Lepton number violation by heavy Majorana neutrino in $B$ decays

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Abstract

Heavy Majorana neutrinos are predicted in addition to ordinary active neutrinos in the models with the seesaw mechanism. We investigate the lepton number violation (LNV) in $B$ decays induced by such a heavy neutrino $N$ with GeV-scale mass. Especially, we consider the decay channel $B^+ \rightarrow \mu^+ N \rightarrow \mu^+ \mu^+ \pi^-$ and derive the sensitivity limits on the mixing angle $\Theta_\mu$ by the future search experiments at Belle II and in $e^+e^-$ collisions at the Future Circular Collider (FCC-ee).
1 Introduction

The discovery of neutrino oscillations, showing the non-zero neutrino masses, has opened the door to physics beyond the Standard Model (SM). The oscillation experiments so far have provided the rather precise values of mass squared differences and mixing angles of active neutrinos [1]. There are, however, unknown properties of active neutrinos, i.e., the ordering and the absolute values of neutrino masses, the violation of CP symmetry in the leptonic sector and the Dirac or Majorana property of neutrinos. In addition, we do not know whether an additional particle is present associated with the origin of neutrino masses.

Heavy neutrino is an well-motivated particle in the models of neutrino masses. One of the most attractive examples is the model with the canonical seesaw mechanism [2–7] where right-handed neutrinos are introduced with Majorana masses. In this case the mass eigenstates are three active neutrinos and heavy neutrinos, and both neutrinos are Majorana particles. Usually, heavy neutrinos are considered to be much heavier than $m_W$ and even close to the unification scale $\sim 10^{16}$ GeV. Such heavy particles are attractive since they can also account for the baryon asymmetry of the universe (BAU) via leptogenesis [8].

On the other hand, heavy neutrinos with masses below $m_W$ are also attractive. Even in this case the seesaw mechanism is still effective by requiring the suppressed Yukawa coupling constants of neutrinos. Furthermore, the BAU can be explained by using the different mechanism [9,10]. Heavy neutrinos with $\sim 100$ MeV are interesting for the supernova explosion [11]. If its mass is around keV scale, it can be a candidate for the dark matter [12]. Further it may explain the origin of pulsar velocities [13–16]. (See, for example, Ref. [17] for astrophysics of heavy neutrinos.) Therefore, heavy neutrinos which are lighter than the electroweak scale are also well-motivated particles beyond the SM. Interestingly, such particles can be tested in terrestrial experiments [18–20].

If neutrinos are Majorana particles, the lepton number of the SM Lagrangian is broken. In this case there appear various phenomena which are absent in the SM. The contribution from heavy Majorana neutrino can be significant depending on its mass and mixing. The well-known example is the neutrinoless double beta decay $(Z,A) \rightarrow (Z+2,A) + 2e^−$. See, for example, a recent review [21] and references therein. When the mass is of the order of 0.1–1 GeV, the contribution from heavy Majorana neutrino can be significant to alter the prediction of the rate solely from active neutrinos.

The LNV process $e^- e^- \rightarrow W^- W^-$ (called as the inverse neutrinoless double beta decay [22]) is another interesting possibility to test the Majorana property of heavy neutrino. Various aspects of this process have been investigated so far. See, for example, the recent analyses in Refs. [23,24] and the references therein. It is a good target of the future lepton colliders such as the International Linear Collider (ILC) [25] and the Compact Linear Collider (CLIC) [26].

Another example is the rare decay of meson like $M^+ \rightarrow \ell^+ \ell^+ M'$ where $M$ and $M'$ are mesons and $\ell$ and $\ell'$ are charged leptons with the same charge [27,36]. See the current experimental limits on these processes in Refs. [19,37]. Heavy Majorana neutrino with an appropriate mass gives a sizable
contribution to these processes, and its mixing receives the upper bounds from the experimental data.

In this paper we discuss the LNV decay of $B$ mesons induced by heavy Majorana neutrino with GeV-scale mass. In particular, we study the testability of the mode $B^+ \rightarrow \mu^+ \mu^+ \pi^-$ by the future experiments. The expected limits on the mixing of heavy neutrino by Belle II \[38\] and the $e^+e^-$ collisions on Z-pole at the future circular collider (FCC-ee) \[39\] will be presented.

## 2 Heavy Majorana neutrino

We consider a heavy Majorana neutrino $N$ with mass $M_N \sim \text{GeV}$ which mixes with ordinary left-handed neutrinos $\nu_{La} (\alpha = e, \mu, \tau)$ as

$$\nu_{La} = U_{ai} \nu_i + \Theta_{\alpha} N,$$

where $U_{ai}$ is the PMNS mixing matrix of active neutrinos $\nu_i (i = 1, 2, 3)$. In this case $N$ have the weak gauge interactions which are suppressed by the mixing $\Theta_{\alpha}$. Here we discuss only one heavy neutrino for simplicity, but the extension to the case with more heavy neutrinos is straightforward by replacing $\Theta_{\alpha} N$ with $\sum_i \Theta_{\alpha i} N_i$.

If heavy neutrinos provide the tiny neutrino masses through the seesaw mechanism, the masses and mixings of heavy neutrinos must satisfy a certain relation to explain the experimental results of the neutrino oscillations. However, we do not specify the origin of $N$ to make a general argument and consider $M_N$ and $\Theta_{\alpha}$ as free parameters in this analysis.

It is possible to test directly heavy neutrino $N$ by various experiments because of the smallness of its mass. Since there is no signal of this particle, the upper bounds on the mixing $|\Theta_{\alpha}|$ are imposed from various experiments depending on its mass \[16-20\]. It is then important to search it by future experiments at the first step. Furthermore, not only the discovery but also the detail study is crucial to reveal the properties of $N$.

In the present analysis we consider the experimental test for the LNV to show the Majorana property of $N$. Especially, we focus on the LNV decay of $B$ meson as a concrete example\[\#1\]

$$B^+ \rightarrow \mu^+ N \rightarrow \mu^+ \mu^+ \pi^-,$$

which is mediated by the on-shell $N$ as shown in Fig. 1. Notice that there is also the charge conjugated process which is implicit from now on. From the kinematical reason we restrict ourselves to the mass region

$$m_B - m_{\mu} > M_N > m_\pi + m_{\mu}.$$

\[\#1\]In this analysis we discuss only the decay into two muons, but the extension to the decays into the like sign leptons with other flavors is straightforward.
In the process \( \mathcal{H} \) the production rate of \( N \) is proportional to \( |\Theta_\mu|^2 \) and the decay rate is also proportional to \( |\Theta_\mu|^2 \), and then the LNV signal is induced as the \( |\Theta_\mu|^4 \) effect. This process has been discussed as an interesting target for Belle and LHCb experiments \([19,33-36]\).

The recent results of the search for \( B^+ \rightarrow \mu^+ \mu^- \pi^- \) are obtained by Belle \([40]\) and LHCb \([41]\). (See also Ref. \([42]\) for the revision of the LHCb limit.) They presented the upper bounds on the mixing \( |\Theta_\mu|^2 \) as shown in Fig. 2. In the same figure we also present various constraints on heavy neutrino which are from Ref. \([19]\). It is found that these bounds on \( |\Theta_\mu|^2 \) are weaker than other constraints on heavy neutrino which are applicable to both Dirac and Majorana cases.

The future prospect of the LHCb search for the LNV decays of \( B \) and \( B_c \) mesons including \( \mathcal{H} \) has been discussed in Ref. \([35]\). The sensitivity on the mixing by using the mode \( B^+_c \rightarrow \mu^+ \mu^- \pi^- \) at LHC run 3, which is better than that of \( \mathcal{H} \), is also shown in Fig. 2. In the present analysis, we then investigate the search for the process \( \mathcal{H} \) at Belle II and FCC-ee.

### 3 Search at Belle II

Let us first consider the search for the LNV decay of \( B^+ \) shown in Eq. \( \mathcal{H} \) at Belle II \([38]\), where \( 5 \times 10^6 \) pairs of \( B \) mesons (at 50 ab\(^{-1}\)) are planned to be produced. In this analysis we take the number of \( B^+ \) as \( N_B = 5 \times 10^6 \) and the energy as \( E_B = m_{B^\pm} \) since the velocity of produced \( B^\pm \)'s is low enough. Let us then estimate the expected number of the signal events below.

First, the partial decay rate of \( B^+ \rightarrow \mu^+ N \) is given by

\[
\Gamma(B^+ \rightarrow \mu^+ N) = \frac{G_F^2 f_{B^\pm}^2 m_{B^\pm}^3}{8\pi} |V_{ub}|^2 |\Theta_\mu|^2 \left[ r_\mu^2 + r_N^2 - (r_\mu^2 - r_N^2)^2 \right] \sqrt{1 - 2(r_\mu^2 + r_N^2) + (r_\mu^2 - r_N^2)^2},
\]

where \( f_{B^\pm} \) is the decay constant, \( V_{ub} \) is the CKM element, and

\[
r_\mu = \frac{m_\mu}{m_{B^\pm}}, \quad r_N = \frac{M_N}{m_{B^\pm}}. \tag{5}
\]

Notice that the rate is enhanced by \( M_N^2/m_\mu^2 \) for \( M_N \gg m_\mu \) because of the helicity suppression effect of this process. The branching ratio of \( B^+ \rightarrow \mu^+ N \) is estimated as

\[
Br(B^+ \rightarrow \mu^+ N) = \frac{\Gamma(B^+ \rightarrow \mu^+ N)}{\Gamma(B^+ \rightarrow \tau^+ \nu_\tau)} \times Br(B^+ \rightarrow \tau^+ \nu_\tau), \tag{6}
\]
where the branching ratio of $B^+ \rightarrow \tau^+ \nu_\tau$ is $Br(B^+ \rightarrow \tau^+ \nu_\tau) = (1.14 \pm 0.27) \times 10^{-4}$ [37]. In order to estimate the number of the signal events the energy distribution of $N$ in $B^+ \rightarrow \mu^+ N$ is important since it determines the decay length of $N \rightarrow \mu^+ \pi^-$. In the present case due to the two-body decay at rest it is simply given by

$$E_N = \frac{m_{B^\pm}^2 + M_N^2 - m_\mu^2}{2m_{B^\pm}}. \quad (7)$$

The number of the signal events is then

$$N_{\text{event}} = 2 N_{B^+} Br(B^+ \rightarrow \mu^+ N) P(N \rightarrow \mu^+ \pi^-; E_N, L_{\text{det}}), \quad (8)$$

where $P(N \rightarrow \mu^+ \pi^-; E_N, L_{\text{det}})$ is the probability that the signal decay $N \rightarrow \mu^+ \pi^-$ occurs inside the detector, which is given by

$$P(N \rightarrow \mu^+ \pi^-; E_N, L_{\text{det}}) = \frac{\Gamma(N \rightarrow \mu^+ \pi^-)}{\Gamma_N} \left[1 - \exp \left(-\frac{M_N \Gamma_N L_{\text{det}}}{E_N}\right)\right], \quad (9)$$

where $\Gamma_N$ is the total decay rate of $N$. We calculate $\Gamma_N$ for the case when $\Theta_\mu \neq 0$ and $\Theta_\nu = \Theta_\tau = 0$ taking into account the possible decay channels by using the expressions for the partial rates in Ref. [18]. On the other hand, the partial rate of $N \rightarrow \mu^+ \pi^-$ is given by

$$\Gamma(N \rightarrow \mu^+ \pi^-) = \frac{1}{16\pi} |\Theta_\mu|^2 |V_{ud}|^2 G_F^2 f_{\pi^\pm}^2 M_N^3 \left[1 - \frac{m_\mu^2}{M_N^2} - \frac{m_{\pi^\pm}^2}{M_N^2} \left(1 + \frac{m_\mu^2}{m_{\pi^\pm}^2}\right)\right] \times \left[1 - 2 \frac{m_{\pi^\pm}^2 + m_\mu^2}{M_N^2} + \frac{(m_{\pi^\pm}^2 - m_\mu^2)^2}{M_N^4}\right]^{1/2}. \quad (10)$$

Here we take $m_{\pi^\pm} = 139.6$ MeV, $f_{\pi^\pm} = 130.4$ MeV and $|V_{ud}| = 0.9743$ [37]. The typical size of the detector is denoted by $L_{\text{det}}$ and we take it as $L_{\text{det}} = 1.5$ m for Belle II detector for simplicity. Note that the factor 2 in Eq. (8) represents the contribution from the charge conjugate process of (2).

We assume that there is no background event and the sensitivity limit on $|\Theta_\mu|^2$ at 95 % C.L. is obtained from $N_{\text{event}} = 3.09$ [48]. The result is shown in Fig. [2]. It is seen that Belle II can probe the LNV effect by heavy neutrino with $M_N \approx 2–3$ GeV and $|\Theta_\mu|^2 = \theta (10^{-5})$ which is consistent with various experimental constraints.\footnote{This issue has also been discussed in Ref. [36]. Although they have not presented the quantitative estimate of the limit, their qualitative result is consistent with ours.}

Interestingly, the sensitivity is better than the test of $B^+_c \rightarrow \mu^+ \mu^- \pi^-$ at LHCb for LHC run 3 [35].

### 4 Search at FCC-ee

Next, we turn to consider the search at the future plan, the $e^+ e^-$ collisions at the Future Circular Collider (FCC-ee). It is planned to produce $10^{12}$-$10^{13}$ $Z$ bosons at the $Z$-pole $\sqrt{s} = m_Z$. The direct
search for heavy neutrino at FCC-ee has been discussed in Ref. \[49\]. The method there cannot clarify
whether heavy neutrino is a Dirac or Majorana particle. Here we shall discuss the sensitivity of the
LNV process (2) aiming to test the Majorana property of heavy neutrino.

The number of $B^+ \rightarrow \mu^+ N$ is estimated as

$$N_{B^+} = N_Z \times Br(Z \rightarrow b \bar{b}) \times f_u ,$$

where $N_Z$ is the number of $Z$ produced at FCC-ee, and $N_Z = 10^{13}$ is assumed in the present analysis.
$Br(Z \rightarrow b \bar{b}) = 0.1512$ \[37\] is the branching ratio of $Z \rightarrow b \bar{b}$ and $f_u = 0.410$ \[50\] is the fraction of $B^+$ from $b$ quark in $Z$ decay. It is then found that $N_{B^+} = 6.20 \times 10^{-2} N_Z$ is much larger than that in the case of Belle II, from which we can expect the much better sensitiv e at FCC-ee. Although the produced $B^+$'s have the energy distribution peaked at $E_{B^+} \sim 40$ GeV (see, e.g., Ref. \[51\]), we shall set $E_{B^+} = m_Z^2$, (12)

for simplicity. In this case the distribution of the energy of $N$ in $B^+ \rightarrow \mu^+ N$ is flat as

$$\frac{1}{\Gamma_{B^+ \rightarrow \mu^+ N}} \frac{d \Gamma_{B^+ \rightarrow \mu^+ N}}{dE_N} = \frac{1}{p_{B^+} \beta_f} ,$$

for the energy range $E_N^+ \geq E_N \geq E_N^-$. Here $p_{B^+} = \sqrt{E_{B^+}^2 - m_{B^±}^2}$ and

$$\beta_f = \sqrt{1 - \frac{2(M_N^2 + m_\mu^2)}{m_{B^±}^2} + \frac{(M_N^2 - m_\mu^2)^2}{m_{B^±}^4}} ,$$

$$E_N^± = \frac{4(m_{B^±}^2 + M_N^2 - m_\mu^2)E_{B^±} \pm 4p_{B^+}m_{B^±}^2 \beta_f}{8m_{B^±}^2} .$$

The number of the signal events (2) is then estimated as

$$N_{\text{event}} = 2 \int_{E_N^-}^{E_N^+} dE_N N_{B^+} Br(B^+ \rightarrow \mu^+ N) \frac{1}{p_{B^+} \beta_1} P(N \rightarrow \mu^+ \pi^-; E_N, L_{\text{det}}) .$$

Now we take $L_{\text{det}} = 2$ m for the probability $P(N \rightarrow \mu^+ \pi^-; E_N, L_{\text{det}})$ in Eq. \[9\].

In Fig.\[2\] we also show the sensitivity limit on the mixing $|\Theta_\mu|^2$ from the LNV decay $B^+ \rightarrow \mu^+ \mu^+ \pi^-$ at FCC-ee with $N_Z = 10^{13}$. As in the previous case we assumed no background event and estimate the limit from $N_{\text{event}} = 3.09$. We can see that FCC-ee improves greatly the sensitivity compared with those of Belle II and LHCb for LHC run 3. For heavy Majorana neutrino with $M_N \approx 4$ GeV the mixing $|\Theta_\mu|^2 \gtrsim 10^{-6}$ can be probed. Thus, FCC-ee can offer the significant test of the LNV by heavy Majorana neutrino.

One might think that the LNV signal might be boosted for $N$ produced in $B_c$ mesons, since the partial rate of $B^+_c \rightarrow N + \mu$ receives a milder suppression factor $|V_{cb}|^2 = 1.69 \times 10^{-3}$ rather than $|V_{ub}|^2 = 1.71 \times 10^{-5}$ \[37\]. The production of $B_c$ in $Z$ decays, however, is hard and the branching ratio is $Br(Z \rightarrow$
\[ B_c^+ + b + \bar{c} = (2.04 - 3.33) \times 10^{-5} \] \[ [52, 53]. \] Thus, the LNV events through \( B_c \) meson is smaller than those through \( B \) and then we shall neglect it in the present analysis. \[ \#3 \]

We should mention that FCC-ee offers another promising test of the LNV induced by heavy Majorana neutrino. \[ \#4 \] It is planned to produce more than \( 2 \times 10^8 \) \( W \) pairs at the center-of-mass energy at the \( WW \) threshold and above. \[ [55] \] In this case the LNV decay \( W^+ \rightarrow \ell^+ N \rightarrow \ell^+ \ell'^+ \pi^- \) can be tested. The sensitivity limit on \( |\Theta_{\mu l}|^2 \) by using this mode is also shown in Fig. 2. It is found that the sensitivity by using \( B^+ \rightarrow \mu^+ \mu^+ \pi^- \) is better than this for the parameter range in which constraints are avoided.

5 Summary

We have discussed the LNV decay of \( B \) meson, \( B^+ \rightarrow \mu^+ \mu^+ \pi^- \), induced by heavy Majorana neutrino. In particular we have estimated the sensitivity limits on the mixing \( |\Theta_{\mu l}|^2 \) by the experimental searches at Belle II and at FCC-ee (at \( Z \)-pole). These facilities can probe the parameter region in which the various experimental constraints on heavy neutrino are avoided. Thus, the LNV \( B \) decay is a significant and promising target for the LNV, which is complementary to the neutrinoless double beta decay.

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References

[1] See, for example, a recent analysis in M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP 1411 (2014) 052 [arXiv:1409.5439 [hep-ph]].

[2] P. Minkowski, Phys. Lett. B 67, 421 (1977).

[3] T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Ibaraki 305-0801 Japan, 1979) p. 95.

[4] T. Yanagida, Prog. Theor. Phys. 64, 1103 (1980).

[5] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by P. van Niewjenhuizen and D. Freedman (North Holland, Amsterdam, 1979) [arXiv:1306.4669 [hep-th]].

\[ \#3 \] The LNV process \( B_c^+ \rightarrow \mu^+ \mu^+ \pi^+ \) is an interesting target for LHCb experiment as discussed in Ref. \[ 35 \]. See also Fig. \[ 2 \]

\[ \#4 \] The Majorana property of heavy neutrino may also be probed from \( e^+ e^- \rightarrow N \nu \rightarrow \ell q \bar{q}' \nu \) by using the angular distribution between \( N \) and the incoming \( e^- \) \[ 54 \]. In addition, the LNV process like \( e^+ e^- \rightarrow Ne^\pm W^\mp \rightarrow \ell^\pm W^\pm \epsilon^\pm W^\mp \) leading to the same-sign dilepton with four hadronic jets is also an interesting target \[ 23 \].
[6] P. Ramond, in *Talk given at the Sanibel Symposium*, Palm Coast, Fla., Feb. 25-Mar. 2, 1979, preprint CALT-68-709 (retroprinted as hep-ph/9809459).

[7] S. L. Glashow, in *Proc. of the Cargése Summer Institute on Quarks and Leptons*, Cargése, July 9-29, 1979, eds. M. Lévy et. al., (Plenum, 1980, New York), p707.

[8] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.

[9] E. K. Akhmedov, V. A. Rubakov and A. Y. Smirnov, Phys. Rev. Lett. 81 (1998) 1359.

[10] T. Asaka and M. Shaposhnikov, Phys. Lett. B 620, 17 (2005) [hep-ph/0505013].

[11] G. M. Fuller, A. Kusenko and K. Petraki, Phys. Lett. B 670 (2009) 281 [arXiv:0806.4273 [astro-ph]].

[12] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. 72, 17 (1994) [hep-ph/9303287].

[13] A. Kusenko and G. Segre, Phys. Lett. B 396 (1997) 197 [arXiv:hep-ph/9701311];

[14] G. M. Fuller, A. Kusenko, I. Mocioiu and S. Pascoli, Phys. Rev. D 68 (2003) 103002 [arXiv:astro-ph/0307267];

[15] M. Barkovich, J. C. D'Olivo and R. Montemayor, Phys. Rev. D 70 (2004) 043005 [arXiv:hep-ph/0402259];

[16] A. Kusenko, B. P. Mandal and A. Mukherjee, Phys. Rev. D 77 (2008) 123009 [arXiv:0801.4734 [astro-ph]].

[17] A. Kusenko, Phys. Rept. 481 (2009) 1 [arXiv:0906.2968 [hep-ph]].

[18] D. Gorbunov and M. Shaposhnikov, JHEP 0710 (2007) 015 Erratum: [JHEP 1311 (2013) 101] [arXiv:0705.1729 [hep-ph]].

[19] A. Atre, T. Han, S. Pascoli and B. Zhang, JHEP 0905 (2009) 030 [arXiv:0901.3589 [hep-ph]].

[20] F. F. Deppisch, P. S. Bhupal Dev and A. Pilaftsis, New J. Phys. 17 (2015) no.7, 075019 [arXiv:1502.06541 [hep-ph]].

[21] H. Päs and W. Rodejohann, New J. Phys. 17 (2015) no.11, 115010 [arXiv:1507.00170 [hep-ph]].

[22] T. G. Rizzo, Phys. Lett. B 116, 23 (1982).

[23] S. Banerjee, P. S. B. Dev, A. Ibarra, T. Mandal and M. Mitra, Phys. Rev. D 92 (2015) 075002 [arXiv:1503.05491 [hep-ph]].

[24] T. Asaka and T. Tsuyuki, Phys. Rev. D 92 (2015) no.9, 094012 [arXiv:1508.04937 [hep-ph]].

[25] H. Baer et al., arXiv:1306.6352 [hep-ph].
[26] E. Accomando et al. [CLIC Physics Working Group Collaboration], hep-ph/0412251.

[27] J. N. Ng and A. N. Kamal, Phys. Rev. D 18 (1978) 3412.

[28] J. Abad, J. G. Esteve and A. F. Pacheco, Phys. Rev. D 30 (1984) 1488.

[29] L. S. Littenberg and R. E. Shrock, Phys. Rev. Lett. 68 (1992) 443.

[30] C. Dib, V. Gribanov, S. Kovalenko and I. Schmidt, Phys. Lett. B 493 (2000) 82 [hep-ph/0006277].

[31] A. Ali, A. V. Borisov and N. B. Zamorin, Eur. Phys. J. C 21 (2001) 123 [hep-ph/0104123].

[32] A. Atre, V. Barger and T. Han, Phys. Rev. D 71 (2005) 113014 [hep-ph/0502163].

[33] G. Cvetic, C. Dib, S. K. Kang and C. S. Kim, Phys. Rev. D 82 (2010) 053010 [arXiv:1005.4282 [hep-ph]].

[34] L. Canetti, M. Drewes and B. Garbrecht, Phys. Rev. D 90 (2014) no.12, 125005 [arXiv:1404.7114 [hep-ph]].

[35] D. Milanes, N. Quintero and C. E. Vera, Phys. Rev. D 93 (2016) no.9, 094026 [arXiv:1604.03177 [hep-ph]].

[36] G. Cvetic and C. S. Kim, Phys. Rev. D 94 (2016) no.5, 053001 [arXiv:1606.04140 [hep-ph]].

[37] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38 (2014) 090001.

[38] SuperKEKB WEB page: http://www-superkekb.kek.jp.

[39] FCC-ee WEB page: http://tlep.web.cern.ch.

[40] D. Liventsev et al. [Belle Collaboration], Phys. Rev. D 87 (2013) no.7, 071102 [arXiv:1301.1105 [hep-ex]].

[41] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 112 (2014) no.13, 131802 [arXiv:1401.5361 [hep-ex]].

[42] B. Shuve and M. E. Peskin, arXiv:1607.04258 [hep-ph].

[43] P. Abreu et al. [DELPHI Collaboration], Z. Phys. C 74 (1997) 57 Erratum: [Z. Phys. C 75 (1997) 580].

[44] J. Badier et al. [NA3 Collaboration], Z. Phys. C 31 (1986) 21.

[45] P. Vilain et al. [CHARM II Collaboration], Phys. Lett. B 343 (1995) 453 [Phys. Lett. B 351 (1995) 387].
[46] A. M. Cooper-Sarkar et al. [WA66 Collaboration], Phys. Lett. B 160 (1985) 207.

[47] A. Vaitaitis et al. [NuTeV and E815 Collaborations], Phys. Rev. Lett. 83 (1999) 4943 [hep-ex/9908011].

[48] G. J. Feldman and R. D. Cousins, Phys. Rev. D 57 (1998) 3873 [physics/9711021 [physics.data-an]].

[49] A. Blondel et al. [FCC-ee study Team Collaboration], arXiv:1411.5230 [hep-ex].

[50] Y. Amhis et al. [Heavy Flavor Averaging Group (HFAG) Collaboration], arXiv:1412.7515 [hep-ex].

[51] K. Ackerstaff et al. [OPAL Collaboration], Phys. Lett. B 420 (1998) 157 [hep-ex/9801026].

[52] C. H. Chang and Y. Q. Chen, Phys. Rev. D 46 (1992) 3845 Erratum: [Phys. Rev. D 50 (1994) 6013].

[53] E. Braaten, K. m. Cheung and T. C. Yuan, Phys. Rev. D 48 (1993) no.11, R5049 [hep-ph/9305206].

[54] F. del Aguila and J. A. Aguilar-Saavedra, JHEP 0505 (2005) 026 [hep-ph/0503026].

[55] M. Bicer et al. [TLEP Design Study Working Group Collaboration], JHEP 1401 (2014) 164 [arXiv:1308.6176 [hep-ex]].
Figure 2: The sensitivity limits on $|\Theta_\mu|^2$ from the LNV decay $B^+ \rightarrow \mu^+ \mu^+ \pi^-$ due to heavy neutrino at Belle II with $N_B = 5 \times 10^{10}$ (magenta dot-dashed line) and at FCC-ee with $N_Z = 10^{13}$ (red solid line). The orange long-dashed line is the limit from $W^+ \rightarrow \mu^+ \mu^+ \pi^-$ at FCC-ee with $N_W = 2 \times 10^8$. For comparison we also show the limit from the LNV decays $B^+_c \rightarrow \mu^+ \mu^+ \pi^+$ at LHCb for LHC run 3 [35] (cyan solid line). The blue dashed lines are the upper bounds from the LNV $B$ decays by LHCb [41] and Belle [40]. The gray region is excluded by search experiments: DELPHI [43], NA3 [44], CHARM II [45], BEBC [46], and NuTeV [47].