Study of microwave drying of wet materials based on one-dimensional two-phase model

Vl V Salomatov\textsuperscript{1,2}, V A Karelin\textsuperscript{1,2}

\textsuperscript{1}Kutateladze Institute of Thermophysics SB RAS, Kutateladze 2, Novosibirsk, 630090, Russia
\textsuperscript{2}Novosibirsk State University, Pirogova 2, Novosibirsk, 630090, Russia

E-mail: salomatov.vv@mail.ru

Abstract. Currently, microwave is one of the most interesting ways to conduct drying of dielectric materials, in particular coal. In this paper, two processes were considered - heating and drying. The temperature field of the coal semi-mass in the heating mode is found analytically strictly with the use of integral transformations. The drying process is formulated as a nonlinear Stephen problem with a moving boundary of the liquid-vapor phase transformation. The temperature distribution, speed and drying time in this mode are determined approximately analytically. Parametric analysis of the influence of the material and boundary conditions on the dynamics of warming up and drying is revealed.

1. Introduction

Wet material drying processes are among the most energy-intensive. At the present stage is a theoretical and practical interest of heat transfer techniques to dried product, based, in particular, to microwave exposure. Important advantage of microwave energy is electromagnetic radiation ability to penetrate into the material to a considerable depth and creating in it the volume distribution of the heat sources without requiring the presence of a heat transfer medium. Microwave exposure is controlled and almost inertialess process that enables instantly change flow of heat. Energy consumption for evaporation during traditional drying types are up to 3.0 kWh / kg \cite{1}. With microwave drying, they are reduced to 1.6-1.8 kWh / kg. The drying time is between 8-20 hours, and under microwave drying under otherwise equal conditions is reduced to 4 hours.

Most large-scale processes are drying coal fuel. Applying electromagnetic microwave radiation will increase the heat of combustion, to reduce the moisture content, ash content and also lower levels of mercury, chlorine, sulfur and other harmful constituents \cite{2}.

The combustion of fuel in the presence of the microwave field increases the rate of combustion, combustion becomes self sustainable, improved ignition mode, increase environmental performance and others.

To wide dissemination of microwave technology in coal energy, which in the world today there is a strong interest, interferes mainly developed of weak scientific foundations interconnected, rather complex electro and thermal processes. Use the full benefits of microwave effects on quality of coal preparation and drying as well as eco-friendly combustion is possible only in the presence of advanced mathematical modeling and analysis tools. The study of these problems is the subject of this work.
In the article analytically solved the problem of wet coal drying under the influence of a plane electromagnetic wave of the microwave range. The phase transition at the border is formulated in the form of Stephen conditions characterizing the consumption of heat for evaporation and drying, which determines the dynamics of the movement of the front. A mathematical model of microwave drying is considered in two stages.

2. Solutions for the heating stage

Stage 1 - the task of microwave warming. At the first stage we study of microwave warm of wet coal regime, when the absorption of microwave radiation is subject of Bouguer's law. By the time the first phase ends when the maximum temperature within the coal reaches the phase transition temperature "water - steam".

The mathematical formulation of the first phase includes the following key assumptions, which give the possibility of obtaining an analytical solution:

1. Internal power microwave heat absorption follows an exponential function of the coordinate - 
   \[ q_v(x) = q_{v_0}e^{-\psi x}. \]
2. Electrophysical and thermal characteristics of the coal substance conditionally constant.
3. The initial conditions are assumed to be zero.
4. There are no heat flows on the boundary surfaces in a first approximation.

This lead us to the system:

\[
\begin{align*}
\frac{\partial T_2(x,t)}{\partial t} &= a_2 \frac{\partial^2 T_2(x,t)}{\partial x^2} + \frac{q_{v_0}}{c_2 \rho_2} e^{-\psi x}, \\
0 &\leq x \leq \infty, \quad t > 0; \\
T_2(x,0) &= 0 \\
\frac{\partial T_2(0,t)}{\partial x} &= 0 \\
\frac{\partial T_2(\infty,t)}{\partial x} &= 0.
\end{align*}
\]

The system of differential equations solved on step 1 with double Laplace transform and cos-Fourier. As a result, we found as the temperature field of the first stage, and the time to achieve the temperature of phase transition "liquid-vapor":

\[
T_2(x,t) = \frac{q_{v_0}}{2c_2 \psi_2} \left\{ \frac{2}{c_2 \rho_2} (a_2 t)^{1/2} \left[ \frac{1}{\pi} e^{-\frac{x^2}{4a_2 t}} - \frac{x}{2(a_2 t)^{1/2}} \text{erfc} \left( \frac{x}{2(a_2 t)^{1/2}} \right) \right] - \frac{1}{\psi_2^2} e^{-\psi x} + \Phi_+(x,t) + \Phi_-(x,t) \right\},
\]

where 
\[
\Phi_{+(-)}(x,t) = \frac{1}{2\psi_2} \exp\left\{ \psi_2^2 a_2 t - (+)\psi_2^2 x \right\} \text{erfc} \left( \psi_2(a_2 t)^{1/2} - (+) \frac{x}{2(a_2 t)^{1/2}} \right).
\]

Time of phase transition can be found from 
\[ T_2(x,t_{pt}) = T_{pt}. \]

Scheme of the problem is shown in figure 1.
3. Solutions for the drying stage

Stage 2 - the problem of microwave drying. The drying process in this article studied as heat transfer medium in the phase-change - known Stephen problem [3]. In our case, we propose a new approach to the microwave drying of coal array of the problem, which is based on a method “heat quasi-stationary approximation” developed by us. It is based on the concept that the movement of the phase transformation front controls part of the quasi-stationary temperature field.

For calculation formulas and the further search for the basic laws of the microwave drying process itself develops as follows. Due to internal heat sources from the microwave absorption of coal mass after reaching the phase transition temperature dries, forming three different areas in their properties in figure 2.

![Figure 1. Scheme of the first stage of the task.](image1)

![Figure 2. Temperature zones in coal layer.](image2)
Zones 1 and 2 are partially dried. Zone 2 is taken fully dried. In the future, the heat from the microwave absorption is spent on evaporation of residual moisture and making the phase transition "liquid - vapor" on moving the borders. It is required to determine the dynamics of the reduction of humidity, temperature distribution in zones 1 and 2, the dynamics of the reduction of humidity.

During these assumptions, the system of equations is:

The equation of thermal conductivity for zone 1 (the time count goes from $t_{be}$):

$$\frac{\partial T_1(x,t)}{\partial t} = a_1 \frac{\partial^2 T_1(x,t)}{\partial x^2} \quad t>0 \quad 0<x<\delta_1$$  \hspace{1cm} (6)

Heat equation for zone 2:

$$\frac{\partial T_2(x,t)}{\partial t} = a_2 \frac{\partial^2 T_2(x,t)}{\partial x^2} + \frac{q_{f0}}{c_2 \rho_2} e^{-\psi_0 \tau} \quad t>0 \quad x>\delta_2$$ \hspace{1cm} (7)

Initial conditions:

$$T_1(x, t) = q_1(x) \quad t_{be} \leq t < 0$$ \hspace{1cm} (8)

$$T_2(x, t) = q_2(x) \quad t_{be} \leq t < 0$$ \hspace{1cm} (9)

Where $q_1(x)$, $q_2(x)$ – known distributions of $T_1(x, t_{be})$, $T_2(x, t_{be})$ in two zones at the time of the onset of evaporation $t_{be}$.

Border conditions

$$\frac{\partial T_1(0, t)}{\partial x} = 0$$ \hspace{1cm} (10)

$$\frac{\partial T_2(\infty, t)}{\partial x} = 0$$ \hspace{1cm} (11)

Stefan's condition on the moving "liquid-vapor" boundaries

$$-\lambda_1 \frac{\partial T_1(\delta_1, t)}{\partial x} = \rho_1 H_u \frac{d\delta_1}{dt},$$ \hspace{1cm} (12)

$$T_1(\delta_1, t) = T_{ph}$$ \hspace{1cm} (13)

$$-\lambda_2 \frac{\partial T_2(\delta_2, t)}{\partial x} = \rho_2 H_u \frac{d\delta_2}{dt},$$ \hspace{1cm} (14)

$$T_2(\delta_2, t) = T_p$$ \hspace{1cm} (15)

In the microwave drying step the problem is solved analytically approximated using effective asymptotic procedures. Dynamics of the phase boundary is found within the framework of the quasi-stationary approximation. This approach is based on the following physical principle. Due to lack of comparability of the transfer rate of the heat pulse and movement of the phase boundary, the evaporation front movement is controlled by the limit on part-time temperature field. The fundamental role given to the quasi-stationary asymptotic behavior for large times. Calculations on the derived dependencies found such important characteristics of the studied processes as a warm-up time, drying speed, drying time and other popular options practices.

$$\delta_1(t) = \frac{1}{4P} \left( C_{L1} - C_{L2} \cdot t + 4M^2 \cdot t^2 \right)$$ \hspace{1cm} (16)

Where $C_{L1}$, $C_{L2}$, $M$ – corresponding constants, depending on initial conditions.

Similarly, there is find a solution for the motion of the second boundary $\delta_2(t)$.
After $\delta_1(t)$ and $\delta_2(t)$ were found – it is possible to find out another parameters:

To calculate the amount of evaporated moisture at the time $t$, we can use the expression:

$$W_w(t) = \rho_w \left( \delta_{\max} - \delta_1(t) \right) + \rho_w \left( \delta_2(t) - \delta_{\max} \right)$$

(17)

Where $\delta_1(t)$ and $\delta_2(t)$- coordinates of the motion of the fronts of phase transformation in 1 and 2 zones, respectively.

Thus, knowing the initial and final moisture of the coal, you can calculate the time to reach any humidity and the total drying time.

The drying speed can be found:

$$V = W'_w(t)$$

(18)

4. Conclusion

The mathematical model of warming up and drying of coal semi-mass is studied in the article, the results of which can serve as a theoretical basis for the technology of intensive microwave effect on moist material. At the stage of heating by the method of the dual integral Laplace transform and cos-Fourier, the problem is solved analytically strictly. At the stage of microwave drying, the problem is solved approximately analytically. The process of electromagnetic drying is considered as a nonlinear Stephen problem with a "liquid-vapor" phase transition. The motion’s dynamics of the phase interface was found within the framework of a quasi-stationary approximation. This approach is based on the following physical principle. Because of the rates incompatibility of transfer of the thermal pulse and the interface’s motion, the movement of the evaporation front is controlled by the time-limited part of the temperature field. The fundamental role is given to the quasi-stationary asymptotics for large times. By calculating the derived dependencies, such important characteristics of the studied processes as the warm-up time, drying speed, drying time and other parameters required by practice have been found.

References

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