Velocity-dependent energy gaps and dynamics of superfluid neutron stars

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ABSTRACT

We show that suppression of the baryon energy gaps, caused by the relative motion of superfluid and normal liquid components, can substantially influence dynamical properties and evolution of neutron stars. This effect has been previously ignored in the neutron-star literature.

Key words: stars: neutron – stars: oscillations – stars: interiors.

1 INTRODUCTION

According to numerous microscopic calculations (e.g., Yakovlev, Levenfish & Shibanov 1999 [Lombardo & Schulze 2001] and references therein), nucleons and hyperons in the internal layers of neutron stars (NSs) can become superfluid at temperatures \( T \lesssim 10^8 \div 10^{10} \) K. Superfluidity has a strong impact on the thermal evolution of NSs, their oscillations, and (most probably) leads to such observational phenomena as glitches (Anderson & Itoh 1975) and pulsar spin precession (Shaham 1977; Link & Cutler 2002). Recent real-time observations (Heinke & Ho 2010) of a cooling NS in Cassiopea A supernova remnant give strong arguments that the star has superfluid core (Shternin et al 2011; Page et al 2011).

The aim of this short note is to point out the importance of one effect related to superfluidity of baryons in NSs that has usually been ignored in the NS literature. In Sec. II we outline the effect. In Sec. III we demonstrate its efficiency. In Sec. IV we discuss possible consequences for the physics of NSs and in Sec. V we conclude. We use the system of units in which \( k_B = h = 1 \).

2 A SIMPLE PROBLEM AND THE PROPOSED EFFECT

Let us consider a degenerate Fermi-liquid composed of identical particles of mass \( m \). Assume that they interact through a weakly attractive potential so that BCS theory (see, e.g., Lifshitz & Pitaevskii 1981) is applicable. Assume also that they pair (become superfluid) in the spin-singlet \(^1S_0\) state at temperatures \( T \) below some critical temperature \( T_c \). The role of elementary excitations in such superfluid Fermi-liquid is played by Bogoliubov excitations (see, e.g., Feynman 1972). In what follows, all equations will be written in a reference frame in which the mean (hydrodynamic) velocity \( V_q \) of Bogoliubov excitations vanishes, \( V_q = 0 \) (i.e., normal liquid component is at rest).

In the absence of superfluid current (when the superfluid velocity \( V_s = 0 \) the energy \( E_p \) of a Bogoliubov excitation with momentum \( p \) near the Fermi surface can be written as

\[
E_p = \sqrt{v_F^2 (|p| - p_F)^2 + \Delta^2},
\]

where \( v_F \) and \( p_F \) are the Fermi-velocity and Fermi-momentum, respectively; and \( \Delta \) is the energy gap given by the standard equation (Lifshitz & Pitaevskii 1980),

\[
1 = -V_0 \sum_p \frac{1 - 2f_p}{2E_p},
\]

where \( V_0 \) is the (constant) pairing potential and

\[
f_p = \frac{1}{e^{E_p/T} + 1}
\]

is the Fermi-Dirac distribution function for Bogoliubov excitations.

If, however, the superfluid current is present (\( V_s \neq 0 \) then fermions pair with momenta \((-p + Q, p + Q)\) rather than with \((-p, p)\), and the total momentum of a Cooper pair is

\[
2Q = 2mV_s.
\]

What will be the equation for the gap? The answer can be found in Bardeen (1962) and is well known in the physics of superconductors. Now, instead of Eq. (2), one should write

\[
1 = -V_0 \sum_p \frac{1 - F_p Q - F_{-p} Q}{2F_p},
\]

Here \( F_p \) is the distribution function for Bogoliubov excitations with momentum \((p + Q)\) in the system with non-zero...
The energy gap $\Delta$ (in units of $\Delta_0$) versus $Q = mV_s$ (in units of $Q_{cr0}$, see Eq. (10)] for a set of temperatures $T/T_c = 0.1, 0.4, 0.6, 0.75, 0.85$, and $0.95$.

\[ V_s, \]

\[ F_{p+Q} = \frac{1}{e^{\varepsilon_{p+Q}/T} + 1}, \tag{6} \]

where

\[ \varepsilon_{p+Q} \approx \frac{pQ}{m} + E_p \tag{7} \]

is the energy of a Bogoliubov excitation with momentum $(p + Q)$. In Eq. (7) we assumed $Q \ll p_F$ which is true in all interesting cases (see, e.g., [Gusakov & Haensel 2005]).

\[ \Delta \sim \frac{pFm}{\pi^2} \ln \left( \frac{\Delta_0}{\Delta} \right) = \sum_p F_{p+Q} + F_{-p+Q} \tag{8} \]

The solution to this equation gives the gap $\Delta$ as a function of $T$ and $Q = |Q|$. First consider two limiting cases in which $\Delta(T, Q)$ vanishes.

(i) if $Q = 0$ then $\Delta = 0$ at

\[ T = T_c \approx 0.567\Delta_0 \quad \text{(the well known BCS result)}; \tag{9} \]

(ii) if $T = 0$ then $\Delta = 0$ at

\[ Q = Q_{cr0} = \frac{e}{2} \frac{\Delta_0 m}{pF}. \tag{10} \]

The latter result is less known but can be found, e.g., in [Alexandrov 2003]. Notice that, the well-known Landau criterion for superfluidity breaking gives $Q_{cr0}^{(Landau)} = \Delta_0 m/pF$ and is not accurate for a superfluid Fermi-liquid.

Some numerical solutions to Eq. (8) are presented in Figs. 1 and 2. Fig. 1 shows the gap $\Delta(T, Q)$ [in units of $\Delta_0$] versus momentum $Q$ [in units of $Q_{cr0}$] for a set of temperatures $T/T_c = 0.1, 0.4, 0.6, 0.75, 0.85$, and $0.95$. One sees that $\Delta$ is quite sensitive to variation of $Q = mV_s$ as long as $T \geq 0.17T_c$. Another important conclusion that can be drawn from Fig. 1 is that (for a given $T$) the maximum critical momentum $Q_{cr}$ strongly depends on temperature.

Fig. 2 illustrates this point more clearly. In the left panel we plot $Q_{cr}$ (in units of $Q_{cr0}$) versus $T$ (in units of $T_c$). The right panel shows the same dependence $Q_{cr}(T)$ but with $Q_{cr}$ measured in units of

\[ Q_{cr}^{(app)}(T) \equiv \frac{e}{2} \frac{\Delta(T, 0) m}{pF}. \tag{11} \]

We see that $Q_{cr}$ changes with $T$ in such a way that $Q_{cr}(T)/Q_{cr}^{(app)}(T)$ is roughly constant.

Therefore, the energy gap $\Delta$ can be a strong function of the momentum $Q = mV_s$ or, in an arbitrary frame, a strong function of the difference $m(V_s - V_q) \equiv m \Delta V$. We will refer to this effect as to the ‘$\Delta V$-effect’. The critical value $\Delta V_{cr}(T)$ of $\Delta V = |V_s - V_q|$, at which superfluidity dies out, is easily estimated by taking $Q_{cr} \sim Q_{cr}^{(app)}$. Then, from Eq. (11), we obtain

\[ \Delta V_{cr}(T) \sim 10^7 \left[ \frac{\Delta(T, 0)}{10^9 K} \right] (\frac{n_0}{n})^{1/3} \text{ cm s}^{-1}, \tag{12} \]

where $\Delta(T, 0)$ is measured in Kelvins; $n_0 = 0.16$ fm$^{-3}$ is the nucleon density in atomic nuclei; $n = pF/(3\pi^2)$ is the particle number density.

3 IMPORTANCE OF THE $\Delta V$-EFFECT FOR NEUTRON STARS

If the difference $\Delta V$ between the baryon superfluid velocities and a normal velocity is comparable to $\Delta V_{cr}$, then the baryon energy gaps can be substantially reduced. A few interesting consequences of this ‘dynamical reduction’ of the gaps are discussed in the next section. Here we illustrate possible importance of the $\Delta V$-effect by considering radial oscillations of a nonrotating superfluid NS whose core is composed of neutrons, protons, and electrons. The main question is at what oscillation amplitude $\Delta V$ becomes comparable to $\Delta V_{cr}$?

For simplicity we (i) assume that neutrons pair in the spin-singlet $(^1S_0)$ state [rather than in the triplet $(^3P_2)$ state] and (ii) neglect the Landau quasiparticle interaction between quasinucleons when calculating $\Delta_n(T, V_{sn} - V_q)$ [here and below the subscripts n, p, and e refer to neutrons, protons, and electrons, respectively].

\[ \Delta_n(T, V_{sn} - V_q). \]

Let us remark that to calculate $\Delta_n$ and $\Delta_p$ as functions of $(V_{sn} - V_q)$ and $(V_{sp} - V_q)$ with allowance for interactions between quasiparticles, one should follow the derivation of [Gusakov & Haensel 2003]. Namely, one should self-consistently...
The NS model used here and all the microphysics input are essentially the same as in (Kantor & Gusakov 2011); we refer the reader to that work for more details. In particular, we consider the star of gravitational mass $M = 1.4 M_\odot$, circumferential radius $R = 12.2$ km, central density $\rho_c = 9.26 \times 10^{14}$ g cm$^{-3}$, and adopt the APR EOS in the NS core (Akmal, Pandharipande & Ravenhall 1998). The model of nucleon superfluidity employed here coincides with the model 3 of (Kantor & Gusakov 2011) and is shown in Fig. 3.

The left panel of Fig. 3 presents nucleon critical temperatures $T_{\text{np}}$ and $T_{\text{pn}}$ versus density $\rho$ in the NS core, the right panel demonstrates the red-shifted critical temperatures $T_{\text{cp}} \equiv T_{\text{cp}} \rho^{-2/3}$ and $T_{\text{cp}} \equiv T_{\text{cp}} \rho^{-1/3}$ ($\nu$ is the metric function) versus radial stellar coordinate $r$ (in units of $R$). The redshifted proton critical temperature is taken to be constant $T_{\text{cp}} = 5 \times 10^9$ K; the redshifted neutron critical temperature varies with $r$ and has maximum $T_{\text{cp}} \equiv 6 \times 10^8$ K in the stellar centre. In the right panel of Fig. 3 we hatch the region occupied by the neutron superfluidity at a redshifted stellar temperature $T^\infty \equiv T^\infty e^{-2/3} = 4 \times 10^8$ K.

To model oscillations of superfluid NSs one has to use the hydrodynamics of mixtures of superfluid Fermi-liquids (Andreev & Bashkin 1973; Andersson & Comer 2007; Gusakov & Andersson 2006; Gusakov 2007). The important parameter of such hydrodynamics is the so-called entrainment matrix $\rho_{ik}$ (Andreev & Bashkin 1973; Borumand, Jovit & Kluzniak 1993; Gusakov & Haensel 2003), or relativistic entrainment matrix $Y_{ik}$ (Gusakov et al. 2009a,b). Both these matrices are very temperature-dependent (Gusakov & Haensel 2003; Gusakov et al. 2009a,b). As a consequence, the eigenfrequencies and eigenfunctions of oscillating superfluid NS also depend on temperature (Gusakov & Andersson 2006; Kantor & Gusakov 2011; Chugunov & Gusakov 2011). Below we consider the first radial oscillation mode of a superfluid NS (see Kantor & Gusakov 2011, particularly figure 3 there).

Figure 4 shows the amplitude of the eigenfunction $\Delta V_n \equiv |V_n - V_q|$ and the critical velocity $\Delta V_{\text{cr}}$ as functions of $r$ (solid and dashed lines, respectively; both in units of $10^7$ cm s$^{-1}$). We plot $\Delta V_n$ and $\Delta V_{\text{cr}}$ for four redshifted stellar temperatures: $T^\infty = 3.0 \times 10^7$ K (black lines), $8.0 \times 10^7$ K (red lines), $2.0 \times 10^8$ K (red lines), and $5.0 \times 10^8$ K (violet lines). The oscillation frequencies $\omega$ of the first radial mode for such temperatures are $\omega/(10^4$ s$^{-1}) \approx 1.702, 1.702, 1.064, \text{ and } 0.516$, respectively.

The vertical dotted lines in Fig. 4 indicate (temperature-dependent) boundaries between the neutron superfluid region and the outer normal region with nonsuperfluid neutrons. In the normal region the functions $\Delta V_n$ and $\Delta V_{\text{cr}}$ are not defined. The oscillation energy of the star is $E_{\text{mech}} = 10^{47}$ erg. For a nonsuperfluid NS this

![Figure 3](image-url)

**Figure 3.** (color online) Left panel: Nucleon critical temperatures $T_{\text{vk}}$ ($k = n, p$) versus density $\rho$. Right panel: Redshifted critical temperatures $T_{\text{vk}}$ versus radial coordinate $r$. See text for details.

![Figure 4](image-url)

**Figure 4.** (color online) Amplitudes of the eigenfunctions $\Delta V_n$ (solid lines) and the critical velocities $\Delta V_{\text{cr}}$ (dashes) versus $r/R$ for the four temperatures $T^\infty = 3.0 \times 10^7$ K (black lines), $8.0 \times 10^7$ K (red lines), $2.0 \times 10^8$ K (blue lines), and $5.0 \times 10^8$ K (violet lines). To plot $\Delta V_n$ we assumed that the energy of oscillations is $E_{\text{mech}} = 10^{47}$ erg. The vertical dotted lines show the (temperature-dependent) boundaries between the inner superfluid and the outer normal regions. See text for details.

In this paper we use the standard (textbook) version of superfluid hydrodynamics in which the independent velocity fields are $V_{\text{sn}}, V_{\text{sp}}$, and $V_q$. Notice, however, that in the NS literature an equivalent form of superfluid hydrodynamics is often used which follows from the convective variational principle formulated by Carter and analyzed, in the nonrelativistic framework, by Prix (2004). In this hydrodynamics (and in the context of npe-matter) the independent velocity fields are $v_i \equiv J_i/\rho_l$, where $J_i$ and $\rho_l$ are, respectively, the mass current density and density for particle species $i = n, p,$ and $e$. These velocities are related with $V_{\text{sn}}, V_{\text{sp}}$, and $V_q$ by the following equations ($i = n, p$): $v_i = \sum_{i=n, p} \rho_i \rho_k V_{ik} + (\rho_l - \sum_{i=n, p} \rho_k) V_q$.

We stress that both velocities $V_{\text{sn}}$ and $V_q$ are calculated self-consistently using the finite temperature superfluid hydrodynamics.
two temperatures, $T$ and $T_{\infty}$, the violet and blue curves). This means that the critical values $\Delta_c$ we plot the neutron energy gap $\Delta_n$ of NSs already at rather modest oscillation amplitudes. (Gusakov, Yakovlev & Gnedin 2005).

It follows from Fig. 4 that $\Delta_n$ can substantially exceed the critical values $\Delta_c$, so that superfluidity is destroyed by oscillations in the large part of the stellar core (see, in particular, the violet and blue curves). This means that the $\Delta V$-effect can greatly influence (or even drive) the dynamics of NSs already at rather modest oscillation amplitudes.

This point is additionally illustrated in Fig. 5, where we plot the neutron energy gap $\Delta_n(T, \Delta V_n)$ versus $r/R$ for two temperatures, $T_{\infty} = 8 \times 10^7$ K (upper panel) and $T_{\infty} = 2 \times 10^8$ K (bottom panel), and a set of oscillation energies $E_{\text{mech}}$. In the upper panel $\Delta_n(T, \Delta V_n)$ is shown for $E_{\text{mech}} = 0, 5 \times 10^77, 10^8$, and $5 \times 10^8$ erg; in the bottom panel $\Delta_n(T, \Delta V_n)$ is shown for $E_{\text{mech}} = 0, 10^7, 10^8$, and $10^9$ erg. The oscillation amplitudes $\varepsilon$ [given by Eq. (13)] for these oscillation energies are presented in Table 1.

Table 1. Oscillation (mechanical) energy $E_{\text{mech}}$ and the corresponding amplitude of oscillations $\varepsilon$, defined by Eq. (13).

| $E_{\text{mech}}/ (10^{47} \text{ erg})$ | 0.0 | 0.1 | 0.5 | 1.0 | 5.0 | 10.0 | 50.0 |
|---------------------------------|-----|-----|-----|-----|-----|------|------|
| $\varepsilon/10^{-4}$           | 0.0 | 1.4 | 3.1 | 4.4 | 9.7 | 14   | 31   |

$E_{\text{mech}} = 10^{47}$ erg these eigenfunctions have already been presented in Fig. 4 (see the red and blue solid lines; the red line corresponds to $T_{\infty} = 8 \times 10^7$ K, the blue line to $T_{\infty} = 2 \times 10^8$ K). If $E_{\text{mech}} = 0$ (no oscillations; see the solid lines in both panels of Fig. 5) the gap $\Delta_n$ is unaffected by $\Delta V_n$ and is entirely determined by the dependence of $T_n$ on $r$ (see Fig. 3). The vertical dotted lines in Fig. 5 indicate boundaries between the inner superfluid and the outer normal regions; these boundaries depend on $E_{\text{mech}}$. Obviously, the higher $E_{\text{mech}}$, the larger $\Delta_n(r)$, and, correspondingly, the smaller the superfluid region and $\Delta_n$. One sees that the gaps are very sensitive to variation of $\Delta V_n$.

4 POSSIBLE APPLICATIONS

As follows from the consideration of the previous section, the $\Delta V$-effect can operate at not too small oscillation amplitudes. All interesting consequences of this effect are related to the reduction of baryon gaps. Let us list some of them:

1. The reduction of the gaps influences the entrainment matrix $\rho_{ik}$ (Gusakov & Haensel 2003), which depends on them. As a result, $\rho_{ik}$ will become a non-linear function of the oscillation amplitude. This will (i) make the oscillation equations nonlinear and hence (ii) affect the eigenfrequencies and eigenfunctions of oscillating NS. Moreover, this will (iii) influence the dissipation processes, because bulk viscosity terms explicitly depend on $\rho_{ik}$. In a rotating star the decrease of the element $\rho_{np}$ of the entrainment matrix will, in addition, (iv) reduce the mutual friction force, which is proportional to $\rho_{np}$ (Alpar, Langer & Sauls 1984). We emphasize that the dependence of $\rho_{np}$ on $T$ and on $\Delta V_n$ and $\Delta V_p$ is a very important effect for mutual friction and related phenomena, which has been neglected in the literature.

How to calculate the entrainment matrix $\rho_{ik}$ taking into account the $\Delta V$-effect? A direct calculation is difficult (but one can perform it in a manner similar to how it was done in Gusakov & Haensel 2003). A good approximation for $\rho_{ik}$ could be to calculate it from the formula (49) of Gusakov & Haensel 2003, making use of the velocity-dependent gaps from Sec. II instead of the gaps $\Delta_n(T, 0)$ and $\Delta_p(T, 0)$. In this way one would obtain, for instance, for $\rho_{np}$

$$\rho_{np} = \frac{3/2}{9 \pi^2 S} \frac{m_n m_p}{m_n m_p} F_1^{np} (1 - \Phi_n) (1 - \Phi_p),$$

where $S = (1 + F_{in}^{np} \Phi_n/3) \left(1 + F_{ip}^{np} \Phi_p/3 \right) - (F_{ik}^{np}/3)^2 \Phi_n \Phi_p$; $m_i, m_i^*, p_i$, and $F_{ik}^{np}$ are the mass of a free particle, Landau effective mass, Fermi momentum and the dimensionless Landau parameters, respectively ($i, k = n, p$). Further, $\Phi_i$ is a simple function of $x_i \equiv \Delta_i (T, \Delta V_i)/T$, specified in Gusakov & Haensel 2003, which changes from 0 at $T = 0$. 

![Figure 5. (color online) Neutron energy gap $\Delta_n(T, \Delta V_n)$ (in units of $10^9$ K) versus $r/R$ for two temperatures $T_{\infty} = 8 \times 10^7$ K (upper panel) and $T_{\infty} = 2 \times 10^8$ K (bottom panel) and some oscillation energies $E_{\text{mech}}$ (indicated in the figure). Vertical dotted lines show $r$ at which neutron superfluidity disappears ($\Delta_n = 0$). The larger $E_{\text{mech}}$ the smaller the superfluid region. See text for details.](image-url)
to 1 at $\Delta_i(T, \Delta V_i) = 0$. One sees from Eq. (13) that $\rho_{\text{np}}$ vanishes whenever $\Delta_n(T, \Delta V_n) = 0$ (and hence $\Phi_n=1$) or $\Delta_p(T, \Delta V_p) = 0$ (and hence $\Phi_p = 1$).

(2) Another important consequence of the $\Delta V$-effect is its impact on kinetic coefficients of NS matter, in particular, on the bulk and shear viscosities.

(i) Bulk viscosity. There are four bulk viscosity coefficients in the npe-matter of NSs (Gusakov 2007). All of them are generated by nonequilibrium beta-processes (direct or modified URCA reactions) and depend on the difference $\Delta \Gamma$ between the direct and inverse reaction rates. $\Delta \Gamma$ is generally a complicated function of $T$, $\Delta_n$, $\Delta_p$, and of the imbalance of chemical potentials $\Delta \mu_\eta = \mu_\eta - \mu_\eta$ (Haensel, Levenfish & Yakovlev 2000, 2001), where $\mu_\eta$ is the chemical potential for particle species $i = n, p, e$. Recently it has been shown by Alford, Reddy & Schwenzer (2012), that if $\Delta \mu > \max\{\Delta_n, \Delta_p\}$ then, even for $T \ll \Delta_n$ and/or $\Delta_p$, the bulk viscosity is not suppressed by the nucleon superfluidity and can be very efficient. It seems that the $\Delta V$-effect of the reduction of the energy gaps $\Delta_n$ and $\Delta_p$ by relative motion of superfluid and normal component is complementary to the effect considered in Alford et al. (2012). Both effects act in unison to increase the bulk viscosity coefficients, and they are of comparable strength. Notice, however, that the effect of Alford et al. (2012) can only affect the bulk viscosity coefficients, while the applicability range of the $\Delta V$-effect is wider; it directly influences the baryon energy gaps and thus all dynamics of NSs.

(ii) Shear viscosity. Neglecting entrainment between baryon species ($\rho_{\text{np}} = 0$), the shear viscosity $\eta$ can be calculated in the same fashion as was done, e.g., in Shternin & Yakovlev (2008) [the results will be the same]. The only difference is that one should use the velocity-dependent gaps $\Delta_i(T, \Delta V_i)$ instead of $\Delta_i(T, 0)$ in all equations $[i = n, p]$. It is interesting that the $\Delta V$-effect can both increase or decrease the shear viscosity. For example, the electron shear viscosity $\eta_e$ decreases with increasing $\Delta V_p$ (that is, with reducing $\Delta_p$), because electrons are better screened by protons when $\Delta_p$ is large (Shternin & Yakovlev 2008). On the other hand, the neutron shear viscosity $\eta_n$ can either decrease or increase with growing $\Delta V_n$ and $\Delta V_p$. The behaviour of $\eta_n$ in that case is determined by the competition of two effects: by the increase of the normal density of neutron Bogoliubov excitations $\rho_{\text{np}}$ and by the reduction of the neutron mean free path $\lambda$ due to more frequent collisions with neutron and proton Bogoliubov excitations (note that $\eta_n$ can be estimated as $\eta_n \sim \rho_{\text{np}} v_F \eta / \lambda$, where $v_F$ is the neutron Fermi-velocity). Similar effects were carefully analyzed in Baiko, Haensel & Yakovlev (2001) in application to the neutron thermal conductivity.

An entrainment between neutrons and protons will strongly modify the derivation of the neutron shear viscosity, even neglecting the $\Delta V$-effect. The main difference will be the equilibrium Fermi-Dirac distribution function for neutron Bogoliubov excitations in a system with superfluid currents. This function was first obtained in Gusakov & Haensel (2003) [see equation (20) there]; it is very different from the standard expression, valid when $\rho_{\text{np}} = 0$. To our best knowledge, a derivation of $\eta_n$ in a system with entrainment has not been attempted in the literature.

(3) Finally, there is a number of important consequences of the fact that the relative velocity $\Delta V$ between the superfluid and normal liquid components cannot be too large in a stationary rotating NS. Here we present two of them.

(i) It is generally accepted that neutron vortices are pinned to atomic nuclei in the NS crust (or to magnetic flux tubes in the NS core). At a certain critical $\Delta V$ they can unpin from the nuclei (or from magnetic flux tubes). However, in some models (e.g., Link 2009) pinning is so strong that the critical relative velocity can be as high as $10^6 \pm 10^7$ cm s$^{-1}$. These values are close to $\Delta V_{cr}$ (see Eq. (12)). Thus, the $\Delta V$-effect can be very important for such models. It can also play a role in explanation of the long-period precession of isolated pulsars (Link 2003) and Samuelsson et al. (2010) a two-stream instability in homogeneous superfluid matter is discussed, that can be triggered once the relative velocity $\Delta V$ reaches some critical value. According to these authors, the critical value is of the order of the sound speeds, i.e., it is much greater than the typical $\Delta V_{cr}$, at which superfluidity completely disappears [see Eq. (12)]. In other words, it is not very probable that this instability is realized in NSs. Notice, however, that under certain circumstances similar instability in rotating NSs can drive the so-called inertial modes unstable at a much lower $\Delta V$ (Prix, Comer & Andersson 2004).

5 CONCLUSION

The baryon energy gaps depend on the relative velocity between the superfluid and normal components ($\Delta V$-effect). We propose, for the first time, that this effect may have a strong impact on the dynamical properties of NSs. We illustrate this point by considering radial oscillations of an NS with superfluid nucleon core and a nonsuperfluid crust. However, we stress that the $\Delta V$-effect should be equally important in the crust of NSs where superfluid neutrons are present, as well as in the interiors of hyperon and quark stars. Although we discussed some immediate applications in Sec. IV, it is clear that more efforts are needed to analyze all possible consequences of this effect on the evolution of NSs.

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