Determination of Effective Heat-Transfer Coefficient for Dualfoil, Based on Full-Scale Cylindrical and Prismatic Cells

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When using a unit-cell model for simulating the behavior of a cell, one generally tracks the temperature change with time, and the resulting changes in the behavior of the current and or the voltage. When heat is lost to the surroundings, external heat transfer needs to be accounted for. Even if you have measured values for the external heat-transfer coefficient, you need to choose the appropriate number to use in the unit-cell model, even if there are only two layers to the model. The value for the coefficient used needs to be modified by the number of layers in the full-scale cell and also limited thermal conduction within the cell.

The dualfoil assumes that the heat generated is uniformly spread over the cell. This relationship between the dualfoil model is used to generate the input file, the effective heat-transfer coefficient.

The electrical loads are specified in an input file and can be loaded manually or taken from the results of running the dualfoil model. If the unit cell subjected to various electrical loads and environmental conditions.

The heat-transfer coefficient $h$ is a parameter that determines how fast heat can be removed from a surface by an external fluid and is used to project the temperature change within a cell based on ambient and cell conditions. To simulate the temperature rise of a full-scale cell using only a portion of the cell (called a unit cell), a modification to the external heat-transfer coefficient must be made. The following analysis develops a correlation between the size of a cylindrical cell and the effective heat-transfer coefficient $h$ that is used in unit-cell modeling simulations.

Analysis

A thermal model is developed using $h$ and cell properties to calculate the temperature of the cell at various locations for a cylindrical cell subjected to various electrical loads and environmental conditions. The electrical loads are specified in an input file and can be loaded manually or taken from the results of running the dualfoil model. If the dualfoil model is used to generate the input file, the effective heat-transfer coefficient $h$ used in dualfoil needs to be related to the actual $h$. This relationship between $h_i$ and $h$ for cylindrical and prismatic configurations is developed below.

The value of the heat-transfer coefficient $h$ is taken to be measured in air at 23 $\degree$C with the cell in either a horizontal or vertical position (to be specified). The unit cell in dualfoil uses an effective heat-transfer coefficient in an attempt to reflect the thermal behavior of the cylindrical cell in operation. The average temperature determined in dualfoil assumes that the heat generated is uniformly spread over the unit-cell thickness ($l_{cell}$). This implies that the thermal conductivity in this configuration is large (or that the thickness is small). This is not the case in the full-scale cell treated in the thermal model. The thermal conductivity reduces the rate of heat flow in the cell. Therefore, a lower, effective heat-transfer coefficient must be used in dualfoil.

Figures 1 and 2 show a typical cell configuration. Reference 6 treats temperature profiles with heat generation in similar configurations. We assume that no heat is transferred through the ends of the cylinder and that the cell is at steady-state. The energy balance can be given by

$$q = -\frac{k_{cell}}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right)$$

where $k_{cell}$ is the thermal conductivity for the unit cell (current collectors, separator, positive, and negative), $q$ is the heat generation per unit volume, $r$ is the radial direction in the cell, and $T$ is the temperature at a point in the radial direction. The boundary conditions are:

$$\frac{dT}{dr} = 0 \text{ at } r = r_i \text{ and }$$

$$-k_{cell} \frac{dT}{dr} = h(T_{r_0} - T_{r_i}) \text{ at } r = r_0,$$

where $T_0$ is the ambient temperature. The possibility of having cooling in an inner core is not considered. Integration of Eq. 1 gives

$$\frac{q}{2} (r^2 - r_i^2) = -k_{cell} r \frac{dT}{dr},$$

where the boundary condition at $r = r_i$ was used to evaluate the integration constant. A second integration gives

$$\frac{q}{2} \left[ \frac{r^2 - r_0^2}{2} - r_i^2 \ln \left( \frac{r}{r_0} \right) \right] = -k_{cell} (T - T_{r_0}).$$

Boundary condition 3 now reads

$$\frac{q}{2} \left( r_0 - r_i^2 \right) = -k_{cell} \frac{dT}{dr} = h(T_{r_0} - T_{r_i}).$$

The energy balance for the unit cell can be expressed as

$$h_i(T - T_{r_0}) = q l_{cell},$$

where $h_i$ is the effective heat-transfer coefficient for the unit cell, and $l_{cell}$ is the cell thickness for the unit cell. The radial position $r$ of the unit cell in the actual cell can vary from $r_i$ to $r_0$.

The relationship between $h_i$ and $h$ follows from comparison of Equations 5, 6, and 7 by letting $T$ in Equation 7 for the unit-cell model be taken to match the temperature at any radial position $r$ as given by the thermal model in Equation 5.

$$h_i(r) = \frac{l_{cell} h}{r_0 - r_i} \left[ \frac{n Na}{2g} + \frac{n}{4 \pi \varepsilon_{cell} (r_0 - r_i) (r_0^2 - r_0^2 + 2 r_0^2 \ln (r/r_0))} \right].$$
The effective heat-transfer coefficient for various locations is given by

\[ h_i(r) = \frac{l_{cell} h}{r_0 - r_i} \left[ 1 + \frac{h}{4 k_{cell} (r_0 - r_i)^2} \right] \]  

The above equation suggests that as the number of layers of the roll increases \((r)\), the effective heat-transfer coefficient \(h_i(r)\) decreases. In addition, smaller values of \(k_{cell}\) decrease \(h_i(r)\) further. Equations 8 or 9 shows that the effective heat-transfer coefficient \(h_i(r)\) depends on \(r\). The equation gives a value of \(h_i\) that should be used in dualfoil. When plotting the temperature of dualfoil as a function of time, the temperatures of the surface and core of the thermal model \(A\) and \(B\) should bracket that of \(r_0\) to reach this condition. Before calculating the extremes of \(h\), we show the fundamental relation between \(h\) and \(h_i(r)\) at \(r = r_0\).

### Table I. Example for various locations.

| Cell Design Parameters | \(l_{cell}\) | \(r_0\) | \(k_{cell}\) |
|------------------------|---------------|----------|--------------|
| \(l_{cell}\)          | 0.0002 m      | 0.02 m   | 0.38 m       |
| \(k_{cell}\)          |               |          | 0.4 W/m-K    |
| \(h_{cell}\)          | 14.00 W/m²-K  | 0.28 W/m²-K |
| \(h_{cell}\)          | 0.238298 W/m²-K | 0.207407 W/m²-K |

The effective and actual heat-transfer coefficients are related by the area of the can and spiral roll, \(A_{cell}/A_{roll}\). The radius of the roll and the thickness of the unit cell determine the length of the roll, \(L\). Let us determine the length \(L\) and use this parameter to determine the basic ratio of unit-cell thickness to roll length.

The following equation is used to approximate the length of coiled foil (coiled rolls are often referred to as Archimedean Spiral geometry) when the thickness of the foil is much smaller than the diameter of the coil (changes in foil properties are not considered here):

\[ L = \frac{\pi (r_o^2 - r_i^2)}{4 h_{cell}} \]

where \(r_0\) is the outer radius of the roll and \(r_i\) is the point where the roll begins.

For the case where the roll begins at \(r_i = 0\), \(L\) is given by

\[ L = \frac{\pi r_o^2}{2 h_{cell}} \]

The area ratio of the container of the roll to the area of the roll is given by

\[ \frac{A_{can}}{A_{roll}} = \frac{2 l_{cell}}{r_0} \]

This quantity is the same as that shown in Equation 7 above. The relationship between \(h\) and \(h_i(r_i)\) can thus be written as

\[ h A_{can}/A_{cell} = h_i(r_i) A_{cell} \quad \text{or} \quad h_i(r_o) = h A_{can}/A_{cell} \]

Substitution of Equation 12 into Equation 13 gives the relationship between \(h_i(r_0)\) and \(h\):

\[ h_i(r_0) = h A_{can}/A_{roll} = h \frac{2 l_{cell}}{r_0} \]

The decrease in the effective heat-transfer coefficient to use in the unit-cell model can be substantial, as shown by Table 1. The effective heat-transfer coefficient for various locations is given by

At \(r = r_0\), \(h_i(r) = h \frac{2 l_{cell}}{r_0} \)

At \(r = 0.707107r_0\), \(h_i(r) = h \frac{2 l_{cell}}{r_0} \frac{1}{(1 + \frac{2k}{4 l_{cell}})} \)

At \(r = 0\), \(h_i(r) = h \frac{2 l_{cell}}{r_0} \frac{1}{(1 + \frac{2k}{4 l_{cell}})} \).

Note that for the examples given, the value of \(h_i\) calculated is substantially lower than that for \(h\). It is for this reason that the appropriate heat-transfer coefficient needs to be used when accounting for thermal changes in a cell.
Selection of the value of the heat-transfer coefficient to use in the unit-cell model depends on the radial position of interest, as illustrated by Equations 16, 17, and 18. This may be the position of the thermocouple if one is comparing predicted temperature rise with experimental measurements. In another possibility, one may want to predict the temperature profile generated from a thermal model and then calculate how this will affect the heat generation and the battery discharge performance without using a full two-dimensional electrochemical and thermal model.

**Discussion**

Figure 3 shows a graph of \( h_r r_0 / 2 l_{cell} \) vs. \( r/ r_0 \) with the reciprocal of the Biot number \( (r_0 h / 4 k_{cell}) \) as a parameter. The Biot number (Bi) is expressed as a dimensionless quantity \((L h / k)\) where \( L \) is the characteristic length, \( k \) is the thermal conductivity, and \( h \) is the heat-transfer coefficient between the material of interest and the adjacent material. The Biot number gives a simple index of the ratio of the heat-transfer resistances inside the material to that at the surface of a material. This ratio determines whether or not there will be a significant temperature distribution within the material due to internal heat generation or changes in external temperature.

In general, small Biot numbers (much smaller than 1) reflect a relatively uniform temperature profile inside the material of interest. Biot numbers much larger than 1 will indicate a nonuniform temperature distribution within the material.

In a future work one might include some plots with an inner core \( r_i \) (insulated or cooled) and a plot with the outer wrap taken into account.

In Figure 3, \( \kappa = r/r_o \). Therefore, \( \kappa = 1 \) is a prismatic cell, while \( \kappa = 0 \) is a cylindrical cell with no core. When the Biot number is very small (that is, the thermal conductivity is effectively very large), the temperature within the cell is nearly uniform, and the correct heat-transfer coefficient to use in dualfoil is the actual heat-transfer coefficient on the outside of the battery (after correction for the number of layers of the unit cells). For small thermal conductivity, the temperature within the cell is nonuniform, and the appropriate heat-transfer coefficient to use with the dualfoil model is smaller than the actual external heat-transfer coefficient, and the more so the smaller the inner radius. Even for cells with cores as small as \( \kappa = 0.5 \), the cell behaves like a prismatic cell.

In Equations 16, 17, and 18 one can make an approximate correction for the foil not starting at \( r = 0 \) by dividing each by the quantity \((1 - r_i^2 / r_o^2)\). The maximum of \( r_i \) is \( r_i = r_0 (1 - 2l_{cell} / r_0)^{0.5} \). For the limit of one wrap of the unit cell, \( r_i = r_0 - l_{cell} \); \( r_o = 0.02 \) mm and \( l_{cell} = 0.0002 \) m, \( r_i = 0.0198 \) m. However, the preceding equation, which equates the can area to the inner area of one wrap of the unit cell material, yields \( r_i = 0.019798989 \) m, a difference of 0.005%. This slight difference is due to the spiral equation.

A further modification needs to be made to \( h \) to account for insulating material that may be used to wrap the cell. The insulating material may be inside and/or outside the metal can container, see Figure 2. We use the simple change to \( h \) as shown below:

\[
h_1 = h \frac{1}{1 + k_{wrap} / h_{wrap}},
\]

where \( k_{wrap} \) is the thermal conductivity of the cell wrap and \( l_{wrap} \) is the wrap thickness. The impact of this addition will be to reduce the heat flow from the cell and tend to make the temperature in the cell more uniform as well as increase the core temperature.

In trying to match experimental data using the dualfoil model, we need to select an \( h \) value that reflects where the thermocouple is placed. Normally, the temperature probe is placed outside the surface of the cell. The value of \( h \) in dualfoil may have to be further adjusted so that the temperature vs. time profile lies between the core and surface temperatures determined by the thermal model. This modification is impacted by the contact of one layer to another.

**Conclusions**

A mathematical model is used to show how much to reduce the external heat-transfer coefficient below the actual value in using the dualfoil or other unit-cell model to match, as well as possible, the actual thermal conditions of the cell. If the internal thermal conductivity...
is effectively small, the interior of the cell will be of nonuniform temperature, and the quality of the match will be compromised. Even with only two layers of the unit cell, the external heat-transfer coefficient needs modification before use in the unit-cell model used to simulate scaled-up systems.

### List of Symbols

| Symbol | Description |
|--------|-------------|
| $A_{\text{can}}$ | vertical area of can, m$^2$ |
| $A_{\text{roll}}$ | area of uncoiled roll, m$^2$ |
| $B_i$ | Biot number for the cell |
| $h$ | heat-transfer coefficient, W/m$^2$-K |
| $h_s$ | effective heat-transfer coefficient for unit-cell model, W/m$^2$-K |
| $k_{\text{cell}}$ | thermal conductivity of unit cell, W/m-K |
| $l_{\text{cell}}$ | unit-cell thickness, m |
| $L$ | height of the cell, m |
| $L$ | length of a cell if unrolled, m |
| $q$ | volumetric heat-generation rate, W/m$^3$ |
| $r$ | radial position, m |
| $r_i, r_o$ | inner and outer radii of the cell, m |
| $T$ | temperature, K |

#### Greek

| Symbol | Description |
|--------|-------------|
| $\kappa$ | ratio of inner to outer cell radius |

#### References

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