Assessment of Quartz Grain Growth and the Application of the Wattmeter to Predict Quartz Recrystallized Grain Sizes

Leif Tokle¹² and Greg Hirth²

¹Structural Geology and Tectonics Group, Geological Institute, Department of Earth Sciences, ETH Zurich, Zurich, Switzerland, ²Department of Earth, Environmental and Planetary Sciences, Brown University, Providence, RI, USA

Abstract Relationships between the recrystallized grain size and stress are investigated for experimentally deformed water-added quartz aggregates. For stresses ≥100 MPa there is a variation in the measured recrystallized grain size for a given stress. This variation correlates with a change in the c-axis fabric in general shear experiments, where samples with larger recrystallized grain sizes for a given stress have dominantly prism <a> c-axis fabrics and samples with smaller recrystallized grain sizes for a given stress have dominantly basal <a> c-axis fabrics. The dislocation creep flow law also changes at conditions where these two c-axis fabrics form (Tokle et al., 2019, https://doi.org/10.1016/j.epsl.2018.10.017). Using the wattmeter model (Austin & Evans, 2007, https://doi.org/10.1130/G23244A.1), different piezometric relationships are quantified for samples that develop prism <a> and basal <a> c-axis fabrics, respectively. The wattmeter model is sensitive to grain growth kinetics; a new grain growth law for quartz is formulated based on reanalysis of microstructures in samples from previous work. The activation enthalpies and water fugacity exponents for our grain growth law and dislocation creep flow laws are the same within error, suggesting the recrystallized grain size versus stress relationships are nearly independent of temperature and water fugacity, consistent with laboratory observations. The wattmeters successfully predict the recrystallized grain size versus stress relationships of all quartzite samples from experiments with added water. These results support the use and extrapolation of the wattmeter model for both experimental and geologic conditions to investigate the stress state and grain size evolution of quartz rich rocks.

Plain Language Summary Stress is a fundamental rheological parameter in understanding how rocks deform; however, stress is hard to quantify in rocks that flow rather than fracture at geologic conditions. The relationship between the recrystallized grain size and stress is established through rock deformation experiments where both the stress and grain size can be measured. Therefore, the stress can be estimated in naturally deformed rocks by measuring the recrystallized grain size and applying the experimentally calibrated relationship. Here we show new evidence for different recrystallized grain size versus stress relationships in quartz, and that these different relationships correlate with the crystallographic orientations of grains in samples deformed in the shear geometry. To quantify these new observations, we use the wattmeter model, which models the recrystallized grain size of a deforming aggregate based on the balance between the rates of grain growth and grain size reduction, to predict the quartz recrystallized grain size. This model is sensitive to grain growth kinetics; therefore, we formulated a new grain growth law for quartz. The calibrated wattmeters predict the recrystallized grain size versus stress relationships in experimentally deformed wet quartzite samples. The wattmeter model can therefore be used to estimate stress magnitudes in naturally deformed quartz-rich rocks.

1. Introduction

Due to the abundance of quartz in the continental crust, quartz rheology is fundamental to our understanding of many geodynamic processes. Grain size is an important parameter in the rheology of many materials, including quartz. In addition, a practical technique to infer paleo-stress magnitudes in the continental crust is the piezometric relationship between recrystallized grain size and stress, expressed as:

\[ d = B\sigma^{-k} \]  (1)
where \( d \) is recrystallized grain size, \( \sigma \) is differential stress, and \( B \) and \( k \) are constants (Stipp & Tullis, 2003; Twiss, 1977). The most widely applied recrystallized grain size piezometer for quartz was empirically developed based on recrystallized grain size measurements from 12 deformation experiments conducted on “as-is” quartzite (i.e., without pre-drying or adding water) in axial compression at temperatures between 800°C and 1,100°C, where \( B \) and \( k \) are 3.631 and 1.26, respectively (Stipp & Tullis, 2003); we refer to this relationship as the S&T piezometer. Stipp and Tullis (2003) quantified a second piezometric relationship at higher stresses (>350 MPa) and smaller grain sizes (\( \lesssim 2 \mu m \)) based on axial compression experiments on as-is novaculite conducted by Bishop (1996). Furthermore, Stipp et al. (2006) conducted axial compression experiments on water-added and pre-dried quartz samples, showing that over the range of measured stresses and grain sizes there was no difference in the piezometric relationship for water-added, as-is, or pre-dried quartz samples. Kidder et al. (2016) conducted deformation experiments on water-added quartzite aggregates in axial compression and showed similar results to those produced by Stipp and Tullis (2003) and Bishop (1996).

Analyses of samples deformed in general shear suggest that the recrystallized grain size piezometer for quartz is not as simple as presented in Stipp and Tullis (2003) and Twiss (1977). Heilbronner and Kilian (2017) reanalyzed as-is and water-added samples from Heilbronner and Tullis (2002, 2006) deformed in the general shear geometry and demonstrated that the general shear samples plot at larger grain sizes than the axial compression experiments for a given equivalent stress (Figure 1a). Richter et al. (2018) deformed water-added quartz aggregates in the general shear geometry at a wider range of experimental conditions than those analyzed by Heilbronner and Kilian (2017) and found that some samples fit the S&T piezometer while others fit the relationship determined by Heilbronner and Kilian (2017) (Figure 1a). Soleymani et al. (2020) deformed water-added silica-gel origin samples in the general shear geometry and found that all of their samples deformed at a constant temperature fit the S&T piezometer.

The details of the recrystallized grain size versus stress relationship have important implications for our understanding of stress in the continental crust. For example, one could fit a single linear regression through the compiled data set in Figure 1, suggesting that the recrystallized grain size is not as strongly dependent on stress as indicated by the S&T piezometer. In contrast, the variability in recrystallized grain size versus stress relationship can be reconciled by applying the wattmeter (Austin & Evans, 2007) and accounting for differences in quartzite flow laws at the experimental conditions. We analyze recrystallized grain sizes and mechanical data from five studies on experimentally deformed wet quartzites (Figures 1c and 1d). Two wattmeters for quartz are developed, utilizing the two flow laws determined by Tokle et al. (2019). As part of this analysis, a new grain growth law for quartz aggregates is formulated, which is a fundamental component of the wattmeter model. Using the newly formulated wattmeters, stress estimates are evaluated at crustal conditions based on published grain size and c-axis fabric data.

### 2. Experimental Data

Recrystallized grain sizes and mechanical data from five different experimental studies are analyzed; two employed the axial compression deformation geometry (Kidder et al., 2016; Stipp et al., 2006) and three employed the general shear geometry (Heilbronner & Kilian, 2017; Richter et al., 2018; Soleymani et al., 2020). In all five studies, experiments were conducted in a solid-medium Griggs apparatus using either an all-salt or molten-salt assembly. In this analysis, the application of the wattmeter using flow laws and grain growth parameters is investigated for water-added (~0.2 wt.% H\(_2\)O) samples. All of the water-added deformation experiments were conducted at temperatures between 700°C and 1,000°C and at confining pressures ranging from 1.06 to 1.58 GPa. In Section 5.3, the wattmeter model parameterized for water-added samples is compared to as-is and pre-dried experiments conducted at temperatures of 800°C–1,100°C and a pressure of 1.5 GPa (Stipp & Tullis, 2003; Stipp et al., 2006).

For general shear experiments, shear stresses and shear strain rates were converted to equivalent stresses \( \sigma_{eq} = 2 \tau \) were \( \tau \) is shear stress) and equivalent strain rates \( \dot{\varepsilon}_{eq} = \dot{\varepsilon} / \sqrt{3} \), where \( \dot{\varepsilon} \) is shear strain); while
the strain in the general shear experiments is dominated by the imposed shear component, the principal stresses are defined by the imposed axial compression configuration of the experiments (with $\sigma_2 = \sigma_3$) leading to $\sigma_{eq} = 2\tau$ (cf., Paterson & Olgaard, 2000). The equivalent stress conversion is different than the one used by Tokle et al. (2019), where $\sigma_{eq} = \sqrt{3}\tau$; therefore, we update the Tokle et al. (2019) flow law parameters using the $2\tau$ equivalent stress conversion (Table 1); this change has a minor influence on the flow law parameters, and does not impact the ideas or interpretations presented in Tokle et al. (2019). The high temperature/low stress flow laws are unaffected by the change in stress conversion. The low temperature/high stress flow laws have been recalculated. For the extrapolated flow law (where both experimental data and geologic observations were used to constrain the flow law parameters): $n = 3.0$, $Q = 115$ kJ/mol, $r = 1.2$, and $A = 8.3 \times 10^{-13}$ MPa$^{−n}$/s; for the lab-based flow law (based only on experimental data): $n = 2.5$, $Q = 110$ kJ/mol, $r = 1.0$, and $A = 6.5 \times 10^{-11}$ MPa$^{−n}$/s. Figures S1 and S2 show the mechanical data and flow laws plotted using the $2\tau$ equivalent stress conversion; Figure S3 the equivalent stresses calculated using both $2\tau$ and $\sqrt{3}\tau$. In this paper, the extrapolated flow law parameters are used, which are consistent with the rheological model proposed by Tokle et al. (2019); however, the strain in the general shear experiments is dominated by the imposed shear component, the principal stresses are defined by the imposed axial compression configuration of the experiments (with $\sigma_2 = \sigma_3$) leading to $\sigma_{eq} = 2\tau$ (cf., Paterson & Olgaard, 2000). The equivalent stress conversion is different than the one used by Tokle et al. (2019), where $\sigma_{eq} = \sqrt{3}\tau$; therefore, we update the Tokle et al. (2019) flow law parameters using the $2\tau$ equivalent stress conversion (Table 1); this change has a minor influence on the flow law parameters, and does not impact the ideas or interpretations presented in Tokle et al. (2019). The high temperature/low stress flow laws are unaffected by the change in stress conversion. The low temperature/high stress flow laws have been recalculated. For the extrapolated flow law (where both experimental data and geologic observations were used to constrain the flow law parameters): $n = 3.0$, $Q = 115$ kJ/mol, $r = 1.2$, and $A = 8.3 \times 10^{-13}$ MPa$^{−n}$/s; for the lab-based flow law (based only on experimental data): $n = 2.5$, $Q = 110$ kJ/mol, $r = 1.0$, and $A = 6.5 \times 10^{-11}$ MPa$^{−n}$/s. Figures S1 and S2 show the mechanical data and flow laws plotted using the $2\tau$ equivalent stress conversion; Figure S3 the equivalent stresses calculated using both $2\tau$ and $\sqrt{3}\tau$. In this paper, the extrapolated flow law parameters are used, which are consistent with the rheological model proposed by Tokle et al. (2019); however,
the same analysis using the laboratory-derived flow laws is completed (shown in Figures S12 and S13). The mechanical data reported in the respective papers are used, and therefore the calibration for the Griggs apparatus developed by Holyoke and Kronenberg (2010) is not applied (see Section 6.1 from Tokle et al., 2019). For consistency in comparing results from general shear experiments, we apply the same shear piston overlap correction employed by Heilbronner and Kilian (2017) and Richter et al. (2018) to the data from Soleymani et al. (2020) (see Table S1).

All of the recrystallized grain sizes from water-added samples analyzed in this study were measured using either electron backscatter diffraction (EBSD) or computer-integrated polarized (CIP) microscopy (Heilbronner & Tullis, 2006; Panozzo Heilbronner & Pauli, 1993). The root mean square (RMS) diameter of the 2D grain size is chosen as well as the grain size measurements by Stipp and Tullis (2003) and Stipp et al. (2006) instead of the reanalyzed grain sizes measured by Cross et al. (2017); this choice provides us with the largest data set (30 experimental datapoints). Cross et al. (2017) reanalyzed 9 of the 21 samples from Stipp and Tullis (2003) and Stipp et al. (2006) and showed that the RMS grain sizes determined from EBSD and CIP were the same within error over the stress range of 34–189 MPa—with the exception of sample W1143 (see Figure 4a in Cross et al. [2017]). The CIP grain size measurements by Stipp and Tullis (2003) and Stipp et al. (2006) were conducted using ultra-thin sections, leading to an effective grain size resolution of ~400–700 nm, which falls between the 200 and 1,000 nm EBSD step size measurements from Cross et al. (2017). The step sizes for EBSD grain size measurements from Kidder et al. (2016), Heilbronner and Kilian (2017), Richter et al. (2018), and Soleymani et al. (2020) range between 150–500, 250–1,000, 100–200, and 150–200 nm, respectively. Thus, we are confident that our compiled data set is not systematically biased by including data acquired by the different techniques.

The recrystallized grain size versus stress data used in this study are listed in Table S1. A mechanical steady state was achieved during the experiments on the samples included in this analysis. It is assumed that the measured recrystallized grain sizes from these samples also reflect the steady state grain size for the conditions where mechanical steady state is achieved. This interpretation is consistent with the analysis of Kidder et al. (2016) who analyzed grain size evolution after changes in stress and/or strain rate. Similarly, Heilbronner and Tullis (2006) show that the recrystallized grain size is roughly constant for samples deformed in general shear to shear strains of 1.5–8 at approximately that same stress.

### 2.1. Variation in the Recrystallized Grain Size Versus Stress Relationship

The recrystallized grain size versus stress data show an increasing range of deviation from the S&T piezometer at stresses greater than ~100 MPa, with select samples showing a larger grain size for a given stress (Figure 1). This deviation is apparent for both water-added and as-is experiments, indicating that the scatter does not reflect a systematic effect of water content (Figure 1b). Figure 1 also shows that deviation from the S&T piezometer cannot be systematically explained by differences in deformation geometry.

The deviation of recrystallized grain size versus stress data from the S&T piezometer correlates with differences in the crystallographic preferred orientation (CPO). As shown in Figures 1d and 1e, samples that develop prism <a> c-axis fabrics plot on, or at smaller grain sizes, than the S&T piezometer, while samples that develop basal <a> c-axis fabrics plot at larger grain sizes than the S&T piezometer. Samples that exhibit a mixture of basal <a>, prism <a>, and rhomb <a> c-axis fabrics plot on, or at larger grain sizes than the S&T piezometer.

The concomitant switch in the c-axis fabric and recrystallized grain size versus stress relationships is analogous to the observation of changes in quartzite flow laws at the same deformation conditions. Tokle et al. (2019) show a concomitant switch in the c-axis fabric and the stress exponent, where the stress exponent changes from $n = 4$ at high temperature/low stresses to $n \approx 3$ at low temperature/high stresses; these changes correlate with the transition from prism <a> c-axis fabrics at high temperature/low stresses to basal <a> c-axis fabrics at low temperature/high stresses (Figures 1d and 1e). In Figure 1e, samples for which c-axis fabrics were not reported are color-coded based on whether they plot on the $n = 4$ or the $n \approx 3$ flow law defined by Tokle et al. (2019), or in the tran-
sitional regime between the two flow laws (Figure 1d). These trends suggest that the slope of the recrystallized grain size versus stress relationship correlates with a change in the stress exponent; this trend, which is predicted for the wattmeter (see below; Austin & Evans, 2007), provided part of the motivation for our analysis.

The S&T piezometer (and the one modified by Cross et al. [2017]) were calibrated from samples deformed at relatively low equivalent stresses (~30–300 MPa). Both studies also acknowledge that at larger stresses and smaller grain sizes the recrystallized grain size versus stress relationship changes based on the experimental observations of Bishop (1996). Inspection of Figure 1 indicates that the piezometric relationships are primarily calibrated using samples where prism <a> c-axis fabrics develop in general shear experiments. The S&T piezometer (as well as the Cross et al. [2017] piezometer) does not fit the majority of data from samples that develop either mixed c-axis fabrics or dominantly basal <a> c-axis fabrics (Figure 1e). Tokle et al. (2019) highlighted that the change in c-axis fabric observed in experimental samples is also observed in natural quartzites deformed at lower temperatures (~≤400°C), suggesting that the empirical recrystallized grain size piezometers may not accurately estimate paleo-stress for samples deformed at mid-crustal temperatures.

3. Reanalysis of a Wet Quartz Grain Growth Law

Improving our understanding of grain growth kinetics for quartz will improve our ability to model quartz grain size evolution. The most widely applied grain growth law for wet quartz was derived by Wightman et al. (2006) based on the grain growth experiments conducted by Tullis and Yund (1982). Wightman et al. (2006) determined their grain growth law using all of the experimental data presented in Tullis and Yund (1982). However, Tullis and Yund (1982) showed that grain growth in all of the experiments using jasper were influenced by the presence of iron oxides (see Figure 2 in Tullis and Yund [1982]). The presence of secondary phases, even at small volume fractions can significantly impact grain growth kinetics (e.g., Evans et al., 2001; Olgaard & Evans, 1986, 1988).

Following these observations, a new grain growth law for wet quartz is formulated. Microstructures in the samples from the grain growth experiments conducted by Tullis and Yund (1982) were reanalyzed, where approximately 20% of the samples used to develop the Wightman et al. (2006) grain growth law exhibit microstructures indicative of either abnormal grain growth (Figure 2b) or second phase pinning due to the presence of oxides (Figures 2c and 2d). Several grain growth experiments conducted by Fukuda et al. (2019) on novaculite were also included in our analysis. These grain growth experiments were conducted at the same starting water content (1–2 wt.%) as Tullis and Yund (1982). All of the experimental grain growth data used in this analysis are listed in Table S2. The grain growth law for wet quartz is defined by Equations 2 and 3 (see Table 2 for symbols and descriptions):

\[
d^p - d_0^p = K t
\]  \hspace{1cm} (2)

\[
K = k_g e^{-Q_g / RT}
\]  \hspace{1cm} (3)

where \(k_g = A_g r_H^{B_g}\) (e.g., Fukuda et al., 2019). The analysis gives grain growth parameters: \(p = 3.0 \pm 0.4\), \(k_g = 0.261f^{1.38}\pm0.3\), and \(Q_g = 134 \pm 21\) kJ/mol, where uncertainties represent 1 standard deviation (Figure 3). The grain growth exponent \(p\) is determined by fitting the grain growth experiments conducted at \(T = 1,000\)°C and \(P = 1.5\) GPa; \(p\) is empirically determined when the slope equals 1 for a linear fit of the data on plots of log\(d^p - d_0^p\) versus log\(t\) (Figure 3a). Figure S6 shows the variation in the slope for values of \(p\) from 2.0 to 4.0. To determine \(A_g\), \(r_H\), and \(Q_g\), a least squares linear regression assuming \(p = 3.0\) is performed. In Figure 3, the derived grain growth law is plotted together with the data, showing that the grain growth law provides a good fit to the experimental data over a range of pressure/water fugacity and temperature. Equation 4 is used to estimate the water fugacity at crustal conditions, where \(a_{H_2O}\) is the water activity and \(A_1, A_3,\) and \(A_3\) are empirically fit constants. Equation 4 was formulated by Shinevar et al. (2015), who empri-
A1 = 5,521 MPa, A2 = 31.28 kJ/mol, and A3 = −2.009 × 10⁻⁵ m⁶. Equation 4 is also used to determine the water fugacity in the flow laws.

\[
f_{H_2O} = a_{H_2O}A_1 \exp \left( \frac{A_2 + PA_3}{RT} \right)
\]  

(4)

Figure 2. Cross-polarized and plane light photomicrographs of quartz grain growth experiments from Tullis and Yund (1982) showing (a) normal grain growth, (b) abnormal grain growth and the presence of oxides as secondary phases in (c) cross-polarized and (d) plane light. The oxides are the black mineral visible in the plane light micrograph.
The Wattmeter

To investigate the origin for different recrystallized grain size versus stress relationships presented in Figure 1, we apply the wattmeter model (Austin & Evans, 2007, 2009). The wattmeter is a relatively simple grain size evolution model that is practical to apply. The wattmeter is defined as a scaling relationship representing a dynamic balance between the rates of grain growth and grain size reduction, where the grain size evolution rate is expressed as (Austin & Evans, 2007, 2009):

\[ \dot{d} = k_{g} e^{\frac{Q_{g}}{RT}} p^{-1} d^{-n-P} = \frac{\beta \sigma \varepsilon d^{2}}{\pi \gamma} \] (5)

The scaling factor \( \lambda \) is defined as the fraction of total energy input during dislocation creep (\( \sigma \varepsilon \)) that is not dissipated as heat, which is then available to change the internal energy through the creation of crystal defects (Austin & Evans, 2007, 2009). See Table 2 for definition of all symbols and their values. Setting the grain size evolution rate to zero (\( \dot{d} = 0 \)), the steady state grain size is defined as:

| Table 2
| List of Symbols and Descriptions |
|---|
| **B** | Preexponential constant for recrystallized grain size piezometer in Equation 1 [\( \mu m \) \( MPa^{b} \)] |
| **k** | Stress exponent for recrystallized grain size piezometer in Equation 1 |
| **d** | Grain size evolution rate [\( \mu m/s^{-1} \)] |
| **Q_{g}** | Activation enthalpy for grain growth [\( kJ/mol \)] |
| **k_{g}** | Grain growth preexponential [\( \mu m^{2}/s^{-1} \)] |
| **A_{g}** | Preexponential constant for grain growth [\( \mu m^{p} s^{-1}MPa^{-n} \)] |
| **r_{g}** | Water fugacity exponent for grain growth |
| **V_{g}** | Activation volume for grain growth [\( cm^{3}/mol \)] |
| **t** | Time [s] |
| **d** | Grain size after time \( t \) [\( \mu m \)] |
| **d_{0}** | Grain size at \( t = 0 \) [\( \mu m \)] |
| **P** | Grain growth exponent |
| **\lambda** | Proportion of the energy associated with dislocation creep stored in the microstructure |
| **\beta** | Fraction of the total mechanical work rate accommodated by dislocation creep |
| **\pi** | Geometric constant [3.14] |
| **\gamma** | Grain boundary energy [\( 1 J/m^{2} \)] |
| **\dot{\varepsilon}** | Strain rate [\( s^{-1} \)] |
| **A** | Material parameter for creep [\( MPa^{-n+r}/s \)] |
| **\sigma** | Differential Stress [\( MPa \)] |
| **n** | Stress exponent |
| **\dot{f}_{H_{2}O}** | Water fugacity [\( MPa \)] |
| **r** | Water fugacity exponent for creep |
| **Q** | Activation enthalpy for creep mechanism [\( kJ/mol \)] |
| **V** | Activation volume [\( cm^{3}/mol \)] |
| **R** | Gas constant [\( 8.314 J/K^{-1}/mol^{-1} \)] |
| **T** | Temperature [K] |

\(^{a}\)Olmsted et al. (2009).
Figure 3. Plots comparing quartz grain growth experiments with our quartz grain growth law. (a) Plot of log\((d^p - d_o)^p\) versus log\(t\) for grain growth experiments at a pressure of 1.5 GPa. (b) Plot of log\((d^p - d_o)\) versus log\(t\) for grain growth experiments at \(T = 1,000°C\) and \(P = 2.0, 1.5, 1.0, 0.5, \) and 0.2 GPa. (c) Plot of log\((d^p - d_o)^p\) versus \(1/RT\). The log\((d^p - d_o)^p\) values in part c come from best fits to the data for a given temperature in Figure S7a at log\(t\) = 5.0. For 900°C and 1,000°C, the dot represents the average log\((d^p - d_o)^p\) value and the vertical lines represent the range in the data in Figure 3a. (d) Plot of log water fugacity versus log \(k_g\). The triangles represent the average log \(k_g\) value with the vertical lines representing the spread in the data. Figure S8 compares the measured grain size and time data with the grain growth law presented in this figure.

\[ d^{1+p} = \frac{k_g \left( \frac{-Q_g}{RT} \right)^p}{\beta \gamma \Sigma \epsilon}. \]

\(Q_g = 134 \pm 21 \text{ kJ/mol}\)

\(k_g = 0.261 f^{1.38 \pm 0.2}\)
By inserting the dislocation creep flow law,

\[ \dot{\varepsilon} = A\sigma^n f'_{H2O}e^{\left(\frac{-Q}{RT}\right)} \]

into the strain rate term and expanding the \( k_g \) term from the grain growth law into Equation 6, the steady state grain size becomes:

\[ d = C\sigma^{-k} f'_{H2O}e^{\left(\frac{-Q'}{RT}\right)} \]

where:

\[ C = \left( \frac{A_g\pi r}{A_0\lambda_p} \right)^{1/p} \]

\[ k' = \frac{n + 1}{p + 1} \]

\[ r' = \frac{r_g - r}{p + 1} \]

\[ Q' = \frac{Q_g - Q}{p + 1} \]

Therefore, with well-established flow laws and grain growth laws, the wattmeter model can be used to investigate recrystallized grain size versus stress relationships. The experimental work by Kidder et al. (2016) shows that the wattmeter successfully predicts the recrystallized grain size for samples in which recrystallization is not complete.

The steady state grain size defined by Equation 8 provides several useful relationships not represented in the classic recrystallized grain size piezometers, where \( B \) and \( k \) in Equation 1 are empirically fit (Stipp & Tullis, 2003) or theoretically estimated (Twiss, 1977). Equation 8 illustrates that the slope, \( k' \) of the recrystallized grain size versus stress relationship is related to the stress exponent (\( n \)) for dislocation creep and the grain growth exponent (\( p \)) from the grain growth law, which indicates that changing \( n \) or \( p \) will change the slope of the recrystallized grain size versus stress relationship. This effect is consistent with our observations illustrated in Figure 1e; assuming the grain growth exponent is constant, the slope of the recrystallized grain size versus stress relationship will be steeper for samples that define the \( n = 4 \) flow law than the \( n = 3 \) flow law. Equation 4 also shows that the steady state grain size for a given stress can depend on temperature and water fugacity if \( Q_g \neq Q \) and/or \( r_g \neq r \), respectively (cf., Austin & Evans, 2007; de Bresser et al., 1998, 2001). From this context, the observation that the quartz grain size piezometers do not show an obvious influence of temperature or water content suggests that \( Q_g \approx Q \) and \( r_g \approx r \), respectively (Kidder et al., 2016; Stipp & Tullis, 2003; Stipp et al., 2006).

5. Discussion

Motivated by our observations in Figure 1, we assess the utility of the wattmeter model in predicting quartz recrystallized grain sizes and their implications for estimating stress at crustal conditions. First, we discuss the comparison between our reformulated grain growth law and the Wightman et al. (2006) and Fukuda et al. (2019) grain growth laws. Second, we illustrate that the two wattmeters derived using the two quartzite flow laws from Tokle et al. (2019) fit the stress versus grain size data for all water-added experiments. Third, we discuss the effects of temperature and water fugacity dependence on predicting the grain size, and their importance for extrapolating empirically derived laboratory relationships to crustal conditions. Fourth, we provide a few remarks on the activation volume for creep and grain growth. Fifth, we discuss the role of basal \(<a>\) slip in quartz. Sixth, we discuss the role of dynamic recrystallization mechanisms and their influence on recrystallized grain size versus stress relationships. Finally, we discuss the application of the calibrated wattmeters for predicting paleo-stresses at crustal conditions.

5.1. Comparison Between Grain Growth Laws

At crustal conditions, our grain growth law is faster than Wightman et al. (2006) and slower than Fukuda et al. (2019) (Figure 4). The growth rate exponent determined in all three studies is the same within error \( \approx 2.9-3.2 \), however all three studies determine significantly different activation enthalpies, with Wightman et al. (2006) determining \( Q_g = 215 \text{ kJ/mol} \), Fukuda et al. (2019) determining \( Q_g = 60 \pm 49 \text{ kJ/mol} \), and our analysis finding \( Q_g = 134 \pm 21 \text{ kJ/mol} \) (Table 3). Thus, extrapolating all three grain growth laws to crustal conditions shows a large variation in estimated grain growth for a given time (Figure 4). The fact that the Wightman et al. (2006) grain growth law predicts the slowest growth is expected. As previously noted, a
significant number of the grain growth experiments conducted by Tullis and Yund (1982) used to formulate the Wightman et al. (2006) grain growth law show microstructural evidence of phase pinning the quartz grain size (Figure 2). Figure S9 shows that samples with secondary phases have systematically smaller grain sizes than samples with no secondary phases for a given time. In addition, the high activation enthalpy determined by Wightman et al. (2006) leads to slower growth rates when extrapolated to crustal temperatures. As for the grain growth law determined by Fukuda et al. (2019), the uncertainty in $Q_g$ is relatively large. This uncertainty partly reflects that most of their 1,000°C grain growth experiments show slower grain growth than 900°C experiments (see Figure 10 in Fukuda et al. [2019]). A similar result was observed in several novaculite samples conducted by Tullis and Yund (1982), where grain growth was faster at 900°C than 1,000°C at 1.5 GPa (as seen in Figure 3a, and the overlap in the range of data shown for 900°C and 1,000°C in Figure 3c). With a smaller activation enthalpy, the Fukuda et al. (2019) law leads to more rapid growth rates when extrapolated to crustal temperatures. The larger activation enthalpy (and lower uncertainty) determined in our analysis results from analyzing the combined data set for the 1–2 wt.% water-added samples, which included the lower temperature 800°C data from Tullis and Yund (1982).

Wightman et al. (2006) compared deformed and undeformed quartz veins from the central Alpine Fault in New Zealand to evaluate quartz grain growth rates at crustal conditions. They posit that recrystallized grains with a grain size of $\sim 1 \mu$m in a deformed quartz vein grew to 80–100 $\mu$m. The grain size of the undeformed portion of the same quartz vein is also 80–100 $\mu$m. For the grain size of both the deformed and undeformed portions of the vein to be within 80–100 $\mu$m after the same period of grain growth, the initial grain size in the undeformed region had to be less than $\sim 70 \mu$m (Figure 4). The central Alpine Fault has a geothermal gradient of $\sim 45^\circ$C/km and an exhumation rate of $\sim 10$ mm/yr (Cross et al., 2015; Kidder et al., 2018). Grain growth was estimated to occur at $\sim 11$ km depth, and therefore at a temperature between 450°C and 500°C, and estimated to last $<1$ Myr (Cross et al., 2015; Kidder et al., 2018). At these conditions,

### Table 3

|                  | $p$ | $k_g$ | $Q_g$ (kJ/mol) |
|------------------|-----|-------|---------------|
| Wightman et al. (2006) | 3.2 | $4.96 P_{34}$ | 215          |
| Fukuda et al. (2019), nov | 2.9 ± 0.4 | $10^{-5.8} J_{19} H_{2O}$ | 60 ± 49     |
| This study       | 3.0 ± 0.4 | $0.261 J_{38 H_{2O}}$ | 134 ± 21    |

Figure 4. Plots of log grain size versus log time in years comparing our quartz grain growth law with the natural grain growth extent and maximum growth time estimates from Wightman et al. (2006), Cross et al. (2015), and Kidder et al. (2018). We also compare our grain growth laws with the Wightman et al. (2006) and Fukuda et al. (2019) grain growth laws. There are two starting grain sizes of 1 and 70 $\mu$m, representing the starting grain size of the deformed and undeformed quartz vein, respectively. The shaded gray region represents the natural grain growth estimates. The minimum growth time of 10 kyr is arbitrary.
the Wightman et al. (2006) grain growth law is too slow; however, both the Fukuda et al. (2019) and our grain growth law show that grain growth from 1 μm to 80–100 μm can be achieved in less than 1 Myr. The Fukuda et al. (2019) grain growth law is significantly faster than our grain growth law. For the grain growth conditions determined for the deformed and undeformed quartz vein, the Fukuda et al. (2019) grain growth law predicts grain growth from 1 to 80 μm in ~30 kyr at 450°C and ~10 kyr at 500°C. In contrast, our grain growth law predicts grain growth from 1 to 80 μm in ~650 kyr at 450°C and ~100 kyr at 500°C (Figure 4), which is more consistent with the estimated duration of growth (i.e., Cross et al., 2015; Kidder et al., 2018).

5.2. Predicting the Recrystallized Grain Size Using the Wattmeter

With our newly formulated grain growth law, the wattmeter accurately predicts the range of recrystallized grain sizes for all of the water-added experimental samples. To illustrate the efficacy of applying the wattmeter, we calculate the steady state grain size using Equation 6 with the known experimental data consisting of stress, strain rate, pressure, and temperature. Remarkably, the wattmeter, with a constant value of the combined term γ/βλ in Equation 6, accurately predicts the recrystallized grain size over the entire range of experimental conditions, including the samples deformed in both the axial and general shear geometry at a wide range of temperatures (Figure 5b). Assuming $\beta = 1$ and $\gamma = 1$ J/m$^2$, this fit indicates a constant value of $\lambda = 0.015$. The assumption of a constant value of $\beta = 1$ (where $\beta$ is the fraction of the total strain rate accommodated by dislocation creep) is supported by the observation that the field boundary (de Bresser et al., 1998, 2001) is both roughly parallel to the piezometric relationship and—based on the relatively sluggish rate of diffusion creep in quartz (e.g., Rutter & Brodie, 2004)—plots at an approximately 50 times lower grain size for a given stress (see Figure 1 in Austin and Evans [2009]). Our analysis also suggests that $\lambda$ has the same value regardless of the c-axis fabric (Figure 5c). Using the Fukuda et al. (2019) grain growth law we calculate a $\lambda$ value of 0.025 (Figure S10). The $\lambda$ values determined for our grain growth law and the Fukuda et al. (2019) grain growth law are considerably smaller than 0.1, which was used by Austin and Evans (2007); however, these values are on the order of $\lambda$ values determined for olivine (Holtzman et al., 2018) and ice (Behn et al., 2020). This observation provides support for applying the wattmeter to predict steady state grain size.

As shown in Figure 6, the wattmeters also successfully predict the variation of grain size versus stress relationships for samples that deform following the different dislocation creep flow laws (with their attendant differences in c-axis fabric). For these figures, the flow law parameters from Tokle et al. (2019) are substituted into the steady state grain size form of the wattmeter (Equations 6 and 8). The wattmeter calculated with the $n = 4$ flow law is nearly identical to the S&T piezometer at all experimental conditions, with a slope of $-1.25$ compared to a slope of $-1.26$ for the S&T piezometer. The wattmeter for the $n = 3$ flow law has a shallower slope of $-1.00$ and predicts larger grain sizes for a given stress than the $n = 4$ wattmeter at stresses larger than ~15 MPa at experimental conditions (Figure 6). Thus, the application of two wattmeters accurately predicts the variation in stress versus grain size data shown in Figure 1. In Figure 6, five samples were deformed at pressures between 1.0 and 1.1 GPa, while the rest of the samples were deformed at pressures between ~1.3 and 1.6 GPa, showing that the difference in predicted grain size for a pressure between ~1.0 and 1.6 GPa is negligible. Stress versus grain size plots comparing the two wattmeters with all of the water-added data used in this study are shown in Figures S12 and S13. The wattmeters are also calculated using the Fukuda et al. (2019) grain growth law and plotted with the water-added data in stress versus grain size space (Figure S14). The code used to plot the wattmeters in stress versus grain size space is provided in the supplemental material.

It is important to highlight that our wattmeters are calibrated for dominantly prism $<a>$ or basal $<a>$ c-axis fabrics and that natural quartzite mylonites can develop a range of c-axis fabrics based on a mixture of prism $<a>$, prism $<c>$, rhomb $<a>$, and basal $<a>$ orientations. When using the wattmeters to estimate stresses for samples that develop intermediate c-axis fabrics, we suggest averaging the stresses predicted by the two wattmeters; at crustal conditions, the variation in stress between the two wattmeters is within the uncertainty of the stresses measured from the experimental samples used to calibrate the wattmeters and piezometers.

5.3. The Influence of Temperature and Water Fugacity

The wattmeter model provides a prediction for how temperature and water fugacity impact the steady state grain size and grain size evolution of quartz. From Equation 8, if $Q_g = Q$ and $r_g = r$, then no effect of tem-
Figure 5. Measured recrystallized grain size versus predicted recrystallized grain size for the S&T piezometer (left column) and wattmeter (right column) by (a) study, (b) temperature in degrees C, (c) c-axis fabric or stress exponent relationship and (d) water content. Pole figures in (c) are from Nachlas and Hirth (2015), Killian and Heilbronner (2017), and Richter et al. (2018).
temperature or water fugacity on the stable grain size is predicted. In contrast, if $Q_g \neq Q$ or $r_g \neq r$ then variations in temperature or water fugacity are predicted to modify the stress versus grain size relationship.

The similar activation enthalpies and water fugacity exponents for both the creep and grain growth laws determined through our analyses suggest only modest effects of temperature and water fugacity on the steady state grain size in quartz. We determined $Q_g = 134 \pm 21$ kJ/mol for grain growth, which is within error of the values of $Q = 125 \pm 15$ kJ/mol for the $n = 4$ flow law and $Q = 115 \pm 15$ kJ/mol for the $n \approx 3$ flow law determined by Tokle et al. (2019). This assessment is consistent with the experimental results of Stipp and Tullis (2003), who

Figure 6. Plots of log grain size versus log equivalent stress comparing the recrystallized grain size versus stress measurements with the wattmeters using the $n = 4$ and $n = 3$ flow laws from Tokle et al. (2019) and the grain growth law formulated in Figure 3 at $P = 1.5$ GPa. The Stipp and Tullis (2003) recrystallized grain size piezometer is plotted for reference. Samples with white fill were conducted at pressures between 1.0 and 1.1 GPa. Figures S11–S13 show the comparison between the recrystallized grain size versus stress measurements and the wattmeters at all temperature and pressure conditions. The recrystallized grain size versus stress measurements are color-coded based on Figures 1d and 1e.
Journal of Geophysical Research: Solid Earth

show no resolvable temperature effect on the recrystallized grain size of quartz at their experimental conditions (and by the data in Figure 5b). Similar observations have been made in other geologic materials where the activation energies are within error for flow laws and grain growth laws in forsterite + 20 vol.% enstatite (Nakakoji & Hiraga, 2018), wet olivine (Speciale et al. (2020), and anorthite (Dresen et al., 1996; Rybacki et al., 2006).

Similarly, our analysis determined a water fugacity exponent of 1.38 ± 0.2 for grain growth that is similar to the values determined by Tokle et al. (2019), with water fugacity exponents of 1.0 for the \( n = 4 \) flow law and 1.20 ± 0.2 for the \( n = 3 \) flow law (errors for the \( n = 4 \) water fugacity exponent are not well-resolved, owing to relatively sparse data). In Figure 5d, the wattmeter model calibrated with water-added samples is compared to data from experiments conducted on as-is and pre-dried samples. The wattmeter accurately predicts the grain size of the as-is samples for grain sizes larger than 2 μm (which are all from experiments conducted on Black Hills quartzite); these samples contain a relatively large (but undersaturated) water content (e.g., Stipp et al., 2006). In contrast, the wattmeter overpredicts the grain size of pre-dried samples, suggesting that the influence of water on either the flow laws or the grain growth law changes at very low water contents. The only other outliers, at the smallest grain sizes, are the unpublished results on experiments conducted on as-is novaculite (Bishop, 1996), which have been plotted in previous studies (e.g., Kidder et al., 2016; Stipp & Tullis, 2003).

In Figure 7, the effects of temperature (noting \( Q_g - Q \)) and water fugacity (noting \( r_g - r \)) on the stress versus grain size relationships are shown at both experimental and crustal conditions for our grain growth law and the Fukuda et al. (2019) grain growth law. At experimental conditions, \( T = 900°C \) and \( P = 1.5 \) GPa, the wattmeters calculated using our grain growth law and the Fukuda et al. (2019) grain growth law are nearly identical, which is expected because both grain growth laws and \( \lambda \) were constrained at or near this experimental condition (Figure 7a). The wattmeters calibrated with our grain growth law have a \( Q_g - Q \) less than 20 kJ/mol and a \( r_g - r \) less than 0.40 for both flow laws. This results in a slight shift in the wattmeters to smaller recrystallized grain sizes for a given stress at crustal conditions (Figure 7b). The wattmeters calibrated with the Fukuda et al. (2019) grain growth law have a large negative value of \( Q_g - Q \), where \( Q_g - Q = -65 \) kJ/mol (for the \( n = 4 \) flow law) and \( Q_g - Q = -55 \) kJ/mol (for the \( n = 3 \) flow law). The water fugacity exponent for the Fukuda et al. (2019) grain growth law \( (r_g = 2.9) \) is also large compared to the values determined for the two flow laws \( (r_{g,n = 4} = 1 \) and \( r_{g,n = 3} = 1.2) \), therefore \( r_g - r = 0.90 \) for the \( n = 4 \) flow law and \( r_g - r = 0.70 \) for the \( n = 3 \) flow law. These differences result in a slight shift in the wattmeters to larger recrystallized grain sizes for a given stress at crustal conditions (Figure 7c).

5.4. A Few Remarks on the Activation Volume for Quartz Rheology

Recent analyses have highlighted that extrapolation of quartzite flow laws can be impacted by neglecting the activation volume term in the activation enthalpy (Lu & Jiang, 2019). Using a compiled data set of select
experimental samples, Lu and Jiang (2019) calculated a large activation volume for dislocation creep in wet ($V = 35.3$ cm$^3$/mol) and dry ($V = 19$–25 cm$^3$/mol) quartz. Determining $V$ using data from water-added experiments conducted at different confining pressures is difficult because both pressure and water fugacity vary, and there is a co-dependent relationship between $r$ and $V$ (e.g., Hirth & Kohlstedt, 2003; Karato & Jung, 2003). Lu and Jiang (2019) analyze three experiments from Kronenberg and Tullis (1984), and estimate $r = 2.7$ and $V = 35.3$ cm$^3$/mol. Because $r$ and $V$ are solved simultaneously, uncertainty in $r$ leads to uncertainty in $V$ and vice versa. In contrast, Chernak et al. (2009) concluded that the activation volume could not be too large based on the similarity of the calculated influence of water fugacity determined from experiments conducted with varying fluid composition at constant pressure and temperature and experiments conducted with varying confining pressure at constant temperature (see Figure 16 in Chernak et al., 2009). Pressure-stepping experiments conducted at a constant strain rate, temperature, and fluid content also show agreement between the mechanical results of Kronenberg and Tullis (1984), Chernak et al. (2009), and Holyoke and Kronenberg (2013).

An additional complicating factor in the analysis by Lu and Jiang (2019) is that the role of water fugacity has been analyzed using data (the three data points from Kronenberg and Tullis [1984]) that follow the two different flow laws determined by Tokle et al. (2019) (see Figure 7a in Tokle et al. [2019]). After accounting for this issue, Tokle et al. (2019) determined $r$ values for both flow laws in the range between 1.0 and 1.5 (neglecting the possible role of the activation volume). Based on Figure 5 from Lu and Jiang (2019), the values of $r$ (1.0–1.5) determined from experiments that follow the $n = 4$ flow law from Tokle et al. (2019) give values of $V$ in the range of 0–12 cm$^3$/mol, consistent with the assessment of Chernak et al. (2009).

The activation volume for quartz grain growth has not previously been calculated. Using our compiled data set for grain growth, we performed linear regression fits of Equations 2 and 3 assuming a grain growth exponent of $p = 3.0$. We determine a low activation volume with a large uncertainty ($V_g = 0.5 \pm 28$ cm$^3$/mol), and a water fugacity exponent ($r_f = 1.4 \pm 1.1$) with a large uncertainty (Table S3). While this analysis suggests similar activation volumes and water fugacity exponents for the dislocation creep flow laws, a more accurate determination of the activation volume and water fugacity exponents will require additional grain growth and deformation experiments focusing on independently determining $r_f$ and $V_g$.

### 5.5. Basal <a> Slip

Basal <a> has long been cited as an active slip system in quartz that is responsible for the development of the peripheral c-axis maxima that forms roughly perpendicular to the shear plane (i.e., the basal <a> fabric, see Figures 1d and 1e). This interpretation has been called into question based on detailed analyses of general shear experiments (Kilian & Heilbronn, 2017). TEM analyses of some natural and experimentally deformed quartzites indicate that basal <a> is not a dominant slip system and/or that the basal <a> dislocations may have limited mobility (e.g., Mainprice & Jaoul, 2009; Mainprice et al., 1986; Morales et al., 2011; Trepied et al., 1980). In contrast, evidence for basal <a> slip is found in some natural samples deformed at lower temperature conditions, where basal <a> is inferred to be the dominant slip system (Law, 2014). For example, the lowest temperature natural sample studied by Halfpenny et al. (2012) shows a strong peripheral c-axis maxima consistent with basal <a> slip; EBSD analyses of this sample show misorientation axes parallel to <m> across subgrain boundaries—consistent with tilt walls formed by basal <a> slip. Similarly, Stüntitz et al. (2017) show evidence that basal <a> slip is favored over prism [c] slip during experiments on natural milky quartz crystals deformed in the O* orientation. While these studies indicate basal <a> is an active slip system in quartz and may relate to the development of the peripheral c-axis maxima, additional microstructural analysis is required to assess the relationship between the role of different slip systems in quartz, especially at low temperature/high stress conditions where basal <a> slip is expected to occur. Nonetheless, since we leverage the observation of a change in flow law with a change in the c-axis fabric, the details of how the peripheral c-axis fabric forms do not affect our observations of consistency between the concomitant changes in the c-axis fabric, the flow laws and the change in the grain size stress relationship modeled with the wattmeter.
5.6. The Role of Dynamic Recrystallization Mechanisms on Piezometric Relationships

Results of previous studies indicate that changes in the piezometric relationship may correlate with changes in the dynamic recrystallization mechanism. However, our analyses—based on the wattmeter—suggest that the conditions where changes in the piezometric relationships occur reflect the influence of different flow laws (that correlate with the change in c-axis fabrics). Stipp and Tullis (2003) observe a change in the piezometric relationship at conditions where the mechanism of dynamic recrystallization changes from subgrain rotation (regime 2) to bulging (regime 1) (see also, Kidder et al., 2016; regimes 1, 2, and 3 refer to the classification from Hirth and Tullis (1992). No significant change in the piezometric relationship was observed at the transition between regimes 2 and 3 (e.g., Heilbronner & Kilian, 2017; Richter et al., 2018; Stipp & Tullis, 2003). In contrast, analyses of recrystallized grain sizes in experimental samples deformed in general shear at conditions that span the transitions from regimes 1–3 show no systematic variation in piezometric relationship (see Figure 10 in Heilbronner and Kilian [2017]). Additionally, recrystallized grain sizes observed for samples deformed in general shear show that the starting grain size has a strong influence on the recrystallized grain size versus stress measurements (Richter et al., 2018), which led these authors to infer that the recrystallized grain size versus stress relationship was influenced by a change or contribution in deformation mechanisms (e.g., a contribution from grain size sensitive creep), rather than the dynamic recrystallization mechanisms. Our results indicate that the change in piezometric relationship observed by Richter et al. (2018) correlates well with the conditions where a change in the flow law is expected—consistent with the relationships shown in Figure 6 (and Figure 1c of Tokle et al. [2019]). These observations suggest that changes in deformation mechanisms in the form of the flow laws and wattmeters have a more direct influence on the piezometric relationship between recrystallized grain size and stress than dynamic recrystallization mechanisms.

6. Applying the Wattmeter at Geologic Conditions

To illustrate the relationship between the c-axis fabric and the different recrystallized grain size versus stress relationships at crustal conditions, plots of stress versus depth are constructed based on the suite of samples from the Whipple Mountain Core complex (WMCC) analyzed by Behr and Platt (2011), as well as a compilation of data from samples where both the c-axis fabric and recrystallized grain size were measured (Gottardi & Teyssier, 2013; Gottardi et al., 2020; Lusk & Platt, 2020; Singleton et al., 2020). As noted by Tokle et al. (2019), the WMCC samples show a transition in the c-axis fabric from dominantly prism <a> at high temperatures/low stresses to dominantly basal <a> at low temperatures/high stresses (Figure 8a). The recrystallized grain size for the WMCC samples was reported as the arithmetic mean (Behr & Platt, 2011), instead of the RMS. Therefore, the data set in Figure 5 is reanalyzed to determine a λ value appropriate for this difference (see Figure S16), finding λ = 0.04 rather than 0.015. The stress versus depth profile calculated using the arithmetic mean-calibrated wattmeters estimates slightly smaller stresses (with a maximum stress of ∼150 MPa) than those predicted by the S&T piezometer or the Cross et al. (2017) sliding resolution piezometer (Figure 8). The stress versus depth plot for the compiled data set using the extrapolated flow laws from Table 1 shows the expected transition in c-axis fabric with depth (i.e., transition from basal <a> to prism <a> with increasing temperature and decreasing stress) for the different studies (Figure 9). The samples analyzed by Singleton et al. (2020) give the highest estimated stresses, in the range of 160–220 MPa for a compressional tectonic environment; the highest estimated stresses for an extensional tectonic environment are in the range of 100–150 MPa (Figure 9a). For comparison the stress estimate from the bottom of the KTB borehole based on measurements of dislocation density is also plotted, which is consistent with the higher end of the estimated stresses (Figure 9a) (Dresen et al., 1997).

Applying the wattmeters using the laboratory fit flow laws in Table 1 results in a decrease in estimated stress for the lower temperature/higher stress conditions by a factor of ∼2 because of the smaller stress exponent in the low temperature/high stress flow law (Figure 9b). This effect decreases the estimated stresses for the Singleton et al. (2020) and Behr and Platt (2011) samples from 160–220 to 80–120 MPa and 100–150 to 50–80 MPa, respectively (Figure 9b). The comparison between the stress versus depth plots for the wattmeters using different flow laws from Table 1 highlights the effect of the stress exponent on estimating stress at geologic conditions using the wattmeter model. Tokle et al. (2019) constrained a stress exponent of n = 3.0 ± 0.3 by assuming that the relationship between the transitions in c-axis fabric and the flow laws

16 of 21
occurs the same way under both laboratory and geologic conditions. As shown in Table 1, the fit of the laboratory data gives a lower stress exponent of $n = 2.5 \pm 0.3$. Richter et al. (2018) find that the stress exponent at higher stress conditions may be even lower ($n = 1.9 \pm 0.6$), and infer a possible influence of grain size sensitive creep. However, Tokle et al. (2019) point out that an apparently lower stress exponent may arise from analyzing data from deformation conditions that span the transition between multiple deformation
mechanisms. Thus, while the application of the wattmeters provides stress versus depth relationships that are consistent with the experimentally calibrated piezometers and the stress estimates from the KTB bore hole at 9,100 m are shown in yellow. The orange triangle represents the stress estimate based on the average dislocation density with the orange band representing the uncertainty in stress based on the uncertainty in dislocation density (see Figure 4 in Dresen et al. [1997]). The solid black line is the frictional stress assuming a vertical maximum principal stress and the dashed black line is the frictional stress assuming a horizontal maximum principal stress defined by Equations 7a and 7c in Zoback and Townend (2001), respectively. The influence of pore fluid pressure in the frictional regime included using the ratio of lithostatic/pore fluid pressure ($\lambda'$). A value of $\lambda' = 0.6$ for a compressional tectonic environment is shown for consistency with Kidder et al. (2012). Data from Behr and Platt (2011), Gottardi and Teyssier (2013), and Gottardi et al. (2020) are representative of an extensional tectonic environment and data from Singleton et al. (2020) and Lusk and Platt (2020) are representative of a compressional tectonic environment. The arithmetic mean-calibrated wattmeter is used to calculate stress for the Behr and Platt (2011) samples while the RMS-calibrated wattmeter is used to estimate stress for Gottardi and Teyssier (2013), Gottardi et al. (2020), Singleton et al. (2020), and Lusk and Platt (2020). The black cross is the representative uncertainty for the stress versus depth estimate from Behr and Platt (2011).

To illustrate the temperature effect resulting from the differences in the activation energy for grain growth and creep ($Q_{gr} - Q$) on the stress versus depth estimates from the WMCC, the wattmeters calculated using our grain growth law and the Fukuda et al. (2019) grain growth law are compared. As shown in Figure 8d, the wattmeters calculated with the Fukuda et al. (2019) grain growth law predict significantly larger stresses, with a maximum stress of 598 MPa. The variation in the stress versus depth profiles calculated using our grain growth law and the Fukuda et al. (2019) grain growth law highlights the sensitivity of extrapolating rheological parameters calibrated at experimental conditions to crustal conditions with over 450 MPa variation in peak stress estimates based on these two models (Figures 8c and 8d), even though both sets of wattmeters accurately predict the stress versus grain size relationships at experimental conditions (Figures S11–S14). Finally, we note that if the grain growth law changes dramatically owing to a transition in the kinetics of grain boundary mobility at higher temperatures (e.g., Guillotop & Poirier, 1979), that such changes could be accounted for using the wattmeter model. A transition in the mobility of grain boundaries has been inferred to explain the apparently large grain sizes observed for some quartz aggregates deformed at high temperature crustal conditions (e.g., Stipp et al., 2010).

As noted above, two empirically derived piezometric relationships have been formulated for quartz based on experimentally deformed samples that exhibit either regime 2/regime 3 microstructures at high

Figure 9. Plot of stress versus depth for the compilation of natural quartzite samples. Intermediate stresses represent the average stress calculated from the $n = 4$ and $n = 3$ wattmeters, representing samples with a mixture of c-axis fabric orientations. Stress estimates based on dislocation density from the KTB bore hole at 9,100 m are shown in yellow. The orange triangle represents the stress estimate based on the average dislocation density with the orange band representing the uncertainty in stress based on the uncertainty in dislocation density (see Figure 4 in Dresen et al. [1997]). The solid black line is the frictional stress assuming a vertical maximum principal stress and the dashed black line is the frictional stress assuming a horizontal maximum principal stress defined by Equations 7a and 7c in Zoback and Townend (2001), respectively. The influence of pore fluid pressure in the frictional regime included using the ratio of lithostatic/pore fluid pressure ($\lambda'$). A value of $\lambda' = 0.6$ for a compressional tectonic environment is shown for consistency with Kidder et al. (2012). Data from Behr and Platt (2011), Gottardi and Teyssier (2013), and Gottardi et al. (2020) are representative of an extensional tectonic environment and data from Singleton et al. (2020) and Lusk and Platt (2020) are representative of a compressional tectonic environment. The arithmetic mean-calibrated wattmeter is used to calculate stress for the Behr and Platt (2011) samples while the RMS-calibrated wattmeter is used to estimate stress for Gottardi and Teyssier (2013), Gottardi et al. (2020), Singleton et al. (2020), and Lusk and Platt (2020). The black cross is the representative uncertainty for the stress versus depth estimate from Behr and Platt (2011).
temperatures/low stresses (Stipp & Tullis, 2003) or regime 1 microstructures at low temperatures/high stresses (Bishop, 1996). However, to our knowledge, only the low stress piezometric relationship has been applied to estimate stresses in naturally deformed quartz-rich rocks, even for samples that exhibit regime 1 microstructures (e.g., Behr & Platt, 2011; Singleton et al., 2020). This is likely because extrapolating the regime 1 piezometer predicts lower stresses at geologic conditions than the regime 2/3 piezometric relationship, which is inconsistent with experimental observations for how the regime transitions occur with changes in temperature or strains rate (Figure S17). In contrast, the wattmeter model fits both the experimental recrystallized grain size versus stress relationships (Figure 5) and provides consistent stress estimates at geologic conditions (Figures 8 and 9) representative for all dislocation creep regimes, with the highest stresses predicted for the lowest temperature conditions where regime 1 microstructures are observed.

Previous applications of the wattmeter model to estimate paleo-stresses in quartz at crustal conditions show that the wattmeter predicts much lower stresses than other independent techniques, such as empirically derived piezometers (Cross et al., 2017; Heilbronner & Kilian, 2017; Stipp & Tullis, 2003), theoretical models (Shimizu, 2008), and geodynamic models (Kidder et al., 2012), especially at low temperature/high stress conditions (Kidder et al., 2016; Lu & Jiang, 2019). In contrast, the wattmeters calibrated with the flow laws determined by Tokle et al. (2019) and our reformulated grain growth law provide peak stress estimates for the WMCC that are also consistent with the stress estimates from the KTB borehole based on measurements of dislocation density (Dresen et al., 1997) and borehole stress (e.g., Zoback & Townend, 2001). The wattmeters are also consistent with the range of experimental observations (such as the lack of a discernable influence of temperature on recrystallized grain size) and provides an explanation for the variation in stress versus grain size relationships in the experimental data (Figure 1).

7. Conclusions

This study investigates the relationships between the recrystallized grain size and stress in experimentally deformed water-added quartz aggregates. The observations of a concomitant switch in the c-axis fabric and stress versus grain size relationship as well as a switch in the c-axis fabric and flow law relationship provides support linking different deformation mechanisms, piezometric relationships, and c-axis fabrics in quartz. By reformulating the quartz grain growth law, we show there is a modest temperature and water fugacity dependence on the steady state grain size. The wattmeter model explains the different stress versus grain size relationships observed at experimental conditions while also providing stress estimates consistent with other piezometric models. The results of this analysis provide support for the use and extrapolation of the wattmeter model to both experimental and geologic conditions to investigate the stress state and grain size evolution of quartz rich rocks.

Data Availability Statement

Data used in this study are available through the ETH Research Collection (https://doi.org/10.3929/ETHZ-B-000473283).

Acknowledgments

This work benefited from discussions with Jan Tullis, Reid Cooper, Whitney Behr, Cameron Meyers, Seth Kruckenber, and Mark Zimmerman. The authors thank Andrew Cross and three anonymous reviewers as well as editor Doug Schmitt for their comments which improved the manuscript. The authors would like to thank David Prior for providing us with the data sets used to develop the grain growth law in Wightman et al. (2006) as well as Steve Kidder for providing several matlab scripts. This research was supported by the National Science Foundation grant EAR-1624178 awarded to G. Hirth.

References

Austin, N., & Evans, B. (2007). Paleowattmeters: A scaling relation for dynamically recrystallized grain size. Geology, 35, 343–346. https://doi.org/10.1130/G23244A.1
Austin, N., & Evans, B. (2009). The kinetics of microstructural evolution during deformation of calcite. Journal of Geophysical Research, 114, B09402. https://doi.org/10.1029/2008JB006138
Behr, W. M., & Platt, J. P. (2011). A naturally constrained stress profile through the middle crust in an extensional terrain. Earth and Planetary Science Letters, 303, 181–192. https://doi.org/10.1016/j.epsl.2010.11.044
Bishop, R. R. (1996). Grain boundary migration in experimentally deformed quartz aggregates: The relationship between dynamically recrystallized grain size and steady state flow stress (B.Sc. Thesis, p. 36). Brown University.
Cernak, L. J., Hirth, G., Selverstone, J., & Tullis, J. (2009). Effect of aqueous and carbonic fluids on the dislocation creep strength of quartz. Journal of Geophysical Research, 114, B04201. https://doi.org/10.1029/2008JB005884
Cross, A. J., Kidder, S., & Prior, D. J. (2015). Using microstructures and TitanIQ thermobarometry of quartz sheared around garnet porphyroclasts to evaluate microstructural evolution and constrain an Alpine Fault geotherm. Journal of Structural Geology, 75, 17–31. https://doi.org/10.1016/j.jsg.2015.02.012
Nakajoji, T., & Hiraga, T. (2018). Diffusion creep and grain growth in forsterite +20 vol% enstatite aggregates: 2. Their common diffusional mechanism and its consequence for weak-temperature-dependent viscosity. Journal of Geophysical Research Solid Earth, 123, 9513–9527. https://doi.org/10.1029/2018JB015819

Olgaard, D. L., & Evans, B. (1986). Effect of second-phase particles on grain growth in calcite. Journal of the American Ceramic Society, 69, 272–277. https://doi.org/10.1111/j.1151-2916.1986.tb07374.x

Olgaard, D. L., & Evans, B. (1988). Grain growth in synthetic marbles with added mica and water. Contributions to Mineralogy and Petrology, 100, 246–260. https://doi.org/10.1007/BF00373591

Olmsted, D. L., Foiles, S. M., & Holm, E. A. (2009). Survey of computed grain boundary properties in face-centered cubic metals: I. Grain boundary energy. Acta Materialia, 57, 3694–3703. https://doi.org/10.1016/j.actamat.2009.04.007

Panozzo Heilbronner, R., & Pauli, C. (1993). Integrated spatial and orientation analysis of quartz c-axes by computer-aided microscopy. Journal of Structural Geology, 15, 369–382. https://doi.org/10.1016/0191-8141(93)90133-u

Rybacki, E., Gottschalk, M., Wirth, R., & Dresen, G. (2006). Influence of water fugacity and activation volume on the flow properties of polycrystalline quartz aggregates. Physical Chemistry Chemical Physics, 8, 2833–2840. https://doi.org/10.1039/b516545a

Stipp, M., Tullis, J., Scherwath, M., & Behrens, H. (2010). A new perspective on paleopiezometry: Dynamically recrystallized grain size distributions indicate mechanism changes. Geology, 38, 759–762. https://doi.org/10.1130/G31162.1

Stipp, M., Tullis, J., & Behrens, H. (2006). Effect of water on the dislocation creep microstructure and flow stress of fine-grained anorthite aggregates. Journal of Structural Geology, 28, 2088–2100. https://doi.org/10.1016/j.jsg.2004.04.006

Treepid, L., Doukhan, J., & Paquet, J. (1980). Subgrain boundaries in quartz—Theoretical-analysis and microscopic observations. Physics and Chemistry of Minerals, 5, 201–218.

Tullis, J., & Yund, R. A. (1982). Grain growth kinetics of quartz and calcite aggregates. The Journal of Geology, 90, 301–318. https://doi.org/10.1086/628681

Twiss, R. J. (1977). Theory and applicability of a recrystallized grain size paleopiezometer. Pure and Applied Geophysics, 115, 227–244. https://www.springer.com/journal/241

Wightman, R. H., Prior, D. J., & Little, T. A. (2006). Quartz veins deformed by diffusion creep-accommodated grain boundary sliding during a transient, high strain-rate event in the Southern Alps, New Zealand. Journal of Structural Geology, 28, 902–918. https://doi.org/10.1016/j.jsg.2006.02.008

Zoback, M., & Townend, J. (2001). Implications of hydrostatic pore pressures and high crustal strength for the deformation of intraplate lithosphere. Tectonophysics, 336, 19–30. https://doi.org/10.1016/s0040-1951(01)00991-9