A novel artificial intelligent control system to suppress the vibration of a FGM Plate

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In this paper, an adaptive neuro-fuzzy sliding-mode-based genetic algorithm (ANFSGA) control system is proposed to control functionally graded material (FGM) plates. The model of the FGM plate is considered by the finite element method based on the classical laminated plate theory. Moreover, to show the performance of the proposed ANFSGA intelligent control system, a traditional sliding-mode control (SMC) system and an adaptive neuro-fuzzy (ANF) SMC system are designed to suppress the vibrations of the FGM plate as a comparison. The proposed genetic algorithm control system uses the ANF SMC system in the crossover and mutation operation. In this way, the online learning ability can be used by adjusting the control parameters to deal with external disturbance. The control objective is to drive the system state to the original equilibrium point and thus, the asymptotically stability of the proposed control system can be achieved.

Keywords: neuro fuzzy; FGM plates; sliding mode; on line; genetic algorithm

1. Introduction

FGMs offer good possibilities for optimizing engineering structures to achieve high performance and material efficiency. It is well known that functionally gradient materials are used in high-temperature conditions. That is why they have successful applications as electronic devices, optical films, anti-wear and anticorrosion coatings and biomaterials.

In the solid mechanical field, many researchers have investigated the application of piezoelectric materials as the sensors and actuators for the purpose of monitoring and controlling which is used in active structural systems.

Advanced reinforced composite structures incorporating piezoelectric sensors and actuators are increasingly becoming important due to the development of adaptive structures.

These structures offer potential benefits in a wide range of engineering applications such as vibration and noise suppression, shape control and precision positioning. Parashkevova, Ivanova, and Bontcheva (2004) developed an optimal design and defined two cost functions of functionally graded plates. Turteltaub (2002) introduced a numerical procedure to determine an optimal material layout of a functionally graded material (FGM) within the context of a transient phenomenon. Numerous approaches have been introduced into the analysis of plates with bonded piezoelectric sensors and actuators. Piezoelectric materials are more promising to use in structural mechanics in a way to develop adaptive structures. This is due to their coupled mechanical and electrical properties.

Reddy (2004) and Robbins and Reddy (1991) investigated the transformed piezoelectric and dielectric coefficients, and presented the finite element model of a piezoelectrically actuated beam by using four different displacement-based one-dimensional beam theories. He, Ng, Sivashanker, and Liew (2001) presented a finite element formulation based on the classical laminated plate theory for the shape and vibration control of the FGM plates with integrated piezoelectric sensors and actuators. Moita, Correia, Martins, Mota Soares, and Mota Soares (2006) presented a finite element formulation based on the classical laminated plate theory for laminated structures with integrated piezoelectric layers or patches, acting as sensors and actuators. Based on the first-order shear deformation theory, Liew, He, and Kitipornchai (2004) developed a generic finite element formulation to account for the coupled mechanical and electrical responses of FGM shells with piezoelectric sensors and actuator layers. The large-scale shell structures with distributed piezoelectric components of complicated geometrical configurations are approximated by the hybrid strain or mixed formulation based on lower order triangular shell finite elements investigated by To and Chen (2007).

In the control field, variable structure control with sliding mode, which is commonly known as sliding-mode control (SMC), is a nonlinear control strategy that is well...
known for its robustness characteristics. Many methods based on sliding mode have been developed to control the dynamic systems; in particular, Bagheri and Moghaddam (2009) developed decoupled adaptive neuro-fuzzy SMC system methods for the chaos control problem in a system without precise system model information. Wai and Lee (2004) investigated a double-inductance double-capacitance resonant driving circuit and a sliding-mode fuzzy-neural-network control system for the motion control of an linear piezoelectric ceramic motor. Bagheri and Javadi Moghaddam (2010) developed artificial intelligence control system for underwater vehicle. Lin and Wai (2003) presented adaptive and fuzzy-neural-network sliding-mode controllers for the motor-quick-return servomechanism. A total sliding-mode-based genetic algorithm control system for a linear piezoelectric ceramic motor driven by a newly designed hybrid resonant inverter is discussed by Wai and Tu (2007).

In this paper, the traditional SMC and the adaptive neuro-fuzzy (ANF) SMC and also the adaptive neuro-fuzzy sliding-mode-based genetic algorithm (ANFSGA) control system are presented to control the FGM plate in a vibration problem. It can be understood that the proposed control system can be easily used to other mechanical and electrical systems.

2. Model description

A cantilevered (CFFF) FGM plate with the integrated sensors and actuators is shown in Figure 1. The piezoelectric actuator layer and the piezoelectric sensor layer are distributed uniformly on the top and bottom layers of the laminated plate, respectively. The region between the two surfaces is made of the combined aluminum oxide and Ti-6A1-4 V materials. The material properties of the FGM plate are graded through the thickness direction according to a volume fraction power law distribution. The material properties can be easily found in the literature (Liew et al., 2004; Touloukian, 1967).

2.1. Mathematical model using the classical laminated plate theory (CLPT)

In the CLPT theory, the displacement field is presented by the following form:

\[ \{u\} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} \begin{bmatrix} \frac{\partial w_0}{\partial x} \\ \frac{\partial w_0}{\partial y} \\ 0 \end{bmatrix} = [H] \{\bar{u}\}, \] (1)

\[ \{\bar{u}\} = \begin{bmatrix} u_0, v_0, w_0, \frac{\partial w_0}{\partial x}, \frac{\partial w_0}{\partial y} \end{bmatrix}^T, \] (2)

\[ [H] = \begin{bmatrix} 1 & 0 & 0 & -z & 0 \\ 0 & 1 & 0 & 0 & -z \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \] (3)

where \{\bar{u}\} is the midplane displacement.

\[ u_0, v_0, w_0 \] are displacements in the \( x, y \) and \( z \) directions, and \( \frac{\partial w_0}{\partial x}, \frac{\partial w_0}{\partial y} \) are rotations of the \( yz \) and \( xz \) planes due to bending.

The strains according to the displacement field in Equation (1) are given by

\[ \{\varepsilon\} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \\ \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \\ 2 \frac{\partial^2 w_0}{\partial x \partial y} \end{bmatrix} - z \begin{bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2 \frac{\partial^2 w_0}{\partial x \partial y} \end{bmatrix}. \] (4)
The equations of equilibrium and electrostatics are given as follows:

\[ \sigma_{ij} + f_{bi} = \rho \ddot{u}_i, \]  
\[ D_{ci} = 0. \]  

In the quasi-static and plane stress formulation analysis, the constitutive relationship for the FGM lamina in the principal material coordinates of the lamina can be considered by the following form:

\[ \sigma_{ij} = c_{ijkl} \varepsilon_{kl} - e_{ijk} E_k, \] \[ D_k = e_{ijk} \varepsilon_{ij} + k_{kl} E_l, \]

where \( E_i = -\Phi_j \) and \( \Phi \) is the electric potential, \( \sigma_{ij} \) denotes stress, \( \varepsilon_{ij} \), \( E_i \) and \( D_i \) are the strain, electric field and the electric displacements respectively. \( c_{ijkl} \) is the elastic coefficients, \( [\varepsilon] \) and \( [k] \) are accordingly, the piezoelectric stress constants and the dielectric permittivity coefficients for a constant elastic strain. The symbol \( \rho \) is the density of the plate which varies according to the following form:

\[ \rho(z) = (\rho_T - \rho_h) \left( \frac{2z + h}{2h} \right)^n + \rho_h. \]

Here, the simple power law distribution method is used, where \( c_{ij}^T \) and \( c_{ij}^h \) are the corresponding elastic properties of the Ti-6Al-4 V and aluminum oxide, \( n \) and \( h \) are the power law index and thickness of the plate, respectively.

According to Hamilton’s principle and by using the above equations, the variational form of the equations of motion for the FGM plate can be written as

\[ \int_{t_0}^{t_1} \int_{\Omega} \left( -\rho \ddot{u}_t - \sigma_{ij} \delta_{ij} + D_i \delta E_i \right) \, d\Omega \, dt \]
\[ + \int_{t_0}^{t_1} \int_{\Omega} f_{b_i} \delta u_i \, d\Omega \, dt + \int_{t_0}^{t_1} \int_{\Omega} f_{c_i} \delta u_i \, d\Omega \, dt \]
\[ + \int_{t_0}^{t_1} \int_{\Omega} \left( f_{b_i} \delta u_i + q \delta \varphi \right) \, d\Omega \, dt = 0 \]

The matrices and vectors are given by

\[ [M_{uu}] [\ddot{u}] + [K_{uu}] [u] + [K_{u\varphi}] [\varphi] = [F_m], \]
\[ [K_{u\varphi}] [u] - [K_{\varphi u}] [\varphi] = [F_q]. \]

Here \( [M_{uu}] \), \( [K_{uu}] \), \( [K_{u\varphi}] \), \( [F_m] \), \( [F_q] \), \( [M_{uu}] \) and \( [N_u] \) are derivative matrices of linear and non-conforming Hermite cubic interpolation functions, respectively (Reddy, 2004).

The electric field vector \( \{E\} \) can be expressed in terms of nodal variables as

\[ \{E\} = -\nabla \varphi = -[B_{\varphi}] [\varphi'], \]

where \( [B_{\varphi}] = \nabla [N_{\varphi}] \). Substituting Equations (7), (8), (14), (16) and (17) into Equation (13), and assembling the element equations yield

\[ [M_{uu}] [\ddot{u}] + [K_{uu}] [u] + [K_{u\varphi}] [\varphi] = [F_m], \]
\[ [K_{u\varphi}] [u] - [K_{\varphi u}] [\varphi] = [F_q]. \]

In this section, a finite element model of the FGM plate as a plant is introduced. The displacements and electric potential at the element level can be defined in terms of nodal variables by the following form:

\[ \{u\} = [H] [N_u] \{u'\}, \]
\[ \{\varphi\} = [N_{\varphi}] \{\varphi'\}, \]

where \([N_u]\) and \([N_{\varphi}]\) are the shape functions, which include linear interpolation functions and non-conforming Hermite cubic interpolation functions. These shape functions can be found in the literature (He et al., 2001; Reddy, 2004). \( \{u'\} \) is the generalized nodal displacements and \( \{\varphi'\} \) is the nodal electric potentials.
where

\[ [A, B, Q] = \int_{z_k}^{z_{k+1}} \left( \begin{array}{c}
[C]^T - [C]^A \left( \frac{2z + h}{2 \lambda} \right) \\
1, z, z^2
\end{array} \right) dz,
\]

Substituting Equation (19) into Equation (18), one can obtain

\[ [K_{\varnothing a}] = \int_v (B_\varnothing)^T [e][B_\varnothing] \, dv \]

\[ = \sum_{elem} \sum_{K=1}^{N_L} (z_{K+1} - z_K) \int_{-1}^{1} \int_{-1}^{1} (B_\varnothing)^T [e][A_\varnothing] \]

\[ - \frac{1}{2} (z_{K+1} + z_K) [B_\varnothing]^T [e][C_\varnothing] \big| J \, d\xi \, d\eta ,
\]

\[ [K_{\varnothing a}] = \int_v (B_\varnothing)^T [k][B_\varnothing] \, dv
\]

\[ = \sum_{elem} \sum_{K=1}^{N_L} (z_{K+1} - z_K) \times \int_{-1}^{1} \int_{-1}^{1} (B_\varnothing)^T [k][B_\varnothing] \big| J \, d\xi \, d\eta ,
\]

\[ \{ F_m \} = \int [N]^T [H]^T \{ f_s \} \, dv + \int [N]^T [H]^T \{ f_s \} \, ds
\]

\[ + [N]^T [H]^T \{ \xi \},
\]

\[ \{ F_q \} = \int_{s_k} [N_\varnothing]^T \{ q \} \, ds,
\]

where

\[ [I] = \left( \int_{z_k}^{z_{k+1}} (\rho_T - \rho_\lambda) \left( \frac{2z + h}{2} \right)^n [H]^T [H] \right) dz,
\]

\[ + \rho_\lambda [H]^T [H] \, dz ,
\]

Using Equation (19), the sensor output is

\[ \{ \varnothing \} = [K_{\varnothing a}]^{-1} [K_{\varnothing a}] (u)_s \]

and the sensor charge due to deformation from Equation (19) is

\[ \{ F_q \} = [K_{\varnothing a}] (u)_s .
\]

For the actuator layer, from Equation (19), \{ F_q \} can be written in the following form:

\[ \{ F_q \} = [K_{\varnothing a}] (u)_a - [K_{\varnothing a}] (\varnothing)_a .
\]

As mentioned above and substituting Equations (32) and (33) into Equation (30), thus Equation (30) can be expressed as

\[ \{ F_q \} = [K_{\varnothing a}] (u) - [K_{\varnothing a}] (\varnothing)_a
\]

substituting Equation (34) into Equation (29) and by using some mathematics operations one can obtain

\[ [M_{uu}] \dot{\vec{u}} + [C_\varnothing] \dot{\vec{u}} + [K_{\varnothing a}] (u) = \{ F_m \} - \{ K_{\varnothing a} \} (\varnothing)_a ,
\]

where \[ [C_\varnothing] = a[M_{uu}] + b[K_{uu}] \] is the damping matrix, \( a \) and \( b \) are Rayleigh’s coefficients.

### 3. Control system

In this section the control objective is to find a control law \{ \varnothing \} so that the desired sensor output \{ \varnothing \}_m(t) as a state can be tracked by the sensor output \{ \varnothing \}_s(t). Here, to suppress the vibration, the fact that the state (mode shape) of the plate goes to equilibrium point should be considered, therefore \{ \varnothing \}_m(t) = 0 and the corresponding sensor output \{ \varnothing \}_s(t) \rightarrow 0. It is noted that the proposed control systems can be used in forced vibrations problem by selecting a proper \{ \varnothing \}_m(t). Based on this strategy, the mode shapes of the plate can be held on arbitrary trajectory or desired mode shape.

#### 3.1. Traditional sliding mode

In this section a traditional sliding-mode control (TSMC) system is designed and fabricated to suppress vibrations of the FGM plate. To achieve the control objective, the following tracking error vector can be defined \( e(t) = [\varnothing]_m(t) - \overline{[\varnothing]}_s(t) \). Moreover, the sliding surface can be expressed as

\[ S(t) = \left( \frac{d}{dt} + \lambda \right)^2 \int_0^t e(\tau) \, d\tau ,
\]

where \( \lambda \) is a positive constant. Note that since the function \( S(t) = 0 \) when \( t = 0 \), there is no reaching phase as in the traditional sliding-mode control (Lin & Hsu, 2004; Slotine...
& Li, 1991). Differentiating \( S(t) \) with respect to time and using Equation (31), one can obtain:

\[
\dot{S} = \ddot{e}(t) + 2\lambda \dot{e}(t) + \lambda^2 e(t),
\]

(37)

\[
\dot{S} = \ddot{\omega}_m - [K_{\omega \omega}]s^{-1}[K_{\omega u}]s[\dot{u}] + 2\lambda(\dot{\omega}_m - \dot{\omega}_s) + \lambda^2(\omega_m - \omega_s),
\]

(38)

Now, \([\dot{u}]\) can be expressed as

\[
[\dot{u}] = [M_u]^{-1}([F_m] - [K_{\omega u}][\omega_m] - [C_{\omega}][\dot{u}] - [K_u][u])
\]

(39)

substituting Equation (39) in Equation (38) one can obtain

\[
\dot{S} = \ddot{\omega}_m - [K_{\omega \omega}]s^{-1}[K_{\omega u}]s[M_u]^{-1}([F_m] - [K_{\omega u}][\omega_m] - [C_{\omega}][\dot{u}] - [K_u][u]) + 2\lambda(\dot{\omega}_m - \dot{\omega}_s) + \lambda^2(\omega_m - \omega_s).
\]

(40)

The aforementioned tracking problem is to design a control law \([\omega]_a\) so that the state remains on the surface \( S(t) = 0 \) for all times. In designing the sliding-mode control system, first of all the equivalent control law \([\omega]_{aeq}\), which will determine the dynamic of the system on the sliding surface, can be found. The equivalent control law is derived from recognizing

\[
\dot{S}|_{[\omega]_a = [\omega]_{aeq}} = 0.
\]

(41)

Substituting Equation (44) into Equation (43) and rearranging yield

\[
[\omega]_{aeq} = -(K_{\omega \omega})^{-1}[K_{\omega u}]s[M_u]^{-1}[K_{\omega u}]s^{-1}(\dot{\omega}_m - [K_{\omega u}][\omega_m] - [C_{\omega}][\dot{u}] - [K_u][u]) + 2\lambda(\dot{\omega}_m - \dot{\omega}_s) + \lambda^2(\omega_m - \omega_s).
\]

(42)

Thus, given \( \dot{S}(t) = 0 \), the dynamics of the system on the sliding surface for \( t > 0 \) is given by

\[
\ddot{e}(t) + 2\lambda \dot{e}(t) + \lambda^2 e(t) = 0.
\]

(43)

In this controller if the system parameters are perturbed or unknown, the equivalent control design cannot guarantee the performance specified by Equation (43). Moreover, the stability of the controlled system may be destroyed. To ensure the system performance designed by Equation (43) under the existence of the uncertainties, a robust controller \([\omega]_{aR}\) is designed by the following form:

\[
[\omega]_{aR} = -k_s \text{sign}(S),
\]

(44)

where \( k_s \) is the gain control. Finally, the TSMC control system can be obtained as

\[
[\omega]_a = [\omega]_{aeq} + [\omega]_{aR}.
\]

(45)

### 3.2. ANF sliding mode

The architecture diagram of the neuro-fuzzy inference mechanism is depicted in Figure 2. The ANF sliding-mode controller is composed of a neuro-fuzzy network with the online learning algorithm.

Let \( \text{input} = [S(t), \dot{S}(t)] \) and \( \text{output} = [\omega]_a \) be the input and output variables to the ANF sliding-mode system, respectively.

#### 3.2.1. Description of ANF

In the proposed controller, the four-layer NN is used (Figure 2). Layers I–IV represent the inputs to the network, the membership functions, the fuzzy rule base and the outputs of the network, respectively.

#### 3.2.2. Layer I: input layer

Inputs and outputs of nodes in this layer are represented as

\[
\text{net}_I^1 = S(t), \quad y_I^1 = f_I^1(\text{net}_I^1) = \text{net}_I^1 = S(t),
\]

(46)

\[
\text{net}_I^2 = \dot{S}(t), \quad y_I^2 = f_I^2(\text{net}_I^2) = \text{net}_I^2 = \dot{S}(t),
\]

(47)

where \( y_I^1 \) and \( y_I^2 \) are outputs of the input layer. In this layer, the weights are unit and fixed.

#### 3.2.3. Layer II: membership layer

In this layer, each node performs a fuzzy set and the Gaussian function is adopted as a membership function

\[
\text{net}_{II}^1 = -\left(\frac{\text{net}_{I,1}^1 - m_{II,1}^1}{\sigma_{II,1}^1}\right)^2, \quad y_{II,1}^1 = f_{II,1}^1(\text{net}_{II}^1)
\]

\[
= \exp(\text{net}_{II}^1),
\]

(48)

\[
\text{net}_{II}^2 = -\left(\frac{\text{net}_{I,2}^2 - m_{II,2}^2}{\sigma_{II,2}^2}\right)^2, \quad y_{II,2}^2 = f_{II,2}^1(\text{net}_{II}^1)
\]

\[
= \exp(\text{net}_{II}^2),
\]

(49)

where \( m_{II,1}^1, m_{II,2}^2 \) and \( \sigma_{II,1}^1, \sigma_{II,2}^2 \) are the mean and the standard deviation of the Gaussian function, respectively. The variables \( x_{II,1}^1 \) and \( x_{II,2}^2 \) are the outputs of layer I.

#### 3.2.4. Layer III: rule layer

This layer includes the rule base used in the fuzzy logic control. Each node in this layer which multiplies the input signals and outputs can be expressed as follows:

\[
\text{net}_{III}^1 = (x_{III,1}^1 \times x_{III,2}^2), \quad y_{III,1}^1 = f_{III,1}^1(\text{net}_{III}^1) = \text{net}_{III}^1
\]

(50)

where \( x_{III,1}^1 \) and \( x_{III,2}^2 \) are the outputs of layer II. The values of link weights between the membership layer and rule base layer are unity.
to minimize the derivative of the Lyapunov function with expressed as follows:

The error expression for the input of Layer IV can be obtained:

\[
\dot{S}(t) = \sum_j \sum_k W_{jk}^{IV} y_{jk}^{III}, \quad \dot{y}_{0i} = f_0^{IV} (\text{net}_0^{IV}) = \frac{a_i}{b_i}, \quad (51)
\]

where \( y_{jk}^{III} \) is the output of the rule layer, \( a_i \) and \( b_i \) are the numerator and the denominator of the function used in the center of area method according to each degree and \( W_{jk}^{IV} \) is the center of the output membership functions used in the fuzzy logic system, respectively. The aim is to adjust the weights of \( W_{jk}^{IV} \), \( m_{1j}^{II} \), \( m_{2k}^{II} \) and \( \sigma_{1j}^{II} \), \( \sigma_{2k}^{II} \). Finally, \( y_{0i}^{IV} \) is the output of the proposed inference system.

The online learning algorithm is a gradient descent search algorithm in the space of network parameters. The Lyapunov function is chosen as \((1/2)S^2(t)\). The aim is to minimize the derivative of the Lyapunov function with respect to time or \(S(t)\dot{S}(t)\).

**3.2.5. Layer IV: output layer**

This layer represents the inference and defuzzification, which are used in the fuzzy logic system. For defuzzification, the center of area method is used. Therefore, the following form can be obtained:

\[
a_i = \sum_j \sum_k W_{jk}^{IV} y_{jk}^{III} \quad b_i = \sum_j \sum_k y_{jk}^{III},
\]

\[
\text{net}_0^{IV} = \frac{a_i}{b_i}, \quad y_{0i}^{IV} = f_0^{IV} (\text{net}_0^{IV}) = \frac{a_i}{b_i}, \quad (51)
\]

where \( \delta_{0i}^{IV} \) is the learning rate for \( W_{jk}^{IV} \). Therefore, the changing of \( W_{jk}^{IV} \) is written as

\[
W_{jk}^{IV} = - \frac{\partial S(t)\dot{S}(t)}{\partial \text{net}_0^{IV}} \frac{\partial \text{net}_0^{IV}}{\partial a_i} \frac{\partial a_i}{\partial W_{jk}^{IV}} = \frac{1}{b_i} \frac{\partial \text{net}_0^{IV}}{\partial a_i}.
\]

Since the weights in the rule layer are unified, only the approximated error term needs to be calculated and propagated by the following equation:

\[
\delta_{jk}^{III} = - \frac{\partial S(t)\dot{S}(t)}{\partial \text{net}_0^{IV}} \frac{\partial \text{net}_0^{IV}}{\partial \text{net}_0^{IV}} \frac{\partial \text{net}_0^{IV}}{\partial \text{net}_0^{IV}} = \frac{1}{b_i} \frac{\partial \text{net}_0^{IV}}{\partial a_i} (W_{jk}^{IV} - y_{0i}^{IV}).
\]

The error received from Layer III is computed as

\[
\delta_{1j}^{II} = \sum_k \left( \frac{\partial S(t)\dot{S}(t)}{\partial \text{net}_0^{IV}} \frac{\partial \text{net}_0^{IV}}{\partial y_{1j}^{III}} \frac{\partial y_{1j}^{III}}{\partial \text{net}_0^{IV}} \right) = \sum_k \delta_{jk}^{III} \delta_{ijk}^{III},
\]

\[
\delta_{2k}^{II} = \sum_j \left( \frac{\partial S(t)\dot{S}(t)}{\partial \text{net}_0^{IV}} \frac{\partial \text{net}_0^{IV}}{\partial y_{2k}^{III}} \frac{\partial y_{2k}^{III}}{\partial \text{net}_0^{IV}} \right) = \sum_j \delta_{jk}^{III} \delta_{ijk}^{III}.
\]

The update laws of \( m_{1j}^{II} \), \( m_{2k}^{II} \) and \( \sigma_{1j}^{II} \), \( \sigma_{2k}^{II} \) also can be obtained by the gradient decent search algorithm, it means:

\[
\dot{m}_{1j}^{II} = - \frac{\partial S(t)\dot{S}(t)}{\partial \text{net}_0^{IV}} \frac{\partial \text{net}_0^{IV}}{\partial m_{1j}^{II}} = \frac{2}{a_{1j}^{II}} \frac{2}{(a_{1j}^{II})^2} = \frac{2}{a_{1j}^{II}} \frac{2}{(a_{1j}^{II})^2}.
\]

3.2.6. Online learning algorithm

The error expression for the input of Layer IV can be expressed as follows:

\[
\delta_{0i}^{IV} = - \frac{\partial S(t)\dot{S}(t)}{\partial y_{0i}^{IV}} \frac{\partial y_{0i}^{IV}}{\partial \text{net}_0^{IV}} = \varsigma_1 S(t),
\]

where \( \varsigma_1 \) is the learning rate for \( W_{jk}^{IV} \).
\[
\dot{m}_{2,ki}^\Pi = -\frac{\partial S(t) \dot{S}(t)}{\partial \text{net}_{2,ki}^\Pi} \frac{\partial \text{net}_{2,ki}^\Pi}{\partial m_{2,ki}^\Pi} = \varsigma_2 \delta_{2,ki}^\Pi \frac{2(x_{2,ki}^\Pi - m_{2,ki}^\Pi)}{(\sigma_{2,ki}^\Pi)^2}, \tag{58}
\]

\[
\dot{\sigma}_{1,ji}^\Pi = -\frac{\partial S(t) \dot{S}(t)}{\partial \text{net}_{1,ji}^\Pi} \frac{\partial \text{net}_{1,ji}^\Pi}{\partial \sigma_{1,ji}^\Pi} = \varsigma_3 \delta_{1,ji}^\Pi \frac{2(x_{1,ji}^\Pi - m_{1,ji}^\Pi)}{(\sigma_{1,ji}^\Pi)^3}, \tag{59}
\]

\[
\dot{\sigma}_{2,ki}^\Pi = -\frac{\partial S(t) \dot{S}(t)}{\partial \text{net}_{2,ki}^\Pi} \frac{\partial \text{net}_{2,ki}^\Pi}{\partial m_{2,ki}^\Pi} = \varsigma_4 \delta_{2,ki}^\Pi \frac{2(x_{2,ki}^\Pi - m_{2,ki}^\Pi)}{(\sigma_{2,ki}^\Pi)^3}, \tag{60}
\]

where \(\varsigma_2, \varsigma_3, \varsigma_4,\) and \(\varsigma_5\) are the learning-rate parameters of the mean and the standard deviation of the Gaussian functions.

### 3.3. ANFSGA control system

In this section a control law-based genetic algorithm is designed to the FGM plate for tracking mode shapes, suppurating vibration and external disturbance rejection. The proposed control system is included in the SMC concept and the neuro-fuzzy sliding-mode-based evolutionary procedure. In order to achieve the control object, the evolutionary spirit of GA is embedded. The neuro-fuzzy approach is used to further ensure the correct evolutionary direction and decide the appropriate evolutionary step. In this section, the control law is made as the chromosome in GA with floating point coding can be considered. This process, which is a real one, is to be replaced by the ANF sliding-mode crossover method. An ANF sliding-mode mutation is used just after selecting the chromosomes. The first step after creating a generation is to calculate the fitness function of each member in the population. If the evolutionary direction is correct, the fittest control action can be obtained. In order to achieve the correct evolutionary direction and to ensure the stable system dynamic, the concept of the SMC system is embedded in the genetic operators to form the direction-based operators with the ANF sliding-mode evolutionary procedure.

Now, a fitness function is defined as an exponential term by the following form (Wai & Tu, 2007):

\[
\text{FIT}(S) = \exp[-\xi \times (S(t)^2 + \dot{S}(t)^2)] \in [0, 1], \tag{61}
\]

![Simulation result of piezoelectric sensor and actuator due to the traditional sliding mode with robust controller.](image)
where $\zeta$ is a positive constant, $S$ is the sliding surface and $\dot{S}$ is the first derivative of $S$ which is defined as Equation (36). The next step after evaluation is to create a new population from the current generation. The selection operation determines which chromosome participates in producing offspring for the next generation. Initially, the population is selected randomly, which means that several control actions are randomly selected from the operational region $[[\emptyset]_{min}, [\emptyset]_{max}]$. After comparing the fitness values of all the individuals, the best one is regarded as the elite. If the fitness value of the new control action is higher than all the previous ones, it will become the new elite.

Crossover operation is used to reshape the GA system, which can produce offspring by charging the features of the parent. In this study, the sliding surface is combined with the crossover operation by the following form:

$$[\emptyset]_{GA,new} = [\emptyset]_{GA, old} + \mu_1 \times S + \mu_2 \times \dot{S},$$

(62)

where $[\emptyset]_{GA,new}$ is the generated offspring, $[\emptyset]_{GA, old}$ is the selected elitist chromosome of the last generation, $\mu_1$ and $\mu_2$ are the positive tuning parameters of $S$ and $\dot{S}$, respectively. Here, the important problem is selecting the tuning parameters. The small tuning step may not satisfy the stability conditions. Therefore, an ANF sliding-mode system is used to produce the tuning coefficients. In this section the ANF sliding-mode mechanism of the previous section is considered to produce $\mu_1$ and $\mu_2$. Let input $= \text{FIT}(S)$ for both ANF sliding-mode mechanisms and output$_1 = \mu_1$ and output$_2 = \mu_2$. For the two systems, different means and the standard deviations of the Gaussian function are used.

To avoid the problem of local optimization an ANF sliding-mode mechanism is used in mutation operation. Traditional mutation methods are not useful to produce better offspring in an online learning ability. Therefore, the stability of the system may be destroyed. If the control action cannot let the system dynamic stay on the sliding surface after fuzzy sliding-mode crossover, the mutation operation will further compel the system dynamic to close the sliding surface by using the fuzzy sliding-mode inference mechanism.

The offspring after mutation operation can be expressed as

$$[\emptyset]_{GA,new} = [\emptyset]_{GA,new} + \mu_m,$$

(63)

Figure 4. Simulation result of piezoelectric sensor and actuator due to the traditional sliding mode with robust controller in the disturbance condition.
where $\mu_m$ is the adjustment of mutation operation. 
$\{\varnothing\}^\Delta_{i_{GA,\text{new}}}$ is the offspring after mutation operation which is produced by the ANF sliding-mode inference mechanism. 
In this situation, the input to the ANF sliding-mode system is the sliding surface or $\text{input} = S(t)$. If the fitness value is lower than a specified value ($\text{FIT}_B$), mutation occurs. On the other hand, if the fitness value is higher than the specified value, the mutation idles.

The main process of the proposed GA-based controller is represented by the following pseudo-code:

**Step 1** Select the size of population $[N]$ and the fitness function $[\text{FIT}(S)]$.
**Step 2** Generate the initial population.
**Step 3** Evaluate the fitness value via (61) and sort the sequence to choose the elite $\{\varnothing\}^\Delta_{i_{GA,\text{old}}}$.
**Step 4** Do ANF sliding-mode crossover operation to generate $\{\varnothing\}^\Delta_{i_{GA,\text{new}}}$ via (62).
**Step 5** Compare the fitness value with the specified value ($\text{FIT}_B$), if it is not lower, then go to Step 7, otherwise follow the chart.
**Step 6** Do ANF sliding-mode mutation operation to generate the $\{\varnothing\}^\Delta_{i_{GA,\text{new}}}$ via (63).

**Step 7** Output control action.
**Step 8** Program complete? If yes then it is the end, if not go to Step 3.

It is noted that in the proposed controller, for the crossover and mutation operations Equations (46)–(51) are used. The adaptive laws and the online learning algorithm are used in Equations (52)–(60).

The chattering phenomenon is a particular problem in the control algorithms. The chattering problem can reduce the control accuracy and destroy the stability of system. To find the smooth control action and reduce chattering phenomena, the following soft limit switching function $f_{\text{SL}}$ is presented as

$$ f_{\text{SL}}(S) = \frac{S(t)^2}{1 + S(t)^2} \tanh(S(t)). \quad (64) $$

4. Simulation results

The finite element model for the FGM plate is based on the general concept of solid mechanics and to ensure the accuracy, it is validated with different values of volume fraction.

![Simulation result of piezoelectric sensor and actuator due to the ANF SMC system.](image_url)
power law exponent $n$ and compared with the results of Bishop (1979) and Praveen and Reddy (1998).

The G-1195N piezoelectric films bond both the top and bottom surfaces of the FGM plate as shown in Figure 1. The plate is square with both length and width set as 0.4 m. The thickness of the plate is set as 5 mm, and each G-1195N piezoelectric layer has a thickness equal to 0.1 mm. The material properties of piezoelectric materials are elastic modulus $E = 63 \times 10^9$ N/m$^2$, Poisson’s ratio $\nu = 0.3$, density $\rho = 7600$ kg/m$^3$, piezoelectric constant $d_{31} = 254 \times 10^{-12}$ (m/V), piezoelectric constant $d_{32} = 254 \times 10^{-12}$ (m/V) and dielectric coefficients $k_{33} = 15 \times 10^{-9}$ (F/m). The material constants of the constituent of the FGM plate are listed as follows: for aluminum oxide, $E = 3.2024 \times 10^{11}$ N/m$^2$, $\nu = 0.2600$, density $\rho = 3750$ kg/m$^3$ and for Ti-6Al-4 V $E = 1.0570 \times 10^{11}$ N/m$^2$, $\nu = 0.2981$, density $\rho = 4429$ kg/m$^3$.

The cantilevered (CFFF) plate is considered as the boundary condition. For the vibration control analysis, 64 (8 × 8) elements are used to model the FGM plate and to simplify the vibration analysis, the modal superposition algorithm is used and the first six modes are considered in this modal space analysis. An initial modal damping for each mode has been assumed to be 0.8%. A unit of force is imposed at point A of the FGM plate (Figure 1) in the vertical direction and is subsequently removed to generate motion from the initial displacement. Power law exponent for FGM plate is selected as $n = 5$.

In the design of proposed control systems, the effect of external disturbance are modeled as

$$\Delta \text{Dis} = Am \begin{bmatrix} \sin(\omega_d) & \cos(\omega_d) & \sin(\omega_d) \\
\cos(\omega_d) & \sin(\omega_d) & \cos(\omega_d) \end{bmatrix}^T,$$

where $Am = 0.000001$ is the amplitude of disturbance, $[\bar{K}_{uu}]$ is the normalized matrix of $[K_{uu}]$ and $\omega_d = 200$ is the frequency of disturbance. Therefore, Equation (35) in the disturbance condition can be rewritten as

$$[\bar{M}_{uu}](\ddot{u}) + [\bar{C}_s](\dot{u}) + [\bar{K}_{uu}](u) = \{F_m\} - [\bar{K}_{u\phi}]_a \{\varphi\}_a + \Delta \text{Dis}$$

here $[\bar{M}_{uu}]$, $[\bar{C}_s]$ and $[\bar{K}_{u\phi}]_a$ are the normalized matrices of $[M_{uu}]$, $[C_s]$ and $[K_{u\phi}]_a$.

The simulation results are shown in Figures 3–8.

![Figure 6](image_url)  
**Figure 6.** Simulation result of piezoelectric sensor and actuator due to the ANF SMC system in the disturbance condition.
The effectiveness of the TSMC system is depicted in Figure 3 with $\lambda = 14.8$, Figure 4 shows that the ability of the TSMC system is improved by selecting $k_b = 1.5$ for the robust term in the disturbance condition.

The plots of the ANF SMC system (Figures 5 and 6 with control parameters $\varsigma_1 = 1.5$, $\varsigma_2 = \varsigma_3 = \varsigma_4 = \varsigma_5 = 0.05$ and $\lambda = 1.2$) show that the rate of the voltage which is applied on the actuator layer is smaller than the TSMC system.

In this study, 10 initial populations are randomly chosen from the reasonable region $[a_{min}, a_{max}] = [-1, 1]$ for the ANFSGA control system. The ability to suppress the vibration and reduce the external disturbance of the ANFSGA control system rather than the ANF sliding mode and also the TSMC system are demonstrated in Figures 7 and 8. Figure 5 shows that the settling time in the response of the ANF SMC system is nearly 0.17, but Figure 7 shows that the response of the ANFSGA control system is nearly 0.05.

In the proposed control system, a small voltage can be used to drive the system states to the equilibrium point. Therefore, it is superior to the ANF sliding mode and TSMC system to suppress the vibrations. The control parameters of the ANFSGA control system in the crossover operation to produce $\mu_1$ are $\varsigma_1 = 0.0001$, $\varsigma_2 = \varsigma_3 = \varsigma_4 = \varsigma_5 = 0.0001$ and also to produce $\mu_2$ are $\varsigma_1 = 0.00005$, $\varsigma_2 = \varsigma_3 = \varsigma_4 = \varsigma_5 = 0.0001$. In the mutation operation, the control parameters of the ANFSGA control system are $\varsigma_1 = 0.0003$, $\varsigma_2 = \varsigma_3 = \varsigma_4 = \varsigma_5 = 0.0002$. The sliding surface parameter is selected as $\lambda = 1.2$. The threshold value to activate the mutation operation is applied as $FIT_B = 0.1$ and the parameter fitness value $\zeta = -16.3$ is used. It can be regarded that the associated fuzzy sets with the Gaussian function for each input signal are divided into NE (negative), ZE (zero) and PO (positive). Moreover, the means of the Gaussian functions are set as $-0.5, 0, 0.5$ and the standard deviations of the Gaussian functions are set as 0.3 for the NE, ZE and PO neurons.

The reasonable region $[a_{min} = -1, a_{max} = 1]$ bounds the power fluctuation. The reasonable region only makes the system use $[a]$ of interval $[a_{min} = -1, a_{max} = 1]$ and then the values close to 1 and $-1$ are produced by crossover and the mutation algorithm. Moreover, stability of the system is kept in this operation. Therefore, based on this method, the operator can set the reasonable region without tuning the control parameters to have a desired response.
5. Conclusion
A general finite element model of the FGM plate has been introduced in this paper. A TSMC system has been designed to suppress the vibration of the FGM plate in the normal and disturbance conditions. The ANF sliding mode and the ANFSGA control system as the intelligent control methods have been successfully designed and effectively used to reduce the disturbance and eliminate the vibrations for the FGM plate. It is noted that, in the proposed controller, no constrained conditions and prior knowledge of the controlled plant have been used in the design process. Therefore, any information of the FGM plate is not utilized in the ANFSGA control system. The proposed controller is a flexible kind of control systems and it can be applied in another engineering applications.

Disclosure statement
No potential conflict of interest was reported by the authors.

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