An SU(5) SUSY Model with R-Parity Violation and Radiatively Induced Neutrino Masses

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The radiatively induced neutrino mass matrix is investigated on the basis of an SU(5) SUSY model. In order to evade the proton decay, an ansatz based on a discrete symmetry $Z_2$ is assumed: although, at the unification scale, we have two types of superfields $\Psi^\pm \equiv \bar{\Psi}^\pm + 10L(\pm)$, which are transformed under the discrete symmetry $Z_2$, the particles $\Psi^\pm$ are decoupled after SU(5) symmetry is broken, so that our quarks and leptons belong to $\Psi^-$. The $R$-parity violating terms for our quarks and leptons $\Psi^-$ are basically forbidden under the symmetry $Z_2$. However, we assume that mixings between members of $\Psi^-$ and those of $\Psi^+$ are in part caused after SU(5) is broken. As a result, the $R$-parity-violating interactions are in part allowed, so that the neutrino masses are radiatively generated, while the proton decay due to the $R$-parity violating terms is still forbidden because the term $d^R_R d^R_R u^c_R$ has $z = -1$.

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I. INTRODUCTION

The origin of the neutrino mass generation is still a mysterious problem in the unified understanding of the quarks and leptons. The Zee model [1] is one of several promising models, because it has only 3 free parameters and it can naturally lead to a large neutrino mixing [2], especially, to a bimaximal mixing [3]. However, the original Zee model is not on the framework of a grand unification theory (GUT). The most attractive idea [4] to embed the Zee model into GUTs is to identify the Zee scalar $h^+$ as the slepton $\tilde{e}_R$ in an $R$-parity-violating supersymmetric (SUSY) model. However, usually, it is accepted that SUSY models with $R$-parity violation are incompatible with a GUT scenario, because the $R$-parity-violating interactions induce proton decay [5].

In the present paper, in order to suppress this kind of proton decay, a discrete symmetry $Z_2$ is introduced. The essential idea is as follows: At the unification scale $\mu = M_X$, we have two types of superfields $\Psi^\pm = \bar{\Psi}^\pm + 10L(\pm)$, which are transformed with $\pm 1$ under the discrete symmetry $Z_2$ (we will call it “$Z_2$-parity” hereafter). We consider that the particles $\Psi^+$ are decoupled after SU(5) symmetry is broken (but $Z_2$ is still unbroken), so that our quarks and leptons $\bar{\Psi}^\pm + 10L$ belong to $\Psi^-$. The $R$-parity violating terms are given by the combinations $\bar{\Psi}^+_L \bar{\Psi}^+_L 10^c_L$, $\bar{\Psi}^-_L \bar{\Psi}^-_L 10^c_L$ and $\bar{\Psi}^+_R \bar{\Psi}^-_R 10^-_L$, so that they basically do not contribute to the quarks and leptons with $Z_2$-parity $z = -1$, because of the $Z_2$ symmetry. However, we assume that mixings of the members of $\Psi^+$ with those of $\Psi^-$ are caused in part after SU(5) is broken. As a result, the $R$-parity-violating interactions are in part allowed, so that the neutrino masses are radiatively generated, while the proton decay due to the $R$-parity violating terms is still forbidden because the term $d^R_R d^R_R u^c_R$ has $z = -1$.

II. Z_2 SYMMETRY AND THE PROTON DECAY

We identify the Zee scalar $h^+$ as the slepton $\tilde{e}^+_R$, which is a member of SU(5) 10-plet sfermions. Then, the Zee interactions correspond to the following $R$-parity-violating interactions

$$\lambda_{ijk} (\bar{\psi}^-)^i_R (\psi^\pm)^j_L (\psi^\pm)^k_R \kappa_{AB}$$

$$= \frac{1}{\sqrt{2}} \lambda_{ijk} \left\{ \epsilon_{\alpha \beta \gamma} (\tilde{e}^+_R)^{\alpha}_L (\tilde{e}^+_R)^{\beta}_L (\bar{u}^c_R)^{\gamma}_k \right.$$  

$$- (\tilde{\nu}^-_L)^{i}_L (\nu^+)_L - (\tilde{\nu}^-_L)^{i}_L (\nu^+_L) + (\tilde{\nu}^-_L)^{i}_L (\bar{\nu}^-_L) \right\},$$  

(2.1)
where $\psi^c \equiv C\psi^T$ and the indices $(i,j,\ldots)$, $(A,B,\ldots)$ and $(\alpha,\beta,\ldots)$ are family-, $SU(5)_{GUT}$- and $SU(3)_{colour}$-indices, respectively. The coefficients $\lambda_{ijk}$ are antisymmetric in $i$ and $j$. On the other hand, in SUSY GUT models, if the interactions (2.1) exist, the following $R$-parity-violating interactions will also exist:

$$
\lambda_{ijk} \left( \bar{\psi}^c_{ij} A^A \right) \left( \bar{\psi}^c_{ik} B^B \right),
= \frac{1}{\sqrt{2}} \lambda_{ijk} \left\{ \varepsilon_{\alpha\beta\gamma} \left( \overline{d^c_R}_{ij} \right) \left( \overline{d^c_R}_{jk} \right) \left( \overline{d^c_R}_{ik} \right) \right\}
- \left( \left[ \overline{\nu}_{L_i}^c \right] \left( \overline{e}_{L_j} \right) \left( \overline{e}_{L_k} \right) \right)
- \left( \left[ \overline{\nu}_{L_i}^c \right] \left( \overline{\nu}_{L_j} \right) \left( \overline{e}_{L_k} \right) \right)
+ \left( \left[ \overline{\nu}_{L_i} \right] \left( \overline{d^c_R}_{ij} \right) \left( \overline{d^c_R}_{jk} \right) \right) \left( \left[ \overline{d^c_R}_{ik} \right] \right)
\right\},
$$

(2.2)

which contribute to the proton decay through the intermediate state $\bar{d}^c_R$. Also, the term $\left( \overline{d^c_R}_{ij} \right) \left( \overline{d^c_R}_{jk} \right) \left( \overline{d^c_R}_{ik} \right)$ in the interactions (2.1) can contribute to the nucleon decay through the intermediate state $\bar{u}$. The upper limits of the coupling constants $\lambda_{ijk}$ from proton decay experiments have been investigated by Smirnov and Vissani [5], and the values must be highly suppressed.

In order to forbid the contribution of the interactions (2.1) and (2.2) to the proton decay, we must consider that in the $R$-parity-violating interactions $\Psi \times \Psi \times 10$, the term $d^c_R d^c_R u^c_R$ is exactly forbidden, while the terms $\nu_R \nu_R e^c_R$ and/or $\nu_R d^c_R d^c_R$ are in part allowed.

For such purpose, we introduce a discrete symmetry $Z_2$, which exactly holds at every energy scale. At the unification scale $\mu = M_X$, we have two types of superfields $\Psi_{L(\pm)} = \Psi_{L(\pm)} + 10_{L(\pm)}$, which are transformed as $\Psi_{L(\pm)} \rightarrow \pm \Psi_{L(\pm)}$ under the discrete symmetry $Z_2$. We consider that the particles $\Psi_{L(\pm)}$ are basically decoupled after the $SU(5)$ symmetry is broken, so that our quarks and leptons (and their SUSY partners) $\Psi_{L(\pm)} + 10_{L(\pm)}$ are regarded as $\Psi_{L(\pm)} = 5_{L(\pm)} + 10_{L(\pm)}$. The $R$-parity-violating terms for quarks and leptons (and their SUSY partners) are basically forbidden under the symmetry $Z_2$ below $\mu = M_X$, because the terms are composed of $5_{L(\pm)} 10_{L(\pm)}$. However, if we assume that mixings between the members of $\Psi_{L(\pm)}$ and those of $\Psi_{L(\pm)}$ in part take place after SU(5) is broken, R-parity-violating interactions $\Psi_{L(\pm)} \Psi_{L(\pm)} \Psi_{L(\pm)}$ become available at the low energy $\mu = m_Z$, too. For example, we assume a mixing:

$$(2,1)L_i = (2,1)L_{(-i)} \cos \theta^A + (2,1)L_{(+i)} \sin \theta^A,$$

between the two components of $SU(2) \times SU(3)$ for the $i$-th family. (Hereafter, we will refer to the mixing (2.3) as a mixing of type $A_i$.) Then, the $R$-parity-violating interactions

$$
\sin \theta^A \lambda_{ijk} \nu_{L_i} \nu_{L_j} \nu_{L_k} \sin \theta^A \lambda_{ijk} \nu_{L_i} \nu_{L_j} \nu_{L_k},
$$

become available from the interactions

$$
\lambda_{ijk} \overline{d^c_R}_{ij} \overline{d^c_R}_{jk} \overline{d^c_R}_{ik},
$$

(2.4)

above the unification scale $\mu = M_X$. Also, we can consider a mixing

$$(2,3)_L k = (2,3)_L (-k) \cos \theta^B + (2,3)_L (+k) \sin \theta^B,$$

(2.6)

between the (2,3) components of $SU(2) \times SU(3)$ for the $k$-th family. (Hereafter, we will refer to the mixing (2.6) as a $B_k$-type mixing.) Then, the $R$-parity-violating interactions

$$
\sin \theta^B \lambda'_{ijk} \nu_{L_i} \nu_{L_j} \nu_{L_k} \sin \theta^B \lambda'_{ijk} \nu_{L_i} \nu_{L_j} \nu_{L_k},
$$

(2.7)

become available from the interactions

$$
\lambda'_{ijk} \overline{d^c_R}_{ij} \overline{d^c_R}_{jk} \overline{d^c_R}_{ik},
$$

(2.8)

On the other hand, note that the interaction

$$
\overline{d^c_R}_R \overline{d^c_R}_R \nu_{L_R},
$$

(2.9)

is exactly forbidden, independently of whether the mixings (2.3) and (2.6) occur or not, because those interactions have the $Z_2$ parity $z = -1$. Therefore, the proton decay due to the $R$-parity-violating terms is exactly forbidden because of the absence of the term $\nu_{L_R} \nu_{L_R} \nu_{L_R}$. On the other hand, the neutrino masses are radiatively generated through the interactions $\nu_{L_i} \nu_{L_j} \nu_{L_k}$ and $\nu_{L_i} \nu_{L_j} \nu_{L_k}$ with $z = +1$. The possible forms of the radiative neutrino mass matrix will be discussed in the next section.

At present, we do not know any reasonable mechanism not only for such a mixing, but also for the decoupling of $\Psi_{L(\pm)}$. In order to make $\Psi_{L(\pm)} = \Psi_{L(\pm)} + 10_{L(\pm)}$ heavy, the $SU(2)_L$ symmetry must be broken, but, of course, we cannot consider a scenario in which $SU(2)$ is broken just after $SU(5)$ is broken. In the present paper, we give only a phenomenological selection rule: if the superfield $\Psi$ can make a five-body $SU(5)$ singlet operator $\Psi \Psi \Psi \Psi \Psi$ with the $Z_2$ parity $z = +1$, then the superfield $\Psi$ can be decoupled below $\mu = M_X$. Obviously, according to this selection rule, the superfield $\overline{d^c_R}_{R}$ can be decoupled below $\mu = M_X$. Similarly, the superfield $10_{L(\pm)}$ is decoupled below $\mu = M_X$. However, note that those operators in the $SU(5)$ singlets are symbolically expressed in terms of $SU(2) \times SU(3)$ components as follows:

$$
\left( \overline{d^c_R}_{R} \right)^5 = [(2,1)L_{(\pm)}]^2 \times [(1,3)L_{(\pm)}]^3,
$$

(2.10)

$$
\left( 10_{L(\pm)} \right)^5 = [(1,3)_R] \times [(1,3)_L]^2 \times [(2,3)_L]^2
$$

(2.11)

and that even if the interchanges $(2,1)L_{(\pm)i} \leftrightarrow (2,1)L_{(-i)}$; and/or $(2,3)_L(-k) \leftrightarrow (2,3)_L(k)$, are caused, the composite operators

$$
\left( \overline{d^c_R}_R \right)^5 = [(2,1)L_{(\pm)}]^2 \times [(1,3)L_{(\pm)}]^3,
$$

(2.12)
\[(10_L)^3 = (1,1)_{L(+)} \times [(1,3)_{L(+)}]^2 \times [(2,3)_{L(-)}]^2
+ \lambda_{ijk} \left[ \bar{\nu}_L d_R \bar{d}_{Lk} + \bar{\nu}_L d_R \bar{d}_{Lk} - (\nu_L \leftrightarrow d_R^c) \right]. \tag{3.2} \]

still have \( z = +1 \). Such interchanges are possible only for the components \((2,1)_L\) and \((2,3)_L\). As a result, only the combination
\[
\tilde{5}_L + 10_L = [(2,1)_L(+ \, +) + (1,3)_L(-)]
+ [(1,1)_L(-) + (2,3)_L(-) + (1,3)_L(-)], \tag{2.14}
\]
for the \( i \)-th family and/or
\[
\tilde{5}_L + 10_L = [(2,1)_L(- \, +) + (1,3)_L(-)]
+ [(1,1)_L(-) + (2,3)_L(+ \, +) + (1,3)_L(-)], \tag{2.15}
\]
for the \( k \)-th family survive below \( \mu = M_X \) as the quarks and leptons (and their SUSY partners).

Of course, the above selection rule cannot be justified within the framework of the minimal SUSY standard model. At present, this is only an ansatz to select which components of \( SU(2) \times SU(3) \) can be interchanged.

### III. RADIATIVE NEUTRINO MASSES

In a SUSY GUT scenario, there are many origins of the neutrino mass generations. For example, the sneutrinos \( \tilde{\nu}_D \) can have vacuum expectation values (VEVs), and the neutrinos \( \nu_{Li} \) acquire their masses thereby (for example, see Ref. [6]). Although we cannot rule out a possibility that the observed neutrino masses can be understood from such compound origins, we do not take such a point of view in the present paper, because the observed neutrino masses and mixings appear to be rather simple and characteristic. We simply assume that the radiative masses are only dominated even if there are other origins of the neutrino mass generations.

In the present scenario, the origins of the radiatively induced neutrino masses are two: one is induced by the \( R \)-parity-violating interactions \( \nu_i \epsilon_D d_R \) and \( \nu_j \epsilon_D d_R \); the other one is induced by \( \nu_L \epsilon_L \tilde{\epsilon}_R \) and \( \nu_L \epsilon_L \tilde{\epsilon}_R \). Note that there is no Zee-type diagrams due to \( H_d^\tau - \tilde{\epsilon}_R \) mixing in this scheme.

First, we discuss the down-quark loop contributions. For simplicity, we assume that the masses \( M_{Li} \) and \( M_{Ri} \) of the squarks \( d_{Li} \) and \( d_{Ri} \) are approximately constant, independently of the flavours, although we consider the flavour-dependent structure for the mass terms \( d_{Li} M_{Li} d_{Ri} \).

Then, the radiatively induced neutrino mass matrix due to the A-type mixing is given by
\[
(M_{\nu})_{ij} = m_0 \lambda_{ikm} \lambda_{jlm} (M_d^3)_{kn} (M_d^2)_{im} + (i \leftrightarrow j), \tag{3.1} \]
where sine-factors have been dropped for simplicity, \( M_{\nu} \) is defined by \( U_{\nu} U_{\nu}^c \), and the coupling constants \( \lambda_{ijk} \) are defined by

Here, we have changed the definition of \( \lambda_{ijk} \) from that in (2.1) as \( \lambda_{ijk} \rightarrow \lambda_{ijk}^a \) for the convenience of the expression of \( M_{\nu} \) defined by \( U_{\nu} U_{\nu}^c \). In the present paper, the unitary matrix \( U_{\nu} \) used to diagonalize the Majorana mass matrix \( M_{\nu} \) is defined as \( U_{\nu}^T M_{\nu} U_{\nu} = D_{\nu} \). Then, the so-called Maki–Nakagawa–Sakata–Pontecorvo [7] matrix (we will simply call it the “lepton mixing matrix”) \( U \equiv U_{MNS} \) is given by \( U = U_{\nu}^T U_{\nu} \). Usually, it is considered that the matrix form of \( M_{\nu}^2 \) is proportional to the form \( M_{\nu} \). Then, the neutrino mass matrix (3.1) becomes, in a more concise form:
\[
(M_{\nu})_{ij} = m_0 s_i A_j \lambda_{ikm} \lambda_{jlm} (M_d^3)_{kn} (M_d^2)_{im}, \tag{3.3}
\]
where we have redefined the common factor \( m_0 \) from that in (3.1). Of course, exactly speaking, for the \( A_i \) mixings with \( \sin \theta_i \)-factors, we should read the expression (3.3) as
\[
(M_{\nu})_{ij} = m_0 s_i A_j \lambda_{ikm} \lambda_{jlm} (M_d^3)_{kn} (M_d^2)_{im}, \tag{3.4}
\]
for the \( B_k \) mixings, we read the expression (3.3) as
\[
(M_{\nu})_{ij} = m_0 s_i B_j \lambda_{ikm} \lambda_{jlm} (M_d^3)_{kn} (M_d^2)_{im}, \tag{3.5}
\]
where \( s_k^B = \sin \theta_k^B \) defined in (2.6). For mixed-type mixings of \( A_i \) and \( B_k \), we read (3.3) as
\[
(M_{\nu})_{ij} = m_0 s_i A_j m_{\nu_i} \lambda_{ikm} + c_i A_j m_{\nu_i} \lambda_{ikm}
+ (s_i A_j m_{\nu_i} \lambda_{ikm} + c_i A_j m_{\nu_i} \lambda_{ikm} (M_d^3)_{kn} (M_d^2)_{im}, \tag{3.6}
\]
where \( c_i^A = \cos \theta_i^A \) and \( c_k^B = \cos \theta_k^B \).

The contributions from the charged lepton loops are essentially the same as (3.3), except for the absence of the \( B \)-type mixing and the replacement \( M_{\nu} \rightarrow M_{\tau} \). For simplicity, we will continue the investigation for the case of the down-quark loop contributions.

For the phenomenological study of the mass matrix (3.3), it is convenient to take the basis on which the down-quark mass matrix \( M_{\nu} \) is diagonal:
\[
U_{L}^{dT} M_{\nu}^d U_{R}^{dT} = D_d \equiv \text{diag}(m_{\nu_1}^d, m_{\nu_2}^d, m_{\nu_3}^d). \tag{3.7}
\]
We consider that, on the basis with \( M_{\nu} = D_d \), the charged lepton mass matrix \( M_{\nu} \) is also approximately diagonal, \( U_{L}^{dT} M_{\nu} U_{R} \approx D_e = \text{diag}(m_{\ell_1}, m_{\ell_2}, m_{\ell_3}) \), so that the unitary matrix \( U_{\nu} \) approximately gives the lepton mixing matrix \( U = U_{\nu}^{dT} U_{\nu}^{c} \). Then, we can express (3.3) as
\[
(M_{\nu})_{11} = m_{\nu_1}^d (\lambda_{133}^d)^2 + m_{\nu_1}^d (\lambda_{122}^d)^2 + \frac{1}{2} m_{\nu_1}^d m_{\nu_2}^d \lambda_{133} \lambda_{122} + m_{\nu_1}^d m_{\nu_3}^d \lambda_{123} \lambda_{132},
(M_{\nu})_{12} = m_{\nu_2}^d (\lambda_{233}^d)^2 + m_{\nu_2}^d (\lambda_{233}^d)^2 + \frac{1}{2} m_{\nu_2}^d m_{\nu_3}^d \lambda_{213} \lambda_{231},
(M_{\nu})_{13} = m_{\nu_3}^d (\lambda_{333}^d)^2 + m_{\nu_3}^d (\lambda_{333}^d)^2 + \frac{1}{2} m_{\nu_3}^d m_{\nu_2}^d \lambda_{312} \lambda_{321},
(M_{\nu})_{21} = m_{\nu_1}^d (\lambda_{133}^d)^2 + \frac{1}{2} m_{\nu_1}^d m_{\nu_2}^d \lambda_{133} \lambda_{123} + m_{\nu_1}^d m_{\nu_3}^d \lambda_{123} \lambda_{132},
+ m_{\nu_2}^d m_{\nu_3}^d \lambda_{133} \lambda_{123} + m_{\nu_3}^d m_{\nu_1}^d \lambda_{121} \lambda_{122}, \tag{3.8}
\]
\[(M_\nu)_{13} = (m_2^d)^2 \lambda_{122} \lambda_{322} + m_3^d m_4^d \lambda_{132} \lambda_{323} + m_4^d m_1^d \lambda_{131} \lambda_{313} + m_2^d m_4^d \lambda_{121} \lambda_{312},
(M_\nu)_{23} = (m_1^d)^2 \lambda_{211} \lambda_{311} + m_3^d m_2^d \lambda_{232} \lambda_{323} + m_3^d m_1^d \lambda_{231} \lambda_{313} + m_2^d m_1^d \lambda_{212} \lambda_{312},
\]

where, for simplicity, we have dropped the common factor \(m_0\). In order to give the best-fit values for the observed neutrino data \([8,9]\)
\[R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \approx \frac{5.0 \times 10^{-5} \text{eV}^2}{2.5 \times 10^{-3} \text{eV}^2} = 2.0 \times 10^{-2},\]

\[\sin^2 2\theta_{\text{solar}} = \sin^2 2\theta_{12} = 0.76,\]
\[\left[\tan^2 \theta_{\text{solar}} = 0.34\right],\]
\[\sin^2 2\theta_{\text{atm}} = \sin^2 2\theta_{23} = 1.0,\]

we must seek a parameter set that gives \((M_\nu)_{22} \simeq (M_\nu)_{33}\) and \((M_\nu)_{11} \simeq (M_\nu)_{13}\) for the expression (3.8). When we consider the \(A_\nu\)-type mixings, we obtain \([10]\)
\[M_\nu \propto \begin{pmatrix} \varepsilon \varepsilon^* & \varepsilon \varepsilon^* & \varepsilon \varepsilon^* \\ \varepsilon \varepsilon^* & 1 + 1 & 1 \\ \varepsilon \varepsilon^* & 1 & 1 \end{pmatrix},\]

for \(\varepsilon = s_1^A/s_2^A = s_1^B/s_3^A\) and \(\lambda_{223}/\lambda_{323} \simeq m_2^d/m_3^d\). Generally, when we consider only \(A_\nu\)-type mixings, the solutions are highly dependent on the fine-tuning among the coefficients \(\lambda_{ijk}\), and, besides, since the mass matrix is too near to a rank-1 matrix, it is difficult to give the solution a small but sizeable value of \(R\) (it leads to an extremely small value of \(R\)). Even if we take the charged lepton loop contributions into consideration, the situation is not improved unless we assume the special parametrization for \(\lambda_{ijk}\) (\(\lambda_{223}/\lambda_{323} \simeq m_2^d/m_3^d\) and so on).

Next, we consider the case of the \(B_\nu\)-type mixings. When we consider a \(B_1\) mixing, we obtain a simple mass matrix form
\[M_\nu = (s_1^B)^2 (m_1^d)^2 \begin{pmatrix} 0 & 0 & \varepsilon (r_3 + r_2) \\ 0 & 1 & 1 + \varepsilon r_3 \\ \varepsilon (r_3 + r_2) & 1 + \varepsilon r_3 & (1 + \varepsilon r_3)^2 \end{pmatrix},\]

\[(3.14)\]

where \(\varepsilon = s_1^A / s_2^B \lambda' \ll 1, r_2 = m_2^d / m_4^d\) and \(r_3 = m_3^d / m_4^d\), and we have assumed \(c_3^A \simeq 1\). This mass matrix can give a reasonable value of \(R\) together with a nearly bi-maximal mixing. For example, for the parameter value \(\varepsilon = 0.000374\), we obtain the following numerical results:
\[m_1^\nu = 0.0822 m_0, \quad m_2^\nu = -0.3276 m_0, \quad m_3^\nu = 2.2604 m_0,\]

\[(3.15)\]

\[U = \begin{pmatrix} 0.8726 & -0.4822 & 0.0776 \\ -0.3925 & -0.5978 & 0.6990 \\ 0.2907 & 0.6404 & 0.7109 \end{pmatrix},\]

\[(3.16)\]

i.e.
\[R = 0.0201,\]

\[\sin^2 2\theta_{12} = 4 U_{11}^2 U_{12}^2 = 0.708,\]

\[\sin^2 2\theta_{23} = 4 U_{23}^2 U_{33}^2 = 0.988.\]

These values are in good agreement with the best fit values (3.9)–(3.11) for the observed neutrino data. Although it is difficult in the original Zee model to give a sizeable deviation of \(\sin^2 2\theta_{12}\) from 1 \([11]\) (it must be \(\sin^2 2\theta_{12} = 1.0\)), the present model can give a reasonable deviation from \(\sin^2 2\theta_{12} = 1.0\). The result
\[U_{13}^2 = 0.00602\]

\[(3.20)\]

is also consistent with the present experimental upper limit
\[|U_{13}|^2 < 0.03,\]

\[(3.21)\]

from the CHOOZ collaboration \([12]\).

**IV. HIGGS SECTORS**

In the present model, the quark and charged lepton mass matrices are generated by the VEVs of the Higgs scalars with \(z = +1\). Therefore, even if \(SU(2)_L\) is broken later, the \(Z_2\) symmetry still exactly holds.

Let us show the mass matrices \(M_f\) for the case of the mixed-type mixing \(A_3\) and \(B_1\) as an example of the explicit forms of \(M_f\):
\[M_e = \begin{pmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_3 a_3 \end{pmatrix},\]

\[(4.1)\]
M_d = \left( \begin{array}{ccc} c_1^B c_1 & c_1^B c_2 & c_1^B c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{array} \right), \quad (4.2)

M_u = \left( \begin{array}{ccc} c_1^B c'_1 & c_1^B c'_2 & c_1^B c'_3 \\ b'_1 & b'_2 & b'_3 \\ a'_1 & a'_2 & a'_3 \end{array} \right). \quad (4.3)

Here, $M_f$ have been defined by $\overline{f}_LM_f f_R$ ($f = u, d, e$). Note that the mass matrix $M_d$ has a form different from $M_f^T$ because of the mixing factors. Usually, if we consider only one type of Higgs scalar of SU(5) 5-plet (5-plet), it is difficult to obtain realistic mass matrices $M_f$ ($f = u, d, e$). Therefore, the present model has a possibility to improve this problem. However, whether we can give reasonable mass matrix forms of $M_f$ or not is a future task to us.

Now, we would like to give a comment on the Higgs sectors. In Sec. II, we have assumed that although the superfield $\overline{\Phi}_L(\pm)$ is decoupled below $\mu = M_X$, the components $(2, 1)(+)$ of $\overline{\Phi}_{L(\pm)}$ can contribute to low energy phenomena through the mixing (2.3). If we consider the Higgs fields $\overline{H}_{d(+)}$ and $\overline{H}_{d(-)}$ with SU(5) 5-plet, and if we assume a situation similar to the matter fields $\overline{\Phi}_L$, then we obtain

$$\overline{H}_d = (2, 1)(+) + (1, \Phi)(-), \quad (4.4)$$

where we have assumed a perfect interchange between $(2, 1)(-)$ and $(2, 1)(+)$, not a mixing. Note that in this scheme, the Higgs scalar component $(3, 1)(-)$ cannot couple to the fermions $\overline{d}_R u'_R$ with $z = +1$ independently of the mixings $A_i$ and $B_i$, so that the scalar $(3, 1)(-)$ cannot contribute to the proton decay and it need not be super-heavy. (Although it couples to the fermions $[\overline{\Phi}_L d_L - \overline{\Phi}_L u_L]$, these interactions cannot contribute to the proton decay.) We can consider a similar mechanism for the Higgs fields $H_u$ with SU(5) 5-plet. The interactions of $(1, 3)(-)$ with $\overline{\Phi}_L d_L$ and $\overline{\Phi}_R u'_R$ are absent, so that the scalar $(1, 3)(-)$ need not be super-heavy. The so-called $\mu$-terms are composed of $H_{u(+)\overline{H}_{d(+)}}$ and $H_{u(-)\overline{H}_{d(-)}}$.

V. CONCLUSION

In conclusion, we have investigated possible forms of radiatively induced neutrino mass matrix under an ansatz [(2.3) and (2.6)] within the framework of the SU(5) SUSY model. We have assumed two types of matter fields $\Psi_L = \overline{\Phi}_L + 10_L, \Psi(\pm)$, which are transformed as $\Psi(\pm) \rightarrow \pm \Psi(\pm)$ under the $Z_2$ symmetry. We have assumed that the $Z_2$ symmetry exactly holds, even if SU(5) is broken. The essential ansatz is in the mixings (2.3) and (2.6). Although the origin of the mixings (2.3) and (2.6) is still an open question, if we admit this ansatz, we can obtain very interesting and simple neutrino mass matrix form, which can give satisfactory numerical results for the observed neutrino data. How we can justify the ansatz is a future task for us.

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| i  | k = 1          | k = 2          | k = 3          |
|----|----------------|----------------|----------------|
| 1  | \( s_1^3 \lambda'_{1j1} \) | 0              | 0              |
| 2  | \( s_1^3 \lambda'_{2j1} \) | 0              | 0              |
| 3  | \( c_3^3 s_1^2 A_{3j1} \) + \( s_3^3 c_1^2 B_{3j1} \) | \( s_3^3 \lambda_{3j2} \) | \( s_3^3 \lambda_{3j3} \) |

TABLE I. Rule of the replacement \( \lambda_{ijk} \) in the mass matrix (3.3) for the case of \( A_3 \) and \( B_1 \) mixings.