Entanglement routers via a wireless quantum network based on arbitrary two qubit systems

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Abstract

A wireless quantum network is generated between multi-hops, where each hop consists of two entangled nodes. These nodes share a finite number of entangled two-qubit systems randomly. Different types of wireless quantum bridges (WQBS) are generated between the non-connected nodes. The efficiency of these WQBS to be used as quantum channels between its terminals to perform quantum teleportation is investigated. We suggest a theoretical wireless quantum communication protocol to teleport unknown quantum signals from one node to another, where the more powerful WQBS are used as quantum channels. It is shown that, by increasing the efficiency of the sources that emit the initial partial entangled states, one can increase the efficiency of the wireless quantum communication protocol.

Keywords: wireless network, entanglement, teleportation

(Some figures may appear in colour only in the online journal)

1. Introduction

Communication and exchange information are the most rapidly developing phenomena. The current technologies that are used to transmit, store and manipulate information are developed during each short period of time. The most challenging of these classical devices is the possibility of communicating and exchanging information securely [1]. However, quantum techniques of manipulating information are developed in parallel to the classical ones and they are more secure than the classical technology [2]. Quantum networks represent one of the most recent developments in the context of quantum communications [3–8]. Several types of these networks have been introduced. For example, the possibility of building a quantum router based on ac control of qubit chains is discussed by Zueco et al [9]. Duan and Monroe [10] have generated a quantum network with trapped ions. Generating a wireless quantum network (WQN) between Josephen qubits is investigated by Sergeenko and Rotoli [11]. Chudzicki and Strauch [12] studied the routing of quantum information in parallel on multidimensional networks of tunable qubits and oscillators. Spin networks have been used by Ross and Kay [13] to route quantum information perfectly. Generating a quantum network between six maximum entangled qubits by Dzyaloshinskii–Moriya (DM) interaction is investigated by Metwally [14]. Moreover, Abdel-Aty et al [15] used DM interaction to generated a quantum network between partially entangled qubits. Cheng et al [16] have introduced a quantum routing mechanism to teleport an unknown quantum state from one quantum device to another by using their model of the wireless wide-area network. Routing quantum information via the XX spin chain has been investigated by Paganelli et al [17]. The concept of distributed wireless quantum communication networks is considered by Tao et al [18]. Recently, Wang et al [19] have proposed a scheme for faithful quantum communication in a quantum wireless multi-hop network, where they assumed that the intermediate nodes share arbitrary pairs of Bell states.

This motivates us to investigate the possibility of generating a WQN between different disconnected hops’ members. This protocol is different from the others, where
we assume that the sending station contains three sources and the first source \( S_1 \) has the ability to emit different types of quantum signals (two-qubit systems). These quantum signals may be maximum entangled states as Bell states [21] or partial entangled states as Werner [24] and X [23] states or generic pure states [22]. However, to be sure that each hop has at least one Werner state, the second source \( S_2 \) supplies all the hops’ nodes with Werner states. The function of the third source \( S_3 \) is supplying the nodes with the required unknown quantum signals to be teleported between the different nodes. Therefore, our protocol is accepted practically and experimentally compared to that introduced by Wang et al [19], which requires ideal environments. Moreover, our protocol can be applied to a wide area due to the possibility of purifying the less entangled bridges as we shall describe in the next sections. Finally, in the protocol of Wang et al [19], all the nodes on the quantum path should co operate to perform quantum teleportation. So, if there are any dishonest or non-cooperative nodes, the teleportation process will fail, while in the current protocol, all these requirements are not needed.

The structure of the paper is described as follows. In section 2, we report the suggested theoretical wireless communication protocol. Section 2.1 is devoted to the distribution of the quantum signals between hops’ members. In section 2.2, we describe how one can generate different wireless quantum bridges (WQBS) to be used as quantum channels to perform quantum teleportation. The efficiency of the generated WQBS to achieve quantum teleportation is discussed in section 2.3. Teleporting unknown quantum signals from one member to another is studied in section 2.4. The concept of purification is described in section 3. Finally, in section 4, we discuss our results.

2. The suggested protocol

As we mentioned earlier, we have three sources. These sources are similar to a source with multiple antennas that transmit different entangled or separable quantum signals to network members located in different hops. This type of transmitter in a classical context is called multiple-input and multiple-output (MIMO) and transmit different signals. Similarly, we called this source QMIMO. The following steps summarize the suggested protocol:

1. Quantum signal distribution.

At the sending station, one antenna of the QMIMO supplies the nodes in each hop with different types of quantum signals, maximum entangled states (MES), partial entangled states (PES), or separable states (SS); meanwhile, the second antenna supplies the other hops with different types of Werner states randomly. The third antenna sends the unknown quantum signals, which are needed to be teleported from one hops’ partner (node) to another. The details of distributing the different quantum signals on the hops’ partners are given in figure 1(a). Figure 1(b) shows the structure of the WQN clearly, where two hops with two nodes are considered. Each hop’s nodes share a class of partially entangled quantum signal of Werner type. Moreover, the nodes share a finite number of partially entangled quantum signals with the other nodes [20].

2. Wireless quantum bridges.

If one of the hop’s partners receives a unknown quantum signal (qubit) and he/she is asked to send it to another member in the WQN, he/she has to generate a WQB to be used as a quantum channel. The two nodes are called quantum neighbors if they share at least one of the Werner states. However, if the two nodes are not quantum neighbors, then the sender generates a WQB with the one nearest to the required member. In figure 2, we show how the non-connected nodes generate a WQB.

3. Bridges’ efficiency.

The WQB’s partners (nodes) check if their WQB has the ability to be used as a quantum channel to perform quantum teleportation or not. If yes, they move to the second step. If not, they send it to the purification lab to improve their efficiency.

4. Teleportation step.

As soon as the WQB is generated, the sender performs the controlled NOT (CNOT) operation and Hadamard gate between his/her qubits followed by Bell measurements. The sender sends his/her results to the receiver, who retrieves the original state by performing a suitable local operation. The details are given in section 2.4.

5. Purification step.

If the generated wireless bridges are not efficient to be used as quantum channels to perform teleportation, then they are sent to the purification lab to increase their entanglement and hence their ability to achieve quantum teleportation.

In the following subsections, the previous steps of the suggested theoretical wireless communications are investigated extensively and we show our idea by different cases.

2.1. Quantum signal distribution

As was mentioned earlier, one antenna of the QMIMO sends different quantum signals to the hops’ nodes. The emitted initial quantum signals to the hops’ members are classified as maximum entangled, partial entangled or SS. The class of maximum entangled states (Bell states) includes \( \rho_{\phi^z} = |\phi^z\rangle \langle \phi^z| \) and \( \rho_{\psi^z} = |\psi^z\rangle \langle \psi^z| \), where \( |\phi^z\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle) \) and \( |\psi^z\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle) \). These entangled states can be described by using Pauli operators as
\[\begin{aligned}
\rho_B = & \left\{ \begin{array}{l}
\frac{1}{4} \left( 1 + \sigma_1^{(i)} \sigma_1^{(j)} + \sigma_2^{(i)} \sigma_2^{(j)} + \sigma_3^{(i)} \sigma_3^{(j)} \right), \\
\frac{1}{4} \left( 1 + \sigma_1^{(i)} \sigma_1^{(j)} - \sigma_2^{(i)} \sigma_2^{(j)} + \sigma_3^{(i)} \sigma_3^{(j)} \right), \\
\frac{1}{4} \left( 1 - \sigma_1^{(i)} \sigma_1^{(j)} + \sigma_2^{(i)} \sigma_2^{(j)} + \sigma_3^{(i)} \sigma_3^{(j)} \right), \\
\frac{1}{4} \left( 1 - \sigma_1^{(i)} \sigma_1^{(j)} - \sigma_2^{(i)} \sigma_2^{(j)} - \sigma_3^{(i)} \sigma_3^{(j)} \right),
\end{array} \right. \\
\end{aligned}\]

where \(\sigma_k^{(i)}\), \(k = 1, 2, 3\) are the Pauli operators for the qubits \(i\) and \(j\), respectively and \(\sigma_1^{(0)} = |0\rangle \langle 0| + |1\rangle \langle 1|\).

\[\begin{aligned}
\sigma_2^{(0)} = i(-|0\rangle \langle 1| + |1\rangle \langle 0|) \quad \text{and} \quad \sigma_3^{(0)} = |0\rangle \langle 0| - |1\rangle \langle 1|. \\
\end{aligned}\]

It is clear that any one of these states can be transferred into another one by using local operations. The second types of the transmitted states from the QMIMO are partial entangled states. In this contribution, we consider two classes: \(X\) and generic pure states, which can be defined as,

\[\begin{aligned}
\rho_{\text{pure}} = & \frac{1}{4} \left\{ 1 + p \left( \sigma_i^{(i)} - \sigma_i^{(j)} \right) \\
& - \sigma_i^{(i)} \sigma_i^{(j)} - q \left( \sigma_2^{(i)} \sigma_2^{(j)} + \sigma_3^{(i)} \sigma_3^{(j)} \right) \right\}, \\
\rho_X = & \frac{1}{4} \left\{ 1 - c_i^{(i)} \sigma_i^{(i)} \sigma_i^{(j)} \\
& - c_2^{(i)} \sigma_2^{(i)} \sigma_2^{(j)} - c_3^{(i)} \sigma_3^{(i)} \sigma_3^{(j)} \right\},
\end{aligned}\]

where \(p = \sqrt{1 - q^2}\). It is clear that if we set \(p = 0\) in \(\rho_{\text{pure}}\), one gets a maximum entangled state (fourth state (equation (2))). This entangled pure state turns into a SS if we set \(p = 1\). However, this type of pure state can be transformed into four equivalent forms by local operations, and consequently, all the maximum entangled states can be obtained from the other forms of these pure states [22]. The degree of entanglement of this pure state is given by Wootter’s concurrence [25] as

\[C_{\text{pure}} = \max \left\{ 0, \frac{1}{2} (1 + 2q) - \frac{1}{2} \right\}.\]

The second type of the transmitted partial entangled states is called \(X\)-state [23], where one can obtain what is called
Werner state [24] $\rho^w$ by setting $c_{11} = c_{22} = c_{33} = x$ and $\rho_{\text{Bell}}$ if we set $x = 1$. The degree of entanglement of the X-state is given by,

$$C_X = \max \left\{ \frac{1}{2} \text{tr} \left( \rho^w C \rho^w C^\dagger - \frac{1}{2} \right) \right\},$$

where $\rho^w$ is a $3 \times 3$ matrix called a cross-dyadic and represents the correlation between the two qubits. The non-zero elements of the cross-dyadic $\rho^w$ are given by $c_{11}, c_{22}$ and $c_{33}$.  

### 2.2. WQBS: entanglement routing

Now, each node in different hops has its own qubits. The aim of this section to generate WQBS between any two non-connected nodes located in different hops. This procedure can be achieved via CNOT operation and Hadamard gate followed by Bell measurements [16] as shown in figure 2.

Let us first consider two hops whose partners share a class of X-state, where the first hop’s nodes share the state $\rho^{(i)}$ and the second hop’s nodes share the state $\rho^{(k)}$. The nodes $(j)$ and $(k)$ perform a CNOT operation followed by Hadamard gate on the qubit $(k)$. After performing Bell measurements on the qubits $(j)$ and $(k)$, the final state is projected into $\rho^{(C)}$ (see figure 2). The final state $\rho^{(C)}$ represents the generated WQBS between the nodes $(i)$ and $(j)$. However, if the nodes of the first and the second hops share X-states, then we call the generated entangled states by XX wireless bridge. In the computational basis, $00, 01, 10, 11$, this bridge can be written as,

$$\rho_{XX} = \begin{pmatrix}
q_{11} & 0 & 0 & q_{14} \\
0 & q_{22} & q_{23} & 0 \\
0 & q_{32} & q_{33} & 0 \\
q_{41} & 0 & 0 & q_{44}
\end{pmatrix},$$

where

$$q_{11} = \frac{1}{2} (A_1 B_1 + A_2 B_2), \quad q_{14} = \frac{1}{2} (A_1 B_3 + A_4 B_4),$$
$$q_{22} = \frac{1}{2} (A_1 B_2 + A_2 B_1), \quad q_{23} = \frac{1}{2} (A_1 B_4 + A_4 B_3),$$
$$q_{32} = q_{23}, \quad q_{33} = q_{22}, \quad q_{41} = q_{14}, \quad q_{44} = q_{11},$$
$$A_1 = B_1 = \frac{1 + c_{33}}{4}, \quad A_2 = B_2 = 1 - \frac{c_{33}}{4},$$
$$A_3 = B_3 = \frac{c_{11} - c_{22}}{4}, \quad A_4 = B_4 = \frac{c_{11} + c_{22}}{4}. \tag{6}$$

From this WQBS, one can obtain the following bridges:

1. If we set $c_{11}^{(i)} = c_{11}^{(j)} = c_{22}^{(j)} = c_{22}^{(i)} = x$ and $c_{11}^{(k)} = c_{22}^{(k)} = c_{33}^{(i)} = c_{33}^{(j)} = x, we get the wireless Werner–Werner quantum bridge (WW-bridge).
2. If we set $c_{11}^{(i)} = c_{22}^{(j)} = c_{33}^{(j)} = c_{33}^{(i)} = x$ and $c_{11}^{(k)} = c_{22}^{(k)} = c_{33}^{(i)} = -1$, we get the wireless Werner–Bell quantum bridge (WB-bridge).
3. If we set $c_{11}^{(i)} = c_{22}^{(j)} = c_{33}^{(j)} = x$ and $c_{11}^{(k)} \neq c_{22}^{(k)} \neq c_{33}^{(i)} \neq 0$ we get the wireless Werner-X quantum bridge (WX-bridge).

However, if the node of one hop initially shares X-state while the nodes of another hop share a class of the generic pure state, then the generated WQB is called an XP bridge. In the computational basis, this bridge can be described by a density matrix of size $4 \times 4$; its elements are given by,

$$q_{11} = A_1 C_1 + A_2 C_2, \quad q_{12} = -(A_1 C_3 + A_2 C_4),$$
$$q_{13} = C_1 (A_1 + A_4), \quad q_{14} = (A_3 C_1 + A_4 C_2),$$
$$q_{21} = q_{21}, \quad q_{22} = A_1 C_2 + A_2 C_1, \quad q_{23} = C_2 (A_3 + A_4),$$
$$q_{24} = q_{24}, \quad q_{31} = q_{31}, \quad q_{32} = q_{32}, \quad q_{33} = q_{33},$$
$$q_{34} = q_{34}, \quad q_{41} = q_{41}, \quad q_{42} = q_{42},$$
$$q_{43} = q_{43}, \quad q_{44} = q_{44} \tag{7}$$

where $A_i, i = 1..4$ are given from (6) and $C_1 = \frac{1 - p}{2}$, $C_2 = \frac{1 + p}{4}$ and $C_3 = \frac{p}{2}$.

Figure 3(a) describes the behavior of the degree of entanglement between the terminals of the generated WQBS. First, we assume that the non-connected nodes share a WW bridge. From this figure, we can see that the entanglement between the terminals of the WW bridge is generated at $x > 0.578$. However, for larger values of $x$, the entanglement between the terminals of the WW bridge increases and reaches its maximum values, i.e., $(E = 1)$ at $x = 1$, namely, the initial states are Bell states. Second, the nodes of the first hop share a class of Werner type, while the second hop’s nodes share a class of X-state, which is defined by $c_{11} = -0.9, c_{22} = -0.8$ and $c_{33} = -0.7$. In this case, the entanglement between the terminals of the WQB (WX) is generated at $x > 41$. As $x$ increases, the entanglement increases to reach its maximum value $(E = 0.7)$ at $x = 1$. Third, the nodes of one hop share a maximum entangled state (Bell types), while the second hop’s nodes share Werner state. In this case, the entanglement between the WB bridge terminals is generated for smaller values of $x(x = 0.34)$.

In figure 3(b), we assume that the hops share two different initial quantum signals. The first hop’s nodes share Werner state, while the nodes of the second hop share a class of pure state. However, if the second hop is supplied with different initial entangled pure states, where we set $p = 0, 0.7, 0.9, 1$, it is clear that, at $p = 0$, which corresponds to Bell state, the two hops entangle together at $x = 0.33$. The degree of entanglement between the terminals of the wireless WP bridge increases as $x$ increases to reach its maximum value $(E = 1)$ at $x = 1$, where the initial two-quantum signals are maximum entangled states. As one increases $p$, namely the second hop is supplied with a less entangled pure state, the entanglement between the two hops appears suddenly for smaller values of $x$. However, the maximum value of the entanglement is reached at $x = 1$, where it is smaller than ‘1’ for larger values of $p$. Starting from a SS, where we set $p = 1$, the two hops generate a WQB at a very small value of $x$, but the degree of entanglement between the bridge’s terminals is very small compared with those depicted for the entangled pure state.
From figure 3, we can conclude that it is possible to entangle different hops, and their partners share arbitrary classes of initial two-qubit systems. The results show that if each hop’s nodes share a pure state even they are initially separable, one can generate entangled WQBS between the hops’ nodes. Using a Werner state with a larger value of its parameter \(x > 0.5\), one can generate WQBS between the hops’ nodes with high degree of entanglement.

2.3. Bridge’s efficiency

In this section, we investigate the efficiency of the generated WQBS, where we discuss the possibility of using them as quantum channels to perform quantum teleportation. The inequality that measures the efficiency of the WQBS to perform quantum teleportation is given by [26]

\[
T_{\text{elp}} = \text{tr}\{\sqrt{\mathcal{C}^\dagger \cdot \mathcal{C}}\} > 1,
\]

where the elements of the cross-dyadic \(\mathcal{C}\) are given by \(c_{mn} = \text{tr}\{\sigma^{(1)}_m \sigma^{(2)}_n \rho_B\}\), \(m, n = 1, 2, 3\) and \(\rho_B\) stands for the state of the WQB. For example, \(c_{11} = \text{tr}\{\sigma^{(1)}_1 \sigma^{(2)}_1 \rho_B\}\), \(c_{12} = \text{tr}\{\sigma^{(1)}_1 \sigma^{(2)}_2 \rho_B\}\) and so on.

The behavior of teleportation inequality (8) is described in figure 4 for different WQBS. In figure 4(a), the behavior of the teleportation inequality is displayed for WW, WX and WB bridges. It is clear that there is the possibility of using the generated wireless bridges as quantum channels to perform quantum teleportation, depending on the initial degree of entanglement. However, the WW bridge is useful for quantum teleportation for \(x \geq 0.8\), and the WX bridge for \(x \geq 0.6\) and the BW bridge for \(x \geq 0.5\). Figure 4(b) describes the behavior of the teleportation inequality (8) for the WP bridge. This figure shows that the possibility of using the WP bridge as a quantum channel increases as \(p\) decreases.

From this figure, we can find the lower values of Werner parameter \((x)\), where the generated WQBS are useful for quantum teleportation. This means that if one can improve the efficiency of the source that sends Werner states, one can increases the efficiency of the generated WQBS.

2.4. Teleportation

In this section, we investigate the possibility of using the more powerful WQBS to teleport an unknown quantum signal given by,

\[
\rho_\alpha = \begin{pmatrix} |\alpha|^2 & \alpha \beta^* \\ \alpha^* \beta & |\beta|^2 \end{pmatrix}, \quad \text{with} \quad |\alpha|^2 + |\beta|^2 = 1,
\]

from one hop’s partners to another. Let us first consider that the nodes use an XX wireless bridge. In this case, the nodes
where $\mu \nu = WW$, $WB$ or $WX$ if the users use $WW$, $WB$ and $WX$ bridges, respectively.

Figure 5 describes the behavior of the fidelity of the teleported quantum signal via the WQB $WW$ (solid-curve), $WB$ bridge (dash--dot curve) and $WX$ bridge (dot curve), where we consider only the bridges that are generated at $x \geq 0.7$ (efficient bridges for teleportation). It is clear that the initial fidelity of the teleported state depends on the initial entanglement. As an example, if the partners use the $WB$ bridge with $(x \geq 0.7)$, the initial fidelity of the teleported quantum signal is large. However, as $x$ increases, the fidelity $F_{\mu \nu}$ increases to reach its maximum value ($F_{\mu \nu} = 1$ at $x = 1$). On the other hand, if the partners use the wireless $WW$ bridge, then the initial fidelity is smaller than the previous case. As the Werner’s parameter $x$ increases, the fidelity increases to become maximum at $x = 1$; namely, the initial states of the two hops’ members turn into Bell states. Finally, the users use the generated $WX$ bridge; then the initial fidelity depends on the degree of entanglement of the $X$-state. However, the fidelity increases as $x$ increases to reach its maximum bounds.

Finally, users can use the WQB ($WP$) to teleport the unknown quantum signal ($\rho$) with a fidelity given by

$$F_{WP} = \alpha^2 \left\{ \hat{\rho}_{11} + \hat{\rho}_{22} + 2 \left( \alpha^2 - \beta^2 \right) \hat{\rho}_{24} + \left( \alpha^* \beta + \beta^* \alpha \right) \left( \hat{\rho}_{11} - \hat{\rho}_{22} \right) + \left( \alpha^* \beta + \beta^* \alpha \right) \left( \left( \alpha^2 - \beta^2 \right) \hat{\rho}_{23} - \hat{\rho}_{14} \right) - \left( \alpha^* \beta + \beta^* \alpha \right) \hat{\rho}_{23} - 2 \hat{\rho}_{12} + \beta^2 \left( \hat{\rho}_{11} + \hat{\rho}_{22} + 2 \left( \alpha^2 - \beta^2 \right) \hat{\rho}_{24} - \left( \alpha^* \beta + \beta^* \alpha \right) \left( \hat{\rho}_{11} - \hat{\rho}_{22} \right) \right) \right\}.$$ (11)

Figure 6 shows the behavior of the fidelity $F_{WP}$. The behavior shows that the initial fidelity depends on the parameter $p$, where for $p = 0$, the pure state turns into a Bell state. Therefore, the initial fidelity of the teleported state $F_{WP}$ is larger. This fidelity reaches its maximum value ($F_{WP} = 1$ at $x = 1$), which means that the two hops share a Bell state. However, as $p$ increases, the initial fidelity of the teleported state decreases, and the maximum bounds are reached at $x = 1$. The maximum bounds decrease as $p$ increases.

### 3. Purification

Quantum purification has been used to distill small number of strongly entangled qubits from a large number of weakly entangled qubits via local operations, classical communication and measurements. The first purification protocol (IBM) was proposed by Bennett et al [28], where they obtain the singlet states from Werner classes. Deutsch et al [29] have suggested the Oxford protocol, which is more efficient than the IBM protocol. Since then, several protocols have been suggested. For example, a more efficient entanglement purification protocol is suggested by Metwally [30], which is more efficient than the IBM and Oxford protocols. Another improvement has been done on the IBM protocol by Feng et al [31]. All the previous protocols have been improved by several versions. Among these improvements is the protocol

![Figure 5](image1.png)  
**Figure 5.** The fidelity of the teleported state ($\rho$) with $\alpha = \beta = \frac{1}{\sqrt{2}}$. The solid, dash and dot curves represent the fidelity $F_{\mu \nu}$ of the teleported state by using the wireless bridges $WW$, $WB$ and $WX$ ($c_{\nu} = -0.9$, $c_{\gamma} = -0.8$, $c_{\xi} = -0.7$), respectively.

![Figure 6](image2.png)  
**Figure 6.** The same as figure 5, but the users use the wireless $WP$ bridge. The solid, dash and dash--dot curves for $p = 0$, $0.7$ and $0.9$, respectively.
introduced by Metwally and Obada [32], where this improved version is based on using the controlled–controlled NOT gate (CCNOT) instead of CNOT.

In this context, we can use one of the previous protocols to distill a WQB with a high degree of entanglement from weakly entangled bridges. In this WQN, we suggested two strategies: the first is the initial partial entangled state can be purified before sending them to the hops’ nodes. In this case, all the users will be supplied by MES, and the protocol turns into the Wang et al protocol [19]. The second possibility is performing a quantum purification protocol on the less entangled bridges (useless bridges for teleportation) to increase their efficiency. However, this will be our next contribution: to find which strategy is better.

4. Conclusion

The possibility of generating WQNs between different hops’ nodes, where it is assumed that these nodes share arbitrary two-qubit systems randomly, is discussed. To achieve quantum communication between the non-connected hops’ nodes, the users have to generate WQBS. The type of these wireless bridges depends on the states that are shared between the terminals of each hop, where we have generated WW, WB, WX and Werner-Pure (WP) bridges. The entanglement of each WQB is quantified by the means of concurrence. It is shown that for a less entangled state, the non-connected hops’ nodes turn into WQBS for larger values of Werner’s parameter \( x \). However, the partially entangled WQBS turn into maximum entangled wireless bridges when the Werner’s parameter is (\( x = 1 \)). The entanglement of the WP bridges depends on the pure and Werner states parameters, where it increases for larger values of Werner parameter and smaller values of the pure state parameter.

The efficiency of the generated WQBS to perform quantum teleportation is discussed for different types of bridges. It is shown that the teleportation inequality is violated for small values of the Werner’s parameter, and consequently, the efficiency of the WQBS to perform quantum teleportation decreases. However, this efficiency of the generated WQBS increases for larger values of Werner’s parameter and smaller values of the pure state parameter.

The more powerful WQBS are used to teleport unknown quantum signals from one node to another, where we consider only the bridges that obey the teleportation inequality. The fidelity of the teleported quantum signal increases by increasing Werner’s parameter or decreasing the pure state parameter for the WP bridge. The maximum value of the fidelity depends on the entanglement of the WQB used.

In conclusion a WQN can be generated between different hops’ nodes sharing arbitrary, different two-qubit states. The efficiency of the WQN and hence its ability to perform wireless quantum communication can be enhanced by controlling the devices that generate these signals.

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