Efficient Optimal Joint Channel Estimation and Data Detection for Massive MIMO Systems

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Abstract—In this paper, we propose an efficient optimal joint channel estimation and data detection algorithm for massive MIMO wireless systems. Our algorithm is optimal in terms of the generalized likelihood ratio test (GLRT). For massive MIMO systems, we show that the expected complexity of our algorithm grows polynomially in the channel coherence time. Simulation results demonstrate significant performance gains of our algorithm compared with suboptimal non-coherent detection algorithms. To the best of our knowledge, this is the first algorithm which efficiently achieves GLRT-optimal non-coherent detections for massive MIMO systems with general constellations.

I. INTRODUCTION

Massive MIMO wireless systems emerge as an important potential technology for next generation wireless communications. Massive MIMO systems aim to meet the growing demand for higher data rates and wider coverage, by using hundreds of antennas at base stations (BS). In [1], the author showed that, in a single-cell massive MIMO system, when the number of receive antennas \( N \to \infty \), we can eliminate the negative effects of fast fading and non-correlated noise. Massive MIMO systems can also greatly boost the energy efficiency of cellular wireless communications [2]. Besides larger capacity and higher energy efficiency, massive MIMO systems can boost the robustness both to unintended man-made interference and to intentional jamming. These advantages make massive MIMO a promising candidate for new 5G wireless communications technologies [3, 20].

In this paper, we consider a typical TDD (Time Division Duplexing) massive MIMO wireless systems. In TDD massive MIMO systems, user terminals equipped with single antennas transmit pilot sequences and information data to base stations in uplink transmissions. Exploiting channel reciprocity in TDD systems, the base stations use channel estimation from uplink transmissions for precoding in downlink data transmissions. One of the most prominent challenges in massive MIMO systems is timely acquiring the channel state information (CSI) for a large number of antenna pairs. In fact, unknown CSI is a bottleneck in achieving the full potentials of massive MIMO systems [3]. Especially in fast fading environments where the channels change rapidly, one would need to dedicate a significant portion of the coherence interval for pilot sequences, leaving few times slots for data transmissions. In multi-cell multi-user massive MIMO systems, due to the scarcity of resources for orthogonal pilot sequences, pilot sequences from neighboring cells will inevitably pollute channel estimations in the current cell, causing the issue of pilot contamination [3, 5]. With the need of obtaining good CSI, pilot contaminations fundamentally limit the achievable data rates in massive MIMO systems.

It is known that joint channel estimation and data detections can greatly alleviate the issue of pilot contamination, and enhance system performance for massive MIMO systems [3, 6-8, 21]. Maximum likelihood (ML) or GLRT-optimal joint channel estimation and data detection algorithms are especially attractive due to their optimality, when the channel statistics are known or unknown. However, existing efficient joint channel estimation and data detection algorithms for massive MIMO systems are suboptimal, and cannot achieve the optimal non-coherent detection performance. It is thus of great interest to design efficient optimal non-coherent data detection algorithms for massive MIMO. Moreover, it is important to obtain the performance limits of ML or GLRT-optimal non-coherent data detection such that they can be used as benchmarks for evaluating low-complexity suboptimal non-coherent data detection algorithms.

For special cases of conventional MIMO systems with few antennas, there exist a few efficient algorithms which can achieve optimal joint channel estimation and data detection. In [10, 12], the authors introduced sphere decoders to achieve the exact ML non-coherent signal detection for constant-modulus constellations (such as BPSK, QPSK). The sphere decoder demonstrates low computational complexity for high signal-to-noise ratio (SNR) and moderate system dimensions [13]. However, the sphere decoder has exponential complexity at low or constant SNR [14], and it only works for non-coherent single input multiple output (SIMO) systems.

Another line of works use the method of channel state space partition for ML or GLRT-optimal non-coherent data detection, attaining polynomial complexity in channel coherence time \( T \) [15, 17, 19]. For example, [17] achieves GLRT-optimal non-coherent detection for pulse-amplitude modulation (PAM) using auxiliary-angle approach of polynomial-time complexity. However, these works are only for single-input single-output systems where the channel coefficient is a single complex variable, or orthogonal space time block coded MIMO systems with a single receive antenna. These algorithms using channel state partition do not work efficiently for general MIMO systems with a few more receive antennas, not to mention massive MIMO systems with hundreds of receive antennas.

In [18], the authors proposed an efficient branch-estimate-and-bound algorithm for GLRT-optimal non-coherent data detection for conventional MIMO systems with general constellations. Although this algorithm from [18] was the only known efficient GLRT-optimal algorithm for general MIMO...
systems with multiple transmit and receive antennas and general constellations, this approach does not have a computational complexity polynomial in channel coherence time, and is only for conventional MIMO systems with few receive antennas.

In this paper, we propose a novel efficient GLRT-optimal joint channel estimation and data detection for massive MIMO systems with general constellations. We show that our algorithm has an expected computational complexity polynomial in the channel coherence time \( T \) for massive MIMO systems. In its essence, our approach is a branch-and-bound method on the residual energy of massive MIMO signals after projecting them onto certain subspaces. To the best of our knowledge, this framework is the first GLRT-optimal non-coherent signal detection algorithm for massive MIMO systems with low computational complexity and optimal performance. Moreover, our algorithm can provide benchmark performance against which we can evaluate suboptimal low-complexity joint channel estimation and data detection algorithms.

The rest of this paper is organized as follows. Section II presents the system model. In Section III, we introduce the new GLRT-optimal non-coherent data detection algorithm. The expected complexity of the algorithm is derived in Section IV. Section V demonstrates the empirical performance and the computational complexity of ML algorithm.

II. JOINT CHANNEL ESTIMATION AND SIGNAL DETECTION FOR MASSIVE MIMO

We consider a TDD massive MIMO wireless system with \( N \) receive antennas at the base station, and \( M \ll N \) user terminals each equipped with a single antenna. We assume a discrete-time block flat fading channel model where the channel coefficients are fixed for a coherence time \( T \). Across different fading blocks, the channel coefficients take independent values from unknown distributions. We model the uplink transmission of this system within one channel block by

\[
X = HS^* + W, \tag{1}
\]

where \( X \in \mathbb{C}^{N \times T} \) is the received signal at the BS, \( S^* \) is an \( M \times T \) matrix representing the transmitted signal, whose entries are independent and identically distributed (i.i.d.) symbols from a modulation constellation \( \Omega \) (\( \Omega \) can be of constant or non-constant modulus, such as 16-QAM), \( W \in \mathbb{C}^{N \times T} \) represents additive noises, and \( H \in \mathbb{C}^{N \times M} \) represents the unknown channel matrix. The elements of \( W \) are i.i.d. random variables following circularly symmetric complex Gaussian distribution \( \mathcal{CN}(0, \sigma^2_w) \). In each channel coherence block, we further assume that the channel coefficients are deterministic with no prior statistical information known about them [9, 10].

Since the channel coefficients take unknown deterministic values, we can formulate the GLRT-optimal joint channel estimation and data detection as a mixed optimization problem over \( H \) and \( S \):

\[
\min_{H, S^* \in \Omega^{M \times T}} \| X - HS^* \|^2, \tag{2}
\]

where \( \Omega^{M \times T} \) represents the signal lattice of dimension \( M \times T \). We remark that the GLRT-optimal detection is equivalent to ML detection for SIMO systems with constant-modulus modulations, and for MIMO systems with equal-energy signaling, when the channel coefficients are known to take i.i.d. circularly symmetric complex Gaussian values [10].

We note that the combinatorial optimization problem in (2) is a least squares problem in \( H \), while an integer least-squares problem in \( S^* \), since each element of \( S^* \) is chosen from a discrete constellation \( \Omega [11] \). Hereby, for any given \( S^* \), the channel matrix \( H \) that minimizes (2) is given by \( H = X(S^*)^\dagger \), where \((\cdot)^\dagger \) denotes the Moore-Penrose pseudoinverse of a matrix. Since \((S^*)^\dagger = S(S*S)^\dagger \), \( H = XS(S*S)^\dagger \). Substituting this into (2), we get

\[
\min_{H, S^*} \| X - HS^* \|^2 = \min_{S^* \in \Omega^{M \times T}} \| X(I - S(S*S)^\dagger S^*) \|^2
\]

\[
= \min_{S^*} \text{tr}(X(I - S(S^*S)^\dagger S^*)X^*)
\]

\[
= \text{tr}(XX^*) - \max_{S^* \in \Omega^{M \times T}} \text{tr}((S^*S)^\dagger S^*XXS), \tag{3}
\]

where \( \text{tr}(\cdot) \) is the trace of a matrix. To simplify the mathematical formulation, we define \( \Omega \) to be a new \( M \)-dimensional constellation, each element of which is an \( M \)-dimensional vector with its entries taking values from \( \Omega \). So the cardinality of \( \Omega \) is \( |\Omega|^M \). Then we can rewrite (3) as

\[
\text{tr}(XX^*) - \max_{S^* \in \Omega^{M \times T}} \text{tr}((S^*S)^\dagger S^*XXS), \tag{4}
\]

where we use \( \text{tr}(XX^*) = \text{tr}(XX^*) \). Now by choosing \( \rho_{\min} \) to be the minimum eigenvalue of \( XX^* \), the minimization problem in (4) can be equivalently represented by the following optimization problem,

\[
\text{tr}(XX^* - \rho_{\min}I) - \max_{S^* \in \Omega^{M \times T}} \text{tr}((S^*S)^\dagger S^*(XX^* - \rho_{\min}I)S), \tag{5}
\]

because \( \text{tr}((S^*S)^\dagger S^*(\rho_{\min}I)S) \) is a constant. Since \( A = XX^* - \rho_{\min}I \) is positive semidefinite, we can factorize \( A = R^*R^\dagger \) using Cholesky decomposition, where \( R^* \) is the lower triangular matrix of Cholesky decomposition. Finally, using the trace property for product of matrices, (5) can be transformed as follows:

\[
\text{tr}(R^*R) - \max_{S^* \in \Omega^{M \times T}} \text{tr}((S^*S)^\dagger R^*RS)
\]

\[
= \min_{S^* \in \Omega^{M \times T}} \text{tr}(R(I - S(S^*S)^\dagger S^*)R^\dagger)
\]

\[
= \min_{S^* \in \Omega^{M \times T}} ||R^* - S(S^*S)^\dagger S^*R^\dagger||^2. \tag{6}
\]

Thus our goal is to minimize (6), based on which our novel algorithm is built. We remark that this approach of transforming the GLRT-optimal detection to (6) is novel, very different from existing approaches for GLRT-optimal detection including the sphere decoder [11] which only works for SIMO wireless systems. We also note that, the channel estimate \( H = X(S^*)^\dagger \) can be used for downlink precoding after solving (6).

III. EFFICIENT GLRT-OPTIMAL JOINT CHANNEL ESTIMATION AND DATA DETECTION ALGORITHM

Finding the optimal solution to (6) is a formidable task, since it requires searching over all the \(|\Omega|^{MT} \) hypotheses in the signal space. The exhaustive search approach provides the optimal solution, however, its complexity grows exponentially in the channel coherence time. In the special case of SIMO systems, the sphere decoder efficiently solves GLRT-optimal
given in Algorithm 1. Algorithm 1 is GLRT-optimal:

\[ M_{S_1^*} = \| R^* - S (S^* S)^T S^* R^* \|^2. \]  

(7)

For a partial matrix \( S_{1:i}^* \), we define its metric by

\[ M_{S_{1:i}^*} = \| R_{1:i}^* - S_{1:i} (S_{1:i}^* S_{1:i})^T S_{1:i}^* R_{1:i}^* \|, \]

(8)

where \( 1 \leq i \leq T \), and \( R_{1:i}^* \) is the first \( i \) rows of \( R^* \). Thus solving (6) is equivalent to finding an \( S^* \) that minimizes \( M_{S_i} \) among all the possible matrix values for \( S^* \).

To develop our algorithm, we have the following lemma about the comparison between \( M_{S_{1:i}} \) and \( M_{S_i} \).

**Lemma III.1.** For every \( i \leq T \) and any matrix value for \( S^* \),

\[ M_{S_{1:i}^*} \leq M_{S_i^*}. \]

**Proof:** We observe that \( M_{S_{1:i}} \) is the residual energy after projecting the columns of \( R^* \) onto the subspace spanned by the columns of \( S \); and \( M_{S_{1:i}^*} \) is the residual energy after projecting the columns of \( R_{1:i}^* \) (the first \( i \) rows of \( R^* \)) onto the subspace spanned by the columns of \( S_{1:i} \) (\( S_{1:i} \) is the just the first \( i \) rows of \( S \)). Since orthogonal linear projections minimize the residual energy among all linear projections, we can show, at the first \( i \) indices, the residual energy \( M_{S_{1:i}^*} \) after applying orthogonal projections \( S_{1:i} (S_{1:i}^* S_{1:i})^T S_{1:i}^* \) to \( R_{1:i}^* \), will be no bigger than these indices’ residual energy (denoted by \( Q \)) after applying \( S(S^* S)^T S \) to \( R^* \). Moreover, for the orthogonal projection \( S(S^* S)^T S \) applied to \( R^* \), the total residual energy \( M_{S_i} \) over \( T \) indices is no smaller than the residual energy \( Q \) over the first \( i \) indices. Because \( M_{S_i} \geq Q \) and \( Q \geq M_{S_{1:i}^*} \), we have \( M_{S_{1:i}^*} \leq M_{S_i} \).

**Theorem III.2.** Algorithm 1 gives the optimal solution to (6).

This theorem is a result of Lemma III.1 and the branch-and-bound search over the signal space.

**A. Metric calculation and initial radius \( r \)**

To compute \( M_{S_{1:i}^*} \), we can have a constant computational complexity independent of \( T \), by recursive calculations over tree structure. The metric in (8) is equivalent to

\[ M_{S_{1:i}^*} = \text{tr}(R_{1:i}^* R_{1:i}^*) - \text{tr}((S_{1:i}^* S_{1:i})^T S_{1:i}^* R_{1:i}^* R_{1:i}). \]  

(9)

From (9), we can calculate the metric \( M_{S_{1:i}^*} \) efficiently. First, the term \( \text{tr}(R_{1:i}^* R_{1:i}^*) \) can be precomputed. Second, after defining a \( T \times M \) matrix \( A_i = R_{1:i} S_{1:i} \), we can update \( A_{i+1} \) sequentially as \( A_{i+1} = A_i + R_{1:i+1} S_{1:i+1} \). Similarly, we can define \( M \times M \) matrix \( B_i = S_{1:i} S_{1:i} \) and then sequentially update \( B_{i+1} = B_i + S_{1:i+1} S_{1:i+1} \). Furthermore, the complexity of calculating \( B_{i+1}^* \) is \( O(M^2) \) using matrix inversion lemma, where \( B_{i+1}^* = (B_i + S_{1:i+1} S_{1:i+1})^{-1} \). The complexity of all these recursive updates do not depend on \( T \) (noting that only \( i \) rows of \( A \) are nonzero).

For large \( N \), we can choose the radius \( r^2 = cN \), where \( c \) is any sufficiently small constant (please the next section for justifications). In fact, one can also use best-first tree search to find the optimal solution while avoiding picking an \( r \) beforehand.

**IV. EXPECTED COMPUTATIONAL COMPLEXITY**

The computational complexity of our tree search based algorithms is mainly determined by the number of visited nodes in each layer. By “visited nodes”, we mean the partial sequences \( S_{1:i} \) for which metric \( M_{S_{1:i}^*} \) is computed. The fewer the visited nodes, the lower computational complexity of our tree search algorithm has. In this section, we show that the expected number of visited nodes will grow linearly with \( T \) under a sufficiently large number of receive antennas. To analyze the expected number of visited nodes, we assume that the channel coefficients are i.i.d. complex Gaussian random.

**Algorithm 1:** ML channel estimation and signal detection algorithm.

input : radius \( r \), matrix \( R \), constellation \( \Xi \) and a \( 1 \times T \) index vector \( I \)

output: The transmitted signal \( S^* \)

1. Set \( i = 1 \), \( I(i) = 1 \) and set \( S^*_{1:i} = \Xi(I(i)) \).
2. (Computing the bounds) Compute the metric \( M_{S_{1:i}^*} \). If \( M_{S_{1:i}^*} > r^2 \), go to 3; else, go to 4;
3. (Backtracking) Find the smallest \( 1 \leq j \leq i \) such that \( I(j) \in \Xi \). If there exists such \( j \), set \( i = j \) and go to 5; else go to 6.
4. If \( i = T \), store current \( S^*_{1:i} \), update \( r^2 = M_{S_{1:i}^*} \) and go to 3; else set \( i = i + 1 \), \( I(i) = 1 \) and \( S^*_{1:i} = \Xi(I(i)) \), go to 2.
5. Set \( I(i) = I(i) + 1 \) and \( S^*_{1:i} = \Xi(I(i)) \). Go to 2. If any sequence \( S^* \) is ever found in Step 4, output the latest stored full-length sequence as the ML solution; otherwise, double \( r \) and go to 1.
variables following distribution $\mathcal{N}(0, 1)$. We also assume that the $M$ users send $M$ orthogonal pilot sequences between time indices $1$ and $M$.

**Theorem IV.1.** Let $M$ be fixed, and let $r^2 = cN$, where $c$ is any sufficiently small positive constant. Then for the tree search algorithm, the expected number of visited points at layer $i$ converges to $|\Xi| = |\Omega|^M$ for $i \geq (M + 1)$, as the number of receive antennas $N$ goes to infinity. The tree search algorithm only visits one tree node at each layer $i < (M + 1)$.

Due to space limitations, we give an outline of the proof of Theorem IV.1 (outline). We first prove that, the tree search algorithm only visits $|\Xi| = |\Omega|^M$ nodes per layer when $X^*X = E[X^*X]$, where the expectation is taken over the distribution of channel coefficients. Then we show that, when $N \to \infty$, $X^*X/N \to E[X^*X]/N$ in probability and that the expected number of visited nodes at layer $i$ ($(M + 1) \leq i \leq T$) approaches $|\Xi|$.

We first note that, the number of visited nodes at layer $i$ ($(M + 1) \leq i \leq T$) is equal to $|\Xi|$, if there is one and only one sequence $S_i^{\ast}(i-1)$ such that $|M_{\bar{S}_i^{\ast}(i-1)}| \leq r^2$. Let us consider the true transmitted sequence $S^\ast$. Then we have

$$E[X^*X] = E[(HS^* + W)^\ast(HS^* + W)] = SE[H^\ast H]S^\ast + E[W^\ast W] + SE[H^\ast W] + E[W^\ast H]S^\ast = NSS^* + N\sigma^2 w_1,$$

where the second equality is from $E[HH^\ast] = NI$ and $E[H^\ast W] = 0$.

Because $SS^\ast$ is of rank $M$ with $M < T$, from (10), the minimum eigenvalue of $E[X^*X]/N$ is $\sigma^2 w_1$, and we know that $|M_{\bar{S}_i^{\ast}(i-1)}| \leq r^2$. For the tree search algorithm (after scaling $A$ by a constant $N$), $A = E[X^*X]/N - \sigma^2 w_1 I = SS^\ast$. From the Cholesky decomposition, we know that $A = SS^\ast = R^\ast R$. This means that the columns of $R^\ast$ span the same subspace as the columns of $S$. Thus the metric $M_{S_i^{\ast} \ast} = 0$, because $\|R^\ast - S(S^\ast S)S^\ast R^\ast\|^2$ is precisely the residual energy after projecting the columns of $R^\ast$ onto the subspace spanned by the columns of $S$. Since $M_{S_i^{\ast} \ast} \leq M_{S_i^{\ast}}$, $M_{S_i^{\ast}} = 0$ for all $i$.

Let us instead consider any signal matrix $S$ such that $S = S$ and $S_i^{\ast M} = S_i^{\ast M}$ (namely $S$ shares the same pilot sequences as $S_i^{\ast}$). For such $S_i^{\ast}$, we can show that $\|R^\ast - S(S^\ast S)S^\ast R^\ast\|^2 > 0$, and that $M_{S_i^{\ast}} > 0$ for the first $i$ such that $S_i^{\ast} \neq S_i^{\ast}$. In fact, $M_{S_i^{\ast}}$ is smaller than

$$D = \min_{i > 0, S \neq S_{1:i}} \|S_{1:i}^{\ast} - S_{1:i}(S_{1:i}^{\ast} S_{1:i}) S_{1:i}^{\ast} S_{1:i}^{\ast}\|^2 > 0.$$
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