Production of quarkonium pairs in high-energy proton-proton collisions

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Abstract. Recently there has been much interest in the pair production of quarkonia (charmonia, bottomonia). There are two main motivations behind these studies: first, these processes may help to differentiate between different proposed production mechanisms via color-octet and color-singlet $Q\bar{Q}$-pair production. Second, the production of quarkonium pairs is expected to receive an important contribution from double parton scattering (DPS) processes. There remain a number of open problems, especially with the CMS and ATLAS data. In the kinematics of these experiments, the leading order of $O(\alpha_s^4)$ is clearly not sufficient. The double parton scattering (DPS) contribution was claimed to be large or even dominant in some corners of the phase space, when the rapidity distance $\Delta y$ between two $J/\psi$ mesons is large. However the effective cross sections $\sigma_{\text{eff}}$ found from empirical analyses are about a factor 2.5 smaller than the usually accepted $\sigma_{\text{eff}} = 15 \text{ mb}$. Here we discuss, which single-parton-scattering mechanisms can mimic the behavior of DPS induced production. Here especially the production of $\chi$-pairs is important.

1 Introduction

Recently, there has been much interest in quarkonium pair production in proton-proton collisions. For example, the cross sections for production of $J/\psi$-pairs were measured at the Tevatron [1] and the LHC [2–5].

These processes serve as a tool to investigate the dynamics of particle production in high-energy proton-proton collisions. In particular it has been suggested that $J/\psi$-pair production can be used to study double-parton scattering (DPS) [6]. Indeed, as has been shown in [7], the DPS contribution is especially large in the charm sector.

However, there remain a number of puzzles, especially with the measurements from ATLAS and CMS. Here the single parton scattering (SPS) leading order of $O(\alpha_s^4)$ (see e.g. [9, 12]) does not describe the data well. Especially in some regions of the phase space, when the rapidity distance $\Delta y$ between two $J/\psi$ mesons is large it falls short of experimental data. If one ascribes all the discrepancy to DPS processes, the normalization of DPS comes out a factor $\sim 2.5$ larger than in other hard processes. It is still an open issue at the moment whether this points to a nonuniversality of DPS effects or whether there are additional single parton scattering mechanisms which can alleviate the tension.
2 How to separate SPS and DPS

In the common hard scattering approach, one would calculate the cross section of production of a pair of quarkonia $a, b$ from a convolution of parton densities (for our purpose gluons will dominate), and a hard cross section for the subprocess $gg \rightarrow ab$ (see the left diagram in Fig. 1). However at high energies, favored by the rise of the gluon distribution at small $x$ there is a sizable contribution from processes in which two or more hard processes proceed in the same proton-proton collision (see the right diagram in Fig. 1). It is common to assume the factorized ansatz for the DPS production cross section, say of two particles $a, b$:

$$\frac{d\sigma_{DPS}(pp \rightarrow abX)}{dy_ady_b d^2p_aT d^2p_bT} = \frac{1}{1 + \delta_{ab}} \frac{1}{\sigma_{eff}} \frac{d\sigma(pp \rightarrow aX)}{dy_a d^2p_aT} \frac{d\sigma(pp \rightarrow bX)}{dy_b d^2p_bT}.$$  \hspace{1cm} (1)

We see that the DPS cross section is written as a product of the inclusive single-particle spectra. There is a prefactor $1/2$ for identical particles. All other correlations, and in fact the normalization is hidden in the “effective cross section” $\sigma_{eff}$. This is not the cross section for a specific process – rather the true parameter is its inverse, which in the simplest model is related to the overlap of parton densities in the transverse plane, $t_N(b)$:

$$\frac{1}{\sigma_{eff}} = \int d^2b T_{NN}^2(b), \quad T_{NN}(b) = \int d^2s t_N(s)t_N(b - s).$$  \hspace{1cm} (2)

We can read off some salient features of DPS from Eq. 1: firstly, while the products of any single hard process in the collinear parton model tend to have transverse momentum that adds close to zero, mesons $a, b$ produced in different scatterings will be completely uncorrelated in azimuth. Secondly, as each of the single particle spectra is a fairly broad function of $y_a, y_b$, the distribution in rapidity distance $\Delta y = y_b - y_a$ will be very broad as well. The effective cross section is usually taken in the ballpark of $\sigma_{eff} = 15$ mb, which is within the line of a fair amount of hard processes, see e.g. a table in [3].

Evidently, whether these features are sufficient discriminators to isolate the DPS contribution depends on the properties of the SPS production mechanism. Let us start from the $gg \rightarrow J/\psi J/\psi$ leading order $O(\alpha_s^4)$ mechanism, given by the so-called “box-diagrams” of Fig. 2. Our calculations are based on $k_T$-factorization [8], in which the incoming gluons are off-shell and carry transverse momenta $\vec{k}_{T1}, \vec{k}_{T2}$. The relevant off-shell matrix elements in the color singlet model have been first obtained by Baranov [9]. For a recent calculation of single-inclusive production of $J/\psi$ and $\chi_c$, see e.g. [10, 11], who show that at not very large $p_T$ not much room is left for color-octet mechanisms. The finite transverse momenta of incoming gluons in $k_T$-factorization entails an azimuthal decorrelation of the two $J/\psi$’s. In fact only after imposing lower cuts on transverse momenta of $J/\psi$’s, $p_T \geq 6$ GeV does the jet-like back-to-back structure reveal itself [12]. Rapidity distance between $J/\psi$’s is a much better discriminator. Indeed, the box-diagrams depicted in Fig. 2 always have a line off-shell by an amount $\propto \hat{s}$, where $\hat{s}$ is the $gg$-cms energy squared. Hence the cross section will drop as a
power $\propto 1/\hat{s}^3$, and will therefore be sharply peaked around $\Delta y = 0$. This is indeed borne out by the calculation [12], see Fig. 3.

![Figure 2](image-url)\(\text{Figure 2.}\) Two sample diagrams for the SPS mechanism of $J/\psi$-pair production in the color-singlet model. In the $k_T$-factorization approach incoming gluons carry longitudinal momentum fractions $x_{1,2}$ and transverse momenta $\vec{k}_{T1}, \vec{k}_{T2}$.

![Figure 3](image-url)\(\text{Figure 3.}\) The distribution of rapidity distance $\Delta y$ between $J/\psi$'s. Dotted line: the sharp peak predicted by the box-diagrams. Dash-dotted: DPS contribution. The other lines correspond to higher order mechanisms and are multiplied by factors to be visible on the linear scale, see Ref. [12] for details.

## 3 Crossed channel gluon exchange mechanisms in double quarkonium production

![Figure 4](image-url)\(\text{Figure 4.}\) Production mechanisms of $J/\psi$-pairs involving $t$-channel gluon exchanges in the color-singlet model. Left: quasidiffractive exchange of two gluons in a symmetric color octet. Right: single gluon exchange with radiation of additional gluons from the quark lines.

Given the sharp distinction between the SPS box-mechanism and DPS it is interesting to investigate if there can be other SPS mechanisms which can mimic the behaviour of DPS. Evidently we are looking for mechanisms that do not die out with the $gg$-cms energy $\hat{s}$. There should hence not be any far off-shell $s$-channel partons be involved. Indeed $t$-channel gluon exchange mechanisms can fulfill our requirements.
For direct $J/\psi$-pair production these cost another power of $\alpha_s^2$, and have been estimated in [12] (see Fig. 4). Firstly, there is the quasidiffractive two-gluon exchange mechanism. The $t$-channel gluons are in a symmetric color-octet state and the amplitude is related to the color-singlet two-gluon exchange relevant in the $\gamma\gamma \to J/\psi J/\psi$ reaction [13]. Besides the $\alpha_s^2$-factor it is also suppressed by a small color-factor. Secondly, there is a possibility of having a single gluon exchanged, but two additional gluons radiated from the quark lines. This mechanism is still much smaller than the quasidiffractive one– for dynamical reasons a vast region of the phase space is blocked for the gluons.

Single $t$-channel gluon exchange in the color singlet model cannot contribute to direct $J/\psi J/\psi$-pairs. It can however contribute to $\chi_c(J_1)\chi_c(J_2)$ pairs, with $J_{1,2} = 0, 1, 2$, which then via their radiative decays can feed down to the $J/\psi J/\psi$-channel.

But the production of quarkonium pairs is also interesting in a broader context. In [14] we considered the production of pairs of $\chi_c$ mesons. While this process is more difficult to measure experimentally it is interesting from the theoretical point of view. For example, in the case of $\chi_c$-pair production, corrections to the factorized ansatz for the double parton scattering may be somewhat easier to calculate than for other, more involved processes [15].

The building block for the amplitudes of Fig. 5 are the gluon fusion vertices $g^* g^* \to \chi_c(J), J = 0, 1, 2$ with off-shell gluons. These can then be used in the $k_T$-factorization approach. From the many distributions that can be found in Ref. [14], let us concentrate on the rapidity difference $\Delta y$ between $\chi_c$’s. Indeed, as shown in Fig. 6, the exchange of gluons leads to broad distributions in $\Delta y$. We also see a deep dip at $\Delta y = 0$ for all final states. This dip could be filled by a possible box-type mechanism, however one can benchmark it against the box-mechanism for direct $J/\psi$-pairs [16]: the $\chi_c$-box cross section is suppressed by a small parameter $\epsilon^2 \sim 10^{-3}$. Here $\epsilon = |R'(0)|^2/(M_s^2|R(0)|^2)$ is expressed through the $J/\psi$ wavefunction at the origin and the derivative of the $\chi_c$ wavefunction. In Fig. 7 we show distributions in $\Delta y$ for the DPS mechanism, using $\sigma_{\text{eff}} = 15 \text{ mb}$. We see, that these distributions are very broad and in the same ballpark as the SPS contribution. Of course there is no minimum at $\Delta y = 0$ for the DPS distributions. Thus we observe similar distributions in $\Delta y$ for single and double parton scattering production of different $\chi_c$-quarkonia states. This shows that both contributions must be included in analysis of future data for $\chi_c J/\psi J/\psi$ production.

Let us finally have a look at ATLAS data [3] on $J/\psi$-pair production (see Fig. 8), including the box-diagram contribution, the quasidiffractive two-gluon exchange, feed-down from $\chi_c$-states, and a DPS contribution using the canonical $\sigma_{\text{eff}} = 15 \text{ mb}$. We show distributions in rapidity distance $\Delta y$ between $J/\psi$’s as well as transverse momentum of the $J/\psi$-pair. We see that although the additional $t$-channel mechanisms behave indeed similar to DPS, still not all corners of the phase space are explained, although the bulk of the cross section agrees well with the box-diagram prediction.
Figure 6. The distribution in rapidity difference between $\chi_c$-mesons for all possible contributions of $\chi_c$’s for the SPS mechanism.

Figure 7. The distribution in rapidity difference between $\chi_c$-mesons for all possible contributions of $\chi_c$’s for the DPS mechanism.

4 Summary

Pair production of quarkonia is a topic that still poses puzzles to theorists. A quantitative understanding of DPS contributions requires not only a reliable formalism for its calculation but also a good understanding of SPS processes that can show similar behavior as DPS in many kinematic variables.

The theoretically simplest case is production of $\chi$-pairs, where we showed that the cross sections for different combinations of $\chi_c$ quarkonia are of a similar size and of the same order of magnitude as the cross section for $J/\psi$ pair production. Moreover SPS and DPS cross sections are in the same ballpark.

It seems that feed-down from $\chi$-pairs does not resolve the discrepancy between different determinations of $\sigma_{\text{eff}}$. For the case of $J/\psi$-pair production it remains to see if additional mechanism like e.g color-octet pairs produced through $t$-channel gluon exchange can change the picture. Another possibility may be a BFKL enhancement of $\chi$-pair production from additional gluon emissions between $\chi$’s (see a contribution by I. Babiarz to these proceedings [17]).
Figure 8. Distributions in rapidity distance between $J/\psi$’s (left) and pair transverse momentum (right). The cuts on $J/\psi$’s are: $|y(J/\psi)| < 2.1$, $p_T(J/\psi) > 8.5$ GeV. In addition cuts on the decay muons were imposed: $|\eta(\mu)| < 2.3$, $p_T(\mu) > 2.5$ GeV, $2.8 < M(\mu\mu) < 3.4$ GeV.

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