Discriminant Models for Uncertainty Characterization in Area Class Change Categorization

ZHANG Jingxiong, YOU Jiong
School of Remote Sensing and Information Engineering, Wuhan University, 129 Luoyu Road, Wuhan 430079, China

Abstract Discriminant space defining area classes is an important conceptual construct for uncertainty characterization in area-class maps. Discriminant models were promoted as they can enhance consistency in area-class mapping and replicability in error modeling. As area classes are rarely completely separable in empirically realized discriminant space, where class inseparability becomes more complicated for change categorization, we seek to quantify uncertainty in area classes (and change classes) due to measurement errors and semantic discrepancy separately and hence assess their relative margins objectively. Experiments using real datasets were carried out, and a Bayesian method was used to obtain change maps. We found that there are large differences between uncertainty statistics referring to data classes and information classes. Therefore, uncertainty characterization in change categorization should be based on discriminant modeling of measurement errors and semantic mismatch analysis, enabling quantification of uncertainty due to partially random measurement errors, and systematic categorical discrepancies, respectively.

Keywords uncertainty; information classes; data classes; discriminant models; conditional simulation; land cover change

CLC number P208

Introduction

The production of area-class maps, such as those depicting land cover change, using an image classification has been one of the most common applications of remote sensing. Uncertainty characterization has become increasingly recognized as an integral component in thematic mapping and change analysis of area classes based on remotely sensed imagery and auxiliary data,[1-5] and descriptors, such as percent correctly classified pixels and Kappa coefficients of agreement, have been devised as thematic accuracy metrics. However, such spatially averaged measures about accuracy offer neither hints about spatial variation in misclassification nor are useful for quantifying error margins in derivatives, such as the areal extents of different land cover types and land cover change statistics. Such limitations originate because spatial dependency is not accommodated in conventional methods for error analysis, leading to biased quantification of standard errors in derived data and analysis results.[6] Therefore, error modeling should be assessed by accommodating spatial and temporal inter-
dependence. \[7\]

Geostatistical simulation is well known for its tendency to propagate errors through complex geoprocessing. Indicator stochastic simulation is conventionally applied to quantify uncertainty in categorical information but suffers from non-invariant generation of realized maps due to order of class labels. To fix this, i.e., to reinforce consistency in area-class mapping and replicability in error modeling, the method of discriminant space was proposed, which follows the concept of phase space in physics. \[8\]

This paper extends discriminant models for uncertainty characterization from area-class mapping to change categorization. First, we describe methods for mapping area classes in the space of discriminant covariates and projected errors from discriminant space to area classes in the geographic space. Then, we discuss Bayesian methods for change categorization. Spatially explicit uncertainty quantification is facilitated by transforming and summarizing stochastically simulated class-defining covariates. Uncertainty due to data and information class semantics can be examined separately from that due to measurement errors.

1 Methods

1.1 Discriminant models\[9\]

The discriminant space, also known as \( Z \) space, is spanned by discriminant covariates \( Z(x) \), a vector field of dimension \( b \), with \( b \) denoting a positive integer and \( x \) denoting a location within the problem domain \( A \). The discriminant model provides a function \( \eta \) linking the \( Z \) values at a point \( x \) to the area class at that point, so that any point in the \( Z \) space maps to a point in the geographic space. Thus, the mapping from measurement to class labels can be expressed as

\[
\hat{C}(x) = \eta(Z(x)) = \arg \max_{k=1,...,K} f_k(Z(x))
\]  

where \( f_k \) calculates the measures of proximity to indicate categorical similarity to class \( k \), with the predicted class \( \hat{C}(x) \) taking the maximum utility.

To map probabilistic distributions of classes in \( Z \) space, kernel density estimation may be used as a nonparametric way of estimating the probability density function of a random variable. Multivariate joint distributions may also be estimated through \( b \)-dimensional kernels. In discriminant space, a linear model may be prescribed for measurement vector \( Z \) at location \( x \): \( Z(x) = m_Z(x) + \delta Z(x) \), which states that \( Z(x) \) is the version of the mean vector \( m_Z(x) \) corrupted by an error vector of \( \delta Z(x) \), which may be zero mean. Discriminant models assume that there exists a mean value \( m_Z(x) \), hence, a mode class label \( C(x) \) at every location, and measurement \( Z(x) \) containing error \( \delta Z \) leads to error-prone area-class \( C* (x) \).

It is important to recognize that empirical discriminant models do not always support a one to one correspondence between \( m_Z(x) \) and \( C(x) \). Thus, data classes \( C_d(x) \) associated with \( m_Z(x) \) by a data-specific membership function \( f_k(m_Z) \) should be discerned from \( Z \) space to ascribe 1-1 relationships between \( Z \) and \( C_d \). Data classes can be used to bridge the gap between measurement and information classes, which is denoted above by \( C \), which pertain to the classification scheme designed for the specific application. The differentiation between data and information classes will be useful for objective quantification of uncertainty due to measurement error (variance) and model imperfection (bias), respectively.

1.2 \( Z \) -based error modeling

Stochastic simulation based on the \( Z \) space works with generating equal-probable realizations of measurement \( Z \), which are input to class models to derive realized area-class maps. \( Z \)-based error modeling can be formulated as summarizing means and variance for \( c^*(x) = \eta(z^*(x)) \) over specific locations or zones in \( A \), where \( c^* \) and \( z^* \) refer to error-contaminated area-class maps (as responses) and measurements (as covariates), respectively.

If \( N \) stands for the number of simulated \( Z \) or \( C \) maps, the process of error simulation is seen as the mapping from equal-probable realizations \( \{z^{*(l)}\}, l=1,\ldots,N \) to equal-probable realizations of area-class maps \( \{c^{*(l)}\}, l=1,\ldots,N \). These simulated area-class maps can be used to compute summary statistics, such as means and standard deviation of individual class proportions or change extents over specific areas, which would otherwise be difficult to compute analytically due to spatial-temporal interdependence.

An elegant result is that realized classes can be ad-
jacent only if they are adjacent on the $Z$ space, fixing the pathology with indicator-based simulation, i.e., non-invariance in realized categorical maps, which arises since class labels at neighboring locations are drawn from arbitrarily ordered class probability intervals.

In practice, simulation of multiple spatio-temporal variables is commonly involved in land cover mapping and change analysis. However, joint simulation of cross-correlated multiple variables is not trivial.\cite{[10]}

Thus, simplification may be made possible by working with transformed variables, which are assumed to be orthogonal at all lags.

1.3 Bayesian algorithms for change categorization

Consider the elaboration of Eq.(1) in the context of change categorization. While image differencing or ratioing is often performed to detect changes in an unsupervised mode, a compound classification according to Bayes theorem takes place when each pair of pixels with observations is analyzed with the aim of finding the best pair of class labels. This Bayes-algorithm is useful as it allows for tabulation of transition and change analysis. However, joint simulation of multiple spatio-temporal variables is commonly involved in land cover map development by scientists at MRLC was used to derive more accurate and useful land cover change data than it would be possible by direct comparison of NLCD 1992 and NLCD 2001. A training set of 3000 pixels was sampled using simple random sampling to represent land cover changes observed, with 1000 changed pixels and 2000 unchanged pixels.

Tasseled cap transformation was performed with the bi-temporal Landsat TM images (P38/R27) flown on July 17, 1992 (time 1) and August 11, 2001 (time 2) of an area of central Montana, USA, located at 46°25'–48°30' N and 108°04'–111°10' W were used as the dataset for the studies. This study area was chosen for its terrain undulation and typicality in land cover. The land cover labels are 1-open water, 2-forest, 3-grassland/shrub, 4-agriculture, and 5-wetlands. The NLCD 1992-2001 Land Cover Change Retrofit product developed by scientists at MRLC was used to derive more accurate and useful land cover change data than it would be possible by direct comparison of NLCD 1992 and NLCD 2001. A training set of 3000 pixels was sampled using simple random sampling to represent land cover changes observed, with 1000 changed pixels and 2000 unchanged pixels.

Change categorization rule may be expressed in terms of class-conditional density function and prior probabilities in Bayesian classification. To relax the requirement for large number of training samples in the estimation of joint class-conditional density $p(Z^1(x), Z^2(x) | v, \omega_j)$, class-conditional independence is usually assumed as a reasonable approximation.\cite{[11]} It is then possible to replace the quantity in the right side of Eq.(2) with

$$p(v, \omega_j | Z^1(x), Z^2(x)) = p(Z^1(x) | v) p(Z^2(x) | \omega_j) p(v, \omega_j)$$

where $p(v | Z^1(x))$ and $p(\omega_j | Z^2(x))$ are single-date posteriori probabilities, which may be estimated using kernel methods in $Z$ space, and $p(v), p(\omega_j)$, and $p(v, \omega_j)$ are priori probabilities and joint probability, respectively.

2 Experiment and analysis

2.1 The study area and dataset

The subsets (500 by 500 pixels) of Landsat 5 TM images (P38/R27) flown on July 17, 1992 (time 1) and August 11, 2001 (time 2) of an area of central Montana, USA, located at 46°25'–48°30' N and 108°04'–111°10' W were used as the dataset for the studies. This study area was chosen for its terrain undulation and typicality in land cover. The land cover labels are 1-open water, 2-forest, 3-grassland/shrub, 4-agriculture, and 5-wetlands. The NLCD 1992-2001 Land Cover Change Retrofit product developed by scientists at MRLC was used to derive more accurate and useful land cover change data than it would be possible by direct comparison of NLCD 1992 and NLCD 2001. A training set of 3000 pixels was sampled using simple random sampling to represent land cover changes observed, with 1000 changed pixels and 2000 unchanged pixels.

Tasseled cap transformation was performed with the bi-temporal Landsat TM images, and bands of brightness and greenness were selected and transformed via Choleski factorization so that Euclidean distance can be computed in lieu of Mahalanobis distance over the transformed space. This resulted in the discriminant covariates $\{Z_1^{(1)}, Z_2^{(1)}\}$ and $\{Z_1^{(2)}, Z_2^{(2)}\}$ at times 1 and 2, respectively.

The trend surfaces were discerned for $Z_1$ and $Z_2$ fields at time 1 and time 2 by averaging over a moving window of $3 \times 3$ pixels, and then, the corresponding residual surfaces for the bi-temporal vectors were obtained, denoted as $\{R_1^{(1)}, R_2^{(1)}\}$ and $\{R_1^{(2)}, R_2^{(2)}\}$. As will be described later, stochastic simulation in the $Z$ space was performed on these de-trended
datasets.

All pixels of the $Z$ space were discretized in a grid of 256 by 256 cells and the numbers of land cover class labels of pixels falling in individual grid cells were summarized. The majority class labels in these grid cells were taken as the labels of data classes so that all pixels were separable in the $Z$ space. This gave rise to maps showing data classes of land cover in 1992 and 2001, respectively, which correspond but do not equal to NLCD land cover classes (considered to be information classes). Data classes for the 3000 training pixels were recorded based on the data classes map derived above. Thus, we had two sets of training data for 1992 and 2001 land cover mapping, one for data classes and the other for information classes.

2.2 Results and analysis

To realize the Bayesian algorithms for change categorization, first, the training samples were used for kernel-based density estimation in the $Z$ space, which resulted in two sets of probability vector maps indicating class occurrences at time 1 ($p(v_i | Z'(x))$) and time 2 ($p(o_j | Z''(x))$), respectively. Training samples were also used for estimating prior probabilities $p(v_i)$, $p(o_j)$, and prior joint probabilities $p(v_i, o_j)$. Then, the land cover change maps were derived from single-date land cover probability maps and properly estimated prior probabilities and prior joint probabilities by using the Bayesian classification rule specified in Eq. (3).

By the discriminant method of error modeling, stochastic simulation of $Z$ surfaces was carried out by using $sgsim$ in GSLIB on the basis of conditional data sets $\{R_1^{(1)}, R_2^{(1)}\}$ and $\{R_1^{(2)}, R_2^{(2)}\}$, respectively. Normal score transforms necessary for $sgsim$ were performed and variogram models derived, which are $1.0*\exp(-h/30)$, $1.0*\exp(-h/35)$, $1.0*\exp(-h/32)$, and $1.0*\exp(-h/30)$, for $R_1^{(1)}$, $R_2^{(1)}$, $R_1^{(2)}$, and $R_2^{(2)}$, respectively. The residual surfaces realized from stochastic simulation were added to the corresponding trend surfaces to simulate equal-probable and error-contaminated $Z$ data. This process was manipulated with 100 realizations.

These simulated $Z$ data were input to an interpolator on the time-specific probability surfaces to produce class probabilities and, hence, land cover realizations. For each time point, 100 realized area-class maps were generated and summarized with respect to class statistics, e.g., means and standard deviation. Also output was a discrete classification output from a pool of 100 realized area-class maps. Such procedures were undertaken for the datasets for time 1 and time 2, respectively. Results based on both data classes and information classes of land cover were obtained.

Results from Bayesian classification with respect to both information classes and data classes are shown in Table 1. In Table 1, results under the heading “Information classes” are those obtained by tallying land cover maps of NLCD classes. The results based on data classes maps are shown under the heading “data classes.”

| 1992 Information classes | 2001 Information classes | 2001 Data classes |
|--------------------------|--------------------------|------------------|
|                          | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| 1                        | 0.0006 | 0 | 0.3000 | 0 | 0 | 0.0002 | 0 | 0.1800 | 0 | 0.00004 |
|                          | (0.0011) | / | (0.8900) | / | / | (0.0005) | / | (0.1000) | / | (0.0001) |
| 2                        | 0.0091 | 15.6700 | 5.4800 | 2.4200 | 0.1100 | 0.0047 | 22.7000 | 5.3000 | 0 | 0.1300 |
|                          | (0.0134) | (7.6400) | (3.2000) | (0.9300) | (0.1000) | (0.0079) | (9.9800) | (3.6900) | / | (0.1500) |
| 3                        | 0.0050 | 0.1400 | 68.6800 | 1.4200 | 0.0015 | 0.0019 | 8.0100 | 57.0900 | 2.1800 | 0.6800 |
|                          | (0.0049) | (0.1500) | (10.000) | (0.2200) | (0.0011) | (0.0016) | (5.9600) | (6.8600) | (0.1300) | (0.5400) |
| 4                        | 0.0002 | 0 | 4.6300 | 0.5300 | 0.1200 | 0.0032 | 0 | 1.6600 | 1.6400 | 0.0973 |
|                          | (0.0004) | / | (2.9600) | (0.4300) | (0.1100) | (0.0052) | / | (1.0600) | (1.0200) | (0.0666) |
| 5                        | 0 | 0 | 0 | 0 | 0.4900 | 0 | 0 | 0.3200 | 0 | 0 |
|                          | / | / | / | / | (0.6200) | / | / | (0.5700) | / | / |
### Table 2  Error statistics in estimated land cover changes (%)

| Change types  | Information classes | Data classes |
|---------------|---------------------|--------------|
|               | Reference mean      | Bias         | Std | RMSE | Reference mean | Bias | Std | RMSE | Differ. |
| 1 - 1         | 0.0936              | -0.0930      | 0.0011 | 0.09304 | 0.0400 | -0.0398 | 0.0005 | 0.03979 | -0.0536 |
| 1 - 2         | 0                   | 0            | 0     | 0     | 0.0012 | -0.0012 | /     | 0.00120 | 0.0012 |
| 1 - 3         | 0.0052              | +0.3000      | 0.8900 | 0.93000 | 0.0304 | +0.1500 | 0.1000 | 0.18000 | 0.0252 |
| 1 - 4         | 0                   | 0            | 0     | 0     | 0.0028 | -0.0028 | /     | 0.00280 | 0.0028 |
| 1 - 5         | 0                   | 0            | 0     | 0     | 0.0008 | -0.0008 | 0.0001 | 0.00077 | 0.0008 |
| 2 - 1         | 0.0164              | -0.0073      | 0.0134 | 0.01525 | 0.0088 | -0.0041 | 0.0079 | 0.00891 | -0.0076 |
| 2 - 2         | 16.6000             | -0.9200      | 7.6400 | 7.69000 | 12.5700 | +10.1400 | 9.9800 | 14.22000 | -4.0300 |
| 2 - 3         | 0.2400              | +5.2400      | 3.2000 | 6.14000 | 3.8800 | +1.4100 | 3.6900 | 3.95000 | 3.6400 |
| 2 - 4         | 0.0988              | +2.3200      | 0.9300 | 2.50000 | 0.0116 | -0.0116 | /     | 0.01160 | -0.0872 |
| 2 - 5         | 0.0192              | +0.8797      | 0.1000 | 0.14000 | 0.0324 | +0.1000 | 0.1500 | 0.18000 | 0.0132 |
| 3 - 1         | 0.0008              | +0.0042      | 0.0049 | 0.00647 | 0.0084 | -0.0065 | 0.0016 | 0.00668 | 0.0076 |
| 3 - 2         | 0.0024              | +0.1300      | 0.1500 | 0.20000 | 3.6700 | +4.3400 | 5.9600 | 7.37000 | 3.6676 |
| 3 - 3         | 73.1100             | -4.4400      | 10.0000 | 10.94000 | 72.0200 | -14.9300 | 6.8600 | 16.43000 | -1.0900 |
| 3 - 4         | 0.0816              | +1.3400      | 0.2200 | 1.36000 | 1.8900 | +0.2800 | 0.1300 | 0.31000 | 1.8084 |
| 3 - 5         | 0.0020              | -0.0005      | 0.0011 | 0.00123 | 0.3200 | +0.3600 | 0.5400 | 0.65000 | 0.3180 |
| 4 - 1         | 0.0064              | -0.0062      | 0.0004 | 0.00621 | 0.0148 | -0.0158 | 0.0052 | 0.01267 | 0.0084 |
| 4 - 2         | 0                   | 0            | 0     | 0     | 0.0488 | -0.0488 | /     | 0.04880 | 0.0488 |
| 4 - 3         | 0.4100              | +4.2300      | 2.9600 | 5.16000 | 4.0500 | -2.3900 | 1.0600 | 2.61000 | 3.6400 |
| 4 - 4         | 7.2200              | -6.7000      | 0.4300 | 6.71000 | 1.2000 | +0.4400 | 1.0200 | 1.12000 | -6.0200 |
| 4 - 5         | 0.1200              | +0.0053      | 0.1100 | 0.11000 | 0.1200 | -0.0235 | 0.0666 | 0.07058 | 0.0000 |
| 5 - 1         | 0                   | 0            | 0     | 0     | 0     | 0        | /     | 0.0000 | 0.0000 |
| 5 - 2         | 0                   | 0            | 0     | 0     | 0.0028 | -0.0028 | /     | 0.00280 | 0.0028 |
| 5 - 3         | 0                   | 0            | 0     | 0     | 0.0612 | +0.2600 | 0.5700 | 0.62000 | 0.0612 |
| 5 - 4         | 0                   | 0            | 0     | 0     | 0.0072 | -0.0072 | /     | 0.00720 | 0.0072 |
| 5 - 5         | 1.9700              | -1.4800      | 0.6200 | 1.61000 | 0.0012 | -0.0012 | /     | 0.00120 | -1.9688 |

Note: Differ. represents differences between information classes and data classes reference means.

Means in Table 1 reveal individual class proportions and transition magnitudes so that their relative abundance or dominance may be interpreted. The rows display the proportions of the five classes in 1992, whereas the columns display the proportions in 2001. Thus, the off-diagonal elements represent the proportion of the landscape that experienced a transition from class \(i\) to class \(j\) between 1992 and 2001 \((i\neq j)\), and the main diagonal elements indicate the proportion of land classes that showed persistence.

Consider the comparison between reference data and Bayesian classification. As shown in Table 1, referring to information classes, “forest to grassland/shrubland” is the dominant transition type (5.48% of total land cover dataset), while “agriculture to grassland/shrubland” is the second major change type (4.63% of total land cover dataset). Results with respect to data classes indicate that “grassland/shrubland to forest” is the dominant transition type (8.01% of total land cover dataset), while “forest to grassland/shrubland” is the second major change type (5.3% of total land cover dataset). For both data classes and information classes, grassland/shrubland remains the unchanged dominant class. This is because the results of Bayesian classification are sensitive to prior class probabilities and their joint probabilities, whose estimation was based on the training samples and might not be representative of the whole study area.

Uncertainty due to measurement error (standard
deviation) and model imperfection (bias) was quantified separately, with the error statistics shown in Table 2. In Table 2, results under the heading “information classes” are those obtained by tallying land cover changes based on information classes, while those based on data classes are shown under the heading “data classes.”

Consider bias between estimated areal extents of different change types and their corresponding reference values. Bias with “+” describes that land cover change is overestimated and that with “−” describes that transition is underestimated. As shown in Table 2, referring to information classes, “forest to grassland/shrubland” is the most overestimated transition, and “agriculture to grassland/shrub” is the second most overestimated transition. Two unchanged types, “agriculture” and “grassland/shrub,” are underestimated greatly. However, referring to data classes, unchanged type “forest” is the most overestimated, while unchanged type “grassland/shrub” is the most underestimated.

As shown in Table 2, biases referring to data classes are generally smaller than those referring to information classes, except for “grassland/shrubland to forest” (+4.34% when referring to data classes, whereas +0.13% when referring to information classes), “grassland/shrubland to wetlands” (+0.36% when referring to data classes, whereas −0.0005% when referring to information classes) and “agriculture to wetlands” (+0.26% when referring to data classes, whereas 0 percent when referring to information classes) transitions and unchanged type “forest” (+10.14% when referring to data classes, whereas −0.92% when referring to information classes). Similarly, standard deviation referring to data classes tends to be smaller than that referring to information classes. Notable exception is with “grassland/shrubland to forest” (5.96% when referring to data classes, whereas 0.15% when referring to information classes). Most of root mean square errors (RMSEs) referring to data classes are smaller than those referring to information classes, except for the transitions “grassland/shrubland to forest” and “grassland/shrubland to wetlands” (7.37% and 0.65% when referring to data classes, whereas 0.2% and 0.00123% when referring to information classes), and unchanged types “forest” (14.22% when referring to data classes, whereas 7.69% when referring to information classes) and “grassland/shrubland” (16.43% when referring to data classes, whereas 10.94% when referring to information classes).

For transitions “grassland/shrubland to forest” and “grassland/shrubland to wetlands” and unchanged type “forest,” bias, standard deviation, and RMSE referring to data classes are bigger than those referring to information classes. This may be explained by the relatively large differences between reference values referring to data classes and information classes.

From error statistics reported in this paper, it is found that great discrepancies exist between those obtained by referring to data classes versus those to information classes and interpretation in terms of data classes seems to be more consistent than otherwise.

3 Conclusion

This paper extends discriminant space-based approaches to analyzing uncertainty from area-class mapping to change detection. By equipping geostatistics with discriminant models, we quantified spatial uncertainty in spatio-temporal land cover information.

There are three merits to use discriminant models to depict uncertainty:

(1) Discriminant models help to obtain data classes that are nonoverlapping in the Z space and made to coincide with information classes for the purpose of discerning effect due to data and information class semantic biases.

(2) Discriminant-based stochastic simulation has demonstrated its utility for propagating uncertainty in area classes to change information through reproducing auto- and cross-covariance in the process variables underlying the landscape dynamics and by honoring conditional data. The proposed methods are applicable for a range of scales and facilitate error propagation in land use and land cover change.

(3) Discriminant models lead to greater interoperability in geoprocessing with both continuous and discrete fields. For continuous variables, biases between estimated and reference means can be used to measure their accuracy, while variance can be used to indicate their precision. Biases and variance should be
used in combination so that error behaviors may be better elucidated.

In summary, this paper has shown that a similar typology can be usefully implemented for analysis of uncertainty in land cover information, which originates from different sources, such as discriminant variables’ deficiency, sampling errors, measurement errors, and various others. Further research should also be directed toward spatial and statistical analysis of the different error statistics, such as those obtained in the studies, so that we may conduct a well-informed and practically meaningful uncertainty management.

References

[1] Foody G M (2002) Status of land cover classification accuracy assessment[J]. Remote Sensing of Environment, 80(1): 185-201
[2] Steele B M, Patterson D A, Redmond R L (2003) Towards estimation of map accuracy without a probability test sample[J]. Ecological and Environmental Statistics, 10: 333-356
[3] Pearson R G, Thuiller W, Araújo M B, et al. (2006) Model-based uncertainty in species range prediction[J]. Journal of Biogeography, 33: 1704-1711
[4] Graham C H, Elith J, Hijmans R J, et al. (2008) The influence of spatial errors in species occurrence data used in distribution models[J]. Journal of Applied Ecology, 45: 239-247
[5] Zhang Jingxiong, Zhang Jinping, Yao Na (2009) Uncertainty characterization in remotely sensed land cover information[J]. Geo-spatial Information Science, 12(3): 165-171
[6] Congalton R G (1988) Using spatial autocorrelation analysis to explore the errors in maps generated from remotely sensed data[J]. Photogrammetric Engineering and Remote Sensing, 54: 587-592
[7] Jager H I, King A W (2004) Spatial uncertainty and ecological models[J]. Ecosystems, 7: 841-847
[8] Goodchild M F, Zhang Jingxiong, Kyriakidis P (2009) Discriminant models of uncertainty in nominal fields[J]. Transactions in GIS, 13(1): 7-3
[9] Zhang Jingxiong, You Jiong, Tang Yunwei (2010) Discriminant models for uncertainty characterization in remotely sensed land cover[C]. Proceedings of the 9th International Symposium on Spatial Accuracy Assessment in Natural Resources and Environmental Sciences, Leicester
[10] Boucher A, Dimitrakopoulos R (2009) Block simulation of multiple correlated variables[J]. Mathematical Geosciences, 41: 215-237
[11] Lee T. Richards J A, Swain P H (1987) Probabilistic and evidential approaches for multisource data analysis[J]. IEEE Transactions on Geoscience and Remote Sensing, GE-25: 283-293
[12] Erik N (1995) A method to test for systematic differences between maps and reality using error matrices[J]. International Journal of Remote Sensing, 16(16): 3147-3156