Estimation of the coefficient of permeability in deep sedimentary ground

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ABSTRACT

Since 1996, pore water pressures have been continuously measured every 10 min at depths of 80 m (GL-80 m), 402 m (GL-402 m) and 403 m (GL-403 m) on the Tsudanuma campus of the Chiba Institute of Technology in Narashino, Japan. For a long period during the measurements, the periodic behavior of pore water pressures in the span of a year was such that pore water pressures tended to become low in summer and high in winter. Furthermore, pore water pressures were greatly affected by groundwater withdrawal by some cities in the vicinity of the field site. In this study, the permeability in deep sedimentary ground was predicted using two main methods: The Theis and Jacob methods based on the aquifer test theory and Creager’s and Hazen’s empirical equations. For the Theis and Jacob methods, pore water pressure data at the field site and groundwater withdrawal data for some cities surrounding the field site were employed to predict the coefficient of permeability k. For Creager’s and Hazen’s empirical equations, the values of k were calculated using the grain size analysis results of 37 soil samples collected at a field site. The results obtained from these methods were examined for their applicability to the prediction of permeability in deep sedimentary ground.

Keywords: deep sedimentary ground, pore water pressure, coefficient of permeability

1 INTRODUCTION

Pollution control is a very important issue, and numerical advection–diffusion models are frequently used to predict groundwater behavior and the spread of contaminants.

Analyses using such models require parameters such as the coefficients of permeability and storage. There are various methods for determining these coefficients, but accurate estimates can be obtained from in situ tests. However, performing in situ permeability tests in deep ground is particularly inefficient as it is time-consuming and expensive. Thus, using soil samples collected from land sites, these parameters can be obtained alternatively from laboratory permeability tests or other estimation methods using soil properties such as grain size and void ratio. The traditional estimation methods for these parameters are usually applicable for shallow ground and have not yet been clarified for deep ground.

In this study, a fundamental investigation was performed to predict the coefficient of permeability in deep ground using pore water pressure observation data and laboratory soil tests on samples collected from a field site.

2 EXPERIMENTAL

In 1992, pore water pressure transducers were first installed at depths of 40 m (GL-40 m) and 80 m (GL-80 m) from the ground surface on the Tsudanuma campus of the Chiba Institute of Technology in Narashino, Japan (Fig. 1). Three additional transducers were installed at depths of 230 m (GL-230 m), 402 m (GL-402 m) and 403 m (GL-403 m) in 1994. Unfortunately, the monitoring of pore water pressures at GL-40 m and GL-230 m had to be discontinued owing to transducer malfunctions. However, the remaining transducers (GL-80 m, GL-402 m and GL-403 m) continued to work properly, and the pore water pressures at those locations have been recorded every 10 min since 1996. At the measurement site, pore water pressure transducers were embedded in the sand layers just above the clay layers (impermeable layers).

In several clay subsoil zones, at depths of about 125 m, 210 m and 300 m, perforated pipes were grouted with cement mortar to ensure water tightness and prevent the ingress of groundwater between clay layers.
The variations in pore water pressures observed in 2005 at GL-80 m and GL-402 m are shown in Fig. 2. As seen in the figure, pressures became low in summer and high in winter. Such seasonal pressure changes have been observed every measurement year. Furthermore, compared with pore water pressure behaviour at GL-80 m, similar seasonal behaviour was observed at GL-402 m with a delay of approximately three months. Sato et al.\(^3\) reported a good correlation between the seasonal change of pore water pressure and groundwater withdrawals. By taking account of the findings based on correlation analyses between pore water pressure data and annual groundwater withdrawals in Chiba prefecture, Itto et al.\(^3\) found that withdrawals by Funabashi and Ichikawa cities primarily influence the pore water pressure at GL-80 m, whereas Kashiwa city influences at GL-402 m.

![Fig. 2. Observation of pore water pressures at GL-80 m and GL-402 m in 2005.](image)

Fig. 3 shows a geological cross section illustrating the Shimoso Plateau in northern Chiba Prefecture (the northern area of the Boso Peninsula). As shown in the figure, the measurement site is marked, and the depths of the pore water pressure transducers are indicated. It can be seen that the transducers at GL-80 m and GL-402 m are embedded in the Narita and upper Kazusa groups of the Quaternary deposit, respectively. In general, the Kazusa group is widely distributed between the Boso and Miura Peninsulas and Tama Hill. The upper layer of the Kazusa group, a sedimentary stratum, overlies the present continental slope. Consequently, the Narita and Shimosa groups, deposited on the Kazusa group, are inclined with respect to the pore water pressure measurement site, as shown in Fig. 3. Fig. 4 illustrates the simplified geographical cross section around the measurement site. As shown in the figure, the measurement site is above the inclined strata, and the measurement points at GL-80 m and GL-402 m are located in the Narita and Kazusa groups, respectively.\(^5\) The different behaviours of pore water pressures at GL-80 m and GL-402 m seem to be influenced by strong contributions of groundwater withdrawal in different regions (Funabashi and Ichikawa for GL-80 m and Kashiwa for GL-402 m). Furthermore, the pore water pressure at GL-402 m decreases approximately three months later than that of GL-80 m during the summer. This is due to the long distance between the measurement site and the groundwater withdrawal region.

![Fig. 3. Geographical west east cross section of the northern Boso peninsula including the pore water pressure measurement site.](image)

![Fig. 4. Inclined strata and depths of groundwater withdrawals.](image)
Fig. 5 shows pore water pressure observations for the past 10 years. Overall, pressures tend to increase year by year. Typically, groundwater is withdrawn for agricultural usage from April to August in the whole Chiba Prefecture, but for other usages, it is withdrawn at a constant rate throughout the year. In addition, the annual withdrawal amount has been gradually decreasing for decades.

Since \( Q_p/4\pi T \) in Eq. (2) and \( S/4T \) in Eq. (3) are constant, the relationship between \( W(\lambda) \) and \( \lambda \) becomes the same as the relationship between \( s \) and \( r^2/t \). The values of \( S \) and \( T \) can be determined by superimposing and sliding a plot of \( s \) vs. \( r^2/t \) over a plot of \( W(\lambda) \) vs. \( \lambda \) in logarithmic scale to obtain the best possible matching of the two plotting curves, as shown in Fig. 6. Let the best matching point of each plot be \( \lambda_{m}, W(\lambda)_{m} \) and \( [(r^2/t)_{m}, S_{m}] \), the transmissivity \( T \), coefficient of permeability \( k \) and coefficient of storage \( S \) can be obtained from the following equations:

\[
T = \frac{Q_p}{4\pi S_m} \quad (5)
\]

\[
k = \frac{100}{60} \left( \frac{T}{D} \right) \quad (6)
\]

\[
S = 4T \frac{\lambda_m}{(r^2/t)_{m}} \quad (7)
\]

where \( D \) is the aquifer thickness.

Fig. 6. Example of matching of a Theis-type curve to time-drawdown data.

### 3 ESTIMATION METHOD FOR COEFFICIENT OF PERMEABILITY

The methods for estimating the coefficient of permeability used in this study are Theis\(^4\) and Jacob\(^4\) methods based on the aquifer test theory and empirical formulas such as Creager’s and Hazen’s equations.

#### 3.1 Theis method

When a pumping test is performed in non equilibrium or unsteady state flow conditions, the governing equation of the Theis problem for a confined aquifer in a polar coordinate system becomes

\[
\frac{\partial s}{\partial t} = \frac{T}{S} \left( \frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} \right) \quad (1)
\]

where \( r \) is the radial distance from the pumping well to the observation well, \( S \) is coefficient of storage, \( s \) is drawdown in the pumping well, \( T \) is transmissivity and \( t \) is the time since pumping started.

The solution of the governing equation with the initial condition (\( s = 0 \) for \( t = 0 \)) and boundary conditions (\( s = 0 \) for \( r = \infty \), \( t > 0 \) and \( Q = Q_p \) for \( r = 0 \), \( t > 0 \)) lead to the following equations:

\[
s = \frac{Q_p}{4\pi T} W(\lambda) \quad (2)
\]

\[
\lambda = \frac{Sr^2}{4Tt} \quad (3)
\]

\[
W(\lambda) = -0.5772 - \ln \lambda + \frac{\lambda^2}{2 \cdot 2!} - \frac{\lambda^3}{3 \cdot 3!} + \cdots \quad (4)
\]

where \( Q_p \) is the well pumping rate and \( W(\lambda) \) is the well function.

#### 3.2 Jacob method

The Jacob method is useful for determining the transmissivity and coefficient of storage for a confined aquifer. The analysis involves matching a straight line to drawdown data plotted as a function of logarithmic time from the start of pumping.

Fig. 7 shows a plot of \( s \) as function of log \((t/r^2)\) on semi-logarithmic axes and a straight line fitted for the late time data. In the figure, by determining the slope \( \Delta s \) of the fitted line and its intercept on the x-axis \((dr^2)_{int}, \) values of \( T, k \) and \( S \) can be obtained from the following equations:

\[
T = \frac{2.3Q_p}{4\pi \Delta s} \quad (8)
\]

\[
k = \frac{100}{60} \left( \frac{T}{D} \right) \quad (9)
\]

\[
S = 2.25 T (t/r^2)_{int} \quad (10)
\]
3.3 Hazen’s and Creager’s empirical equations

Hazen proposed an empirical relationship for the coefficient of permeability as follows:

\[ k = C_h D_{10}^{0.5} \text{ (cm/sec)} \]  \hspace{1cm} (11)

where \( C_h \) is a constant varying from 1 to 1.5 and \( D_{10} \) is the 10% passing grain size (cm).

Similarly, Creager’s equation \( k = 0.359 \cdot D_{20}^{3.27} \text{ (cm/sec)} \) \hspace{1cm} (12)

where \( D_{20} \) is the 20% passing grain size (cm).

There are many traditional empirical equations proposed for estimating permeability using soil properties such as grain size and void ratio. However, Hazen’s and Creager’s equations, applicable for fine to coarse sand as well as uniformly graded sand, can determine soil permeability.

4 COMPARISON OF METHODS FOR ESTIMATING PERMEABILITY

Fig. 8 shows the grain size distribution curves obtained from 37 soil samples collected from the pore water pressure measurement site during installation of the pore pressure transducers. The coefficient of permeability \( k \) was calculated by Eqs. (11) and (12) using the soil grain size at 10% and 20% cumulative weight, respectively, for each soil sample from the grain size distribution curve, as shown in Fig. 8. The values of \( k \) estimated from Eqs. (11) and (12) are shown in Fig. 9 for all of the soil samples. Table 1 also indicates the values of \( k \) at GL-80 m and GL-402 m, where pore water pressures were measured. It can be seen that the values of \( k \) estimated by Hazen’s equation are at most one order of magnitude larger than those of Creager’s for most of the soil samples. At GL-80 m and GL-402 m, the values of \( k \) obtained by Hazen were 2.6 and 6.6 times larger than those obtained by Creager, respectively.

| Method     | \( k \) (cm/s) at GL-80 m | \( k \) (cm/s) at GL-402 m |
|------------|--------------------------|---------------------------|
| Hazen      | \( 4.05 \times 10^{-2} \) | \( 2.45 \times 10^{-2} \) |
| Creager    | \( 1.56 \times 10^{-2} \) | \( 3.70 \times 10^{-3} \) |

Fig. 8. Grain size distributions for 37 soil samples collected at the pore water pressure measurement site.
For the Theis and Jacob methods, values of $k$ at GL-80 m and GL-402 m were estimated using the groundwater withdrawal data for Funabashi and Ichikawa cities, respectively. The data employed were collected in 2004 and 2005\(^6\). The results estimated using the Theis and Jacob methods are shown in Table 2 and Fig. 9.

Table 2. Values of $k$ from the Theis and Jacob methods at GL-80 m and GL-402 m.

| Method | $k$ (cm/s) |
|--------|------------|
|        | GL-80 m    | GL-402 m   |
| Theis  | $1.06 \times 10^{-2}$ | $0.70 \times 10^{-2}$ |
| Jacob  | $2.63 \times 10^{-2}$ | $1.81 \times 10^{-2}$ |

In a comparison between the Theis and Jacob methods, the variation in estimated permeability is within one order of magnitude for both GL-80 m and GL-402 m. The estimated values of $k$ using the Jacob method were 2.5 and 2.6 times larger than those obtained using Theis at GL-80 m and GL-402 m, respectively. Moreover, compared with the empirical equations, the values of $k$ from the Theis and Jacob methods are very close (same order of magnitude) to those estimated by Creager’s and Hazen’s equations. Overall, it can be said that the permeability calculated by the considered empirical equations at the compared depths are nearly the same as those estimated by the Theis and Jacob methods, both of which provide more reliable and accurate results at these depths.

5 CONCLUSIONS

In this study, the permeability in deep sedimentary ground was predicted using two methods: 1) the Theis and Jacob methods based on the aquifer test theory, which produced reliable and accurate results, and 2) Creager’s and Hazen’s empirical equations, which are simple calculations based on grain size distributions. Overall, the permeability obtained by the empirical equations seems to be quite close to those estimated using the Theis and Jacob methods. Discrepancies in permeability estimates between the considered methods are within one order of magnitude at the compared depths. Thus, it can be concluded that the use of empirical equations involving relatively simple and inexpensive laboratory testing is possible for predicting permeability in deep sedimentary ground compared with results obtained from the Theis and Jacob methods, which involve time-consuming and expensive in situ testing.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the support from the JSPS KAKENHI Grant Number 24560668.

REFERENCES

1) Creager, W.P., Justin, J.D., Hinds, J. (1945): Engineering for dams, Earth, Rock-fill, Steel and Timber Dams, III, Wiley, New York, 648-649.
2) Hazen, A. (1930): Water supply, in American Civil Engineering Handbook, Wiley, New York.
3) Itto, Y., Komiya, K., Watanabe, T. (2009): Annual behavior of pore water pressure deep sedimentary ground, Proceedings of the 64th Annual Meeting of JSCE, III-286, 571-572 (in Japanese).
4) Japanese Geotechnical Society (1997): Site investigation methods, 288-304.
5) Komiya, K. (2010): Pore water pressure profiles in deep sedimentary ground and pressure changes due to shield tunneling, Journal of the Japanese Geotechnical Society, 58(4), 28-31 (in Japanese).
6) Ministry of the Environment (2005), National ground environment information directory.
7) Sato, K., Komiya, K., Watanabe, T. (2007): Effect of groundwater pumping on pore water pressure in deep sedimentary ground. Proceedings of 42nd Annual Meeting of JGS (in Japanese).