Boosted High Order Harmonics from Electron Density Singularity Formed at the Relativistic Laser Bow Wave

Jie Mu, Yanjun Gu, Tae Moon Jeong, and Georg Korn

Institute of Physics of the ASCR, ELI Beamlines Project,
Na Slovance 2, 18221 Prague, Czech Republic

Timur Zh. Esirkepov, Alexander S. Pirozhkov, James K. Koga, and Masaki Kando

Kansai Photon Science Institute, National Institutes for Quantum and Radiological Science and Technology,
8-1-7 Umemidai, Kizugawa, Kyoto 619-0215, Japan

Petr Valenta

Institute of Physics of the ASCR, ELI Beamlines Project,
Na Slovance 2, 18221 Prague, Czech Republic and
Czech Technical University in Prague,
Faculty of Nuclear Sciences and Physical Engineering,
Brehova 7, 11519 Prague, Czech Republic

Sergei V. Bulanov

Institute of Physics of the ASCR, ELI Beamlines Project,
Na Slovance 2, 18221 Prague, Czech Republic and
Kansai Photon Science Institute, National Institutes for Quantum and Radiological Science and Technology,
8-1-7 Umemidai, Kizugawa, Kyoto 619-0215, Japan

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Abstract

We demonstrate coherent hard electromagnetic radiation generation from reflection by the electron density singularity formed at the relativistic bow wave in laser plasma via particle-in-cell simulations. Wake and bow waves driven by an intense laser pulse form an electron density singularity at the laser pulse front where they join. A counter-propagating laser pulse is reflected at the electron density modulations moving with relativistic velocity. The reflected electromagnetic pulse is compressed and its frequency is upshifted. Its frequency spectrum contains relativistic harmonics of the driver pulse frequency generated at the bow wave front, all upshifted with the same factor as the fundamental mode of the incident light.
High brightness sources of electromagnetic radiation have attracted a great deal of attention due to the broad range of applications in biology, molecular imaging, material sciences and fundamental science research [1][4].

One of the way towards developing ultra-short, intense electromagnetic pulse source is based on simultaneous laser frequency upshifting and the pulse compression. These two phenomena were considered, in particular, with the wave amplification reflected at the moving relativistic electron slab in Ref. [5]; the reflection at the moving ionization fronts studied in Refs. [6–9]. A high repetition regime allowing one to produce frequency upshifted high intensity quasi-monochromatic electromagnetic radiation proposed in Ref. [10], uses a laser produced breaking wake wave in underdense plasma as the flying mirrors to reflect, compress and focus the counterpropagating laser pulse (for details see review articles [11, 12] and references cited therein). This concept is based on the Einstein prediction [13] according to which, in the head-on wave-mirror collision, the reflected electromagnetic pulse is compressed with its frequency upshifted by a factor $4\gamma_M^2$. Here, $\gamma_M$ is the mirror Lorentz factor $\gamma_M = 1/\sqrt{1 - v_M^2/c^2}$ with $v_M$ and $c$ being the mirror velocity and speed of light in vacuum. The flying mirror can be a dense plasma slab accelerated by a high contrast ultraintense laser pulse in the radiation pressure dominant regime [14] or a laser accelerated thin electron layer [15]. The underdense plasma with an up-ramp profile can lead to emission of electromagnetic pulses from laser wake fields under certain conditions [16], and mitigate the premature wavebreaking due to thermal effects [17]. The oscillating mirrors formed as oscillating electron density modulations at the surface of an overdense plasma are used to generate high order harmonics [18–20].

As is known, a focused intense laser pulse propagating in underdense plasma excites wake waves [21–23], in which electrons are pushed not only along the laser pulse propagation direction but also aside, creating a cavity void of electrons. The transverse motion of the electrons at the cavity walls leads to the transverse wake wave breaking [24], resulting in the electron injection into the wake field accelerating phase. Due to the transverse electron motion at the laser pulse front the laser excites a bow wave [25] causing a large-scale transverse modulation of the electron density and electron singularities formed at the joint of the boundaries of the cavity and bow wave. The electron singularity oscillations driven by the laser field generate high order harmonics. The harmonic frequency reaches the “water window” region as observed in the experiments on high power ultra-short pulse laser interaction
with underdense plasmas and in computer simulations [26] [27].

In this letter, we propose a flying mirror scheme that uses the electron density singularity to reflect the counter-propagating laser pulse for laser frequency upshifting and for producing boosted high order harmonics. Within the framework of this scheme, first an intense driver laser pulse propagates through an underdense plasma to generate the wake and bow wave. At the region where the wake wave cavity wall joins the bow wave the electrons pile up to form a singularity in the electron density distribution moving with relativistic velocity. Second, a counter-propagating source laser pulse is reflected at the electron density singularity. The reflected electromagnetic pulse is compressed and its frequency is upshifted due the double Doppler effect. The frequency spectrum of reflected radiation contains relativistic harmonics generated at the bow wave front, all upshifted with the same factor as the fundamental mode of the incident light. We note that boosted high order harmonics have been seen in the spectrum of electromagnetic waves reflected by relativistic mirrors found with the computer simulations presented in Refs. [14] [28], when the relativistic mirrors were a high density plasma slab and a thin electron layer, respectively. In contrast to these cases, the configuration under consideration has the properties of a relativistic flying mirror [10], of the oscillating relativistic mirror [18], and it inherits the properties of the laser driven oscillating electron spikes whose high efficiency in the high order harmonics generation is demonstrated in Refs. [26] [27]. Figure 1 shows the formation of the cavity and bow wave with a singularity in the electron density distribution, accompanied by the reflected electromagnetic field at $t = 24.3T_d$. Here $T_d = \lambda_d/c$ is the period of the driver laser and $\lambda_d$ is the wavelength of the driver laser.

To study two laser pulse interaction in the underdense plasma under the conditions, when the bow wave is formed, we carry out multi-dimensional particle-in-cell (PIC) simulation using the EPOCH code [29].

In the 3D simulations, the simulation box has the size of $22\lambda_d \times 30\lambda_d \times 30\lambda_d$. A spatial grid of $\Delta x/\lambda_d = 1/30$, $\Delta y/\lambda_d = 1/30$ and $\Delta z/\lambda_d = 1/30$ is used to show the structure of the scheme and the reflected pulse. The fully ionized homogeneous density plasma slab is located at $2\lambda_d \leq x \leq 22\lambda_d$, $0 \leq y \leq 30\lambda_d$, and $0 \leq z \leq 30\lambda_d$. The electron density of the plasma is $n_e = 1.14 \times 10^{19}$ cm$^{-3} \times (1 \, \mu m/\lambda_d)^2$, corresponding to $0.01n_c$. Here $n_c = m_e\omega_p^2/4\pi e^2 = 1.14 \times 10^{21}$ cm$^{-3} \times (1 \, \mu m/\lambda_d)^2$ is the critical plasma density, $e$ and $m_e$
FIG. 1: (Color online) The wake wave cavity and bow wave with a singularity in the electron density distribution (in black and white) and the electromagnetic field reflected by the density spike and the cavity (in red and blue), at $t = 24.3T_d$.

are the charge and mass of electron, $\omega$ is the plasma frequency. The total number of the particles is $5.7 \times 10^8$. The ion response is neglected due to the large ion to electron mass ratio and relatively low electron density.

We adopt the driver laser pulse with a normalized amplitude of $a_d = eE_d/m_e\omega_d c = 6.62$, corresponding to the initial intensity equal to $I_d = 6 \times 10^{19} \times (1 \mu m/\lambda_d)^2$ W/cm$^2$. Here $E_d$ and $\omega_d$ are the electric field and frequency of the driver pulse, and $c$ is the speed of light in vacuum. The laser radiation is linearly polarized with the electric field directed along the $y$ axis. The full width at half maximum (FWHM) beam size is $5\lambda_d \times 6.66\lambda_d \times 6.66\lambda_d$. The driver laser pulse focus is at the left boundary of the simulation box.

The simulation results are shown in Figs. 1, 2, and 3. Fig. 1 is the result of 3D simulation, and Figs. 2 and 3 are the results of 2D simulations with similar parameters. In the 2D simulations, the simulation box has a larger size of $90\lambda_d \times 60\lambda_d$ to investigate the
propagation of the reflected electromagnetic field. A substantially small-step spatial grid with $\Delta x/\lambda_d = 0.005$ and $\Delta y/\lambda_d = 0.005$ is used to resolve the wavelength of the reflected pulse. The plasma slab is located at $10\lambda_d \leq x \leq 90\lambda_d$, $0 \leq y \leq 60\lambda_d$.

Fig. 2 (a) shows the structure of the bow and wake waves, as well as the detailed view of the region where they join, i.e. of the region where the electron density singularity is formed at time $t = 54T_d$. Note that the density singularity located in the $(x, y)$ plane is shown as two singularity points in the 2D simulation results illustrating the density in the $(x, y)$ plane. As shown in Fig. 2 (b), the electron density singularity has near-critical electron density. It is comparable to the electron density at the wake cavity bottom. Figs. 2 (c) and (d) display the longitudinal and transverse momentum $p_x$ and $p_y$ vs the coordinate $x$ for the particles in different regions. Compared with the particle density located in the central range of $29\lambda_d \leq y \leq 31\lambda_d$, including injected fast electrons in the wake wave in blue, the particles around the density singularity area $36\lambda_d \leq y \leq 38\lambda_d$ are shown in red to
have lower momentum in the $x$ direction, but higher in the $y$ direction. The velocity of the density singularity is lower than the injected electrons, but still it is relativistic. Different from the $x$-axis symmetry of the $p_y$ distribution of the central particles, $p_y$ of the singularity particles are mostly above the $x$-axis, indicating that most of the singularity electrons move outside the wake cavity, and a small part of them moves downwards. A large number of the particles within the singularity have negative longitudinal momentum, but the particles localized near the driver laser front have large positive $p_x$.

We note that the electron density singularity is observed to maintain stable structure and constant density for over more than 150 pulse cycles. The electron density in the singularity is approximately equal to the critical plasma density. The singularity moves with the velocity corresponding to substantially large relativistic factor $\gamma = 1/\sqrt{n_e/n_c} = 10$, where $n_e$ is the electron density of the plasma background. The velocity of the density singularity normalized to light speed is $\beta = v/c = \sqrt{1-1/\gamma^2} = 0.995$.

Once the density singularity is generated, the source pulse irradiates it from the opposite direction to the driver laser pulse propagation. Another simulation is launched with a smaller grid of $\Delta x/\lambda_d = 1/1024$ and $\Delta y/\lambda_d = 1/256$ and a moving window. The source pulse is linearly polarized with the electric field $E_z$ directed along the $z$ axis. The driver and source pulses have different polarization for their radiation to be distinguished clearly from each other. The wavelength of the source pulse is longer than the driver pulse wavelength being equal to $\lambda_s = 8\lambda_d$, here $\lambda_s$ is the wavelength of the source pulse. So that the reflected electromagnetic wave with the upshifted frequency can be more easily resolved for limited computing resources. The normalized amplitude of the source pulse is equal to $a_s = 0.05$, corresponding to the intensity of $I_s = 5.35 \times 10^{13}$ W/cm$^2$. It is weak so as not to induce significant nonlinear response of the mirror electrons. The FWHM size of the source pulse is $8\lambda_d \times 33.3\lambda_d$, with transverse size of the spot substantially large to guarantee the reflection of a significant amount of the photons at the density singularity. The source pulse is launched to encounter the singularity at $t = 25T_d$.

The reflected electromagnetic field and its frequency spectrum are shown in Fig. 3. Fig. 3 (a) presents the reflected electric field $E_z$ at $34.5T_d$, and the electron density isopleths at $0.03n_c$ and $0.04n_c$ at the same time. The source pulse is, first, partially reflected by the front part of the wake wave, and then immediately reflected by the density singularity, and after that the source pulse experiences the reflection from the bottom of the wake waves as in the
FIG. 3: (Color online) (a) The electric field $E_z$ after high-pass filter, showing the frequency higher than the second harmonic of the source pulse $2\omega_s$, at $34.5T_d$. The black thin curves present the electron density isopleths at $0.03n_c$ and $0.04n_c$. The red thick curve presents $E_z$ along the axis at $y = 0$. (b) The outer and inner pulses of the reflected electromagnetic field, selected by a Gaussian spatial filter. (c) and (d) represent the frequency spectrum of the outer and inner pulses. Confocal ellipses marked with red dashed curve represent the frequency upshift dependence on the reflection angle $\alpha$. Straight lines marked with black dots represent the angle of the wave vector every 10 degrees.

normal flying mirror. The reflected radiation with the upshifted frequency shown in Fig. 3(a) contains two parts reflected by the two density singularity points in the $(x, y)$ plane. The interference of the two parts can also be seen. Each part of the reflected electromagnetic field contains two pulses, the outer pulse and the inner one. In this regime, the radiation
is reflected from the density singularity which has multiple velocities and reflection angles according to the phase space shown in Figs. 2 (c) and (d). Thus, the outer and inner pulses reflected from different parts of the singularity are shown to have different frequency up-shift.

Fig. 3 (b) presents the upper part of the outer and inner pulses. The black thin curves show the spatial filter we use to select the reflected two pulses and perform the Fourier transformation. The energy of the outer electromagnetic pulse is estimated to be $2.1 \times 10^{-7}$ J, which is 0.53% of the source pulse. The number of photons is $6.4 \times 10^{11}$, which is $4 \times 10^{-4}$ of the source pulse.

We first assume the density singularity as an inclined flat mirror moving with a dimensionless velocity of $\beta = v/c$. Due to the double Doppler effect, the reflected pulse in $E_z$ experiences a frequency upshift

$$\omega_r = \frac{\omega_s}{1 - \beta \cos \alpha} \tag{1}$$

with the cosine of the the angle $\alpha$ between the reflected pulse and horizontal axis

$$\cos \alpha = \frac{[2 \beta + (1 + \beta^2) \cos \phi] \tan^2 \theta - 2 \tan \theta \sin \phi - \cos \phi}{(1 + \beta^2 + 2 \beta \cos \phi) \tan^2 \theta - 2 \beta \tan \theta \sin \phi + 1}. \tag{2}$$

Here $\omega_s$ and $\omega_r$ are the angular frequency of the source pulse and the frequency of the reflected pulse, $\phi$ is the angle between the propagation direction of the source pulse and the horizontal axis, $\theta$ is the angle between the normal to the mirror and the horizontal axis. In this regime, the source pulse propagates along the horizontal axis $x$. Using these relationships one can find the reflected pulse frequency

$$\omega_r = \omega_s \frac{1 + \beta}{1 - \beta \cos \alpha}. \tag{3}$$

The Fourier transformed electric field $E_z$ either for the outer and inner parts of the reflected pulse is presented in Figs. 3 (c) and (d). The peak of the reflected electromagnetic radiation propagates in a specific direction, which is determined by the singularity velocity and the tilt angle at the reflecting time. From the wave vector distribution in Fig. 3 (c), the angle between the reflected outer pulse and the horizontal axis, (i.e. reflection angle of the outer pulse) approximately equals 30°. So the frequency of the reflected radiation should be $\omega_r = 14.4 \omega_s$ according to Eq. (3). From the simulation, we reach the maximum signal at
the frequency
\[ \omega_r = c \sqrt{k_x^2 + k_y^2} = \omega_{s1} \frac{1 + \beta}{1 - \beta \cos \alpha} \approx 11, \]
which corresponds to \( \omega_{s1} \approx 0.76 \omega_s \), here \( \omega_{s1} \) is the frequency of the source pulse in plasma which is downshifted. This is caused by the depletion of the source pulse in plasma. The frequency downshift is more significant due to the low frequency of the source pulse. Nevertheless, the theoretical estimated frequency \( \omega_r = 14.4 \omega_s \) is also included in Fig. 3(c) due to the wide distribution of the harmonic.

The harmonics of the source pulse are boosted to higher frequency for both the outer and inner pulses. The different propagation directions of the harmonics represented by the dark areas located along the straight dotted grids in Fig. 3(c), show the result of different reflection angles. The tilt angle of the mirror \( \theta \) in this regime has a continuous range because the singularity is irregular with a curvature, instead of a flat mirror. Thus, the reflection angle also has a continuous range due to Eq. (2). The frequency upshift depends only on the mirror velocity and the reflection angle, as in Eq. (3). By substituting the reflection angle \( \tan \alpha = k_y/k_x \) and \( \omega_r = \sqrt{k_x^2 + k_y^2} \) into Eq. (3), we can obtain the canonical form of an ellipse in coordinates \((k_x, k_y)\) corresponding to the incident frequency \( \omega_i \). The frequency of the reflected radiation will lie on the ellipses, as the black dashed curves show in Fig. 3(c) and (d). Each ellipse stands for odd instances of \( \omega_i \), corresponding to the harmonic orders.

The peaks of the frequency shown in white circles in Fig. 3(c) are lower than the analytical results of the ellipse corresponding to the frequency of the source pulse \( \omega_s \), because in plasma the incident frequency is downshifted to \( \omega_j = \omega_{s1} = 0.76 \omega_s \) instead of \( \omega_s \). The ellipses corresponding to odd instances of \( \omega_{s1} \) are in good agreement with the peaks in the frequency spectrum of the simulation results.

The reflected radiation has a different frequency spectrum from the harmonics generated by the density singularity itself. Both the self-produced harmonics and the boosted harmonics depend on the oscillating density singularity with periodic structure. The well-separated peaks of the frequency on the ellipses represent separated reflection angles, e.g. there are at least 6 different well-separated reflection angles for the first order harmonic \( \omega_j = \omega_s \). This is due to the additive and destructive interference with respect to reflection angles, caused by the periodic curvature variation of the singularity.

Similar boosted high order harmonics are generated for the inner pulse, as shown in Fig. 3(d). The main reflection angle of the inner pulse approximately equals 40°. There are at
least 5 different well-separated reflection angles for the first order harmonic $\omega_j = \omega_s$. The frequency of the inner pulse is lower than that of the outer pulse in the figures, due to the asymmetry of the transverse momentum $p_y$ on the $x$-axis.

In conclusion, analyzing the properties of a two counterpropagating laser pulse interaction in underdense plasmas we proposed a novel scheme of the relativistic flying mirror for electromagnetic radiation frequency upshifting. The proposed scheme uses the laser pulse reflection at the electron density singularity moving with relativistic velocity. The singularity is formed in the region where the bow wave and the wake wave merge, producing the stable singularity mirror whose property is known from catastrophe theory \[31\].

The source pulse is reflected by the electron density singularity as a flying mirror. The reflected electromagnetic wave has a frequency upshift due to the double Doppler effect. The frequency upshift depends on the mirror velocity and the mirror tilt angle with respect to its velocity direction. High order harmonics are boosted to higher frequency with respect to various reflection angles, due to the periodic curvature variation of the singularity.

This scheme provides a promising way to produce ultra-bright radiation sources. It can also be used to investigate the dynamics in nonlinear physical processes in relativistic plasmas. The study on the electron density singularity geometry also contributes to the understanding of the electron motion in laser and underdense plasma interactions, especially the nature of the density singularity at the joining area of wake waves and bow waves. The electron density singularity as a relativistic electron mirror can be used to investigate black hole physics under laboratory conditions \[32\].

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* Jie.Mu@eli-beams.eu

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