ABSTRACT
In Cooperative Multi-Agent Reinforcement Learning (MARL) and under the setting of Centralized Training with Decentralized Execution (CTDE), agents observe and interact with their environment locally and independently. With local observation and random sampling, the randomness in rewards and observations leads to randomness in long-term returns. Existing methods such as Value Decomposition Network (VDN) and QMIX estimate the mean value of long-term returns while ignoring randomness. Our proposed model QR-MIX introduces quantile regression, modeling joint state-action values as a distribution, combining QMIX with Implicit Quantile Network (IQN). Besides, because the monotonicity in QMIX limits the expression of joint state-action value distribution and may lead to incorrect estimation results in nonmonotonic cases, we design a flexible loss function to replace the absolute weights found in QMIX. Our methods enhance the expressiveness of our mixing network and are more tolerant of randomness and nonmonotonicity. The experiments demonstrate that QR-MIX outperforms prior works in the StarCraft Multi-Agent Challenge (SMAC) environment.

KEYWORDS
Quantile Regression, Randomness, Collaboration, Multi-agent, Reinforcement Learning

In the multi-agent cooperative task, state-of-the-art methods learn the policy of each agent by decomposing the joint value function. For example, in the discrete action space scenario, VDN [24], QMIX [19], Qatten [27] and other methods achieve excellent results in the SMAC testing environment. In the continuous action space scenario, COMIX [7] proposed by Christian A. Schroeder de Witt et al. extends QMIX functionality to support continuous actions, successfully achieving state-of-the-art performance. Actor-Critic-based methods such as COMA [8] and MADDPG [7] perform relatively poorly in comparison with value-based methods.

However, in MARL, the observation of each agent is local; this typically causes bias and uncertainty in its value function. Besides, random sampling of non-deterministic environments results in randomness [6] in state transitions and rewards obtained. QMIX and VDN ignore randomness and model the mean value of the joint state-action value. To resolve these issues, in the single-agent scenario, Will Dabney et al. collectively proposed C51 [1], QR-DQN [6], and IQN [5] which model the value function as a distribution.

In a recent study, Felipe Leno Da Silva et al. [21] have applied Distributional Independent Q-Learning (C51) to multi-agent Robot Soccer Simulation, achieving better results than Independent Q-Learning (IQL) [26]. However, C51 cannot decompose the joint value, leading to poor results in complex multi-agent cooperation scenarios. Therefore, it proves effective to combine joint value decomposition in unison with Distributional RL [1]. However, with this comes a new set of problems. Studies such as QTTRAN [23] have shown that the monotonicity in VDN and QMIX can lead to erroneous value estimation in nonmonotonic cases [15] and also imposes limits on the expressiveness of the joint state-action value distribution. Therefore, we eliminate the hypernet [9] constraint of generating absolute weights in QMIX, and we use the gradient of expectation of the joint state-action value decomposition to each agent’s state-action value as the loss function to decompose the joint state-action value. In comparison with QMIX, this loss imposes fewer constraints on network expressiveness.

Contribution
(1) We propose Quantile Regression Mixer (QR-MIX) which uses Distributional RL to enhance the tolerance of the model for randomness.
(2) We design a flexible loss function to replace the absolute weight inherent in QMIX, allowing for higher tolerance of nonmonotonicity and fewer constraints on neural network expressiveness.
(3) Our experiments demonstrate that QR-MIX outperforms prior works in the SMAC environment [20].
2 BACKGROUND

2.1 Dec-POMDP

A fully cooperative multi-agent task may be described as a decentralized partially observable Markov decision process (Dec-POMDP) composed of a tuple $G = (S, U, P, r, Z, O, N, \gamma)$. $s \in S$ describes the true state of the environment. At each time step, each agent $i \in N := \{1, \ldots, N\}$ chooses an action $u_i \in U$, forming a joint action $u \in U^N$. All state transition dynamics are defined by Function $P(s' | s, u) : S \times U^N \times S \mapsto [0, 1]$. Each agent has independent observation $z \in Z$, determined by observation function $O(s, i) : S \times N \mapsto Z$. All agents share the same reward function $r(s, u) : S \times U^N \mapsto \mathbb{R}$ and $\gamma \in [0, 1)$ is the discount factor. Given that $\pi_i$ is the policy of agent $i$, the objective of the joint agent is to maximize:

$$J(\pi) = \mathbb{E}_{u_1, \ldots, u_N \sim \pi_N} [\sum_{t=0}^{\infty} \gamma^t r^f (s^t, u^t_1, \ldots, u^t_N)]$$

$$f = (1)$$

2.2 IGM

An essential concept of multi-agent function decomposition methods such as VDN [24] and QMIX [19] is Individual-Global-Max (IGM) [23]; given that $\mathbb{E} Q_1$ (single state-action value function), the following conditions hold:

$$\arg \max_u Q_{\text{tot}}(\tau, u) = \left( \arg \max_u Q_1 (\tau_1, u_1) \right)$$

$$\vdots$$

$$\arg \max_u Q_N (\tau_N, u_N)$$

$$f = (2)$$

where $Q_{\text{tot}}$ is the joint state-action value function, $\tau$ is the joint action-observation history, and $u$ is joint actions. This condition ensures that the $Q_{\text{tot}}$ can be decomposed by $Q_i$.

QMIX and VDN use different methods to ensure that IGM conditions are met. VDN is realized through additivity as in Eq. (3), but QMIX is guaranteed through a monotonic network [19], as shown in Eq. (4). In its implementation, QMIX uses a hypernetwork to generate absolute weights for modeling the non-linear relationship between $Q_i$ and $Q_{\text{tot}}$.

(Additivity) $Q_{\text{tot}}(\tau, u) = \sum_{i=1}^{N} Q_i (\tau_i, u_i)$

(Monotonicity) $\frac{\partial Q_{\text{tot}}(\tau, u)}{\partial Q_i(\tau_i, u_i)} \geq 0, \quad \forall i \in N$

$$f = (3)$$

$$f = (4)$$

2.3 Nonmonotonicity

However, in the case of nonmonotonicity [15], VDN and QMIX cannot learn the real optimal action. A simple example of a non-monotonic Q-function is given by the payoff matrix from [18] of the two-player three-action matrix game, as shown in Table 1. QMIX’s approximation (right) results in an incorrect state-action value (Table 2).

Table 1: Payoff matrix

|     | -12 | 0  | 0  |
|-----|-----|----|----|
| -12 | -12 | -12| -12|
| -12 | 0   | 0  | 0  |
| -12 | 0   | 0  | 0  |

Table 2: QMIX’s approximation

|     | -12 | -12 | -12 |
|-----|-----|-----|-----|
| -12 | -12 | -12| -12 |
| -12 | 0   | 0  | 0   |
| -12 | 0   | 0  | 0   |

2.4 Distributional RL

Rather than using a scalar $Q^\pi(s, a)$ as in DQN, Distributional RL [18] takes into account the randomness of $Z^\pi$ by studying its distribution. The distributional Bellman operator for policy evaluation is defined as

$$Z^\pi(s, u) \overset{D}{=} r(s, u) + \gamma Z^\pi (s', u')$$

where $s' \sim P(\cdot | s, u)$ and $u' \sim \pi(\cdot | s')$, and where $A \overset{D}{=} B$ denotes that A and B follow the same distribution. The meanings of $P, r(s, u), \pi$ and $\gamma$ are consistent with those in Dec-POMDP.

With the scalar setting, a distributional Bellman optimality operator can be defined by

$$\mathcal{T} Z(s, u) := r(s, u) + \gamma Z \left( s', \underset{u' \in U}{\arg \max} \mathbb{E} Z (s', u') \right)$$

$$f = (5)$$

2.5 $p$ - Wasserstein Metric

Bellemare et al. [18] have shown that the distributional Bellman operator is a contraction in the $p$-Wasserstein metric. The $p$-Wasserstein distance is the $L_p$ metric of the inverse cumulative distribution function (CDF). The $p$-Wasserstein distance for random variables U and V with CDF $F_U^1$ and $F_V^1$, respectively, is given by

$$W_p(U, V) = \left( \int_{0}^{1} |F_U^1(\omega) - F_V^1(\omega)|^p d\omega \right)^{1/p}$$

$$f = (6)$$

2.6 Huber Quantile Regression

QR-DQN [6] and IQN [5] estimate the quantile values for each of $K$ fixed, uniform probabilities; the random return is approximated by a uniform mixture of K Diracs,

$$Z^\theta(s, u) := \frac{1}{K} \sum_{i=1}^{K} \delta_{\theta_i}(s, u)$$

$$f = (7)$$

where each $\theta_i$ is assigned a fixed quantile target $\theta_i = \frac{1}{K-1}(\omega_i)$.

QR-DQN uses fixed $\omega_i$, whereas IQN samples $\omega_i \sim U([0, 1])$.

IQN and QR-DQN use Huber Quantile Regression [6] for stochastically adjusting quantile estimates and thereby minimize the Wasserstein distance to a target distribution. Given threshold $\kappa$, the regression loss is given by

$$\rho_\kappa(\delta_{ij}) = \mathbb{I} \{ \omega - 1 \{ \delta_{ij} < \kappa \} \} \frac{L_\kappa(\delta_{ij})}{\kappa},$$

$$\text{otherwise}$$

$$f = (8)$$

on the pairwise TD-errors [25]

$$\delta_{ij} = r + \gamma \theta_j (s', \pi (s')) - \theta_i (s, u)$$

$$f = (9)$$

$$f = (10)$$
Figure 1: The overall architecture of QR-MIX. On the right is agent $i$’s recurrent deep Q-network [10], which receives the action-observation history record $\tau_i$ (last hidden states $h_{t-1}^i$, current local observations $o_t^i$, and last action $a_{t-1}^i$). On the left is the mixing network of QR-MIX, which mixes $Q_i(\tau_i, u_i^i)$ together with $s_t$ and $\phi(\omega_i)$. $Z_{tot}(\tau, u, \phi(\omega_i))$ is the joint state-action value quantile corresponding to $\omega_i$.

## 3 QR-MIX

In this section, we propose a new method called Quantile Regression Mixer (QR-MIX). This method combines IQN [5] and QMIX [19] to model the joint state-action value function as a distribution to improve the tolerance of our model for randomness and non-monotonicity. We discuss the benefits of this in detail in Appendix A.

### 3.1 Quantile Mixing Network

IQN is a deterministic parametric function trained to reparameterize samples from a base distribution, e.g. $\omega \sim U([0,1])$, to the respective quantile values of a target distribution. IQN provides an effective way to learn an implicit representation of the state-action value distribution.

In Atari games [16], the state encoding network $\psi$ is usually relatively deep, composed of several layers of convolutional neural networks. Therefore, IQN uses a multiplicative form $\psi \circ \phi(\omega_i)$ to force convolutional features to interact with sample embedding and uses cosine to encode the sample $\omega_i$,

\[ \phi_i(\omega) := \text{ReLU} \left( \sum_{i=0}^{n-1} \cos(\pi i \omega) \omega_j + b_j \right) \]

where $N$ is the cosine embedding dimension and $N'$ is the state embedding dimension. IQN then uses the result of $\psi \circ \phi(\omega_i)$ to predict the state-action value quantile for each $\omega_i$.

However, in QR-MIX, only one layer of the neural network is used for encoding the global state; we therefore use concatenation to interact with sample embedding, as shown in Figure 1. We also simplify cosine embedding as follows:

\[ \phi_i(\omega) := \cos(\pi i \omega), i \in \{1...N\} \]

Figure 1 shows our mixing network architecture. We input the historical observations and actions of the agent as well as the global state and $\phi(\omega_i)$ to this network, which outputs a joint action value quantile for each $\omega_i$, and we then approximate the expectation of the joint state-action value distribution as

\[ E_{Z_{tot}}(\tau, u) := \frac{1}{K} \sum_{i=1}^{K} Z_{tot}(\tau, u, \phi(\omega_i)) \]

where $K$ is the number of samples.

### 3.2 Expected-IGM

**Definition 1.** Expected-Individual-Global-Max (EIGM). For a joint state-action value distribution function $Z_{tot} : \mathcal{T}^N \times \mathcal{U}^N \mapsto \text{Distribution}$, where $\tau \in \mathcal{T}^N$ is joint action-observation history, we assume that there exist individual state-action value functions $Q_i : \mathcal{T} \times \mathcal{U} \mapsto \mathbb{R}^N_{i=1}$, such that the following holds

\[ Q_i(\tau, u) = \text{max}_{a} \min_{\omega_i} Q_i(\tau, u, a, \omega_i) \]
We use quantile regression in Section 2.6 to train Quantiles Mixing when we want to express a distribution instead of a scalar. Also, we weights (SMAC) environment [20] as our testbed. SMAC consists of a set micromanagement tasks and use StarCraft Multi-Agent Challenge In this section, we evaluate QR-MIX in StarCraft II decentralized and we add Network, and we add second-order differential function of the deep learning framework [17].

For calculating the gradient of this loss function, we can use a where \( N \) is the number of \( Q_i \) and \( E \) is approximated by Eq. (13). This loss is designed to impose a penalty for violation of Expected-Monotonicity.

Our method has fewer restrictions on the expressiveness of the mixing network in comparison with absolute weights, especially when we want to express a distribution instead of a scalar. Also, we do not need for every quantile to be monotonic to keep the expectations monotonic; this property further enhances the model’s tolerance for non-monotonic situations.

For calculating the gradient of this loss function, we can use a second-order differential function of the deep learning framework [17].

### 3.3 Loss Function

We use quantile regression in Section 2.6 to train Quantiles Mixing Network, and we add \( L_{em} \) (16) and \( L_{qr} \) (17) together as \( L_{tot} \) (18). Given the number of samples \( K \) and \( K' \),

\[
L_{qr} = \frac{1}{K} \sum_{i=0}^{K-1} \sum_{j=0}^{K'-1} \rho_{ij} \left( \delta_{ij}^{r} \right) \tag{17}
\]

\[
L_{tot} = L_{qr} + \lambda \cdot L_{em} \tag{18}
\]

where \( \lambda \) is a hyperparameter for scaling.

### 4 EXPERIMENT

#### 4.1 Settings

In this section, we evaluate QR-MIX in StarCraft II decentralized micromanagement tasks and use StarCraft Multi-Agent Challenge (SMAC) environment [20] as our testbed. SMAC consists of a set of StarCraft II micro scenarios used for evaluating how effectively independent agents can learn coordination to solve complex tasks. This environment has become a standard benchmark for evaluating state-of-the-art MARL approaches.

SMAC classifies maps into three difficulty levels: Easy, Hard, and Super Hard. Our test includes maps from each difficulty level. We briefly introduce these maps in Table 5 in Appendix B.

Our main evaluation metric is the relationship between the average winning percentage of the evaluation episodes as a function of environment steps observed over the course of training. This progress can be estimated by periodically running a fixed number of evaluation episodes (actually 32) and disabling any exploratory behavior. We repeat each experiment with many independent training runs, and the results include median performance and percentiles ranging from 25% to 75%. We run the experiment 5 times independently in PyMARL [20]. Each independent run takes between 6 and 13 hours using NVIDIA GeForce GTX 1080Ti graphics cards and Intel(R) Core(TM) i7-7820X CPU.

All hyperparameters in QR-MIX are the same as those found in QMIX [19] and VDN [24] in PyMARL [20] with the exception of hyperparameters used in quantile regression [6], which will be shown in the Appendix B.3.

#### 4.2 Validation

The scenario we tested contains maps of three difficulty levels: Easy, Hard and Super Hard. Easy scenarios include 2s, vs. 1sc, c, 1c5s2, and 2s; Hard scenarios include 5m, vs. 6m, 2c, vs. 64, zq, and 3s, vs. 5z; Super Hard scenarios include MMM2 and 3s5z vs. 3z6z. As shown in Table 5, these maps cover various types, including heterogeneous, homogeneous, micro-trick, etc.

We opt for QMIX and VDN, the best performing model in PyMARL, as our baseline. We do not use QTRAN as the baseline due to its inferior performance [20] in SMAC. The poor performance of QTRAN may be caused by the fact that in complex scenarios, the approximate loss function does not meet its theoretical conditions. Table 3 shows the final median performance (maximum median across the testing intervals within the last 250,000 steps of training) of the algorithms tested. It can be seen that QR-MIX achieves the best results on all test maps used, especially for Hard and Super Hard maps.

Figure 2 shows the comparison of learning curves between QR-MIX, QMIX, and VDN. It can be seen that except for the two maps 2s, vs. 1sc and 5m, vs. 6m, the learning speed of QR-MIX is the fastest among all maps. In the Super Hard scenario 3s5z, vs. 3s6z, other methods have not learned effective policies well; however, the median test win rate of QR-MIX slowly improves.

#### 4.3 Ablation Study

In order to analyze the impact of our proposed loss function (16) on performance, we design a comparison method: QR-MIX-ABS, the hypernetwork [9] in QR-MIX-ABS only generates mixing networks with absolute weights.

Figure 3 shows the comparison of the learning curve of QR-MIX, QR-MIX-ABS, and QMIX. In Super Hard scenarios MMM2 and 3s5z, vs. 3s6z, QR-MIX performs significantly better than other methods. The positive weight constraint in QR-MIX-ABS limits
Figure 2: Median win percentage of baselines and QR-MIX. Easy: 2s_vs_1sc, 1c3s5z; Hard: 5m_vs_6m, 2c_vs_64zg; Super hard: MMM2, 3s5z_vs_3s6z.

Table 3: Median performance of the test win percentage in all scenarios

| Scenario     | QR-MIX | QMIX | VDN |
|--------------|--------|------|-----|
| 2s_vs_1sc    | 100    | 100  | 100 |
| 1c3s5z       | 99     | 97   | 91  |
| 2s3z         | 99     | 98   | 97  |
| 5m_vs_6m     | 81     | 69   | 70  |
| 2c_vs_64zg   | 95     | 45   | 25  |
| 3s_vs_5z     | 97     | 88   | 91  |
| MMM2         | 91     | 69   | 0   |
| 3s5z_vs_3s6z | 27     | 1    | 1   |

Table 4: Median performance of the test win percentage for ablation

| Scenario     | QR-MIX | QR-MIX-ABS | QMIX |
|--------------|--------|------------|------|
| 1c3s5z       | 99     | 99         | 97   |
| 3s_vs_5z     | 97     | 96         | 91   |
| MMM2         | 91     | 55         | 69   |
| 3s5z_vs_3s6z | 27     | 3          | 1    |

the expression of joint state-action value distribution in complex scenarios, so it learns more slowly than either QMIX or QR-MIX in MMM2. QR-MIX-ABS learning speed is significantly faster than other methods in 3s_vs_5z, but has a similar learning speed as QR-MIX in the final stages of training.

We show the final learning results in Table 4; QR-MIX is shown to have the best average performance.

5 CONCLUSION AND FUTURE WORK

In this paper, we propose QR-MIX. We use Distributional RL to enhance the tolerance of our model for randomness. Our proposed loss function contains fewer restrictions on neural network expressiveness and has a higher tolerance for nonmonotonic cases. This enhancement in the expressiveness of the mixing network allows for our method to achieve excellent results. Our method can also be combined with other Mixing Network-based model[27] to improve their ability to express randomness.

However, QR-MIX currently does not consider that the state-action value of each agent also has randomness. Therefore, a more ideal method is to decompose the joint state-action value distribution into the state-action value distribution of each agent. Such modeling will make full use of the benefits of Distributional RL; this is one of our proposed future works.

6 RELATED WORK

Cooperative MARL Both policy-based methods and value-based methods have been proposed for training agents under the CTDE paradigm. Value-based methods are focused on learning a joint state-action value estimator, which may be decomposed into individual state-action value functions such as in VDN [24], QMIX [19], QTRAN [21] and Qatten [27]. Policy-based methods are usually based on Actor-Critic frameworks such as COMA [8], MADDPG [7], and MAAC [12]. These methods can be applied to continuous action spaces, but they perform poorly in complex scenarios such as SMAC [20]. According to the study by Christian A. Schroeder de Wit and others [7], joint state-action value function decomposition is a key factor in determining the performance of cooperative MARL. They extend QMIX to the continuous action space with Actor-Critic method and achieve state-of-the-art performance. For a complete review of MARL, refer to the survey [11].

Distributional RL DQN [16] models the action-value function as a scalar. However, environments evaluated in RL typically have high randomness; Therefore, Distributional RL considers the factors of randomness, modeling the action-value function as a distribution,
and achieves excellent results in Atari games. Will Dabney et al. proposes C51 [1], QR-DQN [6], and IQN [5] successively, perfecting the theory of Distributional RL.

**Distributional MARL.** Felipe Leno Da Silva et al. [21] have applied Distributional Independent Q-Learning (C51) to multi-agent robot soccer simulation, achieving better results than IQL. Xueguang Lyu et al. [14] use IQN to reduce the instability resulting from the exploration behaviors of other agents. However, due to lacking the capability to decompose joint state-action value functions, both C51 and IQN are difficult to apply to complex cooperation scenarios. Our work combines Distributional RL and joint state-action value decomposition, achieving excellent performance.

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Table 5: Part of the Maps in SMAC.

| Name            | Ally Units                  | Enemy Units                        | Type                                      | Difficulty |
|-----------------|-----------------------------|------------------------------------|-------------------------------------------|------------|
| 2s3z            | 2 Stalkers & 3 Zealots      | 2 Stalkers & 3 Zealots             | heterogeneous & symmetric                 | Easy       |
| 2s_vs_1sc       | 2 Stalkers                  | 1 Spine Crawler                    | micro-trick: alternating fire             | Easy       |
| 1c3s5z          | 1 Colossi & 3 Stalkers & 5 Zealots | 1 Colossi & 3 Stalkers & 5 Zealots | heterogeneous & symmetric                 | Easy       |
| 5m_vs_6m        | 5 Marines                   | 6 Marines                          | homogeneous & asymmetric                  | Hard       |
| 2c_vs_64zg      | 2 Colossi                   | 64 Zerglings                       | micro-trick: positioning                  | Hard       |
| 3s_vs_5z        | 3 Stalkers                  | 5 Zealots                          | micro-trick: kiting                       | Hard       |
| 3s5z_vs_3s6z    | 3 Stalkers & 5 Zealots      | 3 Stalkers & 6 Zealots             | heterogeneous & asymmetric                | Super Hard |
| MMM2            | 1 Medivac, 2 Marauders & 7 Marines | 1 Medivac, 3 Marauders & 8 Marines | heterogeneous & asymmetric                | Super Hard |

A WHY LEARN A DISTRIBUTION?
(1) The research of Marc G. Bellemare et al. [1] shows that the Bellman Optimality Operator has a high level of instability in the case of function approximation. (2) Due to local observation and a nondeterministic environment, the same state may correspond to different Q-values. (3) There are more Q-value predictions, some of which may be correct. (4) Only the expectations of the Q-value distribution are guaranteed to remain monotonic; this enhances the tolerance for nonmonotonic cases.

B EXPERIMENTAL SETTINGS
We base our experimental settings on SMAC, which may be referred to in the SMAC paper [20].

B.1 Scenarios
SMAC contains a set of StarCraft 2 micro scenarios designed to evaluate how independent agents may learn to coordinate to solve complex tasks. These scenarios are specially designed to require learning one or more micro-management techniques to defeat the enemy. The scenarios used in our experiment are shown in Table 5.

B.2 PyMARL
PyMARL is an open-source framework [20] based on the architecture of SMAC. This framework implements state-of-the-art MARL methods such as COMA, IQL, QMIX, and VDN. We use PyMARL to conduct a performance comparison.

B.3 Architecture and Hyperparameters
As shown in Figure 1, DRQN is the basic architecture of the agent network [10], containing 64 hidden layer dimensions. A fully connected network layer is put before and after the GRU [3]. The mixing network is a 32-unit single hidden layer network that uses ELU [4] as the activation function. We use a 128-unit fully connected network to mix global state and cosine embedding (12). The mixing results are used in the hyper network to generate mixing networks. The hyper network consists of four single hidden layer networks: the dimension of the bottom three hidden layers is 64, and the dimension of the top hidden layer is 32.

We set $K$ and $K'$ equal to 8 to achieve the balance of performance to computational overhead; we set the dimensions of cosine embedding to 32, threshold $\kappa = 1$, and scaling coefficient $\lambda = 0.8$. All agents share a policy network that inputs an ID to distinguish agents. All neural networks are trained using the RMSProp optimizer with 0.0005 learning rates, and we use $\epsilon$-greedy action selection with decreasing $\epsilon$ from 1 to 0.05 over 50000-time steps for exploration. For the discount factor, we set $\gamma = 0.99$. The replay buffer size is 5000 episodes and the minibatch size is 32.

C QR-MIX TRAINING ALGORITHM
QR-MIX training algorithms are provided in Algorithm 1.
Algorithm 1: QR-MIX

Hyperparameters: $K, K', \kappa, \lambda, \gamma, \epsilon$

Initialize replay memory $D$

Initialize $[Q_i], Q_{tot}$, with random parameters $\theta$

Initialize target parameters $\theta^- = \theta$

for episode ← 1 to $M$ do

Observe initial state $s^1$ and observation $o^1 = [O(s^1, i)]_{i=1}^{N}$ for each agent $i$

for $t ← 1$ to $T$ do

With probability $\epsilon$ select a random action $u^t_i$

Otherwise $u^t_i = \text{arg max}_{u_i} Q_i(t^t, u^t_i)$ for each agent $i$

Take action $u^t$, and retrieve next observation and reward $(o^{t+1}, r^t)$

Store transition $(o', u^t, r^t, o^{t+1})$ in $D$

end

Sample a random mini-batch (B) of transition sequences $(T)$ $(o^1, u^1, r^1, ... o^T, u^T, r^T, o^{T+1})$ from $D$

Sample $\omega_i, \omega_j' \sim U([0, 1]), \quad 1 \leq i \leq K, 1 \leq j \leq K'$

Set $\delta_{ij} \leftarrow r + \gamma Z_{\theta^-}(r', u^-, \phi(\omega_j')) - Z_{\theta}(r, u_i, \phi(\omega_i)), \quad \forall i, j, u^- = \left[\text{arg max}_{u_i} Q_i(t^t, u^t_i; \theta^-)\right]_{i=1}^{N}$

Calculate $L_{qr}$ by Eq. (17)

Calculate $L_{em}$ by Eq. (16)

Update $\theta$ by minimizing the loss:

$L_{tot} = E_{\text{mini-batch}} (L_{qr} + \lambda \cdot L_{em})$

Update target network parameters $\theta^- = \theta$ with period $I$

end