Development of a Macro-Model for Magnetorheological Elastomers based on Microscopic Simulations

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Herein, the parametrization of a macro-model for magneto-rheological elastomers based on microscopic simulations is presented. Within a computational homogenization, the effective response of the composite system is calculated and the data are used for parameter identification. The merit of this strategy is the adjustment of the model independent of any macroscopic sample geometry. With the developed model, the magnetostriective behavior of a macroscopic sample is simulated.

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1 Introduction

Magnetorheological elastomers (MREs) are a class of active composites which consist of a polymer matrix filled with micron-sized magnetizable particles. Thus, the effective stiffness or the shape of a specimen can be controlled with an external magnetic field.

Continuum based strategies regarding the modeling of MREs can be divided into microscopic and macroscopic approaches. In the former, the heterogeneous microstructure is explicitly resolved and it is possible to predict the effective behavior of the composite. However, the disadvantage of microscopic continuum models is the computationally expensive numerical solution. In contrast to that, macroscopic models, which consider the MRE as a homogeneous continuum, enable the efficient simulation of real structures. Since such models are phenomenologically motivated, the model parameters have to be determined which is done by using experimental results so far. Due to the inhomogeneous fields inside the MRE sample [2, 3], this procedure however requires several simplifying assumptions.

In this contribution a macro-model for MREs is identified from data generated by a microscopic model combined with a computational homogenization approach. This strategy enables to identify the parameters independent of any sample geometry.

2 Theoretical Framework

The presented approach is based on a general continuum formulation of the coupled magneto-mechanical boundary value problem [1, 2, 5] which is valid on the micro- as well as the macroscale. To link these two scales, a suitable homogenization scheme is applied [4, 5].

2.1 Microscopic Simulations

For the description of the macroscopic MRE-behavior in the isotropic case, a random microstructure has to be taken into account. In order to ensure a statistical representation, the number of embedded particles is chosen to \( N = 300 \), where representative volume elements (RVEs) with the characteristic values \( \phi = \{10, 15, 20, 25, 30, 35, 40\} \% \) are generated. To estimate mean values as well as confidence intervals for the effective magneto-mechanical response, five different microstructures are considered for each particle-volume fraction.

All RVEs are analyzed by means of three pure mechanical load cases (M1 – M3) and seven coupled magneto-mechanical load cases (MM1 – MM7) in the linear magnetic regime, see [2]. Within the microscopic simulations, the constitutive models presented in [1, 5] are used. All performed simulations are realized for the plane strain case.

2.2 Macroscopic Model

In order to enable a separate description of deviatoric and volumetric parts of the MRE behavior, the deformation gradient \( \bar{F} \) is divided into isochoric \( \bar{F} = J^{-1/3} \bar{J}F \) and volumetric contributions according to the Flory-split, where \( J = \det \bar{F} \) denotes the Jacobi determinant. Due to the assumed isotropy, the MREs effective response can be described in terms of the six invariants

\[
\bar{I}_1 = \text{tr} \bar{C}, \quad \bar{I}_2 = \frac{1}{2}(\text{tr}^2 \bar{C} - \text{tr} \bar{C}^2), \quad I_3 = J^2, \quad I_4 = |\bar{H}|^2, \quad \bar{I}_5 = \bar{H} \cdot \bar{C} \cdot \bar{H} \quad \text{and} \quad \bar{I}_6 = \bar{H} \cdot \bar{C}^{-1} \cdot \bar{H}.
\]  \hspace{1cm} (1)
Therein, $C$ and $H$ denote the right Cauchy-Green deformation tensor and the Lagrangian magnetic field, respectively. For a separate discussion, the amended free energy function $\Omega = \Omega^\text{mech}(C) + \Omega^\text{coup}(C, H) + \Omega^\text{mag}(H) + \Omega^\text{free}(C, H)$ is split into mechanic, coupling, magnetic and free space part, where the latter is the Lagrangian counterpart of the magnetic free field energy and is independent of any material properties. With that, the ansatz

$$\Omega = \frac{1}{2} \left[ C(I_1-3) + K(I_2-1) \right] + \mu_0 \left[ \alpha_1 I_5 + \alpha_2 I_6 + \alpha_3 (I_3-1) I_6 \right] + \frac{\mu_0 \beta}{2} I_4 - \frac{\mu_0}{2} J C^{-1} : (H \otimes H)$$

(2)

is chosen for the macro-model, where $\mu_0$ is the permeability of vacuum.

To ensure a high accuracy of the model, it should describe the effective magnetization $m$ as well as the total and mechanical stress tensors $\sigma^{\text{tot}}$ and $\sigma$. The material parameter sets $\kappa = \{\kappa^\text{mech}, \kappa^\text{coup}, \kappa^\text{mag}\}$ are determined within a stepwise algorithm results in three linear optimization schemes. Finally, to achieve a good approximation of all considered fields, the inherently nonlinear step

$$\{\kappa^\text{coup}, \kappa^\text{mag}\} = \arg \left\{ \min_{\kappa^\text{tot}, \kappa^\text{mag}} \sum_{i=1}^{N} \left( W^\sigma_i \left\| \sigma(F_i, H_i, \kappa) - \sigma_i \right\|^2 + W^m_i \left\| m(F_i, H_i, \kappa^\text{coup}, \kappa^\text{mag}) - m_i \right\|^2 \right) \right\}$$

(3)

is realized, where the parameters determined in the previous steps are used as initial values. $W^\sigma_i$ and $W^m_i$ denote weighting factors for the single error sums which are determined as the inverse of the maximum value of $\sigma_{i,k}$ and $m_{i,k}$ for each load case, respectively.

### 3 Results

The fitting procedure is applied to the homogenized data of the load cases M1–M3 as well as MM1–MM3, where Fig. 1(a) exemplarily depicts $\sigma$ for MM1 with a volume fraction of $\phi = 20\%$. The model deviations of the load cases which where not used within the parameter identification procedure are in a similar range as the fitted ones. Thus, the quality of the developed model could be validated.

Finally a circular MRE sample within a surrounding free space is simulated, where special attention is drawn to the magnetostrictive [3, 4], i.e. the elongation or contraction in an external field $H^{\infty}$. As shown in Fig. 1(c), the induced magnetization within the sample is almost homogeneous, whereas the deformation field is strongly inhomogeneous. In case of a rectangular shaped sample, this effect intensifies [2, 3]. This demonstrates again, that the parameter identification for macroscopic MRE models from experimental results requires several assumptions in the fitting process.

If the underlying particle volume fraction of the MRE is varied, a degressive curve for the overall magnetostrictive strain $\varepsilon = \Delta l/l_0$ is observed if the applied local induction $b$ is set constant, see Fig. 1(b). Altogether, the predicted elongation of the sample in the direction of the external field is in qualitative accordance with experiments for isotropic MREs.

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