AN ANALYSIS OF JITTER AND TRANSIT TIMING VARIATIONS IN THE HAT-P-13 SYSTEM

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Abstract

If the two planets in the HAT-P-13 system are coplanar, the orbital states provide a probe of the internal planetary structure. Previous analyses of radial velocity and transit timing data for the system suggested that the observational constraints on the orbital states were rather small. We reanalyze the available data, treating the jitter as an unknown MCMC parameter, and find that a wide range of jitter values are plausible, hence the system parameters are less well constrained than previously suggested. For slightly increased levels of jitter ($\sim 4.5 \text{ m s}^{-1}$), the eccentricity of the inner planet can be in the range $0 < e_{\text{inner}} < 0.07$, the period and eccentricity of the outer planet can be $440 \text{ days} < P_{\text{outer}} < 470 \text{ days}$ and $0.55 < e_{\text{outer}} < 0.85$, respectively, while the relative pericenter alignment, $\eta$, of the planets can take essentially any value $-180^\circ < \eta < +180^\circ$. It is therefore difficult to determine whether $e_{\text{inner}}$ and $\eta$ have evolved to a fixed-point state or a limit cycle or to use $e_{\text{inner}}$ to probe the internal planetary structure. We perform various transit timing variation (TTV) analyses, demonstrating that current constraints merely restrict $e_{\text{outer}} < 0.85$, and rule out relative planetary inclinations within $\sim 2^\circ$ of $i_{\text{rel}} = 90^\circ$, but that future observations could significantly tighten the restriction on both these parameters. We demonstrate that TTV profiles can readily distinguish the theoretically favored inclinations of $i_{\text{rel}} = 0^\circ$ and $45^\circ$, provided that sufficiently precise and frequent transit timing observations of HAT-P-13b can be made close to the pericenter passage of HAT-P-13c. We note the relatively high probability that HAT-P-13c transits and suggest observational dates and strategies.

Key words: celestial mechanics – methods: data analysis – methods: numerical – methods: observational – planetary systems

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1. INTRODUCTION

The HAT-P-13 system (Bakos et al. 2009) was the first extrasolar planetary system to be discovered in which both a transiting planet and an additional confirmed companion were known to coexist. Since this initial discovery, further transit-plus-companion systems have been discovered: CoRoT-7 (Queloz et al. 2009) and HAT-P-7 (Narita et al. 2010) as well as the recent multi-transit systems from Kepler (Steffen et al. 2010; Holman et al. 2010), while a number of other transiting systems display radial velocity (RV) trends symptomatic of outer companions (e.g., HAT-P-11; Bakos et al. 2010).

Such systems are of interest for a number of reasons. The first relates to the observable effects which arise from interactions between the two planets. The gravitational interaction between multiple planets causes the planetary orbits to be perturbed away from Keplerian ellipses. When one of the planets is transiting, these perturbations mean that the duration of, and the period between, successive transits will not be strictly constant. It has been calculated that observations of such transit timing variations (TTVs) and transit duration variations (TDVs) would allow the (inferred) detection of terrestrial-mass planets in hot-Jupiter systems (Holman & Murray 2005; Agol et al. 2005), trojan planets (Ford & Holman 2007), and exoplanet moons (Kipping 2009a, 2009b).

In addition, accurate measurements of transiting systems can allow us to observationally determine a huge range of system parameters (Winn 2009), one such parameter being the tidal Love number of the planet (if the system architecture is convenient—Wu & Goldreich 2002; Marling 2007; Ragazzzini & Wolf 2009). In an investigation by Batygin et al. (2009), it is demonstrated for the coplanar case that the HAT-P-13 system is indeed such a system, and a relationship is found between the Love number and the eccentricity of the inner planet.

However, the determination of the Love number depends on the assumption that the inner planet tends toward a quasi-fixed point in $(e_{\text{inner}}, \eta)$ space, where $e_{\text{inner}}$ is the eccentricity of the inner planet and $\eta = \sigma_{\text{outer}} - \sigma_{\text{inner}}$ is the difference in the alignment of the longitude of pericenters of the outer and inner planets. The recent work by Marling (2010) looked at the evolution of a general non-coplanar, two-planet system, in which the angular momentum of the outer planet dominates the angular momentum budget of the system, and revealed that if such a system the inner planet does not tend to a fixed point, but instead tends to a limit cycle, with $e_{\text{inner}}$ and $\eta$ constantly sampling along a closed trajectory.

The approach to the limit cycle is found to be strongly dependent on the relative inclination between the two planets, $i_{\text{rel}}$: the average value of $e_{\text{inner}}$ around the cycle decreases and the limit cycle amplitude increases with increasing $i_{\text{rel}}$. Limit cycle behavior only exists for $0^\circ < i_{\text{rel}} < 33^\circ$, $46^\circ < i_{\text{rel}} < 54^\circ$, $126^\circ < i_{\text{rel}} < 134^\circ$, and $147^\circ < i_{\text{rel}} < 180^\circ$. For the regions $33^\circ < i_{\text{rel}} < 46^\circ$ and $134^\circ < i_{\text{rel}} < 147^\circ$, $\eta$ circulates and no limit cycle exists. For the region $54^\circ < i_{\text{rel}} < 126^\circ$, the effects of Kozai oscillations combined with tidal dissipation act to move the relative inclinations back toward $i_{\text{rel}} = 54$ or $i_{\text{rel}} = 126$ for prograde or retrograde orbits, respectively, meaning that the system simply cannot exist with $54^\circ < i_{\text{rel}} < 126^\circ$ for a tidal dissipation factor, $Q < 10^6$. For the range of possible $e_{\text{inner}}$ and $R_{\text{inner}}$ (the radius of the inner planet) reported by Batygin et al. (2009), certain relative inclinations can be ruled out, suggesting that (for the prograde case) the system either has $i \lesssim 10^\circ$ or $i \sim 45^\circ$. 


It is apparent from the analyses in both Mardling (2010) and Batygin et al. (2009) that the conclusions one can derive regarding the possible state of the system rather sensitively depend on (1) the measured eccentricity of the inner planet, (2) the relative pericenter of the two planets, and (3) the essentially unknown relative inclination between the two planets. It is our aim in this paper to try and understand in more detail what the current observational constraints on these quantities are.

We start by re-evaluating the published system parameters for HAT-P-13 from Bakos et al. (2009, henceforth B09) as well as those from the expanded analysis by Winn et al. (2010, henceforth W10), performing a Markov Chain Monte Carlo (MCMC) investigation that concentrates on the effect of assumptions about jitter and how to include it within an MCMC analysis. We use jitter as a mathematical parameter to quantify the magnitude of unmodeled variations in the RV observations. Jitter can be due to undetected planets, stellar activity, or unrecognized noise in the instrument or data analysis pipeline. In the case of a complete model for the planetary system and ideal measurements, the jitter would reduce to the "stellar jitter." Stellar jitter is the noise introduced into RV measurements by unknown changes in the surface of the observed star, driven by sun spots, bulk flows, and other inhomogeneities on the stellar surface. Several investigations have examined this phenomenon, trying to correlate the magnitude of the jitter with observable stellar parameters (Saar & Donahue 1997; Saar et al. 1998; Wright 2005). They find that stars of a given type can have a wide range of jitter values, with stars similar to HAT-P-13 having jitters in the range 2–15 m s\(^{-1}\) (see Wright 2005 and Section 3.1 for further details).

In the papers of B09 and W10, subsequent to the MCMC analysis of the fitted planetary parameters, jitter levels of \(\sigma_j = 3.0\) m s\(^{-1}\) and \(\sigma_j = 3.4\) m s\(^{-1}\), respectively, were required in order to give reduced \(\chi^2\) values of 1 (see Section 3 for further discussion). We wish to understand the effects of changing this methodology and in particular to find out to what degree including the jitter as an MCMC model parameter loosens the constraints on quantities such as the eccentricity of the inner planet and the pericenter alignment of the two planets.

In a rigorous Bayesian analysis, the jitter should be treated as a model parameter simultaneously with the mass and orbital parameters. In this paper, we show that correlations between the jitter and orbital parameters can lead to erroneous inferences if the jitter is held fixed during the modeling process. Thus, one should include the jitter in MCMC analyses alongside all of the other parameters that one is attempting to model. In this manner, one can arrive at a consistent statistical interpretation of one’s knowledge of the observables in the system. In a Bayesian analysis, one must explicitly state assumptions for the prior distributions. While unmodeled observations (e.g., stellar color or temperature) can influence the choice of prior (e.g., F stars are more likely to have a jitter exceeding 10 m s\(^{-1}\)), the choice of prior for the jitter must not be influenced by the RVs themselves. For example, choosing a prior for the jitter based on the residuals to a fit results in double counting the RV data and can result in misleadingly small uncertainties.

Further to this MCMC investigation, we go on to add an analysis of the TTVs for HAT-P-13. We do this to try and ascertain whether (1) combining the MCMC analysis with the TTV analysis can further restrict the range of parameter space available to the observed system quantities (eccentricities, alignments, etc.) and (2) we wish to understand whether the TTVs can provide some insight into the relative inclination of the planets (relative planetary inclinations have previously been shown to be important in determining the expected TTVs in some systems; Nesvorný 2009; Payne et al. 2010).

We proceed in this paper as follows. In Section 2, we outline the numerical methods we use to conduct our MCMC and TTV analyses. In Section 3, we look at the effects of jitter in an MCMC analysis of the orbital elements of the planets in the HAT-P-13 system and combine this with TTV constraints to understand whether this allows a more nuanced determination of the system parameters. In Section 4, we look more generally at the TTVs in the HAT-P-13 system, focusing in detail on the effects that relative planetary inclinations may have on the expected TTVs. After this, we move on in Section 5 to consider the potential for future observations (both transit and RV) to better constrain the planetary orbits. Finally, in Section 6 we present a summary and discussion of our conclusions.

### 2. METHODOLOGY

#### 2.1. Radial Velocity and Transit Observation Analysis

We analyze the RV and transit observations using a Bayesian framework following Ford (2005, 2006). We assume priors that are uniform in log of orbital period, eccentricity, argument of pericenter, mean anomaly at epoch, and the velocity zero point. For the velocity amplitude (\(K\)) and jitter (\(\sigma_j\)), we adopt a prior of the form \(p(x) = (x + x_o)^{-1}\log(1 + x/x_o))^{-1}\), with \(K_o = \sigma_j,o = 1\) m s\(^{-1}\), i.e., high values are penalized. For a discussion of priors, see Ford & Gregory (2007). We adopt a likelihood that is the product of two terms corresponding to the RV and transit observations. The likelihood for RV terms assumes that each RV measurement uncertainty.

Instead of modeling each photometric observation, we account for the transit observations by including a likelihood term that is the product of three Bayesian penalties based on the orbital period, transit duration, and ingress time of HAT-P-13b, assuming Gaussian distributions for each measurement with standard deviations taken from the published uncertainties as derived by B09. We use MCMC to calculate a sample from the posterior distribution (Ford 2006). We calculate multiple Markov chains, each with \(\sim 2 \times 10^8\) states. We discard the first half of the chains and calculate Gelman–Rubin test statistics for each model parameter and several ancillary variables. We find no indications of non-convergence. Thus, we randomly choose a subsample (10,000 samples, large enough to give a statistically valid and accurate outcome, but small enough to be computationally tractable) from the posterior distribution for further investigation.

#### 2.2. Transit Timing Variations

To investigate the TTV signature in the HAT-P-13 system, we employ the same basic method as that used in Payne et al. (2010) and Veras et al. (2011). We consider a fiducial system consisting of two planets: a transiting hot-Jupiter planet and an outer, non-transiting planet which perturbs the transit times of the inner planet. Given an initial specification of the planetary masses and orbital elements, we evolve each system forward in time for \(\sim 3.5\) years, corresponding to several hundred transits by the inner planet in a system such as HAT-P-13 (assuming that the inner planet remains transiting throughout the integration). This 3.5 year integration time was also used in the studies of Payne et al. (2010) and Veras et al. (2011) whose investigations
attempted to illuminate potential Kepler mission observations, where the Kepler mission is expected to run for at least 3.5 years. To facilitate any comparison with the method and results of these papers, we chose to also maintain this 3.5 year simulation timescale.

The n-body integrations are performed using a conservative Bulirsch–Stoer integrator, derived from that of MERCURY (Chambers 1999). We use a barycentric coordinate system and limit the time steps to no more than 0.04 times the orbital period. After each time step, we test whether the star–planet separation projected onto the sky (Δ) passed through a local minimum and the planet in question is closer to the observer than the star. Each time these conditions are met, we find the nearby time that minimizes Δ via Newton–Raphson iteration and increment an index ı. If the minimum Δ is less than the stellar radius, then we record the mid-time of the transit, ti. In calculating the observed transit time, one needs to account for the light travel time, δti(i) = −(rpi(t) · ɾbary) / c, where rpi(t) is the barycentric vector of the planet at time ti, ɾbary is a unit vector pointing to the observer, and c is the speed of light. The observable transit time variations are calculated as δt(i) = ti + δti(i) − i P − t0, where the constants P and t0 are determined by linear least-squares minimization of Σi(δt(i)−i P − t0)2. We neglect any motion of the stellar center between the time of light emission and the time of transit.

It should be noted that the TTV investigations in this paper split into two main strands: (1) an investigation in Section 3.2 of the TTVs that would arise in the systems that come out of our RV MCMC analysis and (2) the structured investigation of inclination effects in TTVs for HAT-P-13 carried out in Section 4. The introduction to the respective result sections provides more detail on the precise manner in which each TTV investigation was conducted.

3. MCMC ANALYSIS OF THE RADIAL VELOCITY OBSERVATIONS

3.1. Jitter

In the B09 discovery paper, upon performing an RV fit (in conjunction with an analysis of the observed transit timing constraints) they found that a jitter of 3.0 m s⁻¹ was required in order to give a reduced χ² figure of 1.0. Similarly, W10 found that a jitter of 3.4 m s⁻¹ was required in order to give a reduced χ² figure of 1.0. In the work of Wright (2005), it was found that from a sample of ~30 stars similar to HAT-P-13 (mass, Me = 1.22 Me) that the distribution of jitters was such that the 20th, 50th, and 80th percentiles were found to be 2.6, 4.0, and 6.2 m s⁻¹, respectively, with an upper bound of ~15 m s⁻¹. It is thus plausible that the actual jitter value for HAT-P-13 is substantially different from the value of ~3.0 m s⁻¹ used in B09 and W10. As discussed in the introduction, we feel that it is important that the jitter be placed on an equal footing and analyzed in the same manner as all of the other parameters that one can constrain using the RV data. As such, we include the jitter as a prior in our MCMC analysis (see Section 2.1 for details). We illustrate in Figure 1 the sensitivity of a number of the key parameters to the value of the jitter that is used in the MCMC analysis.

To illustrate the general effect of including jitter as an integral part the MCMC analysis, we look at two different data cuts for HAT-P-13: the first set (S1) uses only the RV data that were included in the discovery paper of B09, while the second set (S2) uses the larger data set used in the later paper of W10. We have analyzed these data sets in such a manner as to afford direct comparison with the original papers: we analyze S1 assuming a two-planet solution (as was done in B09), while we analyze S2 by allowing for the possibility of an additional “slope” in the RV data (see Equation (1) of W10). Additionally, for S2 we also perform a full three-planet analysis. We plot some of the results of these MCMC analyses in Figure 1.

In general, it can be observed from the various plots in Figure 1 that the MCMC routine investigates a wide range of jitter levels and that (as one might expect) the larger the level of jitter it tries, the wider the range of planetary parameters it can accommodate. However, we see that this process is not completely unlimited, as there are few points in any of the plots which have jitters above 30 m s⁻¹, and the majority of them are significantly below this level.

In more detail, we see from the left-hand column of Figure 1 (in which the S1 results are plotted) that if we restrict ourselves to examining jitter ranges 2.0 m s⁻¹ < σj < 3.0 m s⁻¹ (see the green crosses in the print version) in Figure 1 which label regions with σj = 3.0 m s⁻¹ ± 50%, the eccentricity of the inner planet, einner, is constrained to be 0 < einner < 0.07 (best-fit value of einner = 0.017 ± 0.009), where the uncertainty figures cover 68% of the data, i.e., they equate to 1σ error bars), while the difference in the alignment of the longitude of pericenters, ηouter − ηinner, can take on essentially the full range of available values: −180° < η < +180° (best-fit value of η = 15.9° ± 0.6°). If we allow a higher range of possible jitter, then the available parameter space expands, such that if we take all of the MCMC results with 0 m s⁻¹ < σj < 10 m s⁻¹, the eccentricity of the inner planet could take any value in the range 0 < einner < 0.15 (best-fit value of einner = 0.021 ± 0.012), while the eccentricity, eouter, and period, Pouter, of the outer planet also expand to encompass ranges of 0.6 < eouter < 0.95 (best-fit value of eouter = 0.75 ± 0.12) and 410 days < Pouter < 450 days (best-fit value of Pouter = 430.2 ± 3.1 days, respectively). Interestingly, we note that at such high values of σj, the alignment of the longitude of pericenters starts to favor orthogonal values, η ≈ ±90°.

The middle column in Figure 1 details the results of the two-planet + slope analysis of S2, i.e., analogous to the analysis of W10. We see that there are now very few systems in the analysis which have jitter levels below 5 m s⁻¹. If we restrict our attention to the very few systems which have σj = 3.0 m s⁻¹ ± 50%, the eccentricity of the inner planet, einner, is constrained to be 0 < einner < 0.09 (best-fit value of einner = 0.038 ± 0.022), a value very much larger than that found in S1 (or indeed, in the analysis of W10). If we allow a higher range of possible jitter, then the available parameter space expands, such that if we take all of the MCMC results with 0 m s⁻¹ < σj < 10 m s⁻¹, the eccentricity of the inner planet could take any value in the range 0 < einner < 0.2 (best-fit value of einner = 0.06 ± 0.032), while the eccentricity, eouter, and period, Pouter, of the outer planet also expand to encompass ranges of 0.45 < eouter < 0.9 (best-fit value of eouter = 0.64 ± 0.03) and 430 days < Pouter < 490 days (best-fit value of Pouter = 450.6 ± 8.4 days, respectively).

Finally, in the third column of Figure 1, we display the results from our three-planet analysis of S2. The results pertaining to einner, eouter, Pouter, and η as displayed are qualitatively very similar to those of the two-planet + slope analysis, as we find that for σj = 3.0 m s⁻¹ ± 50% the best-fit values are einner = 0.087 ± 0.06, eouter = 0.61 ± 0.15, and Pouter = 455.0 ± 18.0 days. The middle column in Figure 1 details the results of the two-planet + slope analysis of S2, i.e., analogous to the analysis of W10. We see that there are now very few systems in the analysis which have jitter levels below 5 m s⁻¹. If we restrict our attention to the very few systems which have σj = 3.0 m s⁻¹ ± 50%, the eccentricity of the inner planet, einner, is constrained to be 0 < einner < 0.09 (best-fit value of einner = 0.038 ± 0.022), a value very much larger than that found in S1 (or indeed, in the analysis of W10). If we allow a higher range of possible jitter, then the available parameter space expands, such that if we take all of the MCMC results with 0 m s⁻¹ < σj < 10 m s⁻¹, the eccentricity of the inner planet could take any value in the range 0 < einner < 0.2 (best-fit value of einner = 0.06 ± 0.032), while the eccentricity, eouter, and period, Pouter, of the outer planet also expand to encompass ranges of 0.45 < eouter < 0.9 (best-fit value of eouter = 0.64 ± 0.03) and 430 days < Pouter < 490 days (best-fit value of Pouter = 450.6 ± 8.4 days, respectively).
Figure 1. Sensitivity of selected system parameter determinations to assumptions regarding the jitter. The top row has $e_{\text{outer}}$ vs. jitter, the second row has $P_{\text{outer}}$ vs. jitter, the third row has $\eta = \omega_{\text{outer}} - \omega_{\text{inner}}$ vs. jitter, and the bottom row has goodness-of-fit vs. jitter. In the left-hand column, we present results using only the subset of data known at the time of publication by B09, assuming a two-planet fit. In the center and right-hand columns, we present results obtained using the full data set of W10, with the central column assuming two-planets + a linear trend, while the right-hand column assumes a three-planet fit. Systems with $\sigma_j < 2.0 \text{ m s}^{-1}$ are plotted using a red filled circle (gray in the print version), those with $2.0 \text{ m s}^{-1} < \sigma_j < 4.5 \text{ m s}^{-1}$ are plotted using a green cross (gray in the print version), and those with $\sigma_j > 4.5 \text{ m s}^{-1}$ are plotted using black “+” symbols. If one regards the jitter as being unknown and allows a range of values to be sampled by the MCMC routine, we can see that in this sample of plots for various system parameters, there is a much larger range of parameters which can plausibly fit the data than if one assumes $\sigma_j = 3.0 \text{ m s}^{-1}$. The effective chi-square measure plotted here in the final row illustrates the rather large range of fit “qualities” that the different MCMC runs result in and that the two-planet fit to B09 has rather poor fits at low jitter levels.

The goodness-of-fit parameters plotted in the final row are an effective chi-square measure, used here to illustrate the rather large range of fit “qualities” that the different MCMC runs result in. We can see that in the two-planet analysis of B09 in the left-hand column, it is particularly noticeable that the low jitter systems tend to have a worse fit (higher effective chi-square measure) than do the systems with higher jitter. The analyses
of the W10 data show a wide range of effective chi-square measures right across the jitter range sampled by the MCMC routines.

We provide two tables (Tables 1 and 2) in which we list some key statistics for the fitted values of the various system parameters arising from our respective analyses of S1 and S2. We give the median value for each parameter and then also give as uncertainty figures the spread in parameter values which are required to cover 68% of the data in the sample, i.e., corresponding approximately to the 1σ deviation figure as quoted in B09.

We discuss further some of the implications of these jitter figures in Section 3.3.

### 3.2. Combining RV and TTV Data

In their discovery paper, B09 also perform an analysis of the transit timings to look for any evidence of TTVs. They find that the TTVs in the system are restricted to \( \lesssim 100 \) s. We wish to add this constraint directly to the MCMC analysis performed in Section 3. To do this, we take a 10,000-strong subsample of the MCMC systems (see Section 2 for further discussion of methodology) and use these systems as the basis for the TTV investigation.

We take the fitted parameters for each of the subsample systems and use these to set up the planetary masses and orbits in an n-body simulation. Given that the relative inclination of the planets is unconstrained in the RV analysis, we performed the simulations assuming coplanarity. This simulation is then run in the manner described in Section 2.2 and a TTV analysis performed on the inner planet. This allows us to directly investigate whether any particular range of the MCMC subsample can be excluded by considering the observational TTV constraints. Extremely high TTV signals generally arise due to close approaches between objects in the simulation, giving a strong indication that such systems are unstable (see Veras et al. 2011 and Payne et al. 2010 for further discussion of such high TTV signals, and the likely Hill and/or Lagrange stability). However, irrespective of whether any such systems are absolutely stable or
not, we can use the fact that they have TTV amplitudes $>100$s to remove them from the analysis.

We show a sample of the results from the TTV analysis in Figure 2. On the left-hand side, we plot results for a two-planet analysis of the S1 data set, while on the right-hand side we plot results for the two-planet + slope analysis of the S2 data set. The majority of the observable parameters do not show any obvious correlation with the rms TTV amplitude (we provide some examples of such plots in the Appendix). However, the overall RV offset, as well as the RV amplitude, $K$, longitude of pericenter, $\sigma_{\text{outer}}$, and the eccentricity, $e_{\text{outer}}$, of the outer planet all have such a correlation. We could thus hope to use this upper TTV amplitude cutoff of 100 s to limit the allowed range of these variables. While this is certainly possible on a gross scale, we note that when we color code the results according to the assumed jitter (as done in Figure 2), the majority of the low jitter systems tend to fall into the low-ITV area of parameter space. This means that using the TTV amplitude constraints is probably only of some marginal help in restricting the range of plausible system parameters to be considered.

In Figure 3, we go on to plot a selection of the key orbital parameters, using colored symbols to show the regions of parameter space which are favored or ruled out by the combined jitter and TTV amplitude analyses discussed above. As in Figure 2, on the left-hand side we plot results for two-planet analysis of the S1 data set, while on the right-hand side we plot results for the two-planet + slope analysis of the S2 data set. These suggest that for the inner planet, even for a fairly tightly constrained jitter of $2.0-4.5$ m s$^{-1}$, there can be substantial variation in the eccentricity, $e_{\text{inner}}$, such that it can take any value $0 < e_{\text{inner}} < 0.07$. For the outer planet, much greater variations are possible.

(1) Perhaps the strongest result we find from using an observed constraint on the TTV amplitude of $\lesssim 100$s is constraining the eccentricity of the outer planet to be $e_{\text{outer}} < 0.85$, irrespective of assumed jitter values, giving an approximate overall range of $0.5 < e_{\text{outer}} < 0.85$ (best-fit value of $e_{\text{outer}} = 0.017^{+0.009}_{-0.003}$ assuming that the jitter is in the range $2.0-4.5$ m s$^{-1}$). This is clearly comprehensible in the light of the above argument concerning stability: extremely high eccentricity values for the
outer planet can easily lead to close approaches and/or orbit crossing. These results stand for the analyses of both S1 and S2. (2) While the longitude of pericenter of the outer planet ($\omega_{\text{outer}}$) is strongly constrained to be $\sim 180^\circ$ (best-fit value of $\sigma_{\omega_{\text{outer}}} = 177.0^{+0.7}_{-0.9}$ for S1 and $\sigma_{\omega_{\text{outer}}} = 171.2^{+6.3}_{-7.4}$ for S2), the longitude of pericenter of the inner planet can take almost any value, meaning that there is no strong preference to indicate that the planetary pericenters are aligned ($-180^\circ < \eta < +180^\circ$, the best-fit value for S1 of $\eta = 16.0^{+50.2}_{-10.2}$, while for S2 the best-fit value has $\eta = 3.2^{+64.4}_{-82.0}$). (3) In the S1 analysis, the outer planet can have periods $420\,\text{days} < P_{\text{outer}} < 440\,\text{days}$ (best-fit value of $P_{\text{outer}} = 429.7^{+4.3}_{-8.2}$), while in the S2 analysis, the period is pushed to higher values $430\,\text{days} < P_{\text{outer}} < 490\,\text{days}$ (best-fit value of $P_{\text{outer}} = 455.0^{+18.0}_{-13.7}$).

Figure 2. Understanding the constraints given by combining the TTVs and the jitter: I. TTV amplitude vs. system parameters. In the various plots above we show the rms TTV amplitude as a function of various system parameters (planetary eccentricity, etc.). As in Figure 1, systems with $\sigma_j < 2.0\,\text{m s}^{-1}$ are plotted using a red circle (gray in the print version), those with $2.0\,\text{m s}^{-1} < \sigma_j < 4.5\,\text{m s}^{-1}$ are plotted using a green “×” (gray in the print version), and those with $\sigma_j > 4.5\,\text{m s}^{-1}$ are plotted using black “+” symbols. In the left-hand column, we present results using only the subset of data known at the time of publication by B09, assuming a two-planet fit. In the right-hand column, we present results obtained using the full data set of W10, assuming two-planets + a linear trend. We can see that imposing a cut such that the TTV amplitude is $<100\,\text{s}$ helps to eliminate a sizeable proportion of the overall solutions, but note that the majority of the systems with jitter $<3.5\,\text{m s}^{-1}$ tend to naturally fall into areas which have TTV amplitudes $<100\,\text{s}$. The quantities plotted above are the only variables which have a potentially significant correlation with the rms TTV amplitude. We note that comparing the W10 data to the B09 data, once again the TTVs can be used to (approximately) constrain the eccentricity of the outer planet, but now is even worse at constraining the other parameters due to the much larger scatter.

(A color version of this figure is available in the online journal.)
Figure 3. Understanding the constraints given by combining the TTVs and the jitter: II. We plot a variety of the possible system configurations arising from the MCMC analysis and then constrain the possibilities by indicating (1) using blue squares (gray in the print version) the systems which have TTV amplitudes bigger than 100 s and then (2) in other colors indicate the range of jitter assumed for the systems: systems with \( \sigma_j < 2.0 \text{m/s} \) are plotted using a red circle (gray in the print version), those with \( 2.0 \text{m/s} < \sigma_j < 4.5 \text{m/s} \) are plotted using a green cross (gray in the print version), and those with \( \sigma_j > 4.5 \text{m/s} \) are plotted using black addition symbols. As in Figure 1, in the left-hand column we present results using only the subset of data known at the time of publication by B09, assuming a two-planet fit. In the right-hand column, we present results obtained using the full data set of W10, assuming two-planets + a linear trend. We find that the eccentricity of the outer planet is strongly constrained to be \( \lesssim 0.85 \), but that other parameters are largely unaffected/unconstrained.

(A color version of this figure is available in the online journal.)

Note that Table 1 also contains a section in which the figures are analyzed for systems with TTVs less than 100 s only. As an example of the data contained therein, it can be seen that the lower allowed range on the outer planet’s eccentricity from the TTV analysis manifests itself in the table as a reduction in both the upper uncertainty and the median value of the fitted outer eccentricity.

### 3.3. Summary of MCMC Results

Our re-analysis of the RV data and subsequent introduction of a coupled TTV analysis leads to the following conclusions.

Comparing our results with those of the HAT-P-13 discovery paper B09 (where \( \sigma_j = 3.0 \text{ m/s} \) was used), we find that a rather larger range of fits to the RV data is possible. In particular, we find (for the S1 data set used in B09) that even a rather modest level of jitter (\( 2.0 \text{ m/s} \)) will allow \( e_{\text{inner}} = 0.017^{+0.013}_{-0.009} \), \( e_{\text{outer}} = 0.73^{+0.12}_{-0.06} \), \( P_{\text{outer}} = 429.7^{+4.4}_{-3.7} \) days, and relative pericenter alignment, \( \eta = 15.9^{+5.0}_{-8.3} \).

Similarly, when we look at the S2 data set as used in W10, a level of jitter \( 2.0 \text{ m/s} \) will allow \( e_{\text{inner}} = 0.038^{+0.022}_{-0.018} \), \( e_{\text{outer}} = 0.65^{+0.10}_{-0.06} \), \( P_{\text{outer}} = 449.4^{+5.0}_{-4.8} \) days, and relative pericenter alignment, \( \eta = -12.4^{+5.2}_{-8.5} \).
Table 1 contains a summary of all the parameters and the best fits resulting from our analysis of S1, while Table 2 contains a summary of all the parameters and the best fits resulting from our two-planet + slope analysis of S2. We note that within the range of our uncertainties, all of our results are consistent with those of the discovery paper B09, but we do in general find a much broader spread of allowed values for all parameters. Similarly for the comparison of the S2 analysis with the results of W10, our results are consistent with those of W10 (except for the inner eccentricity) but the error bars on our fitted parameters are much broader. To “make the eccentricities consistent,” one has to appeal to even lower jitter values 2.0 m s\(^{-1}\) < \(\sigma_J\) < 3.5 m s\(^{-1}\), by which point the lower 1σ limits from our analysis start to overlap with the upper limits from the W10 analysis.

We should point out that while our prior on the jitter (Section 2.1) does penalize high jitter values, it may be that an even more punitive prior is warranted. This is due to the observed fact that stellar activity (and therefore stellar jitter) declines with age (Isaacson & Fischer 2010). As HAT-P-13 is old (~5 Gyr), it is possible that it has a jitter level that is drawn from a population with lower values than that quoted from the general population in Wright (2005). As such, it is possible that a more limited range of jitters should be allowed. However, we note that (1) our jitter allows for contributions over and above pure “stellar jitter” and (2) we list results for a number of different jitter ranges in Tables 1 and 2 from which we see that even if one only wishes to accept a limited range for the jitter values resulting from the MCMC analysis (e.g., 2.0 m s\(^{-1}\) < \(\sigma_J\) < 4.5 m s\(^{-1}\)), the associated errors on the fitted planetary parameters are still much broader than those presented in the results of either B09 or W10.

Given that any “jitter” values arising from our analysis will intrinsically be “system” jitter (the combined contribution of jitter from stellar sources, instrumental noise, undetected planets, etc.) rather than pure stellar jitter, we are hesitant to produce detailed figures regarding any “best-fit” jitter parameters arising from our analysis. However, for the sake of completeness, we note that the analyses corresponding to the three columns of Figure 1 (S1, S2 assuming two planets + slope, and S3 assuming three planets) find overall system jitter levels of \(\sigma_J \sim 0.5\) ms, \(\sigma_J \sim 1.188\) ms, and \(\sigma_J \sim 0.691\) ms, respectively.

Subsequent to the pure MCMC analysis, if we then introduce an additional TTV analysis step (based upon the output of the MCMC routine), we can somewhat constrain the eccentricity of the outer planet, finding that now 0.6 < \(e_{\text{outer}}\) < 0.85. However, the remainder of the other elements are essentially unconstrained by this additional analysis. Tables 1 and 2 also contain a summary of all the parameters and the best fits resulting from our MCMC + TTV analysis.

We thus conclude this section of our investigation by suggesting that previous methodologies of excluding jitter from an MCMC analysis significantly overestimates the precision with which other system parameters (e.g., planetary orbital elements) can be estimated from RV data. We thus recommend that jitter be included as a model parameter in future MCMC analyses of RV data, allowing a true estimation of system parameters and their uncertainties to be determined.

4. FURTHER INVESTIGATION OF TRANSIT TIMING VARIATIONS

In the coplanar MCMC + TTV analysis of Section 3, it was suggested that the rms TTV amplitude could be of some diagnostic power in constraining certain system parameters (e.g., the eccentricity of the outer planet, \(e_{\text{outer}}\)). In Nesvorný (2009) and Payne et al. (2010), it has been found that planets which are significantly inclined relative to one another can give rather different signal profiles and amplitudes as compared to the coplanar case. This inclination dependence, combined with the differing planetary masses that would be required in an inclined system to satisfy the RV constraints, suggests that a more detailed investigation of the inclination dependence of the TTV signal in the HAT-P-13 case is warranted.

Our ultimate aim is to find whether it might then be possible to use the TTV characteristics to break the degeneracy of the RV \(M_{\text{outer}} \sin i_{\text{outer}} = 15.2 M_J\) relation.

4.1. Examples of Transit Timing Variation Signals for HAT-P-13

We commence this more detailed investigation of the inclination dependence of the TTV signal variation in the HAT-P-13 system by plotting in Figure 4 some sample results which illustrate the effects of varying the orbital inclination of the outer planet. Note that we set \(a_{\text{inner}} = 0.043\) AU, \(e_{\text{inner}} = 0.02\), \(m_{\text{inner}} = 0.85 M_J\), \(a_{\text{outer}} = 1.188\), \(e_{\text{outer}} = 0.691\), and \(m_{\text{outer}} \sin i_{\text{outer}} = 15.2 M_J\) in line with the standard values from B09. Note that as we vary the inclination of the outer planet, \(i_{\text{outer}}\), we also simultaneously vary the mass of the outer planet, \(m_{\text{outer}}\).

Given the work of Mardling (2010) discussed in Section 1, we concentrate our analysis on a few key relative inclinations. For the prograde cases, we concentrate on \(i_{\text{rel}} = 0°\) and \(i_{\text{rel}} = 45°\), as these are thought to be the most likely states at late times. We also examine the symmetric retrograde cases, \(i_{\text{rel}} = 180°\) and \(i_{\text{rel}} = 135°\). Finally, for completeness we also look at a couple of example plots from the “forbidden” region of parameter space \(55° < i_{\text{rel}} < 125°\) from which it is thought that the planets are likely to be excluded due to the combined effects of Kozai forcing and tidal dissipation.

We can immediately see from the \(i_{\text{rel}} = 0°\) and \(i_{\text{rel}} = 45°\) plots in Figure 4 that (1) the expected TTV amplitudes are well below the current observational limits, so either inclination is plausible in that sense, but (2) perhaps more importantly, the profiles have rather different shapes, with the \(i_{\text{rel}} = 45°\) plot exhibiting prominent spikes in TTVs over a relatively short period of time, in addition to having an overall reduced amplitude. We discuss further in Section 5 a plausible observational strategy to extract this information.

We can also see that the completely retrograde case, \(i_{\text{rel}} = 180°\), is very similar in both shape and amplitude to the \(i_{\text{rel}} = 0°\) prograde case, suggesting that they would be rather difficult to distinguish observationally.

For the plots in the “forbidden” \(55° < i_{\text{rel}} < 125°\) region, it seems that the systems in which the planets are close to perpendicular will have extremely high TTV amplitudes (due, no doubt, to the significantly increased \(m_{\text{outer}}\) required in order to satisfy the \(m_{\text{outer}} \sin i_{\text{outer}}\) constraint). These highly distinctive profiles mean that if for some reason a planet were able to occupy this region of parameter space, it would be readily identifiable (and indeed, certain particularly high inclinations may already be ruled out by the TTV constraint—see Section 4.2 below for further discussion).

Finally, we note in passing that in Section 3 it was found that eccentricities for the outer planet were constrained to lie below \(e_{\text{outer}} \sim 0.85\). We show in Figure 5 some details of the TTV plots which result from increasing the eccentricity...
of the outer planet. Keeping the semimajor axes and masses of the planets fixed at the best-fit values from the discovery paper (and with $i_{\text{rel}} = 0^\circ$), we increase $e_{\text{outer}}$ from 0.5 to 0.85. We find that for values of the eccentricity in the range $0.5 < e_{\text{outer}} < 0.8$, increases in eccentricity result in only marginal increases in the TTV amplitude. However, at very high eccentricities ($e_{\text{outer}} \sim 0.85$), the systems start to exhibit “kicks/step-changes” in their behavior as the outer planet starts to significantly perturb the inner orbit (as the outer planet passes through pericenter). We note that while such systems may be stable against collisions, the relatively large perturbations experienced during close pericenter encounters can still lead to significant orbital evolution and hence large TTV amplitude variation (see Payne et al. 2010 and Veras et al. 2011 for more detailed discussions).

### 4.2. Transit Timing Variation Contour Maps

To gain a further insight into the range of TTV signatures possible for HAT-P-13, we keep the mass and orbit of the inner planet fixed and vary the orbit of the more massive outer planet. For each set of configurations—i.e., semimajor axis, $a$, eccentricity, $e$, and inclination, $i$, of the non-transiting planet—we typically conduct five simulations randomizing the angular orbital elements (argument of pericenter, $\omega$, longitude of ascending node, $\Omega$, and initial mean anomaly, $M$). We assume that the known transiting planet has the mass, semimajor axis, and eccentricity of HAT-P-13b ($M_{\text{inner}} = 0.85 \, M_J$, $a_{\text{inner}} = 1.188 \, a_J$, and $e_{\text{inner}} = 0.691$, and $m_{\text{inner}} \sin i_{\text{inner}} = 15.2 \, M_J$). While the pattern of behavior is non-trivial, we can make certain statements: (1) for the most likely prograde cases ($i \sim 0^\circ$ and $i \sim 45^\circ$), there is a significant difference in the predicted TTV profile, with much sharper features being expected for the high inclination cases; (2) the TTV amplitude for these cases is $\sim 20$ s, well within the current observational constraints; (3) systems with planets close to perpendicular are likely to give highly distinctive, high amplitude TTV signals; and (4) the retrograde systems can give rather similar amplitudes and overall profiles to those of systems in prograde orbits (extra care may be needed to analyze and distinguish these cases). It should be noted that the origin of the time axis on this plot is somewhat arbitrary, given that it is a simulation, but the translation to “real life” can easily be done by noting that the deepest troughs in the TTV plots occur as the outer planet approaches the pericenter, e.g., 2010 April 16.

![Figure 4. Variation of the relative planetary inclination in the HAT-P-13 system. The system parameters are as observed, such that $a_{\text{inner}} = 0.043 \, a_J$, $e_{\text{inner}} = 0.02$, $m_{\text{inner}} \sin i_{\text{inner}} = 0.85 \, M_J$, $a_{\text{outer}} = 1.188$, $e_{\text{outer}} = 0.691$, and $m_{\text{outer}} \sin i_{\text{outer}} = 15.2 \, M_J$. While the pattern of behavior is non-trivial, we can make certain statements: (1) for the most likely prograde cases $i \sim 0^\circ$ and $i \sim 45^\circ$, there is a significant difference in the predicted TTV profile, with much sharper features being expected for the high inclination cases; (2) the TTV amplitude for these cases is $\sim 20$ s, well within the current observational constraints; (3) systems with planets close to perpendicular are likely to give highly distinctive, high amplitude TTV signals; and (4) the retrograde systems can give rather similar amplitudes and overall profiles to those of systems in prograde orbits (extra care may be needed to analyze and distinguish these cases). It should be noted that the origin of the time axis on this plot is somewhat arbitrary, given that it is a simulation, but the translation to “real life” can easily be done by noting that the deepest troughs in the TTV plots occur as the outer planet approaches the pericenter, e.g., 2010 April 16.](image)

![Figure 5. TTVs as a function of time for various values of the outer planet eccentricity, $e_{\text{outer}}$, in the prograde coplanar HAT 13 b system. We find from this particular example that increasing $e_{\text{outer}}$ from 0.5 to 0.8 causes only a minor increase in the TTV amplitude, but that going to higher eccentricities ($e_{\text{outer}} \sim 0.85$) leads to the significant amplitude variations seen in Figure 2, potentially associated with unstable planetary orbits. As in Figure 4, it should be noted that the deepest troughs in the TTV plots occur at the time of pericenter passage of the outer planet.](image)
Figure 6. Contour plots in the \((a_{\text{outer}}/a_{\text{inner}}, e_{\text{outer}})\) plane, illustrating the contours of the predicted rms TTV amplitude at different relative inclinations. The horizontal and vertical lines give the approximate observational constraints from Figure 3 \((0.5 < e_{\text{outer}} < 0.85, 420 \text{ days} < P_{\text{outer}} < 440 \text{ days})\). The plots make clear that the TTVs can always be very large toward the top of the allowed eccentricity range, but that there is little variation with semimajor axis. Very high inclinations result in TTVs that are significantly greater than the observational constraints from B09 across the entirety of the parameter space.

As the orbits become retrograde, we find that the TTV amplitudes decrease again, with the results at \(i_{\text{rel}} = 135^\circ\) \((i_{\text{rel}} = 180^\circ)\) being very similar to those seen at \(i_{\text{rel}} = 45^\circ\) \((i_{\text{rel}} = 0^\circ)\), i.e., there is some kind of approximate symmetry about \(i_{\text{rel}} = 90^\circ\).

The clear difference in TTV amplitude as \(i \to 90^\circ\) is a rather different behavior from that observed by Payne et al. (2010) in their general investigation of inclinations effects on TTVs in hot-Jupiter + hot-Earth systems. There they found that TTV amplitudes were very generally highest at \(i_{\text{rel}} = 0^\circ\), with an approximately monotonic decline as \(i \to 180^\circ\), with no special behavior seen at \(i_{\text{rel}} = 90^\circ\). However, it is difficult to make a precise comparison between their results and our results here, as in Payne et al. (2010) the planetary masses were kept constant.
as the inclination was increased, whereas in this investigation we are varying the outer mass as $M_{\text{outer}} = 15.2 \, M_J / \sin i_{\text{outer}}$.

For completeness, we include in Figure 7 a plot in the $(i_{\text{rel}}, e_{\text{outer}})$ plane, again plotting the median rms TTV values from 3.5 year simulations. In these simulations, we fix the semimajor axis of the outer planet and then vary the inclination and eccentricity of the outer planet. We reproduce here the plot for $a_{\text{outer}}/a_{\text{inner}} = 27.2$, i.e., an outer planet of $P_{\text{outer}} = 429$ days. As seen from Figure 7, the upper allowed eccentricity decreases at larger semimajor axes. There is a critical range of inclinations above which the TTV amplitude becomes too great to fit observational constraints. For the current 100 s limits, the excluded range is very narrow $\sim 90^\circ \pm 2^\circ$. Clearly this is of little benefit, as such regions are already excluded in any practical sense, as such a highly inclined object would be so massive as to be a star and hence likely be readily visible, but if future observations constrain the TTV amplitude to be less than 10 s (for example), then the inclination restrictions become much more severe, and all inclinations $\sim 90^\circ \pm 20^\circ$ could be ruled out. Moreover, we see that as the inclination increases from 0$^\circ$ to 45$^\circ$, the ability to constrain the eccentricity using TTV limits varies significantly, e.g., the yellow region mapping out the 10–30s TTV contour narrows rapidly.

We also made similar plots at different semimajor axes, but we find that there is little difference between the results at different semimajor axes, as might be anticipated from Figure 6.

5. FUTURE TRANSIT OBSERVATIONS

5.1. Inner Planet: Transit Timing Scheduling

As noted in Section 4 from Figures 4 and 5, we have a potential means of distinguishing between different relative inclination and/or (outer) eccentricity states of the system. To do so will require a detailed investigation of the inner transit times to an accuracy greater than $\sim 5$ s. In this section, we focus on the time frame close to the pericenter approach of the outer planet. It is at this point that the TTVs induced on the inner planet are predicted to go through a sharp change in values in only a small number of orbits. In Figure 8, we display the TTVs expected for three different system configurations: (1) $i_{\text{rel}} = 0^\circ$ and $e_{\text{outer}} = 0.69$, (2) $i_{\text{rel}} = 45^\circ$ and $e_{\text{outer}} = 0.69$, and (3) $i_{\text{rel}} = 0^\circ$ and $e_{\text{outer}} = 0.73$, in which we have fixed the longitude of ascending node to be $0^\circ$ and chosen the other system parameters to be those of the best-fit value from the far right-hand column of Table 1.

We see that in all three cases, the TTV signal changes sharply during the time between the predicted transit of the outer body (see Section 5.2) and the predicted date of pericenter passage for the outer body. From Julian Date $\sim 2455300$, over the next $\sim 50$ days (i.e., over $\sim 17$ transits of the inner planet), the signal would be expected to switch from being $\sim -30$s to being $\sim +15$s. A similar pattern would be expected to occur at potential transit dates in 2011 and 2012 at or around the dates given in Table 2. If enough observations can be made of high enough precision over this period, it should be possible to (1) at the very least confirm that a TTV variation is seen, (2) give some significantly improved constraints on the magnitude of the TTVs in the system and hence point toward a better understanding of the system parameters, as well as (3) pointing the way toward a subsequent, longer term observational campaign to further track the TTV profile over the course of an entire orbit of the outer planet.

We would thus suggest that an observational campaign targeting the transits of the inner planet could give informative

![Figure 7](image-url)
results about the relative inclination of the planets in the system. We suggest that observations extending over the period of pericenter passage (for the outer planet) will initially be the most useful in determining the nature and amplitude of the TTVs, but that longer term observations can also give important information that allow different inclination states to be distinguished. In addition, it should be emphasized that such observations would need to be of a precision similar to (or better than) that demonstrated in the study of WASP-10 by Johnson et al. (2009, 2010), where the mid-transit times were determined than) that demonstrated in the study of WASP-10 by Johnson et al. (2009, 2010), where the mid-transit times were determined.

We suggest that observations extending over the period of 10–15 transits of the inner planet, then it is plausible that significant inclination information could be extracted.

We caution that the precise date(s) of the pericenter passage for the outer planet are, of course, highly dependent on the values of the fitted orbital parameters \( a_{\text{outer}}, e_{\text{outer}}, \) etc. Given that we have demonstrated in Section 3 that these fitted parameters are rather uncertain, it should be clear that the precise time of pericenter passage is similarly uncertain, and so any scheduled observations should take these uncertainties into account. Moreover, if/when further additional RV observations become available, it would be prudent to repeat the analysis of Section 3 to allow the uncertainties in the planetary parameters to be improved and hence allow any future observational strategy to be refined. We also caution that in the examples shown in Figure 8, we have made specific assumptions regarding the longitude of ascending node of the planets (fixed = 0). If such assumptions are relaxed, then TTV profiles can be generated that differ in the degree and extent to which their amplitude changes over the course of the pericenter passage. We emphasize that in fitting/modeling any putative future observations, such degeneracies would need to be accounted for.

5.2. Outer Planet: Transit Timing Projections

It is clear from Sections 4 and 5.1 that further observations of the precise timings of the transit of the inner planet will be the key to understanding the TTVs, and subsequently the relative inclinations, of the planets around HAT-P-13. However, we feel that is worth emphasizing that a (fortuitous) transit observation of the outer planet would help to pin down the inclination, period, and most other parameters of the planetary system much more precisely (to say nothing of giving the first ground-based observation of a system with two transiting planets!). While the a priori probability of HAT-P-13c transiting is relatively low if all inclination angles are considered equally likely (~1%; see Seagroves et al. 2003; Kane & von Braun 2009), this probability can be significantly increased (to ~8%) for certain particular inclination and observation angles (Beatty & Seager 2010).

If the outer planet does indeed transit, then there remains the issue of knowing when to observe the star to detect the transit. If we use our MCMC data to predict the times of the next few transits, assuming that the outer planet is inclined at 90° to the plane of the sky (as was also assumed in B09), then using the method outlined in Kane (2007) and Kane & von Braun (2009), we find that there is a significant uncertainty in the predicted timing of any potential transit. We note at this point that our potential transit date for the year 2009 (\( T_{\text{outer,Balok}}, \) Tables 1 and 2) is typically a few days later than that derived by B09 (but with a large range of uncertainties) and that our fitted outer period is slightly longer (see Table 1).

Specific predictions require us to make some assumptions about the stellar jitter, so as an example we consider a low-stellar jitter case and select the results in which the overall jitter lies in the range \(2.0 \text{ m s}^{-1} < \sigma_j < 4.5 \text{ m s}^{-1}\) (and take the cases with TTV amplitude less than 100 s). In this case, we find that the predicted transit dates for 2010–2012 are \( T_{\text{outer,2010}} = 2455314.3^{+0.8}_{-0.7}, T_{\text{outer,2011}} = 2455764.3^{+12.3}_{-10.9}, \) and \( T_{\text{outer,2012}} = 2456213.7^{+16.6}_{-15.4}, \) respectively, which translate into 2010 April 27 ± 9.8 days, 2011 July 21 ±12.3 days, and 2012 October 13 ±16.6 days. We display in Figure 9 the scatter in the predicted transit times for 2011 as a function of the assumed jitter and an accompanying histogram to illustrate the spread in predicted transit times, as well as the difference that results from taking a restricted range of jitter values. We do this for both the B09 S1 and W10 S2 data sets. It is clear from the scatter plot that within a given data set (e.g., B09) the distribution is essentially the same as that of the outer period plotted in Figure 1. The histograms suggest that the systems with jitter in the range \(2.0 \text{ m s}^{-1} < \sigma_j < 4.5 \text{ m s}^{-1}\) tend to have predicted transits which are more sharply peaked toward slightly lower values than the distribution which includes all jitter values.

However, when one compares the S1 results to those of S2, we see that there is an offset in the predictions, due to the preference for longer periods for the outer planet in S2, as well as the asymmetry in the parameter space.

We again emphasize that different assumptions regarding the level of jitter in the system act to shift the predicted date (as well as the error bars) of any potential transit. We also stress that the uncertainties on our figures cover only two-thirds of the data: more extreme excursions exist. It would thus seem prudent for any future campaign of transit observations to be undertaken significantly before and after the mid-point of any predicted transit date to allow for the uncertainties, e.g., for the 2011 potential transit, do not just observe on 2011 July 21, but rather take observations over a large range of nights (perhaps starting around July 9 and continuing for ~3 weeks) to give the highest probability of observing any transit that might occur.

On a practical point, we note from the object visibility calculator at “http://catserver.ing.iac.es/staralt/” that HAT-P-13 would be visible for many hours per night in the northern hemisphere during 2012 October, but that observations around 2011 July would be rather more challenging, as the star is likely to only be visible for ~1 hr per night.

5.3. Additional Thoughts on Variations in \(e_{\text{inner}}\) and \(\eta\)

We note from the study by Mardling (2010) of the likely dynamics of the HAT-P-13 system that the secular timescale for \(e_{\text{inner}}\) and \(\eta\) to vary around the limit cycle is of the order a few thousand years (see her Figure 3(b)). We can also see that the amplitudes of the oscillations are ~50° for \(\eta\) and ~5 × 10^{-3} for \(e_{\text{inner}}\), respectively. This would suggest that over the course of a 10 year observational campaign, the change in these quantities would be \(\Delta \eta(10 \text{ yr}) \sim 0.2\) and \(\Delta e_{\text{inner}}(10 \text{ yr}) \sim 5 \times 10^{-5}\), both very small quantities indeed.

6. CONCLUSION

We have performed an investigation of the HAT-P-13 system, concentrating our efforts on re-evaluating the RV analysis, including jitter as an intrinsic part of the MCMC analysis, and then combining this evaluation with an analysis of the TTVs in the system.
which have TTVs less than 100 s. The dates are Julian Dates.

We note that a transit by the outer planet is rendered

1) and the full data sample (but both

2. If we include the current weak TTV constraints (<100 s)
in addition to the RV analysis, then we find that we can
exclude eccentricities for the outer planet larger than \(\sim 0.85\).
If future observations can pin down the rms TTV amplitude
to a much narrower range (<10 s, for example), then this
would strongly constrain the eccentricity of the outer planet
(\(<0.6\) for the case of TTVs less than 10 s). See Section 3.2
for a discussion of the implications of the TTV amplitude.

2. If the methodology for including jitter within an RV
analysis is changed and jitter is included as a model
parameter within the MCMC analysis, a significantly larger
range of parameter space opens up, i.e., the masses and
orbital parameters for the system are significantly less well
defined. As an example, based on the extended RV data
set analyzed in Winn et al. (2010), if the overall system
jitter is in the range \(2.0 \text{ m s}^{-1} < \sigma_j < 4.5 \text{ m s}^{-1}\) rather than
\(\sigma_j \sim 3.4 \text{ m s}^{-1}\), the eccentricity of the inner planet has a
1\(\sigma\) best-fit value of \(e_{\text{inner}} = 0.038^{+0.022}_{-0.018}\), the eccentricity
of the outer planet has a best-fit value of \(0.65^{+0.10}_{-0.06}\), while
the period of the outer planet would have best-fit values of
\(P_{\text{outer}} = 449.4_{-4.8}^{+5.0}\) and the relative pericenter alignment of
the two planets becomes essentially unconstrained, with a
best-fit value of \(\eta = 12.4_{-32.2}^{+32.3}\). With the current data set,
even higher jitter values are plausible, and this would act
to further increase the uncertainty in the observations of
system parameters (see Section 3.1).

3. We find the following.

1. If the methodological constraints (~100 s) in
addition to the RV analysis, then we find that we can
exclude eccentricities for the outer planet larger than \(\sim 0.85\).
If future observations can pin down the rms TTV amplitude
to a much narrower range (<10 s, for example), then this
would strongly constrain the eccentricity of the outer planet
(\(<0.6\) for the case of TTVs less than 10 s). See Section 3.2
for a discussion of the implications of the TTV amplitude.

3. The current TTV constraints already suggest that the planets
in the system do not have relative inclinations in the range
\(88^\circ < i_{\text{rel}} < 92^\circ\), but any future tightening of the TTV
constraint would act to exclude a much wider range of
inclinations centered on \(i_{\text{rel}} = 90^\circ\) (70° < \(i_{\text{rel}} < 110^\circ\)
would be excluded if TTV amplitudes were less than
10 s—see Section 4.2).

4. The TTV profile can in many circumstances act as an efficient
diagnostic tool in determining the relative inclination
between the two planets. In particular, it is easy to
distinguish between systems with \(i \sim 0^\circ\) and \(i \sim 45^\circ\)
(the approximate angles found to be the most likely in the
analysis of Mardling 2010) through suitably timed transit
observations (see Section 4.1). We thus suggest that
an observational campaign targeting the transits of the
inner planet could give informative results about the relative
inclination of the planets in the system. We suggest that
observations extending over the period of pericenter pas-
sage (for the outer planet) will initially be the most use-
ful in determining the nature and amplitude of the TTVs,
but that longer term observations can also give important
information that allow different inclination states to be
distinguished.

5. We note that a transit by the outer planet is rendered more
likely following the exclusion of regions of inclination space
by the analysis of Mardling (2010), supporting the idea of an observational campaign, but the large spread in
allowed orbital periods resulting from our jitter analysis
would suggest that any observations should be conducted
over a \(\sim 3\) week period centered approximately on either
2011 July 21 or 2012 October 13. (Observations in 2011
will be significantly more difficult than in 2012—see
Section 5.)

6. Given the value of transit timing observations near the time of
pericenter of the outer planet, the long orbital period
of the outer planet, and the poor observability during peri-
center passages in 2011, we suggest that observers should con-
sidee observing whenever it would be possible to ob-
serv full ingress or full egress, rather than only obser-
v full transits. Measuring a large fraction of transits/ingress/egress times during this critical time will require ob-
servations from a network of telescopes at multiple
longitudes.

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APPENDIX

For the sake of completeness, we return to consider
Figure 2 in which we demonstrated that the fitted values for
the overall RV offset, as well as the RV amplitude, K_{outer}, longi-
tude of pericenter, σ_{outer}, and the eccentricity, e_{outer}, of the outer
planet all showed a correlation with the rms TTV amplitude. In
Figure 10, we plot P_{outer} for which there is no obvious cor-
relation between the rms TTV amplitude and the fitted system
parameter.

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Figure 10. TTV amplitude vs. system parameters for parameters in which there is no obvious correlation between amplitude and parameter value. In the various plots above, we show the rms TTV amplitude as a function of various system parameters (planetary eccentricity, etc.). As in Figure 1, systems with σ_j < 2.0 m s^{-1} are plotted using a red circle (gray in the print version), those with 2.0 m s^{-1} < σ_j < 4.5 m s^{-1} are plotted using a green cross (gray in the print version), and those with σ_j > 4.5 m s^{-1} are plotted using black addition symbols. As in Figure 1, in the left-hand column we present results using only the subset of data known at the time of publication by B09, assuming a two-planet fit. In the center and right-hand columns we present results obtained using the full data set of W10, with the central column assuming two-planets + a linear trend, while the right-hand column assumes a three-planet fit. We plot results for the period of the outer planet. Unlike the results plotted in Figure 2, we see that there is no obvious correlation between the rms TTV amplitude and the system parameter value.

(A color version of this figure is available in the online journal.)