SECULAR DYNAMICS OF THE TRIPLE SYSTEM HARBORING PSR J0337+1715 AND IMPLICATIONS FOR THE ORIGIN OF ITS ORBITAL CONFIGURATION

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ABSTRACT

We explore secular dynamics of a recently discovered hierarchical triple system consisting of the radio pulsar PSR J0337+1715 and two white dwarfs (WDs). We show that three-body interactions endow the inner binary with a large forced eccentricity and suppress its apsidal precession, to about 24% of the rate due to the general relativity. However, precession rate is still quite sensitive to the non-Newtonian effects and may be used to constrain gravity theories if measured accurately. A small value of the free eccentricity of the inner binary $e_i \approx 2.6 \times 10^{-5}$ and vanishing forced eccentricity of the outer, relatively eccentric binary naturally result in their apsidal near-alignment. In addition, this triple system provides a unique opportunity to explore excitation of both eccentricity and inclination in neutron star–WD binaries, e.g., due to random torques caused by convective eddies in the WD progenitor. We show this process to be highly anisotropic and more effective at driving eccentricity rather than inclination. The outer binary eccentricity and $e_i \approx 5$ exceed by more than an order of magnitude the predictions of the eccentricity–period relation of Phinney, which is not uncommon. We also argue that the non-zero mutual inclination of the two binaries emerges at the end of the Roche lobe overflow of the outer (rather than the inner) binary.

Key words: binaries: close – celestial mechanics – pulsars: individual (PSR J0337+1715) – white dwarfs

Online-only material: color figures

1. INTRODUCTION

Recent discovery of a triple system harboring two white dwarfs (WDs) and a millisecond pulsar PSR J0337+1715 (Ransom et al. 2014) is intriguing because of the complicated evolutionary history that this system must have undergone (Tauris & van den Heuvel 2014, hereafter TvdH14). It also represents an interesting testbed of the Newtonian dynamics that can be explored at very high accuracy.

In particular, high masses of the WD companions make gravitational three-body effects quite significant. A pulsar for which such study has been done previously is PSR 1257+12 (Wolszczan & Frail 1992), but it is orbited by three planetary mass objects (Rasio et al. 1992; Malhotra 1993).

In addition, high timing accuracy of PSR J0337+1715 (Ransom et al. 2014) makes this system well suited for studying orbital dynamics. Mutual gravitational interactions between the orbiting bodies are also being explored in systems of multiple extrasolar transiting planets discovered by Kepler satellite, using the so-called transit timing variations (TTVs; e.g., Mazeh et al. 2013). However, planetary transits usually have timing accuracy of order minutes, while PSR J0337+1715 already yields median arrival time uncertainty of 0.8 $\mu$s in 10 s integrations. Moreover, pulsar timing has the benefit of revealing the variations of orbital parameters over the full orbit, while TTVs inform us only about the system parameters at the moments of mutual conjunctions of eclipsing bodies.

Our present goal is to explore secular evolution of the PSR J0337+1715 system and to provide possible connections to its origin. This system is hierarchical in nature, with the semimajor axis of the outer binary $a_o = 1.76498 \times 10^{13}$ cm far exceeding that of the inner binary $a_i = 4.776 \times 10^{11}$ cm.

The two features of its present-day dynamical architecture are rather intriguing. First, the orbits of the inner and outer binaries are highly coplanar: their mutual inclination $i = 1^\circ 2 \times 10^{-2}$ is small but is certainly non-zero. Second, the orbital ellipses of the inner and outer orbits are quite well aligned: the difference of their apsidal angles is small, $\sigma_i - \sigma_o = 1^\circ 9987$. Understanding the origin of these dynamical peculiarities of the PSR J0337+1715 system will be one of the goals of this work.

2. GENERAL SETUP

We employ the notation in which all quantities related to pulsar, inner, and outer binaries (or WDs) have subscripts “p,” “i,” and “o” correspondingly. Following the conventional approach (adopted in Ransom et al. 2014), we assume the inner binary to consist of the pulsar and the short-period WD, while the outer binary refers to the motion of the outer WD with respect to the barycenter of the inner binary.

We use the parameters of the system presented in Ransom et al. (2014): pulsar and WD masses, $m_p = 1.4378 M_\odot$, $m_i = 0.1975 M_\odot$, $m_o = 0.41 M_\odot$, total mass $m_3 = 2.045 M_\odot$, orbital periods of the inner and outer binaries $P_i = 1.6294$ days and $P_o = 327.26$ days, eccentricities $e_i = 6.918 \times 10^{-4}$ and $e_o = 3.5356 \times 10^{-2}$.

3. SECULAR EVOLUTION

Unlike the PSR 1257+12 and multi-planet systems with largest TTVs observed by Kepler, PSR J0337+1715 is a hierarchical triple and is devoid of mean motion resonances, simplifying interpretation of its dynamics. For clarity, in this work we also ignore the short-period gravitational perturbations and focus on understanding the current configuration of the system in the secular approximation (Murray & Dermott 1999), which consists of averaging the potentials of all orbiting bodies over their relatively short orbital periods. In this approximation, the semimajor axes of the binaries stay constant on timescales longer than $P_i$ and $P_o$. Also, in the limit $e_i, e_o, i \ll 1$ appropriate...
for the PSR J0337+1715 system the evolution of the orbital eccentricities decouples from the inclination evolution.

In this work we focus on the eccentricity evolution. It is well known (Murray & Dermott 1999) that in secular approximation the components of the eccentricity vectors \( e_i = (k_i, h_i) = (e_i \cos \sigma_i, e_i \sin \sigma_i) \) and \( e_o = (k_o, h_o) = (e_o \cos \sigma_o, e_o \sin \sigma_o) \) of the inner and outer binaries vary in time as

\[
\begin{align*}
    h_i(t) &= e_{i,+} \sin (g_i t + \beta_i) + e_{i,-} \sin (g_i t + \beta_-), \\
    k_i(t) &= e_{i,+} \cos (g_i t + \beta_i) + e_{i,-} \cos (g_i t + \beta_-), \\
    h_o(t) &= e_{o,+} \sin (g_o t + \beta_o) + e_{o,-} \sin (g_o t + \beta_-), \\
    k_o(t) &= e_{o,+} \cos (g_o t + \beta_o) + e_{o,-} \cos (g_o t + \beta_-).
\end{align*}
\]

Here \( g_i \) and \( e_{i, \pm}, e_{o, \pm} \) are the eigenvalues and eigenvectors of the matrix \( \mathbf{A} \) given by

\[
\mathbf{A} = \begin{pmatrix} A_i & B_i \\ B_o & A_o \end{pmatrix}
\]

so that

\[
g_\pm = \frac{1}{2} [A_i + A_o \pm \sqrt{(A_i - A_o)^2 + 4B_iB_o}],
\]

and

\[
e_{i,\pm} = -\frac{B_i}{A_i - g_\pm}.
\]

Because the components of the triple have comparable masses, we use the elements of matrix \( \mathbf{A} \) from Ford et al. (2000):

\[
A_i = A_i^{\text{sec}} + \tilde{\sigma}_{\text{GR}}
\approx 2.3403 \times 10^{-10} \text{s}^{-1} = 0:423 \text{ yr}^{-1},
\]

\[
A_i^{\text{sec}} = \frac{3}{4} \sqrt{G} \frac{m_o}{a_0^3} \frac{m_p + m_i}{\sqrt{m_p + m_i}}
\approx 1.6634 \times 10^{-10} \text{s}^{-1} = 0:3 \text{ yr}^{-1},
\]

\[
\tilde{\sigma}_{\text{GR}} = \frac{1}{2} \sqrt{G} \frac{(m_p + m_i)^3/2}{a_i^{3/2} c^2}
\approx 6.769 \times 10^{-11} \text{s}^{-1} = 0:122 \text{ yr}^{-1},
\]

\[
A_o = \frac{3}{4} \sqrt{G} \frac{m_p m_i m_3^{1/2}}{a_0^{7/2} (m_p + m_i)^2}
\approx 1.2958 \times 10^{-11} \text{s}^{-1} = 0:0234 \text{ yr}^{-1},
\]

\[
B_i = -\frac{15}{16} \sqrt{G} \frac{a_i^{5/2} m_o (m_p - m_i)}{a_0^{5/2} (m_p + m_i)^{3/2}}
\approx -4.267488 \times 10^{-12} \text{s}^{-1} = -7.71 \times 10^{-3} \text{ yr}^{-1},
\]

\[
B_o = -\frac{15}{16} \sqrt{G} \frac{a_i^{5/2} m_o (m_p - m_i)}{a_0^{5/2} (m_p + m_i)^{3/2}}
\approx -3.3245 \times 10^{-13} \text{s}^{-1} = -6 \times 10^{-40} \text{ yr}^{-1}.
\]

The diagonal element \( A_i \) of this matrix for the inner WD includes not only the secular contribution \( A_i^{\text{sec}} \) due to gravitational coupling to the outer WD but also the general relativistic (GR) precession term \( \tilde{\sigma}_{\text{GR}} \). All other components of \( \mathbf{A} \) are secular in nature.

With these values we obtain

\[
g_+ \approx 2.34 \times 10^{-10} \text{s}^{-1}, \quad g_- \approx 1.295 \times 10^{-11} \text{s}^{-1},
\]

and secular periods corresponding to these eigenvalues are

\[
P_+ \approx 851 \text{ yr}, \quad P_- \approx 15,382 \text{ yr}.
\]

These numerical estimates clearly illustrate that \( g_+ \approx A_i \) and \( g_- \approx A_o \) because of the smallness of \( B_i \) and \( B_o \).

Expressions (1)–(4) depend on six free parameters—\( \beta_\pm, e_{i,\pm}, e_{o,\pm} \)—but two can be eliminated using the relation (7). The remaining four parameters are fixed using the knowledge of \( k_i, h_i, k_o, h_o \) at a particular moment of time. We use the values provided in Ransom et al. (2014), \( k_i = -9.17 \times 10^{-5} \), \( h_i = 6.857 \times 10^{-3} \), \( k_o = -3.46 \times 10^{-3} \), \( h_o = 3.52 \times 10^{-2} \), which we assume to correspond to time \( t = 0 \). As a result we find

\[
e_{i,+} \approx 2.571 \times 10^{-5}, \quad e_{i,-} \approx 0.0006825,
\]

\[
e_{o,+} \approx -3.8657 \times 10^{-8}, \quad e_{o,-} \approx 0.035356,
\]

\[
\beta_+ \approx 165:42753, \quad \beta_- \approx 95:61955133.
\]

These parameters fully specify secular behavior of the system via Equations (1)–(4).

4. INNER binary

Solution for the eccentricity vector of the inner binary can be represented as the sum of the free and forced components:

\[
e_i = e_i^{\text{free}} + e_i^{\text{forced}},
\]

\[
e_i^{\text{free}} = e_{i,+} \begin{pmatrix} \cos (g_i t + \beta_i) \\
\sin (g_i t + \beta_i) \end{pmatrix},
\]

\[
e_i^{\text{forced}} = e_{i,-} \begin{pmatrix} \cos (g_i t + \beta_-) \\
\sin (g_i t + \beta_-) \end{pmatrix}.
\]

The amplitude of the free eccentricity vector is directly related to the initial conditions and characterizes the “own” eccentricity of the inner binary. Secular precession of \( e_i^{\text{free}} \) at the rate \( g_+ \), as well as the forced eccentricity \( e_i^{\text{forced}} \), arises because of the gravity of the outer WD. They would be absent if the inner binary were isolated.

This decomposition is illustrated in Figure 1, where we show the forced eccentricity vector at \( t = 0 \) (red) and its evolution as
5. OUTER BINARY

For the outer binary the eccentricity vector is decomposed as

$$\mathbf{e}_o = \mathbf{e}_o^{\text{free}} + \mathbf{e}_o^{\text{forced}},$$

(23)

$$\mathbf{e}_o^{\text{free}} = e_{o,-} \begin{pmatrix} \cos (g_{-_o} t + \beta_-) \\ \sin (g_{-_o} t + \beta_-) \end{pmatrix},$$

(24)

$$\mathbf{e}_o^{\text{forced}} = e_{o,+} \begin{pmatrix} \cos (g_{+_o} t + \beta_+) \\ \sin (g_{+_o} t + \beta_+) \end{pmatrix}.$$  

(25)

This decomposition is illustrated in Figure 3. Note that for the outer binary $\mathbf{e}_o^{\text{free}}$ circulates at the rate $g_{-_o}$, while $\mathbf{e}_o^{\text{forced}}$ librates at the rate $g_{+_o}$.

Compared to the inner binary, the relation between the free and forced eccentricity amplitudes is now reversed, and $\mathbf{e}_o^{\text{forced}}$ is completely negligible (by six orders of magnitudes) compared to $\mathbf{e}_o^{\text{free}}$, see inset in Figure 3. As a result, the evolution of $\mathbf{e}_o$ is accurately represented by the free precession at constant rate $g_{-_o}$. This is best seen in Figure 4, where $|\mathbf{e}_o|$ varies at the level of $10^{-6}$, while $\sigma_o$ varies at the $10^{-5}$ level. This is also the reason why the $\mathbf{e}_o^{\text{free}}$ (blue) and $\mathbf{e}_o$ (black) trajectories in Figure 3 fall on top of each other.

The present-day value of the precession rate for the outer WD is

$$\tilde{\sigma}_o \approx 0:0234 \text{ yr}^{-1},$$

(26)

and is not very different from $\tilde{\sigma}_i$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{inner_binary_eccentricity.png}
\caption{Evolution of the inner binary eccentricity vector $\mathbf{e}_i$ (black), decomposed into the forced (red) and free (blue) contributions, for 3400 yr from the present time. Inset focuses on the geometry of the $e_{i,\text{free}}$ precession, assuming that its guiding center is not moving. The values of $e_{i,\text{free}}$ and $e_{i,\text{forced}}$ at present epoch are shown as blue and red line segments. Note that $|e_{i,\text{free}}| \ll |e_{i,\text{forced}}|$. (A color version of this figure is available in the online journal.)}
\end{figure}
Figure 2. Evolution of orbital elements for the inner pulsar on $\sim 10^3$ yr timescales. Precession rate in the upper panel is in deg yr$^{-1}$.

Figure 3. Precession of the eccentricity vector of the outer binary during the period indicated in Figure 4. (A color version of this figure is available in the online journal.)
The small value of the forced eccentricity of the outer binary implies that its eccentricity vector \(e_o\) is always very well aligned with the forced eccentricity vector of the inner binary \(e_{\text{forced}}^i\); see Equations (19)–(21) and (23)–(25). The only significant source of misalignment between \(e_i\) and \(e_o\) is due to the non-zero value of \(e_{\text{free}}^i\). However, because \(e_{\text{free}}^i \ll e_{\text{forced}}^i\), this misalignment is also rather small. Thus, the eccentricity vectors of both binaries are guaranteed to be locked in near-alignment.

As a result, the observed close apsidal alignment between the two binaries at present epoch \(|\varpi_i - \varpi_o| = 1.9987\) does not require unique circumstances such as observing the system at a special moment of time. The inferred misalignment is in fact close to the maximum possible \(\max |\varpi_i - \varpi_o| = \operatorname{asin} \left| e_{\text{free}}^i / e_{\text{forced}}^i \right| \approx 2.16\), given the smallness of \(e_{\text{free}}^i\).

Collinearity of \(e_o\) and \(e_{\text{forced}}^i\) also explains why \(\dot{\varpi}_i\) and \(\dot{\varpi}_o\) are of the same order of magnitude—they would be exactly equal if \(e_{\text{free}}^i\) were zero. However, because \(e_{\text{free}}^i\) is not zero and the period of free precession of the inner binary is shorter than that of the \(e_{\text{forced}}^i\) circulation, we find that \(\dot{\varpi}_i\) exhibits significant variability around the value corresponding to \(\varpi^c\); see Figure 2.

Note that these conclusions rely on the use of secular approximation and strictly speaking apply to the orbital evolution of the system on long (\(\sim P_o\) and longer) intervals. In reality, orbital parameters of both binaries will also oscillate on shorter timescales (e.g., \(P_o\)) due to the short-period terms in the expansion of the disturbing function (Murray & Dermott 1999). We cannot capture such effects with our orbit-averaged approach. However, we expect the amplitude of short-term variations to be small compared to the secular ones. The general behavior of the system should still be reasonably well described by our results.

6.1. Period–Eccentricity Relation

Millisecond radio pulsars with WD companions are known to obey the so-called eccentricity–period relation \((e_b–P_b)\), which states that the eccentricity of the neutron star (NS)–WD binary \(e_b\) increases with its orbital period \(P_b\) (Lorimer 2008). Phinney (1992) suggested that this correlation emerges at the last stage of the Roche lobe overflow (RLOF) leading to the formation of the WD: random density fluctuations in the envelope of the WD progenitor caused by the convective motions induce stochastic variations of the gravitational quadrupole tensor \(Q_{ij}\) of the progenitor. This results in random quadrupole accelerations acting on the NS and drives eccentricity of the binary. Random walk of \(e_b\) is balanced by the tidal dissipation in the WD envelope, which results in a well-defined theoretical correlation between \(e_b\) and \(P_b\). This theory predicts, in particular, the rough equipartition between the kinetic energy of individual convective eddies in the envelope of the WD progenitor and the energy of eccentric motion of the binary.

More recent compilations of the binary pulsar properties (Ng et al. 2014) show that many systems deviate from the \(e_b–P_b\) relation suggested by Phinney (1992) by orders of magnitude in \(e_b\). Nevertheless, the general trend of \(e_b\) increasing with growing \(P_b\) is still observed; see Figure 5, in which we display properties of 72 binary pulsars with both CO and He WD companions. The data are from the ATNF pulsar catalog\(^1\) (Manchester et al.

\(^1\) http://www.atnf.csiro.au/people/pulsar/psrcat/
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Figure 5. Eccentricity–period ($e_b$–$P_b$) relation for binary pulsars with WD companions. Red points are for systems with CO WDs, green are for systems with He WDs. Binaries comprising the system PSR J0337+1715 are indicated with blue points: inner eccentricity $e_i$ (open hexagon), inner free eccentricity $e_{i\text{ free}}$ (filled hexagon), outer eccentricity $e_o$ (filled square), and mutual inclination $i$ (dotted line). Downward arrows are the theoretical upper limits on the mutual eccentricity given by Equations (33) for the inner and (35) for the outer binaries. Solid curve shows the theoretical $e_b$–$P_b$ relation from Phinney & Kulkarni (1994), with dashed curves encompassing 95% deviations. (A color version of this figure is available in the online journal.)

2005) and include only NS–WD systems that do not reside in globular clusters.

The system of PSR J0337+1715 provides two tests of the $e_b$–$P_b$ relation, for both binaries. This is because the system must have undergone two RLOF episodes, within the inner and outer binaries, to arrive to its present configuration with two WDs (TvdH14).

Present-day eccentricity of the inner binary, $e_i \approx 6.9 \times 10^{-4}$, is more than two orders of magnitude larger than $e_b \sim 10^{-6}$ implied by the $e_b$–$P_b$ relation of Phinney (1992); see Figure 5. However, as mentioned in Section 4, almost all of this eccentricity is forced by the gravitational perturbations due to the outer WD. Only the free eccentricity $e_{i\text{ free}} = e_{i+,} \approx 2.6 \times 10^{-5}$, which is much smaller than $e_i$, carries information about the initial conditions for the evolution of the inner binary. Even this low value is still about an order of magnitude higher than the prediction of Phinney (1992); see Figure 5. However, the same figure also shows several other NS–WD systems with $P_b$ of order several days and $e_b$ deviating from this relation upward by at least an order of magnitude. Thus, the outer binary of PSR J0337+1715 is not unique and have been previously known.

6.2. Origin of the Binary Eccentricities

We now explore what do the measurements of binary eccentricities in the PSR J0337+1715 triple tell us about the past history of the system and the $e_b$-excitation mechanisms, focusing on the theory of Phinney (1992).

During the RLOF phase of the inner binary its eccentricity has already been driven by the outer WD. Tidal dissipation in the inner WD damps $e_i$ on the characteristic circularization timescale (Goldreich & Soter 1966; Correia 2009; Correia et al. 2011):

$$ t_{\text{tid}}^e = \frac{2}{21} \frac{k_2}{k_1} \frac{m_i}{m_p} \left(\frac{a_i}{R_i}\right)^5 \approx 1.6 \times 10^5 \text{ yr} \left(\frac{0.227}{R_i/a_i}\right)^5, $$

assuming present-day orbital parameters of the inner binary and $R_i \approx 0.227 a_i$ given by the Roche radius formula of Eggleton (1983).
it was engulfing the inner binary. During this stage that could have lasted for $\sim 17 \text{ Myr (TvdH14))}$ gravitational torques acting on the inner binary must have reduced its inclination to zero: the angular momentum deposited into the disk by the inner binary, regardless of whether it was able to clear out an inner cavity in the disk or not, gets communicated by pressure and viscous stresses out to the outer binary, effectively damping their mutual inclination (Artymowicz 1994; Terquem 1998).

After the complete orbital alignment of the two binary planes, their non-zero mutual inclination could have been excited by the same underlying mechanism as the binary eccentricity, or a completely different one. Here we will focus on the former possibility, i.e., epicyclic excitation by random forces due to stochastically generated quadrupole in the convective envelope of the WD progenitor (Phinney 1992). This mechanism should naturally drive out-of-plane motion.

Indeed, the rms radial and vertical components of the pulsar acceleration due to stochastically varying WD quadrupole are

$$a_{r,\text{rms}} \approx \frac{3}{2} \frac{GQ_{ij,\text{rms}}}{a^3}, \quad a_{z,\text{rms}} = \frac{GQ_{ij,\text{rms}}}{a^3}; \quad (28)$$

where $Q_{ij}$ are the rms values of the relevant diagonal and off-diagonal components of the WD quadrupole tensor. Since non-zero $Q_{ij}$ is due to a superposition of many randomly varying convective eddies, one can show (Phinney 1992) that $Q_{rr,\text{rms}} = (3/4)^{1/2} Q_{zz,\text{rms}}$, so that

$$\frac{a_{z,\text{rms}}}{a_{r,\text{rms}}} = 3^{-1/2}, \quad (29)$$

i.e., inclination is driven somewhat less efficiently than eccentricity.

Excitation of $e_b$ and mutual inclination $i$ by the WD quadrupole is balanced by the tidal dissipation and one expects (Phinney 1992) their equilibrium values to scale as $e_b \propto (a_i \text{tid})^{1/2} a_{r,\text{rms}}, \quad I \propto (a_i \text{tid})^{1/2} a_{z,\text{rms}}$, where $a_i \text{tid}$ is given by Equation (27) and $t_{i \text{tid}}$ is the inclination damping time. Thus, one expects

$$\frac{I}{e_b} \approx \frac{a_{z,\text{rms}}}{a_{r,\text{rms}}} \left( \frac{t_{i \text{tid}}}{t_{i \text{tid}}} \right)^{1/2}. \quad (30)$$

Note that this relation does not make assumptions about the physical nature of the random acceleration; however, the value of $I/e_b$ depends on the knowledge of $t_{i \text{tid}}$.

Because of tidal dissipation, over the long time interval a hierarchical triple system like PSR J0337+1715 tends to converge to an equilibrium configuration in which the inner and outer orbits are coplanar and the inner WD obliquity $\theta_i$—the angle between its spin axis $S_i$ and the orbital angular momentum $L_i$ of the inner binary—is zero. In this state $L_i$ is aligned with the orbital angular momentum of the outer binary $L_o$, which dominates over $L_i$, and the WD rotation is synchronized with the orbital motion (we neglect the eccentricity of the inner binary).

Next we discuss the way in which convergence to this equilibrium state takes place and the value of inclination damping timescale $t_{i \text{tid}}$. We do this for two alternative scenarios of the inclination driving, depending on whether it is excited by processes operating during the RLOF phase of the inner or outer binaries.

### 6.3.1. Excitation of $I$ during the Inner RLOF Phase

TvdH14 advocate an evolutionary scenario in which the inner WD undergoes RLOF after the outer one. We explore whether
the mutual inclination could have been imprinted at the end of this phase, simultaneously with the free eccentricity excitation for the inner binary.

Random torques driven by the convection in the WD progenitor excite both the mutual inclination of the two binaries $\delta I$ (assuming initial coplanarity) and the obliquity of the inner WD $\delta \theta$. Angular momentum conservation ensures that

$$\delta \theta = \frac{|L_e|}{|S_i|} \delta I,$$  

(31)

so that $\delta \theta \gg \delta I$ since $|L_e| \gg |S_i|$ for a synchronized WD progenitor. Obliquity of the WD damp on the synchronization timescale (Correia et al. 2011)

$$t_{\text{sync},i} = \frac{2 \xi Q}{3 k_2} m_i (m_p + m_i) \left( \frac{a_i}{R_i} \right)^3 \approx 600 \text{yr} \frac{Q/k_2 \cdot 0.227}{10^7 \cdot 0.01 \frac{R_i}{a_i}},$$

(32)

(here $\xi$ is the principal moment of inertia of the inner WD normalized by $m_i R_i^2$), which is much shorter than the circularization timescale $t_{\text{tid}}^i$; see Equation (27). Because of the relation (31) mutual inclination decays due to tides on the same short timescale $t_{\text{sync},i}$, simultaneous with the excitation by random quadrupole torques. Substituting $t_{\text{sync},i}$ for $t_{\text{tid}}^i$ in Equation (30) we find

$$\frac{I}{e_i} \approx \chi \left( 7 \xi \frac{m_i + m_p}{m_p} \right)^{1/2} \frac{R_i}{a_i} \approx 0.064 \chi \left( \frac{\frac{e_i}{0.01}}{0.227} \right)^{1/2},$$

(33)

where we defined $\chi \equiv a_{rms}^m/a_{rms}^i$.

Equation (33) implies that even for $\chi = 1$ the mutual inclination $I$ excited during the inner binary RLOF should be much smaller than the free eccentricity of the inner WD, at the level of $\lesssim 10^{-6}$. This is because spin angular momentum of the pulsar companion is much smaller than the orbital angular momentum of the binary. However, the present-day value of $I = i$ is about an order of magnitude higher than $e_{rms}^i$, $i/e_{rms}^i \approx 8.2$, exceeding the prediction (33) by more than two orders of magnitude; see the upper limit at $P_i$ in Figure 5.

The discrepancy can in fact be even worse as the assumption of $\chi \ll 1$ suggested by the estimate (29) may be too optimistic. Indeed, the final value of $i$ gets established at the freeze-out time when $t_{\text{sync},i}$ becomes comparable to the WD evolution timescale at a stage when mass transfer stops and $R_i$ becomes smaller than the Roche radius. Since $t_{\text{sync},i} \ll t_{\text{tid}}^i$ and $t_{\text{sync},i}$ scales with $R_i/a_i$ slower than $t_{\text{tid}}^i$, the inclination freeze-out must occur considerably later than the eccentricity freeze-out of the inner binary. At that stage the convective envelope of the WD progenitor is less massive than at $e_i$ freeze-out, resulting in less vigorous quadrupole fluctuations and $a_{rms}^m$ being likely smaller than $a_{rms}^i$ was when $e_i$ attained its final value.

As a result, one should expect $\chi \ll 1$, exacerbating the discrepancy between the theoretical and measured values of $i$. For these reasons we believe that the present-day mutual inclination could not have been established at the end of the RLOF phase of the inner binary.

In conclusion we would like to address a subtle point related to the energy equipartition between individual convective eddies and the binary epicyclic motion. Phinney (1992) has shown that the mean energy of the random epicyclic motion in the binary plane $E_e = (1/2)\mu v_K^2 e_i^2$ ($\mu$ is the reduced mass, $v_K$ is the Keplerian speed) is of order the kinetic energy of an individual eddy driving $Q_{ij}$ fluctuations. Equation (33) then suggests that the mean energy of the random epicyclic motion normal to the binary plane $E_i = (1/2)\mu v_K^2 I^2 \ll E_e$ and is not in equipartition with individual convective eddies.

This paradox is easily resolved when one notices that inclination variations unavoidably cause much larger obliquity variations of the WD progenitor; see Equation (31). Because both are driven by equal (but opposite) torques, the energy stored in the obliquity wobble of the pulsar companion $E_\theta$ must be larger than $E_i$ by a factor $\delta \theta/\delta I = |L_e|/|S_i| \gg 1$. Using Equations (29) and (33) one then trivially finds that $E_\theta \approx (7/3)E_e$. Thus, the full binary energy $E_\theta + E_i \approx E_\theta$ associated with the out-of-plane torques is in fact in equipartition with the kinetic energy of individual eddies in the WD progenitor envelope.

### 6.3.2. Excitation of $I$ during the Outer RLOF Phase

Results of Section 6.3.1 can be trivially extended to study the possibility of the mutual inclination excitation by the random $Q_{ij}$ fluctuations during the RLOF phase of the outer binary. The same logic applies in that case as well if we consider inner binary as a point mass (which is reasonable for an hierarchical system) and replace $m_p \rightarrow m_p + m_i$ and $m_i \rightarrow m_i$ in all equations. As a result, we find that random fluctuations of the mutual inclination during the outer RLOF get damped on timescale:

$$t_{\text{sync},o} = \frac{2 \xi Q}{3 k_2} m_o m_3 (m_p + m_i) \left( \frac{a_o}{R_o} \right)^3 \approx 1.6 \times 10^5 \text{yr} \frac{Q/k_2 \cdot 0.01}{10^7 \cdot 0.01},$$

(34)

assuming the present-day masses of all components. If we follow TvdH14 and adopt $m_p = 1.3 M_\odot$ and $m_i = 1.12 M_\odot$ (mass of the inner WD progenitor) during the outer binary RLOF, prior to the inner RLOF episode, this timescale does not change significantly, $t_{\text{sync},o} \approx 10^5$ yr. This is shorter than the expected duration of the outer RLOF phase, $\sim 17$ Myr (TvdH14).

Analogously, Equation (33) becomes

$$\frac{I}{e_o} \approx \chi \left( 7 \xi \frac{m_i}{m_i + m_p} \right)^{1/2} \frac{R_o}{a_o} \approx 0.08 \chi \left( \frac{\xi}{0.01} \right)^{1/2} \frac{R_o}{a_o} 0.268,$$

(35)

where we used $R_o = 0.268 a_o$ for $q = m_o/(m_p + m_i) = 0.251$. Adopting the parameters of the inner binary before its RLOF phase, we get $q = m_o/(m_p + m_i) = 0.17$, $R_o = 0.24 a_o$, and essentially the same value of $I/e_o$.

This estimate shows that if the observed outer binary eccentricity $e_o \approx 0.035$ was excited by randomly varying $Q_{ij}$ of the WD progenitor at the end of the RLOF phase, then the same process must have given rise to the mutual inclination $I \approx 2.8 \times 10^{-5} \chi (\xi/0.01)^{1/2}$. This can be easily reconciled with the present-day value of $i = 2.1 \times 10^{-4}$ (see the upper limit on $I$ at $P_o$ in Figure 5) if either $\xi \lesssim 10^{-2}$, which is quite plausible for an extended WD progenitor in long-period orbit, or $\chi \lesssim 1$, which is also expected, as we described in Section 6.3.1.
However, this comparison of $I$ and the current mutual inclination $i$ is meaningful only if $I$ did not change since the outer binary RLOF phase, which may have occurred ~5 Gyr ago (TvdH14). Mutual inclination would decay because for non-zero $I$ the steady state obliquity $\theta_i$ of the inner pulsar companion (WD or its progenitor) does not converge to $\theta_i = 0$. Instead, tidal dissipation in the companion drives the system toward the so-called Cassini state (Colombo 1966; Peale 1969), in which the companion spin vector $\mathbf{S}_i$, the angular momentum of the inner binary $\mathbf{L}_i$, and the total angular momentum of the system $\mathbf{J}$ are coplanar and precess at the same rate $\lambda_{\text{sec}}$. Convergence of the companion spin to this configuration happens on relatively short synchronization timescale $t_{\text{sync}}$ given by Equation (32).

Obliquity in the low-obliquity Cassini state (the one relevant for our purposes) is non-zero and is given in the case of weak dissipation by $\theta_{\text{Cas}} \approx \lambda_i^{-1} I$ (Ward & Hamilton 2004), where the dimensionless parameter

$$\lambda_i = \frac{2}{3} \frac{k_2}{\xi} \frac{m_p (m_p + m_i)}{m_p m_o} \left( \frac{R_i}{a_i} \right)^3 \left( \frac{a_o}{a_i} \right)^3 \approx 1.1 \times 10^4 \frac{k_2}{\xi} \left( \frac{R_i/a_i}{0.227} \right)^3,$$

is the ratio of the pulsar companion spin precession rate to the nodal precession rate of the inner binary. Tidal dissipation affects the value of $\theta_{\text{Cas}}$ (Fabrycky et al. 2007) but for $\lambda_{\text{sec}} t_{\text{tid}} \gg 1$ the effect is small and will be neglected. The numerical estimate in Equation (36) applies during the inner binary RLOF, when $\lambda_i \gg 1$. During other evolutionary phases $R_i/a_i$ is smaller and $\lambda_i$ is lower. But in any case it is reasonable to expect $\lambda_i \gtrsim 1$ so that $\theta_{\text{Cas}} \lesssim I$.

Non-zero obliquity in the presence of tidal dissipation results in a torque acting on the inner binary, which over time damps the mutual inclination $I$ of the two orbits. Its evolution is described by (Correia et al. 2011)

$$\frac{dI}{dt} = \frac{-\sin \theta_i}{t_I},$$

where $t_I \equiv \tau_{\text{tid}} \gg t_{\text{sync}}$. Thus, $\mathbf{S}_i$ settles into a Cassini state before $I$ has a chance to change. Plugging the dissipationless $\theta_{\text{Cas}} \approx \lambda_i^{-1} I$ for the Cassini state into Equation (37) we find that subsequently $I$ decays on a timescale

$$t_{\text{Cas}} = \lambda_i t_I = \frac{4}{9} \frac{Q}{\xi} \frac{m_i + m_p}{m_p} \left( \frac{a_o}{a_i} \right)^3 \left( \frac{a_i}{R_i} \right)^2 \approx 12 \text{ Gyr} \frac{Q/\xi}{10^7} \left( \frac{R_i/a_i}{0.227} \right)^2;$$

see Equation (27).

Now we need to distinguish two possibilities. First, TvdH14 favor the evolutionary scenario in which the inner binary underwent the RLOF phase after the outer one. Given the scaling $t_{\text{Cas}} \propto R_i^{-2}$ one expects fastest $I$ decay to occur during the RLOF. But even then the estimate (38) shows that $t_{\text{Cas}}$ is much longer than the expected duration of the inner RLOF episode, ~2 Gyr (TvdH14). As a result, mutual inclination does not decay during this phase by more than ~20%.

Before and after that phase, the inner WD or its progenitor have $R_i/a_i$ smaller than during the overflow, resulting in much longer $t_{\text{Cas}}$. Thus, $I$ stays constant during these time intervals.

Second, TvdH14 also do not exclude the possibility of the outer WD to be the last one to form via the RLOF, after the inner WD has already formed. In this case $R_i/a_i$ is always very small and $t_{\text{Cas}}$ is much longer than the lifetime of the system.

The high degree of the orbital coplanarity of the system also strongly argues against the outer binary eccentricity $e_o$ being excited by some external process, e.g., gravitational perturbation by a passing star. Such perturbation is expected to excite $I$ on par with $e_o$, so that an encounter driving $e_o$ two orders of magnitude stronger than $i$ seems extremely improbable. Needless to say, a stellar encounter is in any case highly unlikely for an object in the field as excitation of $e_o = 0.035$ would require stellar passage within ~10 AU from the system (Heggie & Rasio 1996).

Finally, Mardling (2010) found that mutual inclination in a hierarchical triple system can be excited starting from a very small value as long as the inner and outer orbits have non-zero eccentricities. We have found, however, that for the parameters of PSR J0337+1715 this mechanism does not affect mutual inclination at an appreciable level over the lifetime of the system.

Thus, we conclude that the combination of arguments presented here and in Section 6.3.1 does not contradict the scenario in which the present-day mutual inclination $i$ was excited by convectively driven random $Q_i$ fluctuations at the end of the outer binary RLOF phase, simultaneous with the excitation of its eccentricity $e_o$. On the other hand, we cannot exclude the possibility that $I$ and/or $e_o$ were produced by a mechanism completely unrelated to the stochastic $Q_i$ variations of the WD progenitor, e.g., as a result of incomplete damping of the mutual misalignment by disk torques during the outer binary RLOF.

This possibility may be hinted at by the large difference between $e_o$ and the prediction of Phinney (1992); see Section 6.1. Finally, the weak decay of mutual inclination after its excitation in the outer binary does not allow us to distinguish between the two alternative evolutionary scenarios presented in TvdH14, in which the outer WD is either the first or last one to form.

### 6.4. Implications for Timing Measurement

Strong Newtonian three-body coupling between the components of the PSR J0337+1715 system should make the measurement of the GR effects in it difficult, even despite the high timing accuracy of this pulsar (Ransom et al. 2014). In particular, as shown in Section 4, apsidal precession of the inner binary is no longer set by the GR precession alone but is instead strongly suppressed by the three-body effects, so that $\delta\varpi \approx 0.24\delta\varpi_{\text{GR}}$.

At the same time, we find that measurement of $\delta\varpi_i$ is quite sensitive to the actual value of $\delta\varpi_{\text{GR}}$. Artificially varying $\delta\varpi_{\text{GR}}$ by 1% results in roughly 4% variation of $\delta\varpi_i$. This is especially surprising given that $\delta\varpi_{\text{GR}}$ provides only a 29% contribution to $A_1$; see Equations (8)–(10). Such a disproportionate response can be understood if one manipulates the expression for $\delta\varpi_i$ using solutions (1)–(2) and the fact that $e_{i,1}/e_{o,1} \gg e_{i,-}/e_{o,-}$, $g_i \approx A_1$, $g_{-} \approx A_0$. As a result, one can write the following expression for the apsidal rate of the inner binary at the present time:

$$\delta\varpi_i(t_0) \approx A_1 + B_i \frac{\mathbf{e}_i \cdot \mathbf{e}_o}{e_i^2},$$

where $\mathbf{e}_i$ and $\mathbf{e}_o$ are the eccentricity vectors of the inner and outer binaries at present day.

This formula clearly shows the near cancellation of the two large contributions (two terms on the right-hand side), since $\delta\varpi_i(t_0) \ll A_1$ according to our estimate (22). Because of that, even a small variation in the value of $A_1$ in Equation (39), e.g.,
due to the deviation of $\tilde{\sigma}_{\mathrm{GR}}$ from the GR prediction (Damour & Taylor 1992), drives large an important role in the dynamics of this hierarchical system. Gravitational perturbations due to the outer WD endow the inner binary with a large forced eccentricity and strongly suppress its apsidal precession rate $\tilde{\sigma}_i$, to about 24% of the rate predicted by the general relativity. At the same time, $\tilde{\sigma}_i$ is still quite sensitive to the non-Newtonian contribution to the precession rate and may be used to constrain alternative gravity theories provided that a good measurement of $\tilde{\sigma}_i$ is available. The small value of the free eccentricity \( e_i^{\text{free}} \approx 2.6 \times 10^{-5} \) of the inner binary compared to the forced one and the vanishing forced eccentricity of the outer, eccentric binary naturally result in apsidal near-alignment of the two binaries.

These results help understand the evolutionary pathways leading to the current dynamical configuration of the system. Free eccentricity of the inner binary exceeds the value predicted by the theoretical $e_i-P_0$ relation of Phinney (1992) by more than an order of magnitude. Thus, both the inner and outer binaries of this hierarchical triple belong to the population of the NS–WD binaries exhibiting eccentricity in excess of that predicted by Phinney (1992).

Moreover, the non-zero mutual inclination $i$ between the two orbits provides us with a unique chance to explore the anisotropy of epicyclic motion excitation in NS–WD binaries. We find it unlikely that $i$ was excited, simultaneous with eccentricity, by random torques due to convective eddies in the WD progenitor during the RLOF in the inner binary.

On the other hand, the smallness of the mutual inclination compared to the outer binary eccentricity argues in favor of simultaneous excitation of $i$ and $e_0$ by random torques during the outer binary RLOF. Inclination driven during this phase should persist until now despite the complexity of the intermediate evolutionary stages. These facts may provide useful clues for understanding epicyclic excitation in the NS–WD binaries in general.

I am grateful to Scott Ransom, Jihad Touma, and Jeremy Goodman for useful discussions and to Cherry Ng for providing the data on theoretical $e_i-P_0$ relation.

7. SUMMARY

We studied secular dynamics of a recently discovered triple system containing radio pulsar PSR J0337+1715 and two WDs. Three-body interactions play an important role in the dynamics of this hierarchical system. Gravitational perturbations due to the outer WD endow the inner binary with a large forced eccentricity and strongly suppress its apsidal precession rate $\tilde{\sigma}_i$, to about 24% of the rate predicted by the general relativity. At the same time, $\tilde{\sigma}_i$ is still quite sensitive to the non-Newtonian contribution to the precession rate and may be used to constrain alternative gravity theories provided that a good measurement of $\tilde{\sigma}_i$ is available. The small value of the free eccentricity \( e_i^{\text{free}} \approx 2.6 \times 10^{-5} \) of the inner binary compared to the forced one and the vanishing forced eccentricity of the outer, eccentric binary naturally result in apsidal near-alignment of the two binaries.

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