Hot electron cooling in Dirac semimetal Cd$_3$As$_2$ due to polar optical phonons

Shrishtil K Dubakaddi$^{1}$ and Tutul Biswas$^{2,3}$

Abstract

A theory of hot electron cooling power due to polar optical phonons $P_{\text{op}}$ is developed in 3D Dirac semimetal (3DDS) Cd$_3$As$_2$ taking account of hot phonon effect. Hot phonon distribution $N_{\text{ph}}$ and $P_{\text{op}}$ are investigated as a function of electron temperature $T_e$, electron density $n_{\text{e}}$, and phonon relaxation time $\tau_{\text{p}}$. It is found that $P_{\text{op}}$ increases rapidly (slowly) with $T_e$ at lower (higher) temperature regime. Whereas, $P_{\text{op}}$ is weakly decreasing with increasing $n_{\text{e}}$. The results are compared with those for three-dimensional electron gas (3DEG) in Cd$_3$As$_2$ semiconductor. Hot phonon effect is found to reduce $P_{\text{op}}$ considerably and it is stronger in 3DDS Cd$_3$As$_2$ than in Cd$_3$As$_2$ semiconductor. $P_{\text{op}}$ is also compared with the hot electron cooling power due to acoustic phonons $P_{\text{ac}}$. We find that a crossover takes place from $P_{\text{ac}}$ dominated cooling at low $T_e$ to $P_{\text{op}}$ dominated cooling at higher $T_e$. The temperature at which this crossover occurs shifts towards higher values with the increase of $n_{\text{e}}$. Also, hot electron energy relaxation time $\tau_e$ is discussed. It is suggested that $\tau_e$ can be tuned to achieve faster or slower energy loss for suitable applications of Cd$_3$As$_2$.

Keywords: optical phonon, Dirac material, electron–phonon interaction

1. Introduction

Recently, theoretically predicted [1, 2] and by now experimentally realized and verified three-dimensional Dirac semimetals (3DDS) have become the rapidly growing field of research interest[3–17]. These 3DDS, the three-dimensional (3D) analogue of graphene, have gapless band feature with linear band dispersion and vanishing effective mass in their low energy states. The cadmium arsenide (Cd$_3$As$_2$), a potential representative of 3DDS, has drawn more attention as it is robust and chemically stable compound in air with ultrahigh mobility $\sim 10^7$ cm$^2$ V$^{-1}$ s$^{-1}$ [5, 6, 9, 10] and giant magneto resistance $\sim 10^7$ cm$^2$ V$^{-1}$ s$^{-1}$ with $\gamma = 0.5$, in the range 20–200 K, which is attributed to the enhanced electron–phonon (el–ph) scattering. theoretical simulation and experimentally realized and demonstrated. Because of the inherent zero energy gap and linear band dispersion, 3DDS can absorb photons in the entire infrared region. These 3DDS have advantage over two-dimensional (2D) Dirac semimetals like monolayer graphene because bulk nature of 3DDS enhances the efficiency of photon absorption.

The experimentally reported low temperature ($\sim 5$ K) high mobilities $\sim 9 \times 10^6$ cm$^2$ V$^{-1}$ s$^{-1}$ [5, 6, 9, 10] and up to $4.60 \times 10^7$ cm$^2$ V$^{-1}$ s$^{-1}$ are higher than the suspended graphene. The measurements of resistivity $\rho$ versus temperature $T$, show $\rho \sim T$ down to low $T$, which is inferred to be due to umklapp processes and electron-optical phonon scattering [10]. High quality 3DDS Cd$_3$As$_2$ microbelts [21] and nanobelts [22] with room temperature electron mobility $\sim 2 \times 10^4$ cm$^2$ V$^{-1}$ s$^{-1}$ have been fabricated. In nanobelt [22], the Hall mobility $\mu_H$ follows the typical relation $\mu_H \sim T^{-\gamma}$ with $\gamma = 0.5$, in the range 20–200 K, which is attributed to the enhanced electron–phonon (el–ph) scattering.

Theoretically, electronic transport properties of 3D Weyl and Dirac semimetals are studied using the semi-classical
Boltzmann transport equation [23, 24]. Considering the electron momentum relaxation processes due to scattering by disorder (short-range and long-range) and acoustic phonons, the latter is shown to dominate electrical conductivity at higher temperature [24]. However, the quantitative comparison between the existing experimental results and theoretical calculations is still lacking.

In order to find the applications of 3DDS in devices operating in the high field region, it is important to investigate the steady state energy relaxation of the hot carriers, in these systems, by emission of phonons as the only cooling channel. In high electric field electrons gain energy and establish their ‘hot electron temperature $T_e$’ which is greater than the lattice temperature $T$. In the steady state these hot electrons transfer their energy to lattice by emission of acoustic (optical) phonons at relatively low (high) temperature. The electron heating affects the device operation significantly, in the high field region, as it governs the thermal dissipation and heat management. To enhance the device efficiency, it is important to reduce the hot electron power loss.

The hot electron energy relaxation by emission of acoustic and optical phonons has been extensively investigated theoretically and experimentally in conventional 3D electronic gas (3DEG) in bulk semiconductors [25–31], 2DEG gas (2DEG) in semiconductor heterostructures [31, 32], monolayer graphene [33] and bilayer graphene [34]. Recently, hot electron cooling is theoretically studied in monolayer MoS$_2$ [35] and quasi-2DEG in gapped Cd$_3$As$_2$ film [36].

There exist theoretical studies of hot 3D Dirac fermion cooling power due to electron-acoustic phonon interaction $P_{ac}$ in Cd$_3$As$_2$ [37, 38]. The deformation potential coupling constant $D$ ($\sim$10–30 eV) [39] determines the strength of electron-acoustic phonon scattering. In the Bloch–Grüneisen (BG) regime the power laws of $P_{ac}$ dependence on electron temperature $T_e$ and electron density $n_e$ are predicted [38].

Experimentally, the phonon mediated hot electron cooling of photoexcited carriers has been investigated in Cd$_3$As$_2$ from pump-probe measurements [11, 14–16]. The cooling process of photoexcited carriers is shown to be through emission of acoustic and optical phonons [11, 14], with relatively low optical phonon energies $\sim$25 meV [14, 40]. The hot electron cooling, apart from relating el–ph scattering to the high field transport properties, it also plays significant role in designing high speed electronic and photonic devices of Cd$_3$As$_2$. Thus, el– ph interaction is a key issue and central to the understanding of devices based on Cd$_3$As$_2$.

It is important to notice that, in 3DDS Cd$_3$As$_2$, although there is strong experimental evidence of photoexcited hot carrier energy relaxation by optical phonon emission [11, 14, 15], the steady state hot electron cooling by emission of optical phonons has not been addressed both theoretically and experimentally. In the present work, we theoretically investigate the hot electron cooling power in 3DDS Cd$_3$As$_2$ by emission of optical phonons $P_{op}$ including the hot phonon effect. Numerical results are obtained as a function of electron temperature, electron density and phonon relaxation time. These results are compared with $P_{ac}$ in bulk Cd$_3$As$_2$ semiconductor and with $P_{ac}$ in 3DDS Cd$_3$As$_2$. This study is expected to provide thermal link between electrons and phonons in 3DDS Cd$_3$As$_2$ for its application in high speed/field devices.

The structure of the paper is shaped in the following way. In section 2 we provide all the theoretical ingredients including hot phonon effect, cooling power due to optical and acoustic phonons in 3DDS Cd$_3$As$_2$, and cooling power for 3DEG in bulk Cd$_3$As$_2$ semiconductor. The obtained results are discussed in section 3. Finally a summary of the present work is given in section 4.

2. Theory

In this section we develop a theory for the cooling power of hot electrons in 3DDS mediated by polar optical phonons. For comparison purpose we shall also provide the results for cooling power due to acoustic phonon in 3DDS as well as that in bulk Cd$_3$As$_2$ semiconductor due to polar optical phonon. Let us start with mentioning the basic properties of the physical system chosen.

2.1. Preliminary informations

We consider the Dirac fermion gas in a 3DDS Cd$_3$As$_2$, in which the low energy excitations are described by the Dirac-like linear dispersion $E_{k} = \hbar v_F |k|$ in the long wavelength continuum limit. Here, $v_F$ is the Fermi velocity, $k$ is the 3D electronic wave vector, and the band index $s$ takes the value $+1 (-1)$ for conduction(valence) band. The corresponding eigenstate is given by $\psi_k = (1/\sqrt{2V}) e^{i k g} r \chi^s$, where $V$ is the volume of the system, $\chi^+ = [\cos(\theta/2) \sin(\theta/2) e^{i \phi}]^T$ and $\chi^- = [\sin(\theta/2) - \cos(\theta/2) e^{i \phi}]^T$ with $\theta$ and $\phi$ are the polar and azimuthal angle in three dimensional $k$-space, respectively. The corresponding density of states is given by $D(E_k) = g_E E_k^2/(2\pi^2 \hbar^3 v_F^3)$, where $g = g_d g_e$, with $g_d(g_e)$ is the spin(valley) degeneracy.

2.2. Hot electron cooling power in 3DDS

In order to formulate a theory for the hot electron cooling power in 3DDS, we work in the ‘hot electron temperature model’ in which the electron gas is assumed to be in equilibrium with itself at an elevated temperature $T_e$ than the lattice temperature $T$. In this model Dirac fermions are assumed to have the usual Fermi-Dirac distribution $f(E_k) = [\exp\{\beta_c(E_k - \mu)\} + 1]^{-1}$ where $\beta_c = (k_B T_e)^{-1}$ and $\mu$ is the chemical potential determined by the electron density $n_e = \int f(E_k) D(E_k) dE_k$. The 3D Dirac fermions are assumed to interact with the 3D phonons of energy $\hbar \omega_q$ and wave vector $q$. The cooling power per electron $P$ (i.e. average electron energy loss rate) due to el–ph interaction can be obtained by using the well known technique described in [25]. It is given by

$$P = \frac{1}{N_e} \sum_q \hbar \omega_q \left( \frac{d N_q}{dt} \right)_{el-ph}. \tag{1}$$
where $N_e$ is total number of electrons and $N_q$ is the non-equilibrium phonon distribution function. The rate of change of $N_q$ due to electron–phonon interaction i.e. $\langle dN_q/dr \rangle_{el-ph}$ is given by using Fermi’s golden rule

$$
\left( \frac{dN_q}{dr} \right)_{el-ph} = \frac{2\pi \hbar}{\hbar} \sum_k |M(q)|^2 \left\{ (N_q + 1) f(E_k + \hbar \omega_q) \times \left[ 1 - f(E_k) - N_q f(E_k) [1 - f(E_k + \hbar \omega_q)] \right] + \delta(E_k + \hbar \omega_q - E_k - \hbar \omega_q) \right\},
$$

where $|M(q)|^2 = |g(q)|^2 F(\theta_{kk'})^2$ is the square of the matrix element for the el–ph interaction. Here, $|g(q)|^2$ is square of el–ph matrix element without chiral wave function and $F(\theta_{kk'})^2 = (1 + \cos \theta_{kk'})/2$ with $\theta_{kk'}$ being the angle between $k$ and $k'$, resulting from the chiral Dirac fermion.

One may also write equation (2) in the following form

$$
\left( \frac{dN_q}{dr} \right)_{el-ph} = \left[ (N_q + 1) e^{-\beta \hbar \omega_q} - N_q \right] \Gamma_q,
$$

where $\Gamma_q$ is given by

$$
\Gamma_q = \frac{2\pi \hbar}{\hbar} \sum_k |M(q)|^2 f(E_k) \left[ 1 - f(E_k + \hbar \omega_q) \right] \times \delta(E_k + \hbar \omega_q - E_k - \hbar \omega_q).
$$

As a result the cooling power (equation (1)) becomes

$$
P = \frac{1}{N_e} \sum_q \hbar \omega_q \left[ (N_q + 1) e^{-\beta \hbar \omega_q} - N_q \right] \Gamma_q.
$$

Our objective is to find hot electron cooling power $P_{op}$ due to optical phonons. The optical phonon energy $\hbar \omega_q = \hbar \omega_0$ is taken to be constant. The summation over $q$ in equation (5) can be converted into integral as

$$
\sum_q \rightarrow (V/2 \pi^2) \int_0^{2\pi} d\phi \int_0^{\pi} \sin \phi d\phi \int_0^{\pi} \sin \psi d\psi
$$

with $\phi$ and $\psi$ are the polar and azimuthal angle of $q$, respectively. Note that the integrations over $\phi$ and $\psi$ give $4\pi$. Defining $E_q = \hbar \nu q$, we find hot electron cooling power as

$$
P_{op} = \frac{\hbar \omega_0}{2\pi^2 N_e \hbar^3 \nu q} \int_0^{2\pi} d\phi \int_0^{\pi} \sin \phi d\phi \int_0^{\pi} \sin \psi d\psi \left[ (N_q + 1) e^{-\beta \hbar \omega_q} - N_q \right] \Gamma_q.
$$

An explicit evaluation of $\Gamma_q$ is given in the section 2.4.

2.3. Hot phonon distribution

Non-equilibrium phonon distribution $N_q$ can be obtained from the Boltzmann equation for phonon

$$
\left( \frac{dN_q}{dr} \right)_{el-ph} + \left( \frac{dN_q}{dr} \right)_{oth} = 0,
$$

where the first term describes the rate of change of the phonon distribution due to electron–phonon interaction while the later one denotes the same due to the other processes namely phonon–phonon interaction, surface roughness scattering etc.

In the relaxation time approximation one can write

$$
\left( \frac{dN_q}{dr} \right)_{oth} = -\frac{N_q - N_q^0}{\tau_p},
$$

where, $N_q^0 = [\exp(\beta \hbar \omega_0) - 1]^{-1}$ with $\beta = (k_B T)^{-1}$ is the phonon distribution at equilibrium and $\tau_p$ is phonon relaxation time due to all other mechanisms.

Hence, the non-equilibrium phonon distribution $N_q$ will be readily obtained from equation (7) as

$$
N_q = N_q^0 + \tau_p \left( \frac{dN_q}{dr} \right)_{el-ph}.
$$

Inserting equation (3) into equation (9) and considering $\hbar \omega_q = \hbar \omega_0$, we find $N_q$ as

$$
N_q = \frac{N_q^0 + \tau_p \Gamma_q e^{-\beta \hbar \omega_0}}{1 + \pi \tau_p \Gamma_q (1 - e^{-\beta \hbar \omega_0})}.
$$

2.4. Evaluation of $\Gamma_q$

Here, we shall provide an explicit evaluation of el–ph scattering rate $\Gamma_q$. We assume the electron-optical phonon interaction via Fröhlich coupling with the corresponding matrix element $|g(q)|^2 = 2\pi \hbar \omega_0 (\varepsilon_\infty - \varepsilon_0^{-1})/|V_q|^2$, where $\varepsilon_\infty (\varepsilon_0)$ is the high frequency (static) dielectric constant of the material. Momentum conservation $k' = k + q$ also allows us to write $|F(\theta_{kk'})|^2$ as

$$
|F(\theta_{kk'+q})|^2 = \frac{1}{2} \left( 1 + \frac{E_k + E_q \cos \theta}{E_k + \hbar \omega_0} \right).
$$

Converting the summation over $k$ into integrals like

$$
\sum_k \rightarrow (V/2 \pi^3) \int dk d\phi d\theta d\omega
$$

equation (4) becomes

$$
\Gamma_q = \frac{2\pi \hbar}{V (2\pi)^3} \int k^2 dk d\phi d\theta (|g(q)|^2 |F(\theta_{kk'+q})|^2)
$$

$$
\times f(E_k) [1 - f(E_k + \hbar \omega_0)] \delta[X(q)],
$$

where the argument of the delta function is

$$
X(q) = (E_k^2 + E_q^2 + 2E_k E_q x)^{\frac{1}{2}} - E_k - \hbar \omega_0
$$

with $x \equiv \cos \theta$. Note that the $\phi$-integral in equation (12) gives $2\pi$ since the integrand is independent of $\phi$. The $x$-integral can be evaluated by the following property of the delta function

$$
\delta[X(x)] = \frac{\delta(x - x_i)}{dx \mid_{x=x_i}},
$$

where the root $x_i$ of equation (13) can be obtained as

$$
x_i = \frac{(\hbar \omega_0)^2 + 2E_k \hbar \omega_0 - E_k^2}{2E_k E_q}.
$$

Hence, after doing the angular integrations, equation (12) becomes
\[
\Gamma_q = g \frac{V|g(q)|^2}{2\pi\hbar^2 \hbar^2} \int_{\hbar^2}^{\infty} dE_k E_k \left[ f(E_k) - f(E_k + \hbar \omega_0) \right]^2 \\
\quad \times f(E_k) \left[ 1 - f(E_k + \hbar \omega_0) \right] (E_k + \hbar \omega_0),
\] (16)

where \( E_k^{\text{min}} = (E_q - \hbar \omega_0)/2 \). This lower limit of \( E_k \)-integral is a consequence of the fact that \(-1 \leq x \leq 1\). Note also that

\[
|F(E_k, E_q)|^2 = \frac{1}{2} \left[ 1 + \frac{(E_k + \hbar \omega_0)^2 + E_k^2 - E_q^2}{2E_k (E_k + \hbar \omega_0)} \right].
\] (17)

2.5. Hot electron cooling power due to acoustic phonons in 3DDS

As we are giving results for hot electron cooling power \( P_{\text{ac}} \) due to acoustic phonons, for the sake of comparison with \( P_{\text{op}} \), the corresponding expression is given by [38]

\[
P_{\text{ac}} = \frac{gD^2}{8\pi \hbar^3 \hbar^2 \hbar^2 v_s v_s} \int_0^{\infty} dE_k \\
\quad \times \left[ \frac{\hbar^2}{2m^*} \frac{1}{m} \frac{\hbar^2}{2m^*} \frac{1}{m} \frac{\hbar^2}{2m^*} \frac{1}{m} \frac{\hbar^2}{2m^*} \frac{1}{m} \frac{\hbar^2}{2m^*} \frac{1}{m} \frac{\hbar^2}{2m^*} \frac{1}{m} \frac{\hbar^2}{2m^*} \frac{1}{m} \right] \chi(q, k) \\
\quad \times [N_q(T_s) - N_q(T)] [f(E_k) - f(E_k + \hbar \omega_q)],
\] (18)

where \( D \) is the acoustic phonon deformation potential constant, \( v_s \) is the acoustical phonon velocity, \( \rho_m \) is the mass density, \( \chi(q, k) = 1 - q^2/(4k^2) \), \( \omega_q = v_s q \), and \( \omega_0^2 = 2v_s k \). The temperature dependent screening function is given by \( \epsilon(q, T) = 1 + \Pi(q, T) \), where \( \Pi(q, T) \) is the finite temperature static polarizability, evaluated explicitly in [24].

2.6. Hot electron cooling power due to optical phonons in 3D semiconductor

With a view to compare the results of the 3DDS Cd3As2 with those in 3D Cd3As2 semiconductor, we give the expression for the \( P_{\text{op}} \) for a 3DEG in bulk Cd3As2 semiconductor. Taking the parabolic energy relation \( E_k = \hbar^2 k^2/(2m^*) \) \( (m^* \) is the effective mass of electron), it is given by

\[
P_{\text{op}} = \frac{m^* \hbar^2}{2\pi^2 \hbar^2} \int_0^{\infty} dE_k E_k^2 \\
\quad \times [(N_q + 1)e^{-\beta \hbar \omega_0} - N_q] \Gamma_q.
\] (19)

In this case \( E_q = \hbar^2 q^2/(2m^*) \). The corresponding el–ph scattering rate \( \Gamma_q \) is given by

\[
\Gamma_q = \frac{g \hbar^2}{2\pi^2 \hbar^2 E_q^2} \int_{E_q}^{\infty} dE_k f(E_k) \left[ 1 - f(E_k + \hbar \omega_0) \right],
\] (20)

where \( E_q = (\hbar \omega_0 - E_k)^2/(4E_q) \).

2.7. Electron energy relaxation time

Some times it is useful to study the hot electron relaxation in terms of corresponding relaxation time \( \tau_e \) (see

| Parameter | Symbol | Value |
|-----------|--------|-------|
| Lattice constant | \( a \) | 4.6 Angstrom |
| Effective mass of electron | \( m^* \) | 0.036 \( m_0 \) |
| Mass density of ion | \( \rho_m \) | 7 \times 10^3 Kg m^-3 |
| Degeneracy | \( D \) | 20 eV |
| For 3DDS | \( g \) | 4 |
| For 3DEG | \( g \) | 2 |
| Sound velocity | \( v_s \) | 2.3 \times 10^3 m s^-1 |
| Fermi velocity | \( v_F \) | 10^6 m s^-1 |
| Deformation potential | \( D \) | 20 eV |
| Optical phonon energy | \( h\omega_0 \) | 25 meV |
| Dielectric constant (High Freq.) | \( \varepsilon_\infty \) | 12 |
| Dielectric constant (Static) | \( \varepsilon_0 \) | 36 |
| Electron density | \( n_e \) | 0.1–3 \( n_0 \) |
| Lattice temperature | \( T \) | 4.2 K |
| Electron temperature | \( T_e \) | (5–300) K |

Table 1. Numerical values of the parameters used in calculations. Note that the knowledge of the lattice constant \( a \) is required to calculate \( \Pi(q, T) \), given in [24].
Figure 1. Non-equilibrium distribution of polar optical phonon \( N_q \) in 3DDS Cd\textsubscript{3}As\textsubscript{2} as a function of phonon wave vector \( q \) for different \( T_e \), \( \tau_p \), and \( n_e \).

Figure 2. Non-equilibrium distribution of polar optical phonon \( N_q \) in 3DEG Cd\textsubscript{3}As\textsubscript{2} semiconductor as a function of phonon wave vector \( q \) for different \( T_e \), \( \tau_p \), and \( n_e \).

Further increase of \( q \), the \( N_q \) is found to decrease first very slowly and then gradually to zero. Similar observations are made in GaAs heterojunctions [43], bilayer graphene [44] and monolayer MoS\textsubscript{2} [35]. Writing \( N_q \) as the hot phonon number \( N_q(T_{ph}) \) given by Bose distribution at an effective hot phonon temperature \( T_{ph} \), it can be shown from equation (10) that, for \( \Gamma_q > \tau_p^{-1} \), \( T_e \gg T \), and \( \beta_q \sim (\hbar \omega_q)^{-1} \) the \( N_q(T_{ph}) \) approaches the \( N_q(T_e) \).

In figure 1(a), \( N_q \) versus \( q \) is shown for different \( T_e = 50 \), 100, 200, and 300 K with \( n_e = n_0 \) and \( \tau_p = 5 \) ps. The phonon number \( N_q \) increases with increasing \( T_e \), as expected on the physical ground that electrons with larger \( T_e \) can emit large number of phonons. The \( N_q \) corresponding to \( T_e = 50 \) K is very small compared to the other \( T_e \) values, with largest value of \( N_q \approx 0.6 \) corresponding to 300K. The width of the maximum is larger for smaller \( T_e \), similar to the findings in monolayer MoS\textsubscript{2} [35].

\( N_q \) dependence on phonon relaxation time \( \tau_p \) is shown in figure 1(b) by plotting \( N_q \) versus \( q \) for \( \tau_p = 0, 1, 2, \) and 5 ps with \( n_e = n_0 \) and \( T_e = 100 \) K. Number of hot phonons is found to be larger for larger relaxation time, as anticipated. Secondly, we notice that the range of \( q \) for which \( N_q \) remains maximum and nearly constant (i.e. width of the maximum) is larger for phonons with larger \( \tau_p \).

The effect of \( n_e \) on hot phonon distribution is shown in figure 1(c) for \( \tau_p = 5 \) ps at \( T_e = 100 \) K. The electron density chosen are \( n_e = 0.5n_0, n_0, 2n_0, \) and \( 3n_0 \). \( N_q \) is found to be larger for larger \( n_e \) because larger density of electrons emit large number of phonons. Moreover, width of the maximum is larger for larger \( n_e \). Similar observation is made in GaN/AlGaN heterostructure [45].

For comparison we have shown \( N_q \) versus \( q \) in figure 2 for different \( T_e, \tau_p \), and \( n_e \) for 3DEG in bulk Cd\textsubscript{3}As\textsubscript{2} semiconductor. We see that maximum of \( N_q \) in this semiconductor is almost same as found for 3DDS Cd\textsubscript{3}As\textsubscript{2} for each of the \( T_e \). But, unlike in 3DDS Cd\textsubscript{3}As\textsubscript{2}, this maximum is spread over relatively a small range of \( q \). The maximum occurs around \( q \approx 1 \times 10^8 \) m\textsuperscript{-1} which is closer to the phase matching value of \( q_0 = (2m\omega_q/h)^{1/2} = 1.54 \times 10^8 \) m\textsuperscript{-1}. In the low \( q \) region \( N_q \) increases rapidly with \( q \), and after reaching the maximum it gradually decreases. Finally it vanishes at relatively larger \( q \) values as compared to 3DDS. From figure 2(c), in which \( N_q \) is shown for different \( n_e \), we see that, for Cd\textsubscript{3}As\textsubscript{2} semiconductor \( N_q \) remains same, unlike the case of 3DDS Cd\textsubscript{3}As\textsubscript{2} (figure...
1(e)), for certain range of q for all n_e. Beyond this, N_q is found to be larger for larger n_e at larger q.

In figure 3(a), we have shown electron cooling power P_{op}, due to optical phonons, as a function of T_e for different electron relaxation time \( \tau_p = 0, 1, 2, \) and 5 ps. The curve for \( \tau_p = 0 \) ps corresponds to P_{op} without hot phonon effect. In low T_e (about <50 K), all the curves show rapid increase of P_{op} with T_e. This is expected because in low T_e region \( \hbar \omega_0/(k_B T_e) \) is large and it decreases significantly with increase of T_e. Consequently the optical phonon emission increases as \( \sim \exp((-\hbar \omega_0)/(k_B T_e)) \). In the higher T_e region, P_{op} increases slowly. This behavior may be approximately put as \( \exp((-\hbar \omega_0)/(k_B T_e)) \) attributing to the exponential growth of occupation of electron states with high enough energy to emit optical phonons. At T_e = 300 K, the \( \hbar \omega_0/(k_B T_e) \) is nearly 1 and P_{op} is nearly constant.

We find that the hot phonon effect reduces P_{op} significantly. However, the hot phonon effect is larger for \( \tau_p < 1 \) ps and relatively smaller for higher values of \( \tau_p \). For comparison, for \( \tau_p = 1 \) ps, at T_e = 100 and 300 K, the reduction factors are R_{op} = 7.37 and 6.86, respectively. For \( \tau_p = 5 \) ps, at T_e = 100 and 300 K, the reduction factors, respectively, are R_{op} = 19.21 and 15.72. This observation indicates that hot phonon effect is sensitive at low T_e and less sensitive at higher T_e.

For comparison, P_{op} is shown as a function of T_e, in figure 3(b), for 3DEG in Cd_3As_2 semiconductor for \( \tau_p = 0, 1, 2, \) and 5 ps. It is found that without hot phonon effect (i.e. \( \tau_p = 0 \)), P_{op} in 3DEG is much larger than that in 3DDS in the temperature regime T_e < 30 K. Above T_e ~ 40 K, the corresponding values of P_{op} are almost same. With hot phonon effect (\( \tau_p \neq 0 \)), P_{op} in 3DEG is larger than P_{op} in 3DDS over the entire range of temperature considered. The difference between the values of P_{op} in 3DEG and 3DDS is huge below T_e ~ 30 K. However, this difference above T_e ~ 40 K is small and it is increasing with \( \tau_p \). In Cd_3As_2 semiconductor, we find for \( \tau_p = 1 \) ps at T_e = 100(300) K, R_{op} = 4.17(3.44). For \( \tau_p = 5 \) ps, it is obtained R_{op} = 9.52(7.29) at T_e = 100(300) K. These values of R_{op} are smaller than that found in 3DDS. This larger reduction of P_{op} in 3DDS Cd_3As_2 indicates that, at a given T_e, hot phonon population is more in this system compared to 3D Cd_3As_2 semiconductor. This can be seen from the more broader maximum of N_q in the former system.

In figures 4(a) and (b) P_{op} is shown as a function of T_e for different n_e in 3DDS (3DEG) Cd_3As_2 taking \( \tau_p = 5 \) ps. In 3DDS Cd_3As_2 (figure 4(a)) P_{op} is found to be smaller for larger n_e. For about T_e < 50 K, P_{op} is found to be more sensitive to n_e and the dependence becomes weaker at higher T_e. However, the situation is different for 3DEG in Cd_3As_2 semiconductor. As depicted in figure 4(b) the n_e sensitivity of P_{op} is more in high T_e regime than that in low T_e range.

In figure 5, we show temperature dependence of hot electron cooling power due to acoustic phonons P_{ac} and optical phonons P_{op} considering hot phonon effect (with \( \tau_p = 5 \) ps) as well as the total P_T = P_{ac} + P_{op}. P_{ac} increases superlinearly in low T_e region and then increases less slowly at higher T_e. It is also found that, in this temperature range, P_{ac} is smaller for smaller n_e, unlike the case of P_{op}. In figure 5(b), we see that there is a crossover from acoustic phonon to optical phonon dominated cooling power. The T_e at which the crossover takes place depends on n_e significantly. It is found that crossover takes place at T_e ~ 25, 30, 39, and 45 K for n_e = 0.5n_0, n_0, 2n_0, and 3n_0, respectively. Optical phonon is the active channel of power dissipation above this T_e. The crossover temperature may depend on \( \tau_p \) also. Considering n_e = 0.5n_0, the crossover T_e = 25 K may be compared with about 20 K in InSb [27] and 35 K in GaAs [28, 30] bulk semiconductors noting that these samples are non-degenerate. The T_e = 25 K above which P_{op} is dominating P_T in Cd_3As_2 is closer to that in bulk InSb as optical phonon energies in these two systems are closer. It is to be noted that in InSb(GaAs) \( \hbar \omega_0 \) is 24.4(36.5) meV. In the
neighborhood of crossover $T_e$, $P_T$ shows a knee like behavior as found in InSb [27] and GaAs [28].

In the Bloch–Grüneisen (BG) regime, $T_e \ll T_{BG} = (2h\nu/k_B)$, where $k_B$ is the Fermi wave vector, the $P_{ac}$ dependence on $T_e$ and $n_e$ are shown to be given by the power laws $P_{ac} \sim T_e^\alpha$ and $n_e^{-\delta}$ where $\alpha = 9(5)$ and $\delta = 5/3(1/3)$ with(out) screening of electron-acoustic phonon interaction [38]. In relatively higher $T_e$ ($> T_{BG}$) regime, disorder assisted $P_{ac}$ calculations show drastic increase of cooling power due to enhanced energy transfer between electrons and acoustic phonons [37].

In figures 6(a) and (b) $P_{op}$ is shown as a function of $n_e$ for different $T_e$ in 3DDS (3DEG) Cd$_3$As$_2$ taking $\tau_p = 5$ ps. In 3DDS Cd$_3$As$_2$, $P_{op}$ is found to decrease weakly with increasing $n_e$. This decrease may be attributed to partial reabsorption of large number of phonons emitted by larger $n_e$. The decrease is, relatively, faster(slower) at low(high) $n_e$. Moreover, it is found that, compared to 3DEG in Cd$_3$As$_2$ semiconductor (figure 6(b)), $P_{op}$ in 3DDS Cd$_3$As$_2$ is less sensitive to $n_e$. This may be attributed to differing density of states. For example, while varying $n_e$ from 0.1$n_0$ to 3$n_0$ $P_{op}$ decreases, at $T_e = 300$ K, by a factor of about 2.75(5) in 3DDS (3DEG) Cd$_3$As$_2$. At $T_e = 100$ K the respective changes are 1.82 and 4.29.

Electron cooling power due to optical phonons in bulk GaAs is shown to be reduced by screening effect [29]. In 3DDS Cd$_3$As$_2$ also we expect the screening to reduce $P_{op}$. Although screening effect is not considered in our $P_{op}$ calculations, the experimental measurements will be able to indicate its necessity.

We would like to mention that our numerical calculations of $P$ can be used to calculate energy relaxation time $\tau_e$ as given in section 2.7. For $n_e = n_0$, the average electron energy is found to be $\langle E(T_e) \rangle = 0.1212$, 0.1235, and 0.1403 eV at $T_e = 4.2$, 100, and 300 K, respectively. Consequently, taking $P_{op} = 3.417(45.364) \times 10^8$ eV s$^{-1}$ at $T_e = 100(300)$ K we find energy relaxation time $\tau_e = 6.73(4.21)$ ps. These values are nearly same as the values of $\tau_e$ obtained for GaAs quantum wells and are much greater than the values in monolayer graphene (see figure 5 of Baker et al [33]) and bilayer graphene (see figure 8 in Huang et al [34]). Consequently, Cd$_3$As$_2$ has an advantage over monolayer and bilayer graphene in the applications to the devices where longer energy relaxation

![Figure 4](image_url)  
Figure 4. Temperature dependence of $P_{op}$ for different values of $n_e$ at $\tau_p = 5$ ps. Left and right panels show the results corresponding to 3DDS Cd$_3$As$_2$ and for 3DEG in Cd$_3$As$_2$ semiconductor, respectively.

![Figure 5](image_url)  
Figure 5. Temperature dependence of $P_{ac}$ and $P_{op}$ in 3DDS Cd$_3$As$_2$ for different values of $n_e$: (a) $P_{ac}$ in the range $T_e = 5$–100 K and (b) $P_{ac}$ and $P_{op}$ in the range $T_e = 5$–300 K, (c) $P_T = P_{ac} + P_{op}$. Note that $P_{op}$ is for $\tau_p = 5$ ps.
time is preferred. For example in photodetectors, photo-thermoelectric devices and energy harvesting devices like hot carrier solar cells. However, simple power laws can be obtained in BG regime, where $P_{ac}$ is the sole contributor to electron cooling power, with regard to $T_e$ and $n_e$ dependence. Using the BG regime results, we find $\tau_e \sim T_e^{-3}(T_e^{-3})$ and $n_e^{4/3}(n_e^{4/3})$ for screened (unscreened) electron-acoustic phonon interaction.

In Table 2, the BG regime power laws are given for $P_{ac}$ and $\tau_e$ dependence on $T_e$ and $n_e$ for various electron gas systems. In Cd$_3$As$_2$, for unscreened electron-acoustic phonon coupling, the predicted $n_e$ independence of $\tau_e$ in BG regime is same as in monolayer graphene (Huang et al in [34]). However, the screened electron-acoustic phonon coupling gives a strong dependence $\tau_e \sim n_e^{4/3}$. This can be exploited to achieve faster energy relaxation (smaller $\tau_e$), by tuning $n_e$ to the lower carrier density, which finds applications in ultrafast electronics and high speed communications.

We would also like to mention here that although we have presented calculations for $T = 4.2$ K and $T_e = 5–300$ K, our theory is valid for higher temperatures also. For this we have calculated $P_{op}$ versus $T_e$ (305–550 K) for different $\tau_p$ taking $T = 300$ K. These calculations are shown in figure 7. We find $P_{op}$ increases rapidly for $T_e$ closer to $T$, then increases slowly at higher $T_e$. The hot phonon effect at higher temperature is similar as found in figure 3(a).

We point out that there are some similarities and differences between dynamic and steady state hot electron energy relaxation. In both the cases the carriers, soon after photo excitation/ application of the field, initially, thermalize in faster relaxation time, on several femtosecond time scale, with the carrier distribution described by the Fermi-Dirac distribution with an elevated electron temperature $T_e$. This is followed by slower hot electron cooling due to energy transfer to phonons. Besides, two temperatures $T_e$ and $T$ are used to explain the phenomenon of

table 2. Temperature($T_e$) and carrier density($n_e$) dependence of $P_{ac}$ and $\tau_e$ in various 3D and 2D materials due to screened and unscreened electron-acoustic phonon scattering are shown.

| System                                      | Screened | Unscreened |
|---------------------------------------------|----------|------------|
| 3DDS                                        | $T_e n_e^{-3/2}$ [35] | $T_e n_e^{-3/2}$ |
| 3DEG in semiconductor: deformation potential coupling | $T_e^{-1/2} n_e^{3/2}$ [46] | $T_e^{-1/2} n_e^{3/2}$ |
| 3DEG in semiconductor: piezoelectric coupling | $T_e^{-1/2} n_e^{3/2}$ [46] | $T_e^{-1/2} n_e^{3/2}$ |
| Monolayer graphene                          | $T_e n_e^{-3/2}$ [35] | $T_e n_e^{-3/2}$ |
| Bilayer graphene and monolayer MoS$_2$      | $T_e^{-1/2} n_e^{3/2}$ [46] | $T_e^{-1/2} n_e^{3/2}$ |
| Conventional 2DEG: deformation potential coupling | $T_e n_e^{-3/2}$ [35] | $T_e n_e^{-3/2}$ |
| Conventional 2DEG: piezoelectric coupling   | $T_e^{-1/2} n_e^{3/2}$ [46] | $T_e^{-1/2} n_e^{3/2}$ |

* Kubakaddi in [33].
* Viljas et al in [33].
* Huang et al in [34].
* Bhargavi and Kubakaddi in [34].
* Ma et al in [32].

In the paper by Kubakaddi in [33], for screened-acoustic phonon interaction the typo error $F(T) \sim T^4$ has to be corrected as $F(T) \sim T^6$.
relaxation. In dynamic process relaxation of $T_e$ versus time delay is qualitatively studied, in two-temperature model by a simple theory and the electron–phonon strength is estimated by getting agreement with the experimental observations [15]. However, the quantitative model for studying the dynamic behavior is still lacking [14]. Moreover, the relaxation regions due to acoustic and optical phonons seem yet to be resolved. In graphene [47], the dynamics of $T_e$, described by $\Delta T_e (= T_e - T) \propto \exp (-t/\tau_e)$, is studied by finding the carrier cooling time $\tau_e$ after photoexcitation, as a function of $T_e$. In the steady state, the cooling power, by emission of acoustic or optical phonons, in as our present study, is quantitatively studied as a function of $T_e$ and $n_e$ by energy balance equation. Experimentally, $T_e$ is measured as a function of dc power input. These studies clearly indicate the independent role and the strength of acoustic and optical phonons, in the cooling process, in different temperature regimes. This is the case even in the study of other electron systems as mentioned in the introduction.

4. Summary

In summary, we have studied optical phonon limited cooling of hot electrons in 3DDS Cd$_3$As$_2$ considering the effect of hot phonon. The dependence of electron cooling power $P_{op}$, due to optical phonon, on electron temperature $T_e$, electron density $n_e$, and phonon relaxation time $\tau_p$ are investigated. $P_{op}$ is found to increase much rapidly with $T_e$ at low temperature regime while this increase becomes much slower in high $T_e$ regime. The dependence of $P_{op}$ on $n_e$ is weak. It shows a slow decrease with the increase of $n_e$. We compare the results with those corresponding to 3DEG in Cd$_3$As$_2$ semiconductor. It is revealed that hot phonon effect is stronger in 3DDS Cd$_3$As$_2$ than in Cd$_3$As$_2$ semiconductor. It is also found that $P_{op}$ is more (less) sensitive to $n_e$ in 3DEG (3DDS). Additionally, $P_{op}$ is compared with the acoustic phonon limited hot electron cooling power $P_{ac}$. A crossover from $P_{ac}$ dominated cooling at low $T_e$ to $P_{op}$ dominated cooling at higher $T_e$ takes place at about $T_e = 25$ K for $n_e = 0.5n_0$. The crossover $T_e$ shifts towards higher temperature for larger $n_e$. We point out that our calculations need to be tested against the experimental data. We suggest for steady state/electric field experiments in $n$-type 3DDS Cd$_3$As$_2$ to which our present calculations will be directly related. We have also shown that the energy relaxation time can be tailored by tuning the electron–phonon interactions to achieve faster or slower energy loss for suitable hot electron applications of Cd$_3$As$_2$ in electronic and optoelectronic devices.

**References**

[1] Wang Z, Sun Y, Chen X Q, Franchini C, Xu G, Weng H, Dai X and Fang Z 2012 Phys. Rev. B [85]195320
[2] Wang Z, Weng H, Wu Q, Dai X and Fang Z 2013 Phys. Rev. B [88]125427
[3] Borisenko S, Gibson Q,Evtushinsky D, Zabolotnyy V, Büchner B and Cava R J 2014 Phys. Rev. Lett. [113]027603
[4] Liu Z K et al 2014 Nat. Mater. [13]677
[5] Neupane M et al 2014 Nat. Commun. [5]3786
[6] Jeon S, Zhou B B, Gynis A, Feldman B E, Kimichi I, Potter A C, Gibson Q D, Cava R J, Vishwanath A and Yazdani A 2014 Nat. Mater. [13]851
[7] He L P, Hong X C, Dong J K, Pan J, Zhang Z, Zhang J and Li S Y 2014 Phys. Rev. Lett. [113]246402
[8] Liu Z K et al 2014 Science [343]864
[9] Liang T, Gibson Q, Ali M N, Liu M, Cava R J and Ong N P 2015 Nat. Mater. [14]280
[10] Zhao Y et al 2015 Phys. Rev. X [5]031037
[11] Weber C P, Arushanov E, Berggren B S, Hosseini T, Kouklin N and Nateprov A 2015 Appl. Phys. Lett. [106]231904
[12] Weng H, Dai X and Fang Z 2016 J. Phys.: Condens. Matter [28]303001
[13] Wang Q et al 2017 Nano Lett. [17]8343
[14] Lu W, Ge S, Liu X, Lu H, Li C, Lai J, Zhao C, Liao Z, Jia S and Sun D 2017 Phys. Rev. B [95]024303
[15] Zhu C, Yuan X, Xiu F, Zhang C, Xu Y, Zhang R, Shi Y and Wang F 2017 Appl. Phys. Lett. [111]091101
[16] Weber C P et al 2017 J. Appl. Phys. [122]223102
[17] Zhu C et al 2017 Nat. Commun. [8]14111
[18] Feng J, Pang Y, Wu D, Wang Z, Weng H, Li J, Dai X, Fang Z, Shi Y and Lu L 2015 Phys. Rev. B [92]081306
[19] Narayanan A et al 2015 Phys. Rev. Lett. [114]117201
[20] Li H, He H, Lu H Z, Zhang H, Liu H, Ma K, Fan Z, Shen S Q and Wang J 2016 Nat. Commun. [7]10301
[21] Chen Z G, Zhang C, Zou Y, Zhang E, Yang L, Hong M, Xiu F and Zou J 2015 Nano Lett. [15]5830
[22] Zhang E, Liu Y, Wang W, Zhang C, Zou P, Chen Z, Zou J and Xiu F 2015 ACS Nano [9]8843
[23] Lundgren R, Laurell P and Fiete G A 2014 Phys. Rev. B [90]165115
[24] Das Sarma S, Hwang E H and Min H 2015 Phys. Rev. B [91]035201
[25] Conwell E M 1967 High Field Transport in Semiconductors (New York: Academic)
[26] Seeger K 2004 Semiconductor Physics: an Introduction (Springer Series in Solid State Sciences) 9th edn (Berlin: Springer)
[27] Sandercock J R 1965 Proc. Phys. Soc. [86]1221
[28] Maneval J P, Zylbersztejn A and Budd H F 1969 Phys. Rev. Lett. [23]848
[29] Kubakaddi S S and Krishnamurthy B S 1975 Phys. Lett. A [54]389

**ORCID iDs**

Tutul Biswas 🌐 https://orcid.org/0000-0002-7807-905X
