IS THE SOLAR CORONA NONMODALLY SELF-HEATED?

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ABSTRACT

Recently it was pointed out that nonmodally (transiently and/or adiabatically) pre-amplified waves in shear flows, undergoing subsequent viscous damping, can ultimately heat the ambient flow. The key ingredient of this process is the ability of waves to grow, by extracting energy from the spatially inhomogeneous mean flow. In this paper we examine this mechanism in the context of the solar coronal plasma flows. “Self-heating” (SH) processes are examined when both viscous damping and magnetic resistivity are at work. We show that if the plasma viscosity is in the favorable range of values the asymptotic SH rate in these flows can be quite substantial.

1. INTRODUCTION

Astrophysical plasma flows are often complex, inhomogeneous, dynamic systems hosting different kinds of collective phenomena. Sometimes interactions between collective phenomena and ambient flows feature a highly sophisticated level of complexity. In general, shear flows (SF) are well-known examples of behavioral complexity - both in neutral and charged fluids, both for terrestrial and astrophysical cases and both on the level of experiments (observations) and theory (simulations) - SF display a wide spectrum of “shear-induced” phenomena. In particular, astrophysical SF are gradually becoming a popular subject of research because it becomes increasingly more clear that most of the cosmic plasmas are flowing with spatially inhomogeneous rates and in most of these flows shear-induced processes may leave a considerable imprint on the observational appearance of the related astronomical objects.

A separate class of shear-induced phenomena is related with the non-self-adjointness of the linear operators governing the dynamics of SF systems (Trefethen et al. 1993). Fluctuations in a “parent” SF obey “non-Hermitian” equations and, therefore, their evolution, their interaction with the ambient flows can not be molded within the standard Hamiltonian formalism. The most striking novelty is the appearance of new, shear-induced nonperiodic modes - so-called Kelvin or shear vortices. Interactions between the modes and between the flow and the modes bring into the game a nonmodal complexity: these “non-normal” modes show a transient increase in amplitude, they are coupled and are transformed into each other, they may feature beating phenomena and may become the subjects of different shear instabilities.

Recently it was pointed out that SF may host yet another interesting non-modal phenomenon related with the combined presence of nonmodality and dissipative processes in the flows. It can formally be considered as a three-phase process (Rogava 2004):

1. waves and/or vortices get excited within a flow;
2. they amplify nonmodally, due to the presence of the shear flow, extracting a part of the flow kinetic energy;
3. high-amplitude waves and/or transiently amplified vortices undergo a viscous decay and/or magnetic diffusion and give, in the form of heat, a part of their energy back to the flow.

As a result of this three-step process the fluctuation gives back to the flow more thermal energy than it had at the moment of its excitation. Repeated continuously, throughout the whole volume of the flow, these elementary processes should lead to a net heating of the flowing plasma. The process was named “self-heating” (SH), because what actually happens is that nonmodality manages to transfer a part of the flow’s regular kinetic energy into a thermal energy; i.e., the heating happens conservatively without any outside source of energy, the flow manages to heat itself.

The possibility of this process was originally surmised in (Rogava 2003a) and later it was examined and specified in detail for the relatively simple case of a neutral fluid SF and sound waves sustained by it (Rogava 2004). It was shown that in the hydrodynamic situation - even when a mixture of sound waves and Kelvin vortices is propagating in a simple, plane-parallel Couette-like SF and, therefore, acoustic waves are able to amplify only linearly, extracting the kinetic energy of the ambient flow - the self-heating rate can be quite high: fluctuations are able to give back to the flow several times more energy than they have initially had.

The purpose of this paper is to explore whether the SH can be efficient in magnetized plasma MHD flows. In this particular study we consider only the case of a plane-parallel flow with a linear velocity profile. However, in order to lay the foundation for a further,
more general, consideration of flows with kinematic complexity we derive our equations for the case of swirling (helical) flows, following the approach developed in (Rogava et al. 2003a and 2003b) (henceforth referred to as R1 and R2) and consider the case of the plane-parallel flow as a particular example.

Our results show that both in the incompressible (when the flow sustains only Alfvén waves) and in the compressible (full spectrum of MHD waves) cases the self-heating is even more efficient than it was in the hydrodynamic case. Our numerical simulations revealed that the values of the dimensionless asymptotic SH rate \( \Xi_\infty \) can be of the order of several tens. This allows us to argue that the SH may easily be one of the robust physical mechanisms contributing to the heating of the solar corona through the viscous and/or resistive damping of nonmodally pre-amplified waves.

It should be noted that nonmodal SH, being efficient per se possesses another attractive feature in the context of its possible relevance for coronal heating: all existing wave heating scenarios face common problem: how is the energy transferred from longer length-scales to shorter ones, where dissipative effects are significant?! SH phenomena, being nonmodal, involve the shear-induced drift of the wave number vector \( \mathbf{k}(t) \), which provides a natural, flow-related mechanism for the gradual decrease of the mode’s length-scale. This process, being linear in plane-parallel SF, may have an exponentially fast nature in geometrically and kinematically more sophisticated cases. Therefore, we envisage that kinematically complex flow patterns, such as solar spicules, macrospicules and tornados, magnetic plumes, might host much more efficient, sometimes even explosive, SH events.

2. MAIN CONSIDERATION

The present study follows the theory of nonmodal phenomena in helical MHD flows, recently developed both for the incompressible (R1) and the compressible (R2) cases. The difference between our current and those studies is in the presence of a dissipation - viscous damping and/or magnetic resistivity - represented in the equations by terms proportional to the coefficients of kinematic viscosity (\( \nu_h \)) and magnetic resistivity (\( \nu_m \)), respectively. Since a nonmodal analysis is essentially linear, the instantaneous values of all physical variables have to be splitted into their equilibrium and fluctuative components: \( \mathbf{B}=\mathbf{B}_0+\mathbf{B}' \), \( \rho=\rho_0+\rho' \), etc; with the subsequent linearization of the equations for perturbations.

The equilibrium, considered in R1 and R2, assumes a homogeneous MHD plasma (\( \rho_0=const \)) flow, embedded in a homogeneous, vertical magnetic field (\( \mathbf{B}_0=[0, 0, B_0=const] \)). The mean flow vector field \( \mathbf{U}_0(r) \) in the R1 was specified as:

\[
\mathbf{U}(r)=[0, r\Omega(r), U(r)],
\]

with \( \Omega(r) = A/r^n \), where \( r = (x^2 + y^2)^{1/2} \) is a distance from the rotation axis, while \( A \) and \( n \) are some constants.

If we take into account the viscous damping and magnetic resistivity then linearized equations for fluctuations can be written as [\( \partial_t + (\mathbf{U}_0 \cdot \nabla) \)]:

\[
\partial_t + (\mathbf{U}_0 \cdot \nabla) \mathbf{U}_0 = 0,
\]

\[
\partial_t \mathbf{b}_1 + (\mathbf{U}_0 \cdot \nabla) \mathbf{b}_0 = -\nabla p + C_A^2 \nabla \times (\mathbf{b}_0),
\]

\[
\nabla \cdot \mathbf{b} = 0,
\]

with \( d \equiv \rho'/\rho_0 \), \( p \equiv p'/\rho_0 \) and \( \mathbf{b} \equiv \mathbf{B}'/B_0 \). Note that for incompressible fluctuations instead of (2) we have:

\[
\nabla \cdot \mathbf{u} = 0,
\]

while for the compressible case the closure of the (2-5) is guaranteed by the equation of state, implying:

\[
p' = C_s^2 \rho',
\]

with \( C_s \) and \( C_A \) being the homogeneous speed of sound and the Alfvén speed, respectively.

The nonmodal method for studying the dynamics of linearized, small-scale fluctuations in kinematically complex flows (Lagnado et al. 1984, Craik and Criminale 1986, Mahajan and Rogava 1999) enables the reduction of the initial set of partial differential equations for the perturbation variables \( F(r, t) \) in the real physical space to initial value problem for the spatial Fourier harmonics (SFH) of the same variables, \( F(k, t) \), defined in the k-space. The key element of this approach is the time variability of the \( k(k) \)'s, imposed by the presence of the shear flow and governed by the following set of equations [\( \partial_t f \equiv f^{(n)} \)]:

\[
k^{(1)} + \mathbf{S}^{(T)} \cdot \mathbf{k} = 0,
\]

which gives, depending on the particular form of the shear matrix S (Mahajan and Rogava 1999), a full evolutionary picture of the temporal drift of the wave number vector field \( k(t) \).

For a helical flow, specified by the equilibrium velocity (1) and with five nonzero components of the traceless shear matrix (specifying the stretching of flow-lines (\( \sigma \)), velocity shear in rotational (\( A_1 \) and \( A_2 \)) and ejectional (\( C_1 \) and \( C_2 \)) components of the velocity, respectively) \( k_z \) stays constant, while the transversal components obey (R1):

\[
k_{x} + \sigma k_x + A_2 k_y + C_1 k_z = 0, \tag{8a}
\]

\[
k_{y} + A_1 k_x - \sigma k_y + C_2 k_z = 0, \tag{8b}
\]
implying that $k_x(t)$ and $k_y(t)$ may have an algebraic, exponential or periodic time dependence.

The ordinary nonautonomous differential equations for the SFH of physical variables can be derived from the set (2–5) and written in the following way:

$$\dot{\mathbf{v}}^{(1)} = \mathbf{k} \cdot \mathbf{v},$$  \hspace{1cm} (9)

$$v_x^{(1)} + (\mathbf{S} \cdot \mathbf{v})_x = -k_z \mathcal{P} + C_A^2(k_z b_x - k_x b_z) - \nu_h |\mathbf{k}|^2 v_x, \hspace{1cm} (10a)$$

$$v_y^{(1)} + (\mathbf{S} \cdot \mathbf{v})_y = -k_y \mathcal{P} + C_A^2(k_x b_y - k_y b_z) - \nu_h |\mathbf{k}|^2 v_y, \hspace{1cm} (10b)$$

$$v_z^{(1)} + (\mathbf{S} \cdot \mathbf{v})_z = -k_z \mathcal{P} - \nu_h |\mathbf{k}|^2 v_z, \hspace{1cm} (10c)$$

$$b_x^{(1)} = (\mathbf{S} \cdot \mathbf{b})_x - v_x - \nu_m |\mathbf{k}|^2 b_x, \hspace{1cm} (11a)$$

$$b_y^{(1)} = (\mathbf{S} \cdot \mathbf{b})_y - v_y - \nu_m |\mathbf{k}|^2 b_y, \hspace{1cm} (11b)$$

$$\mathbf{k} \cdot \mathbf{b} = 0. \hspace{1cm} (12)$$

The total energy of the perturbation consists of a compressional, a kinetic and a magnetic part (the first part is absent in the incompressible limit) and is equal to:

$$E \equiv [C_A^2 \dot{\mathbf{g}}^2 + C_A^2 \mathbf{b}^2 + \mathbf{v}^2]/2. \hspace{1cm} (13)$$

This total energy obeys the following nonautonomous equation:

$$E^{(1)} = (A_1 + A_2)(b_x b_y - v_x v_y) +$$

$$+C_1(b_x b_z - v_x v_z) + C_2(b_y b_z - v_y v_z) +$$

$$+\sigma[(b_x^2 b_y^2) - (v_x^2 v_y^2)] - |\mathbf{k}|^2[\nu_h \mathbf{v}^2 + \nu_m \mathbf{b}^2]. \hspace{1cm} (14)$$

Finally, following the hydrodynamic case (Rogava 2004), we define the asymptotic self-heating rate as the limit

$$\Xi_{\infty} \equiv \lim_{t \to \infty} \Xi(t), \hspace{1cm} (15)$$

of the following function:

$$\Xi(t) \equiv \frac{1}{E(0)} \int_0^t \left[\nu_h \mathbf{v}^2(t') + \nu_m \mathbf{b}^2(t')\right]|\mathbf{k}|^2(t')dt'. \hspace{1cm} (16)$$

In this short paper our purpose is not the study of fully complex helical flows. Rather, we wish to demonstrate the efficiency of the self-heating for relatively simple, plane-parallel velocity patterns of magnetized plasmas. Therefore, we consider only the simplest, plane-parallel flow case both for incompressible and compressible fluctuations.

2.1 Incompressible limit

In this case (6a) holds leading to the algebraic relation: \((\mathbf{k} \cdot \mathbf{v}) = 0\). The flow sustains only Alfvén waves, strongly modified by the presence of the SFH.

The efficiency of the wave heating mechanism strongly depends on temporal scales of the wave excitation/damping. In our case, incompressible MHD disturbances are able to amplify transiently, within a very short time interval, compared to the total timescale of the wave evolution (Chagelishvili et al. 1993). This makes this process a quite efficient mechanism for the excitement of large-amplitude Alfvén waves. However, when dissipation (viscous damping and/or magnetic resistivity) is also taken into account, it is reasonable to expect that under favorable conditions the “pre-amplified” Alfvén waves will eventually give their energy¹ back to the flow in the form of heat.

In order to give an illustrative example we solved the set of equations numerically for different values of the parameters, roughly typical for various structures of the solar atmosphere: \(V_A = 100 \text{ km s}^{-1}\), \(C_x = 0.08 \text{ s}^{-1}\), \(C_y = 0 \text{ s}^{-1}\) and a fixed value of wave number \(k_z = 4 \cdot 10^{-9} \text{ cm}^{-1}\) corresponding to the oscillation period \(T \approx 2.6 \text{ min}\). In all our calculations we used the conventional value of the magnetic resistivity coefficient (see Walsh & Ireland, 2003): \(\nu_m = 10^4 \text{cm}^2 \text{s}^{-1}\). As regards the viscosity coefficient, since the presence or absence of the microturbulence may change it by many orders of magnitude (Ruderman et al. 1998), we were inclined to consider it as a free parameter and we made a numerical analysis for different values.

It was found that when the shear viscosity is determined only by the momentum transfer due to ion diffusion and, therefore \(\nu_h \approx \nu_m\) (Ruderman et al. 1998) the actual timescale of the viscous damping is too large. Therefore, the damping of these waves in a laminar flow with low values of \(\nu_h\) and \(\nu_m\) could hardly account for the spatially confined processes of the coronal heating.

However, if one assumes that a microturbulence can be present in the coronal plasma, it may increase the viscosity coefficient in a very considerable way. In this case the temporal scale of the viscous damping for transiently pre-amplified waves drastically changes. In particular, it was found that when \(\nu_h \geq 10^9 \text{ cm}^2 \text{s}^{-1}\), Alfvén waves with the above-specified parameters damp efficiently within physically reasonable time intervals. In other words, the viscosity coefficient has to be increased by at least a factor of \(10^3\), compared to its laminar value, in order to guarantee effective damping. Still, this is about five orders of magnitude less than values of the \(\nu_h\) necessary for the efficient coronal heating via

¹Which they have just “stolen” from the flow via shear-induced transient amplification!
the conventional wave heating mechanisms (Ruderman et al. 1998). In the case of the SH of transiently pre-amplified Alfvén waves efficient dissipation occurs even for these, relatively “mild” (∼10^9 cm^2 s^-1) values of the viscosity coefficient!

Since the waves were considerably amplified before starting to decay it is reasonable to expect that they give back to the flow much more energy than they initially had at the moment of excitation. This means that the self-heating mechanism works and the plasma gets heated via the combined action of nonmodal transient pre-amplification of waves and their subsequent viscous decay. In panel A of Fig. 1 we show the temporal evolution of the magnetic field component b_y (the values are scaled up by the factor 10^2). In panel B of the same figure the curves showing the temporal behaviour of the normalized total energy E/E(0) of the perturbation (dashed line) and the SH rate function Ξ(t) (solid line) are presented.

2.2 The compressible medium

For compressible perturbations the closure of the set of equations comes from the Eq. (6b). In this case all three MHD waves can exist and in shear flows their nonmodal mutual transformation (Chagelishvili, Rogava and Tsiklauri 1996) may take place. Since we are interested in the perspectives for the nonmodal SH in solar coronal flows, we have to concentrate on the low plasma β case. In this case the slow mode is decoupled from two other MHD wave modes, but the Alfvén and fast magnetosonic modes are coupled and may transform into each other (Rogava, Poedts and Mahajan 2000).

The governing equations were solved numerically for the following set of parameters: C_s = 5 · 10^6 cm s^-1, V_A = 8 · 10^7 cm s^-1, η = 10^4 cm^2 s^-1, and C_x = 0.04 s^-1. In Fig. 2 the temporal evolution of the dimensionless density perturbation is plotted. In addition, we present in Fig. 3 the curves of variation of the normalized total energy E/E(0) of perturbations and the function Ξ(t) of the SH rate.

The evolution of the initially excited Alfvén wave formally may be split into three stages:

1) In the initial stage t ≤ 500 s (κ_x(t) > 0) we have a pure Alfvén mode and its energy slowly decreases in time (thick dashed line in Fig. 3). Here we also show the evolution of the total energy in the shearless limit (C_x = 0) by the thin dashed-dotted line. We can see that at this stage of the evolution, in both cases, the energy evolves similarly: these curves almost coincide with each other. The slow dissipation of the mode is represented by the slight increase of the SH function both in the case of the non-uniform flow (thick solid line) and in the case of the uniform (shearless) flow (thin dotted line).
2) At a certain stage (around the moment of time when \( k_x(t) \approx 0 \)) the Alfvén wave partially transforms into the fast mode. This fact is represented by the excitation of the density perturbation, see Fig. 2. The perturbation starts extracting energy from the mean flow\(^2\) (dashed line in Fig. 3). The presence of the velocity shear leads to the further monotonous decrease of the wavelength of the excited fast mode, while \( k_x(t) \) changes the sign and \( |k| \) starts growing again, and the energy of the fast mode grows adiabatically. But the efficiency of the dissipation grows as well, as far as the length-scales become shorter, and at a certain stage the energy of the fluctuation ceases to increase and starts decreasing - the exponential damping becomes prevalent, it starts dominating over the linear, adiabatic increase of the wave energy.

3) At the final stage of the evolution the dissipation effects prevail and the mode starts damping efficiently (see Fig. 2). Eventually all the energy extracted on the earlier stage by the wave from the flow returns back to the flow in the form of thermal energy (heat). The SH function rapidly grows and reaches its asymptotic value (see plateau in Fig. 3) when the wave is completely dissipated. This value is approximately equal to the doubled maximum value of the normalized wave energy. This situation is different from the previously considered incompressible case, where the amplification of the perturbation had a transient nature and occurred only within a very small time interval. In that case the quantity of the produced heat was roughly equal to the wave energy maximum (see panel B of Fig. 1).

In addition, we studied numerically the temporal evolution of the Alfvén disturbances for three different initial values of \( k_x \) for fixed values of all other parameters. The results show that with the increase of the initial wave number, for given values of dissipation coefficients, the SH rate decreases - the shorter the initial wavelengths are, the sooner the time comes when the damping effects prevail. Therefore, the nonmodal excited fast mode has less time to gain energy from the flow. A more efficient SH in this case would require smaller values of the dissipation coefficient. On the other hand, this process has another limit: if we decrease the value of the initial wavenumber the mentioned critical moment of time comes too late and, therefore, the effective time scale of the wave damping increases too much: the timescale of the heat production becomes much larger than the characteristic timescales related to the coronal situation.

Similarly, we have different regimes when we study the self-heating process for a fixed value of the wavenumber, but for different values of the viscosity coefficient. For a given Alfvén mode there exist values of the viscosity coefficient for which the wave amplification process is very intensive but the effective time of transferring this energy back to the background medium is too large to be interesting in the solar coronal context. The SH process becomes most efficient for larger viscosities. However, further increase of the strength of the dissipation, again, decreases the SH rate, because the excited waves are damped too rapidly, without significant nonmodal amplification by the shear flow. Therefore, there is a limited range of favorable viscosity levels, at which the self-heating might be expected to be most efficient.

3. CONCLUSIONS

The purpose of the present study was to clarify whether the shear-induced amplification of MHD waves coupled with viscous and magnetic-resistive dissipation may lead to a significant self-heating of the SF. In particular, our aim was to see whether this mechanism could be efficient for solar coronal plasma flows and could, arguably, contribute to the heating of the solar corona.

Since the variety of solar plasma flows is quite wide, both in terms of geometry and kinematics, the local formalism employed and developed within this study is apt for an arbitrary velocity pattern with the condition that the involved MHD waves are having length-scales, \( l \), sufficiently smaller than the linear length-scale, \( \ell \), of a “parent” flow pattern. Basically, our consideration follows Rogava et al. (2003a, 2003b) with addition of dissipative effects related to the presence of viscosity and magnetic resistivity.

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\(^2\)This situation drastically differs from the homogeneous flow case, where the transformation of the Alfvén wave into the fast mode does not happen and the energy of the perturbation continuously decreases (dashed dot line in Fig. 3).
In this paper we have examined only the case of the simplest velocity pattern with plane-parallel and linear velocity profile. Both incompressible and compressible limits were investigated. It was found that:

1. Incompressible, Alfvénic, perturbations, in the ideal MHD limit, are known to undergo an algebraic instability ("transient increase") (Chagelishvili et al. 1993; Rogava et al. 1996, Rogava et al. 2003a), which can be quite strong: under favourable conditions the total energy of a fluctuation may increase several hundred times. We found that when dissipation is taken into account these high-amplitude pre-amplified Alfvén waves get damped and give their energy back to the ambient flow. The resulting SH is quite substantial. As it could be expected the asymptotic self-heating rate in this case is quite large: it increases with the decrease of the Alfvén velocity and in the cases considered here could reach a value of the order of several tens. This happened to be a case for waves with the characteristic periods of the order of few minutes.

2. In the compressible case we considered the evolution of an initially excited Alfvén wave (as an example we considered Alfvén mode with period of the order of 30 seconds) in a plane-parallel flow pattern with the value of the plasma-β of the order of $10^{-2}$, typical for the coronal environment. In this case Alfvén waves and fast magnetosonic waves are coupled, which leads to the transformation of initially Alfvénic fluctuations into the fast ones $^3$ (Rogava et al. 2000). We found (see Fig. [3]) that this process, coupled with the presence of viscous and resistive dissipation, ensures the SH of the "parent" flow with the asymptotic rate of the order of several tens.

These results are quite encouraging. We see that even the simplest kinds of magnetized plasma SF’s are able to heat themselves via the agency of nonmodally pre-amplified Alfvén waves. It is both tempting and reasonable to surmise that in flow patterns with a higher degree of kinematic complexity, where nonmodal processes of energy exchange between flows and waves have a more intense nature (Mahajan and Rogava, 1999), the resulting SH can be considerably higher! In certain cases $^4$ individual flow patterns may tend to self-destruction, undergoing self-imposed, catastrophically fast SH of an eruptive, explosive nature. Further studies in this vein are currently being carried out and the results will be reported elsewhere.

The astrophysically relevant conclusion of the present study is that nonmodal self-heating, even for relatively simple kinds of flows, parallel to the magnetic field and with shear profiles across the field, can pay a significant contribution in the (still poorly understood) phenomenon of solar coronal heating.

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$^3$The latter wave, being of the acoustic nature, dissipates quite similarly to plain sound waves, recently shown to be instrumental in the SH of compressible fluid shear flows (Rogava 2004).

$^4$For instance, in flows with a helical mode of plasma motion (Rogava et al. 2003a, 2003b)