Establishment of M1 multipolarity of a 6.5 $\mu_N^2$ resonance in $^{172}$Yb at $E_\gamma = 3.3$ MeV

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Two-step-cascade spectra in $^{172}$Yb have been measured after thermal neutron capture. They are compared to calculations based on experimental values of the level density and radiative strength function (RSF) obtained from the $^{173}$Yb($^3$He,$\alpha\gamma$)$^{172}$Yb reaction. The multipolarity of a 6.5(15) $\rho_N^2$ resonance at $E_\gamma = 3.3(1)$ MeV in the RSF is determined by this comparison.

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Excited nuclei decay often by a cascade of $\gamma$ rays. While the decay between discrete states is determined by the details of the nuclear wavefunctions, unresolved transitions are best described by statistical concepts like a continuous radiative strength function (RSF) and level density. The RSF (reviewed in [1]) provides the mean value of the decay probability for a given $\gamma$-ray energy $E_\gamma$. For hard $\gamma$ rays, ($\sim$ 7–20 MeV), the RSF is governed by the giant electric dipole resonance whose parameters are determined from photoabsorption [2]. The soft tail of the RSF has been investigated by a variety of methods, most notably by primary $\gamma$ rays [3]. Recently, systematic studies of the soft RSF have been performed at the Oslo Cyclotron Laboratory using a method based on sequential extraction. With this method it is possible to obtain the level density and RSF by a deconvolution of a set of primary $\gamma$ spectra from a range of excitation energies [4]. Total RSFs (summed over all multipolarities) of rare earth nuclei can be extracted for $B_\gamma > E_\gamma > 1$ MeV [5]. Their common, most striking feature is a resonance at $E_\gamma \sim 3$ MeV which is believed to be of dipole nature but whose electromagnetic character is unknown. It has been shown for all investigated rare earth nuclei that the total RSF is most readily decomposed into a sum of the Kadomskii-Markushev-Furman (KMF) $E1$ model [6], a spin-flip $M1$ model [7], and the aforementioned soft dipole resonance [3]. The knowledge of the character of this resonance is essential for its theoretical interpretation. Experimentally, it can be determined from a two-step-cascade (TSC) measurement [8].

The TSC method is based on the observation of decays from an initial state $i$ to a final state $f$ via one, and only one, intermediate level $m$ [9,10]. A convenient initial state is that formed in thermal or average resonance capture (ARC): the final state can be any low-lying discrete state. TSC spectra are determined by the branching ratios of the initial and intermediate states (expressed as ratios of partial to total widths $\Gamma$) and by the level density $\rho$ of intermediate states with spin and parity $J_m^\pi$:

$$I_{if}(E_1, E_2) = \sum_{XL,X'L,J_m^\pi} \frac{\Gamma_{XL}^{X'L}(E_1)}{\Gamma_i} \rho(E_m, J_m^\pi) \frac{\Gamma_{XL}^{X'L}(E_2)}{\Gamma_m} + \sum_{XL,X'L,J_m',J_m^\pi} \frac{\Gamma_{XL}^{X'L}(E_2)}{\Gamma_i} \rho(E_m', J_m'^\pi) \frac{\Gamma_{XL}^{X'L}(E_1)}{\Gamma_{m'}}.$$  (1)

The sums in Eq. (1) are restricted to give valid combinations of the level spins and parities and the transition multipolarities $XL$. They arise since one determines neither the ordering of the two $\gamma$ rays, nor the multiplicities of the transitions nor the spins and parities of the intermediate levels, hence one has to include all possibilities. The two transition energies are correlated by $E_1 + E_2 = E_i - E_f$, thus, TSC spectra can be expressed as spectra of one transition energy $E_i$ only. TSC spectra are symmetric around $E_{2\rm{sym}} = (E_i - E_f)/2$; integration over $E_\gamma$ yields twice the total TSC intensity $I_{if}$ if both $\gamma$ rays are counted in the spectra. The knowledge of the parities $\pi_i$ [11] and $\pi_f$ ensures that $I_{if}$ depends roughly speaking on the product of two RSFs around $E_{2\rm{sym}}$, i.e., $f_{E1}^2 + f_{M1}^2$ for $\pi_i = \pi_f$ and $2f_{E1}f_{M1}$ for $\pi_i \neq \pi_f$. $I_{if}$ depends also on the level density. This usually prevents drawing firm conclusions from TSC experiments alone [10]. A combined analysis of Oslo-type and TSC experiments, however, enables one to establish the sum and product, respectively, of all contributions to $f_{M1}$ and $f_{E1}$ at energies of the soft resonance, thus determining its character. For this goal, the partial widths of Eq. (1) are expressed via

$$\Gamma_{x\rightarrow y}(E_\gamma) = f_{XL}(E_\gamma)E_{2\gamma}^{2L+1}D_x$$  (2)

in terms of RSFs and level spacings $D_x$. Eq. (2) actually gives only the average value of the Porter-Thomas distributed partial widths [12]. The total width $\Gamma$ is the sum of all partial widths. Again, the sum is only the sum...
quantities for calculating TSC spectra are based on ex-
assumption and Eq. (3) have been verified from the dis-
equal numbers of positive and negative parity levels. This
σ
where
from average neutron resonance spacing (filled square) \[7\], and
level density from counting of discrete levels (jagged line) \[19\].
Right panel: total RSF (filled circles), fit to the data, and
decomposition into RSFs of different multipolarities (solid
lines). Inclusion of the soft resonance in the fit decreases
significantly improve the fit.

![Graphs showing level density and RSF decomposition](image)

FIG. 1: Left panel: total level density (filled circles), constant-
temperature extrapolation (solid line), level density at \(B_n\)
from average neutron resonance spacing (filled square) \[5\], and
level density from counting of discrete levels (jagged line) \[19\].

of mean values, however, the distribution of total widths
with many components is almost 6-like \[12\]. The level
density for a given spin and parity is calculated from the
total level density by \[13\]

\[
\rho(E_x, J^x) = \rho(E_x) \left( \frac{2 J_x + 1}{2 \sigma^2} \right) \exp \left( \frac{-(J_x + 1/2)^2}{2 \sigma^2} \right),
\]

(3)

where \(\sigma\) is the spin cut-off parameter, and we assume

equal numbers of positive and negative parity levels. This

assumption and Eq. 3 have been verified from the dis-

crete level schemes of rare earth nuclei \[14\]. Thus, all

quantities for calculating TSC spectra are based on ex-

perimental data.

The combined analysis is applied to the nucleus \(^{172}\)Yb

which has been investigated by the \(^{173}\)Yb(\(^3\)He,\(\alpha\gamma\))\(^{172}\)Yb

reaction in Oslo and by the \(^{171}\)Yb(\(n,\gamma\))\(^{172}\)Yb reaction

at the Lujan Center of the Los Alamos Neutron Science

Center (LANSCE). The Oslo data have been reported in

Fig. 1. The parameters for the \(^{172}\)Yb experiment, we used

\(^{71}\)Ge and \(^{29}\)Si are due to \(n\)-capture in the detector and in the glass ampoule,

respectively. SE and DE stands for single and double escape

peaks, respectively. Lower panel: TSC spectra to the \(2^+_1\)

state. The slight asymmetry is due to the energy-dependent

resolution of the detectors.

![Graphs showing energy-summed coincidence spectra](image)

FIG. 2: Upper panel: energy-summed coincidence spectrum

from the \(^{171}\)Yb(\(n,\gamma\))\(^{172}\)Yb reaction. Peaks are labeled by

the energy of the final state. Peaks denoted by \(^{71}\)Ge and \(^{29}\)Si

are due to \(n\)-capture in the detector and in the glass ampoule,

respectively. SE and DE stands for single and double escape

peaks, respectively. Lower panel: TSC spectra to the \(2^+_1\)

state. The slight asymmetry is due to the energy-dependent

resolution of the detectors.

KMF \(E1\) model, a spin-flip \(M1\) model, and a soft dipole

resonance \[3\]. Here, we have improved on the normal-

ization of the level density and the RSF and included an

isoscalar Lorentzian \(E2\) model \[13\] giving

\[
f_{tot} = K(f_{E1} + f_{M1}) + E^2 f_{E2} + f_{soft},
\]

(4)

where \(K\) is a scaling factor of the order of one. Since

quadrupole transitions populate levels within a broader

spin interval than dipole transitions, Eq. 4 is of an ap-

proximative nature. Given the weakness of quadrupole

transitions and the level of experimental uncertainties,

however, this approximation is believed to be sufficient.

The improved data, the fit to the total RSF, and its

decomposition into different multipolarities are given in

Fig. 3. The parameters for the \(E1\) RSF are taken from

\[3\], those for the \(M1\) and \(E2\) RSFs from \[6\], where we

use the \(f_{E1}/f_{M1}\) systematics at \(~7\) MeV giving values

in agreement with ARC work \[20\]. The fit parameters

are: the constant temperature of the KMF model

\(T = 0.34(3)\) MeV, the normalization coefficient \(K =

1.7(1)\), and the three parameters of the soft resonance

\(E = 3.3(1)\) MeV, \(\Gamma = 1.2(3)\) MeV, and \(\sigma = 0.49(5)\) mb

\[21\].

For the \(^{171}\)Yb(\(n,\gamma\))\(^{172}\)Yb experiment, we used \(~1\) g

of enriched, dry \(Yb_2O_3\) powder encapsulated in a glass
ampoule, mounted in an evacuated beam tube and irradiated by collimated neutrons with a time-averaged flux of $\sim 4 \times 10^5$ neutrons/cm$^2$/s at $\sim 20$ m from the thermal moderator. $\gamma$ rays were detected by two 80% and one shielded and segmented $\sim 200\%$ closer Ge(HP) detector, placed at $\sim 12$ cm from the target in a geometry to minimize angular correlation effects and contributions from higher multiplicity cascades. Single and coincident $\gamma$ rays were recorded simultaneously. The experiment ran for $\sim 150$ h yielding $\sim 10^7$ coincidences. The relative detector efficiencies from 1–9 MeV were determined by using new data and a 4% and (ii) by effectively estimating the number of neutron captures during the experiment from secondary singles and (ii) by effectively estimating the number of neutron captures during the experiment from secondary singles.

\[ R = \frac{\sigma_{n,\gamma}^{th}(0^-)}{\sigma_{n,\gamma}^{th}}. \]

These calculations show, however, that only the TSC intensity to the 0$^+_1$ state has a strong dependence on this ratio. Total experimental and calculated TSC intensities are shown in the left panels of Fig. 3. The calculations assuming E1 for the soft resonance do not reproduce the experimental intensities consistently for any value of $R$. Good agreement is achieved assuming M1 with the additional condition of $R \sim 0.4$ for the 0$^+_1$ final state. However, it has to be emphasized that the conclusion of an M1 multipolarity for the soft resonance can be established from the TSC intensities to the 2$^+_1$, 1$^-_1$, and 2$^-_1$ states independently, irrespective of the value of $R$. Possible systematic uncertainties in the absolute normalization cannot change this conclusion, since in the case of the final state 2$^+_1$, one would need a decrease while at the same time, for the 1$^-_1$ final state one would need an increase in the experimental TSC intensities in order to accommodate the E1 hypothesis. The combined $\chi^2_{red}$ for all four TSC intensities as function of $R$ is also given.

The M1 hypothesis yields
the global minimum for $R = 0.4 \pm 0.25$ with $\chi^2_{\text{red}} = 0.1$ whereas the minimal $\chi^2_{\text{red}}$ for the $E1$ hypothesis is $\sim 2.7$ for $R \sim 0.8$. Finally, we show the TSC spectra to two final states compared to calculations using the M1 hypothesis at $R = 0.4$ and the $E1$ hypothesis at $R = 0.8$. No further conclusions have been drawn from this comparison, however.

The integrated strength of the soft resonance is expressed as

$$B(M1 \uparrow) = \frac{9\hbar c}{32\pi^2} \left(\frac{\sigma\Gamma}{E}\right)_{\text{soft}}$$

(5)

giving a value of 6.5(15) $\mu_N^2$ which is entirely determined from the Oslo-type experiment after M1 multipolarity has been established. This is in agreement with the sum-rule approach for soft, orbital M1 strength \cite{12} but is more than twice the strength reported from nuclear resonance fluorescence (NRF) experiments \cite{26}. However, in \cite{10, 28} several limitations in determining $B(M1 \uparrow)$ using NRF are discussed, all resulting in possible underestimation. Concerns are that (i) too few 1$^+$ levels are observed in NRF experiments compared to level density estimates, (ii) the assumption in NRF experiments that the total radiative width equals the sum of the partial radiative widths for transitions to the ground state and the first excited state is not fulfilled, and (iii) the excitation-energy coverage is insufficient. Also in \cite{10} a soft resonance with $B(M1 \uparrow) \sim 7\mu_N^2$ is required in order to reproduce TSC spectra in $^{163}$Dy.

In conclusion, the soft resonance found in the RSF of $^{172}$Yb in Oslo-type experiments has been determined to be of M1 multipolarity by an auxiliary TSC measurement. The strength of the M1 resonance is $B(M1 \uparrow) = 6.5(15) \mu_N^2$ which is entirely determined by the former experiment. This value agrees with a sum-rule approach for orbital strength, but is more than twice the value reported by NRF experiments. Assuming M1 multipolarity for similar soft resonances in other rare earth nuclei gives consistent strengths of $\sim 6 \mu_N^2$ for various even and odd Dy, Er, and Yb nuclei and reduced strengths of $\sim 3 \mu_N^2$ for the more spherical Sm nuclei \cite{28}. The centroids of the resonances increase weakly with mass number.

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