A refined Einstein–Gauss–Bonnet inflationary theoretical framework

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Abstract
We provide a refined and much more simplified Einstein–Gauss–Bonnet inflationary theoretical framework, which is compatible with the GW170817 observational constraints on the gravitational wave speed. As in previous works, the constraint that the gravitational wave speed is \( c_T^2 = 1 \) in natural units, results to a constraint differential equation that relates the coupling function of the scalar field to the Gauss–Bonnet invariant \( \xi(\phi) \) and the scalar potential \( V(\phi) \). Adopting the slow-roll conditions for the scalar field and the Hubble rate, and in contrast to previous works, by further assuming that \( \kappa \kappa' \xi' \ll 1 \), which is motivated by slow-roll arguments, we succeed in providing much more simpler expressions for the slow-roll indices and for the tensor and scalar spectral indices and for the tensor-to-scalar ratio. We exemplify our refined theoretical framework by using an illustrative example with a simple power-law scalar coupling function \( \xi(\phi) \sim \phi^n \) and as we demonstrate the resulting inflationary phenomenology is compatible with the latest Planck data. Moreover, this particular model produces a blue-tilted tensor spectral index, so we discuss in brief the perspective of describing the NANOGrav result with this model as is indicated in the recent literature.

Keywords: modified gravity, Einstein Gauss Bonnet gravity, inflation, string corrected inflation

(Some figures may appear in colour only in the online journal)
1. Introduction

The last in conjunction with the following decades from an astronomical point of view belong to the gravitational wave astronomy branch of physics. Indeed, the LIGO–Virgo collaboration have already established the new branch of physics [1–6], the gravitational wave astronomy branch, and with the upcoming LISA [7, 8] and future space missions BBO [9, 10] and DECIGO [11, 12], it is expected that our perception regarding the early Universe and astrophysical phenomena will be further enlightened. Already, the GW170817 event [3] followed by a kilonova gamma ray burst [4, 5] which arrived almost simultaneously with the gravitational waves, have formed and changed our perception about modified gravity theories since many models which generate tensor perturbations with propagation speed different from that of light’s in the vacuum, are excluded from being phenomenologically viable theories [13–16]. Since there is no reason for the gravitons to acquire mass primordially, and especially during the inflationary era, the GW170817 event also constrains the primordial gravitational waves propagation speed $c_T$. Essentially, the tensor perturbations should also comply with the GW170817 event and thus the propagation speed of the tensor perturbations must be equal to unity in natural units, that is $c_T^2 \approx 1$. In view of this compelling constraint, in a previous work [17] we examined the quantitative implications of the constraint $c_T^2 \approx 1$, for Einstein–Gauss–Bonnet theories [18–58]. As we demonstrated, the most important feature is that the coupling function of the scalar field to the Gauss–Bonnet invariant and the scalar potential must be related and cannot be chosen freely. The resulting theoretical framework yielded a phenomenologically viable era, and also it yielded quite elegant expressions for most of the slow-roll indices. More importantly it offered the possibility of obtaining analytical results assuming simply the slow-roll conditions, however, the functional forms of the scalar potential or the scalar coupling function which could yield viable phenomenologies, were quite involved, and this was the only drawback of the theoretical framework. In this work we shall further refine the theoretical framework we developed in reference [17] by simply imposing one constraint imposed by the slow-roll approximation. Specifically by assuming that the scalar coupling function $\xi(\phi)$ satisfies $\kappa \xi' \xi'' \ll 1$ along with the slow-roll assumption, the resulting theoretical framework is significantly simplified, and all the slow-roll indices and the corresponding observational indices acquire quite simplified final expressions. Moreover, even simple power-law choices for the scalar coupling function result to simple expressions for the scalar potential, the slow-roll indices and the observational indices. In order to illustrate that the reformed GW170817-compatible Einstein–Gauss–Bonnet framework we develop in this paper, leads to phenomenologically viable results, we choose a simple power-law scalar coupling function $\xi(\phi)$ and we perform a thorough analysis for the resulting model, eventually confronting the model with the latest (2018) observational constraints of the Planck collaboration. As we show, focusing on the spectral index of the primordial scalar curvature perturbations, and the tensor-to-scalar ratio, the model is compatible with the Planck 2018 observational data. Moreover, we discover an interesting feature of this power-law model with $\xi(\phi) \sim \phi^\nu$. Specifically, the model can lead to a blue tilted tensor spectral index, for large values of the exponent $\nu$, and this feature is quite interesting with regard to explaining the NANOGrav results on pulsar timing arrays with a blue-tilted inflationary theory. We also discuss in brief this possibility explaining the NANOGrav results with a stochastic gravitational wave background originating by an inflationary gravitational wave background corresponding to a blue-tilted Einstein–Gauss–Bonnet theory.

This paper is organized as follows: In section 2 we present in detail the reformed GW170817-compatible Einstein–Gauss–Bonnet framework. We discuss how the simplified
framework is obtained by assuming that the slow-roll conditions and also the condition $\kappa \xi' / \xi'' \ll 1$ hold true. In section 3, we use a simple power-law model with $x(\phi) \sim \phi^n$ in order to test the phenomenological viability of the model and confront it with the latest (2018) Planck observations. We also discuss the implications of a blue tilt in the tensor spectral index, on the NANOGrav results. Finally, the conclusions follow at the end of the paper.

2. Simplified Einstein–Gauss–Bonnet inflation framework

Consider the Einstein–Gauss–Bonnet action,

$$S = \int d^4x \sqrt{-g} \left( \frac{R}{2\kappa^2} - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) - \frac{1}{2} \xi(\phi) G \right),$$

(1)

where $R$ stands for the Ricci scalar, $\kappa = 1/\kappa$ where $M_p$ is the reduced Planck mass, and $G$ stands for the Gauss–Bonnet invariant in four dimensions, which is $G = R^2 - 4 R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$ with $R_{\alpha\beta}$ and $R_{\alpha\beta\gamma\delta}$ standing for the Ricci and Riemann tensor respectively. We shall assume that the background metric is described by a flat Friedmann–Robertson–Walker (FRW) metric,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dx^i)^2,$$

(2)

with $a(t)$ denoting as usual the scale factor. For the FRW metric, the Gauss–Bonnet invariant takes the form $G = 24 H^2 (\dot{H} + H^2)$, where $H$ is the Hubble rate which in terms of the scale factor of the FRW metric is defined as $H = \frac{\dot{a}}{a}$. Also in the following we shall assume that the scalar field does not have a spatial coordinate dependence. Upon variation of the gravitational action (1) with respect to the metric and with respect to the scalar field, we obtain the gravitational equations of motion,

$$\frac{3H^2}{\kappa^2} = \frac{1}{2} \dot{\phi}^2 + V + 12 \xi H^2,$$

(3)

$$\frac{2\dot{H}}{\kappa^2} = -\dot{\phi}^2 + 4 \dot{\xi} H^2 + 8 \dot{\xi} \dot{H} - 4 \xi H^3,$$

(4)

$$\ddot{\phi} + 3H \dot{\phi} + V' + 12 \xi' H^2 (\dot{H} + H^2) = 0.$$  

(5)

We shall consider the inflationary era of the Einstein–Gauss–Bonnet theory, so we assume that the slow-roll conditions hold true,

$$\dot{H} \ll H^2, \quad \frac{\dot{\phi}^2}{2} \ll V, \quad \ddot{\phi} \ll 3H \dot{\phi}.$$

(6)

Also, in order to render the Einstein–Gauss–Bonnet model compatible with the GW170817 event, the gravitational wave of the tensor perturbations, which reads,

$$c_T^2 = 1 - \frac{Q_t}{2Q_b},$$

(7)

must be equal to unity in natural units. The functions $Q_t$, $F$ and $Q_b$ defined above are $Q_t = 8(\dot{\xi} - H \dot{\xi})$, $Q_t = F + \frac{\dot{\phi}}{\kappa}$, $F = \frac{1}{\kappa^2}$ and $Q_b = -8 \xi H$. Thus in order to have $c_T^2 = 1$, the
condition $Q_1 = 0$ must hold true, which results to the differential equation $\dddot{\xi} = H\dot{\xi}$, which constrains the Gauss–Bonnet scalar coupling function $\xi(\phi)$. This differential equation can be expressed in terms of the scalar field, using $\dot{\xi} = \xi' \dot{\phi}$ and $\frac{d}{d\phi} = \frac{\dot{\phi}}{\dot{\phi}}$, as follows,

$$\dddot{\xi} \dot{\phi}^2 + \xi' \ddot{\phi} = H \xi' \dot{\phi}, \quad (8)$$

and the ‘prime’ hereafter will denote differentiation with respect to the scalar field. By assuming,

$$\xi' \ddot{\phi} \ll \xi'' \dot{\phi}^2, \quad (9)$$

which is strongly motivated by the slow-roll conditions of the scalar field, the constraint of equation (8) becomes,

$$\dot{\phi} \simeq \frac{H \xi'}{\xi''}. \quad (10)$$

Upon combining equations (5) and (10) we obtain,

$$\frac{\xi'}{\xi''} \simeq \frac{1}{3H^2} \left( V' + 12 \xi'' H^4 \right). \quad (11)$$

So far this framework was used in previous works [17, 48–52], at this point we shall differentiate from the previous studies based on the fact the first slow-roll index for this system depends on the ratio $\xi' / \xi''$. Specifically, we shall consider theories for which,

$$\kappa \frac{\xi'}{\xi''} \ll 1, \quad (12)$$

and in addition the following extra condition,

$$12 \xi' H^3 = 12 \frac{\xi'^2 H^4}{\xi''} \ll V, \quad (13)$$

which is strongly related to the constraint (12). Upon combining equations (6), (10) and (13), we can rewrite the equations of motion in the following quite simplified form,

$$H^2 \simeq \frac{\kappa^2 V}{3}, \quad (14)$$

$$\dot{H} \simeq -\frac{1}{2} \kappa^2 \dot{\phi}^2, \quad (15)$$

$$\dot{\phi} \simeq \frac{H \xi'}{\xi''}. \quad (16)$$

In view of equation (14), the condition (13) takes the simpler form,

$$\frac{4 \kappa^4 \xi'^2 V}{3 \xi''} \ll 1, \quad (17)$$
which shall be extensively used in the following analysis. More importantly, the differential equation (11), which essentially connects the scalar coupling function $\xi(\phi)$ with the scalar potential, becomes,

$$\frac{V'}{V^2} + \frac{4\kappa^4}{3} \xi' \simeq 0,$$

which must be obeyed by both the scalar coupling function $\xi(\phi)$ and the scalar potential, and essentially it is very important for the analysis that follows.

Let us proceed to demonstrate how the whole Einstein–Gauss–Bonnet inflationary framework is simplified, so let us start by recalling the definition of the slow-roll indices [18],

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\dot{\phi}}{H\phi}, \quad \epsilon_3 = \frac{\dot{F}}{2HF}, \quad \epsilon_4 = \frac{\dot{E}}{2HE},$$

$$\epsilon_5 = \frac{\dot{F} + Q_a}{2HQ_t}, \quad \epsilon_6 = \frac{\dot{Q}_t}{2HQ_t},$$

with $F = \frac{1}{\kappa^2}$, and also $E$ is defined as follows,

$$E = \frac{F}{\phi^2} \left( \dot{\phi}^2 + 3 \left( \frac{(\dot{F} + Q_a)^2}{2Q_t} \right) \right),$$

and in addition $Q_a, Q_b, Q_c$ and $Q_e$ are defined as follows [18],

$$Q_a = -4\dot{\xi}H^2, \quad Q_b = -8\dot{\xi}H, \quad Q_c = F + \frac{Q_b}{2}, \quad Q_e = -16\dot{\xi}H.$$

By using the simplified equations of motion, and equations (12) and (17), the slow-roll indices take the following simplified form (after some algebra),

$$\epsilon_1 \simeq \frac{\kappa^2}{2} \left( \frac{\xi'}{\xi''} \right)^2,$$

$$\epsilon_2 \simeq 1 - \epsilon_1 - \frac{\xi\xi''}{\xi'^2},$$

$$\epsilon_3 = 0,$$

$$\epsilon_4 \simeq \frac{\xi'}{2\xi''} \frac{E'}{E},$$

$$\epsilon_5 \simeq \frac{\epsilon_1}{\lambda},$$

$$\epsilon_6 \simeq \epsilon_5 (1 - \epsilon_1),$$

where we defined the functions of $\phi$, $E = E(\phi)$ and $\lambda = \lambda(\phi)$ as follows,

$$E(\phi) = \frac{1}{\kappa^2} \left( 1 + 72 \frac{\epsilon_1}{\lambda^2} \right), \quad \lambda(\phi) = \frac{3}{4\xi''\kappa^2V}.$$
Having derived the simple expressions for the slow-roll indices, we can directly find the observational indices of inflation for the model at hand. We shall be interested in the spectral index of the primordial scalar perturbation $n_S$, the spectral index of the primordial tensor perturbations $n_T$ and the tensor-to-scalar ratio $r$, which in terms of the slow-roll indices have the following form, [18],

$$n_S = 1 - 4\epsilon_1 - 2\epsilon_2 - 2\epsilon_4,$$

(29)

$$n_T = -2(\epsilon_1 + \epsilon_6),$$

(30)

$$r = 16\left|\frac{\kappa^2 Q_e}{4H} - \epsilon_1\right| \frac{2c_\lambda^2}{2 + \kappa^2 Q_b},$$

(31)

where $c_\lambda$ is the sound speed defined as follows,

$$c_\lambda^2 = 1 + \frac{Q_a Q_e}{3Q_a^2 + \phi^2 \left(\frac{1}{2\mu} + Q_b\right)}.$$

(32)

Notice the complicated form of the tensor-to-scalar ratio which is not as simple as the one corresponding to the single scalar case. In our case, since the theory is an Einstein–Gauss–Bonnet theory, the tensor-to-scalar ratio depends on the terms $Q_a$, $Q_b$ and $Q_e$, explicitly and via the sound wave speed $c_\lambda$. However, by using the conditions (17) and (12) in conjunction with the slow-roll conditions, after some algebra it can be shown that the final expression of the tensor-to-scalar ratio is,

$$r \simeq 16\epsilon_1,$$

(33)

which is functionally identical to the single scalar field case, with, recall, $\epsilon_1$ given in equation (22). Also by using the functional form of the simplified slow-roll index $\epsilon_6$ from equations (26) and (27), one can easily obtain the final expression for the spectral index of the tensor perturbations,

$$n_T \simeq -2\epsilon_1 \left(1 - \frac{1}{\lambda} + \frac{\epsilon_1}{\lambda}\right),$$

(34)

where the function $\lambda$ is defined in equation (28). A direct comparison of the final expressions for the slow-roll indices (22)–(27), and of the tensor spectral index (34) and the tensor-to-scalar ratio (33), with the expressions obtained in reference [17], can show the simplicity of the expressions obtained in the present work. Finally, the $e$-foldings number has the form,

$$N = \int_{\phi_i}^{\phi_f} \frac{H dt}{\dot{\phi}} = \int_{\phi_i}^{\phi_f} H \frac{d\phi}{\dot{\phi}} = \int_{\phi_i}^{\phi_f} \frac{\xi''}{\xi} d\phi,$$

(35)

which is the same compared to the one presented in reference [17], with $\phi_f$ being the value of the scalar field at the end of the inflationary era, and $\phi_i$ is the value of the scalar field at the beginning of the inflationary era, precisely at the first horizon crossing time instance. Essentially, the slow-roll indices (22)–(27), the tensor spectral index (34) and the tensor-to-scalar ratio (33) are the main results of this work. We shall use this simplified framework to examine whether a viable phenomenology can be obtained. As we will see in the next section, by using relatively simple expressions for the potential and the scalar coupling function, can lead to a viable inflationary era.

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3. Confrontation with observations: a power-law Einstein–Gauss–Bonnet 5 model with blue-tilted tensor spectral index

In this section we shall demonstrate that the simplified Einstein–Gauss–Bonnet theoretical framework we introduced in the previous section, can produce a phenomenologically viable inflationary era. We shall consider the simplest case by choosing the scalar coupling function to have a power-law dependence with respect to the scalar field, and we shall examine in detail the inflationary predictions of the model. As we will show, the model apart from being viable, it also produces a blue-tilted tensor spectral index, and we discuss in brief the possibility of explaining the NANOGrav observations [59, 60] with this blue tilted model. Also, as we show, the resulting blue-tilted tensor spectrum is also compatible with the LIGO–Virgo constraints [1–5]. With regard to the inflationary era observational quantities, we shall be interested in the spectral index of the primordial scalar curvature perturbations $n_S$, the tensor-to-scalar ratio $r$, and the tensor spectral index which for the theoretical framework at hand are given in equations (29), (33) and (34) respectively. Recall that the scalar spectral index and the tensor-to-scalar ratio are constrained by the 2018 Planck data [61] as follows,

$$n_S = 0.9649 \pm 0.0042, \quad r < 0.064.$$  \hfill (36)

Now to proceed with the power-law model, let us assume that the scalar coupling function has the following power-law form,

$$\xi(\phi) = \beta(\kappa\phi)^\nu, \quad \text{(37)}$$

where $\beta$ is a dimensionless parameter, while recall that $\kappa = 1/M_p$. By substituting the scalar coupling (37) in equation (18) by solving the differential equation, we can obtain the scalar potential, which reads,

$$V(\phi) = \frac{3}{4\beta\kappa^\nu + 4\phi^\nu + 3\gamma\kappa^4}, \quad \text{(38)}$$

where $\gamma$ is a dimensionless integration constant. From both equations (37) and (38) it is apparent that both the scalar coupling function and the scalar potential have particularly simple form, in contrast to the more involved cases studied in reference [17]. In addition, for these choices of the scalar functions, the slow-roll indices (22)-(27) acquire particularly simple functional forms, which we quote here,

$$\epsilon_1 \simeq \frac{\kappa^2 \phi^2}{2(\nu - 1)^2}, \quad \text{(39)}$$

$$\epsilon_2 \simeq -\frac{\kappa^2 \phi^2 - 2\nu + 2}{2(\nu - 1)^2}, \quad \text{(40)}$$

$$\epsilon_3 = 0, \quad \text{(41)}$$

$$\epsilon_4 \simeq \frac{\phi (\kappa(2\nu - 4)\alpha(\phi)\zeta(\phi) - 8\beta\nu\zeta(\phi)\kappa^{\nu+5}\phi^\nu)}{2\kappa(\nu - 1)\phi\alpha(\phi)(\zeta(\phi) + 1)}, \quad \text{(42)}$$

$$\epsilon_5 \simeq \frac{2\beta\kappa^4 \nu(\kappa\phi)^\nu}{(\nu - 1)\alpha(\phi)}, \quad \text{(43)}$$
\[ \epsilon_6 \simeq -\frac{\beta \kappa^4 \nu (\kappa \phi)^{\nu} \left( -\kappa^2 \phi^2 + 2\nu^2 - 4\nu + 2 \right)}{\left( \nu - 1 \right)^3 \alpha(\phi)}. \]  

which are quite simple functionally, compared to reference [17]. In the slow-roll index \( \epsilon_4 \), the functions \( \zeta(\phi) \) and \( \alpha(\phi) \) are defined as follows, \( \zeta(\phi) = \frac{288i(\gamma + 2)(\nu + 2 \gamma - 1)}{(\nu - 1)^3 \alpha(\phi)^2} \) and \( \alpha(\phi) = 4\beta \kappa^{1 + \gamma} \phi^\nu + 3\gamma \kappa^4 \). Upon solving \( \epsilon_1 \simeq 0(1) \) we get the value of the scalar field at the end of the inflationary era which reads \( \phi_e \simeq \frac{\sqrt{\nu} - 18}{\kappa} \), and also the value of the scalar field \( \phi_1 \) at the first horizon crossing during the early stages of the inflationary era, can be obtained by using the definition of the \( e \)-foldings number in equation (35). Thus upon solving equation (35) with respect to \( \phi_1 \) we obtain \( \phi_1 = \frac{\sqrt{\nu} - 18}{\kappa} \). Accordingly, by evaluating the scalar spectral index at the first horizon crossing, it reads,

\[ n_S \simeq -1 + \frac{2(\nu - 2)}{\nu - 1} - 2e^{\frac{48}{\nu - 1}} + (\nu - 1)^{2\nu - 3} \nu^2 \]

\[ \times \frac{9 \beta^2 2^{\nu + 6} \left( \beta 2^{\nu + 3} (\nu - 1)^{\nu + 1} + 3\gamma (\nu - 2) e^{48} \right)}{\left( \beta 2^{\nu + 2} (\nu - 1)^{\nu} + 3\gamma e^{48} \right)^3}, \]

while the tensor-to-scalar ratio has the simple form,

\[ r \simeq 16e^{-\frac{24}{\nu}}. \]

Finally, the tensor-spectral index calculated at the first horizon crossing, has the following form,

\[ n_T \simeq \frac{12\gamma e^{\frac{48}{\nu}}}{\beta 2^{\nu + 2} (\nu - 1)^{\nu} + 3\gamma e^{48}} \]

\[ \times \frac{\beta 2^{\nu + 3} (\nu - 1)^{\nu - 1} e^{48} \left( -3\nu + (\nu - 1)\nu e^{48} + 2 \right)}{\beta 2^{\nu + 2} (\nu - 1)^{\nu} + 3\gamma e^{48}}. \]

One can assign several values to the free dimensionless parameters \( \beta, \gamma \) and \( \nu \) and the model can be confronted with the observational data. Assuming \( N \simeq 60 \), there is a large range of free parameters which can generate a phenomenologically viable inflationary era. We shall consider four sets of parameter values for the parameters \( \nu \) and \( \gamma \), which are the following,

Set 1 \( \nu = 20, \quad \gamma = 5768, \) \hspace{1cm} (48)

Set 2 \( \nu = 19, \quad \gamma = 57, \) \hspace{1cm} (49)

Set 3 \( \nu = 19, \quad \gamma = 5, \) \hspace{1cm} (50)

Set 4 \( \nu = 21, \quad \gamma = 7.7 \times 10^8. \) \hspace{1cm} (51)

Let us consider in some detail the parameter set 1. The spectral index for \( \nu \) and \( \gamma \) taking the values specified in set 1 (48) takes the following form,
3. Let us present one indicative case of red-tilted tensor spectrum, for example by choosing $\beta = \beta_{\text{red-tilted}}$ for negative values of the parameter $\beta$. After we performed our analysis, we ended up to the result that the tensor spectral index is $\beta$. Table 1 presents the order of magnitude of the approximations we made in the previous section hold true, later on in this section we discuss this issue in detail and in Table 1 we present the order of magnitude of the approximations we made in the previous section in order to derive the formulas, calculated at the horizon crossing time instance. For the same set of parameter values, namely set 1, for $\beta = 13.722$ we obtain a blue tilted spectrum, and specifically the observational indices read, $n_S = 0.966$, $n_T = 0.0420649$, $r = 0.0289206$. Notice that the tensor-to-scalar ratio is the same, which is expected since it solely depends on the parameters $\nu$ and $N$. We shall further discuss this issue in the end of this section. Now let us proceed to the rest of the parameter sets, starting with set 2, and hereafter we shall focus solely on the blue-tilted spectrum cases. So for set 2 (49) if we choose $\beta = 12.5016$ we get, $n_S = 0.966$, $n_T = 0.0446047$, and the same tensor-to-scalar ratio, thus viability is obtained for a wide range of parameter values. Proceeding with set 3 (50), for $\beta = 1.096$ $n_S = 0.966$, $n_T = 0.0446284$, and the same viability is obtained for a wide range of parameter values. In order to see visually how well do the aforementioned set of values fit the latest Planck data, in Figure 1 we present the marginalized curves of Planck data and the red curve correspond to the above three sets of values for $N = 60$. Now let us consider the approximations issue, which is definitely an important issue to discuss in some detail. In our analysis above all the approximations used in this work, are satisfied for the values of the free parameters that guarantee the inflationary viability of the model, as the reader can convince himself. However in order to make the article more self-contained, let us show here in detail that all the approximations we assumed in this paper, are satisfied during the first horizon crossing, and also we shall calculate the values of all the slow-roll indices during the first horizon crossing in order to show that the slow-roll conditions hold true during the first horizon crossing. Before getting into the details of our analysis, let us discuss an important issue, having to do with the slow-roll era of inflation. All the slow-roll approximations and all the approximations made in this article, are required to hold true at the first horizon crossing. Basically the first horizon crossing is relevant for the present day observations, but as the inflationary era proceeds, the slow-roll approximation and all the rest of the approximations issue, which is definitely an important issue to discuss in some detail. In our analysis above all the approximations used in this work, are satisfied for the values of the free parameters that guarantee the inflationary viability of the model, as the reader can convince himself. However in order to make the article more self-contained, let us show here in detail that all the approximations we assumed in this paper, are satisfied during the first horizon crossing, and also we shall calculate the values of all the slow-roll indices during the first horizon crossing in order to show that the slow-roll conditions hold true during the first horizon crossing. Before getting into the details of our analysis, let us discuss an important issue, having to do with the slow-roll era of inflation. 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However in order to make the article more self-contained, let us show here in detail that all the approximations we assumed in this paper, are satisfied during the first horizon crossing, and also we shall calculate the values of all the slow-roll indices during the first horizon crossing in order to show that the slow-roll conditions hold true during the first horizon crossing. Before getting into the details of our analysis, let us discuss an important issue, having to do with the slow-roll era of inflation. All the slow-roll approximations and all the approximations made in this article, are required to hold true at the first horizon crossing. Basically the first horizon crossing is relevant for the present day observations, but as the inflationary era proceeds, the slow-roll approximation and all the rest of the
The slow-roll approximations will be violated when the slow-roll approximation is violated. This instance is quantitatively when $\epsilon_1 \sim O(1)$. Thus it is natural near the end of inflation, when approximately $\epsilon_1 \sim O(1)$, hence in our analysis we shall concentrate only on values of the approximations at the first horizon crossing, which is relevant for the present day observations, and all the observational indices are calculated at the first horizon crossing, for $\phi = \phi_i = \sqrt{2(\nu - 1)} e^{-N/\nu - 1}$. Let us recall that the modes that exited the Hubble horizon when $\phi = \phi_i$ are the modes that are relevant for present day observations. These are the modes that correspond to $k \sim 0.05 M_{\text{Pl}}^{-1}$, hence basically these modes re-enter the horizon when the CMB was generated, so at $z \sim 1100$. Basically these are highly short-length ancient primordial modes and the CMB gives exact details on these modes. This is how the CMB can constrain inflation itself, through these modes. Let us comment though that unfortunately, to date no tensor modes have ever been observed.

Let us begin with the case set 1, so for $\beta = 13.722$, we have $\phi_i = 1.14238 M_{\text{Pl}}$ and $\phi_f = 26.8701 M_{\text{Pl}}$, and also the slow-roll indices at the first horizon crossing, thus when $\phi = \phi_i$, take the values $(\epsilon_1, \epsilon_2, \epsilon_4, \epsilon_5, \epsilon_6) = (0.001 807 54, 0.050 824, 0.032 752, -0.022 8814, -0.022 844)$. Let us note here that the values of the scalar field are of the Planck scale, and in the end of this section we shall briefly discuss if these values lead to trans-Planckian inconsistencies, the same applies for the rest of the parameter values sets. Regarding set 2 we have $\phi_i = 0.908112 M_{\text{Pl}}$ and $\phi_f = 25.4558 M_{\text{Pl}}$, and also the slow-roll indices at the first horizon crossing, take the values $(\epsilon_1, \epsilon_2, \epsilon_4, \epsilon_5, \epsilon_6) = (0.001 272 63, 0.054 2829, 0.034 6397, -0.023 6169, -0.023 5869)$. Regarding the set 3, we have $\phi_i = 0.908112 M_{\text{Pl}}$ and $\phi_f = 25.4558 M_{\text{Pl}}$, basically these are the same as in set 2 due to the fact that $\nu$ is the same. Also the slow-roll indices at the first horizon crossing, take the values $(\epsilon_1, \epsilon_2, \epsilon_4, \epsilon_5, \epsilon_6) = (0.001 272 63, 0.054 2829, 0.034 607, -0.023 605, -0.023 575)$. Finally for set 4, for $\beta = 19312$, we have $\phi_i = 1.408 19 M_{\text{Pl}}$ and $\phi_f = 28.2843 M_{\text{Pl}}$, and also the slow-roll indices at the first horizon crossing, thus when $\phi = \phi_i$, take the values $(\epsilon_1, \epsilon_2, \epsilon_4, \epsilon_5, \epsilon_6) = (0.002 478 75, 0.047 5212, 0.031 1898, -0.022 2593, -0.022 2041)$. Therefore the slow-roll conditions hold well true at the first horizon crossing. Let us now consider the quantities entering the approximations quoted in equations (6), (9), (12), (13) and (17), which we also quote here for convenience,

$$\frac{\dot{\phi}^2}{2} \ll V, \quad \kappa \frac{\xi'}{\xi''} \ll 1, \quad \frac{4\kappa^2\xi''^2V}{3\xi'} \ll 1.$$

\[ (53) \]
In table 1 we present all the results of our analysis regarding the approximations. As it can be seen, all the approximations we made hold well true at the first horizon crossing. Some values are identical but not accidentally, it is due to the functional form of the corresponding approximation, for example \( \kappa \xi' \) at the first horizon crossing. A notable feature of this simple Einstein–Gauss–Bonnet model is that it yields a blue-tilted tensor spectral index. Although the Planck data do not constrain the tensor spectral index, for red-tilted values, blue-tilted values of the tensor spectral index are constrained by the LIGO–Virgo observational data [3–5]. In the next subsection we discuss in brief the phenomenological outcomes of this model in view of the LIGO–Virgo data, and also the possibility of explaining the NANOGrav results [59, 60].

Moreover, let us note that the blue-tilted tensor spectral index is obtained for several values of the exponent \( \nu \) in equation (37) and basically the blue tilt is determined partially by the values of the parameter \( \nu \), but mainly from the values of \( \beta \). Specifically the negative values of \( \beta \) yield a red-tilted tensor spectral index, while the positive values of \( \beta \) yield a blue-tilt for the tensor spectral index. The parameter \( \nu \) indirectly affects the presence of a blue spectrum since for \( \nu \leq 3 \), only positive values of \( \beta \) are obtained, thus a blue-tilted tensor spectral index is obtained.

Also we need to note that for \( \nu \geq 23 \) the model become incompatible with the Planck data for \( N = 60 \), since the tensor-to-scalar ratio becomes quite larger than the Planck constraints. Finally let us discuss the issue of having large values for the scalar field during inflation and more importantly the fact that the scalar field value increases during inflation. The latter seems to be a model dependent issue, being related to the combined presence of the potential and of the Gauss–Bonnet coupling. There is no general rule to our knowledge that forbids this general behavior. It might be possible that the scalar field slow-rolls the potential and at the value \( \phi_f \) which is larger than \( \phi_i \), the slow-roll ends and the scalar field starts to oscillate to produce reheating. However the combined presence of the scalar potential and the Gauss–Bonnet coupling makes the interpretation difficult. The situation is quite perplexed compared with the simple scalar field rolling down its potential to reach the minimum. Another highly related question is whether the slow-roll approximated theory we developed above is a stable limit of the original theory, or whether it leads to some stable de Sitter vacuum. This is hard to tell in the context of our formalism and the only rigid answer can be given once the complete phase space of the model is studied with the constraint \( c_T^2 = 1 \). We aim to perform this analysis in a separate work focused exactly on this, but a first comment is that during inflation, it would be desirable to have an exact unstable de Sitter attractor or at least some unstable quasi-de Sitter attractor, as was shown for the \( R^2 \) model in \( f(R) \) gravity in reference [62]. With regard to the large values of the scalar field of the order \( \mathcal{O}(20M_p) \), during inflation it is expected that the scalar field takes values of the order of Planck mass but such large values are notable, and should be considered in view of the trans-Planckian aspects of the theory. This study however extends our knowledge and aims of this introductory paper, and we hope to address in the future.

Before closing let us consider another important issue that we did not consider previously, having to do with the amplitude of the scalar perturbations \( P_\zeta(k) \),

\[
P_\zeta(k) = \frac{k^3}{2\pi^2} P_\zeta(k)
\]
evaluated at the pivot scale \( k_* = 0.05 \, \text{Mpc}^{-1} \), which is relevant for CMB observations. This is constrained by Planck to take values in \( P_\zeta(k) = 2.196^{+0.051}_{-0.06} \). The reason we chose four sets of parameter values is not accidental. As we will show now, although sets 1–3 provide a viable phenomenology, they produce too small amplitude for the scalar perturbations, and only set 4 provides a viable amplitude for the scalar perturbations. However, we quote this analysis last,
because this amplitude is more or less ΛCDM connected. Recall that the scalar amplitude for the scalar perturbations \( P_\zeta(k) \) is related to the two point function for the curvature perturbations \( \zeta(k) \) via \( P_\zeta(k) \) appearing in equation (54) as follows,

\[
\langle \zeta(k)\zeta(k') \rangle = (2\pi)^3 \delta^3(k-k')P_\zeta(k).
\] (55)

Now let us see the predictions for \( P_\zeta(k_\ast) \) for the four different sets of parameter values, evaluated at the pivot scale \( k_\ast \). The formula for \( P_\zeta(k_\ast) \) for the Einstein–Gauss–Bonnet model at hand can be found in reference [18] and in the slow-roll approximation it is,

\[
P_\zeta(k) = \left( k \left(-2\epsilon_1 - \epsilon_2 - \epsilon_4\right)\left( 0.57 + \log\left( \frac{1}{1-\epsilon_2}\right) - 2 + \log(2) \right) - \epsilon_1 + 1 \right) \right)^2, \tag{56}
\]

where \( z = e^\frac{1}{3\nu} \), and we used \( k = aH \) at the horizon crossing, and also the conformal time at horizon crossing is \( \eta = -\frac{1}{aH} \). We shall evaluate it at the first horizon crossing, with \( k_\ast = aH \). For set 1, and \( \beta = 13.722 \) we obtain \( P_\zeta(k_\ast) = 0.000402806 \), for set 2 and \( \beta = 12.051 \) we get \( P_\zeta(k_\ast) = 0.0579883 \), for set 3 and \( \beta = 1.096 \) we obtain \( P_\zeta(k_\ast) = 0.661094 \), and for set 4 and \( \beta = 19312 \) we get \( P_\zeta(k_\ast) = 2.19673 \times 10^{-9} \). Obviously only set 4 produces a viable scalar perturbations amplitude, thus it seems that the model’s viability is crucially affected by the values of \( \beta \) and \( \gamma \), while \( \nu \) does not crucially affect the results. Hence, if one also requires the amplitude of the scalar perturbations to be compatible with the Planck data, one must require large values of the parameter \( \gamma \). In fact, when \( \gamma \) takes smaller values, the scalar amplitude of curvature perturbations deviates more and more from the Planck constraints. Furthermore let us comment that the approximations we made and eventually the slow-roll conditions seize to hold true at the end of the inflationary era, as it is expected.

### 3.1. Blue tilt and discussion in view of the NANOGrav results

One of the most astonishing future observations will, hopefully, be the detection of a stochastic primordial gravitational wave background. To date, the frequencies tested already by the LIGO–Virgo collaboration [6] put strong constraints on the stochastic gravitational wave spectrum

\[
\Omega_{\omega \nu}(k) = \frac{1}{\rho_c} \frac{d \rho_{\omega \nu}}{d \log k},
\]

for scales \( k_{\ell \nu} \) in the range \((1.3–5.5) \times 10^{16}, \text{ Mpc}^{-1}\), and specifically,

\[
\Omega_{\omega \nu}(k) \lesssim 1.7 \times 10^{-7}, \tag{57}
\]

at 95% CL. Essentially, these scales \( k_{\ell \nu} \) correspond to quite large wavelengths originating back to the first stages of the radiation era, and possibly during the speculated reheating era. The frequencies of the primordial modes \( k_{\ell \nu} \) are basically 20–84 Hz, the lowest range of the LIGO–Virgo sensitivities, and some serious constraints can be imposed on the tensor spectral index. The cosmic microwave background radiation scales correspond to much lower frequencies, and much large wavelengths that entered the horizon at \( k_\ast = 0.05, \text{ Mpc}^{-1} \), and...
Figure 2. The tensor spectral index constraints from the LIGO–Virgo collaboration (blue curve). Values in the orange shaded area are excluded by the LIGO–Virgo data and the blue curve indicates the maximum upper limit of the allowed values for the tensor spectral index. The green and red curves correspond to values of the tensor spectral index for the power-law Einstein–Gauss–Bonnet model of Equation (37) and (38).

only weakly constrain the tensor spectral index, but the LIGO–Virgo for a power-law tensor spectrum, for $P_T(k_*) = rA_S$, where $A_S$ is the amplitude of the scalar perturbations at $k_*$, constrain the tensor spectral index as follows [63],

$$n_T < \frac{\ln \left( \frac{\Omega_{GW}(k_{LV})}{z_{eq}} \right)}{\ln \left( \frac{A_{S}}{r} \right)} \leq 0.5, \quad (58)$$

where $z_{eq}$ is the redshift at the matter-radiation equality, and $A_S = 2.1 \times 10^{-9}$ by the latest Planck data. Upon taking the LIGO–Virgo constraint into account on the spectrum, in figure 2 we plot the dependence of the constraint (58) from the tensor-to-scalar ratio, and in the shaded area belong the values of $n_T$ which are excluded by the latest LIGO–Virgo observational constraints. The lower limit of the constraint is denoted by the blue curve in figure 2. In figure 2 there are also two curves depicted with red and green color. The green color curve corresponds to the following choice of the free parameters $(\gamma, \beta) = (7.7 \times 10^8, 19.312)$ and $\nu$ varies in the range $1 \leq \nu \leq 23.0919$ with $\nu = 1$ yielding a value $r \sim 0$ for $N = 60$ and $\nu = 23.0919$ yielding $r = 0.07$. As it can be seen in figure 2, the green curve is well fitted in the LIGO–Virgo constraints depicted by the blue curve, for approximately $r \leq 5$. The red curve corresponds to the choices $(\gamma, \beta) = (5, 1.096)$ and $\nu$ again varies in the range $(1, 23.019)$. As it can be seen, only the values of $\nu$ in the range $\nu \in [1, 23.019]$ yield a tensor spectral index compatible with the LIGO–Virgo constraints. We need to note that the overall behavior is qualitatively similar for different choices of the values of the free parameters, and the viability of the model is ensured for quite a large set of the parameter values, but we omitted this analysis for brevity. The behavior depicted in figure 2 is characteristic for all the cases omitted.

In conclusion, the power-law Einstein–Gauss–Bonnet model we developed in this section yields a blue-tilted tensor spectral index, which is compatible with the LIGO–Virgo data at frequencies 20–84 Hz. It is interesting to discuss another interesting possibility for successful phenomenological model building of Einstein–Gauss–Bonnet model, this time in relation to the NANOGrav astonishing result [59, 60]. Recently the NANOGrav collaboration [59, 60]
reported a possible detection of a stochastic gravitational wave background after nearly thirteen years data set of the pulsar timing array. Millisecond pulsars are among the most stable clocks existing in nature. Therefore if several millisecond pulsars are observed simultaneously, the slightly different timing residuals, or even deformations on the arrival time if the signals emitted by them, can originate by some intrinsic or even environmental noises, and more interestingly by a stochastic gravitational waves background. In the NANOGrav collaboration, the timing residuals originating from an array of millisecond pulsars, has been analyzed coherently to separate gravitational waves induced residuals from other possible effects. Although a stochastic gravitational wave background would surely be confirmed by NANOGrav if quadrupolar spatial correlations are also confirmed, the possibility of having a first stochastic gravitational background at frequencies $f \sim 10^{-8}$ Hz is rather exciting. Now taking into account that Big Bang nucleosynthesis corresponds to $f \sim 10^{-11}$ Hz approximately, which furthermore corresponds to a Universe’s temperature $T \sim 0.1$ MeV, or equivalently, redshift $z \sim 3 \times 10^8$, the NANOGrav frequencies correspond to an era during the radiation domination era. These modes re-entered the horizon during the radiation domination era and have a quite short wavelength, although there exist quite shorter wavelengths probed by the BBO or LISA collaboration. In any case, if the NANOGrav data indeed correspond to a stochastic gravitational wave background formed by inflationary gravitational waves, these modes are inflationary tensor modes that re-entered the horizon during the radiation domination era. The NANOGrav result indicated that the strain amplitude of the corresponding signal is $A \sim 10^{-15}$ and the spectrum of the inflationary gravitational waves would be $\Omega_{GW} \sim 10^{-8}$, which is a quite large value when one considers single field inflation models. In fact, due to the fact that single scalar field inflation has a negative tensor spectral index, the NANOGrav result cannot be explained by canonical single field inflation. An exciting possibility was proposed in references [64, 65], and it was claimed that an inflationary theoretical framework which predicts a positive tensor spectral index with a relatively low reheating temperature, can well describe the NANOGrav result. Thus in view of this exciting possibility, the result we obtained in this work can potentially describe the NANOGrav result, since our model predicts a positive tensor spectral index. However a detailed analysis of the gravitational wave power spectrum is required in order to analyze all the different aspects and possibilities of the phenomenological predictions of the model, and we shall address these issues in a future work.

4. Conclusions

In this work we provided a self-consistent and simplified Einstein–Gauss–Bonnet inflationary theoretical framework compatible with the GW170817 event observational data. Particularly, we surgically modified our previous work [17] in order to provide a simpler and more functional theoretical inflationary framework. Our assumptions were the slow-roll conditions, and in addition the conditions (12) and (17), namely $\kappa\dot{\xi}/\ddot{\xi} \ll 1$ and $4\kappa^4\dot{\xi}^2 V/\ddot{\xi} \ll 1$. The first condition is highly motivated by the slow-roll conditions, and specifically due to the fact that the expression $\kappa\dot{\xi}/\ddot{\xi}$ appears in the first slow-roll index. The second condition is imposed solely in the cosmological system in order to extract analytical results, thus it is the only condition imposed by hand. As we demonstrated, the resulting theoretical framework is easy to study analytically, and also provides us with relatively simpler expressions for the slow-roll indices and the corresponding observational indices in comparison to the ones of reference [17]. Accordingly, we used a simple power-law model in order to investigate whether it is possible to obtain viable inflationary models with this framework. As we demonstrated, the power-law model is compatible with the latest (2018) Planck data, and also the observational indices and the slow-roll
indices had particularly elegant and simple forms. A notable feature of this power-law model is that for specific values of the exponent in the scalar coupling function $\xi(\phi) \sim \phi^r$, the tensor spectral index becomes blue-tilted. We discussed this feature of the model in view of the NANOGrav results, and as it was claimed in references [64, 65] such blue-tilted inflationary theory can explain the NANOGrav results, if the signal indeed corresponds to a stochastic gravitational wave background. However, this issue should be further discussed in more detail, and this task stretches beyond the aims and scopes of this paper. The detailed analysis which will be given elsewhere should combine the generation of a successful inflationary era, the generation of a viable dark energy era, and an appropriate estimation of the primordial gravitational wave spectrum and strain amplitude should be given. Also, with regard to the predicted primordial gravitational wave spectrum, this should be compared with the general relativistic one corresponding to a standard scalar field inflationary model, with the consistency relation $n_t = -r/8$ holding true. This detailed analysis is deferred to a future work focused on this topic.

Let us note further that a significant improvement would be to find a way to explicitly calculate $\ddot{\phi}$. This however is not easy in the present context because the only way is by differentiating the approximate relation (16), and if we do this, and substitute in equation (5) one is lead to an inconsistency. So truly it is not self consistent to use the approximate relation and to our opinion this result should not be trusted, even if it is validated. Unfortunately the lack of analyticity is an obstacle. In addition, there are basically two different relations that can define $\dot{\phi}$ by neglecting $\ddot{\phi}$, one is equation (8) and the other one is (5). Thus if we explicitly differentiate the two different equations that yield an expression for $\dot{\phi}$, things will be complicated and there is doubt whether the results should be trustworthy. The lack of analyticity and the presence of both the potential $V(\phi)$ and of the scalar coupling function $\xi(\phi)$ makes things significantly more complicated in comparison to the simple scalar field case. The only way out might be in fact if one does not neglect $\ddot{\phi}$ in equation (8), solve with respect to $\ddot{\phi}$ and substitute the results in equation (5) the scalar field equation of motion. In this case basically the system of equations becomes much much more involved, but we work toward this research line and the noteworthy results will be reported in a future work. Another future perspective is considering the possible trans-Planckian inconsistencies of the models presented in this work. This might be useful due to the large values of the scalar field which are of the order $\mathcal{O}(20M_p)$, during inflation. It is expected that the scalar field takes values of the order of Planck mass, but such large values are notable to say the least, and should be considered in view of the trans-Planckian aspects of the theory. We hope to address these issues in the near future.

Data availability statement

No new data were created or analysed in this study.

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