Chemical Freeze-out and the QCD Phase Transition Temperature

P. Braun-Munzinger\textsuperscript{1}, J. Stachel\textsuperscript{2}, Christof Wetterich\textsuperscript{3}

\textsuperscript{1}GSI Darmstadt, Germany
\textsuperscript{2}Physikalisches Institut, Universität Heidelberg, Germany
\textsuperscript{3}Institut für Theoretische Physik, Universität Heidelberg, Germany

We argue that hadron multiplicities in central high energy nucleus-nucleus collisions are established very close to the phase boundary between hadronic and quark matter. In the hadronic picture this can be described by multi-particle collisions whose importance is strongly enhanced due to the high particle density in the phase transition region. As a consequence of the rapid fall-off of the multi-particle scattering rates the experimentally determined chemical freeze-out temperature is a good measure of the phase transition temperature.

PACS numbers: 25.75.-q

The yield of (multi-) strange hadrons produced in central high energy nucleus-nucleus collisions was proposed two decades ago [1] as a signature of quark-gluon plasma (QGP) formation. Hadron yields observed in such collisions at AGS, SPS, and RHIC energies are found to be described with high precision within a hadro-chemical equilibrium approach [2-10], governed by a chemical freeze-out temperature $T_{ch}$, baryo-chemical potential $\mu$ and the fireball volume $V_{ch}$. A recent review can be found in [11]. Importantly, the data at SPS and RHIC energy comprise multi-strange hadrons including the $\Omega$ and $\bar{\Omega}$. Their yields agree with the chemical equilibrium calculation and are strongly enhanced as compared to observations in pp collisions. The time needed to achieve this equilibrium for $\Omega$ baryons via two-body collisions was estimated [1] to be much longer than reasonable lifetimes of the fireball. The observations were thus interpreted as a sign that the system had reached a partonic phase prior to hadron production [2,14].

In this note we argue that the chemical freeze-out temperature $T_{ch}$ is actually very close to the critical temperature $T_c$ of the QCD phase transition. This observation has a far reaching consequence: Since $T_{ch}$ is measured for different values of $\mu$ the approximate association of $T_c$ with $T_{ch}$ implies that we have experimental knowledge of part of the critical line in the QCD-phase diagram.

Let us first sketch our overall picture and detail our arguments subsequently. Hadro-chemical equilibration is achieved during or at the end of the phase transition. In particular, the number of strange quarks may be established in the plasma phase and/or hadronization of the QGP. During the very early stages of the hadronic phase the relative numbers of strange baryons and mesons $K, K^*, \Lambda, \Sigma, \Xi, \Omega$ are then realized at the thermal equilibrium values according to the Bose-Einstein or Fermi distribution

$$n_j = \frac{g_j}{2\pi^2} \int_0^{\infty} p^2 dp \{ \exp[(E_j(p) - \mu_j)/T] \pm 1 \}^{-1}$$

Here $E_j^2 = M_j^2 + p^2$ and $M_j$ proportional to the vacuum mass of the hadron $j$, $\mu_j$ is the effective chemical potential, and $g_j$ counts degrees of freedom (for details see [11]). The high accuracy of the distribution in reproducing the data suggests that $T_{ch}$ plays effectively the role of a universal temperature, governing simultaneously the chemical and kinetic distributions.

In the hadronic picture the production of multi-strange hadrons can be described by multi-hadron strangeness exchange reactions\textsuperscript{1}. The multi-hadron scattering is substantial, however, only in the immediate vicinity of the critical temperature $T_c$. As $T$ decreases the multi-particle rates drop very rapidly with a high power of the particle density. Below some temperature $T_{ch}$ very close to $T_c$ only two-particle interactions and decays remain as relevant processes for a change in the relative particle numbers. These are too slow in order to equilibrate the distributions or to catch up with the decreasing temperature - chemical freeze-out occurs for $T_{ch} \approx T_c$. We proceed to discuss the three main points of this scenario in the following in more detail.

(A) The QCD phase transition corresponds to a change in the effective degrees of freedom (from hadrons to quarks and gluons) in a narrow temperature interval\textsuperscript{2}. For both the hadronic and quark-gluon phases sufficiently far away from $T_c$ the dominant processes in thermal equilibrium are two-particle scattering and decays. This is

\textsuperscript{1} Production of multi-strange baryons by multi-particle collisions has also been considered by [14]. Their argument focuses on anti-hyperon production at high baryon density, where indeed relatively short equilibration times are obtained. The authors conclude that their approach should not be applicable for RHIC energies, unless the hadronic phase has a rather long lifetime. Furthermore this approach does not take account of the expected rapid change of density near a phase transition which is central for our argument.

\textsuperscript{2} Our central argument will make no distinction between a true phase transition and a rapid "crossover".
consistent with an effective (pseudo-)particle description. Close to $T_c$, however, collective phenomena play an important role. (As an example, near $T_c$ the $\sigma$-resonance may behave like a particle with mass almost degenerate with the pions.) If $T_{ch}$ is close to $T_c$ the chemical equilibration may be described in several equivalent pictures: multi-hadron scattering, time evolving classical fields or hadronization. We emphasize, however, that no picture should contradict a hadronic description. In the end, the chemical equilibrium distribution has to be established by fast processes involving hadrons (not quarks and gluons).

Let us approach the phase transition (or a rapid crossover) from the hadronic phase. For $T$ sufficiently below $T_c$ not much happens on the level of microscopic scattering processes between hadrons. The main effect of an increase of the temperature is an increase of the density. Near $T_c$, however, the density is so high that new dynamics can be associated with collective excitations. It is precisely the behavior of these collective excitations that triggers the transition. On the level of individual hadrons the propagation and scattering of collective excitations is expressed in the form of multi-hadron scattering. Since the collective dynamics becomes dominant only near $T_c$ the same holds for the multi-hadron scattering.

We next argue that the temperature range where multi-hadron processes can dominate is actually very narrow (typically a few MeV). This is due to (i) a rapid increase of the particle densities as a function of $T$ and (ii) a very steep dependence of multi-particle scattering rates on the density of the incoming particles. Evidence for rapid energy density changes near $T_c$ comes from recent lattice QCD results\(^1\). This is shown in Fig. 4 where the observed rapid rise in energy density beyond the simple $T^4$ dependence reflects the large increase in degrees of freedom. The statistical model of a hadron resonance gas\(^2\)\(^3\) exhibits only a moderate increase beyond the $T^4$ behavior (see Fig. 4), determined by an interplay between the relevant number of hadronic states and the increased importance of repulsive interactions modeled by an excluded volume correction\(^3\).

The rate of an individual multi-particle scattering process with $n_{in}$ incoming particles and average density $\bar{n}$ grows as $\bar{n}^n(T)^{n_{in}}$. In addition, also the phase space increases with temperature. In the temperature region of multi-particle scattering dominance one expects that many processes (with different $n_{in}$) become of similar strength, thus increasing again the total rate. For the purpose of demonstration we model the temperature dependence of a typical rate (say for scatterings which change the number of $\Omega$-baryons) by $r \sim \bar{n}^\gamma(T)^\beta$, with $\gamma \gg 1$. Within a narrow temperature interval $\Delta T = 5$ MeV this rate changes by a substantial factor $(1 + \frac{\Delta T}{T})^{\beta\gamma}$ with $\beta = d \ln \bar{n}/d \ln T \gg 1$. This is demonstrated in Fig. 4. At $T_c$, the multi-particle scattering rates are substantial. For $T < T_c$, however, they drop so rapidly that only a small temperature interval around $T_c$ is left where the multi-particle scattering processes can dominate. The evaluation of these multi-particle rates is detailed in point (C) below.

(B) Let us next turn to the issue of chemical equilibrium and argue that two-particle scattering is insufficient to achieve or sustain it. We focus in the following on the analysis of data at RHIC energies and comment at the end on the applicability of our considerations at lower energies. The high accuracy of the statistical model predictions in reproducing experimental particle ratios varying over several orders of magnitude\(^2\)\(^3\)\(^1\) should be taken as an indication that hadro-chemical equilibration has occurred in the system. Further, chemical equilibration according to eq. (1) requires two crucial ingredients: (i) The particle number changing reaction rates must be sufficiently high such that the particle distributions can adapt to a given $T$. (ii) At freeze-out the masses of the different hadrons must be proportional to their vacuum masses. From (ii) we conclude immediately that chemical equilibration and freeze-out occur in the hadronic phase, $T_{ch} \leq T_c$. The particle distribution in the quark-gluon plasma has no memory of the individual hadronic vacuum masses - for example, the relative density of strange particles (at the same given $\mu$ and $T$) would be deter-

\(^3\) Neglecting the repulsive interactions the energy density of a hadronic gas diverges, reflecting the Hagedorn temperature (see dotted line in Fig. 4 and 1).
the dependence on the pion density \( n_\pi \) and on \( T \). The temperature scale is obtained as discussed in the text. The arrow indicates the chemical freeze-out temperature \( T_{ch} \).

To demonstrate that \( T_{ch} \) is close to \( T_c \) we exclude two possible alternative scenarios with \( T_{ch} \) substantially smaller than \( T_c \): First, we show that an extended period of "hadronic chemical equilibrium" with \( T_{ch} < T < T_c \) is quite unlikely. Second, we generalize this argument to demonstrate that "late equilibration" at a temperature close to \( T_{ch} \) but significantly smaller than \( T_c \) is also disfavored.

The first argument is conceptually simple since the condition for chemical equilibrium at a temperature \( T \) substantially smaller than \( T_c \) can be formulated by using the equilibrium rates and distributions. For our purpose complete thermalization of all quantities is not necessary - the "prethermalization" of some rough quantities like relative particle densities and approximate momentum distributions is sufficient, such that it makes sense to speak about temperature and chemical potential (in the sense of eq. 1) and to compute rates in thermal equilibrium.

We first need the relevant time scale to which the two-particle scattering rates have to be compared. In equilibrium this is given by the inverse rate of decrease of temperature \( \tau_T \). For this purpose we assume, guided by recent results from two-pion correlation measurements \cite{19, 20, 21, 22}, two possible scenarios for the evolution of the fireball. In both cases we use the observation that the density decreases by only 30% between chemical and thermal freeze-out and our knowledge of \( T_{ch} = 176 \) MeV. From the two-pion correlation data we obtain, for a central rapidity slice, the transverse and longitudinal radii at thermal freeze-out of 5.75 and 7.0 fm, a longitudinal expansion velocity \( v_\parallel = 1 \), and a transverse expansion velocity \( v_\perp = 0.5 \). The thermal freeze-out radii are to be understood as widths of Gaussian distributions and give a volume of \( V_f = 3650 \) fm\(^3\). Scenario (1) assumes that the shape of the density distributions is the same (i.e. Gaussian) at thermal and chemical freeze-out and that accordingly an increase in density by 30% is equivalent to a 30% decrease in volume corresponding to \( V_{ch} = 2600 \) fm\(^3\). In scenario (2) we use an initial volume of 1450 fm\(^3\) corresponding to a flat distribution over one unit of rapidity. With the assumption of isentropic expansion at the above velocities we find the longitudinal and transverse radius parameters at \( T_{ch} \) as well as the duration of the expansion and the thermal freeze-out temperature \( T_f \). For scenarios (1) and (2) the duration of the expansion in the hadronic phase is \( \tau_f = 0.9 \) and 2.3 fm and the thermal freeze-out temperature is \( T_f = 158 \) and 132 MeV. Consequently, in the hadronic phase near \( T_{ch} \) the rate of decrease in temperature may be estimated as \( |\dot{T}/T| = \tau_T^{-1} = (13 \pm 1)\%/\text{fm} = (7.7 \pm 0.6 \text{fm})^{-1} \). Note that these time scales are entirely consistent with the duration of pion emission, also obtained from the two-pion correlation data, of less than 2 fm. In both scenarios the lifetime of the fireball is rather short, leaving little room for an extended period of "hadronic cooking" at temperatures significantly below \( T_c \).

As a simple example, a decrease in temperature by \( \Delta T = 5 \) MeV reduces the equilibrium ratio of \( \Omega \)-baryons over kaons, \( n_\Omega/n_K \), by a factor \( F_{\Omega K} = 1.13 \) (using thermal model densities of \( \bar{n} \)). Adaptation of the particle distribution to the changing temperature requires \( F_{\Omega K} = \exp(\alpha \tau_f \Delta T) = \exp(\alpha n_\Omega / n_K) / \text{fm} \) with \( \alpha = d \ln(n_\Omega / n_K)/dt \).

Let us define the rate of change of individual particle number densities as

\[
\dot{\rho}_j = \frac{N_j}{V} = \dot{n}_j + n_j \dot{V}/V. \tag{2}
\]

(The last term accounts for the decrease in particle number densities due to the volume change.) Maintaining chemical equilibrium needs

\[
\left| \frac{\rho_\Omega}{n_\Omega} - \frac{\rho_K}{n_K} \right| = \frac{\ln F_{\Omega K} T_{ch}}{\Delta T} = (1.10 - 0.55) / \text{fm} = 0.55 / \text{fm}. \tag{3}
\]

The numerical evaluation of the two terms in the difference agrees well with the direct evaluation of the term involving \( F_{\Omega K} \) in eq. (3). For the difference in the rate of relative density change of \( \Omega \)-baryons to protons we obtain similarly a value of \((1.10 - 0.90) / \text{fm} = 0.20 / \text{fm} \). Both

FIG. 2: Time \( \tau_\Omega = n_\Omega/\rho_\Omega \) needed to bring \( \Omega \) baryons into chemical equilibrium via multi-particle collisions. We display the dependence on the pion density \( n_\pi \) and on \( T \). The temperature scale is obtained as discussed in the text. The arrow indicates the chemical freeze-out temperature \( T_{ch} = 176 \) MeV.
rates are evaluated here at $T_{ch}$. Eq. 3 can be obeyed either by the destruction of $\Omega$-baryons or the production of kaons. Since $\Omega$-baryons decay weakly the effect of decays is completely negligible. A typical two-particle scattering $\Omega + K \to \Xi + \pi^-$ (or $\Omega + \pi \to \Xi + K^-$) yields at most $\tilde{r}\Omega/n\Omega = nK/(\sigma) = 0.018/\text{fm}$. For this estimate we used a (large) strangeness exchange cross section of $3\text{ mb}$ [23]. Clearly both numbers are much too small to maintain equilibrium close to $T_{ch}$. Analagous arguments can be made for the $\Omega/p$ ratio. The reaction $\pi^- + \Sigma^- \to K^- + p$ contributes to the rate of relative density change of protons a value of $0.018/\text{fm}$. One would need about 50 reactions with similar cross section ($12\text{ mb}$, 24) to keep the proton density in equilibrium. For the $\Omega$ baryons there is obviously no way to achieve this. We conclude that two-particle reactions and decays are not fast enough to maintain chemical equilibrium for multi-strange baryons in the hadronic phase near $T_{ch}$. This finding is also supported by studies using cascade codes 25 and rate equations 26.

We next turn to the production rates for situations where thermal equilibrium has not yet been realized, e.g. during hadronization. The production rates should be consistent between different pictures for such processes and permit a hadronic description, at least towards the end of the chemical equilibration process. Therefore our picture does not require a detailed understanding of hadronization. Although our estimates of $\tilde{r}j/nj$ have used thermal distributions for the incoming particles our arguments can be extended to non-thermal situations: there is no reason why the rates should be much larger or the available times longer. In the final approach to chemical equilibrium (needed in order to achieve the high accuracy of the observed thermal description of the data) the densities of incoming particles must already be close to equilibrium. Also only very rough features of the momentum distribution of the incoming particles are needed for an estimate of the magnitude of $\tilde{r}j/nj$. Therefore, the two-particle scattering is also too slow to achieve chemical equilibration in the production of multistrange hadrons. In particular, this applies to a possible picture of chemical equilibration by hadronization: hadronization at $T$ much below $T_c$ would not lead to equilibrium abundances since there is not enough time to produce the multistrange baryons with rates corresponding to $T_c$. (Otherwise the hadronization picture would be in clear discrepancy with an equivalent hadronic picture for which the rates are dominated by two particle scattering.) This closes our argument: At $T_{ch}$ either multi-particle interactions must be important or the cross section must be dramatically larger than in the vacuum. Both possibilities are conceivable only for $T_{ch}$ very close to $T_c$.

(C) The chemical equilibration of hadrons should be accessible to a hadronic description, at least in a rough sense. Consistency of the hadronic picture for equilibration requires that multi-hadron processes changing the numbers of $\Omega$, $\Sigma$ etc. must be fast enough in order to build up the observed particle numbers at $T_{ch}$. (We do not assume in this part thermal equilibrium with detailed balance of individual rates.) Keeping in mind the considerable quantitative uncertainties we now proceed to evaluate rates for scattering processes involving more than two incoming particles and demonstrate the importance of such processes near $T_c$. For an understanding of multi-particle interactions we write for the rate of scattering events per volume with $n_{in}$ ingoing and $n_{out}$ outgoing particles

$$r(n_{in}, n_{out}) = \bar{n}(T)^{n_{in}}|M|^2\phi$$

with

$$\phi = \prod_{k=1}^{n_{out}} \left( \int \frac{d^3p_k}{(2\pi)^3(2E_k)} \right) (2\pi)^4\delta^4 \left( \sum_k p_k^\mu \right).$$

The rate is proportional to $n_{in}$ powers of the particle densities of the incoming particles that we denote for short by $\bar{n}(T)$. For the outgoing particles $\phi$ is the Lorentz invariant phase space factor which we evaluate case-by-case with the program given by 27, and which needs to be weighted (see below) with the thermal probability to find a particular cm energy in the initial state. The magnitude of the squared transition amplitude$^4$ $|M|^2$ is evaluated using measured cross sections. We assume constant $|M|^2$ independent of temperature and density.

Inspection of measured cross section systematics shows that production cross sections of strange particles are at most a few mb, and usually much smaller, as is reflected in the known strangeness suppression factor in hadronic collisions. Strangeness exchange reactions may reach cross sections in the 20 mb range. As cases in point we evaluate, using eq. 4, in the following the rate $r\Omega$ for $\Omega$ production through the reaction $2\pi + 3K \to N\Omega$ and similar rates for $\Xi$ and $\Lambda$ production.

For this case,

$$r\Omega = n_\Omega^5(n_K/n_\pi)^3|M|^2\phi.$$  

The densities used to evaluate this expression are taken from the thermal model predictions which describe the measured yields at RHIC 28 using the excluded volume

$^4$ We use a normalization adapted to incoming fermions and outgoing bosons. For each outgoing fermion the additional factor $2E$ in the phase space integrals is absorbed here in the squared amplitude. For each incoming boson there is an additional factor $2M$ in $|M|^2$ which is essentially cancelled by an additional factor $1/(2E_k)$ in $\phi$. 

correction. At T=176 MeV these are: \( n_\pi = 0.174/\text{fm}^3 \) and \( n_K/n_\pi = 0.172 \). Leaving out the excluded volume correction would increase the densities by approximately a factor of 2. Note that \( n_\pi \) and \( n_K \) stand here for “generic pion and kaon densities”. Indeed, the incoming particles can include all sorts of resonances like \( \rho \) etc. Therefore \( n_\pi \) and \( n_K \) are not the thermal value of individual pion and kaon densities but rather comprise effectively all non-strange and strange degrees of freedom. The 2\( \pi K \) reaction is likely to be an important channel for \( \Omega \) reactions. The function \( f \) is evaluated numerically by a Monte-Carlo program. Its results were cross-checked for massless particles against an analytic evaluation \[28\].

The numerical rate evaluation needs as input both the matrix element and phase space factor. The phase factor \( \phi \) depends on \( \sqrt{s} \) and needs to be weighted by the probability \( f(s) \) that the five-meson-scattering occurs at a given value of \( \sqrt{s} \). For this purpose we assume thermal momentum distributions for the kaons and pions in the entrance channel. The function \( f \) is evaluated numerically by a Monte-Carlo program. Its results were cross-checked for massless particles against an analytic evaluation \[28\].

For an estimate of the matrix element we note that the cross section for \( p + \bar{p} \rightarrow 5\pi \) has been measured. Close to threshold it takes a value of about 40 mb and is falling exponentially with cm energy \( \sqrt{s} \) according to \( \sigma_{5\pi} = 871 \text{mb} \cdot \exp(-\sqrt{s}/1.95/\text{GeV}) \) \[1\]. For the \( \Omega + N \rightarrow 2\pi + 3K \) reaction the threshold is 2.61 GeV and the peak of \( f(s) \) occurs at 3.25 GeV. To evaluate the corresponding matrix element we assume that the cross section at \( \sqrt{s} = 3.25 \) GeV is 6.4 mb, close to the \( pp \rightarrow 5\pi \) cross section at the same energy above threshold. From the known cross section and phase space the matrix element |\(M|\) square can be extracted by the usual formula. Using our normalization convention this yields |\(M|^2 = 9.5 \cdot 10^9/\text{GeV}^6 \). The final result for \( \Omega \) production through this channel is then \( r_\Omega = 1.39 \cdot 10^{-4}/(\text{fm}^4) \) at \( T = 176 \text{ MeV} \) and scales with the 5th power of the pion density. Furthermore, \( f \) scales approximately \[28\] as \( \sqrt{s}^{3/2} \cdot \exp(-\sqrt{s}/T) \), leading to a further increase of \( r_\Omega \) with temperature (and hence density).

We have alternatively evaluated the rate for \( \Omega \) production in a semi-classical approach, in which the standard two-body rate equation \( r = n_1 n_2 \langle \sigma v \rangle \) is generalized to multi-particle collisions. Inspired by the approach taken in cascade codes a reaction takes place if \( n_{\text{inel}} \) particles approach within a volume \( V = 4\pi/3 \langle \sigma_{\text{inel}}/\pi \rangle \). For the inelastic cross section we take a typical value of \( \sigma_{\text{inel}} = 40 \) mb. A particular exit channel (such as \( \Omega N \)) is obtained by multiplying with a probability \( P_x \). For the reaction under consideration this yields

\[
r_\Omega = 8 P_x n_\pi^3(n_K/n_\pi)^3 \sqrt{(\sigma_{1\Omega}/\pi^3)}(v).
\]  

Using for typical relative velocities \( \langle v \rangle = 0.6 \) and \( P_x = 0.10 \) yields \( r_\Omega = 1.4 \cdot 10^{-4}/\text{fm}^4 \), indicating that a similar result as above is obtained with reasonable parameters.

The meaning of this result is as follows: for a density \( n_\pi = 0.174/\text{fm}^3 \), as used above, the final \( \Omega \) density of \( 3.9 \cdot 10^{-4}/\text{fm}^3 \) can be built up within a characteristic time \( \tau_\Omega = n_\Omega/\Omega = 2.2 \) fm. Since the \( \Omega \) yield scales in this approach as \( n_\pi^3 \) and taking into account the temperature dependence of \( f \), already an increase of \( n_\pi \) to 0.2/\text{fm}^3 reduces this time by more than a factor of three. The time \( \tau_\Omega \) is depicted in Fig. 4 as function of the pion density. We have also added a temperature scale in this figure. This scale is obtained as follows: we take the variation of energy density with temperature from the lattice results (see Fig. 1) and assume that the density scales\[6\] as \( \epsilon/T \). Fixing the overall scale by \( n_\pi = 0.174/\text{fm}^3 \) at \( T = 176 \text{ MeV} \) as above this determines the \( T \)-dependence of \( n_\pi \) near \( T_c \). In consequence, close to \( T_c \), the \( \Omega \) equilibration time scales approximately as \( \tau_\Omega \propto T^{-60} \).

We note that the above time of 2.2 fm is a reasonable time for hadronization and phase change, considering the necessary decrease by about a factor of 3 in the degrees of freedom and the concomitant volume increase. A decrease of the pion density by 1/3 only will increase this time to about 27 fm; this time is even much longer than the total lifetime of the fireball as measured by two-pion correlations \[10\] while we should consider here only the time between begin of hadronization and chemical freeze-out.

We have checked the numerics of our approach by noting that, at equilibrium, detailed balance can be used to evaluate the \( 2\pi 3K \rightarrow \Omega N \) rate in the reverse direction\[6\]. Since both sides scale very differently with (overall) density, setting both rates equal determines the equilibrium density (prior to resonance decays) for each temperature. The so-determined equilibrium density is to within 25% equal to that computed independently with our thermal model, lending strong support to our calculations.

Our main results are fairly insensitive to the details of the calculation. As can be seen by inspecting Fig. 11 the energy (and consequently particle) density is very rapidly increasing with temperature near \( T=176 \text{ MeV} \) due to the phase transition. Density values of 20% different from those used in our calculations are reached

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\[5\] Part of the increase in \( \epsilon \) is due to the increase in the number of effective degrees of freedom. Similarly, more relevant channels for five particle scattering open up. As argued above \( n_\pi \) should be interpreted as an average weighted density \( \bar{n} \) of all particles contributing to five particle scattering with \( \Omega \)-production.

\[6\] We thank C. Greiner and I. Shovkowy for pointing this out.
through temperature changes of less than 3 MeV. Possible uncertainties in our rate estimates of even a factor of 2.5 would be compensated by such an increase or decrease in density, indicating the stability of our estimate near $T_c$. Furthermore, our rate estimates above are more likely overestimates because of the use of a comparatively large $\Omega N \to 3K2\pi$ cross section. The resulting larger densities needed for chemical equilibration are, however, easily reached as in our picture the phase transition is passed through from above.

We have, exactly along the lines for $\Omega$ production, evaluated the equilibration times for $\Xi$ and $\Lambda$ production with values of $\tau_\Xi = 0.71\text{fm}/c$ and $\tau_\Lambda = 0.66\text{fm}/c$, indicating that all strange baryons have similar equilibration times with similar density and temperature dependencies. The corresponding time for protons and antiprotons is typically shorter. We conclude that for all particle species the multiparticle rates are sufficient to produce the equilibrium abundances.

It is an interesting question if for some particle species like the pion-proton-antiproton system the hadronic multiparticle rates could be sufficient to maintain (restricted) chemical equilibrium even for some temperature range below $T_c$. In this case the measured chemical temperature for the proton to antiproton ratio should reflect a lower $T<T_{ch}$, which is not the case observationally. This may favour the idea that the relevant prethermalization process could be associated to hadronization.

We note that thermal models have also been used to describe hadron production in $e^+e^-$ and hadron-hadron collisions, leading to temperature parameters close to 170 MeV. Indeed, this suggests that hadronization itself can be seen as a prethermalization process. However, to account for the strangeness undersaturation in such collisions, multi-strange baryons can only be reproduced by introducing a strangeness suppression factor of about 0.5, leading to a factor of 8 suppression of $\Omega$ baryons. In contrast to heavy ion collisions, $\tau_\Omega$ exceeds here the available time. In the hadronic picture this is due to the "absence" of sufficient multi-particle scattering since the system is not in a high density phase due to a phase transition.

Has the critical temperature of the QCD-phase transition been fixed by observation? The answer to this question needs a quantitative estimate of the difference $\Delta T = T_c - T_{ch}$. An accurate determination is difficult since it involves the detailed understanding of equilibration/chemical freeze-out. From Fig. 1 we conclude that a temperature decrease of $\Delta T = 5$ MeV below the critical temperature lowers the five meson contribution to $\tau_\Omega$ by more than a factor of 10. This factor is even larger if scattering processes with more than five incoming mesons dominate at $T_{ch}$. Unless strangeness exchanging rates are unreasonably high at $T_c$, such a sharp drop should make sufficient $\Omega$ production impossible and we conclude $\Delta T \approx 5$ MeV.

The accuracy of the experimental determination of $T_c$ could be limited by a possible temperature dependence of the hadron masses. Indeed, the hadronic yields determine the ratio $T_{ch}/M$ rather than the absolute value of $T_{ch}$. A large uncertainty in $M(T)$ would reflect in a large uncertainty in $T_{ch}$ and we want to limit the size of this effect. Using chiral symmetry arguments one may surmise that the masses of hadrons except for pions and kaons are proportional to the value of the order parameter $\sigma$ responsible for chiral symmetry breaking. As a result, they depend on $T$ and $\mu$ even in the hadronic phase.

$$M_j(T) = h_j(T, \mu)\sigma(T, \mu)$$

Neglecting the $T$-and $\mu$-dependence of the dimensionless couplings $h_j$ these masses scale proportional to $\sigma(T, \mu)$. However, not all hadron masses scale proportional $\sigma(T, \mu)$ - the pions and kaons scale differently. Therefore, too large mass changes are not easily consistent with the universality of chemical freeze-out. We notice that the "observed temperature" $T_{obs} = 176$ MeV is fitted to the vacuum masses. The true freeze-out temperature $T_{ch}$ therefore obeys

$$\frac{\sigma(T_{ch}, \mu)}{T_{ch}} = \frac{\sigma(0, 0)}{T_{obs}}.$$

Since $\sigma$ is expected to decrease for $T>0$ and $|\mu|>0$ (see e.g. [30]) we infer that the true freeze-out temperature $T_{ch} \approx T_c$ is lower than $T_{obs}$. Already a moderate relative change of $\sigma$ by 10% lowers $T_{ch}$ by 18 MeV [31]. Keeping the rather vague character of our "error estimate" in mind we infer the critical temperature for small $\mu$

$$T_c = 176^{+5}_{-18} \text{ MeV}.$$
It was proposed \cite{13, 14, 32} that the observed hadron abundances arise from a direct production of strange (and non-strange) particles by hadronization. How this happens microscopically is unclear. Nevertheless, in order to escape our argument that $T_{ch}=T_c$ one would have to argue that no (even rough) hadronic picture for this process exists at all - this is unlikely since the abundances are determined by hadronic properties (masses) with high precision. Second, one may question if the “chemical temperature” extracted from the abundances is a universal temperature which also governs the local kinetic aspects and can be associated with the critical temperature of a phase transition in equilibrium. Indeed, in a prethermalization process, different equilibrium properties are realized at different time scales. Nevertheless, all experience shows that kinetic equilibration occurs before chemical equilibration. It seems quite hard to imagine that chemical equilibrium abundances are realized at a time when the rough features of kinematic distributions (like relation between particle density and average kinetic energy per degree of freedom) are not yet close to their equilibrium values. Defining the kinetic temperature by the average kinetic energy one expects that the chemical and kinetic temperatures coincide at chemical equilibrium. It seems quite hard to imagine that chemical equilibrium abundances are realized at a time when the rough features of kinematic distributions (like relation between particle density and average kinetic energy per degree of freedom) are not yet close to their equilibrium values. Defining the kinetic temperature by the average kinetic energy one expects that the chemical and kinetic temperatures coincide at chemical freeze-out (with a typical precision at the percent level). More specific properties, like detailed balance of hadronic freeze-out (with a typical precision at the percent level). The critical temperature determined from RHIC for $T_{ch} \approx T_c$ coincides well with lattice estimates \cite{10} for $\mu=0$. The same arguments as discussed here for RHIC energy also hold for SPS energies: it is likely that also there the phase transition drives the particle densities and ensures chemical equilibration. The values of $T_{ch}$ and $\mu$ collected in \cite{11} would thus trace out the phase boundary for these energies. Whether the phase transition also plays a role a lower beam energies is currently an open question.

We thank J. Knoll for helpful discussions concerning phase space integrals. PBM and JS acknowledge the hospitality of the INT Seattle, where part of this work was performed.

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