We describe generalized $D = 11$ Poincaré and conformal supersymmetries. The corresponding generalization of twistor and supertwistor framework is outlined with $OSp(1|64)$ superspinors describing BPS preons. The $k$ BPS states as composed out of $n = 32 - k$ preons are introduced, and basic ideas concerning BPS preon dynamics is presented. The lecture is based on results obtained by J.A. de Azcarraga, I. Bandos, J.M. Izquierdo and the author.$^1$.

1. Introduction

$M$-theory has been proposed as a hypothetical quantum theory describing elementary level of matter, which should incorporate and possibly explain various properties of “new string theory” (for review see e.g.$^2$-$^4$). One of the features of such new theory of fundamental interactions should be the appearance of many extended elementary objects ($p$-(super)branes, $D$-(super)branes etc.) related with each other via duality/dimensional reductions net. Such a variety of basic objects in the theory makes sensible a search for some underlying composite structure.

The basic dynamical degrees of freedom in $M$-theory yet are not known - there were presented only some proposals usually related with $D = 11$ space-time geometry. We postulate that the composite structure of $M$-theory should be formulated in terms of new degrees of freedom related with new geometry. Because $M$-theory is supersymmetric, and supersymmetry reveals more elementary nature of spinorial objects, we shall postulate that the basic fundamental geometric structure in $M$-theory is spinorial.

The only well-known part of the description of $M$-theory is algebraic. Assuming that $M$-theory lives in $D = 11$ (this assumption is consistent with description of $D = 11$ SUGRA as the low energy limit of $M$-theory) we can postulate the following

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basic $D = 11$ $M$-superalgebra

\[ \{Q_r, Q_s\} = Z_{rs}(\Gamma_\mu C)_{rs}P^\mu + CT_{[\mu\nu]}C_{rs}Z^{[\mu\nu]} + (\Gamma_{[\mu_1-\mu_2]}C)_{rs}Z^{[\mu_1...\mu_5]} . \]

where $\mu, \nu = 0, 1, \ldots, 10$, $r, s = 1, \ldots, 32$. The collection of 528 Abelian generators $Z_{rs}$ ($Z_{rs} = Z_{sr}$) describes the generalized momenta in $M$-theory. Introducing dual generalized coordinate space

\[ X_{rs} = (\Gamma_\mu C)_{rs}X^\mu + (\Gamma_{[\mu\nu]}C)_{rs}X^{[\mu\nu]} + (\Gamma_{[\mu_5-\mu_2]}C)_{rs}X^{[\mu_5...\mu_2]} , \]

we obtain large generalized phase space, with coordinates and positions described by the adjoint representations of $Sp(32)$ algebra.

Let us recall the assumption of Penrose twistor formalism in $D = 4$ that basic spinorial degrees of freedom in twistorial theory of elementary particles are described by $N$ twistors ($i = 1 \ldots N$)

\[ t^{(i)} = (\lambda_A^{(i)}, \omega^{(i)\dot{A}}) , \]

where $\lambda_A^{(i)}$, $\omega^{(i)\dot{A}}$ ($A = 1, 2$) are the pairs of $D = 4$ Weyl spinors. The following formula for the composite fourmomentum is assumed

\[ P_{AB} = \sum_{i=1}^{N} \lambda_A^{(i)} \lambda_B^{(i)} , \]

where $P_{AB} = \frac{1}{2} \sigma_{AB}^\mu P_\mu$. We shall propose analogous formula in $D = 11$ for generalized momenta

\[ Z_{rs} = \sum_{i=1}^{N} \lambda_r^{(i)} \lambda_s^{(i)} , \]

where $\lambda_r$ ($r = 1 \ldots 32$) are $D = 11$ real Majorana spinors. In $D = 4$ the twistors are the fundamental representations of the spinorial covering $SU(2, 2)$ of $D = 4$ conformal algebra ($SU(2, 2) = SO(4, 2)$). In $D = 11$ there exists only minimal conformal spinorial algebra describing the classical real algebra $Sp(64)$, containing $D = 11$ conformal algebra

\[ SO(11, 2) \subset Sp(64; R) . \]

In Sect. 2 we shall consider the generalization of $D = 11$ Poincaré and conformal superalgebras, supersymmetrizing the minimal $D = 11$ conformal spinorial algebra. In Sect. 3 we shall introduce in $D = 11$ the generalization of twistor and supertwistor formalism, with the extensions of Penrose-Ferber relations, which relate $OSp(1|64)$ supertwistor space described by real coordinates ($\xi^2 = 0; R = 1 \ldots 65$)

\[ T_R = (\lambda_r, \omega^{\dot{r}}) , \]
with the generalized phase space \((X_{rs}, P_{rs})\) (see (1)). In Sect. 4 we shall describe algebraically \(BPS\) states by \(n = 32 - k\) superspinors \(\Psi\) representing \(D = 11\) generalized super-twistors. These super-twistorial constituents we shall call BPS preons. It appears that our model geometrically corresponds to new type of Kaluza-Klein theory, with discrete internal extension of space-time coordinates.

2. \(D = 11\) Conformal \(M\)-(Super)Algebra

Let us observe that the \(D = 4\) conformal algebra \((P_{\mu}, M_{\mu\nu}, D, K_{\mu})\) is endowed with the following three - grading structure

\[
P_{\mu} \quad M_{\mu\nu} \quad D \quad K_{\mu}.
\]

Grading (8) is determined by the scale dimensions of generators

\[
[D, P_{\mu}] = P_{\mu}, \quad [D, M_{\mu\nu}] = 0, \quad [D, K_{\mu}] = -K_{\mu}
\]

and it is easy to see that the conformal algebra (9) has two Poincaré subalgebras: \((P_{\mu}, M_{\mu\nu})\) and \((K_{\mu}, M_{\mu\nu})\). For \(D = 4\) superconformal algebra \(SU(2, 2; 1) = (P_{\mu\nu}, M_{\mu\nu}, D, A, K_{\mu}; Q_{A}, \bar{Q}_{\bar{A}}, S_{A}, S_{\bar{A}})\) the three-grading (9) is extended to the following five-grading

\[
P_{\mu} \quad Q_{A}, \bar{Q}_{\bar{A}} \quad M_{\mu\nu}, D, A \quad S_{A}, S_{\bar{A}} \quad K_{\mu}.
\]

where consistently

\[
[D, Q_{A}] = \frac{1}{2} Q_{A}, \quad [D, S_{A}] = -\frac{1}{2} S_{A},
\]

\[
[D, \bar{Q}_{\bar{A}}] = \frac{1}{2} \bar{Q}_{\bar{A}}, \quad [D, \bar{S}_{\bar{A}}] = \frac{1}{2} \bar{S}_{\bar{A}}.
\]

and again \(SU(2, 2; 1)\) contains as subsuperalgebras the Poincaré superalgebras \((P_{\mu}, M_{\mu\nu}, Q_{A}; \bar{Q}_{\bar{A}})\) and \((K_{\mu}, M_{\mu\nu}, S_{A}, S_{\bar{A}})\).

The structure of \(D = 11\) generalized superconformal algebra, which we call conformal \(M\)-superalgebra is quite analogous. The \(D = 11\) conformal \(M\)-algebra \(Sp(64)\) can be in analogy to (9) described by the following three-grading

\[
L_{1} \quad L_{0} \quad L_{-1}
\]

\[
Z_{rs} \quad R_{rs} \quad \bar{Z}_{rs}
\]

\[
528 \text{ Abelian} \quad GL(32; R) \quad 528 \text{ Abelian}
\]

We see that \(Sp(64)\) contains two copies of generalized \(D = 11\) Poincaré algebras, described by inhomogeneous \(Sp(32)\) algebras \((Sp(32; R) \subset GL(32; R))\) with 528 Abelian translation generators.
The superextension of $D = 11$ conformal $M$-algebra $OSp(1; 64)$ which we call conformal $M$-superalgebra is described by the following five-grading (see also \cite{15,16})

\[
L_1 \quad L_{1/2} \quad L_0 \quad L_{-1/2} \quad L_{-1} \\
Z_{rs} \quad Q_r \quad R_{rs} \quad S_r \quad \bar{Z}_{rs} ,
\]

(13)

where $(Q_r, S_r)$ are the pair of 32-component supercharges, transforming as fundamental representations of $Sp(32)$, with $R_{rs} \subset Sp(32)$ if $R_{rs} = R_{sr}$. The subalgebras spanned by the generators $(Q_r, Z_{rs})$ and $(S_r, \bar{Z}_{rs})$ describe two copies of $M$-superalgebra given by the relations \cite{[15]}. It should be added that the gradings (12,13) correspond to the grading structure of real Jordanian (super) algebras \cite{17,18}.

### 3. $D = 11$ Supertwistors and Their Relation with Generalized Superspace

Let us recall two basic relations of Penrose twistor theory in $D = 4$ $8 - 11$

(i) relation between the generators of Poincaré algebra and twistor components

\[
P_{A\dot{B}} = \lambda_A \lambda_B ,
\]

(14)

\[
M_{AB} = \lambda_{(A} \bar{\omega}_{B)} , \quad M_{\dot{A}\dot{B}} = \bar{\lambda}_{(\dot{A}} \omega_{\dot{B})}
\]

(15)

where $M_{AB} = \frac{1}{2} (\sigma_{\mu\nu})_{AB} M^{\mu\nu}$ and $M_{\dot{A}\dot{B}} = \frac{1}{2} (\tilde{\sigma}_{\mu\nu})_{\dot{A}\dot{B}} M^{\mu\nu}$! The relations (15) can be extended to all 15 generators of $D = 4$ conformal algebra.

(ii) Penrose incidence relation between twistor and space-time coordinates

\[
\omega_{\dot{A}} = i \lambda_B X^{B\dot{A}} \quad \bar{\omega}^A = -i X^{A\dot{B}} \bar{\lambda}_B
\]

(16)

where $X^{BA} = (X^{A\dot{B}})^*$ describe four real Minkowski coordinates if the $SU(2, 2)$ twistor norm vanishes

\[
(t, t) \equiv i \left( \lambda_A \bar{\omega}^A - \bar{\lambda}_A \omega^A \right) = 0 .
\]

(17)

The relations (14-17) can be supersymmetrized. If we introduce the $D = 4$ supertwistor $(t_\alpha, \eta)$, which is the fundamental representation of $SU(2, 2; 1)$ with complex Grassmann variable $\eta$ $(\eta^2 = \eta = 0$, $\eta \eta = 0$), the relations (14-17) has been extended by Ferber\cite{19} to all generators of $D = 4$ superconformal group $SU(2, 2; 1)$.

The Penrose relations (16-17), firstly supersymmetrized in\cite{19} look as follows

\[
\omega_{\dot{A}} = i \lambda_B Z^{B\dot{A}} \equiv i \lambda_B \left( X^{B\dot{A}} - i \theta^B \theta_{\dot{A}} \right)
\]

\[\text{We recall that } (\sigma_{\mu\nu})_{AB} = \frac{1}{2} [(\sigma_{\mu})_{AB} \tilde{\sigma}_{\nu B} - (\sigma_{\nu})_{AB} \tilde{\sigma}_{\mu B}] = -\frac{i}{4} \epsilon_{\mu\nu\rho\tau} (\sigma^{\rho\tau})_{AB} = [(\tilde{\sigma}_{\mu\nu})_{BA}]^*\]
\[ \omega^A = i \left( X^{AB} + i \theta^A \bar{\theta}^B \right) \lambda_B \]
\[ \eta = \lambda_A \theta^A \quad \bar{\eta} = \bar{\lambda}_A \bar{\theta}^A. \] (18)

For \( D = 11 \) the generalized twistors and supertwistors are real (see 7) and the real \( OSp(1; 64) \) superalgebra \( (R, S = 1 \ldots 64) \)
\[ \{ Q_R, Q_S \} = R_{RS}, \] (19)
can be obtained if we assume that
\[ R_{RS} = T_R T_S \quad Q_R = \frac{1}{\sqrt{2}} T_R \xi, \] (20)
where \( T_R \) describes \( D = 11 \) real twistorial quantum phase space \( (\eta_{RS} = -\eta_{SR} \) is the \( Sp(64) \) antisymmetric metric)
\[ [T_R, T_S] = i \eta_{RS}, \] (21)
supplemented with trivial one-dimensional Clifford algebra relation \( \xi^2 = 1. \)

The relations (19) are extended to \( D = 11 \) as follows:
\[ \omega^r = (X^{rs} - i \theta^r \theta^s) \lambda_s \quad \xi = \theta^r \lambda_r. \] (22)

Relations (22) relate the \( D = 11 \) supertwistor space coordinates (7) with the extended \( D = 11 \) superspace \( (X_{rs}, \theta_s) \), described by 528 bosonic and 32 fermionic coordinates.

**4. BPS States in M-Theory and Composites of BPS Preons**

The \( \frac{k}{32} \) BPS state \( |k\rangle \) can be defined as an eigenstate of generalized momenta generators
\[ Z_{rs} |k\rangle = z_{rs} |k\rangle, \] (23)
such that \( \text{det} z_{rs} = 0 \). The number \( k \) determines the rank of generalized momenta matrix \( z_{rs} \)
\[ \frac{k}{32} \text{BPS state: } \{ \text{rank } z_{rs} = n = 32 - k; \quad 1 \leq k < 32 \}. \] (24)

From (24) follows that the BPS state \( |k\rangle \) preserves a fraction \( \nu = \frac{k}{32} \) of supersymmetries.

We call BPS preon the hypothetical primary object carrying the following generalized momenta
\[ Z_{rs} = \lambda_r \lambda_s. \] (25)

\(^3\)By Bott periodicity this realization is related with twistor framework in \( D = 3 \) (see\(^2\)), also with real structure. In \( D = 5, 6, 7 \) one has to use the extension of Penrose framework to quaternionic twistors (see e.g.\(^2\) for \( D = 6 \)).
The formula (25) corresponds to putting \( n = 1 \) in the relation (3) and describes \( \frac{k}{32} \) BPS state. More general formula (3) describes the generalized momenta of a system composed out of \( n \) BPS preons and it describes (for \( 1 \leq n \leq 32 \)) the \( \frac{k}{32} \) BPS state (we recall that \( k = 32 - n \)).

The number \( n = 32 - k \) of zero eigenvalues of the matrix \( z_{rs} \) determines the number of independent supercharges \( Q_r^{(i)} \), anihilating the BPS state \( |k\rangle \). These supersymmetries, preserving the BPS state, are called in \( p \)-brane theory the \( \kappa \)-transformations. We see that the supersymmetric \( D = 11 \) single BPS preon dynamics should have 31 \( \kappa \)-symmetries. Recently\(^2\) such dynamical superparticle models\(^2\) with fundamental \( OSp(1; 2n) \) superspinor as basic variable has been proposed. It should be recalled here (see e.g.\(^2\)) that in the standard super \( p \)-brane formulations half of the supersymmetries are promoted to \( \kappa \)-transformations, i.e. in \( D = 11 \) we obtain 16 \( \kappa \)-transformations.

Using the \( D = 11 \) supertwistor description with the relations (22) and (25) providing a bridge between BPS preons and generalized space-time, we can formulate three different geometric pictures:

(i) Purely supertwistorial picture, with basic phase space parametrized by BPS preon coordinates \( T^{(i)}_R \) (see (7)). The canonical Liouville one-form describing free action is given by the relation

\[
\Omega_1 = \sum_{i=1}^{n} \left( \omega^{(i)r} d\lambda^{(i)}_r + i\xi^{(i)} d\xi^{(i)} \right),
\]

which can be supplemented by some algebraic constraints.

(ii) Mixed geometric picture, with the components \( \omega^{(i)} \) expressed by means of the relation (22). One obtains from (25)

\[
\Omega_2 = \sum_{i=1}^{n} \lambda^{(i)r}_s \lambda^{(i)r}_s \left( dX^{rs} - i\theta^r d\theta^s \right),
\]

(iii) Generalized space-time picture, with the relation (3) inserted in (27).

\[
\Omega_3 = Z_{rs} \left( dX^{rs} - i\theta^r d\theta^s \right).
\]

The application of these three geometric pictures to the description of \( D = 11 \) dynamics (for \( n > 1 \)) is under consideration.

5. Final Remarks

We mention here two interesting aspects of the presented approach which deserve further attention;

\(^\star\)For \( D = 4, 6 \) and 10 see\(^2,3\).
(i) geometric confinement of BPS preons

Because the space-time coordinates are composed out of preonic degrees of freedom, the $D=11$ space-time point can be determined only in terms of at least 16 preonic set of spinorial coordinates. This is the $D=11$ extension of known property of Penrose theory in four dimensions with two twistors needed for the definition of composite Minkowski space-time points.

(ii) internal symmetries

The formula (3) expresses 528 generalized momenta in terms of $32n$ preonic spinorial coordinates $\lambda^{(i)}_r$ ($i=1, \ldots n$). The internal symmetries can be obtained by interchanging BPS preons. For the case $n=16$ corresponding to the choice of $\nu=\frac{1}{2}$ SUSY one can introduce internal $O(16)$ symmetries, leaving the values of $Z_{rs}$ invariant.

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