Prediction of the dynamic stiffness of boring bars

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Abstract: The productivity of boring operations is limited by chatter vibrations, as an effect of the low dynamic stiffness and damping of these cantilevered structures. The origin of the beforementioned self-excited vibration is mainly related to the dynamic stiffness of the boring bar’s bending mode, which is influenced by the stiffness and damping of the machine tool interface. In this paper a dynamic model is presented, which uses Timoshenko beam theory for the boring bar itself, while the effect of the clamping is experimentally determined applying Receptance Coupling Substructure Analysis (RCSA). The important effect of the boring head at the tool tip is considered on the coupling as well. This way, once the machine interface is characterized, the Frequency Response Functions (FRF) of different boring bars can be predicted.

Keywords: Boring bar, RCSA, FRF prediction.

1. Introduction

In the manufacturing of many parts as valves, engine cylinders or landing gears, internal machining operations are carried out with static or rotatory boring bars. These tools are inherently slender, with a cutter on their free end, while the other end is clamped on the turret or spindle head.

Degradation in surface quality due to vibration and deflection are common problems when this kind of tools are employed. In fact, in the production of good quality long holes, which are needed in industrial cases, the static and dynamic stiffness of boring bars are one of the most important factors. On the one hand, the accuracy of the diameter of the bore to be machined is limited by the static deflection, although it can be compensated by numeric control. On the other hand, the lack of dynamic stiffness leads to self-excited vibrations known as chatter [1,2]. This causes an unacceptable surface finish and a decrease of tool life, being the main productivity limit of the boring process. In fact, chatter makes impossible to accomplish boring operations in large length $L$ to diameter $D$ ratio holes ($L / D > 4$) [3]. Therefore, the main geometrical parameters are fixed, and the topological optimization of the shape of the boring bar cannot provide important improvements. For this reason, the increase of the dynamic stiffness of the boring bars offers not only higher productivity, but also allows the manufacturing of deeper holes which cannot be manufactured by standard tools [3].

The enhancement of the productivity of boring operations has been tackled from different approaches [4]. On the one hand, chatter can be suppressed by process related methods. As the ratio between chatter frequency and tooth passing frequency (lobe number) is usually high for boring operations, process
damping greatly affects the stability of the process [5-7]. However, it is necessary to reduce the spindle speed to increase this kind of damping causing a reduction of productivity. Due to this limitation, the enhancement of the cutting capacity of boring bars has been mainly approached by structural modifications with the aim of increasing the dynamic stiffness.

While steel is the preferred material in the construction of boring bars, materials with high Young modulus as tungsten carbide have been used, although stability improvement is not very high [8]. Combined structures can be used to fit the material properties of each segment of the bar to the mechanical requirements [3]. Rivin [3] proposed a bar made of aluminium in the free end and carbide in the clamped part. Lee and Suh [9] used graphite epoxy in the construction of the bar, obtaining 5 times higher cutting capacity than a steel boring bar.

The introduction of tuned mass dampers (TMD) in the free end of the bar has also been used to enhance the cutting capacity of these tools. It was introduced for the first time for boring bar chatter suppression by Hahn [10] in 1951, based on a Lanchester type damper. Donnies [11] improved the performance with the inclusion of a spring damper. Although the performance of a Lanchester damper is lower than a complete TMD [12], they are able to dampen more than one mode [13]. The tuning of this kind of devices to maximize the stability limit was developed by Sims [14]. As the tuning frequency of these elements is dependent on the natural frequency of the bending mode of the bar, correct modelling of the tool is needed to design properly these auxiliary damping devices. Indeed, in metal cutting the frequency response functions (FRF) obtained in the cutting point are widely employed to predict stability based on frequency domain methods [15,16].

In this paper a model to predict the receptance in the tool tip is presented, which takes into consideration experimentally obtained machine dynamics coupled with an analytical boring bar model. The combined machine and boring bar model is firstly explained, wherein the experimental response of the machine interface and the analytical responses of the boring bar body and the boring head are coupled based on a hybrid receptance coupling substructure analysis (RCSA). The third section shows the experimental validation of the estimated FRFs and paper ends with the conclusions of the work.

2. Modelling of the boring bar
In the present paper, the modelling of the boring bar is based on a hybrid RCSA [17], which allows to couple analytical and/or experimental FRFs of individual substructures to get the response of the final assembly. In the case of the boring bar, three substructures compose the assembly: the machine (M) with the interface (I), the boring bar body (B) and the boring head (H). The connections between parts are located at points A-B and C-D, while the cutting edge is located at point E (see figure 1).

The modelling of the assembly between a boring bar and a machine cannot be done with a simple lumped element based model. RCSA method overcomes this problem, allowing the coupling between the experimental receptance matrix of the machine side ($G_{AA}$) and analytic receptance matrix of the boring bar in free-free conditions ($R_{ij}$), obtaining the resultant FRF at the tool tip ($H_{EE}$). During this article, the receptance matrices denominated with $G_{ij}$ will refer always to the FRFs in the base of the bar considering translational and rotational degrees of freedom, $R_{ij}$ will be reserved for theoretical receptance matrices obtained in free-free conditions also for translational and rotational degrees of freedom, and $H_{ij}$ for the translational receptances in the final assembly of the boring bar.

![Figure 1. Assembly of the boring bar, with the machine (M) + interface (I) of the boring bar, the bar body (B) and the boring head (H).](image-url)
2.1. Experimental characterization of the machine side (\(G_{AA}\))

The substructure formed by the machine and the interface (M+I) can be considered as a not interchangeable element that concentrates the main damping origin of the bending mode of the boring bar. Considering that the prediction of damping is a difficult issue, the experimental characterization of dynamic response of the (M+I) is a efficient approach to predict the dynamics of the boring bar. However, although the translational receptances (\(G_{xAxA}(\omega)\)) are easily evaluated, the responses related to rotational degrees of freedom (\(G_{xApAp}(\omega)\), \(G_{pApAx}(\omega)\), \(G_{pApAp}(\omega)\)) are not directly measurable (see equation (1)). Park et al. [18] proposed an inverse RCSA to determine the stiffness and damping of the toolholder and combining it with an analytic model of the tool. The present work applies this method to perform the dynamic characterization of the machine with the interface (M+I), taking into account the stiffness and damping of the interface of the toolholder. Figure 2 shows the application of the method in which two dummy bars are used, and inverse RCSA is applied [18] to obtain the displacement (\(x_A\)) and rotation angle (\(\psi_A\)) of point A in one of the bending planes.

\[
\begin{pmatrix}
  x_A \\
  \psi_A
\end{pmatrix} =
\begin{pmatrix}
  G_{xAxA}(\omega) & G_{xApAp}(\omega) \\
  G_{pApAx}(\omega) & G_{pApAp}(\omega)
\end{pmatrix}
\begin{pmatrix}
  F_{xA} \\
  M_{xA}
\end{pmatrix}
\]

(1)

![Figure 2. Inverse substructuring to determine machine side (M+I) dynamics using [18]. (a) Short bar. (b) Long bar.](image)

Therefore, the method is based on three experimental measurements to obtain three different receptances considering only displacements. First, a short bar can be used to get the direct transfer function at point 2, assuming that \(G_{xAxA}(\omega) := H_{22}^{\text{short}}(\omega)\) (see figure 2-a). Secondly, a long bar can be inserted wherein direct and crossed responses at points 1 and 2 are measured (\(H_{21}^{\text{long}}(\omega)\) and \(H_{11}^{\text{long}}(\omega)\)). Then, the long bar can be analytically modelled to obtain its receptance matrices in free-free conditions (\(R_{22}, R_{21}, R_{12}, R_{11}\)), where

\[
R_{ij}(\omega) = \begin{bmatrix}
  R_{xixi}(\omega) & R_{xipij}(\omega) \\
  R_{pji}(\omega) & R_{p pij}(\omega)
\end{bmatrix}
\begin{pmatrix}
  m/N & m/Nm \\
  rad/N & rad/Nm
\end{pmatrix}
\]

(2)

Finally, by using all these responses and with the assumption of \(G_{xApAp}(\omega) := G_{pApAx}(\omega)\), RCSA method can be inversely applied [18] and \(G_{xAxA}(\omega)\) and \(G_{pApAp}(\omega)\) can be determined as

\[
G_{xAxA} = G_{pApAx} = \frac{H_{21}^{\text{long}}(G_{xAxA}+R_{x2x2})R_{x1}\psi_2-H_{21}^{\text{long}}R_{x1x2}R_{x2}\psi_2}{H_{21}^{\text{long}}R_{x1x2}-R_{x1x2}R_{x2x1}+R_{x2x2}(R_{x1x1}-H_{11}^{\text{long}})}
\]

(3)

\[
G_{xApAp} = \frac{G_{xAxA}^2R_{x2x1}-G_{xAxA}R_{x2x1}R_{x2}\psi_2+H_{21}^{\text{long}}(G_{xAxA}+R_{x2x2})R_{x2}\psi_2-G_{xAxA}R_{x2x2}R_{x2}\psi_1+G_{xAxA}R_{x2x1}+G_{xApAp}R_{x2x2}+G_{xAxA}(G_{xApAp}+R_{x2x2})(G_{xAxA}+R_{x2x2})}{G_{xAxA}R_{x2x2}-H_{21}^{\text{long}}(G_{xAxA}+R_{x2x2})}
\]

(4)
2.2. Modelling of the bar (\( R_{ij} \))

The receptances of the boring bar in free-free conditions (FRFR) are needed for the application of the RCSA [19]. If the receptances in free-free conditions are directly calculated by using different beam theories or FEM models, a large number of modes is needed to have an accurate prediction of the tool tip receptance. This drawback can be overcome if analytic fixed-free (FXFRT) responses are combined with fixed-boundaries (FXB), as proposed by Mancisidor et al. [20] (see figure 3). This way, the cut-off frequency of the Timoshenko beam model [21] can be avoided, and the mode shapes of FXB solutions fit in RCSA modelling in the final near FXFRT situation.

![Figure 3. Achievement of free-free boundary conditions response (FRFR) by adding clamped point movements (FXB) to fixed-free boundary conditions response (FXFRT).](image)

First, Timoshenko beam model is used to obtain FXFRT responses (\( x(t, z), \psi(t, z) \)). Timoshenko beam model proposes a beam theory which adds the effect of shear distortion and rotary inertia to the well-known Euler-Bernoulli model. The equations of motion for the homogeneous problem are given by

\[
\begin{align*}
\rho \ddot{x}(t, z) - \kappa G \alpha x''(t, z) - \psi'(t, z) &= 0, \\
\rho l \ddot{\psi}(t, z) - EI \psi''(t, z) - \kappa G \alpha x'(t, z) - \psi(t, z) &= 0,
\end{align*}
\]

(5)

where the elastic properties of the material are defined by the elastic modulus \( E \), Poisson coefficient \( \nu \) and density \( \rho \), being \( G = E/(2(1 + \nu)) \) the shear modulus. Length \( L \), cross-section area \( \sigma \) and inertia (meaning \( I_x \) or \( I_y \)), and the Timoshenko shear coefficient \( \kappa \) [22] characterize the geometry of the beam.

The application of constant cross-section and inertia assumption, and the solution for the corresponding modeshape in form of \( X(t, z) = e^{i\omega t}X(z) \), allow the calculation of the undamped FXFRT beam natural frequencies [23]

\[
\Gamma(\omega) = 2 + \frac{\alpha^2(\gamma^4 + \delta^4)\kappa^2 + 2\gamma(\delta^2 - \gamma^2)\kappa \rho \omega^2 + 2 \rho^2 \omega^4}{(G \gamma^2 - \rho \omega^2)(G \delta^2 + \rho \omega^2)} \cos \frac{\omega}{\nu} \frac{L}{\kappa} \frac{\delta \gamma}{\delta \gamma} + \frac{(\delta^2 - \gamma^2) \sin \frac{\omega}{\nu} \frac{L}{\kappa} \frac{\delta \gamma}{\delta \gamma}}{\delta \gamma} = 0,
\]

(6)

leading to define rotational mode shapes as

\[
\Psi_k(z) = -X_k'(z) \frac{\omega^2 \rho}{\kappa G} \int X_k(z) dz.
\]

(9)
Having the equation for the mode shapes defined, boundary conditions may be applied. In case of fixed-free conditions, next equations must be complied:

\[
\begin{align*}
X(z)|_{z=0} &= 0 \\
X'(z)|_{z=0} &= 0 \\
\Psi'(z)|_{z=L} &= 0 \\
X'(z) - \Psi(z)|_{z=L} &= 0
\end{align*}
\]

(10)

The application of these boundary conditions leads to a linear equation from which mode shapes coefficients and natural frequencies \((\omega_{n,k})\) can be solved up to the cut-off frequency \(\omega_{coff} = (\frac{GkA}{\rho l})^{1/2}\) filling the condition \(\beta(\omega) > \alpha(\omega)\) (see \(\delta(\omega)\) in equation (7)).

\[
\begin{align*}
C_{1,k} &= 1, & C_{2,k} &= -\frac{G\gamma_k^2 - \rho \omega_k^2}{\gamma_k (G \delta_k^2 + \rho \omega_k^2) \cos \gamma_k + \gamma_k (G \gamma_k^2 - \rho \omega_k^2) \sin \gamma_k}, \\
C_{3,k} &= -1, & C_{4,k} &= -\frac{G\gamma_k^2 + \rho \omega_k^2}{\gamma_k (G \delta_k^2 + \rho \omega_k^2) \sin \gamma_k + \gamma_k (G \gamma_k^2 - \rho \omega_k^2) \cos \gamma_k}.
\end{align*}
\]

(11)

The mode shapes need to be mass normalized in order to use FXB, taking into account the second moment of inertia \(I\) in the normalization coefficient \(c\). Therefore, mass normalized mode shapes related to deflection \(X(z)\) and angular motion of the cross section are given as:

\[
U_k(z) = c_k X_k(z) \quad \text{and} \quad U_{\psi,k}(z) = c_k \Psi_k(z)
\]

(12)

where \(c_k = \left(\int_0^L \rho A X_k^2(z)dz + \int_0^L \rho I \Psi_k^2(z)dz\right)^{-\frac{1}{2}}\).

Once fixed-free (FXFRT) response is obtained, FXB method can be applied to mimic FRFR behaviour for RCSA, by adding rigid body motion \(\xi(t)\) and rotation \(\theta(t)\) (see equation (13)). Then, the FRF displacement \(\ddot{x}(t, z)\) and rotation \(\ddot{\psi}(t, z)\) are approximated by a combination of rigid motion \(\lambda := [\xi \ \theta]^T\) plus a linear combination of the low frequency modes \(q_k(t)\):

\[
\ddot{x}(t, z) = x(t, z) + \xi(t) - \theta(t)z = [1 \ -z] \lambda(t) + \sum_{k=1}^n U_k(z) q_k(t),
\]

\[
\ddot{\psi}(t, z) = \psi(t, z) + \theta(t) = [0 \ -1] \lambda(t) + \sum_{k=1}^n U_{\psi,k}(z) q_k(t).
\]

(13)

being \(n\) the number of modes calculated for FXFRT. Then, according to [20], free-free condition modelled FRFs \(R_{ij}(\omega)\) between an arbitrary excitation \(z_j\) and sensing \(z_i\) points can be calculated as:

\[
R_{ij}(\omega) = T_i Q_c(\omega) T_j^T
\]

(14)

where the transformation matrix and the core FRF are given as

\[
T_i = \begin{bmatrix}
1 & z_i & \cdots & U_{\psi,k}(z_i) & \cdots \\
0 & -1 & \cdots & U_k(z_i) & \cdots
\end{bmatrix},
\]

\[
Q_c(\omega) = \begin{bmatrix}
-\omega^2 M_r & \cdots & -\omega^2 M_c \\
\cdots & \cdots & \cdots
\end{bmatrix}^{-1}
\]

(15)

In previous equation, the rigid, the cross and the beam dynamics are defined by the matrices originated partly from the transformation \(U\) in equation (12),

\[
M_r = \rho A \begin{bmatrix}
L & -\frac{1}{2} L^2 \\
-\frac{1}{2} L^2 & \frac{1}{3} L^3
\end{bmatrix}, \quad M_c = \rho A \begin{bmatrix}
\cdots & \int_0^L U_k(z)dz & \cdots \\
\cdots & \cdots & \cdots
\end{bmatrix}
\]

(16)

Regarding the damping of the monolithic parts, it has been modelled as structural damping based on a constant loss factor \(\eta\) of the material:

\[
S_p(\omega) = \left(1 + \eta \cdot \text{sgn}(\omega)\right)\text{diag}(\omega_{n,k}^2).
\]

(17)

With the use of equation (13), free-free FRFs can be calculated in order to feed the boring bar model.
2.3. Boring head modelling

Although this element is just a connection element between the insert and the bar, its effect cannot be neglected due to its mass and inertia in the calculation of the FRF at the tool tip. However, its bending can be ignored in the calculation due to its high stiffness, modelling it as a rigid element. By using Newton-Euler equations, free-free response between sensing point $i$ and excitation point $j$, characterized by their distance to the centre of mass ($z_{CM}$) point can be formulated:

$$R_{ij}(\omega) = -\frac{1}{\omega^2} \left\{ \frac{1}{m_H} + \frac{z_{CM,i} z_{CM,j}}{\theta_{CM}} \right\} \left[ \frac{1}{\theta_{CM}} \right], \quad (18)$$

The rigid element is characterized by the head mass $m_H$ and the mass moment of inertia considered at the centre of mass (CM) ($\theta_{CM,x}$ and $\theta_{CM,y}$, accordingly to each of the bending directions).

2.4. Assembly of the boring bar

A summary of RCSA is given in this section, which allows to assembly of the substructures shown in figure 1, whose responses have been obtained in previous subchapters. This method needs to be applied three times. First, an inverse RCSA is used to estimate machine dynamics at point A with the technique presented in [18], as shown in section 2.1. Then, the FRFs of the machine side, boring bar body beam and head mass/inertia are coupled respectively at points A-B and C-D by RCSA (figure 1).

Having $G_{AA}$ stated by the determination of the machine dynamics as shown in equation (1), as a first step the boring bar is connected by FXFRT+FXB model of the beam ($R_{CC}, R_{CB}, R_{BC}, R_{BB}$), estimating the response at point C as

$$G_{CC} = R_{CC} - R_{CB} (R_{BB} + G_{AA})^{-1} R_{BC} \quad (19)$$

Then, the free end of the boring bar body can be considered as base substructure and the predicted free-free dynamics of the rigid head ($R_{EE}, R_{ED}, R_{DE}, R_{DD}$) can be coupled as

$$G_{EE} = R_{EE} - R_{ED} (R_{DD} + G_{CC})^{-1} R_{DE} \quad (20)$$

from which the useful translational receptance can be obtained: $H_{EE} = G_{xExE}$.  

3. Experimental validation of RCSA

The validation of the analytical model used for the estimation of the FRF is carried out in this section. Experimental tests have been carried out in a lathe equipped with Capto C5 toolholder interface.

![Figure 4. (a) dummy bars. (b) measured machine dynamics in the x direction with $H_{11}^{long}(\omega)$, $H_{21}^{long}(\omega)$ and $H_{22}^{short}(\omega)$, and rotational receptances at toolholder interface ($G_{x2x2}(\omega), G_{x2\psi2}(\omega) = G_{y2x2}(\omega)$ and $G_{y2\psi2}(\omega)$) estimated by inverse RCSA.](image-url)
3.1. Determining machine tool dynamics

Following the work of [18], the machine side until Capto toolholder interface has been characterized (located at point A, figure 1). As explained in section 2.1, the dynamic behaviour of the machine can be semi-analytically determined as a function of frequency by the experimental FRF measurements on a long \( (L_{B2}=175\text{mm}) \) and a short bar \( (L_{B1}=39\text{mm}) \): \( H_{22}^{\text{short}}(\omega) \), \( H_{21}^{\text{long}}(\omega) \) and \( H_{11}^{\text{long}}(\omega) \) (see figure 2).

Experimentally obtained FRFs in figure 4 show the first mode of the LB2 bar at 674 Hz on \( H_{21}^{\text{long}}(\omega) \), \( H_{11}^{\text{long}}(\omega) \). However, LB1 short bar shows mostly the dynamics of the machine itself, hence \( G_{x2x2}(\omega) = H_{22}^{\text{short}}(\omega) \) is justified. The existence of machine modes around 200 Hz and 400 Hz is observed.

3.2. Experimental validation of the boring bar model

In order to validate the model described in previous sections, complete assemblies of two boring bars have been tested. These bars have the same cross section, with \( D \) as outer diameter and \( d \) as inner diameter, and material as the short and long dummy bars used for machine characterization, and are mounted with the same boring head. However, they have different boring bar body lengths, 175 and 275 mm. The FRFs at the tool tip have been estimated (figure 5) according to the method explained in section 2, with the geometrical and physical data that can be found in table 1.

Table 1. Boring bar geometric and physical details.

| \( D \) (mm) | \( d \) (mm) | \( \rho \) (kg/m³) | \( E \) (GPa) | \( v \) | \( \eta \) | \( m_H \) (kg) | \( \Theta_{\text{CM,x}} \) (kgm²) | \( \Theta_{\text{CM,y}} \) (kgm²) | \( z_{\text{CM,D}} \) (mm) | \( z_{\text{CM,E}} \) (mm) |
|-------------|-------------|-----------------|-------------|-----|------|--------------|----------------|----------------|--------------|--------------|
| 50          | 6           | 7800            | 210         | 0.3 | 1    | 0.576        | 1.94 · 10⁻⁴    | 1.85 · 10⁻⁴    | 21           | 15           |

\[ \kappa = 6 \left( \frac{(1+v)(1+m^2)^2}{(7+6v)(1+m^2)^2+(20+12v)m^2} \right) = 0.8541, \quad m = \frac{d}{D} \quad [21] \]

Figure 5. Experimental vs estimated FRFs are shown as well as an ideal rigidly clamped case for two bar assemblies. Panel a) and b) show the x and y direction of a LB2=175mm boring bar assembly respectively, while c) and d) show it for a LB2=275mm boring bar assembly.

A good correlation between estimated and experimental FRF is observed. In addition, dashed lines show the FRF of the bar without considering the machine dynamics, simulating a completely stiff clamping. Comparing this FRF with the experimental one, a great effect of the machine dynamics is observed. While completely stiff clamping FRFs show single dominant mode FRFs corresponding to the first bending mode of the bar, experimental FRFs as well as complete model based ones show a more complex dynamic response with coupled modes. Therefore, machine dynamics must be taken into account in the modelling of boring bars to avoid large errors in frequency and compliance estimations.

4. Conclusions

A mathematical model capable of estimating the FRF at the tool tip for boring bar assemblies has been presented. The presented method couples analytically modelled boring bars, taking into consideration the dynamic behaviour of the machine side of the tool clamping, as well as the effect of the boring head...
at the tool tip. This behaviour has been characterized by experimental FRF obtention. The model has been experimentally validated obtaining accurate results, showing that the boring bar dynamics are greatly affected by the machine dynamics. The obtained FRF will be used as an input in order to calculate the stability of the boring process.

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