Acoustic and buoyancy modes throughout stellar evolution – Seismic properties of stars at different stellar ages and masses

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Parameter regions in which stars can become pulsationally unstable are found throughout the Hertzsprung-Russel diagram. Stars of high, intermediate, low and very low masses may cross various instability regions along their paths of evolutionary sequences. In describing them, I give special consideration to hybrid pulsational characteristics that are particularly valuable for asteroseismic investigations, to \( P \) measurements that allow us to directly follow the stellar evolutionary changes in some stars, and to new research results that stand out with respect to previous consensus.

1 Introduction

Stars are gaseous spheres in hydrostatic equilibrium that continuously radiate away energy into space. The hydrostatic equilibrium, when disturbed, can be balanced again by adjusting the pressure; local deviations from a spherical shape can be restored by the action of buoyancy. The energy carried from the centre to the surface of the star can under the right circumstances be temporarily stored and transformed into kinetic energy, giving rise to oscillations about the equilibrium. The properties of these stellar pulsations can vary widely, and many possible combinations of circumstances that are ‘right’ cause pulsating stars to appear all over the Hertzsprung-Russel diagram. A visual representation of many of these classes of pulsators has been updated several times from Fig. 1 in Christensen-Dalsgaard (1998). A very similar and improved recent representation may be found as Fig. 1 (right panel) in the comprehensive and at the same time readily comprehensible summary of the subject of asteroseismology by Handler (2012).

I will discuss a cross-section of these pulsating stars and their seismic properties for different masses and ages. Since it depends on the mass how a star evolves with age, mass ranges are distinguished from the point of view of different end products of stellar evolution: by high-mass stars I mean those with initial masses on the main sequence above eight solar masses, typically bound to end in a supernova explosion. There is a possible exception for stars around eight to ten solar masses that can go through carbon fusion only (‘super-Asymptotic Giant Branch’) before evolving into O/Mg/Ne core White Dwarfs. I designate as intermediate- and low-mass stars those with initial masses between 0.5 and eight solar masses that will evolve into C/O core White Dwarfs via a central helium burning stage. Low-mass stars have masses between 0.08 and 0.5 solar masses and will eventually turn into helium-core White Dwarfs.

I start by summarising criteria by which pulsating stars may be distinguished, before giving examples for the classes of variable stars associated with the different mass ranges.

It is beyond the scope of this article to discuss the effects on the pulsations that arise when the spherical symmetry is already broken in the equilibrium state of a star. This happens when one orientation is preferred over other directions, for example along the rotational axis, the axis of a magnetic field, or the axis defined by the orbital plane in a binary, and can have pronounced consequences depending on the strength of the geometric, temperature or tidal deviations of the spheroid. See the contribution by Hekker (2013) for a discussion on the evolution of pulsational frequencies as a function of magnetic fields in particular.

1.1 Physical seismic properties

Geometry. In the spherically symmetric case, the solutions to the equations describing time-dependent perturbations to the equilibrium state separate into radial, angle-dependent and time-dependent factors for each of the (infinite number of) modes. Depending on the number of nodes of the radial part, a subset of modes may be described as low overtone (with few radial nodes, including the fundamental mode without any radial node) or high overtone (with many radial nodes, including the regime of ‘asymptotic limit’). The modes in a pulsator exhibiting a subset of low overtones may also be called low-order modes, or high-order modes in the case of high overtones. The angular part can be mathematically described by spherical harmonics, with the degree describing the overall number of node planes (zero to
countably infinite) through the spherical surface, and the azimuthal order describing the number of node planes intersecting the equator while observing their orientation. The azimuthal order can therefore take any value (in integer steps) from minus the value of the degree to plus the value of the degree. Stars with modes appearing uniquely with a degree of zero, i.e. no angular node planes, are pure radial pulsators. The occurrence of radial and non-radial modes, i.e. the general case, characterizes the non-radial pulsators. Stars in which many modes are excited at the same time (see below) are sometimes called multi-mode pulsators; they include non-radial modes in p-mode pulsators and are exclusively non-radial modes in g-mode pulsators (see below for p and g modes).

Restoring force. The oscillation frequency of the radial fundamental (f) mode for a star is determined by its structure - actually almost exclusively by the star’s mean density. Although all other modes are also global phenomena, different mode types propagate in distinct typical depths of the star, governed locally by oscillatory behaviour due to either predominantly pressure or predominantly buoyancy as a restoring force. For both branches separately, the overall geometry of a mode, given by its radial order, degree and azimuthal order, determines its oscillation frequency for a given stellar structure. If the restoring force in the structure for this mode is predominantly conveyed via pressure (p), its frequency will be higher than that of the radial fundamental; if the restoring force is predominantly conveyed via buoyancy (gravity, g), its frequency will be lower than that of the radial fundamental. All modes with periods longer than the fundamental radial mode are necessarily non-radial modes. For modes of the same degree, a mixed mode character that bridges the evanescence zone between the gravito- and acoustic-dominated propagation regions in different depths can sometimes also occur.

Driving mechanism. Although every star has an infinite number of such eigenmodes, by far not all modes are excited in all stars. In stars that are stable, all the modes are damped. This is in fact the case for the Sun, where the oscillation caused by any single perturbation is damped away with a characteristic lifetime. The surface convection zone however provides continued perturbations, or noise, at a large range of frequencies, collectively resulting in the phenomenon of stochastic excitation. The eigenmodes reappear endlessly with very low amplitudes and at random phases.

A functioning driving mechanism is required to excite larger amplitudes in at least a subset of the modes. A valve effect in a suitable envelope layer of the star can turn the system into a heat engine. In the most common case, the valve affects the radiative flux and is provided by the opacity κ whenever said opacity – in contrast to its ‘normal’ dependency on temperature and pressure – increases with compression, a behaviour found in partial ionisation zones. The κ effect can be further enhanced by increased ionisation during compression, called the γ effect, hence also the description as κ-γ mechanism.

The partial ionisation zones of H and He i give rise to κ-mechanism-driven pulsations across the Hertzsprung-Russel diagram, ranging from Giant stars at low effective temperatures (Mira type variables), probably across the main sequence (roAp stars), and down to cool DA White Dwarfs (ZZ Ceti variables). Where the location of the partial ionisation zone of He i is at the right depth in the star, we find the classical instability strip containing the various types of Cepheids, among them the δ Cephei, RR Lyra, and δ Scuti variables. Towards higher temperatures still mainly the partial ionisation of the M-shell electrons of Fe group elements cause the instability regions with β Cephei and SPB variables on the main sequence, sdBV γ and sdBV δ on the extreme horizontal branch, and stretching up into the regime of higher luminosities where a sdBV resides. In the region of the highest stellar temperatures, partial ionisation of the K-shell electrons of C and O drives pulsations in the pre-White Dwarf stars of spectral type PG 1159, the GW Vir variables. In several of these settings, the elements providing the opacity are supported in the ‘right’ stellar depth either by radiative levitation or by mass loss that slows down gravitational settling.

The action of the κ mechanism is modified by convection. The pulsation-convection interaction is complex, but when modelled successfully can explain the location of the red edge (lower effective temperature limit) of the instability strip. Two limiting cases, where the time scales of convection and pulsation much differ, lead to important further driving mechanisms in their own right. Both apply to stellar structures with radiative interiors and exterior convection layers. For convection time scales much longer than the pulsation time scale, the faster luminosity perturbations from deeper layers in the radiative flux are ‘ignored’ by the convective layer which is basically inert on these time scales, and the perturbations are effectively blocked at the base of the convection zone. This translates into heating in phase with compression within the convection zone, our previously formulated requirement for driving. The convective blocking mechanism causes the excitation of modes as seen in the γ Dor instability region. For convection time scales much shorter than the pulsation time scale, surplus energy is immediately redistributed within the convection zone, also leading to heating in phase with compression and hence a valid driving mechanism that acts in White Dwarfs of DBV type (V777 Her) and contributes to the driving in the DAV type (ZZ Ceti).

We complete our systematics of driving mechanisms by mentioning the ε mechanism and the strange modes. The ε mechanism acts on the principle of increased heating of a layer (typically in a star’s central region) upon compression due to strongly increasing nuclear fusion rates with increasing temperature. Changing nuclear reaction rates during a pulsational cycle have been predicted to affect a variety of stellar classes, but modes theoretically excited by it will in
general be unobservable as a matter of principle due to the
time scales involved. Strange modes occur in an environ-
ment that is dominated by radiation, i.e. they are relevant
for stars at high masses. The strange modes are also distinct
from the p and g modes.

Different driving mechanisms tend to excite different
types of modes, e.g. while stochastic excitation selects high-
overtones p and g modes in solar-like stars, the k mecha-




Thermodynamics. The discussion of mode geometry,
restoring forces and to some extent even of driving mech-

1.2 Observable seismic properties: examples

General observables. Variable stars are classified strictly by
a combination of observational properties: their pulsation
periods, amplitudes, and their spectral type and luminosity
class. Of course these characteristics are directly related to
the properties discussed above: spectral type and luminos-
ity class correspond to an evolutionary state that determines
the stellar structure, and the combination of driving mech-
anism, mode type and selected modes in this structure di-
rectly determines the observed pulsation periods and ampli-
tudes. Understanding these physical mechanisms in detail
leads to a number of useful diagnostics. Mode identification
is one possible helper in the process. Because of geometric
cancellation, only low-degree modes retain observable
amplitudes (in velocity or intensity) when integrated across
the stellar surface. This limitation of unavoidable integra-
tion does not apply to the Sun.

Solar-like oscillations. Several diagnostics have been in
place well before the first solar-like oscillations could be
observed in a star other than the Sun. The oscillations cor-
respond to high-overtone p modes in the envelope (the g
modes believed to exist in the core are much harder to de-
tect, see below). This together with the validity of the Cow-
ling approximation, where it is appropriate to neglect per-
turbations to the gravitational potential, simplifies the equa-
tions of stellar pulsation such that an analytic asymptotic re-


A major issue in applying these tools to the large num-
ber of solar-like oscillators known today is the derivation
of proper scaling relations for observables such as the fre-
quency of maximum power and the mean frequency spac-
ings, but also the oscillation amplitudes and line widths, that
describe how these quantities vary with mass, effective tem-
perature, luminosity, and chemical composition of the star.
This quest for the correct scaling relations, beyond a power
of the luminosity-to-mass ratio where simply the exponent
needs to be fixed, includes the solar-like oscillations ob-
erved in Subgiant and Red Giant stars, and any empirical
relation proposed must also be corroborated by a theoretical
understanding of it.

For the solar-like oscillations in Red Giants, a further
important diagnostic has emerged that exploits the existence
of modes with mixed character, where the information about
the core is encoded in the g mode part and communicated
through to the p mode part near the surface, which has a de-
tectable amplitude. In particular, the regular period spacing
of these mixed modes is readily observable.

Subdwarf B stars. The Subdwarf B stars may be consid-
ered intermediate objects between solar-like stars and White
Dwarfs. With surface gravities above \( \log g / \text{cm s}^{-2} \approx 5.2 \)
they belong to the compact pulsators. The propagation re-

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quite the opposite from the situation in the Sun, where it is the g modes that live in the core and that are observationally elusive.

In the case of the GW Vir variables, the regular period spacing of the high-order g modes can be explored as diagnostics. For selected GW Vir stars, Córsico et al. (2009) have systematically investigated the differences in their results obtained for the stellar parameters by comparing the observed period spacing with the asymptotic period spacing of their models, and with the average of the computed period spacings, as well as with their results that they obtained through a direct fit procedure to individual periods.

2 High-mass stars

Stars at high masses are intrinsically highly variable. The large radiation pressure leads to dynamic atmospheres with significant mass loss. The light observed from such stars therefore often appears modified by the previously ejected circumstellar material that it passes through. The various physical processes are not always well enough understood to clearly disentangle the origin of an observed variation, e.g. from rotation or pulsations.

2.1 Vicinity of the main sequence

Pulsations in B stars are driven by the Fe-κ mechanism. At two to seven solar masses, i.e. intermediate mass stars, the Slowly Pulsating B (SPB) stars pulsate in high-order g modes. In the seven to twenty solar mass range, the more rapid β Cephei variability results from low-order p and g modes. Some Be stars are also pulsating and closely linked to the SPB and β Cep. Additionally, (very few) rapidly rotating SPB stars exist with their frequency spectra modified accordingly. Making things yet more complicated, slow pulsations are also detected in all β Cephei stars as soon as the sensitivity is high enough, i.e. when the objects are observed from space (see also the discussion on hybrid pulsators). At higher masses, the formerly separately classified ζ Oph variables of spectral type O have been recognised as the high-temperature extension of the β Cep variables. In this regime, strange mode driving due to the increased radiation pressure starts to add to the Fe-κ mechanism and eventually becomes dominant. The theoretical instability region widens and starts to encompass the main sequence above forty solar masses, as well as the supergiant region at low effective temperatures.

2.2 Evolved stages

At later evolutionary stages, the mass loss suffered previously leads to a higher luminosity-to-mass ratio and hence an increasing contribution from strange modes. Above twenty solar masses, a combination of Fe-κ mechanism and strange modes, with unclear amounts of contributions from each, generates more or less irregular variability in stars. The boundaries between pulsations and outbursts start to dissolve. The Periodically Variable Supergiants (PVSG) of spectral types A and B tend towards pulsational characteristics, while the Luminous Blue Variables (LBV) definitely tend towards the outburst characteristics. The locations of LBV stars in the HR diagram may be appreciated in Fig. 4 of Rest et al. (2012). They are believed to represent an intermediate evolutionary stage in the transition to the Wolf-Rayet stars (see Fig. 3 in Moffat et al. 1989 for an evolutionary sequence including the so-called Wolf-Rayet funnel in the HR diagram). Strange modes have been predicted in many types of massive stars, but are expected to have the largest amplitudes in Wolf-Rayet stars. The detection of a periodic signal of potentially pulsational nature in a Wolf-Rayet star observed by the MOST satellite however exhibits an unexpectedly long period (Lefèvre et al. 2005). Although this and similar observations in two further objects are attributed to pulsational instabilities, they are not generally accepted as unambiguous strange mode detections.

Cepheid variability also occurs partly in the high-mass range, spanning two to twenty solar masses for the classical Cepheids, but is entirely discussed in the next section.

3 Intermediate- and low-mass stars

3.1 The classical instability strip

The Cepheid instability strip encompasses a wide range of evolutionary states and masses.

Pre-main sequence stars. Pre-main sequence stars at masses above eight solar masses are not observed as they evolve too quickly. The pre-main sequence stars in the mass range 2-8 solar masses are the Herbig Ae/Be stars, those with masses below two solar masses the T Tauri stars. Both cross the instability region on their way to the main sequence and can become pulsationally unstable, resulting in δ Scuti-like pulsators that populate the same area in the HR diagram as the classical δ Scuti stars (Zwintz et al. 2009). Some T Tauri stars are known to additionally pulsate in high-order g modes.

Apart from this passage through the classical instability strip, it should be noted that Samadi et al. (2005) have predicted solar-like oscillations in pre-main sequence stars from theory. Solar-like oscillations in their own right as well as the evolution of seismic properties of pre-main sequence – main-sequence – Red Giant stars are furthermore treated in many of the other contributions to this issue.

Cepheids. The Cepheids encompass the classical Cepheid variables of δ Cephei type with masses above three solar masses (and their population II counterparts of W Virgines type with masses below one solar mass); the horizontal-branch stars of RR Lyrae type (see also Guggenberger 2013) in the mass range of 0.6-0.8 solar masses; and the Dwarf Cepheids of δ Scuti type with masses from 1.5-2.5 solar masses (and their population II counterparts of SX Phoenicis type, which are typically blue stragglers and whose origin is not always certain for a given
object), found near the main sequence but actually representing evolutionary inhomogeneous classes as described above. An additional type are the anomalous Cepheids, believed to result from binary mergers.

Historically, one of the great mysteries in Cepheid research has been the mass discrepancy problem, paraphrasing the fact that masses derived from evolutionary models on the one hand and pulsational models on the other hand were in disagreement by up to 30%–40%, until the usage of Los Alamos opacities was replaced by OPAL opacities (Rogers & Iglesias 1992), reducing the discrepancy to below 10% (Moskalik et al. 1992). Systematic differences still persist at this level, as shown for example by Pietrzyński et al. (2010) who were able to confirm the pulsational mass of a Cepheid in an eclipsing binary dynamically with a precision of 1%. The evolutionary models therefore require improvement and at the same time demonstrate the continuing need for better opacities along with the reassessment of further input physics such as convection. The need for work on the opacities is a major issue encountered in the analysis of several further types of variables, including both the pulsating main sequence B and the Subdwarf B stars, and last but not least the Sun itself.

**Radial and non-radial pulsations in Cepheids.** The radial pulsations in classical Cepheids come in several flavours: The most common occurrence are single modes, with the star pulsating either in the fundamental (F) or the first overtone mode (O1), but the OGLE survey in particular has added significantly to the zoo. Still in single mode, some Cepheids pulsate in the second overtone (O2), whereas double mode pulsators can show both the combination F/O1 or O1/O2, and then there are some triple mode pulsators as well as Cepheids with Blazhko-like modulations. (For the more common Blazhko effect in RR Lyrae stars, see e.g. Szabó et al. 2010, Molnár 2013, Guggenberger 2013).

Furthermore, non-radial modes have been observed by Soszyński et al. (2008) and Moskalik & Kołaczkowski (2009). Interestingly, these observed non-radial pulsations cannot be explained theoretically by linear instabilities for low degrees (Mulet-Marquis et al. 2007), so significant challenges remain even for the poster child variable stars that are the Cepheids. The precise knowledge of Cepheid parameters is of paramount interest due to their role as distance indicators. Asteroseismologically, however, Cepheids are not as profitable as other pulsators due to the overall sparsity of their modes: As a rule, more, and different, modes are better to differentially sound the inner structure of stars.

### 3.2 Hybrid pulsators

In trying to meet the need to obtain improved local and global stellar parameters, hybrid pulsators have the potential to contribute significantly to breakthroughs.

**Hybrids with solar-like oscillations.** The Sun itself is a prime example for a hybrid, where the envelope acoustic modes are measured, and the quest for gravity modes from the core is considered a major objective. The detection of g modes via a periodic structure in agreement with the predicted period separation for dipole modes has been claimed by García et al. (2007), but Appourchaux et al. (2010) can only conclude that "there is indeed a consensus [. . . ] that there is currently no undisputed detection of solar g modes".

The situation is different for Red Giants, where all modes are mixed modes and carry information from the stellar core. Such mixed modes (Bedding et al. 2010), as well as g-mode period spacings (Beck et al. 2011), are reported from observations. Beck et al. (2012) use an inverse approach to obtain results implying a fast rotation of the core. Forward modelling by Ouazzani et al. (2012) on the other hand suggests that, already at moderate rotation rates, symmetrical patterns arise around axi-symmetric modes that may be mistaken as multiplet splittings. She further cautions that at high core rotation rates the splittings are so far from symmetrical that the correct selection of modes to measure splittings can become very difficult, if not impossible.

Back on the main sequence, Antoci et al. (2011) report the detection of solar-like oscillations in a δ Scuti star, well above the temperature limit empirically determined to mark the transition between opacity-driven and solar-like pulsations by Huber et al. (2011) with some small overlap between the two. The existence of the δ Scuti - solar-like hybrid does not necessarily imply that the location of this transition must be revised. It could also be possible that the long lifetimes are due to energy transfer from the ς mechanism (Antoci, priv. comm.).

Further up the main sequence, hybrid behaviour has been predicted for the β Cep and the SPB stars (Belkacem et al. 2009b). Solar-like oscillations in a β Cep star have subsequently been reported by Belkacem et al. (2009b), but no further such objects turned up in the KEPLER data investigated by Balona et al. (2011b).

**Hybrids on the main sequence.** The δ Scuti variables show low-order g and p modes, the γ Dor variables show high-order g modes. In the overlap region between the two instability strips, four hybrids had been confirmed prior to KEPLER, e.g. with MOST; see for example Handler & Shobbrook (2002) and Rowe et al. (2006), and references therein. Several of these hybrids show intermediate-order gravity modes, challenging theory. The difficulties have only increased so far since the numbers of known hybrids has gone up significantly with detections from CoRoT and KEPLER, across a parameter space that stretches beyond the bounds of both previously established instability regions. The situation to date is that there exist no pure δ Sct or γ Dor regions. The hybrids detected from space fill the δ Sct instability strip and do not simply concentrate on the overlap region (cf. Hareter 2013). Uyttrejohoen et al. (2011) find that 23% of 171 A- and F-type stars are hybrids. While Grigahcène et al. (2010) go ahead and boldly propose a new classification scheme, Uyttrejohoen et al. (2011) stress the importance of first de-
terminating accurate physical parameters of all the stars. They continue by suggesting that a physical mechanism different from the $\kappa$ mechanism and convective blocking effects might be responsible for the occurrence of hybrids well outside the $\gamma$ Dor instability region that must be investigated.

Further peculiar combinations include the detection of $\gamma$ Dor pulsations in a roAP star, and $\gamma$ Dor and $\delta$ Scu pulsations in Ap stars (Balona et al. 2011a).

Beyond the main sequence A-F stars, a strange mode has been reported in a RR Lyrae star (Szabó et al. 2010).

A further important group are the hybrids among B stars showing $\beta$ Cep and SPB pulsations, i.e. low-order p and g modes simultaneously with high-order g modes (see e.g. Balona et al. 2011b, Jerzykiewicz et al. 2005). These hybrids, however, do mostly not occur in the overlap region, but in hotter $\beta$ Cep that exhibit additional high-order g modes. The task ahead for asteroseismology is to understand and take advantage of all these newly discovered objects.

Hybrids among compact pulsators. The compact pulsators comprise evolved objects either beyond the Red Giant Branch or post-Asymptotic Giant Branch phases. On their way to exposure of the compact core, stars evolving through the AGB stage may first become Mira variables. As post-AGB stars, they may turn into RV Tauri variables, which are Supergiants pulsating in the fundamental mode. Hydrogen-deficient post-AGB stars are discussed at the end of this subsection. Hot Subdwarf stars are bound to completely avoid the AGB phase.

The Subdwarf B stars show either rapid (sdBV,) or slow (sdBV,) pulsations, or both: the first hybrids (sdBV,) were found from the ground in the empirically determined short-period/long-period overlap temperature range (e.g. Schuh et al. 2006). From KEPLER data, Østensen et al. (2011, 2010) find that below the temperature boundary region marked by the hybrid sdB pulsators discovered from the ground, all stars pulsate when observed from space. Not unlike the situation for $\beta$ Cep stars when observed from space, where SPB pulsations are always present as soon as the sensitivity is high enough, Østensen et al. (2010) demonstrate that the only rapid pulsator in the KEPLER field also shows a low-amplitude mode in the period region of slow pulsations. Moreover, from the additional detection of rapid pulsations with low amplitudes in slowly pulsating sdB stars, located in the lower-temperature parameter range, they conclude that hybrid behaviour may be common in these stars even outside the boundary temperature region where hybrid pulsators had previously been found.

A very peculiar object among the pulsating Subdwarf B stars is the He-sdB star LS IV $-14^{1}\!116$. When Ahmad & Jeffery (2005) discovered its pulsations, they attributed them to high-order g modes. This has been confirmed by model calculations by Miller Bertolami et al. (2011), with an interesting twist: in their models, the $\epsilon$ mechanism acting in the He-burning shells is driving the pulsations. If confirmed, this would be the first 'discovery' of the $\epsilon$ mechanism in a star that corresponds to an object actually observed in reality.

For completeness, I also mention the Subdwarf O stars where new classes of variables might be emerging. Only one variable sdO is known in the field (Woudt et al. 2006), and it shows very rapid multi-frequency variations. Randall et al. (2012) report on a group of sdOV, with seismic properties closer to those of the field sdOV, in the cluster $\omega$ Cen.

Within the White Dwarf regime, all the instability regions are well separated from each other. While there are hence no hybrids, White Dwarfs have repeatedly been the targets for $\dot{P}$ measurements which are discussed in the following subsection. The pulsating White Dwarf classes are therefore introduced here.

The majority of White Dwarfs, the DAs with hydrogen-dominated atmospheres, move through the ZZ Ceti instability strip furthest down on the cooling sequence.

The hottest pulsating White Dwarfs, the GW Vir variables, belong to the two (evolutionary probably subsequent) spectral classes Wolf-Rayet-type Central Star of Planetary Nebula (WC) and PG 1159, which are believed to emerge from a second passage to the Asymptotic Giant Branch after a (late) helium flash, known as the born-again scenario. This significantly alters their surface composition: Notably, the atmospheres become hydrogen-deficient, with individual differences in the abundances of the dominant elements He, C, O and Ne. The descendants of the [WC] to PG 1159 sequence are the helium-rich DO, and later DB, spectral type objects, unless there were residual traces of hydrogen left in the PG 1159. There are no stars spectroscopically classified as DO among the GW Vir variables, so the alternative designation DOV is misleading.

The White Dwarfs with helium-dominated atmospheres of DB spectral type eventually make their passage through the DBV, or V777 Her, instability strip.

The wide variety of the hydrogen-deficient (pre-)White Dwarfs and their possible evolutionary links have been described by Werner (2011). Among them are the Hot DQ stars, from which stems the most recently introduced class of pulsating White Dwarfs, the Hot DQ pulsators (Montgomery et al. 2008). A significant fraction of them, currently five out of a total of fourteen of these rare objects, pulsate in low-order and low-degree g modes (Dufour et al. 2011, and references therein). Following latest suggestions for their evolutionary origin (Werner 2011), the Hot DQ stars may be the possible descendants of super-AGB stars, and would not have evolved through the ‘normal’ PG 1159 stage as previously considered.

In this context, it is also opportune to refer to a potential additional hydrogen-deficient post-AGB evolutionary sequence, encompassing further variable classes. If the R CrB stars are the result of a binary merger of a He- and a C/O-core White Dwarfs (instead of descending from a born-again track after a late thermal pulse), then they are probably also the progenitors of the extreme helium
stars (eHe), which subsequently evolve into O(He ) White Dwarfs. The R CrB phenomenon is characterised by a very noticeable variable behaviour consisting of irregular, sudden dimmings by up to several magnitudes, believed to result from dust formation episodes, followed by gradual recovery to the previous brightness. The R CrB stars have spectra of types F and G, the eHe stars spectra of type A and B. At lower luminosities corresponding to Giant stars, the hydrogen-deficient carbon stars (HdC) show very similar characteristics as the R CrB stars, minus the R CrB variability. All of these stars have relatively low masses below one solar mass, mostly concentrated in a compact core, but surrounded by an extended envelope that leads to a very high overall luminosity-to-mass ratio. Just as for the massive main sequence or evolved stars, this high luminosity-to-mass ratio leads to pulsational instabilities that are observed, at lower amplitudes than the R CrB phenomenon itself, in practically all R CrB stars and related objects. The contribution of strange modes is significant, in particular for the g-type or radial modes in the R CrB stars, and for the longest-period modes, as well as the short-period non-radial g modes, in the eHe stars. Finally, in between those period ranges, a third variable class is known among the eHe stars that seems to be better characterised as g modes driven by the Fe-κ mechanism.

### 3.3 Secular $\dot{P}$ effects

Period changes in pulsating stars are a common phenomenon, and the Cepheids may again serve as a first example here. A non-zero $\dot{P}$ has been reported for RR Lyrae as early as 1916 by Shapley. The potential of such measurements to constrain stellar evolution scenarios was highlighted by Eddington (1919), when he concluded from the small value of the period shortening observed for $\delta$ Cephei that it was quite inconsistent with the much larger value “demanded by the contraction theory”, leaving him “no escape from the conclusion that the energy radiated by a star comes mainly from some source other than contraction”. Later, Struve (1959) worked out how the period change in Cepheids is due to a change in radius, with increasing or decreasing effective temperature at constant luminosity. The sign of the period change therefore determines the instability crossing mode (from higher to lower temperatures or vice versa), and its absolute value can indicate whether it is the first, second, or third pass, as Turner et al. (2004) have recently shown by comparing the observed and theoretical $\dot{P}$ for a sample of 200 Cepheids. Figuratively, obtaining a $\dot{P}$ value puts an arrow of defined length and at least with a primer on the direction (negative or positive) onto an existing evolutionary track at the current location of the object, with the evolutionary track simultaneously containing its on theoretical values for $\dot{P}$. Many of them could build up something similar to a slope field for evolutionary tracks.

Assuming a detailed model for a pulsating star exists from asteroseismology, one $\dot{P}$ measurement – or better yet more for different modes – can provide an excellent test of the quality of the underlying evolutionary model and potentially constrain it even further. Such measurements are observationally expensive and, depending on the type of star, may require decades of monitoring for a conclusive result. The measured values available with components that actually represent true secular effects are therefore rather rare. They imply that we are following stellar evolution in real time; here are some examples.

For the Subdwarf B stars, a monitoring program is ongoing (Schuh et al., 2010), and results are available for three objects so far (Silvotti et al., 2002, Lutz, 2011), including preliminary evidence on the degree of their core helium exhaustion.

For the prototype of the GW Vir variables, Costa & Kepler (2008) present measurements that illustrate how complex and at the same time rewarding the interpretation of $\dot{P}$ can be. In this region, the PG 1159 stars turn around the `knee' in a log $g$-T eff diagram, and can take on both negative and positive values for $\dot{P}$.

The $\dot{P}$ values are always positive further down the cooling sequence of White Dwarfs, and since the cooling rate has gone down significantly once the ZZ Ceti instability strip is reached, the values become smaller and harder to measure. Kepler et al. (2005) have succeeded in obtaining a significant measurement for one DAV, using it to derive the C/O ratio in the core. Mulally et al. (2008) have extended the survey to a dozen further objects.

### 4 Very-low-mass stars and beyond

#### 4.1 M Dwarfs

In the centre of stars below 0.5 solar masses, the conditions for stable core helium fusion are never reached, so these stars will eventually evolve into He-core white dwarfs. Stars become fully convective below 0.25 solar masses and never evolve into Red Giants. At lifetimes of $5 \times 10^{11}$ years and longer on the main sequence, well beyond the present age of the universe, evolutionary considerations are however of no practical concern for the discussion of presently known classes of pulsating stars in this mass range.

Observationally, M dwarfs appear as quite variable stars due to their significant levels of activity. This intrinsic variability in conjunction with their relative faintness hampers the search for the signature of pulsations, and no detections have been reported so far.

In principle, solar-like oscillations are expected in any star with a surface convection zone, so on the main sequence this applies for all stars from one solar mass down, including the M dwarfs discussed here. The stochastic excitation should be even more effective once the stars become fully convective, but at the same time (in)stability against pulsations also becomes increasingly harder to compute. Although Huber et al. (2011) report some evidence for the hypothesis that increased stellar activity suppresses mode amplitudes via the magnetic fields (albeit predominantly for
Subgiants (stars), it should only be a question of sensitivity to eventually detect the stochastically excited oscillations, and the first radial-velocity searches are underway.

Additionally, as previously for many other types of stars, it has been proposed from theoretical calculations that pulsations may be excited in M Dwarfs through the $\epsilon$ mechanism. As in previous investigations, however, it turns out that the growth rates, describing the time after which the modes reach an observable amplitude, are of the same order of magnitude as the time scales of stellar evolution, i.e. the nuclear time scale. These modes are therefore unobservable as a matter of principle.

### 4.2 Brown Dwarfs

Beyond the bottom of the main sequence, the Brown Dwarfs are completely uncharted territory with respect to pulsations. The difficulties are on the one hand of observational nature – the objects are much fainter still than the M Dwarfs – but it is also theoretically difficult to explore this mass range. At the temperatures and pressures associated with the structures of Brown Dwarfs, the uncertainties in the opacities, in particular the molecular opacities, are currently still so large that the results of any stability analysis cannot be considered very reliable.

### 4.3 Giant Planets

What is true for Brown Dwarfs when it comes to observational difficulties holds even more with respect to planets in general, but the planets in our own Solar System are an exception. In particular, global oscillations have been predicted and searched for in the Gas Giant Jupiter for quite some time, and it looks like they have finally been securely detected last year by Gaulme et al. (2011). Interestingly, the SYMPA instrument, built for and used in that work, has been modelled on similar instruments (GONG and MDI/SOHO) dedicated to solar observations, with observations resulting in a two-dimensional radial velocity map. In this sense this result builds a bridges from our discussion of asteroseismology back to the complementary topic of helioseismology.

### 5 Summary

Variable stars are classified by their periods and amplitudes. In order to fully characterise the pulsational behaviour when the variations are intrinsic, it is necessary to explore properties such as driving mechanism, character of the restoring force, and mode geometry along with the structure (given by the mass and evolutionary state) of the star. For several classes of pulsators, a good understanding of the seismic properties has lead to the development of useful advanced diagnostic tools, several of which have been mentioned here for reference. Among the examples given for recent results is the finding that the Hertzsprung–Russel diagram is teeming with asteroseismic hybrids, which have been described as completely as possible, that await to be explored. Beyond the classical asteroseismology exercise of matching observed periods to those in a set of models, thus deriving stellar parameters, $P\dot{\epsilon}$ measurements have been recalled to provide an additional test for consistency between evolutionary and pulsational models. The observational difficulties as well as challenges encountered in the interpretation of observations have been showcased across a wide range of stellar masses and ages.

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