Linkage of Dirac Neutrinos to Dark U(1) Gauge Symmetry

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Abstract

It is shown how a mechanism which allows naturally small Dirac neutrino masses is linked to the existence of dark matter through an anomaly-free U(1) gauge symmetry of fermion singlets.
Introduction: There is a known mechanism since 2001 [1] for obtaining small Dirac fermion masses. It was originally used [1] in conjunction with the seesaw mechanism for small Majorana neutrino masses, and later generalized in 2009 [2]. It has also been applied in 2016 [3] to light quark and lepton masses.

The idea is very simple. Start with the standard model (SM) of quarks and leptons with just one Higgs scalar doublet \( \Phi = (\phi^+, \phi^0) \). Add a second Higgs scalar doublet \( \eta = (\eta^+, \eta^0) \) which is distinguished from \( \Phi \) by a symmetry yet to be chosen. Depending on how quarks and leptons transform under this new symmetry, \( \Phi \) and \( \eta \) may couple to different combinations of fermion doublets and singlets. These Yukawa couplings are dimension-four terms of the Lagrangian which must obey this new symmetry.

In the Higgs sector, this new symmetry is allowed to be broken softly or spontaneously, such that \( \langle \eta^0 \rangle = v' \) is naturally much smaller than \( \langle \phi^0 \rangle = v \). The mechanism is an analog of the well-known Type II seesaw for neutrino mass. Consider for example the case where the new symmetry is global U(1) which is broken softly. Let

\[
V = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + [\mu^2 \Phi^\dagger \eta + H.c.] + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi),
\]

where \( \mu^2 \) is the soft symmetry breaking term, then \( v, v' \) are determined by

\[
0 = v [m_1^2 + \lambda_1 v^2 + (\lambda_3 + \lambda_4)v'^2] + \mu^2 v',
\]

\[
0 = v' [m_2^2 + \lambda_2 v'^2 + (\lambda_3 + \lambda_4)v^2] + \mu^2 v.
\]

For \( m_1^2 < 0 \) but \( m_2^2 > 0 \) and \( |\mu^2| << m_2^2 \), the solutions are

\[
v^2 \simeq -\frac{m_1^2}{\lambda_1}, \quad v' \simeq \frac{-\mu^2 v}{m_2^2 + (\lambda_3 + \lambda_4)v'^2},
\]

implying thus \( |v'| << |v| \). In Ref. [1], the new symmetry is taken to be lepton number, under which \( \eta \) has \( L = -1 \) but \( \nu_R \) has \( L = 0 \). This choice forbids \( \bar{\nu}_R(\nu_L \phi^0 - l^-_L \phi^+) \), but allows
\[ \nu_R(\nu_L \eta^0 - l_L \eta^+) \]. Hence \( \nu_L \) pairs up with \( \nu_R \) to have a small Dirac mass through \( \nu' \), but \( \nu_R \) itself is unprotected by any symmetry so it may have a large Majorana mass \( M \). The end result is again a small Majorana neutrino mass proportional to \( \nu'^2 / M \). The difference is that \( \nu' \) is naturally small already, so \( M \) does not have to be much greater than the electroweak scale.

To explore further this mechanism, it is proposed that this new symmetry is gauged and that it enforces neutrinos to be Dirac fermions and requires the addition of neutral singlet fermions which become members of the dark sector, the lightest of which is the dark matter of the Universe.

**Dark U(1) Gauge Symmetry**: The minimal particle content of the SM has only \( \nu_L \), not \( \nu_R \), and only the Higgs doublet \( \Phi \). Hence neutrinos are massless. Knowing that they should be massive \[4\], the usual remedy is to add \( \nu_R \) and to assume that it pairs up with \( \nu_L \) through \( \phi^0 \). However, since \( \nu_R \) is a particle outside the SM gauge framework, it has many possible different guises \[5\]. Here it will be assumed that it transforms under a new \( U(1)_D \) gauge symmetry, whereas all SM particles do not. The linkage of \( \nu_R \) to the SM is achieved through a second Higgs doublet \( \eta \) which transforms under \( U(1)_D \) in the same way as \( \nu_R \). Using Eq. (4) with a very large \( m_2 \), a sufficiently small \( \nu' \), say of the order 1 eV, may be obtained for a realistic Dirac neutrino mass.

To be a legitimate and viable theory of Dirac neutrinos in this framework, there are two important conditions yet to be discussed. First, the gauge \( U(1)_D \) symmetry must not be broken in such a way that \( \nu_R \) gets a Majorana mass. Second, there must be additional fermions so that the theory is free of anomalies. The two conditions are also connected because the additional fermions themselves must also acquire mass through the scalars which break \( U(1)_D \).

Since only the new fermions transform under \( U(1)_D \), the two conditions for anomaly
freedom are
\[ \sum_{i=1}^{N} n_i = 0, \quad \sum_{i=1}^{N} n_i^3 = 0, \] (5)
comprising of \( N \) singlets, with \( N \) to be determined. There are some simple solutions:

- (A) \( (3, -2, -2, -2, -2, 1, 1, 1, 1) \).
- (B) \( (4, -3, -3, -3, 2, 2, 1) \).
- (C) \( (5, -4, -4, 1, 1, 1) \).
- (D) \( (6, -5, -5, 3, 2, -1) \).

In the next two sections, solutions (B) and (C) will be examined in more detail because they allow both Dirac neutrinos and an associated dark sector in a consistent framework. Solution (D) will be mentioned briefly.

**Solution (B):** In addition to the seven singlet fermions listed, this scenario requires just the addition of \( \eta \sim 1 \) and a Higgs singlet \( \chi \sim 1 \) under \( U(1)_D \). Consider first the three fermion singlets \( (4, 2, 2) \). They are not connected to one another because there are no scalar singlets transforming as 8, 6, or 4. Consider then \( (-3, -3, -3) \). They are also not connected because \(-6\) is missing. However, \( (4, 2, 2) \) is connected to \(-3, -3, -3\) through \( \chi \sim 1 \) or \( \chi^* \sim -1 \). This means that they form three massive Dirac fermions of magnitude determined by \( \langle \chi \rangle = u \). As for the remaining singlet, it should be identified as \( \nu_R \sim 1 \). It pairs with \( \nu_L \) through \( \eta \sim 1 \). Because there is no scalar transforming as 2, \( \nu_R \) does not get a Majorana mass. It also cannot connect with the other singlets \( (4, 2, 2) \) or \((-3, -3, -3) \). This means that because of the chosen particle content of the model, there are two residual symmetries after the spontaneous breaking of \( U(1)_D \), i.e. the usual lepton number and dark number under which \( (4, 2, 2) \sim 1 \) and \((-3, -3, -3) \sim -1 \).
The analog of Eq. (1) is

\[
V = m_1^2 \Phi \dagger \Phi + m_2^2 \eta \dagger \eta + m_3^2 \chi \dagger \chi + [\mu \chi \dagger \eta + H.c.]
\]
\[
+ \left( \frac{1}{2} \lambda_1 (\Phi \dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\eta \dagger \eta)^2 + \frac{1}{2} \lambda_3 (\chi \dagger \chi)^2 + \lambda_{12} (\Phi \dagger \Phi)(\eta \dagger \eta)
\right)
\]
\[
+ \lambda_{12} (\Phi \dagger \eta)(\Phi \dagger \Phi) + \lambda_{13} (\Phi \dagger \Phi)(\chi \dagger \chi) + \lambda_{23} (\eta \dagger \eta)(\chi \dagger \chi).
\]

(6)

The analog of Eq. (4) is then

\[
v^2 \simeq -\frac{\lambda_3 m_1^2 + \lambda_{13} m_3^2}{\lambda_1 \lambda_3 - \lambda_{13}^2}, \quad u^2 \simeq -\frac{\lambda_1 m_2^2 + \lambda_{13} m_1^2}{\lambda_1 \lambda_3 - \lambda_{13}^2}, \quad v' \simeq \frac{-\mu u v}{m_2^2 + \lambda_2 u^2 + (\lambda_{12} + \lambda_{12}')u^2}.
\]

(7)

As an example, let \( \mu \sim 10 \text{ GeV}, u \sim 2 \text{ TeV}, m_2 \sim 10^8 \text{ GeV} \), then \( v' \sim 0.5 \text{ eV} \), which is of the order of neutrino masses. Since \( m_2^2 > 0 \) and large, \((\eta^+, \eta^0)\) are very heavy. After the spontaneous breaking of \( SU(2)_L \times U(1)_Y \) and \( U(1)_D \), the only physical scalars are \( h = \sqrt{2} \text{Re}(\phi^0) \) and \( H = \sqrt{2} \text{Re}(\chi) \). They form the mass-squared matrix

\[
\mathcal{M}_{hh}^2 = \begin{pmatrix}
2\lambda_1 v^2 - \mu u v'/v & 2\lambda_{13} u v + \mu v' \\
2\lambda_{13} u v + \mu v' & 2\lambda_3 u^2 - \mu u v'/u
\end{pmatrix}.
\]

(8)

Assuming that \( u^2 >> v^2 \), the \( h - H \) mixing is \((\lambda_{13}/\lambda_3)(v/u)\).

Let the Dirac fermion \( \psi \) be the lightest linear combination of the \((4, 2, 2)\) and \((-3, -3, -3)\) dark fermions, with coupling to \( H \) given by \( f H \bar{\psi} \psi \), implying thus \( m_\psi = \sqrt{2} f u \). Its coupling to the \( U(1)_D \) gauge boson \( Z_D \) is assumed to be \( g_D Z_D^\mu \bar{\psi} \gamma_\mu \psi \). Let \( Z_D \) be lighter than \( \psi \), then the relic abundance of \( \psi \) is determined by its annihilation to \( Z_D \), as shown in Fig. 1. This

![Figure 1: Annihilation of $\psi \bar{\psi} \rightarrow Z_D Z_D$.](image)

cross section $\times$ relative velocity is given by

$$\sigma v_{rel} = \frac{g_D^4}{16\pi m^2_\psi}(1 - m^2_D/m^2_\psi)^{3/2}(1 - m^2_D/2m^2_\psi)^{-2}. \quad (9)$$

Setting this value to the canonical $6 \times 10^{-26}$ cm$^3$/s for a Dirac fermion, and assuming $m_\psi = 1$ TeV and $m_D = 800$ GeV, it is satisfied for $g_D = 0.86$, implying $u \simeq 2$ TeV. Once produced, $Z_D$ decays quickly to two neutrinos.

As for the direct detection of $\psi$, it cannot proceed through $Z_D$ because the latter does not couple to quarks or charged leptons. It may proceed through $h - H$ mixing. For $m_\psi = 1$ TeV, the spin-independent cross section of dark matter scattering off a xenon nucleus is bounded by $10^{-45}$ cm$^2$. This puts an upper limit of $4.55 \times 10^{-22}$ on $\lambda_{13}/\lambda_3$ for $u = 2$ TeV.

**Solution (C):** The charge assignments of this scenario are well-known because $(5, -4, -4)$ is identical to $(-1, -1, -1)$ in terms of $\sum n_i$ and $\sum n_i^3$. Hence the former has been used to replace the latter as $B - L$ for three families of quarks and leptons, so that $B - L$ remains anomaly-free. It was first pointed out in 2009 [7] and became the topic of some recent studies [8, 9, 10]. Here they refer only to $U(1)_D$ under which the SM fermions do not transform.

The scalars required for this solution are a second doublet $\eta \sim -4$, and two singlets $\chi_1 \sim 2$ and $\chi_2 \sim 6$. The $(-4, -4)$ fermions are identified as $\nu_R$, so they obtain Dirac masses through $\eta$, again with small $\langle \eta^0 \rangle = v'$. The analog of Eq. (6) is

$$V = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + m_3^2 \chi_1^\dagger \chi_1 + m_4^2 \chi_2^\dagger \chi_2$$

$$+ [f_1 \chi_1^2 \Phi^\dagger \eta + f_2 \chi_2 \chi_1^* \Phi^\dagger \eta + f_3 \chi_2^* \chi_3^3 + H.c.]$$

$$+ \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\eta^\dagger \eta)^2 + \frac{1}{2} \lambda_3 (\chi_1^* \chi_1)^2 + \frac{1}{2} \lambda_4 (\chi_2^* \chi_2)^2$$

$$+ \lambda_{12}(\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_{13}(\Phi^\dagger \eta)(\eta^\dagger \Phi) + \lambda_{14}(\Phi^\dagger \Phi)(\chi_1^* \chi_1) + \lambda_{14}(\Phi^\dagger \Phi)(\chi_2^* \chi_2)$$

$$+ \lambda_{23}(\eta^\dagger \eta)(\chi_1^* \chi_1) + \lambda_{24}(\eta^\dagger \eta)(\chi_2^* \chi_2) + \lambda_{34}(\chi_1^* \chi_1)(\chi_2^* \chi_2). \quad (10)$$
Hence

\[ v' = \frac{-(f_1 u_1 + f_2 u_2)u_1 v}{m_2^2 + \lambda_{23} u_1^2 + \lambda_{24} u_2^2 + (\lambda_{12} + \lambda'_{12}) v^2} \]  

is again suppressed for large \( m_2^2 > 0 \).

The \( 4 \times 4 \) fermion mass matrix spanning \((5,1,1,1)\) has 9 nonzero entries from \( u_1 \) and 6 from \( u_2 \). It is entirely disjoint from \((-4,-4)\). This means that its lightest Majorana mass eigenstate \( \zeta \) is a dark-matter candidate, stabilized with an odd dark parity. After the spontaneous breaking of \( SU(2)_L \times U(1)_Y \) and \( U(1)_D \), excepting the heavy \( \eta \), there are the \( h \) and \( H \) scalars as in Solution (B) as well as another scalar \( H' \) and a pseudoscalar \( A \), corresponding to the linear combination of \((u_2 \chi_1 - u_1 \chi_2)/\sqrt{u_1^2 + u_2^2}\).

Let \( s = \sin \theta = u_1/\sqrt{u_1^2 + u_2^2} \) and \( c = \cos \theta = u_2/\sqrt{u_1^2 + u_2^2} \), with

\[ \lambda'_3 = s^4 \lambda_3 + c^4 \lambda_4 + 2s^2 c^2 \lambda_{34} + 4s^3 cf_3, \quad \lambda'_{13} = s^2 \lambda_{13} + c^2 \lambda_{14}. \]

Then the analog of Eq. (8) is

\[ \mathcal{M}_{hH}^2 = \begin{pmatrix} 2\lambda_1 v^2 & 2\lambda'_{13} uv \\ 2\lambda'_{13} uv & 2\lambda'_3 u^2 \end{pmatrix}. \]  

The \( h - H \) mixing is \((\lambda'_{13}/\lambda'_3)(v/u)\) and the trilinear \( H^3 \) coupling is \((\lambda'_3/\sqrt{2})uH^3\).

Assuming that \( H \) is lighter than the dark-matter Majorana fermion \( \zeta \), the annihilation of \( \zeta \zeta \to HH \) is shown in Fig. 2. The first diagram is also accompanied by its \( u \)-channel

![Figure 2: Annihilation of \( \zeta \zeta \to HH \).](image)
counterpart, which has the same amplitude in the limit that $\zeta$ is at rest. Let $m_\zeta = fu\sqrt{2}$ and $x = m_H/m_\zeta$, then this cross section at rest multiplied by relative velocity is

$$\sigma \times v_{rel} = \frac{f^4\sqrt{1-x^2}}{128\pi m_\zeta^2} \left[ \frac{2}{2-x^2} - \frac{3x^2}{4-x^2} \right]^2. \quad (14)$$

As an example, let $m_\zeta = 1$ TeV and $m_H = 400$ GeV, then the canonical value of $3 \times 10^{-26}$ cm$^3$/s is obtained for $f = 1.05$. This implies $u = 673$ GeV. The limit on $\lambda'_{13}/\lambda'_3$ is then $2.7 \times 10^{-3}$ from XENON data [6]. In this scenario, the $Z_D$ gauge boson has $m_D^2 = 8g_D^2(s^2 + 9c^2)u^2$, so $m_D$ is of order a few TeV.

**Solution (D):** Just as in (B), only one new Higgs doublet $\eta \sim -5$ and one singlet $\chi \sim -5$ are required. The $(-5, -5)$ fermions are identified as $\nu_R$. The $(6, -1)$ and $(3, 2)$ pairs obtain independent masses from $\chi$, so there are two dark-matter components with two stabilizing symmetries.

**Concluding Remarks:** There are two often raised theoretical objections to having a Dirac neutrino. (1) The singlet right-handed neutrino $\nu_R$ is trivial under the SM gauge symmetry, so it should have a Majorana mass. (2) If a symmetry is invoked to forbid (1), then there is still no explanation as to why the Dirac neutrino mass is so small. Here the answers are (1) that there is indeed a symmetry, i.e. a dark $U(1)_D$ gauge symmetry for $\nu_R$ but not the other SM particles, and (2) a small $v'$ from a second Higgs doublet $\eta$ transforming as $\nu_R$ under $U(1)_D$ is the source of this small Dirac mass. It is obtained naturally by a (Type II) seesaw mechanism first pointed out in Ref. [1]. To implement this idea that Dirac neutrino mass is linked to a dark $U(1)_D$ gauge symmetry, a set of singlet fermions is required so that the theory is free of anomalies. The scalars which are used to break $U(1)_D$ must be such that all fermions acquire mass, and two residual symmetries must remain: one is the usual lepton number, the other is a stabilizing dark symmetry.

Three examples are presented. In (B), one singlet fermion is identified as $\nu_R$ whereas the other six form three dark Dirac fermions. The stabilizing symmetry is global U(1). In
(C), two singlet fermions are identified as $\nu_R$ whereas the other four are Majorana fermions. The dark symmetry is $Z_2$ parity. In (B), the lightest dark Dirac fermion $\psi$ annihilates to the $U(1)_D$ gauge boson $Z_D$ to establish its relic abundance. In (C), it is the lightest dark Majorana fermion $\zeta$ annihilating to the $U(1)_D$ breaking scalar $H$. In both cases, direct-search constraints put an upper limit on $h - H$ mixing of order $10^{-4}$. In (D), dark matter consists of two separate Dirac fermion components.

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