Identification of limiting deep drawing ratio using the energy criterion of fracture

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Abstract. The paper describes a method which has been developed for obtaining the limiting deep drawing ratio of titanium alloys and determines the moment of failure of the blank. This method is based on not only the use of Forming Limit Diagram (FLD) in predicting the failure of the blank but also the using method of variable parameters of elasticity in determining the stress-strain state in the deep drawing process.

1. Introduction

One of the most important tasks in the development of the technological process of sheet metal forming is deep drawing. It is the prediction of the moment of failure of the workpiece, that is, the determination of the limiting drawing ratio or the power drawing [1]. The modern development of computing technology is used for engineering calculations in the design of sheet metal stamping processes, and high accuracy to calculate the parameters of the stress-strain state of the blank. This process obtains to predict the failure of the workpiece by using the Forming Limit Diagram (FLD) [2,3,4,5,6,7,8]. This also allows FLD-diagrams to be widely used in determining the limiting capabilities of sheet metal forming processes.

In this paper, we offer an analytical method for constructing FLD diagrams by using strain energy criterion [9].

According to determine the stress-strain state, the method for variable parameters of elasticity accounts not only the thickness change in the deformation process but also the hardening of the material, which for titanium alloys is well described by the power function.

2. Model to the fracture of sheet blanks based on an energy criterion

In the construction of a model for the fracture of without defect isotropic sheet blanks, the energy criterion is widely used as the criterion of limiting deformation in the theoretical prediction. To determine the forming of sheet metal, Swift proposed a diffuse neck criterion [9], which described as diffuse neck starts when the load reaches a maximum along both principal directions. According to the transforming of the expressions obtained results in work [10], we can define an equation that allows us to construct the FLD diagram using the Swift criterion:

\[
4(e_1 - n)(e_2 + 2e_1)^3 - 3(e_1 - 2n)(e_1 + 2e_2)(e_2 + 2e_1)^2 - 3(e_1 + 2n)(e_1 + 2e_2)^2(e_2 + 2e_1) + 2(2e_1 + n)(e_1 + 2e_2)^3 = 0,
\]

(1)
where \( e_1 \), \( e_2 \) are the major and minor logarithmic strains acting in the plane of the sheet blank; \( n \) is the exponent of the power approximation by the hardening curve of the third kind.
\[
\sigma_2 = A e_1^n.
\]

Assuming the experimental and theoretical works, after the occurrence of a diffuse necking, the plastic deformation of the sample continues, and local necking can later appear, which differs from the diffuse not only in size but also in that its appearance and development take place under conditions of plane strain with intensive thinning sample thickness [11].

Hill [12] proposed that the criterion of forming limit should be the moment of local necking at which the increment of the total effort is zero. It is assumed that the local necking will be formed at an angle \( \psi \) to the direction of the major principal stress. The orientation angle of the neck with respect of loading axes can be expressed as follows:
\[
\psi = \tan^{-1}\left(\sqrt{\alpha}\right) \tag{2}
\]

Where \( \alpha = e_2/e_1 \) is the ratio of main strains arising under conditions of simple (monotonic) loading. Due to the square root in equation (2), this equation has a physical meaning only with a negative value of \( e_2 \).

According to the transforming of the expressions obtained in work [12], we define an equation that allows us to construct the FLD diagram using the Hill criterion:
\[
e_1 + e_2 - n = 0. \tag{3}
\]

In practice, a combination of two criteria is used to construct FLD diagrams for energy criteria. The Hill criterion is applied for a negative value of \( e_2 \) by the equation (3) and the Swift criterion for a positive value of \( e_2 \) by the equation (1).

3. Identification of stress-strain state in the deep drawing process

When solving the problem of stress-strain state in the deep drawing process, we can use the method for variable parameters of elasticity [13, 14].

According to the physical relations of the deformation theory of plasticity and the compatibility equation for true strains, we can be written in Euler coordinates [15] and obtain the normal integral equation in stresses, which determines the stress state on the flange with an axisymmetric hood.
\[
\sigma_\varphi = \frac{1}{2} \left( \frac{2}{3} E_{sec} \ln \left[ \frac{1}{\sqrt{\rho^2 + R^2}} \right] + \frac{3}{2} \sqrt{\rho} \exp \left( -\frac{3}{4} E_{sec} \sigma_\rho \right) d\rho \right) + \frac{R \sqrt{\varphi}}{\rho \sqrt{\rho}} \exp \left[ -\frac{3}{2 E_{sec} \rho} \left( \sigma_\varphi - \frac{1}{2 \sigma_\rho} \right) \right] \tag{4}
\]

Where, the index \( R \) indicates that the parameter value is considered at \( \rho = R \), which corresponds to the outer edge of the flange in the deformation process.

Therefore, the drawing process is recommended to be solved in stresses. The solution is carried out by the method of successive approximations using a recurrent scheme by equation (4):
\[
\sigma_\varphi^{(i+1)} = \int_{R}^{\rho} \frac{\left( \sigma_\varphi^{(i)} - \sigma_\rho^{(i)} \right) S(\rho)^{(i)}}{\rho} d\rho + S(R) \sigma_{\rho R}^{(i)} / S(\rho)^{(i)}.
\]

Where, the quantities with the sign of \((i)\) and \((i+1)\) denote respectively their values in the \( i^{th} \) and \((i+1)^{th}\) approximations.
The calculation results do not depend on the option of the initial approximation, but these results take approximately the values of the stresses obtained by the elastic solution.

\[ \sigma^{(0)}_{\rho} = -\ln(R / R_0) \times \frac{E}{2} \left( \frac{R^2}{\rho^2} - 1 \right); \]

\[ \sigma^{(0)}_{\varphi} = \ln(R / R_0) \times \frac{E}{2} \left( \frac{R^2}{\rho^2} + 1 \right). \]

By the conditions of deep drawing, we can express friction on the flange as follows [16, 17]:

\[ \mu_{mp} \left[ R_0^2 - \left( r_0 + r_m \right)^2 \right] q \]

\[ R \cdot S(R) \]

\[ \mu_{mp} \]

\[ \mathrm{where}, \quad \mu_{mp} \quad \text{is the friction coefficient on the flange}; \] \( q \) is the pressure.

After identification of the stresses, we can determine the strain by using the physical relationship of the deformation theory of plasticity.

According to the method for variable parameters of elasticity, after each approximation we can specify the value of the secant modulus as follows:

\[ E_{sec}^{(i+1)} = \frac{A(e^{(i)}_{e})^n}{e^{(i)}_{e}}; \]

Where, \( e^{(i)}_{e} = \frac{2}{\sqrt{3}} \sqrt{\left( \sigma^{(i)}_{\rho} - e^{(i)}_{\rho} \right)^2 + \left( \sigma^{(i)}_{\varphi} - e^{(i)}_{\varphi} \right)^2 + \left( \sigma^{(i)}_{z} - e^{(i)}_{z} \right)^2}. \]

To control the convergence of the process, the values of the secant modules are compared as follows:

\[ E_{sec}^{(i+1)} - E_{sec}^{(i)} \leq \Delta. \]

After reaching the given accuracy, the value of the stresses at the edge of the die can be taken into account the bending, friction, and flatness according to the following formulas [15, 17]:

a) by \( r_0 < \rho \leq (r_0 + r_m) \)

\[ \sigma^{'\rho} = \left[ \sigma_{\rho} + \sigma_{S1} \frac{S_1}{4r_m + 2S_1} \right] \exp \left( \mu_{mp2} \alpha \right), \]

b) by \( \rho = r_0, \sigma_{\rho r_0} = \sigma_0 \) and \( \alpha = \frac{\pi}{2} \) are the radial stresses (tension in the wall Cup) that reach the highest value,

\[ \sigma_{\rho max} = \left[ \sigma + \sigma_{S1} \frac{S_1}{4r_m + 2S_1} \right] \exp \left( \mu_{mp2} \frac{\pi}{2} \right) + \sigma_0 \frac{S_2}{4r_m + 2S_2}. \]

The tangential stresses in the area of the die edge change from some current value to zero, that is the workpiece leaves from the edge of the die, the stress state is close to linear tension. If we assume that the change occurs linearly depending on the angle, we obtain \( r_0 \leq \rho \leq (r_0 + r_m) \).
\[ \sigma_\varphi = \sigma_\varphi \left[ 1 - \frac{2}{\pi} \arcsin \left( \frac{r_0 + r_m - \rho}{r_m} \right) \right]. \]

Where, \( \sigma_\varphi \) is tangential stress by using equation (4).

4. The identification for technological capabilities in the deep drawing process

The curve of strain hardening of a titanium alloy is approximated by a power function, in the form of the J. H. Hollomon’s equation [18]:

\[ \sigma_i = \sigma_{sw} \cdot (e_i / e_u) \eta , \]

where, \( \sigma_{sw} = \sigma_B \cdot \exp \left( e_u \right) \), and \( e_u \) is true (logarithmic) strain at the moment of the necking and \( \sigma_B \) is ultimate strength of material.

The technological process of deep drawing is set by the following parameters: \( r_0 \) is the radius of the die, \( R_0 \) is the radius of the workpiece, \( r_m \) is the edge radius of the die, \( q \) is the specific pressure on the flange, \( \mu_{npl} \) is the coefficient of friction on the flange and \( \mu_{np2} \) is the coefficient of friction on the edge of the die.

Initially, the radius of the workpiece \( R_0 \) is set that the deep drawing process is guaranteed to pass. For example, \( R_0 = 1.25 r_0 \). The paper represents the most complete view of the deep drawing process, which gives the position of the outer edge \( R \). Therefore, another step is given to change the position of the outer edge \( R \). According to the position of the outer edge that is taken by the method for variable parameters of elasticity, we obtain the stress-strain state on the flange of the blank.

Then, we can assume that \( e_\rho = e_1 \) and \( e_\varphi = e_2 \) which determine the position of the points characterizing the deformed state on the flange in the coordinates \( e_1, e_2 \). If all these points lie below Forming Limit Curve (FLC) by equations (1) and (3), then the process continues changing the position of the outer edge \( R \). The calculation in accordance with the method of successive loading [19] is repeated until \( R \) becomes equal, i.e. the deep drawing process will end.

In a case any stage of loading one of the points characterizing the strain state on the flange in the coordinates \( e_1, e_2 \) will be higher than the FLC diagram, then the calculation is stopped and determined the limiting drawing ratio \( m = r_0 / R_0 \) or the power drawing ratio \( k = R_0 / r_0 \).

The results of calculations for the material BT1-1 are presented in the figure. 1. Where, curves 1, 2 and 3 describe the deformed state of the blank at the moment preceding of fracture. The solid thick line corresponds to the FLC constructed by using the equations (1) and (3).
5. The conclusions
The results of this paper represent the methods of identification for the technological capabilities of titanium alloys in the deep drawing process, which allows predicting the friction of the blank by the Forming Limit Diagram (FLD).

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