Nucleon pair approximation description of the low-lying structure of $^{108,109}$Te and $^{109}$I

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The low-lying level schemes and electromagnetic transitions of $^{109}$Te, $^{109}$I, and the neighboring even-even nucleus $^{108}$Te, are calculated within the framework of the $SD$-pair approximation of the nuclear shell model. Good agreement is obtained between the calculated results and experimental data. The favored components of low-lying bands are discussed in the collective nucleon-pair subspace. The weak-coupling picture shown in these nuclei and its relationship with residual quadrupole-quadrupole interaction between valence protons and neutrons are analyzed.

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I. INTRODUCTION

Great experimental and theoretical efforts have been made in recent years to study the structure and decay properties of neutron-deficient tin, tellurium, iodine and xenon isotopes near the $N = Z = 50$ closed shells. Towards the proton drip line, these nuclei become unstable against particle emissions. α decays have been observed in nuclei $^{105-110}$Te and $^{108-113}$I, and the nucleus $^{109}$I is known as a proton emitter. Another feature of particular interest in this mass region is the behavior of the band structure and electromagnetic transition properties in relation to the doubly magic nucleus $^{100}$Sn. The spectroscopic studies suggest a vibrational-like collective character in even-even tellurium nuclei. The manifestation of vibrational collectivity in these nuclei is, however, not supported by $B(E2)$ measurements. Octupole correlations were found in the nuclei $^{108,109}$Te, interpreted by the $\nu h_{11/2}$ and $\pi h_{11/2}$ orbits. Bands built on the $\nu h_{11/2}$ and $\pi h_{11/2}$ orbits are systematically observed in the odd-mass tellurium and iodine isotopes. The low-lying $\nu h_{11/2}$ bands in odd-mass tellurium isotopes follow the same trend as those yrast states in the even-even core $^{108,109}$Te. This was explained in terms of core-particle coupling. The $\pi h_{11/2}$ bands for odd-mass iodine isotopes reflect a decrease in quadrupole deformation moving away from the midshell, with the maximum occurring in $^{117,119}$I.

The measurement of electromagnetic transitions in this nuclear region is a challenging task due to the very small reaction cross-sections leading to the nuclei of interest. So far, the lightest tin, tellurium and iodine nuclei with known reduced transition probabilities are $^{104}$Sn, $^{108}$Te, $^{109}$Te and $^{109}$I. Few theoretical studies have been carried out to analyze the band structures and electromagnetic transitions in the nuclei $^{108,109}$Te and $^{109}$I. Among these works, cranked Woods-Saxon calculations interpreted the band structures of $^{109}$I as being built on the $\pi g_{7/2}$ and $\pi h_{11/2}$ states in a weakly triaxial deformed nucleus. Interacting boson-fermion model calculations discussed the band structures of $^{108}$Te and identified two favored bands built on the $\nu g_{7/2}$ and $\nu h_{11/2}$ neutron quasiparticle states. Shell-model calculations with the realistic CD-Bonn nucleon-nucleon potential on $^{109}$Te and $^{109}$I reproduced the experimental excitation energies, but showed large deviations from the experimental $B(E2)$ strengths in certain cases.

Recent lifetime measurements on $^{108}$Te, $^{109}$Te and $^{109}$I showed that the $B(E2)$ values are approximately equal. This suggests that the additional proton (or neutron) in $^{109}$I (or $^{109}$Te) might have negligible effect on the reduced transition probabilities. Based on shell-model calculations, it was speculated that in these states the additional unpaired nucleon is weakly coupled to the even-even core $^{108}$Te.

The purpose of this paper is to study the low-lying band structures and electromagnetic transitions in the nuclei $^{108,109}$Te and $^{109}$I within the framework of the nucleon pair approximation (NPA). The NPA has been
shown to be a reliable and economic approximation of the shell model. It has been successfully applied to describe even-even, odd-A and odd-odd nuclei with $A \sim 80$ \cite{28}, 100 \cite{29}, 130 \cite{29} and 210 \cite{30}. In this model, the dimension of the collective nucleon-pair subspace is small, thus providing a simple and illuminating picture of the structure of the nuclei under investigation. In particular it allows one to evaluate the probability of the existence of weak-coupling schemes in a straightforward manner.

This paper is organized as follows. In Sec. II we give a brief introduction to the nucleon pair approximation (NPA), including the basis, the Hamiltonian, the transition operators, and the parametrization of our calculations. In Sec. III we present our calculations on the excitation energies of the low-lying states, their dominant configurations and the electromagnetic transition properties. Our summary and conclusion are given in Sec. IV.

II. THEORETICAL FRAMEWORK

For medium-heavy nuclei, the dimension of the shell model configuration space is usually prohibitively large and one must resort to various truncation schemes, e.g. the interacting boson model \cite{31}, the broken pair approximation \cite{32}, the fermion dynamical symmetry model \cite{33} as well as the NPA \cite{27}. In the NPA approach, a collective pair with angular momentum $r$ and projection $M$ is defined as \cite{27}

$$A^{(r)}_{M\sigma} = \sum_{j_\sigma j'_\sigma} y(j_\sigma j'_\sigma r) \left( C_{j_\sigma}^r \times C_{j'_\sigma}^r \right)^{(r)}_M,$$

where $C_{j_\sigma}^r$ is the single-particle creation operator in the $j$ orbit, and $\sigma = \pi$ and $\nu$ is the index of proton and neutron degrees of freedom, respectively. $r = 0, 2, 4, 6, 8$ corresponds to $S, D, G, I$ and $K$ pairs. The numbers $y(j_\sigma j'_\sigma r)$ are the so-called structure coefficients of the nucleon pair with spin $r$.

In an even-even system with $2N$ valence protons or neutrons, we assume that all the valence nucleons are coupled to collective pairs. Our collective nucleon pair subspace is constructed by coupling $N$ collective pairs $r_1, r_2 \cdots r_N$ stepwise,

$$A^{(J_N)}_{M_{J_N}}(r_1 r_2 \cdots r_N, J_1 J_2 \cdots J_N)|0\rangle \equiv A^{(J_N)}_{M_{J_N}}(0),$$

$$\equiv \left(\cdots (A^{r_1} \times A^{r_2})^{(j_2)} \times \cdots \times A^{r_N})^{(j_N)} \right)^{M_{J_N}}_{J_N}|0\rangle .$$

Similarly, for an odd-A system with $2N + 1$ valence protons or neutrons, all valence nucleons are paired except the last nucleon, which can occupy any single-particle level $j$ of the shell model space under consideration. Our

| $j$   | $s_{1/2}$ | $d_{3/2}$ | $d_{5/2}$ | $g_{7/2}$ | $h_{11/2}$ |
|------|---------|---------|---------|---------|---------|
| $\epsilon_{j_\pi}$ | 1.550   | 1.660   | 0.172   | 0.000   | 3.550   |
| $\epsilon_{j_\nu}$ | 1.550   | 1.660   | 0.172   | 0.000   | 3.550   |

TABLE I: Single-particle (s.p.) energies $\epsilon_{j_\sigma}$ (in MeV) and two-body interaction parameters $G_{\sigma}^0$, $G_{\sigma}^2$, $\kappa_{\sigma}$, $\kappa_{\sigma\nu}$. The unit of $G_{\sigma}^0$ is MeV; the units of $G_{\sigma}^2$, $\kappa_{\sigma}$ and $\kappa_{\sigma\nu}$ are MeV/$r_0^2$, $r_0^2 = 1.012 A^{1/3}$ fm$. \sigma = \pi, \nu$ stands for proton and neutron, respectively.

nucleon-pair subspace is given by successively coupling the $N$ nucleon pairs to the unpaired nucleon in a single-$j$ orbit as

$$A^{(J_N)}_{M_{J_N}}(j r_1 r_2 \cdots r_N, J_1 J_2 \cdots J_N)|0\rangle \equiv A^{(J_N)}_{M_{J_N}}(0),$$

$$\equiv \left(\cdots ((C_{j_1}^r \times A^{r_1})^{(J_1)} \times A^{r_2})^{(J_2)} \times \cdots \times A^{r_N})^{(J_N)} \right)^{M_{J_N}}_{J_N}|0\rangle ,$$

where $J_i$ (half integer) denotes the total angular momentum for the first $2i + 1$ nucleons.

As in Ref. \cite{27}, we choose in our calculations a complete set of non-orthonormal but linearly independent many-pair basis states. If the basis states are chosen appropriately, all the eigenvalues of the overlap matrix $\langle 0|A^{(J_N)}_{M_{J_N}} A^{(J_N)}_{M'_{J_N}}|0\rangle$ are non-zero. This practice guarantees that all multi-pair basis states in the NPA calculations are not over-complete.

The NPA Hamiltonian is chosen to have the form

$$H = \sum_{j_\sigma} \epsilon_{j_\sigma} C_{j_\sigma}^\dagger C_{j_\sigma} + \sum_{\sigma} \left( G_{\sigma}^0 \mathcal{P}_{\sigma}^{(0)} \cdot \mathcal{P}_{\sigma}^{(0)} + G_{\sigma}^2 \mathcal{P}_{\sigma}^{(2)} \cdot \mathcal{P}_{\sigma}^{(2)} \right)$$

$$+ \sum_{\sigma} \kappa_{\sigma} Q_{\sigma} \cdot Q_{\sigma} + \kappa_{\pi\nu} Q_\pi \cdot Q_\nu,$$

where $\epsilon_{j_\sigma}$ is the single-particle energy, $G_{\sigma}^0$, $G_{\sigma}^2$, $\kappa_{\sigma}$ and $\kappa_{\pi\nu}$ are the two-body interaction strengths corresponding to monopole, quadrupole pairing and quadrupole-quadrupole interactions between valence nucleons. We have

$$\mathcal{P}_{\sigma}^{(0)} = \sum_{j_\sigma} \sqrt{\frac{2j_\sigma + 1}{2}} (C_{j_\sigma}^\dagger \times C_{j_\sigma})^{(0)},$$

$$\mathcal{P}_{\sigma}^{(2)} = \sum_{j_\sigma j'_\sigma} q(j_\sigma j'_\sigma) \left( C_{j_\sigma}^\dagger \times C_{j'_\sigma}^\dagger \right)^{(2)}_M,$$

$$Q_{\sigma} = \sum_{j_\sigma j'_\sigma} q(j_\sigma j'_\sigma) \left( C_{j_\sigma}^\dagger \times C_{j'_\sigma}^\dagger \right)^{(2)}_M ,$$

where $q(j j') = \sqrt{\frac{(2j+1)^{1/2}}{2\pi}} j^2 C_{j_1/2, j'_1/2}^{20} |n l| |n l'|$. $C_{j_1/2, j'_1/2}^{20}$ is the Clebsch-Gordan coefficient. The
isospin symmetry is not conserved in our NPA Hamiltonian. But it has to be pointed out that the effect of isospin mixture on the low-lying states is small for the nuclei treated here. To illustrate this point, we make comparison of ground state bands for $^{104,106,108}\text{Te}$ between the shell model calculations without (denoted by SM1) and with isospin symmetry (denoted by SM2) in Fig. 1. It is seen that both calculations give rather similar results.

The single-particle energies and two-body interaction parameters in our calculations are shown in Table I. The neutron single-particle energies of $g_{7/2}$ and $d_{5/2}$ orbitals are taken from the experimental excitation energies in $^{101}\text{Sn}$ [8]. There are no experimental data for the remaining orbitals. Their single-particle energies are extracted from a shell model calculation [4]. The proton single-particle energies are taken to be the same as those for neutrons.

There are totally seven parameters for the two-body interactions: $G^0_{\pi}, G^0_{\nu}, G^2_{\pi}, G^2_{\nu}, \kappa_{\pi}, \kappa_{\nu}$ and $\kappa_{\pi\nu}$. For $^{100}\text{Te}$ and $^{109}\text{I}$, we assume the same parameters as their even- even core $^{108}\text{Te}$. We take $G^0_{\pi} = -0.18$ MeV, which is the same value as the one used in Ref. [9]. As the proton number is close to the neutron number in this region we adopt for the strength of the proton interaction the value $G^0_{\pi} = -0.20$ MeV. The remaining five parameters are obtained by fitting to the excitation energies and $B(E2)$ values in nuclei $^{108,109}\text{Te}$ and $^{109}\text{I}$.

The $E2$ transition operator is defined by $T(E2) = e_\pi Q_\pi + e_\nu Q_\nu$, where $e_\pi$ and $e_\nu$ are the effective charges of valence protons and neutrons, respectively. The $B(E2)$ value in unit of $e^2\text{fm}^4$ is given by

$$B(E2, J_i \rightarrow J_f) = \frac{2J_f + 1}{2J_i + 1} (e_\pi \chi_\pi + e_\nu \chi_\nu)^2 r_0^4,$$

with reduced matrix element $\chi_\sigma = \langle \beta_f, J_f | Q_\sigma | \beta_i, J_i \rangle$ ($\sigma = \pi, \nu$) and $r_0^2 = 1.012 A^{1/3}$ fm$^2$. $|\beta_i, J_i\rangle$ is the eigenfunction of $J_i$ state. Our neutron effective charge is taken to be $e_\nu = 1.28e$, the same as for tin isotopes [6]. The proton effective charge $e_\pi = 1.79e$ is obtained by fitting to experimental data.

The $M1$ transition operator is defined by $T(M1) = g_{\pi} l_\pi + g_{\nu} l_\nu + g_{\pi\nu} s_\pi + g_{\pi\nu} s_\nu$, where $l_\sigma$ and $s_\sigma$ are the orbital and spin angular momenta, $g_{\pi}$ and $g_{\nu}$ are the effective orbital and spin gyromagnetic ratios, respectively. The effective spin gyromagnetic ratios are taken to be $g_{\pi} = 5.586 \times 0.7 \mu_N$ and $g_{\nu} = -3.826 \times 0.7 \mu_N$, where the number 0.7 is the conventional quenching factor (see also Ref. [10]). Two sets of effective orbital gyromagnetic ratios are used in this paper. In the first set, we use their free values, i.e. $g_{\nu} = 0 \mu_N$ and $g_{\pi} = 1 \mu_N$. In the other set, we take $g_{\nu} = 0 \mu_N$ and $g_{\pi} = 1.35 \mu_N$, which are the optimized parameters determined by fitting to the experimental data in this region.

Our nucleon pair subspace is constructed by $SD$ pairs of valence protons and neutrons, with respect to the doubly-closed shell nucleus $^{100}\text{Sn}$. We have also investigated the $SDGIK$-pair subspace and found that the $G, I, K$ pairs do not contribute significantly to the low-lying states of the nuclei $^{108,109}\text{Te}$ and $^{109}\text{I}$. As in Ref. [29], we use the BCS pairs as our $S$ pair. The $D$ pair is obtained by using the commutator $D^1 = \frac{1}{2}[Q, S^\dagger]$ [32].
FIG. 2: (Color online) Partial level schemes for low-lying positive-parity states in (a) $^{108}$Te, (b) $^{109}$I and (c) $^{109}$Te. The experimental data of $^{108,109}$Te and $^{109}$I are taken from Refs. [17] and [24], respectively.

FIG. 3: (Color online) Same as Fig. 2 but for low-lying $h_{11/2}$ bands of $^{109}$I and $^{109}$Te in comparison with the ground state band in $^{108}$Te, plotted relative to the bandheads.

It was shown in Ref. [35] that the NPA is an efficient truncation scheme of the shell model for nuclei $^{44,46}$Ca, $^{130,131}$Te and $^{132}$I by using phenomenological as well as realistic interactions. In order to prove the validity of the NPA even in the nuclei to be studied here we also perform the shell model as well as the NPA calculations and compared the results. In Fig. 1 we make comparisons of ground state bands for $^{104}$Te, $^{106}$Te and $^{108}$Te calculated by SD-pair approximation (denoted by $SD$), $SDGIK$-pair approximation (denoted by $SDGIK$), the calculation in the subspace constructed by all possible NPA pairs (denoted by All-pair) and the shell model (denoted by SM1) by taking the same phenomenological interactions in Eq (1). It is shown that the results of the $SD$ reasonably agree with those of the SM1 (especially for the $2^+_1$ state), indicating that the $SD$ pairs are very important in building up low-lying states. For the nucleus $^{108}$Te, our $SD$ NPA calculation noticeably overestimates the excitation energy of the yrast $6^+$ state in comparison with that of the shell model, due to the influence of other pairs, namely $G, I, K$. If all possible NPA pairs are taken into account (see Fig. 4(a) for $^{104}$Te), our results are equivalent to those of the SM1. Moreover, it is to be pointed out that the remarkable feature in Figs. 2 and 3 is that the $SD$-pair approximation agrees well with the experiment. This indicates that the $SD$-pair approximation with the phenomenological interactions is very well fitted to explain low-lying states in these nearly spherical nuclei, especially the energies and $E2$ transition properties of the $2^+_1$ state of concern (see Fig. 4). Tables II and III. A much more sophisticated treatment of the effective interaction may be necessary for calculations in
a larger space with more pairs or with in the shell-model framework in order to get a better agreement with experimental data.

III. CALCULATIONS AND DISCUSSIONS

Our calculated low-lying energy levels and electromagnetic transitions are presented in Figs. 2 and Tables III–IV. The experimental excitation energies of nuclei $^{108,109}\text{Te}$ and $^{109}\text{I}$ are taken from Refs. 17 and 22. The experimental electromagnetic transitions are taken from $^{108}\text{Te}$ 2, $^{109}\text{Te}$ 4 and $^{109}\text{I}$ 5. The experimental $B(E2, 9/2^+ \rightarrow 5/2^+)$ measurement in $^{109}\text{Te}$ is not available at present. We present their theoretical values in Fig. 4 for comparison. The shell model results (SM) are taken from Refs. 4, 13.

One sees in Figs. 2 and 3 that our calculated energies reproduce reasonably well the corresponding experimental values. The relative level schemes of some states in $^{108}\text{Te}$ and $^{109}\text{I}$ are close to each other, suggesting that the additional proton in $^{109}\text{I}$ might be weakly coupled to the even-even core $^{108}\text{Te}$ 2. That is, an unpaired proton in a single-\textit{j} orbit (\textit{πj}) might be coupled to the \textit{λ} state of $^{108}\text{Te}$ (\textit{108}\text{Te}(\textit{λ})) to induce the \textit{k} state of $^{109}\text{I}$, i.e. $|\langle \text{πj}\rangle \otimes (108\text{Te}(\lambda);k)\rangle$. The nucleus $^{109}\text{Te}$ shows a similar pattern.

To understand the structures of these states we analyze them within our collective nucleon-pair subspace.

TABLE II: The $B(E2, J_i \rightarrow J_f)$ values in units of $e^2\text{fm}^4$. The columns $\chi_\sigma$ ($\sigma = \pi, \nu$) show the reduced matrix elements of Eq. (2). The experimental data and shell model results are taken from a Ref. 2, b Ref. 3 and c Ref. 4.

| $J_i$ | $J_f$ | Expt. | NPA-1 | NPA-2 | SM | $\chi_\pi$ | $\chi_\nu$ |
|------|------|------|------|------|---|---------|---------|
| $^{108}\text{Te}$ | | | | | | |
| 2$^+$ | 0$^+$ | 780($^{+40}_{-80}$) | 733 | 680 | 3.429 | 5.009 |
| 4$^+$ | 2$^+$ | | 894 | | 1.633 | 4.215 |
| 11/2$^+$ | 7/2$^+$ | 710(90) | 762 | 1146 | 1.757 | 3.003 |
| 15/2$^+$ | 11/2$^+$ | | 1034 | | 1.625 | 3.724 |
| 15/2$^+$ | 11/2$^+$ | | 880 | | 1.986 | 2.753 |
| 19/2$^+$ | 15/2$^+$ | | 983 | | 1.473 | 3.603 |
| | | | | | | |
| | | | | | | |
| $^{109}\text{I}$ | | | | | | |
| 9/2$^+$ | 5/2$^+$ | 608(392) | 595 | 125 | 2.069 | 2.153 |
| 9/2$^+$ | 5/2$^+$ | | 31 | 683 | 0.443 | 0.550 |
| 9/2$^+$ | 5/2$^+$ | | 56 | 6 | 0.585 | 0.737 |
| 9/2$^+$ | 5/2$^+$ | | 0.46 | 407 | -0.063 | 0.231 |
| 13/2$^+$ | 9/2$^+$ | | 310 | 850 | 0.785 | 2.270 |
| 11/2$^+$ | 7/2$^+$ | | 583 | 810 | 1.804 | 2.254 |
| 15/2$^+$ | 11/2$^+$ | | 719 | | 1.420 | 3.016 |
| 15/2$^+$ | 11/2$^+$ | | 861 | | 1.810 | 2.941 |
| 19/2$^+$ | 15/2$^+$ | | 1005 | | 1.458 | 3.686 |

TABLE III: The $B(M1, J_i \rightarrow J_f)$ values in units of $10^{-3}\mu_N^2$. For the proton effective orbital gyromagnetic ratio, we adopt the value $g_{el} = 1\mu_N$ in “NPA-1” and $g_{el} = 1.35\mu_N$ in “NPA-2”. The experimental data and shell model results are taken from Ref. 4.

| $J_i$ | $J_f$ | Expt. | NPA-1 | NPA-2 | SM |
|------|------|------|------|------|---|
| 7/2$^+$ | 5/2$^+$ | | 3 | 2 | 151 |
| 7/2$^+$ | 5/2$^+$ | | 6 | 5 | 55 |
| 7/2$^+$ | 5/2$^+$ | | 4 | 10 | 2 |
| 7/2$^+$ | 5/2$^+$ | | 0.2 | 13 | 76 |
| 9/2$^+$ | 7/2$^+$ | | 6(1) | 1 | 10 | 162 |
| 9/2$^+$ | 7/2$^+$ | | 24 | 150 | 126 |
| 9/2$^+$ | 7/2$^+$ | | 16 | 15 | 0 |
| 9/2$^+$ | 7/2$^+$ | 137(92) | 57 | 127 | 102 |

TABLE IV: Same as Table III except for magnetic moments $\mu$ (in units of $\mu_N$) predicted in this work.

| $^{108}\text{Te}$ | $^{109}\text{I}$ | | | |
|------|------|------|------|
| 2$^+$ | 0.910 | +1.237 | 7/2$^+$ | +2.282 | +3.592 |
| 4$^+$ | 1.001 | +1.331 | 11/2$^+$ | +2.954 | +4.510 |
| 5/2$^+$ | 1.096 | -1.078 | 5/2$^+$ | +0.568 | +0.593 |
| 9/2$^+$ | +1.582 | +1.888 | 9/2$^+$ | +0.269 | +0.737 |
| 7/2$^+$ | +1.034 | +1.051 | 11/2$^+$ | +2.008 | +2.395 |
As discussed above, the resulting NPA wave function for the $k$ state, i.e. $|^{109}\text{I}(k)\rangle$, and $|^{109}\text{Te}(k)\rangle$, contains many components consisting of $S$ and $D$ pairs plus the unpaired nucleon which may occupy any single-$j$ orbit in Table V. To evaluate the probability that a weakly coupled state is included in the NPA wave function, we evaluate the overlaps $|^{109}\text{I}(k)\rangle\langle\sigma j| \otimes ^{108}\text{Te}(\lambda); k$ and $|^{109}\text{Te}(k)\rangle\langle\nu j| \otimes ^{108}\text{Te}(\lambda); k$. The results are given in Table V. One sees therein that for the states of interest, i.e. those in Figs. 2 and 3, the overlaps are indeed very large. This suggests that these states can be well represented by the weak coupling between a collective state in the even-even “core” and the unpaired nucleon in a single-$j$ orbit. This is also a strong indication of the vibrational-like character of nuclei in this region, where the ground as well as low-lying excited states behave like boson degrees of freedom, practically unperturbed by the presence of the odd nucleon.

The states of $^{109}\text{I}$ shown in Figs. 2(b) and 3(b) are clearly seen to arise from the coupling of the vibrational-like ground state band in $^{108}\text{Te}$ with the unpaired proton in the $\pi g_{7/2}$ (Fig. 2(b)) and $\pi h_{11/2}$ (Fig. 3(b)) orbit. This is indeed confirmed by the large wave function overlap between the corresponding states in Table V.

The analysis of the positive parity bands of $^{109}\text{Te}$ in Fig. 2(c) is more involved. In order to understand the relation between these states and their eventual (if any) weak coupling description, one needs to rely on the calculated overlaps of Table V. One sees that the states $5/2^+$ and $9/2^+_2$, forming the first band in Fig. 2(c), are built upon the coupling of the $\nu d_{5/2}$ orbit with the collective core states. Instead, the states $5/2^+_3$, $9/2^+_1$ and $13/2^+_1$, which form the second band in that Figure, are atypical, because they arise from the coupling of the $\nu g_{7/2}$ orbit with only the $2^+_1$ and $4^+_1$ states in $^{108}\text{Te}$. The third band is again a weak coupling band, because it arises as the coupling of the orbit $\nu g_{7/2}$ with all the states in $^{108}\text{Te}$. The negative parity band in Fig. 3(c) is also a typical weak coupling band, arising from the coupling of the $\nu h_{11/2}$ orbit with the core states. It is to be pointed out that our results largely agree with what was concluded in Refs. 25, 26, 36.

Electromagnetic transition is another sensitive probe of the calculated wave functions. Unfortunately the experimental observations in this mass region are still scarce and the corresponding errors are relatively large. This can be seen in Fig. 4 and Tables II-III where the available experimental $B(E2)$ and $B(M1)$ values as well as a shell model and our own NPA calculations are shown. In Table II we also list our reduced matrix elements $\chi \sigma$ ($\sigma = \pi, \nu$) (see Eq. 2). Our predicted magnetic dipole moments ($\mu$) of some low-lying states are presented in Table IV.

One sees in Fig. 4 and Tables II-III that our results agree quite well with available experimental data except for the $B(E2, 9/2^+_1 \rightarrow 5/2^+_2)$ value in $^{109}\text{Te}$. To explore this further, one might need new experimental data in addition to the available ones at present.

As seen in Fig. 4, the experimental $B(E2)$ transitions in the nuclei $^{108}\text{Te}$, $^{109}\text{I}$ and $^{109}\text{Te}$ are approximately the same. This is consistent with our previous results on the structure of these states, because it indicates that the additional nucleon of $^{109}\text{I}$ (or $^{109}\text{Te}$) is indeed weakly coupled to the even-even core for these states, as suggested.
in Refs. 4, 5.

One also sees in Fig. 4 that the shell model calculations reproduce well the $B(E2, 2^+_1 \rightarrow 0^+_1)$ value in $^{108}$Te, but overestimate the $B(E2, 11/2^+_1 \rightarrow 7/2^+_1)$ value in $^{109}$I, and fail to describe the $B(E2, 9/2^+_2 \rightarrow 5/2^+_1)$ value in $^{109}$Te. Moreover, in Table III one sees that the theoretical $B(E2, 9/2^+_2 \rightarrow 5/2^+_1)$ value of the SM, i.e. 12 e²fm⁴, is much smaller than the corresponding experimental data, i.e. 608(392) e²fm⁴. As this experimental value practically coincides with the SM one for the transition $(9/2^+_1) \rightarrow (5/2^+_1)$ (683 e²fm⁴), in Ref. 4 it was suggested that the ordering of the first two calculated excited $9/2^+$ states in $^{109}$Te are inverted. This is not the case in our present calculation.

It thus seems that, as pointed out in Ref. 4, the presence of the single decoupled valence proton affects the total measured $B(E2)$ strengths in a manner that is not currently well understood. To investigate this point further we perform several attempts, particularly to survey the sensitivity of the $B(E2)$ values upon the different terms of the interaction entering the theory. This is not a trivial task, because at the same time we require that all the other calculated physical quantities, which agreed well with available experimental data, should remain practically unchanged. We could do this very lengthy task because in our NPA truncated space the computing time needed to perform the calculations is relatively short. In this search we finally find that those requirements are fulfilled if one varies the residual proton-neutron quadrupole-quadrupole interaction ($\kappa_{\pi\nu}$) in a range from $-0.01$ to $-0.11$ MeV/$r_0^4$. Our results are shown in Fig. 5. There are two remarkable features in this Figure. First, our calculation reproduces the available experimental data in all the nuclei analyzed here, i.e. $^{108}$Te, $^{109}$Te and $^{109}$I, by using a strength $\kappa_{\pi\nu} \sim -0.05$ MeV/$r_0^4$. Instead, the shell model results 4, 5, which agree with experiment only in the nucleus $^{108}$Te, are reproduced by using $\kappa_{\pi\nu} \sim -0.09$ MeV/$r_0^4$. This suggests that, as speculated in Ref. 4, 5, the quadrupole-quadrupole correlation in the realistic shell model interaction might be too strong for nuclei in this region. It also indicates that $\kappa_{\pi\nu}$ for these three nuclei is not as strong as the values predicted by empirical formulas (see Appendix B in Ref. 27), i.e. $\kappa_{\pi\nu} = -0.08 \sim -0.10$ MeV/$r_0^4$. The second striking feature seen in Fig. 5 is that the $B(E2)$ values in $^{109}$I and $^{109}$Te are very sensitive to $\kappa_{\pi\nu}$. Thus, in panel (b) the transition $^{109}$I($11/2^+_1 \rightarrow 7/2^+_1$) increases rapidly, from about 500 to more than 1200 e²fm⁴, in the range of the Figure. Even more striking is what panel (c) shows for the transition $^{109}$Te($9/2^+_2 \rightarrow 5/2^+_1$), for which the $B(E2)$ value first remains rather constant at about 600 e²fm⁴ to suddenly, at $\kappa_{\pi\nu} = -0.07$ MeV/$r_0^4$, decrease to reach a vanishing value at $\kappa_{\pi\nu} = -0.1$ MeV/$r_0^4$. Finally, in panel (d) the transition $^{109}$Te($9/2^+_2 \rightarrow 5/2^+_1$) increases from zero to about 300 e²fm⁴ when $\kappa_{\pi\nu}$ decreases from $-0.07$ to $-0.11$ MeV/$r_0^4$. This may explain why in the previous shell-model calculation 4, the two $9/2^+ \rightarrow 5/2^+$ transitions are calculated to be inverted.

This behavior of the $B(E2)$ values in $^{109}$Te, which follows a rather smooth curve as a function of the proton-neutron interaction strength, shows a deviation at $\kappa_{\pi\nu} \sim -0.07$ MeV/$r_0^4$ in panels (c) and (d). There is a peculiar-
ity here, namely that the state $^{109}\text{Te}(5/2_1^+)$ is common in the transitions seen in those panels. We analyze the evolution of the structures of the states involved in those transitions as a function of $\kappa_{\pi\nu}$, and found that the dominant NPA configuration of the state $^{109}\text{Te}(5/2_1^+)$ changes from $|(d_{5/2})_0^5 S^0_\nu S^0_\pi\rangle$ to $|(g_{7/2})_0^9 D^0_\nu S^0_\pi\rangle$ when $\kappa_{\pi\nu}$ varies from $-0.07$ to $-0.09\text{ MeV}/r^4_0$. This change does not occur in the states $^{109}\text{Te}(9/2^-_1)$ and $^{109}\text{Te}(9/2^+_2)$.

We complete the analysis of the weak coupling wave functions by evaluating, as a function of $\kappa_{\pi\nu}$, their overlaps with the states $^{109}\text{I}(7/2_1^+)$, $^{109}\text{I}(11/2_1^+)$, $^{109}\text{Te}(5/2_1^+)$ and $^{109}\text{Te}(9/2^+_2)$. These overlaps are shown in Fig. 8. One sees that the overlaps of $^{109}\text{I}$ in Fig. 8(a) decrease with increasing $-\kappa_{\pi\nu}$, and the most rapid changes occur for the state $7/2_1^+$. Instead, the weak coupling description of the state $^{109}\text{Te}(9/2^+_2)$ in Fig. 8(b), is practically independent of $\kappa_{\pi\nu}$. But the most striking feature in this Figure is the very sharp change of the overlap corresponding to the state $5/2^+_1$ when $\kappa_{\pi\nu}$ varies from $-0.07$ to $-0.09\text{ MeV}/r^4_0$. As mentioned above, this abrupt change is a consequence of the evolution of the NPA wave function, which at that value ($-0.09\text{ MeV}/r^4_0$) of the strength the NPA configuration $|(g_{7/2})_0^9 D^0_\nu S^0_\pi\rangle$ becomes dominant. This analysis shows that the value of the proton-neutron interaction strength $\kappa_{\pi\nu}$ is weak. Perhaps most important is that with the weak strength, the theoretical $B(E2, 11/2^+_1 \rightarrow 7/2^+_1)$ value in $^{109}\text{I}$ and $B(E2, 9/2^+_2 \rightarrow 5/2^+_1)$ value in $^{109}\text{Te}$ acquire the experimental value (within the experimental error) of 762 and 595 $e^2\text{fm}^4$, as shown in Fig. 8 and Table III.

IV. SUMMARY

In this paper we have calculated the low-lying level schemes and electromagnetic transition properties of the nuclei $^{108}\text{Te}$, $^{109}\text{Te}$ and $^{109}\text{I}$ within the nucleon pair approximation (NPA) of the shell model. We extract from the NPA wave functions the probabilities that the low-lying bands in $^{109}\text{I}$ and $^{109}\text{Te}$ could be interpreted in terms of the weak coupling between the collective even-even core $^{108}\text{Te}$ and the unpaired particle. We thus find that is indeed the case, as shown in Table VIII. This is consistent with the conjectures put forward in Refs. 1, 5.

We probe the weak coupling pictures of the states by investigating the corresponding $B(E2)$ values, and find that the calculated electromagnetic transitions to the ground states of $^{109}\text{Te}$ and $^{109}\text{I}$ are very sensitive to the residual quadrupole-quadrupole proton-neutron interaction $\kappa_{\pi\nu}$. By comparing with experimental data, we conclude that the proton-neutron interaction is weak and that the states $7/2_1^+$ and $11/2_1^+$ in $^{109}\text{I}$ as well as the states $5/2^+_1$ and $9/2^+_2$ in $^{109}\text{Te}$ are well described by the weak coupling scheme.

We tabulate our calculated $B(E2)$, $B(M1)$ and $\mu$ values for some low-lying states. Except for the transition $B(E2, 9/2^+_1 \rightarrow 5/2^+_1)$, our results agree very well with available experimental data, thus confirming the experimental order of the first two excited $9/2^+$ states in $^{109}\text{Te}$.

The overall agreement between the calculations and experiments regarding the $B(E2)$ and $B(M1)$ values, as well as the energy levels, indicates that the NPA provides an appropriate theoretical framework to describe low-lying states of these nuclei. Experimental data are relatively scarce in this region. We therefore believe that our predictions (e.g., $E2$ and $M1$ transition rates, and magnetic dipole moments) are useful for future studies of these nuclei.

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