Extended beamforming by optimum 2-D sparse arrays

Shogo Nakamura, Sho Iwazaki, and Koichi Ichige
Department of Electrical and Computer Engineering, Yokohama National University, Yokohama-shi 240–8510, Japan
a) koichi@ynu.ac.jp

Abstract: This paper presents an extended beamforming method by two-dimensional (2-D) Sparse Arrays. The authors have proposed optimum 2-D sparse arrays and evaluated them in the sense of direction of arrival (DOA) estimation accuracy. We develop a way of beamforming by the difference co-array of 2-D sparse arrays, and evaluate their beamforming performance as well as bit error rate (BER) characteristics through computer simulation.

Keywords: 2-D sparse array, beamforming, BER characteristics

Classification: Antennas and Propagation

References

[1] H. L. Van Trees, Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory, Wiley, 2002.
[2] W. K. Ma, T. H. Hsieh, and C. Y. Chi, “DOA estimation of quasi-stationary signals with less sensors than sources and unknown spatial noise covariance: A Khatri-Rao subspace approach,” IEEE Trans. Signal Process., vol. 58, no. 4, pp. 2168–2180, Apr. 2010. DOI:10.1109/TSP.2009.2034935
[3] C. L. Liu and P. P. Vaidyanathan, “Hourglass arrays and other novel 2-D sparse arrays with reduced mutual coupling,” IEEE Trans. Signal Process., vol. 65, no. 13, pp. 3369–3383, July 2017. DOI:10.1109/TSP.2017.2690390
[4] Y. Iizuka and K. Ichige, “Optimum linear array geometry for 2q-th order cumulant based array processing,” Proc. Int. Workshop. Compressed Sensing and its application to Radar, Multimodal Sensing, and Imaging, pp. 198–201, May 2016. DOI:10.1109/CoSeRa.2016.7745730
[5] S. Nakamura, S. Iwazaki, and K. Ichige, “Optimum 2-D sparse array and its interpolation via nuclear norm minimization,” Proc. Int. Sympo. Circuits and Systems, pp. 1–5, May 2019. DOI:10.1109/ISCAS.2019.8702741
[6] S. Nakamura, S. Iwazaki, and K. Ichige, “Optimization and hole interpolation of 2-D sparse arrays for accurate direction-of-arrival estimation,” submitted to IEICE Trans. Commun.
[7] S. Iwazaki and K. Ichige, “Extended beamforming by sum and difference composite co-array for real-valued signals,” IEICE Trans. Fundamentals, vol. E102-A, no. 7, pp. 918–925, July 2019. DOI:10.1587/transfun.E102.A.918
[8] B. D. Carlson, “Covariance matrix estimation errors and diagonal loading in adaptive arrays,” IEEE Trans. Aerosp. Electron. Syst., vol. 24, no. 4, pp. 397–401, July 1988. DOI:10.1109/7.7181
[9] S. Nakamura, S. Iwazaki, and K. Ichige, “A note on beamforming method by 2-D sparse arrays,” Proc. IEICE Society Conf., no. B-1-141, Sept. 2019 (in Japanese).
1 Introduction

Two-dimensional (2-D) array antenna plays an important role in radar, sonar, and indoor/outdoor wireless communications [1]. Sparse arrays are nowadays very much attracted which can create virtual difference co-array by the help of Khatri-Rao product [2] and achieve the degree of freedom (DOF) of $O(N^2)$, where $N$ and $O(\cdot)$ denote the number of physical sensors and the order of computational cost, respectively. Hourglass array [3] has been proposed as a 2-D sparse array which can accurately estimate direction of arrivals (DOAs) while preserving small mutual coupling effect. The hourglass array has a property that its difference co-array becomes uniform rectangular array (URA) without hole, therefore we can accurately estimate DOAs by spatial smoothing preprocessing (SSP)-based DOA estimation algorithms like ESPRIT.

The authors have already studied that the array configuration of the hourglass array can further be modified to reduce mutual coupling effect by applying simulated annealing (SA) [4], and confirmed that we can enhance DOA estimation accuracy [5, 6]. Besides, two of the present authors have developed an extended beamforming method for 1-D sparse arrays and achieved very precise beam patterns [7]. However, only the array configuration and DOA estimation accuracy was discussed in [4, 5, 6, 7]. Evaluation of beamforming performance is mandatory for communication applications of sparse array.

In this paper, we present an extended beamforming method for 2-D sparse arrays. Similarly to [7], we introduce diagonal loading (DL) operation [8] and minimum variance distortionless response (MVDR) beamforming to realize its main-beam and null steering that can greatly suppress interference waves. We evaluate the beamforming performance [9] and bit error rate (BER) characteristics of the 2-D sparse arrays through computer simulation.

2 Signal model

Suppose that $D$ uncorrelated signal sources impinge on a 2-D sensor array in an additive white Gaussian noise (AWGN) environment, where the signals and noises are statistically independent. The array aperture and the sensor location are respectively given by $N_x \times N_y$ and $n_d$, where $n = (n_x, n_y) \in \mathbb{Z}^2$ is an integer-valued vector, and $d = \lambda/2$ is the minimum separation between sensors, and $\lambda$ is the wavelength of incoming sources. Assume that the sensor location $n$ forms a set $S$, then the sensor input $x_S$ on $S$ can be modeled in a similar manner with [3] as

$$x_S = \sum_{i=1}^{D} A_i C v_S(\hat{\theta}_i, \hat{\phi}_i) + u_S,$$

$$\hat{\theta}_i = \frac{d}{\lambda} \sin \theta_i \cos \phi_i,$$

$$\hat{\phi}_i = \frac{d}{\lambda} \sin \theta_i \sin \phi_i,$$

where the $i$-th source is with the complex amplitude $A_i \in \mathbb{C}$, the azimuth $\phi_i \in [0, 2\pi]$ and elevation $\theta_i \in [0, \pi/2]$. The element of the steering vector $v_S(\hat{\theta}_i, \hat{\phi}_i)$
corresponding to the sensor at \( n = (n_x, n_y) \) is given by \( e^{j2\pi(\theta_{n_x} + \phi_{n_y})} \). The mutual coupling matrix \( C \) is characterized by its entries \( (C)_{n_1, n_2} \):

\[
(C)_{n_1, n_2} = \begin{cases} 
    c(\|n_1 - n_2\|_2), & \|n_1 - n_2\|_2 \leq B, \\
    0, & \text{otherwise}
\end{cases}
\]  

(4)

where \( n_1, n_2 \in \mathcal{S} \) denote the sensor location, \( B \) is the maximum sensor separation where the mutual coupling effect exists, and \( c(\cdot) \) is the mutual coupling coefficient given by \( c(0) = 1 \) and \( |c(k)/c(\ell)| = \ell/k \) for \( k, \ell > 0 \) \[3\]. Here, the covariance matrix \( R_S \) of the array \( \mathcal{S} \) is given by \( R_S = E[x_S x_S^H] \).

We also define the difference co-array \( \mathbb{D} = \{ n_1 - n_2 \mid n_1, n_2 \in \mathcal{S} \} \) for any 2-D sparse array \( \mathcal{S} \). Then the input vector of the difference co-array \( x_D \in \mathbb{C}^{N_D \times 1} \) can be obtained by vectorizing the matrix \( R_S \) while removing duplicated entries \[3\], where \( N_D \) denotes the number of elements in difference co-array. Note that the physical array itself can resolve up to \( (N - 1) \) signals while that the higher DOF of the difference co-array \( \mathbb{D} \) enables us to identify \( O(N^2) \) uncorrelated signals.

3 Proposed method

We propose a beamforming method for difference co-arrays and introduce modulation and demodulation scheme of extended signals.

3.1 MVDR beamforming with diagonal loading

The physical array output \( y_S = w_S^H x_S \in \mathbb{C} \) is the inner product of the complex weight vector \( w_S \in \mathbb{C}^{N \times 1} \) and the physical array input \( x_S \in \mathbb{C}^{N \times 1} \). The weight \( w_S \) can be calculated as an MVDR beamforming weight by

\[
w_S = \frac{R_S^{-1} v(\theta, \phi)}{v_S(\theta, \phi)^H R_S^{-1} v_S(\theta, \phi)},
\]

(5)

where \( v_S(\theta, \phi) \in \mathbb{C}^{N \times 1} \) denotes the array steering vector of a desired wave.

We develop a beamforming method for the difference co-arrays of 2-D sparse arrays. The beamforming by the virtual array can be accomplished by defining the virtual steering vector \( v_D(\theta, \phi) \) at the location \( (n'_x, n'_y) \) as \( e^{j2\pi(\theta_{n'_x} + \phi_{n'_y})} \), where \( (n'_x, n'_y) \in \mathbb{D} \). Therefore we calculate the virtual array output \( y_D = w_D^H x_D \in \mathbb{C} \) where the steering vector of the difference co-array \( v_D(\theta, \phi) \in \mathbb{C}^{N_D \times 1} \) and

\[
w_D = \frac{R_S^{-1} v_D(\theta, \phi)}{v_D(\theta, \phi)^H R_S^{-1} v_D(\theta, \phi)} \in \mathbb{C}^{N_D \times 1},
\]

(6)

\[
R_D = E[x_D x_D^H] \approx \frac{1}{K} \sum_{k=1}^{K} x_D(k) x_D^H(k) \in \mathbb{C}^{N_D \times N_D}.
\]

(7)

Note that we create the input vector of the difference co-array \( x_D \) for each snapshot so that to be used in demodulation. Its number of snapshots becomes same with that of the physical array \( K \). The covariance matrix \( R_D \) may become singular in case of small number of snapshots \( K < D \), then we cannot directly apply the MVDR beamforming of (6). In such case, we first apply the DL operation \[7\] to the covariance matrix \( R_D \) for rank restoration, i.e.,

\[
R_{DL} = R_D + \delta I_{N_D},
\]

(8)
where $\delta$ is the loading parameter, and $\mathbf{I}_{N_d}$ denotes the $N_d \times N_d$ identity matrix. Using the matrix $\mathbf{R}_{DL}$ and the steering vector $\mathbf{v}_{DL}(\theta, \phi)$, the MVDR beamformer weight $\mathbf{w}_{DL}$ is given as

$$\mathbf{w}_{DL} = \frac{\mathbf{R}_{DL}^{-1}\mathbf{v}_{DL}(\theta, \phi)}{\mathbf{v}_{DL}(\theta, \phi)^H \mathbf{R}_{DL}^{-1}\mathbf{v}_{DL}(\theta, \phi)},$$

and then we have the output: $\mathbf{y}_{DL} = \mathbf{w}_{DL}^H \mathbf{x}_{DL} \in \mathbb{C}$.

### 3.2 Modulation and demodulation of virtual signals

We introduce modulation and demodulation scheme of the signals at virtual array elements. Removing the mutual coupling matrix $\mathbf{C}$ from (1) for simplicity, we can rewrite (1) as

$$\mathbf{x}_B(k) = \mathbf{V} \mathbf{A} + \mathbf{u}_B(k),$$

where $\mathbf{V} = [\mathbf{v}_B(\theta_1, \phi_1), \ldots, \mathbf{v}_B(\theta_d, \phi_d)]$ and $\mathbf{A} = [A_1, \ldots, A_d]^T$. In case we generate input signals of different co-array for each snapshot, the array input vector $\mathbf{x}_D(k)$ at the time index $k$ can be represented as

$$\mathbf{x}_D(k) = \text{vec}(\mathbf{R}_D(k)) = (\mathbf{V}^* \otimes \mathbf{V}) \mathbf{p} + \sigma^2 \cdot \text{vec}(\mathbf{I}_N),$$

where vec(·) is vectorization operator, $\otimes$ is Khatri-Rao product operator [2], $\sigma^2$ denotes the noise power. Besides, the vector $\mathbf{p}$ is given by $\mathbf{p} = [\sigma_1^2, \ldots, \sigma_d^2]^T$, where $\sigma_i^2 = |s_i|^2$ denotes the power of the $i$-th incident signal. The desired signal power $\sigma_d^2$ can be extracted by beamforming, and is written as $\sigma_d^2 = |s_d|^2$ whereas $s_d$ denotes the desired signal. Note that $|s_d|^2$ becomes non-negative real and does not preserve the phase component of the original complex signal $s_d$, therefore any phase modulation are not applicable. Assume the case of BPSK modulation for simplicity, and we modify the signal representation as follows.

In transmitter scheme, BPSK symbol sequence $s_1(k)$ is generated where $s_1(k)$ takes either −1 or 1. Basically we have BPSK transmission signal by applying cosine rolloff filter and multiplying carrier signal to $s_1(k)$. Instead, we employ shift operation $s_2(k) = s_1(k) + M$ before applying the filter, and then the sequence $s_2(k)$ takes either $(−1 + M)$ or $(1 + M)$ where $M$ denotes a positive integer shift parameter. By this operation, the transmitting signal becomes equivalent with ASK signals. Applying cosine rolloff filter to the sequence $s_2(k)$ and then we have $\tilde{s}_2(k)$, then the transmission signal $s(k)$ becomes $s(k) = \tilde{s}_2(k)e^{-j2\pi f_c k}$, where $f_c$ denotes carrier frequency.

In receiver scheme, the beamforming result of the extended array input signal $\mathbf{y}_D(k) = \mathbf{w}_{DL}^H \mathbf{x}_{DL}$ is first filtered, and then take a positive square-root operation $\sqrt{\hat{y}(k)}$. Finally the obtained signal becomes inversely shifted version of $\tilde{s}(k)$ so that the symbol center becomes zero, and then $\hat{s}(k)$ is demodulated.

### 4 Numerical examples

This section evaluates beamforming performance and BER characteristics of 2-D sparse arrays. The arrays to be compared are (a) URA, (b) Hourglass array [3], (c) the optimum array without holes (called “hole-free”), and (d) the optimum array with hole [5, 6], of which the number of physical elements are set to be common.
as 25. The aperture of the URA and that of the other arrays are (a) $5 \times 5$ and (b)–(d) $9 \times 9$, respectively. The total number of elements including virtual array is (a) $9 \times 9 = 81$ for URA, (b), (c) $17 \times 17 = 289$ for hole-free virtual URA, and (d) $269$ for the URA-like difference co-array with 20 holes. Note that the holes in (d) are interpolated by nuclear norm minimization [6] before the beamforming operation.

We assume 1 desired and 23 interference waves to evaluate beamforming performance where the DOAs are equally distributed for azimuth and elevation angles as in Figs. 1 and 2, where the star at $(\theta, \phi) = (45^\circ, 150^\circ)$ indicates desired wave direction while the 23 circles denote interference wave directions. In case of evaluating BER characteristics, we assume 1 desired and 4 interference waves where the DOA of the desired wave is set to $(\theta, \phi) = (45^\circ, 150^\circ)$, and that of the interference waves are given by $(\theta, \phi) = (20^\circ, 30^\circ), (30^\circ, 200^\circ), (80^\circ, 90^\circ), (10^\circ, 300^\circ)$.

Both SNR and SIR are set to 0 dB, but the SNR is changed from $-20$ to 10 dB in BER characteristics evaluation. The other parameters are set to as follows: $K = 500$ snapshots, the DL parameter $\delta = 10^5$, the signal shift parameter $M = 2$, and the mutual coupling parameters $c(1) = 0.3, B = 5$ as in [5, 6]. Note that the DL parameter $10^5$ is determined based on the discussion in [7], where the values larger than $10^5$ can work as well those less than $10^5$ cannot well create null beams.

4.1 Beamforming performance

Figs. 1 and 2 respectively show the beamforming results in cases without/with the DL operation. We see from Figs. 1 and 2 that the sparse arrays (b), (c) and (d) can make more number of null-beams than URA due to larger aperture. However, the beam patterns in cases without DL operation in Fig. 1 cannot well suppress sidelobes because the covariance matrix becomes nearly singular as mentioned in

![Beamforming results in cases without DL operation.](image)
Section 3. The DL operation well improves this problem as in Fig. 2, we see that the large sidelobes in Fig. 1 are quite well suppressed.

4.2 BER characteristics

BER characteristics are also evaluated. Fig. 3 shows the behavior of SNR–BER characteristics in cases of (a) without DL and (b) with DL, respectively. We see from Fig. 3(a) that the demodulation is not at all accomplished.

On the other hand, Fig. 3(b) says that the BER characteristics becomes better as SNR gets larger. We emphasize that the optimum 2-D sparse arrays achieves better BER characteristics than URA and hourglass arrays. Note that the results in Figs. 2(b), 2(c) and 2(d) are almost same because of using the sparse arrays with a same aperture. The proposed optimum array has the advantages that it can suppress mutual coupling effect while improving BER performance.
5 Concluding remarks

This paper presented an adaptive beamforming method for the extended 2-D Sparse Arrays. We developed an MVDR-based beamforming method for the extended 2-D sparse arrays and evaluated its BER performance. We confirmed that the optimum 2-D sparse arrays can form a very minute beams and improve BER performance.