An investigation on the roll force and torque fluctuations during hot strip rolling process

Mahdi Bagheripoor* and Hosein Bisadi

Department of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran

(Received 1 November 2013; accepted 15 February 2014)

Accurate prediction of the roll separating force and roll torque is critical to assuring the quality of the product. Fluctuation of these parameters during the rolling process is even more important because of its effects on rolling setups, geometrical accuracy (especially flatness in strip rolling), and the uniformity of mechanical and microstructural properties of the rolled material. The main approach of the present study is a precision analysis of the roll force and roll torque and their instabilities during the rolling process. In doing so, a coupling multi-variable simulation model for hot strip rolling was built using non-linear thermo-viscoplastic finite element method. Fluctuations of roll force and torque about the steady-state operating point are derived and the effects of main process parameters such as rolling speed and strip reduction on these instabilities are investigated. To check the validity of the employed model, predicted results are compared with experimental data.

Keywords: hot rolling process; finite element method; roll force; roll torque; aluminum alloys

1. Introduction

Considering the market requirements for flat-rolled aluminum products, an increasing demand can be observed for tight dimension tolerance, especially for thickness distribution along the strip length and profile (thickness distribution along the strip width). At the same time, the increased running speed brings out possible vibration problems in the rolling process, especially in a cold strip mill. If the thickness variation of the hot-rolled aluminum strip can be reduced, it will be possible to increase the production speed of the cold strip mill. These claims set new demands on the acceptable rolling forces and torques which, together with the roll pass design, have a significant influence on gage variations, the shape of the product, and power consumption.

Important pieces of information such as separating force, required torque, their time history and variations during the rolling process, and causes of these variations can be utilized for developing and optimizing pass schedules and providing setup data to the mills renting powerful process tools to satisfy the requirement of product development.

Throughout this century, much research has been devoted to the roll force and torque modeling and prediction using different techniques. Conventional models which are mostly based on thermo-mechanical theory were aimed at relating the roll force, roll torque, and power or energy consumption to the rolling parameters such as the roll diameter, initial temperature of the strip, strip thickness, reduction ratio, number of

*Corresponding author. Email: m_bagheripoor@mecheng.iust.ac.ir (M. Bagheripoor)
passes, speed of rolling, and interface friction (Ford & Alexander, 1964; Sims, 1954). Arnold and Whitton in 1975 proposed a formula for roll-separating force based on Sims’ (1954) hot flat rolling theory, which included modifications for projected area of contact and empirical factors (Biswas, 2003). Joun and Hwang (1992) presented a new method of roll force prediction which was based on approximate solutions to the velocity, strain rate and stress distributions in the roll gap. Said, Lenard, Ragab, and Elkhier (1999) determined the roll separating forces and torques via low carbon steel rolling as a function of the area reduction at entry temperatures of 900, 950, 1000, 1050, and 1100 °C. They examined four empirical, mathematical models of the process for their ability to predict the roll separating forces. Yanagimoto, Morimoto, Kurahashi, and Chikushi (2002) proposed a mathematical model for prediction of rolling force and microstructure evolution in hot strip rolling (HSR), by Orowan’s theory, FDM analysis for temperature, and incremental theory for the evolution of microstructure. Bayoumil (2007) developed a kinematic analytical approach for the prediction of rolling force, rolling torque, and forward slip in the hot strip continuous rolling, based on formulating a velocity field in the roll bite zone that expresses the effect of interfacial friction on the distribution of axial velocity and longitudinal stresses across the strip thickness. Chen, Zhang, Sun, Wang, and Song (2012) developed an online rolling force model for tandem cold rolling mill by numerical integration method, whereby the plastic deformation zone was divided into small units perpendicular to the motion of the strip.

The use of the finite element (FE) technique to predict roll forces and torque has been a popular approach for many researchers, and many commercial FE software packages now exist. Kim, Lee, Shin, and Shivpuri (1992) investigated shape rolling process using computer software, TASKS, based on the finite slab-element method and their predicted roll force and torque indicated reasonable agreement with other published results. Combining the FE and boundary element methods, Shangwu, Rodrigues, and Martins (1999) carried out the three-dimensional (3-D) modeling of hot rolling process of flat strips. They predicted rolling force, rolling torque, and contact pressure on the roll for both rigid and flexible roll cases. Kwak, Lee, Hwang, and Kim (2002) used FE simulation to investigate the effect of HSR process variables on some selected non-dimensional parameters characterizing the thermo-mechanical behavior of the strip and based on those parameters derived an online model for the precise prediction of roll force and roll power. Duan and Sheppard (2002) simulated the hot flat rolling of aluminium alloy 3003 by the commercial FEM program FORGE3. They adopted an inverse analysis method to match the calculated rolling force and torque with the measured rolling force and torque by treating the friction law and the friction coefficient as free parameters. A large-deformation constitutive model applicable to the calculation of roll force and torque in heavy-reduction rolling was presented by Byon, Kim, and Lee (2004). 3-D FE analysis coupled with the proposed constitutive models has been carried out to calculate work-piece deformation and the rolling force in their investigation. An approximate model for predicting roll force and torque in plate rolling was proposed by Moon and Lee (2008). Peening effect which unavoidably occurs in plate rolling due to a small ratio of work roll radius over mean thickness of slab being deformed has been considered in their model. Wang, Peng, Xu, and Liu (2010) investigated comprehensive influences of normal stress and shear stress in longitudinal, transverse direction, and altitude to improve the accuracy of the calculation of rolling force. In their study, the rolling force was solved in 3-D finite differential method. A new model for the prediction of the roll force and tension profiles was presented by Kim, Kwak, Shin, and Hwang (2010). Their approach was based on an approximate 3-D theory of plasticity, and
considered the effect of the pre-deformation of the strip. They used a rigid-plastic FE model to examine the prediction accuracy of the proposed model. Byon, Na, and Lee (2013) coupled FE method with a modified constitutive model in which the flow stress equation has been modified according to the working temperature and strain rate at four stands (passes) of the finishing block mill of an actual rod mill, and showed that their model is more precise in prediction of roll force in continuous rod rolling where strain rates are in the range of 100–400 s\(^{-1}\) and temperature is in the range of 900–1050 °C.

Recently, some researchers have started exploring the possibility of utilizing soft computing techniques (particularly neural network and fuzzy set theory) for investigating the roll force and torque in the hot rolling process. As an improvement, Poliak, Shim, Kim, and Choo (1998) examined a proposed linear regression model to predict the roll force in hot plate rolling. Lee and Lee (2002) proposed a long-term learning method using neural network to improve the accuracy of rolling-force prediction in hot rolling mill. They combined neural network method with the conventional learning algorithm in the preCalculation stage to reduce the thickness error at the head-end part of the strip. Neural networks were used by Yang, Linkens, Talamantes-Silva, and Howard (2003) to develop a rolling force and torque prediction model, without the requirement of a physically based or an empirical model. Lee and Choi (2004) adopted an online adaptable network for the rolling force setup and according to their field test results, they deduced that the prediction ability has been improved about 30% by the proposed method than the conventional method (mathematical models). Moussaoui, Selaimia, and Abbassi (2006) discussed the combination of an artificial neural network with analytical models to improve the performance of the prediction model of finishing rolling force in HSR process. Gudur and Dixit (2008) used a radial basis function neural network for prediction of velocity field and neutral point location in the cold flat rolling process that were further refined and post-processed in the FE code to calculate required parameters such as roll force, torque, and equivalent strain and thus provided highly accurate results with less computational time. The method of fuzzy control modification and Elman dynamic recursive network-combined prediction was used by Jia, Shan, and Niu (2008), to predict the rolling force for a tandem cold mill. A multiple radial basis function neural network model to predict rolling force based on wavelet analysis was proposed by Liu, Tong, and Lin (2012). In their model, the multi-resolution wavelet analysis method has been employed to separate the rolling force signal into several sub-signals corresponding to different factors.

In summary, many techniques have been utilized for predicting the roll separating force and torque in the hot rolling process. However, almost none of the above-mentioned efforts investigated the fluctuations of roll force and torque during the process and they have generally focused on the prediction of the mean roll force and torque in the steady-state region of each pass. The roll force and roll torque are the main parameters in controlling the flatness and geometrical accuracy, yield strength and the hardness of the product, the mill stretch and vibrations of the rolling mill, and roll flattening (Lenard, 2002). Therefore, it is worth to make a more detailed investigation on the variation of these parameters and consider their changes in the steady-state region of rolling process. The present study is aimed at a precision analysis of the roll force and torque during the rolling process and understanding the effect of process condition on the instabilities of these parameters. In doing so, the FE approach has been utilized to model HSR of aluminum alloys. The model is capable of considering temperature-dependent thermo-physical properties of rolling material and 3-D rolling conditions as it occurs during HSR. To verify the validity of the analysis, the predicted results are compared with the experimental data.
2. Mathematical model

2.1. Deformation in the roll gap

The thermo-viscoplastic FEM based on the flow formulation of the penalized form of the incompressibility was used to analyze the HSR process. According to the variational principle, the basic equation for the FE formulation is expressed as Equation (1) (Kobayashi, Oh, & Altan, 1989):

$$\int_V \sigma \delta \varepsilon \, dV + \int_V K \dot{\varepsilon}_e \delta \varepsilon \, dV - \int_S F_s \dot{u}_s \, ds = 0$$  \hspace{1cm} (1)

Here, $\sigma$ is the effective stress which is a function of strain, strain rate and temperature, $\dot{\varepsilon}$ the effective strain rate, $F_s$ the surface tractions, $u_s$ the velocity field, $V$ the volume, $S$ the boundary surface of subdomain, $K$ the penalty constant, and $\dot{\varepsilon}_e$ is the volumetric stain rate.

The metal starts flowing or deforming plastically when the applied stress reaches the value of yield stress. For investigating the metal plastic deformation behavior, the material yield stress under uniaxial conditions, as a function of strain, strain rate, and temperature, should also be considered as flow stress. This was done using a hyperbolic sine equation, shown in Equation (2), which relates the steady-state flow stress of the material to the strain rate and temperature under which it is deformed (Brown, Kim, & Anand, 1989):

$$\dot{\varepsilon} = A \exp\left(\frac{Q_{\text{def}}}{R \theta}\right) \left[ \sinh \left( \frac{\sigma}{s} \right) \right]^{\frac{1}{m}}$$  \hspace{1cm} (2)

where $A$, $\zeta$, and $m$ are material constants, $\dot{\varepsilon}$ is the mean equivalent plastic tensile strain rate, $\sigma$ the equivalent flow stress, $Q_{\text{def}}$ the activation energy for deformation, $R$ the universal gas constant, $\theta$ the absolute temperature, and $s$ is a scalar internal variable with dimensions of stress, called the deformation resistance. The coefficients of the hyperbolic sine equation for AA1100 deformation are summarized in Table 1 (Brown et al., 1989).

The flow behavior in the metal interior follows Equation (1). However, the effect of contact friction on metal flow should be considered when the interaction among metal surfaces exists. Contact friction between strip and roll is solved using Coulomb model, which is used mostly for forming simulations. Interfacial friction for the contact area is proportional to the normal force as shown in Equation (3):

$$\tau_{\text{crit}} = \mu P$$  \hspace{1cm} (3)

where $\tau_{\text{crit}}$ is the critical shear stress, $\mu$ the coefficient of friction, and $P$ is the contact pressure. The friction coefficient $\mu$ varies in wide ranges due to different rolling conditions and different options of the authors.

2.2. Heat transfer in the roll gap

Hot rolling is a thermo-mechanical process analyzed by solving the heat conduction equation coupled to a deformation analysis. During the rolling process, the temperature

| Material | $A$       | $Q$ (kJ/mol) | $\zeta$ | $m$   |
|----------|-----------|--------------|---------|-------|
| AA1100   | 1.91E+07  | 175.3        | 7       | .23348 |
distribution in the strip can be calculated using the governing partial differential equation shown in Equation (4) (Wang, Yang, & He, 2008):

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{Q} = \rho c \frac{\partial T}{\partial t} \tag{4}
\]

where \( \rho \) is the density of the rolled strip, \( c \) the specific heat of the rolled metal, \( k \) the thermal conductivity of the rolled strip, and \( \dot{Q} \) represents the volumetric rate of heat generation arising from the deformation. The heat generation term \( \dot{Q} \) is calculated using Equation (5):

\[
\dot{Q} = \eta \sigma \dot{\varepsilon} \tag{5}
\]

where \( \eta \) is the fraction of plastic work converted into heat. A distributed surface flux, \( q_{\text{fric}} \), is also generated from frictional sliding and rises rapidly near the entry and exit regions along the arc of contact inducing a dramatic change in the relative slip. Its overall contribution to the thermal balance in the hot rolling process is low, but if it is considered, its value is determined as follows (Shahani, Setayeshi, Nodamaie, Asadi, & Rezaie, 2009):

\[
q_{\text{fric}} = |\tau v| \tag{6}
\]

where \( \tau \) is the shear stress and \( v \) is the sliding velocity. Since the heat lost from the strip is gained by the work roll in the roll gap, a simultaneous solution of the governing equations of both the strip and the work roll is required. To do so, the heat transfer in the work roll can be written as (Serajzadeh, Karimi Taheri, & Mucciardi, 2002):

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r k_r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( k_r \frac{\partial T}{\partial z} \right) = \rho_r c_r \frac{\partial T}{\partial t} \tag{7}
\]

where \( \rho_r, c_r, \) and \( k_r \) are the density, the specific heat, and the thermal conductivity of the work roll, respectively. At the contact interface between the strip and the work roll, an interfacial heat transfer coefficient is assumed. The boundary conditions for heat transfer between the strip and work roll and surrounding temperature are applied as the equations employed by Kumar, Samarasekera, and Hawbolt (1992), and the interface heat transfer coefficient is determined based on the experimental work that has been conducted by Chun and Lenard (1997).

### 2.3. FE modeling

As it is observed there is a complex interconnection between governing equations. In order to solve these equations, the simulation was carried out by a 3-D coupled thermo-mechanical FE analysis using commercial FE software package Abaqus/Explicit. Combining the merits of both Lagrangian and Eulerian formulations, ALE formulation was developed to handle mesh distortion, mesh entanglement, and special boundary condition changes in hot rolling process. Employing symmetry along the center line, a quarter of the strip was considered in the model. A sensitivity analysis was employed to determine the effect of changing the mesh density on the predicted model results. The work roll geometry is limited to a 90° section with a thickness of 5 mm. In both the strip and work roll, eight-node isoparametric brick elements were employed. The chosen elements were suitable for coupled thermal-stress analysis. The steel work roll was defined as an elastic material with a Young’s modulus of 200 GPa. The large differences in elastic
modulus between the work roll and the strip causes the work roll to behave as a virtually rigid material. The thermo-physical properties of the strip and the steel work roll are taken from Biswas and Mandal (2011) and Shahani et al. (2009), respectively.

3. Results and discussion
3.1. Roll force and torque variations during the process

In this study, the experimental rolling programs of Hum, Colquhoun, and Lenard (1996) are used to investigate the rolling force and torque changes during HSR of commercial pure aluminum. Typical contact pressure distribution in the roll gap is given in Figure 1, in which an AA1100 strip with 6.28 mm thickness and 50 mm width is rolled at 500 °C with a 250 mm diameter work roll. The amounts of reduction and rolling speed are 30.4% and 80 rpm, respectively, and only one-quarter of the strip is considered in the model (as detailed in previous section). It can be seen from this figure that the distribution of the roll pressure is quite uniform after rising from zero rapidly following which a moderate rise is indicated until the peak at the neutral point, after which the pressure drops gradually. As Jing (2001) describes, besides work-hardening and temperature drop, the additional constraint caused by the friction forces (so called resistance to flow in rolling) is responsible for pressure build-up near the natural point. This type of pressure distribution is often referred to as a ‘friction hill’ in which the more homogeneous deformation pattern is expected (Dvorkin et al., 1997). According to the Li and Kobayashi (1982) findings, the amount of reduction, work roll radius, and the strip entry thickness are the most important parameters that shape the rolling pressure distribution.

Having the pressure distribution, the separating force as well as the acting torque can be calculated by integrating over the area of contact. But it is difficult to find the actual pressure distribution theoretically and working with approximate values as the average pressure will not lead to a precise analysis of rolling force and torque. Accurate
information about the rolling force is essential when designing a rolling pass schedule. The rolling force directly determines the thickness precision of rolled products. The rolling force is also the basis for computing the rolling torque. The accurate prediction of both of these parameters is imperative if mill breakdowns are to be avoided and to ensure maximum productivity in terms of geometric and property requirements. The predicted rolling force for the mentioned rolling program, as the material traverses the roll gap, is shown in Figure 2. As expected the rolling force increases gradually when the strip feeds into the mill gap and reaches a relatively steady value when the deformation is in the steady-state regime. Since only a quarter of strip is modeled, the actual rolling force is twice the average value in the steady-state deformation zone. As it shown in Figure 2, the rolling force fluctuates in the steady-state region and it varies between 21 and 27 kN.

Perhaps, the most important feature when designing the pass schedule is the calculation of energy requirement. An underpowered unit is the most obvious risk since this will lead to a reduction in productivity due to stalling. Thus, we must ensure that the motor is more than adequate for any immediate or future workloads. In the present

Figure 2. Rolling force variations during the time.

Figure 3. Rolling torque variations during the time.
The predicted rolling torque for described rolling condition is shown in Figure 3. As the rolling force, the rolling torque reaches a relatively steady value when the deformation is in the steady-state regime. But as it can be seen from Figure 3, the values of rolling torque fluctuate in the steady-state region with a high frequency.

The plots of Figures 2 and 3 indicate that the force and torque time history (in the steady-state region) show relatively large fluctuations about the average value. Such fluctuations of rolling force around the steady-state point cause variations in roll gap profile which may lead to strip flatness problems without a work roll bending system (Hu et al., 2006; Wang, Zhang, Xu, & Zhang, 2011). And the more fluctuations in roll torque leads to the shorter service life of rolling mill components (Pyun et al., 2013). Significant amplitude of the roll force and roll torque fluctuations (than the steady-state point value) can also cause mill vibrations and inhomogeneity in mechanical and microstructural properties of the rolled material. In follows, we will try to consider the effects of main process parameters on the rolling force and torque predictions. It should be noted that in order for the results not to be mesh-dependent, the mesh of the strip was kept the same in all models.

### 3.2. Effects of thickness reduction

Three real rolling pass schedules that are listed in Table 2 are chosen for considering the effects of strip thickness reduction (Hum et al., 1996). A 50 mm width strip of commercial pure aluminum (AA1100) is rolled with a 250 mm diameter work roll in all passes. Figure 4(a) and (b) compared rolling force and torque variations during the time for different thickness reduction amounts. As it is observed form Figure 4(a), the amplitude of rolling force fluctuations in the steady-state region increases with increasing thickness reduction and it varies about 7.3, 8.7, and 11.1% around the average value for 21.36, 30.4, and 39.05% thickness reduction amounts, respectively. Similar results can be deduced from Figure 4(b) and it is seen that rolling torque fluctuations are increased in higher thickness reductions (the amplitude of torque variations varies about 12.5, 13.7, and 14.2% about average value for 21.36, 30.4, and 39.05% reduction amounts, respectively). The most common effect of these phenomena is that there is more thickness variations during the rolled strip length in the higher thickness reduction amounts. It can be caused by surface condition changes such as interfacial friction in the contact area that affect the contact shear stress distribution and stick slip conditions. According to Hum et al. (1996) observations, the amounts of forward slip and friction coefficient rise with increase in thickness reduction.

Figure 5 compares the predicted and published experimental measurements (Hum et al., 1996) of rolling force and torque. Since only a quarter of strip is modeled, the actual rolling forces and torques are twice the average values in the steady-state regions. As it is observed, the predicted values are all lower than the actual ones. However, the relative error for all samples is less than 13.5%, which is known as a good accuracy in
Figure 4. The effect of thickness reduction amount on the variations of (a) roll force and (b) roll torque.

Figure 5. Comparison of: (a) roll force and (b) roll torque predictions with experimental data at different reduction amounts.
metal forming theories. As expected, rolling forces and the torques clearly increase with increasing reduction. There are several mechanisms contributing to this finding, all of which are well known. Increasing the volume of plastic deformation, and hence more mechanical work is the dominant phenomena. The higher reduction also causes the more strain hardening and affects the continuously changing strip temperature distribution and hence, the metal flow behavior during the rolling.

3.3. Effects of rolling speed

Rolling conditions that are summarized in Table 3 are chosen to consider the effects of rolling speed. Roll force and torque variations during the time are shown in Figure 6(a).

![Figure 6](image)

Table 3. Chosen pass schedules for investigation of rolling speed effects.

| Pass no. | Inlet thick. (mm) | Thick. red. (%) | Roll speed (rpm) | Inlet strip temp (°C) |
|----------|------------------|----------------|------------------|----------------------|
| 4        | 6.29             | 30.68          | 20               | 500                  |
| 5        | 6.28             | 30.41          | 60               | 500                  |
| 6        | 6.29             | 31.48          | 100              | 500                  |

Figure 6. The effect of rolling speed amount on the variations of (a) roll force and (b) roll torque.
and (b) for different rolling speeds. The magnitude of the rolling force fluctuations in the steady-state region increases as the rolling speed increases and it varies about 7, 7.9, and 9.7% around the average value for 20, 60, and 100 rpm rolling speed amounts, respectively (Figure 6(a)). For the rolling torque, amounts of variations are about 12.7, 13.1, and 14.6% around the average value for 20, 60, and 100 rpm rolling speeds (Figure 6(b)). Hence, more fluctuations of roll force and torque is expected at higher rolling speeds. The phenomena that contribute to the changes of fluctuation amplitude with rolling speed are the strip forward slip and the contact shear stress distribution which affect stick slip condition in different rolling speeds.

A comparison between the predicted and measured (Hum et al., 1996) rolling forces and torques vs. the rolling speed is shown in Figure 7. The prediction error has a mean of 13.2% for rolling force and 13.4% for rolling torque. As observed, the dependence of the rolling force and torque on the speed of rolling is not quite clear. The lack of speed effect on the forces and torques may be attributed to the balance between the strain rate dependence of the material's resistance to deformation which is increasing with the speed and the lower pass times (which decreases heat flow from the strip to the roll and environment) and higher internal heat generation (due to strain rate rises), both of which result in softening of the material and cause lower forces and torques. Therefore, effect of rolling speed on the rolling force and torque is completely related to the material flow behavior, heat transfer condition, and other process parameters such as initial strip thickness and the amount of reduction which affect the heat generation of plastic work and changing the strip material and process condition will affect the relation of rolling force and torque to the rolling speed.

![Figure 7. Comparison of: (a) roll force and (b) roll torque predictions with experimental data at different rolling speeds.](image-url)
4. Conclusions

A mathematical model based on the FE method has been developed to simulate hot rolling of commercial pure aluminum strips. The major innovation of this work is a precision analysis of the roll force and torque and their fluctuations during the rolling process. The effects of main process parameters such as rolling speed and strip reduction are considered. By comparing the model predictions with the experimental data, the performance of the model was proved with a reasonable accuracy.

The results show that the rolling force and torque values are not stable in the steady-state region and fluctuate around the average value. These fluctuations are increased in higher thickness reduction amounts that can be caused by contact condition changes. Furthermore, the average roll separating force and torque are clearly increased with increasing reduction. Predicted results indicate that rolling force and torque fluctuations rise as the rolling speed increases. However, the dependence of the average rolling force and torque on the speed of rolling is not quite clear.

References

Bayounil, S. (2007). A kinematic analytical approach to predict roll force, rolling torque and forward slip in thin hot strip continuous rolling. Ironmaking & Steelmaking, 34, 444–448.

Biswa, P., & Mandal, N. R. (2011). Effect of tool geometries on thermal history of FSW of AA1100. Welding Journal, 90, 129–135.

Biswas, S. (2003). Simulation of thermo-mechanical deformation in high speed rolling of long steel products. Worcester, MA: Worcester Polytechnic Institute.

Brown, S. B., Kim, K. H., & Anand, L. (1989). An internal variable constitutive model for hot working of metals. International Journal of Plasticity, 5, 95–130.

Byon, S. M., Kim, S. I., & Lee, Y. (2004). Predictions of roll force under heavy-reduction hot rolling using a large-deformation constitutive model. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 218, 483–494.

Byon, S. M., Na, D. H., & Lee, Y. S. (2013). Flow stress equation in range of intermediate strain rates and high temperatures to predict roll force in four-pass continuous rod rolling. Transactions of Nonferrous Metals Society of China, 23, 742–748.

Chen, S. Z., Zhang, D. H., Sun, J., Wang, J. S., & Song, J. (2012). Online calculation model of rolling force for cold rolling mill based on numerical integration. In Proceeding of the control and decision conference, Taiyuan, (pp. 3951–3955).

Chun, M. S., & Lenard, J. G. (1997). Hot rolling of an aluminum alloy using oil/water emulsions. Journal of Materials Processing Technology, 72, 283–292.

Duan, X., & Sheppard, T. (2002). Three dimensional thermal mechanical coupled simulation during hot rolling of aluminum alloy 3003. International Journal of Mechanical Sciences, 44, 2155–2172.

Dvorkin, E. N., Goldschmit, M. B., Cavaliere, M. A., Amenta, P. M., Marini, O., & Stroppiana, W. (1997). 2D finite element parametric studies of the flat-rolling process. Journal of Materials Processing Technology, 68, 99–107.

Ford, H., & Alexander, J. M. (1964). Simplified hot rolling calculations. Journal of the Institute of Metals, 92, 397–404.

Gudur, P. P., & Dixit, U. S. (2008). A neural network-assisted finite element analysis of cold flat rolling. Engineering Applications of Artificial Intelligence, 21, 43–52.

Hu, X. L., Zhang, Q. S., Zhao, Z., Tian, Y., Liu, X. H., & Wang, G. D. (2006). Application of approximation full-load distribution method to pass scheduling on plate mill with hydro-bending system. Journal of Iron and Steel Research, International, 13, 22–26.

Hum, B., Colquhoun, H. W., & Lenard, J. G. (1996). Measurements of friction during hot rolling of aluminum strips. Journal of Materials Processing Technology, 60, 331–338.

Jia, C. Y., Shan, X. Y., & Niu, Z. P. (2008). High precision prediction of rolling force based on fuzzy and nerve method for cold tandem mill. Journal of Iron and Steel Research, International, 15, 23–27.

Jing, L. (2001). Rolling mill roll design. Durham: Durham University.
Joun, M. S., & Hwang, S. M. (1992). An approximate analysis of hot-strip rolling – A new approach. *International Journal of Mechanical Sciences, 34*, 985–998.

Kim, N., Lee, S. M., Shin, W., & Shivpuri, R. (1992). Simulation of square-to-oval single pass rolling using a computationally effective finite and slab element method. *Transactions of ASME Journal of Engineering for Industry, 114*, 329–335.

Kim, Y. K., Kwak, W. J., Shin, T. J., & Hwang, S. M. (2010). A new model for the prediction of roll force and tension profiles in flat rolling. *ISIJ International, 50*, 1644–1652.

Kobayashi, S., Oh, S. I., & Altan, T. (1989). *Metal Forming and the finite-element method*. Oxford: Oxford University Press.

Kumar, A., Samarasekera, I. V., & Hawbolt, E. B. (1992). Roll-bite deformation during the hot rolling of steel strip. *Journal of Materials Processing Technology, 30*, 91–114.

Kwak, W. J., Lee, J. H., Hwang, S. M., & Kim, Y. H. (2002). A precision on-line model for the prediction of roll force and roll power in hot-strip rolling. *Metallurgical and Materials Transactions A, 33*, 3255–3272.

Lee, D. M., & Choi, S. G. (2004). Application of on-line adaptable neural network for the rolling force set-up of a plate mill. *Engineering Applications of Artificial Intelligence, 17*, 557–565.

Lee, D. M., & Lee, Y. (2002). Application of neural-network for improving accuracy of roll-force model in hot-rolling mill. *Control Engineering Practice, 10*, 473–478.

Lenard, J. G. (Ed.). (2002). *Roll force and tension profile prediction in hot-rolling mills*. Oxford: Oxford University Press.

Li, G. J., & Kobayashi, S. (1982). Rigid-plastic finite-element analysis of plate strain rolling. *Journal of Engineering for Industry, 104*, 55–63.

Liu, Y., Tong, C., & Lin, F. (2012). Rolling force prediction based on wavelet multiple-RBF neural network. In *Proceeding of the IEEE international conference on information and automation*, Shenyang, (pp. 941–944).

Moon, C. H., & Lee, Y. (2008). Approximate model for predicting roll force and torque in plate rolling with peening effect considered. *ISIJ International, 48*, 1409–1418.

Moussaoui, A., Selaimia, Y., & Abbassi, H. A. (2006). Hybrid hot strip rolling force prediction using a Bayesian trained artificial neural network and analytical models. *American Journal of Applied Sciences, 3*, 1885–1889.

Poliak, E. I., Shim, M. K., Kim, G. S., & Choo, W. Y. (1998). Application of linear regression analysis in accuracy assessment of rolling force calculations. *Metals and Materials, 4*, 1047–1056.

Pyun, Y. S., Cho, I. H., Suh, C. M., Park, J., Rogers, J., Kayumov, R., & Murakami, R. (2013). Reducing production loss by prolonging service life of rolling mill shear pin with ultrasonic nanocrystal surface modification technology. *International Journal of Precision Engineering and Manufacturing, 14*, 2027–2032.

Said, A., Lenard, J. G., Ragab, A. R., & Elkhier, M. A. (1999). The temperature, roll force and roll torque during hot bar rolling. *Journal of Materials Processing Technology, 88*, 147–153.

Serajzadeh, S., Karimi Taheri, A., & Mucciardi, F. (2002). Unsteady state work-roll temperature distribution during continuous hot slab rolling. *International Journal of Mechanical Sciences, 44*, 2447–2462.

Shahani, A. R., Setayeshi, S., Nodamaie, S. A., Asadi, M. A., & Rezaie, S. (2009). Prediction of influence parameters on the hot rolling process using finite element method and neural network. *Journal of Materials Processing Technology, 209*, 1920–1935.

Shangwu, X., Rodrigues, J. M. C., & Martins, P. A. F. (1999). Three-dimensional simulation of flat rolling through a combined finite element-boundary element approach. *Finite Elements in Analysis and Design, 32*, 221–233.

Sims, R. B. (1954). The calculation of roll force and torque in hot rolling mills. *ARCHIVE: Proceedings of the Institution of Mechanical Engineers 1847–1982 (vols 1–196)*, 168, 191–200.

Wang, P. F., Zhang, D. H., Xu, L., & Zhang, W. X. (2011). Research and application of dynamic substitution control of actuators in flatness control of cold rolling mill. *Steel Research International, 82*, 379–387.

Wang, X., Peng, Y., Xu, L., & Liu, H. (2010). A 3-D differential method for solving rolling force of PC hot strip mill. *Journal of Iron and Steel Research, International, 17*, 36–39.

Wang, X., Yang, Q., & He, A. (2008). Calculation of thermal stress affecting strip flatness change during run-out table cooling in hot steel strip rolling. *Journal of Materials Processing Technology, 207*, 130–146.
Yanagimoto, J., Morimoto, T., Kurahashi, R., & Chikushi, I. (2002). Mathematical modelling for rolling force and microstructure evolution and microstructure controlling with heavy reduction in tandem hot strip rolling. *Steel Research, 73*, 56–62.

Yang, Y. Y., Linkens, D. A., Talamantes-Silva, J., & Howard, I. C. (2003). Roll force and torque prediction using neural network and finite element modelling. *ISIJ International, 43*, 1957–1966.