Modified Slim-Disk Model Based on Radiation-Hydrodynamic Simulation Data:
The Conflict between Outflow and Photon Trapping

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Abstract

Photon trapping and outflow are two key physics issues associated with supercritical accretion flow. We investigated the conflict between these two processes based on two-dimensional radiation-hydrodynamic (RHD) simulation data, and constructed a simplified (radially) one-dimensional model. Mass loss due to outflow, which is not considered in the slim-disk model, reduces the surface density of the flow, and if very significantly, it can totally suppress the photon-trapping effects. If photon trapping is very significant, conversely, outflow can be suppressed because the radiation-pressure force is reduced. To see what actually occurs, we examined the RHD simulation data and evaluated the accretion and outflow rates as functions of the radius. We have found that the accretion rate monotonically decreases, while the outflow rate increases, as the radius decreases. However, the accretion remains constant at small radii, inside several Schwarzschild radii, since the outflow is suppressed by photon trapping effects. To understand the conflict between the photon trapping and outflow in a simpler way, we modeled the radial distribution of the accretion rate from our simulation data, and built up a new (radially) one-dimensional model, which is similar to the slim-disk model, but incorporates mass loss effects due to the outflow. We find that the surface density (and, hence, the optical depth) is greatly reduced even inside the trapping radius, compared to the case without outflow, whereas the effective temperature distribution hardly changes. That is, the emergent spectra do not sensitively depend on the amount of mass outflow. We conclude that the slim-disk approach is valid for interpreting observations, even if the outflow is taken into account. The observational implications of our findings are briefly discussed in relation to ultra-luminous X-ray sources.

Key words: accretion, accretion disks — black hole physics — hydrodynamics — radiation mechanisms:
general — stars: winds, outflows

1. Introduction

Supercritical (or super-Eddington) accretion onto black holes remains one of the most fundamental, classical issues in present-day astrophysics, and is now discussed in wide fields of astrophysics (see Chap. 10 of Kato et al. 2008 for a concise review). It is well known for the case of spherical accretion that there is an upper limit to the luminosity; that is, the Eddington luminosity, \( L_E \). It is thus impossible for gas to accrete onto a black hole at a rate exceeding the critical accretion rate, \( M_{\text{crit}} \equiv \frac{L_E}{(\eta c^2)} \), where \( c \) is the speed of light and \( \eta \) is the efficiency. Then, how about the cases of disk accretion? Is the supercritical accretion (accretion at a rate exceeding the Eddington rate) feasible? This is an enigmatic issue, which has been discussed from the 1970’s by many authors, including Shakura and Sunyaev (1973), but it still remains as a controversial issue because of technical difficulties about the analytical approach. One of the reasons for these technical difficulties stems from the multidimensional properties of the supercritical accretion flow.

There are two key processes that appear when the disk luminosity approaches the Eddington luminosity: photon trapping and radiation pressure-driven outflow. Both are multidimensional effects. At very high luminosity, the accretion rate should also be very high, and so should be the optical depth. Then, the photon-diffusion timescale in the vertical direction may become shorter than the accretion time of gas. If this happens, photons generated deep inside the accretion flow are unable to reach the surface before the material is swallowed by a black hole. This is a photon-trapping effect (Katz 1977; Begelman 1978; Begelman & Meier 1982; Flammang 1984; Blondin 1986; Colpi 1988; Wang & Zhou 1999; Ohsuga et al. 2002). Furthermore, the flow of high luminosities is supported by radiation pressure, which is likely to induce outflow (Bisnovatyi-Kogan & Blinnikov 1977; Meier 1979; Icke 1980; Tajima & Fukue 1998). For a complete understanding of supercritical accretion flow, we need to solve the multidimensional radiation-hydrodynamic (RHD) equations. It has been possible quite recently thanks to the rapid development of high-speed computers.

Ohsuga et al. (2005) were the first to succeed in running global RHD simulations of the supercritical accretion flow until the flow settles down in a quasi-steady phase. They demonstrated the accretion rate can be arbitrarily high, and that the Eddington luminosity can be exceeded, and further that the luminosity increases with an increase of the mass input rate in...
2. Supercritical Accretion Flows

2.1. Basic Considerations

Through studies based on the slim-disk model, two key observational signatures of supercritical flow have become clear: a smaller innermost radius and a flatter temperature profile (Watarai et al. 2000). The former is because of a large amount of accreting material existing even inside the radius of the innermost stable circular orbit (ISCO), emitting a significant amount of radiation (see also Watarai & Mineshige 2003). The latter is due to a suppression of the radiation flux by photon trapping.

The emergent flux distribution of the standard-type disks is determined by the energy balance, which is approximately expressed by

\[ 2\pi R^2 \cdot \sigma T_{\text{eff}}^4 \propto \frac{GM \dot{M}_{\text{acc}}}{R} \propto R^{-1}. \] (1)

This gives the relation, \( T_{\text{eff}} \propto R^{-3/4}. \) Here, \( \sigma, G, M, \) and \( \dot{M}_{\text{acc}} \) represent the Stefan–Boltzmann constant, the gravitational constant, the mass of the central black hole, and the mass-accretion rate, respectively.

In the slim-disk model, by contrast, the advection effects, the relative importance of which increases inward as \( t_{\text{diff}}/t_{\text{acc}} \propto R^{-1} \) (see Chap. 10 of Kato et al. 2008), should be considered. As a result, the flux distribution becomes

\[ 2\pi R^2 \cdot \sigma T_{\text{eff}}^4 \propto \frac{GM \dot{M}_{\text{acc}}}{R} \cdot \frac{t_{\text{acc}}}{t_{\text{diff}}} \propto R^0, \] (2)

which leads to a somewhat flatter temperature profile, \( T_{\text{eff}} \propto R^{-1/2}. \) Here, \( t_{\text{acc}} = -R/\dot{v}_{\text{th}} \) and \( t_{\text{diff}} = 3H \tau/c \) represent the accretion timescale and photon-diffusion timescale (in the vertical direction), respectively, and \( \dot{v}_{\text{th}}, H, \) and \( \tau \) are the radial velocity, scale-height of the disk, and optical thickness of the disk, respectively. The occurrence of photon trapping gives rise to a critical radius, trapping radius (\( R_{\text{trap}} \)), inside of which photon trapping is significant; it is expressed as

\[ R_{\text{trap}} \approx \frac{H}{R} \left( \frac{M_{\text{acc}} c^2}{L_E} \right) r_s, \] (3)

where \( r_s = 2GM/c^2 \) is the Schwarzschild radius.

Caution should be taken concerning the fact that the slim-disk model does not perfectly describe the properties of the supercritical accretion flow, since the outflow, one of the most important properties of supercritical flow, is not considered. The mass accretion rate is, hence, set to be constant in space in the slim-disk formulation. However, it is likely to decrease inward due to mass loss by the outflow. In the limit of significant mass loss by the outflow, the surface density (and thus the optical depth) of the accretion flow will be greatly reduced, which may result in a total suppression of the photon-trapping effects. That is, the slim-disk model breaks down. In the limit of significant photon-trapping, conversely, the (outward) radiation pressure force is weakened (or may become even inward, see Ohsuga & Mineshige 2007), and mass loss by the radiation pressure-driven outflow becomes negligible. Which is the case? Which process dominates?

We wish to note that the slim-disk model is not the only model of supercritical accretion. Shakura and Sunyaev (1973) have already discussed supercritical flow, and considered standard-type disks with large outflow. They assumed that the flux from each radius, \( F(R) \), cannot exceed the value that gives the Eddington luminosity, i.e., roughly \( 2\pi R^2 F(R) = L_E \) within a certain radius, called the spherization radius, \( R_{\text{sph}} \) (see also Begelman 1979). Using the standard-disk relation [equation (1)], we can easily derive

\[ R_{\text{sph}} \approx \left( \frac{M_{\text{acc}} c^2}{L_E} \right) r_s. \] (4)

Inside this radius, the mass-accretion rate decreases inward in proportion to the radius; i.e., \( \dot{M}_{\text{acc}} \propto R. \) This means that the accretion rate vanishes at a very small \( R \), and we again have a somewhat flatter temperature profile, \( T_{\text{eff}} \propto R^{-1/2}, \) since \( 2\pi R^2 \cdot \sigma T_{\text{eff}}^4 \propto \dot{M}_{\text{acc}}/R \propto \text{const}. \)

Comparing equations (3) and (4), one can notice that the critical radii for the photon trapping and outflow are on the same order because \( H \sim R \). In other words, both effects could be equally important in supercritical flow. In addition, the apparent effective temperature profiles are the same for...
both cases. That is, we cannot simply conclude which effect dominates from looking at the observed spectra.

Theoretically speaking, however, there is a big difference. Photon trapping is more important near the equatorial plane, since it takes a longer time for photons to travel from the equatorial plane than from the middle part of the disk to the disk surface, while outflow occurs from the disk surface. This indicates that both effects can be simultaneously significant, but at different heights. We thus need to carefully examine multidimensional simulation data.

2.2. Model and Simulated Flow

We simulated the supercritical accretion flow using a two-dimensional RHD code developed by Ohsuga et al. (2005). The basic equations and the numerical method are described in detail in Ohsuga et al. (2005). Hence, we briefly summarize them. We used spherical coordinates \((r, \theta, \varphi)\), where \(r\), \(\theta\), and \(\varphi\) are the radial distance, the polar angle, and the azimuthal angle, respectively, and set a nonrotating black hole at the origin. After the model of the general-relativistic effects, we adopted a pseudo-Newtonian potential, \(\psi\), given by \(\psi = -GM/(r - r_g)\) (Paczynski & Wiita 1980). As for the flow structure, we assumed the axisymmetry (i.e., \(\partial/\partial\varphi = 0\)) and the reflection symmetry relative to the equatorial plane (with \(\theta = \pi/2\)). To solve radiative transfer, we applied a flux-limited diffusion (FLD) approximation developed by Levermore and Pomraning (1981). This approximation is that the radiative flux and the radiation pressure tensor are expressed in terms of the radiation energy density (Turner & Stone 2001; Ohsuga et al. 2005).

There is a difference in computational domain between the present and Ohsuga et al. (2005)’s calculations. The purpose of this study was to examine any conflict between the outflow and photon trapping. Therefore, we had to simulate the supercritical accretion flow over a wider spatial range. Thus, we set the computational domain of spherical shells of \(3r_g \leq r \leq 1000r_g\) and \(0 \leq \theta \leq \pi/2\), and divided the domain into \(96 \times 96\) grid cells (note that the computational domain of Ohsuga et al. 2005 was \(3r_g \leq r \leq 500r_g\)). We started the calculations with a hot, rarefied and optically thin atmosphere. There was no cold dense disk initially, and we assumed steady mass injection into the computational domain through the outer disk boundary (\(r = 1000r_g\), \(0.45\pi \leq \theta \leq 0.5\pi\)). The injected matter is supposed to have a specific angular momentum corresponding to the Keplerian angular momentum at \(r = 500r_g\) (cf. \(r = 100r_g\) in Ohsuga et al. 2005), so we set the injected mass-accretion rate (mass input rate), \(M_{\text{input}}\), to remain constant. Throughout the present simulation, we assumed \(M = 10M_\odot\), \(\alpha = 0.1\), \(\gamma = 5/3\), \(\mu = 0.5\), and \(Z = Z_\odot\). Here, \(\alpha\), \(\gamma\), \(\mu\), and \(Z\) are the viscous parameter, specific heat ratio, mean molecular weight, and metallicity, respectively.

Figure 1 indicates the color contours of the matter density (left panel) and the radial inflow velocity (right panel) distributions in the meridional section, which were time-averaged over \(t = 190–250\) s in the case of \(M_{\text{input}} = 1000L_\odot/c^2\). In the right panel the radial inflow velocity was normalized by the escape velocity, and the region with white color indicates the outflow region, i.e., \(v_r > 0\). The supercritical accretion flow forms at \(r \leq 350r_g\). The disk accretion of the high-density gas and the outflow of the low-density one are clear. In a previous calculation, the supercritical accretion flow formed at \(r \lesssim 80r_g\). The behavior of the present simulated flow in this region is roughly consistent with that of the previous one (Ohsuga et al. 2005; see also Ohsuga & Mineshige 2007).

2.3. Mass Accretion/Outflow Rate

We assumed that an accretion flow occurs in the region of \(\theta_{\text{disk}} \leq \theta \leq \pi/2\), while the outflow region corresponds to the region above the accretion flow region, and determined an angle \(\theta_{\text{disk}} (>0)\) from the simulation data. That is, \(\theta_{\text{disk}}\) was chosen so as to roughly coincide with the angle between the \(z\)-axis and the boundary separating the region with the radial inflow (negative velocity) from that of the positive velocity (figure 2). On this assumption, we calculated the mass-accretion rate and the cumulative mass-outflow rate as functions of the radius. Since the quasisteady flows form at \(r \leq 350r_g\), we set in this calculation the outer radius of the flows at \(r_{\text{wind}} = 350r_g\).
We then calculated the accretion rate, $\dot{M}_{\text{acc}}(r)$, and the cumulative mass-outflow rate, $\dot{M}_{\text{out}}(r)$, by

$$\dot{M}_{\text{acc}}(r) \equiv \int_{0}^{90^\circ} 4\pi r^2 \rho v_r \sin \theta \, d\theta$$

and

$$\dot{M}_{\text{out}}(r) \equiv \int_{r}^{r_{\text{wind}}} 4\pi r^2 \rho v_r |\theta_{\text{disk}}\sin \theta_{\text{disk}}| \, dr.$$  

In figure 3, we show the accretion rate and the mass-outflow rate as functions of the radius in the simulated flows for $M_{\text{input}} = 1000 L_\odot/c^2$ (left panel) and $M_{\text{input}} = 3000 L_\odot/c^2$ (right panel), respectively. These values were normalized by the Eddington accretion rate, i.e., $L_\odot/c^2$. Here, the disk inclination angle was chosen to be $\theta_{\text{disk}} = 70^\circ$ in the case of $M_{\text{input}} = 1000 L_\odot/c^2$, and $\theta_{\text{disk}} = 60^\circ$ in the case of $M_{\text{input}} = 3000 L_\odot/c^2$. In each panel, the solid, dashed, and dotted curves represent the accretion rate, $\dot{M}_{\text{acc}}$, the cumulative mass-outflow rate, $\dot{M}_{\text{out}}(r)$, and the sum of the two rates, respectively. In particular, the following mass conservation should hold:

$$\dot{M}_{\text{acc}}(r_{\text{wind}}) = \dot{M}_{\text{acc}}(r) + \dot{M}_{\text{out}}(r) = \text{const.}$$

We confirmed that this relation is true within plus or minus 4%. The reason why $\dot{M}_{\text{acc}}(r_{\text{wind}}) < \dot{M}_{\text{input}}$ is that a part of the injected gases accumulates at $r_{\text{wind}} < r < 1000 r_s$, or goes out the outside of the computational domain without accreting immediately.

The accretion rate is not constant spatially due to mass loss by outflows. In the analytical accretion flow model, in which an outflow is considered, the accretion rate decreases inward in proportion to the radius, i.e., $\dot{M}_{\text{acc}} \propto r$ (e.g., Shakura & Sunyaev 1973). That is, the accretion rate should vanish at a very small radius. However, we found that the accretion rate still has a finite value at small radii, inside several Schwarzschild radii. This is because the emergence of outflow is suppressed due to the attenuation of radiation flux by photon trapping. In fact, the radiation flux become even negative (inward) at these radii (Ohsuga & Mineshige 2007). It is difficult to blow off the outflows, and thus the flow is easy to be accreted toward the central black hole.

In the case of $M_{\text{input}} = 1000 L_\odot/c^2$, the outflow blows at $r \lesssim 300 r_s$. When we evaluated the photon trapping radius by equation (3), we found that photon trapping is effective at $r \lesssim 150 r_s$ for a mass-accretion rate of $\dot{M}_{\text{acc}}(r_{\text{wind}} = 350 r_s) = 364 L_\odot/c^2$. In the case of $M_{\text{input}} = 3000 L_\odot/c^2$, in contrast, the outflow and photon trapping are already effective at $r_{\text{wind}} = 350 r_s$. Thus, we confirmed that the critical radii for the photon trapping and outflow were on the same order in the RHD simulations. Note that this is only a rough estimate, since we assumed a constant accretion rate in the derivation.

3. Modified Slim-Disk Model

On the basis of the simulation data analysis presented in the previous section, we try to construct a modified slim-disk model, which incorporates the effects of mass loss by an outflow, in this section. Accretion flow models that incorporate the mass-loss effect have been proposed by several researchers (Lipunova 1999; Kitabatake et al. 2002; Fukue 2004; Poutanen et al. 2007). They evaluated the outflow...
by adopting a spherization radius. In contrast, we construct a model using the accretion rate mentioned in the previous section.

3.1. Basic Equations

We use cylindrical coordinates \((R, \varphi, z)\) in this section. We assume steady, axisymmetric flow and a nonrotating black hole, and adopt a pseudo-Newtonian potential. We use height-integrated quantities, such as \(\Sigma = \int_H^H \rho \, dz = \Sigma_N \rho H\) and \(\Pi = \int_H^H \rho \, dz = 2I_{N+1} \rho H\). Here, \(\Sigma, \Pi, p,\) and \(H\) are the surface density, the height-integrated pressure, the total pressure, and the scale height, respectively. The coefficients \(I_N\) and \(I_{N+1}\) were introduced by Hoshii (1977). The density and the pressure are related to each other by the polytropic relation, \(p = \Pi/\Sigma\). We assign \(N = 3\) throughout the entire calculation (i.e., \(I_1 = 16/35\) and \(I_4 = 128/315\)).

The continuity equation, the radial component of the momentum equation, the angular momentum conservation, the hydrostatic equilibrium in the vertical direction, the energy equation, and the equation of state are written as follows:

\[
\dot{M}_{\text{acc}}(R) = \dot{M}_{\text{acc}}(R_{\text{wind}}) - \dot{M}_{\text{out}}(R),
\]

\[
v_R \frac{d v_R}{dR} + \frac{1}{\Sigma} \frac{d \Sigma}{dR} = \frac{\ell^2 - \ell_i^2}{R^3} - \frac{\Pi}{\Sigma} \frac{d \ln \Omega_K}{dR},
\]

\[
\dot{M}_{\text{acc}}(\ell - \ell_i) = -2\pi R^2 \Omega_{K0}
\]

\[
(2N + 3) \frac{\Pi}{\Sigma} = H^2 \Omega_{K0}^2,
\]

\[
Q_{\text{vis}}^+ + Q_{\text{rad}}^+ + Q_{\text{adv}}^+ = Q_{\text{vis}}^- + Q_{\text{rad}}^-.
\]

\[
\Pi = \Pi_{\text{gas}} + \Pi_{\text{rad}} = \frac{k_B}{\mu m_H} I_{N+1} \Sigma T_c + \frac{2}{3} I_{N+1} a T_c^4 H.
\]

Here, the flow velocities of the radial component and the azimuthal component are expressed by \(v_R\) and \(v_t\), respectively; the angular momentum of the gas is given by \(\ell = R v_t = R^2 \Omega\); \(\Omega\) and \(\Omega_K\) are the angular speed of rotation and the Keplerian angular speed; \(\ell_k\) and \(\ell_i\) are the Keplerian angular momentum and the angular momentum at the inner radius of the flow; \(T_{\text{K0}}(\equiv -\alpha \Pi)\) is a vertically integrated stress tensor with \(\alpha\) being the viscosity parameter (Shakura & Sunyaev 1973); \(\Pi_{\text{gas}}\) and \(\Pi_{\text{rad}}\) are the gas pressure and the radiation pressure; and \(m_H, T_c, k_B,\) and \(a\) are the hydrogen mass, the temperature on the equatorial plane, the Boltzmann constant, and the radiation constant, respectively. The last term on the right-hand side of equation (9) is a correction term resulting from the fact that the radial component of the gravitational force changes with height (Matsumoto et al. 1984).

In the energy equation (12), the viscous heating rate, \(Q_{\text{vis}}^+\), the radiative cooling rate, \(Q_{\text{rad}}^-\), and the advective cooling rate, \(Q_{\text{adv}}^-\), are defined by:

\[
Q_{\text{vis}}^+ = -RT_{\text{K0}} \frac{d \Omega}{dR},
\]

\[
Q_{\text{rad}}^- = 2F = \frac{16\dot{\alpha c a c T_c}}{3\tau},
\]

\[
Q_{\text{adv}}^- = \frac{9}{8} v_R \Sigma T_c \frac{ds}{dR}.
\]

where \(F\) is the radiative flux per unit surface area on the flow surface; \(s\) is the specific entropy; \(\tau\) is the optical thickness of the flow, given by \(\tau = (\kappa_{\text{gas}} + \kappa_{\text{H}})\Sigma; \kappa_{\text{gas}} = 0.4\) is the electron scattering opacity; \(\kappa_{\text{H}}(= 0.64 \times 10^{21} \rho T^{-7/2})\) is the free-free absorption opacity; \(\rho\) and \(T\) are the vertically averaged density and the temperature, respectively. Note that Jiao et al. (2009) point out that the use of equation (11) may lead to an overestimation of the gravity force when the scale height is comparable to the radius, \(H \sim R\).

We wish to note that the difference between the equations of the original slim-disk model and those of our model exists only in the previous equation (8). In the original slim-disk model, the continuity equation is expressed by \(\dot{M}_{\text{acc}}(R) = \text{const}.\) The reason why the forms of equations (9)–(13) do not change is that these expressions are written per unit mass. While Lipunova (1999) explicitly expresses the outflow effect in her equations, the physical meanings of the equations are the same as our own (see also Kitabatake et al. 2002). The difference between the two models and our model is the fact that we constructed a model that realistically considers the outflow effect by using the accretion rate mentioned in the previous section. In the previous study, moreover, the flow is assumed to be Keplerian (Lipunova 1999; see also Poutanen et al. 2007), or to obey a self-similar solution (Kitabatake et al. 2002; see also Fukue 2004), whereas we numerically solved the radial advection, i.e., equations (9) and (12). However, we need to take into account the work exerted on the outflow by the accretion-flow material. This issue is discussed in the next section.

The calculations were performed from the outer radius at \(R = 1 \times 10^4 r_s\) down to the inner radius, \(R \sim 1.0 r_s\). In our modified slim-disk model, we approximated \(\dot{M}_{\text{acc}}(R) = \dot{M}_{\text{acc}}(r)\) and \(\dot{M}_{\text{out}}(R) = \dot{M}_{\text{out}}(r)\) within \(R = 350 r_s\). Since the range of the accretion rate which was obtained by the simulation mentioned in the previous section was \(R \leq 350 r_s\), we assumed that the accretion rate in the range of \(R = 350 - 10^4 r_s\) is constant, i.e., \(\dot{M}_{\text{acc}}(R) = \dot{M}_{\text{acc}}(R_{\text{wind}})\) for \(350 r_s \leq R \leq 10^4 r_s\). In the original slim-disk model, in contrast, \(\dot{M}_{\text{acc}}(R) = \dot{M}_{\text{acc}}(R_{\text{wind}})\) over the entire radius. We set the black-hole mass and the viscous parameter to be \(M = 10 M_\odot\) and \(\alpha = 0.1\), respectively. We also set \(\dot{M}_{\text{acc}}(R_{\text{wind}}) = 364 L_E/c^2\) and \(828 L_E/c^2\).

3.2. Flow Structure

Figure 4 indicates the surface density and the effective temperature as functions of the radius in the modified slim-disk model (dashed lines) and the original slim-disk model (solid lines) for \(\dot{M}_{\text{input}} = 1000 L_E/c^2\) (left panels) and \(\dot{M}_{\text{input}} = 3000 L_E/c^2\) (right panels), respectively. The surface density of the flow is significantly reduced due to the mass loss by outflow in both cases (see figure 3). Hence, the optical depth of the flow, \(\tau(\propto \Sigma)\), is also reduced by a factor of two or three. However, the flow is optically thick over the entire region. Even though the mass-accretion rate decreases as the radius decreases, the trapping radius derived from the mass-accretion rate at any radius exceeds that radius; i.e., photon trapping is effective.

We quantitatively examined the effect of photon trapping; i.e., we calculated the relative importance of the advective cooling in the energy equation. We derived the relation
Fig. 4. Surface density, $\Sigma$, and the effective temperature, $T_{\text{eff}}$, as functions of the radius in the modified slim-disk model (dashed lines) and the original slim-disk model (solid lines) for $M_{\text{input}} = 1000 L_E/c^2$ (left) and $M_{\text{input}} = 3000 L_E/c^2$ (right).

between $t_{\text{diff}}/t_{\text{acc}}$ and $Q_{\text{adv}}^-/Q_{\text{vis}}^+$. The advective cooling rate are expressed by

$$
Q_{\text{adv}}^- \sim -\frac{9}{8} \frac{v_R \Sigma}{R} \left( e - \frac{p}{\rho} \right) = \frac{3}{2} \frac{v_R}{R} H a T_c^4,
$$

(17)

where $e$ is the specific energy. Here, we used $e = a T_c^4/\rho$ and $p = a T_c^4/3$. Therefore, the condition of effective photon trapping ($t_{\text{diff}}/t_{\text{acc}} > 1$) is derived by

$$
t_{\text{diff}}/t_{\text{acc}} = \frac{32}{3} \frac{Q_{\text{adv}}^-}{Q_{\text{rad}}} = \frac{32}{3} \frac{Q_{\text{adv}}^-}{Q_{\text{vis}}^+ - Q_{\text{adv}}^+} > 1.
$$

(18)

As a result, this gives the relation

$$
\frac{Q_{\text{adv}}^-}{Q_{\text{vis}}^+} > \frac{3}{35} \sim 0.09.
$$

(19)

That is, photon trapping is effective at $Q_{\text{adv}}^-/Q_{\text{vis}}^+ > 0.09$.

In figure 5, we show the ratio, $Q_{\text{adv}}^-/Q_{\text{vis}}^+$, for each model. In the case of the original slim-disk model, in which the outflow is not considered, photon trapping is significantly effective at $R \lesssim 100 r_s$. In the case of the modified slim-disk model, however, the advective cooling rate is smaller due to the mass loss by outflow ($Q_{\text{adv}} \propto \Sigma$), and so is the ratio. Hence, the photon-trapping effect is weaker in the model, but the outflow is not strong enough to totally suppress photon trapping at smaller radii ($Q_{\text{adv}}^-/Q_{\text{vis}}^+ \sim 0.4-0.6$). At a larger radius, there is no difference between the two models, because there is no outflow.

In addition, we can understand from figure 4 that, even if outflow effects are taken into account, the effective temperature profiles do not change. They are $T_{\text{eff}} \propto R^{-1/2}$ at a smaller radius and $T_{\text{eff}} \propto R^{-3/4}$ at a larger radius. Why then is the effective-temperature profile unchanged? This is because it is determined by photon trapping at small radii (note that photon trapping is effective at $Q_{\text{adv}}^-/Q_{\text{vis}}^+ \gtrsim 0.09$), while no significant outflow occurs at large radii.

Kitabatake, Fukue, and Matsumoto (2002) constructed a model for supercritical accretion flows with mass loss, by adopting a self-similar treatment proposed by Narayan and Yi (1994). They mention that the effective-temperature profile of the flow is negligibly affected by the occurrence of outflow in the case of advective-dominated flows (see also Fukue 2004). The reason is as follows. When the radiative flux, $F (= \sigma T_c^4/\Pi)$, is reduced by the mass loss, the optical depth, $\tau (\propto \Sigma)$, is reduced at the same time. As a result, the effective temperature of the flow, $T_{\text{eff}} (= T_c r^{-1/4})$, does not depend on the mass loss. This provides another explanation for the flatter temperature distribution.

In the region $R \lesssim 3 r_s$, the advective-cooling rate becomes
negative. The reason is as follows. The optical depth of the
flow, $\tau(\propto \Sigma)$, steeply decreases inward in that region, although
the flow is optically thick (see figure 4). Therefore, the radiative
cooling rate, $Q_{\text{rad}}^-(\propto \tau^{-1})$, steeply increases as the region
moves toward the inside, whereas the viscous heating rate
($Q_{\text{vis}}^+ \propto M_{\text{acc}}$) does not. As a result, the advective-cooling
rate should become negative, since $Q_{\text{adv}}^- = Q_{\text{vis}}^+ - Q_{\text{rad}}^-$.  

4. Discussion

4.1. Brief Summary

In this paper, we first carefully examined the global RHD simulation data of supercritical accretion flow onto black holes in order to examine the conflict between photon trapping and outflow. We have confirmed that both are equally important; i.e., despite significant mass loss by the outflow, the outflow is not strong enough to totally suppress photon trapping. We evaluated the accretion and outflow rates as functions of the radius based on simulation data, and put them into the formulation of the slim-disk model, in which no outflows were considered originally, and hence the mass-accretion rate was considered to remain constant. We compared the resultant flow structure while considering the outflow with the structure without outflow, and found that, although surface density (and, hence, optical depth) of the flow is significantly reduced, the effective temperature profile is negligibly affected by the occurrence of outflow. Therefore, multiblackbody spectra are also negligibly affected. This has a profound implication when one performs a spectral fitting of black-hole objects, notably of ultraluminous X-ray sources (ULXs, see subsection 4.3).

We here put forth reasons why we stick to the one-dimensional model, when multidimensional simulation data are available. Although it has become possible to simulate the flow from first principles, simulations are still subject to numerical errors and limitations arising from the finite mesh spacing. Also, it is not always easy to select the physical processes from the vast simulation data with substantial fluctuations and numerical errors. Further, multidimensional RHD simulations are very time-consuming. That is, it is impossible to perform extensive parameter studies. Therefore, studies based on simplified (one-dimensional) models like the present one should be useful for understanding the physics.

4.2. Work Exerted on Outflow

In our simplified one-dimensional model, we considered the loss of mass, angular momentum, and energy by outflow. These effects are incorporated by a spatial variation of the mass-accretion rate. That is, we considered angular momentum and energy loss carried by outflow material, while assuming that the specific angular momentum and specific energy are the same for the outflow and the accretion flow. There exists, however, another important factor that should be included in the energy equation; that is the work exerted on the outflow by the disk material, $Q_{\text{wind}}^-$. Obviously, the outflow cannot have positive energy to reach infinity without acquiring additional energy by the underlying accretion flow. Poutanen et al. (2007), for example, included this effect by assuming that radiative loss of the accreting material is partly transferred to the outflow.

To see how our results are affected by this additional energy loss, we reduced the specific energy of accretion flow by hand, assuming that $Q_{\text{wind}}^- = Q_{\text{rad}}^-$. The result was that the effective temperature profile hardly varies, even though the amount of mass loss decreases. Our conclusion concerning the unchanged temperature profile is not altered.

4.3. Model for Ultraluminous X-Ray Sources

ULXs are bright, compact X-ray sources found in the off-center region of nearby galaxies. Their luminosities are typically, $L_\text{X} \sim 10^{39-41}$ [erg s$^{-1}$], and hence exceed the Eddington luminosity of a neutron star (Fabbiano 1989). There are two hypotheses about the origin of ULXs. If the luminosity is below the Eddington luminosity, it then follows that the mass of the central black holes of ULXs should be that of an intermediate-mass black hole (IMBH), whose masses range over $10^2-4 M_\odot$. If the luminosity can be above the Eddington luminosity, conversely, stellar-mass black holes can also account for the luminosities. Unfortunately, the kinematic
method, which is very practical for the estimation of the black-hole masses in binary systems, is not applied to the ULXs because no appreciable (optical) line features are observed.

Interestingly, ULXs share some spectral features with the black-hole binaries (Colbert & Mushotzky 1999; Makishima et al. 2000; see review by Done et al. 2007). Therefore, it seems that we can estimate the black-hole mass from the spectral fitting, although the results appear to be controversial. Some authors claim that the ULXs should contain stellar-mass black holes (King et al. 2001; Watarai et al. 2001; Okajima et al. 2006; Vierdayanti et al. 2006). Vierdayanti, Watarai, and Mineshige (2008), for example, claim that the spectral fitting with the conventional spectral model (with blackbody and power law) is not reliable when the power-law spectral components dominate, and demonstrated based on the original slim-disk model that some ULXs exhibit spectral signatures of the supercritical accretion flow (see also Vierdayanti et al. 2006). The present study supports their conclusions, since, even if outflow effects are taken into account, neither effective temperature profiles nor multicolor blackbody spectra are changed.

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