Predicting a new resonance as charmed-strange baryonic analogue of $D_s^*(2317)$

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In this paper, we predict $\Omega_{c0}(1P, 1/2^−)$, which is as charmed-strange baryonic analogue of $D_s^*(2317)$. By an unquenched quark model, we perform a mass spectrum calculation, and find that the mass of $J^P = 1/2^− \Omega_{c0}(1P)$ with predominant light degree of freedom $J^P = 0$ is lowered down to the position below the $\Xi_c K$ threshold, with about 90 MeV mass shift. Future experimental search for the predicted charmed-strange baryon is suggested.

I. INTRODUCTION

Since 2003, the hadron physics has entered a new era with the observation of a series of new hadronic states and the corresponding novel phenomena (see review articles [1–3] for more details), owing to the accumulation of experimental data with high precision. In 2003, the BaBar Collaboration observed a narrow state $D_s^*(2317)$ [4], which decays into $D_s^0 \pi^0$ final state and has resonance parameters $m = 2317.8 \pm 0.5$ MeV and $Γ < 3.8$ MeV with spin parity $J^P = 0^+$ [5]. Later, CLEO, Belle, and BaBar again confirmed this observation [6–11]. Since its mass is about 100 MeV lower than the result of the quenched quark model [12, 13], there exists the so-called famous low mass puzzle for $D_s^*(2317)$. Such a situation not only results in the exotic state explanations including hadronic $DK$ molecular state and compact tetraquark state proposed in Refs. [14–19], but also stimulates theorists to pay more attention to unquenched picture [20–26], where the important role of the coupled-channel effect played in hadron spectroscopy starts to be realized. Later, low mass puzzle phenomena appear in other several typical observed states $D_s^0(2460)$ [24, 26], $X(3872)$ [27–29], and $Λ_s(2940)$ [30], which naturally construct a complete chain from heavy-light meson, charmonium and to heavy-light baryon, where the coupled-channel effect should be emphasized.

Under the unquenched picture for $D_s^*(2317)$, $P$-wave bare state $D_s(0^+)$ can be dressed by the nearby DK channel, which makes the physical mass be lowered down to be consistent with the mass of $D_s^*(2317)$ [20–26]. If replacing the anti-strange quark $\bar{s}$ inside charmed-strange mesonic state by a ss pair, we believe that there should exist a charmed-strange baryonic analogue of $D_s^*(2317)$, which inspires our interest in exploring whether the coupled-channel effect may play an important role in such a new system corresponding to $P$-wave $\Omega_{c0}(1P, 1/2^−)$ system.

As indicated in Fig. 1, the bare mass of $\Omega_{c0}(1P, 1/2^−)$ predicted by the most of quenched models [31–35] is above the $\Xi_c K$ threshold and there exists typical $S$-wave interaction between $\Omega_{c0}(1P, 1/2^−)$ and the $\Xi_c K$ channel. Thus, we have reason to believe that the coupled-channel is obviously effective. In this work, we will adopt an unquenched quark model to quantitatively reflect the existing coupled-channel effects. Before doing realistic calculation for $\Omega_{c0}(1P, 1/2^−)$, we will first study $D_s(0^+)$ with the same framework, by which we can check the reliability of the adopted unquenched quark model. Our calculation will explicitly show that the coupled-channel effect on $D_s(0^+)$ can exactly reproduce the mass of $D_s^*(2317)$. Naturally, when we continue to focus on $\Omega_{c0}(1P, 1/2^−)$, we find that a low mass phenomenon still exists, which makes the physical mass of $\Omega_{c0}(1P, 1/2^−)$ lower than the $\Xi_c K$ threshold. This fact further shows that $\Omega_{c0}(1P, 1/2^−)$ should be a narrow state. The predicted behavior of $\Omega_{c0}(1P, 1/2^−)$ as charmed-strange analogue of $D_s^*(2317)$ can be examined in future experiments.

In 2017, the LHCb and Belle collaborations have reported higher states for the $\Omega_c$ family [36, 37]. By taking this opportunity, we have a further discussion on the possible relation of the predicted charmed-strange baryonic analogue of $D_s^*(2317)$ and these observations.

The paper is organized as follows. After Introduction, we will take the $D_s^*(2317)$ as a sample to test the effectiveness of the unquenched model in Sec. II. Next, in Sec. III, we will employ the same model to calculate the unquenched mass of $\Omega_{c0}(1P, 1/2^−)$ with coupled-channel effect from the $\Xi_c K$ channel. Finally, the paper will end with the discussion and con-
II. TEST THE UNQUENCHED MODEL FOR $D_{s0}^*(2317)$

To make our conclusion for the $P$-wave $\Omega_{c0}(1P, 1/2^+)$ state more reliable, we will test the adopted unquenched model in this section by examining the coupled-channel effect on $D_{s0}^*(2317)$. We will not only illustrate why the mass of $D_{s0}^*(2317)$ shifts down about 80 MeV, but also fix the parameters of the unquenched quark model, which will be used to study the nontrivial coupled-channel effect on the $P$-wave $\Omega_c$ states.

Due to the unquenched effect, the physical $D_{s0}^*(2317)$ state contains both $c\bar{s}$ and $DK$ components, which could be denoted as [38, 39]

$$|D_{s0}^*(2317)\rangle = c_{c\bar{s}}|c\bar{s}(1^3P_0)\rangle + \int d^3p \ c_{DK}(p)|DK,p\rangle. \quad (1)$$

Here, the $c_{c\bar{s}}$ denotes the probability amplitude of the $c\bar{s}$ core in the $D_{s0}^*(2317)$ wave function, and the $c_{DK}(p)$ is the component of the $DK$ channel. The $c\bar{s}(1^3P_0)$ in Eq. (1) represents the conventional $D_s(0^+)$ with radial quantum number $n = 0$ (see Eq. (17) for spatial wave function). Then, the full Hamiltonian of the physical $D_{s0}^*(2317)$ state can be written as [28, 40]

$$\hat{H} = \left( \hat{H}_0 \hat{H}_I \hat{H}_{DK} \right), \quad (2)$$

in the unquenched quark model. The $\hat{H}_0$ is the Hamiltonian in a conventional quark model, by which one obtains the discrete mass spectrum of the bare charmed-strange mesons. The $\hat{H}_{DK}$ refers to the the free Hamiltonian of the continuum states $|DK\rangle$, i.e.,

$$\hat{H}_{DK}|DK,p\rangle = \left( \sqrt{m_D^2 + p^2} + \sqrt{m_K^2 + p^2} \right)|DK,p\rangle, \quad (3)$$

where the interactions between the $D$ and $K$ mesons are neglected. The $\hat{H}_I$ that causes a mixture of the pure $c\bar{s}$ state (bare state) and $DK$ continuum can be borrowed from the quark-pair-creation (QPC) model [41–45]. In the non-relativistic limit, the transition operator $\hat{H}_I$ can be expressed as

$$\hat{H}_I = -3\gamma \sum_{m} \left\langle 1, m; 1, -m|0, 0 \right\rangle \int d^3p \ d^3p_j \ \delta(p_i + p_j)$$

$$\times \mathcal{Y}_{l_{1}} \left( \frac{p_i - p_j}{2} \right) \omega_0^{(i|j)} \phi_0^{(i|j)} \chi_{1,-m}^{(i|j)} \delta \left( \mathbf{p}_i - \mathbf{p}_j \right), \quad (4)$$

where $\omega$, $\phi$, $\chi$ and $\mathcal{Y}$ are the color, flavour, spin, and spatial functions of the quark pair, respectively. The $h_i$ and $d_j$ are quark and anti-quark creation operators, respectively. The dimensionless parameter $\gamma$ describes the strength of a quark-antiquark pair created from the vacuum. Now the amplitude of $c\bar{s}(1^3P_0) \rightarrow DK$ can be denoted as

$$M_{c\bar{s}(1^3P_0)\rightarrow DK}(p) = \left\langle DK,p|\hat{H}_I|c\bar{s}(1^3P_0)\right\rangle, \quad (5)$$

where $p$ represents the momentum of $D$ meson in the center-of-mass frame of $c\bar{s}(1^3P_0)$ state.

With the above preparation, the Schrödinger equation for $D_{s0}^*(2317)$ could be denoted as

$$\left( \begin{array}{c} \hat{H}_0 \\ \hat{H}_I \\ \hat{H}_{DK} \end{array} \right) \left( \begin{array}{c} c_{c\bar{s}(1^3P_0)} \\ c_{DK}\left|DK\right\rangle \end{array} \right) = M \left( \begin{array}{c} c_{c\bar{s}(1^3P_0)} \\ c_{DK}\left|DK\right\rangle \end{array} \right). \quad (6)$$

After diagonalization of Eq. (6), we obtain the following coupled-channel equation

$$M - M_0 - \Delta M(M) = 0. \quad (7)$$

$\Delta M(M)$ is mass shift with definition

$$\Delta M(M) = \text{Re} \int_0^{\infty} p^2 dp \frac{|M_{c\bar{s}(1^3P_0)\rightarrow DK}(p)|^2}{M - \sqrt{M_0^2 + p^2} - \sqrt{M_0^2 + p^2}}. \quad (8)$$

To extract the mass shift of $D_{s0}^*(2317)$ state by Eq (7), one should obtain the bare mass $M_0$ as the first step. In the following, we employ a non-relativistic potential model to calculate the mass spectrum of the bare charmed-strange mesons. The Hamiltonian is given as

$$\hat{H}_0 = \sum_{m} \left( m_i + \frac{p_i^2}{2m_i} \right) + \sum_{i,j} V_{ij}, \quad (9)$$

where $m_i$ and $p_i$ are mass and momentum of the $i$-th constituent quark. The $V_{ij}$ in Eq. (9) is the interaction between quark-quark (or quark-antiquark), which contains one-gluon-exchange (OGE) potentials and confining potentials and could be expanded as

$$V_{ij} = H_{ij}^{\text{conf}} + H_{ij}^{\text{hyp}} + H_{ij}^{\text{so(cm)}} + H_{ij}^{\text{so(tp)}}. \quad (10)$$

The first term of $V_{ij}$ in Eq. (10) is the Cornell potential, which is spin-independent, i.e.,

$$H_{ij}^{\text{conf}} = -\frac{4 \alpha_s}{3} r_{ij} + b r_{ij} + C, \quad (11)$$

where the $\alpha_s$, $b$, and $C$ denote the coupling constant of OGE, the strength of linear confinement, and mass-renormalized constant, respectively. Besides the spin-independent term, $V_{ij}$ also contains spin-spin interaction, i.e., the hyperfine interaction is

$$H_{ij}^{\text{hyp}} = \frac{4 \alpha_s}{3m_i m_j} \left( \frac{8 \pi}{3} \right) \left( \frac{3}{3} \right) S(r_{ij}, s_i, s_j), \quad (12)$$

where

$$\tilde{\delta}(r) = \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2 r^2} \quad (13)$$

is a Gaussian smearing function with a smearing parameter $\sigma$, and

$$S(r_{ij}, s_i, s_j) = \frac{3 s_i \cdot r_{ij} s_j \cdot r_{ij}}{r_{ij}^2} - s_i \cdot s_j \quad (14)$$

The result of the calculations for $\Delta M(M_0)$ is 80 MeV [38, 39].
is a tensor operator. Besides, the color-magnetic term and Thomas-precession piece of the spin-orbit interactions could be expressed as

$$H_{ij}^{\text{color}} = \frac{4\alpha_s}{3r_{ij}} \left(\frac{1}{m_i} + \frac{1}{m_j}\right) \left(s_i + s_j\right) \cdot \mathbf{L}$$  \hspace{1cm} (15)$$

and

$$H_{ij}^{\text{spin}} = -\frac{1}{2r_{ij}} \frac{\partial H_{ij}^{\text{conf}}}{\partial r_{ij}} \left(s_i + s_j\right) \cdot \mathbf{L},$$  \hspace{1cm} (16)$$

respectively.

The parameters in the quenched quark potential model are fixed by the low-lying well established $\pi$, $K$, and $D_s$ mesons, i.e., $m_{\pi/s} = 0.370$ GeV, $m_K = 0.600$ GeV, and $m_{D_s} = 1.880$ GeV, $\alpha_s = 0.578$, $b = 0.144$ GeV$^2$, $\sigma = 1.028$ GeV, and $C = -0.685$ GeV. Using the above parameters, the predicted masses of $1^3S_0$, $1^3S_1$, $1P_1$ ($j_z = 3/2$), and $1^3P_2$ are well consistent with the measured masses of $D_s(1668)$, $D_s^*(2112)$, $D_s(2536)$, and $D_s^*(2573)$, respectively. However, the mass of $D_s(1^{3}P_0)$ is obtained to be 2441 MeV, which is about 76 MeV above the $D_s^*$ threshold and 124 MeV larger than the measured mass of $D_{s0}^*(2317)$. Our result is similar to the previous works [12, 13, 46].

To incorporate the coupled-channel effect for the $D_{s0}^*(2317)$, we adopt a simple harmonic oscillator (SHO) wave function to depict the spatial wave function of a meson, i.e.,

$$\psi_{nlm}(\beta, \mathbf{P}) = \frac{(-1)^{n}\beta_{m}^{l}i^{l}}{\beta_{l}^{n}l^{l}!} \sqrt{\frac{2n!}{\Gamma(n + l + \frac{3}{2})}} L_{n}^{l} \left(\frac{P^{2}}{2}\right)^{\frac{1}{2}} e^{-\frac{\beta_{l}^{n}P^{2}}{2}} \Omega_{l},$$  \hspace{1cm} (17)$$

where $n$, $l$, and $m$ are radial, orbital, and magnetic quantum numbers, respectively. Then, the spatial wave function overlap in Eq. (5) can be calculated analytically. The parameter $\beta$ that denotes the distance scale in momentum space could be extracted from the potential model mentioned above. In Table I, we collect the obtained $\beta$ values. The remaining parameter $\gamma$ is the strength of a $D_s$ pair creation from the vacuum. For $D_s$ meson, we determine $\gamma = 4.1$ from the width of $D_{s2}^*(2573)$.

| States | $\beta$ | States | $\beta$ | States | $\beta$ |
|--------|--------|--------|--------|--------|--------|
| $\pi$  | 0.409  | $D(1S)$ | 0.357  | $D_s(1S)$ | 0.428  |
| $K$    | 0.385  | $D_s^*(1S)$ | 0.307  | $D_s^*(1S)$ | 0.371  |
|        |        | $D(1P)$ | 0.204  | $D_s(1P)$ | 0.237  |

With the above preparations, we could calculate the physical mass of $D_s(1^{3}P_0)$. The numerical results are plotted in Fig. 2. With the contribution from the intermediate $DK$ channel, the mass of the $D_s(1^{3}P_0)$ could be lowered down approximately to the experimental mass of $D_{s0}^*(2317)$. When the coupled-channel effect from $DK$ is considered, the mass of

$$D_{s0}^*(1^{3}P_0)$$ is shifted down from 2441 MeV to 2364 MeV (a little below the $DK$ threshold), with a 77 MeV mass shift. More importantly, the curve for $\Delta M(M)$ has a cusp-like behavior at the $DK$ threshold, and the intersection between $M - M_0$ and $\Delta M(M)$ is very close to the cusp-like position. The cusp-like mass shift is a typical characteristic among nearby threshold states and $S$-wave couplings, which has been studied in many previous works [27, 30, 47–50]. Our results in Fig. 2 vividly describe how a nearby threshold state is affected by its $S$-wave channel.

III. PREDICTION OF A CHARMED-STRANGE BARYONIC STATE $\Omega_{c0}(1P, 1/2^+)$ AS ANALOGUE OF $D_{s0}^*(2317)$

In the following, we will take the same unquench quark model to predict $\Omega_{c0}(1P, 1/2^+)$ state below the threshold of $\Xi_c K$. This state could be regarded as the charmed-strange baryonic analogue of $D_{s0}^*(2317)$. To this end, one should first calculate the discrete mass spectrum of the bare $\Omega_c$ baryons. Here, we take the pairwise quark-quark potential to depict the spin-independent interactions of charm and strange quarks in the $\Omega_c$ system, i.e.,

$$V = \sum_{i<j} \left(\frac{-2\alpha_s}{3r_{ij}} + \frac{b}{2}r_{ij} + C\right).$$  \hspace{1cm} (18)$$

The interactions appearing in Eq. (18) can be regarded as the most direct way to extrapolate the interaction of meson (see Eq. (11)) to the baryon system since the color factor $\langle F_i \cdot F_j \rangle$ of $qq$ in baryon is $\frac{1}{2}$ that of $g\bar{q}q$ in meson. The color-magnetic term and Thomas-precession piece for the spin-orbit interactions of $P$-wave $\Omega_c$ baryons are taken from Ref. [51], which are given
by

\[
H_{ij}^{\text{conf(cm)}} = \frac{2\alpha_s}{3r_{ij}} \left( \frac{r_{ij} \times p_{ij} \cdot s_i - r_{ij} \times p_{ij} \cdot s_j}{m_i^2} - \frac{r_{ij} \times p_{ij} \cdot s_j - r_{ij} \times p_{ij} \cdot s_i}{m_j^2} \right),
\]

and

\[
H_{ij}^{\text{conf(p)}} = -\frac{1}{2r_{ij}} \frac{\partial H_{ij}^{\text{conf}}}{\partial r_{ij}} \left( \frac{r_{ij} \times p_{ij} \cdot s_i - r_{ij} \times p_{ij} \cdot s_j}{m_i^2} - \frac{r_{ij} \times p_{ij} \cdot s_j - r_{ij} \times p_{ij} \cdot s_i}{m_j^2} \right),
\]

respectively. The \( \alpha_s, b, \) and constituent quark masses for calculating the mass spectrum of \( \Omega_c \) baryons have been fixed in the last section. Other parameters in the quark potential model are determined by the well established \( \Xi_c \) and \( \Omega_c \) states, i.e., \( \sigma = 1.732 \text{ GeV} \) and \( C = -0.344 \text{ GeV} \). Finally, we present the predicted masses of \( \Omega_c \) baryons in Fig. 3. The masses of \( \Omega_c \) and \( \Omega_c^* \) are fitted with the experimental results, and the mass of \( \Omega_c(1P) \) is consistent with the previous predictions [31–33, 35, 52, 53].

FIG. 3: The mass spectrum of \( \Omega_c \) baryons. The short black lines denote the masses from the quenched quark model while the observed states are labeled by blue dots. The red dashed lines correspond to the thresholds.

The mass of \( J^P = 1/2^- \Omega_c(1P) \) with predominant \( j_\ell = 0 \) is given by 3042 MeV in the quenched model, which is about 80 MeV higher than the \( \Xi_c \bar{K} \) threshold. Since this \( P \)-wave \( \Omega_c \) state mainly decays into \( \Xi_c \bar{K} \) in \( S \)-wave channel, its property is expected to be similar to the \( D_{s0}^{*}(2317) \) state (see Fig. 1).

Hence, it is necessary to further examine the coupled-channel effect on \( J^P = 1/2^- \Omega_c(1P) \) with predominant \( j_\ell = 0 \). The harmonic oscillator wave function will be adopted in our calculation, where the parameters \( \beta_p \) and \( \beta_s \) are also fixed by the quark potential model. The concrete values of \( \beta_p \) and \( \beta_s \) are presented in Table II. The remaining parameter is the \( \gamma \) value in the QPC model. According to Ref. [54], for different hadron systems, the \( \gamma \) values are allowed to be different. In charmed-strange baryon system, we determine \( \gamma = 8.66 \) by the decay width of well-established \( \Xi_c(2790) \) state.

Two \( J^P = 1/2^- \) states have been predicted in Fig. 1 for the \( P \)-wave \( \Omega_c \) states. One is a predominant \( j_\ell = 0 \) state while another is a predominant \( j_\ell = 1 \) state. In the heavy quark limit, the pure \( \Omega_c(1/2^-)_{j_\ell=0} \) state can only couple with the \( \Xi_c \bar{K} \) while the \( \Omega_c(1/2^-)_{j_\ell=1} \) state is forbidden for the decay channel of \( \Xi_c \bar{K} \). We notice that the predicted mass of \( \Omega_c(1/2^-)_{j_\ell=1} \) state is about 60 MeV below the threshold of its \( S \)-wave channel \( \Xi_c \bar{K} \). Hence, the coupled-channel effect should also be considered for the predominant \( j_\ell = 1 \) \( \Omega_c(1/2^-) \) state. Obviously, it is convenient to perform the calculation in the basis, which is defined in the heavy quark limit (so-called \( j - j \) coupling scheme, see Ref. [34]).

Then two physical \( J^P = 1/2^- \Omega_c \) states that are denoted as \( |1/2^-_1) \) and \( |1/2^-_2) \) are the mixtures of \( j_\ell = 0 \) and \( j_\ell = 1 \) components, i.e.,

\[
\begin{pmatrix}
|1/2^-_1) \\
|1/2^-_2)
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
|j_\ell = 0, 1/2^-) \\
|j_\ell = 1, 1/2^-)
\end{pmatrix},
\]

where \( \theta \) is the mixing angle. In the \( j - j \) coupling scheme, the coupled-channel Schrödinger equation contains two bare states [60] could be written as

\begin{equation}
\begin{pmatrix}
M_{ii=0} \\
\tilde{V}_{1}^\text{spin} \\
\tilde{V}_{1}^\text{spin} \\
M_{ii=1}
\end{pmatrix} \begin{pmatrix}
\int p^2 dp \langle \Omega_{c0}\mid \hat{H}_i\mid \Xi_c K \rangle \\
0 \\
0 \\
\int p^2 dp \langle \Omega_{c1}\mid \hat{H}_i\mid \Xi_c K \rangle
\end{pmatrix} = M \begin{pmatrix}
c_{0} \\
c_{1} \\
c_{\Xi_c K} \\
c_{\Xi_{c0}}
\end{pmatrix},
\end{equation}

where two \( J^P = 1/2^- \Omega_c \) baryons with \( j_\ell = 0 \) and \( j_\ell = 1 \) are denoted as \( \Omega_{c0} \) and \( \Omega_{c1} \), respectively, and the effects of \( S \)-wave channels \( \Xi_c \bar{K} \) and \( \Xi_{c0} \bar{K} \) are considered. The \( M_{ii=0} \) and \( M_{ii=1} \) are bare masses of \( \Omega_{c0} \) and \( \Omega_{c1} \), respectively, which can be obtained by the quark potential model. The off-diagonal element is defined as \( \tilde{V}_{1}^\text{spin} = \langle \Omega_{c0}\mid V^\text{spin}\mid \Omega_{c1} \rangle \), which is her-
mitean and can be directly determined by the quark potential model. Finally, the multi-coupled-channel equation of Eq. (22) can be simplified as (see details in Appendix A)

\[
\begin{pmatrix}
M_{j=0} + \Delta M^0(M) \\
\tilde{V}_{j=1} + \Delta M^1(M)
\end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = M \begin{pmatrix} c_0 \\ c_1 \end{pmatrix},
\]

where

\[
\Delta M^0(M) = \text{Re} \int_0^\infty p^2 dp \frac{|M_{\Xi (2790) \to K (p)}|^2}{M - \sqrt{M_{\Xi}^2 + p^2} - \sqrt{M_K^2 + p^2}},
\]

\[
\Delta M^1(M) = \text{Re} \int_0^\infty p^2 dp \frac{|M_{\tilde{\Xi} (2790) \to K (p)}|^2}{M - \sqrt{M_{\tilde{\Xi}}^2 + p^2} - \sqrt{M_K^2 + p^2}}.
\]

The Eq. (23) can be obviously decomposed into two independent single coupled-channel equations (as Eq. (7)) in the heavy quark limit \( \tilde{V}_{j=0} \to 0 \).

By diagonalizing Eq. (25), the mass of \( J^P = 1/2^- \) \( \Omega_c (1P) \) with predominant \( j_f = 0 \) is predicted to be 2945 MeV, which is shifted down about 97 MeV. The mixing angle is simultaneously obtained as \( \theta = \tan^{-1} \xi = -12.9^\circ \). Since the mixing angle is small, we tentatively call this state as \( \Omega_c^0 (1P, 1/2^-) \), where the superscript "0" denotes that \( \Omega_c (1P, 1/2^-) \) component is dominant. We would like to emphasize that the physical mass of \( \Omega_c^0 (1P, 1/2^-) \) becomes about 20 MeV below the \( \Xi (2790) \) threshold when the nontrivial unquenched effect is incorporated. Then, we conclude that a charmed-strange baryonic analogue of \( D_{s0}^*(2317) \) may exist in the \( P \)-wave \( \Omega_c \) baryon.

It is a problem how to search for the predicted \( \Omega_c^0 (1P, 1/2^-) \). If the mass of \( \Omega_c^0 (1P, 1/2^-) \) is below the \( \Xi (2790) \) threshold, there is no OZI-allowed decay. The radiative decay channels \( \Omega_c^0 (\gamma) \) and hadronic decay processes \( \Omega_c \pi^0 \) are kinematically allowed and should be considered in searches. For the \( \Omega_c^0 (1P, 1/2^-) \to \Omega_c \pi^0 \) decay, it is a typical isospin breaking process, where \( \Omega_c (1P, 1/2^-) \to \Omega_c \pi^0 \) may occur via \( \eta - \pi^0 \) mixing, which results in a suppression factor \( \sim 10^{-4} \) [55]. Another approach of searching for \( \Omega_c (1P, 1/2^-) \) is the radiative decay. Among the heavy flavor baryons, \( \Xi_c^+, \Xi_c^0, \) and \( \Omega_c^+ \) were discovered by the radiative decays since their masses are below their respective lowest strong decay channels [56–58]. Very recently, the Belle Collaboration [59] has seen \( \Xi_c (2790)^0 \) and \( \Xi_c (2815)^0 \) in the radiative decay channel \( \Xi_c \gamma \). This is a great breakthrough because the \( \Xi_c^+, \Xi_c^0, \) and \( \Xi_c^+ \) are 1S states while \( \Xi_c (2790)^0 \) and \( \Xi_c (2815)^0 \) are orbitally excited states. In consideration of the fact that the excited states \( \Xi_c (2970)^0 \) and \( \Xi_c (2815)^0 \) can be seen via radiative decays, it is also probable to discover \( \Omega_c (1P, 1/2^-) \) via \( \Omega_c \gamma \) channels in future Belle II experiments.

The mass of another \( J^P = 1/2^- \) \( \Omega_c \) state, i.e., the predominant \( j_f = 1 \) state, is obtained as 2991 MeV by considering the coupled-channel effect from \( \Xi_c \) channel. Since this state is still above the \( \Xi (2790) \), it is expected to be a conventional resonance. The mixing angle is determined as \( \theta = -10.9^\circ \). Since Eq. (23) is a multi-coupled-channel equation, it is not strange that the mixing angles for the two physical states contain a small difference [60, 61]. For completeness, we should also check the coupled-channel effect for two \( J^P = 3/2^- \) \( \Omega_c (1P) \) states by the following relations

\[
\begin{pmatrix}
(1^-) \gamma_1 \\
(1^-) \gamma_2
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
(\gamma_1) \gamma_1 \\
(\gamma_1) \gamma_2
\end{pmatrix},
\]

Our results indicate that the masses of two \( J^P = 3/2^- \) \( \Omega_c \) states are not significantly affected by the coupled-channel effect. Their masses and mixing angles are given by,

\[
M_{\Omega_c^0 (1P, 3/2^-)} \text{phy} = 3029 \text{ MeV}, \quad \theta = 4.8^\circ; \quad M_{\Omega_c^+(1P, 3/2^-)} \text{phy} = 3058 \text{ MeV}, \quad \theta = 4.1^\circ.
\]

IV. CONCLUSIONS AND DISCUSSION

The observation of \( D_{s0}^*(2317) \) has made theorists realized the importance of coupled-channel effects on mass spectrum study [20–26]. If replacing the \( \bar{s} \) quark in \( D_{s0}^*(2317) \) by an \( ss \) pair, we may naturally conjecture the existence of a new resonance as charmed-strange baryonic analogue of \( D_{s0}^*(2317) \). In this work, we have predicted such a new resonance by an unquenched quark model, where the predicted charmed-strange baryon has mass lower than the \( \Xi (2790) \) threshold, and hence, its OZI-allowed strong decay mode is forbidden. Searching for this predicted charmed-strange baryon will be an interesting task for future experiments like Belle II and LHCb.

We have noticed that the LHCb Collaboration once reported five narrow \( \Omega_c^0 \) states, i.e., the \( \Omega_c (3000)^0 \), \( \Omega_c (3050)^0 \), \( \Omega_c (3065)^0 \), \( \Omega_c (3090)^0 \), and \( \Omega_c (3120)^0 \), in the \( \Xi (2790)^+ \) channel [36]. The former four \( \Omega_c^0 \) states have been confirmed by the Belle Collaboration in the same decay channel while the \( \Omega_c (3120)^0 \) signal has not been reported in Belle [37]. These observed states are about in the range of 3.0–3.1 GeV, which is roughly fitted on predicted mass region of conventional \( \Omega_c (1P) \) states. Thus, some of the observed states could be good candidates of \( \Omega_c (1P) \) states [34, 62–72]. Besides the observed excited states above the \( \Xi (2790) \) threshold, we think there should exist a missing \( P \)-wave state below the \( \Xi (2790) \) threshold as suggested in this work. When the nontrivial coupled-channel effect has been considered, the mass of \( J^P = 1/2^- \) \( \Omega_c (1P) \) with predominant \( j_f = 0 \) should be shifted below the threshold of \( \Xi (2790)^- \) channel. Obviously, this state cannot be found by the measured \( \Xi (2790)^- \) invariant mass spectrum from LHCb and Belle [36, 37]. How to find this predicted charmed-strange baryon will be a challenging opportunity for the Belle II experiment, where this predicted charmed-strange baryon \( \Omega_c^0 (1P, 1/2^-) \) should decays into \( \Omega_c^0 \gamma \).

Before the present study, there exist several typical examples including \( D_{s0}^*(2317) \), \( D_{s0}^*(2460) \), \( X(3872) \), and \( \Lambda_c (2940) \), where the coupled-channel may play a crucial role to understand their low mass phenomena [20–30]. If the predicted charmed-strange baryon as the charmed-strange baryonic analogue of \( D_{s0}^*(2317) \) can be confirmed in future experiments, it
will provide a new example to show the importance of the coupled-channel effect.

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Appendix A

In this Appendix, we will present some details how to obtain Eq. (23) from Eq. (22). By expanding the Eq. (22), we obtain

\[ c_0 \tilde{\mathcal{V}}^{\text{spin}} + c_1 M^{j=1} + \int p^2 dp e_{\Xi,K} \langle \Omega_{cL} | H_{\Xi,K} | \Xi_K \rangle = c_0 M, \]  

(A1)

and

\[ c_0 \langle \Xi_K | H_{\Xi,K} | \Omega_{cL} \rangle + c_{\Xi,K} H_{\Xi,K} = c_{\Xi,K} M, \]  

(A2)

Using Eq. (A2), we have

\[ c_{\Xi,K} = \frac{\langle \Xi_K | H_{\Xi,K} | \Omega_{cL} \rangle}{M - H_{\Xi,K}}, \quad c_{\Xi,K} = c_1 \frac{\langle \Xi_K | H_{\Xi,K} | \Omega_{cL} \rangle}{M - H_{\Xi,K}}. \]  

(A3)

Then Eq. (A1) could be rewritten as

\[ c_0 M^{j=0} + c_0 \int p^2 dp \frac{|\langle \Xi_K | H_{\Xi,K} | \Omega_{cL} \rangle|^2}{M - H_{\Xi,K}} + c_1 \tilde{\mathcal{V}}^{\text{spin}} = c_0 M, \]  

\[ c_0 \tilde{\mathcal{V}}^{\text{spin}} + c_1 M^{j=1} + \int p^2 dp \frac{|\langle \Xi_K | H_{\Xi,K} | \Omega_{cL} \rangle|^2}{M - H_{\Xi,K}} = c_1 M. \]  

(A4)

The above relations are equivalent to the following eigenvalue equation

\[ \left( \begin{array}{cc} M^{j=0} + \Delta M^0(M) & \tilde{\mathcal{V}}^{\text{spin}} \\ \tilde{\mathcal{V}}^{\text{spin}} & M^{j=1} + \Delta M^1(M) \end{array} \right) \left( \begin{array}{c} c_0 \\ c_1 \end{array} \right) = M \left( \begin{array}{c} c_0 \\ c_1 \end{array} \right), \]  

(A5)

where

\[ \Delta M^0(M) = \text{Re} \int_0^\infty p^2 dp \frac{|M_{\Xi,K} - \Xi_K(p)|^2}{M - \sqrt{M_{\Xi,K}^2 + p^2 - \sqrt{M_{\Xi,K}^2 + p^2}}}, \]  

(A6)

\[ \Delta M^1(M) = \text{Re} \int_0^\infty p^2 dp \frac{|M_{\Xi,K} - \Xi_K(p)|^2}{M - \sqrt{M_{\Xi,K}^2 + p^2 - \sqrt{M_{\Xi,K}^2 + p^2}}}. \]  

(A7)

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