Analysis of Low Energy Pion Spectra

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The transverse mass spectra and the rapidity distributions of $\pi^+$ and $\pi^-$ in Au-Au collisions at 2, 4, 6, and 8 GeV by E895 collaboration are fitted using an elliptically expanding fireball model with the contribution from the resonance decays and the final state Coulomb interaction. The ratio of the total number of produced $\pi^-$ and $\pi^+$ is used to fit the data. The resulting freeze-out temperature is rather low ($T_f < 60$ MeV) with large transverse flow and thus resonance contribution is very small. The difference in the shape of $m_t$ spectra of the oppositely charged pions are found to be due to the Coulomb interaction of the pions.

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Pion production just above the threshold energy is quite different from that at very high energies such as RHIC energy since the ratio of $\pi^-$ to $\pi^+$ at very high energies is one which is not the case at low energies. At just above the threshold energy, pions are produced through the production of $\Delta$ resonances and counting all the possible channels of $\Delta$ decay the difference in the composition of isospins in the colliding nuclei appears as the difference in the numbers of the two oppositely charged pions[1, 2, 3], whereas at high energies many channels producing pions are open and small asymmetry in the initial isospin does not matter.

Other features of the pion spectra at low energies are[1, 2, 3, 4, 5, 6, 7]: (1) The transverse momentum spectra both of the $\pi^-$ and $\pi^+$ seem to have two temperatures. Usually the low temperature component in the low momentum region is attributed to the pions decayed from resonances, especially the delta resonance, while the higher temperature component in the mid-momentum region is the thermal ones. (2) Transverse momentum spectra of $\pi^-$ and $\pi^+$ at very small momentum are different in the sense that the $\pi^+$ spectra is convex in its shape while the $\pi^-$ spectra does not show this behavior. This difference in low momentum region is due to the Coulomb effect. The hadronic matter formed during the collision has charge which comes from the initially colliding two nuclei and thus the thermal pions escaping from the system experience the Coulomb interaction. The Coulomb interaction of $\pi^-$ may bend the spectrum in the low momentum region upward and thus it is hard to disentangle the contribution from the delta resonance and the Coulomb interaction in the low momentum region. (3) Width of the rapidity spectra of $\pi^-$ and $\pi^+$ are much wider than those from the isotropically expanding thermal model. The wide width may either come from partly transparent nature of the collision dynamics or the ellipsoidal expansion geometry. In order to fit large rapidity width using expanding fireball model one usually needs large longitudinal expansion velocity.

Even though all those features mentioned above are not new, calculations with all those features put in together comparing each contributions in detail is hard to find. There are claims that the properties of $\Delta$ resonance are modified inside the hadronic matter formed even at this low energy[5, 6]. In order to draw any conclusion, one should have a model which can explain all of the features above mentioned. Lacking one or two features in the calculations, the result may not be conclusive.

In this paper, we analyze the pion spectra in Au+Au collisions at 2, 4, 6, and 8 A-GeV measured by the E895 collaboration[4] using the expanding fireball[8, 9, 10] with the resonance contribution[10] and the final state Coulomb interaction[1, 11, 12, 13]. The geometry of the expansion used is ellipsoidal[8] and can be varied to sphere and cylinder, by taking the transverse size $R$ as a function of the longitudinal coordinate, $z$.

At just above the threshold energy, pions are produced through the production of $\Delta$ resonances and their subsequent decays. The ratio of $\pi^-$ and $\pi^+$ is given from the initial isospin conservation as 
$$\frac{\pi^-}{\pi^+} = \frac{N_2^+}{N_2^-} \sim 1.94$$
for Au+Au collisions[1, 2, 3]. Hence it is expected that at low beam energies near 2 GeV A in Au+Au collisions, the ratio is near 1.94 and then as the beam energy is increased the ratio will decrease eventually to one. In the present calculation, the ratio of normalization constants for $\pi^-$ and $\pi^+$, $R$ is taken as a fit parameter in order to investigate the beam energy dependence of $R$.

We assume that once the pions are produced they rescatter among themselves and thermalize before they decouple from the system. Hence we assume thermalization of the pionic matter. However, the total number of negatively and positively charged pions are not in chemical equilibrium and the ratio is governed by the isospin asymmetry of the initially colliding nuclei. We keep the ratio as a fitting parameter and want to compare with the expected value of 1.94 near threshold of the pion production.

After the formation of a pionic fireball, it expands and cools down until freeze-out when the particle production is described from the formalism of Cooper-Frye[14]. For the equilibrium distribution function we use the Lorentz-boosted Boltzmann distribution function, where $u_\mu$ is the
TABLE I: Fitted values for each parameters.

| $E_{beam}$ (GeV) | V | $\eta_{0}$ | $\rho_{0}$ | T | $p_{t}$ | $\pi^{-}/\pi^{+}$ | $\chi^{2}/n$ |
|------------------|---|-----------|-----------|---|-------|-----------------|---------|
| 2                | 1.41 | 1.12 | 0.88 | 46 | 25 | 1.96 | 1.3 |
| 4                | 0.93 | 1.32 | 0.92 | 57 | 24 | 1.95 | 2.9 |
| 6                | 1.44 | 1.50 | 1.11 | 54 | 18 | 1.40 | 2.4 |
| 8                | 1.62 | 1.58 | 1.12 | 55 | 15 | 1.38 | 1.8 |

For the geometry of the expanding fireball, we assume azimuthal symmetry and further assume boost-invariance collective dynamics along the longitudinal direction. In this case it is convenient to use the coordinates the longitudinal proper time $\tau = \sqrt{t^2 - z^2}$, space-time rapidity $\eta = \tanh^{-1}(z/t)$ and the transverse coordinate $r_{\perp}$. Then the 4-velocity of expansion can be expressed as $u^\mu(x) = \gamma(1, v_{\perp}(x) e_{\perp}, v_{\parallel}(x))$ where $v_{\parallel}(\tau, r_{\perp}, \eta) = \tanh \eta$, which is the result of the longitudinal boost-invariance. In the transverse direction we take a linear flow rapidity profile, $\tanh^{-1} v_{\perp} = \rho(\eta) (r_{\perp}/R_0)$ where $R_0$ is the transverse radius at midrapidity. Here one takes $\rho(\eta) = \rho_0 \sqrt{1 - (\eta^2/\eta_{max}^2)}$ for the elliptic geometry and a constant value of $\rho$, i.e. $\rho(\eta) = \rho_0$ for the cylindrical geometry. As is the same for the SPS energy by Dobbler et. al., the elliptic case fits the pion spectra a little better.

As pions escape from the system at freeze-out, they experience the Coulomb interaction with the charge of the system which are mainly due to the initial protons in the colliding nuclei. The Coulomb effect on the particle spectra are studied in detail in refs. for the static and dynamical cases. Here due to the low beam energy we restrict ourselves only to the static case. The system is expanding rapidly in the longitudinal direction and the change in the longitudinal momentum is negligible. Only the transverse momentum of the charged particles will be shifted by an average amount.

$$p_{c} = \Delta p_{\perp} \sim 2 e^{3} dN_{ch}/dy R_{f},$$ (3)

where $R_{f}$ is the transverse radius of the system at freeze-out. Due to the lack of detailed knowledge, $p_{c}$ is taken as a fit parameter in the present calculation. And in this way the beam energy dependence of $p_{c}$ can be studied.

Since the transverse momentum of the escaping thermal pions are shifted by the amount $p_{c}$, i.e. $p_{t} = p_{t,0} \pm p_{c}$, the invariant cross section can be written in terms of the unshifted momentum $(p_{t,0}, y_{0})$. 

The fitted values for the parameters are tabulated in the Tab. 1 and the results of the fitting are shown in Figs.1-2 for 2 A-GeV. The fitted value for the ratio $R = \pi^{-}/\pi^{+}$ is close to 1.94 at 2 and 4 A-GeV as expected and decreases to 1.38 at 8 A-GeV, which eventually becomes 1 at higher energies such as RHIC energies. In other words, at this very low energy pion isospin is not in chemical equilibrium.
FIG. 2: $\pi^-$ transverse mass spectrum for each rapidity bin($\Delta y = 1.0$) measured by E895 collaboration in Au+Au collisions at 2 A·GeV. Data on the top line is for the rapidity bin $-0.65 < y < -0.55$ and the next one is for $-0.55 < y < -0.45$ scaled by 0.1, etc.

The freeze-out temperature is rather small, $T_f < 60$ MeV and the expansion velocities in both the longitudinal and transverse direction are quite large. The large longitudinal expansion velocity($>0.8c$) is needed to fit the large width of the rapidity distribution. The low freeze-out temperature together with the large transverse expansion fits the transverse spectra of pions quite nicely. If not for the large expansion velocity, one usually gets much larger freeze-out temperature($T_f > 80$ MeV).

Since the freeze-out temperature is small, there are very few resonances at freeze-out, especially $\Delta$, and thus the resonance contribution is negligible. This is reasonable since at this low energy near the pion threshold energy, production of particles with mass larger than pions is rare and their contribution to the pion spectrum is negligible.

The pion transverse momentum spectra looks like that there are two slopes; one for small momentum region and another for the higher momentum region near the pion mass. The smaller slope at lower transverse momentum is usually attributed to the pions from the resonance decay, especially from the $\Delta$ decay[3, 6]. However, present calculation shows that this is not the case at low beam energies.

The shape of the transverse mass spectra of $\pi^-$ and $\pi^+$ are different especially in the small mass region. As the transverse mass $m_t$ decreases, the $m_t$ spectrum of $\pi^-$ increases sharply while that of $\pi^+$ saturates showing the convex shape. This difference is due to the Coulomb interaction of pions leaving the system which has the charge from the initially bombarding nucleons. The change in the transverse momentum due to the Coulomb interaction decreases from 25 GeV/c at 2 A·GeV to 15 GeV/c at 8 A·GeV. This behavior can be understood from the increase of the screening effect since the number of charged pions increase at higher energies. At very high energies such as RHIC energies, the momentum change from the Coulomb interaction will be small.

The emerging picture of pion production at low energy is that the pions are produced through the intermediate $\Delta$ formation and thus they are not in chemical equilibrium in isospin. They make collisions and thermalize to form a fireball which expands and cools until the freeze-out. Since the fireball has charge which is from the initially colliding nucleus, the pions leaving the system experiences the Coulomb interaction which makes the difference of the $m_t$ spectra of the two oppositely charged pions.

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