Dispersive optical systems for scalable Raman driving of hyperfine qubits

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Hyperfine atomic states are among the most promising candidates for qubit encoding in quantum information processing. In atomic systems, hyperfine transitions are typically driven through a two-photon Raman process by a laser field which is amplitude modulated at the hyperfine qubit frequency. Here we introduce a method for generating amplitude modulation by phase modulating a laser and reflecting it from a highly dispersive optical element known as a chirped Bragg grating. This approach is passively stable, offers high efficiency, and is compatible with high-power laser sources, enabling large Rabi frequencies and improved quantum coherence.

We benchmark this approach by globally driving an array of approximately 300 neutral $^{87}\text{Rb}$ atomic qubits trapped in optical tweezers and obtain Rabi frequencies of $2\text{ MHz}$ with photon-scattering error rates of less than $2 \times 10^{-4}$ per $\pi$ pulse. This robust approach can be directly integrated with local addressing optics in both neutral atom and trapped ion systems to facilitate high-fidelity single-qubit operations for quantum information processing.

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I. INTRODUCTION

Trapped neutral atoms and atomic ions are among the most pristine quantum systems for quantum science and engineering. In such systems, quantum bits can be encoded in pairs of atomic levels which are defined in hyperfine ground-state manifolds or on narrow optical transitions from a single ground state to a metastable excited state [1,2]. Hyperfine-encoded qubits are particularly attractive due to their transition frequencies in the several gigahertz range, which can be driven either directly with microwave fields or by two-photon stimulated Raman transitions. While microwaves have been used for high-fidelity control [3,4], Raman transitions offer substantially higher, megahertz-scale Rabi frequencies [5,6] as well as the opportunity for local addressing of individual qubits separated by micrometer length scales.

A variety of experimental approaches have been used to drive stimulated Raman transitions of hyperfine qubits. The conventional approach to Raman driving uses two phase-locked lasers, with a frequency difference equal to the hyperfine splitting [7,8]. Alternatively, mode-locked optical frequency combs have been used in trapped ion systems, wherein pairs of frequency components combine to drive Raman transitions [9–12]. Another approach is based on phase modulation of a single laser to produce low-noise sidebands at the hyperfine frequency [13–15]. This approach necessitates additional interferometric filtering to suppress destructive interference between sideband pairs, resulting in a loss of usable optical power [13,15]. Other approaches based on spatially separated lasers can be used for momentum transfer, but Doppler sensitivity reduces coherence for qubit manipulations [15]. Furthermore, each of these previously demonstrated approaches requires active stabilization due to interferometric sensitivity.

In this paper we demonstrate a method for Raman driving based on phase modulation followed by reflection from a highly dispersive optical element. The dispersive element, a chirped Bragg grating (CBG), changes the relative phases of the phase-modulated sidebands, converting destructive interference to constructive interference and producing amplitude modulation for driving Raman transitions. We show that the dispersive approach offers high-efficiency conversion from phase modulation to amplitude modulation, enables scaling to high optical power, and is passively stable.

This paper is structured as follows. In Sec. II we review how stimulated Raman transitions induced by a multifrequency laser field can be understood purely in terms of laser amplitude modulation. In Sec. III we show how dispersive optics can be used to efficiently convert phase modulation to amplitude modulation for driving Raman transitions. In Sec. IV we describe our Raman laser system in detail and in Sec. V we experimentally benchmark its performance on an array of approximately 300 neutral $^{87}\text{Rb}$ atomic qubits trapped in optical tweezers. These results demonstrate that this robust approach to Raman driving enables scalable optical control.
of hyperfine qubits, with future opportunities to integrate into local optical addressing systems in both neutral atom and trapped ion platforms.

II. LASER AMPLITUDE MODULATION DRIVES STIMULATED RAMAN TRANSITIONS

Stimulated Raman transitions are two-photon processes which drive transitions between two atomic ground states |0⟩ and |1⟩ (split by a qubit frequency ωq) through an intermediate excited state |2⟩. Conventionally, Raman transitions are understood as being driven by a laser field containing two frequency components separated by ωq, resulting in an effective resonant coupling between the states |0⟩ and |1⟩ with Raman Rabi frequency \( \Omega_{\text{R}} \). The highly dispersive volumetric CBG allows to achieve efficient amplitude modulation at the qubit frequency, which resonantly drives the Raman transition. This connection between Raman driving and laser amplitude modulation can be further clarified by directly solving the three-level system dynamics in the presence of a generic time-dependent laser field \( \Omega(t) \) which couples both ground states |0⟩ and |1⟩ to the excited state |2⟩ [Fig. 1(a)].

This system is described by the following Hamiltonian, given in the rotating frame for the excited state |2⟩:

\[
H = \hbar \omega_q \left( |0⟩⟨1| + |1⟩⟨2| + |2⟩⟨1| \right) + \hbar \Delta \left( |0⟩⟨1| + |1⟩⟨2| + |2⟩⟨1| \right) + \text{H.c.} 
\]

If the intermediate detuning \( \Delta \) is large compared to \( \omega_q \) and the amplitude and spectral width of \( \Omega(t) \), we can adiabatically eliminate the excited state, resulting in an effective two-level system (TLS) Hamiltonian for states |0⟩ and |1⟩,

\[
H_{\text{TLS}} = \hbar \omega_q |0⟩⟨1| + \frac{\hbar \Omega_{\text{TLS}}(t)}{2} |1⟩⟨0| + \text{H.c.},
\]

with an effective coupling

\[
\Omega_{\text{TLS}}(t) = \frac{|\Omega(t)|^2}{2 \Delta}
\]

and with the Stark shifts on states |0⟩ and |1⟩ neglected.

FIG. 1. Amplitude modulation for driving Raman transitions. (a) Stimulated Raman transitions in a \( \Lambda \)-type three-level system. Adiabatic elimination of the excited state results in an effective Raman coupling between ground states |0⟩ and |1⟩. (b) Level structure for \(^{87}\text{Rb}\), showing Raman driving of the clock transition from |0⟩ = |F = 1, m_F = 0⟩ to |1⟩ = |F = 2, m_F = 0⟩. This transition is driven by a time-dependent \( \sigma^+ \) polarized field \( \Omega(t) \), which is far detuned by \( \Delta \) from the excited state (not far detuned relative to the splitting between the \( 5S_{1/2} \) and \( 5P_{1/2} \) excited states). (c) Several approaches for Raman driving, including the dispersive approach presented here, operate by converting phase modulation to amplitude modulation. The dispersive approach benefits both from having the highest coherence metric (see Appendix B) and from being passively stable since it does not rely on interferometric filtering. (e) Weakly dispersive elements, such as a conventional chirped Bragg mirror or a 10-m optical fiber, require large modulation depth \( \beta \) to achieve efficient amplitude modulation, according to Eq. (7). The highly dispersive volumetric CBG allows a low \( \beta \) to be used. Here \( \beta < \pi \) marks the experimentally accessible window of modulation depths.

We highlight here that the Hamiltonian from (3) describes a two-level system with splitting \( \omega_q \) and time-dependent coupling \( \Omega_{\text{TLS}} \propto |\Omega(t)|^2 \). From this description, it is apparent that the intensity of the laser field produces an effective field
which couples the two qubit states; laser intensity modulation at the qubit frequency therefore drives the qubit transition, akin to resonant driving of a spin transition directly using microwaves. Interestingly, we note that in real atoms [e.g., level structure for $^{87}\text{Rb}$ as shown in Fig. 1(b)], the effective field which is proportional to the laser intensity takes the form of the fictitious magnetic field associated with vector light shifts (see Appendix A and [17]). Specifically, an off-resonant laser field acts as a fictitious magnetic field given by $B_{\text{fict}} \propto \text{Im} [\epsilon^* \times \epsilon]$ where $\epsilon$ is the polarization vector of the laser field [18,19]. Circularly polarized light, such as with $\epsilon_+ = \hat{x} + i\hat{y}$, induces an effective magnetic field oriented along $\hat{z}$ which couples $\pi$-polarized spin transitions, and amplitude modulation of the laser field at the transition frequency therefore produces a modulated effective magnetic field which resonantly drives such spin transitions. This analysis, which extends previous work focusing on vector light shifts in the form of the fictitious magnetic field associated with vector light shifts (see Appendix A and [17]). Specifically, an off-resonant laser field acts as a fictitious magnetic field given by $B_{\text{fict}} \propto \text{Im} [\epsilon^* \times \epsilon]$ where $\epsilon$ is the polarization vector of the laser field [18,19]. Circularly polarized light, such as with $\epsilon_+ = \hat{x} + i\hat{y}$, induces an effective magnetic field oriented along $\hat{z}$ which couples $\pi$-polarized spin transitions, and amplitude modulation of the laser field at the transition frequency therefore produces a modulated effective magnetic field which resonantly drives such spin transitions. This analysis, which extends previous work focusing on vector light shifts in the context of Zeeman transitions [17,20–24], also clarifies the interplay between laser polarization and Raman transitions. As an example, the above approach illustrates why linearly polarized light along any propagation axis cannot be used to drive Raman transitions since it produces no vector light shifts, which can be equivalently evaluated through summations over dipole matrix elements.

III. EFFICIENT CONVERSION OF PHASE MODULATION TO AMPLITUDE MODULATION WITH DISPERSIVE OPTICS

While laser amplitude modulation is necessary for Raman driving, the most experimentally accessible form of high-frequency laser modulation is phase modulation using electro-optics. Sinusoidal phase modulation produces frequency sidebands according to the Jacobi-Anger expansion

$$\Omega(t) = \Omega_0 e^{i \beta \sin \omega t} = \Omega_0 \sum_{n=-\infty}^{\infty} J_n(\beta) e^{i n \omega t},$$

(5)

where $J_n$ are Bessel functions of the first kind, $\beta$ is the modulation depth, and $\omega$ is the modulation frequency. Since the laser intensity is constant, $|\Omega(t)|^2 = |\Omega_0|^2$, a phase-modulated laser cannot drive hyperfine qubits. This can be seen also as destructive interference between pairs of adjacent sidebands: $\sum_{n=-\infty}^{\infty} J_n(\beta) J_{n+1}(\beta) = 0$.

There are several methods for modifying the sideband spectrum of a phase-modulated laser to produce amplitude modulation [Figs. 1(c) and 1(d)]. These methods are primarily interferometric in nature, since they act selectively on frequency components with only gigahertz-scale separation [15]. For example, one approach is to use a Fabry-Pérot cavity to filter out the carrier ($n = 0$) spectral component [5]. Another method is to use a Mach-Zehnder interferometer to filter out all odd-order sidebands or a Mach-Zehnder intensity modulator in which the phase modulation occurs in one arm of an interferometer [25]. These approaches are inherently inefficient, in that they discard some portion of the laser light by filtering out components; further, they are all sensitive to path-length fluctuations on wavelength scales. Some fiber-based versions of these systems can be more robust, but they are limited to low optical power. Discarding optical power requires detuning the laser system closer to the excited state to achieve the same Rabi frequency, which correspondingly increases the error rate associated with optical scattering [26]. To compare these various approaches, we define a coherence metric $C$ which is proportional to the number of $\pi$ pulses which can be applied before a scattering error (see Appendix B). This metric accounts for how much light is lost in the filtering process as well as how the remaining frequency components interfere and assumes that the detuning $\Delta$ is chosen to obtain the same Rabi frequency for each approach. A high-level comparison of approaches for converting phase modulation to amplitude modulation is presented in Fig. 1(d), with details in Appendix B.

Rather than filtering out specific spectral components from the phase-modulation spectrum, we consider here an approach to change the relative phases of these spectral components using dispersive optics [27]. We consider in particular a dispersive element which has a nonzero group-delay dispersion (GDD), defined as $G = \partial^2 \Phi/\partial \omega^2$. This element imparts a phase shift to frequency components which is quadratic in their frequency; that is, it produces a modified electric field of the form

$$\Omega(t) = \Omega_0 \sum_{n=-\infty}^{\infty} J_n(\beta) e^{i n \omega t} e^{i \alpha n^2},$$

(6)

where $\alpha = G_{\text{eff}} \omega^2/2$ describes the phase curvature as a function of sideband index. The resulting Raman Rabi frequency depends simply on the phase-modulation depth $\beta$ and the dispersion curvature $\alpha$ according to a Bessel function identity (F5) [28]:

$$\Omega_{\text{eff}} \propto |J_1(2\beta \sin \alpha)|.$$

(7)

The Rabi frequency is optimized when the Bessel function $J_1$ is maximized, which occurs when $2\beta \sin \alpha = 1.84$. However, in practice, the electro-optic phase-modulation depth is limited to $\beta \lesssim \pi$, requiring $\alpha \gtrsim \pi/4$ to achieve reasonable efficiency; this corresponds to an enormous dispersion of $G \gtrsim 8.5 \times 10^8$ fs$^2$. For comparison, dispersion in a typical optical fiber is approximately $4 \times 10^8$ fs$^2$/m [28,29]. Even ultrahigh-dispersion chirped Bragg mirrors (mirrors with gradually varying Bragg layer thickness) offer only up to $1300$ fs$^2$ from a single reflection [30] [see Fig. 1(e) and Appendix C for further discussion].

Recently, new optical elements based on volumetric Bragg gratings have enabled a new level of frequency selectivity and dispersion control [31]. These crystals have a weak modulation in their refractive index over a length scale of approximately 1 cm; chirping of the index modulation wavelength as a function of depth produces highly dispersive properties [31]. We use a chirped volumetric Bragg grating with $G = 4 \times 10^8$ fs$^2$ (OptiGrate, CBG-795-95, apodization of 5 mm on both ends). Reflecting twice from the grating doubles the dispersive effect; this allows us to reach optimal conversion to amplitude modulation with a readily accessible phase-modulation depth $\beta \sim 1.3$ rad. Moreover, the dispersive element does not filter out optical power, but instead produces favorable phase relationships between sidebands, resulting in a high coherence metric [Fig. 1(d)]. Finally, the passive stability of the dispersive element simplifies
FIG. 2. Raman laser system using a chirped Bragg grating. (a) Optical setup. The chirped Bragg grating and the first mirror afterward (in the shaded gray region) are mounted on a single rotation mount. Spectral components separate after the first reflection from the CBG, but recombine after the second reflection. A scanning Fabry-Pérot cavity measures the sideband spectrum and a fast photodetector measures the amplitude modulation. (b) The amplitude modulation (measured as the amplitude of the 6.8-GHz peak of the fast photodetector signal on a spectrum analyzer) depends on both the dispersion of the CBG and the phase-modulation depth (see the text). We observe the expected Bessel function relation and can extract the dispersion coefficient. (c) As we scan the laser frequency across the CBG bandwidth, we see a high total reflectivity of the CBG system, measured as the ratio of the power before and after the PBS in (a), across the approximately 50-GHz bandwidth. The resulting fiber-coupled light should ideally show constant amplitude modulation across the whole bandwidth, but in practice we observe variation with laser frequency due to nonuniform CBG dispersion over its bandwidth. While more uniform CBGs can be used, the current device can be angle tuned to maximize amplitude modulation and is insensitive to frequency drifts which are less than 1 GHz. Independent atomic measurements of Raman Rabi frequency and light shifts at the optimal modulation parameters gives a laser amplitude-modulation efficiency of approximately 50%, consistent with expectations (Table I).

experimental implementation. Ultimately, the CBG serves as an element which passively converts phase modulation to amplitude modulation, so the effective Raman Rabi frequency (phase, amplitude, and frequency) is directly inherited from the microwave source of the phase modulator.

IV. RAMAN LASER SETUP

Our Raman laser system [shown in Fig. 2(a) and Appendix D] is sourced from a tapered amplifier system which outputs up to 1.5 W of fiber-coupled optical power at 795 nm (Toptica TA Pro, free-running at 377.2000 THz). This light is phase modulated by a free-space resonant electro-optic modulator (EOM) (Qubig, PM-Rb). The EOM is driven by a 6.8-GHz microwave source, which consists of a frequency-doubled local oscillator (Stanford Research Systems, SG384) that is IQ modulated by an arbitrary waveform generator (Spectrum Instrumentation, DN2.662-04) to achieve arbitrary frequency, phase, and amplitude control of the phase-modulation signal. The laser is then reflected twice from a CBG to convert phase modulation to amplitude modulation, and the output is gated by an acousto-optic modulator (AOM) and coupled into a single-mode fiber. The phase-modulation depth \( \beta \) is measured by a pickoff onto a scanning Fabry-Pérot cavity, and the amplitude modulation is characterized on a fast photodetector [Fig. 2(a)].

The operational bandwidth of the CBG is 50 GHz; angle tuning of the CBG around the 3° target angle of incidence allows shifting of this bandwidth relative to the laser frequency. While the CBG nominally has a uniform dispersion within its bandwidth, we find that in practice the dispersion oscillates within its finite bandwidth [Fig. 2(c)]; for this reason, it is helpful to have fine control of the incident angle and to monitor the resulting amplitude modulation while tuning the angle.

In order for the entire optical setup to remain aligned while angle tuning the CBG, it is important to design the CBG pathway such that the output spatial mode upon the second reflection is independent of the tuning angle. Additionally, the different spectral components of laser light penetrate different depths within the CBG and therefore spatially separate. To recombine these spatial components and ensure overall angle insensitivity, we use a flat-mirror retroreflector to redirect the spatial components back onto the CBG [Fig. 2(a)]. Furthermore, we mount both the CBG and the pickoff mirror which immediately follows on the same rotation stage (with the center of the CBG at the rotation origin) such that the retroreflection condition is met for all tuning angles. The final retroreflection mirror is aligned once and fixed in place prior to further angle tuning. This configuration ensures a single, stable output spatial mode for the light exiting the CBG system that is independent of the CBG angle and therefore maintains subsequent alignment while the angle is tuned.

After optimizing the CBG angle to maximize amplitude modulation (as measured on the fast photodiode), we experimentally measure the dependence of amplitude modulation on the phase-modulation depth to confirm the expected Bessel function relationship from Eq. (7) and extract the dispersion coefficient [Fig. 2(b)]. Finally, at a fixed modulation depth of \( \beta \approx 1.2 \) rad, we measure the amplitude modulation and total reflectivity of the double-bounce CBG system as we scan the laser frequency across the bandwidth of the CBG to assess sensitivity to frequency drifts of the laser [Fig. 2(c)]; we
find that amplitude modulation is stable near optimal points for laser frequency drifts of less than 1 GHz, and the total reflectivity of the entire CBG system exceeds 80% across the 50-GHz bandwidth.

V. BENCHMARKING THE RAMAN LASER SYSTEM ON A NEUTRAL ATOM ARRAY

We test our high-power Raman laser system on neutral $^{87}$Rb atoms which are loaded within an array of 600 optical tweezers in two dimensions using the platform described in Ref. [32] [Fig. 3(a)]. The optical tweezers, which are arranged in a 100 × 200 μm$^2$ rectangle, are linearly polarized and have a wavelength of 810 nm. In each experimental cycle, atoms are loaded and then imaged on an electron-multiplied CCD camera to detect their positions within the array and their final states are read out by a second image after pushing out atoms in $F = 2$ by cycling photons on the $D2$ transition $F = 2 \rightarrow F' = 3$. During loading and imaging, the tweezers have a trap depth of 14 MHz. During Raman driving, the trap depths are lowered to 5 MHz and an 8.5-G magnetic field is applied [25]. Subsequently, this so-called Carr-Purcell-Meiboom-Gill (CPMG) sequence [33] is robust to pulse miscalibrations across the array [Fig. 3(b), top panel], as well as averaged over the middle four rows [Fig. 3(b), bottom panel]. We attribute the decay of Rabi oscillations primarily to inhomogeneity across the array and small (less than 1%) power fluctuations.

For Raman operation with hyperfine qubits, there is a fundamental tradeoff between Raman Rabi frequency (proportional to $\Omega^2 / 2 \Delta$) and incoherent scattering processes (proportional to $\Omega^2 / 4 \Delta^2$). For a given target Rabi frequency, higher optical power enables working at a larger intermediate detuning, increasing the ratio of Rabi frequency to scattering rate [proportional to the coherence metric tabulated in Fig. 1(d)]. To evaluate this coherence limitation for our high-power system, we apply a (π/2) pulse followed by a train of π pulses [Fig. 3(c)]; this CPMG sequence [33] is robust to pulse miscalibrations that limit our observed Rabi coherence time. By varying the total number of π pulses, we observe a $T_1$-type decay from scattering, with a characteristic 1/e scale of 7852 ± 76 pulses. This decay constant sets a lower bound on our scattering-limited π-pulse fidelity of 0.999 873(1).

We globally drive the qubit array and measure Rabi oscillations across the array with frequency $\Omega_{\text{eff}} = 1.95$ MHz. We analyze Rabi oscillations individually for each row of the array [Fig. 3(b), top panel], as well as averaged over the middle four rows [Fig. 3(b), bottom panel]. We attribute the decay of Rabi oscillations primarily to inhomogeneity across the array and small (less than 1%) power fluctuations.

Having established the high Rabi frequency and large number of possible operations in our system, we now explore its utility in preserving coherence across the array, for practical use in quantum information processing protocols. We first benchmark the hyperfine coherence in our optical tweezers by measuring a Ramsey $T_2^* = 1.17(1)$ ms [Fig. 4(a)], limited by the finite atomic temperature (approximately 20 μK) and small differential light shifts in the tweezers (approximately 4 kHz) [34]. By applying a train of π pulses, we dynamically decouple the atomic qubits from noise sources such as the tweezer differential light shifts and extend the coherence time to $T_2 = 303(13)$ ms, showing second-timescale coherence across hundreds of qubits [Fig. 4(b)]. The π pulses are applied according to the XY16-256 pulse sequence (256
total $\pi$ pulses), which is robust against pulse imperfections for generic initial superposition states [35]. The qubit coherence after the variable-time pulse train is presently limited by residual pulse imperfections, residual dephasing (e.g., fast magnetic field noise or noise on tweezer light shifts), and the approximately 0.5-s $T_1$ time associated with off-resonant scattering from the optical tweezers (see Appendix E). Coherence can be further improved by applying more $\pi$ pulses and by using further-detuned optical tweezers (with trap depth held constant, the tweezer differential light shifts decrease as $1/\Delta$) and the $T_1$ exhibits a favorable $\Delta^3$ scaling [26,34]).

Since state-of-the-art Rydberg-based entangling operations are submicrosecond timescale and Raman-based single-qubit rotations are also submicrosecond timescale, the second-scale quantum coherence will allow for a wide variety of deep quantum circuits with hundreds of qubits. Moreover, together with the demonstrated dynamical decoupling sequences, this system should support approaches for quantum algorithms involving dynamic reconfiguration of atom arrays in submilliseconds timescales to change the connectivity of Rydberg or photonic cavity-mediated interactions while preserving coherence [36–38].

VI. CONCLUSION

While several schemes have been used previously to drive Raman transitions, the dispersive approach offers several advantages. First and foremost, the system is passively stable and faithfully maps the microwave signal which drives the EOM to the resulting amplitude modulation of the laser field. In contrast, other schemes require active stabilization of an interferometer, active locking of the repetition rate of a mode-locked laser [9], or stabilization of the frequency offset between two combs [11]. The dispersive approach is additionally more efficient in its use of optical power compared with other approaches using phase modulators. As compared with mode-locked lasers, the experimental simplicity, stability, and low cost make it an attractive alternative.

This dispersive approach can additionally be used for applications in which stimulated Raman transitions are used to couple the atomic spin to motion, such as for Raman sideband cooling or entangling gates in trapped ion systems, akin to the approach taken with mode-locked lasers [9,39]. Finally, local addressing optics could be used to outcouple the amplitude-modulated laser onto individual atoms in the array. Devices such as spatial light modulators, acousto-optic modulators, and electro-optic modulator arrays can enable fast and parallel control of arbitrary single-qubit rotations in large qubit arrays. These operations can be integrated with multiqubit gates based on Rydberg interactions to realize flexible quantum circuits, potentially enabling fully programmable quantum simulations and scalable quantum information processing [1].

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APPENDIX A: DRIVING HYPERFINE TRANSITIONS WITH MODULATED VECTOR LIGHT SHIFTS

As outlined in the main text, a multifrequency laser field may be used to drive Raman transitions between hyperfine states if it exhibits amplitude modulation at the hyperfine frequency $\omega_{hf}$. In this Appendix we clarify the interpretation of this process through the lens of vector light shifts induced by an off-resonant laser.
We consider an off-resonant laser which couples alkali-metal atoms on the D1 and D2 optical transitions from the \(J = 1/2\) ground state to the \(J' = 1/2\) and \(J' = 3/2\) excited manifolds, respectively. The laser has polarization \(\epsilon\), field amplitude \(\mathcal{E}(t)\), and frequency \(\omega(t)\) which is far off-resonance from the D1 and D2 transitions (of frequency \(\omega_{D1}\) and \(\omega_{D2}\)), relative to the hyperfine structure in the excited states.

A traditional analysis of Raman transitions would consider the frequency components of the laser field, encoded in the time-dependent amplitude and frequency \(\mathcal{E}(t)\) and \(\omega(t)\), and would calculate resonant contributions to hyperfine transitions through pairs of components which are separated by \(\omega_{hi}\). Instead, we will consider the laser field to be slowly varying relative to its large detuning from the excited states, as long as its spectral bandwidth \(\Delta\omega\) is small compared to the detuning from the D1 and D2 transitions.

In this regime, the excited states may be adiabatically eliminated and the resulting Hamiltonian for the ground-state manifold consists of a scalar light shift (which acts as the identity within the hyperfine manifold, and which we will thus ignore) and a vector light shift term [17,18,40]

\[
H_{\text{vec}} = \mu_B g_I B_{\text{vec}} \cdot \hat{J},
\]

(A1)

where \(\mu_B\) is the Bohr magneton, \(g_I\) is the Landé factor for the \(5S_{1/2}\) levels, and the effective magnetic field is given by

\[
B_{\text{vec}} \propto |\mathcal{E}(t)|^2 \left( \frac{1}{\omega_{D2} - \omega(t)} - \frac{1}{\omega_{D1} - \omega(t)} \right) \text{Im}[\epsilon^* \times \epsilon].
\]

(A2)

Each term in these expressions offers useful insights into Raman transitions. First, we note that this effective magnetic field takes the same Hamiltonian form in Eq. (A1) as a real magnetic field acting on the hyperfine qubit manifold. Just as how a real magnetic field can be modulated using microwave radiation to match a qubit resonance, transitions can be similarly driven within the hyperfine manifold by modulation of \(B_{\text{vec}}\) at the hyperfine frequency \(\omega_{hi}\) [41].

Second, maximizing the effective (Raman) coupling between hyperfine states is achieved by maximizing the modulation amplitude of the fictitious magnetic field in Eq. (A2). Laser amplitude modulation, consisting of full-scale modulation of \(|\mathcal{E}(t)|^2\), is the ideal approach, as described in the main text. Laser phase modulation, which can be understood equivalently as modulation of the frequency \(\omega(t)\), plays only a weak role due to the small fractional dependence of \(B_{\text{vec}}\) on the laser frequency.

Third, for large detuning from either the D1 or D2 transition, contributions from both states are significant. Tuning the laser frequency \(\omega(t)\) in between the two transitions offers constructive interference from both pathways; conversely, detuning far from both states relative to their splitting leads to destructive interference [10].

Finally, the effective magnetic field depends on the laser polarization as \(B_{\text{vec}} \propto \text{Im}[\epsilon^* \times \epsilon]\) [14]. While calculating the effects of laser polarization on Raman transitions typically relies on summation over transition matrix elements, the vector light shift interpretation offers a useful alternative. As a first example, for \(\sigma^\pm\)-polarized light propagating along the quantization axis, \(\epsilon_\pm = \hat{x} \pm i\hat{y}\), resulting in \(B_{\text{vec}} \propto \hat{z}\); modulation of \(B_{\text{vec}}\) along the \(\hat{z}\) axis therefore drives \(\pi\)-polarized spin transitions within the ground-state manifold. A second example is linearly polarized light, which cannot drive Raman transitions regardless of propagation axis since \(\epsilon^* \times \epsilon = 0\) for linearly polarized \(\epsilon\). Finally, we consider an example of a circularly polarized laser propagating along \(\hat{x}\), orthogonally to the quantization axis. For such a laser, with polarization \(\epsilon = \hat{x} \pm i\hat{y}\), the effective field is oriented along \(\epsilon^* \times \epsilon \propto \hat{x}\). Just as with a real magnetic field of this orientation, the Raman laser in this configuration couples \(\sigma^\pm\) spin transitions within the ground-state levels. These examples highlight that interpreting Raman transitions as being driven by modulated vector light shifts offers useful additional intuition beyond the standard analysis of two-photon transitions.

APPENDIX B: METHODS FOR CONVERTING PHASE MODULATION TO AMPLITUDE MODULATION

1. Definition of coherence metric

To evaluate the various methods for converting phase modulation to amplitude modulation, we consider two main parameters for each approach: (i) \(T\), the fraction of optical power that is transmitted through the conversion system, and (ii) \(\eta_{\text{AM}}\), the amplitude-modulation efficiency of the resulting light. The amplitude-modulation efficiency is defined for a field with normalized total power split into uniformly spaced sidebands as \(\Omega(t) = \sum_n a_n e^{i\omega n t}\), where \(\sum_n |a_n|^2 = 1\). In this context, the amplitude-modulation efficiency measures how the components interfere to produce amplitude modulation: \(\eta_{\text{AM}} = \sum_n a_n^* a_{n+1}\). This efficiency is bounded above by 1 and characterizes the Raman Rabi frequency for a fixed total amount of optical power in the system.

The Raman Rabi frequencies scales according to \(\Omega_{\text{eff}} \propto T \eta_{\text{AM}} / \Delta\), where \(\Delta\) is the detuning from the intermediate excited state. At the same time, the rate of optical scattering depends on the average optical power on the atoms, according to \(\Gamma_{\text{sc}} \propto T / \Delta^2\).

We combine these two parameters into a single metric which best characterizes the coherence properties of each approach. Specifically, we assume a fixed amount of available optical power and we choose the laser detuning \(\Delta\) such that the resulting Raman Rabi frequency \(\Omega_{\text{eff}}\) is fixed. To achieve this, we set \(\Delta \propto T \eta_{\text{AM}}\). For this setting, the optical scattering scales as \(\Gamma_{\text{sc}} \propto 1 / (T \eta_{\text{AM}})^2\). The ratio of Raman Rabi frequency to scattering rate is therefore given by \(\Omega_{\text{eff}} / \Gamma_{\text{sc}} \propto T \eta_{\text{AM}}^2\), which we define as the coherence metric \(C\). The comparison of approaches is summarized in Table I.

To calculate \(T\) and \(\eta_{\text{AM}}\) for each approach, we begin by considering a phase-modulated laser, with (normalized) field:

\[
\Omega(t) = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{i\omega n t}.
\]

(B1)

The total power is \(\sum_n |J_n(\beta)|^2 = 1\). As we evaluate \(T\) and \(\eta_{\text{AM}}\) by considering the filtering of various sidebands, we find that these values can be expressed as simple combinations of Bessel functions through several Bessel function identities (derived in Appendix F).
TABLE I. Comparison of theoretical limits for several approaches for laser amplitude modulation. Here MZ denotes Mach-Zehnder. For approaches based on conversion of phase modulation to amplitude modulation, an overall coherence metric $C$ can be evaluated which is proportional to the number of Rabi oscillations per scattering time, assuming the same total available laser power before filtering. Other approaches, including mode-locked frequency comb lasers, may be compared based on their amplitude-modulation efficiency $\eta^{\text{AM}}(\text{AME})$, which captures the effective coherence given an equal amount of power on the atoms. Mode-locked lasers achieve near-optimal $\eta$ available or filtered in those methods.

| Method | Transmission $T(\beta)$ | AME $\eta^{\text{AM}}(\beta)$ | Coherence metric $C(\beta) = T(\eta^{\text{AM}})^2$ | Optimal phase modulation $\beta^*$ (rad) | Maximum coherence $C(\beta^*)$ |
|--------|--------------------------|------------------|-----------------------------|-----------------|------------------|
| Filter out carrier | $1 - |J_0(\beta)|^2$ | $\frac{2|J_0(\beta)|^2}{T(\beta)}$ | $1 - |J_0(\beta)|^2$ | 3.574 | 0.144 |
| Filter with MZ interferometer | $\frac{1}{2}[1 + J_0(2\beta)]$ | $\frac{1|J_0(2\beta)|^2}{T(\beta)}$ | $\frac{1|J_0(2\beta)|^2}{2[1 + |J_0(\beta)|^2]}$ | 1.664 | 0.174 |
| MZ modulator (half transmission) | $\frac{1}{2}$ | $|J_1(\beta)|^2/2$ | $|J_1(\beta)|^2/2[1 + |J_0(\beta)|^2]$ | 1.841 | 0.169 |
| MZ modulator (minimum transmission) | $[1 - J_0(\beta)]/2$ | $|J_1(\beta)|^2/2[1 - |J_0(\beta)|^2]$ | $|J_1(\beta)|^2/[1 - |J_0(\beta)|^2]$ | 2.718 | 0.097 |

Dispersive element (coefficient $\alpha$)  $1$ $|J_1(2\beta \sin \alpha)|^2$ $1.336 \ (\alpha = 0.76 \text{ rad})$ $0.339$

Two frequency components $\frac{1}{n}$ $\frac{J_0(\beta)}{\sqrt{n}}$ $\cos(\frac{\pi T}{n})$

$N$ uniform sidebands $\left(\frac{\pi}{n}\right)$ $\frac{\pi T}{n}$

$N$ optimal sidebands $\left(\frac{\pi}{n}\right)$ $\frac{\pi T}{n}$

2. Filter out carrier component

In this approach, the phase-modulation frequency $\omega = \omega_\phi/2$, such that frequency components separated by $2\omega$ contribute to the Raman drive of the qubit. After filtering out the carrier, the resulting optical power is

$$T = 1 - |J_0(\beta)|^2.$$  \hfill (B2)

The amplitude-modulation efficiency is

$$\eta^{\text{AM}} = \left| \left( \sum_{n} J_n(\beta)J_{n+2}(\beta) \right) - J_0(\beta)[J_{-2}(\beta) + J_2(\beta)] \right| / T$$  \hfill (B3)

$$= \frac{2J_0(\beta)J_2(\beta)}{1 - |J_0(\beta)|^2}.$$ \hfill (B4)

The first expression sums up all pairs of frequency components separated with $\Delta n = 2$ and then subtracts the contributions from $n = 0$ with $n = \pm 2$. The sum over all pairs is identically 0, and due to evenness of Bessel functions, $J_{-2}(\beta) = J_2(\beta)$. Complex conjugation in the amplitude-modulation efficiency is ignored since the Bessel functions are real valued.

3. Filter with a Mach-Zehnder interferometer

Here we again consider phase modulation with frequency $\omega = \omega_\phi/2$. Passing the laser through a Mach-Zehnder interferometer with a properly chosen path-length difference between arms can result in filtering of all even-index or all odd-index components in the laser. The optical power after filtering out all odd sidebands (a more favorable configuration) is

$$T = \sum_{n \text{ even}} J_n(\beta)^2 = \frac{1}{2}[1 + J_0(2\beta)]$$  \hfill (B5)

due to a Bessel function identity (F13). The amplitude-modulation efficiency in this configuration is also greatly simplified due to a Bessel function (F20) identity

$$\eta^{\text{AM}} = \frac{1}{T} \sum_{n \text{ even}} J_n(\beta)J_{n+2}(\beta) = \frac{1}{T} \left( \frac{1}{2} J_2(2\beta) \right)$$  \hfill (B6)

$$= \frac{J_2(2\beta)}{1 + J_0(2\beta)}.$$ \hfill (B7)

4. Mach-Zehnder modulation

A Mach-Zehnder modulator is an interferometer in which phase modulation occurs in one arm of the interferometer. If the two pathways are balanced in power, the power transmitted in one output mode is given by the relative phase between the two paths:

$$I(\phi) = \sin^2\left(\frac{\phi}{2}\right) = \frac{1}{2}[1 - \cos(\phi)].$$ \hfill (B8)

To modulate the output intensity at the qubit frequency $\omega_q$, the relative phase can be biased either to the half-transmission point and then modulated at $\omega_q$, according to $\phi = \pi/2 + \beta \sin(\omega_q t)$, or to the minimum transmission point and then modulated at $\omega_q/2$, with $\phi = \beta \sin(\omega_q t/2)$. These approaches result in different electric-field components in the output light, but to analyze the Raman performance, we need only analyze the laser intensity.

We begin with the half-transmission configuration. In this case, plugging $\phi = \pi/2 + \beta \sin(\omega_q t)$ into Eq. (B8), we obtain

$$I(t) = \frac{1}{2}[1 + \sin[\beta \sin(\omega_q t)]]].$$ \hfill (B9)

Using a version of the Jacobi-Anger expansion, the right-hand side can be expanded as

$$I(t) = \frac{1}{2} \left( 1 - i \sum_{n \text{ odd}} J_n(\beta)e^{i\omega_q t} \right).$$ \hfill (B10)
The average optical power is given by the time-independent term

\[ T = \frac{1}{2}. \]  \hfill (B11)

This is as expected, since we modulate symmetrically around the half-transmission point.

The amplitude-modulation efficiency is given by the coefficient of the \( e^{i\omega t} \) term, normalized by \( T \):

\[ \eta_{AM} = \frac{1}{T} \frac{J_1(\beta)}{2} = J_1(\beta). \]  \hfill (B12)

Turning instead to the minimum transmission case, we calculate the time-dependent output intensity by plugging \( \phi = \beta \sin(\omega q t/2) \) into Eq. (B8):

\[ I(t) = \frac{1}{2} \left\{ 1 - \cos \left( \beta \sin \left( \frac{\omega q t}{2} \right) \right) \right\}. \]  \hfill (B13)

Again using the Jacobi-Anger expansion, we obtain

\[ I(t) = \frac{1}{2} \left( 1 - \sum_{n \text{ even}} J_n(\beta) e^{i\omega n t/2} \right). \]  \hfill (B14)

We now read off the average optical power by setting all time-dependent terms to zero:

\[ T = \frac{1}{2} \left( 1 - J_0(\beta) \right). \]  \hfill (B15)

As with the half-transmission case, the amplitude-modulation efficiency is the coefficient of the \( e^{i\omega t} \) term, here corresponding to \( n = 2 \), normalized by \( T \):

\[ \eta_{AM} = \frac{1}{T} \frac{J_2(\beta)}{2} = \frac{J_2(\beta)}{1 - J_0(\beta)}. \]  \hfill (B16)

5. Dispersive elements

After reflecting from a dispersive element with uniform dispersion (group-delay dispersion is independent of frequency), the normalized field is described by

\[ \Omega(t) = \sum_{n = -\infty}^{\infty} J_n(\beta) e^{i\omega n t}. \]  \hfill (B17)

The intensity is then given by

\[ |\Omega(t)|^2 = \sum_{k = -\infty}^{\infty} e^{i2\omega t} \sum_{n = -\infty}^{\infty} J_n(\beta) J_{n+k}(\beta) e^{i\omega n+k^2 - \omega n}. \]  \hfill (B18)

Assuming the phase-modulation frequency is a subharmonic of \( \omega_q \), with \( \omega = \omega_q / k \), then we have the following amplitude-modulation efficiency (of order \( k \)):

\[ \eta_{AM} = \left| \frac{J_k(\beta) e^{i\omega q t}}{J_0(\beta)} \right|. \]  \hfill (B19)

Here we use the Bessel function identity (F5) to simplify

\[ \eta_{AM} = |J_0[2\beta \sin(\alpha k)]|. \]  \hfill (B20)

From this we can immediately evaluate the upper bound on efficiency for any choice of \( \beta \) and dispersive parameter \( \alpha \), because the result is simply bounded by the maximum value of \( J_0(z) \). Moreover, we see that modulating directly at \( \omega = \omega_q \) (taking \( k = 1 \)) is optimal, since \( J_1(\zeta) \) has a larger maximum than any higher-order Bessel function, but we also see that this configuration requires the largest dispersive parameter \( \alpha \) to achieve this maximum, due to the \( \sin(\alpha k) \) coefficient within the Bessel function argument.

APPENDIX C: DISPERSIVE OPTICAL ELEMENTS

The group-delay dispersion of an optical element is defined as

\[ \mathcal{G} = \frac{\partial^2 \phi}{\partial \omega^2}, \]  \hfill (C1)

where \( \phi(\omega) \) is the optical phase shift (in radians) accumulated by a frequency component with angular frequency \( \omega \) after the action of the element. The GDD is typically measured in units of \( \text{fs}^2 \), although many optical elements such as fibers have their dispersive properties described in terms of their group-velocity dispersion (GVD), which is GDD per unit length [typical units are \((\text{ps}/\text{nm})/\text{km})\]

Normal materials have dispersion which acts over a broad wavelength range, which plays an important role in ultrafast optics with broadband lasers, where dispersion results in pulse broadening. However, we are interested here in strong dispersion on the scale of approximately 10 GHz in the near infrared. In particular, as described in the main text, we want optical elements with group-delay dispersion of \( 8 \times 10^8 \text{fs}^2 \) to be able to optimally convert phase modulation to amplitude modulation.

Typical optical fibers at 795 nm have GVD of \(-120 (\text{ps}/\text{nm})/\text{km}, or \( 4 \times 10^8 \text{fs}^2/\text{m} \), with attenuation 4 dB/km. To achieve the target GDD, we would require a 20-km fiber, with a resulting 80-dB laser attenuation. Some photonic crystal fibers have been designed to have significantly larger GVD, but with much higher attenuation.

In the ultrafast optics community, after sending short pulses through a long fiber, they reverse the pulse broadening by reflecting the broadened pulse from a chirped Bragg mirror. The highest available chirped Bragg mirrors offer \( \mathcal{G} \sim 2000 \text{fs}^2 \) per reflection. To achieve our target GDD would require approximately 400 000 reflections from such a mirror.

The volumetric chirped Bragg grating that we use offers the enormous \( \mathcal{G} = 4 \times 10^8 \text{fs}^2 \) from a single pass. After reflecting twice from the CBG, we double the GDD to the target level and conveniently also recombine spatial modes of all spectral components in the laser. One caveat is that the CBG has a narrow bandwidth of approximately 50 GHz, which requires angle tuning to match to the bandwidth of the phase-modulated laser. This could also limit reflectivity at large phase-modulation depth due to high-order sidebands being outside the bandwidth, but for \( \beta \approx \pi \) this does not pose an issue. Another factor is that the CBG does not in fact have uniform GDD over its bandwidth, which further requires angle tuning to position the laser frequency at an optimal point within the CBG bandwidth [Fig. 2(c)].
FIG. 5. Annotated optical setup for the Raman laser system: TA, tapered amplifier; HWP, half waveplate; QWP, quarter waveplate; PBS, polarizing beam splitter; EOM, electro-optic modulator; CBG, chirped Bragg grating; and AOM, acousto-optic modulator.

APPENDIX D: OPTICAL SETUP

An annotated image of the optical setup used in this work is shown in Fig. 5. The Toptica TA Pro laser source at 795 nm outputs up to 1.5 W of fiber-coupled light. Half and quarter waveplates align the polarization to be vertical such that it is reflected by a polarizing beam splitter (PBS) into the EOM. A subsequent half waveplate aligns the polarization to be primarily horizontal such that most of the light propagates through the following PBS (with a small amount deflected up and focused into a scanning Fabry-Pérot cavity).

The light transmitted through the PBS reflects from the CBG at an approximately 3° angle from normal; a pickoff mirror separates the reflection from the incoming beam. Both the CBG and pickoff are mounted on the same rotation stage, but they have a fixed relative orientation such that the light is always reflected from the pickoff at a fixed angle upward. For phase-modulated light, the distinct frequency components penetrate different depths into the CBG and therefore spatially separate; all components reflect from the pickoff mirror at the same angle, however, and are all reflected back onto the CBG by a flat retroreflection mirror. All frequency components pass twice through a quarter waveplate to rotate their polarization such that after recombining on the CBG, they reflect downward from the PBS. At this position, the laser is now amplitude modulated. The total reflectivity measured in Fig. 2(e) is the ratio of the output power (after the final PBS reflection) to the input power (before the first entrance of the PBS).

Finally, the laser is focused through an AOM for power stabilization and fast pulsing. The zeroth-order AOM deflection is aligned into a fiber-coupled fast photodetector for monitoring amplitude modulation. The first-order AOM deflection is coupled into a polarization-maintaining optical fiber and delivered to a separate optical table, where it is outcoupled onto the atoms.

APPENDIX E: IDLE POPULATION DECAY AND ATOM LOSS IN OPTICAL TWEEZERS

While qubit dephasing can be mitigated through dynamical decoupling sequences, the ultimate limit to qubit coherence is set by population decay due to scattering from the optical tweezers as well as the finite atom lifetime in the tweezers. In Fig. 6 we show additional data for these two effects, measuring a qubit state population-decay lifetime of 0.45(1) s and a background atom lifetime which is approximately 10 s.

APPENDIX F: BESSEL FUNCTION IDENTITIES

1. Destructive interference of pure phase modulation

The Bessel function identities that describe destructive interference in Raman driving with a phase-modulated laser can be easily derived from the Jacobi-Anger expansion

$$e^{i\beta \sin \omega t} = \sum_{n=-\infty}^{\infty} J_n(\beta)e^{in\omega t}.$$  \hspace{1cm} (F1)

Taking the magnitude squared of both sides, we find

$$1 = \sum_{m,n} J_m(\beta)J_n(\beta)e^{i(m-n)\omega t}.$$  \hspace{1cm} (F2)
(F7) We now apply the Jacobi-Anger expansion for both terms on the right-hand side. Setting this expression equal to the right-hand side of Eq. (F6), we obtain
\[
\left( \sum_{n=-\infty}^{\infty} n^2 J_n(z)e^{in\theta} \right) \left( \sum_{m=-\infty}^{\infty} m^2 J_m(-z)e^{im\theta} \right) = \sum_{k=-\infty}^{\infty} J_k(2z \sin \phi)e^{ik\phi}. \tag{F8}
\]

Expanding the left-hand side as a sum over indices \( n \) and \( m \), we obtain
\[
\sum_{n,m} i^{n+m} J_n(z)J_m(-z)e^{i(n+m)\phi} e^{i(m-n)\theta} = \sum_{k=-\infty}^{\infty} J_k(2z \sin \phi)e^{ik\theta}. \tag{F9}
\]

We will now rewrite the left-hand side with a change in indexing, using \( n \) and \( k' \equiv m - n \), and regroup terms to pull the \( k' \) sum to be the outer sum:
\[
\sum_{k'=-\infty}^{\infty} e^{ik'\theta} \left[ \sum_{n=-\infty}^{\infty} i^{2n} J_n(z)J_{n+k'}(-z)e^{2in\phi} \right] = \sum_{k=-\infty}^{\infty} \left[ J_k(2z \sin \phi) \right] e^{ik\theta}. \tag{F10}
\]

Recalling that \( J_{n+k'}(-z) = (-1)^{n+k'} J_{n+k'}(z) \) and using that \( i^{2n} = (-1)^n \), we simplify
\[
\sum_{k'=-\infty}^{\infty} e^{ik'\theta} \left[ (-1)^n e^{i\theta} \phi \sum_{n=-\infty}^{\infty} J_n(z)J_{n+k'}(z)e^{2in\phi} \right] = \sum_{k=-\infty}^{\infty} \left[ J_k(2z \sin \phi) \right] e^{ik\theta}. \tag{F11}
\]

On both sides of the equation, we have an outer sum over \( k \) (or \( k' \)), with orthogonal functions \( e^{ik\theta} \). We therefore must require that the coefficients are all equal for corresponding \( k = k' \). Rewriting the equality between coefficients, we obtain
\[
J_k(2z \sin \phi) = (-1)^k e^{i\theta} \phi \sum_{n=-\infty}^{\infty} J_n(z)J_{n+k}(z)e^{2in\phi}. \tag{F12}
\]

2. Quadratic phase shifts

Claim.
\[
J_k(2z \sin \phi) = (-i)^k e^{i\theta} \phi \sum_{n=-\infty}^{\infty} J_n(z)J_{n+k}(z)e^{2in\phi}. \tag{F5}
\]

Proof. We begin using the Jacobi-Anger expansion, treating \( \beta = 2z \sin \phi \) as the modulation depth:
\[
e^{i(2z \sin \phi)(\sin \theta)} = \sum_{n=-\infty}^{\infty} J_n(2z \sin \phi)e^{in\theta}. \tag{F6}
\]

Alternatively, instead of expanding the left-hand side using the Jacobi-Anger expansion, we could also multiply the two sine functions, recalling the trigonometric identity \( \sin(x - y) = \frac{1}{2} [\cos(x - y) - \cos(x + y)] \). Plugging this in, we obtain
\[
e^{i(2z \sin \phi)(\sin \theta)} = (e^{i(2z \cos(\phi - \theta))}e^{-i(2z \cos(\phi + \theta))}). \tag{F7}
\]

3. Even sidebands

We can now use (F12) to prove identities regarding a field with only the even sidebands. We first consider the total power in a beam with only the even-index sidebands.

Claim.
\[
T \equiv \sum_{n \text{ even}} J_n(\beta)^2 = \frac{1}{4} \left[ 1 + J_0(2\beta) \right]. \tag{F13}
\]

Proof. We find that the sum over even sidebands is quite similar to a sum over all sidebands, but with a minus sign on
the odd sidebands. To see this we write
\[
\sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta)^2 = \sum_{n \text{ even}} J_n(\beta)^2 - \sum_{n \text{ odd}} J_n(\beta)^2. \tag{F14}
\]
Recalling that the sum of the power in all sidebands must be unity, we know that
\[
\sum_{n \text{ odd}} J_n(\beta)^2 = 1 - \sum_{n \text{ even}} J_n(\beta)^2. \tag{F15}
\]
Plugging this into Eq. (F14), we have
\[
\sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta)^2 = 1 + 2 \sum_{n \text{ even}} J_n(\beta)^2 
= 1 + 2 T. \tag{F16}
\]
The left-hand side now happens to be in a very similar form to the right-hand side of Eq. (F12). In particular, we now write (F12) with \(k = 0, \phi = \pi/2\), and \(z = \beta\):
\[
J_0(2\beta) = \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta)^2. \tag{F17}
\]
Inserting this result into Eq. (F17), we solve for \(T\):
\[
T = \frac{1}{2} [1 + J_0(2\beta)]. \tag{F19}
\]

**Claim.** Now we can apply a similar technique to prove another identity related to the situation of even sidebands:
\[
\sum_{n \text{ even}} J_n(\beta)J_{n+2}(\beta) = \frac{1}{2} J_2(2\beta). \tag{F20}
\]

**Proof.** We begin by directly applying the quadratic dispersion identity (F12) with \(k = 2, \phi = \pi/2\), and \(z = \beta\):
\[
J_2(2\beta) = \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta)J_{n+2}(\beta). \tag{F21}
\]
Again separating in terms of even and odd terms, we obtain
\[
J_2(2\beta) = \sum_{n \text{ even}} J_n(\beta)J_{n+2}(\beta) - \sum_{n \text{ odd}} J_n(\beta)J_{n+2}(\beta). \tag{F22}
\]
Recalling that the sum over all pairs of sidebands is identically 0, we know that
\[
\sum_{n \text{ odd}} J_n(\beta)J_{n+2}(\beta) = - \sum_{n \text{ even}} J_n(\beta)J_{n+2}(\beta). \tag{F23}
\]
We now plug this result in and find
\[
\sum_{n \text{ even}} J_n(\beta)J_{n+2}(\beta) = \frac{1}{2} J_2(2\beta). \tag{F24}
\]
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