We analyze theoretically the quantization of conductance occurring with cold bosonic atoms trapped in two reservoirs connected by a constriction with an attractive gate potential. We focus on temperatures slightly above the condensation threshold in the reservoirs. We show that a conductance step occurs, coinciding with the appearance of a condensate in the constriction. Conductance quantization relies on a collective process involving the quantum condensation of an atom into an elementary excitation and the subsequent quantum evaporation of an atom, in contrast with ballistic fermion transport. The value of the bosonic conductance plateau is strongly enhanced compared to fermions and explicitly depends on temperature. We highlight the role of the repulsive interactions between the bosons in preventing them from collapsing into the constriction. We also point out the differences between the bosonic and fermionic thermoelectric effects in the quantized conductance regime.

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I. INTRODUCTION

In mesoscopic systems, where the motion of quantum particles occurs over distances of the order of their coherence length, transport phenomena exhibit quantum signatures [1]. The quantization of conductance [2] is a hallmark among these effects. It reflects the discrete nature of the transport channels inside a strongly constricted geometry and occurs if the spread in energies of the incident particle distribution is smaller than the energy separation of these channels. It was first observed in electronic transport through a quantum point contact [3] as a series of plateaus in the conductance when the distance between the gate electrodes was increased. In this fermionic case, the conductance quantum \( G_K = e^2/h \) involves fundamental constants only, making it relevant for metrology [4]. Unlike the quantum Hall effect [5], it occurs in the absence of a magnetic field and has been predicted to affect neutral helium atoms [6]. The recently observed universal value for the low-temperature thermal conductance [7–10] is a related effect.

Conductance quantization has recently been observed in ultracold fermionic gases [11]. Atomic gases allow a clean observation in a simple setup involving two reservoirs connected by a constriction within which an attractive gate potential \( E_G < 0 \) is varied (see Fig. 1). Experiments on ultracold fermions aim at simulating electronic systems using neutral particles [11–14]. In the fermionic experiment of Ref. [11], conductance quantization has been observed at temperatures much lower than both the Fermi temperature \( T_F \) and the confinement energy of the constriction, in analogy with the original results on electronic transport [3] where only particles near the Fermi surface take part in transport phenomena. This raises the question of whether conductance quantization also affects bosons. Previous observations in an optical setup [15,16] and predictions with cold matter waves [17] have focused on systems where all particles have the same incident energy, mimicking fermionic transport at the Fermi energy. To our knowledge, the specific role of bosonic statistics in quantized conductance situations has not yet been investigated. Cold atom setups allow the exploration of mesoscopic physics in situations where the Bose distribution plays a key role [18–20]. They are also expected to exhibit the phenomenon of quantum evaporation, whereby an elementary excitation of a superfluid reaches its surface and causes the evaporation of a single atom. This phenomenon had so far been studied experimentally [21,22] and theoretically [23,24] in superfluid \(^4\)He, and we consider it here in the context of superfluid atomic gases.

In this article, we show that conductance quantization occurs with bosonic atoms as well, and that the Bose statistics strongly enhance the value of the conductance step compared to fermions. Unlike for fermions, this value explicitly depends on temperature, and the effect occurs with bosons up to temperatures higher than with fermions. Furthermore, we show that the underlying transmission mechanism is very different from the fermionic case and leads to the occurrence of a single conductance plateau as the gate potential is varied, coinciding with the appearance of a condensate in the constriction. Transmission through the constriction relies on quantum condensation followed by quantum evaporation: an atom impinging on one end of the constriction excites a phonon in the condensate, which travels through the constriction and causes the evaporation of a single atom at its other end. Hence, transport through the constriction involves a collective mechanism, as in Ref. [25]. However, we focus on weakly interacting Bose gases with temperatures \( T \) slightly above the critical temperature \( T_B \) in the reservoirs, so that these contain a thermal gas and no superflow occurs, in contrast to Refs. [25–27] where the condensate is also present in the reservoirs.

The two reservoirs L and R of Fig. 1 can exchange particles via a constriction of length \( l_C \) produced by the potential \( V_C(r,z) \). At its most stringent point \( z = 0 \), we model it by the radial harmonic trap \( V(r,0) = m_0 \omega_0^2 r^2 / 2 \). We assume that the gate potential \( E_G(z) < 0 \) also reaches its maximum value \(|E_G| \) at \( z = 0 \).

II. EQUILIBRIUM STATE

We first state two conditions on the strength of the interatomic interactions which are required for our analysis to hold for bosons. These interactions should be (i) weak...
enough for the reservoir thermodynamics to be dominated by single-particle effects for temperatures \( T \gtrsim T_B \), and (ii) strong enough to avoid a collapse of the system into the attractive constriction. These conditions are compatible and easily realized with bosonic atoms trapped in box-like potentials [28].

(i) The effects of weak interactions in uncondensed Bose gases are well described by Hartree-Fock theory (see Chap. 13 in Ref. [29]). It predicts the chemical potential \( \mu(n, T) = \mu^{(0)}(n, T) + 2g n \), with \( \mu^{(0)} < 0 \) being the ideal-gas value, \( n \) the density, and \( g > 0 \) the interaction strength. In this theory, the Bose distribution reads

\[
\mathcal{f}_B(E) = 1/\sqrt{[\exp(E + 2g n - \mu)/k_B T] - 1} = 1/\sqrt{[\exp(E)/k_B T] - 1},
\]

where the ideal-gas fugacity \( z = \exp(\mu^{(0)}/k_B T) \) and \( E = p^2/2m \). The quantity \( \partial \mathcal{f}_B / \partial \mu |_T \), relevant for linear response, can be replaced by \( \partial \mathcal{f}_B / \partial \mu |_T \) if \( 2g n k_{\text{F}} T \ll N \). Here, \( N \) is the atom number in one reservoir, and the isothermal compressibility \( \kappa_T = \partial N/\partial \mu |_T \) is linked to its ideal-gas value by \( N/\kappa_T = 2g n + N/\kappa^{(0)}_T \). For \( T \gtrsim T_B \), \( \kappa_T k_B T/B_N = \sqrt{\pi/[\zeta(3/2)/\sqrt{1 - z}]}, \) and the condition \( 2g n k_{\text{F}} T \ll N \) means \( 1 - z \gtrsim (g n/k_B T_B)^2 \pi/\zeta(3/2) \). For a uniform gas, this condition is well satisfied for \( T/\sqrt{T_B} \gtrsim 1 \). We focus on box-trap reservoirs which, for Bose gases, are more favorable than the harmonically trapped case, as interactions play a weaker role within uniform gases \( (g n/k_B T_B \approx 0.02) \) than in trapped geometries \( (g n/k_B T_B \approx 0.2) \) [19]. Thus, we can describe the atoms in the reservoirs as an ideal Bose gas with \( g \) negative and small. We take \( \mu/\hbar \omega_0 \approx -0.01 \) in the following.

(ii) Despite the assumption \( T > T_B \), condensation occurs in the constriction [30–32] if the gate potential \( E_{G0} < -\hbar \omega_0 + \mu \) is attractive enough for the energy of the first transverse state in the constriction to match the chemical potential of the gas in the reservoirs. Then, in the absence of interactions, the atoms would collapse into the constriction, impeding the investigation of transport. The presence of weak repulsive interactions between the bosons prevents this collapse by making the presence of too many atoms in the constriction energetically disfavored. Neglecting the dilute thermal cloud, the condensate wave function \( \Psi_0(r) \) at \( z = 0 \), which depends only on the distance \( r \) to the axis, is the lowest energy solution to the Gross-Pitaevskii (GP) equation:

\[
\left( \mu - E_{G0} \right) \Psi_0 = \left( -\frac{\hbar^2}{2m} \Delta_r + \frac{1}{2} m a^2 r^2 + g |\Psi_0|^2 \right) \Psi_0, \tag{1}
\]

where the radial Laplacian satisfies \( \Delta_r \Psi_0 = d(r \Psi_0/dr)/dr \), \( g = 4\pi \hbar^2 a/m \), and \( a \) is the scattering length encoding the interactions. The density \( |\Psi_0|^2 \) at the point \( z = 0 \) is determined by the effective chemical potential \( \mu - E_{G0} > 0 \). Figure 2 shows the linear density \( n_1 = \int 2\pi r dr |\Psi_0|^2 \) as a function of \( E_{G0} \). For \( \mu - E_{G0} < \hbar \omega_0 \),

![FIG. 1. Two reservoirs (L, R) can exchange particles through a smoothly tapered constriction inside which the spatially dependent and attractive gate potential \( E_{G0} \) is varied.](image)

![FIG. 2. Linear condensate density at the center of the constriction as a function of the gate potential. The exact numerical result (thick red line) interpolates in between the Gaussian approximation (dashed blue), valid for \( |E_{G0}| \gtrsim \hbar \omega_0 \), and the Thomas-Fermi result (dotted green), holding for large \( |E_{G0}| \).](image)

III. TRANSPORT PROPERTIES

We focus on small deviations from the equilibrium situation where both reservoirs are characterized by the same chemical potential \( \mu \) and temperature \( T \). An important difference between fermionic and bosonic transport phenomena concerns the energies of the particles undergoing transport. In the linear response regime, these are the energies for which the derivative \( \partial \mathcal{f}_F/B / \partial \mu |_{E,T} \) of the Fermi (or Bose) distribution function with respect to \( \mu \) is non-negligible. For fermions, this derivative is strongly peaked near the Fermi energy \( k_B T_F \) with a width \( \sim k_B T \) [see Fig. 3(b)], confirming the key role of the Fermi surface. These fermions have nonvanishing energies and efficiently traverse even sharply defined constrictions [33]. By contrast, for bosons, the derivative \( \partial \mathcal{f}_B / \partial \mu |_{E,T} \) nearly diverges for the energy \( E = 0 \), and the mobile particles have energies \( |\mu| \approx (1 - z/k_B T_B) \approx |\mu| \) [see Fig. 3(c)]. This divergent behavior leads to the bosonic enhancement of conductance. Our choice of \( T \gtrsim T_B \) means that \( |\mu| \ll k_B T_B \), and we assume in the following that \( k_B T_B \lesssim \hbar \omega_0 \), hence, mobile bosonic atoms have energies \( \ll \hbar \omega_0 \). Low-energy reflections at the ends of the constriction [34] can be made negligible by smoothly connecting it to the reservoirs [35] with a radius of curvature \( R \) which is large compared to the characteristic atom wavelength \( (\hbar^2/m|\mu|)^{1/2} \approx 10 a_0 \). Such a smoothly tapered constriction was already used in the experiment of Ref. [11] where the Fermi momentum \( k_F \) satisfies \( k_F R \approx 100 \).
Introducing the small differences in atom numbers, \( \delta N = N_R - N_L \), and chemical potentials, \( \delta \mu = \mu_R - \mu_L \), between the reservoirs, we define the isothermal conductance \( G \) by the relation \( \delta \mu \delta N = -G \delta \mu \delta \mu \) (we go beyond the isothermal approximation in Sec. V). The Landauer-Büttiker formalism (see Chap. 1 in Ref. [1]) leads to the expression \( hG(E_G) = L_0 \), where for any \( \alpha, L_0 \) reads

\[
L_\alpha = \int_{-\infty}^{\infty} dE \Phi^{F,B}(E - E_G) \left( \frac{E - \mu}{k_B T} \right)^\alpha \left. \frac{\partial f^{F,B}}{\partial \mu} \right|_{E,T}.
\]

This equation holds for both fermionic and bosonic systems. It is applicable whatever the reservoir geometry, encoded in the value of the degeneracy temperature \( T_D = T_{F,B} \) [29], Chap. 10. Equation (2) shows that \( G(E_G) \) is the convolution of two functions, which both depend on the quantum statistics: (i) the transport function \( \Phi^{F,B}(E) \) of the constriction, and (ii) the derivative of the (Fermi or Bose) distribution function \( f^{F,B}(E) = 1/[\zeta^{-1} \exp(E/k_B T) + 1] \) of the reservoirs.

We first summarize the fermionic results of Ref. [11]. Pauli exclusion ensures that the constriction remains empty, so that transmission is a single-particle ballistic effect. The transport function \( \Phi^{F}(E) \), which counts the transport channels whose threshold energies are \( \leq E \), is determined by the most stringent part of the constriction. It reads \( \Phi^{F}(E/h\omega_0) = [E/h\omega_0]/(E/h\omega_0 + 1)\) for \( |\mu| \ll h\omega_0 \) and the integer part of \( x \). It exhibits jumps for energies that are integer multiples \( h\omega_0 \) of the constriction strength, reflecting the opening of additional transport channels [dashed green line in Fig. 3(a)]. These jumps are the cause of the quantization of conductance.

We now consider bosonic atoms. If the gate potential \( E_{G0} > -h\omega_0 + \mu \), the constriction is empty (see Fig. 2). For sufficiently smooth spatial variations of \( V_C(r) \) and \( E_G(z) \), the motion of single thermal particles impinging on it is quasiclassical [35]. These experience a repulsive barrier of height \( h\omega_0 + E_{G0} \), so that low-energy transmission through the constriction is blocked. Instead, for \( E_{G0} < -h\omega_0 + \mu \), the constriction is filled with a condensate whose presence strongly affects the nature of the transport mechanism within the channel. The energies \( |\mu| \ll h\omega_0 \) of the incident atoms are smaller than \( gn_0 \) at the center of the constriction, so that transport is now a collective process. It involves quantum condensation followed by quantum evaporation [21,23,24], which rely on the superfluidity of the condensate and, hence, on the presence of interactions in between the bosons. A thermal atom in a reservoir impinging on the constriction with energy \( E \) condenses into an elementary excitation inside the superfluid with energy \( \epsilon = E - \mu \), which crosses the constriction and evaporates an atom at its other end. We describe this process using the Bogoliubov equations (see Ref. [29], Chap. 12). The condensate density \( n_0(r,z) \), which appears in these equations, varies along the \( z \) axis, and the Bogoliubov equations reduce to the Schrödinger equation in the reservoirs, where \( n_0 = 0 \). Under our assumption of a smoothly tapered constriction, the condensate can locally be described as translationally invariant along the axial direction for each \( z \), and the corresponding Bogoliubov spectrum varies adiabatically with \( z \). The transport properties of the system are dictated by the strongly constricted region near \( z = 0 \), where the condensate density is maximal. There, the density profile \( n_0(r) \) is nearly that of a condensate trapped in the elongated radial harmonic trap \( m\omega_0^2 r^2/2 \) with the effective chemical potential \( \mu - E_{G0} \). The corresponding Bogoliubov excitation spectrum has multiple branches reflecting the 3D geometry [36]. However, the condensate occupies the lowest energy solution of the GP, Eq. (1), hence, its low-energy excitations belong to the first branch. For \( |E_{G0}|/h\omega_0 \gtrsim 1.1 \), the incident atoms have energies \( |\mu| \ll gn_0 \) and the excitations crossing the constriction are phononic. Regardless of the value of \( E_{G0} \), the second branch has the threshold energy \( 2\omega_0 \) [36,37], which is much greater than the incident energies, so that this branch is never involved. The smooth spatial variation of \( V_{cond}(r) \) ensures a full conversion of the incident atoms into excitations of the superfluid.

To sum up, if the constriction is empty because the energy of its first transverse mode is \( > \mu \), bosonic transmission is blocked; instead, if \( E_G \) is sufficiently attractive for the constriction to contain a condensate, transmission is allowed and relies on a collective phenomenon involving quantum condensation and evaporation. These two mechanisms lead to a bosonic transport function which exhibits a single step. Under our assumption of a smoothly tapered constriction, it reads \( \Phi^{B}(E/h\omega_0) = \Theta(E/h\omega_0 - 1) \), where \( \Theta \) is the Heaviside step function [see Fig. 3(a)].

**IV. QUANTIZED CONDUCTANCE**

The conductance \( G(E_G/h\omega_0) \) calculated from Eq. (2) depends on \( T/T_D \) and \( h\omega_0/k_B T_D \). We compare the fermionic and bosonic predictions in Fig. 4 (\( T/T_D = 0.1 \) for fermions and \( T/T_B = 1.2 \) for bosons; \( h\omega_0/k_B T_D = 4 \) in both cases).
The fermionic prediction has the multiple step structure observed in Refs. [3,11] due to the stepwise structure of the ballistic $\Phi^\mu(E)$. By contrast, the bosonic graph exhibits one single step, relating to the single step of $\Phi^\mu(E)$. It occurs for $E_G = -\hbar\omega_0$ and, hence, coincides with the appearance of the condensate in the constriction (see Fig. 2). For bosons, Eq. (2) can be integrated analytically; an analogous result is obtained for the first fermionic step by accounting for a single transport channel. We find

$$hG^{F,B} = \frac{1}{z^2 \exp(E_G + h\omega_0) k_B T} \begin{cases} 1 & \text{if } E_G > -h\omega_0, \\ z^2 & \text{if } E_G \leq -h\omega_0, \end{cases}$$

where the + and − signs respectively apply to fermions and bosons. Equation (3) reveals three differences between fermions and bosons, concerning the step positions, their heights, and the widths of the transition regions between two plateaus: (i) For fermions, the step is centered at $E_G = -\hbar\omega_0 + \mu$, reflecting the key role of the Fermi surface at energies $\sim\mu$. For bosons, the low-energy divergence discussed above causes the step to occur at $E_G = -\hbar\omega_0$. (ii) For ultracold fermions, the fugacity $z \rightarrow \infty$, leading to the step height $1/(z^{-1} + 1) \approx 1$. Instead, for bosons, $z \lesssim 1$ for $T \gtrsim T_B$, leading to the very large step height $1/(z^{-1} - 1) \approx 27$ for $T/T_B = 1.2$. (iii) For fermions, the width of the transition region is $\Delta E_G^F \sim 2k_B T$, whereas the corresponding width for bosons is $\Delta E_G^B \sim 1 - z k_B T \approx |\mu|$. The conductance step is well defined if $\Delta E_G \ll \hbar\omega_0$, hence, Bose systems are greatly favored, as seen on Fig. 4 where $k_B T/h\omega_0$ is ten times as large for bosons than for fermions, but the bosonic step width is quenched by the factor $(1 - z)$. The conductance $G$ is positive, hence, the current $\delta N / N$ opposes the atom number difference $\delta N$, which relaxes to equilibrium as $\delta N = \delta N_0 \exp(-t/\tau_1)$. The decay time $\tau_1 = \kappa T / G$ is proportional to $N$ and is conveniently expressed in units of $\tau_D = N\hbar/k_B T_D$. Its measurement allows for an access to $G(E_G/h\omega_0)$. It has recently been measured with fermions [11], where $\kappa_T k_B T_F/\hbar = 3/2$ at small $T_F$, so that $\tau_1 = 3\tau_D/2 \sim a few seconds for the first conductance plateau. For bosons, the isothermal compressibility diverges as one approaches the critical temperature, but the stronger divergence of $G$ leads to shorter decay times $\tau_1 = \tau_D(1 - z)^{1/2}/\pi(3/2) \sim a few hundred ms for the single conductance plateau.

V. THERMOELECTRIC EFFECTS

The preceding analysis neglects the impact of temperature changes $\delta T(t)$ on the dynamics of $\delta N$. We evaluate this impact using the transport model of Refs. [14,19]:

$$\tau_1 \frac{d}{dt} \left( \frac{\delta N}{N} \right) = \left( -\frac{N k_B}{C_N} \frac{S^{\delta \mu LT}}{-\tau_1/\tau_T} \right) \left( \delta N / N \right) / \delta T / T. \tag{4}$$

The coupling between particle and heat currents, proportional to the Seebeck coefficient $S = -\partial \mu / \partial T|_N - L_1/L_0$, plays a role over times of the order of the thermalization time $\tau_T = C_N/(\hbar L_0 T)$, where $C_N = \tau T S^T / T|_N$ is the heat capacity and the integrals $L_n$ are given by Eq. (2). For short times $t \ll \tau_1$, $\delta T$ is negligible and Eq. (4) reduces to the isothermal limit investigated above. Before the bosonic conductance step or the first fermionic conductance step, both $G$ and the thermal conductance $hL_2$ are small. This leads to times $\tau_T$ which are longer than the characteristic time over which the transport phenomenon can be observed (a few seconds in the case of Ref. [11]) both for bosons and for fermions, which justifies the isothermal analysis presented above. However, $\tau_T$ becomes shorter with increasing $|E_G|$. Starting from the (first) conductance step, thermal effects cause the relaxation of $\delta N(t)$ towards equilibrium to slow down, and an accurate modeling of this relaxation requires two exponential terms accounting for both time scales $\tau_1$ and $\tau_T$. This is illustrated in Fig. 5. Both the fermionic and bosonic plots, resulting from a

![Fig. 4. Quantized conductance for (a) ultracold fermions ($T/T_F = 0.1$) and (b) cold bosons ($T/T_B = 1.2$). In both cases, $\hbar\omega_0/k_B T_D = 4$. For fermions, the thick solid line is the exact solution $G(E_G)$, and the thin dashed curve is the single-transport-channel prediction of Eq. (3). The results have been vertically rescaled by the step heights $1/(z^{-1} \pm 1)$.](image)

![Fig. 5. Atom number difference $\delta N(t)/N$ (thick solid line), its isothermal approximation $\delta N(t)/N|_{\tau_T}$ (thin solid line), and temperature difference $\delta T(t)/T$ (dashed line), following an atom number mismatch $\delta N_0$, for (a) fermions and (b) bosons, obtained by solving Eq. (4) numerically with the parameters of Fig. 4. The value of $E_G$ is chosen at the conductance step for bosons, and at the first conductance step for fermions. All plotted quantities should be multiplied by $\delta N_0/N$.](image)
numerical solution of Eq. (4) for values of $E_G$ corresponding to the (first) conductance step, differ from the isothermal prediction for times $\gtrsim \tau_F$. The coupling between particle and heat currents also yields a thermoelectric effect, whereby an initial atom imbalance $\delta N_0$ yields a transient change in temperature $\delta T(t)$ [14,38]. This thermoelectric effect is weak for bosons. However, for fermions, its amplitude is enhanced for gate potentials corresponding to a step in the particle conductance $G$. This is due to the existence of maxima in the quantity $S/\tau_1$, appearing in the off-diagonal elements of Eq. (4), which have previously been observed in electronic transport experiments [39].

VI. DISCUSSION AND CONCLUSION

The quantization of bosonic conductance involving quantum evaporation precludes its interpretation as the diffraction of atomic matter waves, in contrast with previous studies [3,15,17]. It also requires an attractive gate potential, unlike for fermions where conductance may be scanned by varying the constriction width [3,11].

The bosonic enhancement of conductance near the Bose-Einstein condensate transition is the transport analog of the isothermal compressibility. It is due to the possibility of accommodating multiple bosons in the lowest energy transport channel, which is more populated at temperatures closer to $T_B$. This enhancement signals a departure from the fermionic conductance quantum $G_F = 1/h$ observed both with electrons [3] and with neutral fermions [11]. Its observation in a regime where conductance is not quantized has recently been reported [20]. Both the compressibility $\kappa_T$ and the conductance $G$, which diverge in the ideal-gas model, depend on many-body effects in the critical region near the transition [40], where their characterization remains an open problem from both the theoretical and experimental points of view. The measurement of the relaxation time $\tau_1$ in bosonic systems with temperatures very close to $T_B$ will provide more insight into these two quantities.

Challenging open questions include (i) conductance quantization in 2D bosonic systems, where the quasicondensate enhances the role of interactions [41–43], and (ii) its impact in the presence of a superfluid, whose investigation has been initiated by recent experiments with strongly interacting Fermi gases [13,44,45].

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