Unitarity in the Brout-Englert-Higgs Mechanism for Gravity

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Abstract

Just like the vector bosons in Abelian and non-Abelian gauge theories, gravitons can attain mass by spontaneous local symmetry breaking. The question is whether this can happen in a Lorentz-invariant way. We consider the use of four scalar fields that break coordinate reparametrization invariance, by playing the role of preferred flat coordinates $x, y, z,$ and $t$. In the unbroken representation, the theory has a (negative) cosmological constant, which is tuned to zero by the scalars in the broken phase. Massive spin 2 bosons and a single massive scalar survive. The theory is not renormalizable, so at best it can be viewed as an effective field theory for massive spin 2 particles. One may think of applications in cosmology, but a more tantalizing idea is to apply it to string theory approaches to QCD: if the gluon sector is to be described by a compactified 26 or 10 dimensional bosonic string theory, then the ideas considered here could be used to describe the mechanism that removes a massless or tachyonic scalar and provides mass to the spin 2 glueball states. The delicate problem of removing indefinite metric and/or negative energy states is addressed. The scalar particle has negative metric, so that unitarity demands that only states with an even number of them are allowed. Various ways are considered to adapt the matter section of the theory such that matter only couples to positive metric states, and we succeed in suppressing the main contributions to unitarity-violating amplitudes, but the exact restoration of unitarity in the spinless sector will continue to be a delicate issue in theories of this sort.
1 Introduction: Motivations for this study

There are two, almost disconnected, areas in theoretical physics where it may be worthwhile to consider the spontaneous breakdown of general coordinate reparametrization invariance. One is the study of cosmological models. Massless scalar fields, only interacting with gravity, could exist in principle. If so, they can give mass to the graviton, thus neutralizing the gravitational force at large distances. There have been numerous speculations on such effects in the literature, but they usually suffer from stability and fine-tuning problems. A tiny cosmological constant would be neutralized by these scalars, and they would lead to a universe that has a tendency to remain flat. The cosmological constant problem itself concerns the occurrence of vastly different scales in physics, and this problem would not be resolved with such ideas, so skepticism is justified, but further investigation along these lines could be of interest.

A second motivation for our work could be the study of Quantum Chromodynamics (QCD). The forces generated by the gluon fields take the form of vortex tubes. If we leave out the quarks, these vortices become unbreakable, and behave like bosonic strings. However, bosonic string theory is beset by anomalies in four space-time dimensions. There are no such anomalies in QCD. At best, therefore, QCD can be approximated by a string theory where the strings are disfigured by non minimal self-interactions, even in the $N \to \infty$ limit. Is there no way to describe such self-interacting strings by starting from the powerful string theory approaches that exist today? Since the gluonic sector has no fermionic states, the first candidate to think of as a starting point would be a 26 dimensional bosonic string theory[1], of which 22 dimensions are compactified. However, this theory has tachyonic and massless scalar (spin 0), massless vector (spin 1) and massless tensor (spin 2) solutions[2]. None of these are expected to exist in the pure gluonic sector of QCD.\footnote{Note, that what we have in mind is not to replace QCD by a string theory, but rather to use string theory techniques to do calculations in the sector of confined gluons and, at a later stages, confined quarks in conventional QCD.}

As for the massless vector particles, there is an elegant way to remove these: the Brout-Englert-Higgs (BEH) mechanism.[3] Due to interactions, the tachyonic scalar (in the symmetric representation) develops a vacuum expectation value, after which the various states rearrange: some of the scalar fields combine with the massless vector fields to produce the dynamical spin 1 solutions of a massive spin 1 field, exactly as it happens in the electro-weak sector of the Standard Model.

However, the string theory also has massless spin 2 modes, which somehow disappear in QCD. Naturally, one would like to invoke a mechanism analogous to the BEH mechanism. The author has been aware of this possibility for some time, but only now it was realized how one might proceed to overcome a nasty problem with indefinite metric and/or negative energy states. Stability of the vacuum requires that all fields describing elementary particles have only excitation modes with non-negative energies. This requires not only the absence of tachyonic mass terms, but also non-negative coefficients in all kinetic terms of the Lagrangian, and of course the existence of an unambiguous inverse of the bilinear terms that can serve as propagators. If one attempts to quantize a
theory with negative coefficients in the kinetic terms, one encounters quantum states with indefinite norm, thus violating unitarity, an important consistency condition in quantum field theories[4]. We defer the discussion of this problem to Sections 6 and onwards; let us momentarily assume that the problem can be handled. Thus, we may have achieved an opening towards describing an interacting string theory containing only massive string modes. One will also have to describe self interactions between strings, so it will not be an easy solution, but at least we see how, in principle, massless modes can be avoided. Indeed, one also would avoid supersymmetry this way, so that we avoid the nonexisting fermionic states in the gluonic sector of QCD.

These arguments should not be regarded as opposed to, but rather complimentary to the AdS/CFT approach to solve QCD using superstring theory[5], where the 3+1 dimensional theory is mapped onto a 5 dimensional AdS theory. There, the massless graviton in 5 dimensions is mapped onto a massive graviton in 4 dimensions. The five-dimensional theory may have to be supersymmetric again. What we try to do here is develop a conceptual understanding of what happens in 4 dimensions.

There is in fact another problem standing in the way of a string theoretical approach to QCD. QCD approaches its perturbative approximation in the far ultraviolet domain, where all gluonic states are far from the mass shell. It is this domain, where the basic QCD action is precisely defined, and where one would like to compare QCD field variables with string degrees of freedom. If we can find a match (by some mathematical transformation), then we have the beginning of a systematic procedure to identify string amplitudes with QCD amplitudes. Yet string theory only allows identification of its on-mass-shell states. Possibly, the deeper reason why string theory does not allow a consistent off-shell description is the fact that, in its standard representation, string theory is invariant under coordinate reparametrizations. So, observable coordinates do not exist, which may well be the reason why local fields cannot exist — we would not be able to specify their coordinates in a meaningful way.

However, when the coordinate reparametrization invariance is spontaneously broken, we do have coordinates. Our four scalar fields will serve as such. Thus, being able to provide for gauge-invariant coordinates, perhaps “off shell” amplitudes can now be defined. This could open the door even further towards a consistent way of treating QCD using string theory, while avoiding the unphysical supersymmetric theories that have been put forward until now.

Most of our discussion is limited to 4-dimensional space-time, with asymptotically flat boundary conditions.

I thank E. Kiritsis for an enlightening explanation of these points.
2 Perturbative Einstein-Hilbert gravity with massless scalars

Let us begin by establishing our notation. For a more elaborate description of perturbative (quantum) gravity see Ref. [6]. The pure Einstein-Hilbert action is

\[ S = \int L(x) \, d^4x ; \quad L(x) = \frac{\sqrt{g}}{16\pi G} (R - 2\Lambda) + L_1. \] (2.1)

\( R \) is the Ricci scalar curvature, \( g \) is the determinant of the metric tensor \( g_{\mu\nu} \) (in Euclidean notation), \( L_1 \) is a remainder, to be discussed later. \( \Lambda \) is a possible cosmological constant. In this section, we shall keep \( \Lambda = 0 \). We have the usual definitions:

\[ \Gamma_{\alpha\mu\nu} = \frac{1}{2} (\partial_\mu g_{\alpha\nu} + \partial_\nu g_{\alpha\mu} - \partial_\alpha g_{\mu\nu}) ; \quad \Gamma^\lambda_{\mu\nu} = g^{\lambda\alpha} \Gamma_{\alpha\mu\nu}. \] (2.2)

\[ R^\lambda_{\alpha\mu\nu} = \partial_\mu \Gamma^\lambda_{\alpha\nu} - \partial_\nu \Gamma^\lambda_{\alpha\mu} + \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\alpha\nu} - \Gamma^\lambda_{\nu\sigma} \Gamma^\sigma_{\alpha\mu} ; \] (2.3)

\[ R = g^{\alpha\nu} R^\mu_{\alpha\mu\nu}. \] (2.4)

In perturbation theory, one writes

\[ g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu}, \] (2.5)

where \( \varepsilon \) is taken to be the perturbation expansion parameter, and \( \delta_{\mu\nu} \) is the Lorentz invariant identity matrix. The inverse of this is

\[ g^{\mu\nu} = \delta^{\mu\nu} - \varepsilon h_{\mu\nu} + \varepsilon^2 h_{\mu\alpha} h_{\alpha\nu} - \cdots, \] (2.6)

Coordinate reparametrization consists of the substitution of the coordinates \( x^\mu \) as follows:

\[ x^\mu \rightarrow x^\mu + \varepsilon \eta^\mu(x), \] (2.7)

where \( \eta^\mu(x) \) is the space-time dependent generator of this ‘gauge transformation’. The metric tensor then transforms as

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} + \varepsilon (\eta^\alpha \partial_\alpha g_{\mu\nu} + g_{\alpha\nu} \partial_\mu \eta^\alpha + g_{\mu\alpha} \partial_\nu \eta^\alpha), \] (2.8)

which for the \( h_{\mu\nu} \) fields implies

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} + D_\mu \eta_\nu + D_\nu \eta_\mu, \] (2.9)

where we used the notion of a covariant derivative, while indices are raised and lowered with the metric tensor \( g_{\mu\nu} \):

\[ \eta_\mu = g_{\mu\nu} \eta^\nu; \quad D_\mu \eta_\nu \equiv \partial_\mu \eta_\nu - \Gamma^\alpha_{\mu\nu} \eta_\alpha. \] (2.10)
To expose the physical degrees of freedom describing gravitational radiation (gravitons), one must first fix the gauge freedom. For our purposes here, the radiation gauge works fine:

$$\sum_{i=1}^{3} \partial_i h_{i\mu} = 0 ; \quad \mu = 1, \cdots, 4. \quad (2.11)$$

Subsequently, we expand the action (2.1) in powers of $\varepsilon$. After removing pure derivatives one finds that the terms linear in $\varepsilon$ vanish. If we identify

$$\varepsilon^2 = 16\pi G , \quad (2.12)$$

the second order terms take the form

$$\mathcal{L} = -\frac{1}{2} h_{\alpha\beta} U_{\alpha\beta\mu\nu} h_{\mu\nu} , \quad (2.13)$$

where $U_{\alpha\beta\mu\nu}$ is a space-time operator containing second order derivatives. In momentum space, this operator is found to take the form

$$U_{\alpha\beta\mu\nu} = \frac{1}{2} k^2 (\delta_{\alpha\mu} \delta_{\beta\nu} - \delta_{\alpha\beta} \delta_{\mu\nu}) + k_{\mu} k_{\nu} \delta_{\alpha\beta} - k_{\beta} k_{\nu} \delta_{\alpha\mu} + b^2 \vec{k}_{\beta} \vec{k}_{\nu} \delta_{\alpha\mu} , \quad (2.14)$$

where $\vec{k}$ is $k$ with its time component replaced by 0, and the parameter $b^2$ is sent to infinity, so as to impose Eq. (2.11). To see the physical modes that propagate, in the case that $\varepsilon$ is infinitesimal, it is instructive to rotate the spacelike components of the momentum vector into the positive $z$ direction:

$$\vec{k}_{\mu} = (0, 0, \kappa, 0). \quad (2.15)$$

To find the propagator in this gauge, we first have to symmetrize $U_{\alpha\beta\mu\nu}$ with respect to interchanges $\alpha \leftrightarrow \beta$, $\mu \leftrightarrow \nu$ and $(\alpha\beta) \leftrightarrow (\mu\nu)$. The propagator $\mathbb{P}$ is then solved from

$$U \cdot \mathbb{P} = \mathbb{I} ; \quad \mathbb{I} = \frac{1}{2} (\delta_{\alpha\mu} \delta_{\beta\nu} + \delta_{\alpha\nu} \delta_{\beta\mu} \cdot \quad (2.16)$$

The solution to this tensor equation is

$$P_{\mu\nu\alpha\beta}(k) = \frac{1}{k^2 - i\varepsilon} \left( \hat{\delta}_{\alpha\mu} \hat{\delta}_{\beta\nu} + \hat{\delta}_{\alpha\nu} \hat{\delta}_{\beta\mu} - \frac{2}{n-2} \hat{\delta}_{\alpha\beta} \hat{\delta}_{\mu\nu} \right) +$$

terms containing only $\vec{k}^2$ in their denominators, \quad (2.17)

where $\hat{\delta}$ is defined as

$$\hat{\delta}_{\mu\nu} \equiv \text{diag}(1, 1, 0, 0) , \quad (2.18)$$

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3When (classical) gravitational radiation is described, $\varepsilon$ indeed is exceptionally small, so that this perturbative description is extremely accurate. The reason why higher order $\varepsilon$ corrections are harmless is that the operator $U$ has an inverse (Eq. (2.17)). The physics of these higher order $\varepsilon$ corrections, which include the effects of the energy transported by gravitational waves, is well understood.
and \( n \) is the number of space-time dimensions, \( n = 4 \) being the physical value. Only the part explicitly written in Eq. (2.17) represents excitations that actually propagate. One sees first of all that only the completely transverse components of the field \( h_{\mu\nu} \) propagate: \( \mu, \nu = 1 \) or \( 2 \). Secondly, the diagonal component (the trace) drops out:

\[
P_{\mu\mu\alpha\beta} = 0 \quad \text{since} \quad \delta_{\mu\mu} = n - 2 .
\] (2.19)

Henceforth, \( n = 4 \). Since then traceless, symmetric \( 2 \times 2 \) matrices have only two independent components, we read off that there are only two propagating modes, the two helicities of the graviton. The propagator (2.17) propagates a graviton with the speed of light.

There is one very important feature that we have to mention when treating perturbative gravity. The action is not bounded when we make a Wick rotation to Euclidean space, in contrast with the boundedness of the action for all other field theories in Euclidean space. This means that the bilinear terms in Eq. (2.13) contain parts that have an unconventional sign. This unusual sign does not invalidate gravity as a field theory, because the unphysical parts do not propagate. It does however have important implications: one is that gravity has a potential instability; stationary gravitational fields carry a negative energy density, so that gravitational collapse can occur. Another implication is that, in Euclidean space, one is not allowed to demand the functional integral to go over the real values for all components of the metric \( g_{\mu\nu} \), even after fixing the gauge. One will always encounter components of \( g_{\mu\nu} \) that will have to be integrated over complex contours, so as to ensure convergence of the functional integrals.\(^4\) The complex components of \( g_{\mu\nu} \) are usually in the trace or dilaton components, and in the gauge fixing procedures they act as Lagrange multipliers. Gravity generates Lagrange multiplier fields even in Euclidean space, where all functional integrals would be real Gaussian integrals. In a functional integral, however, Lagrange multipliers must be handled as imaginary fields in order to guarantee convergence. This can be shown to lead to the observation that, even after Wick rotating to Euclidean space, one must treat the dilaton sector of the gravitational metric field as a complex field variable in the functional integration procedure, a fact frequently overlooked in the literature. In our work, this feature causes a considerable complication, which we shall handle in Section 6 and onwards.

Massless scalars are introduced in (2.1) as

\[
L_1 = L_\phi + L_{\text{matter}} , \quad L_\phi = -\frac{1}{2} \sqrt{g} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^a ,
\] (2.20)

where \( a = 1, \cdots, N \) is an internal index. \( L_{\text{matter}} \) will contain all contributions from other kinds of matter. Soon, we shall restrict ourselves to the case \( N = n = 4 \), the dimensionality of space-time. The propagators are the inverse of this bilinear action, or

\[
P^{ab}(k) = \frac{\delta_{ab}}{k^2 - i\varepsilon} .
\] (2.21)

It is imperative that the sign of all terms in the propagator be positive. This is because the residues of the poles define the normalization of the one-particle states. If a residue is not

\(^4\)We are discussing the functional integrals prior to the integrations over momentum space, so the divergence mentioned here is distinct from the usual ultraviolet difficulties of quantum gravity.
one but some number $Z$, then the $S$-matrix elements generated by the Feynman rules will replace the ket-bra product $|k⟩⟨k|$ by $|k⟩Z⟨k|$. One can renormalize these states by a factor $\sqrt{Z}$, but one cannot change the sign of $Z$ this way. One can also note that $Z$ is directly proportional to the probability that the particle with momentum $k$ is produced in some process. This probability must be a positive (or vanishing) number. Having negative $Z$ would necessitate the inclusion of indefinite norm states in the quantum system. In classical field theories, fields with the wrong sign in the kinetic energy part of the action would tend to destabilize the vacuum by allowing for negative energy states. Note that the gravitational propagator, Eq. (2.17) has only positive residues at its poles. This implies, *inter alia*, that conventional gravity as a field theory obeys unitarity, whereas it would have generated indefinite norm gravitons if Newton’s constant had been negative.

The masslessness of the scalars in Eq. (2.20) is protected by a symmetry, the Abelian Goldstone symmetry,

$$\phi^a \rightarrow \phi^a + C^a,$$

where $C^a$ is a set of constants. Indeed, these scalars are allowed to interact only gravitationally, otherwise they would have collected mass terms.

There is also a rotational symmetry, $O(N)$, allowing us to rotate these scalars among one another. One would not expect the scalars to have a Lorentz symmetry $SO(3,1)$, but later we shall explain how this symmetry nevertheless might come about.

### 3 Breaking the symmetry spontaneously

For comparison, consider the case of a vectorial non-Abelian gauge theory,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{1}{2}(D_{\mu}\phi^a D_{\mu}\phi^a) - V(\phi^2),$$

where $D_{\mu}$ is the covariant derivative containing the vector field $A_\mu$, and

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} + \mathcal{O}(gA^2).$$

Furthermore, the function $V(\phi^2)$ describes the self interaction of the Higgs field. In the symmetric mode, gauge fixing could be done also by imposing the radiation gauge:

$$\partial_iA_i = 0,$$

where we sum only over the spacelike components; as in Eq. (2.11), Latin indices only take the values 1,2 or 3. The bilinear part of the action is then

$$-\frac{1}{2}A_\mu U_{\mu\nu}A_\nu = -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial_4 A_4)^2 - \frac{1}{2}b^2(\partial_iA_i)^2,$$

where $b$ tends to infinity so as to obey (3.3). In momentum space$^5$,

$$U_{\mu\nu} = k^2\delta_{\mu\nu} - k_4^2\delta_{\mu 4}\delta_{\nu 4} + b^2\vec{k}_\mu\vec{k}_\nu.$$

$^5$The factors $i^2$ are cancelled by the minus sign in the partial integration.
Choosing \( \vec{k} \) to be in the \( z \)-direction, Eq. (2.15), this matrix is
\[
U_{\mu\nu} = \operatorname{diag}(k^2, k^2, k^2 + b^2, \vec{k}^2).
\] (3.6)

In the limit \( b \to \infty \), its inverse is the vector propagator in the radiation gauge,
\[
P_{\mu\nu}(k) = \frac{\operatorname{diag}(1,1,0,0)}{k^2 - i\varepsilon} + \text{the Coulomb part containing only } 1/\vec{k}^2.
\] (3.7)

Thus, we see that only two helicities for the photon are propagated along the light cone. In addition, we have the scalars with the usual propagators.

The BEH mechanism is now looked at in the following way. Assuming that the potential \( V \) forces the scalar field \( \phi \) to take a non-vanishing vacuum value,\(^6\) it can be used to define a preference gauge. The preference gauge is the gauge in which \( \phi \) is pointing in a constant direction, so that its vacuum value, \( \langle \phi \rangle \), is constant. If in this gauge \( A_\mu = 0 \), then we have \( D_\mu \phi = 0 \). Consequently, the kinetic term of the \( \phi \) field, \(-\frac{1}{2}D_\mu \phi^2\), acts as a mass term for the gauge field in this gauge. Indeed, if we use this scalar to fix the gauge, thus replacing the radiation gauge, the gauge field propagator becomes
\[
P_{\mu\nu}(k) = \frac{\operatorname{diag}(1,1,1,0)}{k^2 + M^2 - i\varepsilon} \quad \text{in the Lorentz frame where } k_\mu = (0,0,0,k),
\] (3.8)
where \( M \propto g\langle \phi \rangle \). We observe that each vector particle (we may have one or more of them), obtains a mass term, borrowing its third degree of freedom from the scalars. The scalars lose one degree of freedom for each vector particle that gets a mass, because it is used to fix the gauge.

Now let us do the same thing for the gravitational theory. For simplicity, we restrict the discussion to the case of a four-dimensional spacetime. Four scalar fields \( \phi^a, a = 1, \ldots, 4 \), will be used to fix the gauge. Fixing the gauge means fixing a preferred coordinate frame. The only way to do this is, to identify these four coordinates. Therefore, we write
\[
\phi^\mu = m \, x^\mu, \quad \mu = 1, 2, 3, 4,
\] (3.9)
where the parameter \( m \) is yet to be determined. This immediately raises a question: the four coordinates are in Minkowski space; if all four scalars are real fields, the kinetic term (2.20) leaves a Euclidean \( O(4) \) symmetry, not the Lorentz group \( O(3,1) \). The residual system, after spontaneous symmetry breakdown, should still exhibit Lorentz invariance, not the Euclidean rotation group.

The answer to this question is postponed for a moment. Here, we simply assume that \( \phi^4 \) is shifted by an imaginary shift \( i \mu t \). For the time being, let us note that the

\(^6\)Strictly speaking, this mechanism has little to do with the vacuum; the value of the scalar field is nearly never exactly vanishing, so it could always be used to fix the gauge. However, if we arrange things such that this value is already unequal to zero at lowest order in perturbation theory, then we see how this mechanism works order by order in the perturbation expansion, which makes it much more transparent. Indeed, if one attempts to describe a non-perturbative version of the BEH mechanism several complications arise that we do not wish to go into.
Goldstone symmetry, (2.22), actually allows the constants $C^a$ to be complex. Shifting the fields $\phi^a$ by complex rather than real numbers, also leaves the action invariant (after partial integrations). We may also note that the quantum mechanical functional integral, $\int \mathcal{D}\phi^a$, allows a shift of the integration contour into the complex plane:

$$\int \mathcal{D}\phi^a = \int \mathcal{D}(\phi^a + C^a),$$

(3.10)

which means that, in the action, we are allowed to shift the fields by any complex parameter $C^a$.

*Shifts of field variables by a complex parameter are allowed if the action itself stays real!*

This is the case if the action is even in these field variables, as it indeed is here. Consequently, we are encouraged to investigate the gauge choice

$$\phi^1 = mx, \quad \phi^2 = my, \quad \phi^3 = mz, \quad \phi^4 = imt. \quad (3.11)$$

This last condition stems from the contour shift $\phi^4 = imt + \delta \phi^4$, after which the real field $\delta \phi^4$ was gaugefixed to be zero.

There may, however, be a more practical solution to this problem that we can mention here. In perturbation theory, we find that, if instead of $\phi^4$ we take $\phi^0$ to be real, the original Lagrangian (2.20) can still be used. The intermediate states in the unitarity condition[4] for the $S$ matrix receive a minus sign. These signs however are multiplicative. There is a plus sign if we restrict ourselves to intermediate states with an even number of $\phi^0$ particles. Fortunately, the Lagrangian (2.20) is even in $\phi^0$. Therefore, the odd states do not occur! We shall refer to this as the ‘pairing mechanism’.

Again, the question is, is this unorthodox procedure legal? In particular, are we allowed to have a time-dependent vacuum expectation value, which also breaks the $\phi^0 \leftrightarrow -\phi^0$ symmetry? We shall check the law as we go along. The penalty for this unorthodox procedure will turn out to be considerable, but we will see what the consequences are and what actually can be done to repair the theory later.

There is another problem with the gauge choice (3.11): it is not free from ambiguities. What if the scalar fields take the same values at different points in space-time? Curiously, this problem will turn out to be related to the previous one. For the time being, let us note that there is a natural way to restrict ourselves to the unambiguous sector of the theory by imposing the condition that the volume element

$$\varepsilon^{\mu\nu\alpha\beta} \partial_\mu \phi^1 \partial_\nu \phi^2 \partial_\alpha \phi^3 \partial_\beta \phi^4 > 0. \quad (3.12)$$

We will see how this condition emerges in the cure for the indefinite metric problem.

### 4 Spontaneously broken gravity

The gauge constraint (3.11) completely removes the four scalars as physical degrees of freedom. We now ask what the new action will look like in this gauge. We expect that
general covariance will appear to be absent, and that the new gravitational field will have \(2 + 4 = 6\) physically propagating modes.

The contribution from \(L_1\), Eq. (2.20), is derived as follows:

\[
\partial_\mu \phi^\nu = m \partial_\mu x^\nu = m \delta^\nu_\mu, \tag{4.1}
\]

\[
L_\phi = -\frac{1}{2} m^2 \sqrt{g} g^{\mu\mu}. \tag{4.2}
\]

Here, because of the \(i\) in the gauge choice (3.11), the summation over the Lorentz indices is the Lorentz covariant one (for simplicity, the Euclidean notation is used throughout). Of course we notice the first violations of general covariance. Writing

\[
g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu}; \quad g^{\mu\mu} = 4 - \varepsilon h_{\mu\mu} + \varepsilon^2 (h_{\mu\alpha})^2 + \mathcal{O}(\varepsilon h)^3; \quad \sqrt{g} = \exp(\frac{1}{2} \text{Tr} \log(\delta_{\mu\nu} + \varepsilon h_{\mu\nu})) = 1 + \frac{1}{2} \varepsilon h_{\mu\mu} - \frac{1}{4} \varepsilon^2 (h_{\mu\alpha})^2 + \frac{1}{8} \varepsilon^2 (h_{\mu\mu})^2 + \mathcal{O}(\varepsilon h^3), \tag{4.3}
\]

we find that

\[
L_1 = -2m^2 - \frac{1}{2} \varepsilon m^2 h_{\mu\mu} + 0 + \mathcal{O}(\varepsilon h^3) \tag{4.4}
\]

It so happens that, in Eq. (4.4), the terms of second order in \(\varepsilon\) cancel out. There is however a part linear in \(h_{\mu\nu}\), which would give rise to ‘tadpole diagrams’ in the quantum theory, and inhomogeneous equations for \(h_{\mu\nu}\) in the classical perturbation expansion. These would cause shifts in the \(h_{\mu\nu}\) fields, which is inadmissible because we had already subtracted the ‘vacuum part’ of the metric field \(g_{\mu\nu}\). What this means is that the non-vanishing expectation values (3.9) would be eliminated by the dynamics of the theory.

However, there is another way to remove the term linear in \(h_{\mu\nu}\). Let us now add a cosmological constant \(\Lambda\) to the action in the symmetric representation. Thus, we have a third term in the total Lagrangian:

\[
L^\Lambda = \frac{-\Lambda}{8\pi G} \sqrt{g} = \frac{-\Lambda}{8\pi G} (1 + \frac{1}{2} \varepsilon h_{\mu\mu} - \frac{1}{4} \varepsilon^2 (h_{\mu\alpha})^2 + \frac{1}{8} \varepsilon^2 (h_{\mu\mu})^2 + \mathcal{O}(\varepsilon h^3)). \tag{4.5}
\]

So we can absorb the linear term by having a (negative) cosmological constant,

\[
\Lambda = -8\pi G m^2, \tag{4.6}
\]

in the unbroken Einstein-Hilbert action (2.1). What we see is that it cancels the energy-momentum tensor of the vacuum values of the scalar fields.

We now get the picture. The starting point is an Einstein-Hilbert action with scalar fields and a negative cosmological constant. We search for a solution that has a vacuum state, which means that the universe obeys asymptotically flat boundary conditions at \(\infty\). Such solutions do exist but require the scalar fields to vary with space and time, because they must generate an energy-momentum tensor to neutralize the effects of the cosmological constant. These space-time dependent vacuum values of the scalar field spontaneously break reparametrization invariance. Now, let us see what this does to the graviton.
The surviving action, including (4.4) and (4.5), is
\[
L = \frac{\sqrt{g} R}{16\pi G} - m^2 + \varepsilon^2 m^2(-\frac{1}{4} h_{\alpha\beta}^2 + \frac{1}{8} h_{\alpha\alpha}^2) + \mathcal{O}(\varepsilon h^3). \tag{4.7}
\]
Let us expand the first term as well, remembering now that no further gauge fixing should be allowed,
\[
\frac{\sqrt{g} R}{16\pi G} = \varepsilon^2 \left(-\frac{1}{4} (\partial_\mu h_{\alpha\beta})^2 + \frac{1}{8} (\partial_\mu h_{\alpha\alpha})^2 + \frac{1}{2} A^2_\mu \right) + \mathcal{O}(\varepsilon h)^3, \tag{4.8}
\]
where
\[
A_\mu = \partial_\alpha h_{\alpha\mu} - \frac{1}{2} \partial_\mu h_{\alpha\alpha}. \tag{4.9}
\]
We go again to Fourier space. We can read off from Eq. (4.7), which does not vanish for any of the components of \( h_{\mu\nu} \) at zero momentum, that there will be no singularity at vanishing \( k^2 \). It therefore makes sense to rotate the 4-vector \( k \) to the Euclidean 4 direction: \( k_\mu = (0, 0, 0, k) \). We ignore the \(-m^2 \) in (4.7), and identify the remainder as a mass term.

The combined Lagrangian, obtained by substituting (4.8) in (4.7) in momentum space, can now be written as
\[
L = (k^2 + m^2)(-\frac{1}{4} h_{\alpha\beta}^2 + \frac{1}{8} h_{\alpha\alpha}^2) + \frac{1}{2} k^2 (h_{\alpha\alpha} - \frac{1}{2} h_{\alpha\alpha} \delta_{\alpha\beta})^2, \tag{4.10}
\]
where we ignored the higher order terms and the \(-m^2 \). Now we split the dynamical variables into irreducible parts. As before, Latin indices \( i, j, k, \ldots \) run from 1 to 3.

\[
\begin{align*}
  h_{ii} &= u, & h_{ij} &= \tilde{h}_{ij} + \frac{1}{3} u \delta_{ij}, & \tilde{h}_{ii} &= 0, \\
  h_{4i} &= h_{4i} = h_i, & h_{44} &= h.
\end{align*}
\tag{4.11}
\]
In terms of the new variables \( \tilde{h}_{ij}, h_i, h \) and \( u \), we find
\[
L = (k^2 + m^2)(-\frac{1}{4} \tilde{h}_{ij}^2 + \frac{1}{8} u^2) - \frac{1}{2} m^2 h_i^2 - \frac{1}{8} m^2 (h - u)^2. \tag{4.12}
\]
The fields \( \tilde{h}_{ij} \) are the 5 components of a massive spin 2 particle. The field \( u \) is clearly also a dynamical field, whereas \( h_i \) and \( h - u \) do not propagate, since they have no pole in the propagator.

5 Massive gravity, the general case

The Lagrangian (4.7) is not the most general Lagrangian for massive spin 2 fields. One can ask instead, what is the most general bilinear Lagrangian for symmetric fields \( h_{\mu\nu} \) such that inverting the bilinear coefficients leads to a propagator that has single poles in the momentum \( k \), corresponding to a spin 2 particle with \( m \), and a spin 0 particle with mass \( \mu \)? This is a fairly complicated exercise, but the result of the calculation is quite elegant and simple to explain.
Let us assume that these poles occur exclusively in those components of the propagator that are orthogonal to the momentum \( k_\mu \). This implies that, in the residue of the pole term of the propagator, all indices of the \( h_{\mu\nu} \) fields must be contracted by the “spacelike Kronecker delta” \( \delta_{\mu\nu} + k_\mu k_\nu/m^2 \), and consequently, in the limit \( m^2 \to 0 \), the propagator diverges inversely with \( m^2 \) as soon as one of the indices of the \( h_{\mu\nu} \) field is parallel to \( k_\mu \) or \( k_\nu \). This implies that, in the limit \( m^2 \to 0 \), the bilinear part of the Lagrangian is invariant under the replacement

\[
h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \eta_\nu + \partial_\nu \eta_\mu ,
\]

where \( \eta \) is infinitesimal. Of course, this is nothing but the gauge transformation (2.9) in the perturbative regime. One concludes that the part of the Lagrangian that contains two derivatives coincides exactly with the bilinear part of the Einstein-Hilbert action, Eq. (4.8). This is indeed what one gets in an explicit calculation.

For the mass terms, there are two possibilities left, so that one may choose two coefficients,

\[
\mathcal{L}^m = -\frac{1}{4}m_1^2 h_{\mu\nu}^2 + \frac{1}{4}m_2^2 (h_{\alpha\alpha})^2.
\]

In the case \( m_1 = m, m_2 = \frac{1}{2}m \), this is the Lagrangian (4.10) obtained from spontaneous symmetry breaking. Note that we anticipate a scalar with an overall wrong sign in its propagator. What are the masses \( m \) and \( \mu \) in terms of \( m_1 \) and \( m_2 \) in the general case?

To find out, we consider the Lagrangian in momentum space and rotate the momentum into the time direction, \( k_\mu = (0, 0, 0, k) \). Again let Roman indices \( i, j, \ldots \) take the values 1,2, and 3 only. Decomposing \( h_{\mu\nu} \) again as in Eqs. (4.11), we find

\[
\mathcal{L} = -\frac{1}{4}(k^2 + m_1^2)(h_{ij})^2 - \frac{1}{2}m_1^2 h_i^2 + \frac{1}{4(m_2^2 - m_1^2)}\left((m_2^2 - m_1^2)h + m_2^2 u\right)^2 + \frac{1}{6}(k^2 + \mu^2)u^2 , \quad \text{with} \quad \frac{2\mu^2}{m_1^2} = \frac{4m_2^2 - m_1^2}{m_1^2 - m_2^2}.
\]

In this expression the various terms were rearranged such that we can easily read off the masses. The fields \( h_i \) and \( h \) have no kinetic terms, and so they do not propagate with the pole of a physical particle. In the absence of source terms or higher order terms, the field equations enforce

\[
h_i = 0 , \quad h = \frac{m_2^2}{m_1^2 - m_2^2} u .
\]

The particle \( u \) does propagate, with the wrong sign in its propagator, and mass \( \mu \) given by Eq. (5.3). The spin 2 fields have mass \( m = m_1 \). We have \( \mu = m = m_1 \) in the case of spontaneous symmetry breaking, where \( m_2^2 = \frac{1}{2}m_1^2 \) and Eq. (5.3) coincides with (4.12).

It is important now to note that, if \( m_1 = m_2 \), the \( u \) field gets an infinite mass. This indeed is the Fierz-Pauli case[7][8] Only in this case, the effects of the unphysical \( u \) particle disappear, and we have an explicitly unitary theory. It would be ideal if we could generate this Lagrangian from a BEH mechanism, but, within our formalism, this is
unlikely. Unfortunately, we could not find a BEH mechanism of this sort. It appears that the above unitary Lagrangian cannot emerge because the matter fields required would have to consist of real fields that generate an energy-momentum in the spontaneously broken phase that is proportional to $g_{\mu\nu}$. To achieve this, unconventional matter fields would be required; scalar fields — or vector fields — will not do.

6 Removing indefinite metric states

We return to Eq. (4.12). The field $u$ stands for a scalar particle, the 6th degree of freedom, as expected, and it has the same mass as the heavy graviton. But, of course, the reader will see what the problem is. The field $u$ has the wrong sign in its propagator:

$$ P^{uu}(k) = \frac{-3}{k^2 + m^2 - i\varepsilon} . \quad (6.1) $$

Consequently, the theory we have arrived at now, will violate unitarity. The wrong sign here is directly related to the indefiniteness of the action in Euclidean space that we referred to in Section 2, and further analysis shows that it is also related to the fact that our fourth scalar field has the wrong sign in its kinetic term if used as a timelike component.

In the symmetric representation, however, the vacuum appears to be stable, due to the pairing mechanism, and one might wonder whence the instability of the vacuum after the shift $\phi^0 \rightarrow mx^\mu + \tilde{\phi}^\mu$. Since the role of the $\phi^0$ field is taken over by the $u$ field, one might suspect that the $u$ field should be taken to be imaginary, not real. Only then we see that two massive particles emerge, one with spin 2 and one with spin 0.

To study a possible pairing mechanism for the $u$ field, let us now concentrate on the other terms in the Lagrangian. The matter fields couple to the gravitational fields through the energy-momentum tensor $T^\mu{}\nu$ of the matter field. We write $L^{\text{matter}}(g_{\mu\nu})$ to indicate the dependence of this contribution on the metric $g_{\mu\nu}$:

$$ g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu} , \quad L^{\text{matter}}(g_{\mu\nu}) = L^{\text{matter}}(\delta_{\mu\nu}) + \frac{1}{2}\varepsilon T^\mu{}\nu h_{\mu\nu} + O(\varepsilon^2) . \quad (6.2) $$

In momentum space, after again rotating the momentum vector into the 4 direction, $k_\mu = (0, 0, 0, k)$, we write (ignoring higher order corrections),

$$ T = T^\mu{}\mu , \quad T^\mu{}\nu = T^\perp{}\nu + \frac{1}{4}T\delta^\mu{}\nu , \quad T^\mu{}\nu_{\perp} = 0 , \quad (6.3) $$

$$ \partial_\mu T^\mu{}\nu = 0 \rightarrow T^4{}\mu = 0 , \quad T^4{}4 = -\frac{1}{4}T , \quad T^{ii} = T , \quad (6.4) $$

$$ T^{ij} = \tilde{T}^{ij} + \frac{1}{3}T\delta^{ij} = \tilde{T}^\perp{}^{ij} + \frac{1}{4}T\delta^{ij} , \quad \tilde{T}^{ii} = 0 . \quad (6.5) $$

Thus, we find that the coupling term is

$$ \frac{1}{2}T^{\mu\nu}h_{\mu\nu} = \frac{1}{2} T^{ij} + \frac{1}{8}Tu = \frac{1}{2} \tilde{T}^{ij}h_{ij} + \frac{1}{6}Tu . \quad (6.6) $$

So, the $u$ field is coupled to the trace of the energy-momentum tensor. It is this coupling that causes violation of unitarity. Why do we have violation of unitarity? In
the symmetric representation, we could see that the pairing mechanism filters out the negative metric states. Unfortunately, that mechanism now fails. We suspect that the reason for this failure is that the gauge choice, Eq. (3.11), also selects out a vacuum state that breaks the \( \phi^0 \leftrightarrow -\phi^0 \) symmetry. Therefore, the vacuum mixes positive and negative metric states; thus, we cannot select out the even states, as was our intention. The pairing mechanism for the \( \phi^0 \) field fails. One may also observe that we chose a vacuum value for \( \phi^4 \) that is time-dependent, and this may imply a breakdown of energy conservation, or, indeed, a new instability of the vacuum.

Imposing now the pairing mechanism for the \( u \) field implies a new constraint on our theory: the \( u \) field should decouple. It was proposed in Refs. [13][14], where similar theories are discussed, that we should limit ourselves to having only “conformal matter”, that is, matter with vanishing trace of the energy momentum tensor, \( T = 0 \). However, in cosmology this would be a somewhat mysterious restriction, whereas in QCD we definitely do not want to consider only scale invariant states. We conclude that our theory requires a modification.

As stated earlier, conventional matter such as scalar, spinor or vector fields (without analytic continuations to complex field values) can never do the work right. This is because we wish to have a flat spacetime after symmetry breakdown, and only a cosmological constant to our disposal to cancel the effects of the energy-momentum distribution \( T_{\mu\nu} \) of our background fields, but ordinary matter never gives a \( T_{\mu\nu} \propto g_{\mu\nu} \) as in dark energy.

One alley that we investigated is whether we can add a dilaton field \( \eta(x) \) to our theory. The Lagrangian is then replaced by

\[
\mathcal{L} \rightarrow \frac{\sqrt{-g}}{16\pi G} \left( R e^{\alpha \eta} - 2\Lambda e^{\beta \eta} \right) + \mathcal{L}_\phi e^{\gamma \eta} + \mathcal{L}_{\text{matter}} e^{\kappa \eta} - \frac{1}{2} \sqrt{g} e^{\lambda \eta} g^{\mu\nu} (\partial_\mu \eta \partial_\nu \eta) - \frac{1}{2} \sqrt{g} m_3^2 \eta^2 ,
\]

(6.7)

where \( \alpha, \beta, \gamma, \kappa, \lambda, \) and \( m_3 \) are constants that can be adjusted. It turns out, however, that there are no possible choices for these constants such that either the coupling to the \( u \) field vanishes or that the kinetic terms all get the correct sign, nor could the mass of the \( u \)-field be sent to infinity. This alley was a blind one.

Next, we investigate whether another observation might lead to solutions to this problem: the scalar fields \( \phi^a \) themselves actually generate an ‘alternative’ metric tensor:

\[
g^{\phi}_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^a .
\]

(6.8)

This is the metric tensor of flat space, in the gauge (3.11). It is a new tensor obeying the same transformation rules as \( g_{\mu\nu} \). Matter could be coupled either to \( g_{\mu\nu} \) or to \( g^{\phi}_{\mu\nu} \), and there is no a priori reason to select any particular choice. In a renormalizable theory, there would be a reason: the derivatives in Eq. (6.8) would render couplings with \( g^{\phi}_{\mu\nu} \) highly non-renormalizable. But, we had to give up renormalizability from the very start, as we are dealing with gravity.

One attempt to resolve the metric problem, based on this idea, is as follows. Impose a constraint on the scalar fields \( \phi \):

\[
g^{\phi} = g ,
\]

(6.9)
which is invariant under coordinate transformations, and can also be written as
\[ \varepsilon^{\mu\nu\rho\sigma} \partial_\mu \phi^1 \partial_\nu \phi^2 \partial_\rho \phi^3 \partial_\sigma \phi^4 = \sqrt{g} . \] (6.10)

In the gauge (3.11), we now have \( g = 1 \), hence \( h_{\alpha\alpha} = u + h = 0 \).

Unfortunately, upon closer inspection, this procedure does not do the job right. Substituting \( u = -h \) in Eq. (4.12), we find that now the \( h \) field has become a dynamical field, with the wrong sign in the kinetic term of its propagator. Since the gauge choice (3.11) was already made, and supposed to be unitary, the \( h \) field is not a ghost here, so it produces physical states with the wrong metric. \( h \) is coupled to \( T^{44} \), which does not vanish.

Alternatively, one could consider the following approach. Let us couple matter not with the metric \( g_{\mu\nu} \) of our gravity sector, but with
\[ g_{\mu\nu}^\text{matter} = g^{-\frac{1}{4}} g_{\mu\nu} (g^\phi)^{\frac{1}{4}} , \] (6.11)
where \( g^\phi \) is the determinant of the metric \( g_{\mu\nu}^\phi \). This means that the trace of \( h_{\mu\nu} \) does not enter in the matter Lagrangian, but is replaced by the trace of the perturbation of \( g_{\mu\nu}^\phi \), and, since in our scalar-field gauge (3.11) this vanishes, matter will not couple to the trace of \( h_{\mu\nu} \). Conversely then, one expects the trace \( T \) of the energy-momentum tensor not to couple to the \( h_{\mu\nu} \) field.

This, however, is not correct.\(^7\) The deeper reason why one has to reexamine the equations is that matter modified by the insertion (6.11) obeys conservation laws that differ from Eqs. (6.3)–(6.5).

Since \( h_{\alpha\alpha} = h + u \), the matter coupling (6.6) will be replaced by
\[ \frac{1}{2} T^{\mu\nu} (h_{\mu\nu} - \frac{1}{4} h_{\alpha\alpha} \delta_{\mu\nu}) = \frac{1}{2} T^{ij}_{\perp} \tilde{h}_{ij} + \frac{1}{24} u T - \frac{1}{8} h T . \] (6.12)
Indeed, the coupling with \( u \) is replaced by a coupling with the combination \( \frac{1}{3} u - h \), but, since the dynamical equations from the Lagrangian (4.12) force \( h = u \) (apart from a contact term), one finds that the \( u \) field couples with matter after all. Let us now, however, replace Eq. (6.11) by
\[ g_{\mu\nu}^\text{matter} = g_{\mu\nu} (g^\phi / g)^\alpha , \] (6.13)
where the coefficient \( \alpha \) is yet to be determined. We then get the coupling
\[ \frac{1}{2} T^{\mu\nu} (h_{\mu\nu} - \alpha h_{\alpha\alpha} \delta_{\mu\nu}) = \frac{1}{2} T^{ij}_{\perp} \tilde{h}_{ij} + (((\frac{1}{6} - \frac{1}{2} \alpha) u - \frac{1}{2} \alpha h) T , \] (6.14)
and this time we can ensure that only the contact term survives. Since the propagator generated by the kinetic term (4.12) enforces \( h = u \) (apart from contact terms), the condition for this to happen is that
\[ \alpha = \frac{1}{6} . \] (6.15)

\(^7\)I thank Luca Vecchi for detecting this error in version # 3 of this paper.
Thus, we can in principle avoid the direct couplings of matter with single $u$ particles so that, at least in the direct channel, unitarity is restored. As long as the $u$ particles are only pair created, they do not violate unitarity. Unfortunately, we cannot exclude odd terms in the $u$ field at higher orders in the pure gravity sector, a difficulty that needs to be investigated further.

Note, that these proposals require $g^\phi$ not to vanish. If we write

$$\phi^\mu = m x^\mu + \bar{\phi}^{\mu},$$

a perturbative expansion gives

$$\sqrt{g^\phi} = m^4 + m^3 \partial_\mu \phi^\mu + \frac{1}{2} m^2 ((\partial_\mu \phi^\mu)^2 - \partial_\mu \phi^\nu \partial_\nu \phi^\mu) + \mathcal{O}(\phi^3),$$

which can be derived elegantly,

$$\sqrt{g^\phi} = \text{det}(\partial_\mu \phi^a) = \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \phi^1 \partial_\nu \phi^2 \partial_\alpha \phi^3 \partial_\beta \phi^4,$$

and this also shows that the expansion terminates beyond the $m^0 \phi^4$ term. Note that this is the volume term (3.12), which now plays a more prominent role.

### 7 Relation to earlier work

Van Dam and Veltman[8] noted in 1970 that there are fundamental differences between pure gravity, where gravitons have only two polarizations, and massive spin 2 theories, where the particle has 5 polarizations; their propagators are different already at the tree level. They considered the unitarity requirement for the propagator, but made no attempt to find the Lagrangian of a field theory that would generate such a propagator. The discontinuity in the amplitudes that they found, is due to the fact that the massless theory requires a gauge constraint, which is not the same constraint as what the theory is driven to when mass terms are added; in fact, the unitary massive theory (with $\mu \to \infty$ in Eq. (5.3)), enforces $u = 0$, or $\partial_\mu \partial_\nu h_{\mu \nu} = \partial^2 h_{\mu \mu}$ (perturbatively), which cannot be used as a gauge constraint because it is perturbatively gauge-invariant (it is the condition $R = 0$ at lowest order).

After completion of the first version of this paper, various responses were received notifying us of other early work and many further references. Omero and Percacci[9] discussed a Higgs phenomenon in quantum gravity, using the Palatini formalism. Their aim is to apply this mechanism to compactify extra dimensions, where the difficulty with the timelike component does not occur. Percacci observes, like we do, that we have not yet achieved a Higgs mechanism that is completely smooth in the UV direction; it is rather like that of a non-linear sigma model.

Of particular interest is the work by Kiritsis et al[5], who are making progress in the AdS/CFT approach. They report that, even though the gluonic sector of QCD is purely bosonic, one nevertheless may consider a superstring here, with extra projection operators excluding the fermionic modes.
Arkani-Hamed et al[10] consider several tensor fields and multiple sets of general coordinate transformations. Like Siegel[2], they view the graviton as a bound state in open string field theory, but they too make no attempt at rigorously discussing unitarity, which would drastically reduce the number of free parameters and the amount of non-locality that seems to characterize these theories.

Chamseddine[11] also considers spontaneous symmetry breaking, but obtains a massive graviton interacting with a massless one. He also does not discuss unitarity in the longitudinal sector.

The BEH mechanism for gravity has been speculated on also in[12]—[16], usually in connection with brane theories and/or cosmology; in our work we focussed on the problems associated with the indefinite metric. A bridge between our observations and the AdS/CFT approach is further discussed in Bandos et al[17], who observe that a gravitational Higgs effect takes place on $p$-branes imbedded in higher dimensions; gravity in the bulk is unbroken, but on the $p$-brane it is massive.

This modified paper is a considerable improvement of its earlier version. A mistaken idea was withdrawn, as explained in Section 6, and we added Section 5 to explain how a Lagrangian for a unitary description of only spin 2 particles looks, which however could not be obtained within our present scheme. In Section 7, further references are briefly described. The author is indebted, among others, to R. Jackiw, M. Duff, I. Bandos, W. Siegel, A. Chamseddine, E. Kiritsis and L. Vecchi for their comments.

8 Conclusion

The equivalent of the Brout-Englert-Higgs mechanism for gravity may exist. In the symmetric representation, four scalar fields are added to the gravitational degrees of freedom, and a negative cosmological constant is added. After assuming space-time dependent vacuum values for the scalar fields, they rearrange to produce a field theory for a massive spin 2 particle and a scalar. The scalar would have unphysical metric, so that it has to be removed from the system. This can be done by modifying the matter part of the Lagrangian, so that the zero spin field decouples. The ghost poles will cancel in the usual way by employing the Faddeev Popov ghost, and using BRST invariance. We do note that the insertion of Eq. (6.13) in the Lagrangian for the matter field introduces higher derivatives there. This is only allowed in perturbation expansion, which however will become more divergent in the ultraviolet. The theory was already non-renormalizable, so this implies once again that we must constrain ourselves to some finite order in the perturbation expansion.

The complications in the longitudinal (scalar) sector always require a special treatment of the volume factor in the metric, normally controlled by $\sqrt{g}$. It is important to realize that this can be done, at least at the level of classical, effective field theory. Our models are quite singular in the ultraviolet region, but not yet all possibilities at getting less divergent versions have been explored. Ideally, one would like to have a scenario where the scalars only play their special role in the infrared domain, where the effects of the
spin 2 mass are important, while they interact only mildly in the far ultraviolet. This, we hope, might be something that could be realized in string theories. Indeed, recent AdS/CFT approaches are pointing in this direction.

Imposing the pairing mechanism to the $u$ field implies, in a sense, that the conformal sector of gravity theory, described by the determinant $g$ of the metric, must be treated in such a way that this determinant is path-integrated in the complex direction:

\[
g_{\mu\nu} = \omega \tilde{g}_{\mu\nu},
\]
\[
det(\tilde{g}_{\mu\nu}) = 1,
\]
\[
\omega = g^{1/4} = e^{i\eta},
\]
\[
\tilde{g}_{\mu\nu} = \text{real}, \quad \eta = \text{real}.
\] (8.1)

Our research is not complete. There are other potential difficulties. So-far, we only considered the coupling of the $u$ field to matter, and concluded that the matter Lagrangian had to be coupled to the conformal part of the metric in an anomalous fashion — the replacement of (6.11) by (6.13), with $\alpha = \frac{1}{6}$, was an important enough correction to warrant the submission of the 4th corrected version of this manuscript. But how can any of these proposals be reconciled with the fact that the gravitons themselves are not conformally invariant? Newton’s constant has a non-trivial dimension. This difficulty is reflected in the fact that, although the $u$ field does not couple to matter, it does couple to itself, and we note that odd powers of $u$ might arise in the gravitational self couplings. Can these be renormalized away, as suggested above? The following consideration casts further doubts on the validity of this assumption.

We can isolate the troublesome indefinite metric component of gravity by splitting the metric tensor $g_{\mu\nu}$ as in Eqs (8.1) but without the $i$:

\[
g_{\mu\nu} = e^{\eta(x)}\tilde{g}_{\mu\nu}(x), \quad \eta = \text{real};
\]
\[
\sqrt{\tilde{g}} = 1, \quad \sqrt{g} = e^{2\eta}.
\] (8.2)

The Einstein-Hilbert Lagrangian then becomes

\[
L_{\text{EH}} = e^{\eta}(\tilde{R} + \frac{3}{2} \tilde{g}^{\mu\nu}\partial_{\mu}\eta\partial_{\nu}\eta).
\] (8.3)

Apart from a normalization $1/\sqrt{3}$, we could identify this $\eta$ field with the field $\phi^0$, needed for the spontaneous breaking in the time direction, and start from the Lagrangian

\[
L = e^{\eta} \left( L(\tilde{g}_{\mu\nu}) - \frac{1}{2} \tilde{g}^{\mu\nu} \left( \partial_{\mu}\phi^i\partial_{\nu}\phi^i - \partial_{\mu}\phi^0\partial_{\nu}\phi^0 \right) \right),
\] (8.4)

where $i$ counts the three spacelike components. This definitely has the right metric. However, if we impose, as we would like to do,

\[
\langle \phi^{\mu} \rangle \rightarrow m x^{\mu}, \quad \eta \rightarrow m t/\sqrt{3},
\] (8.5)

we see that the prefactor for the Einstein term explodes exponentially with time: Newton’s constant becomes exponentially time dependent. This is not a small effect; the time scale in the exponent is of the order $1/m$. This is not flat space-time.
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