NON-THERMAL FEATURES IN THE COSMIC NEUTRINO BACKGROUND

GIANPIERO MANGANO

INFN, Sezione di Napoli
Dipartimento di Scienze Fisiche, Università di Napoli "Federico II",
Monte SANT’Angelo, Via Cintia, I-80126 Napoli, Italy
E-mail:mangano@na.infn.it

Abstract

I review some of the basic information on the Cosmic Neutrino Background momentum distribution. In particular, I discuss how present data from several cosmological observables such as Big Bang Nucleosynthesis, Cosmic Microwave Background and Large Scale Structure power spectrum constrain possible deviations from a standard Fermi-Dirac thermal distribution.

1 The standard picture

The large amount of observations accumulated in the last decades, see [1]-[9] as a highly incomplete list of references, provided an unprecedented improvement in our understanding of the general features of the observable Universe. In fact, this tremendous experimental effort, supplemented by the generic predictions of inflationary models give a nicely consistent picture which is nowadays customary to refer to as the Standard or Concordance Cosmological Model. Despite of the fact that there are still unsolved major problems, as the nature of Dark Matter or the so fine tuned value of the cosmological constant, it is remarkable that the overall picture of the evolution of the Universe can be described in terms of a relatively small number of parameters defining the $\Lambda CDM$ model, namely: 1) the (cold+hot) dark matter density $\omega_{dm} = \Omega_{dm} h^2$, 2) the baryon density $\omega_b$, 3) the value of the cosmological constant or dark energy density $\Omega_{\Lambda}$, 4) the normalized value of the Hubble parameter today $h$, 5) the primordial perturbation spectrum tilt $n_s$ and 6) the normalization $\ln[10^{10} R_{rad}]$ where $R_{rad}$ is the curvature perturbation in the radiation era, 7) the optical depth to reionization $\tau$, 8) the linear theory amplitude of matter fluctuations at 8 $h^{-1}$ Mpc $\sigma_8$.

Of course, more exotic scenarios can be considered by adding other (theoretically motivated) free parameters, such as extra relativistic degrees of freedom, Quintessence equation of state, non spatially flat Universe etc, which might help in improving the agreement of different experimental observations [10]. In general, the present level of precision of data implies that several ideas, both theoretically motivated or suggested by experimental results obtained in different frameworks, can be tested by checking their implications at the cosmological level. As an example, the overall scale of neutrino masses $m_0$ can be bound by comparing data with the expected free
streaming suppression effect of Large Scale Structure (LSS) power spectrum on scales smaller than $l_{nr}$, the horizon when neutrinos become non-relativistic \[ l_{nr} \sim 38.5 \left( \frac{1 \text{ eV}}{m_0} \right)^{1/2} \omega_m^{-1/2} \text{ Mpc} \]

which gives $\sum m_\nu \leq 0.68 \text{ eV}$ \[10\], where the sum is over the three neutrino species. Notice that this bound is stronger than present constraint from earth-based $^3H$ decay experiments and of the order of the sensitivity which will be reached in the near future \[13\].

It is well known that in the framework of the Hot Big Bang model we expect the Universe to be filled by a large amount of neutrinos, with a Fermi-Dirac distribution (in the standard scenario) characterized by a temperature of $1.95 \, ^o K$ and density of $112 \, \text{cm}^{-3}/\text{flavor}$. These relic neutrinos decoupled from the electromagnetic plasma quite early in time, when weak interaction rates became slower than the Hubble rate for (photon) temperatures in the range $2 \div 4 \, \text{MeV}$, just before Big Bang Nucleosynthesis (BBN) took place. Unfortunately, the fact that this Cosmic Neutrino Background (C$\nu$B) has today a very small kinetic energy, of the order at most of $10^{-6} \, \text{eV}$, and that neutrinos only interact via weak interactions, prevents us from any possible direct detection of this background on the Earth (see e.g. \[14, 15\] for a recent review).

Nevertheless, there are several indirect ways to constrain the C$\nu$B by looking at cosmological observables which are influenced either by the fact that neutrinos contribute to the Universe expansion rate at all stages, or also via their interactions with the electromagnetic plasma and baryons before their decoupling. In this respect, one of the most sensitive probe is represented by the values of the light nuclide abundances produced during BBN. Actually, the final yields of Deuterium, $^7\text{Li}$ and in particular of $^4\text{He}$ strongly depend on the number of neutrino species as well as on their distribution in phase space at about $1 \, \text{MeV}$ when the neutron to proton density ratio freezes. In fact, since neutrinos were in chemical equilibrium with the electromagnetic plasma till this epoch we know by equilibrium thermodynamics that they were distributed according to a Fermi-Dirac function, yet BBN can constrain exotic features like the value of their chemical potential \[16, 17\].

Once decoupled, neutrinos affect all key cosmological observables which are governed by later stages of the evolution of the Universe only via their coupling with gravity. A well known example is their contribution to the total relativistic energy density, which affects the value of the matter-radiation equality point, which in turn influences the Cosmic Microwave Background (CMB) anisotropy spectrum, in particular around the scale of the first acoustic peak. Similarly, as already mentioned their number density and their masses are key parameters in the small scale suppression of the power spectrum of LSS.

The main question which will be addressed in the following Sections is in my opinion particularly intriguing: how present (and future) cosmological observations
can prove that indeed neutrinos are thermally distributed? I will first consider the non thermal features in the CνB distribution arising from the neutrino decoupling stage and then discuss the issue on more general grounds.

2 Neutrino decoupling

Shortly after neutrino decoupling the temperature of the electromagnetic plasma drops below the electron mass, favoring $e^\pm$ annihilations that heat the photons. Assuming that this entropy transfer did not affect the neutrinos because they were already completely decoupled, it is easy to calculate the well-known difference between the temperatures of relic photons and neutrinos $T/T_\nu = (11/4)^{1/3} \approx 1.40$. However, the processes of neutrino decoupling and $e^\pm$ annihilations are sufficiently close in time so that some relic interactions between $e^\pm$ and neutrinos exist. These relic processes are more efficient for larger neutrino energies, leading to non-thermal distortions in the neutrino spectra and a slightly smaller increase of the comoving photon temperature. These distortions have been computed by several authors by explicitly solving the related Boltzmann kinetic equations [18]-[26], and result to be very small at the level of few percent, so that their direct observation is presently out of question. However, they should be included in any calculation of observables which are influenced by relic neutrinos. For instance, non-thermal distortions lead to an enhanced energy density of relic neutrinos parameterized in terms of the so-called effective number of neutrinos [27] $N_{\text{eff}}$

$$
\rho_R = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma ,
$$

(2.1)

where $\rho_\gamma$ is the energy density of photons and $\rho_R$ the total radiation energy density. This parameter influences the CMB anisotropies by shifting the matter-radiation equivalence, which translates into a different power around the first acoustic peak via the Integrated Sachs-Wolfe effect. Future CMB experiments, such as PLANCK [28] or CMBPOL [29] are foreseen to be sensitive to even a tiny change in the Universe radiation content, at the level of percent.

As a second main effect, the distortion on neutrino distribution affects the predictions of BBN. On one hand, the increased value of $N_{\text{eff}}$ shifts the freezing of neutron/proton weak processes, and so the eventual yield of $^4\text{He}$. Furthermore, the corresponding thermal averaged cross sections for these processes are also directly affected by any change in the electron neutrino distribution function. Both effects result into a change of the $^4\text{He}$ mass fraction $Y_p$ at the level of $10^{-4}$ [18]-[24], which is quite small, but it has to be taken into account in precise BBN numerical codes [17, 30, 31].

Finally, distortions in the neutrino distribution also modifies the present neutrino number density, so that the contribution of massive neutrinos to the present energy
density of the universe is also changed. Recently, the neutrino decoupling stage has been studied taking into account the effect of flavor oscillations \[32\]. In this case the neutrino (antineutrino) ensemble is described by a $3 \times 3$ density matrix:

$$
\rho(p, t) =
\begin{pmatrix}
\rho_{ee} & \rho_{e\mu} & \rho_{e\tau} \\
\rho_{\mu e} & \rho_{\mu\mu} & \rho_{\mu\tau} \\
\rho_{\tau e} & \rho_{\tau\mu} & \rho_{\tau\tau}
\end{pmatrix}.
$$

(2.2)

The diagonal elements correspond to the usual occupation numbers of the different flavors, while the off-diagonal terms account for neutrino mixing.

The equations of motion for the density matrices can be cast in the form \[33\]

$$
i(\partial_t - H p \partial_p) \rho = \left[ \left( \frac{M^2}{2p} - \frac{8\sqrt{2}G_F}{3m_W^2} E \right), \rho \right] + C[\rho],
$$

(2.3)

where $H$ is the Hubble parameter. The first term in the commutator corresponds to vacuum oscillation and is proportional to $M^2$, the mass-squared matrix in the flavor basis, related to the diagonal one in the mass basis $\text{diag}(m_1^2, m_2^2, m_3^2)$ via the neutrino mixing matrix. As a reference, in \[32\] the best-fit values from ref. \[34\] were considered

$$
\left( \Delta m_{21}^2, \Delta m_{31}^2, \sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13} \right) = (8.1, 2.2, 0.3, 0.5, 0),
$$

(2.4)

along with the $3\sigma$ upper bound on $\theta_{13}, \sin^2 \theta_{13} = 0.047$.

The decoupling of neutrinos takes place at temperatures of the order MeV, when neutrinos experience both collisions, described by the collisional integral $C[\rho]$ and refractive effects from the medium. The latter correspond in Eq. (2.3) to the term proportional to the diagonal matrix $E$, the energy densities of charged leptons.

In Fig. 1, it is shown the evolution of the distortion of the neutrino distribution as a function of $x = m_e R$ for a particular neutrino comoving momentum ($y = kR = 10$), $R$ being the scale factor. At large temperatures or small $x$, neutrinos are in good thermal contact with $e^\pm$. As $x$ grows weak interactions become less effective in a momentum-dependent way, leading to distortions in the neutrino spectra which are larger for $\nu_e$'s than for the other flavors. Finally, at larger values of $x$ neutrino decoupling is complete and the distortions reach their asymptotic values. Fig. 2 shows the asymptotic values of the flavor neutrino distribution, for the cases without oscillations and with non-zero mixing. The dependence of the non-thermal distortions in momentum is well understood and reflects the fact that more energetic neutrinos were interacting with $e^\pm$ for a longer period. Neutrino oscillations reduce the difference between the flavor neutrino distortions, and slightly change the final value of $N_{\text{eff}}$ and of the comoving photon temperature $z = T/R$. Both effects translate into a small change of the $^4\text{He}$ mass fraction, as also reported in Table 1.
For completeness, I also report the value of the neutrino distribution function as obtained by fitting the numerical results for $\theta_{13} = 0$ \[32\]

\[
\begin{align*}
    f_{\nu_e}(y) &= f_{\nu}(y) \left[ 1 + 10^{-4} \left( 1 - 2.2y + 4.1y^2 - 0.047y^3 \right) \right] \\
    f_{\nu_{\mu,\tau}}(y) &= f_{\nu}(y) \left[ 1 + 10^{-4} \left( -4 + 2.1y + 2.4y^2 - 0.019y^3 \right) \right]
\end{align*}
\] (2.5)

These expressions can be easily incorporated into the numerical tools such as CMBFAST \[35\] or CAMB \[36\] used to compute CMB and LSS spectra. In fact, as long as neutrinos are still relativistic the net effect of the phase space distribution distortion is only via the integrated effect provided by $N_{\text{eff}}$, but a more careful analysis of effects when neutrinos become non-relativistic should take into account distortions as a function of neutrino momenta. As an example, it is easy to calculate from (2.5) the present value of neutrino energy density. For the simplest case of (almost) degenerate neutrinos, i.e. for the neutrino squared mass scale $m_0^2$ much larger than the atmospheric squared mass difference $\Delta m_{31}^2$ one gets

\[
\Omega_\nu h^2 = \frac{3m_0}{93.14 \, \text{eV}}, \tag{2.6}
\]

where the value in the denominator is slightly smaller than the analogous result in the instantaneous decoupling limit (94.12).
Figure 2: Frozen distortions of the flavor neutrino spectra as a function of the comoving momentum. In the case with $\theta_{13} \neq 0$ one can distinguish the distortions for $\nu_\mu$ (upper line) and $\nu_\tau$ (lower line). From [32].

Table 1: Frozen values of $z$, the neutrino energy density distortion $\delta \rho_{\nu_\alpha} \equiv \delta \rho_{\nu_\alpha}/\rho_{\nu_0}$, $N_{\text{eff}}$ and $\Delta Y_p$ including flavor neutrino oscillations. From [32]

| $\theta_{13}$ | $z$ | $\delta \rho_{\nu_e}$ | $\delta \rho_{\nu_\mu}$ | $\delta \rho_{\nu_\tau}$ | $N_{\text{eff}}$ | $\Delta Y_p$ |
|---------------|-----|----------------------|----------------------|----------------------|----------------|-------------|
| $\theta_{13} = 0$ | 1.3978 | 0.73% | 0.52% | 0.52% | 3.046 | $2.07 \times 10^{-4}$ |
| $\sin^2 \theta_{13} = 0.047$ | 1.3978 | 0.70% | 0.56% | 0.52% | 3.046 | $2.12 \times 10^{-4}$ |
| Bimaximal ($\theta_{13} = 0$) | 1.3978 | 0.69% | 0.54% | 0.54% | 3.045 | $2.13 \times 10^{-4}$ |

3 Bounds on non-thermal features in the C$\nu$B

Apart from the non-thermal features discussed so far, which are expected in the framework of Standard Model electroweak interactions, neutrino distribution might be quite different if neutrinos also interact with exotic form of matter. Therefore, obtaining information on the C$\nu$B distribution is a way to constrain new physics beyond our present knowledge of fundamental interactions. Thus, it is an interesting issue to understand to what extent present and future cosmological observations can bound such exotic interactions by observing the C$\nu$B momentum distribution.

On completely general ground, we can specify the neutrino distribution $f_\alpha(y)$ by the set of moments $Q_\alpha^{(n)}$ [37]

$$Q_\alpha^{(n)} = \frac{1}{n^2} \left( \frac{4}{11} \right)^{(3+n)/3} T^{3+n} \int y^{2+n} f_\alpha(y) \, dy ,$$

(3.1)

where the $Q_\alpha^{(n)}$ have been normalized to the standard value of the neutrino tempera-
ture in the instantaneous decoupling limit \( T_\nu = (4/11)^{1/3} T \). Notice that all moments can be defined if one assumes that neutrino distribution decays at large comoving momentum as \( \exp(-y) \). This is quite expected since at very high \( y \) the shape of the distribution is ruled by the behavior imprinted by neutrino decoupling as hot relics at the MeV scale.

If we denote by \( P_m(y) \), \( m \) being the degree of \( P_m(y) \),

\[
P_m(y) = \sum_{k=0}^{m} c_k^{(m)} y^k , \tag{3.2}
\]

the set of polynomials orthonormal with respect to the measure \( y^2/(\exp(y) + 1) \)

\[
\int_0^\infty dy \frac{y^2}{e^y + 1} P_n(y) P_m(y) = \delta_{nm} , \tag{3.3}
\]

then it follows

\[
df_\alpha(y) = \frac{y^2}{e^y + 1} \sum_{m=0}^{\infty} F_{\alpha,m} P_m(y) dy , \tag{3.4}
\]

where

\[
F_{\alpha,m} = \sum_{k=0}^{m} c_k^{(m)} Q^{(k)}_{\alpha} T_\nu^{-k} , \tag{3.5}
\]

i.e. a linear combination of moments up to order \( m \) with coefficients \( c_k^{(m)} \).

For a Fermi-Dirac distribution all moments can be expressed in terms of the number density \( Q_{\alpha}^{(0)} \) or, equivalently, as functions of the only independent parameter \( T_\nu \), the first two moments being related to \( N_{\text{eff}} \) and \( \Omega_\nu \) today, respectively

\[
\omega_\nu = \Omega_\nu h^2 = 0.058 \frac{m_0}{eV} \frac{11}{4} T_\nu^{-3} Q_{\alpha}^{(0)} , \tag{3.6}
\]

\[
N_{\text{eff}} = \frac{120}{\pi^2} \left( \frac{11}{4} \right)^{4/3} T_\nu^{-4} \sum_{\alpha} Q_{\alpha}^{(1)} , \tag{3.7}
\]

where we have assumed for simplicity that the three neutrinos share the same distribution.

In the following I consider only the first two moments \( \omega_\nu \) and \( N_{\text{eff}} \) as free and independent parameters, to be constrained using cosmological data [37]. In fact, the only way to decide how many \( Q_{\alpha}^{(n)} \) should be included in the analysis can be only dictated by the sensitivity of the available observational data to the distortion of the neutrino distribution. Presently (and surprisingly), it is already very hard to get quite strong constraints on the first two moments. In case future observations would reach a higher sensitivity on neutrino distribution, it would be desirable to include higher order moments, such as the skewness or the kurtosis, related to \( Q_{\alpha}^{(2)} \) and \( Q_{\alpha}^{(3)} \), respectively.
For the sake of definiteness in \cite{37} a specific example was worked out in details, where neutrinos interact with a light scalar field $\Phi$ with mass $M$ via the interaction lagrangian density

$$L_{int} = \frac{\lambda}{\sqrt{3}} \Phi \sum_i \bar{\nu}_i \nu_i \ . \quad (3.8)$$

This majoron-inspired model have been also considered in \cite{38}-\cite{41} for a very small light $\Phi$ field, even lighter than neutrinos. In this case, neutrino annihilations would results into a neutrinoless cosmological model \cite{41}. On the other hand, for $m_\Phi \geq 2m_0$ the decays of the unstable $\Phi$ particles might lead to non trivial features in neutrino distribution provided

i) decays take place out of equilibrium, \textit{i.e.} for temperatures smaller than the decaying particle mass $M$.

ii) occur after weak interaction freeze-out, otherwise neutrino interactions with the electromagnetic plasma would erase any non thermal features.

Of course, different scenarios can be considered, as for example the case of unstable neutrinos $\nu_h$ decaying into a (pseudo) scalar particle $\Phi$ and a lighter neutrino $\nu_l$,

$$\nu_h \rightarrow \Phi \nu_l \ , \quad (3.9)$$

but I notice that at this stage the main issue is rather to understand how present (and future) data can constrain the non-thermal contribution to the neutrino background, rather than to discriminate among different models. Despite of the fact that in \cite{37} the out of equilibrium $\Phi$ decay case was taken as the reference model, all results are quite general and can be applied to different scenarios as well.

When the $\Phi$ particles decay, the neutrino distribution gets an additional contribution, which in the narrow width limit and in the instantaneous decay approximation at temperature $T_D$ corresponds to a peaked pulse at $y_* = M/(2T_D)$ so that

$$y^2 f(y)dy = y^2 \frac{1}{e^y + 1} dy + \frac{\pi^2 A}{\sqrt{2\pi} \sigma^2} \exp \left\{ - \frac{(y - y_\ast)^2}{2\sigma^2} \right\} \ . \quad (3.10)$$

An example of this non-thermal neutrino spectrum is shown in Figure 3. The parameter $A$ is given by the comoving $\Phi$ number density at decay $A = n_\Phi(T_D)R^3$.

Correspondingly, the lower moments expressed in terms of the parameters of Eqs. \ref{3.7} and \ref{3.6} read

$$\omega_\nu = \frac{m_0}{93.2 \text{ eV}} \left( 1 + 0.99 \frac{2\pi^2}{3\zeta(3)} A \right) \ , \quad (3.11)$$

$$N_{\text{eff}} = 3.04 \left( 1 + 0.99 \frac{120}{7\pi^2} A y_* \right) \ . \quad (3.12)$$
Figure 3: Differential number density of relic neutrinos as a function of the comoving momentum for the non-thermal spectrum in Equation (3.10). The parameters are $A = 0.018$, $y_* = 10.5$ and $\sigma = 1$, which corresponds to $N_{\text{eff}} \simeq 4$. From [37].

For sufficiently weakly interacting $\Phi$ particles, so that they are decoupled from the thermal bath since at least the BBN epoch, the largest value of $A$ can be bound by BBN as a function of the $\Phi$ mass and number density. Large values of $A$ and $M$ in fact, implies a large value of the $\Phi$ energy density during BBN, thus affecting the Hubble expansion rate and the final abundances of $^4$He and Deuterium. Using present data on these two nuclei, see e.g. [30], this translates into $A \leq 0.1$, unless the value of $M$ is order MeV or larger, in which case $A$ is much more constrained, see Fig. 4.

The model described so far can also be tested using CMB and LSS power spectrum. The results are summarized in third column of Table 2. For comparison I show also the corresponding result for the case where neutrino distribution is assumed to be the standard Fermi-Dirac, but the value of $N_{\text{eff}}$ corresponding to extra relativistic degrees of freedom is taken as a free parameter (second column). In particular, the best value of the effective $\chi^2$ for the three models is shown in the first line. Notice that both models do not improve significantly the $\chi^2$ with respect to the standard $\Lambda$CDM, despite of their larger parameter space. Notice that the bound on the parameter $q \equiv \omega_\nu(93.2 \text{ eV}/m_0)$ comes essentially from the BBN prior $A < 0.1$, since the CMB and LSS data alone would be compatible with much larger deviations from a thermal phase-space distribution (up to $A = 1$ at 2-$\sigma$). Similarly, $y_*$ is poorly constrained, and large values for it are still allowed. In the $\Phi$ decay scenario this implies that these decays can take place in highly out of equilibrium conditions, $T_D \ll M$, the only bound being instead on the scalar particle number density at the BBN epoch.

The fact that $A$ and $y_*$ are so poorly constrained is due the existence of a degen-
Figure 4: The 1 \( \sigma \) (thin lines) and 2 \( \sigma \) (thick lines) BBN bounds on the \( \Phi \) number density (normalized to \( T_\nu^3 \)) versus mass \( M \) in MeV. The regions above the contours would be in disagreement with the observed primordial abundances of \( {}^4\)He or D. From [37].

The \( \omega_{dm}, m_0 \) and \( N_{eff} \) for the two models in which the radiation density near the time of decoupling is a free parameter. Values as large as \( \omega_{dm} = 0.23, m_0 = 1.5 \) eV or \( N_{eff} = 9 \) are still allowed at the 2-\( \sigma \) level. An improvement is expected in removing these degeneracies from future experiments, though not particularly dramatic. In Table 3 it is shown the result of a standard Fisher matrix analysis assuming a fiducial model as reported in the first line of the Table. The forecast is based on the foreseen sensitivity of PLANCK combined with the completed SDSS redshift survey with effective volume \( V_{eff} = 1 \) h\(^{-3}\) Gpc\(^3\) and a free bias. While the sensitivity on \( N_{eff} \) is now at the level (or better) than percent, both \( q \) and \( m_0 \) are not very well constrained, since these two parameters are measurable only from the free-streaming effect in the matter power spectrum, while all other parameters have a clear signature in the CMB anisotropies.

4 Conclusions

The role of \( C\nu B \) in cosmology is quite ubiquitous, yet it is quite disappointing that any direct detection of background neutrinos will be very hard to achieve. Through their influence on several cosmological observables several properties of neutrinos can be already constrained, such as their mass, possible finite lifetimes, magnetic moments, etc. On the other hand, possible exotic features in their momentum distribution would also represent quite a unique imprint of new interactions beyond our present understanding of fundamental physics. Unfortunately, present data have not already
Table 2: Minimum value of the effective $\chi^2$ (defined as $-2 \ln L$, where $L$ is the likelihood function) and $1\sigma$ confidence limits for the parameters of the three models under consideration. $\theta$ is the ratio of the sound horizon to the angular diameter distance multiplied by 100, while $\beta$ is the 2dF redshift-space distortion factor. Finally, $q \equiv \omega_\nu(93.2\text{eV}/m_0)$. The last five lines refer to parameters which can be obtained by the set of independent parameters of the first ten lines. From [37].

| Parameter                  | $\Lambda$CDM | $\Lambda$CDM + extra radiation | $\Lambda$CDM+non thermal $\nu$ |
|----------------------------|---------------|---------------------------------|---------------------------------|
| $\chi^2_{\text{min}}$      | 1688.2        | 1688.0                          | 1688.0                          |
| $\ln[10^{10}R_{\text{rad}}]$ | 3.2±0.1       | 3.2±0.1                         | 3.2±0.1                         |
| $n_s$                      | 0.97±0.02     | 0.99±0.03                       | 1.00±0.03                       |
| $\omega_b$                 | 0.0235±0.0010 | 0.0231±0.0010                   | 0.0233±0.0011                   |
| $\omega_{dm}$              | 0.121±0.005   | 0.17±0.03                       | 0.17±0.03                       |
| $\theta$                   | 1.043±0.005   | 1.033±0.006                     | 1.033±0.006                     |
| $\tau$                     | 0.13±0.05     | 0.13±0.06                       | 0.15±0.07                       |
| $\beta$                    | 0.46±0.04     | 0.48±0.04                       | 0.48±0.04                       |
| $m_0$ (eV)                 | 0.3±0.2       | 0.8±0.5                         | 0.7±0.4                         |
| $N_{\text{eff}}$           | 3.04          | 6±2                             | 6±2                             |
| $q$                        | 1             | 1                               | 1.25±0.13                       |
| $h$                        | 0.67±0.02     | 0.76±0.06                       | 0.76±0.05                       |
| Age (Gyr)                  | 13.8±0.2      | 12.1±0.9                        | 12.1±0.8                        |
| $\Omega_\Lambda$          | 0.68±0.03     | 0.67±0.03                       | 0.67±0.03                       |
| $z_{\text{re}}$            | 14±4          | 16±5                            | 18±6                            |
| $\sigma_s$                | 0.76±0.06     | 0.77±0.07                       | 0.77±0.07                       |

reached enough sensitivity to put severe constraints in this respect, despite of the huge improvements in the last years, but this sensitivity is likely to be highly improved in the near future. Waiting for these new exciting times it is worth scrutinizing new theoretical perspectives, keeping in mind as a general warning what Pauli said to his friend W. Baade about his neutrino hypothesis [42] (see D. Haidt in these Proceedings)

"... Today I have done something which no theoretical physicist should ever do: I have predicted something which shall never be detected experimentally..."

5 Acknowledgments

I warmly thank A. Cuoco, J. Lesgourgues, G. Miele, S. Pastor, M. Peloso, O. Pisanti, T. Pinto and P.D. Serpico for all discussions and fruitful collaboration on the topics summarized in this paper.
| name          | $n_s$ | $\omega_b$ | $\omega_m$ | $\Omega_\Lambda$ | $\tau$ | $m_0$ (eV) | $N_{\text{eff}}$ | $q$ |
|--------------|------|-----------|------------|----------------|------|-----------|----------------|---|
| fiducial values | 0.96 | 0.023    | 0.14      | 0.70        | 0.11 | 0.5       | 4.0          | 1.1 |
| $1\sigma$ error | 0.009 | 0.0003 | 0.004    | 0.02      | 0.005 | 0.3       | 0.1          | 0.7 |

Table 3: Expected errors on the parameters of the $\Lambda$CDM + non thermal neutrino model. The first line shows the fiducial values, i.e. the parameter values assumed to represent the best fit to the data. The second line give the forecast for the associated $1\sigma$ errors for PLANCK + SDSS.

References

[1] C.L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003) [arXiv:astro-ph/0302207].

[2] D.N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003) [arXiv:astro-ph/0302209].

[3] R. Rebolo et al., Mon. Not. Roy. Astron. Soc. 353, 747 (2004) [arXiv:astro-ph/0402466]; C. Dickinson et al., Mon. Not. Roy. Astron. Soc. 353, 747 (2004) [arXiv:astro-ph/0402498].

[4] T.J. Pearson et al., Astrophys. J. 591, 556 (2003) [arXiv:astro-ph/0205388]; J.L. Sievers et al., Astrophys. J. 591, 599 (2003) [arXiv:astro-ph/0205387]; A.C.S. Readhead et al., Astrophys. J. 609, 498 (2004) [arXiv:astro-ph/0402359].

[5] C. Kuo et al., Astrophys. J. 600, 32 (2004) [arXiv:astro-ph/0212289]; J.H. Goldstein et al., Astrophys. J. 599, 773 (2003) [arXiv:astro-ph/0212517].

[6] J.A. Peacock et al., Nature 410, 169 (2001) [arXiv:astro-ph/0103143].

[7] W.J. Percival et al., Mon. Not. Roy. Astron. Soc. 327, 1297 (2001) [arXiv:astro-ph/0105252]; Mon. Not. Roy. Astron. Soc. 337, 1068 (2002) [arXiv:astro-ph/0206256].

[8] M. Tegmark et al., Astrophys. J. 606, 702 (2004) [arXiv:astro-ph/0310725].

[9] A.G. Riess et al., Astrophys. J. 607, 665 (2004) [arXiv:astro-ph/0402512].

[10] D.N. Spergel et al, Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, march 2006, [http://lambda.gsfc.nasa.gov](http://lambda.gsfc.nasa.gov).

[11] W. Hu, D.J. Eisenstein and M. Tegmark, Phys. Rev. Lett. 80, 5255 (1998) [arXiv:astro-ph/9712057].

[12] J. Lesgourgues and S. Pastor, [arXiv:astro-ph/0603494](http://arxiv.org/abs/astro-ph/0603494).
[13] A. Osipowicz et al. [KATRIN Coll.], arXiv:hep-ex/0109033; B. Bornschein [KATRIN Coll.], contribution to the Proceedings of AHEP 2003, Valencia (Spain), published in JHEP Proc. AHEP2003/064.

[14] G.B. Gelmini, arXiv:hep-ph/0412305.

[15] C. Hagmann, arXiv:astro-ph/9905258.

[16] A.D. Dolgov et al., Nucl. Phys. B 632, 363 (2002) arXiv:hep-ph/0201287.

[17] A. Cuoco et al., Int. J. Mod. Phys. A 19, 4431 (2004) arXiv:astro-ph/0307213.

[18] S. Hannestad and J. Madsen, Phys. Rev. D 52 (1995) 1764 arXiv:astro-ph/9506015.

[19] S. Dodelson and M.S. Turner, Phys. Rev. D 46 (1992) 3372.

[20] A.D. Dolgov and M. Fukugita, Phys. Rev. D 46 (1992) 5378.

[21] B.D. Fields, S. Dodelson and M.S. Turner, Phys. Rev. D 47 (1993) 4309 arXiv:astro-ph/9210007.

[22] A.D. Dolgov, S.H. Hansen and D.V. Semikoz, Nucl. Phys. B 503 (1997) 426 arXiv:hep-ph/9703315.

[23] A.D. Dolgov, S.H. Hansen and D.V. Semikoz, Nucl. Phys. B 543 (1999) 269 arXiv:hep-ph/9805467.

[24] S. Esposito et al, Nucl. Phys. B 590 (2000) 539 arXiv:astro-ph/0005573.

[25] N. Fornengo, C.W. Kim and J. Song, Phys. Rev. D 56 (1997) 5123 arXiv:hep-ph/9702324.

[26] G. Mangano, G. Miele, S. Pastor and M. Peloso, Phys. Lett. B 534 (2002) 8 arXiv:astro-ph/0111408.

[27] V.F. Shvartsman, Pisma Zh. Eksp. Teor. Fiz. 9 (1969) 315 [JETP Lett. 9 (1969) 184].

[28] PLANCK Coll., www.rssd.esa.int/index.php?project=PLANCK

[29] CMBPOL, http://universe.nasa.gov/program/inflation.html

[30] P.D. Serpico et al., JCAP 0412, 010 (2004) arXiv:astro-ph/0408076.

[31] R.H. Cyburt, Phys. Rev. D 70, 023505 (2004) arXiv:astro-ph/0401091.
[32] G. Mangano, G. Miele, S. Pastor, T. Pinto, O. Pisanti, and P.D. Serpico, Nucl. Phys. B 729, 221 (2005) arXiv:hep-ph/0506164.

[33] G. Sigl and G. Raffelt, Nucl. Phys. B 406 (1993) 423.

[34] M. Maltoni, T. Schwetz, M.A. Tórtola and J.W.F. Valle, New J. Phys. 6 (2004) 122 arXiv:hep-ph/0405172.

[35] U. Seljak and M. Zaldarriaga, Astrophys. J. 469 (1996) 437 arXiv:astro-ph/9603033. See also the webpage http://cmbfast.org

[36] A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. 538 (2000) 473 arXiv:astro-ph/9911177. See also the webpage http://camb.info

[37] A. Cuoco, J. Lesgourgues, G. Mangano, and S. Pastor, Phys. Rev. D 71, 123501 (2005) arXiv:astro-ph/0502465.

[38] S. Hannestad, JCAP 0502, 011 (2005). arXiv:astro-ph/0411475.

[39] S. Hannestad and G. Raffelt, Phys. Rev. D 72, 103514 (2005) arXiv:hep-ph/0509278.

[40] N.F. Bell, E. Pierpaoli, and K. Sigurdson, arXiv:astro-ph/0511410.

[41] J.F. Beacom, N.F. Bell, and S. Dodelson, Phys. Rev. Lett. 93, 121302 (2004) arXiv:astro-ph/0404585.

[42] F. Hoyle, Proc. Roy. Soc. London A 301, 171 (1967).