Adaptive state-feedback synchronization with distributed input: the cyclic case

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Abstract: Using a setting in which the input is communicated among neighbors (without exchanging any distributed observer variables), the problem of synchronizing an acyclic network of linear uncertain agents has been formulated recently as a distributed model reference adaptive control (MRAC) where each agent tries to converge to the model defined by its neighbors. In this work we show how to parametrize the distributed MRAC in cyclic and undirected graphs.

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1. INTRODUCTION

In recent years, cooperative control of multiagent systems has received increasing attention, due to its impact in formation flying, smart energy, traffic, and other areas (Ren et al. (2007); Bullo et al. (2012)). An important problem in cooperative control is to achieve in a distributed way (i.e. using local information) a common behavior for the entire network: this is the so-called synchronization problem (Dorfler and Bullo (2014); Turci et al. (2014); Gibson (2016); Casadei and Astolfi (2017)).

Synchronization has been studied for: uncertain but homogeneous agents (Li and Ding (2015); Ding and Li (2016)), or heterogeneous agents with limited uncertainty (Seyboth et al. (2016, 2015); Li et al. (2014); Mei et al. (2016)). It results that synchronization for agents that are concurrently heterogeneous and uncertain is still a major problem. Recently, to handle heterogeneity and uncertainty, it has been proposed to formulate the synchronization problem as a special model reference adaptive control (MRAC) in which each agent tries to converge to the model defined by its neighbors (Baldi and Frasca (2018); Harfouch et al. (2017a)). This formulation is based on feedback matching gains (used to match each agent to the reference model, or leader) and coupling matching gains (used to match each agent with its neighbors). Adaptive laws for both feedback and coupling gains are derived via Lyapunov analysis.

The distributed MRAC exploits a communication setting in which the input is communicated among neighbors (distributed input approach). This is alternative to another popular approach to synchronization, the distributed observer approach (Cai et al. (2017); Lu and Liu (2017); Baldi (2018)) where, in place of the input, an observation of the leader state is communicated among neighbors. Both the distributed input and the distributed observer approaches include a feedforward action and need to communicate auxiliary variables to the neighbors (inputs and observations, respectively) to reconstruct the reference signal.

Despite these similarities, the distributed observer scheme can handle cyclic graphs and undirected graphs, while the distributed input with MRAC can be applied only to acyclic directed graphs. For this reason it finds main application in platooning, where no cyclic communication occurs Harfouch et al. (2017b).

Even if the distributed observer approach can be used in a larger number of cases, the distributed input approach is relevant because the dimension of the input vector is typically smaller than the dimension of the leader state vector, and thus communication with distributed inputs is less cumbersome. While hierarchical architectures have been proposed to remove cycles in the distributed input approach (Wang et al. (2016)), the open question that motivates this work is: is it possible to design MRAC algorithms based on distributed input with the ability to handle cyclic and undirected graphs? This work gives a positive answer by showing that the same MRAC parameterization derived for the acyclic graph case can be extended to cyclic and undirected graphs.

Notation: The transpose of a matrix or of a vector is indicated with $X^T$ and $x^T$ respectively. A directed graph (digraph) is indicated with the pair $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is a finite nonempty set of nodes, and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is a set of ordered pair of nodes, called edges. The adjacency matrix $A = [a_{ij}]$ of an unweighted digraph is defined as $a_{ii} = 0$ and $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, where $i \neq j$.

2. THE ACYCLIC CASE

Fig. 1. A simple acyclic network
To recall the synchronization results in the acyclic case, let us consider the network in Fig. 1. Three agents, denoted with indices 1, 2 and 3, have uncertain dynamics
\[
\begin{align*}
\dot{x}_1 &= A_1 x_1 + b_1 u_1 \\
\dot{x}_2 &= A_2 x_2 + b_2 u_2 \\
\dot{x}_3 &= A_3 x_3 + b_3 u_3
\end{align*}
\]
where \( x_1, x_2, x_3 \in \mathbb{R}^n \) is the state, \( u_1, u_2, u_3 \in \mathbb{R} \) is the input, and \( A_1, A_2, A_3 \) and \( b_1, b_2, b_3 \) are unknown matrices of appropriate dimensions, with possibly \( A_1 \neq A_2 \neq A_3 \) and \( b_1 \neq b_2 \neq b_3 \). Time index will be omitted whenever obvious. Consider the reference model
\[
\dot{x}_m = A_m x_m + b_m r
\]
where \( x_m \in \mathbb{R}^n \) is the state of the reference model, \( r \in \mathbb{R} \) is its reference input, and \( A_m \) and \( b_m \) are known matrices of appropriate dimensions, with \( A_m \) being Hurwitz so as to have bounded state trajectories \( x_m \).

The synchronization task is achieved when \( x_1, x_2, x_3 \to x_m \) for \( t \to \infty \). Being the system matrices in (1) unknown, the synchronization task has to be achieved in an adaptive fashion. In order to have a well-posed adaptive control problem, the following assumptions should be verified.

**Assumption 1.** [Feedback matching conditions] There exist vectors \( k_1^*, k_2^*, k_3^* \) and scalars \( l_1^*, l_2^*, l_3^* \) such that
\[
\begin{align*}
A_m &= A_1 + b_1 k_1^*, \quad b_m = b_1 l_1^* \\
A_m &= A_2 + b_2 k_2^*, \quad b_m = b_2 l_2^* \\
A_m &= A_3 + b_3 k_3^*, \quad b_m = b_3 l_3^*.
\end{align*}
\]

**Assumption 2.** The signs of the input vector fields, i.e. the signs of \( l_1^*, l_2^*, l_3^* \), are known.

Assumptions 1 and 2 are classical conditions mutuated from MRAC (Tao (2003); Ioannou and Sun (2012)). We deal with the single-input case, although extension to the multi-input is possible following the related multivariable adaptive control theory. A consequence of Assumption 1 is the existence of coupling gains between neighboring agents.

**Proposition 1.** [Coupling matching conditions] There exist vectors \( k_2^*, k_3^*, k_3^* \) and scalars \( l_2^*, l_3^*, l_3^* \) such that
\[
\begin{align*}
A_1 &= A_2 + b_2 k_2^*, \quad b_1 = b_2 l_2^* \\
A_1 &= A_3 + b_3 k_3^*, \quad b_1 = b_3 l_3^* \\
A_2 &= A_3 + b_3 k_3^*, \quad b_2 = b_3 l_3^*.
\end{align*}
\]

**Proof.** To derive (4), we find from (3)
\[
b_1 = b_2 l_2^*/l_1^*, \quad A_1 - A_2 = b_2 k_2^* - b_1 k_1^*,
\]
which gives \( k_2^* = k_2^* - l_2^*/l_1^* k_1^* \) and \( l_2^* = l_2^*/l_1^* \). Similar calculations hold for \( k_3^*, k_3^*, l_3^*, l_3^* \).

The synchronization of agent 1 to the reference model is the well-known model reference adaptive control (Tao, 2003, Chap. 4): it amounts to the controller
\[
u_1(t) = k_1^*(t)x_1(t) + l_1(t)r(t)
\]
with \( k_1, l_1 \) the estimates of \( k_1^*, l_1^* \), and to the adaptive laws
\[
k_1' = -\text{sgn}(l_1^*) \gamma_k b_m^* P e_1 x_1' \\
l_1' = -\text{sgn}(l_1^*) \gamma_l b_m^* P e_1 r
\]
where \( e_1 = x_1 - x_m \), the scalars \( \gamma_k, \gamma_l > 0 \) are adaptive gains, and \( P \) is a positive definite matrix satisfying
\[
P A_m + A_m^* P = -Q, \quad Q > 0.
\]

Proving synchronization exploits the Lyapunov function
\[
V_1(e_1, \dot{k}_1, \dot{l}_1) = e_1^T P e_1 + tr \left( \frac{\dot{k}_1^T k_1}{\gamma_k |l_1^*|} + \frac{\dot{l}_1^2}{\gamma_l |l_1^*|} \right)
\]
and the error dynamics
\[
\dot{e}_1 = A_m e_2 + b_1 (\tilde{k}_1 x_1 + \tilde{l}_1 r).
\]
The details are well known, cf. (Tao, 2003, Chap. 4).

The synchronization of agent 2 to agent 1 should avoid the use of \( r \). This is possible via the controller
\[
u_2(t) = k_2^*(t)x_2(t) + k_3^*(t)(x_2(t) - x_1(t)) + l_2(t)u_1(t)
\]
and the adaptive laws
\[
k_2' = -\text{sgn}(l_2^*) \gamma_k b_m^* P(x_2 - x_1)x_1' \\
k_3' = -\text{sgn}(l_3^*) \gamma_k b_m^* P(x_2 - x_1)(x_2 - x_1)' \\
l_2' = -\text{sgn}(l_2^*) \gamma_l b_m^* P(x_2 - x_1)u_1
\]
where \( k_2, k_3, l_2 \) are the estimates of \( k_2^*, k_3^*, l_2^* \) respectively. The scalar \( l_2^* \) does not need to be estimated, only its sign is needed. The adaptation law in (12) is derived via the dynamics of the error \( e_{21} = x_2 - x_1 \)
\[
\dot{e}_{21} = A_m e_{31} + b_2 (\tilde{k}_{21} x_1 + \tilde{k}_3 e_{21} + \tilde{l}_{21} u_1)
\]
with \( \tilde{k}_{21} = k_2 - k_2^*, \tilde{k}_3 = k_3 - k_3^*, \tilde{l}_{21} = l_2 - l_2^* \). By taking the Lyapunov function
\[
V_{21} = e_{21}^T P e_{21} + tr \left( \frac{k_{21}^T k_{21}}{\gamma_k |l_{21}^*|} + \frac{k_{31}^T k_{31}}{\gamma_l |l_{31}^*|} \right)
\]
we can calculate the time derivative of (14) along (13)
\[
\dot{V}_{21} = -e_{21}^T Q e_{21} + 2(\text{sgn}(l_{21}^*)) b_m^* P e_{21} x_1' + \gamma_k^{-1} k_{21}^T k_{21}^* |l_{21}^*| + 2(\text{sgn}(l_{31}^*)) b_m^* P e_{21} u_1 + \gamma_l^{-1} l_{21}^T l_{21}^* |l_{31}^*|
\]
where we have used (12). Using standard Lyapunov arguments and Barbalat’s lemma we can show \( V_{21} \to 0 \) as \( t \to \infty \) and hence \( e_{21} \to 0 \).

To synchronize agent 3 to agents 1 and 2, let us derive the dynamics of the error \( e_{31} = x_3 - x_1 \) and \( e_{32} = x_3 - x_2 \)
\[
\dot{e}_{31} = A_m e_{31} + b_3 (u_3 - k_{31}^* x_1 - k_{32}^* x_2 - l_{31}^* u_1)
\]
\[
\dot{e}_{32} = A_m e_{32} + b_3 (u_3 - k_{31}^* x_1 - k_{32}^* x_2 - l_{32}^* u_2)
\]
which leads us to the controller
\[
u_3(t) = k_{31}^*(t)x_1(t) + k_{32}^*(t)x_2(t) + k_3^*(t) e_{31}(t) + e_{32}(t)
\]
\[
\dot{l}_{31}(t) u_1(t) + l_{32}(t) u_2(t)
\]
and the adaptive laws
Extending from Fig. 1, let us consider a set of\( 2 \) agents\( \{k_{31}, k_{32}, k_3, l_{31}, l_{32}\} \) respectively. We derive the adaptation law in (18) via the dynamics of the error \( e_{321} = e_{31} + e_{32} \) and the Lyapunov function
\[
V_{321} = e'_{321}P_{e321} + \text{tr} \left( \frac{k'_{31}k_{31}}{\gamma_k |l_{31}^e|} \right) + \text{tr} \left( \frac{k'_{32}k_{32}}{\gamma_k |l_{32}^e|} \right) + \text{tr} \left( \frac{k'_{31}k_{32}}{\gamma_k |l_{32}^e|} + \frac{k'_{32}k_{31}}{\gamma_k |l_{32}^e|} \right).
\]
(19)

It is possible to verify \( \dot{V}_{321} = -e'_{321}Qe_{321} \) and \( e_{321} \to 0 \) using similar Lyapunov arguments as before. This shows that the state of agent 3 converges to the average of the states of agents 1 and 2. Since the states of agents 1 and 2 converge to the state of the reference model, then the state of agent 3 will converge to \( x_m \) as well (cf. Rosa (2018) for the full details). Overall synchronization can be proved via the Lyapunov function \( V_1 + V_{21} + V_{321} \).

Remark 1. The adaptation laws (12) and (18) remind us of the setting of systems stabilizing each other through adaptation (Narendra and Harashangi (2014)), with the peculiar difference that the directed path to the leader makes our adaptation always stable. On the other hand, (Narendra and Harashangi (2015)) discusses instability due to the absence of such leader.

2.1 General acyclic case

Extending from Fig. 1, let us consider a set of \( N \) agents
\[
\hat{x}_i = A_i x_i + b_i u_i, \quad i \in \{1, \ldots, N\}
\]
(20)
where agent 1 is the one that can access the reference \( r \) in (2). Assumptions 1 and 2 are generalized to
\[
A_m = A_1 + b_1 k_{11}^*, \quad b_m = b_1 l_{11}^*
\]
(21)
with known signs of \( l_{11}^* \).

Remark 2. Similarly to Proposition 1, one can verify the existence, for every pair of agents \((i, j)\), of a constant vector \( k_{ji}^* \) and a scalar \( l_{ji}^* \) such that
\[
A_i = A_j + b_j k_{ji}^*, \quad b_j = b_{21} l_{ji}^*.
\]
(22)

For convenience of notation, let us rewrite (2) as
\[
\hat{x}_1 = A_0 x_0 + b_0 u_0
\]
(23)
with \( x_0 = x_m \), \( A_m = A_0 + b_0 k_{00}^*, \quad b_0 l_{00}^* = b_m \), \( u_0 = k_{00}^* x_0 + l_{00}^* r \), where \( k_{00}^*, l_{00}^* \) are known and do not need to be estimated. This gives the controller (equivalent to (6))
\[
\begin{bmatrix}
  u_1(t) = k_{01}^* x_1(t) + k_{11}^* (x_1(t) - x_0(t)) + l_{10}^* u_1(t) \\
  u_2(t) = k_{02}^* x_2(t) + k_{21}^* x_1(t) + l_{12}^* u_2(t)
\end{bmatrix}
\]
(24)
Under the following assumption a synchronization result holds.

Assumption 3. The directed communication graph is acyclic. In addition, the graph contains a directed spanning tree with the leader as the root node.

Theorem 1. Under Assumptions 1-3, consider the linear systems (20), with reference model (23), controllers
\[
u_j(t) = \frac{\sum_{i=0}^{N} a_{ij} k_{ji}^* x_i(t)}{\sum_{i=0}^{N} a_{ij}} + \frac{\sum_{i=0}^{N} a_{ij} l_{ji}^* u_i(t)}{\sum_{i=0}^{N} a_{ij}}
\]
(25)
with the index \( i = 0 \) used for the reference model (i.e. \( a_{00} \neq 0 \) only for the root node), and update laws
\[
\begin{align*}
\dot{k}_{ji}^* &= -sgn(l_{ji}^*) \gamma_k b_{ij}^* P \left( \sum_{i=0}^{N} a_{ij} e_{ji} \right) x_i^* \\
\dot{k}_{ji} &= -sgn(l_{ji}^*) \gamma_k b_{ij}^* P \left( \sum_{i=0}^{N} a_{ij} e_{ji} \right) x_i^* \\
\dot{l}_{ji}^* &= -sgn(l_{ji}^*) \gamma_k b_{ij}^* P \left( \sum_{i=0}^{N} a_{ij} e_{ji} \right) u_i^* 
\end{align*}
\]
(26)
where \( e_{ji} = x_j - x_i \), and \( k_{ji}, l_{ji}^*, l_{ji}^* \) are the estimates of \( k_{ji}^*, k_{ji}^*, l_{ji}^* \) respectively. Then, all closed-loop signals are bounded and \( e_j = x_j - x_m \), \( e_{ji} = x_j - x_i \), with \( i, j \) such that \( a_{ij} \neq 0 \), converge asymptotically to zero.

Proof 1. The proof uses the Lyapunov function
\[
V = \sum_{j=1}^{N} \sum_{i=0}^{N} a_{ij} e_{ji} \left( \frac{N}{\gamma_k l_{ji}^*} \right)^\frac{1}{2} \left( \sum_{i=0}^{N} a_{ij} e_{ji} \right) \left( \sum_{j=1}^{N} \sum_{i=0}^{N} a_{ij} \frac{\dot{l}_{ji}^*}{\gamma_k l_{ji}^*} \right)
\]
(27)
Stability tools are similar as before and left to the reader. □

3. THE CYCLIC AND UNDIRECTED CASE

To understand the effect of cycles and undirected links, let us consider the undirected network of Fig. 2 (left). Let us assume we can calculate the inputs using the same method of Theorem 1. The ideal control actions are
\[
\begin{align*}
u_1 &= k_{11}^* x_1 + l_{11}^* r + k_{12}^* (x_1 - x_2) + k_{12}^* x_2 + l_{12}^* u_2 \\
u_2 &= k_{21}^* (x_2 - x_1) + k_{21}^* x_1 + l_{12}^* u_1
\end{align*}
\]
(28)
In order to unequivocally determine \( u_1 \) and \( u_2 \), the following equation should have a unique solution
\[
\begin{bmatrix}
2 & -l_{12}^* \\
-l_{21}^* & 1
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
= \begin{bmatrix}
k_{11}^* (2x_1 - x_2) + l_{11}^* r + k_{12}^* x_2 \\
k_{12}^* (x_1 - x_2) + k_{12}^* x_1 + l_{12}^* u_1
\end{bmatrix}
\]
From Proposition 1 we have \( l_{12}^* l_{21}^* = 1 \), so the determinant of the square matrix above is \( 2 - l_{12}^* l_{21}^* = 1 \), and the

![Fig. 2. Simple undirected (left) and cyclic (right) networks](image)
ideal inputs \( u_1 \) and \( u_2 \) are well defined. In addition, the synchronization error dynamics with the ideal gains is

\[
\begin{align*}
\dot{e}_1 + \dot{e}_{12} &= 2A_1 x_1 + b_1 k_1^* x_1 + b_1 l_1^* r - A_m x_m - b_m r \\
&\quad - b_1 k_1^* x_2 + k_1^* x_2 + l_1^* u_2 - A_2 x_2 - b_2 u_2 \\
\dot{e}_{21} &= A_2 x_2 + b_2 k_2^* (x_2 - x_1) + b_2 k_2^* x_1 + b_2 l_2^* u_1 \\
&\quad - A_1 x_1 - b_1 u_1 \tag{29}
\end{align*}
\]

which leads to

\[
\dot{e}_1 + \dot{e}_{12} = A_m (e_1 + e_{12}), \quad \dot{e}_{21} = A_m e_{21}. \tag{30}
\]

Let us now consider the directed cyclic network of Fig. 2 (right) and calculate the inputs using the method of Theorem 1

\[
\begin{align*}
2u_1 &= k_1^* x_1 + l_1^* r + k_1^* (x_1 - x_3) + k_1^* l_3^* x_3 + l_1^* u_3 \\
u_2 &= k_2^* (x_2 - x_1) + k_2^* x_1 + l_2^* u_1 \\
u_3 &= k_3^* (x_3 - x_2) + k_3^* l_2^* x_3 + l_3^* u_2. \tag{31}
\end{align*}
\]

In order to unequivocally determine \( u_1, u_2 \) and \( u_3 \), the following equation should have a unique solution

\[
\begin{bmatrix}
2 & 0 & -l_{13}^* \\
-l_{21}^* & 1 & 0 \\
0 & -l_{32}^* & 1
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix} =
\begin{bmatrix}
2k_1^* x_1 + l_1^* r - (k_1^* - k_1^* l_3^* x_3 \\
k_2^* (x_2 - x_1) + k_2^* x_1 \\
k_3^* (x_3 - x_2) + k_3^* l_2^* x_3 + l_3^* u_2
\end{bmatrix}
\]

The determinant of the square matrix above is 2 – \( l_{13}^* l_{21}^* l_{32}^* = 1 \), so even in this case the ideal inputs \( u_1, u_2, \) and \( u_3 \) are well defined. In addition, using similar calculations as in the previous case, it is possible to show that

\[
\begin{align*}
\dot{e}_1 + \dot{e}_{13} &= A_m (e_1 + e_{13}) \\
\dot{e}_{21} &= A_m e_{21}, \quad \dot{e}_{32} = A_m e_{32}. \tag{32}
\end{align*}
\]

Moving beyond the analysis with ideal input, we have that in the presence of parametric uncertainties, the following result, which extends the parametrization in (Baldi and Frasca (2018)) to general graphs, holds.

Theorem 2. Under Assumptions 1 and 2, for any pair of agents \((j, i)\), the dynamics of the synchronization error

\[
\sum_{i=0}^{N} a_{ij} \dot{e}_{ji} = A_m \sum_{i=0}^{N} a_{ij} e_{ji} + b_j . \tag{33}
\]

holds independently of the network connection.

Proof. 2. The parametrization (33) turns out to be independent on the network connection thanks to the fact that all error dynamics can be homogenized to the reference model \((A_m, b_m)\) via appropriate control gains. Therefore, the dynamics can be summed (cf. (29) or (32)) even in the presence of cycles and undirected links. \( \square \)

Given the parametrization (33), one might be tempted to say that the algorithm in Theorem 1 can be used straightforwardly with the Lyapunov function (27). However, some attention must be paid when doing this: in fact, the actual input \( u_j \) may not be well defined for all time instants on general graphs. To explain this point, let us collect all inputs in (25) on the left-hand side, leading to \( U [u_1 \cdots u_N]^T = [\beta_1 \cdots \beta_N]^T \) for an appropriate square matrix \( U \) depending on the estimates \( l_j \). On directed acyclic graphs, \( U \) can be made upper triangular, with positive weights in its main diagonal, thus \( U \) is always invertible. On general graphs, the invertibility of \( U \) depends on \( l_j \).\footnote{A companion paper (Baldi et al. (2018)) shows that appropriate parameter projection can guarantee invertibility of \( U \).}

Despite this difficult analytic aspect, the simulations in the next section show that the algorithm in Theorem 1 can handle networks beyond Assumption 3, and \( U \) turns out to be invertible at all time instants.

Remark 3. The expression after (31) reveals that the agent is ‘juxtaposition’ inverting the matrix \( U \) by only using neighbors’ information. How such inversion is robust to delays is an open problem worth of future investigation.

4. SIMULATIONS

The simulations are carried out on a directed acyclic network, on a directed cyclic network, and on an undirected network, as shown in Fig. 3. Node 1 acts as the leader node and the reference model is indicated as agent 0. The agents are second-order linear agents in the canonical form

\[
\dot{x}_i = \begin{bmatrix}
0 & 1 \\
-a_{ij} & b_i
\end{bmatrix} x_i + \begin{bmatrix}
0 \\
b_i
\end{bmatrix} u_i .
\tag{34}
\]

with coefficients and initial conditions as in Table 1. The matrices are given in terms of \( x_0 = A_m x_0 + b_m r \) for the reference model and \( \dot{x}_i = A_j x_i + b_i u_i \), \( i \in \{1, \ldots, N\} \) for the other agents. The other design parameters are taken as: \( \gamma_k = 3, \gamma_l = 0.3, Q = diag(1,3) \), and all estimated gains are initialized to 0. Also note that \( sgn(l_i^*) = 1, \forall i \). The simulations are carried out for a sinusoidal reference \( r \) of frequency 0.2 rad/s and amplitude 1. For the acyclic network, the resulting synchronization is shown in Figs. 4 and 7. All states converge asymptotically to the state of the reference model.

| Table 1. Coefficients and initial conditions of agents |
|--------------------------------------------------------|
| \( a_1 \) | \( a_2 \) | \( b_1 \) | \( x_0 \) |
|--------------------------------------------------------|
| agent #0 | -0.5 | -1 | 1 | \([1 - 1]'\) |
| agent #1 | -1 | 2 | 1 | \([1 1]'\) |
| agent #2 | -0.75 | 2.5 | 0.5 | \([-1 -1]'\) |
| agent #3 | -1.25 | 2 | 1.25 | \([-1 0]'\) |
| agent #4 | -0.5 | 1 | 0.75 | \([0 1]'\) |
| agent #5 | -0.75 | 1 | 1.5 | \([0 1]'\) |
| agent #6 | -1.5 | 2.5 | 1 | \([-1 1]'\) |

In the directed cyclic graph, two cycles are present (2-3-4 and 4-5-6). Using the same parameters as in the previous simulations, the resulting synchronization is shown in Figs. 5 and 8. It can be seen that synchronization is slightly faster, at the price of a larger magnitude of the input. Finally, for the undirected graph the synchronization is shown in Figs. 6 and 9. It is observed that having bidirectional connections does not necessarily help synchronization: synchronization is slower than in the previous cases.
Fig. 4. Acyclic network: state/input for reference model (dashed) and agents (solid)

Fig. 5. Cyclic network: state/input for reference model (dashed) and agents (solid)

Fig. 6. Undirected network: state/input for reference model (dashed) and agents (solid)

Fig. 7. Acyclic network: state synchronization errors for all agents

Fig. 8. Cyclic network: state synchronization errors for all agents

Fig. 9. Undirected network: state synchronization errors for all agents
5. CONCLUSIONS

We studied synchronization of uncertain agents via a MRAC formulation with distributed input. We showed that the parametrization derived for the acyclic case (Baldi and Frasca (2018)) can be extended to more general graphs. Despite a suitable Lyapunov function exists and ideal inputs (with ideal gains) might be well defined, it is difficult to prove that the actual inputs (with estimated gains) are well defined for all time instant. Simulations suggest so. Future work will include considering unmatched uncertainties (Lymeropoulos and Ioannou (2016)) and switching topologies by using adaptive switching strategies (Sang and Tao (2012); Yuan et al. (2017)).

REFERENCES

Baldi, S. (2018). Cooperative output regulation of heterogeneous unknown systems via passification-based adaptation. *IEEE Control Systems Letters*, 2(1), 151–156.

Baldi, S. and Frasca, P. (2018). Adaptive synchronization of unknown heterogeneous agents: an adaptive virtual model reference approach. *Journal of The Franklin Institute*. doi: https://doi.org/10.1016/j.jfranklin.2018.01.022.

Baldi, S., Rosa, M.R., Frasca, P., and Kosmatopoulos, E.B. (2018). Platoon merging maneuvers in the presence of parametric uncertainty. *7th IFAC Workshop on Distributed Estimation and Control in Networked Systems (NecSys18)*.

Bullo, F., Carli, R., and Frasca, P. (2012). Gossip coverage control for robotic networks: Dynamical systems on the space of partitions. *SIAM Journal on Control and Optimization*, 50(1), 419–447.

Cai, H., Lewis, F.L., Hu, G., and Huang, J. (2017). The adaptive distributed observer approach to the cooperative output regulation of linear multi-agent systems. *Automatica*, 75, 299 – 305.

Casadei, G. and Astolfi, D. (2017). Multi-pattern output consensus in networks of heterogeneous nonlinear agents with uncertain leader: a nonlinear regression approach. *IEEE Transactions on Automatic Control*. doi: 10.1109/TAC.2017.2771316.

Ding, Z. and Li, Z. (2016). Distributed adaptive consensus control of nonlinear output-feedback systems on directed graphs. *Automatica*, 72, 46 – 52.

Dorfler, F. and Bullo, F. (2014). Synchronization in complex networks of phase oscillators: A survey. *Automatica*, 50(6), 1539 – 1564.

Gibson, T.E. (2016). Adaptation and synchronization over a network: Asymptotic error convergence and pinning. In *2016 IEEE 55th Conference on Decision and Control (CDC)*, 2969–2974.

Harfouch, Y.A., Yuan, S., and Baldi, S. (2017a). Adaptive control of interconnected networked systems with networked heterogeneous platooning. In *13th IEEE International Conference on Control Automation (ICCA)*, 212–217.

Harfouch, Y.A., Yuan, S., and Baldi, S. (2017b). An adaptive switched control approach to heterogeneous platooning with inter-vehicle communication losses. *IEEE Transactions on Control of Network Systems*. doi: 10.1109/TCNS.2017.2718359.

Ioannou, P. and Sun, J. (2012). *Robust Adaptive Control*. Dover Publications.

Li, Z. and Ding, Z. (2015). Distributed adaptive consensus and output tracking of unknown linear systems on directed graphs. *Automatica*, 55, 12 – 18.

Li, Z., Duan, Z., and Lewis, F.L. (2014). Distributed robust consensus control of multi-agent systems with heterogeneous matching uncertainties. *Automatica*, 50(3), 883 – 889.

Lu, M. and Liu, L. (2017). Cooperative output regulation of linear multi-agent systems by a novel distributed dynamic compensator. *IEEE Transactions on Automatic Control*, 62(12), 6481–6488.

Lymeropoulos, G. and Ioannou, P. (2016). Adaptive control of networked distributed systems with unknown interconnections. In *2016 IEEE 55th Conference on Decision and Control (CDC)*, 3456–3461.

Mei, J., Ren, W., and Chen, J. (2016). Distributed consensus of second-order multi-agent systems with heterogeneous unknown inertias and control gains under a directed graph. *IEEE Transactions on Automatic Control*, 61(8), 2019–2034.

Narendra, K.S. and Harshangi, P. (2014). Unstable systems stabilizing each other through adaptation. In *2014 American Control Conference*, 7–12.

Narendra, K.S. and Harshangi, P. (2015). Unstable systems stabilizing each other through adaptation - part ii. In *2015 American Control Conference (ACC)*, 856–861.

Ren, W., Beard, R.W., and Atkins, E.M. (2007). Information consensus in multivehicle cooperative control. *IEEE Control Systems Letters*, 27(2), 71–82.

Rosa, M.R. (2018). *Synchronization of uncertain heterogeneous agents: an adaptive virtual model reference approach*. Master’s thesis, Delft University of Technology, Delft, The Netherlands.

Sang, Q. and Tao, G. (2012). Adaptive control of piecewise linear systems: the state tracking case. *IEEE Transactions on Automatic Control*, 57(2), 522–528.

Seyboth, G.S., Dimarogonas, D.V., Johansson, K.H., Frasca, P., and Allgöwer, F. (2015). On robust synchronization of heterogeneous linear multi-agent systems with static couplings. *Automatica*, 53, 392 – 399.

Seyboth, G.S., Ren, W., and Allgower, F. (2016). Cooperative control of linear multi-agent systems via distributed output regulation and transient synchronization. *Automatica*, 68, 132 – 139.

Tao, G. (2003). *Adaptive Control Design and Analysis*. Wiley.

Turci, L.F.R., De Lellis, P., Macau, E.E.N., Di Bernardo, M., and Simões, M.M.R. (2014). Adaptive pinning control: A review of the fully decentralized strategy and its extensions. *The European Physical Journal Special Topics*, 223(13), 2649–2664.

Wang, W., Wen, C., Huang, J., and Li, Z. (2016). Hierarchical decomposition based consensus tracking for uncertain interconnected systems via distributed adaptive output feedback control. *IEEE Transactions on Automatic Control*, 61(7), 1938–1945.

Yuan, S., Schutter, B.D., and Baldi, S. (2017). Adaptive asymptotic tracking control of uncertain time-driven switched linear systems. *IEEE Transactions on Automatic Control*, 62(11), 5802–5807.