A Thermal Model for Three-Core Armored Submarine Cables Based on Distributed Temperature Sensing

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Abstract: This paper presents a procedure for the derivation of an equivalent thermal network-based model applied to three-core armored submarine cables. The heat losses of the different metallic cable parts are represented as a function of the corresponding temperatures and the conductor current, using a curve-fitting technique. The model was applied to two cables with different filler designs, supposed to be equipped with distributed temperature sensing (DTS) and the optical fiber location in the equivalent circuit was adjusted so that the conductor temperature could be accurately estimated using the sensor measurements. The accuracy of the proposed model was tested for both stationary and dynamic loading conditions, with the corresponding simulations carried out using a hybrid 2D-thermal/3D-electromagnetic model and the finite element method for the numerical resolution. Mean relative errors between 1 and 3% were obtained using an actual current profile. The presented procedure can be used by cable manufacturers or by utilities to properly evaluate the cable thermal situation.

Keywords: submarine cable; three-core; armor; finite element method; thermal modeling; distributed temperature sensing

1. Introduction

In recent decades, the number of projects and installed capacities involving submarine power cables have significantly increased. Most of these are devoted to connecting offshore wind power plants (OWPPs) to the onshore grid (e.g., in Europe, the cumulative installed capacity was raised from 2 GW in 2009 to 22 GW in 2019 [1]), while others are related to underwater HV interconnection links (e.g., inland to islands, between islands). Future plans to install new HV OWPPs show that this trend is increasing, as in the case of Europe, where the annual installation rate is expected to be doubling in the following years [1]. In this sense, design features like cable current ratings higher than 1500 A or cable lengths higher than 100 km are nowadays not infrequent [2].

One of the most critical parts of this kind of infrastructure is the power cable; therefore, it must be adequately designed and operated to prevent costly failures and repair costs. In this sense, much effort has been devoted in recent years to increasing the possibilities for its condition monitoring, allowing the early detection of incipient failures or misoperation risk beyond thermal/mechanical limits [3]. Distributed temperature sensing (DTS) is one of these techniques, commercially available since the 1990s for land-extruded HV cables, which has become widespread in recent years for submarine cables [4]. It measures the temperature along part of or the whole cable (typically dozens of km) through optical fiber (OF), either installed inside or outside (in parallel ducts) of the power cable, with a typical measurement spatial resolution of a few meters, with a temperature measurement error between 1 and 2 °C and a measurement time between 10 and 15 min [5]. This technique has different applications for land and submarine HV cables, including the prevention of cable
overheating due to critical hotspots [6,7], cable fault location [8], implementation of real-time thermal rating (RTTR) tools [9–13] for maximizing the steady-state or cyclic loadability of the cable, and more recently, the detection of deburial events in submarine cables [14,15].

However, in most of these applications, the actual temperature of the conductor (or at the cable surface) is required, so it must be adequately derived from the DTS measurements. Since the location of the OF is not standardized [16,17], this involves the use of analytical or numerical adjustment methods, becoming a standard equivalent thermal network-based method (ETN) [10,11,13,18–21], where the spatially distributed DTS temperature and the cable current are employed as real-time inputs for obtaining the temperatures of the different cable components (conductor, screen/sheath, jacket, etc.). Due to the thermal resistances and capacitances involved, better static and dynamic results are obtained when the OF is installed closer to one of the already existing ETN nodes (screen/sheath, jacket, etc.). This is the case in most single-core cables, where the OF is usually embedded in the sheath/screen or attached over the conductor jacket [9,10]. Conversely, for the case of submarine three-core armored cables (TCACs) (Figure 1), the use of ETN-based methods to adjust DTS measurements involves three additional issues:

- The lack of radial symmetry: Customarily, an equivalent single-core ETN has been employed for the thermal current rating of TCACs [18], where the thermal resistance of the fillers must be adequately obtained. However, [18] does not take into account the filler design, hence some corrections have recently been suggested in [22,23] for larger cables;
- The location of the OF: In TCACs, it is usually embedded in the filler employed to support the armor bedding (Figure 1). This particular location is not explicitly included in any of the ETNs of TCACs reviewed by the authors. Moreover, the correlation between conductor temperature and DTS temperature is affected by the filler design (either extruded or made of PP ropes);
- Loss allocation: The ETN requires as inputs the losses in all the cable components (conductors, sheath/screen and armor). However, it is well-known that the IEC 60287 standard [18] overestimates the power losses in this type of cables. In this sense, 2D simulations based on the finite element method (FEM) were extensively employed for validating the performance of the ETN [20]. Nevertheless, both [18] and 2D-FEM models lead to important errors due to the simplifying assumptions considered, where relevant aspects regarding TCAC design are not taken into account, such as the twisting of armor wires and conductors.

Regarding the last issue, 3D-FEM simulations have proven to provide accurate results in the losses’ calculation [24–26], at the expense of high demanding computations. Due to recent advances [26–28], the length of the geometry to be simulated can be strongly reduced, achieving a saving of about 95% in terms of computation time for solving complex TCACs [28]. This improvement has made it possible to develop a fully coupled electrothermal model for simultaneously evaluating the temperature and ampacity of TCACs [29,30].

![Figure 1. Elements in a TCAC.](image-url)
Considering all the above, this work tackles the issues observed in TCACs by proposing a new procedure for improving the accuracy of the equivalent single-core ETN for the estimation of the conductor temperature through DTS measurements, using the power of 3D-FEM simulations.

2. FEM-Based Simulation

Recent advances have helped in reducing the length, \( L \), of TCACs 3D-FEM models by exploiting the symmetries found both in the geometry and the electromagnetic field distribution within this type of cable [26–28], so it is now possible to simulate a cable geometry as short as

\[
L = \frac{CP}{N} = \frac{1}{N \left( \frac{1}{L_a} + \frac{1}{L_c} \right)}
\]  

where \( CP \) is the crossing pitch, \( N \) is the number of armor wires and \( L_a \) and \( L_c \) are, respectively, the armor wire and phase conductor lay length (twisted in opposite directions). In this situation, rotated periodicity boundary conditions can be applied, where the twisting of power cores is taken into account when mapping the source boundary into the destination boundary (Figure 2). This was done through the rotational displacement (\( \theta \)) of the power cores for a model length equal to \( L \), defined as

\[
\theta = 2\pi \frac{L}{L_c}.
\]  

Figure 2. Rotated periodicity in a TCAC.

Through this ultra-shortened 3D model, a reduction of approximately 95\% is achieved in the computation time, taking less than 60 s to solve a complex TCAC in a laptop with 64 GB of RAM memory and a i7 processor [28].

Due to this improvement, it is now possible to develop a fully coupled electrothermal model for evaluating the temperature and ampacity of TCACs [29,30]. To this aim, a detailed 3D-FEM model is required to include the main dimensions and properties of all the elements involved in this type of cable, such as conductors, sheaths, armor wires, fillers, semiconductive screens, and fiber-optics cables. Furthermore, the accuracy of the model improves if heat transfer by radiation and natural convection inside the cable air gaps is included (as recommended in [23,31]). However, this would lead to highly demanding computations if a fully coupled 3D electrothermal FEM model is employed. To overcome this problem, Ref. [30] proposed a new fully coupled hybrid 2D-thermal/3D-electromagnetic model, where the ultra-shortened 3D electromagnetic model presented earlier is coupled...
with a detailed 2D thermal model. This approach is iteratively solved as follows (Figure 3): the electromagnetic losses are obtained in the periodic 3D geometry. Then, they are taken as inputs in the 2D thermal model, where the temperature distribution in all the elements of the power cable is obtained. Eventually, the temperature distribution in the metallic elements is taken as input in the 3D geometry for updating their electrical resistivity, so that electromagnetic losses can also be updated.

**Figure 3.** Sequence for mapping magnitudes between 2D and 3D geometries.

It should be noticed that the 2D thermal problem involves the boundary conditions represented in Figure 4, where forced convection is assumed in the water-soil interface, defined by a convection heat transfer coefficient $h$, where $d_p$ is the burial depth of the cables, $k_{\text{soil}}$ is the soil thermal conductivity, and $\theta_a$ is the ambient temperature. Additionally, for cables with extruded fillers, the FEM model also solves the problem of air convection inside the cable air-gaps, where surface-to-surface heat radiation is defined by the emissivity $\varepsilon$.

**Figure 4.** Temperature distribution in TCAC and surroundings.

As can be seen, in this procedure it is required to map data between 2D and 3D geometries (Figures 3 and 5). This is achieved by using a feature called “general extrusion” in the FEM software (Comsol Multiphysics [32]). This operator must be adequately configured to consider the helical path of phases and armor wires.

Through this approach, the temperature distribution in complex TCACs is obtained in less than 4 min, all of which includes realistic boundary conditions in a detailed geometry (Figure 5).
3. Case Studies

In this work, two cases were considered for analysis: a typical 132 kV, 800 mm$^2$ cable and a 275 kV, 2000 mm$^2$ cable with a higher voltage and larger section. Their main dimensions and parameters are summarized in Figure 6 and Tables 1 and 2, where $\rho$ is the electrical resistivity, $\alpha$ is the temperature coefficient, $\mu_r$ is the magnetic permeability, $k$ is the thermal conductivity and $C$ is the volumetric heat capacity. A complex permeability is employed in the armor wires to take into account hysteresis losses. Furthermore, different filler designs are considered for these cables:

- Homogeneous (Figure 7a) and extruded fillers (Figure 7b) for Cable 1 (denoted as Cable 1H and Cable 1E, respectively);
- Extruded fillers (Figure 7c) and PP ropes (Figure 7d) for Cable 2 (denoted as Cable 2E and Cable 2R, respectively).

![Figure 6. Main dimensions in a TCAC.](image-url)

![Figure 7. Cables and fillers employed: (a) Cable 1H with homogeneous filler; (b) Cable 1E with extruded filler; (c) Cable 2E with extruded filler; (d) Cable 2R with PP ropes.](image-url)
Table 1. Main dimensions of Cables 1 and 2 (Figure 6).

| Parameter                  | Cable 1     | Cable 2     |
|----------------------------|-------------|-------------|
| Voltage (kV)               | 132         | 275         |
| Section (mm²)              | 800         | 2000        |
| $I_{\text{max}}$ : Maximum current (A) | 780         | 1100        |
| $D_c$ : Conductor diameter (mm) | 35          | 54.5        |
| Cond./insul. screen thickness (mm) | 0.85        | 0.85        |
| $D_e$ : Sheath ext. diameter (mm) | 87.6        | 121.5       |
| $t_e$ : Sheath thickness (mm) | 3.7         | 3           |
| $D_{\text{core}}$ : Core diameter (mm) | 92.4        | 126         |
| $D_f$ : External filler diameter (mm) | 199.1       | 271.5       |
| $D_a$ : Armor mean diameter (mm) | 212         | 290         |
| $d_t$ : Armor wire diameter (mm) | 5.6         | 5.6         |
| $N$ : Number of armor wires | 114         | 157         |
| $D_{\text{ext}}$ : External diameter (mm) | 225.6       | 303.6       |
| $L_a$ : Armor lay length (m) | 3.5         | 4.8         |
| $L_c$ : Conductor lay length (m) | 2.8         | 3.8         |
| $R_f$ : Optical fiber position (extrud./ropes) (mm) | 63.8        | 85.68/105.34 |

Table 2. Main material parameters of Cables 1 and 2.

| Cable Element | $\rho$ (Ω · m) | $\alpha$ (1°C) | $\mu_r$ | $k$ (W/(K · m)) | $C$ (MJ/(m³ · K)) |
|---------------|-----------------|-----------------|---------|----------------|------------------|
| Conductor (copper) | $1.724 \cdot 10^{-8}$ | 0.00393         | 1       | 400            | 3.45             |
| Sheath (lead)   | $2.14 \cdot 10^{-7}$ | 0.004           | 1       | 35.5           | 1.45             |
| Armor (steel)   | $1.38 \cdot 10^{-7}$ | 0.0045          | 300 - j100 | 44.5   | 3.8          |
| Insulation (XLPE) | 0             | –               | 1       | 0.286          | 2.14             |
| Screen (PE)     | 0               | –               | 1       | 0.1            | 2.5              |
| Jacket/outer serving (PE) | 0             | –               | 1       | 0.46           | 2.5              |
| Filler (PP)     | 0               | –               | 1       | 0.25           | 2.5              |
| Fiber optics    | 0               | –               | 1       | 1.38           | 1.55             |
| Air             | 0               | –               | 1       | 0.029          | $1 \cdot 10^{-3}$ |
| Soil            | 0               | –               | 1       | 1              | 2.9              |

In the procedure described in the following sections, the hybrid 2D/3D electrothermal model was employed to evaluate the temperature measured by the DTS system, also providing the actual temperature of the conductor and the outer serving for the assessment. This was done for two sets of thermal boundary conditions (Case 1 and Case 2), as summarized in Table 3, characterized by different values of $d_p$, $k_{\text{soil}}$, and $\theta_a$.

Table 3. Thermal boundary conditions.

| Parameter             | Case 1 | Case 2 |
|-----------------------|--------|--------|
| $\theta_{\text{amb}}$ (°C) | 15     | 10     |
| $d_p$ (m)             | 1      | 2      |
| $k_{\text{soil}}$ (W/(m·K)) | 1      | 1.2    |
| $h$ (W/(m²·K))        | 200    | 200    |
| $\varepsilon$         | 0.9    | 0.9    |

For each set of boundary conditions, Cables 1 and 2 are simulated under two different loading conditions:

- In Sections 4, 5 and 6.1 the ETN is adjusted and validated for different stationary loading conditions, where fixed currents ($I_c$), ranging from 50 A to $I_{\text{max}}$ (Table 1), are injected through the conductors in the FEM model.
• Alternatively, in Section 6.2, the ETN is validated for a more realistic scenario, where the per-unit \( (I / I_{\text{max}}) \) 240 h profile represented in Figure 8 is employed in the FEM model (based on data from [33]). The initial temperature in the transient studies was obtained by solving the stationary problem for a particular initial current (1 pu for Cable 1 and 0.25 pu for Cable 2).

![Figure 8. Normalized load profile.](image)

4. Thermal Modeling

The main goal of this study was to obtain a static ETN specifically adjusted for a particular TCAC to estimate the conductor temperature in both stationary and dynamic loading conditions. To this aim, 3D-FEM simulations are performed to provide all the data required for the adjustment. FEM simulations will be also employed for computing the dynamic behavior of the cable temperature, providing “virtual” DTS measurements that serve as inputs for the estimation of the conductor temperature through the adjusted ETN. Thus, this section presents a description of the different stages for the derivation of the ETN proposed for TCACs. A flowchart with the different steps of the process is depicted in Figure 9.

![Figure 9. Flowchart of the proposed method.](image)

4.1. Equivalent Static Circuit

In this work, a static equivalent single-core ETN was considered for the TCAC (Figure 10), as customarily performed in [18]. This circuit is only intended for obtaining a direct relationship between DTS measurements and the conductor temperature, either under stationary or dynamic loading conditions. Thus, since it will not be employed for computing the dynamic behavior of the cable temperature, the corresponding thermal capacitances are not considered. As will be shown subsequently, this fact does not affect the accuracy of the results under dynamic loading conditions.
In the scheme above, \( W_c, W_s \) and \( W_a \) are the heat losses in the conductor, shield, and armor, respectively, all of which depend on the conductor current, \( I_c \), while the corresponding temperatures are \( \theta_c, \theta_s \) and \( \theta_a \). The temperature of the outer surface of the cable is denoted as \( \theta_{out} \), and \( T_{cs}, T_{sa} \) and \( T_{ao} \) are the thermal resistances for each section of the cable, whose values will be obtained subsequently. It should be noticed that dielectric losses have been omitted since they are comparatively much smaller than the other losses.

4.2. Thermal Resistance Calculation

The expressions for the thermal resistances in the considered equivalent circuit are obtained from [18]. The conductor-shield resistance, \( T_{cs} \), includes the contribution of all the three conductors with their corresponding insulation and semiconductor layers, yielding:

\[
T_{cs} = \frac{1}{3} \left[ \frac{\rho_{sem}}{2\pi} \ln\left( \frac{D_c + 2t_{sem}}{D_c} \right) + \frac{\rho_{ins}}{2\pi} \ln\left( \frac{D_s - 2t_s}{D_c + 2t_{sem}} \right) + \frac{\rho_{sem}}{2\pi} \ln\left( \frac{D_s}{D_s - 2t_s} \right) \right]
\]

(3)

where \( \rho_{sem} \) is the thermal resistivity of the semiconductor layer and \( \rho_{ins} \) is that of the insulation layer. In the case of the shield-armor thermal resistance, \( T_{sa} \), a geometric factor, \( G \), is used as follows:

\[
T_{sa} = \frac{\rho_f}{6\pi} \cdot G
\]

(4)

where \( \rho_f \) is the thermal resistivity of the filling material and the derivation of \( G \) is provided in [18]. Finally, for the resistance \( T_{ao} \), the following expression is used:

\[
T_{ao} = \frac{\rho_{ab}}{2\pi} \ln\left( \frac{D_a - d_a}{D_f} \right) + \frac{\rho_{st}}{2\pi} \ln\left( \frac{D_a + d_a}{D_a - d_a} \right) + \frac{\rho_{ser}}{2\pi} \ln\left( \frac{D_{ext}}{D_a + d_a} \right)
\]

(5)

where \( \rho_{ab}, \rho_{st} \) and \( \rho_{ser} \) are the armor bedding, outer serving and steel thermal resistivities, respectively.

4.3. Curve Fitting for Heat Losses

As commented earlier, the power losses employed as inputs in the ETN (Figure 10) are customarily derived from [18], leading to important errors in the case of TCACs. To overcome this issue, the power losses from [18] are here replaced by a simplified heat loss model, which is obtained using the information provided by FEM-based simulations for the different cables considered. The objective is to establish a numerical relation between the current in the conductor, \( I_c \), the temperature in different sections of the cable (conductor, shield and armor), and the corresponding heat losses. The expressions used in this study are those presented below:

\[
W_c = (\alpha_{c0} + \alpha_{c1} \cdot (\theta_c - 20)) \cdot I_c^2 \ (W/m)
\]

(6)

\[
W_s = (\alpha_{s0} + \alpha_{s1} \cdot (\theta_s - 20)) \cdot I_c^2 \ (W/m)
\]

(7)

\[
W_a = (\alpha_{a0} + \alpha_{a1} \cdot (\theta_a - 20)) \cdot I_c^2 \ (W/m)
\]

(8)
where the subscripts \( c \), \( s \) and \( a \) stand for conductor, shield, and armor, respectively, \( W_j \) refers to the heat losses in section \( j \) (with \( j = c, s, a \)), and \( \theta_j \) is the corresponding temperature. The parameters \( \alpha_{ji} \) with \( i = 0, 1 \) are adjusted in the least-squares sense.

For this procedure, the stationary loading conditions described in Section 3 were considered, as the results of the fitting are represented in Figure 11 for Cable 1H. In this case, the FEM-based simulation was carried out with \( \theta_{amb} = 15^\circ \text{C} \), with a burial depth of \( d_p = 1 \text{ m} \) and a soil thermal conductivity of \( k_{soil} = 1 \text{ W/(K \cdot m)} \). To evaluate the goodness of the adjustments, the corresponding coefficient of determination, \( R^2 \), was included in each graphic. It was observed how the parabolic curve accurately matches the sample values obtained from the simulation for the three sections of the cable, with a coefficient \( R^2 \) close to 1 in all cases.

The validity of the adjusted parameters is now assessed in order to verify whether the previously calculated values can be used for different external conditions. Furthermore, with stationary loads, the 3D-FEM simulated and curve-fitting estimated values of the heat losses for \( \theta_{amb} = 10^\circ \text{C} \) are shown in Figure 12, where a perfect match was also obtained.

The good accuracy of the estimated curves was also assessed even for unrealistic external conditions (\( \theta_{amb} = 50^\circ \text{C} \)), concluding that the FEM-based simulation can be substituted by the fitted heat loss curves (6)–(8) in the thermal model that will be presented in Sections 4.4 and 4.5.

Similar parameters (\( \theta_{amb} = 15^\circ \text{C} \), \( d_p = 1 \text{ m} \) and \( k_{soil} = 1 \text{ W/(K \cdot m)} \)) are used for the rest of the cables (1E, 2R and 2E), obtaining the fitted curves of Figure 13.

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4.4. ETN Resolution

In this stage of the procedure, the circuit in Figure 10 was solved using the outer temperature, with \( \theta_{\text{out}} \) as an input (provided by the FEM simulation), together with the Kirchhoff laws:

\[
\begin{align*}
\theta_s &= \theta_c - W_c \cdot T_{cs} \\
\theta_a &= \theta_s - (W_c + W_s) \cdot T_{sa} \\
\theta_{\text{out}} &= \theta_a - (W_c + W_s + W_a) \cdot T_{ae}
\end{align*}
\]

where \( W_c, W_s \) and \( W_a \) are substituted by expressions (6)–(8) with the previously adjusted parameters, so that, for each value of the conductor current \( I_c \), the only unknowns are the temperatures \( \theta_c, \theta_s, \) and \( \theta_a \), yielding a closed system of three equations with three unknowns.

4.5. DTS Treatment

Once the temperatures \( \theta_c, \theta_s, \) and \( \theta_a \) are calculated from the circuit resolution, the heat losses can be directly obtained from Equations (6)–(8). For the location of the DTS, the shield-armor thermal resistance, \( T_{sa} \) is divided into two sections, using a parameter \( d \in [0, 1] \) as a DTS divider, resulting in a circuit represented in Figure 14.

\[
d = \frac{\theta_{\text{DTS}} - \theta_s}{(W_c + W_s) \cdot T_{sa}}
\]
For Cable 1H, the previously considered stationary loading conditions (with \( I_c \) ranging from 50 A to 900 A) are introduced in the FEM software in order to simulate the measurements of \( \theta_{DTS} \), so that an estimation of the parameter \( d \) can be obtained through Equation (12), as the corresponding results are represented in Figure 15. A dependency relation of the parameter \( d \) with respect to the current can be observed, which can be adjusted using the following linear function:

\[
d = a_{d0} + a_{d1} \cdot I_c
\]

The mentioned adjustment was also depicted in Figure 15. The same procedure was applied to the other cables considered in this work. Figure 16 shows the estimation results for the parameter \( d \) in each case, with the corresponding linear adjustments.

Figure 15. Estimation results for parameter \( d \) in Cable 1H.

(a)

(b)

(c)

Figure 16. Estimation results for parameter \( d \) (a) Cable 1E; (b) Cable 2R; (c) Cable 2E.

5. Estimation of the Conductor Temperature Based on DTS Measurements

Using the thermal model presented in the previous section for three-core armored submarine cables, along with the measurements of the conductor current and the temperature of the DTS, it is possible to estimate the temperature of the conductor. Once the adjustment process is completed, the substitution theorem is applied to reduce the circuit in Figure 14 to that represented in Figure 17. Using this circuit, the estimation of \( \theta_c \) involves the following steps:
Step 1: The value of $I_c$ is introduced in Equation (13) to obtain the estimated parameter $d$ in each case;

Step 2: The subsequent circuit equations are considered:

\[ \theta_s = \theta_c - W_c \cdot T_{cs} \]  
\[ \theta_{DTS} = \theta_s - (W_c + W_s) \cdot d \cdot T_{sa} \]  

where the values of $W_c$ and $W_s$ are substituted by their corresponding adjusted expressions in (6) and (7);

Step 3: The resulting system of two equations is solved to obtain $\theta_c$:

\[ \theta_c = \theta_{DTS} + (W_c + W_s) \cdot d \cdot T_{sa} + W_c \cdot T_{cs} \]  

The accuracy in the estimation of the conductor temperature is assessed in the next section for the different cables under study.

6. Numerical Validation

The performance of the proposed technique for thermal modeling and temperature estimation was tested in two different case studies. For each scenario, the results obtained for the different cables will be presented.

6.1. Base Case. Stationary Current Sweep

In the first scenario, the conductor temperature, $\theta_c$, was estimated using the DTS information provided by the FEM software, using the stationary loading conditions described in Section 3 and considering some variations in the environmental conditions, with respect to those used for the adjustment of the proposed thermal model. For Cable 1H, the obtained results are presented in Figure 18, where the estimated values are compared to those extracted from the FEM-based simulation.
It is observed how, in both cases, the estimations of $\theta_c$ extracted from the ETN-based model are very close to the FEM-based values for the whole range of $I_c$, giving evidence of the accuracy of the proposed procedure.

Finally, in Figures 19 and 20, respectively, the resulting estimations for $\theta_{amb} = 10^\circ$C and $\theta_{amb} = 5^\circ$C for Cables 1E, 2R and 2E are included in the base scenario ($d_p = 1$ m and $k_{soil} = 1$ W/(K·m)). In all cases, the estimation error remains under $1^\circ$C, proving the capability of the adjusted DTS-based model to obtain an accurate estimation of the conductor temperature.

6.2. Actual Current Profiles

In this second case, the current dynamic profile presented in Figure 8 was considered for the FEM-based simulation of a more realistic operating scenario. With this profile, different external conditions were considered for the simulations, from which the temperature $\theta_{DTS}$ was acquired and used to estimate the conductor temperature with the adjusted thermal model. The variables in this scenario are:

- Ambient temperature, $\theta_{amb}$;
- Burial depth, $d_p$, of the cable;
- Soil thermal conductivity, $k_{soil}$;
- Initial conductor current with respect to the rated value, $I_0$.

The values of these boundary conditions for each case were presented in Table 3. The estimation results obtained in each case are represented in Figures 21–24 for Cables 1H, 1E, 2R, and 2E, respectively. The measurements are supposed to be taken every 9 minutes and the DTS temperature has also been represented as a reference value.
Figure 21. Estimation results for $\theta_c$ in Cable 1H: (a) Case 1; (b) Case 2.

Figure 22. Estimation results for $\theta_c$ in Cable 1E: (a) Case 1; (b) Case 2.

Figure 23. Estimation results for $\theta_c$ in Cable 2R: (a) Case 1; (b) Case 2.
In these figures, it can be observed that:

• Cables 1H and 1E (Figures 21 and 22) present similar results, with estimated temperatures close to the FEM-based values in both Cases 1 and 2;
• Regarding Cables 2R and 2E (Figures 23 and 24), the differences are slightly higher, especially for abrupt changes in the conductor temperature, such as those at the beginning of Case 2.

Additionally, the mean relative error (MRE) and the maximum absolute error (MAE), defined as

\[
MRE = \frac{1}{N} \sum_{i=1}^{N} \frac{\left| \theta_{FEM,c}^{i} - \theta_{ETN,c}^{i} \right|}{\theta_{FEM,c}^{i}} \cdot 100 \tag{17}
\]

\[
MAE = \max \left( \left| \theta_{FEM,c}^{i} - \theta_{ETN,c}^{i} \right| \right) \tag{18}
\]

are considered to evaluate the overall performance of the estimation. In Equation (17), \( N \) is the number of available readings and \( \theta_{FEM,c}^{i} \) and \( \theta_{ETN,c}^{i} \) are the \( i \)th temperature value obtained with FEM and ETN, respectively. Table 4 summarizes the MREs and MAEs for the different cables and case studies.

Table 4. Errors obtained from the application of the ETN-based model to the 4 cables.

| Cable | Case | MRE (%) | MAE (°C) |
|-------|------|---------|----------|
| 1H    | 1    | 1.3721  | 2.7635   |
| 1H    | 2    | 1.1102  | 2.0211   |
| 1E    | 1    | 1.3977  | 4.1581   |
| 1E    | 2    | 2.7349  | 5.2082   |
| 2R    | 1    | 2.9603  | 6.2721   |
| 2R    | 2    | 2.8867  | 6.1778   |
| 2E    | 1    | 2.4511  | 5.9917   |
| 2E    | 2    | 2.6814  | 6.2395   |

For the whole set of external conditions, the MREs remain under 3%, proving that the proposed model effectively estimates the temperature of the conductor in TCACs. For the most unfavorable cases (Cables 2R and 2E), values of the MAE near 6 °C are noticed. However, although these values could be perceived as high, they correspond to quite limited periods of time, given the fact that the MRE is much lower. Moreover, the purpose of the proposed procedure is to establish, with a static thermal model, an estimation of the conductor temperature which can be used to evaluate the actual state of the cable rather than the typ-
ical option of directly using the temperature provided by the DTS [33]. In this regard, it can be concluded from Figures 23 and 24 that the ETN-based θc (blue line) is much closer to the actual value (red line) of the temperature than θDTS (black dashed line), which is evidence of the good performance of the proposed technique. Using the typical assumption θc ≈ θDTS, there would be an underestimation of the temperature in most cases, which might lead to a reduction in the useful life of the TCAC, with the corresponding economic consequences [6].

Finally, an important aspect to consider is that the location of the DTS simulated in FEM might differ from the actual location of the sensor due to, for example, the tolerances in the production process. To assess the impact of this deviation in the estimation of θc, two additional simulations were performed for Cable 1H, changing the position of the DTS, Rf, in ±5%, with respect to that employed in the adjustment simulation. The obtained estimations are depicted in Figure 25.

![Figure 25](image)

**Figure 25.** Estimation results with deviations in the sensor position: (a) +5%; (b) −5%.

In both cases, the estimation error has slightly increased with respect to that in Figure 21a. In order to quantify the deterioration of the results with the deviation of the sensor position, Table 5 presents the MAEs obtained in different cases.

| Deviation (%) | MAE (°C) |
|---------------|----------|
| +2.5          | 3.6817   |
| −2.5          | 3.2374   |
| +5            | 4.9324   |
| −5            | 4.0399   |
| +10           | 6.4849   |
| −10           | 5.5069   |

In light of this table, it can be noticed that, as expected, the MAE increases with the deviation in the sensor position. However, even with this error, the estimation given by the proposed method is still valid to provide information related to the thermal state of the TCAC.

7. Conclusions

In this paper, a thermal model is proposed for three-core armored submarine cables using DTS. For each cable studied, a set of FEM-based simulations are considered, from which a simplified heat loss model is derived with a curve-fitting technique. Once the thermal resistances are calculated for each section of the cable, a parameter d, depending on the load, is also adjusted with the simulation data, representing the location of the DTS in the equivalent static circuit modeled and isolating a portion of the cable from the outside.
An error assessment study was also made to conclude that for reasonable deviations in the position of the sensor with respect to the simulated location, the estimation does not deteriorate substantially. Two different scenarios were considered to test the performance of the adjusted thermal model:

- With a stationary current sweep, the proposed thermal model accurately estimates the conductor temperature for changing values of the ambient temperature.
- For a more realistic current profile and pronounced deviations in the external conditions, the obtained MREs range from 1.11% for Cable 1H to 2.96% for Cable 2E. In absolute terms, the maximum deviation of the estimated temperature with respect to the simulated value is 2.02 °C for Cable 1H, and 6.27 °C in the most unfavorable case (Cable 2R).

The proposed thermal model allows, as mentioned before, to establish a more accurate evaluation of the state of the submarine cable in terms of the temperature of the conductor for a certain current, which can be used in some dynamic line rating applications. The adjusted cable parameters of the model can be obtained by the corresponding manufacturer or by the utilities, using the procedure described in this paper with FEM-based simulations. It must be noticed that this adjustment only depends on the conductor current, being unaffected by other environmental boundary conditions, so it can be easily integrated into existing RTTR.

Further research might be oriented to enhance the equivalent model, including some dynamic effects in order to improve the estimation of the temperature under sudden variations in the current.

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Abbreviations
The following abbreviations are used in this manuscript:

- DTS Distributed Temperature Sensing
- IEC International Electrotechnical Commission
- ETN Equivalent Thermal Network
- FEM Finite Element Method
- MAE Maximum Absolute Error
- MRE Mean Relative Error
- OF Optical Fiber
- OWPP Offshore Wind Power Plants
- PE Polyethylene
- PP Polypropylene
- RTTR Real-Time Thermal Rating
- TCAC Three-Core Armored Cable
- XLPE Cross-Linked Polyethylene
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