Access Graphs Results for LRU versus FIFO under Relative Worst Order Analysis*

Joan Boyar, Sushmita Gupta, and Kim S. Larsen

University of Southern Denmark, Odense, Denmark
{joan,sgupta,kslarsen}@imada.sdu.dk

Abstract. Access graphs, which have been used previously in connection with competitive analysis to model locality of reference in paging, are considered in connection with relative worst order analysis. In this model, FWF is shown to be strictly worse than both LRU and FIFO on any access graph. LRU is shown to be strictly better than FIFO on paths and cycles, but they are incomparable on some families of graphs which grow with the length of the sequences.

1 Introduction

The term online algorithm [4] is used for an algorithm that receives its input as a sequence of items, one at a time, and for every item, before knowing the subsequent items, must make an irrevocable decision regarding the current item.

The most standard measure of quality of an online algorithm is competitive analysis [17,22,20]. This is basically the worst case ratio between the performance of the online algorithm compared to an optimal offline algorithm which is allowed to know the entire input sequence before processing it and is assumed to have unlimited computational power.

Though this measure is very useful and has driven much research, researchers also observed problems [22] with this measure from the beginning: many algorithms obtain the same (poor) ratio, while showing quite different behavior in practice. The paging problem is one of the prime examples of these difficulties. The paging problem is the problem of maintaining a subset of a large number of pages in a much smaller, faster cache with space for a limited set of \( k \) pages. Whenever a page is requested, it must be brought into cache if it is not already there. In order to make room for such a page, another page currently in cache must be evicted. Therefore, an online algorithm for this problem is often referred to as an eviction strategy.

For a number of years, researchers have worked on refinements or additions to competitive analysis with the aim of obtaining separations between different algorithms for solving an online problem. Some of the most obvious and well-known paging algorithms are the eviction strategies LRU (Least-Recently-Used) and FIFO (First-In/First-Out). One particularly notable result has been the

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separation of LRU and FIFO via access graphs. Access graphs were introduced in [5] with the aim of modelling the locality of reference that is often seen in real-life paging situations [10,11]. An access graph is an undirected graph with all pages in slow memory as vertices. Given such a graph, one then restricts the analysis of the performance of an algorithm to sequences respecting the graph, in the sense that any two distinct, consecutive requests must be neighbors in the graph. Important results in understanding why LRU is often observed to perform better than FIFO in practice were obtained in [5,9], showing that on some access graphs, LRU is strictly better than FIFO, and on no access graph it is worse; all these previous results are with respect to competitive analysis.

Attempts have been made to define new generally-applicable performance measures and to apply measures defined to solve one particular problem more generally to other online problems. A collection of alternative performance measures is surveyed in [12]. Of the alternatives to competitive analysis, relative worst order analysis [6] and extra resource analysis [19] are the ones that have been successfully applied to most different online problems. See [13] for examples of online problems and references to relative worst order analysis results resolving various issues that are problematic with regards to competitive analysis.

Paging has been investigated under relative worst order analysis in [7]. Separations were found, but LRU and FIFO were proven equivalent, possibly because locality of reference is necessary to separate these two paging algorithms. We apply the access graph technique to relative worst order analysis. Note that the unrestricted analysis in [7] corresponds to considering a complete access graph.

Overall, our contributions are the following. Using relative worst order analysis, we confirm the competitive analysis result [5] that LRU is better than FIFO for path access graphs. Since these two quality measures are so different, this is a strong indicator of the robustness of the result. Then we analyze cycle access graphs, and show that with regards to relative worst order analysis, LRU is strictly better than FIFO. Note that this does not hold under competitive analysis. The main technical contribution is the proof showing that on cycles, with regards to relative worst order analysis, FIFO is never better than LRU. Clearly, paths and cycles are the two most fundamental building blocks, and future detailed analyses of any other graph type will likely build on these results.

The standard example of a very bad algorithm with the same competitive ratio as LRU and FIFO is FWF, which is shown to be strictly worse than both LRU and FIFO on any access graph (containing a path of length at least k + 1), according to relative worst order analysis. Using relative worst order analysis, one can often obtain more nuanced results. This is also the case here for general access graphs, where we establish an incomparability result.

None of the algorithms we consider require prior knowledge of the underlying access graph. This issue was pointed out in [15] and [16] in connection with the limitations of some of the access graph results given in [5,14,18] and the Markov paging analogs in [21].

As relative worst order analysis is getting more established as a method for analyzing online algorithms, it is getting increasingly important that the theoretical