DISCRETE AMBIGUITIES IN $B$-DECAY CP ASYMMETRIES
AND THE SEARCH FOR NEW PHYSICS

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The first measurements of CP violation in the $B$ system will probably extract $\sin 2\alpha$, $\sin 2\beta$ and $\cos 2\gamma$. Assuming that the CP angles $\alpha$, $\beta$ and $\gamma$ are the interior angles of the unitarity triangle, this determines the angle set $(\alpha, \beta, \gamma)$ up to a twofold discrete ambiguity. The presence of this discrete ambiguity can make the discovery of new physics difficult: if only one of the two solutions is consistent with constraints from other measurements in the $B$ and $K$ systems, one is not sure whether new physics is present or not. I present examples of this situation, and discuss ways to resolve the discrete ambiguity.

Within the standard model (SM), CP violation is due to the presence of a nonzero phase in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. This phase information can be elegantly displayed using the unitarity triangle, in which the interior (CP-violating) angles are labelled $\alpha$, $\beta$ and $\gamma$. Constraints on the unitarity triangle come from several sources ($|V_{cb}|$, $|V_{cb}/V_{ub}|$, $|\epsilon|$, $B^0_d-\bar{B}^0_d$ and $B^0_s-\bar{B}^0_s$ mixing), and the allowed region is shown in Fig. 1. (In the following, I will refer to this as the “allowed UT region.”) Note that this region is still relatively large – at present the position of the apex of the triangle is not well-established. This is due principally to the large theoretical hadronic uncertainties present in some of the measured quantities.

In the coming years, CP-violating rate asymmetries in $B$ decays will be measured at a variety of machines. These asymmetries probe the angles $\alpha$, $\beta$ and $\gamma$ without hadronic uncertainties, and will therefore allow us to cleanly reconstruct the unitarity triangle. If Nature is kind, these measurements may reveal the presence of new physics. This can happen as follows.

The principal way in which new physics can affect CP asymmetries is by changing the phase of the neutral $B$-$\bar{B}$ mixing amplitudes, thereby shifting...
the CP angles from their SM values. Note, however, that the angles $\alpha$, $\beta$, and $\gamma$ will probably be first probed via CP violation in $B_d^0(t) \rightarrow \pi\pi$ (or $\rho\pi^0$), $B_d^0(t) \rightarrow \Psi K_S$ and $B^\pm \rightarrow DK^\pm$ respectively. If there is new physics in $B$-$\bar{B}$ mixing, only the measurements of $\alpha$ and $\beta$ will be affected. Furthermore, they will be affected in opposite directions\(^5\). That is, instead of extracting $\alpha$ and $\beta$, the decays will measure $\alpha + \theta_{NP}$ and $\beta - \theta_{NP}$. The upshot is that new physics will not be discovered through a violation of the triangle condition $\alpha + \beta + \gamma = \pi$.

However, new physics can be found if the triangle constructed from measurements of the angles is inconsistent with that constructed from measurements of the sides. Unfortunately, there are some potential problems with this procedure. First, because the allowed unitarity-triangle region is still relatively large (see Fig.\(^1\)), it is conceivable that new physics might be present, but the triangle constructed from the CP angles $\alpha$, $\beta$ and $\gamma$ might still lie within the allowed region. Of course, this would suggest that the new-physics angle $\theta_{NP}$ is small. Still, we would like to be able to detect such a new-physics effect.

More importantly, even if $\theta_{NP}$ is large, the $\alpha$-$\beta$-$\gamma$ triangle may still be within the allowed region. This is due to the presence of discrete ambiguities\(^6\). The point is that we don’t actually measure the CP phases $\alpha$, $\beta$ and $\gamma$. Instead, it is the functions $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2\gamma$ (or, equivalently, $\cos 2\gamma$) which are extracted. And if new physics is present, the angles probed are in
From \(\sin 2\tilde{\alpha}\), the CP phase \(\tilde{\alpha}\) can be extracted with a fourfold ambiguity, and similarly for \(\sin 2\tilde{\beta}\) and \(\cos 2\tilde{\gamma}\). Thus, there is a 64-fold ambiguity in the angle set \((\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})\). Of course, in practice, we will assume that \(\tilde{\alpha}, \tilde{\beta}\) and \(\tilde{\gamma}\) are the interior angles of the unitarity triangle, i.e. (i) they are all of same sign, and (ii) \(|\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma}| = 180^\circ\). (Note: negative CP angles correspond to a downward-pointing unitarity triangle, which implies that the bag parameters \(B_K\) and/or \(B_{B_d}\) are negative. This scenario is disfavoured theoretically, but should be checked experimentally.) The key point here is that not all angle sets satisfy these two conditions. In fact, the measurements of \(\sin 2\tilde{\alpha}\), \(\sin 2\tilde{\beta}\) and \(\cos 2\tilde{\gamma}\) determine \((\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})\) up to a twofold ambiguity.

Given this twofold ambiguity, there are then three possibilities:

1. Both candidate solutions are consistent with the allowed UT region.
2. Only one candidate solution is consistent with the allowed UT region.
3. Neither candidate solution is consistent with the allowed UT region.

In practice, situation (1) cannot arise. According to the constraints shown in Fig. 1, the allowed range for \(\beta\) is \(16^\circ \leq \beta \leq 35^\circ\). The twofold discrete ambiguity has \(\beta \rightarrow -\frac{\pi}{2} - \beta\) or \(\beta \rightarrow -\frac{\pi}{2} - \beta\). It is therefore impossible for both solutions to satisfy the constraint on \(\beta\). In addition, situation (3) poses no problem. In this case we will know that new physics is present (though we would still like to know which is the correct solution).

It is situation (2) which is problematic. Is new physics present or not? Below I give some examples of this scenario.

Suppose that the SM values of the CP angles are \((\alpha, \beta, \gamma) = (70^\circ, 20^\circ, 90^\circ)\). This corresponds to a point near the left-hand side of the allowed UT region.

- Suppose \(\theta_{NP} = -50^\circ\). The two solutions are then
  \[
  (\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (20^\circ, 70^\circ, 90^\circ) ,
  (\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (70^\circ, 20^\circ, 90^\circ) .
  \] (2)

The first solution is completely inconsistent with the allowed UT region, but the second solution is consistent. In fact, the second solution is identical to the SM solution, even though there is a large \(\theta_{NP}\)!
Table 1. Construction of the triangle angle set (˜\(\alpha_1, \tilde{\beta}_1, \tilde{\gamma}_1\)) in the presence of new physics.

| Angle(s) flipped | (\(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\)) | (\(\tilde{\alpha} - \pi, \tilde{\beta}, \tilde{\gamma} - \pi\)) | (\(\tilde{\alpha} - \pi, \tilde{\beta} + \pi, \tilde{\gamma}\)) |
|------------------|----------------------------------|----------------------------------|----------------------------------|
| none             | (\(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\)) | (\(\tilde{\alpha} - \pi, \tilde{\beta}, \tilde{\gamma} - \pi\)) | (\(\tilde{\alpha} - \pi, \tilde{\beta} + \pi, \tilde{\gamma}\)) |
| \(\alpha\)      | (\(\tilde{\alpha}, \tilde{\beta} - \pi, \tilde{\gamma} - \pi\)) | (\(\tilde{\alpha} - \pi, \tilde{\beta}, \tilde{\gamma} - \pi\)) | |
| \(\beta\)       | (\(\tilde{\alpha} - \pi, \tilde{\beta}, \tilde{\gamma} - \pi\)) | (\(\tilde{\alpha} - \pi, \tilde{\beta} + \pi, \tilde{\gamma}\)) | |
| \(\alpha, \beta\)| (\(\tilde{\alpha} - \pi, \tilde{\beta} + \pi, \tilde{\gamma}\)) | |

- Suppose \(\theta_{NP} = +130^\circ\). In this case the values of the new-physics-modified CP angles [Eq. (1)] are (\(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\)) = \((-160^\circ, -110^\circ, 90^\circ\)) which is not even a triangle. (Since two CP angles have changed sign here, I refer to this situation as having “two flips.”) Even in this situation, however, there are two solutions which form a triangle. They are

  \[
  (\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (20^\circ, 70^\circ, 90^\circ), \\
  (\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (70^\circ, 20^\circ, 90^\circ).
  \]

  As above, the first solution is inconsistent with the allowed UT region, but the second solution is consistent.

- Suppose \(\theta_{NP} = +90^\circ\). Here the values of the new-physics-modified CP angles are (\(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\)) = \((160^\circ, -70^\circ, 90^\circ\)) which is also not a triangle. (Since one CP angle has changed sign, this situation is referred to as having “one flip.”) The two solutions which do form a triangle are

  \[
  (\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (-20^\circ, -70^\circ, -90^\circ), \\
  (\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (-70^\circ, -20^\circ, -90^\circ).
  \]

  As usual, the first solution is inconsistent with the allowed UT region. The second solution is consistent only if we allow for the possibility that the unitarity triangle might point down, i.e. that the bag parameter \(B_K\) may be negative.

From the above examples, it is clear that we can categorize the various solutions according to the number of flips. This is summarized in Table 1.

The above examples demonstrate that it is possible for one of the two discretely ambiguous solutions to be consistent with the allowed UT region, even in the presence of a large new-physics phase. In order to establish whether such large new physics is present or not, it will be necessary to remove this
discrete ambiguity. This can be done if a different function of $\tilde{\alpha}$, $\tilde{\beta}$ or $\tilde{\gamma}$ is measured. For example, $\cos 2\tilde{\alpha}$ can be obtained through a study of the time-dependent Dalitz plot for $B_0^0(t) \rightarrow \rho \pi$ decays.

Similarly, Dalitz-plot analyses of the decays $B_d^0(t) \rightarrow D^+ D^- K_S$ and $B_d^0(t) \rightarrow D^{\pm} \pi^\mp K_S$ allow one to extract the functions $\cos 2\tilde{\beta}$ and $2(2\tilde{\beta} + \tilde{\gamma})$, respectively.

$\cos 2\tilde{\beta}$ can also be obtained through a study of $B_d^0 \rightarrow \Psi + K \rightarrow \Psi + (\pi^- \ell^+ \nu)$, known as “cascade mixing.” Finally, $\sin 2\tilde{\gamma}$ can be obtained from $B_s^0(t) \rightarrow D^{\pm}_s K^{\mp}$ if the width difference between the two $B_s$ mass eigenstates is measurable.

All of these measurements are quite difficult, but one may be necessary in order to discover new physics. In all cases, the knowledge of the additional function of $\tilde{\alpha}$, $\tilde{\beta}$ or $\tilde{\gamma}$ eliminates the $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ solution. (Recall that in the examples above, the $(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)$ solution was the one which was consistent with the allowed UT region.)

Does this always work? No: if $\theta_{NP}$ is close to 0 or $\pi$, then discrete ambiguity resolution (DAR) will not reveal the presence of new physics. This can be seen from Table I. If $\theta_{NP} \approx 0$, then there will be no flips, and DAR will choose the solution $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$, which is close to the SM solution $(\alpha, \beta, \gamma)$. And if $\theta_{NP} \approx \pi$, then there will be two flips, and DAR will choose $(\tilde{\alpha} - \pi, \tilde{\beta} + \pi, \tilde{\gamma})$, which again is close to $(\alpha, \beta, \gamma)$. In both cases, DAR will choose the solution which is consistent with the allowed UT region, even though (small) new physics is present.

Note, however, that $\theta_{NP} \approx 0$ need not be that small. For example, suppose that the SM values of the CP angles are $(\alpha, \beta, \gamma) = (113^\circ, 17^\circ, 50^\circ)$, and that $\theta_{NP} = -20^\circ$. Then the two discretely ambiguous solutions are

\[
(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (93^\circ, 37^\circ, 50^\circ),
(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (-3^\circ, -127^\circ, -50^\circ).
\]

Here it is the second solution which is inconsistent with the allowed UT region. The first solution, which will be chosen by DAR, is still consistent. Of course, one can reduce the likelihood of this particular scenario occurring by reducing the allowed UT region.

Finally, in most of the above discussion, I have assumed that $B_K$ and $B_{d_s}$ are both positive, as per theoretical expectations. If we relax this assumption — and I remind the reader that the original excitement over measurements of CP violation in the $B$ system was the ability to test the SM without theoretical input — then this has two consequences. First, the unitarity triangle can point up or down. And second, the signs of the extracted functions $\sin 2\tilde{\alpha}$, $\sin 2\tilde{\beta}$, $\cos 2\tilde{\gamma}$, etc. are uncertain.

Unfortunately, in this situation, one can no longer definitively establish the presence of new physics. Consider again the case where $(\alpha, \beta, \gamma) = (113^\circ, 17^\circ, 50^\circ)$, and that $\theta_{NP} = -20^\circ$. Then the two discretely ambiguous solutions are

\[
(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (93^\circ, 37^\circ, 50^\circ),
(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (-3^\circ, -127^\circ, -50^\circ).
\]

Here it is the second solution which is inconsistent with the allowed UT region. The first solution, which will be chosen by DAR, is still consistent. Of course, one can reduce the likelihood of this particular scenario occurring by reducing the allowed UT region.
(70°, 20°, 90°), and \( \theta_{NP} = -50° \). If \( \sin 2\alpha \), \( \sin 2\beta \) and \( \cos 2\gamma \) have the “wrong” sign (i.e. \( B_{bd} < 0 \)), then the two discretely ambiguous solutions are

\[
(\tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\gamma}_1) = (-20°, -70°, -90°),
(\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2) = (-70°, -20°, -90°).
\]

That is, we find the same solutions as before, except that they are negative. Now, however, since (e.g.) \( \cos 2\beta \) also has the “wrong” sign, DAR will choose the \((\tilde{\alpha}_2, \tilde{\beta}_2, \tilde{\gamma}_2)\) solution. And if \( B_K \) is allowed to be negative, then the unitarity triangle points down, and this solution is completely consistent. In other words, if we allow \( B_K \) and \( B_{bd} \) to be either positive or negative, then we lose the ability to unambiguously conclude that new physics is present. It is therefore important to try to experimentally verify the signs of \( B_K \) and \( B_{bd} \).

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