Boundary-layer receptivity to surface non-uniformities leading to generation of Görtler vortices

A.V. Ivanov, Y.S. Kachanov, D.A. Mischenko
Khristianovich Institute of Theoretical and Applied Mechanics, SB RAS, Russia
E-mail: kachanov@itam.nsc.ru

Abstract.
The paper is devoted to experimental investigation of receptivity of a Blasius boundary layer over a concave wall to surface non-uniformities. This receptivity mechanism is responsible for excitation of either unsteady or steady Görtler vortices by surface vibrations and roughness. As the result of this study quantitative receptivity characteristics are obtained, including the coefficients of the corresponding localized receptivity mechanism. These coefficients are obtained in Fourier space and are independent of the particular shape of the surface non-uniformities.

1. Introduction

The state of the art in investigations of Görtler instability is characterized by a significant delay compared to studies of other types of linear instability of boundary-layer flows. In contrast to the Tollmien-Schlichting instability or the cross-flow instability the quantitative agreement of experimental and theoretical results on the Görtler linear-stability characteristics has been obtained only recently by Boiko et al. (2010). The observed lack of verification of theoretical approaches to investigation of linear stages of generation and development of Görtler-instability modes (i.e. of the receptivity and stability problems) does not allow developing of reliable methods for the laminar-turbulent transition prediction in flows over curved surfaces. The progress in verification of the linear stability characteristics archived by Boiko et al. (2010) has provided a basis for subsequent studies transitional flows initiated by the Görtler instability, in particular for studies of various mechanisms of onset of both steady and unsteady Görtler-instability modes, i.e. for solution of receptivity problems. Quantitative experimental solution of such problems assumes determination of the receptivity coefficients, i.e. the coefficients, which characterize the efficiency of excitation of the Görtler-instability modes by various external (with respect to the boundary layer) perturbations.

In many practical situations non-uniformities of aerodynamical surfaces can be considered as one of the most probable and efficient sources for the onset of Görtler vortices. The results of studies of the boundary layer receptivity obtained previously in various 2D and 3D boundary-layer flows testify that surface irregularities (stationary or non-stationary) excite usually very effectively various boundary-layer instability modes. The present experimental study is devoted to investigation of such mechanism in the case of excitation of steady and unsteady Görtler vortices in a boundary layer developing over a concave surface.
2. Experimental setup

2.1. Experimental model and data acquisition

The present experiments were performed at controlled experimental conditions in a low-turbulence wind-tunnel T-324 of the Khristianovich Institute of Theoretical and Applied Mechanics (ITAM) of the Siberian Branch of the Russian Academy of Sciences (Novosibirsk). The measurements have been performed by means of a hot-wire anemometer at free-stream speed of 9.18 m/s. At the present experimental conditions the free-stream turbulence level in the wind-tunnel test section did not exceed 0.02% in the frequency range above 1 Hz.

The boundary layer under study developed over an experimental model, which was designed and manufactured specially for the present experiments (figure 1). In comparison with the previous experimental model used by the authors (Boiko et al., 2010), the present model had an extended length. The model represented a concave cylindrical surface (plate 2) having radius of curvature of $R = 8.37$ m, length of 2.38 m and spanwise width of 0.996 m. An adjustable wall bump (4) was installed above the model in order to eliminate a streamwise pressure gradient and to make a practically gradientless boundary-layer flow. The exactly fixed curvature of the working surface of the model was provided by a rigid-jointed framework consisted of a block of arch-shaped duralumin ribs (5). A Plexiglass plate (of 8 mm thickness) was braced to the framework. The boundary layer developing on this plate was the subject of the present study. The plate leading section was made thinner and had a cylindrical leading edge (with radius of curvature of 1.5 mm). A specially designed disturbance source (figure 2), producing controlled surface non-uniformities, was mounted into the model at a streamwise distance $x = 290.3$ mm from the model leading edge (3 in figure 1).

At the source location the value of the local Görtler number $G^* = \frac{U_e \delta^*}{\nu} \sqrt{\frac{\delta^*}{R}}$ (the main parameter of the Görtler instability problem) was 8.16 (here $\delta^*$ is the boundary layer displacement thickness and $\nu$ is the kinematic air viscosity). The region of the main measurements was located in a range of the curvilinear streamwise coordinate (along the surface) $x = 290$ to 1200 mm, which corresponded to a range of the Görtler numbers $G^* = 8.16$ to 24.73.

A single-wire probe of a hot-wire anemometer (7 in figure 1) was used for measurements of both the mean flow velocity and velocity disturbances. The probe was installed on a three-component traversing mechanism (8), which enabled the probe positioning at any point of the region of measurements. The output signal of the hot-wire anemometer was input to a computer synchronously with a reference signal of the disturbance source via an eight-channel BNC-2120 connector and 16-bit A/D convertor PCI-6035E National Instruments. Similarly, signals of a
platinum resistance temperature detector and an electronic manometer (which measured flow temperature and incident flow velocity, respectively) were also input into the computer.

2.2. Design of the disturbance source

The main part of the source (the surface vibrator) build-in into the experimental model surface (figure 2) simulated controlled unsteady surface non-uniformities. This part of the source represented a set of oscillating membranes (3) mounted flush with the wall and distributed periodically in span. The membranes made of latex rubber (with thickness of 80 microns) were driven pneumatically with the help of several loudspeakers, which were located outside the wind-tunnel test section and connected to the surface vibrators by means of a set of flexible plastic pipes (4). The loudspeakers were controlled by an electronic device through a multi-channel power amplifier and produced air pressure fluctuations, which forced harmonic oscillations of the membranes with amplitudes of several dozens of microns. Variation of the membrane diameter and spanwise step enabled to vary the spanwise wavelength of the surface non-uniformities and, hence, the spanwise scale of the excited Görtler vortices. Two disturbance sources were used in the present experiments, they had spanwise steps of 4 and 6 mm. As far as the neighboring membranes oscillated in antiphase, these steps provided excitation of disturbances with spanwise wavelengths of 8 and 12 mm, respectively, which corresponded to Görtler vortices belonging to a region of the strongest amplification due to the linear instability mechanism (Boiko et al., 2010).

The input harmonic signals (with desired frequency, amplitude, and phase) to each of the speakers were produced by one of eight channels of an electronic unit of the disturbance source VS-II (Borodulin et al., 2000) consisted of a block of specialized digital-to-analog converters (DAC) combined with power amplifiers. Each channel was able to generate its own signal (stored in the DAC ROM and composed of 2048 points) of virtually any desired shape. The signals were produced by a special PC program (uploaded into the ROM prior to the experiment) and played back during the signal generation at a rate between 20 and 200000 Hz controlled by an external clock generator.

2.3. Base-flow parameters and regimes of measurements

The measurements have shown that the base-flow characteristics over the experimental model are independent practically of the spanwise coordinate $z$, i.e. the primary flow is practically two-dimensional. Due to a carefully adjusted shape of the adjustable wall bump ($4$ in figure 1), a flow with practically zero streamwise pressure gradient was made under the present experimental conditions. In order to achieve a maximum accuracy of measurements of streamwise distributions of the mean flow velocity, a special procedure of correction of the measured distributions was applied. This procedure includes determination and accounting for even very weak changes of the flow velocity associated with either weak, quasi-steady variations of the incident wind-tunnel flow velocity, or with weak changes of the flow velocity associated with blockage effect produced by movements of the traverse. Such correction enables us to exclude, in fact, influence of any slow modulation of the incoming flow on the shape of the measured streamwise distributions of the mean flow velocity.

The resulting boundary layer was shown to correspond with high accuracy to the Blasius flow, including both the shape of the wall-normal mean velocity profile and the integral boundary-layer characteristics: (i) the displacement thickness $\delta^*$, (ii) the momentum thickness $\delta^{**}$, and (iii) the shape factor $H = \delta^*/\delta^{**}$ (see figure 3).

In the studied regimes of measurements the excited boundary-layer perturbations had the following main parameters. The amplitudes of the disturbances under study were of several tenth of hundreds of a percent of the free-stream velocity, while it was shown by Ivanov et al.
(2010) that the nonlinear threshold for unsteady Göttler vortices is much higher (of order of 4 to 6%).

There were several goals of the present experiments. One of them was to check the ability of the designed disturbance source to excite in the boundary layer unsteady Göttler vortices with reliably measurable amplitudes. To solve this problem the spanwise periods of the excited modes $\lambda_z$ were selected to be equal to 8 and 12 mm because the corresponding Göttler vortices having dimensionless spanwise periods of $\Lambda = (U_e\lambda_z/\nu)\sqrt{\lambda_z/R} = 149$ and 274 are close to the most quickly amplified modes.

The measurements are performed for unsteady, localized non-uniformities with frequencies in a range $f_1 = 2$ to 14 Hz, which corresponds to the dimensionless frequency parameters of the excited Göttler vortices $F = 2\pi f_1/U_e \cdot 10^6 = 2.27$ to 15.88.

3. Excited boundary-layer disturbances

3.1. Properties of excited modes

Shown in figure 4 is a pair of typical spanwise profiles of amplitudes (a) and phases (b) of streamwise component of the flow-velocity disturbance measured in boundary layer in the end of the studied streamwise range, i.e. in the disturbance source far-field. The measurements are performed at a wall distance $y = y_{max}$, where the streamwise component of velocity fluctuations has its maximum (the dimensionless mean flow velocity is $U/U_e = 0.6$ there). Symbols display experimental data, while solid lines represent their approximation (in the plain of complex amplitudes) with a tenth-order polynomial.

It is seen that the disturbance corresponds to a standing wave in the spanwise direction having the rated spanwise wavelength and frequency. Such shape is typical for unsteady Göttler vortices investigated in detail by Boiko et al. (2010). The measurements have shown that localized surface non-uniformities simulated by the disturbance source result in a rather efficient excitation of low-amplitude unsteady modes of boundary-layer Göttler instability. The spanwise-periodic array of these vortices represents basically a superposition of two modes of the frequency-wavenumber spectrum of boundary-layer disturbances: $(f_1, +\beta_1)$ and $(f_1, -\beta_1)$ perturbed slightly by admixture of other spanwise-wavenumber Fourier harmonics (here $\beta_1 = 2\pi/\lambda_z$ corresponds to the spanwise step of the disturbance source membranes). It is found also that the perturbations with doubled fundamental frequency $(f_2 = 2f_1)$ are practically absent in the flow demonstrating the linearity of the receptivity and instability mechanisms under study.

Characteristic wall-normal profiles of the disturbance amplitudes and phases are presented in figure 5 for a spanwise location of the disturbance amplitude maximum ($z = 70$ mm). Similar to the distributions shown in figure 4, these profiles are obtained in the end of the region of

Figure 3. Integral characteristics of the boundary layer over the experimental model (symbols) and their comparison with theoretical ones calculated for Blasius boundary layer (curves).
measurements, i.e. in the disturbance-source far field. Symbols display the experimental data, lines correspond to calculations performed by means of the linear theory of unsteady Görtler instability by A.B. Boiko (Boiko et al., 2010). Dashed line: Linear locally Parallel Theory (LPT), solid line: Linear non-local Non-parallel Theory (LNT).

It is seen that the disturbance amplitudes and phases agree very well with the two calculations and correspond to the eigenfunctions of the first mode of discrete spectrum of unsteady Görtler instability. This means, first of all, that the used disturbance source leads really to excitation in the boundary layer of disturbances, which represent the most dangerous mode of the Görtler instability problem generation of which we would like to study and we study in reality.

3.2. Downstream evolution of excited disturbances

The measured sets of spanwise distributions of the disturbance amplitudes and phases (like that shown in figure 4) were subjected to spatial Fourier transform and amplitudes and phases of pairs of the frequency-wavenumber harmonics \((f_1, \pm \beta_1)\) of perturbations, having definite absolute value of the spanwise wavenumber, were determined at every streamwise position. [Note that other modes of the frequency-wavenumber spectrum had extremely low, practically zero, amplitudes.] In this way the streamwise distributions of amplitudes and phases of the excited pairs of modes have been obtained.

Several typical examples of amplification curves and streamwise phase distributions are presented in figure 6 for the dimensionless spanwise wavenumbers \(\Lambda = 274\) and two values of the frequency parameters \(F = 9.08\) and \(15.88\). Closed symbols display experimental data, while lines
show results of calculations based on two linear stability theories indicated above (dashed lines: locally-parallel theory (LPT), solid lines: non-local non-parallel theory (LNT)). Additional sets of symbols (open circles) are presented. These symbols correspond to the experimental data obtained directly at a fixed spanwise position (near the spanwise amplitude maximum, as seen in figure 4) without application of the spanwise Fourier analysis.

![Figure 6](image)

Figure 6. Streamwise distributions of disturbance amplitudes (left) and phases (right) of unsteady Görtler vortices. \( \Lambda = 274, F = 9.08 \) (a, b) and \( F = 15.88 \) (c, d). \( U/U_e = 0.6 \). Solid circles: experimental data obtained for modes of frequency-wavenumber spectrum after spanwise-Fourier transforms. Open circles: experimental data measured directly at spanwise amplitude maximum (\( z \approx 70 \text{ mm} \)). Lines: linear stability calculations.

It is seen from figure 6 that at the present experimental conditions both the amplitudes and phases of the excited boundary-layer disturbances develop downstream in a very good agreement with the two linear theories of unsteady Görtler instability. However, the non-parallel theory (LNT) provides somewhat better agreement with observations. This result, obtained on the new experimental model with the help of the new disturbance source corroborates a similar conclusion drawn in previous investigation by Boiko et al. (2010).

The experimental data obtained for the frequency-wavenumber modes (i.e. after spatial Fourier transform) agree very well with those measured directly at a fixed spanwise coordinate (6). This result testifies to a very good spanwise periodicity of the disturbance source properties leading to generation in the boundary layer of almost purely one pair of modes of the frequency-wavenumber spectrum \((f_1, +\beta_1) \) and \((f_1, -\beta_1) \) constituting the spanwise-periodic set of unsteady Görtler vortices.

The very good agreement of the measured streamwise distributions with those calculated by the linear stability theories (figure 6) corroborates also the previously made conclusion that the present disturbance source generates basically the first discrete-spectrum mode of the problem of unsteady Görtler instability (which is the most amplified and dangerous among all Görtler-instability modes). Due to this circumstance, the length of the disturbance-source near-field (see ref. by Boiko et al. (2010)) turned out to be rather short and practically the whole region of measurements corresponds to the source far-field.
4. Receptivity characteristics

4.1. Definition of receptivity coefficients
Following paper by Gaponenko et al. (2002), for a given model geometry and the base-flow parameters the complex receptivity coefficients $\tilde{G}_r(\beta, f) = G_r(\beta, f) \exp[i\varphi_r(\beta, f)]$ can be defined as follows

$$\tilde{G}_r(\tilde{\alpha}_r, \beta, f) = \frac{\bar{B}_{omax}(\beta, f)}{\bar{C}_m(\tilde{\alpha}_r, \beta, f)}$$

Here all complex functions are marked with bar above (similar to vectors); $G_r$ and $\varphi_r$ are real amplitude and phase of the receptivity coefficient; $\bar{B}_{omax} = B_{omax} \exp(i\phi_{omax})$ is the initial complex amplitude of the excited Görtler vortices; $\bar{C}_m = C_m \exp(i\lambda_m)$ is the 2D wavenumber spectrum of the shape of membrane oscillations, which resonant values $\bar{C}_m = C_m(\tilde{\alpha}_r, \beta, f) \exp[i\lambda_m(\tilde{\alpha}_r, \beta, f)]$ are determined for streamwise wavenumbers $\tilde{\alpha}_r$ which correspond to the dispersion relation for the first Görtler-instability mode: $\tilde{\alpha}_r = \alpha_r(\beta, f)$.

The absolute value of the spanwise wavenumber $\beta$ and the disturbance frequency $f$ are free parameters, which were varied during the present experiments, while the initial amplitudes $B_{omax}(\beta, f)$ and phases $\phi_{omax}(\beta, f)$ of the excited Görtler vortices, as well as the amplitudes $C_m(\tilde{\alpha}_r, \beta, f)$ and phases $\lambda_m(\tilde{\alpha}_r, \beta, f)$ of the resonant spectrum of the surface oscillations must be found from the results of the measurements for every spanwise wavenumber and frequency.

Figure 7. Examples of instantaneous shapes of oscillations of four membranes of the disturbance sources with $\lambda_z = 8$ (a) and 12 (b) mm measured in the $(x, z)$-space and their corresponding streamwise-spanwise wavenumber spectra (c and d). $f_1 = 11$ Hz.

4.2. Shapes and wavenumber spectra of surface non-uniformities
The shapes of oscillations of the disturbance source membranes were measured accurately in every regime of excitation by means of a non-contact optical displacement measuring system "OptoNCDT 1605" with a laser sensor LD 1605-2 by "Micro-Epsilon Messtechnik". The
measurements were performed on a square net with a step of 0.02 mm (for \( \lambda_z = 8 \) mm) and 0.025 mm (for \( \lambda_z = 12 \) mm).

Shown in figures 7a, b is the illustration of characteristic instantaneous shapes of oscillations of membranes of the disturbance sources measured at one of studied frequencies (with \( f_1 = 11 \) Hz, \( F = 12.48 \)). The shape represents intermittent humps and valleys formed by the membrane displacements. After one half of the oscillation period the humps and valleys exchange with their places. The shapes of oscillations of various membranes are practically axisymmetric and almost identical to each other demonstrating the uniformity of their characteristics.

The amplitude parts of the corresponding two-dimensional wavenumber spectra of the disturbance-source oscillations are presented in figures 7c, d in the space of streamwise and spanwise wavenumbers. The spectra represent Fourier integrals in the streamwise direction and Fourier series in the spanwise direction. The largest spectral amplitudes are observed, of course, at the rated spanwise wavenumbers \( \beta = \pm \beta_1 \). Here \( \beta_1 = 2\pi/\lambda_z = 0.785 \) rad/mm for figure 7c and 0.524 rad/mm for figure 7d. Only these rated modes have been analyzed in the present study and only for these modes the receptivity coefficients have been obtained.

4.3. Resonant spectra of surface non-uniformities

The procedure of selection of resonant modes in the 2D wavenumber spectrum of surface non-uniformities is illustrated in figure 8. The amplitude parts of the streamwise-wavenumber spectra presented there correspond to two main leafs of the 2D spectra shown in figures 7(c, d) (corresponding to the spanwise wavenumbers \( \pm \beta_1 \)). The spectral amplitudes are presented in figures 8(a, b), while the corresponding spectral phases are shown in figures 8(c, d). It is seen that the modes with positive (solid lines) and corresponding negative (dashed lines) spanwise wavenumbers have practically the same streamwise-wavenumber spectra.

The vertical dashed-dotted lines indicate positions of the resonant spectral modes, i.e. of those harmonics streamwise wavenumber \( \alpha_r \) of which corresponds to the streamwise wavenumbers \( \alpha_r(\beta_1 \pm \delta_1) = +0.0134 \) and +0.0139 of the excited Görtler vortices. These values are estimated at the source position based on streamwise phase distributions like those shown in figure 6(b, d). Despite the Görtler vortices are unsteady, their streamwise wavenumbers are seen to be very small and close to zero (but not equal to zero). Circles indicate the resonant points themselves, the ordinates of which (marked with horizontal dashed-dotted lines) are equal to the amplitudes and phases of the resonant harmonics in the spectrum of surface vibrations.

4.4. Receptivity coefficients

The result of determination of amplitudes and phases of the coefficients of the boundary-layer receptivity to surface non-uniformities for excitation of the first-mode unsteady Görtler vortices are presented in figure 9 versus the disturbance frequency for two studied values of the spanwise wavelengths: \( \lambda_z = 8 \) mm (\( \Lambda = 149 \)) and \( \lambda_z = 12 \) mm (\( \Lambda = 274 \)). An additional scale shown on the top of every plot display the corresponding values of the frequency parameter \( F \). Symbols display the receptivity coefficients for frequencies 2, 5, 7, 11, and 14 Hz obtained according to the definition given in section 4.1. Solid and open symbols display results obtained by upstream extrapolation of experimental amplification curves by means of either locally-parallel (LPT) or non-local, non-parallel (LNT) linear stability theories, respectively. Lines show approximation of experimental points (as well as their extrapolation to the zero frequency) by either the second-order polynomials used for the receptivity amplitudes (figure 9a) or the first-order polynomials used for the receptivity phases (figure 9b). It is seen that the difference between the receptivity amplitudes obtained by the data extrapolation based on two different stability theories is rather small (not more than \( \pm 1% \) for \( \lambda_z = 12 \) mm and \( \pm 2.5% \) for \( \lambda_z = 8 \) mm).

The corresponding receptivity phases are also very close to each other. The coefficients of flow receptivity to surface vibrations are obtained in a broad range of frequencies, including very
Figure 8. Example of streamwise-wavenumber spectra of amplitudes (a, b) and phases (c, d) of surface vibrations obtained for spanwise wavenumbers $\pm \beta_1 = \pm 0.785$ (a, c) and $\pm 0.524$ (b, d) rad/mm. Solid and dashed lines: spectra obtained for positive and negative values of $\beta_1$. Dashed-dotted lines and circles: positioning of resonant modes. $f_1 = 11$ Hz.

Figure 9. Amplitudes (a) and phases (b) of receptivity coefficients to localized surface non-uniformities versus disturbance frequency. Solid and open symbols: upstream extrapolation of experimental amplification curves by means of LPT and LNT, respectively. Lines: approximation of experimental values and their extrapolation to the zero frequency by second-order (a) and first-order (b) polynomials.

low ones. This enables us to obtain rather reliable estimates of the coefficients of receptivity to steady surface non-uniformities ($f = 0$), i.e. to surface roughness. Such estimated can be performed by means of extrapolation of the obtained coefficients to the zero frequency. A similar procedure has been applied successfully by Gaponenko et al. (2002) for the case of excitation
of cross-flow instability modes by surface non-uniformities. The results of such estimates are presented in figures 9a, b by symbols at axis \( f = 0 \).

In general, it turned out that the receptivity coefficient amplitudes for excitation of Görtler modes are rather low (9a). For instance, for excitation of steady Görtler vortices by surface roughness the receptivity amplitudes are several times lower than those for excitation of steady cross-flow vortices. It is seen also that the receptivity amplitudes increases significantly with disturbance frequency. Due to this property, the receptivity to unsteady surface non-uniformities of frequency \( f = 14 \text{ Hz} \) is about 3 to 4 times greater than that to the steady surface roughness. For unsteady perturbations the receptivity measured for \( \lambda_z = 12 \text{ mm} \) is greater than that obtained for \( \lambda_z = 8 \text{ mm} \), especially at high frequencies. Meanwhile, for stationary disturbances no dependence on the spanwise wavelength is observed. Qualitatively similar properties are found for the receptivity phases.

5. Conclusions

The following most important results have been obtained in the present experimental study:

1. A new approach to investigation of receptivity of boundary layer on concave wall to surface non-uniformities with excitation of first discrete-spectrum modes of unsteady (in general) Görtler instability has been developed and used in experiments.

2. For the first time the complex coefficients (amplitudes and phases) of the boundary-layer receptivity to surface non-uniformities with excitation of unsteady Görtler vortices (including steady ones) are obtained experimentally. The independence of these coefficients from the disturbance amplitudes is shown. As far as the receptivity coefficients are determined in Fourier space, they are independent also of the particular shape of surface non-uniformities.

3. The receptivity is found to increase with frequency. At zero frequency no dependence on spanwise wavelength \( \lambda_z \) is observed, while at high frequencies the receptivity amplitudes depend on \( \lambda_z \) significantly.

4. The frequency dependencies of efficiency of the receptivity and instability mechanisms are found to be opposite and are in competition with each other compensating partly each other.

5. The results obtained in the present paper can be used for verification of linear receptivity theories and improvements of methods of prediction of boundary-layer transition characterized by steady and unsteady Görtler instability.

This work is supported by Russian Foundation for Basic Research (Grant 10-01-00109).

References

Boiko, A. V., Ivanov, A. V., Kachanov, Y. S. & Mischenko, D. A. 2010 Steady and unsteady Görtler boundary-layer instability on concave wall. *Eur. J. Mech. B/Fluids* 29 (2), 61–83.

Borodulin, V. I., Kachanov, Y. S. & Koptsev, D. B. 2000 Study of resonant instability wave interaction in self-similar boundary layer with adverse pressure gradient. In *Proc. X Internat. Conf. on Methods of Aerophysical Research*, , vol. 1, pp. 47–52. Inst. Theoret. Appl. Mech., Sib. Branch, RAS, Novosibirsk.

Gaponenko, V. R., Ivanov, A. V., Kachanov, Yu. S. & Crouch, J. D. 2002 Swept-wing boundary-layer receptivity to surface non-uniformities. *J. Fluid Mech.* 461, 93–126.

Ivanov, A. V., Kachanov, Y. S. & Mischenko, D. A. 2010 Investigation of weakly-nonlinear development of unsteady Görtler vortices. *Thermophysics and Aeromechanics* 17 (4), 455–481.