Large- and Small-Scale Constraints on Power Spectra
in \( \Omega = 1 \) Universes

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ABSTRACT

The cold dark matter (CDM) model of structure formation, normalized on large scales, leads to excessive pairwise velocity dispersions on small scales. In an attempt to circumvent this problem, we study three scenarios (all with \( \Omega = 1 \)) which have more large-scale power and less small-scale power than the CDM model: 1) an admixture of cold and hot dark matter; 2) cold dark matter with a non-scale-invariant, power-law primordial power spectrum; and 3) cold dark matter with coupling of dark matter to a long-range vector field. Despite the reduced small-scale power, when such models are evolved in the non-linear regime to large amplitude, the velocities on small scales are actually increased over CDM with the same value of \( \sigma_8 \). This ‘flip-over’, in disagreement with the expectation from linear perturbation theory, arises from the nonlinear coupling of the extra power on large scales with shorter wavelengths. However, due to the extra large-scale power, the recent COBE DMR results indicate smaller amplitudes for these models, \( \sigma_8 \sim 0.5 - 0.7 \), than for CDM (for which \( \sigma_8 \sim 1.2 \)). Therefore, when normalized to COBE on large scales, such models do lead to reduced velocities on small scales and they produce fewer halos compared with CDM. Quantitatively it seems, however, that models that produce sufficiently low small-scale velocities fail to produce an adequate distribution of halos.

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1. INTRODUCTION

Recent observations of fluctuations in the cosmic microwave background by the COBE satellite (Smoot et al. 1992) have placed the gravitational instability of dark matter theory of structure formation on firmer footing. At the same time, the shape of the linear density fluctuation spectrum is now constrained by a variety of observations ranging from galaxy scales (∼ Mpc) up to the very large scales (∼ 1000 Mpc) probed by COBE. The most popular theory of galaxy formation, the cold dark matter model with a scale-invariant spectrum of primordial fluctuations (hereafter CDM), has been studied by many authors using numerical $N$-body experiments over a wide range of scales (e.g. Davis et al. 1985, hereafter DEFW; Melott 1990; Park 1990; Couchman & Carlberg 1992; Gelb 1992, hereafter G92). Using the COBE DMR measurements to fix the density fluctuation amplitude on large scales, it is now necessary to re-evaluate the predictions of the CDM model on smaller scales. In fact, it is difficult to reconcile the COBE measurements, which indicate a rather high amplitude for CDM, with the observed low galaxy pairwise velocity dispersions on scales of a few Mpc. In a search for solutions to this problem, we consider three scenarios, each of which might be considered a variation on the CDM model and has less small-scale and more large-scale power than CDM.

Given current theoretical uncertainties, the amplitudes of the initial power spectra for the models are free parameters; they are related, for example, to the self-coupling of the scalar field that drives inflation. It is common to normalize spectra by setting the $rms$ density fluctuation in spheres of radii 16 Mpc to $\sigma_8$, assuming linear growth of modes independent of their wavenumber $k = 2\pi/\lambda$. (We assume $H_0 = 50\,\text{km}\,\text{s}^{-1}\,\text{sec}^{-1}$ and $\Omega = 1$ throughout.) Thus, for a power spectrum $P(k)$,

$$\sigma_8^2 \equiv \int_0^{\infty} d^3k P(k) W_{TH}^2(kR) ; \quad W_{TH}(kR) = \frac{3}{(kR)^3} (\sin kR - kR \cos kR) ; \quad R = 16\,\text{Mpc} . \quad (1)$$

On small scales, $\sigma_8$ is constrained by the pairwise velocity dispersion, $\sigma_\parallel$, defined as the $rms$ velocity along lines of separation of galaxy pairs: $\sigma_\parallel(r) \equiv \langle (v_\parallel - \langle v_\parallel \rangle)^2 \rangle^{1/2}$, where $v_\parallel = (\vec{v}_2 - \vec{v}_1) \cdot \vec{r}/r$ is the parallel, peculiar velocity and $\vec{r} \equiv \vec{r}_2 - \vec{r}_1$.

The pairwise velocity dispersion of galaxies on small scales was determined from
redshift surveys by Davis & Peebles (1983): $\sigma_{\parallel} (r \sim 3\,\text{Mpc}) \simeq 300 \pm 50\,\text{km}\,\text{s}^{-1}$ (Bean et al. 1983 found $250 \pm 50\,\text{km}\,\text{s}^{-1}$). Researchers have compared this value with $N$-body simulations in order to constrain the CDM model amplitude $\sigma_8$. DEFW found $\sigma_8 = 0.4$ and G92 found $\sigma_8 \lesssim 0.5$ (allowing for galaxy dispersions in simulated clusters to be less than virial estimates). However, the recent COBE measurements imply a larger amplitude for the CDM model: the observed $r_{\text{ms}}$ fluctuation on $10^\circ$, $[\sigma_T(10^\circ)]_{\text{DMR}} = (1.085 \pm 0.183) \times 10^{-5}$, sets $\sigma_8 \simeq 1.17 \pm 0.23$ [Adams et al. 1992; a nearly identical range for $\sigma_8$ is obtained by fitting the COBE angular correlation function $C(\theta)$]. A larger amplitude ($\sigma_8 \simeq 1.29_{-0.65}^{+0.38}$, Adams et al. 1992) is also required for CDM by the large-scale peculiar velocities of galaxies, determined by Bertschinger et al. (1990) using the POTENT reconstruction algorithm. Thus, for CDM, large-scale observations suggest a $\sigma_8$ amplitude roughly twice as large as that indicated by $\sigma_{\parallel}$ on small scales.

A possible way to reconcile large scales with small scales was suggested by Couchman & Carlberg (1992), who found that $\sigma_{\parallel}$ for the resolved halos is a factor $\sim 2$ lower than $\sigma_{\parallel}$ for the mass, an effect known as ‘velocity bias’. However, G92 showed that despite a velocity bias of $b_v \equiv \sigma_{\parallel}(\text{halos})/\sigma_{\parallel}(\text{mass}) \sim 1/2$, high amplitude CDM still produces $\sigma_{\parallel}$ on scales of $r > 1\,\text{Mpc}$ in excess of the observed $\sigma_{\parallel}$. Furthermore, CDM simulations result in an overproduction of halos, with the high-mass halos (from mergers) possibly representing clusters. If so, then dividing up the clusters into halos eliminates the velocity bias altogether (see G92). The alternative approach of using peak particles to represent galaxies (Bardeen et al. 1986) also leads to $b_v \simeq 1$ (DEFW; Quinn, Katz, & Gelb 1992). We therefore claim, that $\sigma_{\parallel}(\text{mass})$ adequately reflects $\sigma_{\parallel}(\text{halos})$, and consequently only focus on $\sigma_{\parallel}(\text{mass})$ in the following analysis.

2. MODELS

We present results from $128^3$ particle simulations using the particle-mesh technique on a $256^3$ grid (Bertschinger & Gelb 1991) in boxes of comoving length $200\,\text{Mpc}$ on a side. This size is large enough to encompass the waves that contribute significantly to $\sigma_8$ and $\sigma_{\parallel}$, although these simulations, with particle mass $m_p = 2.6 \times 10^{11}\,\text{M}_\odot$, are inadequate for resolving individual halos. (We use $51.2\,\text{Mpc}$ boxes, $m_p = 4.4 \times 10^9\,\text{M}_\odot$, at the end of §4 to study halo formation.)
We explore three models with less small-scale power and more large-scale power than the CDM model: (1) models with an admixture of hot and cold dark matter; (2) a cold dark matter model with a non-scale-invariant, power-law primordial spectrum, \( P(k) \propto k^n \), with \( n = 0.7 \); and (3) cold dark matter models in which the dark matter couples to a hypothetical long-range field (Frieman & Gradwohl 1991; Gradwohl & Frieman 1992). In all cases, we assume negligible baryon density, and, with the exception of (2), a scale-invariant \( (n = 1) \) primordial spectrum. In case (3), the force law between objects of mass \( m \) is:

\[
r^2 F/(Gm^2) = 1 + \alpha (1 + r/\lambda_0) \exp(-r/\lambda_0),
\]

so that gravity is effectively retarded for \( \alpha < 0 \) in the case of a vector field. Here, \( \alpha \) is a measure of the relative strength of the new force, and \( \lambda_0 \) is its range. For these ‘alpha’ models, we regenerate the optimal Green’s function (Hockney & Eastwood 1982) for the potential corresponding to the force law of eqn. 2,

\[
\hat{\phi} \propto \left[ \frac{1}{k^2} + \frac{\alpha}{k^2 + (2\pi/\lambda)^2} \right] \hat{\delta};
\]

\( \lambda = \lambda_0 a_0/a \) is the comoving scale at expansion factor \( a \), and \( \hat{\delta} \) is the Fourier transform of the density contrast. (We define \( a = 1 \) when \( \sigma_8 = 1 \) throughout. The comoving scale equals the physical scale at present day expansion factor \( a_0 = \sigma_8 \).) Because of limited force resolution, the retardation effect is washed out at later expansion times — the principle effect enters in the initial conditions (Gradwohl & Frieman 1992) and the early evolution.

Other authors have studied some of these models. Cen et al. (1992) simulated a \( \sigma_8 = 0.5 \) \( n = 0.7 \) cold dark matter model with comparable resolution. The authors argued that this model has lower \( \sigma_\parallel \sim 400 - 500 \text{ km s}^{-1} \) than \( \sigma_8 = 1 \) CDM, but that a velocity bias \( b_v \simeq 1/1.5 \) is still needed to match the observed \( \sigma_\parallel \sim 300 \text{ km s}^{-1} \). However, we do not allow a velocity bias factor for reasons discussed earlier. Davis, Summers, & Schlegel (1992) performed high resolution simulations with cold and hot dark matter (\( \Omega_{HDM} = 0.30 \)) in 14 Mpc boxes at \( \sigma_8 = 0.9 \). We demonstrate that not only is this box too small to adequately measure \( \sigma_\parallel \) (as pointed out by the authors), but that large-scale waves (not present in a 14 Mpc box) at large values of \( \sigma_8 \) can actually increase \( \sigma_\parallel \) on small scales relative to CDM.
3. LINEAR PERTURBATIONS

We present linear realizations of the initial power spectra and linear estimates of $\sigma_\parallel$ for the models. The dimensionless linear power spectra $P(k)(\Delta k)^3$ (all with $\sigma_8 = 1$) are plotted in fig. 1a as a function of comoving wavenumber $k$, where $\Delta k = (2\pi/L)^3$ with $L = 200$ Mpc. We begin all simulations at an expansion factor $a = 1/50$. We use the transfer function of DEFW for models other than the cold plus hot models, and the density-weighted transfer function of van Dalen & Schaefer (1991) for the cold plus hot models. All of the simulations use the same initial random numbers scaled to the appropriate power spectra. Results are shown for six simulations: CDM (standard cold dark matter); C+H13 and C+H30 (cold dark matter mixed with 13% and 30% hot dark matter); TILT7 (cold dark matter with $n = 0.7$); and ALPHA3 and ALPHA5 (cold dark matter alpha models with $\alpha = -0.3$, $\lambda_0 = 100$ kpc; and $\alpha = -0.5$, $\lambda_0 = 500$ kpc). The sharp cutoff corresponds to the three-dimensional Nyquist wavenumber and the solid line is a direct plot of the DEFW CDM power spectrum for discrete $kL/(2\pi)$. The first bin for the simulated models is an average over 18 waves.

The TILT7, C+H13, and ALPHA3 power spectra are similar. However, the alpha simulations will lag behind the others as the modified force law continues to retard the subsequent evolution. (We discuss the nonlinear power spectra of fig. 1b in §4.)

Although it is a poor approximation for the small scales of interest, it is nevertheless instructive to estimate $\sigma_\parallel$ in linear perturbation theory where the velocity and density are related by (Peebles 1980; Lightman & Schechter 1990)

$$\vec{\nabla} \cdot \vec{v} \approx -H_0 \Omega_4^{4/7} \delta ,$$  

where $\delta$ is the density contrast; therefore, $v_k^2 d^3 k \propto H_0^2 P(k)/k^2 d^3 k$. As a result, the linear estimate is

$$\sigma_\parallel^2 (r) \equiv \langle [v(r) - \bar{v}(0)]^2 \rangle = 2[\langle v^2 \rangle - \langle \vec{v}(r) \cdot \vec{v}(0) \rangle]$$

$$= 2H_0^2 \int_{2\pi/\lambda_{\text{max}}}^{\infty} 4\pi k^2 dk P(k) \left[ 1 - \frac{\sin kr}{kr} \right].$$  

(5)
The factor $1 - \sin(kr)/(kr)$ filters out the contribution of long waves to the small-scale pairwise velocity dispersion; for waves with $\lambda \gg r$, the bulk flow does not contribute to the dispersion. This is opposite to the top hat filter $W_{TH}$ in eqn. 1, which filters out contributions from large $kr$.

In fig. 2a we plot linear theory estimates of $\sigma_\parallel$ for the various spectra at $\sigma_8 = 1$. The Davis & Peebles (1983) estimates from the observations are $\sim 300 \pm 50$ km s$^{-1}$ for scales $\sim 1 - 3$ Mpc. The models with reduced small-scale power appear to fare well at these scales, but, as we now show, nonlinear effects radically alter $\sigma_\parallel$.

4. NONLINEAR CALCULATIONS

4.1 PAIRWISE VELOCITY DISPERSIONS

We first study nonlinear power spectra and $\sigma_\parallel$ for the models. In fig. 1b we present nonlinear power spectra at $\sigma_8 = 1$ for several models, computed from our simulations in the 200 Mpc box. For low values of $k$, the spectra agree with their counterparts in fig. 1a, indicating a sufficiently large box size. The models with less small-scale power in the initial conditions continue to have less small-scale power in the nonlinear regime, yet the nonlinear spectral shapes differ from the linear regime and the differences among the various models are significantly reduced, compared to our linear estimates.

The nonlinear $\sigma_\parallel$ for the models are shown in figs. 2b and 2c. Comparing with fig. 2a, it is clear that linear theory is a very poor estimator of both the amplitude and the general characteristics of $\sigma_\parallel$. In fig. 2b $\sigma_\parallel$ is plotted at $\sigma_8 = 0.25, 0.5, 0.75, \text{and} 1$ for CDM and C+H30. At low amplitude, $\sigma_8 \lesssim 0.5$, $\sigma_\parallel$ for C+H30 is lower than $\sigma_\parallel$ for CDM, in agreement with the expectation from linear theory. However, when the models are evolved further, i.e., for $\sigma_8 \gtrsim 0.7$, $\sigma_\parallel$ ‘flips over’: despite its reduced small-scale power, the C+H30 model yields larger $\sigma_\parallel$ than CDM. Fig. 2c shows that this flip-over at high $\sigma_8$ is generic for these models; it is a reflection of the fact that their extra power on large scales couples significantly to small scales. This effect is manifest in the pairwise velocity dispersion, but not the power spectrum, because the former is more sensitive to long wavelengths.

The flip-over in the initial power spectra occurs on scales exceeding $\sim 75$ Mpc (fig. 1a)
and is therefore missed in the 14 Mpc simulations of Davis, Summers, & Schlegel (1992); at $\sigma_8 = 0.9$ (the case studied there) C+H30 actually yields higher $\sigma_\parallel$ than the CDM model. (Originally we used 51.2 Mpc boxes and did not see this effect either.) Cen et al. (1992) may not have elucidated this effect since they only evolved their $n = 0.7$ simulation to $\sigma_8 = 0.5$. Our TILT7 results agree with Cen et al. (1992) at this amplitude.

To compare these results with the observations of $\sigma_\parallel \simeq 300 \pm 50 \text{ km s}^{-1}$, we fix the $\sigma_8$ amplitudes of the models using the DMR data and linear perturbation theory (which is valid on the large scales probed by COBE). As noted in §1, for CDM, COBE yields $\sigma_8 \gtrsim 1$, which implies $\sigma_\parallel$ in excess of 1200 km s$^{-1}$ over scales of a few Mpc. For the other models, COBE’s $\text{rms}$ fluctuations on $10^\circ$, including the DMR errors and cosmic variance for the models, imply $\sigma_8 \simeq 0.78 \pm 0.16$ (for C+H13), 0.69$\pm$0.14 (for C+H30), 0.53$\pm$0.11 (for TILT7), 0.95$\pm$0.19 (for ALPHA3), and 0.51$\pm$0.10 (for ALPHA5). When normalized to COBE, it is possible to find models with reduced $\sigma_\parallel$. One should, however, point out that models with $\sigma_\parallel \sim 400 - 550 \text{ km s}^{-1}$, although favorable over $\sigma_8 = 1$ CDM, may still be inadequate. This is due to the fact that simulation-to-simulation variations in $\sigma_\parallel$ are typically $\lesssim 100 \text{ km s}^{-1}$ (G92), the observed errors in $\sigma_\parallel$ are $\sim 50 \text{ km s}^{-1}$, and Bean et al. (1992) found lower $\sigma_\parallel$ of order 250 $\pm$ 50 km s$^{-1}$ than Davis and Peebles (1993).

In fig. 3a we plot $\sigma_\parallel$ for ‘favorable’ models, subject to the above COBE normalizations. They are: $\sigma_8 = 0.6$ ALPHA5 and $\sigma_8 = 0.5$ TILT7, both consistent with COBE; $\sigma_8 = 0.5$ C+H30, which is somewhat beyond the 1$\sigma$ level; and $\sigma_8 = 0.5$ CDM, which is inconsistent with COBE and only shown here for comparison. We exclude C+H13 and ALPHA3 from our list, as they produce excessive $\sigma_\parallel$. TILT7 and C+H30 both produce small-scale $\sigma_\parallel$ in the $400 - 550 \text{ km s}^{-1}$ range, a factor of $\sim 2$ smaller than $\sigma_8 = 1$ CDM, but still at least 30% above the observed $\sigma_\parallel$. The only model in fig. 3a that matches COBE and small-scale $\sigma_\parallel$ is ALPHA5. It nicely reproduces the observed $\sigma_\parallel$, but as we now demonstrate, suffers from inadequate halo formation.

### 4.2 HALO FORMATION

In addition to the affect on $\sigma_\parallel$, reducing small-scale power can also help alleviate some problems associated with excessive low-mass halo formation in the field and an excessive number density of high-mass objects that plague the CDM model (White et
In order to study halo formation, we simulate models in 51.2 Mpc boxes, again using $128^3$ particles and a $256^3$ particle-mesh grid. (We emphasize again, that although adequate for analyzing the halo distribution, the 51.2 Mpc box simulations cannot be used to compute $\sigma_\parallel$.)

The details of galaxy formation require higher resolution simulations with separate hot and cold dark matter particles (e.g. Davis, Summers, & Schlegel 1992) and separate dark matter and baryonic matter particles in alpha models (the coupling to the vector field only occurs for the dark matter and can lead to a natural bias between dark and baryonic matter, Gradwohl & Frieman 1992). These considerations, however, are not likely to significantly affect mass pairwise velocity dispersions.

Halos in our simulations are identified, by use of the DENMAX algorithm (Bertschinger & Gelb 1991), as local density maxima in the evolved, nonlinear density field. The distribution of halos, characterized by their circular velocities (computed from the enclosed mass within $R =$300 kpc of the DENMAX center, i.e. $\sqrt{GM(<R)/R}$) are then counted in 25 km s$^{-1}$ bins. The results are shown in fig. 3b for the same scenarios as in fig. 3a. The solid line is the observed estimate, using a Schechter (1976) luminosity function coupled with Tully-Fisher (1977) and Faber-Jackson (1976) relationships (assuming that 70% of the halos are spirals and 30% are ellipticals) to relate luminosity to mass. (See G92 and White et al. 1987 for details. The observed estimates as shown tend to overestimate the highest-mass halos compared with complete elliptical surveys, and the overall normalization has an error $\sim 30%$.)

The cases $\sigma_8 = 0.5$ CDM and $\sigma_8 = 0.5$ TILT7 appear to match the observed distribution fairly well. G92 demonstrated, however, that high-mass halos should be divided into clusters of halos so that 1) the simulations contain clusters and 2) extra weight is given to dense systems thereby enhancing the two-point correlation function in biased ($\sigma_8 < 1$) models (White et al. 1987). Therefore, TILT7 actually does much better than CDM—it produces less mid-mass halos and less high-mass halos which, when divided into mid-mass cluster members, make up the mid-mass deficit. C+H30 produces too few halos, which can be remedied by evolving the simulation further at the expense of raising $\sigma_\parallel$. ALPHA5 drastically fails to match the observed distribution for $\sigma_8 = 0.6$. (ALPHA3 does better, but it produces excessive $\sigma_\parallel$.)
5. CONCLUSIONS

For $\sigma_8 > 0.5$ (the precise value depends on the model), the nonlinear coupling of waves in models with more large-scale power and less small-scale power than CDM actually increases $\sigma_{||}$ on small scales, in complete disagreement with linear perturbation theory. At $\sigma_8 = 0.5$, CDM, C+H30, and TILT7 yield $\sigma_{||} \sim 400 - 550 \, \text{km s}^{-1}$ on small scales, but CDM has inadequate large-scale amplitude. C+H30 and TILT7 also generate, consistent with observations, less halos than CDM. The only model which produces $\sigma_{||} \sim 300 \, \text{km s}^{-1}$, and at the same time matches COBE, is ALPHA5 at $\sigma_8 = 0.6$, but it fails to produce a sufficient number density of halos.

The fact that it seems difficult (if not impossible?) to accommodate a low $\sigma_{||}$, and still have enough small-scale power for adequate halo formation, may hint to a basic problem of $\Omega = 1$ cosmogonies. One way out of this apparent impasse is, of course, to lower the matter density of the universe (recall that $v \propto \Omega^{4/7}$), and thereby maintain a low $\sigma_{||}$ with increased small scale power. It is clearly too early to view this problem as a death stroke to $\Omega = 1$ scenarios, more work still needs to be done involving the details of halo formation and biasing.

We summarize our results in Table 1. There are no obvious ‘winners’. The balance of scores can be shifted by varying $\sigma_8$; e.g., C+H30 can be shifted to $+, - , +$ by choosing a higher amplitude. Recent COBE measurements on large scales require the investigation of a myriad of models with more free parameters than CDM. On small scales, in $\Omega = 1$ scenarios, it seems difficult to reconcile low $\sigma_{||}$ with sufficient halo formation, and one may be forced to consider scenarios with $\Omega < 1$ or a cosmological constant.

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| Model    | $\sigma_8$ | COBE | $\sigma_\parallel (r \lesssim 3 \text{ Mpc})$ | Halos |
|----------|------------|------|---------------------------------|-------|
| CDM      | 0.5        | –    | ◦                               | –     |
| C+H30    | 0.5        | +    | ◦                               | ◦     |
| ALPHA5   | 0.6        | +    | +                               | –     |
| TILT7    | 0.5        | +    | ◦                               | +     |

**Table 1 Caption:** Scorecard for the various models with + indicating a favorable score, ◦ indicating a marginal score, and – indicating a disfavorable score.
FIGURE CAPTIONS:

Figure 1: a) Linear realizations (all use equivalent random numbers) of various power spectra normalized to $\sigma_8 = 1$. (C+H30 has less small-scale power than C+H13 and ALPHA5 has less small-scale power than ALPHA3.) b) Nonlinear power spectra at $\sigma_8 = 1$ for some of the simulations shown in fig. 1a. In both a) and b), the solid curve is the analytic form of the DEFW linear CDM power spectrum.

Figure 2: a) Linear theory calculations of $\sigma_\parallel$ versus comoving separation (eqn. 5) for $\sigma_8 = 1$. (C+H30 is lower than C+H13 and ALPHA5 is lower than ALPHA3.) b) Nonlinear calculations of $\sigma_\parallel$ for CDM (short dashed curves) and for C+H30 (long dashed curves) at four values of $\sigma_8$. c) Nonlinear calculations of $\sigma_\parallel$ for CDM (short dashed curves), C+H13 (long dashed curves), ALPHA3 (dot-short dashed curves), and TILT7 (dot-long dashed curves) at $\sigma_\parallel = 0.5$ and 1. The observed estimates from galaxies for $r \sim 1-3$ Mpc are $300 \pm 50$ km s$^{-1}$ (Davis & Peebles 1983) and $250 \pm 50$ km s$^{-1}$ (Bean et al. 1983).

Figure 3: a) Nonlinear calculations (200 Mpc box) of $\sigma_\parallel$. b) Distribution of halos versus circular velocity (in $\Delta V_{circ} = 25$ km s$^{-1}$ bins). The solid curve is the observed estimate. In both a) and b) the cases are: CDM at $\sigma_8 = 0.5$ (short dashed curves), C+H30 at $\sigma_8 = 0.5$ (long dashed curves), ALPHA5 at $\sigma_8 = 0.6$ (dot-short dashed curves), and TILT7 at $\sigma_8 = 0.5$ (dot-long dashed curves).