Birth and growth of nonlinear massive gravity and it’s transition to nonlinear electrodynamics in a system of Mp-branes

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Recently, an interesting mechanism [Phys.Rev.Lett.106:231101,2011] has been proposed which produces all nonlinear terms in massive gravity to all orders . In this work, we reproduce these results in M-theory and consider the process of birth and growth of nonlinear gravity during the construction of Mp-branes. It has been shown that Mp brane are built up of p- M1-branes which each of them are connected to M1-branes of other Mp-brane through a wormhole. In this model, by increasing the number of dimensions, the number of nonlinear terms in relevant action of branes enhances and some theories like lovelock and nonlinear gravity are raised. By compacting M-branes, graviton fields in nonlinear gravity converts to photon fields and thus nonlinear electrodynamics are produced.

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I. INTRODUCTION

A recent interesting idea [1] has been proposed recently which suggest that a four-dimensional non-linear theory of massive gravity, which is a ghost-free in the decoupling limit to all orders, could include explicitly all nonlinear terms of an effective field theory of a massive gravity. There have been implications with different aspects such as cosmological implications of this idea have been studied as a proxy which embodied key features of the non-linear covariant model [2]. In the same context of nonlinear massive gravity, another scenario investigated open Friedmann-Robertson-Walker (FRW) universes obtained by arbitrary matter source and it has been derived three independent branches of solutions. One of them does not allow for any nontrivial FRW cosmologies and the other two branches allow for general open FRW universes governed by the Friedmann equation with the matter source [3]. Another approaches have tried to understand the landscape of vacua in nonlinear massive gravity and considered tunneling between each pair of adjacent vacua. Several aspects are then studied such as the Hawking-Moss (HM) instanton that is located at a local maximum of the potential, and then investigated the dependence of the tunneling rate on the parameters of this gravity [4]. In another approach, it has been shown that all homogeneous and isotropic solutions in nonlinear massive gravity are unstable. It has been shown that there is at least one ghost that cannot be removed from the low energy effective theory [5]. In another aspect, nonlinear massive gravity has been suggested which includes F(R) modifications and inherited the benefits of previous model. This theory is free of the ghost due to the existence of a Hamiltonian constraint accompanied by a nontrivial secondary one [6].

Now, a natural question arises on what is the origin of nonlinear gravity. We try to answer this question in M-theory. In previous studies, it has been shown that Big Bang may be removed in M-theory and replaced by k fundamental strings[7–14]. In this model, k fundamental strings decay to k pairs of M0-branes. Then, these branes join to each other and form a system of M3-brane, anti-M3-brane in additional to a wormhole. This system is named a BIon. Our universe is located on one of these M3-branes and interact with other universe via a wormhole. This wormhole is a channel for flowing energy from another universe into our universe and hence occurring an inflation epoch [7]. In this research, we will extend these calculations and show that by increasing dimensions of branes, they can have more interactions with other branes and more wormholes are formed between them. These wormholes are originated from graviton fields in the relevant action of branes and are the main cause of emergence of nonlinear gravity in four dimensional universe. If these branes are compacted, some graviton fields transit to photon fields and nonlinear electrodynamics are produced.

The outline of the paper is as follows. In section [11], we show that by adding dimensions of branes, more graviton fields are produced which leads to formation of wormholes. These wormholes are the main cause of generation of nonlinear gravity. In section [11], we will argue that by compacting branes, gravitons change to photons and nonlinear electrodynamics is produced. The last section is devoted to summary and conclusion.

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II. EMERGENCE OF NONLINEAR GRAVITY IN AN MP-BRANE

Previously, it has been shown that all branes can be constructed from M0-branes [7]. These branes have no gauge fields and only scalars interact with them. By adding a dimension to the brane, the first gauge field is appeared and attached to M1. This field plays the role of graviton and produces linear gravity. Increasing dimensions, more graviton fields stick to brane and cause to generation of nonlinear gravity. This gravity leads to emergence of a large number of wormholes which connect a brane to other branes. Branes and wormholes in this system form a complicated system of BIons. To begin, we introduce the Lagrangian of nonlinear gravity [1, 2]:

$$L = \frac{M_0^2}{2} \sqrt{-g}(R - \frac{m_g^2}{4} U(g, H))$$

where $U$ is the gravitational potential, $m_g$ is the graviton mass, $H_{\mu\nu}$ is constructed in terms of the metric $g_{\mu\nu}$ and the four Stuckelberg fields $\Phi^\alpha$ by $H_{\mu\nu} = g_{\mu\nu} - \eta_{ab}\partial_\mu \Phi^a \partial_\nu \Phi^b$. The most generic potential, $U$, has the following form [2]

$$U = -4(U_2 + \alpha_3 U_3 + \alpha_4 U_4),$$

where $\alpha_{3,4}$ are two free parameters and

$$U_2 = -\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{\nu\beta\rho\sigma} k^\mu_3 k^\rho_3 = [\kappa]^2 - [\kappa]^2,$$

$$U_3 = -\varepsilon_{\mu\nu\rho\sigma} \varepsilon^{\nu\beta\rho\sigma} k^\mu_3 k^\rho_3 = [\kappa]^3 - 3[\kappa]^2[\kappa] + 2[\kappa]^3,$$

$$U_4 = -\varepsilon_{\mu\nu\rho\sigma} \varepsilon^{\nu\beta\rho\sigma} k^\mu_3 k^\rho_3 = [\kappa]^4 - 6[\kappa]^2[\kappa]^2 + 8[\kappa]^3[\kappa] + 3[\kappa]^2 - 6[\kappa]^4,$$

where

$$k^\mu_\nu = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H_{\nu}^\mu},$$

$$H_{\mu\nu} = h_{\mu\nu} + 2\Pi_{\mu\nu} - \eta^{\alpha\beta}\Pi_{\alpha\mu}\Pi_{\beta\nu},$$

$$\Phi^a = x^a - \eta^{\mu\nu}\partial_\mu \pi,$$

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi.$$

We can show this Lagrangian can be extracted from relevant actions of M-branes in a BIonic system. In fact, only one graviton field couples to M1 and produces a linear gravity. Also, this field generates a wormhole which connect M1-brane to other branes. By joining two M1-branes to each other, one M2-brane is constructed which two gravitons attach to it. These gravitons leads to first signature of nonlinear massive gravity and appearance of two wormholes. By joining M1-branes, dimension of brane increases and more gravitons are produced and cause to the emergence of a lot of wormholes and nonlinear gravity. The action of M1 can be written as [7, 13, 14]:

$$S = -T_{M1}\int d^2\sigma \, STr \left( -\det(P_{mnl}(E_{mnl} + E_{mic}(Q^{-1} - \delta)^{ik}E_{kl}n) + \lambda F_{mnl})\right)^{1/2}$$

where

$$E_{mnl}^{\alpha,\beta,\gamma} = C_{mnl}^{\alpha,\beta,\gamma} + B_{mnl}^{\alpha,\beta,\gamma}, \quad Q = Q_i^i, \quad Q^i_{j,k} = \delta^i_{j,k} + i\lambda[X^i_{\alpha} T^\alpha, X^k_{\beta} T^\beta, X^k_{\gamma} T^\gamma]E_{k'l}^{\alpha,\beta,\gamma},$$

$$F_{abc} = \partial_\alpha A_{bc} - \partial_\beta A_{ca} + \partial_\gamma A_{ab}.$$
\[ [T^\alpha, T^\beta, T^\gamma] = f^{\alpha\beta\gamma}_T T^\eta \]
\[ [X^M, X^N, X^L] = [X^M T^\alpha, X^N T^\beta, X^L T^\gamma] \]  \hspace{1cm} (7)

\[ \lambda = 2\pi l_s^2, \quad G_{mn} = g_{mn}\delta_n^\eta + \partial_m X^\eta \partial_n X^\eta, \quad X^i \] are scalar fields of mass dimension. Here \( a, b, 0, 1, \ldots, p \) are the world-volume indices of the Mp-branes, \( i, j, k = p + 1, \ldots, 9 \) are indices of the transverse space, and \( m, n \) are the ten-dimensional spacetime indices. Also, \( T_{M}\phi = \frac{1}{g_s(2\pi l_s)^{p+1}} \) is the tension ofMp-brane, \( l_s \) is the string length and \( g_s \) is the string coupling. This action has been proposed for the first time in \([7, 13, 14]\) for interactions with three \( \mathfrak{li} \)-algebra in M-theory and is very different of usual action \([15, 19]\) in string theory which obey of two \( \mathfrak{li} \)-algebra.

Scalars and two form gauge fields should satisfy following relations \([7, 13, 14, 20–23]\):

\[ \langle [X^a, X^b, X^i], [X^a, X^b, X^i] \rangle = \frac{1}{2} \epsilon^{abc} \epsilon^{abd} (\partial_a X^d)(\partial_a X^d) \langle [T^\alpha, T^\beta] \rangle = \frac{1}{2} (\partial_a X^i, \partial_a X^i) \]

\[ \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle = (F^{abc})_{\alpha\beta\gamma}(T^{\alpha}, T^{\beta}) \langle [T^\alpha, T^\beta, T^\gamma] \rangle \]

\[ (F^{abc})_{\alpha\beta\gamma}(T^{\alpha}, T^{\beta}) \langle [T^\alpha, T^\beta, T^\gamma] \rangle = (F^{abc})_{\alpha\beta\gamma}(T^{\alpha}, T^{\beta}) \langle [T^\alpha, T^\beta, T^\gamma] \rangle = (F^{abc}, F^{abc}) \]

\[ i, j = p + 1, \ldots, 10 \quad a, b = 0, 1, \ldots, p \quad m, n = 0, \ldots, 10 \]  \hspace{1cm} (8)

In this model, two form gauge field plays the role of tensor state of graviton and we can replace it by graviton and obtain following results:

\[ A^{ab} = g^{ab} = \eta^{ab} \quad \text{and} \quad a, b, c = \mu, \nu, \lambda \Rightarrow \]

\[ F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab} = 2(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}) = 2\Gamma_{\mu\nu\lambda} \]

\[ \langle [X^\rho, X^\sigma, X^\lambda], [X^\lambda, X^\mu, X^\nu] \rangle = \]

\[ [X_\nu, [X^\rho, X^\sigma, X^\lambda]] - [X_\mu, [X^\rho, X^\sigma, X^\lambda]] + \]

\[ [X^\rho, X_\lambda, X_\nu][X^\lambda, X_\mu, X_\nu] - [X^\rho, X_\lambda, X_\mu][X^\lambda, X_\nu, X_\mu] = \]

\[ \partial_\rho \Gamma^\mu_{\lambda\nu} - \partial_\mu \Gamma^\rho_{\lambda\nu} + \Gamma^k_{\lambda\mu} \Gamma^\rho_{\nu k} - \Gamma^k_{\mu\rho} \Gamma^\lambda_{\nu k} = R^\rho_{\sigma\mu\nu} \]  \hspace{1cm} (9)

These equations show that curvature tensor easily can be obtained from the noncommutative relations in M-theory. In fact, by adding dimensions to branes and growing gauge fields on them, the first signature of gravity can be detected. Previously in (Physics Letters B 748 (2015) 328333, Eur.Phys.J. C76 (2016) no.5, 231.), we have proposed a new model in M-theory which allows for constructing all Mp-branes from M1 and M0-branes. In this published model, we have shown that for M0-branes, there is no gauge field and only scalars have attached to it, however by joining M0-branes and formation of M1-branes, gauge fields emerges on it. These M1-branes are linked to anti-M1-branes and form a new system name BIon. For this system metric can be constructed from metrics of two M1-BIon as follows:

\[ \text{Metric of BIon = (Metric M1)}_1 \times \text{(Metric M1)}_2 - \text{(Metric M1)}_2 \times \text{(Metric M1)}_1 \]

Thus metric of BIon can be antisymmetric. On the other hand, tensor mode of graviton has a direct relation with metric of BIon and can be anti-symmetric. Also, we have shown that M-theory is a more complete version of string theory. Usual string theory is a ten dimensional theory and includes several versions like closed string theory and open string theory which each of them considers the universe via itself point of view. However M-theory is an eleven dimensional space-time theory which is more complete than string theory and has many differences with it. In this theory, two form gauge fields have the main role in phenomenological events between branes. In this model, we have argued that scalars in M-theory which move between branes play the role of scalar mode of graviton and are similar to closed strings that play the role of graviton. Also, two form gauge fields play the role of tensor mode of graviton. Using equation (5), we can obtain following relations:

\[ \text{det}(Z) = \delta^{a_1 a_2 \ldots a_n}_{b_1 b_2 \ldots b_n} Z_{a_1}^{b_1} \ldots Z_{a_n}^{b_n} \quad a, b, c = \mu, \nu, \lambda \]

\[ Z_{a_n}^{b_n} = \delta^{b_n}_{a_n} P^{mnl}(E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk} E_{klm} + \lambda E_{mnl}) \]

\[ \text{det}(Z) = \text{det}(P^{mnl}(E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk} E_{klm})) + \lambda^2 \text{det}(F) \]  \hspace{1cm} (10)
This equation helps us to calculate the relevant terms of determinant in action (5) separately. By substituting equations (8) and (9) into above determinants we obtain
\[
\det(F) = \delta^{\mu\nu}_{\rho\sigma} \langle F^{\rho\sigma}_\lambda, F^{\lambda}_{\mu\nu} \rangle = \delta^{\mu\nu}_{\rho\sigma} R^{\rho\sigma}_{\mu\nu}
\] (11)

\[
\det(P^{mn}\{E_{mn} + E_{mij}(Q^{-1} - \delta^{ijk} E_{kln})\}) =
\delta^{\mu\nu}_{\rho\sigma} [g^\rho_\mu g^\nu_\sigma + g^\rho_\nu g^\sigma_\mu (\partial^\mu X^i, \partial^\nu X^j) + ...] +
\delta^{\mu\nu}_{\rho\sigma} [g^\rho_\mu g^\nu_\sigma + g^\rho_\nu g^\sigma_\mu (\partial^\mu X^i, \partial^\nu X^j) + ...]
\left[ (\lambda)^2 \det([X^i_\beta T^\alpha, X^j_\beta T^\beta, X^k_\gamma T^\gamma]) \right]
\]

\[
\delta^{\mu\nu}_{\rho\sigma} \kappa^\rho_\mu \kappa^\nu_\sigma (1 + \frac{1}{m_g^2}) =
\delta^{\mu\nu}_{\rho\sigma} \kappa^\rho_\mu \kappa^\nu_\sigma (1 + \frac{1}{m_g^2})
\] (12)

where \(m_g^2 = [\lambda)^2 \det([X^i_\beta T^\alpha, X^j_\beta T^\beta, X^k_\gamma T^\gamma])\) is the square of graviton mass. As can be seen from this definition, the graviton mass depends on the scalars which interact with branes. This definition helps us to calculate another term of determinant:

\[
\det(Q) \sim (i)^2 (\lambda)^2 \det([X^i_\beta T^\alpha, X^j_\beta T^\beta, X^k_\gamma T^\gamma]) \det(E) \sim
- [(\lambda)^2 \det([X^i_\beta T^\alpha, X^j_\beta T^\beta, X^k_\gamma T^\gamma]) \det(g)] = m_g^2 \det(g)
\] (13)

By inserting equations (11), (12), (13) into the action (5) and doing some approximations, we can obtain:

\[
S_{M1} = -T_{M1} \int d^2\sigma \sqrt{-g} \left[ \delta^{\rho\sigma}_{\mu\nu} \kappa^\rho_\mu \kappa^\sigma_\nu + m_g^2 \delta^{\rho\sigma}_{\mu\nu} R^{\rho\sigma}_{\mu\nu} \right] =
-T_{M1} \int d^2\sigma \sqrt{-g} \left[ R + m_g^2 \delta^{\rho\sigma}_{\mu\nu} R^{\rho\sigma}_{\mu\nu} \right]
\] (14)

where we have used of this fact that \( (\delta^{\rho\sigma}_{\mu\nu} \kappa^\rho_\mu \kappa^\sigma_\nu = R) \) [20]. This equation gives the leading order of terms in massive gravity. Also, it is clear that the first term of Lovelock gravity appears in this action. To achieve to higher order of terms in massive and Lovelock gravity, we should add dimension to branes. For this, using equation (8), we apply following replacement in action of branes.

\[
i, j = a, b \Rightarrow \langle [X^i, X^j, X^k], [X^i, X^j, X^k] \rangle = \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle = \frac{1}{2} \langle \partial_a X^i, \partial_a X^i \rangle
\]

\[
i, j, k = a, b, c \Rightarrow \langle [X^i, X^j, X^k], [X^i, X^j, X^k] \rangle = \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle = \langle F^{abc}, F^{abc} \rangle
\]

\[
Q_i = \delta_i - i \lambda [X_\alpha^i T^\alpha, X_\beta^k T^\beta, X_\gamma^k T^\gamma] E^{\alpha\beta\gamma}_{kij}
\]

\[
Q_i = \delta_i - i \lambda [X_\alpha^i T^\alpha, X_\beta^k T^\beta, X_\gamma^k T^\gamma] + \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle + \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle \rangle E^{\alpha\beta\gamma}_{kij} =
\delta_i + i \lambda [X_\alpha^i T^\alpha, X_\beta^k T^\beta, X_\gamma^k T^\gamma] + \frac{1}{2} \langle \partial_a X^i, \partial_a X^i \rangle + \langle F^{abc}, F^{abc} \rangle \rangle E^{\alpha\beta\gamma}_{kij}
\]

\[
T_{M1} \int d^2\sigma \Rightarrow T_{M_1} \int d^2\sigma
\] (15)

Substituting the relations in equation (15) into action (5) we have:

\[
S = -T_{M_1} \int d^2\sigma \sqrt{-det(O + 2\pi l_s^2 G(F))}
\]
\[ G = \sum_{n=0}^{p} \frac{1}{n!} (-\frac{F}{\beta^2})^n \]
\[ O = \frac{1}{p} \sum_{n} (p-n)!(\frac{Y^n}{n!}) \]
\[ F = \langle F^{abc}, F^{abc} \rangle \quad Y = \langle \partial_a X^i, \partial_a X^i \rangle \quad \beta = \frac{1}{2\pi l_s^2} \]  

(16)

where we have used of the form of nonlinear field \( G \) in \([12, 30, 31]\). At this stage, we show that this action can be written in terms of relevant actions of \( p M1 \)'s. Also, we argue that by adding each dimension, the probability for producing one wormhole between brane and other branes are enhanced. For simplicity, we choose \( X^1 = \sigma \) and \( X^4 = z \), where \( z \) is the transverse direction between branes. Using the above action, the Lagrangian for \( Mp \)-brane can be written as

\[ L = -T_{Mp} \int d\sigma \sigma^p \sqrt{(1 + z'^2)^p + (2\pi l_s^2)^2 G(F)} \]  

(17)

where \( (') \) denotes the derivative respect to \( \sigma \). To obtain the Hamiltonian, we need to derive the canonical momentum density for graviton. For simplicity, we will use of the method in \([24, 25]\) and assume that \( F_{001} \neq 0 \) and other components of \( F \) are zero. We get:

\[ \Pi = \frac{\delta L}{\delta \partial_t A^{01}} = \frac{\sum_{n=0}^{p} \frac{n}{n!} (-\frac{F}{\beta^2})^{n-1} F_{001}}{\sqrt{(1 + z'^2)^p + (2\pi l_s^2)^2 G(F)}} \]  

(18)

Thus the Hamiltonian can be written as:

\[ H = T_{Mp} \int d\sigma \sigma^p \Pi \partial_t A^{01} - L = 4\pi \int d\sigma [\sigma^p \Pi (\partial_t A^{01} - \partial_\sigma A^{00}) - \partial_\sigma (\sigma^2 \Pi) A^{00}] - L \]  

(19)

In the second step integrated by parts, we have the term proportional to \( \partial_\sigma A^{01} \). Using the constraint \( (\partial_\sigma (\sigma^p \Pi) = 0) \), we get \([24]\):

\[ \Pi = \frac{k}{4\pi \sigma^p} \]  

(20)

where \( k \) is a constant. Substituting \( \Pi \) from above equation into equation \([19]\) yields the following Hamiltonian:

\[ H_1 = T_{Mp} \int d\sigma \sigma^p \sqrt{(1 + z'^2)^p + (2\pi l_s^2)^2 \sum_{n=0}^{p} \frac{n}{n!} (-\frac{F}{\beta^2})^n F_1} \]

\[ F_1 = \sqrt{1 + \frac{k^2}{\sigma^2 p}} \]  

(21)

To derive the explicit form of wormhole between branes, we need a Hamiltonian which be only in terms of separation distance between branes. For this reason, we redefine Lagrangian as:

\[ L = -T_{Mp} \int d\sigma \sigma^p \sqrt{(1 + z'^2)^p + (2\pi l_s^2)^2 \sum_{n=0}^{p} \frac{n}{n!} (-\frac{F}{\beta^2})^n F_1} \]  

(22)

With this new form of Lagrangian, we repeat our previous calculations. We can write:

\[ \Pi = \frac{\delta L}{\delta \partial_t A^{01}} = \frac{\sum_{n=0}^{p} \frac{n(n-1)}{n!} (-\frac{F}{\beta^2})^{n-1} F_{001}}{\sqrt{(1 + z'^2)^p + (2\pi l_s^2)^2 \sum_{n=0}^{p} \frac{n}{n!} (-\frac{F}{\beta^2})^n}} \]

(23)
Therefore the new Hamiltonian can be built as:

\[
H_2 = T_{M_p} \int d\sigma^p F_1 \Pi \partial_t A^{01} - L = \int d\sigma [\sigma^p F_1 \Pi (\partial_t A^{01} - \partial_\tau A^{00}) - \partial_\tau (F_1 \sigma^p \Pi A^{00})] - L \tag{24}
\]

where like previous stage, we have in the second step integrated by parts the term proportional to \(\partial_\tau A^{01}\) like the method in [24]. Imposing the constraint \((\partial_\tau (\sigma^p \Pi) = 0)\), we get:

\[
\Pi = \frac{k}{4 F_1 \pi \sigma^p} \tag{25}
\]

By substituting equation (25) into equation (24) we obtain the following Hamiltonian:

\[
H_2 = T_{M_p} \int d\sigma^p \left( (1 + z'^2)^p + (2\pi l_p^2)^2 \sum_{n=0}^{p} \frac{n(n-1)...(n-n)}{n!} (\frac{-F}{\beta^2})^n F_2 \right)
\]

\[
F_2 = F_1 \sqrt{1 + \frac{k^2}{F_1^2 \sigma^{2p}}}
\]

(26)

and by repeating these calculations for \(p\) times, we obtain:

\[
H_p = T_{M_p} \int d\sigma^p (\sqrt{1 + z'^2})^p F_{tot}
\]

\[
F_{tot} = \sqrt{1 + \frac{k^2}{F_{p-1}^2 \sigma^{2p}}} \sqrt{1 + \frac{k^2}{F_{p-2}^2 \sigma^{2p}}} \cdots \sqrt{1 + \frac{k^2}{F_1^2 \sigma^{2p}}} \sqrt{1 + \frac{k^2}{\sigma^{2p}}}
\]

(27)

In the case of \(\frac{k}{\sigma^p} \ll 1\), we can reobtain the Hamiltonian of \(M_p\)-brane by multiplying the Hamiltonians of \(M_1\)’s:

\[
H_p = T_{M_p} \int d\sigma^p (\sqrt{1 + z'^2})^p F_{tot}
\]

\[
k = k^p \rightarrow F_{tot} = \sqrt{1 + \frac{k^2}{F_{p-1}^2 \sigma^{2p}}} \sqrt{1 + \frac{k^2}{F_{p-2}^2 \sigma^{2p}}} \cdots \sqrt{1 + \frac{k^2}{F_1^2 \sigma^{2p}}} \sqrt{1 + \frac{k^2}{\sigma^{2p}}} \approx \left(1 + \frac{k^2}{\sigma^2}\right)^p
\]

\[
H_p = T_{M_p} \int d\sigma^p (\sqrt{1 + z'^2})^p \left(1 + \frac{k^2}{\sigma^2}\right)^p = T_{M_p} \int d\sigma^p (\sqrt{1 + z'^2})^p \left(1 + \frac{k^2}{\sigma^2}\right)^p \Rightarrow
\]

\[
H_p = (T_{M_1})^p \int d\sigma \sqrt{1 + z'^2} \left(1 + \frac{k^2}{\sigma^2}\right)^p = H_1^p
\]

(28)

where we have used of this assumption that \(T_{M_p} = (T_{M_1})^p\). This equation says that each \(M_p\)-brane can be built of \(p\) \(M_1\)-brane. Also, we can show that each \(M_1\)-brane produces a wormhole. To this end, using the above Hamiltonian, we obtain the following equation of motion \(z\) for any \(M_1\):
we should care in antisymmetric form δ wormholes. That by joining p M1-branes, we can construct one Mp-brane. Then this brane may interact with other branes via p branes increases which causes to emergence of different gravity terms in relevant action of branes.

$$-z_{M1}' = \left( \frac{V_1(\sigma)^2}{V_1(\sigma_0)^2} - 1 \right)^{-1/2}$$

$$V_1 = \sigma F_1 = \sigma \sqrt{1 + \frac{k^2}{\sigma^2}}$$

(29)

The solution of this equation is:

$$z_{M1} = \int_{\sigma}^{\infty} d\sigma' \left( \frac{V_1(\sigma')^2}{V_1(\sigma_0)^2} - 1 \right)^{-1/2}$$

(30)

Thus, the separation distance between two branes can be given by:

$$\Delta_{M1} = 2z_{M1} = 2 \int_{\sigma}^{\infty} d\sigma' \left( \frac{V_1(\sigma')^2}{V_1(\sigma_0)^2} - 1 \right)^{-1/2}$$

(31)

where $\sigma_0$ is the throat of wormhole between two M1-branes of two different branes. In fact, these results show that by joining p M1-branes, we can construct one Mp-brane. Then this brane may interact with other branes via p wormholes.

At this stage, we can obtain the relevant action for Mp-branes by multiplying the action of p M1-branes. However, we should care in antisymmetric form δ. We can write:

$$S_{M_p} = -T_{M_p} \int dt L_{M_p}$$

$$L_{M_p} = \det(M) \quad L_{M,i} = L_{M_i} = \det(M_i) \sim M_i$$

$$\det(M) = \delta_{b_1 b_2 \ldots b_n}^a_{a_1 a_2 \ldots a_m} M_{b_1 \ldots b_p} ^{a_1 \ldots a_p} \Rightarrow$$

$$L_{M_p} = \det(M) = \delta_{b_1 b_2 \ldots b_n} ^{a_1 a_2 \ldots a_m} F_{b_1 \ldots b_p}^{a_1 \ldots a_p}$$

$$\delta_{b_1 b_2 \ldots b_n} ^{\sigma \rho \sigma_1 \ldots \rho \sigma_p} \delta_{\mu \rho \nu} = \delta_{\mu \rho \sigma_1 \ldots \rho \sigma_p}$$

$$\sqrt{-g} = \sqrt{-\det(g)} = \sqrt{-\det(g_1 g_2 \ldots g_p)} = \sqrt{-\det(g_1) \det(g_2) \ldots \det(g_p)}$$

$$S_{M_p} = -(T_{M_1})^p \int dt \delta_{b_1 b_2 \ldots b_n} ^{a_1 a_2 \ldots a_m} L_{b_1} ^{a_1} \ldots L_{b_p} ^{a_p} =$$

$$-(T_{M_1})^p \int dt \int d^p \delta_{b_1 b_2 \ldots b_n} \left( \sqrt{-g_1} \left( \delta_{\mu \nu_1 \nu_2} ^{\rho \sigma_1 \sigma_2} \kappa_{\nu_1} + \rho_2^{\mu \nu_1} \kappa_{\nu_1} \right) \right)_{b_1} ^{a_1} \times$$

$$\left[ g_2 \left( \delta_{\mu \nu_2} ^{\rho \sigma_2} \kappa_{\nu_2} + \rho_2^{\mu \nu_2} \kappa_{\nu_2} \right) \right]_{b_2} ^{a_2} \times \ldots \times$$

$$\left[ g_p \left( \delta_{\mu \nu_p} ^{\rho \sigma_p} \kappa_{\nu_p} + \rho_2^{\mu \nu_p} \kappa_{\nu_p} \right) \right]_{b_p} ^{a_p} =$$

$$-(T_{M_p}) \int dt \int d^p \delta \left[ \sqrt{-g} \left( \sum_{n=1}^{p} \delta_{\mu \nu_1 \ldots \nu_n} ^{\rho \sigma_1 \ldots \rho \sigma_n} \kappa_{\nu_1} \ldots \kappa_{\nu_n} \right) + \right.$$

$$\sum_{n=1}^{p} m_g^{2n} \delta_{\mu \nu_1 \ldots \nu_n} ^{\rho \sigma_1 \ldots \rho \sigma_n} R_{\mu \nu_1 \ldots \nu_n} \right.$$

$$\left. + \sum_{n=1}^{p} m_g^{2n} \delta_{\mu \nu_1 \ldots \nu_n} ^{\rho \sigma_1 \ldots \rho \sigma_n} \kappa_{\nu_1} \ldots \kappa_{\nu_n} \right]$$

(32)

In above equation, second term corresponds to Lovelock gravity [27] [28] and third term is related to nonlinear massive gravity. Also, there are other terms that may produce another types of gravity theories. These results show that by increasing the number of dimensions of branes, more wormholes are created and the interaction between branes increases which causes to emergence of different gravity terms in relevant action of branes.
Using equation (33), we can replace all terms in gravity theories by terms in electrodynamics. In ten-dimensional space-time in addition to some energy which is produced as due to compactification of Mp-branes.

Until now, we have shown that Mp-branes are constructed from p M1-branes. Each of these branes are connected by M1-branes of other Mp-brane by a wormhole. By adding the number of dimensions, the number of nonlinear terms in relevant action of branes increases and some theories like lovelock and nonlinear gravity are produced. At this stage, we can show that graviton has a direct relation with photon. In fact, by compacting Mp-branes, it may transit to Dp-brane and some gravitons convert to photons [29–31]. To show this, we use of the method in [7, 23] and define $X^{10} = \frac{R}{l_p^2}$ where $l_p$ is the Planck length. We can write:

$$\left[ X^a, X^b, X^c \right] = F^{abc} \quad \left[ X^a, X^b \right] = F^{ab}$$

$$F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab} \quad F_{ab} = \partial_a A_b - \partial_b A_a$$

$$\Sigma_{a,b,c=0}^{10} (F_{abc}, F_{abc}) = \Sigma_{a,b,c=0}^{10} ([X^a, X^b, X^c], [X_a, X_b, X_c]) =$$

$$-\Sigma_{a,b,c,d}^{10} \epsilon_{a,b,c,d} X^a X^b X^c X_d X_\nu X_\lambda =$$

$$-6 \Sigma_{a,b,a',b'}^{9} \epsilon_{a,b,a',b'} D_{a,b}X^a X^b X_\nu X_\lambda X_{a'} X_{b'} =$$

$$-6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{a,b,a',b'}^{9} \epsilon_{a,b,a',b'} D_{a,b}X^a X^b X_\nu X_\lambda X_{a'} X_{b'} =$$

$$\Sigma_{a,b,c,d}^{10} \epsilon_{a,b,c,d} X^a X^b X^c X_d X_\nu X_\lambda =$$

$$-6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{a,b,a',b'}^{9} \epsilon_{a,b,a',b'} D_{a,b}X^a X^b X_\nu X_\lambda X_{a'} X_{b'} =$$

$$-6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{a,b}^{9} [X^a, X^b]^2 =$$

$$6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{a,b}^{9} [X^a, X^b] [X_a, X_b] = 6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{a,b}^{9} F^a_b F_b^a$$

This equation shows that self energy of gravitons in eleven dimensional space-time is equal to self energy of photons in ten dimensional space-time in additional to some energy which is produced due to compactification of Mp-branes. Using equation (33), we can replace all terms in gravity theories by terms in electrodynamics.

$$\Sigma_{p,\sigma,\mu,\nu=0}^{10} R_{\sigma\mu} = \Sigma_{p,\sigma,\mu,\nu,\lambda=0}^{10} (F^\sigma \sigma, L^\lambda \mu, L^\lambda \nu) = \Sigma_{p,\sigma,\mu,\nu,\lambda=0}^{10} ([X^\rho, X_\sigma, X_\lambda], [X^\lambda, X_\mu, X_\nu]) \Rightarrow$$

$$6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{p,\sigma,\mu,\nu=0}^{9} [X^\rho, X_\sigma] [X_\mu, X_\nu] = 6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{p,\sigma,\mu,\nu=0}^{9} F^\rho_{\sigma} F_{\mu\nu}$$

These relations help us to replace Lovelock theory with nonlinear electrodynamics:

$$\sum_{n=1}^{p} m_{\eta}^2 \delta^{\rho_1 \sigma_1 \ldots \rho_n \sigma_n}_{\mu_1 \nu_1 \ldots \mu_n \nu_n} R_{\mu_1 \nu_1 \ldots \mu_n \nu_n} \Rightarrow$$

$$\sum_{n=1}^{p} m_{\eta}^2 \left( \frac{R^2}{l_p^2} \right) \delta^{\rho_1 \sigma_1 \ldots \rho_n \sigma_n}_{\mu_1 \nu_1 \ldots \mu_n \nu_n} F^{\mu_1 \nu_1 \ldots \mu_n \nu_n} F_{\rho_1 \sigma_1 \ldots \rho_n \sigma_n}$$

This equation shows that by compacting Mp-brane, nonlinear theories like Lovelock gravity converts to nonlinear electrodynamics. In fact, the origins of Lovelock terms and nonlinear electromagnetic field [12, 30, 31] are the same.
We can consider other effects of compactifications of Mp-brane by extending the relations in (34) and writing following relations:

\[ \Sigma_{a,i,j=0}^{10} ([X^a, X^i, X^j], [X_a, X_i, X_j]) \Rightarrow \]
\[ 6(R^2_{l^3 p}) \Sigma_{i,j=0}^9 [X^i, X^j] \]
\[ \Sigma_{a,b,i=0}^{10} ([X^a, X^i, X^b], [X_a, X_i, X_b]) \Rightarrow \]
\[ 6(R^2_{l^3 p}) \Sigma_{i,b=0}^9 [X^i, X^b] \]
\[ \Sigma_{a,b,i=0}^{10} ([X^a, X^b, X^i], [X_a, X_b, X_i]) \Rightarrow \]
\[ 6(R^2_{l^3 p}) \Sigma_{i,b=0}^9 [X^b, X^i] \]

(36)

On the other hand, we can write following mappings for noncommutative brackets in ten dimensional space-time [7]:

\[ \langle [X^a, X^i], [X^a, X^i] \rangle = \frac{1}{2} (\partial_a X^i) (\partial_a X^i) \]
\[ \langle [X^a, X^b], [X^a, X^b] \rangle = \langle F^{ab}, F^{ab} \rangle \]
\[ i, j = p + 1, \ldots, 10 \quad a, b = 0, 1, \ldots, 9 \quad m, n = 0, \ldots, 10 \] (37)

Using the relations in equations (33), (34), (35), (36) and (37) in action (5), we can obtain the relevant action for D3-brane:

\[ S_{D3} = -T_{D3} \int d^4 \sigma \left( \frac{1}{2} \sum_{a=0, \ldots, 3} \sum_{i=4, \ldots, 9} (\partial_a X^i) (\partial_a X^i) + 6(R^2_{l^3 p}) \Sigma_{i,j=4}^9 [X^i, X^j] [X_i, X_j] - \right. \]
\[ \left. \sum_{n=1}^3 n^2 (R^2_{l^3 p}) \delta^{\rho_1 \sigma_1 \ldots \rho_n \sigma_n} F_{\mu_1 \nu_1} \ldots F_{\mu_n \nu_n} F_{\rho_1 \sigma_1} \ldots F_{\rho_n \sigma_n} \right) ^{1/2} \] (38)

This equation is in agreement with relevant action for D3-branes [7], [15], [19] if we replace the nonlinear term of electrodynamics by linear one. As can be seen from the above equation, the nonlinear gravity in M-theory can be replaced by nonlinear electrodynamics in string theory. This is because that the origin of all matters are strings. When strings are attached to Mp-branes, they have the role of gravitons. By compacting Mp-branes and reducing them to Dp-branes, they may play the role of nonlinear photons and in conditions that they are deliberated from branes, they can be regarded as scalars.

IV. SUMMARY AND DISCUSSION

In this paper, we have shown that Mp-branes can be built by joining and growing of p M1-branes. These M1-branes may be connected to M1-branes of other Mp-branes by a wormhole. Thus, each Mp-brane may have p wormholes which originate from nonlinear gravity on it. In fact, for an M1-brane, we only have linear gravity; however by gluing M1-branes and emergence of Mp-branes, the nonlinear terms in relevant action of them are produced and some theories like lovelock and nonlinear gravity are born. By compacting Mp-brane, nonlinear theories converts to nonlinear electrodynamics. This is because that the origin of graviton and photon are strings. When strings glue to Mp-branes, they have the role of gravitons. By compacting Mp-branes and reducing them to Dp-branes, they may play the role of nonlinear photons and in conditions that they are deliberated from branes, they can be regarded as scalars.
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