Measurement of Gilbert damping parameters in nanoscale CPP-GMR spin-valves

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abstract—In-situ, device level measurement of thermal mag-noise spectral linewidths in 60nm diameter CPP-GMR spin-valve stacks of IrMn/ref/Cu/free, with reference and free layer of similar CoFe/CoFeGe alloy, are used to simultaneously determine the intrinsic Gilbert damping for both magnetic layers. It is shown that careful alignment at a "magic-angle" between free and reference layer static equilibrium magnetization can allow direct measurement of the broadband intrinsic thermal spectra in the virtual absence of spin-torque effects which otherwise grossly distort the spectral line shapes and require linewidth extrapolations to zero current (which are nonetheless also shown to agree well with the direct method). The experimental magic-angle spectra are shown to be in good qualitative and quantitative agreement with both macrospin calculations and micromagnetic eigenmode analysis. Despite similar composition and thickness, it is repeatedly found that the IrMn exchange pinned reference layer has ten times larger intrinsic Gilbert damping ($\alpha \approx 0.1$) than that of the free-layer ($\alpha \approx 0.01$). It is argued that the large reference layer damping results from strong, off-resonant coupling to to lossy modes of an IrMn/ref couple, rather than commonly invoked two-magnon processes.

I. INTRODUCTION

Spin-torque phenomena, in tunneling magnetoresistive (TMR) or giant-magnetoresistive (GMR) film stacks lithographically patterned into ~100 nm nanopillars and driven with dc electrical currents perpendicular to the plane (CPP) of the films have in recent years been the topic of numerous theoretical and experimental papers, both for their novel physics as well as potential applications for magnetic memory elements, microwave oscillators, and magnetic field sensors and/or magnetic recording heads. In all cases, the electrical current density at which spin-torque instability or oscillation occurs in the constituent magnetic film layers is closely related to the magnetic damping of these ferromagnetic (FM) films. This paper considers the electrical measurement of thermal mag-noise spectra to determine intrinsic damping at the device level in CPP-GMR spin-valve stacks of sub-100nm dimensions (intended for read head applications), which allows simultaneous R-H and transport characterization on the same device. Compared to traditional ferromagnetic resonance (FMR) linewidth measurements at the bulk film level, the device-level approach naturally includes finite-size and spin-pumping effects characteristic of actual devices, as well as provide immunity to inhomogeneous and/or two-magnon linewidth broadening not relevant to nanoscale devices. Complementary to spin-torque-FMR using ac excitation currents, broadband thermal excitation naturally excites all modes of the system (with larger, more quantitatively modeled signal amplitudes) and allows simultaneous damping measurement in both reference and free FM layers of the spin-valve, which will be shown to lead to some new and unexpected conclusions. However, spin-torques at finite dc currents can substantially alter the absolute linewidth, and so it is necessary to account for or eliminate this effect in order to determine the intrinsic damping.

II. PRELIMINARIES AND MAGIC-ANGLES

Fig. 1a illustrates the basic film stack structure of a prospective CPP-GMR spin-valve (SV) read sensor, which apart from the Cu spacer between free-layer (FL) and reference layer (RL), is identical in form to well-known, present day TMR sensors. In addition to the unidirectional exchange coupling between the IrMn and the pinned-layer (PL), the usual "synthetic-antiferromagnet" (SAF) structure PL/Ru/RL is meant to increase magnetostatic stability and immunity to field-induced rotation of the PL-RL couple, as well as strongly reduce its net demagnetizing field on the FL which otherwise can rotate in response to signal fields. However, for simplicity in interpreting and modeling the spectral and transport data of Sec. III, the present experiment restricts attention to devices with a single RL directly exchange-coupled to IrMn, as shown in Fig. 1b.

The simplest practical model for describing the physics of the device of Fig. 1b is a macrospin model that treats the RL unit magnetization $\hat{m}_\text{RL}$ as fixed, with only the FL magnetization $\hat{m}_\text{FL}(t) \leftrightarrow \hat{m}(t)$ as possibly dynamic in time. As was described previously, the linearized Gilbert equations for small deviations $\hat{m}' = (m'_x, m'_y, m'_z)$ about equilibrium $\hat{m}_0 \leftrightarrow \hat{x}'$ can be expressed in the primed coordinates as a 2D tensor/matrix equation:

$$
\ddot{\hat{D}} + \ddot{G} \cdot \frac{\partial \hat{m}'}{\partial t} + \hat{H}' \cdot \hat{m}' = \hat{h}(t) \equiv p \cdot \frac{\partial \hat{m}'}{\partial \hat{m}}, \quad \hat{h}(t)
$$

$$
\ddot{\hat{D}} \equiv \alpha \frac{L}{\gamma} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \ddot{\hat{G}} \equiv \frac{L}{\gamma} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad p \equiv \frac{(M_s V)_\text{FL}}{\Delta m} 
$$

$$
\dot{\hat{H}}' = (H'_{\text{eff}} \cdot \hat{m}_0') \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{\partial \hat{m}'}{\partial \hat{m}} \hat{H}' \cdot \frac{\partial \hat{m}'}{\partial \hat{m}} 
$$

FIG. 1. (a) Cartoon of prospective CPP-GMR spin-valve sensor stack, analogous to that used for contemporary TMR read head. (b) Cartoon of simplified spin-valve stack used for present experiments, patterned into ~60nm circular pillars using e-beam lithography.
In (1), $\partial H^{\text{eff}} / \partial \hat{m}$ is a 3D Cartesian tensor, $\hat{m} / \hat{m}'$ is a $3 \times 2$ transformation matrix between 3D unprimed and 2D primed vectors (with $\hat{m} / \hat{m}'$ its transpose) which depends only on $\hat{m}_0$, and $h(t)$ is a 3D perturbation field supposed as the origin of the deviations $m'(t)$. The magnetic moment $\Delta m$ is an arbitrary fixed value, but $\Delta m \rightarrow (M_s V)_{FL}$ is a natural choice for Sec. II. Using an explicit Slonczewski\textsuperscript{2} type expression for the spin-torque contribution, the general form for $H^{\text{eff}}(\hat{m})$ is

$$H^{\text{eff}} = -\frac{1}{\Delta m} \frac{\partial E}{\partial \hat{m}} - \eta(\cos \theta) H_{\text{ST}} (\hat{m}_{\text{RL}} \times \hat{m}_{\text{FL}}),$$

(2)

for any free energy function $E(\hat{m})$. A positive electron current density $J_e$ implies electron flow from the RL to the FL. $P_{\text{eff}}$ is the net spin polarization of the current inside the Cu spacer. Oersted-field contributions to $H^{\text{eff}}$ will be neglected here.

With $h(t)=0$ in (1), nontrivial solutions $m'(t)=m'e^{-st}$ require $s$ satisfy $|H-s(\tilde{D}+G)|=0$. The value $J_e=J_e^{\text{crit}}$ when $Re s=0$ defines the critical onset of spin-torque instability. Using (1), the general criticality condition is expressible as

$$\alpha (H'_{y'y'}+H'_{z'z'}) + H'_{y'y'} - H'_{z'z'} = 0$$

(3a)

$$\frac{H'_{y'y'} - H'_{z'z'}}{H_{\text{ST}}} \equiv (1-q^2) \frac{dn}{dq} - 2q \eta(q) (q=\cos \theta)$$

(3b)

$$\Rightarrow J_e^{\text{crit}} = \frac{\Delta m}{(h/2e)P_{\text{eff}}} \alpha (H'_{y'y'}+H'_{z'z'}) (1-q^2) \frac{dn}{dq} - 2q \eta(q)$$

(3c)

where $\alpha$ is the Gilbert damping. The $J_e$-scaling of the terms in (3a) follows just from the form of (2). The result in (3b) was derived earlier\textsuperscript{4} in the present approximation of rigid $\hat{m}_{\text{RL}}$.

With $\theta$ the angle between $\hat{m}_{\text{RL}}$ and $\hat{m}_{\text{FL}}$ (at equilibrium), it follows from (3c) that at a "magic-angle" $\theta^{\text{magic}}$ where the denominator vanishes, $J_e^{\text{crit}} \rightarrow \infty$ and spin-torque effects are effectively eliminated from the system at finite $J_e$. To pursue this point further, explicit results for $\eta(\cos \theta)$ will be used from the prototypical case where the CPP-GMR stack (Fig. 1b) is approximately symmetric about the Cu spacer, which is roughly equivalent to the less restrictive situation where the RL and FL are similar materials with thicknesses that are not small compared the spin-diffusion length. For this quasi-symmetric case, both quasi-ballistic\textsuperscript{6} and fully diffusive\textsuperscript{7} transport models yield the following simple functional forms:

$$\eta(\cos \theta) = \Gamma / \left[ (\Gamma + 1)^2 + (\Gamma - 1)^2 \cos \theta \right]$$

$$r(\cos \theta) = \frac{\Delta R = R(\cos \theta) - R_{\min}}{\Delta R = R_{\max} - R_{\min}}$$

(4)

which also relates $\eta$ to the normalized resistance $r (0 \leq r \leq 1)$ which is directly measurable experimentally. The transport parameter $\Gamma$ is theoretically related to the Sharvin resistance\textsuperscript{6,8} or mixing conductance\textsuperscript{3} at the Cu/FL interface, but will be estimated via $J_e^{\text{crit}}$ measurement in Sec III. Using $\eta(q)$ from (4) in (3), $\theta^{\text{magic}}$ and $r(\theta^{\text{magic}})$ vs. $\Gamma$ curves are shown in Fig. 2.

The "magic-angle" concept also applies to mag-noise power spectral density (PSD) $S_\nu = [I_{\text{bias}} \Delta R (dR/d\theta)]_0 S_0$ at bias current $I_{\text{bias}}$, arising from thermal fluctuations in $\theta$ about equilibrium bias angle $\theta_{\text{bias}}$. Using $\hat{m}_{\text{RL}}(t)/\text{FL}$ near the film plane ($\hat{z}' \equiv \hat{z} = \text{plane-normal}$), and requiring $|I_{\text{bias}}| < |I_{\text{crit}}|$, it can be shown\textsuperscript{5} from fluctuation-dissipation arguments that

$$S_\nu(f) = \frac{4k_B T}{\gamma \Delta \omega} (\tilde{\eta} : \tilde{D} \cdot \tilde{\eta}') y_{y'y}', \tilde{\eta}(\omega) = [\tilde{H}' - i\omega(\tilde{D} + \tilde{G})]^{-1}$$

$$\Rightarrow S_\nu(f) = \frac{4k_B T}{\gamma \Delta \omega} \alpha^2 \left( H'_{z'z'}^{2} + H'_{y'y'}^{2} \right) + \omega^2 \left( \frac{2}{\omega^2 - \omega_0^2} + \frac{\omega^2}{(\omega - \omega_0)^2} \right)$$

(5)

where $\omega_0 = \gamma \sqrt{H'_{y'y'} + H'_{z'z'} - H'_{y'y'} H'_{z'z'}}$ and $\Delta \omega = \gamma \left[ \alpha (H'_{z'z'} + H'_{y'y'}) + H'_{z'y'} - H'_{y'z'} \right]$.

Comparing (5) with (3), it is seen that the spectral linewidth $\Delta \omega$ is predicted to be a linear function of $J_e$, but with $d\Delta \omega / dJ_e \rightarrow 0$ when $\theta_{\text{bias}} \rightarrow \theta^{\text{magic}}$. Since $H'_{z'z'} \gg H'_{y'y'}$ (due to $\sim 10$ kOe out-of-plane demag fields) and $H'_{y'y'} \gg H'_{z'y'}$, $H'_{y'y'}$ can be expected to influence $S_\nu(f)$. Therefore, measurement of $S_\nu(f)$ with $\theta_{\text{bias}} = \theta^{\text{magic}}$ ideally allows direct measurement of the natural thermal-equilibrium mag-noise spectrum $S_\theta(f)$, from which can be extracted the intrinsic (i.e., $J_e$-independent) Gilbert damping constant $\alpha$. This is the subject of Sec. III.
III. EXPERIMENTAL RESULTS

The results to be shown below were measured on CPP-GMR-spin-valves of stack structure: seed-layers/IrMn (60Å)/RL/Cu (30Å)/FL/cap layers. The films were fabricated by magnetron sputtering onto AITIC substrates at room temperature, with 2mTorr of Ar sputter gas. The bottom contact was a ~1-μm thick NiFe layer, planarized using chemical-mechanical polishing. To increase ∆R/R, both the RL and FL were made from (CoFe)₁₀Ge₁₂₀ magnetic alloys. The RL includes a thin CoFe between IrMn and CoFeGe to help maximize the exchange coupling strength, and both RL and FL include very thin CoFe at the Cu interface. The resultant Mₜ product for the RL and FL were about 0.64 emu/cm². After deposition, SV films were annealed for 5 hours at 245°C in 13kOe applied field to set the RL’s exchange pinning direction unidirectionally-square loops with parallel with the magnetometry. After annealing, patterned devices with diameter (measured at the FL) were fabricated using e-beam lithography and Ar ion milling. A 0.2-μm-thick Au layer was used as the top contact to devices.

Fig. 3 illustrates a full measurement sequence. Devices are first pre-screened to find samples with approximate ideal in-plane δR-H loops (Fig. 3a) for circular pillars: non-hysteretic, unidirectionally-square loops with H = H|| parallel with the RL’s exchange pinning direction (+x̂), along with symmetric loops about H = 0 when H = H⊥ is transverse (y-axis). The right-shift in the δR-H loop indicates a large demagnetizing field of ~500 Oe from the RL on the FL.

As shown previously, narrow-band "low"-frequency N-Iₑ measurements (N = √PSD(f = 100 MHz), 1 MHz bandwidth) can reveal spin-torque criticality as the very rapid onset of excess (1/f-like) noise when |Iₑ| exceeds |Iₑₗ_|. N=Iₑ loops are measured with Iₑ sourced from a continuous sawtooth generator (2-Hz) which also triggers 1/2 sec sweeps of an Agilent-E4440 spectrum analyzer (in zero-span, averaging mode) for ~50 cycles. With high sweep repeatability and virtually no Iₑ -hysteresis, this averaging is sufficient so that after (quadratically) subtracting the mean N(Iₑ) ≈ 1nV/√Hz electronics noise, the resultant N-Iₑ loops (Fig. 3b) indicate stochastic uncertainty ≈ 0.1 nV/√Hz.

With cos θ = ±1, it readily follows from (3c) and (4) that

\[ \Gamma = \frac{-I_e(\theta = \pi)}{I_e(\theta = 0)} = \frac{I_{e\text{crit}}}{I_{e\text{crit}}} \]

Hence, to estimate Γ , N=Iₑ are measured with applied fields \( H_0 \approx -0.45, +1.2 \text{kOe} \) (Fig. 3b), which more than sufficient to align \( \hat{m}_{FL} \) antiparallel (AP), or parallel (P) to \( \hat{m}_{RL} \), respectively (see Fig. 3a), thereby reducing possible sensitivity to Oersted field and/or thermal effects. (Reducing \( |H_0| \) by ~200-300 Oe did not significantly change either N=Iₑ curve.) With Iₑ > 0 denoting electron flow from RL to FL, it is readily found from (3) that \( I_{e\text{crit}}^{AP-FL} > 0 \) and \( I_{e\text{crit}}^{P-FL} < 0 \) for the FL. By symmetry, it must follow that \( I_{e\text{crit}}^{AP-RL} < 0 \) and \( I_{e\text{crit}}^{P-RL} > 0 \) for spin-torque induced instability of the RL. This sign convention readily identifies these four critical points by inspection of the N-Iₑ data. To account for possible small (thermal) spread in critical onset, specific values for the \( I_{e\text{crit}} \) (excluding \( I_{e\text{crit}}^{P-FL} \)) are defined by where the N=Iₑ curves cross the 0.2 nV/√Hz line, which is easily distinguished from the ~0.05 nV/√Hz magnetic/thermal background. \( I_{e\text{crit}}^{P-RL} \) is estimated in Fig. 3b (and repeatedly in Figs. 4-7) to be ~ +4.5 mA. Arbitrariness in the value of \( I_{e\text{crit}}^{P-RL} \) from using the 0.2 nV/√Hz criterion is thought to only be of minor significance for \( I_{e\text{crit}}^{AP-FL} \), due to the rounded shape of the AP N=Iₑ curves near this particular critical point, which may in part explain why \( \Gamma_{RL/CU} \) estimated from N=Iₑ is found to be systematically somewhat larger than \( \Gamma_{Cu/FL} \).

However, the key results here are the 0.1-18 GHz broad-band (rms) PSD(f; Iₑ) spectra (Fig. 3c). They are measured at discrete dc bias currents with the same Miteq preamp (and inserts bias-T) used for the N=Iₑ data, the latter being insitu gain-
calibrated vs. frequency (with ≈50Ω preamp input impedance and additionally compensating the present ≈0.7 pF device capacitance) to yield quantitatively absolute values for these PSD($f; I_e$) (each averaged over ~100 sweeps, with PSD($f; I_e = 0$) subtracted post-process). To confirm the real existence of an effective "magic-angle", the applied field $H_m$ was carefully adjusted (by repeated trial and error) in both amplitude and direction to eliminate as much as possible any real-time observed dependence of the raw PSD($f; I_e$) near the FL FMR peak (~6 GHz) on the polarity as well as amplitude of $I_e$ over a sufficient range. This procedure was somewhat tedious and delicate, and initial attempts using a nominally transverse field $H_\perp$ were empirically found inferior to additionally adjusting the direction of the field, here rotated somewhat toward the pinning direction for the RL. Using a mechanically-positioned permanent magnet as a field source, this field rotation was only crudely estimated at the time to be ~20-30° (see also Sec. IV). With both $H$ and bias-point $\theta_{bias}$ "optimized" as such, an $I_e$-series of PSD($f; I_e$) were measured, after which the bias-resistance $R_{bias}$, and finally $R_{min}$ and $R_{max}$ were measured at a common (low) bias of ~10 mV to determine $\eta_{bias}$ (as in (4)).

The key feature of the rms PSD($f; I_e$) in Fig. 3c is that these measured spectra (excluding $I_e = 1.5$ mA) appear essentially independent of both the polarity and magnitude of $I_e$ (after 1mA-normalization), defining a "universal" spectrum curve over the entire 18GHz bandwidth, including the unexpectedly wide, low amplitude RL-FMR peak near 14 GHz (more on this below). Because of the relatively large $\sqrt{\text{PSD}(f; I_e = 0)} \sim 1 \text{nV/\sqrt{Hz}}$ background, these RL peaks were not well discernible during raw spectrum measurements, and were practically revealed only after electronics background noise subtraction. As suggested in Fig. 3c, eventual breakdown of the magic-angle condition was generally found to first occur from spin-torque instability of the FL at larger positive $I_e$.

The spectra Fig. 4 shows the equivalent set of measurements on a physically different (though nominally identical) 60-nm device. They are found to be remarkably alike in all properties to those of Fig. 3, providing additional confirmation that the "magic-angle" method can work on real nanoscale structures to directly obtain the intrinsic $S_0(f; I_e = 0)$ in the absence of spin-torque effects. This appears further confirmed by the close agreement of measured ($r_b, \Gamma_{Cu/FL}$) pairs (from data of Figs. 3,4, and 6) and the macrospin model predictions described in Fig. 2.

To obtain values for linewidth $\Delta \omega$ and then damping $\alpha$ from the measured PSD($f; I_e$), regions of spectra several-GHz wide, surrounding the FL and RL FMR peaks are each nonlinear least-squares fitted to the functional form for $S_0(f; I_e = 0)$ in (5). In particular, the fitting function is taken to be

$$S_V(f) = \frac{S_0}{2\pi} \frac{\omega_0^2 [\omega_0^2 + (\delta H'_y' / H'_z')\omega^2]}{(\omega^2 - \omega_0^2)^2 + (\omega \Delta \omega)^2},$$

with $\omega_0 = \gamma \sqrt{(\delta H'_y' / H'_z')^2 + \omega^2}$, and $H'_y' \rightarrow [((\omega_{\text{peak}} / \gamma)^2 + (\alpha_{\text{fit}} H'_z')^2 / 2)] / H'_z'^2$  

(7)

Three fitting parameters are used: $S_0 = S_V(f = 0)$, $\alpha_{\text{fit}}$, and $\omega_{\text{peak}}$, the latter being already well defined by the data itself. The substitution for $H'_y'$ is accurate to order $\alpha^2$, leaving $H'_z'^2$ as yet unknown. With $H'_z'^2 >> H'_y'^2$ dominated by out-of-plane demagnetizing fields, $S_V(f)$ depends mostly on the product $\alpha_{\text{fit}} H'_z'^2$. For simplicity, fixed values $H'_z'^{FL} = 8 \text{kOe}$ and $H'_z'^{RL} = 10 \text{kOe}$, were used here, based on macrospin calculations that approximately account for device geometry and net $M_s t$ product for FL and RL films. The fitted PSD($f; I_e$) curves, and the values obtained for $\alpha_{\text{FL}}$ and $\alpha_{\text{RL}}$ are also included in Figs. 3c and 4c. These values are notably independent of (or show no significant trend with) $I_e$.

Although the $\alpha_{\text{FL}}^{\text{fit}} \approx 0.01$ repeatedly found from these data is a quite typical magnitude for Gilbert damping in CoFe alloys, the extremely large, 10× greater value of $\alpha_{\text{RL}}^{\text{fit}} \approx 0.1$ is quite noteworthy, since the RL and FL are not too dissimilar in thickness and composition. Although the small amplitude of the RL-FMR peaks in Figs. 3-4 (everywhere below the raw 1 nV/√Hz electronics noise), may suggest a basic unreliability in this fitted value for $\alpha_{\text{RL}}^{\text{fit}}$, this concern is seemingly dismissed by the data of Fig. 5. Measured on a third (nominally identical) device, an alternative "extrapolation-method" was used, in which

![Fig. 4. Analogous measurement set for a different (but nominally identical) 60 nm device. as that shown in Fig. 3.](image-url)
the applied field was purposefully reduced in magnitude (and more transversely aligned than for magic-angle measurements) to increase $r_{\text{bias}}$ and thus align $\mathbf{m}_{\text{FL}}$ to be more antiparallel to $\mathbf{m}_{\text{RL}}$. As a result, spin-torque effects at larger negative-$I_e$ will decrease $\Delta \omega$ and concomitantly enhance RL-FMR peak amplitude (and visa-versa for the FL), bringing this part of the measured spectrum above the raw electronics noise background.

Using the same fitting function from (7), it is now necessary to extrapolate the $\alpha_{\text{RL}}^{\text{fit}}(I_e)$ to $I_e \to 0$ (Fig. 5d) in order to obtain the intrinsic damping. This method works well in the case of the RL since $d\alpha_{\text{RL}}^{\text{fit}}/d |I_e| < 0$ and the extrapolated $I_e = 0$ intercept value of $\alpha_{\text{RL}}$ is necessarily larger than the measured $\alpha_{\text{fit}}(I_e)$, and hence will be (proportionally) less sensitive to uncertainty in the estimated extrapolation slope. As can be seen from Fig. 5d, the extrapolated values for intrinsic $\alpha_{\text{RL}}$ are virtually identical to those obtained from the data of Figs. 3,4. The extrapolated $\alpha_{\text{FL}}$ is also quite consistent as well. The extrapolation data also confirm the expectation (noted earlier following (5)) that linewidth $\Delta \omega$ will vary linearly with $I_e$.

Comparing with Figs. 3c,4c, the spectra in Fig. 5c illustrate the profound effect of spin-torque on altering the linewidth and peak-height of both FL and RL FMR peaks even if the system is only moderately misaligned from the magic-angle condition. By contrast, for other frequencies (where the $\Delta \omega$ term in the denominator of (5) is unimportant), the 1mA-normalized spectra are independent of $I_e$. Being consistent with (5), this appears to verify that this 2nd form of fluctuation-dissipation theorem remains valid despite that the system of (1) is not in thermal equilibrium at nonzero $I_e$. (Alternatively stated, spin-torques lead to an asymmetric $\tilde{H}$, but do not alter the damping tensor $\tilde{D}$ in (1)). The $\alpha$-proportionality in the prefactor of $S_0(f)$ in (5) relatedly shows that the effect of spin-torque on $\Delta \omega$ is not equivalent to additional damping (positive or negative) as may be commonly misconstrued. It further indicates that Oersted-field effects, or other $I_e$-dependent terms in $\tilde{H}$ not contributing to $\Delta \omega$, are insignificant in this experiment.

Analogous to Figs. 4,5, the data of Figs. 6,7 are measured on CPP-GMR-SV stacks differing only by an additional 1-nm thick Dy cap layer deposited directly on top of the FL. The use of Dy in this context (presumed spin-pumping from FL to Dy, but possibly including Dy intermixing near the FL/Dy interface) was found in previous work to result in an $\sim 3\times$ increase in FL-damping, then inferred from the $\sim 3\times$ increase in measured $1/\text{crit}_{\text{FL}}$. Here, a more direct measure from the FL FMR linewidth indicates a roughly similar, $\approx 2.3\times$ increase in $\alpha_{\text{FL}}$ (now using somewhat thicker FL films). This ratio is closely consistent with that inferred from $|I_e|^{\text{fit}}$ data measured in this experiment over a population of devices (see Table 1). Notably, the values found for $\alpha_{\text{RL}}$ remain virtually the same as before.

Finally, Fig.8 shows results for a "synthetic-ferrimagnet" (SF) free-layer of the form FL1/Ru(8A)/FL2. The Ru spacer provides
an interfacial antiferromagnetic coupling of $\approx 1.0$ erg/cm$^2$. Here, FL1 has a thicker CoFeGe layer than used for prior FL films, and FL2 is a relatively thin CoFe layer chosen so that $(M_s t)_{FL1} - (M_s t)_{FL2} \approx 0.64$ erg/cm$^2$. Although having similar static $M-H$ or $R-H$ characteristics to that of the simple FL (of similar net $M_s t$ product) used in earlier measurements, the transport of the SF-FL in regard to spin-torque effects in particular is fundamentally distinct. The basic physics of this phenomenon was described in detail previously. In summary, a spin-torque induced quasi-coresonance between the two natural oscillation modes of the FL1/FL2 couple in the case of negative $I_e$ and $\vec{m}_{FL1} \cdot \vec{m}_{RL} > 0$, can act to transfer energy out of the mode that is destabilized by spin-torque, thereby delaying the onset of criticality and substantially increasing $|I_{P-FL}|$. Indeed, the side-by-side comparison of $N$- $I_e$ loops provided in Fig. 8b indicate a nearly $5 \times$ increase in $|I_{P-FL}|$, despite that $|I_{AP-FL}|$ remains virtually unchanged.

For the SF-FL devices, attempts at finding the magic-angle under similar measurement conditions as used for Figs. 3c, 4c, and 6c were not successful, and so the extrapolation method at similar $r_{bias} \approx 0.4$ was used instead. To improve accuracy for extrapolated- $\alpha_{FL}$, the data of Fig. 8c include measurements for $|I_e| \leq 0.3$ mA (so that $I_e < |I_{P-FL}^\text{crit}|$) for which electronics noise overwhelms the signal from the RF FMR peaks. Showing excellent linearity of $\alpha_{FL}^\text{fit}$ vs. $I_e$ over a wide $I_e$-range, the extrapolated intrinsic $\alpha_{FL} \approx 0.01$ is, as expected, unchanged from before. The same is true for the extrapolated $\alpha_{RL}$ as well.

Table 1 summarizes the mean critical voltages $-RI_{P-FL}^\text{crit}$ (less sensitive to lithographic variations in actual device area) from a larger set of PSD-$I_e$ measurements. The $\approx 2.3 \times$ increase in $|RI_{P-FL}^\text{crit}|$ with the use of the Dy-cap is in good agreement with that of the ratio of measured $\alpha_{FL}$. 

![Fig. 7](image1.png)

![Fig. 8](image2.png)

| stack  | $-I_{P-FL}^\text{crit}$ (mV) | $\alpha_{RL} / \alpha_{FL}$ |
|--------|---------------------------|----------------------------|
| Control| 10.4±0.1                  | 0.011 / 0.11               |
| Dy cap | 24.5±0.5                  | 0.026 / 0.11               |
| SF-FL  | 44.9±2.0                  | 0.011 / 0.12               |

Table 1. Summary of critical voltages (measured over $\approx 8$ devices each) and damping parameter values $\alpha$ for the present experiment. Estimated statistical uncertainty in the $\alpha$-values is $\approx 10\%$. 

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IV. MICROMAGNETIC MODELLING

For more quantitative comparison with experiment than afforded by the 1-macospin model of Sec. II, a 2-macospin model equally treating both \( \mathbf{m}_{\text{RL}} \) and \( \mathbf{m}_{\text{FL}} \) is now considered here as a simpler, special case of a more general micromagnetic model to be discussed below. The values \( M_{s\text{FL}} = 950 \text{ emu/cc} \), \( t_{\text{FL}} = 7 \text{ nm} \), \( M_{s\text{RL}} = 1250 \text{ emu/cc} \), and \( t_{\text{RL}} = 5 \text{ nm} \) will be used as simplified, combined representations (of similar thickness and \( M_{s} \)) to the actual CoFe/CoFeGe multilayer films used for the RL and FL. The magnetic fields are geometrically modeled as 60 nm squares which (in the macrospin approximation) have zero shape anisotropy (like circles), but allow analytical calculation of all magnetostatic interactions. The effect of IrMn exchange pinning on the RL is simply included as a uniform field \( H_{\text{pin}} = [J_{\text{pin}}/(M_{s})_{\text{RL}}] \hat{x} \) with measured \( J_{\text{pin}} \approx 0.75 \text{ erg/cm}^{2} \).

Firstly, Fig. 9b shows simulated \( r_{\text{bias}} \cdot H_{L} \) and \( r_{\text{bias}} \cdot H_{\text{RL}} \) curves computed assuming \( \Gamma = 3.2 \) , roughly the mean value found from the \( J_{\text{FL}} \text{crit} \) data of Sec. III. The agreement with the shape of the measured \( R-H \) is very good (e.g., Figs. 6,7 in particular), which reflects how remarkably closely these actual devices resemble idealized (macrospin) behavior.

Next, Fig. 9d shows simulated PSD curves \( S_{\nu} \) computed (see Appendix) in the absence of spin-torque (i.e., \( H_{\text{ST}} = 0 \) ), but otherwise assuming typical experimental values \( R=19 \Omega \), \( \Delta R/R=9\% \), and \( T=300 \text{K} \), as well as \( \alpha_{\text{FL}} = 0.01 \) and \( \alpha_{\text{RL}} = 0.1 \), so to be compared with the magic-angle spectra of Figs. 3,4. Since (as stated in Sec. III) the experimental field angle was not accurately known, the field angle \( \phi_{H} \) was varied systematically for the simulations, and in each case the field-magnitude \( H \) was iterated until \( r_{\text{bias}} \approx 0.37 \) , approximately matching the mean measured value.

In terms of both absolute values and the ratio of FL to RL FMR peak amplitudes, the location of \( f_{\text{peak}} \) (particularly for the FL), and the magnitude of \( H \) (on average 650-700 Oe from the three magic-angle data in Sec. III), the best match with experiment clearly occurs with \( 30^\circ \leq \phi_{H} \leq 40^\circ \). The agreement, both qualitatively and quantitatively, is again remarkable given the simplicity of the 2-macospin model.

Finally, results from a discretized micromagnetic model are shown in Fig. 10. Based on Fig.9, the value \( \theta_{H} = 35^\circ \) was fixed, and \( H = 685 \text{ Oe} \) was determined by iteration until \( r_{\text{bias}} \approx 0.37 \). The equilibrium bias-point magnetization distribution is shown...
in Fig. 10a for this 416 cell model. Estimated values for exchange stiffness, $A_{FL} = 1.4 \mu\text{erg/cm}^2$ and $A_{RL} = 2 \mu\text{erg/cm}^2$ were assumed. The simulated spectra in Fig. 10b are shown one eigenmode at a time (see Appendix), for the 7 eigenmodes with predicted FMR frequencies below 20 GHz (the 5th mode is at 22.9 GHz). The 1st, 2nd, 5th, and 7th modes involve mostly FL motion, the nearly degenerate 3rd and 4th modes (and the 6th) mostly that of the RL. (The amplitudes of $S_i(f)$ from the 6th or 7th mode are negligible.) For illustration purposes only, Fig. 10b assumed identical damping $\alpha_{FL} = \alpha_{RL} = 0.01$ in each film.

For Fig. 10c, the computation of $S_V(f)$ is more properly computed using either 6 or all 7 eigenmodes simultaneously, which includes damping-induced coupling between the modes. Including higher order modes makes negligible change to $S_v(f < 20$ GHz) (but rapidly increases computation time). As was observed earlier, the agreement between simulated and measured spectra in Figs. 3c,4c is good (with $\alpha_{RL} = 0.1$), and the simulations now include the small, secondary FL-peak near 8 GHz clearly seen in the measured data (including that of Fig. 6c), though it is somewhat more pronounced in the model results. Notably, the computed spectrum near the RL FMR peak more resembles the measurements after removing the 16-GHz 5th mode from the calculation, as this (FL) mode does not appear to be physically present in the Figs. 3c,4c spectra. While it is perhaps expected that higher order modes in a micromagnetic simulation assuming perfectly homogeneous magnetic films would show deviations from real devices with finite grain-size, edge roughness/damage, etc., the situation is actually more interesting. Fig. 11 shows measured spectra on yet another device (again, nominally identical to that of Figs. 3-5) in which the experiment was perhaps slightly off from the optimum magic-angle condition, as evidenced by the very small shift in the 6-GHz FL FMR peak position with polarity of $I_e$. More noteworthy, however, is the clear polarity asymmetry and nonlinear-in-$I_e$ peak-amplitude (for $I_e > 0$ in particular) of both the 8-GHz secondary FL mode and a higher order mode close to 15 GHz. (Both resemble typical spin-torque effect at angles more antiparallel than the magic-angle condition.) This similarity in behavior indicates with near certainty that this 15-GHz mode is also FL-like in origin, and is thus a demonstration of the "missing" 5th mode predicted in Fig. 10. (In hindsight, there is now discernible a small but similar 15-GHz peak in the spectra of Fig. 4c). It is worth remembering that the "magic-angle" argument was based on a simple 1-macrospin model, and so remarkably there appears to be circumstances where this "spin-torque null" actually does apply simultaneously to both the FL and RL, as well as to higher order modes.

V. DISCUSSION

In addition to the direct evidence from the measured spectral linewidth in Figs. 3-8, evidence for large Gilbert damping $\alpha_{RL} >> \alpha_{FL}$ for the RL is also seen in the $I_e^{crit}$ data. As ratios $I_{P-RL}^{crit} / I_{P-FL}^{crit}$ and $I_{AP-RL}^{crit} / I_{AP-FL}^{crit}$ are (from Figs. 3-5 data) both roughly ~7, this conclusion is semi-quantitatively consistent with the basic scaling (from (3c)) that $I_e^{crit} \propto \alpha$. This, as well as the substantial, $2-3\times$ variation of $\alpha_{RL}^{fit}$ with $I_e$ in Figs. 5d, and 7d, appears to rather conclusively (and expectedly) confirm that inhomogeneous broadening is not a factor in the large linewidth-inferred values of $\alpha_{RL}$ found in these nanoscale spin-valves. Large increases in effective damping of "bulk" samples of ferromagnetic (FM) films in contact with antiferromagnet (AF) exchange pinning layers has been reported previously. Excess damping was generally attributed to two-magnon scattering processes arising from an inhomogeneous AF/FM interface. However, the two-magnon description applies to the case where the uniform, $k = 0$ mode is pumped by a rf source to a high magnon level, which then transfers energy to the uniform mode. By contrast, for the nanoscale spin-valve device, the relevant eigenmodes (Fig. 10) are discrete and generally nondegenerate in frequency. Even for a coincidental case of a quasi-degenerate pair of modes (e.g., RL modes #3 and #4 in Fig. 10), both modes are equally excited to thermal equilibrium levels (as are all modes), and have similar intrinsic damping rates to the lattice. Any additional energy transfer via a two-magnon process should flow both ways, making impossible a large (e.g., ~10\times) increase in the effective net damping of either mode.

Two alternative hypotheses for large $\alpha_{RL}$ which are essentially independent of device size are 1) large spin-pumping effect at the IrMn/RL interface, or 2) strong interfacial exchange coupling at the IrMn/RL resulting in non-resonant coupling to high frequency modes in either the RL and/or the IrMn film. However, these two alternatives can be distinguished since the exchange coupling strength can be greatly altered without necessarily changing the spin-pumping effect. In particular, $\alpha_{RL}$ was very recently measured by conventional FMR methods.
by Mews18 on bulk film samples (grown by us with the same RL films and IrMn annealing procedure as that of the CPP-GMR-SV devices reported herein) of the reduced stack structure: seed/IrMn(t_AF)/Cu(t_d)/RL/Cu(30A)/cap. Removal of IrMn, or alternatively a lack of proper seed layer and/or use of a sufficiently thick t_Cu≈30A can each effectively eliminate exchange pinning strength to RL.

The second hypothesis emphasizes the possibility that the energy loss takes place inside the IrMn, from oscillations excited far off resonance by locally strong interfacial exchange coupling to a fluctuating \( \mathbf{m}_{\text{RL}} \). This local interfacial exchange coupling \( J_{\text{ex}} \) can be much greater than \( J_{\text{pin}} \), since the latter reflects a surface average over inhomogeneous spin-alignment (grain-to-grain and/or from atomic roughness) within the IrMn sub-lattice that couples to the RL. Further, though such strong but inhomogenities coupling cannot truly be represented by a uniform \( \mathbf{H}_{\text{pin}} \) acting on the RL, the similarity between measured and modeled values of ~14 GHz for the "uniform" RL eigenmode has clearly been demonstrated here. Whatever are the natural eigenmodes of the real device, the magic-angle spectrum measurements of Sec. III reflect the thermal excitation of all eigenmodes for which "one-way" intermodal energy transfer should be precluded by the condition of thermal equilibrium and the orthogonality of the modes themselves. Hence, without an additional energy sink \( J_{\text{ex}} \) exclusive of the RL/FL spin-lattice system, the linewidth of all modes should arguably reflect the intrinsic Gilbert damping of the FL or RL films, which the data of Sec. III and Table 2 indicate are roughly equal with \( \alpha \sim 0.01 \). Inclusion of IrMn as a combined AF/RL system, would potentially provide that extra energy loss channel for the RL modes.

A rough plausibility argument for the latter may be made with a crude AF/FM model in which a 2-sublattice AF film is treated as two ferromagnetic layers (#1 and #2) occupying the same physical location. Excluding magnetostatic contributions, the free energy/area for this 3-macrospin system is taken to be

\[
(M_{\text{s1}} t_{\text{AF}}) H_{\text{AF}} (\mathbf{m}_{\text{AF1}} \cdot \mathbf{m}_{\text{AF2}} - \frac{1}{2} (K)_{\text{AF}} [ (\mathbf{m}_{\text{AF1}} \cdot \mathbf{x})^2 + (\mathbf{m}_{\text{AF2}} \cdot \mathbf{x})^2 ] - J_{\text{ex}} \mathbf{m}_{\text{AF1}} \cdot \mathbf{m}_{\text{FM1}} + [J_0 - J_{\text{pin}}] \mathbf{m}_{\text{FM1}} \cdot \mathbf{x}
\]

(8)

For IrMn with Neel temperature of \( T_N \approx 700K \), the internal AF exchange field \( H_{\text{AF}} \approx k_B T_N / \mu_B \sim 10^7 \text{Oe} \). With \( t_{\text{AF}} = 60A \), AF uniaxial anisotropy is estimated to be \( K_{\text{AF}} \sim 10^6 \text{erg/cm}^2 \). A rough estimate \( J_{\text{ex}} \sim 8(A/\ell)_{\text{FM}} \) for strong interfacial exchange is obtained by equating interface energy \( J_{\text{ex}} \frac{\pi}{4} \) to the bulk exchange energy \( 4A \mathbf{\theta}^2 / \ell \) of a hypothetical, small angle Bloch wall (0 ≤ \( \theta \leq \phi \)) twisting through the FM film thickness.

Taking \( \ell \approx 5nm \) and \( A \sim 10^6 \text{erg/cm}^2 \) yields \( J_{\text{ex}} \sim 15 \text{erg/cm}^2 \). The value of \( J_0 \sim [1/\ell_{\text{ex}} + 1/(K)_{\text{AF}}]^{-1} \) in the last "field-like" term in (8) is more precisely chosen to maintain a constant eigenfrequency for the FM layer independent of \( J_{\text{ex}} \) or \( K_{\text{AF}} \), thus accounting for the weaker inhomogeneous coupling averaged over an actual AF/FM interface.

As shown in Fig. 12, this crude model can explain a ~10× increase in the FM linewidth provided \( J_{\text{ex}} \gtrsim 5 \times 10^3 \text{erg/cm}^2 \) and \( \alpha_{\text{AF}} \sim 0.05-0.1 \). It is worthily noted20 that for the 2-sublattice AF, the linewidth \( \Delta \omega / \omega_0 = \alpha_{\text{AF}} \sqrt{2J_{\text{ex}}} (H_K \equiv (K / M_s)_{\text{AF}}) \) is larger by a factor of \( \sqrt{2H_{\text{AF}} / H_K} \sim 100 \) compared to high order FM spin-wave modes in cases of comparable \( \alpha \) and \( \omega_0 \) (with \( \omega_0 \approx \gamma \sqrt{2H_{\text{AF}} / H_K} \sim 10^{12} \text{Hz} \) for the AF). Since the "lossy" part of the "low" frequency susceptibility for FM or AF modes scales with \( \Delta \omega \), it is suggested that the IrMn layer can effectively sink energy from the ~14 GHz RL mode despite the ~100× disparity in their respective resonant frequencies. Size-independent damping mechanisms for FM films exchange-coupled to AF layers such as IrMn are worthy of further, detailed study.

\[
\begin{array}{c|c}
\text{sample type} & \alpha_{\text{RL}} = \frac{\gamma}{2} (d \Delta H / d\omega) \\
\hline
\ell_{\text{Cu}} = 0 & 0.011 \\
\ell_{\text{AF}} = 60A, \ell_{\text{Cu}} = 0 & 0.010 \\
\ell_{\text{AF}} = 60A, \ell_{\text{Cu}} = 30A & 0.013 \\
\ell_{\text{AF}} = 0, \ell_{\text{Cu}} = 0 & 0.013 \\
\end{array}
\]

Table. 2. Summary of bulk film FMR measurements18 for reduced film stack structure: seed/IrMn(t_{\text{Cu}})/Cu(t_d)/RL/Cu(30A)/cap. Removal of IrMn, or alternatively a lack of proper seed layer and/or use of a sufficiently thick t_{\text{Cu}}≈30A can each effectively eliminate exchange pinning strength to RL.

![Graph](image-url)
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APPENDIX

As was described in detail elsewhere, the generalization of (1) or (5) from a single macrospin to that for an N-cell micromagnetic model takes the form

\[
\langle \mathbf{D} + \mathbf{G} \rangle \frac{d \mathbf{m}}{dt} + \mathbf{H} \cdot \mathbf{m} = \mathbf{h}(t)
\]

where \( \mathbf{m} \) (or \( \mathbf{H} \)) is an \( 2N \times 1 \) column vector built from the \( N \) 2D vectors \( m^t_{j=1...N} \), and \( \mathbf{D}, \mathbf{G}, \mathbf{H} \) are \( 2N \times 2N \) matrices formed from the \( N \times N \) array of 2D tensors \( D_{jk}, G_{jk}, H_{jk} \). Here, \( D_{jk} = \bar{D} \delta_{jk} \) and \( G_{jk} = \bar{G} \delta_{jk} \), though \( \bar{H}_{jk} \) is nonlocal in cell indices \( j, k \) due to the magnetostatic interaction.

The PSD \( S_Q(f) \) for any scalar quantity \( Q(\mathbf{m}_j) \) is

\[
S_Q(f) = 2 \sum_{j,k=1}^{N} d_j^* \cdot \bar{S}_{jk}(\omega) \cdot d_k^* \cdot \frac{\partial Q}{\partial m_j}
\]

The computations for the PSD of Figs. 9, 10 took \( Q(\mathbf{m}) \) to be

\[
Q = \frac{1}{N_i} \sum_{i=1}^{N_i} \int _{\text{bias}} \Delta R (1 - \mathbf{m}_{i}^{\text{RL}} \cdot \mathbf{m}_{i}^{\text{FL}})
\]

averaged over the \( N_i = N/2 \) cell pairs at the RL-FL interface.

For a symmetric \( \mathbf{H} \) (e.g., \( H_{ST} = 0 \)), the set of eigenvectors \( \mathbf{e} \leftarrow \mathbf{m} \) of the system (A1) can be defined from the following eigenvalue matrix equation

\[
(\mathbf{G}^{-1} \cdot \mathbf{H}) \cdot \mathbf{e}_n = i \omega_n \mathbf{e}_n
\]

The eigenvectors come in \( N \) complex conjugate pairs \( \mathbf{e}_n^+ , \mathbf{e}_n^- \) with real eigenfrequencies \( \pm \omega_n \). With suitably normalized \( \mathbf{e}_n \), matrices \( H_{mn} = \delta_{mn} \) and \( G_{mn} = \delta_{mn} / i \omega_n \) are diagonal in the eigenmode basis. The analogue to (A1) becomes

\[
[\chi^{-1}(\omega)]_{nn} = (1 - \omega^2 / \omega_n) \delta_{mn} - i \omega_n (D_{mn} = \mathbf{e}_m^* \cdot \mathbf{D} \cdot \mathbf{e}_n)
\]

The utility of eigenmodes for computing PSD, e.g., in the computations of Fig. 10, is that only a small fraction (e.g., 7 rather than 416 eigenvector pairs) need be kept in (A4) (with all the rest simply ignored) in order to obtain accurate results in practical frequency ranges (e.g., < 20 GHz). Despite that \( D_{mn} \) is (in principle) a full matrix, the reduction in matrix size for the matrix inversion to obtain \( \chi(\omega) \) at each frequency more than makes up for the cost of computing the \( (\omega_n, \mathbf{e}_n) \) which need be done only once independent of frequency or \( \alpha \)-values.

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