Parameterized post-Newtonian limit of Horndeski’s gravity theory

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Overview

1. Introduction
2. Massive scalar field
3. Massless scalar field
4. Experimental consistency
5. Particular models
6. Conclusion
Overview

1 Introduction

2 Massive scalar field

3 Massless scalar field

4 Experimental consistency

5 Particular models

6 Conclusion
Motivation

So far unexplained cosmological observations:

- Accelerating expansion of the universe
- Homogeneity of cosmic microwave background
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Models for explaining these observations:
- $\Lambda$CDM model / dark energy
- Inflation

Physical mechanisms are not understood:
- Unknown type of matter?
- Modification of the laws of gravity?
- Scalar field in addition to metric mediating gravity?
- Quantum gravity effects?
- Horndeski gravity
  
  G. W. Horndeski '74: Scalar-tensor theory of gravity.
  Most general STG with second order field equations.
  Healthy, ghost-free theory.
  Contains many interesting cases (Galileons, Higgs inflation...).
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Context: scalar-tensor theories of gravity

- Appear as effective theories of more fundamental models:
  - Low-energy limit of string theory
  - Braneworld models
Context: scalar-tensor theories of gravity

- **Appear as effective theories of more fundamental models:**
  - Low-energy limit of string theory
  - Braneworld models

- **“Classical” scalar tensor theories:**
  - **Action:**

\[
S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ A(\phi) R - B(\phi) g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2 \frac{\mathcal{V}(\phi)}{\ell^2} \right] \\
+ S_m \left[ e^{2\alpha(\phi)} g^{\mu\nu}, \chi_m \right].
\]

- **PPN parameters** $\gamma$ and $\beta$ calculated [MH, L. Järv, P. Kuusk, E. Randla '13].
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  - PPN parameters $\gamma$ and $\beta$ calculated [MH, L. Järv, P. Kuusk, E. Randla ’13].
  - Invariance of the action under conformal transformations:
    - Different functions $\mathcal{A}, \mathcal{B}, \mathcal{V}, \alpha$ describe the same theory.
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- Horndeski’s theory: generalizes aforementioned theories.
Gravitational action

- **Action functional** [T. Kobayashi, M. Yamaguchi, J. 'i. Yokoyama '11]:

\[
S = \sum_{i=2}^{5} \int d^4 x \sqrt{-g} \mathcal{L}_i[g_{\mu\nu}, \phi] + S_m[g_{\mu\nu}, \chi_m].
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\]

- **Gravitational Lagrangian with** \( X = -\nabla_\mu \phi \nabla^\mu \phi/2 \):

\[
\begin{align*}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X) \Box \phi, \\
\mathcal{L}_4 &= G_4(\phi, X) R + G_4 X(\phi, X) \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \\
\mathcal{L}_5 &= G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi \\
&\quad - \frac{1}{6} G_5 X(\phi, X) \left[ (\Box \phi)^3 - 3(\Box \phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \right].
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\end{align*}
\]

- Free functions \( K, G_3, G_4, G_5 \).
Field equations

Structure of the field equations:

\[
\sum_{i=2}^{5} G_{\mu\nu}^{i} = \frac{1}{2} T_{\mu\nu}, \quad \sum_{i=2}^{5} \nabla^{\mu} J_{\mu}^{i} = \sum_{i=2}^{5} P_{\phi}^{i}.
\]
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\]

- More convenient: trace-reversed field equations:

\[
\sum_{i=2}^{5} R_{\mu \nu}^{i} = \frac{1}{2} \bar{T}_{\mu \nu} = \frac{1}{2} \left( T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T \right).
\]
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- Geometry tensors:

\[ \mathcal{R}_{\mu\nu}^i = G_{\mu\nu}^i - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} G_{\rho\sigma}^i. \]
Perturbative expansion

- Background solution:
  - Minkowski metric $\eta_{\mu\nu}$
  - Constant scalar field value $\Phi$

Taylor expansion of free functions:

$$K(\phi, X) = \sum_{m, n=0}^{\infty} K(m, n) \psi^m X^n.$$
Perturbative expansion

- Background solution:
  - Minkowski metric $\eta_{\mu\nu}$
  - Constant scalar field value $\Phi$
- Perturbation of dynamical fields:

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \phi = \Phi + \psi, \quad X = -\frac{1}{2} \nabla^\mu \psi \nabla_\mu \psi. \]
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- **Taylor expansion of free functions:**

  $$K(\phi, X) = \sum_{m,n=0}^{\infty} K_{(m,n)} \psi^m X^n.$$ 

- **Expansion coefficients:**

  $$K_{(m,n)} = \frac{1}{m!n!} \left. \frac{\partial^{m+n}}{\partial \phi^m \partial X^n} K(\phi, X) \right|_{\phi=\Phi, X=0}.$$ 

- **Similar expansion for $G_3, G_4, G_5$.**
Post-Newtonian approximation

- Perfect fluid energy-momentum tensor:
  \[ T^{\mu\nu} = (\rho + \rho \Pi + p)u^{\mu}u^{\nu} + pg^{\mu\nu}. \]
  - Four-velocity \( u^{\mu} \).
  - Matter density \( \rho \).
  - Specific internal energy \( \Pi \).
  - Pressure \( p \).
Post-Newtonian approximation

- Perfect fluid energy-momentum tensor:

\[ T^{\mu\nu} = (\rho + \rho \Pi + p)u^\mu u^\nu + pg^{\mu\nu}. \]

- Four-velocity \( u^\mu \).
- Matter density \( \rho \sim O(2) \).
- Specific internal energy \( \Pi \sim O(2) \).
- Pressure \( p \sim O(4) \).

- Slow-moving source matter:

\[ \nu^i = \frac{u^i}{u^0} \ll 1. \]

- Assign velocity orders \( |\nu^i|^n \sim O(n) \) based on solar system.
Post-Newtonian approximation

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- Relevant terms for dynamical fields:
  \[ g_{00} = -1 + h^{(2)}_{00} + h^{(4)}_{00} + \mathcal{O}(6), \quad g_{0j} = h^{(3)}_{0j} + \mathcal{O}(5), \]
  \[ g_{ij} = \delta_{ij} + h^{(2)}_{ij} + \mathcal{O}(4), \quad \phi = \Phi + \psi^{(2)} + \psi^{(4)} + \mathcal{O}(6). \]
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  \[ g_{ij} = \delta_{ij} + h_{ij}^{(2)} + \mathcal{O}(4) , \quad \phi = \Phi + \psi^{(2)} + \psi^{(4)} + \mathcal{O}(6) . \]
- Time dependence only through motion of source matter.
  \[ \Rightarrow \] Assign time derivative \( \partial_0 \sim \mathcal{O}(1) \).
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Spherically symmetric solution

- Static, point-like mass source:
  \[ \rho = M \delta(\vec{x}) , \quad \Pi = 0 , \quad p = 0 , \quad v_i = 0 . \]
Spherically symmetric solution

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- Spherically symmetric metric:
  \[
  g_{00} = -1 + 2G_{\text{eff}}(r)U(r) - 2G_{\text{eff}}^2(r)\beta(r)U^2(r) + \Phi^{(4)}(r) + \mathcal{O}(6) , \\
  g_{0j} = \mathcal{O}(5) , \\
  g_{ij} = [1 + 2G_{\text{eff}}(r)\gamma(r)U(r)]\delta_{ij} + \mathcal{O}(4) .
  \]

- Newtonian potential: \( U(r) = M/r \).
- Gravitational self energy \( \Phi^{(4)}(r) \).
- Effective gravitational constant \( G_{\text{eff}}(r) \).
- PPN parameters \( \gamma(r) \) and \( \beta(r) \).
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  \]
  \[
  g_{0j} = O(5),
  \]
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- Gravitational self energy \( \Phi^{(4)}(r). \)
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- PPN parameters \( \gamma(r) \) and \( \beta(r). \)

- Consistency condition:
  \[ K_{(0,0)} = K_{(1,0)} = 0. \]
Scalar field $\psi^{(2)}$

Scalar field equation at $\mathcal{O}(2)$ is screened Poisson equation:

$$\psi^{(2)}_{,ii} - m_\psi^2 \psi^{(2)} = -c_\psi \rho.$$
Scalar field $\psi^{(2)}$

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$$\psi^{(2)}_{,ii} - m_\psi^2 \psi^{(2)} = -c_\psi \rho .$$

- Solution:

$$\psi^{(2)}(r) = \frac{M}{4\pi r} c_\psi e^{-m_\psi r} .$$
Scalar field \( \psi^{(2)} \)

- Scalar field equation at \( O(2) \) is screened Poisson equation:

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- Solution:

\[
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\]

- Constants:

\[
m_\psi = \sqrt{\frac{-2K_{(2,0)}}{K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G^2_{4(1,0)}}{G_{4(0,0)}}}},
\]

\[
c_\psi = \frac{G_{4(1,0)}}{2G_{4(0,0)}} \left( K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G^2_{4(1,0)}}{G_{4(0,0)}} \right)^{-1}.
\]
Effective gravitational constant $G_{\text{eff}}(r)$

- Metric field equation:

\[ h^{(2)}_{00,ii} = c_1 \psi^{(2)} - c_2 \rho. \]
Effective gravitational constant $G_{\text{eff}}(r)$

- Metric field equation:

$$h_{00,ii}^{(2)} = c_1 \psi^{(2)} - c_2 \rho.$$ 

- Solve and read off effective gravitational constant:

$$G_{\text{eff}}(r) = \frac{1}{8\pi} \left[ c_2 + \frac{c_1 c_2}{m_{\psi}^2} (e^{-m_{\psi} r} - 1) \right].$$
Effective gravitational constant $G_{\text{eff}}(r)$

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- Constants:

\[ c_1 = -2 \frac{G_{4(1,0)} K_{(2,0)}}{G_{4(0,0)}} \left( K_{(0,1)} - 2G_{3(1,0)} + 3 \frac{G_{2(1,0)}}{G_{4(0,0)}} \right)^{-1}, \]

\[ c_2 = \frac{1}{G_{4(0,0)}} \left[ \frac{1}{2} + \frac{G_{4(1,0)}}{2G_{4(0,0)}} \left( K_{(0,1)} - 2G_{3(1,0)} + 3 \frac{G_{2(1,0)}}{G_{4(0,0)}} \right)^{-1} \right]. \]
PPN parameter $\gamma$

- Metric field equation:

$$h^{(2)}_{ij,kk} = \left(c_3 \psi^{(2)} - c_4 \rho\right) \delta_{ij}.$$
PPN parameter $\gamma$

- Metric field equation:

$$h_{ij,kk}^{(2)} = \left( c_3 \psi^{(2)} - c_4 \rho \right) \delta_{ij}.$$ 

- Solve and read off PPN parameter $\gamma$:

$$\gamma(r) = \frac{c_4 + \frac{c_3 c_\psi}{m^2} \left( e^{-m_\psi r} - 1 \right)}{c_2 + \frac{c_1 c_\psi}{m^2} \left( e^{-m_\psi r} - 1 \right)}.$$
PPN parameter $\gamma$

- **Metric field equation:**
  \[
  h_{ij,kk}^{(2)} = \left( c_3 \psi^{(2)} - c_4 \rho \right) \delta_{ij}.
  \]

- **Solve and read off PPN parameter $\gamma$:**
  \[
  \gamma(r) = \frac{c_4 + \frac{c_3 c_\psi}{m_\psi^2} \left( e^{-m_\psi r} - 1 \right)}{c_2 + \frac{c_1 c_\psi}{m_\psi^2} \left( e^{-m_\psi r} - 1 \right)}.
  \]

- **Constants:**
  \[
  c_3 = \frac{2 G_{4(1,0)} K_{(2,0)}}{G_{4(0,0)}} \left( K_{(0,1)} - 2 G_{3(1,0)} + 3 \frac{G_{4(1,0)}^2}{G_{4(0,0)}} \right)^{-1},
  \]
  \[
  c_4 = \frac{1}{G_{4(0,0)}} \left[ \frac{1}{2} - \frac{G_{4(1,0)}^2}{2 G_{4(0,0)}} \left( K_{(0,1)} - 2 G_{3(1,0)} + 3 \frac{G_{4(1,0)}^2}{G_{4(0,0)}} \right)^{-1} \right].
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PPN parameter $\gamma$

- Metric field equation:

$$h_{ij,kk}^{(2)} = \left( c_3 \psi^{(2)} - c_4 \rho \right) \delta_{ij}.$$  

- Solve and read off PPN parameter $\gamma$:

$$\gamma(r) = \frac{2\omega + 3 - e^{-m_\psi r}}{2\omega + 3 + e^{-m_\psi r}}.$$  

- Constants:

$$\omega = \frac{G_{4(0,0)}}{2G_{4(1,0)}^2} \left( K_{(0,1)} - 2G_{3(1,0)} \right),$$

$$m_\psi = \sqrt{\frac{-2K_{(2,0)}}{K_{(0,1)} - 2G_{3(1,0)} + 3G_{4(1,0)}^2/G_{4(0,0)}}}.$$
PPN parameter $\beta$

- Calculate $\beta$ from fourth order solution:

$$\beta(r) = 1 + \frac{1}{(2\omega + 3 + e^{-m_\psi r})^2} \left\{ \frac{\omega + \tau - 4\omega \sigma}{2\omega + 3} e^{-2m_\psi r} \right.$$  

$$+ (2\omega + 3)m_\psi r \left[ e^{-m_\psi r} \ln(m_\psi r) - \frac{1}{2} e^{-2m_\psi r} \right.$$  

$$- (m_\psi r + e^{m_\psi r}) Ei(-2m_\psi r) \right.$$  

$$+ \frac{6\mu r + 3(3\omega + \tau + 6\sigma + 3)m_\psi r^2}{2(2\omega + 3)m_\psi} \left[ e^{m_\psi r} Ei(-3m_\psi r) \right.$$  

$$- e^{-m_\psi r} Ei(-m_\psi r) \right\},$$

Constants $m_\psi, \omega, \tau, \sigma, \mu$. 

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PPN parameter $\beta$

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- (m_\psi r + e^{m_\psi r}) \text{Ei}(-2m_\psi r) \right\} 
+ \frac{6\mu r + 3(3\omega + \tau + 6\sigma + 3)m_\psi^2 r}{2(2\omega + 3)m_\psi} \left[ e^{m_\psi r} \text{Ei}(-3m_\psi r) 
- e^{-m_\psi r} \text{Ei}(-m_\psi r) \right] \right\},$$

- Constants $m_\psi, \omega, \tau, \sigma, \mu$. 

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Limiting cases

- $m_\psi \to 0$, all other constants fixed and finite:

$$
\gamma = \frac{\omega + 1}{\omega + 2}, \quad \beta = 1 + \frac{\omega + \tau - 4\omega\sigma}{(2\omega + 3)(2\omega + 4)^2}.
$$

- $m_\psi \to \infty$, all other constants fixed and finite:

$$
\gamma = \beta = 1.
$$

- $m_\psi \to \infty$, large distance from the matter source:

$$
\gamma = \beta = 1.
$$
Limiting cases

- $m_\psi \to 0$, all other constants fixed and finite:

\[ \gamma = \frac{\omega + 1}{\omega + 2}, \quad \beta = 1 + \frac{\omega + \tau - 4\omega \sigma}{(2\omega + 3)(2\omega + 4)^2}. \]

- $\omega \to \infty$, all other constants fixed and finite:

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Limiting cases

- \( m_\psi \to 0 \), all other constants fixed and finite:
  \[ \gamma = \frac{\omega + 1}{\omega + 2}, \quad \beta = 1 + \frac{\omega + \tau - 4\omega \sigma}{(2\omega + 3)(2\omega + 4)^2}. \]

- \( \omega \to \infty \), all other constants fixed and finite:
  \[ \gamma = \beta = 1. \]

- \( m_\psi r \to \infty \), large distance from the matter source:
  \[ \gamma = \beta = 1. \]
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Consider more restricted theory:

\[ K_{(2,0)} = K_{(3,0)} = 0. \]

⇒ All mass-like terms for \( \psi \) vanish.
Consider more restricted theory:

$$K_{(2,0)} = K_{(3,0)} = 0.$$ 

⇒ All mass-like terms for $\psi$ vanish.

⇒ PPN limit assumes standard form with constant PPN parameters.

PPN parameters:

$$\gamma = \frac{\omega + 1}{\omega + 2}, \quad \beta = 1 + \frac{\omega + \tau - 4\omega\sigma}{4(\omega + 2)^2(2\omega + 3)},$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = \xi = 0.$$ 

⇒ Only $\gamma$ and $\beta$ potentially deviate from observed values.
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Large mass limit

- Asymptotic behavior of exponential integral:

\[
Ei(-x) \approx \frac{e^{-x}}{x} \left( 1 - \frac{1!}{x} + \frac{2!}{x^2} - \frac{3!}{x^3} + \ldots \right).
\]
Large mass limit

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\text{Ei}(-x) \approx \frac{e^{-x}}{x} \left(1 - \frac{1!}{x} + \frac{2!}{x^2} - \frac{3!}{x^3} + \ldots\right).
\]

implies Terms involving \(\sigma, \tau, \mu \sim e^{-2m_\psi r}\) are subleading.

implies Consider simplified PPN parameters for \(m_\psi r \gg 1\):

\[
\gamma(r) = 1 - \frac{2}{2\omega + 3} e^{-m_\psi r} + \mathcal{O}(e^{-2m_\psi r}),
\]

\[
\beta(r) = 1 + \frac{m_\psi r}{2\omega + 3} \ln(m_\psi r) e^{-m_\psi r} + \mathcal{O}(e^{-2m_\psi r}).
\]

- Only depend on constants \(m_\psi, \omega\).
Large mass limit

- Asymptotic behavior of exponential integral:

\[ \text{Ei}(-x) \approx \frac{e^{-x}}{x} \left( 1 - \frac{1!}{x} + \frac{2!}{x^2} - \frac{3!}{x^3} + \ldots \right). \]

⇒ Terms involving \( \sigma, \tau, \mu \sim e^{-2m_\psi r} \) are subleading.

⇒ Consider simplified PPN parameters for \( m_\psi r \gg 1 \):

\[ \gamma(r) = 1 - \frac{2}{2\omega + 3} e^{-m_\psi r} + O(e^{-2m_\psi r}), \]

\[ \beta(r) = 1 + \frac{m_\psi r}{2\omega + 3} \ln(m_\psi r) e^{-m_\psi r} + O(e^{-2m_\psi r}). \]

- Only depend on constants \( m_\psi, \omega \).

⇒ Need experiments with fixed interaction distance \( r \).

- Most stringent bounds from Cassini tracking [B. Bertotti, L. Iess, P. Tortora '03]:

\[ \gamma - 1 = (2.1 \pm 2.3) \cdot 10^{-5} \text{ at } r \approx 7.44 \cdot 10^{-3} \text{AU}. \]
Excluded parameter ranges at $2\sigma$
PPN parameters independent of $r$ for $m_\psi r \ll 1$:

$$\gamma = \frac{\omega + 1}{\omega + 2}, \quad \beta = 1 + \frac{\omega + \tau - 4\omega\sigma}{4(\omega + 2)^2(2\omega + 3)}. $$
Small mass limit

- PPN parameters independent of $r$ for $m_\psi r \ll 1$:

\[ \gamma = \frac{\omega + 1}{\omega + 2}, \quad \beta = 1 + \frac{\omega + \tau - 4\omega\sigma}{4(\omega + 2)^2(2\omega + 3)}. \]

$\Rightarrow$ Possible to use observations where $r$ is not well-defined.

- INPOP13 ephemeris [A. Fienga, P. Exertier, M. Gastineau, J. Laskar, H. Manche, A. Verma '13/'14]:

\[ \gamma - 1 = (-0.3 \pm 2.5) \cdot 10^{-5}, \quad \beta - 1 = (0.2 \pm 2.5) \cdot 10^{-5}. \]
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Still more stringent bounds by including Cassini tracking:

$$-2.5 \cdot 10^{10} \leq \tau - 4\omega\sigma \leq 2.7 \cdot 10^{10} \quad \text{for} \quad \omega = 4.0 \cdot 10^4.$$

Less stringent bounds for larger values of $\omega$. 

How to get better bounds?

- Stringency of bounds depends on interaction distance $r_0$.
  - Experiments with smaller $r_0$ are more sensitive to $m_\psi > 0$.
  - Measure $\gamma$ and $\beta$ at shorter distances.
  - Earth-moon system? Satellite missions?
How to get better bounds?

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- Can lunar laser ranging help?
  - Nordvedt effect depends on $\gamma$ and $\beta$.
  - But: Nordvedt effect concerns motion in solar gravitational field.
  - Interaction distance $r_0 = 1$AU is large.
  - Not the kind of experiment we need.
Overview

1 Introduction

2 Massive scalar field

3 Massless scalar field

4 Experimental consistency

5 Particular models

6 Conclusion
Scalar-tensor gravity with potential

- Gravitational action:

\[ S_G = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( \phi R - \frac{\omega(\phi)}{\phi} \partial_\rho \phi \partial^\rho \phi - 2\kappa^2 V(\phi) \right). \]
Scalar-tensor gravity with potential

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\]

- **PPN parameters** [MH, L. Järv, P. Kuusk, E. Randla ’13]:

\[
\gamma(r) = \frac{2\omega_0 + 3 - e^{-m_\psi r}}{2\omega_0 + 3 + e^{-m_\psi r}},
\]

\[
\beta(r) = 1 + \frac{1}{(2\omega_0 + 3 + e^{-m_\psi r})^2} \left\{ \frac{\Phi \omega_1}{2\omega_0 + 3} e^{-2m_\psi r} + (2\omega_0 + 3)m_\psi r e^{-m_\psi r} \right\} \times \left[ e^{-m_\psi r} \ln(m_\psi r) - (m_\psi r + e^{m_\psi r}) \text{Ei}(-2m_\psi r) - \frac{1}{2} e^{-2m_\psi r} \right] + \frac{3m_\psi r}{2} \left( 1 - \frac{\Phi V_3}{V_2} + \frac{\Phi \omega_1}{2\omega_0 + 3} \right) \left[ e^{m_\psi r} \text{Ei}(-3m_\psi r) - e^{-m_\psi r} \text{Ei}(-m_\psi r) \right].
\]
Non-minimal Higgs inflation

- Gravitational action [F. L. Bezrukov, M. Shaposhnikov '08]:

\[
S_G = \int d^4 x \sqrt{-g} \left( \frac{M_{Pl}^2 - \xi \phi^2}{2} R + X - V(\phi) \right).
\]

PPN parameters:

\[
\gamma = 1 - 4 \xi^2 e^{-m_{\psi} \Phi^2 M_{Pl}^2} + O(\Phi^3 M_{Pl}^3),
\]

\[
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\]

Higgs field:

\[
m_{\psi} = 125 \text{GeV}, \quad \Phi = 246 \text{GeV}.
\]

\[
\Rightarrow \gamma = \beta = 1 \text{ on any astrophysical scale}.
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\[ \gamma = 1 - 4\xi^2 e^{-m_\psi r} \frac{\phi^2}{M_{Pl}^2} + \mathcal{O} \left( \frac{\phi^3}{M_{Pl}^3} \right), \]
\[ \beta = 1 + \left\{ 2\xi^3 e^{-2m_\psi r} - \xi^2 m_\psi r e^{-2m_\psi r} - 2e^{-m_\psi r} \ln(m_\psi r) \right. \]
\[ + 2(m_\psi r + e^{m_\psi r}) \text{Ei}(-2m_\psi r) \right\} \frac{\phi^2}{M_{Pl}^2} + \mathcal{O} \left( \frac{\phi^3}{M_{Pl}^3} \right). \]
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5. Particular models
6. Conclusion
Horndeski’s gravity theory:
- Most general scalar-tensor theory with second order equations.
- Four free functions of $\phi$ and $X = -\nabla^\mu \phi \nabla_\mu \phi / 2$. 

Example theories:
- Classical scalar-tensor gravity with arbitrary potential.
- Models of Higgs inflation.
- Galileons.

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Reproduces and generalizes well-known results.
Many example theories compatible with solar system observations.
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Extend analysis to more general theories:
- Allow time-dependent scalar background field $\dot{\Phi} \neq 0$.
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- Multi-scalar Horndeski gravity.
Outlook

- Extend analysis to more general theories:
  - Allow time-dependent scalar background field $\dot{\Phi} \neq 0$.
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- Take screening mechanisms into account:
  - Vainshtein mechanism.
  - Chameleon mechanism.
  - Symmetron mechanism.