Note on the magnetotransport in the normal state of high-$T_c$ cuprates

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Abstract

Theories of the magnetotransport, based on the quasiparticle of the Fermi-liquid, in the normal state of high-$T_c$ cuprate superconductors, are critically examined and the necessity of the collective transport theory beyond the quasiparticle is discussed.

Keywords: Hall effect, magnetoresistance, spin fluctuations

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I. INTRODUCTION

Anomalous temperature dependence of the magnetotransport coefficients in the normal state of high-$T_c$ cuprate superconductors has been discussed intensively as the evidence for the breakdown of the Fermi-liquid theory. Experimental data are roughly summarized as $\sigma_{xx} \propto T^{-1}$, $\sigma_{xy} \propto HT^{-3}$ and $\Delta \sigma_{xx} \propto H^2T^{-5}$ where $T$ is the temperature and $H$ is the external magnetic field. Thus Kohler’s rule, expected from the ordinary transport theory based on the quasiparticle of the Fermi liquid, is strongly violated in a wide temperature range.

On the other hand, it has been claimed that such a violation of Kohler’s rule can be derived from the quasiparticle transport within the framework of the Fermi-liquid theory if the momentum-dependence of the scattering due to antiferromagnetic spin fluctuation is fully taken into account.

In this paper we show that the quasiparticle-transport theory is inadequate to explain the anomaly in section II and that the collective-transport theory beyond the quasiparticle is necessary in section III.

II. QUASIPARTICLE TRANSPORT

The quasiparticle contribution to the magnetotransport coefficients within the relaxation-time approximation in two dimensions is expressed in terms of the mean free path $l(s)$:

$$\sigma_{xx} = e^2 \int ds \, l(s)[\cos \theta(s)]^2,$$

(1)

$$\sigma_{xy} = e^2 \omega_c \int ds \, l(s) \cos \theta(s) \frac{d}{ds}[l(s) \sin \theta(s)],$$

(2)

$$\Delta \sigma_{xx} = -e^2 \omega_c^2 \int ds \, l(s) \left\{ \frac{d}{ds}[l(s) \cos \theta(s)] \right\}^2,$$

(3)

where $ds$ is the line element along the Fermi surface, $\theta(s)$ is the angle specifying the position on the Fermi surface and $\omega_c$ is the cyclotron frequency. Here the mean free path is
proportional to the transport life-time $\tau_{tr}(s)$. The effect of the vertex correction beyond the relaxation-time approximation is discussed in Appendix A.

In order to obtain analytic results we use a model for the mean free path

$$l(s) = l(\theta) = l_{\text{hot}} \frac{1 + a}{1 + a \cos 4\theta},$$

(4)

employed in the previous studies [11,12] Here $l_{\text{hot}} \equiv l(\theta = 0)$ and $a = (1 - r)/(1 + r)$ with $r = l_{\text{hot}}/l_{\text{cold}}$ and $l_{\text{cold}} \equiv l(\theta = \pi/4)$. From this model we can know qualitative features of the quasiparticle transport, though it is insufficient for quantitative discussions.

For this model replacing $ds$ by $d\theta$ we obtain

$$\sigma_{xx} = 16\pi e^2 l_{\text{hot}} r^{-1/2},$$

(5)

$$\sigma_{xy} = 8\pi e^2 \omega_c l_{\text{hot}}^2 (r + 1)r^{-3/2},$$

(6)

$$\Delta \sigma_{xx} = -\frac{\pi}{8} e^2 \omega_c^2 l_{\text{hot}}^3 (-5r^4 + 52r^3 + 34r^2 + 52r - 5)r^{-7/2}.$$  

(7)

While eqs. (2.5) and (2.6) agree with the previous result, [11] eq. (2.7) corrects the previous result [12] as discussed in Appendix B.

If $r$ does not depend on the temperature, eqs.(2.5)-(2.7) satisfy Kohler’s rule. Although some violation of the rule can be derived from the temperature-dependence of $r$, it is impossible to explain the experimentally observed violation in a *wide* temperature range. Namely it is impossible to obtain $(r + 1)r^{-1/2} \propto T^{-1}$ and $(-5r^4 + 52r^3 + 34r^2 + 52r - 5)r^{-2} \propto T^{-2}$, from the scattering due to antiferromagnetic spin fluctuation, in a *wide* temperature range consistent with the experiments. Such a criticism has also been made by other authors [17] in another context.

Another criticism should be made for the case where the mean free path $l(s)$ is calculated from the phenomenological spin susceptibility. [10–13] In those studies the self-consistency between the susceptibility and the self-energy for electrons has been neglected. The requirement of the self-consistency weaken the temperature dependence of the Hall constant expected from the phenomenology [10–13] as has been shown by the numerical study based on the fluctuation-exchange approximation. [14] Consequently the temperature dependence
of the Hall constant within the relaxation-time approximation is far weaker than that observed by the experiments.

In conclusion, only weak violation of Kohler’s rule can be derived from the momentum-dependence of the transport life-time within the quasiparticle transport described by the Boltzmann equation.

III. COLLECTIVE TRANSPORT

By the discussions in §2 it has become clear that the anomalous temperature dependence observed in the normal state of high-$T_c$ cuprate superconductors cannot be explained within the quasiparticle transport. However, it does not directly lead to the breakdown [1-7] of the Fermi-liquid theory. There exists the collective contribution beyond the quasiparticle within the linear response theory based on the Fermi liquid: for example, the contributions C and D to the Hall conductivity discussed in ref. 18 correspond to the collective ones, while the contributions A and B constitute the quasiparticle transport theory consistent with the Boltzmann equation. The collective contributions arise in the presence of the magnetic field so that Kohler’s rule is strongly violated in a wide temperature range. [19]

The collective transport theory is well established in the case of superconducting fluctuation. [20,21] In the case of antiferromagnetic spin fluctuation relevant to the cuprate superconductors, the collective transport theory has been developed by the present authors [22-26] in parallel with the case of superconducting fluctuation. (See Appendix C.)

The collective transport is necessary in the case of cuprates, since the anomalous phase with broken Kohler’s rule is proximity to the antiferromagnetic insulator where the low-energy excitation is the spin wave and there is no quasiparticle at low energies. It should be noted that the collective contributions are minor ones unless the spin fluctuation has the nesting character. [22-26] The heart of the collective transport for the nested spin fluctuation is that the anomaly is regarded as $4k_F$ singularity of the charge fluctuation triggered by two modes of the spin fluctuation with $2k_F$ singularity. [22] The $4k_F$ singularity is observed by
the diffusive X-ray scattering experiment. [27]

In conclusion, the strong violation of Kohler’s rule should be ascribed to the collective contribution beyond the quasiparticle contribution described by the Boltzmann equation.

IV. SUMMARY

We have shown that the anomalous temperature dependence of the magnetotransport coefficients in the normal state of high-$T_c$ cuprate superconductors is derived, within the framework of the Fermi-liquid theory, from the collective contribution of the nested spin fluctuation beyond the quasiparticle contribution described by the Boltzmann equation.

In addition, other transport coefficients should have the collective contribution from the nested spin fluctuation. For example, the thermo-electric power has such a contribution [23] in parallel with the case of the superconducting fluctuation. [28]

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APPENDIX A: VERTEX CORRECTION

In this Appendix we discuss the effect of the vertex correction. In the case, where the Fermi-liquid theory is applicable, the current vertex, the three-point vertex appearing in Fig. 1, is determined by three contributions, Figs. 1(a)-1(c). [24] This vertex correction leads to the collision term $I(n_k)$ in the Boltzmann equation [30] with obvious notations:

$$I(n_k) = - \sum_{k',q} W(k, k'; k - q, k' + q) \delta(\varepsilon_k + \varepsilon_{k'} - \varepsilon_{k-q} - \varepsilon_{k'+q})$$

$$\times [n_k n_{k'} (1 - n_{k-q})(1 - n_{k'+q}) - (1 - n_k)(1 - n_{k'}) n_{k-q} n_{k'+q}].$$

(A1)

This collision term is rewritten as [33] [36]
\[ I(n_k) = \sum_{k'} Q(k, k') [n_{k'} - n_k], \tag{A2} \]

with obvious notations. The relation of the detailed balance, \( Q(k, k') = Q(k', k) \), is satisfied if the collision term has the form given by eq. (A.1).

In the absence of the magnetic field the Boltzmann equation leads to

\[ \vec{v}_k = \sum_{k'} Q(k, k') [\vec{\Lambda}_k - \vec{\Lambda}_{k'}], \tag{A3} \]

where \( \vec{v}_k \) is the velocity and \( \vec{\Lambda}_k \) has the meaning of the mean free path. Equation (A.3) is equivalent to eq. (6.17) in ref. 29. It is widely recognized [33–36] that \( \vec{\Lambda}_k \) is not proportional to \( \vec{v}_k \) in general, while it is stressed in ref. 14.

In the relaxation-time approximation, [35,36] the right hand side of eq. (A.2) is replaced by \(- [n_k - n^0_k] / \tau_{tr}(k)\) introducing the transport life-time \( \tau_{tr}(k) \) where \( n^0_k \) is the equilibrium value. In this approximation the effect of the vertex correction is partly taken into account. However, it is shown [29,37] that the full account of the vertex correction does not alter the temperature dependence of the resistivity obtained in the relaxation-time approximation.

In the presence of the magnetic field, the linearized Boltzmann equation for \( g_k \equiv n_k - n^0_k \) is given by

\[ e \vec{E} \cdot \vec{v}_k \frac{\partial n^0_k}{\partial \varepsilon_k} + \frac{e}{c} \left( \vec{v}_k \times \vec{B} \right) \cdot \frac{\partial g_k}{\partial k} = \sum_{k'} Q(k, k') [g_{k'} - g_k]. \tag{A4} \]

This equation is rewritten in the matrix form:

\[ \sum_{k'} A_{kk'} g_{k'} = -e \vec{E} \cdot \vec{v}_k \frac{\partial n^0_k}{\partial \varepsilon_k}, \tag{A5} \]

where

\[ A_{kk'} \equiv (\tau_{tr}^{-1})_{kk'} - \frac{e}{c} \left( \vec{v}_k \times \frac{\partial}{\partial k} \right) \cdot \vec{B} \delta_{k,k'}, \tag{A6} \]

with

\[ (\tau_{tr}^{-1})_{kk'} \equiv \frac{1}{\tau_k} \delta_{k,k'} - Q(k, k'), \tag{A7} \]

and
\[ \frac{1}{\tau_k} \equiv \sum_{k'} Q(k, k'). \] (A8)

Since \( g_{k'} \) is obtained by the matrix inversion:

\[ g_{k'} = -\sum_k (A^{-1})_{k'k} e \vec{E} \cdot \vec{v}_k \frac{\partial n_0^k}{\partial \varepsilon_k}, \] (A9)

and the current \( \vec{j} \) is obtained by

\[ \vec{j} = e \sum_{k'} \vec{v}_{k'} g_{k'}. \] (A10)

the transport coefficient \( \sigma_{\mu\nu} \) is given as

\[ \sigma_{\mu\nu} = -e^2 \sum_{kk'} v^{\mu}_{k} (A^{-1})_{k'k} v^{\nu}_{k'} \frac{\partial n_0^k}{\partial \varepsilon_k}, \] (A11)

and this result fully contains the effect of the vertex correction. Our previous result \[26\] is consistent with eq. (A.11) but the momentum-derivative of the transport life-time has been omitted in eq. (3.38) of ref. 18 and thus in eq. (21) of ref. 14.

Using the fluctuation-exchange (FLEX) approximation, the authors of ref. 14 claim that the vertex correction in the presence of strong antiferromagnetic spin fluctuation leads to strong temperature dependence of the transport coefficients unexpected from the relaxation-time approximation. While their study is in the framework of the Fermi-liquid theory, such a claim contradicts with the previous studies \[18,29\] based on the Fermi-liquid theory. In the following we show that the FLEX approximation does not give the correct vertex correction discussed above.

In order to obtain the correct vertex correction, eq. (A.1), consistent with the Pauli exclusion principle for fermions, at least following two conditions should be satisfied.

First, the four-point vertex appearing in Fig. 1 should satisfy the so-called crossing or exchange symmetry. In the previous studies \[18,29\] based on the Fermi-liquid theory this symmetry is assumed. For example, the four-point vertex \( \Gamma_{ph} \) in the particle-hole channel shown in Fig. 2(a) and \( \Gamma_{pp} \) in the particle-particle channel shown in Fig. 2(b) are assumed to be identical. On the other hand, in the FLEX approximation such a symmetry is broken.
For example, the ladder process typical in the FLEX approximation shown in Fig. 3 cannot be identical in the sense discussed above.

Second, only low-energy states described by the singular part of the electron Green function should appear connecting each vertices in the vertex correction of the Fermi-liquid shown in Fig. 1. This limitation ensures that the resistivity is proportional to $T^2$ in the ordinary Fermi-liquid theory. On the other hand, in the FLEX approximation this limitation is violated in the so-called Maki-Thompson process which has no counterpart in the Fermi-liquid theory. Namely, the Maki-Thompson process contains unphysical high-energy states in comparison with the process of Fig. 1(a). While the Maki-Thompson process is treated on equal footing with the so-called Aslamazov-Larkin processes corresponding to Figs. 1(b) and 1(c) in the Fermi-liquid theory and such a treatment guarantees the vanishing resistivity in the absence of Umklapp scattering as stressed in ref. 29, the Maki-Thompson process is overestimated in the FLEX approximation. It is natural to expect that the fact, that the Aslamazov-Larkin processes have little effect on the transport coefficients, found in the FLEX approximation supports the validity of the relaxation-time approximation.

The reason, why the vertex correction in Fig. 1 plays little role besides the introduction of the transport life-time, is obvious from the previous study: the contribution from the two four-point vertices and two Green functions connecting them, and the one from the two Green functions connecting the four-point vertex and the three-point vertex, both of which are related to the imaginary part of the electron selfenergy, cancel out and the remaining effect is mostly taken into account by introducing the transport life-time.

The theoretical results by the FLEX approximation cannot be compatible with the experiments at least by two reasons. First, the magnetotransport anomaly in the normal state of high-$T_c$ cuprate superconductors should be understood in a single framework which can explain the so-called pseudogap phenomena, while the FLEX approximation at present cannot explain the pseudogap. Second, anomalous temperature dependences observed in several experiments cannot be ascribed to the vertex correction.
APPENDIX B: MAGNETOCONDUCTIVITY

The magnetoconductivity in the model used in §2 is given by

\[ \Delta \sigma_{xx} = -16e^2 \omega_c^2 l_0^3 (1 + a)^3 \int_0^\pi d\theta \left\{ \frac{1}{(1 + a \cos \theta)^3} + \frac{a^2 \sin^2 \theta}{(1 + a \cos \theta)^5} \right\}. \]  
(B1)

These two integrals can be evaluated by use of the parameter-derivative of the formulae

\[ \int_0^\pi dx \frac{1}{p + q \cos x} = \pi (p^2 - q^2)^{-1/2}, \]  
(B2)

\[ \int_0^\pi dx \frac{\sin^2 x}{p + q \cos x} = \pi [p - (p^2 - q^2)^{1/2}] q^{-2}, \]  
(B3)

for \( p > |q| \) and the result is given in §2. In the previous study [12] the second integral in eq. (B.1) has been omitted.

APPENDIX C: SIGN OF HALL CONDUCTIVITY

The collective contribution of the Aslamazov-Larkin process of the superconducting fluctuation to the Hall conductivity is proportional to \( N'_F \) where \( N'_F \) is the energy derivative of the density of states at the Fermi energy. (The definition of \( \alpha \) in the right hand side of eq. (2.27) in ref. 20 should be multiplied by minus sign.) On the other hand, the one of the nested spin fluctuation is proportional to \(-N'_F\). The difference of the sign can be understood as follows. In the case of the superconducting fluctuation the diagram for the Hall conductivity of the Aslamazov-Larkin process is derived from Fig. 4(a) by attaching three current vertices to it. In this process the electron with the momentum \( p \) and the fermionic frequency \( \varepsilon_n \) interacts with the hole with \(-p\) and \(-\varepsilon_n\) and feels a reduced magnetic field by the motion of the hole. If we consider free electrons, \( N'_F \) is positive and so that the Hall conductivity due to the superconductive fluctuation is positive, while the Hall conductivity due to the quasiparticle is negative. In the case of nested spin fluctuation the Aslamazov-Larkin process is derived from Fig. 4(b) where the electron with \( p \) and \( \varepsilon_n \) interacts with the electron with \( p + Q \) and \( \varepsilon_n \). Since \( p + Q \sim -p \) with the nesting vector \( Q \), each electron...
moving in the opposite direction feels an induced magnetic field. It should be noted that we need the vertex correction in order to obtain non-vanishing collective contribution in the case of the spin fluctuation. [24]
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FIGURES

FIG. 1. The current-vertex corrections. The open square represents the interaction vertex and the open triangle the current vertex.

FIG. 2. The interaction vertex (a) in the particle-hole channel and (b) in the particle-particle channel.

FIG. 3. The second order vertex (a) in the particle-hole channel and (b) in the particle-particle channel.

FIG. 4. The generating function of the magnetoconductivity (a) for the superconducting fluctuation and (b) for the spin fluctuation.
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