Multidimensional geometrical model of the electrical
and $SU(2)$ colour charge with splitting off
the supplementary coordinates

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The geometrical model of an electrical charge is proposed. This model has
the "nake" charge shunted with "fur - coat" consisting of virtual wormholes.
The 5D wormhole solution in the Kaluza - Klein’s theory is the "nake" charge.
The splitting off the supplementary coordinates happens on the two spheres
(null surfaces) bounding this 5D wormhole. This allows to sew two Reissner -
Nordström’s black holes to it on both sides. Virtual wormholes entrap a part
of the electrical force lines outcoming from "nake" charge. This effect can es-
sentially decrease the charge visible at infinity up to real relation $m^2 < e^2$. The
analogical construction for colour $SU(2)$ gauge charge is made.

I. INTRODUCTION

The existence of an intrinsic electrons structure may be a radical remedy for singularity in
quantum field theory. In due time J.Wheeler [1] - [3] had proposed a model "charge without
charge". Its idea lies in the fact that the wormhole (WH), entrapping the electrical force lines,
is an electrical charge for external observer. The charge sign depends upon how WH part is
observed: with entering or outgoing force lines. In first case this is a negative charge and in
the second case it is a positive charge.

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This J.Wheeler model is 4 dimensional. On the other hand, the 5 dimensional Kaluza-Klein’s theory has the wonderful properties: In this theory the "electrical" field is a pure geometrical object. These two viewpoints on the electrical field has been joined in Ref. \[4\]. The 5D solution in the Kaluza-Klein’s theory is founded in this work, that is Lorentzian WH, and it is bounded by two null surfaces. This WH can be sewed with 2 stationary Reissner-Nordström’s solutions. Thus, the obtaining object is a composite WH with following properties:

1. The intrinsic part is the 5D Lorentzian WH.

2. Two exterior parts are the 4D stationary asymptotical flat regions as a stationary Reissner-Nordström’s solution \((r > r_+, r_+ = m + \sqrt{m^2 - e^2})\) is event horizon, \(m\) and \(e\) are mass and charge of the black hole, respectively). One region contains the entering force lines into 5D WH and the another region contains the outcoming electrical force lines from 5D WH.

3. The sewing of the intrinsic region with each exterior region happens on the event horizons \(r_+\), that guarantees the multidimensional world nonobservability for exterior observer.

4. The 5D supplementary coordinate splits off on the surface sewing. This results in that the \(G_5t\) component of the intrinsic metrical tensor generates the 4D electrical field in the exterior regions.

**II. THE COMPOSITE WORMHOLE AS AN ELECTRICAL CHARGE**

At first we remind the results achieved in Ref. \[4\]. 5D metric has the following WH-like view:

\[
ds^2 = e^{2\nu(r)} dt^2 - e^{2\psi(r)} (d\chi - \omega(r)dt)^2 - dr^2 - a^2(r) \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right),
\]  

(1)
where $\chi$ is 5 supplementary coordinate; $r, \theta, \varphi$ are 3D polar coordinates; $t$ is time. Corresponding 5D Einstein’s equations have the following solution:

$$a^2 = r_0^2 + r^2,$$

$$e^{-2\psi} = e^{2\nu} = \frac{2r_0 r_0^2 + r^2}{q \ r_0^2 - r^2},$$

$$\omega = \frac{4r_0^2}{q \ r_0^2 - r^2},$$

where $r_0$ is a throat of given wormhole; $q$ is a 5D “electrical” charge. It is easy to see that the time component of metrical tensor $G_{tt}(r = \pm r_0) = 0$. This indicates that there is the null surface, as on its $ds^2 = 0$. The sewing of the 5D and 4D physical quantities happens in the following manner:

$$e^{2\nu_0} - \omega_0^2 e^{-2\nu_0} = G_{tt}(\pm r_0) = g_{tt}(r_+) = 0,$$

$$r_0^2 = G_{\theta\theta}(\pm r_0) = g_{\theta\theta}(r_+) = r_+^2,$$

where $G$ and $g$ are 5D and 4D metrical tensors, respectively. $r_+$ is event horizon for Reissner - Nordström’s solution. The quantity marked by (0) sign are taken by $r = \pm r_0$. The sewing $G_{\chi t}$ and 4D electrical field happens in the following manner:

$$\frac{q}{2r_0^2} = \frac{e}{r_+^2}.$$  

The Reissner - Nordström’s condition $m^2 > e^2$ is the basic cause obstructing to interpret such composite WH as an electron model.

The quantum gravity confirms that in microscopic scale the metric fluctuations are so large that the topological fluctuations - wormholes (handles) appear in spacetime. Such fluctuations have to spring up in regions with very strong gravitational field. It just happens near the event horizon surface $r = r_+$ of the sufficiently small black hole.

In a suggested model it is assumed that the virtual wormholes (VWH) arise between two 4D regions with incoming and outcoming force lines and the strong gravitational field near 2
surfaces $r = r_+$. The appearance of this VWH leads that they entrap the part of 4D electrical force lines. If the total cross size of all VWH is of the same order as the cross size of the 5D WH, then the VWH entrap almost all electrical force lines outcoming from 5D WH. In this case the exterior observer can detect electrical charge with the real relation between mass and charge $m^2 < e^2$.

Thus, in the suggested model the real electrical charge consists of ”nake” electrical charge (5D WH) and VWH ”fur - coat” dressed on it. Such ”fur - coat” essentially decreases the electrical charge visible at infinity, i.e. in fact makes its renormalization. This model cannot yet describe the charge quantization and $\hbar/2$ spin of electron. In the first case it is necessary to have the quantum field theory of gravity which we do not yet have. The second case is considered below.

### III. SU(2) COMPOSITE WORMHOLE

The above suggested 5D model of the electrical charge can be generalized to the $SU(2)$ colour charge case. We consider gravity on the total space of a bundle with base being 4D Einstein’s spacetime and fibre being $SU(2)$ gauge group. In short, we cite results achieved in Ref. [5]. In this case the gravity equation looks as a follows:

$$ R_{A\mu} - \frac{1}{2} G_{A\mu} R = 0, \quad (8) $$

$$ R^a_a = R^4_4 + R^5_5 + R^6_6 = 0, \quad (9) $$

where $R_{A\mu}$ is Ricci tensor and $R$ is Ricci scalar; $A = 0, 1, 2, \ldots, 6$ is multidimensional index on the total bundle space; $\mu = 0, 1, 2, 3$ is spacetime index on the base of bundle; $a = 4, 5, 6$ is the coordinate index on the fibre of bundle. The fibre is a symmetrical space hence the metric on the fibre has only one degree of freedom (conformal factor of the fibre metric), from here leads the Eq.(9). In our case the multidimensional metric has the following form:
\[ ds^2 = e^{2\nu(r)} dt^2 - r_0^2 e^{2\psi(r)} \sum_{a=4}^{6} \left( \sigma^a - A^a_{\mu}(r) dx^\mu \right)^2 - dr^2 - a^2(r) \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right). \]  

(10)

The "potentials" \( A^a_{\mu} \) have the following monopole-like form:

\[ A^a_i = \frac{\epsilon^{a-3} x^j}{r^2} [f(r) + 1], \]

(11)

\[ A^a_t = \frac{x^a}{r^2} v(r). \]

(12)

here \( i, j = 1, 2, 3 \) are space indexes. We examine the most interesting case when \( f(r) = 0 \) and the linear dimensions of fibre \( r_0 \) are vastly smaller than the space dimension \( a_0 \) and "charge" \( q \) is sufficiently large:

\[ \left( \frac{q}{a_0} \right)^{1/2} \gg \left( \frac{a_0}{r_0} \right)^2 \gg 1, \]  

(13)

where \( a_0 = a(r = 0) \) is the throat of the WH, \( q \) defines the \( A^a_t \) potential:

\[ v' = \frac{q}{r_0 a_0^2} e^{\nu - 5\psi}, \]  

(14)

In this case we can achieve the approximate solution of the (8)-(9) system in the following form - [3]:

\[ \nu = -3\psi, \]

(15)

\[ a^2 = a_0^2 + r^2, \]

(16)

\[ e^{-4\nu} = \frac{q}{2a_0} \cos \left( \sqrt{\frac{8}{3}} \arctan \frac{r}{a_0} \right), \]

(17)

\[ v = \sqrt{6} \frac{a_0}{r_0 q} \tan \left( \sqrt{\frac{8}{3}} \arctan \frac{r}{a_0} \right). \]

(18)

It is easy to shown that \( ds^2 = 0 \) by \( r = \pm a_0 \), hence \( r = \pm a_0 \) surfaces are null surfaces.

Analogously to above - mentioned 5D case we can construct the composite WH consisting of intrinsic 7D WH ("nake" \( SU(2) \) colour charge) and two 4D Einstein - Yang - Mills black hole filled by chromoelectrical \( A^a_t \) field and pasting to \( r = \pm r_0 \) null surfaces. Also such \( SU(2) \)
composite WH can be dressed a "fur-coat" consisting from VWH, that naturally leads to decreasing the $SU(2)$ field far from the centre. From the viewpoint of a far observer such composite $SU(2)$ WH is the $SU(2)$ colour charge with nonzero rest-mass and very weak $SU(2)$ field.

We remind the following result [6]-[7]. Let G group be the fibre of principal bundle. Then there is the one-to-one correspondence between $G$-invariant metrics on the total space $X$ and the triples $(g_{\mu\nu}, A_\mu^a, h_{\gamma ab})$. Where $g_{\mu\nu}$ is Einstein’s pseudo-Riemannian metric; $A_\mu^a$ is gauge field of the $G$ group; $h_{\gamma ab}$ is symmetric metric on the fibre. These remarks allow to continue the multidimensional "potentials" $A_\mu^a$ into 4D regions as a true gauge potential of Yang-Mills field.

**IV. DISCUSSION**

There are 3 basic problems relating to above constructed WH with electrical (colour) charge:

1. What parts of the force lines outcoming from 5D(7D) WH are entrapped by VWH "fur-coat"?

2. What spin has such composite wormhole?

3. What is the mechanism of supplementary coordinates splitting off?

The first question can be solved only in the framework of future quantum gravity. It is possible that in the second case $h/2$ spin may be obtained using the mechanism similar to "spin isospin" phenomenon. The third question was discussed in Ref. [8]. In this article the viewpoint is proposed, according to which some quantum transition in quantum gravity (splitting off the supplementary coordinates, changing of metric signature, Universe birth) is connected with changing the algorithmic complexity of corresponding gravity objects. This viewpoint is based
on A.N. Kolmogorov’s theory, according to which the probability of appearance of some objects can be connected with algorithm length describing this object.

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