Dynamics of reversed shear Alfvén eigenmode and energetic particles during current ramp-up

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Abstract
Hybrid MHD-gyrokinetic code simulations are used to investigate the dynamics of frequency sweeping reversed shear Alfvén eigenmode (RSAE) strongly driven by energetic particles (EPs) during plasma current ramp-up in a conventional tokamak configuration. A series of weakly reversed shear equilibria representing time slices of long timescale MHD equilibrium evolution is considered, where the self-consistent RSAE-EP resonant interactions on the short timescale are analyzed in detail. Both linear and non-linear RSAE dynamics are shown to be subject to the non-perturbative effect of EPs by maximizing wave-EP power transfer. In the linear stage, EPs induce evident mode structure and frequency shifts; meanwhile, RSAE saturates by radial decoupling with resonant EPs due to weak magnetic shear, and gives rise to global EP convective transport and fast frequency chirping in the non-adiabatic regime. The spatiotemporal scales of phase space wave-EP interactions are characterized by the perpendicular wavelength and wave-particle trapping time. The simulations provide insights into general as well as specific features of the RSAE spectra and EP transport in experimental observations, and illustrate the fundamental physics of wave-EP resonant interaction with the interplay of the magnetic geometry, plasma non-uniformity and non-perturbative EPs. Possible application for understanding the non-adiabatic frequency chirping as convective and relaxation branches is also discussed.

Keywords: energetic particles, reversed shear Alfvén eigenmode, multi-timescale dynamics, hybrid simulation

(Some figures may appear in colour only in the online journal)

1. Introduction
In tokamak plasmas of fusion interest, the energetic/fast particles (EPs), generated by nuclear fusion reactions and/or external power inputs such as the neutral beam injection (NBI) or ion cyclotron resonance heating (ICRH), play crucial roles in plasma heating and current drive; and thus, they must be well confined to achieve better performance. An important mechanism of anomalous EP transport is via the collective resonant excitation of the shear Alfvén wave (SAW) fluctuations [1], in the form of the Alfvén eigenmodes (AEs) [2] or energetic particle continuum modes (EPMs) [3]. If strongly driven,
these fluctuations could induce substantial power loss and localized thermal flux onto the plasma facing components by EP transport before thermalization [4, 5]. During the past several decades, significant achievements in comprehending the basic and general physics aspects on the SAW-EP dynamics have been greatly gained by intense experimental and theoretical researches [1, 6–14]. Nevertheless, more dedicated efforts are needed to incorporate multiple physics ingredients and address situations of practical interest; thereby, validating and deepening the present understanding, and envisioning burning plasma experiments in the near future [1, 15–19].

In this work, connected with both the fundamental physics and experimental observations, we investigate, via numerical simulations, the linear and non-linear SAW-EP dynamics during the plasma current ramp-up phase in a conventional tokamak discharge. In particular, we consider a generalized scenario where high external power is applied to a plasma with relatively low density and current. Due to insufficient current penetration and/or diffusion, a weakly reversed shear magnetic configuration is usually created, where the radial profile of the safety factor $q$ contains an off-axis minimum, $q_{\text{min}}$, which typically resides deeply in the plasma core region. Concurrently, an anisotropic and concentrated population of fast ions is also produced, and could readily excite the reversed shear AE (RSAE, also dubbed Alfvén cascade) [20, 21], in addition to the more common toroidal AE (TAE) [2, 4, 22, 23]. In fact, the RSAE is empirically recognized as a prominent signature of reversed shear plasmas by its characteristic sweeping frequency, which follows the temporal evolution of $q_{\text{min}}$ with a quasi-coherent spectral line for each toroidal mode number $n$ [24] (see [11] for a recent review). In most representative cases, when $q_{\text{min}}$ decreases from a rational value $mn/n$ to $(m − 1/2)/n$, the RSAE exhibits upward sweeping frequency up to the TAE frequency band, $\omega_{\text{TAE}} \simeq v_\Lambda/(2qR)$. Here, $m$ is the (dominant) poloidal mode number, $v_\Lambda$ is the Alfvén speed, and $R$ is the major radius. Indeed, upward frequency sweeping RSAEs with ramping-up plasma current and continuous NBI and/or ICRH have been routinely observed in many devices [20, 25–28]. Of particular importance, the fluctuations dominated by RSAEs and TAEs often cause significant anomalous fast ion transport/losses, as both short timescale intermittent convective pulses [29–31] and accumulative profile flattening on long timescales [32–34]. Motivated by the rich phenomenology of SAW-EP interactions and the importance of unveiling the underlying physics mechanism, many dedicated numerical simulation works [35–42] have been devoted to analyzing various aspects of the observations. However, previous works either focus on the short timescale dynamics with fixed magnetohydrodynamic (MHD) equilibrium profiles; or sacrifice the crucial self-consistent description of the SAW-EP resonant interaction to extend the simulation time span. Accordingly, on one hand, in order to fully capture the frequency sweeping feature of the RSAE, it is necessary to take into account the finite equilibrium evolution in the simulation setup. On the other hand, a self-consistent, non-perturbative description of the SAW-EP interactions is also crucially needed, since the non-perturbative effect of EPs plays a significant role in the RSAE dynamics when the driven fluctuation deviates from the instability threshold [21, 43–45]. Here, the non-perturbative effect corresponds to the important contribution of EPs in determining the mode structure and real frequency on both linear and non-linear dynamics, such that the SAW-EP resonance condition is best maintained and the corresponding power transfer is maximized [1, 3, 44–53]. Indeed, the deviation of RSAE spectra from the MHD limit has been identified in various experiments with high power input [9, 32, 37, 54–57], suggesting the important role of non-perturbative EPs. More specifically, rapid and repetitive frequency chirping within 1 ms, i.e. the characteristic timescale of a few inverse linear growth rates [50, 58], could take place in addition to the much slower frequency sweeping in the timescale of O(100 ms) induced by the equilibrium evolution (see, e.g. figure 2 of [55] and figure 2 of [57]). The physics mechanism of the multi-timescale RSAE frequency chirping and sweeping underlying vast experimental observations is explored by the simulations presented in this paper, where we specify ‘chirping’ as the fast, short timescale and ‘sweeping’ as the slow, long timescale frequency evolution, in the sense described above.

In order to be feasible within the limitation of computational resources and numerical model assumptions, we utilize the timescale separation between the fast SAW-EP interaction and the slow equilibrium evolution, and set up a series of ad hoc simulations to investigate the RSAE-EP dynamics with superimposed small but finite equilibrium changes. To be more precise, several self-similar MHD equilibria representing time slices of ramping-up plasma current in long timescale are used as the only changing input variable for multiple simulation cases, in which the short timescale non-linear SAW-EP interaction is described self-consistently and in which the equilibrium is kept fixed within each simulation case. A schematic viewgraph of the simulation approach and relevant timescales is shown in figure 1. A single toroidal mode number $n = 3$ is considered; the investigated time window is selected as a signature RSAE upward frequency sweeping period up to the TAE frequency range by directly controlling $q_{\text{min}}$ (see figure 1). Furthermore, the simulations are carried out with the hybrid MHD-gyrokinetic code (HMGC) [49, 59], which describes self-consistent non-linear SAW-EP interactions on short timescales with a simple but yet relevant model [60]. Numerical details are described in section 2, along with primary simulation parameters. In order to be representative and realistic in the choice of the parameters, the mid-size HL−2A tokamak [61] is chosen as a convenient reference whenever applicable, where the EP population is modeled as positive-ion-based neutral beam (P-NB) with a single injection pitch angle. Note that we do not consider $a$ priori any specific HL−2A discharge in the present work; meanwhile, the present research can be readily extrapolated to other discharges/devices based on the dimensionless parameters which dictate the physics [44, 45].

For the self-consistent RSAE-EP resonant interaction on short timescales, we further subdivide it into three stages by the dominant physics mechanism; namely, (exponentially) linear growing, non-linear saturation, and post saturation stages. Note that ‘stage’ implies short timescales in this paper. Main results for the three stages are presented and discussed in,
generated hole-clump pairs in the EP phase space\cite{75–78}. On the other hand, for the strongly unstable scenario with significant non-perturbative EP responses considered in this work, the phase space wave-EP resonance structure evolves non-adiabatically in the ‘shortest non-linear timescale’ with $\tau_{NL} \sim O(\tau_B)$\cite{1, 53}. We demonstrate that during the non-linear saturation stage, fast frequency chirping occurs non-adiabatically with $|\dot{\omega}| \sim O(\tau_B^{-1})$. Furthermore, global EP convective transport takes place over radial scale length $\sim O(\lambda_\perp)$\cite{1, 49, 50, 53, 79}, where the wave-EP phase locking could give rise to convective amplification of the fluctuation amplitude by maximizing power transfer\cite{1, 43, 50, 53, 80–86}. The onset of non-adiabatic frequency chirping and intrinsically nonlocal EP transport is induced by radial decoupling of resonant EPs with the non-perturbatively excited fluctuation\cite{1, 53, 87}. An unified theoretical framework incorporating both the adiabatic and non-adiabatic regimes of non-linear SAW-EP dynamics is outlined in\cite{1, 53}, where the one-on-one correspondence of non-adiabaticity and non-perturbativity is addressed.

On the application side, this work provides useful insights to understand the fast frequency chirping of strongly driven fluctuations in both experiments\cite{9, 54–57, 88–90} and various numerical simulations. In the non-adiabatic regime, the simulations show that the non-perturbatively driven fluctuation consists of a ‘convective’ branch maintaining phase locking with the resonant EP radial convection, and ‘relaxation’ branches back to the weakly damped states. For the non-perturbatively excited RSAEs, the non-linear dynamics is importantly regulated by the magnetic geometry and plasma non-uniformity, where the downward chirping convective branch experiences stronger continuum damping\cite{43, 91–93}, and the relaxation branches dominate the post saturation stage (section 5). Hereby, the simulations propose a probable interpretation to the observed fast frequency chirping with strong EP drive, and neatly recover the conventional MHD analysis of the slow frequency sweeping. Meanwhile, more insights into the phase space transport of fast P-NB ions in a typical present-day tokamak are presented. In the inner-core region of the plasma with generally weak magnetic shear, the radial resonance structure of the RSAE with circulating EPs favors the radial decoupling mechanism\cite{1, 45, 53, 87}. Consequently, a strongly driven RSAE could induce significant and global EP phase space profile distortions, with important implications to the following non-linear dynamics. In particular, the saturated fluctuation could be further amplified by extending the phase space resonant range via non-adiabatic frequency chirping, illustrating the fundamental nature of maximizing power transfer in wave-EP resonant interactions. Further conclusion and discussion are given in section 6.

2. Numerical model and parameters

2.1. Simulation model

In this work, the HMGC is used to investigate the self-consistent non-linear SAW-EP interaction via the
non-perturbative pressure coupling formulation [60]. That is, the thermal (bulk) plasmas are described by reduced MHD equations [94] in a simplified toroidal geometry characterized by shifted circular magnetic flux surfaces; the EP kinetic compression enters the momentum equation via the pressure tensor term, which is computed by the corresponding gyro-center distribution function using the particle-in-cell method. EP orbits are solved in the perturbed electromagnetic fields by non-linear Vlasov equations in the drift-kinetic limit, i.e. the finite Larmor radius (FLR) effect is neglected. The detailed model equations are presented in [8, 59, 95, 96], and are thus omitted here for simplicity.

As described above, the hybrid MHD-kinetic model [60] is a simplified but yet relevant numerical tool for the scope of present analysis, since it preserves the crucial physics ingredients such as the equilibrium geometry, plasma non-uniformity and non-perturbative EP response, which are necessary to properly describe non-linear SAW-EP interactions as articulated in [1, 53, 84]. Here, we note that the RSAE spectrum is known to be subject to the kinetic effects of thermal plasmas in the low frequency domain, including the geodesic acoustic coupling and pressure gradient [97, 98] as well as the Landau damping. However, the kinetic extension of HMGC model [85, 96] is not accounted for in this work, as we consider RSAEs near the TAE frequency band in the low-β limit (see figure 1 and section 2.2 below), where β is the ratio of plasma thermal to magnetic pressures. Thereby, although these neglected effects could impact the RSAE frequency and stability property via e.g. up-shifting the SAW continuum [99], they are not expected to significantly alter the essential physics in the linear as well as the saturation stages, which are dominated by the non-perturbative EP drive (sections 3 and 4). On the other hand, more evident and quantitative differences may appear in the longer timescale non-linear dynamics (section 5) by including a kinetic treatment of thermal plasmas (see also discussions in section 6.2).

2.2. MHD parameters

As introduced in section 1, the simulation parameters are chosen to resemble the current ramp-up phase of a present-day tokamak discharge using typical HL-2A parameters. The plasma domain is characterized by circular poloidal cross sections with the major radius \( R_0 = 1.65 \) m and minor radius \( a = 0.40 \) m. The on-axis magnetic field is \( B_0 = 1.3 \) T, where in this paper, the subscript ‘0’ denotes spatially non-uniform quantities evaluated at the magnetic axis. A series of self-similar weakly reversed q profiles with \( q_0 \simeq 2 \) and \( q_a \simeq 4 \) is considered in this work, where 9 simulation cases are presented with \( q_{\min} \) decreasing from 1.94 to 1.86 with a step size \( \Delta q_{\min} = 0.01 \). Figure 2 shows three reference MHD equilibria. We note again that each q profile is used for different cases, whilst the MHD equilibrium is kept fixed within a single simulation. As shown in figure 2, the total plasma current gradually increases, where a ‘bump’ in the current density profile penetrates towards the plasma core. Such a current profile evolution results in slightly decreasing q profiles and an inward shift of \( r_{\text{min}} \), as one can more clearly see in the inset of figure 2. Moreover, consistent with the reduced MHD formalism in the HMGC physics model described in section 2.1, the thermal plasma compressibility is not considered. Bulk ions are assumed to be Deuterium with an on-axis density \( n_0 = 2.0 \times 10^{19} \) m\(^{-3}\), and a parabolic radial profile

\[
\frac{n(r)}{n_0} = 1 - 0.7\left(\frac{r}{a}\right)^2.
\]

Figure 2 shows the SAW continua for the three reference q profiles. Here, \( \omega_{\text{Alf}} \equiv \nu_{\text{Alf}}/R_0 \) is the on-axis Alfvén frequency used for normalizations of time and frequency. One sees that the small variation of q profiles mainly alters the continuum structure in the inner-core region, where the RSAE frequency increases up to the TAE range as a consequence of decreasing \( q_{\min} \).

The coupled reduced MHD equations consider two electromagnetic field variables; namely, the electrostatic potential \( \phi \) and poloidal flux function \( \psi \), which is related to the parallel component of magnetic vector potential \( A_\| \) [59, 94]. HMGC uses Fourier decomposition in the poloidal and toroidal directions, yielding the respective mode numbers \( m \) and \( n \). As noted above, a single \( n = 3 \) perturbation is considered, while neglecting the couplings among modes with different \( n \). The perturbed harmonics are solved by the finite difference method in the radial direction with a mesh \( N_{\text{MHD}} = 256 \), which is sufficient for the long wavelength fluctuations in the present work. In addition, small values of resistivity \( \eta \) and viscosity \( \nu \) are included in the MHD equations for numerical stability reasons. The resistivity corresponds to Lundquist number \( S \equiv 4\pi a^2 \omega_{\text{Alf}}/(\eta v^2) = 2.5 \times 10^6 \); the normalized viscosity is \( \nu /\omega_{\text{Alf}}/a^2 = 10^{-8} \). Nevertheless, the ideal MHD constraint \( \delta E_\| \simeq 0 \) generally applies, and we will refer to a single scalar field \( \phi \) to represent the SAW fluctuations.
The initial distribution function \( m \) is obtained by the fast Fourier transform (FFT) to a temporal series of \( \phi(r,t) \) within a finite time window; i.e. \( t \in [t_{\text{ref}} - \Delta t_{\text{window}}, t_{\text{ref}} + \Delta t_{\text{window}}] \). To minimize the influence of data prior to and after \( \phi(r,t) \), we weight \( \phi(r,t) \mid _{t=t_{\text{ref}}+\Delta t_{\text{window}}} \) by a Hannning function; in addition, the temporal series are padded with zeros on both sides to smooth the \( \omega \) grid. Since we are interested in the fast frequency chirping during the non-linear stage, it is necessary to choose an intermediate value of \( \Delta t_{\text{window}} \) in the spectral analysis [86]; such that one has a good frequency resolution as \( \Delta \omega = \pi/\Delta t_{\text{window}} \), and minimized numerical artifacts associated with the asymmetry of fluctuation amplitude (weight) within the temporal series. In this paper, \( \Delta t_{\text{window}} = 300 \omega_{A0}^{-1} \) is used as a standard value. In addition, we also cross-check the spectral analysis using \( \Delta t_{\text{window}} = 100 \omega_{A0}^{-1} \), to seek better confidence in the relevant timescale of frequency chirping and numerical convergence. More details with examples are discussed in section 4.1.

2.3. EP parameters

The energetic (hot) particles in the simulations are modeled as tangentially injected deuterium P-NB ions in the co-current direction. They are described by a model slowing-down distribution function, with the birth energy \( E_b = 45 \text{ keV} \) and an uniform critical energy \( E_c = 28 \text{ keV} \) [100], yielding the following normalized parameters: \( v_{h1}/v_{A0} = 0.327 \), \( \rho_{h1}/a = 0.059 \). Here, \( v_{h1} \equiv \sqrt{E_b/m_{h1}} \) is a characteristic EP thermal velocity; \( \rho_{h1} \equiv v_{h1}/\Omega_{\text{LH0}} \) is the corresponding Larmor radius; and \( \Omega_{\text{LH0}} = \epsilon_{h1}B_0/(m_{h1}c) \) is the on-axis cyclotron frequency, with \( m_{h1} \) and \( \epsilon_{h1} \) the EP (Deuterium) mass and charge, respectively. The initial distribution function reads

\[
f_{h1}(\psi,E;\alpha; t=0) = \frac{3}{4\pi} \left( \frac{m_{h1}}{2E_c} \right)^{3/2} n_{h1}(\psi\equiv) \Xi(\alpha) \times \left[ (E/E_c)^{3/2} + 1 \right] \ln \left[ (E_b/E_c)^{3/2} + 1 \right],
\]

Here, \( n_{h1}(\psi\equiv) \) is the radial density profile as a function of the equilibrium flux coordinate

\[
n_{h1}(\psi\equiv)/n_{h0} = \exp \left[ -\left( \frac{\rho}{L_n} \right)^{\lambda_n} \right],
\]

with \( n_{h0} = 10^{18} \text{ m}^{-3}, L_n = 0.6 \) and \( \lambda_n = 3 \) (see figure 4(a)), where \( \rho \equiv \sqrt{(\psi_{eq} - \psi_0)/(|\psi_u - \psi_0|)} \) is a radial-like flux coordinate. \( \Xi(\alpha) \) is the pitch angle distribution function

\[
\Xi(\alpha) = \frac{4}{\Delta \sqrt{\pi} \operatorname{erf}[(1 - \cos \alpha)/(\Delta \alpha)] + \operatorname{erf}[(1 + \cos \alpha)/(\Delta \alpha)]},
\]

with \( \alpha \equiv \cos^{-1}(v_{||}/\sqrt{2E/m_{h1}}) \) the pitch angle and \( v_{||} \) the parallel velocity. We use \( \alpha_{eq} = \pi/4 \) and \( \Delta = 0.1 \), i.e. the EPs are predominantly co-circularizing with a relatively narrow pitch angle distribution (see figure 4(b)), where one can readily estimate \( v_{||} = v_{h1} \approx v_{A}/3 \) at the birth energy and injection pitch angle.

As introduced above, besides the MHD equilibrium profiles, all other simulation inputs are unchanged among cases. This is intended to isolate the effect of the small but finite MHD equilibrium shift; however, we note that artificially fixing the initial EP distribution function neglects its self-consistent evolution on long timescales under the effects of the beam source/sink, Coulomb collisions, micro-turbulences and SAW-EP non-linear interactions. Thus, the simplified model distribution function (2) must be evaluated as only a coarse description of a realistic EP population, and differences should be expected in a simulation with continuous source and continuous MHD activity [86]. Here, we note that these effects can be self-consistently recovered within the theoretical framework of the EP phase space zonal structure transport [1, 53, 101, 102], and will be the subject of a future work with a simplified source and collision module.

Note also that the initial distribution (2) is not characterized by the particles’ constants of motion; in particular, the initial radial density function (3) is described by the flux coordinate \( \psi_{eq} \). Due to the relatively large magnetic drift as \( \rho_{h1}/a = 0.059 \) and \( q > 2 \), the initialized particles will deviate from the nominal flux surfaces and promptly relax to a genuine equilibrium distribution; thus, leading to some corrugations in the relaxed distribution functions is \( \sim O(10^{-3}) \) (normalized to the peak value). Since the fluctuation-induced EP transport is generally at least an order of magnitude larger than such uncertainty, we consider that all cases are initialized to the same promptly relaxed distribution [35] as reported in figure 4. It will be referred to as the linear distribution in later comparisons with the perturbed one due to the fluctuation-induced EP transport.
The EP coordinates are pushed in 5D phase space \((r, \theta, \zeta, \mu, \nuq)\) with a time step \(dt = 0.06 \omega_{\text{Al}}^{-1}\), where \(\theta\) is the poloidal angle, \(\zeta\) is the toroidal angle, and \(\mu \equiv m q r_b^2 / (2B)\) is the conserved magnetic moment. The kinetic module uses 128 \times 128 \times 48 grids in the \(r, \theta, \zeta\) directions, and 16 particle markers per cell; a convergence test shows that such a numerical resolution is sufficient for the present analysis. Furthermore, the EP source and Coulomb collisions are not considered in the present numerical model; meanwhile, particles that leave the simulation domain, i.e. \(r/a > 1\), are considered lost. As discussed above, we focus on comparing different initial-value simulations with equilibrium profiles being the only variable in the present work. The effect of source and collisions in replenishing the perturbed phase space resonant range is not crucially important in the short timescales \((\tau_{\text{NL}} \sim O(\tau_\theta))\) at the saturation stage (section 4); meanwhile, it may substantially impact the post saturation dynamics in longer timescales (section 5). The relevance of source and collisions is further discussed in sections 5 and 6.2.

3. Linear properties

3.1. Mode spectra

In the simulations with only varying the equilibrium profile controlled by \(q_{\text{min}}\), we first focus on the RSAE spectra and radial structures excited by the non-perturbative EPs. An overview of the fluctuation frequencies and linear growth rates is presented in figure 5, whilst figure 6 shows the radial mode structure for the case with \(q_{\text{min}} = 1.90\) as a reference one for discussion. We note that the \(m = 6\) poloidal harmonic is dominant in all cases, while the coupling with the adjacent \(m = 5\) one intensifies with decreasing \(q_{\text{min}}\), suggesting that the RSAE is in a gradual transition to a TAE, along with increasing frequency [43, 103]. Moreover, the RSAE spectra and radial structures in the MHD limit are evaluated by the antenna excitation technique [44] and also reported in figures 5 and 6 for comparison, where one can observe the significant difference of the non-perturbatively EP-induced mode structure and complex frequency shift. Here, we note that in the underlying equilibrium magnetic fields, the RSAE exists as a weakly damped AE due to the finite toroidal mode coupling acting as an effective potential well [103]; whereas in the presence of EPs, the non-resonant response of EPs extends the potential well and clearly increases the radial mode width by their global drive [21, 43, 47, 104]. Besides, the mode damping is mostly induced by the finite resistivity (radiative damping) in the MHD limit [43, 48, 105]; in addition, the continuum damping could become important if the fluctuation significantly couples with the SAW continuum due to the EP-induced frequency shift. Note that although the value of resistivity considered herein is generally larger than realistic conditions, all cases are dominated by strong EP drive and clearly deviate from the marginal stability limit with \(\gamma / \gamma_0 \gg 1\) (see figure 5), consistent with the simulation setup outlined in section 1. Furthermore, for \(q_{\text{min}}\) sufficiently close to \((m - 1/2)/n\), there also exists an upper branch RSAE dominated by \(m = 5\) with a frequency below the upper continuum tip [43, 97, 103]; however, they are not observed to be driven unstable by the sub-Alfvénic \((\nuq / v_A < 1)\) EPs in the linear stage. Nevertheless, for the case with \(q_{\text{min}} = 1.86\), the upper branch RSAE becomes relevant in the non-linear stage, and is also indicated in figure 5 for the sake of completeness.

The impact on the radial mode width by the non-perturbative EPs is evident in other cases, where the most significant mode structure deformation is found for the case with \(q_{\text{min}} = 1.86\) (see figure 7). Its frequency also shows the largest difference with respect to the MHD limit; in fact, we note that this linear mode should be understood as an EPM, as it resides inside the continuum rather than the toroidicity-induced frequency gap. Here, we note that the mode transition and the corresponding identification from a RSAE to a TAE or an EPM are gradual and continuous [51, 52], where a small change of parameters by e.g. slightly shifting the continuum with acoustic mode coupling [99], is not expected to significantly alter the mode character. In particular, there is no sharp distinction between an AE and an EPM in this scenario [1, 51–53], since all the fluctuations are strongly driven by the non-perturbative EPs. They are treated similarly in the following discussions from the perspective of the non-perturbative SAW-EP interplay.

3.2. Resonance analysis

More details underlying the mode excitation, and in particular, the EP-induced frequency shift can be unveiled by the wave-particle resonance analysis. In order to identify the resonant EPs responsible for mode destabilization, we first focus on the phase space structure of the SAW-EP power transfer [87]. Figure 8 shows the properties of linear stage power transfer in the EP energy space, which is a convenient identifier of the EPs with a narrow pitch angle distribution. One sees that the peak location \(E_{\text{peak}}\), corresponding to strongly driving EPs which satisfy the resonance condition \(\omega_{\text{res}} \simeq \omega\), shifts to the high energy end with increasing mode frequency. Here,
Figure 5. Frame (a): the mode frequency $\omega$ in the linear stage (red circles), lower branch (upward sweeping) RSAE frequency in the MHD limit $\omega_{AE}$ (green ‘x’ markers), and the corresponding continuum accumulation point frequency $\omega_{cap}$ (blue asterisks) as functions of $q_{\text{min}}$, which is reported decreasingly to emulate the equilibrium temporal evolution. Frame (b): the mode linear growth rate $\gamma$ (red circles) and damping rate in the MHD limit $\gamma_{d;AE}$ (green ‘x’ markers). For the equilibrium with $q_{\text{min}} = 1.86$, the upper branch RSAE frequency and damping rate in the MHD limit are indicated as magenta crosses.

\[
\omega_{\text{res}} = (nq - m + \ell)\omega_1 + n\omega_d
\]

is the orbital resonance frequency \([53, 87]\) for co-circulating EPs; $\bar{q}$ is the orbit averaged value of the safety factor $q$; $\ell$ is an effective transit resonance harmonic, where $\ell = 2$ is found for the EPs with $E \approx E_{\text{peak}}$ in all cases. Note that for circulating EPs with large magnetically drift orbit widths across flux surfaces, the helicity along the orbit is generally different from that of the fluctuation \([106]\). Correspondingly, rather than $\ell = 0$, $\ell = \pm 1$ and/or $\ell = \pm 2$ could be considered as the ‘primary’ transit resonance harmonic with the most efficient power transfer, where a detailed analysis requires a case-by-case study \([53, 107]\). Moreover,

\[
\omega_1 = \frac{2\pi}{\hat{\phi} d\theta/\theta} \approx \frac{\gamma_1}{qR}
\]

is the transit frequency;

\[
\omega_{\text{cap}} = \int \frac{\omega_1}{2\pi} \left( \frac{\zeta}{\theta} - q \right) d\theta
\]

is the toroidal precessional drift frequency, and is generally subdominant to the first term in the right hand side of (5) for well circulating EPs. To the lowest order, the resonance condition with evolving $q$ can be estimated as

\[
\omega \approx \omega_{\text{res}} \sim (nq - m + \ell) \sqrt{\frac{E_{\text{peak}}}{m_H}} \frac{qR}{\sqrt{E_{\text{peak}}/m_H}}.
\]

Thus, $E_{\text{peak}}$ necessarily shifts up with increasing $\omega$ and decreasing $q$ for a fixed $\ell$, such that the RSAE upward frequency sweeping corresponds to a similar sweeping in the resonant EP phase space distribution as shown in figure 8. One could further expect a relay of dominant resonances via higher/lower $\ell$ in the long timescale RSAE frequency sweeping; this is confirmed in similar simulations with smaller/larger $q_{\text{min}}$ (not reported in this paper). Nevertheless, the power transfer via adjacent $\ell$ becomes notable when $E_{\text{peak}}$ approaches the margin of $\Delta E_{\text{res}}$ (see figure 21(a) below for an example). The sweeping in resonant energy and the relaying of $\ell$ indicate that the RSAE could stay in resonance with a substantial EP population during the long timescale frequency sweeping; consequently, the RSAE-induced EP transport is expected to be global in the EP phase space. The variable resonance condition

Figure 6. For the reference case with $q_{\text{min}} = 1.90$, several dominant Fourier harmonics of the electrostatic potential $\phi_{m,n}$ in the linear stage (a), and the corresponding integrated intensity of the FFT spectra $|\hat{\phi}(r,\omega)|^2$ (b). The dotted curves in the frame (a) correspond to the RSAE radial structure in the MHD limit for comparison. The overlaid black curves in the frame (b) are the ideal MHD SAW continua.
the integrated power transfer can be symbolically written as [87]

\[
\text{Power} \sim \int \text{d}r \text{d}v \left( \frac{\gamma}{\epsilon} \right) \langle \cdot \rangle \cdot \nabla \phi \cdot \mathbf{b} \times \nabla \left( \phi - \frac{\gamma}{\epsilon} \delta \epsilon \right) \right) \times \left( \frac{\gamma}{\epsilon} \nabla \log B + \frac{\epsilon}{\nabla H} \cdot \mathbf{b} \times \nabla \phi \right) \right) \tag{9}
\]

following the EP phase space representation in HMGC, with \( \kappa \equiv \mathbf{b} \cdot \nabla \mathbf{b} \) the equilibrium magnetic field curvature and \( \mathbf{b} \equiv \mathbf{B}/B \). (We refer interested readers to [87] for more details.) Here, the finite \( \gamma \) enters the resonance denominator and broadens \( \Delta E_{\text{res}} \); in addition, the energy dependence of power transfer (second row in (9)) also extends \( \Delta E_{\text{res}} \) to \( E \approx E_b \) in all cases, since the power transfer nominator scales as \( E^{5/2} \) [46] and the underlying distribution function (2) roughly scales as \( E^{-3/2} \) above \( E_c \) and up to \( E_b \). Thus, the different directions and scales of the EP-induced frequency shift can be understood from the asymmetry about \( E_{\text{peak}} \) within the broad \( \Delta E_{\text{res}} \); i.e. the shift to the fluctuation frequency is weighted by the contribution of all resonant regions so as to maximize the overall wave-EP power transfer. Furthermore, the variation of \( \gamma \), which reflects the integrated power transfer, is also consistent with the trend of \( \Delta E_{\text{res}} \).

The above qualitative analysis can be further extended by numerically calculating \( \omega_{\text{res}} \), where we utilize test particles to represent the phase space region with effective power transfer. The test particles are evolved in the perturbed electromagnetic fields stored from the self-consistent simulation [87]; thus, they can also be used as markers to analyze the nonlinear dynamics (see section 4.2). Figure 9 shows the result of this exercise, where the \((r, E)\) space is mapped by test particles using the injection pitch angle. One sees that for the sub-Alfvénic EPs, the variation of \( \omega_{\text{res}} \) is quite small in the energy space, such that significant power transfer takes place within a broad phase space range (see figure 10). Note that the broad \( \Delta E_{\text{res}} \) is a peculiar feature for the present scenario with \( \gamma/\omega_c \sim 0.15 - 0.35 \) (for relevant phase space range), due to large density (strong drive) but low energy (sub-Alfvénic) P-NBs. Furthermore, nearly the same linear \( \omega_{\text{res}} \) map is obtained for other cases with small inter-case equilibrium differences; thus, the inter-case shifts of \( E_{\text{peak}} \) and \( \Delta E_{\text{res}} \) can be appreciated from figure 9 by fitting other mode frequencies. Here, as indicated above, for the low frequency cases shown in figure 8, resonances via multiple \( \ell \) contribute to the power transfer and consequently, the broad \( \Delta E_{\text{res}} \).

On the radial resonance structure, figure 9 shows that \( \omega_{\text{res}} \) is weakly varying radially for a fixed \( E \), due to the weak shear dominating the radial derivative of \( \omega_{\text{res}} \) in (5). Thus, the radial scale length of power transfer is regulated by finite mode width (see (9)), as can be compared in figures 6, 9, 10. The underlying radial resonance structure for an isolated resonance (figure 9), together with the radial scale lengths of mode structure and power transfer (figures 6 and 10), suggest that the non-linear saturation of the fluctuations will be dominated by the radial decoupling mechanism [1, 53, 87], as intensively investigated numerically [45, 81, 85, 87, 108, 109]. That
is, the fluctuation-induced radial EP transport is expected to be comparable with the radial mode width, i.e. on meso- or macro-scales, where the plasma non-uniformity is crucial to describe the non-perturbative SAW-EP dynamics. The saturation dynamics is investigated in section 4.

4. Non-linear saturation

We still use the reference RSAE case \( q_{\text{min}} = 1.90 \) for the analysis of non-linear saturation dynamics in this section, where the relevant spatiotemporal scales are of particular interest. The fluctuation frequency chirping and the EP transport are analyzed in sections 4.1 and 4.2, respectively. Meanwhile, section 4.3 integrates the self-consistent wave-EP dynamics, and summarizes the inter-case similarities and differences.

4.1. Frequency chirping

Figure 11 presents several macroscopic variables of the fluctuation around the initial saturation time \( t_{\text{sat}} \), including the volume integrated total fluctuation energy \( W \propto \int \phi^2 \text{d}r \), mode structures and frequency. It can be directly observed that when the fluctuation reaches an appreciable amplitude, the mode structure undergoes a clear deformation from the coherent one in the linear stage; in particular, a splitting in the \((r, \omega)\) space is observed. A primary branch resides at nearly the same radial location with a reduced radial mode width and an upward chirping frequency; in other words, towards the MHD limit discussed in section 3.1. In the meantime, a subdominant one gradually emerges, it instead goes slightly downwards in frequency, and clearly decays in intensity. Here, we note that the actual onset time of the primary branch frequency chirping should be shifted forward in time by a large fraction of the FFT time window \( \Delta t_{\text{window}} = 300 \omega^{-1}_{A0} \), due to the asymmetry of the fluctuation amplitude in the FFT temporal series approaching saturation (see section 2.2). For cross-reference, a spectral analysis using a smaller value \( \Delta t_{\text{window}} = 100 \omega^{-1}_{A0} \) is also reported in figure 11(b). As expected, it shows a similar chirping range and an onset time closer to \( t_{\text{sat}} \) (but is unable to clearly resolve the subdominant branch). Thus, the saturation and frequency chirping of the fluctuation occur in the same non-linear timescale, \( \tau_{\text{NL}} \). As we are interested in the fast frequency chirping, \( \tau_{\text{NL}} \) is quantified as the time interval in which the primary branch fluctuation frequency clearly varies. From figure 11(b), one can estimate \( \tau_{\text{NL}} \approx 135 \omega^{-1}_{A0} \), as well as the chirping rate \( \dot{\omega} \approx 9.1 \times 10^{-5} \omega_{A0}^2 \). Furthermore, the effective wave-particle trapping time \( \tau_B \) is needed to characterize the non-linear chirping dynamics. We note that \( \tau_B \) varies among particles at different locations in the phase space resonance structure, and roughly scales as the inverted square root of the instantaneous fluctuation amplitude \( \phi \). Nevertheless, the phase space averaged value \( \langle \tau_B \rangle \) can be estimated from the oscillation period of \( W \) after saturation, as the wave-trapped EPs exchange energy back and forth with the wave during oscillation in the potential troughs [71]. For the fluctuation amplitude near \( t_{\text{sat}} \), \( \langle \tau_B \rangle \approx 100 \omega^{-1}_{A0} \) as can be estimated from figure 11(a). Thus, it is demonstrated that the fast frequency chirping in \( \tau_{\text{NL}} \approx O(\tau_B) \) is in the non-adiabatic regime with \( |\dot{\omega}| \approx O(\tau_B^{-1}) \), and is crucially related with the non-perturbative EP dynamics [1, 50, 53, 84].
As noted above, the EP transport should be analyzed in the phase space to fully capture its self-consistent interplay with the fluctuations. Following the constancy of $\mu$, we first examine the EP transport in the $(P_\zeta, E)$ space, with $P_\zeta = m_q R V || + e_q R_0 (\psi - \psi_0) / c^5$ the toroidal canonical angular momentum.

\footnote{Note that in HMGC, $\psi$ is defined by the form of magnetic field $B \equiv B_0 \delta r \delta \zeta \nabla \zeta + B_0 \nabla \psi \times \nabla \zeta$; it peaks on the magnetic axis and vanishes on the edge.}

Both $P_\zeta$ and $E$ are orbit invariants in the equilibrium magnetic field; thus, the effective perturbation to the EP distribution function by finite amplitude fluctuations can be readily illustrated by this method, as shown in figure 12 for a ‘slice’ of the EP distribution with a single $\mu$ value. Significant and global EP transport takes place from the inner- to outer-core region, where the redistribution essentially follows lines with constant $C \equiv E - \omega P_\zeta / n$. Here, $C$ is the extended phase space Hamiltonian and is conserved non-linearly to the lowest order in frequency expansion [111]. Further to this, a representative phase space portion with single values of $\mu$ and $C$ is sampled by test particles as indicated in figure 12 [87]. Figure 13 shows the non-linear evolution of these resonant EP samples in the $(P_\zeta, \Theta)$ plane, with $\Theta \equiv -\omega t + n \zeta - m \theta$ the wave-particle phase. Here, we note that during the saturation stage, the resonant EPs are nonlocally convected via a secular variation of $P_\zeta$ within the radial volume of mode structure. In addition, multiple phase space structures corresponding to the instantaneous potential wells of the wave [66, 71] can be observed in figure 13(c). From another perspective, figure 14 plots the orbit of a representative resonant test particle in both configuration and phase spaces. One can readily see a global-scale transport within a few orbital periods [49, 79], as well as the non-linear oscillations in the potential wells from $P_\zeta$. Note that the effective wave-particle trapping time for this particle during the saturation stage is $\tau \approx 170 \omega^{-1}_R$, somewhat larger than $\tau$ estimated above, since it resides near the outer boarder of the phase space resonance structure (see the initial $P_\zeta$ value with figure 13).

On the non-linear spatiotemporal scales, as noted above, the spatial scale of resonant EP transport is dominated by radial convections comparable with the radial mode width. Moreover, at the saturation time, most resonant EPs have not completed the first non-linear oscillation in $P_\zeta$ within the phase space resonance structure. These characteristics make clear that the fluctuation saturates due to radial decoupling with the resonant EPs, whose transport is intrinsically nonlocal [1, 53]. In fact, as anticipated in section 3.2, radial decoupling is suggested by the flat $\omega_{res}$ profile shown in figure 13(a) (or, more generally, figure 9) induced by the weak magnetic shear in the inner-core region. Since the resonance condition $\omega_{res} \approx \omega$ can be satisfied over a broad radial range, the resonant EPs will not be brought out of resonance by a radially localized orbit excursion. Instead, in order to quench the EP drive necessary for mode saturation, they need to be decoupled from the non-uniform mode structure by radial transport comparable with $\lambda_\perp$. Thus, for long wavelength RSAEs in weakly reversed shear plasmas, global-scale EP transport by radial decoupling is generally prevalent. Note that the general validity of this interpretation also applies to RSAEs driven by the magnetically trapped EPs, where the radial resonance structure is quite similar to the present case [45].

Extending the analysis above, the collective EP transport can then be confidently understood following the linear resonance analysis in section 3.2. Indeed, EP transport occurs in similar spatiotemporal scales for other resonant phase space portions, and results in global EP profile relaxations of the
the resonance condition is maintained as much as possible, the convective branch is nevertheless, not strongly driven in the present case, since the EP drive intensity associated with its phase space profile gradients is significantly reduced simultaneously. Meanwhile, it suffers from stronger continuum damping away from the shear reversal radius \([2\pi, 4\pi]\) for a better visualization. Each test particle’s \(P_\zeta\) and the wave-particle phase \(\Theta\) at the LFS equatorial plane of the last completed orbit are represented as a marker in the Poincaré map. The color scales refer to the initial \(P_\zeta\) with respect to a linear resonant value (see frames (a) and (b)). Note that the primary branch mode frequency is used in calculating \(\Theta\).

4.3. Self-consistent wave-EP dynamics

Going back to the frequency spectrum shown in figure 11, the splitting of mode structure can be qualitatively interpreted in terms of the global EP transport as well as their radial decoupling from the non-perturbatively driven fluctuation. Due to the non-uniform radial distribution, a finite amplitude fluctuation induces a net-outward flux of resonant EPs, with decreasing \(E\) and \(v_||\), i.e. decreasing \(\omega_{res}\). If the underlying EP transport is convective and macroscopic, characteristic of non-perturbative SAW-EP interactions in the nonadiabatic regime, the fluctuation tends to modify its radial structure and frequency, so as to stay resonant, i.e. phase locked (\(|\gamma_N\Theta|, |\gamma_H\Theta| < 1\) with the resonant EP flux, such that the corresponding power transfer is maximized [1, 50, 53, 81, 84]. Indeed, the downward chirping branch arises from phase locking with the nonlocal resonant EP convection (dubbed as the convective branch). Note that although the resonance condition is maintained as much as possible by the fluctuation shifting its mode structure and frequency, the convective branch is nevertheless, not strongly driven in the present case, since the EP drive intensity associated with its phase space profile gradients is significantly reduced simultaneously. Meanwhile, it suffers from stronger continuum damping away from the shear reversal radius [43, 58]. Thus, although the convective branch can be (barely) recognized in figure 11, it decays in intensity due to both reduced drive and enhanced damping. On the other hand, accompanied by the global EP transport, the non-perturbative effect as shaping the
perturbative EPs in the non-adiabatic regime, where more consistency illuminates the dominant role played by the non-adiabaticity and non-perturbativity of EP dynamics, illustrating the one-on-one correspondence of frequency chirps non-adiabatically following the self-consistent process. Moreover, the convective EP transport and the saturation time (red dots). Frames (b) and (c) plot the EP density profiles in the linear stage (thin black curve in figure 4(a)) and the saturation time (red dots). Frames (b) and (c) show similar mode frequencies in all cases, consistent with their radial resonance detuning process [111, 112] as the wave-particle phase shift is minimized, and the resonant EPs maintain predominantly near constant phase with the chirping fluctuation during the radial convective transport. Finally, anticipating the analysis for the post saturation dynamics in section 5, we note that in all cases, the fluctuations are dominated by weakly damped AEs after saturation, as shown in figures 11, 16, 17; the convective branches are eventually attenuated by the enhanced damping. It suggests that for the non-perturbatively driven RSAEs, the relaxation branch as a standing wave inside the potential well is preferred over the convective one, due to the spatially non-uniform damping and the fact that global EP drive is generally not strong enough to trigger an avalanche process [50, 53, 84, 113]. More discussions and possible applications of identifying the two branches can be found in section 6.2.

Similar spatiotemporal scales of EP radial transport are observed in all cases, consistent with their radial resonance
structures analyzed and anticipated in section 3.2, as well as the non-perturbative SAW-EP interplay and non-adiabatic frequency chirping illustrated above. However, the collective EP transport has different features at the saturation stage, since it is also related with $\Delta E_{\text{res}}$ and the fluctuation amplitude responsible for the overall transport intensity. For reference, figure 19 shows the initial saturation fluctuation energy $W_{\text{sat}}$ in a qualitative agreement with the trend of $\gamma$. In accordance with the discussions in this section, the EP profile relaxation is similar to the reference case for most low frequency RSAEs; whereas it is clearly weaker in intensity and restricted in the high energy extent when approaching the TAE frequency, especially for the $q_{\text{min}} = 1.86$ case with the most stringent selection of resonant EPs. However, it is important to note that such an inter-case variation is correlated with the parameter $v_{\parallel}/v_A \lesssim 1/3$ in the present simulations (see section 2.3); where the reduction of $\Delta E_{\text{res}}$, and consequently, $W_{\text{sat}}$, is mostly caused by the transition of the dominant resonance harmonic discussed in section 3.2. In more general cases with a somewhat larger $v_{\parallel}/v_A$ value, the RSAFE is expected to maintain similar amplitudes during the transition to a TAE [114]. Furthermore, it should also be noted that the saturation amplitude does not necessarily represent the post saturation fluctuation and EP dynamics, which is subject to the continuing self-consistent SAW-EP interplay. More details in the post saturation non-linear dynamics are investigated in section 5.

5. Post saturation dynamics

We focus on the time evolutions of the fluctuation amplitude and EP confinement property in this section. Here, it is important to note that the fluctuation and EP dynamics in the post saturation stage are, in principle, subject to many effects not included in the present simulation model, such as the fast ion source and thermal ion kinetic response. In particular, even restricted in short timescales, a strong source term could make a substantial difference to the fluctuation dynamics, depending on the self-consistent evolution of the EP distribution function [53, 101]. The fluctuations that are weakly decaying in the present simulations might remain a
be regarded as qualitative only, and we remind the readers the various assumptions of the present simulation model discussed in section 2. Correspondingly, we focus on the general physics mechanisms underlying the simulation results; understanding the relevant physics ingredients could be useful in predicting the long timescale dynamics in a more realistic investigation.

Two examples for the time evolutions of the fluctuation amplitude are shown in figures 11(a) and 17(a); other cases behave very similar to the reference one (figure 11(a)). The common features include that the fluctuation amplitude oscillates with a period ($\gammaH$) before such oscillations are eventually indiscernible in the late non-linear stage; that the radial structure and frequency of the dominant mode are very close to a weakly damped AE state (see figures 16 and 17); and that the amplitude eventually decays. On the other hand, the case with $q_{\text{min}} = 1.86$ is more distinctive that the amplitude has a visible amplification process before the eventual decay. The mechanisms for the non-linear oscillation and relaxation have been discussed above; we focus on the decay and amplification processes in the following.

For the decay process dominating the post saturation dynamics, it is straightforwardly related with the non-linear reduction of the EP drive intensity and the small but finite mode damping reported in figure 5. Indeed, for the reference case, figure 20 plots the time evolution of the EP driving rate $\gamma_{\text{drive}}$, which is estimated by integrating the wave-EP power transfer, weighting over the distribution function and normalizing to the instantaneous total fluctuation energy [44, 87]. One sees that associated with the significant and global phase space profile relaxations during the saturation stage, the residual EP drive is insufficient to overcome the intrinsic damping. Therefore, the amplitude slowly decays, and the EP transport has not significantly increased in the post saturation stage. Here, we note that for the EP population considered in this work, its diamagnetic frequency can be estimated as $\omega_{aH} \sim m_\text{p}qV/\langle B_0 \rangle \sim 0.9\, \omega_{A0}$, i.e. $\omega_{aH}/\omega \sim O(10)$. Due to the relatively low characteristic EP energy, the condition $\omega_{aH}/\omega \gg 1$ for the EP drive dominated by the density gradient (‘universal instability mechanism’) [115, 116] is not strictly applicable to the present cases; a substantial fraction of EP drive is also provided by the velocity space anisotropy, which can be further pinpointed to the sharp pitch angle distribution (4). Correspondingly, the nearly vanishing EP drive should be restored to the significant transport in both configuration and velocity spaces [117] as shown in figure 15. In particular, we note that in the post saturation stage, the outer-core TAE is distinguishable but not strongly driven in all cases (see figure 17), despite a slightly increased density gradient in the outer-core region. In this regard, a transport route by the relay of RSAEs and outer-core TAEs seems likely in a realistic scenario with continuous beam injection. The inner-core RSAEs could efficiently scatter EPs from the plasma core and possibly increase the drive intensity for the outer-core TAEs, which continue to transport the EPs further outwards. This effect is not clearly observed in the present simulations, since the EP velocity space profiles are significantly and globally distorted by the RSAE saturation; besides,
having EPs with $v_{1}/v_{A} \lesssim 1/3$ might also be responsible for the weak TAE drive. In general, the present simulations suggest that the global EP phase space profile evolution, associated with the linear resonance structures and the non-linear saturation mechanism, is crucially responsible for the further non-linear dynamics of the system.

Different phase space profile distortions are also important for the non-linear amplification observed in the case with $q_{\text{min}} = 1.86$. In contrast with other cases, the EP transport is weaker in the initial saturation stage, such that significant free energy is retained in the EP phase space distribution. Via the non-adiabatic frequency chirping illustrated in figure 17, the relaxed fluctuation is able to extend the phase space resonant range and be further amplified, where the highest frequency upper branch RSAE is most strongly driven non-linearly. To show this, figure 21 compares the SAW-EP power transfer in the velocity space during the linear and non-linear growing stages. We can see that since the phase space resonant region in the linear stage is mostly localized at the high energy end, EPs in the essentially unaffected phase space region are able to resonate with the relaxed AE and yield significant power transfer in the non-linear stage. In particular, the upper branch RSAE is strongly driven in the post saturation stage due to its large frequency separation from the dominant linear fluctuations. This feature denotes a significant difference with respect to other cases characterized by smaller linear frequency shift and broader phase space resonant range. Thus, the different non-linear dynamics are, in fact, produced by the different strength of non-perturbative EP effects, which can be estimated from the scales of the EP-induced non-perturbative frequency deviation from the MHD limit [1, 51–53]. Ultimately, this deviation is reflected by the corresponding separation of the linear and non-linear resonant ranges in the EP phase space, and explains the qualitatively different non-linear evolutions of the fluctuation amplitude. Here, we emphasize that the non-linear amplification is induced by the non-adiabatic frequency chirping and the extension of EP phase space resonant range. Thus, it is consistent with the theoretical paradigm of convective amplification [1, 50, 53, 84], where the simulations illuminate the fundamental nature of maximizing power transfer in the non-perturbative SAW-EP dynamics. After the residual free energy in the EP distribution is exhausted, the fluctuation eventually decays, with significantly enhanced EP transport compared with the initial saturation stage. In the time asymptotic limit of short timescale investigations, i.e. when the fluctuation amplitude decays to a negligibly low value, the overall intensity of EP phase space transport is comparable in all cases.

6. Conclusion and discussion

6.1. Summary

In this paper, via hybrid MHD-gyrokinetic code simulations, we analyze the dynamics of the reversed shear Alfvén eigenmode fluctuation and energetic particles during the current ramp-up phase of a conventional tokamak discharge. The simulations consider time slices in the long timescale MHD equilibrium evolution by initializing a series of self-similar $q$ profiles with decreasing $q_{\text{min}}$. Moreover, an anisotropic concentration of fast beam ions is assumed with an idealized model distribution function. RSAEs are found to be strongly driven by the EPs, where the most evident non-linear dynamics takes place in the non-adiabatic regime dominated by the non-perturbative EP response. The most notable findings of this work may be summarized as follows:

- In the linear stage, EPs induce evident mode structure distortions and frequency shifts, which can be explained by the global resonance structure. In particular, the relatively low beam ion energy restricts the linear mode frequency
into a narrow band; besides, it also allows a generally broad effective resonance range in the EP phase space.

- During the saturation stage, the radial decoupling mechanism is shown to be crucial, consistent with the underlying radial resonance structure dominated by the weak magnetic shear. Global EP phase space convective transport takes place, including a flattening in the density profile, an overall slowing-down in the energy space, as well as a broadening in the pitch angle distribution [67]. The non-linear timescale of the saturation dynamics is of the order of the wave-particle trapping time, and the spatial scale of the EP radial transport is comparable with the perpendicular fluctuation wavelength.

- Echoing the significant EP transport, the non-perturbatively driven fluctuation chirps non-adiabatically in frequency. It splits into a downward chirping 'convective' branch via phase locking with the resonant EP convection, and 'relaxation' branches that tend to evolve into the fluctuation structures obtained in the weakly damped MHD limit. Associated with the particular SAW continuum structure near shear reversal, the convective branch suffers from enhanced continuum damping; thus, the relaxation branch dominates in the post saturation stage.

- The self-consistent evolution of the EP distribution function plays an important role in the long timescale dynamics. In the present simulations without external source, all RSAEs eventually decay in the post saturation stage due to almost quenched EP drive, which is crucially induced by the global EP profile relaxations in both configuration and velocity spaces.

- The non-adiabatic frequency chirping could extend the phase space resonance range, giving rise to enhanced fluctuation amplitude and EP transport. This behavior is characteristic of significant non-perturbative EP responses, such that the phase space region dominating the resonance structures in the non-linear stage is only weakly disturbed during the initial saturation.

6.2. Discussions

6.2.1. General consideration. This work could be regarded as an attempt to connect the fundamental physics of wave-EP resonant interaction with realistic experiments. On one hand, the RSAEs are a commonly observed phenomenon in the current ramp-up phase. The weakly driven RSAE spectra are relatively well understood and are successfully applied as the MHD spectroscopy in experiments [20]; however, the dynamics of strongly driven RSAEs and the correlated EP transport require further analysis. In general, the present work suggests that both linear and non-linear dynamics of strongly driven RSAEs are dominated by EPs, which are prone to induce non-perturbative shifts to the RSAE spectra if a distribution function with relatively large density and strong phase space anisotropy is produced. Indeed, besides the obvious differences of the mode frequencies and resultant characteristic resonant EP energy, varying profile across a wide RSAE frequency range does not significantly modify the signature of dominant physics mechanism; all cases show consistent features of the non-perturbative SAW-EP interactions. In other words, ramping-up SAW-EP frequency sweeping of RSAE frequency sweeping in long timescales; meanwhile, the EPs govern the overall fluctuation intensity, and could induce additional twists to the RSAE spectra in short timescales, with a profound impact to their own confinement property. In particular, as discussed in section 5, the self-consistent evolution of the global EP phase space profiles directs the non-linear evolution of the system; moreover, the non-adiabatic frequency chirping could extend the EP phase space resonance region and further tap the free energy.

On the other hand, the strongly driven RSAEs appear to be rare in most experiments with a moderate heating power; the experimental validation requires further investigations to the current ramp-up scenario, especially noting the various important effects not taken into account in the simulation model. For example, the neglected kinetic thermal ion response and EP FLR effect could reduce the intensity of the driven fluctuation [48]. Moreover, this work considers a simplified model where a single toroidal mode number RSAE is predominantly driven by a well isolated resonance, which induces convective EP transport in short timescales. The stability property of the RSAEs and the EP transport might be drastically different in a more realistic simulation with many unstable waves (Alfvén cascades) excited simultaneously [118]. The couplings among these fluctuations impact their non-linear dynamics, and the eventual EP transport could also have a diffusive character due to resonance overlap [30, 45, 112]. Among others, to facilitate inter-case comparisons, the simulations assume an artificially fixed initial EP distribution function. However, the long timescale self-consistent evolution of the EP phase space distribution function is subject to many effects, such as the external source and internal dissipation/equilibration mechanisms, including the fluctuation-induced EP transport investigated herein. In particular, a high power EP source dominates the initial stage of a discharge before a genuine EP equilibrium distribution function is formed in the timescale comparable with the slowing-down time [6, 100]. Thus, although computationally challenging, self-consistent simulations on long timescales with a continuous EP source and continuous MHD activities are crucially needed to further analyze this scenario. In addition, for the RSAE dynamics in the current ramp-up phase, a long timescale simulation would also require a self-consistent treatment of the MHD equilibrium evolution. Since the plasma current generally consists of a non-negligible beam current fraction, a significant EP redistribution could perturb the macro-scale equilibrium current density profile, and impact the RSAE dynamics. Similarly, meso-scale coherent corrugations of the equilibrium, i.e. zonal structures, could be driven by a finite amplitude Alfvén wave, and have a non-linear feedback to its dynamics [119]. Altogether, the present work recalls the importance of taking into account all relevant physics ingredients to properly tackle the crucial and complex issue of EP physics in the multi-scale dynamics of fusion plasmas [1, 84].
6.2.2. Specific interpretation. The present work also illustrates many peculiar properties of weakly reversed shear plasmas in typical present-day tokamaks. For example, the radial scale length of characteristic EP transport induced by a finite amplitude RSAE is expected to be comparable with its perpendicular wavelength due to the prevalence of radial decoupling, where the underlying resonance structure with circulating EPs is dominated by the weak magnetic shear. The general validity of this interpretation is extended by [44, 45] covering a somewhat different parameter regime, where the RSAE is driven by the precessional resonance, \( \omega \sim n_{\|} \), of the magnetically trapped EPs produced by, e.g. high power on-axis ICRH [120]. In particular, [45] shows that even close to marginal stability, the RSAE-EP non-linear dynamics is dominated by radial decoupling due to the weak radial dependence of \( n_{\|} \) [1, 53]. As a result, global-scale EP profile relaxations take place, contributed by the combination of long fluctuation wavelength and large normalized EP orbit width in common present-day devices. Here, we note that most EPs are still confined in the short timescale analysis of each simulation case; i.e. direct EP losses by a single RSAE are typically negligible with low perturbation amplitude near the plasma edge. However, note also that the redistributed EPs remain energetic in the cooler plasma periphery since

\[
\frac{\Delta E/E}{\Delta P_{\perp}/P_{\perp}} \simeq \frac{\omega P_{\perp}}{nE} \ll 1
\]

for EPs. Thus, these EPs are prone to be lost by other mechanisms in a more realistic scenario, and hereby, proposing challenges to the plasma-wall interaction [121]. The ability to effectively scatter EPs makes the RSAE a formidable culprit to impacting the heating and current drive efficiency as well as the first wall power load. In particular, RSAE instability with a flexible frequency is difficult to be controlled from the side of the EP phase space engineering, since the effective resonance range sweeps the EP phase space distribution along with sweeping RSAE frequency. Further analyses are necessary to investigate the RSAE-induced EP transport and real time control strategy in practice [122].

More specifically, the present simulations propose a probable interpretation to the multi-timescale RSAE frequency chirping and sweeping observed in various experiments [9, 54–57]. Qualitatively similar phenomena have been observed with diverse plasma parameters and heating methods, suggesting an unified physics explanation. It is indicated that the fast frequency chirping could be due to the non-perturbatively EP-induced frequency shift and the consequent non-adiabatic convection/relaxation. The reversed shear configuration always favors the weakly damped relaxation branch via the non-uniform continuum damping to the simultaneously generated convective branch; therefore, the slow frequency sweeping due to equilibrium evolution is also demonstrated by the time asymptotic limit of short timescale dynamics. Note that although the eventual state of the relaxed fluctuation is close to the perturbative MHD limit, a perturbative analysis to this scenario would significantly underestimate the overall EP transport intensity without considering the non-perturbative EP induced frequency shift and mode structure extension. Both linear effects induced by EPs maximize the wave-EP power transfer and result in stronger fluctuation intensity than the perturbative limit [49]. In the non-linear dynamics, the existence of a convective branch illustrates that the non-perturbatively driven fluctuation could evidently modify its radial structure and frequency to stay in resonance with the EPs. The corresponding SAW-EP power transfer is maximized in this process, giving rise to enhanced saturation amplitude [45]; moreover, the onset of phase locking greatly facilitates nonlocal EP transport by extending the finite interaction time (\( \sim \tau_b \)) and length (\( \sim \lambda_\perp \)) [1, 53]. Theoretically, the EPs could only be considered as perturbative in the following condition [1, 51–53]

\[
|\Delta \omega_{\text{EP}}| \ll |\Delta \omega_{\text{SAW}}| , \tag{11}
\]

where \( \Delta \omega_{\text{EP}} \) is the EP induced complex frequency shift, and \( \Delta \omega_{\text{SAW}} \) is the frequency difference of an AE with respect to its corresponding continuum accumulation point (see figure 5).

6.2.3. Further outlook. Looking at a broader picture, by investigating the interplay of the magnetic geometry, plasma non-uniformity and non-perturbative EP effects for a case of practical interest, this work illustrates the interconnections among the deviation from marginal stability, non-perturbative SAW-EP interplay, non-adiabatic frequency chirping and non-local EP transport via radial decoupling, where one can also appreciate their relative importance from inter-case comparisons. The simulations also suggest verification/validation practices and possible applications for the theoretical paradigm of non-perturbative wave-EP interactions in the non-adiabatic regime [1, 53]. In fact, fast chirping starting from a frequency that deviates from the expectation of the MHD theory has been repeatedly identified in experiments [9, 54–57, 88–90], manifesting the one-on-one correspondence of non-adiabaticity and non-perturbativity. Thus, the theoretical understanding can be applied to verify/identify the significance of non-perturbative EP effects and search for possible EP confinement degradation from the fast frequency chirping in experimental observations and numerical simulations.

Further to this, this work provides more insights to interpreting the preferred direction of frequency chirping and its consequence on EP transport by characterizing the non-adiabatic frequency chirping as the convective and relaxation branches. The origin of the two branches is conceptually similar to the beam and plasma roots in the 1D uniform beam-plasma system [123]. However, diverse behaviors are observed for SAW-EP dynamics in tokamaks; the identification and quantification for the chirping mechanism as the relative importance of the two branches could be tricky, and require a case-by-case study, especially in scenarios where they cannot be readily separated, e.g. fishbones [88, 90]. In principle, we note that the convective branch is intrinsically nonlocal; it commonly chirps downwards and propagates radially outwards for fluctuations predominantly driven by the universal instability mechanism. The convective branch could transiently dominate in short timescales and give rise to pulsing EP
transport/losses; whilst in long timescales, it generally suffers from strong continuum damping. On the other hand, the relaxation branch has a well-determined chirping direction towards the weakly damped MHD limit. Both branches tend to increase the resonant range in the EP phase space via non-adiabatic frequency chirping; however, the nonlocal convective branch is more of a concern for possibly intense and concentrated EP losses with an avalanche-like phenomenon [1, 50, 53, 58]. The simulations in this work suggest that the relative importance of these two branches is strongly correlated with the significance of the non-perturbative EP effect. Nevertheless, it requires further analyses to other scenarios due to the complication by the interplay of the magnetic geometry, plasma non-uniformity and non-perturbative EP responses, which govern the relative intensity of the two branches via the local and nonlocal EP drive as well as the continuum damping.

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