Metal-Insulator Transition of the Quasi-One Dimensional Luttinger Liquid Due to the Long-Range Character of the Coulomb Interaction

V. S. Babichenko

RSC "Kurchatov Institute", Moscow 123182, Russia.
e-mail: babichen@kurrm.polym.kiae.su

An instability of the quasi-1D Luttinger liquid associated with the metal-insulator transition is considered. It is shown that the homogeneous metal ground state of this liquid is unstable and the charge density wave arises in the system. The wave vector component along the direction of chains equals $|−→k_{∥}|a<<λ$. The interaction between particles is the usual 3D Coulomb interaction $V_{r,r'} = e^2/|r−r'|$ with the static dielectric constant $\epsilon_0$. So, the effective charge of particles is $e^*=e^2/\epsilon_0$, where $e$ is the bare electron charge. The analysis of the role of the long-range Coulomb interaction character is the main goal of the work.

For the first time the different types of instabilities of a quasi-1D Fermi liquid with the short-range bare interaction in the system of the ordered metallic chains in the parquet approximation have been investigated in the work [3]. The quasi-1D Luttinger liquid with the long-range Coulomb interaction has been investigated in the work [4]. In that work only the forward scattering of particles has been taken into account. The influence of the long-range character of the Coulomb interaction in carbon nanotube which can be considered as consisting of two chains has been analyzed in [5]. In this case the Coulomb interaction between two chains does not change the Luttinger character of the 1D electron liquid. The metal-insulator transition of the electron liquid in a strong magnetic field in the parquet approximation has been analyzed in [6]. Note that the long-range character of the Coulomb interaction can change the situation from the parquet to the situation with only one separated channel [9]. The long-range tail of the Coulomb interaction in the electron liquid in a strong magnetic field has been considered in [7]. In the present work the influence of the long-range character of the Coulomb interaction on the properties of the backward scattering in the quasi-1D electron liquid is analyzed. In addition, the influence of the short-range correlations is taken into account too. If the Coulomb interaction is supposed to have only the short-range character, i.e. the electrons interact only at the same chain, the ground state has the properties of the Luttinger liquid. The influence of the long-range part of the Coulomb interaction, i.e. when the interaction between electrons situated at the different chains is essential, results in the instability of the metallic homogeneous ground state of the Luttinger liquid.

The motion of particles is supposed to take place along the 1D semiconductor chains which form the crystal lattice with the lattice constant $λ$ and represent a quasi-1D doped semiconductor. The thickness of the chains $a$ is supposed to be much smaller than the distance between chains $λ$ ($a<<λ$). The interaction between particles is the usual 3D Coulomb interaction $V_{r,r'} = e^2/|r-r'|$ with the static dielectric constant $\epsilon_0$. So, the effective charge of particles is $e^*=e^2/\epsilon_0$, where $e$ is the bare electron charge. The analysis of the role of the long-range Coulomb interaction character is the main goal of the work.

1. The instability of a quasi-1D electron liquid with respect to the metal-insulator transition with the formation of the charge density wave (CDW) is analyzed in the present work. The materials, which can be considered as candidates for the theory proposed below are quasi-1D organic conductors, for example, (TMTSF)$_2$X in strong magnetic fields [1, 2].

The motion of particles is supposed to take place along the 1D semiconductor chains which form the crystal lattice with the lattice constant $λ$ and represent a quasi-1D doped semiconductor. The thickness of the chains $a$ is supposed to be much smaller than the distance between chains $λ$ ($a<<λ$). The interaction between particles is the usual 3D Coulomb interaction $V_{r,r'} = e^2/|r-r'|$ with the static dielectric constant $\epsilon_0$. So, the effective charge of particles is $e^*=e^2/\epsilon_0$, where $e$ is the bare electron charge.
of the system is assumed to obey the inequalities $\frac{1}{a_B} \ll p_F \ll \frac{1}{\lambda}$ where $a_B = 1/m^e e^2$ is the effective Bohr radius and $m^e$ is the electron effective mass. The first part of this inequality means that the non-dimensional constant $\alpha = e^2/v_F$, which defines the value of the effective interaction, is small $\alpha << 1$ where $v_F$ is the Fermi velocity. Because of the condition $\alpha << 1$ the random phase approximation (RPA) is correct for the calculation of the thermodynamic properties, for example, free energy [11]. The second part of the inequality means that the density of the electron liquid is sufficiently small so that the Coulomb correlations are large and the long-range character of the Coulomb interaction is pronounced. Below, for the simplicity of notations, the effective charge $e^*$ will be denoted by $e$.

The electron liquid is supposed to be spinless. This situation can be realized by the switching-on the strong magnetic field directed along the chains. In this case the electron spins are frozen, but the motion of the electrons along the chains are not perturbed by the magnetic field. Moreover, the jumps between chains are suppressed by the magnetic field.

2. The action of a quasi-1D Fermi liquid with the Coulomb interaction has the form

$$S = S_0 + S_{\text{int}}$$

$$S_0 = \sum_{\vec{R}} \int dz dt \bar{\psi}(t, \vec{r}) \left( i \partial_t + \mu - \vec{p}_z^2/2m \right) \psi(t, \vec{r})$$

$$S_{\text{int}} = -\frac{1}{2} \sum_{\vec{R}, \vec{R}'} \int dt dz dz' \left( \bar{\psi}(t, \vec{r}) \psi(t, \vec{r}') \right) V_{\vec{R}, \vec{R}'} \left( \bar{\psi}(t, \vec{r}) \psi(t, \vec{r}') \right)$$

where $z$ is the direction of chains, $\vec{p}_z = -i \partial_z$, $\vec{r} = (z, \vec{R})$, $\vec{R}$ are the discrete coordinates of a chain in the plane perpendicular to the direction of chains. The field $\psi(t, \vec{r})$ is the Fermi field (Grassmann variables). For simplicity, we suppose that the lattice is the square lattice with the lattice constant $\lambda$. The motion of electrons from one chain to another is neglected.

The correlation properties of the Fermi liquid are connected with the correlations of quasi-particles near the Fermi surface. The fields of these quasi-particles are denoted by $\psi^{(\pm)}$ for the part of the Fermi surface with $p_z = +p_F$ and $\psi^{(-)}$ for $p_z = -p_F$.

The Coulomb interaction can be decoupled by the introduction of the virtual plasmon fields of $\phi$ and $\Phi$ [7]. The plasmon field $\phi$ has the small $z$-component of the momentum transfer $k_z$ and corresponds to the forward scattering of quasi-particles and the plasmon field $\Phi$ has the $z$-component of the momentum transfer $k_z$ close to the value $Q = 2p_F$ and corresponds to the backward scattering. The action of quasi-particles can be represented in the form

$$S_L[\Psi, \phi, \Phi] = S_F[\Psi, \phi, \Phi] + S^{(0)}_{\text{forward}}[\phi] + S^{(0)}_{\text{back}}[\Phi],$$

where

$$S_F = \int_0^{\beta} dt \int dz \sum_{\vec{R}} \left( \bar{\psi}^{(\pm)}, \psi^{(-)} \right) \left( \begin{array}{cc} (G^{(\pm)}_{\phi})^{-1} & -i\Phi \\ -i\Phi^+ & (G^{(-)}_{\phi})^{-1} \end{array} \right) \left( \begin{array}{c} \psi^{(\pm)} \\ \psi^{(-)} \end{array} \right)$$

$$S^{(0)}_{\text{forward}}[\phi] = -\frac{1}{2} \int_0^{\beta} dt \int dk_z d^2 k_\perp \phi(t, -k_z, -\vec{k}_\perp) V^{-1}(k_z, \vec{k}_\perp) \phi(t, k_z, \vec{k}_\perp)$$

$$S^{(0)}_{\text{back}}[\Phi] = -\int_0^{\beta} dt \int dk_z d^2 \vec{k}_\perp \Phi^+(t, k_z, \vec{k}_\perp) U^{-1}(\vec{k}_\perp) \Phi(t, k_z, \vec{k}_\perp)$$

In these expressions the momentum $\vec{k}_\perp$ belongs to the elementary cell of the reciprocal lattice formed by the system of chains, $\vec{k}_\perp$ is perpendicular to the chain direction $z$ and $\beta$ is the inverse temperature. The temperature is supposed
equal to zero, but the Matsubara technique is used. The functions $G^{(\pm)}_\phi$ are the Green functions of quasi-particles near the different parts of the Fermi surface with $p_z = \pm p_F$ in the external plasmon field $\phi$, $\bar{p}_z = -i \partial_z$ and $\bar{R}$ are the discrete coordinates of chains in the plane perpendicular to the chain direction.

$$\left(G^{(\pm)}_\phi\right)^{-1} = -\partial_x \mp v_F \bar{p}_z - i\phi(t,z,\bar{R}), \quad (6)$$

The function $V\left(k_z, \bar{k}_\perp\right)$ is the Fourier transformations of the Coulomb potential with the small momentum transfer $k_z$, and $U\left(\bar{k}_\perp\right)$ is the Coulomb potential with the component of the momentum transfer $k_z$ close to the value of $k_z = Q = 2p_F$

$$V\left(k_z, \bar{k}_\perp\right) = \sum_{\bar{b}} \frac{4\pi e^2}{k_z^2 + (\bar{k}_\perp + \bar{b})^2}; \quad U\left(\bar{k}_\perp\right) = \sum_{\bar{b}} \frac{4\pi e^2}{Q^2 + (\bar{k}_\perp + \bar{b})^2} \quad (7)$$

Here $\bar{b}$ are the vectors of the reciprocal lattice so that $\bar{b} = (2\pi n/\lambda; 2\pi m/\lambda)$ where $n, m$ are the integer numbers, and $\bar{k}_\perp$ belongs to the cell of the reciprocal lattice ($-\pi/\lambda < k_{x,y} < \pi/\lambda$). It is convenient to extract the term with $\bar{b} = 0$ in the sums (7). As a result, the sums (7) can be represented in the form of sums of two items. The first one is the summand with $\bar{b} = 0$ and the second is a sum over $\bar{b} \neq 0$ in (7). Thus $U\left(\bar{k}_\perp\right)$ and $V\left(k_z, \bar{k}_\perp\right)$ are represented in the form of the pseudo-potentials having long-range and short-range parts

$$V\left(k_z, \bar{k}_\perp\right) = \frac{4\pi e^2}{k_z^2 + \bar{k}_\perp^2} + V_{\text{core}}\left(\bar{k}_\perp\right) \quad (8)$$

$$U\left(\bar{k}_\perp\right) = \frac{4\pi e^2}{Q^2 + \bar{k}_\perp^2} + U_{\text{core}}\left(\bar{k}_\perp\right) \quad (9)$$

The functions $V_{\text{core}}\left(\bar{k}_\perp\right)$ and $U_{\text{core}}\left(\bar{k}_\perp\right)$ can be considered as independent of $\bar{k}_\perp$ if $|\bar{k}_\perp| << \Lambda$. In the case of the small thickness of chains $a << \lambda$, which is supposed, the dependence of $V_{\text{core}}$ and $U_{\text{core}}$ on the momentum $\bar{k}_\perp$ is weak for $|\bar{k}_\perp| \sim \Lambda$ too and thus can be neglected

$$V_{\text{core}}\left(\bar{k}_\perp\right) \approx U_{\text{core}}\left(\bar{k}_\perp\right) \approx V_{\text{core}}\left(0\right) = U_{\text{core}}\left(0\right) = \frac{4\pi e^2}{\Lambda^2} \quad (10)$$

The value $\bar{\Lambda}$ in (10) has the form

$$\bar{\Lambda}^2 = (2/\pi)\Lambda^2 \ln^{-1}\left(\frac{\Lambda_\infty}{\bar{\Lambda}}\right),$$

where $\Lambda = \pi/\lambda$, $\Lambda_\infty \sim 1/a$, thus $\Lambda_\infty >> \Lambda$ and $\bar{\Lambda} << \Lambda$. The parameter $\Lambda_\infty$ is the cutoff momentum in the sum over $\bar{b}$ in (7). The truncation of the sum (7) is the necessary operation in the connection with the divergency of this sum at the large momentums $\bar{b}$. The Fermi momentum is supposed to be sufficiently small, so the inequality $Q << \bar{\Lambda} << \Lambda$ obeys. The functions $V\left(k_z, \bar{k}_\perp\right)$ and $U\left(\bar{k}_\perp\right)$ (7) as well as $V^{-1}\left(k_z, \bar{k}_\perp\right)$ and $U^{-1}\left(\bar{k}_\perp\right)$ are the periodic functions of $\bar{k}_\perp$ with the periods $\bar{b}_{x,y} = (2\pi/\lambda; 0); (0; 2\pi/\lambda)$ and $\bar{k}_\perp$ in the expressions (8), (9) belongs to the elementary cell of the reciprocal lattice. The function $U^{-1}\left(\bar{k}_\perp\right)$ can be represented in the form

$$U^{-1}\left(\bar{k}_\perp\right) = U_{\text{core}}^{-1}\left(\bar{k}_\perp\right) \left[ 1 - \frac{4\pi^2 U_{\text{core}}^{-1}\left(\bar{k}_\perp\right)}{\bar{k}_\perp^2 + Q^2 + 4\pi e^2 U_{\text{core}}^{-1}\left(\bar{k}_\perp\right)} \right] \quad (11)$$

The coordinate representation of $U^{-1}\left(\bar{k}_\perp\right)$ can be written as
\[ (U^{-1})_{\vec{R}_1, \vec{R}_2} = U_{core}^{-1} (\vec{R}_1 - \vec{R}_2) - 4\pi e^2 (U_{core}^{-1} (0))^2 C (\vec{R}_1 - \vec{R}_2) \]  

where \( C (\vec{R}) \) is

\[ C (\vec{R}) = \int \frac{d^2k_\perp}{(2\pi)^2} \frac{\exp (i\vec{k}_\perp \vec{R})}{k_\perp^2 + Q^2 + 4\pi e^2 U_{core}^{-1} (k_\perp)} \]  

The region of integration in (13) is the elementary cell of the reciprocal lattice. The value \( 4\pi e^2 U_{core}^{-1} (\vec{k}_\perp) \) satisfies the estimations \( 4\pi e^2 U_{core}^{-1} (\vec{k}_\perp) \approx 4\pi e^2 U_{core}^{-1} (0) = \Lambda^2 \ll \Lambda^2 \). Thus, the momentums \( \vec{k}_\perp \), which give the main contribution to the integral (13) for the function \( C (\vec{R}) \), are of the order of \( \Lambda \ll \Lambda \). For this reason, the function \( U_{core}^{-1} (\vec{k}_\perp) \) in the equality (13) can be taken equal to \( U_{core}^{-1} (0) \). Thus, we obtain

\[ C (\vec{R}) = \begin{cases} \frac{1}{2\pi} K_0 (|\vec{R}| \Lambda) & \text{for } |\vec{R}| \Lambda \gg 1 \\ \frac{1}{2\pi} \ln \left( \frac{\Lambda}{\Lambda Q} \right) & \text{for } |\vec{R}| \Lambda \ll 1 \end{cases} \]  

where \( K_0 (x) \) is the cylinder function of imaginary argument and \( \Lambda Q = \sqrt{\Lambda^2 + Q^2} \). The value of \( |\vec{R}_1 - \vec{R}_2| \) in the equality (12) is supposed to obey the inequality \( |\vec{R}_1 - \vec{R}_2| >> 1/\Lambda \). If \( |\vec{R}_1 - \vec{R}_2| >> 1/\Lambda Q \approx 1/\Lambda \), the function \( K_0 (|\vec{R}_1 - \vec{R}_2| \Lambda Q) \) can be replaced by

\[ K_0 (|\vec{R}_1 - \vec{R}_2| \Lambda Q) \rightarrow \frac{2\pi}{\Lambda^2 + Q^2} \delta (\vec{R}_1 - \vec{R}_2) \]  

In this case the equality (12) can be written as

\[ (U^{-1})_{\vec{R}_1, \vec{R}_2} \rightarrow \frac{1}{4\pi e^2} \left( Q^2 - \nabla^2 \right) \delta (\vec{R}_1 - \vec{R}_2) \]  

The expression (15) is correct in the case of the slow change of the fields \( \Phi \) at the scale of the order of the distance between neighboring chains. When the characteristic distances \( |\vec{R}_1 - \vec{R}_2| \) satisfy the inequality \( 1/\Lambda \ll |\vec{R}_1 - \vec{R}_2| \ll 1/\Lambda \), the non-local character of the function \( C (|\vec{R}_1 - \vec{R}_2|) \) in (12) is essential. Using the equality (12), the expression for \( (U^{-1})_{\vec{R}_1, \vec{R}_2} \) can be represented in the form

\[ (U^{-1})_{\vec{R}_1, \vec{R}_2} = \frac{\Lambda^2}{4\pi e^2} \left( \frac{\Lambda^2}{\pi} \delta (\vec{R}_1, \vec{R}_2) - \Lambda^2 C (\delta \vec{R}) \right) \]  

The general property of \( (U^{-1})_{\vec{R}_1, \vec{R}_2} \) is the negative sign of \( (U^{-1})_{\vec{R}_1, \vec{R}_2} \) for \( \vec{R}_1 \neq \vec{R}_2 \) under the condition \( Q \Lambda \ll 1 \).

3. Green functions \( G_{\phi}^{(\pm)} \) can be calculated exactly [8]

\[ G_{\phi}^{(\pm)} (x, y, \vec{R}) = \exp \left( \theta^{(\pm)} (x) - \theta^{(\pm)} (y) \right) G_0^{(\pm)} (x - y, \vec{R}), \]  

where \( x = (t, z) \) and the Fourier transformations of \( \theta^{(\pm)} (x) \) are connected with the plasmon field \( \phi \) by the equation

\[ \theta^{(\pm)} (k, \omega) = -i \frac{\omega \pm v_F k}{\omega^2 + (v_F k)^2} \phi (k, \omega) \]  

In the Eq. (17) the functions \( G_0^{(\pm)} (x - y, \vec{R}) \) are the Green functions of the non-interacting quasi-particles located at the chain \( \vec{R} \).
\[
G_0^{(\pm)} \left( \omega, p_z, \mathbf{R} \right) = \frac{1}{i \omega \mp v_F (p_z \mp p_F)}
\]

Our goal is the calculation of the effective action for the plasmon field \( \Phi \). To obtain the effective action we consider the generation functional \( Z \)

\[
Z = \int D\psi D\bar{\psi} D\phi D\phi^+ D\Phi \exp \left\{ S_L + S_{\text{sources}} \right\},
\]

where \( S_{\text{sources}} \) depends on the sources \( J, \mathcal{J} \) and has the form

\[
S_{\text{sources}} = i \sum_{\mathbf{R}} \int d^2 x \left[ \Phi^+ (x, \mathbf{R}) J (x, \mathbf{R}) + \mathcal{J} (x, \mathbf{R}) \Phi (x, \mathbf{R}) \right]
\]

The integration over the Fermi fields in the expression for the generation potential Eq. (19) with the use of the Eq. (2) gives

\[
Z = \int D\psi D\bar{\psi} D\phi D\phi^+ D\Phi \exp \left\{ S_{\text{Pl}} + S_{\text{sources}} \right\},
\]

where

\[
S_{\text{Pl}} = S_{\text{Det}} [\phi, \Phi] + S^{(0)}_{\text{forward}} [\phi] + S^{(0)}_{\text{back}} [\Phi] + S_{\text{sources}}
\]

\[
S_{\text{Det}} [\phi, \Phi] = \text{SpLn} \left[ \left( \frac{G_0^{(+)} \left( G_0^{(-)} \right)^{-1}}{-i \Phi^+ \left( G_0^{(-)} \right)^{-1}} \right) \right]
\]

Note that for \( \Phi = 0 \) the part of the action \( S_{\text{Det}} [\phi, \Phi] \) can be calculated exactly [8]. As a result, the action \( S_{\text{Pl}} \) can be transformed to the form

\[
S_{\text{Pl}} = S_{\text{Det}} [\Delta] + S_{\text{forward}} [\theta] + S^{\text{int}} [\theta, \Delta] + S_{\text{sources}},
\]

where

\[
S_{\text{Det}} [\Delta] = \text{SpLn} \left[ \left( \frac{G_0^{(+)} \left( G_0^{(-)} \right)^{-1}}{-i \Delta^+ \left( G_0^{(-)} \right)^{-1}} \right) \right]
\]

\[
S_{\text{forward}} [\theta] = - \int d\omega dk_z d^2 k_\perp \left[ \theta (\omega, -k_z, -k_\perp) D_0^{-1} (\omega, k_z, k_\perp) \theta (\omega, k_z, k_\perp) \right]
\]

\[
S^{\text{int}} [\theta, \Delta] = - \int d^2 x \sum_{\mathbf{R}, \mathbf{R}'} e^{-\theta (x, \mathbf{R}) \Delta^+ (x, \mathbf{R}) U^{-1} \left( \mathbf{R}, \mathbf{R}' \right) \Delta (x, \mathbf{R}')} e^{\theta (x, \mathbf{R}')} \]

In these expressions the new fields \( \Delta, \Delta^+ \) are introduced instead of the fields \( \Phi, \Phi^+ \). They obey the equalities

\[
\Delta (x, \mathbf{R}) = \Phi (x, \mathbf{R}) e^{-\theta (x, \mathbf{R})} \quad \Delta^+ (x, \mathbf{R}) = \Phi^+ (x, \mathbf{R}) e^{\theta (x, \mathbf{R})}
\]

The field \( \theta \) is connected with the fields \( \theta^{(\pm)} \) by the following way

\[
\theta (k, \omega, \mathbf{R}) = \theta^{(+)} (k, \omega, \mathbf{R}) - \theta^{(-)} (k, \omega, \mathbf{R}) = -2 i \frac{v_F k}{\omega^2 + (v_F k)^2} \phi (k, \omega, \mathbf{R})
\]
The inverse propagator of the $\theta$-field $D_{\theta}^{-1} \left( \omega, k_z, \vec{k}_\perp \right)$ has the form

$$
D_{\theta}^{-1} \left( \omega, k_z, \vec{k}_\perp \right) = \frac{\left( \omega^2 + (v_F k_z)^2 \right)^2}{8 (v_F k_z)^4} \left[ V^{-1} \left( k_z, \vec{k}_\perp \right) - \Pi_0^{(f)} (\omega, k_z) \right]
$$

(29)

where $\Pi_0^{(f)} (\omega, k_z, \vec{k}_\perp)$ is the zero polarization operator with the small momentum transfer $k_z$ ($k_z << Q$).

Note that the presence of the term $\Pi_0^{(f)} (\omega, k_z, \vec{k}_\perp)$ leads to the divergency of the functional integral for the large values of $\vec{k}_\perp$ and, for this reason, this means the necessity of the consideration of the large momentum $\vec{k}_\perp$ contribution or the lattice formulation of the plasmon action (23). The lattice formulation will be given below in part 5.
Thus the effective action for the plasmon fields can be represented as

\[
S_{Pl}^{(\text{slow})} [\Delta, \theta] = S_{Det} [\Delta] - \int \theta D_0^{-1} \theta \\
- \frac{\lambda^2}{4\pi e^2} \sum \int dtdz \left[ \Delta^+ \left( Q^2 - \nabla^2_{\perp} - \left( \nabla_{\perp} \theta \right)^2 \right) \Delta \right] \\
+ \frac{\lambda^2}{4\pi e^2} \sum \int dtdz \left[ (\Delta^+ \left( \nabla_{\perp} \Delta \right) - \Delta \left( \nabla_{\perp} \Delta^+ \right)) \left( \nabla_{\perp} \theta \right) \right]
\]

Here \( S_{Det} [\Delta] \) is determined by the equality (24). It is convenient to introduce the amplitude and the phase of the field \( \Delta \) in the following way

\[
\Delta = \rho e^{i\chi} \\
\Delta^+ = \rho e^{-i\chi}
\]

Thus we obtain

\[
S_{\text{slow}} = SpLn \left[ \hat{G}^{-1} (\Delta) \right] - \sum \int \theta D_0^{-1} \theta - \frac{\lambda^2}{4\pi e^2} \sum \int \left( Q^2 \rho^2 + \left( \nabla_{\perp} \rho \right)^2 - \rho^2 \left( \nabla_{\perp} (\theta + i\chi) \right)^2 \right)
\]

where \( D_0^{-1} \) is determined by the equality (29) and \( S_{Det} [\Delta] = SpLn \left[ \hat{G}^{-1} (\Delta) \right] \) (Eq. (24)).

We introduce a new variable \( \eta \) instead of \( \theta \)

\[
\eta = \theta + i\chi
\]

After that we have

\[
S_{\text{slow}} = SpLn \left[ \hat{G}^{-1} (\Delta) \right] - \sum \int \eta \left( D_0^{-1} - \frac{1}{4\pi e^2} \nabla_{\perp} \rho^2 \nabla_{\perp} \right) \eta + 2i \sum \int \eta D_0^{-1} \chi
\]

\[
+ \sum \int \chi \left( D_0^{-1} \right) \chi - \frac{\lambda^2}{4\pi e^2} \sum \int \left( Q^2 \rho^2 + \left( \nabla_{\perp} \rho \right)^2 \right)
\]

where the operator \( \nabla_{\perp} \) acts on the fields which are situated at the right side of this operator and \( \nabla_{\perp} \) acts on the fields which are situated at the left side of this operator.

The integration over \( \eta \) gives

\[
S_{\text{slow}} [\Delta] = SpLn \left[ \hat{G}^{-1} (\Delta) \right] - \frac{1}{2} SpLn \left[ D^{-1} (\rho) \right] - \frac{\lambda^2}{4\pi e^2} \sum \int \left( Q^2 \rho^2 + \left( \nabla_{\perp} \rho \right)^2 \right)
\]

where

\[
D^{-1} = D_0^{-1} - \frac{1}{4\pi e^2} \nabla_{\perp} \rho^2 \nabla_{\perp}
\]

The last term in (40) in the approximation of the small value of \( \rho^2 \) can be represented in the form

\[
\int \chi D_0^{-1} (D - D_0) D_0^{-1} \chi = \int \chi D_0^{-1} \frac{D_0^{-1} - D_0^{-1}}{D_0^{-1} - D_0} D_0^{-1} \chi = \frac{1}{4\pi e^2} \int \rho^2 \left( \nabla_{\perp} \chi \right)^2
\]

After that we obtain the following expression for the effective plasmon action
The statistical sum \( Z \) has the maximum value for \( \vec{K}_\perp = (\pi/\lambda, \pi/\lambda) \). Thus the ground state of the system is inhomogeneous with a charge-density wave which has the wave vector along the direction of chains equal to \( 2 \rho_F \) and in the perpendicular direction equal to \( \vec{K}_\perp = (\pi/\lambda, \pi/\lambda) \).

The fluctuations \( \delta \rho \) of the field \( \rho \) can be taken into account by the following representation of the field \( \rho \)

\[
\rho = i \rho_0 + \delta \rho
\]

Note that in the approximation of the slow variation of the fields \( \rho \) and \( \chi \) in the time \( t \) and \( z \)-coordinate the calculation of the first two terms in (42) gives

\[
S_{\text{slow}}[\Delta] = S_{\text{pLn}}[\tilde{G}^{-1}(\Delta)] - \frac{1}{2} S_{\text{pLn}}[D^{-1}(\rho)] - \frac{\chi^2}{4\pi e^2} \sum_{\vec{R}} \int dt dz \left( Q^2 \rho^2 + \left( \nabla_\perp \rho \right)^2 \right)
\]

\[
- \frac{\chi^2}{4\pi e^2} \sum_{\vec{R}} \int dt dz \left( \rho^2 \left( \nabla_\perp \chi \right)^2 \right)
\]

There is nonzero saddle-point field for this action. The saddle-point equations in the case of the constant \( \rho \) as a function of \( t, z \) and \( \vec{R} \) coordinates and for the field \( \chi \) have the form

\[
- \frac{1}{v_F} \int \frac{d\omega d\xi d^2k_\perp}{(2\pi)^4} \frac{d}{\omega^2 + \xi^2 - \rho^2} + \frac{1}{2v_F} \int \frac{d\omega d\xi d^2k_\perp}{(2\pi)^4} \frac{V^{-1}}{D_0^{-1} - \rho^2 V^{-1}} - \frac{Q^2}{4\pi e^2} - \frac{\left( \nabla_\perp \chi \right)^2}{4\pi e^2} = 0
\]

\[
\nabla_\perp^2 \chi = 0
\]

The possible solution of these saddle-point equations can be represented as

\[
\rho = i \rho_0
\]

where \( \rho_0 \) is the constant real number field and the solution for the phase \( \chi \) is

\[
\chi = \vec{K}_\perp \vec{R}
\]

Here \( \vec{K}_\perp \) is an arbitrary vector lying in the cell of the reciprocal lattice. The value of \( \rho_0 \) has the form

\[
\rho_0 = \epsilon_F \exp \left( - \frac{\chi^2 (Q^2 + \vec{K}_\perp^2)}{2\alpha} \right)
\]

5. In this part of the paper the dependence of \( V_{\text{core}}(\vec{k}_\perp) \) and \( U_{\text{core}}(\vec{k}_\perp) \) on \( \vec{k}_\perp \) are taken into account. This is essentially for involving of the short-range fluctuations. If there is an instability of the system in the plasmon channel with the momentum transfer \( Q = 2\rho_F \), the plasmon effective action, obtained after the integration over the field \( \theta \) and dependent on the field \( \Delta \) alone, tends to zero near the instability point. Thus, near the instability point the exponent in the expression (20) can be expanded into a series in action \( S_{\text{Det}}[\Delta] + S_{\text{int}}[\theta, \Delta] \), and after that the integral over the field \( \theta \) can be calculated due to the Gaussian form of this integral. In this case the contribution of \( S_{\text{int}}[\theta, \Delta] \) can be represented in the form

\[
< S_{\text{int}}[\theta, \Delta] >_\theta = - \int d^2x \sum_{\vec{R}, \vec{R}'} \Delta^+(x, \vec{R}) U^{-1}(\vec{R}, \vec{R}') \Delta(x, \vec{R}') C_\theta(\delta \vec{R})
\]
Thus, the part of the effective action \( S_{\text{int}}(\theta, \Delta) >_g \) leads to the renormalization of \( U^{-1}_e(\vec{R}, \vec{R}') \) in (23) and the renormalized quantity \( U^{-1}_{\text{eff}}(\vec{R}, \vec{R}') \) has the form

\[
U^{-1}_{\text{eff}}(\delta \vec{R}) = U^{-1}(\delta \vec{R}) C_{\theta}(\delta \vec{R})
\]

Thus

\[
U^{-1}_{\text{eff}}(\delta \vec{R}) = \frac{1}{4\pi e^2 \pi^2} \bar{\Lambda}^2 \Lambda^2 \delta \vec{R} \cdot \vec{R}' - \frac{\bar{\Lambda}^4}{4\pi e^2 (2\pi)} K_0(\delta \vec{R} | \bar{\Lambda}_Q) \exp\left[f(\delta \vec{R})\right]
\]

The calculation of the Fourier component of \( U^{-1}_{\text{eff}} \) gives

\[
U^{-1}_{\text{eff}}(\vec{k}_\perp) = \frac{1}{4\pi e^2 \bar{\Lambda}^2} \left[ 1 - A\left(\vec{k}_\perp, \xi\right) \left(\frac{\Lambda}{\bar{\Lambda}_Q}\right)^{\gamma\alpha} \left(\frac{\bar{\Lambda}}{\bar{\Lambda}_Q}\right)^2 \right]
\]

where

\[
A\left(\vec{k}_\perp, \xi\right) = \int_0^\Lambda \frac{dJ_0}{\Lambda} \left(\frac{x}{\bar{\Lambda}_Q}\right) K_0(x) x^{\gamma\alpha + 1}
\]

The momentum \( \bar{\Lambda}_Q = \bar{\Lambda}_Q = \sqrt{\bar{\Lambda}^2 + Q^2} \). Assuming \( \Lambda/\bar{\Lambda}_Q \ll 1 \), the above limit of the integral (55) can be replaced by the infinity. Note that \( U^{-1}_{\text{eff}} \) decreases with the increase of the value of \( |\vec{k}_\perp| \). For the small \( |\vec{k}_\perp| \), \( \bar{\Lambda}_Q/\Lambda_Q \ll 1 \).
The definition of \( A \left( \vec{k}_\perp, \xi \right) \) (55) gives \( A \left( \vec{k}_\perp = 0, \xi = 0 \right) = 1 \). Thus in the approximation \( \alpha \xi \ll 1 \) and \( | \vec{k}_\perp | \ll 1/\Lambda \) we can put \( A \left( \vec{k}_\perp, \xi \right) \) equal to unity \( A \left( \vec{k}_\perp, \xi \right) = 1 \) in (54).

The effective action of the plasmon field \( \Delta \) can be represented in the form

\[
S_{eff} = SpLn \left[ \hat{G}^{-1} (\Delta) \right] + < S^{int} [\theta, \Delta] >_\theta
\]

(56)

\[
< S^{int} [\theta, \Delta] >_\theta = - \int d^2x \int \frac{d^2k_\perp}{(2\pi)^2} \Delta^+ \left( x, \vec{k}_\perp \right) U_{eff}^{-1} \left( \vec{k}_\perp \right) \Delta \left( x, \vec{k}_\perp \right)
\]

Expanding \( SpLn \left[ \hat{G}^{-1} (\Delta) \right] \) in the effective action \( S_{eff} \) (56) in field \( \Delta \), we obtain the quadratic action for the \( \Delta \) field and the inverse propagator of this field has the form

\[
\Gamma^{-1} \left( \vec{k}_\perp, \xi \right) = -\Pi_Q + U_{eff}^{-1} \left( \vec{k}_\perp, \xi \right)
\]

(57)

Here \( \Pi_Q = (-\kappa^2/8\pi e^2) \xi \) where \( \kappa^2 = 4\alpha \Lambda^2/\pi^2 \) and \( \xi \) is the logarithmic variable \( \xi = \ln (\epsilon_F/|\omega|) \).

Now we will consider the pole of the propagator \( \Gamma \) when \( \vec{k}_\perp = 0 \). Note that the situation with \( \vec{k}_\perp \neq 0 \) is analogous to that which has been considered in the part 4. For \( \vec{k}_\perp = 0 \) the equation determining the pole \( \xi_0 = \ln (\epsilon_F/|\omega_0|) \) of \( \Gamma \left( \vec{k}_\perp = 0, \xi \right) \) takes the form

\[
-4\pi^2\Pi_Q + \Lambda^2 \left[ 1 - \left( \frac{\Lambda_\infty}{\Lambda_Q} \right) \frac{\gamma_0 \xi}{\Lambda} \right]^2 = 0
\]

(58)

Using the expression for \( \Pi_Q \) and \( \Lambda^2 = (2/\pi)\Lambda^2 \ln^{-1} \left( \frac{\Lambda^2}{\Lambda} \right) \) the Eq. (58) can be represented in the form

\[
\frac{1}{2} \gamma_0 \xi \ln \left( \frac{\Lambda_\infty}{\Lambda} \right) + \left[ 1 - \left( \frac{\Lambda}{\Lambda_Q} \right) \right] \exp \left( \gamma_0 \xi \ln \left( \frac{\Lambda_\infty}{\Lambda_Q} \right) \right) = 0
\]

(59)

It can readily be seen that the right-hand side of the equation (58) is positive for \( \xi = 0 \). However, as \( \xi \) increases, the right-hand side of (58) becomes negative. Thus, the solution of this equation exists and for the supposition \( Q << \Lambda \) has the following value within the logarithmic accuracy

\[
|\omega_0| \sim \epsilon_F \exp \left( \frac{-2Q^2}{\Lambda^2 \gamma_0 \ln \left( \frac{\Lambda_\infty}{\Lambda} \right)} \right) = \epsilon_F \exp \left( \frac{-\Lambda^2 Q^2}{2\alpha} \right)
\]

(60)

Note, that the Eq. (60) is correct when \( \frac{\Lambda^2 Q^2}{2\alpha} \sim Q^2/\kappa^2 << 1 \). The solution (60) coincides with the solution (46) for \( \vec{k}_\perp = 0 \). Here \( \omega_0 \) corresponds to the pole of \( \Gamma \left( \vec{k}_\perp = 0, \xi \right) \). The existence of this pole means the instability of the system. As a result of this instability there is a CDW in the ground state of the system. The z-component of the wave vector of this CDW equals \( 2\rho_F \). As it has been discussed in part 4 the wave vector of this CDW has the component perpendicular to the direction of the chains and equal to \( \vec{k}_\perp = (\pi/\lambda, \pi/\lambda) \). The existence of the CDW results in the existence of the dielectric gap at the Fermi surface. The value of the gap is equal to \( \Delta_g = \rho_0 \), where \( \rho_0 \) is defined by (46).

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