Viewpoint on the “Theory of the superglass phase” and a proof of principe of quantum critical jamming and related phases

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A viewpoint article on the very interesting work of Biroli, Chamon, and Zamponi on superglasses. I further suggest how additional new superglass and "spin-superglass" phases of matter (the latter phases contain quenched disorder) and general characteristics may be proven as a theoretical proof of concept in various electronic systems. The new phases include: (1) superglasses of Cooper pairs, i.e., glassy superconductors, (2) superglass phases of quantum spins, and (3) superglasses of the electronic orbitals. New general features which may be derived by the same construct include (a) quantum dynamical heterogeneities- a low temperature quantum analogue of dynamical heterogeneities known to exist in classical glasses and spin-glasses wherein the local dynamics and temporal correlations are spatially non-uniform. I also discuss on a new class of quantum critical systems. In particular, I outline (b) the derivation of the quantum analogue of the zero temperature jamming transition that has a non-trivial dynamical exponent. We very briefly comment on (c) quantum liquid crystals.

I. INTRODUCTION

In an exciting article in this issue, Biroli, Chamon, and Zamponi, (BCZ) illustrate theoretically the possibility of a “superglass” phase. This new quantum phase forms an intriguing amorphous counterpart to the "supersolid" phase that has seen a surge of interest in recent years. Within a "supersolid" phase, superfluidity can occur without disrupting crystalline order.

So, what are “superglasses”? Glasses are liquids that have ceased to flow on experimentally measurable time scales. By constrast, superfluids flow without any resistance. The existence of a phase characterized by simultaneous glassiness and superfluidity may seem like a clear contradiction of terms. In their article, BCZ prove that this is not so. Interacting quantum particles can indeed form such a “superglass” phase at very low temperature and high density; their work confirms the earlier numerical suggestion of such a phase by Bonnsegi, Prokof’ev, and Svistunov and an investigation by Philips and Wu. The superglass phase is characterized by an amorphous density profile. At the same time, a finite fraction of the particles flow without any resistance- as if they were superfluid. Thus, the "superglass" constitutes a glassy counterpart to the "supersolid" phase.

The approach invoked by BCZ to prove the existence of superglasses is particularly elegant. It relies on the mapping between viscous classical systems whose properties are well known to new many body quantum systems. In realizing the link between classical and quantum systems to gain insight into the quantum many body phases, BCZ nicely add an important new result to earlier investigations that built on such similar insights elsewhere. Chester suggested the existence of a supersolid by relying on such a connection. In a similar fashion, Laughlin invoked a highly inspirational analogy between variational (Jastrow type) wavefunctions describing the fractional quantum Hall system and a well known system of classical charged particles interacting via a logarithmic potential. By using the classical plasma analogy and using known results on it, Laughlin was able to make headway on the challenging many body quantum problem and construct his highly successful wavefunctions. The mapping used by BCZ similarly enables exact results on quantum problem and a detailed correspondence of spatial and temporal correlations between the classical and quantum systems. BCZ apply this mapping to a classical system well known to exhibit glassy dynamics- the Brownian hard sphere problem. The quantum counterpart of the classical hard sphere problem is a natural system containing hard sphere interactions. On the classical side of the correspondence, the hard sphere system has been heavily investigated. When the sphere packing density is slowly varied, the classical Brownian hard sphere system undergoes a transition from a liquid to an ordered crystal at high density. When crystallization is thwarted by a rapid increase of the packing density or by, e.g., a change of the particle geometry, the system cannot order nicely into a crystal and instead jams into a dense amorphous glass. BCZ noticed that when fused with the mapping between classical and quantum systems, information on classical glass forming systems such as the Brownian spheres gives rise to highly non-trivial results. In particular, the glassy phase of the classical system translates into a quantum glass of a Bose system. Similarly, the classical solid maps onto a quantum bosonic crystal. The ensuing phase diagram is provided in Fig. (2) of their article. The spatio-temporal correlations of the (bosonic) quantum dual can be computed by mapping to the classical system. Both the glassy and solid phases harbor a finite Bose-Einstein condensate fraction. Putting all of the pieces together, BCZ provide an important proof of concept of the superglass phase in a simple and precise way. This route may be replicated for classical systems other than the Brownian hard sphere which also display solid and glass phases.

What physical systems may realize the new superglass
phase? Recent experiments on solid Helium 4 exhibit super-solid type features and have led to a flurry of activity. In the simplest explanation of observations, a fraction of the medium becomes, at low temperatures, a superfluid that decouples from the measurement apparatus. However, the required condensate fraction does not simply conform with thermodynamic measurements. Ritner and Reppy further found that the putative super-solid type feature is acutely sensitive to the quench rate for solidifying the liquid. Aoki, Keiderling, and Kojima discovered rich hysteresis and memory effects. All of these features can arise from glassy characteristics along, precisely as in the superglass phase discussed by BCZ. It may indeed well be that a confluence of both superfluid and glassy features (and their effects on elastic properties, e.g., screened finite elastic shear penetration depths) may be at work. There may be new experimental consequences of (super-) glassy dynamics such as this. For instance, such dynamics can manifest disparate relaxation times that may be probed for. Typical glass formers indeed typically exhibit relaxations on two different time scales.

Cold atom systems may provide another realization of a superglass state. Indeed, a supersolid state of cold atoms in a confining optical lattice was very recently achieved. It is natural to expect a superglass analog of these cold atomic systems.

The super-glass phases that BCZ find and the mapping they employ may also have new manifestations elsewhere. We suggest a few of these below.

We may envision lattice extensions of the continuum system investigated by BCZ: a "lattice superglass". For charged bosons (e.g., Cooper pairs) on a lattice, such a charge superglass would correspond to a superconductor with glassy dynamics. In a similar vein, a "lattice supersolid" of Cooper pairs would correspond to a superconductor concomitant with well defined crystalline (i.e., charge density wave) order. Indeed, in some heavy fermion compounds as well as in the cuprate and the newly discovered iron arsenide family of high temperature superconductors, there are some indications of non-uniform meso-scale spatial electronic structures and glassy dynamics. Classical glass formers are known to exhibit dynamical heterogeneities - a non-uniform distribution of local velocities. Using the very same mapping used by BCZ: Quantum dynamical heterogeneities might be suggested in their corresponding quantum counterparts. Such heterogeneities with concurrent non-uniform spatial structures might have realizations in some of the aforementioned electronic systems.

Other realizations may be manifest as spin superglasses. Quantum spin systems in a magnetic field can exhibit a delicate interplay between the formation of singlet states and the tendency of spins to align with the field direction. These systems can be mapped onto a system of bosons with hard core interactions just as in the system investigated by BCZ. In some spin S=1/2 antiferromagnets in an external magnetic field, triplet states with spins aligned along the field direction can be regarded as hard core bosons. In many other systems, interactions between quantum spins may also be mapped onto hard core type bosonic systems. If a solid or glassy phase appears in a classical Brownian system, then a mapping similar to that of BCZ suggests supersolidity/superglassiness in the corresponding quantum spin system. Recently, there has been much work examining supersolidity in such spin systems, e.g. It is highly natural to expect new lattice spin superglass counterparts.

In transition-metal compounds, the fractional filling of the 3d-shells allows for cooperative orbital ordering. This order has been observed in numerous compounds. Similar to the spin and charge degrees of freedom, we may ask whether low temperature Bose condensed glasses of orbitals may appear: an orbital superglass. The work of BCZ allows to investigate this by knowing the dynamics of hard core Bose model derived from a classical counterpart. Orbital states can be described in terms of a S=1/2 pseudo-spin. We may, in turn, map these pseudo-spins to hard core bosons and then investigate the dynamics of these bosons by mapping the system to that of classical Brownian particles on a lattice.

The mapping employed by BCZ also suggests a new quantum critical point in related systems. The classical jamming transition of hard spheres/disks from a jammed system at high density to an unjammed one at lower densities is a continuous transition with known critical exponents, both static and dynamic. Replicating the mapping used by BCZ, we may derive an analog quantum system harboring a zero temperature transition with similar critical exponents. The classical zero temperature critical point ("point J") may rear its head anew in the form of quantum critical jamming of the bosonic systems with dynamical exponents, potentially as high as z = 4.6, as we may ascertain from those reported for the classical jamming system.

All of the above mentioned examples are free of quenched disorder. Glassiness in structural glasses and the Brownian hard sphere system is not triggered by disorder. In classical (and quantum) spin glass system, sluggish dynamics is triggered by quenched disorder. Applying the same mapping of BCZ anew on viscous classical systems with quenched disorder, we may examine quenched super spin-glass analogs of spin-glass systems. In a similar vein, quantum analogs of classical liquid crystalline systems (nematic or smectic phases) first advanced in and further investigated in can be derived by examining the bosonic analog of classical liquid crystalline systems.

The work of BCZ provides important steps in an entirely new field. It proves, as a matter of principle, the existence of a superglass phase in a physical quantum systems. The ramifications of such a phase may be numerous.
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28. We briefly provide more detail. Following Biroli, Chamon, and Zamponi, for a given classical two body potential $V(\vec{x}) = V(|\vec{x}|)$, the resulting many body quantum potential $\mathcal{V}_N(|\vec{x}|)$ will have dominant two body interactions $v_{\text{pair}}$ augmented by (miniscule) three-body terms ($v_{\text{3-body}}$). Rather explicitly,

$$\mathcal{V}_N(|\vec{x}|) = \frac{1}{2} \sum_{i \neq j} v_{\text{pair}}(\vec{x}_i - \vec{x}_j) \quad + \quad \sum_{i,j \neq i', j'} v_{\text{3-body}}(\vec{x}_i - \vec{x}_j, \vec{x}_i - \vec{x}_{i'}) \; ;$$

$$v_{\text{pair}}(\vec{x}) = -\nabla^2 V(\vec{x}) + \frac{1}{2} (\nabla V(\vec{x}))^2$$

$$v_{\text{3-body}}(x, x') = \frac{1}{4} \nabla V(x) \cdot \nabla V(x')$$

$$\frac{1}{4} \frac{\vec{x} \cdot \vec{x'} - r^2 V'(r)V'(r')}{r^2} \; , \quad (1)$$

with $r = |\vec{x}|$ and $D$ the spatial dimensionality. For a classical system of particles with pair potentials of the form

$$V(r) = V_0 \exp(-\lambda |r/\sigma|^2 - 1) \; , \quad (2)$$

with, for large $\lambda$, the parameter $\sigma$ portraying the diameter of spherical particles with hard cores, the corresponding dominant pair term is

$$v_{\text{pair}}(r) = [2\lambda D - 4\lambda^2 r^2] V(r) + 2\lambda^4 r^2 |V(r)|^2 \; . \quad (3)$$

Classical systems with potentials similar to those of Eq. (2) become jammed at sufficiently high density. A classical critical jamming transition with a two-body potential $V(r)$ rigorously implies an identical transition for the quantum system. General spatio-temporal correlation functions for classical systems governed by $V(r)$ can be translated, without any approximations, into corresponding correlators in quantum systems with the many body potential $\mathcal{V}_N(|\vec{x}|)$. S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature (London) 393, 550-553 (1998).