Neutrino Mass Spectra, CP-Violation and Neutrinoless Double-Beta Decay\footnote{Presented by S. Pascoli}

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Assuming 3-$\nu$ mixing and massive Majorana neutrinos, we present some implications of the oscillation solutions of the solar and atmospheric neutrino problems and of the results of the CHOOZ experiment for the predictions of the effective Majorana mass in neutrinoless double beta-decay. If the present or upcoming ($\beta\beta_{0\nu}$)-decay searches give a positive result, the Majorana nature of massive neutrinos will be established. From the determination of the value of the effective Majorana mass, it would be possible to obtain information on the neutrino mass spectrum. With additional information on the absolute value of neutrino masses, it might be possible to infer if CP-parity is violated in the leptonic sector.

1 Introduction

With the accumulation of more and stronger evidences for oscillations of the atmospheric and solar neutrinos, caused by neutrino mixing (see, e.g.,\textsuperscript{3}), the problem of the nature of massive neutrinos emerges as one of the fundamental problems in the studies of neutrino mixing. Massive neutrinos can be Dirac or Majorana particles. In the former case they possess a conserved lepton charge and distinctive antiparticles, while in the latter there is no conserved lepton charge and massive neutrinos are truly neutral particles identical with their antiparticles (see, e.g.,\textsuperscript{3}). Thus, the question of the nature of massive neutrinos is directly related to the question of the basic symmetries of the fundamental particle interactions.

The present and upcoming experiments devoted to study neutrino oscillations will allow to make a big step forward in understanding the patterns of neutrino mass squared differences and

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of $\nu$-mixing but won’t be able to determine the absolute value of $\nu$-masses and to answer the question regarding the nature of massive neutrinos.

The $^3$H $\beta$-decay experiments, studying the electron spectrum, are sensitive to the electron (anti-)neutrino mass $m_1$ and can give information on the absolute value of neutrino masses: the Troitzk and Mainz experiments present bounds read $m_{\nu_e} < 2.5$ eV (at 95% C.L.) and there are prospects to increase the sensitivity of the $^3$H $\beta$-decay experiments to $m_{\nu_e} \sim (0.3 - 1.0)$ eV by the KATRIN experiment.

The problem of the nature of massive neutrinos can be addressed in experiments in which the total lepton charge $L$ is not conserved and changes by two units, $\Delta L = 2$. The process most sensitive to the existence of massive Majorana neutrinos (coupled to the electron) is the neutrinoless double $\beta ((\beta\beta)_{0\nu} - )$ decay of certain even-even nuclei (see, e.g., Refs.): $(A,Z) \to (A,Z + 2) + e^- + e^-$. If the $(\beta\beta)_{0\nu} -$ decay is generated only by the left-handed charged current weak interaction through the exchange of virtual light massive Majorana neutrinos, the probability amplitude of this process is proportional to the “effective Majorana mass parameter”:

$$|\langle m \rangle| \equiv |U_{e1}^2 m_1 + U_{e2}^2 m_2 + U_{e3}^2 m_3|,$$

where $m_j$ is the mass of the Majorana neutrino $\nu_j$ and $U_{ej} = |U_{ej}|e^{i\alpha_j /2}$ is the element of the neutrino (lepton) mixing matrix, with $\alpha_j$ the CP-violating phase. $|\langle m \rangle|$ depends only on two Majorana CP-violating phase differences $\alpha_j \equiv \alpha_j - \alpha_1$, $j = 2,3$. If CP-parity is conserved we have $\alpha_j = k\pi$, $k = 0, 1, 2$... Many experiments are searching for $(\beta\beta)_{0\nu} -$ decay. No indications that this process takes place were found so far. A rather stringent constraint on the value of $|\langle m \rangle|$ was obtained in the $^{76}$Ge Heidelberg-Moscow experiment: $|\langle m \rangle| < (0.35 \pm 1.05)$ eV and in the IGEX one: $|\langle m \rangle| < (0.33 \pm 1.35)$ eV (both at 90% C.L.). Higher sensitivity to the value of $|\langle m \rangle|$ is planned to be reached in several $(\beta\beta)_{0\nu}$-decay experiments of a new generation. The NEMO3, scheduled to start in 2001, and the planned CUORE experiments will have a sensitivity to values of $|\langle m \rangle| \approx 0.1$ eV. A sensitivity to $|\langle m \rangle| \approx 10^{-2}$ eV, is planned to be achieved in the GENIUS and EXO experiments.

2 Predictions for the Effective Majorana Mass Parameter.

Assuming 3-$\nu$ mixing and massive Majorana neutrinos, we present some implications of the neutrino oscillation fits of the solar and atmospheric neutrino data and of the results of the reactor long baseline CHOOZ and Palo Verde experiments for the predictions of the effective Majorana mass parameter $|\langle m \rangle|$, which controls the $(\beta\beta)_{0\nu} -$ decay rate. The present article represents a brief review of the detailed results obtained in Refs. Earlier studies on the subject include, e.g., Refs. The case of 4-$\nu$ mixing has been treated also in detail in Refs. The predicted values of $|\langle m \rangle|$ depend strongly on the type of the neutrino mass spectrum, on the solution of the solar neutrino problem, as well as on the values of the two Majorana CP-violating phases, present in the lepton mixing matrix. We consider here three types of mass spectra: hierarchical ($m_1 \ll m_2 \ll m_3$), with inverted hierarchy ($m_1 \ll m_2 \sim m_3$), and with three quasi-degenerate neutrinos ($m_1 \sim m_2 \sim m_3$).

If the neutrino mass spectrum is hierarchical, only the contributions to $|\langle m \rangle|$ due to the exchange of the two heavier Majorana neutrinos $\nu_{2,3}$ can be relevant and $|\langle m \rangle|$ depends only on one CP-violating phase, $\alpha_3 - \alpha_2$. The effective mass can be given in terms of the oscillating parameters $\Delta m_2^2$, $\Delta m_3^2$, $\sin^2 \theta_\odot$, $|U_{e3}|^2$, which is constrained by the CHOOZ data, as:

$$|\langle m \rangle| \approx \sqrt{\Delta m_2^2 (1 - |U_{e3}|^2) \sin^2 \theta_\odot + \sqrt{\Delta m_3^2 |U_{e3}|^2 e^{i(\alpha_3 - \alpha_2)}}}. \quad (2)$$

For the SMA and LOW-QVO solutions one has $|\langle m \rangle| \approx 4.0 \times 10^{-3}$ eV. For the LMA solution we get $|\langle m \rangle| \approx 8.5 - 10 \times 10^{-3}$ eV if one uses the results of the analyzes in Refs. and . The maximal values of $|\langle m \rangle|$ correspond to CP-conservation and $\nu_2$ and $\nu_3$ having identical CP-parities,
\( \phi_2 = \phi_3 \). For all three solutions there are no significant lower bounds on \( \langle m \rangle \) because mutual compensations between the terms contributing to \( \langle m \rangle \), corresponding to the exchange of different virtual massive Majorana neutrinos, are possible.

In the case of the inverted hierarchy mass spectrum, the dominant contribution to \( \langle m \rangle \) is due to the two heavier Majorana neutrinos \( \nu_{2,3} \) and \( \langle m \rangle \) is determined by \( \Delta m^2_{\text{atm}} \sin^2 2\theta_\odot \), the CP-violating phase \( \alpha_{31} - \alpha_{21} \) and by \( |U_{e1}|^2 \), constrained by the CHOOZ data as:

\[
\langle m \rangle \approx \sqrt{\Delta m^2_{\text{atm}}} \left( 1 - |U_{e1}|^2 \right) \sqrt{1 - \sin^2 2\theta_\odot \sin^2 (\alpha_{31} - \alpha_{21})} / 2.
\] (3)

The effective Majorana mass can be considerably larger than in the case of a hierarchical neutrino mass spectrum: \( |\langle m \rangle| \lesssim (6.8 - 8.9) \times 10^{-2} \text{ eV} \). The maximal \( |\langle m \rangle| \) corresponds to CP-conservation and \( \phi_2 = \phi_3 \) (\( \alpha_{31} - \alpha_{21} = 0 \)); it is possible for all three solutions of the \( \nu_\odot \)-problem. A lower bound of \( |\langle m \rangle| \gtrsim (3 - 4) \times 10^{-2} \text{ eV} \) is present in the case of the SMA solution, while for the LMA and LOW-QVO solutions values of \( |\langle m \rangle| \ll 10^{-2} \text{ eV} \) are possible. Both for the LMA and LOW-QVO solutions there exist relatively large “just-CP-violation” regions (a value of \( |\langle m \rangle| \) in this region would unambiguously signal the existence of CP-violation in the lepton sector, caused by Majorana CP-violating phases) of \( |\langle m \rangle| \). A measurement of \( |\langle m \rangle| \) , \( \Delta m^2_{\text{atm}} \), \( \sin^2 2\theta_\odot \), \( (1 - |U_{e1}|^2) \) can allow to get direct information on the CP-violation in the lepton sector and on the value of the CP-violating phase \( \alpha_{31} - \alpha_{21} \).

For quasi-degenerate neutrinos, we have \( |\langle m \rangle| \sim m \), where \( m \) is the common neutrino mass constrained by the \( ^3 \text{H} \beta \)-decay experiments. \( |\langle m \rangle| \) depends also on \( \theta_\odot \), \( |U_{e3}|^2 \) (constrained by the CHOOZ data), and on two physical CP-violating phases, \( \alpha_{21} \) and \( \alpha_{31} \):

\[
|\langle m \rangle| \approx m \left| \cos^2 \theta_\odot (1 - |U_{e3}|^2) + \sin^2 \theta_\odot (1 - |U_{e3}|^2) \right| e^{i\alpha_{21}} + |U_{e3}|^2 e^{i\alpha_{31}}.
\] (4)

The maximal value of \( |\langle m \rangle| \) is determined by the value of \( m \), \( |\langle m \rangle| \leq m \) and is limited by the upper bounds obtained in the \( ^3 \text{H} \beta \)-decay and in the \( (\beta\beta)_\text{0ν} \)-decay experiments: \( |\langle m \rangle| < (0.33 - 1.05) \text{ eV} \). The existence of a significant lower bound on \( |\langle m \rangle| \) for the LMA and the LOW-QVO solutions depends on the \( \min \left| \cos 2\theta_\odot \right| \). The latter varies with the analysis: using the results of Ref. one finds \( |\langle m \rangle| \gtrsim (0.1 - 0.2) \text{ m} \) (\( |\langle m \rangle| \gtrsim (0.2 - 0.3) \text{ m} \) for the LMA (LOW-QVO) solution. According to the results of the analysis, one can have \( \cos 2\theta_\odot = 0 \) and therefore there is no significant lower bound on \( |\langle m \rangle| \) for both solutions. There exist “just-CP-violation” regions for the LMA and LOW-QVO solutions, in which \( |\langle m \rangle| \) can be in the range of sensitivity of the future \( (\beta\beta)_\text{0ν} \)-decay experiments, while in the case of the SMA MSW solution there is no such region. The knowledge of \( |\langle m \rangle| \), \( m \), \( \theta_\odot \) and \( |U_{e3}|^2 \) would imply a non-trivial constraint on the two CP-violating phases \( \alpha_{21} \) and \( \alpha_{31} \).

3 Conclusions

If \( (\beta\beta)_\text{0ν} \)-decay will be detected by present or upcoming experiments, we will conclude that neutrinos are massive Majorana particles and that the total lepton charge \( L \) is not conserved. The observation of the \( (\beta\beta)_\text{0ν} \)-decay with a rate corresponding to \( |\langle m \rangle| \gtrsim 0.02 \text{ eV} \), which is in the range of sensitivity of the future \( (\beta\beta)_\text{0ν} \)-decay experiments, can provide unique information on the neutrino mass spectrum. A measured value of \( |\langle m \rangle| \gtrsim (2 - 3) \times 10^{-2} \text{ eV} \) would strongly disfavor (if not rule out), (under the general assumptions of 3-neutrino mixing, \( (\beta\beta)_\text{0ν} \)-decay generated only by the charged (V-A) current weak interaction via the exchange of the three Majorana neutrinos, neutrino oscillation solutions of the solar neutrino problem and atmospheric neutrino anomaly) the possibility of a hierarchical neutrino mass spectrum, while a value of \( |\langle m \rangle| \gtrsim (2 - 3) \times 10^{-1} \text{ eV} \) would rule out the hierarchical neutrino mass spectrum, strongly disfavor the inverted hierarchy one and favor the quasi-degenerate spectrum.
From the determination of the value of $|<m>|$ constraints on the absolute value of neutrino masses could be inferred. Having additional information on the value of neutrino masses or the type of neutrino mass spectrum, we could obtain also information on the CP-violation in the lepton sector, and - if CP-invariance holds - on the relative CP-parities of the massive Majorana neutrinos. For the LMA MSW and LOW solutions of the solar-$\nu$ anomaly, it would be possible to find either an allowed range of values for $m_1$ (if $\Delta m^2_{21} = \Delta m^2_{32}$ or $\Delta m^2_{32} = \Delta m^2_{12}$ and $m_1 > 10^{-2}$ eV) or an upper bound on $m_1$ (if $\Delta m^2_{32} = \Delta m^2_{12}$ and $m_1 < 10^{-2}$ eV). With additional information on $m_1$ we could establish if CP is violated in the lepton sector and either the values of the CP-violating phases or the type of CP-parity patterns of the neutrinos. For the SMA MSW solution of the solar-$\nu$ anomaly, we could obtain the value of $m_1$ (if $\Delta m^2_{21} = \Delta m^2_{32}$ or $\Delta m^2_{32} = \Delta m^2_{12}$ and $m_1 > 10^{-2}$ eV) or an upper bound on $m_1$ (if $\Delta m^2_{32} = \Delta m^2_{12}$ and $m_1 < 10^{-2}$ eV). In this case we have no information on the CP-violation in the lepton sector.

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References

1. SuperKamiokande Coll., talk by H. Sobel at Neutrino 2000, to appear in the Proceedings.
2. SuperK Coll., talk by Y. Suzuki at Neutrino 2000, to appear in the Proceedings.
3. S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59, 671 (1987).
4. V. Lobashov et al., talk at Neutrino 2000, to appear in the Proceedings.
5. C. Weinheimer et al., talk at Neutrino 2000, to appear in the Proceedings.
6. A. Aseev et al., Int. Workshop on $\nu$-Masses in Sub-eV Range, Bad Liebenzell, Germany.
7. S. M. Bilenky et al., Phys. Lett. B 94, 495 (1980).
8. L. Wolfenstein, Phys. Lett. B 107, 77 (1981).
9. S. M. Bilenky, N. P. Nedelcheva and S. T. Petcov, Nucl. Phys. B 247, 589 (1984); B. Kayser, Phys. Rev. D 30, 1023 (1984).
10. L. Baudis et al., Phys. Rev. Lett. 83, 41 (1999); H. V. Klapdor-Kleingrothaus et al., submitted for publ. to Phys. Lett. B and talk given at NOW2000, Otranto, Italy, 09/2000.
11. C.E. Aalseth, F.T. Avignone III et al., Physics of Atomic Nuclei 63, 1225 (2000).
12. C. Marquet et al., (NEMO3 Coll.) to appear in Nucl. Phys. B (Proc. Suppl.) 87 (2000); E. Fiorini, Phys. Rep. 307, 309 (1998); H. V. Klapdor-Kleingrothaus et al., J. Phys. G 24, 483 (1998); L. Baudis et al., Phys. Rep. 307, 301 (1998); M. Danilov et al., Phys. Lett. B 480, 12 (2000).
13. G.L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, Phys. Rev. D 62, 113003 (2000).
14. C. Gonzalez-Garcia et al., Phys. Rev. D 63, 033005 (2001).
15. M. Appolonio et al., Phys. Lett. B 466, 415 (1999) [hep-ex/9907037].
16. S.M. Bilenky, S. Pascoli and S.T. Petcov, hep-ph/0102265 (to appear in Phys. Rev. D).
17. S. T. Petcov and A. Yu. Smirnov, Phys. Lett. B 322, 109 (1994); S.M. Bilenky, C. Giunti, C.W. Kim and S.T. Petcov, Phys. Rev. D 54, 4432 (1996); S.M. Bilenky, C. Giunti, W. Grimus, B. Kayser and S.T. Petcov, Phys. Lett. B 465, 193 (1999); F. Vissani, JHEP 06, 022 (1999); V. Barger and K. Whisnant, Phys. Lett. B 456, 194 (1999); H. Minakata and O. Yasuda, Nucl. Phys. B 523, 597 (1998); T. Fukuyama et al., Phys. Rev. D 57, 5844 (1998) and hep-ph/9804262; M. Czakon et al., hep-ph/0010077; H. V. Klapdor-Kleingrothaus, H. Pas and A. Yu. Smirnov, hep-ph/0003219.
18. S.M. Bilenky, S. Pascoli and S.T. Petcov, hep-ph/0104218.