On the possibility of a maximum fundamental density and the elimination of gravitational singularities.

Gustaf Rydbeck
Onsala Space Observatory
Onsala 43497
Sweden
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With this note we want to point out that already in the early days of cosmology it was understood that negative pressure could eliminate gravitational singularities in a natural way e.g. E.B. Gliner, Sov. Phys. JETP 22(1966)378 and M.A. Markov, Pis’ma Zh. Eksp. Teor. Fiz. 36, No 6, 214-216 (20 Sept. 1982). Today, with the discovery of dark energy and the strong evidence in favor of an inflationary start of the Big Bang, the existence of negative pressure is widely accepted. In fundamental physics, phase transitions are generally thought to be reversible (Cf. Ellis, New Astronomy Reviews Volume 46, Issue 11, October 2002, P. 645). It seems likely then that if inflation has occurred, the process should be reversible. I.e. when the increasing density in a collapsing universe or star reaches a certain limit it should go through a phase transition to a medium with an equation of state of the type \( p = \omega \rho \), where \(-1 < \omega < -1/3\). If this phase transition is fundamental, i.e. occurs for all energy densities, a collapse will always reach a minimum radius and bounce. If the phase transition is symmetric, the result will lead to oscillating universes. If however the phase transition is associated with an hysteresis effect, a collapsing star may, succeeding the bounce inflate into a new universe with a subsequent phase transition becomes dominated by ordinary relativistic matter. The aim of this note is study the time development of a model which mimics this process.

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I. INTRODUCTION

If a theory contains singularities, it seems likely that the theory is inconsistent or incomplete (cf Brandenberger [2006]). The singularities which appear in gravitational theories result if densities become infinite. It seems possible and natural then that at some high density \( \rho = \rho_{\text{lim}} \) limit, all energy densities transforms to a medium with an equation of state.

\[
p = \omega \rho_{\text{lim}}
\]

\(-1 < \omega < -1/3\)

The idea is not new. It was proposed by Sakharov [1966] and Gliner suggested in the same year that there could be a high density limit (cf I. Dymnikova and E. Galaktionov [2007]). The idea was again discussed by Markov [1982] and further by Frolov et al. [1990]. A similar route to eliminate singularities is to introduce high curvature modifications in Einsteins equations (cf Mukhanov and Brandenberger [1992] and Brandenberger et al. [1993]). The possibility that quantum effects may cause a collapsing space to bounce as it reaches extreme curvatures has been discussed for many years (Cf Smolin [1994], Martinec [1994], Vereshchagin [2004 a], Singh et al. [2006], Vereshchagin [2007] and Easson and Brandenberger [2008]).

It appears more or less proven that dark energy with \( \omega \approx -1 \) exists. It is thought that inflation was caused by a medium also with \( \omega \approx -1 \) but with much higher density. What is not so appealing is the notion that inflation was preceded by by a singularity (Turner [2008]). The singularity arises naturally if it is assumed that that the inflationary medium is of 'false vacuum' type, since radiative excitations again becomes dominant as the scale factor continues to decrease as we go back in time or becomes dominant in a collapse phase (cf Vereshchagin [2004 a]).

Since media with \( \omega \approx -1 \) is now widely accepted, the existence of a medium with a phase transformation as suggested above seems both likely and appealing. We shall therefore, as Gliner did, hypothesize that something opposite to vacuum exists, a maximum density limit. It is often claimed that the elimination of gravitational singularities requires a change in the the structure of General Relativity (cf page 79 Giovanni [2008]). As is shown here however, an upper density limit gets rid of gravitational singularities in a way which leaves the equations of general relativity unchanged, equations which by now have been tested to quite a high accuracy (cf. e.g. van Straten et. al [2001], Bertotti et. al [2003], and Kramer et.al [2006]). The high curvature modifications referred to above are of course beyond direct observational tests. Since there are indications that the cosmological constant has really been constant (cf Opher and Pelinson [2005]), it seems that the there might as well be a lower limit to the total energy density of space.

Calculations based on present candidates for fundamental field theories such as string or M theory and quantum loop theory appear extremely complex. Yet the corresponding classical (i.e. non-quantum) theories, e.g. Newtons law of gravitation, are usually quite simple. One could therefore expect that the "classical physics version" of the birth of our universe may be describable in terms of a "classical" model equation of state containing a "simple" transition from relativistic energy density to "inflationary like" energy-density where the transition is accompanied by a "hysteresis" effect (section IV). Since a model with a transition to "soft" inflation i.e. with an \( \omega > -1 \) produces to small universes the model transition is limited to case where \( \omega \) approaches -1 for high densities.
II. THE FRIEDMANN EQUATION

In general the pressure \( P_i \) of the medium \( i \) is related to its density \( \rho_i \) by the equation of state
\[
P_i = \omega_i \rho_i c^2
\]  
(2)

If we assume that a volume \( V \) is related to a scale factor \( a \) as \( V \sim a^3 \) and that the density is related to the volume as
\[
\rho = \rho_0 \left( \frac{V_0}{V} \right)^{n/3} = \rho_0 \left( \frac{a_0}{a} \right)^n
\]  
(3)

then \( \omega = n/3 - 1 \). If \( a \) is the scale-factor in an isotropic and homogeneous universe then the time development of \( a \) is given by the Friedmann equation
\[
\ddot{a}^2 = \mathcal{G} \rho a^2
\]  
(4)

where \( \mathcal{G} = 8\pi G/3 \) and \( G \) is the gravitational constant. \( \rho_i \) are the various energy densities. The densities in the equation are the curvature energy density \( \rho_{cu} \), the cold matter energy density \( \rho_m \), the radiative or relativistic energy density \( \rho_{re} \), the dark energy density \( \rho_{de} \) and "inflationary" energy density \( \rho_{ie} \). Of these densities it is generally thought that only the curvature energy density can be negative. The densities may be related to a given time \( t = t_1 \), where \( a_1 = a(t_1) \). For the well-known densities we have that
\[
\begin{align*}
\rho_{cu} &= \rho_{cu,1}(a_1/a)^2 \\
\rho_m &= \rho_{m,1}(a_1/a)^3 \\
\rho_{re} &= \rho_{re,1}(a_1/a)^4
\end{align*}
\]  
(5)

The curvature density \( \rho_{cu,1} \) is related to the curvature \( \kappa_1 \) by
\[
\rho_{cu,1} = -\frac{c^2 \kappa_1}{\mathcal{G}} = -\frac{c^2}{\mathcal{G} r^2_0}
\]  
(6)

If \( \rho_{cu,1} = \rho_0 = \rho_p \), the planck density then the corresponding curvature radius is \( r_0 = \frac{\sqrt{\mathcal{G}}}{l_p} \) where \( l_p \) is the planck length. As already mentioned (cf Opher and Belinsky [2005]), recent observational results indicate that \( \omega_{de} = -1 \), i.e. that the density
\[
\rho_{de} = \rho_{de,1}
\]  
(7)

is constant. The right hand side of eqn. 6 may be considered as a negative gravitational potential energy. It is obvious then that if \( n > 2 \) or \( \omega > -1/3 \) the corresponding potential approaches minus infinity as \( a \rightarrow 0 \), i.e. if the corresponding density is dominant, the universe will "fall" to a point singularity. If \( n < 2 \), the potential approaches minus infinity as \( a \rightarrow \infty \), i.e. if the corresponding density is dominant the universe will "fall" or inflate towards larger scales. If \( n < 0 \), the energy density will increase with \( a \). This energy is called phantom energy. One might further note that if \( n < 2 \) and the curvature energy density is negative, the scale-factor \( a \) is always positive so a singularity cannot develop.

III. THE DE SITTER UNIVERSE WITH POSITIVE CURVATURE

The density components are in this case limited to a constant inflation driving part \( \rho_{ie} \) and a negative curvature part \( \rho_{cu} \) so that the Friedmann equation becomes
\[
\ddot{a}^2 = \mathcal{G}(\rho_{ie,0} a^2 + \rho_{cu,0} a^2)
\]  
(8)

The solution to eqn. 8 is
\[
a(t) = a_0/2 \left( \exp(t/t_0) + \exp(-t/t_0) \right)
\]  
(9)

where an arbitrary time constant is chosen so that \( a(t) \) has the minimum \( a(t) \) for \( t=0 \).
\[
t_0 = (\mathcal{G} \rho_{ie,0})^{-1/2}
\]  
(10)

Further
\[
\begin{align*}
\rho_{ie,0} &= \rho_{ie,1} \\
\rho_{cu,0} &= -\rho_{cu,1}
\end{align*}
\]  
(11)

where \( \rho_{cu,0} \) is the curvature density at \( t = 0 \). The radius of curvature at \( t=0 \) is
\[
\mathcal{R} = \sqrt{-c^2 \mathcal{G} \rho_{cu,0}}
\]  
(12)

This de Sitter Universe with a finite positive curvature term has the advantage that it does not start from a singularity but has it’s origin in a previous collapse and it describes an inflationary origin which is how we think our universe started.

IV. A MODEL FOR THE CREATION OF A UNIVERSE FROM A COLLAPSING STAR

We shall now give a simple model of the process we have discussed above. The major difference between the previous proposals discussed in the introduction and ours is the introduction of “friction”, i.e. a term which depends on the expansion velocity. The term introduces true time evolution to the model, which without this term would just oscillate eternally. Consider a collapsing star where the central region is uniform and isotropic. The time evolution of the metric of this center can then be described by the Friedmann equation (MTW [1973], page 852 eqn. 32.11). The iterative form of the very simple model equation of state for matter at high density used in our computer code is
\[
\rho_{hd,2} = \rho_{hd,1} \left( \frac{a_1}{a_2} \right)^{\left[\frac{(1-\frac{\rho_{hd,1}}{\rho_{lim}})(1-\frac{\rho_{hd,1}}{\rho_{lim}})}{\left(\frac{\rho_{hd,1}}{\rho_{lim}}\right)^2} \right]}
\]  
(13)

where \( \rho_{hd} \) is the density of the relativistic high density medium and \( \rho_{lim} \) the high density limit. \( v_{lim} \) is the velocity at the event horizon distance \( a_{lim} \) of the limit density. Note that the event horizon distance at the density \( \rho_{hd} \) is larger, i.e. \( \frac{\rho_{hd}}{\rho_{lim}} < 1 \) if \( \rho_{hd} < \rho_{lim} \). \( a_1 \) and \( a_2 \) are supposed to be differentially close,
i.e. \( a_2 = a_1 + da \) and \( \rho_{\text{hd},2} = \rho_{\text{hd},1} + d\rho_{\text{hd}} \). The term \((1 - \frac{\omega_\text{c}(a)}{\rho_{\lim}})\) assures that density stops increasing as \( \rho_{\lim} \) is approached. The term \((1 - \gamma \frac{\rho}{\rho_{\lim}})\), where \( c \) is the speed of light is a hysteresis term, i.e. an expansion velocity dependent "friction" term. This term may cause enormous inflation and entropy increase. The "natural" value for the \( \gamma \) constant would be one. This would cause the inflation to be truly enormous (the Hubble time would be tiny in comparison to the duration of the inflation) so we have introduced the \( \gamma \) parameter to limit inflation, but \( \gamma = 1.0 \) is still possible. Turok’s string driven inflation (cf. Turok [1988]) does lead to a density limit, but does not seem to include a velocity dependence, while intuitively I would have expected such a term.

We may now use the Friedmann equation (13) to replace \( \frac{\rho_{\text{c}}}{c} \) in eqn. (13) by \( -\left(\frac{\rho}{\rho_{\text{p}}}ight)^{1/2} \), where \( \pm \) is negative under contraction and positive under expansion.

\[
\rho_{\text{hd},2} = \rho_{\text{hd},1} + a_2 \left(1 - \frac{\omega_\text{c}(a)}{\rho_{\text{lim}}}ight) \left(1 + \gamma \left(\frac{\rho}{\rho_{\text{lim}}}\right)^{1/2}\right)
\]  

(14)

Note that if \( \rho_{\text{hd}} \ll \rho_{\text{lim}} \) the usual equation of state for relativistic densities is obtained. The differential form of eqn. (14) is

\[
\frac{d\rho_{\text{hd}}}{\rho_{\text{hd}}} = -\frac{da}{a}\left(\frac{\rho_{\text{lim}}}{\rho_{\text{p}}}ight) \left(1 + \gamma \left(\frac{\rho}{\rho_{\text{lim}}}\right)^{1/2}\right)
\]

(15)

The pressure becomes

\[
p = \frac{4}{3} \left(1 - \frac{\rho_{\text{hd}}}{\rho_{\text{lim}}}ight) \left(1 + \frac{\rho}{\rho_{\text{lim}}}ight) - 1 \rho_{\text{hd}} c^2
\]

(16)

Assuming that the limit density is the Planck density, the above equation leads to a cosmology described by figure 1 and 2. We have here chosen \( \gamma = 0.8 \). Our model only treats the central part of the collapsing star. The geometry and the time-evolution of the space away from the center where the density is not isotropic and homogeneous may follow a complex evolution which depends on transition region constraints. If we assume however that the central region is sufficiently large then density or rarefaction waves will not have had time to reach the inner region before the expansion velocity of the corresponding scale-factor is larger than \( c \), the speed of light. What happens outside this inner region then becomes unimportant to the expanding cosmological space. It would of course be interesting to know how it connects to the center of the collapsed star, appearing like a black hole from the outside.

Although our model concerns a special case, we think that any collapsing star or "small" universe will necessarily convert to a contracting "near de Sitter space" which then inflates to large scales approximately as described by the model.

It is interesting that for different input densities \( \rho_{\text{hd},1} \) and \( \rho_{\text{hd},2} \) at the same input radius, the output density becomes the same for times larger than \( \sim 1000 \) Planck times while the output radii are approximately related like \( \frac{a_1}{a_2} = \frac{\rho_{\text{hd},1}}{\rho_{\text{hd},2}} \). The reason for this is that an increased input density makes the universe stay slightly longer in the inflationary mode while it leaves this mode with \( \rho_{\text{hd}} \approx \rho_{\text{lim}} \). It would thus appear that our model produces extreme large and very homogeneous and isotropic expanding universes.

Curvature fluctuations of quantum mechanical origin can be produced and inflated to astronomical scales in the "standard way" (cf. e.g. Taylor and Rowan-Robinson [1992] or Lyth and Riotto [1998]).
V. DISCUSSION

It has been suggested that the total energy density of space may have an upper and a lower bound, and that the approach to the upper limit depends on the expansion velocity. The upper bound and the velocity dependence would remove future singularities from collapsing stars, and instead lead to the formation of an exponentially, or probably many, expanding homogeneous and isotropic spaces, i.e. inflationary like situations. Seen from outside the collapsing star a black hole would still form. It is only in the microscopic center of the black hole that things would be different. How this inflating center connects to the surrounding space would require detailed physical calculations which are beyond the scope of this note. Our Universe could be one result from such a collapse. Unfortunately the possibility to experimentally or observationally prove our suggestion seems exceedingly small.

If the initial collapsing star is small, the ensuing universe might not expand sufficiently for the dark energy density to overtake the negative curvature density. The universe will then re-collapse and go through a second bounce. The output universe will now grow to considerably larger radii so that the dark energy term can become dominant. The end product would thus always be a more and more empty and accelerating universe dominated by dark energy.

The removal of singularities would put the problem of vanishing information at the center of black holes (see e.g. Giusto and Mathur [2006]) in a different perspective. I.e. the information is not necessarily destroyed at the center of the black hole, it might instead be expelled into expanding "new spaces" (see Esson and Brandenberger [2008]).

In one particular aspect our proposition contains "new physics" and that is that the medium in the high density limit cannot be "excitable" since the excitation energy might push the total energy density past the density limit (Cf Vereshchagin [2004]). The medium can only be "de-excited" and should perhaps be thought of as a large cosmological constant. Note that this does not imply any limits to curvature fluctuations which would then be the sole information propagator.

Particle physics would concern excitations between an upper and a lower density limit. Markov [1982] in fact suggests that such limits should be a guiding principle in the search for the fundamental field theory. Both string theory (cf Turok [1988]) and quantum loop theory (cf Singh [2006] and Vereshchagin [2007]) do seem to contain upper density limits but may lead to other problems (Cailleteau et al. [2008]) and are both far from settled theories. Will they also lead to a "friction" term?, the term that induces "life" to space, i.e. a space that eternally renews itself by the creation of new inflating universes in the centers of collapsing stars.

Many others have employed negative pressure to avoid the formation of the central singularity in a black hole eg. Mbonye and Kazanas [2005].

VI. SUMMARY

We have pointed out that

- gravitational singularities are naturally eliminated by the introduction of a fundamental high density limit leading to a phase transition to a medium with negative pressure as this limit is approached.

- instead of the formation of singularities at the center of a collapsing star new inflating universes will form. These universes inflate to cosmological scales if a hysteresis effect is introduced in the phase transition.

- the upper density limit and the hysteresis effect leaves the equations of general relativity intact and keeps the energy-density positive, i.e. the weak energy condition is satisfied.

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