Intrinsically polar elastic metamaterials

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The ability to design and fabricate materials with tailored mechanical properties, combined with immunity to damage, is a frontier of materials engineering. For example, materials which are characterized by elastic properties that depend on the position inside a medium are required in applications where structural stability has to be combined with a soft and compliant surface, like in impact protection and cushioning. A gradient in the elastic properties can be built from a single material, varying gradually the bulk porosity of the material or its geometrical structure. However, if such a gradient is built into the material at production, damage or wearing over time might expose unwanted elastic properties. Here, we implement a design principle for a spatially inhomogeneous material based on topological band-theory for mechanical systems. The resulting inhomogeneity is stable against wearing and even cutting the material in half. We show how, by creating a periodic elastic material with topological properties, one can create an intrinsically polar behavior, where a face with a given surface normal is stiff while its opposing face is soft.

The characteristic properties of periodic materials are captured in their phononic spectrum, i.e., their dispersion relation, which contains information ranging from the material’s quasi-static elastic response (at very low frequencies) to its thermal conductivity (at higher frequencies). In topological (polar) materials, the phonon spectrum, and with it the linked material properties, are different on two opposing surfaces of the material. Uniquely to topological materials, the polar behavior is stable and remains preserved when these materials are cut or fractured, as shown in Fig. 1a, b: No matter how much of our material is lost to a fracture, or wearing during use, the asymmetry in the mechanical response remains the same [8–11]. Note, that these topological features are linked to surface modes whose penetration depth introduces a new length-scale and hence the stability with respect to wearing and fracturing is not bound to the microscopic unit cell size. This similarity to the polarity of a dielectric motivates the term of an intrinsically polar elastic metamaterial.

As it turns out, the simplest lattice that lends itself to the design of such a polar metamaterial has another topologically protected feature in its phononic spectrum: Lines of zero-frequency excitation that span throughout the whole Brillouin zone and which cannot be gapped out [12]. Such nodal, or Weyl lines [10, 12–15] are of considerable recent interest in electronic systems [15] as they mediate non-local magneto-transport. In our mechanical setup, we find clear experimental evidence of these Weyl lines and we further capitalize on their presence to fine tune the elastic response.

We design our polar elastic materials as a truss-like, periodic lattice, constituted by a system of rods connected by hinges (a frame). In the classical description of frames, one balances the number of degrees of freedom ($N_f$) of the hinges with the number of constraints ($N_c$) imposed by the rods. Their difference yields the number of zero

\[ N_f - N_c. \]

FIG. 1. Intrinsically polar metamaterial. (a) Schematic representation of an elastically polarized mechanical metamaterial with varying stiffness from top to bottom surface. (b) Due to the intrinsic programmability of its elastic properties, whenever a cut is performed through the material, the polarity of the surfaces are protected by means of topology, retaining the variation of stiffness from top to bottom. (c) Schematic of the basic building block of the employed Pyrochlore lattice. (d) a $5 \times 5 \times 5$ unit cells of the realized metamaterial by means of additive manufacturing.
FIG. 2. **Theory.** (a) Bulk (blue) and surface (green) Brillouin zone for a surface plane with a surface normal \( \hat{z} \). The red line shows the doubly degenerate nodal Weyl lines in the bulk spectrum. (b) Geometry of a regular corner-sharing Pyrochlore lattice. (c) The distorted Pyrochlore lattice. \( \theta_0 \) indicates the equilibrium angle between two neighboring tetrahedra. (d) In a zero mode of the perfect frame no bars are contracted or stretched. However, the angle between rigid tetrahedra can change from \( \theta_0 \rightarrow \theta \). We model non-perfect hinges by a linear force \( \theta_0 - \theta \). (e) Effective phonon spectrum in the zero-mode sector of the perfect frame as a function of high-symmetry lines in the surface Brillouin zone for a finite system in the \( y \)-direction. The color of the dots indicates where the mode lives (bottom or top surface or in the bulk). One can observe that the effective low-frequency modes are distributed equally over the two opposing surfaces. (f) The same plot for open faces in the \( z \)-direction. One can clearly see that no zero modes live on the upper face.

modes \((N_0)\), where parts of the system can move freely without a restoring force. More precisely, the counting due to Maxwell [16] and Calladine [17]

\[
N_0 - N_s = N_f - N_c
\]

accounts also for the number of states of self stress \((N_s)\). A state of self stress corresponds to a combination of stresses on the rods that do not exert net forces on the hinges. In isostatic frames, where \( N_f = N_c \), one might still find pairs of zero modes and states of self stress due to “misplaced” rods.

In the theory of Kane and Lubensky [8, 18], isostatic frames are described by a polarization vector \( \mathbf{R}_c \) akin the electric polarization of dielectrics. The polarization \( \mathbf{R}_c \) can be expressed as a topological invariant of the bulk of a periodic frame and is therefore stable against local deformations of the frame. Moreover, if a periodic isostatic lattice is cut to obtain a finite sample, one necessarily cuts bonds and zero modes will appear on the surface. The polarization \( \mathbf{R}_c \) indicates how these zero modes are distributed over the different surfaces. This renders isostatic frames an optimal starting point for a polar metamaterial. However, we set out to design an asymmetric elastic response, whereas a zero-mode does not lead to any elasticity as there are no restoring forces.

The idealized description of a frame as made out of perfect hinges will always be an approximation. The connections between the rods typically induce further constraints due to friction or elastic forces that favor a certain angle between the connected rods. We show how one can capitalize on this additional forces to obtain an elastic polar response. In our material, we use a stiff polymer to fabricate the rods, and a soft, elastic rubber for the hinges, and use additive manufacturing for the final fabrication of the composite material. We use this large separation of scales between rods and hinges, to derive an effective elastic description of our material, by projecting the angle-forces to the space of zero modes of the idealized frame.

Isostaticity of a periodic frame in three dimensions requires each hinge to be connected to six bars. The simplest regular lattice with this coordination is the py-
rochlore lattice, which is built from corner sharing tetra-
hedrons. In order to leverage a polar response and to use the
topological theory by Kane and Lubensky [8], we dis-
tort the lattice as indicated in Fig. 1c (see App. A). The
distorted pyrochlore lattice has the additional feature of
two nodal Weyl lines of zero frequency excitations in its
bulk spectrum, see Fig. 2a. In principle, these Weyl lines
render the topological polarization $R_t$ ill-defined and in
turn give rise to a number of zero modes on the surface
that depends on the surface momentum [10]. However,
by designing the distortion in a way that some of the flat
planes of the original pyrochlore lattice prevail, one can
force the two Weyl lines to lie on top of each other along
the (1,1,1)-direction, see Fig. 2. This eliminates their
effect on $R_t$ and we can still achieve a maximal polar-
ization between two opposing faces. In particular, for our
choice of distortion, $R_t = s(-1,0,2)$ with $s$ the overall
scale. This results in a polarization where a face with a
surface normal in the positive (negative) z-direction hosts
four (no) zero modes. For faces normal to the $y$-direction
there is a balance of two and two zero modes, cf. App. B
for details.

The design principle of Maxwell frames mandates the
presence of a rotating degree of freedom (i.e., a perfect
hinge) at each intersection point between frame elements
(bars). To design a material following this principle, one
needs to carry out a nontrivial assembly process on the
macro-scale. Moreover, the miniaturization of such pro-
cedure is tedious and impractical. In order to avoid the
assembly process, while retaining a hinge-like behavior,
we follow a different fabrication process. The realiza-
tion of the metamaterial starts by separating the lattice
into two distinct yet interconnected object classes, the
bars and the joints. The bars are made out of beams of
varying length and constant square cross section of width
1.125 mm. The joints are replaced by spheres of radius
1.5 mm. Both beams and spheres are fabricated using ad-
ditive manufacturing technology (Polyjet 3D printing),
that enables the realization of a single structure with
multiple materials simultaneously. We harness this tech-
nology to obtain a hinge-like performance as the theory
requires, by printing each of the different objects (spheres
and beams) out of different materials. In order to en-
sure flexibility at the joint site, the spheres are printed
with a much softer material, TangoBlack, (with density,
$\rho = 1.15$ g/cm$^3$ and Young’s modulus $E_{TB} = 1.8$ MPa)
than the beams, VeroWhite, (with density, $\rho = 1.17$ g/cm$^3$, Young’s
modulus $E_{VW} = 2$ GPa) with $10^3$ times lower stiffness and
similar densities, which we will treat as being equal in
the following. While the soft TangoBlack enables a
description in terms of perfect hinges, its non-vanishing
$E_{TB}$ gives rise to restoring forces on the angles between
the beams. We incorporate these forces in our general-
ization of the theory of isostatic lattice.

To find the effective elastic theory we first find the
zero frequency excitations for the idealized case of perfect
hinges. The equations of motion read

$$\ddot{x} = D_{in} x,$$

(2)

where the vector $x$ contains the degrees of freedom of
the hinges and $D_{in}$ encodes the effect of the bars and is
proportional to $E_{VW}/\rho$. The eigenvectors of $D_{in}$ separate
into two classes

$$M_0 = \{v_1, v_2, \ldots, v_z\} \quad \text{and} \quad M_\perp = \{v_{z+1}, v_{z+2}, \ldots\},$$

(3)
of $z$ zero modes in $M_0$ and its complement $M_\perp$. Any dis-
tortion of the lattice involving modes from $M_\perp$ stretches
or compresses the bars. For the derivation of a low-
frequency sector, we now proceed by projecting the angle-
restoring forces $D_{angl}$ onto the zero mode subspace

$$D_{eff} = M_0^T D_{angl} M_0.$$

(4)

Note that any small deformation of the lattice is now
giving rise to an elastic response encoded by $D_{eff}$, which
is proportional to $E_{TB}/\rho \ll E_{VW}/\rho$. The resulting low-
energy phonon spectrum is shown in Fig. 2. In panel e, we
show the effective low-frequency spectrum for a system
periodic in $x$ and $z$ direction and of finite extent in the
$y$ direction. From the color code we read that on top of
the Weyl bulk modes (lifted to finite frequencies owing
to the angle-forces) we have as many low-energy states
on the bottom and the top face. In panel f the same
situation is shown where $x$ and $y$ direction are periodic.
Clearly, all surface zero modes are concentrated on the
lower face. Note that the penetration depth of the surface
zero-modes decreases with the amount of distortion and
can be much larger than the size of a unit cell [10].

In a cube made out of an isotropic material, such as
most metals, the stiffness measured on its different faces
are identical. If the material is anisotropic, such as wood
or fibers, one should expect directional dependence in
stiffness (i.e., much stronger along the fiber than across it).
In either case, isotropic or anisotropic, the stiffness
along opposing faces is the same, in other words the ma-
terial response along the same axis is symmetric. In order
to test the asymmetric response of the proposed lattice
material, we fabricate a cubic sample made of $5 \times 5 \times 5$
unit cells. Using a standard compression testing machine
(Instron E3000), we indent the cube at the center of a
particular face with a cylindrical probe (16 mm in diam-
eter) moving for a fixed distance (Fig. 3a), while mea-
suring the reaction force on the opposing face (Fig. 3b).
We repeat the same process for all the faces of the cube.
For the two faces along the $y$ axis, $(0, \pm 1, 0)$, the ma-
terial response to the same compression is almost identical
(dashed lines in Fig. 3b). On the contrary, the two $z$ axis
faces, $(0, 0, \pm 1)$, where we expect the asymmetry, the re-
sponse to the indentation is very different at the linear
scale, and keeps on diverging with increasing indentation
value, even in the nonlinear range (solid lines in Fig. 3b).
Since the theory is only concerned with linear phonons,
we focus on the small indentation region, the gray area in
(Fig. 3b). To characterize the asymmetry of the lattice,
we plot the response of each two opposing faces in the
same panel (Fig. 3c, d). In panel d, the slope of the re-
response curve is identical along the $y$ axis, while in panel c,
the two faces along the $z$ axis show a discrepancy in stiffness of $\sim 80\%$. This provides an experimental evidence of the realization of an intrinsically polarized mechanical metamaterial with an asymmetric elastic response.

Let us turn to the effect of the Weyl nodal lines. We experimentally observe their effect by measuring the material response on the two opposing faces perpendicular to the $z$-axis by indenting with different planes parallel to this $z$-axis, cf. Fig. 4. When indenting with a plane, we pick up an elastic response from all modes with surface momenta $k_{\perp}$ perpendicular to that plane, whereas along the plane direction one only gets weight from $k_{\parallel} = 0$. Therefore, when the plane of indentation is oriented perpendicular to the projection of the Weyl lines onto the surface Brillouin zone [the $(1,1)$-direction], we expect maximal participation of the bulk modes and hence a minimal difference between the two faces. Conversely, when indenting parallel to the Weyl lines, we should find a maximally different response.

In order to characterize this peculiar phenomenon of plane dependence, we print a lattice consisting of $7 \times 7 \times 5$ unit cells following the same fabrication process. We indent the printed lattice at the center with a rectangular wedge (10 mm × 80 mm) perpendicular to the $z$ axis, where the centers of both the wedge and the lattice top face coincide. After performing the compression test along the principle axis $x = 0$, we rotate the wedge with an angle $\theta$ in a counterclockwise fashion in increments of $45^\circ$ and repeat the test until we reach a full circle in rotation (Fig. 4a). We repeat the same experiment for the bottom face of the sample along the $z$ axis as well. We post-process the compression-test data for the different angles, by calculating the slope of the indentation-load curve, to obtain the Young’s modulus of the material and present it in a polar plot for both top and bottom faces in the $z$ direction (green and magenta lines in Fig. 4b).

The Young’s moduli on both surfaces at the same angle are similar, except for the plane along $45^\circ$ and $225^\circ$, where the indenter overlaps with the Weyl lines.
normalized difference between the stiffness of the top and bottom surfaces at each angle is given in Fig. 4c. The measured differential stiffness in different planes across the lattice shows more than a factor 5 of variation on top and bottom surfaces along the same axis. This represents an experimental observation of nodal Weyl lines for phonons and a design methodology to intrinsically program exotic properties in materials.

We have realized a new class of lattice materials introducing a measurable intrinsic polarity in elasticity. The proposed lattice is material (metal, ceramic or polymer) and scale (micro or macro) independent, as it retains its property from structure instead of chemical compound. The elastic polarity can be coupled to dynamical, thermal, optical or electronic properties leading to the discovery of new materials with unprecedented properties.

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Appendix A: Lattice structure.

For the lattice characterization we follow the description used in Ref. [10]. The unit cell of the distorted pyrochlore lattice is given by the position of its lattice sites $\boldsymbol{r}_i^s$ and the connecting bond centers $\boldsymbol{r}_i^b$. The lattice sites are obtained by distorting the lattice sites $\boldsymbol{p}_i$ of the ordinary pyrochlore lattice according to

$$
\begin{align*}
\boldsymbol{r}_1^s &= \boldsymbol{p}_1 + x_1 \sqrt{3} \hat{\boldsymbol{e}}_1 - x_2 \hat{\boldsymbol{a}}_3, \\
\boldsymbol{r}_2^s &= \boldsymbol{p}_2 + x_2 \sqrt{3} \hat{\boldsymbol{e}}_2 - x_3 \hat{\boldsymbol{a}}_1, \\
\boldsymbol{r}_3^s &= \boldsymbol{p}_3 + x_3 \sqrt{3} \hat{\boldsymbol{e}}_3 - x_1 \hat{\boldsymbol{a}}_2, \\
\boldsymbol{r}_4^s &= \boldsymbol{p}_4 - z \hat{\boldsymbol{n}},
\end{align*}
$$

(A1)

where

$$
\begin{align*}
\boldsymbol{p}_1 &= \frac{s}{2} (1, 1, 0), \\
\boldsymbol{p}_2 &= \frac{s}{2} (0, 1, 1), \\
\boldsymbol{p}_3 &= \frac{s}{2} (1, 0, 1), \\
\boldsymbol{p}_4 &= \boldsymbol{0},
\end{align*}
$$

with $s$ the overall scale, $\hat{\boldsymbol{a}}_1 = \hat{\boldsymbol{p}}_2 - \hat{\boldsymbol{p}}_1$, $\hat{\boldsymbol{a}}_2 = \hat{\boldsymbol{p}}_3 - \hat{\boldsymbol{p}}_2$, $\hat{\boldsymbol{a}}_3 = \hat{\boldsymbol{p}}_1 - \hat{\boldsymbol{p}}_3$, $\hat{\boldsymbol{e}}_1 = \hat{\boldsymbol{a}}_1 \times \hat{\boldsymbol{n}}$, $\hat{\boldsymbol{n}} = (1, 1, 1)$, $\hat{\boldsymbol{v}} = \hat{\boldsymbol{v}}/|\hat{\boldsymbol{v}}|$ and $\boldsymbol{X} = (x_1, x_2, x_3, z)$ the parametrization. In our implementation we chose $X = 0.15s(-1, 1, 1, -1)$ and $s = 7.5 \text{ mm}$. The lattice vectors defining the full lattice are $\boldsymbol{T}_i = 2\boldsymbol{p}_i$ for $i = 1, 2, 3$.

Appendix B: Surface zero mode count.

The zero mode count $\nu$ per unit cell depends on the surface orientation. For a surface normal equal to a reciprocal lattice vector $\boldsymbol{q}$, it is given by [8]

$$
\nu = \frac{1}{2\pi} \boldsymbol{q} \cdot (\boldsymbol{R}_T + \boldsymbol{R}_c),
$$

(B1)

with the local dipole moment

$$
\boldsymbol{R}_L = 3 \sum_i r_i^a - \sum_i r_i^b
$$

(B2)

and the topological polarization $\boldsymbol{R}_T$.

The latter is given by [8]

$$
\begin{align*}
\boldsymbol{R}_T &= \sum_{i=1}^3 m_i \boldsymbol{T}_i, \\
m_i &= \frac{1}{2\pi i} \int_0^1 d\xi \frac{d}{d\xi} \log \det \mathbf{Q}(\xi \boldsymbol{b}_i + \boldsymbol{k}_\perp),
\end{align*}
$$

(B3)

where $\mathbf{Q}(\boldsymbol{k})$ is the equilibrium matrix, $\boldsymbol{b}_i$ are the reciprocal lattice vectors defined through $\boldsymbol{b}_i \cdot \boldsymbol{T}_j = 2\pi \delta_{ij}$ and $\boldsymbol{k}_\perp \cdot \boldsymbol{b}_i = 0$. Distorting the lattice according to Eq. (A1) ensures that the two oppositely charged Weyl lines along the $(1, 1, 1)$ direction lie on top of each other [10], making $m_i$ independent of $\boldsymbol{k}_\perp$.

The local dipole moment $\boldsymbol{R}_L$ depends on the choice of the unit cell which must be compatible with the surface under consideration. Going through the different configurations we find

$$
\begin{align*}
\boldsymbol{R}_L^{z,\text{top}} &= s(-1, 0, 2), \\
\boldsymbol{R}_L^{z,\text{bottom}} &= s(-1, 0, -2), \\
\boldsymbol{R}_L^{y,\text{top}} &= s(0, 2, -1), \\
\boldsymbol{R}_L^{y,\text{bottom}} &= s(0, -2, -1),
\end{align*}
$$

(B4)

while $\boldsymbol{R}_T = s(-1, 0, 2)$. All in all, this results in four and zero (two and two) zero modes for the top and bottom surface in $z$ ($y$) direction.

For the effective model analysis and manufacturing of the samples, we used an enlarged unit cell with lattice vectors $\boldsymbol{T}_1 = s(2, 0, 0)$, $\boldsymbol{T}_2 = s(0, 2, 0)$, $\boldsymbol{T}_3 = s(0, 0, 2)$, to facilitate being compatible with the different boundaries. This unit cell is four times larger than the original one and hosts two initial unit cells at a given surface, leading to a doubling of the zero mode count as observed in Fig. 2.

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