Generalized Additive Poisson Models for Quantifying Geological Factors Effect on The Earthquake Risk Mapping

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Abstract. Cluster-type point process models are the popular model for modeling the arrangement of locations of earthquake occurrences. When a spatial trend is presence due to e.g. geological factors, the log-linear intensity of the point process is often considered to exploit inhomogeneity due to such factors. However, this could be a major drawback, especially in seismology when the relation between the intensity of earthquake occurrences and environmental covariates is not log-linear. In this paper, we consider the Cauchy cluster process with a log-additive intensity model to quantify two effects in modeling the distribution of locations of major earthquakes in Sulawesi-Maluku: (1) spatial trend due to geological covariates such as subduction zones, faults, and volcanoes and (2) clustering effect due to seismic activities. The Cauchy cluster process could detect the clustering effect even when the aftershocks are extremely distant to the mainshocks while log-additive intensity is a more flexible model to study inhomogeneity due to the environment. To estimate the parameters, we apply two-step estimation procedure, in the first step, we estimate the regression parameter corresponding to effects of the geological variables by maximum composite likelihood by involving penalized iteratively re-weighted least squares (PIRLS) technique, and in the second step, we obtain the cluster estimates by maximum second order composite likelihood. The results show that the active faults and volcanoes are significant covariates that trigger earthquakes, the estimated mainshock intensity is around 78, and the aftershocks are distributed around it with a distance of 15.2 km due to mainshock activity.

1. Introduction
Earthquakes can occur when rocks in the earth’s crust are subjected to tremendous pressure by the movement of the plates that form the basis of continents. Indonesia is a country that is often hit by earthquakes, due to Indonesia’s tectonic conditions which are located at the confluence of the world’s major plates such as the Eurasian Plate, the Pacific Plate, and the Indo-Australian Plate [1]. One of the sources of the earthquake that has been identified is an active subduction zone in the western to eastern parts of Indonesia [1]. According to data recorded by Indonesian Meteorology Climatology Geophysics Council [2], 49.27% of earthquake events in Indonesia occurred in Sulawesi and Maluku. The high level of earthquake vulnerability in these areas needs special attention because earthquake disasters have the potential to cause large amounts of damage, casualties, and losses. Thus, mitigation efforts need to be done to minimize the impact of the earthquake disaster. One of these efforts is to create a map of the earthquake-prone location in Sulawesi and Maluku surrounding areas using a statistical method, namely the spatial point process.
Statistical methods for modeling earthquake events based on the spatial point process have become a popular method for modeling earthquake intensity (see, e.g., [3-5]). Modeling of earthquake intensity using methods on the spatial point process has been carried out by several researchers [7-9]. To have more comprehensive understanding in the spatial distribution of the locations of earthquake occurrences, it is a major importance to involve geological factors in modeling to understand the relationship between earthquakes and their causes. This has been explained by [10] which involved fault geological factors and subduction zones to model the intensity of earthquakes using the Strauss point process. Furthermore, [11] analyzed the relation between earthquake events and geological variables volcano using a hybrid of Gibbs point process models. Thus, in this study, we will involve geological factors in the form of faults, subduction zones, and volcano to model earthquake intensity.

Several studies previously described have modeled the relationship between earthquake intensity and geological factors using a log-linear model, even though the modeling can be formed into more flexible ones. Research for more flexible modeling on spatial point pattern data has been conducted by [12] using Generalized Additive Models. Other research has also been conducted by [13] for spatial point pattern data involving environmental factors by modeling using Generalized Additive Models and showing that the GAM model can describe the effect of environmental variables on intensity functions better than the log-linear model. Furthermore, [13] also shows that the log-linear model is unable to model the dependency of the intensity function on environmental variables, so the log-additive model is used as an alternative to the log-linear model.

In the spatial point process, the Poisson process is a basis for the formation of the other models. However, it has an assumption that must be fulfilled that is commonly called complete spatial randomness (CSR) or homogenous Poisson point process, especially in earthquake events, the point pattern is usually not fulfilled a CSR, but a clustered form in which the point tend to group in a certain area or also called inhomogeneous Poisson point process [14]. Therefore, we propose Generalized Additive Models to capture the heterogeneity effect due to geological factors and consider the Cauchy cluster process model to capture the cluster effect. To estimate the parameters, we apply two-step estimation procedure, in the first step we estimate the regression parameter corresponding to effects of the geological variables by maximum composite likelihood using penalized iteratively re-weighted least squares (PIRLS) technique, and in the second step we obtain the cluster estimates by maximum second order composite likelihood.

2. Methodology

2.1. Spatial point process

A point process is a random process in a continuous space \( S \), with \( S \subseteq \mathbb{R}^2 \) and in this paper it is denoted by \( X \), in practice, we observe only the points contained in a bounded observation window \( B \subseteq S \), e.g. \( X = \{X_1, \ldots, X_n\} \) where \( X_1, \ldots, X_n \in B \) and \( n \) is the observed number of points in \( B \) [15]. Given \( \lambda : B \rightarrow [0, \infty) \) as intensity function of point process \( X \), then

\[
\mu(B) = \int_B \lambda(u; \theta, z(u)) \, du,
\]

where \( \mu(B) \) is an expected value of random variable \( N(B) \), \( N(B) \) is the number of points that fall in \( B \), \( \theta \) represent unknown parameters, \( z(u) = [z_1(u) \ldots z_p(u)]^T \) represent \( p \)-covariates, \( u \) represents the coordinates of the location of a point on \( B \) which consists of longitude and latitude, and \( \lambda(u; \theta, z(u)) \) is the intensity, quantifying the propensity of occurrence at any given point. The second-order intensity of spatial point process is \( \lambda_{(2)} \) and the pair correlation function is defined below

\[
g(r) = \frac{\lambda_{(2)}(u, v)}{\lambda(u) \lambda(v)}
\]

where \( u \in B \) and \( v \in B \) are a pair of points separated by a distance of \( r \), so that \( g(r) \) is defined as the probability of observing it. Also, there are popular methods that can be used to calculate the spatial correlation between points, namely K-function which is defined as follows.
\[
\bar{K}(r) = \frac{1}{W|B|} \sum_{u,v \in B \cap S} \frac{1[\|u - v\| \leq r]}{\hat{\lambda}(u)\hat{\lambda}(v)} e(u, v, r)
\]

where \( W = \frac{1}{|B|} \sum_{u \in x \cap B} \hat{\lambda}(u)^{-1} \) and \( |B| \) as the area of observation window [16].

2.2. Inhomogeneous Poisson point process
The Poisson process can be used as a spatial point process approach if it assumes that there is no interaction between one object and another. In general, a point process \( X \) is a Poisson point process on \( S \subseteq \mathbb{R}^2 \) with intensity function \( \lambda(.) \) if:

- for any \( B \subseteq S \), \( N(B) \) follows the Poisson distribution with mean \( \mu(B) \); and
- for any \( n \in \mathbb{N} \), conditional on \( N(B) = n \), the \( n \) points are independent and identically distributed (i.i.d) with probability density \( f(u) = \frac{\lambda(u)}{\mu(B)} \) [15].

Inhomogeneous Poisson point process has intensity function \( \lambda(u) \) which is not constant and depends on \( u \), the characteristics of the inhomogeneous Poisson point process are:

- the expected number of points falling in \( B \) is \( \int_B \lambda(u)du \); and
- given that \( N(B) = n \), the \( n \) points are i.i.d with \( f(u) = \frac{\lambda(u)}{\mu(B)} \) where \( \mu = \int_B \lambda(u)du \) [16].

The second-order intensity for the Poisson point process is \( \lambda_2(u, v) = \lambda(u)\lambda(v) \), so that pair correlation function for the Poisson point process is \( g_2(u, v) = 1 \). Inference for the Poisson point process model is then based on the log-likelihood for \( \theta \) [16]

\[
\log L(u; \theta) = \sum_{i=1}^{n} \log \lambda(u_i; \theta) - \int_B \lambda(u; \theta)du \quad (4)
\]

In this paper, intensity function \( \lambda(u; \theta) \) follows the log-additive form, in which the details will be explained in the following subsection.

2.3. Generalized additive models
Generalized additive models (GAMs) [17] is a generalized linear models (GLMs) with a linear predictor including a sum of smooth functions of covariates [18]. Thanks to Berman-Turner approximation [16], the Poisson log-likelihood (4) can be linked to the log-likelihood function of GAM with Poisson log-additive model, that is

\[
\log L(\theta) \approx \sum_{j=1}^{n+d} \left( I_j \ln \lambda(u_j; \theta) - \lambda(u_j; \theta)w_j \right) \quad (5)
\]

where \( d \) is the number of dummy points, \( w_j > 0 \) is quadrature weight, and \( I_j \) has a value of 1 if \( u_j \) are data points, and 0 if \( u_j \) are dummy points.

The intensity function is of the form

\[
\lambda(u; \theta) = \exp \left( \theta + \sum_{j=1}^{p} f_j(z_j(u); \theta) \right) \quad (5)
\]

where \( f_j(.) \) is the smooth function represented by regressions spline, and \( z_j(u) \) is covariates or geological factors. A basis representation for \( f_j(.) \) is

\[
f_j(z(u); \theta) = \sum_{i=1}^{k_j} \theta_{i+1+k_{j-1}}b_{ij}(z(u); \theta) \quad (6)
\]

where \( k_j \) is number of basis function, \( \theta \) is the parameter to be estimated, and \( b_{ij}(.) \) is basis functions.

The penalties measure for parameter estimation is:
Pair correlation function and K with process. The mainshocks with parameters of the distribution of aftershock points around Equation (8) is in log additive form, where is a smooth function due to geological factors, so that inhomogeneous Cauchy cluster process could be considered as a model for the earthquake’s intensity, then has intensity function,

\[
\lambda_c(u; \theta) = \exp\left(\theta + \sum_{j=1}^{p} f_j(z_j(u); \theta)\right) k(u - c; \omega)
\]  

(8)

and is inhomogeneous Cauchy cluster process with mainshock process and aftershock processes is probability density function for Cauchy cluster process or probability density function for the distribution of aftershock points around the mainshocks with parameter \(\omega > 0\), and \(X = \bigcup_{c \in C} X_c\) is inhomogeneous Cauchy cluster process with mainshock process and aftershock processes \(X_c\) (see, [20-21]). So that, the intensity model for point process \(X\) is

\[
\lambda(u; \theta) = \kappa \exp\left(\theta + \sum_{j=1}^{p} f_j(z_j(u); \theta)\right) = \exp\left(\theta_0 + \sum_{j=1}^{p} f_j(z_j(u); \theta)\right)
\]  

(9)

with \(\theta_0 = \theta + \log \kappa\) is an intercept.

Cauchy cluster process has probability density function [22], that is

\[
k(u; \omega) = \frac{1}{2\pi\omega^2} \left(1 + \frac{||u||^2}{\omega^2}\right)^{-\frac{3}{2}}
\]  

(10)

Pair correlation function and K-function of Cauchy cluster process are,

\[
g(u; v; \psi) = 1 + \frac{1}{8\pi\kappa\omega^2} \left(1 + \frac{||u - v||^2}{4\omega^2}\right)^{-\frac{3}{2}}
\]  

(11)

\[
\int [f''(z(u); \theta)]^2 dz(u) = \theta^T S \theta
\]  

(7)

where \(S\) is a positive semi-definite matrix [18], for cubic regression spline basis function, the positive semi-definite matrix \(S = D^T B^{-1} D\), matrix elements for \(D\) and \(B\) are defined in Table 1 [18].

| Table 1. Non-zero matrix elements for \(D\) and \(B\). |
|------------------------------------------------------|
| Matrix | Matrix element |
|--------|----------------|
| \(D\)  | \(D_{i,i} = \frac{1}{h_i}\), \(D_{i,i+1} = -\frac{1}{h_i} + \frac{1}{h_{i+1}}\), \(D_{i,i+2} = 1/h_{i+1}\) |
| \(B\)  | \(B_{i,i} = \frac{h_i + h_{i+1}}{3}\), \(B_{i,i+1} = \frac{h_{i+1}}{6}\), \(B_{i+1,i} = \frac{h_{i+1}}{6}\) |

and \(h_i = (z_{i+1}(u)) - (z_i(u)), \) for \(i = 1, \ldots, k_j - 2\).
where $\kappa$ is the intensity of mainshocks process, $\omega$ represents how far the aftershocks are to the mainshocks, and parameter vector $\psi = (\kappa, \omega)^T$ are parameters of spatial interactions between earthquake events and can be represented with pair correlation function or K-function.

2.5. Parameter estimation

To estimate the parameter, two step estimation is performed, where in the first step (Section 2.5.1) $\theta$ is estimated by linking the first order composite likelihood (4) with GAMs (Section 2.3) and in the second step (Section 2.5.2), we estimate $\psi = (\kappa, \omega)^T$ by maximizing the second order composite likelihood.

2.5.1. Estimation of $\theta$

Estimating of the parameters $\theta$ is done by maximizing the penalized log-likelihood function,

$$
\ell_p(\theta) = \log L(\theta) - \frac{1}{2} \sum_{j=1}^{p} \rho_j \theta^T S_j \theta
$$

where $\rho_j$ is the smoothing parameter selected with minimum Generalized Cross Validation (GCV) $v_g = \frac{n \sum_{i} \{ n_i (\tilde{\theta}_i - \hat{\theta}_i)^2 \}}{[n-tr(\lambda)]^2}$ where $A = Y(Y^T Y + S)^{-1} Y^T$ and $Y \equiv b(z(u))$. Estimating parameter $\theta$ by maximizing equation (14) namely using PIRLS [18], given $z_i^{[0]} = g’(\tilde{\mu}_i^{[0]}) \left( N_i(B) - \hat{\mu}_i^{[0]} \right) + \hat{\eta}_i^{[0]}$. $w_i^{[0]} = \frac{1}{\{v(\tilde{\mu}_i^{[0]})g’(\tilde{\mu}_i^{[0]})\}}$ and link function $\eta = \ln \lambda(u; \theta) = \theta_0 + \sum_{j=1}^{p} f_j(z_j(u))$, so that

$$
\hat{\theta} = \arg \max_{\theta} \left( \| \sqrt{W} Z - \sqrt{W} Y \theta \| + \rho \theta^T S \theta \right)
$$

2.5.2. Estimation of $\psi$

Fitting clustered point process models using 2nd order composite log-likelihood was proposed by [23],

$$
\log CL(\psi) = \sum_{u,v \in X} w(u,v) \left[ \log \lambda_2(u,v; \psi) - \log \int_B w(u,v) \lambda_2(u,v; \psi) dudv \right]
$$

where $w$ is a weight function $w(u,v) = 1 \{|u - v| \leq R\}$, and $R > 0$ is an upper limit on the correlation distance of the model. The first derivative of $\log CL(\psi)$ with respect to $\psi$ is given by [16]

$$
\frac{\partial}{\partial \psi} \log CL(\psi) = \sum_{u,v} w(u,v) \frac{\kappa_2(u,v; \psi)}{\lambda_2(u,v; \psi)} - \sum_{u,v} w(u,v) \frac{\langle \kappa_2, w \rangle}{\langle \lambda_2, w \rangle}
$$

with $\kappa_2(u,v; \psi) = \frac{\partial}{\partial \psi} \lambda_2(u,v; \psi)$, and $f(w) = \int_B \int_B w(u,v) f(u,v; \psi) dudv$.

3. Study area and variable description

Sulawesi and Maluku are regions with a meeting zone of multiple plates in Indonesia, known as the triple intersection which can cause a high danger of an earthquake. Earthquakes in Sulawesi-Maluku usually occur in the area around subduction, faults, and volcanoes. Figure 1 shows that there are some major subduction zones (black line) for example Northern Sulawesi, East Molucca, West Molucca, and Wetar Back Arc, according to [24] the main earthquake usually occurs around the subduction zone. There are volcanoes lined up around East Molucca subduction and West Molucca subduction which are named Sangihe Volcanic Arc and Halmahera Volcanic Arc, based on [25] that earthquake also often occur around volcanoes (red triangle). There were 48 faults (grey line) in Sulawesi-Maluku [1] some of which caused the main earthquake, namely Palu Koro and Matano faults, several earthquakes sometimes
occur around the faults [26]. In this paper, our window area is $|B| = [130.98; 150.96] \times [-8.33; 6.33] \ (100 \ km^2)$.

Figure 1. Location of 1415 points of earthquake occurrences (white dot) Sulawesi and Maluku earthquake) with $M \geq 5$ SR during 2009-2019 along with locations of subduction zone (black line), fault (grey line), and volcano (triangle).

Table 2. Covariates description.

| Variable | Description |
|----------|-------------|
| $z_1(u)$ | The distances of coordinate points of the earthquake to the nearest fault |
| $z_2(u)$ | The distances of coordinate points of the earthquake to the nearest subduction |
| $z_3(u)$ | The distances of coordinate points of the earthquake to the nearest volcano |

In this paper, we investigate the inhomogeneity due to geological variables (covariates) and the clustering effect. The calculation of parameter estimates is computationally carried out using the spatstat and mgcv package on R to building the generalized additive Poisson models of earthquake intensity.

4. Results
4.1. Exploratory data analysis

To determine the pattern of observed earthquake location points including the homogeneous or inhomogeneous point process, a homogeneity test is carried out. The homogeneity test can be done using the Chi-Squared test with a quadrature approach, in which the observation window is divided into several square-shaped sections (grids) and tests the Poisson process assumption whether the number of points on each grid is significantly different. The null hypothesis is that the intensity is homogeneous. The observation window is divided into $10 \times 10$ grids, the statistical value of the Chi-Square test obtained is $4657.4$ with 99 degrees of freedom, greater than a critical point at $\alpha = 0.05$, that is $127.69$. 


In addition, the result of the $p-value < 2.2e-16$ is smaller than $\alpha = 0.05$, so that, we rejected the null hypothesis. Thus, the intensity is inhomogeneous.

It is necessary to know the point pattern of an earthquake occurrence to form an earthquake intensity model. The K-function can be used to analyze the spatial correlation in the point pattern data, the results of the K-function for earthquake location data in Sulawesi and Maluku depicted in Figure 2. Figure 2 shows that the locations of the earthquake in Sulawesi and Maluku are not independent, but there is an influence between points. This is indicated by the correction line (Kinhom bord) which is above the correction line of the Poisson process (Kpois) model so that the distribution of the points of the earthquake location tends to be clustered.

![Figure 2. Inhomogeneous K-function for testing the assumption of spatial independence between earthquake points in Sulawesi and Maluku during 2009-2019.](image)

4.2. Estimation parameter result

Estimating the parameters of the earthquake intensity model in Sulawesi and Maluku is computationally carried out using the R package `spatstat`. Geological variables are involved in modeling the earthquake intensity in Sulawesi and Maluku, and the earthquake point pattern follows the inhomogeneous Cauchy cluster process based on subsection 4.1. We assume that the earthquake intensity model is as follows

$$
\lambda(u; \theta) = \exp \left( \theta_0 + f_1(z_1(u)) + f_2(z_2(u)) + f_3(z_3(u)) \right)
$$

(17)

with $\theta_0 = \theta + \log \kappa$, and covariate $z_1(u), z_2(u), z_3(u)$ as described in Table 1. We consider two-step estimation technique employing the `kppm` function of the `spatstat` and the number of knots for cubic regression spline selected based on the minimum GCV by the `mgcv` package is 10. The output of estimation results on R shows, first, most of the distribution of the major earthquakes in Sulawesi-Maluku is not significantly caused by a fault with a significance level of 5%. The fault may not trigger major earthquakes because the majority of the major earthquakes occur in the area around subduction zones and volcanoes as shown in Figure 1. Second, the resulting clustering estimators $\hat{\theta} = (\hat{\kappa}, \hat{\omega})^T$ for the Cauchy cluster process is $\hat{\theta} = (0.269,0.152)^T$, so we could interpret that the aftershocks are distributed around $\hat{\kappa} \times |B| = 78$ estimated mainshocks with scale 15.2 km.

The resulting parameter estimators with 9 knots are presented in Table 3, to quantify the effect of geological variables. In Table 3, we show the parameters that trigger major earthquakes, and the proposed model is...
\[
\lambda(u; \theta) = \exp \left( \theta_0 + f_2(z_2(u)) + f_3(z_3(u)) \right)
\]

with \( \theta_0 = \theta + \log \kappa \).

(18)

Table 3. The resulting parameter estimation that trigger major earthquakes.

| Parameter | Estimate | S.E. | CI 2.5% | CI 97.5% |
|-----------|----------|------|---------|----------|
| \( \theta_0 \) | 0.79 | 0.16 | 0.47 | 1.11 |
| \( \theta_{21} \) | 0.58 | 0.26 | 0.08 | 1.08 |
| \( \theta_{22} \) | 0.50 | 0.25 | 0.01 | 1.00 |
| \( \theta_{23} \) | 0.55 | 0.28 | 0.01 | 1.09 |
| \( \theta_{24} \) | 0.62 | 0.29 | 0.06 | 1.19 |
| \( \theta_{25} \) | -0.70 | 0.36 | -1.40 | -0.01 |
| \( \theta_{26} \) | -0.70 | 0.33 | -1.34 | -0.06 |
| \( \theta_{27} \) | 0.52 | 0.48 | -0.42 | 1.45 |
| \( \theta_{28} \) | -14.94 | 5.60 | -25.91 | -3.98 |
| \( \theta_{31} \) | 0.48 | 0.39 | -0.28 | 1.23 |
| \( \theta_{32} \) | 1.18 | 0.32 | 0.55 | 1.81 |
| \( \theta_{33} \) | -0.0002 | 0.37 | -0.73 | 0.73 |
| \( \theta_{34} \) | -0.40 | 0.35 | -1.08 | 0.28 |
| \( \theta_{35} \) | -1.62 | 0.41 | -2.42 | -0.82 |
| \( \theta_{36} \) | -1.05 | 0.36 | -1.75 | -0.35 |
| \( \theta_{37} \) | -0.48 | 0.54 | -1.52 | 0.57 |
| \( \theta_{38} \) | -15.64 | 5.83 | -27.07 | -4.20 |

A negative value in the parameter estimators for both subduction and volcano means that the closer the area to the subduction or volcano, the more likely the major earthquake to occur. To test the goodness of the model that has been obtained against the data used, an envelopes test plot is given as in Figure 3.

Figure 3. Envelopes Inhomogeneous K-function plot for earthquake intensity models involve covariate subduction and volcanoes using GAMs approach.

The envelopes test plot shows that the intensity model with GAMs approach is appropriate for modelling the dependency between the earthquake intensity and the covariate because the K-function estimation line is in the interval. The predicted intensity maps of the inhomogeneous Cauchy cluster process model.
with GAMs approach for the earthquake risk mapping in Sulawesi and Maluku are presented in Figure 4.

![Figure 4](image)

**Figure 4.** The predicted intensity maps for the earthquake in Sulawesi-Maluku with \( M \geq 5 \) SR during 2009-2019.

The predicted intensity maps show that the area with the lightest color is the area where the intensity of the earthquake is very high. The area is mostly predicted in the northern central especially in the top of between Sulawesi and Maluku, and the southern ocean area. This is because there exist major subduction zones (e.g. North Sulawesi Megathrust, East Molucca, Banda Sea Subduction, and Wetar Back Arc), and volcanoes (e.g. Sangihe, Halmahera, and Timor) in Sulawesi and Maluku.

5. **Concluding remarks**

In this paper, we propose to model the main earthquake (mainshocks) with magnitude \( M \geq 5 \) SR in Sulawesi-Maluku using the generalized additive Poisson models and Cauchy cluster process to detect clustering effect, which involves geological variables such as the subductions, faults, and volcanoes. The clustering effect causes the estimated mainshock intensity is around 78, and the aftershocks are distributed around it with a distance of 15.2 km due to mainshock activity. Among the three geological variables, we detect that subductions and volcanoes are significant variables to trigger the major earthquake, and the earthquake intensity model in equation (18) can be used as a reference model for making earthquake risk maps. For future study, the time sequence and the depth of the earthquake can be considered to be involved in the earthquake intensity modeling process.

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