Nonlinear Trellis Description for Convolutionally Encoded Transmission Over ISI–channels with Applications for CPM

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Abstract—In this paper we propose a matched decoding scheme for convolutionally encoded transmission over intersymbol interference (ISI) channels and devise a nonlinear trellis description. As an application we show that for coded continuous phase modulation (CPM) using a non–coherent receiver the number of states of the super trellis can be significantly reduced by means of a matched non–linear trellis encoder.

Index Terms—ISI–channel; continuous phase modulation; convolutionally encoded transmission; matched decoding; non–coherent differential detection; super trellis

I. INTRODUCTION

Convolutional encoding is an attractive encoding scheme due to its low latency compared to block encoding. When used for transmission with pulse amplitude modulation (PAM) schemes over an ISI–channel the receiver has to perform equalization and decoding either in two separated trellises or jointly in one super trellis when due to strict latency constraints interleaving and by this iterative equalization–decoding is prohibited. We propose a technique to merge the channel encoder with the ISI–channel. Instead of using ring convolutional codes to integrate the encoder into the ISI–channel as proposed in we describe the ISI channel and the convolutional encoder by a single non–linear trellis encoder with binary delay elements.

As an application continuous phase modulation (CPM) schemes are employed. It poses a class of power–efficient constant–envelope modulation schemes [2]. Due to the steady phase transitions the transmit signal of CPM has good spectral properties. Non–coherent receivers for CPM are relatively simple and straightforward to implement when compared to a coherent receiver. Using differential detection and a matched filter at the receiver one can easily retrieve the transmitted symbols by means of a decision–feedback equalization (DFE) or maximum–likelihood sequence estimation (MLSE) using the Viterbi algorithm (VA) comparable to a PAM transmission over ISI–channels. The main disadvantage of differential detection is the spectral shaping of the channel noise. To counteract this performance loss, an additional discrete–time noise whitening filter is introduced which however further extends the overall impulse response of the transmission system [3].

We introduce coded transmission over ISI–channels using the general concept of serially concatenated convolutional encoder, mapper, modulator and ISI–channel in Section II. At this point we show that the m–ary channel encoder and the M–ary transmission scheme over the ISI–channel can be combined to a single m–ary trellis description representing a non–linear encoding. In order to facilitate description we restrict encoder and mapper to m = 2 and M = 2^k; i.e. each output vector of a rate \( \frac{K}{N} \) convolutional encoder is mapped to one PAM–symbol. Section III shows that a complexity reduction can be achieved by combining multiple trellis states into hyperstates [4], [5]. We then describe non–coherent reception of CPM signals including differential detection, matched filtering and noise whitening in Section IV. In Section V we verify via Monte–Carlo simulations that the proposed approach gives exactly the same performance with less states compared to the super trellis of the concatenation of convolutional encoder and the inherent continuous phase encoding of CPM [6], [7].

II. CONVOLUTIONALLY ENCODED TRANSMISSION OVER ISI–CHANNELS

To introduce matched decoding (MD) we transform the convolutionally encoded transmission scheme step–by–step. For example consider the serial concatenation of a rate–\( \frac{K}{N} = \frac{1}{2} \) binary convolutional encoder, a natural mapper, M–ary PAM transmission and an ISI–channel with \( L + 1 \) channel coefficients \( h[k] \) with \( k \) denoting the time index, cf. Fig. I.

A. Derivation of the Matched Encoder

In the conventional approach one would process the receiver input signal first by a MLSE or a symbol–by–symbol trellis equalizer for the FIR filter \( h[k] \) and forward soft– or hard–output symbols of this trellis equalization to the decoder for the channel code. But an optimum receiver however would perform MLSE over the super trellis decoding the binary channel encoder and the ISI channel impulse response \( h[k] \) of length \( L \) jointly. In a straightforward approach the super trellis would have \( Z_{\text{enc}} \cdot M^L \) states, when \( Z_{\text{enc}} \) is the number of states of the convolutional encoder. If the number of
output symbols from the encoder can be related to the size of the modulation alphabet \( M \) so that \( n = \log_2(M) \) holds, the following complexity reduction can achieve the same performance by a trellis description with less states.

To see this, note that in each encoding step, \( n - K \) output symbols of the encoder are redundant and depend on \( K \) input symbols. E.g., in Fig. 1 one of the two channel encoder output symbols contains no further information. We now show, how to combine the binary channel encoder with the \( M \)-ary channel impulse response to form a single binary non–linear trellis encoder.

First, we combine the P/S conversion and the mapper of Fig. 1. For clarity, we restrict ourselves to \( M = 4 \), but note that the concept easily extends to arbitrary \( M = 2^n \). In this example \( M = 4 \), i.e. \( n = 2 \), the upper branch corresponds to the most significant bit (MSB) whereas the lower branch describes the least significant bit (LSB). The natural mapping can be applied by multiplying the MSB by 2 and adding the LSB. The conversion from unipolar binary symbols \( c[k] \) into bipolar symbols \( b[k] \) within an alphabet of size \( M \) can be done with \( b[k] = (c[k] \cdot 2) - 1 \). The resulting system is depicted in Fig. 2. Recall that the mod operation can be represented using the floor function. In terms of Gaussian notation we can thus write

\[
x \mod n = x - n \cdot \left\lfloor \frac{x}{n} \right\rfloor.
\]

In addition we see that the main branch (after the summation of MSB and LSB) has a multiplication and summation which can be moved behind the convolution. With 
\[
C = -\sum_{k=0}^{L} h[k](M - 1)
\] and the Gauss representation of the modulo operation we can sketch the transmission system as depicted in Fig. 3. Note that now the convolution can be moved into the MSB branch and LSB branch, respectively, which enables to use binary delay elements instead of \( M \)-ary ones.

This representation now has \( n \) independent binary branches which all depend on the same \( K \) input values (here \( n = 2 \) and \( K = 1 \)). The calculations in each branch can be combined into a single non–linear filter.

As an example we use the 4-state minimum free distance convolutional encoder with octal representation \( [g_1 \text{oct}; g_2 \text{oct}] = [\text{oct}; \text{oct}] \) as binary generator polynomials for the MSB and LSB branch. The resulting, non–linear trellis encoder is depicted in Fig. 4.

**B. Complexity Comparison**

The main advantage of matched encoding is the reduction of the convolution by the ISI–channel from an \( M \)-ary input sequence into \( \log_2(M) \) binary parallel convolutions in each branch. As the number of convolutions affect the calculation of metrics at the receiver but does not influence the number of resulting MLSE states we will now examine the complexity of trellis equalization for a matched decoding (MD) receiver and the traditional/serial super trellis decoding (STD).

1) **Super Trellis Decoding:** In a super trellis we consider channel encoder states and channel states separately. The channel encoder is defined using generator polynomials with \( \nu \) binary memory elements resulting in \( Z_{\text{enc}} = 2^\nu \) states for the trellis. The channel impulse response can be described using the impulse response \( h[k] \) with \( L + 1 \) filter coefficients. The ISI–channel with \( M \)-ary input symbols has \( Z_{\text{cha}} = M^L \) states resulting in a total number of states in the super trellis of

\[
Z_{\text{STD}} = Z_{\text{enc}} \cdot Z_{\text{cha}} = 2^\nu \cdot M^L = 2^\nu \cdot 2^{(\nu - L)}.
\]
Recall that for \( n \) to less states. The gain of this state reduction can be calculated. The resulting trellis still describes the super trellis but with STD is the reduction of states without loss in performance.

\[
Z_{MD} = 2^n \cdot 2^L. \tag{3}
\]

Recall that for \( n = 2 \) there are two convolutions in parallel for the computation of the hypothesis.

3) Comparison: The main advantage of MD compared to STD is the reduction of states without loss in performance. The resulting trellis still describes the super trellis but with less states. The gain of this state reduction can be calculated to

\[
G_{MD} = \frac{Z_{STD}}{Z_{MD}} = \frac{2^{(n-L)}}{2^L} = 2^{L(n-1)}. \tag{4}
\]

Table I summarizes several examples for different encoder size and channel lengths for the special case of \( n = 2 \) \((M = 4)\). Obviously the gain increases with the length of the ISI–channel.

III. REDUCED–STATE SEQUENCE ESTIMATION

We have shown that the super trellis of convolutionally encoded transmission over ISI–channel can be represented using significantly less states by parallelizing the \( M \)-ary convolution. At this point we can use reduced state sequence estimation (RSSE) to further reduce the number of states at the cost of small loss in Euclidean distance. In RSSE several MLSE states are combined into hyperstates \([4], [5]\) which are then used for decoding. The partitioning is crucial to the performance of RSSE as it reduces the Euclidean distance. We w.o.l.o.g assume that the channel impulse response \( h[k] \) is minimum phase (by application of a proper all–pass filter) allowing use a partitioning comparable to that of delayed decision–feedback sequence estimation (DFSE) \([8]–[11]\).

In this work we apply this DFSE partitioning to use RSSE for MD of convolutionally encoded CPM with non–coherent differential detection and noise whitening as will be described below.

IV. NON–COHERENT CPM RECEPTION

Our application for MD is based on a CPM transmission over an AWGN channel. ISI results from the differential detection and a noise whitening filter which we will discuss shortly. The phase of a CPM transmit signal \( s(t) \) is given as

\[
\theta(a, t) = 2\pi h \sum_{k=-\infty}^{\infty} a[k] \int_{-\infty}^{t} g(t - kT) \, dt
\]

\[
= 2\pi h \sum_{k=-\infty}^{\infty} a[k] q(t - kT)
\]

where \( h = p/q \) is the modulation index \((p \text{ and } q \text{ relative prime})\), \( g(t) \) is the frequency pulse of length \( L \cdot T \) with the symbol duration \( T \) and \( a = \{a[k]\} \) is a sequence of transmit symbols taken from the bipolar \( M \)-ary alphabet \( \{\pm 1; \pm 3; \ldots; \pm (M-1)\} \) \((M \text{ even})\). An integration of \( g(t) \) over time \( t \) gives the phase pulse \( q(t) \) with normalization \( q(t) = \frac{1}{2} \forall t \geq LT \). With an arbitrary phase–offset \( \theta_0 \) and the signal energy per modulation interval \( E_s \). The equivalent complex baseband (ECB) transmit signal is given by

\[
s(t) = \sqrt{\frac{E_s}{T}} \exp \left\{j(\theta(a, t) + \theta_0)\right\}
\]

\[
= \sqrt{\frac{E_s}{T}} \exp \left\{j2\pi h \sum_{k=-\infty}^{\infty} a[k] q(t - kT) + \theta_0\right\}.
\]

A. Differential Detection

Non–coherent demodulation for additive white Gaussian noise (AWGN) channels can be implemented using a differential detection as described in \([3]\). In this paper a differential detection for \( M \)-ary partial–response \((L > 1)\) CPM is used. The received signal \( r(t) \) is first band–limited, then the phase of the signal is extracted and unwrapped by a phase continuation (Phase unwrapping may be implemented by means of sampling at a sufficiently high frequency, reduction of phase differences \( \mod 2\pi \), and subsequent integration). Finally a differentiator provides the phase differences between two subsequent samples separated by \( T_d \). The receiver is shown in Fig. 3 (ECB–domain). For an ideal continuous–time differentiation \( T_d \rightarrow 0 \) holds. This demodulator fully inverts the non–linear CPM encoding. Hence a matched filter \( \gamma^{-}(-t) \) for transition of continuous to \( T \)-spaced discrete time representation may be applied without loss of information on the sequence of data samples. The parameter \( \gamma = \frac{1}{\sqrt{E_s T}} \) is used for normalization with the energy of \( g(t) \) denoted as \( E_g \). Note, that for an AWGN channel the noise after the matched

| Encoder | \( L \) | \( Z_{STD} \) | \( Z_{MD} \) | \( G_{MD} \) |
|---------|-------|-----------|-----------|-----------|
| \[5\text{oct}; 7\text{oct}] \ | 1 | 16 | 8 | 2 |
| \[
2 \quad 64 \quad 16 \quad 4
\]    | 2 | 3 | 256 | 32 | 8 |
| \[
4 \quad 1024 \quad 64 \quad 16
\]   | 4 | 1024 | 64 | 16 |
| \[133\text{oct}; 171\text{oct}] \ | 1 | 64 | 64 | 1 |
| \[
0 \quad 256 \quad 128 \quad 2
\]   | 0 | 256 | 128 | 2 |
| \[
2 \quad 1024 \quad 256 \quad 4
\] | 2 | 1024 | 256 | 4 |
| \[
3 \quad 4096 \quad 512 \quad 8
\] | 3 | 4096 | 512 | 8 |
| \[
4 \quad 16384 \quad 1024 \quad 16
\]   | 4 | 16384 | 1024 | 16 |
filter is non–white and non–Gaussian due to the differential
demodulation. This issue is often referred to as FM–noise (or
\( f^2 \)–noise) [3].

\[
\begin{align*}
r(t) & \xrightarrow{\text{Diff.}} G^*(f) \cdot kT \\
\text{arg} \{ \} & \xrightarrow{\text{phase cont.}} \hat{d}[k]
\end{align*}
\]

Fig. 5: Non–coherent differential demodulation for CPM.

B. Description of Intersymbol Interference

As the modulation with differential demodulation only affects the
noise power spectral density we can replace the
CPM transmission over an AWGN channel with an equivalent
PAM representation with non–white non–Gaussian noise as
depicted in Fig. 6. An optimum receiver for non–coherent
FM–noise/AWGN–noise

\[
a[k] \xrightarrow{G(f)} \gamma G^*(f) \xrightarrow{\text{kt}} d[k]
\]

Fig. 6: PAM representation of CPM with differential demodulation
(with substitute AWGN for derivation of theory).

CPM would require a continuous–time whitening filter in front of
the matched filter to decorrelate the FM–noise. As a noise
whitening filter would not be stable we neglect the non–white
and non–Gaussian characteristics of the noise at this point and
instead develop a theory for AWGN–channel at first. In [3]
a noise–whitening filter is introduced after the “whitened”
matched filter w.r.t AWGN channel which will be described in
detail later on.

In order to achieve a high bandwidth efficiency the common
approach for CPM is to use a Raised Cosine pulse of length
LT (L–RC pulse) with \( L > 1 \) for frequency impulse \( g(t) \) which
results a) in ISI in the sequence \( d[k] \) of \( T \)–spaced samples of the
Matched–Filter output (as \( G(f) \) does not satisfy the \( \sqrt{\text{Nyquist}} \) condition) and b) in less energy in the spectral
side–slopes and thus higher spectral efficiency. Due to ISI
equalization has to be performed at the receiver. The ISI can
be described with the energy spectral density \( \Phi_{gg}(f) \) of the
transmit pulse, given by

\[
\Phi_{gg}(f) = \gamma |G(f)|^2
\]

and after \( T \)–spaced sampling,

\[
\Phi_{gg}[e^{j2\pi ft}] \overset{\text{def}}{=} \sum_{i=-\infty}^{\infty} \Phi_{gg} \left( f - \frac{i}{T} \right)^2 .
\]

Obviously, \( \Phi_{gg}(f) \) does not fulfill the Nyquist criterion for
\( L > 1 \). For \( L–RC \) CPM the ISI interacts with \( 2L - 1 \) symbols
due to \( \Phi_{gg}[e^{j2\pi ft}] \).

At this point a Viterbi algorithm can be used to estimate the
transmitted symbols but the noise power–spectral density is
still non–white due to the FM–noise and the matched filter.

C. “Whitened Matched Filter” for AWGN–Channel

To optimize the performance we now investigate the
whitened matched filter \( WMF \) [12] still assuming AWGN!
This additional discrete–time whitening filter \( H_W(f) \) with an
arbitrary phase \( \phi(f) \) and the power spectral density
\( \Phi_{gg}[e^{j2\pi ft}] \) is described by

\[
H_W(f) = \frac{e^{j\phi(f)}}{\sqrt{\Phi_{gg}[e^{j2\pi ft}]}}.
\]

In combination with the receive filter \( G^*(f) \) the whitening
filter \( H_W(f) \) results in the whitened matched filter which is
a \( \sqrt{\text{Nyquist}} \) function. Using spectral decomposition of the
Z–Transform \( \Phi_{gg}[z] = B[z] \cdot B^*[z^{-1}] \) we can then separate the
power spectral density \( \Phi_{gg}[e^{j2\pi ft}] \) into a causal minimum phase
\( B[z] \) and a non–causal maximum phase

\[
H_{WMF}(f) = \gamma G^*(f)e^{j\phi(f)} \sqrt{\Phi_{gg}[e^{j2\pi ft}]}
\]

After sampling the overall discrete–time transmission can be
summarized using a single filter \( \gamma B[z] \) as depicted in Fig. 7.
But due to the differential demodulation the noise is indeed
not AWGN but FM–noise (\( f^2 \)–noise).

D. Noise Whitening

In this paper we apply a discrete–time noise whitening filter
\( F[z] \) after the “whitened” matched filter in order to reduce the
\( f^2 \) characteristics of the noise (see [3] and Fig. 7). Exact noise
whitening would correspond to integration which would cause
stability problems. Therefor we use suboptimum filtering with
finite length \( L_{NW} \). A measurement of the noise characteristics

\[
a[k] \xrightarrow{\text{FM–Noise}} B[z] \xrightarrow{F[z]} \text{d}_{NW}[k]
\]

Fig. 7: Added \( T \)–spaced FIR noise whitening filter \( F[z] \).

Fig. 8: Auto correlation \( \phi[k] \) of the noise and impulse responds
of the \( T \)–spaced noise whitening filter \( f[k] \) with \( L_{NW} = 10 \).
shows that the autocorrelation \( \phi[k] \) depicted in Fig. 8 (left)
clearly shows correlated noise. We minimize the error variance that results from a prediction filter with coefficients $p[k]$:  
\[
\min_{p[k]} \left\{ \sum_{k=1}^{L_{NW}} p[k] \phi[k] - \phi[k] \right\}^2
\]

This minimization leads to the Yule–Walker equations with the noise prediction coefficients $p[k]$:  
\[
\sum_{k=1}^{L_{NW}} p[k] \phi[k-k] = \phi[k] \quad \forall k = 1, 2, \ldots, L_{NW}
\]  

(12)
The coefficients of the noise whitening filter are then  
\[
F[z] \triangleq \varphi[k] = \begin{cases} 1 & \text{for } k = 0; \\
-p[k] & \text{for } 1 \leq k \leq L_{NW}. 
\end{cases}
\]  

(13)
In the right hand side picture of Fig. 8 the resulting filter coefficients for the correlated noise from Fig. 8 (left) are depicted. At the receiver we then have to equalize for ISI $h[k] \varphi[k] \varphi[z] F[z]$. Varying the length of the noise whitening filter $L_{NW}$, we can trade between the residual colorfulness of the noise and the complexity of the receiver (in the case of MLSE).

V. SIMULATION RESULTS

First we will show that the results for MD are indeed exactly the same as for decoding in the super trellis, cf., Fig. 9. Here a 4-ary CPM transmission with modulation index $h = \frac{1}{4}$ and $L_{CPM} = 3$ is used. The lowpass at the receiver limits the signal to the $B_{90.9\%}$ bandwidth of the CPM signal. The “whitened” matched filter for AWGN is of length 20, whereas the length of the noise whitening filter $F[z]$ is a parameter. The overall ISI described by $h[k]$ is therefore of length $L = L_{CPM} + L_{NW} - 1$. As convolutional encoder, the $[5_{oct}; 7_{oct}]$ code was used. It becomes clear that the performance of STD is equal to that of MD. The simulations are conducted only for $L_{NW} = \{0; 1\}$ due to fact that STD gets overly complex for longer whitening filters very quickly, cf., Table I.

We now investigate the results using MD in combination with RSSE. In Fig. 10 and Fig. 11 the same coded CPM transmission with non–coherent reception scheme, $L_{NW} = 0$ and $L_{NW} = 2$ but with 64–state rate–1 1/2 encoder with $[133_{oct}; 171_{oct}]$ is used. At the receiver decoding is done using a) serial decoding of channel impulse response and channel code b) MLSE decoding using full–state super trellis and c) matched decoding with RSSE. For a) we have two different approaches. The hard decision approach uses a decision–feedback sequence estimation (DFSE) with 4 or 16 states and a full–state Viterbi algorithm with $2^k$ states to decode the convolutional encoder. The soft decision approach comprises a symbol–by–symbol detection using the well–known BCJR algorithm [13], and the soft–input Viterbi algorithm for channel decoding. One can see that even 4-state MD decoding surpasses the performance of serial decoding and RSSE converges STD with increasing number of states.
complexity is described by the number of states. For STD and MD with RSSE the receiver complexity is directly given by the number of states in the super trellis or can be defined, respectively. For concatenated equalization and decoding the receiver complexity is defined as the sum of states in the equalization and the decoding, i.e., \( C_{\text{RSSE}} = Z_{\text{eq}} + Z_{\text{enc}} \), \( C_{\text{STD}} = Z_{\text{eq}} \cdot Z_{\text{enc}} \) with \( Z_{\text{eq}} \) and \( Z_{\text{enc}} \) is the number of states in equalization trellis and the channel decoding trellis, respectively. For RSSE the complexity depends on the partitioning so that \( C_{\text{RSSE}} = 2^r \) with arbitrary \( r \).

In Fig. 12 we compare MD for a transmission scheme with 4 channel encoder states and 16 or 32 states in the ISI trellis (see also Fig. 9). We can see again, that MD performs equally compared to STD, but with much less states. Additionally the receiver complexity for concatenated receiver structures are included. The best performance is achieve with DFSE equalizing a relatively long ISI of length 10. The channel encoding and the ISI-channel are described by their number of states and abbreviated with \( Z_{\text{enc}} / Z_{\text{cha}} \).

**VI. CONCLUSION**

In this paper we have shown that it is possible to reduce the number of states for the super trellis without loss of performance by transforming the \( M \)-ary channel convolution into \( \log_2(M) \) parallel binary convolutions. Here a coded non-coherent CPM transmission is used, but as several other non-interleaved transmission schemes (i.e. QAM over ISI-channel) can be represented as a separate channel encoder and

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