Managing Large-Scale Projects in a Mixed Economy

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Received September 2, 2021; revised November 12, 2021; accepted January 26, 2022

Abstract—The work continues the research by the present authors on the management of industrial and infrastructure systems in accordance with the global trend in the digitalization of the economy. A description of the initial fundamental foundations of the ongoing developments and a review of domestic experience in the use of mathematical models, information and communication technologies, and large amounts of information in the management of systems are given. The issues of centralization and decentralization of control in complex systems are considered. Theoretical constructions of decision making are given to analyze the prospects for the development of partnership between the state and business within the framework of given legal norms. A block of conceptual models corresponding to the level of planning of large-scale organizational systems is given, issues of data preparation and development of algorithmic support, and a combination of macro- and micro-descriptions of economic systems are considered.

Keywords: systems analysis, decision-making, game theory, directive management, economic mechanism, federal law, conceptual model, coordination of interests of state and business, information support

DOI: 10.1134/S0005117922050083

1. INTRODUCTION

The study is aimed at developing theoretical and methodological problems of instrumental support for decision-making in socio-economic systems that combine directive management and market mechanisms, as well as centralization and decentralization.

The main postulate of the work is that systems research should be based on mathematical modeling. An inspiring example for the authors was the experience [1] on the discovery and calculation of the “nuclear winter” effect based on the construction of models of the movement of air masses over the earth and water surfaces. Calculations showed that the destruction of the capitals of the northern hemisphere by nuclear explosions would plunge the hemisphere into subzero temperatures for 25–50 years, and life on this territory would be destroyed. This computational-analytical result stopped the world nuclear arms race and prevented such a war from starting, becoming a global deterrence imperative.

Currently, the main trend in economic development is determined by the digitization of technologies and management systems. These conditions predetermine a special approach to the strategic planning of large-scale systems.
Large-scale systems are a class of complex (large) systems characterized by an integrated (inter-sectoral and inter-regional) interaction of elements that are distributed over a large area and require considerable resources and time for development [2].

The present authors adhere to the definitions and concepts proposed, in particular, by them in their original publications [1–5] and included in the general lexicon of design and control problems.

In general, large-scale complexes have the following properties:

- dynamism—systems are constantly evolving, raw materials are coming in, knowledge is growing, technological operations are being carried out, fixed assets are being transformed, depreciation is taking place, traffic flows are not interrupted, etc.;
- the presence of uncertain and uncontrollable factors—many parameters of the systems of large-scale complexes are uncertain or uncontrollable; this necessitates taking into account risks. On the one hand, this is due to the uncertainty in the development of technologies, and on the other hand, due to taking into account the peculiarities of the behavior of various system components, which sometimes have their own subgoals and tasks, different from the goals of the system;
- multicriteriality—in addition to the task of fulfilling the given restrictions, a complex economic system usually also has a whole set of formally and informally set goals, as well as clearly or vaguely formulated operation and development criteria;
- hierarchy—a complex system of large-scale complexes structurally consists of a set of hierarchically subordinate systems according to territorial and plant-industry characteristics.

Plants belonging to each structural level can be considered both as systems formed from subsystems (plants of lower levels) and as subsystems that are part of a certain system (a plant of a higher level).

Hierarchical systems have three important properties.

1. Each level of the hierarchy has its own language and its own system of concepts or principles.
2. The properties of plants of lower levels are generalized at each level of the hierarchy. The patterns discovered and described for the lower levels can be included in an explanatory (functional) scheme, while gaining a connection with a higher-level plant. Thus, the description at level \( i \) contributes to the explanation (understanding) of the phenomena taking place at level \( i + 1 \).
3. Relationships between levels are asymmetrical. For higher-level plants to function properly, lower-level plants must “work” successfully, but not vice versa.

In economic systems, the upper levels have a priority in actions, but do not have complete information about the lower levels.

The hierarchical structure of large-scale complexes is characteristic of the most diverse systems—commercial enterprises, computer software complexes, social organization, electronic equipment, etc.

The development of various procedures for coordinating interests and interactions in hierarchical systems has been carried out since the 1960s within the framework of sections of the theory of management of socio-economic systems such as the theory of active systems, the information theory of hierarchical systems, the theory of nonzero-sum games, the theory of contracts, the collective choice theory, the team theory, etc.
In particular, in the information theory of hierarchical systems created in the works by the Russian school [6], the main attention is paid to two most important attributes of the hierarchy—priority in the actions of the Principal and awareness of the Principal and lower levels of the hierarchy.

The impressive progress in the development of control systems in the technical field was generated by significant successes in the natural sciences, which developed a methodology of knowledge based on the idea of modeling. A technology was created for constructing models of control processes, analyzing them, and developing control strategies and procedures for adapting the result to practical needs on this basis.

Similar technologies have been developed in economic process control systems. And just as in technical systems, the development of ideas and success was based on the development of models of controlled processes and methods for their study, starting from the simplest economic tables of the physiocrats (F. Quesnay, 1758) and “invisible hand” models that establish the balance of supply and demand in a spontaneous economy (A. Smith) to intersectoral balance models (V. Leontief), state regulation models (J.M. Keynes), technocratic justifications (J.K. Galbraith) and market models with perfect competition (K. Arrow), based on theorems on the existence of fixed points of point-set mappings.

A great advance in the modeling of economic processes was the Kantorovich–Koopmans theory of linear production processes, which is ideally and completely incorporated into digitalization processes, since it is based on normative digital information.

According to this opinion, decision-making in the economy will be based on computing platforms that reflect individual functional industries, which is close to the ideas of A.I. Kitov and V.M. Glushkov on the creation of a nationwide automated economic management system [7].

The main thesis declared in this article is that the models of interaction of active elements are the initial framework of the platforms, and it is necessary to involve the theoretical constructions of control mechanisms in solving the problems of making coordinated decisions.

General control theory currently has powerful formal tools, including modeling tools, mathematical apparatus, computational tools, as well as its own methodology, the components of which are decision theory, operations research, control of dynamic systems, game theory, systems analysis. This methodology will be in demand and will form the main content of digital platforms.

The digital model of an enterprise describes the development of production, the dynamics of material (products, production assets), and financial flows (investments, assets, liabilities, equity), contains input and output variables, and takes into account technological connections, the management system, and economic goals of production [8].

2. EXPERIENCE IN DEVELOPING LARGE-SCALE SYSTEMS

As in the previous publications by the present authors, the main interest is shown in the article in large-scale projects [2, 9]: industry systems, holdings, concerns, financial and industrial groups, transnational corporations, distributed systems for transmitting and processing information, and other complexes.

The presented models and methods for planning the development of the structure of large-scale systems were born thanks to research conducted at Trapeznikov Institute of Control Sciences of the
Russian Academy of Sciences in the field of synthesis of the structures of fuel and energy, as well as industry and regional production and transport systems [3].

The main features of large-scale systems include

- Considerable expenditure of resources and time for the development of systems—the lead time of investment measures can be several years.
- Blurring of boundaries (in the process of development, the composition of the elements of the system and the nature of their relationship with each other and with the external environment change considerably; the territory covered by the system can expand from regional to global scale).
- Close relationship with other large-scale systems and with the environment.
- Comprehensive nature of control (in particular, the coordination of industry, corporate, and regional interests is required).
- Robustness and stability, small deviations in the parameters of development of individual elements and their interrelations have little effect on the development of systems as a whole.

There is experience [4] in the development of software systems for financial analysis and the development of business plans for investment projects. These systems are used by industrial enterprises, banks, design institutions, and consulting centers.

Due to the growth in the size and complexity of production processes and the processes of their management, the complication of the structure of national economic systems puts forward a number of problems related to the scientific substantiation of the construction of such structures.

The structure of a large-scale system is understood in the paper as the composition of its elements with the corresponding relationships in the dynamics of their development and operation.

The continuity of development, expressed in the constant change in the structure of the system, is the most important feature of large-scale systems. The process of development of large-scale systems is irreversible, i.e., as a rule, the integral characteristics that determine the “product” at the “output” of the system do not decrease during the development process as a whole, although the nature of the change in individual elements of the system may be different. At the same time, in the process of development, the composition of the elements of a large-scale system expands and their interrelations become more complicated.

When managing the development of a large-scale system, it is necessary to obtain an interconnected solution of two groups of issues. The first includes the issues of developing the structure of a managed production and transport system, i.e., determining the optimal composition of elements and their relationships, distributing plan targets by elements, etc. The second incorporates the issues of development of the structure of the management system, including the choice of the hierarchy of management and the distribution of management functions performed between the levels and nodes of the system.

Long-term programs for the development of structures of large-scale systems are being developed in many sectors of the national economy. Examples of such systems are the fuel and energy complex as a whole and its individual branches, a complex of nuclear power plants, the national gas supply network, etc.

The main content of the approach proposed in [10] is the methods of planning the developing large-scale systems based on the interaction, in the process of forming a plan for the development of
large-scale systems, of a set of models that describe the system at various levels of detail, including at
the level of aggregated indicators and at the level of individual production and transport elements.

The scheme of formation and use of a complex of investment models is shown in Fig. 1. Calcula-
tions for optimizing programs for the development of large-scale systems begin with the formation
of the goal of the program and possible ways and means to achieve it. As a result, the possible
composition of the program is determined (a list of plants of various types, logical and technological
connections between them, options for their construction, etc.).

At the next stage, the options for the development of individual elements and complexes are
subjected to technological and feasibility study. After the preparation of the necessary information
and the formation of optimization models, the corresponding calculations of development programs
are carried out.

The next stage is to determine the technical and economic characteristics of the obtained version
of the programs used in the analysis and making appropriate decisions.

An analysis of the specifics of planning, the experience of communicating with persons involved
in the development of planning decisions at various levels, showed a wide variety of optimization
problem formulations—from the formation of one global economic criterion to problems with vector
and local optimization criteria.

Therefore, it seems appropriate to take an approach to solving such problems such that the main
attention is paid to the development of models that more closely model the plant in the area of
interest to the user, the creation, if possible, of a complete list (library) of criteria, constraints,
functional relationships, conditions, and other elements of the description of the class of tasks in
question.

This allows the specialist who prepares a particular solution to choose the set of criteria and
constraints that is necessary in the current situation from the list of available ones. Thus, the
questions of forming the structure of the model, completing the model with optimization criteria,
their meaningful interpretation, and correcting the model and initial data become closer to the
specialist who prepares and makes planned decisions.

The use of the results of model experiments in the preparation of management decisions does not
exclude the possibility, and in some cases the need to use other, nonformalized means of evaluating
the relevant aspects of decisions. In some cases, modeling plays a leading role, while in others it
plays an auxiliary role as a tool for quantifying some indicators of the system under study.

As described in the publications [2, 9] by the present authors, when developing large-scale sys-
tems, the problem arises of an interconnected description, analysis, and synthesis of various aspects
of the systems activity—the processes of selecting goals and making decisions, processing informa-
tion, and technological processes.

Usually, for large-scale systems, it is impossible to describe their structural properties and fea-
tures at one level of detail, so such systems should be represented as an interconnected set of
elements of different levels of detail and development stages: the level of goals for the operation of
subsystems and the system as a whole; functions and tasks of the elements of the system; links in
the organizational hierarchy; production and transport facilities; technical means, etc. Moreover,
various means and languages can be used at the indicated stages and levels of describing the struc-
tures of large-scale systems—verbal descriptions, set theory, analytical and graph-analytical as well
as algorithmic procedures, alternative graph formalization, etc.
The principle of sequential synthesis of models of acceptable options for constructing individual elements, parts, and the system as a whole is efficient when synthesizing the structure of complex systems, followed by the choice of the best variant of its implementation and development using the synthesized model of the system structure. This approach allows one to single out typical tasks for synthesizing the structure of complex systems, the detailing of which is determined by the stage and goals of development.
The functional part of large-scale systems is formulated in the form of alternative graphs at the levels of the goals of operation and the functions and functional tasks performed. The organizational part of the system is formulated in a similar way at the levels of control nodes (organizational hierarchy), regions, enterprises, individual plants, production and technological processes, units, as well as networks of computers and computer centers.

On the basis of large-scale system models, it is possible to develop scenarios for the concentration of efforts and resources within the framework of programs for the development and restructuring of systems. It is noted in [11] that “in one of these programs, it is possible to integrate mining companies and enterprises of the military-industrial complex (MIC) into financial-industrial groups in order to gain (with the support of the state) a strong place in the world markets, as well as to rationally use the income received for restoration of the scientific and technical potential of the country, technological renewal of the extractive industries and especially the spread of high technologies from the military-industrial complex to all sectors of the national economy. It is necessary to take into account the possible risks, because in the late 1980s, the interests of the military-industrial complex and the fuel and energy complex were opposite, and the reforms are taking place under the sign of confrontation between the military-industrial complex and the fuel and energy complex.”

3. STRATEGIC PLANNING IN DECENTRALIZED SYSTEMS

The problem of combining centralization and decentralization, in particular, in the analysis of public-private partnerships, is one of the dominant issues in decision theory and has attracted the attention of many researchers [12].

The fundamental studies of large-scale systems in which interactions are hierarchical include the works by Yu.B. Germeier and his students. These works are reviewed in the monograph [13] and in the papers [14–17].

Experience shows that in practice the management of fairly complex organizational systems is carried out according to a hierarchical decentralized principle. An explanation of the effectiveness of the principle was proposed by Yu.B. Germeier and N.N. Moiseev in [6]: if the decision maker delegates some of his decision-making powers to some agents, then by joint efforts it will be possible to process large amounts of information in a timely manner and thereby make management more efficient. It was possible to construct formal mathematical models that allow describing this effect in [18], where the problem of managing an organizational system under conditions of exogenous uncertainty is considered.

In [19, 20], the question of the advisability of decentralizing control depending on the available amount of information about uncertain factors is investigated, a comparative analysis is carried out for centralized and decentralized methods of control, and a basic theoretical statement is established. The following Theorem is proved:

*if the interests of the Principal and controlled systems are poorly coordinated, then the Principal always benefits from centralized management; if the interests are well coordinated, then with limited information processing capacity, the more beneficial for the Principal is decentralization and otherwise, centralization.*

The following conclusion follows from the consideration of substantive economic processes: a market economy is a decentralized system for managing the interaction of economic agents in society.
and it cannot solve all the problems of the socio-economic development of society—rational government regulation is needed.

There is a shift among the analysts in the West in understanding the role of centralized control. As noted in [21]: “The situation must change, because it is difficult to imagine how an exogenous shock of such magnitude as that caused by COVID-19 can be handled with purely market solutions... The coronavirus has already and almost in the blink of an eye succeeded in shifting perceptions of the complex and delicate balance between the private and public spheres in favor of the latter... Everything that happens in the post-pandemic era will force us to rethink the role of governments... They must ensure that partnerships with business and participation of public funds are driven by the public interest and not by profit.”

Centralization and decentralization is manifested in various forms of public-private partnership. In [22], various organizations of economic partnership between the state and business are proposed using the tools and methods of decision theory, control theory, and operations research.

In [23], it is proposed in these formulations that, along with enterprises of various forms of ownership, the participation of a coordinator is also considered (below we will use the term “Principal” to describe the activity of the coordinator). The subject of its regulation is social relations that arise between public authorities and subjects of the industrial sector of the economy when using various tools of state influence on the activities of companies. Both economic incentive measures and state regulation measures (mandatory prescriptions and prohibitions) can be used as tools of state influence on the subjects of industrial activity.

Consider the issue of the activity of a set of enterprises [24] included in the scope of a complex of large-scale systems and an aggregate description of enterprise models (digital twins) included in the digital platform of the complex with a fan-shaped technological graph. First of all, we note that the model reflects a hierarchical management structure.

We confine ourselves to issues of an organizational and technological nature—we will describe a model for assessing the possibility of fulfilling a plan by a group of enterprises for the production of final products with given financial resources. Models of this type (“robust” models) make it possible to evaluate the qualitative aspects of decision-making. First of all, they are effectively used in the development of scenarios for computational experiments for simulation systems based on digital platforms with a large dimensionality of model complexes.

It is noted in [23] that models of contracts between active economic systems have been studied within the framework of these provisions and structures. The 2016 Nobel Prize in Economics was awarded to Oliver Hart (Harvard University, USA) and Bengt Holmström (MIT, USA) for their contribution to contract theory, which is based on hierarchical Principal–Agents interaction models. The applied meaning of such research is, on the one hand, the creation of a mathematical apparatus for the analysis of simplified models that allow one to draw qualitative conclusions and, on the other hand, at the model level the formation of ideas about the subject of research among decision makers at all organizational levels. The authors’ analysis of optimal contracts lays the intellectual foundation for the development of policies and institutions in many areas, from bankruptcy law to political constitutions.

3.1. Decentralized System under the Programs of the Principal

Let us consider here one of the variants of description based on the domestic theory of hierarchical games [19, 20]. We assume that the organizational system consists of three levels: the Principal—
integrated structures (holdings)—enterprises. The operating side in this model will be identified with the Principal.

Suppose that we have some number of enterprises divided into \( K \) integrated structures. The enterprises will be labeled with double indices \( kn \), where \( k (k = 1, \ldots, K) \) is the number of the integrated structure and \( n (n = 0, 1, \ldots, N) \) is the number of the enterprise. Here, in order not to use unnecessary indices, we assume that the number of enterprises in each integrated structure is the same. If this is not the case, then we can assume that \( N + 1 \) is the maximum number of enterprises in the integrated structure, and in those structures in which there are fewer enterprises, fictitious enterprises with very high production costs are added (see below).

Each integrated structure will be identified with its head enterprise, for which we will reserve the index \( n = 0 \).

1. **Technological processes.** We will consider the operation of the system on the time interval \( t = 1, 2, \ldots, T \).

   Let the system be capable of producing \( I \) types of items. Denote \( U = R_I^I \).

   The Principal selects the \emph{volumes and terms} of the order of releases, i.e., for each integrated structure \( k \) it defines a set \( (u^k_1, \ldots, u^k_T) \) of vectors \( u^k_t \in U, \ t = 1, 2, \ldots, T \).

   In addition, the Principal sets \emph{product prices}, i.e., for each time \( t \) it chooses a vector \( p_t \in U \).

   Thus, for example, structure \( k \) at time \( t \) will receive an amount equal to \( p_t u^k_t \) from the Principal for the execution of the order.

2. **Production capacity.** The state of production capacities of enterprise \( kn \) at time \( t \) is described by the vector \( x^{kn}_t \) with nonnegative components. The set of all such vectors will be denoted by \( X \).

   Again, for simplicity of the formulas, we assume that the dimensions of these vectors are the same for all \( t \) and all \( kn \), but they may have zero components.

   At time \( t \), enterprise \( kn \) chooses a vector \( z^{kn}_t \) of newly created capacities. Thus, the dynamics of production capacities is given by the formula

   \[
   x^{kn}_{t+1} = x^{kn}_t + z^{kn}_t, \quad t = 1, \ldots, T, \ k = 1, \ldots, K, \ n = 0, 1, \ldots, N.
   \]

   The initial value \( x^{kn}_0 \) is considered to be a \emph{parameter of the problem}.

   The company pays \( P_t z^{kn}_t \) for creating new capacities. Here the vector \( P_t \) with nonnegative components is a model parameter.

3. **Output and costs.** At time \( t \), enterprise \( kn \) chooses the output volume \( v^{kn}_t \in U \).

   Of course, production is associated with costs. We assume that this connection is described by the function

   \[
   \Phi = (\phi, \psi) : X \times U \rightarrow U \times W.
   \]

   Here \( \phi(x^{kn}_t, v^{kn}_t) \in U \) are the costs of produce manufactured within the military-industrial complex for the production of items in the amount of \( v^{kn}_t \) in the presence of capacities \( x^{kn}_t \) and \( \psi(x^{kn}_t, v^{kn}_t) \in W \) is the cost of purchasing items acquired from outside the military-industrial complex.
Here again, to simplify the formulas, we assume that each enterprise can produce a full range of items, but the costs of producing “noncore” goods (i.e., not corresponding to the available capacities) are taken substantial to exclude them from the calculation.

Assume that enterprises that are part of one integrated structure can exchange their products.

4. Exchanges. The volume \( \omega_{kn}^{k0} \in U \) of products supplied by enterprise \( n \) to enterprise \( m \) at time \( t \) is determined by the head enterprise of the integrated structure.

The controls of the head enterprise of the integrated structure \( k \) must satisfy the constraints

\[
v_t^{k0} + \sum_{n=1}^{N} \omega_t^{k0n} - \sum_{n=1}^{N} \omega_t^{k0n} - \sum_{n=1}^{N} \varphi(x_t^{kn}, v_t^{kn}) \geq u_t^{kn}, \quad t = 1, 2, \ldots, T.
\]

Let \( U_t^{k0}(x_t^{k0}, u_t^{k0}) \) denote the set of tuples (controls)

\[
(z_t^{k0}, v_t^{k0}, \omega_t^{k00}, \ldots, \omega_t^{k0N}, \omega_t^{k10}, \ldots, \omega_t^{kNN}) \in X \times U^{N+1}
\]

satisfying the constraint

\[
v_t^{k0} + \sum_{n=1}^{N} \omega_t^{k0n} - \sum_{n=1}^{N} \omega_t^{k0n} - \sum_{n=1}^{N} \varphi(x_t^{kn}, v_t^{kn}) \geq u_t^{kn} + \sum_{n=1}^{N} \varphi(x_t^{kn}, v_t^{kn})
\]

(here for simplicity of notation we take \( \omega_{kn}^{kn} = 0, n = 0, 1, \ldots, N \)).

The controls of enterprise \( n (n \neq 0) \) included in the integrated structure \( k \) must satisfy the constraints

\[
v_t^{kn} - \varphi(x_t^{kn}, v_t^{kn}) \geq \sum_{m=1}^{N} \omega_t^{kmn} - \sum_{m=1}^{N} \omega_t^{knm}, \quad t = 1, 2, \ldots, T.
\]

Let \( U_t^{kn}(x_t^{kn}, \omega_t^{k00}, \ldots, \omega_t^{kNN}, \omega_t^{k0n}, \ldots, \omega_t^{kNN}) \) be the set of pairs \((z_t^{kn}, v_t^{kn}) \in X \times U\) satisfying the condition

\[
v_t^{kn} - \varphi(x_t^{kn}, v_t^{kn}) \geq \sum_{m=1}^{N} \omega_t^{kmn} - \sum_{m=1}^{N} \omega_t^{knm}.
\]

5. Interests. Let us assume that the goal of enterprises is to maximize profits.

For brevity we denote \( z^{kn} = (z_1^{kn}, \ldots, z_T^{kn}), \pi^{kn} = (v_1^{kn}, \ldots, v_T^{kn}), \bar{p} = (p_1, \ldots, p_T) \).

For \( n (n \neq 0) \) the profit of enterprise \( kn \) is given by the formula

\[
g^{kn}(\bar{p}, z^{kn}, \pi^{kn}) = \sum_{t=1}^{T} p_t v_t^{kn} - \sum_{t=1}^{T} P_t z_t^{kn} - \sum_{t=1}^{T} p_t \varphi(x_t^{kn}, v_t^{kn}) - \sum_{t=1}^{T} \pi_t \phi(x_t^{kn}, v_t^{kn}).
\]

Here \( \pi_t \) is the vector of prices for goods purchased outside the military-industrial complex. This is a parameter of the model.

We assume that the head enterprise of the integrated system is a shareholder of the other enterprises in this system. Therefore, its profits are given by the formula

\[
g^{k0}(\bar{p}, z^{k0}, \pi^{k0}, \bar{p}^{k0}, \bar{p}^{k1}, \ldots, \pi^{kN}) = \sum_{t=1}^{T} p_t v_t^{k0} - \sum_{t=1}^{T} P_t z_t^{k0} - \sum_{t=1}^{T} p_t \varphi(x_t^{k0}, v_t^{k0}) - \sum_{t=1}^{T} \pi_t \phi(x_t^{k0}, v_t^{k0}) + \sum_{n=1}^{N} \alpha^{kn} g^{kn}(z^{kn}, v^{kn}).
\]
In this formula, \( \alpha^{kn} \) is the share of the head enterprise in the capital of enterprise \( kn \). In the model, these are parameters.

As stated above, this model assumes that any enterprise is in principle capable of producing any product in any quantity, albeit such a noncore output can be accompanied by very high costs. Therefore, within the framework of this model, it is natural to interpret the feasibility of the plan as the break-even of all enterprises. Thus, the Principal is assigned the payoff function

\[
 cg(p, z^1, z^2, \ldots, z^K, v^1, v^2, \ldots, v^K) = \min_{1 \leq k \leq N} \min \left[ g^{k0}(z^1, z^2, \ldots, z^K, v^1, v^2, \ldots, v^K), \min_{1 \leq n \leq N} g^{kn}(z^1, v^1, \ldots, v^K) \right].
\]

6. Awareness and sequence of moves. We will assume that all subjects of the system under consideration exactly know its parameters.

We assume that the Principal is the first to make a decision. It selects controls \( \bar{u}^k = (u_1^k, \ldots, u_T^k) \), \( k = 1, \ldots, K \), \( p = (p_1, \ldots, p_T) \), and informs the enterprises about the choice made.

Then the head enterprises make their decisions simultaneously and independently.

Let \( \bar{U}^{k0}(\pi^k) \) denote the set of all tuples \( u^{k0} = (z_i^{k0}, \omega_i^{k0}, \omega_{i,0}^{k0}, \ldots, \omega_i^{k0}, \omega_{i,N}^{k0}, \ldots, \omega_i^{k0}, \omega_{i,K}^{k0}) \), \( t = 1, \ldots, T \), satisfying the conditions

\[
u_t^{k0} \in U_t^{k0}(x_t^{k0}, u_t^k), \quad x_{t+1}^{k0} = x_t^{k0} + z_t^{k0}, \quad t = 1, \ldots, T.
\]

The head enterprise selects a program \( \bar{u}^{k0} \in \bar{U}^{k0}(p, \bar{u}^k) \) and reports its choice to the enterprises of its integrated structure.

After that, the rest of the enterprises make their choice simultaneously and independently. Enterprise \( kn \) chooses its program \( u_{t}^{kn} = (z_t^{kn}, v_t^{kn}) \), \( t = 1, \ldots, T \), from the set \( \bar{U}^{kn}(\bar{u}^{k0}) \) of programs satisfying the conditions

\[
u_t^{kn} \in U_t^{kn}(x_t^{kn}, x_t^{k0}, \ldots, x_t^{k0}, \omega_t^{kn}, \omega_t^{k0}, \ldots, \omega_t^{k0}), \quad x_{t+1}^{kn} = x_t^{kn} + z_t^{kn}, \quad t = 1, \ldots, T.
\]

7. Optimality principle. We will assume that all subjects are cautious about their uncertainty. At the same time, they rely on the rational behavior of their partners.

Under the decision-making scheme described above, when choosing controls, enterprise \( kn \) \((n > 0) \) finds itself in a situation where its payoff depends only on its decision. Therefore, it is natural to assume that it will choose the program \( \bar{u}^{kn} \) from the set

\[
 BR^{kn}(\bar{u}^{k0}) = \left\{ \bar{u}^{kn} \in \bar{U}^{kn}(\bar{u}^{k0}) : g^{kn}(p, \bar{u}^{kn}) = \max_{u^{kn} \in U^{kn}(\bar{u}^{k0})} g^{kn}(p, u^{kn}) \right\}.
\]

The head enterprise \( k0 \) can evaluate this set. Therefore, it is natural for it to choose its program \( \bar{u}^{k0} \) from the set

\[
 cBR^{k0}(\bar{u}^k) = \left\{ \bar{u}^{k0} \in \bar{U}^{k0}(\bar{u}^k) : \min_{u^{k0} \in U^{k0}(\bar{u}^{k0})} \ldots \min_{u^{kN} \in U^{kN}(\bar{u}^{k0})} g^{k0}(p, \bar{u}^{k0}, u^{k0}, \ldots, u^{kN}) \right\}.
\]
It can be shown that if all the parameters $\alpha^{kn}$ are strictly positive, then under the usual assumptions about the choice sets, the maximum in this formula is reached.

Denote $BR^k(\pi^{0}) = BR^{K0}(\pi^{k}) \times \prod_{n=1}^{N} BR^{kN}(\pi^{0})$.

Let the Principal fix programs $\pi^{k} = (u^{k}_{1}, \ldots, u^{k}_{T})$, $k = 1, \ldots, K$, and $p = (p_{1}, \ldots, p_{T})$. Then it can be expected that with the rational behavior of partners it will receive a gain of at least

$$
\min_{(\pi^{10}, \pi^{11}, \ldots, \pi^{K0}, \pi^{K1}, \ldots, \pi^{KN}) \in BR^{K}(\pi^{k})} \min_{(\pi^{10}, \pi^{11}, \ldots, \pi^{K0}, \pi^{K1}, \ldots, \pi^{KN}) \in BR^{K}(\pi^{k})} g^{k0}(\bar{p}, \bar{u}^{10}, \bar{u}^{K0}, \bar{u}^{K1}, \ldots, \bar{u}^{KN}).
$$

**Theorem.** *If the quantity*

$$
\sup_{\pi \in U^{T} (\pi^{10}, \pi^{11}, \ldots, \pi^{KN}) \in BR^{K}(\pi^{k})} \min_{(\pi^{10}, \pi^{11}, \ldots, \pi^{KN}) \in BR^{K}(\pi^{k})} g(\bar{p}, \bar{u}^{10}, \bar{u}^{11}, \ldots, \bar{u}^{KN})
$$

*is positive or nonnegative and the least upper bound in the formula is attained, then the program $(\bar{\pi}^{1}, \ldots, \bar{\pi}^{K})$ is realizable.*

**Remark on mixed mechanisms.** The presentation does not focus on the case when the enterprise can sell surplus products on the domestic market at market prices. Formally, this is taken into account when describing outputs and the interests of enterprises. This allows for the specifics of the mixed management mechanisms in the industry.

As a result, on the basis of this description, a network model has been constructed that takes into account the technological and organizational connections of enterprises in a complex. The formulated network model has the form of a game-theoretic model with a hierarchical structure, with the priority of Principal’s actions and allowance for the connections between the agents of the technological graph of the complex.

The decision-making task of the Principal is formulated taking into account uncertainties and risks; this corresponds to constructing a general system of mathematical support for the strategic planning procedure by the production unit of a large-scale complex.

### 3.2. Decentralized System for Auction Schemes of Assembly Production

Currently, there is a great interest in the creation of systems that are controlled according to the network principle. As noted by K. Schwab [21], the development trend of the modern economy is determined by digitized technologies and network systems. These are systems consisting of relatively autonomous elements—nodes that exchange information with each other and change their state according to some local rules. This presentation contains a description of such a project [25]—we propose a network form of interaction within the association of enterprises engaged in some common production. Interaction involves the organization of multiple local auctions, the results of which within the association determine the distribution of functions to fulfill the next order for production.

1. **Original state.** It is assumed that the association is engaged in the production, purchase, installation, and assembly of products from a certain fixed set. The set of products from this set will be denoted by $\hat{P}$. The enterprises included in the association will be called participants and the totality of these enterprises will be denoted by $L$. 

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It is assumed that the participants are independent enterprises whose activities are not limited by the association. Its composition may vary from order to order. Organizing the association is a way to expand and streamline the market for the products of each of the participants. Entry and exit of an enterprise from the association is a free choice of the enterprise. Participants offer their services in the execution of orders and assume obligations for their implementation, but their well-being does not depend entirely on whether they are selected to participate in the next order or not. It is important for them to take their place in the flow of incoming orders. Participation in the association imposes certain requirements on enterprises, in particular, the need for integration into a common information network.

In addition to the enterprises that make up the set $L$ of members, there is one more distinguished member—the Principal, the central member of the association. The functions of the Principal within the framework of the model and the auction schemes considered below are the coordination of auctions and the distribution of orders between manufacturers of products of rank 1.

In reality, the Principal must also control the execution of orders, sort out inconsistencies that may arise due to the participants’ failure to fulfill their obligations on schedule, distribute the proceeds among the participants, etc.

Let us introduce the necessary notation.

Products will be named by the index $\hat{P} = \{m\}_{m=1}^{M^0}$.

We will consider deliveries of products from manufacturers to each other. The entire set of deliveries by order is described by a three-dimensional matrix $A = (a_{k,r})$, where the superscript corresponds to the product, with the first subscript corresponding to the supplier and the second, to the consignee of the product.

To an arbitrary set of products $W$ from $\hat{P}$ we assign a vector $\bar{W} \subset R^{M^0}$ in which the components $w_n$ correspond to the number (volume) of products $n \in \hat{P}$ from $W$, and vice versa, for the product vector $\bar{W}$ by $W$ we denote the set of nonzero indices of this vector.

The notation $\bar{w}_n$ will denote the vector $\bar{W} \subset R^{M^0}$ with the only nonzero component $n$ equal to $w_n$. Part of products $n \in M^0$ is assembled from other products produced within the association. Associate a product $n$ with the set $G(n) \subset \hat{P}$ of the parts it is assembled from. For a set of products $W \in \hat{P}$ by $G(W)$ we denote the set of parts the products in $W$ are assembled from by $G(W) = \bigcup_{n \in W} G(n)$.

Note that the set $G(W)$ may include some of the products from $W$, since some products from the set $W$ can use others as components. By $\bar{G}(W)$ we will denote the vector of components needed to assemble all components of the vector $\bar{W}$. We emphasize that $G(n)$ includes only the final assembly parts of product $n$. If some part $k \in G(n)$ is itself a composite, then the volumes of components $\bar{G}(\bar{v}_k)$ are not included in the components of the vector $\bar{G}(\bar{v}_n)$.

We have the set of articles $\hat{P}_1 \subset \hat{P}$ that we will conditionally call goods. These are the products that the association produces for external orders or for sale. They form the first, upper level of production. Goods are assembled from parts that make up the set $\hat{P} \setminus \hat{P}_1$.

Part of the items used in the assembly of goods, in turn, are mounted from the parts included in $\hat{P}$, in which we single out and assign to level 2 a subset $\hat{P}_2$ of products that are used only in assembly of products from $\hat{P}_1$. The set $\hat{P}_2 \subset \hat{P} \setminus \hat{P}_1$. 

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In turn, components may be required for assembly of products from $\hat{P}^2$. These are products from the set $\hat{P} \setminus (\hat{P}^1 \cup \hat{P}^2)$. From these products we select those that are used only in assembly of products of the first two levels. We refer them to level 3 and denote by $\hat{P}^3$. The set $\hat{P}^3 \subset \hat{P} \setminus (\hat{P}^1 \cup \hat{P}^2)$. Continuing this way, in total, some limited number of levels will be allocated.

2. Graphical representation of production. The entire structure of production can be visualized as a directed acyclic graph, called a technological graph. The nodes of this graph correspond to products $n \in M^0$; we use the same notation for them. At the top, which is the root level of the graph, there are nodes corresponding to products that are called goods. Let us denote the set of these nodes as well as the set of goods by $\hat{P}^1$.

The arcs emanating from the node $n \in M^0$ of the technological graph are directed to the nodes corresponding to those parts that the product $n$ is assembled from.

Definition 1. Of two adjacent nodes, the direct predecessor is the node from which the arc originates and the direct descendant is the one that it approaches.

Definition 2. The set of direct descendants that make up some subset of the nodes of a technological graph $W$ corresponds to the set $G(W)$ defined above. Let us call it the projection of the set $W$.

The projection of the set $G(W)$ is the set $G(G(W))$, and so on. It is clear that the number of such projections is limited. Moving along the arcs from node to node, we inevitably reach the terminal nodes, which are scattered over various sets $G^n(W)$. These nodes correspond to items whose production does not require components from the set $\hat{P}$. They are purchased ready-made from outside the association or only components produced outside the association are used for their production.

Figure 2 shows an example of a technological graph. Nodes 1 and 2 make up the graph root set $\hat{P}^1$. The set $\hat{P}^2$ includes nodes 3–7 and 13. The set $\hat{P}^3$ includes nodes 8–10, 11, and 13. Nodes 12 and 13 belong to $\hat{P}^4$, node 14 belongs to $\hat{P}^5$. The terminal nodes are nodes 6, 9, 11, 13, and 14.

Definition 3. The rank of a node $n$ of a technological graph is the length of the maximum path (the number of arcs) from the root of the graph to this node plus 1.

In the graph in Fig. 2, the root nodes 1 and 2 have rank 1, nodes 3–7 have rank 2, nodes 8–11 have rank 3, nodes 12 and 13 have rank 4, and node 14 has rank 5.
We will equip the arcs of a technological graph with weights.

**Definition 4.** We associate a product vector \( \bar{W} \) with \( H(\bar{W}) \), which is the vector of parts of all levels required to assemble \( \bar{W} \). The set of parts that make up the vector \( H(\bar{W}) \) will be denoted by \( H(\bar{W}) \) and called the full projection of the vector \( \bar{W} \).

The vector \( H(\bar{W}) \) is the sum of successive projections \( \bar{G}(\bar{W}) + \bar{G}(\bar{G}(\bar{W})) + \bar{G}(\bar{G}(\bar{G}(\bar{W}))) + \cdots \).

Moving down the arcs from node \( n \) and using the weights of the arcs, it is easy to calculate how many components of one type or another are required to assemble product \( n \).

3. **Participants.** Each of the enterprises produces some subset of products from the set \( \mathcal{P} \). The participant \( l \in L \) is described by one of the many adopted production models. In general terms, it looks like this. Enterprise \( l \) has a certain set of resources the current volumes of which are written as a vector \( S_l = (s_{l1}, s_{l2}, \ldots, s_{lp}) \). A vector function \( \Psi^l(\bar{W}) : \mathbb{R}^{M^0} \to \mathbb{R}^{p} \) showing how many resources of each type the enterprise spends in the manufacture of the production vector \( \bar{W} \) is prescribed.

One of the resources that make up the set \( S \) is finance. For definiteness, let this be component \( s_1 \). The consumption of resource \( s_1 \) reflects the financial costs of the enterprise for the manufacture of the production vector \( \bar{W} \). Only the participant’s own expenses for the assembly of products are included in \( s_1 \) but not the cost of the components necessary for this. It is assumed that the cost of components supplied to participant \( l \) will be included in the cost of its products and will be returned to suppliers after the order is completed.

4. **Transport.** Deliveries between participants involve costs and can be carried out both by manufacturers and specialized transport companies. The present paper uses the simplest solution to model transport. Each facility \( l \) is assigned a set of tables, \( T_l = (t_{lrn}) \), \( n \in M^0 \), that indicate the prices of delivery of a unit of produce \( n \) from enterprise \( l \) to other enterprises, i.e., \( r \). These prices are taken into account in the mutual settlements of the participants.

The problem of distribution of produce over the nodes of the technological graph. The Principal is a member of the association of manufacturers with number 0. Suppose the Principal accepts an order \( Z \) for the production of item \( m \in \mathcal{P}_1 \) in the amount of \( v_m \). The production of volume \( v_m \) is distributed among those participants who specialize in assembling these products. After order \( Z \) is completed, the finished products \( m \) arrive at the Principal in the form of deliveries \( y_{l0}^n \), \( l \in L \).

To fulfill the order, supplies of components must be ensured at all levels of the technological graph. Denote the supply matrix by \( Y = (y_{j,l}^n) \), \( n \in M^0 \), \( j, l \in L \). The following balance relations must be met:

\[
\forall l : \bar{G}(\bar{W}_l) = \sum_{j \in L} \bar{Y}_{j,l}.
\]

Here \( \bar{W}_l \) is the vector of products manufactured by participant \( l \), and \( \bar{Y}_{j,l} \) is the vector of deliveries of products from participant \( j \) to participant \( l \).

Apart from the balance relations, the following resource constraints must be met:

\[
\forall j : \Psi^j \left( \sum_{l \in L} \bar{Y}_{j,l} \right) \leq \bar{S}_j.
\]

So far, the model does not consider stocks of components. All supplies are formed from products produced on the basis of current resources of enterprises. The supply vector \( \sum_{l \in L} \bar{Y}_{j,l} \) is the vector of products manufactured by enterprise \( j \) within the current order.
By agreement, component $s_1$ is the money spent on manufacturing products. Summing up the quantities $\psi_1^l \left( \sum_{i \in L} \bar{Y}_{j,l} \right)$ and transportation costs for all shipments, we obtain the cost of manufacturing the ordered volume of products $v_m$. The goal is to minimize this cost,

$$\psi_1^l \left( \sum_{i \in L} \bar{Y}_{j,l} \right) + \sum_{n \in M^0} \sum_{j \in L} \sum_{l \in L} t_{n,l}^n \cdot y_{n,j,l}^n \to \min.$$ 

The association is interested in reducing the cost of its products, as it participates in the competition for orders. The difference between the selling price of the order and its cost forms the profit of participants. If the resource vector functions 1–3 are linear, then problem 1–3 is a linear programming problem.

5. **Overview of the auction scheme.** The modern development of computer technology and linear programming methods makes it possible to solve problems of very large dimensions. If the above-formulated problem of creating an optimal order distribution plan between manufacturers can be adequately represented by linear relationships, then the main problem of forming such a plan will probably not be related to calculations, but to constructing a very cumbersome model and filling it with specific data. This will require constant monitoring of the resources of association members, as well as changing technological conditions of production. For a large enough association, such a centralized approach may be too costly if feasible at all. Therefore, it is natural to decompose the problem by transferring most of the functions of assessing the production capacities of enterprises and developing a general production plan to the enterprises themselves. The idea is that the production plan is developed as a result of auctions in which some participants choose their suppliers from among other participants.

Decentralization with an auction scheme is a way to decompose the general problem of distribution of production presented above, so it must satisfy the same balance and resource constraints and some additional ones due to decomposition. The variables within the model are assumed to be continuous.

To illustrate the scheme, we use the graph shown in Fig. 2.

The set of enterprises manufacturing products from the set $W$ will be denoted by $L(W)$.

Suppose the Principal received an order for manufacturing a certain volume $w_m$ of products $m$. Let it be item 1 in Fig. 2. The Principal has technological graph schemes for all items $n \in M^0$. The Principal publishes this order along with its full projection, the vector $H(\bar{w}_m)$. Based on their resources, the participants decide what volumes of products $n$ that make up the vector $H(\bar{w}_m)$ they would like and are ready to take on. Given these volumes, they publish orders for the supply of parts from the sets $G(n)$. These publications are essentially the announcement of auctions.

Participants from the set $L(G(n))$ must respond to orders for products $G(n)$. They allocate some part of the production volume supposed by them for deliveries to each participant from $L(n)$. Enterprises that manufacture products corresponding to the terminal nodes do not need supplies from other members of the association for their manufacture and can determine the volume of their production based only on their own resources. To plan the production of other products, their manufacturers must know what volumes of components they have. Thus, in Fig. 2, participants $l \in L(10)$ must form pools of items 9 and 10 that serve as parts for item 4. However, before the item pool 10 is assembled, participants from $L(10)$ must decide on the volume of production and supply of these parts to the product manufacturers 4. In order for the auctions to be synchronized,
they must be held in a certain order. This order is established by the Principal in accordance with the technological graph.

**Definition 5.** Let the rank of the auction held by participant \( l \in L(n) \) for the supply of components for product \( n \) be the rank of product \( n \).

**Condition 1.** Higher ranked auctions precede lower ranked auctions.

In the example under consideration, this order is as follows. Enterprises from the set \( L(12) \) summarize the proposals for components from participants from the set \( L(14) \). These offers include the volumes of deliveries and their prices including transportation costs. Offers are limited to the volumes and terms announced in the orders. If for some items on the list the sums of offers exceed the volume of the order, then participants from \( L(12) \) decide between them giving preference to cheaper options. Having formed pools of components, the participants calculate how many products and at what prices they can produce from these components within the volume declared by them at the auction. Thus, in the framework of the example, having formed the pools of components 14, the participants from the \( L(12) \) calculate how many products 12 and at what prices they can manufacture. Having decided on the volume, the manufacturers of products 12 make offers to supply these products to participants from \( L(8) \). This concludes the auctions for items 12.

Further, according to the same scenario, enterprises from \( L(8) \) hold an auction choosing between proposals for components 12 and forming their proposals to participants from the set \( L(3) \). Then the same auction is held by enterprises from \( L(3) \) for components 8.

Enterprises from the set \( L(10) \) hold separate auctions for components from participants from the sets \( L(14) \) and \( L(13) \) and determine the volume and prices of supplies of products 10 to enterprises from \( L(4) \).

After that, auctions are held by enterprises from the set \( L(4) \), separately for proposals made by participants from \( L(9) \) and \( L(10) \).

Finally, enterprises from the set \( L(1) \) hold auctions based on proposals made by participants from the sets \( L(3) \) and \( L(4) \) for components 3 and 4. Based on the results of these auctions, they calculate the volume of products 1 and at what prices they can manufacture and announce the results to the Principal. This is the end of the downward (in terms of product ranks) wave of auctions.

The final phase of laying out the order fulfillment plan begins. The Principal determines which shares of the ordered products \( m \) will be produced by participants from the set \( L(m) \). Of course, the assigned volumes do not exceed the proposals of the participants. Participants from \( L(m) \) distribute the task for the production of components among manufacturers from the set \( L(G(m)) \) in accordance with the assigned volumes, and these tasks do not exceed those volumes that were offered by participants from the set \( L(G(m)) \) at auctions held by participants from \( L(m) \), etc. The ascending wave of tasks reaches the terminal nodes, and this completes the formation of the production plan.

6. **Formalized description of the scheme of auctions.** As above, but in general terms, we consider the situation where the Principal received an order \( Z \) for the production of an item \( m \in M^0 \) in the amount of \( w_m \). The scheme is divided into several stages.

**Stage 1.** The Principal calculates and announces a vector of all \( \bar{H}(\bar{w}_m) \) parts necessary to complete the order to the participants.
Stage 2. Participants choose product vectors $\bar{W}_{2,l} = (w_{1,l}^{2,l}, w_{2,l}^{2,l}, \ldots, w_{M^0,l}^{2,l})$, $l \in L$, that they would like to produce within a given order. Number 2 in superscript indicates the stage at which this production vector is selected. Participant $l$ chooses a point on a subset of the space $\mathbb{R}^{M^0}$ bounded by the resource inequalities

$$\bar{\Psi}^l(\bar{W}_{2,l}) \leq \bar{S}_l$$

and the requirements of order $Z$, i.e., the inequalities

$$w_{n,l}^{2,l} \leq h_n(\bar{w}_m) \forall n \in M^0.$$ 

Then they announce auctions for the supply of components they need in order to produce the vector $\bar{W}_{2,l}$, i.e., for the supply of components of the vector $\bar{G}(\bar{W}_{2,l})$. The auctions declared by participant $l$ will be denoted by $A^l$.

Stage 3. This is the stage of the actual auctions, which are held sequentially from higher to lower ranks. By the time of the auction for the supply of parts $G(n)$ for item $n \in W_{2,l}$, participant $l$ has collected the entire pool of proposals for the supply of these parts and their prices from different manufacturers.

At auction $A^l$ for component $k$, participant $j$ proposes the volume of supplies, which will be denoted by $Y_{3,k}^{3,l}$ (number 3 in superscript indicates the stage).

Delivery is associated with the tabular price function $c_{k,j,l}(\varsigma)$, $\varsigma \in [0, Y_{3,l}^{3,k}]$ depending on the scope of supply $\varsigma$. The price includes the cost of a given volume of products from manufacturer $j$ and transportation costs.

The set of offers from participant $j$ in the auctions of all participants forms a matrix $Y_{j}^{3,l} = (y_{j,k,l}^{3,k})$, where $l \in L \setminus j$, $k \in M^0$. Denote the price matrix $c_{j,l}(\varsigma)$ of participant $j$ by $C_j$.

The entire corpus of supply proposals from all participants, together with price tables, will be denoted as a pair $(\hat{Y}^3, \hat{C})$, $\hat{Y}^3 = \bigcup_{j \in L} Y_{j}^{3,l}$, $\hat{C} = \bigcup_{j \in L} C_j$.

Based on the volumes and prices of components offered to him, participant $l$ corrects the volume of production $w_{n,l}^{3,l}$. This amount should not exceed the amount announced at stage 2, i.e.,

$$w_{n,l}^{3,l} \leq w_{n,l}^{2,l} \forall n \in M^0.$$ 

The volume $w_{n,l}^{3,l}$ must be covered by supplies of components, i.e.,

$$g_k(\bar{w}_{n,l}^{3,l}) \leq \sum_{j \in L} y_{j,k,l}^{3,k} \forall n, k \in M^0.$$ 

Having chosen the volume of production $w_{n,l}^{3,l}$, participant $l$ determines the volumes of his proposals for deliveries to auctions of lower ranks, those where the need for $n$ components is declared, i.e., the participant determines the vector $\bar{Y}_{l}^{3,n} = (y_{l,j,n}^{3,n})$.

Supply offers must not exceed demand, so the following inequalities must hold:

$$y_{l,j,n}^{3,n} \leq g_n(\bar{W}_{2,j}) \forall j \in L, \forall n \in M^0.$$ 

Thus, a set of offers $(\hat{Y}^3, \hat{C})$ is formed sequentially by the auctions.
Stage 4. At this final stage, based on the set of offers \((\hat{Y}^3, \hat{C})\), the production plan, i.e., the distribution of production tasks and deliveries, is drawn up. This is a collection of matrices 
\[
\hat{Y}^4(\hat{Y}^3, \hat{C}) = \left\{ Y_i^4(\hat{Y}^3, \hat{C}) \right\}_{i=1}^L,
\]
where \(Y_i^4(\hat{Y}^3, \hat{C})\) is the matrix of the volumes of components that participant \(l\) will have to supply to other participants.

The deliveries must satisfy the inequalities
\[
y^{4,n}_{j,l}(\hat{Y}^3, \hat{C}) \leq y^{3,n}_{j,l} \quad \forall l, \ j \in L, \ \forall n \in M^0;
\]
i.e., assignments to participant \(j\) according to the plan should not exceed his proposals.

To avoid ambiguity in the formation of the plan, we can agree that if the prices of components from different manufacturers are equal, preference is given to a supplier with a lower serial number \(j \in L\). The procedure for choosing the values of \(y^{4,n}_{j,l}\) is recursive and is organized as an ascending wave along the technological graph.

Producer \(j\) is waiting to receive orders for the production of products \(n\) from all participants \(l\) to whom he sent offers \(y^{3,n}_{j,l}\). It is assumed that he should receive a response from all such participants even if the volume of the assignment is zero.

After that, participant \(j\) knows the entire volume of products \(n\) that he needs to produce. It is \(w^{i,j}_n = \sum_{l \in L} y^{4,n}_{j,l}(\hat{Y}^3, \hat{C})\). Given this volume, participant \(j\) determines the vector of necessary components \(\bar{G}(\bar{w}^{i,j}_n)\). For each component \(k\) of this vector, a pool of components of volume \(\pi^j_k = \sum_{l \in L} y_{l,j} \) has been created at auctions \(A^j\). If the offers \(y^{3,n}_{j,l}\) were correct (see rules 1–3 below), then \(w^{i,j}_n \leq w^{3,j}_n \ \forall n \in M^0\) and the volumes of these pools are sufficient to provide the production of all components of the vector \(\bar{W}^{3,j}\) with components \(k\), i.e., the following inequalities hold:
\[
\sum_{n \in M^0} G_k(\bar{w}^{i,j}_n) \leq \sum_{n \in M^0} G_k(\bar{w}^{3,j}_n) \leq \pi^j_k.
\]

Participant \(j\) chooses the volume \(G_k(\bar{w}^{3,j}_n)\) from the volume \(\pi^j_k\), starting from the cheapest offers. Then participant \(j\) distributes tasks to those producers whose proposals are included in the scope of \(G_k(\bar{w}^{3,j}_n)\), i.e., for each of them he determines the components \(y^{4,k}_{l,j}(\hat{Y}^3, \hat{C})\) of the supply matrix \(Y_i^4(\hat{Y}^3, \hat{C})\).

Components \(k\) can be included in different products of manufacturer \(j\). For \(j\), we can calculate the total cost of deliveries of parts \(k\) from manufacturers \(l\) who received an order from \(j\) to supply them during stage 4. Denote this cost by \(d^{k}_{j}\),
\[
d^{k}_{j}(\hat{Y}^4, \hat{C}) = \sum_{l \in L} c_{l,j}^{k} (y^{4,k}_{l,j}(\hat{Y}^3, \hat{C})).
\]

The formation of the production plan ends when the wave of tasks reaches the terminal nodes.

For the considered scheme, it has been established that it is advisable for auction participants to adhere to the following rules.

Rule 1. When determining the desired production vector \(\bar{W}^{2,j}\) at stage 2, participant \(j\) makes a choice among nondominated admissible vectors, i.e., among nondominated vectors satisfying constraints 4 and 5.
Rule 2. At stage 3 participant \( j \) determines the production volumes \( w_{n}^{3,j} \) equal to \( \min[w_{n}^{2,j}; \pi_{n}^{j}] \), where \( \pi_{n}^{j} = \sum_{l \in L} y_{l,n}^{3,n} \) is the pool of components \( n \) gathered at auctions \( A^{l} \).

Rule 3. When determining his proposals for the supply of item \( n \)—the value \( y_{l,n}^{3,n} \)—at stage 3, participant \( j \) acts as follows:
- if \( w_{n}^{3,j} = H_{n}(\bar{w}_{m}) \), then \( y_{l,n}^{3,n} = G_{n}(\bar{W}_{2,l}) \) \( \forall l \in L \), i.e., if the production volume \( w_{n}^{3,j} \) chosen by participant \( j \) is equal to the total demand for products \( n \) for order \( Z \) (it cannot be greater than \( H_{n}(\bar{V}_{m}) \) due to condition 2), then \( j \) offers each manufacturer \( l \) the volume of products \( n \) declared by \( l \) at auctions \( A^{l} \);
- if \( w_{n}^{3,j} < H_{n}(\bar{V}_{m}) \), then \( j \) chooses values \( y_{l,n}^{3,n} \) such that so that they satisfy the relations

\[
\sum_{l \in L} y_{l,n}^{3,n} = w_{n}^{3,j}
\]

and

\[
y_{l,n}^{3,n} \leq G_{n}(\bar{W}_{2,l}) \quad \forall l \in L.
\]

The described scheme seems to be quite transparent and efficient in the sense of finding an acceptable plan if the number of participants is large and their production resources substantially exceed those needed to fulfill the order. However, it is easy to see that the auction scheme does not guarantee the formation of a plan in all those cases in which a feasible plan exists.

Consider the same production scheme under the assumption that each participant produces at most one item from the set \( \bar{H}(\bar{w}_{m}) \) of items needed to fulfill order \( Z \). This is equivalent to the condition that each enterprise \( l \), having received information about the next order from the Principal, allocates shares of its resources for the production of certain products from the set \( \bar{H}(\bar{w}_{m}) \). As a result, the maximum allowable volume of production of each of the products by participant \( l \), the quantity \( w_{l,n}^{\text{max}} \) is determined, and the inequality \( \bar{\Psi}_{l}(\bar{W}_{2,l}) \leq \bar{S}_{l} \) is replaced by the inequality

\[
w_{n}^{2,l} \leq w_{l,n}^{\text{max}}, \quad n \in M^{0}, \quad l \in L.
\]

Then we can formally assume that each enterprise \( l \) willing to participate in the execution of order \( Z \) is associated with exactly one product \( \bar{n}(l) \in \bar{H}(\bar{V}_{m}) \) and each product \( n \in H(\bar{V}_{m}) \) corresponds to the set \( \bar{L}(n) \) of participants ready to produce it.

**Condition 2.** For each product \( n \in H(\bar{w}_{m}) \) the inequality \( \sum_{l \in L} w_{l,n}^{\text{max}} \geq h_{n}(\bar{w}_{m}) \) holds.

The following assertion holds true.

**Assertion.** This auction scheme leads to a feasible solution if and only if Condition 2 is satisfied.

**Remark.** Let us describe a computer simulation of the auction scheme.

The auction schemes considered above leave questions that are difficult to find answers to by analytical methods, therefore, it seems appropriate to conduct research on the proposed schemes in simulation mode on a computer model. Such a model should be constructed as a network consisting of nodes, each of which corresponds to a member of the association. In general terms, the simulation system is as follows. For each node, its own production model is created, corresponding to the description made above in Sec. 3.2, item 3. A set \( M^{0} \) of products and production graphs for them are selected. Auction rules are modelled. For each node, the initial state of resources is specified,
and the system is offered a sequence of orders for the production of products from the set \( \hat{P}_1 \). The response of the system to an order will be an assembly plan (if the auction scheme finds it) and a change in the state of the nodes based on the results of order fulfillment. Naturally, in the study it is also desirable to implement a solution to the general problem. Comparing the results of solving the general problem and the auction ones, it will be possible to draw conclusions about the effectiveness of the auction scheme.

The proposed model can be used in business planning problems with the possibility of competitive interactions between players.

4. INFORMATION SUPPORT IN LARGE-SCALE PROJECTS OF A MIXED ECONOMY

As noted, an appropriate infrastructure is required to use the described conceptual constructions. This section is devoted to the issues of development of data on which the specialized information and computing complex will be based and which constitute one of the most important sections of the digitalization program. As for the algorithmic tools for performing calculations and computational experiments, the scientific and applied community has an extensive arsenal of optimization methods, solution of game-theoretic problems, and development of simulation systems [5].

In this section we follow the publication [8].

A special place among the country’s large-scale economic systems is occupied by the military-industrial complex, where, along with solving technical and economic problems, it is necessary to make extremely responsible operational decisions on organizational and technological innovations.

Naturally, the defense industry is part of the country’s economic complex, but it also has specifics that must be taken into account in the modeling process. In organizational terms, this is manifested primarily in the forms of management. Management is of a mixed decentralized nature, and it is carried out as a state-planned directive action, as well as market influence.

The emphasis is mainly on the planned nature of government orders through decision-making in the interests of the state at state-owned enterprises, but market mechanisms are also taken into account, in particular, price mechanisms and demand in the production of civilian products and in export deliveries.

In addition, there is a feature in the products. The manufactured products are divided into three groups. The main military products in economic terminology are not of a direct commodity nature for consumption in the domestic market but services that provide protection from exogenous influences. Consumption of promising global scientific achievements in the military-industrial complex has a priority character due to the indicated specific service.

The main military products supplied to the overseas market have the nature of a market commodity. Civilian products are goods of direct consumption and are included in the composition of goods of a market nature.

The term “Development of the defense industry” implies a change over time in the characteristics of the state of the industry and suggests that a positive improvement in the state occurs in the direction of the progress of the components of the complex and is associated with the latest technologies. Development management involves the implementation of influences on the development of the defense industry that ensure the improvement of the characteristics of its state.

As a result, when using mathematical models, it is assumed that all free variables of the model can be selected for improvement in terms of a set of formally specified goals. Thus, from a method-
ological point of view, in order to develop and implement rational control actions, it is necessary to construct models for changing state variables, determine parameters that can affect development, and determine goals and then develop strategies depending on the available information that lead to the progress of the controlled system.

Obviously, the task of managing the development of the defense industry is a complex task that requires the analysis of a large amount of data when making management decisions, including the use of elements of mathematical modeling in the interests of decision making in the process of program-targeted and situational management of the development of the defense industry.

In order to form an efficient tool for information interaction with the interested bodies of the Principal, the integrated structures, and the organizations of the defense industry in the preparation and solution of a wide range of management and production problems in the field of the defense industry, since 2012, the Ministry of Industry and Trade of Russia has developed a concept and is implementing measures to create and develop a Unified Information Space of the Military-Industrial Complex (hereinafter, UIS MIC) [24].

UIS MIC is territorially distributed automated information systems and information resources united by common rules for data exchange and a telecommunications system with secure communication channels.

The implementation of the development concept of the UIS MIC is carried out in the following four key areas (see Fig. 3):

– development of the regulatory framework and organizational support for the UIS MIC;
– formation of the infrastructure of the UIS MIC;
– development of topical databases and services of the UIS MIC;
– creation of an integrated resource and organization of information interaction with external systems.

Creation and development of thematic databases of the UIS MIC is based on the principle of combining centralized and decentralized approaches to the collection, storage, and processing of information.

5. CONCLUSIONS

The development experience is described and examples of the description of the information space and the complex of mathematical models are given, which jointly provide, on the basis of a holistic and interconnected methodology of the systematic approach, the use of algorithms, forecasting software complexes, and decision support systems at different levels of strategic planning of the development of large-scale projects.

The complex of conceptual mathematical models of development management includes:

1. Models of analysis and synthesis of large-scale projects. They allow the specialist who prepares the solution to choose the set of criteria and restrictions that is necessary in the current situation from a list of available ones. Thus, the issues of forming the structure of the model for planning calculations, completing the model with optimization criteria, their meaningful interpretation, and adjusting the model and initial data create a set of tools for strategic planning based on digital platforms.
2. **Basic production model and digital twin.** The basic model of an enterprise describes the development of production, the dynamics of material (products, production assets), and financial flows (investments, assets, liabilities, equity) taking into account technologies and production goals.

3. **The network model** answers the question of what distribution of financial assets and resources between enterprises ensures the implementation of a given strategic plan taking into account the economic interests of the enterprises based on digital platforms.

Further research within the framework of the ideas of this paper corresponds to the logic of the synthesis of large-scale projects described in [3]. First of all, it is necessary to develop a system of indicators for constructing production functions for individual enterprises and integrated structures. This procedure is based on the ideas of processing nonlinear balances of financial flows of the enterprise system [26, 27].
The next step is related to the analysis and design of the economic mechanisms of the entire system taking into account market relations and state regulation, property relations, and a systemic approach [9, 11, 23, 28].

General equilibrium models consider the most aggregated agents and analyze the impact of changes in economic mechanisms; this allows drawing conclusions about the degree and direction of the influence of sets of mechanisms on the activities of enterprises of large-scale projects. Such mechanisms include a program for investing in national projects and developing infrastructure through targeted lending, issuing long-term securities for project activities, and changing the mechanisms for financial interaction with the outside world. In the future, the question of choosing the most efficient mechanisms for the economic activity of individual agents and the system as a whole can be raised.

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*This paper was recommended for publication by A.A. Galyaev, a member of the Editorial Board*