A Design of Compact Wideband Antenna Based on Hybridization of Minkowski Fractal Curves on Hexagonal Patch and Partial Ground Plane with Truncated Corners

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Abstract
In this manuscript, a compact wideband antenna using a partial ground plane has been designed by the hybridization of Minkowski fractal curves on the hexagonal radiating patch. Further, the corners of this partial ground plane have been truncated and the Minkowski curves superimposed on each truncated corner. The L shaped stub has been employed to enhance the performance parameters of the antenna in terms of the number of frequency bands and impedance bandwidth. Different prototypes of an antenna have been compared and found that prototype with L-shaped stub and ground plane with truncated corners using Minkowski curve (proposed prototype) exhibits better antenna performance parameters. The proposed prototype of the antenna reveals the maximum bandwidth of 10.86 GHz (140.10%) with four distinct frequency bands 4.4, 7.1, 10.8, 16.3 GHz. Designed antenna has also been fabricated and tested for authentication of simulated results with measured results and found in reasonable agreement with each other. The proposed antenna uses a low-cost FR4 substrate with a compact size of 24 × 30 × 1.6 mm³. Due to the wider bandwidth, stable radiation pattern, and desired gain at the desired frequency points, the proposed antenna can be used for different wireless applications.

Keywords Wideband antenna · Stub · FR4 · Partial ground plane · Minkowski curve · Fractal

1 Introduction
With the fast growth of mobile communication, one can integrate various types of wireless applications into a single wireless device [1]. These days, the fractal antenna is commonly used, as these can show multiband/wideband behavior in its $S_{11}$ parameter characteristic. This antenna reduces its patch size because of its two very important properties: self-similarity
and space-filling [2, 3]. Fractal antennas are generally utilized for the framework that needs a broadband behavior and high information rate. Presently, the developing interest in remote frameworks requires smaller antennas with more extensive data transmission rate and wide-band qualities with increased reflection coefficient [4, 5]. By utilizing iterative recursive procedure in the antenna geometries, one can make a unique structure [6]. Space-filling and self-similarity are the properties of the fractal that shows novel qualities (wideband and multiband) in antenna design [7, 8]. Space-filling property helps to reduce the size of the antenna. The self-similarity is useful to attain the multiband/broadband behavior [9]. The designing of fractal antennas have used various geometries like Minkowski, Meander, Koch Hilbert, Sierpinski Gasket/Carpet, and Giuseppe Peano etc. For distinct mobile applications, many researchers have joined these fractal geometries to make hybrid fractal antennas.

Siakavara et al. [10] has designed the direct radiating arrays (DRAs) termed as hybrid – fractal antenna array. The main advantage described by the author to use the proposed technique to reduce the cost along with size of the antenna. Azaro et al. [11] have illustrated a hybrid pre antenna obtained by superimposing the Sierpinski and a Meander structure and it can be beneficial for Wi-Fi and GSM frequency bands. Chang et al. [12] have demonstrated a traveling-wave hybrid fractal antenna. Authors have reported the enhanced antenna performance parameters by hybridization of various geometries. Soni et al. [13] have investigated the hybrid Minkowskized fractal antenna. This prototype resonates at six distinct frequencies. Gashatsbi et al. [14] explained a novel hybrid fractal loop antenna using Minkowski and Koch geometries. The fabricated antenna used for the GSM900MHz application and reported a stable radiation pattern. Rajkumar et al. [15] investigated the MIMO multiband hybrid antenna based on a combination of altered dragon curve and inverted Koch curve. After broadly going through the aforementioned literature, in this manuscript, a hybrid fractal antenna has been designed by superimposing the Minkowski curves on the hexagonal radiating patch and truncated corners of the partial ground plane along with L-shaped stub. The proposed HFA is useful for various wireless applications.

2 Steps for the Proposed Antenna Design

This section illuminates an investigation of the parametric study and the different design steps of the simulated proposed HFA. In this manuscript, the hybrid antenna has been obtained by superimposing the Minkowski curve on a hexagonal radiating patch. The process of designing the Minkowski fractal curve and the procedure of combining it with a hexagonal patch is discussed in detail.

2.1 Design Process of Minkowski Fractal Curve

Figure 1a shows the Initiator defined as the structure of the Minkowski fractal curve with 90-degree indentation angle ($\theta$) is by using the straight-line, and Fig. 1b shows the generator structure of the fractal curve. Repeat this iterative process up to 2nd iteration to as delineated in Fig. 1c to get the required fractal (Minkowski) shape. IFS (Iterated Function System) code [16] is used to get desired geometry, and its dimensions are evaluated as [17]:

$$
\text{Indentation Factor}(\rho) = \frac{d}{R_{1/3}}
$$

(1)
where, \( d \) is the depth of indentation and \( R_1/3 \) is the fractional part of \( R_1 \).

IFS are a convenient mathematical tool used for the construction of fractal geometries by applying affine transformations \((K)\) to a basic geometry \((Z)\). Affine conversion contains translation, scaling and rotation and can be characterized as:

\[
K(x) = Zx + t = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \tag{2}
\]

where basic geometry \(Z\) (matrix) is given as:

\[
Z = \begin{bmatrix} \frac{1}{s} \cos \theta & -\frac{1}{s} \sin \theta \\ \frac{1}{s} \sin \theta & \frac{1}{s} \cos \theta \end{bmatrix} \tag{3}
\]

Here ‘a to d’ are the parameters are for the scaling and rotation. The ‘\(x_1, x_2\)’ signifies the coordinate points of \(x\); ‘\(e\) and \(f\)’ are for the translation and shifting respectively. The ‘\(\theta\)’ is the rotation angle; ‘\(t\)’ represents the transformation factor, and ‘\(s\)’ represents the scaling factor.

The Eq. (4) used to obtain fractal geometry with the application of the set of linear transformation on ‘\(Z\)’ and using Hutchinson operator \((K)\)

\[
K(Z) = \bigcup_{n=1}^{N} K_n(Z) \tag{4}
\]

Figure 1c illustrated the renovation of all segment can generate the desired shape of fractal curve. Thus, the proposed curve can be obtained by using above mentioned operator \((K)\) and is represented by:

\[
K_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.333 & 0 \\ 0 & 0.333 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}
\]
where \( K_1, K_2, K_3, K_4 \) and \( K_5 \) represents a set of linear transformation and the originator curve of Minkowski fractal is then attained using \( K(Z) = K_1(Z) \cup K_2(Z) \cup K_3(Z) \cup K_4(Z) \cup K_5(Z) \). This procedure is repeated for obtaining the further iterations of the desired Minkowski curve. The self-similarity index (D), can be obtained using the following equation:

\[
D = \frac{\log(N)}{\log(r)}
\]

(10)

By using the above-mentioned process, the different and similar types of fractal geometries and iterations can be generated. In this paper, two iterations of Minkowski curve have been designed. The designed curve length has been evaluated by taking the value of ‘\( n = 2 \)’ using the following equation:

\[
l = h\left(\frac{N}{r}\right)^n
\]

(11)

Here ‘\( h \)’ represents the height of curve, ‘\( N \)’ designates the number of segments, ‘\( r \)’ represents the number of segments divided on iteration, and ‘\( n \)’ designates the number of iterations.

### 2.2 Proposed Hybrid Fractal Antenna (HFA) Design

The proposed HFA design is a combination of two different geometries. The fractal curve designed in the previous sub-section has been superimposed on each side of the hexagonal patch. After applying the fractal curves on the radiating patch, it has been combined with 50 \( \Omega \) transmission line feed and fractional ground plane (FGP) as shown in Fig. 2a. Consider low-cost FR4 material as a substrate with dielectric constant = 4.4, thickness \( h = 1.6 \) mm, the resonant frequency = 5.65 GHz, loss tangent = 0.02, the mass density of 19,000 kg/m\(^3\) for the proposed HFA. The hexagonal patch radius(\( R = 7 \) mm) has been calculated as using the following equation [18].

\[
R = \frac{F}{\left\{ 1 + \frac{2h}{\pi F_e} \left[ \ln \left( \frac{\pi F_e}{2h} \right) + 1.7726 \right] \right\}^{\frac{1}{2}}}
\]

(12)

where, \( F = \frac{8.791 \times 10^6}{\sqrt{\pi F_e}} \).

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In compact-sized antenna radiators the proper impedance matching is a relatively cumbersome method. Hence, to improve this problem the parametric dimensions of FGP have been changed further to attain better impedance matching characteristics and wider bandwidth (BW) as illustrated in Fig. 2b. Further, Fig. 2c illustrated the corners of the rectangular fractional ground plane have been truncated with side length T2 and T3 for both the lower and upper edges, the side length of truncated corners are the same as the side length of the hexagonal radiating patch. The designed fractal curve is overlaid on the truncated corners of the fractional ground plane as shown in Fig. 2d, to analyze the performance of a proposed antenna. Finally, the inverted L-shaped stub is employed in the geometry of the ground plane as depicted in Fig. 2e to obtain the wider BW and improve the reflection coefficient ($\Gamma$) in the desired frequency range. The optimized dimensions of the proposed HFA are illustrated in Table 1.

![Fig. 2 Design evolution of the proposed HFA](image)

### Table 1  Optimal value of different dimensions of the proposed antenna

| Parameters | Optimized Value (mm) | Parameters | Optimized Value (mm) | Parameters | Optimized Value (mm) |
|------------|----------------------|------------|----------------------|------------|----------------------|
| R          | 7.0                  | $W_1$      | 24.0                 | $X_1$      | 6.1                  |
| $L_F$      | 18.665               | $T_1$      | 14.1                 | $S_{W1}$  | 2.8                  |
| $W_F$      | 1.8                  | $T_2$      | 7.0                  | $S_{T1}$  | 1.0                  |
| $L_1$      | 30.0                 | $T_3$      | 7.0                  | $S_{L1}$  | 4.0                  |
The effects of designed prototypes from the initial stage (prototype: 1) to the final stage (proposed prototype) in terms of S11 and impedance BW have been illustrated in Fig. 3. The designed antenna (prototype: 1 and 2) operates on three frequency bands with a maximum BW of 6.67 and 8.0 GHz. While the other two prototypes exhibit only two frequency bands and reveal the maximum BW of 8.32 and 9.50 GHz respectively. It is very much clear from the above discussion that by modifying the shape of the proposed HFA from the prototype-1 to prototype-4, the impedance BW has been increased. Though the BW has been improved the reflection coefficient (Γ) is degraded and also the number of frequency bands gets reduced. To overcome this problem the final geometry (proposed prototype) has been designed by introducing the inverted L-shaped stub in the geometry of prototype-4, to obtain more frequency bands with improved BW and Γ. It has been revealed that a proposed prototype of the designed antenna exhibits four distinct resonance frequency points with enhanced reflection coefficient and a maximum BW of 10.86 GHz. The mathematical results of all the designed prototype of the proposed antenna are tabulated in Table 2 for the better understanding.

The performance parameter like impedance BW of the proposed HFA has also been analysed by optimizing its different parameters such as ’WF’ feed line width and ‘SW1’ width of stub arm. The ‘WF’ and ‘SW1’ parameters have been varied from 1.0

![Fig. 3 S11 plot for the different prototypes of the proposed HFA](image)

| Antenna Design    | Resonant frequency points (GHz) | Maximum BW (GHz) | BW Ratio | BW (%)  |
|-------------------|---------------------------------|------------------|----------|---------|
| Prototype-1       | 5.0/9.6/19.7                    | 6.67             | 2.80:1   | 94.81   |
| Prototype-2       | 4.7/7.0/10.1                    | 8.0              | 3.78:1   | 116.44  |
| Prototype-3       | 6.1/10.8                        | 8.32             | 3.53:1   | 111.82  |
| Prototype-4       | 6.1/16.3                        | 9.50             | 3.95:1   | 119.19  |
| Proposed Prototype| 4.4/7.1/10.8/16.3               | 10.86            | 5.72:1   | 140.10  |
to 1.8 mm with an increased step size of 0.2 mm as delineated in Figs. 4 and 5 respectively. It can be analysed from both the figures that the proposed HFA exhibits the wider BW and improved $\Gamma$ only at $WF = 1.8$ mm and $SW1 = 1.8$ mm. Thus, the optimal values of the abovementioned parameters have been fixed to 1.8 mm for better performance in terms of BW and $\Gamma$.

The current distribution plot for both the surfaces (radiating patch and ground plane) of the proposed HFA at distinct frequency points are illustrated in Fig. 6. From this figure it can be observed that a strong current is concentrated on inverted L-shaped stub and transmission line along the ground plane at the resonance points of 4.4, 7.1, and 10.8 GHz. The strong current generation on the stub helps in attaining the broad BW.

Fig. 4 $S_{11}$ plot of proposed HFA at distinct values of ‘$WF$’ parameter

Fig. 5 $S_{11}$ plot of proposed HFA at distinct values of ‘$SW1$’ parameter
and improved $\Gamma$ at these frequency bands. Similarly, the $\Gamma$ is improved at 16.3 GHz due to the concentration of strong current at the feed line and lower edge of the hexagonal patch as shown in Fig. 6d. So, from the aforementioned discussion, it is clear that the effects of the truncated ground plane with inverted L-shaped stub play a very important role in the various performance parameters like impedance bandwidth and $S_{11}$ of the proposed HFA.

3 Fabricated Prototype and Results

This section reveals the comparison of simulated and measured results of the proposed HFA and the fabricated prototype of the designed antenna (front and rear view) as illustrated in Fig. 7. The fabricated prototype of the antenna has been tested by using a vector network analyser (VNA) with a frequency range from 1 MHz to 20 GHz and the experimental $\Gamma$ is compared with simulated as shown in Fig. 8. It clears results that they have good agreement with each other, due to the small variations in both the curves (simulated and measured). The simulated antenna exhibits the maximum BW of 10.86 GHz with resonance points of 4.4, 7.1, 10.8, and 16.3 GHz. Whereas, the measured antenna shows the maximum impedance BW of 10.05 GHz with frequency bands of 4.8, 7.0, 10.1, and

![Simulated surface current distribution plot of proposed hybrid antenna at (a) 4.4 GHz, (b) 7.1 GHz, (c) 10.8 GHz and (d) 16.3 GHz frequency points](image-url)
15.5 GHz. There are some variations in simulated and measured results of the proposed HFA due to the environmental conditions, soldering bumps, connector losses physical properties of a fabricated prototype, etc.

The radiation properties of a proposed HFA have been analyzed with the help of its radiation patterns in both the planes. So, the radiation patterns of the proposed antenna have been measured at distinct frequency points and compared with the simulated patterns as illustrated in Fig. 9. It indicates that antenna represents the stable omnidirectional radiation pattern in the E and H-planes at all the frequency bands. The omnidirectional properties of a proposed HFA indicate that it can be utilized for various wireless applications in the operational frequency range. Figure 10 illustrated the simulated and measured peak realized gains of the proposed HFA. Based on simulations and measurements, the antenna exhibits positive gain in the desired frequency range between 2.30 and 13.16 GHz for simulated antennas, and 2.40–12.45 GHz for measured antennas. In the proposed antenna, the maximum gain is 4.54 dB at 2.3 GHz and the gain remains
positive at all the other frequency points. From the above discussion it can be said that due to the stable radiation pattern and gain at all the frequency points in the desired frequency range the proposed antenna can be used for different wireless standards such as LTE 2300/LTE 2500 (2.3–2.4 GHz/2.5–2.69 GHz), Bluetooth (2.4 GHz), WLAN (2.4–2.48 GHz, 5.15–5.35 GHz), WiMAX (3.3–3.7 GHz), ISM (5.725–5.875 GHz), ITU band (7.8–8.4 GHz), TV broadcasting (7.91–8.62 GHz), point-to-point wireless applications (5.92–8.5 GHz) and a radio navigations (15.43–17.3 GHz).

In addition to the earlier discussion, Table 3 shows the comparison in terms of performance parameters and size between with the proposed HFA has been compared with the existing hybrid fractal antennas. It is found that the proposed antenna is compact and exhibits enhanced BW as compared to the antennas proposed in a review of the current literature.
4 Conclusion

The proposed compact wideband antenna using partial ground plane has been designed by hybridization of Minkowski fractal curves on the hexagonal radiating patch. Different structures of an antenna are compared and found that structure with L-shaped stub and ground plane with truncated corners using Minkowski curve exhibits better antenna performance in terms of impedance bandwidth and the number of resonant frequency bands. The proposed structure of a HFA exhibits four distinct frequency bands 4.4, 7.1, 10.8, 16.3 GHz with the maximum BW of 10.86 GHz (140.10%). The proposed HFA is fabricated and the measured results are compared with simulated and found in good agreement with each other. The antenna exhibits a stable radiation pattern at distinct frequency bands and shows the maximum gain of 4.54 dB at 2.3 GHz. Due to the larger BW, stable radiation pattern, and positive gain at the anticipated frequency range, the proposed HFA can
be used for different wireless applications such as Bluetooth, WLAN, WiMAX, ISM, TV broadcasting, and radio navigations.

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**Data Availability** The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

**Declarations**

**Conflict of interest** The authors have no conflicts of interest to declare that are relevant to the content of this article.

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