Assessment of an Alternative Concept for a High-Altitude Wind-Power Generator

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Abstract. To generate power from high-altitude winds, concepts using kites or planes linked to the ground with tether are in development. The most popular high-altitude wind generation concept is one using a flying wing attached to a single tether whose movement generates power by turning a winch. The usual trajectories for power generation consist of a period where the kite distance is increased, and the pulling force enlarged by figure-of-eight movements, interrupted by a pull-back phase where power is consumed. We compare that with a new concept we introduce here. It uses a triplet of tethers whose length sum is kept more or less constant using differential gears, resulting in a trajectory surface. It does not have a pull-back phase and allows to have similar power output in a closed trajectory. Moreover, starting and landing can be achieved without additional equipment when using a soft kite as wing, and keeping the wing flying without any wind is easier. Also the control can be easier, as one has more degrees of freedom in the force direction and the movement of the kite. Its disadvantages are an increased effort for the ground stations and more restrictions on the location. Also the tether’s air drag is increased. Optimal power generation is compared using an example configuration and state with given wind speed under the assumption of an optimal steering of the generators and the kites. This is done for state snapshots, for example trajectories, wind speeds, and kites.

This paper wants to make a contribution to the set of concepts developed for high-altitude airborne wind-power generators. A concept using two ropes has already been published in [1]. However, in that concept two mounts have to be on a line perpendicular to the wind direction, while in our concept, all directions are allowed, and also upward motions generate power. A nice method to calculate the properties of a tethered kite has been published in [2]. Calculation methods for the movement and forces on a kite are described in [3].

1. The New Concept
In figure [1] the principle is explained: a kite is attached to three tethers which are unreeled and retracted with three individual winches (inside the generator housing) connected with a four-way differential gear described in figure [2]. The four-way-differential gear is needed so that the retraction of one rope can be powered by the pulling of another rope without electric energy transfer, and the height is still steerable. It consists of two regular differential gears (c) which effect that the sum of rotation angles $w_1 + w_2 + w_3 + w_4$ is constant. If
(d) is fixed ($w_4$ is constant), it means that also the sum of the tether lengths which are attached to the winches (b) is constant. One sees that for every movement above a certain height, always at least two of the tethers change their length, and so the motor-generators at their winches can be used to produce power. In figure 3 the trajectory surfaces, which result if motor-generator (d) in figure 2 keeps the sum of the three tether lengths constant, are visualized. One sees that only the red surface which is near the attachment points at the ground differs significantly from a spherical shape, so the setup is essentially independent of rotation and wind direction. In figure 4 the intersection of the trajectory surfaces with the ground are shown, together with the three basepoints (one with the ground station and the other two with the guide rollers) Figure 5 shows a possible path for starting the kite.

![Figure 1](image1.png)

**Figure 1.** sketch of the overall system.

![Figure 2](image2.png)

**Figure 2.** sketch of generator mechanics. (a), (d) are electric motor-generators, (b) are winches, (c) are two coupled differential gears. $w_1...w_4$ are the rotation angles of the axes.

![Figure 3](image3.png)

**Figure 3.** trajectory surfaces resulting from keeping sum of the three lengths constant

![Figure 4](image4.png)

**Figure 4.** intersections of the trajectory surfaces with the ground, the ground stations are indicated with crosses.
In phase one, the kite is slowly pulled to an optimal starting position. In phase two it actually starts, the main pulling force being applied by the northern tether, and in phase three it slowly approaches the final trajectory where power can be generated optimally. In case of the base stations placed on three hills, landing is also very simple: the kite is just pulled to the central position, which is in plain air. Of course the hills could be replaced by small towers with a height of \( r/60 + h_{kite} \), assuming a tether tension of twenty times the kite’s weight force, with \( h_{kite} \) being the kite’s height and \( r \) the radius of the enclosing circle of the ground stations. One could argue that an offshore installation is harder with this set-up, however, for high altitudes, the power generation difference between onshore and offshore vanishes (see e.g. [4], section 3.3, figure 4).

2. Optimization problems solved for the evaluation

To evaluate the new concept, we do optimizations for the following three settings:

1. One-tether, angle of attack optimized, distance is allowed to increase,
2. Three tethers, angle of attack fixed, distance (tether length sum) fixed,
3. Three tethers, angle of attack optimized, distance (tether length sum) fixed.

We optimized for power output at given positions with the boundary condition that the kite’s force equilibrium is fulfilled (no acceleration), and the velocity’s direction is in a prescribed direction, or in the 1-tether-case, on the plane formed by the prescribed direction and the tether direction. Additional boundary conditions resulting from the physical setup are applied. We also assume that the ground station is steered in a way that the calculated optimal rope forces are in effect. The losses of the motor-generators and the gears are ignored. To be able to ignore the stretching of the tether, we use an upper limit for the tension of 50% of the tether’s strength. As the breaking elongation of the high-performance tether is 3%, we have a maximum stretch of 1.5% (See section 2.6 for details). For the lift and drag force equation, we assumed the kite being at an angle where the airspeed is perpendicular to its span.

2.1. Definitions

We optimize the following variables:

- \( \vec{F} \) Wind’s force on kite
- \( \vec{v} \) velocity of kite relative to ground
- \( v_d \) velocity component in "desired direction".
- \( v_p \) velocity component in direction showing away from the ground station’s center. (\( \vec{p}/||\vec{p}|| \))
- \( c_L \) Lift ratio (uniquely maps to angle of attack)
The following constants and functions and input variables define the optimization problem:

- $\vec{d}$: desired trajectory direction.
- $\vec{p}$: current position relative to ground station’s center.
- $\vec{p}_i$: ground station’s positions,
- $\vec{t}_i$: the vertices of an equal-sided triangle enclosed in a unit circle
- $\vec{v}_{rw}$: wind at kite’s position.
- $\vec{v}_{rw} := \vec{v}_w - \vec{v}$: - airspeed of the kite (received wind speed)
- $\alpha_{rw}$: rotation of kite around the received wind speed direction
- $\vec{v}$: projection of vector $\vec{p}$ onto the received-wind-speed-perpendicular plane
- $c_{D}(c_{L})$: drag force for lift -force function, taken from tabular data
- $m$: kite’s weight.
- $A$: kite’s area.
- $F_{max}$: maximum load on kite
- $F_{r,i}$: absolute value of tether-parallel force on tether $i$ at the ground station
- $F_{r,i}||$: absolute value of tether-parallel force on tether $i$ caused by gravity
- $F_{r,i,\perp}$: force perpendicular to tether $i$ caused by gravity and air drag, part held at top
- $F_{r,i,\perp,\text{top}}$: part of $F_{r,i,\perp}$ held at top
- $m_r$: tether’s weight per length.
- $d_r$: tether diameter

The calculations for $F_{r,\perp}, F_{r,||}$ are described in section 2.6.

### 2.2. Equations and boundary conditions

\[ c_L \varrho A \frac{||v_{rw}||^2}{2} = \frac{||\vec{v} \times v_{rw}||}{||v_{rw}||} \]  
(lift force)

\[ c_D(c_L) \varrho A \frac{||v_{rw}||^2}{2} = \frac{\vec{v} \cdot v_{rw} ||v_{rw}||}{||v_{rw}||^2} \]  
(drag force)

\[ ||\vec{F}|| < F_{max} \]  
(maximum wing load)

\[ \vec{v} = \vec{d} v_d + \frac{\vec{p}}{||\vec{p}||} v_r \]  
(velocity constraints)

Only for 1r:

\[ \vec{F} = (F_r + F_{r,i}) ||\vec{p}|| - \vec{F}_{r,\perp} \]  
(tether forces)

\[ F_r > 2||\vec{F}_{r,\perp}|| \]  
(minimum tension of tether)

Only for 3r:

\[ \vec{F} = \sum_{i=1}^{3} ((F_{r,i} + F_{r,i,\perp})) ||\vec{p}|| - \vec{F}_{r,i,\perp} \]  
(tether forces)

\[ F_{r,i} > 2||\vec{F}_{r,i,\perp}|| \]  
(minimum tension of tethers)

\[ v_p = 0 \]  
\[ c_L = \text{const} \]  
only 3r.

### 2.3. Objective function

We want to maximize the power output of the system. For 1r this is

\[ P(\vec{p}, \vec{v}, F_r) = \vec{v} \cdot \frac{\vec{p}}{||\vec{p}||} F_r \]
and for 3r it is

\[ P(\vec{p}, \vec{v}, F_r) = \vec{v} \cdot \sum F_{r,i} \frac{\vec{p} - \vec{p}_i}{\|\vec{p} - \vec{p}_i\|} \]

with \( \vec{p}_i \) being the positions of the three ground station points.

2.4. Solving

To solve the optimization problems, one can reduce the dimensions to the following variables and calculate the rest using the equations:

1r three variables: \( v_d, v_p, F_r \).

\( \vec{v} \) is calculated from \( v_d, v_p, \vec{F} \) then calculated from \( F_r \) and \( v \) using the tether force equation, \( c_L \) is calculated from the lift equation from \( \vec{F} \) and \( \vec{v} \) using the tether equations.

Only the drag force equation has to be taken care of by the optimizer.

3r three variables: \( v_d, c_L, \alpha_w \).

\( \vec{v} \) is calculated from \( v_d, v_p, \vec{F} \) then calculated from \( \vec{v} \) and \( c_L \) using the lift force and drag force equations, the tether forces are then calculated from \( \vec{F} \) and \( \vec{v} \) using the tether equations.

Only restrictions are given to the optimizer.

For 1r we used sampling with interval newton algorithm as refinement, for 3r we used subdivision with interval arithmetics with interval newton as refinement.

2.5. Properties used for the kite

We selected some typical kites (Paragliders) which are below 5.0 kg (table 1). From these

| Designation      | m(kg) | m\_carry(kg) | \( \Lambda \) | A[m^2] | c\_L/c\_D(max) |
|------------------|-------|-------------|-------------|--------|---------------|
| Axis comet 3     | 4.5   | 105         | 4.06        | 21.81  | 10            |
| Advance epsilon 7| 4.7   | 80          | 3.59        | 19.3   | 9             |
| Skywalk tequila 2| 4.8   | 80          | 3.73        | 19.6   | 8.2           |
| Trekking Bird 2016 | 4.9  | 105         | 3.96        | 22.35  | 9.3           |
| Skywalk chili 3 XS | 4.9   | 75          | 4.07        | 18.57  | 9.2           |
| Skywalk cayenne 3| 4.9   | 80          | 4.3         | 18.7   | 9.1           |

example values and the example profile, we created an example \( c_L/c_D \)-curve, using a stretch (span-to-depth ratio) \( \Lambda = 4 \), and a factor 0.7 for the lift coefficient, resulting in a maximum glide ratio between 9 and 10. The surface load is 50N/m^2 there (calculated from a payload of 100kg and an area of 20m^2 for typical paragliders) Another curve used for the big config is the profile with an artificial summand of 0.02 added to \( c_D \) instead of decreasing the lift, so again a glide ratio below ten is reached. The profile 798 which we chose to conduct the calculations with is a fat profile reproduced in [6], taken from [7], which has been studied for the critical Reynolds number and has measurement data for drag/lift coefficients for the turbulent range (see figure 6). To do the calculations also for a big kite with surface of 2000m^2 and an eight times higher surface load, we extrapolated its data linearly from the small kite: The eightfold strength results in an eightfold mass per surface, which is then \( \frac{2000 m^2 \cdot 8.5 kg}{20 m^2} = 4t \). We assume using a grid of high-performance tethers (2GPa strength, density 1g/cm^3) as enforcement
The minimum force boundary conditions for the tether result from the analysis of the elongation curve depending on parallel/orthogonal force ratio: For the tether bending we use the formula

\[ 1 - \frac{l_{\text{secant}}}{l_{\text{orig}}} \approx \frac{1}{24} \left( \frac{F_\perp}{F_\parallel} \right)^2 \]

which results from approximating the bent tether with a circular segment whose ratio is \(2\sin(x/2)/x \approx 1 - x^2/24\), and \(l_{\text{secant}}\) being the secant length and, \(l_{\text{orig}}\) the bent tether’s length. The elevation angles at the ends of the segments are \(\arctan(0.5F_\perp/F_\parallel) \approx 0.5F_\perp/F_\parallel\), so the total angle is approximated by \(x = F_\perp/F_\parallel\). For the tether’s air force calculations, we
use the assumption of the wind increasing linearly with the height, which results in a force of

\[ F_{r,\text{air} \perp} = \frac{1}{3} 1.2 \rho_{\text{air}} d l \frac{v_{r,\text{w} \perp}^2}{2}, \quad F_{r,\text{air} \perp, \text{top}} = \frac{3}{4} F_{r,\text{air} \perp} \]

with \( v_{r,\text{w} \perp} \) being the tether-perpendicular part of the kite’s airspeed. The tether’s gravity forces have also to be added, again only the top force is needed for the orthogonal part:

\[ F_{r,\text{g} \perp} = g l m_r \| (p_x, p_y) \| / \| \vec{p} \|, \quad F_{r,\text{g} \perp, \text{top}} = \frac{1}{2} F_{r,\text{g} \perp} \]

\[ F_{r,\text{g} ||} = g l m_r \| p_z \| / \| \vec{p} \|, \quad F_{r,\text{g} ||, \text{top}} = F_{r,\text{g} ||} \]

\( \vec{p} \) is here the vector from a ground station to the kite.

2.7. Trajectories
To create example trajectories with parameter \( t \), we use the vector \((a \cos(t), 1.0, a 0.3 \sin(2t))\), rotated by an elevation angle \((15^0, 30^0)\) around the x-axis and by the angle of the wind direction around the z-axis, as the tangent of a line from the origin which is intersected either with a sphere or with the surface induced by a constant sum of the three tether lengths. The factor 0.3 is a value where the maximum curvature is minimal for that 2d-curve.

3. Results
In figure 7, power has been assessed for an example trajectory at wind speed 3.5 m/s for the small kite. For sake of comparability, the one-tether-trajectory points have all been calculated at the same distance, however, the real trajectory would increase the distance. The trajectory assessed is shown on the left with A,B,C being the ground stations for the new concept and c being the ground station of the old concept. The values for \( 1r \) are calculated allowing the tether length being increased with optimal velocity. In one figure-of-eight cycle, the power curves of the concepts move between the following values:

|         | 3r | 1r |
|---------|----|----|
| min. [kW]| 1.4| 1.3|
| max. [kW]| 2.3| 2.1|

In figure 8, power has been assessed for an example trajectory at wind speed 16 m/s for the big kite. The trajectory in this case held the sum of all three tether length constant and flew the turning points upwards. For sake of comparability, the one-tether-trajectory points have all been calculated starting at the \( 3r \)’s trajectory samples, however their distance would increase in a real trajectory. The trajectory assessed is shown on the left with A,B,C being the ground stations for the new concept and c being the ground station of the old concept. The values for \( 1r \) are calculated allowing the tether length being increased with optimal velocity. In one figure-of-eight cycle, the power curves of the 3 concepts move between the following values:

|         | 3r | 3r | 1r |
|---------|----|----|----|
| min. [MW]| 3.8| 4.0| 6.1|
| max. [MW]| 8.1| 8.4| 8.1|

For the average power of the \( 1r \) concept, of course the power consumption of the pull-back-phase would have to be taken into account which has not been evaluated here.
Figure 7. Optimal power for trajectory points of the small kite, wind speed 3.5 m/s.

Figure 8. Optimal power for trajectory points of the big kite, wind speed 16 m/s.

In figure 9 the power generation and the tether forces are shown using a 20 m², 5 kg paraglider as kite at an example position at different wind speeds for the three configurations 1r, 3r, 3r. In figure 10 we show the possibility to keep the kite airborne if no wind is present, of course using power then. For the heavy kite designed for high wind loads, this seems to take too much power, but for the small light kite it seems to be a feasible solution. The pulling role is always transferred to the next rope of the three ropes in the state two ropes are parallel (in top view).
4. Discussion & Outlook

It has been shown that:

3r is comparable in terms of power output.

3r makes it possible to have a kite with constant angle of attack.

3r makes it possible to land and start the kites without additional equipment.

3r makes it possible to keep the kite airborne when no wind is present.

A key feature of 3r could be a constant power production which could be achieved by steering the distance motor-generator accordingly while keeping a closed trajectory. An optimization achieving this would proof the value of this new concept. To increase the precision of the evaluation, acceleration forces based on the trajectory should be included and whole trajectory optimizations should be done.

References

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Figure 10. Power used for a trajectory in calm air for the big (left) and small kite (right). Because of symmetry the diagrams cover only one third of the trajectory.