Near Infrared and Visible Image Registration Using Whale Optimization Algorithm

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ABSTRACT

This paper reports the use of a nature-inspired metaheuristic algorithm known as ‘whale optimization algorithm’ (WOA) for multimodal image registration. WOA is based on the hunting behaviour of humpback whales and provides better exploration and exploitation of the search space with small possibility of trapping in local optima. Though WOA is used in various optimization problems, no detailed study is available for its use in image registration. For this study, different sets of NIR and visible images are considered. The registration results are compared with the other state-of-the-art image registration methods. The results show that WOA is a very competitive algorithm for NIR-visible image registration. With the advantages of better exploration of search space and local optima avoidance, the algorithm can be a suitable choice for multimodal image registration.

KEYWORDS

Convergence, Image Registration, Meta-Heuristic Algorithms, Near Infrared, Particle Swarm Optimization, Whale Optimization Algorithm

1. INTRODUCTION

Image registration is a process to spatially align two geometrically displaced images, one of them called the source image and the other reference image, so that the corresponding points assume the same coordinates. Registration of images plays a vital role in various fields ranging from medical diagnostics, weather forecasting, crop monitoring, military surveillance to computer vision and artificial intelligence. There have been a number of established theories and techniques described in previous references for image registration (Moigne, Netanyahu, & Eastman, 2011; Brown, 1992; Goshtasby, 2012; Zitova & Flusser, 2003). However, with increase in variety, complexity and heterogeneity of images and in the view of the vast amount of information available, the need for newer of image processing algorithms has become necessary and this field is continuously evolving (Brown, 1992). In addition to the classical analytical techniques like correlation based or frequency plane-based methods (Goshtasby, 2012), optimization based iterative methods also takes a very important part in image registration.

When two images only have translational, rotational or affine differences, an intensity-based rigid transformation algorithm can be used for registration, for example principle axes registration or multiresolution registration methods. However, in presence of additional changes other than the geometric mismatch, for example, topological, non-rigid methods, such as adaptive transformation methods are used. In such cases, a registration may be considered optimal if a criterion of similarity...
or dissimilarity measured is defined, and the optimization algorithm either maximizes (similarity) or minimizes (dissimilarity) the measurement criterion. It is not possible to prescribe a universal method applicable to all types of image registration tasks. This is due to variation in image acquisition process, as the images can be acquired at different times (multi-temporal), from different viewpoint (multi-view) or from different sensing devices (multi-modal) (Argyriou et al., 2015).

For non-rigid image registration, in addition to Gradient-descent algorithms, stochastic optimization methods have been proposed which worked well on multimodal and multi-resolution images (Klein, Staring, & Pluim, 2007). In recent years, research in metaheuristic approaches are gradually growing due to its flexibility and ease of applicability. Swarm Based meta-heuristic algorithms, imitating the behavior of the swarming of a species occurring in nature are frequently being used in the optimization problems in diverse fields of research due to their simplicity and easier implementation than strictly analytical algorithms. Some of the examples in this category are Particle Swarm Optimization (PSO) algorithm (Eberhart & Kennedy, 1995), Ant Colony optimization algorithm (Dorigo & Di Caro, 1999), Cuckoo Search optimization algorithm (Gandomi, Yang, & Alavi, 2013), Firefly algorithm (Yang, 2009) etc. Among all the swarm-based algorithms PSO based image registration algorithms have been reported extensively in literature (Wachowiak et al., 2004; Pramanik, Dalai & Rana, 2015; Wang & Bian, 2012; Madaia & Pournami, 2019) though there have been very few attempts of using other swarm-based algorithms for image registration task (Daniel & Anitha, 2016; Zhang et al., 2013).

In 2016, a new meta-heuristic algorithm known as “Whale Optimization Algorithm (WOA)” was proposed by S. Mirjalili and A. Lewis (Mirjalili & Lewis, 2016), which is based on the hunting behavior (known as bubble-net feeding method) of humpback whales. WOA provides better exploration and exploitation of the search space and deals more effectively with the local optima trapping problem. WOA has been used successfully in various branches of engineering and physics since its inception (Aljarah, Faris, & Mirjalili, 2018; Kaveh, & Ghazaan, 2017; Oliva, El Aziz, & Hassanien, 2017; Mehn & Mirjalili, 2018; Pham et al., 2020). There are several variants of WOA including its combined implementation with other nature-based algorithms (Bentouati, Chaib, & Chettih, 2016; Hu, Bai & Xu,2016); Kaur & Arora, 2018; Aziz, Ewees, & Hassanien, 2018.; Singh & Hachimi, 2018; Bozorgi & Yazdani,2019.; Ling, Zhou & Luo, 2017; Abdel-Basset, Manogaran, El-Shahat & Mirjalili, 2018) To the best of the authors’ knowledge, its use in multi-sensor image registration problem has not been reported in literature till date.

In this study, WOA has been used for the Near Infrared and visible image registration. Near infrared (NIR) image registration ranging from 750 nm to 1400 nm is a growing area of research in many fields including remote sensing and biomedical applications. NIR images possess enhanced contrast between darker and lighter regions. Image registration of NIR and visible images of the same scene can provide more information to an image analyst. For example, the distinct reflective properties of infrared images are used to find the healthy vegetation or the presence of a water body in a remote sensing scene. It is useful in medical field where the registration of two different sensor images can provide detailed information about a patient body part. The paper is divided into following sections. Section 2 explains the concept and selection of similarity metric/fitness function in image registration. Section 3 explains the WOA in detail for application in image registration. Section 4 describes the results of WOA algorithm for several sets of visible and NIR images. Section 5 summarizes the main findings of the study.

2. MUTUAL INFORMATION FUNCTION FOR IMAGE REGISTRATION

If $I$ and $I'$ are the geometrically displaced images used for image registration then the registration problem can be defined as a transformation defined as,
Where \((x, y)\) and \((x', y')\) are the intensity values in the images \(I\) and \(I'\) respectively. Here, \(t\) is a transformation function which is problem oriented. For mutually translated and rotated misaligned images \(t\) is defined as:

\[
t(X,Y,\theta) = \begin{bmatrix} 
\cos\theta & \sin\theta & t_x \\
-\sin\theta & \cos\theta & t_y \\
0 & 0 & 1 
\end{bmatrix}
\]  

Where \(\theta\) is a rotational parameter and \(t_x\) and \(t_y\) are translational parameters along respective directions.

To register two different images, precise information about \(t_x\), \(t_y\) and \(\theta\) is required. All registration techniques broadly built upon the following components: feature space, similarity metric, search space and search strategy (Brown, 1992). The feature space is the representation of the data that is to be used for registration. The similarity metric determines how the matches are evaluated. Choice of similarity metric plays a very important role in the performance of an image registration process.  

Correlation-based methods, which depends on the intensity values of images provides very effective results for same sensor image registration. However, this method is inaccurate for the multimodal image registration problem because the images produced by different sensors may have different range of intensities. On the other hand, mutual information (MI) based similarity measure depends upon the joint probability distribution of two images and has been proved to be a powerful and robust tool. In multimodal image registration MI between two images can be used as a similarity measure providing the common information shared by both the images (Pluim, Maintz & Viergever, 2003; Maes et al., 1997). Higher the value of MI, better is the alignment between the images. For two images \(A\) and \(B\), the value of mutual information can be calculated by the probabilities of the two images. Mutual information is defined as,

\[
M(A, B) = H(A) + H(B) - H(A, B)
\]  

Where \(H(A)\) and \(H(B)\) are the entropy of the individual images and are given by,

\[H(A) = -\sum_a p_A(a) \log p_A(a)\] \hspace{1cm} (4a)

and,

\[H(B) = -\sum_b p_B(b) \log p_B(b)\] \hspace{1cm} (4b)

\(p_A(a)\) and \(p_B(b)\) are the measure of occurrence of the \(a^{th}\) and \(b^{th}\) grey levels respectively. Here, \(H(A, B)\) is the joint entropy of images \(A\) and \(B\) and \(H(A)\) and \(H(B)\) are the individual grey level probabilities. \(H(A, B)\) can be calculated by the following equation:
In this equation, \( p \) is defined as the joint probability distribution of images A and B (Wang & Bian, 2012). In multimodal image registration, the MI value between the images is optimized with the help of an optimization algorithm. In the next section this process is explained.

The search space generally consists of all the classes of transformation and their combinations from which one has to find the optimal transformation that perfectly aligns the images. Each transformation candidate can be evaluated by using the specific similarity measure, which in the present case, is MI. The problem becomes more complicated if more types of distortion other than translation and rotation are present, forcing an increase in the search space size. Design of a registration method is based on a search strategy that tries to reduce the computational cost associated with feature space dimensions and similarity measures. Instead of examining all neighborhood points in the feature space to refine the registration parameters, a nature-based metaheuristic method may be used to refine the parameters without examining all possible values. The randomness incorporated into the search improves the diversity of a metaheuristic algorithm and enables it to jump out of local optima. This increases the efficiency of the algorithms in the search, and their rate of convergence on the global optimum is higher. In the following section the proposed whale optimization-based image registration algorithm is explained.

3. WHALE OPTIMIZATION ALGORITHM FOR IMAGE REGISTRATION

WOA has been used successfully in various optimization problems since its inception [16-19] but so far, its use in image registration problem has not been reported (Aljarah, Faris, & Mirjalili, 2018; Kaveh & Ghazaan, 2017; Oliva, El Aziz, & Hassanien, 2017; Mehne & Mirjalili, 2018; Pham et al., 2020). WOA is a very effective optimization algorithm which provides optimum balance between random and deterministic parts during the algorithm run. For the success of any optimization algorithm proper balance of these two processes is essential otherwise the optimization result may be trapped in local minima (Chen et al., 2012). In the initial phase the algorithm vigorously explores in all directions for all possible solutions in the search space while in the later part it gradually reduces the exploration space around the probable solution. In addition, the search process includes an exploration phase where a random variation in the search agent is included to avoid the local minima problem.

A metaheuristic algorithm searches for the global optimum in the N dimension solution space, where N is the parameters of the solution. In the context of the image registration problem defined in this paper, the search space is three dimensional (Eqn.1), i.e., the three parameters are translation in two-dimensional plane (x and y) and rotation (\( \theta \)). Any arbitrary value of x, y and \( \theta \) defines a possible solution. Global optimum will be the transformation of the reference image that matches the position and the orientation of the target image correctly, which is ideally when the predator meets the prey in the WOA. To construct the mathematical model of image registration, let us consider the number of solutions (predators) that exists in the search space be ‘S’. Thus, each predator would refer to a version of the reference image, arbitrarily rotated and positioned.

The mathematical model of the WOA algorithm has three successive steps. The initial part of the hunting behavior is ‘encircling’ the prey followed by ‘attacking’ the prey and third is ‘exploration’ which introduces occasional randomness in search.

From Eqns. 3, 4 and 5, the MI between the target image and each of the candidate solution images are calculated. Since the optimum solution is not known, the WOA algorithm takes the solution \( P_{\text{best}} \) with highest MI as the current best candidate solution. The rest of the (S-1) candidate solutions are updated according to the following equations,

\[
H(A, B) = -\sum_{a,b} p_{A,B}(a,b) \log p_{A,B}(a,b)
\]
\[ \bar{P}(t+1) = \bar{P}_{\text{best}}(t) - \bar{A}(|\bar{C}.\bar{P}_{\text{best}}(t) - \bar{P}(t)|) \]  

(6)

With

\[ \bar{A} = \tilde{a} (\bar{C} - 1) \]  

(7)

Where, \( P(t) \) indicates the solution at \( t \)th instant, \( P(t+1) \) is the updated solution, \( P_{\text{best}}(t) \) is the best candidate solution so far; \( \bar{A} \) and \( \bar{C} \) are coefficient vectors related through Eqn. (7), \( \tilde{a} \) linearly decreases from 2 to 0 over the course of iterations and \( \bar{C} \) is a random vector in \([0, 1]\); ‘.’ indicates point to point multiplication. So, in the initial phase, the candidate solutions adjust their positions around the best candidate solution.

The second phase of hunting is called the encircling phase, which has two simultaneous approaches described as (i) Shrinking encircling mechanism and (ii) Spiral updating of position. In the case of image registration problem, the randomly generated candidate images slowly orient and shifts themselves to the best candidate image. It can be seen from Eqn. (6) that a decrease in value of \( \tilde{a} \) from 2 to 0 will decrease the distance between the predator and the prey. Setting random values for \( \bar{A} \) in \([-1, 1]\), each candidate solution is generated anywhere between its previous position and the current \( \bar{P}_{\text{best}} \) solution by making use of Eqn. (6) and (7) as shown in fig. 1(a).

The equation governing the spiral updating is given by,

\[ \bar{P}(t+1) = \overrightarrow{D'} e^{bl} \cdot \cos(2\pi l) + \bar{P}_{\text{best}}(t) \]  

(8)

Where, \( \overrightarrow{D'} = |\bar{P}_{\text{best}}(t) - \bar{P}(t)| \), \( b \) is a constant defining the shape of spiral and \( l \) is a random number in the range of \([-1, 1]\). In this case the candidate solution updates itself along a spiral path joining itself with the best solution as shown in fig. 1(b).

The complete hunting pattern of the predator is mimicked by the equations (6) and (8) in the following manner,

\[ \bar{P}(t+1) = \bar{P}_{\text{best}}(t) - \bar{A}(|\bar{C}.\bar{P}_{\text{best}}(t) - \bar{P}(t)|) \]  

\[ \overrightarrow{D'}. e^{bl} \cdot \cos(2\pi l) + \bar{P}_{\text{best}}(t) \]  

\[ p < 0.5 \]  

\[ p > 0.5 \]  

(9)

Where \( p \) is a random number between \([0, 1]\).

The third phase of the hunting mechanism is the exploration phase where a random search for the prey is initiated. This is a key step of the algorithm as this step, the solution will converge to the initial best search agent. Use of \( \bar{A} \) with random values outside the range \([-1, 1]\) forces the candidate solution to move away from the current \( \bar{P}_{\text{best}} \) solution and search for a global optimum. This step can be mathematically expressed as follows:
Where, $\tilde{P}_{\text{rand}}$ is a random transformation of the reference image.

The flow chart of the proposed algorithm for multimodal image registration is shown in fig. 2. The algorithm starts with the random generation of whale population in the search space. As the image registration problem is a maximization problem because the maximum value of fitness function (MI) will provide the best registration results, the initial value of fitness function is taken to be zero. The algorithm generates transformed reference images by using the positions of each whale. The fitness value between these images and the reference image is computed and the best value is taken as the new fitness value. The position of the search agents is updated accordingly. The parameters of the WOA are updated and the new positions of the particles are selected according to constraints. The whole process is continued till the termination criterion is satisfied. The position values of the best particle correspond to the best fitness value is used to generate a transformed source image which is the registered image.

The pseudo code of the proposed WOA is shown as follows:

**Algorithm** Pseudo-code of the WOA algorithm.

Initialize the search agent/Whale Population $X_n$
Set Max iterations, dimensions, upper and lower bounds
Set Initial Fitness for best search agent $M_0=0$

while (k<Maximum Iterations)
Create $X_n$ transformed images by source image using whale populations
Compute fitness between transformed images and reference image $M_n$
if ($M_n>M_0$)
$M_0=M_n$
else
Update Search agent positions
endif
for each search agent
    Update $A,l,a,c$
    Generate $p$
    if ($p<0.5$)
        if ($A\geq1$)
            Update the positions by random search agents
        else if ($A<1$)
            Update the positions by best search agents
        else if ($p\geq0.5$)
            Update the positions by best search agents
    end
end

$$\tilde{P}(t+1) = \tilde{P}_{\text{rand}} - \tilde{A} \left(\frac{\tilde{C} \cdot \tilde{P}_{\text{rand}} - \tilde{P}(t)}{\lVert \tilde{P}(t) \rVert} \right)$$ (10)
Update the positions by spiral equations

end
end
k=k+1
end
return \( M_0 \)
4. EXPERIMENTAL VERIFICATION

4.1 Algorithm Validation

The proposed algorithm was used to register different sets of NIR and visible images. The images were resized to 300×300 pixels from the available image dataset and were converted to gray scale for ease of computation (Brown & Süsstrunk, 2011). Random translational and rotational misalignment in images was introduced to evaluate the performance of the proposed algorithm. Fig. 3 shows six sets (numbered from I to VI) of visible and the NIR images respectively. For all the experiments, a population size of 30 was considered and 500 number of iterations were used for WOA. Standard parameters are used for the experiment (Mirjalili & Lewis, 2016). As explained earlier, Mutual Information (MI) was used as an objective function to measure the similarity between images. The registration results are presented in Fig. 3, where the overlapping of the fixed image and the moving images are shown. Many times, a real image consists of noise due to the sensor and the image capturing environment rendering the registration process quite difficult. To evaluate the performance of the proposed algorithm, a second experiment has been performed where Gaussian noise was introduced in the infrared image and tested against the visible image. The image sets (VII and VIII) with registration results are presented in Fig. 4. It shows the images successfully registered with the Whale Optimization Algorithm.

Fig. 5 shows the convergence curve of the fitness function (MI) with number of iterations. Though each time the algorithm was run for 500 iterations, it can be observed from the table that the maximum value of the metric is achieved within about 100 iterations. Further, it can be seen that the MI function attains different saturation values depending on the similarity between two images in the set.

4.2 Comparison With Standard Algorithms

To evaluate the accuracy of the registration results, we compared our results with well-known state of the art image registration methods such as Speeded-up Robust Feature (SURF) (Bay, Tuytelaars & Van Gool, 2006), Features from Accelerated Segment Test (FAST) (Muja & Lowe, 2009), Binary Robust Invariant Scalable Key point (BRISK) (Leutenegger, Chli & Siegwart, 2011) and Harris feature detector (Harris & Stephens, 1988).

These methods utilize the distinctive image features which are locally invariant to geometric transformations and photometric changes. The SURF method uses the determinant of Hessian Blob detector and Haar wavelet response around the point of interest to detect the features. FAST corner detector uses a Bresenham circle of a fixed radius to classify a corner. BRISK method uses a saliency criterion to detect the point of interest. Octave and intra-octave layers of the scale are used to detect the key points. Harris’ feature detector uses the differential of the corner score with direction and detect the corners. For each of these algorithms number of iterations was considered to be 500 and same set of the images were used.

The comparative evaluation of the mutual information function of the final registered image is presented in Table 1. A high mutual information value is considered as a better registration results and signifies more accuracy in the result. It is apparent that WOA performs well for all the image sets in comparison to the other feature-based image registration methods. The WOA algorithm successfully registered images in this situation and performs better than the other well-known feature-based image registration methods. It is to be noted that presence of noise in set VII and VIII substantially decreases the MI value for all algorithms. However, the proposed WOA performs better than all other algorithms.

5. CONCLUSION

Near infrared imaging is gaining importance in the research community due to sharp contrast between visible and NIR images. An image registration between NIR and visible image provides additional information about a given image scene. Meta-heuristic algorithms are becoming very popular in
image registration process as there is no requirement for feature selection and feature mapping in this case. Whale optimization algorithm is a relatively new meta-heuristic algorithm which is based on the hunting behavior of humpback whales. The algorithm is more robust against local optima trapping which is a major concern for any meta-heuristic algorithm. In this work, an automatic
Figure 4. Image Registration Results for noisy infrared image: (a) & (d) visible images; (b) & (e)Near-infrared images with noise, (c) & (f) registration results.

Figure 5. Convergence of Fitness function with number of iterations

Table 1. Registration Results

| Image Set | SURF     | FAST     | BRISK    | HARRIS   | WOA     |
|-----------|----------|----------|----------|----------|---------|
| I         | 1.4601   | 1.5629   | 1.3144   | 1.552    | 1.6502  |
| II        | 1.0925   | 0.9904   | 1.1122   | 0.9513   | 1.158   |
| III       | 1.5557   | 1.5436   | 1.5381   | 1.5793   | 1.5795  |
| IV        | 1.4369   | 1.4012   | 1.4872   | 1.4597   | 1.496   |
| V         | 1.6148   | 1.664    | 1.5788   | 1.7164   | 1.7574  |
| VI        | 1.7877   | 1.787    | 1.7962   | 1.7741   | 1.7937  |
| VII       | 1.0938   | 1.0645   | 1.0859   | 1.0857   | 1.1093  |
| VIII      | 0.7975   | 0.8511   | 0.6701   | 0.8588   | 0.8823  |
image registration algorithm was developed using whale optimization algorithm for the multimodal image registration with relatively small population of searching agents. The algorithm was applied on different sets of NIR and visible images including images with noise. The results are evaluated and compared with other well-known image registration methods used in the literature. From the experiment outcomes and comparative study, it is evident that WOA effectively demonstrates the capability to register multimodal images. Therefore, we conclude that WOA is a very competitive optimization algorithm for multimodal image registration.
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