Strangeness Form Factors of the Proton in the Chiral Quark Model

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Abstract

The chiral quark model describes the strangeness components of the light quarks as fluctuations into strange mesons and quarks. The single strange pseudoscalar and vector meson loop fluctuations of the constituent $u$- and $d$-quarks give rise to only very small strangeness form factors for the proton. This result is in line with recent experimental results, given their wide uncertainty range.

12.39.-x, 12.39.Ki, 14.20.Dh
I. INTRODUCTION

The HAPPEX experiment [1] shows the combination $G_E^s + 0.39G_M^s$ of the strange charge and magnetic form factors of the proton at $Q^2 = 0.48 \text{(GeV/c)}^2$ to be consistent with $0 (0.023 \pm 0.034 \pm 0.022 \pm 0.026)$. Similarly the SAMPLE experiment [2] shows that $G_M^s$ at $Q^2 = 0.1 \text{(GeV/c)}^2$ is consistent with 0, modulo uncertainties in the calculated value of the weak axial form factor of the nucleon [3]. The experimental result that the strangeness form factors of the proton are small may be used to constrain or test theoretical models for nucleon structure, as the theoretical predictions for these observables have covered a fairly wide range [3–10].

A calculation of the strangeness form factors of the proton based on the chiral quark model is reported here. The approach considers the strangeness component of the proton as loop fluctuations, with intermediate strange mesons and $s$-quarks, of the constituent quarks that form the proton. The constituent quark model approach represents an alternative to the hadronic approach, in which the strangeness components are considered as fluctuations of the nucleon into intermediate strange mesons and hyperons. The chiral quark model approach brings the advantages of much smaller coupling constants and consequently the possibility of a converging loop expansion that takes all baryonic intermediate states into account. Moreover, as the amplitudes of the loops mainly scale with the inverse squared mass of the intermediate meson, heavy meson contributions are suppressed. Examples of this are the desirably small meson loop contributions to the neutron charge radius [11] and the recent demonstration that the strange meson loop contributions to the proton give rise to but a very small strangeness magnetic moment [12].

It is shown here that the kaon and $K^*$ loop fluctuations of the light quarks, that are illustrated in Fig. 1, and which lead to but a small value for the strangeness magnetic moment of the proton, also lead to strange form factors of the proton with very small magnitudes. The results are found to be fairly insensitive to the value of the cut-off scale for the loop integrals, provided that this is taken to be about 1 GeV or larger, ie. values
commensurate with the chiral symmetry restoration scale \( \sim 4\pi f_\pi \sim 1.2 \text{ GeV} \), at which scale
the pseudoscalar mesons are expected to decouple from the constituent quarks.

The magnitude of the strange loop contributions to the strangeness form factors of the
proton is of the order \( (g^2/4\pi^2)(m_q^2/(m_q^2 + m_M^2)) \), where \( g \) is the meson-quark coupling, and
\( m_q \) and \( m_M \) are the masses of the light constituent quarks and strange mesons respectively.
As \( g^2/4\pi \sim 0.7 \) for \( K \) and \( K^* \) mesons, it follows with \( m_q \sim 300 \text{ MeV} \) that the loop amplitude
in the case of kaons is expected to only be about \( \sim 0.06 \), and smaller still in the case of \( K^* \)
mesons. In comparison the expected magnitude of a typical loop amplitude in the case of
the hadronic approach, where \( g^2/4\pi \simeq 10 \) and \( m_q \) is replaced by the proton mass, is more
than an order of magnitude greater. This is also revealed by a comparison of the calculated
values for the strangeness magnetic moment of the proton in Refs. [9] and [12]. It suggests
that a small net loop contribution in the hadronic approach only can result as a consequence
of strong cancellations between several large amplitudes unless strong cut-offs are invoked.

This paper falls into 5 sections. In section 2 the contribution of the strange loop amplitudes
to the proton form factors, that contain intermediate kaons are derived. The corre-
sponding results for the loop amplitudes that involve intermediate \( K^* \) mesons are derived
in section 3. The contribution from loop fluctuations with \( K^* \rightarrow K\gamma \) vertices is derived
in section 4. The numerical results for the strangeness form factors of the proton are calculated
in section 5.

II. KAON LOOP CONTRIBUTIONS

The strangeness form factors of the nucleon are defined as the invariant coefficients of the
matrix elements of the operators \( \bar{s}\gamma_\mu s \) in the proton. In standard notation the strangeness
current is

\[
\langle p'|j^s_\mu(0)|p\rangle = i\bar{u}(p')\left[F^s_1(Q^2)\gamma_\mu - F^s_2(Q^2)\frac{\sigma_\mu\nu q_\nu}{2m_N}\right]u(p).
\] (2.1)

Here \( q = p' - p \), and \( Q^2 = q^2 = q^2 - q_0^2 > 0 \), and \( m_N \) is the proton mass. The two form factors
are calculated here in the constituent quark model from the strangeness loop fluctuations.
illustrated by the Feynman diagrams in Fig. 1, where the meson lines represents $K$ and $K^*$ mesons.

Consider first the kaon loop diagrams. To calculate these, we consider the kaon-quark pseudovector coupling:

$$\mathcal{L} = i \frac{f_{K_{qs}}}{m_K} \bar{\psi} \gamma_5 \gamma_\mu \sum_{a=4}^{7} \lambda^a \partial_\mu K^a \psi.$$ \hfill (2.2)

The pseudovector kaon-quark coupling constant is obtained as \[12\]

$$f_{K_{qs}} = \frac{g_A^q m_K}{2 f_K}, \hfill (2.3)$$

where $g_A^q = 0.87$ for quarks \[13,14\] and $f_K$ is the kaon decay constant ($f_K = 113$ MeV). The numerical value for $f_{K_{qs}}$ is then $f_{K_{qs}} = 1.9$.

The kaon and strange quark current density operators have the form

$$j_\mu = ie \{ \partial_\mu K^+ K + \text{h.c.} \}, \hfill (2.4a)$$

$$\bar{j}_\mu = -\frac{ie}{3} \bar{\psi} s \gamma_\mu \psi s. \hfill (2.4b)$$

The standard convention on the strangeness form factors assigns the $s$-quark a strangeness charge of $+1$ and the kaon, which contains an $\bar{s}$-quark, a strangeness charge of $-1$. In the calculation of the loop amplitudes the $s$-quark current (2.4b) should therefore be multiplied by $-3$ and the kaon current (2.4a) by $-1$.

The pseudovector coupling term (2.2) requires introduction of a contact coupling term for current conservation. This contact current term gives rise to two seagull diagrams, which exactly cancel the corresponding seagull diagrams, which arise in the evaluation of the amplitudes of the loop diagrams in Fig. 1, upon application of the Dirac equation for the external quarks. The remaining loop amplitudes are equivalent to those, which arise if the loop amplitudes are calculated using the pseudoscalar coupling

$$\mathcal{L}_{K_{qs}} = ig_{K_{qs}} \bar{\psi} \gamma_5 \sum_{a=4}^{7} \lambda^a K^a \psi,$$ \hfill (2.5)

where the pseudoscalar coupling constant $g_{K_{qs}}$ is defined as \[12\].
\[ g_{Kqs} = \frac{m_q + m_s}{m_K} f_{Kqs}. \] (2.6)

In these expressions \( m_q \) represents the constituent mass of the light flavor quarks \( (u,d) \) and \( m_s \) represents the strange quark mass. For these masses we shall use the values \( m_q = 340 \) MeV and \( m_s = 460 \) MeV respectively \[12\]. With these mass values we obtain the value \( g_{Kqs}^2/4\pi = 0.75 \) for the (squared) kaon-quark pseudoscalar coupling constant.

The kaon loop contributions to the Dirac form factors \( F_s^1 \) of the quarks are logarithmically divergent. We regularize these loop amplitudes by cutting off the loop momentum integrals smoothly at the chiral restoration scale \( \Lambda_{\chi} = 4\pi f_\pi = 1.2 \) GeV. The cut-off is implemented by replacing the meson propagator \( v(k^2) = 1/(k^2 + m_K^2) \) in the loop diagram, that contains the s-quark current coupling (Fig. 1a), by the propagator multiplied by a dipole form factor

\[ v(k^2) \to \frac{1}{m_K^2 + k^2} \left[ \frac{\Lambda^2 - m_K^2}{\Lambda^2 + k^2} \right]^2. \] (2.7)

Current conservation then demands that the product of the two meson propagators in the loop amplitude that corresponds to the kaon current loop (Fig. 1b) be modified as

\[ \frac{1}{m_K^2 + k_1^2} \frac{1}{m_K^2 + k_2^2} \to \frac{v(k_2^2) - v(k_1^2)}{k_1^2 - k_2^2}. \] (2.8)

The calculation then proceeds by first calculating the strangeness form factors \( F_s^1 \) and \( F_s^2 \) for the constituent quarks. These form factors are the same for the \( u \)- and \( d \)-quarks. The relation of these form factors to the corresponding strangeness form factors of the proton is simple only under the assumption that \( m_q = m_p/3 \), which implies an equipartition of the total proton momentum between the quarks:

\[ F_s^1(Q^2) = 3F_s^{1q}(Q^2), \quad F_s^2(Q^2) = \frac{m_p}{m_q} F_s^{2q}(Q^2). \] (2.9)

From these relations we obtain the charge and magnetic form factors of the proton as

\[ G_E^s(Q^2) = F_1^s(Q^2) - \frac{Q^2}{4m_p^2} F_2^s(Q^2), \] (2.10a)

\[ G_M^s(Q^2) = F_1^s(Q^2) + F_2^s(Q^2). \] (2.10b)
The magnetic form factor at $Q^2 = 0$ then yields the strangeness magnetic moment in units of nuclear magnetons.

Charge conservation requires that $F_1^s(0) = 0$. This requirement is satisfied by subtracting the value of $F_1^s(0)$ from the expression below.

The contribution to the strangeness form factor $F_1^s$ of the proton from the two loop diagrams in Fig. 1 are obtained as

$$F_1^s(Q^2)\{a, K\} = \frac{g_{Kq}^2}{8\pi^2} \int_0^1 dx (1 - x) \int_0^1 dy \left\{ \left[ (m_s - m_q)^2 + 2m_q(m_s - m_q)(1 - x) + m_q^2(1 - x)^2 - Q^2(1 - x)^2(1 - y) \right] K_1(Q^2) + \ln \frac{H_1(\Lambda_s^2)}{H_1(m_K^2)} - x \frac{\Lambda_s^2 - m_K^2}{H_1(\Lambda_s^2)} \right\}, \quad (2.11a)$$

$$F_1^s(Q^2)\{b, K\} = \frac{g_{Kq}^2}{8\pi^2} \int_0^1 dx x \int_0^1 dy \left\{ 2m_q(m_s - m_q)(1 - x)K_2(Q^2) - \ln \frac{H_2(\Lambda_s^2)}{H_2(m_K^2)} + x \frac{\Lambda_s^2 - m_K^2}{H_2(\Lambda_s^2)} \right\}. \quad (2.11b)$$

Here the functions $K_1(Q^2)$ and $K_2(Q^2)$ have been defined as

$$K_1(Q^2) = \frac{1}{H_1(m_K^2)} - \frac{1}{H_1(\Lambda_s^2)} - x \frac{\Lambda_s^2 - m_K^2}{H_1(\Lambda_s^2)}, \quad (2.12a)$$

$$K_2(Q^2) = \frac{1}{H_2(m_K^2)} - \frac{1}{H_2(\Lambda_s^2)} - x \frac{\Lambda_s^2 - m_K^2}{H_2(\Lambda_s^2)}. \quad (2.12b)$$

The denominator functions $H_1(m^2)$ and $H_2(m^2)$ are defined as

$$H_1(m^2) = m_s^2(1 - x) - m_q^2x(1 - x) + m^2x + Q^2(1 - x)^2y(1 - y), \quad (2.13a)$$

$$H_2(m^2) = m_s^2(1 - x) - m_q^2x(1 - x) + m^2x + Q^2x^2y(1 - y). \quad (2.13b)$$

After subtraction of the corresponding values at $Q^2 = 0$ ($F_1^s(0)$) from the form factors $F_1^s(Q^2)$ the integrals remain finite even in the limit $\Lambda_s^2 \to \infty$.

The corresponding contributions to the strangeness Pauli form factors of the quarks is

$$F_2^s(Q^2)\{a, K\} = -\frac{g_{Kq}^2}{4\pi^2} \int_0^1 dx (1 - x)^2 \int_0^1 dy m_q(m_s - m_q)xK_1(Q^2), \quad (2.14a)$$

$$F_2^s(Q^2)\{b, K\} = -\frac{g_{Kq}^2}{4\pi^2} \int_0^1 dx x(1 - x) \int_0^1 dy m_q(m_s - m_q)xK_2(Q^2). \quad (2.14b)$$

These expressions reduce to those given in Ref. [12] in the limit $Q^2 \to 0$, if multiplied by the factor $m_p/m_q$ to give strangeness magnetic moments in units of nuclear magnetons.
In Fig. 2 the kaon loop contributions to the proton strangeness Dirac form factor $F_s^1$ are shown as functions of momentum transfer, after subtraction of the irrelevant constant $F_s^1(0)$. This loop contribution to the proton strangeness form factor is very small and negative, and for $Q^2 \leq 1 \text{ (GeV/c)}^2$ it decreases slowly from 0 to $\sim -0.01$. As shown below, the magnitude of this contribution is smaller than that of the strange vector meson loops.

The contributions from the kaon loop amplitudes to the strangeness Pauli form factor $F_s^2$ are shown in Fig. 3. These contributions, while small, are notably larger than the corresponding vector meson loop contributions that are derived in section 3 below. The momentum dependence of the kaon loop contribution to $F_s^2(Q^2)$ is fairly weak for $Q^2$ values below 1 $(\text{GeV/c})^2$.

III. STRANGE VECTOR MESON LOOP FLUCTUATIONS

The coupling of $K^*$ mesons to constituent quarks is described by the Lagrangian

$$\mathcal{L}_{K^*qs} = ig_{K^*qs} \bar{\psi}_s \left( \gamma_\mu + \frac{m_s - m_q}{m_{K^*}^2} \partial_\mu + i \frac{\kappa_{K^*qs}}{m_s + m_q} \sigma_{\mu\nu} \partial_\nu \right) \sum_{a=4}^7 \lambda^a K^a_\mu \psi + \text{h.c.} \quad (3.1)$$

This coupling is a generalization to fermions of unequal mass of the conventional transverse Proca coupling for vector mesons.

The coupling constants $g_{K^*qs}$ and $\kappa_{K^*qs}$ may be determined from the corresponding couplings of $K^*$ mesons to the baryon octet by the quark model relations [12]:

$$g_{K^*qs} = g_{K^*BB},$$
$$g_{K^*qs}(1 + \kappa_{K^*qs}) = \frac{3}{5} \frac{m_s + m_q}{M} g_{K^*BB}(1 + \kappa_{K^*BB}). \quad (3.2)$$

Here $M$ represents the average of the nucleon and $S = -1$ hyperon ($\Lambda, \Sigma$) masses.

A recent comprehensive boson exchange potential model fit to nucleon-nucleon scattering data gives $g_{K^*BB} = 2.97$ and $\kappa_{K^*BB} = 4.22$, with a liberal uncertainty margin [13]. These values yield $g_{K^*qs}^2/4\pi \simeq 0.7$ and $\kappa_{K^*qs} \simeq 0.21$. The small value of the tensor coupling $\kappa_{K^*qs}$ and its large uncertainty range suggests that it is consistent with 0. At this stage it is therefore justified to neglect the Pauli term in (3.1) altogether.
The current density operator for the $K^*$ mesons takes the form

$$ j_\mu = \pm ie \{ K_{\nu}^* \partial_\mu K_{\nu}^* - K_{\nu}^* \partial_\nu K_{\mu}^* \} + \text{h.c.} \quad (3.3) $$

The contribution to the strangeness from factors $F_1^*\bar{s}$ of the $u$- and $d$-quarks from the $K^*$ meson loops described by the Feynman diagrams in Figs. 1a and b (when the meson line represents a $K^*$ meson) are

$$ F_1^*\bar{s}(Q^2)\{a, K^*\} = \frac{g^2_{K^*\bar{s}q}}{4\pi^2} \int_0^1 dx \int_0^1 dy \left\{ m_s^2 - 4m_qm_sx + m_q^2x^2 - Q^2(x + (1-x)^2y(1-y)) \right\} \bar{K}_1(Q^2) + \ln \frac{H_1(\Lambda^2)}{H_1(m_{K^*}^2)} - x\frac{\Lambda^2 - m_{K^*}^2}{H_1(\Lambda^2)} + \mathcal{O}\left( \frac{1}{m_{K^*}^2} \right). \quad (3.4a) $$

$$ F_1^*\bar{s}(Q^2)\{b, K^*\} = \frac{g^2_{K^*\bar{s}q}}{8\pi^2} \int_0^1 dx \int_0^1 dy \left\{ 6m_q^2x(1-x) - 6m_qm_s(1-x) + Q^2x(1-2xy(1-y)) \right\} \bar{K}_2(Q^2) + 6\left[ \ln \frac{H_2(\Lambda^2)}{H_2(m_{K^*}^2)} - x\frac{\Lambda^2 - m_{K^*}^2}{H_2(\Lambda^2)} \right] + \mathcal{O}\left( \frac{1}{m_{K^*}^2} \right). \quad (3.4b) $$

Here the auxiliary functions $\bar{K}_1(Q^2)$ and $\bar{K}_2(Q^2)$ have been defined as the functions $K_1(Q^2)$ and $K_2(Q^2)$ in Eqs. (2.12), with the replacement of $m_K^2$ by $m_{K^*}^2$.

The terms of order $m_{K^*}^2$ and higher powers of $m_{K^*}^{-2}$ in (3.3) arise from the terms proportional to $m_{K^*}^2$ in the vector meson propagator $(\delta_{\mu\nu} + k_\mu k_\nu/m_{K^*}^2)/(m_{K^*}^2 + k^2)$ and the coupling (3.1). These terms are small in comparison to the terms of Eqs. (3.3) at low values of $Q^2$. The explicit expressions for the contributions of order $m_{K^*}^{-2}$ to $F_1^*\bar{s}$ that are indicated in Eqs. (3.4a) and (3.4b) from the two loop diagram amplitudes illustrated in Figs. 1a and b are

$$ F_1^*\bar{s}(Q^2)\{a, K^*, \mathcal{O}(m_{K^*}^{-2})\} = \frac{g^2_{K^*\bar{s}q}m_q^2}{8\pi^2 m_{K^*}^2} \int_0^1 dx \int_0^1 dy \left\{ m_s^2(1-x)^2 - \frac{m_q^2}{m_q^2}Q^2(1-x)^2y(1-y) ight. $$

$$ + m_s m_q[2x(1-x) + \frac{Q^2}{m_q^2}(1-x)^2(1+2xy(1-y))] + m_q^2x^2(1-x)^2 $$

$$ + Q^2(1-x)^2(2x^2y(1-y) - y(1-y^2) + x(1-3y + 4y^2 - y^3)) $$

$$ + \frac{Q^4}{m_q^2}(1-x)^2(1-y)y(x + (1-x)(1-y)y) \right\} K_1(Q^2) + \left[ \frac{m_q^2}{m_q^2} + 2\frac{m_s}{m_q}(1-3x) $$

$$ + 1 + 6x(1-x) + \frac{Q^2}{m_q^2}(2-3x - 6(1-x)^2y(1-y)) \right]\left[ \ln \frac{H_1(\Lambda^2)}{H_1(m_{K^*}^2)} - x\frac{\Lambda^2 - m_{K^*}^2}{H_1(\Lambda^2)} \right]. $$
Finally the corresponding terms that are proportional to 1/m_q^2 have the expressions:

$$F_{2q}^2(Q^2)\{a, K^*\}, O(m_{K^*}^2) = -\frac{g_{K^*qs}^2}{4\pi^2} \int_0^1 dx (1-x) \int_0^1 dy \frac{m_q^2}{m_{K^*}^2} \left\{ (1-x)^2 (m_s - m_q) (m_s + m_q x)
+ \frac{Q^2}{m_q^2} (m_s - m_q)(1-x)^2 (1-y) y K_1(Q^2)
- 2(1 - \frac{m_s}{m_q})(1 - \frac{3}{2} x) (\ln \frac{H_1(\Lambda_{\chi}^2)}{H_1(m_{K^*}^2)} - \frac{\Lambda_{\chi}^2 - m_{K^*}^2}{x}) \right\}$$

(3.7a)

$$F_{2q}^s(Q^2)\{b, K^*\}, O(m_{K^*}^2) = \frac{g_{K^*qs}^2}{4\pi^2} \int_0^1 dx x \int_0^1 dy \frac{m_q^2}{m_{K^*}^2} \left\{ m_q (1-x)^2 (m_s - m_q x^2)
+ Q^2 x^2 y (2 - 2x + x^2 - \frac{m_s}{m_q}) K_2(Q^2)
- 2 \left[ \frac{m_s}{m_q} - 2(1-x)^2 - x \right] \left[ \ln \frac{H_2(\Lambda_{\chi}^2)}{H_2(m_{K^*}^2)} - \frac{\Lambda_{\chi}^2 - m_{K^*}^2}{x} \right] \right\}$$

(3.7b)
These expressions reduce to those derived in Ref. [12] in the limit $Q^2 \to 0$, once multiplied by $m_p/m_q$ in order to obtain the results in units of nuclear magnetons.

The strange vector meson loop contributions to the strangeness Dirac form factor $F_s^q(Q^2)$ of the proton are obtained after subtraction of the irrelevant values $F_s^q(0)$ from the expressions (3.4) and (3.5) and multiplication of the sum of the remainders by a factor 3 (2.9). This contribution is shown in Fig. 2 along with the corresponding kaon loop contribution. In this case the vector meson loop contribution is larger in magnitude than the kaon loop contribution. Even so the sum of the $K$ and $K^*$ loop contributions remains very small in magnitude, reaching only the value -0.086 around $Q^2 = 1 (\text{GeV}/c)^2$.

The contribution of the strange vector meson loops to the strangeness Pauli form factor is very small because of a near cancellation between the two involved loop diagrams [12]. This contribution is shown in Fig. 3, with the much larger contribution from the kaon loop diagrams.

**IV. THE $K^* - K$ LOOP CONTRIBUTION**

We finally consider the strangeness loop fluctuation, for which the e.m. coupling is to the $K^*K$ transition vertex (Fig. 4). The amplitude of this loop fluctuation may be calculated from the empirically known radiative widths of the $K^*$ mesons. The $K^*K$ transition current vertex has the form

$$\langle K^a(k') | J_\mu | K^{*b}(k) \rangle = -i \frac{g_{K^*K\gamma}}{m_{K^*}} \epsilon_{\mu\nu\lambda\sigma} k^\lambda k'^\nu \delta^{ab} \quad (4.1)$$

The coupling constant $g_{K^*K\gamma}$ depends on the charge state of the strange mesons. It may be determined from the radiative decay widths using the expression [12]

$$\Gamma(K^* \to K \gamma) = \alpha \frac{g_{K^*K\gamma}^2 m_{K^*}}{24\pi} [1 - \left( \frac{m_K}{m_{K^*}} \right)^2]^3. \quad (4.2)$$

Here $\alpha$ is the fine structure constant.

Given the empirical radiative widths $\Gamma(K^{*+} \to K^{+}\gamma) = 50$ keV and $\Gamma(K^{*0} \to K^{0}\gamma) = \ldots$
116 keV \[16\], the corresponding coupling constant values are obtained as \(g_{K^*K^+\gamma} = 0.75\) and \(g_{K^*K^0\gamma} = 1.14\) (with the sign convention of \[12\]).

The \(K^*K\) loop diagrams (Fig. 4) only contribute to the strange Pauli form factors of the quarks. With the convention of assigning the \(K\) and \(K^*\) mesons a “strangeness charge” of \(-1\), the contribution to the strangeness Pauli form factor \(F_{2q}^s(Q^2)\) from these loop diagrams is found to be

\[
F_{2q}^s(Q^2) = -\frac{g_{Kqs}g_{K^*q\gamma} m_q}{2\pi^2 m_{K^*}} \int_0^1 \int_0^1 dx \, x \, dy \left\{ m_q(1-x)(m_s - m_q x) \right\}.
\]

The quantities \(G_\ell\) here have been defined as

\[
G_1 = G(m_K, m_{K^*}), \quad G_2 = G(\Lambda_\chi, m_{K^*}),
\]

\[
G_3 = G(m_K, \Lambda_\chi), \quad G_4 = G(\Lambda_\chi, \Lambda_\chi).
\]

The auxiliary function \(G(m, m')\) is defined as

\[
G(m, m') = m_s^2(1-x) - m_q^2 x(1-x) + m^2 x(1-y) + m^2 x y + Q^2 x^2 y(1-y).
\]

These expressions represent direct generalizations of the corresponding expressions defined in Ref. \[12\] for the case \(Q^2 = 0\).

To account for the different coupling constants \(g_{K^*K\gamma}\) in the case of \(u\)- and \(d\)-quarks, (2.9) is generalized to

\[
F_2^s(Q^2) = \frac{m_p}{m_s} \left[ \frac{4}{3} F_{2u}^s(Q^2) - \frac{1}{3} F_{2d}^s(Q^2) \right].
\]

The \(K^*K\) loop contribution to \(F_2^s(Q^2)\) is shown in Fig. 3. This loop contribution has the opposite sign to that of the diagonal strangeness loop fluctuations.

**V. SAMPLE AND HAPPEX**

The SAMPLE [3] experiment measures the strangeness magnetic form factor of the proton \(G_{sM}^M\) at \(Q^2 = 0.1\) (GeV/c)^2. The HAPPEX [1] experiment measures the combination
\( G_E^s + 0.39G_M^s \) at \( Q^2 = 0.48 \) (GeV/c)^2. The contributions to these observables from the kaon and \( K^* \) loops in Figs. 1 and 4 are shown in Figs. 5 and 6 as a function of momentum transfer.

The cut-off dependence is small \[12\], and for \( \Lambda_\chi = 1.2 \) GeV we obtain \( G_M^s(0.1) = -0.06 \) and \( G_E^s(0.48) + 0.39G_M^s(0.48) = -0.08 \). The latter value is slightly below the uncertainty range of the result of the HAPPEX experiment. Similarly the former value is within the uncertainty range of the result of the SAMPLE experiment, provided that \( G_A^2 \) is small and positive rather than large and negative as originally suggested \[4,17\].

The smallness of the calculated strangeness observables of the proton is an inherent feature of the chiral quark model. The smallness of the measured strangeness observables suggests that this model may provide a useful framework for describing those observables. The calculated momentum dependence of \( G_E^s \) agrees fairly well with that obtained by heavy baryon chiral perturbation theory \[18\]. The strangeness radius is \( \sim 0.02 \) fm^2, which value is also obtained in the baryon loop calculations in Ref. \[10\] if the cut-off scale is set to the chiral symmetry restoration scale. Because of the large negative vector meson contribution to the Dirac form factor \( F_1^s \) the calculated \( G_M^s \) form factor grows more negative with increasing \( Q^2 \), whereas the third order chiral perturbation theory result, which does not consider vector mesons, for \( G_M^s \) is that it increases slowly with momentum transfer. Hitherto QCD lattice calculations have been made only for the strangeness magnetic moment \( G_M^s(0) \), but not for the form factors. The calculated values are negative, with substantial uncertainty limits \((-0.36 \pm 0.20 \) \[12\], \(-0.16 \pm 0.18 \) \[20\]). The chiral quark model value for \( G_M^s(0) \sim -0.06 \) \[12\] falls within the uncertainty range of the latter value.

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FIGURES

FIG. 1. Kaon and $K^*$ loop fluctuations of constituent quarks, which contribute to the strangeness form factors of the proton.

FIG. 2. The Dirac strangeness form factor $F_1^s$ of the proton as a function of momentum transfer.

FIG. 3. The Pauli strangeness form factor $F_2^s$ of the proton as a function of momentum transfer.

FIG. 4. Strangeness fluctuations of the constituent quarks, which involve an intermediate radiative $K^* \rightarrow K \gamma$ transition.

FIG. 5. The Sach’s strangeness form factor $G_E^s$ of the proton as a function of momentum transfer.

FIG. 6. The Sach’s strangeness magnetic form factor $G_M^s$ of the proton as a function of momentum transfer. The SAMPLE experiment gives the experimental value at $Q^2 = 0.1 \text{ (GeV}/c)^2$. 
(a)

(b)

Fig. 1
Fig. 3

$F_2^S$ vs. $Q^2$ [(GeV/c)$^2$]

- $K$
- $K^*$
- $K-K^*$
- Total
Fig. 4
Fig. 5
Fig. 6

$G_M^S$ vs. $Q^2$ [GeV/c]^2