PARTICLE ACCELERATION IN THE DRIVEN RELATIVISTIC RECONNECTION

YURI LYUBARSKY AND MICHAEL LIVERTS
Department of Physics, Ben Gurion University of the Negev, Beer-Sheva, Israel
Received 2008 January 2; accepted 2008 April 20

ABSTRACT

We study compression-driven magnetic reconnection in a relativistic electron-positron plasma. Making use of a 2.5-dimensional particle-in-cell code, we simulated compression of a magnetized plasma layer containing a current sheet within it. We found that the particle spectrum within the reconnecting sheet becomes nonthermal; it can be approximated by a power-law distribution with an index of \(-1\) and an exponential cutoff.

Subject headings: acceleration of particles — magnetic fields — plasmas

1. INTRODUCTION

Magnetic reconnection is one of the important processes for particle acceleration and heating. The production of energetic particles has been extensively studied in nonrelativistic reconnection (Cargill 2001; Drake et al. 2005; Pritchett 2006), mostly in the solar terrestrial context. Magnetic reconnection may also play a major role in relativistic objects such as pulsars (Kirk et al. 2007), magnetars (Thompson & Duncan 1995; Lyutikov 2003), active galactic nuclei (Romanova & Lovelace 1992; di Matteo 1998; Birk et al. 2001), or gamma-ray bursts (Drenkhahn 2002; Drenkhahn & Spruit 2002; Thompson 2006). Genuinely relativistic reconnection occurs if the magnetic energy exceeds the plasma energy, including the rest energy. Then the dissipation of the magnetic field inevitably leads to relativistic energies of the particles (Kirk 2004).

Relativistic reconnection typically occurs in electron-positron plasmas; in this case the physics of the process is considerably simplified compared to electron-ion plasmas, where two significantly different scales are present. Larrabee et al. (2003) calculated self-consistent equilibria of the relativistic electron-positron plasma in the vicinity of the magnetic X-point in the reconnecting current sheet. The particles are accelerated there by the electric field parallel to the X-line. It was found that the particle distribution function is described by a power law \(dn/d\gamma \propto \gamma^{-1}\) with an exponential cutoff. Note that generally only a small fraction of the magnetic energy is released in the vicinity of the X-point. Reconnection in the X-point just allows the magnetic field lines to shrink, thus releasing magnetic energy. For example, in the classical Petschek model, most of the energy is released in slow shocks that stem from the X-point. Therefore, in order to find overall particle distribution function, one has to study particle acceleration/heating in the whole reconnection region; the results will presumably depend on global geometry and boundary conditions. Particle-in-cell (PIC) simulations of spontaneous reconnection in an infinite plane current sheet (Zenitani & Hoshino 2001, 2005, 2007; Jaroschek et al. 2004) confirm that a DC acceleration takes place around the X-point and that the particle energy spectrum in this region is roughly described by a power law with the power index of \(-1\). However, the energy spectrum over the whole simulation domain is significantly steeper, and can be approximated by a power law with index \(-3\) (Jaroschek et al. 2004; Zenitani & Hoshino 2007).

This paper aims to study the particle acceleration in compression-driven reconnection. Our study is motivated by the observation (Lyubarsky 2003, 2005; Pétri & Lyubarsky 2007) that reconnection could be an essential part of the dissipation mechanism at the termination shock in the striped pulsar wind. Pulsars lose their rotational energy predominantly via generation of the Poynting flux–dominated winds. Most of the energy is transferred in the equatorial belt, where the sign of the magnetic field alternates with the pulsar period, forming stripes of opposite magnetic polarity; such a structure is called a striped wind (e.g., review by Kirk et al. 2007). When the striped wind arrives at the termination shock, the plasma is strongly compressed; in the comoving frame, the compression ratio is very large, about the Lorentz factor of the upstream flow. Therefore, the alternating magnetic fields are easily annihilated by the compression-driven reconnection. This conjecture is supported by 1.5-dimensional PIC simulations (Lyubarsky 2005; Pétri & Lyubarski 2007).

In one-dimensional simulations, the magnetic energy is transformed into heat because all the particles gain energy with the same rate. In order to study particle acceleration, multidimensional simulations are necessary, because one can expect the formation of nonthermal tails in the particle spectrum if different particles gain different energy; this could happen if X-points are formed within the current sheet. As a step toward the multidimensional simulations of the shock in a striped wind, we performed 2.5-dimensional PIC simulations of driven magnetic annihilation within a plasma layer containing only two stripes of opposite magnetic polarity. In our simulations, the plasma layer is compressed by an external force, which imitates the compression within the shock structure. We show that within such a structure a nonthermal particle spectrum is formed, which can be roughly described by a power law with slope \(-1\) and an exponential cutoff. We analyze the particle motion within the compressed layer, and show that the particle acceleration in the vicinity of X-points plays a crucial role even though only a small fraction of the energy is released there. This is because in the compressing medium, the larger the particle Larmor radius, the more energy it gains; therefore, particles preaccelerated in the X-point gain more energy than the particles bypassed by the X-point. Thus, a large fraction of the total energy is eventually transferred to particles that passed the X-point.

The paper is organized as follows. In § 2, simulation parameters and methods are introduced. In § 3, the results are shown and discussed. Section 4 gives the conclusions of this work.

2. SIMULATION SETUP

We used a 2.5-dimensional (2D3V; two spatial and three velocity components) fully relativistic electromagnetic particle-in-cell
The evolution of electric and magnetic fields is governed by Maxwell’s equations for components of the field vectors $E_x$, $E_y$, and $B_z$. In a two-dimensional configuration with $B_z = 0$, the components $E_x$ and $E_y$ of the electric field are decoupled from the remaining field components and could arise only due to charge fluctuations (cf. simulations by Karlický [2008] and Zenitani & Hoshino [2008], where the charge separation arises at fluctuations). In an electron-positron plasma with the same distribution functions for both species, such fluctuations are very low, and therefore we take $E_x = E_y = 0$ throughout the simulations. Such a suppression of the electrostatic fluctuations permits the use of relatively small number of particles.

The fields are updated using the leap-frog scheme. A staggered grid mesh system, known as the Yee lattice (Yee 1966), ensures that the change of the magnetic flux through a cell surface equals the negative circulation of the electric field around that surface, and that the change of the electric flux through a cell surface equals the circulation of the magnetic field around that surface minus the current through it. Then the divergence-free condition is maintained to machine accuracy. Here the electric and magnetic fields are in symmetry, except for subtracting the charge flux $J$ in Ampère’s equation. The current density is calculated and subtracted after the particles are moved later in the program (Buneman 1993). Before and after moving (or pushing) the particles, the magnetic field is updated in two half-steps, so that it is available at the same time as the electric field for the particle update. The particles positions and velocities are advanced by Newton-Lorentz three-dimensional equations of motion, which were solved by the Buneman-Boris method (Birdsall & Langdon 1985).

Assuming the reconnecting current sheet to lie in the $y$-$z$ plane, Figure 1a illustrates the configuration of the simulated domain (rectangular grid shown by dashed lines). The system lengths in the $y$- and $z$-directions are $L_y$ and $L_z$, respectively. The domain is divided into 3000($L_y$) x 400($L_z$) cells grid or normalized to electron skin depth $384(L_y \omega_p/c) \times 51(L_z \omega_p/c)$, where the plasma frequency $\omega_p$ is calculated for the particle density in the lobe. Figure 1b demonstrates the initial profiles of density $n(y)$, temperature $T(y)$, and velocity $v_{ph}(y)$ near the current sheet. Outside the central part of the simulation box, the density remains constant up to the box margins $0 < y < 10$ and $2990 < 3000$, where the plasma is absent ($n = 0$).

Particles are loaded in the region $10 < y < 2990$. At the boundaries, two external currents normal to the simulation plane are set (as illustrated in Fig. 1 by solid lines at the gray margins) in the same directions. The magnetic field is directed along $z$; in the region between the currents, it is set initially as

$$B_z(y) = B_0 \tanh \frac{L_y - 2y}{2\Delta},$$

where $\Delta = 25$ is the sheet half width. The initial magnitude of the external currents is

$$J_0 = \frac{cB_0 \gamma}{q},$$

so that the magnetic field vanishes in the region outside the currents on both sides.

The magnetic field (eq. [1]) implies current in the $x$-direction; the current density is determined by Ampère’s law,

$$\nabla \times B = \frac{4\pi J}{c}.$$

This current is generated by the motion of electrons and positrons in opposite directions with velocity

$$v_{ph}(y) = \frac{c}{2q(n(y))} \frac{KB_z(y)}{dy}.$$

The initial spatial distribution of particles is nonuniform along the $y$-direction,

$$n(y) = n_0 + \frac{n_{\text{max}} - n_0}{\cosh \left( (L_y/2 - y)/\Delta \right)}.$$

In the main simulations presented here, the total number of particles is $5 \times 10^6$, giving $n_0 = 4$ particles per cell in the lobe and $n_{\text{max}} = 10$ in the sheet. In order to check the reliability of our simulations, we have also run similar models with a smaller box $300(L_y) \times 400(L_z)$ but with a different number density of particles, where $n$ was multiplied by factor $\alpha = 1-10$. In order to keep the system in mechanical equilibrium (eq. [6]), the magnetic field $B$ was multiplied by $\sqrt{\alpha}$, while other parameters were kept unchanged. We found that the particles’ spatial distribution and energy spectrum during the compression were independent of the number density $n$. 

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**Fig. 1.** (a) Schematics of initial configuration, fields, and charge fluxes. (b) Initial profiles of density $n(y)$, temperature $T(y)$, and velocity $v_{ph}(y)$ near the current sheet. Outside the central part of the simulation box, the density remains constant up to the box margins $0 < y < 10$ and $2990 < 3000$, where the plasma is absent ($n = 0$).
The particles are initialized by the Maxwell distribution, with a temperature depending on \( y \) such that the system is in mechanical equilibrium, i.e.,

\[
nT + \frac{B^2}{8\pi} = \text{const}.
\]

Then

\[
T(y) = \frac{B_0^2 - B(y)^2}{16\pi n(y)\sqrt{1 - \left[\frac{v_B(y)}{c}\right]^2}} + T_0,
\]

where \( T_0 \) is a minimal temperature value, \( T_0 = 0.1\text{m}_e^2c^2 \). The initial state chosen is close to the true kinetic equilibrium, because the initial thickness of the sheet is significantly larger than the initial particle Larmor radius, \( r_L = \gamma m_e c^2/qB_0 \approx 1.75 \) (the thermal velocity \( v \) is calculated using the initial temperature at the center of the sheet, which is \( T \approx 1.22\text{m}_e^2c^2 \)).

The boundary conditions imposed along the \( z \)-axis are periodic. Along the \( y \)-axis, we impose Lindman’s radiation-absorbing condition, which requires that the fields should be able to radiate away into space and should not be reflected (Lindman 1975). The time step is chosen such that \( \omega_c \Delta t \approx 0.2 \), where \( \omega_c = qB_0/m_e \). The plasma in the lobe is magnetically dominated; \( \omega_c/L \) is initially 0.32 (\( \omega_c \) is used to calculate \( \omega_L \)) and decreases in the course of compression.

3. SIMULATION RESULTS

In simulations, the reconnection is driven by compression. We increase the magnitude of the boundary currents on both sides, so that the magnetic field grows and the plasma is compressed toward the midplane of the simulated domain. Note that in the course of compression the plasma layer becomes narrower, so that the pressure balance is restored faster; therefore, in order to save computer time, we can take the compression rate as increasing with time. We choose the boundary currents to grow quadratically with time:

\[
J(t) = J_0 \left(1 + \frac{\nu}{L_y} t\right)^2,
\]

where \( \nu = 1.3 \). We have also run the simulation with the compression rate \( \nu = 0.75 \) (twice slower), but no significant difference in the final spatial distribution or energy spectrum was observed.

As a result of external compression, magnetic field line reconnection occurs in the midplane, where the field reverses sign. Figure 2 demonstrates the evolution of the particles’ spatial distribution. The simulated domain is initially separated into several stripes, which are located at different distances from the midplane. The particles initially located in each stripe are marked with a corresponding color, which allows us to trace the motion of different parts of the plasma in the course of compression. Figure 2a shows the initial distribution of particles, where due to the reflection symmetry, the stripes at the same distances from the midplane are colored identically on both sides. The width of the stripes is the same except for the central, more dense stripe (cyan), which is twice as large as the sheet width. As the boundary currents grow, the plasma is compressed and each stripe shifts toward the midplane i.e., to \( y = 1500 \) (see Fig. 2b). In the lobe region, the particle energy is low; therefore, the Larmor radii are small, and particles are restricted to moving with the magnetic field lines toward the current sheet. Thus, each stripe moves as a whole until it reaches the current sheet. Within the current sheet, the magnetic field lines are reconnected, and as a result particles assemble on the magnetic island, where each stripe forms a corresponding ring, as one can see in Figure 2d.

The evolution of the particle energy spectrum in each stripe is presented in Figure 3. Initially, as shown in Figure 3a, the energy of the particles in the lobe is low, and only in the current sheet are the higher energy particles present. When compressed, the stripes begin to form ring regions in the sheet. When a stripe assembles into a ring, the particle energy spectrum extends to higher energies. Figure 3c shows the spectrum of the magenta stripe, which begins to form a ring at \( t = 7000 \). Figure 3d shows that by the end of simulations, the hard tail in the energy spectrum is formed predominantly by particles that entered the current sheet in the course of the reconnection process. The fraction of particles residing in the sheet from the beginning (cyan curve) is small; therefore, the final spectrum is independent of the initial configuration of the system.

In the course of compression and magnetic reconnection, all the particles gain energy. However, taking two identical particles at the same distance from the sheet center, one can find a significant difference (2 orders of magnitude) in their final energies. This wide range of final energies demands further explanation. We take a number of particles with high and low final energies initially located at the same distance of 300 from the midplane (i.e., \( y = 1200 \) and \( y = 1800 \)) and trace them during the compression process. Figure 4 demonstrates the energy evolution of high final energy (solid curves) and low final energy (dashed curves) test particles. As shown, the high final energy particles...
begin acceleration at approximately the same time ($t \approx 5000$) as the chosen stripe enters the current sheet. The particles from the same stripe that were not accelerated at this time remain cold. In Figure 5, five typical trajectories of high final energy particles are shown during the short time interval ($\Delta t = 100$) when the particle energy has just begun to grow. The simultaneous magnetic field lines are shown by thin lines. One sees that, during the onset of acceleration, all these particles are located in the vicinity of the X-point. After these particles leave the X-point their energy continues to grow (see Fig. 4), whereas the energy of particles that bypassed the X-type region remains low. This is because in the course of compression, the particles gain energy in proportion to the energy available. Therefore, the particles preaccelerated near the X-point continue to gain energy, while those that bypassed the X-point remain cold.

A closer look at the particle trajectory shows that the steepest jumps in energy appear at certain locations within the X-point region. The particle energy grows sharply when the electric field becomes equal to or exceeds the magnetic field.

Fig. 4.—Energy of test particles. Solid line: High final energy particles. Dashed line: Low final energy particles.

Fig. 5.—Magnetic field lines (gray) and five typical trajectories (black) of high final energy particles during the onset of acceleration at $t \approx 5000$ (see Fig. 4). All the trajectories (over the interval $\Delta t = 100$) are within the X-type region.

Fig. 6.—(a) Low (yellow crosses) and high (colors) energy particle positions over the $\chi$ value background at $t = 8260$. (b) Energy evolution of the traced particles.
In order to trace the acceleration sites, the quantity

\[ \chi = \frac{B^2 - E^2}{B^2 + E^2} \]  \hspace{1cm} (9)

was constructed, and is shown in Figure 6a as a color background; here \( \chi \rightarrow -1 \) corresponds to \( E \gg B \) and \( \chi \rightarrow 1 \) corresponds to \( E \ll B \). The locations of the test particles are also presented for high final energy (colored dots) and low final energy (yellow crosses) particles. Figure 6b shows the energy evolution of the traced particles; the vertical dashed line marks the instant \( t = 8260 \) of the snapshot in Figure 6a. One can see that when a test particle (green and blue dots in Fig. 6a) enters the region of low \( \chi \) value, the energy (green and blue curves in the bottom panel) of the particle grows explosively.

The black and cyan particles are already outside the X-point region; they were accelerated in this region earlier, and now gain energy by compression. One sees in Figure 6b that the energy of the cyan particle varies in an oscillatory manner; at the later stage, the green particle exhibits the same oscillatory behavior. In order to explain such oscillations, the electric and magnetic fields at the same time \( t = 8260 \) as in Figure 6a are shown in Figures 7a and 7b, respectively. One can see a small magnetic island in the region where the cyan particle is located in Figure 6a. The island rapidly moves away from the X-point region, as shown in Figure 7b by arrows, and eventually merges into the large island in the upper half of the current sheet. Such islands form occasionally when the magnetic field lines are tearing in the X-point. The electric field within such a rapidly moving island reverses sign according to \( E = r \times B \), being positive in front and negative in back of the island motion, as one can see in Figure 7a. Note that within the island, the electric field remains smaller than the magnetic field (even though the magnitude of the electric field is significantly larger within the island than near the X-point), so the particles are not accelerated there. Low-energy particles just drift together with the island because their Larmor radius is less than the size of the island; however, particles with Larmor radius comparable to the size of the island experience oscillations.

When a small magnetic island appears, an additional X-point arises; in Figure 7b it is at the top of the box (recall that the box is periodic in the \( z \)-direction). An additional low-\( \chi \) region arises there (at the top of Fig. 6a) so that an additional acceleration site appears. In Figure 6a, the red particle moves down, about to enter this region from above; Figure 6b shows that this particle is indeed soon accelerated.

Figure 8 shows the evolution of the overall energy spectrum. One can see that the range of particle energies gradually expands, and the high-energy tail appears as plasma is compressed. The
resulting energy spectrum is well approximated by the expression proposed by Larrabee et al. (2003):

$$\frac{dn}{d\gamma} \propto \frac{1}{\gamma} \exp\left(-\frac{\gamma}{\gamma_0}\right).$$

They showed that this formula fits the energy spectrum in the vicinity of the X-point. In our simulations, the overall particle distribution is well fitted at high energies by equation (10). This is because the high-energy tail is formed only by particles pre-accelerated in the X-point. Outside the X-point, they gain energy predominantly by compression. In this case the final particle energy is proportional to the initial energy, so that the particle energy spectrum that forms near the X-point just shifts toward larger energies, the spectral shape remaining unaltered.

Finally, in order to show that the above magnetic reconnection is driven solely by compression, the same simulation was performed for the uncompressed case with constant boundary currents on both sides. The timescale of the simulation was the same as in the compressed case, \( t = 9990, \) which corresponds to \( \sim 3.5 \) light travel times from the boundary to the center of the box (note that in the lobe the plasma is magnetically dominated, so the Alfvén’s velocity is close to the speed of light). Figure 9 shows the particles’ final at \( t = 9990 \) spatial distribution, corresponding energy spectra, and magnetic field. As one can see compared to Figures 2a and 3a, practically nothing has changed from the initial state; the particles remain at their initial locations along the \( \gamma \)-direction (see Fig. 9a), and the energy spectra in each stripe show no considerable changes since the initial state (see Fig. 9b). The small magnetic field islands observed in the current sheet midplane (Fig. 9c) have already developed at the early stages of the simulation, after which nothing changes. The magnetic energy in the box also remains constant, to within the error of computation, throughout the simulation. These results clearly show that the initial configuration is stable, which means that the magnetic reconnection and the resulting particle acceleration discussed in the present paper occur exclusively due to external compression.

Fig. 8.— Evolution of the total particle energy spectrum. The dotted curves represent the fit (10). Spectrum curves are presented for times \( t = 4000, 6000, 8000, 10000, \) and fit \( \gamma_0 = 4, 16, 25, 35, \) correspondingly.

Fig. 9.— Uncompressed configuration at \( t = 9990, \) showing (a) the particle spatial distribution, (b) the energy spectrum for each of the colored stripes, and (c) the magnetic field lines in the current sheet.

4. CONCLUSIONS

We performed 2.5-dimensional PIC simulations of particle acceleration in the course of compression-driven reconnection in a relativistic electron-positron plasma. Since the plasma is strongly compressed at the front of a relativistic shock, we consider our simulations as a simple model for the particle acceleration at the termination shock in the striped pulsar wind. We found that the X-points play crucial role, even though a relatively small fraction of the total energy is released there. Particles preaccelerated in the vicinity of the X-point take a significant fraction of the released energy, because in the compressing medium, particles with larger Larmor radii gain more energy. This result could be of a more general nature, because for particles with larger Larmor radii, the frozen-in condition is violated more easily, and therefore one can expect that in various situations, particles preaccelerated in the vicinity of X-points could take a good fraction of the dissipated magnetic energy.

In our simulations, the fraction of particles initially confined in the current sheet is small, so that in the final state, the energy spectrum of the particles in the sheet is independent of the initial condition. This spectrum is found to be approximately \( \frac{dn}{d\gamma} \propto \gamma^{-1} \) for \( \gamma < \gamma_0 \), while it is rapidly decreasing for \( \gamma > \gamma_0 \). The maximal energy \( \gamma_0 \) grows in the course of reconnection. The electrons with spectrum \( \frac{dn}{d\gamma} \propto \gamma^{-1} \) emit synchrotron radiation with a flat spectrum. Therefore, our results support the idea that flat radio spectra of plerions may be attributed to particle acceleration at the termination shock in the striped pulsar wind (Lyubarsky 2003).

Note that the occurrence of X-points was questioned by Zenitani & Hoshino (2005, 2007), who showed that in the electron-positron plasma, the tearing instability, which is thought to be responsible for the formation of X-points, develops more slowly than the drift kink instability, which just shifts straight magnetic field line tubes, making the current sheet folded. Their PIC
simulations show that the field dissipation due to the drift-kink instability does not result in nonthermal particle acceleration, the plasma just being heated. This is because the field line tubes remain straight, so that all particles gain energy at the same rate. However, the drift kink instability is suppressed in the presence of the current-aligned magnetic field (the so-called guide field); then magnetic reconnection occurs in X-points and produces many nonthermal particles (Zenitani & Hoshino 2005, 2008).

Therefore, the mechanism identified in this work remains relevant under quite general conditions.

This work was supported by the German-Israeli foundation for scientific research and development under the grant I-804-218.7/2003.

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