Adaptive weighting for estimation of the mean of the merged measurement for multi-target bearing tracking

Quanrui Li,1,2,3,4 Longhao Qiu,1,2,3,4 Bin Qi,1,2,3,4,* and Guolong Liang1,2,3,4

1Acoustic Science and Technology Laboratory, Harbin Engineering University, Harbin 150001, China
2College of Underwater Acoustic Engineering, Harbin Engineering University, Harbin 150001, China
3Key Laboratory of Marine Information Acquisition and Security, Harbin Engineering University, Harbin 150001, China
4Qingdao Haina Underwater Information Technology Co., Ltd, Qingdao 266500, China
*E-mail: qbin@hrbeu.edu.cn

In multi-target tracking, sensor with finite resolution generates merged measurements, which means that a group of targets might produce only one measurement, and such phenomenon could lead to degraded tracking performance if it is not considered. The generalised labelled multi-Bernoulli filter for merged measurement (GLMB-M) provides a promising solution for such problem. However, the merged measurement likelihood it used was modelled as a Gaussian density with its mean being the uniform weighted mean of the measurements generated by the merging targets. Such uniform weighting strategy might fail if the merged measurements deviate severely from the uniform weighted position due to significant difference of the strengths of the merging targets. To avoid such deviation, an adaptive weighting strategy for estimation of the mean of the merged bearing measurement is proposed, which uses the bearing measurement error to obtain the adaptive weighting factors. Simulation results show that the proposed adaptive weighting strategy outperforms the uniform weighting strategy when the strengths of the targets vary greatly, although both strategies provide similar performance when the strengths of the target are equal.

Introduction: Finite sensor resolution is an important issue for practical multi-target tracking (MTT) [1]. Sensors such as radar and sonar with finite resolution generate merged measurements if the targets are in close proximity in the measurement space [2]. It means that a group of targets might produce only one measurement, and thus results in degraded MTT performance if measurement merging is not properly modelled. A number of finite sensor resolution MTT algorithms exist. Broadly speaking, these algorithms can be divided into two categories according to the number of merging targets one could handle. Earlier algorithms generally handle the merging of only two targets, such as the joint probabilistic data association for merged measurement (JP DAM) [2] and multiple hypothesis tracking (MHT) for unresolved measurement in [3]. More recent algorithms generally handle the merging of an arbitrary number of targets, such as the JP DAM for multiple targets in [4], and the linear multi-target integrated splitting filter for finite resolution tracking (LMTISfr) [5], and the generalised labelled multi-Bernoulli filter for merged measurement (GLMB-M) [6].

The GLMB-M filter is a full-featured Bayesian MTT algorithm that intrinsically handles the initialization, maintenance and termination of target tracks, which differs from algorithms [2–4] since themselves alone handle the track maintenance only. However, the merged measurement likelihood GLMB-M filter used is modelled as a Gaussian density with its mean being the arithmetic mean of the measurements generated by the merging targets. Such uniform weighting strategy is also adopted by algorithms [3, 4], and it would fail if the merged measurements deviate severely from the uniform weighted mean due to significant difference of the strengths of the merging targets.

To avoid such deviation, we present in this letter an adaptive weighting strategy for estimation of the mean of the merged bearing measurement. In the following, we show first the merged measurement likelihood with uniform weighted mean, and then illustrate the problem of such weighting strategy under certain circumstance, and then we present the proposed adaptive weighting solution, and simulation results and conclusion are presented at the end.

Merged measurement likelihood with uniform weighted mean: According to [6], if at time step k, a group of N targets whose states are \( x_{k,n} \) generate only one merged measurement \( z_k \), then the merged measurement likelihood is modelled as a Gaussian density with mean \( \bar{z}_k \) and covariance matrix \( R_k \), which means that the merged measurement likelihood,

\[
g(z_k | x_{k,n}^N) = \mathcal{N}(z_k; \bar{z}_k, R_k)
\]

where the mean

\[
\bar{z}_k = \frac{1}{N} \sum_{n=1}^{N} z_{k,n}
\]

in which \( N \) represents the number of the merging targets, and \( z_{k,n} \) is the measurement generated by target state \( x_{k,n} \) separately.

Problem of uniform weighting: The uniform weighting strategy in Equation (2) is appropriate when the strengths of the merging targets demonstrate no significant difference. Nevertheless, such uniform weighting strategy is inappropriate if the strengths of the merging targets are significantly different. We illustrate this via an example. Figure 1 shows the noisy bearing measurements of two targets, where the measurements are direction of arrival (DOA) estimates of the two targets outputted by the MUSIC [7] algorithm. Figure 1a shows the scenario where the strengths of the two targets are equal, whereas Figure 1b shows the scenario where the strength of one target is 6 dB stronger than other one.

Figure 1 clearly shows that due to close proximity of the bearings of the two targets, measurement merging occurs, where two targets produce only one bearing measurement at a given time step (see the green dashed boxes in Figure 1). It also reveals that the mean of the merged bearing measurement depends heavily on the relative strengths of the merging targets. For the equal strengths scenario shown in Figure 1a, the means of the noisy merged bearing measurements are close to the uniform weighted mean (middle line in Figure 1) of the true bearings of the two targets. For the unequal strengths scenario shown in Figure 1b, the merged bearing measurements are close to the true bearings of the 6 dB stronger target, which means that the merged bearing measurements deviate from the uniform weighted mean, especially when merging just occur or is about to come to an end. Such deviation could lead to problems such as premature track termination or track fragmentation.

To avoid such deviation, we propose next an adaptive weighting strategy for the estimation of the mean of the merged bearing measurement, which utilizes the variances of the bearing measurement errors of the merging targets to compute the adaptive weighting factors.

Adaptive weighting using variance of the bearing measurement error: Let target state \( x_{k,n} \) be augmented by \( \xi_{k,n} \), which represents the error variance of the bearing measurement formulating the track of target \( n \), then the likelihood of the merged bearing measurement given augmented states \( \{(x_{k,n}, \xi_{k,n})^N_{n=1}\} \) of the merging targets is modelled as:

\[
g(z_k | (x_{k,n}, \xi_{k,n})^N_{n=1}) = \mathcal{N}(z_k; \bar{z}_k, R_k)
\]

Fig. 1 Illustration of the bearing measurements of two crossing targets with equal and unequal strengths (a) Equal strengths scenario (merged measurements in the middle). (b) Unequal strengths scenario (merged measurements closer to the 6 dB stronger target track)
where the mean,

$$z_n = \sum_{a=1}^{N} \omega_{a,n} x_{a,n}$$  \hspace{1cm} (4)

in which $z_{a,n}$ denotes the bearing measurement generated by state $x_{a,n}$ separately and the adaptive weighting factor,

$$\omega_{a,n} = \frac{1}{\sum_{a=1}^{N} \frac{R_{1-a,n}}{R_{1-a,n}}}$$  \hspace{1cm} (5)

where $R_{1-1,n}$ is the error variance part of the updated state $(x_{1-1,n}, R_{1-1,n})$ of target $n$ at time step $k-1$, and $R_{k}$ in Equation (3) is set to the minimum value among variances $\{R_{1-1,n}\}_{n=1}^{N}$, meaning that,

$$R_{k} = \min(R_{1-1,n})$$  \hspace{1cm} (6)

Note that in Equations (5) and (6), we use variances $\{R_{1-1,n}\}_{n=1}^{N}$ rather than $\{R_{1,n}\}_{n=1}^{N}$ because $\{R_{1,n}\}_{n=1}^{N}$ are unavailable at time step $k$. Since we assume no prior knowledge of the bearing measurement errors, thus a measurement noise adaptive filter such as the adaptive Kalman filter [8] is needed. Therefore, variances $\{R_{1,n}\}_{n=1}^{N}$ are unavailable until the Bayesian update process is completed at time step $k$, because the variance itself is part of the augmented state. In this letter [8], is chosen as the state estimator. Also, according to Equation (3), when merging occurs, only the kinematic part $x_{a,n}$ of the augmented state $(x_{a,n}, R_{a,n})$ is updated, and variance $R_{a,n}$ is set to be equal to $R_{1-1,n}$.

The adaptive weighting strategy Equation (5) uses the fact that the variance of the bearing measurement error is inversely proportional to signal to noise ratio (SNR) according to the Cramer–Rao lower bound of the DOA estimate [9]. Thus if the environmental noise is stationary, then the stronger the target strength is, the higher the SNR becomes, and the smaller the variance of the bearing measurement error turns into and vice versa. Also, targets with close strengths should lead to similar variances of the bearing measurement errors. The modelling of the error variance in Equation (6) reflects our ignorance of the impact of the weaker target on the error variance of the merged bearing measurement and our assumption that the error variances of the merged bearing measurements are unchanged throughout the whole merging process.

From Figure 1a, we observe that the error variances (roughly measured by the distance between the measurement and the true bearing) of the bearing measurements of the two targets prior to merging are roughly the same, thus the adaptive weighting factors given by Equation (5) should be around 1/2; and as Figure 1b shows that the stronger target results into a smaller variance, and thus a larger weighting factor for the stronger target is obtained via Equation (5), hence the mean of the merged bearing measurement given by Equation (4) is closer the true bearing of the stronger target. This implies that compared to the uniform weighting strategy Equation (2), the proposed adaptive weighting strategy Equation (5) could provide a better estimate of the mean of the merged measurement when significant difference of target strengths exist, despite similar estimates are provided when the strengths of the merging targets are equal. Such difference will also be verified by the simulation results provided next.

In the following, performance comparisons of the GLMB-M [6] filter with uniform weighting (GLMB-M-U) and adaptive weighting (GLMB-M-A) strategies are reported in terms of the average cardinality estimate and optimal sub-pattern assignment (OSPA) metric [10].

Simulation results: Consider a multi-target tracking scenario where at most two targets exist at the same time. The sensor is assumed to be a standard 10-element uniformly spaced passive linear array, which collects the bearings of the targets that emit narrow-band signals. We also assume that the bearing measurements are DOA estimates of the targets that are outputs of the MUSIC [7] algorithm and estimates are based on 100 snapshots. The true bearings of the two targets are exactly the same as shown in Figure 1.

The simulation considers two scenarios. In equal target strengths scenario, the strengths of the two targets are set to $-6$ dB, and in unequal target strengths scenario, the strength of the stronger target is $-6$ dB, while the strength of the weaker target is $-12$ dB. The standard deviation of the bearing measurement error is set to $\sigma_{x} = 1.5^\circ$. The probabilities of target survival and detection are set to $p_{Sx} = 0.99$ and $p_{Dx} = 0.9$, respectively. The clutter intensity is set to $c(x_{k}) = 0.01$. The number of kept components of both filters is capped to 100. The order and cutoff value of the OSPA metric is set to $p = 1$ and $c = 10$, respectively.

Figure 2 shows the tracking results of a sample run of both filters for the equal strengths scenario. We observe from Figure 2 that both the GLMB-M-U and GLMB-M-A filters track the targets well, although the state estimates of both filters become less accurate at the merging moments due to the joint update of multiple target states using only one measurement. The average cardinality estimate and OSPA distance in Figure 3 also show that the average performances of both filters are similar and both can provide nearly unbiased cardinality estimates at the merging moments under the equal target strengths scenario.

Figure 4 shows the tracking results of a sample run of both filters for the unequal strengths scenario. From Figure 4a, we observe that the GLMB-M-U filter suffers from severe underestimated target number at the merging moments and track fragmentation, which are caused by the biased estimate of the mean of the merged measurement due to its uniform weighting strategy. Figure 4b shows the GLMB-M-A filter track both targets well and no underestimates of target number are observed when merging occurs. It is because the adaptive mean estimation strategy of the GLMB-M-A filter makes the merging event of the two targets dominating the components of the filter, which leads to the continuation of both target tracks rather than the termination of the weaker track, as is done in the GLMB-M-U filter. Such results are further verified by the average cardinality estimate and OSPA distance in Figure 5.

Conclusion: In finite resolution multi-target bearing tracking, if the strengths of the merging targets demonstrate significant difference,
then applying the strategy where the mean of the merged bearing measurement is estimated using uniform weighting to the GLMB-M filter could lead to severe underestimation of the target number. An adaptive weighting strategy that uses the error variances of the bearing measurements of the merging targets to adaptively compute such mean is proposed. Simulation results show that compared to uniform weighting, the proposed adaptive weighting strategy leads to considerable performance improvement when the strengths of the merging targets demonstrate significant difference, although both strategies handle the merging of targets with equal strengths effectively.

Acknowledgments: This work was supported by the National Key R&D Plan of China (2017YFC0306900), National Defence Basic Scientific Project (JCKY2019604B001), Equipment Pre-research Program (61406190105), Equipment Pre-research of Acoustic Science and Technology Laboratory (6142109180207), and Stable Supporting Fund of Acoustic Science and Technology Laboratory (JCKYS2019604SSJS011).

References

1. Daum, F., Fitzgerald, R.: Importance of resolution in multiple-target tracking. In: SPIE's International Symposium on Optical Engineering and Photonics in Aerospace Sensing, Orlando, FL (1994)
2. Kuo-Chu, C., Bar-Shalom, Y.: Joint probabilistic data association for multitarget tracking with possibly unresolved measurements and maneuvers. IEEE Trans. Autom. Control 29(7), 585–594 (1984). https://doi.org/10.1109/TAC.1984.1103597
3. Koch, W., Keuk, G.V.: Multiple hypothesis track maintenance with possibly unresolved measurements. IEEE Trans Aero. Electron. Syst. 33(3), 883–892 (1997). https://doi.org/10.1109/7.599263
4. Svensson, D., Ulmke, M., Hammarstrand, L.: Multitarget sensor resolution model and joint probabilistic data association. IEEE Trans. Aero. Electron. Syst. 48(4), 3418–3434 (2012). https://doi.org/10.1109/TAES.2012.6524722
5. Musicki, D., Song, T.L., Lee, H.H.: Linear multitarget finite resolution tracking in clutter. IEEE Trans. Aero. Electron. Syst. 50(3), 1798–1812 (2014). https://doi.org/10.1109/TAES.2014.120257
6. Beard, M., Vo, B.T., Vo, B.N.: Bayesian multi-target tracking with merged measurements using labelled random finite sets. IEEE Trans. Signal Process. 63(6), 1433–1447 (2015). https://doi.org/10.1109/TSP.2015.2393843
7. Schmidt, R.: Multiple emitter location and signal parameter estimation. IEEE Trans. Antennas Propag. 34(3), 276–280 (1986). https://doi.org/10.1109/TAP.1986.1143830
8. Sarkka, S., Nummenmaa, A.: Recursive noise adaptive kalman filtering by variational bayesian approximations. IEEE Trans. Autom. Control 54(3), 596–600 (2009). https://doi.org/10.1109/TAC.2008.2008348
9. Friedlander, B.: Wireless direction-finding fundamentals. In: T.E. Tuncer, B. Friedlander (eds.). Classical and Modern Direction-of-Arrival Estimation. Academic Press, Boston, USA (2009)
10. Schuhmacher, D., Vo, B.T., Vo, B.N.: A consistent metric for performance evaluation of multi-object filters. IEEE Trans. Signal Process. 56(8), 3447–3457 (2008). https://doi.org/10.1109/TSP.2008.920469