Dynamic response analysis of an aircraft structure under thermal-acoustic loads

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Abstract. Future hypersonic aircraft will be exposed to extreme combined environments includes large magnitude thermal and acoustic loads. It presents a significant challenge for the integrity of these vehicles. Thermal-acoustic test is used to test structures for dynamic response and sonic fatigue due to combined loads. In this research, the numerical simulation process for the thermal acoustic test is presented, and the effects of thermal loads on vibro-acoustic response are investigated. To simulate the radiation heating system, Monte Carlo theory and thermal network theory was used to calculate the temperature distribution. Considering the thermal stress, the high temperature modal parameters are obtained with structural finite element methods. Based on acoustic finite element, modal-based vibro-acoustic analysis is carried out to compute structural responses. These researches are very vital to optimum thermal-acoustic test and structure designs for future hypersonic vehicles structure.

1. Introduction

One of the problems to be encountered by future hypersonic aircraft will be the extreme combined loading conditions include thermal, acoustic and mechanical loading. Since the coupled interactions between these loadings, the ability to accurately and reasonably analyze and predict the dynamic response is key capabilities for the engineers. Among these loadings, the high temperature and intense fluctuating pressures experienced by the skin panels of hypersonic vehicles are a primary design consideration.

The majority of structural responses under combined thermal-acoustic loads are nonlinear and not many techniques exist for the analysis. Since 1960s, extensive numerical studies have been performed using Fokker-Planck-Kolmogorov equations approaches, perturbation approaches, equivalent linearization approaches and Monte Carlo.

With the development of finite element methods (FEM) and boundary element methods (BEM), they are extended to analyze the structural response under combined acoustic and thermal loads. Behnke utilized the commercial FE code Abaqus to conduct a time domain analysis of an ITPS structure under a combined loading environment. Jeyaraj presented numerical simulation studies on the vibration and acoustic response characteristics of an isotropic rectangular plate in a thermal environment using commercial finite element softwares ANSYS and SYSNOISE. The effect of a thermal environment on the vibration response and consequent acoustic radiation from an isotropic plate is investigated. Later, a composite plate for different values of fiber orientation was further investigated.
Jeyaraj obtained the acoustic radiation characteristics of a heated plate in frequency domain. If the vibration response of the hot structure under acoustic load is needed, it is difficult to apply random acoustic load in frequency domain FEM. In this paper, Based on structure FEM and acoustic FEM, the numerical analysis process for thermal acoustic test are carried out. The random sound load and quartz radiation heating load are applied to the stiffened panel simultaneously. Finally, the accelerations vibration response is obtained.

2. Formulations
One of the most common means of simulating the thermal-acoustic environment is through the use of a progressive wave tube. NASA Langley Research Centre has the progressive wave tube facility, which known as the Thermal Acoustic Fatigue Apparatus (TAFA)\(^\text{11}\). And U.S. Air Force has a similar system named as Combined Environment Acoustic Chamber\(^\text{12}\).

A Diagram of this kind of thermal acoustic facility is shown in Figure 1. Testing articles are heated by quartz lamps and the high-intensity acoustic load is generated by air modulators. The test articles are mounted at one side of the test section and the quartz lamp heater is placed on another side.

![Figure 1. Diagram of thermal acoustic facility.](image)

In this research, the numerical analysis is performed to simulate this thermal acoustic test by using coupled structure finite element methods and acoustic finite element methods. The random response of the plate exposed to thermal and intense acoustic loads are calculated. Figure 2 presents the analysis process used to determine the dynamic response. First, a prestressed modal analysis is carried out. And then, after the computation of the natural frequencies and mode shapes at elevated temperature, the random acoustic load is applied. By vibro-acoustic response analysis, the structure vibration response is finally obtained.

![Figure 2. Analysis process of the thermal-acoustic response analysis.](image)

2.1. Thermal analysis formulations\(^\text{10}\)
During thermal acoustic test, the thermal load is produced by means of radiation heating system. In order to simulate the radiation heating system, the Monte-Carlo theory is adopted to calculate the radiation heat flux arrived at the specific surface\(^\text{13}\). Rays are emitted from each node of the heat source and "traced" around the geometry. The rays simulate the effect of a "bundle" of photons. When
a ray strikes another surface, energy is decremented from the ray and absorbed by the struck surface. The ray is then reflected or transmitted, according to the optical properties on the surface. The radiation exchange factors are computed from view factor data, which is calculated using ray tracing.

The temperature distribution is calculated by the thermal network theory. Considering the transient heat transfer process as the example, if there are \( N \) nodes in the thermal system, the thermal conservation equation of node \( i \) at time \( t \) is

\[
Q_i + \sum_{j=1}^{N} f_{ij} (T_j - T_i) = \left( mc_p \right) \frac{dT_i}{dt}
\]

where \( Q_i \), \( T_i \), \( m \), and \( c_p \) are the heat generation, the temperature, the mass and the thermal capacity of node \( i \), and \( f_{ij} \) is the conductance of nodes \( i \) and \( j \). The conductance which represents a heat flow path through a material is equal to the product of the material’s thermal conductivity and the cross-sectional area of the flow path, and then divided by the length of the path. The conductance which represents heat flow by convection is equal to the product of convection coefficient and the convection area.

2.2. The effect of a thermal environment on structure stiffness

The thermal loads will change the material properties and also induce membrane forces, which will produce the thermal stresses and the thermal strains. Therefore, the thermal preloads will influence the stiffness of the structure. The preloads on the structures due to thermal loads, which obtained from Section 2.1, are calculated using a nonlinear static analysis. Therefore, the stiffness matrix will be modified to

\[
K_s = K_L + K_R + K_o
\]

and

\[
[K_L] = \int_\Omega [B_L]^T [E] [B_L] d\Omega
\]

\[
[K_R] = \int_\Omega ([B_L]^T [E] [B_{NL}] + [B_{NL}]^T [E] [B_L] + [B_{NL}]^T [E] [B_{NL}]) d\Omega
\]

\[
[K_o] = \int_\Omega [dB_{NL}]^T [\sigma] d\Omega
\]

where \([K_L]\) is the linear stiffness matrix, \([K_R]\) and \([K_o]\) include the influence of the thermal stress and thermal strain. And the influence of the change of the material properties is reflected in all the three items.

Since the thermal load is applied to the structure, the stiffness is changed. The modal parameters of the heated structure can be obtained by solving the eigenvalue problems as given below.

\[
((K_L + K_R + K_o) - \omega^2 M) \Phi = 0
\]

Therefore, the effect of the thermal load can be included into structural dynamic characteristics.

2.3. Coupled thermal-acoustic response analysis

In this section, acoustic FEM is used to apply acoustic load accurately to the heated structure. Combining the structural FE model and the acoustic FE model yields the Eulerian FE/FE model for a coupled vibro-acoustic system

\[
\begin{bmatrix}
K_s & K_a \\
0 & K_a
\end{bmatrix} + j\omega
\begin{bmatrix}
C_s & 0 \\
0 & C_a
\end{bmatrix} - \omega^2
\begin{bmatrix}
M_s & 0 \\
-\rho_c K_c^T & M_a
\end{bmatrix}
\begin{bmatrix}
w_f \\
p_f
\end{bmatrix} = \begin{bmatrix}
F_{ai} \\
\end{bmatrix}
\]

The first row is structural equations, and the second is acoustic equations. In contrast with uncoupled equations, these coupled matrices are not symmetric, due to the cross-coupling matrix \( K_c \).

3. Numerical simulations
The test specimen is a stiffened plate constructed of titanium alloy, with fixed boundary conditions. The size of the plate is $288 \times 288 \times 1$ mm. And the stiffener is 5 mm high and 4 mm wide. The temperature dependent material properties are given in Table 1. Besides these temperature dependent material properties, the density and Poisson's ratio are assumed constant in the analysis at $4440$ kg/m$^3$ and 0.342, respectively $^{[10]}$.

Table 1. Thermal physical properties of Titanium alloy TC4 $^{[10]}$.

| Temperature °C | Thermal conductivity $\lambda$/W·m$^{-1}$·°C$^{-1}$ | Expansion coefficient $\alpha$/×10$^{-6}$·°C$^{-1}$ | Specific heat $c$/J·kg$^{-1}$·°C$^{-1}$ | Elastic modulus /Gpa |
|----------------|-----------------------------------------------|--------------------------|-----------------------------------|------------------|
| 20             | 6.8                                          | 9.1                      | 611                               | 123.0            |
| 100            | 7.4                                          | 9.1                      | 624                               | 110.3            |
| 200            | 8.7                                          | 9.2                      | 653                               | 106.2            |
| 300            | 9.8                                          | 9.3                      | 674                               | 97.4             |
| 400            | 10.3                                         | 9.5                      | 691                               | 86.5             |

3.1. Thermal analysis
The temperature distributions of the test article are obtained by SINDA/FLUINT software. In order to get the temperature distribution more accurate, the entire test section of the thermal acoustic facility is modelled. As shown in Figure 3, the model includes the quartz lamp radiation heater, the test section and the reflector, etc. Even the air flow in the progressive wave tube can be considered in this model.

![Figure 3. The model for the thermal analysis.](image)

The calculated temperature distribution is shown in Figure 4. The temperature at the centre of the plate is 400 °C.

![Figure 4. The temperature distribution.](image)
3.2. Thermal modal analysis

Subsequently, the obtained temperature fields are exported, and then import into the MSC.Nastran finite element model to obtain the thermal stress first and then conduct a modal analysis. The plate is modeled with 4201 quadrilateral shell elements (CQUAD4), with 4304 nodes. The linear mode solver (SOL 103) is used to carry out the modal analysis at room temperature, and the implicit nonlinear solver (SOL 400) is used to carry out the modal analysis at high temperature. Both the temperature dependent material properties and the thermal stresses resulted from the temperature distributions are considered.

The change of the modal frequency is presented in Figure 5.

![Figure 5. The change of the modal frequency.](image)

The modal shapes are presented in Figure 6. As shown in Figure 5 and Figure 6, the first two mode shapes are swapped.

![Figure 6. Mode shapes. (a) Mode 1 at room temperature, (b) Mode 2 at room temperature, (c) Mode 3 at room temperature, (d) Mode 1 at 300°C, (e) Mode 2 at 300°C, (f) Mode 3 at 300°C.](image)

3.3. Vibro-acoustic response analysis

This section describes the process of hybrid structure finite element and acoustic finite element Method (FEM-FEM) analysis to get the response of heated structure under acoustic load. The calculation was carried out by commercial software.
The finite element model of the progressive wave tube is shown in Figure 7. The tube is modeled with 4455 hexahedral solid elements (CHEXA). The structure finite element model, which already considered the thermal load, is imported.

![Figure 7. Acoustic model.](image)

The acoustic load, which is used in the thermal-acoustic test, is adopted. The required acoustic load is white noise and its frequency is in the range of 50 to 1000Hz. The overall sound pressure level of acoustic load is 140dB.

In order to carry out random analysis, the definition of the diffuse field is extended to this progressive wave tube. The analytic formulation of the diffuse field cross spectra is based on an infinite series representation of randomly distributed plane waves. Here, the truncated series of plane waves method is used to obtain the random acoustic load. And then, this acoustic load is applied to the intake of the progressive wave tube, as shown in Figure 8.

![Figure 8. Apply the random acoustic load.](image)

In order to get the progressive wave, there should be no reflection for the finite tube. Therefore, the Automatically Matched Layer (AML) property is applied at the end of the tube. After applying the random load and AML property, the propagation of the acoustic field in the tube is obtained by acoustic FEM.

![Figure 9. The accelerations response.](image)
Finally, by using the coupled acoustic FEM and structure FEM analysis, the vibration response, which is evaluated by modal superposition, can be obtained. In Figure 9, the accelerations PSD response positioned at the center of the plate is given. It can be seen that the frequency peaks in the structure response is mainly induced by the first mode. The position of the response peak, which is corresponding to the first mode, is shifted to higher frequency, since the first modal frequency is increased with increased temperature.

4. Conclusions
The numerical simulation of the combined thermal-acoustic test has been carried out. The Monte-Carlo theory is adopted to calculate the radiation heat flux arrived at the specific surface. The temperature distribution is calculated by the thermal network theory. In order to carry out random analysis, diffused acoustic load is applied to the intake of the progressive wave tube. Finally, the dynamic response of a stiffened plate exposed to thermal-acoustic loads is obtained based on coupled structure FEM and acoustic FEM. Since the modal superposition is used, the response is mainly affected by the change of the mode. It can be seen that the highest peak is appeared at the first modal frequency. Since the first modal frequency is increased with increased temperature, the position of the peak is shifted to higher frequency.

This work will be continued. Next, the comparison between the calculation and the experiment test data should be done. And since the majority of the structural dynamic responses are nonlinear and Snap-through phenomena will be occurred in thermal-acoustic combined environments. Future efforts will focus on the development of a nonlinear integrated analysis for the hypersonic vehicle structures experienced coupled thermal and acoustic loads.

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