Insights into phase-slip events via dynamics of current-induced transitions from a superconductive to a resistive state

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Abstract. Phase-slip fluctuations play a central role in a variety of systems and are responsible for dissipation via a loss of coherence. By virtue of these fluctuations, quasi-one-dimensional superconductors acquire a non-zero resistance unlike the bulk superconductors in which the resistance drops to zero upon entering the superconducting phase. The present generation of experiments on superconducting nanowires are capable of characterizing non only the thermally activated phase slips in detail, but also the quantum phase slip processes as well as the crossover from thermal to quantal. The proceeding will report on the recent progress made towards achieving this goal by studying the dynamics of current-induced transitions from superconductive to resistive state and analyzing the switching statistics.

Phase slip processes—in which the relative phase between two regions of space (or space-time) changes or slips by a multiple of $2\pi$—play an important role in a variety of theoretical and experimental settings ranging from cosmic strings in the universe to ultra-cold atomic systems in optical traps. In the Bose-Hubbard model realized using trapped atoms, the damping rate of the atomic motion was experimentally found to be induced by phase-slip processes [1]. In analogy to the phase slips in Josephson junctions or weak links in superconductors, phase slips processes are found to give rise to dissipation events in the flow of superfluid $^4$He and $^3$He through orifices [2, 3, 4]. To give another example, phase slips have also been studied in the context of charge and spin density wave systems [5, 6]. This proceeding aims at gaining insights into phase-slip processes by focusing on their behavior in quasi-one-dimensional superconductors [7, 8, 9, 10].

In the quasi-one-dimensional limit in which the transverse dimension is smaller than the superconducting coherence length, the superconducting order parameter has a constant amplitude and a phase that winds along the length to form a helix. In a phase-slip process [11], the amplitude of the complex-valued order parameter field fluctuates to zero (typically over a length-scale of the order of coherence length) somewhere along the wire, allowing the phase to slip by $2\pi$. In doing so, the system goes from one metastable minimum to another by crossing a potential barrier either by thermal activation or quantum tunneling [12, 13]. Experimental observation of quantum phase slips and the nature of thermal to quantum crossover is one of the central questions that plays an essential role in determining the physics, not only in the case of quasi-one-dimensional superconductors but also for other phase-slip settings e.g. stated in the previous paragraph.
Figure 1. Trace of a typical trajectory of the temperature of the central segment (see text) of the nanowire illustrating the switching caused by thermal runaway. Overlaid on the trace are red lines showing the temperatures $T_{sc}$ (lowest), $T_{sp}$ (middle), and $T_{rs}$ (highest) corresponding to the superconducting minimum, the saddle-point, and the resistive minimum of an effective potential in the Langevin equation. The inset shows a blow-up of the boxed region of the main figure.

The phenomenological Ginzburg-Landau (GL) formalism that describes the superconducting phase in terms of a complex order parameter field provides a natural and direct language to describe phase-slip fluctuations. This framework was used to calculate the rate of thermally activated phase slips more than four decades ago [14] and soon afterwards the time-dependent GL formalism was used to calculate the attempt frequency [15], together constituting the LAMH theory of thermally activated phase slips. The regime of validity of this theory is expected to be in the vicinity of the transition temperature, and indeed in this regime it is successful in characterizing the broad resistive transition. Some of the more recent works have used a variety of theoretical approaches to further characterize phase-slip fluctuations and their implications. For instance, the rate of thermally activated phase slips was reanalyzed using an imaginary time effective action approach [16] and microscopically calculated in the clean limit using Eilenberger equations [17]. The quantum phase slip rate was microscopically calculated [18] and the interaction between quantum phase slips was analyzed [19, 20]. A memory formalism approach was used to study the contribution of quantum phase slips to the resistivity of dirty and inhomogeneous wire [21].

The present generation of experiments on superconducting nanowires are indeed capable of accessing the regime in which quantum fluctuations play an essential role [9, 10]. Since it is difficult to unambiguously establish the existence of quantum phase slips within the linear resistance measurements due to the noise floor, recent experiments have successfully evolved in better measuring and characterizing the entire I-V characteristics for ultra-narrow wires at exceedingly lower temperatures [13, 22, 23, 24, 25, 26]. One of the very recent experiments [26] has in fact been able to measure the distribution of switching currents—the current values at which the nanowire switches from a superconductive (i.e. low resistance state) to higher resistance state—as a function of temperature. Note that it was established that the nanowires in this experiment were in an overdamped regime and did not support phase-slip centers [27]. Here I will report on this experiment and its theoretical underpinning [28, 29].

The experimental set-up consists of a superconducting nanowire fabricated using molecular templating (by using a carbon nanotube or a DNA molecule as a template) to furnish a
Figure 2. Logarithm of the mean switching rate, $\ln \Gamma_{\text{sw}}$, as a function of current for bath temperatures $T_b = 0.1, 0.3, 0.5 \ldots, 1.9$ K, obtained by assuming the rate $\Gamma$ of phase slips to be given by $\Gamma_{\text{LAMH}}$, i.e., the LAMH formula for the rate of thermally activated phase slips. For reference, the thinner black lines indicate the rate $\Gamma_{\text{LAMH}}$.

free-standing superconducting nanowire connected to thermal baths at its two ends. If the Joule heating caused by resistive phase slip fluctuations is not overcome sufficiently rapidly by conductive cooling, it effectively reduces the depairing current, ultimately to below the applied current, thus causing switching to the highly resistive state. Although the discrete phase-slip events take place at random moments of time and are centered at random spatial locations along the wire, one can argue that it is reasonable to assume that all the phase slips and hence the Joule heating takes place within a central segment of length to which a uniform temperature $T$ is assigned and the heat is conducted away through the end segments (within which the heat capacity is ignored).

Within this simplified model, the dynamics of the nanowire is given by a stochastic (Langevin) ordinary differential equation for the time-evolution of the temperature $T$ of the central segment. A typical trajectory of the temperature of the central segment is shown in Figure 1. To gain intuition and illustrates the bistability, one can consider an effective potential description of the Langevin equation by replacing the sum over discrete phase slips by a phase slip rate $\Gamma(T, I)$ (the problem is then similar to that of the motion of an over-damped “particle” of position $T(t)$ at time $t$, moving in the fictitious potential). The switching then involves going from the low temperature ‘superconducting’ minimum to the high temperature ‘resistive’ minimum of the potential.

As illustrated by Figure 1, the switching dynamics is dictated by the competition between discrete heating (since each phase slip heats the wire by a “quantum” of energy $hI/2e$, for a given value of the applied current $I$) and continuous cooling. Thus the mechanism of switching can be physically understood in terms of a thermal runaway i.e. heating by rare sequences of closely-spaced phase slips. The number of phase-slips in such a phase-slip sequence can be easily estimated by evaluating the number $N(T_b, I)$ of phase-slips needed to cause switching in the absence of cooling.

Within this theoretical framework [28, 29] one can successfully understand the seemingly counter-intuitive broadening of the switching current distribution on lowering the bath temperature $T_b$ observed in the experiment [26]. Alternatively one can fit the mean switching rates $\Gamma_{\text{sw}}$ shown in Figure 2, to those extracted from the experiments. As one can see from the figure, $\Gamma_{\text{sw}}$ deviates drastically from the rate of phase slips when the thermal runaway sequence involves multiple phase slips. The two overlap only when a single phase slip is sufficient to cause the switching. Detailed fits of the experimental data amply demonstrate that as low
temperatures, it is essential to phenomenologically incorporate quantum phase slips into the phase slip rate [26, 29]. Moreover, one is able to conclude that the experiment is in fact probing individual quantum phase slips in the low temperature regime.

In conclusion, the recent study of switching dynamics is able to provide insights into the quantum phase slip fluctuations and the crossover from thermal to quantal fluctuations. The understanding of low temperature phase slip behavior is essential to establish whether superconductivity survives upon approaching zero temperature. The question of what is the state of the nanowire at zero temperature is a fundamental one and correspondingly the experimental and theoretical study of possible superconductor-metal or superconductor-insulator quantum phase transitions is an important topic of current interest [30, 31, 23, 32, 33, 34, 35, 36, 37].

Acknowledgments
I would like to gratefully acknowledge my theory collaborators, D. Pekker and P. M. Goldbart and experimental collaborators M. Sahu and A. Bezryadin.

References
[1] McKay D, White M, Pasienski M and DeMarco B 2008 Nature 453 76–79
[2] Avenel O and Varoquaux E 1985 Phys. Rev. Lett. 55 2704–2707
[3] Avenel O and Varoquaux E 1988 Phys. Rev. Lett. 60 416–419
[4] Sato Y, Hoskinson E and Packard R E 2006 Phys. Rev. B 74 144502
[5] Maki K 1995 Physics Letters A 202 313 – 316 ISSN 0375-9601
[6] Glatz A and Nattermann T 2002 Phys. Rev. Lett. 88 256401
[7] Skocpol W J and Tinkham M 1975 Reports on Progress in Physics 38 1049–1097
[8] Tinkham M 1996 Introduction to Superconductivity (McGraw-Hill, New York)
[9] Arutyunov K, Golubev D and Zaikin A 2008 Physics Reports 464 1 – 70 ISSN 0370-1573
[10] Bezryadin A 2008 Journal of Physics: Condensed Matter 20 043202
[11] Little W A 1967 Phys. Rev. 156 396–403
[12] Grabert H and Weiss U 1984 Phys. Rev. Lett. 53 1787–1790
[13] Giordano N 1990 Phys. Rev. B 41 6350–6365
[14] Langer J S and Ambegaokar V 1967 Phys. Rev. 164 498–510
[15] McCumber D E and Halperin B I 1970 Phys. Rev. B 1 1054–1070
[16] Golubev D S and Zaikin A D 2008 Phys. Rev. B 78 144502
[17] Zharov A, Lopatin A, Koshelev A E and Vinokur V M 2007 Phys. Rev. Lett. 98 197005
[18] Golubev D S and Zaikin A D 2001 Phys. Rev. B 64 014504
[19] Zaikin A D, Golubev D S, van Otterlo A and Zimányi G T 1997 Phys. Rev. Lett. 78 1552–1555
[20] Khlebnikov S 2008 Phys. Rev. B 78 014512
[21] Pai G V, Shimshoni E and Andrei N 2008 Phys. Rev. B 77 104528
[22] Tinkham M, Free J U, Lau C N and Markovic N 2003 Phys. Rev. B 68 134515
[23] Rogachev A, Bollinger A T and Bezryadin A 2005 Phys. Rev. Lett. 94 017004
[24] Altomare F, Chang A M, Melloch M R, Hong Y and Tu C W 2006 Phys. Rev. Lett. 97 017001
[25] Zgirski M, Riikonen J, Bollinger A T and Bezryadin A 2008 Phys. Rev. B 78 014511
[26] Shah N, Pekker D, Goldbart P M and Bezryadin A 2009 Nature Physics 5 503–508
[27] Vodolazov D Y, Peeters F M, Piraux L, Matefi-Tempfli S and Michotte S 2003 Physical Review Letters 91 175701 (pages 4)
[28] Shah N, Pekker D and Goldbart P M 2008 Phys. Rev. Lett. 101 207001
[29] Pekker D, Shah N, Sahu M, Bezryadin A and Goldbart P M 2009 Phys. Rev. B 80 214525
[30] Bezryadin A, Lau C N and Tinkham M 2000 Nature 404 971
[31] Lopatin A V, Shah N and Vinokur V M 2005 Phys. Rev. Lett. 94 037003
[32] Shah N and Lopatin A 2007 Phys. Rev. B 76 094511
[33] Refael G, Demler E, Oreg Y and Fisher D S 2007 Physical Review B (Condensed Matter and Materials Physics) 75 014522 (pages 37)
[34] Del Maestro A, Rosenow B, Shah N and Sachdev S 2008 Phys. Rev. B 77 180501
[35] Del Maestro A, Rosenow B, Müller M and Sachdev S 2008 Phys. Rev. Lett. 101 035701
[36] Bollinger A T, Dinsmore R C, Rogachev A and Bezryadin A 2008 Phys. Rev. Lett. 101 227003
[37] Refael G, Demler E and Oreg Y 2009 Phys. Rev. B 79 094524