Resonant Transport in Coupled Quantum Wells: a Probe for Scattering Mechanisms

Y. Berk \(^1\), A. Kamenev\(^2\), A. Palevski\(^1\), L. N. Pfeiffer\(^3\), and K. W. West\(^3\)

\(^1\)School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel

\(^2\)Department of Condensed Matter, The Weizmann Institute of Science, Rehovot 76100, Israel.

\(^3\)AT&T Bell Laboratories, Murray Hill, New Jersey 07974

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Abstract

We present a microscopical theory and experimental results concerning resistance resonance in two tunneling coupled quantum wells with different mobilities. The shape of the resonance appears to be sensitive to the small angle scattering rate on remote impurities and to the electron–electron scattering rate. This allows the extraction of scattering parameters directly from the transport measurements. The negative resonance in a Hall coefficient is predicted and observed for the first time.

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The novel resistance resonance (RR) in two coupled quantum wells (QW) with different mobilities was recently suggested and discovered experimentally [1]. The basic physical idea of this phenomenon is the following. One studies the in–plane resistance of two QW’s as function of a relative position of their energy levels (gate voltage), by contacting to both of them. If the energy levels are far from each other, the tunneling is suppressed and the resulting resistance is given by, $R_{off} \sim (\tau_{1}^{tr} + \tau_{2}^{tr})^{-1}$ (two resistors connected in parallel), where $\tau_{i}^{tr}$’s are the transport mean free times in the corresponding wells. The situation, however, is remarkably different once the system is brought into the resonance (energy levels coincide). In this case the wavefunctions form symmetric and antisymmetric subbands, split by a tunneling gap. As any electron is completely delocalized between two wells, the scattering rate in each of these subbands is $(\tau^{tr})^{-1} = (2\tau_{1}^{tr})^{-1} + (2\tau_{2}^{tr})^{-1}$. The resulting resistance is given by $R_{res} \sim (2\tau^{tr})^{-1}$. If the transport scattering rates of two QW’s are different, one should find the peak in the resistance whose relative amplitude is

$$\frac{R_{res} - R_{off}}{R_{off}} = \frac{(\tau_{1}^{tr} - \tau_{2}^{tr})^{2}}{4\tau_{1}^{tr}\tau_{2}^{tr}} \equiv A. \quad (1)$$

This effect was indeed observed and reported in a number of publications [1–3]. In this letter we present a microscopical model of the RR, which includes elastic scattering on a long–range remote impurity potential. We also report the experimental measurements of the RR shape and its temperature dependence, and we analyze them within the presented theoretical model. In addition, we have calculated and measured the Hall coefficient in coupled QW’s. We show that at the resonant conditions the Hall coefficient exhibits a local minimum, which may be well understood on a basis of a classical two–band model.

The main messages which follow from the present investigation are the following: (i) at low temperature, the width of the RR is determined by a small angle scattering time on remote impurities; (ii) the temperature dependence of the RR indicates that a shape of the resonance is sensitive to the electron-electron interactions, allowing determination of the electron–electron scattering rate; (iii) the resonance in a Hall coefficient is predicted and demonstrated experimentally.
We first present a theoretical model of a transport in two coupled QW’s. In a basis of local states of each QW, the Hamiltonian of the system may be written in the following matrix form:

\[ H = \hat{a}_k^\dagger \hat{H}_{k,p} \hat{a}_p, \]

where

\[ \hat{H}_{k,p} = \delta_{kp} \begin{pmatrix} \epsilon_1 + (p - eA)^2/(2m^*) & \Delta/2 \\ \Delta^*/2 & \epsilon_2 + (p - eA)^2/(2m^*) \end{pmatrix} + \begin{pmatrix} U_1(p - k) & 0 \\ 0 & U_2(p - k) \end{pmatrix}, \]

and \( k, p \) are 2D momentum of the electrons. In the last equation \( \epsilon_i(V_G) \) are bare quantized levels of corresponding wells, which are functions of a gate voltage, \( V_G \). The tunneling coupling (gap), \( \Delta \), is assumed to be in–plane momentum conserved and energy independent. A vector potential of the external field (electric and magnetic) is denoted by \( A = A(r, t) \). Finally, \( U_i(p - k) \) represents the elastic disorder in each layer. We shall assume that impurity potentials in different wells are uncorrelated. Inside each QW an impurity potential has a finite correlation length and may be characterized by the two scattering times: the full one (or small angle) and the transport one

\[ \frac{1}{\tau_i} \propto \int |U_i(p)|^2 d\Omega; \quad \frac{1}{\tau_{i tr}} \propto \int |U_i(p)|^2 (1 - \cos \theta) d\Omega, \]

where the integrations are carried out over the Fermi circles.

Now the model is specified completely and we apply it first to the calculation of a linear conductance. Using the Kubo formula, one has

\[ \sigma = \int d\epsilon \frac{f(\epsilon) - f(\epsilon + \omega)}{2\pi S\omega} \text{Tr}(\hat{I}_p \hat{G}_{p,k}^+(\epsilon + \omega) \hat{G}_{k,p}^- (\epsilon)), \]

where \( \text{Tr} \) stays for both matrix and momentum indexes; \( S \) is an area of the structure. A current operator, \( \hat{I}_p \), is \( e\mathbf{p}/m \) times a unit matrix (if all contacts are attached to the both wells). Retarded and advanced Green functions of a system are defined as

\[ \hat{G}_{p,k}^\pm (\epsilon) = \langle p | (\epsilon - H \pm i\eta)^{-1} | k \rangle. \]

Constructing the perturbation expansion over the impurity potential (the second term in Eq. (2)), and solving the Dyson equation for an average Green function, one obtains to
leading order in \((\epsilon_F \tau_i)^{-1}\), \[4\]

\[
\langle \hat{G}_{\mathbf{p},k}^\pm (\epsilon) \rangle = \delta_{\mathbf{k} \mathbf{p}} \left( \begin{array}{cc} \epsilon - \epsilon_1 - \epsilon_p \pm i/2\tau_1 & \Delta/2 \\ \Delta^*/2 & \epsilon - \epsilon_2 - \epsilon_p \pm i/2\tau_2 \end{array} \right)^{-1}.
\]

Note that a tunneling coupling is taken into account in a non-perturbative fashion, hence the final results should not be restricted by lowest orders in \(\Delta\). The conductivity, according to Eq. \((4)\) is given by a diagram Fig. 1a, where the shaded triangle represents the renormalized current vertex. To evaluate the latter one should solve the matrix integral equation schematically depicted on Fig. 1c, \[4\]. The calculation gives the following result for the zero-frequency resistance \((R = \sigma^{-1})\)

\[
R = \frac{R_1 R_2}{R_1 + R_2} \left[ 1 + A \frac{|\Delta|^2 \tau^\text{tr} \tau^{-1}}{(\epsilon_1 - \epsilon_2)^2 + |\Delta|^2 \tau^\text{tr} \tau^{-1} + \tau^{-2}} \right]; \quad (6)
\]

where \(R_i = (e^2 n_i \tau^\text{tr}_i / m)^{-1}\) are resistances of each well and the asymmetry coefficient, \(A\), is defined by Eq. \((1)\). The result, Eq. \((6)\), is valid if all relevant energies are much less than the Fermi energy, \(\epsilon_F\); this implies that the concentrations of carriers in two QW’s are close to each other, \(|n_1 - n_2| \ll n_i\). For relatively clean case, \(|\Delta|^2 \gg (\tau \tau^{-1})^{-1}\), Eq. \((6)\) confirms our qualitative conclusions, drawn in the beginning. In the dirty case (the opposite limit) the height of the resonance is suppressed. Note, that the width of the resonance depends on the small angle scattering time, \(\tau\), although the resistances of each well are fully determined by the transport times, \(\tau^\text{tr}_i\). The physical nature of this fact is the following. Any elastic scattering process (including the small angle scattering) leads to a mixing between the states of symmetric and antisymmetric subbands (according to classification in clean wells). Not too far from the exact resonance (say \(\epsilon_1 - \epsilon_2 \approx \Delta\)) the wavefunctions of clean wells are already mostly localized in one of the wells (eg. “symmetric” in the upper one and “antisymmetric” in the lower one). In this case they are sensitive only to scatterers in the corresponding well and the resonance is destroyed. The above mentioned mixing changes the situation, making the exact eigenfunction of dirty wells delocalized. As a result the resonance appears
to be broader, than in the case without small angle scattering. The relative amplitude is
determined only by transport quantities and is not affected by the latest.

The Hall coefficient is given by the two diagrams, one of which is depicted in a Fig. 1b. We present here only the result for the short range impurity potential \( \tau = \tau_{tr} \)

\[
R_H = \frac{R_{H,2}R_1^2 + R_{H,1}R_2^2}{(R_1 + R_2)^2} \left[ 1 - A \frac{2\tau_1\tau_2}{\tau_1^2 + \tau_2^2} |\Delta|^2 \frac{(\epsilon_1 - \epsilon_2)^2 + |\Delta|^2 + 3\tau_2^{-2}}{[(\epsilon_1 - \epsilon_2)^2 + |\Delta|^2 + \tau_2^{-2}]^2} \right], \tag{7}
\]

where \( R_{H,i} = (n_i e)^{-1} \) is a Hall coefficient of each QW. We shall discuss the physics of the last expression later, when presenting the experimental results.

The double QW structure was grown on N\(^+\) GaAs substrate by molecular-beam epitaxy
and consisted of two GaAs wells 139 Å width separated by a 40 Å Al\(_{0.3}\)Ga\(_{0.7}\)As barrier. The electrons were provided by remote delta-doped donor layers set back by 250 Å and 450 Å spacer layers from the top and the bottom well correspondingly. In order to obtain the difference in the mobilities, an enhanced amount of impurities was introduced at the upper edge of the top well (Si, \(10^{10} \text{ cm}^{-2}\)). Measurements were done on 10\(\mu\)m-wide and 200 \(\mu\)m-long channels with Au/Ge/Ni Ohmic contacts. Top and bottom gates were patterned using the standard photolithography fabrication method. The top Schottky gate covered 150 \(\mu\)m of the channel. The data were taken using a lock-in four terminal techniques at \(f = 11 \text{ Hz}\). The voltage probes connected to the gated segment of the channel were separated by 100 \(\mu\)m.

The application of the upper gate voltage allows us to sweep the potential profile of the QW’s through the resonant configuration. The variation of the resistance vs. upper gate voltage is plotted in Fig. 2 (circles). The data were obtained at the T=4.2 K for the bottom gate voltage \(V_{GB} = 0.5V\). The resistance resonance is clearly observed at \(V_G \approx -0.6V\).

In order to compare the experimental data with the theoretical formula, Eq. (6), one has to establish the correspondence between the gate voltages and the energy levels, \(\epsilon_i\). The latter was found, using the known density of states and dc electrical capacitances between the QW’s and corresponding gate electrodes. The experimental values of these capacitances were established using the Hall measurements in the regime of the complete
depletion of the top QW, and are given by $C_1 = 4.53 \times 10^{-8} F cm^{-2}$ for the upper gate and $C_2 = 1.79 \times 10^{-8} F cm^{-2}$ for the bottom gate, which are extremely close to the theoretical estimates. The complementary measurements of the resistance and Hall coefficient far from the resonance allow us to determine the following parameters of our structure (as grown, i.e., $V_G = V_{GB} = 0$ and $T = 4.2 K$): $\mu_1 = 47,000 cm^2/V sec \ [8]$, $\mu_2 = 390,000 cm^2/V sec$, $n_1 = 4.7 \times 10^{11} cm^{-2}$, $n_2 = 2.5 \times 10^{11} cm^{-2}$. The quantum mechanical calculation of the tunneling gap results in $\Delta = 0.55$ meV; a very similar value for an identical structure was found experimentally [9]. The single fitting parameter, which was not determined by independent measurement is a small angle scattering rate, $\tau^{-1}$. The best fit (solid line in Fig. 2) was achieved for $\tau^{-1} = 1.3$ meV. This value implies the ratio between transport and small angle scattering times to be equal to 4.7, which is in a very good agreement with the measurements, using Shubnikov–de Haas oscillations, [3]. To our knowledge this is the first time, when the small angle scattering rate was determined in a pure (zero magnetic field) transport experiment.

The same fitting procedure was applied to a set of the resistance resonance data within the temperature range 4.2 – 60K, see Fig. 3. Amongst the independently measured parameters, only the mobility of a clean QW, $\mu_2$, exhibits pronounced temperature dependence, which is consistent with previously reported experimental data [10]. The temperature dependence of the fitting parameter, $\tau^{-1}(T)$, is plotted by circles in an inset to Fig. 3. At low temperature it may be well approximated by the following relation (the solid line in the inset):

$$\tau^{-1}(T) = \tau^{-1} + 3.0 T^2/\epsilon_F,$$

where $\epsilon_F = 10.9$ meV is the Fermi energy and $\tau^{-1} = 1.3$ meV is a zero temperature scattering rate, associated with a small angle scattering on the remote impurities. We tend to attribute the quadratic dependence of the scattering rate on temperature to an electron–electron (e–e) interactions. Indeed, in a clean limit ($\tau^{-1} \ll T \ll \epsilon_F$), the e–e scattering rate is given by $[9] \tau^{-1}_{ee} = \alpha T^2/\epsilon_F$, where dimensionless coefficient $\alpha$ is of order of unity. The
e–e interactions do not change the resistances of each well separately, due to conservation of the total momentum of an electronic system. Therefore, the interactions do not influence the resistance at the resonance and very far from it. In the intermediate region, however, e–e interactions cause mixing between symmetric and antisymmetric subbands, making the resonance broader. In this sense it plays a role very similar to that of a small angle scattering (see discussion after Eq. 6). Following this argument, we assume that the e–e scattering rate, enters the expression in the same way as a small angle one. The last suggestion requires some additional theoretical treatment; however, if verified, it provides a powerful method of measuring of e–e scattering rate.

The Hall effect measurements are necessary to establish parameters of the structure. They are, however, interesting due to a presence of a "negative" resonance in a Hall coefficient. The experimental data for the Hall coefficient, $R_H$, at $T = 4.2$K and magnetic field less than 0.05 T (the region, where a Hall voltage is linear with field) is presented in Fig. 4 (circles). The theoretical curve (see Eq. 7) is also plotted on the same graph by a solid line. The nature of the "negative" resonance may be easily understood using a classical two–band model [9]. According to this model, two bands having concentration of carriers $n_i$ and transport times $\tau_{i\text{tr}}$, exhibit the following Hall coefficient

$$R_H = \frac{1}{e} \frac{n_1(\tau_{1\text{tr}})^2 + n_2(\tau_{2\text{tr}})^2}{(n_1\tau_{1\text{tr}} + n_2\tau_{2\text{tr}})^2}.$$  

(9)

Far from the resonance the role of two bands are played by two QW’s, thus in this case $n_i$ and $\tau_{i\text{tr}}$ are characteristics of uncoupled wells (cf. Eq. 4). In the exact resonance the two bands are symmetric and antisymmetric subbands, which obviously have the same transport times, $\tau^{tr}$, and practically the same concentrations, $n$ ($\Delta \ll \epsilon_F$); thus in the resonance, $R_H = (2en)^{-1}$ (in agreement with Eq. 7). If the concentrations in the two wells differ from each other not too much ($n_1 \approx n_2 \approx n$), the resonance value of the Hall coefficient is strictly less than the off-resonance one. Another prediction of the simple two band model is the dependence of a Hall coefficient on a magnetic field [4]. This was also observed experimentally in a full agreement with a model, confirming that a classical two–
band model is applicable to our structure.

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FIGURES

Fig. 1. Diagrams for a conductivity (a) and a Hall coefficient (b), current vertex renormalization due to a small angle scattering (c). Full circle – bare current vertex; dashed line – impurity scattering.

Fig. 2. Resistance Resonance (RR) curves: circles – experimental data, solid line – theoretical calculation.

Fig. 3. The set of RR curves at different temperatures. The inset shows the variation of $\tau^{-1}(T) - \tau^{-1}(0)$ vs. temperature. The circles denote the values deduced from analysis of experimental data, the solid line represents $3.0 T^2/\epsilon_F$ [meV].

Fig. 4. Hall coefficient vs. gate voltage: circles – experimental data, solid line – theoretical calculation.