Δ-scaling and Information Entropy in Ultra-Relativistic Nucleus-Nucleus Collisions

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Ultra-relativistic heavy ion collisions provide a unique means to search for a new state of matter, the Quark-Gluon Plasma (QGP), where quarks and gluons over an extended volume are de-confined [1-3] and a phase transition between hadrons and the QGP will occur [4-6]. There could be a discontinuity in global features, in particular, the entropy from particle multiplicities, in nuclear collisions associated with the onset of the phase transition. In this Letter we report the first application of the Δ-scaling method and information entropy calculation in ultra-relativistic heavy ion collisions.

In intermediate energy heavy ion collisions, Δ-scaling was proposed by Botet and Ploszajczak [7]. They applied the Δ-scaling law to the INDRA data in intermediate energy heavy ion collisions (Xe+Sn, 25-100 MeV/nucleon) by using Zmax (the maximum of charge in reactions) as order parameter [8] and found that the distributions of Zmax obey the Δ = 1/2 scaling law below 32 MeV/nucleon while they obey the Δ = 1 scaling law above 32 MeV/nucleon. This indicates that a transition from an order phase to the maximum fluctuation phase (disorder phase) occurs around 32 MeV/nucleon. Recently, Ma et al. analysed the multi-fragmentation data for mass around ∼ 36 light nuclei systems. They found a similar change of Δ-scaling occurs when the excitation energy of the system around 5.6 MeV/nucleon. Combination of analysis with other probes of phase transition (e.g., Zipf law [9] and maximum fluctuations [10]) reflects a transition of matter phases with the change of Δ-scaling [11]. Since Δ-scaling is a useful tool to identify the phase transition, we try to make the similar analysis in ultra-relativistic heavy-ion collisions.

Botet and Ploszajczak proposed Δ-scaling to identify the transition in intermediate energy heavy ion collisions [12,13]. The Δ-scaling law is observed when two or more probability distributions P[m] of the stochastic observable m collapse onto a single scaling curve Φ(z) if a new scaling observable is defined by:

\[ z = \frac{m - m^*}{\langle m \rangle^\Delta} \] (1)

This curve is:

\[ \langle m \rangle^\Delta P[m] = \Phi(z) = \Phi\left(\frac{m - m^*}{\langle m \rangle^\Delta}\right) \] (2)

where Δ is a scaling parameter, m* is the most probable value of m, and \( \langle m \rangle \) is the mean of m. When Δ = 1, this kind of scaling law is called the first scaling law that is caused by self-similarity of system. This self-similarity means that, if these distributions with different \( \langle m \rangle \) by a new kind of variable z, they entirely collapse on the same curve. In fact, the famous KNO scaling [14] is the special case that the Δ=1 scaling law holds with a stochastic observable of multiplicity of particles. The INDRA data was explored using the Δ = 1/2 and 1 scaling laws and a phase transition was observed [15]. If we assume that P[m] is a Gaussian distribution, we have:

\[ P[m] = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{m - \mu}{\sigma}\right)^2\right] \] (3)

where \( \mu = \langle m \rangle = m^* \), σ is the width of the Gaussian distribution, they both depend on incident energy. If this Gaussian distribution P[m] obeys Δ-scaling law, we should have:

\[ \mu^\Delta \propto \sigma \] (4)

On the other hand, the information entropy is observable such that it was proposed to describe the fluctuation.
and disorder of a system by Shannon [16]. It is defined by:

\[ H = -\int P(m) \ln P(m) dm \]  

(5)

where \( \int P(m) dm = 1 \). This indicates that the quantity of fluctuation of system and depend on the distribution \( P[m] \). If \( P[m] \) is the Gaussian distribution, we have:

\[ H = \ln \sigma + (1 + \ln 2\pi)/2 \simeq \ln \sigma + 1.419 \]  

(6)

In this work, we investigate the \( \Delta \)-scaling law and the information entropy for p+p, C+C and Pb+Pb in high-energy collisions (20-200 AGeV) with help of LUCIAE 3.0 of Sa and Tai [17]. The head-on collisions are simulated in this work. The LUCIAE is of a Monte Carlo model and is an extension of the FRITIOF [18]. Here the nucleus - nucleus collision is described as the sum of nucleon-nucleon collisions. LUCIAE was improved in the following three aspects: (1) Re-scattering of final hadrons, spectator and participant nucleons are considered in the LUCIAE, because the role of re-scattering [19] can not be neglected in high-energy domain. (2) The LUCIAE implements the Firecracker Model that includes collective multi-gluon emission from the color fields of interacting strings in the early stage of relativistic ion collisions. (3) LUCIAE introduces the suppression of strange quarks and effective string tensor to make some parameters related to product of strange quarks and string tensor in the JETSET depend on incident energy, size of system, centrality etc. Generally the LUCIAE can explain many data very well [20,21].

First, we choose the particle multiplicity produced in each event as stochastic observable \( m \) and simulated 10000 events for p+p at every energy point to obtain normalized multiplicity distributions correspondingly. Then we performed \( \Delta \)-scaling. As a result, we find that the multiplicity distributions are approximately satisfied with the \( \Delta = 1 \) scaling law, as shown in Figure 1.

We calculated \( \langle m \rangle = \sum m_i P(m_i) \) and \( m_{RMS} = \sqrt{\sum (m^2_i - \langle m_i \rangle^2) P(m_i)} \) in terms of these normalized multiplicity distributions directly. While we fitted these distributions with Gaussian distributions to obtain \( \mu \) and \( \sigma \). By comparing \( \mu \) with \( \langle m \rangle \) and \( \sigma \) with \( m_{RMS} \), we found that these distributions are basically Gaussian. Table 1 shows the fitting parameters and information entropy (\( H_{direct} \) or \( H_{gauss} \), which was calculated directly or by Gaussian parameter, respectively (see details in Ref. [22] and in the following).

In order to better investigate the \( \Delta \)-scaling, we defined a coefficient

\[ L = \frac{\langle m \rangle^\Delta}{m_{RMS}}, \]  

(7)

which characterizes the validity of the \( \Delta \)-scaling. We investigate its dependence of incident energy with different \( \Delta \) values. Figure 2 shows that the \( L \)-dependence on incident energy from \( \Delta = 0.5 \) to \( \Delta = 1.6 \). As a result, we find that \( L \) is nearly a constant in whole investigated range (20-200 AGeV) when \( \Delta = 1.35 \), i.e., the system obeys the \( \Delta \)-scaling law most suitably with \( \Delta = 1.35 \) (see Fig.3a). We also deal with another two systems of C+C (5000 events) and Pb+Pb (10000 events). Similarly, their multiplicity distributions basically obey Gaussian distributions. In the same way, the best \( \Delta \)-scaling is obtained with \( \Delta = 1.00 \) for C+C and \( \Delta = 0.80 \) for Pb+Pb, respectively. Figure 3b and 3c illustrate these results.

Through \( \Delta \)-scaling for different systems, we found that the distributions of particle multiplicity obey Gaussian distributions approximately, especially for C+C and Pb+Pb systems and they obey the \( \Delta \)-scaling law in a wide range.
FIG. 2: $L$ as a function of energies.

FIG. 3: $\Delta$-scaling for $p + p$ with $\Delta = 1.35$ (upper panel), for $C + C$ with $\Delta = 1.00$ (middle panel) and for $Pb + Pb$ with $\Delta = 0.80$ (lower panel) for the different beam energies.

FIG. 4: Energy dependence of information entropy for $p + p$, $C + C$ and $Pb + Pb$, respectively.

energy range for a given system, which indicates that no phase transition and no change of reaction mechanism exist. This is expected for the simulated data because the underlying particle production dynamics in the LUCIAE is a smooth function of beam energy without a phase transition.

We calculate the respective information entropy in terms of these distributions of particle multiplicity with the method proposed by Ma in Ref. [22]. Figure 4 shows the dependences of the information entropy on incident energy for the $p + p$, $C + C$ and $Pb + Pb$ systems. The information entropy increases with the incident energy and with the sizes of the system monotonously. Also, the calculated values of $H_{\text{direct}}$ are consistent with the value $H_{\text{gauss}}$ obtained from Eq. (6) in the Gaussian distribution. Again, no indication of phase transition exists.

In addition, we use the distribution of strange particles multiplicity [23] and the distribution of the number of binary collisions [24] as two stochastic observables to make the $\Delta$-scaling plots and extract the information entropy in the same way. The similar $\Delta$-scaling was obtained and a monotonous information entropy was also observed.

In summary, we have demonstrated, for the first time to our knowledge, the $\Delta$-scaling of charged particle multiplicity, strange particle multiplicity and the number of binary collisions using simulated $p + p$, $C + C$ and $Pb + Pb$ collisions from $E_{\text{lab}} = 20$ to $E_{\text{lab}} = 200$ AGeV. The LUCIAE 3.0, which includes the final state interactions, was used for the simulation. The $\Delta$-scaling means that these observables obey a certain kind of universal
laws, regardless of beam energy and collision system. We found that the scaling values $\Delta$ for charged particle multiplicity distributions are 1.35, 1.00 and 0.80 for $p + p$, $C + C$ and $Pb + Pb$ collisions, respectively. Moreover, the information entropy calculated from charged multiplicity distributions increases with the beam energy and with the colliding system size monotonously. Both the $\Delta$-scaling and the entropy values show no dis-continuity as a function of beam energy as expected because the LUCIAE has no change of particle production dynamics, while they are associated with a phase transition in the simulated data. We expect that the $\Delta$-scaling and the entropy variable can be a valuable tool to search for possible dis-continuities in nucleus-nucleus collisions associated with the onset of a QCD phase transition. Further checks for the models which incorporate the QGP phase transition are in progress.

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