Theoretical Formalism of Andreev Reflection for Distinguishing Singlet Pairing and Triplet Pairing

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A theoretical formalism of Andreev reflection is proposed to provide a theoretical support for distinguishing the singlet pairing and the triplet pairing by the point contact Andreev reflection (PCAR) experiments, in contrast to the previous models designed only for the singlet pairing case. We utilize our theoretical curves to fit the data of the PCAR experiment on the unconventional superconductivity in the Bi/Ni bilayer [arXiv:1810.10403], and find the Anderson-Brinkman-Morel state satisfies the main characteristics of the experimental data. Moreover, the Andreev reflection spectra of the Balian-Werthamer state and the chiral p-wave are also presented.

I. INTRODUCTION

Andreev reflection[1] (AR) is the process of electrons from a normal metal (N) transformed into Cooper pairs from a superconductor (S), as the primary mechanism of electron transport across a N-S interface. The AR spectra can be used to study the characteristics of the superconducting gap, including the symmetry and magnitude of the gap. As the quantitative description models about the AR process, the Blonder-Tinkham-Klapwijk (BTK) model[2] has been used to study the isotropic gap of the BCS superconductor, and the model proposed by Kashiwaya and Tanaka[3, 4] can be used to describe the anisotropic gap superconductor by the analysis of the conductance spectra of a N-S junction. The Mazin model[5] can be used to analyze a fully polarized current across a N-S junction quantitatively, and the unified model[6] is valid for quantitative analysis of an arbitrary polarization current. However, the models mentioned above are all designed for the singlet pairing superconductivity, so they can not be used to analyze naturally the triplet pairing superconductivity.

As the well-known triplet pairing, the p-wave state was firstly found in the electrically neutral superfluid $^3$He[7], but it has never been verified experimentally in solids. The triplet pairing superconductors are often associated with the topological superconductivity[8] and applications in quantum computing and spintronics[9, 10]. There are some candidates of the triplet pairing superconductivity, including heavy fermions (e.g., Upt$_3$)[11–15], superconductors with broken inversion symmetry[16, 17] and well known Sr$_2$RuO$_4$[18–20]. Recently the AR spectroscopy of Bi/Ni bilayers[21] might indicate the existence of the p-wave superconductivity in solids. And the Bi/Ni bilayer itself is interesting enough. The single crystal Bi (110), observed bulk superconductivity below 0.53 mK[22], is epitaxially grown on the weak ferromagnetic Ni (100) layer to yield a Bi/Ni bilayer whose superconducting transition temperature is enhanced to 4 K[21]. Therefore, this bilayer system has attracted much research interest[21, 23–27].

In this paper, we propose a theoretical formalism of Andreev reflection by the four-component wave function to naturally obtain the singlet pairing case or the triplet pairing case, which provides a theoretical support for PCAR experiments to distinguish the singlet pairing and the triplet pairing. The Andreev reflection conductance of the singlet pairing superconductivity depends on spin polarization, but the case of triplet pairing superconductivity is not related to spin polarization.

This paper is organized as follows. In Section II our formalism is derived by the generalized BCS theory[28]. In Section III we follow the procedure of the formalism to obtain the results of the Anderson-Brinkman-Morel (ABM) state case. By comparing our theoretical work with the work of PCAR experiments on the unusual superconductivity of the epitaxial Bi/Ni bilayer, we find that the theoretical conductance spectra of the ABM state can describe well the features of the experimental conductance spectra. In Section IV the conductance spectra of the Balian-Werthamer (BW) state case and the chiral p-wave case are presented. In Section V, by some tricks, we present the exact conductance formula for an arbitrary cross-section of the 3D gap by the example of the ABM state and the chiral cross-section of the gap of the ABM state might explain well polar Kerr effect measurements and the time-domain THz spectroscopy in the Bi/Ni bilayer. The discussion and conclusion is given in Section VI.

II. CONDUCTANCE FORMULA

For the N-S junction, the plane wave at the normal metal side can be expressed by the four-component wave function

$$
\psi_N \left( \mathbf{r} \right) = e^{i \mathbf{k}_1 \cdot \mathbf{r}_N} \begin{pmatrix}
    e^{i k_{1x} x} + b e^{-i k_{1x} x} \\
    0 \\
    a_2 e^{i k_{2x} x} \\
    a_1 e^{(\alpha i + \beta) k_{-x}}
\end{pmatrix},
$$

where $\alpha$ and $\beta$ are the branches of the 1D gap in the N side and $\mathbf{k}_{1,2}$ are the wave vectors of the N-S interface.

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where $k_\parallel$ and $k_\perp$ (or $k_-$ and $k_+$ representing the electron-like quasiparticle wave vector and the hole-like quasiparticle wave vector respectively) are wave vector components parallel and vertical to the interface of the N-S junction respectively, the dimensionless real number $\alpha$ is related to spin polarization $P[6]$, i.e. $P = \frac{\alpha^2}{4 + \alpha^2}$, $e^{ik_\perp x}$ is an incident plane wave with spin up, $be^{-ik_\perp x}$ is a normal reflection wave, $a_1 e^{(a_1 x)}$ is a normal AR wave which is evanescent and spin up and $a_2 e^{ik_\perp x}$ is an unconventional AR wave with spin down. The second row representing the electron with spin down part is 0, since we can always choose the spin direction of the incident electron as the positive $s_Z$ direction. Therefore, Eq(1) is without loss of generality. And the coefficients ($a_1$, $a_2$ and $b$) can be solved from the boundary conditions below.

The wave function at the superconductor side is given by

$$\psi_S(r) = e^{i k_\parallel \cdot r_\parallel} \left( \begin{array}{c} u_{k_\parallel}^{(+) +} \\ u_{k_\parallel}^{(-) +} \\ v_{s_\parallel}^{(+) +} \\ v_{s_\parallel}^{(-) +} \end{array} \right) + de^{-i q \cdot x} \left( \begin{array}{c} u_{k_\parallel}^{(-) -} \\ u_{k_\parallel}^{(-) -} \\ v_{s_\parallel}^{(-) -} \\ v_{s_\parallel}^{(-) -} \end{array} \right),$$

where $(+)$ and $(-)$ denote electron-like and hole-like respectively, $u_{k_{s\parallel}}$ and $v_{k_{s\parallel}}$ are coherence factors which originate from the equation[28]

$$a_{k_{s\parallel}} = \sum_{s'} \left( u_{k_{s\parallel}} \alpha_{s'_{s\parallel}} + v_{k_{s\parallel}} \alpha_{s'_{s\parallel}} \right),$$

where $s$ represents the spin index, $a_{k_{\parallel}}$ is the annihilator operator of the electron, $a_{s_{\parallel}}$ is the annihilator operator of the quasiparticle. For simplification, we take $q_\parallel \approx k_\parallel \approx k_F$ where $k_F$ is the Fermi wave vector size at the normal side. We can utilize the four-component notations $a_k = (\hat{a}_{\parallel k}, \hat{a}_{\parallel k}^\dagger, \alpha_{\parallel k}, \alpha_{\parallel k}^\dagger)^T$ and $\hat{a}_k = (\alpha_{\parallel k}, \alpha_{\parallel k}^\dagger, \hat{a}_{\parallel k}, \hat{a}_{\parallel k}^\dagger)^T$ to make Eq(3) compact: $a_k = U_k a_k$ with

$$U_k = \left( \begin{array}{cc} \hat{u}_k & \hat{v}_k \\ \hat{s}_k & \hat{s}_k \end{array} \right)$$

and $U_k U_k^\dagger = 1,$

where $\hat{u}_k$ and $\hat{v}_k$ are $2 \times 2$ matrices, i.e.

$$\hat{u}_k = \left( \begin{array}{cc} u_{k_{\parallel}+} & u_{k_{\parallel}+} \end{array} \right) \hspace{1cm} \text{and} \hspace{1cm} \hat{v}_k = \left( \begin{array}{cc} u_{k_{\parallel}+} & u_{k_{\parallel}+} \end{array} \right).$$

According to Eq(4), we can obtain

$$a_k = U_k^\dagger a_k.$$  

By Eq(6), we can construct Eq(2). $\psi_S(r)$ are combined by the transmitted quasiparticle of the same side of the Fermi surface (i.e. the electron-like quasiparticle) and the transmitted quasiparticle crossing through the Fermi surface (i.e. the hole-like quasiparticle)[2]. The electron-like quasiparticle wave function is $e^{i k_{\parallel} \cdot r_\parallel} e^{iq_{\parallel} x} (u_{k_{\parallel}}^{(+)} u_{k_{\parallel}}^{(+) +})^{T}$ corresponding to the creation operator of the quasiparticle $a_{k_{\parallel}}^\dagger$[29] and we have the relation $\alpha_{k_{\parallel}} = u_{k_{\parallel}+} a_{k_{\parallel}+} + u_{k_{\parallel}+} a_{k_{\parallel}+}^\dagger + v_{k_{\parallel}+} a_{k_{\parallel}+} + v_{k_{\parallel}+} a_{k_{\parallel}+}^\dagger$. It is the same way to obtain the hole-like quasiparticle wave function. Therefore, we can construct Eq(2) using the elements of the matrix $U_k$. In the case of the unitary $\Delta(k)$, the wave function components corresponding to other quasiparticle $\alpha_{k_{\parallel}}$ can be neglected and the reason will be explained below. For the unitary solution[28]

$$\hat{u}_k = \frac{[E_k + \epsilon(k)] \sigma_0}{\{E_k + \epsilon(k)\}^2 + \frac{1}{2} \text{tr} \Delta \Delta^\dagger}^{1/2},$$

$$\hat{v}_k = \frac{-\Delta(k)}{\{E_k + \epsilon(k)\}^2 + \frac{1}{2} \text{tr} \Delta \Delta^\dagger}^{1/2},$$

where $E_k = \left[ \epsilon^2(k) + \frac{1}{2} \text{tr} \Delta \Delta^\dagger \right]^{1/2}$, and $\Delta(k)$ is a matrix. For the singlet pairing case

$$\Delta(k) = i \sigma_y \psi(k) = \left( \begin{array}{cc} 0 & \psi(k) \\ -\psi(k) & 0 \end{array} \right),$$

where $\psi(k)$ is an even function. For the triplet pairing case

$$\Delta(k) = i(d(k) \cdot \sigma) \sigma_y$$

$$= \left( \begin{array}{cc} -d_z(k) & id_y(k) \\ d_z(k) & id_y(k) \end{array} \right),$$

where $d(k)$ is an odd vectorial function. Since $\alpha_{k_{\parallel}} = u_{k_{\parallel}+}^\dagger a_{k_{\parallel}+} + u_{k_{\parallel}+}^\dagger a_{k_{\parallel}+}^\dagger + v_{k_{\parallel}+} a_{k_{\parallel}+} + v_{k_{\parallel}+}_{k_{\parallel}+}^\dagger$ and according to Eq(7), for the unitary solution of superconductivity we have $u_{k_{\parallel}+} = 0$, thus $\alpha_{k_{\parallel}} = u_{k_{\parallel}+}^\dagger a_{k_{\parallel}+} + v_{k_{\parallel}+} a_{k_{\parallel}+}^\dagger + v_{k_{\parallel}+} a_{k_{\parallel}+}^\dagger$. According to Eq(1), we know the spin of the incident electron wave is up, so the transmitted electron is also spin up, i.e. the space of $\alpha_{k_{\parallel}}$ is orthogonal to the incident electron. Then the transmitted electron with spin up in the superconductor will induce the quasiparticle corresponding to the annihilation operator $\alpha_{k_{\parallel}}^\dagger = u_{k_{\parallel}+} a_{k_{\parallel}+} + v_{k_{\parallel}+} a_{k_{\parallel}+}^\dagger + v_{k_{\parallel}+} a_{k_{\parallel}+}^\dagger$ through the way that it plays a role of the component corresponding to $\alpha_{k_{\parallel}}$ of the quasiparticle $\alpha_{k_{\parallel}}$. However, the transmitted electron with spin up is unable to induce the other quasiparticle corresponding to the annihilation operator $\alpha_{k_{\parallel}}^\dagger = u_{k_{\parallel}+} a_{k_{\parallel}+} + v_{k_{\parallel}+} a_{k_{\parallel}+}^\dagger + v_{k_{\parallel}+} a_{k_{\parallel}+}^\dagger$, since there is no transmitted electron with spin down in the superconductor as the inductive factor. So the wave function components corresponding to the quasiparticle $\alpha_{k_{\parallel}}$ can be neglected.

The boundary conditions

$$\frac{\hbar^2}{2m} \left\{ \frac{\partial}{\partial x} \right\} \left|_{x=0^-} \psi(x = 0^+) - \psi(x = 0^-) = 0, \right.$$  

$$\frac{\hbar^2}{2m} \left\{ \frac{\partial}{\partial x} \right\} \left|_{x=0^+} \psi(x = 0^+) + \psi(x = 0^-) + U \psi \right|_{x=0} = 0.$$  

\hspace{2cm} (10)
For the ballistic transport, the normalized conductance with a bias voltage $V$ is

$$\sigma(eV) = \frac{\int dE g^T(eV)}{\int dE g^T(\infty)}$$

where $g^T(eV) = \int_0^1 g\left(eV + \frac{1}{2}E_n - i\frac{1}{2}f\right) df$, $g(E) = 1 + |a_1(E)|^2 + |a_2(E)|^2 - |b(E)|^2$ and $\beta = 1/k_B T$. $k_B$ is the Boltzmann constant. In the simple case, we just consider the Andreev reflection in two dimensions and the corresponding formula is

$$\sigma(eV) = \frac{\int_{-\pi/2}^{\pi/2} g^T(eV) \cos \theta d\theta}{\int_{-\pi/2}^{\pi/2} g^T(\infty) \cos \theta d\theta}.$$  

However, we emphasize that this 2D formula cannot completely describe the Andreev reflection of the superconductor whose gap is anisotropic in three dimensions and is just suitable for isotropic superconductors like the $s$-wave superconductor and the particular cross-section of the 3D anisotropic case. For the 3D anisotropic superconductor, we will give the method by the ABM state case, and the corresponding formula will be presented.

Considering the effect of the additional resistance on the PCAR spectrum, we take the self-consistent method[30] for extracting $dI_{NS}/dV_{AR}$ from the measured $dI_{NS}/dV$.

$$eV_{AR} = \frac{eV}{1 + r_E \sigma(eV_{AR})},$$

$$\left(\frac{dI_{NS}}{dV}\right)_0 = \frac{(1 + r_E)(dI_{NS}/dV_{AR})_0}{1 + r_E(dI_{NS}/dV_{AR})_0},$$

where $V_{AR}$ is the voltage on the region in which the AR process occurs, $V$ is the total voltage, $r_E \equiv R_E/R_{NN}$, $1/R_{NN}$ is the normal metal conductance and $R_E$ is the additional resistance.

Following the procedure above, one can obtain the coefficients $a_1$, $a_2$, $b$, $c$, $d$ for $s$-wave and $d$-wave, which are consistent with Ref. [6] and Ref. [3] respectively. The conductance of $s$-wave and $d$-wave depends on spin polarization, which is the significant characteristic of the singlet pairing case, i.e., the conductance peak of the singlet pairing case will disappear and the conductance within gap energy voltage area will approach to zero with increasing spin polarization[6].

Note that we have presented the part of our theoretical formalism in the supplementary materials of Ref. [23], and our theoretical fitting curves to the experimental data can also be found in Ref. [23]. In this paper a more detailed version of our theoretical formalism is demonstrated. Moreover, some detailed discussions about the theoretical fitting curves to the experimental data can be found in the later sections.

### III. Conductance Spectra for the ABM State

As the triplet pairing, $p$-wave has two kinds of states[28, 31]. The first state is the ABM state[32-34]

![Diagram](image)

FIG. 2. Schematic illustration of the PCAR experimental work on Bi/Ni bilayers[21, 23]. They used epitaxial Bi/Ni bilayers (various thicknesses of Ni (0-7.5nm) and Bi (0-500nm)[21]) and obtained the conductance spectra from three almost mutually perpendicular directions, i.e. A, B and C directions as shown in the diagram. Compared with their experimental conductance spectra, our FIG. 4 (a) and (b) corresponds to the A direction, (c) and (d) corresponds to the B direction, and (e) and (f) corresponds to the C direction. Therefore, the ABM state might be indicated in the bulk of the Bi layer, and the axis of symmetry of the gap of the ABM state might be almost parallel to the B direction.

![Diagram](image)

FIG. 1. The diagram describes the process of transmission and reflection at the N-S junction. The parameter $\theta$ represents the injection angle of the electron and the parameter $\varphi$ represents the angle between the $x$ axis of the $p$-wave ($d$-wave) and the normal direction of the interface.
The relation \[ \hat{\Delta}(k) = \Delta \left( \begin{array}{cc} -e^{i\phi_k} \sin \theta_k & 0 \\ 0 & e^{i\phi_k} \sin \theta_k \end{array} \right). \] (14)

The relation \[ \hat{\Delta}(k) \hat{\Delta}^{-1}(k) = \Delta^2 \sin^2 \theta_k \hat{\sigma}_0 \] implies the ABM state belongs to the unitary solution. Eq.(7) is utilized to obtain its coherence factors which are elements of matrixes \[ \hat{u}_k = \sqrt{\frac{1}{2}(1 + \epsilon(k)/\Gamma_E)} \hat{\sigma}_0, \quad \hat{u}_k = \text{sgn}(\sin \theta_k) e^{i\phi_k} \sqrt{\frac{1}{2}(1 - \epsilon(k)/\Gamma_E)} \hat{\sigma}_z, \quad (u_{1+}^+ = v_{1+}^+ = 0). \]

In general case, the gap of the ABM state is anisotropic in three dimensions, but for getting a feeling we firstly study a cross-section of the gap of the ABM state, which is in the plane fixed by \( \phi_k = 0 \) and \( \phi_k = \pi \). In this cross-section case, the calculated coefficients \( a_1, a_2, b, c, d \) are listed in Table I. Where \( Z = m\Omega/\hbar^2k_F \) and \( \Gamma \) representing the inelastic scattering factor [35]. You will find there is no the parameter \( \phi_k \). The range of \( \theta_k \) is changed from \((0, \pi)\) to \((0, 2\pi)\) for the mathematical convenience that we equivalently replace \( \phi_k \) with extending the range of \( \theta_k \), and the process of Andreev reflection is in the two-dimensional plane as shown by FIG. 1. Other parameters can be illustrated by FIG. 1. The parameter \( \theta \) represents the electron injection angle at the interface from the normal metal side and the parameter \( \varphi \) represents the angle between the \( x \) axis of the \( p \)-wave (\( d \)-wave) and the normal direction of the interface. The transmitted hole-like quasiparticle and the electron-like quasiparticle have different effective pair potentials \( \Delta(\theta+) \) and \( \Delta(\theta-) \), respectively, with \( \theta_+ = \theta \) and \( \theta_- = \pi - \theta \). Due to \( a_1 \equiv 0 \) and from Eq(1), we can easily understand these coefficients are not related to the parameter \( \alpha \). The spin polarization \( P \) only depending on \( \alpha \), i.e. \( P = \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \), these coefficients don’t depend on spin polarization \( P \). Therefore, the conductance of the ABM state is not related to spin polarization, contrary to the singlet pairing case.

Before proceeding to discuss the conductance spectra of the ABM state, we give the physical understanding on the Andreev reflection conductance peak in order to understand well the results of the PCAR experiments on the Bi/Ni bilayer. In the \( d \)-wave case, Satoshi Kashiwaya, etc., proposed that the conductance peak originates from the bound states, which are localized around the N-S interface, and these bound states decay into the bulk [3]. The N-S interface can be regarded as the node of pair potential for the quasiparticles. The tunneling electrons from N to S flow via the bound states just like the resonant tunneling process. Moreover, the conductance peak is formed at the energy levels of the bound states.

For example, as shown in FIG. 3(b), in the \( d \)-wave case with \( \varphi = \pi/4 \) (the definition of \( \varphi \) is explained in FIG. 1), a quasiparticle with negative phase Cooper pair potential \( \Delta^- \) injected from the bulk of the superconductor to the N-S interface is reflected at the interface. Its pair potential phase changes from negative to positive at the interface, and these bound states decay into the bulk [3]. Then at the point of incidence, the Cooper pair potential is equal to zero \((\Delta_1 = 0)\) due to the overlap of the negative phase Cooper pair potential of the incident quasiparticle wave and the positive phase Cooper pair potential of the reflected quasiparticle wave. The reflectivity of quasiparticles is enhanced with increasing the barrier height \( Z \), and the large bar-

| ABM state (2D case) | \( a_1 \) | \( a_2 \) | \( b \) | \( c \) | \( d \) |
|---------------------|--------|--------|--------|--------|--------|
|                     | 0      | \( u_{-q\uparrow}^{(-)} - u_{-q\uparrow}^{(+)} \) | \( Z(\epsilon + u_{q\uparrow\uparrow}^{(-)} - u_{q\downarrow\downarrow}^{(+)} - v_{q\uparrow\downarrow}^{(-)} - v_{q\downarrow\uparrow}^{(+)} \) | \( -i(\epsilon + u_{q\uparrow\downarrow}^{(-)} - u_{q\downarrow\uparrow}^{(+)} - v_{q\uparrow\downarrow}^{(-)} - v_{q\downarrow\uparrow}^{(+)} \) | \( (\epsilon + u_{q\downarrow\uparrow}^{(-)} - u_{q\uparrow\downarrow}^{(+)} - v_{q\uparrow\downarrow}^{(-)} - v_{q\downarrow\uparrow}^{(+)} \) |
rier height limit gives rise to high-density bound states at the interface. There will be a zero-energy peak when the bias voltage makes the Fermi energy of the superconductor slightly higher than the normal metal (vice versa). However, in the $d$-wave case with $\varphi = 0$ as shown in FIG. 3(a), the phase of pair potential $\Delta^-(\Delta^+)$ of a quasiparticle coming from the bulk of the superconductor will not change after it reflects at the interface. Then the overlap of the identical phases of the incident quasiparticle wave and the reflected quasiparticle wave at the point of incidence results in the finite amplitude of pair potential ($\Delta_I \neq 0$), which contradicts the condition that the Cooper pair potential is equal to zero at the N-S interface with the zero bias voltage. This implies that quasiparticles are not reflected at the N-S interface in the $d$-wave case with $\varphi = 0$ and the zero bias voltage. However, a positive bias voltage $\Delta/e$ added, the Fermi energy difference between the superconductor and the normal metal becomes $\Delta$. Therefore, the Cooper pair potential at the N-S interface becomes $\Delta_I = \Delta$, since a hole-like quasiparticle excitation needs at least the energy $\Delta$. A negative bias voltage $-\Delta/e$ added, the Cooper pair potential at the N-S interface becomes $\Delta_I = -\Delta$ since an electron-like quasiparticle excitation excitation needs at least the energy $\Delta$. In the large barrier height limit, the two cases give rise to the high density of bound states at the interface, and thus there will be a peak near the bias voltage $\Delta/e$ (or $-\Delta/e$).

The ABM state is analyzed in the same way. In the case of the cross-section of $\omega = 0$ and $\phi_0 = 0$ (the definitions of $\omega$ and $\phi_0$ are explained in Section V), the phase of the Cooper pair potential $\Delta^+(\Delta^-)$ of a quasiparticle coming from the bulk of the superconductor will not change after it is reflected at the interface, as shown in FIG. 3(c). For the same reason above, there will be a conductance peak near the bias voltage $\Delta/e$ (or $-\Delta/e$) in the large barrier height limit. In the case of $\omega = \pi/2$ and $\phi_0 = 0$, as shown in FIG. 3(d), the Cooper pair potential $\Delta^+(\Delta^-)$ of a quasiparticle coming from the bulk of the superconductor will change sign after it is reflected at the interface, which yields the zero energy conductance peak in the large barrier height limit.

The normalized conductance spectra of the ABM state are shown in FIG. 4. Furthermore, we utilized the theoretical curves of the ABM state to fit the data from the PCAR experiment on the Bi/Ni bilayer[23]. In FIG. 4 the black circles and the blue circles are the experimental data[23] and the red lines are theoretical fitting curves by our theory. The normalized conductance spectra do not depend on spin polarization. (a) and (b) with $\varphi = 0.27\pi$ and $\varphi = 0.3\pi$, respectively, both of them are of a single peak; (c) and (d) with $\varphi = 0.01\pi$ near zero, both of them are of a double peak; (e) and (f) with $\varphi = 0.29\pi$ and $\varphi = 0.21\pi$ respectively, both of them are of a single peak. There is the highly unusual superconductivity experimentally found in epitaxial Bi/Ni bilayers[36, 37]. Moreover, Ref. [21] implies it might be $p$-wave. The PCAR experimental work on Bi/Ni bilayers[23] can be illustrated by FIG. 2. They used the Au tip and the LSMO ($La_{0.67}Sr_{0.33}MnO_3$) tip, which can produce incident electrons with spin unpolarized and highly polarized respectively, to vertically touch the Bi/Ni bilayer surface (i.e. the A direction). Furthermore, they found the conductance is always of a single peak in the A direction, whether the tip is Au or LSMO. In the other two directions parallel to the Bi/Ni bilayer surface, they obtain a double peak from the B direction and a single peak from the C direction, which are also independent of the spin polarization of incident electrons. The features of their experimental conductance spectra are completely consistent with our theoretical conductance spectra. In FIG. 4 the black circles and the blue circles represent the experimental data of the conductance by using the Au tip and the LSMO tip, respectively. The data of (a) and (b) are obtained from the A direction, the data of (c) and (d) are obtained from the B direction, and the data of (e) and (f) are obtained from the C direction. We can easily find that the configuration of the ABM state shown in FIG. 3(c) and FIG. 3(d) satisfies the features of the conductance peaks in the three directions. Moreover, it is the same with our theory that their experimental conductance spectra are also not related to spin polarization. Therefore, the ABM state might be indicated in the bulk of Bi layer of the Bi/Ni bilayer system as shown in FIG. 2. According to FIG. 2, FIG. 3(c), FIG. 3(d) and FIG. 4, we should consider the three-dimensional gap structure of the ABM state in the Bi/Ni bilayer, and its axis of symmetry is almost parallel to the B direction. Since the process of Andreev reflection is in a two-dimensional plane, it is enough that we consider the cross-section of the 3D gap parallel to the surface of the Bi/Ni bilayer when the tip at the B direction. Moreover, we can illustrate the Andreev process by FIG. 1. However, when the tip at the A direction or the C direction, we only obtain the Andreev reflection in the special cross-section of the 3D gap of the ABM state, and we should consider an arbitrary cross-section of the 3D gap. The discussion and the method will be presented in the later section.

IV. CONDUCTANCE SPECTRA FOR THE BW STATE AND THE CHIRAL P WAVE

The second state of $p$-wave is the BW state[34, 38] and its Andreev reflection is restricted to the condition

$$i\alpha \tan(\theta + \varphi) + (\alpha + 2i)\tan(\theta - \varphi) + \tan(\theta + \varphi)|Z + 2[\tan(\theta - \varphi) + \tan(\theta + \varphi)]Z^2 = 0.$$  \hspace{1cm} (15)

From Eq.(15), we find $Z = \alpha = 0$. And for the BW state, we can easily obtain

$$g(E) = \frac{2E}{E + \sqrt{E^2 - \Delta^2}}.$$  \hspace{1cm} (16)

From Eq.(12) we can find that the normalized conductance is isotropic. Therefore, its conductance spectra are
not related to the angle parameter $\varphi$, as shown in FIG 5. In the case of $T = 0K$, the FIG 5(a) is the same with the $s$-wave conductance spectrum at 0K with $Z = 0$. For comparing, FIG 5(b), the normalized conductance spectrum of the BW state at 1.43K with $\alpha = 0$ and $Z = 0$, is presented.

The effective pairing potential $\hat{\Delta}(k)$ for the $p + ip$ state (in two dimensions) is

$$\Delta(k) = \Delta e^{-i\theta_k}\hat{\sigma}_z,$$

where $\theta_k \in (0, 2\pi)$. Because of $\hat{\Delta}(k)\hat{\Delta}^\dagger(k) = \Delta^2\hat{\sigma}_0$, which implies the chiral $p$-wave belongs to the unitary solution, in the same way we can utilize Eq(7) to obtain its coherence factors which are elements of matrixes

$$\hat{u}_k = \frac{1}{2}(1 + \epsilon(k)/E_k)\hat{\sigma}_0,$$

$$\hat{v}_k = \frac{-\hat{\Delta}(k)}{\sqrt{2E_k(E_k + \epsilon(k))}}.$$  

where $E_k = \sqrt{\epsilon^2(k) + \Delta^2}$. Following the same procedure, we can quickly obtain the coefficients $a_1, a_2, b, c,$ $d$, which are essentially the same with the ABM state case, and its conductance is also not related to spin polarization, however, the conductance of the chiral $p$-wave being isotropic as shown in FIG 6.
and you will find they are almost the same with

Choose a coordinate system \(I\) where the \(I_x - I_y\) plane is in the N-S interface and the \(I_z\) axis pointing to the superconductor side is perpendicular to the interface. Without loss of generality, we restrict the axis of symmetry of the gap of the ABM state lying in the \(I_z - I_y\) plane and the angle part of spherical coordinates of the axis of symmetry in the coordinate system \(I\) is \((\theta_n, \varphi_n) \equiv (\omega, 0)\). The wave vector direction of an incident electron is \(k \equiv (k_x, k_y, k_z) \equiv (\sin \theta_k \cos \phi_k, \sin \theta_k \sin \phi_k, \cos \theta_k)\) expressed in the coordinate system \(I\) (the incident electron with wave vector \(k\)). Then rotate the coordinate system \(I\) around \(I_y\) to make the axis \(I_z\) coincide with the axis of symmetry of the gap and in a new coordinate system \(I'\) the wave vector direction \(\hat{k}\) is expressed by \(\hat{k}' \equiv R(\hat{I}_y, -\omega)\hat{k}^T\). Where \(R(\hat{I}_y, -\omega)\) is

\[
R(\hat{I}_y, -\omega) = \begin{pmatrix}
\cos \omega & 0 & -\sin \omega \\
0 & 1 & 0 \\
\sin \omega & 0 & \cos \omega
\end{pmatrix}.
\]

Then we obtain

\[
\hat{k}' = (\sin \theta_k \cos \phi_k \cos \omega - \cos \theta_k \sin \omega, \\
\sin \theta_k \sin \phi_k, \sin \theta_k \cos \phi_k \sin \omega + \cos \theta_k \cos \omega).
\]

The gap equation of the ABM state is

\[
\hat{\Delta}(k) = -\Delta(\hat{k}' + i\hat{k}^T)\hat{\sigma}_z = -\Delta(\sin \theta_k \cos \phi_k \cos \omega - \cos \theta_k \sin \omega + i \sin \theta_k \sin \phi_k)\hat{\sigma}_z.
\]

Now this gap function is linked with the direction of the wave vector of the incident electron. Therefore, the expressions of \(\hat{u}_k\) and \(\hat{v}_k\) are linked to the wave vector of the incident electron

\[
\hat{u}_k = \sqrt{\frac{1}{2} (1 + \epsilon(k)/E_k)} \hat{\sigma}_0,
\]

\[
\hat{v}_k = \frac{-\hat{\Delta}(k)}{\sqrt{2E_k(E_k + \epsilon(k))}}.
\]

The corresponding calculated coefficients \(a_1, a_2, b, c, d\) for the incident electron with the wave vector \(k\) are listed in Table II and you will find they are almost the same with the 2D case.

**TABLE II.** Here \(\hat{u}_k^{(\pm)} = \sqrt{\frac{1}{2} (1 \pm \epsilon^{\pm}(q)/(\|E_k| + i\Gamma)}\hat{\sigma}_0, \hat{v}_k^{(\pm)} = -\Delta(\hat{\theta}_k^{\pm}, \phi_k, \omega)/\sqrt{2(\|E_k| + i\Gamma) \pm \epsilon^{\pm}(q)}\hat{\sigma}_z, \hat{\Delta}(q) = (Z^2 + 1)u^{(+)}_{q^+}v^{(-)}_{q^+} - Z^2u^{(-)}_{q^+}v^{(+)}_{q^+}, \epsilon^{\pm}(q) = \sqrt{(\|E_k| + i\Gamma)^2 - \|\Delta(t(\hat{\theta}_k^{\pm}, \phi_k, \omega)\|^2/2}, \Delta(t(\hat{\theta}_k^{\pm}, \phi_k, \omega) = -\Delta(\sin \theta_k \cos \phi_k \cos \omega - \cos \theta_k \sin \omega + i \sin \theta_k \sin \phi_k), \hat{\theta}_k^{\pm} = \hat{\theta}_k and \hat{\theta}_y = \pi - \hat{\theta}_k.

| \(a_1\) | \(a_2\) | \(b\) | \(c\) | \(d\) |
|---|---|---|---|---|
| ABM state (3D case) | 0 | \(\epsilon^{(+)}/\epsilon^{(-)}\) | \(\epsilon^{(+)}/\epsilon^{(-)}\) | \(\epsilon^{(+)}/\epsilon^{(-)}\) |

**FIG. 6.** The conductance of the chiral \(p\)-wave is isotropic, so there is no the angle parameter \(\varphi\). And it is a single peak and doesn’t depend on spin polarization.

**FIG. 7.** The red circle represents the cross-section of the 3D gap of the ABM state in the case of \(\omega = \pi/2\) and \(\phi_0 = \pi/2\), and this cross-section is the same with the chiral \(p\)-wave in two dimensions.

**V. THE CONDUCTANCE FORMULA FOR ANDREEV REFLECTION OF THE ABM STATE IN AN ARBITRARY CROSS SECTION**

Now let’s demonstrate how to get the exact conductance formula for the 3D gap in an arbitrary cross-section by the example of the ABM state. We need some tricks to link the gap equation to the wave vector direction of the incident electron since the incident electron with different wave vector directions will feel different phases of the gap at the superconductor side. Choose a coordinate system \(I\) where the \(I_x - I_y\) plane is in the N-S interface and the \(I_z\) axis pointing to the superconductor side is perpendicular to the interface. Without loss of generality, we restrict the axis of symmetry of the gap of the ABM state lying in the \(I_z - I_y\) plane and the angle part of spherical coordinates of the axis of symmetry in the coordinate system \(I\) is \((\theta_n, \varphi_n) \equiv (\omega, 0)\). The wave vector direction of an incident electron is \(k \equiv (k_x, k_y, k_z) \equiv (\sin \theta_k \cos \phi_k, \sin \theta_k \sin \phi_k, \cos \theta_k)\) expressed in the coordinate system \(I\) (the incident electron with wave vector \(k\)). Then rotate the coordinate system \(I\) around \(I_y\) to make the axis \(I_z\) coincide with the axis of symmetry of the gap and in a new coordinate system \(I'\) the wave vector direction \(\hat{k}\) is expressed by \(\hat{k}' \equiv R(\hat{I}_y, -\omega)\hat{k}^T\). Where \(R(\hat{I}_y, -\omega)\) is

\[
R(\hat{I}_y, -\omega) = \begin{pmatrix}
\cos \omega & 0 & -\sin \omega \\
0 & 1 & 0 \\
\sin \omega & 0 & \cos \omega
\end{pmatrix}.
\]

Then we obtain

\[
\hat{k}' = (\sin \theta_k \cos \phi_k \cos \omega - \cos \theta_k \sin \omega, \\
\sin \theta_k \sin \phi_k, \sin \theta_k \cos \phi_k \sin \omega + \cos \theta_k \cos \omega).
\]

The gap equation of the ABM state is

\[
\hat{\Delta}(k) = -\Delta(\hat{k}' + i\hat{k}^T)\hat{\sigma}_z = -\Delta(\sin \theta_k \cos \phi_k \cos \omega - \cos \theta_k \sin \omega + i \sin \theta_k \sin \phi_k)\hat{\sigma}_z.
\]

Now this gap function is linked with the direction of the wave vector of the incident electron. Therefore, the expressions of \(\hat{u}_k\) and \(\hat{v}_k\) are linked to the wave vector of the incident electron

\[
\hat{u}_k = \sqrt{\frac{1}{2} (1 + \epsilon(k)/E_k)} \hat{\sigma}_0,
\]

\[
\hat{v}_k = \frac{-\hat{\Delta}(k)}{\sqrt{2E_k(E_k + \epsilon(k))}}.
\]

The corresponding calculated coefficients \(a_1, a_2, b, c, d\) for the incident electron with the wave vector \(k\) are listed in Table II and you will find they are almost the same with the 2D case.
In an arbitrary cross-section of the 3D gap of the ABM state, the normalized conductance with a bias voltage $V$ is

$$
\sigma(V) = \frac{(1 + Z^2)}{2} \left\{ \int_0^{\pi/2} g^T(eV) \cos \theta_k d\theta_k |_{\phi_k = \phi_0} + \int_0^{\pi/2} g^T(eV) \cos \theta_k d\theta_k |_{\phi_k = \phi_0 + \pi} \right\},
$$

where $g^T(eV) = \int_0^1 g \left( eV + \frac{1}{2}\left|\mathbf{E} - \mathbf{E}_\mathbf{f}\right| \right) df$, $g(E) = 1 + |\alpha_1(E)|^2 + |\alpha_2(E)|^2 - |\mathbf{E}|^2$ and $\beta = 1/\sqrt{\mathbf{B}} T$. The parameters $\omega$ and $\phi_0$ will determine the cross-section of the gap of the ABM state, in which the Andreev reflection process occurs. And the Andreev reflection cross-section will be determined by connecting the tip and the sample in the PCAR experiment.

As said above, we should consider the gap structure of the ABM state in three dimensions for the PCAR experiments on the Bi/Ni bilayer. Electrons across the N-S interface, there are four trajectories[2]: electrons reflection, holes reflection, electron-like quasiparticles transmission, and hole-like quasiparticles transmission, as shown in FIG. 1. According to the translation invariance in the direction along the interface[3], these four kinds of trajectories are in the same plane.

As shown in FIG. 2, when the tip is in the B direction, the wave vector $k_B$ of an incident electron can be decomposed into two components, $k_\|$ and $k_\perp$ parallel and perpendicular to the surface touched by the tip respectively. Therefore, the two wave vector components determine the cross-section of $\omega = 0$ and $\phi_0 = \text{const}$, and the pair potential that quasiparticles feel in this cross-section is $\pm \Delta \sin \theta_k e^{i\phi_0} \hat{\sigma}_z$, according to Eq(21). For example, in the case of $k_\|$ being parallel to the surface of the Bi/Ni bilayer, which corresponds to the cross-section of $\omega = 0$ and $\phi_0 = 0$, one can find there is a sign resulting from an effective $\pi$ phase. As discussed above, the symmetric axis of the 3D gap of the ABM state is almost along the B direction. On account of the rotational symmetry of the 3D gap along the B direction, the characteristics of the cross-sections of $\omega = 0$ and $\phi_0 = \text{const}$ are the same with the case of $\omega = 0$ and $\phi_0 = 0$.

For the case of probing in the C direction as shown in FIG. 2, in the same way, we can obtain the Andreev reflection cross-section of $\omega = \pi/2$ and $\phi_0 = \text{const}$. When the $k_\|$ is also parallel to the surface of the Bi/Ni bilayer, the corresponding cross-section is determined by $\omega = \pi/2$ and $\phi_0 = 0$, namely $\Delta \cos \theta_k \hat{\sigma}_z$ according to Eq(21). However, there is no the rotational symmetry of the 3D gap along the C direction. Therefore, the characteristics of other cross-sections of the 3D gap in the C direction are different from the case of $\omega = \pi/2$ and $\phi_0 = 0$. Particularly, when the $k_\|$ is simultaneously perpendicular to the B direction, namely the case of $\omega = \pi/2$ and $\phi_0 = \pi/2$, its cross-section of the 3D gap is $\Delta \hat{\sigma}_z$, according to Eq(21). As shown in FIG. 7 (the red circle represents the cross-section of this particular case), this cross-section is the same as the chiral $p$-wave in 2D. The Andreev reflection spectra of the chiral $p$-wave are shown in FIG. 6, and they are still of a single peak just like the case of $\omega = \pi/2$ and $\phi_0 = 0$. However, they are isotropic, i.e., they do not depend on the angle $\phi$, which is defined in FIG. 1. In the case of $\omega = \pi/2$ and $\phi_0 = \text{const}$, the corresponding conductance spectra are always of a single peak, and the height of the peak becomes lower with the parameter $\phi_0$ increasing from 0 to $\pi/2$. This important single peak characteristic guarantees that one will always obtain a single peak signal from the A direction or the C direction at the experiment as shown in FIG. 2. For convenience, we utilized the cross-section of $\omega = \pi/2$ and $\phi_0 = 0$ to describe the main characteristics of Andreev reflection in the C direction. The analysis of the A direction is completely the same as the C direction.

The physical origin of the zero-bias conductance peak of the ABM state in the cross-section of $\omega = \pi/2$ and $\phi_0 = 0$ is the same as the case of $\omega = \pi/2$ and $\phi_0 = \pi/2$. In these two kinds of cases, when a quasiparticle from the superconductor side moves to the N-S interface, the effective pair potential of the reflected quasiparticle changes a $\pi$ phase compared with the incident quasiparticle, and there is a perfect elastic scattering process for quasiparticles at the interface, due to the complete destructive interference of the effective pair potential.

The chiral cross-section, namely the case of $\omega = \pi/2$ and $\phi_0 = \pi/2$, is very important. We propose that the chiral cross-section of the ABM state might explain the broken time-reversal symmetry (TRS) as determined by polar Kerr effect measurements[26] and the time-domain THz spectroscopy where a fully gapped superconductivity in the bulk of the system is proposed[27]. The Bi layer of the Bi/Ni bilayer has a certain thickness and the PCAR experimental data[23] are obtained from three mutually perpendicular directions of the Bi/Ni bilayer, which indicate the gap of the ABM state in the Bi/Ni bilayer should be understood from a three-dimensional perspective rather than a two-dimensional perspective. Namely, we should regard the superconducting electronic system in the Bi/Ni bilayer as a three-dimensional system instead of a quasi-two-dimensional system. When the measurement is performed in a direction parallel to the chiral cross-section of the ABM state, the characteristics of the chiral $p$-wave are detected, including the broken TRS and a fully gapped superconductivity. Here the chiral $p$-wave, i.e. $p + ip$, has naturally two $p$-wave components with equal magnitudes. This can explain well the time-domain THz spectroscopy[27] where a gap structure with approximately equal magnitudes for two $p$-wave components of the chiral $p$-wave is proposed. In fact, as long as the measurement is not along the axis of symmetry of the 3D gap of the ABM state in Bi/Ni bilayers, the characteristics of the chiral $p$-wave manifest. Moreover, we considering the inhomogeneous sample of the Bi/Ni bilayer, the axes of symmetry of the ABM states in different localities of a sample might only be roughly parallel to each other rather than in the same direction completely. Therefore, the characteristics of the
chiral $p$-wave always manifest in various measurements.

VI. DISCUSSION AND CONCLUSION

In Ref. [23] it is proposed that the axis of symmetry of the gap of the ABM state in the Bi/Ni bilayer is along the direction of the Ni magnetization which is in-plane. And they used an external in-plane magnetic field to change the Ni magnetization to different directions, which indicates a two-fold symmetry of the gap structure. As said above, the chiral cross-section of the ABM state might explain well the broken TRS as determined by polar Kerr effect measurements[26] and the time-domain THz spectroscopy[27]. Therefore, the ABM state might be a good candidate state for the superconductivity of the Bi/Ni bilayer.

In conclusion, we propose a theoretical formalism to demonstrate that the Andreev reflection of the triplet pairing superconductivity is independent of spin polarization, contrary to the singlet pairing case. Our theoretical conductance spectra of the ABM state can explain the main features of the PCAR experimental measurements on epitaxial Bi/Ni bilayers. The superconducting gap of the ABM state might be indicated from the experimental data. In the case of the ABM state, the conductance spectra with different angle parameters $\varphi$ are presented, and the candidate ABM state demonstrates the exact conductance formula for an arbitrary cross-section of the 3D gap. The conductance spectra of the BW state are isotropic and are restricted to the condition $Z = 0$ and $P = 0$. The Andreev reflection conductance spectra of the chiral $p$-wave are also isotropic.

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