ELECTROWEAK MATRIX ELEMENTS AT LARGE $N_C$: MATCHING QUARKS TO MESONS.

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I review some progress made on the problem of calculating electroweak processes of mesons at low energy with the use of an approximation to large-$N_C$ QCD which we call the Minimal Hadronic Approximation. An update of results for the matrix elements of the electroweak penguin operators $Q_7$ and $Q_8$ is also given.

1 Introduction

Due to the disparity in scales $(M_K/M_W)^2 \sim 10^{-4}$ it becomes very useful to employ this ratio as an expansion parameter and construct an Effective Lagrangian in which the heavy $W$ field is integrated out. In practice one treats as heavy all particles above and including the charm quark. The technique for making this construction is very well known. According to this technique, after having integrated out all the heavy degrees of freedom by shrinking the corresponding propagators to a point (see Fig. 1), one is left with a Lagrangian which is a linear combination of ten four-quark operators involving only the light quark fields $u, d, s$ coupled to the gluon and the photon. Denoting these operators by the generic form $Q_j(\mu) \sim \overline{q} \Gamma_j q \overline{q} \Gamma'_j q$, where $\Gamma_j$ and $\Gamma'_j$ are some known Dirac matrices, one has

$$\mathcal{L}_{\text{eff}} = i \overline{q} \not{D} q + \sum_{j=1}^{10} c_j(\mu) \, Q_j(\mu) .$$

(1)

Shrinking a massive propagator to a point modifies the ultraviolet properties of the theory and the Wilson coefficients $c_j(\mu)$ fix this so that the physics is, however, not changed. Therefore the Wilson coefficients only know about short-distance physics and are given by a series expansion in powers of $\alpha_s$ for scales $\mu$ in between $M_W$ and a typical hadronic scale $\Lambda_{\text{had}} \sim O(1 \text{ GeV})$. Since this series has large coefficients of the form $\log M_W/\Lambda_{\text{had}}$ one uses the Renormalization Group to resum them. Presently, due to the effort mainly of two groups, we know the Wilson coefficients up to the next-to-leading order.
1.1 The problem

In fact, the separation made in the Lagrangian in Eq. (1) in terms of Wilson coefficients and operators must be defined by some regularization and depends on the conventions chosen like, e.g., the scale $\mu$ used in the minimal subtraction, the value of the anticommutator $\{\gamma_\mu, \gamma_5\}$ (i.e. whether the NDR or HV prescription), the so-called evanescent operators, etc... However, since the physical result cannot depend on these conventions there has to be a cancelation between the Wilson coefficients and the matrix elements of $Q_j$. This cancelation becomes a highly nontrivial consistency check in any calculation.

Since kaons are light with respect to $\Lambda_{had}$ one still writes a second Effective Lagrangian which is a Chiral Lagrangian, i.e. which is organized in powers of momenta and masses of the light mesons according to chiral symmetry. Meson matrix elements are then computed in terms of masses and couplings of this Chiral Lagrangian such as, e.g., $f_\pi$.

A puzzle then arises. How can the convention dependence, e.g. of $\gamma_5$, cancel between Wilson coefficients and matrix elements of the Chiral Lagrangian which has no explicit $\gamma_5$? Remarkably this problem has been plaguing all analytic calculations of electroweak matrix elements up to date.

The key are the matching conditions. These are conditions on Green’s functions which fix the value of the effective couplings so as to keep the physics unaltered in the transition from the fundamental to the Effective Lagrangian. Within a perturbative context, this much was already understood. However, perturbation theory cannot be valid if quarks are to make mesons. The difficulty now lies in that we use two different languages to describe the same physics: while at short distances we use quark and gluons as our variables, at long distances we use meson fields. Therefore the matching conditions entail
Figure 2. This figure represents a matching condition. Here “ext” stands for zero-momentum insertions of external fields. The gray loops on the right-hand side signify the complete infinite set of gluonic contributions which are leading at large $N_c$ (i.e. planar diagrams with no internal quark loops). A possible factorized contribution has been disregarded for simplicity. For further explanations, see the text.

a conceptually new situation: one has to match an expansion in powers of $\alpha_s$ coming from short distances to an expansion in powers of meson momenta at long distances. In other words, in order to make of the matching conditions a practical tool one has to find a dictionary capable of translating a Green’s function from the language of quarks and gluons to the language of mesons. In principle, if the solution to large-$N_c$ QCD were known, we could use it as this dictionary. In practice, since this solution is not known, we shall restrict ourselves to an approximation to large-$N_c$ QCD that we shall call the “Minimal Hadronic Approximation”.
1.2 Our solution: The Minimal Hadronic Approximation to large-$N_c$ QCD.

What does the matching condition actually look like? The Chiral Lagrangian is generically given by a function $\mathcal{L}_{\text{chiral}}(D_\mu U)$, where $D_\mu U$ represents the covariant derivative in the presence of external fields acting on the usual exponential of the Goldstone fields. (External fields are useful for identifying Green’s functions.) On the other hand, the quark-gluon Effective Lagrangian is given by Eq. (1) where the covariant derivative $D_\mu$ also contains the same external fields as $\mathcal{L}_{\text{chiral}}$ plus the gluon, which is certainly dynamic. In this situation the matching conditions can be expressed pictorially as in Fig. 2.

This Figure expresses the fact that, at large $N_c$, coupling constants in the Chiral Lagrangian contribute to a given Green’s function at tree level (left-hand side of Fig. 2). The higher the number of external fields, the higher the order in the chiral expansion. The matching condition demands that this Green’s function equals that obtained when the Lagrangian of Eq. (1) is used (i.e. first graph on the right-hand side of Fig. 2). This involves the insertion in the Green’s function of the local four-quark operators in the Lagrangian (3) which is actually a mathematically ill-defined operation. To appreciate the need to define this locality one can reexpress this contribution by integrating over a euclidean momentum $Q$ in the loop; and this is divergent. Consequently a prescription has to be given, and here is where the conventions used for the Wilson coefficients come in. I have tried to represent this crucial point in the second term on the right-hand side of Fig. 2 by the script \(^{\text{MS}}\) on both the coefficients $c_j^{\text{MS}}(\mu)$ and the $D-$dimensional integral $\int^\infty_0 d^D Q$. The lesson from Fig. 2 is that at leading order in $1/N_c$ the couplings of the Chiral Lagrangian are given by $D-$dimensional integrals over an euclidean momentum $Q$ of QCD Green’s functions at large $N_c$, with zero-momentum insertions of external fields and in the forward limit for $Q$. Let’s call these QCD Green’s functions generically $\mathcal{G}(Q^2)$. If $\mathcal{G}(Q^2)$ was known for the full range of momentum $Q$ one could go ahead and do the integral. The actual situation, of course, is that hardly ever does one have all this information. Nevertheless, one does have at least the chiral expansion of $\mathcal{G}(Q^2)$ for low values of $Q^2$ and also the large-$Q^2$ expansion of $\mathcal{G}(Q^2)$ given by its OPE, so that the problem becomes rather how to build an interpolating function between these two regimes. This interpolating function is what we have termed the “Minimal Hadronic Approximation” (MHA) to Large-$N_c$ QCD. It turns out that some versions of Vector Meson Domiance are particular examples of MHA, but they not

\(^a\)A possible factorized contribution has been disregarded here for simplicity.
always coincide.

To be specific, let’s imagine that we want to construct the MHA to the Green’s function \( \Pi_{LR}(Q^2) \) in the chiral and large-\( N_c \) limits. This function is defined as:

\[
\Pi_{LR}^{\mu\nu}(q) = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_{LR}(q^2 \equiv -Q^2),
\]

where

\[
\Pi_{LR}^{\mu\nu}(q) = 2i \int d^4x \ e^{iqx} \langle 0 | T\{L^\mu(x)R^\nu(0)\} | 0 \rangle,
\]

and \( R^\mu(L^\nu) = i\gamma^\mu \begin{pmatrix} 1 + \gamma_5 \end{pmatrix} u(x) \). About \( Q^2 = 0 \) chiral perturbation theory yields a Laurent series in \( Q^2 \),

\[
\Pi_{LR}(Q^2) \approx -\frac{f_2^2}{Q^2} - 4L_{10} + \mathcal{O}(Q^2),
\]

while about \( Q^2 = \infty \) the OPE is given by a (possibly asymptotic) series in inverse powers of \( Q^2 \),

\[
\Pi_{LR}(Q^2) \approx \frac{h_1}{Q^2} + \frac{h_2}{Q^4} + \frac{h_3}{Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right),
\]

with \( h_1 = h_2 = 0 \) and \( h_3 = 4\pi \alpha_s \langle \bar{\psi}\psi \rangle (1 + \mathcal{O}(\alpha_s \log Q^2)) \). In order to achieve matching with the Wilson coefficients at the next-to-leading order we do not need to consider the \( \mathcal{O}(\alpha_s \log Q^2) \) in \( h_3 \).

On the other hand, general properties of large-\( N_c \) QCD tell us that \( \Pi_{LR}(Q^2) \) is a meromorphic function given by

\[
\Pi_{LR}(Q^2) = \sum_V \frac{f_{\nu V} M_{V}^2}{Q^2 + M_{V}^2} - \sum_A \frac{f_{\nu A} M_{A}^2}{Q^2 + M_{A}^2} - \frac{f_2^2}{Q^2}.
\]

Obviously, dealing with infinite sums is not a simple matter, particularly when the poles and the residues are unknown. The Minimal Hadronic Approximation is defined by keeping only a finite number of resonances in these sums, whose residues and masses are fixed by matching to the first few terms of both the chiral and the OPE expansions, Eqs. (3,4). Once this is done we have what is known in Mathematics as a rational approximant. This is an interpolating function which is the ratio of two polynomials and that, by construction, has the same low- and high-\( Q^2 \) behavior as the full \( \Pi_{LR}(Q^2) \). There is no a priori condition on how many terms in both the chiral and the OPE expansions one has to match. A sensible choice should probably be made considering whether the particular observable one is looking at weights more the low or the high-\( Q^2 \) tail; in practice, however, one is limited by the availability

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\( ^3 \)I thank A. Pich for discussions on this point.

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of the terms in these two expansions. Modulo this practical limitation, the approximation is clearly systematic and well defined.

Therefore, keeping finite sums in Eq. (5), the matching conditions for our MHA read

\[
-4L_{10} = \sum_{V} f_{V}^{2} - \sum_{A} f_{A}^{2}
\]

\[
0 = f_{\pi}^{2} + \sum_{A} f_{A}^{2} M_{A}^{2} - \sum_{V} f_{V}^{2} M_{V}^{2}, \quad 0 = \sum_{A} f_{A}^{2} M_{A}^{4} - \sum_{V} f_{V}^{2} M_{V}^{4}
\]

\[
4\pi \alpha_{s} \langle \psi \bar{\psi} \rangle^{2} = \sum_{A} f_{A}^{2} M_{A}^{6} - \sum_{V} f_{V}^{2} M_{V}^{6},
\]

where as many terms from Eqs. (3,4) as needed are understood in order to fix the resonance parameters. Eq. (6) is the statement of resonance dominance considered by Ecker et al.\(^7\) Eqs. (7) are nothing but a generalization of the celebrated Weinberg sum rules\(^8\) and Eq. (8) was first considered by Knecht and de Rafael\(^9\) and is the first page of the dictionary we have been looking for: it relates an expression written in terms of quarks and \(\alpha_{s}\) to an expression in terms of mesons\(^6\). In the next section we shall see its usefulness. Finally, our \(\Pi_{LR}(Q^{2})\) in the MHA is like that in Eq. (6) but with a finite set of resonances in the sums whose residues and poles satisfy Eqs. (6,8).

How well does the MHA work? In the case of \(\Pi_{LR}(Q^{2})\) even only one vector and one axial (plus the pion) does quite well in a comparison with ALEPH data\(^6\).\(^6\)

2 An instructive example: the \(\pi^{+}-\pi^{0}\) electroweak mass difference.

Pions are massless in the chiral limit provided the electroweak interactions are switched off. However, the effective operator

\[
\mathcal{L}_{\text{Chiral}} = e^{2} C \text{ Tr } (Q_{R} U Q_{L} U^{\dagger}) = -\frac{2e^{2}C}{f_{\pi}^{2}} \pi^{+} \pi^{-} + \cdots,
\]

where \(Q_{L} = Q_{R} = \text{ diag}(2/3, -1/3, -1/3)\) and \(e\) the electric charge, shows that charged pions do pick up a mass for \(e \neq 0\).

\(^6\)In actual fact what appears in Eq. (6) is the four-quark condensate. Here we have taken the strict large-\(N_{c}\) limit and factorized. This may not always be very precise numerically; see the last section for a potential example of this.
It is useful to think of the matrices $Q_{L,R}$ as arbitrary external fields. If you expand the exponential $U$ in the operator (9) and keep only the unity, this operator gives rise to a “mixed mass term” between the field $Q_L$ and the field $Q_R$. Therefore $C$ can be regarded as a coupling between the two external fields $Q_L$ and $Q_R$ in this Chiral Lagrangian, just like we discussed in the general introduction. Can one determine $C$? The answer is yes. Following our previous discussion of Fig. 2, it is given by a matching condition which in this case reads

$$C = \frac{3}{32\pi^2} \int_0^\infty dQ^2 \left\{ 1 - \frac{Q^2}{Q^2 + M_Z^2} \right\} (-Q^2\Pi_{LR}(Q^2)),$$

where the unity in {...} above comes from the photon and the $\frac{Q^2}{Q^2 + M_Z^2}$ from the $Z$ propagator. Therefore $C$ is of $O(N_c)$. The $\Pi_{LR}(Q^2)$ function stems from the fact that the external fields $Q_{L,R}$ couple to the left- and right-handed quark currents (see Eq. (13) below). Since charged currents are fully lefthanded, there can be no $W$ contribution to the $Q_L \times Q_R$ operator in Eq. (9). In Eq. (10), unlike Fig. 2, there are no Wilson coefficients because the $Z$ propagator has not been shrunk to a point yet, although it will be soon.

Therefore $C$ is known if $\Pi_{LR}(Q^2)$ is known over the full range of momentum, as we discussed in the introduction. Now we may approximate this function by its MHA defined as Eq. (11) with only one $V$ and one $A$ instead of the infinite sums. The net result is that in this MHA one finds the amazingly simple expression

$$-Q^2\Pi_{LR}(Q^2) = \frac{f_\pi^2 M_V^2 M_A^2}{(Q^2 + M_V^2)(Q^2 + M_A^2)},$$

with which the coupling constant $C$ in Eq. (9) can be computed explicitly as

$$C_{\text{MHA}} = \frac{3}{32\pi^2} f_\pi^2 M_V^2 \left[ \frac{M_A^2}{M_A^2 - M_V^2} \log \frac{M_A^2}{M_V^2} - \frac{M_V^2}{M_A^2 - M_V^2} \log \frac{M_A^2}{M_V^2} \right].$$

Assuming $f_\pi \chi^{-\text{limit}} = 87 \pm 3.5 \, \text{MeV}$ and $M_V = 748 \pm 29 \, \text{MeV}$ are large-$N_c$ values, Eqs (6-8) are overconstrained. Since the four-quark condensate (see footnote on page 6) is not very well known, in practice we use only Eqs. (6-7) as true constraints and we take Eq. (8) as a prediction for the condensate.

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\textsuperscript{a}Obviously this cannot be done with the photon propagator.

\textsuperscript{b}These values are extracted from a comparison of Aleph data with this very same MHA.
Notice that these equations give \( M_A \) in terms of \( \pi, M_V \) and \( L_{10} \). It turns out that this leads to \( M_V^2/M_A^2 \simeq 0.50 \pm 0.06 \) (corresponding to the experimental number for \( L_{10} \simeq L_{10}(\mu = M_H) = -5.1 \pm 0.2 \times 10^{-3} \)), which is how the celebrated Weinberg result appears in our approach. Numerically this shifts the charged pion mass by \( \sim 4.88 \text{ MeV} \) to be compared with \( 4.5936(5) \text{ MeV} \) which is the experimental number. One expects a rough 30\% to be a fair estimate of the error in the result of Eq. (12). The photon contribution coincides with the classic result by Low et al.\(^8\).

Our interest in the \( Z \) contribution was not really its numerical result but the following conceptual lesson that can be drawn from it. Let’s imagine that, as is usually done, one first integrates the \( Z \) out by shrinking its propagator to a point. In this case, the Effective Lagrangian due to \( Z \) exchange is given by the four-quark operator

\[
\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{4\pi M_Z^2} \{ Q_{LR} \equiv \overline{\psi}_L \gamma_\mu Q_L q_L \overline{\psi}_R \gamma^\mu Q_R q_R \} + \cdots
\]

where the dots stand for some other terms which are irrelevant in what follows. According to the Effective Lagrangian technique, this operator is valid at the \( M_Z \) scale. As one runs it down to a scale \( \mu \sim \Lambda_{\text{had}} \) the four-quark operator \( Q_{LR}(\mu) \) mixes into the operator

\[
D_{LR}(\mu) \equiv (Q_L)_{ij} (Q_R)_{kl} \overline{q}_L^i(x) \overline{q}_R^j(x)
\]

according to (I keep only the one-loop leading log for simplicity): \(\begin{align*}
Q_{LR}(M_Z^2) &= Q_{LR}(\mu) - \frac{3\alpha_s}{2\pi} \log \frac{M_Z^2}{\mu^2} D_{LR}(\mu^2) .
\end{align*}\)

The \( \pi^+ \) mass can be defined through the matrix element of the Lagrangian \(\begin{align*}
\langle \pi^+ | \mathcal{L}_{\text{Chiral}} | \pi^+ \rangle ,
\end{align*}\), and because of Eqs. (13,14), it receives now two contributions. Firstly let us consider the one that comes from \( Q_{LR}(\mu) \). Closing the quarks in a single loop produces a \( Q_L \times Q_R \) term whose coefficient is again the function \( \Pi_{LR} \). This contribution has a similar structure as the electromagnetic interaction discussed before, i.e.

\[
C(Q_{LR}) = -\frac{3}{32\pi^2} \int_0^\infty dQ^2 \frac{Q^2}{M_Z^2} (-Q^2 \Pi_{LR}(Q^2))
\]

but, because it lacks the photon propagator, it is actually divergent and one has to regularize the integral. It coincides with the \( Z \) contribution in Eq. (11).
if one takes bluntly the limit $M_Z \to \infty$ inside the integral. Going back to a general MHA with $N_{V,A}$ resonances it reads

$$C(Q_{LR}(\mu)) = \frac{3}{32\pi^2} \left( \sum_A f_A^2 M_A^6 \log \frac{M_A^2}{\mu^2} - \sum_V f_V^2 M_V^6 \log \frac{M_V^2}{\mu^2} \right). \quad (17)$$

The contribution to $C$ from the operator $D_{LR}(\mu)$ is simpler due to the explicit power of $\alpha_s$ appearing in (15), which allows one to use factorization at $O(N_c)$ to extract the $Q_L \times Q_R$ contribution. It is simply $(1/4) \langle \bar{\psi} \psi \rangle^2$. Together with the Wilson coefficient in Eq. (15) this yields

$$C(D_{LR}(\mu)) = \frac{3 \alpha_s}{2 \pi} \log \frac{M_Z^2}{\mu^2} \frac{1}{4} \langle \bar{\psi} \psi \rangle^2. \quad (18)$$

The total result is of course $C = C(Q_{LR}(\mu)) + C(D_{LR}(\mu))$ and one observes here the usual difficulty I discussed in the introduction: the necessary cancelation of $\mu$ in $C$ is not seen. The problem is deeply related to the fact that one piece depends on $\alpha_s$ and comes from some perturbative running Eq. (15) whereas the other comes from some matrix element computed with mesons (Eq. 17). However when the matching condition (8) is recalled all pieces fall into place and one can explicitly see the expected cancelation of $\mu$ in the final result for $C$, which is again given by the full $Z$ propagator in Eq. (10). As promised, the condition (8) has played the role of a dictionary.$^f$

3 Electroweak penguins.

There is another instance where our understanding of the $\Pi_{LR}$ function will help us. This is in the $O(p^0)$ calculation of the electroweak penguin $Q_8$, which is defined as

$$Q_8 = -12 e \left( \lambda^{(32)}_{L} \right)_{ij} (Q_R)_{kl} \bar{q}_i^L \gamma_5 q^k_R(x) \bar{q}_l^R \gamma_i^L(x), \quad (19)$$

where $Q_R$ is the same as in the previous section and $(\lambda^{(32)}_{L})_{ij} = \delta_{i3} \delta_{j2}$. This operator is the same as $D_{LR}$ in Eq. (14) except that the $\lambda_L$ matrix replaces now $Q_L$. Therefore following our previous discussion one can see that $Q_8$ bosonizes at $O(p^0)$ as

$$Q_8 = -12 \langle 0|\bar{s}_L s_R(0) \bar{d}_R d_L(0)|0 \rangle \, \text{tr} \left( U \lambda^{(23)}_L U^\dagger Q_R \right)^\dagger, \quad (20)$$

$^f$See our calculation of $B_K$ for a more sophisticated application of MHA showing explicit scheme independence at the next-to-leading-log level.$^{[3]}$
calculations are in progress.\footnote{The Trieste group has also obtained values for $M_{7,8}$,\cite{Doninietal:2015}, although without this matching.}

As it turns out, it is precisely this four quark condensate in Eq. (20) which governs the $1/Q^6$ fall-off of the OPE of $\Pi_{LR}(Q^2)$ in the chiral $SU(3)$ limit. Specifically one finds that

\[-Q^6\Pi_{LR}(Q^2) = 16\pi\alpha_s \left(1 + \frac{\alpha_s}{\pi}\right) <0|\overline{\pi_1} \gamma_{\mu} \pi_2 \gamma_{\nu} d_L|0> + \cdots + O\left(\frac{1}{Q^2}\right), \tag{21}\]

where the $\cdots$ stand for smaller contributions.\footnote{The Trieste group has also obtained values for $M_{7,8}$,\cite{Doninietal:2015}, although without this matching.} The term $\xi$ was not known at the time we wrote our paper,\cite{Babu:2008} but now it is known: $\xi = 25/8$ in NDR, and $\xi = 21/8$ in HV. Using the MHA with just the $\rho$ and the $a_1$ as we did in the case of the $\pi^+ - \pi^0$ mass difference, this large-$Q^2$ fall-off is given by

\[ - Q^6\Pi_{LR}(Q^2) \overset{Q^2 \to \infty}{\longrightarrow} f_A^2 M_A^2 - f_V^2 M_V^2 = f_A^2 M_A^2 M_V^2. \tag{22}\]

Choosing $\alpha_s(2\ GeV) \simeq 0.33 \pm 0.04$ one obtains a value for the four quark condensate in Eq. (22) and with it and Eq. (20) one can compute, e.g., the matrix elements $M_8 \equiv <(\pi\pi)_{I=2}|Q_8|K^0> (2\ GeV)$. One can proceed analogously with the other EW penguin $Q_7$, which is akin to $Q_{LR}$ in Eq. (13), and the corresponding matrix element $M_7$.\cite{Babu:2008} An updated summary of results for these matrix elements is given in Table 1, where I have limited myself to those theoretical approaches which are capable of matching to the Wilson coefficients at short distances.\cite{Babu:2008}

| Refs.          | $M_7$(NDR) | $M_7$(HV) | $M_8$(NDR) | $M_8$(HV) |
|----------------|------------|-----------|------------|-----------|
| Knecht et al.  | 0.11 ± 0.03| 0.67 ± 0.20| 2.34 ± 0.75| 2.52 ± 0.79|
| Narison        | 0.35 ± 0.10| 2.7 ± 0.6  |            |           |
| Cirigliano et al.| 0.16 ± 0.10| 0.49 ± 0.07| 2.22 ± 0.67| 2.46 ± 0.70|
| Bijnens et al. | 0.24 ± 0.03| 0.37 ± 0.08| 1.2 ± 0.9  | 1.3 ± 0.9  |
| Battacharya et al.| 0.32 ± 0.06| 1.2 ± 0.2  |            |           |
| Donini et al.  | 0.11 ± 0.04| 0.18 ± 0.06| 0.51 ± 0.10| 0.62 ± 0.12|

Table 1. Summary of results for $M_{7,8} \equiv <(\pi\pi)_{I=2}|Q_{7,8}|K^0> (2\ GeV)$, in units of $GeV^3$.\footnote{The Trieste group has also obtained values for $M_{7,8}$,\cite{Doninietal:2015}, although without this matching.}
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References

1. See, e.g., A. J. Buras, arXiv:hep-ph/9806471 and references therein.
2. A. J. Buras et al., Nucl. Phys. B 370, 69 (1992) [ibid. B 375, 501 (1992)].
3. M. Ciuchini et al., Nucl. Phys. B 415, 403 (1994) arXiv:hep-ph/9304257.
4. A. J. Buras and P. H. Weisz, Nucl. Phys. B 333, 66 (1990).
5. See, e.g., E. de Rafael, arXiv:hep-ph/9502254 and references therein.
6. G. 't Hooft, Nucl. Phys. B 370, 69 (1992) [ibid. B 375, 501 (1992)].
7. See also M. Knecht and E. de Rafael, AIP Conf. Proc. 602, 14 (2001) [Err., arXiv:hep-ph/0006146, v3].
8. S. Weinberg, Phys. Rev. Lett. 18, 507 (1967).
9. M. Knecht and E. de Rafael, Phys. Lett. B 424, 335 (1998).
10. S. Peris et al., Phys. Rev. Lett. 86, 14 (2001). E. de Rafael, AIP Conf. Proc. 602, 14 (2001). M. F. Golterman and S. Peris, Phys. Rev. D 61, 034018 (2000).
11. M. Knecht et al., Phys. Lett. B 443, 255 (1998).
12. S. Peris and E. de Rafael, Phys. Lett. B 490, 213 (2000) [Err., arXiv:hep-ph/0006146, v3].
13. T. Das et al., Phys. Rev. Lett. 18, 759 (1967).
14. M. Knecht et al., Phys. Lett. B 457, 227 (1999). See also Knecht et al. 15.
15. M. Knecht et al., Phys. Lett. B 508, 117 (2001).
16. S. Descotes, JHEP 0103, 002 (2001) and refs. therein.
17. V. Cirigliano et al., Phys. Lett. B 522, 245 (2001).
18. J. Bijnens et al., JHEP 0110, 009 (2001).
19. S. Narison, private communication to E. de Rafael. See also S. Narison, Nucl. Phys. B 593, 3 (2001).
20. T. Bhattacharya et al., Nucl. Phys. Proc. Suppl. 106, 311 (2002) and S. Sharpe, private communication.
21. A. Donini et al., Phys. Lett. B 470, 233 (1999).
22. S. Bertolini et al., Phys. Rev. D 63, 056009 (2001); S. Bertolini, arXiv:hep-ph/0201218 and references therein.
23. M. Knecht et al., Phys. Rev. Lett. 83, 5230 (1999).
24. M. Knecht and A. Nyffeler, arXiv:hep-ph/0111058; M. Knecht et al., Phys. Rev. Lett. 88, 071802 (2002).