Cluster decay in very heavy nuclei in Relativistic Mean Field

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Exotic cluster decay of very heavy nuclei has been studied in the microscopic Super-Asymmetric Fission Model. Relativistic Mean Field model with the force FSU Gold has been employed to obtain the densities of the cluster and the daughter nuclei. The microscopic nuclear interaction DDM3Y1, which has an exponential density dependence, and the Coulomb interaction have been used in the double folding model to obtain the potential between the cluster and the daughter. Half life values have been calculated in the WKB approximation and the spectroscopic factors have been extracted. The latter values are seen to have a simple dependence of the mass of the cluster as has been observed earlier. Predictions have been made for some possible decays.

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Decay of cluster of nucleons from very heavy nuclei was suggested by Sandulescu et al. in 1980. Subsequently, emissions of various clusters from very heavy nuclei leading to daughters around the magic nucleus $^{208}$Pb have been observed experimentally. This new type of radioactivity is analogous to alpha decay where the decaying particle tunnels through the potential barrier. A lot of work has been done by various authors to calculate the half life for these exotic decay processes. Super-Asymmetric Fission Model (SAFM) has been used in and Preformed Cluster Model in to describe the various observed decays.

In the present work, we have used the microscopic SAFM. In most of the previous works in this model, the potentials that have been used were phenomenological in nature. Even in cases where the potential has been constructed from nuclear densities, the densities themselves were obtained using phenomenological prescriptions. However, microscopic densities obtained from mean field approaches may be expected to provide a better description of the densities and hence that of the process of decay. With this in mind, we have chosen Relativistic Mean Field (RMF) to calculate the densities and study cluster decay in the present brief report.

There has been a number of calculations for half life of cluster decay under the SAFM and we cite only a few recent ones. Basu has studied the nuclear cluster radioactivity also in the framework of a SAFM using a phenomenological density and the realistic M3Y interaction. Bhagwat and Gambhir have used densities from RMF calculations using the force NL3. In the present work, we have used another form of the Lagrangian density for RMF to study the structure of the ground state of the nuclei. We also have selected a form of the interaction with an exponential density dependence, DDM3Y1, different from that used in Ref. This particular form of the interaction is consistent in the sense that it can also explain the low energy scattering in the double folding approach.

The interaction potential between the decaying cluster and the daughter nucleus has been obtained in the present work applying the double folding model by folding the densities of the cluster and the daughter nucleus using some suitable nuclear interaction along with the Coulomb interaction. As already mentioned, usually the densities are obtained from phenomenological description while in the present work, we have employed RMF to obtain the densities of the cluster and the daughter nucleus.

RMF is now a standard tool in low energy nuclear structure, successfully reproducing various features such as ground state binding energy, deformation, radius, excited states, spin-orbit splitting, neutron halo, etc. It is well known that in nuclei far away from the stability valley, the single particle level structure undergoes certain changes in which the spin-orbit splitting plays an important role. Being based on the Dirac Lagrangian density, RMF is particularly suited to investigate these nuclei because it naturally incorporates the spin degrees of freedom. In the case of exotic decays, certain neutron rich clusters such as $^{26}$Ne, $^{30}$Mg and $^{34}$Si are emitted. We expect RMF
to successfully describe the nucleon densities in these clusters. There exist different variations of
the Lagrangian density as well as a number of different parameterizations in RMF. Recently, a
new Lagrangian density has been proposed\[8\] which involves self-coupling of the vector-isoscalar
meson as well as coupling between the vector-isoscalar meson and the vector-isovector meson. The
corresponding parameter set is called FSU Gold\[8\]. This Lagrangian density has earlier been em-
ployed to obtain the proton nucleus interaction to successfully calculate the half life for proton
radioactivity\[9\]. In this work also, we have employed this force.

In the conventional RMF+BCS approach for even-even nuclei, the Euler-Lagrange equations are
solved under the assumptions of classical meson fields, time reversal symmetry, no-sea contribution,
etc. Pairing is introduced under the BCS approximation. Since the nuclear density as a function
of distance is very important in our calculation, we have solved the equations in co-ordinate space. The
strength of the zero range pairing force is taken as 300 MeV-fm for both protons and neutrons.

The microscopic density dependent M3Y interaction (DDM3Y) may be obtained from a finite
range nucleon nucleon interaction by introducing a density dependent factor. This class of in-
teractions has been employed widely in the study of nucleon-nucleus as well as nucleus-nucleus
scattering, calculation of proton radioactivity, etc. In this work, we have employed the interaction
DDM3Y1 which has an exponential density dependence

\[
v(r, \rho_1, \rho_2, E) = C(1 + \alpha \exp (-\beta(\rho_1 + \rho_2)))(1 - 0.002E)u^{M3Y}(r)
\]

used in Ref. 10 to study alpha-nucleus scattering. It uses the direct M3Y potential \(u^{M3Y}(r)\) based
on the \(G\)-matrix elements of the Reid\[11\] NN potential and reproduces the saturation properties of
cold nuclear matter. The weak energy dependence was introduced\[12\] to reproduce the empirical
energy dependence of the optical potential. The parameters used have the standard values \(\alpha = 0.2845, \beta = 3.6391\) and \(\beta = 2.9605\text{fm}^2\). Here \(\rho_1\) and \(\rho_2\) are the densities of the \(\alpha\)-particle and
the daughter nucleus. and \(E\) is the energy of the \(\alpha\)-particle per nucleon in MeV. This interaction
has been folded with the theoretical densities of cluster and the daughter nuclei in their ground
states using the code DFPOT[13]. The assault frequency has been calculated from the decay
energy following Gambhir et al[14]. The centrifugal barrier has been included in the cases where
the \(l\) value for the decay is known to be non-zero.

The decay probabilities have been calculated in the WKB approximation for penetration of the
barrier by the cluster. No theoretical calculation can reproduce the \(Q\)-values for the decay very
accurately. As the decay probability changes very rapidly with \(Q\)-value, we have taken the \(Q\)-values
(and the decay energies) from experimental measurements.

The spectroscopic factors have been calculated as the ratios of the calculated half lives to the
experimentally observed half lives. This represents the preformation factor and may be considered
as the overlap of the actual ground state configuration and the configuration representing the
cluster coupled to the ground state of the daughter. Obviously it is expected to be much less than
unity. The results for the spectroscopic factors are presented in TABLE 1. Theoretical calculations
have been performed for spectroscopic factors for some of these nuclei. Our results are comparable
to those values. For example, our calculated value for \(S\) of \(^{212}\)Po is \(1.88 \times 10^{-2}\) as compared to
theoretical value \(2.5 \times 10^{-2}\) deduced in 15. A value of \(3.1 \times 10^{-2}\) was obtained by Mohr\[16\] in a
double folding model calculation using density from experimentally known charge distribution.

It has been suggested\[17\] that in the case of decay of heavy clusters, the spectroscopic factor may scale as

\[
S = (S_\alpha)^{(A-1)/3}
\]

where \(A\) is the mass of the heavy cluster and \(S_\alpha\) is the spectroscopic factor for the \(\alpha\)-decay. Thus
a plot of \(\log_{10} S\) against \(A\) should be a straight line. In Fig. 1 we have plotted the negative of
\(\log_{10} S\) for the decays where both the parent and the daughter are even-even nuclei against the
mass number of the cluster and plotted a best fit line. One can see that the points fall nearly on a
straight line with the \(S_\alpha\) value given by \(1.93 \times 10^{-2}\). This is comparable to the value \(1.61 \times 10^{-2}\)
based on Poenaru et al[18].

We have also extended our study to decays where both the parent and the daughter nuclei have
odd mass though the number of observed decays is rather small. The corresponding curve is shown
in Fig. 2. Here the \(S_\alpha\) value given by \(1.35 \times 10^{-2}\).

With such a good linear fit of the logarithm of spectroscopic factors with mass numbers, we have extended our scheme to calculate the half life of some other possible decays where unambiguous
lifetime measurements are not yet available possible or where there are possibilities of some decay taking place. Our results obtained with the fitted values of $S$ from eqn. (1) are tabulated in TABLE II along with whatever experimental data are available. Except in the case of the decay of $^{233}\text{U}$, the results are consistent with experimental observations. Even in the case of $^{233}\text{U}$, the error is small.

To summarize, we have studied exotic cluster decay of very heavy nuclei in the microscopic SAFM. The densities of the cluster and the daughter nuclei have been obtained from RMF. The microscopic interaction DDM3Y1 and the Coulomb interaction have been folded with these densities to obtain the potential between the cluster and the daughter. The lifetime have been calculated in the WKB approximation and the spectroscopic factors have been extracted. The spectroscopic factors are seen to follow a simple rule as have been observed earlier. Predictions have been made for some possible decays.

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TABLE I: Spectroscopic factor (S) of cluster decay obtained in the present calculation.

| Parent | Cluster | Q (MeV) | log_{10} T_{ex} (s) | S |
|--------|---------|---------|----------------------|---|
| $^{212}$Po | $^4$He | 8.95 | -6.52 | $1.88 \times 10^{-2}$ |
| $^{213}$Po | $^4$He | 7.83 | -5.44 | $1.67 \times 10^{-2}$ |
| $^{214}$Po | $^4$He | 7.83 | -3.78 | $3.45 \times 10^{-2}$ |
| $^{215}$At | $^4$He | 8.178 | -4.00 | $3.13 \times 10^{-2}$ |
| $^{217}$Fr | $^{14}$C | 31.317 | 15.52 | $1.50 \times 10^{-8}$ |
| $^{223}$Ra | $^{14}$C | 32.396 | 13.39 | $1.55 \times 10^{-8}$ |
| $^{224}$Ra | $^{14}$C | 33.05 | 11.00 | $1.64 \times 10^{-7}$ |
| $^{225}$Ac | $^{14}$C | 30.477 | 15.20 | $3.45 \times 10^{-2}$ |
| $^{226}$Ra | $^{14}$C | 30.54 | 15.92 | $1.04 \times 10^{-7}$ |
| $^{227}$Ra | $^{14}$C | 30.477 | 17.34 | $8.14 \times 10^{-8}$ |
| $^{228}$Th | $^{20}$O | 44.72 | 20.72 | $8.37 \times 10^{-11}$ |
| $^{230}$U | $^{14}$C | 32.929 | 15.47 | \n| $^{232}$Th | $^{16}$O | 46.481 | 18.28 | \n| $^{234}$Th | $^{18}$O | 45.727 | > 16.8 | 18.23 |
| $^{236}$Th | $^{24}$Ne | 54.497 | 29.96 | \n| $^{238}$Th | $^{24}$Ne | 55.945 | 30.16 | \n| $^{238}$U | $^{24}$Ne | 55.964 | 28.57 | \n| $^{233}$U | $^{28}$Mg | 74.11 | > 27.6 | 26.56 |
| $^{237}$Np | $^{30}$Mg | 74.817 | > 27.6 | 27.92 |
| $^{240}$Pu | $^{34}$Si | 91.19 | > 27.6 | 26.56 |

TABLE II: Half life values of cluster decay obtained in the present calculation.

| Parent | Cluster | Q (MeV) | log_{10} T_{ex} (s) | log_{10} T_{th} |
|--------|---------|---------|----------------------|-----------------|
| $^{224}$Th | $^{14}$C | 32.929 | 13.68 | \n| $^{226}$Th | $^{14}$C | 30.596 | 18.28 | \n| $^{224}$Th | $^{16}$O | 46.481 | 15.47 | \n| $^{226}$Th | $^{18}$O | 45.727 | > 16.8 | 18.23 |
| $^{232}$Th | $^{24}$Ne | 54.497 | 29.96 | \n| $^{236}$Th | $^{24}$Ne | 55.945 | 30.16 | \n| $^{238}$Th | $^{24}$Ne | 55.964 | 28.57 | \n| $^{233}$U | $^{28}$Mg | 74.11 | > 27.6 | 26.56 |
| $^{237}$Np | $^{30}$Mg | 74.817 | > 27.6 | 27.92 |
| $^{240}$Pu | $^{34}$Si | 91.19 | > 27.6 | 26.56 |
FIG. 1: Negative of logarithm of spectroscopic factors ($S$) as a function of cluster mass number $A$ for even-even parents and daughters.
FIG. 2: Negative of logarithm of spectroscopic factors ($S$) as a function of $A$ for odd mass parents and daughters.