Standard Quantum Mechanics without observers

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I like to think the moon is there even if I am not looking at it.

Albert Einstein

I. INTRODUCTION

Standard Quantum Mechanics (QM) in its various formulations relies essentially on the existence of quantum measuring devices as quasi-classical systems. The central role of the measuring device is manifest in the Projection Postulate, which states that the observed quantum system is found to be in an eigenstate of the operator associated to the observable we measure, and prescribes as well as the probability for each outcome to occur, according to the Born rule. The world is considered to be divided in two parts, one is classical or quasi-classical, and includes the apparatus, and the other one is the observed quantum system. Bohr prescribed that the apparatus is a classical system. John von Neumann treated the apparatus like a quantum system which behaves quasi-classically, but gave a central role to the observer, and the split between apparatus and the observed system persisted [35]. This split remains true for modern approaches to quantum measurement [6]. Moreover, since the Standard QM leads to macro superpositions like Schrödinger cats, something is needed to project large systems to make them appear classical. In the Copenhagen Interpretation, this is achieved by the observer, whose sensory organs or maybe consciousness acts like measuring devices. It seems that the measurement process and the observer play a fundamental role in the theory. This leads to some foundational problems. One is the following:

Problem 1. Can Quantum Mechanics be formulated without relying on measuring devices and/or observers?

Even if a formulation which solves Problem 1 can be found, it still needs to be able to describe quantum measurements, to be able to connect the theory with the experiments. But since a measuring apparatus is a quasi-classical system, another problem appears:

Problem 2. What criterion should a quantum system satisfy to be considered classical?

Another problem is that the space of quantum states is vastly large, and most of the states it includes can be rather seen as superpositions of quasi-classical systems. In particular, the Schrödinger equation predicts that, even if we start with a quasi-classical measuring device, during the measurement it evolves into a superposition. Hence, there is a closely related problem:

Problem 3. Why does the world appear to be classical at the macro level?

Since the postulates of Standard QM in the usual formulations do not address Problems 2 and 3, relying on measuring devices in the Projection Postulate is in some sense circular: on the one hand, the Projection Postulate makes the world appear classical, on the other hand, the Projection Postulate requires a quasi-classical measuring device.

The presence of the Projection Postulate in Standard QM, without specifying exactly the conditions that make the projection happen, is another problem:

Problem 4. Under what conditions does the projection occur?

Closely related, and due to the fact that Standard QM does not specify when exactly between the preparation and the measurement the projection takes place, is the following problem:

Problem 5. Can the quantum state be well defined at all times?
These problems are often considered to be of interest just to the philosophy of physics, in particular to ontology, and addressing them is usually done by the so-called “interpretations” of QM, still considered by many working physicists not serious science.

But it is preferable for a theory in physics to be able to describe precisely both its states and its dynamics. In this article, I propose a formulation of Standard QM which addresses the above mentioned problems. This formulation is still based on the Schrödinger dynamics and a projection postulate, like Standard QM, but without invoking observers or measuring devices, and at the same time having a clear prescription of what is classical and what is the state of the system at any given time.

Possible answers to at least some of these problems are already proposed by various interpretations of QM, and are extensively developed and researched. For example, in the objective collapse theories quantum states collapse into well localized, hence quasi-classical, states [15, 16]. This is accomplished by modifying the dynamics of QM with randomly occurring collapses. We do not know yet if a modification of the dynamics is needed. It is believed that decoherence can solve these problems without modifying the dynamics [22, 30, 40, 41]. But since at best decoherence can explain the emergence of the classical world by branching, even if this solution will turn out to work, it requires to be accompanied by an interpretation where branching occurs, like Everett’s [12, 13], or Consistent Histories [17, 27]. Even de Broglie-Bohm’s theory [4, 5] needs to make use of decoherence of the universal pilot wave, and in fact Bohm described decoherence before it was discovered. Despite the extended work done and the progress made with the decoherence program, it is not clear yet that decoherence alone can fully solve these problems [1, 24, 29]. It is considered that a decoherence-based solution of Problems 2 and 3 requires a preferred basis to emerge, which then can be applied to solve Problem 1 and the other problems.

The formulation proposed here does not necessarily compete with the various interpretations of QM, but it rather redefines Standard QM, so that the mentioned problems are addressed in a very straightforward manner. Problem 3 is only provisionally solved, by imposing the criterion of classicality proposed to address Problem 2 as a fundamental principle, rather than deducing it from other principles. An approach addressing these problems may seem at first sight a completely different theory from Standard QM. But I claim that the difference is not that large, the formulation proposed here being rather a minor strengthening and refinement of Standard QM. In fact, the main difference consists in using the criterion of classicality (from the proposed solution of Problem 2) to replace the Projection Postulate with a version which does not require observers or measuring devices, and applies to the total wavefunction.

Along with the Heisenberg and Schrödinger pictures, shown by Schrödinger to be equivalent [31], it is possible to formulate Quantum Mechanics on the phase space [18, 26, 36, 37]. This extends to the Projection Postulate [3]. There are also original alternative formulations like [11] and [9].

I now summarize how I use the phase-space representation of QM in the formulation proposed in this article. The Postulates are proposed and discussed in Sec. §II. We start with a brief review of the classical phase space (in §II.A) and quantization (in §II.B), to fix the notations. Then, in §II.C and §II.D, we discuss the first three postulates, which are also present in the Standard QM, but will be expressed in terms of the phase space:

Postulate 1 (Quantum state). The state of a quantum system is represented by a time dependent Wigner function on the classical phase space.

Postulate 2 (Composite system). For composite systems, the phase space is the direct sum of the phase spaces associated with the component systems.

Postulate 3 (Observable). An observable of the total system is represented by a real-valued function on the classical phase space (its Weyl symbol).

The next postulate, discussed in §II.E, establishes that in the quantum world, the macro states are quasi-classical, in a way that relies on the classical macro level:

Postulate 4 (Macro level). For any time, there is a coarse-graining region of the classical phase space within which the quantum state of the total system is quasi-restricted.

We will clarify exactly what “quasi-restricted” means and why it is used, rather than “strict restriction”.

The dynamics within a coarse-graining region is just the Schrödinger dynamics, translated to the phase space (§II.F):

Postulate 5 (Dynamics). The evolution of the quantum state of the total system within the same coarse-graining region is given by the Liouville-von Neumann equation.

Finally in §II.G, the Projection Postulate is replaced with a version that does not rely on measuring devices or observers:

Postulate 6 (Transition). When the Wigner function of the total system propagates from one coarse-graining region to others, it transitions to only one of these regions, with a probability proportional to the squared amplitude corresponding to that region.

The phase-space formulation proposed here is translated into the more popular Hilbert space formulation in §II.H. In Sec. §III it is shown that the standard Projection Postulate can be recovered from this formulation. A somewhat more detailed discussion of the interplay between dynamics and projections is given in Sec. §IV. A discussion of the proposed formulation of Standard QM, as well as of possible shortcomings and remaining open problems, takes place in Sec. §V.
II. POSTULATES

A. Classical phase space

We review the phase space of a classical system of \( m \) particles in the 3D-space \( \mathbb{R}^3 \). The classical configuration space is the \( n \)-dimensional space \( \mathbb{R}^n \), where \( n = 3m \).

The configuration of a system is characterized by the positions \( x \in \mathbb{R}^n \). The evolution depends not only of the configuration \( x \in \mathbb{R}^3 \), but also of the velocities \( v_j = \dot{x}_j = \frac{d x_j}{d t} \) or the momenta \( p_j = m_j v_j \), so the classical state is fully specified by a point in the phase space \( \mathbb{R}^{2n} = \mathbb{R}^{6m} \).

The state (or phase) of a system is characterized by both positions and momenta \( z = (x, p) \in \mathbb{R}^{2n} \).

The phase space \( \mathbb{R}^{2n} \) is naturally endowed with a symplectic structure

\[
J = \begin{pmatrix} O_n & I_n \\ -I_n & O_n \end{pmatrix},
\]

which satisfies \( J^2 = -I \). It defines the symplectic product between \( z = (x, p) \) and \( z' = (x', p') \),

\[
\sigma(z, z') = z^T J z' = x \cdot p' - p \cdot x'.
\]

A classical observable is a real function on the phase space, \( A : \mathbb{R}^{2n} \to \mathbb{R} \). One defines the Poisson bracket between two classical observables \( A, B \) by

\[
\{A, B\}(x, p) := (\partial_x A \partial_p B - \partial_x B \partial_p A)(x, p).
\]

The dynamics is given by Hamilton’s equations,

\[
\begin{align*}
\dot{x}(t) &= \partial_p H(x, p, t) \\
\dot{p}(t) &= -\partial_x H(x, p, t),
\end{align*}
\]

where the Hamilton function is a scalar function defined on \( \mathbb{R}^{2n} \times \mathbb{R} \).

B. Quantization

Quantization associates to each classical observable a Hermitian operator acting on the Hilbert space \( \mathcal{H} = L^2(\mathbb{R}^n, \mathbb{C}) \) consisting of the square-integrable complex functions on the configuration space \( \mathbb{R}^n \). The Hilbert space \( \mathcal{H} \) is endowed with the Hermitian scalar product

\[
\langle \psi | \phi \rangle := \int_{\mathbb{R}^n} \overline{\psi(x)} \phi(x) \, d x.
\]

To the constant observable \( A(z) = 1 \), quantization associates the identity operator \( \hat{I} \), to the position \( x_j \) and to the momentum component \( p_j \) it associates the operators \( \hat{x}_j | \psi \rangle = x_j | \psi \rangle \) and \( \hat{p}_j | \psi \rangle = -i \hbar \partial_j | \psi \rangle \). Since the classical observables \( x_j \) and \( p_j \) commute, and their corresponding quantum operators do not commute, we also need to specify a rule to choose a particular ordering of the products of such observables. In the following we will assume the Weyl quantization rule, which uses the symmetrized product and is the most common,

\[
(x_j)'(p_j)' \mapsto \left( (\hat{x}_j)'(\hat{p}_j)\right)^{\text{sym}}.
\]

The quantum states are represented by rays in \( \mathcal{H} \), but in case they are mixed or the information is incomplete, they can be represented more generally as density operators, which are self-adjoint operators \( \rho \) on \( \mathcal{H} \), whose diagonal elements are non-negative and add up to 1 in any orthonormal basis.

C. Quantum states

Let \( \hat{\rho} \) be a density operator on \( \mathcal{H} \), representing the state of the quantum system. If the system is in a pure state, then \( \hat{\rho} = |\psi\rangle \langle \psi| \), where \( |\psi\rangle \in \mathcal{H} \) is the unit vector representing the state of the system.

The Wigner phase-space function of \( \hat{\rho} \), which will be called in the following the Wigner function, is defined as

\[
W_\rho(x, p) := \frac{1}{(2\pi \hbar)^n} \int_{\mathbb{R}^n} e^{-\frac{i}{\hbar} p \cdot x'} \rho(x + \frac{x'}{2}, p) \, d x'.
\]

The density operator can be obtained from its Wigner function by

\[
\hat{\rho} = \int \int \int \left| x + \frac{x'}{2} \right|^2 e^{\frac{i}{\hbar} p \cdot x'} W_\rho(x, p) \left| x + \frac{x'}{2} \right|^2 p \, d x' d p d x.
\]

In particular, for a pure state \( \hat{\rho} = |\psi\rangle \langle \psi| \),

\[
W_\psi(x, p) = \frac{1}{(2\pi \hbar)^n} \int_{\mathbb{R}^n} e^{-\frac{i}{\hbar} p \cdot x'} \psi^* \left( x + \frac{x'}{2} \right) \psi \left( x + \frac{x'}{2} \right) \, d x'.
\]

The Wigner function is real-valued, but can take negative values, and for this reason it cannot be a probability distribution, but it can be a quasiprobability distribution, from which the correct probability distributions in the position and momentum bases can be recovered as marginal distributions. It is non-negative everywhere only for Gaussian states.

The recovery property holds: if \( \psi(0) \neq 0 \), the state \( |\psi\rangle \) can be recovered from the Wigner function,

\[
\psi(x) \psi^*(0) = \int_{\mathbb{R}^n} W_\psi \left( \frac{x}{2}, p \right) e^{\frac{i}{\hbar} p \cdot x} \, d p.
\]

We introduce the following postulates:

Postulate 1 (Quantum state). The state of a quantum system is represented by a time dependent Wigner function on the classical phase space.

Postulate 2 (Composite system). For composite systems, the phase space is the direct sum of the phase spaces associated with the component systems.
D. Observables

Let $S(\mathbb{R}^n)$ be the space of smooth complex functions $f$ so that, for any multiindices $\alpha, \beta$, $x^\alpha \partial_x^\beta f$ is bounded in $\mathbb{R}^n$. Under the Weyl quantization rule (6), to any complex function $A \in S(\mathbb{R}^{2n})$ we associate the operator $\hat{A}$ defined by

$$\langle x | \hat{A} | x' \rangle = \frac{1}{(2\pi \hbar)^n} \int_{\mathbb{R}^n} e^{i\frac{x' - x}{\hbar}} p A \left( \frac{x + x'}{2}, p \right) \, dp. \tag{11}$$

The operator $\hat{A}$ acts on a quantum state $|\psi\rangle$ by

$$\hat{A} \psi(x) = \int_{\mathbb{R}^n} \langle x | \hat{A} | x' \rangle \psi(x') \, dx'. \tag{12}$$

The Weyl symbol $A(x, p)$ can be obtained from $\hat{A}$:

$$A(x, p) = \int_{\mathbb{R}^n} e^{-i\frac{x' - x}{\hbar} p} \left( \frac{x + x'}{2} \right) A \left( \frac{x - x'}{2}, p \right) \psi(x') \, dx'. \tag{13}$$

The Weyl correspondence $A \leftrightarrow \hat{A}$ is a linear bijection. We call $\hat{A}$ the Weyl symbol of the operator $A$. Since $\hat{A}^* = \hat{A}^\dagger$, $A$ is real iff $\hat{A}$ is Hermitian.

We introduce the following postulate:

**Postulate 3 (Observable).** An observable of the total system is represented by a real-valued function on the classical phase space (its Weyl symbol).

The Weyl symbol of a density operator $\hat{\rho}$ is $(2\pi \hbar)^n W_\rho(x, p)$. Hence, the Weyl symbol $\pi_{|\psi\rangle}$ of the projector $\pi_{|\psi\rangle} := |\psi\rangle \langle \psi|$ is

$$\pi_{|\psi\rangle}(x, p) = (2\pi \hbar)^n W_\psi(x, p). \tag{14}$$

The mean value of $\hat{A}$ is given by

$$\langle \hat{A} | \psi \rangle = \int_{\mathbb{R}^{2n}} A(z) W_\psi(z) \, dz. \tag{15}$$

In particular, by applying (15) to $\pi_{|\psi\rangle} = |\psi\rangle \langle \psi|$, we find

$$|\langle \psi | \psi \rangle|^2 = (2\pi \hbar)^n \int_{\mathbb{R}^{2n}} W_\psi(z) W_\psi(z) \, dz. \tag{16}$$

The *Moyal product* of two observables $A, B \in S(\mathbb{R}^{2n})$ is the Weyl symbol of the product $\hat{A}\hat{B}$, and it is given by

$$(A \star B)(z) = (\pi \hbar)^{-2n} \int_{\mathbb{R}^{2n}} e^{-\frac{i}{\hbar} \sigma(s', s'')} A(s + z') B(z + s') \, ds' \, dz' \, ds''. \tag{17}$$

The local (on the phase space) form of the Moyal product operator is

$$a = e^{\frac{i}{\hbar} \sigma(\overline{s}, \overline{z})}. \tag{18}$$

E. The quasi-classicality of the macro world

In order to define what is understood for a quantum state to be quasi-classical, we will rely on the idea that the macro world, even if we know it to be quantum, looks like the classical macro world. To formalize this idea, we will use the quasi-projection operators proposed by Omnès [28]. We will also take into account the insights of de Gosson regarding the phase space and quantum blobs [8], based on the principle of space and quantum blobs [8], based on the principle of space and quantum blobs [8], based on the principle of space and quantum blobs [8], based on the principle of space and quantum blobs [8], based on the principle of space and quantum blobs [8].

To define the classical macro level, we assume that the classical theory to which we applied the quantization procedure has a definite set of observables $\mathcal{M}$, which will be called *macro observables*. They are in general aggregate functions of other observables: averages, integrals or sums, volumes, densities etc, and are important for example in Statistical Mechanics. The macro observables partition the phase space into *coarse-graining regions* where the macro observables take constant (or indistinguishable) values. Each of these regions of the phase space contains (micro) states that cannot be distinguished macroscopically. Even for a classical theory there are ambiguities in defining exactly the coarse-graining, but given the success of Statistical Mechanics, in particular in reducing thermodynamics to mechanics, we will assume that both the macro observables and the coarse-graining can be defined unambiguously at least in principle.

To be able to transfer the classical coarse graining of the phase space to the quantized theory, we will require the coarse-graining regions to be unions of *quantum blobs*. This is necessary because

1. a quantum blob is the smallest symplectic invariant region of the phase space compatible with the uncertainty principle, and

2. we want to exclude regions containing only functions that are not Wigner functions of quantum states, being too localized or having negative quasi-probability.

Quantum blobs, introduced by de Gosson [8], define the most fine-grained partition of the phase space consistent with Quantum Mechanics. They are in a one-to-one correspondence with the squeezed coherent states from Standard QM. Quantum blobs were also used in the formulations or interpretations of quantum mechanics, e.g. in [9] and [11].

Let $\mathcal{R}$ be the partition (coarse-graining) of the phase space $\mathbb{R}^{2n}$, satisfying $R \subset \mathbb{R}^{2n}$, $\mathbb{R}^{2n} = \bigcup_R \mathcal{R}$, and $R \cap R' = \emptyset$ if $R \neq R'$. For each region $R$ we define the characteristic function $\chi_R : \mathbb{R}^{2n} \rightarrow \mathbb{R}$,

$$\chi_R(z) = \begin{cases} 1, & \text{if } z \in R \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

But if we try to contain the wavefunction in a small region of space, the Schrödinger dynamics will spread it
outside that region very fast. For this reason, we will require that the Wigner function is highly peaked inside a coarse-graining region \( R \), rather than to have its support completely contained in \( R \). Hence, instead of the characteristic function of \( R \), \( \chi_R(z) \), we will use its convolution product with some highly peaked function \( \varphi(z) \) centered at \((0,0)\) in the phase space. Due to the relation with quantum blobs, the natural choice is \( \varphi(z) = (z|0,0) \), where \((0,0)\) is the coherent state located at \((0,0)\), which is a Gaussian. We will use the following convolution:

\[
\Pi_R = \chi_R \ast \varphi. \tag{20}
\]

Since the characteristic functions \( \chi_R \) for all regions in \( \mathcal{R} \) add up to the constant function identically equal to 1 on the phase space, the functions \( \Pi_R \) from eq. (20) form a partition of unity, and the corresponding operators \( \hat{\Pi}_R \) form, because of linearity, a resolution of the identity operator \( \hat{I} \).

In [28], Omnès introduced the operators \( \hat{\Pi}_R \) corresponding to the Weyl symbols \( \Pi_R \), and proved that they form a set of quasi-projectors. An equivalent definition he proposed is

\[
\hat{\Pi}_R := \frac{1}{(2\pi \hbar)^n} \int_{\mathbb{R}^{2n}} |x, p\rangle \langle x, p| \, dx \, dp, \tag{21}
\]

where \(|x, p\rangle\) is the coherent state centered in the phase space at \((x, p)\), i.e. a normalized Gaussian state whose average is the point \((x, p) \in \mathbb{R}^{2n}\).

No two distinct coherent states are orthogonal, so they cannot form a basis of the Hilbert space \( \mathcal{H} \), but they form an orthonormal system, by providing a resolution of the identity operator,

\[
\hat{I} = \frac{1}{(2\pi \hbar)^n} \int_{\mathbb{R}^{2n}} |x, p\rangle \langle x, p| \, dx \, dp = \sum_{R \in \mathcal{R}} \hat{\Pi}_R. \tag{22}
\]

Despite the fact that two distinct coherent states always overlap, the operators (21) are quasi-projectors, i.e. they behave as projectors in a very good approximation, due to the fact that the coarse-graining regions \( R \) are large enough [28]. In particular, they are almost idempotent \( \hat{\Pi}_R^2 \approx \hat{\Pi}_R \), and the product of two of them corresponding to distinct coarse-graining regions almost vanishes exponentially. The approximations are of the order of \( \hbar^2 \). Moreover, the quantum time evolution \( e^{-\frac{i}{\hbar}Ht}\hat{\Pi}_Re^{\frac{i}{\hbar}Ht} \) of a quasi-projector \( \hat{\Pi}_R \) also approximates well the quasi-projector corresponding to the classical evolution of the region \( R \), due to a result by Hagedorn [20] (see [28]).

Omnès also shown that there exists a complete sets of actual projectors (i.e. idempotent, orthogonal, and adding-up to the identity operator \( \hat{I} \))

\[
\hat{\Pi}_R^+ \approx \hat{\Pi}_R \tag{23}
\]

that are close to the \( \hat{\Pi}_R \) within a similar approximation.

**Definition II.1.** We call the operators \( \hat{\Pi}_R \) from eq. (23) classicality quasi-projectors, and the operators \( \hat{\Pi}_R^\perp \) from eq. (23) classicality projectors. We say that a Wigner function \( W_\psi \) is quasi-restricted to \( R \) if \( \hat{\Pi}_R |\psi\rangle \approx |\psi\rangle \) (which is to say that \( \hat{\Pi}_R^\perp |\psi\rangle \approx |\psi\rangle \)).

It is therefore justified to take as a criterion of quasi-classicality

**Criterion 1.** A quantum state \(|\psi\rangle\) is quasi-classical if there is a coarse-graining region \( R \in \mathcal{R} \) so that the Wigner function \( W_\psi \) is quasi-restricted to \( R \).

The postulate of quasi-classicality is therefore

**Postulate 4 (Macro level).** For any time, there is a coarse-graining region of the classical phase space within which the quantum state of the total system is quasi-restricted.

**F. Dynamical law**

The Schrödinger equation is

\[
i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle, \tag{24}
\]

where \( \hat{H} \) is the Hamiltonian operator. Since our system is the entire universe, we take \( \hat{H} \) to be time independent. The time evolution of a Wigner function \( W_\psi \), corresponding to the Schrödinger evolution (24), is given by the Liouville-von Neumann equation

\[
\frac{\partial W_\psi(z,t)}{\partial t} = -\{\{W_\psi(z,t), H(z)\}\}, \tag{25}
\]

where

\[
\{\{A, B\}\} := \frac{1}{i\hbar} (A \ast B - B \ast A) \tag{26}
\]

is the (Groenewold-)Moyal bracket, and \( H(z) \) is the Weyl symbol of the Hamiltonian operator \( \hat{H} \).

Postulate 4 introduced classicality directly, rather than attempting to derive it. Whatever experiment the observer performs, the outcome can be observed only when it produces a macroscopic difference, i.e. when the system moves from a coarse-graining region of the phase space to another one. As long as there is no macroscopically observable difference, there should be no observable projection. This justifies the postulate that, as long as the Wigner function of the state \(|\psi\rangle\) remains in the same coarse-graining region on the phase space, its dynamics is given by the Liouville-von Neumann equation, no projection being invoked. We formulate this postulate on the phase space:

**Postulate 5 (Dynamics).** The evolution of the quantum state of the total system within the same coarse-graining region is given by the Liouville-von Neumann equation.
Suppose that at some time $t_0$ the Wigner function $W_0(x, p, t_0)$ is quasi-restricted to the coarse-graining region $R_0 \in \mathcal{R}$, and at a future but not very distant time $t_1 > t_0$ equation (25) its evolution according to Postulate 5 will spread it over the regions $R_1, \ldots, R_k \in \mathcal{R}$. On the Hilbert space, this corresponds to the unitary evolution of $|\psi_0\rangle$ until, at $t_1$, it becomes $|\psi_1\rangle = \hat{U}(t_1, t_0) |\psi_0\rangle$, which is a linear combination of states quasi-restricted to the regions $R_1, \ldots, R_k \in \mathcal{R}$. In other words,

$$|\psi_1\rangle = \sum_{j=1}^{k} c_j \hat{\Pi}_{R_j} |\psi_1\rangle \approx \sum_{j=1}^{k} c_j \hat{\Pi}_{R_j}^{\perp} |\psi_1\rangle,$$  \hspace{1cm} (27)

where

$$c_j = \frac{\langle \psi_1 | \hat{\Pi}_{R_j} | \psi_1 \rangle}{\langle \psi_1 | \psi_1 \rangle}.$$  \hspace{1cm} (28)

As Postulate 4 specifies, at $t_1$ the Wigner function describing the system will be in only one of the regions $R_1, \ldots, R_k \in \mathcal{R}$, say $R_j$, $j \in \{1, \ldots, k\}$. We now introduce the Born rule, stating that the probability for the system to end out in the region $R_j$ is $|c_j|^2$.

**Postulate 6 (Transition).** When the Wigner function of the total system propagates from one coarse-graining region to others, it transitions to only one of these regions, with a probability proportional to the squared amplitude corresponding to that region.

In Sec. §III we will show that the standard Projection Postulate is a consequence of the postulates proposed here.

Due to the blurry boundaries of the quasi-projectors (21) and their eigenstates, the separation between dynamics (Postulate 5) and transitions (Postulate 6) is not completely strict. There is an interplay between the two, which will be discussed in Sec. §IV. We will see that the same holds in the standard QM, even though apparently its usual formulations strictly separate dynamics and projections.

**III. RECOVERING THE STANDARD PROJECTION POSTULATE**

A fundamental role in the usual formulations of Standard QM is played by the Projection Postulate. Let $\mathcal{S}$ be the observed quantum system, and $\mathcal{M}$ the measuring apparatus, having as Hilbert spaces $\mathcal{H}_S$ and $\mathcal{H}_M$. Let $\hat{A}$ be a Hermitian operator on $\mathcal{H}_S$, representing the observable measured by the measuring device $\mathcal{H}_M$. We assume for simplicity that the spectrum of $\hat{A}$ is non-degenerate, and $N = \dim \mathcal{H}_S$. The measuring apparatus $\mathcal{M}$ is assumed, by definition, to be such that it has among its possible states the following:

1. A quasi-classical state $|\text{ready}\rangle_\mathcal{M}$, corresponding to the system $\mathcal{M}$ being prepared to observe.

2. A number of $N$ quasi-classical states $|\text{outcome} = \lambda_j\rangle_\mathcal{M}$, corresponding to the measuring apparatus $\mathcal{M}$ indicating that the outcome of the measurement is $\lambda_j$, for $j \in \{1, \ldots, N\}$.

All these states are assumed to be mutually orthogonal.

The measuring device is also constructed so that, if the observed system is already in an eigenstate $|j\rangle$ of $\hat{A}$ before the measurement, the composed system evolves under the Schrödinger dynamics like this:

$$|\text{ready}\rangle_\mathcal{M} \otimes |j\rangle \mapsto |\text{outcome} = \lambda_j\rangle_\mathcal{M} \otimes |j\rangle,$$  \hspace{1cm} (29)
for all \( j \in \{1, \ldots, N\} \).

Then, Schrödinger’s equation predicts that if the observed system is in the initial state

\[
|\psi\rangle = \sum_{j=1}^{N} c_j |j\rangle ,
\]

then the composed system evolves into

\[
|\text{ready}\rangle_{\mathcal{M}} \otimes |\psi\rangle \mapsto \sum_{j=1}^{N} c_j |\text{outcome} = \lambda_j\rangle_{\mathcal{M}} \otimes |j\rangle .
\]

With these settings, the Projection Postulate consists of the following two parts:

**The Projection Postulate Part 1.** The result of one of the eigenvalues \( \lambda_j \) of \( \hat{\mathcal{A}} \), and the state of the total system is, after the measurement, projected to \( |\text{outcome} = \lambda_j\rangle_{\mathcal{M}} \otimes |j\rangle \).

**The Projection Postulate Part 2** (The Born rule). The probability to obtain the outcome \( \lambda_j \) is \( |c_j|^2 \).

Since the Projection Postulate is among the fundamental postulates of Standard QM, this makes the theory rely on the existence of a quasi-classical state – the measuring apparatus, or on the existence of an observer. Some of the founders of QM even thought that the measuring device is projected into a quasi-classical state by the very observer conducting the measurement. This made researchers like Heisenberg [21], von Neumann [35], Wigner [38], Stapp [32, 33], and others [2] think that consciousness plays a fundamental role in QM.

By contrast, the formulation proposed in this article does not need to appeal to an observer or even to a measuring device in its fundamental postulates. Now we need to show that indeed the theory obtained from these postulates is the same as Standard QM. In §IIIH we have seen that most of the postulates of Standard QM are equivalent to postulates from the formulation presented here. It remains to show that we can derive the Projection Postulate Part 1 & Part 2.

To show this, let us go back to the system composed of the observed system \( \mathcal{S} \) and the measuring apparatus \( \mathcal{M} \). Their phase spaces are \( \mathcal{P}_\mathcal{S} \) and respectively \( \mathcal{P}_\mathcal{M} \). The total system has the Hilbert space \( \mathcal{H}_\mathcal{M} \otimes \mathcal{H}_\mathcal{S} \), and the corresponding classical phase space is \( \mathcal{P}_\mathcal{M} \oplus \mathcal{P}_\mathcal{S} \).

A central remark is that the observed system \( \mathcal{S} \) is not directly observed: whatever we learn about its state by measurement, comes in the form of a change of the macrostate of the measuring device \( \mathcal{M} \). Not only the measuring device, but the total system is in a quasi-classical state before the measurement, and it ends out in a quasi-classical states after the measurement, due to the Postulate 4. Whatever can be said about the observed system is inferred from the classical states of the total system before and after the measurement. On the total phase space \( \mathcal{P}_\mathcal{M} \oplus \mathcal{P}_\mathcal{S} \), the following coarse-graining regions associated to the systems \( \mathcal{M} \) and \( \mathcal{S} \) are relevant:

1. A coarse-graining region \( \mathcal{R}_{\text{ready}} \), corresponding to the system \( \mathcal{S} \) being prepared to be measured, and the system \( \mathcal{M} \) being prepared to measure it.

2. A number of \( N \) coarse-graining regions \( \mathcal{R}_{\text{outcome} = \lambda_j} \), corresponding to the system \( \mathcal{S} \) being in the eigenstate \( |j\rangle \), and the measuring apparatus \( \mathcal{M} \) indicating that the outcome of the measurement is \( \lambda_j \), for \( j \in \{1, \ldots, N\} \).

In addition, we know that the measuring device is, by construction, such that the only way the composed system can evolve by Schrödinger dynamics is by Schrödinger dynamics is in a superposition of the form from eq (31). In our phase-space formulation, this translates into the fact that the Wigner function of the total system evolved from the coarse-graining region \( \mathcal{R}_{\text{ready}} \) to the union of the coarse-graining regions \( \mathcal{R}_{\text{outcome} = \lambda_j} \). By Postulate 6, the total system has to transition to only one of the coarse-graining regions \( \mathcal{R}_{\text{outcome} = \lambda_j} \), with the probability \( |c_j|^2 \), with \( c_j \) from eq. (31). Hence, the standard Projection Postulate follows as a consequence of the postulates proposed here.

**IV. THE INTERPLAY BETWEEN DYNAMICS AND TRANSITIONS**

In this section I will explain the interplay between dynamics and transitions, which is a bit more complex than the standard narrative of unitary evolution interrupted once in a while by projections.

But first, let us apply the discussions in Sec. §II.G and §III to the situation when the observed system is an excited atom, and that the measuring apparatus is a detector which either detected the decay or not. So there are two coarse-graining regions relevant here, say \( \mathcal{R}_{\text{excited}} \) and \( \mathcal{R}_{\text{decayed}} \). As the system is contained in region \( \mathcal{R}_{\text{excited}} \), the measuring device registers no decay. But as the time goes, the Wigner function spreads out of region \( \mathcal{R}_{\text{excited}} \), leaking into region \( \mathcal{R}_{\text{decayed}} \). The quantum Zeno effect [25] implies that the monitoring of the excited atom while the system’s Wigner function only spreads very little into region \( \mathcal{R}_{\text{decayed}} \) projects it back to the excited state, preventing or delaying the decay [23, 39]. By contrast, monitoring it after its Wigner function spread enough into region \( \mathcal{R}_{\text{decayed}} \) results in an enhancement of the decay rate [7]. This exemplifies the interplay between dynamics and transitions, which will be explained now.

The quasi-projectors \( \hat{\Pi}_R \) used to define quasiclassically project the Wigner state only approximately to the coarse-graining regions. Due to the use of the convolutions \( \Pi_R = \chi_R \ast \varphi \) with the Gaussian function \( \varphi \), instead of the characteristic functions \( \chi_R \) of the coarse-graining regions, the projection is distorted around the boundaries of the regions. This means that the separation between dynamics (Postulate 5) and transitions (Postulate 6) is not exact. The Wigner functions have “tails”, of very low amplitudes, that go outside of the
coarse-graining region $R$. This means that even for the times when the Wigner function of the system is included in a coarse-graining region $R$, projections by $\tilde{\Pi}_R$ happen, albeit with a very small effect.

Here is a more detailed explanation. For each coarse-graining region $R$, there is an internal region $R^o \subset R$, defined by $R^o = R^o(1)$, i.e. $R^o$ is the set of all $z \in R$ for which $\Pi_R(z) = 1$. Due to the fact that $R$ is much larger than the width of the Gaussian function $\varphi$, the region $R^o$ is approximately the same as $R$. Inside $R^o$, the Wigner function $W_\psi$ satisfies the Liouville-von Neumann equation (25), independently on the fact that $W_\psi$ does not vanish completely outside $R^o$. The reason is that the Hamiltonian function $H(x,p)$ in eq. (25) acts on the Wigner function $W_\psi$ through the Moyal product (18), which is local on the phase space. This means that the dynamics is not affected by the quasi-projection $\tilde{\Pi}_R$ for $z \in R^o$, and Postulate 5 holds exactly for these points. But for $z \notin R^o$, the dynamics is distorted by the projection.

The Wigner function may have a very small amplitude outside the region $R$, because Postulate 4 and the classicality quasi-projector $\tilde{\Pi}_R$ allow this. Even when $W_\psi$ is restricted to the region $R$, its tiny “tails” slightly spread outside of $R^o$ and then outside of $R$, and activate Postulate 6 (our version of the Projection Postulate). But since the amplitude of $W_\psi$ is very small outside of $R$, the probability to project the state on another region $R^o \neq R$ is small, and the quantum Zeno effect implies that the chosen quasi-projector $\tilde{\Pi}_R$ is significantly more often the preferred one, and it projects the Wigner function $W_\psi$ back into region $R$. Note that even if the Wigner function is quasi-constrained to the same region $R$ for a certain amount of time, the quantum Zeno effect does not imply that it remains unchanged. The reason why the Wigner function evolves even when it is quasi-restricted to $R$ is that the operator $\tilde{\Pi}_R$ has a very high degeneracy in the eigenvalue $\lambda = 1$. So the quantum Zeno effect does not apply for $z \in R^o$, and Postulate 5 indeed holds exactly there.

This interplay between dynamics and transitions, when the Zeno effect takes place, leads to the following truncated Schrödinger equation, valid when the Wigner function is quasi-restricted to the coarse-graining region $R$:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \tilde{\Pi}_R \hat{H} |\psi(t)\rangle. \tag{32}$$

The meaning of eq. (32) is that, if after an infinitesimal time interval the Wigner function $W_\psi$ leaves very slightly the region $R$, it is mostly projected back inside of $R$.

However, $W_\psi$ continues to evolve towards the boundary of region $R$. As it accumulates at the boundary of region $R$, where the function $\Pi_R$ overlaps with the functions $\Pi_R$, $j \in \{1, \ldots, k\}$ from Sec. §II G, it becomes more probable that Postulate 6 allows $W_\psi$ to transition to another coarse-graining region $R_j$.

This may seem different from the usual formulations of Standard QM, where the common understanding is that only when quantum measurements happen, the Projection Postulate applies [10, 35]. But in reality this alone does not explain the fact that the macro level is quasi-classical, in particular it does not justify the existence of a quasi-classical measuring apparatus in the first place. The Copenhagen Interpretation, the default companion of the Standard QM, explains this by the presence of the observer, whose sensory organs (or consciousness?) act like measuring devices. The observer is the one who makes the measuring device be quasi-classical, and the one who, by monitoring the measuring device, maintains it to be quasi-classical rather than to evolve into a superposition. So even in Standard QM we have this interplay between the dynamics and the projection. In our formulation, this interplay is visible, and even for the case when the system is expected to simply follow the evolution equation, this happens strictly only inside region $R^o \subset R$, and for other parts of the phase space the evolution involves a continuous projection, expressed in eq. (32).

V. DISCUSSION

In this section I discuss what is achieved by the formulation of Standard QM proposed here, and what are some open problems, and possible criticism or objections.

**Problem 1. Can Quantum Mechanics be formulated without relying on measuring devices and/or observers?**

The formulation of Standard QM proposed here does not use the Projection Postulate 1 & 2 as fundamental, replacing it with Postulate 6, which makes no reference to measuring devices. Nevertheless, we have seen in Sec. §III that it can be derived from them.

**Problem 2. What criterion should a quantum system satisfy to be considered classical?**

We argued for Criterion 1 as the criterion of classicality, based on [28]. While Omnès applied it mainly to define the projectors for the Consistent Histories Interpretation, it applies much more generally.

**Problem 3. Why does the world appear to be classical at the macro level?**

This question received here only a provisional answer: “because Postulate 4 requires quantum states to satisfy Criterion 1”. The task of this article was to remove the measuring device and the observer from the formulation of Standard QM. But a solution to Problem 3 was not provided here. A fully satisfactory solution requires most likely an extension of Standard QM, or a so-called “interpretation”. As explained in Sec. §I, even if we appeal to decoherence, it should be done in conjunction with an interpretation where the branching of the wavefunction is present, like Everett’s interpretation, Consistent Histories, or the de Broglie-Bohm interpretation. For one of its problems, that of the preferred basis, one may find that Criterion 1 is more appropriate.
**Problem 4.** Under what conditions does the projection occur?

In this formulation, the solution to Problem 4 is given by Postulate 6. In fact, following the discussion in Sec. §IV, we have seen that projection occurs to some extent continuously, the coarse-graining region being chosen according to the Born rule. This is related to Problem 3, so hopefully a solution will address both of them properly. Again, the formulation presented here only offers a provisional answer, and a full answer most likely requires an extension of Standard QM.

**Problem 5.** Can the quantum state be well defined at all times?

The answer to this question is yes. The Wigner function is, in the formulation proposed here, well defined at all moments of times, provided that the coarse-graining is well defined. While here we are not interested in the problem of ontology, this makes possible to assign an ontology to the wavefunction or to the Wigner function. The wavefunction is defined on the configuration space, while the Wigner function on the phase space. But if by “ontology” we mean something defined on the physical 3-dimensional space, it is possible to represent the wavefunction in terms of fields defined on the 3-dimensional space, and in [34] was given such a representation, which serves at least as a proof of concept. Therefore, since the formulation of Standard QM allows quantum states to be well defined at all times, it also allows an ontology on the 3-dimensional physical space, as shown by the construction made in [34].

There is a place on this formulation where additional verifications may be needed: the classicality quasi-projectors, because they

1. involve the existence of tails, and

2. they are only approximately projectors, even if this approximation is small [28].

The existence of tails mean that there is an “infinitesimally” small but non-vanishing probability that the system tunnels or fluctuates in a way that is unexpected in Classical Mechanics. This can be addressed similarly to the way the problem of tails is addressed in the GRW interpretation [15], or we can specify that the Wigner function is in a certain quasi-classical macro state when it is highly peaked in the corresponding coarse-graining region. But the fact that the classical macro level is approximate rather than exact was to be expected, given that Quantum Mechanics is not the same as Classical Mechanics, even if at the macro level it approximates it very well. In fact, as explained in Sec. §IV, without the slight overlapping of distinct functions $\Pi_R$ and $\Pi_R'$ around the boundaries it would not be possible for the quantum state to transition from one coarse-graining region to another.

Another point where things are not completely defined is the classical coarse-graining, which was not precisely defined here, but it was assumed that there is a well defined coarse-graining. This is a problem for the formulation proposed here to the extent that it is a problem for Classical Statistical Mechanics.

A possible development of the formulation presented here is to extend equation (32) to also include the transitions, by making it stochastic.

Even though the standard Projection Postulate was derived from the Postulates presented here, one may object that this new formulation is not merely a reformulation of Standard QM, but an entirely different theory. It is difficult to assess this, given the different interpretations of Standard QM and of the Copenhagen Interpretation itself.

However, I think the two formulations are pretty much the same, maybe the one proposed here is slightly more restrictive. For example, it simply does not allow macro *Schrödinger cats* or *Wigner friends* [38], due to the Postulate 4. This prevents the potential problems attributed to Standard QM recently by by Frauchiger and Renner [14], by appealing to thought experiments based on Wigner’s friend.

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