PARTICLE FILTER APPROACH TO FAULT DETECTION AND ISOLATION IN NONLINEAR SYSTEMS

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ABSTRACT
This paper introduces the particle-filtering (PF) based framework for fault diagnosis in non-linear systems and noise and disturbances being Gaussian. In this paper, we use the sequential Monte Carlo filtering approach where the complete posterior distribution of the estimates are represented through samples or particles as opposed to the mean and covariance of an approximated Gaussian distribution. We compare the fault detection performance with that using the extended Kalman filtering and investigate the isolation performance on a nonlinear system.

Index Terms— Recursive Bayesian approach, Extended Kalman filter, particle filter, Parameter estimation, fault detection

1. INTRODUCTION
Problem of fault detection (FD) in dynamic systems has received considerable attention for many years due to increasingly complex systems and reliability[2]. Different types of approaches appearing in literature, as can be seen from a large number of survey papers[1],[2],[3],[4],[5],[6]. The problem of fault detection can be roughly divided into two major categories: First, we need to estimate the unknown and unmeasurable state variable of model and generate residuals on the basis of the available observations and a model of the system. Secondly, we need to decide on the occurrence of a fault based on the residuals generated[2]. For the stochastic systems, much of development in fault detection schemes has relied on the system being linear and disturbances being Gaussian. In such cases, the Kalman filter (KF) is known to be optimal and employed for state estimation. The innovations errors from the KF are used as the residuals, based on which statistical hypothesis tests are carried out for fault detection (see e.g. willsky,[4]and Darouach et al.,[5]). However, in comparison with linear systems, the literature addressing fault detection (FD) for nonlinear stochastic systems is not simple, the main reason being that the estimation of the state vector of a nonlinear stochastic system is not easy. Sub-optimal solutions use some form of approximation for non linear systems will additive Gaussian noise and disturbance by employing linearisation techniques[9]. In this case, the Kalman filter is usually replaced by the extended Kalman filter (EKF) as a residual generator. However, EKF is only an approximation method for non linear filtering, there are no general results to guarantee that such approximation will work in most case. When the nonlinearity systems is strong and non-Gaussian distributions, the performance of EKF will descend or even it becomes divergent[9][10][13]. for that the fault detection for nonlinear stochastic systems is known as a difficult problem and very few results are available[8][15]. General solution of the state estimation problem is described by the Bayesian recursive relations. The closed form solution of the Bayesian recursive relations is available for a few special cases (Gaussian or non-Gaussian). During the 1990s, the particle filter (PF) has dominated in recursive nonlinear state estimation, has attracted much attention and widely applied in many fields (see e.g. (Gordon and al.[7], Bolviken and al.[8], Doucet and al.[10], Benhmida and al.[9]). The PF solves the Bayesian recursive relations using Sequential Monte Carlo methods. These methods allow for a complete representation of the posteriori probability density function of the states, so that any statistical estimates, such as the minimum mean squared error estimate (MMSE) and the maximum a posteriori probabilities (MAP) can by easily computed. In year 2000 Kadirkamanathan and al.[2] to introduce Sequential Monte Carlo methods into field of fault detection and isolation (FDI). Different types of approaches appearing in literature of FDI problem[1][3] for solving of general nonlinear systems with known sets of possible faults. In this later, the particle filter, is combined with the innovation based fault detection techniques to develop a fault detection and isolation scheme. The paper is organized as follows: section 2 states the problem of interest. In section 3 we treat the Recursive Bayesian approach. An Innovation-based fault detection of the stochastic system and detected to update step of the Extended Kalman Filter in section 4. The particle filter based detection and isolation schemes are described in sections 5, 6 and 7. Finally, we illustrate in section 8 the simulation results on a highly non linear system witch demonstrate the effectiveness of the Particle Filter.

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2. PROBLEM FORMULATION

The problem of fault detection (FD) consists of making the decision on the presence or absence of faults in the monitored system. In this paper, the dynamics of the system considered is assumed known and given by discrete time nonlinear stochastic system given by the state equation (1) and the measurement equation (2):

\[ x_{k+1} = f_k(x_k, u_k, w_k) \]  
\[ y_k = h_k(x_k, v_k) \]

where \( x_k \in \mathbb{R}^n \) is the state vector of the system, \( u_k \in \mathbb{R}^m \) is the input vector, \( y_k \in \mathbb{R}^p \) is the vector of measurements, \( w_k \in \mathbb{R}^w \) and \( v_k \in \mathbb{R}^v \) are uncorrelated white noises sequences of zero-mean and covariance matrices are \( Q_k = E[w_k w_k^T] > 0 \) and \( R_k = E[v_k v_k^T] > 0 \), respectively. The functions \( f_k : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^w \rightarrow \mathbb{R}^n \) is the state transition function at time \( k \), and \( h_k : \mathbb{R}^n \times \mathbb{R}^w \rightarrow \mathbb{R}^p \) is the measurement function at time \( k \), can be both nonlinear or linear and assumed known.

Denoted by \( D_k = \{(u_i, y_i) | i = 1, ..., k\} \) the input/output data observed up to the time instant \( k \).

The type of faults of interest here are the failure type where the system parameter values to in new value reflected in a change in the state transition function, \( f_k(\cdot) \) at time \( k \) and the measurement function, \( h_k(\cdot) \) at time \( k \) such fault can be detected using the state observer approach. The idea is to enumerate.

3. RECURSIVE BAYESIAN APPROACH

This section gives a brief informal introduction to the basic recursive Bayesian approach. Recursive Bayesian state estimation of discrete-time nonlinear stochastic systems has been the subject of a considerable research interest over the last three decades. Two good surveys on nonlinear recursive estimation are provided by Sorenson, 1988[11] and Kulhavsy, 1996[12]. Another good reference for nonlinear recursive and nonrecursive estimation is Jazwinski, 1970 [13]). The idea of the Bayesian approach to state estimation problems involve the construction of the probability density function of the current state \( x_{k+1} \), based on the input/output data \( D_k \) observed up to instant \( k + 1 \). More precisely, to estimate the conditional probability density function \( p(x_{k+1} | D_{k+1}) \). In general, no accurate true estimator exists, the minimum mean squared error estimate (MMSE) and the maximum a posteriori probabilities (MAP) estimate for nonlinear stochastic systems, even if the noises are assumed to be Gaussian or non-Gaussian as follows[10],

\[ \hat{x}_{k+1 | k+1}^{\text{MMSE}} = E[x_{k+1} | D_{k+1}] \]  
\[ \hat{x}_{k+1 | k+1}^{\text{MAP}} = \arg \max_{x_{k+1}} p(x_{k+1} | D_{k+1}) \]  

Our aim is to estimate recursively in time the distribution \( p(x_{k+1} | D_k) \) and its associated features including \( p(x_{k+1} | D_{k+1}) \). From a Bayesian perspective the probability density functions \( p(x_{k+1} | D_k) \) may be obtained, recursively, in two stages: prediction and update.

Prediction: the prediction stage involves using the equation (1) to obtain the priori probability density function of the state at time \( k \) via Chapman-Kolmogorov equation,

\[ p(x_{k+1} | D_k) = \int_{\mathbb{R}^n} p(x_{k+1} | x_k) p(x_k | D_k) dx_k + 1 \]

Update: At time \( k + 1 \) a measurement \( D_{k+1} \) available, and this may be used to update the priori via Bayes rule

\[ p(x_{k+1} | D_{k+1}) = \frac{p(D_{k+1} | x_{k+1}) p(x_{k+1} | D_k)}{p(D_{k+1} | D_k)} \]

where the \( p(D_{k+1} | D_k) \) the normalizing constant

\[ p(D_{k+1} | D_k) = \int_{\mathbb{R}^n} p(D_{k+1} | x_{k+1}) p(x_{k+1} | D_k) dx_{k+1} \]

from the equations (5) and (7), We obtain the equation (6)

\[ p(x_{k+1} | D_{k+1}) = \frac{p(D_{k+1} | x_{k+1}) p(x_{k+1} | D_k)}{p(D_{k+1} | D_k)} \]

There is however a severe problem in Bayesian state estimation for nonlinear systems is that these equations (5) and (8) cannot be readily evaluated because they involve high-dimensional integrations[10]. The most important special case is when the system is linear, i.e., \( f_k(\cdot) \) and \( h_k(\cdot) \) are linear and assume that the noise and the initial state are Gaussian, i.e., \( w_k \sim N(0, Q_k) \), \( v_k \sim N(0, R_k) \). The solution is provided by the Kalman filter. We also describe how, when the analytic solution is intractable, Extended Kalman filter, and Particle filter to approximate the optimal Bayesian solution[17].

4. INNOVATION-BASED FD DESIGN

One of the main difficulties in fault detection of the stochastic system described by (1) and (2) is due to the presence of unknown and unmeasured state variables \( x \). The idea is to generate estimates of the states and the predicted outputs from these state estimates. The residuals or innovation from the output prediction are used in a measure which changes significantly under a failure type fault. For nonlinear Gaussian system, the states are estimated using an extended Kalman filter [13] an approximate sub-optimal estimate probability density functions described in section 3, obtained by linearisation, is recursively given according to be Gaussian, i.e.,

\[ p(x_k | D_k) \approx N(\hat{x}_{k | k}, P_{k | k}) \]

Aside from (9), we have the following correlations of the priori probability density functions described in section 3, obtained by linearisation, is recursively given according to be Gaussian, i.e.,

\[ p(x_{k+1} | D_k) \approx N(\hat{x}_{k+1 | k}, P_{k+1 | k}) \]

\[ p(x_{k+1} | D_{k+1}) \approx N(\hat{x}_{k+1 | k+1}, P_{k+1 | k+1}) \]
where.

Prediction:

\[ \hat{x}_{k+1|k} = f_k(\hat{x}_{k|k}, u_k) \]  (12)

\[ P_{k+1|k} = \Phi_k P_{k|k} \Phi_k^T + Q_k \]  (13)

Correction

\[ \hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(y_k - h_k(\hat{x}_{k+1|k})) \]  (14)

\[ K_{k+1} = P_{k+1|k} \Psi_k^T (\Psi_k P_{k+1|k} \Phi_k^T + R_k)^{-1} \]  (15)

\[ P_{k+1|k+1} = \Phi_k P_{k|k+1} \Phi_k^T + K_{k} \Psi_k P_{k+1|k} \]  (16)

where \( Q_k \) and \( R_k \) are the variance of the disturbance and noise respectively and \( \Phi_k = \frac{\partial f_k(x_k, u_k)}{\partial x_k} \).

The residual or the innovation is then,

\[ r_k = y_k - \hat{y}_k \]  (17)

where the predicted output based on the EKF state estimate is given by

\[ \hat{y}_k = h_k(\hat{x}_{k+1|k}) \]  (18)

It is well-known that, under fault free or normal operation, the innovations are zero mean Gaussian with covariance

\[ Q_k = \Psi_k^T P_{k+1|k} \Psi_k + Q_k \]  (19)

Any faults or changes in system dynamics can therefore be detected by a change in the weighted squared residual (WSR) measure

\[ l_k = r_k^T Q_k^{-1} r_k \]  (20)

This however can lead to false alarms occurring at a particular instant due to disturbances and noise and a more robust decision function for fault detection is the weighted sum squared residual (WSSR) for each particle (6) defined as follows:

\[ d_k = \sum_{j=k-W+1}^{k} l_j = \sum_{j=k-W+1}^{k} r_j^T Q_j^{-1} r_j \]  (21)

where W is the length of the sliding window within which the residual measure is summed. The window length W should be chosen in accordance with the requirement for detection time and the fault alarm is set at time k when the condition [4]

\[ d_k > \epsilon \]  (22)

is satisfied, \( \epsilon \) being the threshold. When using extended Kalman filter to estimate the states and hence \( Q_k \), the measure \( L_k \) will thus consist of fluctuations which can in turn lead to higher false alarm rates and also to faults not being detected.

5. SHORT INTRODUCTION TO THE BASIC PARTICLE FILTER ALGORITHM

This section gives a brief informal introduction to the basic particle filter algorithm(also known as Monte Carlo filter) [10][17], they effectively provide a good approximation to probability density function. The basic idea of Particle filter is to approximate the probability density function of \( x_k \) at each instant \( k \) with the sum of Dirac functions, and to make them evolve at each time instant based on the latest observed data \( D[4] \). For more complete and general presentations, the reader is referred to [7][10].

In order to develop the details of the algorithm, the particle filter at the initial instant \( k = 0 \), randomly \( N \) point in \( R^n \) following the probability density function \( p(0) \) of initial state vector. Denote these \( N \) points with the vectors \( x^0_0, \ldots, x^N_0 \). The probability density function at each particle \( r_k \) is rewritten as

\[ p(x_k|D_k) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x_k - x^i_k) \]  (23)

where \( \delta(.) \) is the Dirac -delta function(\( \delta(x) = 1 \) for \( x = 1 \) and \( \delta(x) = 0 \) otherwise)

Recursively, at instant \( k \geq 0 \), with

\[ p(x_k|D_k) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x_k - x^i_k) \]  (24)

already estimated, the distribution of \( x_{k+1} \) is first predicted with the state equation (1), leading to approximation \( p(x_{k+1}|D_k) \). For this purpose, each particle \( x^i_k \) for \( i = 1, \ldots, N \), is propagated following the state equation (1) to the position \( f_k(x^i_k, u_k) \) and perturbed by a random vector \( w^i_k \). Let us consider system (1) in each particle which is rewritten as

\[ x^i_{k+1} = f_k(x^i_k, u_k) + w^i_k \]

Then at each instant \( k + 1 \),

\[ p(x_{k+1}|D_k) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x_{k+1} - x^i_{k+1}) \]  (25)

Now the data observed at instant \( k + 1 \) is used to estimate \( p(x_{k+1}|D_{k+1}) \). According to the bayes rule (6), each particle \( x^i_{k+1} \) is weighted by its likelihood \( \beta^i_{k+1} \). Samples \( x^i_{k+1} \) are drawn from a (chosen) importance density function \( q(x^i_{k+1}|x_k, D_{k+1}) \), and the weights \( \beta^i_{k+1} \) are updated, using the current measurement \( D_{k+1}[10] \)

\[ \beta^i_{k+1} = \beta^i_{k} \frac{p(D_{k+1}|x^i_{k+1})p(x^i_{k+1}|x_k)}{q(x^i_{k+1}|x_k, D_{k+1})} \]  (26)

The weights are given by,

\[ \beta^i_{k+1} = \frac{\beta^i_{k+1}}{\sum_{j=1}^{N} \beta^j_{k+1}} \]  (27)

where \( \beta^i_{k+1} \) are the normalized weights. Here, we chose the importance density \( q(x^i_{k+1}|x_k, D_{k+1}) \) equal to the state transition probability density function \( p(x^i_{k+1}|x_k) \). The PF algorithm is summarized in Algorithm [10].
The Basic Particle Filter Algorithm

- **Step 1: Initialization**
  For $i = 1, ..., N$, draw the states $x_i^0$ from the prior $p(x_i^0)$. Assign weight $\beta^0_i = \frac{1}{N}$

  \[ p(x_0|D_0) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x_0 - x_i^0) \]

- **Step 2: Importance sampling**
  For $i = 1, ..., N$, sample $x_{k+1}^i \sim q(x_{k+1}^i|x_k^i, D_k)$.
  For $i = 1, ..., N$, evaluate the importance weights up to a normalizing constant:

  \[ \beta_{k+1}^i = \frac{\phi^i_{k+1} p(D_{k+1}|x_{k+1}^i) p(x_{k+1}^i|x_k^i)}{q(x_{k+1}^i|x_k^i, D_k)} \]

  For $i = 1, ..., N$, normalize the importance weights:

  \[ \hat{\beta}_{k+1}^i = \frac{\beta_{k+1}^i}{\sum_{j=1}^{N} \beta_{k+1}^j} \]

- **Step 3: resampling**
  Draw $N$ particles $x_{k+1}^i$ with high/low importance $\hat{\beta}_{k+1}^i$, respectively to obtain $N$ random samples $x_{k+1}^i$ approximately distributed according to $p(x_{k+1}|D_{k+1})$.
  For $i = 1, ..., N$, set $\beta_{k+1}^i = \hat{\beta}_{k+1}^i = \frac{1}{N}$

- **Step 4: Output prediction**
  The output of the algorithm is a set of sample that can be used to approximate the posterior distribution as follows

  \[ p(x_k|D_k) \approx \hat{p}(x_k|D_k) = \frac{1}{N} \sum_{i=1}^{N} \beta_{k+1}^i \delta(x_k - x_k^i) \]

6. PARTICLE FILTER FOR FAULT DETECTION

For the purpose of fault detection, the method proposed in this paper is to design several particle filters, each assuming a different subset of the possible faults formulated in system described by (1) and (2). The advantages of using the complete probability density function of the system state in a fault detection is bound to be superior than one which uses approximations, such as in the extended Kalman filter (EKF). Our approach is precisely to replace the EKF based estimation scheme by the particle filter, and the weighted squared residual (WSSR) measure by an appropriate innovations likelihood measure as the fault detection criteria [1][3]. The following algorithm describes the complete particle filter based fault detection scheme:

Particle filter based fault detection Algorithm

- **Step 1: State prediction**

7. AUGMENTED STATES MODEL FOR FAULT ISOLATION

In the previous section is a fault detection scheme which cannot readily be extended to fault isolation. The idea is to view the parameters as additional states, or more precisely, to augment the state vector with the parameter vector $\theta = [x_k^T, \theta^T]^T$ and rewrite the state space model in terms of $\zeta_k$. The following set of equations result:

\[ \begin{bmatrix} x_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} f_k(x_k, \theta_k; u_k) + w_k \\ \theta_{k+1} + \nu_k \end{bmatrix} \]  
\[ y_k = h_k(x_k, \theta_k) + v_k \]  

where, a random walk model $\theta_{k+1} = \theta_k + w_k$, with $w_k$ a zero mean Gaussian white noise, is introduced for parameter evolution to allow the exploration of the parameter space [2]. Given the above system representation (30) and (31), the particle filter outlined in this section 5, can be used to obtain the sample-based joint probability function density of the state $x$ and parameter vector $\theta$. Such an estimate based by particle filter algorithm is given by:

\[ z_{k+1}^{\text{MSE}}(k+1) = \sum_{j=1}^{N} \beta_{k+1}^j z_{k+1}^j \]
The estimate is essentially a weighted average of the particles representing the underlying distribution[8][6]. The parameter estimates $\hat{\theta}_k$ can be compared to the nominal values $\theta_0$ as a means for fault detection and its deviation $\tilde{\theta}_k = \theta_0 - \hat{\theta}_k$

8. NUMERICAL EXAMPLE

An example is presented in this section to illustrate the operation of the particle filter based fault detection and isolation proposed in this paper. The considered system is described by the following dynamical equations,

$$f_k(x_{1,k}, x_{2,k}, \theta_1, \theta_2, u_k) = \left[ \theta_1 x_{1,k} x_{2,k} + \theta_2 x_{2,k} + \frac{u_k}{\theta_1} x_{1,k} \right] + \frac{\theta_2}{2} u_k$$

$$h_k(x_{1,k}, x_{2,k}) = x_{1,k} - x_{2,k}$$

The initial joint state vector $\hat{z}_{0|0} = [0, 0, 0, 1]$. The measurement and process noise signals are both set to be Gaussian with covariance matrices,

$$R = 0.1$$

$$Q_k = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 \\
0 & 0 & 0.05 & 0 \\
0 & 0 & 0 & 0.05 \end{bmatrix}$$

The input and observation shown in Fig.1. The time index is incremented as $k = 0, ..., 200$. The nominal values of the parameters are $\theta_1 = 0.5$ and $\theta_2 = 0.3$. In the simulation, the length of the data window for WSSR calculation is $N_1 = 30$ and the number of particles in the particle filter is $N = 600$. The fault is simulated to occur at time $k = 100$ at which time the parameter $\theta_2$ jumps from a value of $0.5\theta_1$ to $0.5\theta_1$ while $\theta_1$ remains unchanged. The weighted sum squared residual (WSSR) results for the EKF is shown in Figure 2 and the negative log likelihood for the particle filter in Figure 3. The EKF based approach fails to detect the occurrence of the fault around $k = 100$ as evidenced by Figure 2 where there is no significant change in WSSR. The detection performance is unacceptable. On the other hand, the particle filter detects the fault at $k = 104$ at a threshold value of $\varepsilon = 2$. The change in log likelihood is quite pronounced following the onset of the fault. The combined fault detection isolation (FDI) scheme proposed in section 7 was also applied to the above example.

Figure 2. Weighted sum squared residual (WSSR)-EKF

Figure 3. Parameter $\theta_1, \theta_2$ estimates from the EKF and true values.

Figure 5 shows that the change in the parameter $\theta_2$ is tracked following an initial transient and the estimate for the other parameter $\theta_1$ hover around the nominal parameter. The PF successfully track the system parameter, while this is not seen for EKF (Figure 3). Thus we can safely conclude that the fault in the system is due to the change in the parameter $\theta_2$.

9. CONCLUSIONS

This paper considers the Particle filter based approach to fault detection and isolation scheme is developed. This approach is applicable to general non-linear systems with
The fault detection performance compared with that using the EKF. The results from simulation show that the detectability of the particle filtering approach is superior to the EKF based scheme, especially in the case where the system model is highly nonlinear. The fault isolation scheme is also shown to identify the parameter associated with the fault and the level of the fault.

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