Pattern Shape Optimization of a Two Piece Brassiere Cup to Improve Its Design Efficiency*

Kotaro YOSHIDA†, Hidefumi WAKAMATSU†, Eiji MORINAGA†, Eiji ARAI†, Seiiichiro TSUTSUMI‡ and Takahiro KUBO§

A method to design the two-dimensional shapes of patterns of two piece brassiere cup is proposed. A brassiere cup consists of several patterns and their shapes are designed by repeatedly making a paper cup model and then checking its three-dimensional shape. This process requires much trial and error and would benefit from an improvement in design efficiency. The form of a brassiere is characterized by two curves: the ridge line and the lower edge of the cup. If their three-dimensional shapes are determined, the two-dimensional shape of a pattern must be designed so that its edges coincide with individual curves. When the two-dimensional shape of an edge of a paper pattern and the three-dimensional shape of a target curve are given, the surface of the pattern is uniquely determined. This means that the surface of the pattern is obtained from one edge and one target curve and another surface of the pattern is obtained from the other edge and the other target curve. Then, the pattern can be designed by optimizing both shapes of edges so that both surfaces completely overlap each other. It was experimentally verified that the edges of a designed pattern formed the target shapes.

1. Introduction

Curved surfaces - in particular, the developable surfaces formed by deforming flat surfaces without expanding or contracting are widely used in the shipbuilding field, the automotive field, and so on. In such fields, it is very important to design the two-dimensional shapes of plates which can form the required three-dimensional shape by being bent and joined. In this paper, we focus on production of brassieres, which is related to design of the two-dimensional shapes of plates. Many kinds of brassieres are developed to meet various demands, such as to enhance a woman’s breast size, to create cleavage, or to minimize breast movement. For such demands, the cup shape of a brassiere is very important. A brassiere cup is formed by several pieces of cloth called patterns.

The form of a brassiere cup can be characterized by two curves as shown in Fig. 1. One is the ridge line of the cup corresponding to the outline of a bust on a transverse plane. The other is the lower edge of the cup corresponding to the boundary between a breast and a body. This indicates that design of the cup is equal to design of these lines.

The form of the cup is fixed by fashion designers and pattern makers. Fashion designers draw an image of the brassiere and pattern makers determine the shapes of patterns, to make the image substantial without geometric discrepancy. Pattern makers of the brassiere cup must determine the shapes of patterns considering deformation of a breast and the cup itself when the brassiere is worn. Conventionally, they check the shapes with a paper model of the cup and modify them repeatedly to realize the designed three-dimensional shape. In order to improve the design efficiency of brassieres, it is required to reduce such
trial and error as much as possible.

As a cup model of a brassiere is made of paper, each pattern can be assumed to be inextensible. This means that the cup model consists of several developable surfaces. Many researches with respect to modeling of the shapes of developable surfaces have been done[1–6]. Especially in studies on brassieres, Wakamatsu et al. proposed a method to predict the three-dimensional shape of a paper cup model when the two-dimensional shapes of patterns are given[7,8]. In[7,8], to improve brassiere's design efficiency, the three-dimensional shape of a virtual paper model is determined by minimizing its potential energy. As a result, evaluation of the patterns becomes possible without actually creating a paper model. However, even if the shape of a paper model is obtained, it remains unknown how designers modify the two-dimensional pattern shape when the cup shape doesn’t satisfy designers’ demands. Thus, repetitive modification of the patterns is still required to obtain the target three-dimensional shape of the cup. Ito et al. developed a paper model CAD system by the theory of developable surfaces[9]. If a three-dimensional curve means that the cup model consists of several developable surfaces. Many researches with respect to modeling of the shapes of developable surfaces have been done[1–6]. Especially in studies on brassieres, Wakamatsu et al. proposed a method to predict the three-dimensional shape of a paper cup model when the two-dimensional shapes of patterns are given[7,8].

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2. Modeling of A Deformed Pattern

In this section, we aim to explain that a deformed pattern can be uniquely determined using several parameters and their functions. As shown in Fig. 2, a two piece brassiere cup is composed of an upper pattern, an lower pattern, and a lower wire. The design of the brassiere is determined by the three-dimensional line joining the upper pattern and the lower pattern. This line is referred to as the joint line in this paper. Determining the shape intended by a designer is equivalent to determining the shape of this line. Therefore, the goal of this paper is to propose a method to derive the two-dimensional shape of the lower cup when the shape of the lower wire and the joint line are given. Note that each pattern to be designed is made of paper not cloth. Therefore, we can assumed that it is inextensible.

First, we formulate the two-dimensional shape of the lower pattern. As shown in Fig. 3, let $s$ be the distance along the lower edge and $L_L$ be the total length of the lower edge. The $v$-axis is set on the line connecting the start and end points of the lower edge of the lower pattern, and the $w$-axis is set so as to be perpendicular to that line. Let $\mu_L$ and $\lambda_L$ be the angle between the tangential direction of the lower edge and the $v$-axis and the curvature of the lower edge, respectively. $\mu_L$ is obtained as follows when $\lambda_L$ is given:

$$\mu_L = \mu_{L_0} + \int_0^s \lambda_L ds,$$

where $\mu_{L_0}$ represents the angle of the lower edge at $s = 0$. The surface coordinates $v_{LE} = (v_L(s), w_L(s))^T$ representing the two-dimensional shape of the lower edge are expressed as follows:

$$v_L(s) = \int_0^s \cos \mu_L ds,$$

$$w_L(s) = \int_0^s \sin \mu_L ds.$$

If the curvature of the lower edge $\lambda_L(s)$ and the angle at its start point $\mu_{L_0}$ are given, the two-dimensional shape of the lower pattern can be determined. The two-dimensional shape of the upper edge of the lower pattern also can be determined by the curvature $\lambda_U(u)$ and the angle at the start point $\mu_{U_0}$.

Next, we formulate the deformed shape of the lower pattern. The object coordinate system $P_L(s)$-$\xi_L\eta_L\zeta_L$ is set on the lower edge of the lower pattern so that the $\xi_L$-axis coincides with the tangential direction of the lower edge and the $\eta_L$-axis always coincides with the normal direction of the pattern as shown in Fig. 4.
pressed as follows:

The orientation of the object coordinate system is changed by deformation of the pattern. Then, the infinitesimal displacement vector of each axial direction can be described by use of the infinitesimal rotational ratios \( \omega^L \) and \( \omega^L \), and the geodesic curvature \( \lambda_L \) of the lower edge as follows:

\[
\begin{pmatrix}
\xi_L' \\
\eta_L' \\
\zeta_L'
\end{pmatrix} =
\begin{pmatrix}
0 & \omega^L_L & -\lambda_L \\
-\omega^L_L & 0 & \omega^L_L \\
\lambda_L & -\omega^L_L & 0
\end{pmatrix}
\begin{pmatrix}
\xi_L \\
\eta_L \\
\zeta_L
\end{pmatrix},
\]

where a prime mark denotes a derivative of \( s \). Note that \( \omega^L_L \) and \( \omega^L_L \) are represented as shown Fig. 5.

Then, the tangent vector of the lower edge is expressed as follows:

\[
\zeta_L(s) = \zeta_{L0} + \int_0^s (\lambda_L \xi_L - \omega^L_L \eta_L) \, ds,
\]

where \( \zeta_{L0} \) represents the tangent vector at the start point of the lower edge. Further, by integrating eq. (4), the spatial coordinates \( x_L(s) \) can be obtained as follows:

\[
x_L(s) = x_{L0} + \int_0^s \zeta_L(s) \, ds,
\]

where \( x_{L0} \) represents the position at the start point of the lower edge.

In general, a curved surface \( x(s,t) \) is specified by the maximum normal curvature \( \kappa_1 \) and its direction \( d_1 \), and the minimum normal curvature \( \kappa_2 \) and its direction \( d_2 \). They are referred to as principal curvatures and principal directions. The surface is also characterized by the following Gaussian curvature \( K \) and the mean curvature \( H \) defined using coefficients of the first and the second fundamental forms \( E,F,G,L,M, \) and \( N \):

\[
K = \kappa_1 \kappa_2 = \frac{LN - M^2}{EG - F^2},
\]

\[
H = \frac{\kappa_1 + \kappa_2}{2} = \frac{EN - 2FM + GL}{2(EG - F^2)}.
\]

Note that these coefficients are expressed by functions of \( s \) and \( t \). When \( s \) is equal to the arc length of the deformed pattern edge and the direction of \( t \) always cross at right angles, coefficients of the first fundamental form are represented as following equations:

\[
E = \zeta_L \cdot \xi_L = 1,
\]

\[
F = \zeta_L \cdot \xi_L = 0,
\]

\[
G = \xi_L \cdot \xi_L = 1.
\]

Also, coefficients of the second fundamental form are represented as following equations:

\[
L = \zeta_L \cdot \eta_L = -\omega^L_L,
\]

\[
M = \zeta_L \cdot \eta_L = \omega^L_L.
\]

\( N \) means the curvature of the \( \xi_L \)-axis direction and the partial derivative of the \( \xi_L \) with respect to \( t \) is required to calculate it. However, \( t \) isn’t defined clearly in this paper. So, \( N \) cannot be expressed by any functions of \( s \). Then, Gaussian curvature \( K \) and the mean curvature \( H \) are described, with \( N \) remaining as it is, by

\[
K = -\omega^L_L N - \omega^L_L, \\
H = \frac{-\omega^L_L + N}{2}.
\]

If the minimum normal curvature \( \kappa_2 \) at any point on a surface is equal to zero, this surface can be flattened without its expansion or contraction. Then, \( K = 0 \) and \( 2H = \kappa_1 \). Furthermore, \( N \) and \( \kappa_1 \) are calculated as follows:

\[
N = -\frac{\omega^L_L}{\omega^L_L},
\]

\[
\kappa_1 = -\frac{\omega^L_L + \omega^L_L}{\omega^L_L}.
\]

Such surface is referred to as a developable surface.
eratrix. Let $\alpha_L$ be the angle between the generatrix and the $\xi_L$-axis at a point $P_L(s)$ as shown in Fig. 6 and let $\beta_L$ be $\tan\alpha_L$. This angle $\alpha_L$ is referred to as the rib angle in this paper. By use of this angle, principal directions are described as follows:

\[ d_1 = \xi_L \cos \alpha_L + \eta_L \sin \alpha_L, \]
\[ d_2 = -\xi_L \sin \alpha_L + \eta_L \cos \alpha_L. \]

As a pattern only bends in the principal direction $d_1$, an infinitesimal displacement vector of the normal direction of the pattern $d\eta_L = -\omega_L^2 \xi_L ds + \omega_L^2 \xi_L ds$ is parallel to the $d_1$ direction.

Then, the relationship between the infinitesimal rotational ratios and the rib angle can be represented as follows:

\[ \frac{\omega_L^2}{\omega_L^2} = \tan \alpha_L = \beta_L. \]

Therefore, $\omega_L^2$ can be represented as a function of $\omega_L^2$ and $\beta_L$ as follows:

\[ \omega_L^2 = -\beta_L \omega_L^2. \]

By substituting eq. (15) into eq. (12), a principal curvature $\kappa_1$ is also described as follows using $\omega_L^2(s)$ and $\beta_L(s)$:

\[ \kappa_1 = -\omega_L^2 (1 + \beta_L^2). \]

In general, a developable surface can be determined by the value of principal curvature and its direction. Therefore, the shape of a pattern can be uniquely determined using two functions $\beta_L$ and $\omega_L^2$ [7].

3. Design Method of The Pattern Shape

3.1 Surface Determination from The Lower Wire

In the previous section, we explained that the developable surface can be determined using two functions: $\omega_L^2$ and $\beta_L$. In this section, we present how these two functions can be represented using some parameters characterizing the given three-dimensional curve and two-dimensional pattern shape. Let us consider joining of the lower edge of the lower pattern to the lower wire. We assume that the shape of the lower wire is given as a planar curve. This means that the curvature of the lower wire is predetermined. Furthermore, the lower wire is assumed to curve in the $zy$-plane. Let $s$ be the distance along the lower wire. The object coordinate system $P_{LW}(s)\xi_{LW}\eta_{LW}\zeta_{LW}$ is set on the lower wire. Then, the infinitesimal displacement vector of each axial direction can be described using the curvature $\kappa_{LW}$ of the lower wire as follows:

\[ \begin{pmatrix} \xi_{LW}' \\ \eta_{LW}' \\ \zeta_{LW}' \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\kappa_{LW} \\ 0 & 0 & 0 \\ \kappa_{LW} & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_{LW} \\ \eta_{LW} \\ \zeta_{LW} \end{pmatrix}. \]

The shape around a point $P_{LW}(s)$ on the lower edge of the lower cup is coincided with that around a point $P_{LW}(s)$ to form a cup as shown in Fig. 7. First, the $\zeta_{LW}$-axis and the $\xi_L$-axis are aligned. Fig. 8 shows a vertical plane to the $\zeta_L$-axis after joining of the edge and the wire.

![Fig. 6 Definition of the rib angle](image)

![Fig. 7 Joining of the lower edge of the lower pattern to the lower wire](image)

Let $\delta_L$ be an angle between the $\xi_{LW}\eta_{LW}$-plane and the $\zeta_L\xi_L$-plane at the point $P_{LW}(s)$ as shown Fig. 8. From eq. (18), the displacement of the tangent vector of the lower wire whose length is $ds$ is described by

\[ \Delta(\zeta_{LW}ds) = \kappa_{LW} \xi_{LW} \eta_{LW} ds^2. \]

From eq. (3), the displacement of the tangent vector of the lower edge is represented as follows by using the unit vector of the $\xi_{LW}$-axis direction and that of $\eta_{LW}$-axis direction:

\[ \Delta(\zeta_{LW}ds) = (\lambda_L \xi_L - \omega_L^2 \eta_L)ds^2 \]
\[ = (\lambda_L \cos \delta_L + \omega_L^2 \sin \delta_L) \xi_{LW} ds^2 \]
\[ + (\lambda_L \sin \delta_L - \omega_L^2 \cos \delta_L) \eta_{LW} ds^2. \]

In order that both displacement coincide with each other, the following equations must be satisfied:

\[ \lambda_L \cos \delta_L + \omega_L^2 \sin \delta_L = \kappa_{LW}, \]
\[ \lambda_L \sin \delta_L - \omega_L^2 \cos \delta_L = 0. \]

Note that $\xi_{LW}$ coincides with a principal normal vector of the curve in this case, which means the angle $\delta_L$ is the angle between $\xi_L$ and a principal normal vector. From eq. (21), the normal curvature $\omega_L^2(s)$ and the angle $\delta_L$ can be represented as follows:
This means that the lower pattern must tilt at the angle \( \delta_L \) and bend with the curvature \( \omega^L_\xi(s) \) to join its lower edge to the lower wire at the point \( P_L(s) \).

Next, let us consider the torsion of the lower pattern. From eq. (22), the angle \( \delta_L \) can change when the edge curvature and/or the wire curvature change. As shown in Fig. 9, let \( \delta_L + d\delta_L \) be the angle between the \( \xi_{LW} \)-plane and the \( \zeta_L \xi_L \)-plane at a point \( P(s+ds) \).

Fig. 9 Torsion of the lower pattern at point \( P_L(s+ds) \) on the lower edge

This means that the \( \zeta_L \xi_L \)-plane rotates \( d\delta_L \) around the \( \zeta_L \)-axis while the coordinate system moves \( ds \) along the lower wire. It is equal to \( \omega^L_\xi ds \). Therefore, the following equation is obtained:

\[
\omega^L_\xi = \delta_L = -\frac{\lambda_L' \kappa_{LW} - \lambda_L \kappa'_{LW} \xi_{LW} - \lambda_L' \kappa_{LW} - \lambda_L \kappa'_{LW} \xi_{LW}}{\kappa_{LW} \sqrt{\kappa_{LW}^2 - \lambda_L^2}}.
\] (22)

Then, \( \beta_L \) can be expressed as follows:

\[
\beta_L = \frac{\lambda_L' \kappa_{LW} - \lambda_L \kappa'_{LW} \xi_{LW}}{\kappa_{LW} (\kappa_{LW}^2 - \lambda_L^2)}. \] (23)

Thus, the normal curvature \( \omega^L_\xi(s) \) and the tangent of the rib angle \( \beta_L(s) \) of the lower pattern can be described by use of the curvature of the lower edge \( \lambda_L(s) \) and the curvature of the lower wire \( \kappa_{LW}(s) \). Consequently, the shape of the lower pattern can be determined uniquely when \( \lambda_L(s) \) and \( \kappa_{LW}(s) \) are given.

3.2 Surface Determination from The Joint Line

Next, let us consider joining of the upper edge of the lower pattern and the joint line. We assume that the shape of the joint line is given as a three-dimensional curve. Let \( u \) be the distance along the joint line. The object coordinate system \( Q_{UW}(u) - \xi_{UW} \eta_{UW} \zeta_U \) is set on the joint line. Then, the infinitesimal displacement vector of each axial direction can be described as follows:

\[
\begin{pmatrix}
\dot{\xi}_{UW} \\
\dot{\eta}_{UW} \\
\dot{\zeta}_{UW}
\end{pmatrix} =
\begin{pmatrix}
0 & \omega^U_{\xi} & -\omega^U_{\eta} \\
-\omega^U_{\eta} & 0 & \omega^U_{\zeta} \\
\omega^U_{\eta} & -\omega^U_{\zeta} & 0
\end{pmatrix}
\begin{pmatrix}
\xi_{UW} \\
\eta_{UW} \\
\zeta_{UW}
\end{pmatrix}.
\] (25)

where a dot mark denotes a derivative of \( u \), and \( \omega^U_{\xi} \) and \( \omega^U_{\eta} \) represent the infinitesimal rotational ratios around the \( \xi_{UW} \)-axis and the \( \eta_{UW} \)-axis respectively. Rotation around the \( \xi_{UW} \)-axis is assumed to be equal to zero constantly. Let \( \kappa_{UW} \) be the curvature of the joint line. It is represented as follows:

\[
\kappa^2_{UW} = \omega^2_{\xi} + \omega^2_{\eta}.
\] (26)

The shape around a point \( Q_U(u) \) on the upper edge of the lower pattern is coincided with that around a point \( Q_{UW}(u) \) to form a cup as shown in Fig. 10. Fig. 11 shows a vertical plane to the \( \zeta_U \)-axis after joining of the edge and the line. Then, from Fig. 11, the following equation must be satisfied:

\[
\omega^2_{\xi} + \omega^2_{\eta} = \omega^2_{\xi} + \lambda^2_U.
\] (27)

Fig. 10 Joining of the upper edge of the lower pattern to the joint line

In Fig. 11, let \( \varphi_U \) be the angle between \( \eta_{UW} \) and a principal normal direction and \( \delta_U \) be the angle between \( \xi_U \) and a principal normal direction. In the case of the upper edge joining, not only the orientation of the coordinate system \( Q_{U}(u) - \xi_U \eta_U \zeta_U \) but also the direction of the curvature of the joint line \( \kappa_{UW} \) changes while the coordinate system moves along the joint line.
Therefore, as shown in Fig. 12, \( \omega^U_\zeta(u) \) is described as follows:

\[
\omega^U_\zeta = \lambda_U \kappa_U W - \kappa_U W \lambda_U - \omega^U_\eta \omega^U_W - \omega^U_W \omega^U_\xi - \omega^U_\eta \omega^U_\xi - \omega^U_W \omega^U_\xi \]

\[
\kappa^2_U W \sqrt{\kappa^2_U W - \lambda^2_U},
\]

then, \( \beta_U \) can be expressed as follows:

\[
\beta_U = \frac{-\lambda_U \kappa_U W - \kappa_U W \lambda_U}{\kappa_U W (\kappa_U^2 W - \lambda_U^2)} + \frac{\omega^U_\eta \omega^U_W - \omega^U_W \omega^U_\xi - \omega^U_\eta \omega^U_\xi - \omega^U_W \omega^U_\xi}{\sqrt{\kappa^2_U W - \lambda^2_U}},
\]

![Fig. 11 Bend of the lower pattern at point Q_U(u) on the upper edge](image1)

![Fig. 12 Torsion of the lower pattern at point Q_U(u+du) on the upper edge](image2)

Consequently, the shape of the lower pattern can be determined uniquely when the curvature of the upper edge \( \lambda_U(u) \) and the curvature of the joint line \( \kappa_U W(u) \) are given.

### 3.3 Optimization of The Edge Shapes to Form A Unique Surface

We propose a method to design the shape of a lower pattern of a two piece brassiere cup. Let us assume that the curvature of the lower wire \( \kappa_{LU} \) and the curvature of the joint line \( \kappa_{LUW} \) are given. If the curvature of the lower edge of the lower pattern \( \lambda_L \) is determined, the curved surface of the lower pattern, which corresponds to a developable surface, can be uniquely obtained from \( \kappa_{LUW} \) and \( \lambda_L \). If the curvature of the upper edge \( \lambda_U \) is determined, the surface of the pattern can be uniquely obtained from \( \kappa_{LUW} \) and \( \lambda_U \). In order that these two surfaces coincide with each other, any generatrix determined by \( \kappa_{LUW} \) and \( \lambda_L \) shown by an upward dash-dot arrow in Fig. 13 and that determined by \( \kappa_{LUW} \) and \( \lambda_U \) shown by a downward dash-dot arrow in Fig. 13 must be aligned.

Therefore, by optimizing \( \lambda_U \) and \( \lambda_L \) in order that any generatrix extending from the lower edge and that extending from the upper edge are aligned, we can design the shape of the lower pattern. If it is assumed that any generatrix from one edge does not intersect with another generatrix from the same edge, the intersection point of a generatrix from the lower edge with the upper edge is uniquely determined. That is, if the distance along the lower edge \( s \) is given, the distance along the upper edge \( u \), which is paired up with \( s \), is determined. Therefore, \( u \) can be expressed as a function of \( s \): \( u = f(s) \). Note that this function \( f \) also depends on \( \lambda_L \). Similarly, \( s \) is expressed as a function of \( u \): \( s = g(u) \). Then, for any \( s \) and \( u \), \( s = g(f(s)) \) and \( u = f(g(u)) \) must be satisfied to form a unique developable surface. Therefore, in this paper, the following function \( \sigma \) is introduced as the objective function:

\[
\sigma = \int_0^{L_L} (s - g(f(s)))^2 ds + \int_0^{L_U} (u - f(g(u)))^2 du.
\]

By obtaining the design variables \( \lambda_L \) and \( \lambda_U \) that...
minimize $\sigma$, the shape of the lower pattern is derived. Let us consider determining $f(s)$ and $g(u)$. When the distance along the lower edge $s$ is given, the surface coordinates of the lower edge $\mathbf{v}_{LE}$ is determined. Also, when the distance along the upper edge $u^*$ is given, the surface coordinates of the upper edge $\mathbf{v}_{UE}$ is determined. Let $\psi(s,u^*)$ be an angle between a $v$-axis and a straight line connecting $\mathbf{v}_{LE}$ and $\mathbf{v}_{UE}$ as shown Fig. 14. Then, it is represented as follows:

$$\psi(s,u^*) = \tan^{-1}\frac{w_U(u^*) - w_L(s)}{v_U(u^*) - v_L(s)}. \quad (31)$$

We introduce the following equation:

$$\varepsilon(s,u^*) = \psi(s,u^*) - \left(\alpha_L(s) - \mu_L(s) + \frac{\pi}{2}\right). \quad (32)$$

When the straight line corresponds to a generatrix, that is, when $u^* = f(s)$, $\varepsilon(s,u^*) = 0$. From the shape characteristics of the lower pattern that a line expanding from any point on the lower edge $P_L(s)$ intersects with the upper edge at one point, $\varepsilon(s,u^*) > 0$ if $u^* < f(s)$ and $\varepsilon(s,u^*) < 0$ if $u^* > f(s)$. Namely, $\varepsilon(s,u^*)$ is a monotone decreasing function with respect to $u^*$. Therefore, we can determine numerically the distance $u^*$ satisfying $u^* = f(s)$ for each $s$ with the bisection method. By linearly interpolating such $u^*$ for various $s$, the function $f(s)$ is obtained approximately. The function $g(u)$ also can be obtained with a similar procedure.

$$\varepsilon(s,u^*) = \psi(s,u^*) - \left(\alpha_L(s) - \mu_L(s) + \frac{\pi}{2}\right). \quad (32)$$

![Fig. 14 Definition of $\psi$](image)

Next, let us consider constraints in order to form a unique developable surface. A constraint such that the upper edge and the lower edge are connected at the end point is described as follows:

$$\mathbf{v}_{LE}(L_L) = \mathbf{v}_{UE}(L_U). \quad (33)$$

The spatial distance between a point on the lower wire $P_{LW}(s)$ and a point on the joint line $Q_{UW}(f(s))$ always coincide with the surface distance between a point on the lower edge $P_L(s)$ and a point on the upper edge $Q_U(f(s))$ because both correspond to the length of the same generatrix. Therefore, the following constraints must be satisfied:

$$d_{LE}(s) - d_{LW}(s) = 0 \hspace{1em} \forall s \in [0,L_L], \quad (34)$$

$$d_{UE}(u) - d_{ UW}(u) = 0 \hspace{1em} \forall u \in [0,L_U], \quad (35)$$

where $d_{LE}(s), d_{LW}(s), d_{UE}(u), d_{UW}(u)$ are defined as these equations:

$$d_{LE}(s) = |\mathbf{v}_{LE}(s) - \mathbf{v}_{UE}(f(s))|, \quad (36)$$

$$d_{LW}(s) = |\mathbf{x}_{LW}(s) - \mathbf{x}_{UW}(f(s))|, \quad (37)$$

$$d_{UE}(u) = |\mathbf{v}_{UE}(u) - \mathbf{v}_{UE}(g(u))|, \quad (38)$$

$$d_{UW}(u) = |\mathbf{x}_{UW}(u) - \mathbf{x}_{LW}(g(u))| \quad (39)$$

Furthermore, a position vector from the point $P_L(s)$ to the point $Q_U(f(s))$, which corresponds to a generatrix, must be aligned with the direction $\mathbf{d}_2(s)$ as shown in Fig. 15. This constraint is described as follows:

$$\mathbf{x}_{UW}(f(s)) - \mathbf{x}_L(s) = d_{LE}(s)\mathbf{d}_2(s) \quad \forall s \in [0,L_L]. \quad (40)$$

![Fig. 15 Constraints that with respect to the direction of a generatrix](image)

From eq. (22) and eq. (27), the following constraints must be satisfied:

$$-\kappa_{LW} \leq \lambda_L \leq \kappa_{LW}, \quad \quad -\kappa_{UW} \leq \lambda_U \leq \kappa_{UW}. \quad (41)$$

Therefore, the shape of the lower pattern can be designed by minimizing the objective function described by eq. (30) under constraints by eqs. (33), (35), (40), (41). In this paper, in order to satisfy constraints described by eq. (41) necessarily, we introduce functions $\gamma_L(s)$ and $\gamma_U(u)$ representing $\lambda_L$ and $\lambda_U$ as follows:

$$\lambda_L = \frac{2\kappa_{LW}}{\pi} \tan^{-1} \gamma_L, \quad \lambda_U = \frac{2\kappa_{UW}}{\pi} \tan^{-1} \gamma_U. \quad (42)$$

$\gamma_L$ and $\gamma_U$ are represented by the weighted sum of the base functions by using Ritz method [10], as follows:

$$\gamma_L = \sum_{i=1}^{n} a_i \ell_i^L(s), \quad \quad \gamma_U = \sum_{i=1}^{n} a_i \ell_i^U(u). \quad (43)$$

In this paper, the base functions are expressed as follows based on Fourier series expansion:
\[ e^L_i = \begin{cases} \frac{1}{L^L} & (i = 1) \\ \sin \left( \frac{(i-1)\pi}{2L^L} \right) & (i = 2k - 1) \\ \cos \left( \frac{(i-1)\pi}{2L^L} \right) & (i = 2k) \quad (k = 2, \ldots, 8), \end{cases} \] 

\[ e^U_i = \begin{cases} \frac{1}{L^U} & (i = 1) \\ \sin \left( \frac{(i-1)\pi}{2L^U} \right) & (i = 2k - 1) \\ \cos \left( \frac{(i-1)\pi}{2L^U} \right) & (i = 2k) \quad (k = 2, \ldots, 8). \end{cases} \] 

From the above, the optimization problem that minimizes \( \sigma \) can be reduced to a nonlinear programming problem on the weight vectors: \((a^L_1, \ldots, a^L_n), (a^U_1, \ldots, a^U_n)\). Therefore, by solving this problem, it is possible to obtain the shape of the pattern. In this paper, optimization problem was solved using the Nelder-Mead method and the multiplier method.

4. Experimental Verification

To verify the validity of the proposed design method, we compared the computed shape of the lower pattern and its measured shape. In the case of an actual brassiere cup, the shapes of the lower wire and the joint line are not represented as functions of distances along those lines. So, we designed the shapes of the lower wire and the joint line for this experiment. The curvature of the lower wire \( \kappa_{LW} \) and the infinitesimal rotational ratios of the joint line \( \omega^L_{UW} \) and \( \omega^U_{UW} \) were as follows:

\[ \kappa_{LW} = 2.9, \] 

\[ \omega^L_{UW} = 5.5 \left( 1 - \frac{u}{L^U} \right), \] 

\[ \omega^U_{UW} = -0.4 \left( 1 + \cos \left( \frac{u}{L^U} \right) \right). \] 

And the total length of the lower wire \( L^L \) and that of the joint line \( L^U \) were set to 1.08 and 1.0, respectively. Their designed shapes are shown in Fig. 16. The lower wire corresponds to a circular arc. The joint line overhangs in the z-axis negative direction near the origin point imitating the actual joint line of a brassiere cup.

![Fig. 16 The designed shape of lower wire and joint line](image)

![Fig. 17 The optimized curvatures of the lower edge and upper edge](image)
By using eqs. (1), (2), the optimized shape of the lower pattern can be obtained as shown in Fig. 18. In Fig. 18, it is shown that the curvature of the lower edge is reversed near the left endpoint to realize overhang of the pattern. And Fig. 19 shows its simulated shape when its lower edge is joined to the lower wire. In Fig. 19, curved lines represent the shape of the lower wire and the joint line and straight lines represent generatrices. It is shown that a developable surface can be obtained without generatrices intersecting each other by solving this optimization problem.

Next, we made the lower cup shown in Fig. 18 with paper, and joined it to the lower wire. After that, we measured the actual shape of the upper edge of the pattern and compared it with the computed shape of the joint line described by eqs. (47), (48). Fig. 20 shows the comparison between them. Curved lines represent the given shapes of the lower wire and the joint line and dots represent the measured shape of the upper edge.

The maximum errors are 5.6% of the maximum height in the $x$-axis direction of the pattern in the $x$-axis positive direction and 0.73% in the $x$-axis negative direction. The maximum errors are 4.1% of the maximum width in the $y$-axis direction of the pattern in the $y$-axis positive direction and 1.6% in the $y$-axis negative direction. These errors are caused by mainly mechanical factors such as the gravity exerted on the paper model and/or restoring force of a paper and an adhesive tape used to make the paper model. However, except for endpoints, the measured shape of the upper edge of the lower pattern, which is designed by use of our proposed method, coincide with the given shape of the joint line. Therefore we conclude that our proposed method in this paper is useful for efficient design of paper patterns of a two piece brassiere cup.

5. Conclusion

In this paper, a method to design the two-dimensional developed shapes of patterns of a two piece brassiere cup was proposed when its target three-dimensional shape is given. As a cup model for check of the shapes of patterns is made of paper, it is assumed that the surface of the model is composed of several developable surfaces. When the two-dimensional developed shape of an edge of a paper pattern and the target three-dimensional shape of the edge are
given, the surface of the pattern is uniquely determined. Therefore, the lower pattern can be designed by optimizing the shapes of the upper and the lower edges so that determined developable surfaces from the shape of the joint line and the lower wire coincide with each other. It was experimentally verified that the edges of a designed lower pattern using the proposed method formed the target shapes. The upper cup also can be designed when the shape of the joint line and the upper edge of a brassiere cup. Our proposed method will be useful for efficient design of not only patterns of a brassiere cup but also plate parts of a structure consisting of developable surfaces.

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Authors

Kotaro Yoshida
Kotaro Yoshida received his B.Eng. degree from Osaka University in 2017, respectively. He is doing a master's degree at Osaka University.

Hidefumi Wakamatsu (Member)
Hidefumi Wakamatsu received his B.Eng., M.Eng. and Ph.D. degrees from Osaka University, Japan, in 1993, 1994 and 2001, respectively. From 1995 to 2006, he worked as a Research Associate, Osaka University. He then became an Associate Professor at Division of Materials and Manufacturing Science, Osaka University. His research interests include handling of flexible object. He is a member of RSJ, JSME, JSPE, JWS and TMSJ.

Eiji Morinaga (Member)
Eiji Morinaga received his B.Eng., M.Eng. and Ph.D. degrees in Mechanical Engineering from Osaka University, Japan, in 2000, 2002 and 2005, respectively. From 2005 to 2007, he worked as a Designated Researcher at Center for Advanced Science and Innovation, Osaka University. He then became an Assistant Professor at Division of Materials and Manufacturing Science, Osaka University. His research interests include system design and integration in product design and manufacturing. He is a member of JSME, JSPE, JWS, JIEP and TMSJ.

Eiji Arai (Member)
Eiji Arai received his B.Eng., M.Eng. and Dr.Eng. degrees from The University of Tokyo, Japan, in 1975, 1977 and 1980, respectively. He worked as a Research Associate at Kobe University from 1980 to 1984, an Associate Professor at Shizuoka University from 1984 to 1992, an Associate Professor at Tokyo Metropolitan University from 1992 to 1995, and a professor at Osaka University from 1995 to 2018, respectively. He then became an Emeritus Professor of Division of Materials and Manufacturing Science. His research interests include intelligent CAD/CAM for mechanical design. He is a member of JSME, JSPE, JWS, TMSJ, etc.
Seiichiro Tsutsumi received his B.Eng., M.Eng., Ph.D.(Agr.) and Ph.D.(Eng.) degrees from Kyushu University, Japan, in 1997, 1999, 2002 and 2007, respectively. He worked as a Assistant Professor at Tohoku University and Kyushu University from 2002 and 2004, respectively. He then joined at Joining and Welding Research Institute, Osaka University from 2011 as an Associate Professor. His research interests include plasticity and fracture of solids. He is a member of JWS, JSCE, JSME, JASNAOE, etc.

Takahiro Kubo received his B.A. degree in Sport Sciences from Waseda University, Japan, in 2007, and M.A. degree in Arts and Sciences from Tokyo University, Japan, in 2010. He then joined Wacoal Corp. His position is in Human Science Research Center.