We study the $q$-deformed oscillator algebra acting on the wavefunctions of non-compact D-branes in the topological string on conifold. We find that the mirror B-model curve of conifold appears from the commutation relation of the $q$-deformed oscillators.
1. **Introduction**

The topological string is an interesting playground to study the gauge/string duality via the geometric transition \[1,2,3\]. It is also interesting to study how the target space geometry is quantized in this context. Recently, it is realized that the A-model side is described by a statistical model of crystal melting \[4,5\], while the B-model side is reformulated as matrix models \[6,7\]. In both cases, a spectral curve appears either as the limit shape of molten crystal or from the loop equation of matrix model. It is expected that the spectral curve should be viewed as a “quantum Riemann surface” in the sense that the coordinates of this curve become non-commutative at finite string coupling \(g_s\). It is argued that the natural language to deal with this phenomenon is the \(D\)-module \[8,9\].

In this paper, we study the non-commutative algebraic structure in the mirror B-model side of the topological string on the resolved conifold \(\mathbf{O}(-1) \oplus \mathbf{O}(-1) \to \mathbb{P}^1\). As noticed in \[10\], there is an underlying \(q\)-deformed oscillator (or, \(q\)-oscillator for short) structure in the wavefunction of non-compact D-branes on conifold. We study the representation of \(q\)-oscillators in terms of non-commutative coordinates and show that the mirror curve of conifold appears from the commutation relation of the \(q\)-oscillators.

This paper is organized as follows. In section 2, we construct the \(q\)-oscillators \(A_\pm\) acting on the D-brane wavefunctions in terms of variables obeying the commutation relation \([p, x] = g_s\). In section 3, we show that the commutation relation of \(q\)-oscillators \(A_\pm\) is nothing but the mirror curve of resolved conifold. In section 4, we revisit the computation of the partition function of Chern-Simons theory using the \(q\)-oscillators. We conclude in section 5 with discussion.

1.1. **Our Notations**

Here we summarize our notations and elementary formulas used in the text. We denote the string coupling as \(g_s\), and the Kähler parameter of the base \(\mathbb{P}^1\) of resolved conifold as \(t = g_s N\). Then we introduce the parameters \(q\) and \(Q\) by

\[
q = e^{-g_s}, \quad Q = q^N = e^{-t}.
\]

We also introduce the canonical pair of coordinates \(x\) and \(p\) satisfying

\[
[p, x] = g_s.
\]
We use the representation such that \(x\) acts as a multiplication and \(p\) acts as a derivative \(p = g_s \partial_x\). In particular, when acting on a constant function \(1\), we get

\[
x \cdot 1 = x, \quad p \cdot 1 = 0, \quad e^{ax} \cdot 1 = e^{ax}, \quad e^{bp} \cdot 1 = 1,
\]

where \(a, b\) are c-number parameters. More generally, \(e^{bp}\) shifts \(x\) when acting on a function \(f(x)\)

\[
e^{bp} f(x) = f(x + bg_s).
\]

We frequently use the commutation relations such as

\[
e^{ax + bp} = e^{ax} e^{bp} q^{-\frac{1}{2}ab}, \quad e^{bp} e^{ax} = e^{ax} e^{bp} q^{-ab},
\]

which follow from the relation \(e^A e^B = e^B e^A e^{[A,B]} = e^{A+B} e^{\frac{1}{2}[A,B]}\) which is valid when \([A, B]\) is a c-number.

2. Operator Representation of D-brane Wavefunctions

In this section, we consider an operator representation of the wavefunction of D-branes on conifold. The wavefunction of a D-brane on conifold in the standard framing is given by \([11–17]\)

\[
Z_N(x) = \prod_{k=0}^{\infty} \frac{1 - q^{k + \frac{1}{2}} e^{-x}}{1 - Q q^{k + \frac{1}{2}} e^{-x}} = \prod_{n=1}^{N} (1 - q^{n - \frac{1}{2}} e^{-x}) = \sum_{r=0}^{N} \left[ \frac{N}{r} \right] q^{\frac{r^2}{2}} (-1)^r e^{-rx}, \tag{2.1}
\]

where the \(q\)-binomial is defined as

\[
\left[ \frac{N}{r} \right] = \frac{(q)^N}{(q)_r (q)_{N-r}}, \quad (q)_r = \prod_{n=1}^{r} (1 - q^n). \tag{2.2}
\]

In order to rewrite the wavefunction \(Z_N(x)\) in the operator language, let us recall the \(q\)-binomial formula for the variables \(z\) and \(w\) obeying the relation \(wz = qzw\)

\[
(z + w)^N = \sum_{r=0}^{N} \left[ \frac{N}{r} \right] z^r w^{N-r}. \tag{2.3}
\]

Applying this formula for \(z = -e^{-x+p}\) and \(w = e^p\), we find

\[
(e^p - e^{-x+p})^N = \sum_{r=0}^{N} \left[ \frac{N}{r} \right] (-1)^r e^{rx+rp} e^{(N-r)p} = \sum_{r=0}^{N} \left[ \frac{N}{r} \right] q^{\frac{r^2}{2}} (-1)^r e^{-rx} e^{Np}. \tag{2.4}
\]
In the last step, we used the commutation relation (1.3). By comparing (2.1) and (2.4), we see that the D-brane wavefunction is written as the operator \((e^p - e^{-x+p})^N\) acting on the constant function “1”, according to the rule in (1.3). Namely, the D-brane wavefunction has a simple expression

\[
Z_N(x) = A_+^N \cdot 1 \tag{2.5}
\]

where \(A_+\) is given by

\[
A_+ = e^p - e^{-x+p}. \tag{2.6}
\]

Using the commutation relation (1.3), \(A_+\) is also written as

\[
A_+ = (1 - q^{\frac{1}{2}} e^{-x}) e^p = e^p (1 - q^{\frac{1}{2}} e^{-x}). \tag{2.7}
\]

One can see that the product form of \(Z_N(x)\) in (2.1) easily follows from our simple expression \(Z_N(x) = A_+^N \cdot 1\). By repeatedly using the relation (1.5), we can change the ordering so that \(e^p\) comes to the rightmost position

\[
A_+^2 = (1 - q^{\frac{3}{2}} e^{-x}) e^p (1 - q^{\frac{3}{2}} e^{-x}) e^p = (1 - q^{\frac{3}{2}} e^{-x}) (1 - q^{\frac{1}{2}} + 1 e^{-x}) e^{2p},
\]

\[
A_+^3 = A_+^2 (1 - q^{\frac{3}{2}} e^{-x}) e^p = (1 - q^{\frac{3}{2}} e^{-x}) (1 - q^{\frac{1}{2}} + 1 e^{-x}) (1 - q^{\frac{1}{2}} + 2 e^{-x}) e^{3p},
\]

\[
\vdots
\]

\[
A_+^N = (1 - q^{\frac{3}{2}} e^{-x}) (1 - q^{\frac{1}{2}} + 1 e^{-x}) \cdots (1 - q^{\frac{1}{2}} + N e^{-x}) e^{Np}. \tag{2.8}
\]

When acting on “1”, the last expression of \(A_+^N\) gives the product form of wavefunction in (2.1).

3. \(q\)-Oscillators and D-brane Wavefunctions

In this section, we consider the \(q\)-oscillator structure of the wavefunction \(Z_N(x)\). The \(q\)-oscillator structure for the Rogers-Szegö polynomials and the Stieltjes-Wigert polynomials, which are related to our wavefunction \(Z_N(x)\) by a change of framing [15,16,18], was studied in [19,20].

From the definition \(Z_N(x) = A_+^N \cdot 1\), it follows that \(A_+\) acts as the raising operator

\[
A_+ Z_N(x) = Z_{N+1}(x). \tag{3.1}
\]

Next consider the operator lowering the index of \(Z_N(x)\). From the relation

\[
e^{-p}Z_N(x) = Z_N(x - g_s) = (1 - q^{-\frac{1}{2}} e^{-x}) Z_{N-1}(x), \tag{3.2}
\]
and
\[ Z_N(x) = (1 - q^{N-\frac{1}{2}}e^{-x})Z_{N-1}(x), \quad (3.3) \]

one can see that the operator \( A_- \) defined by
\[ A_- = \frac{q^{\frac{1}{2}}e^x(1 - e^{-p})}{1 - q}, \quad (3.4) \]

lowers the index of \( Z_N(x) \) as desired:
\[ A_- Z_N(x) = \frac{1 - q^N}{1 - q} Z_{N-1}(x). \quad (3.5) \]

Note that \( A_- \) annihilates the constant function “1”
\[ A_- \cdot 1 = 0. \quad (3.6) \]

This implies that the constant function “1” can be identified as the vacuum of \( q \)-oscillator
\[ 1 \leftrightarrow |0\rangle. \quad (3.7) \]

Now let us see that \( A_+ \) and \( A_- \) obey the \( q \)-oscillator algebra. From (3.1) and (3.3), we find
\[ A_+ A_- Z_N(x) = \frac{1 - q^N}{1 - q} A_+ Z_{N-1}(x) = \frac{1 - q^N}{1 - q} Z_N(x), \quad (3.8) \]
\[ A_- A_+ Z_N(x) = A_- Z_{N+1}(x) = \frac{1 - q^{N+1}}{1 - q} Z_N(x). \]

It follows that \( A_+ \) and \( A_- \) satisfy the \( q \)-oscillator algebra
\[ [A_-, A_+] = q^\hat{N}, \quad A_- A_+ - qA_+ A_- = 1. \quad (3.9) \]

Here the operator \( q^\hat{N} \) is defined as
\[ q^{\hat{N}} A_\pm = q^{\pm 1} A_\pm q^{\hat{N}}, \quad q^{\hat{N}} \cdot 1 = 1, \quad q^\hat{N} Z_N(x) = q^N Z_N(x). \quad (3.10) \]

We can directly compute the algebra of \( A_\pm \) using the commutation relations (1.5) without acting them on the wavefunction as above. From the expression of \( A_\pm \) in terms of variables \( x, p \) in (2.6) and (3.4), we can show that \( A_\pm \) satisfy
\[ A_- A_+ = 1 + \frac{q(1 - q^{-\frac{1}{2}}e^x)(1 - e^{-p})}{1 - q}, \]
\[ A_+ A_- = \frac{(1 - q^{-\frac{1}{2}}e^x)(1 - e^{-p})}{1 - q}, \quad (3.11) \]
\[ [A_-, A_+] = 1 - (1 - q^{-\frac{1}{2}}e^x)(1 - e^{-p}), \quad A_- A_+ - qA_+ A_- = 1. \]
Therefore, we arrive at the expression of \( q^\hat{N} \) in terms of \( x \) and \( p \) as
\[
q^\hat{N} = 1 - (1 - q^{-\frac{1}{2}}e^x)(1 - e^p). \tag{3.12}
\]
One can check that the RHS of (3.12) satisfies the defining properties of \( q^\hat{N} \) (3.10). When acting on \( Z_N(x) \), the relation (3.12) leads to the following constraint on the wavefunction \( Z_N(x) \):
\[
1 - (1 - q^{-\frac{1}{2}}e^x)(1 - e^p) = Q. \tag{3.13}
\]
This agrees with the known mirror B-model curve for the conifold [21,22]. In other words, we find an interesting interpretation of the mirror curve (3.13): it represents the \( q \)-oscillator relation \([A_-, A_+] = q^\hat{N}\) written in the canonical variables \( x, p \).

### 3.1. Wavefunction of anti-D-Brane

In contrast to the ordinary oscillator, in the case of \( q \)-oscillator we can consider the formal inverse of \( A_+ = e^p(1 - q^{-\frac{1}{2}}e^{-x}) \)
\[
A_+^{-1} = (1 - q^{-\frac{1}{2}}e^{-x})^{-1}e^{-p}. \tag{3.14}
\]
Repeating the similar calculation as in (2.8), we find
\[
A_+^{-N} = (1 - q^{-\frac{1}{2}}e^{-x})^{-1}e^{-p}(1 - q^{-\frac{1}{2}}e^{-x})^{-1} \cdots (1 - q^{-\frac{1}{2}}e^{-x})^{-1}e^{-p} = (1 - q^{-\frac{1}{2}}e^{-x})^{-1}(1 - q^{-\frac{1}{2}}e^{-x})^{-1} \cdots (1 - q^{-\frac{1}{2}}e^{-x})^{-1}e^{-Np} = \prod_{n=1}^{N} (1 - q^{n-\frac{1}{2}-N}e^{-x})^{-1}e^{-Np}. \tag{3.15}
\]
From this expression, we see that
\[
Z_{-N}(x) = A_+^{-N} \cdot 1 = \frac{1}{Z_N(x-t)}. \tag{3.16}
\]
As argued in [12], this is interpreted as the wavefunction of anti-D-brane, up to a shift of \( x \). \( Z_{-N}(x) \) has another interpretation as the wavefunction of D-brane ending on a different leg of the toric diagram of conifold [13,16,17]. The constraint equation \([A_-, A_+] = q^\hat{N}\) for \( Z_{-N}(x) \) reads
\[
1 - (1 - q^{-\frac{1}{2}}e^x)(1 - e^p) = Q^{-1}, \tag{3.17}
\]
which can be rewritten as
\[
1 - (1 - q^{-\frac{1}{2}}e^{x-t})(1 - e^{-p}) = Q. \tag{3.18}
\]
This is the same form as the equation for \( Z_N(x) \) (3.13) under the replacement \((x, p) \rightarrow (x-t, -p)\). This is consistent with the wavefunction behavior of D-brane amplitude under the change of polarization [23,13].
4. Closed String Partition Function and the $q$-Oscillators

In this section, we consider the partition function of closed topological string on conifold from the viewpoint of $q$-oscillators. Although our computation is essentially the same as [24], we emphasize that the $q$-oscillator structure makes the computation more transparent. There is essentially no new result in this section, but we include this for completeness.

As shown in [25,24], the closed string partition function of conifold is given by the $U(N)$ Chern-Simons theory on $S^3$. Later, it was noticed in [24] that the same partition function is written as the log-normal matrix model

$$Z_{\log} = \int_{N \times N} dM e^{-\frac{1}{2g_s} \text{Tr}(\log M)^2}.$$  \hspace{1cm} (4.1)

The orthogonal polynomial associated with this log-normal measure is known as the Stieltjes-Wigert polynomial $S_N(x)$ [26], which is given by

$$S_N(x) = \sum_{k=0}^{N} \binom{N}{k} q^{k^2 + \frac{1}{2}k} (-1)^k e^{-kx}.$$  \hspace{1cm} (4.2)

In the following we will show that $S_N(x)$ is related to $Z_N(x)$ by the following change of framing

$$x \rightarrow x - p + \frac{1}{2}g_s, \quad p \rightarrow p.$$  \hspace{1cm} (4.3)

This is realized by the conjugation by the operator $U = e^{-\frac{g_s^2}{2} + \frac{1}{2}p}$

$$U \cdot 1 = 1 \quad \leftrightarrow \quad U|0\rangle = |0\rangle.$$  \hspace{1cm} (4.5)

In terms of the conjugated $q$-oscillators

$$B_+ = U A_+ U^{-1} = e^{p} - q^\frac{3}{2} e^{-x + 2p} = e^{p} - q^\frac{3}{2} e^{-x} e^{2p}$$

$$B_- = U A_- U^{-1} = \frac{q^\frac{3}{2} e^{x} (e^{-p} - e^{-2p})}{1 - q}.$$  \hspace{1cm} (4.6)

the Stieltjes-Wigert polynomial is written in the same form as $Z_N(x) = A_+^N \cdot 1$

$$S_N(x) = B_+^N \cdot 1.$$  \hspace{1cm} (4.7)
Using the commutation relation (1.3), one can see that (1.7) agrees with the expression (4.2), as promised

\[
S_N(x) = \sum_{k=0}^{N} \left[ \binom{N}{k} (-q^\frac{k}{2} e^{-x+2p})^k e^{(N-k)p} \right] = 1 = \sum_{k=0}^{N} \left[ \binom{N}{k} q^{k^2 + \frac{k}{2}} (-1)^k e^{-kx} \right].
\] (4.8)

In order to calculate the partition function (4.1), we need the norm of the function \( \Psi_N(x) \) defined by

\[
\Psi_N(x) \equiv \langle \det(e^{-x} - M) \rangle = (-1)^N q^{-N^2 - \frac{N}{2}} S_N(x),
\] (4.9)

which is known as the FZZT wavefunction [23,27]. We see that the \( q \)-oscillator representation (4.7) simplify the computation of the norm.

The log-normal measure associated with (4.1) becomes Gaussian under the change of variable \( y = e^{-x} \)

\[
\int_{0}^{\infty} \frac{dy}{2\pi} e^{-\frac{1}{2} y (\log y)^2} = \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-\frac{x^2}{2y_s} - x}.
\] (4.10)

The inner product with respect to this measure is defined as

\[
\langle f, g \rangle \equiv \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-\frac{x^2}{2y_s} - x} f(x)g(x).
\] (4.11)

Let us consider the adjoint of \( B_+ \) with respect to this measure

\[
\langle f, B_+ g \rangle = \langle (B_+)^\dagger f, g \rangle.
\] (4.12)

Using the representation of \( B_+ \) in terms of \( x, p \) (4.6), the action of \( B_+ \) on a function \( g(x) \) reads

\[
B_+ g(x) = g(x + g_s) - q^{\frac{3}{2}} e^{-x} g(x + 2g_s).
\] (4.13)

Then the inner product \( \langle f, B_+ g \rangle \) is written as

\[
\langle f, B_+ g \rangle = \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-\frac{x^2}{2y_s} - x} f(x) \left[ g(x + g_s) - q^{\frac{3}{2}} e^{-x} g(x + 2g_s) \right]
\]

\[
= \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-\frac{x^2}{2y_s} - x} q^{-\frac{1}{2}} e^{\frac{x}{2}} \left[ f(x - g_s) - f(x - 2g_s) \right] g(x).
\] (4.14)

From this equation, we find that the adjoint of \( B_+ \) is proportional to \( B_- \) (4.6)

\[
(B_+)^\dagger = q^{-\frac{1}{2}} e^{\frac{x}{2}} (e^{-p} - e^{-2p}) = (q^{-1} - 1) B_-.
\] (4.15)
Now it is straightforward to calculate the norm of $S_N(x)$. In order to do that, it is convenient to use the bra-ket notation

$$1 \leftrightarrow |0\rangle$$

$$S_n(x) = B_+^n \cdot 1 \leftrightarrow |n\rangle = B_+^n|0\rangle.$$  \hfill (4.16)

Noticing that $B_-$ satisfies the same relation as $A_-$ (3.3) when acting on the state $|n\rangle$

$$B_-|0\rangle = 0, \quad B_-|n\rangle = \frac{1 - q^n}{1 - q}|n - 1\rangle,$$  \hfill (4.17)

and using the relation between $(B_+)^\dagger$ and $B_-$ (4.15), we find

$$(B_+)^\dagger|n\rangle = q^{-1}(1 - q^n)|n - 1\rangle.$$  \hfill (4.18)

Now let us compute the norm $\langle n|n\rangle$. First, the norm of unit function $1 = |0\rangle$ is given by

$$\langle 0|0\rangle = \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-\frac{x^2}{2gs} - x} = \sqrt{\frac{gs}{2\pi q^{\frac{1}{2}}}}.$$  \hfill (4.19)

The norm of higher state $|n\rangle$ is determined by the following recursion relation

$$\langle n|n\rangle = \langle n - 1|(B_+)^\dagger|n\rangle = q^{-1}(1 - q^n)\langle n - 1|n - 1\rangle.$$  \hfill (4.20)

Finally, the norm of $|n\rangle$ is found to be

$$\langle n|n\rangle = \langle 0|0\rangle q^{-n} \prod_{k=1}^{n} (1 - q^k) = \sqrt{\frac{gs}{2\pi q^{n - \frac{1}{2}}}} \prod_{k=1}^{n} (1 - q^k).$$  \hfill (4.21)

This agrees with the known result of the norm of $S_n(x)$ with respect to the measure (4.10) \cite{26}.

The partition function of matrix model (4.1) is given by the product of the norm of $\Psi_n(x)$ in (4.9)

$$h_n = \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-\frac{x^2}{2gs} - x}\Psi_n(x)^2 = q^{-2n^2 - n}\langle n|n\rangle = \sqrt{\frac{gs}{2\pi q^{2(n + \frac{1}{2})^2}} \prod_{k=1}^{n} (1 - q^k)}.$$  \hfill (4.22)

Then the partition function of matrix model (4.1) is given by

$$Z_{\log} = \prod_{n=0}^{N-1} h_n = \eta(\tilde{q})^N q^{-\frac{N^3}{3} + N} \prod_{n=1}^{\infty} \left(1 - Q q^n \right)^n.$$  \hfill (4.23)
where $\eta(\tilde{q})$ denotes the $\eta$-function

$$
\eta(\tilde{q}) = \sqrt{\frac{g_s}{2\pi}} q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n) = \tilde{q}^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - \tilde{q}^n),
$$

(4.24)

with

$$
\tilde{q} = e^{-\frac{4\pi^2}{g_s}}.
$$

(4.25)

On the other hand, the partition function of the $U(N)$ Chern-Simons theory on $S^3$ is

$$
Z_{CS} = \left(\frac{g_s}{2\pi}\right)^N \prod_{k=1}^{N-1} (q^{-\frac{k}{2}} - q^{\frac{k}{2}})^{N-k} = \eta(\tilde{q})^N q^{-N^3_{12}} + \frac{N}{24} \prod_{n=1}^{\infty} \left(1 - \frac{Q q^n}{1 - q^n}\right)^n,
$$

(4.26)

which agrees with $Z_{\log}$ up to terms in the free energy which are polynomial in $t$.

We note in passing that $Z_{\log}$ is written as the norm of Fermi sea state $|\Psi\rangle$

$$
Z_{\log} \propto \langle \Psi | \Psi \rangle, \quad |\Psi\rangle = \frac{1}{\sqrt{N!}} |0\rangle \wedge |1\rangle \wedge \cdots \wedge |N-1\rangle.
$$

(4.27)

5. Discussion

In this paper, we find that the mirror B-model curve of resolved conifold has an interesting interpretation as the $q$-oscillator relation $[A_-, A_+] = q^N$ itself. It would be interesting to find the physical origin of this algebraic structure.

Recently, the partition function of the Donaldson-Thomas theory of the non-commutative version of conifold is calculated by Szendrői

$$
Z_{NC} = \prod_{n=1}^{\infty} \left(1 - \frac{Q^{-1} q^n}{1 - q^n}\right)^n \prod_{n=1}^{\infty} \left(1 - \frac{Q q^n}{1 - q^n}\right)^n.
$$

(5.1)

The last factor of $Z_{NC}$ in (5.1) is the same as the Chern-Simons partition function in (4.26), but the first factor is different

$$
\eta(q)^N = q^{\frac{N}{24}} \prod_{k=1}^{\infty} (1 - q^k)^N = q^{\frac{N}{24}} \prod_{n=N+1}^{\infty} (1 - Q^{-1} q^n)^n \prod_{n=1}^{\infty} (1 - q^n)^{-n}.
$$

(5.2)

This difference is discussed in the context of the wall-crossing phenomena [30,31,32]. It is tempting to identify the extra factor in $Z_{NC}$ as the effect of anti-D-branes [33]

$$
\prod_{n=1}^{N} (1 - Q^{-1} q^n)^n \sim \prod_{n=1}^{N} \frac{1}{|\langle -n \rangle - n|}.
$$

(5.3)
However, one should not take this relation literally, since both sides of (5.3) vanish when $N$ is an integer. We leave this as an interesting future problem.

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