The hierarchy of the preferred scales in the fractal universe

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Abstract

The fractal structure with a power index of 2 is considered within the framework of the universe with the linear law of evolution. The fractal structure arises due to the linear evolution of the scale of mass. Potential fluctuations connect two scales and thus define the preferred scales. The hierarchy of the preferred scales is developed which includes universe, superclusters, clusters of galaxies, galaxies, star clusters and stars.

1 Introduction

It has been suggested that the universe can be viewed as a fractal [1], [2] where the density of the matter obeys the law

$$\rho \propto r^{-D}.$$ (1)

If the fractal power index $D = 2$, all the objects in the universe are self-similar, since the gravitational potential do not change with radius $r$

$$\varphi = \frac{Gm}{r} = G\rho r^2 = \text{const.}$$ (2)

Fractal galaxy distribution was discussed in [3], [4], [5], [6]. This can be described in terms of the radial density run

$$N(< R) = \int_0^R dr \sum_i \delta(r - r_i) \propto R^D$$ (3)

where $N(< R)$ is the average number of galaxies within radius $R$ from any given galaxy. The conventional point of view [7], [8], [9], [10] is that, on scales $< 20 \, h^{-1} \, \text{Mpc}$, galaxies obeys $D \approx 1.2 - 2.2$. On scales $> 20 \, h^{-1} \, \text{Mpc}$, the fractal power index increases with scale towards the value $D = 3$ on scales of about $100 \, h^{-1} \, \text{Mpc}$. On the contrary Pietronero and collaborators [11], [12], [13] claimed that galaxies have a fractal distribution with constant $D \approx 2$ on all scales.

In the model of the universe with the linear law of evolution [14], the density of the matter obeys the law (1) with the power index $D = 2$. Such a law arises due to the linear evolution of the scale of mass with time. Beneath, within the framework of this model, the fractal structure of the universe will be considered.
2 The universe with the linear law of evolution

Let us consider the model of the homogeneous and isotropic universe \[14\] based on the premise that the coordinate system of reference is not defined by the matter but is a priori specified. Take the coordinate system of reference in the form
\[
dl^2 = a(t)^2 d\tilde{l}^2, \quad t
\]
where \(d\tilde{l}^2\) is the Euclidean metric, and \(t\) is the absolute time. The scale factor of the universe is a function of time. Specify the evolution law of the scale factor as linear when the scale factor grows with the velocity of light
\[
a = ct.
\]
Consider the universe as a particle relative to the coordinate system of reference. The total mass of the universe relative to the coordinate system of reference includes the mass of the matter and the energy of its gravity. Adopt the total mass of the universe equal to zero, then the mass of the matter is equal to the energy of its gravity
\[
c^2 = \frac{Gm}{a}.
\]
Allowing for eq. \((5)\), from eq. \((6)\) it follows that the mass of the matter changes with time as
\[
m = \frac{c^2a}{G} = \frac{c^3t}{G},
\]
and the density of the matter, as
\[
\rho = \frac{3c^2}{4\pi Ga^2} = \frac{3}{4\pi Gt^2}.
\]
So the universe with the linear law of evolution has the fractal structure with the power index \(D = 2\). The fractal structure arises due to the linear evolution of the scale of mass with time and hence due to the linear dependence of the scale of mass on the distance \(M \propto t \propto r\).

3 The permanent hierarchy of scales

In view of eq. \((8)\), every distance defines its own density
\[
\rho_i \sim r_i^{-2}.
\]
The objects of the radii \(r_i\) are arranged in the permanent hierarchy of scales. This hierarchy arises due to the evolution of the scale of mass. Since galaxies and clusters of galaxies approximately obey the law \((9)\), the formation of these is not caused by the growth of the density fluctuations by gravitational instability. Since stars do not obey the law \((9)\), it is naturally to think that stars are formed due to the growth of the density fluctuations by gravitational instability. In this case the radius \(r_i\) defines the size of the region from which the star forms by gravitational instability.
In view of eq. (9), the Jeans length for the region of the radius \( r_i \) is given by

\[ \lambda_{Ji} \propto \rho_i^{-1/2} \propto r_i. \]  

(10)

So all the regions of the radii \( r_i \) are of scale invariance from the viewpoint of growth of density fluctuations by gravitational instability.

Potential fluctuations connect two scales, the scale of homogeneity and the scale of fluctuations

\[ \frac{\delta M_i}{M_i} = \frac{\delta \varphi}{\varphi}. \]  

(11)

Here \( M_i \) is the scale of homogeneity, and \( \delta M_i \) is the scale of fluctuations. The size of fluctuations \( \delta r_i \) is given by

\[ \frac{\delta r_i}{r_i} = \left( \frac{\delta M_i}{M_i} \right)^{1/2}. \]  

(12)

In the epoch of recombination \( z = 1400 \), the potential fluctuations are of order of the cosmic microwave background (CMB) anisotropy \( \delta \varphi/\varphi \sim \delta T/T \sim 10^{-5} \) \[15\]. Hence the size of fluctuations is of order \( \delta r_i \sim r_i \times 10^{-2.5} \).

Before recombination, the Jeans length is of order of the size of the region \( \lambda_{Ji} \sim r_i \). Hence the size of fluctuations is less than the Jeans length. After recombination, the Jeans length decreases \[16\] and becomes of order \( \lambda_{Ji} \sim r_i \times 10^{-4} \). Hence the size of fluctuations becomes more than the Jeans length, and the density fluctuations grow by gravitational instability.

4 The hierarchy of the preferred scales

Consider the hierarchy of the preferred scales arranged in the following way. Let potential fluctuations connect two scales

\[ \frac{M_i}{M_j} = \frac{\delta \varphi}{\varphi}. \]  

(13)

Here \( M_i \) is the scale of homogeneity, and \( M_j \) is the scale of fluctuations. \( M_j \) being the scale of fluctuations relative to the scale \( M_i \), in turn, defines another scale of homogeneity.

Develop the hierarchy of the preferred scales starting from the scale defined by the mass and the radius of the universe. Determine the modern age of the universe within the framework of the universe with the linear law of evolution \[14\]. Since density of the relativistic matter is defined by its temperature as

\[ \rho \sim T^4, \]  

from eq. (8) it follows that the temperature of the relativistic matter changes with time as

\[ T \sim a^{-1/2} \sim t^{-1/2}. \]  

(15)

In view of eq. (15), the modern age of the universe is given by

\[ t_0 = \alpha t_{Pl} \left( \frac{T_{Pl}}{T_0} \right)^2 \]  

(16)
where $\alpha$ is the electromagnetic coupling, the subscript $Pl$ corresponds to the Planck period, the subscript 0 corresponds to the modern period. Calculations yield the value $t_0 = 1.06 \times 10^{18}$ s. In view of eq. (3), the mass of the universe is $M_U = 4.29 \times 10^{56}$ g. This value corresponds to the relativistic matter. To transit to the usual matter it is necessary to multiply the value by a factor of 2. In view of eq. (5), the radius of the universe is $r_U = 3.18 \times 10^{28}$ cm.

The potential fluctuation $\delta \varphi/\varphi$ can be determined from the CMB spectrum. The size of the potential fluctuation $\Delta r$ represents the feature in the CMB spectrum. In the fractal universe, the multipole in the CMB spectrum is given by

$$\ell_{eff} = \left(\frac{\Delta r}{r}\right)^{-1} = \left(\frac{\Delta M}{M}\right)^{-1/2} = \left(\frac{\delta \varphi}{\varphi}\right)^{-1/2}.$$  \hspace{1cm} (17)

Anisotropy measurements on degree scales pin down the feature in the CMB spectrum. The position of the feature is $\ell_{eff} = 263$ \cite{17}, $\ell_{eff} = 260$ \cite{18}. Adopt the value $\ell_{eff} = 260$. This corresponds to the potential fluctuation $\delta \varphi/\varphi = 1.48 \times 10^{-5}$.

With the use of the above determined mass and radius of the universe and potential fluctuation, develop the hierarchy of the preferred scales.

$$M_1 = 8.6 \times 10^{56} \text{ g} \quad r_1 = 3.2 \times 10^{28} \text{ cm}$$
$$M_2 = 1.3 \times 10^{52} \text{ g} \quad r_2 = 1.2 \times 10^{26} \text{ cm}$$
$$M_3 = 1.9 \times 10^{47} \text{ g} \quad r_3 = 4.7 \times 10^{23} \text{ cm}$$
$$M_4 = 2.8 \times 10^{42} \text{ g} \quad r_4 = 1.8 \times 10^{21} \text{ cm}$$
$$M_5 = 4.1 \times 10^{37} \text{ g} \quad r_5 = 7.0 \times 10^{18} \text{ cm}$$
$$M_6 = 6.1 \times 10^{32} \text{ g} \quad r_6 = 2.7 \times 10^{16} \text{ cm}$$

Here the second scale can be identified with superclusters, the third scale can be identified with clusters of galaxies, the fourth scale can be identified with galaxies, the fifth scale can be identified with star clusters, the sixth scale can be identified with stars. The radius $r_6 = 2.7 \times 10^{16}$ cm corresponds to the size of the region from which the star forms by gravitational instability.

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