Structure Functions at small-$x$ from String Theory

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Main idea and results

Using AdS/CFT to study DIS from mesons

DIS at strong coupling + $\mathcal{N} = 2$ SYM / D3D7 brane model: holographic mesons

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Dual description by coupling brane excitations to a current from the boundary

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Hadronic tensor from superstring theory

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Calculations at small-$x$ (Bjorken parameter)

Our results

- Eight structure functions $F_i(q^2, x)$ for polarized vector mesons at small-$x$.
- Callan-Gross type relations: $F_2 \sim 2xF_1$ and $b_2 \sim 2xb_1$.
- Similar results for different D$p$-brane models $\rightarrow$ general results? and for QCD?

1 M. Kruczenski et all (2003).
Some Definitions

- $q = k - k'$ is the momentum transfer and $y \equiv \frac{P \cdot q}{P \cdot k}$ is the fractional energy loss of the lepton.

- $x \equiv -\frac{q^2}{2P \cdot q}$ is the Bjorken parameter ($0 \leq x \leq 1$).

- Small $x$ regime:
  
  \[ e^{\sqrt{\lambda}} \ll x \ll 1/\sqrt{\lambda}. \]

DIS is the study of the lepton-hadron scattering when $q^2 \to \infty$, with $x$ fixed.

The differential cross section is given

\[
\frac{d\sigma}{dx\,dy\,d\phi} = \frac{e^2}{16\pi^2 q^4} y l^{\mu\nu}(k, k', \epsilon) W_{\mu\nu}(P, q, \zeta),
\]

where the leptonic tensor $l^{\mu\nu} \approx 2 \left( k^\mu k'^\nu + k'^\nu k^\mu - g^{\mu\nu} k \cdot k' - i\epsilon^{\mu\nu\alpha\beta} q_\alpha s_{\beta}^{lep} \right)$ is easy to obtain from pQED. Current conservation implies $q_\mu l^{\mu\nu} = q_\nu l^{\mu\nu} = 0$. 
The Holographic picture

**Figure 2:** Picturing the relation between the DIS process and the holographic scattering one.

\[ h^{MN} = \frac{1}{2} \left( A^M v^N + A^N v^M \right) \]

The perturbation induced by the electro-magnetic currents \( J^\mu \) from the boundary is a metric fluctuation of the form

where \( A^m \) is a \( U(1) \) gauge field and \( v^i \) is a killing vector on the sphere \(^2\).

These fluctuations will interact with **scalar or vector brane fields** (transversal or longitudinal brane fluctuations)

\(^2\)J. Polchinski and M. Strassler (2002)
Finally, we obtain the following results for the structure functions (at leading order)

\[ g_1 = -2g_2 = \frac{1}{4x^2}(l_1 + l_0), \quad F_1 = \frac{1}{12x^2}l_1, \quad F_2 = \frac{1}{6x}(l_1 + l_0), \]

\[ b_1 = \frac{1}{4x^2}l_1, \quad b_2 = -\frac{3}{2}b_3 = 3b_4 = \frac{1}{2x}(l_1 + l_0), \]

where

\[ l_1 \propto \left(\frac{q}{\Lambda}\right)^{-2\Delta+2} \frac{1}{\sqrt{gN}} l_{1,2\Delta+3}, \quad l_0 = \propto \left(\frac{q}{\Lambda}\right)^{-2\Delta+2} \frac{1}{\sqrt{gN}} l_{0,2\Delta+3}, \]

and since \( \frac{l_{0,2\Delta+3}}{l_{1,2\Delta+3}} = \frac{2\Delta+3}{\Delta+2} \) one recovers relations of the Callan-Gross type:

\[ F_2 = 2xF_1 \left(1 + \frac{l_0}{l_1}\right), \quad b_2 = 2xb_1 \left(1 + \frac{l_0}{l_1}\right). \]
Conclusions and future work

- DIS scattering of leptons from spin-0 and spin-1 mesons at small $x$ at strong coupling and in the large $N$ limit has been investigated in terms of superstring theory (in the large $N$ limit).

- For polarized vector mesons the $8$ structure functions were obtained, along with the Callan-Gross type relations\(^3\)
  \[ F_2(x) \sim 2xF_1(x) \text{ and } b_2(x) \sim 2xb_1(x). \]

- This results have similarities for all the different $Dp$-brane models. This could be a signal of a universal behaviour for confining gauge theories with a dual description in terms of probe $Dp$-branes.

- **Future work**: calculating the differential cross section for DIS and apply this techniques to other process, using the OPE of vertex operators and other techniques in order to describe the string theory scattering.

Thank you for listening! Any questions?

\(^3\)M. Schvellinger, E. Koile and S. Macaluso (2011,2013) found $F_2 = 2F_1$ and $b_2 = 2b_1$ at $x \sim 1$. 
Thank you for listening! Any questions?
Some important steps

- Using the optical theorem to relate DIS and forward Compton scattering, in order to relate our calculations and the hadronic tensor.

- Calculate the leading amplitude and finding an effective action $S_{\text{eff}}$ (first in flat space-time).

- Obtaining the field solutions on the curved background ($AdS_5 \times S^3$) with boundary conditions.

- Inserting these solutions in the $S_{\text{eff}}$ and folding the amplitude in $AdS$ (since the interaction can be considered local) \(^4\).

- Using the AdS/CFT dictionary $\rightarrow n_\mu n_\nu \tilde{W}^{\mu\nu} \sim \frac{\delta^2}{\delta(b.c.)^2} S_{\text{eff}}^{AABB}$.

- Comparing the resulting $W^{\mu\nu}$ with the most general one and extract the structure functions $F$, $b$ and $g$.

\(^4\)Polchinski et all (2006)
\( \mathcal{N} = 2 \) SYM gauge theory:

- It is derived from \( \mathcal{N} = 4 \) SYM with 8 broken susy.
- It has matter in the fundamental rep. and mesons, like QCD. However recall that QCD is logarithmically running in the UV.
- At high energies it becomes conformal, so we can use the gauge-string duality.
- It possesses a mass gap of order \( m/\lambda \) with \( m \propto L \rightarrow \) light mesons at strong coupling.

The D3D7 brane model\(^5\):

- \( N_f << N \) probe D7-branes are introduced in an \( AdS_5 \times S^5 \) background:

\[
\begin{align*}
D3 & : \quad 0 \ 1 \ 2 \ 3 \ - \ - \ - \ - \ - \\
D7 & : \quad 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ - \ - 
\end{align*}
\]

- Mass comes from separation in the 89-plane (L).
- In the interaction region the induced metric can be approximated by \( AdS_5 \times S^3 \).
- We will use \( N_f = 1 \) and some other models\(^6\): \( D4D8\overline{D8} \) and \( D4D6\overline{D6} \).

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\(^5\) Kruczenski et al. (2003), A. Karch and E. Katz (2002)

\(^6\) Sakai and Sugimoto (2004) y citar el otro
Formally, both for scalar and vector mesons one should compute the two open vs two closed strings amplitude from a vertex operator worldsheet integral on the disk of the form\(^7\)

\[
A_{\text{string}}(h_1, h_4, \epsilon_2, \epsilon_3) = \int_{\partial H_+} dx \int_{\partial H_+} dy \int_{z \in H_+} dz \int_{\bar{z}} \int_{w \in H_+} dw \bar{w} \langle c(z) \bar{c}(\bar{z}) \times V_c^{(-1, -1)}(z, \bar{z}; h_{1\mu\nu}, k_1) : : V_c^{(0, 0)}(w, \bar{w}; h_{4\mu\nu}, p_4) : (c(x) - c(y)) : V_o^{(0)}(x, \epsilon_2\mu, k_2) : : V_o^{(0)}(y, \epsilon_3\mu, k_3) : \rangle.
\]

This gives a sum of terms with an \(\alpha'\)-independent kinematic term and a pre-factor that carries the \(\alpha'\) dependence of the form

\[
A^2_{4o2c} = P^2_{1o2c} K^2_{1o2c} + P^2_{2o2c} K^2_{2o2c} + \ldots.
\]

For us, \(|t| << 1 << s\). Only the one that has a \(\frac{1}{t}\) pole in the pre-factor and a kinetic term that can be obtained from supergravity is important in this regime.

\(^7\)S. Stieberger (2009).
Figure 3: The $s$, $u$- and $t$-channel diagrams together with the contact interaction. They all appear in the supergravity calculation.
We begin with the action
\[ S_{DBI} = - T_7 \int d^8 \xi \sqrt{-\det (\hat{g}_{ab} + (2\pi \alpha') G_{ab})}, \]
where \( G_{ab} = \partial_a B_b - \partial_b B_a \) and choose the static gauge
\[ H_{ab} \rightarrow \hat{g}_{ab} \equiv g_{ab} + 2g_{l(a} \partial_{b)} X^l + g_{lJ} \partial_a X^l \partial_b X^J, \]
\[ g_{ab} = \eta_{ab} + 2\kappa h_{ab} , \quad g_{IJ} = \delta_{IJ} + 2\kappa h_{IJ} , \quad g_{al} = 2\kappa h_{al}. \]

By expanding (1) we get the usual propagator for \( B_a \) and the interaction lagrangians
\[ L_{BB} = - \frac{T_7}{4} G^{ab} G_{ab} , \quad L_{hBB} = T_7 \kappa \left[ \frac{1}{4} h G^{ab} G_{ab} + h^{ab} G_{bc} G_a^c \right] , \]
\[ L_{hhBB} = T_7 \kappa^2 \left[ \frac{1}{8} h^2 G^{ab} G_{ab} + 2hh^{ab} G_{bc} G_a^c - \frac{1}{4} h^{ab} h_{ab} G^{cd} G_{cd} \right. \]
\[ - h^{ab} h^{cd} G_{bc} G_{da} - 2h^{ab} h_{bc} G^{cd} G_{da} \].
When $x$ is not exponentially small the string size is the interaction can be considered local and so we can fold our flat spacetime results into the AdS interaction. Thus, from the effective action our starting point we consider our starting point

$$n_\mu n_\nu \text{Im}_{\text{exc}} T^{\mu\nu} = \frac{\pi \alpha'}{8} \sum_{m=1}^{\infty} \int d\Omega_3 \, dr \, \sqrt{-g} v_i v^i G^{m q}(P) G^n_q(P) F^*_m(q) F^p_n(q) \delta \left( m - \frac{\alpha' \tilde{s}}{4} \right),$$

where all indices are contracted with the full $p+1$-dimensional metric. The integration in $x^0, \ldots, x^3$ has already been used to set the momenta by conservation, and the solutions are

$$A_\mu(q) = n_\mu f(r) e^{iq \cdot x}, \quad f(r) = \frac{q R^2}{r} K_1 \left( \frac{q R^2}{r} \right)$$

$$A_r(q) = -i q \cdot n \frac{f'(r)}{q^2} e^{iq \cdot x}, \quad B_r(P) = 0$$

$$B^l_\mu(P) = \frac{\zeta_\mu}{\Lambda} \frac{c_i^l}{\Lambda R^3} \left( \frac{r}{\Lambda R^2} \right)^{-\Delta} Y_l(\Omega_3) e^{iP \cdot x} = \frac{\zeta_\mu}{\Lambda} X^l(P)$$

The last lines come from the asymptotic form of the solutions in the $D3D7$ model.

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8 J. Polchinski et al (2006)
9 M. Kruczenski et al (2003).
Substituting the solutions one gets

\[
\text{Im}_{\text{exc}} T^{\mu\nu} = \frac{\pi \alpha'}{8} \sum_{m=1}^{\infty} \int d\Omega_3 dr \delta \left( m - \frac{\alpha' s}{4} \right) \times
\]

\[
P^\mu P^\nu \times \left[ \frac{R^4 |X|^2}{r^4 \Lambda^2} \left( \frac{R^4}{r^4} q^2 f^2 + (f')^2 \right) (\zeta \cdot \zeta^*) \right] + \left[ \frac{1}{2} (\zeta^{*\mu} \zeta^\nu + \zeta^{*\nu} \zeta^\mu) \right.
\]

\[
+ \frac{1}{2} (\zeta^{*\mu} \zeta^\nu - \zeta^{*\nu} \zeta^\mu) \left] \times \frac{R^4 |X|^2}{r^4 \Lambda^2} \left[ \frac{R^4}{r^4} P^2 q^2 f^2 + P^2 (f')^2 + \frac{\Sigma^2}{r^2} q^2 f^2 + \frac{\Sigma^2 r^2}{R^4} (f')^2 \right]
\]

\[
+ \eta^{\mu\nu} \times \frac{R^4 |X|^2}{r^4 \Lambda^2} \left[ \frac{R^4}{r^4} f^2 (\zeta \cdot \zeta^*)(P \cdot q)^2 + \left( \frac{R^4}{r^4} P^2 + \frac{\Sigma^2}{r^2} \right) f^2 (q \cdot \zeta)(q \cdot \zeta^*)
\]

\[
+ (\zeta \cdot \zeta^*) \left( \frac{r^2 \Delta^2}{R^4} (f')^2 + 2 \frac{\Delta}{r} ff' (P \cdot q) \right) \right] ,
\]
This parameter region characterized by \( x \ll \exp -\sqrt{gN} \) is more complicated because it goes deeper into string theory issues:

- The locality approximation breaks down. This is because the \((\alpha' \tilde{s})^{\alpha' \tilde{t}}\) that we had set to 1 cannot be neglected because we must take into account the transverse momentum transfer. Thus one has to include
  \[
  m^{\alpha' \tilde{t} / 2} \sim (\alpha' \tilde{s})^{\alpha' \tilde{t} / 2} \sim x^{-\alpha' \tilde{t} / 2} \sim x^{-\alpha' \nabla^2 / 2}.
  \]

- The differential operator acts on the solutions as a diffusion operator in the \( r \) direction, coming from the growth of the strings.

- The odd \( q^2 \)-dependence both in the structure functions obtained in the previous regimes \( x \gg 1/\sqrt{gN} \) and \( 1/\sqrt{gN} \gg x \gg \exp -\sqrt{gN} \) is fixed, and so is the divergent moment issue.

In the last ten years a lot of progress has been made in this direction, specially in the spin zero (glueball) case. 10

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10 J. Polchinski et al. (2006), R. Brower et al. (2007 and 2012), and references therein.