LOCAL DEMANDS ON STERILE NEUTRINOS

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In a model independent manner, we explore the local implications of a single neutrino oscillation measurement which cannot be reconciled within a three-neutrino theory. We examine this inconsistency for a single region of baseline to neutrino energy $L/E$. Assuming that sterile neutrinos account for the anomaly, we find that the local demands of this datum can require the addition to the theory of one to three sterile neutrinos. We examine the constraints which can be used to determine when more than one neutrino would be required. The results apply only to a given region of $L/E$. The question of the adequacy of the sterile neutrinos to satisfy a global analysis is not addressed here. Finally, using the results of a 3+2 analysis, we indicate values for unknown mixing matrix elements which would require two sterile neutrinos due to local demands only.

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1. Introduction

The experimental evidence for neutrino oscillations is overwhelming. What is more, most of the data can be understood using a three neutrino model. In this, the three different flavors of neutrinos are related to three mass eigenstates, $\nu_f = U_{\text{MNS}} \nu_m$, where $U_{\text{MNS}}$ is the unitary matrix of Maki, Nakagawa, and Sakata. Recalling the standard parameterization of the mixing matrix, it is apparent that the parameters in the model are three mixing angles, one Dirac CP phase, and two independent mass-squared differences. Solar neutrino experiments indicate a mass-squared difference on the order of $8 \times 10^{-5}$ eV$^2$, and the KamLAND long baseline experiment confirms this independent of a solar neutrino flux model. Studies of atmospheric neutrinos indicate a second mass-squared difference around $2 \times 10^{-3}$ eV$^2$ which is consistent with the results from the K2K experiment. The neutrino appearance result from the LSND experiment requires a mass-squared
difference of at least $10^{-1}$ eV$^2$. With respect to the standard three-neutrino theory, these three mass scales are incompatible.

As most experiments, save LSND, can be explained within the standard three-neutrino framework, the LSND result is perplexing; however, should the MiniBoone experiment confirm the results of LSND, the existence of a third mass-scale will be a reality. Attempts have been made to explain all data by the introduction of new physics. Violation of CPT symmetry for the three neutrinos would allow for additional mass scales. Alternatively, one could introduce into the theory additional neutrinos. These additional neutrinos would have to be sterile in order to escape the limits placed upon the neutrino number by the invisible decay width of the Z boson. The inclusion of one or more sterile neutrinos permits the introduction of additional mass scales. Additionally, one can combine the notions of CPT violation and sterile neutrinos in order to attempt to fit the experimental data.

We shall consider only the addition of sterile neutrinos. The simplest extension of the three neutrino model is the inclusion of one sterile neutrino. With four neutrinos, one has three independent mass-squared differences. There are two main divisions of mass-squared differences. One, referred to as the 3+1 scheme, is the straightforward extension of the three-neutrino scheme. The added sterile neutrino is separated from the working three-neutrino model with a large mass-squared difference designed to accommodate the LSND result. In the 2+2 scheme, the large mass-squared difference separates two pair of neutrinos whose mass-squared differences accommodate the solar and atmospheric data. Both schemes have at one time been promising explanations; however, given present data, neither scheme seems to provide a compelling description. In short, short baseline experiments are in tension with the result of the LSND experiment in the 3+1 scheme. The 2+2 scheme is also disfavored due to the results of the solar and atmospheric experiments. We note that the predictions of the 2+2 scheme are slightly better if small, but non-zero, mixing angles are included in the analysis.

As the confidence in a four neutrino scenario is underwhelming, five neutrino scenarios have been investigated. It has been shown that the tension among the short-baseline experiments can be relieved somewhat with a 3+2 neutrino scheme. Future experiments could require the addition of even more mass scales and, hence, sterile neutrinos. There is no theoretical limit on the number that might eventually be required, assuming that cosmological constraints can be avoided.

In this brief note, we wish to examine in a model independent manner the local implications of a single anomalous neutrino oscillation measurement. As our concern is local, we shall not consider the whole of the world’s data. Our only appeal to global phenomenology is to define a measurement to be an anomaly if it cannot be reconciled within the standard three-neutrino framework. For instance, we class the LSND measurement as anomalous; however, considerations within this paper would not lead us to a five neutrino theory because the need for this number of neutrinos
comes from a global analysis of data. Instead, we are interested in whether one can place a local theoretical limit on the effective number of sterile neutrinos needed for such an observation. In the interest of parsimony, one would like to have as few additional sterile neutrinos as necessary.

2. Contraction dilation

We postulate a local region in which the existence of a sterile neutrino(s) is suspected to result in an oscillation measurement irreconcilable within a three-neutrino theory. As such, we consider a fixed ratio of baseline to neutrino energy. We ignore matter effects. If indeed a sterile neutrino is of consequence in this region, then the MNS matrix which relates three mass eigenstates to the three active flavors must be replaced by non-unitary mixing relation $T$. In fact, the matrix $T$ can be shown to be a contraction; that is, $T^\dagger T \leq 1$, where 1 is the identity. There are an infinite number of ways that $T$ may be dilated into a unitary matrix which mixes mass eigenstates and active/sterile neutrinos. This implies that an infinite number of additional neutrinos could be involved in creating this anomalous oscillation measurement. On the other hand, there exists a canonical manner by which one may minimally dilate the contraction. To be definite, let us say that $T$ acts on the space $V$. We define the defect of the contraction to be

$$D_T = (1 - T^\dagger T)^{1/2}. \quad (1)$$

The defect, a positive contraction, indicates, in some sense, how far from unitary is $T$. It can be shown that the following commutation relation holds

$$TD_T = D_T T. \quad (2)$$

With this fact, one can prove that the contraction can be minimally dilated to the unitary operator

$$U = \begin{pmatrix} T & D_{T^\dagger} \\ D_T & -T^\dagger \end{pmatrix}, \quad (3)$$

where $U$ acts upon the larger space $\hat{V} := V \oplus D_{T^\dagger}V$. The dimension of this larger space can be calculated

$$\dim \hat{V} = \dim V + \text{rank} D_{T^\dagger}. \quad (4)$$

The original space $V$ contains three mass eigenstates so its dimension is three. It is thus the rank of the defect which tells us the minimal number of mass eigenstates needed locally at a particular anomalous measurement. Trivially, we have

$$0 \leq \text{rank} D_{T^\dagger} \leq 3. \quad (5)$$

If the rank of the defect is zero, then $T$ is actually unitary so that one requires no additional sterile neutrinos. However, for non-zero defect, we see that one needs locally at least one sterile neutrino to account for the particular anomaly. In addition, the local demands of the measurement require at most three extra neutrinos.
Obviously, this upper limit is a minimum when considering a global analysis of the oscillation data.

This above argument is based upon the non-unitarity of the mixing matrix which relates the active flavors of the neutrino with mass eigenstates. This matrix is not directly measurable; rather, it is inferred from measurements. The inference depends upon the number of neutrinos put into the theory from the start, as such it is model dependent. We cannot conclude whether a single anomaly truly does require more than one sterile neutrino from the above arguments alone, for it is conceivable that the contraction $T$ can always be chosen in such a manner as to have a defect whose rank is one. This would indicate that no more than one sterile neutrino would be necessary to explain a single anomaly. In fact, this is not the case. We show this now.

3. Establishing a lower bound

Given a neutrino source of flavor $\alpha$, one can, at least in principle, measure the flux of flavor $\beta$ neutrinos at the baseline and energy in question. We denote this ratio of the measured flux to the source flux by $P_{\alpha\beta}$. We use lower case Greek letters to indicate the active neutrino flavors. We can imagine that all flavors, $\beta = e, \mu, \tau$, can be measured. For ease of notation, we define the sum over these measured flavors to be

$$c_\alpha := \sum_\beta P_{\alpha\beta}. \tag{6}$$

Barring exotic physics, the total measured fluxes of active flavors at the detector must be less than the flux of the source; hence, the following inequality is satisfied

$$0 \leq c_\alpha \leq 1. \tag{7}$$

If $c_\alpha$ is unity, then this particular series of measurements would not indicate any disagreement with respect to a three neutrino theory; otherwise, for this quantity less than one, the measurement suggests the need for a sterile neutrino or some other physics which we do not consider.

In principle, one can measure the sum $c_\alpha$ independently for each active flavor $\alpha = e, \mu, \tau$ at the prescribed ratio of baseline to neutrino energy. This is not an experimental reality at the moment; however, in our thought experiment we are permitted such indulgences. Each of these measurements satisfy the inequality in (7) independently. As we account for the deficit in active flavors by invoking sterile neutrinos, then the total flux of sterile neutrinos at the detector for a given $\alpha$–flavor source is

$$\sum_{j=1}^N P_{\alpha{s_j}} = 1 - c_\alpha, \tag{8}$$

where the sum is performed over the $N$ sterile neutrinos. The index $s_j$ indicates the $j$th sterile neutrino. We do not limit the number of sterile neutrinos a priori.
The inclusion of the sterile neutrinos allows us to treat the ratio of fluxes $P_{\alpha x}$ as a probability. Considering a fixed sterile neutrino flavor $s_j$ at the detector, we have the following limit

$$\sum_{\alpha} P_{\alpha s_j} \leq 1.$$  \hspace{1cm} (9)

This inequality is valid for each sterile neutrino. As such, the double sum over the three active flavors at the source and all sterile neutrinos at the detector yields the inequality

$$\sum_{j=1}^{N} \sum_{\alpha} P_{\alpha s_j} \leq N.$$  \hspace{1cm} (10)

The expression on the left-hand side of this inequality can be written in terms of the active flavor deficits by using Eq. (8); the result follows easily by summing over the active flavors at the source

$$\sum_{\alpha} \sum_{j=1}^{N} P_{\alpha s_j} = 3 - \sum_{\alpha} c_{\alpha}.$$  \hspace{1cm} (11)

Interchanging the sums, we find a lower limit on the number of sterile neutrinos required to account for a particular anomaly

$$N \geq 3 - \sum_{\alpha} c_{\alpha}.$$  \hspace{1cm} (12)

Clearly, if one finds that the sum over the measured active flavors $c_{\alpha}$ is less than two, then one needs more than a single sterile neutrino to account for this anomaly.

4. Discussion

The limit established in Eq. (12) can be thought of as a means to characterize the degree of the anomaly for a given baseline to neutrino energy ratio. The burden of information needed to establish the order of an anomaly is extremely high; the oscillation data for all flavors is needed. In light of this, we consider the application of our treatment to the LSND experimental results in the context of a five neutrino fit of oscillation data.

As stated in the introduction, the results of the LSND experiment cannot be reconciled in the framework of a three neutrino theory with all other existing neutrino oscillation data. From a muon anti-neutrino beam, the decay at rest (DAR) experiment indicates the appearance of electron anti-neutrinos consistent with a $\bar{\nu}_\mu - \bar{\nu}_e$ oscillation probability of $(0.264 \pm 0.081)\%$ at an average baseline to neutrino energy around 1 m/MeV. Three-neutrino analyses neglecting the LSND result provide a compelling explanation of the world’s data however, their prediction of $P_{\mu e}$ at the LSND DAR baseline is on the order of $10^{-7}$. Hence, we categorize this baseline as one resulting in an anomalous measurement. Invoking sterile neutrinos as a possible explanation, trivially one must have at least one sterile neutrino. Of
interest, however, is data that requires $N > 1$. The prescription in Eq. (12) requires knowledge of all active-neutrino oscillation channels at this value for $L/E$, a total of nine independent measurements. Assuming that CP symmetry is not violated, then one may reduce the number of needed measurements to six as $P_{\alpha\beta} = P_{\beta\alpha}$. We shall assume that CP is conserved.

Other than $\nu_{\mu} - \nu_e$ oscillation probability, there is no other data at the LSND baseline. There are other short baseline experiments that do have values of $L/E$ in the neighborhood of the LSND measurement which are consistent with no neutrino oscillations and with the three neutrino analyses. Of these experiments, limitations are placed upon $\nu_e$ disappearance by the Bugey [16], CHOOZ [6], and Palo Verde [17] experiments. The disappearance of $\nu_\mu$ is constrained by the CCFR84 [18] and CDHS [19] experiments. Finally, the KARMEN [20] and NOMAD [21] experiments show no evidence of $\nu_\mu - \nu_e$ oscillation at values of $L/E$ less than that of the LSND experiment. At the LSND baseline, the null-result data of the other experiments indicate that most likely $P_{ee} \lesssim 1$ and $P_{\mu\mu} \lesssim 1$. There is no data in the neighborhood for $\nu_\tau$ channels. Given the dearth of data, it is more useful at this point to apply the limit in Eq. (12) to a neutrino analysis.

We shall focus upon the 3+2 neutrino analysis presented in Ref. [23]. In general, 3+N analyses are tractable models to work with as they are a straightforward extension of three neutrino analyses as in Ref. [7]. For all practical purposes, the three-neutrino fit to the data remains intact while the additional $N$ sterile neutrinos are assumed to have much larger masses which are of consequence for short baselines only. Assuming CP is conserved, the probability, in vacuo, that a neutrino of flavor $\alpha$ and energy $E$ will be detected as a neutrino of flavor $\beta$ a distance $L$ from the source is

$$P_{\alpha\beta}(L/E) = \delta_{\alpha\beta} - 4 \sum_{j,k=1}^{3+N} U_{\alpha j} U_{\alpha k} U_{\beta k} U_{\beta j} \sin^2(\varphi_{jk})$$

(13)

where $\varphi_{jk} := \Delta_{jk} L/4E$ with $\Delta_{jk} := m_j^2 - m_k^2$. As in Ref. [7] we have mass-squared differences $\Delta_{21} \sim 8 \times 10^{-5} \text{eV}^2$ and $\Delta_{31} \sim 2 \times 10^{-3} \text{eV}^2$. Indicated above, these yield oscillation probabilities which are inconsequential for the baselines in the neighborhood of the LSND region so that terms involving these mass-squared differences can be neglected. Additionally, the mass of each sterile neutrino is assumed to be large in relation to other neutrinos so that at short baselines one may approximate $m_1 \approx m_2 \approx m_3$, or in terms of mass-squared differences $\Delta_{41} \approx \Delta_{42}$, and so on. Using these approximations and the unitarity of the mixing matrix $U$, one may approximate the oscillation probability [18] for short baselines given a 3+2 theory

$$P_{\alpha\beta}(L/E) \approx \delta_{\alpha\beta} - 4 U_{\alpha 5} U_{\alpha 4} U_{\beta 4} U_{\beta 5} \sin^2(\varphi_{54})$$

$$- 4 \left( \delta_{\alpha\beta} - \sum_{j=4}^{5} U_{\alpha j} U_{\beta j} \right) \sum_{k=4}^{5} U_{\alpha k} U_{\beta k} \sin^2(\varphi_{k1}).$$

(14)
In this case, only two independent mass-squared differences and six mixing matrix elements are relevant.

The 3+2 neutrino analysis presented in Ref. 23 agrees relatively well with all SBL data, including LSND. In the context of this particular analysis, we wish to determine whether the LSND result is a first or second order anomaly. That is, we shall evaluate the limit in Eq. 12 at the LSND baseline to determine if the local measurement requires $N > 1$. As SBL data concerns only electron and muon neutrino oscillations, analyses can only address four elements of the mixing matrix: $U_{e4}, U_{e5}, U_{\mu4},$ and $U_{\mu5}$. In considering all flavor oscillations, we see from Eq. 14 that the matrix elements $U_{\tau4}$ and $U_{\tau5}$ remain indeterminable. As such, in evaluating the required number of sterile neutrinos for the local anomaly, these two matrix elements will remain unknown, and we will examine the allowed parameter space for $U_{\tau4}$ and $U_{\tau5}$ in order to determine what values, if any, require $N > 1$.

The best fit mass-squared differences in the SBL analysis 23 are $\Delta_{41} = 0.92$ eV$^2$ and $\Delta_{51} = 22$ eV$^2$. Additionally, the best fit mixing matrix elements are $U_{e4} = 0.121, U_{e5} = 0.036, U_{\mu4} = 0.204,$ and $U_{\mu5} = 0.224$. From Ref. 3, it is apparent that the larger mass-squared differences $\Delta_{51} = \Delta_{54} - \Delta_{41}$ lie outside the resolution of the LSND experiment. Given this, we shall set $\langle \sin^2(\varphi_{51}) \rangle = \langle \sin^2(\varphi_{54}) \rangle = 0.5$. From the best fit parameters, oscillation probabilities at the LSND baseline of 1 m/MeV can be determined from Eq. 14:

$$P_{ee} = 0.95, P_{\mu\mu} = 0.78, \text{ and } P_{e\mu} = 0.29\%.$$  

In the absence of CP violation, we may simplify the sum

$$\sum_{\alpha} c_\alpha = P_{ee} + P_{\mu\mu} + P_{\tau\tau} + 2(P_{e\mu} + P_{e\tau} + P_{\mu\tau}). \quad (15)$$

The oscillation probabilities $P_{\alpha\tau}$ depend upon the unknown matrix elements $U_{\tau4}$ and $U_{\tau5}$. Evaluating the right hand side of Eq. 12 at the LSND baseline, we have

$$3 - \sum_{\alpha} c_\alpha = 3.01 U_{\tau4}^2 + 1.79 U_{\tau5}^2 - 0.34 U_{\tau4} U_{\tau5} - 3.39 U_{\tau4}^4 - 2.00 U_{\tau5}^4 - 3.39 U_{\tau4}^2 U_{\tau5}^2 + 0.27. \quad (16)$$

In general, this sum is positive for the allowed values of $U_{\tau4}$ and $U_{\tau5}$. We shall now explore regions in parameter space where this sum is greater than unity, indicating the need for two sterile neutrinos in this region.

Unitarity of the mixing matrix imposes some constraints upon the allowed values of matrix elements $U_{\tau4}$ and $U_{\tau5}$. For one, the following inequality must be satisfied

$$U_{\tau j}^2 \leq 1 - U_{e j}^2 - U_{\mu j}^2, \quad (17)$$

for $j = 4, 5$. Employing the best fit parameters from above, we have $|U_{\tau4}| \leq 0.97$ and $|U_{\tau5}| \leq 0.97$. Additionally, unitarity requires

$$U_{\tau4}^2 + U_{\tau5}^2 \leq 1. \quad (18)$$

In fact, the goal of a $3 + N$ neutrino analysis is to use the $N$ sterile neutrinos to accommodate the LSND anomaly while keeping the relatively successful three
Fig. 1. The interior of the solid circle contains the allowed values for the unknown mixing matrix elements $U_{\tau 4}$ and $U_{\tau 5}$. The shaded regions indicate the values of these parameters which require more than one sterile neutrino for $L/E = 1$ m/MeV. The interior of the dashed circle contains the parameters which satisfy $U_{\tau 4}^2 + U_{\tau 5}^2 \leq 0.5$. We shall consider this constraint as well.

The symmetry of the two disconnected regions in Fig. 1 is no surprise as Eq. 16 shows that measurements of $\mathcal{P}_{\tau \beta}$ oscillations at short baselines can only determine the relative algebraic sign of $U_{\tau 4}$ and $U_{\tau 5}$. Perhaps more noteworthy is the fact that a portion of the shaded region lies within the more restrictive dashed circle. As an example, we shall focus upon one such point: $U_{\tau 4} = 0.6$ and $U_{\tau 5} = -0.3$. A quick calculation shows that $U_{\tau 4}^2 + U_{\tau 5}^2 = 0.45$ and $3 - \sum \epsilon_\alpha = 1.008$. At the LSND baseline of 1 m/MeV, we may determine the unknown oscillation probabilities for these parameters. We find $\mathcal{P}_{\tau \tau} = 0.17$, $\mathcal{P}_{\mu \tau} = 0.032$, and $\mathcal{P}_{e \tau} = 0.015$. Summing up neutrino fit intact; therefore, mixing among active flavor and sterile states should not be too great. As such, the additional restriction adopted in the 3+2 analysis in Ref. 23 would limit the unknown matrix elements to $U_{\tau 4}^2 + U_{\tau 5}^2 \leq 0.5$. We shall consider this constraint as well.

In Fig. 1 the shaded region in the plot indicates which values of $U_{\tau 4}$ and $U_{\tau 5}$ make the expression in Eq. 16 greater than unity. The limits on the magnitude of the mixing parameters established in Eq. 17 are satisfied. From the inequality in Eq. 18, the allowed values for the matrix elements lie within the solid circle. The interior of the dashed circle contains the parameters which satisfy the more rigorous requirement $U_{\tau 4}^2 + U_{\tau 5}^2 \leq 0.5$. From the limit in Eq. 12, the shaded regions in the plot contain matrix elements which require the existence of two sterile neutrinos at this local measurement. All other acceptable values of $U_{\tau 4}$ and $U_{\tau 5}$ outside these regions require only one sterile neutrino; as such, the two sterile neutrinos exist merely to satisfy global phenomenological constraints.
these probabilities over the active flavors, one has \( c_\tau = 0.22 \). We recall that if each \( c_\alpha \) is unity then no sterile neutrinos are needed locally. As our choice of parameters require \( N > 1 \), it is expected that \( c_\tau \) should be small, indicating a large coupling of \( \nu_\tau \) to the sterile neutrinos.

5. Conclusion

In summation, we began with the assumption that sterile neutrinos account for an oscillation datum which is irreconcilable with a three-neutrino theory. Given that, we demonstrated that the local demands of this datum can require from one to three additional neutrinos. We also provide guidance as to how the local measurements might be used to indicate when more than one neutrino is required. These additional neutrinos are not the result of an attempt at a global fit of data; rather, they are needed to account for the vanishing of multiple active flavors. We made no attempt to determine whether these additional sterile neutrinos, mandated by local considerations, were amenable to the global phenomenology of all existing data. Such questions were outside the scope of our initial questions. Global considerations, if not already consistent with the local demands, would necessarily increase the demands beyond that of the single local measurement.

Given the absence of \( \nu_\tau \) oscillation data at short baselines, one cannot use experimental data to evaluate the limit in Eq. (2) in order to determine if the LSND baseline is a region which locally requires more than one sterile neutrino. However, as an example of practical application, we use the best fit results of a 3+2 neutrino analysis of SBL data to determine what values of the mixing matrix elements \( U_{\tau 4} \) and \( U_{\tau 5} \) would indicate the local need for two sterile neutrinos. Examining one such point in the parameter space \( U_{\tau 4} = 0.6 \) and \( U_{\tau 5} = -0.3 \), we see that \( \nu_\tau \) would need to be strongly coupled to the sterile neutrinos. Most likely, this would create tension in the fit of longer baseline data, resulting in the need for additional sterile neutrinos to accommodate global phenomenology.

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