Numerical simulation of the interaction between two bubbles

R Han, X L Yao and A M Zhang
College of Shipbuilding, Harbin Engineering University, #515 Chuanhai Building, Nantong Street No.145, Nangang District, Harbin 150001, China
E-mail: zhangaman@hrbeu.edu.cn

Abstract. Different evolution patterns of two bubbles may be observed for different values of the phase difference and the inter-bubble distance. Based on potential flow theory, a boundary element method (BEM) is adopted to simulate the interaction between two bubbles and the toroidal bubble after the jet impact is also investigated by placing a vortex ring within the bubble. Meanwhile, some numerical techniques are used to deal with problems like coalescence and collapse. Typical phenomena like coalescence, jet towards and jet away are investigated numerically in this paper. The mechanisms underlying the various phenomena are given through the analysis of the velocity and pressure fields.

1. Introduction
The dynamic character of bubble is of much importance to engineering applications, like gas bubble gun used to raise drilling speed in the field of ocean engineering, high-speed jet formed by underwater explosion bubble for the military and the jet of bubble used to remove the calculus in medical treatment. A wide range of applications make bubble dynamics deserve attention. However, complicated motion mechanisms of non-spherical bubble make the difficulties for theory research; therefore numerical simulation and experimental observations are widely used to study bubble dynamics. In aspect of numerical simulation, a boundary element method has been verified repeatedly [1-3], and a vortex ring model is introduced by Wang [4] to simulate the toroidal bubble after the jet impact; in aspect of experimental observations, researchers often use spark-generated bubble [5, 6] or laser-induced bubble [7] to study bubble dynamic behaviors, observing the phenomena like jet, collapse and toroidal bubble. With further investigations in bubble dynamics, extensive research about multiple-bubble cases has been made, such as the interaction between two bubbles and a rigid wall [7] and the interaction between a vertical column of two bubbles and free surface [8,9]. The two-bubble interactions depending on the phase differences and the inner-bubble distances have been classified in a graph by Fong et al. [5], and phenomena like coalescence, jet towards and jet away have been observed in a series of experiments involving multiple spark-generated bubbles. Rungsiyaphornrat et al. [10] have simulated the merging of two gaseous bubbles numerically and the coalescence criterion in their work is established by the measurement of explosion experiments.

Based on potential flow theory, a boundary element method (BEM) is adopted to simulate the interaction between two identical-sized bubbles in this paper and numerical techniques are also used to deal with problems like coalescence and collapse. A vortex ring model is established to further simulate the interaction between the toroidal bubble and the other singly connected one after the jet impact.
impact. Typical phenomena like coalescence, jet towards and jet away are investigated numerically in this paper, and bubble profile is presented together with the velocity and pressure fields, aiming to provide mechanisms underlying the bubble motion.

2. Theory and numerical techniques

2.1. Theory

BEM simulations are carried out in this paper to investigate the two-bubble interaction in a free field. Due to the short duration of the interaction and the large Reynolds numbers, the velocity potential in the flow field satisfies the boundary integral equation [11]

$$\sigma \Phi(p) = \int_S \left( \frac{\partial \Phi(q)}{\partial n} G(p,q) - \Phi(q) \frac{\partial}{\partial n} G(p,q) \right) dS$$

(1)

where the normal derivative is defined as $\frac{\partial}{\partial n} = \mathbf{n} \cdot \nabla$ and $n$ is the outward normal, $p$ and $q$ are the fixed and the source point located at the interface, $\sigma$ is the solid angle at the fixed point $p$ with a value of $\sigma = 2\pi$, $G(p,q) = 1/|\mathbf{r}(p) - \mathbf{r}(q)|$ is the Green’s function, $S$ is all the boundaries of the fluid domain.

Since the influence from the motion of bubble internal gas during its overall process can be neglected, the bubble internal pressure is related to the initial state and its volume. The liquid pressure outside the bubble wall is supposed to be equal to the pressure within the bubble, it can be written as [4]

$$P = P_c + P_0 \left( \frac{V_0}{V} \right)^k$$

(2)

where $P_c$ is the constant vapor pressure inside the bubble, $P_0$ and $V_0$ are the initial pressure and volume of the bubble, and $k$ is the ratio of the specific heat for the gas related to gas component which equals 1.25 [12].

Scaling factors are used to obtain the dimensionless form of the equations. The bubble’s maximum radius $R_n$, $P_{nf} = P_n - P_c$, $R_n(\rho / P_{nf})^{1/2}$, $R_n(P_{nf} / \rho)^{1/2}$ are used for length, pressure, time, velocity potential and buoyancy, respectively. Therefore, the dimensionless form of the bubble state equation can be written as [13]

$$\frac{d \Phi}{dt} = 1 + \frac{1}{2} \left( \frac{\nabla \Phi}{V} \right)^2 - \delta^2 z - \varepsilon \left( \frac{V_0}{V} \right)^k$$

(3)

where $\delta = \sqrt{\rho g R_n / P_{nf}}$ is defined as buoyancy parameter with a value of $\delta = 0$ without the influence of gravity in this paper and $\varepsilon = \rho_c / P_{nf}$ is defined as the strength parameter inside the bubble. In addition, we define dimensionless inter-bubble distance as $d_{in} = D_{in} / R_n$, in which $D_{in}$ is the distance between the initial bubble centroid. The phase difference is written as

$$\Delta \theta = 2\pi \left( \frac{t_1}{t_{osc,1}} - \frac{t_2}{t_{osc,2}} \right)$$

[5], in which $t$ is the elapsed time from the first initiation of a bubble and $t_{osc}$ is the oscillation time.

The kinematic boundary condition governing the bubble motion is [4]

$$\frac{d \mathbf{r}}{dt} = \nabla \Phi$$

(4)
The idealized impact model (one point impact) \cite{14} is used in this paper, and a vortex ring model \cite{4} is introduced after the jet impact. The vortex ring is placed within the toroidal bubble, the strength of which is equal to the potential difference of the jet tip and the opposite bubble surface just before the jet impacts. The total potential of the bubble surface is now decomposed into two parts

$$\Phi = \Phi_v + \Phi_r$$

where $\Phi_v$ is the potential due to the circulation of the vortex ring and $\Phi_r$ is a remainder. The details on obtaining $\Phi_v$ are available in Ref. \cite{14}, while $\Phi_r$ satisfies the boundary integral equation, by which the corresponding velocity $u_r$ can be computed. The total velocity $u = u_v + u_r$ is used to update the position of the node on bubble surface, and the update of $\Phi_r$ can be obtained by

$$\frac{d\Phi_r}{dt} = u \cdot \nabla \Phi_r - \frac{|u|^2}{2} + 1 - \delta^2 z$$

2.2. Numerical techniques

Three evolution patterns of two bubbles are simulated numerically in this paper. In the case of coalescence, it should be judged whether the coalescence criterion is satisfied and the jointing of nodes is performed with some numerical technique. The coalescence criterion mentioned above is based on Ref. \cite{10} and Ref. \cite{15}, and we reckon that the minimal film thickness between bubbles satisfies the coalescence criterion when $\varepsilon$. Some nodes are now eliminated and the potential distribution over the bubble surface remain unchanged for all other nodes. In the coalescence case, the motion of the bubble can be solved by equation (1)-(4), for it only relates to the calculation of singly connected bubble.

3. Computational results

Different phenomena of two bubbles can be observed for different values of the phase difference and the inter-bubble distance \cite{5}. Three typical phenomena- coalescence, jet towards and jet away- are investigated numerically in this paper. Meanwhile, the velocity and pressure fields at certain moment are calculated and discussions are made through the analysis of the flow field.

3.1. Coalescence

Coalescence of two bubbles is investigated with $\varepsilon = 200, R_0 = 0.1159, d_m = 0.9$, and the motion of two in-phase bubbles is simulated. Both the velocity and pressure fields are presented to explain the evolution patterns. Bubble profile at four different times is shown in Figure 1.

Two bubbles are incepted simultaneously and they expand rapidly due to the internal high pressure. The interplay causes the closer walls of bubbles to be flattened, as shown in Figure 1(a). At $T=0.2744$, the minimal film thickness between bubbles satisfies the coalescence criterion and numerical technique is used for obtaining the coalesced bubble. The pressure field at frame $T=0.2744$ states the bubble overexpansion, and it’s noted that the subsequent expansion of the coalesced bubble will be stopped by the high pressure of flow field. The collapse of the coalesced bubble is observed at $T=1.7829$. At this moment, the larger pressure gradient of the flow field around the top and the bottom of bubble leads to a faster collapse of both the top and the bottom than that of the side wall. The fluid near to the axis of symmetry is rapidly drawn due to the fast contraction of both the top and the bottom, and then two high-pressure regions are formed as a result of the fluid impact on the symmetric axis. The downward jet of the top surface and the upward jet of the bottom surface impact in the middle of the coalesced bubble at $T=2.2573$, which are caused by the two high-pressure regions. It can be seen that a central high-pressure region is formed due to the jet impact, which can redirect the incoming fluid to form a jet pointing to the outside wall of the toroidal bubble. At the same time, the toroidal bubble proceeds to collapse. At $T=2.4080$, the jet impacts the outside wall and the bubble continues to collapse (the bubble profile and the flow field are shown in Figure 1(d)).
3.2. Jet towards

The parameters for jet towards case are $\varepsilon = 20, R_0 = 0.2695, d_{in} = 3$. Two bubbles are created at the same time and calculation results of three different times are shown in Figure 2 to account for the evolution patterns of the bubbles.

Two in-phase bubbles expand due to the internal high pressure, and the pressure inside the bubble decreases gradually in the expansion phase. The expansion is about to be restrained once the internal pressure is less than the pressure of the flow field. As shown in Figure 2(a), the pressure in fluid domain is greater than that inside the bubble at $T=0.4111$, which restrains the expansion. It can be found that the pressure of the region between two bubbles is less than that of other regions in fluid domain, which accounts for the later collapse of the closer walls of bubbles. At $T=1.2039$, the bubbles attain the maximum volume with collapse of the distant walls and expansion of the closer walls (see Figure 2(b)). It can be seen that the asymmetric pressure field around one bubble causes different collapse velocities of the nodes distributed over the bubble surface, so the top of the upper bubble and the bottom of the lower bubble tend to collapse relatively faster, leading to jet towards. The bubble profile and the flow field at the moment of impact are shown in Figure 2(c), and two high-pressure regions can be obviously observed.

Figure 1. Coalescence case
3.3. Jet away

The parameters for jet away case are $\varepsilon = 20, R_o = 0.2695, d_w = 2$, and the phase difference is $\Delta \vartheta = 2.9679$. The bubbles are generated sequentially with bubble 1 at $T=0$, and then bubble 2 at $T=1.0099$. Calculation results of three different times are shown in Figure 3.

Bubble 1 expands rapidly after its inception at $T=0$ and its expansion will be restrained when the time comes to generate bubble 2 at $T=1.0099$. At $T=1.0214$, bubble 1 comes into collapse phase due to the restraint from bubble 2. As shown in Figure 3(a), the bottom of bubble 1 has the tendency to form an upward jet (a jet away from bubble 2), while the elongated shaped bubble 2 is obtained because the overexpansion of bubble 1 provides a low-pressure boundary to bubble 2. See velocity and pressure fields in Figure 3(a), the expansion of bubble 2 has been restrained by the flow fluid, however, the top of bubble 2 continues to expand due to the low-pressure boundary provided by bubble 1 and it’s clear that both the collapse of bubble 1 and the elongation of bubble 2 are continued. In the collapse phase of bubble 1, fluid is rapidly drawn into the region between the two bubbles, leading to the formation of a stagnation point sometime. A high-pressure region is formed here to promote the jet of bubble 1 and
to restrain the expansion of bubble 2. At \( T=1.8181 \), the jet of bubble 1 impacts the upper wall and the obvious flattening of the top of bubble 2 can be observed, as shown in Figure 3(b). At this moment, bubble 2 is in the collapse phase and the contraction of its top surface is fastest under the effects of the high-pressure region. The interaction between the toroidal bubble 1 and bubble 2 is simulated afterwards. Two high-pressure regions are formed after the jet impact \([16]\), so the fast contraction of the top surface of bubble 2 proceeds with fluid continually drawn into the narrow region between the two bubbles. Similar to bubble 1, the downward jet of bubble 2 is also promoted by the high-pressure region. The jet proceeds with the continued contraction of bubble 2, while bubble 1 rebounds rapidly. When it comes to \( T=2.5073 \), the jet of bubble 2 impacts the bottom surface and bubble 1 rebounds obviously (see Figure 3(c)).

4. Conclusions
Based on potential flow theory, axisymmetric numerical investigation has been carried out for the evolution patterns of two identical-sized bubbles. Three typical phenomena- coalescence, jet towards and jet away- are simulated numerically. Meanwhile, velocity and pressure fields in these cases are calculated, which accounts for the bubble evolution to some extent. The following conclusions are obtained.

1) After the coalescence of two identical-sized bubbles, the jets of the top and the bottom surface impact in the middle of the coalesced bubble. Afterwards, the bubble becomes toroidal and a jet pointing to its outside wall is formed, leading to the formation of two toroidal bubbles.

2) The existence of an in-phase bubble with the same size will restrain the collapse of the closer walls of bubbles. The asymmetric pressure field around one bubble leads to the earlier and relatively faster collapse of the distant walls, so the two bubbles jet towards each other.

3) When two bubbles are not in-phase, the later incepted bubble 2 restrains the expansion of the earlier bubble 1 and bubble 1 will jet away from bubble 2. A high-pressure region between the two bubbles is formed due to the collapse of bubble 1, which helps bubble 2 to jet away from bubble 1.

References
[1] Voinov O V and Voinov V V 1975 Sov. Phy. Dokl. 20 179
[2] Blake J R and Gibson D C 1981 J. Fluid Eng. 111 123
[3] Blake J R, Taib B B and Doherty G 1986 J. Fluid Mech. 170 479
[4] Wang Q X, Yeo K S, Khoo B C and Lam K Y 1996 Comput. Fluids 25(7) 607
[5] Fong S W, Adhikari D, Klaeboer E and Khoo B C 2008 Exp. Fluids 46 705
[6] Zhang A M, Wang S P, Bai Z H and Wang C 2011 Acta Phys. Sin. 43(1) 71 (in Chinese)
[7] Blake J R, Robinson P B, Shima A and Tomita Y 1993 J. Fluid Mech. 255 707
[8] Robinson P B, Blake J R, Kodama T, Shima A and Tomita Y 2001 J. Appl. Phy. 89(12) 8225
[9] Pearson A, Cox E, Blake J R and Otto S R 2004 Eng. Anal. Bound. Elem. 28 295
[10] Rungsiyaphornrat S, Klaeboer E, Khoo B C and Yeo K S 2005 Comput. Fluids 32 1049
[11] Newman J N 1977 Marine Hydrodynamics (1st Ed.) (London: MIT Press) 131
[12] Cole R H. 1948 Underwater Explosions (1st Ed.) (Prinston University Press) 164
[13] Wang C and Khoo B C 2004 J Comput. Phys. 194 451
[14] Best J P 1993 J. Fluid Mech. 251 79
[15] Best J P 1994 The rebound of toroidal bubbles Bubble Dynamics and Interface Phenomena (Kluwer) ed. Blake I R, Boulton-Stone J M, Thomas N H 405-412
[16] Li S, Zhang A M and Han R Wang C and Khoo B C 2004 J. Comput. Phys. 194 451