Superconductivity and a condensate of ordered zero-point oscillations

B.V. Vasiliev

1 About isotopical effect in superconductors

Superconductivity was experimentally discovered about 100 years ago. Its theoretical explanation was the object of many studies. But so far the superconductivity is not be totally understandable. The existing theories of superconductivity describe the general properties of superconductors: their behavior in magnetic fields, features of the behavior near the critical temperature, etc. But they are not applicable to the determination of specific properties of individual superconductors.

Since 50-ies it is generally accepted to describe the phenomenon of superconductivity by BCS theory, which assumes that the Cooper pairing of electrons is a result of electron-phonon interaction. It is generally accepted to think that the electron-phonon interaction is unique mechanism which can give a correct description for superconductivity phenomenon since the isotope effect exists. Indeed, if an effect depends on the ion mass, it is essential to think that it is based on some oscillator (phonon) process. But it must be noted that the quantitative explanation of the isotope-effect is complicated by the fact that for some superconductors masses of the isotope are effecting on the critical temperature, and in some - no, and the degree of this influence is not always the same. With a few I type superconductors the situation is easier - for them this effect can be considered as equal. This - Zn, Sn, In, Hg, Pb. For them, the isotope-effect can be described by the relation:

$$\sqrt{M_i} T_c = \text{const},$$

(1)

Where $M_i$ is the isotope mass.

It is important that another explanation for the isotopic effect in superconductors can be given. In the last decades it has shown experimentally that the isotopic substitution in metals can directly lead to a change of the crystal cells parameters. It is the consequence of the fact that the zero-point oscillations of atoms in some crystal lattices are anharmonic (and they are harmonic in another crystals). It leads to the dependence of the electron density from the isotopic mass, i.e. it changes the Fermi-energy and other important characteristics of an electronic system of metals and thus it may affect on the properties of the superconductor.
As it is following from the results of measurements which was carried out on Ge, Si, diamond and light metals, such as Li\cite{2},\cite{3},\cite{4} there are root dependence of force constants from the mass of the isotope, which is required for the Eq.(1). The same dependence of force constants from the mass of the isotope was found in the tin \cite{4}. Unfortunately, there are no direct measurements of an influence of the isotopic substitution on the electronic properties of metals, such as the electronic specific heat and the Fermi-energy.

Nevertheless, it seems that the existence of influence of isotope substitution on the critical temperature of the superconductors is not necessarily to be clearly indicated on the work of the phonon mechanism in the formation of pairs. They can be coupled by other interactions, and isotopic substitution causes a changing in the Fermi-energy and thus it can affect on the critical parameters of the superconductor.

The existing of superconductivity is the result of ordering in the system of conduction electrons.

It is shown \cite{1}, that the interaction of a zero-point oscillations of the collectivized electrons can play a role of the physical mechanism which is leading to this ordering.

\footnote{Researchers prefer to use crystals, where the isotope-effects are large and it is easier to obtain appropriate results}
2 The condensate of ordered zero-point oscillations of electron gas

J. Bardeen was first who turned his attention on a possible link between superconductivity and zero-point oscillations [5].

The special role of zero-point vibrations exists due to the fact that in metals all movements of electrons have been freezed out except of these oscillations.

At a decreasing of temperature, the conducting electrons lose the thermal excitations and, if there is a mechanism of combination of electrons into pairs, which obey to Bose-Einstein statistics, they tend to form condensate on the level of minimum energy.

Thus, the ordering in a gas of the conduction electrons can exists as a result of a work of two mechanisms (see Fig. 1).

2.1 The electron pairing

First, in the electron gas should occur an energetically favorable electron pairing.

The pairing of electrons can occur due to the magnetic dipole-dipole interaction.

In order to the magnetic dipole-dipole interaction could merge two electrons in the singlet pair at the temperature of about 10K, the distance between this particles must be small enough:

\[ r < \left( \frac{\mu^2}{kT_c} \right)^{1/3} \approx a_B, \]

where \( a_B = \frac{\hbar^2}{m_e e^2} \) is the Bohr radius.

Ie two collectivized electrons must be localized in a volume of one lattice site. It is in an agreement with the fact that the superconductivity can occur only in metals with two collectivized electrons per atom, and can not exist in the monovalent alkali and noble metals.

The pairing of electrons above \( T_c \) ([6], [7]) indicates that the pairing is a necessary but not sufficient condition for the existence of superconductivity.

2.2 The condensate of zero-point oscillations

The condensation of the electron pairs, as bosons, is the additional necessary condition of the superconductivity arising.

An additional necessary condition is the property of the electron pairs, as bosons, condense at a lower energy level, which should occur at the expense of their interactions.

As at low temperatures, the zero-point oscillations are exist only, a further lowering of energy can occur at the arising of a coherence zero-point oscillations, ie ordering of their amplitudes, frequencies and phases.
The lowering of electron energy at their pairing due to magnetic dipole-dipole interaction

The lowering of electron energy due to the ordering of their zero-point oscillations

The condensate of ordered zero-point oscillations

Рис. 1: The schematic representation of the energy levels of conducting electrons in a superconducting metal
Let an electron gas has density $n_e$ and its Fermi-energy $E_F$. Each electron of this gas is fixed inside a cell with linear dimension $\lambda_F$:

$$n_e = \frac{1}{\lambda_F^3}. \quad (3)$$

If to neglect the interactions of the electron gas, then its Fermi-energy can be written as [8]:

$$E_F = \left(3\pi^2\right)^{2/3} \frac{\hbar^2}{2m_e n_e} n_e^{2/3} \simeq \frac{e^2 a_B}{\lambda_F^2}. \quad (4)$$

However, a conduction electron interacts with the ion at its zero-point oscillations. If to consider the ions system as a positive background uniformly spread over cells, the electron inside the cell has potential energy:

$$E_p \simeq -\frac{e^2}{\lambda_F}. \quad (5)$$

An electron will perform inside the cell zero-point oscillations with an amplitude of $a_0$ and with the energy:

$$E_k \simeq \frac{1}{2} \frac{\hbar^2}{m_e a_0^2}. \quad (6)$$

In accordance with the virial theorem [12], if a particle executes a finite motion, its potential energy $E_p$ should be associated with its kinetic $E_k$ by the simple relation $|E_p| = 2E_k$. In this regard, we find that the amplitude of the zero-point oscillations of the electron in the cell:

$$a_0 \simeq \sqrt{2\lambda_F a_B}. \quad (7)$$

Coming from the quantization condition

$$m_e a_0^2 \Omega_0 = \hbar, \quad (8)$$

one can determinate the frequency of zero-point oscillations $\Omega_0$ and their wavelength $L = \frac{2\pi}{\Omega_0}$.

These zero-point oscillations form an oscillating electric dipole moment of the electron with the amplitude value:

$$d_\perp = ea_0 \quad (9)$$

The electron interaction via their dipole moments should lead at sufficiently low temperatures to a forming of an ordered condensate of the zero-point oscillations.

The identity of the electron pairs leads to the equality of the frequencies and amplitudes of zero-point vibrations.

The oscillations of the electron pairs, which are located at distances which are equal to an integer number of wavelengths $L$, will be in-phase, or, more exactly will be shifted on (integer number)$\cdot 2\pi$. 

5
Since the ordering of the oscillations should occur due to electromagnetic interaction of oscillating dipoles, the energy minimum should correspond to the antiphase mode of neighboring oscillators at which the distance between they \( \Lambda_0 \) should be equal to half the wavelength \( L \), which is induced by oscillating dipoles:

\[
\Lambda_0 = \frac{L}{2} = \frac{c}{4\pi\Omega_0}.
\]

Hence

\[
\Lambda_0 \simeq \frac{\lambda_F}{\pi \alpha}
\]

and the relation between the density of the ordered condensate and the density of Fermi-gas, from which it is formed:

\[
\frac{n_0}{n_e} = \frac{\lambda_F^3}{\Lambda_0^3} \simeq (\pi \alpha)^3 \simeq 10^{-5}.
\]

The using of the above equations allows us to find the linear size of the volume of the pair localization

\[
\Lambda_0 = \frac{L}{2} \simeq \frac{1}{\pi \alpha (n_e)^{1/3}}
\]

This linear dimension equals \( 10^{-6} \text{cm} \) in the order of value and plays in this model a role which is similar to the Pippard’s coherence length in the BCS.

The comparison of these calculated values with measured data is shown in Table(5.1.3).

As result of the particle electromagnetic interaction, the density of electron system energy decreases on the value:

\[
\Delta_0 \simeq \frac{d_0^2}{\Lambda_0^3}
\]

or at taking into account Eq.(7) we obtain relation between the particle density and the value of gap of condensate spectrum:

\[
\Delta_0 \simeq 2\pi^2 \alpha^2 \frac{\hbar^2}{m_e n_0^{2/3}}
\]

and

\[
n_0 = \frac{1}{\Lambda_0^3} = \left( \frac{m_e}{2\pi^2 \alpha^2 \hbar^2 \Delta_0} \right)^{3/2}.
\]

It should be noted that these ratios differ from corresponding expressions for the Bose-condensate, which are obtained in many courses (see eg [8]): the expressions for the ordered condensate of zero-point oscillations have the coefficient \( \alpha \) on the right side.

From Eq.(15) we can obtain

\[
\frac{\Delta_0}{E_F} \simeq 1.5 \cdot \pi^2 \alpha^3,
\]
\[
\frac{T_e}{T_F} \simeq 3.2 \cdot 10^{-6}.
\] (18)

The comparison of this relation with measured data is shown in Table 5.1.4.
3 The critical parameters of the zero-point oscillations condensate.

3.1 The temperature dependence of the energetic distribution of condensate particles.

The phenomenon of condensation of zero-point oscillations in the electron gas has the characteristic features.

The evaporation of condensate into the normal state should be classified as order-disorder transition.

This condensate can be destroyed by heating as well as by the application of a sufficiently strong magnetic field. So between the critical temperature and critical magnetic field condensate there must be a link which will appear in superconductors, if the superconductivity occurs at the ordering of zero-point oscillations.

Let us assume that at a given temperature \( T < T_c \) the system of vibrational levels of conducting electrons consists of two levels - basic level which is characterized by anti-phase oscillations of the electron pairs at the distance \( \Lambda_0 \), and excited, characterized by in-phase oscillation of the pairs.

Let the population of basic level is \( N_0 \) particles and the excited level has \( N_1 \) particles.

Two electron pairs with in-phase oscillation have high energy of interaction and cannot form the condensate. The condensate can be formed only the particles that make up the difference between the populations of levels \( N_0 - N_1 \).

In dimensionless form, this difference defines the order parameter:

\[
\Psi = \frac{N_0}{N_0 + N_1} - \frac{N_1}{N_0 + N_1}.
\] (19)

In the theory of superconductivity, by definition, the order parameter is determined by the value of the energy gap

\[
\Psi = \frac{\Delta_T}{\Delta_0}.
\] (20)

At taking a counting of energy from the level \( \varepsilon_0 \), we obtain

\[
\frac{\Delta_T}{\Delta_0} = \frac{N_0 - N_1}{N_0 + N_1} \approx \frac{e^{\Delta_T/kT} - 1}{e^{2\Delta_T/kT} + 1} = \text{th}(2\Delta_T/kT).
\] (21)

Passing to dimensionless variables \( \delta = \frac{\Delta_T}{\Delta_0} \), \( t = \frac{kT}{kT_c} \) and \( \beta = \frac{2\Delta_0}{kT_c} \) we have

\[
\delta = \frac{e^{\beta\delta/t} - 1}{e^{\beta\delta/t} + 1} = \text{th}(\beta\delta/t).
\] (22)

This equation describes the temperature dependence of the energy gap in the spectrum of zero-point oscillations. It coincides in form with other equations describing other physical phenomena, which are also characterized by the
The temperature dependence of the value of gap in the energetic spectrum of bosons calculated on Eq. (22).

existence of the temperature dependence of order parameters [8], [9]. For example, such as the concentration of the superfluid component in liquid helium or the spontaneous magnetization of ferromagnetic materials. This equation is common for all order-disorder transitions (the phase transitions of II-type in the Landau classification).

The solution of this equation, obtained by the iteration method, is shown in Fig. (2).

This decision is very accurately coincides with the known transcendental equation of the BCS, which was obtained by the integrating of the phonon spectrum, and is in a quite satisfactory agreement with the measurement data.

After numerical integrating we can obtain the averaging value of the gap:

$$
\langle \Delta \rangle = \Delta_0 \int_0^1 \delta dt = 0.852 \ \Delta_0 . \quad (23)
$$
3.1.1 The critical parameters of a zero-point oscillations condensate and superconductivity

To convert the condensate in the normal state, we must put up the half of its particles into the excited state (the gap collapses under this condition). To do this, taking into account Eq. (23), the unit volume of condensate should get the energy:

\[
\mathcal{E}_T \simeq \frac{1}{2} n_0 \langle \Delta_0 \rangle \approx \frac{0.85}{2} \left( \frac{m_e}{2\pi^2\alpha\hbar^2} \right)^{3/2} \Delta_0^{5/2}, \quad (24)
\]

On the other hand, we can get the normal state of an electrically charged condensate at an applying of a magnetic field of critical value \( H_c \) with the density of energy:

\[
\mathcal{E}_H = \frac{H_c^2}{8\pi}, \quad (25)
\]

As a result, we obtain

\[
\frac{1}{2} n_0 \langle \Delta_0 \rangle = \frac{H_c^2}{8\pi}, \quad (26)
\]

The comparison of the critical energy densities \( \mathcal{E}_T \) and \( \mathcal{E}_H \) for type I superconductors are shown in Fig. 3. The obtained agreement between energies \( \mathcal{E}_T \) (Eq. (24)) and \( \mathcal{E}_H \) (Eq. (25)) can be considered as quite satisfactory for type I superconductors [10, 11]. A similar comparison of data for type-II superconductors gives the results differ in approximately two times. The correction this calculation, apparently, has not make sense. The purpose of these
calculations was to show that the description of superconductivity as the effect of the condensation of ordered zero-point oscillations is in accordance with the available experimental data. And this goal can be considered quite reached.
3.2 The zero-point oscillations and critical magnetic field of superconductors

The direct influence of the external magnetic field of the critical value applied to the electron system is too weak to disrupt the dipole-dipole interaction of two paired electrons:

\[ \mu_B H_c \ll kT_c. \]  

(27)

To violate the superconductivity enough to destroy the ordering of the electron zero-point oscillations. In this case it is enough not very strong magnetic field to violate their.

With using of Eqs. (26) and (14), we can express the gap through the critical magnetic field and the magnitude of the oscillating dipole moment:

\[ \Delta_0 \approx \frac{1}{2} e a_0 H_c. \]  

(28)

The properties of zero-point oscillations of the electrons should not be depended on the characteristics of the mechanism of association and the existence of electron pairs. Therefore, one should expect that this equation should be valid for type I superconductors, as well as for type-II superconductors (for type-II superconductor \( H_c = H_{c1} \) is the first critical field).

A satisfaction of this condition is illustrated on the Fig. (4).
Рис. 4: The comparison of the calculated energy of the superconducting pairs in the critical magnetic field with the value of the superconducting gap. On this figure are marked: triangles - type-II superconductors, squares - type-I superconductors. On vertical axis - logarithm of the product of the calculated value of the oscillating moment of the electron on the critical magnetic field is plotted. On abscissa - the value of the gap is shown.
4 The critical temperature of superconductor and its electronic specific heat

4.1 The electron states density and specific heat

Let us consider the process of heating the electron gas in a metal. At a heating, the electrons from levels slightly below the Fermi-energy are raised to higher levels. As a result, the levels closest to the Fermi level, from which at low temperature electrons was forming bosons, become vacant.

At critical temperature $T_c$, all electrons from the levels of energy bands from $\mathcal{E}_F - \Delta$ to $\mathcal{E}_F$ are moved to higher levels (and the gap collapses). At this temperature superconductivity is destroyed completely.

This band of energy can be filled by $N\Delta$ particles:

$$N\Delta = 2 \int_{\mathcal{E}_F - \Delta}^{\mathcal{E}_F} F(\mathcal{E}) D(\mathcal{E}) d\mathcal{E}. \tag{29}$$

Where $F(\mathcal{E}) = \frac{1}{e^{(\mathcal{E}-\mu)/kT} + 1}$ is the Fermi-Dirac function, $D(\mathcal{E})$ is number of states per an unit energy interval, deuce front of the integral arises from the fact that there are two electron at each energy level.

To find the density of states $D(\mathcal{E})$, one needs to find the difference in energy of the system at $T = 0$ and finite temperature:

$$\Delta \mathcal{E} = \int_{0}^{\infty} F(\mathcal{E}) \mathcal{E} D(\mathcal{E}) d\mathcal{E} - \int_{0}^{\mathcal{E}_F} F(\mathcal{E}) \mathcal{E} D(\mathcal{E}) d\mathcal{E}. \tag{30}$$

At calculation of the density of states $D(\mathcal{E})$, we must take into account that two electrons can be placed on each level. Thus, from the expression of the Fermi-energy

$$\mathcal{E}_F = \left(3\pi^2\right)^{2/3} \frac{\hbar^2}{2m_e} n_e^{2/3}, \tag{31}$$

we obtain

$$D(\mathcal{E}_F) = \frac{1}{2} \frac{d n_e}{d\mathcal{E}_F} = \frac{3n_e}{4\mathcal{E}_F} = \frac{3\gamma}{2k^2\pi^2}, \tag{32}$$

where

$$\gamma = \frac{\pi^2 k^2 m_e}{4\mathcal{E}_F} = \frac{1}{2} \left(\frac{\pi}{3}\right)^{3/2} \left(\frac{k}{\hbar}\right)^2 m_e n_e^{1/3}, \tag{33}$$

is the Sommerfeld constant.

At the using of similar arguments, we can calculate the number of electrons, which populate the levels in the range from $\mathcal{E}_F - \Delta$ to $\mathcal{E}_F$. For an unit volume of material, Eq. (29) can be rewritten as:

$$n\Delta = 2kT \cdot D(\mathcal{E}_F) \int_{0}^{\mathcal{E}_F} \frac{dx}{e^x + 1}. \tag{34}$$

\footnote{It should be noted that because on each level can be placed two electrons, the expression for the Sommerfeld constant Eq. (33) contains the additional factor 1/2 in comparison with the usual formula in the literature.}
At taking into account that for superconductors $\Delta_0/kT_c = 1.76$, as a result of numerical integration we obtain

$$\int_{-\Delta_0/kT_c}^{0} \frac{dx}{(e^x + 1)} = [x - ln(e^x + 1)]_{-1.76}^{0} \approx 1.22. \quad (35)$$

Thus, the density of electrons, which throw up above the Fermi level in a metal at temperature $T = T_c$ is

$$n_e(T_c) \approx 2.44 \left(\frac{3\gamma}{k^2\pi^2}\right) kT_c. \quad (36)$$

Where the Sommerfeld constant $\gamma$ is related to the volume unit of the metal.
5 The Sommerfeld constant and critical temperature superconductors

5.1 The type-I superconductors

The performed calculations make it possible to find the direct dependence of the critical temperature superconductor from the electronic specific heat - the experimentally measurable parameter of a solid-state.

The density of superconducting carriers at $T = 0$ has been calculated earlier (Eq.(16)).

The comparison of the values $n_0$ and $n_e(T_c)$ is given in the Table (5.1.1) in Fig.(5). (The necessary data for superconductors are taken from the tables [10], [11]).

| superconductor | $n_0$ | $n_e(T_c)$ | $2n_0/n_e(T_c)$ |
|----------------|------|-----------|----------------|
| Cd             | $6.11 \times 10^{17}$ | $1.48 \times 10^{18}$ | 0.83 |
| Zn             | $1.29 \times 10^{18}$ | $3.28 \times 10^{18}$ | 0.78 |
| Ga             | $1.85 \times 10^{18}$ | $2.96 \times 10^{18}$ | 1.25 |
| Al             | $2.09 \times 10^{18}$ | $8.53 \times 10^{18}$ | 0.49 |
| Tl             | $6.03 \times 10^{18}$ | $1.09 \times 10^{19}$ | 1.10 |
| In             | $1.03 \times 10^{19}$ | $1.94 \times 10^{19}$ | 1.06 |
| Sn             | $1.18 \times 10^{19}$ | $2.14 \times 10^{19}$ | 1.10 |
| Hg             | $1.39 \times 10^{19}$ | $2.86 \times 10^{19}$ | 0.97 |
| Pb             | $3.17 \times 10^{19}$ | $6.58 \times 10^{19}$ | 0.96 |

Table (5.1.1). The comparison of the number of superconducting carriers at $T = 0$ with the number of thermally activated electrons at $T = T_c$.

From the data obtained above, one can see that the condition of destruction of superconductivity after heating for superconductors of type I can really be written as the equation:

$$n_e(T_c) \simeq 2n_0$$  \hspace{1cm} (37)

Eq.(37) gives us the possibility to express the critical temperature of the superconductor through its Sommerfeld constant:

$$\Delta_0 \simeq C\gamma^2,$$  \hspace{1cm} (38)

where the constant

$$C \approx 31 \frac{\pi^2}{k} \left[ \frac{\alpha \hbar^2}{km_e} \right]^3 \approx 6.55 \times 10^{-22} \frac{K^4 cm^6}{erg}.$$  \hspace{1cm} (39)

The comparison of the temperature calculated by Eq.(38) (corresponding to complete evaporation of electrons with energies in the range from $E_F - \Delta_0$ up to $E_F$) and the experimentally measured critical temperature superconductors is given in Table (5.1.2) and in Fig.(6).
Рис. 5: The comparison of the number of superconducting carriers at $T = 0$ with the number of thermally activated electrons at $T = T_c$.

Рис. 6: The comparison of the calculated values of critical temperature superconductors with measurement data. On abscissa the measured values of the critical temperature superconductors of type I are shown, on the vertical axis the function $C\gamma^2$ defined by Eq. (38) is shown.
At taking into account Eqs. (33) and (31), we can express the value of the Fermi energy through the measured Sommerfeld constant:

$$\mathcal{E}_F(\gamma) = \frac{p_F^2(\gamma)}{2m_e} \approx \left(\frac{12\gamma}{k^2}\right)^2 \left(\frac{\hbar^2}{2m_e}\right)^3$$

(40)

and rewrite the value of the gap:

$$\Delta_0 \approx 1.7\pi^2\alpha^3\mathcal{E}_F(\gamma).$$

(41)

This expression for the value of a gap is in a good agreement with the previously obtained Eq. (17).

The de Broglie wavelengths of Fermi electrons expressed through the Sommerfeld constant

$$\lambda_F = \frac{2\pi\hbar}{p_F(\gamma)} \approx \frac{\pi}{3} \frac{k^2m_e}{\hbar^2\gamma}.\quad (42)$$

are shown in Table 5.1.3. For comparison, the de Broglie wavelengths of the superconducting pairs (Eq. (10)) and the ratio of the density of superconducting carriers to the density of fermions are given in this table. It can be seen that the ratio of these densities on the order of value are close to $10^{-5}$ really in full compliance with the previously obtained Eq. (12).
In accordance with Eq. (18), the ratio $T_c/T_F \simeq 3 \cdot 10^{-6}$. The calculated values of $T_F$, measured values of $T_c$ and their ratios are shown in Table 5.1.4.

| superconductor | $T_c$,K | $T_F$,K | $T_c/T_F$ |
|---------------|--------|--------|----------|
| Cd            | 0.51   | $1.81 \cdot 10^3$ | $2.86 \cdot 10^{-6}$ |
| Zn            | 0.85   | $3.30 \cdot 10^5$ | $2.58 \cdot 10^{-6}$ |
| Ga            | 1.09   | $1.65 \cdot 10^5$ | $6.65 \cdot 10^{-6}$ |
| Tl            | 2.39   | $4.67 \cdot 10^5$ | $5.09 \cdot 10^{-6}$ |
| In            | 3.41   | $7.22 \cdot 10^5$ | $4.72 \cdot 10^{-6}$ |
| Sn            | 3.72   | $7.33 \cdot 10^5$ | $5.08 \cdot 10^{-6}$ |
| Hg            | 4.15   | $1.05 \cdot 10^6$ | $3.96 \cdot 10^{-6}$ |
| Pb            | 7.19   | $1.85 \cdot 10^6$ | $3.90 \cdot 10^{-6}$ |

The agreement between the previously obtained estimation Eq. (18) and these data can be considered as satisfactory.
5.1.1 The relationship $\Delta_0/kT_c$

Coming from Eq.(47) and taking into account Eqs.(16), (36) and (38), we obtain

$$\frac{\Delta_0}{kT_c} \simeq 1.35.$$  \hspace{1cm} (43)

The obtained estimation of relationship $\Delta_0/kT_c$ has satisfactory agreement with measured data [10], which for type I superconductors are listed in Table (5.1.1).

| superconductor | $T_c$,K | $\Delta_0$,mev | $\frac{\Delta_0}{kT_c}$ |
|----------------|---------|----------------|-------------------------|
| Cd             | 0.51    | 0.072          | 1.64                    |
| Zn             | 0.85    | 0.13           | 1.77                    |
| Ga             | 1.09    | 0.169          | 1.80                    |
| Tl             | 2.39    | 0.369          | 1.79                    |
| In             | 3.41    | 0.541          | 1.84                    |
| Sn             | 3.72    | 0.593          | 1.85                    |
| Hg             | 4.15    | 0.824          | 2.29                    |
| Pb             | 7.19    | 1.38           | 2.22                    |

Table 5.1.1
5.2 The estimation of properties of type-II superconductors

The situation is different in the case of type-II superconductors. In this case the measurements show that these metals have the electronic specific heat on an order of magnitude greater than the calculation based on free electron gas gives.

The peculiarity of these metals associated with the specific structure of their ions. They are transition metals with unfilled inner d-shell (see Table 5.2).

It can be assumed that this increasing of the electronic specific heat of these metals should be associated with a characteristic interaction of free electrons with the electrons of the unfilled d-shell.

| superconductors | electron shells |
|-----------------|-----------------|
| Ti              | 3d² 4s²         |
| V               | 3d³ 4s²         |
| Zr              | 4d² 5s²         |
| Nb              | 4d³ 5s²         |
| Mo              | 4d⁴ 5s²         |
| Tc              | 4d⁵ 5s²         |
| Ru              | 4d⁶ 5s²         |
| La              | 5d¹ 6s²         |
| Hf              | 5d² 6s²         |
| Ta              | 5d³ 6s²         |
| W               | 5d⁴ 6s²         |
| Re              | 5d⁵ 6s²         |
| Os              | 5d⁶ 6s²         |
| Ir              | 5d⁷ 6s²         |

Table 5.2. The electron shells of elementary type-II superconductors.

Since the heat capacity of the ionic lattice of metals at low temperatures under consideration is negligible, only the electronic subsystem is active thermally.

At \( T = 0 \), the maximum kinetic energy has the electrons at the Fermi level. When heated, these electrons gain additional kinetic energy:

\[
\mathcal{E}_k = \frac{m_e v^2}{2},
\]

Only fraction of the heating energy transferred to the metal is consumed to increase the kinetic energy of the electron gas in transition metals.

Another part of the energy will be spent on the magnetic interaction of a moving electron.

At a contact with the electron d-shells, a moving free electron induces on it the magnetic field of the order of value:

\[
H \approx \frac{e}{v^2 c}.\]

21
The magnetic moment of d-electron is approximately equal to the Bohr magneton. Therefore the energy of magnetic interaction between the moving electron of conductivity and the d-electron is approximately equal to:

\[ E_\mu \approx \frac{e^2 v}{2\mu c^2}. \]  

(46)

This energy is not connected with the process of destruction of superconductivity.

Whereas in metals with filled d-shell (type-I superconductors), the whole energy of the heating increases the kinetic energy of conductivity electrons, only a small part of the heating energy is spent on it in transition metals:

\[ \frac{E_k}{E_\mu + E_k} \approx \frac{1}{\alpha^{3/2}} \sqrt{\frac{2\Delta_0}{m_e c^2}}. \]  

(47)

Therefore, whereas the dependence of the gap in type-I superconductors from the heat capacity is defined by Eq.(38), it is necessary to take into account the relation Eq.(47) in type-II superconductors for the determination of this gap dependence. As a result of this estimation, we obtain:

\[ \Delta_0 \simeq C\gamma^2 \left( \frac{E_k}{E_\mu + E_k} \right) \simeq \frac{2}{m_e c^2} \left( \frac{6\pi \alpha^{3/4} k^3}{k^2 m_e^{3/2}} \right)^4 \gamma^4. \]  

(48)

The comparing of the results of these calculations with the measurement data (Fig.(7)) shows that for the majority of type II superconductors the estimation Eq.(48) can be considered quite satisfactory.
5.3 Alloys and high-temperature superconductors

To understand the mechanism of high temperature superconductivity is important to establish whether the high-$T_c$ ceramics are the I or II type superconductors, or they are a special class of superconductors.

To address this issue, one can use the above established dependence of critical parameters from the electronic specific heat and the fact that the specific heat of superconductors I and II types are differing considerably.

There are some difficulty on this way - one do not known confidently the density of the electron gas in high-temperature superconductors. However, the density of atoms in metal crystals does not differ too much. It facilitates the solution of the problem of a distinguishing of I and II types superconductors at using of Eq. (38).

For the I type superconductors at using this equation, we get the quite satisfactory estimation of the critical temperature (as was done above, see Fig. 6). For the type-II superconductors this assessment gives overestimated value due to the fact that their specific heat has additional term associated with the polarization of d-electrons.

Indeed, such analysis gives possibility to share all of superconductors into two groups, as is evident from the figure (8).

It is generally assumed to consider alloys $Nb_3Sn$ and $V_3Si$ as the type-II superconductors. The fact seems quite normal that they are placed in close surroundings of Nb. Some excess of the calculated critical temperature
Рис. 8: The comparison of the calculated parameter $C\gamma^2$ with the measurement critical temperatures of elementary superconductors and some superconducting compounds.

over the experimentally measured value for ceramics $Ta_2Ba_2Cu_3O_{10}$ can be attributed to the fact that the measured heat capacity may make other conductive electrons, but nonsuperconducting elements (layers) of ceramics. It is not a news that it, as well as ceramics $YBa_2Cu_3O_7$, belongs to the type-II superconductors. However, ceramics $(LaSr)_2Cu_4$, Bi-2212 and Tl-2201, according to this figure should be regarded as type-I superconductors, which is somewhat unexpected.

The understanding of the mechanisms of the superconducting state is important to open the way for solution of technological problems, which constituted, however, the dream of the last century - to fabricate a superconductor which easy in producing (in the sense of a ductility) and has high critical temperature.

To move in this direction, it is important, in the first place, to understand the mechanism limiting the superconducting properties.

Consider a superconductor carrying a current. The value of this current is limited by the critical speed of the carrier $v_c$.

At a comparing the equation Eqs. (70) and (41), we can find that the critical velocity of superconducting carriers

$$v_c \approx \frac{\pi}{2}\alpha v_F$$

is about a hundred times smaller than the Fermi velocity.

Thus the critical velocity has a value which seems to be equal to the speed of sound $v_s$, since according to Bohm-Staver relation [19] the speed of sound

$$v_s \approx \frac{k\Theta_D}{E_F}v_F \approx 10^{-2}v_F.$$
This makes it possible to consider the destruction of superconductivity as a overcome of the sound barrier for superconducting carriers. If they were moving without friction at speeds which less than the speed of sound, after the overcoming of the sound barrier they acquire a friction mechanism. As a result of this assumption we get

\[ T_c \sim v_s^2. \] (51)

If so, then to obtain superconductors with a high critical temperature one must synthesize they that have a high speed of sound. It is in agreement with the fact that ceramics compared to metals and alloys, have higher elastic moduli. However, this same circumstance leads to a contradiction with the processability of the material. In order that a material was malleable it must have a relatively low elastic constants (at room temperature). A solution to this paradoxical problem remains yet elusive.
6 About the London penetration depth

6.1 The traditional approach to calculation of the London penetration depth

It is commonly accepted to consider the theory the London penetration depth (see, for example [11]) in several steps:

Step 1.
At first, the action of an external electric field on free electrons is considered. In accordance with the Newton’s law, free electrons gain acceleration in an electric field \( E \):

\[
a = \frac{eE}{m_e}.
\]  

(52)

The obtained directional movement of the "superconducting" electron gas with the density \( n_s \) creates the current with the density:

\[
j = en_s v.
\]  

(53)

where \( v \) is the carriers velocity.

After differentiating on time and substituting in Eq.(52), one obtains the first London’s equation:

\[
\frac{d}{dt} j = en_s a = \frac{n_s e^2}{m_e} E.
\]  

(54)

Step 2.
After application of operations \( \text{rot} \) to both sides of this equation and with using of the Faraday’s law of electromagnetic induction \( \text{rot} E = -\frac{1}{c} \frac{dB}{dt} \) one gets the relation between current density and magnetic field:

\[
\frac{d}{dt} \left( \text{rot} j + \frac{n_s e^2}{m_e c} B \right) = 0.
\]  

(55)

\[
\text{rot} j + \frac{n_s e^2}{m_e c} B = 0,
\]  

(56)

Step 3. At selecting of the stationary solution of Eq.(55)

\[
\text{rot} j + \frac{n_s e^2}{m_e c} B = 0,
\]  

(57)

and after simple transformations, one concludes that there is a so-called London penetration depth of the magnetic field in a superconductor:

\[
\lambda_L = \sqrt{\frac{m_e c^2}{4 \pi e^2 n_s}}.
\]  

(58)
6.2 The London penetration depth and the density of superconducting carriers

The London penetration depth is one of the measurable characteristics of superconductors, and for many of them it equals to a few hundred Angstroms \[15\]. In the table (1) the measured values of \(\lambda_L\) are given in the second column.

| superconductor | \(\lambda_L, 10^{-6}\text{cm}\) measured \[15\] | \(n_s\) according to Eq.(58) | \(n_e\) | \(2\frac{n_s}{n_e}\) |
|---------------|------------------------|-------------------|--------|-----------------|
| Tl            | 9.2                    | 3.3 \times 10^{21} | 1.05 \times 10^{24} | 0.06          |
| In            | 6.4                    | 6.9 \times 10^{21} | 1.2 \times 10^{23}  | 0.12          |
| Sn            | 5.1                    | 1.1 \times 10^{22} | 1.4 \times 10^{23}  | 0.14          |
| Hg            | 4.2                    | 1.6 \times 10^{22} | 8.5 \times 10^{22}  | 0.36          |
| Pb            | 3.9                    | 1.9 \times 10^{22} | 1.3 \times 10^{23}  | 0.28          |

However, if to use these experimental data and to calculate the density of superconducting carriers \(n_s\) in accordance with the Eq.(58), it will give three orders of magnitude larger (see the middle column of Tab.(1).

Indeed, only a small fraction of the free electrons can combine into the Cooper pairs. It is only those electrons which energies are in the thin strip of the energy spectrum near \(E_F\). We can expect therefore that the concentration of superconducting carriers among all free electrons of the metal should be at the level \(\frac{kT}{E_F} \approx 10^{-4}\). While these concentrations, if to calculate them from Eq.(58), are in \(2 \div 3\) orders of magnitude higher (see last column of the Table (1).

The reason for this discrepancy, apparently, in the using of nonequivalent transformation. At the first stage in Eq.(52), the straight-line acceleration in a static electric field is considered. At this moving, there is no current circulation. Therefore the application of the operation rot in Eq.(55) in this case is not correct. It does not lead to the Eq.(57):

\[
\frac{\text{rot} \ j}{n_e c^2} \neq -1, \quad (59)
\]

but it leads to a pair of equations:

\[
\frac{\text{rot} \ j}{n_e c^2} = 0 \quad (60)
\]

and to the uncertainty:

\[
\frac{\text{rot} \ j}{n_e c^2} \neq 0 \quad (61)
\]
6.3 The adjusted estimation of the London penetration depth

To avoid this incorrectness, let us consider a balance of magnetic energy in a superconductor in magnetic field. This magnetic energy is composed by an energy of penetrating external magnetic field and an magnetic energy of moving electrons.

6.3.1 The magnetic energy of a moving electron

With the using of formulas of [13], let us estimate the ratio of the magnetic and kinetic energy of the electron (the charge of $e$ and the mass $m_e$) at the moving rectilinearly with velocity $v \ll c$.

The density of the electromagnetic field momentum is expressed by the equation:

$$ g = \frac{1}{4\pi c} |EH| $$

(62)

At the moving with velocity $v$, the electric charge carrying the electric field with intensity $E$ creates a magnetic field

$$ H = \frac{1}{c} |Ev| $$

(63)

with the density of the electromagnetic field momentum (at $v \ll c$)

$$ g = \frac{1}{4\pi c^2} |E[vE]| = \frac{1}{4\pi c^2} (vE^2 - E(v \cdot E)) $$

(64)

As a result, the momentum of the electromagnetic field of a moving electron

$$ G = \int_V gdV = \frac{1}{4\pi c^2} \left( v \int_V E^2 dV - \int_V E \cdot E \cos \vartheta dV \right) $$

(65)

Where the integrals are taken over the entire space, which is occupied by particle fields, and $\vartheta$ is the angle between the particle velocity and the radius vector of the observation point. At the calculating the last integral in the condition of the axial symmetry with respect to $v$, the contributions from the components of the vector $E$, which is perpendicular to the velocity, cancel each other for all pairs of elements of the space, if they located diametrically opposite on the magnetic force line. Therefore according to Eq. (65), the component of the field which is collinear to $v$

$$ \frac{E \cos \vartheta \cdot v}{v} $$

(66)

can be taken instead of the vector $E$. Taking this into account, going over to spherical coordinates and integrating over angles, we obtain

$$ G = \frac{v}{4\pi c^2} \int_r^{\infty} E^2 \cdot 4\pi r^2 \, dr $$

(67)
If to limit the integration of the field by the Compton electron radius \( r_C = \frac{\hbar}{m_e c} \), then \( v \ll c \), we obtain:

\[
G = \frac{v}{4\pi c^2} \int_{r_C}^{\infty} E^2 \cdot 4\pi r^2 \, dr = \frac{\nu e^2}{c^2 r_C}.
\]  

(68)

In this case at taking into account Eq. (63), the magnetic energy of a slowly moving electron pair is equal to:

\[
\mathcal{E} = \frac{vG}{2} = \frac{\nu^2 e^2}{c^2 2r_C} = \frac{\alpha m_e v^2}{2}.
\]  

(69)

From this equality we obtain an expression relating the energy with the size of the condensate localization of its particles (at \( T = 0 \)):

\[
\Delta_0 \simeq 2\alpha \frac{\nu^2 e^2}{2m_2} \simeq \frac{\alpha}{m_2} \left( \frac{2\pi \hbar}{\Lambda_0} \right)^2,
\]  

(70)

where \( m_2 = 2m_e \) is mass of electron pair.

It should be noted that this implies the dependence of the gap on the density of particles in the condensate which is coinciding with the previously obtained formula ( ref D-2).

### 6.3.2 The magnetic energy and the London penetration depth

The energy of external magnetic field into volume \( dv \):

\[
\mathcal{E} = \frac{H^2}{8\pi} \, dv.
\]  

(71)

At a density of superconducting carriers \( n_s \), their magnetic energy per unit volume in accordance with (69):

\[
\mathcal{E}_H \simeq \alpha n_s \frac{m_2 \nu^2}{2} = \frac{\alpha m_e j_s^2}{2n_s e},
\]  

(72)

where \( j_s = 2en_s v_s \) is the density of a current of superconducting carriers.

Taking into account the Maxwell equation

\[
\text{rot} \mathbf{H} = \frac{4\pi}{c} j_s,
\]  

(73)

the magnetic energy of moving carriers can be written as

\[
\mathcal{E}_H \simeq \frac{\lambda^2}{8\pi} (\text{rot} H)^2,
\]  

(74)

\[^3\text{Such effects as the pair generation force us to consider the radius of the "quantum electron" as approximately equal to Compton radius [14].}\]
where we introduce the notation

$$\tilde{\Lambda} = \sqrt{\frac{\alpha m_e c^2}{4\pi n_s e^2}}. \quad (75)$$

In this case, part of the free energy of the superconductor connected with an application of a magnetic field is equal

$$F_H = \frac{1}{8\pi} \int_V \left( H^2 + \tilde{\Lambda}^2 (\text{rot}H)^2 \right) dv. \quad (76)$$

At minimization of the free energy, after simple transformations we obtain

$$H + \tilde{\Lambda}^2 \text{rot} \text{rot} H = 0, \quad (77)$$

and thus $\tilde{\Lambda}$ is the magnetic field penetration depth into the superconductor.

In view of Eq. (16) from Eq. (75) we can estimate the values of London penetration depth (see table (6.3.2)). The consent of the obtained values with the measurement data can be considered quite satisfactory.

| superconductors | $\lambda_L, 10^{-6}$ cm measured | $\tilde{\Lambda}, 10^{-6}$ cm calculated | $\tilde{\Lambda}/\lambda_L$ |
|-----------------|----------------------------------|-----------------------------------------|-------------------------|
| Tl              | 9.2                              | 11.0                                    | 1.2                     |
| In              | 6.4                              | 8.4                                     | 1.3                     |
| Sn              | 5.1                              | 7.9                                     | 1.5                     |
| Hg              | 4.2                              | 7.2                                     | 1.7                     |
| Pb              | 3.9                              | 4.8                                     | 1.2                     |

The resulting refinement may be important for estimates in frame of Ginzburg-Landau theory, where the London penetration depth is used at comparison of calculations and specific parameters of superconductors.


7 About superfluidity of liquid helium

The main features of superfluidity of liquid helium became clear few decades ago [16], [17]. L.D.Landau explains this phenomenon as the manifestation of a quantum behavior of the macroscopic object.

However, the causes and mechanism of the formation of superfluidity are not clear still. There is not an explanation why the λ-transition in helium-4 occurs at about 2 K.

The related phenomenon - superconductivity, which can be regarded as superfluidity of the conduction electrons - can be quantitatively described if to consider it as the consequence of ordering of the electron gas zero-point oscillations.

Therefore it seems as appropriate to consider superfluidity from the same point of view.

The atoms in liquid helium-4 are electrically neutral, have no dipole moments and do not form molecules. Yet some electromagnetic mechanism should be responsible for phase transformations of liquid helium (as well as in other condensed substance where phase transformations are related with changes of energy of the same scale).

In liquid helium, the atom density is \( n_4 \approx 2 \cdot 10^{22} \text{ cm}^{-3} \), so that a single atom trapped adjacent atoms in a volume with linear dimensions roughly equal to \( \lambda \approx 2 \cdot 10^{-8} \text{ cm} \). In this volume, the atom makes a zero-point oscillations with the amplitude of \( a_0 \approx \lambda \). It permits for helium to remain the liquid state even at \( T = 0 \).

The radius of the atom \( r_a \), defined by the first Bohr orbit, is about \( \frac{\hbar^2}{2n e^2} \), which is approximately equal to \( 3 \cdot 10^{-9} \text{ cm} \), ie almost in an order of magnitude smaller.

The frequency of zero-point oscillations \( \omega_0 \) of atom in liquid helium can be determined from the condition of quantization:

\[
m_4a_0^2\omega_0 \approx \hbar, \tag{78}
\]

where \( m_4 = 6.7 \cdot 10^{-24} \text{ g} \) is the mass of the He-4 atom.

At zero-point oscillations, the atoms collide through their electron shells, and at a short time of collision they reverse the direction of their motion. At same time the nuclei of atoms are affected by inertia. Let us assume that the collision time is

\[
\tau \approx \frac{r_a}{\omega_0a_0}. \tag{79}
\]

In this case, the inertial force \( F \approx m_4\left(\frac{n_4e\omega_0}{2}\right)^2 \) will act on the nucleus periodically, and it will move the nucleus relative to the center of the negative charged shell on the distance \( \delta_\omega \), ie it leads to an existence of oscillating dipole electric moment of atom \( d_\omega = e\delta_\omega \).

To determine the polarizability of helium under action of the inertia force, it is possible to use the Clausius-Mossotti equation [18], describing the phenomenon of the polarizability under action of an external electric field.

\[
\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{n_4\alpha}{3} \tag{80}
\]
Where $\alpha$ is the polarizability of He-atom, $\varepsilon \approx 1.055$ is the dielectric permeability of liquid helium at $T \to 0$.

In accordance with this equation under the action of applied to a nucleus the oscillating force $F$, the atom obtains the oscillating dipole electric moment with the amplitude depending on the polarizability [18]:

$$d_\omega = e\delta_\omega \approx 18\pi \alpha \frac{F}{\varepsilon} \quad (81)$$

After numeral calculation, we obtain

- $\alpha \approx 2.5 \cdot 10^{-24} cm^3$,
- $\delta_\omega \approx 7 \cdot 10^{-11} cm$,
- and $d_\omega \approx 4 \cdot 10^{-20} cm^5/s$.

At relatively high temperature, the zero-point oscillations are independent. The interaction energy of oscillating dipole moment of neighboring atoms

$$E \approx \frac{2d_\omega^2}{\lambda^3} \quad (82)$$

leads at the sufficient low temperature to an ordering in the system, at which the coherence in zero-point oscillations is established throughout the whole ensemble of particles.

The substitution of numerical values gives the value of the critical temperature of this ordering

$$T_c = \frac{\varepsilon}{k} \approx 1 K \quad (83)$$

Thus, it can be seen that the ordering energy of oscillating dipoles is consistent with the energy $\lambda$-transition in helium-4 in order of value. It is difficult to make successfully more accurate calculations in this way because in the first the collision time of two atoms in the liquid can be estimated very roughly only.

A simile explanation can be given to the transition to a superfluid state of helium-3. The difference is that the electromagnetic interaction should order the magnetic moments of the nuclei He-3 in this case. We can estimate the temperature at which this ordering is happens. Due to the zero-point oscillation, the electron shell creates on the nucleus an oscillating magnetic field:

$$H_\omega \approx \frac{d_\omega}{\delta_\omega} \cdot \frac{\omega_a a}{c} \quad (84)$$

Because the value magnetic moments of the nuclei He-3 is approximately equal to the nuclear Bohr magneton $\mu_{nB}$, the ordering in their system must occur below the critical temperature

$$T_c = \frac{\mu_{nB} H_\omega}{k} \approx 10^{-3} K. \quad (85)$$

It is in a good agreement with the data of measuring.

32
8 Conclusion

It is generally accepted to thought that the existence of the isotope effect in superconductors leaves only one way for the phenomenon of superconductivity explanation - based on the phonon mechanism. However, there are experimental data for believing that the isotope effect in superconductors may be a consequence of another effect, and therefore a nonphonon mechanism may be in the basis of the superconductivity.

A satisfactory agreement with the measured data can be obtained if we consider the superconductivity of both - type-I and type-II - as a result of a condensation of ordered zero-point oscillations of the electrons.

The density of superconducting carriers and the critical temperature of the superconductor are determined by the peculiarities of the interaction of the zero-point oscillations, and the critical magnetic field of a superconductor is defined mechanism for the destruction of the coherence of zero-point oscillations of the electrons.

The evaluations show that the critical parameters of superconductors are depending on the Sommerfeld constant (or the Fermi-energy) and does not depend on the electron-phonon interaction.

The temperature dependence of the energy gap determined by the mechanism, which is standard for the order-disorder transitions.

There are the both - I and II - type superconductors among the high-temperature superconducting ceramics.

The general conclusion which is obtained from agreement of calculation results and measuring data in that, the superconductivity of elementary metals is result of ordering of their electron systems, ie it is based on the nonphonon mechanism.
Список литературы

[1] Vasiliev B.V.: Physica C, 471, 277-284 (2011)

[2] Kogan V.S.: Physics-Uspekhi, 78, 579 (1962) (in Russian)

[3] Inyushkin A.V.: Chapter 12 in "Isotops" (reductor Baranov V.Yu), PhysMathLit, 2005 (In Russian).

[4] Wang D.T. et al: Phys.Rev.B, 56, N 20, p 13167 (1997).

[5] Bardeen J.: Phys.Rev., 79, p. 167-168 (1950).

[6] Shablo A.A. et al: Letters JETPh, v.19, 7, p.457-461 (1974)

[7] Sharvin D.Iu. and Sharvin Iu.V.: Letters JETPh, v.34, 5, p.285-288 (1981)

[8] Landau L.D. and Lifshits E.M.: Statistical Physics, 1, 3rd edition, Oxford: Pergamon, (1980)

[9] Kittel Ch.: Introduction to Solid State Physics, Wiley (2005)

[10] Pool Ch.P.Jr: Handbook of Superconductivity, Academic Press, (2000)

[11] Ketterson J.B. and Song S.N.: Superconductivity, CUP, (1999)

[12] Vasiliev B.V. and Luboshits V.L.: Physics-Uspekhi, 37, 345, (1994)

[13] Abragam-Becker: Teorie der Elektizität, Band 1, Leupzig-Berlin, (1932)

[14] Albert Messiah: Quantum Mechanics (Vol. II), North Holland, John Wiley and Sons. (1966)

[15] Linton E.A.: Superconductivity, London: Mathuen and Co.LTDA, NY: John Wiley and Sons Inc., (1964)

[16] Landau L.D.: JETP, 11, 592 (1941)

[17] Khalatnikov I.M.: Introduction into theory of superfluidity, Moscow, Nauka, (1965)

[18] Fröhlich H.: Theory of dielectrics, Oxford, 1957.

[19] Ashcroft N.W., Mermin N.D.: Solid state physics, v 2., Holt, Rinehart and Winston, (1976)