Pseudo-Dirac neutrinos via mirror-world and depletion of UHE neutrinos

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Abstract

We propose a possible particle physics explanation of the non-observation of muon neutrino events at IceCube coincident with GRB gamma ray at the rates predicted by the standard Bahcall-Waxman model, in terms of neutrino oscillations. Our model is based on assuming that (a) all neutrinos are pseudo-Dirac particles and (b) there exists a mirror world interacting gravitationally with the observed world. This scenario has three sterile neutrinos associated with each flavour of ordinary neutrinos. Very tiny mass splitting between these neutrinos is assumed to arise from lepton number violating dimension five operators suppressed by the Planck scale. We show that if a mass splitting of $10^{-15} \text{eV}^2$ is induced between the four mass eigenstates of a given species, then its flux will be suppressed at IceCube energies by a factor of 4 which could be the explanation of the IceCube observation that the muon neutrino flux is lower than expected. Hierarchies in mass splitting among different flavours may result in different amount of suppression of each flavour and based on this, we predict a difference in the flavour ratios of the observed neutrinos which is significantly different compared to the standard prediction of $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$ which could serve as a test for this model.

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I. INTRODUCTION

Gamma Ray Burst (GRB) are believed \cite{1-3} to be responsible for accelerating the charged cosmic rays to very high energies and a sizable fluence of neutrinos is expected from GRB's through the interactions of protons with photons in the fireball \cite{4}. The $p\gamma$ interaction produce charged pions: $p + \gamma \rightarrow \Delta^+ \rightarrow n + \pi^+$ and subsequently the decays $\pi^+ \rightarrow \mu^+\nu_\mu$ and $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$ produce muon and electron neutrinos coincident in direction with $\gamma$-rays. The energy spectrum of these neutrinos are expected to peak in the range $10^5 - 10^7\text{GeV}$ and in km$^3$ sized detectors like IceCube and ANTARES about 10-100 neutrino events per year, coincident with GRB photons, were predicted \cite{5-9}. Contrary to expectations, no muon neutrino events coincident with GRB photons have been detected in measurements at IceCube \cite{10,11} which looks for the CC produced muons from $\nu_\mu$ through the Cerenkov radiation of upward going muons. An experimental upper bound on the muon neutrino flux from GRB’s smaller than the Waxman-Bahcall prediction \cite{5,8,9} by a factor 3.7 has been established with the IceCube observations \cite{11}. Other experiments probing the ultra-high-energy regime, such as ANITA\cite{12} have not seen any evidence of PeV energy neutrinos in association with GRB events either. A recent observation \cite{13} of a GRB at a low redshift of $z \sim 0.34$ and an estimated isotropic electromagnetic energy of $10^{54}$ makes it the most energetic of GRB’s emission seen at $z < 0.5$. No neutrinos coincident with this GRB event within 100 s and 3.5° were seen at IceCube \cite{14}. A search for muon neutrinos associated with GRB’s performed with the ANTARES detector \cite{15} shows no events over the atmospheric neutrino background.

There are two PeV neutrino events which have been seen at IceCube \cite{16}. They are expected to be of cosmogenic origin \cite{17}. There are a further 26 events seen at IceCube at energies between 0.02-0.3 PeV \cite{18}. These 28 events may be initiated by neutrinos from dark matter decay \cite{19,20} or the neutrinos may have a astrophysical origin \cite{21}. There is only one muon track in the 10 events between energies 0.1-1.15 PeV \cite{20} and the paucity of muon neutrinos continues to be evident even in these non-GRB related events suggesting possible non-astrophysical origin for the muon neutrino depletion.

In the light of the IceCube non-observation of the GRB neutrinos the standard GRB fireball or internal shock model parameters have been revised \cite{22-26} in a full numerical calculation by taking into account in greater detail dilution of charged pions and kaons in the expanding fireball and due to multi-pion and kaon production. The new neutrino flux limits are consistent with the upper bound put from IceCube \cite{10,11,14}. Other ways of explaining the paucity of IceCube neutrinos without overthrowing the GRB models is to explain it by oscillations of the flavor neutrinos into sterile ones. There exist strong constraints on the possible oscillations of active to sterile neutrinos from the terrestrial experiments \cite{27,29} and from nucleosynthesis \cite{30,32}. But due to very high energy and long distances, the relevant mass scale probed through the active sterile oscillations of UHE are completely different and are as yet unconstrained. For example, consider the neutrinos from GRB’s (at cosmological distances $L = 1000 - 3000\text{Mpc}$) and in the range $E = 10^5 - 10^7\text{GeV}$. These active UHE
neutrinos ($\nu_\alpha$, $\alpha = e, \mu, \tau$) can significantly oscillate to sterile neutrinos ($\nu'$) if the $(\text{mass})^2$ difference between them is

$$\Delta m^2_{\nu_\alpha \nu'} \geq 0.8 \times 10^{-15} \text{eV}^2 \left( \frac{1000 \text{Mpc}}{L} \right) \left( \frac{E}{10^7 \text{GeV}} \right)$$

and mixing between them is significant. A pair of active and a sterile neutrino with the above $(\text{mass})^2$ difference may be regarded as a pseudo-Dirac neutrino which in some limit can be considered a Dirac particle respecting some unbroken Lepton number. Small violation of this symmetry not only splits them but also mix them maximally. UHE neutrinos can thus be used to probe the pseudo-Dirac nature of neutrinos [33–39]. The maximal mixing within a pseudo Dirac pair can bring a suppression in the original flux by a factor of 1/2. Note that this is over and above the suppression in muon neutrino flux resulting from the active to active oscillation which is already included in the the Waxman-Bahcall prediction [5] of the neutrino flux. This falls significantly short of the suppression by 3.7 observed at IceCube. If $\Delta m^2_{\nu_\alpha \nu'}$ is in the range $10^{-18} - 10^{-16} \text{eV}^2$ the interference terms in the oscillation probability does not average to zero for the range of red-shifts of the observed GRBs and the suppression factor can be lower than 1/2 [38]. Other explanations offered for the IceCube muon neutrino deficit are neutrino spin flip in a magnetic field [40] and neutrino decays over cosmological distances [39, 41].

We discuss here how a suppression by a factor of $\sim 1/4$ can be achieved in the flux of UHE (a) if all neutrinos are pseudo-Dirac and (b) if there exists a mirror world [42–44] interacting gravitationally with the observed world. Global lepton number breaking through the gravitational interactions provides a source in this scenario which can split the mass eigenstates of all the Dirac neutrinos and make them pseudo-Dirac.

II. PSEUDO-DIRAC NEUTRINOS VIA MIRROR WORLD

First we consider the case of a single flavour say, $\nu_\mu$. Assume that $\nu_\mu$ is a Dirac particle and is accompanied by a mirror “muon neutrino” also a Dirac particle. They together consist of four two component left handed states labeled as $\nu'_{\mu a}$ with $a = 1...4$. Of these, $\nu'_{\mu 1}$ is active and others are sterile states. Their mass matrix to the leading order is given by

$$\mathcal{M}^0_\mu = m_{\nu_\mu} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

We have assumed the same mass for the both the Dirac states. All the zeros in the above mass matrix are protected by Lepton numbers in our and the mirror worlds. Breaking of these symmetries is assumed to induce small entries in places of zeros. Thus for example, non-zero values for 11 and its mirror symmetric 33 elements are induced by the conventional dimension five Weinberg operator in our and mirror world respectively and may arise from
the seesaw mechanism or gravity induced effects. Following the mirror world scenario\textsuperscript{42}, we assume that ordinary and mirror worlds communicate only gravitationally with each other, thus $(\nu'_{\mu 1}, \nu'_{\mu 3})$ get coupled only gravitationally through the dimension-5 operator\textsuperscript{42–44}:

$$L_{\text{comm}} = \frac{\lambda_{13}}{M_P} (\nu'_{\mu 1} \phi)(\nu'_{\mu 3} \phi')$$

(3)

where $\phi$ and $\phi'$ are the neutral components of the Higgses in our and the mirror world respectively. The contribution of this dimension 5 operator to the mass matrix is

$$\epsilon_{13} \equiv \frac{\lambda_{13} v^2}{M_P}$$

(4)

where we have taken $v = \langle \phi \rangle \simeq \langle \phi' \rangle \simeq 174\text{GeV}$. The sterile partners $(\nu'_{\mu 2} \text{ and } \nu'_{\mu 4})$ may couple to different set of Higgs $\eta$ and $\eta'$ and assuming that these Higgs vevs are at the TeV scale, one can have a gravitational mixing term from all the sterile pairs. For example $(\nu'_{\mu 2}, \nu'_{\mu 4})$ will mix via the operator

$$L'_{\text{comm}} = \frac{\lambda_{24}}{M_P} (\nu'_{\mu 2} \eta)(\nu'_{\mu 4} \eta')$$

(5)

where $\eta$ and $\eta'$ are the neutral components of the Higgses in our and the mirror world respectively. The contribution of this dimension 5 operator to the mass matrix is

$$\epsilon_{24} = \frac{\lambda_{24}}{M_P} \langle \eta \rangle \langle \eta' \rangle$$

(6)

Taking similar terms for mixing of all sterile pairs, we can write the mass matrix in the $\nu'_{\mu a}$ basis as

$$\mathcal{M}_\mu = \begin{pmatrix}
\epsilon_{11} & m_{\nu_{\mu}} & \epsilon_{13} & \epsilon_{14} \\
m_{\nu_{\mu}} & \epsilon_{22} & \epsilon_{23} & \epsilon_{24} \\
\epsilon_{13} & \epsilon_{23} & \epsilon_{11} & m_{\nu_{\mu}} \\
\epsilon_{14} & \epsilon_{24} & m_{\nu_{\mu}} & \epsilon_{22}
\end{pmatrix}$$

(7)

We have assumed that the ordinary and mirror worlds are symmetric if their mixing is neglected and thus assumed $\epsilon_{33} = \epsilon_{11}$ and $\epsilon_{22} = \epsilon_{44}$ in eq. (7). We also assume that $\epsilon$ parameters are $\ll m_{\nu_{\mu}}$ and neutrinos remain pseudo-Dirac. Let

$$V_4^T \mathcal{M}_\mu V_3 \equiv \text{diag}(m_{\mu 1}, m_{\mu 2}, m_{\mu 3}, m_{\mu 4})$$

(8)

Eigenvalues $m_{\mu a}$ are given to leading orders in $\epsilon$ as

$$m_{\mu 1} \simeq \frac{1}{2} (2m_{\nu_{\mu}} + \epsilon_{11} + \epsilon_{13} + \epsilon_{14} + \epsilon_{22} + \epsilon_{23} + \epsilon_{24})$$

$$m_{\mu 2} \simeq \frac{1}{2} (-2m_{\nu_{\mu}} + \epsilon_{11} + \epsilon_{13} - \epsilon_{14} + \epsilon_{22} - \epsilon_{23} + \epsilon_{24})$$

$$m_{\mu 3} \simeq \frac{1}{2} (2m_{\nu_{\mu}} + \epsilon_{11} - \epsilon_{13} - \epsilon_{14} + \epsilon_{22} - \epsilon_{23} - \epsilon_{24})$$

$$m_{\mu 4} \simeq \frac{1}{2} (-2m_{\nu_{\mu}} + \epsilon_{11} - \epsilon_{13} + \epsilon_{14} + \epsilon_{22} + \epsilon_{23} - \epsilon_{24})$$

(9)
Diagonalizing matrix is given to the same order by

\[ \mathcal{V}_4 \equiv \mathcal{V}_4^0 \mathcal{V}_4 \approx \frac{1}{2} \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & y_1 & 0 & y_2 \\ -y_1 & 1 & y_2 & 0 \\ 0 & -y_2 & 1 & y_3 \\ -y_2 & 0 & -y_3 & 1 \end{pmatrix}, \]  

(10)

where

\[ y_1 \equiv \frac{1}{4m_{\nu_\mu}}(\epsilon_{11} + \epsilon_{13} - \epsilon_{22} - \epsilon_{24}), \]
\[ y_2 \equiv \frac{1}{4m_{\nu_\mu}}(\epsilon_{23} - \epsilon_{14}), \]
\[ y_3 \equiv \frac{1}{4m_{\nu_\mu}}(\epsilon_{11} - \epsilon_{13} - \epsilon_{22} + \epsilon_{24}). \]  

(11)

The role of \( \epsilon_{ab} \) is essentially to split all four degenerate states and mixing between them is essentially determined by the \( \epsilon_{ab} \) independent matrix \( \mathcal{V}_4^0 \) in eq.(10). In the following, we shall assume that all the parameters \( \epsilon_{ab} \) have the same typical magnitude given by

\[ \epsilon_{ab} \equiv \epsilon \sim \frac{\lambda}{M_P}v^2 \approx 2.4 \times 10^{-6}\lambda \text{ eV}, \]  

(12)

where \( v \sim 174\text{GeV} \). Then it follows from eq.(10) that a typical scale responsible for the long wavelength oscillations of muon neutrinos is given by

\[ \Delta_2 \approx 2m_{\nu_\mu} \epsilon \approx 4.5 \times 10^{-8}\text{ eV}^2\lambda \left( \frac{m_{\nu_\mu}}{0.009 \text{ eV}} \right). \]  

(13)

The oscillation length associated with this scale and energy \( E = 10^5 - 10^7 \text{ GeV} \) is smaller than the typical distance of the UHE sources, see eq.(11) and the effect of \( \Delta_2 \) gets averaged out resulting in suppression of the muon neutrino flux. The averaged survival probability is essentially determined by \( \mathcal{V}_4^0 \) in eq.(10) and is given by:

\[ P_{\mu\mu} = 4 \left( \frac{1}{2} \right)^4 = \frac{1}{4}. \]  

(14)

This reduction is over and above the flux reduction which takes place due to averaged oscillations between active flavours and one needs to generalize the above formulation to take this effect into account. We do this in the next section.

### III. THREE GENERATIONS

In the following, we shall assume a straightforward generalization of the above scenario and require that all three active neutrinos and their mirror partners are pseudo-Dirac. We are thus dealing with 12 left handed states \( \nu'_a \), \( \alpha = e, \mu, \tau, a = 1..4 \) in this case and mixing among them would now be governed by a \( 12 \times 12 \) matrix. The mass matrix \( (7) \) for a single
flavour is generalised to a $12 \times 12$ matrix for the three flavours as follows. The mass $m_{\nu_{\mu}}$ is replaced by a $3 \times 3$ mass matrix $m^{\alpha\beta}$ in the flavour space $\alpha, \beta = e, \mu, \tau$,

$$m_{\nu_{\mu}} \rightarrow m^{\alpha\beta} \equiv m$$

(15)

This matrix can be diagonalised by the usual bi-unitary transformation:

$$U_L^T m U_R = {\text{diagonal}} (m_1, m_2, m_3) \equiv d_{\nu}.$$  

(16)

Here, the matrix $U_L$ governing the left handed mixing can be identified with the usual MNSP matrix $U$. Each of the the parameters $\epsilon_{ab}$ appearing in eq. (10) now gets replaced by a $3 \times 3$ matrices in flavour space

$$\epsilon_{ab} \rightarrow \epsilon_{ab}^{\alpha\beta} \equiv \epsilon_{ab}$$

(17)

These matrices are generated by dim. 5 operators as before, e.g. $\epsilon_{13}^{\alpha\beta}$ arise from gravitational mixing between neutrinos in our universe and the mirror universe:

$$\frac{\lambda_{13}^{\alpha\beta}}{M_P} (\nu'_{\alpha 1}\phi)(\nu'_{\beta 3}\phi')$$

(18)

which gives the $3 \times 3$ mixing matrix in flavour space,

$$\epsilon_{13}^{\alpha\beta} = \frac{\lambda_{13}^{\alpha\beta} v^2}{M_P} \approx 2.5 \times 10^{-6} \lambda_{13}^{\alpha\beta} \text{eV}.$$

(19)

Taking similar terms for mixing of all pairs, we write the $12 \times 12$ mass matrix in the $(\nu'_{\alpha,1}, \nu'_{\alpha,2}, \nu'_{\alpha,3}, \nu'_{\alpha,4})$ basis as

$$\mathcal{M} = \begin{pmatrix}
\epsilon_{11} & m & \epsilon_{13} & \epsilon_{14} \\
m & \epsilon_{22} & \epsilon_{23} & \epsilon_{24} \\
\epsilon_{13}^T & \epsilon_{23}^T & \epsilon_{11} & m \\
\epsilon_{14}^T & \epsilon_{24}^T & m & \epsilon_{22}
\end{pmatrix}$$

(20)

Note that each entry above is a $3 \times 3$ matrix in the generation space. This matrix can be diagonalised by the following steps. We first diagonalise the $m$ blocks by the matrix,

$$U' = \begin{pmatrix}
U_L & 0 & 0 & 0 \\
0 & U_R & 0 & 0 \\
0 & 0 & U_L & 0 \\
0 & 0 & 0 & U_R
\end{pmatrix}$$

(21)

with the transformation,

$$\mathcal{M}' = U'^T \mathcal{M} U' = \begin{pmatrix}
\bar{\epsilon}_{11} & d_{\nu} & \bar{\epsilon}_{13} & \bar{\epsilon}_{14} \\
d_{\nu} & \bar{\epsilon}_{22} & \bar{\epsilon}_{23} & \bar{\epsilon}_{24} \\
\bar{\epsilon}_{13}^T & \bar{\epsilon}_{23}^T & \bar{\epsilon}_{11} & d_{\nu} \\
\bar{\epsilon}_{14}^T & \bar{\epsilon}_{24}^T & d_{\nu} & \bar{\epsilon}_{22}
\end{pmatrix}$$

(22)
where

\[ \tilde{e}_{ab} \equiv U_L^T \epsilon_{ab} U_L \quad \text{for} \quad ab = 11, 13, \]
\[ \tilde{e}_{ab} \equiv U_L^T \epsilon_{ab} U_R \quad \text{for} \quad ab = 14, \]
\[ \tilde{e}_{ab} \equiv U_R \epsilon_{ab} U_L \quad \text{for} \quad ab = 23, \]
\[ \tilde{e}_{ab} \equiv U_R^T \epsilon_{ab} U_R \quad \text{for} \quad ab = 22, 24. \] (23)

The diagonal elements \((\tilde{e}_{ab})_{ii}\) in flavour space serve to split the masses \(m_{\nu_i}\) of the \(i^{th}\) flavour. The off-diagonal entries give corrections to them and also mix sterile states of different flavours. Since mixing of an active neutrino of one flavour with a sterile neutrino associated with a different flavour is strongly constrained from experiments, we shall assume that off-diagonal entries of each of the matrices \(\tilde{e}_{ab}\) are small compared to the diagonal ones and take these matrices as diagonal:

\[ \tilde{e}_{ab}^D \equiv \text{diagonal}(\epsilon_{ab}^1, \epsilon_{ab}^2, \epsilon_{ab}^3), \] (24)

where \(\tilde{e}_{ab}^D\) are now diagonal \(3 \times 3\) matrix for each \(ab\). In this approximation, eq. [22] can be written as

\[ \mathcal{M}' \approx \begin{pmatrix} \epsilon_{11} & \epsilon_{13} & \epsilon_{14} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{24} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{34} \end{pmatrix} \left( \begin{array}{ccc} d_{\nu 1} & \delta_{D}^{1} & d_{\nu 4} \\ d_{\nu 2} & \delta_{D}^{2} & d_{\nu 5} \\ d_{\nu 3} & \delta_{D}^{3} & d_{\nu 6} \end{array} \right), \] (25)

To the first order in \(d_{\nu}^{-1}\), the matrix \(\mathcal{M}'\) is now diagonalised by,

\[ V \equiv V^D V \approx \frac{1}{2} \begin{pmatrix} I & -I & -I & I \\ I & I & -I & -I \\ I & -I & I & -I \\ I & I & I & I \end{pmatrix} \begin{pmatrix} I & y_1 & 0 & y_2 \\ 0 & y_1 & I & y_2 \\ -y_2 & I & 0 & y_3 \\ -y_3 & I & 0 & -y_2 \end{pmatrix}, \] (26)

where \(I\) (0) denotes \(3 \times 3\) identity (null) matrix. The diagonal \(3 \times 3\) matrices \(y_{1,2,3}\) are given by expressions analogous to eq. [11]:

\[ y_1 \equiv \frac{1}{4}(\epsilon_{11}^D + \epsilon_{13}^D - \epsilon_{22}^D - \epsilon_{24}^D)d_{\nu}^{-1}, \]
\[ y_2 \equiv \frac{1}{4}(\epsilon_{23}^D - \epsilon_{14}^D)d_{\nu}^{-1}, \]
\[ y_3 \equiv \frac{1}{4}(\epsilon_{11}^D - \epsilon_{13}^D - \epsilon_{22}^D + \epsilon_{24}^D)d_{\nu}^{-1}. \] (27)

The 12 eigenvalues of \(\mathcal{M}\) are given to leading order in \(\tilde{e}_{ab}\) by equation analogous to [9]:

\[ m_1 \approx \frac{1}{2}(2d_{\nu} + \tilde{e}_{11} + \tilde{e}_{13} + \tilde{e}_{14} + \tilde{e}_{22} + \tilde{e}_{23} + \tilde{e}_{24}) \]
\[ m_2 \approx \frac{1}{2}(-2d_{\nu} + \tilde{e}_{11} + \tilde{e}_{13} + \tilde{e}_{14} + \tilde{e}_{22} - \tilde{e}_{23} + \tilde{e}_{24}) \]
\[ m_3 \approx \frac{1}{2}(2d_{\nu} + \tilde{e}_{11} - \tilde{e}_{13} - \tilde{e}_{14} + \tilde{e}_{22} - \tilde{e}_{23} - \tilde{e}_{24}) \]
\[ m_4 \approx \frac{1}{2}(-2d_{\nu} + \tilde{e}_{11} - \tilde{e}_{13} + \tilde{e}_{14} + \tilde{e}_{22} + \tilde{e}_{23} - \tilde{e}_{24}). \] (28)
The mixing matrix $\mathcal{U}$ between the 12 gauge eigenstates $(\nu_{a1}', \nu_{a2}', \nu_{a3}' , \nu_{a4}')$, $(\alpha = e, \mu, \tau)$ and the mass eigenstates $(\nu_1, \nu_2, \nu_3, \nu_4)$, $i = 1 \cdots 3$ is given by the product of $\mathcal{U}'$, eq.(21) and $\mathcal{V}$, eq.(26). To zeroth order in $\epsilon_{ab}$, one can approximate $\mathcal{V}$ by $\mathcal{V}^0$ and $\mathcal{U}$ is approximately given by

$$\mathcal{U} \equiv \mathcal{U}' \mathcal{V} \simeq \frac{1}{2} \begin{pmatrix} U_L - U_L & U_L \\ U_R & U_R - U_R \\ U_L - U_L & U_L - U_L \\ U_R & U_R \\ U_R & U_R - U_R \end{pmatrix}$$

In this approximation, the three flavour eigenstates $\nu_\alpha \equiv \nu_{a1}'$, are given in terms of 12 mass eigenstates $\nu_{ia}$ from the above equation by

$$\nu_\alpha \equiv \nu_{a1}' = U_{ia}^\dagger \nu_{ia} \equiv \frac{1}{2} U_{ia} (\nu_1 - \nu_2 - \nu_3 + \nu_4)$$

with $U_L \equiv \mathcal{U}$ denoting the MNS matrix. The mass $m_{ia}$ of each component $\nu_{ia}$ is given by $(m_{ia})_{ii}$ from eq.(28). The splitting among the mirror partners of a given mass eigenstate $\nu_{i,a}$ is then given by $\Delta^i_{ab} = m^2_{ia} - m^2_{ib}$. The corresponding oscillation probabilities follow from the time evolution of the state $\nu_i$ defined in eq. (30):

$$P_{\alpha\beta}(L) = \frac{1}{16} \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\beta j} U_{\alpha j} e^{-i(m^2_{ij} - m^2_{jj}) \frac{L}{2E}}$$

$$\times \left(1 + e^{i\chi^i_{12}} + e^{i\chi^i_{13}} + e^{i\chi^i_{14}}\right) \left(1 + e^{-i\chi^i_{12}} + e^{-i\chi^i_{13}} + e^{-i\chi^i_{14}}\right)$$

where $\chi^i_{ab} \equiv \Delta^i_{ab} \frac{L}{2E}$. The four states $\nu_{ia}$ for a given $i$ are degenerate when $\Delta^i_{ab}$ are small and $\chi^i_{ab}$ can be neglected as in typical short baseline experiments. In this limit, $\nu_i$ defined in eq.(30) behave as a single mass eigenstate and one recovers the standard mixing and oscillations of the flavour states. The $\Delta^i_{ab}$ induce observable long wavelength oscillations between active and sterile states. For long baselines with $(E/L > 10^{-12} eV^2)$ the exponential factor $e^{-i(m^2_{ij} - m^2_{jj}) \frac{L}{2E}}$ in (31) averages to zero if $i \neq j$. The oscillation probability in the long baseline experiments then can be written as,

$$P_{\alpha\beta}(L) = \frac{1}{8} \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 \left(2 + \cos \chi^i_{12} + \cos \chi^i_{13} + \cos \chi^i_{14}\right)$$

$$+ \cos \chi^i_{23} + \cos \chi^i_{24} + \cos \chi^i_{34})$$

Typical scales associated with these splittings can be written assuming normal mass hierarchy as

$$\Delta^1_{ab} \simeq 4\lambda_1 m_1 \frac{v^2}{M_p^2} \ll 9 \times 10^{-8} \lambda_1 \text{ eV}^2,$$

$$\Delta^2_{ab} \simeq 4\lambda_2 \sqrt{\Lambda_0} \frac{v^2}{M_p^2} = 9 \times 10^{-8} \lambda_2 \text{ eV}^2,$$

$$\Delta^3_{ab} \simeq 4\lambda_3 \sqrt{\Lambda_{\text{atm}}} \frac{v^2}{M_p^2} = 4.7 \times 10^{-7} \lambda_3 \text{ eV}^2,$$
where $\lambda_{1,2,3}$ denote the gravitational couplings in three sectors controlling the strength of the dim. 5 operators. These are constrained from two major considerations. The number of neutrino species in equilibrium at the time of BBN is severely constrained. Requirement that a sterile neutrino does not equilibrate at that time through large angle oscillations to active one implies that their (mass)$^2$ difference must obey $30, 32$ \[ \Delta m^{2}_{\nu_{\alpha}\nu'} \leq 10^{-9} \text{eV}^2. \] A stronger constraint exist on $\Delta^1_{ab}$. In the approximation of neglecting mixing between active and sterile partners of different generations, $\Delta^1_{ab}$ control the solar neutrino oscillations. The corresponding oscillation length for MeV neutrino is of the order of the Sun-Earth distance for $\Delta^1_{ab} \sim 10^{-12} \text{eV}^2$. Such $\Delta^1_{ab}$ can modify the LMA solution and detailed fits in case of pseudo-Dirac neutrinos [44] imply a bound $\Delta^1_{ab} < 1.8 \times 10^{-12} \text{eV}^2$ at 3\(\sigma\). One expects similar but somewhat stronger bound when mirror partners are also present. This bound can be satisfied either by choosing $m_1 \ll \sqrt{\Delta^1_{ab}}$ or in case of $m_1 \sim O(\sqrt{\Delta^1_{ab}})$ by choosing $\lambda_1 \leq 10^{-5}$.

We will assume that all the splittings among a given flavour $\Delta^{ii}_{ab}$ for different pairs of $ab$ are equal and in this case the oscillation probability (32) reduces to the simple form

\[ P_{\alpha\beta}(L) = \frac{1}{4} \sum_{i} |U_{\alpha i}|^2 |U_{\beta i}|^2 (1 + 3 \cos \chi_i) \] (34)

where $\chi_i \equiv \chi^i_{ab} \forall a, b$. One can now work out the observed flux ratios of UHE neutrinos using this $P_{\alpha\beta}$. The flux $\Phi_\beta = (\phi_e, \phi_\mu, \phi_\tau)$ in a flavour $\beta$ is given by

\[ \Phi_\beta = P_{\beta\alpha} \Phi^0_\alpha \] (35)

where $\Phi^0_\alpha$ denotes the initial flux. For $\Phi^0_\alpha \sim \frac{\phi_0}{3} (1, 2, 0)$ one obtains

\[ \Phi_\beta \sim \frac{\phi_0}{12} \sum_{i} |U_{\beta i}|^2 (|U_{ei}|^2 + 2|U_{\mu i}|^2)(1 + 3 \cos \chi_i). \] (36)

One recovers the standard value $\Phi^S_\beta = \frac{\phi_0}{3} \sum_{i} |U_{\beta i}|^2 (|U_{ei}|^2 + 2|U_{\mu i}|^2)$ with only three Dirac neutrinos in the limit $\chi_i = 0$. The deviation in flux compared to the standard value is thus given by

\[ \delta \Phi_\beta \equiv \Phi_\beta - \Phi^S_\beta = -\frac{\phi_0}{2} \sum_{i} |U_{\beta i}|^2 (|U_{ei}|^2 + 2|U_{\mu i}|^2) \sin^2 \frac{\chi_i}{2}. \] (37)

For maximal atmospheric mixing and $\theta_{13} \simeq 0$, $|U_{ei}|^2 + 2|U_{\mu i}|^2 = 1$ for every $i$ and the above simplifies to

\[ \delta \Phi_\beta = -\frac{\phi_0}{2} \sum_{i} |U_{\beta i}|^2 \sin^2 \frac{\chi_i}{2}. \] (38)

This is to be compared with the corresponding formula [34, 39] obtained for the pseudo-Dirac neutrinos in the absence of the mirror neutrinos:

\[ \delta \Phi_\beta = -\frac{\phi_0}{3} \sum_{i} |U_{\beta i}|^2 \sin^2 \frac{\chi_i}{2}. \] (39)
The presence of pseudo-Dirac mirror partners now lead to stronger deviation in $\Phi_\beta$ from the canonical value $1/3$. The values of $\delta \Phi_\beta$ depend both on the values of $|U_{\beta i}|$ which are now reasonably well-known and on the hierarchies in $\Delta_i$ given typically by eq.(33). There exist two interesting ranges of $\Delta_i$ which can effect the oscillations of UHE in physically different ways:

(A) Strong mass hierarchies among neutrinos $m_1 \ll m_2 \simeq \sqrt{\Delta_{\odot}} < m_3 \simeq \sqrt{\Delta_{\text{atm}}}$ (or equivalently $\lambda_1 \ll \lambda_2 \simeq \lambda_{2,3}$ in eq.(33)) such that $\Delta_1$ in eq.(33) is $< 10^{-16}$ eV$^2$ but $\Delta_{2,3} > 10^{-16}$ eV$^2$. The $\Delta_1$ in this case does not induce the appreciable oscillations of the UHE neutrinos while effects of $\Delta_{2,3}$ can be averaged out. This corresponds to taking $\chi_1 = \cos \chi_2 = \cos \chi_3 = 0$ in eq.(36) and one obtains

$$\phi_\beta \approx \frac{\phi_0}{12} (1 + 3|U_{\beta 1}|^2)$$

which translates to

$$\phi_e : \phi_\mu : \phi_\tau \approx 2 : 1 : 1$$

for the tri-bimaximal mixing. The corresponding number for the current best fit values of mixing angles is $2.12 : 1 : 1.09$. While fluxes in all three flavours are suppressed compared to the canonical value $1/3$, the suppression of the electron neutrinos flux is less.

(B) The alternative possibility corresponds to a milder hierarchy characterized by $m_1 \approx \sqrt{\Delta_{\odot}}$ and all $\lambda_i$ similar in magnitude such that $\Delta_1$ is suppressed compared to $\Delta_{2,3}$ to satisfy the solar bound but all of them still are bigger than the oscillation scale $\sim 10^{-16}$ eV$^2$ of the UHE neutrinos. This case corresponds to taking $\cos \chi_i = 0$ for all $i$ in eq.(36) and all the flavours are suppressed by a factor of 4 compared to the canonical value of $1/3$.

Although no neutrinos of any flavour have been seen from GRB’s, the fact that of the nine IceCube events in the 0.15 -1.15 PeV range there is only one with a muon track suggests a preferential suppression of muon neutrinos, which indicates the hierarchical mass splitting of scenario (A).

In our model we have introduced 9 extra neutrinos which can potentially be in conflict with the BBN constraints on the effective number of species of light particles during nucleosynthesis. Of these extra neutrinos, $\nu_2^\alpha, \nu_4^\alpha$ ($\alpha = e, \mu, \tau$) are sterile and can decouple much before the time of BBN, their temperatures will be smaller than the radiation bath and they will not contribute to the Hubble expansion at the time of BBN. In our model intergenerational mixing between an active neutrino of one flavour with sterile neutrinos associated with other is negligible. As a result, $\nu_2, \nu_4$ will not equilibrate with the active $\nu_1, \nu_3$ species by oscillation. There is no equilibrium attained by $\nu_1 \leftrightarrow \nu_{2,3,4}$ oscillations of the same flavour as the mass splittings is $\leq 10^{-9}$eV$^2$ [30,32]. However $\nu_3^\alpha$ are ‘active’ in the mirror world and they could count as three extra neutrino degrees of freedom if their temperature were to be identical to the temperature of the active neutrinos in our world. One way to avoid this doubling of neutrino degrees is to assume that the mirror world couplings to the inflaton are slightly different and the reheating temperature of the mirror world following inflation is lower than reheat temperature of our universe [46]. The effective neutrino degrees of freedom observed by Planck [47] at the time of matter-radiation equality is $N_{eff} = 3.30 \pm 0.27$
at 68% CL. If there are \( N_m \) species of mirror neutrinos with temperature \( T_m \) then they will count as

\[
N_{\text{eff}} = 3.046 + N_m \left( \frac{T_m}{T_\nu} \right)^4
\]  

(41)

neutrinos. We see that in order that \( \Delta N_{\text{eff}} < 0.3 \) with \( N_m = 9 \) extra mirror neutrino species the mirror neutrino temperature \( T_m < 0.43T_\nu \) at the time of matter-radiation equality to evade the Planck bound. Another way in which sterile neutrinos, including mirror ones, can evade the stringent Planck bound is if there is an annihilation of MeV dark matter preferentially into photons such that the photon temperature relative to the neutrino temperature is raised after neutrino decoupling and prior to \( z_{eq} \). Scenarios for evading the Planck constraint on sterile neutrinos via dark matter models have been discussed in ref. [48, 49]

IV. CONCLUSIONS

In this paper we have studied the neutrino fluxes and flavour ratios of \( 10^5 - 10^7 \) GeV neutrinos originating from the GRB in a scenario with three sterile neutrinos for each flavour having tiny splitting as given in eq. (1) among them. It is shown that in this scenario GRB neutrinos of all flavours or muon and tau flavour can get suppressed by factor of 1/4 as required by IceCube result. This suppression can result in the presence of maximal mixing among a neutrino and three sterile partners as given in eq. (10). Such mixing can arise if all the neutrinos are pseudo-Dirac and there is a mirror world replicating our own interactions. As far as \( \Delta_\nu \) is concerned, it is required to be < \( 10^{-12} \) eV\(^2\) [44]. This leads to an interesting possibility. UHE neutrinos are also expected at energies of the GZK limit cosmic rays with energies of \( 10^9 \) GeV and their sources are closer at distances of 100 Mpc. Thus the (mass)\(^2\) difference required for significant conversion of these neutrinos should be \( \geq 10^{-12} \) eV\(^2\). Thus it is possible that electron neutrinos from GRB get depleted but one from the nearby sources and high energy remain undepleted. Similar thing can happen for other flavours also if \( \lambda_\mu, \lambda_\tau \) in eq. (33) are such that \( \Delta_\mu, \tau \) also fall in the range \( 10^{-12} - 10^{-15} \) eV\(^2\). These could serve as discriminating tests of models of pseudo-Dirac neutrinos like the one discussed in this paper.

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[1] E. Waxman, Phys. Rev. Lett. 75, 386 (1995) astro-ph/9505082.
[2] M. Vietri, Astrophys. J. 453, 883 (1995) astro-ph/9506081.
[3] M. Milgrom and V. Usov, Astrophys. J. 449, L37 (1995) astro-ph/9505009.
[4] P. Meszaros and M. J. Rees, Astrophys. J. 405, 278 (1993).
[5] E. Waxman and J. N. Bahcall, Phys. Rev. Lett. 78, 2292 (1997) astro-ph/9701231.
[6] J. P. Rachen and P. Meszaros, AIP Conf. Proc. 428, 776 (1997) astro-ph/9811266.
[7] E. Waxman and J. N. Bahcall, Phys. Rev. D 59, 023002 (1999) hep-ph/9807282.
[8] D. Guetta et al., Astropart Phys. 20, 429 (2004).
[9] M. Ahlers, M.C. Gonzalez-Garcia and F. Halzen, Astroparticle Phys. 35, 87 (2011).
[10] R. Abbasi et al. [IceCube Collaboration], Phys. Rev. Lett. 106, 141101 (2011) arXiv:1101.1448 [astro-ph.HE].
[11] R. Abbasi et al. [IceCube Collaboration], Nature 484, 351 (2012) arXiv:1204.4219 [astro-ph.HE].
[12] A. G. Vieregg et al., Astrophys. J. 736, 50 (2011); arXiv:1102.3206.
[13] Y. -Z. Fan, P. H. T. Tam, F. -W. Zhang, Y. -F. Liang, H. -N. He, B. Zhou, R. -Z. Yang and Z. -P. Jin et al., arXiv:1305.1261 [astro-ph.HE].
[14] http://gcn.gsfc.nasa.gov/gcn3/14520.gcn3
[15] S. Adrin-Martinez, A. Albert, I. A. Samarai, M. Andr, M. Anghinolfi, G. Anton, S. Anvar and M. Ardid et al., arXiv:1307.0304 [astro-ph.HE].
[16] M. G. Aartsen et al. [IceCube Collaboration], arXiv:1304.5356 [astro-ph.HE].
[17] V. Barger, J. Learned and S. Pakvasa, arXiv:1207.4571 [astro-ph.HE].
[18] N. Whitehorn, C. Kopper, and N. Neilson. (for IceCube Collaboration), Talk at IceCube Particle Astrophysics Symposium, at Madison, Wisconsin, USA, May 15, 2013.
[19] B. Feldstein, A. Kusenko, S. Matsumoto and T. T. Yanagida, arXiv:1303.7320 [hep-ph].
[20] V. Barger and W. -Y. Keung, arXiv:1305.6907 [hep-ph].
[21] R. Laha, J. FBeacom, B. Dasgupta, S. Horiuchi and K. Murase, arXiv:1306.2309 [astro-ph.HE].
[22] S. Hummer, P. Baerwald and W. Winter, Phys. Rev. Lett. 108, 231101 (2012) arXiv:1112.1076 [astro-ph.HE].
[23] Z. Li, arXiv:1112.2240 [astro-ph.HE]. Z. Li, Phys. Rev. D85,02730(2012), arXiv:1210.6594 [astro-ph.HE].
[24] H. -N. He, R. -Y. Liu, X. -Y. Wang, S. Nagataki, K. Murase and Z. -G. Dai, Astrophys. J. 752, 29 (2012) arXiv:1204.0857 [astro-ph.HE].
[25] B. Zhang and P. Kumar, Physical Review Letters, vol. 110, Issue 12, id. 121101 (2013) arXiv:1210.0647 [astro-ph.HE].
[26] S. Gao, K. Kashiyama and P. Meszaros, arXiv:1305.6055 [astro-ph.HE].
[27] K. N. Abazajian et al., [arXiv:1204.5379] [hep-ph].
[28] C. Giunti and M. Laveder, Phys. Rev. D 84, 073008 (2011) [arXiv:1107.1452] [hep-ph].
[29] E. Giusarma, M. Corsi, M. Archidiacono, R. de Putter, A. Melchiorri, O. Mena and S. Pandolfi, Phys. Rev. D 83, 115023 (2011) [arXiv:1102.4774] [astro-ph.CO].
[30] X. Shi, D. N. Schramm and B. D. Fields, Phys. Rev. D 48, 2563 (1993) [astro-ph/9307027].
[31] K. Enqvist, K. Kainulainen and M. J. Thomson, Nucl. Phys. B 373, 498 (1992).
[32] D. P. Kirilova and M. V. Chizhov, Nucl. Phys. B 591, 457 (2000) [hep-ph/9909408].
[33] M. Kobayashi and C. S. Lim, Phys. Rev. D 64, 013003 (2001) [hep-ph/0012266].
[34] J. F. Beacom, N. F. Bell, D. Hooper, J. G. Learned, S. Pakvasa and T. J. Weiler, Phys. Rev. Lett. 92, 011101 (2004) [hep-ph/0307151].
[35] A. de Gouvea, W.-C. Huang and J. Jenkins, Phys. Rev. D 80, 073007 (2009) [arXiv:0906.1611] [hep-ph].
[36] A. Esmaili, Phys. Rev. D 81, 013006 (2010) [arXiv:0909.5410] [hep-ph].
[37] J. Barry, R. N. Mohapatra and W. Rodejohann, Phys. Rev. D 83, 113012 (2011) [arXiv:1012.1761] [hep-ph].
[38] A. Esmaili and Y. Farzan, JCAP 1212, 014 (2012) [arXiv:1208.6012] [hep-ph].
[39] S. Pakvasa, A. Joshipura and S. Mohanty, Phys. Rev. Lett. 110, 171802 (2013) [arXiv:1209.5630] [hep-ph].
[40] J. Barranco, O. G. Miranda, C. A. Moura and A. Parada, Phys. Lett. B 718, 26 (2012) [arXiv:1205.4285] [astro-ph.HE].
[41] P. Baerwald, M. Bustamante and W. Winter, JCAP 1210, 020 (2012) [arXiv:1208.4600] [astro-ph.CO].
[42] R. Foot and R. R. Volkas, Phys. Rev. D 52, 6595 (1995) [hep-ph/9505359].
[43] Z. G. Berezhiani and R. N. Mohapatra, Phys. Rev. D 52, 6607 (1995) [hep-ph/9505385].
[44] V. Berezinsky, M. Narayan and F. Vissani, Nucl. Phys. B 658, 254 (2003) [hep-ph/0210204].
[45] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D 86, 073012 (2012) [arXiv:1205.4018] [hep-ph]; G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunno, Phys. Rev. D 86, 013012 (2012) [arXiv:1205.5254] [hep-ph]; M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, JHEP 1212, 123 (2012) [arXiv:1209.3023] [hep-ph].
[46] Z. G. Berezhiani, A. D. Dolgov and R. N. Mohapatra, Phys. Lett. B 375, 26 (1996) [hep-ph/9511221].
[47] P. A. R. Ade et al. [Planck Collaboration], [arXiv:1303.5076] [astro-ph.CO].
[48] C. M. Ho, R. J. Scherrer and , Phys. Rev. D 87, 023505 (2013) [arXiv:1208.4347] [astro-ph.CO]; C. M. Ho, R. J. Scherrer , Phys. Rev. D 87, 065016 (2013) [arXiv:1212.1689] [hep-ph].
[49] G. Steigman, Phys. Rev. D 87, 103517 (2013) [arXiv:1303.0049] [astro-ph.CO].