Dynamical Evolution of the Extra Dimension in Brane Cosmology

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Abstract

The evolution of the extra dimension is investigated in the context of brane world cosmology. New cosmological solutions are found. In particular, solutions in the form of waves travelling along the extra dimension are identified.

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1 Introduction

The brane world scenario [1, 2] stipulates that our four-dimensional Universe (the brane) is embedded in a higher dimensional space-time (the bulk). This approach differs from the usual Kaluza-Klein ideas in that the size of the extra dimensions could be large. The concept of large extra dimensions might have phenomenological consequences in particle physics [3, 4, 5]. In particular, it could lead to a solution of the hierarchy problem (the problem of why the electroweak energy scale and the Planck energy scale are far apart from one another).

Another important ingredient of the brane world scenario is that matter is confined to the brane and the only communication between the brane and the bulk is through gravitational interaction (or some other dilatonic matter). Newton’s law of gravitational interaction, as we know it in our Universe, would then arise as a very good approximation in this context. The brane world picture relies on a $\mathbb{Z}_2$ symmetry and is inspired from string theory and its extensions [6].

There are, however, numerous questions that one would like to address in the brane world scenario. The first, and most difficult, question regards the mechanism by which matter is forced to live on the brane only. In the absence of any answers to this question, one would accept it as a hypothesis and looks for experimental evidence for this scenario. The natural laboratory for testing the ideas of this new theory (and of string theory in general, see [7, 8, 9] for reviews) would be in cosmology. It is therefore essential to understand the theoretical implications of a brane world based cosmology [10] (see [11, 12] for some pedagogical reviews).

The most striking feature of brane world cosmology [13, 14, 15, 16, 17, 18, 19, 20, 21] is the fact that the square of the Hubble parameter on the brane is proportional to the square of the energy density of the brane. This is in contrast to the situation, described by the Friedmann equation, in the standard four-dimensional cosmology. This proportionality between the Hubble parameter and the energy density is due primarily to the requirement that the included metric on the brane is that of Friedmann-Robertson-Walker (FRW).

Another important point in the brane world scenario regards the size of the extra dimension. As mentioned previously, this size can be arbitrarily large. It is therefore crucial to have an idea on how this extra dimension evolves. This problem can be naturally examined in the context of brane world cosmology. In most cosmological studies of the brane world, the extra dimension is taken to be constant. In this note, we will allow the size of the extra dimension to evolve dynamically. Some interesting solutions are consequently found.

We start, in section 2, by a brief review of the equations of motion behind the brane world cosmology. The assumptions used in this scenario are also revisited. It is shown that there is a great deal of freedom in choosing the bulk metric. In section 4, we make an ansatz for the form of the size of the extra dimension and determine some new cosmological solutions.

2 A review of brane cosmology

The model studied here is that of a single brane (our Universe) embedded in a five-dimensional spacetime (the bulk) whose coordinates are $x^\mu = (t, r, \theta, \phi, y)$ with $\mu, \nu, \ldots = 0, \ldots, 4$. The brane is located at $y = 0$. Our starting point is the five-dimensional Einstein’s
equations

\[ \mathcal{E}_{\mu\nu} \equiv \alpha \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) - \frac{\Lambda}{2} g_{\mu\nu} - T_{\mu\nu} = 0 \quad . \]  

Here \( \alpha \) is the gravitational coupling constant and \( \Lambda \) is a possible cosmological constant. The five-dimensional metric is specified by the line element \[ ds^2 = -A(t, y)^2 dt^2 + B(t, y)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2(\theta) d\phi^2 \right) \right] + C(t, y)^2 dy^2 \quad . \]  

The functions \( A(t, y), B(t, y) \) and \( C(t, y) \) depend on the variable \( y \) through its modulus \( |y| \) only. This is in order to realise the \( Z_2 \) symmetry; a crucial point in the brane world scenario. The parameter \( k \) \((k = -1, 0, 1)\) is the spatial curvature of a maximally symmetric three-dimensional metric. To complete the cosmological setting, the energy-momentum tensor \( T_{\mu\nu} \) is taken to have the form

\[ T_{\mu\nu} = \frac{1}{C} \text{diag} \left[ -\rho(t), p(t), p(t), p(t), 0 \right] \delta(y) \quad . \]  

This choice is compatible with the metric and the matter is indeed located on the brane only. It is clear that our description breaks down whenever \( C(t, y) = 0 \). We require, also, the matter on the brane to obey the equation of state

\[ p = \omega \rho \quad , \]  

where \( \omega \) is a constant.

The equations of motion lead to four equations: \( \mathcal{E}_{00}, \mathcal{E}_{11} = \mathcal{E}_{22}, \mathcal{E}_{33}, \mathcal{E}_{44} \) and \( \mathcal{E}_{04} \). These are, respectively, given by\(^1\)

\[ \frac{1}{C} \rho \delta(y) = 3\alpha \left\{ -\frac{1}{BC^2} [B'' + 2B'\delta(y)] + \frac{B'}{BC^2} \left( \frac{C'}{C} - \frac{B'}{B} \right) + \frac{\dot{B}}{BA^2} \left( \frac{\dot{B}}{B} + \frac{C}{C} \right) + \frac{k}{B^2} \right\} + \frac{\Lambda}{2} \quad , \]

\[ \frac{1}{C} p \delta(y) = \alpha \left\{ 2 \frac{1}{BC^2} [B'' + 2B'\delta(y)] + \frac{1}{AC^2} [A'' + 2A'\delta(y)] + \frac{B'}{BC^2} \left( \frac{B'}{C} - 2\frac{C'}{C} \right) \right. \]

\[ + \left. \frac{A'}{AC^2} \left( 2\frac{B'}{B} - \frac{C'}{C} \right) - \frac{\dot{B}}{BA^2} \left( \frac{\dot{B}}{B} + 2\frac{C}{C} \right) + \frac{A}{A^3} \left( \frac{\dot{C}}{C} + 2\frac{\dot{B}}{B} \right) \right. \]

\[ - \frac{2\dot{B}}{BA^2} - \frac{\dot{C}}{CA^2} - \frac{k}{B^2} \right\} - \frac{\Lambda}{2} \quad , \]

\[ 0 = 3\alpha \left\{ \frac{B'}{B} \left( \frac{B'}{B} + \frac{A'}{A} \right) + \frac{C^2 B}{BA^2} \left( \frac{A}{A} - \frac{\dot{B}}{B} \right) + \frac{C^2 B}{BA^2} - \frac{kC^2}{B^2} \right\} - \frac{\Lambda}{2} C^2 \quad , \]

\[ 0 = 3\alpha \left\{ \frac{BA'}{BA} + \frac{C B'}{CB} \right\} \left[ 2\Theta(y) - 1 \right] \quad . \]  

\(^1\text{Notation:}\) If \( f(|y|) \) and \( h(|y|) \) are two functions, then \( \frac{df}{dy} = f' \frac{dy}{dy} = f'[\Theta(y) - 1] \), where \( \Theta(y) \) is the Heaviside function and \( f' \) denotes the derivative of \( f \) with respect to its argument \( |y| \). Consequently, \( \left( \frac{df}{dy} \right) \left( \frac{dh}{dy} \right) = f'h' \) and \( \frac{d^2 f}{dy^2} = f'' + 2f' \delta(y) \), where \( f'' \) is the second derivative of \( f \) with respect to \( |y| \). We also use a dot to denote a derivative with respect to the time coordinate \( t \).
Matching the delta functions on both sides of the first two equations leads to

\[
\rho = -6\alpha \frac{B_0'}{B_0 C_0},
\]

\[
p = -\frac{2}{3} \rho + 2\alpha \frac{A_0'}{A_0 C_0}.
\]  

(2.6)

Here the subscript 0 means that the functions are evaluated at \( y = 0 \) (that is \( A_0 = A(t, 0) \), and so on). Once this matching is carried out, the delta function contributions cancel out and the equations become valid everywhere. Notice also that the obtained equation of state is not of the form \( p = \omega \rho \) but a time dependent one.

It is remarkable that the first three equations in (2.5) can be expressed in terms of the single function \[10\]

\[
F(t, y) = \frac{(BB')^2}{C^2} - \frac{(B\dot{B})^2}{A^2} - kB^2.
\]  

(2.7)

Assuming that \( \mathcal{E}_{04} \) (that is, the last equation in (2.5)) is satisfied, then the components \( \mathcal{E}_{00} \) and \( \mathcal{E}_{44} \) of the equations of motion can be cast, respectively, in the form

\[
F' = \frac{\Lambda}{3\alpha} B^3 B',
\]

\[
\dot{F} = \frac{\Lambda}{3\alpha} B^3 \dot{B}.
\]  

(2.8)

On the other hand, the component \( \mathcal{E}_{11} \) of the equations of motion (with the help of \( \mathcal{E}_{00}, \mathcal{E}_{44} \) and \( \mathcal{E}_{04} \)) can be written as

\[
\frac{\partial}{\partial t} \left( \frac{F'}{B'} \right) = \frac{\Lambda}{\alpha} \dot{B} B^2.
\]  

(2.9)

This last equation is identically satisfied due to (2.8).

Therefore, by integration of (2.8), one obtains the first integral of motion

\[
\frac{(BB')^2}{C^2} - \frac{(B\dot{B})^2}{A^2} - kB^2 = \frac{\Lambda}{12\alpha} B^4 + C,
\]  

(2.10)

where \( C \) is a constant of integration. This last equation can be used to determine the unknown function \( A \). Indeed, this is given by

\[
A^2 = \dot{B}^2 \left[ \frac{(B')^2}{C^2} - k - \frac{\Lambda}{12\alpha} B^2 - \frac{C}{B^2} \right]^{-1}.
\]  

(2.11)

Therefore \( A \) is entirely expressed in terms of \( B, C \) and their derivatives. Substituting for \( A \) in the last equation of (2.5), yields the differential equation

\[
\dot{B} \frac{\partial}{\partial |y|} \left\{ \ln \left[ \frac{(B')^2}{C^2} - k - \frac{\Lambda}{12\alpha} B^2 - \frac{C}{B^2} \right] \right\} = 2B' \dot{C}.
\]  

(2.12)

This is the only equation that the two unknown functions \( B \) and \( C \) have to satisfy. Hence, it has many solutions in general. Furthermore, one can use it to exclusively fixe the \( y \) dependence of \( B \) and \( C \) (and consequently \( A \)) but not their time dependence.
3 The brane equations

The most crucial relation in brane world cosmology is equation (2.11). It leads, when evaluated at \( y = 0 \) together with the use of the matching conditions (2.6), to the expression

\[
A_0^2 = \frac{\dot{B}_0^2}{B_0^2} \left\{ \frac{\rho^2}{36\alpha^2} - \frac{k}{B_0^2} - \frac{\Lambda}{12\alpha} - \frac{C}{B_0^4} \right\}^{-1} .
\] (3.1)

This is the time dependent value of \( A_0 \) as determined by the equations of motion. If one imposes the temporal gauge \( A_0 = 1 \), then one obtains the Friedmann like relation [10]

\[
H^2 \equiv \frac{\dot{B}_0^2}{B_0^2} = \frac{\rho^2}{36\alpha^2} - \frac{k}{B_0^2} - \frac{\Lambda}{12\alpha} - \frac{C}{B_0^4} .
\] (3.2)

The unusual proportionality between \( H^2 \) and \( \rho^2 \), together with the presence of the term involving \( C \), are the main characteristics of brane world cosmology [10]. This equation, however, is obtained only when one supposes that \( A_0 = 1 \). This assumption is equivalent to demanding that the metric on the brane is a FRW metric at all times. It would be, therefore, desirable to consider other gauges and to determine their cosmological implications. However, this is not the issue of this note.

Let us first examine the time dependence of the two functions \( B \) and \( C \). We start by exploring the consequences of imposing the equation of state (2.4). Notice first that using the \( E_{04} \) component of the equations of motion at \( y = 0 \), leads to the conservation equation

\[
B_0\dot{\rho} + 3\dot{B}_0 (\rho + p) = 0 .
\] (3.3)

It is, therefore, natural to interpret \( B_0 \) as the scale factor of our Universe. Moreover, if the equation of state \( p = \omega \rho \) holds then one obtains for the energy density

\[
\rho = \beta B_0^{-3(1+\omega)} ,
\] (3.4)

where \( \beta \) is an integration constant. On the other hand, for the matching equations (2.6) to yield the equation of state \( p = \omega \rho \), we must have

\[
2\alpha \frac{A'_0}{A_0 C_0} = \gamma \rho ,
\] (3.5)

where \( \gamma \) is a constant and we have the identification

\[
\omega = -\frac{2}{3} + \gamma .
\] (3.6)

In order for the energy density \( \rho \) to decrease when \( B_0 \) grows, one must have \( \gamma > -1/3 \). We are of course assuming that the Universe is expanding.

Using the component \( E_{04} \) of the equations of motion and the expression of \( \rho \) in (2.6), equation (3.5) can be cast in the form

\[
\frac{\partial}{\partial t} \left[ \ln \left( \frac{B_0^6 B_0^{3\gamma}}{C_0} \right) \right] = 0 .
\] (3.7)
Therefore
\[ C_0 = \lambda B'_0 B_0^{3\gamma} \],
where \( \lambda \) is a constant. Furthermore, by comparing the two expressions of \( \rho \) in (3.4) and (2.6), we deduce that
\[ \beta B_0^{-(1+3\gamma)} = -6\alpha \frac{B'_0}{B_0 C_0} \].
Replacing for \( C_0 \) in the last equation fixes the constant \( \lambda \) to
\[ \lambda = -\frac{6\alpha}{\beta} \]

The equation of state is, therefore, given by \( p = \omega \rho \) provided that the function \( C \) obeys the brane condition \( C_0 = \lambda B'_0 B_0^{3\gamma} \).

4 Solutions with a dynamical fifth dimension

One of the simplest cases studied so far corresponds to setting \( C(t, y) = 1 \) at all times [10]. We would like to present in this section another solution in which the fifth dimension is evolving. This is in the spirit of dynamical compactification in Kaluza-Klein theories [22]. Let us assume that \( C \) is a function of \( B \) only. Namely,
\[ C(t, y) = C(B(t, y)) \] .
This leads immediately to the relation
\[ 2B' \frac{\dot{C}}{C} = \dot{B} \frac{\partial}{\partial |y|} \left( \ln C^2 \right) \] .

Therefore, equation (2.12) can be integrated and one obtains the first order differential equation
\[ (B')^2 = \xi C^4 + C^2 \left[ k + \frac{\Lambda}{12\alpha} B^2 + \frac{C}{B^2} \right] \] ,
where \( \xi(t) \) is a function of integration (it will be shown later that \( \xi \) is a constant). This equation determines \( B \) once the function \( C(B) \) is known. Its right hand side is a functional of \( B \) only and therefore can be formally integrated as
\[ E(B) = \int dB \left\{ \xi C^4 + C^2 \left[ k + \frac{\Lambda}{12\alpha} B^2 + \frac{C}{B^2} \right] \right\}^{-1/2} \]
\[ E(B) = \pm |y| + \zeta(t) \] ,
where \( \zeta(t) \) is a function of integration. The last equation determines \( B \) in terms of \( |y|, \xi(t) \) and \( \zeta(t) \).

Let us now impose the constraint \( C_0 = \lambda B'_0 B_0^{3\gamma} \). Replacing for \( B'_0 \) using (4.3), one find the following expression for \( C_0^2 \)
\[ C_0^2 = \frac{1}{\xi} \left\{ \frac{1}{\lambda^2 B_0^{6\gamma}} \left[ k + \frac{\Lambda}{12\alpha} B_0^2 + \frac{C}{B_0^2} \right] \right\} \] .
In order for this expression of $C_0$ to be in accordance with our assumption in (4.1), the function $\xi(t)$ must be constant in time. Since $C_0(B_0)$ is obtained from $C(B)$ by simply replacing $B$ by $B_0$, we deduce that

$$C^2 = \frac{1}{\xi} \left\{ \frac{1}{\lambda^2 B^{6\gamma}} - \left[ k + \frac{\Lambda}{12\alpha} B^2 + \frac{C}{B^2} \right] \right\} \ .$$

Finally, we should mention that the function $A$ in (2.11) is, in this case, given by

$$A^2 = \frac{B^2}{\xi C^2} \ .$$

The $y$ dependence of $B$ is therefore determined by evaluating the integral

$$E(B) = \sqrt{\xi} \lambda^2 \int dB \frac{B^{(1 + 6\gamma)}}{\sqrt{B^2 - \lambda^2 k B^{(2 + 6\gamma)} - \frac{\lambda^2 A}{12\alpha} B^{(4 + 6\gamma)} - \lambda^2 C B^{6\gamma}}} \ .$$

It is clear that this integral depends on the the values of $\gamma$ and the other parameters $k$, $\Lambda$ and $C$.

As an illustration, we will consider here the case corresponding to $\gamma = 0$ and $\Lambda \neq 0$. This means that the matter on the brane obeys the equation of state $p = -2\rho/3$ and is therefore not an ordinary matter. The integral can be calculated and we find, using (4.4), the following expression for $B$

$$B^2 = 2\sqrt{3\alpha C} - \left( \frac{3\alpha}{\lambda^2 A} \right)^2 \left( 1 - \lambda^2 k \right)^2 \sinh \left\{ -\frac{\Lambda}{3\alpha \xi \lambda^2} \left[ \pm |y| + \zeta - a \right] \right\} + \frac{6\alpha}{\lambda^2 A} \left( 1 - \lambda^2 k \right) \ ,$$

where $a$ is a constant of integration.

The case $\gamma = 0$ and $\Lambda = 0$ is also worth studying. In this case, we find the following expression for $B^2$

$$B^2 = \frac{1}{(1 - \lambda^2 k)} \left\{ \lambda^2 C + \frac{(1 - \lambda^2 k)}{\xi \lambda^4} (\pm |y| + \zeta - b)^2 \right\} \ ,$$

where $b$ is a constant of integration. Here also we are considering the case where $(1 - \lambda^2 k)$ does not vanish.

Cosmological solutions:

The time dependence of $B$, for $\gamma = 0$ and $\Lambda \neq 0$, is determined by solving the constraint $A_0^2 = 1$. Equation (4.11) yields in this case

$$B_0^2 \dot{B}_0^2 = \left( \frac{1}{\lambda^2} - k \right) B_0^2 - \frac{\Lambda}{12\alpha} B_0^4 - C \ .$$

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We will assume that \((\frac{1}{\lambda^2} - k)\) is different from zero. The solution to the above equation is given by

\[
B_0^2 = \frac{6\alpha}{\Lambda} \left(\frac{1}{\lambda^2} - k\right) + a_1 \exp\left(\sqrt{-\frac{\Lambda}{3\alpha}}t\right) + a_2 \exp\left(-\sqrt{-\frac{\Lambda}{3\alpha}}t\right),
\]

(4.12)

where \(a_1\) and \(a_2\) are two constants related by

\[
a_1 a_2 = \left(\frac{3\alpha}{\Lambda}\right)^2 \left(\frac{1}{\lambda^2} - k\right)^2 - \frac{3\alpha C}{\Lambda}.
\]

(4.13)

The expression found for \(B_0^2\) must agree with that obtained from (4.9) upon setting \(y = 0\). This requirement leads to

\[
2\sqrt{-a_1 a_2} \sinh \left[\sqrt{-\frac{\Lambda}{3\alpha \xi^2}} (\zeta - a)\right] = a_1 \exp\left(\sqrt{-\frac{\Lambda}{3\alpha}}t\right) + a_2 \exp\left(-\sqrt{-\frac{\Lambda}{3\alpha}}t\right).
\]

(4.14)

This last equation allows for the determination of the unknown function \(\zeta(t)\).

It is clear that \(\zeta\) depends on the choice one makes for the two constants \(a_1\) and \(a_2\). An interesting case corresponds to taking \(a_2 = -a_1\) and such that

\[
a_1^2 = \frac{3\alpha C}{\Lambda} - \left(\frac{3\alpha}{\Lambda}\right)^2 \left(\frac{1}{\lambda^2} - k\right)^2.
\]

(4.15)

The function \(\zeta(t)\) is in this case linear and is given by

\[
\zeta(t) = \lambda \sqrt{\xi t} + a.
\]

(4.16)

The importance of this special case stems from the fact that \(B^2(t, y)\) is a function of the combination \((\pm|y| + \lambda \sqrt{\xi t})\). Therefore, it has the form of a wave travelling in the \(y\) direction.

Similarly, the case corresponding to \(\gamma = 0\) and \(\Lambda = 0\) leads to

\[
B_0^2 = \left(\frac{1}{\lambda^2} - k\right) t^2 + b_1 t + b_2,
\]

(4.17)

where the two constants \(b_1\) and \(b_2\) are related by

\[
\frac{1}{4} b_1^2 = \left(\frac{1}{\lambda^2} - k\right) b_2 - C.
\]

(4.18)

Finally, the comparison of this last expression with that obtained from (4.10) upon setting \(y = 0\), yields

\[
\zeta(t) = \pm \lambda \sqrt{\xi} \left(t + \frac{b_1}{2} \frac{\lambda^2}{(1 - \lambda^2 k)}\right) + b.
\]

(4.19)

This shows also that \(B^2(t, y)\) is a wave travelling in the \(y\) direction.

In conclusion, we have in this note revisited the scenario of brane world cosmology. We have found a class of solutions in which the fifth dimension evolves dynamically. It is interesting to notice that these solutions are in the form of waves travelling in the \(y\) direction. This has been shown for some special values of the parameters \(\gamma\) and \(\Lambda\). However, by a change
of the time coordinate, one can show that these waves are always present. Indeed, let the new time coordinate be $\tau = \zeta(t)$, then the metric corresponding to our solution is given by

$$
\frac{1}{\xi C^2} \left( \frac{dB}{d\tau} \right)^2 d\tau^2 + B^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2(\theta) d\phi^2 \right) \right] + C^2 dy^2,
$$

(4.20)

where $C^2$, as a functional of $B$, is given by the expression written in (4.6). On the other hand, $B$ is a function of the variable $\pm |y| + \tau$ only. This can be seen from the second equation of (4.4).

If we now demand that $\tau$ is the cosmological time and that at $y = 0$ (on the brane) the metric is a FRW metric, then the time evolution of $B_0$ is determined by solving the equation

$$
\left( \frac{dB_0}{d\tau} \right)^2 = \xi C_0^2.
$$

(4.21)

This is precisely equation (3.2) where $t$ is simply replaced by $\tau$.

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**References**

[1] L. Randall and R. Sundrum, *An Alternative to Compactification*, Phys. Rev. Lett. **83** (1999) 4690-4693.

[2] L. Randall and R. Sundrum, *A Large Mass Hierarchy from a Small Extra Dimension*, Phys. Rev. Lett. **83** (1999) 3370-3373.

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The Hierarchy Problem and New Dimensions at a Millimeter*, Phys. Lett. **B429** (1998) 263-272.

[4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New Dimensions at a Millimeter to a Fermi and Superstrings at a TeV*, Phys. Lett. **B436** (1998) 257-263.

[5] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phenomenology, Astrophysics and Cosmology of Theories with Sub-Millimeter Dimensions and TeV Scale Quantum Gravity*, Phys. Rev. **D59** (1999) 086004.

[6] P. Hořava and E. Witten, *Eleven-Dimensional Supergravity on a Manifold with Boundary*, Nucl. Phys. **B475** (1996) 94-114;

P. Hořava and E. Witten, *Heterotic and Type I String Dynamics from Eleven Dimensions*, Nucl. Phys. **B460** (1996) 506-524.

[7] F. Quevedo, *Lectures on string/brane cosmology*, Class. Quant. Grav. **19** (2002) 5721-5779.

[8] D. A. Easson, *The Interface of Cosmology with String and M(ILLENNIUM) Theory*, Int. J. Mod. Phys. **A16** (2001) 4803-4843.
[9] J. E. Lidsey, D. Wands and E. J. Copeland, *Superstring Cosmology*, Phys. Rept. **337** (2000) 343-492.

[10] P. Binétruy, C. Deffayet and D. Langlois, *Non-conventional cosmology from a braneworld*, Nucl. Phys. **B565** (2000) 269-287;
P. Binétruy, C. Deffayet and D. Langlois, *Brane cosmological evolution in a bulk with cosmological constant*, Phys. Lett. **B477** (2000) 285-291.

[11] D. Langlois, *Brane cosmology: an introduction*, Prog. Theor. Phys. Suppl. **148** (2003) 181-212.

[12] P. Brax and C. van de Bruck, *Cosmology and Brane Worlds: A Review*, Class. Quant. Grav. **20** (2003) R201-R232.

[13] J. M. Cline, C. Grojean and G. Servant, *Cosmological Expansion in the Presence of an Extra Dimension*, Phys. Rev. Lett. **83** (1999) 4245.

[14] C. Csaki, M. Graesser, C. Kolda and J. Terning, *Cosmology of One Extra Dimension with Localized Gravity*, Phys. Lett. **B462** (1999) 34-40.

[15] T. Nihei, *Inflation in the five-dimensional universe with an orbifold extra dimension*, Phys. Lett. **B465** (1999) 81-85.

[16] N. Kaloper, *Bent Domain Walls as Braneworlds*, Phys. Rev. **D60** (1999) 123506.

[17] E. E. Flanagan, S.-H. H. Tye and I. Wasserman, *Cosmological Expansion in the Randall-Sundrum Brane World Scenario*, Phys. Rev. **D62** (2000) 044039.

[18] P. Kanti, I. I. Kogan, K. A. Olive and M. Pospelov, *Cosmological 3-Brane Solutions*, Phys. Lett. **B468** (1999) 31-39.

[19] C. van de Bruck, M. Dorca, C. J. A. P. Martins and M. Parry, *Cosmological consequences of the brane/bulk interaction*, Phys. Lett. **B495** (2000) 183-192.

[20] L. Anchordoqui and K. Olsen, *Comments on Brane World Cosmology*, Mod. Phys. Lett. **A16** (2001) 1157-1169.

[21] P. Bowcock, C. Charmousis and R. Gregory, *General brane cosmologies and their global spacetime structure*, Class. Quant. Grav. **17** (2000) 4745-4764.

[22] T. Appelquist, A. Chodos and P. G. O. Freund, *Modern Kaluza-Klein Theories*, Frontiers in Physics Series Vol. 65 (Addison-Wesley, Reading, MA, 1986).