Spin networks and the cosmological constant in the AdS/CFT correspondence

Carlos Silva

Instituto Federal de Educação Ciência e Tecnologia do Ceará (IFCE),
Campus Tianguá - Av. Tabelião Luiz Nogueira de Lima, s/n - Santo Antônio, Tianguá - CE

(Dated: October 5, 2020)

We introduce a generalization of the recently proposed holographic duality between spin networks and superstrings and show that it can provide a solution to the cosmological constant problem.

One of the most intricate problems in modern physics consists of the fact that the observed value of the cosmological constant is much smaller than the value given by particle physics (about 122 orders of magnitude). Such an unconformity between theoretical results and observations has been described as the "largest discrepancy between theory and experiment in all of science" [1], and as the "worst theoretical prediction in the history of physics" [2]. Such a problem has consequences for different aspects of our universe, e.g., for the structure formation and consequently for the existence of life. In this case, an universe as ours, with conditions to support life, would be very improvable, and a fine-tuning mechanism for its initial conditions becomes necessary [3].

It is expected that a theory of quantum gravity must shed some light on the issue of the cosmological constant [4]. However, the road to a quantum description of the gravitational phenomena has been a tough challenge. In this sense, the main approaches for such purpose, superstring theory [5], and loop quantum gravity (LQG) [6], seem to address different aspects of reality.

In LQG the fundamental objects are spin networks described in the same unified quantum way through vibrations of strings moving within a spacetime which is smooth and continuous. Particularly, the gravitational interaction is described in terms of vibrations of closed strings. Other conflicts arise: string theory requires that spacetime must have 10 dimensions, and needs the existence of supersymmetry. LQG, on the other hand, is based on a brane-world spacetime.

In the equation above $\tilde{r}_H = H^{-1}$ corresponds to the physical radius of the universe horizon, for the situation where we have a flat FLRW metric induced on the brane:

$$T = (2\pi \tilde{r}_H)^{-1}.$$  \hfill (1)

In addition to the discrepancy between these theories, the lack of any phenomenological evidence for both of them has led to the incorrect prediction of the cosmological constant could be presented as a phenomenological criterion for deciding among candidates to a quantum gravity theory. However, no satisfactory solution to the cosmological constant problem has appeared in string theory or LQG so far.

On the other hand, recently a holographic duality has been established between string theory and LQG, where it has been demonstrated that $U(1)$ spin networks, as they appear in loop quantum cosmology (LQC), corresponds to the holographic duals of closed strings living in a ten-dimensional spacetime [7]. As the first application of this result, the long-standing problem of the Big Bang singularity in AdS/CFT was solved.

In the present work, we shall investigate how such results could be extended from the case of a single brane, approached in [8], for the case where we have a stack of $N$ coincident identical branes. The results found out will lead us to a matrix theory that describes a fuzzy sphere. Based on such results, we obtain a value for the cosmological constant that agrees with observations.

The work is organized as follows: in the next two sections, we shall review the results found out in [7] for the case of a single brane. After, we shall generalize such results to the case of $N$ branes, and show how a fine-tuning for the cosmological constant can be obtained in this scenario. Throughout the paper, we shall take $G = c = \hbar = k_B = 1$.

- Cosmology and the Big Bang singularity avoidance on a brane:

In the reference [7], the cosmological evolution of the braneworld spacetime was considered to be driven by a flux of Hawking radiation through the universe horizon, with temperature given by [8–10]:

$$T = (2\pi \tilde{r}_H)^{-1}.$$  \hfill (1)

In the equation above $\tilde{r}_H = H^{-1}$ corresponds to the physical radius of the universe horizon, for the situation where we have a flat FLRW metric induced on the brane:

$$ds_{\text{FLRW}}^2 = h_{ab} dx^a dx^b + \tilde{r}^2 d\Omega_3^2,$$  \hfill (2)

where $h_{ab} = \text{diag}(-1, a^2)$, $H = \dot{a}/a$, and $\tilde{r} = a(t)r$.

A perfect fluid energy-momentum tensor to the matter-energy content of the brane is chosen, as usual. It will provide us an amount of energy that crosses the universe horizon during a time $dt$ as [11]:

$$dQ = A\psi = A(\rho + p)\tilde{r}H \left(1 + \frac{\rho}{\sigma}\right) dt,$$  \hfill (3)
with \( A = 4\pi r_H^2 \), and \( \sigma \) corresponding to the brane tension.

From a global point of view, the universe horizon enters in the bulk. Consequently, one must take into account effects from extra dimensions to the calculation of its entropy. However, by considering the black hole holographic conjecture (BHHC) [12], if one considers only regimes above the AdS length scale, the extra-dimensional effects can be implemented as semiclassical corrections to the induced theory on the brane. In this case, in the reference [13], the entropy

\[
S = \sqrt{\frac{A^2 - A_I^2}{4}} ,
\]

with \( A_I = 4\sqrt{2}\pi l^2 \), was considered as that associated with the universe horizon. In the equation above, the parameter \( l \) carries information about the higher dimensional effects present in the induced theory.

Following these lines, the first law of thermodynamics, \( dQ = TdS \), and the temperature [14], will give us

\[
\dot{H} = 4\pi \frac{\sqrt{A^2 - A_I^2}}{A} \rho \left( 1 + \frac{\rho}{\sigma} \right) .
\]

By the use of the continuity equation, one obtains yet:

\[
\frac{8\pi}{3} \frac{d\rho}{dt} \left( 1 + \frac{\rho}{\sigma} \right) = \frac{A}{\sqrt{A^2 - A_I^2}} \frac{d(H^2)}{dt} .
\]

which, by integration, gives us the following Friedmann equation:

\[
H^2 = \frac{4\pi}{A_I} \cos(\Theta) ,
\]

In the expression above \( \Theta = \pm \left[ \frac{2A}{\sqrt{2\pi} l} \left( 1 + \frac{1}{2} \beta^2 \right) - \alpha \right] \), with \( \alpha \) as a phase constant.

The result in the Eq. (7) provide us with an effective density term in the form of a harmonic function of the classical density. Consequently, a scenario where a bounce arises in the place of the Big Bang singularity is obtained, in a very different way from that given by the usual braneworld cosmology [15].

In this point, we observe that if one expands the right-hand side of the equation (7) in a Taylor series, by discarding higher-order terms in \( A_I \) and \( 1/\sigma \) (the higher energy corrections), one obtains

\[
H^2 = A(\alpha) \rho^2 + B(\alpha) \rho + C(\alpha) ,
\]

with

\[
A(\alpha) = \frac{4\pi}{9} \left( \frac{3 \sin(\alpha)}{\sigma} - 2A_I \cos(\alpha) \right) ,
\]

\[
B(\alpha) = \frac{8\pi}{3} \sin(\alpha) ,
\]

\[
C(\alpha) = \frac{4\pi}{A_I} \cos(\alpha) .
\]

The result expressed in the Eq. (9) can be written as

\[
H^2 = \frac{8\pi}{3} \rho_{tot} \left( 1 - \frac{\rho_{tot}}{\rho_c} \right) ,
\]

with \( \rho_{tot} = \rho + \lambda \), where \( \lambda \) as the vacuum energy density, and

\[
\rho_c^{-1} = \frac{1}{6} \left( 2A_I \cos(\alpha) - \frac{3 \sin(\alpha)}{\sigma} \right) ,
\]

\[
1 - \frac{2\lambda}{\rho_c} = \sin(\alpha) ,
\]

\[
\cos(\alpha) = \frac{2A_I \lambda}{3} \left( 1 - \frac{\lambda}{\rho_c} \right) .
\]

Besides, the Raychaudhuri equation can be written as

\[
\dot{H} = -4\pi(\rho_{tot} + p_{tot}) \left( 1 - \frac{2\rho_{tot}}{\rho_c} \right) ,
\]

with \( p_{tot} = p - \lambda \).

We note that, by taking the limit where \( A_I \to 0 \) in the Eqs. (11), (12), and (13), by substituting the results in the Eqs. (10) and (14), one can get the usual braneworld cosmology Friedmann equations [13], (remembering that \( \Lambda < \rho_c \)).

On the another standpoint, if one takes \( l = \beta l_{AdS} \) in \( A_I \), with \( \beta \neq 0 \), we obtain

\[
\frac{1}{\rho_c} = \frac{1}{2\sigma} \left( 1 + \frac{\pi}{2\beta^2} \right)^{-1/2}
\]

\[
= \frac{1}{2\sigma} \left( 1 + \frac{3}{8\beta^2 \sigma} \right)^{-1/2} ;
\]

and

\[
\lambda = 2\sigma \left[ \left( 1 + \frac{\pi}{2\beta^2} \right)^{1/2} - 1 \right] .
\]

\[
= 2\sigma \left[ \left( 1 + \frac{9}{16\pi l_{AdS}^2 \sigma} \right)^{1/2} - 1 \right] .
\]

where we have used \( l_{AdS} = \sqrt{3/4\pi \sigma} \).

By discarding higher order terms in \( 1/\sigma \), one gets for the universe critical density
\[ \rho_c \approx 2\sigma , \]  
\[ \lambda \approx \frac{9}{16\pi^4\sigma} , \]  
where we shall leave it to fix the length scale \( l \) later.

- **Spins networks on a brane as the holographic duals of closed strings.**

The Eqs. (10) and (14) possess semiclassical corrections similar to that are present in LQC equations [14–16]. It will bring us consequences for the description of the microscopic degrees of freedom associated with the induced gravitational theory on the brane.

In fact, by using the techniques developed by Singh and Soni in [17], it is possible to obtain from the Raychaudhuri equation (14), the following Hamiltonian for gravity on the brane:

\[ H_{\text{grav}} = \frac{-3V}{32\pi\xi^2} \left( e^{i\sqrt{\Delta}} - e^{-i\sqrt{\Delta}} \right) , \]  
(19)

where \( p \) corresponds to the conjugate momentum to the volume \( V \).

Besides,

\[ \xi = (3/(32\pi\rho_c))^{1/2} ; \quad \Delta = 6\pi/\rho_c . \]  
(20)

In the equation above, \( \xi \) and \( \Delta \) have dimensions of length squared, and \( \rho_c \) is a constant-energy density to be determined by the underlying theory [17]. From the results of the last section, one can write

\[ \Delta = \frac{3\pi}{\sigma} . \]  
(21)

The key detail here is that the Hamiltonian (19) is not defined in terms of the conjugate momentum \( p \), but in terms of its complex exponentials. It is because such exponentials correspond to holonomies, the basic components of spin networks in LQG. The appearance of such holonomies will not occur if one considers the case of the usual braneworld cosmology [17]. However, in the present scenario, it tells us that the microscopic description of the braneworld spacetime can be naturally provided by polymer quantization, the usual quantization method in the braneworld spacetime can be provided by the polymer structure, defined by a graph.

\[ \gamma_{\sqrt{\Delta}} = \{ x \in \mathbb{R} \mid x = n\sqrt{\Delta}, \forall n \in \mathbb{Z} \} , \]  
(24)

which possess the form of a regular lattice. Such a polymer structure, which is similar to LQC spin networks [19], provides a discreteness in the position \( x \), where the discreteness parameter is given by \( \sqrt{\Delta} \). As we can observe from the Eq. (21) such a discreteness parameter will be defined by the brane tension.

As an important result, superselection rules for the braneworld gravitational sector will be imposed, in such a way the universe will evolve through discrete increments of the scale factor \( a \) (or some object defined as a function of it, such as an area or volume). Such superselection rules will also affect the bulk physics.

To see this, one remembers that the brane couples gravitationally to the bulk by emitting or absorbing closed strings, whose couplings, \( g_s \), can be related with the brane tension as \( g_s \sim 1/\sigma [1] \). Consequently, one finds out, from the Eq. (21)

\[ g_s \sim \Delta . \]  
(25)

In this way, the discrete evolution of the braneworld spacetime will define the string spectrum in the bulk.

The quantum gravity effects will drive the bounce to occur before a regime below the AdS length scale be reached by the universe. In this way, scenarios, where the BHHC should not be valid, will be avoided. Actually, from the Eq. (21), the discreteness parameter \( \sqrt{\Delta} \), corresponding to the universe minimal size, is given by \( \sqrt{\Delta} \approx 8.89 \ l_{\text{AdS}} \).

In addition, we have that, from the bulk standpoint, the Big Bang singularity will be produced by the uncontrolled backreaction of the dilaton field, as it diverges due to the vanishing of the closed string coupling [20–22]. However, from the Eq. (25) we have that the discrete spacetime evolution on the brane will constraint \( g_s \) to have only nonvanishing finite values. In this way, the string modes that could lead to the dilaton divergency and, as a consequence, to the Big Bang singularity, will be cut out.
Another interesting point is that the polymer structures found out in the present section, that will correspond to the quantum gravity degrees of freedom of a brane, differ from the LQC ones only by the fact they are defined by the brane tension and not by the Barbero-Immirzi parameter. However, as it was pointed out in [3], such a result can shed some light on a longstanding problem in LQG, the so-called Immirzi ambiguity [23], since the brane tension can be dynamically determined [3]. In this way, such results match the idea proposed by several authors that a possible solution to the Immirzi ambiguity could be found out in a dynamical determination of the Barbero-Immirzi parameter [24][25].

- The cosmological constant for the case of a N coincident branes universe.

Now we shall generalize the results found out in the last section to the case where the matter-energy content of the universe is sourced by a stack of N identical and coincident branes.

In such a situation, the brane tension will be transformed from the single brane case as:

\[ \sigma \rightarrow N \sigma. \] (26)

Consequently, using the Eq. (18), we obtain the following expression for the universe vacuum energy density:

\[ \lambda_{\text{renorm}} = \frac{9}{16 \pi^2 l^4 N} = \frac{\lambda}{N}. \] (27)

The result above tells us that the value of the vacuum energy density will depend on the number of branes in the stack. Since, in the AdS/CFT context, such a number must be large, a small vacuum energy density must be obtained in the present scenario.

In order to investigate how much small such energy density would be, we shall investigate the microscopic description of the stack of branes. In this sense, in order to work in a situation where the universe quantum state is not fixed a priori, we shall consider that the branes picked up to build the stack are in unknown quantum spin network states. However, in this situation, we must observe that, since the branes will fill the same spacetime region, i.e., they will be coincident, the spin networks describing them must be in the same global quantum state. The only situation quantum mechanics will allow constructing such a set of unknown quantum states, is to connect them through quantum entanglement.

We observe yet that, even though the branes have been considered to be identical at the classical level, it is not necessary that such a coincidence will occur from the quantum point of view. Actually, it is enough that the entanglement among the branes quantum states be chosen in a way that their classical outputs be identical.

In order to construct such an entangled quantum state, we can take advantage of holonomies introduced in the last section which, as has already been demonstrated, can be used as mediators of quantum entanglement [27]. However, to use such objects to entangle different branes in the stack, it is necessary to observe that there will be several possibilities to weave the entanglement network between them. Consequently, the holonomies we shall use to establish the entanglements must carry some information about the branes they are connecting.

To add such kind of information, we shall equip the holonomies with a pair of indices \((ij)\), where \(i\) will be related with the source brane, and \(j\) with the target brane, i.e:

\[ h \rightarrow h_{ij}. \] (28)

In this way, the quantization of the geometry of a stack of branes will be performed by promoting the holonomies introduced in the last section to \(N \times N\) unitary matrices, where the diagonal elements will carry information about the branes themselves, and the elements out of the diagonal will carry information about the connections among the branes. In this way, the most general quantum state representing a stack of \(N\) branes will be given by a \(U(N)\) matrix theory.

From the geometrical point of view, such a matrix theory describes a fuzzy sphere. In this context, the \(U(N)\) group corresponds to the diffeomorphism group on such manifold [28], with \(N\) given by the number of area quanta on the fuzzy sphere.

By considering such facts, now we shall take \(N\) as the number of area quanta encoded on the universe boundary

\[ N = \frac{A_{UH}}{l^2_p}. \] (29)

In the expression above, \(A_{UH}\) corresponds to the area of the observable universe horizon, and \(l^2_p\) is the Planck area.

Now, by taking into account the result for the renoramlized vacuum energy density found out in the equation [27], we can obtain

\[ \lambda_{\text{renorm}} = 1.024029207 \times 10^{-124} \lambda, \] (30)

where we have considered the observable universe radius (the particle horizon radius) as \(14.6 \text{Gpc}\).

The result above depends on the choice of the length scale \(l\) in the Eq. (15). Since \(l\) carries information about extra dimensions \(l\), the most natural choice for it is \(l = l_{(10)}\), where

\[ l_{(10)} = \frac{1}{(2 \pi^3 \sigma)^{\frac{1}{2}}}. \] (31)

corresponds to the 10-dimensional Planck length [2].

In this case, we obtain

\[ \lambda = 11.10330495 \rho_p, \] (32)
where $\rho_P$ is the Planck energy density.

In this way, for the renormalized vacuum energy density, one gets

$$\lambda_{\text{renorm}} = 1.137010857 \times 10^{-123} \rho_P.$$  \hfill (33)

Moreover, for the cosmological constant, we shall have, for $\rho_P = 1$

$$\Lambda = 8\pi\lambda = 2.857619964 \times 10^{-122},$$  \hfill (34)

which coincides with the value of the cosmological constant value given by observations [29].

- **Remarks and conclusions**

In the present work, we have shown that a solution to the cosmological constant problem can be obtained from generalization of the string/loop duality proposed in [3]. To do this, we have considered that the matter-energy content of our universe is sourced by a stack of $N$ coincident identical branes.

In this situation, the theory that will describe the quantum state of the universe corresponds to a $U(N)$ matrix theory, which from the geometrical point of view, corresponds to a fuzzy sphere. In such a context, the number $N$ of branes in the stack will be constrained by the number of quanta of area on the universe horizon. Based on such results, the correct value for the cosmological constant, as given by observations, can be found out.

It is interesting to note that an $U(N)$ matrix theory can be used to build a generalized spin network state as those appear in full LQG [30]. In this way, the discussions introduced in the present work can pave the way for the generalization of the string/loop duality found out in [7], in a way that close string states in the bulk will appear as duals of full spin network states that describes the braneworld geometry [31].

The present results give us the first observational evidence for the string/loop duality proposed in [7] and show that such a duality has the potential not only to unify the results already obtained by string theory and LQG, but also to solve problems that none of them have managed to solve so far.

[1] R. J. Adler, B. Casey and O. C. Jacob, Am. J. Phys. 63, 620-626 (1995) doi:10.1119/1.17850
[2] M. P. Hobson, G. P. Efstathiou and A. N. Lasenby, “General relativity: An introduction for physicists,” Cambridge, UK: Univ. Pr. (2006) 572 p
[3] S. Weinberg, Phys. Rev. Lett. 59, 2607 (1987).
[4] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[5] K. Becker, M. Becker and J. H. Schwarz, New York, Cambridge University Press (2007).
[6] C. Rovelli, “Quantum gravity,” doi:10.1016/B978-044451560-5/50015-4
[7] C. Silva, Phys. Rev. D 102, no.4, 046001 (2020) doi:10.1103/PhysRevD.102.046001 [arXiv:2008.07270 [gr-qc]].
[8] X. H. Ge, Phys. Lett. B 651, 49 (2007).
[9] Y. Gong and A. Wang, Phys. Rev. Lett. 99 211301 (2007).
[10] R. G. Cai, L. M. Cao and Y. P. Hu, Class. Quant. Grav. 26 155018 (2009).
[11] R. G. Cai and S. P. Kim, JHEP 0502 050 (2005).
[12] R. Emparan, A. Fabbri and N. Kaloper, JHEP 0208 043 (2002).
[13] R. Maartens, Living Rev. Rel. 7 7 (2004).
[14] V. Taveras, Phys. Rev. D 78 064072 (2008).
[15] M. Bojowald, “Absence of singularity in loop quantum cosmology,” Phys. Rev. Lett. 86 5227 (2001).
[16] A. Ashtekar, T. Pawlowski and P. Singh, “Quantum nature of the big bang,” Phys. Rev. Lett. 96 141301 (2006).
[17] P. Singh and S. K. Soni, Class. Quant. Grav. 33 no.12, 125001 (2016).
[18] A. Corichi, T. Vukasinac and J. A. Zapata, Phys. Rev. D 76, 044016 (2007)
[19] J. Mielczarek, Springer Proc. Phys. 157, 555 (2014)
[20] Due the S-duality, it is equivalent to the situation where the string coupling goes to infinity.
[21] N. Engelhardt and G. T. Horowitz, Phys. Rev. D 93 (2016) no.2, 026005
[22] D. Bak, “Dual of big-bang and big-crunch,” Phys. Rev. D 75 (2007), 026003
[23] C. Rovelli and T. Thiemann, Class. Quant. Grav. D 57, 1009 (1998)
[24] T. Jacobson, Class. Quant. Grav. 24 4875 (2007).
[25] V. Taveras and N. Yunes, Phys. Rev. D 78 064070 (2008).
[26] S. Mercuri, Phys. Rev. Lett. 103 081302 (2009).
[27] J. Mielczarek, Universe 5, no.8, 179 (2019) [arXiv:1810.07100 [gr-qc]].
[28] W. Taylor, NATO Sci. Ser. C 556, 91-178 (2000) [arXiv:hep-th/0002016 [hep-th]].
[29] N. Aghanim et al. [Planck Collaboration], “Planck 2018 results. VI. Cosmological parameters,” [arXiv:1807.06200 [astro-ph.CO]]
[30] F. Girelli and E. R. Livine, Class. Quant. Grav. 22, 3295-3314 (2005) doi:10.1088/0264-9381/22/16/011 [arXiv:gr-qc/0501075 [gr-qc]].
[31] Carlos Silva, in preparation.