Dynamic Scaling and Two-Dimensional High-$T_c$ Superconductors

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There has been ongoing debate over the critical behavior of two-dimensional superconductors; in particular for high $T_c$ superconductors. The conventional view is that a Kosterlitz-Thouless-Berezinskii transition occurs as long as finite size effects do not obscure the transition. However, there have been recent suggestions that a different transition actually occurs which incorporates aspects of both the dynamic scaling theory of Fisher, Fisher, and Huse and the Kosterlitz-Thouless-Berezinskii transition. Of general interest is that this modified transition apparently has a universal dynamic critical exponent. Some have countered that this apparent universal behavior is rooted in a newly proposed finite-size scaling theory; one that also incorporates scaling and conventional two-dimensional theory. To investigate these issues we study DC voltage versus current data of a 12A thick YBa$_2$Cu$_3$O$_{7-\delta}$ film. We find that the newly proposed scaling theories have intrinsic flexibility that is relevant to the analysis of the experiments. In particular, the data scale according to the modified transition for arbitrarily defined critical temperatures between 0 K and 19.5 K, and the temperature range of a successful scaling collapse is related directly to the sensitivity of the measurement. This implies that the apparent universal exponent is due to the intrinsic flexibility rather than some real physical property. To address this intrinsic flexibility, we propose a criterion which would give conclusive evidence for phase transitions in two-dimensional superconductors. We conclude by reviewing results to see if our criterion is satisfied.

I. INTRODUCTION

There have been many reports of a two-dimensional (2D) Kosterlitz-Thouless-Berezinskii (KTB) phase transition in the cuprate superconductors. Repaci et al. studied the “most two dimensional” high temperature superconducting sample possible, a single unit cell thick film of YBa$_2$Cu$_3$O$_{7-\delta}$. They concluded that there was no phase transition because a population of unbound vortices exists in this system well below the temperature where a conventional KTB transition would occur. This conclusion was shown to be in agreement with theory because the requirement that the sample be much smaller than the perpendicular penetration depth was not met.

In contradiction, Pierson et al. and Ammirata et al. reanalyzed Repaci et al.’s data (along with those from others) by using the general scaling ideas of Fisher, Fisher, and Huse (FFH) and concluded that finite-size effects do not obscure the KTB transition. They further concluded that important details of the KTB dynamics were not included in its original formulation for superconductors by Halperin and Nelson. In this paper we will refer to this Pierson and Ammirata analysis as the modified KTB scaling analysis.

Later theoretical work by Colonna-Romano et al. and Medvedyeva et al. and Holzer et al. suggests that the KTB transition can be obscured by finite-size effects. This work, in distinction from the earlier scaling analysis, seems to support Repaci et al.’s original conclusions. That being said, all of these theoretical works do however find it intriguing that the modified KTB scaling analysis produces the large value of $z \approx 6$ for a variety of 2D systems.

In addition, Medvedyeva et al. and have proposed a finite-size scaling form that is also based on FFH-scaling applied to 2D superconductors. They argue that it is this finite-size scaling, as opposed to the modified KTB scaling, that is the correct scaling form for the data of Repaci et al. They further argue that under certain conditions their finite-size scaling is of the same form as the modified KTB scaling analysis of Pierson and Ammirata. Since this would mean that the modified KTB transition only appears to occur while their finite-size scaling is the correct physical description, Medvedyeva et al. label this a “ghost” transition.

The question is: Does the reoccurring large value of $z \approx 6$ suggest a common origin; possibly resulting from a modified KTB transition or perhaps from a “ghost” transition based on finite-size scaling features in the vicinity of a KTB transition?

The work we present here indicates that the results of $z \approx 6$ from the modified KTB scaling analysis are not linked to fundamental aspects of a KTB transition. This is due to the fact that an opposite concavity criterion, similar to one proposed for 3D transitions in magnetic fields by Strachan et al., is not satisfied by the experimental data. Instead, we argue that the agreement between the many systems analyzed by Pierson et al. and Ammirata et al. in Refs. 17 and 14 (having $z \approx 6$) stems...
from the fact that a single experimental data set has a tremendous amount of flexibility when analyzed through modified KTB scaling.

In our work, we have carefully re-analyzed Repaci’s data. We are able to successfully collapse the experimental data to the modified KTB scaling theory using a wide range of critical temperatures and exponents; something discussed in the theoretical work of Medvedyeva et al. only above the transition but which we investigate in depth on experimental measurements at all temperatures. In fact, we find that this modified KTB scaling is still achievable when the critical temperature is defined to lie well outside of the temperature range of the measurements and for z values much larger than 6. This indicates that a single data set could be found to agree with any sufficiently large z value, which could easily explain the agreement amongst the various 2D systems. We also find that the modified KTB scaling analysis is further weakened by its strong dependence on the voltage resolution limit of the experiment, as was theoretically demonstrated by Holzer et al. and in much the same way that the conventional vortex glass scaling is.

In distinction from modified KTB scaling, we find that a “ghost” transition based on the finite-size scaling of Medvedyeva et al. does not contain the same flexibility in determining the z. Despite this possible success of dynamic-scaling applied to a 2D superconductor, we find that the Medvedyeva et al. finite-size scaling theory entails its own flexibility and we argue that the experimental data fall within this realm of flexibility.

In a related issue, Medvedyeva et al. argue that Repaci et al.’s measurements show vestiges of a conventional KTB transition; one that would have occurred in the film had finite-size effects not obscured it. They determine this obscured KTB transition to be at a temperature 10K higher than the modified transition temperature of Pierson et al. and Ammirata et al., a view that is in accord with the original conclusions of Repaci et al.

We address this issue in Sec. VI, where we use our opposite concavity criterion to motivate a criterion for determining a universal jump in the super-electron density at a conventional KTB transition. We demonstrate the need for our criterion by showing how one might be led to determine that a KTB transition exists, surprisingly, directly between the modified TKT (of Pierson et al. and Ammirata et al.) and the higher-temperature obscured TKT (of Medvedyeva et al.). We argue that this perplexing predicament can be clarified by using our KTB concavity criterion. We conclude by reviewing theoretical and experimental results to see if they satisfy our KTB criterion. We find intriguing agreement with only one experimental data set in the literature on a high-Tc film and we compare this to the measurements of Repaci et al.

II. SCALING APPLIED TO THE KOSTERLITZ-THOULESS-BEREZINSKII TRANSITION

The dynamic scaling approach of Pierson et al. and Ammirata et al. is based on work by Fisher, Fisher, and Huse (FFH). FFH predict that the voltage, V, across a superconductor with an applied current, I, at temperature, T, should vary as

\[ V = I \xi^{D-2-z} \chi_{\pm} \left( \frac{\xi^{D-1}}{T} \right), \]  

(2.1)

where ξ is the superconducting coherence length, D is the dimensionality, and z is the dynamic exponent. The two unspecified functions, χ±, apply above (+) and below (−) the transition temperature TC.

Repaci’s 12Å thick film is two dimensional; for two dimensions Eq. (2.1) becomes

\[ V = I \xi^{-z} \chi_{\pm} \left( \frac{\xi}{T} \right) = I \xi^{-z} \chi_{\pm} (x). \]  

(2.2)

If a factor of (Iξ/T)z is factored out of χ± and the remaining function is named ε±, we can rewrite Eq. (2.2) as

\[ \frac{I}{T} \left( \frac{I}{V} \right)^{1/z} = \varepsilon_{\pm} (I / T). \]  

(2.3)

Eq. (2.3) is sometimes preferred for analysis of data because ξ, which is expected to diverge at TC, is present only in the argument to the scaling function, ε±.

At TC the right-hand side of Eq. (2.3) becomes ε±(∞) since ξ diverges. For an applied I the measured V should be some non-zero finite value at TC, so that the left-hand side of Eq. (2.3) is a non-zero finite value at TC. Therefore, ε±(∞) must also be non-zero and finite at TC. Furthermore, for a non-zero applied current the left-hand side of Eq. (2.3) should be continuous and smooth since non-analytic points, i.e., critical points, are only approached as I → 0. This requires that ε+=∞ = ε−∞ = A, where A is a non-zero finite constant. By setting T = TC and substituting A in for the right-hand side of Eq. (2.3) we can solve for the I − V relation

\[ V \propto I^{z+1}, \]  

(2.4)

valid at TC. It is important to note that Eq. (2.4) is also valid at temperatures and currents which make the argument, x, of ε±(x) large since this causes Eq. (2.3) to go to the same limiting form. Above TC, χ+ is expected to be constant in the x → 0 limit of Eq. (2.2). In this limit, Eq. (2.2) becomes

\[ \frac{V}{I} = R_L \propto \xi^{-z}. \]  

(2.5)
Having reviewed the well-known FFH scaling results for \( D = 2 \), we now discuss a proposed connection between them and the KTB transition.

According to KTB theory one expects power law \( I - V \) relations for \( T < T_{KT} \) of the form \( V \sim I^{n(T)} \) due to current induced unbinding of vortex pairs. This occurs because vortex pairs with size greater than \( r_c \) and under the influence of an applied 2D current density, \( J_{2D} \), are repelled rather than attracted, where (in SI units)

\[
 r_c = \frac{\Phi_0}{2\pi(\pi r_c^2)\mu_0 J_{2D} \lambda_L}. \tag{2.6}
\]

In the above relation, \( \Phi_0 \) is the flux quantum, \( \lambda_L \) is the bare (unrenormalized) penetration depth for a thin film, \( \mu_0 \) is the permeability of vacuum, \( \epsilon_0 \) is the permittivity of vacuum, and \( \epsilon_\infty \approx \epsilon(r_c) \) is the dielectric constant which takes into account screening of pairs of size \( r_c \) by other much smaller pairs. As long as \( r_c \) is large enough such that \( \epsilon(r_c) \) is approximately constant, i.e., \( \epsilon_\infty \), one expects \( V \sim T^{\alpha(T)} \).

Harris et al.\textsuperscript{31} and, recently, Pierson et al.\textsuperscript{17} and Ammirata et al.\textsuperscript{14} have proposed using the KTB correlation length,

\[
 \xi = e^{\sqrt{b/\left[(T - T_{KT})/T_{KT}\right]}}, \tag{2.7}
\]

in FFH’s results with \( T_{KT} \) replacing \( T_c \). This is a surprising proposition because the KTB correlation length only has the form of Eq. (2.7) above \( T_{KT} \), \( \xi_+ \approx e^{\sqrt{b/\left[(T - T_{KT})/T_{KT}\right]}}, \) which we demonstrate with the finite-size scaling of Medvedyeva et al.\textsuperscript{25} One of the main points of that work is the apparent existence of the scaling form

\[
 \frac{V}{IR(T)} = h(ILg_L(T)), \tag{2.10}
\]

for smaller values of \( ILg_L(T) \). In this relation, \( R(T) \) is the linear resistance of the low current ohmic tail at temperature \( T \), \( L \) is the size of the 2D sample, and \( g_L(T) \) is an unspecified function of \( T \) and \( L \) that permits a data collapse.

Having outlined the pertinent theory and shown the incompatibility between KTB theory and FFH scaling through Eq. (2.9), we will now demonstrate how the modified KTB scaling analysis may lead one to believe that the two theories are compatible.

### III. THE MODIFIED KTB SCALING ANALYSIS

Fig. 1 shows a log – log plot of Repaci et al.’s\textsuperscript{20} \( I - V \) isotherms from a 12Å thick laser ablated \( YBa_2Cu_3O_{7-\delta} \) film. Straight lines on this plot indicate power law behavior, with the power equal to the slope of the line. The dashed line on the left has a slope of 1, characteristic of ohmic response. Note that at low currents many isotherms have ohmic tails.

Following the analysis of Pierson et al.\textsuperscript{17} and Ammirata et al.\textsuperscript{14}, the solid line labelled “\( T_{KT} \)” (Modified \( T_{KT} \)) in Fig. 1 is a fit to the isotherm which seems to separate those curves with ohmic tails from those without. Pierson et al. and Ammirata et al. define this isotherm to be the critical temperature, \( T_{KT} \approx 17.6\,\text{K} \).

We note that \( T_{KT}^M \) occurs when \( V \approx I^{6.9} \) in the data. This disagrees with the conventional KTB analysis,\textsuperscript{23} where the critical isotherm is cubic, \( V \approx I^3 \), assuming that the length probed by the applied current is large enough so that \( \epsilon(r) \) is completely renormalized. The only isotherm of Fig. 1 which is cubic at high currents is near 27K, denoted as \( T_{KT}^H \), which we demonstrate with the solid line fit (at high currents) with slope = 3. However, this isotherm has an ohmic tail at low currents, a signature of free vortices. Medvedyeva et al.\textsuperscript{25} have recently suggested that a conventional KTB transition would have
occurred near this $T_{KT}^H$ if finite-size effects had not obscured it; a view in accord with the original arguments of Repaci et al.\textsuperscript{20}

Repaci et al. originally argued\textsuperscript{20} that the finite 2D penetration depth caused these free vortices below $T_{KT}^H$. This is supported by Eq. (2.6), assuming that the finite penetration depth becomes important once $r_c \approx \lambda_{\perp}$. The ohmic tails of Fig. 1 set in roughly when $I \approx 1 \times 10^{-4} \text{A}$, which corresponds to $J_{2D} = \frac{I}{W} \approx \frac{1 \times 10^{-4} \text{A}}{2 \times 10^{-3} \text{m}}$, where $W$ is the $200 \mu\text{m}$ width of the film. A low density of vortex pairs implies $\epsilon(r_c) \approx \epsilon_{\infty} \approx \epsilon_0$, so that we can use Eq. (2.6) with $\epsilon_0$ replacing $\epsilon_{\infty}$. Substituting $\lambda_{\perp}$ for $r_c$ in Eq. (2.6), we find $\lambda_{\perp} \approx 23 \mu\text{m}$, which is a reasonable (order of magnitude) value for a unit cell thick YBCO film.

Power law dependence towards ohmic behavior. Pierson et al.\textsuperscript{17} and Ammirata et al.\textsuperscript{14} argued that the sample is driven normal at these high currents and so they omit these data (denoted as smaller symbols in Fig. 1) in their analysis. This could be a reasonable assumption if the critical current density, $J_c$, for Repaci’s film is assumed equal to the value for a $12 \mu\text{m}$ thick single crystal\textsuperscript{15} ($J_c \approx 10^{10} \frac{\text{A}}{\text{m}^2}$). This gives $I_c \approx 2 \times 10^{-3} \text{A}$, which is approximately where the high current down turn takes place. Following Pierson’s and Ammirata’s prescription, we will also omit these data from the analysis of this paper.

To find $b$ (Eq. (2.7)) from theory one uses Eq. (2.5) in the form

$$\log_{10}(R_L) = \{-z\sqrt{b}\log_{10}(e)\} \sqrt{T_{KT}^M/[T - T_{KT}^M]} + C,$$

with $C$ a constant. $\log_{10}(R_L)$ vs. $\sqrt{T_{KT}^M/[T - T_{KT}^M]}$ with $T_{KT}^M = 17.6K$ is plotted in the inset of Fig. 2(a). According to modified KTB scaling theory this should be linear, which it is, with slope $-z\sqrt{b}\log_{10}(e)$.

Using the three parameters, $T_{KT}^M$, $b$, and $z$, Refs. 17 and 14 demonstrated that Repaci’s data collapse to Eq. (2.3). [They also allowed for slight variation of these parameters in order to optimize the scaling collapse. This accounts for the slight deviation of the power-law fit (using their $z$ exponent) from the data in Fig. 1.]

The scaling collapse is shown in Fig. 2(a), where the higher current data is towards the right and the lower temperatures are higher along the vertical axis. All measurements taken below 17.6K fall on the higher of the two curves ($\varepsilon_-$), while the rest are described by $\varepsilon_+$. There are two regions where the data seem to deviate slightly from the scaling functions; these are indicated with arrows in Fig. 2(a). The solid arrow points to data which seem to bend away from the scaling function at high currents. It is possible to argue that not enough of the high current data (where, presumably, the sample is driven normal) have been discarded from the analysis, as discussed above. The same may be argued for the region indicated in Fig. 2(a) by the dashed arrow. Thus, one might conclude that there is good agreement with a modified KTB scaling theory since the low current data seems to scale. This was the conclusion in Refs. 17 and 14.

**IV. FURTHER ANALYSIS**

When the data of Fig. 1 are examined closely, a problem with the modified scaling analysis becomes apparent. $T_{KT}^M$ occurs at the beginning of the region where the voltmeter’s sensitivity is no longer adequate to follow the $I - V$ curves down to currents sufficiently low for detecting the ohmic tail. Since this ohmic tail is taken

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{\textit{I} - \textit{V} isotherms for a 12A thick YBa$_2$Cu$_3$O$_7$-$\delta$ film (from Ref. 20). The dashed line has a slope of one, while the solid line labelled $T_{KT}$ represents the critical power law $I - V$ curve according to the analysis of Refs. 14 and 17. The solid diamonds are the $I_{so}$ values where the isotherms change from non-ohmic to ohmic behavior.}
\end{figure}

Pierson et al.\textsuperscript{17} and Ammirata et al.\textsuperscript{14} however, proposed that this same data is well described by their modified KTB scaling theory. Since this modified analysis relaxes the constraint on the power of the critical isotherm (\textit{i.e.} the $z$ exponent), their modified $T_{KT}$ ($T_{KT}^M$) can occur at much lower temperatures. Thus, Pierson et al.\textsuperscript{17} and Ammirata et al.\textsuperscript{14} argued that the ohmic tails below 27K result from $T$ being above their modified $T_{KT}^M$, rather than a finite-size effect.

A close look at the $I - V$ curves in Fig. 1 reveals that at high currents the isotherms near $T_{KT}^M$ bend away from the form

$$\log_{10}(R_L) = \{-z\sqrt{b}\log_{10}(e)\} \sqrt{T_{KT}^M/[T - T_{KT}^M]} + C,$$
as evidence for being above $T_{KT}^M$ in the modified KTB theory, its detection is crucial in determining the critical temperature.

To quantify this, we determine the current, $I_{co}$, at which the crossover from non-ohmic to ohmic behavior occurs as a function of temperature. $I_{co}$ is defined as the point where the slope is a fraction, $f$, between the maximum slope of the isotherm and the minimum. (The minimum is 1 for isotherms with an ohmic tail.) We chose $f = 0.9$ although other choices, such as $f = 0.7$, do not make significant changes (with $I_{co}$ lowering only by a factor of about 1.7).

The solid diamonds in Fig. 1 are these crossover currents. The $I_{co}$ below 32K are roughly constant and slightly greater than $10^{-4}$A. This is important because it is the current where Pierson et al.’s$^{17}$ and Ammirata et al.’s$^{14}$ critical isotherm of Fig. 1 meets the 1nV resolution level of the voltmeter. A straight line through these points intersects the critical isotherm of Refs. 17 and 14 at the resolution limit of the experiment. (Repaci et al.$^{20}$ realized this as well, which they demonstrated through plots in their Fig. 3.)

There are two possible explanations for this. The first is that $T_{KT}^M$ is, by remarkable coincidence, precisely the temperature where the voltmeter runs out of resolution, and the $I - V$ curves below 17.6K do not have ohmic tails. The second is that the curves below 17.6K have ohmic tails beyond the voltmeter’s resolution. The latter implies that there is no phase transition at 17.6K, and the agreement between Repaci et al.’s data and the modified KTB transition is due to the modified scaling analysis being much too flexible.

A. Further analysis for $T_{KT}^M < 17.6K$

To distinguish between these two possibilities, we reanalyze the data using several lower values of modified $T_{KT}^M$. We find the $z$ value at each $T_{KT}^M$ by setting the slope of the steepest portion of that isotherm equal to $z + 1$ (Eq. (2.4)). Once $z$ is found for a modified $T_{KT}^M$, we determine a value for $b$ exactly as is done in Sec. III. As with Pierson et al.’s$^{17}$ and Ammirata et al.’s$^{14}$ analysis, we allow for a slight change in $b$ and $z$ in order to optimize the data collapse.

Starting with a $T_{KT}^M$ of 13.5K, we find $z = 8.5$ and $b = 14.5$. For this $T_{KT}^M$ all the data that scales in Fig. 2(a) also scales in Fig. 2(b). The only difference in the scaling collapse is that some data from $\varepsilon_-$ have shifted to $\varepsilon_+$. As $T_{KT}^M$ is further lowered, more data is shifted. When $T_{KT}^M$ reaches the low temperature limit of the experiment, at about 10K (Fig. 2(c)), there ceases to be anything falling on the scaling function $\varepsilon_-$, which is expected since there is no further data below this temperature.

We can lower $T_{KT}$ still further and still obtain a data collapse. These scaling collapses are accompanied by much larger $z$ and $b$ values, as is noted in Fig. 2.

B. Further analysis of ohmic tails

The inset of Fig. 2(b) shows the $R_L$ plot for $T_{KT}^M = 13.5K$. In this plot $\log_{10}(R_L)$ deviates from linearity at about 33K, denoted $T_d$. (Note that higher temperatures are located towards the upper left.) Since linear depen-
dence of $\log_{10}(R_L)$ vs. $(T_{KT}^M/(T - T_{KT}^M))^{1/2}$ is only expected near $T_{KT}^M$, where critical fluctuations become important, this does not imply a deviation from the modified KTB scaling theory. In fact, the region of temperature ($\Delta T_{KT} = T_d - T_{KT}^M$) where critical fluctuations are important according to the $\log_{10}(R_L)$ plots in Fig. 2 is relatively independent of the choice of $T_{KT}^M$.

We will denote the lowest temperature at which $R_L$ is measurable as $T_{cut}$, which is 23K for Repaci’s measurements. Although Eq. (3.1) is non-analytic at $T_{KT}^M$, it is analytic at $T_{cut} > T_{KT}^M$. Thus, we can Taylor expand Eq. (3.1) about $T_{cut}$ as

$$\log_{10}(R_L) = C - a_0(T_{cut} - T_{KT}^M)^{-\frac{1}{2}} + \frac{a_0}{2}(T_{cut} - T_{KT}^M)^{-\frac{3}{2}}(T - T_{cut})$$

$$- \frac{3a_0}{8}(T_{cut} - T_{KT}^M)^{-\frac{5}{2}}(T - T_{cut})^2 + \cdots$$

$$a_0 = z\sqrt{bI_{KT}}\log_{10}(e).$$

(4.2)

Given any arbitrary analytic function of temperature for the linear $I - V$ behavior, $f_L(T)$, of the form

$$\log_{10}(f_L(T)) = c_0 + c_1(T - T_{cut}) + c_2(T - T_{cut})^2 + \cdots,$$

this can be recast into the form of Eq. (4.1), as follows.

First we note that regardless of the values of $T_{KT}^M$, $z$, $T_{cut}$, and $b$, we can always find a $C$ such that $c_0 = C - a_0(T_{cut} - T_{KT}^M)^{-\frac{1}{2}}$. Likewise, for any $T_{KT}^M$, $z$, and $T_{cut}$ we can always find a $b$ which permits $c_1 = \frac{a_0}{2}(T_{cut} - T_{KT}^M)^{-\frac{3}{2}}$.

Since this agreement will exist only close to $T_{KT}^M$, which is the theoretical expectation, this test will always confirm KTB scaling, modified or conventional. (This is the flexibility in this analysis implied in the discussion by Mooij.36)

The other three parameters are nominally determined directly through the measurements, as is the case for $T_{KT}^M$ and $z$, or its limits, as with $T_{cut}$. However, these three parameters have some flexibility, especially when selecting only certain cuts of the data (as was done at high currents with Pierson et al. and Ammirata et al.’s analysis of Repaci’s data) or when performing “eye ball” data collapses where $T_{KT}^M$ and $z$ can both be varied independently. This flexibility allows for better apparent agreement with scaling theory over an even larger temperature range.

Having demonstrated the large flexibility of the modified scaling analysis for $T_{KT}^M < 17.6K$, we now examine higher transition temperatures, $T_{KT}^M > 17.6K$.

C. Further analysis for $T_{KT}^M > 17.6K$

Fig. 3(a) shows the results of scaling for $T_{KT}^M = 21K$. The collapse is much worse than those presented in Fig. 2. As the modified $T_{KT}^M$ is increased the data collapse becomes even worse, as is seen in Fig. 3(b) for $T_{KT}^M = 24K$. The deviations shown in Figs. 3(a) and 3(b) cannot be attributed to driving the sample normal since these departures all occur for the lowest currents. These data clearly do not scale for higher modified $T_{KT}^M$.

![Fig. 3](image-url)

FIG. 3. (a) and (b) show failed scaling collapses for $T_{KT}^M$ equal to 21 and 24K respectively. (c) and (d) show the same scaling attempts when resolution limits are imposed upon the data. (c) has a voltage floor of 100nV imposed while (d) has one at the 10µV level.
However, when we remove the lowest voltage data shown in Fig. 1, to mimic a less sensitive experiment, and repeat the scaling analysis, we find that the highest transition temperature for which collapse occurs increases. This is shown in Figs. 3(c) and 3(d) where the measurement floors are set to 100nV and 10µV, respectively. For the 100nV cutoff, the highest \( T_{KT}^{M} \) which gives good collapse is about 21K while for the 10µV cutoff it is raised up to 24K. Measurements with even worse sensitivity would allow us to define still higher transition temperatures.

### D. Implications of further analysis

The preceding discussion in this section shows that the modified scaling analysis alone is insufficiently restrictive. Any trial value of \( T_{KT}^{M} \) below 17.6K yields a good scaling collapse, and higher \( T_{KT}^{M} \) would also work if the voltmeter were less sensitive. This has important implications because it could easily account for the apparent universal result of \( z \approx 6 \) that Pierson et al.\(^{17}\) and Ammirata et al.\(^{14}\) have determined on various systems, and in which there has been much recent interest.\(^{24–27,32}\)

These implications can be understood if we consider a comparison of several hypothetical measurements of the same sample made with varying sensitivities. If one would attempt to attribute a low universal value of \( z \) to all these measurements it would be found that the less sensitive ones would show good agreement with scaling (like those of Figs. 2(a-d), 3(c), and 3(d)) while the more sensitive measurements would not (like those of Figs. 3(a), and 3(b)). However, as the value of \( z \) is increased, one would arrive at values that could collapse all the measurements and universality could be claimed amongst the various measurements. If various 2D-superconducting systems have the same flexibility of analysis that we find for the Repaci et al. data (a completely reasonable assumption) then the same arguments could be applied amongst measurements made on the various systems. Thus, by choosing a sufficiently large \( z \) value, our analysis implies that one should always be able to obtain universal agreement amongst different measurements and systems.

Our analysis in this paper investigates in depth the single measurement of Repaci et al. It would therefore be an interesting test of our hypothesis to see the results of other groups rigorously examining the flexibility in analysis of their data.

Having demonstrated that relying on the modified KTB scaling alone is clearly unsatisfactory, we now investigate the alternative finite-size scaling method of Medvedyeva et al.\(^{25}\) Eq. (2.10); a scaling method that also applies FFH ideas to the KTB transition.

### E. Finite-size scaling

To examine the data through Medvedyeva et al.\(^{25}\) theory we will ignore the size dependance of the scaling since the measurements are made on the same sample. We thus rewrite Eq. (2.10) as

\[
\frac{V}{IR(T)} = h(Ig(T)), \quad (4.4)
\]

with \( g(T) \) unspecified by the theory. In the work of Medvedyeva et al. they determine that a \( g(T) \) given by

\[
g(T) \propto R(T)^{-\alpha}, \quad (4.5)
\]

with \( \alpha \approx 1/6 \) provides a good collapse of the Repaci data in the low current regime. We have reproduced this scaling collapse in Fig. 4(a), where the solid lines are Repaci et al.\’s data at 23.0K and above.

Although this seems to show good agreement with the scaling theory, there is intrinsic flexibility in this analysis. To demonstrate this flexibility, we suppose we have any group of non-linear \( I - V \) isotherms made on the same sample which may or may not be applicable to the scaling theory of Medvedyeva et al. The simplest scenario would be that the \( I - V \) curves at any single temperature are analytic and can be expanded in the form

\[
V = R(T)I + R_3(T)I^3 + R_5(T)I^5 + \ldots, \quad (4.6)
\]

where we have kept only odd terms in the expansion since the simplest behavior is to assume the voltage changes sign upon reversing the applied current. (We further point out that the measurement techniques of Repaci et al. assume this antisymmetry.)

We can recast Eq. (4.6) into the form of Eq. (4.4) by dividing by \( R(T)I \), which yields

\[
\frac{V}{IR(T)} = 1 + \left( I \sqrt{\frac{R_3(T)}{R(T)}} \right)^2 + \left( \frac{R_5(T)}{R(T)} \right) I^4 + \ldots \quad (4.7)
\]

The above relation will always satisfy the Medvedyeva et al. scaling requirements at low currents up to the second term in the expansion by identifying \( \sqrt{\frac{R_3(T)}{R(T)}} \) as \( g(T) \). Since the form of \( g(T) \) is completely unspecified by the scaling, it can always be determined by \( \sqrt{\frac{R_3(T)}{R(T)}} \) and thus this finite-size scaling should always hold for analytic \( I - V \) behavior.

To test whether this is the sort of trivial agreement being seen in the data collapse of Fig. 4(a), we superimpose a function of the form \( \frac{V}{IR(T)} = 1 + cI^2 \), which is simply the first two terms of the expansion of Eqs. (4.6) and (4.7); with \( c \) a constant fitting parameter that determines the unspecified \( g(T) \). Clearly, this function fits the data collapse well and indicates that the apparent success of this scaling could be due to the intrinsic flexibility of the analysis.
A better test of the Medvedyeva et al. scaling is to subtract one from \( V_{IR} \). Since the low current region of the data collapse is expected to be one this should give a more sensitive examination of the data at small currents, i.e., the only region where the Medvedyeva et al. finite-size scaling is expected to be valid. This is plotted in Fig. 4(b), where we find that the region over which the collapse is achieved goes approximately as \( I^2 \); i.e., parallel to the solid line. This means that we are essentially just collapsing the second (i.e., quadratic) term of the expansion in Eq. (4.7). This is further indication that the successful scaling is due to trivial analytic behavior as in Eq. (4.6) or Eq. (4.7).

Having noted that, we point out that Medvedyeva et al. originally proposed that their scaling is essentially the same as the modified KTB scaling analysis under certain conditions, most notably that \( R(T) \) behaves according to Eq. (4.5) with \( \alpha \) a temperature independent constant equal to \( 1/\zeta \). (This analogy is of course only valid experimentally in the low current and high temperature regime due to the need for a low current linear resistance value in Eq. (2.10).) Under these conditions Medvedyeva et al. argue that their finite-size scaling behavior can appear to have a transition. They call this a “ghost” transition because it should only appear to occur whereas they argue that their finite-size scaling is actually the correct physical description.

The fact that we find no significant agreement between the Medvedyeva et al. finite-size scaling and the measurements analyzed here means support for a “ghost” transition is also lacking experimentally; since the latter depends on the existence of the former. That said, we find however that the parameters of an assumed “ghost” transition are not flexible when we analyze the measurements. By using the various scaling tests discussed in Ref. 25 on the Repaci data, we find that the value of \( 1/\alpha \) in Eq. (4.5) is well restricted to lie between five and eight. The fact that the second term in the Taylor expansion of Eq. (4.7) has this restricted behavior may indicate some sort of underlying physical basis. Further studies on experiments with varying sizes might be suitable for investigating this possibility, as one could utilize the full finite-size scaling form of Eq. (2.10), as has been done for the simulations of Refs. 25 and 26.

If the Medvedyeva et al. scaling had succeeded in Fig. 4 over regions that the simple analytic behavior could not have described, then this would have been evidence for non-trivial agreement with theory. Thus, this is a sort of criterion that could be used to determine whether the finite-size scaling of Medvedyeva et al. is non-trivial.

Having motivated a criterion for the finite-size scaling, in the next section we will address the subtle issue of a criterion for determining a non-trivial modified KTB transition. This criterion will later be used in Sec. VI to motivate a different criterion for determining the existence of a conventional KTB transition.

### V. AN EXPERIMENTAL CRITERION FOR OBSERVING THE MODIFIED KTB TRANSITION

In the previous section we showed that data collapses and \( R_L \) fits alone are not sufficient to indicate a modified KTB phase transition. It remains a possibility that such a phase transition is present, but the scaling analysis does not uniquely determine it. What is needed is an unambiguous signature for a phase transition—something to differentiate between true and false scaling agreements.

Each isotherm in Fig. 1 collapses onto only a small portion of \( \varepsilon_+ \) in Fig. 2(a). In the low current direction of the collapses the isotherms are cut off by the sensitivity floor of the experiment. For the region below the sensitivity floor, one of two possibilities would occur if more
sensitive measurements were made. These measurements would either collapse onto the scaling function, indicating a real transition, or deviate, indicating that the original collapse was simply a product of the experimental resolution.

To see what a real transition would look like, we can use the scaling functions obtained from higher-voltage data to predict behavior at voltages smaller than the experimental resolution. We do this by choosing a temperature $T$, which specifies the appropriate scaling function to use from a data collapse, like the one in Fig. 2(a). Since $T_{KT}^M$ and $b$ are found from the previous fits, choosing a value of $I$ determines $I/\xi T$, and thus specifies a point on the $x$-axis. This $x$ value and the scaling function determine a $y$ value. The only unknown in $y$ is $V$, which is thus determined. We emphasize that if $I$ is large enough, this procedure just returns the measured value of $V$, but if $I$ is small enough, it gives an extrapolated small value for $V$ which is beyond the resolution of the voltmeter. To perform this extrapolation we used a data collapse with $z = 6$, which is the value suggested by Pier-son et al. The other two parameters which collapse the data for this $z$ are $T_{KT}^M = 18.183K$ and $b = 5.37$.

The results of the extrapolation are shown in Fig. 5(a). Our first observation is that the extrapolated data from a modified KTB scaling analysis do not show the $V \sim I^{n(T)}$ behavior expected below $T_{KT}^M$ down to $10^{-20}V$. Instead, we find that the voltage goes to zero faster than a power of $I$ as $I \to 0$, clearly apparent in the negative concavity for $T < T_{KT}$. This is further testament to the incompatibility of conventional KTB theory and FFH scaling, as outlined earlier in this paper.

Following the arguments of Ref. 28, we observe next that the extrapolated curves display a signature not seen in the measured data. Isotherms with equal |$(T - T_{KT}^M)/T_{KT}^M$| have opposite concavities at the same current level. We demonstrate this in Fig. 5(a). The dashed vertical line (constant current) is drawn between isothersms 1.5K on either side of the critical temperature. The lines tangent to these isothersms clearly show the opposite concavity. Two other pairs of isothersms are also shown in Fig. 5(a) with opposite concavities at the same current levels. Fig. 5(b) clearly demonstrates that a true data collapse should show this property whether the voltage resolution is $10^{-20}$ or at $10^{-12}V$. In either case $T_{KT}^M$ would be restricted to lie well within the two inner curves at 16.7K and 19.7K, and would thus be independent of the resolution limit of the experiment.

Repaci et al.’s data in Fig. 1 has positive concavity in all the $I - V$ isothersms above 19.5K (and over a 12.5K range) at approximately $1.5 \times 10^{-4}A$. For a transition to exist, this trend must cease since, below $T_{KT}^M$, isothersms are not expected to have ohmic tails. To show that this trend ceases it is necessary to see negative concavity for an isotherm below $T_{KT}^M$, while one above, at the same current, and with equal |$(T - T_{KT}^M)/T_{KT}^M$| has a positive concavity. We require that the relevant temperature scale of the transition, |$(T - T_{KT}^M)/T_{KT}^M$|, be equal since both isothersms must be in the critical region, excluding the possibility of comparing critical to non-critical behavior.

![Fig. 5](image-url) Simulated log($V$) vs. log($I$) isothersms. The temperatures of the isothersms are marked in (a) with a dashed vertical line drawn between one of the pairs. The other dashed lines in (a) are tangents drawn to these isothersms at the location of intersection with the vertical marker. (b) shows the same curves but with a sensitivity floor equal to 1pV.

Repaci et al.’s data shows no evidence of this necessary signature about $T_{KT}^M$. We conclude that the measurements are not consistent with scaling.

Having proposed a criterion for determining a modified KTB transition, we use it in the next section to help motivate a criterion for determining a conventional KTB transition.

### VI. Applicability of Analysis to Conventional KTB Transition

The quintessential characteristic of a KTB transition is the universal jump in the fully renormalized superfluid density from
at $T_{KT}$ to zero at $T_{KT}^{-}$ (see for example Ref. 37). In the above equation, $n_{\text{C}}^{*}$ is the bare Cooper pair density per area while $m^{*}$ is the Cooper pair mass.

This jump has been reported in many superconducting systems for both finite frequency and DC $I - V$ measurements. However, the jump is only expected in the limit of $\omega \rightarrow 0$ and $I \rightarrow 0$. At finite frequencies and currents the jump is somewhat rounded which makes determination of the KTB transition less obvious. In fact, it has recently been suggested that many reports of the KTB transition do not show the universal jump or transition at all.

A. Motivating a KTB concavity criterion

To address this concern, we propose a criterion for determining whether this jump exists (as opposed to a possible finite size effect) in DC $I - V$ measurements.

We motivate a criterion by employing the one developed in the last section with one change. Since a KTB transition predicts $V \propto I^{2(T)}$ below $T_{KT}$, we expect that the below $T_{KT}$ isotherms have zero concavity on a log–log plot at the same applied currents, while ones above have positive concavity. We will refer to this criterion as the “KTB concavity criterion,” to distinguish it from the “opposite concavity criterion,” of the last section and Ref. 28. This KTB concavity criterion is further supported by a consideration of the important length scales of the problem, as we will show below in Sec VI C.

B. What is the need for the KTB criterion?

First we will consider the case of a finite-size effect at temperatures much below $T_{KT}$. In this regime there should not be a cubic $I - V$ power law because we are far below the real $T_{KT}$ for an infinite 2D sample. However, we will now argue that evidence for a cubic $I - V$ power law will always be found in this finite size dominated regime despite the fact that a KTB transition does not exist here.

In this low temperature regime we would expect $V \propto I^{2(T)}$ at all finite applied currents such that $\lambda_{L} > r_{c} > \xi_{GL}$, where $\xi_{GL}$ is the approximate normal core size and also the smallest separation for a bound pair. $V \propto I^{2(T)}$ because, as $T$ is lowered below $T_{KT}$, renormalization effects due to fluctuating vortex pairs diminishes quite rapidly and so $\epsilon(r_{c}) \approx \epsilon_{\infty} \approx \epsilon_{0}$.

At sufficiently low temperatures, as the current is lowered the measurement floor is reached before $r_{c}$ becomes comparable in size to $\lambda_{L}$ or the sample size. Therefore, in this regime we only expect to find the power-law dependence expected below $T_{KT}$. As higher temperature $I - V$ curves are investigated, but still below the true $T_{KT}$, lower currents produce measurable voltages.

According to Eq. (2.6), this means that larger length scales could be probed, i.e., larger $r_{c}$. When $r_{c}$ becomes comparable to $\lambda_{L}$ or the sample size we would expect ohmic tails due to the existence of free vortices. Since these finite-size induced ohmic tails will emerge smoothly (due to the statistical nature of unbinding) from isotherms with $a(T) > 3$, one must necessarily pass an $I - V$ curve which seems to have the power of 3. Thus, one would conclude that a rounded jump in $\rho_{\sigma \sigma} / \epsilon_{\infty}$ is measured; the sort typical of a true KTB transition, even though no true transition occurs.

1. Illustrating need for criterion with example

This is illustrated in Fig. 1 by fitting the power of 2.9 (i.e., approximately 3) to the low-current regime of the curve at 22.5K ($T_{KT}$). In Fig. 6 we show a blow up of Fig. 1 with power law fits to the low current portions of the $I - V$ curves. Over three decades in voltage we seem to have very good power-law fits, as expected for the KTB transition. We draw further connection with the conventional KTB analysis by plotting in Fig. 7 the $a(T)$ derived from these power-law fits. Remarkably, both plots seem to indicate, in the usual way, that a universal jump in the superconducting electron density occurs.

![Graph showing the blow up of Fig. 1 with power law fits](attachment:image.png)
This is a significant observation in that the temperature where we have just determined a conventional KTB transition to lie (22.5K) is midway between 17.6K ($T_{KT}^M$) where Pierson et al.\cite{pierson17} and Ammirata et al.\cite{ammirata14} argue for a modified KTB transition, and 27K ($T_{KT}^2$) where Medvedyeva et al.\cite{medvedyeva25} argue that a conventional KTB transition would have existed had finite-size effects not obscured it. Furthermore, according to Repaci et al. and Medvedyeva et al. the regime we focus on in Fig. 6 is dominated by finite size effects and should not support a conventional KTB transition. Thus, despite the fact that various scenarios have been proposed that are incompatible with a KTB transition at 22.5K, we do in fact find very good agreement with theory for one. In light of this perplexing situation we ask if there are other criterions that we could use to determine whether our agreement with KTB theory is in fact valid?

2. **How about other criterions?**

To start with, the universal jump condition shown in Fig. 7 is clearly not sufficient as we have just seen. Another possibility is the fit of the linear resistive tails; however, we have already explained in Sec. IV B that these fits are far too flexible. A third possibility is the fit of the linear resistive tails; Fig. 7 is clearly not sufficient as we have just seen. Another argument in support of the KTB concavity criterion

Since finite-size effects should dominate when $v_0$ is roughly of size $\lambda_\perp$, we can substitute $\lambda_\perp$ into Eq. (2.6) and solve for the current where we cross over to these effects, i.e.

$$j_{co}^{\lambda_\perp} \approx \frac{\Phi_0}{2\pi\mu_0\lambda_\perp^2}. \quad (6.2)$$

Typically we should expect $\lambda_\perp$ to be only weakly temperature dependent in this regime and, thus, we should also expect $j_{co}^{\lambda_\perp}$ to be roughly constant as a function of temperature. In the case where $\lambda_\perp$ is temperature dependent, $\lambda_\perp$ should grow as temperature increases. Thus, we conclude that $j_{co}^{\lambda_\perp}$ will decrease or remain constant as $T$ increases for finite-size induced unbinding.

For a true KTB transition, where the ohmic tails are due to a finite correlation length when $T > T_{KT}$, $j_{co}^{KT}$ should have the completely opposite temperature dependence, i.e., increasing as $T$ grows. A rough argument supporting this goes as follows:

At temperatures sufficiently close to and above $T_{KT}$, we expect changes to occur at large length scales determined by the diverging $\xi_\perp$. That is, the largest pairs are the first to unbind in going up through $T_{KT}$. Thus, when probing at sufficiently small scales, i.e., with sufficiently large $j_{2D}$ and for $T > T_{KT}$, we should not expect large deviations from the $E \sim j_{2D}^3$ behavior expected at $T_{KT}$. We approximate this behavior near $T_{KT}$ for large driving currents as

$$E \approx C_{Ej}j_{2D}^3, \quad (6.3)$$

where $C_{Ej}$ is a constant and $E$ is the electric field. However, we also expect the temperature dependence of the ohmic tails to vary as

$$\frac{E}{j_{2D}} = \rho \approx \frac{\rho_0}{\xi_\perp^2} = \rho_0 e^{-2\sqrt{b(T-K_T)}}, \quad (6.4)$$

according to Eq. (2.5) with $z = 2$. Eliminating $E$ between Eqs. (6.3) and (6.4), we obtain a rough estimate for the crossover current between power law and ohmic behavior very close to $T_{KT}$, i.e.,

$$j_{co}^{KT} \approx \left(\frac{C_{Ej}}{\rho_0}e^{-2\sqrt{b(T-K_T)}}\right). \quad (6.5)$$
This equation says that this crossover should occur at increasing values of current as the temperature is raised above $T_{KT}$. In the crossover regime between power law and ohmic $\log V - \log I$ curves we expect positive concavity. Since the extent of this regime will increase with temperature there should be some isotherms with $T < T_{KT}$ which have zero concavity at the same applied current. This is the extension of the concavity criterion applied to a true KTB transition discussed earlier in this section.

On the other hand, the crossover and its associated positive concavity due to finite size effects would occur at either a constant applied current or a decreasing current as temperature is raised above $T_{KT}$. This implies that we would not see positive concavity for higher temperature isotherms while lower ones maintain zero concavity. The occurrence of either behavior can be used to determine the cause of the ohmic tails. Again, since the relevant temperature scale is $\left| \frac{T - T_{KT}}{T_{KT}} \right|$ we should be comparing isotherms above and below $T_{KT}$ with this value equal, not to confuse critical with non-critical behavior.

Despite our rough arguments, it is instructive to take another look at the $I_c$ plotted on Fig. 1. At low temperatures we see that the crossover current is roughly constant as a function of temperature. According to our arguments this would indicate that this results from a finite size effect, in agreement with Repaci et al.’s conclusion. Furthermore, as temperature is raised above about 32 K, we begin to clearly see that this crossover current increases. Interestingly, the increase in $I_c$ sets in roughly where the high current regime of the $I - V$ curves has a power of approximately 3, i.e., $T \approx T_{KT}^h \approx 27.5 K$. These observations give further support to Repaci et al.’s and Medvedyeva et al.’s view that finite-size effects obstruct the KTB transition from occurring.

D. Putting the KTB concavity criterion to the test

As a test of our criterion we review a theoretical work and three simulations which report $I - V$ isotherms near a KTB transition. First we consider the renormalization-group analysis of current induced vortex pair unbinding by Sujani et al. The results of their analysis, which is based on a model without finite-size effects included, clearly demonstrate a positive concavity for higher temperature isotherms (see Fig. 5(b) in Ref. 52), while isotherms below $T_{KT}$ and with equal $\left| \frac{T - T_{KT}}{T_{KT}} \right|$ do not show concavity.

We next consider the simulations of $I - V$ curves by Holzer et al. which contain both a true KTB transition plus finite size effects. At higher currents their $I - V$ curves are dominated by characteristics of a true KTB transition. In this regime we see that our KTB concavity criterion is clearly satisfied. However, at lower currents their $I - V$ characteristics are dominated by size effects and the concavity criterion is no longer satisfied.

A third test is the numerical simulations of Colonna-Romano et al. Like the $I - V$ curves of Holzer et al., they too show true KTB behavior at high currents and finite-size effects at lower currents. Again, in the KTB regime our criterion is satisfied while in the finite-size dominated region it is not.

Our final check uses the $I - V$ simulations of Medvedyeva et al. Like the simulations of Colonna-Romano et al. and Holzer et al., these demonstrate finite-size effects at low currents which do not satisfy the KTB concavity criterion. At high currents, there is a slight opposite concavity which may be due to the saturation of pair unbinding. That is, all the pairs tend to be unbound at high currents, which should cause $I - V$s to become ohmic in this regime and thus have negative concavity.

Based on these four works, it seems that evidence of a KTB transition is found when the KTB concavity criterion is satisfied. Having motivated the KTB concavity criterion and found support for it from theoretical studies and simulated $I - V$ curves, we now review the literature to see if it is satisfied experimentally.

1. Testing experimental results with criterion

First we review those reports of a KTB transition in conventional 2D superconductors and Josephson junction arrays. In this case we find that the criterion is clearly satisfied for the superconductors in Refs. 46 and 45 and for the arrays in Ref. 44, while for many other reports the criterion is not satisfied. Thus, our criterion is consistent with a KTB transition in at least some conventional type II superconductors and arrays.

Having shown that the KTB concavity criterion can be experimentally satisfied, we now address the cuprate superconductors.

Whether a KTB transition exists in 2D cuprates has been an unresolved issue (see discussion in Ref. 36). Furthermore, many papers report $I - V$ critical isotherms with $a(T_{KT})$ greater than 6, which is essentially the basis for Pierson et al.’s proposal that $z > 5$. However, we find only one report of $I - V$ measurements of a 2D cuprate superconductor, Ref. 12, which satisfies our criterion.

To demonstrate the difference between the data of Ref. 12 and the those of Repaci et al. we plot $d \log V/d \log I$ vs current for both sets in Fig. 8. In Fig. 8(a) we make this plot for the $I - V$ regime of Repaci et al.’s data shown in Fig. 6. Despite the fact that the typical KTB analysis shown in Fig. 7 indicates there is a transition, these data fail our KTB concavity criterion. This is evident from Fig. 8(a) where all the $d \log V/d \log I$ have positive slope (i.e., positive concavity for $I - V$ curves) both above and below the $T_{KT}$ determined through Figs. 6 and 7. This is in contrast to a similar plot for the data of Ref. 12 in Fig. 8(b). Here we see that the low-temperature
$d \log V / d \log I$ plots level off between three and four; a value in reasonable accord with the conventional KTB theory.\textsuperscript{23} This could explain the poor fit of this data to the modified KTB scaling analysis (see Ref. 17) which had been analyzed using the unusually large power at the critical isotherm of approximately 7, as opposed to 3.

![Graph](image)

FIG. 8. (a) are the derivatives of the $I-V$ curves in the regime examined in Fig. 6. (b) are the derivatives\textsuperscript{35} from $I-V$ measurements of Ref. 12.

We also emphasize that the axes of Figs. 8(a) and 8(b) have the same span; $d \log V / d \log I$ goes from one to six while $\log_{10} I$ spans 1.8. The only difference in the two plots is that the range of Matsuda et al.’s data (Ref. 17) is centered at higher applied currents. Since one generally expects the length scale probed to be smaller for larger applied current densities, this may simply indicate that the data of Matsuda et al. is in a regime where finite-size effects are not probed.

### VII. SUMMARY

We find that the modified KTB scaling analysis\textsuperscript{14,17} gives inconclusive results when applied to measurements of a 12Å thick YBa$_2$Cu$_3$O$_{7-\delta}$ film. The choice of a modified critical temperature and exponents is arbitrary to within factors of 3 or more and the results depend significantly on experimental sensitivities. We argue that this flexibility in analysis could be the source for the large and apparently universal $z$ exponent values.

We propose a criterion necessary to ensure that a scaling analysis is not afflicted with the same problems as the modified KTB scaling analysis. We use this to motivate a different criterion for determining a conventional KTB transition. We find that some two-dimensional superconductors satisfy our criterion; however, many, in particular most of those in the cuprates, do not.

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There is a technical issue about determining zero concavity as opposed to positive concavity. Since any curve with positive concavity will, over a short enough length of line, seem to have zero concavity, it is important that the same range of current used to determine positive concavity for a high temperature isotherm is also used to determine a zero concavity isotherm below $T_{KT}$.

To probe length scales smaller than $\xi_{GL}$ would no longer investigate the inter-vortex interactions which are desired.

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This assumes that at fixed DC current the dissipation (and thus voltage) is greater as temperature increases. This reasonable assumption is supported by the $I - V$ curves in Fig. 1. We can also see from this data that the higher temperature isotherms intersect with the $10^{-9}$V resolution limit at lower currents.

The data in Fig. 8(b) have been obtained by digitizing the data of Ref. 12 with the program DataThief. The data points in Fig. 8(b) are calculated by taking every fourth $I - V$ data point from Ref. 12 and then fitting a quadratic function to groupings of three successive points of the remaining $I - V$ data. We use the derivative of the quadratic function at the current value of the middle data point as the derivative values for Fig. 8(b). This procedure has the effect of smoothing out the noise in the measurements, but does not seem to influence the concavity properties significantly; i.e., other smoothing techniques we tried yielded very similar results.