Finding Better Topologies for Deep Convolutional Neural Networks by Evolution

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Abstract—Due to the nonlinearity of artificial neural networks, designing topologies for deep convolutional neural networks (CNN) is a challenging task and often only heuristic approach, such as trial and error, can be applied. Evolutionary algorithm can solve optimization problems where the fitness landscape is unknown. However, evolutionary algorithm is computing resource intensive, which makes it difficult for problems when deep CNNs are involved. In this paper we propose an evolutionary strategy to find better topologies for deep CNNs. Incorporating the concept of knowledge inheritance and knowledge learning, our evolutionary algorithm can be executed with limited computing resources. We applied the proposed algorithm in finding effective topologies of deep CNNs for the image classification task using CIFAR-10 dataset. After the evolution, we analyzed the topologies that performed well for this task. Our studies verify the techniques that have been commonly used in human designed deep CNNs. We also discovered that some of the graph properties greatly affect the system performance. We applied the guidelines learned from the evolution and designed new network topologies that outperform Residual Net with less layers on CIFAR-10, CIFAR-100 and SVHN dataset.

Index Terms—evolutionary algorithm, deep convolutional neural network, knowledge inheritance

I. INTRODUCTION

Deep convolutional neural networks (CNN) have been one of the most important research topics in recent years, ever since its overwhelming victory in ImageNet challenge 2012 [27], [21]. Part of the success of deep CNNs owns to the improvement of computing facilities and the availability of large annotated datasets [27]. Due to the nonlinearity, analytically studying neural networks is difficult. However, scientists have found various practical techniques to improve the performance of deep CNNs, for example: using a deeper network structure [30], [37], [38]; better optimization techniques [35], [39]; better initialization of trainable variables [6]; more effective activation functions [25], [22]; regularization [28]; batch normalization [14].

The network topology is one of the most important aspects that affect the performance of a deep CNN. Over the last couple of years, numerous new network topologies have been proposed with the target of either improving the accuracy or reducing the computational complexity. One indisputable trend is that deep CNNs are getting deeper and more complicated. AlexNet [18] has 8 layers. VGG Net extends the AlexNet structure to 16 and 19 layers [30]. GoogLeNet uses inception modules and has 22 layers [36]. The inception structure is further enhanced with different variations in Inception Net [38]. Residual Net [8] and DenseNet [11] use cross-layer connections to deal with the gradient explosion/vanishing problems in training deep CNNs. Squeeze Net [13] has a bottleneck structure that efficiently reduces the number of trainable variables in the network. Developing network structures is a difficult task due to the lack of mathematical tools and theoretical understanding of the neural networks. New network structures are always heuristically designed and evaluated by experiments. Finding the optimal topology of deep CNNs is a very difficult task, if by any means possible.

Inspired by nature, evolutionary algorithms find solutions of an optimization problem using mechanisms such as mutation, reproduction and selection [40]. They can be applied to difficult problems when the underlying landscape of the fitness function is unknown. Ever since the emerging of artificial neural networks (ANN), evolutionary algorithms have been used to train a neural network or finding a better topology of neural networks [4]. [23], [34], [29], [33], [45], [44]. Extraordinary computational demand is the major obstacle when applying an evolutionary algorithm to optimize the network topology, because a large number of individuals must be evaluated. Training a deep CNN is a very time consuming and resource-intensive task because of the large model, low convergence speed and complicated hyperparameter tuning. Early stopping is commonly used to reduce the training time [44], [34]. However, this compromise does not solve the entire problem and the number of individuals can be evaluated is normally low. Rather than understanding the topology of a better deep CNN, previous research mainly aimed at finding an optimal network for a specific task. They entangled the network topology optimization, network training and hyperparameter tuning together. To the best of our knowledge, no detailed study has been made to understand the impact of the network topology to the performance of a deep CNN.

In this paper, we present our research of finding better deep CNN topologies for the image classification task using evolutionary algorithm. Our algorithm tries to use as less heuristic knowledge as possible. We employ the concept of knowledge inheritance and knowledge learning, so that the evolution can be executed in a more efficient way. We found that the top-performing networks during the evolution have similar patterns and substructures of the human designed networks. Taking deep CNNs as directed graphs, we studied graph properties of the top-performing topologies. Based on our observations, we find principles and guidance for designing new network topologies.

The rest of the paper is organized as the following: previous works about evolutionary algorithms for ANNs are reviewed in Section II; details of our evolutionary algorithm are in Section III; Section IV shows the results of our evolutionary algo-
rithm applied to the image classification task using CIFAR-10 dataset; Section VI presents the analysis of the top-performing topologies during the evolution and shows the experimental results of some networks designed based on our analysis; and conclusion and the future work are presented in Section VII.

II. PREVIOUS WORK

Ever since the impressive performance in the ImageNet challenge 2012, deep CNNs have gained tremendous attention among researchers [27], [21], [4]. Soon, people found that deep and complex networks perform better than shallow and simple networks. Designing better network topologies has been a major task to further improve the performance. Some successful topologies of human designed networks are: LeNet [20], AlexNet [18], VGG Net [36], Inception Net [38], GoogLeNet [30], Residual Net [8], DenseNet [11] and SqueezeNet [13].

Other than the heuristic approaches, evolutionary algorithms have also been used to find better network topologies. Evolutionary algorithms are based on Darwinian-like evolutionary process, where the following basic rules are applied [40]:

- A large number of individuals that represent possible solutions of a problem are evaluated.
- The survival of an individual is decided by the fitness of the individual.
- The offspring generated from the survived individuals are similar but not identical to their parents.

Evolutionary algorithms can be used to optimize a fitness function when an analytical solution is difficult to achieve.

Evolutionary algorithms have been studied and used in ANNs for many decades [41]. They have been proved to be effective in: finding weights of the connections; finding better network topologies; finding hyperparameters such as learning rule and batch size. Two of the most important algorithms are NeuroEvolution of Augmenting Topologies (NEAT) [34] and HyperNEAT [33] developed by Stanley et al. NEAT is a genetic algorithm that is based on genetic operators such as crossover, mutation and selection. Because there is more than one way to encode a network topology by a genetic representation, crossover of two different genomes that represent a same network topology will cause the competing convention problem—critical information get lost [34]. To deal with this problem, NEAT encodes a network topology using genes with historical markings. These markings record how the network has been constructed from the origin. To prevent the evolution being dominated by local optimal, NEAT promotes innovations with speciation. It defines a measurement of the distance of two genomes by the number of their excess and disjoint genes. Individuals are speciated by a compatibility distance of two genomes.

The survival of an individual is decided by the fitness of the individual. The offspring generated from the survived individuals are similar but not identical to their parents.

Network topology is defined using the concept of modules and blueprints. The structure of the blueprints is based on human-designed network topologies. In [44], Zoph et al. used a RNN network as a controller to generate structure and hyperparameters of a deep CNN. To promote a complex topology, anchor point is introduced such that any two layers can be connected. The authors applied reinforcement learning method to train the controller network. In [27], the method was extended by increasing the number of hyperparameters and generated more complex network topologies. The generated networks achieved the state-of-the-art accuracy on CIFAR-10 and ImageNet datasets. These evolutionary strategies are based on heuristic knowledge gained from previous experiments. However due to the fundamental difficulties of theoretically analyzing a deep CNN topology, one could argue that whether the existing heuristic knowledge is correct and sufficient. Have we already explored enough to find better topologies of deep CNNs?

Our study focuses on the topology of deep CNNs. The evolutionary strategies are designed with the following principles:

- Evolution can be executed with limited computing resources and sufficient number of individuals shall be evaluated.
- Use as less heuristic human knowledge as possible.
- Focus on the topologies of deep CNNs. Hyperparameters that are not related to the topologies shall not be involved during the evolution.

Next we describe the details of our evolutionary algorithm.

III. METHODOLOGY

Let \( y = f(x; \theta) \) be the decision function that a deep CNN represents, where \( x \) is input variable, \( y \) is output variable and \( \theta \) represents the parameters. A metric function \( d(y, \hat{y}) \) defines the distance between the output variable \( y \) and the ground truth value \( \hat{y} \). The target of the learning is to find the optimal \( \theta^* \) that minimizes the expected loss of the output, such that

\[
\theta^* = \arg \min_{\theta} E(d(f(x; \theta), \hat{y})).
\]

For a deep CNN, the network topology determines the function \( f \). Parameter \( \theta \) includes all trainable variables such as weights and bias of the neurons. With a given \( f \), backpropagation is normally used to find \( \theta^* \) by optimizing the loss function. Our target is to find better topologies for deep CNNs using evolutionary algorithm.

A. Network topology

We adopt the canonical artificial neuron model that has been commonly used in ANNs [3]. Let \( x_i \) be the input of a neuron and \( y \) be the output of it. An artificial neuron is modeled as

\[
y = a \left( \sum_{i=1}^{k} w_i x_i + b \right),
\]

where \( w_i \) is the weight for the input \( x_i \), \( b \) is the bias, \( a(\cdot) \) is the activation function and \( k \) is the number of precedent neurons. The summation \( \sum_{i=1}^{k} w_i x_i \) is called propagation since it propagates the output of the precedent neurons to this
neuron. For a convolutional neural network, the propagation is done in the convolutional manner [20]. Instead of seeing all neurons in the previous layer, each neuron sees $k$ of them, where $k$ is the size of a kernel (filter). Note that some deep CNNs do not adopt this commonly accepted model. For example, the propagation is separated from the activation function in pre-activation Residual Net [9].

We model a deep CNN by a directed graph $G(N, E)$, where $N$ is the set of nodes and $E$ is the set of edges. We use a source node to represent the input and a sink node to represent the output of the network. Except the sink node and the source node, each internal node in graph $G$ performs summation, activation and pooling operation to the output of its predecessor nodes. We name the internal nodes convolutional nodes (corresponding to convolutional layer in some literatures [18], [20], [36]). The output of a convolutional node is called a feature map. A convolutional node consists identical neurons—they have the same activation function, the weights and the bias. Each convolutional node has the following model parameters: activation function $a(\cdot)$, number of channels $C$, and the types pooling operation $P$. To focus on the effect of a network topology, we use only ReLU as the activation function, which is usually used in deep CNNs [5]. The types of pooling operation can be “None” or “Max-pooling” (of stride 2). If “Max-pooling” is used in a convolutional node, the size of the output feature map is half of the size of the input feature map.

Except those edges that connect a convolutional node and the sink node, the edges in graph $G$ apply weight operation in the propagation function to the output of the predecessor nodes in a convolutional manner. The size of the kernel is defined by $k$. If the size of the feature map of the predecessor node is different than the input size of the current node, proper stride will be applied. The edges that connect a convolutional node to the sink node always operate in a fully connected manner.

Note, the deep CNN graph is a directed graph with one source node and one sink node. The graph must be acyclic to guarantee that it represents a feed-forward neural network. Every internal node in this graph must be in one of the path between the source node and the sink node.

**B. Mutations**

Our evolution starts from the simplest network topology that contains a source node, a sink node and a convolutional node. Every time an individual is reproduced, a mutation is randomly selected from the predefined mutations and applied to the topology of its parent. The following mutations may happen during the evolution:

1) Double the number of channels of a convolutional node
2) Add a new convolutional node to the graph. Two nodes are randomly selected and linked to the new node. By default, the kernel size of the new edges is 3. Proper stride is applied if the sizes of the feature maps of two connected nodes differ.
3) Connect two nodes in the graph by a new edge. Kernel size is random selected from 1, 3, 5, 7, 9.
4) Prune an edge. After the edge is removed, the nodes and the edges that are not in a path between the source node and the sink node are removed from the graph.
5) Insert a node to an edge. If the node is inserted to the edge that connects the sink node, max-pooling is applied to the predecessor node of the inserted node. The number of the channel of the inserted node is same as its predecessor node.

**C. Reproduction**

NEAT incorporates sexual reproduction where crossover happens on the chromosomes of the parents. Crossover is important to create variations in offspring thus enhance the exploration during the evolution. However, in nature, sexual reproduction appears much later than the asexual reproduction. Early organisms, in particular the single cell organisms, reproduce asexually. The genetic traits of these organisms are simple and a few mutations are sufficient to bring variation. Since our evolution starts from the simplest topology, we mimic the evolution of the early organisms and apply asexual reproduction. Because it is difficult to merge two different graphs to reproduce a new graph, any harsh rules of combining two graphs would dramatically limit the search space. Asexual reproduction is able to search solutions in the whole topology space. However, the offspring may be “close” to its parent thus the evolution can be slow when the graph is complicated.

To reproduce an offspring in our evolution, a mutation is randomly selected from one of the 5 possible mutations defined in Section II-B. If the network generated by a mutation is invalid, for example the source node and the sink node is not connected or the graph is cyclic, another mutation is randomly selected. The procedure repeats until a valid graph is reproduced.

**D. Selection**

Selection is the stage during the evolution in which individuals are selected according to its fitness and the survived individuals reproduce offspring. For the classification task, we evaluate the fitness of each individual by the classification accuracy.

We use stochastic rank-proportional selection strategy as our selection method [15]. The probability that an individual can reproduce is proportional to its rank in the whole population. We applied Boltzmann distribution as the model and the probability mass function is defined as:

$$p(k) = \frac{(1 - e^{-\lambda})e^{-\lambda k}}{1 - e^{\lambda N}},$$

where $k$ is the rank of an individual, $N$ is the size of the population and $\lambda$ is the shape parameter that balance the exploration and exploitation. When $\lambda$ is large, $p(k)$ becomes flat, thus the individuals with low fitness has more chance to be selected and the exploration is enhanced. If $\lambda$ is small, the system will concentrate on the best-performing individuals and will be in favor of exploitation.
E. Knowledge inheritance and knowledge learning

The biggest challenge of using evolutionary algorithm to deep CNNs is the difficulty of evaluating each individual, since the deep CNN has to be trained. Even using a powerful GPU, it normally takes hours or even days to fully train a deep CNN. This requires extremely large computational resource and very long evolving time, since a large number of individuals have to be evaluated. A computing platform with hundreds of GPUs were used in previous research [44], [45]. To deal with this problem, we incorporate the concept of knowledge inheritance and knowledge learning.

During the evolution, the network topology is implicitly encoded and evolved by mutations. We consider the factors except the network topology that impact the performance of an individual as knowledge. The fundamental difference between knowledge and gene traits is that an individual is free to utilize, alter and contribute to the knowledge it gains. We further divide knowledge into two categories: inheritable knowledge and learnable knowledge. As the name suggests, inheritable knowledge is inherited through the evolution and is useful to the offsprings in the same evolutionary branch. The weights, bias and the other learnable parameters of the deep CNN are inheritable knowledge. Learnable knowledge is the information collected from the whole population and can be beneficial to every individual. We treat learning related hyperparameters, such as learning rate, batch size and optimization method, as the learnable knowledge.

We first explain how the inheritable knowledge is applied to an individual using an example. Suppose an individual is reproduced by adding an edge to the graph of its parent. Let $\theta$ be the inheritable knowledge that the offspring individual receives. $\theta$ contains weights, bias and other learnable variables of the deep CNN. Let $f_o(x; \theta_p)$ be the decision function of the parent and $f_o(x; \theta_p, \theta_o)$ be the decision function of the offspring, where $\theta_o$ contains weights of the newly added edge. For a neural network, it is obvious that $f_o(x; \theta_p) = f_o(x; \theta_p, \theta_o = 0)$, since $\theta_o$ appears only in the summation terms of $f_o$. We assume that the gradient descent method is used to find the optimum of Eq. 1. Let $\theta_p^*$ be the optimal solution for the parent individual. For any $\theta_p$, we have $f_o(x; \theta_p^*, \theta_o = 0) \leq f_o(x; \theta_p, \theta_o = 0)$. Thus, the $\theta_p^*$ is a reasonable initialization to optimize $f_o$. With this observation, after the topology of an individual is generated by mutating its parent, we assign the values from its parents as the initialization of the learnable variables of the offspring network. For edges that do not appear in its parent network, the weights are randomly initialized. This can be understood as knowledge inherited from the parent. This concept is similarly to the fine-tune techniques that are often used in deep learning systems [42].

A big challenge of using evolutionary algorithm with deep CNNs is that the performance of a deep CNN is greatly affected by the training related hyperparameters, such as learning rate, batch size, dropout rate and optimization method. In practice, given a network topology, the hyperparameters are either tuned manually [5] or programmatically [1]. Previous research treats these training related hyperparameters as vari-

ables of the fitness function and use evolutionary algorithm to find a better solution. We argue that the performance of a deep CNN is determined by its decision function, but the training related hyperparameters and training procedures affects the speed and the difficulty of finding the optimal or a suboptimal solution. An optimization method should be robust to the surface of a fitness function. Similarly, a good fitness function has a surface that a solution can be easily found by an optimization method. We consider the training related hyperparameters as the knowledge that can be learned from the population and can be taught to an individual. We apply a Bayesian Approach (BA) to learn this information from the population [24], [2] with the assumption that the parameters are mutually independent. Algorithm 1 shows the details of finding the optimal value of a training related hyperparameter. Note that all training related hyperparameters are discrete in our system.

Note that the concepts of inheritable knowledge and learnable knowledge are major differences of our evolutionary strategy comparing to previous evolutionary algorithms used on deep CNNs [34], [33], [8], [7], [45], [23].

F. Evolution

As mentioned in Section III-B, our evolution starts from an individual with the simplest network topology that contains the source node, the sink node and a convolutional node. Each individual runs independently. Because of the randomness involved in training a deep CNN, our evolution allows individuals with identical network topology and training related hyperparameters to be reproduced. Algorithm 1 shows the pseudo-code of an individual. Before an individual finishes, it spouces new individuals according to the computing capacity. The whole evolution procedure is terminated manually when the fitness stops improving.

IV. EVOLVING ON CIFAR-10 DATASET

We used our evolution algorithm described in Section III to find effective CNN topologies for the image classification task. CIFAR-10 database was used because of its popularity in evaluating the performance of deep learning algorithms [17]. The size of the dataset is large enough for the evaluation purpose and the size of each image is appropriate so that computational complexity is acceptable. Due to the limitation of the computing resources that are available to us, we have to apply certain constraints.

Algorithm 1 Bayesian approach of finding training related hyperparameters

initialize equal probability to the values of a parameter
repeat
  • update the distribution from the population
  • sample the value of the parameter using current distribution
  • evaluate the individual
  • if the individual is selected
    – update the population
Algorithm 2 Pseudo-code of the individuals during the evolution

select an individual from the population as the parent based on Eq.5
apply a mutation as specified in Section III-B
determine the training related hyperparameters as described in Algorithm 1
inherit trainable parameters from its parent
train and evaluate the individual
if the individual is selected as described in Section III-D:
  • update the population
spouse new individuals according to the computing capacity

A. Computational constraints

• Each individual is trained with one epoch. Normally hundreds of epochs are required to train a deep CNN. It would be impossible to evaluate a deep CNN if trained with one epoch and random initialization. For evolutionary algorithms, the training of each individual has to be compromised. For example, in [44], the authors trained each individual using 20 epochs. As described in Section III-E, the knowledge inheritance can greatly reduce the difficulty of training a deep CNN. We are able to evaluate a network when it is trained with one epoch. It should also be noted that with this constraint, the evolution encourages topology that learn fast. Same behavior is observed in [23].
• The number of simultaneously running individuals is limited by the computing capacity. However, the size of the population is not affected this constraint. Each individual runs independently and the evaluation result is stored in a centralized database.
• The maximum amount of training time for each individual is fixed. If an individual is not fully trained within the time limit, its accuracy will be evaluated using the unfinished training result. Similar idea is applied in [32]. This constraint also encourages a topology that learns fast.
• The maximum amount of memory required to evaluate each individual is fixed. If an individual cannot be evaluated within the given memory, it will be discarded. This rule promotes a topology that consumes less computing resources.

B. Evolutionary results

We applied the proposed evolutionary algorithm described in Section III to find better network topologies using CIFAR-10 dataset. The size of the population is 1000 and the maximum number of simultaneously running individuals is 200. Each individual uses one CPU core and 16GB memory. Maximum training time for each individual is 12 hours. We trained each individual using the training set of CIFAR-10 and calculated the validation accuracy using the validation set. To balance the accuracy of the training set and the validation set, we evaluate each individual by taking the mean of the training accuracy and the validation accuracy. During the evolution, the system evaluated over 37k individuals.

Fig. 1 shows the training accuracy and validation accuracy during the evolution. The training accuracy and the validation accuracy get saturated as the topologies become very complicated. Close to the end, the system took many hours to evaluate an individual. Because of the constraint of the training time, many individuals had been evaluated even before one epoch was finished.

Fig. 2 shows the top 4 best-performing topologies among all the evaluated individuals.

V. Discussion

A. Statistics of the top-performing networks

We first analyze some graph properties of the top-performing topologies. For comparison, we generated the same number of networks using the proposed evolutionary algorithm except that individuals were selected randomly.

We choose the best 1000 graphs from the CIFAR-10 evolved networks and the last 1000 generated graphs from randomly evolved networks. The average values of the following graph properties are calculated: the number of nodes, the number of edges, graph density, graph algebraic connectivity, average values of the number of channels (convolutional nodes), the longest distance from the source node to the sink node (in short, longest distance), the shortest distance from the source node to the sink node (in short, shortest distance). Refer [26] for details of these graph properties. Note that the longest distance of a deep CNN is also called depth, because the early deep CNN topologies have layered structure. Table I shows the comparison of these graph properties between the CIFAR-10 evolved networks and the randomly evolved networks. It also shows the average number of the generations of the individuals.

B. Network growth pattern

As the statistics in Table I show, the number of the nodes, the number of the edges and the average channels of the CIFAR-10 evolved networks is significantly greater than those of the randomly evolved networks. This indicates that bigger networks—more nodes, more edges and larger number of channels—perform better than smaller networks, thus they have better chance to be selected during the evolution. The
Table I

### Average Values of Some Graph Properties of the CIFAR-10 Evolved Networks and Randomly Evolved Networks

|                  | CIFAR-10 evolved | randomly evolved |
|------------------|------------------|------------------|
| nodes            | 9.66             | 6.51             |
| edges            | 13.1             | 9.12             |
| density          | 0.159            | 0.270            |
| connectivity     | 0.396            | 0.721            |
| channels         | 21.4             | 3.81             |
| longest dist.    | 8.67             | 5.70             |
| shortest dist.   | 5.60             | 3.73             |
| generations      | 33.2             | 11.8             |

Figure 2. Top 4 best-performing topologies. The text in the node shows the node name, pooling flag and the number of channels. The number next to an edge shows the size of the kernel.

Figure 3. Evolution history of the topology (a) in Fig. 2. The numbers below the networks are the numbers of generations.

The number of generations of the CIFAR-10 evolved networks is also clearly greater than the random evolved network. It indicates that there is a dominant trend towards a better network topology. The individuals with a better network topology consistently outperform other individuals. Thus this branch gets continuously evolved and the number of generations grows rapidly.

It should also be noted that the density and the algebraic connectivity of the CIFAR-10 evolved network is smaller than the randomly evolved networks. This indicates that adding new edges does not greatly improve the performance comparing to adding new nodes, in particular in the early stage of the evolution. Next we try to understand the how the network grows during the evolution and try to explain the differences of the other properties.

1) Going deeper is the first priority: Fig. 3 shows the topologies of some ancestors of the best-performing individual during the evolution (individual α in Fig. 2).

It is obvious that the network topology simply evolves by increasing the depth (the longest distance between the source node and the sink node) in the early stage of the evolution. The network has a layered structure that there is almost no cross-layer links. This explains the observations shown in Table I that the longest distance and the shortest distance of the evolved networks are significantly greater than the random evolved networks.
It is found that the network 18 in Fig. 3 is the common ancestor of the top 1000 best-performing individuals. This aligns well with the progress of the research in deep CNN that the accuracy improves by simply using a deeper network structure, for example: LeNet (5 layers, 1990), AlexNet (8 layers, 2012), VGG Net (16 and 19 layers, 2014). However, after network 18 (the 6-layer network), the depth increasing slows down. The evolution goes towards adding more links to increase the density and the connectivity of the networks.

2) Connectivity of a deep CNN: According to Table I, the average generation of the top 1000 individuals is 33.2, much larger than the longest distance of the network which has the value of 8.67. Note that the 6-layer network 18 (the 18th generation) in Fig. 3 is the common ancestor of the top 1000 individuals. This indicates that the depth of the network in the last 15 generations only increased by 2.67, comparing to 4 in the first 18 generations.

The density and the algebraic connectivity of network 18 is 0.142 and 0.198 respectively. Obviously, when a network reaches to a certain depth, the density and connectivity becomes a critical factor that affect the performance. We also noticed that the average of the shortest distance between the source node and the sink node is 5.60, which is close to the depth of network 18. This shows that the shortest distance between the source node and the sink node stays at 6 even the network gets complicated. This interestingly coincides the phenomenon of “six degrees of separation” that is well-known in social networks and other types of networks [26].

Gradient vanishing is a commonly accepted explanation of the difficulty in training deep CNNs [10]. In backpropagation, the weights of the deep CNN is updated using gradient descend and the gradients are backpropagated during training. If a network is deep, the gradients of some weights may shrink and be close to zero thus prevent the network from updating. Different approaches have been invented to overcome or ease the problem. One of the major approaches is the shortcut links between layers, for example, Residual Net [8], DenseNet [11] and FractalNet [19]. The links between the layers actually shorten the path from the source node to the sink node. This effectively avoids the difficulties of training a deep CNN caused by gradient vanishing. The results show that this technique is selected naturally by the evolution.

3) Efficiency of a deep CNN: As stated in Section III, the evolution is in favor of the networks that are light and learn fast. The top-performing networks shows the following patterns that can reduce the computational complexity.

- The first node that is connected to the source node is always a pooling node. This lowers the feature map size and greatly reduces the computational complexity of the network.
- Heavy nodes (with large number of channels) are always on the top of the network. The heaviest node is always above at least two pooling nodes.
- Kernel size 3 and 1 are dominant. Since the computational complexity of the convolution is proportional to the square of the kernel size, small kernel sizes are preferred. These graph pattern are also very commonly used in human designed networks [18], [36], [8], [11], [13].

As stated in Section V-E and V-A, the evolution encourages networks that learn fast. To validate this, we trained the top 10 networks using the training related hyperparameters learned during evolution, random initialization and 400 epochs. All the evolved networks get validation accuracy above 80% within the first 6 epochs, which is clearly faster than 12 epochs that is reported in [23] where a different evolutionary strategy was used.

C. Performance evaluation of the evolved topologies

Because of the limitation of the computing resources during the evolution, the networks were only trained with one epoch. The accuracies shown in Fig. 1 does not reflect the true potentials of the evolved topologies. We took the top 10 evolved networks and completed the training using random initialization and 400 epochs. The training related hyperparameters are learned during the evolution as described in Section III-E. Table II shows the validation error rate of these networks. The values reported are the median values of 5 experiments with random initialization.

Table II shows that the top-performing networks perform similarly with different network topologies. Note that none of the shortest distance of the these graphs is greater than 7.

As discussed earlier, we terminated the evolution when the networks became very complicated. The evolution has not found to the optimal topologies. However, we can design new topologies using the knowledge we gained as we discussed in Section V-A. From the topology 1 in Fig. 2, we take the induced subgraph that contains the source node, $n_1$, $n_2$, $n_4$, $n_6$, $n_7$ and $n_8$ as the building block. To keep the subgraph symmetric, we set the channel of the source node and $n_1$ to be 64. Fig 4 shows the building block. We construct the following networks:

- **EVO-44**: We stack 7 building blocks by connecting the output of a lower block to the input of the upper block. Pooling is applied to output node of the 2nd, the 4th and the 6th block. The depth of this network is 44, thus named EVO-44. The shortest distance of the graph is 23.
- **EVO-44a**: We stack 7 building blocks similarly as EVO-44. To reduce the distance between the source node and the sink node, we add edges to connect $n_7$ to the $n_8$ in the adjacent upper block, and $n_1$ to $n_6$ in the adjacent upper block. If the sizes of the feature map of the two connected nodes are different, a suitable stride is used. With these extra edges, the shortest distance of the graph 13.
- **EVO-44b**: We stack 7 building blocks in the same way as EVO-44. We connect $n_7$ to all $n_8$ in the upper blocks and $n_1$ to all $n_6$ in all the upper blocks. The shortest distance of the graph is 5.
- **EVO-91**: We stack 15 building blocks in the same way as EVO-44. The pooling is applied to the output node of the 4th, the 8th and the 12th block. The depth of this deep CNN is 91. The shortest distance of the graph is 47.
- **EVO-91a**: We build the deep CNN similar to how EVO-44a is built except that 15 build blocks are used. The pooling is applied to the output of the 4th, the 8th and the 12th block. The shortest of the graph is 25.
### Table II

| Topology | Depth | Shortest Dist. | Error Rate |
|----------|-------|----------------|------------|
| 1        | 10    | 6              | 7.84%      |
| 2        | 9     | 7              | 7.72%      |
| 3        | 8     | 4              | 7.50%      |
| 4        | 8     | 6              | 7.63%      |
| 5        | 9     | 7              | 7.37%      |
| 6        | 9     | 7              | 7.65%      |
| 7        | 9     | 6              | 7.54%      |
| 8        | 8     | 4              | 8.07%      |
| 9        | 9     | 8              | 7.41%      |
| 10       | 9     | 9              | 7.66%      |

### Table III

|                      | CIFAR-10 | CIFAR-100 | SVHN |
|----------------------|----------|-----------|------|
| NIN                  | 8.8% [43]| 33.7% [43]| 2.35 [13]|
| DSN                  | 8.2% [43]| 34.6% [43]| 1.92 [13]|
| Highway              | 7.5% [13]| 32.4% [43]| -    |
| All-CNN [31]         | 7.25%    | 33.71%    | -    |
| ResNet-56 [8]        | 6.97%    | -         | -    |
| ResNet-110 [12]      | 6.43%    | 25.2% [13]| 2.01 [13]|
| pre-act-ResNet-110 [9]| 6.37%   | -         | -    |
| DenseNet-100 [12]    | 3.74%    | 19.3%     | 1.59%|
| EVO-8                | 7.84%    | 30.0%     | 2.47%|
| EVO-44               | 5.99%    | 29.0%     | 2.03%|
| EVO-44a              | 5.74%    | 26.8%     | 1.99%|
| EVO-44b              | 5.65%    | 25.8%     | 1.97%|
| EVO-91               | 6.35%    | 34.0%     | 2.02%|
| EVO-91a              | 5.71%    | 28.7%     | 1.96%|
| EVO-91b              | 5.19%    | 24.6%     | 1.85%|

- **EVO-91b**: We build the deep CNN similar to how EVO-44b is built except that 15 build blocks are used. The pooling is applied to the output of the 4th, the 8th and the 12th block. The shortest distance of the graph is 5.

![Basic building block for our designed networks](image)

We evaluated our designed networks on CIFAR-10, CIFAR-100 and SVHN dataset. On the two CIFAR datasets, the standard image augmentation (flipping and clipping) is used. The maximum number of epoch is 400. Training related hyperparameters are learned during evolution as specified in Section III-F: Adam [16] optimization method is used; initial learning rate is 0.0005; no dropout is used; no weight decay is used. Learning rate is reduced by a factor of 5 on every 100 epochs. On SVHN dataset, no data augmentation is applied. The maximum number of epoch is 180. Learning related hyperparameters are same as those used for CIFAR datasets.

Table III shows the error rate of our designed networks tested on CIFAR-10, CIFAR-100 and SVHN datasets. All values reported on our topologies are the median values of 5 experiments with random initialization. Error rate of some other human designed network are also shown for comparison. The results in Table III show that extra links that connect the nodes between different blocks improve the performance. Due to the large shortest distance between the source node and the sink node, the accuracy of EVO-91 is clearly worse than the other networks. The EVO-91b achieves the best performance by increasing the depth of the network and keeps the shortest distance of the source node and sink node low.

Comparing to Residual Net model, the constructed topologies achieve better results with less layers. It should be noted that DenseNet gives better performance with a much higher graph density. However, it should be noted that the shortest distance between the source node and the sink node of the DenseNet-100 is 6, which is also coordinate to the top-performing network during the evolution as discussed in Section V-A.

### D. Limitations

First, even though the knowledge inheritance greatly alleviates the difficulties of training a deep CNN, computing resource is significantly limit the scope of our evolution. We had to stop the training of each individual at very early stage (one epoch). This becomes more severe when networks are getting complicated.

Second, our evolution is based on the canonical neuron model. This is sufficient to study the network topology, but not likely to find the best-performing deep CNN. For example, previous research showed that pre-activation structure, which separates the propagation and activation in Eq. 2, can greatly improve the performance of Residual Net [2]. Zoph et al. evolved a network with many different neuron models and achieved the state-of-the-art results on CIFAR-10 and ImageNet dataset [45].

Our evolutionary algorithm evolves slowly when network topology is getting big and complicated. For example, the changes generated from the mutations defined in Section III-B is relatively minor when the network is big. More advanced mutations and other techniques are required to increase the exploration when the evolution reaches to a certain stage.

### VI. Conclusion

In this paper, we present an evolutionary algorithm to find better deep CNN topologies. Our evolutionary strategy is based on asexual reproduction. One of the 5 predefined mutations is applied to a parent to generate an offspring. Boltzmann distribution is used for the stochastic rank-proportional selection strategy. To deal with the difficulty of training deep CNNs, we exploit the concept of knowledge inheritance and
each individual is trained with one epoch. Training related hyperparameters are learned during the evolution in virtue of the concept of knowledge learning.

We applied the proposed algorithm to the image classification problem using CIFAR-10 dataset and studied the topologies that performed well during the evolution. Our studies verify some generally accepted techniques in human designed networks. The studies also show that the shortest distance from the source node to the sink node, graph density and graph connectivity are also major factors that affect the performance of a deep CNN. Based on these findings, we designed new topologies and evaluated their performance using CIFAR-10, CIFAR-100 and SVHN datasets. The experiment results confirm the efficiency of the guidelines we learned from the evolution. Our designed topologies can outperform Residual Net with less layers.

We also discussed the limitations of our evolution experiments. Although our evolutionary algorithms employ different techniques to speed up the training of deep CNNs, computing resource still greatly hinder the evolution since complex topologies cannot be effectively evaluated. Also better evolutionary strategy should be considered when the topologies are getting complicated, similar to nature where complex organisms always adapt complicated reproduction mechanism. This will be the main direction of our future research. We make the software package and all the topologies that had been evaluated during the evolution publicly available\footnote{\texttt{<URL to be released later>}.}. We hope our research will be beneficial for the society to get better understandings of the topologies for the deep CNNs and design more efficient networks.

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