Heralded Generation of Macroscopic Superposition States in a Spinor Bose-Einstein Condensate

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We propose to generate macroscopic superposition states of a large number of atoms in the ground state of a three-mode spinor Bose-Einstein condensate. The ground state is protected by a finite energy gap, is immune to phase noise, and the measurement of the number of particles in one mode heralds the coherent preparation of the other two modes simultaneously empty and filled. Highly entangled macroscopic superposition states are generated with large probability also taking into account quasi-adiabatic preparation of the ground state. All the key ingredients necessary to realize our proposal are experimentally available.

Introduction.—Macroscopic superposition states (MSSs) are a striking prediction of quantum theory [1] as well as a holy grail in quantum science and technology. Prominent examples, such as GHZ states of many qubits or NOON states of bosonic particles, set a benchmark in quantum metrology [2] and can find important application in quantum information processing [3]. Furthermore, they strongly couple to the environment and may thus provide a fundamental testbed of decoherence processes unveiling the transition from quantum to classical [4]. However, it is extremely challenging to generate genuine MSSs, i.e. for a large number of particles.

For photons, there exist several proposals [5–7] for the creation of NOON states (|N⟩_a|0⟩_b + |0⟩_a|N⟩_b)/√2, where a and b are two polarization or spatial modes simultaneously empty and filled with N particles. So far, these states have been created up to N = 5 photons [8–10] – for instance mixing a coherent and a squeezed-vacuum state at a balanced beam splitter and post-selecting the total number of photons in output [5, 8] – and N = 10 indistinguishable nuclear spins [11]. Maximally entangled GHZ states (|(↑⟩^⊗N + |↓⟩^⊗N)/√2 of N spin-1/2 have been generated up to N = 8 photons [12] combining several parametric down-conversion sources, and up to N = 14 trapped ions using one-axis twisting dynamics [13–16]. Such a nonlinear evolution [16] has also been realized in systems of a large number of particles [2] using atom-atom interaction in Bose-Einstein condensates (BECs) or cavity-assisted light-matter interaction in cold-thermal atom ensembles. Yet, the generation of MSSs with this method [17–19] requires long evolution times and, so far, spin-squeezed [20, 22] and slightly non-Gaussian states [24] have been successfully identified. MSSs can be also generated adiabatically, for instance preparing the ground state of a BEC trapped in a double-well potential with attractive interaction [25, 26]. However, due to the degeneracy of the spectrum, this method is extremely fragile to symmetry-breaking perturbations.

Here we demonstrate that MSSs of a large number of atoms can be created in the ground state of a ferromagnetic spin-1 BEC. After the preparation of the ground state, MSSs are generated stochastically with large (up to 90%) probability and heralded by the measurement of the number of atoms in one of the three modes of the BEC. The state creation is nondestructive – the MSS can be manipulated after its generation – and scalable with the number of particles. Remarkably, the ground state is protected by a finite energy gap and lives in a decoherence-free subspace robust against phase noise. Furthermore, even without any post-selection, the ground state of the spinor BEC has a Fisher information scaling at the Heisenberg limit and, following [27], can thus find key applications in quantum sensing [28–31]. The state preparation requires crossing over a quantum phase transition point [32], as recently demonstrated experimentally [33, 34] – note however that the states discussed here have not been addressed in these experiments. We thus believe, supported by numerical simulations, that our method can provide a feasible scheme to generate MSSs of a large number of particles in current experiments.

The model.—A spin-1 BEC of N particles with magnetic sublevels m_F = {0, ±1} is described in the single-mode approximation [35, 36] by the Hamiltonian (energy is in units of |λ|)

$$\hat{H} = \left( \frac{q}{|\lambda|} + \frac{1}{2} - \hat{N}_0 \right) (\hat{N}_{+1} - \hat{N}_{-1}) - (\hat{a}_{m_F}^\dagger \hat{a}_{m_F}^2 + \text{h.c.}),$$

(1)

where \(\hat{a}_{m_F}\) and \(\hat{a}_{m_F}^\dagger\) are bosonic creation and annihilation operators, respectively, and \(\hat{N}_{m_F} = \hat{a}_{m_F}^\dagger \hat{a}_{m_F}\) are the associated number operators. The interaction coefficient \(\lambda\) depends on the scattering lengths, atom mass, and trapping potential [35, 37]. In the following we consider ferromagnetic interaction \(\lambda < 0\) as in ^87Rb atoms [38]. We have \(q = (\Delta E_1 + \Delta E_{-1})/2\), where \(\Delta E_{m_F} = E_{m_F} - E_0\) is the relative energy shift of the \(m_F = \pm 1\) mode with respect to \(m_F = 0\), which can be tuned by an external...
magnetic field or near-resonant microwave dressing \[30\]. Spin-changing collisions, described by the last part of Eq. (1), preserve the total magnetization \(\hat{D} = N_{+1} - N_{-1}\). The Hilbert space of states with \(\hat{D}|\psi\rangle = 0\) is spanned by the Fock state basis \(|k\rangle \equiv |k\rangle_{+1}|N - 2k\rangle_0|k\rangle_{+1}\) with \(N = N_{-1} + N_0 + N_{+1}\) the total, fixed, number of particles in the three modes. To have a better insight to Eq. (1), let us introduce symmetric (\(g\)) and antisymmetric (\(h\)) combinations of operators \(\hat{a}_{\pm 1}^\dagger\) with \(\hat{h} = (\hat{a}_{+1}^\dagger - \hat{a}_{-1}^\dagger)/\sqrt{2}\). We can form two non-commuting sets of collective pseudospin-\(\frac{1}{2}\) operators:

\[
\begin{align*}
\hat{S}_x &= \frac{\hat{a}_{+1}^\dagger \hat{g} + \hat{g}^\dagger \hat{a}_0}{2} - \frac{\hat{a}_{-1}^\dagger \hat{g}^\dagger \hat{a}_0}{2}, \\
\hat{S}_y &= \frac{\hat{a}_{+1}^\dagger \hat{g} - \hat{g}^\dagger \hat{a}_0}{2i}, \\
\hat{S}_z &= \frac{\hat{a}_{+1}^\dagger \hat{a}_0 - \hat{a}_{-1}^\dagger \hat{g}^\dagger \hat{a}_0}{2}.
\end{align*}
\]

Thus \(\hat{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)\) describes the two-mode system composed by modes \(\hat{a}_0\) and \(\hat{g}\) and \(\hat{A} = (\hat{A}_x, \hat{A}_y, \hat{A}_z)\) corresponds to modes \(\hat{a}_{+1}\) and \(\hat{h}\). We will also consider the two-mode system composed by modes \(\hat{a}_{+1}\) and \(\hat{a}_{-1}\), \(J = (\hat{J}_x, \hat{J}_y, \hat{J}_z)\), where \(\hat{J}_x = (\hat{a}_{+1}^\dagger \hat{a}_{-1} + \hat{a}_{-1}^\dagger \hat{a}_{+1})/2\), \(\hat{J}_y = (\hat{a}_{+1}^\dagger \hat{a}_{-1} - \hat{a}_{-1}^\dagger \hat{a}_{+1})/2i\) and \(\hat{J}_z = (\hat{a}_{+1}^\dagger \hat{a}_{-1} - \hat{a}_{-1}^\dagger \hat{a}_{+1})/2\). Equation (1) rewrites as

\[
\hat{H} = -2\left(\hat{S}_x^2 + \frac{q}{3|\lambda|}\hat{S}_z\right) - 2\left(\hat{A}_y^2 + \frac{q}{3|\lambda|}\hat{A}_z\right).
\]

up to constant terms \[2\].

The Hamiltonian \(\hat{H}\) has, as a function of the ratio \(q/|\lambda|\), three quantum phases \[22, 24\] in the thermodynamic limit (\(N \rightarrow \infty\), \(|\lambda| \rightarrow 0\) and \(N|\lambda| = \text{const}\.) The quantum phase transition point \(q = \pm q_c\), where \(q_c = 2N|\lambda|/0\), are identified by an energy gap that closes as \(N^{-1/3}\) \[22\], see Fig. 1(a). For \(q > q_c\), we have a polar (P) phase, where the ground state has all particles in \(m_f = 0\), \(|\psi_{g>0}^0\rangle = |0\rangle_0 |N\rangle_0 |0\rangle_{+1}\). Tracing out the \(h\) (or the \(g\)) mode provides \(|N\rangle_0 |0\rangle_0 |0\rangle_{+1}\) that corresponds to a coherent spin state polarized along the \(S_z\) (\(A_z\)) axis, see Fig. 1(b). For \(|q| < q_c\), we have a broken-axisymmetry (BA) phase, where all three modes are populated, with \(\langle N_0\rangle = \frac{N}{4}\left(1 + \frac{q}{q_c}\right)\) \[2\]. In particular, at \(q = 0\) \[3\], the Hamiltonian \(\hat{H}\) takes the form \(\hat{H} = -2S_x^2 - 2S_y^2\). MSSs can be heralded by measuring the number of particles in \(h\) (or in the \(g\) mode). A simple intuition can be gained by considering the case in which the measurement of particles in \(h\) yields \(N_h = 0\). In this case, we may argue (see a detailed analysis below) that the contribution of \(\hat{A}_y\) to \(\hat{H}\) can be disregarded. The ground state of \(\hat{H} \approx -2S_x^2\) is a NOON state along the \(S_x\) axis, see Fig. 1(b): \(\exp(-i\frac{\pi}{2}\hat{S}_y)(|N\rangle_0 |0\rangle_0 + |0\rangle_0 |N\rangle_0)/\sqrt{2}\), corresponding (up to a rotation) to a superposition of \(N\) atoms in the symmetric \(g\) mode and \(0\) atoms in the \(m_f = 0\) mode, and viceversa. As shown below, the result \(N_h = 0\) has the highest probability. States heralded by other measurement results will be analyzed in the following. For \(q < -q_c\), we identify a Twin-Fock (TF) phase, where the condensate in \(m_f = 0\) is completely depleted and the ground state is given by \(|\psi_{g<0}^0\rangle = |N/2\rangle_{-1} |0\rangle_0 |N/2\rangle_{+1}\) with exactly \(N/2\) particles in \(m_f = \pm 1\) \[32, 34\]. Tracing out the \(m_f = 0\) modes provides \(|N/2\rangle_{-1} |N/2\rangle_{+1}\) that corresponds to a Twin-Fock state \[39\], see Fig. 1(b).

To further characterize the ground state, we determine its interferometric sensitivity as well as its multiparticle entanglement. Both quantities are captured by the quantum Fisher information (QFI) \(F_Q[|\psi\rangle, \hat{R}]\), which characterizes the sensitivity of the state \(|\psi\rangle\) under unitary rotations generated by \(\hat{R}\) \[40\]. A plot of the QFI is shown.
in Fig. 1(c). At \( q = 0 \) we have
\[
F_Q[|\psi_{gs}^{q=0}\rangle, \hat{R}_{\text{opt}}] = N(N+1)/2, \tag{4}
\]
where the optimal rotation operator, \( \hat{R}_{\text{opt}} \), is given by an arbitrary linear combination of \( \hat{S}_x \) and \( \hat{A}_y \). In the TF phase,
\[
F_Q[|\psi_{gs}^{\langle q \rangle=\langle q \rangle}\rangle, \hat{R}_{\text{opt}}] = N(N+2)/2, \tag{5}
\]
where \( \hat{R}_{\text{opt}} \) is an arbitrary linear combination of \( \hat{J}_x \) and \( \hat{J}_y \). In both cases, only two of the three modes are involved in the optimal rotation and \( F_Q[|\psi\rangle, \hat{R}_{\text{opt}}]/N \) scales as the number of entangled particles \([2,42]\). Hence, we observe that, in addition to the Twin-Fock state, the ground state at \( q = 0 \) possesses large entanglement useful for metrology. This can be exploited with coherent manipulations of the state on the \( \mathbf{S} \)-Bloch sphere using rf pulses \([43,44]\) as has been recently demonstrated with an atomic clock \([27]\).

**Heralded generation of MSSs.**—In the following we focus on the ground state at \( q = 0 \) \([35]\) and analyze (without any approximation) the state \( |\phi_{N_h}\rangle \) of the \( g \)-0 modes obtained after measuring \( N_h \) particles in the \( h \) mode. The probability to measure \( N_h \) particles in mode \( h \) is shown in Fig. 2(a). The highest probability is found for \( N_h = 0 \), as discussed above, and only even values of \( N_h \) are detectable. The state heralded by the measurement result \( N_h = 0 \) is (with a fidelity of more than 99\%) a rotated NOON state in the \( g \)-0 modes, \( |\phi_{N_h=0}\rangle \approx \exp(-i\pi/2\hat{S}_y)(|N\rangle_g|0\rangle_0 + |0\rangle_g|N\rangle_0)/\sqrt{2} \), see Fig. 2(b) and (c). Moreover, MSSs (given by a coherent superposition of highly populated zero and \( g \) modes) are heralded not only for \( N_h = 0 \), but also for values of \( 0 \leq N_h \leq N/2 \), see Fig. 2(b). The population of the \( m_F = 0 \) and \( g \) modes start to overlap around \( N_h \approx N/2 \). Remarkably, as the probability to detect \( N_h \leq N/2 \) is about 90\%, we can conclude that the measurement of \( N_h \) prepares a MSS in the \( g \)-0 modes in 90\% of the cases, regardless of the total number of atoms. To emphasize the coherence of the states \( |\phi_{N_h}\rangle \) obtained after heralding projection, we show, in Fig. 2(c), their QFI. Values \( F_Q[|\phi_{N_h}\rangle, \hat{S}_x] \sim (N-N_h)^2 \) are found up to \( N_h \approx N/2 \) \([15]\).

**Robustness and experimental feasibility.**—We propose to address the stochastic generation of MSSs by (i) initially preparing a condensate in the P phase, at \( q > q_c \), (ii) slowly decreasing \( q \) down to \( q = 0 \) and (iii) measuring the number of particles in the \( h \) mode. Preparing the polarized state allows us to address the subspace of zero magnetization \( (\hat{D}|\psi\rangle = 0) \), which is conserved during the adiabatic process. This entails stability with respect to phase noise generated by \( \hat{D} \): for any obtained state \( |\psi\rangle \) and any time-dependent noise parameter \( \phi(t) \), \( \exp(-i\phi(t)\hat{D})|\psi\rangle = |\psi\rangle \). The linear coupling to the magnetic field and its fluctuations (linear Zeeman shift) become irrelevant. Furthermore, after passing though the quantum phase transition point at \( q = q_c \), the energy gap re-opens, see Fig. 1(a). Crucially, at \( q = 0 \) the energy gap between the ground state and the first excited state assumes its largest value in the BA phase. This gap
state preparation of the less demanding in terms of BEC stability.

izes the non-adiabaticity of the process. In Fig. 3(a) we show the probability distribution of the particles (bars) for the state prepared at $|\psi_{gs}(t)|^2$, with $F_Q(\tau) = F_Q[|\psi(\tau)|, S_+]$ and $F_Q^{gs} = N(N + 1)/2$, Eq. (4), as a function of $\tau$. The dashed line is the inverse of the energy gap at $q = q_c$. Solid lines are a guide to the eye. (b) Probability to detect $N_h$ particles (bars) for the state $|\psi(\tau)|$ prepared at $\tau = 0.06$. The dashed lines are cumulative probability thresholds. (c) Heralded state after a rotation $\exp(-i\tau^2 S_y)$. We show after measuring $N_h$ particles in the $g$ mode obtained after measuring $N_g$ particles in the $h$ mode. The inset shows the Husimi distribution of the heralded state at $N_h = 150$, as an example. In all panels $N = 500$.

plot the overlap (blue dots) between the evolved state $|\psi(\tau)|$ and the ground state $|\psi_{gs}|$ at $q = 0$. We also show how the ratio between the QFI of the evolved $|\psi(\tau)|$ and the full adiabatic value, Eq. (3). These results are remarkable: even for a relatively fast ramping, faster than the inverse critical gap and thus producing small values of the fidelity $|\langle \psi_{gs}|\psi(\tau)\rangle|^2$, the QFI is hardly affected by the finite ramping speed and we obtain quite high values, comparable with the one of the adiabatic ground state. In Fig. 3(b) and (c) we show for the heralded state generation for a ramping time $\tau = 0.06$, that is feasible experimentally [33, 34] taking into account that $|\lambda| \sim 1 \div 10$ mHz and ramping times can be of the order of $\sim 10$ seconds. Figure 3(b) shows the distribution of particles in the $h$ mode, while Fig. 3(c) shows the heralded states (in terms of distribution of particles in the $g$ mode after measurement of $N_h$ particles in the $h$ mode), as in Fig. 2(b). MSSs are found about 80% of the cases (a value slightly lower than in the full adiabatic case). While, as expected, the finite ramping time broadens the particle distribution, the effect is not dramatic and MSSs are clearly reachable for the experimental ramping time and for a wide range of $N_h$. Finally, we point out that the $h$ and $g$ modes can be addressed after a $\pi/2$ balanced beam splitter between the $m_f = \pm 1$ with a two-frequency microwave pulse, as demonstrated in Ref. [31]. The measurement of number of particles in the $h$ mode is thus obtained after a $\pi/2$ coupling between $m_f = \pm 1$ (i.e. applying $\exp(-i(\pi/2)J_z)$) and the measurement of number of particles in one of the $m_f = \pm 1$ modes. We also emphasize that the state preparation is symmetric in the $h$ and $g$ modes: the measurement of number of particles in the $g$ mode would prepare MSSs in the $h$ mode with the same distributions discussed above. Finally, the generation of MSSs does not rely on single-particle resolution in the heralding detection, as is the case in many protocols: a small, finite, detection noise does not remove the macroscopic superposition features.

Conclusions.—We have discussed a method to create macroscopic superposition states of a large number of atoms in a spinor Bose-Einstein condensate. These states are prepared stochastically with high probability and are heralded by the measurement of the number of particles in one of the modes (namely the symmetric or the antisymmetric combination of hyperfine $\pm 1$ states). Moreover, even without post-selection, the state generated by quasi-adiabatic evolution is highly entangled and has a quantum Fisher information showing Heisenberg scaling with the number of atoms. The crucial advantage of our method is the robustness with respect to experimental phase noise and the presence of a finite energy gap which protects the highly entangled state. Finally, we point out that all ingredients necessary for the state generation discussed in this manuscript are experimentally available following Refs. [27, 33, 34, 41, 43, 44]. Our proposal thus provides a crucial step forward for the generation of

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**FIG. 3.** Stochastic generation of MSSs via quasi-adiabatic evolution. We consider the state $|\psi(\tau)|$ prepared at $q = 0$ in a ramping time $\tau$ starting from the P phase. (a) Fidelity with the ground state at $q = 0$, $|\langle \psi_{gs}|\psi(\tau)\rangle|^2$ (blue circles), and ratio between the QFI of the evolved state and the one for the ground state, $F_Q(\tau)/F_Q^{gs}$ (black dots), with $F_Q(\tau) = F_Q[|\psi(\tau)|, S_+]$ and $F_Q^{gs} = N(N + 1)/2$, Eq. (4), as a function of $\tau$. The dashed line is the inverse of the energy gap at $q = q_c$. Solid lines are a guide to the eye. (b) Probability to detect $N_h$ particles (bars) for the state $|\psi(\tau)|$ prepared at $\tau = 0.06$. The dashed lines are cumulative probability thresholds. (c) Heralded state after a rotation $\exp(-i\tau^2 S_y)$. We show after measuring $N_h$ particles in the $g$ mode obtained after measuring $N_g$ particles in the $h$ mode. The inset shows the Husimi distribution of the heralded state at $N_h = 150$, as an example. In all panels $N = 500$. 

A quasi-adiabatic state preparation similar to the one proposed here has been demonstrated experimentally in [33, 34] with the aim of addressing the TF phase [32, 34], which requires decreasing $q$ beyond $q = 0$ and crossing the second quantum phase transition point at $q = -q_c$. Preparing the state at $q = 0$ may be even less demanding in terms of BEC stability.

We have numerically simulated the quasi-adiabatic state preparation of the $q = 0$ ground state with realistic parameters: we assume a BEC prepared at $t = 0$ at $q/q_c = 2$ in the P phase, and decrease $q$ linearly as $q(t)/q_c = 2 - t/\tau$, where the ramping time $\tau$ characterizes the non-adiabaticity of the process. In Fig. 3(a) we protect $|\psi_{gs}=0\rangle$ against perturbations, e.g. due to finite temperature.
macroscopic superposition states of many atoms, a highly sought but still missing ingredient of quantum state engineering for quantum technologies.

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