Defining relations of creep theory for a polymer composite material

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Abstract. Simplified defining relationships are being developed that will make it easier to determine the mechanical characteristics of PCM. The creep strain of a PCM in a plane-stress state is considered. The constitutive relations for PCM are obtained using the asymptotic analysis method.

For the case of a complex stress state, the defining relations contain a large number of arguments (SSS invariants) of a different order of smallness, then they will be simplified by asymptotic analysis methods. To determine the parameters included in the constructed relationships, new approaches (parametric identification methods) to solving inverse problems will be adapted or developed.

Keywords: creep, asymptotic analysis, polymer composite material.

1 Introduction

Due to the constant tightening of the operating conditions of load-bearing structural elements (increasing loads, expanding the range of permissible temperatures, increasing the duration of operation), there is an increase in requirements for the characteristics of materials.

Polymer composite materials (PCM) are highly efficient materials, their specific strength is several times higher compared to traditional materials. PCMs have pronounced inelastic properties, wherein the most significant contribution to the total strain is given by creep, moreover, the creep in PCM appears at normal temperatures. Therefore, it is necessary to determine the rheological characteristics of the material and their consideration when deforming the construction.

The defining relations in the general case, written for strain even for a plane-stress state will contain three stress state invariants as arguments. Their determination requires a large number of different experiments and special methods for their processing.

There are works [1-4], in which various hypotheses are used to reduce them or they are simplified based on analysis using features of material properties. The analysis of the work [5] on the experimental study of the fibrous composites deformation processes shows that, when loaded along the fibers, they behave almost like linearly elastic materials. During the course of shear, creep begins to manifest even at times calculated in minutes and even seconds.

In this paper, we will use an approach based on a decreasing number of arguments by asymptotic analysis of the creep defining relations [6]. It is based on using a small parameter of the ratio of the creep strain rate along with the reinforcement of the shear strain rate. This allows, in a first approximation, to leave only one-dimensional functions found from experiments. The constitutive relations for the PCM will be obtained using the asymptotic analysis method [7-11], in which the
defining relations were studied for the nonlinear theory of elasticity, the theory of plasticity and viscoelasticity.

Below we will consider PCMs, which are orthotropic bodies in a plane-stressed state.

1.1 The main relationships

To describe the creep strains of fibrous composites, the model of a viscoelastic (hereditarily elastic) material, which agrees well with experiments and is considered the most adequate for experiments. A peculiarity of the hereditarily elastic model is that after removal of the load, the creep strain disappears in the limit. However, literature analysis and performed experiments [5] show that, nevertheless, part of the creep strain cannot be restored. This suggests that this part of the creep strain should not be described by the relations of the hereditary theory of elasticity, but by the relations of the creep theory, according to which, after unloading, the accumulated creep deformations are irreversible.

Let’s write the creep law for PCM according to the theory of hardening in the form:

\[ \varepsilon_{ij}^{cr} = f(\sigma_{ij}, q). \]  

(1)

where \( \varepsilon_{ij}^{cr} \) is the creep strain rate, \( \sigma_{ij} \) is stress, \( q = q(e_i^{cr}) \) is the hardening parameter.

First, we consider a variant of the flow theory that is, the case when

\[ f(\sigma_{ij}) = \text{const}. \]  

(2)

Let’s write the power of energy dissipation during creep in the form:

\[ L = \sigma_{ij} \varepsilon_{ij} = L(\sigma_{ij}). \]  

(3)

Assuming \( L(\sigma_{ij}) = \text{const} \), we obtain in the stress space the surface of constant dissipation. The surfaces of constant dissipation do not intersect and they are embedded in each other [12]. We consider a certain loading path from point \( M \) to point \( M_1 \), moreover, this path intersects each of the surfaces of constant dissipation once (Figure 1). Then, during loading, the dissipation power does not decrease and let the power of the additional impacts be non-negative [12]:

\[ (\sigma_{ij} - \sigma_{ij}^{0}) \dot{\varepsilon}_{ij} \geq 0. \]  

(4)

Figure 1. Surfaces of constant dissipation.

From figure 1 it can be seen that the strain rate vector is directed normal to the surface, therefore

\[ \dot{\varepsilon}_{ij} = h(\sigma_{ij}) \frac{\partial L}{\partial \sigma_{ij}}. \]  

(5)

Relation (5) is an analog of the Drucker postulate in the case of a steady creep [12].

If there exists a function \( \Phi(\sigma_{ij}) \) such that

\[ \frac{\partial \Phi}{\partial \sigma_{ij}} = h(\sigma_{ij}) \frac{\partial L}{\partial \sigma_{ij}}, \]  

(6)

then the function \( \Phi \) is called the creep rate potential.
If condition (4) is satisfied, then it follows from it that the surface $L = \text{const}$ is convex, then

$$\Phi(\sigma_0) = \text{const}$$

is also convex.

To simplify the analysis of defining relations, we introduce a redesignation (single indexing):

$$\zeta_1 = \zeta_{11}, \quad \zeta_2 = \zeta_{22}, \quad \zeta_3 = 2\zeta_{12}, \quad \tau_1 = \sigma_{11}, \quad \tau_2 = \sigma_{22}, \quad \tau_3 = \sigma_{12}.$$  

(7)

We write the expression for the increments of creep strain rates

$$d\zeta_i = \frac{\partial^2 \Phi}{\partial \tau_i \partial \tau_j} d\tau_j = B^i_j d\tau_j,$$

(8)

where $B^i_j$ is the creep strain compliance matrix.

Given the convexity of the surface $\Phi(\sigma_0) = \text{const}$, it follows that the matrix $B^i_j$ will be positive definite, i.e.

$$B^i_j d\tau_j d\tau_j > 0.$$  

(9)

In the case of hardening theory (1), we also accept relation (8). Then, similarly, we can obtain relation (9) for the case of flow theory with hardening.

2 Materials and methods

2.1 Analysis and simplification of relations

Let the orthotropy axes coincide with the coordinate axes. PCM is characterized by a high yield to shear creep deformations and strains across the fibers in comparison with strain compliance along the fibers. This means that $B_{21} = B_{22} \leq B_{13}$. Let be $\eta^2 = \max(B_{11} / B_{22}) \leq 1$, $\theta^2 = \max(B_{22} / B_{11}) \leq 1$.

Since $\eta, \theta$ is small, the elements of the matrix $B_{ij} / B_{33}$ have a different order of smallness. From the condition that the $B_{ij}$ matrix is positive definite, for example, from the condition

$$B_{21} > 0,$$

(10)

it follows that $B_{21} < B_{11} B_{22} \leq B_{33}^2 \eta^2 \theta^4$. Similarly, $B_{13}^2 < B_{33}^2 \eta^2 \theta^2$, $B_{23}^2 < B_{33}^2 \theta^2$.

We introduce the notation for estimating the order $B_{ij}$:

$$B_{12} \sim B_{33} \eta^{3+},$$

$$B_{33} \sim B_{33} \eta^{\theta^{2}},$$

$$B_{23} \sim B_{33} \theta^{2},$$

$$...$$

Express $\eta, \theta$ through one small parameter $a$:

$$\eta = a^n, \quad \zeta = a^s.$$  

(12)

Then we can write the following relation for the elements of the matrix B:

$$B \sim B_{33} \begin{bmatrix} a^{2n+2n} & a^{n+r+sm} & a^{mq+np} \\ a^{n+rs+sm} & a^{2n} & a^{kn} \\ a^{mq+np} & a^{kn} & 1 \end{bmatrix}.$$  

(13)

Here $k, p, q, r, s \geq 1$. From relation (13) it follows that the velocities $\xi_j$ depend strongly on some $\tau_i$, and are insignificant on some. To simplify the creep model, we discard terms with small factors and restrict ourselves to the first terms of the series. Then it follows that the creep strain rates can be represented in the form of the following asymptotic series:

$$\zeta_1 = a^{2n+2n} \varphi_{11}(\tau_1, \tau_2, \tau_3) + a^{n+r+sm} \varphi_{12}(\tau_2, \tau_3) + a^{mq+np} \varphi_{13}(\tau_3) + o(a^\delta) B_{33},$$

$$\zeta_2 = a^{n+rs+sm} \varphi_{21}(\tau_1, \tau_2, \tau_3) + a^{2n} \varphi_{22}(\tau_2, \tau_3) + a^{kn} \varphi_{23}(\tau_3) + o(a^\delta) B_{33},$$

$$\zeta_3 = a^{mq+np} \varphi_{31}(\tau_1, \tau_2, \tau_3) + a^{kn} \varphi_{32}(\tau_2, \tau_3) + o(a^\delta) B_{33},$$

(14)

Since the creep strain compliance matrix $B^i_j$ is positive definite, for example, from the condition

$$B_{21} > 0,$$

(10)

it follows that $B_{21} < B_{11} B_{22} \leq B_{33}^2 \eta^2 \theta^4$. Similarly, $B_{13}^2 < B_{33}^2 \eta^2 \theta^2$, $B_{23}^2 < B_{33}^2 \theta^2$. 

In the case of hardening theory (1), we also accept relation (8). Then, similarly, we can obtain relation (9) for the case of flow theory with hardening.
where \( \beta = \min(2n + 2m, n + mn + sm, m + np) \), \( \gamma = \min(2n, kn, n + mn + sm) \), \( \delta = \min(kn, m + np) \). Here \( \varphi_{ij} \) are related by the relation (8):

\[
\frac{\partial \zeta_i}{\partial \tau_j} = \frac{\partial \zeta_j}{\partial \tau_i} = \frac{\partial^2 \Phi}{\partial \tau_i \partial \tau_j}.
\]  

(15)

Then, taking into account (15), relation (14) takes the form:

\[
\begin{align*}
\zeta_1 &= \alpha^{2n+2m}\varphi_{11}(\tau_1), \\
\zeta_2 &= \alpha^{2n}\varphi_{22}(\tau_2), \\
\zeta_3 &= \varphi_{33}(\tau_3^2). \\
\end{align*}
\]  

(16)

3 Results

3.1 Special cases of plane stress

Case 1. Let PCM in the direction \( Ox^1 \) have no creep strain, i.e. \( \zeta_1 = 0 \). This means that \( m \to \infty \). Then, for \( s \neq n \), relations (16) take the form

\[
\begin{align*}
\zeta_1 &= 0, \\
\zeta_2 &= \alpha^{2n}\varphi_{22}(\tau_2), \\
\zeta_3 &= \varphi_{33}(\tau_3^2) \cdot \tau_3. \\
\end{align*}
\]  

(17)

If the shear and strain compliance of the transverse fibers are of the same order, then \( \zeta_1 \approx 1 \). Therefore, a strong simplification (16) cannot be achieved.

\[
\zeta_2 = \varphi_{22}(\tau_2, \tau_3^2), \\
\zeta_3 = \varphi_{33}(\tau_2, \tau_3^2)
\]  

(18)

To such PCMs related to unidirectionally reinforced materials.

Case 2. If the fibers in PCM can have creep strain, and the shear compliance is much greater than the compliance in the direction of the axis \( Ox^2 \), then we obtain

\[
\begin{align*}
\zeta_1 &= \varphi_{11}(\tau_1), \\
\zeta_2 &= \varphi_{22}(\tau_2), \\
\zeta_3 &= \varphi_{33}(\tau_3^2) \cdot \tau_3.
\end{align*}
\]  

(19)

An example of such materials is PCM fabric.

Case 3. If the compliance in two orthogonal directions \( Ox^1, Ox^2 \) is equal greater than the compliance by shear, then

\[
\begin{align*}
\zeta_1 &= \varphi_{11}(\tau_1, \tau_2), \\
\zeta_2 &= \varphi_{22}(\tau_1, \tau_2), \\
\zeta_3 &= \varphi_{33}(\tau_3^2) \cdot \tau_3.
\end{align*}
\]  

(20)

4 Conclusion

Thus, an asymptotic analysis of PCM creeps ratios allows us to write simplified models of PCM creep strain. Defining relations contain functions that have a smaller dimension than the original ones. This is important in their experimental determination.

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