Pose Tracking Algorithm of an Endoscopic Surgery Robot Wrist

L Wang¹, H L Yin¹ and Q Meng²

¹Chinese-German Institute of Automatic Control Engineering, Tongji University
²Shanghai University of Electric Power, China

E-mail: leiwang@mail.tongji.edu.cn

Abstract. In recent two decades, more and more research on the endoscopic surgery has been carried out [2]. Most of the work focuses on the development of the robot in the field of robotics and the navigation of the surgery tools based on computer graphics. But the tracking and locating of the EndoWrist is also a very important aspect. This paper deals with the tracking algorithm of the EndoWrist’s pose (position and orientation). The linear tracking of the position is handled by the Kalman Filter. The quaternion-based nonlinear orientation tracking is implemented with the Extended Kalman Filter. The most innovative point of this paper is the parameterization of the motion model of the Extended Kalman Filter.

1. Introduction

In recent years, the research and development of robotically assisted minimally invasive surgery (RMIS) has been playing an increasingly important role in the surgical areas worldwide [1]. An endoscopic operation is a minimally invasive surgery. The realization of an endoscopic operation system includes many key technologies, such as medical robotics, medical images, telepresence system, application of Augmented Reality (AR) system and the motion tracking of the robot, and so on. This paper focuses on the tracking algorithm of the instrument tips called EndoWrist. The tracking of the EndoWrist pose runs separate Kalman filters for the position and orientation components. The tracking of the position is linear and is handled with discrete Kalman filter. To deal with the tracking of the orientation, it is more complicated. Comparing several methods of representation, we choose quaternion representation for the orientation, because quaternions are free from singularities, numerically well-conditioned. The dynamic character of the orientation is nonlinear and therefore the Extended Kalman filter is implemented for the tracking of the orientation. As the innovative point of this paper, the parameterization of the motion model of the Extended Kalman filter for the orientation is detailed introduced.

2. Tracking of position

Since the motions in x-y-z three directions are independent and the analysis-methods are identical in form, only the one-dimensional position tracking along the axis x is described. The EndoWrist has a non-zero mass and therefor the motion holds the physical rules of inertia:

\[ x = x_0 + \dot{x} \cdot t + \frac{1}{2} \ddot{x} \cdot t^2 \]  

(1)
That means that, in addition to position, also the current velocity and acceleration are important parameters for the system dynamics.

2.1. Introduction of Kalman Filter
Since 1960, the Kalman Filter has been the subject of extensive research and application [3-4]. Discreted Kalman Filter is a well-suited tool for the estimation of position. It addresses the general problem of trying to estimate the state of a dynamic process that is governed by the linear motion equation:

\[ X_k = A_{k,k-1}X_{k-1} + B_{k,k-1}u_{k-1} + w_{k-1}, \quad X \in R^n \quad \text{and} \quad p(w) \sim N(0,Q) \quad (2) \]

With a measurement that is:

\[ Z_k = H_{k,k-1}X_k + v_k, \quad Z \in R^m \quad \text{and} \quad p(v) \sim N(0,R) \quad (3) \]

The random variables \( w, v \) represent the process and measurement noise respectively. They are assumed to be independent of each other, white and with Gaussian probability distributions. In the system of this paper, the control input \( u \) is 0.

With the time update step and measurement update step according to the predictor-corrector principle, the state vector \( X \) is estimated [4].

2.2. Parameterization for the Kalman Filter of position estimation
In the discrete Kalman Filter for position tracking, the state vector consists of three elements: position, velocity and acceleration (for better readability, the index \( k \) and \( k-1 \) is dropped):

\[ X = [x, \dot{x}, \ddot{x}]^T. \]

From the motion equation (1), we have the state Transitions matrix \( A \):

\[
A = \begin{bmatrix}
1 & \Delta t & \frac{1}{2} \Delta t^2 \\
0 & 1 & \Delta t \\
0 & 0 & 1
\end{bmatrix}.
\]

Where \( \Delta t \) is the time interval.

The used A.R.T. tracking system in this work provides position measurement and therefore the measurement matrix is: \( H = [1 \ 0 \ 0] \). Process and measurement noise covariance matrix \( Q \) and \( R \) are set in this system manually.

3. Tracking of orientation

3.1. Representation of orientation
There are several methods for representation of the orientation, such as matrix representation, Euler Angles and quaternions. Compared to other methods, quaternions have the advantages that they are free from singularities, numerically well-conditioned and the operations product and interpolation can be performed easily. Quaternions are chosen here.

3.2. Introduction of quaternions
Quaternions were developed as a generalization of complex numbers in three dimensions. They consist of a scalar (real) and a vector (imaginary) part.

\[ q = \omega + ix + jy + kz, \quad \omega, x, y, z \in R \quad (4) \]

The basic quaternion algebra and the characters of quaternion are detailed analysed in many literatures. Here, only the points used in this research work are introduced.

A rotation by an angle \( \theta \) and an axis \( e \) can be constructed as an unit quaternion:

\[ q_{\theta, e} = [\cos(\frac{1}{2} \theta), \sin(\frac{1}{2} \theta) \cdot \vec{e}] \quad (5) \]
The twice rotations with the first angle $\theta_1$, the first axis $e_1$ and the second angle $\theta_2$, the second axis $e_2$ are equivalent to one rotation with the quaternion:

$$q = q_{\theta_1,e_2} \otimes q_{\theta_2,e_1}$$  \hspace{1cm} (6)

This formula is important for the derivation of the orientation dynamic model.

The quaternion’s derivative is computed by the following equation:

$$\dot{q} = \frac{1}{2}(q \otimes \omega)$$  \hspace{1cm} (7)

The multiplication between $q$ and $\omega$ is a quaternion standard multiplication, and $\omega$ is written as a quaternion with the real term set to zero and the imaginary terms set to the $\omega_x$, $\omega_y$ and $\omega_z$ angular velocity values respectively [7].

3.3. Extended Kalman Filter for the orientation

For the estimation of the orientation, the 7 by 1 state vector is used, which holds the orientation quaternion and angular velocity terms.

$$X = [q_w,q_x,q_y,q_z,\omega_x,\omega_y,\omega_z]$$

The nonlinearity of the quaternion’s derivative (equation(7)) results in the nonlinearity of the dynamic model of the orientation.

Kalman Filter is suited only for linear systems. Extended Kalman Filter handles nonlinear situations, by first-order linearizing around the current estimate of the state vector. The linearization is done by computing Jacobians of the nonlinear functions.

In the nonlinear situations, both the state transition and the measurements may be described not by matrices, but by arbitrary functions:

$$X_k = f(X_{k-1},u_{k-1},w_{k-1})$$  \hspace{1cm} (8)

$$Z_k = h(X_k,v_k)$$  \hspace{1cm} (9)

3.3.1. Motion model

Given of the previous state estimate and the $\Delta t$ of the time interval. The corresponding rotation quaternion is

$$\Delta q_{k-1} = [\cos(\frac{1}{2}\theta_{\Delta k-1}),\sin(\frac{1}{2}\theta_{\Delta k-1}) \cdot \hat{e}_{\Delta k-1}]$$  \hspace{1cm} (10)

From the rotation equivalence equation (6), we have

$$q_k = \Delta q_{k-1} \otimes q_{k-1}$$  \hspace{1cm} (11)

It is supposed that the motion is with constant angular velocity.

$$\omega_k = \omega_{k-1}$$  \hspace{1cm} (12)

With equation (11) and (12), the nonlinear state transition function $f$ of the motion model can be computed. With the linearization of the function $f$, we have the Jacobian matrix $A_{k,k-1}$, which describes the linear state transition (for better readability, the index $k$ and $k-1$ are dropped):

$$A =  
\begin{bmatrix}
1 & -\frac{1}{2}\omega_x & -\frac{1}{2}\omega_y & -\frac{1}{2}\omega_z & -\frac{1}{2}q_w & -\frac{1}{2}q_z & -\frac{1}{2}q_y \\
\frac{1}{2}\omega_x & 1 & \frac{1}{2}\omega_z & \frac{1}{2}\omega_y & \frac{1}{2}q_w & \frac{1}{2}q_y & \frac{1}{2}q_z \\
\frac{1}{2}\omega_y & \frac{1}{2}\omega_z & 1 & \frac{1}{2}\omega_x & \frac{1}{2}q_w & \frac{1}{2}q_z & \frac{1}{2}q_y \\
\frac{1}{2}\omega_z & \frac{1}{2}\omega_y & \frac{1}{2}\omega_x & 1 & \frac{1}{2}q_w & \frac{1}{2}q_z & \frac{1}{2}q_y \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 
\end{bmatrix}$$
3.3.2. Measurement Attributes

The output quaternion signals of the A.R.T. system is unit quaternion. Therefore the measurement function consists of a quaternion normalization:

\[ Z_k = \text{normalize}(q_k) \]  

The corresponding measurement Jacobian \( H_k \) (index \( k \) is omitted):

\[
H = \begin{bmatrix}
D - q_x^2 & -q_y q_x & -q_z q_x & -q_w q_x & 0 & 0 & 0 \\
-q_y q_x & D - q_y^2 & -q_z q_y & -q_w q_y & 0 & 0 & 0 \\
-q_z q_x & -q_z q_y & D - q_z^2 & -q_w q_z & 0 & 0 & 0 \\
-q_w q_x & -q_w q_y & -q_w q_z & D - q_w^2 & 0 & 0 & 0
\end{bmatrix}
\]

\[ (L \cdot D) \]

where

\[ D = q_x^2 + q_y^2 + q_z^2 + q_w^2, \quad L = \sqrt{D}. \]

4. Results

The research is performed with a scenario of the motion of real EndoWrist instead of the real robot. A dummy is built in form of the endoscope. The dummy is a 45cm long wooden body and the retro reflective A.R.T. markers are fixed on plastic threads at the upper end of the tool. The EndoWrist point is at the other tip of the tool. The calibration between the measured point and the EndoWrist is determined. With the pose of the measured point and the calibration, the pose of the EndoWrist can be computed. The figures show the results of the linear Kalman filter for the position and the extended Kalman filter for the orientation.

**Figure 1.** Result of the position tracking.

![Figure 1](image1)

**Figure 2.** Result of the orientation tracking.

The figures show the position signal along the \( x \)-axis, \( q_x \) for the orientation signals and the relevant filtered signals. The measured and the filtered curves almost overlap with each other. The difference
between them are printed at the right side. From the figures we can see that the filter errors of Kalman filter and Extended Kalman filter are much smaller than the pose signals. That is to say, Kalman filter can track the pose of the EndoWrist well. The filter errors depend on the choice of the noise parameters $Q$ and $R$.

**References**

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