Mean-field and fluctuation analyses of a forced turbulence simulated by the lattice Boltzmann method

W. Sakikawa and O. Narikiyo

Department of Physics, Kyushu University, Fukuoka 810-8560, Japan

(July 16, 2003)

On the basis of the lattice Boltzmann method we have done a numerical experiment of a forced turbulence in real space and time. Our new findings are summarized into two points. First in the analysis of the mean-field behavior of the velocity field using the exit-time statistics we have verified Kolmogorov’s scaling and Taylor’s hypothesis for the first time in the simulation for the Navier-Stokes equation. Second in the analysis of the intermittent velocity fluctuations using a non-equilibrium probability distribution function and the wavelet denoising we have clarified that the coherent vortices sustain the power-law velocity correlation in the non-equilibrium state.

I. INTRODUCTION

Understanding turbulence has not been attained over millenniums. However, new approaches to turbulence [1] have been invented successively.

Recently turbulence data in nature and laboratories are analyzed from new view points. One [2] is based on the exit-time statistics and discusses Kolmogorov’s scaling [1]. The other [3] on the basis of the Rényi-Tsallis statistics [4,5] discusses the probability distribution functions (PDF).

Our study starts from the application of the above two schemes to our numerical simulation data and aims to obtain some new insight of the turbulence.

Before reporting our study of the turbulence we briefly review here our strategy approaching intermittency problems in other non-equilibrium systems. In the numerical study of the critical spin-state of the Ising model [6] we have observed that the degree of the non-Gaussianity of the Rényi-Tsallis PDF represents that of the non-equilibrium. On the other hand, in the numerical study of a supercooled liquid [7] near the glass transition we have observed that the spatial distribution of the coherently moving regions is well analyzed by the singularity spectrum of multifractal. Since the Rényi-Tsallis statistics can describe the systems with multifractality, the above two observations are closely related.

In these two systems the correlation length of the fluctuations becomes divergently large and we can expect the scale-invariance. A scale-invariant system has multifractal nature. On the other hand, we can also expect the scale-invariance in the turbulent systems at high Reynolds number [1]. Thus we adopt the same strategy to intermittency problem in turbulence.

The next section describes our simulation method. In the following sections the simulation data are analyzed at the mean-field level at first and subsequently taking fluctuations into account. In the final section a brief summary is given.

II. LATTICE BOLTZMANN SIMULATION

We simulate a forced turbulence in real space and time and adopt the lattice Boltzmann method [8,9] as one of the easiest ways for such a purpose.

The distribution function $f_i(\vec{r},t)$ of the particle with the velocity $\vec{c}_i$ at position $\vec{r}$ and time $t$ obeys the lattice Boltzmann equation [8,9]

$$f_i(\vec{r} + \vec{c}_i, t + 1) - f_i(\vec{r}, t) = -\omega[f_i(\vec{r}, t) - f_0^i(\vec{r}, t)],$$

(1)

where we have adopted the Bhatnagar-Gross-Krook (BGK) model for the collision term and the time step has been chosen as unity. The position $\vec{r}$ of the particle is restricted on the cubic lattice and the lattice spacing is chosen as unity. The local equilibrium distribution $f_0^i(\vec{r}, t)$ is assumed to be given as

$$f_0^i(\vec{r}, t) = w_i \rho[1 + 3(\vec{c}_i \cdot \vec{u}) + \frac{9}{2}(\vec{c}_i \cdot \vec{u})^2 - \frac{3}{2}(\vec{u} \cdot \vec{u})],$$

(2)

The density $\rho$ and the velocity $\vec{u}$ of the fluid are given by the sum of the contributions of the particles as $\rho = \sum_i f_i(\vec{r}, t)$ and $\rho \vec{u} = \sum_i f_i(\vec{r}, t) \vec{c}_i$, respectively. Using the multiscale analysis the Navier-Stokes equation, which is supposed to
be able to describe turbulence, is derived from the present BGK lattice Boltzmann equation and the corresponding viscosity \( \nu = (1/\omega - 1/2)/3 \).

In the following we use a 15-velocity model and \( \epsilon_i \) is chosen as \( \epsilon_i = (0, 0, 0), \epsilon_i = (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1) \) for \( i = 2, 3, \cdots, 7 \) and \( \epsilon_i = (\pm 1, \pm 1, \pm 1) \) for \( i = 8, 9, \cdots, 15 \). Then the weight factors \( w_i \) for the local equilibrium distribution is determined as \( w_i = 2/9, w_i = 1/9 \) for \( i = 2, 3, \cdots, 7 \) and \( w_i = 1/72 \) for \( i = 8, 9, \cdots, 15 \). The relaxation frequency \( \omega \) is chosen as \( \omega = 1.94 \).

In order to simulate forced turbulence we add the forcing term \( \vec{g}_i \cdot \vec{F}(\vec{r}, t) \) due to an applied body force \( \vec{F}(\vec{r}, t) \) to the right hand side of Eq. (1). In order to reproduce the Navier-Stokes equation we should choose as \( \vec{g}_i = \epsilon_i/10 \) for the present 15-velocity model [8]. For simplicity we apply a solenoidal force \( \vec{F} = (0, F_y(z, t), 0) \), and \( F_y(z, t) \) is a Gaussian white noise whose variance \( \sigma^2 \) is a function of the lattice coordinate in \( z \)-direction, \( \sigma^2 = [0.01 \times \sin(2\pi k_F z/L)] \), where \( L \) is the linear dimension of the simulation region of cubic box. We have chosen as \( L = 200 \) so that \( 1 \leq x, y, z \leq 200 \). We have adopted the periodic boundary condition. Although we have chosen as \( k_f = 4 \), the results of the simulation is insensitive to the choice.

In the initial state of the simulation the fluid density is uniform, \( \rho = 1 \), and the spatial distribution of the fluid velocity \( \vec{u} \) is chosen to be random.

In Fig. 1 we show a snapshot of the vorticity field, \( \vec{\omega} = \nabla \times \vec{u} \), in order to visualize a turbulent structure. We see an inhomogeneous distribution of the vortices.

The Reynolds number \( Re \) for our simulation [8,9] is estimated as \( Re = \sqrt{\langle u'^2 \rangle} L/2\pi \nu \) and time-dependent as shown in Fig. 2. Here \( \langle \cdots \rangle \) represents the spatial average. In our simulation we can realize a turbulent state for relatively small Reynolds number, since we add random force.

### III. MEAN-FIELD BEHAVIOR

In this section we discuss the mean-field aspects of the velocity field.

Turbulence is one of the typical phenomena with multiscale motions. Each phenomena of a scale strongly couples with all the other scales of turbulent motion. In order to analyze such a system a scale-dependent entropy, the so-called \( \epsilon \)-entropy, works well [2]. For example, the time series of the velocity fluctuation in turbulence leads to a non-trivial scaling relation of the \( \epsilon \)-entropy, \( h(\epsilon) \propto \epsilon^{-3} \), expected from Kolmogorov’s scaling [2]. The existing experimental data are consistent with this scaling [2]. In this section we try to show the consistency of the scaling with our numerical experiment simulating the Navier-Stokes equation.

We focus our attention to the time series of the \( y \)-component \( u_y(\vec{r}, t) \) of the fluid velocity \( \vec{u}(\vec{r}, t) \) as shown in Fig. 3, since our system is anisotropic due to the forcing. This turbulent signal is characterized by the exit-time \( \tau(\epsilon; \vec{r}) \) at position \( \vec{r} \). At the exit-time \( t = \tau(\epsilon; \vec{r}) \) the first exit satisfying the condition, \( |u_y(\vec{r}, t_0 + \tau) - u_y(\vec{r}, t_0)| > \epsilon/2 \), occurs. Here the time \( \tau \) is measured from \( t_0 \) and \( t_0 = 1000 \) in our simulation regarding the states at the first 1000 time-steps as transitional.

The mean of the exit-time, \( \tau(\epsilon) \equiv \langle \tau(\epsilon; \vec{r}) \rangle \), is related to the \( \epsilon \)-entropy, \( h(\epsilon) \), as [2]

\[
h(\epsilon) \propto \frac{1}{\tau(\epsilon)}, \tag{3}
\]

In the region where Kolmogorov’s scaling, the 4/5 law [1,2], holds the velocity difference between two points behaves as

\[
\langle |\vec{u}(\vec{r} + \vec{R}, t_0) - \vec{u}(\vec{r}, t_0)| \rangle \propto |\vec{R}|^{1/3}.
\tag{4}
\]

Using Taylor’s hypothesis [1,2], we obtain

\[
\langle |\vec{u}(\vec{r}, t_0 + \tau) - \vec{u}(\vec{r}, t_0)| \rangle \propto \tau^{1/3}, \tag{5}
\]

as a mean-field description. In spite of the absence of the global flow in our simulation Taylor’s hypothesis is valid in a local sense [11] as shown in the following. Namely in the mean-field description the velocity difference in temporal and spatial directions have the same fractal scaling exponent. With Eq. (5) the definition of the exit-time leads to \( \tau(\epsilon)^{1/3} \propto \epsilon \). From Eq. (3) we can conclude that \( h(\epsilon) \propto \epsilon^{-3} \) in the above mean-field description [2].

In accordance with the above scaling analysis our simulation data [10] for \( 1/\tau(\epsilon) \) shown in Fig. 4 have the scaling region for \( \epsilon > \epsilon_0 \) where \( \epsilon_0 \sim 0.04 \). Thus by our simulation we have established that \( h(\epsilon) \propto \epsilon^{-3} \). For \( \epsilon \ll \epsilon_0 \) the exit in the simulation occurs within one time step and the curve saturates at small \( \epsilon \). These behaviors are consistent with the experimental data [2].

2
In Fig. 5 the result of the similar analysis in spatial direction is shown. In place of \( \tau(\epsilon) \) we measure \( r(\epsilon) \) where at the exit-length \( r = r(\epsilon; \vec{r}) \) the first exit satisfying the condition, \( |u_y(x+r, y, z, t) - u_y(x, y, z, t)| > \epsilon/2 \), occurs at a time \( t \). Here \( r(\epsilon) = (r(\epsilon; \vec{r})) \). Because of the smallness of the number of the available data in space direction, 200, in comparison with that in time direction, 7000, the scaling region is narrow. Since \( 1/\tau(\epsilon) \) and \( 1/r(\epsilon) \) have the same scaling exponent, Taylor’s hypothesis is confirmed to be valid.

In conclusion we have confirmed Kolmogorov’s scaling and Taylor’s hypothesis at the same time. In contrast to the evaluation of the energy spectrum, which is usually employed for testing Kolmogorov’s scaling and determined by the two-point correlation function, Kolmogorov’s scaling is easily observed in the \( \epsilon \)-entropy, since it is a mean-field description and fluctuations are averaged out. Such a mean-field scaling is a unifractal description and fluctuations can be taken into account in a multifractal analysis as shown in the following.

IV. FLUCTUATIONS: QUALITATIVE OBSERVATION

We have discussed the mean-field aspects of the velocity field in the preceding section. Next we discuss the fluctuations of the velocity field. In this section we make a qualitative observation using the correlation maps.

In Fig. 6 we show the space correlation. The correlation decays as the length scale is increased. In Fig. 7 we show the time correlation. The correlation decays as the time scale is increased. At small length or time scale the strong correlation leads to a non-equilibrium state which is identified as the coherent vortex in the following section.

We see a multiscale structure in the correlation map, Fig. 8, which is one of the characteristics of turbulence neither periodic nor random.

V. FLUCTUATIONS: NON-EQUILIBRIUM NATURE

We have discussed the qualitative aspects of the velocity fluctuation in the preceding section. Next we try to quantify the fluctuation by two measures. One is the non-equilibrium parameter used in this section and the other is the singularity spectrum in the next section.

In order to analyze the distribution of the fluctuation we use the Rényi-Tsallis PDF \([4,5]\)

\[
P(X) = P_0 \cdot \left[1 + \frac{q-1}{2\sigma_X^2}X^2\right]^{-\frac{1}{q-1}},
\]

for a variable \( X \). Here the parameter \( q \) represents the degree of non-equilibrium [6]. To be precise, the deviation of \( q \) from unity is the measure for the degree of non-equilibrium, since the equilibrium Gaussian distribution is expressed by Eq. (6) in the limit of \( q \to 1 \).

The PDF in Eq. (6) is derived from the Rényi or Tsallis entropy by using the maximum entropy principle [12]. The Tsallis entropy is non-extensive, while the Rényi entropy is extensive. The functional form of the distribution is independent of the choice of the entropy, the Rényi or Tsallis, so that our result is independent of the extensivity of the entropy.

We show the PDF for the velocity difference between two points in Fig. 9 and the vorticity difference in Fig. 10. The \( q \)-value of the PDF is length-scale dependent. Similarly the PDF for the velocity or vorticity difference between two times at a point is time-scale dependent. In any case the degree of non-equilibrium is larger at smaller spatio-temporal scales where the correlation survives. As shown later on, the formation of the coherent vortex structure sustains the strong correlation leading to non-Gaussian PDF. At larger scales the contribution of the structureless incoherent background among vortices dominates. This incoherency leads to Gaussian PDF. The Gaussian PDF is a parabola in the semi-logarithmic plane as Fig. 9 and Fig. 10.

Although our data in Fig. 9 are not enough to discuss the scaling between \( q \) and \( r \) quantitatively, we have to derive such a relation in our future study. Already a scaling relation between \( q \) and \( r \) has been proposed [13] under the assumption of a cascade picture. On the other hand, another scheme with a \( r \)-independent \( q \)-value has been proposed [12] under the assumption of another cascade picture. In contrast to these two approaches our analysis has nothing to do with cascades.

In Fig. 11 we show the PDF for the vorticity itself taken from a snapshot of the vorticity field. This PDF is also non-Gaussian. The Gaussian components can be removed from the snapshot by using the wavelet denoising technique [14]. It can be seen from the coincidence between the left and right panels in Fig. 12 that the non-Gaussian components corresponds to the vortices. Here the Gaussian component is filtered out by the wavelet denoising in the right panel. This visualization is not new, since it has already been done in the study of the coherent vortex simulation [14]. Our new contribution is to clarify the origin of the scale-dependent \( q \)-value in terms of the coherent
vortex structure. The strong correlation leading to the non-Gaussian PDF at smaller scales is sustained in the inner region of the vortices, while the PDF at larger scales than the vortex size tends to Gaussian. In other words the inner region is coherent and the outer is incoherent.

VI. FLUCTUATIONS: MULTIFRACTAL NATURE

We have discussed the local nature of the vortex as the elementary excitation in the preceding section. In this section we discuss the spatial distribution of the vortices in a global point of view.

First we make a qualitative observation. Already as seen in Fig. 12 the spatial distribution of the coherent vortices is intermittent. In Fig. 13 we also see a intermittent behavior for the derivative of the vorticity. Similar intermittent behavior is observed in experimental data [15].

Next we quantify the intermittency by introducing the measure, \( \mu_k(l) \), defined as the probability of finding a coherent vortex in the \( k \)-th cubic box of the linear dimension \( l \) in a snapshot of the vorticity field. The total number of the boxes \( N_L = (L/l) \times (L/l) \) where \( L \) is the linear dimension of the simulation region. The measure behaves as

\[
\mu_k(l) \propto l^{\alpha_k},
\]
for small \( l/L \). The number density \( N(\alpha) \) for the exponent \( \alpha_k \) is defined as

\[
N(\alpha) \equiv \sum_k \delta(\alpha - \alpha_k) \propto l^{-f(\alpha)}.
\]

This relation defines the singularity spectrum, \( f(\alpha) \), which is the key quantity to qualify multifractal systems. Using the normalized \( q \)-th moment \( \mu_k(q,l) \) of the measure \( \mu_k(l) \),

\[
\mu_k(q,l) = \{ \mu_k(l) \}^q / \sum_{k'=1}^{N_L} \{ \mu_{k'}(l) \}^q,
\]
the singularity spectrum \( f(\alpha) \) is obtained by the following formulae, [16]

\[
\alpha(q) = \sum_{k=1}^{N_L} \mu_k(q,l) \ln \mu_k(l) / \ln(l/L),
\]
and

\[
f(\alpha(q)) = \sum_{k=1}^{N_L} \mu_k(q,l) \ln \mu_k(q,l) / \ln(l/L).
\]

In Fig. 14 we show the singularity spectrum for the coherent vortex. For simplicity we have used the definition \( N_k/N_L = l^{n_k} \) where \( N_k \) is the number of lattice points satisfying the condition \( \omega_z > \langle \omega_z \rangle \). The spectrum is similar to the typical one measured for the energy-dissipation rate by experiments [15] or simulations [17].

In the last part of this section we give some speculations. As seen in our previous study [7] the width of the singularity spectrum depends on the degree of intermittency so that we expect some Reynolds-number dependence of the spectrum. It has been discussed by many authors [18] and should be clarifyed in future systematic study. In our present study the density of the vortex is dilute as seen in Fig. 12. Thus the singularity spectrum in Fig. 14 describes the spatial distribution of relatively free vortices. As the Reynolds number is increased the density increases [19]. In this case the interaction among vortices becomes important and the correlation length of the vorticity fluctuation becomes large. While we can observe the vortex only as an individual elementary excitation in our numerical experiment, some collective excitation is expected to dominate at higher Reynolds number. In the limit of divergently large Reynolds number the correlation length becomes divergently large so that we can expect full scale-invariance. We can find a resemblance to the case of the scaling theory [20] in polymers where an ideal scaling relation is realized for dense solutions where polymers are strongly entangled. In the limit of high Reynolds number each boxes counting the coherent vortex will be filled by almost equal number of vortices so that intermittency will disappear and unifractal Kolmogorov’s scaling will prevail.
VII. SUMMARY

Numerical simulation data, in real space and time, for a forced turbulence on the basis of the lattice Boltzmann method have been analyzed by unifractal and multifractal schemes.

Our new findings are summarized into two points. First in the unifractal analysis using the exit-time statistics we have verified Kolmogorov’s scaling and Taylor’s hypothesis at the same time. Second in the analysis using the Rényi-Tsallis PDF and the wavelet denoising we have clarified that the coherent vortices sustain the power-law velocity correlation in the non-equilibrium state.

Finally in the multifractal analysis it is clarified that the intermittent distribution of the coherent vortices in space-time is described as a multifractal.

This work was supported in part by a Grand-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

[1] U. Frisch, Turbulence, Cambridge University Press, Cambridge, 1995.
[2] M. Abel, L. Biferale, M. Cencini, M. Falcioni, D. Vergni, A. Vulpiani, Physica D 147 (2000) 12.
[3] F.M. Ramos, R.R. Rosa, C.R. Neto, M.J.A. Bolzan, L.D.A. Sá, H.F.C. Velho, Physica A 295 (2001) 250.
[4] C. Beck, F. Schlögl, Thermodynamics of chaotic systems, Cambridge University Press, Cambridge, 1993.
[5] C. Tsallis, Nonextensive Statistical Mechanics and Its Applications (eds. S. Abe, Y. Okamoto), Springer-Verlag, Berlin, 2001, p.3.
[6] W. Sakikawa, O. Narikiyo, J. Phys. Soc. Jpn. 71 (2002) 1200.
[7] W. Sakikawa, O. Narikiyo, J. Phys. Soc. Jpn. 72 (2003) 450.
[8] D.A. Wolf-Gladrow, Lattice-Gas Cellular Automata and Lattice Boltzmann Models, Springer-Verlag, Berlin, 2000.
[9] S. Succi, The Lattice Boltzmann Equation for Fluid Dynamics and Beyond, Clarendon Press, Oxford, 2001.
[10] The first simulation data for a smaller system \((100^3)\) to confirm Kolmogorov’s scaling has been reported by us in an unpublished preprint, cond-mat/0208094.
[11] M.H. Jensen, G. Paladin, A. Vulpiani, Phys. Rev. A 45 (1992) 7214.
[12] T. Arimitsu, N. Arimitsu, Physica A 295 (2001) 177.
[13] C. Beck, Physica A 277 (2000) 115.
[14] M. Farge, K. Schneider, N. Kevlahan, Phys. Fluids 11 (1999) 2187.
[15] C. Meneveau, K.R. Sreenivasan, P. Kailasnath, M.S. Fan, Phys. Rev. A 41 (1990) 894.
[16] M. Schreiber, Computational Physics, (eds. K.H. Hoffmann and M. Schreiber), Springer-Verlag, Berlin, 1996, p.147.
[17] I. Hosokawa, K. Yamamoto, J. Phys. Soc. Jpn. 59 (1990) 401.
[18] K.R. Sreenivasan, More Things in Heaven and Earth: A Celebration of Physics at the Millenium, (ed. B. Bederson), Springer-Verlag, New York, 1999, p.644.
[19] S. Kida, H. Miura, J. Phys. Soc. Jpn. 67 (1998) 2166.
[20] P-G. de Gennes, Scaling Concepts in Polymer Physics, Cornell University Press, Ithaca, 1979.

FIG. 1. The snapshot of the \(\omega_z\)-component of the vorticity, \(\omega_z\), in the \(xy\)-plane at \(z = 100\) and \(t = 4000\).

FIG. 2. The Reynolds number \(Re\) as a function of time \(t\).

FIG. 3. The \(u^y\)-component of the fluid velocity \(u^y\) as a function of time \(t\) at \(\vec{r} = (100, 100, 100)\).

FIG. 4. The inverse of the mean exit-time \(\tau(\epsilon)\). The exits are measured at \(200^3\) lattice points for \(1000 \leq t \leq 8000\). Here the dots and straight line represent the simulation data and \(\epsilon^{-3}\) law, respectively.

FIG. 5. The inverse of the mean exit-length \(r(\epsilon)\). The exits are measured at \(200^3\) lattice points at \(t = 6000\). Here the dots and straight line represent the simulation data and \(\epsilon^{-3}\) law, respectively.

FIG. 6. The space correlation of the velocity fluctuation at \(t = 4000\). The horizontal and vertical axes represent \(u^y(x, y, z, t)\) and \(u^y(x, y + y_{\text{corr}}, z, t)\), respectively. The space difference \(y_{\text{corr}}\) is taken as \(y_{\text{corr}} = 1\) for the left, \(y_{\text{corr}} = 4\) for the center and \(y_{\text{corr}} = 30\) for the right.
FIG. 7. The time correlation of the velocity fluctuation at \( \vec{r} = (100, 100, 100) \). The horizontal and vertical axes represent \( u_y(\vec{r}, t) \) and \( u_y(\vec{r}, t + t_{corr}) \), respectively for \( 1000 \leq t \leq 8000 \). The time difference \( t_{corr} \) is taken as \( t_{corr} = 10 \) for the left, \( t_{corr} = 100 \) for the center and \( t_{corr} = 500 \) for the right.

FIG. 8. The correlation map for the velocity difference between two times \( t_i \) and \( t_j \) for \( 1000 \leq t_0 + t_i \leq 8000 \) and \( 1000 \leq t_0 + t_j \leq 8000 \) with \( t_0 = 1000 \). Namely \( 0 \leq i \leq 7000 \) (horizontal axis) and \( 0 \leq j \leq 7000 \) (vertical axis). The dot is marked at \( (i, j) \) if \( D(i, j) < \Theta \) where \( D(i, j) = |u_y(\vec{r}, t_i) - u_y(\vec{r}, t_j)| \) and \( \Theta = 0.2 D_{max} \) with \( D_{max} \) being the maximum of \( D(i, j) \). The position \( \vec{r} \) is fixed at \( (100, 100, 50) \).

FIG. 9. The unnormalized PDF \( P(X) \) for the velocity difference \( X = |u_y(x, y + \Delta y, z, t) - u_y(x, y, z, t)| \) between two points \( (x, y + \Delta y, z) \) and \( (x, y, z) \) at \( t = 4000 \). The difference in \( y \)-direction is chosen as \( \Delta y = 1, 4, 30 \). The data are fitted by the Rényi-Tsallis PDF with \( q = 1.218, 1.164, 1.100 \) for \( \Delta y = 1, 4, 30 \), respectively.

FIG. 10. The unnormalized PDF \( P(X) \) for the vorticity difference \( X = |\omega_y(x, y + \Delta y, z, t) - \omega_y(x, y, z, t)| \) between two points \( (x, y + \Delta y, z) \) and \( (x, y, z) \) at \( t = 6000 \). The difference in \( y \)-direction is chosen as \( \Delta y = 1, 4, 30 \). The data are fitted by the Rényi-Tsallis PDF with \( q = 1.240, 1.209, 1.179 \) for \( \Delta y = 1, 4, 30 \), respectively.

FIG. 11. The unnormalized PDF \( P(X) \) for the vorticity \( X = |\omega_y(x, y, z, t)| \) at \( t = 6000 \). The data are fitted by the Rényi-Tsallis PDF with \( q = 1.392 \).

FIG. 12. The snapshot of the \( z \)-component of the vorticity, \( \omega_z \), in the \( xy \)-plane at \( z = 100 \) and \( t = 4000 \). In contrast to Fig. 1 the regions satisfying the condition, \( \omega_z > \langle \omega_z \rangle + 2 \sigma \equiv \Theta \), are depicted in the left panel. Here \( \langle \omega_z \rangle \) is the spatial average and \( \sigma \) is the standard deviation. In the right panel the high vorticity regions satisfying the same condition, \( \tilde{\omega}_z > \Theta \), are depicted where \( \tilde{\omega}_z \) is the non-Gaussian component after the wavelet denoising filtering out the Gaussian component. By the restriction of the algorithm of the wavelet transform we have used only 128\times 128 lattice points.

FIG. 13. The spatial distribution of the vorticity gradient, \( \Delta \omega_z = \omega_z(x + 1, y, z, t) - \omega_z(x, y, z, t) \), in \( x \)-direction (\( 1 \leq x \leq 200 \)) with \( y = z = 100 \) at \( t = 6000 \).

FIG. 14. The singularity spectrum of the coherent vortex with \( L/L = 1/10 \). The data correspond to \(-90 \leq q \leq 95 \).
\[ P(X) \]

- \( \Delta y = 1, \; q = 1.240 \)
- \( \Delta y = 4, \; q = 1.209 \)
- \( \Delta y = 30, \; q = 1.179 \)
