3D-PIC simulation of the electron beam interaction with modulated density plasma

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Abstract. In this paper, the three-dimensional (3D3V) numerical model is developed for the simulation of enhanced electromagnetic emission due to the beam injection into the modulated density plasma. The proposed model is fully kinetic and based on Particle-In-Cell method. The results of numerical experiments for different beam and plasma parameters are presented. Comparison of the radiation efficiency for the flat plasma layer and circle plasma column with different plasma density modulation amplitude are obtained. Numerical experiments have been performed using computer systems with parallel architecture.

1. Introduction

Regardless of the history of half a century both theoretical and numerical research, the problem of the description of beam-plasma interactions is still actual for different physical problems. The investigation of various mechanisms of electromagnetic radiation generation in the plasma is important for the controlled thermonuclear fusion and radiometric diagnostics. The generation of electromagnetic waves due to an interaction of high-current relativistic electron beam with plasma is one of the promising solutions to obtain the terahertz electromagnetic radiation.

The practical application of this study can be reflected in the solution of the problem of remote sensing of objects, transmission of information in the terahertz range and processing of materials with the help of powerful terahertz radiation.

One of the perspective directions in the terahertz radiation obtaining is the generation of electromagnetic radiation near the plasma frequency when the electron beam interacts with the plasma. Experiments on GOL-3 facility (Budker Institute of Nuclear Physics of the SB RAS, Novosibirsk, Russia) showed the electromagnetic radiation efficiency increase (∼ 1%) in the regime when the transverse dimension of the system is comparable with the length of the emitted waves [1]. In the linear theory, the mechanism of a plasma antenna for generating terahertz radiation was proposed [2]. It is assumed that the inhomogeneity of the plasma density plays a key role in the conversion of the beam energy into electromagnetic radiation. Theoretical estimation predicts obtaining up to 5% efficiency in the continuous injection of a beam into a plasma channel with a previously created longitudinal density modulation. The determination of the most suitable plasma and beam parameters for radiation generation is a difficult task.

The main problem of linear theories is that they do not take into account many factors such as
instability which plays an important role in the real plasma. Therefore it is necessary to create a numerical model that takes into account nonlinear effects.

In the article [3] authors studied electromagnetic emission using simulations with a two-dimensional model for plasma. They studied how efficiently the emission generation can be in the initially inhomogeneous flat thin plasma and found the regimes in which relativistic electron beam can generate electromagnetic radiation propagating transversely or with the angle to the magnetic field. We also studied such processes with the two-dimensional model in work [4].

In this article, the three-dimensional parallel model of beam-plasma interactions is presented. Our aim is the simulation of the electromagnetic emission enhanced by the electron beam injection into the inhomogeneous magnetized plasma with parameters available in laboratory beam-plasma experiments at the GOL-3 mirror trap.

2. Computational model
The kinetic approximation with the real electron to ion mass ratio is used to simulate the radiation generation due to the beam-plasma interaction. The mathematical model of the plasma and beam particles dynamics is based on the Vlasov-Maxwell equation system [5]

\[
\frac{\partial f_\alpha}{\partial t} + v \frac{\partial f_\alpha}{\partial r} + \kappa_\alpha (E + [v, B]) \frac{\partial f_\alpha}{\partial p} = 0, \tag{1}
\]

\[
\frac{\partial E}{\partial t} = \text{rot} B - j, \quad \frac{\partial B}{\partial t} = - \text{rot} E, \tag{2}
\]

\[
\text{div} E = \rho, \quad \text{div} B = 0. \tag{3}
\]

\[
\rho = \sum_\alpha q_\alpha \int f_\alpha(t, r, p) dp, \quad j = \sum_\alpha q_\alpha \int v f_\alpha(t, r, p) dp. \tag{4}
\]

\[
\kappa_e = -1, \quad \kappa_i = m_e/m_i, \quad q_e = -1, \quad q_i = 1. \tag{5}
\]

There \( f_\alpha(t, r, p) \) is the particle distribution function of the species \( \alpha (\alpha = e - \text{the beam and the plasma electrons, } \alpha = i - \text{the plasma ions}) \); \( m_\alpha \) is the mass; \( v \) is the speed, \( p \) is the momentum, \( r \) is the coordinate; \( E \) is the electric field, \( B \) is the magnetic field; \( j \) is the electric current density, \( \rho \) is the electric charge density. These equations are in dimensionless variables. To proceed to dimensionless variables, the following quantities are used: the speed of light \( c = 3 \cdot 10^{10} \text{ sm/sec} \), the electron mass \( m_e = 9.1 \cdot 10^{-28} \text{ g} \), the plasma density \( n_0 = 10^{14} \text{ sm}^{-3} \), the time \( t = \omega_{pe}^{-1} \), where the plasma electron frequency is \( \omega_{pe} = 5.6 \cdot 10^{11} \text{c}^{-1} \).

The Vlasov equations are solved with the Particle-in-Cell method [6]. The characteristics of the Vlasov equation (coordinate \( r_\alpha \) and momentum \( p_\alpha \)) describe the trajectories of the particles moving in self-consistent electromagnetic field

\[
\frac{dp_\alpha}{dt} = \kappa_\alpha (E + [v_\alpha, B]), \tag{6}
\]

\[
\frac{dr_\alpha}{dt} = v_\alpha, \quad p_\alpha = \frac{v_\alpha}{\sqrt{1 - v_\alpha^2}}.
\]

For the equations of motion (6) we use the Leap-frog scheme [7]. It provides the second order of approximation in space and time.

For electromagnetic fields calculation, Finite-Difference Time-Domain or Yee’s method is used [8]. The electric and magnetic fields are calculated on the staggered grid that yields second-order accuracy with respect to coordinate and time. The rotor and the divergence operators are defined on the Yee cell. These grid operators are constructed in such a way that the equations are self-consistently performed in the absence of the charge. When the charge density and the
current is not equal to zero, one should calculate the current density and the charge density so that the continuity equation is satisfied

$$\frac{\partial \rho}{\partial t} + \text{div} \mathbf{j} = 0.$$  \hspace{1cm} (7)

In our calculation the charge density and the current density are defined by the particle’s momenta and coordinates according to [7, 9]. It allows us to satisfy the continuity equation automatically.

3. The problem statement
On the figure 1 the computational domain is shown. It has a parallelepiped shape. The lengths of the computational domain in each direction are $L_x$, $L_y$, $L_z$. At the center of the domain is the plasma layer or column bounded by vacuum. We considered two cases: flat plasma layer (case 1) and circle plasma column (case 2).

![Figure 1. The simulation domain](image)

The hydrogen plasma is bounded by vacuum from both y-sides and in case 2 plasma is bounded by vacuum from both z-sides also. The plasma ions and electrons are distributed in the domain with the modulated density $n_i(x) = n_e(x) = n_0 + \delta n \cos(qx)$, $n_0 = \text{const}$, $\delta n$ is the perturbation amplitude. The magnetic field $\mathbf{B} = (B_x, 0, 0)$ provides plasma confinement.

X-boundary conditions are open for the particles and for the fields ($\mathbf{E}(0, y, z) = \mathbf{E}(L_x, y, z)$, $\mathbf{B}(0, y, z) = \mathbf{B}(L_x, y, z)$ etc.), so the particles of beam and plasma can leave the domain through the bounds. In case 1 z-boundary conditions are periodic for fields and for particles. On X,Y-boundaries (and Z for case 2) first-order Mur absorbing boundary condition for fields [10] are set. The electron beam have the same shape as plasma and it continuously injected into the plasma with the average momentum $p_0$ through the left boundary. At the initial moment of
time the particles of the beam is absent in the domain. The plasma and the beam electrons have Maxwell momentum distribution (for the plasma electrons $p_0 = 0$)

$$f(p_x, p_y, p_z) = \left(\frac{1}{\sqrt{2\pi m_e T}}\right)^3 \exp\left(-\frac{(p_x - p_0)^2 + p_y^2 + p_z^2}{2m_e T}\right).$$ (8)

All our simulations have been performed for the hydrogen plasma electron density $n_e = 10^{14} \text{cm}^{-3}$. The electron beam density $n_b$ is determined by $n_b/n_0 = 0.02$. The electron beam have the average velocity $v_b = 0.9 c$ and the temperature $T_b = 100 \text{eV}$. The plasma electron temperature is $T_e = 12 \text{eV}$. The ion temperature is zero. The external magnetic field $B_x$ is determined by the ratio of the Larmor frequency ($\Omega = eB_x/m_ec$) and the plasma frequency ($\omega_p = \sqrt{4\pi n_0 e^2/m_e}$) $\Omega/\omega_p = 0.6$.

The size of the computational domain and the space steps (measured in $[c/\omega_p]$) are $L_x = 83 \div 166$, $L_y = 60$, $L_z = 2.4$ in case of flat plasma layer (case 1) and $L_z = L_y$ in case 2, $h_x = h_y = h_z = 0.2 \div 0.4$. The plasma layer or column width $L_{plas} = 6.4$. The time step $\tau = 0.5h_x [\omega_p^{-1}]$. The number of each type particles per cell is 40.

A parallel Lagrangian decomposition algorithm to carry out the computational experiments is developed. The computational field, charge and current densities grid of all processors is common and synchronized each step. The particles are distributed equally among the processors.

4. Simulations results
Let us consider some results of computer simulations. Figure 2 shows the numerical experiment results for the electric field $E_x$. The first column corresponds to the initially homogeneous flat plasma layer. The second column corresponds to the flat plasma layer with the ion density modulation amplitude $\delta n = 0.2$ (xy-plane). The third and fourth columns correspond to the circle plasma column with the ion density modulation amplitude $\delta n = 0.2$ (xy- and yz-planes).

**Figure 2.** The time history of electric field maps $E_x$ (t=60 - top row, t=120 - bottom row). a) Field $E_x$ of the initially homogeneous flat plasma layer (xy-plane). b) Field $E_x$ of the flat plasma layer with the ion density modulation amplitude $\delta n = 0.2$ (xy-plane). c) Field $E_x$ of the circle plasma column with the ion density modulation amplitude $\delta n = 0.2$ (xy-plane and yz-plane).
Plasma in this test has the longitudinal ion density modulation with the period \( \lambda_q = 2\pi v_b/\omega_b \approx 5.93 \), \( \delta n = 0.2 \). At the beginning, the electric field equals to zero. Further, beam injects into the plasma and leads to the field inhomogeneity. As a result of the interaction of the beam with the modulated plasma oblique electromagnetic waves are pumping. The generated electromagnetic radiation propagates in a vacuum perpendicular to the motion of the beam in cases of initially inhomogeneous plasma particles distribution. In the case of the flat plasma layer the waves have a flat shape, and in the case of the plasma column the waves are circles. Also it is clear that the electric field amplitude of the flat beam-plasma layer is larger in magnitude than the field amplitude of the beam-plasma column. This is due to the fact that the radiation from the plasma column is scattered along the radius in different directions.

**Figure 3.** The time dependence of the radiation power \((P_{Rad})\) values for the O+X-modes of generated electromagnetic waves in the units of the power of the electron beam \((P_{Beam})\) for the plasma density modulation amplitude \(\delta n = 0 \div 0.2\).

**Figure 4.** The time history of electric field maps \(E_x\) \((t=80 - top\ row, t=190 - bottom\ row)\). a) The first column corresponds to the plasma with the ion density modulation period \((xy-coordinates)\ \lambda_q \approx 5.93\). b) The second column corresponds to the plasma with the ion density modulation period \((xy-coordinates)\ \lambda_q \approx 2.96\).
The plane wave propagating in the plasma can generate only ordinary electromagnetic waves ($E_x, E_y, B_z$, O-mode). However, nonuniformity of the transverse wave structure also gives rise to extraordinary waves ($B_x, B_y, E_z$). At figure 3 shown the time dependence of the radiation power ($P_{Rad}$) values for the O+X-modes of generated electromagnetic waves in the units of the power of the electron beam ($P_{Beam}$) for the different plasma density modulation amplitudes. For the calculation of this value, we used only radiation absorbed by the $y$ and $z$ bounds. As seen from the figure, in the case of inhomogeneous plasma the radiation efficiency increases, then saturates. Over time the ions move. It brings distortion into the density. And with the time radiation generation significantly decrease due to the changing of the ion density.

Changing the wave number of density modulation affects the direction of the radiation. Let us vary $q$ to $2q$. The period of ion density modulation decreases to the value $\lambda_q \approx 2.96$. $E_x$ for this simulation shown at figure 4 at the right column. It is seen that the radiation goes with the angle to the plasma column. It has smaller magnitude then fundamental emission and rapidly scattered.

5. Conclusion

The 3D3V PIC model and the computational code with the beam injection for the simulation of the beam-plasma interaction are developed and tested. The results of the three-dimensional electron beam interaction with the modulated density plasma are presented. Conducted computer simulations with the parameters close to the condition of laboratory experiments showed enhanced electromagnetic emission near the first harmonic of the plasma frequency. The three-dimensional model makes possible the simulation of the beam with different width and transverse configuration. Since the plasma and the beam shape affect the efficiency of the radiation, the possibility is important for comparison with physical experiments.

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