The Supersymmetric Flavor Problem for Heavy First-Two Generation Scalars at Next-to-Leading Order

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Abstract

We analyze in detail the constraints on SUSY-model parameters obtained from $K - \bar{K}$ mixing in the hypothesis of a splitted SUSY spectrum. FCNC contributions from gluino-squark-quark interactions are studied in the so-called mass insertion approximation. We present boundaries on mass insertions and on SUSY mass scales. We improve previous results by including the NLO-QCD corrections to $\Delta S = 2$ effective Hamiltonian and the complete set of B-parameters for the evaluation of hadronic matrix elements. A full set of magic-numbers, that can be used for further analyses of these models, is also given. We find that the inclusion of NLO-QCD corrections and the B-parameters change the results obtained at LO and in the Vacuum Insertion Approximation by an amount of about $25 - 35\%$. 
1 Introduction

It is well known that SuperSymmetry (SUSY) introduces many new sources of Flavor Changing Neutral Currents (FCNC) which give strong constraints on the construction of extensions of the Standard Model (SM).

A common feature of these models is that FCNC effects are induced by SUSY breaking parameters that mix different flavors. In the literature several ideas have been proposed in order to suppress unwanted FCNC effects. For instance, in models where SUSY breaking is induced by gauge interactions SUSY breaking parameters are flavor blind or they are dominated by the dilaton multiplet of string theory. Alternatively, flavor symmetries are used to provide either a sufficient degeneracy between the first-two generation of sfermions or alignment between quark and squark mass matrices.

Here we want to investigate the hypothesis that the average squark mass of the first-two generations is much higher then the rest of the spectrum of (s-)particles. Throughout the paper we indicate the average mass of the heavy scalar squarks as $M_{sq}$ and the typical mass scale of gauginos and of the other light sparticles as $m_{\tilde{g}}$. Small Yukawa couplings of the first-two generations of scalars to the Higgs doublets, together with masses of the rest of the supersymmetric spectrum close to the weak scale allow a natural electroweak symmetry breaking (EWSB). This scenario has very interesting phenomenological signatures and can be easily realized in string theory.

We consider gluino-squark-mediated FCNC contributions to $\Delta M_K$ and $\epsilon_K$ in the neutral $K$-system. The effect of the most general squark mass matrix for this class of models is studied. In some cases further restrictions on the squark masses are required and other contributions can be more important. In particular chargino-squark-quark interactions should be also considered. We postpone a discussion with the inclusion of these effects to a subsequent work.

We work in the so-called mass insertion approximation. In this framework one chooses a basis for fermions and sfermions states where all the couplings of these particles to neutral gauginos are flavor diagonal and FC effects are shown by the non-diagonality of sfermion propagators. The pattern of flavor change, for the K-system, is given by the ratio

$$ (\delta_{ij})_{AB} = \frac{(m_{\tilde{d}}_{ij})^2_{AB}}{M_{sq}^2}, $$

where $(m_{\tilde{d}}_{ij})^2_{AB}$ are the off-diagonal elements of the $\tilde{d}$ mass squared matrix that mixes flavor $i, j$ for both left- and right-handed scalars $(A, B =$Left, Right), see e.g. [11]. The sfermion propagators are expanded as a series in terms of the $\delta$'s and the contribution of the first term of this expansion is considered.

The supersymmetric flavor problem consists in building viable models in which FCNC are suppressed without requiring excessive fine tuning of the parameters.

In models with a splitted spectrum of s-particles, in which the average mass of the lightest $(m_{\tilde{g}})$ is in the electroweak or TeV region, two scenarios are possible:
1. for reasonable values of $M_{sq}$, the suppression of FCNC requires small $\delta$’s values. Thus, by fixing $M_{sq}$, one can find constraints on $\delta$’s values. See refs. [11, 12] and for a very recent NLO analysis ref. [13];

2. for natural values of the $\delta$’s, say $\mathcal{O}(1)$ or order of the Cabibbo angle, $\mathcal{O}(0.22)$, one finds that the only way to get rid of unwanted FCNC effects, is by having the squarks of the first-two generations heavy enough. Thus, by fixing the $\delta$’s, one can find constraints on the minimal values for $M_{sq}$. Large values of $M_{sq}$, however, induce large values for the GUT masses of the third generation of squarks via Renormalization Group Equations (RGE). Consequently there can be fine-tuning problems for the $Z$-boson mass. We study this issue in sec. 4. This point of view was adopted in refs. [16, 17].

In the past, several phenomenological analyses were carried out, which relied on some approximations. For instance, the work of ref. [13] does not include QCD radiative corrections and makes use of Vacuum Insertion Approximation (VIA) for the evaluation of hadronic matrix elements. Leading order QCD corrections to the evolution of Wilson coefficients were, instead, considered in the papers of refs. [12, 17]. These authors found that QCD corrections are extremely important. For example in [17] the lower bound on the heavy squark mass is increased by roughly a factor three.

In this work we discuss both the cited scenarios and improve previous analyses including the Next-to-Leading Order (NLO) QCD corrections to the most general $\Delta F = 2$ effective Hamiltonian [18] and the lattice calculation of all the B-parameters appearing in the $K - \bar{K}$ mixing matrix elements that have been recently computed [20]. We find this very interesting for several reasons. First of all we find that the inclusion of these effects leads to sizeable deviation from the previous computations. The results obtained using only LO-QCD corrections and Vacuum Insertion Approximation are corrected by about $25 - 35\%$. Furthermore, the uncertainties of the final result due to its dependence on the scale at which hadronic matrix elements and quark masses are evaluated is much reduced.

Predictions for any model can be tested using the so-called magic numbers we provide. These numbers allow to obtain the coefficient functions at any low energy scale once the matching conditions are given at a higher energy scale. The magic-numbers will be useful, e.g., when a complete NLO analysis of SUSY contributions to $\Delta F = 2$ processes (which should include also chargino exchange effects) will be implemented in the future.

A complete NLO calculation should be comprehensive also of the $\mathcal{O}(\alpha_s)$ corrections to the Wilson coefficients at the scale of the SUSY masses running in the loops. So far, we miss this piece of information for gluino-squark contributions. We can argue the smallness of these corrections from the smallness of $\alpha_s$ at such scales. This uncertainty can be removed only by a direct computation.

The paper is organized as follows. In sec. 2 we introduce the formalism concerning the operator basis, the Wilson coefficients and the Renormalization Group Equations (RGE). In sec. 3 constraints on the $\delta$’s are derived. The problem of consistency of the squark spectrum

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1 The matching conditions for charged-Higgs and chargino contributions have been recently computed [22].
for given entries of the $\delta$’s is considered in sec. [4]. Finally our conclusions are written in sec. [4] and all the magic-numbers are given in the appendix.

2 Effective Hamiltonian and hadronic matrix elements

In this section we describe the framework in which the basic calculations have been performed. We follow the discussion of ref. [12] in the case $M_{sq} \gg m_{\tilde{g}}$. Throughout the paper (unless otherwise explicitly specified), we assume that the average mass of gluinos and of the squarks of the third generation are of the same order of magnitude.

The three steps needed to use the Operator Product Expansion (OPE) (matching of the effective theory, perturbative evolution of the coefficients and evaluation of hadronic matrix elements) are treated in detail in the following subsections.

2.1 Operator basis and matching of the effective theory

In order to apply the OPE one has to calculate coefficients and operators of the effective theory. One first integrates out the heavy scalars of the first-two generations at the scale $M_{sq}$. This step produces $\Delta S = 1$ (of the form $\bar{d}\tilde{g}_{\gamma}s$) as well as $\Delta S = 2$ operators, at the same order $1/M_{sq}^2$. When also gluinos are integrated out at $m_{\tilde{g}}$, $\Delta S = 1$ operators generate $\Delta S = 2$ contributions that are proportional to $m_{\tilde{g}}^2/M_{sq}^4$, and so can be neglected.

The final basis of operators is:

\[
Q_1 = \bar{d}^\alpha \gamma^\mu (1-\gamma_5) s^\alpha \bar{d}^\beta \gamma^\mu (1-\gamma_5) s^\beta, \\
Q_2 = \bar{d}^\alpha (1-\gamma_5) s^\alpha \bar{d}^\beta (1-\gamma_5) s^\beta, \\
Q_3 = \bar{d}^\alpha (1-\gamma_5) s^\beta \bar{d}^\beta (1-\gamma_5) s^\alpha, \\
Q_4 = \bar{d}^\alpha (1-\gamma_5) s^\alpha \bar{d}^\beta (1+\gamma_5) s^\beta, \\
Q_5 = \bar{d}^\alpha (1-\gamma_5) s^\beta \bar{d}^\beta (1+\gamma_5) s^\alpha, 
\]

(2)
together with operators $\tilde{Q}_{1,2,3}$ which can be obtained from $Q_{1,2,3}$ by the exchange $(1-\gamma_5) \leftrightarrow (1+\gamma_5)$.

The Wilson coefficients at the matching scale $M_{sq}$ are (see e.g. [11, 12]):

\[
C_1 = -\frac{\alpha_s^2}{216M_{sq}^2} \left( 24xf_6(x) + 66\tilde{f}_6(x) \right) (\delta_{12}^d)^2_{LL}, \\
C_2 = -\frac{\alpha_s^2}{216M_{sq}^2} 204f_6(x)(\delta_{12}^d)^2_{RL}, \\
C_3 = \frac{\alpha_s^2}{216M_{sq}^2} 36xf_6(x)(\delta_{12}^d)^2_{RL}, \\
C_4 = -\frac{\alpha_s^2}{216M_{sq}^2} \left[ (504xf_6(x) - 72\tilde{f}_6(x)) \right] (\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}
\]
\[
C_5 = -\frac{\alpha_s^2}{216M_{sq}^2} \left[ (24x f_6(x) + 120 f_6(x))(\delta_{12}^{d})_{LL}(\delta_{12}^{d})_{RR} - 180 f_6(x)(\delta_{12}^{d})_{LR}(\delta_{12}^{d})_{RL} \right],
\]

where \( x = (m_{\tilde{g}}/M_{sq})^2 \) and

\[
f_6(x) = \frac{6(1 + 3x) \ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5},
\]
\[
\tilde{f}_6(x) = \frac{6x(1 + x) \ln x - x^3 - 9x^2 + 9x + 1}{3(x - 1)^5}.
\]

The coefficients for the operators \( \tilde{Q}_{1,2,3} \) are the same as those of \( Q_{1,2,3} \) with the replacement \( L \leftrightarrow R \). The authors of refs. [12, 16, 17] use the matching coefficients directly in the limit \( x \to 0 \). However, we have contemplated also the extreme case of \( m_{\tilde{g}} \sim M_{sq}/2 \), so that we keep the whole expression. Of course, the value of the coefficients is the same as that of refs. [12, 16, 17] in cases where \( x \ll 1 \).

As we said, NLO-corrections to these coefficients have not been computed yet. We assume they are negligible, in view of the smallness of \( \alpha_s(M_{sq}) \) and of the fact that similar corrections turned out to be rather small in the SM, the two Higgs doublet model, and for the chargino contribution in the constrained MSSM [22]. Our effective Hamiltonian is so affected by a residual renormalization scheme dependence because of the missing piece of \( O(\alpha_s(M_{sq})) \) in the matching.

### 2.2 Evolution of Wilson coefficients and running of \( \alpha_s \)

In order to evolve the Wilson coefficients between \( M_{sq} \) and the scale at which hadronic matrix elements are evaluated (\( \mu=2 \text{ GeV} \)), one has to account for the presence of all particles whose mass is intermediate between the two scales, both in the \( \beta \)-function of \( \alpha_s \) and in the Anomalous Dimension Matrix (ADM) of the operators.

For what concerns the former one has [25]:

\[
\beta(\alpha_s) = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 + O(\alpha_s^4),
\]
\[
\beta_0 = \frac{1}{3} \left( 11N_c - 2n_f - 2N_c n_{\tilde{g}} - \frac{1}{2} n_{\tilde{q}} \right),
\]
\[
\beta_1 = \frac{1}{3} \left( 34N_c^2 - 13N_c^2 - 3 \right) n_f - 16N_c^2 n_{\tilde{g}} - \frac{4N_c^2 - 3}{2N_c} n_{\tilde{q}} + 3 \left( \frac{3N_c^2 - 1}{2N_c} n_{\tilde{g}} \right),
\]

where \( N_c = 3 \) for color SU(3) and \( n_f \) is the number of fermion flavors. The terms proportional to \( n_{\tilde{g}} \) and \( n_{\tilde{q}} \) represent, respectively, the gluino and light scalar contributions. \( n_{\tilde{g}}=1 \) and \( n_{\tilde{q}}=4 \) when one evolves between \( M_{sq} \) and \( m_{\tilde{g}} \) and to \( n_{\tilde{g}} = n_{\tilde{q}}=0 \) evolving from \( m_{\tilde{g}} \) to a lower mass scale.
In ref. [18] the Anomalous Dimension Matrix of the operators was computed at NLO. In that reference, since all SUSY particles are taken to be heavy, only loops with fermions and gluons were considered. This result must be modified taking into account that, from $M_{sq}$ to $m_{\tilde{g}}$, also the squarks of the third generation and gluinos can run in the loops. As a matter of fact, for the K-system, light third generation squarks and gluinos can enter two-loops ADM only via the renormalization of the gluon propagator. An explicit calculation shows that the required modification consists in considering the ADM of ref. [18] as a function of $n_f + N_c n_{\tilde{g}} + n_{\tilde{q}}/4$ when one evolves between the heavy squark and gluino mass scales and as a function of $n_f$ below the latter scale. This substitution is no more true if the squarks of the first-two generations are light too.

The value of the Wilson coefficients at the hadronic scale $\mu = 2$ GeV where matrix elements are computed can then be easily calculated. Following ref. [18] one evolves between two scales according to:

$$C(\mu) = \tilde{N}[\mu] \tilde{U}[^{\mu}, M] \tilde{N}^{-1}[M] \tilde{C}(M),$$

$$\tilde{N}[\mu] = \hat{1} + \frac{\alpha_s(\mu)}{4\pi} \hat{J}(\mu),$$

$$\tilde{U}[\mu, M] = \left[ \frac{\alpha_s(M)}{\alpha_s(\mu)} \right]^{\hat{\gamma}^{(0)}T / (2\beta_0)}.$$

(8)

where $\hat{\gamma}^{(0)}$ is the LO-ADM and $\tilde{C}(\mu)$ are the Wilson coefficients arranged in a column vector. This formula is correct up to the NLO. $\tilde{U}[\mu, M]$ gives the LO evolution already computed in ref. [12] while $\hat{J}$ gives the NLO corrections calculated in ref. [18]. $\hat{J}$ depends both on the number of active particles at the scale $\mu$, and on the renormalization scheme used for its computation. We have used $\hat{J}$ in the same scheme used for the lattice calculation of hadronic matrix elements, that is the so-called Landau-RI scheme. In this way the renormalization scheme dependence of the final result, at the scale at which hadronic matrix elements are evaluated, cancels out at this perturbative order. As already been stressed, for a complete scheme independence of our result one should include also the NLO corrections of the Wilson coefficients at the high matching scale.

We provide here the full set of Wilson coefficients at $\mu=2$ GeV as functions of $M_{sq}$ and $m_{\tilde{g}}$ (the so-called magic-numbers). We find

$$C_i(\mu) = \sum_{r,j=1}^{5} \left[ b_{ij}^{(r)} + \frac{\alpha_s(m_{\tilde{g}})}{4\pi} c_{ij}^{(r)} \right] \alpha_s^{a_r}(m_{\tilde{g}}) C_j(m_{\tilde{g}}),$$

$$C_i(m_{\tilde{g}}) = \sum_{r,j=1}^{5} \left[ d_{ij}^{(r)} + \frac{\alpha_s(m_{\tilde{g}})}{4\pi} e_{ij}^{(r)} + \frac{\alpha_s(M_{sq})}{4\pi} f_{ij}^{(r)} \right] \left( \frac{\alpha_s(M_{sq})}{\alpha_s(m_{\tilde{g}})} \right)^{a_{r}'} C_j(M_{sq}).$$

(9)

The complete expression of $a_r$, $a_r'$, $b_{ij}^{(r)}$, ..., is given in the appendix. Eq. (11) is useful for testing predictions for any model, once the two scales are fixed. The magic numbers for the evolution of $C_{1-3}$ are the same as the ones for the evolution of $C_{1-3}$. Eq. (11) and the formulae of the appendix, can be used with B-parameters evaluated at $\mu = 2$ GeV (see eq. (12)), in order to determine the contribution to $\Delta M_K$ and $\epsilon_K$ at NLO in QCD for any
model of new physics in which the new contributions with respect to the SM originate from the extra heavy particles. It is sufficient to compute the values of the coefficients at the matching scales $M_{sq}$ and $m_{\tilde{g}}$ and put them in eq. (3).

2.3 Hadronic Matrix Elements

The hadronic matrix elements of the operators of eq. (2) in the Vacuum Insertion Approximation (VIA) are:

\begin{align}
\langle K^0|Q_1|\bar{K}^0\rangle_{VIA} &= \frac{1}{3} M_K f_K^2, \\
\langle K^0|Q_2|\bar{K}^0\rangle_{VIA} &= -\frac{5}{24} \left( \frac{M_K}{m_s + m_d} \right)^2 M_K f_K^2, \\
\langle K^0|Q_3|\bar{K}^0\rangle_{VIA} &= \frac{1}{24} \left( \frac{M_K}{m_s + m_d} \right)^2 M_K f_K^2, \\
\langle K^0|Q_4|\bar{K}^0\rangle_{VIA} &= \left[ \frac{1}{24} + \frac{1}{4} \left( \frac{M_K}{m_s + m_d} \right)^2 \right] M_K f_K^2, \\
\langle K^0|Q_5|\bar{K}^0\rangle_{VIA} &= \left[ \frac{1}{8} + \frac{1}{12} \left( \frac{M_K}{m_s + m_d} \right)^2 \right] M_K f_K^2,
\end{align}

where $M_K$ is the mass of the $K$ meson and $m_s, m_d$ are the masses of the $s$ and $d$ quarks respectively. An analogous definition holds for $\tilde{Q}_{1,2,3}$.

Hadronic matrix elements can be evaluated non-perturbatively introducing B-parameters, defined as follows:

\begin{align}
\langle K^0|Q_1(\mu)|\bar{K}^0\rangle &= \frac{1}{3} M_K f_K^2 B_1(\mu), \\
\langle K^0|Q_2(\mu)|\bar{K}^0\rangle &= -\frac{5}{24} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_2(\mu), \\
\langle K^0|Q_3(\mu)|\bar{K}^0\rangle &= \frac{1}{24} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_3(\mu), \\
\langle K^0|Q_4(\mu)|\bar{K}^0\rangle &= \frac{1}{4} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_4(\mu), \\
\langle K^0|Q_5(\mu)|\bar{K}^0\rangle &= \frac{1}{12} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_5(\mu),
\end{align}

where $Q_i(\mu)$ are the operators renormalized at the scale $\mu$. The B-parameters for $\tilde{Q}_{1,2,3}(\mu)$ are the same as those of $Q_{1,2,3}(\mu)$.

In the computation of $B_i$ for the operators 2-5, smaller contributions of higher order in chiral expansion, coming from axial current, have been neglected. A detailed explanation of the reasons of this approximation can be found in ref. [20]. The definition of B-parameters
in eq. (11) takes explicitly into account this approximation and using it the low scale ($\mu$) dependence of the final result is explicitly canceled in the product of coefficient functions and hadronic matrix elements.

The B-parameter of the first operator is usually addressed as $B_K$ and has been extensively studied on the lattice and used in many phenomenological applications (see, e.g. [23, 24]). We have considered its world average [23]. The other $B_i$ have been taken from ref. [20] (for another determination of these $B_i$, calculated with perturbative renormalization see ref. [24]).

All the B-parameters are evaluated at a scale of 2 GeV in the LRI renormalization scheme:

\[
\begin{align*}
B_1(\mu = 2 \text{ GeV}) &= 0.60 \pm 0.06, \\
B_2(\mu = 2 \text{ GeV}) &= 0.66 \pm 0.04, \\
B_3(\mu = 2 \text{ GeV}) &= 1.05 \pm 0.12, \\
B_4(\mu = 2 \text{ GeV}) &= 1.03 \pm 0.06, \\
B_5(\mu = 2 \text{ GeV}) &= 0.73 \pm 0.10. \\
\end{align*}
\]

So far in the literature all phenomenological analyses on this subject have used the VIA and have computed Wilson coefficients and quark masses at a scale variable between 0.5-1 GeV. We will see this represents in some cases quite a rough approximation.

Finally we give in table (1) all the numerical values of the physical constants we have considered. All coupling constants and $\sin^2 \theta_W(M_Z)$ are meant in the $\overline{\text{MS}}$-scheme [26].

| Constants          | Values       |
|--------------------|--------------|
| $\alpha_{em}(M_Z)$ | $1/127.88$   |
| $\alpha_s(M_Z)$    | 0.119        |
| $M_K$              | 497.67 MeV   |
| $f_K$              | 159.8 MeV    |
| $m_d(2 \text{ GeV})$ | 7 MeV       |
| $m_s(2 \text{ GeV})$ | 125 MeV     |
| $m_c$              | 1.3 GeV      |
| $m_b$              | 4.3 GeV      |
| $m_t$              | 175 GeV      |
| $\sin^2 \theta_W(M_Z)$ | 0.23124    |

Table 1: Constants used for phenomenological analysis.
3 Constraints on the $\delta$’s

We are ready to provide a set of constraints on SUSY variables coming from the $K_L - K_S$ mass difference, $\Delta M_K$ and the CP violating parameter $\epsilon_K$ defined as

$$\Delta M_K = 2 \text{Re} \langle K^0 | H_{\text{eff}} | K^0 \rangle,$$

$$\epsilon_K = \frac{1}{\sqrt{2 \Delta M_K}} \text{Im} \langle K^0 | H_{\text{eff}} | K^0 \rangle.$$ (13)

The parameter space is composed of two real and four complex entries, that is $M_{sq}$, $m_{\tilde{g}}$ and $(\delta^d_{12})_{LL}$, $(\delta^d_{12})_{LR}$, $(\delta^d_{12})_{RL}$, $(\delta^d_{12})_{RR}$.

Neglecting interference among different SUSY contributions, we give upper bounds on the $\delta$’s, at fixed values of $M_{sq}$ and $m_{\tilde{g}}$, with the condition $M_{sq} > m_{\tilde{g}}$. In this way one gets a set of constraints on individual $\delta$’s. Indeed, since we are interested in model independent constraints, it is meaningful to study the interference of cancellation effects only in specific models.

The physical condition used to get the bounds on the $\delta$’s is that the SUSY contribution (proportional to each single $\delta$) plus the SM contribution to $\Delta M_K$ and $\epsilon_K$ do not exceed the experimental value of these quantities. For what concerns the SM contribution to $\Delta M_K$, we assume that the values of the CKM elements $V_{cd}$ and $V_{cs}$ are unaffected by SUSY. This implies the (very reasonable) hypothesis that SUSY does not correct significantly tree level weak decays. The value of the SM contribution to $\epsilon_K$, instead, depends on the phase of the CKM matrix. This phase can be largely affected by unknown SUSY corrections and can be treated as a free parameter. We put the CKM phase to zero so that the experimental value of $\epsilon_K$ is completely determined by SUSY. Finally, to be even more conservative, we subtract one standard deviation to the values of the B-parameters.

The final results are shown in tabs. 2-7 for gluino masses of 250, 500, 1000 GeV. We consider the heavy squark masses expected in some common models (see e.g. [5, 6, 7, 8]).

The constraints that come from the four possible insertions of the $\delta$’s are presented: in the first and second rows only terms proportional respectively to $(\delta^d_{12})_{LL}$ and $(\delta^d_{12})_{LR}$ are considered; in the last two rows the contribution of operators with opposite chirality, $RR$ and $RL$, is also evaluated by assuming $(\delta^d_{12})_{LR} = (\delta^d_{12})_{RL}$ and $(\delta^d_{12})_{LL} = (\delta^d_{12})_{RR}$.

In each column of the table we show the bounds on the $\delta$’s in the various approximations that one can use for their determination: without QCD correction and in VIA, with LO-QCD corrections and in VIA, with LO-QCD corrections and with lattice B-parameters and, eventually, with NLO-QCD corrections and lattice B-parameters. Comparing the values of our constraints at LO-VIA with those found from the authors of ref. [12] we find some differences. The reason is twofold. On the one hand they do not consider the SM contribution to $\Delta M_K$ and on the other they evaluate the hadronic matrix elements at a scale $\tilde{\mu}$ such that $\alpha_s(\tilde{\mu}) = 1$. This latter choice may be questionable because at this scale strong interactions break perturbation theory.

The combination of B-parameters and NLO-QCD corrections change the LO-VIA results by about $25 - 35\%$. As expected [12], the tightest constraints are for the cases $(\delta^d_{12})_{LL} =$
$$(\delta_{12}^d)_{RR} \text{ and } (\delta_{12}^d)_{LR} = (\delta_{12}^d)_{RL}$$.

In these cases the coefficients proportional to $(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}$, $(\delta_{12}^d)_{LR}(\delta_{12}^d)_{RL}$ dominate the others.

We have checked that the uncertainties of the results due to higher perturbative orders, are sizeable, being, in some cases up to 10%.

4 Constraints on squarks spectrum

In this section, following the discussion of ref. [16], we provide a different kind of constraints.

For fixed values of the $\delta$’s and of the average light sparticle mass, $m_{\tilde{g}}$, it is possible to calculate the minimum value of $M_{sq}$ necessary to suppress the FCNC at an experimentally acceptable level. Here we give constraints on $M_{sq}$ and we discuss about their consistency. Using Renormalization Group Equations, one finds that a too large $M_{sq}$ can drive to zero or negative values the average mass of the third generation of sfermions, $m_{\tilde{f}}$, at the TeV scale ($m_{\tilde{f}}(\sim 1 \text{TeV})$). To circumvent this problem, a minimum value for $m_{\tilde{f}}(\mu_{\text{GUT}})$ at the GUT scale has to be chosen. If $m_{\tilde{f}}(\mu_{\text{GUT}})$ is too high (say more then 3-4 TeV), however, a too large fine-tuning of the SUSY parameters is required in order to account for the observed mass of the $Z$-boson and severe naturalness problems arise [14, 15]. This problem was studied in refs. [16, 17].

One obtains constraints about the consistency of models with a splitted mass spectrum following three steps:

- determining the minimum value of $M_{sq}$ necessary to suppress FCNC. This is discussed in subsec. 4.1;
- computing the maximum value of $M_{sq}$ allowed by positiveness of light scalar masses and fine-tuning. More about this in subsec. 4.2;
- combining the previous two results one can determine regions of allowable values of $M_{sq}$ that satisfy both the requests of the previous points. We comment about that in subsec. 4.3.

4.1 Minimum values for heavy squark mass

In order to obtain constraints on $M_{sq}$ one has to specify a value for the $\delta$’s. We consider the cases

$$
\begin{array}{cccc}
(\delta_{12}^d)_{LL} & (\delta_{12}^d)_{LR} & (\delta_{12}^d)_{RL} & (\delta_{12}^d)_{RR} \\
I & \mathcal{K} & 0 & 0 \\
II & 0 & \mathcal{K} & 0 \\
III & \mathcal{K} & 0 & \mathcal{K} \\
IV & 0 & \mathcal{K} & \mathcal{K} & 0
\end{array}
$$

(14)
where $\mathcal{K}$ can take the values (1, 0.22, 0.05). We have chosen these entries to leave aside possible accidental cancellations. The cases in which $\mathcal{K}=1$, are, of course, extreme cases: one may wonder about the consistency of mass insertion approximation as the neglected terms are of order $O(1)$. However these cases have already been studied in the literature, (see e.g. [16, 17]), and we report them for completeness. The results so obtained just give an estimate of the mass scales that are involved and can be trusted if other corrections do not provide accidental cancellations. This can be checked only by a direct calculation.

The assumptions made for the SM contribution and the B-parameters are the same as in section 3.

In order to monitor the effect of the different corrections on the final result we show in figs. [1, 2] the lower bound obtained for the cases I and III with $\mathcal{K} = 0.22$ (the other cases give similar results). As we see, B-parameters and NLO-QCD corrections play a significant role in the final computation and the correction they provide with respect to the LO-VIA results are of the order of $(25 - 35)\%$. In particular, in case I, fig. 1, B-parameters provide the most important corrections with respect to LO-VIA results. In case III, fig. 2, instead, corrections to LO-VIA results are dominated by the NLO-QCD perturbative contributions.

The case $(\delta_{12})_{LL} = (\delta_{12})_{RR}$ was also considered in refs. [16, 17]. The differences, at LO and without B-parameters, among our result and the ones of refs. [16, 17] come from our

Figure 1: Lower bounds on $M_{sq}$ from $\Delta M_K$ with various approximations for the case I with $\mathcal{K} = 0.22$. In this case the larger correction to LO-VIA come from the B-parameters.
Figure 2: Lower bounds on $M_{sq}$ from $\Delta M_K$ with various approximations for the case III with $K = 0.22$. In this case the larger correction to LO-VIA come from NLO perturbative corrections.

inclusion of the SM contribution, from the value of the strange quark mass and from the scale at which hadronic matrix elements are evaluated. We agree with them for the same choice of parameters.

Notice that, if the imaginary parts of the $\delta$’s are of the same order of their real parts, there are much stronger constraints coming from $\epsilon_K$ than from $\Delta M_K$ (namely by a factor $\sim 7.7$). To be conservative, we consider in this section only constraints coming from the real parts of the $\delta$’s.

The final results are shown by the (colored) continuous lines in figs. 3, 4, 5. The minimum value of $M_{sq}$ depends strongly both on $K$ and on the case one considers (I, II, III or IV, see eq. 14). Notice that $(\delta_{12}^{d})_{LR}$, $(\delta_{12}^{d})_{RL}$, (which enter the cases II and IV), are “naturally” small in the MSSM. However, since we would like to do a model–independent analysis, we have made no particular assumption on them. In all graphs the strongest constraints come from the case $(\delta_{12}^{d})_{LR} = (\delta_{12}^{d})_{RL} \neq 0$. Much lower constraints are generally obtained in cases I and II. In fig. 5 the case I has not been drawn since no constraint can be derived.
Figure 3: The full (colored) lines give the lower bounds on $M_{sq}$ necessary to suppress FCNC and with $K=1$ for the various cases. An upper bound on $M_{sq}$ is derived in order to satisfy fine-tuning requirements and it is shown by the dashed line. The two kinds of constraints are not compatible in this case.

4.2 RGE for the masses of the third generation of scalars

It is well known that large values of $M_{sq}$ can drive the mass of the third generation of scalars to negative values, via RGE [16]. Let us consider the two-loop RGE’s for the mass, $m_{\tilde{f}}$ of the third generation of scalars, $\tilde{f}$. In the $\overline{DR}'$ scheme (see e.g. ref. [27]), with two generations of heavy scalars, one has

$$\mu \frac{d}{d\mu} m_{\tilde{f}}(\mu) = -\frac{8}{4\pi} \sum_i \alpha_i(\mu) C_i^f (m_G^2)_{ij}(\mu) + \frac{32}{(4\pi)^2} \sum_i \alpha_i^2(\mu) C_i^f M_{sq}^2, \quad (15)$$

where $C_i^f$ is the Casimir factor for $\tilde{f}$ in the SU(5) normalization, the sums are over the gauge groups SU(3), SU(2), U(1) and $m_G$ denotes the gaugino masses. In eq. (15), Yukawa couplings are neglected: these couplings drive the light masses to even lower values and so, in this respect, our choice is a conservative one. Moreover, the introduction of Yukawa interactions requires further assumptions on SUSY parameters (see e.g. [17]) that we do not discuss in this paper.

The solution of eq. (15) between a Grand Unification (GUT) scale $\mu_{\text{GUT}} \sim 2 \cdot 10^{16}$ GeV,
and $\mu \sim 1$ TeV can be easily written as

$$m_f^2(\mu) = m_f^2(\mu_{\text{GUT}}) - \sum_i \frac{16}{4\pi \beta_i^0} \left[ \alpha_i(\mu_{\text{GUT}}) - \alpha_i(M_{\text{sq}}) \right] C_{i}^{f} M_{\text{sq}}^2 +$$

$$\sum_i \frac{2}{\beta_i^0} \left[ m_G^2(\mu_{\text{GUT}}) - (m_G^2)_i(M_{\text{sq}}) \right] C_{i}^{G} +$$

$$\sum_i \frac{2}{\beta_i^0} \left[ (m_G^2)_i(M_{\text{sq}}) - (m_G^2)_i(\mu) \right] C_{i}^{f}, \tag{16}$$

where $\beta_i^0$ are the $\beta$-functions LO coefficients of the $i$-th gauge coupling. In eq. (16) we have considered a common gaugino mass, $m_G$ at the GUT scale, while for what concerns the couplings we have evolved them starting backward from $\mu = M_Z$. Note that in eq. (16) the contribution of the heavy scalars has been decoupled at $M_{\text{sq}}$.

Eq. (16) can be used in order to derive consistency constraints on the values of $M_{\text{sq}}$ and $m_G^2(\mu_{\text{GUT}})$ once the values of $m_f^2(\mu)$ and of $m_f^2(\mu_{\text{GUT}})$ are fixed. The latter can be determined according to the following requirements. First, $m_f^2(\mu)$ must be at least positive, such as to leave color and electric symmetries unbroken. The value of $m_f^2(\mu_{\text{GUT}})$ determines the amount of fine-tuning necessary in order to achieve the electroweak symmetry breaking. Following ref. [14] the necessary fine-tuning scales approximately as $10\% \times (0.3 \text{ TeV}/m_{\tilde{Q}_3}(\mu_{\text{GUT}}))^2$ for the squark doublet of the third generation $\tilde{Q}_3$. We have calculated the constraints on $M_{\text{sq}}$. 

Figure 4: The same as fig. 3 with $K=0.22$. Cases I and II are now compatible with fine-tuning requirements.
Figure 5: The same as fig. 3 with $\mathcal{K}=0.05$. Case I is not drawn since no lower bound on $M_{sq}$ can be obtained in this case. Cases II and III are now compatible with fine-tuning requirements.
and $m_{\tilde{Q}_3}^2(\mu_{\text{GUT}})$ coming from eq. (14) in the case $\tilde{f} = \tilde{Q}_3$ choosing for $m_{\tilde{Q}_3}^2(\mu_{\text{GUT}})$ the value of $(3.5 \text{ TeV})^2$. The latter choice corresponds to a fine-tuning of more than 0.1%.

At fixed values of $m_{\tilde{Q}_3}^2(\mu)$ and $m_{\tilde{Q}_3}^2(\mu_{\text{GUT}})$ (which depend on $M_{sq}$ and $m_G$) one can plot the upper value of $M_{sq}$ as function of $m_G$. The result is the (black) dashed line of figs. 3, 4, 5. One finds that $M_{sq}$ can not be much larger than about 25 TeV. Of course this is just an estimate of this limiting value. The inclusion of Yukawa couplings, of more severe fine-tuning requirements and of other effects can only lower this limit.

4.3 Final remarks

In figs. 3, 4, 5 we combine the constraints derived in the two previous subsections. These figures (together with tables tabs. 2-7) suggest that also models with a splitted mass spectrum need further assumptions to be phenomenologically viable, e.g. one has to introduce flavor symmetry or dynamical generation of degenerate scalar masses [16].

In particular, without these further hypotheses, most of the cases which we have considered, face fine tuning problems. In particular, values of $K \sim \mathcal{O}(1)$ are hardly acceptable. Although $K \sim \mathcal{O}(0.22)$ and $K \sim \mathcal{O}(0.05)$ have better chances they must be treated carefully.

5 Conclusions

In this work we analyze in detail the constraints on SUSY-models parameters coming from $K - \overline{K}$ oscillations in the hypothesis of a splitted SUSY spectrum. FCNC contributions coming from gluino-squark-quark interactions, working in the so-called mass insertion approximation, have been considered. We provide boundaries on mass insertions and on SUSY mass scales and we discus their consistency. Previous results including NLO-QCD corrections to $\Delta S = 2$ effective Hamiltonian, B-parameters for the evaluation of hadronic matrix elements have been improved. A full set of magic-numbers is provided, that can be used for further analyses.

We have discussed the residual uncertainty of our results coming from our ignorance of NLO-QCD corrections to the matching coefficients.

Our analysis confirms that a splitted sparticle mass spectrum does not explain easily FCNC suppression without some amount of fine-tuning. These problems can be solved only if further assumptions in these kind of models are made, e.g. flavor symmetry or dynamical generation of degenerate scalar masses [16].

In order to perform a complete analysis of SUSY-FCNC effects chargino contributions should be included. It is also interesting to extend this kind of analysis to $\Delta B = 2$ processes, once the calculation of B-parameters for the $B - \overline{B}$ system parameters (which is in progress [19]) will be completed.
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Appendix

We give here the numerical values for the magic numbers of eq. (3). Only the non-vanishing entries are shown:

\[
\begin{align*}
    a^{(r)} &= (0.29, -1.1, 0.14, -0.69, 0.79) \\
    a'^{(r)} &= (0.46, -1.8, 0.23, -1.1, 1.3) \\
    b^{(r)} &= (1.5, 0, 0, 0, 0) \\
    b^{(r)}_{22} &= (0.0048, 1.1, 0, 0) \\
    b^{(r)}_{23} &= (0, -0.0073, 0, 0, 0) \\
    b^{(r)}_{32} &= (0, -0.23, 0.47, 0, 0) \\
    b^{(r)}_{33} &= (0.34, 0, 0, 0) \\
    b^{(r)}_{44} &= (0, 0, 0, 0.52, -0.017) \\
    b^{(r)}_{45} &= (0, 0, 0, 0.99, -2.2) \\
    b^{(r)}_{54} &= (0, 0, 0, -0.00051, 0.020) \\
    b^{(r)}_{55} &= (0, 0, 0, -0.00096, 2.5) \\
    c^{(r)}_{11} &= (-3.4, 0, 0, 0, 0) \\
    c^{(r)}_{22} &= (0, -0.12, 2.8, 0, 0) \\
    c^{(r)}_{23} &= (0, 0.12, 1.2, 0, 0) \\
    c^{(r)}_{32} &= (0, 6.3, 2.0, 0, 0) \\
    c^{(r)}_{33} &= (0, -6.2, 0.88, 0, 0) \\
    c^{(r)}_{44} &= (0, 0, 0, -5.7, 1.8) \\
    c^{(r)}_{45} &= (0, 0, 0, -26, -27) \\
    c^{(r)}_{54} &= (0, 0, 0, 0.0086, -0.77) \\
    c^{(r)}_{55} &= (0, 0, 0, 0.040, 12) \\
    d^{(r)}_{11} &= (1.0, 0, 0, 0, 0) \\
    d^{(r)}_{22} &= (0, 0, 1.0, 0, 0) \\
    d^{(r)}_{23} &= (0, 0, 0, 0, 0) \\
    d^{(r)}_{32} &= (0, -0.67, 0.67, 0, 0) \\
    d^{(r)}_{33} &= (0, 0, 0, 0, 0) \\
    d^{(r)}_{44} &= (0, 0, 0, 1.0, -0.015) \\
    d^{(r)}_{45} &= (0, 0, 0, 1.9, -1.9) \\
    d^{(r)}_{54} &= (0, 0, 0, -0.081, 0.0081) \\
    d^{(r)}_{55} &= (0, 0, 0, -0.015, 1.0) \\
    e^{(r)}_{11} &= (-1.5, 0, 0, 0, 0) \\
    e^{(r)}_{22} &= (0, 0.70, -4.9, 0, 0) \\
    e^{(r)}_{23} &= (0, -1.1, 0, 0, 0) \\
    e^{(r)}_{32} &= (0, -27, -9.0, 0, 0) \\
    e^{(r)}_{33} &= (0, 40, 0, 0, 0) \\
    e^{(r)}_{44} &= (0, 0, 0, 22, 0.46) \\
    e^{(r)}_{45} &= (0, 0, 0, 41, 58) \\
    e^{(r)}_{54} &= (0, 0, 0, 0.15, -0.12) \\
    e^{(r)}_{55} &= (0, 0, 0, 0.29, -15) \\
    f^{(r)}_{11} &= (1.5, 0, 0, 0, 0) \\
    f^{(r)}_{22} &= (0, 0, 4.2, 0, 0) \\
    f^{(r)}_{23} &= (0, 0, 1.1, 0, 0) \\
    f^{(r)}_{32} &= (0, 0, 3.2, 0, 0) \\
    f^{(r)}_{33} &= (0, -41, 0.70, 0, 0) \\
    f^{(r)}_{44} &= (0, 0, 0, -23, 0.40) \\
    f^{(r)}_{45} &= (0, 0, 0, -72, -28) \\
    f^{(r)}_{54} &= (0, 0, 0, 0.18, -0.21) \\
    f^{(r)}_{55} &= (0, 0, 0, 0.57, 15)
\end{align*}
\]
Table 2: Limits on $\text{Re}(\delta_{12})_{AB}$ from $\Delta M_K$ with gaugino masses of 250 GeV.

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|---------------|-------------|--------|-----------|------------|
| 2             | $3.1 \times 10^{-2}$ | $3.6 \times 10^{-2}$ | $4.9 \times 10^{-2}$ | $4.9 \times 10^{-2}$ |
| 5             | $7.5 \times 10^{-2}$ | $8.8 \times 10^{-2}$ | $0.12$ | $0.12$ |
| 10            | $0.15$ | $0.18$ | $0.25$ | $0.24$ |

$\sqrt{\text{Re}(\delta_{12}^d)^2_{LL}}$

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|---------------|-------------|--------|-----------|------------|
| 2             | $2.1 \times 10^{-2}$ | $1.4 \times 10^{-2}$ | $1.8 \times 10^{-2}$ | $1.6 \times 10^{-2}$ |
| 5             | $9.8 \times 10^{-2}$ | $6.5 \times 10^{-2}$ | $8.2 \times 10^{-2}$ | $7.2 \times 10^{-2}$ |
| 10            | $0.34$ | $0.22$ | $0.28$ | $0.25$ |

$\sqrt{\text{Re}(\delta_{12}^d)^2_{LR}}$

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|---------------|-------------|--------|-----------|------------|
| 2             | $6.6 \times 10^{-3}$ | $3.5 \times 10^{-3}$ | $3.5 \times 10^{-3}$ | $2.8 \times 10^{-3}$ |
| 5             | $1.5 \times 10^{-2}$ | $7.7 \times 10^{-3}$ | $7.9 \times 10^{-3}$ | $6.4 \times 10^{-3}$ |
| 10            | $3.0 \times 10^{-2}$ | $1.5 \times 10^{-2}$ | $1.5 \times 10^{-2}$ | $1.2 \times 10^{-2}$ |

$\sqrt{\text{Re}(\delta_{12}^d)^2_{LR} = \text{Re}(\delta_{12}^d)^2_{RL}}$

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|---------------|-------------|--------|-----------|------------|
| 2             | $1.1 \times 10^{-2}$ | $5.2 \times 10^{-3}$ | $5.1 \times 10^{-3}$ | $4.1 \times 10^{-3}$ |
| 5             | $4.1 \times 10^{-2}$ | $1.6 \times 10^{-2}$ | $1.6 \times 10^{-2}$ | $1.3 \times 10^{-2}$ |
| 10            | $0.10$ | $3.6 \times 10^{-2}$ | $3.4 \times 10^{-2}$ | $2.7 \times 10^{-2}$ |

Table 3: Limits on $\text{Re}(\delta_{12}^d)_{AB}$ from $\Delta M_K$ with gaugino masses of 500 GeV.

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|---------------|-------------|--------|-----------|------------|
| 2             | $3.8 \times 10^{-2}$ | $4.5 \times 10^{-2}$ | $6.1 \times 10^{-2}$ | $6.1 \times 10^{-2}$ |
| 5             | $8.1 \times 10^{-2}$ | $9.6 \times 10^{-2}$ | $0.13$ | $0.13$ |
| 10            | $0.16$ | $0.19$ | $0.26$ | $0.26$ |

$\sqrt{\text{Re}(\delta_{12}^d)^2_{LL}}$

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|---------------|-------------|--------|-----------|------------|
| 2             | $1.6 \times 10^{-2}$ | $1.1 \times 10^{-2}$ | $1.3 \times 10^{-2}$ | $1.2 \times 10^{-2}$ |
| 5             | $6.3 \times 10^{-2}$ | $4.2 \times 10^{-2}$ | $5.3 \times 10^{-2}$ | $4.7 \times 10^{-2}$ |
| 10            | $0.21$ | $0.14$ | $0.17$ | $0.15$ |

$\sqrt{\text{Re}(\delta_{12}^d)^2_{LR} = \text{Re}(\delta_{12}^d)^2_{RL}}$

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|---------------|-------------|--------|-----------|------------|
| 2             | $9.6 \times 10^{-3}$ | $4.6 \times 10^{-3}$ | $4.5 \times 10^{-3}$ | $3.6 \times 10^{-3}$ |
| 5             | $1.7 \times 10^{-2}$ | $8.5 \times 10^{-3}$ | $8.7 \times 10^{-3}$ | $7.0 \times 10^{-3}$ |
| 10            | $3.2 \times 10^{-2}$ | $1.6 \times 10^{-2}$ | $1.6 \times 10^{-2}$ | $1.3 \times 10^{-2}$ |

$\sqrt{\text{Re}(\delta_{12}^d)^2_{LL} = \text{Re}(\delta_{12}^d)^2_{RR}}$

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|---------------|-------------|--------|-----------|------------|
| 2             | $8.6 \times 10^{-3}$ | $4.4 \times 10^{-3}$ | $4.4 \times 10^{-3}$ | $3.6 \times 10^{-3}$ |
| 5             | $3.2 \times 10^{-2}$ | $1.4 \times 10^{-2}$ | $1.4 \times 10^{-2}$ | $1.1 \times 10^{-2}$ |
| 10            | $8.8 \times 10^{-2}$ | $3.4 \times 10^{-2}$ | $3.2 \times 10^{-2}$ | $2.6 \times 10^{-2}$ |
| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|----------------|------------|--------|----------|-----------|
|                | $\sqrt{|\text{Re}(\delta_{12}^d)^2_{LL}|}$ |        |          |           |
| 2              | $5.9 \times 10^{-2}$ | $6.9 \times 10^{-2}$ | $9.4 \times 10^{-2}$ | $9.3 \times 10^{-2}$ |
| 5              | $9.6 \times 10^{-2}$ | 0.11   | 0.15     | 0.15      |
| 10             | 0.17       | 0.21   | 0.28     | 0.28      |
|                | $\sqrt{|\text{Re}(\delta_{12}^d)^2_{LR}|}$ |        |          |           |
| 2              | $1.4 \times 10^{-2}$ | $9.7 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $1.1 \times 10^{-2}$ |
| 5              | $4.6 \times 10^{-2}$ | $3.0 \times 10^{-2}$ | $3.9 \times 10^{-2}$ | $3.4 \times 10^{-2}$ |
| 10             | 0.14       | $8.8 \times 10^{-2}$ | 0.11     | $9.8 \times 10^{-2}$ |
|                | $\sqrt{|\text{Re}(\delta_{12}^d)_{LR} = \text{Re}(\delta_{12}^d)_{RL}|}$ |        |          |           |
| 2              | $4.2 \times 10^{-3}$ | $9.8 \times 10^{-3}$ | $7.8 \times 10^{-3}$ | $6.0 \times 10^{-3}$ |
| 5              | $2.2 \times 10^{-2}$ | $1.1 \times 10^{-2}$ | $1.1 \times 10^{-2}$ | $8.5 \times 10^{-3}$ |
| 10             | $3.6 \times 10^{-2}$ | $1.8 \times 10^{-2}$ | $1.8 \times 10^{-2}$ | $1.4 \times 10^{-2}$ |
|                | $\sqrt{|\text{Re}(\delta_{12}^d)^2_{LL} = \text{Re}(\delta_{12}^d)^2_{RR}|}$ |        |          |           |
| 2              | $8.0 \times 10^{-3}$ | $4.2 \times 10^{-3}$ | $4.3 \times 10^{-3}$ | $3.5 \times 10^{-3}$ |
| 5              | $2.5 \times 10^{-2}$ | $1.2 \times 10^{-2}$ | $1.2 \times 10^{-2}$ | $9.7 \times 10^{-3}$ |
| 10             | $6.8 \times 10^{-2}$ | $2.9 \times 10^{-2}$ | $2.9 \times 10^{-2}$ | $2.3 \times 10^{-3}$ |

Table 4: Limits on $\text{Re}(\delta_{12}^d)_{AB}$ from $\Delta M_K$ with gaugino masses of 1000 GeV.

| $M_{sq}$ [TeV] | No-QCD, VIA | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|----------------|------------|--------|----------|-----------|
|                | $\sqrt{|\text{Im}(\delta_{12}^d)^2_{LL}|}$ |        |          |           |
| 2              | $4.0 \times 10^{-3}$ | $4.7 \times 10^{-3}$ | $6.4 \times 10^{-3}$ | $6.4 \times 10^{-3}$ |
| 5              | $9.7 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $1.6 \times 10^{-2}$ | $1.6 \times 10^{-2}$ |
| 10             | $2.0 \times 10^{-2}$ | $2.4 \times 10^{-2}$ | $3.2 \times 10^{-2}$ | $3.2 \times 10^{-2}$ |
|                | $\sqrt{|\text{Im}(\delta_{12}^d)^2_{LR}|}$ |        |          |           |
| 2              | $2.7 \times 10^{-3}$ | $1.8 \times 10^{-3}$ | $2.3 \times 10^{-3}$ | $2.1 \times 10^{-3}$ |
| 5              | $1.3 \times 10^{-2}$ | $8.4 \times 10^{-3}$ | $1.1 \times 10^{-2}$ | $9.4 \times 10^{-3}$ |
| 10             | $4.5 \times 10^{-2}$ | $2.9 \times 10^{-2}$ | $3.7 \times 10^{-2}$ | $3.2 \times 10^{-2}$ |
|                | $\sqrt{|\text{Im}(\delta_{12}^d)_{LR} = \text{Im}(\delta_{12}^d)_{RL}|}$ |        |          |           |
| 2              | $8.6 \times 10^{-4}$ | $4.5 \times 10^{-4}$ | $4.6 \times 10^{-4}$ | $3.7 \times 10^{-4}$ |
| 5              | $2.0 \times 10^{-3}$ | $1.0 \times 10^{-3}$ | $1.0 \times 10^{-3}$ | $8.3 \times 10^{-4}$ |
| 10             | $3.9 \times 10^{-3}$ | $2.0 \times 10^{-3}$ | $2.0 \times 10^{-3}$ | $1.6 \times 10^{-3}$ |
|                | $\sqrt{|\text{Im}(\delta_{12}^d)^2_{LL} = \text{Im}(\delta_{12}^d)^2_{RR}|}$ |        |          |           |
| 2              | $1.4 \times 10^{-3}$ | $6.7 \times 10^{-4}$ | $6.6 \times 10^{-4}$ | $5.4 \times 10^{-3}$ |
| 5              | $5.4 \times 10^{-3}$ | $2.1 \times 10^{-3}$ | $2.0 \times 10^{-3}$ | $1.6 \times 10^{-3}$ |
| 10             | $1.4 \times 10^{-2}$ | $4.7 \times 10^{-3}$ | $4.5 \times 10^{-3}$ | $3.6 \times 10^{-3}$ |

Table 5: Limits on $\text{Im}(\delta_{12}^d)_{AB}$ from $\epsilon_K$ with gaugino masses of 250 GeV.
| $M_{sq}$ [TeV] | No-QCD, VIA   | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|--------------|-------------|--------|-----------|------------|
|              | $\sqrt{\text{Im}(\delta_{12}^d)^2_{LL}}$ |        |           |            |
| 2            | $5.0 \times 10^{-3}$ | $5.9 \times 10^{-3}$ | $8.0 \times 10^{-3}$ | $7.9 \times 10^{-3}$ |
| 5            | $1.1 \times 10^{-2}$ | $1.3 \times 10^{-2}$ | $1.7 \times 10^{-2}$ | $1.7 \times 10^{-2}$ |
| 10           | $2.1 \times 10^{-2}$ | $2.5 \times 10^{-2}$ | $3.4 \times 10^{-2}$ | $3.3 \times 10^{-2}$ |
|              | $\sqrt{\text{Im}(\delta_{12}^d)^2_{LR}}$ |        |           |            |
| 2            | $2.0 \times 10^{-3}$ | $1.4 \times 10^{-3}$ | $1.8 \times 10^{-3}$ | $1.6 \times 10^{-3}$ |
| 5            | $8.3 \times 10^{-3}$ | $5.5 \times 10^{-3}$ | $7.0 \times 10^{-3}$ | $6.2 \times 10^{-3}$ |
| 10           | $2.7 \times 10^{-2}$ | $1.8 \times 10^{-2}$ | $2.2 \times 10^{-2}$ | $2.0 \times 10^{-2}$ |
|              | $\sqrt{\text{Im}(\delta_{12}^d)^2_{LL}} = \text{Im}(\delta_{12}^d)^2_{RL}$ |        |           |            |
| 2            | $1.3 \times 10^{-3}$ | $6.0 \times 10^{-4}$ | $5.9 \times 10^{-4}$ | $4.7 \times 10^{-4}$ |
| 5            | $2.2 \times 10^{-3}$ | $1.1 \times 10^{-3}$ | $1.1 \times 10^{-3}$ | $9.1 \times 10^{-4}$ |
| 10           | $4.2 \times 10^{-3}$ | $2.1 \times 10^{-3}$ | $2.1 \times 10^{-3}$ | $1.7 \times 10^{-3}$ |
|              | $\sqrt{\text{Im}(\delta_{12}^d)^2_{LL}} = \text{Im}(\delta_{12}^d)^2_{RR}$ |        |           |            |
| 2            | $1.1 \times 10^{-3}$ | $5.8 \times 10^{-4}$ | $5.8 \times 10^{-4}$ | $4.7 \times 10^{-4}$ |
| 5            | $4.2 \times 10^{-3}$ | $1.9 \times 10^{-3}$ | $1.8 \times 10^{-3}$ | $1.5 \times 10^{-3}$ |
| 10           | $1.1 \times 10^{-2}$ | $4.4 \times 10^{-3}$ | $4.2 \times 10^{-3}$ | $3.4 \times 10^{-3}$ |

Table 6: Limits on $\text{Im}(\delta_{12}^d)_{AB}$ from $\epsilon_K$ with gaugino masses of 500 GeV.

| $M_{sq}$ [TeV] | No-QCD, VIA   | LO-VIA | LO, $B_i$ | NLO, $B_i$ |
|--------------|-------------|--------|-----------|------------|
|              | $\sqrt{\text{Im}(\delta_{12}^d)^2_{LL}}$ |        |           |            |
| 2            | $7.7 \times 10^{-3}$ | $9.0 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $1.2 \times 10^{-2}$ |
| 5            | $1.3 \times 10^{-2}$ | $1.5 \times 10^{-2}$ | $2.0 \times 10^{-2}$ | $2.0 \times 10^{-2}$ |
| 10           | $2.3 \times 10^{-2}$ | $2.7 \times 10^{-2}$ | $3.7 \times 10^{-2}$ | $3.6 \times 10^{-2}$ |
|              | $\sqrt{\text{Im}(\delta_{12}^d)^2_{LR}}$ |        |           |            |
| 2            | $1.8 \times 10^{-3}$ | $1.3 \times 10^{-3}$ | $1.6 \times 10^{-3}$ | $1.4 \times 10^{-3}$ |
| 5            | $6.0 \times 10^{-3}$ | $4.0 \times 10^{-3}$ | $5.0 \times 10^{-3}$ | $4.5 \times 10^{-3}$ |
| 10           | $1.8 \times 10^{-2}$ | $1.1 \times 10^{-2}$ | $1.5 \times 10^{-2}$ | $1.3 \times 10^{-2}$ |
|              | $\sqrt{\text{Im}(\delta_{12}^d)^2_{LR}} = \text{Im}(\delta_{12}^d)^2_{RL}$ |        |           |            |
| 2            | $5.5 \times 10^{-3}$ | $1.3 \times 10^{-3}$ | $1.0 \times 10^{-3}$ | $7.8 \times 10^{-4}$ |
| 5            | $2.9 \times 10^{-3}$ | $1.4 \times 10^{-3}$ | $1.4 \times 10^{-3}$ | $1.1 \times 10^{-3}$ |
| 10           | $4.7 \times 10^{-3}$ | $2.3 \times 10^{-3}$ | $2.3 \times 10^{-3}$ | $1.9 \times 10^{-3}$ |
|              | $\sqrt{\text{Im}(\delta_{12}^d)^2_{LL}} = \text{Im}(\delta_{12}^d)^2_{RR}$ |        |           |            |
| 2            | $1.0 \times 10^{-3}$ | $5.5 \times 10^{-4}$ | $5.6 \times 10^{-4}$ | $4.6 \times 10^{-3}$ |
| 5            | $3.3 \times 10^{-3}$ | $1.6 \times 10^{-3}$ | $1.6 \times 10^{-3}$ | $1.3 \times 10^{-3}$ |
| 10           | $8.9 \times 10^{-3}$ | $3.8 \times 10^{-3}$ | $3.7 \times 10^{-3}$ | $3.0 \times 10^{-3}$ |

Table 7: Limits on $\text{Im}(\delta_{12}^d)_{AB}$ from $\epsilon_K$ with gaugino masses of 1000 GeV.
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