Conceptual aspects of QCD factorization in hadronic $B$ decays

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Abstract: I review the meaning of “QCD factorization” in hadronic two-body $B$ decays and then discuss recent results of theoretical (rather than phenomenological) nature: the proof of factorization at two loops; the identification of “chirally enhanced” power corrections; and the role of annihilation contributions.

1. Introduction

Hadronic, two-body $B$ decays are highly interesting observables for flavour physics, since they depend on CKM matrix elements, including the CP-violating phase of the CKM matrix, and potential other flavour-changing interactions. They also present a formidable challenge for theory, since they involve three fundamental scales, the weak interaction scale $M_W$, the $b$-quark mass $m_b$, and the QCD scale $\Lambda_{\text{QCD}}$. From the point of view of fundamental physics, the sensitivity to the weak interaction scale is probably most interesting, but since this physics is weakly coupled, it is straightforwardly computable, given a particular model of flavour violation. Most theoretical work therefore concerns strong-interaction corrections. The strong-interaction effects which involve virtualities above the scale $\mu \sim m_b$ are well understood. They renormalize the coefficients of local operators $O_i$ in the weak effective Hamiltonian. Assuming the Standard Model of flavour violation, the amplitude for the decay $B \to M_1M_2$ is given by

$$A(B \to M_1M_2) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) \langle M_1M_2|O_i|B\rangle(\mu), \quad (1.1)$$

where $G_F$ is the Fermi constant. Each term in the sum is the product of a CKM factor $\lambda_i$, a coefficient function $C_i(\mu)$, which incorporates strong-interaction effects above the scale $\mu \sim m_b$, and a matrix element of an operator $O_i$. In extensions of the Standard Model, there may be further operators and different flavour-violating couplings, but the strong-interaction effects below the scale $\mu$ are still encoded by matrix elements of local operators.

The theoretical problem is therefore to compute these matrix elements. Since they depend on $m_b$ and $\Lambda_{\text{QCD}}$, one should take advantage of the fact that $m_b \gg \Lambda_{\text{QCD}}$ and compute the short-distance part of the matrix element. The remainder then depends only on $\Lambda_{\text{QCD}}$, and to leading order in $\Lambda_{\text{QCD}}/m_b$ turns out to be much simpler than the original matrix element. In this talk I summarize some conceptual aspects of our recent work on this problem. The discussion of the phenomenology of some particular decay modes is omitted here, but can also be found in Refs. 1, 2, 3.
2. QCD Factorization

2.1 The Physical Picture

Factorization is a property of the heavy-quark limit, in which we assume that the $b$ quark mass is parametrically large. The $b$ quark is then decaying into a set of very energetic partons. How these partons and what is left of the $B$ meson hadronize into two mesons depends on the identity of these mesons.

The simplest case is $\bar{B}_d \to D^{+}\pi^-$, when the $D$ meson is also taken to be parametrically heavy. The spectator quark and other light degrees of freedom in the $B$ meson have to rearrange themselves only slightly to form a $D$ meson together with the charm quark created in the weak interaction. The other two light quarks are very energetic and for them to form a pion they must be highly collinear and in a colour-singlet configuration. Soft interactions decouple from such a configuration and this allows it to leave the decay region without interfering with the $D$ meson formation. The probability of such a special configuration to form a pion is described by the leading-twist pion light-cone distribution amplitude (LCDA) $\Phi_\pi(u)$. The $B \to D$ transition is parameterized by a standard set of form factors. I have repeated essentially the argument of Ref. 4 in favour of the conventional factorization picture, but it is important that this can be converted into a quantitative scheme to compute higher order corrections. For example, if the light quark-antiquark pair is initially formed in a colour-octet state, we can still show that soft gluons decouple, if this pair is to end up as a pion. This implies that the pair must interact with a hard gluon, and hence this provides a calculable strong-interaction correction to the basic mechanism discussed above. (A correction of this type was computed already in Ref. 5, but it seems to me that the generality and importance of the result went unnoticed.) An important element in demonstrating the suppression of soft interactions (except for those parameterized by the $B \to D$ form factor) is the assumption that the pion LCDA vanishes linearly as the longitudinal momentum fraction approaches the endpoints $u = 0, 1$. This assumption can be justified by the fact that it is satisfied by the asymptotic distribution amplitude $\Phi_u(u) = 6u(1-u)$, which is the appropriate one in the heavy-quark limit.

As a consequence

$$\int_0^{\Lambda_{QCD}/m_b} du \, u^n \Phi_\pi(u) \sim \left(\frac{\Lambda_{QCD}}{m_b}\right)^{n+2} (n > -2).$$

(2.1)

This guarantees the suppression of soft endpoint contributions.

Note that the above discussion relies crucially on the spectator quark going to the heavy meson in the final state. If, as in the case of a $D^0\pi^0$ final state, the spectator quark must be picked up by the light meson, the amplitude is suppressed by the $B \to \pi$ form factor. But since the $D$ meson’s size is of order $1/\Lambda_{QCD}$, the $D^0$ formation and $B \to \pi$ transition cannot be assumed to not interfere and factorization is violated.

The case of two light final state mesons is the most interesting one. The dominant decay process is indeed the same as for the case of the $D^+\pi^-$ final state, but this implies that the light meson that picks up the spectator quark is formed in a very asymmetric configuration in which the spectator quark carries a tiny fraction $\Lambda_{QCD}/m_b$ of the total energy. Such a configuration is suppressed, see (2.1), and this suppression is equivalent to the well-known $(\Lambda_{QCD}/m_b)^{3/2}$-suppression of heavy-to-light form factors at large recoil. Owing to this suppression there exists a competing process, in which a hard gluon is exchanged with the spectator quark, propelling it to large energy, thus avoiding the penalty factor of (2.1). If the hard gluon connects to the quark-antiquark pair emanating from the weak decay vertex to form the other light meson, this gives rise to another contribution to the factorization formula. (If the gluon connects to the $b$ quark or the quark that forms the light meson together with the spectator quark, we can consider this as a hard-scattering contribution to the heavy-to-light form factor.) This further contribution, called “hard-spectator interaction”, can be computed with standard methods for light-cone-dominated reactions 4, 5.

There also exist “annihilation” contributions, defined as those diagrams, in which the spectator fermion line connects to the weak decay vertex.
These contributions are suppressed by the factor
\[
\frac{\int d\xi \Phi_B(\xi)}{\int d\xi \Phi_B(\xi)/\xi} \equiv \frac{\lambda_B}{M_B} \sim \frac{\Lambda_{QCD}}{m_b},
\]  
(2.2)

where \( \Phi_B(\xi) \) is the \( B \) meson LCDA and \( \xi \sim \Lambda_{QCD}/m_b \) the light-cone momentum fraction of the spectator quark. Hence annihilation contributions can be neglected in the heavy-quark limit (but see the later discussion).

2.2 The Factorization Formula

We consider weak decays \( B \rightarrow M_1M_2 \) in the heavy-quark limit. The formal expression of the previous discussion is given by the following result for the matrix element of an operator \( \mathcal{O} \) in the weak effective Hamiltonian, valid up to corrections of order \( \Lambda_{QCD}/m_b \):

\[
\langle M_1M_2|\mathcal{O}_i|\bar{B}\rangle = \sum_j F_j^{B\rightarrow M_1}(m_{1.2}^2) \int_0^1 du T_j^{M_1}(u) \Phi_{M_1}(u)
\]

\[+ (M_1 \leftrightarrow M_2)
\]

\[+ \int_0^1 d\xi d\eta d\nu T_j^{1/2}(\xi,\eta,\nu) \Phi_B(\xi) \Phi_{M_1}(\nu) \Phi_{M_2}(u)
\]

\[\text{if } M_1 \text{ and } M_2 \text{ are both light},
\]

\[
\langle M_1M_2|\mathcal{O}_i|\bar{B}\rangle = \sum_j F_j^{B\rightarrow M_1}(m_{1.2}^2) \int_0^1 du T_j^{M_2}(u) \Phi_{M_2}(u)
\]

\[\text{if } M_1 \text{ is heavy and } M_2 \text{ is light}.
\]

(2.3)

Here \( F_j^{B\rightarrow M_1}(m_{1.2}^2) \) denotes a \( B \rightarrow M_{1.2} \) form factor, and \( \Phi_X(u) \) is the light-cone distribution amplitude for the quark-antiquark Fock state of meson \( X \). \( T_j^{M_1}(u) \) and \( T_j^{1/2}(\xi,\eta,\nu) \) are perturbatively calculable hard-scattering functions; \( m_{1.2} \) denote the light meson masses. Eq. (2.3) is represented graphically in Fig. 1. (The fourth line of (2.3) is somewhat simplified and may require including an integration over transverse momentum in the \( B \) meson starting from order \( \alpha_s^2 \).)

Eq. (2.3) applies to decays into two light mesons, for which the spectator quark in the \( B \) meson can go to either of the final-state mesons. An example is the decay \( B^- \rightarrow \pi^0K^- \). If the spectator quark can go only to one of the final-state mesons, as for example in \( \bar{B}_d \rightarrow \pi^+K^- \), we call this meson \( M_1 \) and the second form-factor term on the right-hand side of (2.3) is absent. The factorization formula simplifies when the spectator quark goes to a heavy meson [see (2.4)], such as in \( \bar{B}_d \rightarrow D^+\pi^- \). In this case the hard interactions with the spectator quark can be dropped because they are power suppressed in the heavy-quark limit. In the opposite situation that the spectator quark goes to a light meson but the other meson is heavy, factorization does not hold as discussed above.

As an example, consider the matrix element \( \langle \pi^+\pi^-|\langle \bar{u}b\rangle_{V-A}(\bar{d}u)_{V-A}|\bar{B}\rangle \). In leading order the conventional factorization result \( i f_{\pi} F_+^{B\rightarrow \pi}(0)M_B^2 \) is obtained. It is convenient to introduce “factorized operators” \( j_1 \otimes j_2 \), whose matrix elements are defined by \( \langle \pi\pi|j_1 \otimes j_2|B\rangle \equiv \langle \pi|j_1|B\rangle \langle \pi|j_2|0\rangle \). The benefit of this notation is that the result of the factorization formula can be expressed in terms of these factorized operators, and this gives a compact representation of the result for all \( \pi\pi \) final states. We then find, including the order-\( \alpha_s \) corrections,

\[
\langle \bar{u}b\rangle_{V-A}(\bar{d}u)_{V-A} = \langle \bar{u}b\rangle_{V-A} \otimes \langle \bar{d}u\rangle_{V-A}
\]

\[+ \frac{1}{3} \langle \bar{u}b\rangle_{V-A} \otimes \langle \bar{d}u\rangle_{V-A}
\]

\[+ \alpha_s \frac{9\pi}{2}\left[ V + H \right] \langle \bar{d}b\rangle_{V-A} \otimes \langle \bar{d}u\rangle_{V-A}
\]

\[+ P \sum_{q=u,d} \langle \bar{q}b\rangle_{V-A} \otimes \langle \bar{d}q\rangle_{V-A},
\]

(2.5)

where

\[
V \equiv \int_0^1 du \Phi_\pi(u) \left[ 6 \ln \frac{m_2^2}{\mu^2} - 18 + \frac{3(1-2u)}{1-u} \ln u - 3\pi i \right],
\]

(2.6)
\[
P \equiv \int_0^1 du \Phi_\pi(u) \left[ \frac{2}{3} \ln \frac{m_0^2}{\mu^2} + \frac{2}{3} \right] + 4 \int_0^1 dz (1-z) \ln[-z(1-z)u - i\epsilon] \tag{2.7}
\]

account for the hard spectator scattering, the second line of (2.3), which includes vertex (\(V\)) and penguin (\(P\)) contractions and

\[
H \equiv \frac{4\pi^2}{3} M_B \lambda_B F^{H+\pi}(0) \int_0^1 du \Phi_\pi(u) \times \int_0^1 dv \left[ \Phi_\pi(v) + \frac{2\mu_\pi}{m_\pi} \Phi_p(v) \right], \tag{2.8}
\]

2.3 Implications of Factorization

The significance and usefulness of the factorization formula stems from the fact that the non-perturbative quantities which appear on the right-hand side of (2.3) are much simpler than the original non-leptonic matrix element on the left-hand side. This is because they either reflect universal properties of a single meson state (light-cone distribution amplitudes) or refer only to a \(B \to \text{meson} \) transition matrix element of a local current (form factors). Factorization is a consequence of the fact that only hard interactions between the (BM) system and \(M_2 \) survive in the heavy quark limit. As a result we can say that

- conventional factorization gives the correct limit, when \(a_\pi\) and \(\Lambda_{QCD}/m_\pi\) corrections are neglected, provided the spectator quark does not go to a heavy meson;

- radiative corrections to conventional factorization can be computed systematically, the result is, in general, non-universal, i.e. there is no reason to suppose that the parameters \(a_i\) should be the same, say for \(D\pi\) and \(\pi\) final states;

- the problem of scheme-dependence in the conventional factorization ansatz is solved in the same way as in any other next-to-leading order computation with the weak effective hamiltonian;

- all strong interaction phases are generated perturbatively in the heavy quark limit, as form factors have no imaginary parts;

- many observables of interest for CP violation become accessible in this way, the current limiting factors being our poor knowledge of \(\lambda_B\) and that of power corrections.
3. Discussion

In this section I discuss some aspects that concern factorization beyond the one-loop correction to conventional factorization: the validity of factorization in higher orders of perturbation theory [Sect. 3.1], the issue of final state rescattering [Sect. 3.2] and various sources of power corrections [Sects. 3.3-3.5].

3.1 Factorization in higher orders

A proof of the factorization formula (2.4) for decays into a heavy and a light meson has been given at the two-loop order \([3.1]\). Some of the arguments used there have straightforward extensions to all orders, but a technical all-order “proof” has not yet been accomplished. Nonetheless, the arguments used to prove infrared finiteness at two-loop order may be sufficiently convincing to make infrared finiteness at all orders plausible.

It has to be demonstrated that the hard-scattering kernels are infrared finite. To state this more precisely, we write the factorization formula for a heavy-light final state schematically as

\[ A \equiv \langle \pi^- D^+ | O | \bar{B}_d \rangle = F_{B \to D}(0) \cdot T \cdot \Phi_\pi, \]  

(3.1)

where the \(\ast\) represents the convolution and \(O\) represents a four-quark operator. In order to extract \(T\), one computes \(A\), \(F_{B \to D}\) and \(\Phi_\pi\) in perturbation theory and uses (3.1) to determine \(T\). We therefore rewrite (3.1) in perturbation theory,

\[ A^{(0)} + A^{(1)} + A^{(2)} + \cdots = \left( F^{(0)}_{B \to D} + F^{(1)}_{B \to D} + F^{(2)}_{B \to D} + \cdots \right) \cdot \left( T^{(0)} + T^{(1)} + T^{(2)} + \cdots \right) \ast \left( \Phi^{(0)}_\pi + \Phi^{(1)}_\pi + \Phi^{(2)}_\pi + \cdots \right), \]  

(3.2)

where the superscripts in parentheses indicate the order of perturbation theory, and then compare terms of the same order. Thus up to two-loop order

\[ F^{(0)}_{B \to D} \cdot T^{(0)} \ast \Phi^{(0)}_\pi = A^{(0)}, \]  

(3.3)

\[ F^{(0)}_{B \to D} \cdot T^{(1)} \ast \Phi^{(0)}_\pi = A^{(1)} \]  

(3.4)

\[ -F^{(1)}_{B \to D} \cdot T^{(0)} \ast \Phi^{(0)}_\pi - F^{(0)}_{B \to D} \cdot T^{(0)} \ast \Phi^{(1)}_\pi = \text{infrared finite}. \]  

(3.5)

By perturbative expansion of the \(B \to D\) form factor, we mean the perturbative expansion of the matrix element of \(\bar{c} \Gamma b\), evaluated between on-shell \(b\)- and \(c\)-quark states. By perturbative expansion of the pion light-cone distribution amplitude, we mean the perturbative expansion of the light-cone matrix element which defines the LCDA, but with the pion state replaced by an on-shell quark with momentum \(uq\) and an on-shell antiquark with momentum \(\bar{u} \bar{q}\).

The zeroth order term in (3.3) is trivial. The two terms that need to be subtracted from \(A^{(1)}\) at first order exactly cancel the “factorizable” contributions to \(A^{(1)}\). The first order term in (3.3) therefore states that \(T^{(1)}\) is given by the “non-factorizable” diagrams. (Here we use “non-factorizable” in the conventional sense, i.e. to denote the diagrams with gluon exchange between the \((BD)\) system and the pion.) If \(T^{(1)}\) is to be infrared finite, the sum of these diagrams must be infrared finite, which is indeed the case as seen by the explicit one-loop calculation.

The second order term (3.3) has a more complicated structure. The last three terms on the right-hand side exactly cancel the “factorizable” corrections to the two-loop amplitude \(A^{(2)}\). The remaining two terms that need to be subtracted from \(A^{(2)}\) are non-trivial. The infrared divergences in the sum of “non-factorizable” contributions to \(A^{(2)}\) must then be shown to have precisely the right structure to match the infrared divergences in \(F^{(1)}_{B \to D}\) and \(\Phi^{(1)}_\pi\), such that

\[ A^{(2)}_{\text{non-fact.}} = -F^{(0)}_{B \to D} \cdot T^{(1)} \ast \Phi^{(1)}_\pi \]  

(3.6)

Eq. (3.4) is verified by first identifying the regions of phase space which can give rise to infrared singularities. In general these arise when massless lines become soft or collinear with the direction of \(q\), the momentum of the pion. This requires that one or both of the loop momenta in a two-loop diagram become soft or collinear. Rather than
computing the 62 “non-factorizable” two-loop diagrams (excluding self-energy insertions and field renormalization), the Feynman integrands corresponding to these diagrams in those momentum configurations that can give rise to singularities are then analyzed in all possible combinations: one momentum soft or collinear, the other hard; both momenta soft or collinear; one momentum soft, the other collinear.

The analysis of Ref. [3] shows that infrared divergences cancel in the soft-soft, collinear-collinear and soft-collinear region as required for the validity of (3.4). The non-cancelling divergence in the hard-soft region factorizes into the form

$$A_{\text{hard-soft}}^{(2)} = f_{B \to D} \cdot T^{(1)} \Phi_{\pi}^{(0)},$$

(3.7)

where $f_{B \to D}$ is precisely the soft contribution to the $B \to D$ form factor at the one-loop order; this cancels the second of the two subtraction terms in (3.6). The infrared divergent hard-collinear contributions sum up to the expression

$$A_{\text{hard-collinear}}^{(2)} = F_{B \to D}^{(0)} \cdot C_F \frac{\alpha_s}{\pi} \ln \frac{\mu_{UV}}{\mu_{IR}}$$

$$\times \int_0^1 du \, dv \, u \, T^{(1)}(w) \, V(w, u) \, \Phi_{\pi}^{(0)}(u),$$

(3.8)

with $V(w, u)$ the ERBL evolution kernel [4, 5]. The infrared singular contribution to the (perturbative, one-loop) pion distribution amplitude is determined by

$$\Phi_{\pi}^{(1)}(w) = C_F \frac{\alpha_s}{\pi} \ln \frac{\mu_{UV}}{\mu_{IR}} \int_0^1 du \, V(w, u) \, \Phi_{\pi}^{(0)}(u),$$

(3.9)

and by combining the previous two equations we find that $A_{\text{hard-collinear}}^{(2)}$ is precisely equal to the remaining subtraction term in (3.6). It follows from (3.9) that $T^{(2)}$ is free of infrared singularities.

### 3.2 Rescattering and Parton-Hadron Duality

Final-state interactions are usually discussed in terms of intermediate hadronic states. This is suggested by the unitarity relation (taking $B \to \pi\pi$ for definiteness)

$$\text{Im} \, A_{B \to \pi\pi} \sim \sum_n A_{B \to n} A_{n \to \pi\pi},$$

(3.10)

where $n$ runs over all hadronic intermediate states. In many discussions of final state rescattering the sum on the right hand side of this equation is truncated by keeping only elastic rescattering. It is clear that this approximation is incompatible with the heavy-quark limit, in which the opposite limit of an arbitrarily large number of intermediate states should be considered. Decays of $B$ mesons lie somewhere in between these limiting cases, but only the heavy-quark limit provides a controlled approximation to the problem. In my opinion, in view of the factorization results, proponents of the elastic scattering limit and methods inspired by Regge physics now need to justify better their motivation for choosing this particular ansatz.

The heavy-quark limit, and the dominance of hard rescattering in this limit, suggest that the sum in (3.10) is interpreted as extending over intermediate states of partons. In this picture the sum over all hadronic intermediate states is approximately equal to the contribution of a quark-anti-quark intermediate state of small transverse extension. The approximation could be further improved by including $q\bar{q}g$ intermediate states etc. This is similar to the QCD description of $e^+e^- \to$ hadrons at large energy; the total cross section of this reaction is well represented by the production cross section of a $q\bar{q}$ pair, even though the production of every particular final state cannot be computed with perturbative (or any known) methods. There is a limit to the accuracy of such kinds of descriptions, which is discussed under the name of “parton-hadron” duality. Quantifying this accuracy is a formidable, unsolved problem. The same assumption forms the basis for the application of the operator product expansion to inclusive non-leptonic heavy-quark decays and there have been some quantitative studies in this context, though in the two-dimensional ’t Hooft model [20, 21, 22]. Parton-hadron duality is also an implicit assumption in applying perturbative QCD techniques to jet observables and hadron-hadron collisions at large momentum transfer. It is probably safe to conclude that the accumulated experience suggests that violations of parton-duality are subdominant effects in the heavy-quark limit, and this is all we need to justify our theoretical framework.
3.3 Higher Fock States and Non-factorizable Contributions

The factorization formula needs only the leading-twist LCDAs of the mesons. Higher Fock components of the mesons appear in higher orders of the collinear expansion. The collinear expansion is justified as long as the additional partons carry a finite fraction of the meson’s momentum in the heavy-quark limit. Under this assumption, it is easy to see that adding additional partons to the Fock state increases the number of off-shell propagators in a given diagram. This implies power suppression in the heavy-quark expansion.

Soft contributions are also power-suppressed, but it seems difficult to classify them systematically. Again the decay \( B_d \to D^+\pi^- \) is the simplest case, and I briefly consider the situation, where the “non-factorizable” gluon, i.e. the gluon exchanged between the pion and the \((BD)\) system, is soft. In this case, the “q\(q\) Fock state” cannot be described by a light-cone wave function, but such a contribution still receives a power suppression in the heavy-quark limit. The important point is that soft gluons couple very weakly to the \( q\bar{q} \) pair. The coupling can be evaluated (the \( q\bar{q} \) pair being very energetic), and the result is

\[
\langle D^+\pi^- | (\bar{c}T^A b)_{V-A} (\bar{d}T^A u)_{V-A} | \bar{B}_d \rangle_{\text{soft}} =
\]

\[
- \frac{1}{m_Q} \int_0^{1/m_Q} ds \langle D^+ | \bar{c}\gamma^\mu (1 - \gamma_5) g_{\mu\nu} (sn)n^\nu b | \bar{B}_d \rangle
\]

\[
\times \int_0^1 du \frac{f_2 \Phi_\pi(u)}{8N_c \langle u \bar{u} \rangle},
\]

where \( q = En \) is the momentum of the pion. This depends on a non-local higher-dimension \( B \to D \) form factor, but comparing this with the leading order, conventional factorization expression,

\[
\langle D^+\pi^- | (\bar{c}b)_{V-A} (\bar{d}u)_{V-A} | \bar{B}_d \rangle_{\text{LO}} =
\]

\[
\langle D^+ | \bar{c}\gamma^\mu (1 - \gamma_5) b | \bar{B}_d \rangle_{\text{LO}} f_2 E_n \int_0^1 du \Phi_\pi(u),
\]

we conclude on dimensional arguments that the soft non-factorizable correction is suppressed by one power of \( \Lambda_{QCD}/m_b \). (Note that similar considerations for the \( J/\psi K \) final state lead to a local \( B \to D \) form factors, because the non-locality is then cut off at a distance \( 1/m_c \).)

Note that power corrections come without a factor of \( \alpha_s \) and we may expect them to be as large as the computable perturbative corrections in general. An important point, however, is that there exists a systematic framework that allows us to classify both effects as corrections.

3.4 “Chirally enhanced” Corrections

There are two reasons why the hard spectator interaction in \( (2.3) \) is particularly sensitive to power-suppressed corrections. The first reason is that the short-distance scale is not \( m_b \) (as is the case for the form factor term in \( (2.3) \)), but \( (m_b \Lambda_{QCD})^{1/2} \). To see this, note the gluon virtuality is

\[
k_g^2 = -\bar{q}q M_B^2 + \text{terms of order } \Lambda_{QCD}^2,
\]

which on average is around \( 1 \text{ GeV}^2 \). To arrive at \( \bar{q}q \) I have neglected the terms of order \( \Lambda_{QCD}^2 \) and this might not be a particularly good approximation. However, there is no (known) systematic way of treating such terms, which amongst other things are sensitive to the off-shellness of the spectator quark in the \( B \) meson (and hence to higher Fock components of the \( B \) meson), and so we must neglect these terms together with many other power corrections.

There is an enhancement of power-suppressed effects for decays into light pseudoscalar mesons connected with the curious numerical fact that

\[
2\mu_\pi \equiv \frac{2m_\pi^2}{m_u + m_d} = -\frac{4\langle \bar{q}q \rangle}{f_2^2} \approx 3 \text{ GeV}
\]

is much larger than its naive scaling estimate \( \Lambda_{QCD} \). (Here \( \langle \bar{q}q \rangle = \langle 0 | \bar{u}u | 0 \rangle = \langle 0 | \bar{d}d | 0 \rangle \) is the quark condensate.) These “chirally enhanced” corrections have originally been discussed in connection with \( V + A \) penguin operators in the weak effective hamiltonian, but they affect the hard spectator interaction more severely.

Consider first the contribution of the operator \( \mathcal{O}_6 = \langle \bar{d}_ib_j | V-A | \bar{u}_j u_i \rangle_{V+A} \) to the \( \bar{B}_d \to \pi^+\pi^- \) decay amplitude. The parameter \( \mu_\pi \) arises already in the naively factorized matrix element:

\[
\langle \pi^+\pi^- | (\bar{d}_ib_j)_{V-A} (\bar{u}_j u_i)_{V+A} | \bar{B}_d \rangle =
\]

\[
iM_B^2 F_+^{B \to \pi}(0)f_\pi \times \frac{2\mu_\pi}{m_b}.
\]
This is formally a \( \Lambda_{QCD}/m_b \) power correction but numerically large due to \((3.14)\). We would not have to worry about such terms if they could all be identified and the factorization formula \((2.3)\) applied to them, since in this case higher-order perturbative corrections would not contain non-factorizing infrared logarithms. However, this is not the case.

After including radiative corrections, the matrix element on the left-hand side of \((3.15)\) is expressed as a non-trivial convolution with the pion light-cone distribution amplitude. The terms involving \( \mu_\pi \) can be related to two-particle twist-3 (rather than leading twist-2) distribution amplitudes, conventionally called \( \Phi_\pi(u) \) and \( \Phi_\sigma(u) \). The distribution amplitude \( \Phi_\pi(u) \) does not vanish at the endpoint. As a consequence the hard spectator interaction contains an endpoint divergence. In other words, the “correction” relative to \((3.15)\) is of the form \( \alpha_s \times \) logarithmic divergence, which we interpret as being of the same order as \((3.15)\). It turns out, however, that the \( \alpha_s \) correction to the \( V + A \) operator (giving rise to the parameter \( a_6 \) in conventional notation) is free of this potential problem.

As a consequence the most important effect of the chirally enhanced twist-3 LCDAs (with exception of the leading order result for \( a_6 \)) is on the matrix elements that contribute to the coefficients \( a_1 \) to \( a_5 \). An example of this is shown in \((2.8)\). Substituting the asymptotic LCDAs, \( H \) is rewritten as

\[
H = \frac{12\pi^2 f_B f_\pi}{M_B \lambda_B F_\pi^B(0)} \left[ 1 + \frac{2\mu_\pi}{3m_b} \int_0^1 dv \right],
\]

which exhibits the problem of dealing with the endpoint-divergent integral. I should emphasize that this divergence is not in contradiction with the factorization formula \((2.3)\), in fact it is expected at the level of power-suppressed effects. But from the phenomenological point of view it is somewhat disappointing that these effects are sizeable and can introduce a substantial uncertainty into the hard spectator interaction. The complete set of chirally enhanced terms has been estimated up to now only in Ref. \([3]\), where it was assumed that the divergent integral can be replaced by a universal constant. The variation of this constant constitutes the largest theoretical uncertainty, but it is also shown that the predictivity of the approach is not lost. (Some chirally enhanced terms have been included in Refs. \([24, 25]\), but the correction \((2.8)\), \((3.16)\) to \( a_1 \) to \( a_5 \), which contains the endpoint divergence, has been omitted \([24]\) or computed incorrectly \([25]\) in these papers.) As in a related situation for the pion form factor \([26]\) one might argue that the endpoint divergence is suppressed by a Sudakov form factor. However, it is likely that when \( m_b \) is not large enough to suppress these chirally enhanced terms, then it is also not large enough to make Sudakov suppression effective especially since the short-distance scale is not large enough to build up a strong logarithmic evolution.

3.5 Annihilation Topologies

My final concern in this section are the annihilation topologies (Fig. 2). The hard part of these diagrams would amount to another contribution to the second hard-scattering kernel, \( T_i^{\mu}(\xi, u, v) \), in \((2.3)\). The soft part, if unsuppressed, would violate factorization. However, a straightforward power-counting analysis shows that all annihilation topologies are \( 1/m_b \) corrections to the decay amplitudes in the heavy-quark limit \([2]\). This statement also applies to diagram d, in which case the largest term comes from an endpoint contribution.

As for the hard spectator interaction at leading power in the heavy-quark expansion, there exist “chirally enhanced” contributions from the annihilation topologies related to the corresponding twist-3 light meson LCDAs. It has recently been noted \([14, 18]\) in the context of the hard-scattering approach that these could be non-negligible. To illustrate this effect, I consider the annihilation correction to the coefficient \( a_6 \) in the effective transition operator defined in Ref. \([4]\). Note that \( a_6 \) is the coefficient of a power correc-
tion (though chirally enhanced), and I am considering now a power correction to $a_6$. For two identical final state mesons, say two pions, only the diagrams a and b contribute to $a_6$. To simplify the result, I assume the LCDAs to be the asymptotic ones and obtain for the sum of leading order and annihilation contribution:

$$a_6 \simeq \left( C_6 + \frac{C_5}{N_c} \right) \left[ 1 + \frac{\alpha_s}{9\pi} \frac{96\pi^2 f_B f_\pi}{M_B^2 F_B^{\pi\pi}(0)} \right] \times \left( X_l^2 - \frac{X_t}{2} \right),$$

(3.17)

where $X_l$ is the divergent integral $\int_0^1 dv/v$. Although power-suppressed, the correction is enlarged by a numerical factor and a logarithmic endpoint divergence from each of the two final mesons. We can exhibit this more transparently by comparing the annihilation correction to the generic leading-power hard spectator correction:

$$H_{\text{ann}} \approx \frac{\lambda_B}{M_B} \times 8 \left( X_l^2 - \frac{X_t}{2} \right),$$

(3.18)

suggesting that in this particular case the annihilation topologies are more important than the generic hard spectator interaction. A complete analysis of annihilation contributions to light-final states (extending the analysis \cite{2} for the $D\pi$ case) is currently in progress.

### 4. Conclusion

The QCD factorization approach described in this talk constitutes a powerful and systematic approach to non-leptonic decay amplitudes, based on familiar methods of perturbative QCD, and the assumption that the $b$ quark mass is large. It does not render trivial the problem of accurately computing these amplitudes, but it appears to me that the outstanding issues now are more of numerical than of conceptual character: the best way of dealing with chirally enhanced power corrections; the role of annihilation; the size of power corrections in general and their impact on strong interaction phases; the role of hard scattering in heavy-to-light form factors (and, related to this, the importance of Sudakov form factors).

It is probable that experimental data will be needed to shed light on some of these issues.

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