Analysis of the hidden-charm pentaquark molecular states with strangeness and without strangeness via the QCD sum rules

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Abstract

In this article, we investigate the $D\Sigma_c$, $\bar{D}\Xi'_c$, $\bar{D}\Sigma'_c$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma'_c$ and $\bar{D}^*\Xi'_c$ pentaquark molecular states with strangeness and without strangeness via the QCD sum rules at length, pay much attention to the light flavor $SU(3)$ breaking effects, and make predictions for new pentaquark molecular states besides assigning the $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$ self-consistently. We can search for those pentaquark molecular states in the decays of the $\Lambda^{(p)}_c$, $\Xi^{(p)}_c$ and $\Xi^{(p)}_{c*}$ in the future. Furthermore, we discuss the higher dimensional vacuum condensates in details.

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Key words: Pentaquark molecular states, QCD sum rules

1 Introduction

In 2015, the LHCb collaboration explored the $\Lambda^0_c \rightarrow J/\psi K^- p$ decays and observed two pentaquark candidates $P_c(4380)$ and $P_c(4450)$ in the $J/\psi p$ mass spectrum with the preferred quantum numbers $J^P = \frac{3}{2}^-$ and $\frac{1}{2}^+$, respectively [1]. The Breit-Wigner masses and widths are $M_{P_c}(4380) = 4380 \pm 8 \pm 29$ MeV, $M_{P_c}(4450) = 4449.8 \pm 1.7 \pm 2.5$ MeV, $\Gamma_{P_c}(4380) = 205 \pm 18 \pm 86$ MeV, and $\Gamma_{P_c}(4450) = 39 \pm 5 \pm 19$ MeV, respectively. In 2019, the LHCb collaboration re-investigated a data sample of the $\Lambda^0_c \rightarrow J/\psi K^- p$ decays, which was an order of magnitude larger than that previously analyzed, and observed a narrow pentaquark candidate $P_c(4312)$ in the $J/\psi p$ mass spectrum, and confirmed the structure $P_c(4450)$, which are consisted of two narrow overlapping peaks $P_c(4440)$ and $P_c(4457)$ [2]. The measured Breit-Wigner masses and widths are

\begin{align}
P_c(4312) : & \quad M = 4311.9 \pm 0.7^{+6.8}_{-0.6} \text{ MeV}, \quad \Gamma = 9.8 \pm 2.7^{+3.7}_{-4.5} \text{ MeV}, \\
P_c(4440) : & \quad M = 4440.3 \pm 1.3^{+4.1}_{-1.7} \text{ MeV}, \quad \Gamma = 20.6 \pm 4.9^{+8.7}_{-10.1} \text{ MeV}, \\
P_c(4457) : & \quad M = 4457.3 \pm 0.6^{+4.1}_{-1.7} \text{ MeV}, \quad \Gamma = 6.4 \pm 2.0^{+5.7}_{-1.9} \text{ MeV}.
\end{align}

(1)

Very recently, the LHCb collaboration reported an evidence of a hidden-charm pentaquark candidate $P_{cs}(4459)$ with strangeness $S = -1$ in the $J/\psi^*\Lambda$ invariant mass spectrum with a statistical significance of $3.1\sigma$ in the $\Xi^{(p)}_{c*} \rightarrow J/\psi K^- \Lambda$ decays using the $pp$ collision data corresponding to a total integrated luminosity of 9 fb$^{-1}$ collected with the LHCb experiment at centre-of-mass energies of 7, 8 and 13 TeV [3], the Breit-Wigner mass and width are

\begin{align}
P_{cs}(4459) : & \quad M = 4458.8 \pm 2.9^{+1.7}_{-1.1} \text{ MeV}, \quad \Gamma = 17.3 \pm 6.5^{+8.0}_{-5.7} \text{ MeV},
\end{align}

(2)

but the spin and parity have not been determined yet. The $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$ lie slightly below the thresholds of the meson-baryon pairs $D\Sigma_c$, $D\Sigma_c^*$, $\bar{D}^*\Sigma_c$ and $\bar{D}^*\Xi_c$, respectively, the nearby meson-baryon thresholds are shown clearly in Table [1] because the $D$ and $D^*$ mesons, the $\frac{1}{2}^+$ flavor antitriplet ($\Lambda_c^*(2286)$, $\Xi_c^*(2468)$, $\Xi_c^*(2471)$), and the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ flavor sextets ($\Omega_c(2695)$, $\Xi_c^*(2579)$, $\Sigma_c(2453)$) and ($\Omega_c^*(2766)$, $\Xi_c^*(2646)$, $\Sigma_c^*(2518)$) have been well established [4].

As expected, the $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$ have been tentatively assigned to be the $D\Sigma_c$, $D\Sigma_c^*$, $\bar{D}^*\Sigma_c$ and $\bar{D}^*\Xi_c$ pentaquark molecular states respectively based on the contact-range effective field theory [5, 6], one-boson exchange potential model [7, 8, 9].

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(quasipotential) Bethe-Salpeter equation \[10\ \[11\ \[12\ \[13\], effective Lagrangian approach \[14\ \[15\], effective-range expansion and resonance compositeness relation \[10\], Lippmann-Schwinger equation \[17\ \[18\], the QCD sum rules \[19\ \[20\ \[21\ \[22\ \[23\], etc.

At first glance, \(M_{D_s^0} + M_{\Xi_c^0} - M_{D_s^*},_{\bar{p}}(4459)\) = 19 MeV, \(M_{D^0} + M_{\Sigma^+_c} - M_{D^*},_{\bar{p}}(4312)\) = 6 MeV, it is odd that the \(D^0 \Xi^0_{c}\) molecular state which involving exchanges of the strange mesons is more tightly bound than the \(D^0 \Sigma^+_c\) molecular state which involving exchanges of the non-strange mesons, we have to introduce the coupled-channel effects \[7\ \[8\ \[9\ \[10\ \[11\ \[12\ \[13\ \[14\ \[15\ \[16\ \[17\ \[18\].

In the QCD sum rules, we usually choose the local currents to interpolate the tetraquark or pentaquark molecular states having two color neutral clusters \[19\ \[20\ \[21\ \[22\ \[23\], which are not necessary to be the physical mesons and baryons, and the tetraquark or pentaquark molecular states are not necessary to be loosely bound, they can be compact objects and can lie below or above the corresponding meson-meson or meson-baryon pairs \[24\].

On the other hand, the \(P_c(4312), P_c(4380), P_c(4440), P_c(4457)\) and \(P_{cs}(4459)\) can also be tentatively assigned to be the diquark-diquark-antiquark type (or diquark-triquark type) pentaquark states in the diquark-model through exploring their masses \[25\ \[26\ \[27\ \[28\ \[29\ \[30\ \[31\] via the effective Hamiltonian, or investigating their masses \[32\ \[33\ \[34\ \[35\ \[36\ \[37\] and electromagnetic properties \[38\] via the QCD sum rules.

In Ref.\[40\], we suggest the hadronic dressing mechanism to compromise the pentaquark and pentaquark molecule interpretations based on the calculations of the QCD sum rules, the pentaquark states maybe have a diquark-diquark-antiquark type pentaquark core with the typical size of the \(qqq\) type baryon states, the strong couplings to the meson-baryon pairs lead to some pentaquark molecule Fock components, and the pentaquark states maybe spend a rather large time as the molecular states. We can choose either the diquark-diquark-antiquark type currents or color-singlet-color-singlet type five-quark currents to interpolate the pentaquark states.

In the scenario of the pentaquark molecular states, the \(P_c(4312), P_c(4380), P_c(4440), P_c(4457)\) are assigned to the \(D\Sigma_c, \bar{D}\Sigma_c^*, \bar{D}^*\Sigma_c \) pentaquark molecular states, which involve charmed baryon states in flavor sextets \(6_f\ \[5\ \[6\ \[7\ \[8\ \[9\ \[10\ \[11\ \[12\ \[13\ \[14\ \[15\ \[16\ \[17\ \[18\ \[19\ \[20\ \[21\ \[22\ \[23\ \[24\ \[25\ \[26\ \[27\ \[28\ \[29\ \[30\ \[31\ \[32\ \[33\ \[34\ \[35\ \[36\ \[37\ \[38\ \[39\ \[40\\) via the QCD sum rules.

Thus the \(P_c(4312), P_c(4380), P_c(4440), P_c(4457)\) belong to different flavor multiplets, in the present work, we will focus on the flavor sextets \(6_f\).
2 QCD sum rules for the pentaquark molecular states

We write down the two-point correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ in the QCD sum rules,

$$
\begin{align*}
\Pi(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J(x) \bar{J}(0) \} | 0 \rangle, \\
\Pi_{\mu\nu}(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_\mu(x) \bar{J}_\nu(0) \} | 0 \rangle, \\
\Pi_{\mu\nu\alpha\beta}(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_{\mu\nu}(x) \bar{J}_{\alpha\beta}(0) \} | 0 \rangle,
\end{align*}
$$

where the currents $J(x) = J^{D \Sigma_c^+}(x)$, $J^{D \Xi_c^0}(x)$, $J_\mu(x) = J^{D \Sigma_c^0}(x)$, $J^{D \Xi_c^{*0}}(x)$, $J_{\mu\nu}(x) = J^{D \Sigma_c^{*+}}(x)$, $J^{D \Xi_c^{*+}}(x)$,

$$
\begin{align*}
J^{D \Sigma_c^+}(x) &= \bar{c}(x)i\gamma_5u(x)\varepsilon^{ijk}u_T^\dagger(x)C\gamma_\alpha d_j(x)\gamma^\alpha\gamma_5c_k(x), \\
J^{D \Xi_c^0}(x) &= \bar{c}(x)i\gamma_5u(x)\varepsilon^{ijk}u_T^\dagger(x)C\gamma_\alpha s_j(x)\gamma^\alpha\gamma_5c_k(x),
\end{align*}
$$

$$
\begin{align*}
J^{D \Sigma_c^0}_\mu(x) &= \bar{c}(x)i\gamma_5u(x)\varepsilon^{ijk}u_T^\dagger(x)C\gamma_\alpha d_j(x)c_k(x), \\
J^{D \Xi_c^{*0}}_\mu(x) &= \bar{c}(x)i\gamma_5u(x)\varepsilon^{ijk}u_T^\dagger(x)C\gamma_\alpha s_j(x)c_k(x),
\end{align*}
$$

$$
\begin{align*}
J^{D \Sigma_c^{*+}}_{\mu\nu}(x) &= \bar{c}(x)\gamma_\mu u(x)\varepsilon^{ijk}u_T^\dagger(x)C\gamma_\alpha d_j(x)c_k(x) + (\nu \leftrightarrow \mu), \\
J^{D \Xi_c^{*+}}_{\mu\nu}(x) &= \bar{c}(x)\gamma_\mu u(x)\varepsilon^{ijk}u_T^\dagger(x)C\gamma_\alpha s_j(x)c_k(x) + (\nu \leftrightarrow \mu),
\end{align*}
$$

the $i, j, k$ are color indices. In the present work, we choose the color-singlet-color-singlet type currents $J(x)$, $J_\mu(x)$ and $J_{\mu\nu}(x)$ to interpolate the pentaquark molecular states with the spin-parity $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ and $\frac{5}{2}^-$, respectively. The currents couple potentially to the pentaquark molecular states having two color neutral clusters, one has the same quantum numbers as the charmed mesons, the other has the same quantum numbers as the charmed baryons, they are not physical mesons and baryons, as we choose the local five-quark currents, while the mesons and baryons are spatial extended objects and have mean spatial sizes $\sqrt{\langle r^2 \rangle} \neq 0$, for example, $\sqrt{\langle r^2 \rangle_{E, \Sigma_c^{*+}}} = 0.48$ fm, $\sqrt{\langle r^2 \rangle_{M, \Sigma_c^{*+}}} = 0.83$ fm, $\sqrt{\langle r^2 \rangle_{M, \Xi_c^{*0}}} = 0.81$ fm from the lattice QCD, where the subscripts $E$ and $M$ stand for the Electric and magnetic radii, respectively [41]. $\sqrt{\langle r^2 \rangle_{M, \Sigma_c^{*+}}} = 0.77$ fm, $\sqrt{\langle r^2 \rangle_{M, \Xi_c^{*0}}} = 0.52$ fm, $\sqrt{\langle r^2 \rangle_{M, \Xi_c^{*+}}} = 0.81$ fm, $\sqrt{\langle r^2 \rangle_{M, \Xi_c^{*0}}} = 0.55$ fm, $\sqrt{\langle r^2 \rangle_{M, \Xi_c^{*0}}} = 0.79$ fm from the self-consistent $SU(3)$ chiral quark-soliton model [42], $\sqrt{\langle r^2 \rangle_{D^+}} = 0.43$ fm, $\sqrt{\langle r^2 \rangle_{D^0}} = 0.55$ fm.
from the light-front quark model \[43\]. In the present work, though we refer the color-singlet-color-singlet type pentaquark states as the pentaquark molecular states, they have the average spatial sizes as that of the typical heavy mesons and baryons, and are compact objects. For example, a loosely bound $D^0\Sigma^+_c$ molecular state with the physical meson $D^0$ and baryon $\Sigma^+_c$ should have average spatial size $\sqrt{\langle r^2 \rangle} \geq 1.36$ fm, which is too large to be interpolated by the local currents.

The currents $J(0)$, $J_\mu(0)$ and $J_{\mu
u}(0)$ couple potentially to the $\frac{1}{2}^+, \frac{3}{2}^+$ and $\frac{5}{2}^+$ hidden-charm pentaquark molecular states $P^+_\frac{1}{2}$, $P^+_\frac{3}{2}$ and $P^+_\frac{5}{2}$, respectively,

\[
\begin{align*}
0,J(0)|P^-_{\frac{1}{2}}(p)\rangle &= \lambda^{\frac{1}{2}} U^{-}(p, s), \\
0,J(0)|P^+_{\frac{1}{2}}(p)\rangle &= \lambda^{\frac{1}{2}} i\gamma_5 U^{+}(p, s), \\
0,J_\mu(0)|P^-_{\frac{1}{2}}(p)\rangle &= f^\pm_\mu p_\mu U^{-}(p, s), \\
0,J_\mu(0)|P^+_{\frac{1}{2}}(p)\rangle &= f^\pm_\mu p_\mu i\gamma_5 U^{+}(p, s), \\
0,J_\mu(0)|P^-_{\frac{3}{2}}(p)\rangle &= \lambda^{\frac{3}{2}} U^{-}_{\mu}(p, s), \\
0,J_\mu(0)|P^+_{\frac{3}{2}}(p)\rangle &= \lambda^{\frac{3}{2}} i\gamma_5 U^{+}_{\mu}(p, s), \tag{8}
\end{align*}
\]

\[
\begin{align*}
0,J_{\mu\nu}(0)|P^-_{\frac{1}{2}}(p)\rangle &= g^\pm_\mu p_\nu U^{-}(p, s), \\
0,J_{\mu\nu}(0)|P^+_{\frac{1}{2}}(p)\rangle &= g^\pm_\mu p_\nu i\gamma_5 U^{+}(p, s), \\
0,J_{\mu\nu}(0)|P^-_{\frac{3}{2}}(p)\rangle &= f^\pm_\mu \left[ p_\mu U^{-}_{\mu}(p, s) + p_\nu U^{-}_{\nu}(p, s) \right], \\
0,J_{\mu\nu}(0)|P^+_{\frac{3}{2}}(p)\rangle &= f^\pm_\mu i\gamma_5 \left[ p_\mu U^{+}_{\mu}(p, s) + p_\nu U^{+}_{\nu}(p, s) \right], \\
0,J_{\mu\nu}(0)|P^-_{\frac{5}{2}}(p)\rangle &= \sqrt{2}\lambda^{\frac{5}{2}} U^{-}_{\mu\nu}(p, s), \\
0,J_{\mu\nu}(0)|P^+_{\frac{5}{2}}(p)\rangle &= \sqrt{2}\lambda^{\frac{5}{2}} i\gamma_5 U^{+}_{\mu\nu}(p, s), \tag{9}
\end{align*}
\]

where the $U^{\pm}(p, s)$, $U^{-}_{\mu}(p, s)$ and $U^{\pm}_{\mu}(p, s)$ are the Dirac and Rarita-Schwinger spinors \[22, 32, 33, 34, 35, 36\]. At the hadron side of the correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$, we isolate the ground state contributions from the hidden-charm pentaquark molecular states with the spin-parity $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ and $\frac{5}{2}^+$ respectively without contaminations according to the current-hadron couplings shown in Eqs. (8) - (10), and get the hadronic representation \[22, 32, 33, 34, 35, 36\],

\[
\begin{align*}
\Pi(p) &= \lambda^{\frac{1}{2}} (p^2) \not\!p + \Pi^\alpha_\frac{1}{2}(p^2), \\
\Pi_{\mu\nu}(p) &= \lambda^{\frac{3}{2}} (p^2) \not\!p g_{\mu\nu} - \Pi^\alpha_\frac{3}{2}(p^2) g_{\mu\nu} + \cdots, \\
\Pi_{\mu\nu\alpha\beta}(p) &= \lambda^{\frac{5}{2}} (p^2) \not\!p (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + \lambda^{\frac{1}{2}} (p^2) (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + \cdots, \tag{11}
\end{align*}
\]
There are other spinor structures, which are not shown explicitly, we choose the components corresponding to the spinor structures \( \bar{\phi}, 1, \gamma_5 g_{\mu\nu}, g_{\mu\nu} \) and \( \bar{\psi} (g_{\mu\alpha} g_{\nu\beta} + g_{\nu\beta} g_{\mu\alpha}), g_{\mu\alpha} g_{\nu\beta} + g_{\nu\beta} g_{\mu\alpha} \) in the correlation functions \( \Pi(p), \Pi_{\mu\nu}(p) \) and \( \Pi_{\mu\nu\alpha\beta}(p) \) respectively to investigate the \( J^P = \frac{1}{2}^+, \frac{3}{2}^+ \) and \( \frac{5}{2}^+ \) pentaquark molecular states.

Now we take a not long digression to discuss the isospins of the interpolating currents. From Eqs.\((14)-(17)\), we can see clearly that the currents \( J^{D\Sigma^-}(x), J^{\bar{D}\Sigma^+}(x), J^{D^{\ast}\Sigma^-}(x) \) and \( J^{\bar{D}^\ast\Sigma^+}(x) \) without strangeness have the same isospin structures, while the currents \( J^{D\Xi^-}(x), J^{D^{\ast}\Xi^-}(x), J^{\bar{D}^{\ast}\Xi^+}(x) \) and \( J^{\bar{D}^{\ast}\Xi^-}(x) \) with strangeness also have the same isospin structures, and they can be transformed into each other with the simple replacements \( d \leftrightarrow s \). It is a good object to explore the light flavor \( SU(3) \) breaking effects. We can rewrite the currents \( J^{D^{\ast}\Sigma^-}(x) \) and \( J^{\bar{D}^{\ast}\Xi^+}(x) \) in terms of the isospin eigenstates,

\[
J^{D^{\ast}\Sigma^-}(x) = J^0_D(x) J^{\Sigma^-}_{\ast}(x) = \frac{1}{\sqrt{3}} J^{\Sigma^-}_{D\Sigma_c}(x) + \sqrt{\frac{2}{3}} J^{\Sigma^-}_{D\Sigma_c}(x), \\
J^{\bar{D}^{\ast}\Xi^+}(x) = J^0_{D}(x) J^{\Xi^+}_{\ast}(x) = J^{\Xi^+}_{D\Xi_c}(x),
\]

where

\[
J^{\Sigma^-}_{D\Sigma_c}(x) = \frac{1}{\sqrt{3}} J_D(x) J^{\Sigma^-}_{\ast}(x) - \frac{\sqrt{2}}{3} J_D(x) J^{\Sigma^+}_{\ast}(x), \\
J^{\Xi^+}_{D\Xi_c}(x) = \frac{\sqrt{2}}{3} J_D(x) J^{\Sigma^-}_{\ast}(x) + \frac{1}{\sqrt{3}} J_D(x) J^{\Sigma^+}_{\ast}(x),
\]

the \( J^0_{D}(x), J^0_{D^{\ast}}(x), J^{\Sigma^-}_{\ast}(x), J^{\Sigma^+}_{\ast}(x) \) and \( J^{\Xi^+}_{\ast}(x) \) are the standard currents for the mesons and baryons, respectively, the superscripts \( \frac{1}{2}, 1 \) and \( \frac{3}{2} \) stand for the isospin of the interpolating currents. Now let us estimate the isospin breaking effects,

\[
\langle 0 | T \left\{ J^{D^{\ast}\Sigma^-}(x) J^{\bar{D}^{\ast}\Sigma^+}(0) \right\} | 0 \rangle = \frac{1}{3} \langle 0 | T \left\{ J^{\Sigma^-}_{D\Sigma_c}(x) J^{\Sigma^-}_{D\Sigma_c}(0) \right\} | 0 \rangle + \frac{2}{3} \langle 0 | T \left\{ J^{\Xi^+}_{D\Xi_c}(x) J^{\Xi^+}_{D\Xi_c}(0) \right\} | 0 \rangle,
\]

\[
\langle 0 | T \left\{ J^{D^{\ast}\Sigma^-}_{D}(x) J^{\bar{D}^{\ast}\Sigma^+}_{D^{\ast}}(0) \right\} | 0 \rangle = \frac{2}{3} \langle 0 | T \left\{ J^{\Sigma^-}_{D\Sigma_c}(x) J^{\Sigma^-}_{D\Sigma_c}(0) \right\} | 0 \rangle + \frac{1}{3} \langle 0 | T \left\{ J^{\Xi^+}_{D\Xi_c}(x) J^{\Xi^+}_{D\Xi_c}(0) \right\} | 0 \rangle,
\]

where the current \( J^{D^{\ast}\Sigma^-}_{D}(x) J^{\bar{D}^{\ast}\Sigma^+}_{D^{\ast}}(0) \) is the isospin breaking effects between the vacuum matrix elements \( \langle 0 | T \{ J^{D^{\ast}\Sigma^-}_{D}(x) J^{\bar{D}^{\ast}\Sigma^+}_{D^{\ast}}(0) \} | 0 \rangle \) and \( \langle 0 | T \{ J^{D^{\ast}\Sigma^-}_{D}(x) J^{D^{\ast}\Sigma^+}_{D}(0) \} | 0 \rangle \) at the quark-gluon level are suppressed by a factor \( \frac{1}{N_c} \), which are of minor importance and can be neglected safely in the large \( N_c \) limit. Then we can estimate (or obtain the conclusion tentatively) that the currents \( J^{\Sigma^-}_{D\Sigma_c}(x) \) and \( J^{\Xi^+}_{D\Xi_c}(x) \) couple potentially to the spin-1 \( \bar{D}\Sigma_c \) pentaquark molecular states with almost degenerated masses but different pole residues, thereafter, we will not distinguish the isospins \( I = \frac{1}{2}, \frac{3}{2} \) as we are only interested in the molecule masses, just like in the previous works [19, 20, 21, 22]. Furthermore, the current \( J^{\Xi^+}_{D\Xi_c}(x) \) has the quantum numbers \( I = 1 \) and \( I_3 = 1 \), we can add the superscripts \( I_3 \) to distinguish the components in the isospin triplet and singlet,

\[
J^{1,1}_{D\Xi_c}(x) = J^0_{D}(x) J^{\Xi^+}_{\ast}(x), \\
J^{1,0}_{D\Xi_c}(x) = \frac{1}{\sqrt{2}} J^0_{D}(x) J^{\Xi^+}_{\ast}(x) + \frac{1}{\sqrt{2}} J^{-1}_{D^{\ast}}(x) J^{\Xi^-}_{\ast}(x), \\
J^{1,-1}_{D\Xi_c}(x) = J^{-1}_{D^{\ast}}(x) J^{\Xi^-}_{\ast}(x), \\
J^{0,0}_{D\Xi_c}(x) = \frac{1}{\sqrt{2}} J^0_{D}(x) J^{\Xi^+}_{\ast}(x) - \frac{1}{\sqrt{2}} J^{-1}_{D^{\ast}}(x) J^{\Xi^-}_{\ast}(x),
\]
they couple potentially to the pentaquark molecular states with almost degenerated masses, again the isospin breaking effects are suppressed by the factor $\frac{1}{4\pi^2}$, thereafter, we will not distinguish the isospins $I = 1$ and $0$.

Now let us go back to the correlation functions. It is straightforward to obtain the spectral densities at hadron side through dispersion relation,

$$
\frac{\text{Im} \Pi_j^1(s)}{\pi} = \lambda_j^{-2} \delta \left( s - M_j^2 \right) + \sum_{\tau} \rho_{\tau H}^j(s),
$$

$$
\frac{\text{Im} \Pi_j^0(s)}{\pi} = M_\tau \lambda_j^{-2} \delta \left( s - M_j^2 \right) - M_\tau \lambda_j^{-2} \delta \left( s - M_j^2 \right) = \rho_{\tau H}^j(s),
$$

where $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, we add the subscript $H$ to represent the hadron side, then we introduce the weight functions $\sqrt{s} \exp \left( \frac{-s}{T^2} \right)$ and $\exp \left( \frac{-s}{T^2} \right)$ to obtain the QCD sum rules at the hadron side,

$$
\int_{s_0}^{s_\infty} ds \left[ \sqrt{s} \rho_{\tau H}^j(s) + \rho_{\tau H}^j(s) \right] \exp \left( \frac{s}{T^2} \right) = 2M_\tau \lambda_j^{-2} \exp \left( \frac{M_j^2}{T^2} \right),
$$

where the $s_0$ are the continuum threshold parameters and the $T^2$ are the Borel parameters.

It is also straightforward to accomplish the operator product expansion in the deep Euclidian space-time. For technical details in performing the operator product expansion for the correlation functions in exploring the multiquark states with hidden-charm, one can consult Refs. [32, 33, 34, 35, 36, 37, 38, 39]. If we contract the quark fields in the correlation functions in Eq. (19) with the Wick's theorem, we can observe clearly that there are two heavy quark propagators and three light quark propagators, if each heavy quark line emits a gluon and each light quark line contributes a quark-antiquark pair, we obtain a quark-gluon mixed operator $g_q G_{\mu\nu} g_q G_{\alpha\beta} q\bar{q} q\bar{q} q\bar{q}$ with $q = u, d$ or $s$, which is of dimension 13, and leads to the vacuum condensates $\langle q\bar{q}\rangle \langle q\bar{q}\sigma \bar{q}Gq \rangle^2$ and $\langle \bar{q}\bar{q}\rangle^3 \langle \bar{q}\bar{q}\bar{q}G \rangle$. It is better to take account of the vacuum condensates up to dimension $n = 13$ at least. On the other hand, the quark-gluon operators can be counted by the fine structure constant $\alpha_s = \frac{4\pi}{\alpha}$ with the orders $O(\alpha_s^k)$, where $k = 0, \frac{1}{2}, 1, \frac{3}{2}, \cdots$. In the present work, we take the truncations $n \leq 13$ and $k \leq 1$ in a consistent way, the quark-gluon operators of the orders $O(\alpha_s^k)$ with $k \leq 1$ are given full considerations, while in previous work [22], we neglected the vacuum condensates $\langle \bar{q}\bar{q}\rangle \langle \bar{q}G \rangle$, $\langle \bar{q}\bar{q}\rangle^2 \langle \bar{q}G \rangle$ and $\langle \bar{q}\bar{q}\rangle^3 \langle \bar{q}G \rangle$ due to their small contributions. Furthermore, we take account of the light flavor $SU(3)$ mass-breaking effects by including the contributions of the order $O(m_s)$ in a consistent way.

Now we take a short digression to discuss the higher dimensional vacuum condensates. In the QED, we deal with the perturbative vacuum, the vacuum expectation values of the normal-ordered electron-photon operators can be set be zero, for example, $\langle 0 | : e e : | 0 \rangle = 0$, $\langle 0 | : e \sigma \cdot F e : | 0 \rangle = 0$, $\langle 0 | : e \bar{e} : | 0 \rangle = 0$, etc.

In the QCD, we deal with the non-perturbative vacuum, and have to resort to non-zero vacuum expectation values of the normal-ordered quark-gluon operators to describe the hadron properties in a satisfactory way, for example, $\langle 0 | : q_\alpha q_\beta : | 0 \rangle = 0$, $\langle 0 | : \bar{q}_\alpha q_\beta : G_{\mu\nu} : | 0 \rangle = 0$, $\langle 0 | : \bar{q}_\alpha q_\beta : \bar{q}_\gamma q_\delta : | 0 \rangle = 0$, etc, where the $i, j, m$ and $n$ are color indexes, the $\alpha, \beta, \lambda$ and $\tau$ are Dirac spinor indexes. We usually parameterize the vacuum matrix elements in terms of $\langle 0 | : q_\alpha q_\beta : | 0 \rangle = \hat{q}_\alpha \hat{q}_\beta$, $\langle 0 | : \bar{q}_\alpha q_\beta : G_{\mu\nu} : | 0 \rangle = \hat{g}_{\alpha \beta} G_{\mu\nu}$, $\langle 0 | : \bar{q}_\alpha q_\beta : \bar{q}_\gamma q_\delta : | 0 \rangle = \hat{g}_{\alpha \beta} \hat{g}_{\gamma \delta}$, etc, where the $\hat{A}$ are the Gell-mann matrices. Except for the quark condensates, which indicate spontaneous breaking of the Chiral symmetry through the Gell-Mann-Oakes-Renner relation $f_K^2 m_K^2 = -2(m_u + m_d)\langle q\bar{q} : | 0 \rangle$, other vacuum condensates, such as $\langle 0 | : g_{\alpha \beta} : G_{\mu\nu} : | 0 \rangle$, $\langle 0 | : \bar{q}\bar{q} q\bar{q} q\bar{q} : | 0 \rangle^2$, etc, are just parameters introduced by hand to describe the non-perturbative vacuum.

We can parameterize the non-perturbative properties in one way or the other, then confront them to the experimental data on the multiquark states to obtain the optimal values. In the
QCD sum rules for the multiquark states, the $\phi(\bar{q}q)^2$ play an important role, and influence the convergent behaviors of the operator product expansion and the pole contributions remarkably, therefore influence the predictions remarkably, large values of the $\phi$ maybe destroy the platforms [51]. for example, in the present case, if we take the value $\phi = 2(3)$ in the QCD sum rules for the $D\Sigma_c$ pentaquark molecular state, we can obtain the uncertainty $\delta M_P = -0.10(-0.16)$ GeV, which is of the same order of the total uncertainty from other parameters, and a very bad platform (in other words, no platform at all). In calculations, we observe that the optimal value is $\phi = 1$, vacuum saturation (factorization) works well in the QCD sum rules for the multiquark states [32][33][34][35][36][37][44][45][46][47][48][49][51].

In the QCD sum rules for the $\bar{q}q, q\bar{Q}, QQ$ mesons, the $\phi(\bar{q}q)^2$ are always companied with the fine-structure constant $\alpha_s$, and play a tiny role, the deviation from vacuum saturation (factorization) $\phi = 1$, for example, $\phi = 2 \sim 3$, cannot make much difference in the numerical predictions, although in some cases the values $\phi > 1$ can lead to better QCD sum rules [52][53].

Once the corresponding analytical spectral densities $\rho_{j,QCD}^1(s)$ and $\rho_{j,QCD}^0(s)$ at the quark-gluon level are obtained, we can take the quark-hadron duality below the continuum thresholds $s_0$ and introduce the weight functions $\sqrt{s}$ and eliminate the pole residues $\lambda_j^-$ with $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ to obtain the QCD sum rules:

$$2M_-\lambda_j^- \exp \left( -\frac{M_j^2}{T^2} \right) = \int_{4m_c^2}^{s_0} ds \left[ \sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp \left( -\frac{s}{T^2} \right), \tag{21}$$

the explicit expressions of the spectral densities $\rho_{j,QCD}^1(s)$ and $\rho_{j,QCD}^0(s)$ at the quark level are neglected for simplicity.

We differentiate Eq. (21) with respect to $\tau = \frac{1}{T}$, then eliminate the pole residues $\lambda_j^-$ with $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ to obtain the QCD sum rules for the masses of the pentaquark molecular states,

$$M_j^2 = -\frac{\int_{4m_c^2}^{s_0} ds \left[ \sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp (-\tau s)}{\int_{4m_c^2}^{s_0} ds \left[ \sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp (-\tau s)}, \tag{22}$$

where the spectral densities $\rho_{j,QCD}^1(s) = \rho_{j,QCD}^1(s)$ and $\rho_{j,QCD}^0(s) = \rho_{j,QCD}^0(s)$.

3 Numerical results and discussions

We take the standard values of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01)$ GeV$^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle$, $\langle \bar{q}g_s\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $\langle \bar{s}g_s\sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$, $\langle \bar{q}s \rangle = (0.8 \pm 0.1)$ GeV$^2$, $\langle \bar{q}q \bar{q}q \rangle = 0.012 \pm 0.004$ GeV$^4$ at the energy scale $\mu = 1$ GeV [51][55][56], and take the $\overline{MS}$ masses $m_c(m_c) = (1.275 \pm 0.025)$ GeV and $m_s(\mu = 2$ GeV) = $(0.095 \pm 0.005)$ GeV from the Particle Data Group [3]. Furthermore, we take account of the energy-scale dependence of the quark condensates, mixed
quark condensates and \( \overline{MS} \) masses according to the renormalization group equation \[ 57 \].

\[
\langle \bar{q}q \rangle (\mu) = \langle \bar{q}q \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{2\pi} \log \frac{\mu}{\Lambda}} ,
\]

\[
\langle \bar{s}s \rangle (\mu) = \langle \bar{s}s \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{2\pi} \log \frac{\mu}{\Lambda}} ,
\]

\[
\langle \bar{q}g_\alpha \sigma Gq \rangle (\mu) = \langle \bar{q}g_\alpha \sigma Gq \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{3\pi} \log \frac{\mu}{\Lambda}} ,
\]

\[
\langle \bar{s}g_\alpha \sigma Gs \rangle (\mu) = \langle \bar{s}g_\alpha \sigma Gs \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{3\pi} \log \frac{\mu}{\Lambda}} ,
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{3\pi} \log \frac{\mu^{2\pi}}{\Lambda}} ,
\]

\[
m_s(\mu) = m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{12}{3\pi} \log \frac{\mu^{2\pi}}{\Lambda}} ,
\]

\[
\alpha_s(\mu) = \frac{1}{b_0} \left[ 1 - \frac{b_1}{b_0} \log t + \frac{b_2}{b_0^2} \left( \log^2 t - \log t - 1 \right) + b_0 b_2 \right],
\]

where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33-2\pi}{12\pi} \), \( b_1 = \frac{153-19\pi}{24\pi} \), \( b_2 = \frac{2857-1303\pi+442\pi^2}{128\pi^2} \), \( \Lambda = 213 \text{ MeV} \), \( 296 \text{ MeV} \) and \( 339 \text{ MeV} \) for the flavors \( n_f = 5, 4 \), and \( 3 \), respectively \[ 4, 57 \].

In the present work, we investigate the hidden-charm pentaquark molecular states with strangeness and without strangeness, it is better to choose the flavor numbers \( n_f = 4 \), and evolve all the input parameters to typical or special energy scales \( \mu \), which satisfy the energy scale formula or modified energy scale formula \[ 32, 33, 34, 35, 44, 45, 46, 47, 48, 49 \] with the updated value of the effective (or constituent) charmed quark mass \( M_c = 1.85 \text{ GeV} \) \[ 48 \]. By comparing with the constituent quark masses based on analysis of the \( J/\psi \) and \( \Upsilon \) mass spectrum with the famous Cornell potential \[ 59 \], we introduce an uncertainty \( M_c = 1.85 \pm 0.01 \text{ GeV} \). Furthermore, we take account of the light flavor \( SU(3) \) mass breaking effects, and prefer the modified energy scale formula \( \mu = \sqrt{M_{X/Y/Z/P}^2 - (2M_c)^2} - k M_s \) with the s-quark numbers \( k = 0, 1, 2, 3 \) and the effective s-quark mass \( M_s = 0.2 \text{ GeV} \), which was proved work well \[ 60 \]. Compared to the constituent quark mass, the effective s-quark mass \( M_s = 0.2 \text{ GeV} \) seems too small, as the effective u/d-quark masses \( M_{u/d} \) serve as a milestone and have been absorbed into the energy scales \( \mu \), the value \( M_s = 0.2 \text{ GeV} \) embodies the net \( SU(3) \) mass-breaking effects.

We can rewrite the energy scale formula in the form,

\[
M_{X/Y/Z/P}^2 = (\mu + k \delta)^2 + \text{Constants} ,
\]

where the light flavor \( SU(3) \) mass-breaking effects \( \delta \) have the value \( M_s \), the Constants have the value \( 4M_c^2 \), they are all fitted by the QCD sum rules. The \( \mu \) and \( M_s \) embody the light degrees of freedom, while the \( 4M_c^2 \) embodies the heavy degrees of freedom, the hidden-charm tetraquark and pentaquark (molecular) states can be divided into both the heavy and light degrees of freedom \[ 32, 33, 34, 35, 44, 45, 46, 47, 48, 49 \]. The predicted tetraquark and pentaquark (molecular) masses and the pertinent energy scales of the QCD spectral densities have a Regge-trajectory-like relation \[ 51 \].

In Ref.\[ 22 \], we explore the \( D\Sigma_c, D\Sigma_c^* \), \( \bar{D}^*\Sigma_c \) and \( \bar{D}^*\Sigma_c^* \) pentaquark molecular states with the QCD sum rules at length, and obtain the conclusion that the energy scale formula \( \mu = \sqrt{M_{X/Y/Z/P}^2 - (2M_c)^2} \) can enhance the pole contributions at the hadron side remarkably and improve the convergent behaviors of the operator product expansion remarkably. In fact, we take the energy scale formula as a constraint on the predicted molecule masses, which should be obeyed
in the QCD sum rules. In the present work, we consider the light flavor SU(3) mass-breaking effects and resort to the modified energy scale formula \( \mu = \sqrt{M^2_{X/Y/Z}/\rho - (2M_c)^2} - k M_s \) to choose the best energy scales of the spectral densities at the quark-gluon level, and search for the best Borel parameters and continuum threshold parameters to satisfy the two fundamental criteria of the QCD sum rules, i.e. pole dominance at the hadron side and convergence of the operator product expansion at the QCD side, via trial and error.

Then we obtain the Borel parameters, continuum threshold parameters \( s_0 \), optimal energy scales of the spectral densities at the quark-gluon level, and pole contributions of the ground state pentaquark molecular states, which are shown plainly in Table 2. From the table, we can see clearly that the contributions from the ground states are about or larger than \((40 - 60)\%\), the pole dominance criterion is satisfied very well. For the conventional hadrons, the QCD spectral densities \( \rho(s) \sim s^n \) with \( n \leq 1 \) and 2 for the mesons and baryons, respectively, it is easy to satisfy the pole dominance criterion, as the integral,

\[
\int_{\Delta^2}^s ds s^n \exp \left(-\frac{s}{T^2}\right)
\]  

(25)

converges quickly even if we choose a large Borel parameter \( T^2 \), where the \( \Delta^2 \) is the threshold, the uncertainty originates from the continuum threshold parameter \( s_0 \) is small. For the multiquark states, the QCD spectral densities \( \rho(s) \sim s^n \) with \( n \leq 4 \) and 5 for the tetraquark and pentaquark (molecular) states, respectively, it is very difficult to satisfy the pole dominance criterion, as the integral,

\[
\int_{\Delta^2}^s ds s^n \exp \left(-\frac{s}{T^2}\right)
\]  

(26)

converges very slowly even if we choose a rather small Borel parameter \( T^2 \). In general, we expect to choose \( T^2 = O(M^2) \), the integral (or continuum state) is suppressed by a fact \( \exp \left(-\frac{M^2}{T^2}\right) \sim \exp \left(-\frac{M^2}{T^2}\right) \sim e^{-1} \). Thus, for the multiquark states, we have to resort to a much stringent suppression of the continuum states, \( T^2 \ll M^2 \). One may wonder that such a small Borel parameter might lead to a bad convergent behavior in the operator product expansion.

In Fig 1 we plot the absolute values of the \( D(n) \) for the central values of the input parameters shown in Table 2 where the \( D(n) \) are defined by

\[
D(n) = \frac{\int_{4M_c^2}^{s_0} ds \rho_n(s) \exp \left(-\frac{s}{T^2}\right)}{\int_{4M_c^2}^{s_0} ds \rho(s) \exp \left(-\frac{s}{T^2}\right)}
\]  

(27)

the \( \rho_n(s) \) are the QCD spectral densities for the vacuum condensates of dimension \( n \), and the total spectral densities \( \rho(s) = \sqrt{s} \rho_{QCD}(s) + \rho^0_{QCD}(s) \). From the figure, we can see clearly that although the largest contributions do not come from the terms \( D(0) \) in some cases, the vacuum condensates \( \langle \bar{q}q \rangle/\langle \bar{q}q \rangle \) and \( \langle \bar{q}q \rangle/\langle \bar{s}s \rangle \) with the dimension 6 serve as a milestone, the absolute values of the contributions \( |D(n)| \) with \( n \geq 6 \) decrease monotonically and quickly with the increase of the dimensions \( n \), the value \( |D(13)| \approx 0 \), the operator product expansion converges very well. The two basic criteria of the QCD sum rules are all satisfied.

In calculations, we observe that the predicted molecule masses increase monotonically and slowly with the increase of the continuum threshold parameters \( s_0 \) if we fix the Borel parameter \( T^2 \); on the other hand, larger continuum threshold parameters mean larger pole contributions. We truncate the continuum threshold parameters \( s_0 \) by requiring about the same pole contributions \((40 - 60)\%\) in all the QCD sum rules so as to reduce the uncertainties originate from the continuum threshold parameters \( s_0 \).

In previous work [22], we neglected the vacuum condensates \( \langle \bar{q}q \rangle/\langle \bar{q}q \rangle GG \), \( \langle \bar{q}q \rangle^2/\langle \bar{q}q \rangle GG \) and \( \langle \bar{q}q \rangle^3/\langle \bar{q}q \rangle GG \), which are of dimension 7, 10 and 13, respectively, due to their small contributions.
data and theoretical works are still needed to identify the P and besides reproducing the masses of the existing pentaquark candidates molecular states should be narrow. The large width $\Gamma$ distributions and are kinematically suppressed in the phase space, the widths of those pentaquark molecules without strangeness and with strangeness according to variations of the Borel parameters, where the regions between the two short vertical lines are the Borel windows. From the figure, we can see clearly that there appear rather flat platforms in the Borel windows, the uncertainties from the Borel parameters are just supplementary parameters, not physical quantities. Furthermore, in the figure, we also present the experimental values of the masses of the $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$ from the LHCb collaboration [1][2][3].

The pentaquark (molecule) candidates $P_c(4312)$, $P_c(4380)$, $P_c(4440)$ and $P_c(4457)$ are observed in the $J/\psi p$ mass spectrum, their isospins are $I = \frac{1}{2}$, while pentaquark (molecule) candidate $P_{cs}(4459)$ is observed in the $J/\psi \Lambda$ mass spectrum, its isospin is $I = 0$. The present calculations support assigning the $P_c(4312)$ as the $\bar{D}\Sigma_c$ pentaquark molecular state with the quantum numbers $J^P = \frac{1}{2}^+$ and $I = \frac{1}{2}$, assigning the $P_c(4380)$ as the $\bar{D}\Sigma_c^*$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^+$ and $I = \frac{1}{2}$, assigning the $P_c(4440)$ as the $D^*\Sigma_c$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^-$ and $I = \frac{1}{2}$, assigning the $P_{cs}(4459)$ as the $D^*\Sigma_c^*$ pentaquark molecular state with the quantum numbers $J^P = \frac{5}{2}^-$ and $I = \frac{1}{2}$, assigning the $P_{cs}(4459)$ as the $D\Xi_c^*$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^-$ and $I = 0$ do to the uncertainties, see Table 3 and Fig.2. For example, it is marginal to assign the $P_c(4457)$ as the $D^*\Sigma_c^*$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^-$ and $I = \frac{1}{2}$, as the $P_c(4457)$ lies at the bottom of the predicted mass of the $D^*\Sigma_c^*$ pentaquark molecular state, see Fig.2 G.

From Tables 2 and 3, we can see that the modified energy scale formula $\mu = \sqrt{\frac{M_{X/Y/Z}\mu}{kM_s} = (2M_{c})^2 - kM_s}$ with the s-quark numbers $k = 0, 1, 2, 3$ and the effective s-quark mass $M_s = 0.2$ GeV is satisfied very well [60]. On the other hand, the predicted masses for the pentaquark molecular states without strangeness and with strangeness have the relation, $M_{P_{cs}} - M_{P_c} \approx m_s - m_q \approx 0.13 \sim 0.15$ GeV, which is consistent with the light-flavor $SU(3)$ breaking effects for the heavy baryons in the flavor sextet $6_f$, $M_{\Xi_c^*} - M_{\Xi_c} \approx M_{\Sigma_c^*} - M_{\Sigma_c} \approx m_s - m_q \approx 0.13$ GeV from the Particle Data Group [4].

The present calculations indicate that there maybe exist the $\bar{D}\Sigma_c$ ($D\Xi_c$), $\bar{D}\Sigma_c^*$ ($D\Xi_c^*$), $D^*\Sigma_c$ ($D^*\Xi_c$) and $D^*\Sigma_c^*$ ($D^*\Xi_c^*$) pentaquark molecular states with the $J^P = \frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{3}{2}^-$ and $\frac{5}{2}^-$, respectively, which lie near the corresponding $\bar{D}\Sigma_c^*$ ($D\Xi_c^*$), $\bar{D}\Sigma_c^*$ ($D\Xi_c^*$), $D^*\Sigma_c^*$ ($D^*\Xi_c^*$) and $D^*\Sigma_c^*$ ($D^*\Xi_c^*$) thresholds, respectively, see Table 3. The two-body strong decays to the corresponding open-charm meson-baryon pairs, such as $\bar{D}\Sigma_c$ ($D\Xi_c$), $\bar{D}\Sigma_c^*$ ($D\Xi_c^*$), $D^*\Sigma_c$ ($D^*\Xi_c$) and $D^*\Sigma_c^*$ ($D^*\Xi_c^*$), with the fall-apart mechanism directly, can only take place through the higher tails of the mass distributions and are kinematically suppressed in the phase space, the widths of those pentaquark molecular states should be narrow. The large width $\Gamma_{P_c(4380)} = 205 \pm 18 \pm 86$ MeV maybe indicate that the $P_c(4390)$ maybe correspond to two or more unresolved structures. More experimental data and theoretical works are still needed to identify the $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$ unambiguously.

In the present work, we make predictions for the masses of new pentaquark molecular states besides reproducing the masses of the existing pentaquark candidates $P_c(4312)$, $P_c(4380)$, $P_c(4440)$,
Table 2: The optimal energy scales $\mu$, Borel parameters $T^2$, continuum threshold parameters $s_0$ and pole contributions (pole) for the hidden-charm pentaquark molecular states.

| $J^P$ | $\mu$(GeV) | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | pole |
|-------|------------|----------------|------------------|------|
| $\bar{D}\Sigma_c$ | $\frac{1}{2}^-$ | 2.2 | 3.1 - 3.5 | 5.02 ± 0.10 | (42 - 64)% |
| $D\Xi'_c$ | $\frac{1}{2}^-$ | 2.2 | 3.2 - 3.6 | 5.14 ± 0.10 | (42 - 63)% |
| $D\Sigma_c^*$ | $\frac{3}{2}^-$ | 2.4 | 3.2 - 3.6 | 5.08 ± 0.10 | (43 - 64)% |
| $D\Xi_c^*$ | $\frac{3}{2}^-$ | 2.4 | 3.3 - 3.7 | 5.21 ± 0.10 | (43 - 64)% |
| $D^*\Sigma_c$ | $\frac{3}{2}^-$ | 2.5 | 3.3 - 3.7 | 5.16 ± 0.10 | (41 - 62)% |
| $D^*\Xi'_c$ | $\frac{3}{2}^-$ | 2.5 | 3.4 - 3.8 | 5.29 ± 0.10 | (41 - 61)% |
| $D^*\Sigma_c^*$ | $\frac{5}{2}^-$ | 2.6 | 3.4 - 3.8 | 5.22 ± 0.10 | (40 - 60)% |
| $D^*\Xi_c^*$ | $\frac{5}{2}^-$ | 2.6 | 3.5 - 3.9 | 5.35 ± 0.10 | (40 - 60)% |

Table 3: The predicted masses and pole residues of the hidden-charm pentaquark molecular states with the possible assignments, where the double-? denotes that such assignments are not excluded due to the uncertainties.

$P_c(4457)$ and $P_{cs}(4459)$. We can search for the non-strange $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $D^*\Sigma_c$ and $D^*\Sigma_c^*$ pentaquark molecular states with the isospin $I = \frac{1}{2}$ (or $I = \frac{3}{2}$) and with the spin-parity $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{3}{2}^+$ and $\frac{5}{2}^-$, respectively in the $\Lambda_b^0$ decays,

$$\Lambda_b^0 \rightarrow p J/\psi K^-, n J/\psi K^0, n J/\psi \bar{K}^0, p \eta_c K^-, n \eta_c \bar{K}^0, n \eta_c \bar{K}^0,$$

$$\Delta^+ J/\psi K^-, \Delta^0 J/\psi K^0, \Delta^0 J/\psi \bar{K}^0, \Delta^+ \eta_c K^-, \Delta^0 \eta_c K^0, \Delta^0 \eta_c \bar{K}^0, \Delta^0 \eta_c \bar{K}^0,$$  

(28)

and search for the strange $\bar{D}\Xi'_c$, $\bar{D}\Xi'_c$, $\bar{D}\Xi_c^*$ and $\bar{D}\Xi_c^*$ pentaquark molecular states with the isospin $I = 0$ (or $I = 1$) with the spin-parity $J^P = \frac{1}{2}^+$, $\frac{3}{2}^-$, $\frac{3}{2}^+$ and $\frac{5}{2}^-$, respectively in the $\Xi_b^0$ and $\Xi_b^-$ decays,

$$\Xi_b^0 \rightarrow \Sigma^+ J/\psi K^-, \Sigma^0 J/\psi K^0, \Lambda^0 J/\psi K^0, \Sigma^+ \eta_c K^-, \Sigma^0 \eta_c K^0, \Lambda^0 \eta_c \bar{K}^0,$$

$$\Sigma^+ J/\psi K^-, \Sigma^0 J/\psi \bar{K}^0, \Sigma^0 J/\psi \bar{K}^0, \Sigma^+ \eta_c K^-, \Sigma^0 \eta_c \bar{K}^0, (29)$$

$$\Xi_b^- \rightarrow \Lambda^0 J/\psi K^-, \Sigma^0 J/\psi K^0, \Sigma^+ J/\psi \bar{K}^0, \Lambda^0 \eta_c K^-, \Sigma^0 \eta_c K^0, \Sigma^+ \eta_c \bar{K}^0,$$

$$\Sigma^0 J/\psi K^-, \Sigma^0 J/\psi \bar{K}^0, \Sigma^0 \eta_c K^-, \Sigma^+ \eta_c \bar{K}^0.$$  

(30)
where the pentaquark molecular states besides reproducing the masses of the existing pentaquark candidates of the spectral densities at the quark-gluon level, and make predictions for the masses of new molecular states with strangeness and without strangeness via the QCD sum rules at length by carrying out the operator product expansion up to the vacuum condensates of dimension 13 in a consistent way, and take the modified energy scale formula to choose the best energy scales.

4 Conclusion

In this article, we investigate the $\bar{D}\Sigma_c$, $D\Xi'_c$, $\bar{D}\Xi'_c$, $\bar{D}^*\Sigma_c$, $D^*\Xi'_c$, $\bar{D}^*\Sigma_c$ and $D^*\Xi'_c$ pentaquark molecular states with strangeness and without strangeness via the QCD sum rules at length by carrying out the operator product expansion up to the vacuum condensates of dimension 13 in a consistent way, and take the modified energy scale formula to choose the best energy scales of the spectral densities at the quark-gluon level, and make predictions for the masses of new pentaquark molecular states besides reproducing the masses of the existing pentaquark candidates $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$. The present calculations support assigning the $P_c(4312)$ as the $\bar{D}\Sigma_c$ pentaquark molecular state with the quantum numbers $J^P = \frac{1}{2}^-$ and $I = \frac{1}{2}$, assigning the $P_c(4380)$ as the $\bar{D}\Sigma_c$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^-$ and $I = \frac{1}{2}$, assigning the $P_c(4440/4457)$ as the $D^*\Sigma_c$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^-$ and $I = \frac{1}{2}$, assigning the $P_{cs}(4459)$ as the $\bar{D}\Xi'_c$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^-$ and $I = 0$; but cannot exclude the possibilities of assigning the $P_c(4457)$ as the $D^*\Sigma_c$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^-$ and $I = \frac{1}{2}$ and assigning the $P_{cs}(4459)$ as the $\bar{D}\Xi'_c$ pentaquark molecular state with the quantum numbers $J^P = \frac{3}{2}^-$ and $I = 0$ due to the uncertainties. In calculations, we observe that the predicted masses of the pentaquark molecular states without strangeness and with strangeness have mass gap about $0.13 \sim 0.15$ GeV, which is consistent with the light-flavor $SU(3)$ breaking effects of the heavy baryons in the flavor sextet $6_f$. We can search for both the old and new pentaquark molecular states in the decays of the $\Lambda_b^0$, $\Xi_b^0$ and $\Xi_b^-$ in the future to preform more robust investigations and shed light on the nature of the $P_c$ and $P_{cs}$ states.

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Figure 2: The masses of the pentaquark molecular states with variations of the Borel parameter $T^2$, where the A, B, C, D, E, F, G and H denote the pentaquark molecular states $D\Sigma_c$, $D\Xi_c$, $D^*\Sigma_c$, $D\Xi_c^*$, $D^*\Sigma_c^*$, $D^*\Xi_c$, and $D^*\Xi_c^*$, respectively.
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