Research on trajectory smoothing algorithm of dynamic geometry software

Ying Wang¹,², Ruxian Chen¹,², *, Ailing Rao³, Xiaowei Zhong², Yong Huang²

¹Software Engineering Institute of Guangzhou, Guangzhou, China
²Institute of Computing Science and Technology, Guangzhou University, Guangzhou, China
³Taibo Primary school, Zixi, China

*Corresponding author: chrxian@e.gzhu.edu.cn

Abstract. As a kind of mathematical geometry education software, dynamic geometry software is an application software that can interactively create and manipulate geometric objects. Its vivid image changes and colorful geometric objects provide teachers with an efficient Geometry teaching tool. Comparing and analyzing the existing geometry software, we found that there is a curve smoothness problem in its trajectory drawing. In this article, we propose the uniform point selection method based on the principle of the trajectory generation geometric method. Under the premise of ensuring rendering efficiency, extensive case studies have shown that the smoothness of the trajectory curve generated by this method has been improved.

Keywords: Dynamic Geometry Software, Loci Generation, Smoothness.

1. Introduction
Dynamic Geometry Software (DGS) is an educational software that allows users to create geometric objects and perform a series of operations on these geometric objects. In the process of user operation, its geometric object can still maintain the original geometric structure unchanged [1]. When the teacher uses this kind of educational software for geometry teaching, it not only makes the students' observation objects more vivid, but also creates a dynamic experimental environment for the students. The educational value of dynamic geometry software has been affirmed by the mathematics education industry at home and abroad. In the process of its development and application development, many high-efficiency and high-quality software systems have been developed, and they have become practical tools for geometry teaching around the world, such as the domestic super sketchpad [2] and the American geometric sketchpad software "The Geometer's SketchPad" [3], Cabri in France [4], Cinderilla in Sweden [5], GeoGebra in the United States [6] and so on.

In addition to the drawing of basic geometric objects such as points, surfaces, lines and circles, the dynamic geometry software also contains some advanced drawing functions, such as the tracking and geometric transformation of geometric objects. Among them, the trajectory function is a more distinctive function. The trajectory function can not only generate colorful graphics with geometric properties, but also effectively improve the trajectory teaching methods and methods in middle school mathematics.
In geometry, trajectory refers to the figure formed by moving points that meet certain conditions. It has two characteristics: (1) The purity of the trajectory: all points on the trajectory meet the given conditions. (2) The completeness of the trajectory: any point not on the trajectory does not meet the given conditions.

There are two main methods for generating trajectory curve in dynamic geometry software: interactive method and symbolic method. The interactive method refers to the geometric method, and the relative symbolic method is the algebraic method. According to the interactive definition, when the driving point is moving, the user wants to obtain the trajectory of the tracking point movement. The computer obtains the coordinate data of the tracking point in a certain interval through the bottom calculation, and then sequentially connects the coordinate points to draw the trajectory curve. The trace is displayed on the screen. The actual design and development process of dynamic geometry software is realized on the basis of this kind of algebraic geometry calculation. We first use computer processing equipment to perform calculations and deal with algebraic problems at the bottom, and then use computer output equipment to display geometric figures and their relationships. The symbolic method realizes the trajectory function by using algebraic expressions or polynomials to express geometric figures on the basis of the computer screen coordinate system, and is expressed in the form of an implicit equation [12]. Specifically, we use polynomial equations to express the corresponding geometric relationship, calculate the trajectory equation of the tracking point, and then use the curve approximation algorithm to draw the corresponding trajectory curve.

**Table 1** Comparison of trajectory generation methods

| Methods          | Advantages and disadvantages                                                                 |
|------------------|---------------------------------------------------------------------------------------------|
| Geometric method | • Advantages: The realization and principle of the method are simple; the dynamic reaction trajectory generation process has geometric characteristics;  |
|                  | • Disadvantages: the effect of the trajectory curve is general; there needs to be a constraint relationship between the driving point and the driven point; it cannot be used for post-processing; |
| Algebraic method | • Advantages: good curve effect; can be used for subsequent processing; does not require strict relationship between driving point and driven point; |
|                  | • Disadvantages: The realization and principle of the method are relatively complicated; the trajectory after algebraization has no intuitive geometric characteristics; |

By comparing the two methods of geometric interactive method and algebraic notation, the interactive method is relatively simple and easy to implement. It intuitively embodies the characteristics of the "moving point" in the trajectory definition, but the trajectory curve is a linear interpolation of the acquired trajectory points Generated, the curve effect is general. The algebraic notation method has advantages in curve rendering, but the involved mathematical calculations are complicated and difficult to implement. At the same time, because this method transforms geometric figures into algebraic expressions, it cannot intuitively reflect the state of track jog. Therefore, the current dynamic geometry software adopts the geometric method to realize the trajectory drawing.

2. Problem description

From the principle of the geometric method of trajectory generation, it can be known that the generation of trajectory curve in dynamic geometric software is formed by connecting and drawing many trajectory points in sequence. Therefore, if too few trace points are drawn, smoothness problems will occur, that is, a broken line segment will appear when the trace curve is drawn on the device screen. In the figure below, C is a point on D1D2, and then use D1 on circle A and D2 on circle B as the driving points. D1 moves on circle A for two weeks, and D2 moves on circle B for 7 times. The trajectory of C, when point C is close to D1, the curve appears smooth due to the shorter trajectory...
curve. When dragging point C close to D2, because the trajectory curve grows, but the trajectory points do not increase, a broken line segment appears in the trajectory curve.

![Figure 1 Smooth curve and unsmooth curve](image1)

According to the principle of drawing the trajectory by the geometric method, it can be known that when the trajectory points are obtained, the running path of the driving point is equally divided, and then the geometric relationship between the driving point and the driven point is used to obtain several trajectory point data. Finally, we use the linear interpolation method to complete the trajectory drawing. However, the "uniform motion" of the driving point is not necessarily "uniform motion" when it drives the motion of the driven point. This is because when the given trajectory points are too few, the trajectory curve drawn on the computer screen appears as a broken line in some areas, as shown in Figure 2. In the left picture of Figure 2, D is a point on circle O, point A in circle O, T is the intersection of OD and the vertical line of AD, D is the active point as the trajectory of T, and the trajectory of point T is the trajectory of point O and point A is the focal ellipse. In order to explain the problem more clearly, we reduce the total number of track points to 10. During the "uniform movement" of the driving point, the movement of point T is not uniform, with more distribution in some areas and less in other areas. At this time, the smoothness of the curve will appear in the area with fewer track points, as shown in the right figure of Figure 2.

![Figure 2 The uniform movement of the driving point results in a non-smooth curve](image2)

3. Algorithm design

Aiming at the uneven distribution of trajectory points in the geometric method of trajectory generation, when acquiring trajectory points, first take as many points as possible, and then compare the geometric factors such as curvature, angle change, distance between the trajectory points, and remove the too close distance and the angle change. Small trajectory points, so as to filter out a set of trajectory points with a certain range of curvature, and then through linear interpolation, connect each point in sequence
to complete the corresponding trajectory curve. On the basis of not affecting the smoothness of the trajectory curve, the efficiency of trajectory drawing is increased by reducing unnecessary points. The method of uniformly picking points is a method of increasing the number of samples. The steps of the algorithm are as follows:

1: Obtain enough track point samples
2: Calculate the angle change, rate of change and distance between three adjacent track points
3: Determine whether the angle change size, rate of change, and distance are within a given interval
4: Delete eligible track points
5: Linear interpolation method to draw the filtered track points

When acquiring trajectory points, the active point selection is to equally divide its motion path, and then calculate the coordinate information of the trajectory point through geometric relations. We use a screening method from more to less to delete the dense part of the points, so that the track points are relatively evenly distributed. The main goal is to draw as few trajectory points as possible on the premise that the trajectory curve is smooth, so as to improve efficiency.

By calculating the geometric information between adjacent trajectory points, it reflects whether the trajectory points are dense, so as to filter out the trajectory points that can be deleted. When expressing the smoothness of the trajectory curve, three influencing factors are considered: the angle change between consecutive points, the distance, and the curvature of discrete points. The angle change is the change in the slope between two straight lines. The coordinates of two points are known. According to the calculation method of the slope: \[ k = \frac{y_2 - y_1}{x_2 - x_1} \]. The slope can be calculated. Connecting three adjacent points can determine two straight lines. Find the magnitude of the slope change, which is the magnitude of the angles \( \alpha, \beta, \theta \) marked in Figure 3.

![Figure 3](image)

**Figure 3** Changes in the angle between track points

In addition, how fast the angle changes also affects the uniformity of the trajectory points. Since the distance between the trajectory points is small, if \(|\theta - \beta| \approx |\beta - \alpha|\), then the point BCDE distribution is relatively uniform. In addition, the distance between points is easy to find.

The method of uniformly picking points generated by the trajectory curve is as follows:
Uniform point selection method for trajectory curve generation

Input:
The total number of original trajectory points is n+1, the sample array \( P (P_0, P_1, \ldots, P_n) \), given the condition KC

Output: \( (m \leq n) \)
Homogenized trajectory point sample array \( R (R_0, R_1, \ldots, R_m) \)

1: \( i \leftarrow 0 \)
2: \( j \leftarrow 1 \)
3: \( R \leftarrow \{null\} \)
4: \( \text{while } j-2 \leq n \)
5: \( \text{do} \)
6: \( R_i \leftarrow P_i \)
7: \( \text{Calculate the geometric information } G_i \text{ between three consecutive nodes } P_i P_{i+1} \)
8: \( P_{i+2} \)
9: \( \text{if } G_i > KC \text{ then} \)
10: \( \text{delete node } P_{i+1} \text{ adjust node array } P \)
11: \( \text{else} \)
12: \( j++ \)
13: \( i++ \)
14: \( \text{end if} \)
15: \( \text{return } R; \)

4. Experimental analysis
According to the trajectory generation method implemented by the above design, we test in the mobile device simulator. Specifically, we use the control variable method, comparison method, and observation method to compare the effects of different trajectory generation methods in the unified software, and then compare the effects of trajectory curves in different software to verify the effectiveness and feasibility of the optimized method Sex. As shown in the figure, drawing the same graph, the uniform picking method is more smooth than the original method.

![Figure 4](image)

Figure 4 Comparison of test results between the original method and the uniform point method

While improving the smoothness, it is also necessary to consider the issue of drawing efficiency. The above figure is an example to compare the two drawing time-consuming. When there are not many trajectory points, the uniform point selection method consumes more time, but there is no obvious difference in the dynamic operation process. However, as the total number of trajectory points increases, the original method needs to draw more trajectories, and the total time-consuming trend increases obviously.
Figure 5 Time-consuming comparison between the original method and the uniform point method

5. Conclusion
The uniform point selection method proposed in this paper is to divide the motion path of the trajectory points into equal parts, and then calculate the coordinate information of the trajectory points through geometric relations. Aiming at the uneven distribution of the trajectory points, increase the distribution of the sparse part of the trajectory points. Improve the smoothness of the trajectory. Compared with the original method, this method can effectively improve the smoothness of the curve, which has been verified in multiple experiments. But this method has room for improvement, especially in terms of drawing efficiency.

Acknowledgments
This work is supported by the Innovation Projects of Universities in Guangdong Province (No. 2020KTSCX215).

References
[1] Bantchev B B. A brief tour to dynamic geometry software [J]. CaMSP February 2015 Newsletter, 2010.
[2] Hauser K, Ng-Throw-Hing V. Fast smoothing of manipulator trajectories using optimal bounded-acceleration shortcuts[C]//2010 IEEE international conference on robotics and automation. IEEE, 2010: 2493-2498.
[3] The Geometer’s Sketchpad, http://www.dynamicgeometry.com/General_Resources/.html.
[4] Cabri Geometry II, http://www-cabri.imag.fr/cabri2/accueil-e.php
[5] Cinderella, http://www.cinderella.de/tiki-index.php
[6] GeoGebra, http://www.geogebra.org
[7] Li X, Sun Z, Cao D, et al. Real-time trajectory planning for autonomous urban driving: Framework, algorithms, and verifications[J]. IEEE/ASME Transactions on mechatronics, 2015, 21(2): 740-753.
[8] Rauch H E, Tung F, Striebel C T. Maximum likelihood estimates of linear dynamic systems[J]. AIAA journal, 1965, 3(8): 1445-1450.
[9] Vakanski A, Mantegh I, Irish A, et al. Trajectory learning for robot programming by demonstration using hidden Markov model and dynamic time warping[J]. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 2012, 42(4): 1039-1052.
[10] Botana F. Interactive versus Symbolic Approaches to Plane Loci Generation in Dynamic Geometry Environments[C]// International Conference on Computational Science. Springer-Verlag, 2002:211-218.
[11] Liu Y, James J Q, Kang J, et al. Privacy-preserving traffic flow prediction: A federated learning approach[J]. IEEE Internet of Things Journal, 2020, 7(8): 7751-7763.
[12] Kaess M, Ranganathan A, Dellaert F. iSAM: Incremental smoothing and mapping[J]. IEEE Transactions on Robotics, 2008, 24(6): 1365-1378.