A sufficient condition for Gaussian departure in turbulence

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The interaction of two isotropic turbulent fields of equal integral scale but different kinetic energy generates the simplest kind of inhomogeneous turbulent field. In this paper we present a numerical experiment where two time decaying isotropic fields of kinetic energies $E_1$ and $E_2$ initially match over a narrow region. Within this region the kinetic energy varies as a hyperbolic tangent. The following temporal evolution produces a shearless mixing. The anisotropy and intermittency of velocity and velocity derivative statistics is observed. In particular the asymptotic behavior in time and as a function of the energy ratio $E_1/E_2 \to \infty$ is discussed. This limit corresponds to the maximum observable turbulent energy gradient for a given $E_1$ and is obtained through the limit $E_2 \to 0$. A field with $E_1/E_2 \to \infty$ represents a mixing which could be observed near a surface subject to a very small velocity gradient separating two turbulent fields, one of which is nearly quiescent. In this condition the turbulent penetration is maximum and reaches a value equal to 1.2 times the nominal mixing layer width. The experiment shows that the presence of a turbulent energy gradient is sufficient for the appearance of intermittency and that during the mixing process the pressure transport is not negligible with respect to the turbulent velocity transport. These findings may open the way to the hypothesis that the presence of a gradient of turbulent energy is the minimal requirement for Gaussian departure in turbulence.

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I. INTRODUCTION

A turbulent shearless mixing layer is generated by the interaction of two homogeneous isotropic turbulent (HIT) fields, see definition diagrams in figures 1 and 2 and the flow visualizations in figure 3. This kind of mixing is characterized by the absence of a mean shear, so that there is no production of turbulent kinetic energy and no mean convective transport. The turbulence spreading is caused only by the fluctuating pressure and velocity fields. The inhomogeneous statistics are typically due to the presence of the gradients of turbulent kinetic energy and integral scale. The shearless turbulence mixing was first experimentally investigated by Gilbert (1980)1 and by Veeravalli and Warhaft (1989)2 by means of passive grid generated turbulence. Later on, numerical investigations were carried out by Briggs et al. (1996)3 and Knaepen et al. (2004)4, and more recently by Tordella and Iovieno (2006, 2007)5 6. All these studies considered a decaying turbulent mixing.

In all studies, apart from that of Gilbert, where the turbulent energy ratio was very low, the mixing layer was observed to be highly intermittent and the transverse velocity fluctuations seen to have large skewness. Across the mixing the distributions of the second, third and fourth order moments collapse when the mixing layer width is used as lengthscale 2 3 6.

In passive grid laboratory experiments the gradients of integral scale and kinetic energy are intrinsically linked.

In past studies the ratio of the integral scale of the interacting turbulence fields was in the range 1.3 1 - 4.3 2 with a ratio of kinetic energies in the range 1.5 1 - 23 2. In numerical 5 or active grid experiments these two parameters can be independently varied.

In the present study, a mixing configuration in which the integral scale is homogeneous is considered. The ratio of the turbulent kinetic energies has been chosen as the sole control parameter and is varied from 1.5 to $10^6$, $Re_\lambda$ of the high turbulent energy field is 45. The aim of this study is to show the intermittent behavior of such a configuration that in the past was considered to have almost Gaussian velocity statistics. This interpretation was motivated by the absence of both a kinetic energy production and an integral scale variation, two typical sources of intermittency and was also supported by laboratory observations carried out in the absence of a sufficiently high kinetic energy gradient 1. Another aim of this numerical experiment is to reach the asymptotic condition where the kinetic energy ratio $E = E_1/E_2$ goes to infinity. This last condition is relevant in applications concerning the diffusion of a turbulent field in a region of quiescent fluid, where extreme bursts of rate of strain and vorticity can be expected 2. The presence of such events is shown by high values of skewness and kurtosis.

A description of the numerical experiment is given in section II. Data on the degree of anisotropy observed in the second and third order velocity moments are described in section III, where an interpretation based on Yoshizawa's hypothesis is also given. In section IV we present the two types of asymptotics considered: the temporal asymptotics of the second and third order velocity moments, and the asymptotics with respect to the turbu-
responses, see fig. 1. Tests were performed on a 4
tral method [8]. The computational domain is a par-
fully dealiased (3/2 - rule) Fourier-Galerkin pseudospec-
in section V.

The concluding remarks are presented
other moments of the velocity derivative is also discussed in
al. (1996)[3], and Knaepen et al. (2004)[4]. The match-
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domains with 256 × 128² points. Further tests with a 8π(2π)² parallelepiped with 512 × 128² points
were used to obtain an estimate of the numerical accuracy. The Taylor-microscale Reynolds number \( \bar{Re}_\lambda \), cor-

domaining on which the initial condition is built up is a linear
superposition of the two isotropic fields as indicated in
equations (1) and (2). The initial condition, the two isotropic turbulent
fields are matched by means of a hyperbolic tangent func-
tion, This transition layer represents 1/40 of the 4π do-
main, and 1/80 of the 8π domain. The matched field is

\[
u(x) = u_1(x)p(x) + u_2(x)(1 - p(x))
\]

(1)

\[
p(x) = \frac{1}{2} \left[ 1 + \tanh \left( a \frac{x}{L} \right) \tanh \left( a \frac{x - L/2}{L} \right) \times \tanh \left( a \frac{x - L}{L} \right) \right]
\]

(2)

where the suffixes 1, 2 indicate high and low energy sides
of the mixing respectively, \( x \) is the inhomogeneous direc-
tion, \( L \) is the width of the computational domain in the
\( x \) direction. Constant \( a \) in (2) determines the initial
mixing layer thickness \( \Delta \), conventionally defined as the
distance between the points with normalized energy values
0.25 and 0.75 when the low energy side is mapped to
zero and the high energy side to one. When \( a = 12\pi \) the
ratio \( \Delta/L \) is about 0.026, for \( a = 20\pi \) the ratio \( \Delta/L \) is
about 0.015. These values have been chosen so that this
initial thickness is large enough to be resolved but small
enough to have large regions of homogeneous turbulence
during the simulations. This technique of generating the
transition layer is analogous to that used in Briggs et al. (1996)[3],
and Knaepen et al. (2004)[4]. The matching
in which the initial condition is built up is a linear
superposition of the two isotropic fields as indicated in
equations (1) and (2). A set of statistical properties of the
high kinetic energy HIT field is shown in Table 1. Since
the low energy field \( u_2 \) is obtained by multiplying the
initial velocity field \( u_1 \) by a constant, the numerical ex-
periment carried out by mixing these fields is a turbulent
mixing with different energies but of equal integral scale.

It should be noted that, by doing so, the mean pressure
along the mixing direction is not constant. However, the
mean pressure gradient is opposite to the gradient of tur-
bulent kinetic energy and thus no mean velocity field is
generated, see the appendix. Examples of the shearless
mixing obtained in this way for direct numerical simul-
ations can be found in [3] and [4]. The initial spectra of
the two HIT fields are shown in figure [3]. In this figure
the temporal decay of the two isotropic turbulent fields is
shown together with, as a reference, the decay of the
homogeneous and isotropic turbulence simulated in one
of the computational domain used to simulate the turbulent
shearless mixing (2π)² × 8π, 128² × 512). In figure [4] the
estimate of the time instant where the self-similar decay
of the mixing starts is also shown.

Let us now consider the flow symmetry. It can be seen

II. NUMERICAL EXPERIMENT

Navier-Stokes equations are numerically solved with a
fully dealiased (3/2 - rule) Fourier-Galerkin pseudospec-
tral method [3]. The computational domain is a par-
allelepiped with periodic boundary conditions in all di-
rections, see fig. 1. Tests were performed on a 4π(2π)²
parallelepiped domain with 256 × 128² points. Further
tests with a 8π(2π)² parallelepiped with 512 × 128² points
were used to obtain an estimate of the numerical accuracy. The Taylor-microscale Reynolds number \( \bar{Re}_\lambda \), cor-

FIG. 1: Scheme of the flow. Direction \( x \) is the mixing di-
rection. The high energy (\( E_1 \)) and low energy (\( E_2 \)) regions
are separated by mixing layers of conventional thickness \( \Delta(t) \)
defined by mapping the low energy side of the mixing layer
to zero and the high energy side to one. \( \Delta(t) \) is equal to the
distance between the points with normalized energy values
0.25 and 0.75 [2], [5].

FIG. 2: Scheme of the flow. Reference frame: \( y_1, y_2 \) are nor-
mal to \( x \), that is the direction of the flow inhomogeneity. The
flow is homogenous in all planes normal to this direction.
FIG. 3: (Color online) Visualization at two time instants of contours of kinetic energy $E(x, y_1, y_2, t)/E_1(0)$ in a plane at constant $y_2$, $E_1/E_2 = 6.77$, $Re_\lambda = 45$: (a) $t/\tau = 0.8$, (b) $t/\tau = 2.5$.

that a shearless mixing is a flow in which only one direction of inhomogeneity is present, as a consequence any plane normal to the inhomogeneous direction is homogeneous. This corresponds to a cylindrical symmetry. See the reference frame scheme in figure 2.

The time integration is carried out by means of a four-stage fourth-order explicit Runge-Kutta scheme. Statistics are obtained by averaging over planes normal to the inhomogeneous direction, see figure 2.

The initial conditions were generated from the homogeneous and isotropic turbulent field produced by Wray in 1998 [9], which is a classic data set often used in literature.

A posteriori, it is possible to obtain numerical accuracy estimates. The raw data by Wray has an inhomogeneity level on the kinetic energy of about $\pm 8\%$ and skewness and kurtosis values slightly different from those of the statistical equilibrium ($0.02 \pm 0.12$ instead of 0 and $2.8 \pm 0.2$ instead of 3, respectively). As far as our set of direct numerical simulations is concerned, the increase in width of the computational domain from $4\pi$ to $8\pi$ (from 256 to 512 grid points) allowed an estimate of the relative accuracy to be obtained. For the maximum values of the distributions across the mixing, the accuracy is of about $5\%$ for the skewness, and of about $8\%$ for the kurtosis.

In figure 5, which summarizes the results regarding the maximum values reached by the velocity skewness and kurtosis within the mixing and the results about the penetration, it can be seen that the simulations with initial $\Delta/L = 1/40$ and $1/80$ yield data which collapse in a satisfactory way. On checking the symmetry of the numerical solutions, which, due to the periodicity of the
FIG. 4: Turbulent kinetic energy decay of the two interacting isotropic turbulent fields (E₁ high energy, E₂ low energy) at Reₐ = 45 and, in the inset, the corresponding initial energy spectrum. Data from a homogeneous and isotropic turbulence (E) simulated in a (2π)² × 8π domain (128² × 512) have been shown for comparison. The initial spectrum is equal to the spectrum of the high energy region in the mixing.

TABLE I: Statistical properties of the high energy HIT field

|   | E₁ | S₁ | K₁ |
|---|----|----|----|
|   | 1.01 ± 0.08 | 1.6 · 10⁻² ± 0.12 | 2.85 ± 0.2 |

Velocity derivative statistics

|   | S∂u/∂x | K∂u/∂x | S∂u/∂y₁ | K∂u/∂y₁ |
|---|--------|--------|--------|--------|
|   | −0.42 ± 0.08 | 3.61 ± 0.2 | −0.40 ± 0.08 | 3.53 ± 0.2 |

FIG. 5: Anisotropy of the turbulent second and third order moments at the centre of the mixing layer. The horizontal dashed line in part (a) indicates the isotropic reference value, the horizontal dotted line in part (b) indicates the estimate of the asymptote value.

III. ANISOTROPY AND YOSHIZAWA’S HYPOTHESIS

In isotropic turbulence the normalized second order moment of the velocity components, normalized with the sum $\bar{u}² + \bar{v}₁² + \bar{v}₂²$, is 1/3, whilst the third order moment is zero. In the present flow the field anisotropy develops during the mixing process. The value of the normalized moments vary in time and reach an asymptotic value after few time units, see figure 4. The time unit τ is defined as $τ = (t/0)/E₁^{1/2}(0)$, where t is the integral scale, here uniform across the mixing, and E₁ is the turbulent kinetic energy of the high energy side of the mixing.

An initial turbulent energy gradient $\nabla E = (E₁ - E₂)/(2Δ)$ corresponds to each value of $E = E₁/E₂$. The width Δ is defined by mapping the low energy side of the mixing layer to zero and the high energy side to one, and it is equal to the distance between the points with energy values 0.25 and 0.75, as in the paper by Veeravalli & Warhaft 2 (in the following referred to as V&W). The turbulent energy gradients can be normalized by the value of the high energy field, and by the value of the mixing thickness $Δ(t)$. It should be noticed that by doing so, the normalized gradient value has the upper limit of 0.5, which is reached in the limit for E₂ going to zero.

In figure 5(a) the time evolution inside the mixing of the second order moment $u^2/(u^2 + v^1_2 + v^2_2)$ is shown. After a linear growth the curves bend toward the asymptotic value, which is in the range 0.37-0.39 for a kinetic energy ratio growing from 4 to $10⁴$ (this corresponds to a normalized gradient of turbulent kinetic energy from 0.37 to 0.50, or, by supposing a mixing in air with a Reₐ = 45 in the high energy side, to a dimensional gradient from 1.8 to 2.4 m/s²).
As a consequence of the cylindrical symmetry of this mixing, it follows that the second moment orders $v_1^2/(u^2 + v_1^2 + v_2^2)$ and $v_2^2/(u^2 + v_1^2 + v_2^2)$ are equal and range from 0.315 to 0.305 when $\varepsilon$ varies from 4 to $10^4$. The anisotropy level, defined as the difference between the second-moment values referred to the isotropic value, can be considered mild (16% for $\varepsilon = 12$, 25% for $\varepsilon = 10^4$) given that the accuracy in the original data base used to build the initial condition is of about 8% as far as both the homogeneity and isotropy are concerned. It should be considered that this level of initial accuracy of homogeneity and isotropy is excellent in nominal HIT numerical fields. In higher resolution fields $(1024^3)$ the accuracy is analogous [10].

Figure 5(a) indicates that the value 0.39 for $\bar{u}^2/(\bar{u}^2 + \bar{v}_1^2 + \bar{v}_2^2)$ is reached by increasing $\varepsilon$ from 12 to $10^4$. This value can be considered as an approximation of the asymptotic value attainable by increasing the turbulent energy gradient.

It is important to note that in literature concerning the shearless mixing, almost all authors report a near homogeneity of the HIT data base used to build the initial condition [9] is of about 8% as far as the non perfect homogeneity and isotropy is excellent in nominal HIT numerical fields. In higher resolution fields $(1024^3)$ the accuracy is analogous [10].

The anisotropy of the third-order velocity moments is more enhanced than that of the second-moments. This can be observed in figure 5(b), where the ratio $\bar{u}^3/(\bar{u}^3 + \bar{v}_1^3 + \bar{v}_2^3)$ is represented. The estimate of the temporal asymptotic value we obtained is $0.53 \pm 0.03$ and does not depend on $\varepsilon$. If the level of anisotropy is defined as the difference between the third moments divided by their mean, an anisotropy of 80% is obtained. This means that, for all the energy ratios, nearly one half of the turbulent kinetic energy flow across the mixing is due to the self transport of $\bar{u}^3$. Let us note that at the initial instant, when the mixing process starts, the quantity $\bar{u}^3/(\bar{u}^3 + \bar{v}_1^3 + \bar{v}_2^3)$ is not defined because both the numerator and the denominator are not defined. This is numerically verified through the large dispersion of the initial values associated to different $\varepsilon$. Of course, this dispersion is also due to the non perfect homogeneity of the HIT data base used to build the initial condition, see section II. The data dispersion is however reduced as the mixing process advances. After 6 times scales is less than 10%.

It is possible to analyze this result by means of simplifying hypotheses currently found in literature - (a) the pressure transport is almost proportional to the convective transport associated to the fluctuations (Lumley 1978 [11], Yoshizawa, 1982, 2002, [12], [13]), - (b) the dissipative scales are nearly isotropic [14], and - (c) the second order moments are almost isotropic as observed in shearless turbulent mixings and also confirmed by the present numerical experiment, as discussed above.

Let us now consider the one point second order moment equations

\[
\partial_t \bar{u}^2 + \partial_x \bar{u}^3 = -2\rho^{-1} \partial_x \bar{p} \bar{u} + 2\rho^{-1} \bar{p} \bar{u}_x \bar{u} - 2\varepsilon_u + \nu \partial_x^2 \bar{u}^2
\]

\[
\partial_t \bar{v}_i^2 + \partial_x \bar{v}_i^3 = 2\rho^{-1} \bar{p} \partial_y \bar{v}_i - 2\varepsilon_{v_i} + \nu \partial_x^2 \bar{v}_i^2, \quad i = 1, 2 \quad (4)
\]

where $u$ is the fluctuating velocity in the inhomogeneous direction $x$, $v_1, v_2$ are the fluctuation components in the plane normal to $x$ and $\varepsilon_{u_1}, \varepsilon_{v_i}$ are the dissipation terms in the mixing and normal directions, respectively.

The pressure strain terms $\bar{p} \partial_x u$ and $\bar{p} \partial_y v_i$ in the absence of a mean flow, are of the order of $\varepsilon (\bar{p} \partial u_j - \frac{2}{3} \bar{p} \partial v_j)$, see for instance Monin & Yaglom, 1971 [15] (Volume 1, equation 6.12, page 379), where $\varepsilon$ is the total dissipation and $b$ is the turbulent kinetic energy per unit of mass. Since, as previously explained, experiments show no appreciable difference in the second order moments in the mixing, see condition (c) above, the pressure strain terms are neglected.

Condition (a) implies that we can write

\[
-\bar{p} \bar{u} = \alpha \rho \bar{u}^2 + 2\nu \bar{u}_x^2
\]

(5)

for any value of position $x$ along the mixing and for any time instant $t$. The difference between equation (3) and equation (4) gives

\[
\partial_t (\bar{u}^3 - \bar{v}_1^3) + \partial_x (\bar{u}^3 - \bar{v}_2^3) \approx -2\rho^{-1} \partial_x \bar{p} \bar{u} - 2(\varepsilon_u - \varepsilon_{v_1}). \quad (6)
\]

By condition (b) $\bar{v}_2^2 \approx \bar{v}_1^2$ and by condition (c) $\varepsilon_u \approx \varepsilon_{v_1}$. Thus, the unsteady term on the left hand side as well as the second term on the right hand side can be neglected and it follows that

\[
\partial_x (\bar{u}^3 - \bar{v}_1^3) \approx -2\rho^{-1} \partial_x \bar{p} \bar{u}.
\]

(7)

Integration of (7) with respect to $x$ leads to

\[
\bar{u}^3 - \bar{v}_1^3 \bar{u} \approx -2\rho^{-1} \bar{p} \bar{u} + C,
\]

but, considering that all quantities in this equation vanish outside the mixing (i.e. for $x \rightarrow \pm \infty$), the integration constant $C$ is equal to zero. Thus

\[
\bar{u}^3 - \bar{v}_1^3 \bar{u} \approx -2\rho^{-1} \bar{p} \bar{u},
\]

(8)

By inserting the previous relation into (5), it is possible to write

\[
\bar{v}_1^3 \bar{u} = \beta \bar{u}^3, \quad \beta = \frac{1 - \alpha}{1 + 2\alpha}.
\]

(9)

Then, by defining $\Phi$ the proportion of the turbulent kinetic energy flow associated to the $u$ fluctuation, it follows that

\[
\Phi = \frac{\bar{u}^3}{\bar{u}^3 + 2\bar{v}_1^3} = \frac{1}{1 + 2\beta}.
\]

(10)
We have computed the constant α for the present experiments and found, in asymptotic temporal condition and for $\mathcal{E} \in [12, 10^4]$, an average value of 0.37±0.03. This gives $\beta \sim 0.36$ and $\Phi \sim 0.58$. This last value contrasts with our numerical experimental value of $\Phi = 0.53 \pm 0.03$ shown in fig. 5b.

We have verified that α and β remain almost constant during the decay and when varying the shearless mixing parameter $\mathcal{E}$, a fact which confirms that the pressure transport correlation is almost proportional to the convective transport associated to the fluctuations and confirm the Yoshizawa hypothesis that when the turbulent field does not posses a unidirectional mean flow, the velocity turbulent transport term is not dominating the pressure transport [12, 13, 17]. In the present mixing both the advection and the production rate of the turbulent energy are zero and thus the turbulent transport (velocity and pressure) rate is of the same order of the dissipation rate.

## IV. INTERMITTENCY ASYMPTOTIC BEHAVIOR

In this section we consider the asymptotic behavior with regards to the variation of the parameter that controls this kind of shearless mixing layer, that is the initial energy ratio $\mathcal{E} = E_1/E_2$ between the high energy turbulent field 1 and the low energy turbulent field 2. As stated above, this ratio is unequivocally linked to the turbulent kinetic energy gradient. In this work, $\mathcal{E}$ was varied between 1.5 and $10^6$. The two external fields show, for moderate values of $\mathcal{E}$, decay exponents which are very close, so that the two homogeneous turbulences external to the mixing decay in a similar way and the value of $E_1/E_2$ remains quite constant during the time interval considered [2, 3, 6].

After few initial eddy turnover times $\tau = t(0)/E_1^{1/2}(0)$, where $t$ is the initial integral scale (homogeneous through the whole domain) and $E_1(0)$ is the initial energy of the high energy side, a true mixing layer begins to emerge from the initial conditions and reaches a self-similar state. This means that all normalized moments distributions across the mixing collapse to a single curve when the position is normalized with the mixing layer thickness, which is defined as the distance between the points with normalized energy $(E - E_2)/(E_1 - E_2)$ equal to 1/4 and 3/4, see sketch in fig.1. This definition has been used in many previous works on shearless mixing [2, 3, 6].

Results from numerical simulations show that the mixing layer is highly intermittent in the self-similar stage of decay, and its intermittency is dependant on $\mathcal{E}$. In order to analyze the flow intermittency, moments of the component $u$, that is the component in the direction of the flow of turbulent kinetic energy, were computed (the averages are computed by integrating over planes at $x = const$). A particular focus was placed on the skewness $S = \overline{u^3}/(\overline{u^2})^{3/2}$ and kurtosis $K = \overline{u^4}/(\overline{u^2})^{2}$.

The velocity fluctuation $u$ is responsible for the energy transport across the mixing. The skewness distribution is a principal indicator of intermittent behavior. It vanishes in homogeneous isotropic turbulent flows and thus it remains close to zero in the fields external to the mixing. The skewness takes a positive value within the mixing layer. Figure 6(a) shows the time evolution of the maximum of the skewness for various energy ratios ranging from 12 to $10^4$. (a) Numerical experiments at $Re_\lambda = 45$. Empty symbols refer to simulations in a $(2\pi)^2 \times 8\pi$ domain, the others to simulations in a $(2\pi)^2 \times 4\pi$ domain. The dashed line is the value of the reference skewness in a simulation of homogeneous and isotropic turbulence carried out on the same computational domain, bars represent the maximum fluctuations of this skewness. (b) Laboratory data at $Re_\lambda = 44.5$ (perforated plate experiment) and $Re_\lambda = 78.1$ (bar grid experiment) from wind tunnel experiments where a spatial decay is observed [2]. The time in laboratory experiments has therefore been computed using Taylor’s hypothesis, as $t = d/U$, where $d$ is the distance from the grid and $U$ is the mean velocity across the grids.

![FIG. 6: Temporal evolution of the maximum of the skewness](image)
of evolution. Figure 7(b) shows the time evolution of the maximum of the skewness in the V&W experiments, the 3.3:1 perforated plate experiment, where $E = 6.27$, and the 3:1 bar grid experiment, where $E = 6.19$. Since in the laboratory all the statistics decay in space, we have estimated an equivalent temporal decay by using Taylor’s hypothesis. The corresponding time in laboratory experiments has therefore been computed using Taylor’s hypothesis, as $t = d/U$, where $d$ is the distance from the grid and $U$ is the mean velocity across the grids.

FIG. 7: Temporal evolution of the maximum of the kurtosis $K = \overline{u^4}/(\overline{u^2})^2$ in the mixing for various energy ratios ranging from 12 to $10^4$. (a) Numerical experiments at $Re_\lambda = 45$. Empty symbols refer to simulations in a $(2\pi)^2 \times 8\pi$ domain, the others to simulations in a $(2\pi)^2 \times 4\pi$ domain. The dashed line is the value of the reference kurtosis in a simulation of homogeneous and isotropic turbulence carried out on the same computational domain, bars represent the maximum fluctuations of this kurtosis. (b) Laboratory data at $Re_\lambda = 44.5$ (perforated plate experiment) and $Re_\lambda = 78.1$ (bar grid experiment) from wind tunnel experiments where a spatial decay is observed. The time in laboratory experiments has therefore been computed using Taylor’s hypothesis, as $t = d/U$, where $d$ is the distance from the grid and $U$ is the mean velocity across the grids.

FIG. 8: (a) Maximum of the skewness and (b) estimate of the asymptotic kurtosis value as a function of the initial energy ratio (the horizontal dashed line indicates the Gaussian reference value), (c) normalized position of the maximum of the skewness in the mixing layer as a function of the initial energy ratio. Note that one can expect a higher level of intermittency in the data of Sreenivasan (1998) since these experiments had non-unity integral scale ratio ($\ell_1/\ell_2 \simeq 1.5$).

ues of $S_{max}$ approaching those of [2]. Note that in the laboratory experiment the ratio of macroscales is about 1.5 (this value is estimated by considering the finiteness of $Re_\lambda$ according to Sreenivasan (1998) [16]). This agrees with the finding that if the gradient of kinetic energy and macroscale are concurrent the mixing process is enhanced. In fact, one sees here that an higher energy...
gradient, $\mathcal{E} = 12$, produces the same skewness than the gradient of scale associated with the lower energy gradient, $\mathcal{E} = 6.19$, in the V&W experiment. In our numerical experiment, for the higher $\mathcal{E}$ ratios, we note a sort of damped oscillation that appears beyond the first maximum. This seems also to be shown by the 3 : 1 bar grid experiment, see figure 8 (b).

The value of maximum skewness inside the mixing layer as a function of the energy ratio is depicted in figure 8 (a). For values of $E_1/E_2$ lower than $10^2$ it scales almost linearly with the logarithm of the energy ratio, which is in fair agreement with the scaling exponent of 0.29 found in [8].

Figure 7 shows the temporal evolution of the maximum of the skewness inside the layer. Here again the comparison between our numerical data and the data of the V& W experiment is presented. The numerical and the laboratory results contrast well for comparable values of $E_1/E_2$. A high peak is shown at the end of the formation time interval where the mixing process develops. This peak is followed by a decrease, that could be interpreted as the fact that the more extreme intermittent turbulent events take place at the end of the formation interval and before the self-similarity sets in. In the numerical experiments which last more time scale units than those in the laboratory, the decrease is followed by another damped increase-decrease cycle, as in the skewness case. The time asymptotic values were estimated by averaging over the last cycle. Note that data in figures 8 and 7 from laboratory experiments were obtained in the presence of concurrent gradients of integral scale and kinetic energy. Also in the kurtosis case, it can be observed that a higher energy gradient produces the same intermittency than a gradient of scale associated with a lower energy gradient [8].

The distribution of the peak of kurtosis inside the mixing is shown in figure 8 (b). From this figure it can be noted that the kurtosis reaches very high values, much higher than the value of 3, that is the Gaussian reference value indicated in the figure by the dashed line. The kurtosis asymptote is in fact close to 10.5, which indicates the presence in the mixing layer of extremely intense intermittent events.

A similar behavior of the skewness and kurtosis maxima can be seen in the mixing penetration, defined as the instantaneous position along the $x$ direction of the maximum of the skewness normalized with the instantaneous mixing layer thickness $\Delta(t)$, see figure 8 (c). The penetration becomes constant in the self-similar evolution. The penetration physically highlights the region of maximum intermittency, which is located in the low energy side of the mixing layer. An increase of the energy ratio enhances the penetration of the high energy side into the low energy side. An asymptotic value of about $1.2\Delta$ is obtained for $\mathcal{E} \rightarrow \infty$, which gives an indication of the penetration of an isotropic turbulent field into a quiescent field.

An alternative measure of the anisotropy is given by the velocity gradient statistics. We have computed the third and fourth order moments of both the longitudinal velocity derivative $\partial u/\partial x$ and transverse velocity derivative $\partial u/\partial y_i$, (no summation over $i$). These are so defined

$$S_{\partial u/\partial x} = \frac{\langle (\partial u/\partial x)^3 \rangle}{\langle (\partial u/\partial x)^2 \rangle^{3/2}}$$

$$S_{\partial u/\partial y_i} = \frac{\langle (\partial u/\partial y_i)^3 \rangle}{\langle (\partial u/\partial y_i)^2 \rangle^{3/2}}, \quad i = 1, 2$$

$$K_{\partial u/\partial x} = \frac{\langle (\partial u/\partial x)^4 \rangle}{\langle (\partial u/\partial x)^2 \rangle^2}$$

$$K_{\partial u/\partial y_i} = \frac{\langle (\partial u/\partial y_i)^4 \rangle}{\langle (\partial u/\partial y_i)^2 \rangle^2}, \quad i = 1, 2$$

The averages are computed by integrating over planes at $x = \text{const}$. Figure 7 shows the time evolution of the peak of the longitudinal and transverse velocity derivative skewness and kurtosis within the mixing. The figure includes, for comparison, the values measured in the two homogeneous and isotropic turbulent fields outside the mixing and the values deduced from figures 5 and 6 of the review by Sreenivasan and Antonia (1997) for $Re_\lambda = 45$.

We observe that the temporal evolution of all these velocity derivative statistics during the mixing decay presents an initial transient which is very similar to that shown by the velocity statistics, the transient length is the same in the two cases and there is no lag. The maximum values are always reached at $t/\tau \sim 4$ and increase with $\mathcal{E}$. The longitudinal derivative moments are always larger than the transverse derivative moments, the difference decreases with the increase of $\mathcal{E}$. For instance, for $\mathcal{E} = 10^4$, absolute values as high as $4 - 5$ are reached for the skewness, while values of 55 and 38 are measured for the longitudinal and transverse kurtosis, respectively. The anisotropy picture yielded by these velocity derivative correspond to that of a a higher intermittency along the inhomogeneous direction than across it.

V. CONCLUSIONS

We considered the simplest kind of turbulent shearless mixing process which is due to the interaction of two isotropic turbulent fields with different kinetic energy but the same spectrum shape. This mixing is characterized by the absence of advection, production of turbulent kinetic energy and an integral scale gradient. Such a situation can be seen as the simplest form of turbulence inhomogeneity that can lead to a departure from Gaussianity. The study was carried out by means of Navier-Stokes direct numerical simulations based on a fully dealiased Fourier-Galerkin pseudospectral method of integration. The data base was analyzed through single-point statistics involving the velocity and pressure fluctuations.

We determined the temporal asymptotic behavior of the self-similar state. We also obtained the asymptotics for very high energy ratios between the isotropic turbulent fields which, through their interaction, initiate the
mixing process. The infinite limit of the turbulent energy ratio corresponds to the interaction of a region of isotropic turbulence with a relatively still fluid. In this limit the turbulent energy gradient reaches the maximum observable value associated to a given energy in the high energy side of the mixing. In this limit the mixing penetration is maximum and is as deep as 1.2 times the mixing thickness.

We observed the intermittency and anisotropy of the mixings. Anisotropy was found to be mild for second order moments, on the contrary it was very intense in third and fourth order moments. The time asymptotic behavior of the anisotropy was almost independent of the third and fourth order moments. The time asymptotic order moments, on the contrary it was very intense in mixings. Anisotropy was found to be mild for second thickness.

The anisotropy observed through the third and fourth order moments of the velocity derivatives (longitudinal and transverse) is also very intense, but depends on the turbulent energy ratio.

Despite having no gradient of integral scale, no mean shear and thus no advection and no production of turbulent kinetic energy, all mixings showed a departure from a Gaussian state for any turbulent energy ratio. This signifies that the absence of these flow properties does not imply a condition of no intermittency. On the contrary the intermittency is highly dependant on the turbulent energy ratio between the two interacting fields. The intermittency has a constant asymptote when this ratio approaches to infinity, which is consistent with the maximum value of the turbulent energy gradient that can be asymptotically attained in this limit. It is deduced that the presence of a gradient of turbulent kinetic energy is a sufficient condition for the onset of intermittency. For any turbulent energy ratio we verified that the pressure transport is not negligible with regard to the velocity transport as in recirculating turbulent flows.

In conclusion, by assuming that the interaction of two isotropic turbulent fields with different kinetic energy but the same integral scale is the non-homogeneous turbulent flow with the lowest level of dynamical complexity, we propose the hypothesis that the existence of a gradient of turbulent energy is the minimal requirement for Gaussian departure in turbulence, since there is experimental evidence that it is a sufficient condition to promote intermittency.

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FIG. 9: (Color online) Temporal evolution of the peak of the velocity derivative statistics within the mixing. (a) Longitudinal velocity derivative kurtosis. (b) Longitudinal velocity derivative skewness. (c) Transverse velocity derivative skewness. (d) Transverse velocity derivative kurtosis. The instantaneous values in the high and low homogeneous regions external to the mixing are shown with the red (dashed) and green (dash-dotted) lines respectively. The violet (dotted) line is the reference value for $Re_\lambda = 45$ deduced from figures 5 and 6 in ref. [18].

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APPENDIX A: MEAN PRESSURE FIELD IN THE TURBULENT SHEARLESS MIXING FLOW

The shearless turbulent mixing that we have studied is a flow where the average momentum is zero since the initial condition and the boundary conditions are such as to not generate a mean flow. It should be noted that the laboratory configuration, at least, those to date, is somehow different. In fact, when the two interacting homogeneous isotropic turbulent fields are generated by grids placed in a wind tunnel, a mean (homogeneous, i.e. shearless) flow is present in the normal direction to the mixing. However, it is true that an acceleration along the mixing direction could emerge if the initial gradients of mean pressure and turbulent kinetic energy do not compensate. As in the laboratory situation, this mean flow would remain homogeneous, thus also in this case the mixing would be shearless (i.e. devoid of the production of turbulent kinetic energy). Let us first consider the averaged Navier-Stokes equations without the introduction of any model. The mean momentum equation is:

$$\partial_t U_i + \partial_j U_i U_j = -\left(1/\rho\right)\partial_i P - \partial_j u_i u_j + \nu \nabla^2 U_i \quad (A1)$$

where the capital letters denote mean quantities, the small letters fluctuations, and the overline denotes the statistical average. For $t = 0$ we have $U_i = 0$ and the only non zero derivative is in the $x$ direction, so that these equations reduce to

$$\partial_t U = -(1/\rho)\partial_x P - \partial_x u^2, \quad (A2)$$

where $U$ is the mean velocity in the mixing direction. It can be seen that if the initial pressure gradient term balances the gradient of the part of the initial turbulent kinetic energy associated to the fluctuations in the $x$ direction ($u^2 = 2/3K$), the acceleration term $\partial_t U$ is zero. In such a situation, a mean field will be absent. On the contrary, for example in the hypothetical case of an initial kinetic energy gradient facing a zero pressure gradient, a mean homogeneous (without shear) flow will be generated.

In the present numerical experiment the initial velocity field is first introduced. Then, as is standard practice, the code builds the pressure field by using the Poisson equation obtained from the divergence of the momentum balance. Periodicity conditions plus a condition fixing the average pressure $p_0$ value in the entire domain are used. Since the field is incompressible, the divergence of

$$\nabla^2 P/\rho = -\partial_i \partial_j u_i u_j - \partial_i \partial_j U_i U_j \quad (A3)$$

At $t = 0$ the fluctuating velocity field is statistically uniform apart from in the $x$ direction (note: it remains so during the mixing process). By also considering the symmetries of the initial velocity field, and in particular the fact that, outside the mixing, the field is uniform, we obtain

$$\partial^2_{xx} P/\rho = -\partial^2_{xx} u^2, \quad \partial_x P/\rho = -\partial_x u^2. \quad (A4)$$

Consequently, by coming back to (A2), one can see that no mean acceleration is generated at $t = 0$. Figure 10 shows the terms in equation (A2) - the pressure and turbulent kinetic energy gradients and $\partial_t U$ - in two instants. We have considered the field configuration observed in the laboratory experiment by Veeravalli and Warhaft (1989, 3:1 perforated plate experiment,
air flow at standard conditions), which is actually the field configuration that we tried to reproduce in this numerical experiment. In particular, we have estimated the dimensional values of the pressure gradients and pressure difference between the high turbulent energy and low energy regions of the mixing. If \( (dP/dx)_{\text{max}} \) is the maximum value of the mean pressure gradient and \( \Delta P \) = the pressure difference between the two homogeneous regions, we have at the initial instant of the simulations:

\[
E_1/E_2 = 6.6, \quad (dP/dx)_{\text{max}} = 1.39\text{Pa/m},
\]

\[
\Delta P = 2.30 \times 10^{-2} \text{ Pa}, \quad 2\Delta = 2\text{cm}
\]

\[
E_1/E_2 = 40, \quad (dP/dx)_{\text{max}} = 1.60\text{Pa/m},
\]

\[
\Delta P = 2.71 \times 10^{-2} \text{ Pa}, \quad 2\Delta = 2\text{cm}
\]

\[
E_1/E_2 = 60, \quad (dP/dx)_{\text{max}} = 1.62\text{Pa/m},
\]

\[
\Delta P = 2.72 \times 10^{-2} \text{ Pa}, \quad 2\Delta = 2\text{cm}
\]

It can be observed that these pressure differences are very small. As a consequence, measurements in the laboratory should be very difficult.