‘Forbidden’ intersubband optical transitions in quantum well structures in a tilted magnetic field

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Abstract

In this paper, we report on the behavior of intersubband optical absorption and emission spectrum in quantum well structures with an asymmetrical potential profile in a quantizing magnetic field tilted with respect to the layers in the considered structure. The analysis of the effect of both the magnetic field value and orientation and potential profile of a quantum well structure on the intensity of the ‘forbidden’ ($\Delta n \neq 0$) intersubband optical transitions is presented. We also present the approximate analytical expression for the estimation of the contribution to the absorption/emission coefficient of intersubband transitions with different $\Delta n$. 

1. Introduction

Application of a quantizing magnetic field to a quantum well leads to a substantial change in the single-electron spectrum. Each of the continuous 2D subbands of the quantum well is split into a discrete set of equidistant Landau levels (LL). The discrete nature of the Landau spectrum leads to an essential transformation of electron scattering [1–4] and relaxation processes [5–7], which can be used to improve the characteristics of THz and far-infrared optoelectronic devices [1–16].

The other appealing aspect is a possibility of tuning the frequency of optical transitions between Landau levels of different subbands [17, 18]. The frequency of transition between $n_1^1$th LL in $\nu_1$th subband (LL ($\nu_1$, $n_1$)) and $n_2^1$th LL in $\nu_2$th subband (LL ($\nu_2$, $n_2$))

$$\hbar \omega = \Delta \epsilon + \hbar \omega_c (n_1 - n_2)$$

(1)

(where $\Delta \epsilon$ is intersubband spacing and $\omega_c = eB/m_{\text{e}}c$ is the cyclotron frequency) can be tuned by varying magnetic field strength, if $\Delta n = n_1 - n_2 \neq 0$. However, there is the selection rule ($\langle \Delta n = 0 \rangle$) which forbids any transitions with $\Delta n = 0$. This selection rule can be violated by tilting the magnetic field with respect to the layers of quantum well structure [17, 19]. The numerical calculations carried out in the [19] for some fixed quantum well structure while varying the magnitude and direction of the magnetic field, demonstrated that the intensity of peaks induced by transitions with $\Delta n \neq 0$ can exceed that of the peak with $\Delta n = 0$, attaining the values close to those in the absence of a magnetic field. On the contrary in the [17], the potential profile of the quantum well structure was varied, while fixing the magnetic field magnitude and direction, and the importance of the potential asymmetry for the effect was established.

So both the magnetic field and potential profile are the critical parameters determining the intensity of the ‘forbidden’ transitions.

The primary aim of our present investigation is to study the violation of the selection rule ‘$\Delta n = 0$’ in a tilted magnetic field while simultaneously taking into account of both the critical parameters which determines the effect: tilting angle and the potential profile of the quantum well structure. We also aimed to obtain analytical expressions which give the transition intensities for different $\Delta n$, structure parameters as well as the magnetic
field strength and direction. The expressions make it possible to estimate the contribution of intersubband transitions with different \( \Delta n \) to the absorption/emission coefficient and to understand the nature of the observed features in the optical spectra.

2. Optical transitions in a quantizing tilted magnetic field

We consider the intersubband optical transitions of electrons in the quantum well structures with a potential profile \( U(z) \) in a magnetic field \( \mathbf{B} = B_{||} e_y + B_{\perp} e_z \) tilted with respect to growth axis \( z \) by an angle \( \theta \).

In the Landau gauge \( A = (B_{||} z - B_{\perp} y) e_x \), the electron envelope-function Hamiltonian in parabolic approximation has the form [18]

\[
\hat{H} = \hat{p}_z - \frac{1}{2m(z)} \hat{p}_z + U(z) + \frac{m_w}{m(z)} \left[ \left( \omega_\parallel z - \omega_\perp y \right) \hat{p}_x + \frac{m_w}{2} (\omega_\parallel z - \omega_\perp y)^2 \right]
\]  

(2)

where, \( m(z) = \begin{cases} m_w, & z \in \text{well} \\ m_0, & z \in \text{barrier} \end{cases} \) is the electron effective mass, \( \omega_\parallel = eB_{||} / (m_w c) \) and \( \omega_\perp = eB_{\perp} / (m_w c) \) are the cyclotron frequency for longitudinal \( (B_{||}) \) and transverse \( (B_{\perp}) \) components of the applied magnetic field respectively.

Since the Hamiltonian operator commutes with the operator \( \hat{\mathbf{p}}_z \) of momentum projection on the \( z \)-axis, i.e., \( \hat{H}\hat{\mathbf{p}}_z - \hat{\mathbf{p}}_z \hat{H} = 0 \), we can construct a basis from the stationary states with definite values \( \hbar k_x \) in this momentum projection. The wave-functions of such basis has the form [18]

\[
\psi(x, y, z) = \frac{\exp(ik_x x)}{\sqrt{L}} \psi(y - k_x \ell_z', z),
\]

(3)

where \( L \) is the cross section of the sample, \( \ell_z' = \sqrt{\hbar / (m_\omega z)} = \sqrt{\hbar c / (eB_{z})} \) is the magnetic length for the magnetic field component \( B_z \). The electron energies \( E \) and functions \( \psi(y, z) \) are the eigenenergies and the eigenfunctions of the two-dimensional Hamiltonian [18]

\[
\hat{H}_{2D} = \hat{p}_z - \frac{1}{2m(z)} \hat{p}_z + U(z) + \frac{m_w}{m(z)} \left[ \hat{p}_y^2 + \frac{m_w \omega_\perp^2}{2} y^2 + \frac{m_w \omega_\parallel^2}{2} z^2 - m_w \omega_{\parallel} |\omega_\parallel z| y \right].
\]

(4)

Since the Hamiltonian (4) is independent on \( k_x \), the electron energy levels are degenerate with respect to \( k_x \), the matrix of the Hamiltonian (2) in the basis of wave functions (3) is diagonal in \( k_x \), and the matrix element for \( k_{x1} = k_{x2} \)

\[
\left\langle \psi_1(y - k_x \ell_z', z) | \hat{H} | \psi_2(y - k_x \ell_z', z) \rightangle = \langle \psi_1(y, z) | \right.
\]

(5)

does not depend on \( k_x \).

In a magnetic field perpendicular to the structure layers \( (B_{||} = 0) \), the electron energy levels and wave functions are given by the expressions [20]

\[
E^{(0)}_{\nu, n} = \varepsilon_\nu + \hbar \omega_\perp (n + 1/2)
\]

(6)

and

\[
\psi^{(0)}_{\nu, n}(x, y, z) = \Phi_\nu(x) \Phi_\nu(y z),
\]

(7)

where \( \Phi_\nu(y) \) is the wave function for \( \nu \)th energy level of the linear harmonic oscillator of the cyclotron frequency \( \omega_\perp \), \( \varepsilon_\nu \) and \( \Phi_\nu(z) \) are \( \nu \)-th energy level and wave function in the 1D quantum well

\[
U(z) = \left( 1 - \frac{m_w}{m(z)} \right) \hbar \omega_\parallel n + \frac{1}{2}.
\]

In the considered structures \( (\text{GaAs}/\text{AlGaAs}) \), the effective mass in the barrier is larger than that in the well. Therefore, the addition to the quantum well potential \( U(z) \) leads to an increment of barrier height with the Landau level number \( n \) [19]. However, we consider sufficiently deep subbands, therefore, this effect is neglected.

In the basis (7), the matrix element of Hamiltonian for electron in a tilted magnetic field has the form

\[
\left\langle \nu_1, n_1 | \hat{H}_{2D} | \nu_2, n_2 \rightangle = [\varepsilon_{\nu_1} + \hbar \omega_\parallel (n_1 + 1/2)] \delta_{\nu_1, \nu_2} \delta_{n_1, n_2} + \frac{m_w \omega_\parallel^2}{2} \left( \frac{z}{\sqrt{m_w}} \right)^2 n_2 \delta_{\nu_1, \nu_2} \delta_{n_1, n_2}
\]

\[
- m_w \hbar \omega_\parallel \sqrt{\frac{m_w}{2\hbar^2}} \left( \frac{z}{\sqrt{m_w}} \right)^2 \delta_{\nu_1, \nu_2} \times \left[ \sqrt{n_2 + 1} \cdot \delta_{n_1, n_2+1} + \sqrt{n_2} \cdot \delta_{n_1, n_2-1} \right]
\]

(8)
Here we assumed that \( \varphi_0(z) \) is mostly localized within the quantum well, and, therefore we can write

\[
\left( \frac{m_w}{m(z)} \right)_{\nu_1,\nu_2} = \int dq_0^w \frac{m_w}{m(z)} z^3 \varphi_0^w(z) \approx \int dq_0^w \frac{m_w}{m(z)} z^3 \varphi_0^w(z) = \langle z^3 \rangle_{\nu_1,\nu_2},
\]

(9)

The matrix (8) can be analytically diagonalized, if the couplings between the subbands (elements with \( \nu_1 = \nu_2 \)) are neglected. In this approximation, the energy levels and wave functions are given by the analytic expressions [21–23]

\[
E_{(\nu,n)} = \varepsilon_\nu + \hbar \omega (n + 1/2) + \frac{m_w \omega_s^2}{2} (\langle z^3 \rangle_\nu - \langle z^2 \rangle_\nu),
\]

(10)

and

\[
\psi_{(\nu,n,k)}(r) = \varphi_\nu(z) \Phi_k(y - \langle z \rangle_{\nu,n,k}) \cos \theta.
\]

(11)

In the dipole approximation [24], the Fermi rule leads to the following expression for the number of electron transition events per unit time (absorption or emission intensity) from the Landau level \( i = (\nu_1, n_1) \) to the Landau level \( f = (\nu_2, n_2) \), due to the absorption or emission of photons

\[
W_{i \to f}^{\text{abs}}(\omega) = I(\omega) L^2 \frac{4\pi^2 e^2}{c\hbar} \frac{(E_f - E_i)^2}{2(2\hbar)^2} \frac{2}{\beta L^2} \times \sum_{k} |eD|^2 [N_f - N_i] \delta(E_f - E_i \mp \hbar\omega),
\]

(12)

Here the negative sign represents the absorption, and the positive sign represents the emission. \( I(\omega) \) is the intensity of the incident radiation, \( \hbar \omega \) is the photon energy, \( \eta \) is the refractive index, and \( \epsilon \) is the polarization vector of radiation,

\[
D = \langle \nu_f, n_f, k_f | r | \nu_i, n_i, k_i \rangle
\]

(13)

dipole matrix element, \( N_i \) is the population of the Landau level \( i \) (2D concentration of electrons on the Landau level), \( \beta = 1/(\pi \ell^2) \) is the degeneracy of the Landau level.

The expression (12) gives the following expressions for the power of the absorbed

\[
P_{i \to f}^{\text{abs}}(\omega) = \hbar \omega [W_{i \to f}^{\text{abs}}(\omega) - W_{f \to i}^{\text{abs}}(\omega)] \text{ or emitted } P_{i \to f}^{\text{emis}}(\omega) = \hbar \omega [W_{f \to i}^{\text{abs}}(\omega) - W_{i \to f}^{\text{abs}}(\omega)],
\]

(14)

and thus, the absorption \( \alpha_{i \to f}^{\text{abs}}(\omega) = p_{i \to f}^{\text{abs}}(\omega)/I(\omega)L^2 \) and emission \( \alpha_{i \to f}^{\text{emis}}(\omega) = p_{i \to f}^{\text{emis}}(\omega)/I(\omega)L^2 \) coefficients are

\[
\alpha_{i \to f}^{\text{abs}}(\omega) = \frac{4\pi^2 e^2}{c\hbar} \frac{(E_f - E_i)^2}{2(2\hbar)^2} \frac{2}{\beta L^2} \times \sum_{k} |eD|^2 [N_f - N_i] \delta(E_f - E_i \mp \hbar\omega),
\]

(15)

Since the component of the dipole matrix element parallel to the structure layers \( D_0 \propto \langle \varphi_\nu(z) | \varphi_\nu(z) \rangle = 0 \) for intersubband optical transitions, we consider the electromagnetic wave polarization along the structure growth axis.

The substitution of wave-functions (11) into (13) leads to the following analytical expression for the \( z \)-component of the dipole matrix element

\[
|D_0|^2 = T_{(\nu,\nu)}^{(0)} [\varphi_\nu(z)],
\]

(16)

\[
T_{(\nu,\nu)}^{(0)} = [\varphi_\nu(z) | \varphi_\nu(z)]^2,
\]

(17)

\[
T_{(\nu,\nu)}^{(M)} = \exp \left( -\frac{\xi^2}{2} \right) \times \begin{cases} \frac{2^{n_1} n_1!}{2^{n_f} n_f!} \xi^{2(n_f - n_i)} \left[ L_{n_f}^{n_i - n_f} \left( \frac{\xi^2}{2} \right) \right]^2, & \text{for } n_i \leq n_f \\ \frac{2^{n_1} n_1!}{2^{n_f} n_f!} \xi^{2(n_f - n_i)} \left[ L_{n_f}^{n_i - n_f} \left( \frac{\xi^2}{2} \right) \right]^2, & \text{for } n_i \geq n_f \end{cases}
\]

(18)

\[
L_n^m(x) = \sum_{i=0} \frac{(-1)^i}{(n - i)!(m + i)!} x^i.
\]

(19)
is the associated Legendre polynomial,
\[
\xi = \frac{\langle \hat{z} \rangle_{\eta} - \langle \hat{z} \rangle_{\eta}}{\epsilon_+} \tan \theta = \sqrt{-\frac{\epsilon \hbar^2}{c}} \left( \langle \hat{z} \rangle_{\eta} - \langle \hat{z} \rangle_{\eta} \right) \frac{B_y}{B_z},
\]
and \(\langle \hat{z} \rangle_{\eta} = \langle \hat{z} \rangle_{\eta}\).

Lyo et al also obtained expressions similar to (16–20) for tunneling transitions in a magnetic field [22]. Since \(|D_s|^2\) does not depend on \(k_x\), the sum
\[
\frac{2}{\beta L_x} \sum_{k_x} |D_s|^2 = |D_s|^2.
\]

As it is seen from the obtained expressions, the contribution of the longitudinal component \(B_y\) of the magnetic field is determined by the parameter \(\xi\) which is proportional to the difference \(\langle \hat{z} \rangle_{\eta} - \langle \hat{z} \rangle_{\eta}\) of the mean coordinates of an electron along the structure growth axis in the final and the initial states. This results in diverse effects on the absorption and emission depending on the symmetry of the quantum well structure potential.

3. Symmetric well potential

In the case of the symmetric potential profile of the structure, \(U(-z) = U(z)\), the subband wave functions \(\varphi_{\eta}\) are either even or odd. Therefore, the mean value of the coordinate \(\langle \hat{z} \rangle_{\eta}\) is the same for all subbands, and the parameter \(\xi = 0\). As a result, we get
\[
\delta_{\nu_j, \nu_i} = \delta_{\nu_j, \nu_i}.
\]

Thus, in a symmetric quantum well structure the selection rule \(\Delta n = n_f - n_i = 0\) still holds for the zero order in the intersubband coupling, i.e., intersubband transitions are only possible between Landau levels with the same numbers.

Moreover, some transitions remain forbidden even if the coupling between the subbands is completely taken into account. Indeed, in the case of \(U(-z) = U(z)\), the Hamiltonian (4) does not change under inversion of the coordinates in the \((y, z)\) plane: \(\hat{H}_{2D}(y, -z) = \hat{H}_{2D}(y, z)\). Consequently, the exact electron wave-function \(\psi(y, z)\) is even or odd with respect to the simultaneous inversion of both \(y\) and \(z\) coordinates:
\[
\psi(-y, -z) = \pm \psi(y, z).
\]
As a result, the dipole matrix element is exactly equal to zero for the transitions between states with the same parity with respect to the inversion in \((y, z)\) plane.

Since \(\varphi_{\eta}(-z) = (-1)^{n+1}\varphi_{\eta}(z)\) and \(\Phi_{\eta}(z) = (-1)^n\Phi_{\eta}(z)\), the expansion \(\psi(y, z) = \sum_{\nu_j = 1} \sum_{\nu_i = 1} C_{(\nu_j, \eta) (\nu_i, \eta)} \varphi_{\nu_j}(z) \Phi_{\nu_i}(z)\) of an even wave-function in the basis (7) includes only terms with different parity of \(\nu_j\) and \(\nu_i\), and that of an odd wave-function \(\psi(y, z)\)—only terms with the same parity of \(\nu_j\) and \(\nu_i\). Thus, the dipole matrix element between the states \((\nu_j, \eta)\) and Landau Levels \((\nu_i, \eta)\), calculated with using the exact wave-functions is equal to zero in the cases when: (1) the parity of \(\nu_j, \nu_i\) is the same, also, the parity of \(\nu_j, \eta\) is a like; (2) \(\nu_j, \eta\) have dissimilar parity, and, \(\nu_j, \eta\) also do not have the same parity. For example, the transitions between the ground Landau level \((1, 0)\) and any odd-numbered Landau level in the second subband \((2, 1), (2, 3), (2, 5)\) and so on are always forbidden even if the coupling between subbands is exactly taken into account.

For the remaining transitions, the coupling between the subbands leads to a violation of the selection rule \(\Delta n = n_f - n_i = 0\). To estimate the magnitude of this violation, we consider an infinitely deep quantum well. In this case, the matrix (8) of the Hamiltonian (4) takes the form
\[
\langle \nu_j, \eta | \hat{H}_{2D} | \nu_i, \eta \rangle = \delta_{\nu_j, \nu_i} \cdot \hat{H}_{(\nu_i, \eta)(\nu_j, \eta)}
\]
where \(\delta_{\nu_j, \nu_i} = \hbar^2 \pi^2 \sqrt{2m_0 a^2}\) is ground subband energy, \(a\) is the quantum well width,
\[
\hat{H}_{(\nu_i, \eta)(\nu_j, \eta)} = [\nu_f^2 + \hbar^2 (n + 1/2)] \delta_{\nu_j, \nu_j} \delta_{\nu_i, \nu_i} + \left( \frac{\hbar \omega}{\epsilon_i} \right)^2 \left( \frac{\nu_f^2}{\frac{\nu_i^2}{2}} \right) \delta_{\nu_i, \nu_i} + \left( \frac{\hbar \omega}{\epsilon_i} \right)^2 \left( \frac{\nu_i^2}{\frac{\nu_f^2}{2}} \right) \delta_{\nu_i, \nu_i} - \left( \frac{\hbar \omega}{\epsilon_i} \right)^2 \left( \frac{\nu_f^2}{\frac{\nu_i^2}{2}} \right) \delta_{\nu_i, \nu_i} - \left( \frac{\hbar \omega}{\epsilon_i} \right)^2 \left( \frac{\nu_i^2}{\frac{\nu_f^2}{2}} \right) \delta_{\nu_i, \nu_i} \times \left( \frac{\nu_i^2}{\frac{\nu_f^2}{2}} \right) \delta_{\nu_i, \nu_i}.
\]

Thus, the diagonalization of the matrix (8) of the Hamiltonian (4) reduces to that of the matrix \(\hat{H}_{(\nu_i, \eta)(\nu_j, \eta)}\). The eigenvectors of this matrix coincide with the coefficients \(C_{(\nu_i, \eta)}\) of the expansion of the electron wavefunction \(\psi(y, z)\) with respect to the basis (7), and its eigenvalues \(E_{\nu_i}/\epsilon_i\) are the electron energy \(E\) in the units of \(\epsilon_i\). Since the matrix elements \(\langle \nu_i/\epsilon_i \rangle^2 \delta_{\nu_i, \nu_j}\) do not depend on the width \(a\) of the quantum well, the magnetic field and the quantum well width enter into the matrix only in the form of the ratios \(\hbar \omega_2/\epsilon_i\) and \(\hbar \omega_1/\epsilon_i\). Consequently, the dimensionless energy \(E/\epsilon_i\) and the coefficients of the wave function expansion depend on the width of the quantum well and the magnetic field strength only via these ratios.
The dipole matrix element is expressed in term of \( C \) as

\[
D_{(\nu_i, n_i) \rightarrow (\nu_f, n_f)} = a \sum_{\nu_i, \nu_f} C_{\nu_i, n_i}(\nu_i, n_i) C_{\nu_f, n_f}(\nu_f, n_f) \frac{\hbar}{a \rho_{\nu_f, \nu_i}},
\]

and, hence, the ratio \( D / a \) depends on the magnetic field and the quantum well width only in terms of the ratios of the Landau energies to the confined energy \( \Delta \varepsilon = \varepsilon_2 - \varepsilon_1 = 3\varepsilon \). In figure 1, the ratio of the module of the dipole matrix elements between (1, 0) and (2, n) LLs to the width of the GaAs quantum well is shown as a function of \( \hbar\omega_l / \varepsilon_1 \) for the fixed value \( \hbar\omega_l / \varepsilon_1 = 1/3 \). As it is seen, considering the coupling between the subbands leads to a violation of the selection rule. However, this effect is very small even when the intersubband spacing is equal to the Landau energy.

4. Asymmetric well potential

A completely different situation occurs in the structures with an asymmetric potential profile. The asymmetry leads to the subband wave-functions no longer having a definite parity. As a consequence, the mean coordinates \( \langle z \rangle \) in different subbands are not the same, and the parameter \( \xi \) becomes nonzero, which results in a violation of the \( \langle \Delta n = 0 \rangle \) selection rule already in the zero order of coupling between the subbands.

To see the extent of the selection rule violation, we compare the amplitudes of absorption/emission coefficients for transitions with \( \Delta n = 0 \) and \( \Delta n 
eq 0 \). Let us consider the following transitions: absorption from ground Landau level of the lowest subband and emission from zeroth Landau level of the upper subband. The absorption/emission coefficients for the transition \( (\nu_f, 0) \rightarrow (\nu_f, n) \) from the ground Landau level of the \( \nu_f^{th} \) subband to the \( n^{th} \) Landau level of the \( \nu_f^{th} \) subband are

\[
\alpha_{\nu_f,0}^{abs} - (\nu_f, n) (\omega) = \frac{4\pi^2\varepsilon^2}{\epsilon h \eta} T_{\nu_f,0}^{(0)} [N_{\nu_f,0} - N_{\nu_f, n}] \times h\omega_{\nu_f,0}^{abs} (\omega) \delta (E_f - E_i - \hbar\omega),
\]

where

\[
E_f(\xi) = \left( \frac{\xi}{2n!} \right) \exp \left( -\frac{\xi}{2} \right).
\]

The transition frequency \( h\omega_{\nu_f,0}^{emis} - (\nu_f, n) = |E_{\nu_f, n} - E_{\nu_f, 0}| \) is

\[
h\omega_{\nu_f,0}^{emis} - (\nu_f, n) = \Delta \varepsilon \pm \hbar\omega_0 \Delta n + \hbar\omega_0 \gamma \frac{\xi^2}{2}
\]
Here $\Delta \varepsilon = |\varepsilon_{f} - \varepsilon_{i}|$ is the intersubband spacing,

$$
\gamma = \frac{\left| \langle \delta z \rangle_{f} \right|^2}{(\Delta z)^2} = \frac{\left| \langle \delta z \rangle_{i} \right|^2}{(\Delta z)^2},
$$

(29)

Thus, the ratio of amplitudes for the transition with $D \neq n_0$ and $D = n_0$ (i.e., the ratio of the intensities of the corresponding lines in the absorption or the emission spectrum) is

$$
\frac{\alpha_{abs}^{(\Delta n=0)}(\omega)}{\alpha_{abs}^{(\Delta n=\pm1)}(\omega)} = \left[ 1 \pm \frac{\hbar \omega_{\Delta n}}{\Delta \varepsilon + \hbar \omega_{\gamma} \frac{\xi^{2n}}{2!n!}} \right] \frac{\xi^{2n}}{2!n!}.
$$

(31)

In the case of absorption (the positive sign), the expression in brackets always exceeds 1, and therefore

$$
\frac{\alpha_{abs}^{(\Delta n=0)}(\omega)}{\alpha_{abs}^{(\Delta n=\pm1)}(\omega)} \geq \frac{\xi^{2n}}{2!n!}.
$$

(32)

The parameter $\xi \propto B_{||}/\sqrt{B_{\perp}}$, and therefore it can be varied by changing the magnetic field strength and orientation. Accordingly, when $|\xi| > \sqrt{2/n!}$ the right part of (32) exceeds 1, i.e., the transition intensity for $\Delta n = 0$ dominates.

In case of the emission (the negative sign):

$$
\frac{\alpha_{em}^{(\Delta n=0)}(\omega)}{\alpha_{em}^{(\Delta n=\pm1)}(\omega)} \geq \left[ 1 - \frac{\hbar \omega_{\Delta n}}{\Delta \varepsilon} \right] \frac{\xi^{2n}}{2!n!},
$$

(33)

and the transition intensity for $\Delta n = 0$ exceeds that for $\Delta n = 0$, when $\xi > \sqrt{2/n!} \frac{\Delta \varepsilon}{\Delta \varepsilon - \hbar \omega_{\gamma} n}$.

Thus, we conclude that in both the considered cases, the intensities of peaks with $D \neq n_0$ can exceed those with $D = n_0$.

The effect of selection rule violation in tilted magnetic field is caused by the shift of the initial and final wave functions with respect to each other in the plane of layers. This relative shift can qualitatively explain the decrease...
in the transition intensity for $\Delta n = 0$. For the transitions with $\Delta n = 0$, the in-plane wave functions are initially orthogonal, and the corresponding matrix elements are equal to zero. When the parallel component of the magnetic field arises, these in-plane wave functions are no longer orthogonal due to their relative shift and the matrix elements become non-zero, that is, start to increase. With the increasing strength of the parallel component, the relative shift of the wave function also increases. Finally, for high values of the parallel component, the matrix elements start to go down. The details of the process are described by the formulas obtained.

The dependence of the line intensity on $x$ is determined by the product of transition frequency and function $F_D(x)$. The value of the transition frequency is a monotonous function of $x$, while the function $F_D(x)$ has an explicit maximum when $x = D_n^2$. It leads to the following behavior of the line intensities. When $x = 0$, there is only one line in the spectrum that corresponds to the transitions with $\Delta n = 0$. If $\xi$ becomes non-zero, the lines from $\Delta n = 0$ appear and their intensities grow with the increase of $\xi$. When $\xi \approx \sqrt{2}$, the intensity of a line with $\Delta n = 1$ reaches the maximum. The intensity of this line decreases with further increase in $\xi$. However, the line amplitude from $\Delta n = 2$ rises and reaches the maximum when $\xi \approx 2$, then goes down, and so on. Besides, the intensity of the line corresponding to $\Delta n = 0$ decreases close to exponentially.

This behavior is illustrated in figure 2, where the absorption spectra caused by the transitions between Landau levels of the first and second subband are shown for an asymmetric structure consisting of two tunnel-coupled quantum wells GaAs/Al_{0.3}Ga_{0.7}As of different widths (12 and 5 nm), separated by the 2 nm barrier (figure 3). The calculation was performed for the fixed component $B_z$, and the parameter $\xi$ is varied via $|B|$. The dependence of line intensities on $B_z$ when $B_L = 5$ T (a) and $B_L = 1$ T (b).

**Figure 3.** (a) The potential profile of the asymmetrical double quantum well and (b) respective subband wave functions.

**Figure 4.** The line intensity dependence on $|B|$ when $B_L = 5$ T (a) and $B_L = 1$ T (b).
5. Conclusion

The nature of the violation of the selection rule (\[\Delta n = 0\]) for optical transitions between Landau levels of different subbands in tilted magnetic field is studied. Neglecting the intersubband coupling, the analytical expression is obtained for the absorption/emission intensities, which makes it possible to estimate the contribution of the ‘forbidden’ intersubband transitions with different \(\Delta n\). The magnitude of the violation of the selection rule for transitions with \(\Delta n \neq 0\) is shown to be essentially determined by the symmetry of the potential profile of the structure. In a symmetric structure, the selection rule remains in the zeroth approximation of intersubband coupling. Moreover, some transitions are forbidden in the tilted magnetic field even with taking account of intersubband coupling. For other transitions with \(\Delta n \neq 0\), the intersubband coupling gives rise to the violation of the selection rule in the tilted magnetic field, but the degree of violation and the magnitude of the corresponding matrix elements are insignificant in the considered range of magnetic field strength. In asymmetric structures, the selection rule \(\Delta n = 0\) is significantly violated in zero order of the intersubband coupling, and the intensity of the transitions with \(\Delta n \neq 0\) can even exceed those with \(\Delta n = 0\) for an appropriate choice of the strengths of the magnetic field components and the structure potential profile. The intensity of the ‘forbidden’ intersubband transitions is traced as a function of the magnitude and orientation of the magnetic field.

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