On the lightlike Lorentz violation

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Abstract

We consider the lightlike spontaneous Lorentz invariance violation (SLIV) appearing through the zero "length-fixing" constraint put on a gauge vector field, $A^2_\mu = 0$, and discuss its physical consequences in the framework of a conventional QED and beyond. Again, as in the timelike and spacelike SLIV cases, $A^2_\mu = \pm M_A^2$ ($M_A$ is a scale of SLIV), while this constraint leads to an emergence of the Nambu-Goldstone modes collected in physical photon, the SLIV itself is still left unobservable unless gauge invariance in the theory is broken. At the same time, a crucial difference with the two former cases is that the asymmetrical vacuum corresponding to the lightlike Lorentz violation appears infinitely degenerated with all other vacua including the symmetrical one. We show that this degeneracy can be lifted out by introducing an extra gauge vector field being sterile with respect to an ordinary matter, though having some potential couplings with the basic $A_\mu$ field. A slight mixing of them makes the underlying gauge invariance to be partially broken due to which physical Lorentz invariance occurs broken as well. This may cause a variety of the Lorentz violating processes some of which are briefly discussed.
1 Introduction

In some analogy with the nonlinear \(\sigma\)-model \[1\] for pions, one may think a similar ”length-fixing” constraint put on a gauge vector field

\[ A_\mu^2 = n^2 M_A^2, \quad n^2 \equiv n_\mu n^\mu = \pm 1 \]  

(1)
could lead to a spontaneous Lorentz invariance violation (SLIV) with a proposed scale \(M_A\) and a direction determined by an unit vector \(n_\nu\). This approach has had a long history, dating back to the model of Nambu \[2\] for QED with a nonlinearly realized Lorentz symmetry for the underlying gauge vector field. Note that a correspondence with the nonlinear \(\sigma\) model for pions may appear rather suggestive in view of the fact that pions are the only presently known Nambu-Goldstone (NG) particles whose theory, chiral dynamics\[1\], is given by the nonlinearly realized chiral \(SU(2) \times SU(2)\) symmetry rather than an ordinary linear \(\sigma\) model. The constraint (1) means in essence that the vector field \(A_\mu\) develops some constant background value

\[ < A_\mu(x) > = n_\mu M_A \]  

(2)

and the Lorentz symmetry \(SO(1,3)\) breaks down to \(SO(3)\) or \(SO(1,2)\) depending on the timelike \((n^2 = +1)\) or spacelike \((n^2 = -1)\) nature of SLIV. This \(\sigma\) model type ansatz could in principle provide some dynamical approach to quantum electrodynamics with photon appearing as massless NG boson \[3\]. Generally, such an intriguing possibility first discussed in \[4\] appears to be very attractive over the last fifty years having been considered in many different contexts (for some later developments see \[5, 6, 7, 8, 9\]).

Returning to the above \(\sigma\) model type ansatz, one can see, however, that in sharp contrast to the nonlinear \(\sigma\) model for pions, the nonlinear QED theory, due to the underlying gauge invariance, ensures that all the physical Lorentz violating effects turn out to be unobservable. It was shown, at least at the tree level \[2\] and one-loop approximation \[10\], that the nonlinear constraint (1) implemented as a supplementary condition into the standard QED Lagrangian with a charged fermion field \(\psi(x)\)

\[ L_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma \partial + m)\psi - e A_\mu \bar{\psi} \gamma^\mu \psi \]  

(3)
appears in fact as a possible gauge choice for the vector field \(A_\mu\), while the \(S\)-matrix remains unaltered under such a gauge convention. In fact, this nonlinear QED contains a plethora of Lorentz and \(CPT\) violating couplings when it is expressed in terms of the emergent NG modes \((a_\mu)\) stemming from the corresponding parametrization

\[ A_\mu = a_\mu + n_\mu (M_A^2 - n^2 a^2)^{\frac{1}{2}}, \quad n_\mu a_\mu = 0 \quad (a^2 \equiv a_\mu a^\mu). \]  

(4)

being put and expanded in the Lagrangian \[3\] in powers of \(a^2/M_A^2\). However, the contributions of all these Lorentz violating couplings to physical processes completely cancel out among themselves. So, while the constraint (1) leads to an emergence of the zero NG modes collected in physical photon, the SLIV itself, is shown to be superficial as it affects only the gauge of the vector potential \(A_\mu\) at least in the the approximation mentioned above. Later
this result was also confirmed for the spontaneously broken massive QED \cite{[11]}, non-Abelian
theories \cite{[12]}, tensor field gravity \cite{[13]} and other cases. Also, it has been shown \cite{[14]} that
the only way to observe physical Lorentz violation in all the constrained vector or tensor
field theories may appear when the underlying gauge symmetry happens to be really broken
rather than merely constrained by some gauge condition. Nonetheless, in all cases Lorentz
invariance appears in fact spontaneously broken regardless of whether it is observable or not.
Indeed, this violation is normally hidden in gauge degrees of freedom of a vector field in a
gauge invariant theory. However, when these superfluous degrees are eliminated by gauge
symmetry breaking, the Lorentz violation becomes observable. In this connection, we will
use later on the terms Lorentz violation and physical Lorentz violation in order to distinguish
these two cases.

Here, we consider the lightlike Lorentz violation appearing through the zero ”length-
fixing” constraint put on a gauge vector field
\[ A^2_{\mu} = 0 \]  
and discuss its physical consequences in the framework of a conventional QED and beyond.
This case has not previously been considered in detail in the literature and, therefore, we
try, in some sense, to fill this gap. We show that again, as in the timelike and spacelike
Lorentz violation \cite{[1]}, the constraint \cite{[15]} put into the QED Lagrangian leads to the essentially
nonlinear theory containing a variety of Lorentz and CPT violating couplings in terms of
the corresponding Nambu-Goldstone modes. Nonetheless, all the Lorentz violating effects,
due to the starting QED gauge invariance, turn out to be strictly cancelled in all the lowest
order processes in the ordinary QED framework. At the same time, a crucial difference with
the former two cases is that the asymmetrical vacuum corresponding to the lightlike Lorentz
violation appears infinitely degenerated with all other vacua including the symmetrical one.
We show that this degeneracy can be lifted out by introducing an extra gauge vector field
being sterile with respect to an ordinary matter, though having some potential couplings
with the basic \( A_\mu \) field. A slight mixing of them makes the underlying gauge invariance to be
partially broken due to which physical Lorentz invariance occurs broken as well. This may
cause a variety of the Lorentz violating processes some of which are briefly discussed.

The paper is organized in the following way. In section 2 we present a general discussion
of the lightlike Lorentz violation. Then in section 3 we discuss the problem of the vacuum
degeneracy being generic for this type of violation and consider the QED model extension with
an extra gauge vector field that allows to lift out this degeneration. In section 4 we present
a conventional QED accompanied by the lightlike vector field constraint with all the basic
interactions appearing in the emergent framework. We discuss various low-order physical
processes and show that all Lorentz violating contributions in them are completely cancelled
out. In next section 5 we consider the QED model with two constrained gauge vector fields
and describe their potential mixing mechanism which may lead to physical Lorentz violation
that is then demonstrated by an example of the modified Moller scattering. And finally, in
section 6, we present our conclusion.

\footnote{Many interesting aspects of the constrained QED and other theories has also been studied in \cite{[15]}\cite{[16]}.}
2 General discussion

Let us take a closer look at the lightlike Lorentz violation provided by the constraint (5). In this case, it is clear that the vacuum expectation value (VEV) of the vector field has to be simultaneously developed on at least two components, one of which is bound to be the time one. Accordingly, we can use the following parametrization for the vector field $A_{\mu}$ with the emergent NG modes

$$A_{\mu} = a_{\mu} + \frac{n_{\mu} a^{2}}{\sqrt{2} M_{A}} - \frac{n'_{\mu} a^{2}}{\sqrt{2} M_{A}}$$

(6)

where the Lorentz violation scale is given by the VEV parameter $M_{A}$, while its direction is determined by the unit vectors $n_{\mu}$ and $n'_{\mu}$ taken in the form

$$n_{\mu} = (1, 0, 0, 0), \quad n'_{\mu} = (0, 0, 0, 1), \quad n^{2} = 1, \quad n'^{2} = -1$$

(7)

For all practical purposes, one can make an obvious simplification for what follows. Assuming, as usual, the Lorentz violation scale is rather high, we can expand the $A_{\mu}$ field parametrization (6) in the series of the $a^{2}/M_{A}^{2}$ so to have in the lowest approximation

$$A_{\mu} = a_{\mu} + \frac{n_{\mu} + n'_{\mu}}{\sqrt{2}} M_{A} - \frac{n_{\mu} - n'_{\mu}}{\sqrt{2}} \frac{a^{2}}{2M_{A}}$$

(8)

The vacuum direction is now presented by the pure ’lightlike’ vector

$$N_{\mu} = (n_{\mu} + n'_{\mu})/\sqrt{2}, \quad N_{\mu}^{2} = 0$$

(9)

being orthogonal to the emergent NG modes, as one can immediately confirm requiring the lightlike constraint (5) to be fulfilled.

According to directions determined by the unit vectors $n_{\mu}$ and $n'_{\mu}$ these NG modes are

$$a_{\mu} = a_{1}, \quad a_{2}, \quad N_{\pi a_{0}} (\pi = 0, 3)$$

(10)

satisfying the orthogonality condition

$$N_{\mu} a_{\mu} = 0$$

(11)

within the taken accuracy $O(a^{2}/M_{A}^{2})$. Meanwhile, the effective Higgs component $h$ is given by the equation

$$h = N_{\mu} A_{\mu} = -a^{2}_{\mu}/2M_{A}$$

(12)

which means that, in contrast to the timelike and spacelike SLIV, the Higgs component does not contain the constant part, while its field contained part appears rather small.

One can readily confirm that the NG mode spectrum (10) corresponds to the broken generators

$$\mathcal{J}^{[03]}, \quad (\mathcal{J}^{[01]} - \mathcal{J}^{[31]})/\sqrt{2}, \quad (\mathcal{J}^{[02]} - \mathcal{J}^{[32]})/\sqrt{2}$$

(13)
as follows from the projection of the $SO(1, 3)$ Lorentz generators

$$\left( \mathcal{J}^{[\alpha\beta]} \right)_\mu^\nu = i \left( \delta_\alpha^\nu g^{\beta\mu} - \delta_\beta^\nu g^{\alpha\mu} \right)$$

(14)
on to the vacuum direction given by the VEV vector $N_\mu = (n_\mu + n'_\mu) / \sqrt{2}$. The remained subgroup is given then by the generators

$$\mathcal{E}^{(1)} = (\mathcal{J}^{[01]} + \mathcal{J}^{[31]}) / \sqrt{2}, \quad \mathcal{E}^{(2)} = (\mathcal{J}^{[02]} + \mathcal{J}^{[32]}) / \sqrt{2}, \quad \mathcal{E}^{(3)} = \mathcal{J}^{[12]}$$

(15)

having the commutation relations

$$[\mathcal{E}^{(1)}, \mathcal{E}^{(2)}] = 0, \quad [\mathcal{E}^{(3)}, \mathcal{E}^{(a)}] = -i \epsilon^{[3ab]} \mathcal{E}^{(b)} \quad (a = 1, 2)$$

(16)

as can be directly shown. Remarkably, this algebra formally coincides with an algebra of the so-called Euclidean $E(2)$ symmetry group which is usually related to Poincare group (with $\mathcal{E}^{(1)}$ and $\mathcal{E}^{(2)}$ appearing as the translation generators) rather than Lorentz one. However, as we can see, just this symmetry subgroup now appears solely due to the lightlike SLIV

$$SO(1, 3) \to E(2)$$

(17)

To demonstrate SLIV dynamically, one normally considers the standard vector-fermion interaction which with a taken an accuracy $O(a^2 / M_A^2)$ in (8) leads to

$$A_\mu \bar{\psi} \gamma_\mu \psi = a_\mu \bar{\psi} \gamma_\mu \psi - \frac{a_\mu^2 / 2M_A}{\sqrt{2}} (\bar{\psi} \gamma_0 \psi + \bar{\psi} \gamma_3 \psi) + \frac{M_A}{\sqrt{2}} (\bar{\psi} \gamma_0 \psi - \bar{\psi} \gamma_3 \psi)$$

(18)

The last large term here is in fact fictitious and can be removed by the proper fermion field transformation

$$\psi \to \psi e^{iM_A (n+n') x / \sqrt{2}}$$

(19)

while the terms which are suppressed by a large scale $M_A$ might lead in principle to Lorentz violation. However, this only happens when gauge invariance in the theory is at least partially broken.

### 3 Asymmetrical vacuum choice

Let us note that the scale of the lightlike Lorentz violation $M_A$ introduced above in the parametrization (6) is in fact completely arbitrary. Therefore, the asymmetrical vacuum is infinitely degenerated in itself, apart from that it is degenerated with the symmetrical vacuum, $\langle A_\mu \rangle = 0$, as well. We show now that this degeneracy can be completely lifted out provided that there is some interaction coupling in the Lagrangian which includes the vector field $A_\mu$ itself rather than its square. Such an arrangement would be possible if there was some extra vector field in the theory with which the $A_\mu$ field could mix. Then, if this extra field, hereinafter referred to as $B_\mu$, develops the VEV by itself, this may determine the
VEV of the basic $A_{\mu}$ field as well, so that the above degeneracy will be appear lifted. While generally one might speculate about possible nature of the extra vector field $B_{\mu}$, we propose for uniformity that this is a massless gauge field as well, which is related to some new local $U(1)'$ symmetry in the same way as the basic $A_{\mu}$ field is related to the gauge $U(1)$ symmetry in QED. Moreover, this $B_{\mu}$ field is also proposed to be constrained by the nonlinear vector field condition
\[ B_{\mu}^2 = n^2 M_B^2, \quad n^2 = \pm 1 \] (20)
developing, however, timelike or spacelike Lorentz violating VEVs rather than the lightlike one, as the basic $A_{\mu}$ field does. Whether the $B_{\mu}$ field is connected to some (hidden) matter or it is sourceless by itself, but one way or another, it has to be largely sterile with respect to an ordinary matter in order not to significantly influence the conventional QED results. Nonetheless, a slight mixing of the $A_{\mu}$ and $B_{\mu}$ fields in their common potential will make, as we show later, the QED gauge invariance to be partially broken that leads to the lifting of vacuum degeneracy, from the one hand, and physical Lorentz violation, from the other.

In line with the above, we will assume that the vector $A_{\mu}$ and $B_{\mu}$ fields develop their VEVs according to the virtually general Lagrange-multiplier potential
\[ U(A, B) = \frac{\rho_A}{4} (A^2)^2 + \frac{\rho_B}{4} (B^2 - n^2 M_B^2)^2 + \frac{\lambda}{2} (AB - M_{AB}^2)^2 \] (21)
where $\rho_A(x)$ and $\rho_B(x)$ are introduced as the corresponding Lagrange multipliers providing the existence of the constraints (3) and (20), while $\lambda$ is an intersecting coupling constant ($\lambda > 0$); $M_B$ and $M_{AB}$ are the corresponding violation scales. This type of the potential with the quadratic Lagrange multipliers might be considered as the $\sigma$-model type limit of a usual polynomial potential containing all the possible coupling between the $A_{\mu}$ and $B_{\mu}$ fields.

One way or another, one can see that variation of the $U$ under $\rho_A$ and $\rho_B$ immediately leads to the vector field constraints causing condensation of the vector fields $A_{\mu}$ and $B_{\mu}$. This will be expressed, as before, through the vector $A_{\mu}$ parametrization (6) with the lightlike Lorentz violation, while for the vector $B_{\mu}$ we will take the parametrization corresponding to the timelike SLIV
\[ B_{\mu} = b_{\mu} + n_{\mu} \sqrt{M_B^2 - b_{\mu}^2} \simeq b_{\mu} + n_{\mu} M_B - n_{\mu} \frac{b_{\mu}^2}{2 M_B} \] (nb = b_0 = 0) (22)

*This potential can be written in the form
\[ U(A, B) = \frac{\lambda_A}{4} (A^2)^2 + \frac{\lambda_B}{4} (B^2 - n^2 M_B^2)^2 + (\lambda_{AB}' A^2 + \lambda_{AB}' B^2) AB + \frac{\lambda_{AB}}{2} (AB - M_{AB}^2)^2 \]
where the second line terms are proposed to lift the light-like vacuum degeneracy mentioned above ($\lambda_{A,B}$, $\lambda_{A,B}'$, $\lambda_{AB}$ and $M_{AB}^2$ are the corresponding (positive) coupling constants and mass parameters). One can see that in a special $\sigma$-model type limit (11) for the individual vector field couplings in the potential, $\lambda_A \rightarrow \infty$ and $\lambda_B \rightarrow \infty$, the nonlinear constraints $A^2 = 0$ and $B^2 = n^2 M_B^2$ will appear. So, without loss of generality, the above potential $U(A, B)$ could eventually include only these constraints being taken through the Lagrange multiplier terms plus the second line intersecting terms as true potential terms (the first term in this line can be included into the second one, once the constraints is implemented into the Lagrangian). In fact, the final form of this potential is presented in (21). Note that such a type of the quadratic Lagrange multiplier potential has been also discussed in the literature (15).
where \( n_\mu \) is the unit timelike Lorentz vector introduced above in (7) and \( b_\mu \) fields correspond, as usual, the emergent NG modes.

Let us note the \( B_\mu \) is in fact independent vector field gauging its own \( U(1)' \) symmetry and does not interact with the charged fermion field \( \psi(x) \) in our QED model as the \( A_\mu \) does. Without the last term in (21) each vector field carries its own Lorentz type symmetry, so that only this mixing term reduces this extended symmetry of the potential to the conventional Lorentz one

\[
SO(1,3)_A \times SO(1,3)_B \rightarrow SO(1,3)
\]  

We can see that just the last term in (21) is proposed to lift the lightlike vacuum degeneracy mentioned. Indeed, just the intersecting mass parameter \( M_{AB} \) appears to determine the scale of the lightlike violation \( M_A \) in the parametrization (9). In fact, the stable minimum of the potential \( U \) (21) is provided by the combined VEV of the vector fields \( A_\mu \) and \( B_\mu \)

\[ \langle AB \rangle = M_{AB}^2 \]  

Using then the above parametrizations for the vector fields (8, 22) containing their VEV

\[ \langle A_\mu \rangle = (n + n')_\mu M_A/\sqrt{2}, \quad \langle B_\mu \rangle = n_\mu M_B \] 

one readily finds

\[ \langle AB \rangle = (\langle A_\mu \rangle) (\langle B^\mu \rangle) = (n^2) M_A M_B/\sqrt{2} = M_{AB}^2, \quad M_A/\sqrt{2} = M_{AB}^2/M_B \]  

that, therefore, fixes the scale \( M_A \) and, as a result, degeneracy of the lightlike Lorentz violation scale appears completely lifted out.

However, for such a lifting some extra price has to be paid, as is shown below. Actually, putting the vector fields \( A_\mu \) and \( B_\mu \) expressions (8, 22) into the potential (21) one has in terms of the NG fields \( a_\mu \) and \( b_\mu \)

\[
U(a,b) = \frac{\lambda}{2} \left[ (n'b)M_A/\sqrt{2} + (na)M_B + O(a^2, b^2, ab) \right]^2
\] 

which means that some hyperbolic mixture of the \( a \) and \( b \) modes acquires the large mass

\[ b' = b_3 \cosh \theta - a_0 \sinh \theta, \quad \tanh \theta = \sqrt{2} M_B/M_A \]  

acquires the large mass

\[ m_b = \lambda (M_A^2/2 - M_B^2)^{1/2} \] 

while the conjugated combination

\[ a' = a_0 \cosh \theta - b_3 \sinh \theta \]  

is left massless.

Let us note that the vacuum direction is not given more by the vector \( N_\mu \) (9), as in the one vector field case. Rather, it is given by a similar mixture of the vectors \( N_\mu \) and \( n_\mu \) (7)

\[ N^0_\mu = N_\mu \cosh \theta - n_\mu \sinh \theta \] 

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which naturally goes to the old vector $N_\mu$ once the vector field mixing disappears. The vector $N_\mu^\theta$ properly acts on the combined NG spectrum involved, as can be directly shown. Indeed, now one has in total the five massless NG modes, whose collection can be written (using the constant vectors $N_\mu = (1, 0, 0, 1)$ and $n'_\mu = (0, 0, 0, 1)$ and properly enlarging the low-case indices) in the form

$$G_m = a_1, a_2, (N_\mu \cosh \theta - n'_\mu \sinh \theta)a', b_1', b_2' \quad (m = 1, 2, \bar{\mu}, 1', 2'; \; \bar{\mu} = 0, 3) \quad (32)$$

So, one can readily confirm that the "orthogonality" condition for them is well satisfied

$$N^\theta_m G_m = 0 \quad (33)$$

The NG modes $G_m$ correspond to the broken generators

$$\mathcal{J}^{[31]}, \mathcal{J}^{[32]}, \mathcal{J}^{[03]}, \mathcal{J}^{[01]}, \mathcal{J}^{[02]} \quad (34)$$

respectively. Thus, only the generator $\mathcal{J}^{[12]}$ is conserved that means the starting Lorentz $SO(1, 3)$ symmetry is finally broken to the plane rotation symmetry $SO(2)$ rather than the $E(2)$ symmetry as in the one-vector field case.

The mass mixing in the zero mode potential $\mathcal{V}$ leads to the significant modification of the $a$ and $b$ mode interactions that may lead in general to physical Lorentz violation being considered in section 5. For an extreme mass scale hierarchy $M_A >> M_B$ this mixing appears very small, thus leading to an approximate separation of the $A$ and $B$ field sectors with their own Lorentz symmetries $\mathcal{V}$. In a decoupling limit, the vacuum vector $\mathcal{V}$ approaches to the old form, $N_\mu^\theta \to N_\mu$ and our QED model contains only massless $a$-modes $\mathcal{V}$ corresponding to the pure lightlike SLIV with the remained $E(2)_a$ symmetry appearing from violation of the Lorentz group $SO(1, 3)_A$ in $\mathcal{V}$. The massive $b_3$ and massless $b_{1,2}$ modes are practically sterile in the model possessing the remnant symmetry $SO(3)_b$ originated from the breaking of the second Lorentz group $SO(1, 3)_B$ in $\mathcal{V}$. Actually, this remnant $SO(3)_b$ is in fact explicitly broken to the $SO(2)_b$ symmetry due to the mass splitting in the $b$ mode triplet mentioned above.

4 QED with lightlike vector field constraint

We start with the constrained QED model with one vector field. However, since (as was discussed above) the lightlike SLIV can not be consistently formulated in the one-vector field framework, we then consider in next section its extension including two vector fields.

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3 Noticeably, in contrast to the equality of the $a$ mode components, $a_3 = a_0$, stemming from the orthogonality condition $\mathcal{V}$ in the one-vector case, now one has instead $a_3 = (\cos \theta - \sin \theta)a'$, while $a_0 = a' \cos \theta + b' \sin \theta$. This follows from comparison of the new mode components $\mathcal{G}_0$ and $\mathcal{G}_3$ in $\mathcal{V}$ and the fact that, unlike the $a_0$, the starting mode $a_3$ does not mix ($a_3 = \mathcal{G}_3$). Thus, the mode $a_3$ is left a true zero mode, whereas the $a_0$ mode goes to the superposition with a heavy state $b'$. 

7
4.1 Basic interactions

Putting the $A_{\mu}$ parametrization into the conventional QED Lagrangian and remaining only lowest order interaction terms we come to the emergent QED model for the zero vector $a_{\mu}$ modes

$$L_{EQED} = L_0 + \frac{1}{\sqrt{2}M_A}L_1 + a_{\mu}J^\mu + L_m$$

(35)

where

$$L_0 = -\frac{1}{4}f_{\mu\nu}^2 + \frac{[a_{\mu}(n + n')^\mu]^2}{2\alpha},$$

$$L_1 = -\frac{a_{\mu}^2}{2}(n_{\nu} - n_{\nu}') (\partial^\mu f_{\mu\nu} + J_{\nu})$$

(36)

and $f_{\mu\nu}$ stands for the conventional stress-tensor of the $a_{\mu}$ field identified with physical photon. Here, we also included the vector field source current $J_\mu$ determined by some matter field(s) whose kinetic terms are presented in the Lagrangian part $L_m$. For certainty, one can assume this matter field to be some charged fermion. As to the vector NG modes $a_{\mu}$, they are presented in the Lagrangians $L_0$ and $L_1$ in the above including the corresponding axial gauge fixing term (gauge appearing the limit $\alpha \rightarrow 0$). Their propagator is in fact determined by the lightlike vacuum vector, $(n + n')_{\mu}$, thus leading to its conventional form

$$D_{\mu\nu}(a) = \frac{-i}{k^2} \left( g_{\mu\nu} - \frac{(n_{\mu} + n'_{\mu}) k_{\nu} + (n_{\nu} + n'_{\nu}) k_{\mu}}{(n + n')k} \right)$$

(37)

Note that the conventional $k_{\mu}k_{\nu}$ part in the bracket, usually included in a propagator in axial gauge, is now absent because this part is proportional to $(n_{\mu} + n'_{\mu})^2$, which is zero. One can immediately confirm that the propagator $D_{\mu\nu}$ satisfies the both orthogonality conditions

$$(n_{\mu} + n'_{\mu}) D_{\mu\nu} = 0, \quad k_{\mu}D_{\mu\nu} = 0$$

(38)

though the latter is only satisfied on the photon mass shell ($k^2 = 0$).

Actually, the whole picture with the lightlike SLIV is very similar to that of the spacelike and timelike ones, where physical Lorentz invariance violation does not occur. We will confirm the same for the lightlike SLIV as well unless the QED model includes some gauge invariance violating terms. Such a possibility may appear, for example, if model contains the second vector field, as will be discussed in next section.

4.2 Physical processes

We start with the SLIV processes which occur in the $1/M_A$ order and, as some simple example, consider a possible modification to the conventional Compton effect which turns out to leave Lorentz invariance unbroken. Then we briefly discuss some other processes appearing in higher orders and argue that there is no physical Lorentz violation either.

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Note that, though the VEV scale $M_A$ of the vector field $A_{\mu}$ may be arbitrary in the one-vector field model we will, however, consider $M_A$ as a precisely fixed scale in a view of lifting this degeneracy in the two-vector field model extension discussed in the previous section.
4.2.1 Compton scattering

For modified Compton scattering we have two diagrams in the $1/M_A$ order. The first one is related to the longitudinal photon exchange between the conventional matter current $J_\mu$ (the term $a_\mu J_\mu$ in the total Lagrangian $L_{\text{tot}}$ (35) in the above) and the Lorentz violating three-photon vertex $\Gamma_{\mu\nu\rho}$ (determined by the new Lagrangian part $L_1$), while the second one is the contact diagram (following again from the $L_1$). So, matrix element for the first diagram reads as

$$M_1 = \xi_\mu(q)\xi_\nu(p)\Gamma_{\mu\nu\rho}(q,p,k)D^{\rho\lambda}(k)J_\lambda$$

where $\xi_\mu(q)$ and $\xi_\nu(p)$ are polarization vectors for the ingoing and outgoing photons, respectively. So, plugging the expressions for the propagator $D_{\rho\lambda}$ and the three-photon vertex $\Gamma_{\mu\nu\rho}$, and using the matter current conservation we readily come

$$M_1 = \frac{\xi_\mu(q)\xi_\nu(p)}{\sqrt{2}M_A}(n - n')_\lambda J_\lambda$$

which appears to be exactly opposite to the second (contact) diagram matrix element

$$M_2 = -\frac{\xi_\mu(q)\xi_\nu(p)}{\sqrt{2}M_A}(n - n')_\lambda J_\lambda$$

Therefore, one has a total cancellation for physical Lorentz violation in the modified Compton scattering

$$M_2 + M_1 = 0$$

4.2.2 Other processes

Many other tree level Lorentz violating processes related to the emergent $a_\mu$ modes (interacting with each other and the matter fields in the theory) appear in higher orders in the basic SLIV parameter $1/M_A$, by iteration of couplings presented in our basic Lagrangian (35), or from a further expansion of the effective vector field (6) inserted into the starting total Lagrangian (3). Again, as in the Compton scattering considered above, their amplitudes are essentially determined by an interrelation between the longitudinal photon exchange diagrams and the corresponding contact interaction diagrams, which appear to cancel each other, thus eliminating physical Lorentz violation in the theory.

Most likely, the same conclusion could be expected for SLIV loop contributions as well. Actually, as in the massless QED case considered earlier [10], the corresponding one-loop matrix elements in our lightlike QED theory may either vanish by themselves or amount to the differences between pairs of similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of the external four-momenta of the particles involved) that, in the framework of dimensional regularization, could lead to their total cancellation.

So, physical Lorentz invariance in QED with the lightlike vector field constraint is eventually maintained provided that the underlying gauge symmetry in the theory remains unbroken.
5 QED with two constrained vector fields

5.1 Towards physical Lorentz violation

As was discussed earlier in section 3, in the two-vector field model where the vacuum degeneracy is lifted, the emergent photon is influenced by the degree of mixing with the second vector field. This is depended on the hierarchy between the VEVs $M_A$ and $M_B$ of the starting vector fields $A_\mu$ and $B_\mu$ containing the zero modes $a_\mu$ and $b_\mu$, respectively. As follows, the bigger their mixing parameter (28)

$$\delta = \tanh \theta = M_B / \sqrt{2} M_A$$

(42)

is the stronger Lorentz violating effects appear for the $a_\mu$ modes associated with physical photon. So, for a realistic scenario we have to propose $\delta << 1$. Remarkably, while this mixing is caused by the gauge noninvariant intersecting term in the potential (21, 27), the size of the mixing itself does not depend on the corresponding coupling constant $\lambda$. This constant seems to be naturally small since its vanishing increases symmetry of the potential, as one can see from (23). However, even for a very small constant $\lambda$ this mixing could be the same, and also the massive $b'$ mode (28) might be still heavy due to the proposed large $A_\mu$ field VEV $M_A$.

In the small $\delta$ approximation and with an accuracy $O(a^2/M_A^2, b^2/M_B^2)$ taken everywhere, one can readily find that the one-vector field Lagrangian (35) is only disturbed by terms of the type

$$\Delta L(a) = -\frac{\delta^2}{2} m_b^2 (na)^2 - \delta m_b^2 (na)(n'b)$$

(43)

appearing as a result of mixing of the starting vector $A$ and $B$ fields in the two field Lagrangian $L(A, B)$. This Lagrangian, apart from their potential terms in (21), contains the $B$ field kinetic terms as well that for the emergent NG modes $b_\mu$ looks as

$$L(b) = -\frac{1}{4} b^2_{\mu\nu} + \frac{(b_\mu n^\mu)^2}{2 \beta} - \frac{b^2_A}{2 M_B} n_\mu \partial^\mu b_\nu - \frac{1}{2} m_b^2 (n'b)^2$$

(44)

where we also included the corresponding axial gauge fixing term (gauge appearing in the limit $\beta \to 0$) and the lowest order interaction term. So, we have now both zero modes $a_\mu$ and $b_\mu$ in the combined two-vector field theory. In terms of them, the starting $A - B$ mixing provides, as on can see, the large mass to the $(n'b)$ component of the $b_\mu$ field, while the small one to the $(na)$ component of the $a_\mu$ field. Also, there is still left some mixing term of these components that is presented in (43). The point is, however, that due to the gauge invariant $F^2_{\mu\nu}$ structure of the $a$ and $b$ mode kinetic terms one can not diagonalize these modes simultaneously both in their mass and kinetic sectors. For this reason, we remain them unrotated in the Lagrangians (43, 44) treating their mixing term as some small perturbation provided that the above $\delta$ parameter or VEV ratio $M_B/M_A$ is really small. Thus, we will calculate their physical masses through the oscillations provided by this mixing term. One can readily see that these oscillations practically will not change the large mass of the $(n'b)$ component in (44), while may significantly disturb the small mass term of the $(na)$ component in (43).
As matter of fact, we basically have the same emergent QED picture as in the one field case which is only disturbed by the small mixing of the a modes (associated with photons) with the b modes being essentially separated from physical matter. Nonetheless, this mixing causes, as we see below, physical Lorentz violation through the a mode propagation. Meanwhile, the b modes containing the massive \((n'b)\) component in the Lagrangian (44) have the total propagator of the form

\[
    ik^2 D_{\mu \nu}^m(b) = g_{\mu \nu} - \frac{k^2 [(k_\mu n_\nu + k_\nu n_\mu)(nk) - n^2 k_\mu k_\nu]}{k^2 - m_b^2 [1 - (n'k/nk)^2]} \frac{1}{(nk)^2}
\]

(45)

where the kinematical tensor \(P_{\mu \nu}\) given by

\[
    P_{\mu \nu} = k_\mu k_\nu + [n'_\mu(nk) - n_\mu(n'k)][n'_\nu(nk) - n_\nu(n'k)] + k_\mu [n'_\nu(n'k) - n_\nu(nk)] + k_\nu [n'_\mu(n'k) - n_\mu(nk)]
\]

(46)

may cause the physical Lorentz violation depending on the mass \(m_b\) involved. One can readily see that in the massless limit this propagator goes to the standard vector boson propagator \(D_{\mu \nu}(b)\) taken in the axial gauge \((nb = 0)\). However, even in the above extended form it still satisfies the conventional condition \(n^\mu D_{\mu \nu}^m(b) = 0\) and also the conditions \(n'^\mu D_{\mu \nu}^m(b) = 0\) and \(k'^\nu D_{\mu \nu}^m(b) = 0\) when being taken on the \(k^2 = 0\) shell. Indeed, the same conditions are also satisfied by the polarization vector \(\xi^\mu_k(k)\) of the \(b_\mu\) field.

The total a mode propagator \(D_{\mu \nu}^{tot}(a)\), in turn, contains two type of contributions stemming from the mass term and mixing term in \(\Delta L(a)\) in (43), respectively. The first lead exactly to the same type of propagator \(D_{\mu \nu}^m(a)\) as is the \(D_{\mu \nu}^m(b)\) in (45), though with a proper replacement of the unit vector \(n_\mu\) by \(n_\mu + n'_\mu\) and mass \(m_b^2\) by \(\delta^2 m_b^2\). The second causes the oscillation of the \(na\) mode into the \(n'b\) mode and back whose sum over an infinite number of such oscillations

\[
    D_{\mu \nu}^{osc}(a) = [n^3 D_{\lambda \mu}(a)] \left[ \delta^2 m_b^4 D^{n'a}(b) \right] \left[ n'^\alpha D_{\mu \nu}(a) \right] \sum_{r=0}^\infty \left[ D(a) \delta^2 m_b^4 D^{n'b}(b) \right]^r
\]

(47)

(where \(D(a)\) and \(D^{n'b}(b)\) stand for \(n'^\alpha n'^\beta D_{\alpha \beta}(a)\) and \(n''\alpha n''\beta D_{\alpha \beta}(b)\), respectively) can then be expressed in terms of a geometrical progression, while its tensorial structure will be determined by the end vectors \(n'^3 D_{\lambda \mu}(a)\) and \(n''D_{\mu \nu}(a)\). Once all these simple, though lengthy, calculations are carried out using the propagators (47) and (45), the joint contribution, \(D_{\mu \nu}^{tot}(a) + D_{\mu \nu}^{osc}(a)\), leads eventually to the total a field propagator

\[
    ik^2 D_{\mu \nu}^{tot}(a) = ik^2 D_{\mu \nu}(a) - \delta^2 \frac{m_b^2}{k^2 - m_b^2 [1 - (n'k/nk)^2]} \frac{P_\mu P_\nu}{(nk + n'k)^2}
\]

(48)

which contains the one-field propagator part (57) and we are accustomed to plus small Lorentz violating addition with the kinematical vector \(P_\mu\)

\[
    P_\mu = k_\mu + n'_\mu(nk) - n_\mu(n'k)
\]

(49)

One can readily confirm the total propagator \(D_{\mu \nu}^{tot}(a)\) satisfies the same conditions (48) as its one-vector field part \(D_{\mu \nu}\). Remarkably, this propagator contains the second pole related
to the massive mode \((n'b)\). In its massless limit one comes to the standard \(a\) and \(b\) field propagators \(D_{\mu\nu}(a)\) and \(D_{\mu\nu}(b)\) taken in axial gauge, and Lorentz invariance is recovered. On the contrary, for the heavy \((n'b)\) mode, when \(k^2 << m_b^2\), as it may actually appear for known physical processes with the longitudinal \(a\) mode exchange, the Lorentz violating effects in the leading order does not really depend on the mass \(m_b\), but rather only on the mixing \(\delta\) parameter. The main thing, however, is that the total propagator \(D_{\mu\nu}^{\text{tot}}(a)\), due to the two-vector mixing part in (48), leads to the nonzero Lorentz breaking result when it is sandwiched between the conserved currents. A real driving force standing behind this physical Lorentz violation is indeed a small mixing of the \(a\) and \(b\) modes.

5.2 Phenomenological aspects

5.2.1 A Møller scattering primer

In this connection, as some demonstrative example may be considered the Møller electron-electron scattering whose conventional amplitude being in the lowest order determined by the longitudinal photon exchange is well known. Thus, one can use this example to take into account the effects of the modified propagator (48) for photon field. Normally, one could consider this scattering in the center of mass system with electrons have the equal initial energies \(E\) and oppositely directed 3-momenta \(-\rightarrow p\) and \(-\rightarrow q\). However, in this case, since Lorentz breaking correction in the propagator is proportional to the transferred energy, one will have zero effect (more precisely, it happens when there appears \(kn = 0\) but \(kn' \neq 0\) in the propagator). Instead, we could take the case when the interacting electrons have different energies but their 3-momenta are directed to each other. So, for initial and final momentum we would have the following configuration

\[
\begin{align*}
p_\mu &= (E_p, p \cdot \overrightarrow{l}), & q_\mu &= (E_q, -q \cdot \overrightarrow{l}) \\
p'_\mu &= (E'_p, p' \cdot \overrightarrow{l'}), & q'_\mu &= (E'_q, q' \cdot \overrightarrow{l'})
\end{align*}
\]

where \(\overrightarrow{l}\) and \(-\rightarrow l\) is direction of the initial 3-momentum and \(\overrightarrow{l_p}\) and \(\overrightarrow{l_q}\) denotes direction for the final 3-momentum. For high energies one can neglect the electron mass in the leading order, thus having for the final energy \(E'_p\)

\[
E'_p = \frac{2E_pE_q}{E_p(1 - l \cdot l_p) + E_q(1 + l \cdot l_p)}
\]

With the chosen kinematics \([50\,51]\) the total differential cross section for an elastic \(e - e\) scattering can be presented as

\[
\frac{d\sigma^{\text{tot}}}{d\Omega} = \frac{d\sigma}{d\Omega} + \delta^2 \frac{d\sigma'}{d\Omega}
\]

which contains the Lorentz violating part \(d\sigma'/d\Omega\) as well. An exact formula for this term is generally too long to be displayed, but for the case when initial electron energies are close to each other, \(E_p - E_q \ll E_p + E_q\), it is significantly simplified

\[
\frac{d\sigma'}{d\Omega} = -4\alpha^2 \left[ (l \cdot l_p)^2 + 5 \right] \frac{[l \cdot l'(n')^2] + 4 (l \cdot l_p)(l \cdot l')(l \cdot n')(l \cdot n') (E_p - E_q)^2}{(E_p + E_q)^2 [((l + l_p) \cdot n')^2][(l + l_p) \cdot n']^2}
\]
Note that its high energy behavior is given by $1/E^4$, while the standard cross section part, $d\sigma/d\Omega$, behaves as $1/E^2$. The further simplification in (53) is related to a choice of a scattering plane which is still arbitrary. For example, when the starting 3-momentum $\vec{p}$ is taken perpendicular to the unit vector $n'$, which means $l \cdot n' = 0$, this cross section becomes even simpler

$$
\frac{d\sigma'}{d\Omega} = -4\alpha^2 \frac{(l \cdot b)^2 + 5 (E_p - E_q)^2}{(E_p + E_q)^2 (l \cdot n')^2 (E_p + E_q)^2} \tag{54}
$$

Many other interesting kinematical configurations are also possible to be set at an experiment.

A comparison of the Lorentz violating cross section parts taken for different kinematics may give a clue for an actual physical Lorentz violation in Möller scattering. One could also try to derive some limit on the underlying parameter $\delta$ (or ratio of the VEVs of the starting $A$ and $B$ fields) using an accuracy in the experimental determination of the above cross sections. Particularly, for uncertainty of the $10^{-10}$ order in it one could have $\delta \sim 10^{-5}$ that would mean that if the VEV mass scale $M_A$ is taken to be of the order of the grand unified scale, $10^{15}$ GeV, then the mass $M_B$ has to be about $10^{10}$ GeV, that would be quite admissible from any point of view.

### 5.2.2 Sterile Nambu-Goldstone modes

Basically, in contrast to $a_\mu$ field associated with photon, the massless $b_\mu$ field modes stemming from the second vector field $B_\mu$ are staying sterile in the theory since they have not directly coupled with the matter fermion current $J_\mu$ involved. Nevertheless, one might think that they could interact with the matter through the $b$ mode oscillation into the $a$ mode (as was discussed in section 5.1), thus creating some effective vertex $b_\mu J^\mu$. The point is, however, that such a vertex with the real $b_\mu$ field turns to zero because of the mass shell conditions for its propagator $D^m_{\mu\nu}(b)$ and polarization vector $\xi^\mu(k)$ mentioned above. At the same time, while the single $b$ modes do not interact with a matter, their pair production is quite possible through both potential and kinetic sectors of the theory. However, such processes, say, the $b$ pair production in electron-positron collisions appears extremely suppressed at laboratory energies being much smaller than the Lorentz violation scale $M_A$. Particularly, the ratio of the cross section $\sigma(\text{ee} \rightarrow \text{bb})$ to the conventional one, $\sigma(\text{ee} \rightarrow \text{µµ})$, appears to be given approximately by $E^2/M_A^2$ that, say, for energy $E = 1$ TeV and scale $M_A$ taken again around $10^{15}$ GeV is only $10^{-24}$. So, the zero $b$ modes are pretty sterile and the emergent QED is not practically influenced by an existence of the second vector field $B_\mu$ in the theory. The only real effect of this field is eventually manifested through the Lorentz violating corrections which appear in the photon propagator.

This and some other issues, where physical Lorentz violation could be manifested, will be addressed in more details later on.

### 6 Conclusion

We considered the lightlike Lorentz violation appearing through the zero ”length-fixing” constraint put on a gauge vector field, $A^2_\mu = 0$, and discuss its physical consequences in
the framework of a conventional QED and beyond. Again, as in the timelike and spacelike Lorentz violation, \( A_\mu^2 = \pm M^2 \), while putting this constraint into the Lagrangian leads to an emergence of the zero Nambu-Goldstone modes collected in physical photon, the SLIV itself, is shown to be superficial as it affects only the gauge of the vector field \( A_\mu \). Actually, this field being expressed in terms of the zero NG modes involved leads to the essentially nonlinear theory containing a variety of Lorentz and \( CPT \) violating couplings. Nonetheless, all the Lorentz violating effects, due to the underlying gauge invariance, turn out to be strictly cancelled in the lowest order processes, as was demonstrated for some processes in section 4.

At the same time, the lightlike spontaneous Lorentz violation case is certainly different from spacelike and timelike SLIV cases. Particularly, in contrast to the former two cases the asymmetrical vacuum corresponding to the lightlike Lorentz violation, \( SO \rightarrow E(2) \), appears to be infinitely degenerated with all other vacua including the symmetrical one. We show that this degeneracy can be completely lifted by introducing some extra gauge vector field in the model. As a result, while the timelike and spacelike Lorentz violation effects are left hidden within gauge degrees of freedom of a photon, the lightlike one with extra vector field leads to physical Lorentz violation. This is related to the fact that such an extension makes gauge invariance broken in the model due to which physical Lorentz invariance occurs broken as well that was demonstrated by an example of the modified Moller scattering in the previous section.

The two-vector field mixing mechanism for physical Lorentz violation proposed here can be also applied to any other type of SLIV, particularly, to the timelike or spacelike Lorentz violation cases. Some special issue being crucial for a present consideration is that the second constrained vector field \( B_\mu \) related to its own gauge symmetry \( U(1)' \) is proposed. Whether the massless \( B_\mu \) field is connected to some (hidden) matter or it is sourceless by itself, but one way or another, it has to be largely sterile with respect to an ordinary matter in order not to significantly influence the conventional QED results. Meanwhile, the corresponding NG modes could interact with the matter through the \( b \) mode oscillation into the \( a \) modes. Although the single \( b \) modes do not interact with a matter, their pair production turns out quite possible. Whereas such processes, say, the massless \( b \) pair production in electron-positron collisions appears extremely suppressed at laboratory energies, they might be important in the early universe.

Another and more conventional scenario could be to make these \( b \) mode rather heavy instead that would seem to be more appropriate from the phenomenological point of view. Remarkably, this might be reached if the extra \( U(1)' \) were spontaneously broken so that the constrained \( B_\mu \) field could acquire the heavy mass, just as was shown in [11], though in some other context. A very attractive accommodation for this two-vector mixing mechanism would be then the Standard Model extension possessing the starting symmetry \( SU(2) \times U(1) \times U(1)' \) where the present model could be included.

These and related questions are planned to be considered in detail elsewhere.
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