Linear Cosmological Perturbations in D-brane Gases

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Abstract

We consider linear cosmological perturbations on the background of a D-brane gas in which the compact dimensions and the dilaton are stabilized. We focus on long wavelength fluctuations and find that there are no instabilities. In particular, the perturbation of the internal space performs damped oscillations and decays in time. Therefore, the stabilization mechanism based on D-brane gases in string theory remains valid in the presence of linearized inhomogeneities.

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I. INTRODUCTION

It is well known that the theory of big-bang nucleosynthesis together with the observational abundances of light elements provide a limit to the change in the size of internal space following the production of these elements. Therefore, string theory should be able to explain why the extra dimensions are stabilized following nucleosynthesis? Intuitively, winding branes are expected to resist the expansion and this may offer an explanation. An accommodation of this idea in early string theory can be found in the paper of Brandenberger and Vafa [1], and recently the stabilization of toroidal extra dimensions by winding strings is verified by explicit calculations in [2, 3] (see also [4]). The model of [1] is developed in [5] to include other extended objects in string theory and this scenario is now known as brane gas cosmology (BGC) (see e.g. [6]-[31] for recent work).

It was also claimed in [1] that the dimensionality of space-time can be explained by the annihilation of winding strings in a three dimensional subspace leading decompactification. From the naive intersection probability of hypersurfaces embedded in $d$-spatial dimensions one can infer that $d = 2p + 1$ is the critical value which allows annihilation of $p$-branes. Remarkably, for strings $d = 3$ and according to [1] that is why a three dimensional subspace is decompactified. As an additional consequence of this argument, in BGC all higher dimensional branes are expected to be annihilated in the early universe. Although these claims are supported numerically in [32], there are also some caveats. It is observed in [15] that the number of decompactified dimensions depends on initial conditions and the outcome of three large dimensions is argued to be statistically insignificant [25]. In [26] it is shown that the interaction rates of strings are not strong enough to sustain thermal equilibrium. Moreover, D-brane anti-D-brane annihilation rate turns out to be small compared to the expansion rate of a FRW universe including the higher dimensional D-branes [12]. Therefore, it seems that the dimensional counting argument of [1] is not sufficient alone for the annihilation process and it is plausible to have surviving branes other than strings or membranes.

In the light of these results, we think that late time models with higher dimensional branes should also be studied in the context of BGC. Such models may be useful in solving two difficulties with string gases. Firstly, as pointed out in [7], strings may not be able to stabilize volume modulus, or further, there may not be any winding strings in the spectrum depending on the topology of internal space (consider e.g. a sphere). Secondly, it turns out
to be difficult to stabilize the dilaton and keep up the stability of extra dimensions [23],
which may cause problems in canonical Einstein frame after compactification.

In [24], we have considered a gas of D6-branes wrapping over the six-dimensional internal
space and demonstrated dynamical stabilization of the volume modulus and the dilaton. The
aim of this paper is to consider long wavelength perturbations in this model to see the validity
of the conclusions under linearized inhomogeneities. For the string gas the perturbation
analysis has been carried out in [18, 19]. The plan of the paper is as follows. In the next
section we derive linearized equations for the scalar metric and dilaton perturbations in the
generalized longitudinal gauge. In section III, we focus on long wavelength fluctuations and
solve the equations. We briefly conclude in section IV.

II. PERTURBATION EQUATIONS

We start with ten-dimensional dilaton gravity coupled to matter Lagrangian $L_m$
representing brane winding and momentum modes:

$$ S = \int d^{10}x \sqrt{-g} \, e^{-2\phi} \left[ R + 4(\nabla \phi)^2 + e^{a\phi} L_m \right]. $$

(1)

The field equations that follow from the above action can be found as

$$ R_{\mu\nu} + 2 \nabla_\mu \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \left[ R + 4 \nabla^2 \phi - 4 (\nabla \phi)^2 \right] = e^{a\phi} T_{\mu\nu}, $$

(2)

$$ R + 4 \nabla^2 \phi - 4 (\nabla \phi)^2 = \frac{(a - 2)}{2} e^{a\phi} L_m, $$

(3)

$$ \nabla^\mu T_{\mu\nu} = \frac{(a - 2)}{2} L_m \nabla_\nu \phi - (a - 2) T_{\nu\lambda} \nabla^\lambda \phi, $$

(4)

where $T_{\mu\nu}$ is the matter energy-momentum tensor:

$$ T_{\mu\nu} = - \frac{1}{\sqrt{-g}} \frac{\partial}{\partial g^{\mu\nu}} \left( \sqrt{-g} L_m \right). $$

(5)

Eqns. (2) and (3) are obtained by directly varying the action with respect to the metric and
the dilaton. Although for matter equations one in principle needs Lagrangian $L_m$ explicitly,
in our case (4) can be deduced from the consistency of (2) and (3).

It is well known that one particular combination of the field equations gives a constraint
on initial data. Let $n^\mu$ to denote the unit normal vector field to an initial value hypersurface.
One can easily show that (2) contracted with $n^\mu n^\nu$

$$ \left( R_{\mu\nu} + 2 \nabla_\mu \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \left[ R + 4 \nabla^2 \phi - 4 (\nabla \phi)^2 \right] - e^{a\phi} T_{\mu\nu} \right) n^\mu n^\nu = 0 $$

(6)
does not contain any second order time derivatives. Firstly, it is well from the initial value formulation of general relativity that \( n^{\mu n^{\nu}}(R_{\mu \nu} - g_{\mu \nu}R/2) \) involves only first order time derivatives of the metric (see e.g. [33]). On the other hand, the dangerous terms with two derivatives acting on dilaton in (6) can be reexpressed straightforwardly as

\[
\nabla^2 \phi + n^{\mu n^{\nu}} \nabla_\mu \nabla_\nu \phi = h^{\mu \nu} \nabla_\mu \nabla_\nu \phi,
\]

where \( g_{\mu \nu} = h_{\mu \nu} - n_{\mu}n_{\nu} \) and \( D_\mu = h^{\nu}_{\mu} \nabla_\nu \) is the spatial covariant derivative along constant time hypersurface. It is clear that (7) contains only first order time derivatives and thus (6) can be viewed as a constraint on initial data.

Assuming a metric of the following form

\[
ds^2 = -e^{2A}dt^2 + e^{2B}dx^i dx^j + e^{2C} d\Sigma^2_p,
\]

(8)

\((i = 1, \ldots, m)\) the energy-momentum tensor for a gas of branes winding over the compact manifold \( \Sigma_p \) can be found as (see e.g. [22])

\[
T_{\hat{b}\hat{b}} = T_w e^{-mB} + T_m e^{-mB-(p+1)C},
\]

\[
T_{\hat{i}\hat{j}} = 0,
\]

\[
T_{\hat{a}\hat{b}} = -T_w e^{-mB} \delta_{ab} + \frac{T_m}{p} e^{-mB-(p+1)C} \delta_{ab},
\]

where \( T_w \) and \( T_m \) are constants corresponding to winding and momentum modes. In this paper we take \( \Sigma_p \) to be Ricci flat. From (9) an equation of state \( p_\mu = \omega_\mu \rho \) can be deduced where \( \omega_i = 0, \omega_a = -1 \) for winding modes and \( \omega_i = 0, \omega_a = 1/p \) for momentum modes. Note that the equation for momentum modes is identical to radiation confined in the compact space. One can check from (5) and (9) that

\[
\mathcal{L}_m = -2\rho,
\]

(10)

which is precisely the Lagrangian for hydrodynamical matter (see e.g. section 10.2 of [34]). In general we have \( \nabla^\mu T_{\mu \nu} \neq 0 \) due to the coupling of \( \mathcal{L}_m \) to dilaton.

To focus on D-branes one should set

\[
a = 1,
\]

(11)
which is required to have the correct $g_s = e^\phi$ dependence in the low energy effective action (1). Recall that we also have

$$m + p = 9,$$

where $m$ and $p$ are the dimensions of the observed and the compact dimensions, respectively.

In [24], we have found a particular solution to (2)-(4)

$$ds^2 = -dt^2 + (t)^{8/(m+3)} dx^i dx^i + e^{2C_0} d\Sigma_p^2,$$

$$e^\phi = e^{\phi_0} (t)^{2(m-3)/(m+3)},$$

$$e^{\phi_0} = \frac{4(p+2)p}{T_w (p+1)(m+3)^2}, \quad e^{(p+1)C_0} = \frac{(p+2)T_m}{p T_w},$$

which has some appealing properties. For instance, it is possible to argue analytically\(^1\) and support the argument numerically that for arbitrary initial conditions (within a given ansatz) the fields approach (13) asymptotically. Moreover, for $m = 3$, the dilaton becomes a constant and thus the canonical Einstein and string frames become identical up to a constant scaling.

Let us now consider linearized inhomogeneities on (13). We are interested in the case where the fluctuations are independent of compact coordinates\(^2\) and choose the generalized longitudinal gauge (see [18]) in which the scalar metric perturbations only appear in diagonal entries. Therefore, the perturbed solution has the form:

$$ds^2 = -e^{2A}(1 + 2\delta A) dt^2 + e^{2B}(1 + 2\delta B) dx^i dx^i + e^{2C}(1 + 2\delta C) d\Sigma_p^2,$$

$$e^\phi = e^\phi(1 + \delta \phi),$$

where $A$, $B$, $C$ and $\phi$ denote the background values depending only on $t$ and

$$\delta A = \delta A(t, x^i), \quad \delta B = \delta B(t, x^i), \quad \delta C = \delta C(t, x^i), \quad \delta \phi = \delta \phi(t, x^i).$$

Let us remind that in the longitudinal gauge $\delta A$, $\delta B$ and $\delta C$ coincide with diffeomorphism invariant quantities.

In obtaining the linearized field equations we find it convenient to use (6) together with a particular combination of (2) and (3)

$$R_{\mu\nu} + 2\nabla_{\mu} \nabla_{\nu}\phi = \frac{1}{2} g_{\mu\nu} e^\phi \rho + e^\phi T_{\mu\nu}.$$

\(^1\) As discussed in [30] and [31], it is possible to give analogous arguments for intersecting brane configurations with or without dilaton.

\(^2\) This is indeed a good approximation since the extra dimensions are assumed to be very small.
As we pointed out above, (6) can be interpreted as a constraint on initial data. Substituting (9) and (14) in these equations and keeping only the linear terms one gets

\[2\ddot{\phi} - m\dot{B} - p\ddot{C} + e^{-2B}\sum_k \delta A_{kk} + 2\left[m\ddot{B} + m\dot{B}^2 - 2\dot{\phi}\right]\delta A + \sum_k \delta A_{kk} + 2\left[m\ddot{B} + m\dot{B}^2 - 2\dot{\phi}\right]\delta A + \frac{1}{2} e^\phi \delta \phi + \frac{1}{2} e^\phi \delta \phi,\]  

(17)

\[(m - 1)\delta B_i - 2\ddot{\phi}_i - \left[(m - 1)\dot{B} - 2\dot{\phi}\right]\delta A_i + p\dot{\phi}C_i - p\dot{B}\ddot{C}_i + 2\dot{B}\ddot{\phi}_i = -e^{\phi + B}\delta T_{bi},\]  

(18)

\[\ddot{B} + e^{-2B}\left[2\ddot{\phi}_{ii} - \delta A_{ii} - p\ddot{C}_{ii} - (m - 2)\delta B_{ii}\right] - e^{-2B}\sum_k \delta B_{kk} + 2\left[m\ddot{B} - \dot{\phi}\right]\delta B - \dot{B}\delta A + p\dot{B}\dot{C} - 2\dot{B}\ddot{\phi} + 2\left[2\dot{B}\dot{\phi} - \dot{\phi} - m\ddot{B}\dot{B}\right]\delta A = \frac{1}{2} e^\phi \delta \phi + \frac{1}{2} e^\phi \delta \phi,\]  

(19)

\[2\ddot{\phi} - \delta A - p\ddot{C} - (m - 2)\delta B_{ij} = 0, \quad i \neq j\]  

(20)

\[\ddot{C} - e^{-2B}\sum_k \ddot{C}_{kk} + \left[m\ddot{B} - 2\dot{\phi}\right]\ddot{C} = \frac{1}{2} e^\phi \delta \phi + \frac{1}{2} e^\phi \delta \phi + \frac{1}{2} e^\phi T_{aa}\delta \phi + e^\phi T_{aa},\]  

(21)

\[m(m - 1)\ddot{B} - 2m\dot{\phi}\]  

\[\ddot{B} + \left[m\ddot{B} - 2p\dot{\phi}\right]\ddot{C} + \left[4m\dot{B}\dot{\phi} - 4\dot{\phi} - m(m - 1)\dot{B}\dot{B}\right]\delta A + \left[4\dot{\phi} - 2m\ddot{B}\right]\ddot{\phi} + e^{-2B}\sum_k \left[2\ddot{\phi}_{kk} - (m - 1)\delta B_{kk} - p\ddot{C}_{kk}\right] = \frac{1}{2} e^\phi \delta \phi + \frac{1}{2} e^\phi \delta \phi,\]  

(22)

where (17)-(21) follow from (16), and (22) can be obtained from (6). In the above equations the dot and the subindex \(i\) denote partial differentiations with respect to \(t\) and \(x^i\), respectively, and we have not used the summation convention.

Note that in (18) we introduce a non-diagonal component for the linearized energy momentum tensor \(\delta T_{bi}\). Although for long wavelength fluctuations this equation is satisfied identically, it is in general necessary to add \(\delta T_{bi}\) since the perturbed metric functions depend on both \(t\) and \(x^i\), which gives non-zero \(R_{0i}\) (indeed (18) is the (0\(i\)) component of (16)). Moreover, one can see that without \(\delta T_{bi}\) the matter equations constrain the metric functions too much, see (24) below. The appearance of this non-diagonal term for winding modes can be deduced from the Born-Infeld (BI) action as follows. When the metric functions depend only on \(t\), a static brane winding \(\Sigma_p\) and located at a constant position \(x^i = x^i_0\) becomes an extremum of the BI action (i.e. the embedding coordinates become harmonic maps).

Using this solution in the energy-momentum tensor obtained from BI action and smearing the delta function singularity by taking a continuum average for a gas of branes, one gets the winding part of (9) [9]. However, when the metric is perturbed with \(t\) and \(x^i\) dependent functions, the static brane solution should be modified \(x^i = x^i_0 + \delta x^i(t)\) and this correction

\(^3\text{It is possible to show that (6) and (16) are equivalent to (2) and (3) with } a = 1.\)
induces non-zero $\delta T_{0i}$.  

Linearizing the matter equation (4) we find that $\nu = a$ component is identically satisfied while $\nu = 0$ and $\nu = i$ equations give

$$
\delta \dot{\rho} + (m \dot{B} + p \dot{C}) \delta \rho + (m \dot{B} + p \dot{C}) \rho + \dot{C} T^a_a + \dot{C} \delta T^a_a + e^{A-B} \delta T^0_j = 0, \quad (23) 
$$

$$
\delta \dot{T}_{0i} + [(m + 1) \dot{B} + p \dot{C} - \dot{\phi}] \delta T_{0i} + e^{A-B} [\delta C T^a_a + \rho \delta \phi_i - \rho \delta A_i] = 0. \quad (24)
$$

Eq. (23) has actually two separate parts corresponding to winding and momentum modes. In obtaining $\delta \rho$ and $\delta T_{ab}$ from (9), one should also consider the variations $\delta T_w$ and $\delta T_m$. Therefore, there are seven variables which are $\delta T_w$, $\delta T_m$, $\delta T_{0i}$, $\delta A$, $\delta B$, $\delta C$ and $\delta \phi$. Equations (23) and (24) determine the evolution of source fields $\delta T_w$, $\delta T_m$ and $\delta T_{0i}$, which in turn should be substituted in (17)-(22). It is possible to solve (20) as

$$
2 \delta \phi - \delta A - p \delta C - (m - 2) \delta B = 0, \quad (25)
$$

which reduces the number of unknown functions by one. On the other hand, (18) and (22) are not dynamical evolution equations since they only contain first order time derivatives and thus merely restrict the solution space. Ignoring them, it is easy to see that the number of equations and unknowns are the same.

III. LONG WAVELENGTH FLUCTUATIONS

Let us now focus on long wavelength perturbations. As pointed out in [18] an instability on such scales would be a problem for the stabilization mechanism. In this regime all spatial derivatives can be ignored in field equations which in turn implies that (18) and (24) are identically satisfied. On the other hand, from (23) one obtains

$$
\delta \dot{T}_w = \delta \dot{T}_m = 0. \quad (26)
$$

Since constant shifts of $T_w$ or $T_m$ alter the solution (13), they are not honest perturbations and one should set $\delta T_w = \delta T_m = 0$. In other words, (26) gives the zero modes which must be ignored. Solving $\delta \phi$ from (25)\(^4\) and using the background values from (13), we find that

\(^4\) For long wavelength fluctuations (25) can be viewed as the residual gauge symmetry fixing.
(17), (19), (21) and (22) respectively become

\[ \ddot{\delta A} - 2\dot{\delta B} + \frac{12\dot{\delta A}}{(m+3)t} - \frac{8m\dot{\delta B}}{(m+3)t} - \frac{10p\dot{\delta A}}{(m+3)^2t^2} + \frac{2p(m+2)\delta B}{(m+3)^2t^2} = 0, \]  

(27)

\[ \ddot{\delta B} - \frac{8\dot{\delta A}}{(m+3)t} + \frac{20\dot{\delta B}}{(m+3)t} - \frac{10p\dot{\delta A}}{(m+3)^2t^2} + \frac{2p(m+2)\dot{\delta B}}{(m+3)^2t^2} = 0, \]  

(28)

\[ \ddot{\delta C} + \frac{12\dot{\delta C}}{(m+3)t} + \frac{2p(p+2)\dot{\delta C}}{(m+3)^2t^2} = 0, \]  

(29)

\[ \frac{12\dot{\delta A}}{(m+3)t} + \frac{4(m-6)\dot{\delta B}}{(m+3)t} + \frac{20p\dot{\delta A}}{(m+3)^2t^2} - \frac{4p(m+2)\delta B}{(m+3)^2t^2} = 0. \]  

(30)

Note that \( \delta C \) decouples from \( \delta A \) and \( \delta B \). As a consistency check of these equations, we find that the time derivative of the constraint (30) is proportional to four times (27) plus \( m \) times (28) and thus identically satisfied, as it should be. This does not mean however that (30) is empty. Rather, one should first solve (27)-(28) and then use (30) to narrow the solution space.

Eq. (29) can easily be solved for \( \delta C \) which gives

\[ \delta C(t) = c_1 \frac{\cos(a \ln t)}{t^b} + c_2 \frac{\sin(a \ln t)}{t^b}, \]  

(31)

where \( a, b \) are positive numbers depending on \( m \) (e.g. for \( m = 3 \), \( a = \sqrt{29/12}, b = 1/2 \)). On the other hand, the coupled equations (27) and (28) give

\[ \delta A = (m+2) k_1 + k_2 t^{-(7+m)/(m+3)} + k_3 (m-2) t^{(-9+m)/(m+3)} + k_4 (29 + 11m) t^2 \]

\[ \delta B = 5k_1 - k_2 t^{-(7+m)/(m+3)} + k_3 t^{(-9+m)/(m+3)} + k_4 (23 + m) t^2 \]  

(32)

where \( k_1, \ldots, k_4 \) are arbitrary constants. Substituting these solutions into (30), we observe that it yields

\[ k_4 = 0, \]  

(33)

and thus the growing mode in (32) is eliminated by the constraint equation. As a result, we find that there are no instabilities to this order.

IV. CONCLUSIONS

In this paper, we consider long wavelength fluctuations on the background of a D-brane gas in ten dimensional dilaton gravity constructed in [24]. The most attractive feature of the solution is that it yields stabilized internal dimensions. Moreover, when the observed space
is three dimensional the dilaton also becomes a constant. The main result of this paper is that the long wavelength perturbation of the internal space performs damped oscillations. Other fluctuations, including the dilaton, are found to decay to a constant. Therefore, the stabilization mechanism based on D-brane winding and momentum modes remains valid in the presence of linearized inhomogeneities.

Technically, the perturbation analysis for D-branes is similar to the string gases studied in [18]. The main difference is that in our case there appears to be a growing mode which is eliminated by the constraint equation. Although the presence of this mode would not affect the internal space, it would ruin the stabilization of dilaton for $m = 3$, which is crucial in the effective field theory obtained after compactification.

Higher dimensional D-brane gases offer an analogous scenario to string gases for the stabilization of the extra dimensions. Moreover, D6 branes happen to fix dilaton in three dimensions. This mechanism can be useful in topologically non-trivial compactifications where the winding strings do not appear in the spectrum or they are not capable of fixing volume modulus. Of course, the toy model we consider is immature and it should be developed in many different directions. However, the results in BGC show that string theory has the right ingredients to solve the stabilization problem of extra dimensions in cosmology.

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