The Spatial Distribution of Liquefaction Susceptibility by Logistic Regression Model Adapted for Adapazari, Turkey

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ABSTRACT
A logistic regression model has been developed for evaluation of soil liquefaction by the use of cone penetration test (CPTu, PCPT) on data collected from Adapazari, Turkey. The model inputs are the clean sand equivalent normalized cone tip resistance \( q_{c\text{eq},N} \) and cyclic stress ratio corrected for moment magnitude of 7.5 earthquake \( (\text{CSR}_{7.5=7.5}) \) that was experienced in 1999. Liquefaction probabilities \( (P_L) \) are obtained for each district of the city for which CPTu data is available with the proposed logistic regression model. Average liquefaction probabilities of the depth interval 0-6 m and coordinates (Longitude, Latitude) of CPT soundings were plotted to construct a liquefaction probability map by longitude and latitude. In order to show the effect of depth in liquefaction potential, the obtained liquefaction probability contours were reconstructed by dividing 0-6m depth into three narrow sublayers of 0-2m, 2-4m and 4-6m wherein liquefaction was observed during the earthquake. For each depth interval, liquefaction probabilities of the districts are compared with the observed liquefied and non-liquefied sites in the city after 1999 Adapazari Earthquake.

Keywords: Liquefaction, CPT, logistic regression, probability of liquefaction.

1. INTRODUCTION
Liquefaction has been investigated extensively during the past 50 years. Conditions triggering liquefaction and evaluation of liquefaction potential have been the main issues of research since the first evidence of liquefied areas were reported for Alaska Mw=9.2 and...
Niigata, Japan M_w=7.5 during the 1964 quakes. The developed evaluation models use in-situ test results such as the standard penetration test (SPT) and cone penetration test (CPT) in order that the resistance of soil against liquefaction is defined [1, 2, 3, 4, 5, 6, 7, 8]. The dynamic response of the earthquake is defined as cyclic shear stress ($\tau_{cyc}$) shown in Eq. 1:

$$\tau_{cyc} = 0.65 \frac{a_{\text{max}}}{g} \sigma_{0v} r_d$$  \hspace{1cm} (1)

The effect of dynamic loading is represented in the developed evaluation models as the ratio of $\tau_{cyc}$ and the effective vertical stress, which is called the cyclic stress ratio (CSR) defined as [1]:

$$\text{CSR} = \frac{\tau_{cyc}}{\sigma_{0v}} = 0.65 \frac{a_{\text{max}}}{g} \left( \frac{\sigma_{0v}}{\sigma_{0v}'} \right) r_d$$  \hspace{1cm} (2)

where $a_{\text{max}}$ is the peak ground acceleration; $\sigma_{0v}$ and $\sigma_{0v}'$ are the total and effective vertical stresses respectively; $r_d$ is the stress reduction factor. Most evaluation models follow similar procedures to determine the liquefaction potential. The main objective is to find a boundary using the cyclic loading and the resistance of soil against this load, which reasonably separates the liquefied and non-liquefied data as shown on a simple model in Figure 1.

![Figure 1 - Simple model for evaluation of liquefaction [9]](image)

The deterministic models establish the Cyclic Resistance Ratio (CRR) and compare it with the existing CSR to determine whether liquefaction has materialized. Probabilistic models
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have also been developed to seek the onset of liquefaction [10, 11, 12]. Logistic regression model is one of the commonly used methods in probabilistic evaluation of liquefaction. Liao et al. [10] applied logistic regression to liquefaction for the first time using corrected SPT blow count \((N_{1,60})\) values. Toprak et al. [11] developed a logistic regression model for liquefaction evaluation with both SPT and CPT data. Since the data from CPTu soundings are more numerous than SPT for a similar borehole depth due to the smaller sampling interval of CPT (2cm/sec whereas one reading per 1.5m in SPT), CPT data have been commonly used in evaluation of liquefaction. Lai et al. [12] proposed a logistic regression model using CSR corrected for 7.5 moment magnitude \((CSR_{M=7.5})\) and normalized cone tip resistance \((q_{c,1N})\). In this study, a logistic regression model has been developed by using \(CSR_{M=7.5}\) and sand equivalent normalized cone tip resistance \((q_{c,1N,cs})\) in order that the liquefaction susceptibility can be determined as well as the probability of liquefaction. With the developed logistic regression equation, probability of liquefaction for the districts of Adapazari has been obtained by using the CPT soundings carried out after the 1999 earthquake \((M_w=7.4)\). Liquefaction probabilities and available coordinates (Longitude; Latitude) of CPTs have been used to create a liquefaction probability map for the City. The map has been constructed as a two-dimensional planar system which indicates the contours of liquefaction probability. The liquefaction probabilities of each district have been compared with the liquefied and non-liquefied sites in order that the performance of proposed logistic regression equation for liquefaction probability can be tested.

2. LOGISTIC REGRESSION

Logistic regression is used to determine the probability of occurrence of events having binary response. Liquefaction is a convenient topic for the application of the logistic regression because the outputs are either “liquefaction occurs” or “liquefaction does not occur”. The output can be represented as the indicator binary variable \(Y\) which consists of “1” for “liquefaction” and “0” for “no liquefaction” in order that the regression can be performed. \(Y\) is related to the vector of explanatory variables \(X = [X_1, X_2, \ldots, X_m]^T\). The probability of occurrence is then defined with the help of \(n\) number of observations \((X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)\). In the case of liquefaction, the explanatory variables are determined by considering the representation of cyclic loading effects and the resistance of the soil to these effects. The liquefaction probability \(P_L\) with logistic regression is the expected value \(Y\) against \(X\) as follows;

\[
P_L(X) = P[Y = 1 \mid X] = E[Y \mid X] \tag{3}
\]

Since \(P_L\) is a probability function, the condition \(0 < P_L(X) < 1\) must be fulfilled, and the regression equation is obtained by solving the logit transformation function \(Q_L\), which has the range \(-\infty\) to \(\infty\) defined in Eq. 4 [13]:

\[
Q_L = \text{logit}(P_L) = \ln \left( \frac{P_L}{1-P_L} \right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_m X_m \tag{4}
\]

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\( \beta_0, \beta_1, \beta_2, \ldots, \beta_m \) are the logistic regression coefficients to be determined with the use of \( n \) number of observations. The primary assumptions of logistic regression are that the function \( Q_L \) and the explanatory variables \( X_i \) are linearly dependent and these variables are normally distributed. The liquefaction probability is then defined in Eq. 5 as:

\[
P_L(X) = \frac{1}{1 + \exp(-Q_L)} = \frac{1}{1 + \exp(-[\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_m X_m])}
\]  

(5)

Regression coefficients are determined by maximizing the logarithm of likelihood function which consist of the variable vector \( X \) and the regression coefficients. Likelihood function for liquefaction probability is defined in Eq. 6:

\[
L(X; B) = \prod_{j=1}^{m} [P_L(X)]^{y_j} [1 - P_L(X)]^{(1-y_j)}
\]  

(6)

where \( B \) is the regression coefficient vector.

The goodness-of-fit of the logistic regression has been evaluated by modified likelihood ratio index (MLRI) denoted by \( \rho^2 \) as [14]:

\[
\rho^2 = 1 - \frac{\ln[L(\hat{\beta})] - (m + 1)}{\ln[L(0)]}
\]  

(7)

where \( \ln[L(\hat{\beta})] \) is the maximum value of log-likelihood function, \( \ln[L(0)] \) is the log-likelihood value for \( \beta=0 \) and \( m \) is the number of independent variables. \( \rho^2 \) ranges from 0 to 1, and the regression fits well if it converges to 1. Additionally, there is an assumption after Liao et al. [10] that the regression model is well fitted for \( \rho^2 \) values greater than 0.4 [12].

3. DETERMINATION OF LOGISTIC REGRESSION VARIABLES

The vector \( X \) should contain the variables which represent the cyclic loading effect and the soil resistance against this loading. Cone tip resistance (\( q_c \)) has been used in this study as the strength parameter. The correction of \( q_c \) for overburden is given as the normalized cone tip resistance at 1 atm pressure \( q_{c+\text{N}} \) [4]:

\[
q_{c+\text{N}} = C_N \frac{q_c}{P_a} \quad C_N = \frac{P}{\sigma_{0v}} \leq 1.70
\]  

(8)

where \( C_N \) is the overburden correction factor and \( P_a \) is the atmospheric pressure (95.76 kPa). Another correction to cone tip resistance is made for the fines content (FC).
the characteristics of soil is the soil behaviour type index ($I_c$) in CPTu [3] by which the frictional or cohesive behaviour of the soil can be identified. In addition, the effect of fines content is generally represented by $I_c$ for CPTu-based liquefaction evaluations. $q_{c1N}$ has been transformed into sand equivalent normalized cone tip resistance ($q_{c1N,cs}$) by using Eq. 9 [2, 4, 5]:

$$q_{c1N,cs} = K_f q_{c1N}$$ (9)

where $K_f$ is the fines content correction factor which is a function of $I_c$. The cyclic loading effect is generally expressed as $CSR$ after the Simplified Procedure in both deterministic and probabilistic studies. Therefore, $CSR$ has been used as a loading parameter in the vector $X$. $CSR$ has been corrected for the earthquake moment magnitude. The correction has been performed by using

$$CSR_M = \frac{CSR_{M=7.5}}{MSF}$$ (10)

where $CSR_M$ and $CSR_{M=7.5}$ are the cyclic resistance ratio for moment magnitude of $M$ and $M_{w}=7.5$, respectively. $MSF$ is the magnitude scaling factor which may be calculated using the relation proposed by Youd et al. [5] by

$$MSF = \left(\frac{M}{7.5}\right)^{-2.56}$$ (11)
The data used for $q_{cIN,CS}$ has been demonstrated by a box plot given in Figure 2 as to the assumption of logistic regression. The range of $q_{cIN,CS}$ used in the logistic regression model varies from 10 to 200.

In connection with the assumptions involved in logistic regression, the use of square root of $q_{cIN,CS}$ has been preferred [12]. The goodness-of-fit of $q_{cIN,CS}$ and $\sqrt{q_{cIN,CS}}$ to the normal distribution is presented with the probability plots in Figure 3a and b, respectively.

Several researchers have expressed the opinion that the use of natural logarithm of $CSR$ as explanatory variable increases the goodness-of-fit to the normal distribution [10, 11, 12]. In this study, the variable $CSR$ has been used as the form of previous studies ($\ln(CSR_{M=7.5})$) in order that the main assumptions of logistic regression are fulfilled.

4. DEVELOPMENT OF LOGISTIC REGRESSION MODEL

Instead of taking the partial derivatives of the logarithm of likelihood function given in Eq. 6 to obtain the regression coefficients, a MATLAB function has been used for this purpose. It is titled “glmfit Generalized Linear Model Regression” (manual of the MATLAB function is available on https://www.mathworks.com/help/stats/glmfit.html ). The data of the explanatory variables have been obtained from the CPT soundings which were carried out in Adapazari, Turkey after the 1999 [15]. The locations of each CPT soundings and boreholes are depicted in Figure 4.

The total number of $q_c$ data obtained from CPT tests is 9379. The corrections and normalization of $q_c$ have been performed by Eqs. 8 and 9. For the earthquake effect, peak ground acceleration ($a_{max}$) and moment magnitude ($M_w$) of 1999 Adapazari Earthquake records have been used to derive the value of CSR using Eq. 1. The corrections for CSR have
been applied with Eqs. 10 and 11. The indicator binary variable \( Y \) has been formed using the “Adapazari Criteria” [16, 17] for liquefaction susceptibility. Fine-grained soil is susceptible to liquefaction whenever the following set of rules are satisfied:

- Liquid Limit \( (w_L) \leq 33\% \),
- Liquidity Index \( (I_l) \geq 0.9 \) (instead, use \( w_d/w_L \) for NP soils)
- Clay Fraction \( (D_{<2 \mu m}) \leq 10\% \),
- Average grain size \( (D_{50}) > 0.02 \text{mm} \)

Adapazari Criteria has been used as indicator binary variable since it has been proven that the liquefaction susceptibility is reflected more accurately than by other studies such as the “Chinese Criteria”. Thus, the variable \( Y \) consists of 2709 liquefied \( (Y=1) \) and 6670 non-liquefied \( (Y=0) \) data. It should be added here that the majority of liquefaction sites have been identified after the earthquake. The database contains physical properties of soil for liquefaction susceptibility and values of explanatory variables from CPT results and earthquake record (Table 1).

Figure 4 - CPT soundings and borehole locations of the studied area [18]
The logit function $Q_L$ can be written with the help of the developed regression coefficients as

$$Q_L = 10.4559 - 0.9934 \sqrt{q_{c1N,cs}} + 2.476 \ln(CSR_{M=7.5})$$

The relationship of liquefaction probability with logistic regression coefficients is then defined by:

$$P_L = \frac{1}{1 + \exp(-[10.4559 - 0.9934 \sqrt{q_{c1N,cs}} + 2.476 \ln(CSR_{M=7.5})])}$$

The goodness-of-fit of the developed logistic regression model has been found as $\rho^2=0.95$. The effect of the selection of variable forms to the goodness-of-fit has been presented by comparing MLRIs of the two relations using $\sqrt{q_{c1N,cs}}$ and $q_{c1N,cs}$ as explanatory variables in Table 2.
Since the logit transformation with explanatory variables $\sqrt{q_{\text{culn}_{cs}}}$ and $\ln(\text{CSR}_{M=7.5})$ have greater $\rho^2$ value according to the comparison given in Table 2, the development of logistic regression model and further studies have been carried out with the probability function of Eq. 13. The boundaries of different liquefaction probabilities have been defined with liquefied and non-liquefied data on $q_{\text{culn}_{cs}}$ versus $\text{CSR}_{M=7.5}$ plane (Figure 5).

![Figure 5 - The developed logistic regression model](image)

In addition to the evaluation of liquefaction potential, the developed logistic regression model provides the possibility of determining the probability of liquefaction with the available inputs of $q_{\text{culn}_{cs}}$ and $\text{CSR}_{M=7.5}$. The boundary of 50% liquefaction probability in the developed logistic regression model is analogous with the boundaries defined in deterministic models since it effectively separates the liquefied and non-liquefied data. Furthermore, the boundaries of 99% and 1% liquefaction probabilities indicate that the zones where liquefaction is most likely and where it has not developed, respectively.

**5. DETERMINATION OF LIQUEFACTION PROBABILITIES IN ADAPAZARI**

The performance of the developed logistic regression model has been tested by obtaining the liquefaction probabilities for each district of Adapazari. The probabilities have been calculated using the CPT data whose coordinates are made available using the relationship in Eq. 13. The data used in developing the proposed model has been excluded, in order that
the soundness of the model can be verified. The liquefaction probabilities for each CPT profile has been plotted on the 2D-planar system of equal liquefaction probability contours for 6m depth along the coordinates (Figure 6a) to secure simplicity of comparison for the liquefied and non-liquefied sites (Figure 6b).

In Figure 6b, the liquefied sites are seen to concentrate near the districts TI and YG whose probabilities of liquefaction are the highest at 60-65% as shown in Figure 6a. The non-liquefied sites such as OZ, TE, PA and YD have liquefaction probabilities between 5 and 10%. Although Figure 6a gives analogous information about the liquefaction potential with Figure 6b, it does not reflect the effect of depth on liquefaction because it has been constructed by taking the average of probabilities for the entirety of 6m depth. Therefore, the liquefaction probability contours have been re-established for three sub-intervals of 0-2, 2-4 and 4-6m in order to reflect the effect of depth on liquefaction potential (Figure 8, 9, and 10).

The liquefaction probabilities in the established contours are the highest for shallow depths and diminish with increasing depth. The depth of 0-2 m has the highest liquefaction probabilities whereas 4-6 m depth interval has the lowest, which indicates that the liquefaction has materialized mostly within a few meters from the ground level where Holocene layers as young as 100 years are known to have been sedimented. Figure 7 depicts the typical CPT profile of the liquefied sites at the studying region. In this figure, the resistance increases reasonably at 5 to 6 meters from ground level after which liquefaction cannot be initiated. The soil type becomes clayey after that depth which prevents the triggering of liquefaction.
In Figure 8, the liquefaction probability in the districts where liquefaction was observed to be widespread, have been found to be between 80 and 95%, which confirms that liquefaction occurred mostly near the ground surface. Moreover, the liquefaction probabilities and confirmed liquefied sites match well, confirming that the liquefaction probability estimation by the developed logistic regression model provides reliable results.
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Figure 8 - Equal probability contour map for depth interval 0-2 m

Figure 9 - Equal probability contour map for depth interval 2-4 m
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The liquefaction probabilities for the coordinates corresponding to the liquefied districts are even lower than for the 0-2 m depth interval in Figure 9 which depicts the sub-layer 2-4m. However, some of the districts have probabilities higher than those obtained for 0-2 m. This may be because either the ground water level (GWL) is within the 2-4 m depth interval or there exist non-liquefiable layers above 2 m. For instance, a location in district TH has 0% liquefaction probability for 0-2 m whereas a probability approximately 50% is indicated for 2-4 m. Nevertheless, the liquefaction probabilities generally do not exceed 70-75%.

Liquefaction probabilities for most of the districts are within 5-30% for 4-6m depth interval. Rare locations in Figure 10 have probabilities of 50-70% for which the liquefaction has occurred along the whole profile. The liquefaction probabilities generally decrease from 0-2 m to 4-6 m depths.

![Figure 10 - Equal probability contour map for depth interval 4-6 m](image_url)

The summary of Figures 8, 9 and 10 are given in tabular form in Table 3. In this table, probabilities of liquefaction have been compared with the available observations of liquefaction for each district. The liquefaction susceptibility is represented as “YES” for liquefaction, “NO” for no liquefaction, and “YES/NO” for the districts where some profiles have liquefied whereas some have not. In the table, cell colours have the same meaning with the colour-scale of the developed probability contours. Red coloured cells indicate liquefied sites, whereas non-liquefied districts have green filled cells. The districts denoted as “YES/NO” as liquefaction condition are assigned yellow coloured cells. In connection with the liquefaction probabilities, those cells with 50% or greater probability are red. Green cells indicate probability of liquefaction below 50%. The cells having both less than and higher...
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than 50% probability are coloured yellow. The colours in Table 3 show that the liquefaction condition in each district and the liquefaction probability for the corresponding district match well. For some of the districts such as KP, “no-liquefaction” is indicated; however, the probabilities remain above 50% for 0-2m depth. The situation emanates from the fact that the soil along the profile may fall within the gray “test” region in according to the Adapazari Criteria. Moreover, some districts such as TH and YC indicate “YES” for liquefaction but probabilities over 50% are encountered for rare depth intervals. This is because liquefaction occurred at a restricted depth of the whole profile.

Table 3 - The liquefaction probabilities for districts of Adapazari

| Districts | $P_l$ (%) for depth | Liquefied? |
|-----------|---------------------|------------|
| AK        | 84-92  39-63  10-27 | YES        |
| CM        | 6-83   18-57   3-60 | YES        |
| ER        | 2-11   0.3-11  65  | NO         |
| HO        | 74     71       65  | YES        |
| IS        | -      19       24  | NO         |
| KO        | 93     69       34  | YES        |
| KP        | 54     29       10  | NO         |
| KU        | 20     27       26  | NO         |
| MP        | 62-82  0.2-48  | YES        |
| OR        | 48-93  28-87   13-51 | YES/NO    |
| OZ        | 9-30   14-24   | NO         |
| PA        | 2-8    6-14    | NO         |
| SA        | 7-54   16-58   0-44 | YES/NO    |
| SK        | 18-77  27-72   21-38 | YES/NO    |
| SM        | 5-45   6-25    0-18 | NO         |
| SV        | 30     54       50  | NO         |
| TE        | 7-34   7-34    | NO         |
| TH        | 28     42-48   39-75 | YES        |
| TI        | 58-93  52-82   13-63 | YES        |
| TK        | 49-78  55-68   6-42 | YES/NO    |
| TZ        | 16     6-25    6-22 | NO         |
| YA        | 59     76       50  | YES        |
| YC        | 27     25-54   0.4-23 | YES        |
| YD        | 24-27  24-47   12-34 | NO         |
| YG        | 60-91  51-82   15-56 | YES        |
| YM        | 4      | NO         |
| YT        | 1.4    1.4      8   | NO         |
6. COMPARISON WITH OTHER LOGISTIC REGRESSION MODEL

The proposed model has been compared with another logistic model in order to confirm that a new model is required for the study of liquefaction susceptibility. The results of a recent study in logistic regression modelling using CPT results [12] have been used for comparison of the findings of this research. Liquefaction probabilities at each district have been calculated using both the developed model in this study and the compared logistic regression equations found in literature. Figure 11 illustrates the discrepancy of liquefaction probabilities of the currently developed and the compared model.

![Figure 11 - Comparison of PL from developed equation (Eq. 13) and the proposed equation by [12]](image)

The equation of the regression model used for comparison gives more conservative values of probabilities than the model developed for this study. It has been found that the use of the previously developed model may result in the underestimation of liquefaction susceptibility in observed liquefaction sites. In contrast, the probabilities derived from this developed model which are above 50% have been verified at the liquefied districts. The maps also illustrate the comparison of average liquefaction probability contours for 6m depth with both developed (Figure 12a) and compared (Figure 12b) models.

The probability contour map from the developed model has indicated values higher than 50% for the liquefied sites whereas the contours from the compared model has less than 50% probability values which do not match with the available observations made at liquefied sites.
7. CONCLUSION

- A logistic regression model for evaluation of liquefaction susceptibility has been developed with $\sqrt{q_{cx}}$ and $\ln(CSR_{M=7.5})$ as explanatory variables, derived from 9379 $q_c$ data of 26 CPT soundings performed in most districts of Adapazarı City after the 1999 earthquake as well as $a_{max}$ and $M_w$ values recorded for the earthquake. The developed logistic regression model is as follows:

$$P_L = \frac{1}{1 + \exp(-[10.4559 - 0.9934q_{cx} + 2.476\ln(CSR_{M=7.5})])}$$

- The goodness-of-fit for the proposed relationship has indicated a high MLRI, at $\rho^2=0.95$.
- Average liquefaction probability contours have been constructed for the 0-6 m depth interval for each district in the city.
- The probability contours for sub-layers of the depth of investigation (0-2 m, 2-4 m, and 4-6 m) have been established in order that the change in liquefaction probabilities by depth is reflected.
- It has been determined that the highest liquefaction probabilities are within the top 2m layers. This is due to the finding that the layers through this thickness have been deposited during the floods of the past 500 years.
- The probability of liquefaction has been obtained at 80-95% only for the districts of TI and YG for which the occurrence of liquefaction has been confirmed in situ. Thus, the obtained probabilities with the developed model gives matching results with the observed liquefaction cases.
For the districts of AK, CM, OR and SK where liquefaction has been triggered in rare cases, the probabilities have been obtained to be both below and above 50%. The remaining districts where the liquefaction has not occurred have the probability less than 50%.

Among the compared total of 27 districts of the city, the probability of liquefaction has risen above 50% for only 9 districts, which suggests that liquefaction occurred in about 30% of the city during the 1999 Earthquake.

The necessity for adopting a new liquefaction evaluation approach for Adapazari has been confirmed by comparing the developed model with another logistic regression model from literature. The developed model gives more accurate results than the previous logistic models. In addition, Adapazari Criteria have been used as liquefaction indicator for development of the model which has been proven to represent the properties of ground failure in the region studied.

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