Upsilon decay widths in magnetized asymmetric nuclear matter

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Abstract

The in-medium partial decay widths of $\Upsilon(4S) \rightarrow B\bar{B}$ in magnetized asymmetric nuclear matter are studied using a field theoretic model for composite hadrons with quark (and antiquark) constituents. $\Upsilon(4S)$ is the lowest bottomonium state which can decay to $B\bar{B}$ in vacuum. The medium modifications of the decay widths of $\Upsilon(4S)$ to $B\bar{B}$ pair in magnetized matter arise due to the mass modifications of the decaying $\Upsilon(4S)$ as well as of the produced $B$ and $\bar{B}$ mesons. The in-medium masses of the open bottom meson in magnetized nuclear matter are computed from their interactions with the nucleons and the scalar mesons within a chiral effective model. The mass modification of the $\Upsilon(4S)$ arises due to the medium modification of a scalar dilaton field, which is introduced in the model to simulate the gluon condensates of QCD. The effects of the anomalous magnetic moments for the proton and neutron are taken into consideration in the present investigation. The presence of the external magnetic field is observed to lead to different mass modifications within the $B(B^+,B^0)$ as well as the $\bar{B}(B^−,\bar{B}^0)$ doublets, even in isospin symmetric nuclear matter. This is due to the difference in the interactions of the proton and the neutron to the electromagnetic field. The charged $B^\pm$ mesons have additional contributions from the Landau energy levels, leading to positive shifts in their masses in the presence of a magnetic field. In the presence of an external magnetic field, there are contributions to the masses of the $B$, $\bar{B}$ mesons and $\Upsilon(4S)$ state (longitudinal component) due to the pseudoscalar meson-vector meson (PV) mixing ($B − B^*$, $\bar{B} − \bar{B}^*$ and $\Upsilon(4S) − \eta_b(4S)$ mixings), which are also considered in the present study. The PV mixing effects are observed to be the dominant contributions to the mass shifts of these mesons, which lead to appreciable modifications in the decay widths of $\Upsilon(4S)$ to the neutral ($B^0\bar{B}^0$) and the charged ($B^+\bar{B}^−$) pairs in the presence of a magnetic field. These should have observable consequence in the production of open bottom mesons and bottomonium states at LHC and RHIC, where huge magnetic fields are produced in ultra-relativistic peripheral heavy ion collisions.

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I. INTRODUCTION

The study of hadrons, and more recently of heavy flavour hadrons [1] has been a topic of intense research due to the relevance to high energy nuclear collision experiments. Due to the isospin asymmetry in the heavy colliding nuclei in the high energy heavy ion collision experiments, it is important to study the isospin asymmetry effects on the properties of the hadrons resulting from these collisions. The magnitudes of the magnetic fields created in the non-central ultra-relativistic heavy ion collision experiments have been estimated to be huge \( (eB \sim 2m^2_\pi \text{ at RHIC, BNL, and } eB \sim 15m^2_\pi \text{ at LHC, CERN}) \) [2–5]. This has triggered extensive studies on the effects of strong magnetic fields on the hadron properties which should show up in the experimental observables of the heavy ion collision experiments. The effects of magnetic fields have been studied on the in-medium properties of the heavy flavour mesons [6–11, 11–16]. However, the time evolution of the magnetic field created in non-central ultra relativistic heavy ion collision experiments [17–21], which depends on the electrical conductivity of the medium and needs solutions of the magneto-hydrodynamic equations, is still an open question.

The heavy flavour mesons have been studied extensively in the literature, using the QCD sum rule approach [22–38], potential models [39–49], the coupled channel approach [50–56], the quark meson coupling (QMC) model [57–65], heavy quark symmetry and interaction of these mesons with nucleons via pion exchange [66], heavy meson effective theory [67], studying the heavy flavour meson as an impurity in nuclear matter [68]. The mass modifications of the charmonium states have been studied from the medium change of the scalar gluon condensate calculated in a linear density approximation in Ref.[69], using leading order QCD formula [70–72]. Within a chiral effective model [73–75], generalized to include the interactions of the charm and bottom flavoured hadrons, the in-medium heavy quarkonium (charmonium and bottomonium) masses are obtained from the medium changes of a scalar dilaton field, which mimics the gluon condensates of QCD [76–78]. The mass modifications of the open heavy flavour (charm and bottom) mesons within the chiral effective model have also been studied from their interactions with the baryons and scalar mesons in the hadronic medium [79, 77, 79, 83]. The chiral effective model, in the original version with three flavours of quarks (SU(3) model), has been used extensively in the literature, for the study of finite nuclei [74], strange hadronic matter [75], light vector mesons [84], strange pseudoscalar mesons, e.g. the kaons and antikaons [85–88] in isospin asymmetric hadronic matter, as well as for the study of bulk matter of neutron stars [89]. Using the medium changes of the light quark condensates and gluon condensates calculated within the chiral SU(3) model, the light vector mesons (\( \omega, \rho \) and \( \phi \)) in
(magnetized) hadronic matter have been studied within the framework of QCD sum rule approach [90, 91]. The kaons and antikaons have been recently studied in the presence of strong magnetic fields using this model [92]. The model has been used to study the partial decay widths of the heavy quarkonium states to the open heavy flavour mesons, in the hadronic medium [77] using a light quark creation model [93], namely the $^3P_0$ model [94–97] as well as using a field theoretical model for composite hadrons [98, 99]. Recently, the effects of magnetic field on the charmonium partial decay widths to $D\bar{D}$ mesons have been studied using the $^3P_0$ model [100] as well as using the field theoretic model of composite hadrons [101]. In the present work, the effects of magnetic field on the partial decay width of $\Upsilon(4S)$ to $B\bar{B}$, which arise due to their mass modifications, have been investigated within field theoretical model of composite hadrons. The masses of the open bottom ($B$ and $\bar{B}$) mesons as well as the upsilon state are studied in the magnetized nuclear matter using an effective chiral model. Within the model, the medium Modifications of the $B$ and $\bar{B}$ mesons arise due to their interactions with the nucleons and the scalar mesons. On the other hand, the mass modification of the $\Upsilon$ state is calculated from the medium change of a scalar dilaton field, which mimics the gluon condensates of QCD. For the charged $B^\pm$ mesons, there are additional Landau energy level contributions, which are not present for the neutral open bottom ($B^0$ and $\bar{B}^0$) mesons. In the presence of the magnetic field, the effects of the pseudoscalar meson-vector meson (PV) mixings [14, 101–104] on the bottomonium and open bottom mesons are also taken into account in the present study. These are observed to lead to more significant modifications to the masses of the open bottom mesons (due to $B - B^*$ and $\bar{B} - \bar{B}^*$ mixings), as compared to the mass modification of the longitudinal component of $\Upsilon(4S)$, due to mixing with $\eta_b(4S)$, for the values of magnetic field considered in the present work. The dominant contributions from the PV mixing to the masses of the $B$ and $\bar{B}$ mesons (along with the Landau level contributions to the charged $B^\pm$ mesons) as well as for $\Upsilon(4S)$ meson (due to $\Upsilon(4S) - \eta_b(4S)$ mixing) are observed to lead to significant modifications to the partial decay widths of $\Upsilon(4S)$ to the charged and neutral $BB$ pairs.

The outline of the paper is as follows: In section 2, we discuss briefly the computation of the in-medium masses of the bottomonium state ($\Upsilon(4S)$) and the open bottom ($B$ and $\bar{B}$) mesons in magnetized nuclear matter, using a chiral effective model. The Landau level contributions (for the charged $B^\pm$ mesons) as well as effects of the PV mixings on the masses of the $B$, $\bar{B}$ and $\Upsilon(4S)$ are studied in the presence of the external magnetic field. Section 3 gives a brief description of the field theoretic model of composite hadrons with quark (and antiquark) constituents used to calculate the partial decay widths of $\Upsilon$ to $BB$. The decay width is calculated within the model using the
explicit constructions for the bottomonium state Υ(4S) as well as the $B$ and $\bar{B}$ mesons in terms of the constituent quark and antiquark operators and the matrix element of the quark antiquark pair creation term of the free Dirac Hamiltonian. The in-medium decay width is calculated from the medium modifications of the masses of the decaying bottomonium state, as well as the produced $B$ and $\bar{B}$ mesons in the magnetized asymmetric nuclear matter. In section 4, we discuss the results obtained for these in-medium decay widths in (asymmetric) nuclear matter in presence of strong magnetic fields. In section 5, we summarize the findings of the present study.

II. IN-MEDIUM MASSES OF Υ(4S) AND OPEN BOTTOM MESONS

The masses of the $B$ and $\bar{B}$ mesons in magnetized nuclear matter have been studied using a chiral effective model in Ref. [10]. The model is a generalization of a chiral SU(3) model, to include interactions of the open heavy flavour mesons with the light hadrons. The model incorporates the broken scale invariance of QCD through a scalar dilaton field, $\chi$, which mimics the gluon condensate of QCD. The Lagrangian density of the chiral effective model, in the presence of a magnetic field, is given as [74]

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{W} \mathcal{L}_{BW} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{0} + \mathcal{L}_{\text{scalebreak}} + \mathcal{L}_{SB} + \mathcal{L}_{\text{mag}}^{B\gamma},$$

where, $\mathcal{L}_{\text{kin}}$ corresponds to the kinetic energy terms of the baryons and the mesons, $\mathcal{L}_{BW}$ contains the interactions of the baryons with the meson, $W$ (scalar, pseudoscalar, vector, axialvector meson), $\mathcal{L}_{\text{vec}}$ describes the dynamical mass generation of the vector mesons via couplings to the scalar fields and contains additionally quartic self-interactions of the vector fields, $\mathcal{L}_{0}$ contains the meson-meson interaction terms, $\mathcal{L}_{\text{scalebreak}}$ is a scale invariance breaking logarithmic potential given in terms of a scalar dilaton field, $\chi$, and $\mathcal{L}_{SB}$ describes the explicit chiral symmetry breaking. The term $\mathcal{L}_{\text{mag}}^{B\gamma}$ describes the interaction of the baryons with the electromagnetic field, which includes a tensorial interaction $\sim \bar{\psi}_{i}\sigma^{\mu\nu}F_{\mu\nu}\psi_{i}$, whose coefficients account for the anomalous magnetic moments of the baryons [9] [11].

Using the mean field approximation, i.e., writing the meson fields as classical fields, the masses of the $B$ and $\bar{B}$ mesons are calculated by solving the dispersion relations obtained from the interaction Lagrangian of these mesons with the nucleons and the scalar mesons. The values of the non-strange ($\sigma$) and strange ($\zeta$) scalar-isoscalar meson, the scalar-isovector ($\delta$) meson fields, and, the dilaton field, $\chi$ are obtained by solving their coupled equations of motion. These are calculated for given values of the magnetic field, the baryon density, $\rho_{B}$, and, the isospin asymmetry parameter $\eta$. 
defined as \( \eta = (\rho_n - \rho_p)/(2\rho_B) \), where, \( \rho_p \) and \( \rho_n \) are the number densities of proton and neutron.

The dispersion relations for the \( B \) and \( \bar{B} \) mesons are obtained from the Fourier transformations of the equations of motion of these mesons. These are given as

\[
-\omega^2 + \vec{k}^2 + m_{B(\bar{B})}^2 - \Pi_{B(\bar{B})}(\omega, |\vec{k}|) = 0,
\]

where \( \Pi_{B(\bar{B})} \) denotes the self energy of the \( B \) (\( \bar{B} \)) meson in the medium, which are given in terms of the scalar fields, the number and scalar densities of the nucleons [10] [82]. In the presence of a magnetic field, the number and scalar densities of the proton have contributions from the Landau energy levels [9] [10]. The charged \( B^\pm \) mesons have contributions from the Landau energy levels in the presence of an external magnetic field [105] [106]. Retaining only lowest Landau level (LLL) contribution the mass of the \( B^\pm \) meson is given as

\[
m_{B^\pm}^{\text{eff}} = \sqrt{m_{B^\pm}^* + |eB|},
\]

whereas for the neutral (\( B^0 \) and \( \bar{B}^0 \)) mesons, the effective masses are given as

\[
m_{B^0(\bar{B}^0)}^{\text{eff}} = m_{B^0(\bar{B}^0)}^*.
\]

It might be noted here that equation (3) refers to the mass of a spin zero particle in a magnetic field due to the lowest Landau level contribution, ignoring its internal structure [105]. In equations (3) and (4), \( m_{B^\pm, B^0, \bar{B}^0}^* \) are the masses calculated using the chiral effective model, as the solutions for \( \omega \) at \( |\vec{k}| = 0 \), of the dispersion relations for these mesons given by equation (2).

The mass shift of the heavy quarkonium state is proportional to the change in the gluon condensate in the medium. This is the leading order result of a study of the heavy quarkonium state in a gluon field, assuming the distance between the heavy quark and antiquark (bound by a Coulomb potential) to be small as compared to the scale of gluonic fluctuations [70–72]. The dilaton field \( \chi \) of the scale breaking term \( \mathcal{L}_{\text{scalebreak}} \) in the chiral effective model is related to the scalar gluon condensate of QCD and this relation is obtained by equating the trace of the energy momentum tensor in the chiral effective model and in QCD. The mass shift of the bottomonium state in the magnetized nuclear matter is hence computed from the medium change of the dilaton field from vacuum value, calculated within the chiral effective model, and is given as [78]

\[
\Delta m = \frac{4}{81}(1 - d) \int d|\vec{k}|^2 \langle |\frac{\partial \psi(\vec{k})}{\partial \vec{k}}|^2 \rangle \frac{|\vec{k}|}{|\vec{k}|^2/m_b + \epsilon} \left( \chi^4 - \chi_0^4 \right),
\]

where

\[
\langle |\frac{\partial \psi(\vec{k})}{\partial \vec{k}}|^2 \rangle = \frac{1}{4\pi} \int |\frac{\partial \psi(\vec{k})}{\partial \vec{k}}|^2 d\Omega.
\]
In equation (5), $d$ is a parameter introduced in the scale breaking term in the Lagrangian, $\chi$ and $\chi_0$ are the values of the dilaton field in the magnetized medium and in vacuum respectively. The wave functions of the bottomonium states, $\psi(k)$ are assumed to be harmonic oscillator wave functions, $m_b$ is the mass of bottom quark, $\epsilon = 2m_b - m$ is the binding energy of the bottomonium state of mass, $m$.

A. Pseudoscalar meson-Vector meson (PV) mixing

In the presence of a magnetic field, there is mixing between the pseudoscalar meson and vector mesons, which modifies the masses of these mesons [14, 15, 101–104]. The PV mixing leads to a drop (rise) in the mass of the pseudoscalar (longitudinal component of the vector meson). The mass modifications have been studied using an effective Lagrangian density of the form $\mathcal{L}_{PV}\gamma \sim \tilde{\mathcal{F}}_{\mu\nu}(\partial^\mu P)^V\nu$ [14, 104] for the heavy quarkonia [14, 101], the open charm mesons [102] and strange ($K$ and $\bar{K}$) mesons [103].

In the present work, we estimate the modifications to the masses of the pseudoscalar and vector mesons ($Q_1\bar{Q}_2$ bound states) due to mixing of these states in the presence of a magnetic field, using the Hamiltonian [16, 104].

$$H_{\text{spin-mixing}} = -\sum_{i=1}^{2} \mathbf{\mu}_i \cdot \mathbf{B}, \quad (7)$$

which describes the interaction of the magnetic moments of the quark (antiquark) with the external magnetic field. In the above, $\mathbf{\mu}_i = g|e|q_i\mathbf{S}_i/(2m_i)$ is the magnetic moment of the $i$-th particle, $g$ is the Lande g-factor (taken to be $2(-2)$ for the quark(antiquark)), $q_i, S_i, m_i$ are the electric charge (in units of the magnitude of the electronic charge, $|e|$), spin and mass of the $i$-th particle [14, 104]. This interaction leads to a drop (increase) of the mass of the pseudoscalar (longitudinal component of the vector meson) given as [16]

$$\Delta M^{PV} = \frac{\Delta E}{2} \left( 1 + \Delta^2 \right)^{1/2} - 1, \quad (8)$$

where $\Delta = 2g|eB|(q_1/m_1 - (q_2/m_2))/\Delta E$, $\Delta E = m_V - m_P$ is the difference in the masses of the pseudoscalar and vector mesons. As we shall see, the masses of the open bottom ($B$ and $\bar{B}$) mesons are observed to have dominant contributions from the PV ($B - B^*$ and $\bar{B} - \bar{B}^*$) mixings, which, in turn, affect appreciably the partial decay widths $\Upsilon(4S) \rightarrow B\bar{B}$ in presence of an external magnetic field.
We use a field theoretical model for composite hadrons with quark (and antiquark) constituents to investigate the effects of strong magnetic fields on the decay widths of bottomonium states to $B\bar{B}$. The model has been used for calculating the in-medium partial decay widths of the charmonium (bottomonium) states to $D\bar{D}$ ($B\bar{B}$) in hot strange hadronic matter, as well as for studying the effects of magnetic fields on the decay widths of charmonium states decaying to $D\bar{D}$. In the model, the decay width is calculated using explicit constructions for the heavy quarkonium as well as the open heavy flavour (charm, bottom) mesons, and the quark antiquark pair creation term of the free Dirac Hamiltonian, written in terms of the constituent quark field operators. The decay amplitude is multiplied with a strength parameter for the light quark pair creation, which is fitted to the vacuum partial decay widths of the lowest quarkonium state ($\psi(3770)$ for charmonium state and $\Upsilon(4S)$ for bottomonium state) which can decay to $D\bar{D}$ or $B\bar{B}$ in vacuum. The present paper investigates the partial decay width $\Upsilon(4S)$ to $B\bar{B}$ in magnetized isospin asymmetric nuclear matter. The modifications to the decay width are computed from the mass modifications of the $\Upsilon(4S)$ as well as $B$ and $\bar{B}$ mesons in the magnetized matter. These mass shifts are calculated within the chiral effective model using equations (5) and (2) [10, 12]. For the charged $B^\pm$ mesons there are (lowest) Landau level contribution, which modifies their masses as given by equation (3). The PV mixing effects lead to a drop (rise) in the mass of the pseudoscalar (longitudinal component of the vector meson) by $\Delta M_{PV}$ given by equation (8). As we shall see later, the mass modifications of the $B$, $\bar{B}$ mesons and $\Upsilon(4S)$ due to the PV mixing are quite significant, leading to appreciable modification in the decay widths of $\Upsilon(4S) \rightarrow B\bar{B}$ in the magnetized medium.

For $\Upsilon(4S)$ decaying at rest to $B(p)\bar{B}(-p)$, the decay width has already been calculated using the field theoretical model for composite hadrons with constituent quarks and antiquarks, in Ref. [99]. For the sake of completeness, we briefly describe the computation of the decay width of $\Upsilon(4S) \rightarrow B(p)\bar{B}(-p)$ in this section. The results obtained for these decay widths in magnetized nuclear matter will be described in the next section. For the decay of $\Upsilon(4S)$ at rest to $B(p)\bar{B}(-p)$, the magnitude of $|p|$ is given as

$$|p| = \left( \frac{m_\Upsilon^2}{4} - \frac{m_B^2 + m_{\bar{B}}^2}{2} + \frac{(m_B^2 - m_{\bar{B}}^2)^2}{4m_\Upsilon^2} \right)^{1/2}. \quad (9)$$

In the above, $m_\Upsilon$, $m_B$ and $m_{\bar{B}}$ are the in-medium masses of the bottomonium state ($\Upsilon(4S)$ in the present investigation), $B$ and $\bar{B}$ mesons respectively.

The field operator for a constituent quark for a hadron at rest, (as the case for the bottomonium
state decaying at rest) at time, t=0, is written as

\[ \psi(x) = (2\pi)^{-3/2} \int \left[ U(k)q_r(k)u_r e^{ik \cdot x} + V(k)\bar{q}_s(k)v_s e^{-ik \cdot x} \right] dk, \]  

(10)

where, the operator \( q_r(k) \) annihilates a quark with spin \( r \) and momentum \( k \), whereas, \( \bar{q}_s(k) \) creates an antiquark with spin \( s \) and momentum \( k \), and these operators satisfy the usual anticommutation relations

\[ \{ q_r(k), q_s(k') \} = \{ \bar{q}_r(k), \bar{q}_s(k') \} = \delta_{rs} \delta(k - k'). \]  

(11)

In equation (10), \( U(k) \) and \( V(k) \) are given as [107],

\[ U(k) = \begin{pmatrix} f(|k|) \\ \sigma \cdot k g(|k|) \end{pmatrix}, \quad V(k) = \begin{pmatrix} \sigma \cdot k g(|k|) \\ f(|k|) \end{pmatrix}, \]  

(12)

and, \( u_r \) and \( v_s \) are the two component spinors for the quark and antiquark. The functions \( f(|k|) \) and \( g(|k|) \) satisfy the constraint \( f^2 + g^2 = 1 \), as obtained from the equal time anticommutation relation for the four-component Dirac field operators. These functions, for the case of free Dirac field of mass \( M \), are given as,

\[ f(|k|) = \left( \frac{k_0 + M}{2k_0} \right)^{1/2}, \quad g(|k|) = \left( \frac{1}{2k_0(k_0 + M)} \right)^{1/2}, \]  

(13)

where \( k_0 = (|k|^2 + M^2)^{1/2} \). In the above, \( M \) is the constituent quark mass, which has been assumed to be momentum independent [98, 99] in the present work. We also take the low momentum expansions for the the functions \( f(|k|) \sim 1 - (g(|k|)^2 k^2/2) \) and \( g(|k|) \sim 1/2M [98, 99]. \)

For a hadron in motion (as for the case for the outgoing \( B \) and \( \bar{B} \) mesons), the field operators for quark annihilation and antiquark creation, for \( t=0 \), are obtained by Lorentz boosting the field operator of the hadron at rest. The field operators for the constituent quark and antiquark of a hadron with four momentum \( p \), are given as [109]

\[ Q^{(p)}(x, 0) = (2\pi)^{-3/2} \int dk S(L(p))U(k)Q_r(k + \lambda p)u_r \exp(i(k + \lambda p) \cdot x) \]  

(14)

and,

\[ \hat{Q}^{(p)}(x, 0) = (2\pi)^{-3/2} \int dk S(L(p))V(-k)\hat{Q}_s(-k + \lambda p)v_s \exp(-i(-k + \lambda p) \cdot x). \]  

(15)

In the above, \( \lambda \) is the fraction of the energy of the hadron at rest, carried by the constituent quark (antiquark), and, the Lorentz boosting factor \( S(L(p)) \) is given as

\[ S(L(p)) = \left( \frac{p^0 + m_h}{2m_h} \right)^{1/2} + \frac{\alpha \cdot p}{(2m_h(p^0 + m_h))^{1/2}}, \]  

(16)
where, \( \alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \) are the Dirac matrices, and, \( m_h \) is the mass of the hadron with momentum \( p \), which is the \( B(\bar{B}) \) meson here.

The explicit construct for the state for the bottomonium state \( \Upsilon(4S) \) with spin projection \( m \) at rest, assuming harmonic oscillator wave functions for the bottomonium states, is given as

\[
|\Upsilon_m(0)\rangle = \int d\mathbf{k} b^\dagger_r(\mathbf{k})u^\dagger_r u^\dagger \Upsilon(4S)(\mathbf{k}) \sigma_m \tilde{b}^i_s v_s(-\mathbf{k})|\text{vac}\rangle,
\]

where, \( b^\dagger_r(\tilde{b}^i_s) \) creates a bottom quark (antiquark) of spin \( r(s) \) and color \( i \), \( S_m \equiv \frac{1}{2} \sigma_m \) gives the spin projection of the bottomonium state, and, the harmonic oscillator wave function of \( \Upsilon(4S) \) is given as

\[
u_{\Upsilon(4S)}(\mathbf{k}) = -\frac{1}{\sqrt{6}} \left( \frac{R^2_{\Upsilon(4S)}}{\pi} \right)^{3/4} \left( 1 - 2R^2_{\Upsilon(4S)}k_1^2 + \frac{4}{5}R^4_{\Upsilon(4S)}k_1^4 - \frac{8}{105}R^6_{\Upsilon(4S)}k_1^6 \right)
\times \exp \left[ -\frac{1}{2}R^2_{\Upsilon(4S)}k_1^2 \right].
\]

In the above, \( R_{\Upsilon(4S)} \) is the strength parameter of the harmonic oscillator wave function, \( u_{\Upsilon(4S)} \). The factor \( \frac{1}{\sqrt{6}} \) in equation (18) refers to normalization factor arising from degeneracy factors due to color (3) and spin (2) of the quarks and antiquarks. The \( B^0(\bar{B}^0) \) states, with finite momenta, are explicitly given as

\[
|B^0(p')\rangle = \int d^2r(k_3 + \lambda_1 p') u^\dagger_r B(k_3) \tilde{b}^i_s v_s(-k_3 + \lambda_2 p') dkd3k.
\]

and

\[
|\bar{B}^0(p)\rangle = \int b^i_s(k_2 + \lambda_2 p') u^\dagger_r \bar{B}(k_2) \tilde{d}^i_s v_s(-k_2 + \lambda_1 p) dkd2k,
\]

where,

\[
u_B(k) = \frac{1}{\sqrt{6}} \left( \frac{R_B^2}{\pi} \right)^{3/4} \exp \left( -\frac{R_B^2 k^2}{2} \right),
\]

is the wave function of \( B(\bar{B}) \) with \( R_B \) as the strength parameter of the harmonic oscillator wave function. The charged states \( B^- (B^+) \) mesons at finite momenta are obtained by replacing the \( d^\dagger(d) \) in the states for \( B^0 (\bar{B}^0) \) by \( u^\dagger(u) \). In the above, \( \lambda_1 \) and \( \lambda_2 \) correspond to the fractions of the energy of the hadron carried by the constituent quark (antiquark). These are determined by assuming that the binding energy of the hadron shared by the quark (antiquark) is inversely proportional to the quark (antiquark) mass [98, 99, 108].
The matrix element of the quark-antiquark pair creation part of the Dirac Hamiltonian density, between the initial and the final states, \( M_{fi} \) is evaluated to calculate the partial decay width for the reaction \( \Upsilon(4S) \rightarrow \bar{B}^0(p) + B^0(-p) \). The expression obtained for the partial decay width of the bottomonium state, \( \Upsilon(4S) \) decaying at rest to \( B^0 \bar{B}^0 \) pair, after averaging \( |M_{fi}|^2 \) over spin, is given as \([98]\),

\[
\Gamma(\Upsilon(4S) \rightarrow B^0 \bar{B}^0) = \frac{\gamma_2}{2} \frac{1}{2\pi} \int \delta(m_{\Upsilon(4S)} - p_{B^0}^0 - p_{\bar{B}^0}^0)|M_{fi}|^2 \times 4\pi|p_{B^0}|^2 d|p_{\bar{B}^0}| \\
= \gamma_2 \frac{8\pi^2}{3} |p|^3 \frac{p_{B^0}^0 p_{\bar{B}^0}^0}{m_{\Upsilon(4S)}} A^{\Upsilon(4S)}(|p|)^2 \tag{22}
\]

In the above, \( p_{B^0}^0 = (m_{B^0}^2 + |p|^2)^{1/2} \), \( p_{\bar{B}^0}^0 = (m_{\bar{B}^0}^2 + |p|^2)^{1/2} \), and, \( |p| \) is the magnitude of the momentum of the outgoing \( B^0(B^0) \) mesons. The decay of \( \Upsilon(4S) \) to \( B^+ B^- \) proceeds through a \( u\bar{u} \) pair creation and the decay width \([22]\) is modified to

\[
\Gamma(\Upsilon(4S) \rightarrow B^+ B^-) = \gamma_2 \frac{8\pi^2}{3} |p|^3 \frac{p_{B^+}^0 p_{B^-}^0}{m_{\Upsilon(4S)}} A^{\Upsilon(4S)}(|p|)^2 \tag{23}
\]

In the above, \( p_{B^\pm}^0 = (m_{B^\pm}^2 + |p|^2)^{1/2} \), and, \( |p| \) is the magnitude of the momentum of the outgoing \( B^\pm \) mesons. The expression for \( A^{\Upsilon(4S)}(|p|) \) is given as

\[
A^{\Upsilon(4S)}(|p|) = 6c_{\Upsilon(4S)} \exp[(a_{\Upsilon(4S)} b_{\Upsilon(4S)}^2 - R_B^2 \lambda_2^2)|p|^2] \times \left( \frac{\pi}{a_{\Upsilon(4S)}} \right)^{3/2} \times \left[ \frac{F_0^{\Upsilon(4S)}}{2a_{\Upsilon(4S)}} + \frac{3}{2a_{\Upsilon(4S)}} \cdot F_1^{\Upsilon(4S)} + \frac{15}{4a_{\Upsilon(4S)}} \cdot F_2^{\Upsilon(4S)} + \frac{105}{8a_{\Upsilon(4S)}} \cdot F_3^{\Upsilon(4S)} + \frac{105 \times 9}{16a_{\Upsilon(4S)}} \cdot F_4^{\Upsilon(4S)} \right] \tag{24}
\]

In the above equation, \( F_i^{\Upsilon(4S)}(|p|) \)'s, \( i = 0, 1, 2, 3, 4 \), have been computed in Ref. \([99]\), which for sake of completeness, we quote in the following.

\[
F_0^{\Upsilon(4S)} = \frac{1}{2}(b_{\Upsilon(4S)} - 1)(b_{\Upsilon(4S)} - \lambda_2)(3b_{\Upsilon(4S)} + \lambda_2 - 4)g^2 |p|^2 \\
\times \left( 1 - 2R_{\Upsilon(4S)}^2 b_{\Upsilon(4S)}^2 |p|^2 + \frac{4}{5} R_{\Upsilon(4S)}^4 b_{\Upsilon(4S)}^4 |p|^4 - \frac{8}{105} R_{\Upsilon(4S)}^6 b_{\Upsilon(4S)}^6 |p|^6 \right) \tag{25}
\]
\[ F_1^{T(4S)} = \frac{g^2}{6} \left( 9(b_{T(4S)} - 1) - 2(3b_{T(4S)} - \lambda_2 - 2) \right) \\
+ \frac{g^2 |p|^2 R_{T(4S)}^2}{3} \left[ (-5b_{T(4S)} + 3)(3b_{T(4S)} + \lambda_2 - 4)(b_{T(4S)} - \lambda_2) \\
- 9b_{T(4S)}^2(b_{T(4S)} - 1) + 2b_{T(4S)}(3b_{T(4S)} - \lambda_2 - 2)(3b_{T(4S)} - 2) \right] \\
+ \frac{4g^2 |p|^4 R_{T(4S)}^4 b_{T(4S)}^2}{15} \left[ (7b_{T(4S)} - 5)(3b_{T(4S)} + \lambda_2 - 4)(b_{T(4S)} - \lambda_2) \\
+ \frac{9}{2} (b_{T(4S)} - 1)b_{T(4S)}^2 - b_{T(4S)}(5b_{T(4S)} - 4)(3b_{T(4S)} - \lambda_2 - 2) \right] \\
- \frac{8g^2 |p|^6 R_{T(4S)}^6 b_{T(4S)}^4}{105} \left[ \frac{1}{2} (9b_{T(4S)} - 7)(3b_{T(4S)} + \lambda_2 - 4)(b_{T(4S)} - \lambda_2) \\
+ \frac{3}{2} b_{T(4S)}^2(b_{T(4S)} - 1) - \frac{1}{3} b_{T(4S)}(3b_{T(4S)} - \lambda_2 - 2)(7b_{T(4S)} - 6) \right] \tag{26} \]

\[ F_2^{T(4S)} = \frac{1}{3} g^2 R_{T(4S)}^2 (-9b_{T(4S)} - 2\lambda_2 + 5) \\
+ \frac{4}{5} g^2 R_{T(4S)}^4 |p|^2 \left[ b_{T(4S)}^2(7b_{T(4S)} - 5) \\
+ \frac{1}{6} (3b_{T(4S)} + \lambda_2 - 4)(b_{T(4S)} - \lambda_2)(7b_{T(4S)} - 3) \\
- \frac{2}{15} b_{T(4S)}(3b_{T(4S)} - \lambda_2 - 2)(21b_{T(4S)} - 10) \right] \\
+ \frac{4}{5} g^2 R_{T(4S)}^6 |p|^4 b_{T(4S)}^2 \left[ - \frac{1}{7} b_{T(4S)}^2(9b_{T(4S)} - 7) \\
- \frac{4}{15} b_{T(4S)}(b_{T(4S)} - \lambda_2)(3b_{T(4S)} + \lambda_2 - 4) \\
- \frac{1}{3} (b_{T(4S)} - 1)(b_{T(4S)} - \lambda_2)(3b_{T(4S)} + \lambda_2 - 4) \\
+ \frac{2}{105} b_{T(4S)}(3b_{T(4S)} - \lambda_2 - 2)(45b_{T(4S)} - 28) \right], \tag{27} \]

\[ F_3^{T(4S)} = \frac{2g^2}{15} R_{T(4S)}^4 (15b_{T(4S)} + 2\lambda_2 - 5) \\
+ \frac{4}{5} g^2 R_{T(4S)}^6 |p|^2 \left[ - \frac{4}{5} b_{T(4S)}^3 - (b_{T(4S)} - 1)b_{T(4S)}^2 \\
- \frac{2}{21} b_{T(4S)}(b_{T(4S)} - \lambda_2)(3b_{T(4S)} + \lambda_2 - 4) \\
- \frac{1}{21} (b_{T(4S)} - 1)(b_{T(4S)} - \lambda_2)(3b_{T(4S)} + \lambda_2 - 4)) \\
+ \frac{2}{105} b_{T(4S)}(3b_{T(4S)} - \lambda_2 - 2)(27b_{T(4S)} - 10) \right], \tag{28} \]
\[ F_4^{\Upsilon(4S)} = -\frac{4g^2R_{\Upsilon(4S)}^6}{35 \times 9} (21b_{\Upsilon(4S)} + 2\lambda_2 - 5). \]  

In the above, the parameters \( a_{\Upsilon(4S)} \) and \( b_{\Upsilon(4S)} \) are given as

\[ a_{\Upsilon(4S)} = \frac{1}{2} R_{\Upsilon(4S)}^2 + R_B^2, \quad b_{\Upsilon(4S)} = R_B^2 \lambda_2 / a_{\Upsilon(4S)}, \]  

with \( R_{\Upsilon(4S)} \) as the radius of the bottomonium state, \( \Upsilon(4S) \), and,

\[ c_{\Upsilon(4S)} = \frac{1}{6\sqrt{6}} \left( \frac{\sqrt{35}}{4} \right) \left( \frac{R_{\Upsilon(4S)}^2}{\pi} \right)^{3/4} \left( \frac{R_B^2}{\pi} \right)^{3/2}. \]

The parameter \( \gamma_{\Upsilon} \), has been introduced in the expressions for the decay widths of \( \Upsilon(4S) \rightarrow B^0\bar{B}^0(B^+B^-) \), which is a measure of the production strength of the \( B\bar{B} \) pair from the \( \Upsilon \)-state through light quark antiquark pair (\( dd \) or \( uu \)) creation. In the present investigation of the bottomonium decay widths, the parameter, \( \gamma_{\Upsilon} \) is fitted from the vacuum decay width for the channel \( \Upsilon(4S) \rightarrow B\bar{B} \) (\( \Upsilon(4S) \) is the lowest \( \Upsilon \)-state which decays to \( B\bar{B} \) in vacuum). The decay widths of the bottomonium state depend on the magnitude of \( B(\bar{B}) \) meson momentum, \( |p| \) as a polynomial function multiplied by an exponential factor, as can be seen from the expressions given by equations (22) and (23). The medium modification of the decay width is due to the mass changes of the bottomonium state and the open bottom mesons, which are incorporated in \( |p| \) given by equation (9).

When we include the PV mixing effect (for the \( \Upsilon(4S) \) with the pseudoscalar meson, \( \eta_b(4S) \)), the mass of the longitudinal component of \( \Upsilon(4S) \) is modified, and, the expression for the decay width of \( \Upsilon(4S) \rightarrow B\bar{B} \) is given as

\[ \Gamma^{PV}(\Upsilon(4S) \rightarrow B(p)\bar{B}(-p)) = \gamma_{\Upsilon}^2 \frac{8\pi^2}{3} \left[ \left( \frac{2}{3} |p|^3 P_B^0(|p|) P_B^0(|p|) \frac{m_{\Upsilon(4S)}}{m_{\Upsilon(4S)}} A^{\Upsilon(4S)}(|p|)^2 \right) \right. \]

\[ + \left. \left( \frac{1}{3} |p|^3 P_B^0(|p|) P_B^0(|p|) \frac{m_{PV}}{m_{\Upsilon(4S)}} A^{\Upsilon(4S)}(|p|)^2 \right) \left( |p| \rightarrow |p|(m_{\Upsilon(4S)} = m_{\Upsilon(4S)}) \right) \right]. \]  

In the above, the first term corresponds to the transverse polarizations for the bottomonium state, \( \Upsilon(4S) \), whose masses remain unaffected by the mixing of the pseudoscalar and vector bottomonium states. The second term in (32) corresponds to the longitudinal component of the \( \Upsilon(4S) \) state whose mass is modified due to mixing with the pseudoscalar meson, \( \eta_b(4S) \), in the presence of the magnetic field.
FIG. 1: (Color online) The decay widths for the bottomonium state $\Upsilon(4S) \rightarrow B \bar{B}$ plotted as functions of density for $\eta = 0$ and $\eta = 0.5$ and for $eB = 4m^2$. The decay widths for the sub-processes (i) $\Upsilon(4S) \rightarrow B^+ B^-$ and (ii) $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$, as well as (iii) the total of these processes are shown. These decay widths are compared with the zero magnetic field case, shown as dotted lines. The mass modifications due to PV mixing effects have not been considered.

IV. RESULTS AND DISCUSSIONS

The in-medium decay widths of $\Upsilon(4S) \rightarrow B \bar{B}$, are calculated using a field theoretic model for composite hadrons in the present investigation. The model was used to study the partial decay widths of charmonium states to $D \bar{D}$ in (magnetized) hadronic matter [98, 101], as well as, of
FIG. 2: (Color online) The decay widths for the bottomonium state $\Upsilon(4S) \to B\bar{B}$ plotted as functions of density for $\eta = 0$ and $\eta = 0.5$ and for $eB = 10m_{\pi}^2$. The decay widths for the sub-processes (i) $\Upsilon(4S) \to B^+B^-$ and (ii) $\Upsilon(4S) \to B^0\bar{B}^0$, as well as (iii) the total of these processes are shown. These decay widths are compared with the zero magnetic field case, shown as dotted lines. The mass modifications due to PV mixing effects have not been considered.

bottomonium states to $B\bar{B}$ in hadronic matter, at zero magnetic field [99]. The present work accounts for the effects of an external magnetic field for the study of the in-medium decay widths of $\Upsilon(4S)$ to $B\bar{B}$, which is the lowest bottomonium state which can decay to $B\bar{B}$ in vacuum. The anomalous magnetic moments for the nucleons have been taken into account [110,117] in the present work.
FIG. 3: (Color online) The masses of the $B^+, B^-, B^0$ and $\bar{B}^0$ (and the longitudinal components of their vector meson counterparts $B^{*+}, B^{*-}, B^{*0}$ and $\bar{B}^{*0}$) modified due to the PV mixing effects (along with LLL contributions for the charged mesons) are plotted for $\rho_B = 0$ as functions of $eB/m_{\pi}^2$ in (a), (b), (c) and (d) respectively. The dotted lines illustrate the masses when the LLL (for the charged mesons) and PV effects are not taken into account.

In the present calculations, we take the parameters as follows. The values of the constituent masses (in MeV) of the $u(d)$ and bottom quarks are taken to be $m_{u(d)}=330$, $m_b=5360$. The harmonic oscillator strength of $\Upsilon(4S)$ is fitted from its observed decay width of 0.272 keV, to be $R_{\Upsilon(4S)}^{-1} = 638.6$ MeV [99]. For the $B(\bar{B})$ mesons, this strength is taken as $R_B^{-1}=875.6$ MeV, assuming $R_B = R_D(m_D/m_B)$, with the harmonic oscillator strength for $D$ meson obtained as fitted from the partial decay widths of $\psi(3770)$ and $\psi(4040)$ to the open charm mesons [69, 98]. The value of the strength parameter, $\gamma_\Upsilon$ is obtained to be 5.6, as fitted from the observed decay widths of $\Upsilon(4S) \to B^+B^-$ and $\Upsilon(4S) \to B^0\bar{B}^0$ as 10.516 MeV and 9.984 MeV respectively [99].
FIG. 4: (Color online) The masses of the $B^+, B^-, B^0$ and $\bar{B}^0$ (and the longitudinal components of their vector meson counterparts $B^{*+}, B^{*-}, B^{*0}$ and $\bar{B}^{*0}$) modified due to the PV mixing effects (along with LLL contributions for the charged mesons) are plotted for $\rho_B = \rho_0$ in magnetized symmetric ($\eta=0$) nuclear matter as functions of $eB/m_+^2$ in (a), (b), (c) and (d) respectively. The dotted lines illustrate the masses as calculated within the chiral effective model, which show marginal dependence on the magnetic field.

The mass modification of $\Upsilon(4S)$ due to its mixing with $\eta_b(4S)$ in the presence of a magnetic field is considered in the present study. The harmonic oscillator strength of $\eta_b(4S)$, is calculated to be $R_{\eta_b(4S)}^{-1} = 642.627$ MeV from a linear interpolation along with $\Upsilon(3S)$ and $\Upsilon(4S)$ in their mass vs $(R)^{-1}$ plot, where the values of $R^{-1}$ for $\Upsilon(3S)$ and $\Upsilon(4S)$ as fitted from their leptonic decay widths are 779.75 MeV and 638.60 MeV respectively [99].

The decay widths $\Upsilon(4S) \rightarrow B\bar{B}$, in magnetized nuclear matter, along with the decay widths for the sub-processes (i) $\Upsilon(4S) \rightarrow B^+B^-$ and (ii) $\Upsilon(4S) \rightarrow B^0\bar{B}^0$, are plotted in figures [1] and
FIG. 5: (Color online) The masses of the $B^+, B^-, B^0$ and $\bar{B}^0$ (and the longitudinal components of their vector meson counterparts $B^{*+}, B^{*-}, B^{*0}$ and $\bar{B}^{*0}$) modified due to the PV mixing effects (along with LLL contributions for the charged mesons) are plotted for $\rho_B = \rho_0$ in magnetized asymmetric ($\eta=0.5$) nuclear matter as functions of $eB/m_{\pi}^2$ in (a), (b), (c) and (d) respectively. The dotted lines illustrate the masses as calculated within the chiral effective model, which show marginal dependence on the magnetic field.

For magnetic fields, $eB = 4m_{\pi}^2$ and $eB = 10m_{\pi}^2$ respectively. The decay width to the charged $B\bar{B}$ mesons accounts for the Landau level contributions to the masses of $B^\pm$ mesons. However, the decay widths plotted in figures 1 and 2 do not consider the effects due to PV mixing on the masses of the decaying particle, $\Upsilon(4S)$, as well as the $B$ and $\bar{B}$ mesons. The density dependences of these widths are shown in these figures for the isospin symmetric nuclear matter ($\eta=0$) as well as for the case of asymmetric nuclear matter (with $\eta=0.5$). These are compared with the case of zero magnetic field, shown as dotted lines.
In the presence of the magnetic field, the proton, being electrically charged, has contributions from the Landau energy levels. The effects of the magnetic field are also taken into account in the present work by considering the anomalous magnetic moments of the nucleons. In isospin symmetric nuclear matter ($\rho_p = \rho_n$), the presence of magnetic field thus introduces differences in the mass modifications between the $B^0$ and $B^+$ within the $B$ doublet as well as between $B^-$ and $\bar{B}^0$ within the $\bar{B}$ doublet. The difference in the masses have additional positive mass shifts for the charged $B^{\pm}$ in the magnetized matter, arising from the lowest Landau level (LLL). The positive
FIG. 7: (Color online) The decay widths for Υ(4S) to (i) $B^+B^-$, (ii) $B^0\bar{B}^0$, and the total of these two processes ((i)+(ii)) are plotted as functions of $eB/m_{\pi}^2$ for $\rho_B=0$, as well as, for $\rho_B = \rho_0$ for symmetric ($\eta=0$) and asymmetric ($\eta=0.5$) nuclear matter in the presence of a magnetic field. In (a), (c) and (e), these decay widths are shown for the case when PV mixing effects are not taken into account, but the lowest Landau level (LLL) contributions are taken into account for the charged $B^\pm$ mesons. Subplots (b), (d) and (f) show the decay widths including the PV mixing effects, where the dotted lines correspond to the case when the mass modification of Υ(4S) from PV mixing is not considered.

mass shifts of the $B^\pm$ due to LLL are observed to lead to suppression of the decay of Υ(4S) to $B^+B^-$, as compared to the decay to the neutral $B^0\bar{B}^0$, as can be seen from the figures\ref{fig:1} as well as\ref{fig:2}. For $eB = 4m_{\pi}^2$, as can be seen from figure\ref{fig:1}, at zero density, the decay to $B^+B^-$ is negligible ($\sim 1.7$ MeV) and the total decay width for Υ(4S) $\rightarrow B\bar{B}$ ($\sim 12.4$ MeV) is due to the decay mode to the neutral $B\bar{B}$ pair ($\sim 10.6$ MeV). On the other hand, in vacuum, the partial decay widths for Υ(4S) $\rightarrow B^+B^-$ and $B^0\bar{B}^0$, are 10.5 MeV and 10 MeV respectively, with total decay width
of 20.5 MeV. In isospin symmetric nuclear matter in presence of magnetic field of $eB = 4m^2_\pi$, the in-medium decay widths for the channels $\Upsilon(4S) \rightarrow B^+B^-$, as well as $\Upsilon(4S) \rightarrow B^0\bar{B}^0$, are observed to have an initial increase with density, followed by a drop leading to vanishing of the decay widths at around $2.1 \rho_0$. Above this density, there is again observed to be an increase in the decay widths followed by decrease with subsequent vanishing of these decay widths at around a density of $3.8\rho_0$. These behaviours were observed for the case of zero magnetic field, and the values of the densities where the decay widths vanish, were observed to be the same. In the asymmetric nuclear medium (with $\eta=0.5$), the partial decay width for the decay mode $\Upsilon(4S) \rightarrow B^+B^-$ is observed to vanish at a density of around $3\rho_0$, whereas for the neutral $B\bar{B}$ pair, the decay width is zero at densities of $2 \rho_0$ as well as $3.5\rho_0$.

The effects of the magnetic field on the decay widths of $\Upsilon(4S) \rightarrow B\bar{B}$ are observed to be much larger, as might be seen from figure 2 which is plotted for $eB = 10m^2_\pi$. The value of the density above which the decay of $\Upsilon(4S) \rightarrow B^+B^-$ becomes possible, is observed to be around $0.4$ ($0.5$) $\rho_0$ for symmetric (asymmetric) nuclear matter for $eB = 10m^2_\pi$. The effects of the isospin asymmetry on the decay widths of $\Upsilon(4S) \rightarrow B\bar{B}$ are observed to be much larger at higher densities for the higher magnetic field of $eB = 10m^2_\pi$, as can be seen from figure 2. The decay of $\Upsilon(4S) \rightarrow B^+B^-$ is suppressed as compared to the decay to $B^0\bar{B}^0$ at low densities, which is due to the increase in the masses of the charged $B^\pm$ mesons arising from the Landau level contributions.

The masses of the open bottom pseudoscalar mesons ($B^+, B^-, B^0$ and $\bar{B}^0$) mesons in the magnetized nuclear matter are calculated using the chiral effective model. The masses of these mesons within the model are obtained from their interactions with the scalar mesons and the nucleons. The number and scalar densities of the proton, which is electrically charged, have contributions from the Landau energy levels in the presence of a magnetic field [9, 10]. The calculations for the masses of the $B$ and $\bar{B}$ in the chiral effective model are performed using the mean field approximation, i.e., neglecting the effects of the Dirac sea. The magnetic field dependence of the masses of the open bottom mesons ($B$, $\bar{B}$, $B^*$ and $\bar{B}^*$) for $\rho_B = \rho_0$ for the isospin symmetric ($\eta = 0$) and asymmetric ($\eta=0.5$) nuclear matter, are shown in figures 4 and 5. The masses of the open bottom vector mesons ($B^*$ and $\bar{B}^*$ mesons), which have the same quark-antiquark constituents as $B$ and $\bar{B}$ mesons, are assumed to have identical mass shifts as the shifts in the masses of the $B$ and $\bar{B}$ mesons calculated within the chiral effective model. This is in line with the mass modifications of hadrons within the QMC model, which arise due to the modification of the scalar density of the light quark (antiquark) constituent of the hadron [11]. To
obtain the masses of the $B^*$ and $\bar{B}^*$ mesons in the magnetized nuclear matter, we assume,

$$m_{B^*(\bar{B}^*)}^* - m_{B^*(\bar{B}^*)}^{(vac)} = m_{B(\bar{B})}^* - m_{B(\bar{B})}^{(vac)}$$  \hspace{1cm} (33)$$

In figures 4 and 5 for $\rho_B = \rho_0$ and $\eta = 0$ and 0.5, the masses of the $B$ and $\bar{B}$ mesons, as calculated using the chiral effective model are shown as dotted lines. These are observed to have marginal dependence on the magnetic field. This is due to the fact that the scalar fields ($\sigma, \delta, \zeta$ and $\chi$) have negligible dependence on the magnetic field for a given value of the baryon density and isospin asymmetry parameter. In the mean field approximation, we thus have the quark condensates, which are related to the scalar fields as

$$m_u\langle\bar{u}u\rangle = \frac{1}{2}m_\pi^2 f_\pi (\sigma + \delta), \quad m_d\langle\bar{d}d\rangle = \frac{1}{2}m_\pi^2 f_\pi (\sigma - \delta),$$  \hspace{1cm} (34)$$

to have marginal dependence on the magnetic field. In the presence of a magnetic field, the magnetic catalysis, which is the phenomenon of enhancement of the quark condensates with increase in the magnetic field, arises from the magnetic field dependent Dirac sea contribution [118]. In Ref. [118], within the Walecka model, in isospin symmetric nuclear matter in the presence of a magnetic field, the mass of the nucleon ($M_N^* = M_N - g_N\sigma\sigma$) is also observed to be substantially modified as a result of the magnetic catalysis, due to the modification of the scalar field, $\sigma$ (which is proportional to the quark condensates). This effect is not observed in the present calculations which uses the mean field approximation, and, hence, neglects the Dirac sea contributions. The density effects on the quark condensates ($\sim (\sigma \pm \delta)$) and gluon condensates ($\sim \chi^4$) are the dominant medium effects, as compared to the effects from the isospin asymmetry and the magnetic field of the medium [9111].

In the presence of the magnetic fields, there is mixing between the pseudoscalar meson and longitudinal component of vector meson. The pseudoscalar meson-vector meson (PV) mixing gives rise to modifications of their masses, with a drop (increase) in the mass of the pseudoscalar (longitudinal component of the vector) meson. The studies of the heavy quarkonium state in the presence of an external magnetic field within a QCD sum rule approach [1315] as well as solution of the Schrödinger equation for the heavy quarkonium bound state using an an effective potential [16] show that the mass modifications of these mesons due to PV mixing are important. In Refs. [1314], a phenomenological Lagrangian interaction $L_{PV\gamma} = (g_{PV}/m_{av})e\tilde{F}_{\mu\nu}(\partial^{\mu}P)^{V^\nu}$ (with $m_{av}$ as the average of the masses of the pseudoscalar and vector mesons), was used for the charmonium states to study the PV ($J/\psi - \eta_c, \psi' - \eta_c'$) mixing effects in the presence of an external magnetic field. In the field theoretical model of composite hadrons with quark (and antiquark) constituents
as used in the present work, the masses of the charmonium states have been studied accounting for the PV \((J/\psi - \eta_c, \psi' - \eta_c', \text{and } \psi(3770) - \eta_c')\) mixing effects in magnetized (nuclear) matter \(^{101}\) using the above phenomenological Lagrangian interaction \(^{13-15}\). The decay widths of \(\psi(3770) \rightarrow D\bar{D}\) have been studied in Refs. \(^{101, 102}\), which are observed to be modified significantly due to the contributions of the \(\psi(3770) - \eta_c'\) mixing to the mass of the charmonium state \(\psi(3770)\), as well as due to the PV \((D - D^* \text{ and } \bar{D} - \bar{D}^*)\) mixing contributions to the masses of the open charm mesons \(^{102}\). The parameter \(g_{PV}\) of the phenomenological Lagrangian density was calculated from the observed decay width of \(V \rightarrow P\gamma\) in vacuum. In the present study, we shall consider the PV mixing effects, in addition to the lowest Landau level contribution to the masses of the charged open bottom mesons, for the study of the in-medium partial decay width of \(\Upsilon(4S) \rightarrow B\bar{B}\). Due to lack of data on radiative processes \((V \rightarrow P\gamma)\) in the bottom sector, we shall use the interaction Hamiltonian given by equation (7) to study the PV mixing effects on the masses of the open bottom mesons and \(\Upsilon(4S)\) state, and hence on the decay width for \(\Upsilon(4S) \rightarrow B\bar{B}\).

Strong magnetic fields are produced in non-central ultra-relativistic heavy ion collision experiments and the produced matter is extremely dilute. In the following, we shall consider the effects of magnetic field on the masses of the open bottom mesons and the \(\Upsilon(4S)\) state for \(\rho_B = 0\) and \(\rho_B = \rho_0\), and study their effects on the decay width of \(\Upsilon(4S) \rightarrow B\bar{B}\). We shall take into account the contributions due to the PV mixing effects (in addition to the LLL contributions for the charged open bottom mesons) to the masses calculated in the chiral effective model in the magnetized nuclear matter.

The masses of the open bottom (pseudoscalar and vector) mesons are plotted for \(\rho_B=0\) in figure 3. The effects of the lowest Landau level (LLL) for the \(B^\pm\) as well as \(B^{*\pm(\parallel)}\) mesons are observed to lead to an increase in their masses. However, the effects due to the PV \((B - B^* \text{ and } \bar{B} - \bar{B}^*)\) mixings on the masses of these mesons are observed to dominate over LLL contributions. These, along with modifications of the mass of \(\Upsilon(4S)\) (longitudinal component) due to \(\Upsilon(4S) \parallel - \eta_b(4S)\) mixing, have significant effects on the decay widths of \(\Upsilon(4S) \rightarrow B\bar{B}\) as can be seen from figure 7. For \(\rho_B=0\), the decay widths are plotted in (a) and (b). In figure 7(a), the effects of PV mixing are not taken into account, whereas, these effects are considered in 7(b). The vacuum mass of \(\Upsilon(4S)\) is 10579.4 MeV. The pseudoscalar meson \(\eta_b(4S)\) is not yet observed experimentally. We take the mass of \(\eta_b(4S)\) to be 10573 MeV, as calculated from a relativistic potential model \(^{47}\). When the PV mixing effects are not taken into consideration, the decay width of \(\Upsilon(4S) \rightarrow B^+B^-\) is observed to decrease with increase in magnetic field and becomes zero at around \(eB = 4.5m_\pi^2\) for \(\rho_B = 0\), as can be seen from figure 7(a). This is due to the fact that the masses of the charged \(B^\pm\) increase
due to the LLL contributions. On the other hand, the decay width for the neutral $B\bar{B}$ for $\rho_B = 0$, remains same as its vacuum value, when the PV mixing effects are not considered, as can be seen from figure 7(a). When the PV effects are taken into account, the decay width of $\Upsilon(4S)$ to $B^+B^-$ is observed initially to increase with increase in the magnetic field followed by a drop reaching a value of around 21.8 MeV at around $eB/m_B^2 \sim 6$. As the magnetic field is further increased, there is observed to be a steep rise with the decay width reaching a value of around 177.6 (254) MeV at $eB/m_B^2 = 8.5(9)$. A similar behaviour for the decay width of $\Upsilon(4S) \rightarrow B^+B^-$ is observed for the case when the $\Upsilon(4S) - \eta_b(4S)$ mixing contributions are not taken into account (shown as dotted lines), however, still accounting for the mass modifications of the open bottom mesons due to PV mixing. For the decay width $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ there is observed to be an increase with the magnetic field, with the rise being steeper for $eB$ larger than $8m_B^2$. The main difference between the two channels is due to the absence (presence) of the Landau level contributions for the neutral (charged) $B\bar{B}$ mesons. The behaviour of the decay width is determined by the value of the center of mass momentum, $|p|$, which depends on the masses of the $\Upsilon(4S)$, $B$ and $\bar{B}$ mesons, through equation (9). The dependence of the decay widths of $\Upsilon(4S) \rightarrow B\bar{B}(B^0\bar{B}^0, B^+B^-)$ are through a polynomial term multiplied by an exponential term, as can be seen from the expression of the decay widths.

The in-medium masses of the $B$, $\bar{B}$, $B^*$ and $\bar{B}^*$ mesons at $\rho_B = \rho_0$ in symmetric and asymmetric nuclear matter in presence of magnetic field are plotted as functions of $eB/m_B^2$ in figures 4 and 5 respectively. The mass of the $B^{*\pm}$ due to the lowest Landau level contribution (n=0) is given as

$$m^{eff}_{B^{*\pm}} = \sqrt{m_{B^{*\pm}}^2 + (-gS_z + 1)|eB|},$$  \hspace{1cm} (35)

whereas the masses of the neutral $B^{*0}$ and $\bar{B}^{*0}$ are given as

$$m^{eff}_{B^{*0},\bar{B}^{*0}} = m_{B^{*0},\bar{B}^{*0}}.$$  \hspace{1cm} (36)

Equation (35) refers to the mass of a charged vector particle due to lowest Landau level, ignoring its internal structure. As can be seen from equation (35), the mass depends on the z-component of the spin, $S_z$. For $S_z = 1$, the mass squared of the vector particle decreases by $|eB|$, if we take the gyromagnetic ratio to be $g = 2$. In the presence of a magnetic field, however, there is (PV) mixing between the longitudinal component ($S_z = 0$) of the $B^*(\bar{B}^*)$ meson and the pseudoscalar meson $B(\bar{B})$, which gives rise to mass modification of the $B(\bar{B})$ meson. The mass of the longitudinal component ($S_z=0$) of the charged $B^*(\bar{B}^*)$ meson due to the lowest Landau level (LLL) contribution
is given by

\[ m_{B^\pm (\perp)}^{\text{eff}} = \sqrt{m_{B\pm}^2 + |eB|}. \] (37)

As has already been mentioned, the masses of the \( B^* \) and \( \bar{B}^* \) mesons in the nuclear medium, occurring in equations (36) and (37), are obtained from the in-medium masses of the \( B(\bar{B}) \) mesons, \( m_{B(\bar{B})}^* \), using equation (33). The LLL contributions lead to an increase in the masses of the charged \( B^\pm \) as well as of the longitudinal component of the charged \( (B^{\ast \pm}) \) mesons. As can be seen from figures 3 and 4, the \( B - B^* \) and \( \bar{B} - \bar{B}^* \) mixings lead to significant modifications to the masses of these mesons at \( \rho_B = 0 \), and at \( \rho_B = \rho_0 \) in the symmetric as well as asymmetric nuclear matter.

In figure 6, the effects of the PV mixing on the masses of the longitudinal component of \( \Upsilon(4S) \) and the pseudoscalar meson \( \eta_b(4S) \) are plotted as functions of \( eB/(m_{\pi}^2) \). These are shown for the symmetric and asymmetric magnetized nuclear matter for \( \rho_B = \rho_0 \) and compared with the masses for \( \rho_B = 0 \). The modifications of the mass of \( \Upsilon(4S) \) due to PV mixing is observed to affect significantly the decay width of \( \Upsilon(4S) \) to \( B^+B^- \) and \( B^0\bar{B}^0 \), as can be observed from figure 7.

The decay width of \( \Upsilon(4S) \) to the charged open bottom meson pair \( (B^+B^-) \), as can be seen from figure 7, is observed to have an initial increase with magnetic field, followed by a drop leading to vanishing of the decay width and again a rise as the magnetic field is further increased. The vanishing of the decay width (so called nodes) is similar to the behaviour of the decay width with density, for isospin symmetric matter (see figures 1 and 2). The in-medium behaviour of the decay widths of \( \Upsilon(4S) \rightarrow B^+B^- \) and \( \Upsilon(4S) \rightarrow B^0\bar{B}^0 \), are determined through the center of mass momentum, \( |p| \). These decay widths, given by equations (23) and (22), have a polynomial term multiplied by an exponential term, dependence on \( |p| \) (given by equation (9)) which is expressed in terms of the in-medium masses of the \( \Upsilon(4S) \), \( B \) and \( \bar{B} \) mesons. The suppression of the decay channel to \( B^+B^- \) as compared to the \( B^0\bar{B}^0 \) should show as the charged open bottom mesons to be suppressed as compared to the neutral \( B\bar{B} \) mesons.

The modification of the mass of \( \Upsilon(4S) \), due to mixing with \( \eta_b(4S) \) is observed to be quite appreciable. The mass of \( \eta_b(4S) \) is calculated using equation (5), from the change in the dilaton field within the chiral effective model. The values of the masses (in MeV) of \( \Upsilon(4S) \) and \( \eta_b(4S) \) are obtained to be 10535.69 and 10531.22 at \( \rho_B = \rho_0 \) in isospin symmetric \((\eta=0)\) and 10542.66 and 10537.88 in asymmetric \((\eta=0.5)\) nuclear matter for \( B = 0 \). The modifications of these masses are negligible as the magnetic field is increased, when the PV mixing is not taken into account. Considering the PV \((\Upsilon(4S) - \eta_b(4S))\) mixing, at \( \rho_B = \rho_0 \), for \( eB = 10m_{\pi}^2 \), the mass of the \( \Upsilon(4S) \) (\( \eta_b(4S) \)) is obtained as 10545.9 (10521.46) MeV for isospin symmetric nuclear matter (\( \eta=0 \)), and
10552.5 (10528) MeV in asymmetric nuclear matter (with \( \eta = 0.5 \)). The appreciable contribution
due to the mass modification of \( \Upsilon(4S) \) from PV mixing, can be seen from the dotted lines in (b), (d)
and (f) of figure 7, which correspond to the cases when PV mixing is not taken into account for the
mass of \( \Upsilon(4S) \), which are observed to be significantly modified when the PV mixing contributions
for \( \Upsilon(4S) \) are taken into account. The PV mixing effects of the open bottom mesons as well as
of the \( \Upsilon(4S) \) state on their masses are thus the most important effects due to the presence of the
magnetic field for the study of the in-medium decay width of \( \Upsilon(4S) \rightarrow B\bar{B} \) in the magnetized
(nuclear) matter.

V. SUMMARY

In the present work, we have used a composite model for hadrons with quark (and antiquark)
constituents, to calculate the in-medium decay widths of \( \Upsilon \rightarrow B\bar{B} \) in nuclear matter in the presence
of strong magnetic fields. The effects of the isospin asymmetry are observed to be large at high
densities, and more prominent for higher values of the magnetic fields. In the presence of strong
magnetic fields, the decay widths of \( \Upsilon(4S) \) to the charged \( B\bar{B} \) are observed to be suppressed
as compared to the decay to neutral \( B^0\bar{B}^0 \) pair, when the PV mixing effects are not taken into
consideration. This is due to the positive mass shifts of the charged \( B^+ \) and \( B^- \) mesons arising
from the lowest Landau level contributions. The PV (\( B - B^* \) and \( \bar{B} - \bar{B}^* \)) mixing effects are
observed to lead to appreciable modifications to the masses of the \( B \) and \( \bar{B} \) mesons, which, in
turn, are observed to have dominant modifications to the partial decay widths of \( \Upsilon(4S) \rightarrow B\bar{B} \).
The mass shift of \( \Upsilon(4S) \) due to mixing with \( \eta_b(4S) \) is observed to be quite appreciable and has
significant effect on the decay width. The significant modifications of the decay widths of \( \Upsilon(4S) \)
to the charged and neutral \( B\bar{B} \) should show in the production of these mesons and \( \Upsilon(4S) \) in
peripheral ultra-relativistic heavy ion collision experiments, e.g. at RHIC and LHC, where the
produced magnetic field is huge.
Acknowledgements

Amruta Mishra acknowledges financial support from Department of Science and Technology (DST), Government of India (project no. CRG/2018/002226).

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