DISK-OUTFLOW COUPLING: ENERGETICS AROUND SPINNING BLACK HOLES

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ABSTRACT

The mechanism by which outflows and plausible jets are driven from black hole systems still remains observationally elusive. This notwithstanding, several observational evidences and deeper theoretical insights reveal that accretion and outflow/jet are strongly correlated. We model an advective disk-outflow coupled dynamics, incorporating explicitly the vertical flux. Inter-connecting dynamics of outflow and accretion essentially upholds the conservation laws. We investigate the properties of the disk-outflow surface and its strong dependence on the rotation parameter of the black hole. The energetics of the disk outflow strongly depend on the mass, accretion rate, and spin of the black holes. The model clearly shows that the outflow power extracted from the disk increases strongly with the spin of the black hole, inferring that the power of the observed astrophysical jets has a proportional correspondence with the spin of the central object. In the case of blazars (BL Lacs and flat spectrum radio quasars, FSRQs), most of their emission are believed to be originated from their jets. It is observed that BL Lacs are relatively low luminous than FSRQs. The luminosity might be linked to the power of the jet, which in turn reflects that the nuclear regions of the BL Lac objects have a relatively low spinning black hole compared to that in the case of FSRQs. If extreme gravity is the source that powers strong outflows and jets, then the spin of the black hole, perhaps, might be the fundamental parameter to account for the observed astrophysical processes in an accretion powered system.

Key words: accretion, accretion disks – black hole physics – galaxies: jets – hydrodynamics – galaxies: active – X-rays: binaries

Online-only material: color figures

1. INTRODUCTION

High resolution observations show strong outflows and jets in black hole accreting systems, both in active galactic nuclei (AGNs) or quasars (Begelman et al. 1984; Mirabel 2003) and microquasars (SS433, GRS 1915+105; Margon 1984; Mirabel & Rodriguez 1994, 1998). Extragalactic radio sources show evidence of strong jets in the vicinity of spinning black holes (Meier et al. 2001; Meier 2002). Outflows are also observed in neutron star low mass X-ray binaries (LMXBs; Fender et al. 2004; Migliari & Fender, 2006) and also in young stellar objects (Mundt 1985). It has been argued (Ghosh & Mukhopadhyay 2009 and Ghosh et al. 2010, hereinafter GM09 and GM10, respectively, and references therein) that outflows and jets are more prone to emanate from strong advective accretion flows; the said paradigm is more susceptible for super-Eddington or super-critical (Fabrika 2004, Begelman et al. 2006, GM09, G10, and references therein), and the accretion disk is precisely “radiation trapped” as in ultra-luminous X-ray (ULX) sources. It has been confirmed by radiation hydrodynamic simulation at super-critical accretion rate (Okuda et al. 2009) as well to explain luminosity and mass outflow rate of relativistic outflows from SS433. The under-luminous accreting sources, having high sub-critical accretion flow, were explained by an advection dominated accretion flow (ADAF) model (Narayan & Yi 1994). The promising outcome of this model lies in the large positive value of the Bernoulli parameter because of the small radiative energy loss. It leads to conceive that the gas in the inflowing disk is susceptible to escape, leading to strong unbounded flows in the form of outflows and jets. This also signifies that even in the absence of the magnetic field and radiation pressure, outflows are plausible from strongly advective accretion flow if the system is allowed to perturb. Nevertheless, the definitive understanding of the origin of outflows/jets is still unknown. It remains one of the most compelling problems in high energy astrophysics.

Whatever might be the reason for the origin of outflow and then jet from the disk, one aspect is, however, definite that strong outflows producing relativistic jets are powered by extreme gravity. Although it seems paradoxical, the strength and the length scale of observed astrophysical jets vary directly with the strength of the central gravitating potential. Observationally, it is evident that strong outflows and relativistic jets are more
powerful in observed AGNs and quasars, harboring supermassive black holes, compared to that in black hole X-ray binaries (XRBs). In addition, the jets observed in AGNs and quasars have greater length scale compared to that seen in stellar mass black hole systems. One of the most important signatures of relativistic gravitation is the spin of the central object. It is presumably believed that the spin (practically specific angular momentum) of the neutron star is less than that of a black hole, and thus the observed jet from black holes is much stronger and powerful than that of neutron star sources. Earlier, Blandford & Znajek (1977) demonstrated that if there is a magnetic field associated with the black hole due to threading of magnetic field lines from the disk and the angular momentum of the Kerr black hole is large enough, then the energy and the angular momentum can be extracted from the underlying black hole by a purely electromagnetic mechanism, which can thus be expected to power the jet in an AGN. These imply that the spin might play a significant role in powering jets, both in microquasars and in AGNs; thus, it can act as a fundamental parameter in an accretion powered system.

Most of the studies of accretion disk and studies of related outflow/jet have evolved separately, assuming these two to be apparently dissimilar objects. However, several observational inferences (for details see GM09, G10, and references therein) and improved understanding of accretion flow and outflow reveal that accretion and outflow/jet are strongly correlated. The unifying scheme of disk and outflow is essentially governed by conservation laws: conservation of matter, energy, and momentum. Hence, in modeling the accretion and outflow simultaneously in any accretion powered system, the following aspects should be taken into account: (1) the effect of relativistic central gravitational potential including its spin, (2) proper mechanism of the origin of outflow/jet from the disk, and (3) appropriate hydrodynamic equations (considering that the accreting gas be treated as a continuum fluid), capturing the information about the intrinsic coupling between inflow and outflow which are governed by the conservation laws in a strongly advective paradigm.

Recently, GM09 and G10 made an endeavor to explore the dynamics of the accretion-induced outflow around black holes/compact objects in a 2.5 dimensional paradigm. The authors formulated the disk-outflow coupled model in a more self-consistent way by solving a complete set of coupled partial differential hydrodynamic equations in a general advective regime through a self-similar approach in an axisymmetric, cylindrical coordinate system. They explicitly incorporated the information of the vertical flux in their model. However, they restricted their study to the Newtonian regime, thus neglecting the requisite effect of general relativity, especially the effect of spin of the central object. Based on the model, the authors computed the mass outflow rate and the power extracted by the outflow from the disk self-consistently, without proposing any prior relation between the inflow and outflow.

In this paper, we propose a new model for the accretion-induced outflow by extending the work of GM09 and G10, by incorporating the general relativistic effect of the central potential without limiting ourselves to a self-similar regime. As the definitive mechanism of launching of outflows/jets is still evasive, we do not embrace any specific mechanism of outflow like inclusion of the magnetic field into our model equations. Nevertheless, the importance of the magnetic field cannot be, in principle, discarded to explain the launching and collimation of jets off the accretion disk. Here, owing to our inability to solve coupled partial differential MHD equations in a 2.5 dimensional advective regime, we neglect the influence of the magnetic field in our model equations. However, the implicit coupling between the inflow and outflow, dictated by the conservation equations, have been taken into account appropriately. We arrange our paper as follows. In the next section, we present the formulation of our model. Section 3 describes the computational procedure to solve the model equations of the accretion-induced outflow. In Sections 4 and 5, we study the dynamics and the energetics of the flow, respectively. Finally, we end up in Section 6 with a summary and discussion.

2. MODELING THE CORRELATED DISK-OUTFLOW SYSTEM

We formulate the disk-outflow coupled model by considering a geometrically thick accretion disk, which is strongly advective as strong outflows/jets are more likely to eject from a thick/puffed up region of the accretion flow (GM09; G10). The vertical flow is explicitly included in the system. The basic features of the model are similar to that in GM09. We adopt the cylindrical coordinate system to describe a steady, axisymmetric accretion flow. The dynamical flow parameters, namely, radial velocity ($v_r$), specific angular momentum ($\lambda$), vertical velocity or outflow velocity ($v_z$), adiabatic sound speed ($c_s$), mass density ($\rho$), and pressure ($P$) depend both on radial and vertical coordinates. We have already highlighted the importance of the spin of black hole to power the outflow/jet and its presumed relative effect on the observation of various AGN classes. Thus, we have included the effect of the spin in our model. As the spin of the black hole is a signature of pure general relativity, i.e., Einstein’s gravitation, its effect on the accretion flow, especially in the inner region of the disk, is mimicked approximately with the use of pseudo-general-relativistic or pseudo-Newtonian potential (PNP). Because of the disk-outflow system to be geometrically thick and the flow to be 2.5 dimensional (not confined to the equatorial plane), we use the PNP of Ghosh & Mukhopadhyay (2007), which is a pseudo-Newtonian vector potential, to capture the inner disk properties of the accretion flow around a Kerr black hole approximately.

At the first instant, we neglect the effect of viscosity in our system. One of the reasons behind it is the unavailability of the effective computational technique to solve coupled partial differential viscous hydrodynamic conservation equations for the compressible flow. To make the inviscid assumption more arguable, it can also be noted that the angular momentum transport in the accretion flow, for which the necessity of turbulent viscosity is invoked, can alternatively take place purely by outflow. At this extreme end, the outflow extracts angular momentum from the disk allowing the matter to get accreted toward the black hole and hence the inviscid assumption can be adopted. This is in essence similar to the Blandford & Payne (1982) mechanism to extract energy and angular momentum from the magnetized disk, where the extraction of the angular momentum and energy is essentially done by the outflow, and not due to the viscous dissipation. The outflow originates from just above the equatorial plane of the disk and this lower boundary is maintained at $z = 0$, when $v_z = 0$, unlike other work where the outflow is hypothesized to effuse out from the disk surface (e.g., Xie & Yuan 2008). Our model is effectively valid only in the region where disk and outflow are coupled, i.e., the region from where essentially the outflow emanates from the accretion flow. Hence our study will remain confined within this predefined
region. Further, we neglect the contribution of the magnetic field as argued in Section 1.

We circumvent the idea of vertical integration of the flow equations. The validity and the reliability of the height integrated equations is normally gratifying in the geometrically thin limit. In those circumstances, the flow velocities are likely to be more or less independent of the disk scale height, which is not the case of the present paradigm of interest. We further consider that the disk to be non-self-gravitating, assuming that the mass of the disk to be much less than that of the black hole. The radial and vertical coordinates are expressed in units of \( GM/c^2 \), flow velocities in \( c \), time in \( GM/c^2 \), and the specific angular momentum in the unit of \( GM/c \). Here \( G, M, \) and \( c \) are gravitational constant, mass of the black hole, and speed of light, respectively. The steady state, axisymmetric disk-outflow are gravitational constant, mass of the black hole, and speed of light, respectively. The steady state, axisymmetric disk-outflow coupled equations in cylindrical geometry in the inviscid limit are then as follows.

(a) Mass transfer:

\[
\frac{1}{r} \frac{\partial}{\partial r}(r \rho v_r) + \frac{\partial}{\partial z}(\rho v_z) = 0, \tag{1}
\]

where the first term is the signature of accretion and the second term attributes to outflow. As the outflow starts from just above \( z = 0 \) surface, within the inflow region itself, we make a reasonable hypothesis that within the prescribed disk region the variation of the dynamical flow parameters with \( z \) is much less than that with \( r \), allowing us to choose \( \partial A/\partial z \approx O(A/\partial z) \), for any parameter \( A \). Strictly speaking, the weak variation with \( z \) ensures that the outflow originating from the mid-plane of the disk does not disrupt the disk structure, thus allowing a smooth accretion flow toward the black hole. Thus, Equation (1) then reduces to

\[
\frac{1}{r} \frac{\partial}{\partial r}(r \rho v_r) + \frac{\rho v_z}{z} = 0. \tag{2}
\]

(b) Radial momentum balance:

\[
v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{\lambda^2}{r^3} + F_{Gr} + \frac{1}{\rho} \frac{\partial P}{\partial r} = 0, \tag{3}
\]

where \( F_{Gr} \) is radial component of the gravitational force. As discussed earlier, here \( F_{Gr} \) is the radial force corresponding to the PNP in cylindrical coordinate system given by Ghosh & Mukhopadhyay (2007) containing the information of spin of the black hole. With the similar argument as above the equation then reduces to

\[
v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{\lambda^2}{r^3} + F_{Gr} + \frac{1}{\rho} \frac{\partial P}{\partial r} = 0, \tag{4}
\]

where \( v_{z0} \) is the radial velocity at the mid-plane of the disk.

(c) Azimuthal momentum balance:

\[
\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \rho v_r v_\phi) + \frac{\partial}{\partial z}(\rho v_\phi v_z) = 0, \tag{5}
\]

where \( v_\phi \) is the azimuthal velocity of the flow. The first term of this equation signifies the radial transport of the angular momentum, while the second term describes the extraction of angular momentum due to mass loss in the outflow. If we consider that the net angular momentum extracted by the outflow is \( \lambda_j \), and the remaining angular momentum retained by the disk is \( \lambda_d \), then total angular momentum \( \lambda = \lambda_j + \lambda_d \) can be assumed to remain constant throughout the flow within our predefined disk-outflow region by the virtue of an inviscid flow. Therefore,

\[
\lambda = \text{constant}. \tag{6}
\]

(d) Vertical momentum balance:

\[
v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} + F_{Grz} + \frac{1}{\rho} \frac{\partial P}{\partial z} = 0, \tag{7}
\]

where \( F_{Grz} \) is the vertical component of the gravitational force, described by Ghosh & Mukhopadhyay (2007). Following previous arguments, this reduces to

\[
v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} + F_{Grz} + \frac{1}{\rho} \frac{P - P_0}{z} = 0, \tag{8}
\]

where \( P_0 \) is the pressure of the flow at the mid-plane of the disk. If there is no outflow, then \( v_z = 0 \), and Equation (8) reduces to the well-known hydrostatic equilibrium condition in the disk, and the hydrostatic disk scale height can be obtained. Similarly, one can customarily extend the vertical momentum equation to compute the disk scale height when there is an outflow coupled with the disk. Thus we will use Equation (8) to obtain the scale height of the disk-outflow coupled system. We can reasonably assume that at height \( h \) (i.e., at \( z = h \)), the pressure of the disk is much less compared to that at the equatorial plane to prevent any disruption of the disk, permitting a steady structure of accretion flow. The variation of density along the \( z \)-direction is approximately kept constant (\( \rho_0 \sim \rho \)) which in turn means that the disk-outflow coupled region is weakly stratified. Considering the above facts, Equation (8) is simplified to obtain the scale height as

\[
v_r \frac{\partial v_z}{\partial r} \bigg|_{h} + v_z^2 \frac{1}{h} + F_{Grz} h - \frac{P_0}{h \rho} = 0, \tag{9}
\]

where \( h \) is thus the root of the above transcendental equation. Thus, Equations (2), (4), (6), and (9) will simultaneously have to be solved to obtain the dynamics and the energetics of the accretion-induced outflow.

3. SOLUTION PROCEDURE AND THE DISK-OUTFLOW SURFACE

We assume that the accretion flow follows an adiabatic equation of state \( P = K \rho^{\gamma+1/n} \) where \( n = 1/(\gamma - 1) \), and \( n \) and \( \gamma \) are the polytropic and the adiabatic indices, respectively. Note that constant \( K \) carries the information of entropy (e.g., Mukhopadhyay 2003) of the flow. Thus for an adiabatic flow, \( c_s = (\gamma P)^{1/2} \). In an earlier work (GM09), a reasonable assumption was made that \( h \sim r/2 \), which can be approximately accepted for a geometrically thick, 2.5 dimensional disk structure. Presuming that the outflow velocity is not likely to exceed the sound speed at the disk-outflow surface (the outer boundary of the accretion flow in the \( z \)-direction), we propose a simplified relation between \( c_z \) and \( v_z \) as

\[
v_z \lesssim 2 \left( \frac{z}{r} \right) c_z. \tag{10}
\]
Hence, at $z = h \sim r/2$, we obtain

$$v_z \lesssim c_s.$$  \hspace{1cm} (11)

This implies that the outflow can ideally come out off the disk surface which can appropriately be termed as the sonic surface in the vertical direction. With this notion in mind let us generalize this particular scaling between $v_z$ and $c_s$, pertaining to our formalism as

$$v_z = t \left( \frac{z}{r} \right)^\mu c_s,$$  \hspace{1cm} (12)

where $t$ and $\mu$ are the constants which will be determined by substituting Equation (12) in our model conservation equations described in Section 2. The index $\mu$ measures the degree of subsonic nature of the vertical flow within the disk-outflow coupled region (i.e., in between the mid-plane and the surface of the accretion flow).

Generalizing the procedure adopted in earlier works for 1.5 dimensional disks (Chakrabarti 1996; Mukhopadhyay & Ghosh 2003), we solve Equations (2), (4), (6), and (9). Using Equation (12) along with Equation (6) and combining Equations (2) and (4), we obtain

$$\frac{\partial v_r}{\partial r} = \frac{\frac{\lambda c_0 v_r}{r} - \frac{4}{3} \left( \frac{v}{v_{zc}} \right)^\mu c_0 v_r}{v_r - \frac{\lambda c_0}{r} + \left( \frac{v}{v_{zc}} \right)^\mu c_0 v_r} = \frac{N}{D}.  \hspace{1cm} (13)$$

Equation (13) shows that to guarantee a smooth solution at the “critical point,” $N = D = 0$. From the critical point condition we obtain $v_{rc} = c_{sc}$, at $r = r_c$. Here subscript $c$ is referred to as the critical point. The radius $r_c$ is also called the “sonic radius” since no disturbance created within this radius can cross the radius (also known as sound horizon) and escape to infinity. Conditions at critical/sonic radius give

$$v_{rc} = c_{sc} = -\frac{t}{2\varepsilon} \left( \frac{z}{r_c} \right)^\mu c_0 v_{sc}$$

$$+ \left[ \left( \frac{t}{2\varepsilon} \left( \frac{z}{r_c} \right)^\mu c_0 v_{sc} \right)^2 + r_c F_{Grc} - \frac{\lambda c_0}{r_c} \right]^{1/2},  \hspace{1cm} (14)$$

where $c_{00} = v_{00} = (r_c F_{Grc} - \frac{\lambda c_0}{r_c^2})^{1/2}$ is the sound speed or the radial velocity of the flow at the sonic radius in the disk mid-plane ($z = 0$). $F_{Grc}$ is the radial gravitational force at the sonic radius, and $F_{G0c}$ is the corresponding value in the disk mid-plane.

Now at sonic location $\frac{\partial v_r}{\partial r_c} = 0$. Hence by applying l’Hospital’s rule, Equation (13) reduces to

$$\frac{\partial v_r}{\partial r_c} = -\left[ \frac{2n}{c_{sc}} \left( \frac{B + \sqrt{B^2 - 4AC}}{2A} \right) + v_{zc} + \frac{t}{z} \left( \frac{z}{r_c} \right)^\mu c_0 \right].  \hspace{1cm} (15)$$

where $A = F_1(r, z)$, $B = F_2(r, z)$, and $C = F_3(r, z)$, are complicated functions of $r$ and $z$ at sonic location; $F_1$, $F_2$, and $F_3$ have been explicitly given in the Appendix. Equations (13) and (15) are then solved with an appropriate boundary condition to obtain $v_r$ and $c_s$ as functions of $r$. The value of specific angular momentum in our flow always remains constant and is same as $\lambda_c$, the value at sonic radius, by virtue of Equation (6).

Until now, we have emphasized velocity profiles of the accretion-induced outflow at any arbitrary $z$. We have, however, mentioned before that the intrinsic coupling of inflow and outflow is confined within a specified region, from where the outflow emanates. The disk-outflow inter-correlated region is bounded vertically from the mid-plane ($z = 0$) to an upper surface above which any inflow of matter ceased to exist. We aim at precisely investigating the nature and dynamics of the flow at different layers within this disk-outflow coupled region. Therefore, we first need to calculate the scale height of the accretion flow, based on our proposed model, and obtain the disk-outflow surface (upper boundary). It is to be noted that the transcendental Equation (9) cannot be independently used to calculate the scale height $h$ or its general variance with $r$, for unknown variables $v_z$ and $c_s$. Nevertheless, we can easily obtain the scale height $h$ at $r_c$, say $h_c$, from Equation (9) as radial velocity and sound speed at the sonic location are known. Thus at $r_c$, Equation (9) simplifies to

$$\left( \frac{h_c}{r_c} \right)^\mu c_{sc} h_c = \mu c_{sc} h_c - \mu c_{sc} h_c + \left( \frac{\partial c_s}{\partial h} \right)_{c_{sc}} h_c$$

$$+ F_{Gsc} h_c - \frac{c_{0c}}{r_c} = 0,  \hspace{1cm} (16)$$

where $\partial c_s/\partial h_{c_{sc}}$ is $\partial c_s/\partial h$ at the sonic location in the plane with scale height $h$. Then $h_c$ obtained from Equation (16) can be inserted into Equation (15) to obtain $\partial v_r/\partial r_c$ at scale height $h_c$. The disk scale height $h$ without any outflow is seen to be linearly increasing with $r$ (approximately by dimensional analysis). For a 2.5 dimensional disk-outflow system, it seems that $h$ and $r$ can also be linearly connected, based on the order of magnitude analysis (GM09). However, unlike the thin disk, strong gas pressure gradient in the disk-outflow coupled system leads to a geometrically thick 2.5 dimensional disk. Therefore, for the present purpose we make a generalized scaling of $h$ with $r$ as

$$h \sim \delta r_c  \hspace{1cm} (17)$$

where $\delta$ is a dimensionless arbitrary constant or any numerical variable (discussed in detail later). The impressionable choice about the factor $\delta$ would be that it should contain precisely the information of the nature of the flow. The only physical information we can extract related to the scale height from our model equations is $h_c$. Thus, we rationally demand an approximate expression for the dimensionless parameter $\delta$ as

$$\delta \sim \frac{h_c}{r_c},  \hspace{1cm} (18)$$

It is found that the value of $\delta$ circumvents around 1/2, for the entire range of spin parameter of the black hole, which here acts as a normalization constant. Thus eventually, Equations (13), (15), (17), and (18) are combined together to obtain $v_r$ and $c_s$ along the scale height $h$ for an accretion-induced outflow. It also appears that under all these circumstances, and for a physically acceptable flow, the constant $t$ and the index $\mu$ of Equation (12) connecting $v_z$ and $c_s$ yield to be $\sim 1$ and $\sim 3/2$, respectively.

### 3.1. Construction of Disk-outflow Surface

After establishing the scale height $h$, we in principle can obtain the profiles of the dynamical parameters at different layers of the disk, i.e., at $z = \ell h$, where $0 \leq \ell \leq 1$, within our prescribed disk-outflow region. We find that the radial velocity profile $v_r$ along all layers of disk for any arbitrary spin parameter
exhibits some unusual, yet very interesting behavior. For any specific $z$ and spin parameter $a$, $v_r$ attains a negative value at a radius greater than a certain distance $r = r_b$. The magnitude of $r_b$ decreases with the increase of $\ell$, which indicates that the positive trait of $v_r$ is greater for lower $z$. The negative value of $v_r$ does not necessarily reflect any unphysical behavior. If the outflow originates from any particular layer $\ell h$ at a radius $r \geq r_b$, it then appears that the coupling between the disk and the outflow ceased to exist. It infers that along the layer $\ell h$, both accretion and outflow simultaneously occur up to the radial distance $r_b$, and beyond which the solution (with negative $v_r$) reflects the truncation of the disk (along the specified layer). Let us consider different layers of $z$. As we ascend to layers from a lower to a higher $z$, the truncation of the disk-outflow region along these layers occurs at a radius smaller than that of the lower $z$. As accretion is the source of the outflow, we can ostensibly conclude that the region from where the outflow matter originates in the disk shrinks as we go to higher latitudes. Note that $r_b$ corresponds to zero $v_r$. Therefore, we restrict our study up to the boundary $r_b$. With this insight, we simply compute $r_b$ along various layers of the disk and construct a surface in the $r-z$ plane connecting all $r_b$ for different layers. The enclosed region bounded by this surface is defined by the positivity of $v_r$, where both the accretion and outflow simultaneously persist and are intrinsically coupled to each other. We attribute the outer surface of the region as disk-outflow surface ($h_{surf}$) and above this layer no inflow takes place. The arrows in the diagram shown in Figure 1 reveal the direction of flow. The arrow exactly at $r_b$ points vertically upward, which indicates that just at $r_b$ the flow pattern exclusively corresponds to vertical motion and any accretion flow ceased to exist. However, more realistic disk-outflow surface could be visualized with a thick-solid line shown in Figure 1.

As in our system we have neglected the viscosity and any radiative loss, and thus the flow is considered to be strongly advective; presumably, it is gas pressure dominated. This notwithstanding, various scattering processes will indeed produce radiation in the system, whose contribution could be insignificant. Hence, $\gamma \sim 3/2$ is reasonably an appropriate choice in our model. Note that this is in essence similar to the choice of the earlier authors who modeled gas dominated low mass ADAFs (e.g., Narayan & Yi 1994). We would also like to clarify that $\gamma = 5/3$ corresponds only to the case of zero angular momentum. Figure 2 shows the profiles of disk-outflow surface for various spin parameters $a$ of the black hole. All the input parameters corresponding to each value of $a$ are listed in Table 1. It is seen that the disk-outflow region and the peak of the surface, namely, $R_{j\nu}$, shifts to the vicinity of the black hole for a higher spin.

The marginally stable orbit in a Keplerian accretion flow defines the inner edge of the disk (with zero torque condition). However, the orbit is more or less an artifact of the flow. For a transonic advective disk (as is the present case), an a priori definition of the inner edge is somewhat fuzzy, due to which we can pretend to choose that the disk extends to the black hole horizon. In the presence of an outflow intrinsically coupled to the disk, there is supposed to be an inner boundary beyond which any outflow and then jet will ceased to exist. We define this inner boundary of the disk-outflow coupled region in explaining the energetics of the flow. Also, with the increase of the spin of the black hole, it is seen that the disk-outflow coupled region shrinks considerably, attaining a steeper nature. Literally speaking, the spin of the black hole directly influences the nature and region of the outflow. The tendency of the outflow region to get contracted to the inner radius suggests that the outflow is more likely to emanate exuberantly from inner region of the disk for rapidly spinning black holes which have more gravitating power. Therefore, with the increase in spin, the disk-outflow region becomes more dense and more susceptible to eject the matter with a greater power.

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1 The relationship between $\gamma$ and $\beta$, the ratio of gas pressure to the total pressure, is given by $\beta = \frac{2\sqrt{\gamma+1}}{\gamma+1}$ (GM09).
4. DYNAMICS AND NATURE OF THE FLOW

As the system is gas pressure dominated and strongly advective, it is more receptive to strong outflows. Figures 3 and 4 describe the variation of flow parameters as functions of the radial coordinate \(r\) along the disk-outflow surface for various spins of the black hole. With the increase of the spin of the black hole, \(v_r\) increases at the inner region of the disk. It is seen that along the disk-outflow surface, \(v_r\) becomes zero beyond a certain distance \(R_{\text{in}}\), which is also termed as zero \(v_r\) surface illustrated in Section 3.1, which is explained in detail in the next section in order to understand the outflow power. The zero \(v_r\) surface extends up to more inner region of the disk with the increase of spin of the black hole, indicating the fact that outflows and then jets originate from more inner region of the disk for rapidly spinning black holes. The increase of \(c_v\) with spin (Figure 4) indicates that the temperature of the disk-induced outflow is higher for rapidly spinning black holes. It is found from Figure 4 that the maximum temperature of the accretion-induced outflow varies from \(\sim 10^{11}\) to \(10^{12}\) \(K\) corresponding to zero to maximal spin of the black hole. The truncation of the curves in the inner region symbolizes the inner boundary of the disk-outflow region as mentioned in Section 3.1.

5. ENERGETICS OF THE FLOW

Accretion by a black hole is the primary source of energy of the mass outflow from the inner disk region. The energetics of the accretion-induced outflow are mainly attributed to the mass outflow rate and the power extracted by the outflow from the disk. The derivation of the mass outflow rate had been elaborated in G10. We follow the same procedure here, though including the spin information of the black hole. The mass outflow rate is given by

\[
\dot{M}_j(r) = -4\pi r \rho(h_{\text{surf}}) v_z(h_{\text{surf}}) dr + c_j, \quad (19)
\]

where the constant \(c_j\) is determined by an appropriate boundary condition. From the adiabatic conditions, \(\rho\) can be written in terms of \(c_j\), which is already determined as a function of \(r\). The corresponding proportionality constant is determined at a radius, outside which the contribution to the mass outflow rate is negligible \((v_z \sim 0)\). However, the total mass accretion rate \(\dot{M}\) (which is sum of the inflow rate and the outflow rate) can be obtained by integrating the continuity equation along the radial and vertical directions. Hence, the constant is computed easily by supplying the values of \(v_r\) and \(c_v\) at that radius in the expression of \(\dot{M}\). As we have discarded any possible small amount of the outflow at that radius, the actual value of the constant, and hence the outflow power, could be slightly over estimated.

\(\dot{M}_j(r)\) in Equation (19) refers to the rate at which the vertical mass flux ejects from the disk-outflow surface. Figure 5 shows the variation of \(\dot{M}_j\) profiles with the spin of the black hole. It is seen that with the increase of spin of the black hole, \(\dot{M}_j\) increases. All the profiles have been shown considering a supermassive black hole of mass \(\sim 10^8 M_\odot\) with a mass accretion rate at infinity, \(\dot{M} \sim 10^{-2} M_{\text{Edd}}\), where \(M_{\text{Edd}}\) is the Eddington mass accretion rate \(\sim 1.44 \times 10^{32} \text{ g} \text{s}^{-1}\). Low mass accretion rate (sub-Eddington accretion flow) is in conformity with our gas pressure dominated advective disk paradigm. It is found that \(\dot{M}_j\) increases with the increase of mass of the black hole and mass accretion rate as well. The truncation of the curves at an inner radius indicates the inner boundary of the disk-outflow coupled region, explained in detail in the next paragraph.

In computing the power extracted by the outflow from the disk, we follow the same procedure as in G10. Thus, the power of the outflow is given by

\[
P_j(r) = \int 4\pi r \left[ \frac{v_z^2}{2} + \frac{\gamma P}{\gamma - 1} + \phi_G \right] \rho v_z dr, \quad (20)
\]

which is the total power removed from the disk by the outflow along the disk-outflow surface. In measuring the net power of any astrophysical jet, it appears that the computed power \(P_j\) will then be the initial power of the jet. Figure 6 depicts the variation of the power \(P_j\) with \(r\) for various spin parameters of the black hole. If we meticulously investigate the nature of the power profiles, we observe that when the radial distance is less than a certain value, \(P_j\) begins to decrease. The decreasing trend of \(P_j\) infers that the characteristic flow is bounded (integrand of Equation (20) carrying the information of the vertical energy flux becomes negative). This occurs owing to the fact that in
the extreme inner region of disk, due to its strong gravitating power the starved black hole sucks all of the matter in its sphere of influence, even if there is any outflow emanating from the disk. We identify this inner transition radius as $R_{jt}$, beyond which no outflow occurs. We attribute $R_{jt}$ as the inner boundary of the disk-outflow region. In all of the previous profiles, the said truncation of the curves is ascribed to $R_{jt}$. We describe the dynamical variables in our study within the region between an outer boundary and $R_{jt}$, and within this prescribed region strong outflows are most plausible to originate, intrinsically coupled to the disk. The power profile shows that with the increase in $a$, $P_j$ increases, and $R_{jt}$ gradually shifts to the vicinity of the black hole, indicating the fact that outflow region moves further inward. We show the variation of total $P_j$ extracted from the disk-outflow region with the spin of the black hole in Figure 7(a). Considering a black hole of mass $\sim 10^8 M_\odot$, as seen in AGNs and quasars, accreting with $M \sim 10^{-2} M_{\text{Edd}}$, it is seen that $P_j \sim 10^{41}$ erg s$^{-1}$ for $a = 0$. For a maximally spinning black hole ($a = 0.998$) with same parameters, the computed $P_j \sim 10^{43}$ erg s$^{-1}$. Thus, there is an increase of 2 orders of
magnitude of \( P_j \) with the increase of spin of the black hole from 0 to 0.998. In an earlier work, Donea & Biermann (1996) showed that the power extracted by the outflow/jet from the disk increases with \( a \). However, they did not compute the power extracted from the disk explicitly. The numerical simulations by De Villiers et al. (2005), in a different accretion paradigm including magnetic field, also concluded that the jet efficiency is possible to increase with the increase of spin of the black hole from 0 to 0.998. Figure 7(b) shows the variation of \( R_{jt} \) with \( a \). In describing the disk-outflow surface in Section 3.1, we have articulated the impact of spin on the disk-outflow coupled region. The geometry of the surface for a particular \( a \) is identified with a parameter \( R_{js} \) (see Figure 2). We show exclusively the variation of \( R_{js} \) with the spin of the black hole in Figure 7(c), which too reveals that the fast rotating black hole retracts the outflow region toward it, a sign of a pure relativistic gravitation.

6. DISCUSSION

The Blandford–Znajek process (Blandford & Znajek 1977) is still one of the most promising mechanisms to drive powerful jets in AGNs and XRBs. Although the exact mechanism of formation of the jet in the vicinity of the black holes is still elusive, and whatever might be the reason for the origin of jet, the said work is significant mostly due to two underlying reasons: (1) extreme gravity is indispensable to effuse strong unbounded flows in the vertical direction from the inner region of the accretion disk, and (2) the spin of the black holes, which is purely a relativistic effect, is essential to power strong outflows and jets.

In the present study, we have neither laid importance to the nature of the outflow nor invoked any aspect for the origin of outflows or jets. Indeed, the distinctive or definitive understanding of the formation of strong outflows or jets is still unknown. This notwithstanding, the most distinguishable and obvious picture in this case is that outflows and jets observed in AGNs and XRBs can only originate in an accretion powered system, where the accretion of matter around relativistic gravitating objects like black holes acts as a source, and outflow and then jet takes the form of one of the possible sinks (the other sink is the central nucleus). The dynamics of the outflowing matter should then be intrinsically coupled to the accretion dynamics macroscopically through the fundamental laws of conservation (of matter, momentum, and energy). The outflow is unbounded and the total energy just at the base of the outflow should be positive. The accretion flow should be bounded in the vicinity of the central object as the central potential is attractive. However, the strength of the unbounded flows in the form of jets is distinctly proportional to the attractiveness of the central gravitational potential field. This paradox is well manifested in the observed universe, as relativistic jets are more populated around extreme gravitating objects like black holes. Also noticeably, length scale of jets increases from microquasars to quasars, which are supposedly harboring stellar mass and supermassive black holes, respectively.

Thus, in any theoretical modeling of the accretion and outflow, it can be presumably argued that the mathematical equations governing the dynamics of the inflow and outflow should inherently be correlated and evolved self-consistently without any ad hoc proposition, as accretion and outflow should not be treated as dissimilar objects. Second, the relativistic gravitational effect of the black hole should be incorporated, as the nature of gravity is the cornerstone to both the accretion and the unbounded outflow. To capture this essential physics, in our present study to understand the connection between disk and outflow, we have incorporated the general relativistic effect of the spinning black hole through a pseudo-Newtonian approach. Although pseudo-Newtonian formalism is an approximate method to mimic the space–time geometry of the Kerr black hole, it captures the important salient features of the corresponding metric, and thus can be used to examine the nature of outflows from the inner region of the disk.

In Section 2, we have described the general disk-outflow coupled hydrodynamic equations in the inviscid limit following GM09. The necessity to simplify our model equations for an inviscid flow is discussed in Section 2. Although GM09 explored the 2.5 dimensional accretion-induced outflow for a fully viscous system, they used a self-similar approach in order to solve the necessary partial coupled differential equations. In addition, they neglected the most indispensable effect of relativistic

![Figure 7](image-url)
gravitation or, precisely, the effect of spin of the black hole. 
In this paper, we have solved the disk-outflow model equations 
in a more general 2.5 dimensional paradigm, while trying to 
limit our assumptions to the least possible extent theoretically. 
One of the important premises we have made is the relationship 
between $v_\phi$ and $c_\phi$, which we have established empirically. We 
have not used the height integrated equations which are mostly 
valid in the circumstances where the dynamical fluid param-
eters are likely to be independent of $z$. Without presuming the 
fact that the outflow originates from the surface of the disk, as 
most of the authors do, we have logically constructed a disk-
outflow surface with properly defined boundary conditions. One 
of the most important computations we have done is to evaluate 
the mass outflow rate and the power of the outflow extracted 
from the disk self-consistently in the inviscid limit, unlike the 
previous works (e.g., Donea 1996; Blandford & Begelman 1999; 
Xie & Yuan 2008). We have found that the 
spin of the black hole plays a crucial role in determining the 
structure, dynamics, and the energetics of the outflow coupled 
to the disk. With the increase of the spin, the outflow region 
extends further inward and then the disk-outflow region shrinks 
and compresses (see Figure 2). As a result, the outflow and 
then any plausible jet is likely to eject out from an extreme 
inner region of the flow around a rapidly spinning black hole 
with a greater efficiency. Note that the higher spin results in the 
system to get more compressed with a greater outflow power. 
Therefore, the efficiency of outflow and jet is directly related to 
the disk scale height and hence the disk-outflow surface. Pre-
viously, in a different context, McKinney & Gammie (2004), 
while examining the electromagnetic luminosity of a Kerr black 
hole, assumed the ratio of the disk height to the radius ($h/r$) 
constant, irrespective of the black hole spin. However, as seen 
in the present work, the constant $h/r$ for different spins of the 
black hole may not be an obvious choice. 
The power extracted by the outflow from the disk not only 
depends directly on the mass of the black hole and the initial 
mass accretion rate of the flow, but also on the spin of the 
black hole. With our model, keeping the black hole mass and 
the accretion rate the same, the power of the outflow increases 
with the spin of the black hole and the computed power differs 
in 2 orders of magnitude between non-rotating and maximally 
rotating black holes. We have restricted our study vertically up 
to the region where the inflow and the outflow are least coupled, 
i.e., the disk-outflow surface. Above this, surface accretion 
ceased to exist and probably the outflow gets decoupled from 
the disk, accelerates, and eventually forms relativistic jet. The 
modeling of the astrophysical jet is altogether a different issue 
and is coupled to each other. Limited observational 
inputs put irremediable constraint on the boundary conditions as 
well as the fundamental scaling parameters, governing the 
coupled dynamics of the accretion and outflow. The inadequacy of 
an effective mathematical tool to handle partial coupled differ-
ential hydrodynamic equations for compressible flow motivates 
us to invoke approximations and assumptions. Despite this fact, 
one needs to explore the possibilities to examine the accretion-
induced outflow. These questions then arise: what are the valid 
asumptions and to what extent can they be considered? In the 
present work, we have addressed these questions in the follow-
ing way. 
1. As extreme gravity is the most important aspect to effuse 
jet from the disk, we have incorporated its effect through a 
pseudo-general-relativistic potential. 
2. The disk and outflow should not be treated as dissimilar 
objects, and hence their correlated dynamics should be es-
sentially governed by the conservation laws. The energetics 
of the accretion-induced outflow would then be evaluated 
self-consistently as is shown in our work. 
3. Any unbounded flow in the form of outflow is more 
plausible to emanate from a hot, puffed up region of the 
accretion flow (may be low/hard state of the black hole). 
Hence we have formulated our model in a 2.5 dimensional, 
strongly advective paradigm, surpassing the simplicity of 
height integration. 
4. We have approximated our system to an inviscid limit, the 
reason for which has been given in Section 2. Nevertheless, in 
in any future work, viscosity should be incorporated into 
the flow to make it more realistic. 
5. We do not account for the mechanism of the formation 
of strong jets, like those due to magnetic field or strong 
radiation pressure, as the definitive mechanism of the 
formation of jets is still unknown. Indeed, it is beyond the 
scope of this paper to solve MHD equations in the present 
scenario. However, assuming that the outflow resides, 
and is coupled to the disk, the governing conservation 
equations of matter, momentum, and energy should be 
treated accordingly. 
6. The power of the outflow and jet is expected to directly 
depend on the spin of the black hole. The spin, which is the 
signature of general relativity, can make an impact on the 
nature of the observed AGN classes, and thus the spin of the 
black hole has been incorporated in our study. As spin
of the black hole has a direct impact on the flow parameters, we obtain different outflow power for different spin.

What can be the most defined parameter for the accretion powered system—mass accretion rate, mass of the central star/black hole, or the spin of the central star/black hole? Our analysis has shown that the power extracted by the outflow from the disk proportionately increases with the spin of the black hole. It infers that if extreme gravity is essential to power the jet, then the strength and the length scale of the observed astrophysical jets directly depend on the spin of the black hole. If it is believed that the distant quasars harbor massive black holes of the same mass scale and accrete matter with similar rate, then perhaps spin can be the guiding parameter for the different observed AGN classes. We can end with the specific question: are the observed AGN classes astrophysical laboratories to measure the spin of the supermassive black holes?

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APPENDIX

Equation (15) consists of complicated terms, where \( A \), \( B \), and \( C \) are given by following equations:

\[
A = 4n + 2, \quad (A1)
\]

\[
B = \left( F_{Gr0c} - \frac{\lambda_c^2 c}{r_c^3} \right) \left( \frac{1}{2n} + 1 \right) \frac{v_{0c}}{r_c} \left( 1 - \frac{1}{2n} \right) - \frac{1}{z} \left( \frac{z}{r_c} \right)^{\mu} v_{0c} \left( 1 + \frac{1}{n} \right) + \frac{2v_{0c}}{r_c} + \frac{2}{z} \left( \frac{z}{r_c} \right)^{\mu} v_{0c}, \quad (A2)
\]

and

\[
C = \left( F_{Gr0c} - \frac{\lambda_c^2 c}{r_c^3} \right) \frac{v_{0c}}{2nr_c^2} + \frac{v_{0c}}{2nr_c^2} - \frac{1}{n} \left( \frac{z}{r_c} \right)^{\mu} \times v_{0c} - \frac{1}{n} \left( \frac{z}{r_c} \right)^{\mu} \frac{v_{0c}}{r_c} - \frac{1}{n} \left( \frac{z}{r_c} \right)^{\mu} \frac{v_{0c}}{r_c} \]

\[
+ \frac{v_{0c}}{2nr_c^2} + \frac{\mu}{2n} \left( \frac{z}{r_c} \right)^{\mu} \frac{v_{0c}}{r_c} + \frac{1}{2n} \left( \frac{z}{r_c} \right)^{\mu} \times v_{0c} \left( -B_0 + \sqrt{B_0^2 - 4A_0C_0} \right) + \frac{v_{0c}}{r_c}, \quad (A3)
\]

where

\[
A_0 = 2 + 4n, \quad (A4)
\]

\[
B_0 = \left( F_{Gr0c} - \frac{\lambda_c^2 c}{r_c^3} \right) \left( \frac{1}{2n} + 1 \right) \frac{v_{0c}}{r_c} \left( 1 - \frac{1}{2n} \right) + 2v_{0c}, \quad (A5)
\]

and

\[
C_0 = \left( F_{Gr0c} - \frac{\lambda_c^2 c}{r_c^3} \right) \frac{1}{2nr_c^2} + \frac{v_{0c}}{nr_c^2} + \frac{1}{2n} \left( \frac{\partial F_{Gr0c}}{\partial r} \right) + \frac{3\lambda_c^2 c}{r_c^3}. \quad (A6)
\]

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