Computational Treatment of Transient MHD Slip Flow of an Arrhenius Chemical Reaction in a Convectively Heated Vertical Channel

Muhammed M. Hamza1, Muhammad Z. Shehu2, Halima Usman1 and Emmanuel Omokuhaule3

A numerical computational treatment of transient electrically conducting fluid with an Arrhenius chemical reaction in the presence of Navier slip and Newtonian heating is obtained by using implicit finite difference scheme. A transverse magnetic field is applied to the flow direction due to the exothermic nature of the fluid. Numerical computation shows that, higher values of Frank-Kamenetskii parameter (λ) and Biot number (Br) significantly influence the transport phenomenon. Irrespective of smaller or larger time, Magnetic parameter (M) reduces velocity of the fluid as well as wall shear stress.

Keywords: Transient; Magnetohydrodynamic (MHD) flow; Arrhenius chemical reaction; Newtonian heating; Navier slip.

1. Introduction

At the present time, flow of MHD has a wide area of usage from defense to automotive industry, from medical field to basic science, such as chemistry and biology. Preventing the development of MHD applications is due to a delay in malignant tumor therapy, which results in less bleeding in the case of severe injuries, magnetic reinforcement of some visual and numerous other diagnostic tastes [10]. In addition, MHD flow and heat transfer from fluid along elastic surface have more than one engineering application. Many industries, such as metallurgical operation, thread dragged and refrigerating of continuous strip through an inclusive fluid and refinement of molten metal from non-metallic applications, use these techniques. Many researchers for example [1-10] tried to examine the MHD flow and heat transfer pertaining the stretchable surface under diverse boundary condition.

In the real world, the flow through parallel have numerous applications in aerospace, chemical, civil, environmental, mechanical and biomechanical engineering. Many researchers with sufficiently great interest in studying the slip effect in different geometries for different fluid [11-14]. Nandeppanavar et al. [15] carried out a detailed examination of slip flow and heat transfer with non-linear, Navier boundary condition using a stretchable surface. Besides, the study of a magnetic field second order slip and thermal radiation effect on nano fluid flow have been examined by Hakeem et al. [16] using a stretching or shrinking sheet. The boundary layer flow of a nanofluid with the influence of nanomaterial migration and second order slip has been studied by Zhu et al. [17]. Some published work on this topic can be found in reference [18-19]. Exothermic chemical reaction flow has application both in industry and engineering, thus in polymer expulsion, atomic reactor plant, geophysics and energy storage system. In respect to this application, the array solution to the reactive variable viscosity of Couette flow under Arrhenius chemical kinetic in the present of exothermic reaction was examined by Gbadeyan and Hassan [20]. Inherent irreversibility of hydro magnetic third grade reactive Poiseuille flow of a variable viscosity in porous media in convective cooling was studied by Salawu and Fatunmbi [21]. They examined the sensitized, Arrhenius and Bimolecular chemical kinetic of a single step exothermic reaction.

On the other hand, a careful discussion on slip flow outrage has been expressed in particular because of its near-term applicability in mother science and technology. Investigations on free
convection slip flow in a vertical channel with different boundary condition has been studied by many researchers [22-24]. Most recently, Hamza et al. [25] investigated the steady state MHD free convection slip flow of an exothermic fluid in a convectively heated vertical channel. The goal of this study is to extend Hamza et al. [25] work by incorporating time dependent MHD flow of an Arrhenius chemical kinetic in a convectively heated vertical channel. This research is significant because it uses the implicit finite difference scheme to analyze the influence of an Arrhenius chemical reaction in MHD time dependent free convection flow in a convectively heated vertical channel. This type of results is use in Bioengineering and diverse clinical therapeutic interventions which including drug permeation through human skin, it may also be relevant in paper manufacturing sector as well petroleum industries to ascertain the flow of oil through rock layers.

2. Formulation of the problem

Consider a time dependent free convection flow of electrically conducting and chemically reacting fluid in a vertical channel with Navier slip condition and Newtonian heating. The flow is induced by the convective heating introduced on the lower surface of the channel wall as well as the reactive property of the fluid. Following Hamza [24] and Hamza et al. [25], the non-dimensional governing equations under the Boussinesq’s approximation can be written as:

\[
\frac{\partial u'}{\partial t'} + \mathbf{v} \cdot \nabla u' = \nabla^2 u' + \frac{\sigma R^2 B^2}{\rho} \nabla \theta' \tag{1}
\]

\[
\frac{\partial \theta'}{\partial t'} = \frac{k}{\rho C_p} \left[ \frac{\partial^2 \theta'}{\partial y'^2} + \frac{Q C_o A}{\rho C_p} e^{\frac{E}{RT_i}} \right] \tag{2}
\]

The initial and boundary conditions for the present problem are

\[
\begin{align*}
& t' \leq 0: u' = 0, \quad T' = T_i, \quad 0 \leq y' \leq H \\
& t' > 0: u' = \frac{Q C_o A}{\rho C_p} e^{\frac{E}{RT_i}}, \quad k \frac{\partial \theta'}{\partial y'} = h \left[ \frac{T_i - T'}{T_i} \right], \quad \text{at } y' = 0 \\
& u' = 0, \quad T' = T_i, \quad \text{as } y' \to H \\
\end{align*}
\tag{3}
\]

where \( \beta \) is the coefficient of thermal expansion, \( Q \) is the heat of reaction, \( A \) is the rate constant, \( E \) is the activation energy, \( R \) is the universal gas constant, \( \nu \) is the kinematic viscosity, \( C^{\infty}_o \) is the initial concentration of the reactant species, \( g \) is the gravitational force, \( C^p \) is the specific heat at constant pressure, \( k \) is the thermal conductivity of the fluid, while \( \rho \) is the density of the fluid.

To solve equations (1) and (2), we employ the following dimensionless variables and parameters

\[
y' = \frac{y}{H}, \quad t' = \frac{t \mu_i}{H^2}, \quad \theta' = \frac{E(T_i - T)}{RT_i}, \quad \dot{e} = \frac{RT_i}{E}, \quad \dot{u}' = \frac{u' \mu_i E g \beta H^2}{RT_i^2}, \quad \lambda = \frac{Q C_o A E H^2}{RT_i^2} \left( \frac{e^x}{x^2} \right), \quad M = \frac{\sigma R^2 H}{\nu \rho}, \quad Pr = \frac{\mu \beta C}{k}
\]

Using (4), the equations (1) to (2) take the following form:

\[
\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial y^2} + \theta - MU \tag{5}
\]

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \frac{\lambda}{Pr} \left( \frac{\partial}{\partial y} \left( \frac{\partial \theta}{\partial y} \right) \right) \tag{6}
\]

The initial and boundary conditions in dimensionless form are

\[
\begin{align*}
& \frac{\partial U}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = 0, \quad \text{at } y = 0 \\
& U = 0, \quad \theta = 0, \quad \text{at } y = 1 \\
\end{align*}
\tag{7}
\]

where \( M, Pr, \lambda \) and \( E \) are magnetic parameter, Prandtl number, Frank-Kamenetskii parameter and activation energy parameter respectively. The other non-dimensional quantities are the skin friction (\( C_f \)), and the heat transfer rate, (\( Nu \)) which are given as:

\[
C_f = \left. \frac{\partial U}{\partial y} \right|_{y=0.1}, \quad Nu = \left. \frac{\partial \theta}{\partial y} \right|_{y=0.1}
\]

where \( C_f \) is the skin friction and \( Nu \) is the Nusselt number.

3. Numerical Solutions

The set of partial differential equations (5) and (6) with the boundary conditions (7) are solved numerically using implicit finite difference scheme. We used forward difference formulas for all time derivatives and approximate both the second and first derivatives with second order central differences. The equations corresponding to the first derivative and last grid points are modified to incorporate the boundary conditions.
The implicit finite difference equation corresponding to equations (5) and (6) are as
\[ -r_1 y_j^{(N-1)} + (1 + 2r_1) y_j^{(N-1)} - r_1 y_j^{(N-1)} = r_j^{(N)} + (1 - 2r_2 - M \Delta t) y_j^{(N)} + r_1^{(N)} + \Delta t \theta_j^{(N)} \]
\[ -r_1 \theta_j^{(N-1)} + (Pr + 2r_1) \theta_j^{(N-1)} - r_1 \theta_j^{(N-1)} = r_2 \theta_j^{(N)} + (Pr - 2r_2) \theta_j^{(N)} + r_2 \theta_j^{(N)} + \lambda \Delta t \exp \left( \frac{\theta_j^{(N)}}{1 + \epsilon \theta_j^{(N)}} \right) \]

where \( r_1 = \xi \Delta t / \Delta y \), \( r_2 = (1 - \xi) \Delta t / \Delta y \) and \( 0 \leq \xi \leq 1 \). We chose \( \xi = 1 \) so that we are free from choosing smaller or larger time steps.

4. Results and Discussion

Free convection flow of viscous reactive fluid has been considered in a convectively heated vertical channel with Navier slip boundary condition. The numerical computation results are displayed in Figures (1) to (5). The governing parameters controlling the flow systems are: the Prandtl number (Pr), Biot number (Br), Navier slip parameter (\( \lambda \)), activation energy (\( \epsilon \)), Frank-Kamenetskii parameter (\( \lambda \)), and non-dimensional time (\( t \)). Unless otherwise stated, the following values are assigned to the governing parameters: Pr = 0.71, Br = 0.1, \( \lambda = 0.1 \), \( \gamma = 0.1 \), \( \epsilon = 0.01 \), M = 1 and \( \theta = 1 \).

Figures 1a and b show the impact of the reaction of Arrhenius kinetic parameter (\( \lambda \)) and non-dimensional time (\( t \)) on temperature and velocity distribution respectively. It has been discovered that when the value of \( \lambda \) and \( t \) increase, so does the temperature and velocity of the fluid increases. According to Hamza [24], increasing \( \lambda \) and \( t \) improves the strength of the chemical reaction and viscous heating source factor in the temperature equation, which improves both temperature and velocity sequentially.

Figures 2a and b present the impact of the local Biot number (Br) and non-dimensional time (\( t \)) on temperature and velocity profile sequentially. By increasing the Br and \( t \), Figures 2a and b grows. The presence of Newtonian heating and Navier slip on both temperature and velocity lead to the enhancement of both temperature and velocity at lower plate of the channel. The findings of Hamza [24], and Makinde and Aziz [26] concord with the present results.

Figures 3a and b are graphed to show the variation of M and \( t \) on velocity profiles. The graphical results revealed that a rise in M indicates a decrease in fluid velocity at small time or larger time. As M is increased, a resistive drag force is created that acts in the opposite direction of the transportation, lowering fluid velocity while when \( t \) is increased, the opposite development is noticed. Figures 4a and b represent the effect of shear stress against M for different values of \( t \) at \( y = 0 \) and \( y = 1 \) respectively. At both plates, skin friction decreases as M increase. Also increasing \( t \) lead to the substantial increase in the wall shear stress at both plates. Figures 5a and b illustrate the influence of \( \lambda \) on skin friction for different values of \( t \) at the plate \( y = 0 \) and \( y = 1 \) respectively. Increasing value of \( \lambda \) and \( t \) significantly enhance the skin friction at both plates. Table 1 represents the comparison between the present work and the work of Hamza [24]. When M = 0, the result of Hamza [24] is recovered.

Table 1: Comparison between the present work and the work of Hamza [24].
Fig 2: Influence of Br and t on Temperature and Velocity profiles

Fig 3: Influence of M and t on Velocity profiles

Fig 4: Influence of M and t on wall shear stress
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Fig5: Influence of and t on wall shear stress

Table 1: Comparison between the present work and the work of Hamza [24]

| Y | $\lambda$ | Hamza [24] | Present Work | M =1 | M = 2 |
|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.0272 | 0.0272 | 0.0264 | 0.0241 |
| 0.1 | 0.2 | 0.0514 | 0.0514 | 0.0498 | 0.0455 |
| 0.1 | 0.3 | 0.0758 | 0.0758 | 0.0735 | 0.0671 |
| 0.1 | 0.4 | 0.1004 | 0.1004 | 0.0973 | 0.0890 |
| 0.1 | 0.5 | 0.1253 | 0.1253 | 0.1214 | 0.1110 |
| 0.1 | 0.6 | 0.1503 | 0.1503 | 0.1457 | 0.1332 |
| 0.1 | 0.7 | 0.1756 | 0.1756 | 0.1703 | 0.1557 |

5. Conclusion

A numerical approach on transient flow of electrically conducting fluid in a vertical channel is studied along with the impact of pertinent parameters such as MHD, chemical reaction parameter and non-dimensional time. By utilizing implicit finite difference scheme, the non-dimensional governing equations are solved numerically. Some of the major findings are:

(i) Both chemical reaction parameter and Biot number significantly increases the velocity and temperature of the fluid.

(ii) Magnetic parameter decreases the velocity of the fluid.

(iii) It is noticed that, increasing non-dimensional time greatly influence the transport phenomenon.

Conflicts of Interest

The authors declare that there is no conflict of interest.

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