Spectral Approach in Vibrations of Overhead Transmission Lines

Maciej Dutkiewicz 1, Marcela R. Machado 2

1 Faculty of Civil, Environmental Engineering and Architecture, University of Science and Technology, 85-796 Bydgoszcz, Poland
2 Department of Mechanical Engineering, University of Brasilia, 70910-900, Brasilia, Brazil
macdut@utp.edu.pl

Abstract. In the paper, the description of the type of vibrations of overhead transmission lines and theoretical background of spectral element method is presented. The main goal of this paper is to develop a model of the transmission line via spectral element method. The methodology of analysis is presented. Due to the accuracy of the results and the efficiency of the method, it is recommended to analyse the vibrations of the systems in various configurations of boundary and initial conditions. Numerical analysis investigates the natural frequency of the model due to different parameters, such as a tension force and the area of the cross-section of the conductor.

1. Introduction

That is the huge need to design and construct the overhead transmission lines with the respect of a wide range of load cases acting on these slender structures. There is some theoretical and experimental research on overhead transmission lines [1-3]. The most important load is the wind. This load cause three main types of conductor vibrations: Aeolian vibrations with a frequency from 3 to 150 Hz and amplitudes lower than the conductor diameter, galloping with a frequency from 0.1 to 2 Hz and amplitudes from ±0.1 to 1 of conductor sag, wake induced vibrations with a frequency from 0.15 to 10 Hz and amplitudes from 0.5 to 80 times the conductor diameter [4-6]. The majority of common wind-induced vibrations are Aeolian vibrations. These vibrations are generated as a result of vortices shed in the conductor wake under the sustained wind of low speed from 1 to 7 m/s – they occur mainly in the vertical plane. Vibrations of conductors, both single and in a bundle, form standing waves with forced nodes and intermediate nodes located along the span at intervals depending on the frequency of free vibrations. When the conductor wind flow is laminar, alternately shedding vortices are formed in two points of the suction zone and make the conductor move perpendicularly towards the wind direction. The alternate shedding of vortices is regular. As a result, a so-called Karman vortex street is formed. When the frequency of the shedding of vortices is approximately equal to one of the frequencies of free vibrations of a conductor, a ‘lock-in’ phenomenon occurs. During this frequency synchronisation, the conductor is in the resonance state. Aeolian vibrations occur on single conductors and conductors in a bundle. Although these vibrations are hardly noticeable due to low amplitude values (lower than the conductor diameter), they are very important, since they can lead to fatigue destruction of a conductor in points of high stress concentrations. Galloping is an aero elastic self-excitation phenomenon characterised by low frequencies and high amplitudes, and it refers to single conductors and conductors in a bundle, with one or two loops of standing and running waves, or their combination in a conductor
Standing waves may have one or more loops (up to 10) over the span length. However, a small number of loops is predominant.

This is very important to develop the easy and fast methodology for design, taking into consideration all loads and uncertainties. Nowadays we see the wide development of new materials and solutions to raise the conductivity but at the same time, we observe that conductors' durability is changed and require the permanent update analysis. Spectral element method seems to be such fast and easy tool which fulfils these requirements.

The Spectral Element Method (SEM) is a meshing method similar to Finite Element Method (FEM), where the approximated element shape functions are substituted by exact dynamic shape functions obtained from the exact solution of governing differential equations. Therefore, a single element is sufficient to model any continuous and uniform part of the structure. This feature reduces significantly the number of elements required in the structure model and improves the accuracy of the dynamic system solution. At the same time, there are some drawbacks like the unavailability of exact wave solutions for most complex and 2D and 3D structures. In these cases, approximated spectral element modelling can be used and may still provide very accurate solutions. Although SEM ensures exact frequency-domain it is not true for time-domain solutions, because errors due to aliasing or leakage are inevitable in the use of the inverse-DFT process. Thus, special attention in obtaining the inverse-DFT is required. In recent years some researchers were performed with use of SEM. The extensive study of the fundamentals and a variety of new applications such as composite laminated, periodic lattice, damage detection was presented in [7]. The wave behaviour in composites and inhomogeneous media are studied in [8]. Studies related to structural damage detection have been developed in [9]. Other works using wave propagation and SEM to detect damage under the presence of structural randomness can also be found in references [10-12].

In the paper, the description of the type of vibrations of overhead transmission lines and theoretical background of spectral element method was presented. The aim of this paper is to develop a model of the transmission line via spectral element method. Numerical analysis investigates the natural frequency of the model due to different parameters, such as a tension force and the area of the cross-section of the conductor. The model is validated by comparing with the analytical beam solution.

2. Theoretical assumptions of the spectral analysis

Considering a simplified cable model, as shown in Figure 1, the governing differential equation for the undamped free vibration is given by Clough and Yu [13-14]:

\[
EI \frac{\partial^4 v}{\partial x^4} - T \frac{\partial^2 y}{\partial x^2} + \rho A \frac{\partial^2 v(x, t)}{\partial t^2} = 0
\]

For a simply supported beam under axial force the natural frequency can be written as [15]:

![Figure 1. Analysed model](image-url)
\[
\omega_n = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{\rho A}} \left( n^4 + \frac{n^2 T L^2}{\pi^2 EI} \right)^{\frac{1}{2}}, \quad n = 1, 2, \ldots
\]

where \( \rho A \) is mass per unit length, \( EI \) the uniform bending rigidity, \( L \) cable length, \( T \) is tension force, and \( v(x, t) \) is the cable displacement as a function of the position \( x \) and time \( t \).

The undamped Euler-Bernoulli beam equation of motion subjected to axial force and under bending vibration is given by equation (1). Figure 2 shows an elastic two-node element with a uniform rectangular cross-section subjected to an axial force, where the properties are assumed to be deterministic variables. A structural internal damping is introduced into the beam formulation by adding into Young’s modulus weighted by a complex damping factor \( i \eta i = \sqrt{-1} \), \( \eta \) is the hysteretic structural loss factor, to obtain \( E = E (1 + i \eta) \).

![Figure 2. Two-node spectral element](image)

By considering a constant coefficient, a displacement solution can be assumed of the form:

\[
v(x, t) = v_0 e^{-i(kt + \omega t)}
\]

where \( v_0 \) is an amplitude, \( \omega \) is the frequency and \( k \) is the wavenumber. Substituting it at Eq. (1), the dispersion equation is given by

\[
k^4 EI + k^2 T - \omega^2 \rho A \omega = 0
\]

There are two distinct wave modes in the positive direction \( (k^2) \), which is positive-going waves with wave numbers given as

\[
k_1 = \sqrt{-\frac{T}{2EI} + \sqrt{\left(\frac{T}{2EI}\right)^2 + \frac{\rho A \omega^2}{EI}}}
\]

\[
k_2 = -\sqrt{-\frac{T}{2EI} - \sqrt{\left(\frac{T}{2EI}\right)^2 + \frac{\rho A \omega^2}{EI}}}
\]

The general solution for the Euler-Bernoulli beam spectral element subjected to axial load of length \( L \), can be expressed in the form
\[ v(x, \omega) = a_1 e^{-ikx} + a_2 e^{-kx} + a_3 e^{-ik(L-x)} + a_4 e^{-k(L-x)} = s(x, \omega) a \tag{7} \]

where

\[ s(x, \omega) = \left\{ e^{-ikx}, e^{-kx}, e^{-ik(L-x)}, e^{-k(L-x)} \right\} \tag{8} \]

\[ a(x, \omega) = \{a_1, a_2, a_3, a_4\}^T \tag{9} \]

The spectral nodal displacements and slopes of the beam element are related to the displacement field at node 1 (x=0) and node 2 (x=L), by

\[
\begin{bmatrix}
    v_1 \\
    \phi_2 \\
    v_2 \\
    \phi_2
\end{bmatrix} =
\begin{bmatrix}
    v(0) \\
    v'(0) \\
    v(L) \\
    v'(L)
\end{bmatrix}
\tag{10}
\]

By substituting equation (7) into the right-hand side of equation (10) and written in a matrix form gives

\[
\begin{bmatrix}
    s(0, \omega) \\
    s'(0, \omega) \\
    s(L, \omega) \\
    s'(L, \omega)
\end{bmatrix} a = G(\omega) a
\tag{11}
\]

Where

\[
G(\omega) = d = \begin{bmatrix}
    1 & 1 & e^{-ikL} & e - kL \\
    -ik & -k & ie^{-ikL} & e - kL \\
    e - ikL & e - kL & 1 & 1 \\
    -ie - ikLk & e - kLk & ik & k
\end{bmatrix}
\tag{12}
\]

The frequency-dependent displacement within an element is interpolated from the nodal displacement vector \( d \), by eliminating the constant vector \( a \) from equation (7) and using equation (11) it is expressed as

\[
v(x, \omega) = g(x, \omega) d
\tag{13}\]

where the shape function is

\[
g(x, \omega) = s(x, \omega)G^{-1}(\omega) = s(x, \omega)\Gamma(\omega)
\tag{14}\]

The dynamic stiffness matrix for the spectral beam element under axial tension can be determined as:
\[ S^e(\omega) = EI \left[ \int_0^L \mathbf{g}''(x)^T \mathbf{g}''(x) \, dx - k^4 \int_0^L \mathbf{g}(x)^T \mathbf{g}(x) \, dx \right] \]  

where \( \mathbf{g}'' \) express the spatial partial derivative. By solving the integral the dynamic stiffness matrix is

\[ S^e(\omega) = \frac{EI}{\Delta} \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix} \]  

where \( \Delta = \cos(kL)\cosh(kL) - 1 \) and the components of element matrix (eq.16) are given as

\begin{align*}
    s_{11} &= -k^3(\cos(kL)\sinh(kL) + \sin(kL)\cosh(kL)) \\
    s_{12} &= -k^2\sin(kL)\sinh(kL) \\
    s_{13} &= k^3(\sin(kL) + \sinh(kL)) \\
    s_{14} &= k^2(\cos(kL) - \cosh(kL)) \\
    s_{22} &= k(\cos(kL)\sinh(kL) - \sin(kL)\cosh(kL)) \\
    s_{23} &= k^2(\cosh(kL) - \cos(kL)) \\
    s_{24} &= k(\sin(kL) - \sinh(kL)) \\
    s_{33} &= -k^3(\cos(kL)\sinh(kL) + \sin(kL)\cosh(kL)) \\
    s_{34} &= k^2\sin(kL)\sinh(kL) \\
    s_{44} &= k(\cos(kL)\sinh(kL) - \sin(kL)\cosh(kL))
\end{align*}  

As far as structure beam is uniform without any sources of discontinuity, it can be represented by a single spectral element with very accurate solutions [16]. However, if there exist sources of discontinuity such as the point loads the beam should be spatially discretized into spectral elements. Analogous to Finite Element Method (FEM) [17], the spectral elements can be assembled to form a global structure matrix system [7].

3. Numerical analysis

On the basis of the formulas presented in equations (16-18), it is possible to obtain the Frequency Response Function (FRF) of the overhead transmission conductor. For the numerical tests, it is assumed a pinned-pinned boundary condition. The beam structures are made with aluminium whose mechanical properties are: \( E = 74 \text{GPa} \), \( \rho = 2700 \text{ kg/m}^3 \), \( \eta = 0.01 \). The geometry properties are: length of \( L = 100 \) m and circular section area of \( A = [50, 70, 120, 240, 300] \text{ mm}^2 \). With regard to the excitation and measured position, it is at position \( L_1 = 33 \) m from the node 1, the cable is excited by an impulsive unitary force.
Two values of tension force of 1 kN, 100 kN and 1000 kN were assumed in the tests. In all tests, two spectral elements on the span were used.

In order to verify the proposed model, the 10 firsts resonance picks of the FRF are compared with the natural frequency analytical formulation (eq. 2). Once, the structure is excited with a unitary force we can expect that the resonance picks are close to the natural frequency of the system. Figure 3 shows the FRF measured at point \( L_1 = 33 \) m from the node. The conductor has the circular Area = 50 mm\(^2\) and Tension force of \( T = 10 \) kN. A zoom images at 0 to 1.7 Hz frequency band to better visualization of the firsts resonance pick. Table 1 summarized the 10-first's analytical natural frequency and resonance pick obtained with the SEM model. By comparing the results, analytically and numerically calculated frequencies were closed with exception of the 10\(^{th}\). In the numerical model, the 10\(^{th}\) mode has not excited by this chosen point excitation, while that the 11\(^{th}\) is the same for both methods.

**Figure 3.** FRF measured at point \( L_1 \), for a conductor of Area=50 mm\(^2\) and Tension force of \( T=10 kN \) (LHS). Zoom image in 0 to 1.7 Hz frequency band. (RHS)

| \( \omega_n \) (Hz) | 0.13 | 0.2722 | 0.4085 | 0.5450 | 0.6816 | 0.8186 | 0.9560 | 1.0937 | 1.2319 | 1.3707 | 1.51 |
|------------------|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
| Resonance        | 0.14 | 0.27   | 0.41   | 0.55   | 0.68   | 0.82   | 0.96   | 1.09   | 1.23   |        | 1.51 |

Table 1. Comparison between 11 firsts analytical natural frequency with resonance pick obtained with the SEM model
Figure 4. FRF of the conductor with circular area=50mm² and tension force of:
(a) T=1kN; (b) T=100kN; (c) T=1000kN

Figure 5. FRF of the conductor tension force equals to 1000kN and circular area of: (a) A=70mm²; (b) A=120mm²; (c) A=240mm²; (d) A=300mm².
Figure 6. FRF of the conductor with tension force equals to 1000kN, circular area of 50mm², and excitation-measured position in: (a) L₁ = 1m; (b) L₁ = 2m; (c) L₁ = 5m; (d) L₁ = 10m.

Figure 4 shows the FRFs of the conductor with circular area of 50mm², excited and measured at the same point (L₁) and tension force of: (a) T=1kN; (b) T=100kN; (c) T=1000kN. The number of resonance picks change decrease as the conductor cable tension force increasing, it because as the axial tension increase more rigidity the conductor became which increasing the first resonance picks values in the frequency axis. Figure (5) shows is the FRFs obtained with a tension force equals to 1000kN and circular area of: (a) A=70mm²; (b) A=120mm²; (c) A=240mm²; (d) A=300mm². Again, as the conductor increase the circular area and maintain the tension force less rigidity the cable became and the resonance pick number increase in the frequency band without big changes on the FRF shape. Figure (6) presents the FRFs for a tension force equals to 1000kN, circular area of 50mm², and excitation-measured position in: (a) L₁ = 1m; (b) L₁ = 2m; (c) L₁ = 5m; (d) L₁ = 10m. Different of the last tests some resonance picks have omitted in case (c) and (d) in reason of the excitation point was not able to excite whole modes of the structure. Therefore, the numerical model of overhead transmission line via SEM presented great efficiency and accuracy in this case of study.

4. Conclusions
The paper presents numerical model of overhead transmission line via Spectral Element Method. In order to validate the proposed model, the resonance response (picks) obtained with the SEM is compared with the natural frequency of overhead transmission calculated analytically. The accuracy of the results and the efficiency of the method is underline for various configurations of boundary and initial
conditions. Numerical analysis investigates the frequency response function of the structure for different configurations of cable circular section and applied tension force.

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