Cross section measurements of $e^+ e^- \rightarrow K^+ K^- K^+ K^-$ and $\phi K^+ K^-$ at center-of-mass energies from 2.10 to 3.08 GeV

M. Ablikim,1 M. N. Achasov,10,d P. Adlarson,59 S. Ahmed,15 M. Albrecht,4 M. Alekseev58a,58c A. Amoroso58a,58c F. F. An,1 Q. An,55,43 Y. Bai,42 O. Bakina,27 R. Baldini Ferroli,23a Y. Ballossino,24a Y. Ban,35 K. Begzsuren,5 J. V. Bennett,5 N. Berger,26 M. Bertani,23 D. Bettoni,24a F. Bianchi,58a,58c J. Biernat,59 J. Bloms,52 I. Boyko,27 R. A. Briere,5 H. Cai,60 X. Cai,1,47 A. Calcaterra,23a G. F. Cao,47 N. Cao,47 S. A. Cetin,47 J. Chai,58c J. F. Chang,143 W. L. Chang,51 G. Chelkov,137,47 D. Y. Chen,4 G. Chen,1 H. S. Chen,147 J. C. Chen,1 M. L. Chen,147 S. J. Chen,3 Y. B. Chen,52 W. Cheng,58c G. Cibinetto,54a F. Cossio,38c X. F. Cui,34 L. H. Dai,1,47 J. P. Dai,143 X. C. Dai,1,47 A. Dbyessy,1 D. Dedovich,27 Z. Y. Deng,1 A. Denig,26 I. Denysenko,27 M. Destefanis,58a,58c D. De Mori,58a,58c Y. Ding,33 C. Dong,34 J. Dong,143 L. Y. Dong,1,47 M. Y. Dong,1,43,47 Z. L. Dou,33 X. X. Du,63 J. Z. Fan,45 J. Fang,143 S. S. Fang,1,47 Y. Fang,1 R. Farinelli,24a,24b H. Wang,37 K. Wang,1,43 L. L. Wang,1 L. S. Wang,1 M. Wang,37 M. Z. Wang,35 Meng Wang,1,47 P. L. Wang,1 R. M. Wang,61 D. Lu,1,47 J. G. Lu,1,43 Y. Lu,1 Y. P. Lu,1,43 C. L. Luo,32 M. X. Luo,62 P. W. Luo,44 T. Luo,9,j X. L. Luo,1,43 S. Lusso,58c X. Y. Shan,55,43 M. Shao,55,43 C. P. Shen,23b D. Dedovich,27 Q. An,55,43 Y. Bai,42 O. Bakina,27 R. Baldini Ferroli,23a Y. Ballossino,24a Y. Ban,35 K. Begzsuren,5 J. V. Bennett,5 N. Berger,26 M. Bertani,23 D. Bettoni,24a F. Bianchi,58a,58c J. Biernat,59 J. Bloms,52 I. Boyko,27 R. A. Briere,5 H. Cai,60 X. Cai,1,47 A. Calcaterra,23a G. F. Cao,47 N. Cao,47 S. A. Cetin,47 J. Chai,58c J. F. Chang,143 W. L. Chang,51 G. Chelkov,137,47 D. Y. Chen,4 G. Chen,1 H. S. Chen,147 J. C. Chen,1 M. L. Chen,147 S. J. Chen,3 Y. B. Chen,52 W. Cheng,58c G. Cibinetto,54a F. Cossio,38c X. F. Cui,34 L. H. Dai,1,47 J. P. Dai,143 X. C. Dai,1,47 A. Dbyessy,1 D. Dedovich,27 Z. Y. Deng,1 A. Denig,26 I. Denysenko,27 M. Destefanis,58a,58c D. De Mori,58a,58c Y. Ding,33 C. Dong,34 J. Dong,143 L. Y. Dong,1,47 M. Y. Dong,1,43,47 Z. L. Dou,33 X. X. Du,63 J. Z. Fan,45 J. Fang,143 S. S. Fang,1,47 Y. Fang,1 R. Farinelli,24a,24b H. Wang,37 K. Wang,1,43 L. L. Wang,1 L. S. Wang,1 M. Wang,37 M. Z. Wang,35 Meng Wang,1,47 P. L. Wang,1 R. M. Wang,61 D. Lu,1,47 J. G. Lu,1,43 Y. Lu,1 Y. P. Lu,1,43 C. L. Luo,32 M. X. Luo,62 P. W. Luo,44 T. Luo,9,j X. L. Luo,1,43 S. Lusso,58c X. R. Lyu,47 F. C. Ma,31 H. L. Ma,1 L. L. Ma,37 M. M. Ma,1,47 Q. M. Ma,1 X. N. Ma,34 X. X. Ma,1,43 Y. M. Ma,37 F. E. Maas,15 M. Maggiora,58c S. Maldaner,26 S. Malde,53 Q. A. Malik,58c M. Mangoni,23b Y. J. Mao,35 Z. P. Mao,58c S. Marcella,58c Z. X. Meng,49 J. G. Messchendorp,29 G. Mezzadri,24a J. Min,43 T. J. Min,34 R. E. Mitchell,22 X. H. Mo,143,47 J. M. Mo,147,47 W. K. Moon,147,47 F. M. Muro,59 A. Mustafa,58c S. Nakhtoul,147,43,47 Y. Nefedov,27 F. Nerling,11,46d Z. Ning,1,43 S. Nisar,63 S. L. Niu,58c S. L. Olsen,47 Q. Ouyang,65 S. Pacetti,23b Y. Pan,58c M. Papenbrock,59 P. Patteri,23a M. Pelizaeus,58a,58c H. P. Peng,55,43 K. Peters,11,g J. Pettersson,59 J. L. Ping,147 R. G. Ping,147 A. Pitka,147 R. Poling,58a,58c V. Prasad,34 M. Q. Qi,34 S. Qian,1,43 C. F. Qiao,47 N. Qin,60 X. P. Qiu,13 X. X. Qi,58c Z. H. Qiu,1,43 J. F. Qiu,1,43 S. Q. Qu,54 K. H. Rashid,57,43 F. Redmer,26 M. Richter,4 S. A. Cetin,46b,a A. Zafar,57 Y. Zeng,20 B. X. Zhang,1 B. Y. Zhang,1,43 C. C. Zhang,1 H. H. Zhang,44 H. Y. Zhang,1,43 K. Zhang,1,47 L. Zhang,45 S. F. Zhang,33 T. J. Zhang,38,h X. Y. Zhang,37 Y. Zhang,55,43 Y. H. Zhang,1,43 Y. T. Zhang,55,43 Yang Zhang,1 Yao Zhang,1

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(BESIII Collaboration)

1. Institute of High Energy Physics, Beijing 100049, People’s Republic of China
2. Beihang University, Beijing 100191, People’s Republic of China
3. Beijing Institute of Petrochemical Technology, Beijing 102617, People’s Republic of China
4. Bochum Ruhr-University, D-44780 Bochum, Germany
5. Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
6. Central China Normal University, Wuhan 430079, People’s Republic of China
7. China Center of Advanced Science and Technology, Beijing 100190, People’s Republic of China
8. COMSATS University Islamabad, Lahore Campus, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan
9. Fudan University, Shanghai 200443, People’s Republic of China
10. G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia
11. GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany
12. Guangxi Normal University, Guilin 541004, People’s Republic of China
13. Guangxi University, Nanning 530004, People’s Republic of China
14. Hangzhou Normal University, Hangzhou 310036, People’s Republic of China
15. Helmholtz Institute Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
16. Henan Normal University, Xinxiang 453007, People’s Republic of China
17. Henan University of Science and Technology, Luoyang 471003, People’s Republic of China
18. Hunan Normal University, Changsha 410081, People’s Republic of China
19. Hunan University, Changsha 410082, People’s Republic of China
20. Indian Institute of Technology Madras, Chennai 600036, India
21. Indiana University, Bloomington, Indiana 47405, USA
22. INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy
23. INFN and University of Perugia, I-06100, Perugia, Italy
24. INFN Sezione di Ferrara, I-44122, Ferrara, Italy
25. Institute of Physics and Technology, Peace Ave. 54B, Ulaanbaatar 13330, Mongolia
26. Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
27. Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
28. Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
29. KVI-CART, University of Groningen, NL-9747 AA Groningen, The Netherlands
30. Lanzhou University, Lanzhou 730000, People’s Republic of China
31. Liaoning University, Shenyang 110036, People’s Republic of China
32. Nanjing Normal University, Nanjing 210023, People’s Republic of China
33. Nanjing University, Nanjing 210093, People’s Republic of China
34. Nankai University, Tianjin 300071, People’s Republic of China
35. Peking University, Beijing 100871, People’s Republic of China
36. Shandong Normal University, Jinan 250014, People’s Republic of China
37. Shandong University, Jinan 250100, People’s Republic of China
38. Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
39. Shanxi University, Taiyuan 030006, People’s Republic of China
40. Sichuan University, Chengdu 610064, People’s Republic of China
41. Soochow University, Suzhou 215006, People’s Republic of China
42. Southeast University, Nanjing 211100, People’s Republic of China
43. State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People’s Republic of China
44. Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
45. Tsinghua University, Beijing 100084, People’s Republic of China
46a. Ankara University, 06100 Tandogan, Ankara, Turkey
46b. Istanbul Bilgi University, 34060 Eyup, Istanbul, Turkey
We measure the Born cross sections of the process $e^+e^\rightarrow K^+K^-K^+K^-$ at center-of-mass (c.m.) energies, $\sqrt{s}$, between 2.100 and 3.080 GeV. The data were collected using the BESIII detector at the BEPCII collider. An enhancement at $\sqrt{s} = 2.232$ GeV is observed, very close to the $e^+e^-\rightarrow \Lambda\bar{\Lambda}$ production threshold. A similar enhancement at the same c.m. energy is observed in the $e^+e^-\rightarrow \phi K^+K^-$ cross section. The energy dependence of the $K^+K^-K^+K^-$ and $\phi K^+K^-$ cross sections differs significantly from that of $e^+e^-\rightarrow \phi\pi^+\pi^-$. DOI: 10.1103/PhysRevD.100.032009

I. INTRODUCTION

The $\phi(2170)$ resonance, denoted previously as $Y(2175)$, was first observed by BABAR in the process $e^+e^-\rightarrow \phi f_0(980)\rightarrow \phi\pi\pi$ [1] via initial-state radiation (ISR) and was confirmed by Belle [2]. BES [3] and BESIII [4,5] also observed the $\phi(2170)$ in the $\phi f_0(980)$ invariant-mass spectrum. The discovery of $s\bar{s}$ bound states is of interest for the understanding of the strangeonium spectrum, which is less well understood than for example the hidden-charm states ($c\bar{c}$). The CLEO Collaboration found the first evidence for $Y(4260)\rightarrow K^+K^-J/\psi$ [6] above the $D\bar{D}$-production threshold. A similar process, $e^+e^-\rightarrow \phi K^+K^-$, potentially allows the study of strangeoniumlike vector states above the $K\bar{K}$-production threshold.

Many theoretical interpretations have been proposed for the $\phi(2170)$, such as a $s\bar{s}\bar{g}$ hybrid [7], a $2^3D_1$ $s\bar{s}$ state [8], a tetraquark state [9,10], a $\Lambda\bar{\Lambda}$ bound state [11,12], or a three-quark meson system $\phi K^+K^-$ [13]. The $1^- s\bar{s}\bar{g}$ hybrid can decay to $\phi\pi\pi$, with a cascade $s\bar{s}\bar{g}\rightarrow (s\bar{s})(gg)\rightarrow \phi\pi\pi$ [14], whereby $s\bar{s}\bar{g}\rightarrow \phi f_0(980)$ may make a significant contribution. However, none of the theoretical models has so far been able to describe all experimental observations in all aspects. Searching for new decay modes and measuring the line shapes of their production cross sections will be very important.

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helpful for interpreting the internal structure of the $\phi(2170)$ resonance.

The BABAR Collaboration measured the $e^+e^- \rightarrow K^+K^-K^+K^-$ cross sections and observed an enhancement around 2.3 GeV [15,16]. In addition, the BES Collaboration observed the $f_0(980)$, $f_2(1525)$ and $f_0(1790)$ in the invariant-mass distribution of $K^+K^-$ pairs in events in which the other $K^+K^-$ pair has an invariant mass close to the nominal $\phi$ mass [17]. An enhancement at $\sqrt{s} = 2.175$ GeV was seen in the line shape of the process $e^+e^- \rightarrow \phi f_0(980)$ [16], but due to poor statistics, no strong conclusion could be drawn from the data. Torres et al. have performed a Faddeev calculation for the three-meson system $\phi K^+K^-$ and obtained a peak around 2.150 GeV/$c^2$ [13]. These observations stimulate experimentalists to study the energy dependence for the production of the $\phi K^+K^-$ and $K^+K^-K^+K^-$ final states.

Using a data sample corresponding to an integrated luminosity of 650 pb$^{-1}$ collected at center-of-mass (c.m.) energies from 2.0 GeV to 3.08 GeV [18], we present in this paper the results of a study of the reaction $e^+e^- \rightarrow K^+K^-K^+K^-$ and its dominant intermediate process $e^+e^- \rightarrow \phi K^+K^-$. The optimization of event-selection criteria, the determination of detection efficiencies and the estimates of potential backgrounds are performed based on Monte Carlo (MC) simulations taking the various aspects of the experimental setup into account. The GEANT4-based [21] MC simulation software, which includes the geometric and material description of the BESIII detector, the detector response and digitization models, and the detector running conditions and performances, is used to generate the MC samples.

For the background study, the $e^+e^- \rightarrow q\bar{q}$ process is simulated by the MC event generator CONEXC [22], while the decays are generated by EVTGEN [23,24] for known decay modes with branching fractions set to Particle Data Group (PDG) world-average values [25] and by LUARLW [26] for the remaining unknown decays. MC samples of $e^+e^- \rightarrow e^+e^-$ and $\mu^+\mu^-$ processes are generated by BABAYAGA 3.5 [27]. The signal MC samples from the phase-space models (PHSP) of $e^+e^- \rightarrow K^+K^-K^+K^-$ and $e^+e^- \rightarrow \phi K^+K^-$ are generated at c.m. energies corresponding to the experimental values, where the line shape of the production cross section of the two processes is taken from the BABAR experiment [16] and the signal detection efficiency is obtained by weighting the MC-generated PHSP sample to data according to the observed invariant-mass distribution.

III. EVENT SELECTION AND BACKGROUND ANALYSIS

A. $e^+e^- \rightarrow K^+K^-K^+K^-$

To improve the detection efficiency, candidate events are required to have three or four charged tracks. Charged tracks are reconstructed from hits in the MDC within the polar angle range $|\cos \theta| < 0.93$ and are required to pass the intersection point within 10 cm along the beam direction and within 1 cm in the plane perpendicular to the beam. For each charged track, the TOF and the $dE/dx$ information are combined to form particle identification (PID) confidence levels (C.L.) for the $\pi$, $K$, and $p$ hypotheses. The particle type with the highest C.L. is assigned to each track. At least three kaons are required to be identified. The primary vertex of the event is reconstructed by three kaons. For events with four identified kaons, the combination with the smallest chi-square of the vertex fit is retained.

Figure 1 shows the momentum distribution of the three identified kaons for $\sqrt{s} = 2.125$ GeV after applying the

![Momentum spectrum of the three identified kaons at $\sqrt{s} = 2.125$ GeV. The black dots with error bars are data, the dashed (red) histogram is from $e^+e^- \rightarrow q\bar{q}$, the solid (green) histogram is from $e^+e^- \rightarrow e^+e^-$, the hatched (black) histogram is from $e^+e^- \rightarrow \mu^+\mu^-$, and the dotted (blue) histogram is the sum of all MC samples.](image-url)
above-mentioned selection criteria. The peak on the right-
side of the spectrum stems from reducible QED back-
ground, dominated by the processes $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$. To suppress this background, the momenta
of the identified particles are required to be less than 80% of
the mean momentum of the colliding beams ($p_{\text{beam}}$).

**B. $e^+e^- \rightarrow \phi K^+K^-$**

For $e^+e^- \rightarrow \phi K^+K^-$ with $\phi \rightarrow K^+K^-$, the final state is
$K^+K^-K^+K^-$. The selection criteria for three or four kaons
are the same as described in the previous subsection. In
addition to the primary-vertex fit of the three kaons, a one-
constraint (1C) kinematic fit is performed under the
hypothesis that the $KK^-$ missing mass corresponds to the
kaon mass. For events with four reconstructed and
identified kaons, the combination with the smallest chi-
square of the 1C kinematic fit ($\chi^2_{\text{IC}}(K^+K^-\text{KK}_{\text{miss}})$) is
retained and required to be less than 20. In the following,
the $K_{\text{miss}}$ momentum is that obtained from the 1C kinematic
fit and is used in invariant-mass calculations.

The open histogram in Fig. 2 shows the invariant-mass
distribution for all $K^+K^-$ pairs for the selected
$K^+K^-K^+K^-$ events (four entries per event) for data taken
at $\sqrt{s} = 3.080$ GeV. The hatched histogram in the same
figure corresponds to the distribution of the pair with a
mass closest to the nominal $\phi$ mass. A prominent peak near
the $\phi$ mass is seen in both histograms and indicates that
the $\phi K^+K^-$ channel dominates the $K^+K^-K^+K^-$ final states.

**IV. SIGNAL YIELDS**

The signal yields of $e^+e^- \rightarrow K^+K^-K^+K^-$ are obtained
from unbinned maximum-likelihood fits to the $K^+K^-K$
recoil-mass [$M_{\text{recoil}}(K^+K^-K)$] data. The signal is described
by the line shape obtained from the MC simulation
convolved with a Gaussian function, where the Gaussian
function describes the difference in resolution between data
and MC simulation. The background shape is parametrized
by a second-order Chebyshev polynomial function. The
parameters of the Gaussian function and the Chebyshev
polynomial function are left free in the fit. The correspond-
ing fit result for data taken at $\sqrt{s} = 3.080$ GeV is shown in
Fig. 3.

To determine the signal yields of the $e^+e^- \rightarrow \phi K^+K^-$
process, an unbinned maximum-likelihood fit is performed
to the $M(K^+K^-)$ spectra. The probability density function
of the $M(K^+K^-)$ spectra for the $\phi$ is obtained from a
$P$-wave Breit-Wigner function convolved with a Gaussian
function that accounts for the detector resolution. The
$P$-wave Breit-Wigner function is defined as

$$ f(m) = |A(m)|^2 \cdot p, \quad (1) $$

$$ A(m) = \frac{p^\ell}{m^2 - m_0^2 + i m \Gamma(m)} \cdot \frac{B(p)}{B(p')}, \quad (2) $$

$$ B(p) = \frac{1}{\sqrt{1 + (Rp)^2}}, \quad (3) $$

$$ \Gamma(m) = \left( \frac{p}{p'} \right)^{2\ell+1} \left( \frac{m_0}{m} \right) \cdot \Gamma_0 \frac{B(p)}{B(p')}, \quad (4) $$

where $m_0$ is the nominal $\phi$ mass as specified by the PDG, $p$ is
the momentum of the kaon in the rest frame of the $K^+K^-$
system, $p'$ is the momentum of the kaon at the nominal mass
of $\phi$, and $\Gamma_0$ is the width of the $\phi$. The angular momentum
($\ell$) is assumed to equal one, which is the lowest allowed
given the parent and daughter spins, $B(p)$ is the Blatt-
Weisskopf form factor, and $R$ is the radius of the centrifugal
barrier, whose value is taken to be 3 GeV/$c^{-1}$ [28].

The background shape is described by an ARGUS
function [29]. The parameters of the Gaussian function

![FIG. 2. Invariant-mass distribution at $\sqrt{s} = 3.080$ GeV for all $K^+K^-$ pairs in selected $e^+e^- \rightarrow K^+K^-K^+K^-$ events (open histogram), and for the combination in each event closest to the $\phi$-meson mass (hatched).](image)

![FIG. 3. The fit to the $M_{\text{recoil}}(K^+K^-K)$ mass spectra at $\sqrt{s} = 3.080$ GeV. The black dots with error bars are data, the solid (red) curve shows the result of the best fit, and the dashed (blue) curve shows the result for the background.](image)
and the ARGUS function are left free in the fit. The corresponding fit result for data taken at $\sqrt{s} = 3.080$ GeV is shown in Fig. 4.

The same event selection criteria and fit procedure are applied to the other 19 data samples taken at different c.m. energies. The number of events for these samples are listed in Tables I and II.

V. SELECTION EFFICIENCY

A. $e^+ e^- \rightarrow \phi K^+ K^-$

The detection efficiency is obtained by MC simulations of the $\phi K^+ K^-$ channel using PHSP. It is found that data deviate strongly from the PHSP MC distributions, as demonstrated by the histograms in Fig. 5, which show the non-$\phi$ pair $K^+ K^-$ invariant-mass distributions. Here, $\phi$ candidates are selected in the signal region and background from the sideband region shown in Fig. 4. The signal region is defined as $|M(K^+ K^-) - m_{\phi}| < 3 \sigma$, where $m_{\phi}$ is the nominal $\phi$ mass from PDG and $\sigma$ is the $\phi$ width convolved with detector resolution. The sideband region is $1.050 \, \text{GeV}/c^2 < M(K^+ K^-) < 1.130 \, \text{GeV}/c^2$. The background in Fig. 5 is the distribution of the invariant mass of the remaining pair in the sideband event, and the data points are the invariant mass of the remaining pair of the $\phi$ candidates minus the background. To obtain a more accurate detection efficiency, the MC-generated events are weighted according to the observed $K^+ K^-$ (non-$\phi$ pair) invariant-mass distribution, where the weight factor is the ratio of the $K^+ K^-$ mass distribution between data and PHSP MC. The weighted PHSP MC distribution is

| $\sqrt{s}$ (GeV) | $\mathcal{L}$ (pb$^{-1}$) | $N_{\text{obs}}$ | $(1 + \delta)$ (e%) | $\sigma^B$ (pb) |
|----------------|-----------------|----------------|----------------|--------------|
| 2.1000         | 12.2            | 12.9±6.1       | 0.8346         | 5.7          | 45.3±21.4    |
| 2.1250         | 109             | 309.6±31.5     | 0.8555         | 9.6          | 70.6±7.2     |
| 2.1500         | 2.84            | 15.8±5.9       | 0.8714         | 13.7         | 94.7±35.4    |
| 2.1750         | 10.6            | 84.5±15.6      | 0.8835         | 18.8         | 97.3±18.0    |
| 2.2000         | 13.7            | 137.7±18.7     | 0.8898         | 21.7         | 105.8±14.4   |
| 2.2324         | 11.9            | 260.0±22.3     | 0.8543         | 27.2         | 191.8±16.5   |
| 2.3094         | 21.1            | 682.3±28.0     | 0.9388         | 40.82        | 84.4±3.5     |
| 2.3864         | 22.5            | 934.6±32.0     | 0.9515         | 46.78        | 93.1±3.2     |
| 2.9600         | 66.9            | 2838.7±57.4    | 0.9534         | 47.53        | 93.7±1.9     |
| 2.5000         | 1.10            | 55.3±8.0       | 0.9741         | 55.13        | 93.8±13.6    |
| 2.6444         | 33.7            | 1819.9±47.0    | 1.0044         | 58.92        | 91.2±2.4     |
| 2.6646         | 34.0            | 1817.6±47.1    | 1.0049         | 58.77        | 90.5±2.3     |
| 2.7000         | 1.03            | 44.2±7.3       | 1.0173         | 60.40        | 69.6±11.5    |
| 2.8000         | 1.01            | 37.2±7.3       | 1.0424         | 62.50        | 56.6±11.1    |
| 2.9000         | 105             | 4366.4±76.1    | 1.0686         | 62.22        | 62.4±1.2     |
| 2.9500         | 15.9            | 629.1±29.5     | 1.0799         | 61.43        | 59.5±2.8     |
| 2.9810         | 13.7            | 555.6±28.1     | 1.0864         | 61.98        | 51.4±2.6     |
| 3.0000         | 15.9            | 557.3±28.1     | 1.0860         | 62.17        | 52.0±2.6     |
| 3.0200         | 17.3            | 591.4±29.2     | 1.0854         | 62.21        | 50.7±2.5     |
| 3.0800         | 126             | 3693.7±73.1    | 1.0185         | 60.59        | 47.4±0.9     |

| $\sqrt{s}$ (GeV) | $\mathcal{L}$ (pb$^{-1}$) | $N_{\text{obs}}$ | $(1 + \delta)$ (e%) | $\sigma^B$ (pb) |
|----------------|-----------------|----------------|----------------|--------------|
| 2.1000         | 12.2            | 12.9±6.1       | 0.8346         | 5.7          | 45.3±21.4    |
| 2.1250         | 109             | 309.6±31.5     | 0.8555         | 9.6          | 70.6±7.2     |
| 2.1500         | 2.84            | 15.8±5.9       | 0.8714         | 13.7         | 94.7±35.4    |
| 2.1750         | 10.6            | 84.5±15.6      | 0.8835         | 18.8         | 97.3±18.0    |
| 2.2000         | 13.7            | 137.7±18.7     | 0.8898         | 21.7         | 105.8±14.4   |
| 2.2324         | 11.9            | 260.0±22.3     | 0.8543         | 27.2         | 191.8±16.5   |
| 2.3094         | 21.1            | 682.3±28.0     | 0.9388         | 40.82        | 84.4±3.5     |
| 2.3864         | 22.5            | 934.6±32.0     | 0.9515         | 46.78        | 93.1±3.2     |
| 2.9600         | 66.9            | 2838.7±57.4    | 0.9534         | 47.53        | 93.7±1.9     |
| 2.5000         | 1.10            | 55.3±8.0       | 0.9741         | 55.13        | 93.8±13.6    |
| 2.6444         | 33.7            | 1819.9±47.0    | 1.0044         | 58.92        | 91.2±2.4     |
| 2.6646         | 34.0            | 1817.6±47.1    | 1.0049         | 58.77        | 90.5±2.3     |
| 2.7000         | 1.03            | 44.2±7.3       | 1.0173         | 60.40        | 69.6±11.5    |
| 2.8000         | 1.01            | 37.2±7.3       | 1.0424         | 62.50        | 56.6±11.1    |
| 2.9000         | 105             | 4366.4±76.1    | 1.0686         | 62.22        | 62.4±1.2     |
| 2.9500         | 15.9            | 629.1±29.5     | 1.0799         | 61.43        | 59.5±2.8     |
| 2.9810         | 13.7            | 555.6±28.1     | 1.0864         | 61.98        | 51.4±2.6     |
| 3.0000         | 15.9            | 557.3±28.1     | 1.0860         | 62.17        | 52.0±2.6     |
| 3.0200         | 17.3            | 591.4±29.2     | 1.0854         | 62.21        | 50.7±2.5     |
| 3.0800         | 126             | 3693.7±73.1    | 1.0185         | 60.59        | 47.4±0.9     |
The combined detection efficiency is given by
\[ \phi \text{f}_K^+K^- \text{PHSP MC} \text{ and the solid (red) histograms are the weighted MC samples.} \]

by a QED calculation [30] and by taking the line shape of the Born cross section measured by the \textit{BABAR} experiment into account. The vacuum polarization factor \((1 + \delta')\) is taken from a QED calculation with an accuracy of 0.5% [31], and \(e\) is the detection efficiency. The branching fraction of the intermediate process \(\phi \to K^+K^-\) \((49.2 \pm 0.5\%)\) [25] is taken into account in the determination of the cross section of \(e^+e^- \to \phi K^+K^-\).

Both \(e\) and \((1 + \delta)\) are obtained from MC simulations of the signal reaction for each c.m. energy. In the \textsc{conexc} generator, the cross section for the ISR process \((\sigma_{e^+e^-\to\gamma\chi})\) is parametrized using
\[ \sigma_{e^+e^-\to\gamma\chi} = \int d\sqrt{s'} \frac{2\sqrt{s'}}{s} W(s,x) \frac{\sigma^B(\sqrt{s'})}{[1 - \Pi(\sqrt{s'})]^2}, \]
where \(\sqrt{s'}\) is the effective c.m. energy of the final state with \(s' = s(1-x)\), \(x\) depends on the energy of the radiated photon according to \(x = 2E_{\gamma}/\sqrt{s}\), \(W(s,x)\) is the radiator function and \(\Pi(\sqrt{s'})\) describes the vacuum polarization.
(VP) effect. The latter includes contributions from leptons and quarks. The detection efficiency and the radiative-correction factor depend on the input cross section, and are determined by an iterative procedure, in which the line shape of the cross section from BABAR is used initially, and the updated Born cross section is obtained according to the simulation. We repeat the procedure until the measured Born cross section does not change by more than 0.5%.

The values of \( \mathcal{L} \), \( N_{\text{obs}} \), \( (1 + \delta) \) and \( \epsilon \) are listed in Table I, together with the measured cross section at each energy point. Figures 7(a) and 7(b) show the line shapes of cross sections for \( e^+e^- \rightarrow K^+K^-K^+K^- \) and \( e^+e^- \rightarrow \phi K^+K^- \), respectively.

**VII. SYSTEMATIC UNCERTAINTY**

Several sources of systematic uncertainties are considered in the measurement of the Born cross sections. These include the luminosity measurements, the differences between the data and the MC simulation for the tracking efficiency, PID efficiency, kinematic fit, the fit procedure, the MC simulation of the ISR-correction factor and the vacuum-polarization factor, as well as uncertainties in the branching fractions of the decays of intermediate states.

(a) Luminosity: The integrated luminosity of the data samples used in this analysis are measured using large-angle Bhabha scattering events, and the corresponding uncertainties are estimated to be 1.0% [18].

(b) Tracking efficiency: The uncertainty of the tracking efficiency is investigated using a control sample of the \( e^+e^- \rightarrow K^+K^-\pi^+\pi^- \) process [32]. The difference in tracking efficiency between data and the MC simulation is estimated to be 1% per track. Hence, 3.0% is taken as the systematic uncertainty for the three selected kaons.

(c) PID efficiency: To estimate the uncertainty in the PID efficiency, we study \( K^\pm \) PID efficiencies with the same control samples as those used in the tracking efficiency. The average difference in PID efficiency between data and the MC simulation is found to be 1% per charged track. Therefore, 3.0% is taken as the systematic uncertainty for the three selected kaons.

(d) Kinematic fit: The uncertainty associated with the kinematic fits comes from the inconsistency of the track helix parameters between data and the MC simulation. The helix parameters for the charged tracks of MC samples are corrected to eliminate the inconsistency, as described in Ref. [33], and the agreement of \( \chi^2 \) distributions between data and the MC simulation is significantly improved. We take the differences of the selection efficiencies with and without the correction as the systematic uncertainties.

(e) Fit procedure: A fit to mass spectrum of the recoiling kaon is performed to determine the signal yields of the \( e^+e^- \rightarrow K^+K^-K^+K^- \) process, and the two kaon invariant mass \( M(K^+K^-) \) is fitted to determine the number of \( e^+e^- \rightarrow \phi K^+K^- \) events. The following three aspects are considered when evaluating the systematic uncertainty associated with the fit procedure.

(1) **Fit range:** The \( M(K^\pm) \) spectrum of the recoiling kaon is fitted by varying the range from \( (0.3, 0.7) \) GeV/c\(^2\) to \( (0.31, 0.69) \) GeV/c\(^2\). The \( M(K^+K^-) \) spectrum is fitted in the region from 0.98 to 1.15 GeV/c\(^2\). An alternative fit range, from 0.98 to 1.20 GeV/c\(^2\), is considered. The differences between the yields are treated as the systematic uncertainty from the fit range.

(2) **Signal shape:** The signal shape of the mass spectrum of the recoiling kaon is described by a shape obtained from a MC simulation convolved with a Gaussian function. The uncertainty related to this line shape is estimated with an alternative fit using the same line-shape function, but fixing the width of the Gaussian function to a value differing by one standard deviation from the width obtained in the nominal fit. The signal shape of the \( \phi \) is described by...
a \textit{P}-wave Breit-Wigner function convolved with a Gaussian function. An alternative fit with a MC shape convolved with a Gaussian function is performed. The difference in yield between the various fits is considered as the systematic uncertainty from the signal shape.

(3) Background shape: The background shape of the mass spectrum for the recoiling kaon is described as a second-order Chebyshev polynomial function. A fit with a first-order Chebyshev polynomial function for the background shape is used to estimate its uncertainty. The

### TABLE III. Relative systematic uncertainties (in \%) for the cross section of $e^+e^- \rightarrow K^+K^-K^+K^-$. The uncertainties are associated with the luminosity ($L$), tracking efficiency (Tracking), PID efficiency (PID), fit range (Range), signal and background shape (Sig. shape and Bkg. shape), the initial-state radiation factor (ISR), the vacuum-polarization correction factor (VP), the weighted detection efficiency ($\epsilon$), MC statistics (MC) and others. The total uncertainty is obtained by summing the individual contributions in quadrature.

| $\sqrt{s}$ (GeV) | $L$ | Tracking | PID | Range | Sig. shape | Bkg. shape | ISR | VP | $\epsilon$ | MC | Others | Total |
|------------------|-----|----------|-----|-------|------------|------------|-----|----|--------|-----|--------|-------|
| 2.1000           | 1.0 | 3.0      | 3.0 | 3.2   | 0.3        | 3.2        | 0.1 | 0.5| 2.3    | 1.2 | 1.0    | 6.9   |
| 2.1250           | 1.0 | 3.0      | 3.0 | 0.8   | 1.9        | 0.1        | 0.5 | 0.5| 0.6    | 0.9 | 1.0    | 5.1   |
| 2.1500           | 1.0 | 3.0      | 3.0 | 3.8   | 1.6        | 4.4        | 0.7 | 0.5| 2.6    | 0.7 | 1.0    | 8.0   |
| 2.1750           | 1.0 | 3.0      | 3.0 | 1.9   | 7.3        | 0.3        | 0.5 | 0.5| 1.3    | 0.6 | 1.0    | 8.9   |
| 2.2000           | 1.0 | 3.0      | 3.0 | 0.1   | 0.6        | 7.6        | 0.5 | 0.5| 1.1    | 0.5 | 1.0    | 9.0   |
| 2.2324           | 1.0 | 3.0      | 3.0 | 0.7   | 0.1        | 0.6        | 0.4 | 0.5| 0.7    | 0.5 | 1.0    | 4.7   |
| 2.3094           | 1.0 | 3.0      | 3.0 | 1.4   | 2.1        | 4.5        | 0.4 | 0.5| 0.6    | 0.4 | 1.0    | 6.9   |
| 2.3864           | 1.0 | 3.0      | 3.0 | 0.2   | 0.0        | 1.2        | 0.0 | 0.5| 0.5    | 0.3 | 1.0    | 4.7   |
| 2.3960           | 1.0 | 3.0      | 3.0 | 3.5   | 3.5        | 4.5        | 0.4 | 0.5| 0.3    | 0.3 | 1.0    | 7.7   |
| 2.5000           | 1.0 | 3.0      | 3.0 | 0.8   | 0.3        | 2.7        | 0.3 | 0.5| 2.0    | 0.3 | 1.0    | 5.7   |
| 2.6444           | 1.0 | 3.0      | 3.0 | 0.3   | 0.1        | 0.7        | 0.1 | 0.5| 0.3    | 0.3 | 1.0    | 4.6   |
| 2.6464           | 1.0 | 3.0      | 3.0 | 0.0   | 0.1        | 0.2        | 0.5 | 0.5| 0.3    | 0.3 | 1.0    | 4.5   |
| 2.7000           | 1.0 | 3.0      | 3.0 | 0.2   | 0.2        | 7.5        | 0.3 | 0.5| 1.7    | 0.3 | 1.0    | 8.9   |
| 2.8000           | 1.0 | 3.0      | 3.0 | 1.1   | 1.9        | 3.8        | 0.3 | 0.5| 1.8    | 0.2 | 1.0    | 6.5   |
| 2.9000           | 1.0 | 3.0      | 3.0 | 0.4   | 0.2        | 0.5        | 0.0 | 0.5| 0.1    | 0.2 | 1.0    | 4.6   |
| 2.9500           | 1.0 | 3.0      | 3.0 | 0.9   | 0.4        | 0.8        | 0.3 | 0.5| 0.4    | 0.3 | 1.0    | 4.7   |
| 2.9810           | 1.0 | 3.0      | 3.0 | 0.2   | 0.7        | 1.6        | 0.1 | 0.5| 0.4    | 0.2 | 1.0    | 4.9   |
| 3.0000           | 1.0 | 3.0      | 3.0 | 0.4   | 0.7        | 0.4        | 0.2 | 0.5| 0.3    | 0.2 | 1.0    | 4.6   |
| 3.0200           | 1.0 | 3.0      | 3.0 | 1.9   | 0.8        | 1.1        | 0.0 | 0.5| 0.3    | 0.2 | 1.0    | 5.1   |
| 3.0800           | 1.0 | 3.0      | 3.0 | 0.8   | 0.3        | 0.1        | 0.0 | 0.5| 0.1    | 0.3 | 1.0    | 4.6   |

### TABLE IV. Summary of relative systematic uncertainties (in \%) related to the cross section measurements of $e^+e^- \rightarrow \phi K^+K^-$. See Table III for a description of the various items. $B$ refers to the uncertainty in the branching fraction $\phi \rightarrow K^+K^-$. The...
background shape for \( \phi \)-mass distribution is described by an ARGUS function. The fit with a function of \( f(M) = (M - M_\text{c})^c (M_\text{b} - M)^d \), where, \( M_\text{c} \) and \( M_\text{b} \) are the lower and upper edges of the mass distribution, is used to estimate this uncertainty.

(f) **ISR factor:** The cross section is measured by iterating until \((1 + \sigma) e \) converges, and the difference between the last two iterations is taken as the systematic uncertainty associated with the ISR-correction factor.

(g) **VP factor:** The uncertainty on the calculation of the VP factor is 0.5% [31].

(h) **Branching fraction:** The experimental uncertainties in the branching fraction for the process \( \phi \rightarrow K^+K^- \) are taken from the PDG [25].

(i) **Weighted detection efficiency:** The detection efficiencies obtained in different processes are combined using the previously described method. The combined uncertainty is calculated by accounting for the statistical variation, by one standard deviation, of the signal yields.

To obtain a reliable detection efficiency of \( e^+e^- \rightarrow \phi K^+K^- \), the PHSP MC sample is weighted to match the distribution of the background-subtracted data. To consider the effect on the statistical fluctuations of the signal yield in the data, a set of toy-MC samples, which are produced by sampling the signal yield and its statistical uncertainty of the data in each bin, are used to estimate the detection efficiencies.

(j) **MC statistics:** The uncertainty is estimated by the number of the generated events, whereby the weighting factor has been taken into account.

(k) **Other systematic uncertainties:** Other sources of systematic uncertainties include the trigger efficiency, the determination of the start time of an event, and the modeling of the final-state radiation in the simulation. The total systematic uncertainty due to these sources is estimated to be less than 1.0%. To be conservative, we take 1.0% as its systematic uncertainty.

Assuming all of the above systematic uncertainties, shown in Tables III and IV, are independent, the total systematic uncertainties are obtained by adding the individual uncertainties in quadrature.

**VIII. SUMMARY AND DISCUSSION**

In summary, using data collected with the BESIII detector taken at twenty c.m. energies from 2.100 to 3.080 GeV, we present measurements of the processes \( e^+e^- \rightarrow K^+K^-K^+K^- \) and \( \phi K^+K^- \) and we obtain the corresponding Born cross sections. The Born cross sections of the process \( e^+e^- \rightarrow K^+K^-K^+K^- \) are in good agreement with the results by BABAR, but with improved precision. The Born cross sections for the channel \( e^+e^- \rightarrow \phi K^+K^- \) are measured for the first time at twenty energy points. Both data sets reveal anomalously high cross sections at \( \sqrt{s} = 2.232 \) GeV.

A previous analysis on a much smaller dataset [17] has demonstrated that the \( K^+K^-K^+K^- \) final state exhibits resonant substructure. It is difficult to disentangle these contributions from other final states, and we make no attempt to do so.

By examining the \( \phi K^+K^- \) cross section as a function of c.m. energy, an enhancement at \( \sqrt{s} = 2.232 \) GeV, i.e., near the \( \Lambda \Lambda \) production threshold, is observed. The cross section of \( e^+e^- \rightarrow \Lambda \Lambda \) is also found to be anomalously high at the threshold [34]. In the case of charged baryons one would expect a Coulomb enhancement factor, which, however, is absent in the of the electrically neutral \( \Lambda \). It has been suggested that a narrow resonance, very close to the threshold, might provide an explanation [35]. BABAR has observed an enhancement at 2.175 GeV and a sharp peak at 2.3 GeV, corresponding to \( \phi K^+K^- \) final states with \( K^+K^- \) invariant masses smaller than 1.06 GeV/c\(^2\) and within a mass interval of 1.06–1.2 GeV/c\(^2\), respectively. The intriguing \( \phi(2170) \) resonance [13] has a relatively wide width and it is very close to the kinematical threshold, but not close enough to be related to the observed anomaly. Alternatively, the enhancement at 2.232 GeV could be explained by an interference effect of different resonances. More data in the vicinity would be helpful to understand the anomaly.

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