Generally Covariant Conservative Energy-Momentum for Gravitational Anyons *

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Abstract

We obtain a generally covariant conservation law of energy momentum for gravitational anyons by the general displacement transform. The energy-momentum currents have also superpotentials and are therefore identically conserved. It is shown that for Deser’s solution and Clément’s solution, the energy vanishes. The reasonableness of the definition of energy-momentum may be confirmed by the solution for pure Einstein gravity which is a limit of vanishing Chern-Simons coupling of gravitational anyons.

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I. Introduction

Gravity in 1+2 dimensional spacetime has been a popular subject of discussion from a decade ago\cite{1}-\cite{4}. Though some of the models are toy models, the studies may shed light on the understanding of not only quasi-(1+2) dimensional physics (e.g. QHE and high $T_c$), but also the realistic 1+3 dimensional gravity. In any gauge theory in odd dimensions, there exists a special term, i.e. the Chern-Simons term, which can be incorporated into the model Lagrangian. The concept of gravitational anyons is a simple non-Abelian generalization of the U(1) Chern-Simons theory to non-compact gauge group\cite{1}\cite{2}. Using his solution to the linearized field equations, Deser studied the mass and spin of the gravitational anyons\cite{4}. The conclusion states that the gravitational and inertial quantities are not equal to each other in general and thus the equivalence principle is violated. Hence, there exist much difference between gravitational anyons and 1+3 Einstein gravity.

It seems to us that in order to understand the difference, we should have a well-defined definition of gravitational conservative quantities, or in other words, we should have generally covariant conservation laws. In our previous work\cite{7}, we have obtained a generally covariant conservation law of angular-momentum for gravitational anyons. As suggested in\cite{7}, the present paper is to study the generally covariant conservation law of energy-momentum in the approach proposed in\cite{8}. The paper is arranged as follows. In section II, we give a general description of the scheme for establishing generally covariant conservation laws in general relativity. In section III, we use the general displacement transform and the scheme to obtain a generally covariant conservation law of energy-momentum. In this section, we will use a first order Lagrangian instead of the original one which is in second order. In section IV, we calculate the total energy-momentum for Deser’s, Cleément’s solutions, and a solution in the vanishing Chern-Simons coupling limit. The last section is devoted to some remarks and discussions.
II. General Scheme for Conservation Laws in General Relativity

As in 1+3 Einstein gravity, conservation laws are also the consequence of the invariance of the action corresponding to some transforms. In order to study the covariant energy-momentum of more complicated systems, it is benifical to discuss conservation laws by Noether theorem in general. Suppose that the spacetime is of dimension $D = 1 + d$ and the Lagrangian is in the first order formalism, i.e.

$$I = \int_G \mathcal{L}(\phi^A, \partial_\mu \phi^A) d^D x$$

where $\phi^A$ denotes the generic fields. If the action is invariant under the infinitesimal transforms

$$x'\mu = x\mu + \delta x\mu \quad \phi'\mu(x') = \phi^\mu(x) + \delta\phi^\mu(x)$$

(it is not required that $\delta\phi^\mu |_{\partial G} = 0$), then following relation holds[8]-[10].

$$\partial_\mu (\mathcal{L} \delta x^\mu + \frac{\partial \mathcal{L}}{\partial \phi^A} \delta^0 \phi^A) + [\mathcal{L}]_{\phi^A} \delta^0 \phi^A = 0$$

where

$$[\mathcal{L}]_{\phi^A} = \frac{\partial \mathcal{L}}{\partial \phi^A} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^A}$$

and $\delta^0 \phi^A$ is the Lie variation of $\phi^A$

$$\delta^0 \phi^A = \phi'^A(x) - \phi^A(x) = \delta\phi^A(x) - \partial_\mu \phi^A \delta x^\mu$$

If $\mathcal{L}$ is the total Lagrangian of the system, the field equations of $\phi^A$ is just $[\mathcal{L}]_{\phi^A} = 0$. Hence from eq.(3), we can obtain the conservation equation corresponding to transform eq.(2)

$$\partial_\mu (\mathcal{L} \delta x^\mu + \frac{\partial \mathcal{L}}{\partial \phi^A} \delta^0 \phi^A) = 0$$

It is important to recognize that if $\mathcal{L}$ is not the total Lagrangian, e.g. the gravitational part $\mathcal{L}_g$, then so long as the action of $\mathcal{L}_g$ remains invariant under transform eq.(2), eq.(3) is still valid yet eq.(6) is no longer admissible because of $[\mathcal{L}_g]_{\phi^A} \neq 0$.

Suppose that $\phi^A$ denotes the Riemann tensors $\phi^A_\mu$ and Riemann scalars $\psi^A$ (for gravitational
anyons, they are dreibein $e^a_\mu$, SO(1,2) connection $\omega^a_\mu$, the Lagrangian multiplier $\lambda^a_\mu$ and the matter field $\psi^A$. Eq.(3) reads (suppose that $L_g$ does not contain $\psi^A$)

$$\partial_\mu(L_g \delta x^\mu + \frac{\partial L_g}{\partial \phi^A_\nu} \delta_0 \phi^A_\nu) + [L_g]_{\phi^A_\mu} \delta_0 \phi^A_\mu = 0$$

(7)

Under transforms eq.(2), the Lie variations are

$$\delta_0 \phi^A_\nu = -\delta x^\alpha_\mu \phi^A_\alpha - \phi^A_{\nu,\alpha} \delta x^\alpha$$

(8)

where the dot “,” denotes partial derivative. So eq.(7) reads

$$\partial_\mu[L_g \delta x^\mu - \frac{\partial L_g}{\partial \phi^A_\lambda} (\phi^A_{\lambda,\nu} + \phi^A_{\nu,\lambda} \delta x^\nu)] - [L_g]_{\phi^A_\mu} (\phi^A_{\lambda,\nu} + \phi^A_{\nu,\lambda} \delta x^\nu) = 0$$

(9)

Comparing the coefficients of $\delta x^\nu$, $\delta x^\mu$, and $\delta x^\nu$, we may obtain an identity

$$\partial_\lambda ([L_g]_{\phi^A_\mu} \phi^A_\nu) = [L_g]_{\phi^A_\mu} \phi^A_{\lambda,\nu}$$

(10)

Then eq.(9) can be written as

$$\partial_\mu[L_g \delta x^\mu - \frac{\partial L_g}{\partial \phi^A_\lambda} (\phi^A_{\lambda,\nu} + \phi^A_{\nu,\lambda} \delta x^\nu)] - \partial_\mu [L_g]_{\phi^A_\mu} \phi^A_{\lambda,\nu} = 0$$

(11)

or

$$\partial_\mu ([L_g \delta x^\mu - \frac{\partial L_g}{\partial \phi^A_\lambda} \phi^A_{\nu,\lambda} - [L_g]_{\phi^A_\mu} \phi^A_{\nu}) \delta x^\nu - \partial_\mu [L_g]_{\phi^A_\mu} \phi^A_{\lambda,\nu} = 0$$

(12)

By definition, we introduce

$$\tilde{I}^\mu_\nu = -(L_g \delta x^\mu - \frac{\partial L_g}{\partial \phi^A_\lambda} \phi^A_{\nu,\lambda} - [L_g]_{\phi^A_\mu} \phi^A_{\nu})$$

(13)

$$\tilde{Z}^\lambda_\nu = \frac{\partial L_g}{\partial \phi^A_{\lambda,\mu}} \phi^A_\nu$$

(14)

Then eq.(12) gives

$$\partial_\mu(\tilde{I}^\mu_\nu \delta x^\nu + \tilde{Z}^\lambda_\nu \delta x^\nu) = 0$$

(15)

So by comparing the coefficients of $\delta x^\nu$, $\delta x^\nu$, and $\delta x^\nu$, we have the following from eq.(15)

$$\partial_\mu \tilde{I}^\mu_\nu = 0$$

(16)

$$\tilde{I}^\lambda_\nu = -\partial_\mu \tilde{Z}^\lambda_\mu$$

$$\tilde{Z}^\mu_\lambda = -\tilde{Z}^\lambda_\nu$$

(17)

Eq.(16)-(17) are fundamental to the establishing of conservation law of energy-momentum.
III. Conservation Law of Energy-momentum for Gravitational Anyons

3.1. General Displacement Transform

In 1+3 Einstein gravity, a generally covariant conservation law of energy-momentum was obtained by means of what we usually call the *general displacement transform*

\[ \delta x^\mu = e_a^\mu \epsilon^a \quad (\epsilon^a = \text{const.}) \]  

This really represents an infinitesimal displacement while

\[ \delta x^\mu = b^\mu = \text{const.} \]

does not because \( x^\mu \) can be any coordinates. For instance, it can be spherical coordinates, in which case, the resulting conservation law, if exists, should be that of angular-momentum other than energy-momentum. Using the invariance of the action with respect to eq. (18) and Einstein equations, the following general covariant conservation law of energy-momentum is obtained

\[ \nabla_\mu (T^\mu_a + t^\mu_a) = 0 \]  

and it was shown that there exist superpotentials \( V^\mu_\nu_a \)

\[ T^\mu_a + t^\mu_a = \nabla_\nu V^\mu_\nu_a \]

\[ V^\mu_\nu_a = \frac{c^4}{8\pi G} \left[ e_\nu^a \epsilon_c \epsilon_a \omega_b^c + (e_\nu^a \epsilon_b^c - e_b^c \epsilon_\nu a) \omega^b \right] \]

This definition of energy-momentum has the following main properties:

1. It is a covariant definition with respect to general coordinate transforms. But the energy-momentum tensor is not covariant under local Lorentz transforms, this is reasonable because of the equivalence principle.

2. For closed system, the total energy-momentum does not depend on the choice of Riemann coordinates and transforms in the covariant way

\[ P'_a = L_a^b P_b \]
under local Lorentz transform $\Lambda^a_{\ b}$ which is constant $L^a_{\ b}$ at spatial infinity.

3). For a closed system with static mass center, the total energy-momentum is $P_a = (Mc, 0, 0, 0)$.

4). For a rather concentrated matter system, the gravitational energy radiation is

$$-\frac{\partial E}{\partial t} = \frac{G}{45c^5}(\dot{D})^2$$  \hspace{1cm} (24)

5). For Bondi’s plane wave, the energy current is

$$t^\mu_0 = (\frac{1}{4\pi}\beta^2, \frac{1}{4\pi}\beta^2, 0, 0)$$  \hspace{1cm} (25)

6). For the solution of gravitational solitons, we can obtain finite energy while the Landau-Lifshitz definition leads to infinite energy [12].

7). In Ashtekar’s complex formalism of general relativity, the energy-momentum and angular-momentum constitute a 3-Poincare algebra and the energy coincides with the ADM energy [10].

With these foundations, we next use 1+2 dimensional transform eq. (18) to obtain conservative energy-momentum for gravitational anyons.

3.2 The Energy-momentum for Gravitational Anyons

We take the Lagrangian for gravitational anyons to be in the first order ($\omega^a_\mu = \frac{1}{2} e^{abc} \omega^b_{\mu bc}, \omega^a_{\mu bc}$ is the SO(1,2) connection.)

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m$$  \hspace{1cm} (26)

where

$$\mathcal{L}_g = -\frac{1}{\kappa} \epsilon^{\mu\nu\alpha} \omega^\alpha_\mu \partial_\nu e_{\alpha a} + \frac{1}{2\kappa} \epsilon^{\mu\nu\alpha} e_{\alpha a} e^{abc} \omega^b_\mu \omega^c_{\nu c} + \mathcal{L}_{c-s} + \epsilon^{\mu\nu\alpha} \lambda^\alpha_\mu (\partial_\nu e_{\alpha a} + e_{abc} \omega^b_\nu e^c_\alpha)$$  \hspace{1cm} (27)

$$\mathcal{L}_{c-s} = \frac{1}{2\kappa\mu} \epsilon^{\mu\nu\alpha} (\omega^\mu_\alpha \partial_\nu e^\alpha_\mu + \frac{1}{3} \epsilon_{abc} \omega^a_\mu \omega^b_\nu e^c_\alpha)$$  \hspace{1cm} (28)

and $\mathcal{L}_m$ denotes the matter part. The field equations for $e^a_\mu, \omega^a_\mu$ and $\lambda^a_\mu$ are $[\mathcal{L}]_{e^a_\mu} = 0$, $[\mathcal{L}]_{\omega^a_\mu} = 0$ and $[\mathcal{L}]_{\lambda^a_\mu} = 0$, i.e.

$$\frac{1}{2\kappa} \epsilon^{\mu\nu\alpha} e^{abc} \omega^b_\mu \omega^c_{\nu c} + \epsilon^{\mu\nu\alpha} \lambda^b_\mu e_{abc} e^c_\alpha + \frac{1}{\kappa} \epsilon^{\mu\nu\alpha} \partial_\mu \omega^a_\nu + \epsilon^{\mu\nu\alpha} \partial_\nu \lambda^a_\mu = -[\mathcal{L}_m]_{e^a_\mu}$$  \hspace{1cm} (29)

$$\epsilon^{\mu\nu\alpha} (\partial_\nu e_{\alpha a} + e_{abc} \omega^b_\nu e^c_\alpha) = 0$$  \hspace{1cm} (30)
Energy-momentum, anyons

\[
\frac{1}{\kappa} \epsilon^\nu{}^\mu{}^\alpha (\partial_\mu e^a_\alpha + \epsilon^a_\beta \omega^b_\mu e^c_\alpha) + \frac{1}{\kappa \mu} \epsilon^\nu{}^\mu{}^\alpha (\partial_\mu \omega^a_\alpha) \\
+ \frac{1}{2} \epsilon^{abc} \omega^a_\mu \omega^b_\alpha e^c_\mu + \epsilon^\nu{}^\mu{}^\alpha \lambda_{b\mu} e^{abc} e_{ac} = -[L_m]_{\omega^a_\mu}^a
\]\n
(31)

Using eq.(30), eq.(31) can be rewritten as

\[
\frac{1}{\kappa \mu} \epsilon^\nu{}^\mu{}^\alpha (\partial_\mu \omega^a_\alpha + \frac{1}{2} \epsilon^{abc} \omega^a_\mu \omega^b_\alpha e^c_\mu) + \epsilon^\nu{}^\mu{}^\alpha \lambda_{b\mu} e^{abc} e_{ac} = -[L_m]_{\omega^a_\mu}^a
\]\n
(32)

These equations are the same as those given in [13].

From eq.(14)

\[
\tilde{Z}^\mu_\nu = \frac{\partial L_\phi}{\partial e^a_{\lambda,\mu}} e^a_\nu + \frac{\partial L_\phi}{\partial \omega^a_{\lambda,\mu}} \omega^a_\nu + \frac{\partial L_\phi}{\partial \lambda^a_{\lambda,\mu}} \lambda^a_\nu \\
= -\frac{1}{\kappa} \epsilon^{\mu\alpha\lambda} \omega^a_{a\alpha} e^a_\nu + \frac{1}{2\kappa \mu} \epsilon^{\mu\alpha\lambda} \omega^a_{a\alpha} \omega^a_\nu + \epsilon^{\mu\alpha\lambda} \lambda^a_{a\alpha} e^a_\nu
\]\n
(33)

For transform eq.(18), eq.(15) implies

\[
\partial_\mu (\tilde{I}^\mu_\nu e^a_\nu + \tilde{Z}^\mu_\nu e^a_{a,\lambda}) = 0
\]\n
(34)

Define

\[
\tilde{I}^\mu_a = -\partial_\lambda \tilde{Z}^\mu_a, \quad \tilde{Z}^\mu_a = \tilde{Z}^\mu_\nu e^a_\nu
\]\n
(35)

we then have

\[
\partial_\mu \tilde{I}^\mu_a = 0
\]\n
(36)

Since \([L_\phi]_{e^a_\mu} = -[L_m]_{e^a_\mu}^a\), and \(T^\mu_a = -\frac{1}{e}[L_m]_{e^a_\mu}^a\), we have from eq.(13)

\[
\tilde{I}^\nu_\nu = -(L_\phi \delta^\nu_\nu - \frac{\partial L_\phi}{\partial e^a_{\lambda,\mu}} e^a_{\lambda,\nu} - \frac{\partial L_\phi}{\partial \omega^a_{\lambda,\mu}} \omega^a_{\lambda,\nu} - [L_\phi]_{\omega^a_{\mu}} \omega^a_\nu - [L_\phi]_{\lambda^a_{\mu} \lambda^a_\nu} + e T^\mu_a e^a_\nu
\]\n
(37)

Define \(t^\mu_a\) by

\[
\frac{\tilde{I}^\mu_a e^a_\nu + \tilde{Z}^\mu_a e^a_{a,\lambda}}{e(T^\mu_a + t^\mu_a)} = e(T^\mu_a + t^\mu_a)
\]\n
(38)

Then we have

\[
e(T^\mu_a + t^\mu_a) = e \nabla_\lambda \tilde{Z}^\mu_a
\]\n
(39)

where \(\tilde{Z}^\mu_a = e Z^\mu_a\). Eq.(39) is the desired general covariant conservation law of energy-momentum for gravitational anyons. The total energy-momentum is

\[
P_a = \int e(T^\mu_0 + t^\mu_0) d^2 x = \int \partial_\mu \tilde{Z}^\mu_0 d^2 x
\]\n
(40)
3.3 The iso(1,2) Algebra

The pure Einstein case is restored by setting $\mu \to \infty$ and $\mathcal{L}_m = 0$. In this limit, we have $\lambda^a_\mu = 0$ and the superpotential is simply

$$\tilde{Z}^{\mu \nu}_a = -\frac{1}{\kappa} \epsilon^{\mu \nu \alpha} \omega_{\alpha a}$$

(41)

and the total energy-momentum is

$$P_a = \frac{1}{\kappa} \oint_{\partial \Sigma} \omega_a$$

(42)

where $\partial \Sigma$ is the spatial infinity which is 1 dimensional. From the angular-momentum

$$J_a = -\frac{1}{\kappa} \oint_{\partial \Sigma} e_a = \frac{1}{\kappa} \int e^{ij} \epsilon_{abc} \omega^b_i \epsilon^c_j d^2x$$

(43)

and the Poisson brackets given in [7]

$$\{\omega^a_i(x), e^b_j(y)\} = \kappa \epsilon_{ij} \eta^{ab} \delta^2(x - y)$$

$$\{\omega^a_i(x), \omega^b_j(y)\} = \{e^a_i(x), e^b_j(y)\} = 0$$

(44)

we have

$$\{J_a, J_b\} = -\frac{1}{\kappa^2} \left\{ \oint e_a(x), \int e^{ij} \epsilon_{bcd} \omega^c_i \epsilon^d_j d^2y \right\} = \epsilon_{abc} J^c$$

(45)

$$\{P_a, P_b\} = 0$$

(46)

$$\{J_a, P_b\} = \left\{ \frac{1}{\kappa} \int e^{ij} \epsilon_{acd} \omega^c_i \epsilon^d_j d^2x, \frac{1}{\kappa} \oint \omega_b(y) \right\} = \epsilon_{abc} P^c$$

(47)

Thus the iso(1,2) algebra can be restored.

IV. Examples

We now consider the special case that $[\mathcal{L}_m]_{\omega^a_\mu} = 0$. Using the identities in 3-dim Riemann geometry

$$R_{\alpha \beta \gamma \delta} = g_{\alpha \gamma} \bar{R}_{\beta \delta} + g_{\beta \delta} \bar{R}_{\alpha \gamma} - g_{\alpha \delta} \bar{R}_{\beta \gamma} - g_{\beta \gamma} \bar{R}_{\alpha \delta}$$

$$\bar{R}_{\mu \nu} = R_{\mu \nu} - \frac{1}{4} g_{\mu \nu} R$$

$$R^\alpha_{\gamma \delta} = -\epsilon^{\alpha \beta \mu} \epsilon_{\gamma \delta \nu} G^\nu_{\mu}$$

$$G^{\mu \nu} = R^{\mu \nu} - \frac{1}{2} g^{\mu \nu} R$$
we have
\[-\frac{1}{\kappa \mu} \epsilon^{\mu \alpha \beta} (\partial_\alpha \omega_{\beta a} + \frac{1}{2} \epsilon_{abc} \omega^b_\alpha \omega^c_\beta) = \frac{1}{\kappa \mu} e G^\mu_a \]

(48)

So
\[\lambda^a_\mu = -\frac{1}{\kappa \mu} \tilde{R}^a_\mu \]

(49)

Substitute into eq.(29), we have
\[G^\mu_a + \frac{1}{\mu} C^\mu_a = -\kappa T^\mu_a \]

(50)

where $C^\mu_a$ is the Cotton tensor. For Deser’s solution [4].

\[e_0^0 \simeq 1, \quad e_1^1 = e_2^2 \simeq \sqrt{\frac{m \kappa^2}{\pi}} \ln \frac{r}{2} r \]

\[e_0^0 \simeq -\frac{\kappa}{\mu} \left( \frac{m + \mu \sigma}{2 \pi} \right) \frac{x}{r^2}, \quad e_1^0 \simeq \frac{\kappa^2 (m + \mu \sigma)}{2 \pi} \frac{y}{r^2} \]

\[e_2^1 \simeq -\frac{\kappa^4 (m + \mu \sigma)^2}{4 \pi^2 \mu^2} \frac{xy}{r^4 \ln^{1/2} r} \]

we can obtain the asymptotical behaviour of the spin connection
\[\omega^{\mu ab} \simeq \frac{1}{rf(r)} \]

(52)

where $f(r)$ represents some monotonically increasing functions of $r$. Thus we have
\[\oint_{\partial \Sigma} \omega^{\mu ab} dx^\mu = 0 \]

(53)

Thus the total energy-momentum vanishes. For Clément’s [14] self-dual exact solution
\[ds^2 = A^{-1} [dt - (\omega_0 + A) d\theta]^2 - dr^2 - Ad\theta^2 \]

(54)

\[A = a + ce^{-\mu r} \]

where $a, c$ are constants and $\mu$ should be positive since $r \in (0, +\infty)$. In terms of rectangular coordinates, it takes the following form
\[ds^2 = A^{-1} dt^2 - 2(\omega_0 A^{-1} + 1)(\frac{x^2}{r^2} dt dy - \frac{y^2}{r^2} dt dx) \]
\[+ \{[A^{-1}(\omega_0 + A)^2 - A] \frac{x^2}{r^4} - 1\} dy^2 + \{[A^{-1}(\omega_0 + A)^2 - A] \frac{y^2}{r^4} - 1\} dx^2 \]
\[ - [A^{-1}(\omega_0 + A)^2 - A] \frac{2xy}{r^4} dydx \]  

(55)

We obtain the following asymptotical dreibein

\[
\begin{align*}
    e_0^0 &= \frac{1}{\sqrt{a}}, & e_1^0 &= \sqrt{a}(1 + \frac{\omega_0}{a}) \frac{y}{r^2} \\
    e_2^0 &= -\sqrt{a}(1 + \frac{\omega_0}{a}) \frac{x}{r^2}, & e_1^1 &= 1 + \frac{ay^2}{2r^4} \\
    e_2^1 &= \frac{axy}{r^4}, & e_2^2 &= 1 + \frac{ax^2}{2r^4}
\end{align*}
\]

(56)

Hence it can be shown that

\[
\lim_{r \to \infty} r\omega_{\mu ab} = 0
\]

Thus

\[
\int_{\partial \Sigma} \omega_{\mu ab} dx^\mu = 0
\]

So the total energy vanishes also. In the limit, \( \mu \to \infty \), eq.(50) has a solution with dreibein and spin connection

\[
\begin{align*}
    e^0 &= dt + \frac{\kappa J}{2\pi r^2} r \times dr & e = (1 - \frac{\kappa m}{2\pi}) dr + \frac{\kappa m}{2\pi r^2} (r \cdot dr) \\
    \omega^0 &= \frac{\kappa m}{2\pi r^2} r \times dr & \omega^i = 0
\end{align*}
\]

(57)

we have

\[
P_a = (m, 0, 0)
\]

(58)

which is the same as in [16].

V. Discussions

As an end, we make some discussions. First, general covariance is a fundamental demand for conservation laws in general relativity. Our definition eq.(39) (40) of energy-momentum is coordinate independent. As the definition of angular-momentum [7], under local SO(1,2) transform \( e^a \to \Lambda^a_{\ b}(x)e^b \), where \( \Lambda^a_{\ b}|_{\partial \Sigma} = L^a_{\ b} = \text{const.} \), we have \( P^a \to L^a_{\ b}P^b \). Second, it is worth while noting that for Deser’s solution, the source stress-energy tensor of which is given
a priori (the energy-momentum vanishes while Deser’s gravitational mass vanishes only when $m + \sigma \mu = 0$). This is quite different from the solution eq.(57) in the limit $\mu \to \infty$. The reason is that, though the form of eq.(51) agrees with eq.(57), the fall-off is substantially different. Remember that in Deser’s solution, the metric is linearized $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, which is a good approximation on condition that $|h_{\mu\nu}| \ll 1$. Yet in Deser’s solution, $h_{ij} = \phi \delta_{ij}$ and $\phi \sim \ln r$, so $h_{\mu\nu}$ does not satisfy the condition. We expect a solution with the same $T^{\mu\nu}$ as Deser’s while without the difficulty.

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References

[1] S.Deser, R. Jackiw & G.’t Hooft Annls of Phys. 152(1984):220;

[2] S. Deser R. Jackiw & S.Templeton Annl.of Phys. 140(1982):372

[3] S.Deser & J. G. McCarthy. Nucl. Phys.B344(1990):747

[4] S.Deser Phys. Rev.Lett.64 No.6(1990): 611
   S.Deser Class. Quan. Grav.(supplement)9 (1992):61

[5] E. Witten Nucl. Phys. B 311 (1988):46.

[6] E. Witten Nucl. Phys. B 323 (1989):113.

[7] S.S. Feng Nucl.Phys. B 468 (1996):163.

[8] Y.S. Duan & J.Y. Zhang Acta Physica Sinica 19 (1963):589.

[9] Y.S. Duan, J.C. Liu & X.G. Dong Gen. Rel. Grav. 20 (1988):5.
[10] S.S. Feng & Y.S. Duan *Gen. Rel. Grav.* **27** (1995):887.

[11] Y.S. Duan & Y.T. Wang *Sci. Sin.* (Chinese Edition) **4A** (1983):343.

[12] Z.D. Yan & M.L. Ge *Kexue Tongbao* **5** (1987) :343.

[13] P. Valtancoli *Class. Quan. Grav.* **10** (1993):245.

[14] G. Clément *Class. Quan. Grav.* suppl 9 (1992) :2635,2615.

[15] P.de Sousa Gerbert *Nucl. Phys.* **B 346** (1990):440.

[16] Dongsu Bak, D. Cangemi & R. Jackiw *Phys. Rev.* **D 49** (107) (1994):5173.