Exceedance Probability Forecasting via Regression for Significant Wave Height Forecasting

Vitor Cerqueira\textsuperscript{a,*}, Luis Torgo\textsuperscript{a}

\textsuperscript{a}Faculty of Computer Science, Dalhousie University, 6050 University Ave, Halifax, NS B3H 1W5, Canada

Abstract

Significant wave height forecasting is a key problem in ocean data analytics. Predicting the significant wave height is crucial for estimating the energy production from waves. Moreover, the timely prediction of large waves is important to ensure the safety of maritime operations, e.g., passage of vessels. We frame the task of predicting extreme values of significant wave height as an exceedance probability forecasting problem. Accordingly, we aim at estimating the probability that the significant wave height will exceed a predefined threshold. This task is usually solved using a probabilistic binary classification model. Instead, we propose a novel approach based on a forecasting model. The method leverages the forecasts for the upcoming observations to estimate the exceedance probability according to the cumulative distribution function. We carried out experiments using data from a buoy placed in the coast of Halifax, Canada. The results suggest that the proposed methodology is better than state-of-the-art approaches for exceedance probability forecasting.

Keywords: Exceedance Probability, Actionable Forecasting, Binary Probabilities, Time Series Forecasting

1. Introduction

Significant wave height (SWH) forecasting is a key problem in ocean data analytics, and several methods have been developed for tackling it (Huang et al., 1998; Duan et al., 2016; Ali et al., 2020). Forecasting the ocean wave conditions is valuable for multiple operations. The main motivation is related to renewable energy, where forecasts are used to estimate energy production. Moreover, these forecasts are also useful for managing the safety of maritime operations. Predicting impending large waves (extreme values of SWH) is important for coastal disaster prevention and protection of wave energy converters, which should be

\textsuperscript{*}Corresponding author

Email addresses: vitor.cerqueira@dal.ca (Vitor Cerqueira), ltorgo@dal.ca (Luis Torgo)

Preprint submitted to Elsevier

June 22, 2022
shut down to prevent their damage ([Li et al., 2012](#)). Moreover, the passage of vessels requires a minimum depth of water for their movement. The occurrence of large waves reduces the depth of water and this minimum may not be met. In summary, accurately predicting the SWH conditions is not only important to estimate energy production but it also enables a more efficient movement of vessels, which reduces costs and improves the reliability of ports.

We frame the prediction of impending large values of a time series as an exceedance probability forecasting problem. Exceedance probability forecasting denotes the process of estimating the probability that a time series will exceed a predefined threshold in a predefined future period. This task is usually relevant in domains where extreme values (i.e., the tail of the distribution) are highly relevant, such as floods, earthquakes, and hurricanes ([Lambert et al., 1994](#), [Kunreuther, 2002](#), [Frangopol & Kim, 2014](#)).

A common example of an exceedance forecasting problem is in economics. If the inflation rate is predicted to exceed a given threshold with sufficient probability, then central banks may be pushed to increase interest rates ([Granger & Pesaran, 2000](#)). In our work, we hypothesize that forecasting the probability of impending large SWH values may be valuable for a better decision-making in maritime operations.

Figure 1 shows a SWH time series with an hourly granularity. The red horizontal dashed line represents an example exceeding threshold above which a specified event occurs. The objective is to predict these events early in time.

**Figure 1:** Significant wave height time series with an hourly granularity. The red horizontal dashed line represents the exceeding threshold.

Exceedance forecasting is typically formalized as a binary classification problem, where the target variable denotes whether or not the threshold will be exceeded. Ensemble-based regression approaches can also be applied, in which the exceedance probability is estimated according to the ratio of individual predictions that exceed the threshold.
In this paper we propose a novel approach to exceedance probabilistic forecasting. The method is based on the numeric predictions for the value of future observations produced by a forecasting model. However, the proposed method does not rely on an ensemble-based forecasting model, though it could be one. Essentially, we leverage the cumulative distribution function of the Normal distribution to estimate the exceedance probability. The location parameter of the distribution is derived from the output of the forecasting model, while the dispersion is computed on the training data. In the experiments, we also test heavy-tailed distributions which may be more suitable to model extreme values.

We resort to a case study related to SWH forecasting to validate the proposed approach. The results suggest that coupling a forecasting model with the proposed mechanism based on the cumulative distribution function is better for exceedance probability forecasting relative to a classification method or an ensemble-based forecasting model.

In summary, the contributions of this work are the following:

- A novel approach for estimating exceedance probability, which is based on the cumulative distribution function;
- The application of the method to estimate the exceedance probability of the height of waves (SWH), which is important for managing maritime operations.

The code to for the methods used in the paper is publicly available.\footnote{https://github.com/vcerqueira/exceedance_wave} The rest of the paper is organized as follows. In Section 2 we overview the related work. In Section 3 we define the predictive problem, followed by Section 4 where we formalize the proposed method. The case study used in this work is described in Section 5. Afterwards, we described the experiments carried out to validate the proposed method in Section 6. We discuss the results and conclude the paper in Section 7.

2. Related Work

In this section we overview previous works related to ours. We start by reviewing the literature on significant wave height forecasting (Section 2.1). Afterwards, we describe approaches for probabilistic forecasting focusing on exceedance probability estimation (Section 2.2).

2.1. Significant Wave Height Forecasting

Ocean waves represent an important source of renewable energy. Information regarding the significant wave height is a key input to estimate the production of energy from waves. This is the main motivation behind several works that forecast SWH time series.
Ali et al. (2020) use buoy data from eastern Australia. They test several regression approaches, including a model tree and multivariate adaptive regression splines, to forecast short-term (30 minutes in advance) values of SWH. They conclude that a hybrid model, optimized using a method based on least squares, provided the best performance.

Duan et al. (2016) attempt to forecast SWH time series several hours in advance. Unlike a short-term forecast, an horizon of a few hours is important to ensure the safety of maritime operations, e.g. passage of vessels. They use the empirical mode decomposition (Huang et al., 1998) to decompose the time series into several simpler components, which are then modelled using an autoregressive approach. This decomposition leads to better forecasting performance relative to an autoregressive approach without this process.

Shamshirband et al. (2020) compared different regression algorithms for significant wave height forecasting, including artificial neural networks, support vector regression, and extreme learning machines (Huang et al., 2006). They concluded that all methods perform comparably, though the extreme learning machine model showed a slight advantage.

Several authors attempt to apply different neural network architectures for SWH predictions, including vanilla recurrent neural networks (Sadeghifar et al., 2017), feedforward neural networks (Jain & Deo, 2007), gated recurrent units (Wang et al., 2021), LSTMs (Wang et al., 2021), or graph neural networks (Chen et al., 2021). A comparison of these architectures is not available. But there is a tendency for recurrent architectures, which is expected due to the sequential nature of the data.

Fasuyi et al. (2020) and Kuppili (2021) also deal with smart buoy data, but their goal is to interpolate data across different buoys located in different places. When a buoy is shut down for maintenance, the data from other near buoys can be used to interpolate the missing values of that buoy.

The works mentioned so far are similar to each other in the sense that all attempt to forecast the future values, or interpolate from near buoys, of SWH time series using machine learning regression models. It is difficult to compare the performance of models across different works due to the usage of different data and performance estimation methods. However, it is clear that larger forecasting horizons lead to a poorer forecasting performance, as expected.

Mendez et al. (2006) also investigate the extreme values of SWH data. However, their objective is to study the statistical properties of the observations which exceed the threshold rather than try to predict them. In this work, we are interested in the future dynamics of SWH data. However, rather than predicting each instance as accurately as possible our goal is to estimate exceedance probability. Thus, we aim at predicting when extreme values are likely to occur.

2.2. Exceedance Probability Forecasting

Uncertainty in the forecasts of predictive models is unavoidable. Conveying this uncertainty is critical for better decision-making. For forecasting a numeric quantity uncertainty quantification can be accomplished with prediction
For binary events we can simply output the probability of that event occurring. An instance of a binary event is exceedance, which, as mentioned before, refers to when a time series exceeds a predefined threshold.

Slud & Kedem (1994) apply a logistic regression to model the exceedance probability of rainfall. In their model, the lagged observations of the time series are used as predictor variables in an auto-regressive fashion.

Taylor & Yu (2016) use an exceedance probability approach for financial risk management. They predict whether the returns of financial assets will exceed a threshold. This is accomplished by using a method called CARL (conditional auto-regressive logit), which is based on logistic regression. Later, CARL was extended to handle problems involving multiple thresholds by Taylor (2017). The extended model was used to predict wind ramp events, where the tails in both sides of the distribution are important to predict.

The alternative to using a classification approach to estimate the exceedance probability is to use an ensemble of forecasts. Accordingly, the probability is computed as the ratio of forecasts above the threshold. This type of approach is commonly used in fields such as meteorology (Lewis 2005), where different forecasts are obtained by considering different initial conditions (Ferro 2007). Toth et al. (2003) provide a comprehensive description of ensemble-based approaches listing their main properties.

Estimating the exceedance probability for different thresholds enables the construction of exceedance probability curves (Kunreuther 2002), which may be helpful for better decision-making.

Exceedance probability forecasting is related to extreme values prediction. The goal of forecasting extreme values is to accurately predict the numeric value of extreme observations in time series. For example, Polson & Sokolov (2020) attempt to forecast energy load spikes using a deep learning approach. On the other hand, in exceedance probability the goal is simplified to predict whether or not a given threshold is exceeded, i.e., the magnitude of exceedance is not a major concern.

Our goal in this paper is to estimate the exceedance probability with SWH time series data. Since large waves are important to anticipate, we design an approach to estimate the probability of their occurrence. To our knowledge, this is the first attempt at framing SWH forecasting as an exceedance probability problem.

3. Problem Definition

In this section, we define the predictive task we address in this work. We start by formalizing the significant wave height forecasting task (Section 3.1), followed by the definition of exceedance probability (Section 3.2).
3.1. Time Series Forecasting

A univariate time series represents a temporal sequence of values \( Y = \{y_1, y_2, \ldots, y_n\} \), where \( y_i \in \mathcal{Y} \subset \mathbb{R} \) is the value of \( Y \) at time \( i \) and \( n \) is the length of \( Y \). The general goal behind forecasting is to predict the value of the upcoming observations of the time series, \( y_{n+1}, \ldots, y_{n+h} \), given the past observations, where \( h \) denotes the forecasting horizon.

We formalize the problem of time series forecasting based on an auto-regressive strategy. Accordingly, observations of a time series are modelled based on their past recent lags. We construct a set of observations of the form \((X, y)\) using time delay embedding according to the Takens theorem (Takens, 1981). In each observation, the value of \( y_i \) is modelled based on the most recent \( q \) known values: \( X_i = \{y_{i-1}, y_{i-2}, \ldots, y_{i-q}\} \), where \( y_i \in \mathcal{Y} \subset \mathbb{R} \), which represents the observation we want to predict, \( X_i \in \mathcal{X} \subset \mathbb{R}^q \) represents the \( i \)-th embedding vector. For each horizon, the goal is to build a regression model \( f \) that can be written as \( y_i = f(X_i, Z_i) \), where \( Z_i \) represents additional covariates known at the \( i \)-th instance. These covariates can be, for example, explanatory time series related to the phenomenon. In such case, each of the explanatory time series is also represented at each time step \( i \) with an embedding vector based on the past recent values.

3.2. Exceedance Probability Forecasting

The probability of exceedance \( p_i \) denotes the probability that the value of a time series will exceed a predefined threshold \( \tau \) in a given instant \( i \). Typically, \( p_i \) is modelled by resorting to a binary target variable \( b_i \), which can be formalized as follows:

\[
b_i = \begin{cases} 
1 & \text{if } y_i \geq \tau, \\
0 & \text{otherwise.} 
\end{cases}
\]

Essentially \( b_i \) takes the value of 1 if the threshold \( \tau \) is exceeded, or 0 otherwise. This naturally leads to a classification problem where the goal is to build a model \( g \) of the form \( b_i = g(X_i, Z_i) \). Accordingly, \( p_i \) represents the conditional expectation of \( b_i \) which we estimate based on Bernoulli density. Logistic regression is commonly used (Taylor & Yu, 2016) to this effect.

Since the exceedance events are rare, the number of cases for the positive class (i.e. when \( b_i = 1 \)) are significantly smaller than the number of cases for the negative class (\( b_i = 0 \)). This leads to an imbalanced classification problem (Branco et al., 2016), which hinders the learning process of algorithms.

The alternative to using a classification model is to use a forecasting ensemble \( F \) composed by a set of \( M \) models: \( F = \{f_1, f_2, \ldots, f_M\} \). These models can be combined for forecasting using the uniformly weighted average as follows:

\[
\hat{y}_i = \frac{1}{M} \sum_{k=1}^{M} f_k(X_i, Z_i)
\]

We can also use the ensemble \( F \) to estimate the exceedance probability according to the ratio of ensemble members that forecast a value that exceeds
the threshold $\tau$:

$$p_i = \frac{1}{M} \sum_{k=1}^{M} I(f_k(X_i, Z_i) \geq \tau)$$

where $I$ denotes the indicator function. Hereafter, we will refer to this approach as an ensemble-based direct method for exceedance probability. Ensemble-based approaches are commonly used in some fields related to environmental science, for example, meteorology or hydrology [Toth et al. 2003].

4. Methodology

In this section, we describe the proposed approach for estimating the exceedance probability (Section 4.1). We also list the main advantages of the method relative to state-of-the-art approaches (Section 4.2).

4.1. Exceedance Probability via Cumulative Distribution Function

The goal of this work is to predict $p$, the probability that the SWH will exceed a specified threshold $\tau$. As mentioned in the introduction, large waves pose safety concerns regarding maritime operations, and it is important to anticipate them.

According to the overview of the literature we put forth in Section 2, there are two main approaches for estimating the exceedance probability: (i) using a binary classification method, or (ii) using a forecasting (regression) ensemble. In this work, we introduce a novel strategy for exceedance probability forecasting based on a time series forecasting model.

The proposed method leverages the numeric predictions produced by a forecasting model regarding the value of a future observation. Note that, while we use a forecasting (i.e. regression) approach, our method does not require an ensemble, although it could be one. Essentially, the method we describe is agnostic to the forecasting model making the predictions.

Let $\hat{y}_i$ denote the forecast for a future $i$-th instance. We assume that $\hat{y}_i$ can be modelled according to a Normal distribution with mean $\hat{y}_i$ and standard deviation $\sigma_y$: $N(\hat{y}_i, \sigma_y^2)$. Here, $\sigma_y$ is computed according to the standard deviation of the training data.

In these conditions, we can estimate $p_i$, the exceedance probability at the $i$-th time step, using the cumulative distribution function (CDF) of $N(\hat{y}_i, \sigma_y^2)$:

$$p_i = 1 - CDF_{N(\hat{y}_i, \sigma_y^2)}(\tau)$$

(2)

When evaluated at the threshold $\tau$, the CDF represents the probability that the respective random variable will take a value less than or equal to $\tau$. In effect, we take the complementary value to obtain the probability that the random variable will exceed $\tau$.

We formalized this method using a Normal distribution because it works well in the case study we will present afterwards. Notwithstanding, the most appropriate distribution depends on the input data.
4.2. Methodological Advantages

Aside from performance considerations, which we will delve into in Section 6, the proposed method may be preferable relative to a classifier for two main reasons. The first reason is integration because the same forecasting model can be used to predict both the upcoming values of SWH (for estimating energy production) and to estimate exceedance probability (for risk management). The second reason is threshold flexibility: a classification approach fixes the threshold during training, and it cannot be changed during inference. On the other hand, a regression model follows a lazy approach regarding the threshold. Therefore, in a given instant we can use the same model to estimate the exceedance probability for a different threshold – this enables the plotting of an exceedance probability curve.

These advantages are applicable to an ensemble-based forecasting approach. However, the high computational costs and lack of transparency often preclude the application of these strategies Lakkaraju et al. (2016); Cerqueira et al. (2021).

5. Case Study

In this section we present the case study we use in the experiments. First, we describe the data sets and its source (Section 5.1). Then, we explain the preprocessing steps carried out before applying the predictive models to the time series (Section 5.2).

5.1. Data Set

We conducted our study using data collected from the Environment and Climate Change Canada Buoy (ECCC Buoy). This buoy is located 13 kilometers away in the open sea close to Halifax, Canada. The data from this buoy was previously used for an interpolation task by Fasuyi et al. (2020), as we described in Section 2.1.

The time series is regularly sampled at an hourly frequency and spans from 11-02-2000 15:00:00 to 01-04-2020 11:00:00. The time series is multivariate and includes the following variables:

- SWH: Significant wave height (in meters) which represents the variable we want to predict;
- PWP: Peak wave period (in seconds), which is the wave period carrying the maximum amount of energy;
- CWH: Maximum zero crossing wave height (in meters);
- WDIR: Wind direction (in degrees);
- WSPD: Horizontal wind speed (in meters per second);
- GSPD: Gust wind speed (in meters per second);
• DRYT: Buoy dry bulb temperature (in degree Celsius);
• SSTP: Approximated sea surface temperature (in degree Celsius).

Figure 2 shows the SWH time series. For visualization purposes we only plot the response variable (SWH). The total number of observations is 125,890. However, there is a considerable number of missing observations, especially in the period spanning February 2016 to October 2019.

![SWH Time Series](image)

Figure 2: SWH time series captured from the ECCC buoy.

In Figure 3, we show an histogram which illustrates the distribution of the SWH time series. The distribution is right-skewed where the heavy tail represents the large waves we attempt to predict.

5.2. Data Preparation

As exploratory data analysis, in Figure 4 we show the Pearson correlation of each pair of variables in the data set. Overall, the main variable SWH is highly correlated with CHW (0.91), and moderately correlated with the wind and gust speed variables (WSPD and GSPD) with values 0.41 and 0.42, respectively.

We constructed a set of observation for training the predictive models according to the formalization in Section 3.1. The target variable $y$ represents the value of SWH observed $h$ steps (hours) ahead in the future. The respective binary target variable $b$ is computed according to whether $y$ takes a value above $\tau$ in $h$ hours. In order to measure the predictability of exceeding events in different time frames, we analysed multiple forecasting horizons. Specifically, we experiment with an horizon from 1 to 24 hours in advance. For example, with $h = 6$ we attempt to predict whether the threshold will be exceeded in 6 hours.

In terms of the threshold $\tau$, we adopt a data-driven approach to set this value. Specifically, we set the threshold to be the 95th percentile of the available SWH time series. We follow this approach, as opposed to a judgemental
one (e.g., based on some domain expert advice), because it can be generalized to different data sets or domains. In effect, at each iteration of the cross-validation procedure (which is detailed in the next section), we compute $\tau$ using the training data. The average threshold across all iterations was 3.17 meters.

Regarding the predictor variables, we apply time delay embedding to all variables in the time series with an embedding size (number of lags) of five. This means that, at a given instant, $y$ (or $b$) is modelled according to the past five values of each of the eight variables described above. This leads to a total of 40 explanatory variables.

We also performed a data validation step. The objective is to predict exceedance events. Therefore, if exceedance is occurring in the explanatory variables ($X$) it means that the event is already happening and prediction is not necessary in that moment. In this context, we remove the instances where exceedance is occurring in the explanatory variables because the model will not be used in this case.

6. Experiments

This section describes the experiments which were designed to address the following research questions:

1. **RQ1**: Which approach is better for forecasting the future numeric values of the significant wave height time series?

2. **RQ2**: How does the proposed CDF-based method performs for estimating the exceedance probability relative to state-of-the-art approaches?
Figure 4: Heatmap showing the pairwise correlation across all variables in the data set.

3. **RQ3**: What is the impact of the forecasting horizon in the results obtained?

4. **RQ4**: Is the proposed method sensitive to different distributions?

### 6.1. Experimental Design

We carry out a Monte Carlo cross-validation procedure to evaluate the performance of models Picard & Cook (1984). This estimation method, which is also referred to as repeated holdout, has been shown to provide competitive forecasting performance estimates with univariate time series Cerqueira et al. (2020). Monte Carlo cross-validation is applied with 10 folds. The training and test sizes in each fold are 50% and 20% of the size of the available data, respectively. In each fold, a point is randomly selected from the available data (constrained by the size of training and testing data). This point then represents the end of the training set, and the start of the testing set Cerqueira et al. (2020).

We evaluate the exceedance probability performance of each approach using the area under the ROC curve (AUC) as evaluation metric. Besides, we also apply the coefficient of determination to measure the forecasting performance of the regression approaches, i.e., the performance at forecasting the value of future observations.
6.2. Methods

In this subsection we overview the approaches used to estimate the exceedance probability. These can be broadly split into classification methods (Section 6.2.1) and forecasting (regression) methods (Section 6.2.2).

6.2.1. Classification Methods

We apply the following three classification methods for estimating exceedance probability:

- **Random Forest Classifier (RFC):** A standard binary classifier built using a Random Forest [Breiman (2001)];
- **Logistic Regression (LR):** Another standard classifier but created using the Logistic Regression algorithm. We specifically test this method because it is a popular one for exceedance probability tasks (c.f. Section 2.2);
- **RFC+SMOTE:** Exceedance typically involves imbalanced problems because the exceeding events are rare. We created a variant of RFC, which attempts to cope with this issue using a resampling method. Resampling methods are popular approaches for dealing with the class imbalance problem. In effect, we test a classifier which is fit after the training data is pre-processed with the SMOTE resampling method [Chawla et al. (2002)];

6.2.2. Forecasting Methods

We also test three different regression approaches. These include a Random Forest regression method (RFR) and a LASSO linear model, which are two widely used regression algorithms. The third approach is an heterogeneous regression ensemble (HRE). An ensemble is referred to as heterogeneous, as opposed to homogeneous, if it is comprised by base models trained with different learning algorithms. The following learning algorithms were used to train the base models of HRE: a rule-based model based on the Cubist algorithm, a bagging ensemble of decision trees (Bagging), ridge regression (Ridge), elastic-net (ElasticNet), LASSO, multivariate adaptive regression splines (MARS), light gradient boosting optimized using random search (LGBM); extra trees (ExtraTree), AdaBoost (AdaBoost), Random Forests (RFR), partial least squares regression (PLS), principal components regression (PCR), and nearest neighbors (KNN).

We tested different parameters settings and the total number of initial models is 50. The complete list of parameters is shown in Table 1. Using a validation set we trimmed the ensemble by removing half of the models with worse performance. The trimming process has been shown to improve forecasting performance of ensembles [Jose & Winkler (2008); Cerqueira et al. (2019)]. Usually, a dynamic combination rule improves the performance of forecasting ensembles relative to a static one. Notwithstanding, the simple average is a solid benchmark [Cerqueira et al. (2019)].

We apply the regression approaches in two different ways in order to obtain the estimates for exceedance probability. The first is a Direct approach: the
Table 1: Summary of the learning algorithms

| ID   | Algorithm                  | Parameter      | Value              |
|------|----------------------------|----------------|--------------------|
| MARS | Multivar. A. R. Splines    | Degree         | {1, 3}             |
|      |                            | No. terms      | {10, 20}           |
|      |                            | Threshold       | {0.01, 0.001}      |
| LASSO| LASSO Regression            | L1 alpha        | {0.25, 0.5, 0.75, 1} |
| Ridge| Ridge Regression            | L2 alpha        | {0.25, 0.5, 0.75, 1} |
| ElasticNet | ElasticNet Regression     | Default         | -                  |
| KNN  | k-Nearest Neighbors         | No. of Neighbors| {1, 5, 10}         |
|      |                            | Weights         | {Uniform, Distance}|
| RFR  | Random Forest Regression    | No. trees       | {50, 100}          |
|      |                            | Max depth       | {default, 3}       |
| ExtraTree | Extra Trees Regression    | No. trees       | {50, 100}          |
|      |                            | Max depth       | {default, 3}       |
| Bagging | Bagging Regression         | Base estimator  | {Decision Stump, Decision Tree} |
|      |                            | No. estimators  | {50, 100}          |
| RBR  | Rule-based Regression       | No. iterations  | {1, 2, 3}          |
| LGBM | LightGBM                   | Booster         | {DART, GOSS, GBDT} |
| AdaBoost | AdaBoost Regression       | Base estimator  | {Decision Stump, Decision Tree} |
|      |                            | Learning rate   | {0.3, 0.7}         |
| PCR  | Principal Comp. Regr.      | No. of components| {2, 5}            |
| PLS  | Partial Least Regr.        | No. of components| {2, 5}            |
| OMP  | Orthogonal Matching Pursuit| Default         | -                  |

Exceedance probability $p$ is computed according to the ratio of ensemble members which predict a value above the threshold $\tau$. The Direct approach is only valid for ensembles, in this case HRE (an heterogeneous ensemble) and RFR (an homogeneous ensemble). The second approach is using the cumulative distribution function (CDF) according to our proposed method described in Section 4. We denote which approach is used by appending the name to the regression model. For example, HRE+CDF represents the heterogeneous ensemble applied with the cumulative distribution function for estimating exceedance probability.
6.3. Results

In this subsection we analyse the results of the experiments. First, we analyse the performance of each regression method (RFR, LASSO and HRE) for forecasting the future values of the time series (RQ1). The results are shown in Figure 5, which represents a boxplot of the $R^2$ scores of each method across the forecasting horizon. The horizontal red line denotes the mean value for the respective method. Overall, the ensembles show a similar forecasting performance, though HRE has a slight advantage. Effectively, despite the additional computational costs, the heterogeneous ensemble HRE outperforms RFR for forecasting. On the other hand, LASSO underperforms relative to the other two models.

![Figure 5: Distribution of $R^2$ scores across the forecasting horizon for each regression approach.](image)

We then evaluated all methods according to their ability to estimate exceedance probability. Figure 6 shows the distribution of AUC for each method across the forecasting horizons (up to 24 hours). The best performing models are RFR+CDF and HRE+CDF, respectively. These results indicate that coupling a forecasting model with the mechanism based on the CDF for estimating exceedance probability provides a better performance relative to binary classification strategies (RFC, RFC+SMOTE, and LR) and ensemble-based approaches (RFR+Direct and HRE+Direct). While HRE shows a slight advantage relative to RFR for forecasting the future numeric values of the time series, for estimating exceedance probability the outcome is reversed. Notwithstanding, the difference in AUC is negligible.

Another interesting result to note is that HRE+Direct provides poor estimates regarding exceedance probability, while RFR+Direct is competitive with classification methods. This outcome suggests that building an ensemble with weak models (RFR is an ensemble of decision trees) is better than an ensemble of strong models, such as HRE.

Pre-processing the training data with a resampling method (SMOTE) improved the average AUC of the RFC, though the difference is negligible. Finally,
the linear models LR and LASSO+CDF also underperform relative to the other methods, except for HRE+Direct. These results answer the research question RQ2, regarding the competitiveness of the performance of a CDF-based method for estimating exceedance probability.

Figure 6: Distribution of the AUC for each method across the forecasting horizons.

Figure 7 shows the AUC score for each method across the forecasting horizons. There is an generalized tendency for decreasing scores as the forecasting horizon increases (RQ3). This is expected as predicting further into the future represents a more difficult task.

In the previous experiments, we used the CDF of the Normal distribution \( N(\hat{y}_i, \sigma^2_y) \) at each \( i \)-th instance.

We studied the sensitivity of the proposed method to different distributions.
We focused on continuous distributions with location and scale parameters, which we set to \( \hat{y}_i \) and \( \sigma_y \), respectively. Besides the Normal, we tested the following distributions: Gumbel, Cauchy, Laplace, Logistic, and Lognormal. These have heavier tails relative to the Normal, which may be important because the tails are usually relevant in exceedance probability forecasting tasks. We present the results in Figure 8, which shows the average AUC across the forecasting horizon. The results are similar across all distributions, and differences in AUC were only detected after 5 decimal places (RQ4).

Finally, we draw the exceedance probability curve in Figure 9 for a given test instance to exemplify the dynamics of the probabilities across different thresholds. The predicted and observed value for that instance are represented as the red and blue vertical bars, respectively. In this example, we compute the exceedance probability using the proposed CDF-based approach. The point forecast is about 2.55. According to our normality assumption, the exceedance probability at that point is 50%. We then compute this probability for different points, which leads to a curve with the form of the Normal CDF. Given the predicted value, it is almost certain that the threshold 1 will be exceeded. The threshold 2 will be exceeded with a 75% probability. The actual value (around 3.3) had a probability of about 24% of occurring according to the model.

7. Final Remarks

We proposed a new method to estimate exceedance probability using the numeric predictions obtained from a forecasting model. This approach is an alternative to using a binary classification model or an ensemble-based approach. The main conclusion is that the proposed method provides better exceedance probability estimates relative to the alternatives listed above for a case study.
in the ocean data domain (SWH forecasting). This outcome has an important practical impact, because users (e.g., decision-makers or analysts responsible for maritime operations) can rely on a forecasting model to predict not only the value of future observations but also quantify the probability of an exceedance event (in this case, a large wave).

A second important conclusion is that an heterogeneous regression ensemble (HRE), whose members are trained with several different learning algorithms, did not provide better exceedance probability estimations than a Random Forest (RFR).

While exceedance probability forecasting usually refers to binary events (whether or not the exceedance occurs), multiple thresholds can also be defined. An example of this is the triple barrier method used in quantitative trading (e.g., (Baía & Torgo, 2017; de Prado, 2020)). Stock market traders may be interested in predicting buy, sell, or hold signals according to price movements. Exceeding a positive threshold in terms of predicted price returns can be used as a buy signal; the opposite (predicted price returns below a negative threshold) can represent a sell trigger. If none of the thresholds is met, then the trader should hold the current position.

We transformed the time series into a trainable format using time delay embedding on each variable, as we formalized in Section 3. During our experiments, we also tested two preprocessing steps: a differencing operation to remove trend and stabilize the mean, and a Box-Cox power transformation to stabilize the variance. Since neither of these processes improved performance
we excluded them in the final experiments.

Finally, the proposed approach settled on the cumulative Normal distribution function. However, the data showed a clear positive skewness. This lead us to try different distributions designed for extreme values. We did not find significant different between different distributions. Notwithstanding, we believe that the most appropriate distribution strongly depends on the input data.

Acknowledgments

The work of L. Torgo was undertaken, in part, thanks to funding from the Canada Research Chairs program

References

Ali, M., Prasad, R., Xiang, Y., & Deo, R. C. (2020). Near real-time significant wave height forecasting with hybridized multiple linear regression algorithms. Renewable and Sustainable Energy Reviews, 192, 110003.

Baía, L., & Torgo, L. (2017). A comparative study of approaches to forecast the correct trading actions. Expert Systems, 34, e12169.

Branco, P., Torgo, L., & Ribeiro, R. P. (2016). A survey of predictive modeling on imbalanced domains. ACM Computing Surveys (CSUR), 49, 1–50.

Breiman, L. (2001). Random forests. Machine learning, 45, 5–32.

Cerqueira, V., Torgo, L., & Mozetič, I. (2020). Evaluating time series forecasting models: An empirical study on performance estimation methods. Machine Learning, 109, 1997–2028.

Cerqueira, V., Torgo, L., Pinto, F., & Soares, C. (2019). Arbitrage of forecasting experts. Machine Learning, 108, 913–944.

Cerqueira, V., Torgo, L., Soares, C., & Bifet, A. (2021). Model compression for dynamic forecast combination. arXiv preprint arXiv:2104.01830.

Chawla, N. V., Bowyer, K. W., Hall, L. O., & Kegelmeyer, W. P. (2002). Smote: synthetic minority over-sampling technique. Journal of artificial intelligence research, 16, 321–357.

Chen, D., Liu, F., Zhang, Z., Lu, X., & Li, Z. (2021). Significant wave height prediction based on wavelet graph neural network. In 2021 IEEE 4th International Conference on Big Data and Artificial Intelligence (BDAI) (pp. 80–85). IEEE.

Duan, W.-y., Huang, L.-m., Han, Y., & Huang, D.-t. (2016). A hybrid end-ar model for nonlinear and non-stationary wave forecasting. Journal of Zhejiang University-SCIENCE A, 17, 115–129.
Fasuyi, J. W., Newport, J., & Whidden, C. (2020). A machine learning redundancy model for the herring cove smart buoy. *Journal of Ocean Technology, 15*.

Ferro, C. A. (2007). Comparing probabilistic forecasting systems with the brier score. *Weather and Forecasting, 22*, 1076–1088.

Frangopol, D., & Kim, S. (2014). Prognosis and life-cycle assessment based on shm information. In *Sensor technologies for civil infrastructures* (pp. 145–171). Elsevier.

Gneiting, T., & Katzfuss, M. (2014). Probabilistic forecasting. *Annual Review of Statistics and Its Application, 1*, 125–151.

Granger, C. W., & Pesaran, M. H. (2000). Economic and statistical measures of forecast accuracy. *Journal of Forecasting, 19*, 537–560.

Huang, G.-B., Zhu, Q.-Y., & Siew, C.-K. (2006). Extreme learning machine: theory and applications. *Neurocomputing, 70*, 489–501.

Huang, N. E., Shen, Z., Long, S. R., Wu, M. C., Shih, H. H., Zheng, Q., Yen, N.-C., Tung, C. C., & Liu, H. H. (1998). The empirical mode decomposition and the hilbert spectrum for nonlinear and non-stationary time series analysis. *Proceedings of the Royal Society of London. Series A: mathematical, physical and engineering sciences, 454*, 903–995.

Jain, P., & Deo, M. (2007). Real-time wave forecasts off the western indian coast. *Applied ocean research, 29*, 72–79.

Jose, V. R. R., & Winkler, R. L. (2008). Simple robust averages of forecasts: Some empirical results. *International journal of forecasting, 24*, 163–169.

Khosravi, A., Nahavandi, S., Creighton, D., & Atiya, A. F. (2011). Comprehensive review of neural network-based prediction intervals and new advances. *IEEE Transactions on neural networks, 22*, 1341–1356.

Kunreuther, H. (2002). Risk analysis and risk management in an uncertain world 1. *Risk Analysis: An International Journal, 22*, 655–664.

Kuppili, A. S. (2021). Forecasting meteorological variables and anticipating climatic aberrations of an oceanic buoy using a neighbour buoy, .

Lakkaraju, H., Bach, S. H., & Leskovec, J. (2016). Interpretable decision sets: A joint framework for description and prediction. In *Proceedings of the 22nd ACM SIGKDD international conference on knowledge discovery and data mining* (pp. 1675–1684).

Lambert, J. H., Matalas, N. C., Ling, C. W., Haines, Y. Y., & Li, D. (1994). Selection of probability distributions in characterizing risk of extreme events. *Risk Analysis, 14*, 731–742.
Lewis, J. M. (2005). Roots of ensemble forecasting. *Monthly weather review, 133*, 1865–1885.

Li, G., Weiss, G., Mueller, M., Townley, S., & Belmont, M. R. (2012). Wave energy converter control by wave prediction and dynamic programming. *Renewable Energy, 48*, 392–403.

Méndez, F. J., Menéndez, M., Luceño, A., & Losada, I. J. (2006). Estimation of the long-term variability of extreme significant wave height using a time-dependent peak over threshold (pot) model. *Journal of Geophysical Research: Oceans, 111*.

Picard, R. R., & Cook, R. D. (1984). Cross-validation of regression models. *Journal of the American Statistical Association, 79*, 575–583.

Polson, M., & Sokolov, V. (2020). Deep learning for energy markets. *Applied Stochastic Models in Business and Industry, 36*, 195–209.

de Prado, M. M. L. (2020). *Machine learning for asset managers*. Cambridge University Press.

Sadeghifar, T., Nouri Motlagh, M., Torabi Azad, M., & Mohammad Mahdizadeh, M. (2017). Coastal wave height prediction using recurrent neural networks (rnn) in the south caspian sea. *Marine Geodesy, 40*, 454–465.

Shamshirband, S., Mosavi, A., Rabczuk, T., Nabipour, N., & Chau, K.-w. (2020). Prediction of significant wave height; comparison between nested grid numerical model, and machine learning models of artificial neural networks, extreme learning and support vector machines. *Engineering Applications of Computational Fluid Mechanics, 14*, 805–817.

Slud, E., & Kedem, B. (1994). Partial likelihood analysis of logistic regression and autoregression. *Statistica Sinica*, (pp. 89–106).

Takens, F. (1981). Dynamical systems and turbulence, warwick 1980: Proceedings of a symposium held at the university of warwick 1979/80. chapter Detecting strange attractors in turbulence. (pp. 366–381). Berlin, Heidelberg: Springer Berlin Heidelberg.

Taylor, J. W. (2017). Probabilistic forecasting of wind power ramp events using autoregressive logit models. *European Journal of Operational Research, 259*, 703–712.

Taylor, J. W., & Yu, K. (2016). Using auto-regressive logit models to forecast the exceedance probability for financial risk management. *Journal of the Royal Statistical Society: Series A (Statistics in Society), 179*, 1069–1092.

Torgo, L., & Ohashi, O. (2011). 2d-interval predictions for time series. In *Proceedings of the 17th ACM SIGKDD international conference on Knowledge discovery and data mining* (pp. 787–794).
Toth, Z., Talagrand, O., Candille, G., & Zhu, Y. (2003). Probability and ensemble forecasts. *Forecast verification: A practitioner’s guide in atmospheric science, 137*, 163.

Wang, J., Wang, Y., & Yang, J. (2021). Forecasting of significant wave height based on gated recurrent unit network in the taiwan strait and its adjacent waters. *Water, 13*, 86.