The 2016 Vela glitch: A key to neutron star internal structure and dynamics

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ABSTRACT
High resolution, pulse to pulse observation of the 2016 Vela glitch and its relaxation provided an opportunity to probe the neutron star internal structure and dynamics with unprecedented detail. We use the observations of this glitch to infer superfluid characteristics in the framework of the vortex creep model. The glitch rise time constraint of 12.6 seconds put stringent limits on the angular momentum exchange between the crustal superfluid and the observed crust. Together with the observed excess acceleration in the rotation rate as compared to the post-glitch equilibrium value this discriminates crustal superfluid-crust lattice and core superfluid-crustal normal matter coupling time-scales. An evident decrease in the crustal rotation rate immediately before the glitch is consistent with the formation of a new vortex trap zone that initiates the large scale vortex unpinning avalanche.

Key words: stars: neutron — pulsars: general — pulsars: individual: PSR B0833-45 (Vela)

1 INTRODUCTION

The Vela pulsar, with large glitches $\Delta \nu/\nu \sim 10^{-6}$ at intervals of ~1000 days, has been an emblematic source for establishing and testing glitch models thanks to dedicated monitoring programs (Cordes et al. 1988; Buchner & Flanagan 2008; Shannon et al. 2016; Palfreyman et al. 2018). Until recently glitch models took neutron stars as two component systems. One component in these models is the crustal superfluid, which is the agent of the glitch itself as well as the site of post-glitch recovery. The other component is the normal matter which includes crustal normal matter, electrons throughout the star and effectively includes the superfluid core, which is coupled to the normal matter on time-scales shorter than the observational bounds for the glitch rise time. Until the 2016 Vela glitch, best constraints for the glitch rise time were obtained for the 2000 and 2004 glitches, with upper limits of 40 s and 30 s, respectively (Dodson et al. 2002, 2007). High resolution, pulse to pulse observations of the 2016 Vela glitch (Palfreyman et al. 2018) have brought this upper limit to much lower values. Ashton et al. (2019) used these data and employed Bayesian techniques to bring a 12.6 seconds upper limit to the glitch rise time at 90% confidence level. Their analysis also resolved the peak of the glitch increase in the observed crustal angular velocity which decays to a lower level with a $\leq 1$ minute exponential relaxation timescale immediately after the glitch. This promptly decaying component contains more than half of the total glitch amplitude. Unresolved promptly decaying components were barely evident for the 2000 and 2004 glitches.

These results point to a three component neutron star model: angular momentum is first transferred from crustal superfluid to the crustal normal matter in less than 12.6 s and thereafter shared with the core superfluid within a minute. Ashton et al. (2019) also reported definitive evidence for an apparent decrement in the crustal rotation rate right before the glitch, a behaviour never resolved before from any glitches. In this paper we evaluate these observations in terms of the vortex creep model and explore a glitch scenario accounting for them. We propose that the crustal rotational velocity decrease prior to the 2016 glitch marks the formation of a new vortex trap, and that the peak glitch amplitude and its prompt relaxation are signatures of glitch rise due to fast coupling of the crustal superfluid first to the crustal normal matter, followed by the gradual coupling of crustal normal matter to the core superfluid. In §2 we summarize the main observational features of the 2016 Vela glitch. In §3 we propose a scenario within the vortex creep model and then obtain constraints on model parameters. We discuss our results in §4.

2 OBSERVATIONAL FEATURES OF THE 2016 VELA GLITCH

The study of the detailed analysis of the 2016 Vela glitch is based on single pulse to pulse observations conducted at Mount Pleasant radio telescope by Palfreyman et al. (2018). This glitch occurred on 12 December 2016. The fractional changes in the pulsar frequency and spin-down rate are $\Delta \nu/\nu = 1.431(2) \times 10^{-6}$ and $\Delta \nu/\dot{\nu} = 73.354 \times 10^{-3}$, respectively (Xu et al. 2019). Palfreyman et al. (2018) detected pulse morphology and polarization level variations starting 20 rotations before the glitch and extending through...
the first 4.4 seconds after the glitch. They interpreted this as a tran-
sient change in the magnetospheric state associated with the glitch. 
Ashton et al. (2019) reanalysed observational data presented by 
Palfreyman et al. (2018) with Bayesian techniques. Xu et al. (2019) 
reported on the longer term post-glitch $\Delta \nu$ and $\Delta \nu$ recoveries based 
on analysis of timing data from the Kunming 40-m radio telescope. 
Basu et al. (2020) also reported on the $\Delta \nu$ recovery of this glitch 
with timing data from the Oooty radio telescope. These observations 
and analysis led to the following conclusions:

(i) Prior to the glitch there was a decrease in the rotation rate by 
$\Delta \nu = 5.40^{+3.98}_{-2.05}$ $\mu$Hz for $\sim 100$ seconds (Ashton et al. 2019). This 
is comparable to the glitch size itself.

(ii) There was a temporary change in pulse shape and one 
missed pulse at the time of the glitch (Palfreyman et al. 2018).

(iii) The tightest limit so far was obtained for the glitch rise time 
$t_{\text{rise}} < 12.6$ s (Ashton et al. 2019).

(iv) The peak initial spin-up in the glitch, $\Delta \nu_{\text{peak}} = 17.77^{+13.68}_{-7.99}$ $\mu$Hz promptly relaxed with an exponential decay time $t_{\text{dec}} = 53.96^{+24.02}_{-14.62}$ seconds (Ashton et al. 2019).

(v) After the prompt decay of the initial peak, the remaining fre-
quency step of $\Delta \nu = 16.01(5)$ $\mu$Hz relaxed with two short time-scale 
exponential decay terms with time constants 1 and 6 days, and a 
long term healing of $\Delta \nu$ with a constant $\Delta \nu$ (Xu et al. 2019).

The Vela pulsar glitched once again on 1 February 2019 (Gan-
cio et al. 2020).

3 VORTICES AND ANGULAR MOMENTUM EXCHANGE

Elements of the vortex unpinning and vortex creep model for 
glitches are, in time order:

(i) Possible vortex trap formation and quake triggered events 
lead to the vortex unpinning avalanche.

(ii) Crust breaking as trigger may have magnetospheric signa-
tures like pulse shape and emission behaviour changes.

(iii) Glitches themselves are vortex unpinning events which first 
transfer angular momentum from crustal superfluid to normal matter 
nuclei and electrons in the crust.

(iv) The crustal superfluid plus normal matter in the crust then 
couples via electrons to the core superfluid on still very short time-
scales.

(v) Once the core superfluid is coupled to the normal matter in 
the crust, the core superfluid + normal matter system behaves as 
an effective crust which contains most of the moment of inertia of 
the star. This effective crust relaxes back with crustal superfluid as 
the continuous vortex creep process builds up again towards the 
steady-state pre-glitch conditions.

All except (v) were not observed in the Vela pulsar before. 
(i) trap formation and triggering quake were surmised in the Crab 
pulsar (Gügercinoğlu & Alpar 2019) and (ii) glitch induced mag-
etospheric changes observed for PSR J1119–6127 (Akbal et al. 
2015). So far for the Vela and all other pulsars only the post-glitch 
recovery (v) was fitted with creep response models.

3.1 Formation of vortex traps leading to crustal rotation rate 
decrease and a quake triggering the glitch

The observed decrement in the crustal rotation rate prior to the 
glitch is one of the striking properties of the 2016 Vela glitch. Such 
a behaviour had never been seen before from any pulsar glitch. The 
2000 and 2004 Vela pulsar glitches (Dodson et al. 2002, 2007) had 
the previous smallest uncertainties in the glitch occurrence time. 
For the former glitch high cadence arrival time data were not avail-
able prior to the glitch, while for the latter the immediate pre-glitch 
data were noisy.

Glitches involving superfluid vortex unpinning may be trig-
gered by crustquakes. We propose that the slow-down prior to the 
2016 glitch is a signal of the formation of a new vortex trap in asso-
ciatiion with crust breaking, which then provided the site where 
the glitch was triggered. The motion of broken crustal plates would 
lead to extra vortex pinning, creation of vortex free regions and 
duced motion of clusters of pinned vortices. This idea was first pro-
posed by Alpar et al. (1996) in order to account for the persistent 
shifts from the Crab glitches (Lyne et al. 1993), i.e. the glitch asso-
ciated permanent increases in the observed spin-down rate which 
do not recover subsequently. Further support for the glitch trigger 
involving both crust breaking and its induced effects on the con-
figuration of pinned vortices came from the realization that neither 
pure crustquakes nor pure vortex unpinning and creep recovery 
models could explain the intervals between the Crab pulsar glitches 
(Alpar et al. 1996). This idea was further applied to the Crab pulsar’s largest glitch by Gügercinoğlu & Alpar (2019).

Vortex density in the pinned superfluid is unlikely to be uniform. 
Depending on the distribution of pinning centers of various 
strengths, and on the history of the pinned superfluid, vortex traps 
which have high vortex density regions surrounded by vortex dep-

tion regions will be formed, as first suggested by Cheng et al. 
(1988). A vortex trap is formed when the local lag between the 
crustal superfluid and the crust angular velocities increases from 
the steady-state lag $\omega_{\text{cr}}$ to a value above the critical lag $\omega_{\text{cr}}$ for 
unpinning or re-pinning, thereby allowing extra vortex pinning in 
the trap. The extra (re-pinned) vortex density leads to the forma-
tion of a vortex free region as part of the trap: Due to the extra 
pinned vortices the local superfluid velocity in the surrounding 
regions becomes so large that the lag is sustained at a value above 
the critical value for unpinning and no pinning sites are effective in 
the surrounding region. Therefore the vortex creep process is not 
sustained in the newly and irreversibly formed vortex free part of 
the trap, leading to the observed slow-down of the crust before the 
glitch.

For the change $\delta \Omega_{\text{trap}}$ in superfluid rotation rate in the newly 
formed vortex trap - vortex free regions to the observed crust we 
use the estimate (Gügercinoğlu & Alpar 2016)

$$\delta \Omega_{\text{trap}} = \frac{f^p}{\rho R} \frac{K T}{E_p} \ln \left( \frac{2 \Omega_{\text{cr}} v_0}{|\Omega| R} \right),$$

(1)

where $T$ is the temperature of the inner crust, $\kappa \approx 2 \times 10^{-3}$ $\text{cm}^2$ $\text{s}^{-1}$ is the quantum of vorticity, $\Omega_{\text{cr}}$ is the superfluid angular velocity 
($\approx \Omega_{\text{cr}}$, the rotation rate of the neutron star since the lag $\omega$ is small), $f_p$ is the pinning force per unit length of a vortex line, $E_p$ is the 
pinning energy, $\rho$ is the matter density, $R$ is the distance from the 
rotation axis, $|\Omega|$ is the magnitude of the spin-down rate, and $v_0$ is 
the microscopic vortex velocity around nuclei. The permanent 
establishment of a vortex free region means that the corresponding 
region no longer transfers superfluid angular momentum to the 
crust because it no longer sustains vortices or vortex creep. The 
crust is then spinning down at a slightly higher rate leading to a 
decrease $\Delta \nu_\nu$ in the observed spin frequency of the crust

$$\frac{\Delta \nu_\nu}{2\pi} = \frac{1}{2\pi} \frac{l_{\text{trap}}}{T} (\omega_{\text{cr}} - \omega_{\text{free}}),$$

(2)

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Table 1. Pinning parameters and increase in the lag needed for unpinning in various layers of crustal superfluid. Entries in the first three columns are taken from Seveso et al. (2016) ($\beta = 3$ model). The last column is calculated from Eq. (1) using a temperature $kT = 6.5$ keV.

| $\rho$ (10$^{13}$ g cm$^{-3}$) | $E_p$ (MeV) | $f_p$ (10$^{-14}$ dyne s$^{-1}$) | $\omega_{\text{et}} - \omega_{\text{cr}}$ (rad s$^{-1}$) |
|---|---|---|---|
| 0.15 | 0.21 | 3.2 | 0.101 |
| 0.96 | 0.29 | 3.1 | 0.012 |
| 3.4 | 2.74 | 85.5 | 0.011 |
| 7.8 | 0.72 | 18.4 | 0.004 |
| 13 | 0.02 | 0.6 | 0.003 |

\[ \Delta E \approx 5.40^{+3.39}_{-2.05} \text{ MeV} \] (Ashton et al. 2019).

The establishment of a new vortex trap is a redistribution of vortices, by typical microscopic motions over the distance to nearby vortices at the average vortex spacing $\ell_v = (2\Omega_v/\kappa)^{-1/2}$. The time-scale for the trap formation and the accompanying persistent increase in the spin-down rate and decrease in the rotation rate of the crust is then the time taken by a vortex to move one inter-vortex spacing $\ell_v$ at the typical microscopic speeds $v_0$ and is given by (Alpar et al. 1989)

\[ \tau_v = \frac{\ell_v}{v_0} \exp \left( \frac{E_p}{kT} \right). \]

By taking $v_0 \approx 10^7$ cm s$^{-1}$ (Güercinoğlu & Alpar 2016), $kT = 6.5$ keV (Vigano et al. 2013) and for $E_p = 0.17$ MeV pertaining to densities $\rho \approx 10^{14}$ g cm$^{-3}$, Eq. (4) gives $\tau_v \approx 100$ s. This agrees with the observed time-scale of the immediate pre-glitch slowdown (Ashton et al. 2019). Having determined the location of the trap region with $E_p = 0.17$ MeV corresponding to densities $\rho \approx 10^{14}$ g cm$^{-3}$ the fractional moment of inertia of the newly formed trap constraint (3) becomes $I_{\text{trap}}/I = 8.58^{+5.38}_{-3.26} \times 10^{-3}$ for $\Delta \nu = 5.40^{+3.39}_{-2.05}$ MHz.

Since Eq. (4) depends exponentially on the pinning energy $E_p$ of the crustal region where vortex avalanche starts, $\tau_v$ can be unobservably short for smaller $E_p$. The pinning energy $E_p$ decreases with increasing density at the higher densities in the inner crust. Glitches originating in the inner crust superfluid at layers denser and deeper than the location of the vortex trap triggering the 2016 glitch would have durations of the spin-down event immediately preceding the glitch that are (exponentially) shorter than the ~100 s event. Such glitches originating from deeper layers would also be larger glitches as the unpinned vortices would be moving through more crustal superfluid and therefore cause more angular momen-
tum transfer. This is consistent with the fact that the 2016 glitch, the first case of immediate pre-glitch data resolved at this level, is one of the smaller glitches, below the median for this pulsar (Xu et al. 2019), suggesting that the Vela pulsar’s larger glitches originate from the inner crust at densities comparable to and larger than the density range $\rho \sim 10^{14}$ g cm$^{-3}$.

### 3.2 Effects of the glitch triggering quake on the magnetosphere

The 2016 glitch was accompanied by short-lived changes in the Vela pulsar’s electromagnetic signature. There was a pulse shape change (broadening) and notably a null state starting 3.3 s before the glitch with a missed pulse within 0.2 s, and a loss of linear polarization in the next two pulses extending less than 4.4 s at the time of the glitch (Palfreyman et al. 2018).

Changes in the pulsar’s signature associated with the glitches are very rarely observed, so far only from the 2007 glitch of PSR J1119–6127 [except for the magnetar glitches (Kaspi & Beloborodov 2017)]. If the broken crustal plate triggering the glitch happens to extend to the neutron star surface, the glitch will influence the magnetosphere through its coupling to the magnetic field lines anchored in the conducting crust, leading to changes in the electromagnetic signature of the pulsar and the dipole spin-down torque. This idea was first applied by Akbal et al. (2015) to the 2007 peculiar glitch of PSR J1119-6127 (Weltevrede et al. 2011; Antonopoulou et al. 2015), which exhibited a clear change in the pulsar signature and a possible change in the dipole spin-down torque. The 2016 glitch is the first glitch observed from the Vela pulsar with a glitch precursor associated increase in the spin-down rate as well as a change in the pulse morphology. Bransgrove et al. (2020) have pursued the idea of a crust breaking trigger for the 2016 Vela glitch and presented its consequences for the magnetospheric changes. In PSR J1119–6127 the 2007 glitch also led to magnetospheric activity change but in that case additional pulse components emerged after reactivation of the radio emission, a behaviour extending to about three months. Akbal et al. (2015) interpreted peculiarities associated with the 2007 glitch of PSR J1119–6127 as a result of crust breaking extending to the surface in the polar cap which brings about magnetospheric activity changes by twisting of the magnetic field lines on the scale of the broken plate motion, 10 - 100 m. This large length scale disturbance was associated by Akbal et al. (2015) with the 3 month duration of the glitch induced transient changes (intermittency and RRAT-like behaviour). Bransgrove et al. (2020) have shown that the ~4 seconds short time-scale transient magnetospheric activity change coincident with the 2016 glitch may arise from ephemeral electron/positron discharge due to bouncing seismic waves and energy pumped into the magnetosphere at correspondingly high frequencies on short time and length scales by a quake occurring in the crust.

### 3.3 Angular momentum transfer from superfluid to normal matter in the crust and the limit on the glitch rise time

Glitches are thought to be arising from rapid angular momentum transfer from crustal superfluid to the crust mediated by vortex lines. One such mechanism is kelvon wave excitation on vortex lines and their coupling with the lattice phonons (Epstein & Baym 1992; Jones 1992). As shown by Graber et al. (2018) this mechanism leads to very effective coupling of the superfluid to the crust, consistent with the 12.6 seconds upper limit obtained by Ashton.
et al. (2019). The crustal superfluid-crustal normal matter coupling time can be expressed in terms of a drag coefficient \( R \) by

\[
\tau_{\text{mf}} = \left( \frac{1 + R^2}{R} \right) \frac{I_{\text{cs}}/I_{\text{cn}}}{2 \Omega},
\]

where \( I_{\text{cs}} \) and \( I_{\text{cn}} \) are the moments of inertia of the crustal superfluid and the normal matter in the crust, respectively. Delsate et al. (2016) have obtained the ratio \( I_{\text{cs}}/I_{\text{cn}} \approx 8.35 \). For kelvon wave coupling, the drag coefficient \( R \) can be expressed in terms of the crustal parameters as (Graber et al. 2018; Gugercino˘glu & Alpar 2016)

\[
R = 1.14 \left( \frac{v_0}{10^7 \text{ cm s}^{-1}} \right)^{-1/2} \left( \frac{a}{63 \text{fm}} \right)^{-3} \left( \frac{R_N}{7 \text{fm}} \right)^{5/2}.
\]

Here \( v_0 \) is the microscopic vortex line velocity around nuclei, \( R_N \) is the nuclear radius and \( a \) is the lattice constant. The range of \( v_0 \) values was most recently obtained by Gugercino˘glu & Alpar (2016). For instance, at baryon density \( n_B = 7.89 \times 10^{-2} \text{ fm}^{-3} \), the corresponding parameters are \( a = 29.2 \text{ fm}, R_N = 7.2 \text{ fm}, v_0 = 4.58 \times 10^6 \text{ cm s}^{-1} \) yielding \( \tau_{\text{mf-kelvon}} \approx 1.1 \text{ s} \); while for the lower density \( n_B = 3.73 \times 10^{-3} \text{ fm}^{-3} \), \( a = 81.2 \text{ fm}, R_N = 6.6 \text{ fm}, \) and \( v_0 = 6.4 \times 10^5 \text{ cm s}^{-1} \) Eqs. (5) and (6) give \( \tau_{\text{mf-kelvon}} \approx 0.14 \text{ s} \). Thus, throughout the neutron star crust kelvon waves bring about very effective coupling on time-scales of order \( \tau_{\text{mf-kelvon}} \approx 0.14 \text{ s} \). A general relativistic treatment is likely to lead to even tighter coupling (Sourie & Chamel 2020). This very fast coupling implies that we will not be able to discriminate the coupling between the crustal superfluid and the normal matter crust components through post-glitch timing observations. This is because the construction of pulse templates used in time of arrival analysis involves binning of many individual cycles. Timing analysis for the Vela pulsar is possible on time-scales longer than at least a few seconds (Palfreyman et al. 2018). Short \( \tau_{\text{mf-kelvon}} \) also sets the scale for the detection of transient gravitational wave emission associated with the glitches if this becomes feasible with future instruments (Melatos et al. 2015; Keitel et al. 2019).

Sourie & Chamel (2020) considered 12.6 seconds upper limit as the glitch rise time with the interpretation that angular momentum transfer occurs as a consequence of unpinning of the vortex lines from magnetic flux tubes in the outer core.

### 3.4 Prompt relaxation of the peak spin-up due to crust-core coupling

Pulsar glitch observations prior to the 2016 Vela glitch showed that post-glitch relaxation involves only at most a few percent of the moment of inertia of the star on time-scales longer than hours. The implication was that the bulk of the star’s moment of inertia, which lies in the core superfluid was already coupled to the crust on time-scales less than a minute. This is explained by spontaneous magnetization of vortex lines in the core neutron superfluid by dragging superconducting proton currents, leading to very effective electron scattering off spontaneous magnetized vortex lines (Alpar et al. 1984b). Thanks to the high resolution, pulse to pulse observations (Palfreyman et al. 2018) it now becomes possible to distinguish items (iii) and (iv) in Section 2. The former corresponds to the crustal superfluid-normal matter crust coupling which must have taken place within \( \tau_{\text{pert}} = 12.6 \text{ s} \). We associate the latter, the prompt \( \tau_{\text{pert}} \approx 54 \text{ s} \) decay following the initial peak of the glitch, with crust-core coupling as taking place by sharing the initial angular momentum imparted to the crustal normal matter with the core superfluid on the time-scale \( \tau_{\text{core}} \) of the Alpar et al. (1984b) mechanism.

Thus, the glitch is observed on the shortest time-scales in a sequence of angular momentum transfers between the three components: the crustal superfluid that presents a vortex unpinning event, the crust normal matter as the first recipient of the angular momentum transfer and the core superfluid which subsequently takes part in sharing of the angular momentum. The frequency increase \( \Omega(t) \) at the time of the glitch is determined from the angular momentum exchange between the crustal superfluid (with moment of inertia \( I_{\text{cs}} \)) and the crustal normal matter (with moment of inertia \( I_{\text{cn}} \)) first, and is given by

\[
\frac{\Delta \Omega(0)}{\Omega_c} = \frac{I_{\text{cs}}}{I_{\text{cn}}} \frac{\delta \Omega_c}{\Omega_c},
\]

with \( \delta \Omega_c \) being the change in the superfluid angular velocity due to unpinning of vortex lines. After the sharing of the glitch spin-up with the core superfluid on the relaxation time-scale \( \tau_{\text{core}} \) given by Eq. (9) below, the immediate post-glitch magnitude of the “effective” crust comprised of the crust and the core superfluid is (Gugercino˘glu & Alpar 2014)

\[
\frac{\Delta \Omega(t)}{\Omega_c} = \frac{f I_{\text{cs}}}{I - I_{\text{cs}}} \frac{\delta \Omega_c}{\Omega_c},
\]

where the factor \( f \approx 1 \) designates the fraction of the crustal superfluid that participated in the glitch event.

At times between glitches, the crustal superfluid where vortex lines can be pinned to nuclei [as well as the superfluid in the outer core where vortex lines can pin to a toroidal arrangement of quantized flux lines (Gugercino˘glu & Alpar 2014)] support continuous angular momentum transfer by vortex creep. After the motion of unpinned vortices during the glitch, the conditions driving creep are deeply offset leading to temporary decoupling of the creep regions in the crust (with moment of inertia \( I_{\text{cs}} \)) and the outer core (with moment of inertia \( I_{\text{tor}} \)) from the whole neutron star (with moment of inertia \( I \)) on longer time-scales.

The crust-core coupling time-scale is given by (Sidery & Alpar 2009; Alpar et al. 1984b)

\[
\tau_{\text{core}} = 3.4 \times 10^{-16} \left( \frac{P}{10^{14} \text{ g cm}^{-3}} \right)^{-1/6} \left( \frac{\delta m^*}{m_p} \right)^{1/2} \left( \frac{m_p^*}{m_p} \right)^{1/2},
\]

where \( P \) is the proton fraction, \( \rho \) is the mass density, \( m_p^* (m_p) \) is the effective (bare) proton mass, \( \delta m^* = m_p - m_p^* \), and \( P \) is the rotation period in seconds. As angular momentum transfer between the core and the observed crust is weighed by the moment of inertia \( dI = \rho(r) r^4 \sin^2(\theta) \) in spherical coordinates \((r, \theta, \phi)\) with respect to the rotation axis, the effective coupling time will be dominated by the outer core layers with \( \rho \sim 2 \times 10^{14} \text{ g cm}^{-3} \). Comparing Eq (9) with the relaxation time fitted to the observations yields \( m_p^*/m_p \approx 0.7 \) at \( \rho \geq 2 \times 10^{14} \text{ g cm}^{-3} \) in agreement with theoretical calculations in this density range. Figure 1 shows the run of \( \tau_{\text{core}}(\rho) \) using the \( m_p^*/m_p \) results of Chamel (2008) and employing the A18 + J + UXIX* equation of state (Akmal et al. 1998). The prompt relaxation time-scales discussed in item (iv) of Section 2 are indicated with arrows. Relaxation time \( \tau_{\text{pert}} = 53.96 \pm 24.02 \) s corresponds to the core superfluid with densities above \( 7.4 \times 10^{14} \text{ g cm}^{-3} \) the crust-core coupling times are shorter than the glitch rise time constrained to \( < 12.6 \text{ s} \) and thus were already coupled to the normal matter crust when the glitch was resolved. The amplitude of the prompt decay component of the glitch
I

where the angular velocity lag

\[ \Omega = \Omega_c - \Omega_c \]

exceeds the critical threshold \( \omega_c \) for unpinning of vortex lines. Collective unpinning of vortices initiates an avalanche which bring about a glitch as confirmed by numerical simulations (Warszawski & Melatos 2011; L"onnborn et al. 2019). In between glitches these vortex lines can not remain absolutely pinned. At finite temperature, vortex lines have probabilities proportional to Boltzmann factors to overcome the pinning potential barriers and jump between adjacent pinning sites, with a bias for radically outward motion, as dictated by spin-down of the pulsar under the external braking torque. The superfluid manages to spin down in the presence of pinning as a result of the flow of the vortex lines thermally activated against pinning energy barriers \( E_p \). This process is called “vortex creep” (Alpar et al. 1984a).

The dynamical coupling between the superfluid and the crust provided by vortex creep is an exponential function of the lag \( \omega \), which acts as the driving force for the vortex current, analogous to the voltage in electric circuits. Glitches due to sudden vortex unpinning (discharge) (Anderson & Itoh 1975) are analogous to capacitive discharges superposed on and interacting with the continuous process of vortex creep. In some parts of the pinned superfluid the response of vortex creep is linear in the glitch induced changes, leading to exponential relaxation (Alpar et al. 1989; Gügercinoglu & Alpar 2014).

Most parts of the pinned superfluid sustain nonlinear creep with a response that is highly non-linear in the glitch induced changes. In the conventional notation of papers on glitches and creep (Alpar et al. 1984a, 1989, 1996), the moment of inertia of the non-linear creep regions that are affected by the glitch is denoted by \( I_A \). The notation \( I_B \) is used for the moment of inertia of the vortex free “capacitive” regions surrounding the vortex traps. Like the space between capacitor plates, such vortex free regions do not sustain any vortices and therefore do not contribute to the vortex creep. As discharged vortices move through these regions in the glitch, they contribute to the angular momentum transfer to the crust, observed as the glitch, in proportion to the moment of inertia \( I_B \).

A glitch involves unpinning of a very large number of vortices whose sudden motion through the superfluid decreases the superfluid angular velocity by \( \delta \Omega \) and transfers angular momentum to the crust normal matter which spins up by \( \Delta \Omega_c \). The aftermath of the glitch is analogous to voltage drop: as the angular velocity lag decreases the creep process coupling the superfluid to the normal matter crust weakens; indeed non-linear creep can temporarily stop. With the corresponding superfluid region decoupled from the crust, the external torque is acting on less moment of inertia so that the spin-down rate increases. This behaviour persists until steady-state creep conditions are reestablished after a waiting time \( t_0 = \Omega_0/|\delta \Omega| \).

In the Vela pulsar, after the exponential recoveries are over, the observable quantities associated with the glitches are related to the vortex creep model parameters by the following basic equations (Alpar & Baykal 2006):

\[
I_c \Delta \Omega_c = (I_A/2 + I_B) \delta \Omega_c, \tag{11}
\]

\[
\frac{\Delta \Omega_c(t)}{\Omega_c} = \frac{I_A}{I_c} \left( 1 - \frac{t}{t_0} \right), \tag{12}
\]

\[
\frac{\Delta \Omega_c}{\Omega_c} = \frac{I_A \Omega_c^2}{I_c} \frac{t}{\delta \Omega_c}. \tag{13}
\]

The model parameters \( I_A, I_B, \) and \( \delta \Omega \) can be obtained from these equations on using the observed glitch parameters without making a detailed fit to the data.

Xu et al. (2019) fit the long-term post-glitch data for 416 days starting 2 days after the glitch with the following function:

\[
\Delta \nu(t) = \Delta \nu_{d1} e^{-t/t_1} + \Delta \nu_{d2} e^{-t/t_2} + \Delta \nu_p + \Delta \nu_{p1} + \Delta \nu_{p2}, \tag{14}
\]

Here \( \Delta \nu = \Delta \nu_{d1} + \Delta \nu_{d2} + \Delta \nu_p \) is the component of the glitch, excluding the prompt relaxation, as discerned and extrapolated from the data starting at \( t = 2 \) days after the glitch. Results of their data fit are included in Table 2. Their fit contains two exponential decay terms with time constants 1 and 6 days, respectively, and a decrease in |\( \nu \)|
with a constant positive $\dot{v}$. Spin-down rate evolution with constant, positive, large $\dot{v}$ is a standard feature of the inter-glitch behaviour of the Vela pulsar (Alpar et al. 1984a; Akbal et al. 2017). It is also common in the inter-glitch timing behaviour of other pulsars with large glitches (Yu et al. 2013; Alpar & Baykal 2006). Using the long term post-glitch timing fit parameters of Xu et al. (2019), we obtain from Eqs. (11), (12), and (13) the model parameter values
\[
\frac{I_A}{I_c} = (6.42 \pm 1.93) \times 10^{-3},
\]
\[
\frac{I_B}{I_c} = (1.13 \pm 0.45) \times 10^{-2},
\]
\[
\delta\Omega_e = (6.91 \pm 2.07) \times 10^{-3} \text{ rad s}^{-1}.
\]

We use these parameter values as determined from the long term timing behaviour (Xu et al. 2019) as input in a treatment of the full range of timing data (Palfreyman et al. 2018; Ashton et al. 2019) starting from the 12.6 s gap containing the actual occurrence of the glitch. In the vortex creep model the post-glitch behaviour of the spin-down rate can be expressed as (Alpar et al. 1996; Gügercioğlu 2017)
\[
\Delta \dot{\nu}(t) = -\frac{I_{\text{tor}}}{I_c} \frac{\Delta \nu}{\tau_{\text{nl}}} \exp\left[-\frac{(\tau_{\text{nl}}/t_0) \ln \left(1 + (e^{\Delta \nu/\tau_{\text{nl}}} - 1)e^{-t_0/\tau_{\text{nl}}} \right)}{1 - e^{-t_0/\tau_{\text{nl}}}}\right],
\]
where non-linear creep relaxation time-scale is given by (Alpar et al. 1984a)
\[
\tau_{\text{nl}} \equiv \frac{kT}{\frac{\Omega_e}{\Omega}} \frac{\nu_{\text{nl}}}{\nu_{\text{p}}},
\]
and the recoupling time-scale for creep against pinning to flux lines in the toroidal field region (Gügercioğlu & Alpar 2014) is
\[
\tau_{\text{tor}} \approx 60 \left(\frac{\nu_{\text{nl}}}{10^{-2} \text{ rad s}^{-1}}\right)^{-\frac{3}{2}} \left(\frac{T}{10^8 \text{ K}}\right)^{-\frac{1}{2}} \left(\frac{R}{10^6 \text{ cm}}\right)^{\frac{1}{2}} \left(\frac{m_p}{m_p}\right)^{-\frac{1}{2}} \left(\frac{\rho}{10^{14} \text{ g cm}^{-3}}\right)^{-\frac{1}{2}} \left(\frac{B_\phi}{10^{14} \text{ G}}\right)^{\frac{1}{2}} \times
\]
\[
\left(\frac{1}{10^{-10} \text{ rad s}^{-2}}\right)^{-\frac{1}{2}} \times \left(\frac{1}{10^8 \text{ K}}\right)^{-\frac{1}{2}} \left(\frac{R}{10^6 \text{ cm}}\right)^{-\frac{1}{2}} \left(\frac{m_p}{m_p}\right)^{-\frac{1}{2}} \left(\frac{\rho}{10^{14} \text{ g cm}^{-3}}\right)^{-\frac{1}{2}} \left(\frac{B_\phi}{10^{14} \text{ G}}\right)^{\frac{1}{2}} \times
\]
\[
with B_\phi is the magnitude of the toroidal component of the magnetic field. In Eq. (18) the second term reduces to $\dot{\nu} t$ for $t \geq \tau_{\text{nl}}$. The exact expression in Eq. (18) provides a better fit at intermediate time-scales $t \leq \tau_{\text{nl}} \approx 45$ days. The fit to the data is shown in Figure 2. Model parameter obtained from the data are shown in Table 2.

$I_B/I_c$ is interpreted as the fractional size of the vortex free region. This parameter inferred from post-glitch timing data agrees with our estimate above, in Eq. (3), for the new vortex trap formed just before the glitch, obtained from the pre-glitch slow-down episode. The total crustal superfluid moment of inertia participating in the glitch is $I_c = I_A + I_B \approx 1.69 \times 10^{-2} I_c$.

\section{Discussion and Conclusion}

The vortex unpinning and creep model for pulsar glitches involves angular momentum exchange between three components of the neutron star. The glitch itself is due to sudden unpinning and outward motion of vortices in the crustal superfluid, as a result of an instability which is still not understood in detail. This event first transfers angular momentum to the crust normal matter, a solid lattice of nuclei which can pin and interact with vortices. The angular momentum is then shared with the core superfluid via electrons. After these two short time-scale angular momentum exchanges the core superfluid is effectively part of the crust and normal matter system to which it is tightly coupled. Finally the vortex creep process, offset by the glitch induced changes in the rotation rates of the crust superfluid and the effective crust, relaxes back towards steady-state creep on post-glitch and inter-glitch timescales of hours to a few years.

In all glitches observed from the Vela and other pulsars prior to the Vela pulsar’s 2016 glitch, the initial angular momentum transfer from the crustal superfluid to the crust normal matter, and then to the core superfluid were not resolved by the observations, which were modelled in terms of only two components, the crustal superfluid and the effective crust including the core superfluid. Observation of the 2016 Vela glitch by Palfreyman et al. (2018) and its subsequent reanalysis by Ashton et al. (2019) resolved the early timing signatures of the glitch for the first time.

Just prior to the 2016 glitch the rotation rate of the Vela pulsar was found to decrease. We have evaluated this observation as an

\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameter & Value & Reference \\
\hline
$\Delta \nu (\text{Hz})$ & 1.60085(9)\times 10^{-5} & Xu et al. (2019) \\
$\Delta \nu (\text{s}^{-2})$ & -1.0(3)\times 10^{-3} & Xu et al. (2019) \\
$\Delta \nu (\text{s}^{-3})$ & 1.416(13)\times 10^{-21} & Xu et al. (2019) \\
$\Delta \nu_{\text{gt}} (\text{Hz})$ & 7.7(5)\times 10^{-8} & Xu et al. (2019) \\
$\tau_{\text{gl}} (\text{days})$ & 0.96(17) & Xu et al. (2019) \\
$\tau_{\text{gt}} (\text{days})$ & 6.05(7)\times 10^{-8} & Xu et al. (2019) \\
$t_B (\text{days})$ & 6.0(5) & Xu et al. (2019) \\
$t_B (\text{days})$ & 782 & Gancio et al. (2020) \\
$I_A/I_c$ & 3.37\times 10^{-3} & this work \\
$I_B/I_c$ & 1.35\times 10^{-2} & this work \\
$I_{\text{tor}}/I_c$ & 45 & this work \\
$t_0 (\text{days})$ & 9 & this work \\
$t_0 (\text{days})$ & 781 & this work \\
\hline
\end{tabular}
\caption{Post-glitch parameter values from data fit with Eq.(14) by Xu et al. (2019) and inferred parameters from Eq. (18) with the vortex creep model. The vortex creep model fit to the data from Xu et al. (2019) is shown in Figure 2.}
\end{table}
indicating formation of new vortex traps by a crustquake which thereafter triggers the vortex unpinning avalanche that constitutes the glitch. Around the glitch ephemeral changes in the electromagnetic signature were detected, which we interpret as the aftermath of the crust breaking event on the magnetospheric conditions.

The 2016 Vela glitch has revealed the best constraint on the spin-up time-scale of the observed crust, placing an upper limit of 12.6 seconds for the glitch rise time. We have interpreted this as an upper limit on the time-scale of angular momentum transfer from the unpinned vortices to the nuclei forming the crust solid. After the peak glitch spin-up the crustal rotation rate promptly relaxed within a minute. This is interpreted as the gradual coupling of the core superfluid to the normal matter crust via the electrons’ scattering off magnetised vortex lines.

These observations yield important information on the neutron star internal structure and dynamics. With the 2016 Vela pulsar glitch we are able for the first time to discuss the sequence of angular momentum transfer between the three components, the crustal superfluid, the crust normal matter (and electrons throughout the star), and the core superfluid. We have taken the sequence of events in chronological order, starting with (i) the formation of a vortex trap that triggered the glitch, proceeding through (ii) short-lived magnetospheric changes associated with crust breaking at the glitch, which (iii) transfers angular momentum from crustal superfluid to the solid crust via the interactions of unpinned vortices with the nuclei forming the crust lattice. This is followed by (iv) the gradual coupling of the core superfluid to the crust and finally by (v) the longer term relaxation of the crustal superfluid, which is the only process addressed by modelling the post-glitch and interglitch timing data for previous glitches. We make the following inferences for this sequence of developments:

(i) There are strong indications that formation of new vortex traps, coupled to breaking of the solid crust, may be triggering the vortex unpinning glitch. Gitch associated changes in the electromagnetic signature of PSR J1119-6127 (Akbal et al. 2015) and the early timing data from the largest Crab pulsar glitch (G{"u}gercinoğlu & Alpar 2019) were interpreted in terms of such crust breaking. There are no previous indications of a trigger event in glitch associated timing data of the Vela pulsar prior to the 2016 glitch. But the precursor slow-down event extending over 100 seconds just prior to this glitch (Ashton et al. 2019) is below the resolution of timing observations for the previous glitches. We have evaluated this pre-glitch decrease in the crustal rotation rate in terms of formation of a new vortex trap which initiated the glitch. In a crustal superfluid region vortex creep is continuously transferring angular momentum to the crust. If a vortex trap forms as a result of some superfluid dynamical instability and/or an agent like crust breaking in a quake, then the angular momentum transfer from this new trap ceases irreversibly so that a decrease in the crustal rotation rate follows. The time-scale \( \tau_v = \left( \frac{\ell_v}{\ell_0} \right) \exp \left( \frac{D_0}{k T} \right) \) of thedecline is the time-scale of re-establishing the pinned vortex distribution, estimated as the vortex transit time over the inter-vortex spacing \( \ell_v \) at the microscopic vortex speed \( v_0 \) taking into account the pinning and unpinning process. In the inner crust, for \( \rho \lesssim 10^{14} \text{ g cm}^{-3} \) this vortex transit time agrees with \( \approx 100 \) seconds, the observed duration of the slow-down event before the glitch. The formation of new traps also triggers the unpinning event that is observed as the glitch. This means that the fluctuations in the vortex number density and local superfluid velocity arising during the vortex trap formation raise the angular velocity lag between the crustal superfluid and the crust normal matter from its steady-state creep value to the critical value for vortex unpinning and thus initiate the glitch.

Estimating the change in the local lag and in the superfluid rotation rate, together with the observed pre-glitch slow-down of the crust one obtains the fractional moment of inertia of the newly formed trap regions to be \( I_{\text{trap}}/I = 8.58 \times 10^{-3} \). Since the vortex unpinning avalanche starts at moderate depths corresponding to our density estimate of \( \rho \lesssim 10^{14} \text{ g cm}^{-3} \) and thus the outward motion of unpinned vortices does not cover the entire crust superfluid, the glitch magnitude is not expected to attain a large value. This is indeed the case as the 2016 glitch is amongst the moderate size events observed for the Vela (Xu et al. 2019).

(ii) This is the first Vela glitch resolved to display changes in electromagnetic signature of the pulsar. These changes were recently addressed by Bransgrove et al. (2020) in terms of a wave transmission model for seismic activity deep inside the crust which released energy to high frequency magnetospheric modes and induced temporary electron/positron discharge in the magnetosphere. There are very few previous examples of glitch associated changes in pulsar signature. In PSR J1119-6127 emergent additional pulse components extending to about three months were observed following the 2007 glitch. Akbal et al. (2015) interpreted these observations in terms of a quake involving crust plates extending to the surface which bring about low frequency plastic motion of the magnetic field lines on the scale size of the broken plate, \( D \sim 10 \sim 100 \) m, and their subsequent relaxation to a new configuration on a three month time-scale. For the 2016 Vela glitch a quake occurring deep inside the crust is invoked to induce high frequency oscillations leading to the observed changes of duration \( \approx 4.4 \) s in the magnetosphere, while most of the released quake energy is drained to the core (Bransgrove et al. 2020). The underlying physical reason for the qualitative differences in glitch associated pulsar behaviour between the 2016 Vela glitch and the 2007 glitch of PSR J1119-6127 is the location of thequake. A quake in the inner crust is indicated for the 2016 glitch so that its effects reach the surface and magnetospheric field lines anchored in the surface via high frequency elastic waves as envisaged by Bransgrove et al. (2020). This is consistent with our inference of an inner crust location, at a density \( \rho \lesssim 10^{14} \text{ g cm}^{-3} \) for the formation of vortex traps.

(iii) The observed 12.6 seconds upper limit for the glitch rise time has implications for the efficiency of angular momentum exchange mechanism coupling the crustal superfluid to the normal matter in the crust. As the unpinned vortices move through the crust, they interact with crustal nuclei and phonons via kelvons which are excitations of the vortices. We have obtained coupling times \( \tau_{\text{mf-kelvon}} \sim 0.1 \sim 1 \) s applying the results of Graber et al. (2018) in Eqs. (5) and (6) for a range of values of the microscopic vortex velocity around nuclei given by G{"u}gercinoğlu & Alpar (2016). The 12.6 s upper limit for the glitch rise time is not likely to be improved substantially in future glitches since timing analysis is fundamentally limited by the requirement of pulse templates to be constructed from a train of many individual pulses. The coupling time \( \tau_{\text{mf-kelvon}} \sim 0.1 \sim 1 \) s is therefore unlikely to be resolved in glitches electromagnetically. An interesting speculation is that \( \tau_{\text{mf-kelvon}} \sim 0.1 \sim 1 \) s might be detected with future detectors if it sets the scale for transient gravitational wave emission associated with glitches.

(iv) The peak glitch spin up observed at 12.6 s after the last pre-glitch data continues to relax as the crustal superfluid-normal matter system transfers angular momentum back to the core superfluid. The coupling is mediated by magnetized vortex-electron scattering (Alpar et al. 1984b). The coupling time \( \tau_{\text{core}} \) as a function of density is shown in Figure 1. We find that the core superfluid regions with densities \( \gtrsim 7.4 \times 10^{14} \text{ g cm}^{-3} \) with coupling times.
toroidal flux lines, pinning and creep. Eq. (20) inferred for the superfuid (inner) core where there are no τ from the 2016 to the 2019 glitch quite well. The non-linear creep implies I net = ∆I τ / τ ≈ Φ τ = ∆I τ / τ. This form is an approximation which is not valid on the short time-scales within a few days after the glitch. In the 9 early Vela glitches the uncertainties in the date of the glitch were a few days or longer and immediate post-glitch data were sparse or lacking. Applying the same terms employed by Chau et al. (1993) to the 2016 Vela glitch does not fit the earliest data points well, and also leads to an increase in moments of inertia of the corresponding creep components that is hard to accommodate with crustal superfluid alone. We have found that the exact non-linear creep response given in Eq. (18) plus a single exponentially decaying component with time constant τ = 9 days satisfactorily describes the 2016 post-glitch data. We associate this term with the response of vortex creep against toroidal flux lines of the proton superconductor in the outer core (Gügercinoğlu 2017). The amplitude of this term constrains the extent of the toroidal field region to a fractional moment of inertia of I τ / I = 1.2 × 10^−2. The 9 day relaxation time gives information about the interaction between the superfluid vortex lines and the magnetic flux lines. The dominant response of the core superfluid-proton superconductor system to the long term post-glitch recovery comes from the regions with ρ ≲ 8 × 10^14 g cm^−3. We estimate the critical lag for unpinning of vortex lines from magnetic flux tubes as δτ = 0.08 rad s^−1 and vortex-flux tube pinning energy per junction as E = 2 MeV (Gügercinoğlu & Alpar 2016). The range of densities inferred for the outer core toroidal field region is consistent with the range ρ ≳ 3 × 10^15 g cm^−3 from Eq. (20) inferred for the superfluid (inner) core where there are no toroidal flux lines, pinning and creep.

The non-linear creep response term yields the time-scale τ = 45 days, and the time τ = 781 days to the next glitch. This estimate matches the observed interval τ = 782 days (Gancio et al. 2020) from the 2016 to the 2019 glitch quite well. The non-linear creep relaxation time T = 45 days for Ep = 0.17 MeV and kT = 6.5 keV implies δτ = 0.01 rad s^−1 by Eq. (19), characterizing conditions for superweak pinning in the inner crust (Alpar et al. 1989).

The total crustal superfuid moment of inertia that participated in the glitch is I / I = 5.2 × 10^−2, below the constraint brought by crustal entrainment effect if the crust is crystalline even for quite large mass entrainment enhancement factor < m_e / m_n = 5.1 (Delsate et al. 2016), for a 1.4Ms neutron star with a thick crust (Basu et al. 2018). [If the neutron star crust is actually disordered the entrainment effect for the crust superfluid does not bring a significant constraint on the crustal superfuid moment of inertia (Sauls et al. 2020).]

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