Bright and dark states of light: The quantum origin of classical interference

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Classical theory asserts that several electromagnetic waves cannot interact with matter if they interfere destructively to zero, whereas quantum mechanics predicts a nontrivial light-matter dynamics even when the average electric field vanishes. Here we show that in quantum optics classical interference emerges from collective bright and dark states of light, i.e., entangled superpositions of multi-mode photon-number states. This makes it possible to explain wave interference using the particle description of light and the superposition principle for linear systems.

The quest for understanding what light is, and what its properties are, comes from the Greek school of philosophy a few centuries BC. Since then the subject has been extensively studied, with prominent contributions from Newton and Huygens, the former defending the corpuscular and the latter advocating the wave character of light [1, 2]. This dispute remained unresolved until, among others, Young performed experiments on light diffraction [3, 4], and Maxwell developed a unified theory of electromagnetism which includes a wave equation for the light field [5]. This effectively removed any doubts about the wave aspect of light. However, in 1905, Einstein explained the photoelectric effect by reintroducing the idea of light particles [6] (later reinterpreted by Lamb and Scully [7]). Since then, and with the advent of quantum physics, light is associated with both properties, wave and particle. Depending on the experiment, one or the other aspect manifests itself: the interference of delocalized waves on the one hand and the propagation of particles along well-defined trajectories on the other hand. Although this is textbook knowledge by now, it was highly debated at its time. For example, Millikan argued that the particle aspect “flies in the face of the thoroughly established facts of interference” [8].

Here we resolve Millikan’s objection and show that the interference between independent radiation modes, usually taken as an undoubted signature of the wave character of radiation, has in fact a purely corpuscular explanation. For this we adopt a microscopic description of the measurement process in terms of energy exchange between the radiation and a sensor atom, as formally derived by Glauber [9]. We then introduce the entangled dark and bright states of light, i.e., a new kind of subradiant and superradiant states for collective modes of light fields, analogous to those which appear in atomic systems [10]. With this new model we are able to interpret the optical double-slit experiment in terms of particle states only. We discuss the rather counterintuitive result that a vanishing photon-detection probability at locations of destructive interference does not prove the absence of photons but instead indicates a photonic state that is perfectly dark for the employed sensor atom. We also show that the collective states of the light fall into three distinct classes: perfectly dark, maximally superradiant, and intermediate. Interestingly, classical interference, fully destructive or constructive, is described by a superposition of perfectly dark or maximally superradiant states only (see the illustration in Fig.1(a)), but any intermediate quantum state has no counterpart in classical theory.

According to the quantum theory of optical coherence introduced by Glauber [9], the statistical properties of a given field can be derived from its electrical field operator

$$E(r, t) = E^+(r, t) + E^-(r, t),$$

(1)

with $E^+(r, t)$ and $E^-(r, t)$ the positive and negative frequency parts, respectively. Quantum mechanically, these parts are proportional to the photon annihilation and creation operators, that is, $E^+(r, t) \propto a$ and $E^-(r, t) \propto a^\dagger$. Still according to [9], the probability of a photon from a single mode in a given state $|\Psi\rangle$ being absorbed by a sensor atom is proportional to

$$\langle \Psi | E^-(r) E^+(r) | \Psi \rangle \propto \langle \Psi | a^\dagger a | \Psi \rangle.$$  

(2)

This expression comes from the energy-change interaction between the field and the sensor, described by the Hamiltonian (in the rotating-wave approximation)

$$H = E^+(r, t) \sigma^+ + E^-(r, t) \sigma^-,$$

(3)

with $\sigma^+$ ($\sigma^-$) the raising (lowering) operators that induce transitions between ground $|g\rangle$ and excited $|e\rangle$ states of the sensor atom. From Eq. (2) one can easily see that, for single mode fields, only zero-intensity (vacuum) fields are unable to excite the sensor.

For two-mode fields, and from a quantum perspective, the situation is much more interesting. To see this, we evaluate the probability of exciting the sensor using the eigenstates of the positive-frequency operator with null eigenvalues, that is, $E^+(r, t)|\Psi\rangle = 0$ or, equivalently, $H|\Psi\rangle |g\rangle = 0$. For a single mode, only the vacuum state satisfies this condition. But as shown below, multi-mode fields allow for a plethora of such states, even states with many photons in each mode. In the context of cavity quantum electrodynamics, the states which carry photons but are unable to excite an atom were dubbed “generalized ground states” by Alsing, Cardimona, and Carmichael [11], but here we decide to name them perfectly dark states (PDSs) since the sensor cannot see the field whenever it is in such a state.
To address the multi-mode case, we consider two modes, A and B, represented by their respective annihilation (creation) operators \( a \) and \( b \) (\( a^\dagger \) and \( b^\dagger \)), and a relative phase between them given by \( \theta \). In this case, the positive frequency operator to excite the sensor atom, for any \( N \). We name it the perfectly dark state (PDS) for the subspace of \( N \) photons

\[
|\psi_0^N(\theta)\rangle = \sqrt{\frac{N!}{2^N}} \sum_{m=0}^{N} \frac{(-1)^m e^{im\theta}}{\sqrt{m!}(N-m)!} |m, N-m\rangle_{a,b},
\]

In atomic system it was coined subradiant by Dicke since \( H|\psi_0^N\rangle = 0 \). Oppositely, the \( n = N \) state

\[
|\psi_N^N(\theta)\rangle = e^{-iN\theta} \sqrt{\frac{N!}{2^N}} \sum_{m=0}^{N} \frac{e^{im\theta}}{\sqrt{m!}(N-m)!} |m, N-m\rangle_{a,b},
\]

comes with a transition rate \( g\sqrt{2N} \), which is \( \sqrt{2} \) times that of the single-mode result: \( H_{JC}|g\rangle|N\rangle = g\sqrt{N}|e\rangle|N-1\rangle \), with \( H_{JC} \) denoting the standard Jaynes-Cummings Hamiltonian [14]. State (7) is the analogue of the symmetric superradiant mode, studied by Dicke in the atomic decay cascade [10, 15], and represents the most superradiant of the states with \( N \) photons. We here refer to this state as a maximally superradiant state (MSS) or bright state. Finally, states within the range \( 0 < n < N \) have intermediate transition rates. In contrast to the two-level-atom case, the present Hilbert space is unbounded, even for a finite number of field modes, since each one can support an arbitrary number of photons [13] (see Fig. 1b).

As we have just seen, dark states are eigenstates of the operator \( E^{(+)}(r,t) \) with null eigenvalue, which implies that they are undetectable by usual sensors such as those described by two-level atoms. On the other hand, the bright states couple stronger than in the single-mode case. Thus, a natural question arises concerning the connection between the quantum mechanical dark (bright) states and the classical effects of destructive (constructive) interference between radiation fields, as in regions of destructive interference no light is detected, while in regions of constructive interference light scattering is enhanced. To address this question we consider, for simplici-
ity, the case of two modes with $\theta = 0$. Then, one can easily show that in-phase coherent states decompose exclusively on the MSS subspace:

$$|\alpha, \alpha\rangle = e^{-|\alpha|^2} \sum_{N=0}^{\infty} \frac{2^N N!^2}{\sqrt{N!}} \alpha^N |\psi_N^N\rangle.$$  

(8)

This implies an enhanced absorption by a factor of 2 ($H|\alpha, \alpha\rangle|g\rangle = 2g|\alpha, \alpha\rangle|e\rangle$) as compared to a single coherent state ($H|\alpha\rangle|g\rangle = g|\alpha\rangle|e\rangle$). The two-mode MSS quantum state therefore corresponds to the constructive interference of classical in-phase fields.

On the other hand, two coherent fields with opposite phases decompose in terms of PDSs only:

$$|\alpha, -\alpha\rangle = e^{-|\alpha|^2} \sum_{N=0}^{\infty} \frac{2^N N!^2}{\sqrt{N!}} \alpha^N |\psi_N^N\rangle.$$  

(9)

This state gives a suppressed interaction $H|\alpha, -\alpha\rangle|g\rangle = 0$, which can be interpreted either as belonging to the PDS subspace or, classically, as a destructive interference for two fields with the same amplitude but opposite phases. However, not every destructively interfering field is undetectable. This can be seen by considering two modes in the state

$$|\Upsilon\rangle = (|0\rangle_a + |1\rangle_a) (|0\rangle_b - |1\rangle_b) / 2.$$  

(10)

In each of the modes we have a non-zero average electric field but, as they are out of phase with each other, the average resulting electric field is zero. According to the classical interpretation of interference, such a field would be undetectable but, according to Glauber’s theory [16], it is detectable, in the sense that it will induce a non-trivial dynamics for the sensor/atom. This can be easily explained using the description in terms of bright and dark states, since such state can be written in the form $|\Upsilon\rangle = [(|\psi_0^0\rangle - \sqrt{2}|\psi_0^2\rangle) + (|\psi_2^0\rangle - |\psi_0^2\rangle) / \sqrt{2}]/2$, which shows a projection onto the detectable subspace of bright states.

A key phenomenon for evidencing the wave nature of light comes from Young’s double slit experiment. The fundamental result is that both classical and single-photon coherent sources produce the same fringe pattern, despite the very different nature of these fields. To revisit this experiment using the collective basis, one can consider two equally weighted light modes emerging from two slits and in the far-field limit. Without loss of generality, we assume both waves with wave vectors $k_1$ (mode $a$) and $k_2$ (mode $b$), with $|k_1| = |k_2| = k$. Then, $k_1 \cdot r_1$ and $k_2 \cdot r_2$, with $r_1$ the vector connecting the $i$th slit with the sensor position, are the phases acquired by the respective fields when propagating from slits 1 and 2, respectively, to the detection point (see Fig. 1).

For a single photon impinging on a double slit, the field in the plane of interest and the detection process can be described following Scully and Zubairy [17], i.e., by replacing the two slits with two source atoms, the first at the position $d_1$ and the second at the position $d_2$ (the positions of the slits). Apart from a normalization factor which depends on the radiation pattern of the two ‘slit’ atoms, $g(k)$, the field is given by the state (see SM for details)

$$|S\rangle = \frac{1}{\sqrt{2}} (e^{-ik_1 d_1}|1, 0\rangle_{a,b} + e^{-ik_2 d_2}|0, 1\rangle_{a,b}) \otimes_{k_2,k_1}|0_k\rangle.$$  

(11)

Then, the probability for a photon at position $r$ is given by the first-order intensity correlation function $C^{(1)}(r, r, 0) = \langle \psi|E(-)E(+)|\psi\rangle = \langle 0|E(+)|\psi\rangle^2$, with $E(\pm) \propto ae^{ik_1 r} + be^{ik_2 r} \propto a + be^{i\theta}$, and $\theta = (k_2 - k_1) \cdot r$. As described above, for this collective measurement operator the dark and bright states can be written as $|\psi^0_1(\theta)\rangle = (|1, 0\rangle_{a,b} - e^{-i\theta}|0, 1\rangle_{a,b}) / \sqrt{2}$ and $|\psi^1_1(\theta)\rangle = (|1, 0\rangle_{a,b} + e^{-i\theta}|0, 1\rangle_{a,b}) / \sqrt{2}$, respectively (apart from the other vacuum modes of the electromagnetic field). With such equations we can rewrite Eq. (11), up to a global phase factor, as

$$|S\rangle = \cos(\delta\phi/2)|\psi^1_1(\theta)\rangle - i\sin(\delta\phi/2)|\psi^0_1(\theta)\rangle,$$

(12)

where $\delta\phi = -(k_2 \cdot d_2 - k_1 \cdot d_1) + \theta = k_2 \cdot r_2 - k_1 \cdot r_1$, i.e., $\delta\phi$ represents the phase difference of the two light paths from the slits to the sensor atom. Clearly, at any detector position ($\delta\phi$) we may have a bright, a dark, or a superposition of bright and dark states, which implies that the photon can be at any position. In other words, apart from the $g(k)$ distribution, the average number of photons as a function of $\delta\phi$ is constant, i.e., $\langle a^\dagger a + b^\dagger b(\delta\phi)\rangle = 1$. However, the sensor atom can detect the photon only at positions where the bright state survives.

On the other hand, when a coherent state is sent to a double slit, part of the light goes through slit 1 at position $d_1$, and part of it goes through slit 2 at position $d_2$. In this case, we do not have a superposition state, but rather a product state of the two modes $k_1$ and $k_2$ in coherent states with amplitude $\alpha$ (assumed the same for both slits) and a relative phase which depends on the position of the slit, i.e.,

$$|e^{-ik_1 d_1 \alpha}, e^{-ik_2 d_2 \alpha}\rangle_{a,b} = e^{-|\alpha|^2} \sum_{N=0}^{\infty} C_N |\chi^N(\delta\phi)\rangle,$$

(13)

with $C_N = \sqrt{2^N N!^2} e^{-iN\delta\phi \pm iN\theta}$ and the phase-dependent state

$$|\chi^N(\delta\phi)\rangle = \frac{\sqrt{N!}}{2^N} \sum_{m=0}^{N} \frac{e^{-im\delta\phi \pm iN\theta}}{\sqrt{m!(N-m)!}} |m, N-m\rangle,$$

(14)

with $\delta\phi$ defined right after Eq. (12). Such state corresponds to a MSS when the two modes are in phase, $|\chi^N(2l\pi)\rangle = e^{iN\theta}|\psi^N_0(\theta)\rangle$, and to a PDS when in opposite phases, $|\chi^N((2l+1)\pi)\rangle = |\psi^0_1(\theta)\rangle$, with $l = 0, \pm 1, \pm 2, \ldots$. In this case, the average number of photons is independent of the phase difference, i.e.,

$$\langle a^\dagger a + b^\dagger b(\delta\phi)\rangle = 2|\alpha|^2,$$

(15)

meaning that photons are present at every point on the screen, contrary to the standard classical description of interference,
which states that no light arrives at points of destructive interference. The single-photon state $|S\rangle$, which decomposes as a sum of PDSs or MSSs only (see Eq. (12)), has the same feature, as discussed above. Therefore, the decomposition either in only PDSs or only MSSs explains why single-photon fields and classical fields exhibit the same fringe pattern.

The particular case just discussed actually points towards a more general result: as proven in the SM, states of light composed of only PDSs or MSSs exhibit the same interference patterns as those derived from linear (classical) optics or, equivalently, for coherent states. However, this is not the case for the general collective states $|\psi^N_n(\theta)\rangle$, Eq. (5), or superpositions with both dark and bright states, an effect which could be useful to discriminate between quantum and classical states of light without the need of employing field-field correlations [17, 18]. This can be exemplified by considering a Mach-Zehnder interferometer (MZI), see SM for details, with the input state as the intermediate entangled state $|\psi_2^\ominus\rangle = (|0,2\rangle_{a,b} - |2,0\rangle_{a,b})/\sqrt{2}$ (mode $a$ ($b$) as the first (second) input port of the MZI), a state which is generated by impinging two photons on the two input ports of a 50/50 beam splitter [19]. With this input, the two output ports of the MZI contain the same average number of photons independent of the relative phase $\varphi$ within the MZI, i.e., $\langle n_a\rangle_{|\psi_T^\ominus\rangle} = \langle n_b\rangle_{|\psi_T^\ominus\rangle} = 1$ (see SM). This means that the fringes completely disappear, and the visibility of an interference pattern goes to zero. The situation is completely different for classical fields, where the intensity in one of the output ports of the MZI goes to zero depending on the relative phase $\varphi$. In this sense, the reduction of the visibility in the quantum case stems from the quantum nature of the field.

In light of our discussion, we can conclude that the single-mode case has a unique PDS, namely the vacuum state $|0\rangle$. It may sound trivial since there is no photon to excite the detector, yet its interest lies in its uniqueness: any other state excites the sensor atom. Thus, our interpretation in terms of dark and bright states provides a new way to explain why single-mode Fock states $|N\rangle$ with $N > 1$ do excite the sensor atom, even for zero mean electric field. On the other hand, the multimode case is fundamentally different since it possesses an infinite family of dark states with an arbitrarily large number of photons, which do not couple to the sensor atom in the ground state. This scenario is analogous to the case of two-level atoms where single-atom (spontaneous) emission always occurs, whereas the emission from a couple of atoms in the dark state $|\ominus\rangle = (|e,g\rangle - |g,e\rangle)/\sqrt{2}$ is suppressed. Alternatively, dark states can be produced using single emitters with a multilevel structure [20], in particular in electromagnetically induced transparency [21]. In addition, the two-mode case also predicts bright and intermediate states, the latter having no correspondence in classical physics. Although the discussions above were developed for only two modes of the radiation field, their generalization to the case $N$ modes of the field is straightforward (see SM).

From an experimental perspective, the two-mode light-matter interaction discussed in the present work suggests an implementation in optical cavities coupled to a two-level atom [22, 23], trapped ions where a single emitter can be coupled to its two vibrational modes [24, 25], or superconducting circuits [26]. We visualize many possibilities, drawing inspiration from the diverse applications that appear in the context of super- and subradiance in atomic systems. For example, one could employ photonic superradiant states to further enhance light emission in high-brightness light sources. On the other hand, as the dark states do not interact with matter, they could in principle be employed as decoherence-free photonic quantum memories. Finally, by taking advantage of the fact that bright states do interact with atoms and dark states do not, one could use such states to imprint a conditional phase on an atomic system. With this, one could employ such collective mode states to implement single-shot logic operations in crossed-cavity setups [23], such as two- and three-qubit CNOT and Fredkin gates, thus allowing for universal quantum computing with traveling modes [27].

In conclusion, we have discussed how a description of multi-mode light in terms of maximally superradiant or perfectly dark collective states offers a natural interpretation for constructive and destructive interference. Remarkably, this Dicke-like bosonic basis applies to classical and non-classical states of light, thus going beyond the simple classical approach of average fields. We have shown that, from a quantum perspective, interference is intimately related to the coupling of light and matter which differs for the bright and dark states. This is completely different in the classical description where no assumption on the matter is necessary to describe the sum of electromagnetic fields. One can interpret this as a manifestation of the quantum-measurement process where the expectation value of an observable depends on the properties of the measuring apparatus [28, 29]. Within this framework, we have interpreted the double-slit experiment and the interference of light waves in general in terms of bright and dark states, i.e., using only the corpuscular description of the light and the quantum-superposition principle.

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[1] I. Newton, *Opticks, or, a treatise of the reflections, refractions, inflections & colours of light* (Courier Corporation, 1952).

[2] C. Huygens, *Treatise on Light: In which are Explained the Causes of that which Occurs in Reflexion, & in Refraction. And Particularly in the Strange Refraction of Iceland Crystal* (MacMillan and Company, limited, 1912).

[3] T. Young, *Miscellaneous Works of the Late Thomas Young…*, Vol. 2 (John Murray, 1855).

[4] A. Rubinowicz, *Nature* 180, 160 (1957).

[5] J. D. Jackson, Classical electrodynamics (1999).

[6] A. Einstein, *American Journal of Physics* 33, 367 (1965).

[7] In 1968, Lamb and Scully provided a semiclassical model, which treats the electromagnetic field classically and the matter quantum mechanically, to explain the photoelectric effect without the need for the corpuscular aspect of light [30].

[8] R. A. Millikan, *Phys. Rev.* 7, 355 (1916).

[9] R. J. Glauber, *Phys. Rev.* 130, 2529 (1963).

[10] R. H. Dicke, *Phys. Rev.* 93, 99 (1954).

[11] P. M. Alsing, D. A. Cardimona, and H. J. Carmichael, *Phys. Rev. A* 45, 1793 (1992).

[12] C. E. Máximo, T. B. B. ao, R. Bachelard, G. D. de Moraes Neto, M. A. de Ponte, and M. H. Y. Moussa, *J. Opt. Soc. Am. B* 31, 2480 (2014).

[13] M. Delanty, S. Rebic, and J. Twamley, Superradiance of harmonic oscillators (2011), arXiv:1107.5080.

[14] E. Jaynes and F. Cummings, *Proceedings of the IEEE* 51, 89 (1963).

[15] M. Gross and S. Haroche, *Physics Reports* 93, 301 (1982).

[16] R. J. Glauber, *Phys. Rev.* 131, 2766 (1963).

[17] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, 1997).

[18] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, 1995).

[19] C. K. Hong, Z. Y. Ou, and L. Mandel, *Phys. Rev. Lett.* 59, 2044 (1987).

[20] A. Piñeiro Orioli and A. M. Rey, *Phys. Rev. Lett.* 123, 223601 (2019).

[21] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, *Rev. Mod. Phys.* 77, 633 (2005).

[22] C. Hamsen, K. N. Tolaži, T. Wilk, and G. Rempe, *Nature Physics* 14, 885 (2018).

[23] M. Brekenfeld, D. Niemietz, J. D. Christesen, and G. Rempe, *Nature Physics* 647 (2020).

[24] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, *Rev. Mod. Phys.* 75, 281 (2003).

[25] A. Mokhberi, M. Henrich, and F. Schmidt-Kaler (Academic Press, 2020) pp. 233–306.

[26] A. Blais, A. L. Grimsmo, S. M. Girvin, and A. Wallraff, *Rev. Mod. Phys.* 93, 025005 (2021).

[27] L. O. R. Solak, D. Z. Rossatto, and C. J. Villas-Boas, arXiv preprint arXiv:2308.14881 (2023).

[28] M. Srinivas and E. Davies, *Optica Acta: International Journal of Optics* 28, 981 (1981).

[29] A. V. Dodonov, S. S. Mizrahi, and V. V. Dodonov, *Phys. Rev. A* 72, 023816 (2005).

[30] W. E. Lamb Jr and M. O. Scully, *The photoelectric effect without photons*, Tech. Rep. (1968).