STATISTICAL PROPERTIES OF SORET AND DUFOUR EFFECTS: RESULTS ON HEAT AND MASS TRANSFERS

Alias Jedi¹, Nor Ashikin Abu Bakar²

¹Department of Mechanical and Manufacturing Engineering, Faculty of Engineering and Built Environment, The University Kebangsaan Malaysia, Bangi 43600 Malaysia
²Institute of Engineering Mathematics, The University Malaysia Perlis, Arau 02600 Malaysia

¹aliasjedi@ukm.edu.my, ²ashikinbakar@unimap.edu.my

Corresponding Author: Alias Jedi

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Abstract

This article is investigated the data on heat and mass transfer using Soret and Dufour act as independent variables. From the investigation, it is revealed that the heat transfer rate increases when slip parameter and Soret number increase, while Dufour number decreases through solving ordinary differential equation (ODE). Statistical correlation coefficient were used to see the relationship effect of Soret number and Dufour number on the heat and mass transfer. The correlation coefficient's results of the parameters and to the local Nusselt/Sherwood number are found to be statistically significant.

Keywords: Statistical Thermodynamics, Nanofluid, stretching/shrinking sheet, Soret/Dufour effects, Brownian motion, thermophoresis

I. Introduction

Nanofluid is a new technology combining base fluid and nano-sized particles. Choi [XVI] discovered that nanofluid influence the thermal conductivity. Through reviewed, Minea [I] found that nanoparticle enhance nanofluid's thermal conductivity. There are many researchers have employed numerical technique to explore convection heat transfer of nanofluid in details. Hashim et al. [V] used the mathematical nanofluid model that takes into consideration two slip mechanisms which are Brownian motion and thermophoresis. He et al. [XVIII] further explored the boundary layer flow theory in nonequilibrium supersonic. It seems Zaimi et al. [VII] was study boundary layer flow of a nanofluid over a stretching sheet by assuming nonlinearity and extend by Jedi et al. [IV] to the case with suction effect. Sabir et al. [XVII] however study the chemical reaction in Cass on nanofluid. Sobamowo [VIII] studied the free convection flow of a Cassonnanofluid with thermal

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The effects of Brownian motion was discovered by Devi [XIV]. Waini et al. [VI] and Rahman et al. [XII] studied the stretching/shrinking sheet in a nanofluid. Further, such effects arise in many applications such as in chemical engineering, heat insulation, geothermal systems, drying technology and catalytic reactors. Ali et al. [II] investigated the Soret and Dufour in nanotechnology research. The thermal diffusion and diffusion thermo effects on Marangoni convection boundary layer flow of Self-rewetting fluid was analysed numerically by Tsang and Sun [XV]. Mahabaleshwar et al. [X], [IX], Abdal et al.[III], Jamaludin et al. [III] and Noor et al. [II] continued to study the MHD convective with various effects. Motivation by the work done by Zaimi et al. [IV], we extend these studies to the cases of Soret and Dufour with the partial slip effect.

II. Methodology

The extension of the governing equations, considering method by Zaimi et al. [VII], are;

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \sum \frac{D_k}{\eta_1} \frac{\partial C}{\partial y}, \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_0 \frac{\partial^2 C}{\partial y^2} + \frac{D_k}{\eta_1} \frac{\partial^2 T}{\partial y^2}, \tag{4}
\]

subject to

\[
u \frac{\partial u}{\partial y} + v = \nu_{xy}, \quad T = T_w, \quad C = C_w \quad \text{at} \ y = 0, \tag{5}
\]

By introducing the relation (6) into the equations (2)-(4), we obtain

\[
f'' + \left( \frac{2n}{n+1} f' \right)^2 = 0, \tag{7}
\]

The dimensionless variables are:

\[
u = \frac{av(n+1)}{2} \left[ f'(\eta) + \frac{n-1}{n+1} \eta f''(\eta) \right], \tag{6}
\]

\[
\eta = y \left( \frac{av(n+1)}{2} \right)^{1/2} x^{(r-\eta)/2}, \quad \theta(\eta) = \frac{T - T_w}{T_w - T_c}, \quad \phi(\eta) = \frac{C - C_c}{C_w - C_c}.
\]
where prime denote the differentiation with respect to \( \eta \). The boundary conditions are now transformed to

\[
\begin{align*}
  f(0) = S, & \quad f'(0) = \varepsilon + \sigma f'(0), \quad \theta(0) = 1, \quad \phi(0) = 1, \\
  f'(\eta) \to 0, & \quad \theta(\eta) \to 0, \quad \phi(0) \to 0 \quad \text{as} \quad \eta \to \infty.
\end{align*}
\]

With

\[
V_w = \left( \frac{a(n+1)}{2b} \right)^{1/2} S,
\]

velocity slip parameter, \( \sigma \) is

\[
\sigma = N_t \rho \alpha \left( \frac{a(n+1)}{2b} \right)^{1/2}.
\]

Where;

\[
\Pr = \frac{\nu}{\alpha}, \quad \Le = \frac{\nu}{D_b}, \quad \Nb = \frac{(\rho_1)_p (C_e - C_b)}{(\rho_1)_p}, \quad \Pr = \frac{(\rho_1)_p (T_e - T_b)}{(\rho_1)_p T_b},
\]

\[
D_p = \frac{D_{K_p}(C_e - C_b)}{C_p(T_e - T_b)}, \quad \Sr_p = \frac{D_{K_p}(T_e - T_b)}{C_p(T_e - T_b)}
\]

\[
C_j = \frac{\tau_n}{\rho U^2}, \quad N_t = \frac{q_n}{k(T_e - T_b)}, \quad S_l = \frac{q_n}{D_b(C_e - C_b)},
\]

and,

\[
\tau_n = \frac{\partial T}{\partial \eta}, \quad q_n = \frac{\partial \theta}{\partial \eta}, \quad q_n = \frac{\partial \phi}{\partial \eta}
\]

Applying similarity variables in (6),

\[
C_j \text{Re}^{12} = f'(0), \quad N_t \text{Re}^{12} = \theta(0), \quad S_l \text{Re}^{12} = -\phi(0),
\]

The Nomenclature and Greek Symbols of the above equations (1) - (16) are in Appendix I.

III. Results and Discussion

The stretching/shrinking sheet with partial slip for Soret and Dufour were analysed. Ordinary differential equations (7)-(9) with (10) are solved using the
shooting method. The comparison table for the skin friction coefficient with results found by Zaimi et al. [IV] in Table 1 act as a benchmark that the numerical results are in perfect agreement.

**Table 1: Effect of S and $\varepsilon$**

| S  | $\varepsilon$ | Local Nusselt | Present results |
|----|---------------|---------------|----------------|
| 2.5| -1            | -             | 7.2280502[8.2273622] |
|    | -0.5          | 6.991104[7.887191] | 6.991104[7.8871915] |
|    | 0             | -             | 6.8519856[7.5982374] |
|    | 1             | -             | 6.6924610[7.2967475] |
|    | 2             | 6.607723[7.151258] | 6.6077233[7.1512588] |
| 3  | -1            | -             | 8.4794183[9.6275893] |
|    | -0.5          | 8.330163[9.323738] | 8.3301634[9.3237389] |
|    | 0             | -             | 8.2223828[9.0974233] |
|    | 1             | -             | 8.0764801[8.8204727] |
|    | 2             | 7.984141[8.661916] | 7.9841415[8.6619165] |

a. [ ] second solution

Figures 1a and 1b display different values of Soret number $Sr$, respectively when $Nb,Nt = 0.5$, $Du = 0.1$, $\sigma = 0.1$, $Le = 1$, $S = 2.5Pr = 1$ and $n = 2$. The values of $-\theta'(0)$ in Figure 1a and $-\phi'(0)$ in Figure 1b are seem to increase as the Soret number $Sr$ increases. The increasing of Soret number in the fluid flow will reduce the boundary layer thicknesses which are lead to increase both the heat and mass transfer rates on the surface. The dual similarity solution exist when $\varepsilon_c = -1.5972$ where $\varepsilon_c$ is the critical value of $\varepsilon$ and no similarity solution is found for $\varepsilon < \varepsilon_c = -1.5972$ where the boundary layer separation occurs. Results for different values of Dufour number $Du$ given that $Nb$ and $Nt = 0.5$, $Sr = 0.1$, $\sigma = 0.1$, $Le = 1$, $S = 2.5$, $Pr = 1$ and $n = 2$ are depicted in Figures 2a and 2b, respectively. The presence of Dufour effect do not effect the range of dual solutions exist for $\varepsilon$. From the investigation, the dual solution exist up to $\varepsilon < \varepsilon_c = -1.5972$ and no solution can be found when $\varepsilon < \varepsilon_c = -1.5972$ where as beyond this value, the boundary layer has separated from the surface. As seen in Figures 2a, the value of $-\theta'(0)$ decreases when Dufour number $Du$ increases. While in Figures 2b, the reverse trend is observed in Fig. 2b. where the value of $-\phi'(0)$ increases when $Du$ increases. The increasing of Dufour number in the fluid flow will increase the thermal boundary layer thickness, but to reduce the nanoparticle concentration boundary layer thickness.
The results for $\theta'(0)$ and $\phi'(0)$ is higher for first solution. Figs. 3a - 6b are the results for temperature profiles $\theta(\eta)$ and concentration $\phi(\eta)$ profiles.

IV. Statistical Properties

Statistical analysis is used to study the effects of Suret and Dufour. The relationship between Suret/Dufour and heat/mass transfer is done by finding the values of correlation, $r$. The purpose of $r$ is to measure the relationship between physical parameter and local Nusselt/Sherwood number. The value of $r$ will be used to obtain the probable error, $P.E (r)$. The probable error;

$$P.E (r) = 0.6745 \frac{1-r^2}{\sqrt{n}}$$  \hspace{1cm} (17)

$n$ denotes as the number of observations taken from the local Nusselt/Sherwood number. The value $r > 0.7$ indicates strong liner relationship between parameter variables. If $r$ less than $P.E$ the correlation coefficient is not significant. Furthermore, the correlation is significant when the value ratio between correlation with probable error is more than 6. Table II shows that, the values of $Sr$ and $Du$ for the local Nusselt/Sherwood number are fulfilled the relation of the value ratio between correlation with probable error, which is indicated that all the values are greater than 6. Conclusion can be made that the correlation coefficient between physical parameter and local Nusselt/Sherwood number are statistically significant

Table 2: The values ratio of correlation with probable error

| Local Nusselt | Local Sherwood |
|--------------|----------------|
| $Du$  | $Sr$  | $Du$  | $Sr$  |
| $R$ | 0.9842 | 0.9451 | 0.9445 |
| $P.E(r)$ | 0.0045 | 0.0154 | 0.0155 |
| $r/P.E(r)$ | 218.6652 | 61.4755 | 60.8118 |

Fig. 1a: Variation of $-\theta'(0)$ versus $\varepsilon$ with $Sr$

Fig. 1b: Variation of $-\phi'(0)$ versus $\varepsilon$ with $Sr$
Fig. 2a: Variation of $-\theta'(0)$ versus $\varepsilon$ with $Du$

Fig. 2b: Variation of $-\phi'(0)$ versus $\varepsilon$ with $Du$

Fig. 3a: Effects of $Sr$ to the temperature profile when $\varepsilon = -1$ (shrinking)

Fig. 3b: Effects of $Sr$ to the concentration profile when $\varepsilon = -1$ (shrinking)

Fig. 4a: Effects of $Sr$ to the temperature profile when $\varepsilon = 1$ (stretching)

Fig. 4b: Effects of $Sr$ to the concentration profile when $\varepsilon = 1$ (stretching)
Fig. 5a: Effects of $Du$ to the temperature profile when $\varepsilon = -1$ (shrinking)  

Fig. 5b: Effects of $Du$ to the concentration profile when $\varepsilon = -1$ (shrinking)  

Fig. 6a: Effects of $Du$ to the temperature profile when $\varepsilon = 1$ (stretching)  

Fig. 6b: Effects of $Du$ to the concentration profile when $\varepsilon = 1$ (stretching)  

V. Conclusion  
Soret/Dufour gives the effect on temperature/nanoparticle concentration profiles. The increasing of the parameters Soret and Dufour lead to an decreasing the heat and mass transfers rate. From statistical output, the correlation coefficient are statistically significant.

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### Appendix I.

- $u$ velocity components ($x$ direction) $v_x > 0$ suction
- $v$ velocity components ($y$ direction) $v_y < 0$ injection
- $T$ fluid temperature $\varepsilon > 0$ stretching
- $T_w$ surface temperature $\varepsilon < 0$ shrinking
- $T_\infty$ ambient temperature $n = 1$ linear
- $C$ nanoparticle concentration $n \neq 1$ nonlinear
- $C_w$ nanoparticle volume fraction (at the plate) $\eta$ similarity variable
- $C_\infty$ nanoparticle volume fraction (far from the plate) $f(\eta)$ dimensionless stream function
- $\nu$ kinematic viscosity $\theta(\eta)$ fluid temperature
- $D_B$ Brownian diffusion $\phi(\eta)$ fluid concentration
- $D_T$ thermophoresis diffusion $Pr$ Prandtl number
- $\alpha = k/(\rho c)_f$ is the thermal diffusivity of the fluid, $Le$ Lewis number
- $\tau = (\rho c)_f/(\rho c)_f$ is the ratio of effective heat capacity $Nb$ Brownian motion
- $\rho_f$ fluid density $Nt$ thermophoresis
- $\rho_p$ particles density $Du$ Dufour number
- $c$ volumetric volume expansion $Sr$ Soret number
- $D_B$ Brownian diffusion $C_f$ skin friction
- $D_T$ thermophoresis diffusion $Nu_s$ local Nusselt number
- $D_m$ mass diffusivity $Sh_s$ local Sherwood number
- $K_T$ thermal diffusion ratio $\mu$ dynamic viscosity
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