Ferromagnetism of spinor atomic condensates in the double well

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Abstract. We show that the ferromagnetic and antiferromagnetic spinor condensates display distinct dynamical behaviors in double well. For the ferromagnetic case, strong symmetry-breaking dynamics can be induced by a small initial distortion. Contrarily, the $\sigma = \pm 1$ components tend to be well mixed in the antiferromagnetic condensate. The tunneling coupling plays an important role in distinguishing ferromagnetism and antiferromagnetism.

1. Introduction
Recent developments on ultracold atomic gases with multiple components have greatly stimulated the interest in itinerant ferromagnetism in both bosons and fermions.

Itinerant ferromagnetism in Fermi systems has been intensively studied in the context of condensed matter physics. Nevertheless, the mechanism of itinerant-fermion ferromagnetism is still under exploration. So far, the most acceptable explanation for this is based on the Stoner theory which suggest that the itinerant-electron ferromagnetism arises from the strong Coulomb repulsion. Ultracold atomic fermions provide an ideal test-bed for the Stoner theory since interactions between atoms can be tunable by Feshbach resonance [1]. An experimental group from MIT has claimed that the itinerant ferromagnetic (FM) phase is realized in Fermi gases of an equal mixture of $^6$Li atoms in the lowest two hyperfine states [2]. Theoretical investigations on the observable experimental signature of the FM transition in a single trap [3] or a double-well trap [4] have been performed within a local density approximation.

The $^{87}$Rb gas comes as the first example of the ferromagnetic Bose system [5, 6]. It is already suggested that the spinor Bose gas with FM couplings undergoes a FM transition with the critical temperature never below the Bose-Einstein condensation temperature [7]. This is different from the Fermi gas in which the FM transition can not occur unless the FM coupling exceeds the Stoner point. Magnetic domains in $^{87}$Rb condensates have been directly observed [8]. Meanwhile, the magnetization in each domain also appears to oscillate. The coherent dynamics of domain formation is also theoretically studied [9].

Quite recently, spinor condensate in double well is attracting fast growing research interest. New results on quantum spin tunneling and Josephson oscillation [10, 11] and non-Abelian Josephson effect [12] are reported. Moreover, the symmetry breaking and restoring dynamics in two-component mixture are proposed [13]. This paper studies magnetic phenomena in double-well spin-1 condensates. We find that the tunneling coupling plays an important role in distinguishing ferromagnetism and antiferromagnetism. In FM condensates, strong separation
of $\sigma = \pm 1$ components is observed. On the other hand, the $\sigma = \pm 1$ components seems well mixed in the antiferromagnetic condensate.

2. The method
Observation of spin population dynamics, i.e., the evolution of spin population in different Zeeman sublevels, has ever been the main tool to investigate Bose magnetism. For antiferromagnetic (AFM) condensates, the spin population displays very good harmonic oscillation [14]. However, spin dynamics in FM condensates seem rather complicated. It was suggested that the population oscillation is damped due to the formation of inner structures [15].

The dynamical behaviors of the double-well spinor condensate is examined through numerical simulations from the time-dependent Gross-Pitaevskii (GP) equations,

\[ i\hbar \frac{\partial}{\partial t} \psi_\pm = [H_0 + c_2 (\rho_+ + \rho_0 - \rho_-)] \psi_\pm + c_2 \psi_\mp \psi^*_\pm, \]
\[ i\hbar \frac{\partial}{\partial t} \psi_0 = [H_0 + c_2 (\rho_+ + \rho_-)] \psi_0 + 2c_2 \psi_\mp \psi^*_0, \]

where $H_0 = -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}} + c_0 (\rho_+ + \rho_0 + \rho_-)$, and the $c_0$ ($c_2$) term represents contributions of the spin-independent (-dependent) interaction. The double-well potential is given by

\[ V_{\text{ext}} = \frac{\mu}{2} \left( \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right) + V_0 \cos^2 (\pi x/q_0). \] 

As in Ref. [11], the parameters in Eq. (2) are chosen as $\omega_x = 2\pi \times 100 \text{Hz}$, $\omega_y = 2\pi \times 66 \text{Hz}$, $\omega_z = 2\pi \times 90 \text{Hz}$, $q_0 = 5.2 \text{\mu m}$, and $V_0 = 3500 \text{hHz}$. The dynamical equations are transformed into one-dimensional ones since the double well is created only along one direction. The condensate wave function is expressed as $\psi_\sigma(x, t) = \sqrt{n_\sigma(x, t)} e^{i\theta_\sigma(x, t)}$, where $\sigma = \pm 1, 0$ is the spin index.

Consider a system with conserved total population $N = \sum \sigma N_\sigma$ and total spin $M = N_+ - N_-$, where $N_\sigma = \int dx |\psi_\sigma|^2$ is the total number of the $\sigma$ component. In the following discussions, this parameter is normalized as $n_\sigma = N_\sigma/N$. To characterize the tunneling dynamics, we define the number of each component in the left well, $N_\sigma^L = \int_0^L dx |\psi_\sigma|^2$, and the right well, $N_\sigma^R = \int_0^R dx |\psi_\sigma|^2$. Then $M^{\sigma} = (N_\sigma^L - N_\sigma^R)/(\frac{1}{2}N)$ refers to the magnetization in each well.

Dynamical behaviors of the condensate depend strongly on interactions. In following numerical calculations, the effective interactions are deduced based on the two-mode approximation [11]. The condensed wave function is given by

\[ \varphi_\sigma(x, t) = \varphi_\sigma^L(t) \phi_\sigma^L(x) + \varphi_\sigma^R(t) \phi_\sigma^R(x), \]

and $\phi_\sigma^L, R$ stands for the wave function mainly localized in the left (right) well. The phase $\theta_\sigma^{L, R}$ is considered to be a global variable in each well. The effective spin-independent (dependent) interactions read

\[ U_{0(2)} = \frac{1}{4} \int |\phi_\sigma^L|^4 dx = \frac{1}{4} \int |\phi_\sigma^R|^4 dx, \]

and the tunneling coupling is given by

\[ K = \int \phi_\sigma^L \left( -\frac{\hbar^2}{2\mu} \nabla^2 + V_{\text{ext}} \right) \phi_\sigma^R dx. \]
3. Results and discussions
We start with symmetric spin configurations, $n_1 = n_{-1}$ and $m^L = m^R = 0$. Meanwhile, the initial phase of each component is set to be 0 and the total numbers in both wells are equal. In this case, the total number in each well keeps almost unchanged, $n^L + n^R + n^- = n^L + n^R + n^-$, but spin dynamics remains free. $n_0$ is assumed to have a small value at the beginning. To stir up spin dynamics, a small distortion is introduced to the original state, giving $n^L(0) = [1 - n_0(0)]/4 \pm \delta^+$, $n^R(0) = [1 - n_0(0)]/4 \pm \delta^-$, and $\delta^+ \approx 1$. Thus, spin populations in two wells become slightly imbalanced and the initial magnetization $m(0) = m^L(0) = -m^R(0) = \delta^+ + \delta^-$. To perform numerical calculations, initial populations of the original state are set to be $\{n^+ : n_0 : n^-\} = \{0.995: 0.01: 0.995\}$. The distortion is chosen as $\delta^\pm = 0.025$ for the AFM case and $\delta^\pm = 0.005$ for the FM case.

Let us look at interactions of alkali atoms. $c_2$ is generally one or two orders of magnitude less than $c_0$. Taking the $^{87}$Rb for example, $U_2 \approx -0.1K$ and $U_0 \approx 20K$ when $N = 2500$. In our numerical simulations, we set $U_2/K = \pm 2$ for AFM and FM cases respectively. Fig. 1 and 2 illustrate the numerical results respectively.

As shown in Fig. 1 and Fig. 2, $n^L_0$ can oscillate from zero to almost one in both cases. It means that the participation of $\sigma = 0$ component significantly affects all dynamical behaviors. Nevertheless, the distinction between the antiferromagnetism and ferromagnetism is still remarkable. In AFM case, the population oscillation is nearly harmonic and the spin $\sigma = \pm 1$ components are always well-mixed. Consequently, the $m$ oscillation is modulated by the population dynamics. $m$ oscillates much faster than $n_0$ and its value keeps small all the time, $|m(t)| < 0.05$. On the other hand, the FM behaviors are rather complicated. Both the population dynamics and the $m$ oscillation are inclined to be anharmonic. It is worth noting that the variation of $m$ is quite large, sustaining the symmetry-breaking dynamics.

![Figure 1](image1.png) ![Figure 2](image2.png)

**Figure 1.** Evolution of spin populations and the magnetization in the left well, obtained numerically from GP equations. $\delta^\pm = 0.025$. The involved parameters are set as $U_0 = 20K$, $U_2 = 2K$ and $K \approx 0.09$.

**Figure 2.** Evolution of spin populations and the magnetization in the left well, obtained numerically from GP equations. $\delta^\pm = 0.005$. The involved parameters are set as $U_0 = 20K$, $U_2 = -2K$ and $K \approx 0.09$.

At last, we examine the boundary between FM and AFM dynamics. As is well-known,
the single-well trapped spinor condensate displays ferromagnetism when $U_2 < 0$, while antiferromagnetism when $U_2 > 0$ [6]. For the double-well spinor condensate, our finding shows that its magnetic properties are not only determined by $U_2$, but also related to the tunneling coupling $K$. It seems that the ferromagnetism does not manifest unless $|U_2/K| > 1$. The tunneling coupling plays an important role in distinguishing ferromagnetism and antiferromagnetism.

Considering that the total atom number in each well is almost unchanged but there exists spin current through the barrier, we can construct a simplified effective Hamiltonian for the condensate in the left well,

$$\mathcal{H}_m^L \approx -2K\sqrt{1-m^2}\cos\alpha + U_2m^2,$$

where $\alpha = (\theta^L - \theta^R)/2$. The first term is originated from the quantum tunneling and the second term denotes the magnetic exchange energy. The ground state for the AFM case lies at the $(m = 0, \alpha = 0)$ point, which means that the $\sigma = \pm 1$ components tend to be mixed up. On the contrary, the ground state for the FM condensate is located at point $(m = \pm \sqrt{1-(K/U_2)^2}, \alpha = 0)$. $m$ goes to a finite value under the FM condition $|U_2/K| > 1$, that is to say, the condensate in each well becomes magnetized.

4. Summary
In summary, we explore the manifestation of distinct magnetic phenomena of spin-1 Bose condensates in a symmetric double-well. The ferromagnetic condensate is expected to display strong symmetry-breaking dynamics, with the amplitude evolving quickly from an almost negligible distortion of the original symmetric spin configuration. On the contrary, the distortion just results in a mild oscillation in the antiferromagnetic condensate. We also suggest that the condensate with ferromagnetic couplings may not exhibit ferromagnetic behaviors unless the effective coupling is stronger than a critical value which is relevant to the tunneling coupling.

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