Anti-localization of graphene under the substrate electric field

K.-I. Imura(a), Y. Kuramoto and K. Nomura

Department of Physics, Tohoku University - Sendai 980-8578, Japan

received 17 June 2009; accepted 16 December 2009
published online 22 January 2010

PACS 73.23.-b – Electronic transport in mesoscopic systems
PACS 73.63.-b – Electronic transport in nanoscale materials and structures
PACS 73.20.Fz – Weak or Anderson localization

Abstract – A simple criterion is provided on how the (anti-)localization properties of graphene are determined in the presence of inter-valley scattering, Kane-Mele topological mass term, and Rashba spin-orbit interaction (SOI). A set of (pseudo) time-reversal operations show that the number of effective internal degrees of freedom, such as spin and pseudo-spins distinguishing the sublattice and the valley, is the crucial parameter for localization. It is predicted that the perpendicular electric field due to the gate voltage of the substrate drives the system to anti-localization by enhancing the Rashba SOI.

Copyright © EPLA, 2010

Introduction. – Graphene has a strong tendency not to localize [1–3], reflecting its linear spectrum [4]. In ordinary two-dimensional metals, scaling either to weak localization (WL) or to weak anti-localization (AL) is controlled by impurity scattering with spin-orbit interaction [5]. If time reversal symmetry (TRS) is broken by an external or internal magnetic field, the system exhibits neither WL nor AL [6], and belongs to the unitary class [7]. Graphene shows the three localization classes even without magnetic impurities, but with potential scatterers.

In the literature, theory predicts that graphene under doping shows AL provided inter-valley scattering can be neglected [1–3]. Together with the absence of backward scattering, AL in graphene is a clear manifestation of the Berry phase $\pi$ [8]. Inter-valley scattering, on the other hand, drives the system from AL to WL [1]. Experiments [9,10] show also a unitary behavior. The absence of WL may be attributed to ripples [11,12]. It is natural to ask how the localization properties are influenced by modification of the linear spectrum by a finite mass, whose magnitude is under debate in photoemission experiments [13,14], and theoretically [15,16]. Another modification is due to the Rashba SOI which inevitably appears in graphenes on substrates. Kane and Mele have proposed that graphene with intrinsic spin-orbit interaction realizes a $Z_2$ topological insulator [16]. This paper demonstrates another striking aspect of graphene under doping that crossovers between different symmetry class of localization are controlled simply by the number of effective internal degrees of freedom.

The inter-valley coupling depends on the range of impurity potential [17]. Long-range scatterers (LRS) do not involve inter-valley scattering, whereas short-range scatterers (SRS) activates the valley spin. Localization properties of a disordered Kane-Mele model have been studied numerically [18,19]. Since tight-binding approximation is employed there, inter-valley scattering is always present, and cannot be controlled. Our approach is indeed complementary to refs. [18,19]; we employ the lowest-order WL theory, but analyze systematically each element that influences the localization behavior.

Symmetry considerations. – Our results are summarized in fig. 1, which shows localization classes WL, AL, and U depending on the inter-valley scattering, types of the mass term, and the Rashba SOI. Without these effects, the system becomes massless and belongs to AL, as is the case of single Dirac cone [1,3]. Note that, in the presence of inter-valley scattering, crossover to AL occurs by switching on the Rashba SOI. Physically, the crossover is driven by controlling the gate voltage of the substrate on which the system is placed. Our phase diagram is also relevant to intrinsic single-valley systems [20], such as the one realized in HgTe/CdTe quantum well [21]. We predict that such system shows a crossover from unitary to orthogonal (WL) symmetry class, on activating the real spin degrees of freedom by a (Rashba-like) off-diagonal interaction between real spin $\uparrow$ and $\downarrow$ sectors.

(a)E-mail: imura@cmpt.phys.tohoku.ac.jp
Table 1: Three-story structure of Kane-Mele $Z_2$ topological insulator, and its WL properties under doping. LRS is equivalent to the single-valley model. The parity of $N_s$, the number of activated spin degrees of freedom, determines its WL properties: standard WL (orthogonal) or AL (symplectic). Broken TRS leads to U (unitary) behavior.

| Masslessness | LRS (single valley) | SRS ($K$-$K'$ coupled) |
|--------------|---------------------|-------------------------|
| i) massless graphene: | $N_s = 1 (AB) \rightarrow AL$ | $N_s = 2 (AB, KK') \rightarrow WL$ |
| $H_1 = p_z \sigma_z + p_y \sigma_y$ | | |
| ii) mass terms: $H_2 = H_1 + H_{\Delta m}$ | unitary | a) unitary |
| a) topological: $H_{\Delta} = -\Delta \sigma_z \tau_z s_z$ | no $1/g$-correction | b) $N_s = 2 (AB, KK') \rightarrow WL$ |
| b) ionic: $H_m = m \sigma_z$ | | |
| iii) Rashba spin-orbit interaction: $H_3 = H_2 - \lambda_R (\sigma_x \tau_z s_y - \sigma_y s_z)/2$ | a) $N_s = 2 (AB, \text{real spin}) \rightarrow WL$ | $N_s = 3 (AB, KK', \text{real spin}) \rightarrow AL$ |
| | b) unitary | |

$N_s = 1(AB) \rightarrow AL$: Scattering matrix $S_{\Sigma} = 1$, TRS broken. $N_s = 2 (AB, KK') \rightarrow WL$: Scattering matrix $S_{\Sigma} = 2$, TRS broken. $N_s = 3 (AB, KK', \text{real spin}) \rightarrow AL$: Scattering matrix $S_{\Sigma} = 3$, TRS broken.

$H_{\Delta}$, we consider also ionic mass term $H_m = m \sigma_z$. In the ionic mass case, contributions to $\sigma_{xy}$ from the $K$- and $K'$-valleys cancel [22]. In the topological mass case, this cancellation of anomaly does not occur any longer [23], and a quantized spin Hall effect with preserved TRS occurs. In the absence of Rashba term $H_2$, the Hamiltonian $H_{KM}$ is diagonal in the real spin $s$ space, implying that the latter is actually inactive.

We take the doped and disordered Kane-Mele model for our weak-coupling perturbative theory. The Kane-Mele model has a three-story structure, depicted in table 1: i) graphene in the massless limit, ii) topological mass term, encoding Kane-Mele spin-orbit interaction, iii) Rashba term, playing the role of activating the real spin degree of freedom. Note that the Kane-Mele model has also valley degrees of freedom, corresponding to two Dirac points of graphene: $K$ and $K'$. The topological mass term is induced by chiral-symmetry-breaking staggered chemical potential. The Kane-Mele model possesses three types of pseudo or real spins, represented by Pauli’s matrices, $\vec{\sigma}$, $\vec{\tau}$ and $\vec{s}$, operating in different subspaces: $\vec{\sigma}$ acts on the sublattice spin $A-B$, $\vec{\tau}$ on the valley spin $K-K'$, and $\vec{s}$ on the real spin. In the continuum limit, the Kane-Mele Hamiltonian,

$$H_{KM} = H_1 + H_{\Delta} + H_R,$$

consists of the following three elements: i) $H_1 = \hbar v_F (p_z \sigma_z + p_y \sigma_y)$, ii) $H_{\Delta} = -\Delta \sigma_z \tau_z s_z$, iii) $H_R = -\lambda_R (\sigma_x \tau_z s_y - \sigma_y s_z)/2$, each describing the corresponding floor of the three-story structure. For comparison with $A, B, C, \bar{s}$, $\bar{C}$, $T$ are complex conjugation. $T_x$ and $T_y$ represent the genuine TRS operation. Effective TRS of the system is, therefore, determined by the transformation property of the mass term (see table 2). When a mass term is odd against TRS, the system shows the unitary behavior. The four unitary phases in fig. 1 correspond to the four minus signs in table 1. If some (pseudo or genuine) TRS exists in the system, its weak-localization property is determined by the number $N_s$ of the activated spin degrees of freedom. One can verify $T_{\Sigma}^2 = 1$ if $N_s$ is even, whereas $T_{\Sigma}^2 = -1$ if $N_s$ is odd. The former (latter) corresponds to the orthogonal (symplectic) class in the random matrix theory [7], and leads to constructive (destructive) interference between two scattering processes transformed from one to the other by $T_{\Sigma}$.

In the presence of a mass term, irrespective of its type, one finds unitary behavior for LRS. Scattering matrix elements are diagonal in $\vec{\sigma}$ space, and $\vec{\sigma}$ is the only active spin ($N_s = 1$, TRS broken). In the massless case,
Table 2: Time reversal operations $T_{\sigma}$, relevant in the subspace spanned by activated spins. Transformation property of a mass term $\mathcal{O} = \sigma_3 \mathcal{O}_{s_3} T_{\sigma s_3}$, under $T_{\sigma}$: $T_{\sigma} T_{\sigma s_3} = \pm \mathcal{O}$. The sign appears in the table. $U$ refers to unitary class.

| activated spins | $\sigma_3$ | $\sigma_3 \tau_3 s_3$ | $\bar{\sigma}_3 \tau_3 s_3$ | $\bar{\sigma}_3 \bar{\tau}_3 s_3$ |
|-----------------|-----------|---------------------|---------------------|---------------------|
| relevant TRS operation | $T_{\sigma}$ | $T_{\sigma \tau}$ | $T_{\sigma s_3}$ | $T_{\sigma \tau s_3}$ |

a pseudo TRS operation $T_{\sigma}$ mimics the role of genuine TRS [24]. Once TRS is effectively restored, the system’s WL property is determined by the parity of $N_\sigma$. The mass term, on the other hand, breaks the (pseudo) TRS. SRS activate the valley spin $\tau$ ($N_\tau = 2$), since its matrix elements involve off-diagonal terms in this subspace. In the case of intrinsic mass term, this leads to standard WL, since activation of the valley spin $\tau$ restores the pseudo TRS. In the case of topological mass term, the system stays unitary, since restoration of TRS needs also the activation of real spin $\tau$. The latter is embodied by the Rashba SOI. In the presence of both SRS and Rashba SOI, we predict AL, since $N_\sigma = 3$.

Weak-localization corrections. – Starting with the case of $\lambda R = 0$, let us go into some details of diagrammatic calculations. In the presence of the topological mass term $H_\Delta$, the spinor part of the conduction band eigenstates reads

$$|K\alpha\rangle = \begin{pmatrix} \cos \frac{\theta_\alpha}{2} \\ e^{i\phi_\alpha} \sin \frac{\theta_\alpha}{2} \end{pmatrix}, \quad |K'\alpha\rangle = \begin{pmatrix} e^{i\phi_\alpha} \sin \frac{\theta_\alpha}{2} \\ -\cos \frac{\theta_\alpha}{2} \end{pmatrix},$$

where $\alpha$ specifies a three-dimensional fictitious momentum $\vec{p} = (p_x, p_y, -\Delta)$, and the polar angles $\theta, \phi$ satisfy $\cos \theta = -\Delta/\sqrt{p_x^2 + p_y^2 + \Delta^2}$, $\cos \phi = p_x/\sqrt{p_x^2 + p_y^2}$.

LRS have a potential range much larger than the interatomic distance, and do not couple $K$ and $K'$. The system cannot see the difference between two types of mass term, both showing unitary behavior, i.e., the diffusion-type singularity is cut off by a cooperon’s lifetime. This unitary phase shows a crossover to the well-established symplectic behavior of graphene in the single Dirac cone [1,3].

SRS involve inter-valley scattering, allowing for distinguishing the two different types of mass term: topological and ionic. The scattering matrix elements involve a projection operator in the AB sublattice space, $\mathcal{P}_{A,B}$. As for the singular contribution, one can focus on the two types of diagrams shown in fig. 2(a). The “trans” component $\gamma_t$ reads explicitly

$$\gamma_t = 2\pi\nu n_\alpha u_\alpha^3 (K \beta |P_{A \tau}| K \alpha) (K \beta' |P_{A \tau}| K' \alpha') + 2\pi\nu n_\beta u_\beta^3 (K \beta' |P_{B \tau}| K \alpha) (K \beta |P_{B \tau}| K' \alpha') = -e^{i(\phi_\alpha - \phi_\beta)} \eta_S \sin^2 \theta/2 = -\gamma_c,$$

where we have introduced $\eta_S = 2\pi\nu (n_\alpha u_\alpha^3 + n_\beta u_\beta^3)/2$, with $\nu$, $n_\alpha n_\beta$ and $u_\alpha u_\beta$ being, respectively, the density of states, the impurity density and the typical strength of scattering potential at the A (B) sites. $\gamma_c$ has an additional minus sign, which plays the role of driving the crossover from symplectic to orthogonal behavior in the massless limit [1]. The Bethe-Salpeter equation (BSE) takes the form of two coupled equations:

$$\begin{pmatrix} \Gamma'_{c} \\ \Gamma'_{t} \end{pmatrix}_{\alpha\beta} = \begin{pmatrix} \gamma_{c} & \gamma_{\tau} \\ \gamma_{\tau} & \gamma_{c} \end{pmatrix} \Pi_{\mu} \begin{pmatrix} \Gamma'_{c} \\ \Gamma'_{t} \end{pmatrix}_{\mu\beta}. \quad (5)$$

After diagonalization, one finds, $\Gamma_c + \Gamma_t = 0$, and $|1-(\gamma_{c}^{(1)}-\gamma_{\tau}^{(1)})\Pi S| (\Gamma_c - \Gamma_t) = \gamma_c - \gamma_t$, where $\Pi S \simeq \gamma_S (1 - \gamma_S Dq^2)$ with $\gamma_S = 1/\eta_S$ being the scattering time. We also introduced $\gamma_{c}^{(1)}$ in the light of a general expression: $\gamma = \sum_{l} \gamma_{c}^{(l)} e^{i(l\phi_\alpha)}$. Cancellation between $\gamma_{c}$ and the bare $\gamma_{\tau}$ is incomplete, giving a finite lifetime. The system shows a unitary behavior, irrespective of the preserved TRS of underlying Hamiltonian $H = H_1 + H_2$, $1/\eta^2$-singularity is recovered in the limit $E \to \infty$ (orthogonal class).

In the case of ionic mass term, a complete cancellation between the self-energy and the bare vertex function occurs, driving a crossover from unitary to orthogonal symmetry class. This situation is relevant to the intrinsic single-valley system [20,21] in the presence of off-diagonal interaction.

Rashba SOI appears when inversion symmetry with respect to the 2D plane is broken, say, by a perpendicular electric field. Rashba SOI lifts the two-fold real spin degeneracy of the two conduction and two valence bands. An accidental degeneracy occurs on top of the valence bands, whereas the conduction bands $E_{u\pm}$ are split by $2\lambda R$: $E_{u\pm} = \pm p_x^2 + p_y^2 + (\Delta - \lambda R^2)/2 \pm \lambda R/2$. In order to parametrize the corresponding eigenspinors, it is convenient to introduce fictitious 3D momenta: $\vec{p}_{u\pm} = (p_x, p_y, \Delta \pm \lambda R/2)$, and $\cos \theta_{u\pm} = (\Delta \pm \lambda R^2)/\sqrt{p_x^2 + p_y^2 + (\Delta \pm \lambda R^2)^2}$. In terms of these parameters, the eigenspinors corresponding to $E_{u\pm}$ read

$$|K(\pm)\rangle \propto \begin{pmatrix} e^{-i\phi} \sin \frac{\theta}{2} \\ \cos \frac{\theta}{2} \\ \mp i \cos \frac{\theta}{2} \end{pmatrix} \sin \theta/2 / \sin \theta/2,$$

where we omitted the normalization factor $1/\sqrt{2}$. Rashba coupling imposes a stronger constraint on the choice of our basis. As a result, the matrix element, such as, $\langle K(\beta) |K(\alpha) \rangle = e^{i(\phi_\beta - \phi_\alpha)} \sin^2(\theta_{\beta}/2) + \sin^2(\theta_{\beta}/2)$, becomes real (no Berry phase).

When $E > \Delta + \lambda R$, inter-branch matrix elements plays a role. They modify the scattering time $\tau_{\pm}$ for the $|K u \pm\rangle$ branches as

$$\frac{1}{\tau_{\pm}} = \frac{\eta_S}{2} \sin^4 \frac{\theta_\pm}{2} + 2 \cos^4 \frac{\theta_\pm}{2} + 2 \sin^2 \frac{\theta_\pm}{2} \sin^2 \frac{\theta_\pm}{2}. \quad (7)$$
As for particle-particle ladders, such as fig. 2(b), four electron states $\alpha, \beta, \alpha', \beta'$ can, in principle, take either of the two channel indices, $|K-\rangle$ or $|K+\rangle$, generating eight types of diagrams in total. However, a simplification is possible at this level: since we are interested only in the $1/q^2$-singular part of cooperon diagrams, we need $p_\alpha + p_\alpha' = q \approx 0$. This means that $\alpha$ and $\alpha'$ must belong to the same branch. We can thus safely focus on such diagrams as $\gamma_{++}$ and $\gamma_{-+}$ depicted in fig. 2(b), or even simpler $\gamma_{--}$ and $\gamma_{+-}$. Other $\gamma$'s such as figs. 2(c) and (d) are irrelevant to $1/q^2$ singularity. Explicit form of relevant $\gamma$'s are given as

$$
\gamma_{\pm\pm} = \eta_l \left[ \cos^2(\phi_\alpha - \phi_\beta) \sin^4 \frac{\theta_\pm}{2} + 2 \cos^2(\phi_\alpha - \phi_\beta) \sin^2 \frac{\theta_\pm}{2} \cos^2 \frac{\theta_\pm}{2} + \cos^4 \frac{\theta_\pm}{2} \right]
$$

$$
\gamma_{\mp\mp} = -\eta_l \sin^2(\phi_\alpha - \phi_\beta) \sin^2 \frac{\theta_\pm}{2} \sin \frac{\theta_\pm}{2}.
$$

These four types of diagrams satisfy coupled BSE. But its $4 \times 4$ coupling matrix is shown to be block diagonalizable. One can, e.g., decouple $\Gamma_{--}$ and $\Gamma_{+-}$, from the remaining part. The former two obey coupled equations, which take symbolically the following form:

$$
\begin{bmatrix}
\Gamma_{--} \\
\Gamma_{+-}
\end{bmatrix} = \begin{bmatrix}
\gamma_{--} & \gamma_{-+} \\
\gamma_{+-} & \gamma_{++}
\end{bmatrix} \begin{bmatrix}
\Gamma_{--} \\
\Gamma_{+-}
\end{bmatrix},
$$

where $\Pi_{\pm \mp} \approx \tau_{\pm \mp} (1 - \tau_{\pm \mp} Dq^2)$ with $\tau_{\pm \mp}$ given in eq. (7). To identify the singular contribution, we employ the expansion into different angular momentum contributions, both for $\gamma$'s and $\Gamma$'s as $\Gamma = \sum_\ell \Gamma^{(\ell)} e^{i(\phi_\alpha - \phi_\beta)}$, and pick up only the $\ell = 0$ component. The following identity, ensures that the dressed cooperons show indeed $1/q^2$-singularity at the $l = 0$ channel. Rashba SOI thus drives the system to standard WL, whenever the Fermi level is above the gap.

If one adiabatically switches off the Rashba term, the simplification we have made for justifying eq. (9) is no longer valid. In the limit of vanishing $\lambda_R$ we cannot simply neglect such diagrams as figs. 2(c), (d). They could contribute equally to the $1/q^2$ singularity if the singularity ever appears. Relations such as $p_\alpha + p_\alpha' = q \approx 0$ can be satisfied in these diagrams. In the coupled BSE, the cooperons acquire more channels to couple to, and one can no longer decouple $\Gamma_{--}$ and $\Gamma_{+-}$ as eq. (9). As a result, the cancellation property between particle-particle ladders and the self-energy such as eq. (10) is lost. The loss of cancellation property immediately leads to the absence of WL. On the contrary, if one starts from the situation where Rashba SOI is absent in the first place, it is possible to choose a basis in which that two (degenerate) channels are decoupled.

As for the ionic mass, Rashba SOI plays no role and the system stays unitary. The same applies to the case of ripples [12], the latter possessing the same symmetry properties as the ionic mass.

We finally consider the most realistic case of SRS in the presence of Rashba SOI ($\lambda_R \neq 0$). One has to use (6) instead of (3), and develop a similar analysis. One finds in this case the cancellation between the self-energy and $\gamma$ at the $l = 0$ channel. But here additional minus sign analogous to eq. (4) appears due to inter-valley scattering, driving the system to AL.

**Concluding remarks.** – Let us estimate the strength of gate electric field required for observing the crossover to AL. For the crossover to be experimentally accessible, Rashba SOI needs to be the order of $\sim 1 K$. This corresponds to the electric field of order $\sim 1 \text{ V/nm}$ [15], a value

![Fig. 2: Particle-particle ladders. (a) Relevant diagrams in the presence of short-range scatterers (SRS). “cis” and “trans” refer to specific configurations of the valleys. (b)–(d) Bare diagrams involving inter-branch processes ($\lambda_R \neq 0$, no SRS). (b) $\gamma_{+-}$ contributes to $1/q^2$-singularity, whereas such diagrams as (c) and (d) are irrelevant to the singularity, since $p_\alpha + p_\alpha'$ cannot be smaller than the order of $\lambda_R$.](Image)
attainable in double-gated graphene devices [25]. The crossover to AL will be observed for a sample with insignificant ripples. A similar crossover due to Rashba SOI has been observed in another context in InGaAs/InAlAs quantum wells [26]. In the case of graphene, crossovers occur not only from WL, but also from the unitary class [9].

In conclusion, we have identified key elements responsible for rich localization properties of graphene that are controllable by the substrate electric field. The criterion on whether the system is driven to WL or AL is simply given by the number of active (pseudo-)spin degrees of freedom.

***

K-II and KN acknowledge support from KAKENHI: Grant-in-Aid for Young Scientists (B) K-II: 19740189; KN: 20740167.

REFERENCES

[1] Suzuura H. and Ando T., Phys. Rev. Lett., 89 (2002) 266603; McCann E., Kechedzhi K., Falko V. I., Suzuura H., Ando T. and Altshuler B. L., Phys. Rev. Lett., 97 (2006) 146805.
[2] Khveshchenko D. V., Phys. Rev. Lett., 97 (2006) 036802; Aleiner I. L. and Efetov K. B., Phys. Rev. Lett., 97 (2006) 236801; Altland A., Phys. Rev. Lett., 97 (2006) 236802.
[3] Nomura K., Koshino M. and Ryu S., Phys. Rev. Lett., 99 (2007) 146806; Bardarson J. H., Tworzydlo J., Brouwer P. W. and Beenakker C. W. J., Phys. Rev. Lett., 99 (2007) 106801.
[4] Geim A. K. and Novoselov K. S., Nat. Mater., 6 (2007) 183.
[5] Hikami S., Larkin A. I. and Nagaoka Y., Prog. Theor. Phys., 63 (1980) 707.
[6] Chakravarty S. and Schmid A., Phys. Rep., 140 (1986) 193; Bergman G., Phys. Rep., 107 (1984) 1.
[7] Dyson F. J., J. Math. Phys. (N.Y.), 3 (1962) 140.
[8] Ando T., Nakanishi T. and Saito R., J. Phys. Soc. Jpn., 67 (1998) 2857.
[9] Morozov S. V., Novoselov K. S., Katsnelson M. I., Schedin F., Ponomarenko L. A., Jiang D. and Geim A. K., Phys. Rev. Lett., 97 (2006) 016801; Tan Y. W., Zhang Y., Stormer H. L. and Kim P., Eur. Phys. J., 148 (2007) 15.
[10] Wu X. S., Li X. B., Song Z. M., Berger C. and de Heer W. A., Phys. Rev. Lett., 98 (2007) 136801.
[11] Morpurgo A. F. and Guinea F., Phys. Rev. Lett., 97 (2006) 196804.
[12] Nomura K., Ryu S., Koshino M., Mudry C. and Furusaki A., Phys. Rev. Lett., 100 (2008) 246806.
[13] Bostwick A., Ohta T., Seyller T., Horn K. and Rotenberg E., Nat. Phys., 3 (2007) 36.
[14] Zhou S. Y., Gweon G. H., Fedorov A. V., First P. N., de Heer W. A., Lee D. H., Guinea F., Castro Neto A. H. and Lanzara A., Nat. Mater., 6 (2007) 770.
[15] Min H., Hill J. E., Sinitsyn N. A., Sahu B. R., Kleinman L. and MacDonald A. H., Phys. Rev. B, 74 (2006) 165410; Huertas-Hernando D., Guinea F. and Brataas A., Phys. Rev. B, 74 (2006) 155426; Yao Y., Ye F., Qi X. L., Zhang S. C. and Fang Z., Phys. Rev. B, 75 (2007) R041401.
[16] Kane C. L. and Mele E. J., Phys. Rev. Lett., 95 (2005) 146802; 226801.
[17] Nakanishi T. and Ando T., J. Phys. Soc. Jpn., 68 (1999) 561.
[18] Onoda M., Avishai Y. and Nagaosa N., Phys. Rev. Lett., 98 (2007) 076802.
[19] Obuse H., Furusaki A., Ryu S. and Mudry C., Phys. Rev. B, 76 (2007) 075301; 78 (2008) 115301.
[20] Bernevig B. A., Hughes T. L. and Zhang S. C., Science, 314 (2006) 1757.
[21] König M., Wiedmann S., Brüne C., Roth A., Buhmann H., Molenkamp L. W., Qi X. L. and Zhang S. C., Science, 318 (2007) 766.
[22] Semenoff G. W., Phys. Rev. Lett., 53 (1984) 2449.
[23] Haldane F. D. M., Phys. Rev. Lett., 61 (1988) 2015.
[24] Ludwig A. W. W., Fisher M. P. A., Shankar R. and Grinstein G., Phys. Rev. B, 50 (1994) 7526.
[25] Oostinga J. B., Heersche H. B., Liu X., Morpurgo A. F. and Vandersypen L. M. K., Nat. Mater., 7 (2007) 151.
[26] Koga T., Nitta J., Akazaki T. and Takayanagi H., Phys. Rev. Lett., 89 (2002) 046801.