Compact U(1) Gauge Theory on Lattices with Trivial Homotopy Group

C. B. Lang

Institut für Theoretische Physik
Universität Graz
A-8010 Graz, Austria

and

T. Neuhaus

Fakultät für Physik
Universität Bielefeld
D-33615 Bielefeld, Germany

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Abstract

We study the pure gauge model on a lattice manifold with trivial fundamental homotopy group, homotopically equivalent to an $S_4$. Monopole loops may fluctuate freely on that lattice without restrictions due to the boundary conditions. For the original Wilson action on the hypertorus there is an established two-state signal in energy distribution functions which disappears for the new geometry. Our finite size scaling analysis suggests stringent upper bounds on possible discontinuities in the plaquette action. However, no consistent asymptotic finite size scaling behaviour is observed.
1 Motivation

Pure gauge lattice QED is a prototype model of gauge interactions of abelian nature and was among the first gauge systems studied in Monte Carlo simulations \[1, 2, 3\]. It is also a starting points of the discussion of the more realistic lattice field theory containing dynamical fermion degrees of freedom. Unfortunately the status of nonperturbative studies of full QED is still controversial. In this situation it is important to clarify the fixed point properties i.e., the critical properties and critical exponents of the pure gauge system.

From the early beginnings this field theoretic lattice model has posed many interesting questions about its continuum limit. The compact formulation of the pure gauge model has a confining strong coupling phase and a spin-wave (Coulombic) phase. The order of the phase transition has been controversial throughout the years. However, it was generally accepted that the formulation with the Wilson action on a finite lattice leads to a clear two-state signal at the transition, indicating a first order deconfinement phase transition \[4\].

Adding extra terms to the action introduces new couplings. In this larger space the nature of the phase transition may change. Allowing for an adjoint coupling $\gamma$ (multiplying the plaquette variable in the adjoint representation \[5\]) a clear increase of the energy gap towards positive $\gamma$ was observed \[6\]. Based on a detailed study of discontinuities of the plaquette expectation value in the $\beta - \gamma$ plane and, using a power law scaling ansatz for the vanishing of discontinuities along the first order phase transition line, the location of a possible change of the order of the deconfinement phase transition to a second order phase transition line was estimated \[4, 6\]. From that extrapolation the infinite volume location of the so-called tricritical point in pure gauge compact QED was expected to lie at slightly negative values of the coupling $\gamma$. Thus the theory on the Wilson line was thought to have a weak first order phase transition, however close to a tricritical point in the extended parameter space.

Let us stress here, that up to now no direct simulation of pure compact QED did unanimously show the existence of a second order phase transition point. It is well known from statistical mechanics systems that sizeable crossover effects from a discontinuous to a continuous behaviour are present on finite lattices in the vicinity of a tricritical point. Such effects are also present in pure gauge compact QED even at reasonably large negative val-
ues of $\gamma$ and they spoil attempts to extract critical exponents of the system using finite size scaling methods. However, due to the central role of the model as a starting point of full QED and due to its prototype character in lattice gauge studies, it would be very important to know e. g. the value of the correlation length divergence exponent $\nu$. A possible determination then could tell us whether $D = 4$ pure gauge compact QED might be in the universality class of the free theory with $\nu = 0.5$, or not. We may remark here that MCRG studies of the model showed in the vicinity of the Wilson line a flow of couplings, which in spite of the two state signal resembled the flow diagram of a second order critical point quite well [7].

The nature of the deconfinement phase transition appears to be inherently connected to topological excitations present in this formulation (where the gauge fields assume their values in the group $U(1)$, not in the algebra). Formulated on the dual lattice these are closed loops of monopoles. In several investigations a strong correlation between the behaviour of the monopole loop density with the internal energy variable was observed [3, 8, 9, 10, 11]. If the monopole loops are forbidden or suppressed by further terms in the action [9, 11] a shifting of the phase transition towards smaller values of the plaquette coupling $\beta$ was found.

Here we concentrate on a modification of the simulation, which appears to be softer than introducing additional terms to the action, and which should become irrelevant in the thermodynamic limit. Instead of choosing periodic (hypertorus) boundary conditions we work on a lattice topology homotopic to the surface $S^4$ of a 5D hypersphere. In the infinite volume limit the two versions should lead to equivalent results.

There is a subtle difference, which, however, may be important. The monopole loops are nonlocal objects. The Monte Carlo update algorithms are intrinsically local (except for scarce attempts towards multigrid updating). This is related to the gauge symmetry nature of the field variables. On the hypertorus closed loops lie in equivalence classes defined by the fundamental homotopy group of the torus: $\Pi_1(T^4) = \mathbb{Z}_4^4$. This puts some restrictions on the fluctuation behaviour of such loops. Hypercooling a hot configuration shows that monopole loops wrapped around the torus may get stuck like rubberbands [8]. Recently this behaviour was studied and classified in more detail [12]. On the other hand, the fundamental homotopy group of the surface of a sphere is (for $D > 2$) trivial: $\Pi_1(S^4) = \mathbb{I}$. There monopole loops may contract and disappear without restriction.
Preliminary results with half the statistics and for $\gamma = 0$ have been published earlier [13].

2 Action and Lattice Geometry

The action on the lattice $\Lambda$ is given

$$S = \beta \sum_{P \in \Lambda} \text{Re}(U_P) + \gamma \sum_{P \in \Lambda} \text{Re}(U_P^2)$$

(1)

where $U_P$ denotes the usual plaquette variable and we consider all plaquettes with equal weight (see, however the discussion at the end).

For the construction we choose the 4-manifold as the boundary of a 5D hypercube $H_5(L)$ with linear extend $L$ ($L^5$ sites). We denote this boundary by $SH_4(L)$; it consists of all sites which have at least one cartesian coordinate value 1 or $L$. The manifold $SH_4(L)$ has

- sites: $10(L-1)^4 + 20(L-1)^2 + 2$
- links: $40(L-1)^4 + 40(L-1)^2$
- plaquettes: $60(L-1)^4 + 20(L-1)^2$
- 3-cubes: $40(L-1)^4$
- 4-cubes: $10(L-1)^4$

(2)

thus ratios of e.g. $n_{\text{links}}/n_{\text{sites}}$ for the $SH_4(L)$ lattice approach for large $L$ asymptotically the values for the hypercubic geometry. Whereas the hyper-torus lattice $T_4(L)$ is self-dual, $SH_4(L)$ is obviously not. For an identification of the monopole loops we did construct the dual manifold $SH'_4(L)$ (which has the same homotopy properties) as usual, identifying the dual sites with the centers of the hypercubes of the original lattice. This was done to check the dynamics of the updating algorithm.

We define the volume of the lattice $V \equiv |\Lambda|$ as the number of plaquettes and we use $V^{\frac{4}{3}}$ as the linear size variable; as finite size scaling asymptotic this definition is as arbitrary as any other. In the computer programs the geometrical properties have been implemented by tables providing vectorization possibilities.
3 Simulation and Coupling Parameters

We performed long runs with an integrated statistics of 500K up to 1500K sweeps for each lattice size, using a combination of 3-hit Metropolis updating with one overrelaxation step.

Lattices $SH(L)$ were studied for $L = 4, 6, 8, 10$ having roughly the same number of variables as $6^4 \ldots 16^4$ lattices with torus geometry. Most of our data was taken for values of $\gamma = 0$ and $\beta$ close to the transition.

Simulations were done for various coupling values close to the pseudocritical values. For given lattice size the histograms from different couplings were combined to one multi-histogram according to the technique of Ferrenberg and Swendsen [14]. Here we introduce statistical mechanics notations and define the partition function by

$$Z_L(\beta) = \sum_E \rho_L(E) \exp(-\beta E), \quad (3)$$

where we denote the internal energy function by $E = -\sum_P \text{Re}(U_P)$. Note that $E$ is extensive and negative for positive $\beta$-values. Correspondingly we can introduce energy densities $e = E/V$, which up to a negative sign correspond to the plaquette operator. Here $\rho(E)$ denotes the density of states for each value of $E$ and is determined from the combined individual distributions. In our simulation we determine expectation values of the specific heat and two cumulants [15, 16]

$$C_V(\beta, L) = \frac{1}{V} \langle (E - \langle E \rangle)^2 \rangle,$$

$$V_{BCL}(\beta, L) = -\frac{1}{3} \frac{\langle (E^2 - \langle E \rangle^2)^2 \rangle}{\langle E^2 \rangle^2}, \quad (4)$$

$$U_4(\beta, L) = \frac{\langle (E - \langle E \rangle)^4 \rangle}{\langle (E - \langle E \rangle)^2 \rangle^2}.$$

Our definition of $V_{BCL}$ differs by an additive constant $\frac{2}{3}$ from that of [15]. The specific heat $C_V$ develops a maximum, while $V_{BCL}$ and $U_4$ both develop minima as functions of $\beta$ for given lattice size. We define pseudocritical couplings through the peak positions of these quantities and peak values of the considered quantities by their values at pseudocriticality.

We observed quite long autocorrelation lengths at the corresponding pseudocritical points for each lattice size, with peak values of $\tau_{int}$ ranging from
15 (for $L = 4$) up to 750 (for $L = 10$). This was one of the reasons for doubling the statistics and introducing overrelaxations steps as compared to the preliminary results presented in [13].

4 Results and scaling analysis

4.1 Pseudocritical couplings

Table 1 gives the pseudocritical couplings and the values of the cumulants at the peak positions.

In fig. 1 we compare the multihistograms resulting from all data for given $L$, that is $h_L(\beta, e = E/V) \equiv \rho_L(E) \exp(-\beta E)$ at the values of the peak positions of $C_V$. These are the effective probability distributions for the action at these couplings. We find no indication of double peaks, in distinct contrast to results for the same action but standard hypertorus geometry [4, 10, 13, 17].

We then performed runs at $\gamma = 0.2$ where torus-results had identified even more prominent two-state signals. An analogous multihistogram analysis there showed pronounced two-state signals for the theory on $SH$ lattices even on the smallest lattice sizes. Whenever this situation occurs, tunneling becomes scarce on larger lattices and the multihistogram analysis becomes impracticable. In this case one should rely on multicanonical simulations [18]. For that reason we did not attempt simulations for $SH(L > 6)$ at this value of $\gamma$.

Returning to $\gamma = 0$ we display in fig. 2 as a function of the variable $1/V$ a comparison of pseudocritical $\beta$-values on $SH$ lattices, namely $\beta(C_V), \beta(V_{BCL})$ and $\beta(U_4)$, with results from simulations on the standard hypertorus geometry for $\beta(C_V)$ [14]. We find a strikingly different finite size scaling behaviour when comparing both sets of data. The pseudocritical $\beta$-values of the $SH$ lattices approach the infinite volume critical $\beta_c$-value from above, while data on the hypertorus do so from below. Furthermore our data indicate in case of $\beta(V_{BCL})$ a non-monotonic behaviour on the $SH$ lattice, while on the standard hypertorus the approach seems to be monotonic for all quantities.

In principle finite size scaling theory gives the asymptotic approach of the considered pseudocritical $\beta$-values. If the deconfinement phase transition is a continuous phase transition the homogeneous free energy density $f_{hom}(\beta)$ =
\( f_{\text{hom}}(\frac{\xi}{L}) \) scales with the scaling variable; with the usual definition of the correlation length divergence exponent \( \nu \) one has for large volume systems the asymptotic behaviour

\[
\beta_{c,V}^{\frac{1}{4}} = \beta_c + aV^{-\frac{1}{4\nu}}.
\]  

(5)

Alternatively, if the phase transition is of discontinuous nature (first order), we expect an expansion of the pseudocritical \( \beta \)-values in powers of the inverse volume \( \frac{1}{V} \)[20, 21]

\[
\beta_{c,V} = \beta_c + \frac{a}{V} + \mathcal{O}(1/V^2).
\]  

(6)

In fig. 2 we display a fit to the \( \beta(C_V) \)-values on the standard hypertorus according to the scaling law (6) including \( 1/V^2 \) corrections (dashed curve in the figure). This fit gives a consistent description of the data and results in the infinite volume \( \beta_c \)-estimate \( \beta_c = 1.01102(9) \). In the current context this demonstrates the consistency of the data with a finite size scaling behaviour of a discontinuous transition in the considered volume size interval. As already mentioned, this was anticipated for the case of the hypertorus geometry. That fit alone is not sufficient, though. A definite statement on whether the deconfinement phase transition is of first order type would require the calculation of its latent heat and interface tension in the thermodynamic limit. Such an analysis is beyond the scope of our present work. Later on we argue, indeed, that evidence for a weaker first order or even continuous phase transition can be obtained from \( SH \) lattices.

Returning to the theory on \( SH \) lattices we note that by construction the theory is not homogeneous. On the surface of the 5D hypercube there are inhomogeneities related to its edges i.e., the theory on the \( SH \) lattice exhibits less symmetries than formulated on a hypertorus. In general one then expects additional finite size scaling violations which may be parametrized by a contribution \( f_{\text{inhom}}(\beta) \) to the free energy density on the \( SH \) lattice. In the thermodynamic limit \( f_{\text{inhom}}(\beta) \) will vanish.

In absence of a rigorous theory we may estimate the dimensionality of the additional terms by considering quantities sensitive to the inhomogeneity of the lattice. For instance we might consider ratios of the number of plaquettes to the number of links [2], or the number of links which are contained in 4 or 5 different plaquette variables as compared to the standard case where each link is contained in 6 different plaquette variables. These considerations suggest a dimensionality of the inhomogeneous free energy density part \( \propto 1/V^{\frac{1}{2}} \).
The usual ansatz for the partition function (the indices $o$ and $d$ denote ordered and disordered phase) of a discontinuous system, neglecting interfacial effects, is

$$Z(\beta) = \exp(-V f_d) + A \exp(-V f_o).$$

(7)

Here the quantity $A$ parametrizes the asymmetry. The free energy densities $f_o(\beta)$ and $f_d(\beta)$ can be expanded in powers of $\Delta \beta = \beta - \beta_c$

$$f_o = f_{o,\text{hom}} + f_{o,\text{inhom}} = c + e_o \Delta \beta + V^{-\frac{1}{2}}(\eta_o + \sigma_o \Delta \beta) + O((\Delta \beta)^2)$$

(8)

(and correspondingly for $f_d$). In the infinite volume limit $f_o = f_d = c$ at $\Delta \beta = 0$. At finite volume we find at the metastability point, where both contributions have same weight,

$$\ln A + e_o \Delta \beta + V^{-\frac{1}{2}}(\eta_o + \sigma_o \Delta \beta) = e_d \Delta \beta + V^{-\frac{1}{2}}(\eta_d + \sigma_d \Delta \beta) + O((\Delta \beta)^2).$$

(9)

Solving for $\Delta \beta$ we then expect the finite size scaling law of (6) to be replaced by the form

$$\beta_{c,V} = \beta_c + \frac{a}{V^{\frac{1}{2}}} + \frac{b}{V} + O(V^{-\frac{3}{2}}),$$

(10)

if the phase transition was of discontinuous nature. It turns out that the dominant term now is $O(1/V^{\frac{1}{2}})$.

In fig. 2 we show fits of the pseudocritical $\beta$-values for the $SH$ lattices according to (10) (dotted curves). These fits result in a positive contribution proportional to $1/V^{\frac{1}{2}}$ and a negative contribution proportional to $1/V$, which actually is of the same order of magnitude as in case of the hypertorus. Thus the observed non-monotonic behaviour observed in the quantity $\beta(V_{BCL})$ could be explained by the interplay of a contribution related to the inhomogeneity, entering with the opposite sign, as compared to a contribution $\propto 1/V$, which may be attributed to the homogeneous part of the free energy density. The obtained infinite volume $\beta_c$-estimates from these fits $\beta_c(CV) = 1.01133(13)$, $\beta_c(V_{BCL}) = 1.01134(15)$ and $\beta_c(U4) = 1.01125(16)$ are consistent with the result from the hypertorus, though a tendency towards values slightly higher by about one standard deviation is noticeable. Again we emphasize that this analysis is merely a statement on the consistency of different finite size scaling fits in the considered and limited volume.
size interval i.e., both regularisations on the hypertorus and the SH lattice support a unique infinite volume $\beta_c$-value. As far as the SH lattice is concerned, we conclude, that the behaviour of the pseudocritical $\beta$-values is strongly affected by contributions stemming from the inhomogeneity of the system. Consequently we have to pay special attention to the effect of these contributions when in the later analysis we study finite volume estimators of energy gaps at the deconfinement phase transition.

We have also analyzed the SH lattice pseudocritical $\beta$-values assuming the finite size scaling form (5) corresponding to a critical point. Including data with $L \geq 8$ into the fit we obtain $\nu$-values of about $\nu \approx 0.53(20)$ and $\beta_c$-values consistent with the above determinations. However, as discussed such a value of $\nu$ close to 0.5 can just be mimicked by contributions related to the inhomogeneity of the system (10). We therefore cannot trust these fits, as long as these contributions are sizable and likely to be present. This observation also indicates that an unambiguous extraction of critical exponents of the deconfinement phase transition, if it were in fact a continuous transition, would require much larger lattice sizes, or alternatively, a formulation of the theory with smaller finite size scaling violations.

\section{4.2 Cumulant peak values}

The peak values of the specific heat and cumulants defined in (4) give in the thermodynamic limit unambiguous signals on the order of the considered phase transition. If the phase transition is of discontinuous nature one finds in the thermodynamic limit $L \to \infty$ on general grounds

\begin{align}
\frac{C_{V,\text{max}}}{V} &= \frac{(e_o - e_d)^2}{4}, \\
V_{BCL,\text{min}} &= -\frac{(e_o^2 - e_d^2)^2}{12(e_o e_d)^2}, \\
U_{4,\text{min}} &= 1.
\end{align}

(11)

Here $e_o$ and $e_d$ denote the discontinuous values of the energy density at the critical point. In the context of the present paper $-e_o$ corresponds to the plaquette expectation value in the Coulomb phase, $-e_d$ to its expectation value in the confinement phase. For a continuous phase transition $e_o = e_d$. In this case $U_{4,\text{min}}$ can be different from 1.
Again finite size scaling theory predicts for discontinuous and homoge-
neous systems, that finite size corrections to above quantities on finite vol-
umes follow asymptotically an expansion in powers of the inverse volume
$1/V$, similar to the expansion written down in (6). In fig. 3a we display a
comparison of results on the hypertorus (triangles) with results on $SH$
lattices (circles) for the quantity $C_{V,max}(L)/V$ as a function of the parameter
$1/V$. This quantity can be directly interpreted as a finite volume estimator
of a possible infinite volume discontinuity in the internal energy or the pla-
quetter expectation value, correspondingly. We observe for both geometries
a deviation from the leading asymptotic finite size scaling with terms pro-
portional $1/V$ i.e., higher order terms in the corresponding expansions are
important. It is therefore rather doubtful, whether a fit to the data, even
including higher orders, could lead to a unambiguous determination of the
infinite volume discontinuities.

Here we are not that ambitious. We note, however, that the $SH$
data exhibit much lower values for the finite volume estimator of the discontinuity
on comparable volume sizes. Thus, if the observed finite size scaling on the
$SH$ lattice stays monotonic in the variable $1/V$ up to to the thermodynamic
limit, then the $SH$ results predict rather stringent upper bounds on the values
of possible discontinuities at the phase transition point. In light of the above
discussion on possible effects related to the inhomogeneous part of the free
energy density we have to be rather careful. We therefore performed in
our analysis several fits to the observable $C_{V,max}(L)/V$ including $1/V^{\frac{1}{2}}, 1/V$
and also corrections proportional to $1/V^{\frac{3}{2}}$. In none of these cases we found
indication of a non-monotonic behaviour. The dotted curve of fig. 3a displays
as an example such a fit with the form

$$\frac{C_{V,max}}{V} = \frac{(e_o - e_d)^2}{4} + \frac{a}{V^{\frac{1}{2}}} + \frac{b}{V}.$$ (12)

Thus we convert the value obtained on the $SH(10)$-lattice, $C_{V,max}(10)/V =
.0000528(13)$, to an upper bound on the possible discontinuity in $e_d - e_o$ in
the thermodynamic limit

$$e_d - e_o \leq 0.0145(2),$$ (13)

which then should be satisfied by pure gauge compact QED with the Wilson
action in the hypertorus geometry as well as on $SH$ lattices. We remark here
that such a bound is more stringent than any gap observed or extrapolated from torus results \[3\]; simulations on the hypertorus on $16^4$ lattices typically result in estimates of the discontinuity of about 0.030 on that lattice size. In a study of the discontinuity in the $\beta - \gamma$ plane of couplings a power law ansatz suggested an infinite volume value of the above discontinuity of about 0.016 \[3\]. However the statement here is that (13) constitutes a bound and that the true value of the discontinuity is likely to be still smaller. Thus compact pure gauge QED on $SH$ lattices provides evidence that the first order deconfinement phase transition on the Wilson line is at least weaker than previously thought.

In fig. 3b we also display a comparison of results on the hypertorus (triangles) with results on $SH$ lattices (circles) for the quantity $V_{BCL,\min}$. The remarks concerning deviations from asymptotic finite size scaling in the preceding paragraph are appropriate here as well. Performing various fits of the already mentioned forms (dotted curve in fig. 3b) we do not find a non-monotonic behaviour. Again the observed monotonic behaviour of the quantity $V_{BCL,\min}$ lead to an upper bound on possible discontinuities. On the $SH(10)$ lattice (cf. table 1) we obtain the inequality

$$e_d - e_o \leq -0.022(1)\ e_d. \quad (14)$$

The plaquette expectation value in the disordered state $-e_d$ can be easily bounded from above; numerical simulation results give $-e_d \leq 0.66$. Inserting this number we again obtain $e_d - e_o \leq 0.0145(2)$, which is precisely the bound derived in (13). Finally we display in fig. 3c the quantity $U_4$. It does not exhibit asymptotic finite size scaling behaviour for either version of the theory and it is not possible to decide whether it approaches a value of 1 (for a first order phase transition) or larger (continuous transition).

All together our findings indicate that asymptotic finite size scaling for both formulations, on the hypertorus and on the $SH$ lattice, is very far away from the lattice sizes, which we are able to investigate in our present studies. This is not surprising for the theory on the hypertorus as we know from studies in much simpler models (like Potts models) with first order phase transitions, that asymptotic behaviour may set in only on very large lattices \[22\] and in general little is known about the onset of finite size scaling for gauge theories. On $SH$ lattices additional finite size corrections are caused by the inhomogeneity.
5 Discussion

The studied geometry – the boundary of a 5D hypercube – has less symmetry than the hypertorus. Asymptotically it amounts essentially to a sum over a set of ten hypercubes (like in 2D it corresponds to the six faces of the cube, the boundary contributions become negligible). On finite lattices the reduced translational and lattice rotational symmetry gives rise to large additional contributions of finite size scaling violating operators. Thus the asymptotic scaling behaviour sets in only on very large lattice sizes. A possible cure for the observed finite size scaling violations may be the introduction of weight factors i.e., considering the metric for a truly curved topology like the $S_4$. Studies in this direction including also terms proportional to $\gamma$ into the action are under progress and we expect from those studies clarification on the critical properties of pure gauge compact QED close to the Wilson line.

It remains a remarkable fact that the clear two-state signal of earlier Monte Carlo simulations disappears when one changes the lattice geometry with respect to their properties at the boundary. This finding corresponds to a rather small bound on possible discontinuities in the plaquette operator, which, based on the observed monotonic behaviour of finite volume discontinuities, is suggested by the present analysis. It also remains a remarkable fact, that the introduced soft modification of the theory leads in comparison to the results for the theory on a hypertorus to such large effects. This may be related to the influence of closed loops for the dynamics of the phase transition and may be a special property of gauge theories containing the compact $U(1)$ gauge group. One may wonder whether this observation can be of relevance for other systems with loop excitations, including systems with fermionic fields.

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### Tables

**Table 1:** Peak positions and values of the measured cumulants.

| $L$ | $C_{V,\text{max}}$ | $V_{BCL,\text{min}} \times 10^3$ | $U_{4,\text{min}}$ |
|-----|-------------------|-------------------------------|------------------|
| 4   | 2.21(1)           | -1.631(7)                     | 2.739(8)         |
| 6   | 4.96(13)          | -0.45(11)                     | 2.625(30)        |
| 8   | 10.08(32)         | -0.23(8)                      | 2.455(43)        |
| 10  | 20.91(62)         | -0.17(5)                      | 2.263(37)        |

| $L$ | $\beta(C_V)$ | $\beta(V_{BCL})$ | $\beta(U_4)$ |
|-----|--------------|------------------|--------------|
| 4   | 1.01536(22)  | 1.01194(29)      | 1.01688(37)  |
| 6   | 1.01370(16)  | 1.01326(18)      | 1.01401(19)  |
| 8   | 1.01263(6)   | 1.01257(6)       | 1.01273(7)   |
| 10  | 1.01216(1)   | 1.01215(1)       | 1.01218(2)   |
Figure 1: The multihistograms $h_L(\beta(C_V), e)$ resulting from all data for given $L$ at the values of the pseudocritical points as defined by the peak positions of the specific heat. Shown are data on $SH(4), SH(6), SH(8)$ and $SH(10)$ lattices.
Figure 2: The pseudocritical $\beta$-values as a function of the parameter $1/V$. We compare results for $\beta(CV)$ (circles), $\beta(V_{BCL})$ (triangles) and $\beta(U4)$ (crosses) on the $SH$ lattice with results from simulations for $\beta(CV)$ (triangles down) on the hyperbolic. The data on the hyperbolic have been obtained in multicanonical simulations [19] on lattices ranging in volume between $6^4$ and $12^4$. On the hyperbolic we include also a data point from [10] on the $16^4$ lattice. The dashed curve represents a fit according to (8), the dotted curves represent fits according to (10). The full symbols represent infinite volume extrapolations of $\beta_c$. 

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Figure 3: The peak values of (a) specific heat $C_V$ and of (b) the cumulants $V_{BCL}$ and (c) $U_4$ vs. $1/V$. We compare data on the hypertorus (triangles) with data on $SH$ lattices (circles). The dotted curves are explained in the text.
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Figure 1: The multihistograms $h_L(\beta(C_V), \epsilon)$ resulting from all data for given $L$ at the values of the pseudocritical points as defined by the peak positions of the specific heat. Shown are data on $SH(4), SH(6), SH(8)$ and $SH(10)$ lattices.
Figure 2: The pseudocritical $\beta$-values as a function of the parameter $1/V$. We compare results for $\beta(CV)$ (circles), $\beta(V_{BCL})$ (triangles) and $\beta(U_4)$ (crosses) on the SH lattice with results from simulations for $\beta(CV)$ (triangles down) on the hypertorus. The data on the hypertorus have been obtained in multicanonical simulations [19] on lattices ranging in volume between $6^4$ and $12^4$. On the hypertorus we include also a data point from [10] on the $16^4$ lattice. The dashed curve represents a fit according to (6), the dotted curves represent fits according to (10). The full symbols represent infinite volume extrapolations of $\beta_c$. 

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Figure 3: The peak values of (a) specific heat $C_V$ and of (b) the cumulants $V_{BCL}$ and (c) $U_4$ vs. $1/V$. We compare data on the hypertorus (triangles) with data on $SH$ lattices (circles). The dotted curves are explained in the text.