New approach for the identification of anisotropy material parameters using hydraulic bulge tests

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Abstract. This paper put forwards a new method for the identification of the flow curves of anisotropic sheet metals using hydraulic bulge tests throughout circular and elliptical dies. This method is based on the membrane equilibrium equation, numerical simulation to derive the pole thickness and experimental data involving the measurement of only the polar deflection versus the applied hydraulic pressure curves. The experimental curves of four hydraulic bulge tests were obtained: one bulge test through circular die, and three bulge tests through elliptical dies, where the major axe of the elliptical die was oriented in three directions with respect the rolling direction of the sheet metal. These results were used in the identification of flow parameters and anisotropy coefficients of Hill48’ yield criterion. Then, FEA was utilized to perform sensitivity analysis of material parameters on the different hydraulic bulge test responses. The case study was a sheet metal of low carbon steel DC04. The identified material parameters were used in the numerical simulation of hydraulic bulge tests for the accuracy verification of the proposed method. It was shown a good agreement between model’s prediction and experimental results.

1. Introduction
The hydraulic bulge test is biaxial tensile test, which can be used as a good complementary or as an alternative to the simple tensile test for material behaviour characterization. Indeed, the homogeneous plastic deformation in simple tension is limited by the appearance of necking which manifests relatively "early" compared to that observed in the hydraulic bulge test. Extrapolation of the hardening curve obtained from simple tensile test for large deformations does not correctly predict the formability and springback in the sheet metal forming process. Therefore, the hydraulic bulge test has attracted the attention of many researchers to be used for the determination of stress-strain curves of sheet metals with the isotropic material assumption [1, 2]. The analysis of the hydraulic bulge test through circular and elliptical dies was reported in some published works in the literature [3, 4]. Several methods were reported on the determination of flow curves using hydraulic bulge tests based on analytical formulation to express the polar thickness and the curvature radius at the pole bulge. A simple model was proposed by Hill [5] to calculate the polar thickness; later improved by Chakrabarty [6] by introducing the hardening effects. Kruglov proposed a simple formula to compute the polar strain [7]. Later, Lăzărescu developed a modified version of Kruglov’s model by considering non-uniform strains on the dome surface [2]. Panknin proposed a formula for the determination of polar radius considering the die fillet...
radius [8]. Some works reported on the application of digital image correlation (DIC) to measure the polar radii [9, 10] and to detect necking and fracture in bulge tests [11]. However, few researches were dedicated for the parameter identification of the anisotropic behaviour of sheet metals using simultaneously hydraulic bulge tests through circular and elliptical dies.

The main goal of this paper is to present a new methodology for an accurate determination of the biaxial stress-strain curve and anisotropy coefficients using hydraulic bulge tests through circular and elliptical die cavity. This methodology combines an analytical model with numerical simulation of the bulge tests to determine the polar thickness as a function of bulge height instead of using approximated formulas. To capture the anisotropy effect, the hydraulic bulge tests through elliptical die are carried out by turning the major axis of the ellipse with a given angle relative to the rolling direction (RD). Numerical simulation of hydraulic bulge tests was compared to experimental results to validate the proposed identification methodology.

2. Description of the analytical model
The hydraulic bulge test consists of applying a pressure using fluid to deform a tightly clamped blank at its extremities through die cavity. Figure 1 shows the schematic drawing used for stress analysis at the pole of the bulging zone for an hydraulic bulge test through elliptical die cavity.

![Figure 1. Schematic drawing for stress analysis: a) geometry of hydraulic bulge test through an elliptical die cavity; b) equilibrium of infinitesimal element at the pole of the bulged zone.](image)

The analysis of the hydraulic bulge test is commonly based on the equilibrium of infinitesimal element at the pole of the bulge (see, Figure 1 b). It is assumed that the great axis of the elliptical die is along the rolling direction of the metal sheet. The plastic strains along the three principal axes were determined using the blank thickness at the pole of the bulged area.

\[
\varepsilon_\theta = \ln \frac{h}{t_0}; \quad \varepsilon_\phi = -\frac{\beta}{1+\beta} \varepsilon_\theta; \quad \varepsilon_\rho = -\frac{1}{1+\beta} \varepsilon_\theta; \quad \beta = \frac{\varepsilon_\theta}{\varepsilon_\phi}
\]

where \(t, t_0\) define the instant and the initial pole thickness. \(\beta\) represents the plastic strain ratio. The biaxial stresses \(\sigma_\rho\) and \(\sigma_\phi\) along \(\phi\) and \(\theta\) directions define the stress components along the two directions of the major and the minor axes of the elliptical die cavity, respectively. They are given as follows.
\[
\sigma_\theta = \frac{P}{t} \left( \frac{1}{\rho_\theta} + \frac{\alpha}{\rho_\phi} \right)^{-1}; \quad \sigma_\phi = \frac{\alpha P}{t} \left( \frac{1}{\rho_\phi} + \frac{\alpha}{\rho_\theta} \right)^{-1}; \quad \alpha = \frac{\sigma_\phi}{\sigma_\theta}
\]  

(2)

where \( P \) is the applied pressure, \( \rho_\theta, \rho_\phi \) define the curvature radii along the major and minor axes, respectively. \( \alpha \) represents the stress ratio. The curvature radii along the major axis \( a \) and minor axis \( b \) of the elliptical die were obtained by the Panknin expressions [8], as given in equation (3). In these relations and for better accuracy, the fillet radius of the die was taken into account. Koç has demonstrated that using the Panknin relations, the measured polar radii were closer to the ones obtained by stepwise or digital image correlation technique [10]. Therefore, Panknin relations are considered in this work to accurately evaluate the polar radii.

\[
\rho_\phi = \frac{(a + r_f)^2 + h^2 - 2r_f h}{2h}; \quad \rho_\theta = \frac{(b + r_f)^2 + h^2 - 2r_f h}{2h}
\]  

(3)

To compute the polar thickness as a function of the bulge height, Hill [5], Chakrabarty [6] and Kruglov [7] have proposed analytical relationships. However, different prediction was obtained for the pole thickness. This latter has a significant effect on the accuracy of the determination of the biaxial stress-strain curves. Therefore, the numerical simulation of polar thickness vs. bulge height was carried out and compared to those obtained using analytical formulas. A noticeable difference was revealed. Therefore, the cornerstone on which is based this identification methodology is that the polar thickness was obtained using numerical simulation of the hydraulic bulge test using circular die cavity. The equivalent stress-strain curves were calculated using Hill48 yield criterion for the different hydraulic bulge tests. Equations (4) and (5) express only the equivalent stress and strain obtained for the hydraulic bulge test through the circular die cavity. For each hydraulic bulge test, the equivalent stress-strain curve was determined and not presented here. Figure 2 displays the flow-chart of the computation of the equivalent stress-strain curve using the experimental pressure vs. bulge height curve, as a continuous measurement data.

\[
\frac{1}{\sigma} = \left\{ \frac{3}{2} \frac{r_\phi \sigma_\phi^2 + r_\theta \sigma_\theta^2 + r_\theta r_\phi (\sigma_\theta - \sigma_\phi)^2}{r_\phi r_\theta (1 + \frac{1}{r_\phi} + \frac{1}{r_\theta})} \right\}^{\frac{1}{2}}
\]  

(4)

\[
\varepsilon = \left\{ \frac{2}{3} \left( 1 + \frac{r_\theta + r_\phi}{r_\theta r_\phi} \right) \left( \frac{1}{r_\theta} \frac{1}{r_\theta} \frac{1}{r_\phi} \frac{1}{r_\phi} \frac{1}{r_\phi} \left( \frac{1}{r_\phi} - \frac{1}{r_\phi} \right)^2 + \frac{1}{r_\phi} \left( \frac{1}{r_\phi} - \frac{1}{r_\phi} \right)^2 + \left( \frac{1}{r_\phi} - \frac{1}{r_\phi} \right)^2 \right) \right\}^{\frac{1}{2}}
\]  

(5)

Figure 2. Flow-chart for the computation algorithm of the equivalent stress-strain curves using hydraulic bulge test throughout circular and elliptical die cavity.
The curvature radii were calculated using Panknin formula and the polar thickness was obtained using numerical simulation of the hydraulic bulge test. As it will be shown later, it was demonstrated that the polar thickness as a function of bulge height, was insensitive to anisotropy coefficients and to the die geometry. Therefore, the simple tensile test result along rolling direction was used as input data to run the numerical simulation of the hydraulic bulge thought circular die to derive the polar thickness.

3. Experiments

Experimental hydraulic bulge tests were carried out using a simple stand-alone apparatus which has been manufactured for this purpose. Circular and elliptical die cavities were used to apply throughout them a fluid pressure in order to bulge sheet metal blanks. These latter were firmly constrained at their borders using draw bead implemented at the flange contour of the blanks to ensure their tightness and insure a pure stretching. DC04 metal sheet blank of 1 mm thickness was used as an application for the proposed parameter identification methodology. Figure 3 shows the four samples deformed until blanks burst and they are labelled Test-0, test-1, Test-2 and Test-3. The experimental burst pressure measured for the bulged sample through the circular die cavity was around of 12.4 MPa and the fracture pressure for those bulged using elliptical die cavity was around an average value of 15.1 MPa. The diameter of the circular die cavity is \(d_c = 91\) mm, and the die fillet radius is \(r_f = 6\) mm. The samples 1, 2 and 3 correspond to the hydraulic bulge tests performed using elliptical die cavity for which the length of the major axis \(a =110\) mm, the length of the minor axis \(b = 74\) mm and the die fillet radius is \(r_f = 6\) mm. In addition, uniaxial tensile tests according to ASTM-E8 standards were performed to obtain the uniaxial stress-strain curves and the Lankford coefficients \(r_{0\circ}, r_{45\circ}, \) and \(r_{90\circ}\) along three directions: 0°, 45° and 90° with respect to the rolling direction (RD). Figure 4 displays the experimental bulge pressure vs. bulge height at the pole obtained for the four samples. The measuring device was composed of pressure gage to monitor the applied pressure and a displacement transducer probe was placed at the centre of the blank to measure the bulge height during the bulging process. It is worth noting that for the test repeatability, these curves represent the average results of three hydraulic bulge replication tests performed for each test case. Only curve plots for a bulge height higher than almost 5 mm were shown.

![Figure 3](image)

**Figure 3.** Four hydraulic bulge samples deformed until fracture: the sample labelled Test-0 was bulged using circular die cavity and the samples labelled Test-1, Test-2 and Test-3 were hydroformed using elliptical die.

![Figure 4](image)

**Figure 4.** Experimental hydraulic bulge pressure vs. bulge height curves obtained at the pole of the four bulge tests, using circular and elliptical die cavity.
4. Numerical models

Numerical simulation of hydraulic bulge tests was performed using ABAQUS® Standard commercial software. The blank was discretised using shell element (S4R) and the die was modelled as analytical rigid body. The draw bead was modelled by a firm clamp at the flange of the blank. It is worth to note that three-dimensional (3D) finite element model was also built using eight note solid elements with reduced integration (C3D8R) and 2 elements were stacked across the blank thickness. Equal simulation results were obtained, but with very short computation time for the 2D finite element model compared to 3D model. Thus, the model using shell elements was definitely considered to run numerical simulation of the hydraulic bulge tests. Taking account of the material, geometry and boundary condition symmetries, one quarter of the FE model was adopted to run numerical simulation for the bulge tests: Test-0, Test-1 and Test-2. The total number of elements was 462. However, for the Test-3, which represents the bulge test with elliptical die cavity in the 45° direction, the whole finite element model composed of 1848 shell elements was used. For the material behaviour, it was assumed as an isotropic elastic behaviour defined by the generalized Hooke’s law. Besides, an anisotropic plastic material behaviour, which is described by Hill’48 yield criterion and obeying to the isotropic hardening law, being defined by Swift equation. The Hill’48 yield criterion is expressed as follows:

\[ F(\sigma_{xx} - \sigma_{zz})^2 + G(\sigma_{zz} - \sigma_{yy})^2 + H(\sigma_{yy} - \sigma_{xx})^2 + 2L\sigma_{zz}^2 + 2M\sigma_{yy}^2 + 2N\sigma_{xx}^2 = \overline{\sigma}^2 \]  

(6)

where \( \sigma_{ij} \) (i,j = 1..3) are the Cauchy stress tensor components and defined in the material’s orthotropic coordinate system. The parameters \( F, G, H, L, M, N \) are the Hill48’s anisotropy coefficients and \( \overline{\sigma}^2 \) is the yielding stress. In the case of the plane stress condition, and assuming that the yield stress is identified to the uniaxial tensile stress along the RD of the sheet metal, thus the relation \((G+H=1)\) is deduced. \( L \) and \( M \) parameters are set to the isotropic value, which is equal to 1.5. The Hill48’s anisotropy parameters can be expressed as a function of the Lankford anisotropy coefficients, \( r_{0}, r_{45°}, \) and \( r_{90°}, \) using the following equations:

\[ F = \frac{r_{0}}{r_{0}\left(1 + r_{0}\right)}; \quad G = \frac{1}{1 + r_{0}}; \quad H = \frac{r_{0}}{1 + r_{0}}; \quad N = \frac{(r_{0} + r_{90})\left(2r_{45} + 1\right)}{2r_{0}(1 + r_{0})} \]  

(7)

The Swift empirical relation was used to model isotropic strain-hardening behaviour.

\[ \overline{\sigma} = K(\varepsilon_{0} + \varepsilon)^{n} \]  

(8)

where \( K, \varepsilon_{0}, \) and \( n \) are the Swift’s law constants. Figure 5 shows the numerical simulations of the hydraulic bulge tests through circular and elliptical die cavity, when the major axis coincided with 0°, 90° and 45° with respect to the RD. For a balance between better accuracy and the computing time, the mesh was refined close to the centre of the blank, and a coarse mesh was considered elsewhere.

5. Sensitivity of material parameters

Sensitivity analysis was carried out to investigate the effect of the material parameters (e.g., the hardening exponent \( n \) and the anisotropy coefficients \( r_{0}, r_{45°}, r_{90°} \)) variation on the pressure and polar thickness \( \) vs. the bulge height. These responses were determined using numerical simulation of the four hydraulic bulge tests. The aim of this parametric sensitivity analysis is to assess the relevance of responses obtained from hydraulic bulge tests through circular and elliptical die cavity. Figure 6 shows the effect of anisotropy coefficient on the simulated bulge test responses. It is revealed that the anisotropy coefficients \( r_{0} \) and \( r_{90°} \) have an effect on the pressure – bulge height curves, noticeably for the \( r_{90°} \) coefficient. However, the \( r_{45°} \) anisotropy coefficient was insensitive to the bulge responses obtained for all the bulge tests. As consequence, the accuracy of the identification of the \( r_{45°} \) anisotropy coefficient using bulge test performed through the elliptical die cavity will be less evident. Figure 7 shows that the anisotropy coefficient has no significant effect on the pole thickness. It is worth noting that anisotropy coefficients were varied and no effect was revealed on the pole thickness for all bulge tests. Thus, they are not shown here. Figure 7a shows only one case studied, where it depicts the
thickness at the pole of the bulged area obtained using the numerical simulation of the hydraulic bulge tests; Test-0, Test-1, Test-2 and Test-3.

![Numerical simulation of hydraulic bulge tests](image)

**Figure 5.** Numerical simulation of the hydraulic bulge tests using circular die cavity for Test-0 and elliptical die cavity for Tests 1, 2 and 3 (von Mises stress contours).

One may conclude that the predicted thickness calculated using three different anisotropy coefficients was insensitive to these coefficients. However, it is clear that the work-hardening exponent has a significant effect on the polar thickness, as exhibited in Figure 7b. It is also noticed that this influence is observable for a bulge height higher than 10 mm. This exponent has also an influence on the pressure vs. bulge height for hydraulic bulge tests through the circular and the elliptical die cavity (see, Figure 7c). Therefore, the thickness vs. bulge height curve, which is essential to compute the equivalent stress-strain curves for the four hydraulic bulge tests (i.e., Test-0, Test-1, Test-2 and Test-3) was generated using numerical simulation of the hydraulic bulge test through circular die cavity.

6. Parameter identification

The mild steel DC04 is used as an application of the proposed identification methodology to determine the anisotropy coefficients and flow curve of this material. This method was decoupled in two steps to identify the material parameters. In the first step, the anisotropy coefficients of the Hill’48 yield criterion were identified. In the second step, Swift law parameters were determined, as the anisotropy coefficients were already identified in the first step.

6.1 Identification of anisotropy parameters

Hill48’s anisotropy coefficients were identified using the equivalent stress-strain curves obtained from the hydraulic bulge tests and the computation algorithm given in Figure 2. The parameter identification was converted as an optimization problem, for which an objective function was defined. The objective function is given in equation (9) and it defines the gap between four equivalent stress-strain curves calculated for each bulge test. The minimization of this function $f$ leads to the convergent solution of the parameter identification problem. This solution represents the set of the identified anisotropy parameters.

$$
f = \sum_{i=1}^{4} (\bar{\sigma}_{cr} - \bar{\sigma}_{ell_i})^2 + (\bar{\sigma}_{cr} - \bar{\sigma}_{el_c})^2 + (\bar{\sigma}_{cr} - \bar{\sigma}_{el_c})^2 \tag{9}$$
The optimization method used to solve this non-linear problem and to avoid trapping into local minima, the simulated annealing (SA) algorithm, as a heuristic optimization method was used to reach a global minimum. SA algorithm adopts an iterative movement according to the variable temperature parameter which imitates the annealing transaction of the metals. The base of the annealing process is to generate random points in the neighbourhood of the current best point and evaluate the problem functions there. This strategy evolves iteratively until the value of objective function is less than a given tolerance. Figure 8 displays the flow-chart of the algorithm used in the parameter identification of the anisotropy coefficients based on the four responses of the hydraulic bulge tests.

6.2 Identification of work-hardening parameters
The identification of Swift’s parameters was determined by curve fitting procedure. These parameters were determined using the equivalent stress-strain curve. This curve was derived from the hydraulic bulge test which is performed through circular die cavity. Table 1 lists the identified Hill’48 anisotropy coefficients along with the Swift work-hardening parameters using the proposed identification methodology. Moreover, for comparison purposes, the Hill’48 anisotropy coefficients calculated using Lankford anisotropy coefficients measured using uniaxial tensile tests with respect to three directions (\(r_{0}\), \(r_{45}\) and \(r_{90}\)) to RD were also given in this table. It is clearly seen that the set of the identified
parameters: anisotropy coefficients and work-hardening parameters are in good agreement with those obtained using uniaxial tensile tests. However, the attained equivalent strains using hydraulic bulge tests are higher than those obtained for tensile test. Figure 9 shows the equivalent stress-strain curves obtained for the four hydraulic bulge tests using circular and elliptical die cavity. It shows that at algorithm convergence, the four responses are overlapping and in good agreement, even a small discrepancy was noticed. Figure 10 displays the comparison between \( r \)-values calculated using the Hill’48 anisotropy parameters and the Lankford coefficients measured using uniaxial tensile tests along the three directions relative to RD. It is worth noting that despite the observed difference, these results are generally in good agreement with those obtained using uniaxial tensile tests. This could be likely explained by the weak ability of Hill’48 yield criterion for accurate description of the DC04 material behaviour. Moreover, it is interesting to mention that the Test-3 has a shallow sensitivity with respect to the \( r_{45^\circ} \) anisotropy coefficient, as probably was not expected in the foremost step of this study.

![Figure 7. sensitivity analysis obtained for the four bulge tests: a) of the anisotropy coefficients on the pole thickness; b) of the hardening exponent on the thickness at the pole; c) of the hardening exponent on the bulge response: pressure vs. bulge height.](image)

![Figure 8. Flow chart algorithm used in the identification of anisotropy parameters.](image)
Table 1. Comparison between the identified parameters using hydraulic bulge tests and those determined by simple tensile tests.

| DC04 | Swift parameters | Hill’s anisotropy coefficients |
|------|------------------|-------------------------------|
|      | $K$ (MPa) $Y_0$ (MPa) $n$ $F$ $G$ $H$ $N$ |
| Hydraulic Bulge tests | 692.3 209.3 0.271 0.344 0.425 0.575 1.231 |
| Tensile test | 601.8 181.2 0.209 0.361 0.371 0.629 1.363 |

Figure 9. Flow curves obtained using hydraulic bulge test through circular and elliptical die cavity along with stress-strain curve measured from uniaxial tensile test along the rolling direction.

Figure 10. Comparison between $r$-values identified using hydraulic bulge tests through circular and elliptical die cavity and the Lankford coefficients (i.e., experimental values).

Figure 11. Comparison between experimental and identified pressure vs. bulge height curves obtained using circular and elliptical die cavity.
7. Experimental verification
The identified parameters: anisotropy coefficients and work-hardening parameters were used as inputs into finite element model to simulate the hydraulic bulge tests through circular and elliptical die cavity to assess the accuracy of the identification procedure. This could be considered as a first attempt in the validation and verification of the proposed identification method along with the chosen plasticity yield criterion to describe the mechanical behaviour of the sheet metal DC04. Figure 11 shows the comparison between experimental and numerical responses obtained for the hydraulic bulge tests (Test-0, Test-1, Test-2 and Test-3) carried out using circular and elliptical die cavity. A good agreement was revealed for all the considered tests, despite a small discrepancy between numerical and experimental results. However, these findings globally demonstrate an accurate prediction of pressure vs. pole height owing to this simple and robust identification procedure.

8. Conclusions
New approach was proposed for material parameter identification of anisotropy and flow curves using hydraulic bulge tests through circular and elliptical die cavity. The use of the experimental continuous measurement of the applied pressure vs. the bulge height is the chief advantage of this parameter identification methodology. The instantaneous thickness was obtained by numerical simulation, in contrast to previous works, where the thickness was either determined using analytical formulas or using DIC technique. It was observed that the thickness evolution at the pole as a function of the bulge height, which was calculated for the four bulge tests, was found insensitive to the anisotropy coefficients. However, the curvature radii were calculated using Panknin relation and taking account of the die fillet radius. The experimental validation shows that the predicted results of the numerical simulation using the identified material parameters were in good agreement with the experimental data. Nevertheless, further work is needed to systematically investigate the sensitivity analysis of the input parameters on the different outputs of bulge tests. Furthermore, the application of more metal sheets diversity and using non-quadratic yield criteria is the purpose of the forthcoming research work.

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