Harmonic potential and hadron masses

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Abstract

The quark-gluon sea in the hadrons is considered as periodically correlated. Energy levels of Schrödinger equation with harmonic potential is used for describing the spectrum of hadron masses. In the considered cases the effective potential operating on each particle of ensemble, under certain conditions becomes square-law on displacement from an equilibrium point. It can become an explanation of popularity of oscillator potential for the description of a spectrum of masses of elementary particles. The analysis shows that levels of periodic potential better agreed to the spectrum of hadron masses, than levels of other potentials used for an explanation of a spectrum of masses.

1 Introduction

The description and explanation of elementary particles spectroscopy (spectrum of masses) is one of the important problems of modern physics. For today there is a number of theories and models applied to the description as interaction between quarks, and a spectrum of masses of the elementary particles made of quarks or partons. Phenomenological potentials in non relativistic case are considered in [1, 2, 3, 4]. Relativistic approach on the basis of Dirac equation is considered in [12]. However, despite the great achievements in these areas, still there is a number of problems, in particular, at an explanation of a mass spectrum. The basic problem is obtaining of highest levels both mesons, and baryons. For an explanation of shift of masses of the higher D mesons found out in [13, 15] it is used the dynamical QCD mechanism [14] in which the shift of masses calculated by introducing of a mixing angle of the connected channels. In the present work the new approach for the masses of hadrons is considered. Such approach is proved by presence of partons in hadrons, which named a quark-gluon sea. Thus there is a natural question of the average effective field operating on particles of the quark-gluon sea. We will notice that in case of light quarks \( m_u = 2 - 3 \text{mev}, m_d = 6 - 8 \text{mev} \) the mass of a proton does not satisfy to virial theorem. If to consider a share of the neutral partons,
presumably gluon, (50 percent of mass of a proton) as potential energy and to add its half there are 25 percents more. It shows that the proton is not simply bounded condition of 3 quarks. As well known the liquid state of quark-gluon matter is observed in RHIC experimental results and it is clear that near phase transition point the quark-gluon sea density can become periodical. In this Letter we would like to report our study of such new state of quark-gluon sea and their effect on the hadron masses. Obtained results can be used for studying of usual substances near transition point too.

2 Energy spectrum of periodical potential

Under certain conditions in the quark-gluon sea can exist short range correlations, which means that the density and correlation functions become periodical. Therefore we can choose the mean field in the sea periodical with amplitude $U_0$ and wave number $k = 2\pi/r_0$

$$U = U_0(1 - \varepsilon \cos kx)$$

where the $\varepsilon$ describe the depth of correlation. The Shrodinger equation with this potential for particle with mass $m$ and energy $E$ after substitution of new variable $z = kx/2$ become a Mattieu equation

$$\psi'' + (a - q \cos 2z)\psi = 0$$

$$a = (E - U_0)4m/(\hbar k)^2$$

$$q = \varepsilon U_04m/(\hbar k)^2$$

In case of small oscillation amplitude $kx \ll 1$ and we can restricted by second power of $x$ in the Taylor expansion of the potential. This equivalent to oscillator approximation. In general case the analyse of eigenvalues of Mattieu equation is necessary. These eigenvalues are depend on $q$ and number of level. When $q$ not so big and $r > 7$ the eigenvalues equal

$$a_n \approx b_n + 1 \approx n^2 + \frac{q^2}{2(n^2 - 1)} + \frac{(5n^2 + 7)q^2}{32(n^2 - 1)(n^2 - 4)} + \ldots$$

As seen the second term

$$\delta_n = q^2/2(n^2 - 1).$$

is inverse proportional to number of level and is similar to levels of Coulomb potential $1/r$. In certain physical cases this can explain the using of Coulomb potential. This term is small if $q \ll 1$, but become considerable when $q > 1$. Notice that such amendment can be used successfully for explanation of mass level shift in many cases of elementary particle mass. This can be realized when $\varepsilon \ll 1$. When $q \gg 1$ the eigenvalues $a_n, b_n$ number $n$ of Mattieu functions with period $\pi$, equal

$$a_n \sim b_n + 1 \sim -2q + 2W\sqrt{q} - \Delta_n$$

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\[ \Delta_n = \frac{W}{2^7\sqrt{q}} + \frac{W^2 + 1}{8} + \frac{1}{W} \left( \frac{3}{2^7\sqrt{q}} + \frac{33}{2^{17}q^{3/2}} \right) \]

As seen the general term in this expression is \( \sim W \sim n + 1/2 \) as levels of oscillator. But the negative amendment becomes considerable for levels of number \( n > 3 \). And the higher levels becomes lower of oscillator levels approximately on value

\[ \Delta_n \approx W^2/8 \]

The Mattieu equation is the specific type of Floque equations with periodic coefficients. The stable and finite solutions are solutions with \( a_n < a < b_n + 1 \) [9], which are the energy bands. The solutions with other energies are unstable and infinite.

## 3 Three dimensional spectrum

In this chapter we elucidate the influence of dimensionality on the potential levels. Three dimensional Schrödinger equation have a form

\[ \frac{\hbar^2}{2m} \Delta \psi + [E - U] \psi = 0 \]

where \( \Delta \) - is the 3-dimensional Laplace operator, \( \hbar \) is the Plank constant. If the potential is central symmetric after substitution

\[ \psi(r, \theta, \phi) = R(r)Y_{lm}(\theta, \phi) \]

one can find

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) - \frac{l(l + 1)}{r^2} R + \frac{2m}{\hbar^2} [E - U(r)] R = 0 \]

where \( Y_{lm} \) - are the eigenfunctions with azimuth number \( l \) and magnetic number \( m \). As seen from additional second term in our approach the energy levels are degenerated. But consideration of 3-dimensionality to take off the degeneracy and split the levels. The ground state \( l = 0 \) is not degenerate and is equivalent to one dimensional case. The case \( l > 0 \) we consider qualitative in the approximation \( r = r_{\text{eff}} \) in the potential. In this approximation appears additional term in the expression of energy \( \delta E_l = \hbar^2 l(l+1)/2m^2 r_{\text{eff}}^2 \), which split the levels with \( l \neq 0 \). Calculation by perturbation theory give amendment with \( r_{\text{eff}} = r_n \), where \( r_n \) is the mean square radius of the level number \( n \). It is not difficult to show that splitting is equidistant in coincide with experimental data. It is known [3], that the highest level is splitted to three levels. So this is the fourth level with \( l = 0, 1, 2 \). The whole classification of hadrons by using of oscillator levels can be used in our case too.
4 Application to mass spectrum

Calculations show that spectrum ($\sim n^2$) can be applicable to both mesons and baryons. The meson masses are raised approximately by law

$$m_n = m_1 + m_2 n^2$$  \hspace{1cm} (11)

In this sequence are included long life mesons with π in the beginning, π, K, η, Φ, D, ψ, ψ'. The mass agreement is good if one chooses $U_0 = m_1 \approx 15\text{Mev}$, but multiplier of $n^2$ is approximately 125 Mev. The calculation of characteristic length parameter $r_0$ gives a value $r_0 = 0.25\text{Fm}$ if we choose the quark mass $m_q = 2\text{Mev}$ [15]. In case of higher mass of the quark this is decreased more. But the charge radius of mesons is about 0.67 Fm [15]. But we must remember that meson charge radius is scattering length, but is not the particle real physical radius. The comparison of high levels of mesons, for example ψ, as quark-anti quark system, shows that high levels do not agree to oscillator levels [5]. Possible, they agree with (6), where the condition $q \gg 1$ is satisfied and levels have shift down [5]. The relative amendment $\delta_n = \Delta n / 2W\sqrt{q} = W/16\sqrt{q}$ of levels $n = 4 - 6$ is about 5-10 percents, if $\sqrt{q} = 5 - 10$, which give a good agreement of levels to real masses of particles. Calculation shows, that this possible, if the characteristic length two times longer. Possible, this is a second harmonic of some periodic potential. This spectrum is better agreed to real mass spectrum than an energy spectrum of any other potential. Notice that obtained levels $a_n, b_n + 1$ are very close each other, which means possible the degeneration on spin due to quarks are fermions. Mass levels of the same form (11) describe baryon masses if we add to nucleon $N$ mass this expression with $m_1 = 155\text{Mev}$, and $m_2 = 25\text{Mev}$. This row include $N, \Lambda, \Sigma, \Theta, \Omega$. The characteristic length is about 0.85 Fm, which is close to the charge radius of proton [15]. So the characteristic lengths are close to particles characteristic radii. But we must remember, that these radii are scattering lengths, but these are not the particle real physical radii. Taking into account the relation for energy

$$E_n = \frac{(\hbar k)^2}{4m} a_n + U_0$$ \hspace{1cm} (12)

it is clear that mesons and baryons mass formulae differs in $1/m^2_0$ and $U_0$. In our approach the agreement of calculated masses to real hadron masses considerably improves. Thus, the assumption of periodicity of potential removes set of problems and gives more adequate picture of potential inside hadrons. Notice, that the second term in case $q = 1 - 4$ can be used for obtaining of low bound states and energy levels similar to very strong Coulomb force. But the rich and complicated spectrum of hadron masses, most likely, means presence of periodic potential with several Fourier components. This explains also taking place well-known violation of strong interaction by middle strong interaction which is approximately ten times weaker than the strong. And, well-known that growth of mass of particles on linear law relative to level number in some cases is observed. Only the periodic potential depending on parameters of a problem
has the spectrum creating a basis for the description of all such cases. Let’s notice that at \( q \gg 1 \) eigenvalues of energy of periodic potential more coincide to masses of systems a quark-anti quark, including the higher levels which is not possible to do by any other method. All other methods are introducing new parameters for obtaining of levels agreement, for example mixing angle et al.

5 Summary

Studying of an energy spectrum of the equation of Schrodinger with harmonic potential shows that it suitable for an explanation of a spectrum of masses more than other potentials used. The spectrum of harmonic potential with small amplitude give sequence of masses of meson and baryon nonets. The additional term is inverse proportional to square of level number and is similar to levels of Coulomb potential. The spectrum of harmonic potential with great amplitude is similar to a oscillator spectrum, but there is a negative amendment to levels which grows quadratic on number of level and reduces high levels. Such spectrum explains also high levels of mesons, considered as pair a quark-anti quark, charmonium and etc. The account of 3 dimensionality leads to splitting of levels by removal of degeneration on the orbital moment. Thus, the assumption of periodicity of an effective field inside hadrons due to periodic spatial distribution of partons gives good agreement with a spectrum of hadron masses . But for a complete description, probably, it is necessary to consider few harmonics. Such approach, most likely, will create possibility of a uniform method to describe the whole spectrum of hadron masses as it have the energy levels both quadratic and linear on level number in dependence of task parameters. Notice that we use only two mass parameters instead of two parameters \((\alpha_s, k)\) used by others. Beside this the high levels are obtained too. Any other method uses new parameters, for example, mixing angle, for obtaining higher levels. It is very surprising that the periodical potential exist in the hadrons. But as known the liquid state of quark-gluon sea is observed at RHIC and under certain conditions constituents of this sea can arranged periodically near the phase transition point. Our results can be used for better understanding of RHIC results and hadron structure.

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