Top quark Kaluza-Klein mode mixing in the Randall-Sundrum bulk Standard Model and Constraint from $\Delta \rho$

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The Randall-Sundrum (RS) scenario with all the standard model (SM) fermions and gauge bosons in the bulk is phenomenologically studied. Even though the simple assumption of universal bulk fermion mass $m_\psi$ leads to the same Kaluza-Klein (KK) mass spectrum for all the SM fermions and thus suppresses new contributions to Flavor-Changing-Neutral-Current (FCNC) and the $\rho$ parameter, large Yukawa coupling of the top quark generates its KK mode mixing and breaks the degeneracy: unacceptably large contribution to $\Delta \rho$ occurs. With a different bulk fermion mass to SU(2) singlet bottom quark, we demonstrate that there exists some parameter space to satisfy the $\Delta \rho$ constraint.

1. Introduction

Extra dimensional models have drawn a lot of interest since it can leave distinct phenomenological signatures at future colliders. In the original RS scenario, the SM fields are assumed to be confined to our brane. Phenomenological signatures come from KK gravitons with electroweak scale masses and couplings to matter, characterized by $\Lambda_\pi$. However, the small size of the RS-bulk allows that the SM fields may also be in the bulk. In Ref. [1], it is demonstrated that placing the SM gauge fields in the RS-bulk while confining the fermions to our brane is strongly constrained by the current precision electroweak data so that $\Lambda_\pi$ is pushed up to about 100 TeV. This is disfavored as a solution of the gauge hierarchy problem. If both the SM gauge and fermion fields are in the bulk, their phenomenological signatures are very sensitive to the bulk fermion mass $m_\psi$.

In the early study of the RS-bulk SM, Yukawa interactions with the Higgs field have been ignored due to small quark masses compared to the KK mass scale. Then a simple assumption of universal bulk fermion mass suppresses their contributions to FCNC as well as the $\rho$ parameter. This is because the common $m_\psi$ leads to the same KK mass spectra for all the SM fermions. The degeneracy of the up-type (and down-type) quark KK modes operates the GIM cancellation KK-level by level: With the minimal flavor violation assumption that at the tree level the flavor mixing comes only through the CKM matrix, FCNC is suppressed as in the SM. Since two constituents of SU(2)–doublet have also the same KK mass, their contribution to the $\rho$ parameter vanishes. However, Yukawa interaction with the Higgs field mixes the fermion KK tower members, which can be substantial for the top quark. Recently it has been shown that the large mixing in the top quark KK sector leads to unacceptably large contribution to the $\rho$ parameter and raises $\Lambda_\pi$ above 100 TeV. To accommodate those electroweak precision data, a ‘mixed’ scenario was proposed with the third generation fermions on the TeV brane but the other generations in the bulk. However, the first excited KK mode of gauge bosons should be heavier than 11 TeV due to the strong constraints from precision measurements: It is hard to probe the new physics effects at LHC. In addition, the obvious discrimination of fermions according to genera-
tion may lead to potentially dangerous FCNC due to the absence of GIM mechanism.

It is worthwhile to keep the original framework where all the fermions are in the bulk, and to question other unsubstantiated assumptions. Relaxing the universal bulk fermion mass assumption, we assign a different bulk fermion mass $m'_n$ to the SU(2)–singlet bottom quark field, and see whether there exists some parameter space to accommodate the $\rho$ constraint\[^{[2]}\]. We show that this $m'_n$ allows some limited parameter space where the degeneracy between the top and bottom quark KK modes is retained, suppressing their contribution to the $\rho$ parameter.

2. Extended Bulk SM in the RS scenario

2.1. Original set-up

Let us review the KK solution of a bulk fermion with arbitrary Dirac bulk mass in the RS scenario\[^{[3],[4],[11]}\], which causes a subtle problem when discussing the bulk SM. The five-dimensional action of a Dirac fermion $\Psi$ with the bulk mass $m_\psi$ is

$$S = \int d^4x \int d\phi \sqrt{-g} \left\{ E^A_\Delta \left[ \frac{1}{2} \bar{\Psi} \gamma^\Delta (D_\Delta - \bar{D}_\Delta) \Psi \right] + m_\psi \text{sign}(\phi) \bar{\Psi} \Psi \right\}. $$

With the KK expansion of $\Psi$

$$\Psi_{L,R}(x, \phi) = \sum_{n=0}^{\infty} \psi^{(n)}_{L,R}(x) \frac{e^{2\phi}}{\sqrt{F_\phi}} f^{(n)}_{L,R}(\phi), \tag{1}$$

Eq. (1) becomes the action for a tower of massive Dirac fermions

$$S = \sum_{n=0}^{\infty} \int d^4x \left\{ \bar{\psi}^{(n)} \gamma^\nu \left( \frac{i}{2} \bar{\psi}^{(n)} - M_\nu^{(n)} \bar{\psi}^{(n)} \right) \right\}. \tag{2}$$

Note that $Z_2$-symmetric action constrains $\bar{\Psi} \Psi = \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L$ to be $Z_2$-odd. If $f^{(n)}_L$ is a $Z_2$-even function $\chi^{(n)}$ then $f^{(n)}_R$ should be a $Z_2$-odd function $\tau^{(n)}$ and vice versa. The order one parameter $\nu \equiv m_\psi/k$ determines the $\chi^{(n)}$ and $\tau^{(n)}$ as some combination of Bessel functions.

Let us discuss the physical implication of the parameter $\nu$. Note that the canonically re-scaled zero mode of $Z_2$-even bulk fermion is proportional to $e^{(1/2+n)kr_c|\phi|}$. For $\nu \ll -1/2$ the fermion bulk wave functions are localized toward the Planck brane: The magnitudes of its gauge couplings with KK gauge bosons are quite small. Numerically for $\nu \lesssim -0.5$ the couplings are too small to be probed at high energy colliders\[^{[3],[10]}\]. For $\nu \gg 1$ the SM fermions become localized closer to the TeV brane: The model approaches the RS model with the gauge fields only in the bulk, which is phenomenologically disfavored\[^{[3]}\]. To be specific, if $\nu \gtrsim -0.3$, the large contribution to the precision electroweak data pushes $M_{A}^{(2)}$ up to about 6 TeV, beyond the direct production at any planned collider. Therefore, we consider the parameter space of $\nu$ between $-0.5$ and $-0.3$.

Another subtle point when placing the SM fermions in the AdS$_5$ bulk is that the fermion field contents should be doubled. In the SM, a fermion field with left-handed chirality and that with right-handed chirality belong to different representations of a gauge group. In the RS-bulk SM, however, if a fermion which belongs to a specific representation of a gauge group has a single chirality, the bulk wave function cannot be determined. For each generation, we introduce four five-dimensional Dirac fields, an SU(2)–doublet fermion field $Q = (q_u, q_d)^T$ and two SU(2)–singlet fermion fields, $u$ and $d$, with weak hypercharges $Y = 1/6, 2/3,$ and $-1/3$ respectively.

Since the SM fermion should correspond to the KK zero mode, we assign $Z_2$-even wave function $\chi^{(n)}$ to the left-handed SU(2)–doublet and the right-handed SU(2)–singlet, $u(x, \phi)$ and $d(x, \phi)$. The charged current interactions, mediated by the bulk $W$ boson, connect $q_u$ and $q_d$:

$$S \supset \int d^4x \frac{g}{\sqrt{2}} \sum_{l=0}^{\infty} \sum_{n,m=0}^{\infty} q_{uL}^{(n)} W^{+(l)} d_{dL}^{(m)} C_{nml}^{ffW} \right\} + h.c.,$$

where $g = g_3/\sqrt{2\pi r_c}$, $C_{nml}^{ffW}$ and $H_{nml}^{ffW}$ denote the couplings of the $m$-th and the $n$-th fermion states to the $l$-th $W$ boson in the unit of the SM
coupling. It is defined by
\[
C_{nlm}^{WW} = \sqrt{2\pi} \int_{-\pi}^{\pi} d\phi \, e^{\sigma} \chi^{(n)}(\phi) \chi^{(m)}(\phi) \chi^{(l)}(\phi),
\]
\[
H_{nlm}^{WW} = \sqrt{2\pi} \int_{-\pi}^{\pi} d\phi \, e^{\sigma} \tau^{(n)}(\phi) \chi^{(l)}(\phi).
\]

2.2. Minimally Extended Model

Now every SM fermion possesses its KK tower. We remind the reader that in the RS background the fermion KK mass spectrum is determined by the bulk fermion mass \(m_{\psi}\). A simple assumption of universal bulk fermion mass results in the same KK mass spectrum for all the SM fermions. However, there is another mass source, Yukawa interaction. This Yukawa mass relates SU(2)–doublet fermion masses to the corresponding mass of fermions in the bulk.

2.3. Constraints from the \(\rho\) parameter

The \(\rho\) parameter is defined by the difference between the \(W\) and \(Z\) boson self-energy functions re-scaled by each mass. It has been known to play a special role among precision measurements since it is sensitive to heavy fermions beyond SM. With the Higgs mass below 1 TeV, the current electroweak precision data constrain \(\Delta \rho < 2 \times 10^{-3}\) at 95% CL (with \(\Delta \rho \equiv \rho - \rho_{SM}\)), which restricts new SU(2)–doublet fermion mass spectrum to satisfy

\[
\sum_{i} \Delta m_{i}^{2} \leq (115 \text{ GeV})^{2}.
\]
Thus different mass spectra for the top and bottom quark KK modes lead to dangerous contributions to $\Delta \rho$. First let us demonstrate that the RS-bulk SM with the universal bulk fermion mass assumption ($\nu = \nu'$) cannot satisfy this $\Delta \rho$ constraint. In Fig. 1, we show top and bottom KK mass differences up to the fifth KK excitation states as a function of parameter $\nu$, which is too large to satisfy Eq. (4).

Now let us allow $\nu \neq \nu'$ and see whether there exists a parameter space to satisfy Eq. (4). In Fig. 2, we show top and bottom KK mass differences up to the fifth KK excitation states as a function of parameter $\nu$, which is too large to satisfy Eq. (4).

In summary, the minimal RS-bulk SM with a common bulk fermion mass has disastrous contribution to $\Delta \rho$. First let us demonstrate that the RS-bulk SM with the universal bulk fermion mass assumption ($\nu = \nu'$) cannot satisfy this $\Delta \rho$ constraint. In Fig. 1, we show top and bottom KK mass differences up to the fifth KK excitation states as a function of parameter $\nu$, which is too large to satisfy Eq. (4).

Now let us allow $\nu \neq \nu'$ and see whether there exists a parameter space to satisfy Eq. (4). In Fig. 2, we shows, with the fixed $\nu' = -0.6$, the top-bottom quark mass differences for the first five excited modes. We find that for example the $\nu = -0.39$ case with $\nu' = -0.6$ gives vanishing mass differences below 20 GeV for the first five KK excited states. A remarkable point is that the $\Delta m$ decreases for higher KK modes; the contribution of higher KK modes becomes less important.

In conclusion, the minimal RS-bulk SM with a common bulk fermion mass has disastrous contribution to the $\rho$ parameter due to quite large mass shifts between the top and bottom quark KK modes. We relax the universal bulk fermion mass, and let the SU(2)-singlet bottom quark field have a different bulk fermion mass $m_{\psi}$. It is shown that for example if $m_{\psi}/k \simeq -0.4$, the degeneracy of the top and bottom quark KK mode is good enough to suppress the new contribution to $\Delta \rho$.

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