A Separable Pairing Force for Relativistic Quasiparticle Random Phase Approximation

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We have introduced a separable pairing force, which was adjusted to reproduce the pairing properties of the Gogny force in nuclear matter. This separable pairing force is able to describe in relativistic Hartree-Bogoliubov (RHB) calculations the pairing properties in the ground state of finite nuclei on almost the same footing as the original Gogny interaction. In this work we investigate excited states using the Relativistic Quasiparticle Random Phase Approximation (RQRPA) with the same separable pairing force. For consistency the Goldstone modes and the convergence with various cutoff parameters in this version of RQRPA are studied. The first excited 2+ states for the chain of Sn-isotopes with Z = 50 and the chain of isotones with N = 82 isotones are calculated in RQRPA together with the 3− states of Sn-isotopes. Comparing with experimental data and with the results of original Gogny force we find that this simple separable pairing interaction is very successful in depicting the pairing properties of vibrational excitations.

PACS numbers: 21.30.Fe, 21.60.Jz, 24.30.Cz, 24.30.Gd

I. INTRODUCTION

At present Density Functional Theory (DFT) based on the mean-field concept has been widely used for all kinds of quantum mechanical many-body systems. In nuclear physics the relativistic mean field theory based on DFT has been of great success in describing the properties of many nuclei all over the periodic table [1]. Conventional DFT with a functional \( E[\rho] \) depending only on the single particle density \( \rho \) can be applied in nuclear physics practically only in a few doubly closed shell nuclei. The vast majority of nuclei and in particular those far away from the \( \beta \) stability line the inclusion of particle-particle (\( pp \)) correlations is essential for a quantitative description of structure phenomena. In the framework of DFT pairing correlations are taken into account in the form of Bogoliubov theory for the ground states and in Quasiparticle Random Phase Approximation (QRPA) for the excited states. Although monopole pairing or density dependent delta pairing interactions are widely used because of their simplicity, a cutoff parameter has to be introduced by hand. In order to avoid the complicated problem of a pairing cutoff the finite range Gogny force has been applied [4, 5]. Its parameters have been adjusted very carefully in a semi-phenomenological way to characteristic properties of the microscopic effective interactions and to experimental data [6, 7]. Over the years the relativistic Hartree-Bogoliubov theory (RHB) [8] with finite Gogny pairing force has turned out to be a very successful way to describe pairing correlations in nuclei. The price we have to pay for the advantages is much more numerical efforts involved, especially in calculations of deformed nuclei and in applications for excited states.

As presented in Refs. [8, 9], we have introduced a new separable form of the pairing force for RHB theory. A similar ansatz has been used in the pairing channel of non-relativistic Skyrme calculations in Refs. [10, 11]. The parameters of our separable force are adjusted to reproduce the pairing properties of the Gogny force in nuclear matter. It preserves translational invariance and it has finite range. Applying well known techniques of Talmi and Mosinsky [12, 13, 14, 15] this pairing interaction can be used in relativistic and in non-relativistic Hartree-Bogoliubov or Hartree-Fock-Bogoliubov calculation of finite nuclei. It avoids the complicated problem of a cutoff at large momenta or energies inherent in other zero range pairing forces. In Ref. [8] it has been shown that with this force the pairing properties of ground states can be well depicted on almost the same footing as with the original Gogny pairing interaction.

For excited states it is important to combine the RPA and the RHB in a consistent way in order to describe the excitations in unstable nuclei near drip-line, especially when the pairing correlations play a crucial role. Recently Ring et al. [16, 17] have used time-dependent relativistic mean field theory to derive the fully self-consistent relativistic random phase (RRPA) and relativistic quasiparticle random phase approximation (RQRPA). For the pairing channel the finite range Gogny force D1S is used. Excited states are calculated in a consistent framework using the same density functional. It has been shown in several applications that RQRPA provides an excellent tool for the description of the multipole response of stable as well as of unstable and weakly bound nuclei far from stability. These investigations have been devoted to low-lying collective excitations [18, 19, 20], to giant resonances [21, 22, 23, 24, 25], to spin-isospin resonances [17, 26], and to new exotic modes in stable [27] and unstable nuclei [28, 29, 30, 31, 32].

Of course, in the case of spherical nuclei, the calculation of QRPA-matrix elements of the original Gogny force is possible and computer codes are available [17]. Although the evaluation of such matrix elements for the
new separable force is much faster, nonetheless the application of this force for QRPA-calculations in spherical nuclei does not bring an essential advantage. This is, however, no longer true for QRPA-calculations in deformed nuclei. Here one has to do with several thousands two-quasiparticle configurations and several millions of matrix elements, in particular in relativistic applications where the Dirac sea has to be treated properly. A separable force is also of considerable advantage in all cases where the RPA-problem cannot be solved by diagonalization, as for instance for energy dependent self energies in the treatment of complex configurations by particle-vibrational coupling. In this case one has to work at fixed energy and to solve the linear response equations at fixed energy. It is well known that the dimension of the coupled linear response equations scales with the number of separable terms and not with the number of two-quasiparticle configurations. Therefore a separable force brings essential advantages in all these cases.

So far, the separable pairing force has been used only in static applications and in this case was very successful. It is not clear from the beginning, whether one can also reproduce the dynamic properties of full Gogny pairing in such a simple way, because, in fact, as shown in Fig. 6 of Ref. both forces are not fully identical. In particular there is the problem of Goldstone modes connected with translational symmetry. It is well known that these modes depend in a very delicate way on the properties of the residual interaction. Only in the case of full self-consistency these modes decouple fully from the rest of the spectrum. This is particular important for isoscalar dipole excitations, where the large strength of the spurious translational mode can contaminate the low-lying E1-spectrum considerably. Translational invariance is one of the essential advantages of the new pairing force as compared to older separable pairing forces such as monopole, quadrupole, or other multipole pairing forces. However, the new force is presented as a sum over separable terms and translational invariance is strictly fulfilled only for an infinite number of separable terms. As it has been shown in Ref. , in static applications this series converges quickly and one needs only 8 separable terms to get convergence. It is not clear, wether this number is large enough for a proper treatment of the Goldstone modes.

This paper is devoted to an investigation of all these open questions. The new separable pairing interaction is implemented in the relativistic QRPA program and details for the calculation of the new pp-matrix elements are presented. In order to test the numerical implementation of the QRPA equation with the separable pairing interaction we study the Goldstone modes and the consistency of the method. In addition we investigate the question whether the dynamic properties of pairing correlations in vibrational excitations can be reproduced with the new pairing force. As it is known the first $2^+$ excited states in semi-magic nuclei are very sensitive to the pairing gap.

Therefore we investigate the isoscalar quadrupole excitations in Sn-isotopes and in $N = 82$ isotones in the RHB+RQRPA approach with the new pairing force and compare the first $2^+$ states with those obtained with the full Gogny pairing force. Furthermore we calculate $3^-$ excitations in Sn-isotopes and investigate the sensitivity of the isoscalar octupole states to the pairing properties.

The paper is arranged as follows. The theoretical formalism of RHB+RQRPA with the separable form of the pairing interaction is presented in Sec. II. The consistency of the method as well as the Goldstone spurious modes are investigated in Sec. III. The isoscalar quadrupole in Sn-isotopes and in $N = 82$ as well as the isoscalar octupole states in Sn-isotopes are calculated in the RHB+RQRPA approach, which are discussed in Sec. IV. Finally we shall give a brief summary in Sec. V.

II. THEORETICAL FORMALISM

We start with the $^1S_0$ channel gap equation in symmetric nuclear matter at various densities,

$$\Delta(k) = - \int_0^{\infty} \frac{k'^2 dk'}{2\pi^2} \langle k | V_{sep}^* | k' \rangle \frac{\Delta(k')}{2E(k')} ,$$

where

$$\langle k | V_{sep}^* | k' \rangle = -Gp(k)p(k')$$

is the separable form of the pairing force introduced in Ref. with a Gaussian ansatz $p(k) = e^{-a^2k^2}$. A The two parameters $G$ and $a$ are fitted to the density dependence of the gap at the Fermi surface $\Delta(k_F)$ in nuclear matter. Comparing with the Gogny D1S force , we obtain the parameter set of $G = 728$ MeV·fm$^3$ and $a = 0.644$ fm.

The RQRPA is constructed in the canonical single-nucleon basis, where the wave functions of the RHB model have BCS form (for details see Ref. ). In these calculations the same interactions are used in the RHB calculation for the nuclear ground state and in the RQRPA equations for the excited states, as well in the particle-hole ($ph$) as in the $pp$-channel. Since the interaction in the $ph$-channel is identical to earlier calculations , we discuss here only the derivation of the matrix elements of the separable interaction of Eq. used in the $pp$-channel of the RQRPA equation in finite nuclei. First, we transform the separable force Eq. from momentum space to coordinate space and obtain

$$V(r_1,r_2,r'_1,r'_2) = -G \delta(R - R') P(r)P(r') \frac{1}{2}(1 - P^\sigma) ,$$

where $R = \frac{1}{2}(r_1 + r_2)$ and $r = r_1 - r_2$ are the center of mass and relative coordinates respectively, and $P(r)$ is obtained from the Fourier transform of $p(k)$,

$$P(r) = \frac{1}{(4\pi a^2)^{3/2}} e^{-\frac{r^2}{4a^2}} .$$

The term $\delta(R - R')$ in Eq. insures the translational invariance. It also shows that this force is not completely
separable in coordinate space. However, in the basis of harmonic oscillator functions the matrix elements of this force can be represented by a sum of separable terms which converges quickly (for details see Ref. [8]).

In the pairing channel we need the two-particle wave functions coupled to angular momentum \( J \) and the projector \( \frac{1}{2}(1 - P^2) \) restricts us to the quantum numbers of total spin \( S = 0 \) and total orbital angular momentum \( \lambda = J \). These wave functions are usually expressed in terms of the laboratory coordinates \( r_1 \) and \( r_2 \) of the two particles, while the separable pairing interaction in Eq. (3) is expressed in the center of mass coordinate \( \mathbf{R} \) and the relative coordinate \( \mathbf{r} \) of the pair. Therefore we transform to the center of mass frame by using the well known Talmi-Moshinsky brackets [12,13,14] in the notation of Baranger [15]

\[
|n_1l_1,n_2l_2;\lambda\mu\rangle = \sum_{Nl} M_{n_1n_2}^{Nl}|NL, nl;\lambda\mu\rangle ,
\]

where

\[
M_{n_1n_2}^{Nl} = \langle NL, nl, \lambda|n_1l_1, n_2l_2 , \lambda\rangle
\]

are the Talmi-Moshinsky brackets with the selection rule

\[
2N + L + 2n + l = 2n_1 + l_1 + 2n_2 + l_2 .
\]

Here we need these brackets only for the case \( \lambda = J \). We therefore can express the two-body function with the quantum numbers \( S = 0 \) and \( \lambda = J \) in terms of center of mass and relative coordinates by the sum

\[
|12\rangle_J = \frac{\hat{j} j_2}{s} \left\{ \frac{j_2}{l_1} \frac{l_2}{J} \right\} \sum_{NLnl} \sum_{l_1l_2} M_{n_1l_1n_2l_2}^{Nl}\times R_{NL}(R, b_R)R_{nl}(r, b_r)|\lambda = J\rangle|S = 0 \rangle,
\]

where \( \hat{j} = \sqrt{2J + 1} \) and \( s = \frac{1}{2J} \) and \( R_{NL}(R, b_R) \), \( R_{nl}(r, b_r) \) are radial oscillator wave functions for the center of mass and relative coordinates with the oscillator parameters \( b_R = \frac{b}{\sqrt{2}} \) and \( b_r = b \sqrt{2} \). Finally we find the pairing matrix elements of the interaction \( V_{1212'}^{Nl} = \langle n_1l_1j_1, n_2l_2j_2|V|n_1l'j_1', n_2l'j_2', j_2'\rangle \) in Eq. (3) as a sum over the quantum numbers \( N, L, N', L', n, l, n', l' \) in Eq. (5). The integration over mass coordinates \( R \) and \( R' \) leads to \( N = N', L = L' \). Further restrictions occur through the fact that the sum contains integrals over the relative coordinates of the form

\[
\int R_{nl}(r, b_r)Y_{lm}(\hat{r})P(r)d^3r.
\]

They vanish unless \( l = 0 \) and \( L = J \). The quantum numbers \( n \) and \( n' \) are determined by the selection rule [7] and are left with a single sum of separable terms

\[
V_{1212'}^{ppJ} = G \sum_{N} V_{12}^{N} \times V_{12'}^{N} .
\]

III. VERIFICATION OF CONSISTENCY

In the following investigations we solve the RHB+QRPA equations with this separable pairing force. The Dirac spinors are expanded in a spherical oscillator basis with \( N_F = 18 \) major oscillator shells [35]. This leads to a very large number of two-quasiparticle(2qp) pairs and to a huge dimension of the corresponding QRPA matrix. In the practical numerical calculations a cutoff energy has to be adopted and 2qp-pairs with an energy larger than this cutoff are neglected. In relativistic RPA we have two types of 2qp-pairs and therefore two cutoff energies: \( E_{Cp} \) is the maximum value of the 2qp-energy for positive energy states, and \( E_{Ca} \) is the maximum absolute value of the 2qp-energy for states with one fully or partially occupied state of positive energy and one empty negative-energy state in the Dirac sea.

To test the numerical implementations of the RHB+QRPA equations with the separable form of the pairing interaction we first check the Goldstone modes [39]. As it is known, the Goldstone modes (often called spurious excitations) connected with symmetry violations in the mean field wave function have zero excitation energy and decouple from the physical states for RPA or QRPA-calculations based on a self-consistent mean field solution and using the interaction derived as the second derivative of the energy density functional [35]. The importance of a consistent treatment of pairing correlations

\[
V_{12}^{N} = \sqrt{\frac{4\pi}{\hat{J}_1\hat{j}_2}} \left\{ \frac{j_2}{l_1} \frac{l_2}{J} \right\} \times M_{n_1l_1n_2l_2}^{Nl}\int_{0}^{\infty} R_{nl}(r, b_r)P(r)r^2dr.
\]

For a Gaussian ansatz of \( P(r) \) in Eq. (9) this integral can be evaluated analytically and we find

\[
V_{12}^{N} = \frac{1}{\beta/2} \frac{21/4}{\pi^{3/4}} \left( \frac{\hat{j}_1\hat{j}_2}{s} \right) \sum_{Nl} \sum_{l_1l_2} M_{n_1l_1n_2l_2}^{Nl} \times \frac{(1 - \alpha^2)^n}{(1 + \alpha^2)^{n+3/2}} M_{n_1l_1n_2l_2}^{Nl} \sqrt{2n + 1} !\]

where the parameter \( \alpha = a/b \) characterizes the width of the function \( P(r) \) in terms of the oscillator length \( b \) and \( n \) is given by the selection rule [7]: \( n = n_1 + n_2 + \frac{1}{2}(l_1 + l_2 - J) - N \). The results of the RHB + QRPA model will depend on the choice of the effective RMF Lagrangian in the \( ph \)-channel, as well as on the treatment of pairing correlations. In this work the NL3 effective interaction [35] is adopted for the RMF Lagrangian. In the pairing channel we use a separable form in Eq. (3) adjusted to the pairing part of the Gogny force D1S and compare the results of RQRPA calculations with the full Gogny force D1S in the pairing channel.
in QRPA calculations has been demonstrated in the non-relativistic \cite{40,41} and in the relativistic \cite{17,42} framework. The zero-energy Goldstone modes also provide a rigorous check of the consistency. Two kinds of spurious states have been investigated: one corresponds to the violation of particle number in the monopole resonance with the quantum numbers $J^p = 0^+$; another is connected with the violation of translational invariance and the spurious center of mass motion in the dipole resonance with the quantum numbers $J^\pi = 1^-$. There should be no response to the number operator since it is a conserved quantity, i.e., the Nambu-Goldstone mode associated with the nucleon number conservation should have zero excitation energy. It is observed that the spurious state of the number operator in the nucleus $^{22}$O disappears when the pairing interaction is treated consistently in the RHB and RQRPA with our separable pairing force.

For sufficiently large values of the two cutoff parameters $E_{Cp}$ and $E_{Ca}$ the response to the corresponding generator of the broken symmetry should vanish for all non-vanishing energies. The investigation of the convergence of the RQRPA results as a function of these two cutoff parameters provides a very sensitive verification for the numerical performance of the code. In Fig. 1(a) we show how the response to the neutron number operator in $^{120}$Sn varies with the energy for various values of the cutoff parameter $E_{Cp}$ in the range from 40 – 280 MeV. Here the choice $E_{Ca} = 1800$ MeV includes almost the entire negative-energy Dirac bound spectrum, which is large enough to yield a convergent result in the usual RRPA calculations. For $E_{Cp} = 180$ MeV the Nambu-Goldstone $0^+$ mode converges to $\leq 0.1$ MeV. The choice of the cutoff parameter $E_{Ca}$ has also a pronounced influence on the calculated isoscalar monopole response. In Fig. 1(b) we show the peak energies of the isoscalar giant monopole resonance (ISGMR) in $^{120}$Sn as a function of $E_{Ca}$. It saturates for $E_{Ca} \geq 1300$ MeV.

In the dipole channel a large configuration space is necessary to bring the spurious $1^-$ state to zero excitation energy. In Fig. 1(c) we illustrate the convergence of the energy of the $1^-$ spurious state in $^{120}$Sn. The excitation energy of the spurious state is plotted as a function of the energy cutoff parameter $E_{Cp}$ for a fixed value of $E_{Ca} = 1800$ MeV. We find that the excitation energy goes to zero slightly slower in the case of the separable pairing force than that with the full Gogny force. This might be explained by slightly different ranges of the two pairing forces.

As a consequence we use in the following calculations for the solution of the self-consistent RHB equations the values $E_{Cp} = 180$ MeV and $E_{Ca} = 1800$ MeV. This leads to a dimension in the order of 3500 $2qp$-pairs for the RQPRPA matrix.

As we see from the Eq. (10) the separable pairing interaction is not fully separable in the spherical harmonic oscillator basis. We have a sum over the quantum number $N$ characterizing the major shells of the harmonic oscillator in the center of mass coordinate. In order to study the convergence with the parameter $N_0$, we show in Fig. 1(d) the $1^-$ spurious state in $^{120}$Sn as a function of $N_0$. We find that for nuclei around the line of $\beta$-stability ($^{120}$Sn), $N_0 = 5$ is already large enough to bring the spurious $1^-$ state to zero excitation energy. In the previous investigation in Ref. 8 it was found that a somewhat larger value of $N_0 = 8$ is needed to obtain convergence for the ground state properties.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(a) The response of the neutron number operator in $^{120}$Sn for five values of the $2qp$ cutoff energy parameter $E_{Cp}$. (b) The excitation energy of the ISGMR in $^{120}$Sn as a function of the cutoff energy parameter $E_{Ca}$. The red circles and black squares correspond to the results with the separable pairing force and the Gogny force, respectively. (c) The position of the spurious $1^-$ state in $^{120}$Sn as a function of the $2qp$ cutoff energy parameter $E_{Cp}$. The notations are the same as in (b). (d) The position of the spurious $1^-$ state in $^{120}$Sn as a function of the number $N_0$ of separable terms in Eq. (10).}
\end{figure}
Again very similar results are found for the Gogny force of $N = 82$ isotopes. The $B(E2)$ values are slightly larger than those calculated by Ansari [19] using the original Gogny force $D1S$ in the mass number. We compare the results obtained using values for the chain of Sn-isotopes as a function of the excitation energies and the $B(E2)$ values for protons obtained with RHB calculations for $N = 82$ isotones. The experimental values of the pairing gap in even-even nuclei are calculated by the odd-even mass difference with the three-point formula [43]. It is shown that the agreement between theoretical predictions and experimental data is reasonable, except for the nucleus $^{140}$Ce. Our results for the RHB-calculations show that the nucleus $^{140}$Ce with the charge number $Z = 58$ has a closed sub-shell for protons: the $\pi g_{7/2}$ state is fully occupied and we observe in the single particle spectrum for protons a large energy gap at the Fermi surface. Therefore a very small pairing gap and a large excitation energy of the lowest $E2$ state were predicted, which is inconsistent with the experimental data.

We also investigate in the RHB+RQRPA approach excited octupole states of spherical Sn-isotopes. In Fig. 3 we plot the first and second excited $J^e = 3^-$ states, and the peak energy of the giant octupole resonance for the Sn-isotopes as functions of the neutron number. Strong low-lying $3^-$ states are found also in the Sn-isotopes, which are consistent with experimental observations. We can also observe that the results for the $3^-$ states calculated with the separable pairing force are very close to those obtained with the Gogny force in Ref. [20]. The calculations with the separable pairing force yield slightly larger values of the first and second excited states than those with the Gogny force as we have seen it in the case of low-lying quadrupole excitations, while both of

IV. RESULTS AND DISCUSSION

First we investigate the lowest $2^+$ excitations in Sn-isotopes for which experimental data are available using the relativistic parameter set NL3 in the $ph$-channel. In Fig. 2 we plot the $E2$ excitation energies and the $B(E2)$ values for the chain of Sn-isotopes as a function of the mass number. We compare the results obtained using the new separable interaction [2] with the calculations by Ansari [19] using the original Gogny force $D1S$ in the pairing channel. The results obtained with the separable pairing interaction are slightly larger than those calculated with the Gogny force. However, the discrepancy stays very small within a few percent, and the behavior of the $E2$ and the $B(E2)$ values along the chain of isotopes is well reproduced. These small deviations can be understood by the fact that the one term separable pairing interaction, as we discussed in Ref. [8], yields in the ground states slightly larger pairing gaps and therefore stronger pairing fields than those found with the original Gogny force. Therefore, as expected, also the effect of the pairing fields in the excited states is slightly increased in the case of the separable pairing force. This conclusion is consistent with the pairing properties of the ground states described in the RHB calculations.

In the top panel of Fig. 3(a) we show the average pairing gaps for protons obtained with RHB calculations for the chain of the $N = 82$ isotones from $^{122}$Zr to $^{152}$Yb. We find that the separable pairing interaction describes the average pairing gaps of finite nuclei on almost the same footing as the Gogny force $D1S$, although the pairing gaps calculated with the separable force are always slightly larger than those with the Gogny force. In the middle and bottom panels of Fig. 3 we display the $E2$ excitation energies and the $B(E2)$ values for the chain of $N = 82$ isotones as functions of the proton number. Again very similar results are found for the Gogny force and for its separable form in the pairing channel. Experimental data for the pairing gaps, the excitation energies of the lowest $2^+$ states, and the corresponding $B(E2)$ values are also plotted in Fig. 3 for the chain of $N = 82$ isotones. The experimental values of the pairing gap in even-even nuclei are calculated by the odd-even mass difference with the three-point formula [43]. It is shown that the agreement between theoretical predictions and experimental data is reasonable, except for the nucleus $^{140}$Ce.
them give almost the same peak energies of the giant octupole resonances. This is due to the fact that pairing collections have rather little influence on the giant resonances, but a strong effect on the low-lying excitations in semi-magic nuclei. In comparison with the experimental data [44], the RHB+RQRPA calculations both with Gogny and separable pairing forces can well describe the first $3^-$ states. This again illustrates that both the Gogny pairing force and its separable approximation describe the pairing properties of excited states on almost the same footing.

V. CONCLUSION

In summary we have presented first results for RHB+RQRPA calculations in finite nuclei based on a new separable force in the pairing channel. This separable force is translational invariant and has finite range. It contains 2 parameters which are adjusted to reproduce the pairing gap of the Gogny force in nuclear matter. In the RHB calculations for finite nuclei the two-body matrix elements in the $pp$-channel are evaluated using well known techniques of Talmi and Moshinsky. The separable form of the pairing interaction can describe the pairing properties of finite nuclei in the ground states on almost the same footing as its corresponding pairing Gogny interaction [8]. Similar techniques are used for the evaluation of the two-body matrix elements in the $pp$-channel for RQRPA calculations. The numerical implementations of the RQRPA with this separable pairing force are verified by checking for the separation of Goldstone modes connected with symmetry violations in the mean field solutions. The numerical convergence of the RQRPA calculations as a function of various cutoff parameters is shown for the nucleus $^{120}$Sn. We presented applications to the lowest $2^+$ states and the corresponding reduced transition rates in Sn isotopes and $N = 82$ isotones. The isoscalar octupole excitations in Sn isotopes are also investigated. We found excellent agreement of our results in comparison with those obtained by using the full Gogny force in the $pp$-channel and with available experimental data.

Therefore we can conclude that this simple separable pairing interaction can also be applied in future applications of the RHB+RQRPA approach in nuclei far from stability instead of the complicated Gogny force. In particular this will allow us to use realistic, finite range pairing forces also in cases where the numerical complexity forced us up to now to neglect pairing correlations completely, as for instance in recent investigations of magnetic dipole modes based the tilted axis cranking approach [45], or to restrict us to very simple monopole or zero range forces in the pairing channel, as for instance in relativistic QRPA calculations in deformed nuclei [33, 46]. There are also many extensions of relativistic density functional theory beyond mean field, which were so far only possible with rather simple pairing forces, such as applications using projection [47] onto subspaces with good symmetries, generator coordinate methods (GCM) [48, 49], or investigation of complex configurations in the framework of particle-vibrational coupling (PVC) [34, 50]. All these methods require a more realistic description of pairing correlations in the future. Investigations in this direction are in progress.

Acknowledgments

This research has been supported by the National Natural Science Foundation of China under Grant Nos 10875150, 10775183 and 10535010, the Major State Basis Research Development of China under contract number 2007CB815000, the Bundesministerium für Bildung und Forschung (BMBF), Germany under project 06 MT 246 and the DFG cluster of excellence “Origin and Structure of the Universe” (www.universe-cluster.de).

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