Consistency Management of Normal Logic Program by Top-down Abductive Proof Procedure

Ken Satoh
Hokkaido University, N13W8, Sapporo, Japan
ksatoh@db-ei.eng.hokudai.ac.jp

Abstract

This paper presents a method of computing a revision of a function-free normal logic program. If an added rule is inconsistent with a program, that is, if it leads to a situation such that no stable model exists for a new program, then deletion and addition of rules are performed to avoid inconsistency. We specify a revision by translating a normal logic program into an abductive logic program with abducibles to represent deletion and addition of rules. To compute such deletion and addition, we propose an adaptation of our top-down abductive proof procedure to compute a relevant abducibles to an added rule. We compute a minimally revised program, by choosing a minimal set of abducibles among all the sets of abducibles computed by a top-down proof procedure.

Introduction

Knowledge base is always subject to change since an environment around the knowledge base is not guaranteed to be stable forever and even some error might be included at the initial stage. Therefore, study of revision of knowledge base is very important (Fagin et al., 1983; Katsuno and Mendelzon, 1991). Gärdenfors and Rott, 1993; Kakas and Mancarella, 1990; Alferes et al., 1990; Witteveen and van der Hoek, 1997; Inoue and Sakama, 1993). (Fagin et al., 1983) and (Katsuno and Mendelzon, 1991) consider a revision of monotonic theories and there are a lot of researches in this direction (see (Gärdenfors and Rott, 1993) for a survey). (Kakas and Mancarella, 1990) and (Inoue and Sakama, 1993) consider an update of nonmonotonic theories to derive a given goal or a given observation. (Alferes et al., 1990) and (Witteveen and van der Hoek, 1997) consider a revision of nonmonotonic theories which is more related to a revision of monotonic theories studies (Fagin et al., 1983; Katsuno and Mendelzon, 1991); they consider a revision when inconsistency arises at addition of rules.

In this paper, we follow the latter approach. Revision of nonmonotonic theories is especially important for AI, since it is very rare that commonsense reasoning can be represented as a monotonic theory. However, revision of nonmonotonic theories is more complicated than revision of monotonic theory. In monotonic theory, if some addition of knowledge or observation leads to inconsistency, then we can avoid inconsistency by deleting a part of knowledge base. On the other hand, we might add a piece of assumptions since deletion leads to inconsistency.

Consider the following program.

\[
\begin{align*}
\text{runs}(X) & \leftarrow \text{car}(X), \sim \text{broken}(X). \\
\text{car}(c_1) & \leftarrow. \\
\text{car}(c_2) & \leftarrow.
\end{align*}
\]

\sim means “negation as failure”. The first rule says that if X is a car and X is not known to be broken, X should run. Since there is no information about \text{broken}(c_1) and \text{broken}(c_2) in the current program, \text{runs}(c_1) and \text{runs}(c_2) are derived. Suppose, however, that we add a rule \( \bot \leftarrow \text{runs}(c_1) \) meaning that car a does not run. Then, we have no stable model, that is, we are in an inconsistent situation. To fix this inconsistency, we have (at least) two possible ways.

1. We simply discard the default rule of car a:

\[
\begin{align*}
\text{runs}(c_1) & \leftarrow \text{car}(c_1), \sim \text{broken}(c_1).
\end{align*}
\]

2. We derive \text{broken}(c_1) since we assumed \sim \text{broken}(c_1), then contradiction would occur and thus, we have a reason to assume \text{broken}(c_1).

The first revision is contraction widely used in belief revision of monotonic theories (Fagin et al., 1983) but the second is special for nonmonotonic theories such as normal logic programs. In monotonic theories, addition of formula can not help to restore consistency, but in nonmonotonic theories addition can help. This phenomena were firstly observed in Doyle’s justification TMS and he introduced dependency-directed backtracking. Moreover, in monotonic theories, contraction or deletion of formula can not produce any inconsistency, whereas in nonmonotonic theories, deletion can cause inconsistency.

\[\bot\] means contradiction.

\[\bot\]
Therefore, we need more functions for revision in non-monotonic theories than monotonic ones. In this paper, we propose a top-down procedure using abduction to compute revision for a normal logic program when there exists no stable model.

Our idea of using abduction for revision is as follows. We introduce two kinds of abducibles one of which represents a deletion of each retractable rule and the other of which represents an addition of each addable rule. For a retractable rule, we add negation of a corresponding abducible in the body of the rule so that if an instance of abducible is assumed then an instance of rule corresponding with the instance of abducible is no longer applicable. For an addable rule, we add a corresponding abducible in the body of the rule so that if an instance of abducible is assumed then an instance of rule corresponding with the instance of abducible becomes applicable.

Then, in order to compute such abducibles to specify revision, we show that we can use a modification of Satoh and Iwayama’s query evaluation procedure on stable models (Satoh and Iwayama, 1992) which is a combination of integrity constraint checking (Sadri and Kowalski, 1988) and abductive procedure (Kakas and Marecalle, 1990). Instead of starting with a subprocedure which show a derivation of positive literals, we start with a subprocedure for rule consistency checking to derive abducibles to specify revision. This procedure traverses rules of a program which is related with addition or deletion of a rule and we guarantee that a minimal revision can be found by selecting a minimal set of abducibles among all the sets of abducibles computed by the rule consistency checking procedure.

Revision of Normal Logic Program

Firstly, we define a revision framework as follows. In this paper, we consider a function-free normal logic program. We use domain closure axiom and unique name axiom so that constants in the language for a program are finite and denote distinct objects. We can easily extend our results to function-free extended logic programs by translating an extended logic program into a normal logic program proposed by (Gelfond and Lifschitz, 1991).

Definition 1 A rule $R$ is of the form:

$$H \leftarrow P_1, ..., P_j, \sim N_1, ..., \sim N_h$$

where $H, P_1, ..., P_j, N_1, ..., N_h$ are atoms.

We call $H$ the head of the rule $R$ denoted as head($R$) and $P_1, ..., P_j, \sim N_1, ..., \sim N_h$ the body of the rule denoted as body($R$). If $H = \bot$, we sometimes call the rule an integrity constraint.

Let $T$ and $T_{bck}$ be sets of rules. A revision framework $R$ is a pair, $(T, T_{bck})$ where $T$ can be divided into two sets of rules $T_{pst}$ and $T_{imp}$. We call $T_{pst}$ a persistent part of $R$ and $T_{imp}$ a temporal part of $R$ and $T_{bck}$ a backup part of $R$.

$T$ expresses the current logic program which consists of $T_{pst}$ and $T_{imp}$. $T_{pst}$ is an unchanged part which should always be satisfied such as integrity constraints whereas any part of $T_{imp}$ can be retracted and any part of $T_{bck}$ can be added to restore consistency. Usage of $T_{bck}$ is inspired by back-up semantics proposed by (Witteveen and van der Hoek, 1997).

We use stable model semantics for the above program.

Definition 2 Let $P$ be sets of rules. We denote a set of ground rules obtained by replacing all the variables in every rule of $P$ by every element in the language as $\Pi_P$.

Definition 3 Let $M$ be a set of ground atoms and $\Pi_P^M$ be the following program.

$$\Pi_P^M = \{H \leftarrow B_1, ..., B_i\}$$

where $H = B_1, ..., B_i, \sim A_1, ..., \sim A_h$, $\in \Pi_P$ and $A_i \notin M$ for each $i = 1, ..., h$.

Let $\min(\Pi_P^M)$ be the least model of $\Pi_P^M$. A stable model for a logic program $P$ is $M$ iff $M = \min(\Pi_P^M)$ and $\bot \notin M$.

We say that $P$ is consistent if $P$ has a stable model.

Now, we define a revised program in a revision framework.

Definition 4 Let $R$ be a revision framework, $(\langle T_{pst} \cup T_{imp} \rangle, T_{bck})$. Let $R_{new}$ be a rule. Then, a revised program w.r.t. $R$ and $R_{new}$ is $(\langle T_{pst} \cup \{R_{new}\} \rangle \cup (\Pi_{T_{imp}} - O) \cup I)$ such that

- $O \subseteq \Pi_{T_{imp}}$
- $I \subseteq \Pi_{T_{bck}}$
- $(\langle T_{pst} \cup \{R_{new}\} \rangle \cup (\Pi_{T_{imp}} - O) \cup I)$ is consistent.

We say for such $O$ and $I$ that a pair $(O, I)$ accomplishes revision of $R_{new}$ to $R$.

A minimally revised program w.r.t. $R$ and $R_{new}$ is $(\langle T_{pst} \cup \{R_{new}\} \rangle \cup (\Pi_{T_{imp}} - O) \cup I)$ such that there is no revised program $(\langle T_{pst} \cup \{R_{new}\} \rangle \cup (\Pi_{T_{imp}} - O') \cup I')$ such that $I' \subseteq O$ and $O' \subseteq O$ where $\subseteq$ is a strict inclusion.

Example 1 Let

- $T_{pst}$ be $\{c(c_1) \leftarrow ., c(c_2) \leftarrow .\}$ and $T_{imp}$ be $\{r(X) \leftarrow c(X), \sim b(X)\}$ and $T_{bck}$ be $\{b(X) \leftarrow c(X), \sim r(X)\}$ and $R_{new} = \{\bot \leftarrow r(c_1)\}$.

Then, $\Pi_{T_{imp}} \cup \{\{R_{new}\}\} \cup T_{imp} \cup T_{bck} = \{r(c_1) \leftarrow c(c_1), \sim b(c_1), r(c_2) \leftarrow c(c_2), \sim b(c_2)\}$ and $\Pi_{bck} \cup \{\{R_{new}\}\} \cup T_{imp} \cup T_{bck} = \{b(c_1) \leftarrow c(c_1), \sim r(c_1), b(c_2) \leftarrow c(c_2), \sim r(c_2)\}$.

For the above revision, we have the following two minimally revised programs:

1. a program accomplished by $(O_1, I_1)$ where $O_1 = \{r(c_1) \leftarrow c(c_1), \sim b(c_1)\}$ and $I_1 = \{\}$; $(\langle T_{pst} \cup \{R_{new}\} \rangle \cup r(c_2) \leftarrow c(c_2), \sim b(c_2)\}$.
2. a program accomplished by $(O_2, I_2)$ where $O_2 = \{\}$ and $I_2 = \{b(c_1) \leftarrow c(c_1), \sim r(c_1)\}$; $(\langle T_{pst} \cup \{R_{new}\} \rangle \cup T_{imp} \cup b(c_1) \leftarrow c(c_1), \sim r(c_1)\}$. 


There are other non-minimally revised programs, for example, one accomplished by \(O_1, I_2\) or by \(\{ O_1 \cup \{ r(c_2) \leftarrow c(c_2), \sim b(c_2) \}, I_2 \cup \{ b(c_2) \leftarrow c(c_2), \sim r(c_2) \} \}\).

In the above example, we follow Giordano's approach (Giordano and Martelli, 1990), where contrapositives of default rules are in the back-up part, but we can actually assume any rules which we think are appropriate for back-up rules when inconsistency occurs.

To compute a revised program, we use a translation from a specification to an abductive logic program and compute a consistent generalized stable model for the translated program which denotes deletion and addition of rules.

**Definition 6** (Kakas and Mancarella, 1994). An abductive framework is a pair \((P, A)\) where \(A\) is a set of predicate symbols, called abducible predicates and \(P\) is a set of rules each of whose head is not in \(A\). We call a ground atom for a predicate in \(A\) an abducible.

The semantics of abductive framework is based on a generalized stable model (Kakas and Mancarella, 1990).

The following is a definition of a generalized stable model which can manipulate abducibles in abductive logic programming.

**Definition 6** Let \((P, A)\) be an abductive framework and \(\Theta\) be a set of abducibles. A generalized stable model \(M(\Theta)\) is a stable model of \(P \cup \{ H \leftarrow \} \Theta\\) where \(\Theta\) is a name of a rule.

We say that a model \(M(\Theta)\) is a generalized stable model with a minimal set of abducibles \(\Theta\) if there is no generalized stable model \(M(\Theta')\) such that \(\Theta'\) is a proper subset of \(\Theta\).

Now, we define a translation of a revision framework into an abductive framework as follows.

**Definition 7** Let \(R\) be a revision framework \((\{T_{pst} \cup T_{imp}, T_{bck}\}, X_{bck})\). We firstly give a name to every rule in \(T_{imp}\) and \(T_{bck}\) such as

\[
\phi : H \leftarrow P_1, \ldots, P_j, \sim N_1, \ldots, \sim N_h,
\]

where \(\phi\) is a name for the rule.

A translation for a consistency management of \(R\) (denoted as \(\tau(R)\)) is a set of the following translation from \(R\) to an abductive framework \((P, A)\) where

- \(A = \{ \phi^- | \phi\) is a name of a rule in \(T_{imp}\} \cup \{ \phi^+ | \phi\) is a name of a rule in \(T_{bck}\} \)
- We add every rule in \(T_{pst}\) into \(P\).
- We translate every rule in \(T_{imp}\) with a name \(\phi\)

\[
\phi : H \leftarrow P_1, \ldots, P_j, \sim N_1, \ldots, \sim N_h
\]

into the following rule in \(P\):

\[
H \leftarrow P_1, \ldots, P_j, \sim N_1, \ldots, \sim N_h, \sim \phi^-(x)
\]

where \(x\) is a tuple of variables in the clause.

- We translate every rule in \(T_{bck}\) with a name \(\phi\)

\[
\phi : H \leftarrow P_1, \ldots, P_j, \sim N_1, \ldots, \sim N_h
\]

into the following rule in \(P\):

\[
H \leftarrow P_1, \ldots, P_j, \phi^+(x), \sim N_1, \ldots, \sim N_h
\]

The following shows that an revised program corresponds with a generalized stable model.

**Theorem 1** Let \(R\) be a revision framework \((\{T_{pst} \cup T_{imp}, T_{bck}\}, X_{bck})\) and \(R_{new}\) be an added clause. \((\{T_{pst} \cup \{R_{new}\}\} \cup \{T_{imp} - T_{del}\} \cup T_{new}\) is a (minimally, resp.) revised program if and only if there is a generalized stable model of \(\tau((\{T_{pst} \cup \{R_{new}\}\} \cup T_{imp}, T_{bck}))\) with a (minimal, resp.) set of abducibles \(\Theta\) s.t.

- \(T_{del} = \{ R(\phi^-((x)\theta)) \in \Theta \)

where \(\phi\) is a name of \(R \in T_{imp}\).

- \(T_{new} = \{ R\theta((\phi^+(x)\theta)) \in \Theta \)

where \(\phi\) is a name of \(R \in T_{bck}\)

- \{head(R) \leftarrow body(R), \sim (EQ(\theta_1)), \ldots, \sim (EQ(\theta_k)) \}

\(\phi^-((x)\theta_i) \in \Theta\) where \(\phi\) is a name of \(R \in T_{imp}\) and \(EQ(\theta_i) = (x_1 = (x_1\theta_i)) \wedge \ldots \wedge (x_k = (x_k\theta_i))\)

We say that \(\Theta\) (minimally, resp.) realizes revision of \(R_{new}\) to \(R\).

Note that in the above theorem, we delete whole rules related to inconsistency and then add modified rules with negation of conjunctions of disequity in the body of the deleted rules. The modified rules are logically equivalent to rules in Definition 6 since we assume domain closure axiom and unique name axiom.

**Example 2** Consider the revision framework in Example 1. Let us give names to the rules in \(T_{imp}\) and \(T_{bck}\) as follows:

- \(\phi_1 : r(X) \leftarrow c(X), \sim b(X)\).
- \(\phi_2 : b(X) \leftarrow c(X), \sim r(X)\).

Then \(\tau((\{T_{pst} \cup \{R_{new}\}\} \cup T_{imp}, T_{bck}))\) is:

- \(A = \{ \phi_{1^-}, \phi_{2^+} \}\).
- \(P\) becomes as follows:

\[
c(c_1) \leftarrow \sim.
\[
c(c_2) \leftarrow \sim.
\[
r(X) \leftarrow c(X), \sim b(X) \sim \phi_{1^-}(X).
\[
b(X) \leftarrow c(X), \phi_2^+(X), \sim r(X)\).

Then, we have two generalized models with minimal abducibles:

1. \(\Theta = \{ \phi_{1^-}(c_1) \}\). Then, a minimally revised program is:

\(\{T_{pst} \cup \{R_{new}\}\} \cup X \neq a, c(X), \sim b(X)\).

2. \(\Theta = \{ \phi_{2^+}(c_1) \}\). Then, a minimally revised program is:

\(\{T_{pst} \cup \{R_{new}\}\} \cup T_{imp}\) and \(b(c_1) \leftarrow c(c_1), \sim r(c_1)\).

**Computing Revision by Abduction**

To compute a revision, it is sufficient to compute a generalized stable model of \(\tau((\{T_{pst} \cup \{R_{new}\}\} \cup T_{imp}, T_{bck}))\). But, if we concern a minimal revision,
we need to compute all the generalized stable models and then compare sets of abducibles in these models to choose minimal sets of abducibles. For this purpose, it is desirable to restrict sets of abducibles to be compared as small as possible. This can be done if we compute only revision related to $R_{new}$. For example, suppose that some of temporary rules are not relevant to inconsistency of addition of $R_{new}$. If we naively compute all the generalized stable models, then we have to compare all the combination of in/out of abducibles for these irrelevant rules.

In order to avoid this kind of redundancy, we modify Satoh and Iwayama’s query evaluation procedure on stable models (Satoh and Iwayama, 1992). Basically, we change the order of application of subprocedures so that we can use the procedure for consistency checking.

We impose rules in a revision framework must be range-restricted, that is, any variable in a rule $R$ must occur in $pos(R)$. However, any rule can be translated into range-restricted form by inserting a new predicate “dom” describing Herbrand universe for every non-range-restricted variable in the rule.

Before showing our procedure to compute revision, we need the following definitions. Let $l$ be a literal. Then, $\bar{l}$ denotes the complement of $l$.

**Definition 8** Let $P$ be a logic program. A set of resolvents w.r.t. a ground literal $l$ and $T$, resolve$(l, P)$ is the following set of rules:

$\text{resolve}(l, P) =
\{(A ← L_1, ..., L_k)\} \mid l \text{ is negative and } \not\exists H \leftarrow L_1, ..., L_k \in P \text{ and } \bar{T} = H \theta \text{ by a ground substitution } \theta\}
\{(H ← L_1, ..., L_i, L_{i+1}, ..., L_k)\}
\{(H ← L_1, ..., L_i, L_{i-1}, ..., L_k)\}
\not\exists H \leftarrow L_1, ..., L_i \in P \text{ and } l = L_i \theta \text{ by a ground substitution } \theta\}$

**Definition 9** Let $P$ be a logic program. A set of deleted rules w.r.t. a ground literal $l$ and $P$, del$(l, P)$, is the following set of rules:

$\text{del}(l, P) = \{(A ← L_1, ..., L_k)\} \mid H ← L_1, ..., L_k \in P \text{ and } \bar{T} = H \theta \text{ by a ground substitution } \theta\}$

**Definition 10** Let $P$ be a logic program and $P^-$ be an abducible-and-negation-removed program obtained by removing all integrity constraints in $P$ and all the negative literals and integrity in the body of remaining rule and $\text{min}(P^-)$ be the least minimal model of $P^-$. We define a relevant ground program $\Omega_P$ for $P$ as follows:

$\Omega_P = \{H ← B_1, ..., B_k, \sim A_1, ..., \sim A_m \mid P \in \Pi_P \text{ and } B_i \in \text{min}(P^-) \}$

We briefly explain our procedure. Our procedure consists of 4 subprocedures, rule$_{\text{con}}(R, \Delta)$, derive$(p, \Delta)$, literal$_{\text{con}}(l, \Delta)$, and deleted$_{\text{con}}(R, \Delta)$ where $p$ is a non-abducible atom and $\Delta$ is a set of ground literals already assumed and $l$ is a ground literal and $R$ is a rule. rule$_{\text{con}}(R, \Delta)$, literal$_{\text{con}}(l, \Delta)$, and deleted$_{\text{con}}(R, \Delta)$ return union of $\Delta$ and a set of ground literals which are assumed during the execution of the subprocedures. derive$(p, \Delta)$ return the above union and a substitution for $p$ which are made during the execution of derive$(p, \Delta)$.

In the procedure, we have a select operation and a fail operation. The select operation expresses a non-deterministic choice among alternatives. The fail operation expresses immediate termination of an execution with failure. Therefore, a procedure succeeds when its inner calls of subprocedures do not encounter fail. We say a subprocedure succeeds with (a substitution $\theta$ and) a set of assumptions $\Delta$ when the subprocedure successfully returns ($\theta$ and $\Delta$).

Our procedure firstly starts from rule$_{\text{con}}(R_{new}, \{\})$. rule$_{\text{con}}(R_{new}, \{\})$ checks the consistency of a rule $R_{new}$ with a program $P \cup \{R_{new}\}$. We can show the consistency of addition of $R_{new}$ by showing one of the following.

1. A literal $l$ in body$(R_{new})$ can be falsified. To do so, we invoke subprocedure literal$_{\text{con}}$ for $l$.

2. Every positive literal $p$ in body$(R_{new})$ can be made true and every negative and every abducible literal $l$ can be consistently assumed and head$(R_{new})$ consistent. To do so, we invoke subprocedure derive for $p$ and literal$_{\text{con}}$ for $l$ and head$(R_{new})$.

The informal specification of the other 3 subprocedures is as follows.

1. literal$_{\text{con}}(l, \Delta)$ checks the consistency of a ground literal $l$ with $P \cup \{R_{new}\}$ and $\Delta$. To show the consistency for assuming $l$, we add $l$ to $\Delta$; then, we check the consistency of resolvents and deleted rules w.r.t. $l$ and $P \cup \{R_{new}\}$.

2. derive$(p, \Delta)$ searches a rule $R$ of $p$ in a program $P \cup \{R_{new}\}$ whose body can be made true with a ground substitution $\theta$ under a set of assumptions $\Delta$. To show that every literal in the body can be made true, we call derive for non-abducible positive literals in the body. Then, we check the consistency of other literals in the body with $P \cup \{R_{new}\}$ and $\Delta$. Note that because of the range-restrictedness, other literals in $R$ become ground after all the calls of derive for non-abducible positive literals.

3. deleted$_{\text{con}}(R, \Delta)$ checks if a deletion of $R$ does not cause any contradictions with $P \cup \{R_{new}\}$ and $\Delta$. To show the consistency of the implicit deletion of $R$, it is sufficient to prove that the head of every ground instance $R \theta$ in $\Omega_{P \cup \{R_{new}\}}$ can be made either true or false.

Now, we describe a complete specification of the subprocedures in Figure 1 and Figure 2. In Figures, we denote a set of non-abducible positive literals, non-abducible negative literals, and abducibles (either negative or positive) in a rule $R$ as pos$(R)$, neg$(R)$ and abd$(R)$ respectively, and we denotes empty substitution as $\varepsilon$ and $\theta, \sigma$ expresses a composition of two substitutions $\theta$ and $\sigma$.
**Rule con** \((R, \Delta)\): A rule; \(\Delta\): A set of literals

\[\begin{align*}
\Delta_0 &= \Delta, \ i := 0 \\
\text{for every ground rule } R\theta &\in \Omega_{P \cup \{R_{\text{new}}\}} \text{ do} \\
\text{begin} \\
\quad \text{select case (a) or case (b)} \\
\quad (a) \text{ select } l \in \text{body}(R\theta) \\
\quad \quad \text{if } l \in \text{pos}(R\theta) \cup \text{abd}(R\theta) \text{ and} \\
\quad \quad \quad \text{literal con}(l, \Delta_i) \text{ succeeds with } \Delta_{i+1} \\
\quad \quad \quad \text{then } i := i + 1 \text{ and continue} \\
\quad \quad \text{elseif } l \in \text{neg}(R\theta) \text{ and} \\
\quad \quad \quad \text{derive}(l, \Delta_i) \text{ succeeds with } (\varepsilon, \Delta_{i+1}) \\
\quad \quad \quad \text{then } i := i + 1 \text{ and continue} \\
\quad (b) \Delta_0^l := \Delta_i, \ j := 0 \\
\quad \text{for every } l \in \text{body}(R\theta) \text{ do} \\
\quad \quad \text{if } l \in \text{pos}(R\theta) \\
\quad \quad \quad \text{and derive}(l, \Delta_i) \text{ succeeds with } (\varepsilon, \Delta_{i+1}) \\
\quad \quad \quad \text{then } j := j + 1 \text{ and continue} \\
\quad \quad \text{elseif } l \in \text{neg}(R\theta) \text{ and} \\
\quad \quad \quad \text{literal con}(l, \Delta_i^j) \text{ succeeds with } \Delta_{i+1} \\
\quad \quad \quad \text{then } j := j + 1 \text{ and continue} \\
\quad \text{end} \\
\quad \text{if literal con}(\text{head}(R\theta), \Delta_i^j) \text{ succeeds with } \Delta_{i+1} \\
\quad \quad \text{then } i := i + 1 \text{ and continue} \\
\quad \text{return } \Delta_i \\
\text{end (rule con)}
\end{align*}\]

**Literal con** \((l, \Delta)\): A ground literal; \(\Delta\): A set of literals

\[\begin{align*}
\text{if } l \in \Delta \text{ then return } \Delta \\
\text{elseif } l = \bot \text{ or } \overline{l} \in \Delta \text{ then fail} \\
\text{else} \\
\text{begin} \\
\Delta_0 := \{l\} \cup \Delta, \ i := 0 \\
\text{for every } R \in \text{resolve}(l, P \cup \{R_{\text{new}}\}) \text{ do} \\
\quad \text{if rule con}(R, \Delta_i) \text{ succeeds with } \Delta_{i+1} \\
\quad \quad \text{then } i := i + 1 \text{ and continue} \\
\text{end} \\
\text{for every } R \in \text{del}(l, P \cup \{R_{\text{new}}\}) \text{ do} \\
\quad \text{if deleted con}(R, \Delta_i) \text{ succeeds with } \Delta_{i+1} \\
\quad \quad \text{then } i := i + 1 \text{ and continue} \\
\text{end} \\
\text{return } \Delta_i \\
\text{end (literal con)}
\end{align*}\]

**Derive** \((p, \Delta)\): A non-abducible atom; \(\Delta\): A set of literals

\[\begin{align*}
\text{begin} \\
\quad \text{if } p \text{ is ground and } p \in \Delta \text{ then return } (\varepsilon, \Delta) \\
\quad \text{elseif } p \text{ is ground and } \sim p \in \Delta \text{ then fail} \\
\quad \text{else} \\
\quad \quad \text{begin} \\
\quad \quad \quad \text{select } R \in P \cup \{R_{\text{new}}\} \\
\quad \quad \quad \text{s.t. } \text{head}(R) \text{ and } p \text{ are unifiable with an mgu } \theta \\
\quad \quad \quad \text{if such a rule is not found then fail} \\
\quad \quad \quad \text{if } \Delta_o := \Delta, \ \theta_0 := \theta, \ B_0 := \text{pos}(R\theta), \ i := 0 \\
\quad \quad \quad \text{while } B_i \neq \{\} \text{ do} \\
\quad \quad \quad \quad \text{begin} \\
\quad \quad \quad \quad \quad \text{take a literal } l \text{ in } B_i \\
\quad \quad \quad \quad \quad \text{if derive}(l, \Delta_i) \text{ succeeds with } (\sigma_i, \Delta_{i+1}) \\
\quad \quad \quad \quad \quad \quad \text{then } \theta_{i+1} := \theta_i \sigma_i, \ B_{i+1} := (B_i - \{l\}) \sigma_i, \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad i := i + 1 \text{ and continue} \\
\quad \quad \quad \quad \quad \text{end} \\
\quad \quad \quad \text{end} \\
\quad \quad \text{if literal con}(p, \Delta_i) \text{ succeeds with } \Delta' \\
\quad \quad \text{then return } (\delta, \Delta') \\
\quad \text{end} \\
\text{end (derive)}
\end{align*}\]

**Delete con** \((R, \Delta)\): A rule; \(\Delta\): A set of literals

\[\begin{align*}
\text{begin} \\
\quad \text{if } l \in \Delta \text{ then return } \Delta \\
\Delta_o := \Delta, \ i := 0 \\
\text{for every ground rule } R\theta \in \Omega_{P \cup \{R_{\text{new}}\}} \text{ do} \\
\quad \text{begin} \\
\quad \quad \text{select case (a) or case (b)} \\
\quad \quad (a) \text{ if derive}(\text{head}(R\theta), \Delta_i) \text{ succeeds with } (\varepsilon, \Delta_{i+1}) \\
\quad \quad \quad \text{then } i := i + 1 \text{ and continue} \\
\quad \quad (b) \text{ if literal con}(\sim \text{head}(R\theta), \Delta_i) \text{ succeeds with } \Delta_{i+1} \\
\quad \quad \quad \text{then } i := i + 1 \text{ and continue} \\
\quad \text{end} \\
\quad \text{return } \Delta_i \\
\text{end (delete con)}
\end{align*}\]

Figure 1: The definition of rule con and literal con

Figure 2: The definition of derive and delete con
The following theorem on correctness for rule checking can be derived from correctness on query evaluation procedure of Satoh and Iwayama, 1992.

**Theorem 2** Let \((P, A)\) be a consistent abductive framework. Suppose \(\text{rule}_\text{con}(R_{\text{new}}, \{\})\) succeeds for \(P\) with \(\Delta\), then there is a generalized stable model \(M(\Theta)\) for \((P \cup R_{\text{new}}, A)\) such that \(\Theta\) includes positive abducibles in \(\Delta\).

The above theorem only guarantees that \(R\) is consistent with \(P\) and the procedure produces some abducibles included in a generalized stable model. To compute revision, however, we must have the stronger result that \(\Delta\) includes all the necessary \(\phi^+\)'s and \(\phi^-\)'s. Actually, we can guarantee this by the following theorem.

**Theorem 3** Let \(\mathcal{R}\) be a revision framework \((\langle T_\text{pst} \cup T_\text{tmp}, T_{\text{back}} \rangle)\) such that \(T_\text{pst} \cup T_\text{tmp}\) is consistent and \(R_{\text{new}}\) be an added rule. Suppose \(\text{rule}_\text{con}(R_{\text{new}}, \{\})\) succeeds for \(\tau(\mathcal{R})\) with \(\Delta\), then, a set of positive abducibles in \(\Delta\) realizes revision of \(R_{\text{new}}\) to \(\mathcal{R}\).

The following theorem means that if we can search exhaustively in selecting the rules or cases and there is a generalized stable model whose abducibles minimally realizes a revision for addition of rule \(R_{\text{new}}\), then we can find such a set of abducibles by our procedure. Note that this property is always guaranteed if a program is a finite propositional program or has finite constant symbols and no function symbols.

**Theorem 4** Let \(\mathcal{R}\) be a revision framework \((\langle T_\text{pst} \cup T_\text{tmp}, T_{\text{back}} \rangle)\) and \(R_{\text{new}}\) be an added rule. Suppose that every selection of rules or cases terminates for \(\text{rule}_\text{con}(R_{\text{new}}, \{\})\) with either success or failure for \(\tau(\mathcal{R})\). If \(\Theta\) minimally realizes revision of \(R_{\text{new}}\) to \(T\), then there is a selection of rules and cases such that \(\text{rule}_\text{con}(R_{\text{new}}, \{\})\) succeeds with \(\Delta\) where a set of abducibles in \(\Delta\) is equivalent to \(\Theta\).

Note that we cannot guarantee that positive abducibles of every \(\Delta\) always corresponds with a minimal revision. This problem is inherited from Satoh’s procedure in that it is not guaranteed for the procedure to produce a minimal abducibles. However, using the procedure, we can restrict sets of abducibles related with inconsistency and, thus, considered sets of abducibles to choosing minimal sets are smaller than sets of abducibles from a naive calculation of all the generalized stable models.

**Example 3** Consider the revision framework in Example 2. The following are two sequences of main calls of subprocedures for \(\text{rule}_\text{con}(R_{\text{new}}, \{\})\) to \(\tau(U)\) shown in Example 2. In the following, \(rc, lc, dr\) and \(dc\) corresponds with \(\text{rule}_\text{con}, \text{literal}_\text{con}, \text{derive}\) and \(\text{delete}_\text{con}\) respectively, and indexes in the front express a nesting structure of the calls. Note that existence of “\(c(\varphi) \leftarrow \cdot\)” in \(T_\text{pst}\) does not influence these derivations.

**Sequence 1** (for \(I_1, O_1\) in Example 2)
\[
rc((\bot \leftarrow r(\varphi)), \{\})
\]
\[1. \] \(lc(\sim r(\varphi)), \{\})\) or \(1.1.1.1.1.1. \]
\[1.1. r(c(\varphi), \{\sim r(\varphi)\})
\]
\[1.1.1.1. \]
\[1.1.1.1.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim b_{1^*}(\varphi)), \{c(\varphi), \sim r(\varphi)\})
\]
\[1.1.1.2.1.1. \]
\[1.1.1.1.1.1. \] \(dr(c(\varphi), \{\sim r(\varphi)\})
\]
\[1.1.1.1.2. \]
\[1.1.1.1.2.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim b_{1^*}(\varphi)), \{c(\varphi), \sim b(\varphi), \sim r(\varphi)\})
\]
\[1.1.1.1.2.1.1. \]
\[1.1.1.1.2.1.1.1. \] \(dr(c(\varphi), \{\sim b(\varphi), \sim r(\varphi)\})
\]
\[1.1.1.1.2.1.1.2. \]
\[1.1.1.1.2.1.1.2.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim b_{1^*}(\varphi)), \{c(\varphi), \sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.2.1.1.2.1.1. \]
\[1.1.1.1.2.1.1.2.1.1.1. \] \(dr(c(\varphi), \{\sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.2.1.1.2.1.1.2. \]
\[1.1.1.1.2.1.1.2.1.1.2.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim \phi_{1^*}(\varphi)), \{c(\varphi), \sim b(\varphi), \sim r(\varphi)\})
\]
\[1.1.1.1.2.1.1.2.1.1.2.1.1. \]
\[1.1.1.1.2.1.1.2.1.1.2.1.1.1. \] \(dr(c(\varphi), \{\sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.2.1.1.2.1.1.2.1.1.2. \]
\[1.1.1.1.2.1.1.2.1.1.2.1.1.2.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim \phi_{1^*}(\varphi)), \{c(\varphi), \sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.2.1.1.2.1.1.2.1.1.2.1.1. \]
\[1.1.1.1.2.1.1.2.1.1.2.1.1.2.1.1.1. \] \(dr(c(\varphi), \{\sim \phi_{1^*}(\varphi)\})
\]

**Sequence 2** (for \(I_2, O_2\) in Example 2)
\[
rc((\bot \leftarrow r(\varphi)), \{\})
\]
\[1. \] \(lc(\sim r(\varphi)), \{\})\) or \(1.1. \]
\[1.1. r(c(\varphi), \{\sim r(\varphi)\})
\]
\[1.1.1. \]
\[1.1.1.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim b_{1^*}(\varphi)), \{c(\varphi), \sim r(\varphi)\})
\]
\[1.1.1.1.1. \]
\[1.1.1.1.1.1. \] \(dr(c(\varphi), \{\sim r(\varphi)\})
\]
\[1.1.1.1.1.2. \]
\[1.1.1.1.1.2.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim b_{1^*}(\varphi)), \{c(\varphi), \sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.1.2.1.1. \]
\[1.1.1.1.1.2.1.1.1. \] \(dr(c(\varphi), \{\sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.1.2.1.1.2. \]
\[1.1.1.1.1.2.1.1.2.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim \phi_{1^*}(\varphi)), \{c(\varphi), \sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.1.2.1.1.2.1.1. \]
\[1.1.1.1.1.2.1.1.2.1.1.1. \] \(dr(c(\varphi), \{\sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.1.2.1.1.2.1.1.2. \]
\[1.1.1.1.1.2.1.1.2.1.1.2.1. \] \(rc((r(\varphi) \leftarrow \sim b(\varphi), \sim \phi_{1^*}(\varphi)), \{c(\varphi), \sim \phi_{1^*}(\varphi)\})
\]
\[1.1.1.1.1.2.1.1.2.1.1.2.1.1. \]
\[1.1.1.1.1.2.1.1.2.1.1.2.1.1.1. \] \(dr(c(\varphi), \{\sim \phi_{1^*}(\varphi)\})
\]
Related Work

There are works on calculation method of updates (Kakas and Mancarella, 1990, Inoue and Sakama, 1995). (Kakas and Mancarella, 1990) propose a top-down procedure to compute view updates in a database for proving a given goal, but it is not applicable to updating a normal logic program in general. (Inoue and Sakama, 1995) provide an extended logic program to an extended update framework (Inoue and Sakama, 1995) to an added rule. In (Sakama and Inoue, 1999), translation from an update framework (Inoue and Sakama, 1995) to an extended logic program is provided. Differences between our translation and their translation are as follows.

- They give a translation to compute an update to explain a goal whereas we consider a revision to avoid inconsistency of addition of a rule.
- They introduce a new predicate symbol in stead of abducibles. This makes their translation rather complex. If we translate our translated abductive logic program to a new normal logic program by a method proposed in (Satoh and Iwayama, 1991), the new normal logic program would be the same as their program.
- They consider addition/deletion of the whole rules to derive a given observation. That is, instead of considering deletion/addition of parts of $\Pi_{tmp}/\Pi_{bck}$ in Definition 3, they propose deletion/addition of parts of $T_{tmp}/T_{bck}$. At least, however, to handle exception of integrity constraints in software engineering (Satoh, 1998), we believe that our fine-grained approach is better since we would like to keep consistent part of integrity constraints for further checking of other data when some instances cause inconsistency. See the detailed discussion in (Satoh, 1998).

There are many procedures to compute stable models, generalized stable models or abduction. If we use a bottom-up procedure for our translated abductive logic program to compute all the generalized stable models naively, then sets of abducibles to be compared would be larger since abducibles of irrelevant temporary rules and addable rules with inconsistency will be considered. Therefore, it is better to compute abducibles related with inconsistency. To our knowledge, top-down procedure which can be used for this purpose is only Satoh and Iwayama’s procedure since we need a bottom-up consistency checking of addition/deletion of literals during computing abducibles for revision. This task is similar to integrity constraint checking in (Sadri and Kowalski, 1988) and Satoh and Iwayama’s procedure includes this task.

Conclusion

In this paper, we propose an abductive top-down procedure to compute a minimal revised program which traverses only relevant parts of the program to the added rule. It is done by translating a revision framework of a normal logic program into an abductive logic program.

As a future work, we would like to find an efficient method of computing a minimal revision directly by combining our top-down procedure and ATMS-like method of memorizing justifications of revisions.

Acknowledgments

This research is partly supported by Grant-in-Aid for Scientific Research on Priority Areas, “Principles for Constructing Evolutionary Software”, The Ministry of Education, Japan. We also thank the anonymous referees for valuable comments on this paper.

References

(Alferes et al., 1996) Alferes, J. J., Pereira, L. M. and Przymusinski, T. C. 1996. Belief Revision in Non-Monotonic Reasoning and Logic Programming. Fundamenta Informaticae vol.1, pages 1 - 22.

(Fagin et al., 1983) Fagin, R., Ullman, J. D. and Vardi, M. Y. 1983. On the Semantics of Updates in Databases. in Proceedings of PODS’83, pages 352 – 356.

(Gärdenfors and Rott, 1995) Gärdenfors, P., and Rott, H. 1995. Belief Revision. Handbook of Logic in Artificial Intelligence and Logic Programming Vol. 4, pages 36 – 132.

(Gelfond and Lifschitz, 1991) Gelfond, M. and Lifschitz, V. 1991. Classical Negation in Logic Programs and Disjunctive Databases. New Generation Computing, 9, pages 365 – 385.

(Giordano and Martelli, 1990) Giordano, L. and Martelli, A. 1990. Generalized Stable Models, Truth Maintenance System and Conflict Resolution. in Proceedings of ICLP’90, pages 427 – 441.

(Inoue and Sakama, 1995) Inoue, K. and Sakama, C. 1995. Abductive Framework for Nonmonotonic Theory Change. in Proceedings of IJCAI-95, pages 204 – 210.

(Inoue and Sakama, 1998) Inoue ,K. and Sakama, C. 1998. Specifying Transactions for Extended Abduction. in Proceedings of KR’98, pages 394 – 405.

(Inoue, 2000) Inoue ,K. 2000. A Simple Characterization of Extended Abduction. Unpublished Manuscript.

(Kakas and Mancarella, 1990) Kakas, A. C. and Mancarella, P. 1990. Generalized Stable Models: A Semantics for Abduction. in Proceedings of ECAI’90, pages 385 – 391.

Recently, Inoue, 2000 independently proposes exactly the same technique as our translation to show correspondence their extended abduction and ordinary abduction.
(Kakas and Mancarella, 1990) Kakas, A. C. and Mancarella, P. 1990. Database Updates Through Abduction. in *Proceedings of VLDB’90*, pages 650 – 661.

(Katsuno and Mendelzon, 1991) Katsuno, H., and Mendelzon, A. O. 1991. Propositional Knowledge Base Revision and Minimal Change. *Artificial Intelligence*, 52, pages 263 – 294.

(Sadri and Kowalski, 1988) Sadri, F. and Kowalski, R. 1988. A Theorem-Proving Approach to Database Integrity. *Foundations of Deductive Database and Logic Programming*, (J. Minker, Ed.), Morgan Kaufmann Publishers, pages 313 – 362.

(Sakama and Inoue, 1999) Sakama, C., and Inoue, K. 1999. Updating Extended Logic Programs through Abduction. *Proc. of the 5th International Conference on Logic Programming and Nonmonotonic Reasoning (LP-NMR’99)*, pages 147 – 161.

(Satoh and Iwayama, 1991) Satoh ,K. and Iwayama, N. 1991. Computing Abduction by Using the TMS. in *Proceedings of ICLP’91*, pages 505 – 518.

(Satoh and Iwayama, 1992) Satoh ,K. and Iwayama, N. 1992. A Query Evaluation Method for Abductive Logic Programming. in *Proceedings of JICSLP’92*, pages 671 – 685.

(Satoh, 1998) Satoh, K. 1998. Computing Minimal Revised Logic Program by Abduction. in *Proc. of the International Workshop on the Principles of Software Evolution(IWPSE98)*, pages. 177 – 182.

(Witteveen and van der Hoek, 1997) Witteveen, C. and van der Hoek, W. 1997. A General Framework for Revising Non-monotonic Theories. *Logic Programming and Non-Monotonic Reasoning*, (J. Dix et al, Eds.) LNAI 1265, Springer, pages. 258 – 272.