Type II Solutions on $\text{AdS}_3 \times S^3 \times S^3$ with Large Superconformal Symmetry

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Abstract

New local solutions in type II supergravity that are foliations of $\text{AdS}_3 \times S^3 \times S^3$ over an interval and preserve at least large $\mathcal{N} = (4,0)$ supersymmetry are found. Some cases have compact internal space, some not and one experiences an enhancement to $\mathcal{N} = (4,4)$. We present two new globally compact solutions with D brane and O plane sources explicitly, one in each of IIA and IIB. The former appears to be part of a broader family of global solutions with D8/O8’s back-reacted on $\text{AdS}_3 \times S^3 \times S^3 \times S^1$. The later has an internal geometry bounded between D5’s and O5’s.
1 Introduction

The AdS-CFT correspondence has by now shown itself to be a powerful tool to probe the dynamics of theories on both sides of the correspondence. Since its inception it has stimulated progress constructing many CFT’s and their dual AdSd+1 solutions, in many cases embedded into 10 dimensions. One area where progress on the CFT side has somewhat outpaced the other is the AdS3-CFT2 correspondence. This is not to say that progress on the gravity side has not been made (see [1–15] for an incomplete list), merely that there is much more yet to be studied.

Two dimensional CFT’s play an important role in physics, in string theory and beyond so there is clear motivation to construct holographic duals. The barrier to this is that when embedded into 10 dimensional supergravity, their internal space is 7-dimensional which is rather large. Progress can be made tractable by assuming extended supersymmetry - in this case the dual geometry will realise an additional R-symmetry reducing the number of undetermined dimensions. An interesting feature of superconformal field theories in 2 dimensions is that a relatively large number of superconformal algebras exist for each number of preserved supercharges, with each preserving a distinct R-symmetry. Those of that can be embedded into 10 and 11 dimensions where classified in [16]. Given this, and the recent G-structure classification of N = 1 AdS3 solutions in type II supergravity [17], the time seems right to begin to seriously explore the possibilities.
In this work, the focus will be AdS$_3$ solutions preserving at least $\mathcal{N} = (4, 0)$ supersymmetry with the large superconformal algebra

$$\mathfrak{sl}(2) \oplus \mathfrak{so}(4)$$

The first term here is realised by AdS$_3$, the second is an SO(4) R-symmetry that will be realised by the internal space. There are several ways to arrange for this to happen with products of 2 and 3-spheres. Here it will be assumed that the R-symmetry is realised by a warped $S^3 \times S^3$ factor in the metric$^1$. Generically, such solutions will have a flavour SO(4) in addition to the R-symmetry$^2$. The reasons to make this choice are two fold: i) In short, it is the easiest example to look at. However this simplicity will allow for a complete local description of all such solution in type II supergravity$^3$. ii) With $S^3 \times S^3$ there is the possibility of an enhancement to large $\mathcal{N} = (4, 4)$ supersymmetry (a maximal case for AdS$_3$ [20]). These are rare, indeed the only example the author was aware of before this work was $AdS_3 \times S^3 \times S^3 \times S^1$ - this is now no longer the case.

The method used here to find new solutions with large $\mathcal{N} = (4, 0)$ supersymmetry shall be to first construct spinors on the internal space that manifestly transform under the action of SO(4), and then to find every geometry with an $S^3 \times S^3$ factor consistent with them. This follows the line of reasoning of the earlier works [21–25], where many of the technical details exploited here were originally worked out. Here it will be possible to give the explicit local form of every type II solution consistent with the SO(4) spinor. Given the results here and [17], where AdS$_3$ solutions with exceptional R-symmetries were studied, it has become clear that such R-symmetry based spinor constructions are a powerful tool to study AdS$_3$ solutions with extended supersymmetry.

The outline of the paper is as follows: In section 2 we explicitly construct general spinors that transform in the fundamental representation of one of the two available independent SO(4) isometries on $S^3 \times S^3$, that are also singlets under the action of the other - this ensures we are consistent with large $\mathcal{N} = (4, 0)$ supersymmetry. In section 3 we use G-structure techniques to extract geometric conditions from the SO(4) spinors that all solutions should obey, and in sections 4 and 5 we find all local solutions that follow. The most interesting of these are clearly those that can be used to construct global solutions with compact internal

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$^1$Solutions with a similar local foliation were constructed in [18, 19] by utilising Romans $F(4)$ gauged supergravity.

$^2$We shall impose that this entire SO(4)$\times$SO(4) is preserved by the remaining physical fields also

$^3$The same methods could also be used to probe the space of M theory solutions, but that is left for future work.
space. We explicitly construct two such examples (though more are probably possible): A new \( N = (4, 0) \) massive IIA solution in section 4.1 and a new \( N = (4, 0) \) IIB solution in section 5.1. The former is constructed by gluing two locally non compact solutions together with a D8 brane defect. Further new locally non compact solutions preserving \( N = (4, 4) \) and \( N = (4, 0) \) can be found in sections 4.2 and 5.2 respectively. In section 4.2 we speculate that these might also be made compact by using (this time) smeared Dp brane defects for \( p < 8 \), however a detailed study is beyond the scope here.

2 Realising an SO(4) R-Symmetry on \( S^3 \times S^3 \)

We are interested in large \( N = (4, 0) \) AdS\(_3\) solutions preserving an SO(4) R-symmetry. As such the 10 dimensional Majorana spinors may be written as

\[
\begin{align*}
\epsilon_1 &= \sum_{I=1}^{4} \zeta^I \otimes v_+ \otimes \chi^I_1, \\
\epsilon_2 &= \sum_{I=1}^{4} \zeta^I \otimes v_- \otimes \chi^I_2
\end{align*}
\] (2.1)

where \( \zeta^I \) and \( \chi^I_{1,2} \) are 4 independent spinors on AdS\(_3\) and the internal 7 dimensions respectively and \( v_\pm \) is an auxiliary 2 vector, that is always required when decomposing an even dimensional spinor in terms of 2 odd ones - \( \pm \) refers to chirality, so the upper/lower signs are taken in IIA/B. The spinors on AdS\(_3\) are Killing, so obey the equation

\[
\nabla_{a\text{AdS}_3} \zeta^I = \frac{\mu}{2} \gamma^a_{\text{AdS}_3} \zeta^I
\] (2.2)

where \( \mu = \pm 1 \) parametrise a spinor that is charged under the SL(2)\(_L/R\) subgroup of SO(2,2)\(\cong\)SL(2)\(_L\) \(\times\) SL(2)\(_R\) and a singlet under SL(2)\(_R/L\).

As we want an SO(4) R-symmetry, \( \chi^I_{1,2} \) should transform in fundamental of this group and solutions should admit a local description with SO(4) realised geometrically. There are several ways to do this, but from the perspective of finding solutions, the simplest way to realise this R-symmetry is to decompose the internal space as a foliation of \( S^3_1 \times S^3_2 \) over an interval in which the physical fields have support only. This gives us SO(4)\(_1\) \(\times\) SO(4)\(_2\) to work with allowing for enhancements to \( N = (4, 4) \) supersymmetry with SO(4) \(\times\) SO(4) R-symmetry whenever the physical fields obey certain constraints. The Killing spinor equation for \( S^3 \) is

\[
\nabla_{i\text{S}^3} \xi = \frac{i \nu}{2} \gamma_{i\text{S}^3} \xi
\] (2.3)

where \( \nu = \pm 1 \) parametrise a spinor charged under the SU(2)\(_L/R\) subgroup of SO(4) \(\cong\) SU(2)\(_L\) \(\times\) SU(2)\(_R\) and are singlets under the SU(2)\(_R/L\). As shown in [22], in the Hopf fibration
frame of \( S^3 \), the doublets of \( SU(2)_{L/R} \) are simply

\[
\xi^a = \left( \begin{array}{c} \xi \\ \xi^c \end{array} \right)^a
\]  

(2.4)

where \( \xi^c \) is the Majorana conjugate of \( \xi \). On \( S^3 \) there are two sets of one-forms that are charged under \( SU(2)_{L/R} \) and dual to the corresponding Killing vectors. We parametrise these in a unified language as \( K_i \) such that

\[
dK_i + \frac{\nu}{2} \epsilon_{ijk} K_j \wedge K_k = 0, 
\]

(2.5)

where the sign of \( \nu \) determines the relevant \( SU(2) \) as before - these are of course the \( SU(2)_{R/L} \) invariant 1-forms. The spinoral Lie derivative of the \( SU(2)_{L/R} \) doublet along along the \( SU(2)_{L/R} \) Killing vectors is

\[
\mathcal{L}_{K_i} \xi^a = i\nu (\sigma_i)^a_b \xi^b. 
\]

(2.6)

while acting on the \( SU(2)_L \) doublet with the \( SU(2)_R \) Killing vector, or vice-versa, yields zero. We can use this relation to form a spinor transforming in the fundamental of \( SO(4)_{L/R} \cong SO(3)_{1L/R} \times SO(3)_{2L/R} \) and as a singlets under \( SO(4)_{R/L} \) depending on the sign of \( \nu \) - when we couple this to an \( AdS_3 \) spinor as in (2.1), the result will be a spinor realising the large \( \mathcal{N} = (4,0) \) superconformal algebra

\[
\mathfrak{sl}(2) \oplus \mathfrak{so}(4) 
\]

(2.7)

as required.

In [24] it was established how to form an \( SO(3) \) triplet from products of two \( SU(2) \) doublets - when the doublets are both formed from \( S^3 \) Killing spinors, there is only one such triplet (for each sign of \( \nu \)), namely

\[
\eta_i = (\sigma_2 \sigma_i)_{ab} \xi^a_1 \otimes \xi^b_2, 
\]

(2.8)

it turns out that this is also Majorana. We define diagonal and anti-diagonal \( SO(3) \) Killing vectors as

\[
K^+_i = K^1_i + K^2_i, \quad K^-_i = K^1_i - K^2_i, 
\]

(2.9)

then it is a simple exercise in Pauli matrix manipulations to establish that

\[
\mathcal{L}_{K^+_i} \eta_j = \nu \epsilon_{ij}^k \eta_k, \quad \mathcal{L}_{K^-_i} \eta_j = 0, 
\]

(2.10)
so that $K_i^+$ realises the Lie algebra of SO(3). We can parameterise a basis for the Lie algebra of SO(4) in block form as

$$(T_i^+)_{ij} = \begin{pmatrix} \varepsilon_{ijk} & 0 \\ 0^T & 0 \end{pmatrix}, \quad (T_i^-) = \begin{pmatrix} 0_{3\times3} & c_i \\ -c_i^T & 0 \end{pmatrix},$$

(2.11)

where $c_1 = (1,0,0)^T$, $c_2 = (0,1,0)^T$, $c_3 = (0,0,1)^T$. It is then clear that 3 components of the SO(4) spinor one wishes to construct are simply the SO(3) triplet as these give rise to the top left blocks of (2.11) under $K_i^+$ and $K_i^-$. The 4th component should be a singlet under the action of $K_i^+$ - such a spinor was also provided in [24], and for $S^3$ is once more unique and Majorana

$$\eta_4 = -i(\sigma_2)_{ab}\xi_1^a \otimes \xi_2^b.$$  

(2.12)

So there is exactly one SO(4) spinor (for each sign of $\nu$) we can define on $S^3 \times S^3$, namely

$$\eta^I = (\mathcal{M}_I)_{ab}^a \otimes \xi_2^b, \quad \mathcal{M}_I = (\sigma_2\sigma_1, \sigma_2\sigma_2, \sigma_2\sigma_3, -i\sigma_2)_I,$$

(2.13)

It is not hard to confirm that

$$\mathcal{L}_{K_i^\pm}\eta^I = \nu(T_i^\pm)^I J \eta^J,$$

(2.14)

so the spinorial Lie derivative of the SO(4) spinor along the SO(4) Killing vectors does indeed realise the associated Lie algebra. Given that they must be Majorana, and satisfy $|\chi_{1,2}^I|^2 = e^A$ component by component, the most general 7d spinors preserving an SO(4) R-symmetry can be parametrised as

$$\chi_I = e^{\frac{A}{2}} \begin{pmatrix} \sin(\alpha_1 + \alpha_2) \\ i \cos(\alpha_1 + \alpha_2) \end{pmatrix} \otimes \eta^I, \quad \chi_2^I = e^{\frac{A}{2}} \begin{pmatrix} \sin(\alpha_1 - \alpha_2) \\ i \cos(\alpha_1 - \alpha_2) \end{pmatrix} \otimes \eta^I.$$  

(2.15)

Now since each component of $\chi_{1,2}^I$ can be mapped into every other through the action of the R-symmetry (2.14), which we assume the physical fields also respect, we need only explicitly solve the supersymmetry conditions of an $\mathcal{N} = 1$ sub-sector to know that $\mathcal{N} = (4,0)$ is preserved. In what follows, we shall take this sub-sector to be,

$$\chi_1 = \chi_1^4, \quad \chi_2 = \chi_2^4,$$

(2.16)

but we stress that as long as SO(4)$_1 \times$SO(4)$_2$ is preserved by a solution, this choice is totally arbitrary.$^4$ Additionally, if one finds a solutions with metric, dilaton and fluxes that do not depend on the signs of $\mu, \nu$ then there exists a second independent 10 dimensional spinor of the form (2.1) charged under the second copies of SO(4) and SL(2) at our disposal. This enhances supersymmetry to large $\mathcal{N} = (4,4)$ - as we shall see, this will indeed happen in some instances.

$^4$This would no longer be the case if we were to break SO(4)$_1 \times$SO(4)$_2$ to some subgroup by for instance fibring one $S^3$ over the other as in [24] - then different choices of $\chi_{1,2}$ would lead to different amounts of supersymmetry preserved.
3 Supersymmetric Bi-Spinors Conditions

A solution of type II supergravity respecting the isometry group of AdS$_3$ can always be decomposed as

\[ ds^2 = e^{2A} ds^2(\text{AdS}_3) + ds^2(M_7), \]

\[ F = f + e^{3A} \text{Vol(AdS}_3) \wedge \star_7 (f), \quad H = h_0 \text{Vol(AdS}_3) + H_3, \]

where the dilaton $\Phi$ and warp factor $e^{2A}$ have support on $M_7$ only, and likewise the fluxes. As explained in [17], if one insists on preserving at least $N = 1$ supersymmetry and a nontrivial RR sector one must fix $h_0 = 0$.

- supersymmetry is then implied by

\[ d_H (e^{A-\Phi} \Psi_\pm) = 0, \]

\[ d_H (e^{2A-\Phi} \Psi_\pm) - 2 \mu e^{A-\Phi} \Psi_\pm = \pm \frac{e^{3A}}{8} \star_7 \lambda(f), \]

\[ e^{-\Phi} (f, \Psi_\pm) - 2 \mu \text{Vol}_7 = 0. \]

with the upper/lower signs taken in IIA/B, for

\[ e^{-A} \chi_1 \otimes \chi_2^\dagger = \Psi_+ + i \Psi_-, \quad |\chi_1|^2 = |\chi_2|^2 = e^A, \]

where $\chi^i$ are two 7 dimensional Majorana spinors and $(\cdot, \cdot, \cdot)$ is the Mukai pairing in 7 dimensions.

In the previous section an $\mathcal{N} = 1$ spinor was constructed, (2.16), on $M_7 = \mathbb{R} \times S^3 \times S^3$. The Supersymmetry preserved by any solution one can construct from this spinor will be least $\mathcal{N} = (4, 0)$ with the remaining independent spinors generated through the action of the SO(4) R-symmetry. Specifically one has

\[ \chi_1 = -ie^\frac{4}{2} (\sigma_2)_{ab} \left( \sin(\alpha_1 + \alpha_2) \right) \otimes \xi^a_1 \otimes \xi^b_2, \quad \chi_2 = -ie^\frac{4}{2} (\sigma_2)_{ab} \left( \sin(\alpha_1 - \alpha_2) \right) \otimes \xi^a_1 \otimes \xi^b_2 \]

as this is a tensor product of spinors in each factor of the foliated internal space it should be clear that the 7 dimensional bi-spinors $\Psi_\pm$ can be expressed in terms of wedge products of bi-spinors on the interval and two 3-spheres - two this end it is useful to know the bi-spinors

\[ ^6 \text{This assumption can be made without loss of generality because, while solutions with } f = 0 \text{ and } h_0 \neq 0 \text{ do exist, they all lie in the common NS sector of type II supergravity. As such, when viewed as solutions in IIB, they are all S-dual to solutions with } f \neq 0 \text{ and } h_0 = 0. \]
on $S^3$ as these are the only non-trivial building blocks one requires. One can show, [22],
that the matrix bilinear following from the SU(2) spinor doublet of (2.4) are
\[
\xi^a \otimes \xi^{b\dagger} = \frac{1}{2} \left( (1 - i e^{3C} \text{Vol}(S^3)) \delta^{ab} + \left( \frac{1}{2} e^C K_i - \frac{i}{8} e^{2C} \epsilon_{ijk} K_j \wedge K_k \right) (\sigma^i)^{ab} \right)
\]  
where $e^{2C}$ is a warp factor on a round $S^3$, and $K_i$ can be either the L or R invariant forms.
The internal space of the solutions we are interested in will decompose as a foliation of
$S^3 \times S^3$ over an interval, thus the most general metric one can write is
\[
ds^2 = e^{2k} dr^2 + e^{2C_1} ds^2(S^3_1) + e^{2C_2} ds^2(S^3_2)
\]  
with $e^{2C_i}, e^{2k}$ functions of $r$ only, and where diffeomorphism invariance can be used to fix
$e^{2k}$ to any (non zero) value we choose. Making use of all these ingredients it is then a rather
simple exercise in the exploitation of Fierz identities to construct $\Psi_\pm$. These can be most
succinctly written in terms of an SU(3)-structure as
\[
\Psi_+ = \text{Re} \left[ e^{i\alpha_2} e^{-iJ} - e^k dr \wedge \Omega \right], \quad \Psi_- = \text{Im} \left[ - e^{i\alpha_2} e^k dr \wedge e^{-iJ} + \Omega \right]
\]  
where the specific SU(3)-forms are
\[
J = \frac{1}{4} e^{C_1+C_2} \left( K_1^1 \wedge K_1^2 + K_2^1 \wedge K_2^2 + K_3^1 \wedge K_3^2 \right),
\]
\[
\Omega = \frac{1}{8} e^{i\alpha_1} \left( e^{C_1} K_1^1 + ie^{C_2} K_1^2 \right) \wedge \left( e^{C_1} K_2^1 + ie^{C_2} K_2^2 \right) \wedge \left( e^{C_1} K_3^1 + ie^{C_2} K_3^2 \right).
\]  
At this point, in principle, once could blindly plug (3.6) into (3.2) and find every solution
that is consistent with the metric and spinor - but one needs to take a little more care if one
wants to ensure that SO(4) x SO(4) symmetry and $\mathcal{N} = (4,0)$ supersymmetry is preserved.
The issue is the fluxes, (3.2) only assumes $\mathcal{N} = 1$ supersymmetry is unbroken so the second
condition will generically give flux components that break (super)symmetry. To mitigate
this issue we demand that all fluxes must decompose in a basis of the invariant forms of
SO(4)xSO(4), namely
\[
dr, \quad \text{Vol}(S^3_1), \quad \text{Vol}(S^3_2), \quad (3.8)
\]  
and their wedge products, with functional support on the interval only - this greatly in-
creases the number of independent conditions in (3.2) that give rise to purely geometric
constraints and allows for the exact local form of all solutions consistent with an SO(4) x
SO(4) isometry to be found in the following sections. We study type IIA in section 4 and
type IIB in section 5, in both instances we fix the NS 3-form as
\[
H = c_1 \text{Vol}(S^3_1) + c_2 \text{Vol}(S^3_2),
\]  
for constants $c_i$ without loss of generality.
4 All Local Solutions in Type IIA

In this section we find all $\mathcal{N} = (4,0)$ solutions with an $\text{SO}(4) \times \text{SO}(4)$ isometry in type IIA supergravity. There are two independent forms of local solution we study in sections 4.1 and 4.2 that (generically) preserve $\mathcal{N} = (4,0)$ and $\mathcal{N} = (4,4)$ respectively. We show how the former can be used to construct new compact global solutions, and provide a hint as to how one might do the same with the latter.

Upon plugging the bi lines of (3.6) into (3.2) one quickly realises two zero form constraints
\[
\left(\cos \alpha_1 e^{C_1} - \sin \alpha_1 e^{C_2}\right) = (\mu \cos \alpha_1 e^{C_1} - \nu e^A \sin \alpha_2) = 0
\] (4.1)
these are very useful as they cannot be solved when any of $\cos \alpha_1$, $\sin \alpha_1$ or $\sin \alpha_2$ are set to zero, as this would require us to do the same to one of the warp factors. We can then take (4.1) as a general definitions for $e^{C_i}$ in IIA and eliminate these factors from the rest of the supersymmetry constraints, after some work we find the additional conditions
\[
\alpha_1' = F_2 - F_0 \cos \alpha_2 = 0, \quad c_1 \cos^4 \alpha_1 + c_2 \sin^4 \alpha_1 = \mu^2 c_2 \sin^3 \alpha_1 + \nu^3 \cos \alpha_1 e^{2A} \sin(2\alpha_2) = 0,
\] (4.2)
\[
(e^{5A-\Phi} \sin^3 \alpha_2)' - 2\mu e^{4A+k-\Phi} \cos \alpha_2 \sin^2 \alpha_2 = 0,
\] (4.3)
\[
(e^{3A} \sin^3 \alpha_2)' - \frac{3\mu}{2} e^{2A+k} \cos \alpha_2 + \frac{3}{4} e^{3A+k-\Phi} \sin^2 \alpha_2 F_0 = 0,
\] (4.4)
where the last of these comes from imposing that $F_0$ is constant - the rest of the Bianchi identities then follow rather trivially. Clearly there are two cases, $F_0 = 0$ and $\cos \alpha_2 = 0$.

4.1 Case I: Compact Solutions from D8/O8’s Back-Reacted on $\text{AdS}_3 \times S^3 \times S^3 \times S^1$

For Case I we set
\[
\cos \alpha_2 = 0, \quad \sin \alpha_2 = s, \quad s = \pm 1,
\] (4.5)
then (4.3) implies also
\[
c_1 = c_2 = 0,
\] (4.6)
so there is no NS flux turned on. We can solve (4.4) by fixing
\[
e^{5A-\Phi} = q L^5
\] (4.7)
where $L$ and $q$ are constants and we fix diffeomorphism invariance with
\[
e^{A+k} = q L.
\] (4.8)
We can use (4.1) to define $e^{C_1}, e^{C_2}$ which leaves (4.5) to solve. This becomes simply

$$(L^4 e^{-4A})' = \nu F_0,$$

which is solved by

$$L^4 e^{-4A} = H_8, \quad H_8 = F_0 \nu r + c$$

(4.10)

for $c$ another constant - ie the warp factor of a D8 brane or O8 hole depending on the sign of $F_0$ and $\nu$. We then fix

$$s = \mu = \nu = \pm 1$$

(4.11)

and find the following general form for local solutions in massive IIA

$$
ds^2 = \frac{1}{\sqrt{H_8}} \left( L^2 ds^2(\text{AdS}_3) + \frac{L^2}{\cos^2 \alpha_1} ds^2(S^3_1) + \frac{L^2}{\sin^2 \alpha_1} ds^2(S^3_2) \right) + \sqrt{H_8} q^2 dr^2,

F_4 = 2q^2 H_8 \left( L^2 \text{Vol}(\text{AdS}_3) + \frac{L^2}{\cos^2 \alpha_1} \text{Vol}(S^3_1) + \frac{L^2}{\sin^2 \alpha_1} \text{Vol}(S^3_2) \right) \wedge dr,

e^{-\Phi} = q H_8^{\frac{5}{2}}.

(4.12)$$

Clearly when $F_0 = 0$ we recover the standard solution on $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ which preserves $\mathcal{N} = (4,4)$ supersymmetry - in this limit none of the physical fields depend on $\mu = \nu = \pm 1$ confirming the enhanced supersymmetry. The generic local solution is D8 branes or O8 planes or both \footnote{The near horizon geometry of a D8 brane is indistinguishable from the geometry near an O8 plane.} backreacted on this. As the warp factor now depends on $\nu$ supersymmetry is broken to $\mathcal{N} = (4,0)$ in the presence of the backreated D8/O8 system - which is by no means a surprise.

When $F_0 \neq 0$ the internal space of (4.12) is non compact, if we assume $F_0 > 0$ and $\nu = 1$, then the warp factor does bound the interval from below at $r = -\frac{c}{F_0}$ where the behaviour is consistent with a D8/O8 system wrapped on $\text{AdS}_3 \times S^3 \times S^3$, however the interval is not bounded from above and $r \to \infty$ is at infinite distance. Before giving up though, one should remember that this is only a local solution - which is to say that all coordinate patches of a global solution can be expressed in the form (4.12). One can try to make a compact solution by gluing a second mirrored copy of (4.12) onto the first in the spirit of [26]. At the point where the local patches connect there should be a D8 brane defect where $F_0$ jumps, but the metric and dilaton are continuous. The most simple way to arrange for this is to glue the patches together at $r = 0$ and have $F_0$ flip from positive to negative as one crosses $r = 0$ from below, ie one takes the warp factor to be

$$H_8 = c + |F_0|r, \quad r < 0, \quad H_8 = c - |F_0|r, \quad r > 0,$$

(4.13)
so that the metric and dilaton are continuous without the need to further tune constants, and only $F_0$ jumps. This does indeed bound $r$ to the interval $\mathcal{I}$ between two D8/O8 systems at $r = \pm \frac{c}{|F_0|}$ and one is now able to quantise the fluxes without issue. In units where $g_s = \alpha' = 1$ one requires that the following charges are integer valued

$$n_0 = 2\pi F_0, \quad N_2 = -\frac{1}{(2\pi)^5} \int_{S_3^1 \times S_3^2} \star F_4, \quad N_4^i = \frac{1}{(2\pi)^3} \int_{S_3^2} \int_{r \in \mathcal{I}} F_4. \quad (4.14)$$

This is not hard to achieve by tuning $L, q$ and $\tan \alpha$ and the curvature of the solution is under parametric control. A standard computation leads to a finite central charge of the form

$$c \sim N_2 \frac{N_4^1 N_4^2}{N_4^1 + N_4^2} \quad (4.16)$$

which is independent of $F_0$ and actually has the same $N$-dependence as the central charge of $\text{AdS}_3 \times S^3 \times S^3 \times S^1$. This is consistent with what one expects from a CFT with large superconformal algebra.

This all sounds rather positive, however to be sure this solution really exists, and preserves supersymmetry, we need to check the Bianchi identities at the D8 brane defeat are satisfied and that the D8 brane is calibrated [29–31]. The Bianchi identities in the presence of the defect require

$$df = \frac{N_8}{2\pi} \delta(r) e^{2\pi f_g} \wedge dr \quad (4.17)$$

where $f_g$ is a gauge field on the world volume of the D8 brane. We find ourselves in a far simpler scenario than [26], because there is no NS flux and the only RR flux that shifts across the defeat is $F_0$ as

$$\Delta F_0 = 2|F_0|. \quad (4.18)$$

Comparing this with the integrated form of $(4.17)$, we find that the Bianchi identity requires simply

$$N_8 = 4\pi |F_0| = 2|n_0|, \quad f_g = 0. \quad (4.19)$$

It is also not hard to confirm that the brane is supersymmetric - this is so whenever the DBI action of a given brane satisfies a so called calibration condition. Here the DBI action of the D8 should equal the integral of $e^{2A - \Phi} \text{Vol}(\text{AdS}_3) \wedge \Psi_6$ - a quick computation shows

$^7$The holographic central charge for an $\text{AdS}_3$ solution in 10 dimensions goes like

$$c \sim \int_{M_7} e^{A - 2\Phi} \text{Vol}(M_7). \quad (4.15)$$

See section 4.6 of [8] for a computation of the central charge for $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ and it’s non-Abelian T-dual
this to indeed be the case. Thus we have constructed a bone-fide $\mathcal{N} = (4, 0)$ solution in massive IIA.

The result of gluing the two local solutions together is essentially a global solution with an orientifold under which a circle parameterised by $r$ becomes a segment. At the two ends there are two O8-planes with different charges and tensions. One can interpret this as two O8$_-$’s with $k$ and $16-k$ D8’s, or when $k = 8$ as an O8$_-$ and an O8$_+$, the later being similar to what appears in the recently constructed classical dS solutions in [27].

It would be interesting to find the local solution of (4.12) as a near horizon limit of some intersecting brane set-up. This should be in some sense a trivial extension of the realisation of AdS$_3 \times S^3 \times S^3 \times S^1$ in terms of D2 and D4 branes - however to the author’s knowledge this first step is currently absent from the literature (see [2] for every near horizon realisation of AdS$_3 \times S^3 \times S^3 \times S^1$ except D2-D4), and finding it is beyond the scope here.

Finally let us stress that there is not any particular need to place the D8 brane defect at $r = 0$ and so there appears to be no restriction to gluing together an arbitrary number of local solutions of the form (4.12), with a D8 brane at each intersection in the vein of [28]. One could potentially construct infinitely many globally compact solutions in this fashion so it would be interesting to study this possibility in more detail.

4.2 Case II: A New Local $\mathcal{N} = (4, 4)$ Solution with O2 Plane

For Case II we set

$$F_0 = 0. \quad (4.20)$$

To avoid falling into a sub-case of the previous section one must demand $\cos \alpha_2 \neq 0$ which requires the same of $c_1, c_2$ without loss of generality. We can solve the first condition of (4.3) with

$$c_1 = c \sin^4 \alpha_1, \quad c_2 = -c \cos^4 \alpha_1 \quad (4.21)$$

and take the second condition as the definition of $e^A$. Using this, and by taking a linear combination of (4.4) and (4.5) such that $e^k$ is eliminated one finds $(\tan \alpha_2 e^{-2\Phi})' = 0$ which is solved by

$$\tan \alpha_2 e^{-2\Phi} = q^2 \quad (4.22)$$

where $q$ is a constant. At this point it is useful to use diffeomorphism invariance to fix $e^k$ in terms of another arbitrary function $f(r)$ such that

$$32\nu^3 e^{A+k} = c\mu \sin^3 \alpha_1 f' \quad (4.23)$$
the remaining conditions (4.4)-(4.5) both then reduce to \( f' = (\sec \alpha_2)' \) which one can solve without loss of generality with

\[
\cos \alpha_2 = \frac{1}{\sqrt{f}}. \tag{4.24}
\]

As the left hand side of this expression is bounded between 0 and 1, we must have that \( 1 < f < \infty \), a sensible choice is then

\[
f = \frac{1}{\cos^2 r}, \tag{4.25}
\]

so that we simply have \( \alpha_2 = r \). This leads to a completely determined local solution of the form

\[
ds^2 = L^2 \left[ \frac{1}{\cos r \sin r} ds^2(\text{AdS}_3) + \frac{\sin^3 r}{\cos^5 r} dr^2 + \frac{\sin r}{\cos r} \left( \frac{1}{\cos^2 \alpha} ds^2(S^3_1) + \frac{1}{\sin^2 \alpha} ds^2(S^3_2) \right) \right],
\]

\[
H = 2L^2 \left( \frac{\tan \alpha}{\cos^2 \alpha} \text{Vol}(S^3_1) - \frac{\cot \alpha}{\sin^2 \alpha} \text{Vol}(S^3_2) \right), \quad qe^\Phi = \sqrt{\tan r}, \tag{4.26}
\]

\[
F_4 = 2L^3 \left[ 4 \frac{\sin 4r}{\sin^2 2r} \text{Vol}(\text{AdS}_3) + q \frac{\tan r}{\cos^2 r} \left( \frac{1}{\cos^2 \alpha} \text{Vol}(S^3_1) + \frac{1}{\sin^2 \alpha} \text{Vol}(S^3_2) \right) \right] \wedge dr,
\]

where we have introduced

\[
L^2 = \frac{c}{2} \cos^3 \alpha \sin^3 \alpha, \tag{4.27}
\]

and fixed

\[
\sin \alpha_1 = \nu \sin \alpha, \quad \mu = \nu = \pm 1. \tag{4.28}
\]

to simplify expressions. Notice that none of the physical fields depend on \( \nu = \pm 1 \) - so this solution experiences an enhancement of supersymmetry to large \( \mathcal{N} = (4, 4) \).

The internal radial coordinate is bounded as \( 0 < r < \frac{\pi}{2} \), with the lower bound a singularity of the metric. The behaviour close to \( r = 0 \) is intriguing, indeed after redefining \( r = \sqrt{y} \) the behaviour is that of O2 planes at the base of a cone over \( S^3 \times S^3 \), which is rather novel. More disappointing is the behaviour close to \( r = \frac{\pi}{2} \) where the metric is actually regular but the dilaton is infinite, which does not appear to be physical behaviour. Worst still perhaps, is that \( r = \frac{\pi}{2} \) is at infinite proper distance, so the internal space is non-compact. One way to see this is with the central charge, this goes like

\[
c \sim \lim_{r \to \frac{\pi}{2}} \tan^4 r \tag{4.29}
\]

which is clearly divergent. Thus any putative CFT dual will have a continuous operator spectrum, a sign that it is sick.

One might be able to cure this issue as before by gluing two copies of (4.26) together. As \( F_0 = 0 \), one can no longer achieve this with D8 branes, however, since this sort of
gluing does work with D8 branes, T-duality and S-duality informs us that at the very least, it should be possible to glue solutions together with other types of branes when they are smeared over all but one of their co-dimensions - the options here are D2 and NS5 branes. As this may be a way of constructing new holographic duals to well defined CFTs with large $\mathcal{N} = (4,4)$ supersymmetry it would certainly be interesting to peruse this possibility in future.

Finally, since this solution has no Romans mass turned on, it can be lifted to M-theory. It is possible then that this lifted solution and the lift of $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ are special cases of a broader class of $\mathcal{N} = (4,4)$ solutions with $\text{AdS}_3 \times S^3 \times S^3$ foliated over a Riemann surface in M-theory. If this is true, then it should be possible to construct many more such solutions.

## 5 All Local Solution in Type IIB

In this section we find the local form of all solutions preserving at least an SO(4) R-symmetry on $S^3 \times S^3$. In section (5.1) we find a new compact solution containing D5’s and O5’s, while in section (5.2) we find a solution that back-reacts D5’s on $\text{AdS}_3 \times S^3 \times S^3 \times S^1$, but is non compact.

Once again we begin by plugging the bi linears of (3.6) into (3.2). It is immediate to establish the following zero form constraints

\[
c_1 \cos \alpha_2 = c_2 \cos \alpha_2 = \mu \sin \alpha_2 = 0
\]

which means that for $\text{AdS}_3$ solutions we must set

\[
c_1 = c_2 = \sin \alpha_2 = 0
\]

ans so all flux components but $f_3$ and $f_7$ are set to zero. We will thus parametrise the 3-form it in terms of two constants $c_3, c_4$ as

\[
f_3 = c_3 \text{Vol}(S^3_1) + c_4 \text{Vol}(S^3_2).
\]

Given this, after some massaging, it is possible to extract the following algebraic constraints

\[
\cos \alpha_2 = s,
\]

\[
\nu e^A (c_1 e^{c_1} - c_2) - s \mu e^{c_1 + c_2} = 0,
\]

\[
e^A (c_3 \sin \alpha_1 e^{3c_2} - c_4 \cos \alpha_1 e^{3c_1}) + 2 \mu e^{3c_1 + 3c_2 - \Phi} = 0,
\]
where $s = \pm 1$. Using these we can simply the differential constraints to

\begin{align}
(\alpha_1)' &= \cos \alpha_1 \sin \alpha_1 (e^{C_1-C_2})' = 0, \\
(e^{2A+2C_1+2C_2-\Phi})' &= 2sve^{2A+C_1+C_2+k-\Phi}(\cos \alpha_1 e^{C_2} + \sin \alpha_1 e^{C_1}), \\
(e^{3A+2C_1-C_2-\Phi} \cos \alpha_1)' &= 2e^{2A+C_1+C_2+k-\Phi}(sve^{A} + \mu \sin \alpha_1 e^{C_1}), \\
(e^{3A+2C_1-C_2-\Phi} \sin \alpha_1)' &= 2e^{2A+C_1+C_2+k-\Phi}(sve^{A} - \mu \cos \alpha_1 e^{C_2}), \\
(e^{3A+3C_2-\Phi} \cos \alpha_1)' &= 2\mu e^{2A+3C_2+k-\Phi} \sin \alpha_1 = c_3 e^{3A-3C_1+3C_2+k}, \\
(e^{3A+3C_1-\Phi} \sin \alpha_1)' &= 2\mu e^{2A+3C_1+k-\Phi} \cos \alpha_1 = c_4 e^{3A+3C_1-3C_2+k},
\end{align}

which are overdetermined, but this makes finding a solution easier.

It appears that there are 3 cases, $\cos \alpha_1 = 0$, $\sin \alpha_1 = 0$ and $(e^{C_1-C_2})' = 0$, however there is no physical difference between the first two of these as one is mapped to the other by relabelling the spheres - thus there are two physically distinct cases.

### 5.1 Case I: A New Compact Solution Bounded Between D5’s and O5’s

For case I we set

\[ \cos \alpha_1 = t = \pm 1 \]  \hspace{1cm} (5.13)

and take (5.5)-(5.6) to define the AdS warp factor and dilaton. Substituting this back into (5.7)-(5.12) one is left with just two independent conditions

\[ sve^{C_2+k} - t(e^{C_1+C_2})' = \nu c_3 e^k + c_4 ste^{3C_1}(e^{-2C_2})' = 0, \]  \hspace{1cm} (5.14)

by taking a linear combination of these one can eliminate $e^k$ and find in general that

\[ e^{2C_1} = \frac{c_3 e^{2C_2}}{c_4 + c_3 q e^{4C_2}}, \]  \hspace{1cm} (5.15)

for $q$ an integration constant. This leave a single condition to solve, which one can trivialise by fixing

\[ \nu e^k = \frac{2\sqrt{c_3 c_4 s f'(r)}}{(c_4 + c_3 q e^{4C_2})^{\frac{3}{4}}}, \]  \hspace{1cm} (5.16)

for $f$ an arbitrary, non constant, function, one then finds $(e^{C_2})' = f'$ which can be solved with ease. At this point it is helpful to fix

\[ s = t = \mu = \nu = \pm 1, \]  \hspace{1cm} (5.17)
and write out the warp factor of the two $S^3$'s explicitly

\[ e^{2c_1} = \frac{c_3 f^2}{c_4 + c_3 q f^4}, \quad e^{2c_2} = f^2. \]  

(5.18)

Clearly these expressions either blow up or shrink to zero at 3 points, $f = 0$, $f = \infty$ and $c_4 + c_3 q f^4 = 0$. One can show that the first two of these are indeed singularities of the metric, but the third is just a coordinate singularity - indeed one can remove it by fixing

\[ f = \frac{L}{\sqrt{\tan r}}, \quad q = \frac{c_4}{c_3 L^4}, \]  

(5.19)

The general local solution for this class then takes the form

\[
 ds^2 = L^2 \left[ \frac{\cos r}{\sin r} \left( ds^2(\text{AdS}_3) + ds^2(S^3_2) \right) + \frac{c_3 \sin r}{c_4 \cos r} \left( \sin^2 r dr^2 + \cos^2 r ds^2(S^3_1) \right) \right], \\
 F_3 = c_4 \left( \nu \text{Vol}(\text{AdS}_3) + \text{Vol}(S^3_2) \right) + c_3 \text{Vol}(S^3_1), \quad e^{-\Phi} = \frac{c_4}{2L^2} \tan r, 
\]

(5.20)

where the Freund-Rubin term in $F_3$ clearly depends on $\nu$ so only $\mathcal{N} = (4, 0)$ supersymmetry is preserved. The internal radius is bounded as $0 < r < \frac{\pi}{2}$ and this time the local metric is already compact without the need to glue patches together. At the end points of the interval there are singularities, however these have an obvious physical origin. It should not be hard to see that close to $r = 0$ the metric becomes that of O5 planes wrapped on $\text{AdS}_3 \times S^3_2$, while at $r = \frac{\pi}{2}$ it behaves as D5’s wrapped on $\text{AdS}_3$ and either of the two 3-spheres.

Flux quantisation requires that the following charges are quantised

\[ N_5^i = \frac{1}{(2\pi)^2} \int_{S^3_i} F_3, \quad N_1 = -\frac{1}{(2\pi)^6} \int_{\mathbb{R} \times S^3_1 \times S^3_2} *F_3 \]  

(5.21)

this can be simply achieved by fixing

\[ N_5^i = 4c_{i+2}, \quad N_1 = \frac{c_3^2 L^4}{c_4 \pi^2}. \]  

(5.22)

and the radius about the singularities for which the supergravity approximation does not hold can be made arbitrarily small by making $L$ large. We once more compute the central charge and find that it is finite with the following

\[ c \sim N_1 N_5^2. \]  

(5.23)

This behaviour is markedly different from that of $\text{AdS}_3 \times S^3 \times S^3 \times S^3$, which is rather interesting, but for which the author has no immediate explanation. This solution is a well defined AdS dual to an as yet to be determined SCFT with large $\mathcal{N} = (4, 0)$ superconformal symmetry so is well deserving of further detailed study - but this is beyond the scope here.
5.2 Case II: D5’s Back-Reacted on AdS$_3 \times S^3 \times S^3 \times S^1$

In case II we assume

$$0 < \sin \alpha_1 < 1, \quad s = \mu = \nu = \pm 1. \quad (5.24)$$

Due to (5.7) this means that the two 3-sphere warp factors can only differ by a constant, thus we introduce a new function, $H$ and constants $b_1, b_2$ such that

$$e^{C_i} = b_i H. \quad (5.25)$$

We again use (5.5)-(5.6) as definitions for $e^{A}, e^{\Phi}$ and substitute for these quantities in (5.7)-(5.12) - they once more reduce to just two conditions that may be easily solved with

$$b_1 = b\sqrt{c_3}, \quad b_2 = b\sqrt{c_4}, \quad \nu e^{C+k} = bq, \quad H = c + \nu \lambda_1 r, \quad (5.26)$$

where we introduce

$$\lambda_1 = \frac{q \cos \alpha_1}{\sqrt{c_3}} - \frac{q \sin \alpha_1}{\sqrt{c_4}}, \quad \lambda_2 = \frac{\cos \alpha_1}{\sqrt{c_4}} + \frac{\sin \alpha_1}{\sqrt{c_3}}, \quad (5.27)$$

to ease notation and $b, c$ are constants. The general local form of solutions can then be written as

$$ds^2 = L^2 H \left( ds^2(\text{AdS}_3) + c_3 \lambda_2 ds^2(S^3_1) + c_4 \lambda_2 ds^2(S^3_2) \right) + \frac{L^2 q^2 \lambda_2^2}{H} dr^2,$$

$$F_3 = \frac{1}{\lambda_2^2} \text{Vol}(\text{AdS}_3) + c_3 \text{Vol}(S^3_1) + c_4 \text{Vol}(S^3_2), \quad e^{\Phi} = 2L^2 \lambda_2^2 h, \quad b = L \lambda_2 \quad (5.28)$$

The warp factor $H$ depends on $\nu$ so this solution generically experience no enhancement beyond $N = (4, 0)$. Similar to section 4.1 however, when one sets $\lambda_1 = 0$ $\nu$ drops out of all expressions and supersymmetry is enhanced to $N = (4, 4)$ - this is because the solution becomes $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ in this limit. The generic solution has D5 branes backreacted on this. The attentive reader will not that $H$ is not the warp factor of a D5 brane, however, if one assumes $\nu = 1$, then the interval is bounded from below at

$$r = -\frac{c}{\lambda_1}, \quad (5.29)$$

where the near horizon geometry of a D5 brane wrapped on either $S^3$ is recovered. The interval is not however bounded from above and $r = \infty$ is at infinite proper distance, so the metric is non compact.

On might wonder about the possibility of making the solution compact by glueing two copies of (5.28) together with D5 branes smeared on $S^3$ - the issues are essentially the same as for Case II in IIA.
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