ON THE ERDŐS–LAX INEQUALITY CONCERNING POLYNOMIALS

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Abstract. If $P(z)$ is a polynomial of degree $n$ which does not vanish in $|z| < k$, where $k \leq 1$, then N. K. Govil [On a theorem of S. Bernstein, Proc. Nat. Acad. Sci., 50 (1980), 50–52] proved that

$$\max_{|z|=1} |P'(z)| \leq \frac{n}{1 + k^n} \max_{|z|=1} |P(z)|,$$

provided $|P'(z)|$ and $|Q'(z)|$ attain maximum at the same point on $|z| = 1$, where $Q(z) = z^n P(1/z)$. In this paper, we obtain certain refinements and generalizations of this inequality and related results.

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