloak” was coined by Jensen Li and J.B. Pendry in [5] for such transformation.

The development of cloaking devices is very actual direction in spectroscopy, medicine, image analysis, etc. There are many implementations of devices based on various principles and working in the different frequency ranges [6, 7]. But the operation of the cloak in the terahertz frequency range hasn’t been clearly understood. In this simulation of carpet cloaking device was proposed in terahertz frequency range. The special constitutive parameters were obtained by grid generation using difference approximation method.

2. Analytical results
For our problem the refractive index profile was calculated. The quasi-conformal mapping is used for transformation from the virtual space (with coordinates ($\xi$, $\eta$)) to the physical space (with coordinates ($x$, $y$)) because this mapping minimizes anisotropy [5]. The cloaking area was chosen as a symmetric domain. In the virtual space it seems as a rectangular domain divided into squares with the aspect ratio $\xi/\eta = 1$. The distortion function $f(x)$ for mapping in general case [8] looks like:
It was chosen more simply case which corresponds to the quasi-conformal mapping where distortion function has the following form:

$$f(\xi, \eta) = M.$$  \hfill (1)

Figure 1. Propagation of wave in (a) virtual space; (b) physical space.

After the mapping the physical space will be analogously to the virtual space and it differs by only the lower boundary which is defined by the special function. This domain is divided into quadrangular cells. The refractive index profile associated with geometry has following form:

$$n^2 = \frac{1}{\sqrt{\text{det} \ g}} = \frac{1}{|\xi||\eta|},$$  \hfill (3)

where $g$ is the metric tensor in the physical space. The anisotropy factor $\alpha$ is defined by [5]:

$$\alpha + \frac{1}{\alpha} = \frac{\text{Tr}(g)}{\sqrt{\text{det} \ g}}.$$  \hfill (4)

According to the optimal ratio [5] between refractive index and anisotropy, the refractive index was calculated.

3. Grid generation

The two-dimensional transformation between virtual space and physical space can be obtained by grid generation. In an orthogonal grid all off-diagonal components of the metric tensor $g$ are equal to zero:

$$g_{ij} = \bar{a}_i \cdot \bar{a}_j = \frac{\partial x}{\partial \xi} \frac{\partial x}{\partial \eta} + \frac{\partial y}{\partial \xi} \frac{\partial y}{\partial \eta} = 0,$$

with $i \neq j$ \hfill (5)

$$\sqrt{\text{det} \ g} = \frac{\partial x}{\partial \xi} \frac{\partial x}{\partial \eta} + \frac{\partial y}{\partial \xi} \frac{\partial y}{\partial \eta} = \sqrt{g_{11}g_{22}} = h_\xi h_\eta,$$

where $g_{11}$, $g_{22}$ are diagonal elements of metric tensor $g$; $h_\xi$, $h_\eta$ is Lamé coefficients.

A 2-D orthogonal grid must satisfy the Beltrami equations [8]:

$$\begin{cases}
\frac{\partial x}{\partial \xi} = \frac{\partial y}{\partial \eta} \\
\frac{\partial y}{\partial \xi} = -\frac{\partial x}{\partial \eta}
\end{cases} \quad (6)
$$

Here, the distortion function $f$ has the following form:

$$f = \frac{h_\eta}{h_\xi} = \sqrt{\left(\frac{\partial x}{\partial \eta}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2}.$$  \hfill (7)
If \( f \) is the constant equal to \( M \) then the grid is quasi-conformal. In the case \( M = 1 \) (conformal mapping) the Beltrami equations (5) becomes the Cauchy-Riemann equations.

The discretization of typical grid cell was calculated by definition of the unknown point at the center of the control cell, as shown in Fig. 2.

Figure 2. (a) The typical control cell used in discretization; (b) the typical control cell in the virtual space; (c) the typical control cell in the physical space.

An optimal (small enough value of anisotropy and not large enough value of refractive index) map is generated by minimizing the following functional:

\[
\Phi = \iint_{D} \left[ f(u_{x}^{2} + v_{x}^{2}) + \frac{1}{f}(u_{y}^{2} + v_{y}^{2}) \right] \, d\xi \, d\eta,
\]

where \( u(\xi, \eta), v(\xi, \eta) \) describe the mapping. The difference approximation method of such variation problem [9] was used in this paper.

Two different domains in the physical space with nonlinear distortion of the low boundary and the linear one were considered. The transformed orthogonal grid in the physical space for two cases was obtained. The color maps demonstrate permittivity profile \( \varepsilon \) (Fig. 3).

Figure 3. (a) Physical space with nonlinear distortion. (b) Physical space with linear distortion.

4. Numerical simulation

In this part the numerical simulation of effectiveness of the designed cloak was presented. For the simulation of wave propagation Finite Element Method (FEM) was used for solving Maxwell's
equations. The cloak with sizes of 8x4 mm$^2$ was investigated at the frequency of 0.1 THz. The nonmagnetic cloak was considered ($\mu = 1$). The permittivity of the cloak varies from 0.99 to 2.75. The $E_z$-field distribution is presented in Fig. 4.

![Field distributions](image)

**Figure 4.** (a) The rectangular cloak with nonlinear boundary around the object; (b) the rectangular medium ($\varepsilon=2.25$) with the nonlinear boundary around the object; (c) the rectangular cloak with the linear boundary around the object; (d) the rectangular medium ($\varepsilon=2.25$) with nonlinear boundary around the object.

5. Conclusions
The simulation of carpet cloaking device in the terahertz frequency range was proposed. The effective constitutive parameters of the physical system which corresponds to wave propagation in the virtual space were calculated. The special transformation of grid was calculated by the quasi-conformal mapping using MATLAB software. Finite element method was applied for 2D electromagnetic wave simulation. The distribution of electromagnetic wave, which propagates through this cloak, was obtained for two types of lower boundaries with the different curvature (linear and nonlinear). The nonideal cloaking effect was caused by the coarse grid.

6. Acknowledgments
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7. References
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