Instability of gravitational baryogenesis with fermions

E. Arbuzova\textsuperscript{a,b} and A. Dolgov\textsuperscript{a,c}

\textsuperscript{a}Novosibirsk State University, Novosibirsk, 630090, Russia
\textsuperscript{b}Department of Higher Mathematics, Dubna State University, Dubna, 141980, Russia
\textsuperscript{c}ITEP, Bol. Cheremushkinsaya ul., 25, 117218 Moscow, Russia
E-mail: arbuzova@uni-dubna.ru, dolgov@fe.infn.it

Received March 7, 2017
Revised May 14, 2017
Accepted May 22, 2017
Published June 1, 2017

Abstract. The derivative coupling of baryonic current to the curvature scalar in gravitational baryogenesis scenarios leads to higher order equations for gravitational field. It is shown that these equations are strongly unstable and destroy standard cosmology. This is a generalization of our earlier results obtained for scalar baryons to realistic fermions.

Keywords: baryon asymmetry, cosmology of theories beyond the SM, modified gravity

ArXiv ePrint: 1702.07477
1 Introduction

During the last decade scenarios of gravitational baryogenesis (GBG) [1] gained considerable popularity [2–11]. They present a modification of the good old scenario of spontaneous baryogenesis (SBG) [12–15] with the substitution instead of the (pseudo)goldstone field $\theta$ the curvature scalar $R$ coupled to the non-conserved baryon current:

$$\mathcal{L}_{\text{GBG}} = \frac{f}{m_0^2} (\partial_\mu R) J^\mu_B,$$

where $m_0$ is a constant parameter with dimension of mass and $f$ is dimensionless coupling constant which is introduced to allow for an arbitrary sign of the above expression.

GBG scenarios possess the same interesting and nice features of SBG, namely generation of cosmological asymmetry in thermal equilibrium without necessity of explicit C or CP violation in particle physics. However, an introduction of the derivative of the curvature scalar into the Lagrangian of the theory results in high order gravitational equations which are strongly unstable. The effects of this instability may drastically distort not only the usual cosmological history, but also the standard Newtonian gravitational dynamics. In our recent paper [16] we discovered such instability for scalar baryons and here we found similar effect for the more common spin one-half baryons (quarks).

2 Equations of GBG

We start from the action in the form:

$$A = \int d^4x \sqrt{-g} \left[ \frac{m_{\text{Pl}}^2}{16\pi} R - \mathcal{L}_m \right]$$

with

$$\mathcal{L}_m = \frac{i}{2} (\bar{Q} \gamma^\mu \nabla_\mu Q - \nabla_\mu \bar{Q} \gamma^\mu Q) - m_Q \bar{Q} Q$$

$$+ \frac{i}{2} (\bar{L} \gamma^\mu \nabla_\mu L - \nabla_\mu \bar{L} \gamma^\mu L) - m_L \bar{L} L$$

$$+ \frac{g}{m_X^2} \left[ (\bar{Q} Q^c) (\bar{Q} L) + (\bar{Q}^c Q) (\bar{L} Q) \right] + \frac{f}{m_0^2} (\partial_\mu R) J^\mu_B + \mathcal{L}_{\text{other}},$$
where $Q$ is the quark (or quark-like) field with non-zero baryonic number, $L$ is another fermionic field (lepton), $\nabla_\mu$ is the covariant derivative of Dirac fermion in the tetrad formalism (see e.g. lectures [17]), $J^\mu = \bar{Q} \gamma^\mu Q$ is the quark current with $\gamma^\mu$ being the curved space gamma-matrices, $\mathcal{L}_{\text{other}}$ describes all other forms of matter. The four-fermion interaction between quarks and leptons is introduced to ensure the necessary non-conservation of the baryon number with $m_X$ being a constant parameter with dimension of mass and $g$ being a dimensionless coupling constant. In grand unified theories $m_X$ is usually of the order of $10^{14}$–$10^{15}$ GeV.

The Lagrangian (2.2) leads to the following equations of motion for quarks:

$$i\gamma^\mu \nabla_\mu Q = m_Q Q - \frac{f}{m_0^2} (\partial_\mu R) \gamma^\mu Q + \frac{g}{m_X^2} \left[ 2Q^c (\bar{Q}L) + (\bar{Q} Q^c) L \right],$$
$$i\nabla_\mu Q \gamma^\mu = -m_Q \bar{Q} + \frac{f}{m_0^2} (\partial_\mu R) \bar{Q} \gamma^\mu + \frac{g}{m_X^2} \left[ 2\bar{Q}^c (\bar{L}Q) + \bar{L}(\bar{Q} Q^c) \right],$$  \hspace{1cm} (2.3)

and leptons:

$$i\gamma^\mu \nabla_\mu L = m_L L - \frac{g}{m_X^2} (\bar{Q}^c Q) \bar{Q},$$
$$i\nabla_\mu \bar{L} \gamma^\mu = -m_L \bar{L} + \frac{g}{m_X^2} (\bar{Q} Q^c) \bar{Q}.$$  \hspace{1cm} (2.4)

Note, that the fermionic part of Lagrangian (2.2), taken at the equations of motion of quarks and leptons, (2.3) and (2.4), does not vanish due to the interaction between them:

$$\mathcal{L}_m [\text{Eqs of motion}] = -\frac{g}{m_X^2} \left[ (\bar{Q} Q^c)(\bar{Q}L) + (\bar{Q} Q^c)(\bar{L}Q) \right],$$  \hspace{1cm} (2.5)

in contrast to the case of free fermions.

Taking variation of the action (2.1) over metric, $\delta A/\delta g^{\mu\nu}$, we obtain the equations for gravitational field in the form:

$$\frac{m_{Pl}^2}{8\pi} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = T_{\mu\nu}^m,$$  \hspace{1cm} (2.6)

where energy-momentum tensor, $T_{\mu\nu}^m$, is defined as:

$$T_{\mu\nu}^m = \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta A_m}{\delta g^{\mu\nu}}$$  \hspace{1cm} (2.7)

with

$$A_m = \int d^4 x \sqrt{-\tilde{g}} \mathcal{L}_m.$$  \hspace{1cm} (2.8)

The gravitational equations of motion, obtained this way, can be written as:

$$\frac{m_{Pl}^2}{8\pi} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = -g_{\mu\nu} \mathcal{L}_m + \frac{i}{4} \left[ (\bar{Q} (\gamma_\mu \nabla_\nu + \gamma_\nu \nabla_\mu) Q - (\nabla_\nu Q \gamma_\mu + \nabla_\mu Q \gamma_\nu) Q \right] + \frac{i}{4} \left[ (\bar{L} (\gamma_\mu \nabla_\nu + \gamma_\nu \nabla_\mu) L - (\nabla_\nu L \gamma_\mu + \nabla_\mu L \gamma_\nu) L \right]
- \frac{2f}{m_0^2} \left[ R_{\mu\nu} + g_{\mu\nu} D^2 - D_\mu D_\nu \right] D_\alpha J^\alpha
+ \frac{f}{2m_0^2} (J_\mu \partial_\nu R + J_\nu \partial_\mu R),$$  \hspace{1cm} (2.9)

where $D_\mu$ is the usual tensor covariant derivative in the background metric.
Taking trace of equation (2.9) with respect to $\mu$ and $\nu$ we obtain:

\begin{equation}
-\frac{m^2_{Pl}}{8\pi} R = m_Q\bar{Q}Q + m_L\bar{L}L + \frac{2g}{m_X} \left[ (\bar{Q}Q')(\bar{Q}L) + (\bar{Q}'Q)(\bar{L}Q) \right] - \frac{2f}{m_0} (R + 3D^2)D_\alpha J^\alpha + T_{\text{other}},
\end{equation}

where $T_{\text{other}}$ is the trace of the energy momentum tensor of all other fields. At relativistic stage, when masses are negligible, we can take $T_{\text{other}} = 0$. The average expectation value of the interaction term in eq. (2.10), which is proportional to $g$, is also small, especially at $T < m_X$, so the contribution of all matter fields may be neglected.

We see in what follows, that the kinetic equation (3.1) leads to an explicit dependence of the current divergence, $D_\alpha J^\alpha$, on $R$, if the current is not conserved. As a result we obtain high (fourth) order equation for $R$, as is discussed in the next section.

We can use an alternative representation of the quark field:

\begin{equation}
Q^2 = \exp(i f R/m_0^2) Q
\end{equation}

analogously to what is done in our paper [28]. The substitution of $Q^2$ instead of $Q$ results in the elimination of the term $f J^\mu \partial_\mu R/m_0^2$ in the Lagrangian (2.2) but the dependence on the curvature reappears in the interaction term as:

\begin{equation}
\frac{2g}{m_X} \left[ e^{-3i f R/m_0^2} (\bar{Q}_2 Q_2')(\bar{Q}_2 L) + e^{3i f R/m_0^2} (\bar{Q}_2 Q_2')(\bar{L} Q_2) \right].
\end{equation}

Nevertheless we obtain the same fourth order equation for the evolution of curvature, as in the case of the non-rotated field $Q$.

In what follows we study solutions of eq. (2.10) in cosmology in homogeneous and isotropic FRW background with the metric:

\begin{equation}
ds^2 = dt^2 - a(t)^2 dr^2.
\end{equation}

In this background the curvature is a function of time only and the covariant derivative acting on a vector $V^\alpha(t)$, which has only time component, has the form:

\begin{equation}
D_\alpha V^\alpha = (\partial_t + 3H)V^t,
\end{equation}

where $H = \dot{a}/a$ is the Hubble parameter.

### 3 Kinetic equation

Let us consider e.g. the reaction $q_1 + q_2 \leftrightarrow \bar{q}_3 + l_4$, where $q_1$ and $q_2$ are quarks with momenta $q_1$ and $q_2$, while $\bar{q}_3$ and $l_4$ are antiquark and lepton with momenta $q_3$ and $l_4$. We use the same notations for the particle symbol and for the particle momentum. The kinetic equation for the variation of the baryonic number density $n_B \equiv J^t$ through this reaction in the FRW background has the form:

\begin{equation}
(\partial_t + 3H)n_B = f_B^{\text{coll}},
\end{equation}
where the collision integral for space and time independent interaction is equal to:

\[
I_{B}^{\text{coll}} = -3B_q(2\pi)^4 \int d\nu_{q_1,q_2} d\nu_{\bar{q}_3,l_4} \delta^4(q_1 + q_2 - q_3 - l_4) \left[ |A(q_1 + q_2 \to \bar{q}_3 + l_4)|^2 f_{q_1} f_{q_2} - |A(\bar{q}_3 + l_4 \to q_1 + q_2)|^2 f_{\bar{q}_3} f_{l_4} \right],
\] (3.2)

where \(A(a \to b)\) is the amplitude of the transition from state \(a\) to state \(b\), \(B_q\) is the baryonic number of quark, \(f_a\) is the phase space distribution (the occupation number), and

\[
d\nu_{q_1,q_2} = \frac{d^3q_1}{2E_{q_1}(2\pi)^3} \frac{d^3q_2}{2E_{q_2}(2\pi)^3},
\] (3.3)

where \(E_q = \sqrt{q^2 + m^2}\) is the energy of particle with three-momentum \(q\) and mass \(m\). The element of phase space of final particles, \(d\nu_{\bar{q}_3,l_4}\), is defined analogously.

We neglect the Fermi suppression factors and the effects of gravity in the collision integral. This is generally a good approximation.

The calculations are strongly simplified if quarks and leptons are in equilibrium with respect to elastic scattering and annihilation. In this case their distribution functions take the form

\[
f = \frac{1}{e^{(E/T-\xi)} + 1} \approx e^{-E/T+\xi},
\] (3.4)

where \(\xi = \mu/T\) is dimensionless chemical potential, different for quarks, \(\xi_q\), and leptons, \(\xi_l\).

The assumption of kinetic equilibrium is well justified since it is usually enforced by very efficient elastic scattering. Equilibrium with respect to annihilation, say, into two channels: \(2\gamma\) and \(3\gamma\), implies the usual relation between chemical potentials of particles and antiparticles, \(\bar{\mu} = -\mu\). However, if we use the original representation for the quark fields, when they satisfy equations of motion (2.3), the conclusion of kinetic equilibrium is not evident because the quark evolution depends upon \(R(t)\), which may be quickly varying, as we see in what follows. At first sight the equilibrium distribution may not be able to keep pace with a fast variation of \(R\). This problem is absent in the representation (2.11), since \(R(t)\) neither enters the equation of motion, nor the amplitudes of elastic scattering and annihilation.

In representation (2.11) the baryonic number density is given by the expression:

\[
n_B = \int \frac{d^3q}{2E_q(2\pi)^3} (f_q - f_{\bar{q}}) = \frac{gsB_q}{6} \left( \mu + \frac{\xi^3}{\pi^2} \right) = \frac{gsB_qT^3}{6} \left( \xi + \frac{\xi^3}{\pi^2} \right),
\] (3.5)

where \(T\) is the cosmological plasma temperature, \(g_s\) and \(B_q\) are respectively the number of the spin states and the baryonic number of quarks.

Since the transition amplitudes, which enter the collision integral, are obtained by integration over time of the Lagrangian operator (2.12), taken between the initial and final states, the energy conservation delta-function in eq. (3.2) would be modified due to time dependent factors \(\exp[\pm 3i\int R(t)/m_0^2]\). In the simplest case, which is usually considered in gravitational (and spontaneous) baryogenesis, a slowly changing \(\dot{R}\) is taken, so we can approximate \(R(t) \approx \dot{R}(t) t\). For a constant \(\dot{R}\) the energy is not conserved but the energy
conservation condition is trivially modified, as

\[
\delta[E(q_1) + E(q_2) - E(q_3) - E(l_4)] \to \delta[E(q_1) + E(q_2) - E(q_3) - E(l_4) - 3f \dot{R}(t)/m_0^2].
\] (3.6)

Thus the energy is non-conserved due to the action of the external field \(R(t)\). Delta-function (3.6) is not precise, but the result is pretty close to it, if \(\dot{R}(t)\) changes very little during the effective time of the relevant reactions.

If the dimensionless chemical potentials \(\xi_q\) and \(\xi_l\), as well as \(f \dot{R}(t)/m_0^2/T\), are small, and the energy balance is ensured by the delta-function (3.6), the collision integral can be approximated as:

\[
I_{coll}^B \approx C I g^2 T^8 \left[ \frac{3f \dot{R}(t)}{m_0^2 T} - 3\xi_q + \xi_l \right],
\] (3.7)

where \(C_I\) is a positive dimensionless constant. The factor \(T^8\) appears for reactions with massless particles and the power eight is found from dimensional consideration. Because of conservation of the sum of baryonic and leptonic numbers \(\xi_l = -3\xi_q/3\).

The case of an essential variation of \(\dot{R}(t)\) is analogous to fast variation of \(\dot{\theta}(t)\) studied in our paper [28]. Clearly, it is much more complicated technically. Here we consider only the simple situation with quasi-stationary background and postpone more realistic time dependence of \(R(t)\) for the future work.

For small chemical potential the baryonic number density (3.5) is equal to

\[
n_B \approx \frac{g_s B_q}{6} \xi_q T^3,
\] (3.8)

and if the temperature adiabatically decreases in the course of the cosmological expansion, according to \(\dot{T} = -HT\), equation (3.1) turns into

\[
\dot{\xi}_q = \Gamma \left[ \frac{9f \dot{R}(t)}{10m_0^2 T} - \xi_q \right],
\] (3.9)

where \(\Gamma \sim g^2 T^5/m_X^4\) is the rate of B-nonconserving reactions.

If \(\Gamma\) is in a certain sense large, this equation can be solved in stationary point approximation as

\[
\xi_q = \xi_{eq} - \frac{\dot{\xi}_{eq}}{\Gamma},
\] (3.10)

where

\[
\xi_{eq} = \frac{9}{10} \frac{f \dot{R}}{m_0^2 T}.
\] (3.11)

This is the main conclusion of this section. If we substitute \(\xi_{eq}\) (3.11) into eq. (2.10) we arrive to the fourth order equation for \(R\), as it is described in the next section.

- 5 -
4 Curvature instability

According to the comment below eq. (2.10), the contribution of thermal matter into this equation can be neglected and we arrive to the very simple fourth order differential equation:

\[
\frac{d^4 R}{dt^4} = \lambda^4 R, \tag{4.1}
\]

where \( \lambda^4 = C_\lambda m_{\text{Pl}}^2 m_0^4 / T^2 \) with \( C_\lambda = 5/(36\pi f^2 g_s B_q) \). Deriving this equation we neglected the Hubble parameter factor in comparison with time derivatives of \( R \). It is justified a posteriori because the calculated \( \lambda \) is much larger than \( H \).

Equation (4.1) has the exponential solutions \( R \sim \exp(\mu_n t) \) with

\[
\mu_n = \lambda \exp(in\pi/2) \tag{4.2}
\]

with \( n = 0, 1, 2, 3 \). Evidently this equation has extremely unstable solution with instability time by far shorter than the cosmological time. This instability would lead to an explosive rise of \( R \), which may possibly be terminated by the nonlinear terms proportional to the product of \( H \) to lower derivatives of \( R \). Correspondingly one may expect stabilization when \( HR \sim \dot{R} \), i.e. \( H \sim \lambda \). Since

\[
\dot{H} + 2H^2 = -R/6, \tag{4.3}
\]

\( H \) would also exponentially rise together with \( R \), \( H \sim \exp(\lambda t) \) and \( \lambda H \sim R \). Thus stabilization may take place at \( R \sim \lambda^2 \sim m_{\text{Pl}}^2 m_0^4 / T \). This result is much larger than the normal General Relativity value \( R_{\text{GR}} \sim T_{\text{matter}} / m_{\text{Pl}}^2 \), where \( T_{\text{matter}} \) is the trace of the energy-momentum tensor of matter.

If \( \dot{R} \) is still slow, such that the energy balance condition (3.6) is fulfilled, but \( \dot{R} / (m_0^2 T) \) is large, then the asymmetry would also become large and the approximation of Boltzmann statistics becomes invalid. Nevertheless the equilibrium solution which annihilates the collision integral remains the same, (3.11), but for \( \xi \gtrsim 1 \) the cubic terms in the baryonic density (3.5) becomes essential and instead of equation (4.1) we obtain

\[
\frac{d^2}{dt^2} \left[ \dot{R} \left( 1 + \frac{3}{\pi^2} \left( \frac{9f \dot{R}}{10m_0^2 T} \right)^2 \right) \right] = \lambda^4 R. \tag{4.4}
\]

If the lower order derivatives of \( R \) still can be neglected we arrive to the simpler equation

\[
\frac{d^4 R}{dt^4} = \lambda^4 R \left[ 1 + \frac{243}{10\pi^2} \left( \frac{f \dot{R}}{m_0^2 T} \right)^2 \right]^{-1}, \tag{4.5}
\]

from which it is evident that the rise of \( R \) should terminate when \( \dot{R} \sim m_0^2 T / f \).
5 Discussion and conclusion

The considered here effect of strong instability in high order differential equations with small coefficient \( \epsilon \) in front of the highest derivative term is well known in mathematics but might be unexpected for a physicist. Even more surprising than the instability is a discontinuity of the limit \( \epsilon \to 0 \). If we take \( \epsilon = 0 \) from the very beginning, then the instability does not appear and the theory is reduced to the normal lower order one, while with any small but non-zero \( \epsilon \) the equation of motion has solutions which are absent for \( \epsilon = 0 \). Moreover, the smaller is \( \epsilon \), the faster is the rise of the unstable solution. Surprising at first sight, this is very well established fact, as one can check playing with simple model examples of higher order differential equations.

There is an apparent counterexample known in quantum field theory, namely, the decoupling of heavy modes. The low energy limit of a normal field theory is not sensitive to existence of very high mass particles. However, this is true only for stable equations of motion. The equations of motion may be higher order but the key words is “stable”. In the case considered in the present work the condition of stability is violated. The instability may be present at any \( \epsilon \), even very large one, but, we repeat, that the rate of the instability development is faster at smaller \( \epsilon \).

The found here effect of strong instability of high order differential equations with small coefficient, \( \epsilon \), in front of the highest derivative term, follows from the well known Lyapunov stability theory. In its classical form this theory is applied to the solutions of a system of generally non-linear first order differential equations. The infinitesimal variation of the solution under scrutiny leads to a homogeneous system of linear differential equations satisfied by these small variations. The properties of the solutions are determined by the eigenvalues of the characteristic determinant of the emerging system of these linear equations. If the eigenvalues are all negative or imaginary, then the solution is stable. Small fluctuations around it do not rise with time. Real positive eigenvalues induce an exponential rise of small fluctuations with time, leading to a strong deviation from the original solution. In this work we studied the issue of stability of the gravitational equations of motion which are reduced to the linearized fourth order equation (4.1). This equation can be trivially transformed to the classical Lyapunov system, though it is not necessary. All Lyapunov eigenvalues can be determined directly from eq. (4.1). One of the eigenvalues \( \mu_n \) (4.2) is positive and huge, so the rise of \( R \) is really fast.

For the subsequent discussion is convenient to consider the toy model governed by the linear 4th order equation of the form:

\[
\epsilon \frac{d^4 R}{dt^4} + cR = 0.
\] (5.1)

The case \( \epsilon = 0 \) in this toy example corresponds to the General Relativity limit.

Equations of similar kind arise in the models of F(R) modified gravity, suggested for the description of the observed cosmological acceleration. The effects established in these works are similar to those found in the present paper. Usually the coefficients in front of the highest derivative in such equations are assumed to be small. By assumption, in our toy model the approach to GR is expected, when \( \epsilon \to 0 \). If the equation of motion of modified theory is stable with respect to small perturbations, the limiting transition to GR is realized without problems. However, if the equation is unstable there is no transition to GR when \( \epsilon \) tends to zero, remaining small but non-vanishing. So the theories with \( \epsilon = 0 \) and with \( \epsilon \) arbitrary close to zero are very much different. There is no continuous limit as \( \epsilon \to 0 \).
Usually the modified gravitational equations are of higher order with respect to unmodified GR ones. If the former are unstable, the phenomenological implications of modified theories may be endangered. The applicability of such theories is determined by the characteristic time of the instability, $t_{\text{in}}$. If $t_{\text{in}}$ is sufficiently long, one may not worry about this instability, since it evolves very slowly to be observed. So we are in some kind of the crunch: large $\epsilon$ means large deviation from GR, while small $\epsilon$ may lead to fast instability if the equation is unstable. Such phenomenon was observed in $F(R) \sim 1/R^n$ [18, 19] theories as found in the work [20]. Accordingly to cure this type of instability the function $F$ was modified in the works [21–23]. However, further modification was found to be necessary because of emergence of past [24] and future singularities [25, 26].

The kind of instability found in our work on gravitational baryogenesis is mathematically the same as that discovered in the examples presented above. In all the unstable cases, studied in the published works, the limiting transition to GR with vanishingly small modification was not possible because of exponentially fast instability. The instability may be terminated due to non-linear terms in the equation but usually the stabilization takes place at the values of $R$ much larger than the canonical GR values. Though initially the corrections to GR may be very small, proportional to the small coefficient $\epsilon$, they rise exponentially and soon destroy GR. In this sense there is no limiting transition to GR at small $\epsilon$.

Returning to the 4th order equation (5.1), we see that the eigenvalues are $\mu_n = (-c/\epsilon)^{1/4}$. This equation has 4 roots, $n = 0, 1, 2, 3$, see eq. (4.2), and at least one of them has positive real part, independently on the sign of $c/\epsilon$. There is a known pathology in $\lambda \phi^4$-theory with negative $\lambda$. But in this example the equation of motion is second order and so is not directly related to our case.

There is another example of instability of high order equations, the so called Ostrogradsky instability. His original work is difficult to find but there is a good review of this instability in the paper [27]. In particular, an example of the Lagrange theory with higher derivative is considered there. The corresponding fourth order equation of motion (29) of the paper [27] is stable in the Lyapunov sense due to a special relation between the coefficients of the equation. In our case there is no such a relation and judging by the value of $\mu_n$ our equation is strongly unstable, in the sense that the instability is developed in very short time. Possibly the solution is stabilized by the nonlinearity of the equation, but at very high curvature.

So the simple version of the gravitational baryogenesis does not work, because of the found here instability. However, we cannot exclude that some further modification of this scenario may possibly cure its sickness.

In this work we have described only the basic features of the new effect of instability in gravitational baryogenesis with fermions. For a more accurate analysis numerical solution will be necessary. We plan to perform it in another work. The problem is very complicated technically, because the assumption of slow variation of $\dot{R}$ quickly becomes broken and the collision integral should be evaluated in time dependent background. This is by far not so easily tractable as the usual stationary one. We will also take into account finite integration limits over time. The technique for solving kinetic equation in non-stationary background is developed in ref. [28].

To conclude we have shown that gravitational baryogenesis in the simplest versions discussed in the literature is not realistic because the instability of the emerging gravitational equations destroys the standard cosmology. Some stabilization mechanism is strongly desirable. Probably stabilization may be achieved in a version of $F(R)$ modified gravity or by an introduction the formfactor $g(R)$ into the coupling (1.1), such that $g(R)$ drops down with rising $R$. 


Acknowledgments

We acknowledge the support of the Grant of President of Russian Federation for the leading scientific Schools of Russian Federation, NSh-9022-2016.2.

References

[1] H. Davoudiasl, R. Kitano, G.D. Kribs, H. Murayama and P.J. Steinhardt, Gravitational baryogenesis, Phys. Rev. Lett. 93 (2004) 201301 [hep-ph/0403019] [nSPIRE].
[2] G. Lambiase and G. Scarpetta, Baryogenesis in f(R): Theories of Gravity, Phys. Rev. D 74 (2006) 087504 [astro-ph/0610367] [nSPIRE].
[3] H.M. Sadjadi, A Note on Gravitational Baryogenesis, Phys. Rev. D 76 (2007) 123507 [arXiv:0709.0697] [nSPIRE].
[4] G. Lambiase, S. Mohanty and L. Pizza, Consequences of f(R)-theories of gravity on gravitational leptogenesis, Gen. Rel. Grav. 45 (2013) 1771 [arXiv:1212.6026] [nSPIRE].
[5] G. Lambiase, S. Mohanty and A.R. Prasanna, Neutrino coupling to cosmological background: A review on gravitational Baryo/Leptogenesis, Int. J. Mod. Phys. D 22 (2013) 1330030 [arXiv:1310.8459] [nSPIRE].
[6] M. Fukushima, S. Mizuno and K.-i. Maeda, Gravitational Baryogenesis after Anisotropic Inflation, Phys. Rev. D 93 (2016) 103513 [arXiv:1603.02403] [nSPIRE].
[7] A. Maleknejad, Gravitational leptogenesis in axion inflation with SU(2) gauge field, JCAP 12 (2016) 027 [arXiv:1604.06520] [nSPIRE].
[8] J.I. McDonald and G.M. Shore, Leptogenesis and gravity: baryon asymmetry without decays, Phys. Lett. B 766 (2017) 162 [arXiv:1604.08213] [nSPIRE].
[9] S.D. Odintsov and V.K. Oikonomou, Gauss-Bonnet gravitational baryogenesis, Phys. Lett. B 760 (2016) 259 [arXiv:1607.00545] [nSPIRE].
[10] V.K. Oikonomou, S. Pan and R.C. Nunes, Gravitational Baryogenesis in Running Vacuum models, arXiv:1610.01453 [nSPIRE].
[11] S.D. Odintsov and V.K. Oikonomou, Loop Quantum Cosmology Gravitational Baryogenesis, Europhys. Lett. 116 (2016) 49001 [arXiv:1610.02533] [nSPIRE].
[12] A.G. Cohen and D.B. Kaplan, Thermodynamic Generation of the Baryon Asymmetry, Phys. Lett. B 199 (1987) 251 [nSPIRE].
[13] A.G. Cohen and D.B. Kaplan, Spontaneous baryogenesis, Nacl. Phys. B 308 (1988) 913 [nSPIRE].
[14] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Spontaneous baryogenesis at the weak phase transition, Phys. Lett. B 263 (1991) 86 [nSPIRE].
[15] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Diffusion enhances spontaneous electroweak baryogenesis, Phys. Lett. B 336 (1994) 41 [hep-ph/9406348] [nSPIRE].
[16] E.V. Arbuzova and A.D. Dolgov, Intrinsic problems of the gravitational baryogenesis, Phys. Lett. B 769 (2017) 171 [arXiv:1612.06206] [nSPIRE].
[17] I.L. Shapiro, Covariant derivative of fermions and all that, arXiv:1611.02263 [nSPIRE].
[18] S. Capozziello, S. Carloni and A. Troisi, Quintessence without scalar fields, Recent Res. Dev. Astron. Astrophys. 1 (2003) 625 [astro-ph/0303041] [nSPIRE].
[19] S.M. Carroll, V. Duvvuri, M. Trodden and M.S. Turner, Is cosmic speed-up due to new gravitational physics?, Phys. Rev. D 70 (2004) 043528 [astro-ph/0306438] [nSPIRE].
[20] A.D. Dolgov and M. Kawasaki, *Can modified gravity explain accelerated cosmic expansion?*, *Phys. Lett. B* 573 (2003) 1 [astro-ph/0307285] [arXiv:0706.2041] [SPIRE].

[21] A.A. Starobinsky, *Disappearing cosmological constant in f(R) gravity*, *JETP Lett. 86* (2007) 157 [arXiv:0705.1158] [SPIRE].

[22] W. Hu and I. Sawicki, *Models of f(R) Cosmic Acceleration that Evade Solar-System Tests*, *Phys. Rev. D* 76 (2007) 064004 [arXiv:0705.1158] [SPIRE].

[23] S.A. Appleby and R.A. Battye, *Do consistent F(R) models mimic General Relativity plus Λ?*, *Phys. Lett. B* 654 (2007) 7 [arXiv:0705.3199] [SPIRE].

[24] S.A. Appleby, R.A. Battye and A.A. Starobinsky, *Curing singularities in cosmological evolution of F(R) gravity*, *JCAP* 06 (2010) 005 [arXiv:0909.1737] [SPIRE].

[25] A.V. Frolov, *A Singularity Problem with f(R) Dark Energy*, *Phys. Rev. Lett.* 101 (2008) 061103 [arXiv:0803.2500] [SPIRE].

[26] E.V. Arbuzova and A.D. Dolgov, *Explosive phenomena in modified gravity*, *Phys. Lett. B* 700 (2011) 289 [arXiv:1012.1963] [SPIRE].

[27] R.P. Woodard, *Ostrogradsky’s theorem on Hamiltonian instability*, *Scholarpedia* 10 (2015) 32243 [arXiv:1506.02210] [SPIRE].

[28] E.V. Arbuzova, A.D. Dolgov and V.A. Novikov, *General properties and kinetics of spontaneous baryogenesis*, *Phys. Rev. D* 94 (2016) 123501 [arXiv:1607.01247] [SPIRE].