Synthesis of an optimal algorithm for processing random signals during phase direction finding

A O Zhukov1,2,3, I N Valyaev4, V P Kovalenko4, Z N Turlov4, M K Bondareva4 and I N Kartsan5,6,7

1 Institute of Astronomy of the Russian Academy of Sciences, 48, street Pyatnitskaya, Moscow, 119017, Russia
2 FGBSI «The Federal Center of Analyzis», 33, Talalikhina St., building 4, Moscow, Russia
3 Russian Technological University, 78, Vernadskogo Av., Moscow, 119454, Russia
4 JSC "Special Design Bureau of the Moscow Energy Institute", 14, Krasnokazarmennaya St., Moscow, 111250, Russia
5 Marine Hydrophysical Institute, Russian Academy of Sciences, 2, Kapitanskaya St., Sevastopol, 299011, Russia
6 Reshetnev Siberian State University of Science and Technology, 31, Krasnoyarsky Rabochy Av., Krasnoyarsk, 660037, Russia

E-mail: kartsan2003@mail.ru

Abstract. A new method of correlation signal processing with symmetric heterodination is presented, which is a development of the two-band processing method. The proposed processing method allows you to use all the advantages of two-band processing when receiving signals of any type, from broadband to monochromatic.

1. Introduction

The principle of operation of phase angle meters is based on the use of antenna systems with spaced in space [1-5]. The cosine of the guide angle (the angle between the direction finder base and the direction to the radiation source) is determined by measuring the phase difference of the useful signal received by the direction finder antennas. The operation algorithm of the phase direction finder is as follows

$$\cos \theta = \frac{\lambda \alpha}{2 \pi D}$$  \hspace{1cm} (1)

where: $\alpha$ - phase difference of the received oscillations; $\lambda$ - wavelength of the received oscillations; $D$ - direction finder base, distance between receiving antennas.

The main provisions of the theory of synthesis of optimal parameters-measuring devices for random radio signals are presented in the works of G P Tartakovsky, Ya D Shirman, V N Manzhus, G Van-Tris [6-8].

2. Parameter estimation model for phase direction finding of a random signal

The space-coherent signal received by a spaced (multi-position) receiving system is modeled by a selective function of a random process whose characteristics depend on the parameter being measured.
the calculations will receive the amplitude of signals in the receiving channels that the signal is received by the same type of antennas in synchronous guidance mode and to simplify mutual spectral densities of the interference reception channels, interchannel correlation of interference is usually absent. In this case, the matrix of conditions is defined as

$$\begin{align*}
X(t) &= \|x_iX_0(t - t_i)e + j2\pi f_0(t - t_i)\|
\end{align*}$$

(2)

Surveillance is conducted against the background of stationary, generally correlated interference. It is assumed that are receiving an arbitrary Gaussian signal on the background of arbitrary Gaussian interference.

In the absence of a priori information about the measured parameter, the synthesis of the structure of optimal processing is carried out by the criterion of maximum plausibility [7, 8]. Estimates of the parameter $$\hat{\alpha}$$ determined as the roots of the maximum likelihood equation.

$$\frac{\partial \ln l(\hat{\alpha})}{\partial \alpha} = 0 \quad \text{if} \quad \alpha = \hat{\alpha}$$

(3)

where: \(\ln l(\hat{\alpha})\) - logarithm of the plausibility function of the accepted implementation \(y\).

Mathematical expectations of both the direction finding and the interference random process are equal to zero. Random processes will be characterized by matrices of mutual spectral densities \(S(f)\) for signal oscillations and \(N(f)\) for interference oscillations. For the signal model under consideration (2), when the oscillations received by the spaced antenna system differ only in time delay, the matrix of mutual spectral densities of signals can be reduced to the product of equivalent vectors \(S(f)=X(f)X^T(f)\), composed of the values of

$$\begin{align*}
X(f) &= \sqrt{S_0(f)}\|x_i e^{-j2\pi(f+f_0)t_i}\|
\end{align*}$$

(4)

where: \(S_0(f)\) - spectral density of the envelope signal.

According to [7] the expression for the logarithm of the likelihood function has the form of

$$\ln l(\alpha) = \frac{1}{2} \int_{-\infty}^{\infty} |W(t)|^2 dt - T \int_{-\infty}^{\infty} \ln[1 + X^T(f)R(f)] df$$

(5)

Scalar function of time characterizes the effect of spatial and temporal processing of incoming vibrations.

$$W(t) = \int_{-\infty}^{\infty} W(f) e^{j2\pi ft} df$$

(6)

It is the result of Fourier transformation of scalar spectral density

$$W(f) = \frac{R^T(f)Y(f)}{\sqrt{1+X^T(f)R(f)}}$$

(7)

Spectral density \(W(f)\) determines the processing operations of separate harmonic components of the vector-column \(Y(f)\) of the received vibrations with the help of the vector \(R(f)\).

The vector-column \(R(f)\) is the solution of the generalized Fredholm equation and under the introduced conditions is defined as [6]

$$R(f) = N^{-1}(f)X(f)$$

(8)

In most practical cases, the interference has the form of white noise with the same intensity in the reception channels, interchannel correlation of interference is usually absent. In this case, the matrix of mutual spectral densities of the interference takes the form of \(N(f)=N_0I\), where \(I\) - unit matrix. We believe that the signal is received by the same type of antennas in synchronous guidance mode and to simplify the calculations will receive the amplitude of signals in the receiving channels \(x_i\) equal to one. Under these conditions \(R(f)=X(f)/N_0\)

$$W(f) = \sqrt{\frac{S_0(f)}{N_0[S_0(f)+N_0(f)])}}\left[y_1(f)e^{j2\pi(f+f_0)t_1} + y_2(f)e^{j2\pi(f+f_0)t_2}\right]$$

(9)
The first multiplier of expression (9) represents the frequency response of the optimal filter of the received vibrations.

The expression of the logarithm of the plausibility function of the additive mixture of a random signal with white, not correlated on the channels of reception, the noise is converted to a view.

\[
\ln l(\alpha, t_1, t_2) = \frac{1}{2} \int_{-\tau/2}^{\tau/2} \left| \bar{Y}'_n(t)X_\epsilon(t, \alpha) \right|^2 dt + \text{const}
\]  

(10)

Will convert the logarithm of the plausibility function with the replacement of the module square by the product of complex conjugate values.

\[
\ln l(\alpha) \cong Z_{11}(0) + Z_{22}(0) + Z_{12}(\tau)e^{-j\alpha} + Z_{21}(\tau)e^{j\alpha}
\]  

(11)

where: \( Z_{ij}(s) \) - the result of mutual correlation processing; \( \tau = t_2 - t_1 \) - relative delay in receiving signals; \( \alpha = 2\pi f_0 \tau \) - measured parameter.

Neglecting the first two summands, which do not depend on the measured parameter and considering that \( Z_{21} = Z_{12} \) gets the final expression for the logarithm of the plausibility function

\[
\ln l(\alpha) = Z e^{-j\alpha} + Z(\tau)e^{j\alpha}
\]  

(12)

where:

\[
Z = \frac{1}{2} \int_{-\tau/2}^{\tau/2} \bar{y}_{n1}(t)y_{n2}^*(t)dt
\]

Expected signal \( X(t)e^{j2\pi f_g t} \) can be represented as a work \( X_1(t)e^{j2\pi f_0 t} \cdot X_2(t)e^{j2\pi f_0 t} \). Such signal model takes into account the peculiarities of correlation-filter processing. Thus, at \( X_1(t) = 1, X_2(t) = X(t) \), the heterodynation of the received signals by monochromatic frequency oscillation \( f_g \) and filtration at the intermediate frequency \( f_{pr} = f_0 - f_g \) is implemented. The bandpass filters are tuned to the intermediate frequency.

Optimal phase difference estimation of signals received by the spaced antenna system is the solution of maximum plausibility equation (3). Using an expression for the logarithm of the plausibility function (12), you can obtain

\[
\frac{\partial \ln l}{\partial \alpha} \bigg|_{\alpha = \hat{\alpha}} = -jze^{-j\hat{\alpha}} + jze^{j\hat{\alpha}} = 0
\]  

(13)

Using Euler formulas for \( e^{\pm j\hat{\alpha}} \) and considering that \( z + z^* = 2Rez \) and \( z - z^* = 2Re(-jz) \), will find

\[
\hat{\alpha} = \arctg \frac{Re(-jz)}{Rez}
\]  

(14)

The operation (14) of obtaining the estimation of phase difference of random signals is realized according to the scheme shown in figure 1.

**Figure 1.** Optimal treatment structure for phase direction finding.
Signals received by the antennas are converted into mixers by frequency using a monochromatic oscillator with a frequency $f_r$. The converted signals are filtered by optimal bandpass filters with frequency response (9). The signals are then fed into correlation processing, multiplication and time averaging (11). To obtain $Re(\phi(z))$, a phase shifter of $90^\circ$ is used.

Optimal estimation $\hat{\alpha}$ of the measured parameter is obtained at the solver output implementing the algorithm (14).

The synthesis resulted in the structure of a well-known phase direction finder that uses mutual correlation processing of signals [2, 7-10]. This structure is also optimal for direction finding of coherent signals. The difference lies in the fact that due to the noise nature of random signals in the resulting structure uses bandpass filters, rather than consistent, as in the case of coherent signal.

The resulting structure is optimal when you can ignore the delay of envelope signals based on the direction finder. Otherwise, using the expression for mutual correlation processing of received signals (11), you can see that in each direction finder channel it is necessary to enter delays the value of which is equal to $t_1-t_c$. Such delays should ensure combination of useful fluctuations in time, i.e. compensate for spatial delay of received signals.

The scheme shown in figure 1 is not the only possible one. Depending on the representation of expression (13), you can get different schemes of optimal meters.

The processing can be modified by conversion in channels to different frequencies with the help of the so-called forked heterodyne [11, 12]. In this case, the expected signal is represented as:

$$X(t) \begin{bmatrix} e^{-j2\pi f_0 t_1} \\ e^{-j2\pi f_0 t_2} \end{bmatrix} e^{j2\pi f_0 t} = \begin{bmatrix} e^{j2\pi f_r t} \\ e^{j2\pi (f_r+F) t} \end{bmatrix} X(t) \begin{bmatrix} e^{-j2\pi f_0 t_1} \\ e^{-j2\pi f_0 t_2} \end{bmatrix} e^{j2\pi f_{np} t} \tag{15}$$

It is taken into account here that the heterodyne frequency of the second receiver channel is greater than the heterodyne frequency of the first receiver channel by $F$ value. The maximum plausibility equation (13) takes the following form

$$ze^{-j(2\pi Ft+\hat{\alpha})} - z^* e^{j(2\pi Ft+\hat{\alpha})} = 0 \tag{16}$$

It should be noted that the result of the mutual correlation processing is isolated at the difference frequency of the forked heterodyne, and the integration is replaced by narrow-band filtering (11).

To illustrate the possibility of obtaining a different structure of the optimal measurer than the one presented in figure 1, we convert (16) to the following form

$$Im \left[ ze^{-j(2\pi Ft+\hat{\alpha})} \right] = 0 \tag{17}$$

Operation (18) corresponds to phase detection of the correlation processing result. If the output effect of correlation processing is written as $z = |z| e^{j\arg z}$, then

$$\hat{\alpha} = \arg z - 2\pi Ft = \phi(z) \tag{18}$$

Operation (18) to obtain the optimal estimation of the signal phase difference corresponds to the algorithm of the ideal phasometer. The phase difference is measured relative to the reference signal of the difference frequency. The symbol $\phi(z)$ indicates the phase of $z$ oscillation relative to the difference frequency. The structural diagram of the direction finder with forked heterodyne is shown in figure 2. Here is provided compensation for the spatial delay of the received oscillations.

Despite the variety of possible schematic implementations of the optimal processing algorithm, all of them have the same potential measurement accuracy because they are described by the same expression for the plausibility function.

As a model, let us consider the work of the structural processing scheme shown in Figure 2.

The expected $X_m(a)e^{j2\pi f_0 t}$ signal is converted by frequency using a forked heterodyne. The heterodyne frequency is selected below the expected signal frequency $f_0 = f_{pr} + f_c$. As a result of the conversion, the input signal is transferred to an intermediate frequency, where it is filtered by bandpass filters.
After conversion with heterodyne, which frequency is lower than the signal frequency, the spectrum of the intermediate frequency signal completely corresponds to the input $X_{pr}(\alpha) = X_{in}(\alpha)$. 

**Figure 2.** Signal processing with fork heterodyne.

We will assume that the signal received by the second antenna is delayed relative to the signal received by the first antenna. 

After filtering in the bandpass filter, the signal of the first channel is delayed by a compensating delay line for the $\tau_0$ time, due to which the intermediate frequency signal receives additional phase shift $\alpha_0 = 2\pi f_{pr} \tau_0$. 

The solution of the maximum likelihood equation gives the result (18) where 

$$Z = \frac{1}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} y_{n1}(t - \tau_0) y_{n2}(t) dt$$

is the result of mutual correlation processing of oscillations.

By carrying out similar transformations it is possible to obtain a measurement algorithm similar to (18)

$$\hat{a} - \alpha_0 = \arg Z - 2\pi Ft; \quad \hat{a} = \varphi(z) + \alpha_0$$

(19)

The measurement operation of the parameter (19) is implemented according to the scheme figure 3.

**Figure 3.** Signal processing taking into account channel delay.

The essential difference between the obtained structure with compensating delay and the scheme in figure 2. is the need to take into account the $\tau_0$ channel delay. In general, the value of this delay is a
random value, because it is determined by instability of real delay lines characteristics and receiving channels difference-phase characteristics. All this reduces the accuracy of phase measurements.

Let us consider the case of signal transformation with heterodyne, the frequency of which is higher than the frequency of received oscillations, i.e. $f_0 = f_g - f_{pr}$. For the expected signal of the intermediate frequency

$$X_{pr0}(\alpha)e^{j2\pi f_{pr}t} = X_{in}^*(\alpha)e^{-j2\pi f_{0}t} \cdot \left| e^{j2\pi(f_{g}+F)t} \right|$$ (20)

After conversion by heterodyne, whose frequency is higher than the frequency of the signal, the intermediate frequency signal is a complex conjugate input oscillation, and the frequency spectrum of the intermediate frequency oscillation "turned over" relative to the spectrum of input oscillation [13-16]. To account for this, the upper index "n" is entered. After filtering and delay, get the oscillations

$$X_n^*(\alpha) = \left| e^{-j\alpha/2}e^{-j\alpha_0/2}e^{j\alpha/2}e^{j\alpha_0/2}e^{j2\pi Ft} \right|$$ (21)

In this case, the solution of the maximum likelihood equation looks like

$$z_n e^{-(j2\pi Ft + \hat{\alpha} + \alpha_0)} - z_n^* e^{j(2\pi Ft + \hat{\alpha} + \alpha_0)} = 0$$ (22)

where $z_n = \frac{1}{T} \int y_{n1}^*(t - \tau_0) y_{n2}^*(t) dt$, and the algorithm for measuring the phase difference of the received vibrations

$$\hat{\alpha} = \phi(z_n) + \alpha_0$$ (23)

Comparison (19) and (23) shows that when heterodyning signals with a frequency higher than the signal frequency to get an estimate of the measured parameter must be subtracted from the phase of the result of correlation processing to subtract phase raids at the expense of channel delay, rather than add, as in the case of heterodyning a frequency below the signal frequency. This allows you to exclude the influence of channel delay on the evaluation of the parameter by implementing the following algorithm

$$\hat{\alpha} = \frac{\phi(z) + \phi(z_n)}{2}$$ (24)

Operation (24) of optimal phase difference measurement can be carried out by the device of correlation signal processing [12] with time separation of measurement results, figure 4. This device provides, at the expense of signal switchers of heterodyne, the temporary separation of the results of mutual correlation processing of straight and inverted signals.

In the first switching mode the input signals are converted by frequency heterodyne 1, the frequency of which is lower than the signal frequency. At the phasometer output get an estimate of phase difference after mutual correlation processing of direct signals $\phi(z)$. In the second mode of switching the input signals are converted by the frequency heterodyne 2, whose frequency is selected above the frequency of the received oscillations. In this mode at the phasometer output get an estimate of the phase difference after mutual correlation processing of upside-down signals $\phi(z_n)$. The received estimations come to the adder, whose work is synchronized with the switches clock frequency.

Summator implements the operation (24). At its output, we obtain an estimate of phase difference of the accepted implementations. At such processing knowledge of channel delay parameters of signals is not required, therefore instability of characteristics of receiving channels, channels after mixers does not decrease accuracy of measurements. Potential accuracy of the phase direction finder with time division of measurements is defined as follows

$$\sigma_e = \sqrt{\frac{\sigma_z^2 + \sigma_n^2}{2}}$$ (25)
where $\sigma_z^2$ and $\sigma_n^2$ - measurement dispersions in mutual correlation processing of straight and inverted signals.

Figure 4. Signal processing with symmetrical heterodyne.

Given that the $\sigma_z^2 = \sigma_n^2 = \sigma_\alpha^2$, the resulting error is equal to $\sigma_e = \sigma_\alpha / \sqrt{2}$. It should be taken into account here that the time required to obtain an estimate for the time division of measurements is twice as long as the time required to obtain an estimate for a conventional phase direction finder. Therefore, with the same measurement time, the potential accuracy of a direction finder with a switch is equal to the potential accuracy.

5. Conclusion
In aperture synthesis systems, dual-band signal processing is widely used to ensure interferometer response phase independence from the intermediate delay frequency input. But this method of processing is applicable only when receiving very broadband signals, because it is based on the upper and lower side bands after heterodynization. It is not possible to separate the processing results of the upper and lower bands separately, and as a result, the output response of the interferometer has an additional envelope type $\cos[\omega_p r (\tau - \tau_0)]$. The presence of this additional multiplier in the correlation function of the processed signal imposes more stringent restrictions on the accuracy of spatial delay compensation, $\delta \tau = \tau - \tau_0$.

Acknowledgements
Project № 2686.2020.8 "Models, methods and means for obtaining and processing information about space objects in a wide spectral range of electromagnetic waves"

References
[1] Bychkov S I 1967 Space radio technical complexes (Moscow: Soviet Radio)
[2] Pestryakov V B 1968 Phase-based radio engineering systems (Moscow: Soviet Radio)
[3] Chebotarev A S, Zhukov A O, Makhnenko Yu Yu and Turlov Z N 2011 Monitoring of spacecraft based on the use of correlation-phase direction finders (Moscow: FIZMATLIT)
[4] Valyaev I N and Zhukov A O 2015 Mathematical model of the correlation-phase radio direction finder (In the book: XL Academic readings on cosmonautics dedicated to the memory of
academician S. P. Korolev and other outstanding Russian scientists-pioneers of space exploration: Theses Collection) p 262

[5] Zhukov A O and Okunev E V 2016 Radiotechnical monitoring of the environment by spatially distributed information systems In the collection: Mints Readings Proceedings of the 3rd all-Russian scientific and technical conference of young designers and engineers, dedicated to the 70th anniversary of the Radio Engineering Institute named after academician A. L. Mints and the 70th anniversary of FIZTECH 293-303

[6] Tartakovsky G P 1963 Issues of statistical theory of radar (Moscow: Soviet Radio)

[7] Shirman Ya D and Manzhus V N 1981 Theory and Technique of Radar Information Processing on the Background of Interference (Moscow: Radio and Communication)

[8] Van-Tris G 1977 Theory of detection, estimation and modulation (Moscow: Soviet Radio)

[9] Zhukov A O, Turlov Z N and Valyaev I N 2014 Monitoring of spacecraft based on the use of correlation-phase direction finders Intelligence and Technology 3 64

[10] Vinokurov V I and Vakker R A 1972 Questions of processing of complex signals in correlation systems (Moscow: Soviet Radio)

[11] Begun I Z and Bjalyi L I 1964 Mutual correlation of random processes with mutually displaced spectra at the outputs of linear channels vol 5 (Moscow: Voprosy elektroniki)

[12] Gonorovsky I S 1964 Radio engineering circuits and signals (Moscow: Soviet Radio)

[13] Zhukov A O, Valyaev I N, Kovalenko V P, Turlov Z N, Novikov A S, Kartsan I N and Malanina Y N 2020 Journal of Physics: Conference Series 1515(2) 022044

[14] Zhukov A O, Valyaev I N, Kovalenko V P, Turlov Z N, Chebotarev A S, Kartsan I N and Shumakova N A 2020 Journal of Physics: Conference Series 1515(3) 032070

[15] Zhukov A O, Valyaev I N, Kovalenko V P, Turlov Z N, Minin I V, Zaverzaev A A, Kartsan I N and Malanina Y N 2020 Journal of Physics: Conference Series 1515(2) 022051

[16] Gubanov V S, Finkelstein A M and Friedman P A 1983 Introduction to radio astronomy (Moscow: Nauka)