Simulation of Snaking and Buckling in Hot Sheet Rolling

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In hot sheet rolling, the sheet rear end often snakes, contacts the inlet side guide, buckles, and goes into the roll gap, whereas the overlapped rear end of the sheet is squeezed. Although a number of researches on the simulation of the sheet snaking are reported, no researches have been performed to simulate both the sheet snaking and the sheet buckling. In this study, a combined method to simulate the sheet snaking by the rigid-plastic FEM and to analyze the sheet buckling by the elementary theory of buckling was proposed. First, the method in which the in-plane lateral load and the in-plane bending moment were assumed at the surface of the simulation region by the rigid-plastic FEM was proposed. Next, the amount of snaking at the sheet rear end simulated by the rigid-plastic FEM agreed with that analyzed by the elementary theory of rolling. Finally, the effects of rolling conditions on the occurrence of squeezing, such as the difference in the sheet thickness in the direction of the roll axis, the difference in the roll gap in the direction of the roll axis, and the amount of the sheet off-center, were clarified.

KEY WORDS: sheet snaking; sheet buckling; simulation; hot sheet rolling.

1. Introduction

In hot sheet rolling, the phenomenon called the sheet squeezing described below sometimes occurs. First, the sheet rear end snakes after passing through the roll gap of the i-th rolling mill. Next, the sheet rear end contacts the inlet side guide of the \((i+1)\)th rolling mill and buckles. Finally, the sheet rear end which overlaps itself due to the buckling is squeezed into the roll gap of the \((i+1)\)th rolling mill. It is obvious that the sheet squeezing results from the sheet buckling. Therefore, in this study, the sheet squeezing is assumed to occur due to the sheet snaking and the sheet buckling.

When the sheet squeezing occurs, the rolls of the \((i+1)\)th rolling mill are damaged significantly.\(^3\) Hence, a number of researches on the sheet snaking\(^2, 11\) which is the first half of the sheet squeezing have been performed. However, few researches on the sheet squeezing,\(^12\) in other words, both the sheet snaking and the sheet buckling, have been performed. In the reference 12), the in-plane lateral load which the sheet rear end receives from the inlet side guide, is not calculated analytically but estimated using an empirical formula. Therefore, to the best of my knowledge, no researches on both the sheet snaking and the sheet buckling have been performed analytically.

In this study, a combined method to simulate the sheet snaking by the rigid-plastic FEM and to analyze the sheet buckling by the elementary theory of bucking is proposed. The simulation of the sheet squeezing is performed by the combined method, and the effects of several rolling conditions on the occurrence of the sheet squeezing are clarified analytically.

2. Method of Simulation

2.1. Outline

Figure 1 shows the flow chart of the entire simulation. First, the sheet rear end is assumed to pass through the roll gap of the i-th rolling mill. Next, the in-plane lateral load which the sheet rear end receives from the inlet side guide of the \((i+1)\)th rolling mill is calculated from the simulation of the sheet snaking by the rigid-plastic FEM. Finally, the buckling load of the sheet is calculated from the analysis of the sheet buckling by the elementary theory of buckling. When the in-plane lateral load is larger than the buckling load of the sheet, the sheet squeezing is assumed to occur. In contrast, when the in-plane lateral load is smaller than the buckling load of the sheet, the sheet squeezing is not assumed to occur.

The calculations described above are iterated until the sheet rear end enters into the roll gap of the \((i+1)\)th rolling mill.

2.2. Simulation of Sheet Snaking by Rigid-plastic FEM

2.2.1. Outline

It requires an enormous amount of computation time to perform the finite-element simulation of the sheet between the i-th rolling mill and the \((i+1)\)th rolling mill. Hence, the finite-element simulation of the sheet only in the roll gap of the \((i+1)\)th rolling mill is performed. In the simulation, an in-plane lateral load and an in-plane bending moment

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are assumed to be applied to the sheet at the entrance cross section of the simulation region. The velocity of the sheet rear end is calculated on the assumption that the sheet which exists upstream of the simulation region is rigid.

Figure 2 shows the flow chart of each step in the simulation of the sheet snaking by the rigid-plastic FEM.

First, the simulation is performed on the assumption that no in-plane lateral load is applied at the sheet rear end, and the in-plane lateral velocity at the sheet rear end $v_{y1}$ is calculated. When the sheet rear end does not contact the inlet side guide, the simulation in the next step is performed. In contrast, when the sheet rear end contacts the inlet side guide, the simulation in the next step is performed after the two simulations described below are performed.

First, the simulation is performed on the assumption that a trial in-plane lateral load $F_y2$ is applied at the sheet rear end, and the in-plane lateral velocity at the sheet rear end $v_{y2}$ is calculated. Next, the simulation is performed on the assumption that an optimized in-plane lateral load $F_y2 \times v_{y1}/(v_{y1} - v_{y2})$ is applied at the sheet rear end. As is examined in the following chapter, the ratio of the change in the in-plane lateral velocity to the change in the in-plane lateral load is almost constant regardless of the magnitude of the in-plane lateral load. Hence, a negligible in-plane lateral velocity at the sheet rear end $v_{y3}$ is obtained. In other words, the simulation result in which the boundary condition that the sheet rear end contacts the inlet side guide is satisfied, is obtained.

2.2.2. Boundary Condition on Velocity at Entrance Cross Section

Figure 3 shows the boundary condition on the velocity at the entrance cross section of the simulation region. The coordinate axis in the rolling direction is $x$, the coordinate axis in the direction of the roll axis is $y$, and the coordinate axis in the direction of the sheet thickness is $z$. The origin is located at the center of the plane which contains the upper and lower roll axes, and $b$ denotes the width of the sheet.

The sheet which exists upstream of the simulation region is assumed to be rigid. When the rigid sheet is not constrained, the following three types of boundary conditions on the force should be satisfied at the entrance cross section of the simulation region; (a) the axial load, i.e., the back tension $F_x$ is equal to zero, (b) the in-plane lateral load $F_y$ is equal to zero, and (c) the in-plane bending moment $M_z$ is equal to zero.

Figure 3(a) shows the boundary condition on the velocity for the axial load. The velocity in the rolling direction at the entrance cross section is assumed to be a constant value $v_{xB}$. This assumption is conventionally used in the simulation of sheet rolling. Figure 3(b) shows the boundary condition on the velocity for the in-plane lateral load. The velocity in the direction of the roll axis at the entrance cross section is assumed to be a constant value $v_{yB}$. Figure 3(c) shows the boundary condition on the velocity for the in-plane bending moment. The velocity in the rolling direction at the entrance cross section is assumed to change linearly in the direction of the roll axis, and is assumed to be $\pm \Delta v_{yB}/2$ at the edges of the sheet.

The velocity at the entrance cross section is assumed to be the summation of these three kinds of velocities. The result calculated from the simulation using the rigid-plastic FEM to which this assumption is applied satisfies the following three types of boundary conditions on the force at the entrance cross section; (a) the axial load, i.e., the back tension $F_x$ is equal to zero, (b) the in-plane lateral load $F_y$ is equal to zero, and (c) the in-plane bending moment $M_z$ is equal to zero.
2.2.3. Introduction of In-plane Lateral Load and In-plane Bending Moment

To consider the axial load, i.e., the back tension in the simulation, the term \(-F_x v_x\) which is the product of the load \(F_x\) and the velocity \(v_x\) is supplemented to the conventional functional.\(^{13}\) In a similar manner, to consider the in-plane lateral load and the in-plane bending moment in the simulation, the term \(-F_y v_y\) which is the product of the load \(F_y\) and the velocity \(v_y\) and the term \(-F_y L v_x / b\) which is the product of the moment \(F_y L\) and the angular velocity \(v_x / b\) are supplemented to the conventional functional; the following functional is supplemented to the conventional functional,

\[
\Phi = -F_x v_x - F_y v_y - F_y L v_x / b
\]  

(1)

where \(L\) is the distance between the sheet rear end and the entrance cross section of the simulation region. The supplement of the product of the moment and the angular velocity to the conventional functional has previously been performed. For instance, in the reference \(^{14}\), the out-of-plane bending moment at the exit cross section has been calculated using the supplement of the product of the moment and the angular velocity in order that no curvature in the vertical direction of the angle after rolling is obtained in angle rolling.

To use Eq. (1), the velocity in the direction of the roll axis at the entrance cross section should be constrained to be a constant value, and the velocity in the rolling direction at the entrance cross section should be constrained to change linearly in the direction of the roll axis. When the in-plane lateral load \(F_y\) in Eq. (1) is unboundedly brought close to zero, it is obvious that the simulation result of \(F_y \approx 0\) is obtained. Hence, to impart these constraints implies to assume that the in-plane lateral load \(F_y\) is equal to zero and to assume that the in-plane bending moment \(F_y L\) is equal to zero. In a similar manner, to impart the constraint that the velocity in the rolling direction at the entrance cross section is a constant value implies to assume that the back tension \(F_x\) is equal to zero.

2.2.4. Velocity of Sheet in Upstream Region

The method of the analysis shown in the reference \(^{2}\) is referred. The velocity at the entrance cross section of the simulation region, calculated using the rigid-plastic FEM is used. The velocity at the entrance cross section the coordinate of which is \((x, y)\) is assumed to be \((v_x, v_y)\). The \(x\)-coordinate of the center of rotation \(x_{\text{rot}}\) and the angular velocity of the center of rotation \(\omega\) are calculated as follows.

\[
x_{\text{rot}} = y \frac{v_x}{v_y} + x
\]  

(2)

\[
\omega = \frac{v_y}{y}
\]  

(3)

The \(y\)-coordinate of the center of rotation \(y_{\text{rot}}\) is assumed to be zero. The sheet which exists upstream of the simulation region, is dealt with. The velocity of the sheet in the upstream region the coordinate of which is \((x, y)\), is assumed to be \((v_x, v_y)\) and is calculated as follows using Eqs. (2) and (3).

\[
v_y = (x_{\text{rot}} - x) \omega
\]  

(5)

2.2.5. Boundary Condition on Velocity at Exit Cross Section

The following boundary condition on the velocity at the exit cross section of the simulation region, is assumed; the velocity in the rolling direction is a constant value \(v_x\), whereas the velocity in the direction of the roll axis and the velocity in the direction of the sheet thickness are zero.

2.3. Analysis of Sheet Buckling by Elementary Theory of Buckling

2.3.1. Deflection and Stress Distribution

Figure 4 shows the sheet for the buckling analysis and the boundary conditions. The coordinate axis in the rolling direction is \(x\), the coordinate axis in the direction of the roll axis is \(y\), and the coordinate axis in the direction of the sheet thickness is \(z\). The boundary conditions on the velocity at the entrance cross section are:\n
(a) Axial load

(b) In-plane lateral load

(c) In-plane bending moment

Fig. 3. Boundary condition on velocity at entrance cross section of simulation region.
axis is \( y \), and the coordinate axis in the direction of the sheet thickness is \( z \). The origin is located at the center of the exit cross section of the analysis region. The exit cross section of the analysis region is made to coincide with the entrance cross section of the simulation region by the rigid-plastic FEM. Hence, although the directions of the coordinate axes coincide with the directions of the coordinate axes in the simulation of the sheet snaking, the origin is not made to coincide with the origin in the simulation of the sheet snaking to simplify the analysis.

The fixed end is assumed at \( x = 0 \) which is the exit cross section of the analysis region, whereas the free end is assumed at \( x = L \) which is the sheet rear end and at \( y = \pm b/2 \) which are the edges of the sheet. To further simplify the analysis, the sheet snaking, \( i.e., \) the rotation of the sheet about the \( z \)-axis is not considered. The in-plane lateral load which the edge of the sheet rear end receives from the inlet side guide is denoted by \( F_y \). The deflection of the sheet, which results from the in-plane lateral load \( F_y \), in the direction of the sheet thickness \( w \) is assumed as follows with reference to the boundary conditions of the sides of the sheet.

\[
w = \left( 1 - \cos \frac{\pi x}{2L} \right) \left( 1 - \cos \frac{\pi (y + b/2)}{2b} \right) \tag{6} 
\]

The stress distribution in the sheet due to the in-plane lateral load \( F_y \) is assumed as follows. The shear stress \( \tau_{xy} \) is assumed in order that the shear force due to the shear stress \( \tau_{xy} \) at \( x = x \) coincides with the in-plane lateral load \( F_y \), whereas the bending stress \( \sigma_i \) is assumed in order that the in-plane bending moment due to the bending stress \( \sigma_i \) at \( x = x \) coincides with the in-plane bending moment due to the in-plane lateral load \( F_y \).

\[
\sigma_x = -\frac{12F_y}{Lb^3} (x + L) y \tag{7} 
\]

\[
\sigma_y = 0 \tag{8} 
\]

\[
\tau_{xy} = -\frac{F_y}{Lb} \tag{9} 
\]

Here, \( t \) is the thickness of the sheet.

### 2.3.2. Buckling Condition

When the deflection of the sheet is infinitesimal, the condition of buckling, \( i.e., \) the condition of neutral equilibrium is expressed as follows.\(^{15} \)

\[
D \int_L \int_{-b/2}^{b/2} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2\nu \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \, dx \, dy 
\]

\[
= -\frac{1}{2} \int_L \int_{-b/2}^{b/2} \sigma_i \left( \frac{\partial w}{\partial x} \right)^2 + \sigma_y \left( \frac{\partial w}{\partial y} \right)^2 + 2\nu \sigma_i \left( \frac{\partial w}{\partial x} \right) \left( \frac{\partial w}{\partial y} \right) \, dx \, dy 
\]

\[
= \frac{F_y}{8b} \left[ 2 - \frac{3(7 - 2\pi) \left( \pi + 2 \right) \left( \pi - 2 \right)}{\pi^2} \right] 
\]

\[
\frac{D \int_L \int_{-b/2}^{b/2} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2\nu \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \, dx \, dy}{\pi^3 \left( 3\pi - 8 \right) b} + \frac{\pi^3 \left( 3\pi - 8 \right) L}{b^3} + 2\nu \frac{\pi^2 \left( 4 - \pi^2 \right)}{Lb} + 2(1 - \nu) \frac{\pi^4}{Lb^3} \tag{11} 
\]

Here, \( D \) is the flexural rigidity of the sheet and is equal to \( E t^3 / 12(1 - \nu^2) \), where \( E \) denotes the Young’s modulus of the sheet and \( \nu \) denotes the Poisson’s ratio of the sheet.

When Eqs. (6), (7), (8), and (9) are substituted into Eq. (10) and intricate calculations are performed, the following equation is finally obtained.

The relationship between the distance \( L \) between the sheet rear end and the entrance cross section of the simulation region by the rigid-plastic FEM and the in-plane lateral load \( F_y \) which is the buckling load, is calculated using Eq. (11).

### 3. Simulation Results

Table 1 shows the standard simulation condition. The rolling condition is established with reference to the rolling condition indicated in the reference 12). The sheet thickness before rolling and the sheet thickness after rolling are defined at \( y = 0 \) which is the central section in the direction of the roll axis. Because this study is a fundamental study, the sheet is assumed to be rigid perfectly plastic in the simulation of the sheet snaking, whereas the sheet is assumed to be elastic in the analysis of the sheet buckling. Although the roll is assumed to be rigid, the mill modulus of the rolling mill is not assumed to be infinite but assumed to be a certain value, which is not inconsistent with the mill modulus of the conventional rolling mill. When the simulation condition is not indicated explicitly in the simulation result in this chapter, the simulation condition shown in Table 1 is used.

**Figure 5** shows the finite-element meshes for the standard simulation condition and the coordinate axes. The symmetry of the sheet in the direction of the sheet thickness is assumed in the simulation, and the simulation of the sheet only in the roll gap is performed. The number of the finite elements in the rolling direction is six, the number of the finite elements in the direction of the roll axis is 20, and the number of the finite elements in the direction of the sheet thickness is one.

First, the simulation in a steady state is performed and the shape of the contact surface between the sheet and the roll is converged. The sheet thickness at \( y = 0 \) which is the

![Fig. 4. Sheet for buckling analysis and boundary conditions.](image-url)
The trial in-plane lateral load at the sheet rear end is demonstrated. According to the table, despite the magnitude guide is 1000 mm which is equal to the sheet width. The velocity on the roll surface in the direction of the roll axis is approximately ten minutes on average.

In this section, the mill modulus of the rolling mill is assumed to be infinite, and the width of the inlet side guide is assumed to be infinite. The difference in the roll gap in the direction of the roll axis is changed from 5 μm/m to 15 μm/m. With increasing the displacement of the sheet rear end in the rolling direction, the displacement of the sheet rear end in the direction of the roll axis, i.e., the amount of the sheet snaking increases. With increasing the difference in the roll gap in the direction of the roll axis, the displacement of the sheet rear end in the direction of the roll axis, i.e., the amount of the sheet snaking increases.

According to the elementary theory of rolling, the displacement of the sheet rear end in the direction of the roll axis $u^s_{\text{fr}}$ is provided as follows:

$$u^s_{\text{fr}} = \frac{1}{2b} \frac{\Delta h_1}{h_0} \left( \frac{v_{y1} - v_{y2}}{v_{y1}} \right)$$

Here, $h_0$ denotes the average sheet thickness after rolling in the direction of the roll axis, $\Delta h_1$ denotes the difference in the sheet thickness after rolling in the direction of the roll axis, and $u^s_{\text{fr}}$ denotes the displacement of the sheet rear end in the rolling direction.

The relationship between the displacement of the sheet rear end in the rolling direction and the displacement of the sheet rear end in the direction of the roll axis calculated using Eq. (12) is supplemented in Fig. 6. The difference in the sheet thickness after rolling in the direction of the roll axis $\Delta h_1$ is changed from 5 μm to 15 μm. Since the sheet width $b=1$ m and the mill modulus of the rolling mill is infinite, the differences in the roll gap in the direction of the roll axis of 5, 10, and 15 μm/m are identical to the differences in the sheet thickness after rolling in the direction of the roll axis of 5, 10, and 15 μm/m, respectively. The amount of snaking at the sheet rear end calculated from the simulation by the rigid-plastic FEM agrees with the amount of snaking at the sheet rear end calculated from the analysis by the elementary theory of rolling. Therefore, the validity of the simulation of the sheet snaking by the rigid-plastic FEM is confirmed.

### Table 1. Standard simulation condition.

| Parameter                        | Value |
|----------------------------------|-------|
| Sheet thickness before rolling (mm) | 2     |
| Sheet thickness after rolling (mm) | 1.6   |
| Sheet width (m)                  | 1     |
| Roll diameter (mm)               | 600   |
| Distance between two roll chocks (m) | 2     |
| Distance between two rolling mills (m) | 5     |
| Mill modulus (MN/mm)             | 5     |
| Flow stress of sheet (MPa)       | 200   |
| Friction shear factor between roll and sheet | 1     |
| Young’s modulus of sheet (GPa)   | 150   |
| Poisson’s ratio of sheet         | 0.3   |

### Table 2. Effect of trial in-plane lateral load at sheet rear end on optimized in-plane lateral load at sheet rear end and in-plane lateral velocity at sheet rear end.

| $v_{y1}$ (m/s) | $F_{z1}$ (N) | $v_{y2}$ (m/s) | $F_{z2} \times v_{y1} / (v_{y1} - v_{y2})$ (N) | $v_{y3}$ (m/s) |
|----------------|--------------|----------------|-----------------------------------------------|----------------|
| 0.014821       | 2 000        | 0.01268        | 11 611                                       | −0.000059      |
| 0.014821       | 4 000        | 0.009713       | 11 606                                       | −0.000053      |
| 0.014821       | 6 000        | 0.007155       | 11 599                                       | −0.000044      |
| 0.014821       | 8 000        | 0.004590       | 11 590                                       | −0.000031      |
| 0.014821       | 10 000       | 0.002019       | 11 577                                       | −0.000015      |
| 0.014821       | 12 000       | 0.000562       | 11 562                                       | 0.000005       |

### 3.1. Amount of Sheet Snaking

Figure 6 shows the relationship between the displacement of the sheet rear end in the rolling direction and the displacement of the sheet rear end in the direction of the roll axis. In this section, the mill modulus of the rolling mill is assumed to be infinite, and the width of the inlet side guide is assumed to be infinite. The difference in the roll gap in the direction of the roll axis is changed from 5 μm/m to 15 μm/m. With increasing the displacement of the sheet rear end in the rolling direction, the displacement of the sheet rear end in the direction of the roll axis, i.e., the amount of the sheet snaking increases. With increasing the difference in the roll gap in the direction of the roll axis, the displacement of the sheet rear end in the direction of the roll axis, i.e., the amount of the sheet snaking increases.

According to the elementary theory of rolling, the displacement of the sheet rear end in the direction of the roll axis $u^s_{\text{fr}}$ is provided as follows:

$$u^s_{\text{fr}} = \frac{1}{2b} \frac{\Delta h_1}{h_0} \left( \frac{v_{y1} - v_{y2}}{v_{y1}} \right)$$

Here, $h_0$ denotes the average sheet thickness after rolling in the direction of the roll axis, $\Delta h_1$ denotes the difference in the sheet thickness after rolling in the direction of the roll axis, and $u^s_{\text{fr}}$ denotes the displacement of the sheet rear end in the rolling direction.

The relationship between the displacement of the sheet rear end in the rolling direction and the displacement of the sheet rear end in the direction of the roll axis calculated using Eq. (12) is supplemented in Fig. 6. The difference in the sheet thickness after rolling in the direction of the roll axis $\Delta h_1$ is changed from 5 μm to 15 μm. Since the sheet width $b=1$ m and the mill modulus of the rolling mill is infinite, the differences in the roll gap in the direction of the roll axis of 5, 10, and 15 μm/m are identical to the differences in the sheet thickness after rolling in the direction of the roll axis of 5, 10, and 15 μm/m, respectively. The amount of snaking at the sheet rear end calculated from the simulation by the rigid-plastic FEM agrees with the amount of snaking at the sheet rear end calculated from the analysis by the elementary theory of rolling. Therefore, the validity of the simulation of the sheet snaking by the rigid-plastic FEM is confirmed.

### 3.2. Buckling Load

Figure 7 shows the relationship between the distance
from the sheet rear end to the roll axis and the buckling load. With increasing the distance from the sheet rear end to the roll axis, the buckling load decreases drastically, and the buckling load becomes a minimum value when the distance is in the vicinity of 2 m. Then, with increasing the distance from the sheet rear end to the roll axis, the buckling load increases gradually. With increasing the sheet thickness, the buckling load increases.

3.3. Effect of Width of Inlet Side Guide

Figure 8 shows the relationship between the distance from the sheet rear end to the roll axis and the buckling load or the in-plane lateral load in various widths of the inlet side guide. With the advance of rolling, the distance from the sheet rear end to the roll axis decreases. Hence, the coordinate of the horizontal coordinate axis is determined in order that the coordinate which indicates the distance from the sheet rear end to the roll axis moves from left to right with the advance of rolling. Figures 8(a) and 8(c) show the simulation results in the cases that the differences in the sheet thickness before rolling in the direction of the roll axis are 10 μm and 2 μm, respectively. Figures 8(b) and 8(d) show the simulation results in the cases that the differences in the roll gap before rolling in the direction of the roll axis are 10 μm and 2 μm, respectively.

With decreasing the distance from the sheet rear end to the roll axis, the in-plane lateral load decreases, whereas with increasing the width of the inlet side guide, the in-plane lateral load increases. Hence, with increasing the width of the inlet side guide, the distance from the sheet rear end to the roll axis at which the in-plane lateral load coincides with the buckling load decreases.

In the reference 12), the in-plane lateral load is assumed not to depend on the width of the inlet side guide, and the relationship between the distance from the sheet rear end to the roll axis and the in-plane lateral load is described using an empirical formula. However, it is obvious from Fig. 8 that the in-plane lateral load depends on the width of the inlet side guide. Hence, the assumption in the reference 12) that the in-plane lateral load does not depend on the width of the inlet side guide is confirmed to be inappropriate.

In the case that the difference in the sheet thickness before rolling is 10 μm or the difference in the roll gap before rolling is 10 μm, when the sheet rear end contacts the inlet side guide, the in-plane lateral load is larger than the buckling load despite the magnitude of the width of the inlet side guide. In other words, the endpoint of the curve which represents the in-plane lateral load is above the curve which represents the buckling load. Hence, the sheet squeezing occurs in these cases.

In the case that the difference in the sheet thickness before rolling is 2 μm and the width of the inlet side guide is 1 000 mm, when the sheet rear end contacts the inlet side guide, the in-plane lateral load is smaller than the buckling load, i.e., the sheet squeezing does not occur. However, when the width of the inlet side guide is infinite, the maximum displacement of the sheet rear end in the direction of the roll axis is 19 mm. Hence, it is meaningless to use the inlet side guide the width of which is 1 000 mm, since the gap between the edge of the sheet rear end before rolling and the inlet side guide is 18 mm, which is almost identical to the maximum displacement of the sheet rear end in the direction of the roll axis. In the case that the difference in the roll gap before rolling is 2 μm and the width of the inlet side guide is not 1 0036 mm, when the sheet rear end contacts the inlet side guide, the in-plane lateral load is larger than the buckling load, i.e., the sheet squeezing occurs. Hence, in the case that...
the difference in the roll gap before rolling is 2 \( \mu \text{m} \), the sheet squeezing is assumed to occur.

According to Fig. 8, in the case that the width of the inlet side guide is identical to the sheet width and the sheet rear end contacts the inlet side guide, the sheet squeezing does not occur when the in-plane lateral load is smaller than the buckling load, whereas the sheet squeezing occurs when the in-plane lateral load is larger than the buckling load. Hence, to judge whether the sheet squeezing occurs or not, it is sufficient to perform the simulation in which the width of the inlet side guide is identical to the sheet width. Hence, in the following sections, the simulation results in the case that the width of the inlet side guide is identical to the sheet width are demonstrated.

3.4. Effect of Difference in Sheet Thickness

Figure 9 shows the relationship between the distance from the sheet rear end to the roll axis and the buckling load or the in-plane lateral load in various differences in the sheet thickness before rolling in the direction of the roll axis. With increasing the difference in the sheet thickness before rolling, the in-plane lateral load increases.

In the case that the difference in the sheet thickness before rolling is 1 \( \mu \text{m} \) or 2 \( \mu \text{m} \), the in-plane lateral load is smaller than the buckling load when the sheet rear end contacts the inlet side guide, i.e., the sheet squeezing does not occur. In the case that the difference in the sheet thickness before rolling is 5 \( \mu \text{m} \) or 10 \( \mu \text{m} \), the in-plane lateral load is larger than the buckling load when the sheet rear end contacts the inlet side guide, i.e., the sheet squeezing occurs.
3.5. Effect of Difference in Roll Gap

Figure 10 shows the relationship between the distance from the sheet rear end to the roll axis and the buckling load or the in-plane lateral load in various differences in the roll gap before rolling in the direction of the roll axis. With increasing the difference in the roll gap before rolling, the in-plane lateral load increases.

In the case that the difference in the roll gap before rolling is 1 μm/m, the in-plane lateral load is smaller than the buckling load when the sheet rear end contacts the inlet side guide, i.e., the sheet squeezing does not occur. In the case that the difference in the roll gap before rolling is 2 μm/m, 5 μm/m, or 10 μm/m, the in-plane lateral load is larger than the buckling load when the sheet rear end contacts the inlet side guide, i.e., the sheet squeezing occurs.

According to Figs. 9 and 10, when the value of the difference in the sheet thickness before rolling is identical to the value of the difference in the roll gap before rolling, the following result is obtained. The in-plane lateral load in the case that the difference in the roll gap before rolling is assumed, is larger than that in the case that the difference in the sheet thickness before rolling is assumed. Hence, the probability of the occurrence of the sheet squeezing in the case that the difference in the roll gap before rolling is assumed, is larger than that in the case that the difference in the sheet thickness before rolling is assumed. For instance, the sheet squeezing does not occur in the case that the difference in the sheet thickness before rolling is 2 μm/m, whereas the sheet squeezing occurs in the case that the difference in the roll gap before rolling is 2 μm/m.

3.6. Effect of Amount of Sheet Off-center

Figure 11 shows the relationship between the distance from the sheet rear end to the roll axis and the buckling load or the in-plane lateral load in various amounts of sheet off-center.
sheet off-center, i.e., the deviation between the center of the sheet before rolling in the direction of the sheet width and the center of the roll in the direction of the roll axis. With increasing the amount of the sheet off-center, the in-plane lateral load increases. The direction of the sheet off-center coincides with the direction of the rotation of the sheet rear end. With increasing the difference in the roll gap in the direction of the roll axis due to the increase in the amount of the sheet off-center, the in-plane lateral load increases.

In the case that the amount of the sheet off-center is 1 mm or 2 mm, the in-plane lateral load is smaller than the buckling load when the sheet rear end contacts the inlet side guide, i.e., the sheet squeezing does not occur. In the case that the amount of the sheet off-center is 5 mm or 10 mm, the in-plane lateral load is larger than the buckling load when the sheet rear end contacts the inlet side guide, i.e., the sheet squeezing occurs.

4. Conclusions

The simulation of the sheet squeezing in hot sheet rolling was performed and the following results were obtained.

(1) A combined method to simulate the sheet snaking by the rigid-plastic FEM and to analyze the sheet buckling by the elementary theory of bucking was proposed.

(2) A method to assume the in-plane lateral load and the in-plane bending moment on the surface of the simulation region by the rigid-plastic FEM was proposed.

(3) The amount of the sheet snaking calculated from the simulation by the rigid-plastic FEM agreed with that calculated from the analysis by the elementary theory of buckling.

(4) The effects of the rolling conditions such as the difference in the sheet thickness, the difference in the roll gap, and the amount of the sheet off-center on the occurrence of the sheet squeezing were clarified.

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