Benchmarking Quantum State Transfer on Quantum Devices
using Spatio-Temporal Steering

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Quantum state transfer (QST) provides a method to send arbitrary quantum states from one
system to another. Such a concept is crucial for transmitting quantum information into the quantum
memory, quantum processor, and quantum network. The standard criteria of QST are based on the
fidelity between the prepared and received states. However, a non-vanishing fidelity is obtained even
when the received quantum states are described using the classical model, called the local-hidden
state (LHS) model. With the above definition of the classical QST process, in this work, we quantify
the non-classicality of a QST process by measuring the spatio-temporal steerable. We show that
the spatio-temporal steerable is preserved when the perfect QST process is successful. Otherwise,
it decreases under imperfect QST processes. Therefore, the failure of the LHS model implies not
only the achievement of spatio-temporal steering but also QST non-classicality. We then apply the
spatio-temporal steerable measurement technique to benchmark quantum devices including the
IBM quantum experience and QuTech quantum inspire under QST tasks. The experimental results
show that the spatio-temporal steerable decreases as the circuit depth increases, and the reduction
agrees with the noise model, which refers to the accumulation of errors during the QST process.
Moreover, we show that the no-signaling in time condition could be violated because of the intrinsic
non-Markovian effect of the devices.

I. INTRODUCTION

A reliable quantum state transfer (QST) from the sender to receiver is an important protocol for both
quantum communication and scalable quantum computation [1, 2]. For instance, from the perspective of communication, it can be used to store (read) the quantum information into (out of) the quantum memories and processor [3–5]. Moreover, QST can be employed to entangle two systems without the bridge in the architecture of the superconducting qubit (in general, the communication line in the quantum network) by transmitting one of the entangled systems constructed by the bridge into the desired quantum device [6–8]. To implement the QST, one can rely on the SWAP operation [9] or the quantum teleportation [10] between the sender and the receiver. For hybrid quantum systems e.g., phonons in ion traps [11], spin chain [12, 13], electro-optic [14], and bosonic quantum systems [15], interaction between the sender and receiver through the communication line is required.

A similar concept is known as spatial steering, which states that the quantum states can be remotely prepared using entangled pairs. It was first proposed by Schrödinger [16] against the famous thought experiment called the Einstein-Podolsky-Rosen paradox [17]. The mathematical formulation of spatial steering was proposed recently [18–21]. Spatial steering plays a crucial role in many quantum information tasks, such as the channel discrimination problem [22–24], one-sided quantum key distribution [25], measurement incompatibility [21, 26–29], and no-cloning principle [30]. Similar to the analogy between Bell and Leggett-Garg (LG) inequalities [31–34], temporal steering [35–37] is also proposed as the temporal analogue of spatial steering. Such a non-classical temporal quantum correlation can be used to quantify the non-Markovianity [38, 39], witness quantum scrambling [40], and certify quantum key distribution [37]. Recently, spatio-temporal steering, which is defined similarly to the Bell-LG inequality [41], was proposed [42] to certify the nonclassical correlations in a quantum network [43].

In this work, we employ the spatio-temporal steerability to quantify the non-classicality of the QST process. More specifically, because the received assemblage (an ensemble of the quantum states) can be described by the local-hidden state (LHS) model, which is a free resource in the context of the resource theory of steering [44, 45], the QST process can be simulated by the classical strategy. Here, the classical strategy represents that the received assemblage can be achieved by the receiver’s ontic states together with some stochastic maps without using the QST process. We further show that the “perfect” QST process preserves spatio-temporal steerability.

We then utilize the quantification of spatio-temporal steerability (or the QST non-classicality) to benchmark noisy intermediate-scale quantum devices [46] including the IBM quantum experience [47] and QuTech quantum inspire [48]. Such quantum devices can now be applied to implement some quantum algorithms [49–52]. In general, benchmarks of quantum devices provide us with a simple method to evaluate the performance of the quantum devices under certain quantum information tasks, e.g., benchmarking the shallow quantum circuits [53], the non-classicality for qubit arrays [54], quantum chemistry [55],
II. BENCHMARKING THE QUANTUM STATE TRANSFER WITH SPATIO-TEMPORAL STEERING

In this section, let us briefly recall the quantum state transfer (QST) and the spatio-temporal steering (STS) scenario [42] in terms of the language of the quantum information science. We will also discuss the similarities between them and demonstrate how to quantify the non-classicality of the QST process in the context of STS.

A. Quantum State Transfer

The protocol for QST is depicted by a sender (Alice) who prepares an arbitrary quantum state $\rho_{A_0}$ and a receiver (Bob) who then receives the transferred state $\rho_B$. Without loss of generality, the state transfer process can be described using a global quantum channel $\Lambda_t$. The result before the maintenance violates the no-signaling in time condition [59–63], which is possibly due to the gate error and the non-Markovian effect in the bridge of the superconducting qubit [64–66].

$$\rho_B = \text{Tr}_A [\Lambda_t (\rho_{A_0} \otimes \sigma_{B_0})], \quad (1)$$

where $\sigma_{B_0}$ is the initial state of Bob. Here, we use the subscript $t$ to represent the time which will be used later. The process of the QST is perfect if the fidelity of the prepared and received states under a suitable unitary operation is unity [1]. We note that Eq. (1) can be easily expanded to the multiparty scenario in which intermediate qubits between Alice and Bob act as a bridge helping the transformation. For instance, the states can be perfectly transferred in a spin chain model with $XY$ coupling [2, 12] or with $XYZ$ coupling [13]. We will experimentally present an explicit example in the cloud based on the $XY$ interaction to implement the QST process in Sec. III B.

B. Spatio-Temporal Steering

In the STS scenario, a bipartite system is shared by Alice and Bob. At initial time $t = 0$, Alice performs local measurements labeled as $x$ with the corresponding outcomes labeled as $a$. After Alice’s measurement, the bipartite system is then sent into a global quantum channel $\Lambda_{t}$. Finally, Bob receives a set of quantum states denoted as $\{\hat{\rho}_{a|x}(t)\}_{a,x}$. Without loss of generality, one can use the terminology in the standard spatial steering [18, 19] which is termed as the assemblage $\{\hat{\rho}_{a|x}(t) := PA(a|x)\hat{\rho}_{a|x}(t)\}_{a,x}$ to characterize the spatio-temporal steerability. Here, $PA(a|x)$ describes the probability of obtaining the output $a$ conditioned on Alice’s choice of measurement $x$, and $\hat{\rho}_{a|x}(t) = \rho_{a|x}(t)/PA(a|x)$ is the conditional quantum state received by Bob. According to quantum theory, when Alice and Bob share an initial separable state, i.e. $\sigma_{A_0} \otimes \sigma_{B_0}$, the states received by Bob after the channel can be expressed as

$$\hat{\rho}_{a|x}(t) = \text{Tr}_A [\Lambda_t (\rho_{a|x}(0) \otimes \sigma_{B_0})], \quad (2)$$

where $\rho_{a|x}(0) = (M_{a|x}\sigma_{A_0}M_{a|x}^\dagger)/PA(a|x)$, and the $\{M_{a|x}\}$ is considered to be a set of projective measurements. Next, we consider the initial states to be $I/2$ and $|0\rangle/0$ for Alice and Bob, respectively.

We call the assemblage $\{\hat{\rho}_{a|x}(t)\}_{a,x}$ spatio-temporal unsteerable if it agrees with the local-hidden-state (LHS) model [18, 67], namely

$$\hat{\rho}_{a|x}^{LHS}(t) = \sum_{\lambda} P(\lambda) PA(a|x,\lambda)\sigma(\lambda) \quad \forall \ a, x, \quad (3)$$

such that the assemblage can be constructed by an ensemble of ontic states $\{P(\lambda), \sigma(\lambda)\}_{\lambda}$ together with the stochastic map $\{PA(a|x,\lambda)\}_{\lambda}$, which maps the local hidden variable $\lambda$ to $a|x$. In other words, an assemblage can be described by the LHS model, whenever it can be explained classically. Because the set of LHS models forms a convex set, we can, in general, quantify the spatio-temporal steerability by the notion of the spatio-temporal steering robustness $STSR$ [42, 68], which is defined as follows:

$$STSR(\{\hat{\rho}_{a|x}(t)\}) = \min_{r,\{\tau_{a|x}\}: \{\hat{\rho}_{a|x}^{LHS}(t)\}} r,$$

s.t.

$$\frac{1}{1 + r} \hat{\rho}_{a|x}(t) + \frac{r}{1 + r} \tau_{a|x} = \hat{\rho}_{a|x}^{LHS}(t) \quad \forall \ a, x. \quad (4)$$

The optimal solution $r^*$ in Eq. (4) can be interpreted as the minimal amount of noisy assemblage $\{\tau_{a|x}\}$ required to destroy the spatio-temporal steerability of the underlying assemblage $\{\hat{\rho}_{a|x}(t)\}$. The optimization problem can be computed by the semidefinite program presented in Appendix A.

To obtain the spatio-temporal steerability quantum mechanically [42, 68], the assemblage should satisfy the no-signaling in time (NSIT) condition [60, 62, 69], that is, the underlying assemblage obeys the following condition:

$$\sum_{a} \hat{\rho}_{a|x}(t) = \sum_{a} \hat{\rho}_{a|x'}(t) \quad \forall \ x \neq x'. \quad (5)$$

and quantum devices [56–58]. Our experimental results show that the degrees of QST non-classicality decreases as the circuit depth increases. In addition, the decrease agrees with the noise model, which describes the accumulation of noise (qubit relaxation, gate error, and readout error) during the QST process. In general, the results for the IBM quantum experience show that it outperforms QuTech quantum inspire from the viewpoint of STS. The result before the maintenance violates the no-signaling in time condition [59–63], which is possibly due to the gate error and the non-Markovian effect in the bridge of the superconducting qubit [64–66].
Once the NSIT condition is violated, the obtained spatio-temporal steerability can be explained classically. Actually, one can always violate the spatio-temporal steering inequality using additional classical communication from Alice to Bob. A similar situation has been reported as a communication loophole in the spatio-temporal scenario [70] and the clumsiness loophole in the spatio-temporal quantum correlations [63, 66]. Recent work [68] has further suggested that the signaling effect can be quantified using the trace distance, and such a quantification is related to the STSR, where Alice is only allowed to measure two different bases. Here, we generalize the quantification of the signaling effect to arbitrary measurement inputs (available for Alice) called

$$D = \max_x \frac{1}{2} || \sum_a \varrho_{a|x}(t) - \sum_a \varrho_{a|x'}(t) ||_1 \quad \forall \ x \neq x' \,. \quad (6)$$

The proof is given in Appendix C.

C. Quantifying the non-classicality of QST using STS

We can observe that Bob’s received states for STS and QST processes are basically in the same form [see Eq. (1) and Eq. (2)], indicating that a QST process can be discussed from the viewpoint of STS. Because the QST task is perfect if and only if the fidelity between the prepared state and the received state up to an unitary operation is unity [1], the perfect QST must reliably transfer a steerable assemblage from Alice to Bob and lead to the unchanged spatio-temporal steerability. In addition, from the perspective of QST, the spatio-temporal unsteerable assemblage, which can be described using an LHS model in Eq. (3), can also be seen as the classical strategy for state transfer, where Bob can locally generate the received states so that the classically transferred state reads $\tilde{\varrho}_{a|x}(t)$. Based on such insights, the STSR, which is used to quantify the spatio-temporal steerability, can also be used to quantify the non-classicality of QST.

The advantage of employing the STSR is twofold. First, we can show that the STSR of the underlying assemblage is invariant under an arbitrary unitary transformation $\hat{U}$; i.e.,

$$STS\mathcal{R}(\{\varrho_{a|x}\}) = STSR(\{\hat{U}\varrho_{a|x}\hat{U}^\dagger\}) \quad (7)$$

(see Appendix B for the derivation). Accordingly, the local unitary operation for verifying the fidelity after the QST process is unnecessary when considering the STS scenario. Second, because steering-type scenarios are one-sided device independent [19, 21], it means that to certify the QST non-classicality from the perspective of STS, we do not have to characterize Alice’s measurement operators $\{M_{a|x}\}$ and the post-measurement states $\varrho_{a|x}$ (the prepared states) before sending the system into global operation $A_x$. Instead, Alice’s measurement results are only summarized in the probability distribution $P_A(a|x)$, and the transferred states for Bob are unknown to him before the state tomography is performed. Therefore, to certify the QST non-classicality, we do not have to verify all possible prepared states, where for single qubit system, all possible states are described by all points in the Bloch sphere.

III. EXPERIMENTAL REALIZATION

In this section, we provide a scalable circuit, which can be used to implement the $n$-qubit QST as shown in Fig. 4. Alice prepares the states in $Q_1$, and Bob receives the transferred states in $Q_n$. To calculate the spatio-temporal steering robustness ($STSR$), we introduce the preparation method of the assemblage, the quantum state transfer process, and both of their circuit implementations. Moreover, we discuss the ideal theoretical results and model the noise effect by introducing extra qubit decoherence described by the Lindblad master equation.

A. State preparation

Because the IBM quantum experience does not allow one to access the post-measured states after Alice’s measurements, we prepare six eigenstates of Pauli matrices being Alice’s post-measurement states with indexes $x \in \{1, 2, 3\}$ and $a \in \{0, 1\}$. Note that one can use the ancilla qubit, the CNOT operation, and the measurement operation on the ancilla qubit to replace the measurement operation on the system qubit [66]. Nevertheless, we consider the state preparation to avoid further errors from the CNOT operation. Note that the gate fidelity and the execution time of the CNOT operation are both at least 10 times larger than the single qubit operation. Thus, to decrease the errors, the number of CNOT operations should be as less as possible. The initial state of the qubits on IBM quantum experience is always in $|0\rangle$. We can prepare $\tilde{\varrho}_{a|x}$ by applying the corresponding $u_3(\delta, \phi, \xi)$ operation at $Q_1$, mathematically as follows:

$$\tilde{\varrho}_{a|x} = u_3(\delta, \phi, \xi)|0\rangle\langle 0|u_3^\dagger(\delta, \phi, \xi) \quad (8)$$

with the matrix representation of the $u_3(\delta, \phi, \xi)$ operation being

$$u_3(\delta, \phi, \xi) = \left( \begin{array}{cc} \cos \frac{\delta}{2} & -e^{i\phi} \sin \frac{\delta}{2} \\ e^{i\phi} \sin \frac{\delta}{2} & e^{i(\xi+\phi)} \cos \frac{\delta}{2} \end{array} \right) \quad (9)$$

Because we prepare the above states uniformly, $P_A(a|x) = 0.5 \quad \forall \ a, x$, and the corresponding assemblage can be obtained by performing Pauli measurements on the maximally mixed state $\mathbb{1}/2$. The above assemblage satisfies the NSIT condition in Eq. (5) and can maximize the spatio-temporal steering robustness [21, 71].
B. Quantum state transfer process

We consider a QST process described by an n-qubit chain, as shown in Fig. 1, with each qubit labelled as $Q_l$, where $l = 1, 2, \ldots, n$. In this process, Alice prepares the states in $Q_1$, and after the QST process, Bob will receive the transferred states in $Q_n$. We consider a QST procedure, which involves several iterations of quantum operations. For each iteration, we turn on the qubit-qubit interaction between $Q_l$ and $Q_{l+1}$, and then, turn it off when the QST from $Q_l$ to $Q_{l+1}$ is accomplished. Here, the “closed” interaction can be represented by the identity operator in the interaction Hamiltonian. The interaction Hamiltonian between $Q_l$ and $Q_{l+1}$ \[ H_{l,l+1} = \hbar J (\sigma_l^+ \sigma_{l+1}^- + \sigma_l^- \sigma_{l+1}^+) \] (10) where $J$ is the coupling strength between $Q_l$ and $Q_{l+1}$. $\sigma_l^+ (\sigma_l^-)$ is the raising (lowering) operator acting on $Q_l$. The corresponding time evolution unitary operator can then be written as \[ U_{l,l+1}(t) = \exp(-iH_{l,l+1}t) \]

\[ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos (Jt) & -i \sin (Jt) & 0 \\ 0 & i \sin (Jt) & \cos (Jt) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{l,l+1} \]

where the matrix representation of the unitary operator is expanded in the computational basis for $Q_l$ and $Q_{l+1}$. Therefore, when the two qubit state is initialized in $|0\rangle \otimes |0\rangle$, the reduced state $\rho'$ for $Q_{l+1}$ after the evolved time $\Delta t = \pi/(2J)$ reads as follows:

\[ \rho' = \text{Tr}_l [U_{l,l+1}(\Delta t) (\rho \otimes |0\rangle \langle 0|) U_{l,l+1}^\dagger (\Delta t)] = S^l \rho S^l \]

where $S = \text{diag}(1, i)$ is a unitary operator. Obviously, the state $\rho$ is perfectly transferred from $Q_l$ to $Q_{l+1}$ because the fidelity between the prepared and received states under the local unitary operation $S$ is unity. We note that the effective dynamics of the $U_{l,l+1}(\Delta t)$ is identical to the iSWAP operation. If one of the subsystems is $|0\rangle (|1\rangle)$, the iSWAP operation can be viewed as a SWAP operation together with a $S^l (S^r)$ operation. Also note that while considering the XYZ interaction Hamiltonian in Ref. [13], the evolution operator is proportional to the SWAP operation.

Accordingly, to transfer Alice’s prepared states from $Q_l$ to $Q_n$, $n-1$ times of the aforementioned two-qubit operations are required. The total QST process can be described using the following unitary operation

\[ \hat{U}_{l,n} = \prod_{l=n-1}^{1} U_{l,l+1} \] (13)

Finally, the local unitary $S^{n-1}$ is applied on $Q_n$, and Bob obtains the states that are the same as Alice’s prepared states. However, based on Eq. (7), the local unitary $S^{n-1}$ is unnecessary when considering the STS scenario. Usually, for digital quantum processors, e.g. the IBM quantum experience and QuTech quantum inspire considered in this work, the $S^{n-1}$ operation comprises a sequence $S$-gate. Therefore, getting rid of these operations dramatically decreases the circuit depth of the process when $n$ is large; i.e. the accumulated errors from the sequence of operations are reduced.

The circuit implementation of the evolution operator $U_{l,l+1}(\theta)$ in Eq. (11) is shown in Fig. 2a. Here, we replace $t$ with $\theta/2J$. To implement the controlled rotation $X$ (CRX) in IBM quantum experience, one has to decompose it with two CNOT operations and three $u_3$ operations. Thus, there are a total of four CNOT operations in the evolution operator $U_{l,l+1}(\theta)$. As mentioned above, we would like to decrease the number of the CNOT operations to decrease the inevitable errors. Thus, we consider an alternative unitary operator $\hat{U}_{l,l+1}(\theta)$, which reduces one CNOT operation, as

\[ \hat{U}_{l,l+1}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \\ 0 & 0 & \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ 0 & 1 & 0 & 0 \end{pmatrix}_{l,l+1} \] (14)

where the circuit implementation of $U_{l,l+1}$ is shown in Fig. 2b. We can replace $\hat{U}_{l,l+1}$ with $U_{l,l+1}$; thus, Eq. (12) still holds, where

\[ \rho' = \text{Tr}_l[U_{l,l+1}(\frac{\theta}{2J}) (\rho \otimes |0\rangle \langle 0|) U_{l,l+1}^\dagger (\frac{\theta}{2J})] = \text{Tr}_l[U_{l,l+1}(\theta) (\rho \otimes |0\rangle \langle 0|) U_{l,l+1}^\dagger (\theta)] \] (15)

For this implementation, the QST process is perfect only when $\theta = \pi$, that is, $\rho$ and $\rho'$ are related by unitary
operation $S$. We refer to the cases where $\theta \neq \pi$ as imperfect QST processes because the transferred states cannot be transformed to the prepared states through a suitable unitary transformation.

$$Q_l \xrightarrow{R_x(\theta)} Q_{l+1} \quad \text{(a) } U_{l,l+1}(\theta)$$  $$Q_l \xrightarrow{R_x(\theta)} Q_{l+1} \quad \text{(b) } U_{l,l+1}(\theta)$$

FIG. 2. Circuit decomposition of $U_{l,l+1}(\theta)$ and $U_{l,l+1}(\theta)$. $R_x(\theta)$ is the rotation $X$ operation with rotating angle $\theta$.

C. Ideal theoretical results

Figure 3 shows theoretical predictions of $STSR$ with respect to the parameter $\theta$ for different qubit numbers $n \in \{2, 3, 4, 5\}$. We can observe that for fixed $n$, the value of $STSR$ for the perfect QST case ($\theta = \pi$) is always larger than those for the imperfect QST cases ($\theta \neq \pi$). This is because for a fixed $n$, the assemblages for the $\theta = \pi$ case and those for the $\theta \neq \pi$ cases are, in general, related by a unitary transformation and a completely positive and trace-preserving map (CPTP), respectively. It has been proved that $STSR$ monotonically decreases whenever the underlying assemblage is sent into a CPTP map [44].

Moreover, for fixed $\theta$, the value of the $STSR$ monotonically decreases with increasing qubit number $n$. As shown in Fig. 1 and Fig. 4, increasing $n$ means increasing the number of the iterations required in the QST process. As described in Eq. (15), for each iteration, the input state $\rho$ and the output state $\rho'$ can also be generally related by a CPTP map. Therefore, when increasing the number $n$, the prepared assemblage will be iteratively sent into the CPTP maps, which results in a decrease of the $STSR$ [44].

D. Noise simulation

Because the quantum devices nowadays suffer from noise due to the interactions with environments [73, 74], we model the noise effect by introducing extra qubit decoherence (dephasing and relaxation) described by the following Lindblad master equation (similar discussions can be found in the Ref. [66]):

$$\dot{\rho}(t) = \sum_l^n \frac{\gamma_{T_1}}{2} \left[ 2\sigma_-\rho(t)\sigma_+^l - \sigma_+^l\sigma_-\rho(t) - \rho(t)\sigma_+^l\sigma_-^l \right]$$

$$+ \sum_l^n \frac{\gamma_{T_2}}{2} \left[ 2\sigma_1\rho(t)\sigma_2^l - \sigma_2^l\sigma_1^l\rho(t) - \rho(t)\sigma_2^l\sigma_1^l \right].$$

(16)

where $\rho$ denotes the density operator for the total n-qubit system, and the coefficients $\gamma_{T_1} = 1/T_1$ and $\gamma_{T_2} = (1/T_2^1 - 1/2T_2^z)/2$ are the qubit relaxation and decoherence rates for $Q_1$, respectively. Here, $T_1$ and $T_2^z$ represent the relaxation and dephasing time for $Q_1$, respectively. The relaxation and the coherence time for each qubit and the operation-execution time are all public in IBM quantum experience [47]. We use these public information together with the master equation to model the decoherence effect for real devices, such that the final reduced state $\rho_n$ of $Q_n$ can be obtained when tracing out other qubits.

We further consider the measurement (readout) errors, which is not described in the aforementioned master equation. To insert such errors, we briefly recall how to obtain measurement errors in IBM quantum experience. In the measurement-error calibration, we always measure the system in computational basis while initializing the qubit with two basis states, $|0\rangle$ and $|1\rangle$. For the ideal situation, the measurement outcome is 0 (1) with certainty when the qubit is initialized in $|0\rangle$ ($|1\rangle$). Therefore, measurement errors $\Gamma$ can be determined by the average probability of preparation in $|0\rangle(|1\rangle)$ with the opposite outcome 1 (0). We model such errors by sending the quantum state into the bit-flip channel before measurement; i.e.,

$$\rho_n \rightarrow (1 - \Gamma)\rho_n + \Gamma X \rho_n X^\dagger.$$ 

(17)

The above channel changes the population of the quantum state $\rho_n$ with probability $\Gamma$. Notably, once the state is $|0\rangle$ or $|1\rangle$, the population obtained by the above is the same as the one in the measurement-error calibration. Finally, we emphasize that since both Eq. (16) and Eq. (17) can be described by CPTP maps, $STSR$ can only decrease [44] after introducing our noisy model.
Preparation Quantum Channel $\Lambda_n(\theta)$

$U_{1,2}(\theta)$

$U_{2,3}(\theta)$

$U_{n-1,n}(\theta)$

Tomography

$Q_1 |0\rangle$

$Q_2 |0\rangle$

$Q_3 |0\rangle$

$\vdots$

$Q_{n-1} |0\rangle$

$Q_n |0\rangle$

FIG. 4. Circuit implementation of QST. Here, we prepare the initial states by implementing the $u_3$ operation in qubit $Q_1$. The states can be transferred to the $Q_2$ by applying the $U_{1,2}(\theta)$ operation decomposed with a CRX followed by a CNOT operation. We then iterate the $U_{l,l+1}(\theta)$ operation on the $n$-qubit chain. Finally, the states will be transferred to $Q_n$, which can then be obtained by the procedure of quantum state tomography.

FIG. 5. The experimental values of the $STSR$ under the QST. The evolution operators transfer the state from $Q_1$ to $Q_n$. The experimental results of (a) and (c) are implemented in Mar 2020 after maintenance, whereas those for (b) and (d) were complete in Jan 2020 before the maintenance. In (a), we show the spatio-temporal steerability as quantification of the QST process. When $\theta = \pi$, the ideal evolution circuit, corresponding to the perfect QST process, provides the maximal spatio-temporal steerability. Although the experimental $STSR$ cannot reach the theoretical prediction, the trend of the experimental results functions similarly with the ideal case. Moreover, one can observe that the $STSR$ well fitted the noisy model for (a) and (c). One can see that the signaling effect in (d) actually dominates $STSR$ in (b) by Eq. (6).
IV. EXPERIMENTAL RESULTS

We prepare the eigenstates of Pauli matrices in $Q_1$ using the corresponding single-quantum operation $u_3$, which rotates the $|0\rangle$ to the prepared states (see Sec. III A). The global evolution $\Lambda_n(\theta)$ is then applied as shown in Fig. 4 with different qubit numbers $n \in \{2, 3, 4, 5\}$. After sending the system into the channel $\Lambda_n(\theta)$, we can reconstruct the reduced density matrices on $Q_0$ by standard state tomography. Here, the measurement results are obtained through 8000 shots for each procedure in the state tomography.

In Fig. 5, we present experimental data with $\theta \in \{\frac{m\pi}{14} | m = 0, 1, ..., 14\}$ obtained from different dates on March 2020 and January 2020 for the same device named IBMQ boeblingen. The experiment shown in Fig. 5 (a) and (c) was completed right after maintenance, whereas the results in Fig. 5 (b) and (d) were obtained before the maintenance. We also provide the noise simulation mentioned in Sec. III D and the violation of the NSIT described in Eq. (6). One can find that the value of the $STSR$ at $\theta = \pi$, where the perfect QST occurs for the ideal case, decreases as the qubit number $n$ increases. The reduction agrees with the noise simulations, suggesting that the QST non-classicality is suppressed because of accumulation of noise. Additionally, it seems that the overall $STSR$ or the QST non-classicality for the result before the maintenance are larger than that after maintenance. However, by observing Fig. 5(c) and Fig. 5(d), we can clearly find that the $STSR$ before the maintenance, as shown in Fig. 5(b), is actually dominated by the signaling effect, which cannot be regarded as a genuine quantumness. Therefore, benchmarking non-classicality of a quantum device requires both $STSR$ and the condition of NSIT.

Furthermore, there exists intrinsic non-Markovianity in the quantum processors [64, 75]. The non-Markovianity is a possible source of the violation of NSIT shown in the presented experimental results because the existence of the non-Markovian effect implies that the operations shown in Fig. 4 could not be divisible. In other words, the global evolution $\Lambda_n(\theta)$ could depend on the state preparation operation $u_3$ and could result in the violation of Eq. (5).

Finally, the experimental results of the perfect QST from the other quantum devices based on spin qubits (QuTech spin-2) in silicon [76, 77] and the superconducting transmon qubits (QuTech starmon-5) are also presented in Table I. Because QuTech devices do not support the generalized $u_3$ and CNOT operation (one has to decompose the arbitrary quantum operations by a serious single quantum operations and the CZ operations), we only consider the perfect QST case which can be decomposed by $H, S, Z$, and CNOT operations. The qubit relation time ($T_1$) and the coherence time ($T_2$) given by IBM quantum experience [47] is about six to eight times longer than those given by QuTech quantum inspire [48]. Generally speaking, IBM quantum experience outperforms QuTech quantum inspire when both the $STSR$ and singalling effect are considered. This could be because of the unwanted operation decompositions on the CRX and the CNOT operation such that the noise effect and circuit depth increase. The signaling effect in QuTech spin-2 dominates the result, just like IBMQ’s results before maintenance. We also present $STSR$ under the perfect QST process on other IBM quantum devices (in Appendix D).

| Devices          | Transference routes | $STSR$ | Signaling |
|------------------|--------------------|--------|-----------|
| IBMQ             | $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ | 0.216  | 0.015     |
| boeblingen (Mar, 2020) | 3                  | 0.202  | 0.018     |
|                   | 4                  | 0.173  | 0.019     |
|                   | 5                  | 0.129  | 0.026     |
| QuTech starmon-5  | $0 \rightarrow 2 \rightarrow 4$ | 0.170  | 0.051     |
| (May, 2020)      | 3                  | 0.054  | 0.035     |
| QuTech spin-2     | $0 \rightarrow 1$  | 0.103  | 0.100     |
| (May, 2020)      |                    |        |           |

V. DISCUSSION

In this work, we propose a method based on spatio-temporal steering, which can be seen as the generalization of the spatial steering in the context of non-locality, to quantify the non-classicality of the QST process. More specifically, we have shown that the spatio-temporal steerability is invariant under the process of the perfect QST, whereas the reduction during the process of the QST is imperfect. Therefore, the spatio-temporal steering can be used to quantify the non-trivial QST process. Not only did we realize a proof-of-principle experiment but also performed a benchmark experiment of the QST process on IBM quantum experience and QuTech quantum inspire. Our experimental results show that the degrees of QST non-classicality decrease as the circuit depth increases. In addition, the decrease agrees with the noise model, which describes the accumulation of noise (qubit relaxation, gate error, and readout error) during the QST process. The experimental results from the IBMQ boeblingen before the maintenance shows that the spatio-temporal steerability is actually dominated by the signaling effect. Such signaling effect could be caused by the intrinsic non-Markovianity for the quantum devices.
This work also raises some open questions. Can we characterize the non-Markovian effect? Can we implement the QST process with less CNOT operations? In our work, we have to use three CNOT operations to implement QST, while the number of the CNOT operations is the same as the operation decomposition of the SWAP operation.

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Appendix A: Semidefinite Programming for Spatio-Temporal Steering Robustness

Here, we briefly describe the semidefinite program (SDP) of the spatio-temporal steering robustness \( STSR \) in Eq. (4) which is first introduced in [42]. We also note that the SDP of the \( STSR \) is identical to that in the spatial and temporal steering scenarios [21, 22, 39].

Given an assemblage \( \{ \varrho_{a|x} \} \), the primal SDP of \( STSR \) can be formulated as follows:

\[
\begin{align*}
\min_{\{ \sigma_\lambda \}} \quad & \operatorname{Tr} \sum_\lambda \sigma_\lambda - 1 \\
\text{s.t.} \quad & \sum_\lambda D_\lambda(a|x)\sigma_\lambda - \varrho_{a|x} \geq 0 \quad \forall \ a, x , \\
& \sigma_\lambda \geq 0 \quad \forall \ \lambda ,
\end{align*}
\]

(A1)

where \( D_\lambda(a|x) \) is the deterministic strategy function. The dual formulation of Eq. (A1) is given by

\[
\begin{align*}
\max_{\{ F_{a|x} \}} \quad & \operatorname{Tr} \sum_{a,x} F_{a|x} \varrho_{a|x} - 1 \\
\text{s.t.} \quad & \mathbb{1} - \sum_{a,x} D_\lambda(a|x)F_{a|x} \geq 0 \quad \forall \ \lambda , \\
& F_{a|x} \geq 0 \quad \forall \ a, x .
\end{align*}
\]

(A2)

Here, \( F_{a|x} \) is the steering witness that distinguishes the steerable assemblage from the unsteerable ones. We note that when the strong duality of \( STSR \) holds, the results of the above SDPs are equivalent.

Appendix B: Proof of Eq. (7)

In this section, we show that given an assemblage \( \{ \varrho_{a|x} \} \), the \( STSR \) is invariant under unitary transformation using the strong duality mentioned in Appendix A. More specifically, we show \( STSR(\{ \varrho_{a|x} \}) = STSR(\{ \varrho_{a'|x} \}) \), where \( \varrho'_{a|x} = U\varrho_{a|x}U^\dagger \) with \( U \) being an arbitrary unitary operator.

Because the dual formulation of SDP in Eq. (A2) of \( STSR \) is strongly feasible, given an assemblage \( \{ \varrho_{a|x} \} \), one can always find the optimal spatio-temporal steering witness \( \{ F^*_{a|x} \} \) satisfying both constraints in Eq. (A2):

\[
STSR(\{ \varrho_{a|x} \}) = \operatorname{Tr} \sum_{a,x} F^*_{a|x} \varrho_{a|x} - 1.
\]

With the above, we now apply a unitary transformation \( U \) on the given assemblage \( \{ \varrho_{a|x} \} \). The dual formulation of \( STSR(\{ \varrho'_{a'|x} \}) \) can be expressed as follows:

\[
STSR(\{ \varrho'_{a'|x} \}) = \max_{\{ F'_{a'|x} \}} \operatorname{Tr} \sum_{a,x} F'_{a'|x} \varrho'_{a'|x} - 1 \geq \operatorname{Tr} \sum_{a,x} (UF'_{a'|x}U^\dagger)(U\varrho_{a|x}U^\dagger) - 1 = \operatorname{Tr} \sum_{a,x} UF'_{a'|x} \varrho_{a|x}U^\dagger - 1 = \operatorname{Tr} \sum_{a,x} F^*_{a'|x} \varrho_{a|x} - 1 = STSR(\{ \varrho_{a|x} \}) .
\]

The inequality holds because \( \{ UF^*_{a'|x}U^\dagger \} \) is not the optimal solution of SDP. Nevertheless, it is indeed a valid solution because it satisfies both constraints in Eq. (A2):

\[
\mathbb{1} - \sum_{a,x} D_\lambda(a|x)F^*_{a'|x} = \mathbb{1} - \sum_{a,x} D_\lambda(a|x)UF^*_{a'|x}U^\dagger = U(\mathbb{1} - \sum_{a,x} D_\lambda(a|x)F^*_{a'|x})U^\dagger \geq 0 \quad \forall \ \lambda ,
\]

\[
F'_{a'|x} = UF^*_{a'|x}U^\dagger \geq 0 \quad \forall \ a, x .
\]

Therefore, we arrive at the bound relation; i.e.,

\[
STSR(\{ \varrho'_{a'|x} \}) \geq STSR(\{ \varrho_{a|x} \}) .
\]

(B1)

A similar argument can also be applied to the primal SDP in Eq. (A1) of \( STSR \). Given an assemblage, one can always find the optimal \( \{ \sigma^*_\lambda \} \) that satisfies both constraints in Eq. (A1):

\[
STSR(\{ \varrho_{a|x} \}) = \operatorname{Tr} \sum_{\lambda} \sigma^*_\lambda - 1.
\]

By applying a unitary transformation \( U \) on the given assemblage \( \{ \varrho_{a|x} \} \), the primal SDP of \( STSR(\{ \varrho'_{a'|x} \}) \) can
then be expressed as follows

\[
\text{STS}_R(\{\theta_a|x\}) = \min_{\{\sigma_x\}} \text{Tr} \sum_{\lambda} \sigma \lambda' - 1
\]

\[
\leq \text{Tr} \sum_{\lambda} U \sigma \lambda' U^\dagger - 1
\]

\[
= \text{Tr} \sum_{\lambda} \sigma \lambda' - 1
\]

\[
= \text{STS}_R(\{\theta_a|x\}).
\]

The inequality holds because \(\{U \sigma \lambda' U^\dagger\}\) is not the optimal solution of the SDP. Nevertheless, it is indeed a valid solution because it satisfies both constraints in Eq. (A1):

\[
\sum_{\lambda} D_\lambda(a|x)\sigma \lambda - \theta a|x = \sum_{\lambda} D_\lambda(a|x)U \sigma \lambda' U^\dagger - U \theta a|x U^\dagger
\]

\[
= U(\sum_{\lambda} D_\lambda(a|x)\sigma \lambda - \theta a|x)U^\dagger
\]

\[
\geq 0 \quad \forall \ a, x,
\]

\[
\sigma \lambda' = U \sigma \lambda' U^\dagger \geq 0 \quad \forall \ \lambda.
\]

Therefore, we arrive at another bound relation which is given as

\[
\text{STS}_R(\{\theta a|x\}) \leq \text{STS}_R(\{\theta a|x\}). \tag{B2}
\]

There are some similar properties of Eq. (B2) that have been discussed in Ref. [44].

By combining Eq. (B1) and Eq. (B2), we find that \(\text{STS}_R\) of the assemblage is invariant under unitary transformation:

\[
\text{STS}_R(\{U \theta a|x U^\dagger\}) = \text{STS}_R(\{\theta a|x\}), \tag{B3}
\]

; thus, we have completed the proof that perfect state transfer implies invariance of \(\text{STS}_R\). \(\square\)

Appendix C: Proof of Eq. (6) in the main text

Following the similar structure in Ref. [68], given an STS assemblage \(\{\theta a|x\}\), the constraint of the primal problem of \(\text{STS}_R\) in the context of the SDP in Eq. (A1) can be expressed as follows:

\[
\sum_{\lambda} D_\lambda(a|x)\sigma \lambda - \theta a|x \geq 0 \quad \forall \ \ x \in \mathcal{X}, \quad a. \tag{C1}
\]

where \(\mathcal{X} = \{1, 2, 3, ..., n\}\). Because \(\Sigma \alpha D_\alpha(a|x) = 1\), we arrive at the following:

\[
\sum_{\lambda} \sigma \lambda \geq \sum_{\alpha} \theta a|x \quad \forall \ x \in \mathcal{X}. \tag{C2}
\]

Now, if we consider the classical assemblage which we refer to for all elements of the assemblage only containing the diagonal entries in the computational basis, the summation of the assemblages can be written as

\[
\sum_{\alpha} \theta a|x = \begin{pmatrix} \gamma x & 0 \\ 0 & 1 - \gamma x \end{pmatrix} \quad \forall \ x \in \mathcal{X}. \tag{C3}
\]

We can choose the local hidden assemblage which satisfies Eq. (C2) as

\[
\sum_{\lambda} \sigma \lambda = \begin{pmatrix} \gamma_{\max} & 0 \\ 0 & 1 - \gamma_{\min} \end{pmatrix}, \tag{C4}
\]

where \(\gamma_{\max} = \max\{\gamma x\}\), and \(\gamma_{\min} = \min\{\gamma x\}\). To show that the above assemblage is the optimal solution in Eq. (A1) (especially the positive semidefinite constraint), one can add a non-negative value \(\epsilon\) into the diagonal entries in Eq. (C4). It is trivial to see that \(\text{Tr}(\Sigma \alpha \sigma \lambda)\) provides the minimum solution when \(\epsilon = 0\). Therefore, we get

\[
\text{STS}_R = \text{Tr} \sum_{\lambda} \sigma \lambda - 1
\]

\[
= \gamma_{\max} - \gamma_{\min}
\]

\[
= \max_{x} \frac{1}{2} \| \sum_{\alpha} \theta a|x - \sum_{\alpha} \theta a|x' \|_1 \quad \forall \ x \neq x', \tag{C5}
\]

and hence, we have completed the proof. \(\square\)

Appendix D: The experimental results from different IBMQ devices

In Table II, we show \(\text{STS}_R\) under the process of the perfect QST with different IBMQ devices: 20-qubits almaden, 20-qubits boeblingen, 28-qubits cambridge, 5-qubits london, and 27-qubits paris. The circuit implementations for all are the same as the one introduced in the main text (see Sec. III A and Sec. III B). One can see that the different chips shows different performances of the QST. We thus can benchmark each chips under the QST tasks.

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TABLE II. Experimental results of the perfect QST ($\theta = \pi$) from different IBMQ devices.

| Devices   | Transference routes | $n$ | $STS$ | $SR$ | Signaling |
|-----------|---------------------|-----|-------|------|-----------|
| almaden   | 0 $\rightarrow$ 1 $\rightarrow$ 2 $\rightarrow$ 3 $\rightarrow$ 4 | 2   | 0.169 | 0.026 |           |
| (Mar, 2020) | 3   | 0.130 | 0.021 |       |           |
|           | 4   | 0.086 | 0.019 |       |           |
|           | 5   | 0.040 | 0.021 |       |           |
| almaden   | 5 $\rightarrow$ 6 $\rightarrow$ 7 $\rightarrow$ 8 $\rightarrow$ 9 | 2   | 0.133 | 0.025 |           |
| (Mar, 2020) | 3   | 0.040 | 0.025 |       |           |
|           | 4   | 0.016 | 0.015 |       |           |
|           | 5   | 0.018 | 0.018 |       |           |
| boeblingen | 0 $\rightarrow$ 1 $\rightarrow$ 2 $\rightarrow$ 3 $\rightarrow$ 4 | 2   | 0.216 | 0.015 |           |
| (Mar, 2020) | 3   | 0.202 | 0.018 |       |           |
|           | 4   | 0.173 | 0.019 |       |           |
|           | 5   | 0.129 | 0.026 |       |           |
| boeblingen | 0 $\rightarrow$ 1 $\rightarrow$ 2 $\rightarrow$ 3 $\rightarrow$ 4 | 2   | 0.231 | 0.184 |           |
| (Jan, 2020) | 3   | 0.198 | 0.153 |       |           |
|           | 4   | 0.170 | 0.140 |       |           |
|           | 5   | 0.140 | 0.138 |       |           |
| boeblingen | 5 $\rightarrow$ 6 $\rightarrow$ 7 $\rightarrow$ 8 $\rightarrow$ 9 | 2   | 0.059 | 0.024 |           |
| (Mar, 2020) | 3   | 0.133 | 0.025 |       |           |
|           | 4   | 0.116 | 0.030 |       |           |
|           | 5   | 0.033 | 0.030 |       |           |
| boeblingen | 15 $\rightarrow$ 16 $\rightarrow$ 17 $\rightarrow$ 18 $\rightarrow$ 19 | 2   | 0.025 | 0.021 |           |
| (Mar, 2020) | 3   | 0.032 | 0.027 |       |           |
|           | 4   | 0.010 | 0.010 |       |           |
|           | 5   | 0.005 | 0.005 |       |           |
| cambridge  | 0 $\rightarrow$ 1 $\rightarrow$ 2 $\rightarrow$ 3 $\rightarrow$ 4 | 2   | 0.017 | 0.017 |           |
| (Jul, 2020) | 3   | 0.006 | 0.006 |       |           |
|           | 4   | 0.009 | 0.008 |       |           |
|           | 5   | 0.011 | 0.011 |       |           |
| london    | 0 $\rightarrow$ 1 $\rightarrow$ 3 $\rightarrow$ 4 | 2   | 0.203 | 0.022 |           |
| (Oct, 2019) | 3   | 0.190 | 0.027 |       |           |
|           | 4   | 0.154 | 0.029 |       |           |
| paris     | 0 $\rightarrow$ 1 $\rightarrow$ 2 $\rightarrow$ 3 $\rightarrow$ 5 | 2   | 0.208 | 0.074 |           |
| (Jul, 2020) | 3   | 0.197 | 0.087 |       |           |
|           | 4   | 0.148 | 0.039 |       |           |
|           | 5   | 0.085 | 0.061 |       |           |

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