Flatness-based nonlinear control strategies for trajectory tracking of quadcopter systems

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Quadcopter unmanned vehicles

| Breguet-Richet Gyroplane No.1 | Oemichen No.2 | DJI Phantom 4 |
|-------------------------------|---------------|---------------|
| 1907                          | 1920          |               |

- **Breguet-Richet Gyroplane No.1**: 1907
- **Oemichen No.2**: 1920
- **DJI Phantom 4**
### Quadcopter unmanned vehicles

| Model                          | Year | Image |
|-------------------------------|------|-------|
| Breguet-Richet Gyroplane No.1 | 1907 | ![Breguet-Richet Gyroplane No.1](image) |
| Oemichen No.2                 | 1920 | ![Oemichen No.2](image) |
| DJI Phantom 4                 |      | ![DJI Phantom 4](image) |
| Bitcraze Crazyfile 2.0        |      | ![Bitcraze Crazyfile 2.0](image) |
| Bell Boeing V–22 Osprey       |      | ![Bell Boeing V–22 Osprey](image) |
| Ambulance Drone               |      | ![Ambulance Drone](image) |

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Quadcopter trajectory tracking

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The forthcoming findings are done in collaboration with:

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Outline

1. Quadcopter modelling
   - Kinematics
   - Dynamics

2. Differential flatness characterization

3. B-spline basis functions

4. B-spline parametrizations

5. Flat output description of the quadcopter system

6. Control design for trajectory tracking

7. Simulation results

8. Conclusions and future developments
Kinematics of quadcopter

Euler ZYX ($\psi, \theta, \phi$) rotation sequence:

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & c\theta & s\theta \\
0 & -s\phi c\theta & c\phi c\theta + s\phi s\theta
\end{pmatrix}
\begin{pmatrix}
\dot{\psi} \\
\dot{\theta} \\
\dot{\phi}
\end{pmatrix} = \omega
\]

The angular velocity $\omega$ physically measured by the gyroscope is expressed in terms of 3 Euler angles $\eta = (\phi, \theta, \psi)$ as:

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & c\theta & s\theta \\
0 & -s\phi c\theta & c\phi c\theta + s\phi s\theta
\end{pmatrix}
\begin{pmatrix}
\dot{\psi} \\
\dot{\theta} \\
\dot{\phi}
\end{pmatrix} = \dot{\eta}
\]
Kinematics of quadcopter

Euler ZYX ($\psi, \theta, \phi$) rotation sequence:

$$\begin{pmatrix}
  c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\
  c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\
  -s\theta & s\phi c\theta & c\phi c\theta
\end{pmatrix}$$

The angular velocity $B \rightarrow \omega$ physically measured by the gyroscope $B \rightarrow \omega \equiv (\omega_x, \omega_y, \omega_z)^T$ is expressed in terms of 3 Euler angles $\eta \equiv (\phi, \theta, \psi)^T$ as:

$$B \rightarrow \omega = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & c\phi & s\phi \\
  0 & -s\phi & c\phi
\end{pmatrix} \begin{pmatrix}
  \dot{\phi} \\
  \dot{\theta} \\
  \dot{\psi}
\end{pmatrix} = \omega \dot{\eta}$$
**Kinematics of quadcopter**

**Euler ZYX \((\psi, \theta, \phi)\) rotation sequence:**

\[
I_B R = R_Z(\psi)R_Y(\theta)R_X(\phi) = \begin{bmatrix}
c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\
c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\
-s\theta & s\phi c\theta & c\phi c\theta
\end{bmatrix}
\]

The angular velocity \(B\overrightarrow{\omega}\) physically measured by the gyroscope \(B\overrightarrow{\omega} \triangleq (\omega_x \omega_y \omega_z)^T\) is expressed in term of 3 Euler angles \(\eta \triangleq (\phi, \theta, \psi)^T\) as:

\[
B\overrightarrow{\omega} = \begin{bmatrix}
1 & 0 & -s\theta \\
0 & c\phi & s\phi c\theta \\
0 & -s\phi & c\phi c\theta
\end{bmatrix} \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = W\dot{\eta}
\]
Forces and torques created by the 4 propellers

Aerodynamic forces and torques (Spakovszky (2007)):

\[ f_i = K_T \omega_i^2 \]
\[ \tau_i \approx (-1)^i b \omega_i^2 \]

Total thrust force and torques acting on the quadcopter (Formentin and Lovera (2011)):

\[ T = \sum_{i=1}^{4} f_i = K_T \sum_{i=1}^{4} \omega_i^2 \]
\[ \tau_\phi = L f_4 - L f_2 = L K_T (\omega_4^2 - \omega_2^2) \]
\[ \tau_\theta = L f_3 - L f_1 = L K_T (\omega_3^2 - \omega_1^2) \]
\[ \tau_\psi = \sum_{i=1}^{4} \tau_i = b (-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2) \]
Newton-Euler formalism

Translation equation:

\[ m\ddot{\xi} = m\vec{g} + (l_B R) \vec{T} + \vec{F}_D, \]

where \( \xi \triangleq (x, y, z)^T \) represents the position of the quadcopter in the IF, \( m \) is the system mass, \( \vec{T} \) is the thrust force in BF and \( \vec{F}_D \) is the external perturbation force.

Rotation equation:

\[ B \dot{I} B \dot{\vec{n}} + B \vec{n} \times (B \dot{B} \vec{n}) = \tau_\eta, \]

where \( B I \) is a diagonal inertial matrix, \( \tau_\eta \triangleq (\tau_\phi, \tau_\theta, \tau_\psi)^T \) gathers the roll, pitch and yaw torques and \( \times \) denotes the cross-product of two vectors.
Outline

1 Quadcopter modelling

2 Differential flatness characterization
   - Differential flatness
   - Constructive details

3 B-spline basis functions

4 B-spline parametrizations

5 Flat output description of the quadcopter system

6 Control design for trajectory tracking

7 Simulation results

8 Conclusions and future developments
Flat systems and their trajectories

Consider the continuous nonlinear system Prodan et al. (2013a)

\[ \dot{x}(t) = f(x(t), u(t)), \]

it is called differentially flat if there exist \( z(t) \) s.t. the states and inputs can be algebraically expressed in terms of \( z(t) \) and a finite number of its derivatives:

\[
\begin{align*}
    x(t) &= \Phi_0(z(t), \dot{z}(t), \ldots, z^{(q)}(t)), \\
    u(t) &= \Phi_1(z(t), \dot{z}(t), \ldots, z^{(q)}(t)), \\
    z(t) &= \gamma(x(t), u(t), \dot{u}(t), \ldots, u^{(q)}(t))
\end{align*}
\]

- For any linear and nonlinear flat system, the number of flat outputs equals the number of inputs Lévine (2009)
- for linear systems, the flat differentiability is implied by the controllability property Sira-Ramírez and Agrawal (2004)
Constructive details

Differentially flat systems are well suited to problems requiring trajectory planning $\Rightarrow$ it reduces the problem of trajectory generation to finding a trajectory of the flat outputs:

- assume an interval $t \in [0, T]$
- boundary conditions $x(0) = x_0$, $x(T) = x_f$, $u(0) = u_0$, $u(T) = u_f$
- choose a basis function $\Lambda(t) = [\ldots \Lambda^i(t) \ldots ]$
- parametrize the flat output $z(t) = \sum_{i=1}^{N_{\alpha}} \alpha_i \Lambda^i(t)$
- and its derivatives $z^{(q)}(t) = \sum_{i=1}^{N_{\alpha}} \alpha_i \Lambda^{(q)}(t)$
- obtain coefficients $\alpha_i$ from the boundary conditions
- go back to $x(t)$ and $u(t)$
Constructive details – II

There are several issues:

- there are many basis functions but not all of them all well-suited
  - polynomials ($t^i$): poor numerical performance, their dimension depends on the number of conditions imposed on the inputs, states and their derivatives
  - Bézier basis functions: their dimension depends on the number of conditions imposed on the inputs, states and their derivatives
  - B-spline basis functions: their degree depends only up to which derivative is needed to ensure continuity

- state and input constraints are not enforced: we impose constraints at the boundaries (or even in intermediate points) but what happens “in-between”?

- shortcomings which are not usually taken into account:
  - imposition of avoidably strict constraints;
  - the trajectory passing through two consecutive way-points intersects one (or more) obstacles.
Illustrative example

3-DOF model of an airplane in which the autopilot forces coordinated turns (zero side-slip) at a fixed altitude:

\[
\begin{align*}
\dot{x}(t) &= V_a(t) \cos \Psi(t), \\
\dot{y}(t) &= V_a(t) \sin \Psi(t), \\
\dot{\Psi}(t) &= \frac{g \tan \Phi(t)}{V_a(t)}
\end{align*}
\]

\[
\begin{align*}
\Psi(t) &= \arctan \left( \frac{\dot{z}_2(t)}{\dot{z}_1(t)} \right), \\
V_a(t) &= \sqrt{\dot{z}_1^2(t) + \dot{z}_2^2(t)}, \\
\Phi(t) &= \arctan \left( \frac{1}{g} \frac{\dot{z}_2(t) \dot{z}_1(t) - \ddot{z}_2(t) \ddot{z}_1(t)}{\sqrt{\dot{z}_1^2(t) + \dot{z}_2^2(t)}} \right).
\end{align*}
\]

with

- states are the position \((x(t), y(t))\) and the heading (yaw) angle \(\Psi(t) \in [0, 2\pi]\) rad
- inputs signals are the airspeed velocity \(V_a(t)\) and the bank (roll) angle \(\Phi(t)\)
- \(z(t) = [z_1(t) \quad z_2(t)]^T = [x(t) \quad y(t)]^T\) is the flat output
Outline

1. Quadcopter modelling
2. Differential flatness characterization
3. B-spline basis functions
   - B-spline functions
   - B-spline curves
4. B-spline parametrizations
5. Flat output description of the quadcopter system
6. Control design for trajectory tracking
7. Simulation results
8. Conclusions and future developments
B-spline functions

A B-spline of order \( d \) is characterized by a *knot-vector* Gordon and Riesenfeld (1974), Patrikalakis and Maekawa (2009)

\[ \mathbb{T} = \{\tau_0, \tau_1, \ldots, \tau_m\}, \]

of non-decreasing time instants \((\tau_0 \leq \tau_1 \leq \cdots \leq \tau_m)\) which parametrizes the associated basis functions \( B_{i,d}(t) \):

\[
B_{i,1}(t) = \begin{cases} 
1, & \text{for } \tau_i \leq t < \tau_{i+1} \\
0, & \text{otherwise}
\end{cases},
\]

\[
B_{i,d}(t) = \frac{t - \tau_i}{\tau_{i+d-1} - \tau_i} B_{i,d-1}(t) + \frac{\tau_{i+d} - t}{\tau_{i+d} - \tau_{i+1}} B_{i+1,d-1}(t)
\]

for \( d > 1 \) and \( i = 0, 1, \ldots, n = m - d \).

The B-splines partition the unity, in the sense that

i) \( B_{i,d}(t) \geq 0 \)

ii) \( \sum_{i=0}^{n} B_{i,d}(t) = 1 \) for all \( t \in [t_0, t_m] \).
B-spline curve generation

Considering a collection of control points

\[ \mathbb{P} = \{p_0, p_1, \ldots, p_n\}, \]

we rewrite the knot-vector as

\[ T = (\tau_0, \tau_1, \ldots, \tau_{d-1}, \underbrace{\tau_d, \tau_{d+1}, \ldots \tau_{n-1}, \tau_n, \tau_{n+1}, \tau_{n+d}}_{\text{d equal knots}}, \underbrace{\tau_{n+1}, \tau_{n+d}}_{\text{n–d+1 internal knots}}) \]

and define a B-spline curve as a linear combination of the control points and the B-spline basis functions

\[ z(t) = \sum_{i=0}^{n} B_{i,d}(t)p_i = \mathbf{P} \mathbf{B}_d(t) \]
B-spline curve properties

P1) $z(t)$ is $C^\infty$ in any $t \notin T$ and $C^{d-1}$ in any $t \in T$;

P2) the control points with a knot of multiplicity $d - 1$ coincides with the B-spline curve, thus making it a **clamped B-spline curve**;

P3) at a time instant $\tau_i < t < \tau_{i+1}$, $z(t)$ depends on the B-splines $B_{i-d+1,d}(t), \ldots, B_{i,d}(t)$; hence, $z(t)$ lies in the convex hull generated by points $p_{i-d+1}, \ldots, p_i$;

P4) as a corollary to the previous property: the B-spline curve $z(t)$ lies within the union of all convex hull formed by all $d$ successive control points;

P5) the ‘$r$’ order derivatives of B-spline basis functions can be expressed as linear combinations of B-splines of the same order (due to relations $B_{d}^{(r)}(t) = M_{r}B_{d-r}(t)$ and $B_{d-r}(t) = L_{r}B_{d}(t)$ with matrices $M_{r}$, $L_{r}$ are of appropriate dimensions and content).
Illustrative example – II

- take parameters $n = 6$ and $d = 4$
- and knot vector $\tau_0 = \tau_1 = \tau_2 = \tau_3 = 0$, $\tau_4 = 0.25$, $\tau_5 = 0.50$, $\tau_6 = 0.75$ and $\tau_7 = \tau_8 = \tau_9 = \tau_{10} = 1$
- construct $z(t) = \sum_{i=0}^{n} B_{i,d}(t)p_i = P B_d(t)$

4th order B-spline basis functions:

B-spline curve and control polygon:
Outline

1. Quadcopter modelling
2. Differential flatness characterization
3. B-spline basis functions
4. B-spline parametrizations
   - Constrained parametrization
5. Flat output description of the quadcopter system
6. Control design for trajectory tracking
7. Simulation results
8. Conclusions and future developments
Constrained parametrization – I

Let us consider a collection of \( N + 1 \) way-points and time stamps associated to them Prodan (2012):

\[
\mathcal{W} = \{w_k\} \text{ and } \mathcal{T}_\mathcal{W} = \{t_k\},
\]

for any \( k = 0, \ldots, N \).

The goal is to construct a flat trajectory which passes through each way-point \( w_k \) at the time instant \( t_k \), i.e.,

find a flat output \( z(t) \) such that

\[
x(t_k) = \Theta(z(t_k), \ldots z^{(r)}(t_k)) = w_k, \ \forall k = 0 \ldots N.
\]

\[
\tilde{\Theta}(B_d(t_k), P) = w_k, \ \forall k = 0 \ldots N,
\]

where \( \tilde{\Theta}(B_d(t), P) = \Theta(PB_d(t), \ldots, PM_rL_rB_d(t)) \) is constructed along property (P5).
Constrained parametrization – II

A nonlinear MPC iterative approach has been extensively studied in De Doná et al. (2009), Suryawan (2012):

\[
P = \arg \min_P \int_{t_0}^{t_N} \| \tilde{\Xi}(B_d(t), P) \|_Q dt
\]

s.t. \( \tilde{\Theta}(B_d(t_k), P) = w_k, \forall k = 0 \ldots N \)

with \( Q \) a positive symmetric matrix.

- The cost \( \tilde{\Xi}(B_d(t), P) = \Xi(\tilde{\Theta}(B_d(t), P), \tilde{\Phi}(B_d(t), P)) \) can impose any penalization we deem to be necessary (length of the trajectory, input variation, input magnitude, etc).
- In general, such a problem is nonlinear (due to mappings \( \tilde{\Theta}(\cdot) \) and \( \tilde{\Phi}(\cdot) \)) and hence difficult to solve.
Illustrative example – III

We take as cost to be minimized the length of the curve since we would like to have the shortest path which respects the constraints, i.e.,

\[ \tilde{\Xi}(B_d(t), P) = \|z'(t)\|. \]

This translates into the integral cost:

\[
\int_{t_0}^{t_N} \|z'(t)\| dt = \int_{t_0}^{t_N} (PM_1B_{d-1}(t))^T PM_1B_{d-1}(t) dt
\]

\[
= \sum_{i,j} ([PM_1]_i)^T \left( \int_{t_0}^{t_N} B_{i,d-1}(t)B_{j,d-1}(t) dt \right) [PM_1]_j
\]

| n  | 5   | 7   | 9   | 12  | 16  |
|-----|-----|-----|-----|-----|-----|
| cost| 5.34| 4.58| 4.62| 4.39| 4.43|
Outline

1. Quadcopter modelling
2. Differential flatness characterization
3. B-spline basis functions
4. B-spline parametrizations
5. Flat output description of the quadcopter system
6. Control design for trajectory tracking
7. Simulation results
8. Conclusions and future developments
Flat output description of the quadcopter system

The flat output vector is considered as:

\[ z = [z_1 \ z_2 \ z_3 \ z_4]^T = [x \ y \ z \ \tan \left( \frac{\psi}{2} \right)]^T, \]

The states and inputs described in term of \( z \):

\[ x = [\xi^T \ \eta^T]^T = [x \ y \ z \ \phi \ \theta \ \psi]^T \]
\[ u = [T \ \tau_\phi \ \tau_\theta \ \tau_\psi]^T \]
\[ \phi = \arcsin \left( \frac{2z_4 \ddot{z}_1 - (1 - z_4^2) \ddot{z}_2}{(1 + z_4^2) \sqrt{\ddot{z}_1^2 + \ddot{z}_2^2 + (\ddot{z}_3 + g)^2}} \right) \]
\[ \theta = \arctan \left( \frac{(1 - z_4^2) \ddot{z}_1 + 2z_4 \ddot{z}_2}{(1 + z_4^2)(\ddot{z}_3 + g)} \right) \]
\[ \psi = 2\arctan(z_4) \]
\[ T = m \sqrt{\ddot{z}_1^2 + \ddot{z}_2^2 + (\ddot{z}_3 + g)^2} \]
\[ \tau_\eta = B \ I \left( W \ddot{\eta} + \dot{W} \dot{\eta} \right) + (W \dot{\eta}) \times (B \ W \dot{\eta}) \]
Intensively studied trajectory generation problem

Specifications which need to be taken into account at the off-line and on-line stages:

- internal dynamics of the system
- state and input constraints fulfillment
- optimization problem such that a certain objective is minimized/maximized (e.g., length curve, total energy, dissipating energy, wind effects)
- trajectory reconfiguration mechanisms
- obstacle avoidance specifications
- multi-trajectory generation

Stoican et al. (2015), Prodan et al. (2013a), Chamseddine et al. (2012), Suryawan et al. (2011), De Doná et al. (2009), Formentin and Lovera (2011); Sydney et al. (2013)
Outline

1. Quadcopter modelling
2. Differential flatness characterization
3. B-spline basis functions
4. B-spline parametrizations
5. Flat output description of the quadcopter system
6. Control design for trajectory tracking
   - General control scheme
   - Flat angle tracking (FAT)
   - Flat position tracking (FPT)
   - Combined flat angle and position tracking (CFAPT)
7. Simulation results
8. Conclusions and future developments
General control scheme

\[ \ddot{\xi} = f_\xi(\eta, T) \]
\[ \dot{\omega} = f_\eta(\dot{\eta}) \]
\[ \ddot{\omega} = f_\omega(\omega, \tau_\eta) \]

\[ C_\eta(\xi, \xi_{ref}) \]

\[ C_\tau(\eta, \eta_{ref}) \]

\[ \xi_{ref} \rightarrow B' \rightarrow \eta_{ref} \rightarrow B'' \rightarrow \ddot{\xi} = f_\xi(\eta, T) \rightarrow \dot{\omega} = f_\eta(\dot{\eta}) \rightarrow \ddot{\omega} = f_\omega(\omega, \tau_\eta) \]

Quadcopter dynamics
Control design for trajectory tracking

General control scheme

Computed torque based control law

Consider the general dynamics of mechanical system:

\[ M(\Phi)\ddot{\Phi} + V(\Phi, \dot{\Phi}) = \tau_\Phi \]

Computed torque concept Craig (2005) considers:

\[ \tau_\Phi = \alpha \tau' + \beta \]

where the model-based portions \( \alpha, \beta \) and the servo portion \( \tau' \) are given as:

\[
\alpha = M(\Phi), \quad \beta = V(\Phi, \dot{\Phi}) \\
\tau' = \ddot{\Phi}_r + K_p E + K_d \dot{E} + K_i \int E \, dt
\]

with \( E = \Phi_r - \Phi \).

The controlled system becomes:

\[
M(q)\ddot{\Phi} + V(\Phi, \dot{\Phi}) = M(q) \left( \ddot{\Phi}_r + K_p E + K_d \dot{E} + K_i \int E \, dt \right) + V(\Phi, \dot{\Phi})
\]

\[ \Rightarrow \ddot{E} + K_d \dot{E} + K_p E + K_i \int E \, dt = 0 \]
Consider the rotational equation of the quadcopter:

\[ B I W \ddot{\eta} + B I \dot{W} \dot{\eta} + (W \dot{\eta}) \times (B I W \dot{\eta}) = \tau_\eta \]

Applying the computed torque based control law, the input angle torques can be considered as:

\[ \tau_\eta = B I W \left( \ddot{\eta}_r + K_{p\eta} E_\eta + K_{d\eta} \dot{E}_\eta + K_{i\eta} \int E_\eta \, dt \right) + B I \dot{W} \dot{\eta} + (W \dot{\eta}) \times (B I W \dot{\eta}) \]
Consider the roll, pitch, yaw angles and input thrust $T$ in terms of the flat output $z$:

$$
\phi = \arcsin \left( \frac{2z_4 \ddot{z}_1 - (1 - z_4^2) \ddot{z}_2}{(1 + z_4^2) \sqrt{\ddot{z}_1^2 + \ddot{z}_2^2 + (\ddot{z}_3 + g)^2}} \right) = \Gamma_\phi(\ddot{z}_1, \ddot{z}_2, \ddot{z}_3, z_4)
$$

$$
\theta = \arctan \left( \frac{(1 - z_4^2) \ddot{z}_1 + 2z_4 \ddot{z}_2}{(1 + z_4^2)(\ddot{z}_3 + g)} \right) = \Gamma_\theta(\ddot{z}_1, \ddot{z}_2, \ddot{z}_3, z_4)
$$

$$
\psi = 2 \arctan(z_4) = \Upsilon_\psi(z_4)
$$

$$
T = m \sqrt{\ddot{z}_1^2 + \ddot{z}_2^2 + (\ddot{z}_3 + g)^2} = \Gamma_T(\ddot{z}_1, \ddot{z}_2, \ddot{z}_3)
$$
Attitude controller (cont.)

The feedback linearization based control law for attitude controller:

\[
\phi_{\text{ref}} = \Gamma_\phi(\ddot{z}_1^*, \ddot{z}_2^*, \ddot{z}_3^*, z_4), \\
\theta_{\text{ref}} = \Gamma_\theta(\ddot{z}_1^*, \ddot{z}_2^*, \ddot{z}_3^*, z_4), \\
\psi_{\text{ref}} = \Upsilon_\psi(\bar{z}_4), \\
T = \Gamma_T(\ddot{z}_1^*, \ddot{z}_2^*, \ddot{z}_3^*),
\]

where the corrective term \(\xi^* \triangleq [z_1^* \ z_2^* \ z_3^*]^\top\) is given as:

\[
\xi^* = \xi_{\text{ref}} + K_{d\xi} \int \epsilon_\xi \, dt + K_{p\xi} \int \int \epsilon_\xi \, dt + K_{i\xi} \int \int \int \epsilon_\xi \, dt,
\]
Flat angle tracking

\[ T \]

\[ \eta \]

\[ C_T(\eta, \eta_{ref}) \]

Rotation controller

\[ \tau_\eta \]

\[ \xi = f_\xi(\eta, T) \]
\[ \omega = f_\eta(\dot{\eta}) \]
\[ \dot{\omega} = f_\omega(\omega, \tau_\eta) \]

Quadcopter dynamics

\[ \eta \]
The angle torques are provided to the quadcopter as the insertion at point C based on the rotational equation:

\[
\tau_\eta = ^B I \left( W_\eta \ddot{\eta}_{ref} + \dot{W}_\eta \dot{\eta}_{ref} \right) + (W_\eta \dot{\eta}_{ref}) \times (^B I W_\eta \dot{\eta}_{ref}),
\]

where \( W_\eta = W(\eta_{ref}) \).
Combined flat angle and position tracking

\[ C_\eta(\xi, \xi_{\text{ref}}) \]

Attitude controller

\[ C_T(\eta, \eta_{\text{ref}}) \]

Rotation controller

\[ \frac{\ddot{\xi}}{f_\xi(\eta, T)} \quad \frac{\omega}{f_\eta(\dot{\eta})} \quad \frac{\dot{\omega}}{f_\omega(\omega, \tau_\eta)} \]

Quadcopter dynamics
Outline

1 Quadcopter modelling
2 Differential flatness characterization
3 B-spline basis functions
4 B-spline parametrizations
5 Flat output description of the quadcopter system
6 Control design for trajectory tracking

7 Simulation results
   - Trajectory generation
   - Trajectory tracking

8 Conclusions and future developments
Waypoint trajectory generation

Generate the trajectory passing through 5 way points at specific time instants while minimize the length of the trajectory.

Simulation of wind disturbances:

|          | Nominal | Moderate | Strong  |
|----------|---------|----------|---------|
| km/h     | 0       | 15.3     | 20.5    |

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Waypoint trajectory generation

Flat trajectory and the associated control points

Flat reference of the angles
Waypoint trajectory generation

Flat reference of the thrust force

Flat reference of the angle torques
Simulation of combined flat angle and position tracking strategy in strong wind condition (20.5 km/h)
Trajectory tracking

Crazyfile 2.0 numerical data for simulations

Flat angle tracking in moderate wind gush

Flat position tracking in moderate wind gush

Combined flat angle and position tracking in strong wind gush
Simulation parameters and IAE results

- Controller parameters:

| Control Scheme                  | \( K_p \)         | \( K_d \)         | \( K_i \)         |
|---------------------------------|--------------------|--------------------|--------------------|
| Rotation controller             | diag \{225, 225, 225\} | diag \{30, 30, 30\} | diag \{0, 0, 0\}   |
| used in FAT, CFAPT              |                    |                    |                    |
| Attitude controller             | diag \{25, 25, 9\} | diag \{10, 10, 6\} | diag \{1, 1, 0.3\} |
| used in FPT, CFAPT              |                    |                    |                    |

Table: Parameters of rotation and attitude controller

- Fixed sampled time of 0.01s and solver ode4
- Integral of Absolute magnitude of the Error (IAE) over the position:

\[
IAE_\xi = \int_{t_0=0}^{t_f=10} ||\xi_r - \xi|| dt
\]

|                  | Nominal Wind | Moderate Wind | Strong Wind |
|------------------|--------------|---------------|-------------|
| FAT              | 0.0182       | 38.7929       | *           |
| FPT              | 0.4702       | 0.4819        | 0.4914      |
| CFAPT            | 0.0278       | 0.3750        | 0.5980      |

Table: IAE results of 3 control strategies
Outline

1. Quadcopter modelling
2. Differential flatness characterization
3. B-spline basis functions
4. B-spline parametrizations
5. Flat output description of the quadcopter system
6. Control design for trajectory tracking
7. Simulation results
8. Conclusions and future developments
Conclusions and future developments

Conclusions:
- Quadcopter modeling using Newton-Euler formalism
- Novel flat output representation
- Trajectory generation problem
- Feedback linearization based control designs for trajectory tracking
- Extensive simulations for different wind conditions

Future development:
- MPC/NMPC implementations
- Bounded/stochastic disturbances considerations
- Trajectory reconfiguration mechanisms
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