A Few Notes on Non-perturbative Parameters
in Heavy Quark Expansion

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Abstract
Non-perturbative parameters to the order $O(\frac{1}{m_Q})$ in HQET expansion, $\overline{\Lambda}$, $\lambda_1$ and $\lambda_2$, as well as the masses, $m_b$ and $m_c$, are estimated phenomenologically from $B$ and $D$ meson system spectroscopy. We found relatively large value of $m_b$ and quite small value of $\overline{\Lambda}$ compared to previous estimates.

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The heavy quark effective theory (HQET) provides a systematic expansion in $\frac{1}{m_Q}$ to study physics of heavy hadrons containing a heavy quark \[1\], and has been applied to various area of phenomenology \[2, 3\]. In $m_Q \to \infty$ limit the effective theory has a spin–flavor symmetry (heavy quark symmetry). Applied to the heavy hadron spectra, the symmetry leads to sum rules relating masses of heavy hadrons with different heavy flavors. E.g. using the sum rules, the masses of some beauty hadrons have been predicted in terms of the masses of known charm hadrons \[4\].

The observed mass splitting of the ground-state spin doublet is 141 (or 46) MeV for $D$ (or $B$) mesons. It implies that the $\frac{1}{m_Q}$ corrections to the leading order are important for the realistic heavy hadron spectra, especially, for charm hadrons. Furthermore, the parameters of the order $O(\frac{1}{m_Q})$ represent the contributions from the kinetic and chromomagnetic operator \[4\] and are the fundamental parameters of HQET, which play a crucial role in the description of power corrections to the leading order. For example, the leading power corrections to $B \to X_q + e\bar{\nu}$ decay rate are completely determined in terms of the parameters \[3\], which parameterize the nonperturbative effects. However, unfortunately, the parameters have not been calculated theoretically in an unambiguous way. Therefore, to estimate them from phenomenological analyses is quite meaningful. In this short note we study the spectra of heavy mesons to the order $O(\frac{1}{m_Q})$ in the framework of heavy quark expansion, and estimate the parameters of the order $O(\frac{1}{m_Q})$ from phenomenological analyses.

From the effective Lagrangian in HQET, the mass of a heavy hadron can be written as

$$M_h = m_Q + \Lambda + \frac{1}{m_Q} (a + \langle \bar{s}_Q \cdot \vec{j}_l \rangle b),$$

where $\Lambda$ is the “mass” of light degree of freedom from the binding energy and masses of the light quarks. $a$ and $b$ are the parameters which characterize the effects of the heavy flavor symmetry breaking and spin symmetry breaking at the order $O(\frac{1}{m_Q})$, respectively:

$$\Lambda = \lim_{m_Q \to \infty} (M_h - m_Q),$$

$$a \equiv -\frac{1}{2}\lambda_1,$$

$$b \equiv 2\lambda_2.$$

Evidently $\Lambda$, $a$ and $b$ do not depend on heavy quark mass, $m_Q$. In Eq. (1) $\bar{s}_Q$ is the heavy quark spin, and $\vec{j}_l$ is the spin of the light system. The measurements of $D$ and $B$ meson masses are now quite accurate, and from Eq. (1) we have

$$\overline{M}_D \equiv \frac{1}{4}(M_D + 3M_D^*) = m_c + \Lambda + \frac{a}{m_c} \simeq (1973 \pm 2) \text{ MeV},$$

(2)
\[ \Delta M_D \equiv M_{D^*} - M_D = \frac{b}{m_c} \simeq (141 \pm 1) \text{ MeV}, \tag{3} \]

\[ \overline{M}_B \equiv \frac{1}{4}(M_B + 3M_{B^*}) = m_b + \overline{\Lambda} + \frac{a}{m_b} \simeq (5313 \pm 2) \text{ MeV}, \tag{4} \]

\[ \Delta M_B \equiv M_{B^*} - M_B = \frac{b}{m_b} \approx 46 \text{ MeV}, \tag{5} \]

where we also show the experimental values from PDG96 \cite{PDG96}.

Eqs. (2-5) have five unknown independent quantities, \( (m_c, m_b, \overline{\Lambda}, a \text{ and } b) \), with four independent observables. We choose \( m_b \) as a free parameter\(^3\), so that we find the other parameters as functions of \( m_b \):

\[ m_c = r \times m_b \simeq 0.326 \times m_b, \tag{6} \]

\[ \overline{\Lambda} = \left( \overline{M}_D + \frac{\Delta \overline{M}}{1 - r} \right) - (1 + r) \times m_b \simeq (6928 \pm 2) \text{ MeV} - 1.326 \times m_b, \tag{7} \]

\[ a = r \times m_b \cdot \left( m_b - \frac{\Delta \overline{M}}{1 - r} \right) \simeq 0.326 \times m_b \cdot (m_b - (4955 \pm 3) \text{ MeV}), \tag{8} \]

\[ b = \Delta M_B \times m_b \simeq 46 \text{ MeV} \times m_b, \tag{9} \]

where

\[ r \equiv \frac{m_c}{m_b} = \frac{\Delta M_B}{\Delta M_D} \simeq 0.326, \]

\[ \Delta \overline{M} \equiv \overline{M}_B - \overline{M}_D \simeq (3340 \pm 2) \text{ MeV}. \]

It is obvious that Eqs. (6-9) are not the definitions of the parameters, but phenomenological relations satisfying the experimental results of Eqs. (2-4).

If we fix \( m_b = 4800 \text{ MeV} \), which is deduced from a QCD analysis of the \( \Upsilon \) system, we get

\[ m_c \simeq 1565 \text{ MeV}, \quad \overline{\Lambda} \simeq 563 \text{ MeV}, \quad a \simeq -155 \text{ MeV} \times m_c, \quad b \simeq 0.221 \text{ GeV}^2. \]

Note that the value of \( a \) becomes negative. This negative value for \( a \), certainly, can not be acceptable because the parameter \( a \) is essentially the expectation value of the kinetic energy operator of the heavy quark in the rest frame of the hadron in HQET \cite{HQET1, HQET2}, and should be positive. This implies that the effective mass of the \( b \) quark inside the \( B \) meson is different, probably within order \( O(\Lambda_{QCD}) \), from that inside the \( \Upsilon \) system if the expansions of heavy hadron masses in \( \frac{1}{m_Q} \) are valid.

In Fig. 1 we show the values of \( m_c \) (in GeV), \( \overline{\Lambda} \) (in GeV), \( a \equiv -\frac{1}{2}\lambda_1 \) (in GeV\(^2\)) and \( b \equiv 2\lambda_2 \) (in GeV\(^2\)) as functions of \( m_b \). As shown in Eq. (8), only for \( m_b > 4958 \text{ MeV} \) the

\(^3\)Here the parameter \( m_b \) is phenomenologically defined only through Eq. (1) as an expansion parameter. It might be a pole mass or a running mass or else depending on given extra assumptions. In any case, however, it together with other parameters must satisfy Eq. (1) and the experimental constraints of Eqs. (2-5).
value of $a$ becomes positive. Similarly, only for $m_b < 5223$ MeV the value of $\Lambda$ can be positive from Eq. (7). The shaded region, $4958 < m_b < 5223$ (in MeV), represents for both $a > 0$ and $\Lambda > 0$. From the shaded region we can find the region of parameter space allowed in HQET expansion to the order $\mathcal{O}(\frac{1}{m_Q})$:

$$m_b = [4958, 5223] \text{ MeV},$$

$$m_c = [1616, 1703] \text{ MeV} \quad \text{(or} \quad (m_b - m_c) = [3342, 3520] \text{ MeV}),$$

$$\Lambda = [0, 356] \text{ MeV},$$

$$a \equiv -\frac{1}{2} \lambda_1 = [0, 0.461] \text{ GeV}^2,$$

$$b \equiv 2 \lambda_2 = [0.228, 0.240] \text{ GeV}^2.$$

[II] Previously the values of parameters have been estimated, e.g. see Ref. [7], as:

$$m_b = (4.71 \pm 0.07) \text{ GeV},$$

$$m_b - m_c = (3.39 \pm 0.04) \text{ GeV},$$

$$\Lambda = (0.57 \pm 0.07) \text{ GeV}, \quad \text{(see [8])}$$

$$-\lambda_1 = (0.3 \pm 0.2) \text{ GeV}^2, \quad \text{(see [9, 10])}$$

$$\lambda_2 \approx 0.12 \text{ GeV}^2.$$

Compared to our numerical bounds, Eqs. (10-14), previous estimations give relatively small value of $m_b$ and quite large $\Lambda$, however at the same time, very compatible values for $(m_b - m_c)$, $\lambda_1$ and $\lambda_2$. As is well known, the heavy quark mass difference, $(m_b - m_c)$, and spin symmetry breaking, $\lambda_2$, are precisely determined within 5% from $B$ and $D$ meson spectroscopy. There are intrinsic uncertainties for the definitions of $m_Q$, $\Lambda$ and $\lambda_1$ related to higher order perturbative corrections [4, II]. However, we note that Eqs. (1-9) are perfectly valid independent of the definition of parameters in heavy quark mass expansion up to $\mathcal{O}(\frac{1}{m_Q})$. Therefore, the bounds on the parameters, Eqs. (10-14), and the experimental constraints, Eqs. (2-5), have to be satisfied simultaneously as pre-requirements: e.g. a parameter set chosen from central values of the previous estimates

$$(m_b, m_c, \Lambda, -\lambda_1, \lambda_2) = (4.71 \text{ GeV}, 1.32 \text{ GeV}, 0.57 \text{ GeV}, 0.3 \text{ GeV}^2, 0.12 \text{ GeV}^2)$$

can not simultaneously satisfy the experimental results, Eqs. (2-5), and should be discarded as a heavy quark expansion parameter set, even though an individual parameter can have
value outside or inside the bound depending on its proper definition. We also note that if the value of $m_b$ is chosen as $\sim 4.7$ GeV, the only parameter set consistent with Eqs. (2-5) is given as

$$(m_b, m_c, \Lambda, -\lambda_1, \lambda_2) \sim (4.7 \text{ GeV}, 1.5 \text{ GeV}, 0.7 \text{ GeV}, -0.8 \text{ GeV}^2, 0.11 \text{ GeV}^2),$$

which does not satisfy the bounds at all.

Recently, it has been an important subject to obtain an accurate value of the kinetic energy, $\mu_\pi^2 (\equiv -\lambda_1)$, of the heavy quark inside $B$-meson. Ball \textit{et al.} \cite{9} calculated using the QCD sum rule approach and obtained $\mu_\pi^2 \sim 0.50$ GeV$^2$ for $B$-meson, while Neubert \cite{9} obtained $-\lambda_1 \sim 0.1$ GeV$^2$. It should be noted that those two derivations differ in the choice of the 3-point correlation functions used to estimate the matrix elements of interest. Bigi \textit{et al.} \cite{9} derived an inequality between the expectation value of the kinetic energy operator of the heavy quark inside the hadron and that of the chromomagnetic operator, $\mu_\pi^2 \geq \frac{3}{4}(M_V^2 - M_P^2)$, which gives $\mu_\pi^2 \geq 0.36$ GeV$^2$ for $B$-meson system. However, Kapustin \textit{et al.} \cite{9} showed later that this lower bound could be significantly weakened by higher order perturbative corrections to the 3-point functions. Hwang \textit{et al.} \cite{9} also calculated the value by applying the variational method to the relativistic Hamiltonian, and obtained $\mu_\pi^2 \sim 0.44$ GeV$^2$. Similarly de Fazio \cite{9} computed the matrix elements of the kinetic energy operator by means of a QCD relativistic potential model, and found $\mu_\pi^2 \sim 0.46$ GeV$^2$. Besides the theoretical calculations of $\mu_\pi^2$, Gremm \textit{et al.} \cite{10} extracted the average kinetic energy by comparing the prediction of the HQET \cite{9} with the shape of the inclusive $B \to X l \nu$ lepton energy spectrum, and obtained $-\lambda_1 = 0.19 \pm 0.10$ GeV$^2$.

If we choose the previously estimated average $\lambda_1$ value \cite{4, 9, 11}, $-\lambda_1 = (0.3 \pm 0.2)$ GeV$^2$, we can find quite narrow region of parameter space from the given experimental bounds, Eqs. (2-9), and Fig. 1:

$$m_b = (5.05 \pm 0.07) \text{ GeV}, \quad (m_b - m_c) = (3.41 \pm 0.04) \text{ GeV}, \quad \Lambda = (0.23 \pm 0.08) \text{ GeV}. \quad (15)$$

The experimental value of semileptonic decay width of $B$ meson can give the value $m_b \eta_{QCD}^{1/5}$ \cite{12}, with QCD correction $\eta_{QCD}$, by comparing with theoretical inclusive semileptonic decay width. After using $B_{sl}$ and $\tau_B$ from PDG96 \cite{9} and using $\eta_{QCD} = 0.77 \pm 0.05$ \cite{13}, we can get $m_b = (5.1 \pm 0.2)$ GeV, which is in good agreement with our result. It is interesting to note that the value of hard pole mass $m_b$ obtained by Chernyak \cite{14} is very close to that in Eq. (15). Finally we note that higher order corrections of $\mathcal{O}(\frac{1}{m_Q^2})$ can change the bounds of our estimation. However, as shown in \cite{4, 9}, the correction to $m_b$ is about $\sim \pm 0.004$ GeV,
and we can safely conclude that the higher order effect can not affect our results at all.

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Figure 1: The values of $m_c$ (in GeV), $\Lambda$ (in GeV), $a \equiv -\frac{1}{2}\lambda_1$ (in GeV$^2$) and $b \equiv 2\lambda_2$ (in GeV$^2$) as functions of $m_b$. The shaded region, $4958 < m_b < 5223$ (in MeV), represents for both $\Lambda > 0$ (see Eq. (7)) and $a > 0$ (see Eq. (8)).