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(Dated: April 28, 2013)

A single bit memory system is made with a brownian particle held by an optical tweezer in a double-well potential and the work necessary to erase the memory is measured. We show that the minimum of this work is close to the Landauer’s bound only for very slow erasure procedure. Instead a detailed Jarzynski equality allows us to retrieve the Landauer’s bound independently on the speed of this erasure procedure. For the two separated subprocesses, i.e. the transition from state 1 to state 0 and the transition from state 0 to state 0, the Jarzynski equality does not hold but the generalized version links the work done on the system to the probability that it returns to its initial state under the time-reversed procedure.

PACS numbers: 05.40.-a, 05.70.-a, 05.70.Ln, 89.70.Cf

The connection between thermodynamics and information is nowadays a widely studied problem [1–5]. The main questions concern the amount of energy necessary in order to perform a logical operation in a given time and how the information entropy is related to the free energy difference between the initial and final state of this logical operation. In this context the Landauer’s principle [6] is very important as it states that for any irreversible logical operation the minimum amount of entropy production is \(-k_B \ln(2)\) per bit commuted by the logical operation, with \(k_B\) the Boltzmann constant. Specifically a logically irreversible operation is an operation for which the knowledge of the output does not allow to retrieve the initial state, examples are logical AND, OR and erasure. In a recent paper [8] we have experimentally shown that indeed the minimum amount of work necessary to erase a bit is actually associated with this Landauer’s bound which can be asymptotically reached for adiabatic transformations. The question that arises naturally is whether this work corresponds to the free energy difference between the initial and final state of the system. To answer to this question it seems natural to use the Jarzynski equality [7] which allows one to compute the free energy difference between two states of a system, in contact with a heat bath at temperature \(T\). When such a system is driven from an equilibrium state \(A\) to a state \(B\) through any continuous procedure, the Jarzynski equality links the stochastic work \(W_{st}\) received by the system during the procedure to the free energy difference \(\Delta F = F_B - F_A\) between the two states:

\[
\langle e^{-\beta W_{st}} \rangle = e^{-\beta \Delta F}
\]

(1)

Where \(\langle \cdot \rangle\) denotes the ensemble average over all possible trajectories, and \(\beta = \frac{1}{k_B T}\) (see eq. 2 for the precise definition of the work \(W_{st}\)).

In this letter we analyze the question of the application of eq. 1 for estimating the \(\Delta F\) corresponding to the erasure operation in our experiment, in which a colloidal particle confined in a double well potential is used as a single bit memory. We will show that a detailed Jarzynski Equality is verified, retrieving the Landauer limit independently of the work done on the system.

The setup has already been described in a previous article [8] and we recall here only the main features.

A custom-built vertical optical tweezers is used to realize a two-state system: a silica bead (radius \(R = 1\mu m\)) is trapped at the focus of a laser beam (wavelength 1024nm) which is rapidly switched (at a rate of 10kHz) between two positions (separated by 1.45\(\mu m\)) using an acousto-optic deflector. A disk-shaped cell (18mm in diameter, 1mm in depth) is filled with a solution of beads dispersed in bidistilled water at low concentration. The bead used for the experiment is trapped by the laser and moved into the center of the cell (with gap \(\sim 80\mu m\)) to avoid all interactions with other beads. The trapped bead feels a double-well potential with a central barrier varying from 2\(k_B T\) to more than 8\(k_B T\) depending on the power of the laser (see figure 1, a and b). The left well is called “0” and the right well “1”. The position of the bead is tracked using a fast camera with a resolution of 108\(\mu m\) per pixel, which after treatment gives the position with a precision greater than 10\(\mu m\). The trajectories of the bead are sampled at 502Hz.

The logical operation performed by our experiment is the erasure procedure. This procedure brings the system initially in one random state (0 or 1 with same probability) in one chosen state (e.g. 0). It is done experimentally in the following way.

At the beginning the laser power is high (48mW) so that the central barrier is more than 8\(k_B T\) and the characteristic jumping time (Kramers Time) is about 3000s, which is long compared to the time of the experiment, and the bead is trapped in one well-defined state (0 or 1). The laser power is first lowered (in a time \(T_{lowering} = 1s\) to 15mW so that the barrier is about 2.2\(k_B T\) and the jumping time falls to 10s. A viscous drag force linear in time is induced by displacing the cell with respect to the laser using a closed-loop 3-axis NanoMax stage. The force is given by \(f = -\gamma v\) where \(\gamma = 6\pi R\eta\) (\(\eta\) is the vis-
about the initial state. For one bit of memory [6], it corresponds to a change in the entropy of the system \( \Delta S = -k_B \ln(2) \). The procedure can arbitrarily be decomposed in two kinds of sub-procedures: one where the bead starts in one well and ends in the other (e.g. \( 1 \rightarrow 0 \)) and one where the bead is initially in the same well where it should be at the end of the procedure (e.g. \( 0 \rightarrow 0 \)).

The two accessible quantities are \( x(t) \), which is measured, and \( f(t) \) which is imposed by the displacement of the cell. The derivatives are estimated using the discretization \( \dot{x}(t + \Delta t/2) \approx [x(t + \Delta t) - x(t)]/\Delta t \). Starting from these quantities it is possible to measure the stochastic work \( W_{st} \) done during the erasure procedure.

For a colloidal particle confined to one spatial dimension and submitted to a conservative potential \( V(x, \lambda) \), where \( \lambda = \lambda(t) \) is a time-dependent external parameter, one can define the stochastic work received by the system along a single trajectory [5]:

\[
W_{st}[x(t')] = \int_0^{t'} \frac{\partial V}{\partial \lambda} \dot{\lambda} dt
\]

Here the potential is \( V(x, t) = U_0(x, I(t)) - x \times f(t) \), where \( U_0 \) is due to the optical trapping and \( I(t) \) is the intensity of the laser (see figure 1). If the bead does not jump from one well to the other during the modulation of the height of the barrier this part of the procedure does not contribute to the work received by the bead. Then the work can be computed only when the external force is applied (between \( t = 0 \) and \( t = \tau \)). The force is directly the control parameter, and considering that \( f(t = 0) = 0 = f(t = \tau) \), it follows that the stochastic work is equal to the classical work \( W \):

\[
W_{st}[x(\tau)] = \int_0^{\tau} -x \dot{f} dt = \int_0^{\tau} f \dot{x} dt = W[x(\tau)]
\]

The two integrals have been calculated for all the trajectories of all the procedures tested. Amongst all of them, the maximal difference \( |W_{st} - W| \) observed was of 0.06\( k_B T \), which is negligible.

We now analyse the results of our experiments. For every chosen duration \( \tau \), the maximal force \( f_{max} \) was set to different values (typically between 1 and \( 6 \times 10^{-14} N \)). For each set of parameters (\( \tau, f_{max} \)), the procedure was repeated several hundreds of times to be able to compute statistical values. For each \( \tau \), the value of \( f_{max} \) is optimized in order to be as small as possible and give a proportion of success \( P_S > 90\% \).

The trajectories where the information is erased, i.e. the ones where the bead ends where it was supposed to be (e.g. in state 0), are selected. The mean of the work received \( \langle W \rangle_{=0} \) and the logarithm of the mean of its exponential \( -\ln(\langle e^{-\beta W} \rangle_{=0}) \) are calculated, where \( \langle \cdot \rangle_{=0} \) stands for the mean on all the trajectories ending in 0. We call the value \( -\ln(\langle e^{-\beta W} \rangle_{=0}) \) the effective free energy difference \( \Delta F_{eff} \). The error bars

The ideal erasure procedure is a logically irreversible operation because the final state gives no information on the double-well potential so that the bead ends always in the same well (e.g. state 0) independently of where it started. At the end, the force is stopped and the central barrier is raised again to its maximal value (in a time \( T_{rising} = 1s \)). Between two successive procedures the system is left to equilibrate for 4 s. The experimental procedure is sketched in figure 2. A procedure is fully characterized by its duration \( \tau \) and the maximum value of the force applied \( f_{max} \). Its efficiency is characterized by the “proportion of success” \( P_S \), which is the proportion of trajectories where the bead ends in the chosen well (e.g. 0), independently of where it started.
on this value are estimated by computing the mean on the data set with 10% of the points randomly excluded, and taking the maximal difference in the mean value observed by repeating this operation 1000 times. The results are shown in figure 3. The mean work \( \langle W \rangle_{\rightarrow 0} \) decreases when the duration of the procedure increases. For the optimized values of the force, it follows a law \( \langle W(\tau) \rangle_{\rightarrow 0} = k_B T \ln(2) + B/\tau \) where \( B \) is a constant, which is the behavior for the theoretical optimal procedure [9]. A least mean square fit gives \( B = 8.45 k_B T s \).

The effective free energy difference \( \Delta F_{eff} \) is always close to the Landauer limit \( k_B T \ln(2) \), independently of the value of the maximal force or the procedure duration.

![FIG. 3: Mean of the work (•) and effective free energy difference (×) for different procedures. The over-forced procedures (red) have a proportion of success \( P_S \sim 95\% \), the optimized procedures (black) have \( P_S > 91\% \), the under-forced procedures have \( P_S > 83\% \) (except the last point, that has \( P_S \approx 75\% \)). The fit (blue line) is: \( \langle W(\tau) \rangle_{\rightarrow 0} = k_B T \ln(2) + B/\tau \) with \( B \) a constant.](image)

The mean of the exponential can be computed on the sub-procedures by sorting trajectories in function of the initial position of the bead. Specifically:

\[
\langle e^{-\beta W} \rangle_{\rightarrow 0} = \frac{M_{10} + M_{00}}{2} \tag{4}
\]

where:

\[
M_{10} = \langle e^{-\beta W} \rangle_{1 \rightarrow 0} \quad \text{and} \quad M_{00} = \langle e^{-\beta W} \rangle_{0 \rightarrow 0} \tag{5}
\]

The values \( M_{10} \) and \( M_{00} \) are plotted in fig. 4. The sum \( M_{10} + M_{00} \) is always close to 1, which corresponds to the fact that \( \Delta F_{eff} \) is close to \( k_B T \ln(2) \), but \( M_{00} \) decreases with \( \tau \) whereas \( M_{10} \) increases consequently.

These results can be understood in the following way: Since the memory erasure procedure is made in a cyclic way and \( \Delta S = -k_B \ln(2) \) it is natural to await \( \Delta F \) to be close to \( k_B T \ln(2) \). But the \( \Delta F \) that appears in the Jarzynski equality is the difference between the free energy of the system in the initial state (which is at equilibrium) and the equilibrium state corresponding to the final value of the control parameter: \( F(\lambda(\tau)) - F(\lambda(0)) \).

Since the height of the barrier is always finite there is no change in the equilibrium free energy of the system between the beginning and the end of the procedure. Then \( \Delta F = 0 \), which implies \( \langle e^{-\beta W(\tau)} \rangle = 1 \). Nevertheless Vaikuntanathan and Jarzynski [10] have shown that when there is a difference between the actual state of the system (described by the phase-space density \( \rho_1 \)) and the equilibrium state (described by \( \rho^0 \)), the Jarzynski equality should be modified:

\[
\langle e^{-\beta W(\tau)} \rangle \approx \frac{\rho^0(x, \lambda(\tau))}{\rho(x, t)} e^{-\beta \Delta F(t)} \tag{6}
\]

Where \( \langle . \rangle_{(x,t)} \) is the mean on all the trajectories that pass through \( x \) at \( t \).

In our procedure, the selection of the trajectories where the information is actually erased, corresponds to fix \( x \) to the chosen final well (e.g. state 0) at the time \( t = \tau \). It follows that \( \rho(0, \tau) \) is directly \( P_S \), the proportion of success of the procedure, and \( \rho^0(0, \lambda(\tau)) = 1/2 \). Then:

\[
\langle e^{-\beta W(\tau)} \rangle_{\rightarrow 0} = \frac{1/2}{P_S} \tag{7}
\]

Similarly for the trajectories that end the procedure in the wrong well (e.g. state 1) we have:

\[
\langle e^{-\beta W(\tau)} \rangle_{\rightarrow 1} = \frac{1/2}{1 - P_S} \tag{8}
\]

Taking into account the Jensen’s inequality, i.e. \( \langle e^{-x} \rangle \geq e^{-\langle x \rangle} \), we find that equations 7 and 8 imply:

\[
\langle W \rangle_{\rightarrow 0} \geq k_B T [\ln(2) + \ln(P_S)]
\]

\[
\langle W \rangle_{\rightarrow 1} \geq k_B T [\ln(2) + \ln(1 - P_S)]
\]

![FIG. 4: Mean of the exponential of the work, for the sub-procedures \( 1 \rightarrow 0 \) (blue) and \( 0 \rightarrow 0 \) (red). For readability questions, only one value is shown for each \( \tau \): it corresponds to the procedure with the highest proportion of success \( P_S \).](image)
Notice that the mean work dissipated to realize the procedure is simply:

\[
\langle W \rangle = P_S \times \langle W \rangle_{\rightarrow0} + (1 - P_S) \times \langle W \rangle_{\rightarrow1}
\]  

(10)

where \(\langle \cdot \rangle\) is the mean on all trajectories. Then using the previous inequalities it follows:

\[
\langle W \rangle \geq k_B T \left[ \ln(2) + P_S \ln(P_S) + (1 - P_S) \ln(1 - P_S) \right]
\]  

(11)

which is indeed the generalization of the Landauer’s limit for \(P_S < 1\). In the ideal case where \(P_S = 1\), all the trajectories end in state 0 at time \(\tau\), and finally:

\[
\langle e^{-\beta W} \rangle = \langle e^{-\beta W} \rangle_{\rightarrow0} = 1/2.
\]  

(12)

Since this result remains approximatively verified for proportions of success close enough to 100\%, it explains why in the experiment we find \(\Delta F_{eff} \approx k_B T \ln(2)\).

To understand the evolution of the probability that the memory returns to its initial state under the time-reversed procedure, these results are important because they clarify the use of the Jarzinsky equality in irreversible operations.

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ACKNOWLEDGEMENTS

We thank David Lacoste, Krzysztof Gawedzki, Luca Peliti and Christian Van den Broeck for very useful and interesting discussions. This work has been partially supported by ESF network "Exploring the Physics of Small Devices."

[1] C. H. Bennett. Int. J. Theor. Phys. 21, 905-940 (1982).
[2] K. Maruyama, F. Nori and V. Vedral. Rev. Mod. Phys. 81, 1-23 (2009).
[3] C. Van den Broeck. Nature Phys. 6, 937-938 (2010).
[4] M. Bauer, D. Abreu and U. Seifert. J. Phys. A: Math. Theor. 45, 162001 (2012).
[5] U. Seifert. Rep. Prog. Phys. 75, 126001 (2012).
[6] R. Landauer, IBM J. Res. Develop. 5, 183-191 (1961).
[7] C. Jarzynski, Phys. Rev. Lett. 78, 2690-2693 (1997).
[8] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz. Nature 483, 187-189 (2012)
[9] E. Aurell, K. Gawedzki, C. Mejia-Monasterio, R. Mohayaee, and P. Muratore-Ginanneschi. J. Stat. Phys. 147, 487-505 (2012).
[10] S. Vaikuntanathan and C. Jarzynski, Euro. Phys. Lett., 87, 60005 (2009).
[11] R. Kawai, J. M. R. Parrondo, and C. Van den Broeck. Phys. Rev. Lett. 98, 080602 (2007).
[12] T. Sagawa, M. Ueda. Phys. Rev. Lett. 104, 090602 (2010).
[13] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki and M. Sano. Nature Phys. 6, 988-992 (2010).