Heat Transfer in the Flow of a Cold, Two-Dimensional Draining Sheet over a Hot, Horizontal Cylinder

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Abstract

An accurate and comprehensive numerical solution to the parabolic free boundary problem arising from thin film flow with both velocity and temperature distribution at large Reynolds numbers is obtained using a modified Keller box method. A detailed numerical solution scheme is presented to establish the flow and heat transfer characteristics of the draining film flow. A solution is obtained for selecting representative values for the basic parameters of heat transfer in the flow of a cold two-dimensional draining sheet over a hot horizontal cylinder. These indicate the underlying features of developing film thickness, velocity, and temperature distribution. Good agreement with the approximations is obtained, providing basic confirmation of the validity of the numerical results presented. Numerical solutions, supplemented with parameter values associated with specified configurations and operating conditions, can be readily invoked to establish the details of engineering practice.

Keywords: Heat transfer, thin film flow, large Reynolds number, modified Keller box method, tube.

1. Introduction

The flow of a thin liquid film over a horizontal cylinder under gravity frequently occurs in a variety of industrial heat transfer applications, such as heat exchange in desalination apparatus [1], power and process condensers [2], and film cooling [3]. To understand the operation, particularly the efficiency of these processes, it is important to study such processes in detail.

An approximate solution was obtained by Abdelghaffer et al. [4] using the Pohlhausen integral momentum technique, assuming an approximate velocity profile across the film thickness. A numerical solution was obtained by Hunt [5] using a modified Keller box method that accommodated the outer free boundary.

In this paper, an accurate and comprehensive numerical solution of both velocity and temperature distributions is obtained. This solution can be applied to a variety of problems [6-9]. Specifically, heat transfer in the flow of a cold two-dimensional draining sheet over a hot horizontal cylinder is investigated.

2. Modelling

The problem to be examined concerns film cooling that occurs when a cold vertically draining sheet strikes a hot horizontal cylinder. Although a sheet of fluid draining under gravity is accelerated and thinned upon impact [10,11], it is reasonable to model the associated volume flow as a jet of uniform velocity $U_0$, temperature $T_0$, and semi-thickness $H_0$, as illustrated in Figure 1. The film Reynolds number can be defined as $Re = \frac{U_0 a}{\nu}$ where $\nu$ is the kinematic viscosity and $a$ is the radius.
3. Governing Equations

The flow under investigation is modelled as a steady two-dimensional flow of an incompressible fluid. In the absence of viscous dissipation, the equations expressing the conservation of mass, momentum, and energy are consequently

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left( \rho u \right) + \frac{\partial}{\partial y}\left( \rho v \right) = 0, \\
\frac{\partial}{\partial x}\left( \rho u \right) + \frac{\partial}{\partial y}\left( \rho v \right) + \frac{\partial p}{\partial x} = 0, \\
\frac{\partial}{\partial x}\left( \rho \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y}\left( \rho \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2}, \\
\frac{\partial}{\partial x}\left( \rho \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y}\left( \rho \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{1}{Re} \frac{\partial^2 v}{\partial y^2},
\]

where \( \rho \) is the density, \( u \) and \( v \) are the velocity components, and \( Re \) is the Reynolds number. The dependent variables are transformed to a stream function \( \psi \) introduced

\[
x = \theta, \quad y = \frac{\psi}{\psi + \xi + 1 - \eta}, \\
\theta = \frac{\psi}{\psi + \xi + 1 - \eta}, \quad \eta = \frac{\psi}{\psi + \xi + 1 - \eta},
\]

subject to boundary conditions

\[
f = 0, \quad u = 0, \quad \psi = 0 \quad \text{on} \quad \eta = 0, \quad 0 \leq \xi \leq \frac{\sqrt{\pi}}{\xi}, \\
f = \gamma, \quad v = 0, \quad w = 0 \quad \text{on} \quad \eta = 1, \quad 0 \leq \xi \leq \frac{\sqrt{\pi}}{\xi}, \\
h = \gamma, \quad f = f_0(\eta), \quad \psi = \phi_0(\eta) \quad \text{on} \quad \xi = 0, \quad 0 < \eta \leq 1,
\]

where the initial profiles \( f_0(\eta) \) and \( \phi_0(\eta) \) are found by putting \( \xi = 0 \) and \( h = \gamma \) into (10) and solved subject to the conditions \( f = u = \psi = 0 \) on \( \eta = 0 \) and \( u = \psi = 1 \) on
\[ \eta = 1. \] This needs to be found numerically. This solution has been successfully tested against previously reported results [14-19].

5. Results

In Figure 2, the film thickness distribution around the cylinder is plotted from the numerical solution of \( F_e = 1 \), \( \gamma = 1 \) and the individual points calculated by Abdelghaffer et al. [4] using approximate theory are included in the graph. The agreement is seen to be remarkably good.

6. Conclusion

A detailed numerical solution scheme is presented to establish the flow and heat transfer characteristics of the draining film flow. Solutions are obtained to select the representative values of the basic parameters for the horizontal cylinder case. These indicate the underlying features of developing film thickness, velocity, and temperature distribution. Numerical solutions, supplemented by parameter values associated with specified configuration and operating conditions, can be readily invoked to establish the details of engineering practice.

Fig. 2. Film thickness for numerical solution and approximation at \( F_e = 1 \) and \( \gamma = 1 \)

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