The potential of IORIO as relativity probe; the impact of the mismodeling in the Jovian gravity field multipoles

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Abstract

The IORIO (In-Orbit Relativity Iuppiter Observatory) concept consists of a fast orbiter of Jupiter ($P_b \approx 0.12$ d), endowed with a Juno-type Doppler ranging device, accurate to $\sigma_\rho \approx 0.003$ mm s$^{-1}$, originally proposed to measure the post-Newtonian range-rate signature proportional to $GSJ_2c^{-2}$. It could be made as large as $\Delta\rho_{gm} \approx 0.2 - 0.02$ mm s$^{-1}$, depending on the orientation of the orbital plane. Also other general relativistic effects could be measurable, including also the one proportional to $GMJ_2c^{-2}$, never put to the test so far. A major source of systematic error is our imperfect knowledge of the magnitude and orientation of the Newtonian multipolar expansion of the Jovian gravity field. We numerically investigate its biasing impact on the general relativistic signatures by determining the required level of improvement of each Newtonian multipole with respect to its present-day accuracy based on the analysis of just some perijove passages of the spacecraft Juno, currently orbiting Jupiter. If, on the one hand, the accuracy of most of the Jovian multipoles up to degree $\ell = 12$ should be improved by a factor of about $50 - 500$, with a peak of 1,000 for $J_{10}$, in order to reach the size of the $GSJ_2c^{-2}$ effect, on the other hand, the spin pole position, impacting mostly the $J_2$ classical signal, would require a relatively more modest improvement by a factor of 100. The improvements needed to reach the size of the $GMJ_2c^{-2}$, $GSc^{-2}$ signals are smaller, amounting to a factor of $\approx 5 - 50$ or so. Remarkably, most of the competing Newtonian signals have quite different temporal signatures with respect to the post-Newtonian ones, making, thus, potentially easier disentangling them.

keywords General relativity and gravitation; Experimental studies of gravity; Experimental tests of gravitational theories; Lunar, planetary, and deep-space probes; Satellite orbits

1. Introduction

Recently, Iorio (2018) proposed a spacecraft-based mission targeted to Jupiter, tentatively named IORIO (In-Orbit Relativity Iuppiter Observatory, or IOvis Relativity In-orbit Observatory), aimed to measure a novel general relativistic effect, proportional to $GSJ_2c^{-2}$, induced by the post-Newtonian gravitomagnetic spin-octupole moment (Panhans & Soffel 2014) of the gaseous giant. By assuming a Juno-type probe in a much faster ($P_b \approx 0.12$ d) and almost circular jovicentric orbit, in Iorio (2018) it was shown that, by suitably choosing the orientation of its orbital plane in space, it would be possible to raise the relativistic range-rate signature to the $\Delta\rho \approx 0.2 - 0.02$ mm s$^{-1}$ level; the current accuracy of the Juno Doppler measurements is at the $\sigma_\rho \approx 0.003$ mm s$^{-1}$ level over 1,000 s (Iess et al. 2018).

In this paper, we want to dig more thoroughly into the potential of the IORIO concept as a tool to measure even more general relativistic features of motion ranging from the standard
Schwarzschild-like one to the so far never tested gravitoelectric effect proportional to $GMJ_2c^{-2}$ (Soffel et al. 1988; Soffel 1989; Brumberg 1991), including also the gravitomagnetic Lense-Thirring frame-dragging (Lense & Thirring 1918). Moreover, we will investigate also the impact of the Newtonian part of the multipolar expansion of the Jovian gravity field, acting as a major source of systematic errors on the proposed relativistic measurements. We will work numerically by integrating the equations of motion of the Earth, Jupiter and the probe with respect to the International Celestial Reference Frame (ICRF) (Petit, Luzum & et al. 2010), and producing simulated time series of the Earth-spacecraft range-rate signatures for each of the Newtonian and post-Newtonian accelerations considered. In addition to a quantitative evaluation of the biasing level due to the size of the present-day uncertainties in the Jovian spin axis orientation and gravity field multipoles (Iess et al. 2018; Durante et al. 2018), a crucial feature which will be investigated in detail is the temporal patterns of both the Newtonian and the post-Newtonian range-rate signatures in order to see if they are different or too similar to make difficult disentangling them in real data reductions. The analytical expressions of the long-term orbital precessions proportional to $GS c^{-2}$, $GMJ_2c^{-2}$, $GS J_2c^{-2}$, valid for an arbitrary orientation of the spin axis of the primary and for quite general orbital configurations of the orbiter, can be found in Iorio (2011, 2015, 2018).

The paper is organized as follows. In Section 2, we investigate the consequences of the mismodeling in the Newtonian potential coefficients of Jupiter, while Section 3 is devoted to the impact of the uncertainty in the Jovian spin axis orientation. Our findings and conclusions are summarized in Section 4. Appendix A collects the symbols and definitions of the quantities used throughout the text, while the tables and the figures are displayed in Appendix B.

2. The impact of the mismodeling in the Jovian gravity field’s multipoles

Our most accurate knowledge of the Newtonian multipolar expansion of the Jovian gravity field comes from the analysis of the Doppler measurements of the ongoing Juno mission, accurate to $\sigma_\rho \simeq 0.003$ mm s$^{-1}$ over 1,000 s, collected at some selected perijove passages. At the time of writing, results were published in the peer reviewed literature (Iess et al. 2018, Tab. 1) just for two perijove passes (PJ03 and PJ06), while data from PJ08, PJ10, PJ11 are still under analysis (Durante et al. 2018). The total perijove passages of Juno dedicated to the gravity field determination should be 25. The spacecraft is scheduled to deorbit into the planet’s atmosphere on July 2021. Table displays, among other things, the best estimates and the associated realistic uncertainties for the even and odd zonal coefficients $J_\ell$, $\ell = 2, 3, 4, \ldots, 12$, and the tesseral and sectorial multipoles $C_{2,1}$, $S_{2,1}$, $C_{2,2}$, $S_{2,2}$.

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1 D. Durante, private communication.

2 See https://www.jpl.nasa.gov/missions/juno/ on the Internet.
Figures 1 to 15 depict the numerically integrated Newtonian and post-Newtonian range-rate time series for a given orbital configuration of the probe which, as it will be shown below, should make the detection of the relativistic signals more favorable. We numerically integrated the equations of motion of the Earth, Jupiter and the probe in Cartesian rectangular coordinates referred to the ICRF with and without the disturbing Newtonian or post-Newtonian accelerations which, from time to time, are under investigation. For each acceleration, both the runs, spanning 1 d, shared the same set of initial conditions which, for the Earth and Jupiter, were retrieved from the WEB interface HORIZONS maintained by JPL, NASA, for a given initial epoch which, in the present case, is January 1, 2020. After each run, a numerical time series of the Earth-probe range-rate $\dot{\rho}(t)$ was produced; $\dot{\rho}_{\text{pert}}(t)$ includes the perturbing acceleration, while $\dot{\rho}_N(t)$ is the purely Newtonian one due to only the monopoles of the Sun and Jupiter. In order to single out the effect of the perturbing acceleration considered, the difference of both the time series was computed obtaining the curves for $\Delta \dot{\rho}(t) = \dot{\rho}_{\text{pert}}(t) - \dot{\rho}_N(t)$ displayed in Figures 1 to 15. In order to better visualize the temporal patterns of the various effects, the classical signatures were computed by using fictitious values $C^*$ of the Newtonian gravity field coefficients able to make their magnitudes roughly equal to those of the post-Newtonian time series of interest. If such figures $C^*$ for the Jovian multipoles are smaller than their present-day uncertainties listed in Table 1, they can be interpreted as a measure of how much they should still be improved with respect to their current levels of accuracy in order to make the size of the Newtonian signatures at least equal to the relativistic ones. If, instead, $C^*$ are larger than their present mismodeling, they can be viewed as a measure of the relative accuracy with which a given relativistic signal would be impacted right now. See Table 2 for a complete list of such improvement factors for all the Newtonian multipoles considered here in connection with the various relativistic effects. It turns out that the largest improvements—of the order of $\approx 50 - 500$, with a peak of 1,000 for $J_{10}$—would be required to bring the Newtonian signals to the level of the post-Newtonian gravitomagnetic effect proportional to $GS J_2 c^{-2}$. Interestingly, a much smaller improvement would be required to make the size of the classical multipole signatures comparable with the post-Newtonian gravitoelectric and gravitomagnetic effects proportional to $GM J_5 c^{-2}$, $GS c^{-2}$. As far as the Schwarzschild-type signature is concerned, the current level of accuracy in almost all the Jovian multipoles, with the exception of $J_{10}$, $S_{2,1}$, $S_{2,2}$, would yield a bias at the $\approx 1 - 10\%$ level. A very important feature of all the curves displayed in Figures 1 to 15 is that the relativistic ones exhibit neatly different temporal patterns with respect to the Newtonian ones, making, thus, easier to detect them. It would not be so for different orbital geometries of the probe.

3. The impact of the uncertainty in the Jupiter’s pole position

The position of the Jovian spin axis, determined by its right ascension $\alpha$ and declination $\delta$ with respect to the ICRF (Durante et al. 2018), enters the Newtonian accelerations induced by the gravity field multipoles in a nonlinear way. It can be easily realized, e.g., by inspecting the analytical expressions of the long-term precessions of the Keplerian orbital elements due to some
even and odd zonal harmonics calculated by Iorio (2011); Renzetti (2013, 2014) for an arbitrary orientation of $\hat{S}$. Thus, the uncertainties $\sigma_\alpha$, $\sigma_\delta$ has an impact on the general relativistic effects of interest through the Newtonian multipolar signatures. The latest determinations of $\alpha$, $\delta$ along with the associated realistic uncertainties, of the order of $\sigma_\alpha$, $\sigma_\delta \approx 0.1$ arcsec (Durante et al. 2018), are listed in Table I.

Figure 16 depicts the numerically simulated mismodeled range-rate signals due to the first four even zonals of Jupiter induced by the present-day errors $\sigma_\alpha$, $\sigma_\delta$. They were obtained as described in the previous Section by using the nominal values of the even zonals and taking the differences between the time series computed with $\delta_{\text{max}} = \delta + \sigma_\delta$, $\delta_{\text{min}} = \delta - \sigma_\delta$ (red curves) and $\alpha_{\text{max}} = \alpha + \sigma_\alpha$, $\alpha_{\text{min}} = \alpha - \sigma_\alpha$ (blue curves), respectively. It turns out that the largest residual signals are due to the uncertainty in the declination. The largest one occurs for $J_2$, with an amplitude which can reach $\Delta \dot{\rho}_{J_2} \lesssim 60$ mm s$^{-1}$. The signatures of the odd zonals are completely negligible. It can be shown that an improvement of $\sigma_\delta$ by a factor of 100 with respect to the current value of Table I would bring the size of the Newtonian $J_2$-induced range-rate time series to the same level of the post-Newtonian one proportional to $G S J_2 c^{-2}$. Such an improvement seems to be quite feasible in view of the fact that it already occurred from the analysis of PJ03, PJ06 (Iess et al. 2018, Tab. 1) to that of PJ08, PJ10, PJ11 (Durante et al. 2018). In any case, as already noticed in the previous Section, the temporal pattern of the classical $J_2$ signal is different from the relativistic ones.

4. Summary and overview

We explored the potential of the recently proposed In-Orbit Relativity Jupiter Observatory (IORIO), which should orbit Jupiter along an almost circular, 0.12 d path, as an effective probe for measuring several general relativistic features of motion in the field of the gaseous giant with accurate Earth-spacecraft Doppler measurements. In particular, we looked at the post-Newtonian gravitoelectric and gravitomagnetic effects proportional to $G M J_2 c^{-2}$, $G S J_2 c^{-2}$, which have never been put to the test so far, and at the standard Lense-Thirring and Schwarzschild signatures, proportional to $G S c^{-2}$, $G M c^{-2}$, respectively.

The experimental uncertainties in the values of both the Newtonian coefficients of the multipolar expansion of the Jovian gravity field and in the orientation of the spin axis of Jupiter would induce mismodeled range-rate signatures in the Doppler measurements of the spacecraft acting as sources of competing systematic biases for the post-Newtonian signals of interest. At present, just 5 of the planned 25 perijove passes dedicated to mapping the planet’s gravity field of the ongoing Juno mission, scheduled to end in July 2021, have been analyzed so far. Thus, if and when IORIO will be finally implemented, it will benefit of the analysis of the entire Juno data record yielding a much more accurate determination of the Jovian gravity field coefficients and pole position than now.

For a given orbital configuration of the spacecraft, we numerically simulated its mismodeled
Newtonian range-rate signatures due to the gravity field coefficients and the spin axis position of Jupiter currently determined by Juno, and the predicted post-Newtonian signals. We determined the level of improvement of the Jovian multipoles and pole position with respect to their present-day accuracies still required to bring the competing classical effects to the level of the various relativistic ones. It turned out that the most demanding requirements pertain the measurability of the $GS J_2 c^{-2}$ signature, implying improvements by a factor of $\approx 50 \sim 500$ for most of the Jovian gravity coefficients considered, with a peak of 1,000 for $J_{10}$. The other relatively small post-Newtonian effects, proportional to $GMJ_{2c^{-2}}$, $GS c^{-2}$, require less demanding improvements by a factor of just $\approx 5 \sim 50$ or less. The Schwarzschild signature would be measurable right now at a $\approx 1 \sim 10\%$ level, apart from the impact of $J_{10}$, $S_{2,1}$, $S_{2,2}$. As far as the Jupiter’s spin axis is concerned, an improvement by a factor of 100 would be required for its declination $\delta$ to make the size of the $J_2$-induced signature to the same level of the post-Newtonian $GS J_2 c^{-2}$ one. The uncertainty in the declination $\alpha$ is less important. The range-rate signals due to the odd zonals are affected by the errors in the pole position at a negligible level. A fundamental outcome of our analysis consists of the fact that the temporal patterns of the relativistic signatures turned out to be quite different from the classical ones, making, thus, easier, in principle, to separate the post-Newtonian from the Newtonian effects.

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Appendix A  Notations and definitions

Here, some basic notations and definitions used throughout the text are presented (Brumberg 1991; Bertotti, Farinella & Vokrouhlický 2003; Kopeikin, Efroimsky & Kaplan 2011; Poisson & Will 2014).

$G$ : Newtonian constant of gravitation

c : speed of light in vacuum

$M$ : mass of Jupiter

$\mu \equiv GM$ : gravitational parameter of Jupiter

$S$ : magnitude of the angular momentum of Jupiter

$\hat{S} = \{\hat{S}_x, \hat{S}_y, \hat{S}_z\}$ : spin axis of Jupiter with respect to the ICRF

$\alpha$ : right ascension (RA) of the Jovian spin axis
\( \delta \): declination (DEC) of the Jovian spin axis

\( \dot{S}_x = \cos \delta \cos \alpha \): x component of the Jovian spin axis with respect to the ICRF

\( \dot{S}_y = \cos \delta \sin \alpha \): y component of the Jovian spin axis with respect to the ICRF

\( \dot{S}_z = \sin \delta \): z component of the Jovian spin axis with respect to the ICRF

\( R_e \): equatorial radius of Jupiter

\( J_{\ell}, \ell = 2, 3, 4, \ldots \): Newtonian zonal multipole mass moments of Jupiter

\( C_{2,1}, S_{2,1}, C_{2,2}, S_{2,2} \): tesseral and sectorial multipole mass moments of degree \( \ell = 2 \) of Jupiter

\( f \): true anomaly of the spacecraft

\( a \): semimajor axis of the spacecraft

\( n_b \equiv \sqrt{\mu/a^3} \): Keplerian mean motion of the spacecraft

\( P_b \equiv 2\pi/n_b \): orbital period of the spacecraft

\( e \): eccentricity of the spacecraft

\( I \): inclination of the orbital plane of the spacecraft to the Earth’s mean equator at the epoch J2000.0

\( \Omega \): longitude of the ascending node of the spacecraft with respect to the ICRF

\( \omega \): argument of pericenter of the spacecraft with respect to the ICRF

**Appendix B  Tables and figures**
Table 1: Relevant physical parameters of Jupiter. Most of the reported values come from Soffel et al. (2003); Petit, Luzum & et al. (2010); Iess et al. (2018); Durante et al. (2018) and references therein. In particular, the values and the uncertainties of $\alpha$, $\delta$ determining the Jovian pole position at the epoch J2017.0 come from Durante et al. (2018), while the multipoles of the gravity potential are retrieved from Iess et al. (2018, Tab. 1).

| Parameter | Units | Numerical value |
|-----------|-------|-----------------|
| $G$       | kg $^{-1}$ m$^3$ s$^{-2}$ | $6.67259 \times 10^{-11}$ |
| $c$       | m s$^{-1}$ | $2.99792458 \times 10^{8}$ |
| $\mu$     | m$^3$ s$^{-2}$ | $1.26713 \times 10^{17}$ |
| $S$       | kg m$^2$ s$^{-1}$ | $6.9 \times 10^{38}$ |
| $\alpha$  | deg | $268.057132 \pm 0.000036$ |
| $\delta$  | deg | $64.497159 \pm 0.000045$ |
| $R$       | km | $71,492$ |
| $J_2$     | ($\times 10^{-6}$) | $14,696.572 \pm 0.014$ |
| $J_3$     | ($\times 10^{-6}$) | $-0.042 \pm 0.010$ |
| $J_4$     | ($\times 10^{-6}$) | $-586.609 \pm 0.004$ |
| $J_5$     | ($\times 10^{-6}$) | $-0.069 \pm 0.008$ |
| $J_6$     | ($\times 10^{-6}$) | $34.198 \pm 0.009$ |
| $J_7$     | ($\times 10^{-6}$) | $0.124 \pm 0.017$ |
| $J_8$     | ($\times 10^{-6}$) | $-2.426 \pm 0.025$ |
| $J_9$     | ($\times 10^{-6}$) | $-0.106 \pm 0.044$ |
| $J_{10}$  | ($\times 10^{-6}$) | $0.172 \pm 0.069$ |
| $J_{11}$  | ($\times 10^{-6}$) | $0.033 \pm 0.112$ |
| $J_{12}$  | ($\times 10^{-6}$) | $0.047 \pm 0.178$ |
| $C_{2,1}$ | ($\times 10^{-6}$) | $-0.013 \pm 0.015$ |
| $S_{2,1}$ | ($\times 10^{-6}$) | $-0.003 \pm 0.026$ |
| $C_{2,2}$ | ($\times 10^{-6}$) | $0.000 \pm 0.008$ |
| $S_{2,2}$ | ($\times 10^{-6}$) | $0.000 \pm 0.011$ |
Table 2: Improvement factors (if greater than 1) required to each of the Jovian multipole coefficients with respect to their current accuracy levels (see Iess et al. (2018, Tab. 1) and Table I 1) to make the size of the corresponding Newtonian signatures equal to the magnitude of the general relativistic ones; see Figures 1 to 15. Figures k smaller than 1 in a given row imply that the current level of accuracy in the multipole of that row would allow right now to measure the corresponding relativistic effects with the relative accuracies as good as k themselves. For example, in the second row corresponding to $J_3$, there are two figures smaller than 1; it means that the present-day accuracy in $J_3$ would yield a mismodeled Newtonian signal impacting, say, the Schwarzschild-like one at 1.67%. Instead, the accuracy of $J_3$ should be improved by a factor of 12.5 with respect to its current level in order to induce a mismodeled Newtonian signature having, at least, the same magnitude of the relativistic effect proportional to $GS J_2 c^{-2}$. From Table I 1 it should be noted that the values of $J_{11}$, $J_{12}$, $C_{2,1}$, $S_{2,1}$, $C_{2,2}$, $S_{2,2}$ are statistically compatible with zero.

| Multipole | GS $J_2 c^{-2}$ | GM $J_2 c^{-2}$ | GS $c^{-2}$ | GM $c^{-2}$ |
|-----------|-----------------|-----------------|-------------|-------------|
| $J_2$     | 70              | 5               | 2.5         | 0.11        |
| $J_3$     | 12.5            | 1.1             | 0.5         | 0.0167      |
| $J_4$     | 33              | 3.3             | 1.4         | 0.033       |
| $J_5$     | 10              | 0.8             | 0.58        | 0.01        |
| $J_6$     | 100             | 5               | 2.5         | 0.12        |
| $J_7$     | 50              | 4.5             | 3.3         | 0.067       |
| $J_8$     | 33              | 4               | 2.2         | 0.05        |
| $J_9$     | 100             | 10              | 3.3         | 0.15        |
| $J_{10}$  | 1,000           | 33              | 33          | 20          |
| $J_{11}$  | 500             | 28.6            | 20          | 0.7         |
| $J_{12}$  | 500             | 50              | 28.6        | 1.1         |
| $C_{2,1}$ | 50              | 4               | 2.8         | 0.1         |
| $S_{2,1}$ | 500             | 20              | 12.5        | 12.5        |
| $C_{2,2}$ | 333             | 28.6            | 18.2        | 0.4         |
| $S_{2,2}$ | 500             | 33              | 20          | 1           |
Fig. 1.— Simulated range-rate signatures $\Delta \dot{\rho}$, in $\text{mm s}^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian first even zonal harmonic $J_2$ of Jupiter after 1 d. In each panel, a fictitious value $J_2^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, for which the actual value of $J_2$ is, instead, used, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $(GSJ_2c^{-2}; J_2^* = 2.0 \times 10^{-10})$. Upper-right corner: post-Newtonian gravitoelectric moment $(GMJ_2c^{-2}; J_2^* = 2.8 \times 10^{-9})$. Lower-left corner: Lense-Thirring effect $(GSc^{-2}; J_2^* = 5.6 \times 10^{-9})$. Lower-right corner: Schwarzschild $(GMc^{-2}; J_2^* = 1.26 \times 10^{-7})$. The present-day actual uncertainty in the Jovian first even zonal is $\sigma_{J_2} = 1.4 \times 10^{-8}$ (Jess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50 \text{ deg}$, $\Omega_0 = 140 \text{ deg}$, $\omega_0 = 149.43 \text{ deg}$, $f_0 = 228.32 \text{ deg}$.
Fig. 2.— Simulated range-rate signatures $\Delta \hat{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian first odd zonal harmonic $J_3$ of Jupiter after 1 d. In each panel, a fictitious value $J_3^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment ($GSJ_2c^{-2}; J_3^* = 8.0 \times 10^{-10}$). Upper-right corner: post-Newtonian gravitoelectric moment ($GMJ_2c^{-2}; J_3^* = 9.0 \times 10^{-9}$). Lower-left corner: Lense-Thirring effect ($GS c^{-2}; J_3^* = 2.0 \times 10^{-8}$). Lower-right corner: Schwarzschild ($GMc^{-2}; J_3^* = 6.0 \times 10^{-7}$). The present-day actual uncertainty in the Jovian first odd zonal is $\sigma_{J_3} = 1.0 \times 10^{-8}$ (less et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 \, R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 3.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian second even zonal harmonic $J_4$ of Jupiter after 1 d. In each panel, a fictitious value $J_4^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $\left( GS J_2 c^{-2}; J_4^* = 1.2 \times 10^{-10} \right)$. Upper-right corner: post-Newtonian gravitoelectric moment $\left( GM J_2 c^{-2}; J_4^* = 1.2 \times 10^{-9} \right)$. Lower-left corner: Lense-Thirring effect $\left( GS c^{-2}; J_4 = 2.8 \times 10^{-9} \right)$. Lower-right corner: Schwarzschild $\left( GM c^{-2}; J_4 = 1.2 \times 10^{-7} \right)$. The present-day actual uncertainty in the Jovian second even zonal is $\sigma_{J_4} = 4 \times 10^{-9}$ [Iess et al. 2018, Tab. 1]. The adopted orbital configuration for the probe is $a_0 = 1.015 \, R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 4.— Simulated range-rate signatures \( \Delta \dot{\rho} \), in mm s\(^{-1}\), of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian second odd zonal harmonic \( J_5 \) of Jupiter after 1 d. In each panel, a fictitious value \( J_5^* \) is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment \( (GS J_2 c^{-2}; J_5^* = 8.0 \times 10^{-9}) \). Upper-right corner: post-Newtonian gravitoelectric moment \( (GMJ_2 c^{-2}; J_5^* = 9.6 \times 10^{-9}) \). Lower-left corner: Lense-Thirring effect \( (GS c^{-2}; J_5^* = 1.36 \times 10^{-8}) \). Lower-right corner: Schwarzschild \( (GMc^{-2}; J_5^* = 6.4 \times 10^{-7}) \). The present-day actual uncertainty in the Jovian second odd zonal is \( \sigma_{J_5} = 8 \times 10^{-9} \) [Less et al. 2018, Tab. 1]. The adopted orbital configuration for the probe is \( a_0 = 1.015 \, R_j, e_0 = 0.0049, I_0 = 50 \, \text{deg}, \Omega_0 = 140 \, \text{deg}, \omega_0 = 149.43 \, \text{deg}, f_0 = 228.32 \, \text{deg} \).
Fig. 5.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian third even zonal harmonic $J^*_6$ of Jupiter after 1 d. In each panel, a fictitious value $J^*_6$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $(GS J^*_2 c^{-2}; J^*_6 = 9.0 \times 10^{-11})$. Upper-right corner: post-Newtonian gravitoelectric moment $(GM J^*_2 c^{-2}; J^*_6 = 1.8 \times 10^{-9})$. Lower-left corner: Lense-Thirring effect $(GS c^{-2}; J^*_6 = 3.6 \times 10^{-9})$. Lower-right corner: Schwarzschild $(GM c^{-2}; J^*_6 = 7.2 \times 10^{-8})$. The present-day actual uncertainty in the Jovian third even zonal is $\sigma_{J^*_6} = 9 \times 10^{-9}$ (Iess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 6.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian third odd zonal harmonic $J_7$ of Jupiter after 1 d. In each panel, a fictitious value $J_7^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment \( (GS_{J_2}c^{-2}; J_7^* = 3.4 \times 10^{-10}) \). Upper-right corner: post-Newtonian gravitoelectric moment \( (GM_{J_2}c^{-2}; J_7^* = 3.74 \times 10^{-9}) \). Lower-left corner: Lense-Thirring effect \( (GS_{c}c^{-2}; J_7^* = 5.1 \times 10^{-9}) \). Lower-right corner: Schwarzschild \( (GMc^{-2}; J_7^* = 2.55 \times 10^{-7}) \). The present-day actual uncertainty in the Jovian third odd zonal is $\sigma_{J_7} = 1.7 \times 10^{-8}$ [Iess et al. 2018, Tab. 1]. The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 7.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian fourth even zonal harmonic $J^*_{8}$ of Jupiter after 1 d. In each panel, a fictitious value $J^*_{8}$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $(GS J_2 c^{-2}; J^*_{8} = 7.5 \times 10^{-10})$. Upper-right corner: post-Newtonian gravitoelectric moment $(GM J_2 c^{-2}; J^*_{8} = 6.25 \times 10^{-9})$. Lower-left corner: Lense-Thirring effect $(GS c^{-2}; J^*_{8} = 1.125 \times 10^{-8})$. Lower-right corner: Schwarzschild $(GM c^{-2}; J^*_{8} = 5.0 \times 10^{-7})$. The present-day actual uncertainty in the Jovian fourth even zonal is $\sigma_{J_{8}} = 2.5 \times 10^{-8}$ (Iess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian fourth odd zonal harmonic $J_9$ of Jupiter after 1 d. In each panel, a fictitious value $J_9^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $\left( G S J_2 c^{-2}; J_9^* = 4.4 \times 10^{-10} \right)$. Upper-right corner: post-Newtonian gravitoelectric moment $\left( G M J_2 c^{-2}; J_9^* = 4.4 \times 10^{-9} \right)$. Lower-left corner: Lense-Thirring effect $\left( G S c^{-2}; J_9^* = 1.32 \times 10^{-8} \right)$. Lower-right corner: Schwarzschild $\left( G M c^{-2}; J_9^* = 3.0 \times 10^{-7} \right)$. The present-day actual uncertainty in the Jovian fourth odd zonal is $\sigma_{J_9} = 4.4 \times 10^{-8}$ (Jess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 9.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian fifth even zonal harmonic $J_{10}$ of Jupiter after 1 d. In each panel, a fictitious value $J_{10}^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment ($GS J_2 c^{-2}; J_{10}^* = 6.9 \times 10^{-11}$). Upper-right corner: post-Newtonian gravitoelectric moment ($GMJ_2 c^{-2}; J_{10}^* = 2.07 \times 10^{-9}$). Lower-left corner: Lense-Thirring effect ($GS c^{-2}; J_{10}^* = 2.07 \times 10^{-9}$). Lower-right corner: Schwarzschild ($GMc^{-2}; J_{10}^* = 3.45 \times 10^{-9}$). The present-day actual uncertainty in the Jovian fifth even zonal is $\sigma_{J_{10}} = 6.9 \times 10^{-8}$ (Iess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $\epsilon_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 10.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian fifth odd zonal harmonic $J_{11}$ of Jupiter after 1 d. In each panel, a fictitious value $J_{11}^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $(GS J_2 c^{-2}; J_{11}^* = 2.24 \times 10^{-10})$. Upper-right corner: post-Newtonian gravitoelectric moment $(GM J_2 c^{-2}; J_{11}^* = 3.92 \times 10^{-9})$. Lower-left corner: Lense-Thirring effect $(GS c^{-2}; J_{11}^* = 5.6 \times 10^{-9})$. Lower-right corner: Schwarzschild $(GM c^{-2}; J_{11}^* = 1.568 \times 10^{-7})$. The present-day actual uncertainty in the Jovian fifth odd zonal is $\sigma_{J_{11}} = 1.12 \times 10^{-7}$ (Jess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 11.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian fifth odd zonal harmonic $J_{12}^*$ of Jupiter after 1 d. In each panel, a fictitious value $J_{12}^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $(G S J_{2} c^{-2}; J_{12}^* = 3.56 \times 10^{-10})$. Upper-right corner: post-Newtonian gravitoelectric moment $(G M J_{2} c^{-2}; J_{12}^* = 3.56 \times 10^{-9})$. Lower-left corner: Lense-Thirring effect $(G S c^{-2}; J_{12}^* = 6.23 \times 10^{-9})$. Lower-right corner: Schwarzschild $(G M c^{-2}; J_{12}^* = 1.602 \times 10^{-7})$. The present-day actual uncertainty in the Jovian fifth odd zonal is $\sigma_{J_{12}} = 1.78 \times 10^{-7}$ (Less et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 12.— Simulated range-rate signatures $\Delta \rho$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian tesseral coefficient $C_{2,1}$ of Jupiter after 1 d. In each panel, a fictitious value $C_{2,1}^* = 3.0 \times 10^{-10}$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $(GS J_{2c}^{-2}; C_{2,1}^* = 3.0 \times 10^{-10})$. Upper-right corner: post-Newtonian gravitoelectric moment $(GMJ_{2c}^{-2}; C_{2,1}^* = 3.75 \times 10^{-9})$. Lower-left corner: Lense-Thirring effect $(GS c^{-2}; C_{2,1}^* = 5.25 \times 10^{-9})$. Lower-right corner: Schwarzschild $(GMc^{-2}; C_{2,1}^* = 1.5 \times 10^{-7})$. The present-day actual uncertainty in the Jovian tesseral coefficient is $\sigma_{C_{2,1}} = 1.5 \times 10^{-8}$ (Less et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 13.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian tesseral coefficient $S_{2,1}$ of Jupiter after 1 d. In each panel, a fictitious value $S_{2,1}^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment ($GS J_2 c^{-2}$, $S_{2,1}^* = 5.2 \times 10^{-11}$). Upper-right corner: post-Newtonian gravitoelectric moment ($G M J_2 c^{-2}$, $S_{2,1}^* = 1.3 \times 10^{-9}$). Lower-left corner: Lense-Thirring effect ($GS c^{-2}$, $S_{2,1}^* = 2.08 \times 10^{-9}$). Lower-right corner: Schwarzschild ($G M c^{-2}$, $S_{2,1}^* = 2.6 \times 10^{-8}$). The present-day actual uncertainty in the Jovian tesseral coefficient is $\sigma_{S_{2,1}} = 2.6 \times 10^{-8}$ (Jess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 \, R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 14.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian sectorial coefficient $C_{2,2}$ of Jupiter after 1 d. In each panel, a fictitious value $C_{2,2}^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment $\left(GSJ_2 c^{-2}; C_{2,2}^* = 2.4 \times 10^{-11}\right)$. Upper-right corner: post-Newtonian gravitoelectric moment $\left(GMJ_2 c^{-2}; C_{2,2}^* = 2.8 \times 10^{-10}\right)$. Lower-left corner: Lense-Thirring effect $\left(GSc^{-2}; C_{2,2}^* = 4.4 \times 10^{-10}\right)$. Lower-right corner: Schwarzschild $\left(GMc^{-2}; C_{2,2}^* = 2.0 \times 10^{-8}\right)$. The present-day actual uncertainty in the Jovian sectorial coefficient is $\sigma_{C_{2,2}} = 8.0 \times 10^{-9}$ (Iess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 \, R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
Fig. 15.— Simulated range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the nominal post-Newtonian accelerations considered in the text and by the Newtonian sectorial coefficient $S_{2,2}^*$ of Jupiter after 1 d. In each panel, a fictitious value $S_{2,2}^*$ is used in the Newtonian signature just for illustrative and comparative purposes. Indeed, it is suitably tuned from time to time in order to bring the associated classical signature to the level of the nominal post-Newtonian effect of interest, so to inspect the mutual (de)correlations of their temporal patterns more easily. Upper-left corner: post-Newtonian gravitomagnetic spin-octupole moment ($GJ_2c^{-2}; S_{2,2}^* = 2.2 \times 10^{-11}$). Upper-right corner: post-Newtonian gravitoelectric moment ($GMJ_2c^{-2}; S_{2,2}^* = 3.3 \times 10^{-10}$). Lower-left corner: Lense-Thirring effect ($GS c^{-2}; S_{2,2}^* = 5.5 \times 10^{-10}$). Lower-right corner: Schwarzschild ($GMc^{-2}; S_{2,2}^* = 1.1 \times 10^{-8}$). The present-day actual uncertainty in the Jovian sectorial coefficient is $\sigma_{S_{2,2}} = 1.1 \times 10^{-8}$ (Jess et al. 2018, Tab. 1). The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg
Fig. 16.— Numerically simulated impact of the present-day errors $\sigma_\alpha = 0.13$ arcsec, $\sigma_\delta = 0.16$ arcsec (Durante et al. 2018) in the position of the spin axis of Jupiter on the range-rate signatures $\Delta \dot{\rho}$, in mm s$^{-1}$, of a hypothetical Jovian orbiter induced by the Newtonian accelerations due to the first four even zonals $J_2$, $J_4$, $J_6$, $J_8$ after 1 d. It turns out that the uncertainties in the Jupiter’s spin axis affect the odd zonals signatures in a completely negligible way. The adopted orbital configuration for the probe is $a_0 = 1.015 R$, $e_0 = 0.0049$, $I_0 = 50$ deg, $\Omega_0 = 140$ deg, $\omega_0 = 149.43$ deg, $f_0 = 228.32$ deg.
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