A non-local geometrodynamics model based on general relativity and the notion of quantum foam

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Abstract

In this paper we introduce a new type of (3 + 1) spacetime metric, that is non-local and based on the notion of quantum foam. We investigate it and show how quantum effects can be obtained.

Keywords: Field theory; Geometrodynamics; General relativity; Quantum foam

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1 Introduction

When discussing on relativity theory we immediately think on its unique effects such as time dilation, length contraction, and gravitational force. But, this theory is important in every manner of physics. Since, in principle, this theory shapes the foundations of our reality which are obtained by a unique dimension, the spacetime dimension. Therefore, every theory that attempts to describe any kind of physical phenomena must obey to relativity theory. From this insight we learn that any model should not contradict the notion of Lorentz invariance, which claims that there is no, in any circumstances, preferred reference frames. In the last century a vast number of researches was published, trying to develop a quantum theory that fits with the special relativistic concepts, the quantum field theory (QFT). After establishing
this theory, a natural attempt was to derive a quantum theory that is consistent with general theory of relativity. The major difference between quantum theory and general relativity is that, unlike the first, general relativity is not a renormalizable theory.

Einstein-Cartan theory was an attempt to unify all the forces in nature, in the concept of geometrodynamics. This theory is a natural generalization of general relativity, which consists a non-symmetric tensors. The theory defines the non-symmetric Riemannian tensor $R_{\mu \nu}$ through the non-symmetric Christoffel symbols $\Gamma_{\alpha \beta}^{\mu}$. For infinitesimal vector shift

$$\delta A^\mu = -\Gamma_{\alpha \beta}^{\mu} A^\alpha dx^\beta,$$

the non-symmetric Riemannian tensor $R_{\mu \nu}$ is

$$R_{\mu \nu} = R_{\alpha \upsilon \mu}^{\alpha} = -\frac{\partial \Gamma_{\mu \nu}^{\alpha}}{\partial x_\alpha} + \Gamma_{\mu \nu}^{\alpha} \Gamma_{\alpha \beta}^{\beta} + \frac{\partial \Gamma_{\mu \nu}^{\alpha}}{\partial x_\upsilon} + \Gamma_{\mu \nu}^{\alpha} \Gamma_{\alpha \beta}^{\beta}.$$

The problem with this theory is that it describes only the gravity and the classical electromagnetic forces. Another famous theory that describes physics through the geometry of spacetime is the theory of quantum foam [17, 20, 26]. This theory constructs gravitation and electromagnetism forces from curved spacetime solely, and deals with the quantization of Einstein-Maxwell geometrodynamics equations. Another interesting models that relates geometry and quantum theory can be found in a vast number of papers [3, 5, 6, 22, 25]. For instance, the Berry phase model, which is a geometric phase associated with any cyclic evolution [7, 14].

In this paper we introduce a novel model, based on both, general relativity and quantum foam, that describes nature from geometrodynamics perspective and provide a geometrical interpretation to the superposition state of the quantum particles.

In Section 2 we introduce the model and thoroughly discuss on its properties. Section 3 examine the quantization of the model and the non-local state of the spacetime metric. In Section 4 we examine the spin of particles. Section 5 offers a discussion about the paper.

## 2 The spacetime metric

General relativity considers a symmetric metric $g_{\mu \nu}$ that describes the curvature and properties of the spacetime manifold. We now generalize this
metric, as follows:

Suppose that the universe consists a global metric $r_{\mu\nu}$, i.e. a metric that consists all of our universe, which is non-local in space and time with a consistent creation and annihilation of compact spacetime manifolds in the Planck length, we call them "foamions". Therefore, $r_{\mu\nu}$ is influenced from different spatial and time regions. Relativity theory forbids superluminal signaling. Thus, we suppose that in non-local points of $r_{\mu\nu}$, this metric becomes stochastic in time in the case that a strong measurements was taken. This probabilistic structure protects on the non-existence of such superluminal signals.

From the global metric we define the metric of each particle (throughout the paper, particle means any microscopic or macroscopic object) in the universe, as follows:

\[ Z_{\mu\nu} = g_{\mu\nu} + \hbar r_{\mu\nu}, \]

where $g_{\mu\nu}$ is the local pseudo-Riemannian metric, and $\hbar$ is the Planck constant.

The metric is then the sum of $g_{\mu\nu}$ and $\hbar r_{\mu\nu}$

\[ ds^2 = Z_{\mu\nu}dx_\mu dx_\nu \]

\[ = g_{\mu\nu}dx_\mu dx_\nu + \hbar r_{\mu\nu}dx_\mu dx_\nu. \]

The Christoffel symbols are then

\[ \Gamma^\rho_{\mu\nu} = \frac{1}{2}Z^{\rho\alpha} \left( \frac{\partial Z_{\alpha\nu}}{\partial x^\mu} + \frac{\partial Z_{\alpha\mu}}{\partial x^\nu} + \frac{\partial Z_{\mu\nu}}{\partial x^\alpha} \right). \]

The proposed model assumes that the Lagrangian of the matter is always a local classical Lagrangian $L_m$. The stress-energy tensor $T_{\mu\nu}$ takes the form

\[ T_{(h),\mu\nu} = Z_{\mu\nu}L_m - 2\frac{\delta L_m}{\delta Z_{\mu\nu}}. \]

From the stress-energy tensor we can derive the energy

\[ E = \int_V T_{(h),00}dV = \int_V g_{00}L_m + \lambda_{h,00}, \]

where

\[ \lambda_{h,\mu\nu} = \int_V h r_{\mu\nu}L_m - 2\frac{\delta L_m}{\delta Z_{\mu\nu}}dV. \]
The momentum is, straightforwardly,

$$P_{\mu} = \int_v T_{(h),0\mu} dV$$

$$= \int_v g_{0\mu} \mathcal{L}_m + \lambda_{h,0\mu}, \mu = 1, 2, 3. \tag{5}$$

### 2.1 The Lagrangian of $Z_{\mu\nu}$

The Einstein-Hilbert Lagrangian take the form

$$\mathcal{L} = \frac{1}{2\kappa} \mathcal{R} \sqrt{- \det (Z_{\mu\nu})} = \frac{1}{2\kappa} \mathcal{R} \sqrt{- \det (g_{\mu\nu} + \hbar r_{\mu\nu})}, \tag{6}$$

where $\kappa$ is the Einstein constant.

In the case that $Z_{\mu\nu}$ is a weak field, i.e. $Z_{\mu\nu} = g_{\mu\nu} - \hbar \varepsilon r_{\mu\nu}$, where $g_{\mu\nu}$ is approximated to the Minkowski metric $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, and $\varepsilon$ is small parameter. The Lagrangian (6) takes a simplified form:

$$Z_{\mu\nu} = \eta_{\mu\nu} - \hbar \varepsilon r_{\mu\nu}.$$ 

Then, the Lagrangian $\mathcal{L}$ takes the form

$$\mathcal{L} = \frac{1}{2\kappa} \mathcal{R} \sqrt{- \det (Z_{\mu\nu})} = \frac{1}{2\kappa} \mathcal{R} \sqrt{- \det (\eta_{\mu\nu} - \hbar \varepsilon r_{\mu\nu})},$$

using the well known identity, $\det (\eta_{\mu\nu} - \hbar \varepsilon r_{\mu\nu}) = -1 + \hbar \varepsilon \text{Tr} (r_{\mu\nu}^\ast) + O ((\hbar \varepsilon)^2)$, where $r_{\mu\nu}^\ast = \text{diag}(-1, 1, 1, 1) \cdot r_{\mu\nu}$. We clearly have

$$\mathcal{L} = \frac{1}{2\kappa} \mathcal{R} \sqrt{1 - \hbar \varepsilon \text{Tr} (r_{\mu\nu}^\ast) - O ((\hbar \varepsilon)^2)}$$

$$\approx 1 - \frac{\hbar \varepsilon}{4\kappa} \mathcal{R} \text{Tr} (r_{\mu\nu}^\ast).$$

Since a constant has no effect on the equation of motion, therefore we define the Lagrangian $\tilde{\mathcal{L}}$ such that

$$\tilde{\mathcal{L}} \approx \text{CTr} (r_{\mu\nu}^\ast), \tag{7}$$

where $C = -\hbar \varepsilon \mathcal{R} / 4\kappa$ acts as the coupling of the Lagrangian. The Yang-Mills Lagrangian density is a special case of (7).
3 Quantization and superposition of $r_{\mu\nu}$

The non-local global metric of the universe, $r_{\mu\nu}$, consists of foam ions in a quantum foam that act as non-static potentials in spacetime at the Planck scale. Therefore, for a very short time, $\tau^*$, the particle may not follow these potentials. For measurements with time $\tau \geq \tau^*$ the values of the particles seem to follow quantum mechanics. This is no more than approximation of the model, since, in fact, in a very short time $\tau^*$ the quantum particle takes values behind the eigenvalues of the quantum operators, since in this short time everything is continuous. In the case that the annihilation and creation of the foam ions are massless, $\tau^*$ is the very small value

$$\tau^* = \frac{\ell_h}{c},$$

where $\ell_h$ is the Planck length (the length of each foamion), $c$ is the speed of light. Therefore, the model claims that for measurements in such scale of time, the values of the particles, e.g. energy, will take values behind the eigenvalues of the quantum Hamiltonian.

Since the time of the measurements is $\tau \gg \tau^*$, the quantization method is a very good approximation to the influence of $r_{\mu\nu}$ on the particles. There exist a vast number of quantization methods discussed in the literature [2,11,11,12,15]. Another approximation of $r_{\mu\nu}$ is by considering this metric in the sense of loop quantum gravity [9,23], since we have a pseudo quantum-like spacetime. For a quantized $r_{\mu\nu}$, we write it as $r_{\mu\nu}^Q$. Notice that as we quantize $r_{\mu\nu}$ we get also a quantized $Z_{\mu\nu}$ since $g_{\mu\nu}$ is a deterministic metric, and thus $r_{\mu\nu}$ provides the quantum-like behavior. Taking $\hbar \to 0$, any quantization of $r_{\mu\nu}$ will not reflect on $Z_{\mu\nu}$, since in this case $Z_{\mu\nu} = g_{\mu\nu}$. The quantization of $r_{\mu\nu}$ also reflects on the quantization of the Lagrangian $\mathcal{L}$, and in the weak field such quantization can be obtained for the Yang-Mills Lagrangian.

We note that since quantization of the $r_{\mu\nu}$ is no more than approximation to the behavior of the foam effect in spacetime, renormalization is a method that solves infinite quantities of such approximated theory. Furthermore, we notice that the quantization of the spacetime metric leads to a spin-2 particle. If indeed the foamions travel in the speed of light, they can be associated with the graviton particle, a massless spin-2 particle.
3.1 Superposition state of $r_{\mu\nu}$

From the non-locality property of metric $r_{\mu\nu}$ we learn that this metric can be superposed into multiple states since each point can be in more than one place at the same time. Then, for a Planck length region $(t', x')$ (we call it a "point") the particle that is placed in this point remains local, although this point, $(t', x')$, is now non-local, i.e. it is in $n > 1$ different states, because $r_{\mu\nu}$ is in a superposition state. It means that in the coherent state, or in the case of interaction that makes the particle superposed in terms of quantum theory, is, in fact, in a single region, while this region is in more than one number of regions. The collapse of such superposition state occurs during measurements which make the decoherence into a single state.

In the case that at least a single point in spacetime $(t', x')$ is placed in $n > 1$ different regions simultaneously, this can be described by the following superposed metric

$$r_{\mu\nu} = \left( r_{\mu\nu}^{(1)} \quad r_{\mu\nu}^{(2)} \quad \ldots \quad r_{\mu\nu}^{(n)} \right),$$

(9)

and straightforwardly,

$$Z_{\mu\nu} = \left( g_{\mu\nu} + \hbar r_{\mu\nu}^{(1)} \quad g_{\mu\nu} + \hbar r_{\mu\nu}^{(2)} \quad \ldots \quad g_{\mu\nu} + \hbar r_{\mu\nu}^{(n)} \right).$$

The superposed stress-energy tensor is then

$$T_{(\hbar)\mu\nu} = Z_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta Z_{\mu\nu}}.$$

(10)

This shows the superposition of $r_{\mu\nu}$, since any particle that is in $(t', x')$ is, in fact, in $n$ possible states, and thus it is equivalent to the superposition state of this particle in the case of a single point in spacetime, i.e. a single state of the particle in superposed non-local point in spacetime is equivalent to a superposition of the particle with a local spacetime.

The most natural and intuitive interpretation to the collapse of the superposed state of $r_{\mu\nu}$ into a single outcome $r_{\mu\nu}^{(i)}$ comes from the Penrose interpretation [18, 19], which claims that the superposition state exist until the difference of spacetime curvature attains a significant level, and then the collapse occur. The collapse of $r_{\mu\nu}$ to $r_{\mu\nu}^{(i)}$ will take the probability $P_{r_i}$, such that $\sum_{i=1}^{n} P_{r_i} = 1$. The motivation behind taking into account probabilistic outcome come from the fact that relativity theory forbids a transmission of any kind of information faster than the speed of light, and furthermore, it
was shown that any quantum deterministic theory will violate Lorentz invariance \cite{13}, it is reasonable to suppose that the collapsed state is, in fact, a probabilistic outcome. In this way we consider uncertainty principles of the outcomes such that the no-superluminal signaling is preserved. Therefore, for each probability $P_{r_i}$ of $r_{\mu\nu}^{(i)}$, the expected value is

$$\langle r_{\mu\nu} \rangle = r_{\mu\nu} P_r = \sum_{i=1}^{n} r_{\mu\nu}^{(i)} P_{r_i}.$$ 

Since we oblige to follow quantum mechanics in flat spacetime, in the case of a weak gravitational field, the probability function should correspond with the probability function $|\psi(x, t)|^2$ of the quantum operators, where $\psi(x, t)$ is the wavefunction of the particle.

The superposed energy values and momentum are, respectively,

$$E = \int_v T_{(b),00}dV = \left( E_1 \ E_2 \ \ldots \ E_n \right), \quad (11)$$

and

$$P^\mu = \int_v T_{(b),\mu\nu}dV = \left( P_1^\mu \ P_2^\mu \ \ldots \ P_n^\mu \right), \ \mu = 1, 2, 3. \quad (12)$$

The vectors $E$ and $P^\mu$ are the particles energy levels and momentum, respectively. We note that since our probability function corresponds with the quantum theory one (for the weak gravitational field), we can derive the variance and the covariance of the characteristics of the particles: momentum, position, energy etc..

### 4 The Spin angular momentum

For weak field, the spacetime metric can be approximated to the Minkowski one, which represents the metric of special relativity, and thus special relativity should be taking into account. Unlike general relativity, special relativity reveals us that particles can have an intrinsic angular momentum called "spin". This property can be observed in the rest frame of the spinning particle. It exist not only for microscopic, but also for macroscopic objects.
Therefore, it is not claimed to be related directly to the notion of spin in quantum mechanics.

We argue that this type of spin can be related to the spin of the microscopic particles. First, notice that the quantum effects of non-locality and the quantum foam (which represents the "quantization" of the spacetime metric) can be achieved from \( r_{\mu\nu} \), and thus properties such as energy and angular momentum are quantized and can be superposed (which means on a non-local state of \( r_{\mu\nu} \)). Hence, the spin of the particles is also approximately quantized with non-local sense, e.g. a spin of electron can be "up" and "down" in the same time. Unlike microscopic particles, for macroscopic ones recall that the metric is simply \( g_{\mu\nu} \), and therefore the spin property is "classical" in the sense of locality and continuity of its values. The relativistic spin and the quantum spin can be shown in a related mathematical description using the Pauli–Lubanski pseudovector (see, for instance, [8], page 273).

5 Discussion

In this paper we introduced a non-local theory based on general relativity and on the notion of quantum foam. The model assume that each particle in the universe is described by the sum of two metrics, a local general relativistic metric and a global one that is influence on each particle in the universe. This global metric is non-local, i.e. there is "action at a distance", and it consists of foamions, a compact spacetime manifolds in the Planck length. This model try to understand quantum theory and general relativity from a single scheme, the spacetime metric and its properties. Several questions that should be addressed in a future work are: how to understand more deeply the properties of such foamions?, and how to describe Bosons and Fermions in the formulation of such metric?, furthermore, can quantum properties of black holes be understood from the model?. It is believed that these questions can be addressed and reveal a new insightful about the geometrodynamics of such particles and their structure. Moreover, the main role and the properties of the foamions should be more profoundly discussed.

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