Wave dynamics of a six-dimensional black hole localized on a tensional three-brane

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Abstract

We study the quasinormal modes and the late-time tail behavior of scalar perturbation in the background of a black hole localized on a tensional three-brane in a world with two large extra dimensions. We find that finite brane tension modifies the standard results in the wave dynamics for the case of a black hole on a brane with completely negligible tension. We argue that the wave dynamics contains the imprint of the extra dimensions.

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The study of wave dynamics outside black holes has been an intriguing subject for the last few decades (for a review, see [1, 2, 3]). We have got the schematic picture regarding the dynamics of waves outside a spherical collapsing object. A static observer outside a black hole can indicate three successive stages of the wave evolution. First the exact shape of the wave front depends on the initial pulse. This stage is followed by a quasinormal ringing, which describes the damped oscillations under perturbations in the surrounding geometry of a black hole with frequencies and damping times of the oscillations entirely fixed by the black hole parameters. The quasinormal modes (QNM) is believed as a unique fingerprint to directly identify the black hole existence. Detection of these QNM is expected to be realized through gravitational wave observation in the near future [1, 2]. Despite the potential astrophysical interest, QNM could also serve as a testing ground of fundamental physics. It is widely believed that the study of QNM can help us get deeper understandings of the AdS/CFT [3, 4], dS/CFT [5] correspondences, loop quantum gravity [6] and also the phase transition of black holes [7] etc. At late times, quasinormal oscillations are swamped by the relaxation process. This relaxation is the requirement of the black hole no hair theorem [8].

It is of interest to extend the study of wave dynamics on usual black holes in general relativity to black holes obtained in string theory, since this could help us to get the signature of the string. Attempt on this respect has been carried out in [9]. String theory predicts quantum corrections to classical General Relativity, and the Gauss-Bonnet terms is the first and dominating correction among the others. The investigation on wave evolutions in the Gauss-Bonnet black holes has been done in [10]. String theory predicts the existence of the extra dimension. Recent developments on higher dimensional gravity resulted in a number of interesting theoretical ideas such as the brane world concept. The essence of this string inspired model is that Standard Model fields are confined to a three dimensional hypersurface, the brane, while gravity propagates in the full spacetime, the bulk. It has been argued that the extra dimension could imprint in the wave dynamics in the branworld black holes [11, 12]. Besides the wave dynamics, other arguments for detecting extra dimensions in the branworld black hole Hawking radiation have also been proposed [11, 15, 16, 17].

In general it is very hard to obtain exact solutions of higher-dimensional Einstein’s equations describing black holes on brane with tension. Recently, a metric describing a black hole located on a three-brane with finite tension, embedded in locally flat six-dimensional spacetime was constructed in [13]. In spherically symmetric six-dimensional Schwarzschild gauge, it is described as

\[
\begin{align*}
    ds^2 &= -(1 - \frac{M_{BH}}{4\pi^2 M_{pl}^4 r}) dt^2 + (1 - \frac{M_{BH}}{4\pi^2 M_{pl}^4 r})^{-1} dr^2 + r^2 \left[ d\theta^2 + \sin^2 \theta (d\phi^2 + \sin^2 \phi (d\chi^2 + b^2 \sin^2 \chi d\psi^2)) \right],
\end{align*}
\]
where \( b \) measures the deficit angle about the axis parallel with the three-brane in the angular direction \( \psi \) and is related to the brane tension \( \lambda \) by
\[
b = 1 - \frac{\lambda}{2\pi M_s^4}.
\]
\( M_{BH} \) is the ADM mass of the usual six-dimensional Schwarzschild black hole and \( M_s \) is the fundamental mass scale of six-dimensional gravity. The position of the black hole horizon is located at
\[
r_h = \left( \frac{M_{BH}}{4\pi^2 M_s^4 b} \right)^{1/3}.
\]
(2)

For convenience, we can define \( M \equiv \frac{M_{BH}}{4\pi^2 M_s^4} \) and then the Hawking temperature of the black hole can be written as
\[
T_H = \frac{3}{4\pi} \left( \frac{b}{M} \right)^{1/3}.
\]
Obviously, the presence of the brane tension changes the relation between the black hole mass and horizon radius and leads to the existence of a deficit angle in the angular direction \( \psi \) which means that the geometry of the metric asymptotes a conical bulk space at large distance \( (r \gg r_H) \). When the brane tension \( \lambda = 0 \) (i.e. \( b = 1 \)), this metric reduces to the six-dimensional Schwarzschild metric.

Finite brane tension modifies the standard results if we compare with the black hole on a brane with negligible tension. This has been observed in the semiclassical description of the black hole decay process[14]. It has been found that the brane tension alters significantly the power output of small black holes located on the brane and the power emitted in the bulk diminishes as the tension increases. This distinct signature is argued to be observed in high energy collisions. The question we want to ask is that whether the wave dynamics of black holes can show signatures of information about brane tension, whether this could complement collider searches for these information through Hawking radiation. This is the main motivation of the present work.

We will concentrate on the scalar perturbation in the background of the black hole localized on a tensional three-brane in a world with two large extra dimensions. The equation of motion for the massless scalar field is described by
\[
\frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu}) \Psi(t, r, \theta, \phi, \chi, \psi) = 0..
\]
(3)

Seperating the variable \( \Psi(t, r, \theta, \phi, \chi, \psi) = e^{i\omega t + R(r)} \Theta(\theta) \Phi(\phi) \Gamma(\chi) \Xi(\psi) \) and expressing the tortoise coordinate
\[
r_* = \int \frac{1}{1 - \frac{\lambda}{M_s^4}} dr,
\]
(4)
we can get the wave equation of the scalar field in the metric (11),
\[
\frac{d^2}{dr_*^2} R(r) + [\omega^2 - V(r)] R(r) = 0,
\]
(5)
\[
\frac{1}{\sin^3 \theta} \frac{d}{d\theta} \left( \sin^3 \theta \frac{d}{d\theta} \Theta(\theta) \right) + \left( \eta_4 - \frac{\eta_1}{\sin^2 \theta} \right) \Theta(\theta) = 0,
\]

(6)

\[
\frac{1}{\sin^2 \phi} \frac{d}{d\phi} \left( \sin^2 \phi \frac{d}{d\phi} \Phi(\phi) \right) + \left( \eta_3 - \frac{\eta_2}{\sin^2 \phi} \right) \Phi(\phi) = 0,
\]

(7)

\[
\frac{1}{\sin \chi} \frac{d}{d\chi} \left( \sin \chi \frac{d}{d\chi} \Gamma(\chi) \right) + \left( \eta_2 - \frac{\eta_1}{\sin^2 \chi} \right) \Gamma(\chi) = 0,
\]

(8)

\[
\frac{1}{b^2} \frac{d^2}{d\psi^2} \Xi(\psi) + \eta_1 \Xi(\psi) = 0,
\]

(9)

with

\[
V(r) = \left(1 - \frac{M}{br^3}\right) \left(\frac{\eta_4 + 2}{r^2} + \frac{4M}{br^5}\right).
\]

(10)

Here parameters \(\eta_1, \eta_3, \eta_2, \text{ and } \eta_4\) denote the eigenvalues of the equations (6)-(9) respectively. In terms of quantum theory, they are determined by four quantum numbers \((L, l_2, l_1, m)\) of the system. In the case \(b = 1\) (i.e., the brane tension \(\lambda = 0\)), the angular equations (6)-(9) reduce to those in the spherically symmetric cases. Their solutions can be expressed as the expansion in the Gegenbauer functions with the eigenvalues \(\eta_4 = L(L + 3), \eta_3 = l_2(l_2 + 2), \eta_2 = l_1(l_1 + 1)\) and \(\eta_1 = m^2\) respectively. In this case, the eigenvalue \(\eta_4\) is independent of the angular number \(m\) and is defined only by the quantum number \(L\). However when \(b \neq 1\), the spherical symmetry is broken and then \(\eta_4\) depends on the quantum numbers \(L\) and \(m\).

In order to study the QNM and the late-time tail behaviors of the external perturbations in the black hole spacetime (1), the first step for us is to determine eigenvalue \(\eta_4\) in the equation (10). Obviously, the eigenvalue \(\eta_1\) in the equation (9) is \(m^2\). We restrict to \(m > 0\) here and rewrite equation (8) as

\[
\frac{1}{\sin \chi} \frac{d}{d\chi} \left( \sin \chi \frac{d}{d\chi} \Gamma(\chi) \right) + \left( \eta_2 - \frac{m^2}{\sin^2 \chi} \right) \Gamma(\chi) + \frac{1}{\sin^2 \chi} (m^2 - \frac{m^2}{b^2}) \Gamma(\chi) = 0.
\]

(11)

We limit ourselves to the case where the deviation of the parameter \(b\) from unity is very small which is physically justified for small brane tension. Then the third term on the left-hand-side of the equation above can be regarded as a perturbation. Using the perturbation theory, we have

\[
\Gamma(\chi) = P_l^{m}(\cos \chi) + \gamma S_l^{m}(\cos \chi) + O(\gamma^2),
\]

\[
\eta_2 = \eta_2^{(0)} + \gamma \eta_2^{(1)} + O(\gamma^2),
\]

(12)

where \(\gamma\) is a dimensionless parameter denoting the perturbation scale. For convenience, we set \(\gamma = 1\) throughout our paper. Substituting the variables (12) into the angular equation (11), we can obtain

\[
(D_0 + \eta_2^{(0)}) P_l^{m}(\cos \chi) = 0,
\]

(13)

\[
(D_0 + \eta_2^{(0)}) S_l^{m}(\cos \chi) + (D_1 + \eta_2^{(1)}) P_l^{m}(\cos \chi) = 0,
\]

(14)
with
\[
D_0 = \frac{1}{\sin \chi} \frac{d}{d\chi} \left( \sin \chi \frac{d}{d\chi} \right) - \frac{m^2}{\sin^2 \chi},
\]
\[
D_1 = \frac{1}{\sin^2 \chi} (m^2 - \frac{m^2}{b^2}).
\] (15)

From the zeroth order equation (13), we have
\[
\eta_2^{(0)} = l_1(l_1 + 1).
\] (16)

Multiplying equation (14) by \(P_m l_1 (\cos \chi)\) from the left and integrating it over \(\chi\), we get
\[
\eta_2^{(1)} = \frac{m(2l_1 + 1)(1 - b^2)}{2b^2}.
\] (17)

Changing the wave function \(P_m l_1 (x)\) to \(\tilde{C}_h(x,y) = \frac{d}{dy}C_h(x)\) (where \(C_h(x)\) is the Gegenbauer function) in the equation (12) and repeating the operations above, we can obtain the eigenvalues of equations (7) and (6)
\[
\eta_3 = l_2(l_2 + 2) + \frac{m(2l_2 + 2)(1 - b^2)}{2b^2},
\]
\[
\eta_4 = L(L + 3) + \frac{m(2L + 3)(1 - b^2)}{2b^2}.
\] (18) (19)

The formula (19) tells us that when \(b \neq 1\) the eigenvalue \(\eta_4\) of the angular equation (6) depends not only on the multiple moment \(L\), but also on the parameter \(b\) and angular number \(m\). Moreover, the eigenvalue \(\eta_4\) increases with the increase of the parameters \(m\) and the brane tension \(\lambda\). The perturbation result is in agreement with the numerical result obtained in [14].

To observe the fundamental QNM of the scalar perturbation in the background of a six-dimensional black hole on the brane with tension, we can adopt the third-order WKB approximation. The formula for the complex quasinormal frequencies \(\omega\) in this approximation is
\[
\omega^2 = [V_0 + (-2V_0'')^{1/2} \Lambda] - i(n + \frac{1}{2})(-2V_0'')^{1/2}(1 + \Omega),
\] (20)

where
\[
\Lambda = \frac{1}{(-2V_0'')^{1/2}} \left\{ \frac{1}{8} \left( \frac{V_0^{(4)}}{V_0''} \right) \left( \frac{1}{4} + \alpha^2 \right) - \frac{1}{288} \left( \frac{V_0'''}{V_0''} \right)^2 \right\},
\]
\[
\Omega = \frac{1}{(-2V_0'')^{1/2}} \left\{ \frac{5}{6912} \left( \frac{V_0'''}{V_0''} \right)^4 (77 + 188\alpha^2) \right\}
\]
\[
- \frac{1}{384} \left( \frac{V_0''' V_0^{(4)}}{V_0'''} \right) (51 + 100\alpha^2) + \frac{1}{2304} \left( \frac{V_0^{(4)}}{V_0''} \right)^2 (67 + 68\alpha^2)
\]
\[
+ \frac{1}{288} \left( \frac{V_0''' V_0^{(5)}}{V_0'''} \right) (19 + 28\alpha^2) - \frac{1}{288} \left( \frac{V_0^{(6)}}{V_0''} \right) (5 + 4\alpha^2) \right\},
\] (21)
\[ \alpha = n + \frac{1}{2}, \quad V_{0}(s) = \frac{d^{3}V}{dr_{*}^{2}} \bigg|_{r_{*}=r_{*}(r_{p})}, \]

\( n \) is the overtone number.

Setting \( M = 2 \) and substituting the effective potential (10) into the formula above, we can obtain the quasinormal frequencies of scalar field in the black hole localized on a tensional three-brane. The fundamental QNM frequencies for \( L = 5 \) are listed in Table 1. In Fig.1, we display the variation of the QNM frequencies with brane tension. For a tensionless brane, where \( b = 1 \), the result coincides with that of the six-dimensional Schwarzschild black hole [19]. Increasing the brane tension, which corresponds to decreasing the parameter \( b \), the absolute value of the imaginary part of quasinormal frequency decreases. The dependence of the real part of quasinormal frequency on the brane tension is very complicated. For each fixed \( L \), the real part of quasinormal

| \( b \) | \( \omega \) (\( m = 0 \)) | \( \omega \) (\( m = 1 \)) | \( \omega \) (\( m = 2 \)) | \( \omega \) (\( m = 3 \)) | \( \omega \) (\( m = 4 \)) | \( \omega \) (\( m = 5 \)) |
|---|---|---|---|---|---|---|
| 0.90 | 2.847856-0.380539i | 2.898630-0.380473i | 2.948528-0.38042i | 3.045869-0.38302i | 3.093389-0.380355i | 3.093389-0.380252i |
| 0.92 | 2.868797-0.383337i | 2.908446-0.383285i | 2.947561-0.383237i | 3.024271-0.383147i | 3.061905-0.383105i | 3.093389-0.380355i |
| 0.94 | 2.889436-0.386005i | 2.918480-0.386057i | 2.947236-0.386026i | 3.003922-0.385952i | 3.031867-0.385919i | 3.061905-0.383105i |
| 0.96 | 2.909785-0.388814i | 2.928706-0.388789i | 2.947055-0.388765i | 2.984748-0.388718i | 3.003195-0.388696i | 3.031867-0.385952i |
| 0.98 | 2.929853-0.391495i | 2.939103-0.391483i | 2.948323-0.391471i | 2.966678-0.391459i | 2.975813-0.391436i | 3.003195-0.388696i |
| 1.00 | 2.949650-0.394141i | 2.949650-0.394141i | 2.949650-0.394141i | 2.949650-0.394141i | 2.949650-0.394141i | 2.949650-0.394141i |

TABLE I: The fundamental \((n = 0)\) quasinormal frequencies of scalar field in the background of a six-dimensional black hole on the brane with tension, where \( L = 5 \).
frequency decreases with the increase of the brane tension for small values of $m$, while increases with the increase of the brane tension for big values of $m$. For $L = 5$, the property is shown in Fig.1 and the value $m = 2$ is the turning point separating the increasing and decreasing behavior of the real part frequency with the increase of the brane tension. The $m$ values’ involvement in the dependence of the real part of quasinormal frequency on the brane tension has been observed for other chosen $L$. The qualitative property shown in Fig.1 holds while the turning point value $m$ changes.

We have also showed the dependence of the QNM frequencies on the angular number $m$ for fixed brane tension. In Fig.2, we see that the real part frequency increases while the absolute value of the imaginary part frequency decreases with the increase of $m$ for fixed brane tension. This has not been observed in the usual six-dimensional Schwarzschild black hole with $b = 1$.

Therefore in this black hole spacetime the quasinormal frequencies depend not only on the parameter $b$ but also on angular number $m$. This result can be explained by the fact that due to the presence of the parameter $b$, the angular eigenvalues $\eta_4$, the event horizon radius and symmetry of the black hole are changed from those of the usual spherical cases.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Variation of the real part (the left) and the magnitude of the imaginary part (the right) of quasinormal frequencies with the angular number $m$ for fixed $L = 5$ and $b = 0.9$.}
\end{figure}

Now we extend our discussion to the late-time tail behavior of the scalar perturbation in the background of a six-dimensional black hole on a brane with tension. In Ref. [18], Ching et al. made a complete study on the late-time tail in black hole spacetimes for an external perturbation dominated by the evolution equation with a potential in the form

$$V(x) \sim \frac{\nu(\nu + 1)}{x^2} + \frac{c_1 \log x + c_2}{x^\alpha}, \quad x \to \infty.$$  \hspace{1cm} (22)

After a careful study of the contribution of the branch cut to the Green’s function, they found that the late-time behavior of the external perturbation in general is dominated by a power-law or by a power-law times a
logarithm, and its decay rate depends on the leading term at very large spatial distances. Making use of their conclusion, Cardoso [19] studied the late-time tail of scalar fields in the D-dimensional Schwarzschild spacetime for the case $c_1 = 0$. When $\nu$ is an integer, the main contribution to the late-time tail comes from the $\frac{c_2}{x^\alpha}$ term and then the form of the tail is given by

$$\Psi \sim t^{-(2\nu + 2\alpha - 2)}, \quad \alpha \text{ odd integer } < 2\nu + 3,$$

$$\Psi \sim t^{-(2\nu + \alpha)}, \quad \text{all other real } \alpha.$$  \hspace{2cm} (23)

When $\nu$ is not an integer, the late-time tail takes the form

$$\Psi \sim t^{-(2\nu + 2)}, \quad \text{non-integer } \nu.$$  \hspace{2cm} (24)

The reason is that the main contribution to the late-time tail comes from the $\nu(\nu + 1)x^2$ in this case. In the background of a six-dimensional black hole on a brane with tension, the effective potential of the evolution equation for scalar perturbation can be rewritten as

$$V(r_*) = (1 - \frac{M}{br^3})(\frac{\nu(\nu + 1)}{r^2} + \frac{4M}{br^5}),$$  \hspace{2cm} (25)

where

$$\nu = \frac{2L + 3}{2} \sqrt{1 + \frac{m(1 - b^2)}{2(2L + 3)b^2} - \frac{1}{2}}.$$  \hspace{2cm} (26)

When $r_* \to \infty$, we find that it has the asymptotical form

$$V(r_*)_{r_* \to \infty} = \frac{\nu(\nu + 1)}{r_*^2} - \frac{2ML(L + 3)}{br_*^5}. $$  \hspace{2cm} (27)

Obviously, this potential is a special case of the potential (22), i.e. $c_1 = 0$. Thus, comparing with the results in [18, 19] we can directly obtain the form of the late-time behavior. For the case when $\nu$ is not an integer, in terms of equation (26) we find that the late-time tail is described by

$$\Psi \sim t^{1-(2L + 3)}\sqrt{1 - \frac{m(1 - b^2)}{2(2L + 3)b^2}}.$$  \hspace{2cm} (28)

In the case when $\nu$ is an integer, it is easy to obtain that $2\nu + 3 > 2L + 5 > 5 = \alpha$ and the late time tail of the scalar wave propagation has the form

$$\Psi \sim t^{7-(2L + 3)}\sqrt{1 - \frac{m(1 - b^2)}{2(2L + 3)b^2}}.$$  \hspace{2cm} (29)

From equations (29) and (30), we find that in both cases for the larger multiple moment $L$, angular quantum number $m$ and brane tension $\lambda$, the decay of the scalar field at very late time becomes more quickly. The
dependence of the late time tail on the brane tension is consistent with that observed in the power output result in [14]. The bigger brane tension may diminish the power emission of the black hole so that it is easier for the perturbation outside the black hole to die out. Moreover, we have also found that the decay rate of late-time tail in the integer $\nu$ case is larger that in the non-integer $\nu$ situation.

In summary, we have studied the QNM and the late-time tail behavior of scalar perturbation in the background of a black hole localized on a tensional three-brane in a world with two large extra dimensions. Our results show that with the nonzero brane tension, the properties of the wave dynamics are different from those of the usual spherical black holes where the brane tension is completely negligible. We argue that significant modifications of the standard wave dynamics due to the presence of the brane tension compared to that of the black hole on a brane with negligible tension could serve as signatures, complementing the information through the evaporation of a black hole off a tense brane in the future collider searches.

Recently a non-negligible black hole-brane interaction was discussed in [20]. If the brane tension is small, as taken in this work, then by exciting degrees of freedom of the scalar field propagating in the background of a black hole, one would also excite the degrees of freedom of the brane itself. These degrees of freedom can also influence the wave dynamics. It would be interesting to study this influence in detail in the future.

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