Assessment of single passage simulations for the aeroelastic stability of a multistage centrifugal compressor

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Abstract. In this paper, the aeroelastic flutter stability of an impeller is investigated numerically with single passage and full annulus URANS simulations carried out on one side on a single stage configuration including a strut and the impeller and on the other side on a multistage configuration in which two additional diffusers are considered downstream. The robustness of single passage simulations relying on the use of multiple frequency phase-lagged boundary conditions is put to the test especially in the case of the multistage configuration for which both blade passage and aeroelastic effects have to be taken into account. Comparisons between single passage simulations on the single and multistage configurations indicate that the impeller-diffuser interaction acts locally on the impeller fluid-structure interface and has only little effect on the generalized aerodynamic forces. Full annulus simulations have also been carried out to assess the accuracy and efficiency of single passage simulations which introduce additional hypotheses of periodicity in the flow. Full annulus simulations are competitive in terms of wall clock since the convergence to the periodic state is faster than with single passage simulations involving the Fourier approximation of the flow field.

1. Introduction

The numerical modelization of the aeroelastic stability of bladed rows is commonly assessed for a single isolated bladed row. Recent works have however highlighted the need to consider the interactions with adjacent bladed rows to improve the accuracy of this prediction [1, 2]. In this case, the flexible bladed row of interest experiences multiple sources of unsteadiness induced on one side from its proper vibration and on the other side from perturbations induced by the passage of adjacent bladed rows of the (multi)stage configuration. All these unsteady perturbations can be taken into account accurately when the full 360° annulus containing all blade passages are included in the multistage numerical model. Such models involve however a very large number of degrees of freedom and can not be run routinely for design purposes.

Assuming that the main unsteady perturbations are periodic, single passage simulations involving only one blade passage by row can be conducted if appropriate boundary conditions on azimuthal and stage interfaces are defined. The unsteady periodic perturbations induced by the vibration and the blade passage effects can indeed be approximated by a superposition of rotating waves, or spinning modes, with their own frequency and azimuthal wave number.

Phase-lagged boundary conditions taking advantage of this periodicity have been introduced for a long time [3] and later improved [4, 5] to deal with a single perturbation using Fourier
approximations of the flow fields on the boundaries. The generalization of these boundary conditions to multiple unsteady perturbations has been introduced by He [4]. These Multiple Frequency Phase-Lagged (MFPL) boundary conditions have then been considered for multistage 3D configurations to capture the blade passage effects [6, 7] and to assess the aeroelastic stability of a single stage flexible contrafan [8]. In these cases, only two unsteady sources of unsteadiness are involved in each blade row.

The present paper focuses on the use of MFPL boundary conditions in the case of a 3D multistage research centrifugal compressor whose impeller is flexible. Two single passage configurations including a single stage or multiple stages are considered. In the last case, up to three unsteady perturbations are experienced by the impeller: two blade passage effects and the vibration effect. Both single and multistage single passage models are compared in terms of pressure fluctuations monitored at several locations through the vein and also in terms of Generalized Aerodynamic Forces (GAF) which are of prime importance for the evaluation of the aeroelastic stability. Results from these single passage models are also compared to reference numerical results obtained from the full 360° annulus multistage model which does not rely on the MFPL boundary conditions and consequently does not assume any periodicity of the flow.

2. Numerical models for the aeroelastic problem

2.1. Single stage and multistage configurations

Two single passage configurations presented in Fig. 1 are considered to compare the influence of downstream blade rows on the impeller GAFs: the single stage configuration contains a strut and the impeller, whereas the multistage configuration has two additional diffusers downstream. The impeller is the only rotating part and is composed of a main blade and a splitter blade shown respectively in blue and orange in Fig. 1. The single stage configuration has an artificially extended outlet to avoid spurious interactions with the impeller wakes.

![Figure 1: Single stage (left) and multistage (right) single passage configurations](image)

The fluid volume is discretized with 5.3 M cells for the single stage configuration and 6 M cells for the multistage configuration. The full 360° configuration contains 56.6 M cells. In order to monitor the computations and to analyze locally the spectral content of the perturbations propagating through the different blade rows, 38 probes are defined in the channel and on the blade surfaces around the half of the vein height.

2.2. Finite Volumes fluid dynamics model

The flow through the engine is computed with elsA CFD software [9] which solves the Unsteady Reynolds Average Navier-Stokes (URANS) equations formulated with an Arbitrary Lagrangian-
Eulerian framework to deal with mesh deformation induced by the structural vibration of the impeller. The transport equations written in the rotating frame of reference using the relative velocity read for each cell after numerical discretization with Finite Volumes:

$$\frac{d}{dt}(VW) = -\sum_{i=1}^{6} F(W, W_i, s) \cdot n_i + VT$$

with $W$ the vector of conservative and turbulent variables, $F$ the numerical flux approximation through the $i$-th cell face with normal $n_i$, $s$ the grid velocity due to the mesh deformation, $T$ the numerical source term approximation and $V$ the control cell volume. Fluxes are approximated here by the 2nd order Roe’s scheme with minmod limiter and Harten entropic correction. The fluid is a perfect gas and turbulent effects are modeled with the $k - \ell$ Smith model [10].

Adiabatic wall boundary conditions are applied on the hub, the shroud and the blade surfaces. A subsonic injection boundary condition is prescribed to the inlet surface of the air intake with aerodynamic conditions corresponding to the standard atmosphere at sea level ($p_{in} = 101,325$ Pa and $T_{in} = 288.15$ K). The outlet boundary condition is a constant prescribed pressure for the stage configuration and a radial equilibrium of pressure for the multistage configuration. The outlet pressure $p_{out}$ is set so as to obtain a similar operating point with all configurations.

Azimuthal boundary conditions are periodic joins for steady computations but MFPL boundary conditions are used in the unsteady case. Blade rows interfaces are treated as mixing planes in the steady case but are replaced by stage MFPL boundary conditions in the unsteady case. For 360° simulations, azimuthal boundaries reduce to matching join boundary conditions between the different passages and stage interfaces are treated as mismatched sliding interfaces.

Unsteady simulations are time-integrated with Gear’s algorithm and the physical time step is set to sample one revolution of the impeller at the rotation speed $\Omega$ with 2,000 iterations. At least 15 Gear subiterations are performed at each time step. Aeroelastic computations are conducted with a harmonic motion prescribed to the flexible fluid-structure interface $\Gamma$ of the impeller blade and hub surfaces. The displacements on $\Gamma$ are defined with the restriction of the structural mode shape $\varphi_{\Gamma}$ to the fluid-structure interface as:

$$u_{\Gamma}(t) = \Re\{\varphi_{\Gamma} q(t)\} \quad \text{with} \quad q(t) = q^* e^{j\omega t}$$

with $q^*$ the amplitude of motion, $\omega$ the pulsation of the prescribed motion and $\epsilon = \pm 1$ to generate forward/backward displacement waves. The maximum physical prescribed motion is about 0.4 mm. The prescribed displacement is propagated throughout the fluid volume and the mesh is updated at each time step as a result of a mesh deformation problem that considers the fluid mesh as an artificial elastic structure [11].

2.3. Finite Elements structural dynamics model

The modal shapes used to define the structural displacements Eq. (2) for aeroelastic computations are computed with ANSYS around the pre-stressed static equilibrium state $u_s$ induced by the rotation $\Omega$. The linear elastic structure is assumed to be tuned so that cyclic symmetry boundary conditions are applied to left and right azimuthal boundaries $L_l$ and $L_r$ and only one passage has to be modeled. The structural eigenmode basis $\Phi_{n_d}$ is therefore computed for each value of the nodal diameter $n_d$ defining the azimuthal mode shape periodicity associated to the interblade phase angle $\sigma_{n_d} = n_d \beta$ with $\beta = 2\pi/N_s$ the angular sector of a single passage and $N_s$ the number of passages of the whole blade row. The structural eigenproblem discretized by Finite Elements with cyclic symmetry boundary conditions reads [12]:

$$\begin{cases}
K(u_s, \Omega)\Phi_{n_d} = M\Phi_{n_d}\omega^2 \\
\Phi_{n_d}|_{L_l} = \Phi_{n_d}|_{L_r} e^{j\sigma_{n_d}}
\end{cases} \quad \text{for} \ n_d = 0, \ldots, N_s$$
with $K$ and $M$ the stiffness and mass matrices, $\Phi_{nd} = [\varphi_{nd}^{(1)}, \ldots, \varphi_{nd}^{(n_m)}]$ the eigenmode basis and $\omega = \text{diag}(\omega_{nd}^{(1)}, \ldots, \omega_{nd}^{(n_m)})$ the diagonal matrix containing the eigenpulsation of vibration associated to the modes shapes. Because of the cyclic symmetry boundary conditions the mode shapes are complex conjugate $\Phi_{-nd} = \overline{\Phi}_{nd}$.

For aeroelastic computations with a prescribed harmonic motion, a single mode shape for a given nodal diameter is considered at once and the restriction of the mode shape to the fluid-structure interface $\Gamma$ is interpolated from the structural mesh to the fluid mesh: $\varphi_{\Gamma} = \mathcal{F}\{\varphi_{nd}^\Gamma|\}$ with $\mathcal{F}\{\}$ the transfer operator. For 360° aeroelastic computations the mode shape has to be duplicated for all passages; for that purpose the cyclic symmetry condition is used and the duplication on each passage involves the specific nodal diameter value the mode shape has been computed for; the mode shape on each sector $S_k$ is computed from the mode shape on the reference sector $S_0$ with the relation $\varphi_{\Gamma}(S_k) = \varphi_{\Gamma}(S_0)e^{jk\pi n_d}$. The mode shapes are azimuthally shifted to each other and the multiplication by the complex exponential $e^{j\omega t}$ defining the physical displacement in Eq. (2) therefore produces a rotating wave of displacement.

2.4. Inputs for multiple frequency phase-lagged boundary conditions

Single passage computations with MFPL boundary conditions require user inputs to characterize the periodic phenomena which contribute possibly to the flow unsteadiness on azimuthal and stage interfaces. In the present case only blade passage effects from the directly adjacent blade rows are considered and an additional perturbation due to the vibration is considered in the impeller domain and in the strut and diffuser since the vibration wave propagates upstream and downstream. The blade passage effects as well as the vibration wave can be represented as azimuthally rotating waves, or spinning modes, of the form [8, 13]:

$$w_p(x, r, \theta, t) = \sum_{k=0}^{N_{h,p}} \Re \left\{ \hat{w}_{p,k}(x, r)e^{jk(\omega_p t + \kappa_p \theta)} \right\}$$

(4)

with their own pulsation $\omega_p$ and wave number $\kappa_p$ such that each spinning mode satisfies a phase-lagged relationship $w_p(x, r, \theta - \beta, t) = w_p(x, r, \theta, t - \tau_p)$ based on the phase shift $\sigma_p = \kappa_p \beta = \omega_p \tau_p$. The vector of continuous conservative variables on MFPL boundary conditions is finally approximated as the superposition of the different spinning modes: $w(x, r, \theta, t) = \sum_{p=1}^{N_{h,p}} w_p(x, r, \theta, t)$. The Fourier coefficients $\hat{w}_{p,k}$ associated to each spinning mode are evaluated iteratively during the numerical simulation on each MFPL boundary condition by means of the moving average procedure for multiple perturbations [4, 5].

Table 1 summarizes for each blade row the set of spinning modes denoted $\mathcal{SP} = \{\mathcal{SP}_{R_i>R_j}^{id}, \ldots, \mathcal{SP}_{R_j>R_i}^{id}\}$ such that $\text{card}(\mathcal{SP}) = N_{sp}$; the superscript “id” indicates the type of perturbation (“BPE” for Blade Passage Effect and “vib” for the vibration) and the subscript $R_j > R_i$ indicates that the perturbation is induced by the blade row $R_j$ and is observed in blade row $R_i$. Each spinning mode $\mathcal{SP}_{R_j>R_i}^{id}$ corresponding to a rotating wave Eq. (4) is defined by its pulsation $\omega_p$, its wave velocity $c_p = \omega_p / \kappa_p$ and the number of harmonic $N_{h,p}$ which are detailed in Table 1 for each blade row. The number of harmonics is chosen to have a sufficient sampling for the highest harmonics with respect to the time step used for the simulation. Note that the pulsation associated to the vibration spinning mode $\mathcal{SP}_{R_i>R_j}^{vib}$ for $i \neq j$ is shifted due to the change of relative frame when the perturbation is observed in the adjacent blade rows. No spinning mode is defined in the second diffuser $R_4$ since there is no relative motion between $R_3$ and $R_4$ and the vibration wave induced by $R_2$ is supposed to vanish before reaching $R_4$. Note finally that the spinning mode set $\mathcal{SP}$ contains only “primary” spinning modes and additional modes corresponding to interactions between these modes could be added [8].
2.5. Aerodynamic damping estimation

The aeroelastic stability of the impeller is assessed in a decoupled way for each eigenmode considered separately. Following Carta’s energy method [14], the aerodynamic damping is defined as the ratio of the aerodynamic work over a vibration cycle to the maximal kinetic energy and reduces for a prescribed modal-based motion to:

\[
\zeta = \frac{-1}{2\pi\gamma q^2} \int_{t_0}^{t_0+T_{vib}} \Re \{ \overline{q}(t)f_{ag}(t) \} \, dt \tag{5}
\]

with \( \gamma = \overline{\varphi^T K \varphi} \) and \( \overline{q} \) the generalized stiffness and velocity, \( T_{vib} = \frac{2\pi}{\omega} \) and \( f_{ag}(t) = \overline{\varphi^T f_d(t)} \) the GAF defined as the projection of the aerodynamic force on \( \Gamma \) on the structural eigenmode.

The stability of the whole impeller blade row requires the evaluation of the global GAF on the whole fluid-structure interface \( \Gamma = \bigcup_{k=0}^{N_s} \Gamma_k \). For single passage computations the GAF is only available on \( \Gamma_0 \) and has to be approximated for all other sectors. This can be achieved by means of a Fourier approximation of the GAF involving the different frequencies associated to the spinning modes considered, just like for the aerodynamic field [15]: \( f_{ag}(t) \approx \sum_{p=1}^{N_{h,p}} \sum_{k=0}^{N_{h,p}} \hat{f}_{ag,p,k} e^{j\omega p dt} \). This approximation can then be used to get an analytical expression of the integral term in Eq. (5) for the determination of the aerodynamic damping.

3. Results and discussion

Aeroelastic computations are run with a prescribed harmonic motion defined by a single mode of interest but several nodal diameter values \( n_d = 0, \pm 1, \pm 2 \) are investigated. Simulations are run for the three configurations considered here: single stage/single passage, multistage/single passage and multistage/multipassage 360°.

The time histories of a selection of probes located at the mid-chord of each blade and around the half of the vein height are presented in Fig. 2 for \( n_d = 2 \); every second column is a zoom of the last part common to all simulations. Simulations are run for only 7 revolutions of the impeller with the multistage 360° configuration but up to 15 revolutions are performed for the multistage/single passage configuration since the convergence of the Fourier coefficients for the MFPL boundary conditions is quite long. The qualitative inspection of the time histories (included those for the other nodal diameters not shown here) brings out the following points:

- both single passage models work poorly upstream in \( R_1 \) (probes #01 & #07)
- the agreement between multistage simulations (360° vs single passage) is quite fair in \( R_3 \) (probes #14 & #20) and becomes satisfactory when moving to \( R_3 \) and \( R_4 \) (probes #30 &

### Table 1: List of the spinning modes for the MFPL boundary conditions in each blade row.

| Blade Row | Spinning Modes | Pulsation \( \omega_p \) | Wave Nb. \( \kappa_p \) | Velocity \( c_p \) | \( N_{h,p} \) |
|-----------|----------------|-----------------|-----------------|-----------------|-------------|
| Strut \( R_1 \) | \( \mathcal{S}^BPE_{R_1} > R_1 \) | \( N_{s,2} \Omega \) | \( N_{s,2} \) | \( \Omega \) | 32 |
| | \( \mathcal{S}^P_{R_1} \) | \( \epsilon \omega_{n_d} + n_d \Omega \) | \( n_d \) | \( (\epsilon \omega_{n_d} + n_d \Omega) / n_d \) | 8 |
| Impeller \( R_2 \) | \( \mathcal{S}^BPE_{R_2} > R_2 \) | \( N_{s,1} \Omega \) | \( N_{s,1} \) | \( -\Omega \) | 48 |
| | \( \mathcal{S}^P_{R_2} \) | \( \epsilon \omega_{n_d} + n_d \Omega \) | \( n_d \) | \( \epsilon \omega_{n_d} / n_d \) | 24 |
| Diffuser \( R_3 \) | \( \mathcal{S}^BPE_{R_3} > R_3 \) | \( N_{s,2} \Omega \) | \( N_{s,2} \) | \( \Omega \) | 40 |
| | \( \mathcal{S}^P_{R_3} \) | \( \epsilon \omega_{n_d} + n_d \Omega \) | \( n_d \) | \( (\epsilon \omega_{n_d} + n_d \Omega) / n_d \) | 8 |
| Diffuser \( R_4 \) | — | — | — | — | — |
Figure 2: Comparison of the time histories of static pressure recorded by a selection of probes in each blade row for \( n_d = 2 \).

#37). The single stage single passage model becomes inaccurate as moving downstream towards \( R_3 \) since the blade passage effects \( R_3 > R_2 \) is not accounted for.

- the levels of fluctuation predicted by single passage simulations are overall underestimated with respect to those from 360° simulations.

The Fourier transform of the probes time histories are compared in Fig. 3. For the different probes in each blade row \( R_i \), the blade passing frequencies \( F_{i,i+1} = N_{s,i,i+1} \) and vibration frequencies \( F_{vib}^j \) potentially experienced by each probe are indicated by the gray vertical lines along with some linear combinations of frequencies \( kF_{i,i+1} \pm F_{vib}^j \) for \( k = 1, 2 \). In blade row \( R_1 \) the frequency content captured by single passage models is not satisfying (see probes #01 & #07) since the main peak of response occurs for an interaction \( F_{2,i+1} + F_{vib}^j \) between the vibration frequency and the blade passage frequency which is not taken into account by single passage models. The agreement is better in blade row \( R_2 \) in terms of frequency content. The main peaks corresponding to the vibration and blade passage effect \( R_3 > R_2 \) are detected and the associated levels of fluctuation are satisfactory. Finally in blade rows \( R_3 \) and \( R_4 \) the agreement is reasonable concerning the blade passage effects \( R_2 > R_3 \) but the level of fluctuation due to the propagated vibration wave is not correctly evaluated at the frequency \( F_{vib}^3 \).

Finally the global GAFs on the whole impeller are reconstructed for single passage models and compared to those of the 360° model for different nodal diameters, see Fig. 4a. The agreement is qualitatively good in terms of frequency content (essentially driven by the vibration frequency) and magnitude but single passage simulations induce a phase shift with respect to the 360° simulation for the nodal diameter \( n_d = 0 \). Once the global GAFs are available, the aerodynamic damping can be evaluated from the GAFs using Eq. (5) and the values obtained with the
different models are compared in Fig. 4b. The agreement is very good for the nodal diameter value $n_d = 2$ but deteriorates slightly for $n_d = 0$ and $n_d = -2$ because of the small phase shift barely visible in the time histories of the approximated GAFs.

4. Conclusion
In this paper, single and multiple stage single passage simulations have been compared to full 360° multistage simulations on a research centrifugal compressor engine. Single passage simulations are able to provide a qualitatively good trend of the aerodynamic damping, even with the single stage model. Indeed, although the blade passage effect $R_3 > R_2$ which is not accounted for with the single stage model is locally significant (see probes #20 in Fig. 3), the global GAF is well approximated since it mainly depends on the vibration frequency. Full 360° multistage simulations should however be preferred if an accurate estimation of the aerodynamic damping is required since a small phase shift in the GAF time history induces significantly different damping values. When sufficient computational resources are available 360° multistage models are competitive in terms of wall clock since the convergence is slower with single passage models because of the Fourier coefficients convergence on MFPL boundary conditions. Ongoing work is currently being carried out to investigate the sensitivity of the aerodynamic damping with respect to the temperature which has been observed experimentally [16].
(a) Comparison of the GAF for different nodal diameters

(b) Aerodynamic damping

Figure 4: Comparison of the real part of the GAF (left) and aerodynamic damping (right) computed with single/multistage single passage models vs multistage 360° model.

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