About the applicability limits of the Tymoshenko model and the principle of two-dimensional similarities in problems of elastic plastic bending and stability of densely perforated plates and shells

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Abstract. The applicability limits of the Tymoshenko model are determined for the elastic-plastic bending problems of densely perforated plates by the finite element method. The boundaries of applicability of the two-dimensional similarity principle are investigated for stability problems of densely perforated elastoplastic cylindrical shells under axial compression. Cylindrical densely perforated shell is represented as a set of cyclically repeating structural elements. The principle of two-dimensional similarity makes it possible to reduce the number of these elements while maintaining the porosity, thickness and length of the shell. The calculations were carried out by the theory of the Timoshenko-type shells and continuum theory. The nature of the change in the applicability of this principle is shown in the article, depending on the thickness of the shell. The values of the critical load are determined with a decrease in the number of structural elements for different values of porosity. Based on these calculations, the applicability limits of the two-dimensional similarity principle are estimated for stability problems to axial compression of a densely perforated cylindrical shell by the continuum theory and theory of the Timoshenko-type shells.

1. Introduction
Densely perforated plates and shells are used to create hydraulic and gas distributing structures. Direct modeling of the stress-strain state of such structures is impossible or extremely difficult because it requires very high power of computing resources. The existing simplified methods of calculation are based on the averaging principle, which allows us to move from a perforated structure to a continuous material with effective elastic moduli and a yield surface [1-5]. Applicability of the averaging method for densely perforated plates and shells for elastic deformation is shown in [6-8]. For elastoplastic deformation this approach is not applicable, because does not take into account the heterogeneity of deformation mode in a representative volume. To solve the problem, we can apply the principle of two-dimensional similarity, which takes into account the heterogeneity of deformation mode in a representative volume and allows us to save computational resources in comparison with direct three-dimensional modeling by reducing the number of holes while maintaining the porosity, thickness and length of the shell. In the article the results of the investigation of the applicability limits of this
principle are described for the bending and stability problems of elastic plastic densely perforated plates and shells.

2. Applicability limits of the theory of Tymoshenko-type plates in solving bending problems of densely perforated structural elements

The applicability limits of the theory of Timoshenko-type plates for densely perforated structures are determined by the example of a cyclically repeating structural element under the action of a bending moment. For simplicity of representation of results, the pitch of the perforation in both directions of the plate is equal. In this case, the structural element is a square prism of thickness $h$ with side length equal to $a$ (Figure 1). At the center of the element is a hole of diameter $d_o$.

![Figure 1. Structural element](image)

A bending moment was applied on one of the faces of the structural element. The value of the moment was chosen in such a way that the maximum plastic deformations near the hole were of the order of 10%. The symmetry condition was imposed on the opposite face. The value of the angle of rotation of the face was considered as a result of the study. The elastic and elastoplastic model of the structural element is considered.

Structural elements with different porosity and thickness are considered. Porosity is the ratio of the hole area to the total area of the face plane of the structural element.

$$\gamma = \frac{\pi d_0}{4a^2} \cdot \text{porosity.}$$  \hspace{1cm} (1)

Dependences of the angle of rotation on porosity and thickness are obtained in the elastic and elastoplastic models of bending. All problems are solved by the finite element method using the theory of the Timoshenko-type plates and the continuum theory. Three-dimensional results were used as a reference result. The applicability limit of the theory of plates and shells was determined by a 5% difference between the result and the reference solution.

Figure 2 shows the curves describing the applicability limits of the theory of the Timoshenko-type plates in the problems of elastoplastic bending of the structural element of densely perforated plates.
Figure 2. Applicability limits of the theory of Timoshenko-type plates in problems of elastoplastic bending of densely perforated plates.

The parameters under each of the graphs determine the field of applicability of the theory of Timoshenko-type plates in the bending problems of densely perforated plates. It is obvious that the applicability limit for elastic model is much larger than for elastoplastic. The range of applicability expands with decreasing porosity.

3. Investigation of the applicability limits of the two-dimensional similarity principle for the stability problem of densely perforated cylindrical shells under axial compression

Calculations of the stability of a cylindrical densely perforated shell with radius R under axial compression were carried out. The simulation is performed by a nonstationary elastoplastic continuum theory and the theory of Timoshenko-type shells. A densely perforated cylindrical shell is considered, in which one end is fixed, and a movable clamping with an axial velocity is determined at the other end. The value of the velocity was determined in such a way that the inertia forces were small. The sector of a cylindrical shell with two rows of structural elements is considered. In this case, it is possible to obtain axisymmetric forms of stability loss with small hole sizes. The shell with two rows of holes in the circumferential direction made it possible to see the appearance of non-axisymmetric shapes. The number of structural elements varied along the axis. The porosity, thickness and length of the shell are unchanged. The sector angle changed in proportion to the increase in the size of the structural element. The finite element mesh was the same inside one structural element in all considered variants, and the number of finite elements varied in proportion to the number of shell holes.

The relative error in determining the critical load is shown in figure 3. The change in the ratio $h/d_0$ corresponds to a decrease in the number of holes along the shell axis at a constant thickness, length, and porosity of the shell. The value of the critical force is the value of the maximum axial load. The figure 3 shows two curves corresponding to different thickness values of the shell ($h/D$) with an unchanged diameter. The porosity is constant and equal to 0.1. A short cylindrical shell was considered with a ratio $L/D = 0.4$. Reference result is the value of the critical load at the maximum ratio $h/d_0$ for each of the values of thickness. It is corresponds to the largest number of holes along the shell. Difference of the critical load from the reference value by more than 5% determined the limit of
applicability of the two-dimensional similarity principle. The relative error was calculated using the formula:

\[ \delta = \left| \frac{F_{h} - F_{0}}{F_{0}} \right| \cdot 100\% - \text{relative error}, \]

\( F_{0} \) – reference value of critical load,
\( F_{h} \) – critical load value for ratio \( h/d_0 \).

\[ \text{Figure 3. Relative error of the critical load as a function of } h/d_0 \text{ for densely perforated shells of different thickness} \]

Obviously, the application of the two-dimensional similarity principle does not depend on the thickness of the shell. For \( h/D = 0 \), the shell degenerates into a plate. Thus, the two-dimensional similarity principle is effective for solving problems of plate stability.

Figure 4 shows the relative error of the critical load, depending on the number of structural elements along the axis for different values of the shell porosity. The parameter of thickness is constant and equal to \( h/D = 0.00078 \). The calculations are made using the theory of the Tymoshenko-type shells and the continuum theory. For each porosity value, the reference solution is the value of the critical load at the highest ratio \( h/d_0 \), using three-dimensional modeling.

The theory of Tymoshenko-type shells for porosity greater than 0.5 is not applicable, because it gives an error of more than 5% in comparison with the standard three-dimensional solution.

The principle of two-dimensional similarity makes it possible to reduce the number of structural elements by 5 times with a porosity value of less than 0.1. For a porosity value of 0.65, the number of structural elements may be reduced by a factor of 2. In this case, the critical load varies less than 5%. With a porosity of more than 0.25, the change in the size of the structural element transforms the axisymmetric form of stability loss into a non-axisymmetric form.
Figure 4. Relative error of the critical load as a function of $h/d_o$ for different values of porosity.
4. Conclusion
With the use of finite element analysis, the applicability limits of the theory of Timoshenko-type plates are determined by estimating the difference in the value of the angle of rotation (5%) from the solution of the problem of elastoplastic complete three-dimensional bending of the structural element. It is determined that the applicability limit for elastic setting is much wider than for elastoplastic. The range of applicability expands with decreasing porosity.

The applicability limits of the Tymoshenko model and the two-dimensional similarity principle are investigated for stability problems of densely perforated elastoplastic cylindrical shells under axial compression. The results obtained with full three-dimensional modeling were used as a reference result. The applicability of the Tymoshenko model for cylindrical shells is limited by the porosity values of 0.5. The principle of two-dimensional similarity makes it possible to reduce the number of structural elements 5 times for porosities 0.1 and 2 times for porosities 0.65.

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