A simplified method for evaluating the liquefaction of sandy soil confined by a lattice-type deep mixing wall

Akira Ishikawa i), Yasuhiro Shamoto ii) and Takumi Kimura iii)

i) Senior research engineer, Institute of technology, Shimizu corporation, 3-4-17 Echujima, Kotu-ku, Tokyo 135-8530, Japan.
ii) Research fellow, Institute of technology, Shimizu corporation, 3-4-17 Echujima, Kotu-ku, Tokyo 135-8530, Japan.
iii) Group Leader, Design and proposal supervising division, Shimizu corporation, 2-16-1, Kyobashi, Chuo-ku, Tokyo 104-8370, Japan.

ABSTRACT

A simplified method for evaluating the liquefaction of sandy soil confined by a lattice-type deep mixing wall is proposed in this paper. The method calculates the cumulative excess pore water pressure from the initial shear strain using an equivalent shear-stiffness model created using a homogenization method. The proposed method uses parameters whose physical meaning is clearly defined, and can evaluate not only the effect of variations in the external forces but also the physical properties of the ground. This paper first explains the proposed idea and then presents the calculation procedure using an actual site plan.

Keywords: liquefaction, homogenization method, cumulative damage, ground improvement, grid shape

1 INTRODUCTION

Lattice-type soil improvement methods prevent liquefaction by creating a rigid lattice-shaped mixing wall with improvements using cement in the ground. Former methods do not consider the strength of the seismic forces. Thus, Taya et al.(2008) have proposed a new design method to solve this problem. However, the generality of their method, which is based on a two-dimensional (2D) numerical analysis under certain ground conditions, is difficult to prove.

In this paper, we propose a simple design method for lattice-type ground improvement to prevent soil liquefaction Ishikawa et al.(2012). The method uses the relationship between initial shear strain and excess pore water pressure based on the concept of the degree of cumulative damage during liquefaction. To easily calculate the initial shear strain of the interstitial ground after improvement, we used the homogenization theory Terada and Kikuchi(2003). Because the proposed method uses parameters whose physical meaning is clearly defined, the differences in ground conditions and external forces due to the intensity of an earthquake can be assessed reasonably.

2 PROPOSED METHOD

2.1 Experimental results of liquefaction

Fig. 1 shows typical results obtained from a liquefaction element test Zhang et al.(1997). To apply a shear stress \( \tau / \sigma_v' \) to a specimen as a constant repeated load, excess pore water pressure \( \Delta u \) is gradually accumulated until liquefaction is reached. While shear strain \( \gamma \) is initially very small, it increases rapidly as liquefaction is reached. In this paper, this small shear strain before excess pore water pressure accumulates and which depends on the stiffness of the soil skeleton, is defined as “initial shear strain \( (=\gamma_1) \)”. Initially, our method obtains \( \gamma_1 \) and the resulting \( \Delta u \) relationship in each sand layer in-situ at the site. The following sections explain the calculation method.

![Fig. 1. Typical liquefaction element test results.](image)

a) Shear stress–strain relationship of alluvial sand

As \( \gamma_1 \) is the shear strain before the excess pore water pressure rises, it can be expressed by the constitutive
law of sand as shown in Eq. (1).

\[ \gamma = \frac{\tau}{G_0} \left( 1 + \left( \frac{2}{\gamma G_0} \right)^{\beta} \right) - 1 \]

(1)

Here, \( h_{\text{max}} \) is the maximum attenuation constant in typical alluvial sand ranging from 0.2 to 0.25, \( G_0 \) is the initial shear stiffness, and \( \gamma_{\text{c}} \) is the criteria shear strain.

b) Relationship between the repeated shear stress ratio and the degree of cumulative damage during liquefaction

The liquefaction strength can be used to express the relationship between \( \tau_\text{d}/\sigma' \) and the number of repetitions \( N_l \) before liquefaction. \( R_{20} \) is the \( \tau_\text{d}/\sigma' \) when the number of repetitions to reach liquefaction is 20. In the case of alluvial or landfill loose sand, \( R_l \) and \( N_l \) have an almost linear relationship on both logarithmic axes as shown in Fig. 2. This relationship can be expressed by the following equation.

\[ R_l = R_{20} \times \left( \frac{N_l}{20} \right)^k \]

(2)

In the above equation, \( k \) is an experimental constant, usually ranging from -0.1 to -0.25 in loose sand. From Eq. (2), we can calculate \( N_l \) for a given \( R_l \). Because liquefaction approaches \( 1/N_l \) in one iteration, the degree of cumulative damage \( R_n \) can be expressed as the following equation.

\[ R_n = \frac{20}{N_l} \]

(3)

c) Relationship between the degree of cumulative damage and the excess pore water pressure ratio

The relationship between \( R_n \) and \( \Delta u/\sigma' \) can be expressed by the following equation according to De Alba et al. (1976).

\[ \frac{\Delta u}{\sigma'} = \frac{2}{\pi} \cdot \text{Arc} \sin \left( R_n \frac{1}{\sigma'_{\text{eq}}} \right) \]

(4)

Here, \( \sigma'_{\text{eq}} \) is an experimental constant. In the case of loose sand its value is approximately 0.7.

d) Change in excess pore water pressure under repeated shear strain

As Eq. (2) defines the \( R_l \) and \( N_l \) relationship, we can specify the \( \gamma_l \) and \( N_l \) relationship through Eq. (1). Also, the relationships between \( N_l \) and \( R_n \) and \( \Delta u/\sigma' \)

are defined by Eqs. (3) and (4), respectively. Using Eqs. (1) through (4), we can specify the \( \gamma_l - \Delta u/\sigma' \) relationship. Fig. 3 shows example \( \gamma_l - \Delta u/\sigma' \) curves using typical sand parameters. From this figure, we can see that a small liquefaction strength leads to liquefaction under a small initial shear strain. We define \( \gamma_l \) reaching liquefaction, i.e., \( \Delta u/\sigma' = 1 \), as \( \gamma_{l_\text{eq}} \). For example, when liquefaction strength \( R_{20} \) is 0.25, \( \gamma_{l_\text{eq}} \) is 0.1% as shown in Fig. 3.

2.2 Simple evaluation method for sand liquefaction constrained by a lattice-type improvement

a) Concept of evaluation

When the same earthquake external force is applied, \( \gamma_1 \) in the interstitial soil constrained by a high rigidity lattice improvement is smaller than that of the original in-situ soil (Fig. 4). If \( \gamma_1 \) is constrained by less than \( \gamma_{l_\text{eq}} \), subsequent excess pore water pressure is suppressed and liquefaction does not occur.

\[ \gamma_1 \]

\[ \frac{\Delta u}{\sigma'} \]

\[ \text{Original in-situ sandy ground} \]

\[ \text{Improved ground} \]

Fig. 4. Concept of evaluation.

\( \gamma_1 \) in the interstitial soil is easily determined using the equivalent shear stiffness of lattice-type improvement, \( G_{\text{eq}} \). Here, \( G_{\text{eq}} \) is a shear stiffness, which is 13 or 23 components of homogenized elastic coefficient \( D_{ijkl}^H \) calculated by the homogenization method. \( D_{ijkl}^H \) is used to calculate \( \gamma_1 \) because it can reveal reasonable overall mechanical properties of the soil.
composite ground.

\[ G_{eq} = G^{\prime \prime} \left( = G^{\prime \prime}_{ij} \right) \]  (5)

\[ D^{\prime \prime}_{ij} = \frac{1}{2} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} \left( \tau^{\prime \prime}_{\gamma} - \frac{\partial \chi_{ij}^{\prime \prime}}{\partial x} \right) dy \]  (6)

Here, \( Y \) represents the area of the unit periodic structure (unit cell) of improved ground (Fig. 4). \( D_{ijkl}^{ij} \) is the elastic modulus tensor of the unit cell. \( \chi_{ijkl}^{ij} \) is a \( Y \)-periodic characteristic displacement tensor, the so-called characteristic deformation function. Details of the \( D_{ijkl}^{ij} \) calculation are available in reference 3).

b) Evaluation procedure

The evaluation procedure is presented below. Fig. 5 shows a flow chart of the procedure.

1. Calculation of equivalent cyclic shear stress ratio \( \tau_e / \sigma_v' \)

Shear stress \( \tau_e \) generated in the ground during an earthquake can be calculated from an earthquake response analysis. However, it can also be simply determined using Eq. (7). Note that this equation is used as a simple liquefaction evaluation method.

\[ \frac{\tau_e}{\sigma_v'} = 0.1 \times (M - 1) \times \frac{\alpha_{max}}{g} \times \frac{\sigma_v}{\sigma_v'} \times \left( 1 - 0.015 \times z \right) \]  (7)

Here, \( M \) is the magnitude of earthquake, \( \alpha_{max} \) is the earth's surface maximum acceleration before improvement, \( z \) is the depth, \( g \) is the gravitational acceleration, \( \sigma_v' \) is the vertical effective stress, and \( \sigma_v \) is the vertical total stress.

2. Calculation of initial shear strain of the constrained ground

Initial shear strain \( \gamma_i \) is calculated by the following equation.

\[ \gamma_i = \frac{\tau_e}{G_{eq}} \]  (8)

Using the calculated \( \gamma_i \), we recalculate \( G_{eq} \) to use the \( G/G_0-\gamma \) relationship of sand. This cycle is repeated several times until \( \gamma_i \) converges to \( \gamma_{i_{eq}} \). Here, \( \gamma_{i_{eq}} \) is defined as the equivalent initial shear strain resulting in improved ground during earthquake.

3. Calculation of the excess pore water pressure ratio \( \Delta u / \sigma_v' \)

\( \Delta u / \sigma_v' \) occurring in the ground was obtained to substitute \( \gamma_{i_{eq}} \) to the \( \gamma - \Delta u / \sigma_v' \) relationship as previously described. If \( \Delta u / \sigma_v' \) is less than 1.0, the ground does not liquefy. On the other hand, the possibility of liquefaction of the original in-situ ground was evaluated from the initial shear strain of the original ground \( \gamma_{i_{o}} \), which was obtained to substitute \( \tau_o \) in Eq. (1).

c) Liquefaction safety factor before and after improvement

To substitute \( \gamma_{i_{eq}} \) and \( \gamma_{i_{o}} \) in Eq. (1), we can calculate the corresponding shear stress ratios \( \tau_e / \sigma_v' \) and \( \tau_o / \sigma_v' \) of sand, respectively. As liquefaction safety factors, \( FL \) values, are calculated as the ratio of \( \tau_{eq} \) and \( \tau_e \) to \( \tau_{o} \), they can be calculated by the following equations (Fig. 5).

(FL value before improvement)

\[ FL_b = \frac{\tau_{eq}}{\tau_e} \]  (9)

(FL value after improvement)

\[ FL_a = \frac{\tau_{eq}}{\tau_{eq}} \]  (10)

3 AN EXAMPLE DESIGN USING THE PROPOSED METHOD

In this section, we show an example design of lattice-type improvement using the proposed method. By comparing the results of a three-dimensional (3D)
effective stress analysis, we verified the preventive effect of the excess pore water pressure ratio due to the improvement.

3.1 Ground overview

Fig. 7 shows the ground histogram from our example. The building of interest is a rectangular plane about 40 m × 35 m. The ground is composed of loose fine sand and soft silt or clay layers whose N values are from 1 to 11.

Table 1 lists the calculated FL values of the original fine sand layer at depths indicated by the arrows in Fig. 7. When the maximum surface acceleration \( \alpha_{\text{max}} \) is \( 3.5 \times 10^{-2} \) m/s\(^2\), the minimum FL value is 0.39. We assume liquefaction would occur in these layers during a strong earthquake.

![Fig. 7. Ground histogram used for our example design.](image)

Table 1. FL values from the simplified liquefaction evaluation method.

| Depth (m) | Ground condition | N-value | \( F_c \) | \( V_s, G \) | \( V_s, I \) | \( G_0 \) | \( \gamma_{fr} \) | \( h_{\text{max}} \) | \( R_{20} \) |
|----------|------------------|---------|----------|----------|----------|--------|--------|----------|--------|
| 3.4      | Fine sand        | 130     | 36.3     | 0.049    | 0.35    |
| 7.3      | Fine sand        | 130     | 63.6     | 0.891    | 0.4     |
| 18.3     | Fine sand        | 130     | 120.7    | 0.726    | 0.41    |

![Fig. 8. Relationship between initial shear strain and excess pore water pressure ratio at each depth](image)

3.2 Liquefaction evaluation of the interstitial ground using the proposed method

The possibility of liquefaction after improvement was examined using the proposed method. Table 2 lists the parameters for each depth. The shear wave velocity of the ground \( V_{s, G} \) was determined from PS logging data in the borehole. Improvement was set to \( V_{s, I} = 500 \) m/s with a replacement ratio \( R = 25\% \). Initial shear stiffness of the ground \( G_0 \) was calculated by \( V_{s, G} \). \( \gamma_{fr} \) and \( h_{\text{max}} \) were set based on the element test results. \( R_{20} \) was set using the simplified liquefaction evaluation method.

Table 2. Calculation parameters.

| Depth (m) | \( \sigma'_v \) | \( V_{s, G} \) | \( V_{s, I} \) | \( G_0 \) | \( \gamma_{fr} \) | \( h_{\text{max}} \) | \( R_{20} \) |
|----------|------------|----------|----------|--------|--------|----------|--------|
| 3.4      | 36.3       | 130      | 29       | 0.023  | 0.17   |
| 7.3      | 63.6       | 130      | 500      | 0.032  | 0.16   |
| 18.3     | 120.7      | 190      | 65       | 0.05   | 0.18   |

3.3 Relationship between the initial shear strain and excess pore water pressure ratio at each depth

Fig. 8 shows the \( \gamma_{lq} - \Delta u/\sigma'_v \) relationship at each depth. \( \gamma_{lq} \) ranged from 0.055 to 0.11%. The initial shear strains before and after improvement are shown in Fig. 8. \( \gamma_{lq} \) is larger than \( \gamma_{lq} \), while \( \gamma_{eq} \) is smaller than \( \gamma_{lq} \), and therefore the soil does not liquefy.
pore water pressure ratio of each depth.

From the shear stress values corresponding to \( \gamma_{1,0} \), \( \gamma_{1,lq} \), and \( \gamma_{1,eq} \), FL values before and after improvements were calculated, as shown in Fig. 9. We could confirm that the FL value of unimproved ground is almost the same FL value calculated by the simple liquefaction evaluation method, shown in Table 1.

\[
\begin{align*}
\text{(G.L. – 3.4 m)} & \\
F_{Lb} &= \frac{\tau_0}{\sigma_0} = 0.17/0.35 = 0.49 & (11) \\
F_{La} &= \frac{\tau_0}{\sigma_{eq}} = 0.17/0.075 = 2.3 & (12) \\
\text{(G.L. – 7.3 m)} & \\
F_{Lb} &= 0.4, F_{La} = 2.3 & (13) \\
\text{(G.L. – 18.3 m)} & \\
F_{Lb} &= 0.44, F_{La} = 2.3 & (14)
\end{align*}
\]

### 3.4 Comparison of the 3D effective stress analysis

Finite element method (FEM) mesh sizes are 110 m in the shaking direction, 120 m in the orthogonal direction, and 104 m depth. The size of the lattice-type improvement is 36 m \( \times \) 45 m \( \times \) 31 m, and the improvement ratio is \( R = 28 \% \). This replacement ratio was determined using the calculated results of the proposed method. The bottom boundary condition is set as a viscous boundary of \( V_s \_G = 400 \text{ m/s} \) and \( \rho_t = 1.9 \text{ t/m}^3 \). The side boundary condition is set as a roller boundary assuming that the displacement in the horizontal direction is not constrained.

The input earthquake wave used is the JMS Kobe EW wave, which is normalized to the maximum acceleration amplitude of approximately 3 m/s².

The constitutive law used in the analysis is a modified R-O model that takes the changes in the confining pressure into account. The bowl model proposed by Fukutake (2010) was taken as the relationship of the dilatancy associated with the shear strain. The parameters and their values used in the analysis, except for the \( SV \) curve representing the liquefaction strength, were the same as in Table 2. The improvement was assumed to be a linear elastic body with shear stiffness \( V_s \_I = 500 \text{ m/s} \).

Fig. 12 shows the maximum deformation and the maximum acceleration distribution in the cross-section model of the line A-A’ (see Fig. 8). The maximum acceleration of ground surface of point Q, where the influence of improvement is small, was 3.3–3.4 m/s². In addition, the maximum deformation angle of the improved area in the vertical direction is approximately 1/1000, which means that improvement during shaking does not shake as a locking mode. The above results agree with the calculated conditions of the proposed method.

Fig. 13 shows the distribution of the maximum deformation and excess pore water pressure ratio. The
figure also shows the plane distribution of $\Delta u/\sigma_v'$ at G.L. – 7 m in the fine sand layer. The $\Delta u/\sigma_v'$ value reaches 1 outside the improved area, while $\Delta u/\sigma_v'$ in the region of the improved area remains at 0.1–0.6 and the soil is not liquefied.

$\Delta u/\sigma_v'$ distributions from the numerical analysis are slightly larger than the calculated results of the proposed method. We assume this is because the proposed method uses $G_{eq}$ as a one-dimensional (1D) model. This means that ground improvement should construct the whole plane in the improvement depth. Because the lattice-type improvement is constructed in a semi-infinite area, i.e., 36 m $\times$ 45 m (a relatively narrow area), there is a difference between the geometrical conditions used by the proposed method and the 3D analysis. Correcting for the equivalent stiffness considering the effect of finite improvement area is a future challenge.

4 CONCLUSIONS

The following conclusions are reached in this paper.

1. Using the concept of damage accumulation to liquefaction, we derived a unique relationship between the initial shear strain $\gamma_i$ and the excess pore water pressure ratio $\Delta u/\sigma_v'$.

2. We proposed a liquefaction evaluation method for lattice-type improved ground using the above $\gamma_i-\Delta u/\sigma_v'$ relationship and the equivalent shear stiffness $G_{eq}$ calculated using the homogenization method. This proposed method initially calculates $\gamma_i$ in an interstitial ground and then evaluates liquefaction using the $\gamma_i-\Delta u/\sigma_v'$ relationship.

3. A design example showed and compared the results from a 3D effective stress analysis to verify the preventive effect of excess pore water pressure ratio. $\Delta u/\sigma_v'$ in an interstitial ground ranged 0.1–0.6. This result shows that the lattice-type improvement is effective against liquefaction. The $\Delta u/\sigma_v'$ predicted by the proposed method is slightly lower than the results of the numerical analysis. We assume this is because the proposed method treats the improved area as infinite and uses a 1D model, but in a real and numerical model, the improved area is a finite plane.

REFERENCES

1) De Alba, P., Seed, H.B. and Chang, C.K. (1976): Sand liquefaction in large-scale simple shear tests, *Proceedings. GED, ASCE*, Vol.102, No.GT9, 909-927.

2) Fukutake, K. (2010): Modeling the soil behavior during liquefaction and analysis of a tank installed on soil reinforced by chemical grouting and subjected to liquefaction, Research report of Shimizu corporation, Vol.87, 69-76.

3) Ishikawa, A., Shamoto, Y., Jang, G. and Kimura, T. (2012): A simplified method for evaluating the liquefaction of sandy soil confined by a lattice-type deep mixing wall, *Journal of Japan Society of Civil Engineers Ser.C (Geosphere Eng.)*, Vol.68, No.2, 297-304. (In Japanese)

4) Taya, Y., Uchida, A., Yoshizawa, M., Onimaru, S., Yamashita, K. and Tsukuni, S. (2008): Simple method for determining lattice intervals in grid-form ground improvement, *Japanese Geotechnical Journal*, Vol.3, No.3, 203-212. (In Japanese)

5) Terada, K. and Kikuchi, N. (2003): Introduction of homogenization method, Maruzen Print Co. Ltd. (In Japanese)

6) Zhang, J., Shamoto, Y. and Tokimatsu, K. (1997): Moving critical and phase-transformation stress state lines of saturated sand during undrained cyclic shear, *Soils and Foundations*, Vol.37, No.2, 51-59.