Comparison between steady-state characteristics of isotropic and anisotropic doubled-layered porous journal bearings under coupled stress lubrication

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Abstract. A theoretical comparison between the steady-state characteristics of isotropic and anisotropic finite hydrostatic double-layered porous journal bearings dealing with the effects of slip flow at the fine porous layer-film interface and percolation of additives into pores under the coupled stress fluid lubrication is presented. Based on the Beavers-Joseph’s criterion for slip flow, the modified Reynolds equation applicable to finite porous journal bearings lubricated with coupled stress fluids have been derived. The governing equations for flow in the coarse and fine layers of porous medium incorporating the percolation of polar additives of lubricant and the modified Reynolds equation are solved simultaneously using finite difference method satisfying appropriate boundary conditions to obtain the steady-state performance characteristics for both isotropic and anisotropic porous bearing. The comparisons between isotropic and anisotropic doubled-layered porous bearings for various steady-state characteristics are exhibited in the form of graphs which may be useful for design of such bearings.

1. Introduction
Hydrostatic porous bearings have been widely used in industry due to their more even pressure distribution in the film region in addition to their low cost. The merits of these bearings have invoked motivation in undertaking extensive research studies [1-5] on their performance. But these research studies are confined to the bearings with a single layer of porous surface possessing several disadvantages due to lower stability and load capacity owing to seepage into the boundary wall.

For eliminating these difficulties, several researchers [6-7] have suggested use of double-layered porous surfaces in bearings. Few literature [8-9] have been reported so far dealing with the performance of two-layered porous bearings. Saha and Majumder [8] presented the static characteristics and stability characteristics of two-layered hydrostatic porous oil journal bearings without the effect of the slip. Kumar et al. [9] presented a theoretical analysis on the finite double-layered hydrostatic porous oil journal bearing, where the steady-state characteristics are analysed including slip effect. But these studies were confined to the porous bearings lubricated with Newtonian fluids.

In modern science, to fulfil the desires for advance machines, the use of lubricants blended with high polymer additives is getting more and more importance. On the other side, polymeric additives containing lubricants exhibit non-Newtonian behaviour. Various micro-continuum theories have been developed to properly describe the rheological behaviour of non-Newtonian fluids. Among these continuum theories, stoke’s coupled stress fluid model [10] has been extensively used to analyse the performance of various fluid film bearing systems. Rheological effects of coupled stress fluid on the dynamic performance of hydrostatic porous journal bearings including slip effect has been presented by Guha [5].

Till date, there is no literature available that addresses the comparison between the steady-state characteristics of isotropic and anisotropic externally pressurized double-layered porous journal bearings with slip flow including percolation of additives into pores under coupled stress lubrication.
So, the present study deals with a most general modified form of Reynolds equation applicable to coupled stress lubrication in film region and porous region with the slip effect [11] at the porous-film interface of a finite double layered porous journal bearing and percolation of additives’ micro-structure into the pores. Using the steady-state film pressure distribution, the non-dimensional load capacity, end flow and frictional parameter have been obtained at various parametric conditions for both isotropic and anisotropic bearings. Comparison between the isotropic and anisotropic bearing performance under these parameters, has been exhibited in the form of graph.

2. Theoretical analysis
A schematic configuration of externally pressurized double-layered porous journal bearing is shown in Figure 1.

Figure 1. Schematic configuration of externally pressurized double-layered porous journal bearing.

2.1. Non-dimensional governing equations
The non-dimensional governing equations of steady-state pressures in the porous mediums incorporating the additives effects into the pores of an anisotropic doubled-layered porous journal bearing are expressed as follows:

For the coarse layer:

\[ \chi_{xc}K_{xc}^2 \frac{\partial^2 \bar{p}_c'}{\partial \theta^2} + \left( \frac{R}{H} \right)^2 \frac{\partial^2 \bar{p}_c'}{\partial \varphi^2} + \left( \frac{D}{L} \right)^2 \chi_{xc}K_{xc}^2 \frac{\partial^2 \bar{p}_c'}{\partial z^2} = 0 \]  \hspace{1cm} (2.1)

For the fine layer:

\[ \chi_{xf}K_{xf}^2 \frac{\partial^2 \bar{p}_f'}{\partial \theta^2} + \left( \frac{R}{H} \right)^2 \frac{\partial^2 \bar{p}_f'}{\partial \varphi^2} + \left( \frac{D}{L} \right)^2 \chi_{xf}K_{xf}^2 \frac{\partial^2 \bar{p}_f'}{\partial z^2} = 0 \]  \hspace{1cm} (2.2)

Non-dimensional modified Reynold’s equation appropriate to the film region of an anisotropic doubled-layered porous journal bearing lubricated with the couple-stressed fluid with the effect of velocity slip at the fine porous layer- film interface is represented by the following equation:

\[
\frac{\partial}{\partial \theta} \left[ f(\bar{h}, \sigma_{xf}, I, \gamma_{xf}) \frac{\partial \bar{p}}{\partial \theta} \right] + \left( \frac{D}{L} \right)^2 \frac{\partial}{\partial z} \left[ f(\bar{h}, \sigma_{xf}, I, \gamma_{xf}) \frac{\partial \bar{p}}{\partial z} \right] = \Lambda_s \frac{\partial}{\partial \theta} \left[ \bar{h}(1 + \xi_{ox}) \right] + \frac{\beta}{(1 - \gamma_{xf})} \frac{\partial \bar{p}_f'}{\partial \gamma}_{\gamma=0} \]  \hspace{1cm} (2.3)

2.2. Numerical solution procedure
With the help of appropriate boundary conditions [8,9], equations (2.1-2.3) are solved by finite-difference method with Gauss-Sedial iterative procedure using the over-relaxation scheme. The following convergence criterion has been set to achieve the accuracy of pressure values:

\[ \frac{|(\sum p_{i,j})_{M-1}-(\sum p_{i,j})_M|}{|\sum p_{i,j}|_M} \leq 0.0001, \]  

(M is the number of iteration)

Here, a comparison has been made between the present results with no-slip and non-Newtonian case and results from the reference.

Table 1. Comparison of steady-state analysis of two-layered porous bearing lubricated with Newtonian fluid with no-slip condition. Data available from the reference [9]

| β   | ε₀₀ | Present | Ref[12] | Present | Ref[12] | Present | Ref[12] |
|-----|-----|---------|---------|---------|---------|---------|---------|
| 0.001| 0.2 | 0.0456 | 0.0449 | 11.3051 | 11.291 | 0.0615 | 0.0610 |
| 0.4  | 0.1059 | 0.1052 | 7.1032 | 5.0281 | 0.1185 | 0.1184 |
| 0.6  | 0.2247 | 0.2240 | 2.7742 | 2.7232 | 0.1763 | 0.1760 |
| 0.8  | 0.5995 | 0.5980 | 1.6160 | 1.4357 | 0.2337 | 0.2331 |
| 0.05 | 0.2 | 0.1073 | 0.1037 | 5.2409 | 5.2162 | 0.4046 | 0.3684 |
| 0.4  | 0.2346 | 0.2283 | 2.7559 | 2.6299 | 0.3934 | 0.3584 |
| 0.6  | 0.4023 | 0.3951 | 1.8965 | 1.8421 | 0.3766 | 0.3505 |
| 0.8  | 0.6135 | 0.6032 | 1.6318 | 1.5902 | 0.3879 | 0.3670 |

3. Steady-state characteristics
By using the known static film pressure distribution in the clearance zone of the bearing, the following steady-state characteristics in terms of non-dimensional load capacity, end flow rate and the frictional parameter are obtained as follows:

3.1. Load carrying capacity
Non-dimensional components of load carrying capacity along the radial and tangential directions are given by

\[ \bar{W}_r = \frac{W_r}{LDP_s} = -\frac{1}{2} \int_0^{\theta_c} \bar{p} \times \cos \theta \, d\theta \, dz \]
\[ \bar{W}_t = \frac{W_t}{LDP_s} = +\frac{1}{2} \int_0^{\theta_c} \bar{p} \times \sin \theta \, d\theta \, dz \]

Non-dimensional total load-carrying capacity

\[ \bar{W} = \left[ (\bar{W}_r)^2 + (\bar{W}_t)^2 \right]^{1/2} \]

3.2. End flow rate
The non-dimensional volume rate of flow from the ends of the bearing can be obtained from

\[ \bar{Q}_c = \frac{Q \mu L}{C^2 DP_s} = \int_0^{\theta_c} \left[ \bar{h}^3 \left\{ 1 + \frac{\xi_z}{(1 - \gamma_{sf})} \right\} - 6\bar{h}^2 \xi \phi \tanh \frac{\bar{h}}{2l} \right] - 12\bar{h}^2 \left\{ \frac{\bar{h}}{2l} - 2 \right\} \tanh \frac{\bar{h}}{2l} \left( \frac{\bar{h}}{2l} \right) \right|_{\theta} \frac{d\bar{p}}{d\bar{z}} \, d\theta \]

3.3. Coefficient of friction
Due to cavitation of lubricant beyond \( \theta = \theta_c \), the bearing clearance is separated into two regions viz. non-cavitation region and cavitation zone (\( \theta \leq \theta_c \)).

Non-dimensional frictional force for the non-cavitation portion is given by
\[ \bar{F}_{s_1} = \frac{F_{s_1}}{L C_p s} = \int_0^1 \int_0^{\theta_c} \frac{1}{h} (1 - \xi_{0x}) \frac{\partial \bar{p}}{\partial \theta} \left[ \frac{1 + \frac{\xi_x}{3(1 - \gamma_{xf})}}{2} - \bar{I} \xi_{0x} \tanh \left( \frac{\bar{h}}{2l} \right) \right] d\theta d\bar{z} \]

Non-dimensional frictional force for the cavitation zone (\(\theta_c < \theta \leq 2\pi\)), is given by
\[ \bar{F}_{s_2} = \frac{F_{s_2}}{L C_p s} = \int_0^1 \int_{\theta_c}^{2\pi} \frac{1}{h} (1 - \xi_{0x}) \frac{\partial \bar{p}}{\partial \theta} \left[ \frac{1 + \frac{\xi_x}{3(1 - \gamma_{xf})}}{2} - \bar{I} \xi_{0x} \tanh \left( \frac{\bar{h}}{2l} \right) \right] d\theta d\bar{z} \]

Total non-dimensional frictional force \( \bar{F}_s = \bar{F}_{s_1} + \bar{F}_{s_2} \)

Frictional parameter is given by \( \mu_f \left( \frac{R}{C} \right) = \frac{F_s}{W} \)

**4. Results and discussion**

It is clearly seen that the equations (2.1), (2.2) and (2.3) are controlled by the parameters namely, \(L/D\), \(H/R\), \(\alpha, \beta, \Lambda_s, K_{nc}, K_{nf}(n = x, z)\), \(\gamma_{nc}, \gamma_{nf}(n = x, y, z)\), \(\epsilon_0\) and \(\bar{I}\). Among these parameters, the effect of the most significant parameters, namely eccentricity ratio \(\epsilon_0\), bearing feeding parameter \(\beta\) and percolation factor \(\gamma_{xf}\) on the steady-state characteristics of both isotropic and anisotropic bearings, are studied with the help of graphical plotting. The variation of bearing number \(\Lambda_s\) is taken as the abscissa, the variation of steady-state characteristics is taken as the ordinate, and the influence parameters \(\epsilon_0, \beta, \gamma_{xf}\) are varied on the plotting zone. In this analysis \(L/D, K_{xc}, K_{xf}\) are fixed at 1.0 and \(K_{xc}, K_{xf}\) are fixed at 0.8.

**Figure 2.** Variation of load with \(\Lambda_s\) for different values of \(\epsilon_0\).

**Figure 3.** Variation of flow rate with \(\Lambda_s\) for different values of \(\epsilon_0\).

**Figure 4.** Variation of frictional parameter with \(\Lambda_s\) for different values of \(\epsilon_0\).

Variation in load capacity with \(\Lambda_s\) is shown in Figure 2, when \(\epsilon_0\) is taken as a parameter. \(\epsilon_0\) plays a major role in generating the load capacity of the isotropic and anisotropic bearings. The observation of the figure reveals that load capacity increases with \(\Lambda_s\) and the increasing rate is more rapid with \(\Lambda_s\) for higher values of \(\epsilon_0\) for both type of bearings. This is due to the predominant hydrodynamic action at higher speeds. It is also observed by comparing the isotropic bearing with the anisotropic bearing ones that the effect of anisotropic is insignificant even at higher bearing numbers. The similar observation has been reported in reference [9].

Effect of eccentricity ratio on end flow at various values of \(\Lambda_s\) is demonstrated in Figure 3 for both isotropic bearing as well as anisotropic bearing. An increase in \(\epsilon_0\) increases the end flow rate for any value of \(\Lambda_s\) for both type of bearings. This may be due to higher interfacial pressure gradient at higher eccentricity ratio causing greater fluid flow across the interfacial boundary.

Figure 4 exhibits the variation of frictional parameter with respect of \(\Lambda_s\) for various values of \(\epsilon_0\). Frictional parameter reduces due to the increase in \(\epsilon_0\) for both isotropic and anisotropic bearings.
When $\gamma_{0}$ is increased more, the decreasing tendency of the frictional parameter curves is less predominant at any value of $\Lambda_{s}$.

Influence of bearing feeding parameter ($\beta$) on the load capacity is depicted in Figure 5 for various values of $\Lambda_{s}$. It is found that for higher values $\Lambda_{s}$, load capacity reduces with increase in $\beta$ for both isotropic and anisotropic bearings. This is due to the fact that a higher value of $\beta$ means a higher permeability coefficient resulting in the lower resistance to the flow of lubricant in the fine porous layer. The similar observation has been reported in references [2,9].

The disparity of end flow rate with $\Lambda_{s}$ for various values of $\beta$ is shown in Figure 6 for both isotropic bearing as well as anisotropic bearing. In general, end flow decreases with increases in $\beta$ under high bearing number. But in case of low bearing number, reverse trend is observed.

The effect of $\beta$ on the frictional parameter with respect to $\Lambda_{s}$ is exhibited in Figure 7. It is observed that at higher values of $\Lambda_{s}$, an increase in $\beta$ increases the frictional parameter for both isotropic and anisotropic bearings. This may be due to the significant depletion of load carrying capacity with increase in $\beta$ at the higher range of values of $\Lambda_{s}$.

Influence of the bearing number, $\Lambda_{s}$ on the load capacity of bearing is shown in Figure 8, when the percolation factor ($\gamma_{yf}$) is taken as a parameter. From the figure, it is observed that for a specific value of $\Lambda_{s}$, load capacity decreases due to the increase of $\gamma_{yf}$ for both isotropic and anisotropic bearing. This is due to fact that for a value of $\tilde{l}$, an increase in $\gamma_{yf}$ results in increase in the permeability factor

\[ \beta^2, 45, 50, 25, 20, 45, 15, 15, 45, 35, 15, 40, 25, 40, 30, 50, 50, 15, 30, 10, 5, 10, 10, 45, \]

\[ \text{Figure 5. Variation of load with } \Lambda_{s} \text{ for different values of } \beta. \]

\[ \text{Figure 6. Variation of flow rate with } \Lambda_{s} \text{ for different values of } \beta. \]

\[ \text{Figure 7. Variation of frictional parameter with } \Lambda_{s} \text{ for different values of } \beta. \]

\[ \Lambda_{s} = 0.1, l = 0.3, \epsilon_{0} = 0.4, \gamma_{yf} = 0.3 \]

\[ \text{Figure 8. Variation of load with } \Lambda_{s} \text{ for different values of } \gamma_{yf}. \]

\[ \text{Figure 9. Variation of flow rate with } \Lambda_{s} \text{ for different values of } \gamma_{yf}. \]

\[ \text{Figure 10. Variation of frictional parameter with } \Lambda_{s} \text{ for different values of } \gamma_{yf}. \]
Variation of end flow rate with $\Lambda_s$ is shown in Figure 9, when $\gamma_{yf}$ is taken as a parameter. It is found that end flow increases with $\gamma_{yf}$ for lower values of $\Lambda_s$. But at higher values of $\Lambda_s$, the reverse trend follows.

The effect of $\gamma_{yf}$ on the frictional parameter can be analysed from the Figure 10. An increasing nature of the frictional parameter is observed for any value of $\Lambda_s$ when $\gamma_{yf}$ is increased for both isotropic and anisotropic bearing. Picture also reveals that the increasing rate of frictional parameter is higher in case of anisotropic bearing compare to isotropic bearing for higher values of percolation factor.

5. Conclusions
From the above steady-state analysis dealing with the comparison between the isotropic and anisotropic double-layered porous bearings under coupled stress lubrication, the following conclusions are obtained:

Due to the increase in eccentricity ratio, load carrying capacity and end flow rate increase for both isotropic and anisotropic bearings. Due to the increase in load carrying capacity, bearing stability also enhanced. But the effect of anisotropic is insignificant even at higher bearing numbers.

Due to the increase in bearing feeding parameter, load carrying capacity and end flow rate decrease for both isotropic and anisotropic bearings. Due to the increase in bearing feeding parameter, permeability of fine layer enhanced and load capacity decreases. But the effect of anisotropic is insignificant even at higher bearing numbers.

Due to the increase in percolation factor of fine porous layer, non-dimensional load capacity decreases and frictional parameter increases at any value of bearing number for both isotropic and anisotropic bearings. But the rate of decreasing or increasing is more in case of anisotropic bearings.

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