Quantum-like formalism for cognitive measurements

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Abstract

We develop a quantum formalism (Hilbert space probabilistic calculus) for measurements performed over cognitive systems. In particular, this formalism is used for mathematical modeling of the functioning of consciousness as a self-measuring quantum-like system. By using this formalism we could predict averages of cognitive observables. Reflecting the basic idea of neurophysiological and psychological studies on a hierarchic structure of cognitive processes, we use \( p \)-adic hierarchic trees as a mathematical model of a mental space. We also briefly discuss the general problem of the choice of an adequate mental geometry.

1 Introduction

Since the creation of quantum mechanics, there are continuous discussions on possible connections between quantum and mental phenomena. During the last hundred years, a huge number of various proposals and speculations have been presented. We shall mention just a few of them.

The philosophic system of Whitehead [1]-[3] was the first attempt to establish a quantum–mental (or more precisely mental \(\rightarrow\) quantum) connection. In Whitehead’s philosophy of the organism “quantum” was some feature of basic protomental elements of reality, namely actual occasions, see
[1], especially p. 401-403. See also A. Shimony [4] for modern reconsideration of quantum counterpart of Whitehead’s philosophy of organism. It is especially important for us to underline that all protomental elements of reality have quantum temporal structure: “The actual entity is the enjoyment of a certain quantum of physical time.” – [1], p. 401.

The extended discussion on quantum–mental connection was induced by attempts to solve the problem of quantum measurements, see e.g. [5]-[12]. The most extreme point of view is that physical reality is, in fact, created by acts of observations. This kind of considerations is especially closely related to the so called orthodox Copenhagen interpretation of quantum mechanics.\(^1\) By this interpretation a wave function provides the complete description of an individual quantum system. An act of measurement induces collapse of the wave function. The problem of measurement is still unsolved within a quantum framework (at least on the basis of the conventional interpretation of quantum mechanics, see also section 10). Among various attempts to provide a reasonable explanation of wave function reduction, one should mention attempts to use consciousness as the determining factor of reductions of wave functions, see e.g. Wigner [8].

There were also various attempts reduce an act of thinking to quantum collapse, see e.g. Orlov [13] (quantum logic of consciousness); see also Penrose [14], [15]: “I am speculating that the action of conscious thinking is very much tied up with the resolving out of alternatives that were previously in linear superposition.”

In fact, Penrose worked in the reductionist approach, see e.g. [16] (and compare e.g. [17]-[20]): It seems we could not reduce cognitive phenomena to the physical activity of neurons. It might be that we could reduce it to activity of quantum systems. Roughly speaking an act of thinking is reduced to the collapse of a wave function in quantum gravity. Our thinking ability is based on collapses of superpositions of two mass states.

The idea of quantum-physical reduction for cognitive processes is quite popular within the quantum community. We also mention the investigations of H. Stapp [21] who used the Copenhagen (Heisenberg-potentiality) approach to quantum mechanics. He also used a quantum reductionist approach: “Brain processes involve chemical processes and hence must, in principle, be treated quantum mechanically.” We should also mention quantum field reductionist models, by Jibu and Yasue [22], [23] (based on Umezawa [24]), Vitiello et al. [25]. These quantum field models look more attractive (at least for me). At the moment there is no idea how make the great jump\(^1\)

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\(^1\)We mention Berkeley’s idealism as one of sources for such a point of view to physical reality.
from individual gravitational collapses to global acts of cognition. Quantum field models are more useful to provide such a global structure connecting individual quantum collapses to global acts of thinking.

However, it seems that reductionism as the general methodology of studying the brain is less and less popular in cognitive sciences. After the period of large hopes associated with new possibilities to study neuronal firings, there is strong disillusionment in the possibility of some physical reduction of mental processes. This is one reason for the quite strong critical attitude against quantum models in cognitive sciences. In the extreme form this criticism is expressed in the following form:

"The only common thing between quantum and mental is that we have no idea how to understand any of these phenomena."

Another thing that induces prejudice against quantum-reduction theories among neurophysiologists is that a quantum micro description contains many parameters that are far from magnitudes of corresponding brain’s parameters (e.g. temperature, time scale and so on). Thus creators of all quantum reductionist models of brain’s functioning become immediately involved in hard battles with these parameters (e.g. high temperature of brain). Of course, it may be that all these parameter-problems are just technical temporary problems. It may be that in future even the decoherence problem, see, for example, [15], would be solved. Nevertheless, there are doubts about the possibility of the direct application of quantum physical theory to cognitive phenomena.

My personal critical attitude with respect to traditional quantum cognitive models is merely based on absence of a realistic bio-physical model that would explain the transition from quantum processes in the microworld to cognitive processes. Of course, apart from the plethora of theoretical models such that proposed by Hameroff, Nanopoulos, Mavromatos, Mershin and many others there are also very concrete experimental papers such as those by Wolf pertaining to exactly that point (see section 10 for the details). Nevertheless, at the moment we still do not have a transition model that would be accepted by majority of neurophysiologists and psychologists. 2

2On one hand, absence of such a realistic bio-physical model could be simply a consequence of the huge complexity of this problem. If it is really the case, then we could expect that in the future such a model would be finally created. On the other hand, it might occur that such a model does not exist at all, since it might be that cognition is not generated by quantum processing of information in the microworld. As we have already mentioned, the latter view has become very popular over the last years. So absence of a realistic transition model has become a dangerous problem for the whole quantum cognition enterprise. My personal position on the possibility of quantum reduction is based on quantum as well as neurophysiological experience. On one hand, I totally agree with neurophysiologists that there are very strong arguments that cognitive information
Finally, we discuss the holistic approach to cognitive phenomena based on Bohmian-Hiley-Pylkkänen theory of active information. By considering the pilot wave as a kind of information field they presented interesting models of cognitive processes, see [26]-[28], see also author’s work [29]. In the latter paper there was proposed a mathematical model of field of consciousness. This field is not defined on physical space-time. This is a pure information structure. In principle, such a field can be considered as a mathematical representation of Whitehead’s field of feeling [1].

Consciousness-information models also were developed in books of M. Lockwood [30], and J. A. Barrett [31] (who use a many-minds version of many-worlds interpretation of quantum mechanics) and in the author’s paper [32] devoted to quantum information reality.

Over the last few years I tried to split, see [33]-[36], the quantum formalism into two more or less independent parts:

1) really quantum (quanta, Planck constant, discreteness);
2) Hilbert space probabilistic formalism.

Pioneer investigations of M. Planck and A. Einstein on foundations of quantum theory (black body radiation and photoelectric effect) were merely investigations on discreteness (quantization) of energy. Quantum probabilistic (Hilbert space) formalism was developed later (Born, Jordan, Heisenberg, Dirac [37]-[38]). It was created to describe statistics of elementary particles. Due to such a historical origin, the Hilbert space probabilistic calculus is always related to processes in the microworld.

However, careful analysis, [33]-[36], demonstrated that Hilbert space probabilistic calculus (Born, Jordan, Heisenberg, Dirac, see e.g. [37]-[38]) is a purely mathematical formalism that gives the possibility to work with context-dependent probabilities, i.e., probabilities depending on complexes of physical conditions (contexts) related to concrete measurements. Therefore we could apply the Hilbert space probabilistic formalism, quantum-like formalism, not only to the description of statistical micro phenomena, but also to various phenomena outside micro world. One of such possibilities is to apply quantum-like formalism to describe statistical experiments with

is prosessed on neuronal macro-level. On the other hand, I am strongly impressed by the idea of interference of cognitive information proposed by quantum physicists (e.g. Orlov, Penrose, Stapp, Hameroff and many others). I would like to explore this idea and at the same time escape the problem of transition from quantum processes in the microworld to cognitive processes. I use the probabilistic calculus of quantum formalism (Hilbert space formalism) to describe probabilistic amplitude thinking. In particular, in such a model, the brain would be able to create ‘interference of minds.’ However, in my model elementary units of processing of cognitive information are macroscopic neuronal structures and not quantum micro systems.

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cognitive systems. Here a quantum-like formalism describes probabilistic distributions depending on neural, cognitive and social contexts.

Such an approach has no (at least direct) relation to reductionist quantum models. We are not interested in statistical behaviour of micro systems forming a macro system, the brain. Therefore this approach does not induce such a problem as the transition from micro to macro (temperature, decoherence and so on). We just use the Hilbert space probabilistic formalism to describe cognitive measurements. As in the ordinary quantum formalism, mental observables are realized as symmetric operators in the Hilbert space of square integrable functions $\phi(q)$ depending on the mental state $q$ of a cognitive system. By using the Hilbert space scalar product we calculate averages of mental observables. Of course, this cognitive model is a purely statistical one. It could not provide a description of individual thought-trajectories.

One of the reasons for using quantum-like formalism to describe statistics of measurements over cognitive systems is that cognitive systems (as well as quantum) are very sensitive to changes of contexts of experiments - complexes of physical and mental conditions ([33]-[36], compare to Heisenberg [38] or Dirac [37]). Quantum-like formalism might be used to describe external measurements (in neurophysiology, psychology, cognitive and social sciences) over ensembles of cognitive systems or neural ensembles in a single brain.\footnote{Thus we extend to cognitive sciences Heisenberg’s viewpoint to the role of disturbances in producing of quantum interference.}

As well as in quantum experiments with elementary particles, preparation of a statistical ensemble (of rats or people) plays the crucial role in cognitive measurements. Thus, as in ordinary quantum theory, it is meaningless to speak about a measurement without specifying a preparation procedure preceding this measurement. In cognitive sciences we also should follow Bohr’s recommendation to take into account the whole experimental arrangement. The main experimental evidence of quantum-like structure of statistical data obtained in neurophysiology, psychology, cognitive and social sciences should be interference of probabilities, see [33]-[36] and section 10.

Moreover, our quantum-like formalism can be used not only for describing external cognitive experiments, but also modeling of mentality. The basic assumption of our model is that the brain has the ability to “feel” probabilistic amplitude $\phi(q)$ of information states produced by hierarchic neural pathways in brain (and the whole body). There is also a model of consciousness that creates its context by performing self-measurements over extremely sensitive neural contexts.

One of the fundamental problems in foundations of cognitive quantum-like formalism is the choice of a mathematical model for a mental configura-
tion space on which a wave function is defined. We shall discuss this problem in the details in section 2. We now only remark that the Euclidean physical space (within which the physical brain is located) does not look attractive as a model of mental space. Instead of this conventional model of space, we develop cognitive quantum-like formalism on the space of information strings that could be performed by chains of hierarchically ordered neurons. Such a configuration space is geometrically represented by a hierarchical $p$-adic tree. In fact, this idea was already discussed in the authors’s paper [32] (see also [39]-[44]). However, in [32] we did not use the standard Hilbert space formalism. We used a generalization of quantum probabilistic calculus based on $p$-adic probabilities. In the present paper we use the standard Hilbert space formalism on $p$-adic trees. In fact, the mathematical formalism of $p$-adic quantum mechanics is well developed, see Vladimirov, Volovich, Ze- lenov [45], [46], see also [47]. We ”simply” apply this formalism to cognitive phenomena.

In the ordinary quantum mechanics, we could go beyond the statistical application of quantum formalism. One of the most attractive possibilities is to use the pilot wave Bohmian formalism. As we have already remarked, the idea to use Bohmian mechanics in cognitive sciences was already well discussed (Bohm-Hiley-Pilkkänen [26]-[28] and author [29]). It is rather surprising that it seems to be impossible to create a variant of the pilot wave extension of quantum-like mental formalism presented in this paper. Formally we can introduce quantum-like mental potential and force. However, there is no possibility to derive the equation of motion (a kind of Newton equation) that would describe trajectories of individual mental states (describe ”flows of mind”). In our formalism, this is a consequence of the mathematical structure of the model. However, it may be that there are some deep cognitive features behind this mathematical result.

We start with some preliminary considerations on the choice of the geometry of a mental space.

2 Where is consciousness located?

The problem of location (or nonlocality) of consciousness (as well as more primitive cognitive processes) is widely discussed in philosophic, neurophysiological and psychological literature, see e.g. [48]-[56]. There is large variety of views starting with such a primary question:

‘Is consciousness located in human brain?’

Both philosophic and neurophysiological discussions are, in fact, related to one fixed geometry, namely the Euclidean one. It seems that such an
approach was originated (at least in philosophy) by Kant [55]. For him, the space was the absolute Euclidean space. He also pointed out that the idea of space is the primary idea. Nothing could be even imagine without any relation to space. Since Kantian space is identified with the Euclidean space, we have to look for a place of consciousness in this space. It seems that this is the starting point of the main stream of modern philosophic, neurophysiological and psychological investigations. However, despite enormous efforts to find the place of consciousness, there is more and more evidence that consciousness cannot be located in physical space. What is wrong? I think the choice of geometry. I think that the use of the Euclidean geometry is not adequate to this problem.

In fact, the idea that different natural phenomena are in general described by using different geometries is well established in physics, especially general relativity and string theory. Following Chalmers [56], we consider consciousness as a kind of natural phenomenon. First we must find an adequate model of a mental space. Then we get the possibility to describe cognitive (and conscious) phenomena. Let us imagine that we would like to describe electromagnetic processes without using a mathematical model of the electromagnetic field distributed on the Euclidean space. This seems to be impossible. 4

We have already mentioned the use of various geometries in general in physics, e.g. in general relativity and string theory. However, these models are mainly locally-Euclidean (Euclidean manifolds).5 The use of such manifolds could not solve the problem of cognitive nonlocality (in particular, nonlocality of psychological functions). One of possibilities is to proceed in a quantum-like way and use noncommutative mental coordinates, see B. Hiley [28]. Another possibility is to try to find a model of classical mental configuration space (probably as the basis of a quantum-like model). Since [39]-[46], we use a purely information model of mental space, namely the space of all possible information strings that could be produced by hierarchically ordered chains of neurons. One of the simplest models of such a space is a hierarchical (homogeneous) $p$-adic tree $\mathbb{Z}_p$, where $p$ is a natural number. It gives the number of branches leaving each vertex of this tree. We remark that in mathematical models $p$ is typically a prime number, see [45], [47]. But this

4Sometimes (especially in philosophy) one uses the expression “to explain consciousness”. I do not think that we can “explain” it. In the same way we can not “explain”, e.g., the electromagnetic field. We can only describe mathematically and understand it via such a description, compare to Penrose [15], p. 419.

5Even the use of superspace in superstring theory as well as in superfield theory cannot be considered as a fundamental change of geometry. Locally, superspace is still a real continuous manifold, see e.g. [57] for the details.
is not so important for our cognitive considerations.

3 Classical mental states produced by one-layer brain

3.1. $p$-adic coding. We consider the simplest hierarchic “brain” consisting of just one hierarchic chain of neurons:

$$\mathcal{N} = (n_0, n_1, ..., n_N, ...).$$

In a mathematical model it is convenient to consider an infinite chain.

In the simplest model each neuron can perform only one of two states: $\alpha_j = 1$ (firing) and $\alpha_j = 0$ (off).

In more complex models each neuron $n_j$ can perform $p$ different levels of activation: $\alpha_j = 0, 1, ..., p - 1$. For example, such a coding can be obtained by using frequencies of firing of neurons as basic elements of coding. Frequencies of firing are a better basis for the description of processing of information by neurons than simple on/off. This has been shown to be the fundamental element of neuronal communication in a huge number of experimental neurophysiological studies (see e.g. [58], [59] on mathematical modeling of brain functioning in the frequency domain approach).

One of possible $p$-adic coding models is the following one. A $p$-adic structure associated with frequency coding is generated in the following way. There exists some interval (of physical time) $\Delta$ (unit of “mental time”, see section 10 for further consideration). Then $\alpha_j$ is equal to the number of oscillations of the neuron $n_j$ (in the hierarchic chain $\mathcal{N}$) that are performed during the interval $\Delta$. Here $p - 1$ (where $p = p_\Delta$) is the maximally possible number of oscillations during the period $\Delta$ that can be performed by neurons in the chain $\mathcal{N}$. Thus in our model the $p$-adic structure of the brain of a cognitive system $\tau$ (that uses a frequency neural code) is related to the time scale of the functioning of the brain, see also section 10.

We must mention one mathematical fact that may be have some cognitive interpretation. The case $p = 2$ is a very exceptional one in $p$-adic analysis, see e.g. [45], [47]. We can speculate that the transition from 2-adic coding (firing/off) to more complex $p$-adic, $p > 3$, coding (e.g. frequency coding) was the evolutionary jump. Cognitive systems in the $p$-adic model exhibit essentially richer mental behaviour in the case $p > 2$ than in the case $p = 2$, see [47], [39]-[44] on classical mental dynamics.

3.2. Hierarchy and ultrametricity. It is supposed that neurons in a layer $\mathcal{N}$ are hierarchically ordered: $n_0$ is the most important (igniting), $n_1$ is
less important and so on. The $\mathcal{N}$ is able to produce information strings of the form:

$$x = (x_0, x_1, ..., x_N, ...), \ x_j = 0, 1, ..., p - 1.$$  

We denote the set of all such strings by the symbol $\mathbb{Z}_p$. The hierarchic structure in the chain $\mathcal{N}$ induces a tree representation of $\mathbb{Z}_p$. Information strings are represented by branches of such a tree.

![Figure 1: The 2-adic tree](image)

The distance between two branches, $x$ and $y$, is defined in the following way. Let $l$ be the length of the *common root* of these branches. Then the $p$-adic distance between $x$ and $y$ is defined as

$$\rho_p(x, y) = \frac{1}{p^l}.$$  

Thus if $x = (x_j)$ and $y = (y_j)$ and $x_0 = y_0, ..., x_{l-1} = y_{l-1}$, but $x_l \neq y_l$, then $\rho_p(x, y) = \frac{1}{p^l}$. This is a metric on the set of branches $\mathbb{Z}_p$ of the $p$-adic tree. Two branches are close with respect to this metric if they have a sufficiently long common root. We remark that $\mathbb{Z}_p$ is complete with respect to the $p$-adic metric $\rho_p$.

The $p$-adic metric gives a topological representation of the hierarchic structure in neural chains. The distance between information strings $x$ and $y$ approaches the maximal value $\rho_p(x, y) = 1$ if $x_0 \neq y_0$. Thus the state (e.g. the frequency of firing) of the first neuron $n_0$ in a hierarchic chain $\mathcal{N}$ plays the most important role. States $x_j$ of neurons $n_j$, where $j \to \infty$, have a practically negligible contribution into the geometry of the $p$-adic space.
The $p$-adic metric is a so called ultrametric, i.e., it satisfies the strong triangle inequality:

$$\rho_p(x, y) \leq \max[\rho_p(x, z), \rho_p(z, y)], x, y, z \in \mathbb{Z}_p.$$ 

The strong triangle inequality can be stated geometrically: each side of a triangle is at most as long as the longest one of the two other sides. This property implies that all triangles are isosceles. Ultrametricity is a very important feature of $p$-adic geometry. In fact, ultrametricity is the exhibition of hierarchy. Recently it was proved in general topology that in general case ultrametricity induces a treelike representation and vice versa, see [60]. In many particular cases such a relation between ultrametricity and hierarchy was used in theory of spin glasses, see e.g. [61]-[63].

There exists a natural algebraic structure on this tree: addition, subtraction and multiplication of branches. It is based on the representation of information strings by so called $p$-adic numbers:

$$x = x_0 + x_1 p + \ldots + x_N p^N + \ldots$$

This is the ring of $p$-adic integers. In particular, this is a compact additive group. Thus there exists the Haar measure $dx$ (an analogue of the ordinary linear measure on the straight line).

We set $B_r(a) = \{x \in \mathbb{Z}_p : \rho_p(x, a) \leq r\}$ and $S_r(a) = \{x \in \mathbb{Z}_p : \rho_p(x, a) = r\}$, where $r = 1/p^j, j = 0, 1, 2, \ldots$ and $a \in \mathbb{Z}_p$. These are, respectively, balls and spheres in the metric space $\mathbb{Z}_p$. In particular, $\mathbb{Z}_p = B_1(0)$. Each ball has the structure of the homogeneous $p$-adic tree (scaling of the basic tree given by $\mathbb{Z}_p$).

As in every ultrametric space, all these sets (balls and spheres) have a topological structure which seems to be rather strange from the point of view of our Euclidean intuition: they are open and closed at the same time. Such sets are called clopen. Another interesting property of $p$-adic balls is that two balls have nonempty intersection iff one of these balls is contained in another. Finally we note that any point of the $p$-adic ball can be chosen as its center. Thus the ball is not characterized by its center and radius.

3.3. Mental space. We choose the space $Q = \mathbb{Z}_p$ as a mental configuration space. Points $q \in Q$ are called mental classical-like states (or simply mental states) or mental positions.

Thus a mental state $q \in Q$ describes activity of neurons in a hierarchically ordered chain of neurons. This is a kind of information state. Such a state could not be considered simply as the representation of physical (electro-chemical) activity of neurons in a chain. There are two information parameters that play important roles in our model.
First there is the hierarchic structure in a neural chain. Neurons in a chain “do not have equal rights.” The igniting neuron \( n_0 \) is the bandmaster of the orchestra \( \mathcal{N} \). This orchestra is rigidly hierarchic. The next neuron \( n_1 \) in the \( \mathcal{N} \) is less important than \( n_0 \) and so on. I think that the presence of such a hierarchy plays an important role in creation of cognition and may be even consciousness.

Another information parameter is a natural number \( p \) that determines the coding system of (one layer) brain \( \mathcal{N} \). If we follow to the frequency approach to functioning of neural networks in brain, then the parameter \( p \) gives the maximal number of oscillation for a neuron in a chain \( \mathcal{N} \) during the unit interval \( \Delta \) of mental time. The \( \Delta \) is an interval of physical time that in our model determines the neural code of \( \mathcal{N} \), see section 10.

In our model a mental state provides only cognitive representation and not the contents of consciousness. All unconscious processes are performed on the level of mental states. We remark that in a multi-layer brain, see section 5.1, various unconscious cognitive processes can be performed parallelly. Nonlinear dynamical models of such processes were studied in [47], [39]-[44]. One of the distinguishing features of \( p \)-adic nonlinear dynamics is the absence of chaotic behaviour. In general \( p \)-adic dynamical systems are essentially more regular than real ones. Moreover, they are very stable with respect to random perturbations (in particular, noises), [43]. Typically a \( p \)-adic random dynamical system has only deterministic attractors, see [43]. We remark that dynamics in spaces of \( p \)-adic numbers depends crucially on the parameter \( p \) (determining the neural code of a brain \( \mathcal{N} \)). As it was discovered in [47], the same dynamical system (e.g. given by a monomial \( x^n \)) can demonstrate completely different behaviours for e.g. \( p = 2 \), or \( p = 3 \), or \( p = 1999, \ldots \).

We are now going to consider a quantum-like model based on the \( p \)-adic mental configuration space \( Q = \mathbb{Z}_p \). In particular, this model might be used to describe the transition from unconscious representation of cognitive information to conscious one.

## 4 Quantum-like formalism for one layer brain

### 4.1. Hilbert space probabilistic formalism for mental observables.

We consider the space of square integrable functions \( L_2(Q, dx) \), where \( Q = \mathbb{Z}_p \):

\[
\phi : \mathbb{Z}_p \to \mathbb{C}, \|\phi\|^2 = \int_{\mathbb{Z}_p} |\phi(x)|^2 dx < \infty.
\]

The space \( \mathcal{H} = L_2(Q, dx) \) is chosen as the space of mental quantum-like states.
These states are represented by normalized vectors $\phi \in \mathcal{H} : \|\phi\| = 1$. The $\mathcal{H}$ is a complex Hilbert space with scalar product

$$
(\phi, \psi) = \int_Q \phi(x) \bar{\psi}(x) dx .
$$

(1)

Mental observables are realized as self-adjoint operators $A : \mathcal{H} \to \mathcal{H}$. As in the ordinary quantum formalism, by fixing a quantum-like state $\phi \in \mathcal{H}$ in general we do not fix the concrete value $A = \lambda$ of a mental observable $A$. It is only possible to find the average of $A$ in the state $\phi$:

$$
<A>_{\phi} = \int_Q A(\phi)(x) \bar{\phi}(x) dx .
$$

(2)

However, if $\phi \in \mathcal{H}$ is an eigenfunction of $A$ corresponding to the eigenvalue $\lambda$, i.e., $A\phi = \lambda \phi$, then we can be sure that we shall obtain the value $A = \lambda$ with probability 1.

The concrete representations of mental observables by self-adjoint operators is a very important and nontrivial problem. This problem could not be solved by trivial generalization of an ordinary quantum formalism. We start with this surprising remark: it seems to be impossible to define mental position, $q$, observable. Formally the difficulty is purely mathematical: we could not multiply a $p$-adic number $q \in \mathbb{Q}$ with a complex number $\phi(q)$. Therefore the standard Schrödinger’s definition of the position operator could not be generalized to the cognitive case. Of course, we could try to find some mathematical tricky (“non natural”) definitions of mental position operator. However, it might be that this mathematical difficulty is evidence of some important feature of cognitive systems. It might be that:

Even in principle it is impossible to measure mental states $Q$ of brain.

In particular, we could not prepare a statistical ensemble of brains having the fixed mental state (there are no mental state eigenfunctions).

We can only find the probability that a mental state $q$ belongs to some (measurable) subset $O$ of the mental space $Q : P(q \in O) = \int_O |\phi(x)|^2 dx$.

**Example 4.1.** Let us consider the quantum like state $\phi \equiv 1$ (the uniform probability distribution of mental states). Then $P(q \in B_r(a)) = r$. Thus (as it could be expected) the probability to find this cognitive system in the mental state $q$ belonging to a small ball around any fixed point $a$ is small.

**4.2. External mental measurements.** An important class of mental observables is given by measurements that are performed by external systems over a cognitive system $\tau$. In particular, in section 6 we shall introduce a neuron-activation observable that arises naturally in neurophysiological measurements. Besides neurophysiological mental observables, we can consider e.g. psychological or social mental observables. In experiments with
people such a mental observable $A$ can be given just by a question $A$. Here $A$ takes two values: $A = 1$, yes, and $A = 0$, no. In experiments with animals values of $A$ give possible reactions of animals to experimental conditions. In principle an external system that performs a measurement over a cognitive system $\tau$ need not be conscious nor even cognitive. It can be, for example, a magnetic resonance device performing a measurement of neural activity.  

4.3. Consciousness. I would not like to reduce mental measurements to external measurements. It is natural to try to describe consciousness as a continuous flow of mental self-measurements. The idea that cognitive representation of information in the brain becomes conscious in the process of self-measurements is not so new, see e.g. Orlov [13] for a quantum logic model of self-measuring consciousness: "... the volitional act of a free choice plays in this theory a role analogous to the role of the measurement act in quantum mechanics (with the important difference that the brain "measures" itself). Consciousness is a system which observes itself and evaluates itself – being aware, at the same time, of doing so."

The crucial point of our consideration is that we use a quantum-like ideology, instead of the traditional quantum one. In our model the configuration space is the state space of macroscopic neural networks. Thus we need not go deeply into the microworld to find the origin of consciousness (e.g. no collapses of mass-superpositions and so on). So we need not apply to quantum gravity (or even superstring theory).

In our model it is supposed that each cognitive system $\tau$ developed the ability to feel the probability distribution $P(q)$ of realization of the hierarchic information string $q$ by its neural system. Such an ability is basically transferred from generation to generation. However, for each $\tau$ it is permanently developed in the process of brain's functioning. This probability distribution $P(q)$ has an amplitude $\phi(q)$ that can be mathematically described by a normalized vector in the Hilbert space $\mathcal{H} = L_2(Q, dx)$. As usual, $P(q) = |\phi(q)|^2$.

As was already discussed, the appearance of the quantum-like probabilistic formalism (instead of classical Kolmogorov probabilistic formalism) is a general consequence of sensibility of $P(q)$ to changes in the neural context. Here

$$\phi(q) = \sqrt{P(q)} e^{i\theta(q)}.$$  

Here $\theta(q)$ is a phase parameter. It appears automatically in transformation of probabilities from one mental representation (see 5.4) to another, see [33]-[36]. We shall illustrate the role of $\theta(q)$ on the example of transition from...
mental-position to motivation representation, section 5.4.

In our model “feeling” of the probability distribution is performed on an unconscious level. In particular a cognitive system does not feel consciously the evolution of the mental amplitude $\phi(t, q)$.

Moreover, we suppose that each conscious cognitive system $\tau$ has the ability to perform self-measurements. Results of these measurements form the contexts of consciousness. I do not try to develop such a model of consciousness in the present paper. The main aim of this paper was to present quantum-like formalism corresponding to hierarchic neural networks. In principle, the reader can use only a restricted viewpoint of mental observables as external measurements over cognitive systems. We just consider a possible scheme of functioning of such a (quantum-like) self-measuring consciousness.

4.4. Random dynamical quantum-like consciousness. Let us denote the set of all operators representing mental observables participating in the creation of the contents of consciousness by the symbol $\mathcal{L}_{\text{cons}}(\mathcal{H})$. Let us consider a random dynamical system (RDS: see, for example, [64] for general theory) that at each instant of (mental) time chooses randomly some set of commutative operators $A_1, \ldots, A_m \in \mathcal{L}_{\text{cons}}(\mathcal{H})$. The contents of consciousness at this instant of time is created by the simultaneous measurement of $A_1, \ldots, A_m$.

One of the main distinguishing features of the RDS-model is that a RDS in the space of mental observables can have long range memory. Such a feature of RDS is very important to create a realistic mathematical model of functioning of consciousness. Our consciousness does not consist of discrete moments but there is flow of consciousness. We remember something about our earlier conscious experiences, see e.g Whitehead’s analysis of this problem [1], p. 342-343: “Whenever there is consciousness there is some element of recollection. It recalls earlier phases from the dim recesses of the unconscious. Long ago this truth was asserted in Plato doctrine of reminiscence. No doubt Plato was directly thinking of glimpses of eternal truths lingering in a soul derivative from timeless heaven of pure form. Be that as it may, then in a wider sense consciousness enlightens experience which precedes it, and could be without it if considered as a mere datum. Hume, with opposite limitations to his meaning, asserts the same doctrine...But the immediate point is the deep-seated alliance of consciousness with recollection both for Plato and for Hume.”

5 Motivation observable

5.1. Multi-layers hierarchic brain. To consider nontrivial examples of mental observables, it is convenient to study a ”brain” having a more
complex mental space. Such a brain consists of a few hierarchic $p$-adic trees. We consider a layer of neurons

$$\mathcal{N} = (... , n_{k}, ..., n_{0}, ..., n_{l}, ...)$$

that goes in both directions (in the mathematical model it is infinite in both directions). Each neuron $n_{j}, j = 0, \pm 1, \pm 2, ..., $ can be the igniting neuron for right hand side hierarchic chain: $\mathcal{N}_{j} = (n_{j}, ..., n_{l}, ...)$. The corresponding mental space $\mathbb{Z}^{(j)}$ consists of all information strings

$$x = (x_{j}, x_{j+1}, ..., x_{l}, ...), x_{l} = 0, 1, ..., p - 1$$

(in particular, $\mathbb{Z}_{p} = \mathbb{Z}^{(0)}$). Each space has the structure of the homogeneous $p$-adic tree. These spaces are ordered by inclusion: $\mathbb{Z}^{(j+1)} \subset \mathbb{Z}^{(j)}$. We consider union of all these space $\mathbb{Q}_{p} = \bigcup_{j=-\infty}^{\infty} \mathbb{Z}^{(j)}$. Geometrically this space is represented as a huge collection of trees ordered by the inclusion relation. On this space we can introduce the structure of ring: addition, subtraction and multiplication of branches of trees. If the coding parameter $p$ is a prime number (i.e., $p = 2, 5, 7, ..., 1997, 1999, ...$), then $\mathbb{Q}_{p}$ is a field, i.e., division of branches also is well defined. In this case $\mathbb{Q}_{p}$ is a number field (of $p$-adic numbers). Arithmetical operations are performed by using $p$-adic number representation of branches:

$$x = \sum_{i=j}^{\infty} x_{i} p^{i}, j = 0, \pm 1, \pm 2, ... \quad (3)$$

Metric on $\mathbb{Q}_{p}$ is defined in the same way as on $\mathbb{Z}_{p}$. In particular, each tree $\mathbb{Z}^{(j)}$ coincides with a $p$-adic ball $B_{r}(0)$, where $r = 1/p^{j}$. We shall also use $p$-adic absolute value: $|x|_{p} = \rho_{p}(x, 0)$. To calculate it, we have to find in the chain $\mathcal{N}$ the first (from the left hand side) firing neuron $n_{j}$ ($x_{j} \neq 0$, but $x_{l} = 0$ for all $l < j$) and set $|x|_{p} = 1/p^{j}$.

The $\mathbb{Q}_{p}$ is a locally compact field. Hence, there also exists the Haar measure $dx$.

We now choose $Q = \mathbb{Q}_{p}$ as a model of a mental configuration space; consider the Hilbert $\mathcal{H} = L_{2}(Q, dx)$ of square integrable functions $\phi : Q \to C$ as the space of quantum-like mental states.

5.2. Motivation magnitude observable. It would be interesting to consider the following quantity (more precisely, qualia): motivation $\xi$ to change the mental state $q$. Unfortunately, for the same reasons as for the mental state observable we could not introduce a motivation observable. However, we can introduce an observable $M_{\xi}$ that will give the magnitude of a motivation. It is impossible to prepare a brain with the fixed motivation $\xi$,
but we could prepare a brain with the fixed amplitude of a motivation (that
gives a measure of motivation’s strength). Such $M_ξ$ must be a kind of deriva-
tive with respect to the mental state (coordinate) $q$. Such a generalization of
derivative is given by Vladimirov’s operator $D$, see [45], defined with the aid
of the $p$-adic Fourier transform.\(^8\)

**$p$-adic Fourier transform:**

$$\tilde{\phi}(ξ) = \int_Q \phi(x)e(ξx)dx, ξ ∈ Q,$$

where $e$ is a $p$-adic character (an analogue of exponent): $e(ξx) = e^{2πiξx}$. Here, for a $p$-adic number $a$, $\{a\}$ denotes its fractional part, i.e., for $a = \frac{a_m}{p^m} + ... + \frac{a_1}{p} + a_0 + ... + a_k p^k + ...$ (where $a_j = 0, 1, ..., p - 1$, and $a_m ≠ 0$) we have

$$\{a\} = \frac{a_m}{p^m} + ... + \frac{a_1}{p}.$$

Vladimirov’s operator of order $α > 0$ is defined as

$$D^α(ϕ)(x) = \int_Q |ξ|^α \tilde{ϕ}(ξ)e(−ξx)dξ.$$

We remark that $D^αD^β = D^{α+β}$. We define the motivation magnitude ob-
servable $M$ as

$M_ξ = hD$

Here $h = \frac{1}{p^m}$ is some normalization constant. The $h$ plays the role of
the Planck constant in ordinary quantum mechanics. At the moment it is not clear: “Can we expect that there exists a kind of universal constant $h$, the mental Planck constant?” I am quite sceptical that such a universal
normalization constant really exists. It is more natural to suppose that $h$
would depend on a class of cognitive systems under consideration. In fact,
by finding $h$ (the level of motivation discretization) we find the basis $p$ of the
coding system.

To calculate averages of the momentum magnitude operator $M_ξ$ for dif-
ferent quantum-like mental states, it is natural to use the Fourier transform.
By analogy with ordinary quantum mechanics we could say: to move from
position to momentum representation.

**Example 5.1.** Let a quantum-like state $ϕ$ is such that its Fourier trans-
form $\tilde{ϕ}(ξ)$ is uniformly distributed over the ball $B_r(0), r = 1/p^l$. Here

$$< M_ξ >_ϕ = p^l \int_{B_r(0)} |ξ|^p dξ = \frac{1}{p^{l-1}(p+1)}.$$

\(^8\)We remark that it is impossible to define the derivative for maps from $Q_p$ to $R$, see [47].
5.3. Wholeness of mental observables. It is important to remark that (in the opposite to the ordinary quantum momentum) the $M_\xi$ is nonlocal operator. It can be represented as an integral operator, see [45]:

$$D(\phi)(x) = \frac{p^2}{p + 1} \int_Q \frac{\phi(x) - \phi(y)}{|x - y|^2} dy .$$

To find $M_\xi(\phi)(x)$ in some fixed point $x$, we have to take into account values of $\phi$ in all points of the mental configuration space. Thus motivation psychological function can not be localized in some particular neural substructure of brain.

This example is a good illustration of the mathematical description of nonlocality of psychological functions in our $p$-adic quantum-like model. One of the main distinguishing features of this model is nonlocality of derivation operator (Vladimirov’s operator). Hence the corresponding psychological function is produced by the whole neural system of the body (as indivisible system).

5.4. Psychological functions as quantum-like representations.

The mathematical description of the motivation psychological function by using a new representation in the Hilbert state space is the basic example that can be generalized to describe all possible psychological functions. We remark that the motivation representation is, in fact, a new system of quantum-like mental coordinates. In the case of motivation a new system of coordinates was generated by a unitary operator in the Hilbert state space, namely Fourier transform.

In the general case each psychological function $F$ is represented mathematically by choosing a system of coordinates in the Hilbert state space, mental representation. Thus we can identify the set of all psychological functions with the set of all unitary operators: $U \rightarrow F_U$ and $F \rightarrow U_F$. All mental observables $A$ represented by self-adjoint operators that can be diagonalized by using the concrete $U$-representation can be related to the corresponding psychological function $F_U$. For example, concerning visual function observables of shape and colour can be diagonalized in the visual representation of the state Hilbert space.

In such a model all psychological functions coexist peacefully in the neural system. The evolution of the quantum-like mental state $\phi(t, x)$ (see section 9) induces the simultaneous evolutions of all mental functions (in this state). This is unconscious evolution. Thus a conscious system $\tau$ are not consciously aware about simultaneous evolution of the various psychological functions. Only by the acts of self-measurements of some mental observables $A^{(j)}_F$ that are diagonal in the $F$-representation the $\tau$ becomes aware about some features of the corresponding psychological function $F$. 

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5.5. Free mental waves. We remark that Vladimirov’s operator $D$ has a system of (generalized) eigenfunctions that is similar to the system of free-wave eigenfunctions in ordinary quantum mechanics, where $\phi_\xi(x) = e^{i\xi x}/h$ corresponds to the fixed value $\xi$ of momentum. In the mental framework:

$$M_\xi e(h\xi x) = |\xi|_p e(h\xi x).$$

Here we have used the fact [45]:

$$De(\xi x) = |\xi|_p e(\xi x).$$

We remark that in the ordinary quantum formalism the $h$ is placed in denominator, $\xi x/h$, and in the $p$-adic quantum formalism it is placed in the nominator, $h\xi x$. This is a consequence of the fact that $1/h$ is large in $\mathbb{R}$ and $h$ is large in $\mathbb{Q}_p$.

The function $\phi_\xi(x) = e(h\xi x)$ is a kind of free mental wave corresponding to the fixed value $\xi$ of the motivation. As $|\phi_\xi(x)| = 1$ for all $x \in \mathbb{Q}$, the probability to find a cognitive system in the mental state $x$ does not depend on $x$. By analogy with the ordinary quantum mechanics we would like to interpret this mathematical fact in the following way: By fixing the magnitude of motivation (strength of willing) we could not localize the mental state. However, we will see soon that such an analogy (between material and mental states) can not be used.

A free mental wave $\phi_\xi$ gives a good example illustrating of the role of the phase $\theta(x)$ of the mental amplitude. Here we have

$$\theta(x) = 2\pi \{\xi x\}. \tag{4}$$

Thus if a cognitive system $\tau$ has the fixed motivation $\xi$ and the mental probability distribution $P(x)$ is uniform, then the phase of the corresponding mental amplitude is determined by (4). Thus in general the phase $\theta(x)$ of a mental amplitude $\phi(x)$ is not the pure product of neural activity. This phase contains information on the transition from one mental representation to another.

5.6. Privacy of motivation states. The wave $\phi_\xi(x)$ is not determined uniquely by the observable $M_\xi$. The main distinguishing feature of $p$-adic quantum mechanics (discovered by Vladimirov, [45]) is the huge degeneration of the spectrum of the momentum and energy operators. In particular, beside eigenfunctions $\phi_\xi(x)$, the $M_\xi$ has an infinite set of other eigenfunctions corresponding to the eigenvalue $\lambda = |\xi|_p (= p^k$ for some $k = 0, \pm 1, \pm 2, \ldots)$.

Each $\lambda = p^k, k = 0, \pm 1, \pm 2, \ldots$ corresponds to an infinite series of eigenfunctions (distinct from the free mental wave $\phi_\xi(x)$) belonging to $L_2(\mathbb{Q}, dx).$ These eigenfunctions are well localized (concentrated in balls) in the mental configuration space.

\footnote{We remark that free mental waves $\phi_\xi(x)$ are so called generalized eigenfunctions. They are not square integrable. Thus they do not belong to the space of quantum-like mental states $\mathcal{H} = L_2(\mathbb{Q}, dx)$. One could speculate that such non-normalizable free mental waves may be related to altered consciousness events such as e.g. hallucinations.}
This is very natural from the mental point of view. It would be quite strange if the only quantum-like mental state with the fixed motivation magnitude is the state $\phi_\xi$ characterized by totally indefinite distribution of mental states $q$. By intuitive reasons there must be quantum-like mental states characterized by the fixed $M_\xi = \lambda$ that are concentrated on a special class of mental states (a kind of special mental activity).

One of the most important distinguishing features of quantum-like mental theory is that the motivation magnitude operator $M_\xi$ has a discrete spectrum (except of one point, see further considerations). Hence the magnitude of the motivation does not change continuously.

There exists only one point of the spectrum of the operator $M_\xi$ that is not its eigenvalue: $\lambda = 0$. It is the limit point of the eigenvalues $\lambda_k = p^k$, $k \to \infty$. There is no eigenfunction $\phi_0$ belonging to the state space $\mathcal{H}$. Thus our model brain could not be (alive, awake?) in the stationary quantum-like mental state having the motivation of zero magnitude.

Another distinguishing feature is infinite degeneration of spectrum. This purely mathematical result can have important implications for the problem of correspondence between mental and physical worlds. In fact, due to this huge degeneration, we could not uniquely determine the mental state of a cognitive system by fixing the motivation magnitude $M_\xi$.

6 Neuron-activation observable

As we have already discussed, we could not introduce a mental state observable $q$. However, in the same way as for the motivation we can introduce an operator of the $p$-adic magnitude of a mental state:

$$M_q \phi(x) = |x|_p \phi(x).$$

Spectral properties of this operator are similar to spectral properties of the operator $M_\xi$: discreteness and infinite degeneration of spectrum. Eigenfunctions of $M_q$ (belonging to $\mathcal{H} = L_2(Q, dx)$) are localized in $p$-adic balls–trees. Therefore:

There exist stationary states of $M_q$ that are characterized by activation of the fixed tree of mental states.

Unfortunately, $M_q$ could not be used to fix such a tree (as a consequence of infinite degeneration of spectrum).

The operators of position and motivation magnitudes, $M_\xi$ and $M_q$, do not commute (as operators of position and momentum in ordinary quantum mechanics):

$$[M_q, M_\xi] = M_q M_\xi - M_\xi M_q = \hbar J,$$
where \( J \neq 0 \) is an integral operator [45]. Thus we get a mental uncertainty relation, compare to [32]:

For any quantum-like mental state \( \phi \), it is impossible to measure motivation and position magnitudes with an arbitrary precision.

By measuring the motivation magnitudes we change position magnitudes and vice versa. This can also be expressed mathematically by using the \( p \)-adic Fourier transform. We denote by \( \Omega_r(x) \) the characteristic function of the ball \( B_r(0) \) (it equals to 1 on the ball and 0 outside the ball). We have [45], p. 102, \( \tilde{\Omega}_r(\xi) = \frac{1}{r^p} \Omega_1(\xi) \). If the state of mind is concentrated on the ball-tree \( B_r(0) \), then motivations are concentrated on the ball-tree \( B_r^1(0) \).

As in the case of the \( M_\xi \)-observable, the point \( \lambda = 0 \) belongs to a non-discrete spectrum of the \( M_q \) observable. Thus there is no stationary quantum-like mental state \( \phi \) corresponding to zero magnitude of \( q \). A cognitive system is not alive (awake?) in such a state.

To understand better the mental meaning of the \( M_q \)-observable, it is useful to consider a new mental observable:

\[
A = - \log_p M_q.
\]

If, \( \phi \in \mathcal{H} \) is an eigenstate of the \( M_q \) corresponding to the eigenvalue \( \lambda = |q|_p = \frac{1}{p^k} \), then \( \phi \) also is an eigenstate of \( A \) corresponding to the eigenvalue \( \mu = k \) and vice versa. Thus the discrete part of the \( A \)-spectrum coincides with the set of integers \( \mathbb{Z} \). The \( A \) gives the position of the igniting neuron in a layer of neurons. It is called neuron-activation observable. We note that there is an interesting relation between neuron-activation observable and entropy.

Let us consider the quantum-like state \( \phi(q) = \sqrt{p+1}|q|_p \Omega_1(q) \). Here \( \sqrt{p+1} \) is just the normalization constant. The corresponding probability distribution \( P(q) = (p+1)|q|_p \) on the tree \( \mathbb{Z}_p \) and equals to zero outside this tree. The entropy of this probability distribution

\[
E_p = -\int_{\mathbb{Z}_p} \log_p P(q)P(q)dq = <A>_{\phi} - \log_p(p+1).
\]

7 Complex cognitive systems; evolution

We now consider a cognitive system consisting of \( n \) hierarchic layers of neurons. It can be an individual brain as well as a system of brains. The mental space of this cognitive system is

\[
Q = Q_p \times \cdots Q_p
\]

(\( n \) times). For each mental coordinate \( q_j, j = 1, 2, \ldots, n \), we introduce the motivation magnitude operator \( M_j = hD_j \), where \( D_j \) is Vladimirov operator for \( q_j \). We introduce kinetic mental energy (free energy of motivations) as
\[ H = h^2 \Delta, \]

where \( \Delta = \sum_{j=1}^{n} D_j^2 \) is Vladimirovian (a \( p \)-adic analogue of the Laplacian).

We note that free mental waves \( \phi_\xi(x) = e(h\xi x) \) are eigenfunctions of this operator with eigenvalues \( \lambda = |\xi|^2_p \). As in the cases of the \( M_\eta \), \( M_\zeta \)-observables, there is an infinite family of other eigenfunctions distinct from free mental waves. These functions are localized on the mental configuration space (describing fixed ranges of ideas). The spectrum is discrete: \( \lambda = p^k, k = 0, \pm 1, \pm 2 \). Thus the kinetic mental energy is changed only by jumps. The \( \lambda = 0 \) is the only point that belongs to the non discrete spectrum of the operator of the kinetic mental energy.

Interactions between brain’s layers as well as interactions with the external world are described by the operator of the potential mental energy. It is given by a real valued function (potential) \( V(q_1, \ldots, q_n) \). The total mental energy is represented by the operator:

\[ H = h^2 \Delta + V. \]

We note that a mental potential \( V(q_1, \ldots, q_n) \) can change crucially spectral properties of the mental energy observable. If \( V \) depends only on \( p \)-adic magnitudes \( |q_j|_p \) of mental coordinates and \( V \to \infty, |q_j|_p \to \infty \), and \( V \) is bounded from below (e.g., nonnegative), then the spectrum of \( H \) (that is discrete) has only finite degeneration. Thus the ”state of mind” of a free cognitive system could not be determined by fixing the mental energy. However, by using additional mental (information) potentials we could (at least in principle) do this.

The ground mental energy state \( \lambda_0 \) is not degenerated at all. In the latter case by fixing the minimal value of the mental energy \( H = \lambda_0 \) we can determine the ”state of mind”, namely the \( \lambda_0 \)-eigenstate. Even for other eigenvalues we can try to determine the ”state of mind” if the degeneration of spectrum is not so large. It is interesting to remark that mathematical results [45] imply that degeneration of eigenvalues (distinct from the ground energy) increases (as \( p^2 \)) with increasing of \( p \). If we connect the complexity of a cognitive system with the coding base \( p \), then we obtain that, for complex cognitive systems (e.g., \( p = 1999 \)), it is practically impossible to determine the ”state of mind” corresponding to the fixed value of mental energy.

8 Entanglement of psychological functions

8.1. Classical viewpoint to localization of psychological functions.
The problem of neural localization of psychological functions split the neurophysiological community, see e.g. A. R. Damasio [53]: “One held that psychological functions such as language or memory could never be traced to a particular
region of brain. If one had to accept, reluctantly, that the brain did produce the
mind, it did so as a whole and not as a collection of parts with special functions.
The other camp held that, on the contrary, the brain did have specialized parts
and those parts generate separate mind functions.” Both adherents of whole-
ness and localization of psychological functions have a lot of experimental
evidences supporting their views. A kind of peaceful unification of these
two views to localization of psychological functions is given by our model
of coding of cognitive information by hierarchic pathways activity. As each
pathway $\mathcal{N}$ is hierarchic, there are a few neurons in the pathway that play
the most important role. Their location in some domain $U$ of the brain de-
termines localization of a psychological function containing $\mathcal{N}$. However, $\mathcal{N}$
goes throughout many other brain (and body) regions. So $U$-localization is
only a kind of fuzzy localization.

However, I do not think that this is the end of the localization story.
We suppose that cognition involves not only classical dynamics of neural
networks, but also quantum-like processing described by the evolution of
quantum-like wave function, see section 9. The latter gives the amplitude
of probability distribution of realization of classical mental states (hierar-
chic strings of e.g. frequencies of firings). Such a quantum-like processing
of cognitive information would automatically create psychological functions
that do not have even fuzzy localization. Such functions are induced via
entanglement of localized psychological functions.

8.2. Entanglement. Let $U_1, ..., U_k$ be some neural structures – ensem-
bles of hierarchic neural pathways – specialized for performing psychological
functions $F_1, ..., F_k$. Consider corresponding Hilbert spaces of quantum-like
mental states: $\mathcal{H}_j = L_2(Q_j, dx_j)$, where $Q_j = Q_{p_j}$ and $dx_j$ is the Haar
measure on $Q_j$. Let $e_{F_j}$ be the orthonormal basis in $\mathcal{H}_j$ corresponding to the
function $F_j$.

Let us consider the Hilbert space of quantum-like mental states of the
composite neural system, $U = U_1 \cup ... \cup U_k : \mathcal{H} = L_2(Q, dx)$, where $Q = Q_1 \times
... \times Q_k$. Here a normalized state $\phi(q_1, ..., q_k)$ gives the amplitude of probability
that $U_1$ produces $q_1, ..., U_k$ produces $q_k$. We consider in $\mathcal{H}$ the orthonormal
basis $e$ obtained as the tensor product of bases $e_{F_j}$. The $e$ describes a mental
representation corresponding to the psychological function $F = (F_1, ..., F_k)$
produced by classical combination of psychological functions $F_1, ..., F_k$. Let us
now consider some other basis $\tilde{e}$ containing nontrivial linear combinations of
vectors of the $e$. This basis gives the mental representation of a psychological
function $G$ that could not be reduced to classical combination of psychological
functions $F_j$. We call $G$ an entanglement of psychological functions $F_j$. Of
course, $G$ is produced by the collection $U$ of neural structures $U_j$. But $G$
arises as nontrivial quantum-like combination of psychological functions $F_j$.22
We remark that entanglement of psychological functions has nothing to do with entanglement of quantum states of individual micro systems in the brain (compare to conventional reductionist quantum models of brain functioning). Entanglement of psychological functions is entanglement of probabilistic amplitudes for information states of macroscopic neural systems, see section 10 for further discussion.

9 State-evolution

We want to describe the evolution of a quantum-like mental state (mental wave function) $\phi(t,x)$. The first natural and rather nontrivial problem is the choice of the evolution parameter $t$. This problem was discussed in detail in [32]. It was shown that there are different natural possibilities to describe the evolution of mental states: ”mental time”, ”psychological time” as well as ordinary physical time evolution, see also section 10. In this paper we consider the evolution with respect to physical time $t$ belonging to the real line $\mathbb{R}$. To derive the evolutional equation for $\phi(t,x)$, we proceed in the same way as Schrödinger in ordinary quantum mechanics. We start with a free mental wave $\phi_\xi(x) = e(h\xi x), \xi, x \in \mathbb{Q}_p$. We have:

$$H_0 \phi_\xi(x) = |\xi|^2_\mathbb{P} \phi_\xi(x),$$

where $H_0 = h^2 D^2$ is the operator of the mental energy for a free system.

The $\phi_\xi(x)$ is a stationary state corresponding to mental energy $E = |\xi|^2_\mathbb{P}$. Such a wave evolves as

$$\phi_\xi(t,x) = e^{itE} \phi_\xi(x).$$

We note that this function is a combination of two essentially different exponents: ordinary exponent and $p$-adic character. This function satisfies to the evolutional equation:

$$ih \frac{\partial \phi}{\partial t}(t,x) = h^2 D^2 \phi(t,x). \quad (5)$$

This is Schrödinger’s mental equation for a free cognitive system. If we introduce a mental potential $V(x)$, then we get general Schrödinger’s mental equation:

$$ih \frac{\partial \phi}{\partial t}(t,x) = h^2 D^2 \phi(t,x) + V(x)\phi(t,x). \quad (6)$$

If the initial quantum-like state $\psi(x) = \phi(0,x)$ is known, then by using (6) we can find $\phi(t,x)$ at each instant $t$ of physical time. Under quite general conditions [45], the operator $H = h^2 D^2 + V(x)$ is a self-adjoint operator. Therefore (6) is the standard Schrödinger’s equation in the Hilbert space $\mathcal{H}$.
for one rather special class of operators $H$. There also are mathematical results on analytical properties of solutions and correctness of Cauchy problem [47].

**Remark 9.1.** (Bohmian theory) We can try to develop an analogue of Bohmian (pilot wave) approach. As in ordinary Bohmian mechanics, we can define a quantum-like mental potential

$$W_\phi(t, x) = -\frac{\hbar^2}{R} D^2 R, \text{ where } R(t, x) = |\phi(t, x)|.$$  
Equation 7

This potential has the same properties as the ordinary quantum potential: (a) $W_\phi(t, x)$ does not depend on the absolute magnitude of $\phi$; (b) $W_\phi(t, x)$ depends on the second variation of the magnitude of $\phi$. However, (in the opposite to ordinary Bohmian mechanics) we could not describe evolution of an individual mental state (position) $q(t)$ by using Newton’s equation with additional potential $W$. At first glance, this is a purely mathematical difficulty. But I think that this mathematical fact has deep cognitive meaning, namely that the dynamics of quantum-like state $\phi(t, x)$ does not determine the dynamics of classical mental states. Very different flows of classical mental states (hierarchically ordered neural flows) can produce the same wave $\phi(t, x)$. In our model only this wave determines results of mental measurements. Thus (in our model) it seems to be impossible to find a one to one correspondence between mental behaviour and neural activity. The flow of consciousness does not uniquely correspond to neural dynamics in the brain.

## 10 Discussion

### 10.1. Why quantum-like formalism?

One of the main reasons to expect that mental observables (including mental self-observables) should be described by the quantum-like (Hilbert space probabilistic) formalism is the very high sensivity of neural structures to changes of contexts of measurement. Such a sensivity implies the violation of rules of classical probabilistic calculus and induces a so called quantum probabilistic calculus, see [33], [35] for the detailed analysis. The main distinguishing feature of this quantum probabilistic calculus is interference of probabilities of alternatives. Therefore a quantum-like structure of mental observables should imply interference effects for such observables. In [65] a general scheme of mental measurements was proposed that could be used to find the interference effect. It may be that the corresponding statistical data have already been collected somewhere. We need only to extract the interference effect.

Another reason for quantum-like considerations is the discrete structure of information processing in brain. It is natural to describe this exchange by quanta of information by a quantum-like formalism. In particular, in
our model we automatically obtained that basic mental observables such as e.g. mental energy have discrete spectra. We underline that philosophy of organism of Alfred Whitehead was one of the first philosophic doctrines in that fundamental proto-mental elements of reality, namely actual occasions, had quantum (in the sense of discreteness) structure. The philosophy of organism was based on one-substance cosmology, see [1], p. 26, “Descartes and Locke maintained a two-substance ontology – Descartes explicitly, Locke by implication. Descartes, the mathematical physicists, emphasized his account to of corporeal substance; and Locke, the physician and the sociologist, confined himself to an account of mental substance. The philosophy of organism, in its scheme for one type of actual entities, adopts the view that Locke’s account of mental substance embodies, in a very special form, a more penetrating philosophic description than Descartes’ account of corporeal substance.”

10.2. Quantum-like statistical behaviour and consciousness. In our model of consciousness as the process of (quantum-like) self-measurements over hierarchic neural structures the quantum structure plays an important, but not determining role. There are many sensitive physical systems (not only microscopic, but also macroscopic) that could exhibit quantum-like behaviour, see [33]-[35] for the details. Thus to be quantum-like is not the sufficient condition to be conscious. There must be something else that is crucial in inducing consciousness. This consciousness determining factor may be quantum as well as classical (or a very special combination of classical and quantum factors).

It seems that the crucial point might be the ability to “feel” the ensemble probability distribution of information strings produced by neural activity. My conjecture is that such a feeling is the basis of mentality. In such a model a cognitive system reacts not to firings of individual neurons or even large populations of neurons, but to integral probability distributions of firings. If this is the really the case, then quantum-like probabilistic formalism would appear automatically, since this is the most general theory of transformations of context depending probabilities [35]-[39].

10.3. Why $p$-adic space? On the classical level the main distinguishing feature of our model is the ultrametric $p$-adic structure of the classical mental space. As we have already mentioned in section 3, ultrametricity is simply a topological representation of hierarchy. Hence, the main classical feature of the model is its very special hierarchic structure. I think that the presence of such a hierarchic structure is the very important condition of cognition and consciousness. In principle, it is possible to consider general ultrametric cognitive models. I restrict myself to consideration of $p$-adic models, since there is the possibility to connect $p$-adic hierarchic model with frequency domain models.
However, the presence of the $p$-adic hierarchy is not sufficient to induce consciousness (nor even cognition). For example, spin glasses have hierarchic structures that in some cases could also be mathematically described by $p$-adic numbers, see [62], [63]. The crucial point may be a complex system of interconnections between the huge ensemble of hierarchic neural structures in brain.

10.4. Individual and ensemble interpretations

The large diversity of physical interpretations of the mathematical formalism of quantum mechanics is one of many serious problems in quantum foundations. Different interpretations provide totally different views to physical reality (including the absence of such a reality at all), see e.g. [10]-[12], [15].

As a consequence of the great success of the books of R. Penrose on the quantum approach to mind, neurophysiologists, psychologists, cognitive scientists, and philosophers are now quite familiar with one very special interpretation, namely Penrose’s quantum gravity improvement of the conventional interpretation of quantum mechanics.

The first question is: Why does the conventional interpretation need some improvements at all?

This was well explained in book [15]. Conventional quantum theory with the orthodox Copenhagen interpretation has many problems including numerous mysteries and paradoxes (e.g. [15], p. 237: ... yet it contains many mysteries. ... it provides us with a very strange view of the world indeed.”; or R. Feynman: “It is all mysterious. And the more you look at it the more mysterious it seems.”).

Unfortunately, all these mysteries and paradoxes related to the interpretation of quantum mechanics were automatically transmitted to cognitive sciences. Some people enjoy this and they are happy to speak about mental nonlocality or mental collapse. It is the general attitude to couple the mystery of consciousness with some still unclear aspects of interpretation of quantum mechanics. On the other hand, many realistically thinking neurophysiologists, psychologists, cognitive scientists, and philosophers dislike to use all such tricky quantum things as superposition of (e.g. position) states for an individual system, collapse, nonlocality, death of reality in the cognitive framework. I strongly support this viewpoint.\(^{11}\)

\(^{10}\)Neurophysiologists, psychologists, cognitive scientists, and philosophers are lucky that R. Penrose does not support orthodox views to quantum theory. So in his books [14], [15] this theory was not presented in the rigid orthodox form.

\(^{11}\)Of course, discussing all these intriguing problems of interpretation of quantum mechanics we should not forget that (as it was mentioned by one of referee’s of this paper): “The interpretation of the formalism is, in the end, largely irrelevant to the description of the phenomena under observation or the mathematics used to treat them.”
There is no any possibility to go deeply into foundations of conventional quantum theory. I think that the crucial point is the individual interpretation of a wave function. The wave function is associated with an individual quantum system (in the orthodox approach – it gives the complete description). For example, the individual interpretation induces such a mysterious thing as superposition (e.g. position) states for an individual system. On the other hand, individual superposition immediately implies that “Quantum theory provides a superb description of physical reality on a small scale...” [15], p.237. As the superposition of states for individual macroscopic objects (e.g. cars) was never observed, conventional quantum theory should be applied on so called quantum scale. In particular, all cognitive models based on conventional quantum theory should go deeply beyond the macroscopic neural level, see [15], p.355: “It is hard to see how one could usefully consider a quantum superposition consisting of one neuron firing, and simultaneously not firing.” Therefore all such models suffer of the huge gap between quantum micro and neural macro scales. Of course, there are various attempts to solve these problem. For example, in [15] it was proposed to use quantum coherence to produce some macro states by coherence of large ensembles of quantum systems.

Finally, we mention the quantum gravity improvement of conventional quantum theory, [14], [15]. This is really an improvement and not a cardinal change of conventional quantum ideology. It is an attempt to explain reduction as “gravitationally induced state-vector reduction.” It would not be useful to discuss the role of such an improvement of physical theory in a biological journal. However, for cognitive models, the use of quantum gravity arguments looks as just increasing of conventional quantum mystification. There is a new huge gap between quantum scale and Planck scale \(10^{-33}\) cm). It is even less believable that the mind is induced by superpositions of mass states.

Quantum-like approach to cognitive modeling used in this paper is based on so called ensemble interpretation of quantum mechanics, see e.g. L. Ballentine [11]. By this interpretation (that was strongly supported by A. Einstein) a wave function is associated not with an individual physical system, but with a statistical ensemble of systems. The statistical approach has its advantages and disadvantages. In particular, there is no mystery of state reduction. Individual systems are not in superposition of different states. Superposition of wave functions is a purely statistical property of various ensembles of physical systems. One of the main problems of the statistical approach was the impossibility to get interference of probabilities on the basis of classical ensemble probability. Recently it was done in author’s works [33]-[36] by taking into account context dependence of probabilities. The absence of the mysterious superposition for individual systems and operation
with ensembles gives the possibility to apply the Hilbert space probabilistic formalism, quantum-like theory, to ensembles of macroscopic systems. We agree with R. Penrose that an individual neuron could not be in superposition of two states, but two ensembles of neurons (as well as the same ensemble at distinct moments) could demonstrate features of superposition.

10.5. **Neural code and structure of mental space.** Suppose that the coding system of a cognitive system is based on a frequency code. There exists an interval of physical time $\Delta$ such that a classical mental state (mental position) produced by a hierarchic chain of neurons is a sequence with coordinates given by numbers of oscillations for corresponding neurons during the interval $\Delta$. This $\Delta$ depends on a cognitive system and even on a psychological function inside the same brain, namely $\Delta = \Delta_{r,F}$. Thus in our model the problem of the neural code is closely related to the problem of time-scaling in neural systems. For different $\Delta$, we get different coding systems, and, consequently, different structures of mental spaces. The corresponding natural number $p$ that determines the $p$-adic structure on the mental space is defined as the maximal number of oscillations that could be performed by neurons (in hierarchic chains of neurons working for some fixed psychological function) for the time interval $\Delta$. The coding that is based on e.g. the 2-adic system induces the 2-adic mental space that differs crucially from the 5-adic (or 1997-adic) mental space induced by the 5-adic (or 1997-adic) system. As it was remarked in section 3, by changing the $p$-adic structure we change crucially dynamics. Hence, the right choice of the time scaling parameter $\Delta$ and corresponding $p = p_\Delta$ plays the important role in the creation of an adequate mathematical model for functioning of a psychological function.

10.6. **Mental time.** There might be some connection between the time scale parameter $\Delta$ of neural coding and *mental time*. There are strong experimental evidences, see e.g. K. Mogi [66], that a moment in mental time correlates with $\approx 100$ ms of physical time for neural activity. In such a model the basic assumption is that the physical time required for the transmission of information over synapses is somehow neglected in the mental time. A moment in mental time is subserved by neural activities in different brain regions at different physical times.

10.7. **Quantum-like models with $p$-adic valued functions.** A series of works of the author and his collaborators, see, for example, [47], developed the formalism of quantum mechanics in which not only the classical configuration space, but also wave functions are $p$-adic. Originally this formalism was developed for high energy physics, namely, for theory of $p$-adic strings. Later I used this formalism for cognitive modeling, see e.g. paper [29] on $p$-adic cognitive pilot wave model ("conscious field model")
giving the very special realization of Bohm-Hiley-Pylkkänen ideas on active
information. From the mathematical point of view the $p$-adic valued formal-
ism looks more attractive than the complex valued formalism developed by
Vladimirov and Volovich, see e.g. [45]. In particular, here operators of men-
tal position and motivation are well defined. However, there is a difficulty
that induces strong prejudice against this $p$-adic valued formalism, namely
the appearance of $p$-adic valued probabilities. Despite very successful math-
ematical development of the theory with $p$-adic valued probabilities [47], it
is clear that we cannot use it for ordinary measurements over physical and
cognitive systems. In such measurements we always observe ordinary prob-
abilities. Thus $p$-adic valued quantum-like formalism could not be used to
describe traditional mental measurements over a cognitive system performed
by external systems. As it was pointed out in [32], such $p$-adic probabilities
(stabilization of frequencies in $p$-adic topology and chaotic behaviour of these
frequencies in ordinary real topology) might appear in anomalous phenom-
ena. In principle, such probabilities might be related to the functioning of
consciousness. It might be that consciousness uses self-measurements follow-
ing to $p$-adic valued quantum-like theory. However, in the present paper we
would not like to study such a model of consciousness.

Finally, we mention the fundamental work of M. Pitkänen [67] that also
contains a $p$-adic model of consciousness. However, M. Pitkänen used an
orthodox Copenhagen interpretation of quantum mechanics (state reduction,
superposition of states for individual quantum systems and so on). Another
fundamental aspect of his approach is the proposal to formulate space-time
geometry by using both real and $p$-adic space-time regions.

10.8. Real and $p$-adic spaces. At first glance, in our model there is
no direct connection between real continuous space that is traditionally used
to describe classical states of material objects and $p$-adic hierarchic (tree-
like) spaces that was proposed to describe classical mental states of brain. So
we follow to Descartes doctrine. Such an approach was strongly critisized
from many sides. In particular, such a theory is not coherent, see Whitehead
[1]. Of course, it would be nice to develop some classical and corresponding
quantum-like models based on real/$p$-adic space. The real and $p$-adic parts of
material–mental space would describe the physical brain and “mental brain”,
respectively. Our first point is that, in general, we could not work with the
fixed $p$-adic structure. As we have already discussed, different cognitive sys-
tems and psychological functions can be based on different $p$-adic mental
spaces. Thus in a general model we have to use all $p$-adic spaces simulta-
neously. We remark that a mathematical topological structure unifying real
and all $p$-adic numbers (for prime $p$) is well known. This is so called adelic
space, see [45] on physical models over adels. The next natural step would
be to apply adelic quantum-like formalism to measurements over material and cognitive systems. In adelic quantum-like model “the disastrous separation of body and mind, characteristic of philosophical systems which are in any important respect derived from Cartesianism” (see [1], p.348) could be avoided, since adelic amplitudes would depend both on body (real) and mind (p-adic) variables.

10.9. Microtubules. Are neurons really the basic elements for hierarchic mental coding? At the moment there is strong neurophysiological evidences that this is really the case. Nevertheless, we should not totally reject other possibilities. In particular, over the last 20 years S. Hameroff and his collaborators, see e.g. [68], have been developing a model of consciousness based on quantum processes in microtubules. Hameroff’s approach is a traditional quantum reductionist approach. Thus our paper has nothing to do with it. However, the general idea that microtubules play an important role in information processing in brain should be considered very seriously in a quantum-like approach. Of course, in such a model the main role would be played by hierarchic organization of microtubules on a classical level. Quantum-like formalism can be used to describe the corresponding mental amplitude. Finally, we mention some other fundamental papers on the quantum brain [69]-[71].

10.10. Non-reductionism. The basic question of all quantum reductionist models of consciousness is “How is it that consciousness can arise from such seemingly unpromising ingredients as matter, space, and time? – [15], p. 419. In our model, consciousness has no direct relation to matter. It is a feature of very special hierarchic configuration of information described by the mental amplitude $\phi(x)$. By answering to Penrose’s question “The physical phenomenon of consciousness?”, [15], p.406, I say: “Consciousness is a bio-physical as well as a bio-information phenomenon.”

10.11. Quantative measure of consciousness. I was extremely fascinated by Baars’ idea to consider consciousness as a variable [72]. The main problem is to find some numerical representation of such a consciousness-variable. In our model, such a variable should be in some way connected with the basic probability distribution $P(t, x) = |\phi(t, x)|^2$. This is the probability that the concrete hierarchic configuration of firings (e.g. configuration of frequencies) is realized in brain at the moment $t$. Hence, if sufficiently many hierarchic chains of neurons produce $x$, then $P(t, x)$ is sufficiently large. The value of $P(t, x)$ by itself cannot be taken as a quantative measure of mentality.

For instance, suppose that $P(t, x) \equiv 1$ for all $x$. This is the uniform distribution on the $p$-adic space. We could not expect that such an amplitude with uniform activation of all classical mental states corresponds to a high
level of mentality. Conscious behaviour corresponds to a mixture of various motivations. Such a mixture is characterized by variation of the probability distribution \( P(t, x) \). I propose the following numerical measure of consciousness (at mental state \( \phi(t, x) \)):

\[
\mathcal{M}_{\text{consciouness}} = \int_{Q_p} \left( |D_x P(t, x)|^2 + \left| \frac{\partial P(t, x)}{\partial t} \right|^2 \right) dx
\]

**10.12. Neural groups.** The fundamental role that internally organized groups of neurons (and not individual neurons) play in processing of information in brain was discussed in details in Edelman’s theory of neural groups selection (TNGS), [73]. Our model in that neural pathways are used as the neural (classical) basis for processing of information in brain is closely related to TNGS. Of course, we understand that our model may be oversimplified. It may that the basic units should be not chains, but whole trees of neurons.

**10.13. Does consciousness benefit from long neural pathways?** Finally, we discuss one of the greatest mysteries of neuroanatomy, see, for example, [17], [52]-[54], [73], [15]. It seems that in the process of neural evolution cognitive systems tried to create for each psychological function neural pathways that are as long as possible. This mystery might be explained on the basis of our neural pathway coding model. If such a coding be really the case, then a cognitive system \( \tau \) gets great benefits by extending neural pathways for some psychological function as long as possible. For example, let the neural code basis \( p = 5 \) and a psychological function \( F \) is based on very short pathways of the length \( L = 2 \). Then the corresponding mental space contains \( N(5, 2) = 2^5 = 32 \) points. Let now \( p = 5 \) and \( L = 10000 \). Then the corresponding mental space contains huge number of points \( N(5, 10000) = 10^{20} \) points. On the latter (huge) mental space mental amplitudes having essentially more complex behaviour (and, consequently, the measure of consciousness) can be realized. It might be that this mental space extending argument can be used to explain spatial separation of various maps in the brain, see e.g. Edelman [73].

**Summary**

The crucial difference between my model and other classical as well as quantum models of cognition is the use of the \( p \)-adic configuration space, mental space, instead of a (traditionally used) continuous real physical space. This is a kind of information model of brain (even on the ‘classical level’). However, we could not

\[\text{12In particular, the free mental wave } \phi_\xi(x) \text{ induces such a probability distribution. In such a state a cognitive system has the fixed motivation } \xi. \text{ By proceeding with a fixed motivation (aim, task) a cognitive system } \tau \text{ performs not conscious, but merely AI-behaviour (for example, realization of a program given by the string of digits } \xi).\]
say that there exist two totally different worlds – mental and physical. Connection with physical space is performed via the use of the special topology, namely the ultrametric $p$-adic topology. In fact, the use of the concrete number system, $p$-adic numbers, is not so important. It is just the simplest (but, of course, very important) model in that the special spatial structure, namely hierarchic treelike structure (e.g., hierarchic neuronal structures that generate the hierarchic frequency coding of cognitive information), is represented by an ultrametric topological space. Once we did such a transition from the physical space to an ultrametric space, we can forget (at least at the first stage of modelling) about physical space and work in the ultrametric mental space. We understand well that at the moment our fundamental postulate on ultrametric topology of information brain has only indirect confirmations in neurophysiology and psychology, namely hierarchic processing of cognitive information, see e.g. [17],[52],[54],[72],[73]. In principle, on the basis of hierarchy we could reconstruct the corresponding ultrametric topology. However, the problem of creating of the detailed map of hierarchic structures in brain is far from its solution. Therefore, it is quite natural to try to start simulation with the simplest ultrametric space – the $p$-adic one.

The next crucial step is the use of the quantum-like formalism to describe mathematically the process of thinking in that brain operates with probability distributions. Here a mental state is mathematically described by a probability distribution. This formalism is based on the standard quantum probabilistic calculus in a Hilbert space. However, in the opposite to orthodox quantum views, a quantum state (normalized vector belonging to a Hilbert space) describes not an individual microscopic quantum system (e.g., an electron), but a statistical ensemble of macroscopic neuronal structures. Thus our model differs crucially from quantum cognitive models making possible macroscopic and macrotermal quantum coherence, see e.g. [67]. In our model, each individual neuron is (as everybody would agree) purely classical, but ensembles of neurons could demonstrate quantum-like collective behaviour.

Our model can be called a model of probabilistic quantum-like thinking on the ultrametric space of hierarchic neuronal pathways.

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References

1. A. N. Whitehead, *Process and Reality: An Essay in Cosmology*. Macmillan Publishing Company, New York (1929).
2. A. N. Whitehead, *Adventures of Ideas*. Cambridge Univ. Press, London (1933).
3. A. N. Whitehead, *Science in the modern world*. Penguin, London (1939).
4. A. Shimony, On Mentality, Quantum Mechanics and the Actualization of Potentialities. In R. Penrose, M. Longair (Ed.) *The large, the small and the human mind*. Cambridge Univ. Press, New York (1997).
5. E. Schrödinger, *Philosophy and the Birth of Quantum Mechanics*. Edited by M. Bitbol, O. Darrigol. Editions Frontières (1992).
6. J. von Neumann, *Mathematical foundations of quantum mechanics*. Princeton Univ. Press, Princeton, N.J. (1955).
7. W. Heisenberg, *Physics and philosophy*. Harper & Row, Harper Torchbooks, New York (1958).
8. E.P. Wigner, The problem of measurement. *Am. J. Phys.*, 31, 6 (1963); *Symmetries and reflections*. Indiana Univ. Press, Bloomington (1967).
9. N. D. Mermin, Is the moon there when nobody looks? Reality and quantum theory. *Phys. Today*, 38-41, April 1985.
10. A. Peres, *Quantum Theory: Concepts and Methods*. Kluwer Academic Publishers (1994).
11. L. E. Ballentine, *Quantum mechanics*. Englewood Cliffs, New Jersey (1989).
12. B. d’Espagnat, *Conceptual foundations of Quantum Mechanics*. Perseus Books, Reading, Mass. (1999).
13. Y. F. Orlov, The wave logic of consciousness: A hypothesis. *Int. J. Theor. Phys.* 21, 1, 37-53 (1982).
14. R. Penrose, *The emperor’s new mind*. Oxford Univ. Press, New-York (1989).
15. R. Penrose *Shadows of the mind*. Oxford Univ. Press, Oxford (1994).
16. P. M. Churchland, *Matter and consciousness*. MIT Press, Cambridge (1999).
17. A. Clark, *Psychological models and neural mechanisms. An examination of reductionism in psychology*. Clarendon Press, Oxford (1980).
18. K. Lorenz, *On aggression*. Harcourt, Brace and World, New York (1966).
19. B. F. Skinner, *Science and human behaviour*. Macmillan Co., New York (1953).
20. R. Dawkins, The selfish gene. Oxford University Press, New York (1976).
21. H. P. Stapp (1993) *Mind, matter and quantum mechanics*. Springer-Verlag, Berlin-New York-Heidelberg.
22. M. Jibu, K. Yasue, A physical picture of Umezawa's quantum brain dynamics. In *Cybernetics and Systems Research*, ed. R. Trapp (World Sc., London, 1992).
23. M. Jibu, K. Yasue, *Quantum brain dynamics and consciousness*. J. Benjamins Publ. Company, Amsterdam/Philadelphia.
24. H. Umezawa, *Advanced field theory: micro, macro, and thermal physics*. American Institute of Physics, New-York (1993).
25. G. Vitiello, *My double unveiled - the dissipative quantum model of brain*. J. Benjamins Publ. Company, Amsterdam/Philadelphia (2001).
26. D. Bohm, and B. Hiley, *The undivided universe: an ontological interpretation of quantum mechanics*. Routledge and Kegan Paul, London (1993).
27. B. Hiley, P. Pylkkänen, Active information and cognitive science – A reply to Kieseppä. In: Brain, mind and physics. Editors: Pylkkänen, P., Pylkkö, P., Hautamäki, A.. IOS Press, Amsterdam (1997).
28. B. Hiley, Non-commutative geometry, the Bohm interpretation and the mind-matter relationship. To appear in Proc. CASYS 2000, Liege, Belgium, 2000.
29. A. Yu. Khrennikov, Classical and quantum mechanics on p-adic trees of ideas. *BioSystems*, 56, 95-120 (2000)
30. M. Lockwood, *Mind, Brain and Quantum*. Oxford,Blackwell (1989).
31. J. A. Barrett, The quantum mechanics of minds and worlds. Oxford Univ. Press, 1999.
32. A. Yu. Khrennikov, Classical and quantum mechanics on information spaces with applications to cognitive, psychological, social and anomalous phenomena. *Found. Phys.* 29, 1065-1098 (1999).
33. A. Yu. Khrennikov, Origin of quantum probabilities. Proc. Conf. ”Foundations of Probability and Physics”, *Quantum Probability and White Noise Analysis*, 13, 180-200, WSP, Singapore (2001).
34. A. Yu. Khrennikov, Linear representations of probabilistic transformations induced by context transitions. *J. Phys.A: Math. Gen.*, 34, 9965-9981 (2001).
35. A. Khrennikov, Ensemble fluctuations and the origin of quantum probabilistic rule. *J. Math. Phys.*, **43**, N. 2, 789-802 (2002).

36. A. Yu. Khrennikov, *Hyperbolic Quantum Mechanics*. Preprint: quant-ph/0101002, 31 Dec 2000.

37. P. A. M. Dirac, *The Principles of Quantum Mechanics* Claredon Press, Oxford (1930).

38. W. Heisenberg, *Physical principles of quantum theory*. (Chicago Univ. Press, 1930).

39. A. Yu. Khrennikov, Human subconscious as the p-adic dynamical system. *J. of Theor. Biology* **193**, 179-196(1998).

40. A. Yu. Khrennikov, p-adic dynamical systems: description of concurrent struggle in biological population with limited growth. *Dokl. Akad. Nauk* **361**, 752-754(1998).

41. S. Albeverio, A. Yu. Khrennikov, P. Kloeden, Human memory and a p-adic dynamical systems. *Theor. and Math. Phys.*, **117**, N.3, 385-396 (1998).

42. A. Yu. Khrennikov, Description of the operation of the human subconscious by means of p-adic dynamical systems. *Dokl. Akad. Nauk* **365**, 458-460(1999).

43. D. Dubischar D., V. M. Gundlach, O. Steinkamp, A. Yu. Khrennikov, A p-adic model for the process of thinking disturbed by physiological and information noise. *J. Theor. Biology*, **197**, 451-467 (1999).

44. A. Yu. Khrennikov, p-adic discrete dynamical systems and collective behaviour of information states in cognitive models. *Discrete Dynamics in Nature and Society* **5**, 59-69 (2000).

45. V. S. Vladimirov, I. V. Volovich, E. I. and Zelenov, *p-adic Analysis and Mathematical Physics*, World Scientific Publ., Singapore (1994).

46. I. V. Volovich, *p-adic string*. *Class. Quant. Grav.*, **4**, 83–87 (1987).

47. A. Yu. Khrennikov, Non-Archimedean analysis: quantum paradoxes, dynamical systems and biological models. Kluwer Academic Publ., Dordrecht(1997).

48. L. Bianchi, The functions of the frontal lobes. *Brain*, **18**, 497-530(1895).

49. I. P. Pavlov, *Complete Works*. Academy of Science Press, Moscow (1949).

50. W. Bechterew, *Die Funktionen der Nervencentra*. Fischer, Jena (1911).

51. H. Eichenbaum, R. A. Clegg, and A. Feeley, Reexamination of functional subdivisions of the rodent prefrontal cortex. *Exper. Neurol.* **79**, 434-451 (1983).
52. J. M. D. Fuster, The prefrontal cortex: anatomy, physiology, and neuropsychology of the frontal lobe. (1997)
53. A. R. Damasio, Descartes’ error: emotion, reason, and the human brain. Anton Books, New York (1994).
54. H. Damasio, A. R. Damasio, Lesion analysis in neuropsychology. Oxford Univ. Press, New-York (1989).
55. I. Kant, Critique of pure reason. Macmillan Press, 1985.
56. D. J. Chalmers, The conscious mind: in search of a fundamental theory. Oxford Univ. Press, New York (1996).
57. A. Yu. Khrennikov, Supernalysis. Nauka, Moscow, (1997) (in Russian). English translation: Kluwer Academic Publ., Dordreht (1999).
58. F. C. Hoppensteadt, An introduction to the mathematics of neurons: modeling in the frequency domain. Cambridge Univ. Press, New York (1997).
59. F. C. Hoppensteadt and E. Izhikevich, Canonical models in mathematical neuroscience. Proc. of Int. Math. Congress, Berlin, 3, 593-600 (1998).
60. A. J. Lemin, The category of ultrametric spaces is isomorphic to the category of complete, atomic, tree-like, and real graduated lattices LAT. Algebra universalis, to be published.
61. M. Mezard, G. Parisi, M. Virasoro, Spin-glass theory and beyond. World Sc., Singapure (1987).
62. G. Parisi, N. Sourlas, p-adic numbers and replica smmery breaking. The European Physical J., 14B, 535-542 (2000).
63. V. A. Avetisov, A. H. Bikulov, S. V. Kozyrev, Application of p-adic analysis to models of breaking of replica symmetry. J. Phys. A: Math. Gen., 32, 8785-8791 (1999).
64. L. Arnold, Random dynamical systems. Springer Verlag, Berlin-New York-Heidelberg (1998).
65. A. Khrennikov, On cognitive experiments to test quantum-like behaviour of mind. quant-ph/0205092 (2002).
66. K. Mori, On the relation between physical and psychological time. Proc. Int. Conf. Toward a Science of Consciousness, p. 102, Tucson, Arizona (2002).
67. M. Pitkänen, TGD inspired theory of consciousness with applications to biosystems. Electronic version: http://www.physics.helsinki.fi/~matpitka
68. S. Hameroff, Quantum coherence in microtubules. A neural basis for emergent consciousness? J. of Consciousness Studies, 1, 91-118 (1994); Quantum computing in brain microtubules? The Penrose-Hameroff Orch Or model of consciousness. Phil. Tr. Royal Sc., London, A, 1-28 (1998).
69. N. E. Mavromatos, D. V. Nanopoulos, I. Samaras, K. Zioutas, Advances in structural biology, 5, 127-134 (1998).
70. A. Mershin, D. V. Nanopoulos, E. M. C. Skoulakis, Quantum brain? Preprint quant-ph/0007088 (2000).

71. A. Priel, N. Wolf, S. Hameroff, Dynamical properties of dendric arrays of microtubules - relevance to consciousness. Proc. Int. Conf. Toward a Science of Consciousness, p. 69, Tucson, Arizona (2002).

72. B. J. Baars, In the theater of consciousness. The workspace of mind. Oxford University Press, Oxford (1997).

73. G. M. Edelman, The remembered present: a biological theory of consciousness. New York, Basic Books, 1989.

74. A. Khrennikov, Quantum-like formalism for cognitive measurements. Proc. Int. Conf. Toward a Science of Consciousness, p. 272, Tucson, Arizona (2002).