Fixed-time Leader-Following Formation Control of AUVs Without Velocity Measurements

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1 Introduction

Autonomous underwater vehicles (AUVs) have played important roles in various underwater applications, e.g., ocean sampling, oceanographic survey and inspection, seafloor mapping and modeling, submarine mine neutralization, offshore oil and gas exploration [1, 9, 10, 26, 28]. To improve efficiency and to extend task capacity, many tasks in these applications involve collaboration of multiple AUVs in a certain formation [2, 3, 4, 5, 6, 7, 11, 19]. Formation control is therefore very important and there are various methods widely in use, among which the leader-follower scheme is the most popular due to simplicity and scalability. Some outstanding works on leader-following control can be seen in the literature [1, 16].

What are the challenging issues in formation control of AUVs? Some well-known issues include energy limitation [2], communication constraint [11], uncertainties (modeling error, parameter variation, and external disturbance) [20], sensor and actuator constraints [29], etc. The first issue concerned in this paper is the low convergence speed of the formation tracking control system, which is partly due to actuator constraints. Most AUVs formation control methods in existence can only guarantee asymptotic convergence of the tracking errors. Namely, the convergence speed is at best exponential, which implies that the tracking errors will converge to zero with infinite settling time [11, 20]. Clearly, it would be a great advance if AUVs can track a given formation in finite time. Actually, some research attention has been paid to finite-time formation control of various vehicles (e.g., mobile robots, surface vessels and aircrafts) in recent years [12, 21]. However, no similar work on formation control of underwater vehicles has been seen yet, although a finite-time trajectory
The main contributions of this paper are summarized as follows:

(1) A unified novel framework for AUVs formation control with both full state feedback and velocity estimation is developed. The proposed control algorithm can ensure not only a given settling time regardless of the initial conditions of the system, but also a better tracking performance and faster convergence speed of the AUVs formation system than the existing methods [11,29].

(2) The velocity observer itself is also fixed-time convergent, which can estimate the actual AUV velocity with zero error in a designated fixed time regardless of the initial states of the system. In this sense, it is superior to both the linear observer in [20,33] and the finite-time observer in [23].

This paper is organized as follows. Section II gives some preliminaries and the problem formulation. Section III contains the fixed-time formation control method with full velocity and position feedback. Section IV gives the fixed-time velocity observer and the formation control scheme with velocity observer. Simulation studies are given in Section V, which is followed by the conclusions and future works in Section VI.

Notations: Throughout this paper, \( \mathbb{R}^n \) and \( \mathbb{R}^{n \times n} \) denote the \( n \)-Euclidean space and the set of all \( n \times n \) real matrices, respectively; \( \| \cdot \| \) stands for either the Euclidean vector norm or the spectral norm of a matrix; \( \lambda_{\text{max}}(X) \) (\( \lambda_{\text{min}}(X) \)) represents the maximum (minimum) eigenvalue of matrix \( X \); \( X > 0 \) (\( X < 0 \)) means that matrix \( X \) is positive (negative) definite; For two positive definite diagonal matrices \( A \) and \( B \) with the same dimension, \( A > B \) (\( A < B \)) means each diagonal element satisfies \( a_{ii} > b_{ii} \) \( (a_{ii} < b_{ii}) \); \( \frac{1}{\alpha} \) means \( \frac{\alpha}{\alpha - 1} \); \( 1 \) denotes the diagonal matrix with all elements equal to one.

2 Preliminaries and Problem Formulation

2.1 Preliminaries

Lemma 1 [22,27] For system \( \dot{x}(t) = f(x(t)) \) along a trajectory \( x(t) \), if there exists a positive definite function \( V : \mathbb{R}^n \rightarrow \mathbb{R}_+ \cup \{0\} \) such that

(1) \( V(x) = 0 \Leftrightarrow x = 0 \);

(2) \( \dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x) + \vartheta \) for some \( \alpha, \beta, p, q > 0 \), with \( 0 < p < 1 \) and \( q > 1 \);

then, the origin of the system is globally fixed-time stable with \( \vartheta = 0 \), and the settling time \( T \) can be estimated by

\[
T \leq T_{\text{max}} := \frac{1}{\alpha(1 - p)} + \frac{1}{\beta(q - 1)}. \tag{1}
\]

For \( \vartheta \in (0, \infty) \), the system trajectory \( x(t) \) is practically fixed-time stable, and the residual set of the solution of
the system can be given by
\[
\lim_{t \to T^+} x|V(x)| \leq \min \left\{ \frac{1}{\alpha \theta (1-p)} + \frac{1}{\beta \theta (q-1)} \right\}
\]
(2)

where \( \theta \) is a scalar and satisfies \( 0 < \theta < 1 \). The time needed to reach the residual set is bounded as
\[
T \leq T_{\text{max}} := \frac{1}{\alpha \theta (1-p)} + \frac{1}{\beta \theta (q-1)}
\]
(3)

**Lemma 2** \[14\] Let \( \epsilon_1, \epsilon_2, \ldots, \epsilon_N \geq 0 \). Then
\[
\sum_{i=1}^{N} \epsilon_i^\delta \geq \left( \sum_{i=1}^{N} \epsilon_i \right)^{\delta} , 0 < \delta \leq 1,
\]
\[
\sum_{i=1}^{N} \epsilon_i^\delta \geq N^{1-\delta} \left( \sum_{i=1}^{N} \epsilon_i \right)^{\delta} , 1 < \delta \leq \infty.
\]

Now, consider a command filter \[14\] as described below:
\[
\dot{z}_1 = f_{z2}, \quad \dot{z}_2 = -2k_1 f_{z2} - f(z_1 - \alpha_1),
\]
where \( \alpha_1 \) is the input signal, \( z_1 \) and \( z_2 \) are the filter output, \( f > 0 \) and \( \zeta \in (0, 1] \) are the filter gains to be determined, which denote the natural frequency and the damping ratio of the command filter, respectively. Choose appropriate parameters \( f \) and \( \zeta \), the following lemma holds.

**Lemma 3** \[14\] If \( \alpha_1 \) and \( \alpha_1 \) are simply bounded for all \( t \in [0, +\infty) \) and \( z_1(0) = \alpha_1(0), z_2(0) = 0 \), then for any \( \mu > 0 \), there exist \( f_1 > 0 \) and \( \zeta \in (0, 1] \) such that \( |z_1 - \alpha_1| \leq \mu \), and \( |z_1|, |z_2|, |\dot{z}_2| \) are bounded.

**Remark 1** Since the signal passing through the filter will have a phase offset, that is, there is a difference between the filtered signal and the original signal, which leads to the existence of filtering error \( |z_1 - \alpha_1| \).

**Lemma 4** \[37\] Consider a nonlinear system
\[
\dot{x} = f(x), \quad f(0) = 0, \quad x(0) = x_0,
\]
where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \) is the state, \( f(x) : \mathbb{R}^n \to \mathbb{R}^n \) is a nonlinear function. If \( f(x) \) is a homogenous vector function in the bi-limit with associated triples \( (r_0, k_0, f_0) \) and \( (\infty, k_\infty, f_\infty) \), moreover, if the original system \( \dot{x} = f(x) \) and the approximating systems \( \dot{x} = f_0(x), \dot{x} = f_\infty(x) \) are globally asymptotically stable, then we have the following results:

1. The origin of \( \dot{x} = f(x) \) is fixed-time stable when condition \( k_\infty > k_0 \) holds;
2. Let \( d_{V_0} \) and \( d_{V_\infty} \) be real numbers such that \( d_{V_\infty} > \max_{1 \leq i \leq n} r_{0,i} \) and \( d_{V_\infty} > \max_{1 \leq i \leq n} r_{\infty,i} \). There exists a continuous and positive definite function \( V : \mathbb{R}^n \to \mathbb{R}^+ \) such that the function \( \frac{\partial V}{\partial x} \) is homogeneous in the bi-limit with associated triples \( (r_0, d_{V_0} - r_{0,i}, \frac{\partial V}{\partial x}) \) and \( (\infty, d_{V_\infty} - r_{\infty,i}, \frac{\partial V}{\partial x}) \), and the function \( \frac{\partial V}{\partial x} f(x) \) is negative definite, and satisfies
\[
\frac{\partial V}{\partial x} f(x) \leq -k_v \Gamma \left( \frac{d_{V_0} + k_v}{V^{d_{V_0} + k_v}}, \frac{d_{V_\infty} + k_v}{V^{d_{V_\infty} + k_v}} \right),
\]
where \( k_v \) is positive constant and function \( \Gamma : \mathbb{R}^2_+ \to \mathbb{R}_+ \) is defined as \( \Gamma(a, b) = \frac{a}{1+a}(1+b), a, b \in \mathbb{R}_+ \).

2.2 Problem formulation

2.2.1 The AUV dynamics

Consider \( N \) AUVs with a global leader, labeled as \( l \), and \( N-1 \) followers, labeled as \( f_1 \) to \( f_{N-1} \). Each follower is equipped with a sensor to measure its own position and receive that of the leader in the global coordinate frame \( \{E\} \). In addition, each follower can receive velocity vector of the leader in the body coordinate frame \( \{B\} \). Assume that each AUV \( i, i = 1, \ldots, N \), has fixed attitudes and the translational dynamics is given by \[10\]:

\[
\begin{cases}
\dot{p}_i = J_i(\Theta_i)v_i \\
M_i \dot{\psi}_i = -D_i(v_i) + g_i(\Theta_i) + \tau_i + d_i
\end{cases}
\]
(8)

where \( p_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3 \) is the position vector, \( \Theta_i = [\phi_i, \theta_i, \psi_i]^T \in \mathbb{R}^3 \) denotes the generalized attitude in Euler angles of roll \( \phi_i \), pitch \( \theta_i \), and yaw \( \psi_i \), in frame \( \{E\} \), \( J_i(\Theta_i) \) denotes the kinematic transformation matrix from frame \( \{B\} \) to \( \{E\} \). For short, throughout the paper, we will denote \( J(\Theta) \) as \( J. \) \( v_i = [u_i, v_i, \omega_i]^T \in \mathbb{R}^3 \) is the generalized linear velocities, \( \tau_i = [\tau_{u_i}, \tau_{v_i}, \tau_{\omega_i}]^T \in \mathbb{R}^3 \) is the control forces, \( d_i = [d_{u_i}, d_{v_i}, d_{\omega_i}]^T \in \mathbb{R}^3 \) denotes the environmental disturbance forces and moments due to waves, wind and ocean current, respectively. \( M_i, D_i(v_i) \) and \( g_i(\Theta_i) \) are the inertia matrix, the damping matrix, and the restoring force vector, respectively, with
\[
M_i = \text{diag} \{ m_{u_i}, m_{v_i}, m_{\omega_i} \},
\]
(9)
\[
D_i(v_i) = \text{diag} \{ d_{L_{1u}}, d_{L_{2u}} + d_{Q_{21}}|v_i|, d_{L_{3u}} + d_{Q_{31}}|\omega_i| \},
\]
(10)
\[
g_i(\Theta_i) = [(W_i - B_i)s_{\theta_i}, -(W_i - B_i)c_{\Theta_i}, s_{\phi_i}, -(W_i - B_i)c_{\Phi_i}]^T.
\]
For simplicity, the dynamic part (i.e., the second equation in (8)) can be rewritten as:

\[
m_{u_i} \dot{u}_i = (d_{L1}u_i + d_{Q1}|u_i|u_i - (W_1 - B_i)s_\phi u_i) + \tau_{u_i} + d_{\dot{u}_i},
\]

\[
m_{v_i} \dot{v}_i = (d_{L2}v_i + d_{Q2}|v_i|v_i + (W_1 - B_i)c_\phi \dot{q}_u v_i) + \tau_{v_i} + d_{\dot{v}_i},
\]

\[
m_{\omega_i} \dot{\omega}_i = (d_{L3}\dot{\omega}_i + d_{Q3}|\omega_i|\omega_i + (W_1 - B_i)c_\phi \dot{q}_\omega \dot{\omega}_i) + \tau_{\omega_i} + d_{\dot{\omega}_i}.
\]

(12)

Let \( \varphi_i = [\varphi_{u_i}, \varphi_{v_i}, \varphi_{\omega_i}]^T \), where each element is a time-varying function satisfying the following assumption.

Assumption 1 \( \square \) For each element in \( \varphi_i \), there is a known and positive constant \( \varphi_* \) satisfying

\[
|\varphi_*(s_1(t), t)| - |\varphi_*(s_2(t), t)| \leq \varphi_* |s_1(t) - s_2(t)|,
\]

(13)

where \( s = u, v, \omega \) and let \( \varphi_i = \text{diag}(\varphi_u, \varphi_v, \varphi_\omega) \).

Assumption 2 Assume that the environmental disturbance terms satisfy \( |d_{u_i}| \leq d_{u_i \text{ max}} \), \( |d_{v_i}| \leq d_{v_i \text{ max}} \), \( |d_{\omega_i}| \leq d_{\omega_i \text{ max}} \), where \( d_{u_i \text{ max}}, d_{v_i \text{ max}} \) and \( d_{\omega_i \text{ max}} \) are unknown bounded positive constants.

2.2.2 The objective

In this paper, we consider an interesting scenario in which the \( N - 1 \) follower vehicles can each independently follow the leader. Namely, the entire formation can be decomposed into \( N - 1 \) subformations, each of which comprises a follower AUV and the leader, as shown in Fig. 1. In each subformation, the follower AUV tries to maintain a desired distance relative to the global leader.

Without loss of generality, the objective here can be described as designing a control law \( \tau_{fi} \) for follower AUV \( fi \), so that AUV \( fi \) and the leader can achieve a formation given in \( \{ E \} \).

Namely, the distance between the follower and the leader reaches a desired value in finite time, i.e.,

\[
\lim_{t \to T} \|p_i - p_f - d_{fi}\| \leq \delta_{hi},
\]

(14)

and

\[
\|p_i - p_f - d_{fi}\| \leq \delta_{hi}, \forall t \geq T,
\]

where \( \delta_{hi} \) is an arbitrary small positive constant and \( 0 \leq T < \infty \) is the given convergence time.

3 Formation Control with Velocity Measurement

Here, a fixed-time formation control method based on full velocity and position feedback will be given. The controller is designed using the command filter technique and adaptive backstepping technique for each follower AUV to achieve the formation control objective. In what follows, we will drop the subscript \( i \) for short.

The controller design process is divided into two steps, i.e., the kinematics control part and the dynamics control part.

Step 1 (Kinematic controller design): Introduce a position tracking error vector \( e_1 = [e_{1x}, e_{1y}, e_{1z}]^T \in \mathbb{R}^3 \), which is defined by

\[
e_1 = p_l - p_f - d_{lf},
\]

(15)

where \( d_{lf} = [d_{lfx}, d_{lfy}, d_{lfx}]^T \in \mathbb{R}^3 \) denotes the desired relative distance vector between the AUVs. Using (8), we can derive the time derivative of (15) as

\[
\dot{e}_1 = Jv_l - Jv_f,
\]

(16)

where \( v_f \) is the filtered intermediate control vector to be given later.

Note that the controller design procedure is essentially of backstepping type and the use of differential term \( \dot{v}_f \) will be unavoidable in the dynamic controller part (to be given later). Since differential operation is
difficult to implement in practice, here a command filter in the form of (5) is used to generate the virtual control signal and its differential term needed. For notation convenience, denote the output of the command filter as $v_f^d$ and $v_f^d$. And the input of the filter, called the nominal kinematic controller, is denoted as $v_f$. We now choose the nominal kinematic controller $v_f^+$ as:

$$v_f^+ = J^{-1}(ke_1 + \alpha e_1 + \beta e_1^p - \lambda_1 \xi + Jv_1),$$  
(17)

where $k_1 \in \mathbb{R}^{3 \times 3}$, $\lambda_1 \in \mathbb{R}^{3 \times 3}$, $\alpha \in \mathbb{R}^{3 \times 3}$, and $\beta \in \mathbb{R}^{3 \times 3}$ are positive definite diagonal matrices with $k_1 > 2\lambda_1$, $\lambda_1 > 1$, $p > 1$ and $0 < q < 1$ are ratios of two positive odds, and $\xi = [\xi, \xi, \xi, \xi]^T$ denotes the filtered signal to deal with this issue. In doing so, define the filtered compensating signal to be designed later.

The nominal function $v_f^+$ is then passed through a command filter (5) to generate the needed stabilizing control signal $v_f^d$ and its derivative $v_f^{dd}$.

The command filter may have errors if not appropriately designed and calibrated, which will in turn affect the performance. Here we introduce a compensating signal to deal with this issue. In doing so, define the filtered error $w_1 = v_f^d - v_f^d$. Then, the filtered compensating signal $\xi$ is generated by the following system:

$$\dot{\xi} = \begin{cases} -\lambda_1 \xi - \alpha \xi^p - \beta \xi^q - f_1 \xi + w_1, & \text{if } \|\xi\| > \theta_1 \\ 0, & \text{if } \|\xi\| \leq \theta_1 \end{cases}$$  
(18)

where $\theta_1 > 0$ is a small constant to be chosen, $f_1 = \frac{\|\xi\|_1 + \lambda_2 \xi_1^q}{\|\xi\|^2}$, $\lambda_2 \in \mathbb{R}^{3 \times 3}$ is a positive definite matrix to be chosen such that $\lambda_2 > \lambda_1 + 1$, and $\xi(0) = 0$.

To show that the designed controller in (17) can guarantee stabilization of the position tracking errors, define the following candidate Lyapunov function:

$$V_1 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} \xi^T \xi.$$  
(19)

It is obvious that $V_1$ in (19) is continuously differentiable, positive definite and radially unbounded.

According to (8), (16), (17), (18), Young’s inequality and Lemma 2, we have

$$\dot{V}_1 = e_1^T e_1 + \xi^T \xi 
= e_1^T (-k_1 e_1 - \alpha e_1^p - \beta e_1^q + \lambda_1 \xi) + \xi^T (-\lambda_1 \xi - \alpha \xi^p - \beta \xi^q - f_1 \xi + w_1) 
\leq -k_1 e_1^T e_1 - \alpha e_1^T e_1^p - \beta e_1^T e_1^q + \frac{\lambda_1}{2} (e_1^T e_1 + \xi^T \xi) - \lambda_1 \xi^T \xi - \alpha \xi^T \xi^p - \beta \xi^T \xi^q - \|w_1 e_1\| 
\leq -\frac{1}{2} \lambda_2 \|w_1\|^2 + \frac{1}{2} \xi^T \xi + \frac{1}{2} \xi^T \xi + w_1^T \xi_1 
\leq -\frac{1}{2} \lambda_2 \|w_1\|^2 + \frac{1}{2} \xi^T \xi + w_1^T \xi_1 
\leq -\frac{1}{2} \lambda_2 \|w_1\|^2 + \frac{1}{2} \xi^T \xi + w_1^T \xi_1 
\leq -\frac{1}{2} \lambda_2 \|w_1\|^2 + \frac{1}{2} \xi^T \xi + w_1^T \xi_1.
(20)

According to Lemma 1, we know that the position tracking errors are globally fixed-time stable.

Step 2 (Dynamic controller design): Consider velocity tracking error vector $e_2 = [e_{2u}, e_{2v}, e_{2w}]^T \in \mathbb{R}^3$ defined by

$$e_2 = v_f - v_f^d,$$  
(21)

whose time derivative is obtained by using (8) as:

$$\dot{e_2} = M_f^{-1} \{-D(v_f) v_f + g_f(\Theta_f) + \tau_f + d_f - M_f \dot{v}_f^d\}.$$  
(22)

Here, the ideal form of nonlinear controller $\tau_f = [\tau_{fu}, \tau_{fv}, \tau_{fw}]^T$ is designed as

$$\tau_f = -k_2 e_2 - \alpha e_1^p - \beta e_1^q - \dot{v}_f^d - M_f \dot{v}_f^d - d_f,$$  
(23)

where $k_2 \in \mathbb{R}^{3 \times 3}$ is a positive definite diagonal matrix, with $k_2 \geq \frac{1}{2}$, which is to be determined and $\dot{v}_f$ is given in (12).

In practice, $\varphi$ is very hard to obtain accurately. Hence, for implementation consideration, a radial basis function neural networks (RBFNNs) is employed to approximate $\varphi$ as follows:

$$\varphi(\bullet) = W^T \sigma(Z) + \epsilon(Z), \quad \forall \Omega \in \mathbb{R},$$  
(24)

where $Z \in \mathbb{R}^{3 \times 1}$ and $W \in \mathbb{R}^{3 \times 3}$ are the input and weight of the NN, $l > 0$ is the node number and $\Omega \in \mathbb{R}^{3 \times 1}$ is a compact set. $\epsilon(Z)$ is the approximation error having $\|\epsilon(Z)\| \leq \epsilon$. Since the activation function $\sigma(Z)$
is bounded, there exists a positive constant $\sigma^* \in \mathbb{R}$ such that $\|\sigma(Z)\| \leq \sigma^*$. So, the actual input can be written as:

$$\tau_f = -k_2 e_2 - \alpha e_2^p - \beta e_2^q + \epsilon_f^2 - \tilde{W}_T^* \sigma(v_f) - \tilde{d}_{f \text{max}},$$

(25)

where $\tilde{W}_f = [\tilde{W}_{u_f}, \tilde{W}_{v_f}, \tilde{W}_{\omega_f}]^T$ is an estimate of the NN weights $W$, $\tilde{d}_{f \text{max}} = [\tilde{d}_{u_f \text{max}}, \tilde{d}_{v_f \text{max}}, \tilde{d}_{\omega_f \text{max}}]^T$ is an estimate of the upper bound of external disturbances $d_{f \text{max}}$, whose update law are taken to be

$$\dot{\tilde{W}}_{sf} = \Gamma_*(\sigma_*(v_f)e_{2*} - k_{3*}\tilde{W}_{sf}), \quad * = u, v, \omega$$

$$\dot{\tilde{d}}_{f \text{max}} = A_* [e_{2*} - k_{4*}\tilde{d}_{f \text{max}}], \quad * = u, v, \omega$$

(26)

where $\Gamma_* \in \mathbb{R}^{T \times T}$. $A_* \in \mathbb{R}^{T \times T}$ are positive-definite design matrices, $k_{3*}$ and $k_{4*}$ are design constants.

Now, we show that the designed controller in (25) can guarantee stabilization of velocity tracking errors. Introduce the following radially unbounded candidate Lyapunov function:

$$V_2 = \frac{1}{2} e_f^T M_f e_2 + \frac{1}{2} \sum_{s = u, v, \omega} \tilde{W}_{sf}^T \Gamma_*^{-1} \tilde{W}_{sf}$$

$$+ \frac{1}{2} \sum_{s = u, v, \omega} A_*^{-1} \tilde{d}_{f \text{max}} \tilde{d}_{f \text{max}},$$

(27)

where $\tilde{W}_{sf} = \tilde{W}_{sf} - W_{sf}$ and $\tilde{d}_{f \text{max}} = \tilde{d}_{f \text{max}} - d_{f \text{max}}$. It is obvious that $V_2$ in (27) is continuously differentiable, positive definite and radially unbounded.

The time derivative of $V_2$ is given by

$$\dot{V}_2 = e_f^T M_f \dot{e}_2 + \frac{1}{2} \sum_{s = u, v, \omega} \tilde{W}_{sf}^T \Gamma_*^{-1} \tilde{W}_{sf}$$

$$+ \frac{1}{2} \sum_{s = u, v, \omega} A_*^{-1} \tilde{d}_{f \text{max}} \tilde{d}_{f \text{max}}.$$

(28)

According to (12), (22) and (25), we can obtain the following inequation:

$$e_f^T M_f \dot{e}_2$$

$$= e_f^T (\varphi_f + \tau_f + d_f) - M_f \dot{\epsilon}_f$$

$$= e_f^T \{W^T \sigma(v_f) + e(v_f) + d_f - W^T \sigma(v_f) - d_{f \text{max}}$$

$$+ \{ -k_2 e_2 - \alpha e_2^p - \beta e_2^q + \dot{\epsilon}_f \} - \dot{\epsilon}_f \}$$

$$= e_f^T (-W \sigma(v_f) - d_{f \text{max}} + e(v_f)) - k_2 e_2^2$$

$$- \alpha e_2^p e_2 - \beta e_2^q e_2^q$$

$$\leq -k_2 e_2^2 e_2 - \alpha e_2^p e_2 - \beta e_2^q e_2^q$$

$$+ M_f^{-1} \left( \frac{\sigma^{2}}{2} \| \tilde{W} \|^2 + \frac{3}{2} \tilde{W}^2 e_2 + \frac{1}{2} \| d_{f \text{max}} \|^2 + \frac{1}{2} \| \epsilon_f \|^2 \right).$$

Based on (28) and considering the following facts by completion of squares:

$$-W_{sf}^T W_{sf} \leq -\frac{\| W_{sf} \|^2}{2} + \frac{\| W_{sf} \|^2}{2},$$

$$-d_{sf \text{max}}^T d_{sf \text{max}} \leq -\frac{\| d_{sf \text{max}} \|^2}{2} + \frac{\| d_{sf \text{max}} \|^2}{2},$$

(30)

we can obtain the following inequation:

$$W_{sf}^T \Gamma_*^{-1} \dot{W}_{sf} + \dot{d}_{sf \text{max}} A_*^{-1} \dot{d}_{sf \text{max}}$$

$$= W_{sf}^T \left( \sigma_*(v_f)e_{2*} - k_{3*} \tilde{W}_{sf} \right) + \dot{d}_{sf \text{max}} \left( e_{2*} - k_{4*} \tilde{d}_{sf \text{max}} \right)$$

$$\leq W_{sf}^T \sigma_*(v_f)e_{2*} + \dot{d}_{sf \text{max}}^T e_{2*} - k_{3*} \| \tilde{W}_{sf} \|^2 + \frac{k_{3*} \| W_{sf} \|^2}{2}$$

$$- k_{4*} \| d_{sf \text{max}} \|^2 + \frac{k_{4*} \| d_{sf \text{max}} \|^2}{2}.$$
Then, combining (8), (30), (31) and Young’s inequality, the time derivative of $V_2$ is obtained as

$$V_2 = e_t^T M_f e_2 + \frac{1}{2} \sum_{s = u} \tilde{W}_{t, f} \Gamma_s^{-1} \tilde{W}_s + \frac{1}{2} \sum_{s = u} A_s^{-1} \tilde{d}_{s, \text{max}} \dot{d}_{s, \text{max}} = \sum_{s = u} k_{s, 3} \| \tilde{W}_{s, f} \|^{p + 1} + \sum_{s = u} k_{s, 4} \| \tilde{d}_{s, \text{max}} \|^{q + 1} + \sum_{s = u} k_{s, 4} \| d_{s, \text{max}} \|^{q + 1}$$

From Lemma 1, we know that the velocity tracking errors are practically fixed-time stable.

Finally, we proceed to show that the entire tracking errors system composed by (15) and (21) can be stabilized with the designed kinematic and dynamic controllers. Consider the following Lyapunov function for the whole system:

$$V = V_1 + V_2 = \frac{1}{2} (e_t^T e_1 + e_t^T e_2 + \xi^T \xi + \sum_{s = u} \tilde{W}_{t, f} \Gamma_s^{-1} \tilde{W}_s) + \frac{1}{2} \sum_{s = u} \Gamma_s^{-1} \tilde{d}_{s, \text{max}} \dot{d}_{s, \text{max}}$$

It is obvious that $V$ is continuously differentiable, positive definite and radially unbounded. According to (20), (25), (27), (28), (32), Young’s inequality and Lemma 2, we have

$$\dot{V} \leq -\alpha \frac{1}{M_f} - \beta \frac{1}{M_f} + \alpha_0 e_2 + \beta_0 \tilde{W}_s$$

where

$$\alpha_0 = \min \left\{ \alpha \left( \frac{2}{M_f} \right)^{\frac{1}{p + 1}}, 2 \frac{1}{p + 1} k_{s, 4} \Gamma_s^{-\frac{1}{p + 1}}, 2 \frac{1}{q + 1} k_{s, 4} A_s^{-\frac{1}{q + 1}} \right\}$$

and

$$\beta_0 = \min \left\{ \beta \left( \frac{2}{M_f} \right)^{\frac{1}{p + 1}}, 2 \frac{1}{p + 1} k_{s, 4} \Gamma_s^{-\frac{1}{p + 1}}, 2 \frac{1}{q + 1} k_{s, 4} A_s^{-\frac{1}{q + 1}} \right\}$$

Therefore, we have the following result.

**Theorem 1** For a group of autonomous underwater vehicles with kinematics and dynamics in (8) under dynamic control law (29) with update law (26), and kinematic control law (17) with command filter (5) and error compensator (15), the following results hold:

1. The formation tracking control can be achieved in a fixed time, denoted as $T_2$.
2. Signals $e_1$, $\xi$, $e_2$, $\tilde{W}$ and $\dot{d}_{s, \text{max}}$ in the closed-loop formation control system are all practically fixed-time stable.
**Proof:** The (34) can be rewritten as:

\[
\dot{V} \leq -\hat{\alpha}V^{\gamma_1} - \hat{\beta}V^{\gamma_2} + \vartheta_0,
\]

where

\[
\gamma_1 = \frac{p + 1}{2} > 1, 0 < \gamma_2 = \frac{q + 1}{2} < 1
\]

Moreover, assume that there exists an unknown constant \(\Delta\) and a compact set \(D\) such that

\[
D = \{(\tilde{W}, \tilde{d}_{f,\text{max}}) | \tilde{W} \leq \Delta, \tilde{d}_{f,\text{max}} \leq \Delta\}
\]

Then, we have

\[
\dot{V} \leq -\hat{\alpha}V^{\gamma_1} - \hat{\beta}V^{\gamma_2} + \vartheta,
\]

where

\[
\vartheta = \sum_{s=1}^{\omega} k_{s+} ||\Delta||^p + 1 + \sum_{s=1}^{\omega} k_{s+} ||\Delta||^q + 1 + \sum_{s=1}^{\omega} k_{s+} ||\Delta||^p + 1 + \sum_{s=1}^{\omega} k_{s+} ||W_x||^2 + \sum_{s=1}^{\omega} k_{s+} ||d_{f,\text{max}}||^2.
\]

(37)

Again, from Lemma 1, we have that the closed-loop formation tracking system is practically fixed-time stable, and the residual set of the solution of system (32) is calculated as

\[
\lim_{t \to T_s} x ||x|| \leq \min \left\{ \hat{\alpha}^{\gamma_1} \left( \frac{\vartheta}{1 - \theta} \right)^{\frac{1}{\gamma_1}}, \hat{\beta}^{\gamma_2} \left( \frac{\vartheta}{1 - \theta} \right)^{\frac{1}{\gamma_2}} \right\}
\]

(38)

with \(x = \{e_1, e_2, \xi, \tilde{W}, \tilde{d}_{f,\text{max}}\}\) and the settling time \(T_s\) satisfies

\[
T_s \leq T_{\text{max}} := \frac{1}{\hat{\alpha} \theta (\gamma_1 - 1)} + \frac{1}{\hat{\beta} \theta (1 - \gamma_2)}.
\]

(39)

This completes the proof.

**Remark 2** According to (39) we can see the maximum convergence time only depends on the controller parameters and parameter \(\theta\). Therefore, the presented method allows one to arbitrarily choose the convergence rate of the AUV formation, which makes it feasible for us to meet strict settling time requirements in practical applications. Moreover, the fixed-time algorithm can ensure a fixed settling time regardless of the initial states of AUVs.

| Table 1 Relationship between parameters and convergence rate |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(\hat{\alpha} \uparrow\) | \(\hat{\beta} \uparrow\) | \(\gamma_1\) | \(\gamma_2\) | \(\theta\) | convergence rate \(\uparrow\) |
| \(\hat{\alpha} \downarrow\) | \(\hat{\beta} \downarrow\) | \(\gamma_1\) | \(\gamma_2\) | \(\theta\) | convergence rate \(\uparrow\) |
| \(\hat{\alpha} \downarrow\) | \(\hat{\beta} \downarrow\) | \(\gamma_1\) | \(\gamma_2\) | \(\theta\) | convergence rate \(\uparrow\) |

**Remark 3** By selecting the controller parameters (i.e. \(\hat{\alpha}, \hat{\beta}, \gamma_1, \gamma_2\) and \(\theta\) to satisfy corresponding constraints as mentioned in the above discussions, we can guarantee a bound of the settling time as given in (39), which determines a certain convergence rate. Generally, the relation between the convergence rate and these parameters is shown in Table 1. We can find a set of optimal parameters to yield a minimum settling time by simply introducing a search algorithm like the seeker optimization algorithm and cuckoo search algorithm in [8,17].

**Remark 4** From the stability analysis and the definition of convergence time (39), we can see that the values of controller parameters do not affect the stability of the closed-loop system, although they may influence the convergence time. For a practical formation tracking control problem, the desired convergence time cannot be too short, otherwise, the closed-loop system may be unstable due to saturation constraint of the thruster. In addition, a feasible convergence time should also take the transient performance into consideration in the design procedure according to the maximum maneuver capability.

4 Formation Controller without Velocity Measurement

In many practical situations, it is difficult or even impossible to obtain the accurate velocity measurements due to technology limitation or environment disturbances. When the velocity of AUVs is not easily measurable, the state feedback control method in Section III cannot be implemented. Thus, finding a control method based on velocity estimation and position measurements is of great interest. Here we first give a fixed-time observer to estimate the AUV velocity \(v_f\), and then, an observer-based formation control method is presented, see Fig.
4.1 Fixed-time velocity observer

Before giving the velocity observer, a coordinate transformation is introduced.

Coordinate Transformation

Define the actual velocity vector in the frame \{E\} as follows:

\[
\chi = Jv_f,
\]

where \( \chi = [\chi_u, \chi_v, \chi_w]^T \in \mathbb{R}^3 \). Using this and the AU-V dynamics in (8), we have the following transformed dynamics:

\[
\begin{aligned}
\dot{\tilde{p}} &= \chi \\
\dot{\bar{\chi}} &= f(p, \chi) + \bar{\tau} 
\end{aligned}
\]

where \( f(p, \chi) = JM^{-1}(-D(J^{-1}\chi)J^{-1}\chi + g(\Theta) + d) \) and \( \bar{\tau} = JM^{-1}\tau \).

Velocity Observer Design

To estimate the velocity vector \( \chi = [\chi_u, \chi_v, \chi_w]^T \), we introduce the following observer:

\[
\begin{aligned}
\dot{\hat{p}} &= \hat{\chi} + \bar{\tilde{k}}_1\hat{e}_1 + \bar{\tilde{k}}_2\hat{e}_1^2 + \bar{\tilde{k}}_3\hat{e}_2^2 \\
\dot{\hat{\chi}} &= \bar{\tilde{\tau}} + \bar{\tilde{k}}_4\hat{e}_1 + \bar{\tilde{k}}_5\hat{e}_1^2 + \bar{\tilde{k}}_6\hat{e}_2^2 + f(p, \chi),
\end{aligned}
\]

where \( \hat{e}_1 = p - \tilde{p}, \bar{\tilde{k}}_i \in \mathbb{R}^{3 \times 3}, (i = 1, 2, 3, 4, 5, 6) \) are positive gain matrices and the parameters \( \bar{\tilde{e}}_i, \bar{\tilde{e}}_i, (i = 1, 2) \) are given by

\[
\frac{1}{2} < \bar{\tilde{e}}_1 < 1, \bar{\tilde{e}}_2 > 1, \bar{\tilde{e}}_1 = 2\bar{\tilde{e}}_1 - 1, \bar{\tilde{e}}_2 = 2\bar{\tilde{e}}_2 - 1.
\]

**Theorem 2** The states \((\hat{p}, \hat{\chi})\) of the velocity observer in (12) with parameters defined in (13) will globally converge to the real states \((p, \chi)\) in a fixed time \(T_0\) if Assumption 1 holds and each element in \(\bar{\tilde{k}}_i, i = 1, 2, 3, 4, 5, 6\) satisfies the following inequalities:

\[
2\bar{\tilde{k}}_3 > \bar{\tilde{k}}_2, 2\bar{\tilde{k}}_6 > \bar{\tilde{k}}_3, \frac{\bar{\tilde{k}}_3}{\bar{\tilde{k}}_1} - \frac{3}{2} \rho > 0, \bar{\tilde{k}}_1 - \frac{2\bar{\tilde{k}}_4}{\bar{\tilde{k}}_1} - \frac{\bar{\tilde{k}}_1}{2} - \frac{\bar{\tilde{k}}_1(\bar{\tilde{k}}_3 + \bar{\tilde{k}}_1)}{2} > 0.
\]

Namely, the observation error system as defined below is fixed-time stable:

\[
\begin{aligned}
\dot{\hat{e}}_1 &= -\bar{\tilde{k}}_1\hat{e}_1 - \bar{\tilde{k}}_2\hat{e}_1^2 - \bar{\tilde{k}}_3\hat{e}_2^2 + \hat{e}_2 \\
\dot{\hat{e}}_2 &= -\bar{\tilde{k}}_4\hat{e}_1 - \bar{\tilde{k}}_5\hat{e}_1^2 - \bar{\tilde{k}}_6\hat{e}_2^2 + \hat{e}_f,
\end{aligned}
\]

where \( \hat{e}_2 = \chi - \hat{\chi} \) and \( \hat{e}_f = f(p, \chi) - f(p, \hat{\chi}) \).

**Proof:** See Appendix

4.2 Fixed-time control with velocity observer

Here in this subsection, we present a velocity observer-based formation control method for AU-Vs. Replacing \( v_f \) in (25) by its estimated value \( \hat{v}_f \) in (42) yields the following control law:

\[
\tau_f = M_f \{-k_2(\hat{v}_f - v_f) - \alpha(\hat{v}_f - v_f^d)^2 - \beta(\hat{v}_f - v_f^d)^3 + \hat{v}_f^d \}
\]

\[
-W^T\sigma(\hat{v}_f - d^f_{max}),
\]

with all control parameters are the same with those in Theorem 1. For the velocity observer-based control law, we have the following result.

**Theorem 3** Consider system (10) under the control law (46) with \( \hat{v}_f \) generated by observer (47). The closed-loop system comprising (17) and (22) under Assumptions 1 and 2 is practically fixed-time stable if the controller parameters \( k_i, i = 1, 2, 3, 4 \), \( \lambda_1, \lambda_2, \alpha, \beta, p, q, \Lambda, \Gamma \) and the observer parameters \( \bar{\tilde{k}}_i, i = 1, 2, 3, 4, 5, 6 \), \( \bar{\tilde{e}}_i, \bar{\tilde{e}}_i, i = 1, 2 \) are selected as in Theorems 1 and 2, respectively. Furthermore, the convergence time is bounded by \( T \leq T_0 + T_s \).

**Proof:** It follows from Theorem 2 that there exists a finite time \( T_0 \) uniform in initial estimation errors \( \hat{e}_1(0) \) and \( \hat{e}_2(0) \) such that \( v_f(t) = \hat{v}_f(t) \) for \( t \geq T_0 \). As a result, the control law in (46) coincides with state feedback control law (23) for all \( t \geq T_0 \). Furthermore, if the system trajectory under control law (46) does not escape during interval \( t \in [0, T_0] \), it follows from Theorem 1 that there exists a finite time \( T_0 \) uniform in \( V(T_0) \) to ensure the fixed-time stability of the formation tracking system. Therefore, the condition that the closed-loop system under control law (46) does not escape in finite time is sufficient to derive the conclusion.
of Theorem 3. To complete the proof, let us consider the following Lyapunov function:
\[ V_3 = \frac{1}{2} \| \dot{e}_2 \|^2, \tag{47} \]
whose derivative along the trajectory of (22) under control law (40) is given by
\[ \dot{V}_3 = e_T^T (-k_2 \dot{e}_2 - \alpha e_2^p - \beta \dot{e}_2^q) = -\dot{e}_2 e_2 - \alpha e_2^p - \beta \dot{e}_2^q. \tag{48} \]
Since \( \dot{e}_2 = e_2 - J^{-1} \dot{e}_2 \), it follows that \( e_T^T \dot{e}_2 = e_T^T \| e_2 - J^{-1} \dot{e}_2 \| \cdot \text{sign}(e_2 - J^{-1} \dot{e}_2) \) for any \( \zeta > 0 \). To prove convergence of the system, two different cases are discussed.

**Case 1:** Assume that \( \| e_2 \| > \| J^{-1} \dot{e}_2 \| \), which implies sign \( e_2 - J^{-1} \dot{e}_2 = \text{sign}(e_2) \). Therefore, one has \( e_T^T \dot{e}_2 = \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \| \zeta \) for any \( \zeta > 0 \). Then, \( \dot{V}_3 \) in (48) can be rewritten as
\[ \dot{V}_3 = e_T^T (-k_2 (e_2 - J^{-1} \dot{e}_2)) - \alpha (e_2 - J^{-1} \dot{e}_2)^p - \beta \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \|^q. \tag{49} \]

**Case 2:** Otherwise, if \( \| e_2 \| \leq \| J^{-1} \dot{e}_2 \| \), since \( -e_T^T \dot{e}_2 \leq \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \| \zeta \) holds for any \( \zeta > 0 \) and \( e_2, \dot{e}_2 \in \mathbb{R}^3 \), we can have that
\[ \dot{V}_3 \leq k_2 \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \| + \alpha \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \|^p + \beta \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \|^q. \tag{50} \]
Taking into account \( \| e_2 \| \leq \| J^{-1} \dot{e}_2 \| \) and the well-known inequality \( \| a - b \| \leq (\| a \| + \| b \|)^s \), inequality (50) satisfies
\[ \dot{V}_3 \leq k_2 \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \| + \alpha \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \|^p + \beta \| e_2 \| \| e_2 - J^{-1} \dot{e}_2 \|^q \leq 2k_2 \| J^{-1} \dot{e}_2 \|^2 + \alpha 2^p \| J^{-1} \dot{e}_2 \|^p + \beta 2^q \| J^{-1} \dot{e}_2 \|^{q+1}. \tag{51} \]

Then, we will show that in both cases there exists a positive constant \( M \) such that \( \dot{V}_3 \leq M \) at any time. Since Theorem 2 ensures fixed-time convergence of \( \dot{e}_2 \), which implies boundness of \( \dot{e}_2 \), it follows that there exists a least upper bound of the right hand side of (51).

Denote the least upper bound by \( M = \sup \{ 2k_2 \| J^{-1} \dot{e}_2 \|^2 + \alpha 2^p \| J^{-1} \dot{e}_2 \|^{p+1} + \beta 2^q \| J^{-1} \dot{e}_2 \|^{q+1} \} \), it follows from (49) and (51) that \( \dot{V}_3 \leq M \). Therefore, \( \dot{V}_3 \) as well as the system states \( e_1, e_2 \) cannot escape in any finite time interval. From the above analysis, it can be concluded that the formation tracking system with velocity observer-based control law (46) does not escape in any finite time interval. Following the analysis at the beginning of the proof, one has that the closed-loop system under (17), (18), (12) and (16) is practically fixed-time stable. This completes the proof.

**Remark 5** Compared with asymptotic formation control method in (14) and finite time trajectory tracking control method in (23), the presented control scheme can achieve fixed-time convergence and higher control precision.

**Remark 6** The proposed method guarantees that the formation objective can be achieved within an arbitrary time. For practical AUVs subject to actuator saturations, the method is still useful but some amendments are needed and the settling time might be larger. Specifically, one can modify the system model using the auxiliary system technique in (18) or the adaptive approximation method in (23) to address the saturation issue.

## 5 Numerical Simulations

In this section, numerical simulations are given to verify the effectiveness of the proposed method. Without loss of generality, consider the following time-varying forces/moment disturbance in frame \{E\}:
\[ d = \begin{cases} 2 + 1.8 \sin(0.7t) & + 1.2 \sin(0.05t) \text{ N} \\ -0.8 + 2 \sin(0.1t) & + 1.5 \cos(0.06t) \text{ N} \\ 0.5 + 2 \sin(0.5t) & + 1.5 \cos(0.6t) \text{ N} \end{cases}. \tag{52} \]

### 5.1 State feedback formation control

Here the simulations are done for the proposed state feedback control law. Consider two AUVs with identical dynamics, structure and the following same set of model parameters [24]: \( M = \text{diag}(175.4, 140.8, 140.8) \), \( D(v) = \text{diag}(150 + 90|v|, 90 + 90|v|, 150 + 120|v|) \), \( W = 1148 \), \( B = 1108 \), and \( \psi = \pi/4 \), \( \theta = \pi/12 \), \( \phi = -\pi/8 \).

The desired relative distance between the AUVs in \{E\} is chosen as \( d_{f} = [2, 6, 6]^T \). The trajectory of the leader AUV is governed by:
\[ u_l = v_l = \omega_l = \begin{cases} 2 \text{ m/s} & \text{if } 0 \leq t < 10 \\ 2.5 \text{ m/s} & \text{else} \end{cases} \tag{53} \]
with the initial position \( p_1(0) = [0, 0, 0] \). The follower AUV’s initial position and velocity are given as \( p_f(0) = [0, -2, -4] \) and \( v_f(0) = [0, 0, 0] \).

The parameters used in the simulations are \( k_1 = \text{diag}(2, 4, 2) \), \( k_2 = \text{diag}(5, 15, 5) \), \( \lambda_1 = \text{diag}(2, 2, 2) \), \( \lambda_2 = \text{diag}(2, 2, 2) \), \( \lambda_3 = \text{diag}(2, 2, 2) \), \( \lambda_4 = \text{diag}(2, 2, 2) \), \( \lambda_5 = \text{diag}(2, 2, 2) \), \( \lambda_6 = \text{diag}(2, 2, 2) \), \( \lambda_7 = \text{diag}(2, 2, 2) \), \( \lambda_8 = \text{diag}(2, 2, 2) \), \( \lambda_9 = \text{diag}(2, 2, 2) \), and \( \lambda_{10} = \text{diag}(2, 2, 2) \).
diag(4, 4, 4), $\alpha = \text{diag}(1, 1, 1)$, $\beta = \text{diag}(2, 2, 2)$, $p = 5/3$, $q = 3/7$, $f_a = 2000$, $\zeta = 4/8$, $\Gamma = \text{diag}(0.1, 0.1, 0.1)$, $k_3 = \text{diag}(0.01, 0.01, 0.01)$, $\Delta = \text{diag}(0.1, 0.1, 0.1)$, and $k_4 = \text{diag}(0.01, 0.01, 0.01)$. The maximum settling time calculated is $T_{\text{max}} \approx 3$ s.

From Fig. 3 we can see that, under control law (25), the desired formation can be achieved within the maximum settling time. The trajectories of two AUVs are depicted in Fig. 3a. The position tracking errors $e_{1x}$, $e_{1y}$ and $e_{1z}$, and the velocity tracking error $e_2$ are, respectively, shown in Fig. 3b and 3c, all of which converge to zero after 2.8 seconds. Fig. 3d depicts the actual control input $\tau_f$ of the follower AUV.

To demonstrate that the settling time of the proposed method is independent of the initial states, we give simulations results under different initial states of the follower AUV $p_a = [0, -2, -4]^T$, $p_b = [1, -3, -5]^T$. For comparisons, the proposed method are compared with the finite-time controller and the controller in [20]. The results can be seen in Fig. 4, where the error $e_1 = e_{1x} + e_{1y} + e_{1z}$ is considered. Fig. 4 shows trajectories of two AUVs and the position tracking error $e_1$ under the proposed control law [25], the finite time controller and controller in [20] with different initial positions $p_a$ and $p_b$, respectively. From Fig. 4, one can see that the desired formation can all be achieved. From Fig. 4b, we can see that the fixed time controller has the same convergence time with different initial positions, while the finite time controller has different convergence times, the larger the initial position deviation, the larger convergence time, and the system under controller in [20] has larger overshoot and lower convergence speed.

5.2 Observer-based formation control

We first show that the states of the follower AUV can be estimated by the velocity observer (42) with parameters $k_1 = \text{diag}(2, 2, 2)$, $k_2 = k_3 = \text{diag}(5, 5, 5)$, $k_4 = k_5 = k_6 = \text{diag}(8, 8, 8)$, $\tau_1 = 0.8$, $\tau_2 = 1.2$, $\tau_1 = 0.6$ and $\tau_2 = 1.4$. Figs. 5-7 illustrate the simulation results.

Fig. 3. Formation tracking results with velocity measurement.

Fig. 4. Formation tracking results under different controllers with different initial conditions $p_a$ and $p_b$.

Fig. 5. The actual trajectory $p_f$ and the observed trajectory $\hat{p}_f$. 
Fig. 6. Position vector \( (p_f, \hat{p}_f) \) and velocity vector \( (v_f, \hat{v}_f) \).

Fig. 7. Position observation errors \( p_f - \hat{p}_f \) and velocity observation errors \( v_f - \hat{v}_f \).

5.3 Demonstration in the MSS simulator

We further demonstrate the method using the well-known high fidelity Marine System Simulator (MSS) [15], developed by the Department of Marine Technology, Norwegian University of Science and Technology. The MSS integrates hydrodynamics, structural mechanics, marine machinery, electric power generation and distribution, navigation and automatic control of marine vessels of various types (e.g., surface vessels, hydros, semi-subs). The simulator can better capture the hydrodynamic effects, generalized Coriolis and centripetal forces, nonlinear damping and current forces, and generalized restoring forces. It is composed of powerful modules including the environmental module, the vessel dynamics module, the thruster and shaft module, and the vessel control module.

The simulations are conducted based on the Naval Postgraduate School AUV in the MSS. To make the simulations closer to the practical situation, an ocean disturbance is introduced via the Gaussian Random Process in [52]. An input saturation is also considered, which is described by

\[
sat(\tau_f) = \begin{cases} 
\tau_f \text{sign}(\tau_{\text{max}}), & \text{if } |\tau_f| \geq \tau_{\text{max}} \\
\tau_f, & \text{else}
\end{cases}
\]

The simulations results are given in Figs. 9-11, which show, respectively, formation tracking control performance without velocity measurement, under input saturation and ocean disturbances. We can clearly see that, the proposed method is effective in all the three cases with the MSS simulator. For the case with saturations, the formation is achieved with a settling time 6 seconds slightly larger than the calculated value 3 seconds for
the nominal case without input saturation.

In this paper, the fixed-time formation tracking problem for two AUVs with and without velocity measurements has been studied. In contrast to the existing finite-time control methods, the fixed-time control scheme is independent of the initial conditions and has a more rapid convergence and higher accuracy. Using the fixed-time control theory, a fixed time state feedback controller has been proposed for the formation system. The command filter technique is incorporated to the backstepping scheme, together with an error compensator to eliminate the filtering error and an adaptive NN is introduced to overcome the model uncertainties and external disturbance. To obtain the velocity information for feedback, a coordinate transformation and a global fixed-time convergent state observer has been developed. Then, a fixed-time control scheme with velocity observer has been derived by combining the corresponding state feedback controller and the fixed-time convergent state observer together. Rigorous proof is shown that the formation control can be achieved in a fixed time regardless of the initial states while guaranteeing all tracking errors of the closed-loop formation control system are practically fixed-time stable. Simulation results illustrate the effectiveness of the proposed control scheme. Future work will focus on the extension of the results to formation control of heterogeneous AUVs considering the transmission limitations of underwater acoustic communication systems.

6 Conclusion

In this paper, the fixed-time formation tracking problem for two AUVs with and without velocity measurements has been studied. In contrast to the existing finite-time control methods, the fixed-time control scheme is independent of the initial conditions and has a more rapid convergence and higher accuracy. Using the fixed-time control theory, a fixed time state feedback controller has been proposed for the formation system. The command filter technique is incorporated to the backstepping scheme, together with an error compensator to eliminate the filtering error and an adaptive NN is introduced to overcome the model uncertainties and external disturbance. To obtain the velocity information for feedback, a coordinate transformation and a global fixed-time convergent state observer has been developed. Then, a fixed-time control scheme with velocity observer has been derived by combining the corresponding state feedback controller and the fixed-time convergent state observer together. Rigorous proof is shown that the formation control can be achieved in a fixed time regardless of the initial states while guaranteeing all tracking errors of the closed-loop formation control system are practically fixed-time stable. Simulation results illustrate the effectiveness of the proposed control scheme. Future work will focus on the extension of the results to formation control of heterogeneous AUVs considering the transmission limitations of underwater acoustic communication systems.

Appendix

PROOF THE THEOREM 2

We now give the proof, which contains three steps. First, we prove that the closed-loop system (43) with the parameters provided in (44) is globally asymptotically stable, which are followed by the proof that its approximating systems in 0-limit and ∞-limit are also globally asymptotically stable.

Step 1: In this step, we show that the error system (45) is globally asymptotically stable. Consider the following candidate Lyapunov function:

\[
\dot{V} = \frac{2\bar{k}_5}{1 + \bar{\epsilon}_1} (\bar{e}_1^T \bar{e}_1)^{\frac{1 + \bar{\epsilon}_1}{2}} + \frac{2\bar{k}_6}{1 + \bar{\epsilon}_2} (\bar{e}_1^T \bar{e}_1)^{\frac{1 + \bar{\epsilon}_2}{2}} + \frac{1}{2} (\bar{e}_2 - \bar{k}_3 \bar{e}_1)^T (\bar{e}_2 - \bar{k}_3 \bar{e}_1) + \frac{1}{2} \bar{e}_2^T \bar{e}_2.
\]

(54)

It is obvious that \( \dot{V} \) in (54) is continuously differentiable, positive definite and radially unbounded.
According to (45), the time derivative of $\dot{V}$ is

$$\dot{V} = 2k_5 (e_1^T)^{i_1} \dot{e}_1 + 2k_6 (e_1^T)^{i_2} \dot{e}_1$$
$$+ \dot{e}_2^T \dot{e}_2 + (\dot{e}_2 - k_5 \dot{e}_1)^T (\dot{e}_2 - k_5 \dot{e}_1)$$
$$= -2k_5 k_6 (e_1^T)^{-\frac{i_1+i_2}{2}} - k_5 k_6 (e_1^T)^{-\frac{i_1+i_2}{2}}$$
$$- 2k_3 k_5 (e_1^T)^{-\frac{i_2+i_1}{2}} - 2k_3 k_6 (e_1^T)^{-\frac{i_2+i_1}{2}}$$
$$- k_1 k_5 (e_1^T)^{-\frac{i_1+i_2}{2}} - 2k_3 k_6 (e_1^T)^{-\frac{i_2+i_1}{2}}$$
$$- k_4 (2 \dot{e}_2 - k_1 \dot{e}_1)^T \dot{e}_1 + (2 \dot{e}_2 - k_1 \dot{e}_1)^T \dot{e}_f$$
$$- k_1 (\dot{e}_2 - k_1 \dot{e}_1)^T (\dot{e}_2 - k_1 \dot{e}_1)$$
$$+ \dot{e}_1 \dot{e}_2 - k_1 \dot{e}_1)^T (k_3 e_1^{i_1} + k_5 e_1^{i_2})$$

(55)

On the basis of Assumption 1, we can obtain that

$$\langle 2 \dot{e}_2 - k_1 \dot{e}_1 \rangle^T \dot{e}_f$$
$$\leq (2 \dot{e}_2 - k_1 \dot{e}_1)^T \dot{e}_2$$
$$\leq \varrho (e_1^T)^{i_2} (e_2 - k_1 \dot{e}_1)^T \dot{e}_2$$
$$\leq \varrho (e_1^T)^{i_2} e_2 + \varrho (e_2 - k_1 \dot{e}_1)^T \dot{e}_2$$
$$\leq \frac{3}{2} \varrho (e_1^T)^{i_2} e_2$$

(56)

From the class Cauchy inequality, one obtains

$$- k_4 (\dot{e}_2 - k_1 \dot{e}_1)^T \dot{e}_1$$
$$= \frac{k_4}{k_1} (\dot{e}_2 - k_1 \dot{e}_1)^T (\dot{e}_2 - k_1 \dot{e}_1 - e_2)$$
$$= \frac{k_4}{k_1} (\dot{e}_2 - k_1 \dot{e}_1)^T (e_2 - k_1 \dot{e}_1 - e_2)$$
$$- \frac{k_4}{k_1} (\dot{e}_2 - k_1 \dot{e}_1)^T (e_2 - k_1 \dot{e}_1)$$
$$+ \frac{k_4}{2k_1} (\dot{e}_2 - k_1 \dot{e}_1)^T (\dot{e}_2 - k_1 \dot{e}_1)$$
$$+ \frac{k_4}{2k_1} (\dot{e}_2 - k_1 \dot{e}_1)^T (\dot{e}_2 - k_1 \dot{e}_1)$$

(57)

and

$$- k_4 e_2^T \dot{e}_1$$
$$= - \frac{k_4}{k_1} e_2^T \dot{e}_2 + \frac{k_4}{k_1} e_2^T (\dot{e}_2 - k_1 \dot{e}_1)$$
$$\leq - \frac{k_4}{k_1} e_2^T \dot{e}_2 + \frac{k_4}{k_1} (\dot{e}_2 - k_1 \dot{e}_1)^T (\dot{e}_2 - k_1 \dot{e}_1)$$
$$+ \frac{k_4}{4k_1} e_2^T \dot{e}_2$$

(58)

Also, from (43), and Young's inequality, we have that

$$k_1 (\dot{e}_2 - k_1 \dot{e}_1)^T (k_2 e_1^{i_2} + k_5 e_1^{i_1})$$
$$\leq k_1 k_2 (\dot{e}_2 - k_1 \dot{e}_1)^T e_1^{i_2} + k_1 k_5 (\dot{e}_2 - k_1 \dot{e}_1)^T e_1^{i_1}$$
$$\leq \frac{2}{2} k_1 k_2 \dot{e}_2 - k_1 \dot{e}_1)^T (\dot{e}_2 - k_1 \dot{e}_1) + \frac{k_1 k_2}{2} (e_1^T)^{i_1+i_2}$$
$$+ \frac{k_1 k_3}{2} (e_1^T)^{i_2+i_1}$$.

Then, substituting (56), (57), (58) and (59) into (55), we can have

$$\dot{V} \leq -2k_2 k_5 (e_1^T)^{i_1+i_2} - (k_1 k_5 - \frac{k_1 k_2}{2}) (e_1^T)^{i_1+i_2}$$
$$- 2k_3 k_6 (e_1^T)^{i_1+i_2} - (k_1 k_6 - \frac{k_1 k_2}{2}) (e_1^T)^{i_1+i_2}$$
$$- 2k_3 k_6 (e_1^T)^{i_1+i_2} - 2k_3 k_6 (e_1^T)^{i_1+i_2}$$
$$- \epsilon_1 e_2^T \dot{e}_2 + \epsilon_2 (\dot{e}_2 - k_1 \dot{e}_1)^T (\dot{e}_2 - k_1 \dot{e}_1)$$

(60)

where $\epsilon_1 = \frac{k_5}{4k_1} - \frac{3}{2} \varrho$, $\epsilon_2 = \frac{k_1 - \frac{2k_4}{k_1} - \frac{\varrho}{2} - \frac{k_1 (2k_5 + k_3)}{2}}{k_1}$.

With parameters $k_1$ selected according to (43), we have that $\epsilon_1 > 0$ and $\epsilon_2 > 0$. Then it follows from [57] that the error system in [44] is globally asymptotically stable.

Step 2: In this step, we prove that the approximating system of (45) in the 0-limit is homogeneous of a negative degree $l_0 < 0$ and is globally asymptotically stable. The system (45) is written as

$$\dot{\hat{e}}_1 = \epsilon_2 - k_3 e_1^{i_1} + \hat{g}_1 (\epsilon_1, \epsilon_2),$$
$$\dot{\hat{e}}_2 = -k_5 e_1^{i_1} + \hat{g}_2 (\epsilon_1, \epsilon_2),$$

(61)

where $\hat{g}_1 (\epsilon_1, \epsilon_2) = -k_1 \dot{e}_1 - k_3 e_1^{i_2}$ and $\hat{g}_2 (\epsilon_1, \epsilon_2) = -k_4 \dot{e}_1 - k_6 e_1^{i_1} + e_f$.

Since $\frac{1}{2} < \epsilon_1 < 1$ and $\epsilon_2 = 2 \epsilon_1 - 1$, one can see that the following system is homogeneous with degree $l_0 = -1$ with respect to $(r_1, r_2)$, where $r_1 = \frac{1}{1-r}$, $r_2 = 1 - r_1$, $r_1 = \epsilon_2 < \epsilon_1 < 1$.

$$\dot{\hat{e}}_1 = \epsilon_2 - k_3 e_1^{i_1},$$
$$\dot{\hat{e}}_2 = -k_5 e_1^{i_1}.$$

(62)

In addition, due to the fact that $0 < \epsilon_1, \epsilon_1 < 1, \epsilon_2 > 1$, and $\epsilon_2 > 1$, we have that $l_0 + r_1 = r_2 = \epsilon_2 r_1 < r_1 < \epsilon_2 r_1$ and $l_0 r_2 = \epsilon_2 r_1 < \epsilon_2 r_2$.

Hence, we have that

$$\lim_{\epsilon_1 \to 0} \hat{g}_1 (e_1^{i_1}, e_2^{i_2} e_2)$$
$$\leq \lim_{\epsilon_1 \to 0} -k_1 \dot{e}_1 - k_3 e_2^{i_2} e_1^{i_2} = 0,$$

$$\lim_{\epsilon_1 \to 0} \hat{g}_2 (e_1^{i_1}, e_2^{i_2} e_2)$$
$$\leq \lim_{\epsilon_1 \to 0} \frac{||k_4 e_1^{i_2} e_1 + k_6 e_2^{i_2} e_2|| + \varrho ||e_2^{i_2} e_2||}{e_2^{i_2} e_2} = 0.$$

Note that in the second inequality we have used Assumption 1. Therefore, the system in (62) is the approximating system of (45), which is homogeneous with a negative degree $l_0$.

We now show the approximating system is globally asymptotically stable. A candidate Lyapunov function
is chosen as: $\dot{V}_0 = \frac{k_5}{1+r_2} (e_1^T \dot{e}_1)^{\frac{1}{1+r_1}} + \frac{1}{2} e_2^T e_2$ with derivative $\dot{V}_0 \leq -k_2 k_5 (e_1^T \dot{e}_1)^{\frac{1}{1+r_1}} \leq 0$, which implies that $\dot{V}_0$ is non-increasing. We have that $\lim_{t \to \infty} \dot{V}_0 dt = 0$ exists and is finite. Since $V_0(t) \leq V_0(0)$, the errors $e_1$ and $e_2$ are bounded for $t > 0$. From (62), one can obtains that $\dot{e}_2$ is also bounded, implying that $V_0$ is uniformly continuous. According to Barbalat’s Lemma, it is concluded that $\lim_{t \to \infty} \dot{V}_0 = 0$, which implies that $\lim_{t \to \infty} k_5 k_2 (e_1^T \dot{e}_1)^{\frac{1}{1+r_1}} = 0$ and $\lim_{t \to \infty} k_5 k_2 (e_1^T \dot{e}_1)^{\frac{1}{1+r_1}} = 0$, which implies that $\lim_{t \to \infty} e_1 = 0$. Then we have that $\lim_{t \to \infty} k_5 k_2 (e_1^T \dot{e}_1)^{\frac{1}{1+r_1}} = 0$ and $\lim_{t \to \infty} \dot{e}_1 = 0$ with respect to (62) is globally asymptotically stable.

Step 3: This step shows that the approximating system of (45) in the infinite limit is homogenous of a positive degree $\ell_\infty > 0$ and is globally asymptotically stable. Similarly, rewriting system (45) as follows:

$$
\begin{align*}
\dot{e}_1 &= \dot{e}_2 - k_3 e_2^2 + \int f_1(e_1, e_2), \\
\dot{e}_2 &= -k_4 e_1^2 + \int f_2(e_1, e_2),
\end{align*}
$$

where $f_1(e_1, e_2) = -k_1 e_1 - k_2 e_1^2$ and $f_2(e_1, e_2) = -k_4 e_1 - k_5 e_1^2 + f$. Noting that $\epsilon_2 > 1, \epsilon_2 = 2\epsilon_2 - 1$, one can see that the following system (65) is homogeneous with degree $\ell_\infty = 1$ with respect to $(d_1, d_2)$, where $d_1 = \frac{1}{\epsilon_1 - 1}, d_2 = \frac{1}{\epsilon_2 - 1}$.

$$
\begin{align*}
\dot{e}_1 &= \dot{e}_2 - k_3 e_2^2, \\
\dot{e}_2 &= -k_4 e_1^2.
\end{align*}
$$

Furthermore, due to $0 < \epsilon_1, \epsilon_1 < 1, \epsilon_2 > 1$ and $\epsilon_2 > 1$, we have that $l_\infty + d_1 = d_2 = \epsilon_2 d_1 > d_1 > \epsilon_1 d_1$ and $l_\infty d_2 = \epsilon_2 d_1 > d_1 > \epsilon_1 d_1$. Therefore, based on Assumption 1, we can calculate that

$$
\begin{align*}
\lim_{\epsilon \to \infty} \frac{\int f_1(e_1^T e_1, d_2^T e_2)}{e_1^T + d_2^T} &\leq \lim_{\epsilon \to \infty} \frac{k_5 k_2 e_2^T e_2}{e_1^T + d_2^T} = 0, \\
\lim_{\epsilon \to \infty} \frac{\int f_2(e_1, e_2)}{e_1^T + d_2^T} &\leq \lim_{\epsilon \to \infty} \frac{k_5 k_2 e_2^T e_2}{e_1^T + d_2^T} = 0.
\end{align*}
$$

This means that system (65) is the approximating system of (45), which is also homogenous.

In order to show that the approximating system is stable, the following candidate Lyapunov function is chosen as: $\dot{V}_\infty = \frac{k_6}{1+r_2} (e_1^T \dot{e}_1)^{\frac{1}{1+r_1}} + \frac{1}{2} e_2^T e_2$, whose derivative is $\dot{V}_\infty \leq -k_2 k_5 (e_1^T \dot{e}_1)^{\frac{1}{1+r_1}}$. With the same analysis as in Step 2, we can prove that system (65) is globally asymptotically stable by using Barbalat’s Lemma.

Further, the mathematical expression of the settling time is given. According to Lemma 4, we have that there exists a continuous and positive candidate Lyapunov function $V^*$ which satisfies that

$$
\dot{V}^* \leq -k_1 \Gamma (V^*) \leq 2 \epsilon^2 f + \frac{1}{2} \epsilon^2 \gamma^2 (V^* - \gamma^2 (V^*)^2 < 0 \text{ if } \gamma^2 < 1 + V^* \gamma^2, \text{ which implies that } V^* \leq 1 \text{ can be satisfied in a finite time } T_1 \leq \frac{1}{k_1 \epsilon^2} \text{ for any initial states satisfying that } V^*(0) \geq 1. \text{ Further, for } V^* \leq 1, \text{ it obtains that } V^* \leq \Gamma (V^*) \leq V^* \gamma^2 \leq \gamma^2 (V^*)^2 \text{ for } \epsilon \leq \gamma^2 (V^*)^2 \leq 1, \text{ then Eq. (67) can be simplified as } \dot{V}^* \leq \frac{k_1 k_5}{2} \epsilon^2 f + \frac{1}{2} \epsilon^2 \gamma^2 \text{ which implies that } V^* = 0 \text{ can be obtained in a finite time } T_2 = \frac{2}{k_1 (\epsilon^2 - 1)}. \text{ Thus, based on the analysis above, we can obtain that the velocity estimate can be achieved in a fixed time } T_0 \leq T_1 + T_2, \text{ which is only related with the designed parameters with regardless of the initial states of the closed-loop system. This completes the proof.}

Declarations

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References

1. Li J, Du J, Sun Y, et al. Robust adaptive trajectory tracking control of underactuated autonomous under-
water vehicles with prescribed performance [J]. International Journal of Robust and Nonlinear Control, 2019, 29(14): 4629-4643.
2. Peng Z, Wang D, Wang H, et al. Distributed coordinated tracking of multiple autonomous underwater vehicles [J]. Nonlinear Dynamics, 2014, 78(2): 1261-1276.
3. Liu H, Lyu Y, Lewis F, et al. Robust time-varying formation control for multiple underwater vehicles subject to nonlinearities and uncertainties [J]. International Journal of Robust and Nonlinear Control, 2019, 29(9): 2712-2724.
4. Shojaei K. Three-dimensional tracking control of autonomous underwater vehicles with limited torque and without velocity sensors [J]. Robotics, 2018, 36(3): 374-394.
5. Gao Z, Guo G. Velocity free leader-follower formation control for autonomous underwater vehicles with line-of-sight range and angle constraints [J]. Information Sciences, 2019, 486: 359-378.
6. Shojaei K. Three-dimensional neural network tracking control of a moving target by underactuated autonomous underwater vehicles [J]. Neural Computing and Applications, 2019, 31(2): 509-521.
7. Li J, Du J, and Chang W. Robust time-varying formation control for underactuated autonomous underwater vehicles with disturbances under input saturation [J]. Ocean Engineering, 2019, 179: 180-188.
8. Blik L, Verstraete H R, Verhaegen, M, et al. Online optimization with costly and noisy measurements using random Fourier expansions [J]. IEEE Transactions on Neural Networks and Learning Systems, 2018, 29(1): 167-182.
9. Mahapatra S, and Subudhi B. Design of a steering control law for an autonomous underwater vehicle using nonlinear state feedback technique [J]. Nonlinear Dynamics, 2017, 90(2): 837-854.
10. Cui R, Yang C, Li Y, et al. Adaptive neural network control of AUVs with control input nonlinearities using reinforcement learning [J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2017, 47(6): 1019-1029.
11. Cui R, Ge S S, How B, et al. Leader-follower formation control of underactuated autonomous underwater vehicles [J]. Ocean Engineering, 2010, 37(17): 1491-1502.
12. Du H, Zhu W, Wen G, et al. Finite-time formation control for a group of quadrotor aircraft [J]. Aerospace Science and Technology, 2017, 69: 609-616.
13. Defoort M, Polyakov A, Demesure G, et al. Leader-follower fixed-time consensus for multi-agent systems with unknown non-linear inherent dynamics [J]. IET Control Theory & Applications, 2015, 9(14): 2165-2170.
14. Nonlinear Dynamics, 2014, 78(2): 1261-1276.
15. Fossen T I. and Pérez. Marine Systems Simulator (MSS). URL: https://github.com/cybergalactic/MSS, 2004.
16. Gao Z, and Guo G. Adaptive formation control of autonomous underwater vehicles with model uncertainties [J]. International Journal of Adaptive Control and Signal Processing, 2018, 32(7): 1067-1080.
17. Gandomi A H, Yang X S, and Alavi A H. Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems [J]. Engineering with computers, 2013, 29(1): 17-35.
18. Gao Z, and Guo G. Command filtered path tracking control of saturated ASVs based on time-varying disturbance observer [J]. Asian Journal of Control, 2020, 22(3): 1197-1210.
19. Hu Z, Ma C, and Zhang L. Formation control of impulsive networked autonomous underwater vehicles under fixed and switching topologies [J]. Neurocomputing, 2015, 147: 291-298.
20. Park B S. Adaptive formation control of underactuated autonomous underwater vehicles [J]. Ocean Engineering, 2015, 96: 1-7.
21. Jin X. Fault tolerant finite-time leader-follower formation control for autonomous surface vessels with LOS range and angle constraints [J]. Automatica, 2016, 68: 228-236.
22. Jiang B, Hu Q, and Friswell M I. Fixed-time attitude control for rigid spacecraft with actuator saturation and faults [J]. IEEE Transactions on Control Systems Technology, 2016, 24(5): 1892-1898.
23. Li S, Wang X, and Zhang L. Finite-time output feedback tracking control for autonomous underwater vehicles [J]. IEEE Journal of Oceanic Engineering, 2015, 40(3): 727-751.
24. Li S, and Wang X. Finite-time consensus and collision avoidance control algorithms for multiple AUVs [J]. Automatica, 2013, 49: 3359-3367.
25. Milli P, Orihuela L, Jurado I., et al. Formation control of autonomous underwater vehicles subject to communication delays [J]. IEEE Transactions on Control Systems Technology, 2014, 22(2): 770-777.
26. Peng Z, and Wan, J. Output-feedback path-following control of autonomous underwater vehicles based on an extended state observer and projection neural networks [J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2018, 48(4): 535-544.
27. Polyakov A. Nonlinear feedback design for fixed-time stabilization of linear control systems [J]. IEEE Transactions on Automatic Control, 2012, 57(8): 2106-2110.
28. Ríos P, Carreras M, Ribas, D, et al. Visual inspection of hydroelectric dams using an autonomous underwater vehicle [J]. Journal of Field Robotics, 2010, 27(6): 759-778.
29. Shojaei K. Neural network formation control of underactuated autonomous underwater vehicles with saturating actuators [J]. Neurocomputing, 2016, 194: 372-384.
30. Tian B, Zuo Z, Yan X, et al. A fixed-time output feedback control scheme for double integrator systems [J]. Automatica, 2017, 80: 17-24.
31. Wondergem M, Lefeber E, Pettersen K Y, et al. Output feedback tracking of ships [J]. IEEE Transactions on Control Systems Technology, 2011, 19(2): 442-448.
32. Zuo Z. Nonsingular fixed-time consensus tracking for second-order multi-agent networks [J]. Automatica, 2015, 54: 305-309.
33. Zhang L J, Qi X, and Pang Y J. Adaptive output feedback control based on DRFNN for AUV [J]. Ocean Engineering, 2009, 36(9-10): 716-722.
34. Zuo Z, and Tie L. Distributed robust finite-time nonlinear consensus protocols for multi-agent systems [J]. International Journal of Systems Science, 2016, 47(6): 1366-1375.