Isospin Asymmetry in the Pseudospin Dynamical Symmetry

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Pseudospin symmetry in nuclei is investigated considering the Dirac equation with a Lorentz structured Woods-Saxon potential. The isospin correlation of the energy splittings of pseudospin partners with the nuclear potential parameters is studied. We show that, in an isotopic chain, the pseudospin symmetry is better realized for neutrons than for protons. This behavior comes from balance effects among the central nuclear potential parameters. In general, we found an isospin asymmetry of the nuclear pseudospin interaction, opposed to the nuclear spin-orbit interaction which is quasi isospin symmetric.

In some heavy nuclei a quasi-degeneracy is observed between single-nucleon states with quantum numbers \((n, \ell, j = \ell + 1/2)\) and \((n - 1, \ell + 2, j = \ell + 3/2)\) where \(n, \ell, \text{and } j\) are the radial, the orbital, and the total angular momentum quantum numbers, respectively. This doublet structure is better expressed using a “pseudo” orbital angular momentum quantum number, \(\tilde{\ell} = \ell + 1\), and a “pseudo” spin quantum number, \(\tilde{s} = 1/2\). For example, for \([ns_{1/2}, (n - 1)d_{3/2}]\) one has \(\tilde{\ell} = 1\), for \([np_{3/2}, (n - 1)f_{5/2}]\) one has \(\tilde{\ell} = 2\), etc. Exact pseudospin symmetry means degeneracy of doublets whose angular momentum quantum numbers are \(j = \ell \pm \tilde{s}\). This symmetry in nuclei was first reported about 30 years ago [1], but only recently has its origin become a topic of intense theoretical research.

In recent papers [2, 3, 4, 5, 6] possible underlying mechanisms to generate such symmetry have been discussed. We briefly review the main points of these studies.

Blokhin et al. [2] performed a helicity unitary transformation in a nonrelativistic single-particle Hamiltonian. They showed that the transformed radial wave functions have a different asymptotic behavior, implying that the helicity transformed mean field acquires a more diffuse surface. Application of the helicity operator to the nonrelativistic single-particle wave function maps the normal state \((l, s)\) onto the “pseudo” state \((\tilde{l}, \tilde{s})\), while keeping all other global symmetries [3]. The same kind of unitary transformation was also considered earlier by Bahri et al. [4] to discuss the pseudospin symmetry in the nonrelativistic harmonic oscillator. They showed that a particular condition between the coefficients of spin-orbit and orbit-orbit terms, required to have a pseudospin symmetry in that non-relativistic single particle Hamiltonian, was consistent with relativistic mean-field (RMF) estimates.

Ginocchio [5], for the first time, identified the pseudospin symmetry as a symmetry of the Dirac Hamiltonian. He pointed out that the pseudo-orbital angular momentum is just the orbital angular momentum of the lower component of the Dirac spinor. Thus, the pseudospin symmetry started to be regarded and understood in a relativistic way. He also showed that the pseudospin symmetry would be exact if the attractive scalar, \(S\), and the repulsive vector, \(V\), components of a Lorentz structured potential were equal in magnitude: \(S + V = 0\). Under this condition, the pseudospin symmetry was identified as a SU(2) symmetry of the Dirac Hamiltonian [6].

In RMF models, nuclear saturation is explained by a cancellation between a large scalar \((S)\) and a large vector \((V)\) fields [3]. Typical values for these fields in heavy nuclei are of the order of a few hundred MeV (with opposite signs), their sum providing a binding potential of about 60 MeV at the nucleus center. Therefore, a natural claim of Ginocchio was to regard the quasi-degenerate pseudospin doublets in nuclei as arising from the near equality in magnitude of the attractive scalar, \(S\), and the repulsive vector, \(V\), relativistic mean-fields, \(S \sim -V\), in which the nucleons move. When the doublets are degenerate, the shape of the lower components of the Dirac spinor for the two states in the doublet is the same [6].

More recently, Meng et al. [7, 8] showed that pseudospin symmetry is exact when \(dS/dr = 0\) where \(\Sigma = S + V\). They also related the onset of the pseudospin symmetry to a competition between the centrifugal barrier and the pseudospin-orbit potential.

Actually, since in nuclei \(S(r), V(r) \to 0\) when \(r \to \infty\), the conditions \(\Sigma = 0\) and \(d\Sigma/dr = 0\) are equivalent. However, these conditions cannot be realized in nuclei, because they imply that no bound states exist [9]. Nevertheless, as we will show, there is a correlation between the pseudospin splitting and the depth of \(\Sigma\), its surface diffuseness and its radius. Depending on the actual choice of
these parameters, fitted to describe a nucleus, the pseudospin symmetry may show up. Thus, one may argue that pseudospin symmetry is a dynamical symmetry in nuclei, in the sense of Arima’s definition of a dynamical symmetry \cite{11}, (i) a symmetry of the Hamiltonian which is not geometrical in nature; or (ii) a ordered breaking symmetry from dynamical reasons. This is consistent with the findings of Ref. \cite{12}, associating the pseudospin symmetry with a particular relation between the coefficients of spin-orbit and orbit-orbit terms in RMF models.

To establish that correlation, we perform a model calculation with a Lorentz structured potential of Woods-Saxon type in the Dirac equation. The scalar and vector components of this potential are the mean-field central nuclear potentials. The form of the potential is

\[
U(r) = \frac{U_0}{1 + \exp((r - R)/a)},
\]

where \(U(r)\) stands for either the vector or the scalar potential. Although this is not a full self-consistent relativistic potential, the use of (1) as a nuclear mean-field potential is realistic enough to be applied to nuclei and enables us to study the splittings of pseudospin partners not only as a function of the mentioned depth, \(U_0\), but also of the diffuseness, \(a\), and the radius, \(R\).

In nuclear relativistic mean-field theory, each nucleon is described by a Dirac Hamiltonian of a particle of mass \(m\) in an external scalar, \(S\), and vector, \(V\), potentials:

\[
H = \alpha \cdot p + \beta(m + S) + V,
\]

where \(\alpha\) and \(\beta\) are the usual Dirac matrices. The Dirac Hamiltonian is invariant under an SU(2) transformation for two cases: \(S = V\) and \(S = -V\) \cite{13, 14}.

We define \(\Delta = V - S\) and denote the upper and lower components of the Dirac spinor by \(\Psi_+ = \frac{1 + \gamma_5}{2}\Psi\). Assuming that \(S\) and \(V\) are radial functions, the Dirac equation can be decoupled into two equations for the lower and upper components, respectively:

\[
\nabla^2 F_i + \frac{\Sigma'}{E - m - \Sigma} \left( F_i' + \frac{1 - \kappa_i}{r} F_i \right) + (E - m - \Delta)(E - m - \Sigma) F_i = 0
\]

\[
\nabla^2 G_i + \frac{\Delta'}{E + m - \Delta} \left( G_i' + \frac{1 + \kappa_i}{r} G_i \right) + (E + m - \Delta)(E - m - \Sigma) G_i = 0,
\]

where the primes denote derivatives with respect to \(r\). The spinors \(\Psi_\pm\) have been factorized in radial and angular parts: \(\Psi_+ = i G_i(r) \Phi_+^r(\theta, \phi)\) and \(\Psi_- = -F_i(r) \Phi_-^r(\theta, \phi)\), with \(i\) standing for the quantum numbers of the single particle state. The property \(\sigma \cdot \mathbf{L} \Phi_\pm^r = -(1 \pm \kappa_i) \Phi_\pm^r\) was used, \(\sigma\) being the Pauli matrices.

Before we present our results, let us briefly discuss how the pseudospin symmetry gets broken. The commutator of the SU(2) generators of pseudospin symmetry with the Hamiltonian \cite{15} is given by \cite{16, 17, 18, 19}:

\[
\left[H, S_i\right] = \begin{pmatrix} [\Sigma, \tilde{s}_i] & 0 \\ 0 & 0 \end{pmatrix},
\]

where \(\tilde{s}_i = \frac{\sigma \cdot p}{p} \cdot s_i \), \(\sigma \cdot p \cdot p_i - s_i\) and \(s_i = \sigma_i/2\). The breaking of the pseudospin symmetry can thus be related to the commutator \([\Sigma, s_i]\). Requiring \([\Sigma, s_i] = 0\) is equivalent to the previous condition \(d\Sigma/dr = 0\), when \(\Sigma\) is a radial function.

Now we turn to the presentation of our results obtained using the Woods-Saxon potentials \cite{13} in eqs. (3) and (4). There are altogether six parameters for \(\Sigma\) and \(\Delta\), namely the central depths, \(\Sigma_0\) and \(\Delta_0\), two radii and two diffuseness parameters. We observed that the pseudospin splitting is not sensitive to \(R\) and \(a\) of the \(\Delta\) potential, and, accordingly, set the same radius, \(R\), and surface diffuseness, \(a\), for both potentials. We first fitted these parameters to the neutron spectra of \(^{208}\)Pb, obtaining a good agreement with the results of Ref. \cite{20} for the same set of pseudospin doublets: \((1i_{1/2}, 2g_{9/2}); (2f_{5/2}, 3p_{3/2})\) and \((1h_{9/2}, 2f_{7/2})\).

Keeping \(\Sigma_0\) and \(\Delta_0\) fixed, we varied \(a\) and \(R\) in order to see how the energy splittings of the pseudospin doublets change with the surface diffuseness and the radius. This dependence is presented in Fig. 1. As \(a\) increases, the splittings of the pseudospin doublets decrease. On the contrary, as \(R\) increases the pseudospin splittings increase. The same figure shows that the splittings are more dependent on \(R\) than on \(a\). If \(a\) increases and/or \(R\) decreases enough, the pseudospin doublet partners cross each other, inverting the sign of the energy splitting, and are driven apart if \(a\) (\(R\)) further increases (decreases). This inversion of pseudospin partner splittings as \(a\) increases, \(E_{n-1,i+1/2} < E_{n,i-1/2}\) changing to \(E_{n-1,i+1/2} > E_{n,i-1/2}\), is observed experimentally and was also found in Refs. \cite{21, 22}.

We now keep \(a\), \(R\) and \(\Delta_0\) fixed to study the sensitivity of the pseudospin doublets with \(\Sigma_0\). The results are presented in Fig. 2 and one observes that, as \(|\Sigma_0|\) decreases, the splitting also decreases. This is in accordance with Ginocchio predictions for pseudospin symme-

![FIG. 1: Pseudospin energy splittings for the neutron pseudospin partners (2f_{5/2}, 3p_{3/2}) and (1h_{9/2}, 2f_{7/2}) in ^{208}\)Pb as a function of \(R\) for several values of \(a\).](image-url)
try breaking due to the finiteness of the $\Sigma$ mean field. We also found that, for deeper levels, an inversion of pseudospin energy splitting occurs for sufficiently low $|\Sigma_0|$. Comparing Figs. 1 and 2, we see that the splittings of pseudospin partners are more sensitive to the radius and diffuseness than to the central depth of the potential.

We note that the radius dependence is relevant for comparing different nuclei, especially certain isotopes [7]. However, we found an interesting correlation between $|\Sigma_0|$ and $R$. Varying $|\Sigma_0|$ and $R$ but keeping $a$ and the product $|\Sigma_0|R^2$ fixed, the pseudospin doublet splittings remain almost constant. Therefore, only two parameters, out of $(\Sigma_0, R, a)$, are free.

The variation of the other free parameter, $\Delta_0$, does not qualitatively change the splittings. Since the $\Delta$ potential is related to the effective mass of the nucleons, the basic effect is to slightly change the nucleon separation energies, and especially the spin-orbit coupling, as discussed below. We have also performed the calculation for calcium isotopes, and found the same dependence of the pseudospin splitting with $R$, $a$ and $\Sigma_0$.

We apply now the systematics to some nuclei studied in the literature. Recently, the pseudospin symmetry in Zr and Sn isotopes was investigated as a function of the number of nucleons [8]. The form of the $\Sigma$ potential in dependence of the radial distance for such nuclei, as $A$ increases, is given in Ref. [7], starting with $^{108}$Sn and going up to $^{170}$Sn. In [7] it is shown that the central neutron depth, $\Sigma_0$, of Sn isotopes varies from about $-65$ MeV to $-54$ MeV, $R$ from 5.6 fm to 6.8 fm and the surface diffuseness increases. For the proton, the central depth, $\Sigma_0$, varies from $-50$ MeV to $-60$ MeV, $R$ from 5 fm to 6.5 fm, while the surface diffuseness also increases, now slightly more than in the neutron case. Using [8], good fits to the neutron central potentials of the Sn isotopes were obtained, with $\Sigma_0$ from $-68.6$ MeV to $-55.7$ MeV, $R$ from 5.6 fm to 6.7 fm and $a$ from 0.6 fm to 0.76 fm, in agreement with Ref. [7].

This information on $\Sigma$ for Sn isotopes allows us to analyze the behavior for neutron pseudospin doublets going from $A = 100$ to $A = 170$. As $A$ increases, the central depth $|\Sigma_0|$ decreases and the surface diffuseness increases, both effects favoring the pseudospin symmetry, as shown in Figs. 1 and 2. However, the radius increase with $A$ can partially offset those effects. Since the values of $|\Sigma_0|R^2$ are roughly constant for neutrons [7], the correlation between these two parameters, mentioned above, implies that the effects of increasing $R$ and decreasing $|\Sigma_0|$ in the neutron central potential, when $A$ increases, balance each other. Thus, the dominant effect comes from the increasing $a$, slightly favoring the pseudospin symmetry.

We extended our analysis to the proton spectra. For this case, as mentioned above, $|\Sigma_0|$ increases as $A$ increases for Sn isotopes [7], therefore disfavoring pseudospin symmetry. Moreover, $R$ increases, also disfavoring the pseudospin splitting. Hence, at least for the proton spectra, and in opposition to the neutron spectra, we do not have any balance effect coming from $\Sigma_0$ and $R$ dependences. Thus, we have a competition between the effects of surface diffuseness (favoring the symmetry), and of depth and radius of the central potential (both disfavoring the symmetry). From our systematics described before we expect that effects coming from the depth and radius of the central potential override the surface diffuseness dependence. Therefore, although the proton spectra along the Sn isotopic chain was not presented in Ref. [7], our systematics predicts that the splitting increases for the proton pseudospin partners. The different behavior of the splitting for protons and neutrons can be inferred by looking at the value of $|\Sigma_0|R^2$ for the two cases as $A$ changes: the change is considerably higher for protons.

The reason for the different behavior of the central potential depth for neutrons and protons as a function of $A$, lies on the $\rho$ meson interaction, which is repulsive for neutrons and attractive for protons, as explained by mean-field model calculations [7]. The inclusion of this interaction, which is important in asymmetric nuclei, changes the vector part of the nuclear potential $V$ to

$$ V = V_\omega \pm \frac{g_\rho}{2} \rho_0, $$

with $+$ and $-$ signs for protons and neutrons respectively; $\rho_0$ is the time component of the $\rho$ field, which is proportional to the number of protons minus the number of neutrons, and $V_\omega$ comes from the vector-isoscalar $\omega$ meson. The increase of the parameters $R$ and $a$ with $A$ can be traced back to the known nuclear radius $A^{1/3}$ dependence and to the excess of neutrons on the surface (the neutron skin effect). Hence, we may conclude that, for a given nucleus, the parameters $\Sigma_0$, $a$ and $R$ for protons and neutrons are different. Then, an isospin asymmetry in the pseudospin interaction is expected to take place in agreement with the analysis of the mean-field nuclear parameters for protons and neutrons done in [8]. In particular, since $\rho_0$ is negative for heavy nuclei, the vector...
potential (6) is bigger (and, thus, $|\Sigma|$ smaller) for neutrons than for protons. From our previous analysis, for a neutron rich nucleus, the pseudospin symmetry for neutron spectrum is favored, in agreement with the results presented by Lalazissis et al. [13].

The systematics discussed here seems to be quite general and a comment on how it affects the spin-orbit splitting is pertinent. Variations of central depth and surface diffuseness of the nuclear potential do not change as much spin-orbit splittings as they do for the pseudospin-orbit splittings. The reason is that to change substantially spin-orbit splitting one needs a significant relativistic treatment of Ref. [2] to analyze the origin of the pseudospin-orbit splittings. The reason is that to change substantially spin-orbit splitting one needs a significant relativistic content, i.e. a large lower component in the Dirac spinor [13]. The spin-orbit splitting is completely correlated with the nucleon effective mass [19]. From Eq. (4), we see that it will depend strongly on $\Delta$. In RMF models this potential carries a quite large scale when compared with $\Sigma$ ($\Delta_0$ is around 650-750 MeV, while $|\Sigma_0|$ is around 50-70 MeV). Therefore, the $\rho$ meson potential, $V_\rho$, in the range 4-8 MeV, becomes irrelevant compared to $\Delta$. This was verified by recent numerical calculations [17, 20, 21], showing that the difference between the values of $\Delta$ for protons and neutrons (and, therefore, between the corresponding values of the spin-orbit term) is very small. This explains why the spin-orbit interaction is roughly isospin symmetric.

On the contrary, the pseudospin splitting can change when there is little relativistic content, i.e. a small lower component. This may explain why the nonrelativistic treatment of Ref. [2] to analyze the origin of the pseudospin symmetry works well in explaining the small pseudospin splitting. The pseudospin-orbit interaction depends on $\Sigma$, as one can see from Eq. (3). Since $V_\rho$ cannot be neglected compared to $\Sigma$, the different sign contribution of $V_\rho$ in (6) for protons and neutrons has a relevant effect on $\Sigma$, leading to the isospin asymmetry in the pseudospin interaction.

The systematics observed in our model calculation helps us to understand the origin of the pseudospin symmetry and explains the quasi-degeneracy of single-particle states occurring in the spectra of some finite nuclei. The new radioactive nuclear beam facilities will provide more data to which these results can be applied.

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