R-mode stabilization in neutron stars with hyperon cores

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Abstract. R-mode oscillations of neutron stars are known to be generally unstable with respect to the emission of gravitational waves. This instability can be suppressed by dissipative processes in the neutron star core. The standard minimal nucleonic model of the core cannot stabilize r-modes for many observed neutron stars, at the same time the probability to observe them unstable should be very low. One of the solutions to this problem is to account for possible presence of hyperons in the stellar core. Hyperons strongly enhance bulk viscosity in the core, which effectively suppresses the r-mode instability. We revisit the bulk viscosity calculations in the non-superfluid hyperon matter and briefly discuss the effect of baryon superfluidity. Our results suggest that the hyperon solution to the r-mode problem is more promising than it was thought before.

1. Introduction

R-modes exist only in rotating neutron stars (NSs), and are unstable with respect to the gravitational wave emission [1]. This instability can be damped by the shear viscosity \( \eta \) and bulk viscosity \( \zeta \) (appearing due to non-equilibrium weak particle transformations) in the NS core. Rotation frequencies \( \nu \) and internal (redshifted) temperatures \( \tilde{T} \), where damping is not sufficient to stabilize a NS, are called the instability window (IW). Its lower edge, the critical frequency curve \( \nu(\tilde{T}) \), depends on the mass \( M \) and the equation of state (EOS) of the star. About 20 fast-spinning NSs in the low-mass X-ray binaries (LMXBs) have \( \nu \) and \( \tilde{T} \) well inside the ‘standard’ IW for NSs with nucleonic npe\( \mu \) EOSs, but they show no unstable behavior [1, 2]. This problem can be solved by accounting for hyperons (\( \Lambda, \Sigma^-, \Xi^-, \) etc.) in the NS core. Then non-leptonic weak processes (NLWPs) launch and enhance \( \zeta \) [3–5] (in npe\( \mu \) matter \( \zeta \) is provided by Urca processes, which are less efficient at \( \tilde{T} \) of interest [3]). Previous calculations of IWs for hyperon NSs found that hyperons can barely stabilize r-modes in the range of \( \nu \) and \( \tilde{T} \) interesting from point of view of observations [5]. In this work we show, using modern hyperonic EOS (in our case, hyperons are mainly presented by \( \Lambda \) and \( \Xi^- \)) and calculating NLWP rates following [4], that r-modes could be made stable for moderately massive NSs in relevant range of \( \nu \) and \( \tilde{T} \).
2. Bulk viscosity in non-superfluid hyperon matter

There are five NLWPs, which change strangeness by unity, $|\Delta S| = 1$, in the $npe\mu\Lambda\Xi^-$-matter:

$$n + p \leftrightarrow \Lambda + p, \quad n + n \leftrightarrow \Lambda + n, \quad n + \Xi^- \leftrightarrow \Lambda + \Xi^-, \quad \Lambda + n \leftrightarrow \Xi^- + p. \quad (1)$$

We follow the general formalism discussed in [3] to calculate NLWP rates. The non-equilibrium rate for an arbitrary process $\alpha = 1 + 2 \leftrightarrow 3 + 4$ is $\Delta \Gamma_\alpha = \lambda_\alpha \Delta \mu_\alpha$ (the number of direct minus inverse reactions per unit volume per unit time), where $\Delta \mu_\alpha = \mu_1 + \mu_2 - \mu_3 - \mu_4 \ll kT$ ($\mu_i$ is the chemical potential for particle species $i$). In the strongly degenerate matter the rate $\lambda_\alpha$ is

$$\lambda_\alpha \approx \frac{5.1 \times 10^{45} g_{\alpha}^{(\text{max})} - g_{\alpha}^{(\text{min})}}{\text{erg cm}^3 \text{s}} T_8^2 \Theta_\alpha W_\alpha, \quad W_\alpha = \int \prod_{i=1}^4 d\Omega_i \delta(\Delta \mathbf{p}_i) \sum_{\text{spins}} |\mathcal{M}_\alpha|^2 64s_{\alpha}G_{F}^2 m_N^2 \int \prod_{i=1}^4 d\Omega_i \delta(\Delta \mathbf{p}_i). \quad (2)$$

Here $T_8 = T/(10^8 \text{K})$ is the local temperature, $g_{\alpha}^{(\text{min})} = \max\{|p_{F1} - p_{F3}|, |p_{F2} - p_{F4}|\}$ and $g_{\alpha}^{(\text{max})} = \min\{p_{F1} + p_{F3}, p_{F2} + p_{F4}\}$ are the minimum and maximum possible momentum transfers ($p_{Fi}$ is the Fermi momentum of particles $i$), and $\Theta_\alpha = \Theta(g_{\alpha}^{(\text{max})} - g_{\alpha}^{(\text{min})})$ is the Heaviside function. Further, $W_\alpha$ is the angle-averaged matrix element $\mathcal{M}_\alpha$. In the expression for $W_\alpha$ the natural units $\hbar = c = 1$ are used, $\Delta \mathbf{p}_\alpha = \mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4$, $\delta(\mathbf{x})$ is the 3-dimensional Dirac delta, $G_F$ is the Fermi weak interaction constant, $m_N$ is the nucleon mass, $s_{\alpha}$ is the symmetry factor ($s_{nn} \leftrightarrow s_{n\Lambda} \leftrightarrow s_{\Lambda\Lambda} = 2$, otherwise = 1). For each species $|\mathbf{p}_i| = p_{Fi}$ due to strong degeneracy of matter in the NS core.

The main channel for NLWP is the meson exchange [4, 6]. Here we employ the one-meson exchange (OME) interaction model. We consider only the lightest possible mesons, $K$ for $n\Lambda \leftrightarrow \Lambda\Lambda$ and $\pi$ for the other NLWPs. We neglect exchanges of heavier mesons and many-meson interactions. The Feynman diagram for the direct contribution to $\mathcal{M}_\alpha$ of an arbitrary NLWP $1 + 2 \leftrightarrow 3 + 4$, mediated by a meson $M$, is shown in Figure 1. The exchange contribution can be obtained by the permutation $1 \leftrightarrow 2$. Since we disregard mesons heavier than the lightest ones and assume that the strangeness change in the weak vertex is $|\Delta S| = 1$, we have no exchange diagrams for $n\Xi^- \leftrightarrow \Lambda\Xi^-$ and $\Lambda n \leftrightarrow \Xi^- p$. Phenomenological dimensionless weak $A_{ij}^M$, $B_{ij}^M$ and strong $g_{ij}^M$ couplings can be found in [7, 8] and refs. therein (we use the $\gamma^5$ definition from [8]). To calculate $\mathcal{M}_\alpha$, we follow the relativistic mean-field approach to the quasiparticle wavefunctions [9]. Namely, the species $i$ is described by the free-like relativistic bispinor $u_i$ but with the Dirac and Landau effective masses instead of the proper mass and energy, respectively. All wave functions are normalized to one particle per unit volume. Then ($h = c = 1$)

$$\mathcal{M}_\alpha^{\text{dir}} = G_F m_{\pi}^2 \bar{u}_3 (A_{13}^M + B_{13}^M \gamma^5) u_1 D_M(q_{13}) \bar{u}_4 g^M_{24} \gamma^5 u_2, \quad \mathcal{M}_\alpha = \mathcal{M}_\alpha^{\text{dir}} - \mathcal{M}_\alpha^{\text{dir}}|_{1\leftrightarrow 2}. \quad (3)$$

Here $m_{\pi}$ is the bare pion mass, $D_M(q) = - (q^2 + m_{M}^2)^{-1}$ is the meson propagator, and $q_{13} = p_3 - p_1$ is the momentum transfer with $m_{M}$ being the bare meson mass. This expression for $D_M$ is non-relativistic; it gives a lower estimate for an actual propagator value, obtained with account for relativity and in-medium effects. More accurate calculations of $D_M$ are debatable and are beyond the scope of the present paper.
The bulk viscosity $\zeta$ in the $\Lambda\Xi^-$ hyperonic matter can be derived in the same way as in section II of [10], which considered $\Sigma^-$-$\Lambda$ composition. If any strong process operates in the NS core (e.g., $\Xi^- p \leftrightarrow \Lambda\Lambda$ in our case), NS matter is almost in equilibrium with respect to it because its rate is much greater than that of NLWPs. Then for all $\alpha$’s we have

$$\Delta \mu_\alpha = \mu_n - \mu_\Lambda = \Delta \mu.$$  

Due to the quasineutrality condition the baryon number density $n_b$ and strangeness fraction $x_S = \sum_i S_i n_i / n_b$ ($n_i$ is the number density for particles $i$) are the only independent thermodynamic parameters varying in the course of oscillations. Then

$$\zeta = \zeta_{\text{max}} \frac{2\lambda / \lambda_{\text{opt}}}{1 + (\lambda / \lambda_{\text{opt}})^2}, \quad \zeta_{\text{max}} = \frac{n_b}{2\omega} \frac{\partial P}{\partial x_S} \frac{\partial \Delta \mu}{\partial n_b} \left( \frac{\partial \Delta \mu}{\partial x_S} \right)^{-1}, \quad \lambda_{\text{opt}} = n_b \omega \left( \frac{\partial \Delta \mu}{\partial x_S} \right)^{-1}. \tag{4}$$

Here $\lambda = \sum_\alpha \lambda_\alpha$ is the total rate of non-equilibrium NLWPs in the normal matter, $\omega$ is the local oscillation frequency, and $P$ is the pressure. Thermodynamic derivatives are related to the zeroth Landau parameters of nucleon-hyperon matter and can be calculated when EOS is specified.

Notice that the $\zeta - \lambda$, and, consequently, $\zeta - T$ dependence is non-monotonic and has a maximum at some $T$. At a given $T$ our $\lambda$ is greater than what is used in [5], thus we have the maximum of $\zeta$ being shifted to lower $T$ with respect to [5], while at high $T$ $\zeta$ is lowered. This is in agreement with results of [4].

3. Results

In the core we use FSU2H EOS model from [11], but with $\Sigma^-$ potential in the symmetric nuclear matter being equal to 40 MeV. This EOS is calibrated to the up-to-date (hyper)nuclear data. It predicts that a typical NS with $M = 1.4 M_\odot$ have $R = 13.3$ km. Hyperons appears as follows: $\Lambda$ at $M = 1.38 M_\odot$, $\Xi^-$ at $1.69 M_\odot$, and $\Sigma^-$ at $1.91 M_\odot$. The maximum mass is $1.993 M_\odot$. Effective masses and Landau parameters for this EOS, necessary to calculate derivatives in (4), are computed as discussed in [12].

Using equations (2) and (3), we find $W_{np \leftrightarrow \Lambda p} \sim W_{\Lambda n \leftrightarrow \Xi^- p} \sim 1$, $W_{nn \leftrightarrow \Lambda n} \sim 0.5$, and $W_{n\Xi^- \leftrightarrow \Lambda\Xi^-} \sim 0.08$ are roughly constants, while $W_{n\Lambda \leftrightarrow \Lambda\Lambda}$ varies from $\sim 0.1$ at low densities to...
Figure 3. Critical frequency curves for the NSs of different masses; $\nu$ is the rotation frequency, $\tilde{T}$ is the redshifted internal temperature. (a) All NLWPs are included in the total rate $\lambda$ (no baryon superfluidity). (b) Only $n\Lambda \leftrightarrow \Lambda\Lambda$ is open, other NLWPs are strongly suppressed by the superfluidity of charged baryons. Observational data are taken from [2].

$\sim 0.3$ in the center of the NS with the maximum mass. However, these values strongly depend on EOS and can vary by $\sim 50\%$ depending on a model.

The rate of each NLWP from the set (1) is shown in Figure 2(a). In the absence of baryon superfluidity the main contribution to $\lambda$ comes from $np \leftrightarrow \Lambda p$. When baryons are superfluid (typically, charged hyperons pair at larger temperatures than uncharged ones), NLWPs are suppressed; then $nn \leftrightarrow \Lambda n$ and $n\Lambda \leftrightarrow \Lambda\Lambda$ become the leading contributors. The coefficients $\zeta_{\text{max}}$ (the maximum possible bulk viscosity at a given density) and $\lambda_{\text{opt}}$ (the total NLWP rate at which $\zeta_{\text{max}}$ is reached) are shown in Figure 2(b). The chosen frequency $\omega$ is typical for the NS r-modes.

Figure 3 shows the critical frequency curves for the quadrupole r-mode of NSs with different masses. See details of calculations in [2, 5]. The shear viscosity is taken from [13]. In Figure 3(a) all NLWPs contribute to $\lambda$ (the case of non-superfluid baryons), while in Figure 3(b) only $n\Lambda \leftrightarrow \Lambda\Lambda$ is open. The latter case roughly mimics the effects of superfluidity, since $n\Lambda \leftrightarrow \Lambda\Lambda$ is least suppressed by baryon pairing due to (probably) low critical temperature of $\Lambda$-hyperons [14]. Evidently, in both cases hyperon $\zeta$ can stabilize r-modes for moderately massive NSs ($M \lesssim 1.9M_\odot$) in the whole $\nu, \tilde{T}$-range occupied by the observed NSs in LMXBs.

4. Conclusion

We revisit the bulk viscosity calculations in hyperon NS cores and apply the results to construct the r-mode IW. The modern EOS, calibrated to the up-to-date nuclear-physics data is used. The full set of NLWPs in the $npe\mu\Lambda\Xi^-$ matter is considered for the first time. We use the OME weak interaction model (the most effective channel for NLWPs). In spite of many simplifications in our study (no heavy meson exchanges, underestimated meson propagator, no superfluidity), our results indicate that the hyperon solution to the r-mode puzzle is a viable scenario, in contrast to the previous IW calculations [5]. The main reason is that [5] used contact $W$ boson-exchange channel for NLWPs, which is less efficient than OME. This shifts the maxima of critical frequency curves to temperatures, which are lower than in [5] and closer to what is inferred from LMXB observations.
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