Semi-inclusive deeply inelastic (anti)neutrino nucleus scattering

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The (anti)neutrino nucleus scattering plays a very important role in probing the hadronic structure as well as the electroweak phenomenologies. To this end, we calculate the jet production semi-inclusive deeply inelastic (anti)neutrino nucleus scattering process. The initial (anti)neutrino is assumed to be scattered off by a target particle with spin 1. Due to the limitation of the factorization theorem, calculations are carried out in the quantum chromodynamics parton model framework up to tree level twist-3. We consider both the neutral current and the charged current processes and write them into a unified form due to the similar interaction forms. Considering the angular modulations and polarizations of the cross section, we calculate the complete azimuthal asymmetries. We also calculate the intrinsic asymmetries which reveal the imbalance in the distribution of the intrinsic transverse momentum of the quark. We find that these asymmetries can be expressed in terms of the transverse momentum-dependent parton distribution functions (TMD PDFs) and the electroweak couplings. With the determined couplings, these asymmetries can be used to extract the TMD PDFs and further to study the hadronic structures.

I. INTRODUCTION

Our understanding of the parton-level structure in the nucleon comes predominantly from the lepton \((e^-,\mu^-)\) deeply inelastic scattering (DIS) measurements in which parton distribution functions (PDFs) are extracted with very high precision. This process provides a great opportunity to understand the parton model and/or the factorization theorem. It will still play an important role in the future Electron-Ion collider \([1-3]\) experiment to explore the spin and three-dimensional structure of the nucleon over wide kinematic regions. In addition to the charged lepton DIS, (anti)neutrino nucleus scattering is also important in studying the nucleon and/or nucleus structures. On the one hand, it provides information on the flavor separation which can not be realized in the charged lepton (SI)DIS experiments alone. On the other hand, the (anti)neutrino nucleus scattering can be used to study the EMC effect \([4]\) since high \(Z\) nuclei are usually used in the scattering experiments because of the very weak interaction between the (anti)neutrino and the nucleus. The DIS reaction is proposed mainly for the study of strong interactions but it has abilities to study electroweak physics by considering the neutral current and charged current interactions, i.e., interactions are mediated by the \(Z^0\) and \(W^\pm\) bosons. For example, precision measurements of the weak mixing angle \([5]\) (parity violating asymmetries \([6]\), charge asymmetries and left-right asymmetries) can be used to determine the running of \(\sin^2 \theta_W\) as a function of \(Q^2\), which is helpful in finding hints of new physics. Furthermore, measurements of the neutrino oscillation, CP violation, mass hierarchy and other topics rely heavily on accurate measurements of different neutrino scattering processes for different kinematic regions, such as the quasielastic scattering, inelastic scattering and the deeply inelastic scattering \([7, 8]\).

Measurable quantities in DIS are expressed in terms of PDFs which reveal the longitudinal momentum distributions of quarks and gluons or the one-dimensional structure of the nucleon. To explore the three-dimensional structures or the transverse momentum-dependent (TMD) PDFs, we need to consider the semi-inclusive DIS (SIDIS) where a final current region jet or a hadron is also measured in addition to the scattered lepton. Comparing to the hadron production SIDIS, the jet production one has two distinct features. First, jet production reaction does have simpler forms and not introduce extra uncertainties came from fragmentation functions. This is helpful to improve the measurement accuracy. However, due to the conservation of the helicity, jet production SIDIS can not access the chiral-odd TMD PDFs. Second, the final jet in SIDIS can be a direct probe of analyzing properties of the quark transverse momentum in the \(VN\) collinear frame. Here \(V\) denotes the propagator vector boson \((\gamma^*, Z^0, W^*)\). In this frame, the transverse momentum of jet is equal to that of the quark if higher order gluon radiation is neglected. Therefore, the jet production SIDIS is a good candidate to explore nucleon three dimensional imaging \([9-18]\).

The electron induced jet production SIDIS for both the neutral current and the charged current cases are available in refs. \([19, 20]\). In this paper, we calculate the (anti)neutrino induced jet production SIDIS process. It can provides not only information on the flavor separation and EMC effects, as mentioned before, but also additional ways to measure the weak mixing angle. The initial (anti)neutrino is assumed to be scattered off by a target particle with spin 1. Both the neutral current and the charged current interactions are considered. Due to the limitation of the factorization theorem, we here only consider the leading order or tree level reaction. Calculations are carried out up to twist-3 level in the QCD parton model by applying the collinear expansion formalism \([21-23]\). Higher twist effects are often significant for semi-inclusive reaction processes and TMD observables. For the case of twist-3 corrections, they often lead to azimuthal asymmetries which are different from the leading twist ones \([24-26]\).

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ies of higher twist effects will give complementary or even
direct access to the nucleon structure or hadronization mecha-
nism. We therefore present the results of azimuthal asym-
metries and intrinsic asymmetries after obtaining the differen-
tial cross section. The intrinsic asymmetry which was intro-
duced in Ref. [27] would reveal the imbalance in the distribu-
tion of the quark intrinsic transverse momentum. We present a
rough numerical estimation to illustrate the intrinsic asymme-
tries in the context. These asymmetries provide a set of new
quantities for analyzing the (TMD)PDFs and the electroweak
couplings in the jet production SIDIS process. It is helpful to
understand the hadronic structures and the $V-A$ theory. For
the charged current scattering process, we further define the
plus and minus cross sections to study TMD PDFs.

To be explicit, we organize this paper as follows. In sec. II,
we present the kinematics and conventions used in the calcula-
tions. In sec. III, we calculate the hadronic tensor in the parton
model framework and show the explicit expression at twist-3
level. The differential cross sections and corresponding mea-
surable quantities for both the neutral current and charged cur-
cent cases are respectively presented in sec. IV and sec. V. A
brief summary will be shown in sec. VI.

II. THE KINEMATICS AND CONVENTIONS

In this paper, we consider the semi-inclusive (anti)neutrino
SIDIS in the standard model (SM) framework. Both the neu-
tral current and charged current interactions for neutrino and
antineutrino scattering are included, i.e., we calculate the fol-
dowing differential cross sections,

$$d\sigma_{\nu N}^{NC}, \quad d\sigma_{\bar{\nu} N}^{NC}, \quad d\sigma_{\nu N}^{CC}, \quad d\sigma_{\bar{\nu} N}^{CC},$$

where $NC, CC$ denotes the neutral current and the charged
current and $\nu N, \bar{\nu} N$ denote the neutrino nucleus scattering
and antineutrino nucleus scattering. To be explicit, we label the
(anti)neutrino SIDIS as,

$$l(l') + N(p, S) \rightarrow l'(l') + q(k') + X,$$

where $l$ can be a neutrino or an antineutrino and $l'$ is the cor-
responding final neutral or charged lepton. $N$ denotes the target
particle, either a nucleon with spin $S = 1/2$ or a nucleus with
spin $S = 1$. $q$ is used to denote the produced jet which is taken
as a quark in this paper for simplicity, i.e., we do not consider
the inner structure of the jet. Momenta of the incident parti-
cles are shown in the parentheses. The kinematic variables for
the SIDIS are

$$x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot l}, \quad s = (p + l)^2,$$

where $Q^2 = -q^2 = -(l - l')^2$.

Although, the neutral current and charged current in the SM
have different couplings, they have the same forms. We there-
fore can write down the differential cross sections in a unified
form,

$$d\sigma_r = \frac{\alpha_r^2}{\pi Q^2} A_r L_{\nu^r}(l, l') W_{\nu^r}^{\mu \nu}(q, p, k') \frac{d^3 l' d^3 k'}{E_{l'} 2E_{k'}},$$

where subscript $r = Z, W$ for the neutral current and charged
current (anti)neutrino scattering processes, respectively. The
hadronic tensor is given by

$$W_{\nu^r}^{\mu \nu}(q, p, k') = \sum_x (2\pi)^3 \delta^4(p + q - k' - px) \times \langle N|J_{\nu^r}^{(0)}(0)|k'; X\rangle\langle X|J_{\nu^r}^{(0)}(0)|N\rangle,$$

where $J_{\nu^r}^{(0)}(0)$ is the current in the neutral current or
charged weak current, $J_{\nu^r}^{(0)}(0) = \bar{\psi}(0)\gamma^\mu\nu(0)\psi(0)$ with $\Gamma_{\nu^r} = \gamma^\nu(1 - \gamma^5)/2$. It is convenient to consider the $k'_\perp$-dependent hadronic tensor which is given by

$$W_{\nu^r}^{\mu \nu}(q, p, k'_\perp) = \int \frac{dk'_\perp}{(2\pi)^2 2E_{k'_\perp}} W_{\nu^r}^{\mu \nu}(q, p, k'_\perp).$$

In terms of the variables in Eq. (2.3), we have $d^3 l'/2E_{l'} = ysdx dy dz/4$, where $\psi$ is the quark intrinsic transverse momentum. Therefore the cross section can be rewritten as

$$\frac{d\sigma_r}{dxdydzdk'_\perp} = \frac{\alpha_r^2}{2Q^2} A_r L_{\nu^r}(l, l') W_{\nu^r}^{\mu \nu}(q, p, k'_\perp).$$

The propagator factor for the neutral current and charged
current are respectively given by

$$A_Z = \frac{Q^4}{[(Q^2 + M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^4 2\theta_W},$$
$$A_W = \frac{Q^4}{[(Q^2 + M_W^2)^2 + \Gamma_W^2 M_W^2] 16 \sin^4 \theta_W},$$

where $\Gamma_{Z,W}$ and $M_{Z,W}$ are widths and masses for $Z^0$ boson and $W^\pm$ boson, respectively. $\theta_W$ is the weak mixing angle. The leptonic tensor is defined as

$$L_{\mu \nu}(l, l') = 2 [\epsilon_{\mu l'} \epsilon_{\nu l'} - (l \cdot l') \epsilon_{\mu \nu}] \pm i \lambda_\epsilon \epsilon_{\mu \nu l'}. $$

III. THE HADRONIC TENSOR IN THE QCD PARTON
MODEL

The hadronic tensor given in the previous section contains
nonperturbative information and thus can not be calculated
with perturbative theory. To obtain the cross section, we
would decompose the hadronic tensor with Lorentz invariants
and subsequently write it down in terms of the structure func-
tions which are similar to that, e.g., $F_2(x, Q^2)$, in the inclusive
In the QCD parton model, the hadronic tensor is expressed by the gauge-invariant TMD PDFs in the SIDIS process. At the tree level, we need to consider the contributions from the series of diagrams shown in Fig. 1. The dashed lines denote $Z^0$ boson or the $W^\pm$ boson for the neutral and charged current interactions, respectively. One only needs to consider the handbag diagram, (a), for calculating the leading twist (twist-2) hadronic tensor. For higher twist ones, multiple gluon scattering diagrams, (b) and (c), should be included.

We note that in this section we only calculate the hadronic tensor for the neutral current process and the subscript $Z$ is neglected. For the charged current process it can be obtained similarly, we do not repeat here for simplicity.

At the leading order of the QCD, the hadronic tensor at twist-3 can be written as

$$W_{\mu\nu}(q, p, k'_\perp) = \sum_{j, c} \tilde{W}^{(1, c)}_{\mu\nu}(q, p, k'_\perp),$$

(3.1)

after applying the collinear expansion formalism [23]. Here $j$ denotes the number of exchanging gluons and $c$ denotes different cuts, $c = R, L$. After simple algebraic calculations, the hadronic tensor for 0, 1 exchanging gluons are respectively given by

$$W^{(0)}_{\mu\nu}(q, p, k'_\perp) = \frac{1}{2} \Tr \left[ \tilde{h}^{(0)}_{\mu\nu}(x, k'_\perp) \right],$$

(3.2)

$$W^{(1, c)}_{\mu\nu}(q, p, k'_\perp) = \frac{1}{4p \cdot q} \Tr \left[ \tilde{h}^{(1, c)}_{\mu\nu}(x, k'_\perp) \right],$$

(3.3)

where $\tilde{h}$’s denote the hard parts or partonic scattering amplitudes and they are written as

$$\tilde{h}^{(0)}_{\mu\nu} = \frac{1}{p^+} \Gamma^\mu_\rho \Gamma^\nu_\sigma,$$

$$\tilde{h}^{(1, c)}_{\mu\nu} = \frac{1}{p^+} \Gamma^\mu_\rho \Gamma^\nu_\sigma.$$  

(3.4)

As mentioned in the previous section, $\Gamma^\rho_\mu$ are different for the neutral current and the charged current interactions. But the correlators are not affected because they are QCD quantities. The gauge-invariant quark-quark and quark-gluon-quark correlators are defined as

$$\Phi^{(0)}_\mu(x, k'_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{i p^+ y^-} \langle \gamma^\mu N | \bar{N} \gamma^5 \tilde{A}_\perp \gamma_5 \tilde{A}_\perp | N \rangle,$$

(3.5)

$$\Phi^{(1)}_\mu(x, k'_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{i p^+ y^-} \langle \gamma^\mu N | \bar{N} \gamma^5 \tilde{A}_\perp \gamma_5 \tilde{A}_\perp | N \rangle,$$

(3.6)

where $D_\mu(y) = -i\delta^\mu_\nu + gA^\mu(y)$ is the covariant derivative. $\tilde{A}_\perp(x, y)$ is the gauge link obtained from the collinear expansion procedure, which guarantees the gauge invariance of the correlators.

The quark-quark and quark-gluon-quark correlators cannot be calculated with perturbative theory but can be decomposed in terms of the Dirac Gamma matrices and corresponding coefficient functions because they are $4 \times 4$ matrices in Dirac space. In the jet production SIDIS process where the fragmentation functions are not considered, helicity does not flip and therefore only the chiral-even TMD PDFs are involved. This is the reason that jet production SIDIS cannot access to the chiral-odd TMD PDFs. We have

$$\Phi^{(0)}_\mu = \frac{1}{2} \left[ \gamma^\mu \Phi^{(0)}_{\alpha\alpha} + \gamma^\nu \gamma_\nu \Phi^{(0)}_{\alpha\alpha} \right],$$

(3.7)

$$\Phi^{(1)}_\mu = \frac{1}{2} \left[ \gamma^\mu \Phi^{(1)}_{\alpha\alpha} + \gamma^\nu \gamma_\nu \Phi^{(1)}_{\alpha\alpha} \right].$$

(3.8)

The TMD PDFs are obtained by decomposing the coefficient functions. For $\Phi^{(0)}_\mu$ and $\Phi^{(1)}_\mu$, one has

$$\Phi^{(0)}_\mu = p^+ \bar{n}_\alpha \left( f_1 + S_{LL} f_{1LL} - \frac{k_L \cdot S_T}{M} f_{1T} \right)$$

$$+ \frac{k_L \cdot S_{LT}}{M} f_{1LT} + \frac{k_L \cdot S_{TT}}{M^2} f_{1TT} \right) - k_L \bar{\Lambda}_0 f_{1L}$$

$$- M S_{T_0} f_T + M S_{LT_0} f_{LT} + M S_{TT_0} f_{TT} - k_L \bar{\Lambda}_0 f_L$$

$$= p^+ \bar{n}_\alpha \left( f_1 + S_{LL} f_{1LL} - \frac{k_L \cdot S_T}{M} f_{1T} \right)$$

(3.9)

For $\Phi^{(1)}_\mu$ and $\tilde{\Phi}^{(1)}_\mu$, we have

$$\Phi^{(1)}_\mu = p^+ \bar{n}_\alpha \left[ k_{L_\beta} (f^+_{1L} + S_{LL} f_{1LL}) - M S_{T_0} f_{1T}$$

$$+ M S_{LT_0} f_{LT} + M S_{TT_0} f_{TT} - k_L \bar{\Lambda}_0 f_{1L}$$

$$- S_{LT_0} f_{LT} + S_{TT_0} f_{TT} - k_L \bar{\Lambda}_0 f_{1L}$$

(3.10)

$$\tilde{\Phi}^{(1)}_\mu = p^+ \bar{n}_\alpha \left[ k_{L_\beta} (f^+_{1L} + S_{LL} f_{1LL}) - M S_{T_0} f_{1T}$$

$$+ M S_{LT_0} f_{LT} + M S_{TT_0} f_{TT} - k_L \bar{\Lambda}_0 f_{1L}$$

(3.11)
\[ \Phi^{(1)}_{pt} = i p^* \tilde{\eta}_a \left[ k_{1\rho} (s_{\alpha}^+ + S_{LLL}s_{\alpha}^+) + MS_{\tau\rho} s_{\tau\rho} \right] + M S_{LT\rho} s_{\rho\tau} + \delta_{TT\rho} s_{\rho\tau} + \lambda_1 k_{1\rho} \tilde{s}^b_{\rho} + \frac{k_{2\rho} (s_{\rho}^+ - S_{LT\rho} s_{\rho\tau} - S_{TT\rho} s_{\rho\tau})}{M} \right], \]  

(3.12)

where \( S_{TT}^{k_{1\rho}} \equiv S_{\rho\tau}^{k_{1\rho}} k_{1\tau}, S_{TT}^{k_{2\rho}} \equiv S_{\rho\tau}^{k_{2\rho}} k_{2\tau}, \) and \( S_{TT}^{k_{1\rho}} \equiv S_{\rho\tau}^{k_{1\rho}} S_{TT}^{k_{1\tau}}. \) We note that \( \frac{1}{M} S_{TT}^{k_{1\rho}} \) behaves as a Lorentz vector like \( S_{LT}, \) \( \frac{1}{M} S_{TT}^{k_{1\rho}} \) and \( \delta_{TT\rho} \) behave as axial vectors like \( S_\rho. \) Subscript \( d \) denotes that the TMD PDFs are defined via the quark-gluon-quark correlator.

We can also use the following relations obtained from the QCD equation of motion \( \partial_\mu \Phi = 0 \) to eliminate the independent higher twist TMD PDFs.

\[ x p^* \Phi^{(1)}_d = -g^{\mu\nu}_{\rho\tau} Re \Phi^{(-)}_d - g^{\mu\nu}_{\rho\tau} Im \Phi^{(1)}_d, \]  

(3.13)

\[ x p^* \Phi^{(0)}_d = -g^{\mu\nu}_{\rho\tau} Re \Phi^{(-)}_d - g^{\mu\nu}_{\rho\tau} Im \Phi^{(1)}_d. \]  

(3.14)

By inserting Eqs. (3.9)-(3.12) into Eqs. (3.13) and (3.14), we can get the relationships between the twist-3 TMD PDFs defined via the quark-quark correlator and those defined via the quark-gluon-quark correlator. They can be written in a unified form, i.e.,

\[ f^{k\tau}_S - g^{k\tau}_S = -x \left( f^{k}_S - i g^{k}_S \right), \]  

(3.15)

where \( k = 1, S = L, T, LL, LT \) and \( TT. \)

Substituting the Lorentz decomposition expressions of the parton correlators into the hadronic tensor expression in Eqs. (3.2)-(3.3), by carrying out the traces we can obtain the results for the hadronic tensor up to twist-3. The relevant traces we need are

\[ p^+ \text{Tr} \left[ \gamma_{\mu} h^{(0)}_{5\mu} \right] = -4 c_1^2 g_{\mu\nu} - 4 i c_2^2 \epsilon_{\mu\nu\rho}, \]  

(3.16)

\[ p^+ \text{Tr} \left[ \gamma_{\mu} g^{(0)\mu} \right] = -4 c_1^2 g_{\mu\nu} + 4 i c_2^2 \epsilon_{\mu\nu\rho}, \]  

(3.17)

\[ \text{Tr} \left[ \delta^{(1)}_{5\rho} \right] = -8 c_1^2 g_{\mu\nu} \tilde{n}_\rho - 8 i c_2^2 \epsilon_{\mu\nu\rho} \tilde{n}_\rho, \]  

(3.18)

\[ \text{Tr} \left[ \delta^{(1)\rho} \right] = -8 c_1^2 g_{\mu\nu} \tilde{n}_\rho - 8 i c_2^2 \epsilon_{\mu\nu\rho} \tilde{n}_\rho. \]  

(3.19)

Here the \( \mu\nu\)-symmetric tensor is defined as \( g_{\mu\nu} = g_{\mu\nu} - g_{\nu\mu}. \)

Instead of writing down the complicated calculations, we here only show the final results. We first present the leading twist hadronic tensor:

\[ \tilde{W}_{12} = - \left( c_1^2 g_{\mu\nu} + \epsilon_{\mu\nu\rho} \right) \left( f_1 + S_{\mu\nu} f_{1L} - \left( k_{1\mu} - \frac{S_{TT}}{M} f_{1T} \right) \right) + \left( k_{1\mu} - \frac{S_{LT}}{M} f_{1L} - \frac{S_{TT}}{M^2} f_{1TT} \right) \right), \]  

(3.20)

The twist-3 hadronic tensor has two origins, one is the quark-quark correlator and the other is quark-gluon-quark correlator. After using the equation of motion in Eq. (3.15) to eliminate these independent TMD PDFs, we get the complete hadronic tensor at twist-3 level,
where \( \vec{q}' = q' + 2x p' \). From the equalities, \( q' \cdot \vec{q} = q' \cdot k_\perp = 0 \) and \( q' \cdot S_T = q' \cdot S_{LT} = q' \cdot S_{TT}^k / M = 0 \), we see clearly that the full twist-3 hadronic tensor satisfies current conservation law,
\[
q'_\mu \bar{W}^{\mu \nu} = q_\nu \bar{W}^{\mu \nu} = 0.
\]

We emphasise that the hadronic tensor for the neutral current and the charged current interactions is different. Since the correlators are QCD quantities and remain unchanged. Therefore the difference lies in the hart parts \( h_i \)'s. To be more precise, the difference is the weak couplings, \( c_i^q, c_i' \). For charged current interaction process, according to our definition, \( c_i^q = c_i'^q = 1 \). Therefore, we can only calculate the hadronic tensor or the following differential cross section for the neutral current interaction and obtain the charged current one by setting \( c_i^q = c_i'^q = 1 \).

IV. RESULTS OF THE NEUTRAL CURRENT PROCESS

In expressing the cross section, we choose the VN collinear frame, in which momenta related to this neutrino SIDIS process take the following forms:
\[
\begin{align*}
p^\perp &= (p^+, 0, \vec{0}_\perp), \\
p' &= \left( 1 - y \frac{x p^+}{2x p'^+}, \frac{Q^2}{2x p'^+} - \frac{Q \sqrt{1 - y}}{y}, 0 \right), \\
n' &= \left( -x p^+, \frac{Q^2}{2x p'^+}, \vec{0}_\perp \right), \\
k'^\perp &= k'^\perp = |k_\perp| (0, 0, \cos \varphi, \sin \varphi).
\end{align*}
\]

In this frame, the transverse momenta of the jet (\( k_\perp' \)) and that of the quark (\( k_\perp \)) inside the nucleon are the same. We do not distinguish them in the following calculations. This is also the precondition that we calculate the intrinsic asymmetries in the following context. The transverse vector polarization is parametrized as
\[
S_T^\perp = |S_T| (0, 0, \cos \varphi_S, \sin \varphi_S).
\]

For the tensor polarization dependent parameters, we parametrize and define them as in Ref. [28], i.e.,
\[
\begin{align*}
S_T^v &= |S_T| \cos \varphi_T, \\
S_T^s &= |S_T| \sin \varphi_T, \\
S_{TT}^{xx} &= -S_{TT}^{yy}, \quad |S_T| \cos 2\varphi_T, \\
S_{TT}^{xy} &= S_{TT}^{yx} = |S_T| \sin 2\varphi_T,
\end{align*}
\]

where
\[
\begin{align*}
|S_{LT}| &= \sqrt{(S_{LT}^x)^2 + (S_{LT}^y)^2}, \\
|S_{TT}| &= \sqrt{(S_{TT}^{xx})^2 + (S_{TT}^{yy})^2}.
\end{align*}
\]

After making Lorentz contraction of the hadronic tensor and the leptonic tensor, we obtain the differential cross section of the neutrino SIDIS process. We first define four functions of \( y \) which will be often used:
\[
\begin{align*}
A(y) &= y^2 - 2y + 2, \\
B(y) &= 2(2 - y) \sqrt{1 - y}, \\
C(y) &= y(2 - y), \\
D(y) &= 2y \sqrt{1 - y}.
\end{align*}
\]

In this section, we only calculate results for the neutral current neutrino nucleus scattering process.

A. The differential cross section

Similar to dealing with the hadronic tensor, we divide the cross section into two parts, one is the leading twist part, the other is the twist-3 part. Substituting the leading twist hadronic tensor and the leptonic tensor into Eq. (2.7) yields the leading twist cross section. It is given by
\[
\frac{d\sigma_{NC}^{\nu N \rightarrow \mu N \bar{A}}}{dx dy d\phi d^2k'_\perp} = \frac{a_{em}^2 A_Z}{yQ^2} \left\{ \begin{array}{l}
T_0^q(y)(f_1 + S_{LL} f_{1L}) \\
- T_0^q(y) \lambda_4 g_{1L} \\
+ |S_T| \kappa_{LM} \sin(\varphi - \varphi_S) T_0^q(y) f_{1T}^1 \\
- \cos(\varphi - \varphi_S) T_0^q(y) g_{1T}^1 \\
- |S_{LT}| \kappa_{LM} \sin(\varphi - \varphi_{LT}) T_0^q(y) f_{1T}^2 \\
- \cos(\varphi - \varphi_{LT}) T_0^q(y) g_{1T}^2 \\
- |S_{TT}| \kappa_{LM} \sin(2\varphi - 2\varphi_{TT}) T_0^q(y) f_{1T}^3 \\
- \cos(2\varphi - 2\varphi_{TT}) T_0^q(y) g_{1T}^3 \end{array} \right\},
\]

where subscript \( r2 \) denotes leading twist. Here we have defined \( \kappa_{LM} = |k_\perp| / M \), and
\[
\begin{align*}
T_0^q(y) &= c_i^q A(y) + c_i^q C(y), \\
T_0^q(y) &= c_i'^q A(y) + c_i'^q C(y),
\end{align*}
\]

to simplify the expression.

Substituting the twist-3 hadronic tensor and the leptonic tensor into Eq. (2.7) yields the twist-3 cross section. It is given by
\[
\begin{align*}
\frac{d\sigma_{NC}^{\nu N \rightarrow \mu N \bar{A}}}{dx dy d\phi d^2k'_\perp} &= \frac{a_{em}^2 A_Z}{yQ^2} \left\{ \begin{array}{l}
\lambda_4 \kappa_{LM} \\
\kappa_{LM} \sin(\varphi_T^2(y)) f_T^1 \\
- \cos(\varphi_T^2(y)) g_T^1 \\
\lambda_4 \kappa_{LM} \cos(\varphi_T^2(y)) f_T^2 \\
\kappa_{LM} \sin(\varphi_T^2(y)) g_T^2 \\
\kappa_{LM} \sin(2\varphi_T^2(y)) f_T^3 \\
+ \sin(2\varphi_T^2(y)) g_T^3 \end{array} \right\}
\end{align*}
\]
\[-\cos(2\varphi - \varphi_S)T_3^q(y)\frac{k_{1,M}^2}{2} g_{LT}^+ \]
\[+|S_{LT}| \sin \varphi_{LT} T_3^q(y) g_{LT} + \cos \varphi_{LT} T_3^q(y) f_{LT} \]
\[+ \sin(2\varphi - \varphi_{LT}) T_3^q(y) \frac{k_{1,M}^2}{2} g_{LT}^+ \]
\[+ \cos(2\varphi - \varphi_{LT}) T_3^q(y) \frac{k_{1,M}^2}{2} f_{LT}^+ \]
\[+ |S_{TT}| \sin(\varphi - 2\varphi_{TT}) T_3^q(y) k_{1,M} g_{TT} \]
\[- \cos(\varphi - 2\varphi_{TT}) T_3^q(y) k_{1,M} f_{TT} \]
\[- \sin(3\varphi - 2\varphi_{TT}) T_3^q(y) \frac{k_{1,M}^2}{2} g_{TT}^+ \]
\[- \cos(3\varphi - 2\varphi_{TT}) T_3^q(y) \frac{k_{1,M}^2}{2} f_{TT}^+ \bigg] \right) , \quad \text{(4.13)} \]

where subscript \(t3\) denotes twist-3. The twist suppression factor is defined as \(s_M = M/Q\). We have also defined
\[T_0^q(y) = c_1^L B(y) + c_1^D D(y) , \quad \text{(4.14)} \]
\[T_1^q(y) = c_1^L B(y) + c_1^D D(y) . \quad \text{(4.15)} \]

Equations (4.10) and (4.13) give the differential cross section of the neutral current neutrino nucleus scattering. To obtain the cross section corresponding to the antineutrino nucleus scattering, we only need to replace \(T_0^q(y)\) by \(T_{0,1,2,3}^q(y)\) which are defined as
\[T_0^q(y) = c_1^A A(y) - c_3^C C(y) , \quad \text{(4.16)} \]
\[T_1^q(y) = c_1^A A(y) - c_3^C C(y) . \quad \text{(4.17)} \]
\[T_2^q(y) = c_1^B B(y) - c_3^D D(y) , \quad \text{(4.18)} \]
\[T_3^q(y) = c_1^B B(y) - c_3^D D(y) . \quad \text{(4.19)} \]

**B. The azimuthal asymmetries**

One of the important measurable quantities in the high energy nucleon reaction is the azimuthal asymmetry. Azimuthal asymmetries can be measured to extract TMD PDFs and further to study the three dimensional imaging of the nucleon. In this subsection, we calculate the azimuthal asymmetries induced by the final jet. We first introduce the definition
\[\langle \cos \varphi \rangle_{U,U} = \frac{\int d\sigma \cos \varphi d\varphi}{\int d\sigma} , \quad \text{(4.20)} \]
for the unpolarized and the longitudinally polarized target case, and
\[\langle \cos(\varphi - \varphi_S)\rangle_{U,T} = \frac{\int d\sigma \cos(\varphi - \varphi_S) d\varphi d\varphi_S}{\int d\sigma d\varphi d\varphi_S} , \quad \text{(4.21)} \]
for the transversely polarized target case. \(d\varphi\) is used to denote \(\frac{d\sigma}{dx dy \, dE_{K^*}}\), and \(d\varphi_S \approx d\psi\) when the direction is chosen to be the direction of \(S\) in case of a transversely polarized target in which integration corresponds to take the average over the out going electron’s azimuthal angle [26, 29]. The subscripts such as \(U, T\) denote the polarizations of the lepton beam and the target, respectively.

At the leading twist, there are 6 transversely polarized dependent azimuthal asymmetries which are given by
\[\langle \sin(\varphi - \varphi_S)\rangle_{U,T} = \frac{T_1^q(y) g_{TT}^+}{2 T_0^q(y) f_{TT}} , \quad \text{(4.22)} \]
\[\langle \cos(\varphi - \varphi_S)\rangle_{U,T} = -\frac{T_1^q(y) g_{TT}^+}{2 T_0^q(y) f_{TT}} , \quad \text{(4.23)} \]
\[\langle \sin(\varphi - \varphi_{LT})\rangle_{U,T} = -\frac{T_0^q(y) g_{LT}^+}{2 T_0^q(y) f_{LT}} , \quad \text{(4.24)} \]
\[\langle \cos(\varphi - \varphi_{LT})\rangle_{U,T} = -\frac{T_0^q(y) g_{LT}^+}{2 T_0^q(y) f_{LT}} , \quad \text{(4.25)} \]
\[\langle \sin(2\varphi - \varphi_{TT})\rangle_{U,T} = -\frac{T_0^q(y) g_{TT}^+}{2 T_0^q(y) f_{TT}} , \quad \text{(4.26)} \]
\[\langle \cos(2\varphi - \varphi_{TT})\rangle_{U,T} = \frac{T_0^q(y) g_{TT}^+}{2 T_0^q(y) f_{TT}} , \quad \text{(4.27)} \]
In addition to the TMD PDFs, azimuthal asymmetries also depends on \(T\) functions shown in Eqs. (4.11) and (4.12). This implies that TMD PDFs and weak couplings can be determined simultaneously by global fit.

Higher twist effects often lead to azimuthal asymmetries which are different from the leading twist ones. In the jet production SIDIS process, we obtain 18 azimuthal asymmetries at twist-3, which are given by
\[\langle \cos \varphi \rangle_{U,U} = -x_M k_{1,M} \frac{T_2^q(y) f_{LT}^+}{T_0^q(y) f_{LT}} , \quad \text{(4.28)} \]
\[\langle \sin \varphi \rangle_{U,U} = -x_M k_{1,M} \frac{T_2^q(y) g_{LT}^+}{T_0^q(y) f_{LT}} , \quad \text{(4.29)} \]
\[\langle \cos \varphi \rangle_{U,U} = -x_M k_{1,M} \frac{T_2^q(y) f_{LT}^+ - \lambda_T T_3^q(y) g_{TT}^+}{T_0^q(y) f_{LT}} , \quad \text{(4.30)} \]
\[\langle \sin \varphi \rangle_{U,U} = -x_M k_{1,M} \frac{T_2^q(y) g_{LT}^+ + \lambda_T T_3^q(y) f_{LT}^+}{T_0^q(y) f_{LT}} , \quad \text{(4.31)} \]
\[\langle \cos \varphi \rangle_{U,U} = -x_M k_{1,M} \frac{T_2^q(y) (f_{LT}^+ + S_{LT} f_{LT})}{T_0^q(y) f_{LT}} , \quad \text{(4.32)} \]
\[\langle \sin \varphi \rangle_{U,U} = -x_M k_{1,M} \frac{T_2^q(y) (g_{LT}^+ + S_{LT} g_{LT})}{T_0^q(y) f_{LT}} , \quad \text{(4.33)} \]
\[\langle \cos \varphi \rangle_{U,U} = -x_M k_{1,M} \frac{T_2^q(y) g_{TT}^+}{T_0^q(y) f_{TT}} , \quad \text{(4.34)} \]
\[\langle \sin \varphi \rangle_{U,U} = -x_M k_{1,M} \frac{T_2^q(y) f_{TT}^+}{T_0^q(y) f_{TT}} , \quad \text{(4.35)} \]
\[\langle \cos(2\varphi - \varphi_S)\rangle_{U,T} = x_M k_{1,M} \frac{T_2^q(y) g_{TT}^+}{2 T_0^q(y) f_{TT}} , \quad \text{(4.36)} \]
of the transverse momentum distribution.\(x\)

**FIG. 2:** The quark (jet) transverse momentum in the \(x-y\) plane. The difference of the red hemisphere and the blue one gives the imbalance of the transverse momentum distribution.

\[
\langle \sin(2\varphi - \varphi_3) \rangle_{UL} = -x k_M k_{LM}^2 T_2^T(y) f_T^L / 2T_0^0(y) f_1, \quad (4.37)
\]

\[
\langle \cos \varphi_{LT} \rangle_{UL} = -x k_M T_2^T(y) f_T^L / T_0^0(y) f_1, \quad (4.38)
\]

\[
\langle \sin \varphi_{LT} \rangle_{UL} = -x k_M T_3^T(y) g_{LT}^L / T_0^0(y) f_1, \quad (4.39)
\]

\[
\langle \cos(2\varphi - \varphi_{LT}) \rangle_{UL} = -x k_M k_{LM}^2 T_2^T(y) f_T^L / 2T_0^0(y) f_1, \quad (4.40)
\]

\[
\langle \sin(2\varphi - \varphi_{LT}) \rangle_{UL} = -x k_M k_{LM}^2 T_3^T(y) g_{LT}^L / 2T_0^0(y) f_1, \quad (4.41)
\]

\[
\langle \cos(\varphi - 2\varphi_{LT}) \rangle_{UL} = x k_M k_{LM} T_2^T(y) f_T^L / T_0^0(y) f_1, \quad (4.42)
\]

\[
\langle \sin(\varphi - 2\varphi_{LT}) \rangle_{UL} = -x k_M T_3^T(y) g_{LT}^L / T_0^0(y) f_1, \quad (4.43)
\]

\[
\langle \cos(3\varphi - 3\varphi_{LT}) \rangle_{UL} = x k_M k_{LM} T_3^T(y) g_{LT}^L / 2T_0^0(y) f_1, \quad (4.44)
\]

\[
\langle \sin(3\varphi - 2\varphi_{LT}) \rangle_{UL} = x k_M k_{LM} T_3^T(y) g_{LT}^L / 2T_0^0(y) f_1. \quad (4.45)
\]

These azimuthal asymmetries can be measured to extract the corresponding distribution functions.

C. The intrinsic asymmetries

As mentioned in the introduction that the final jet in SIDIS can be a direct probe of analyzing properties of the quark transverse momentum in the \(VN\) collinear frame because the transverse momentum of jet is equal to that of the quark. In this part we calculate the intrinsic asymmetry which was first introduced in Ref. [27] to explore the transverse momentum distribution of the quark in a nucleon.

In the \(VN\) collinear frame, the \(z\) direction is defined by the momentum of the nucleon, the transverse momentum of the incident quark (jet) is therefore in the \(x-y\) plane. It can be decomposed as follows

\[
k^\perp_x = k_\perp \sin \varphi, \quad (4.46)
\]

Considering Eq. (4.46), it is possible to explore the difference of the momentum in the \(x\) direction, i.e., \(k^\perp_x(+x) - k^\perp_x(-x)\), see Fig. 2. We assume that this difference would be induced by the intrinsic transverse moment of the quark or the radiation of the gluon in the nucleon. In order to explore this difference, we here introduce the definition of the intrinsic asymmetry:

\[
A^\perp = \frac{\int_{\pi/2}^{\pi/2} d\varphi \, d\hat{\sigma} - \int_{-\pi/2}^{\pi/2} d\varphi \, d\hat{\sigma}}{\int_{-\pi/2}^{\pi/2} d\varphi \, d\hat{\sigma}_{U,U} d\varphi}. \quad (4.48)
\]

Here \(d\hat{\sigma}\) is used to denote \(d\sigma / d\mu dB d^2p_T^\perp\). Subscript \(U, U\) denotes the unpolarized cross section. From Eq. (4.47), we can define the asymmetry in the \(y\) direction in the similar way but with different integral intervals:

\[
A^y = \frac{\int_0^\pi d\varphi \, d\hat{\sigma} - \int_{-\pi}^0 d\varphi \, d\hat{\sigma}}{\int_0^\pi d\varphi \, d\hat{\sigma}_{U,U} d\varphi}. \quad (4.49)
\]

According to the definition in Eq. (4.48) and (4.49), we calculate these asymmetries which can be written as in the following forms:

\[
A_{U}^{NC,x} = -\frac{4x k_M k_{LM}^2}{\pi^2} T_2^T(y) f_T^L / T_0^0(y) f_1, \quad (4.50)
\]

\[
A_{U}^{NC,y} = -\frac{4x k_M k_{LM}^2}{\pi^2} T_3^T(y) g_{LT}^L / T_0^0(y) f_1, \quad (4.51)
\]

\[
A_{L}^{NC,x} = \frac{4x k_M k_{LM}^2}{\pi^2} T_3^T(y) g_{LT}^L / T_0^0(y) f_1, \quad (4.52)
\]

\[
A_{L}^{NC,y} = -\frac{4x k_M k_{LM}^2}{\pi^2} T_3^T(y) g_{LT}^L / T_0^0(y) f_1, \quad (4.53)
\]

\[
A_{LL,x} = -\frac{4x k_M k_{LM}^2}{\pi^2} T_3^T(y) g_{LT}^L / T_0^0(y) f_1, \quad (4.54)
\]

\[
A_{LL,y} = -\frac{4x k_M k_{LM}^2}{\pi^2} T_3^T(y) g_{LT}^L / T_0^0(y) f_1. \quad (4.55)
\]

where superscript \(NC\) denotes the neutral current.

We present the numerical values of \(A_{NC,x}^{U,U}\) in Fig. 3 and Fig. 4. We take the Gaussian ansatz for the TMD PDFs, i.e.,

\[
f_1(x, k_\perp) = \frac{1}{\pi\Delta^2} f_1(x) e^{-k_\perp^2 / \Delta^2}, \quad (4.56)
\]

\[
f^\perp(x, k_\perp) = \frac{1}{\pi\Delta^2} f_1(x) e^{-k_\perp^2 / \Delta^2}, \quad (4.57)
\]

where \(f_1(x)\) are taken from CT14 [30] and the faction is taken as \(x = 0.3\) for illustration. In order to determine \(f^\perp(x, k_\perp)\), we have used the Wanzura-Wilczek approximation (neglecting quark-gluon-quark correlation function, \(g = 0\) [24, 26]. In the numerical estimation, only the up and down quarks are taken into account. The widths of the unpolarized distribution
Without showing the tedious calculation process, we just give current (anti)neutrino nucleus scattering process. In this section we present the differential cross section into two parts, one is the leading twist part, the other is the twist-3 part. Without showing the tedious calculation process, we just give

\[ \Delta_u^2 = 0.5 \text{ GeV}^2 \] as \( \Delta_u^2 = 0.5 \text{ GeV}^2 \). We find that the intrinsic asymmetry \( A_{NC}^{CC} \) decreases with respect to the energy and it is more sensitive to \( \Delta_u^2 \) than \( \Delta_u^2 \).

V. RESULTS OF THE CHARGED CURRENT PROCESS

In this section we present the differential cross section, azimuthal asymmetries and intrinsic asymmetries in the charged current (anti)neutrino nucleus scattering process.

A. The differential cross section

As before, we divide the differential cross section into two parts, one is the leading twist part, the other is the twist-3 part. Without showing the tedious calculation process, we just give

\[ \frac{d\sigma^{CC}}{dxdy\phi d^2k_L} = \frac{\alpha_m A_W}{2yQ^2} T_0(y) \left( f_1 + S_{LL} f_{1LL} \right) - \lambda h g_L \]

\[ + |S_T| k_{\perp M} \left[ \sin(\phi - \phi_S) f_{1T}^+ - \cos(\phi - \phi_S) g_{1T}^+ \right] \]

\[ - |S_{LT}| k_{\perp M} \left[ \sin(\phi - \phi_{LT}) g_{1LT}^+ + \cos(\phi - \phi_{LT}) f_{1LT}^+ \right] \]

\[ - |S_{TT}| k_{\perp M} \left[ \sin(2\phi - 2\phi_{TT}) g_{1TT}^+ \right] \]
where $T_0(y) = A(y) + C(y)$. We note that the charged current neutrino nucleus scattering would select the negative charged quark flavors ($d, s, \cdots$) and the CKM matrix element should be inserted in the Eq. (5.1). The twist-3 differential cross section is given by

$$
\frac{d\sigma_{CC}}{dxdydf^T} = -\frac{\alpha_{em}^2 A_w}{2yQ^2} 2xk_M T_2(y) \left\{ \right.
\lambda_h k_{\perp M} \\
\times \left[ \sin \varphi f_{LT}^+ - \cos \varphi g_{LT}^+ \right] \\
+ k_{\perp M} \sin \varphi (f_{LT}^+ + S_{LT} g_{LT}^+) \\
+ k_{\perp M} \sin \varphi (g_{LT}^+ + S_{LT} f_{LT}^+) \\
+ \left| S_{LT} \right| \sin \varphi f_{LT} - \cos \varphi g_{LT} \\
+ \sin(2\varphi - \varphi_s) k_{\perp M}^2 f_{LT}^+ \\
- \cos(2\varphi - \varphi_s) k_{\perp M}^2 g_{LT}^+ \\
+ \left| S_{LT} \right| \sin \varphi g_{LT} f_{LT} + \cos \varphi f_{LT} g_{LT} \\
+ \sin(2\varphi - \varphi_s) k_{\perp M}^2 g_{LT}^+ \\
+ \cos(2\varphi - \varphi_s) k_{\perp M}^2 f_{LT}^+ \\
+ \left| S_{LT} \right| \sin(\varphi - 2\varphi_T) k_{\perp M} g_{TT} \\
- \cos(\varphi - 2\varphi_T) k_{\perp M} f_{TT} \\
- \sin(3\varphi - 2\varphi_T) k_{\perp M}^3 g_{TT} \\
- \cos(3\varphi - 2\varphi_T) k_{\perp M}^3 f_{TT} \left\} \right. \right). \tag{5.2}
$$

where $T_2(y) = B(y) + D(y)$. Equations (5.1) and (5.2) give the differential cross section of the charged current neutrino nucleus scattering. To obtain the corresponding antineutrino nucleus scattering, we only need to replace $T_{0,2}(y)$ by $t_{0,2}(y)$ which are defined as

$$
t_0(y) = A(y) - C(y), \tag{5.3}
t_2(y) = B(y) - D(y). \tag{5.4}
$$

We also note here the antineutrino nucleus scattering selects the positive charged quarks ($u, c, \cdots$) and the corresponding CKM matrix element should be inserted in the expression which is neglected here for simplicity.

**B. The azimuthal asymmetries**

With the same method given in the previous section, we here show the azimuthal asymmetries in the charged current process. For the leading twist asymmetries, we have

$$
\langle \sin(\varphi - \varphi_S) \rangle_{UL} = k_{\perp M} T_0(y) f_{LT}^+ \frac{T_0(y) f_{LT}^+}{2T_0(y) f_1^+}, \tag{5.5}
$$

$$
\langle \cos(\varphi - \varphi_S) \rangle_{UL} = -k_{\perp M} T_0(y) g_{LT}^+ \frac{T_0(y) g_{LT}^+}{2T_0(y) f_1^+}. \tag{5.6}
$$

$$
\langle \sin(\varphi - \varphi_L) \rangle_{UL} = -k_{\perp M} T_0(y) g_{LT}^+ \frac{T_0(y) g_{LT}^+}{2T_0(y) f_1^+}. \tag{5.7}
$$

$$
\langle \cos(\varphi - \varphi_L) \rangle_{UL} = -k_{\perp M} T_0(y) f_{LT}^+ \frac{T_0(y) f_{LT}^+}{2T_0(y) f_1^+}. \tag{5.8}
$$

$$
\langle \sin(2\varphi - \varphi_{TT}) \rangle_{UL} = -k_{\perp M} T_0(y) g_{LT}^+ \frac{T_0(y) g_{LT}^+}{2T_0(y) f_1^+}, \tag{5.9}
$$

$$
\langle \cos(2\varphi - \varphi_{TT}) \rangle_{UL} = k_{\perp M} T_0(y) f_{LT}^+ \frac{T_0(y) f_{LT}^+}{2T_0(y) f_1^+}. \tag{5.10}
$$

For the twist-3 asymmetries, we have

$$
\langle \cos \varphi \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) f_{LT}^+ \frac{T_2(y) f_{LT}^+}{T_0(y) f_1^+}, \tag{5.11}
$$

$$
\langle \sin \varphi \rangle_{UL} = -xk_{\perp M} g_{LT}^+ \frac{T_2(y) g_{LT}^+}{T_0(y) f_1^+}. \tag{5.12}
$$

$$
\langle \cos \varphi \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) f_{LT}^+ \frac{T_2(y) f_{LT}^+}{T_0(y) f_1^+}. \tag{5.13}
$$

$$
\langle \sin \varphi \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) g_{LT}^+ \frac{T_2(y) g_{LT}^+}{T_0(y) f_1^+}. \tag{5.14}
$$

$$
\langle \cos(\varphi - \varphi_L) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) (f_{LT}^+ + S_{LT} g_{LT}^+), \tag{5.15}
$$

$$
\langle \sin(\varphi - \varphi_L) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) (g_{LT}^+ + S_{LT} f_{LT}^+), \tag{5.16}
$$

$$
\langle \cos \varphi \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) f_{LT}^+ \frac{T_2(y) f_{LT}^+}{T_0(y) f_1^+}, \tag{5.17}
$$

$$
\langle \sin \varphi \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) g_{LT}^+ \frac{T_2(y) g_{LT}^+}{T_0(y) f_1^+}. \tag{5.18}
$$

$$
\langle \cos(\varphi - \varphi_S) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) g_{LT}^+ \frac{T_2(y) g_{LT}^+}{T_0(y) f_1^+}, \tag{5.19}
$$

$$
\langle \sin(\varphi - \varphi_S) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) f_{LT}^+ \frac{T_2(y) f_{LT}^+}{T_0(y) f_1^+}. \tag{5.20}
$$

$$
\langle \cos(\varphi - \varphi_L) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) f_{LT}^+ \frac{T_2(y) f_{LT}^+}{T_0(y) f_1^+}, \tag{5.21}
$$

$$
\langle \sin(\varphi - \varphi_L) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) g_{LT}^+ \frac{T_2(y) g_{LT}^+}{T_0(y) f_1^+}. \tag{5.22}
$$

$$
\langle \cos(2\varphi - \varphi_{TT}) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) g_{LT}^+ \frac{T_2(y) g_{LT}^+}{2T_0(y) f_1^+}, \tag{5.23}
$$

$$
\langle \sin(2\varphi - \varphi_{TT}) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) f_{LT}^+ \frac{T_2(y) f_{LT}^+}{2T_0(y) f_1^+}. \tag{5.24}
$$

$$
\langle \cos(\varphi - \varphi_{TT}) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) g_{LT}^+ \frac{T_2(y) g_{LT}^+}{2T_0(y) f_1^+}, \tag{5.25}
$$

$$
\langle \sin(\varphi - \varphi_{TT}) \rangle_{UL} = -xk_{\perp M} k_{\perp M} T_2(y) f_{LT}^+ \frac{T_2(y) f_{LT}^+}{2T_0(y) f_1^+}. \tag{5.26}
$$
The intrinsic asymmetries in the charged current neutrino nucleus scattering process can also be obtained by using the definition given in Eqs. (4.48) and (4.49). The explicit results are given by

\[
A_{CC,x}^U = \frac{-4xMK_{LM}^2}{\pi} \frac{T_2(y)g_{TT}^\perp}{T_0(y)f_1},
\]

(5.29)

\[
A_{CC,y}^U = \frac{-4xMK_{LM}^2}{\pi} \frac{T_2(y)g_{TT}^\perp}{T_0(y)f_1},
\]

(5.30)

D. The plus and minus cross section

The neutrino and antineutrino nucleus scattering would select different flavors. Then it is helpful to define the plus and minus cross sections. For simplicity, we consider the \(k'_T\) - integrated cross section, i.e., the inclusive cross section.

\[
d\sigma^P_{in} = \frac{d\sigma_{in}(\bar{\nu})}{dxdy\psi} + \frac{d\sigma_{in}(\nu)}{dxdy\psi},
\]

(5.35)

\[
d\sigma^M_{in} = \frac{d\sigma_{in}(\bar{\nu})}{dxdy\psi} - \frac{d\sigma_{in}(\nu)}{dxdy\psi},
\]

(5.36)

where subscript \(in\) denotes inclusive, \(P, M\) denote the plus and minus cross sections calculated by Eqs. (5.35) and (5.36), respectively. The explicit expressions of the plus and minus cross sections are

Using the same parameterizations given in the previous section in Eqs. (4.56) and (4.57), we present the numerical values of \(A_{CC,x}^U\) in Fig. 5. Since only the down quark is taken into account, the CKM matrix element \(U_{ud}\) is approximated as 1. From Fig. 5, we find that intrinsic asymmetries \(A_{CC,x}^U\) and \(A_{CC,x}^N\) have the same behaviors and the same orders of magnitude. Furthermore, they both decrease with respect to the energy.
\[ \frac{d\sigma_{in}}{dy} = \frac{2\alpha_s^2A_W^2}{2yQ^2} \left\{ \left[ (t_0(y)g_{LT}^U(x) + T_2(y)g_{LT}^D(x)) \right] \left[ \left( t_0(y)f_{1L}^U(x) - T_0(y)f_{1L}^D(x) \right) + \lambda_0 \left( t_0(y)g_{LT}^U(x) - T_0(y)g_{LT}^D(x) \right) \right] - 2xkM \left[ \sin \varphi_S \left( t_2(y)f_{1L}^U(x) - T_2(y)f_{1L}^D(x) \right) \right] \right\} \], (5.38)

The superscripts \( D, U \) denote the \( d \)-type (\( d, s, \cdots \)) and \( u \)-type (\( u, c, \cdots \)) quarks. The CKM matrix elements are not shown in these expressions.

Assuming the target is longitudinally polarized, we would introduce the spin asymmetry as

\[ A_y = \frac{d\sigma_{in\sigma}(+\sigma) - d\sigma_{in\sigma}(-\sigma)}{d\sigma_{in\sigma}(+\sigma) + d\sigma_{in\sigma}(-\sigma)}, \] (5.39)

where the subscript \( \sigma \) denotes the target polarization. Therefore, we have

\[ A_L^P = \frac{t_0(y)g_{LT}^U(x) + T_0(y)g_{LT}^D(x)}{t_0(y)f_{1L}^U(x) + T_0(y)f_{1L}^D(x)}, \] (5.40)

\[ A_L^M = \frac{t_0(y)g_{LT}^U(x) - T_0(y)g_{LT}^D(x)}{t_0(y)f_{1L}^U(x) - T_0(y)f_{1L}^D(x)}, \] (5.41)

\[ A_{LL}^P = \frac{t_0(y)f_{1L}^U(x) + T_0(y)f_{1L}^D(x)}{t_0(y)f_{1L}^U(x) + T_0(y)f_{1L}^D(x)}, \] (5.42)

\[ A_{LL}^M = \frac{t_0(y)f_{1L}^U(x) - T_0(y)f_{1L}^D(x)}{t_0(y)f_{1L}^U(x) - T_0(y)f_{1L}^D(x)}. \] (5.43)

Since we have calculated the differential cross sections for both the neutral current and charged current interactions, we can define the Paschos-Wolfenstein ratio \cite{36} for the semi-inclusive scattering process:

\[ R_d^\gamma = \frac{d\sigma_{\text{NC}_{\gamma N}}(+\sigma) - d\sigma_{\text{NC}_{\gamma N}}(-\sigma)}{d\sigma_{\text{NC}_{\gamma N}(+\sigma) + d\sigma_{\text{NC}_{\gamma N}(-\sigma)}. \] (5.44)

where subscript \( d \) denotes the differential cross section. It was first measured by the NuTeV collaboration \cite{37}. By using the results obtained in the previous section, we have

\[ R_d^\gamma = \frac{2A_y}{A_W} \frac{T_0(y)f_1(x) - t_0(y)f_1(x)}{T_0(y)f_{1L}^D(x) - t_0(y)f_{1L}^U(x)}. \] (5.45)

At low energy limit, \( R_d^\gamma \) in Eq. (5.45) can be approximated as

\[ R_d^\gamma = \frac{1}{2} \frac{T_0(y)f_1(x) - t_0(y)f_1(x)}{T_0(y)f_{1L}^D(x) - t_0(y)f_{1L}^U(x)}. \] (5.46)

With further approximation, \( f_1^u = f_1^d \), we obtain the final result:

\[ R_d^\gamma = \frac{1}{2} - \sin^2 \theta_W = R^\gamma. \] (5.47)

Thus, Eq. (5.45) provides an additional method for measuring the weak mixing angle.

VI. SUMMARY

The (anti)neutrino nucleus scattering is important in exploring the nucleon and/or nucleus structures and it also provides information for neutrino experiments in studying CP violation, mass hierarchy and other related topics. However, we limit ourselves by only calculating the (anti)neutrino induced jet production SIDIS process in this paper. Due to the limitation of the factorization theorem, our calculations are carried out at the leading order twist-3 level. We only show the explicit calculation of the neutral current neutrino scattering process. The results for the neutral current antineutrino scattering, the charged current neutrino scattering and the charged current antineutrino scattering can be obtained by replacing the corresponding coefficients. To obtain the results of the antineutrino nucleus scattering for the neutral current reaction, we only need to replace \( T_{0123}^q(y) \) in the different from the \( f_1^d \). For the charged current reaction, we only need to replace \( T_{02}^q(y) \) by \( f_1^d(y) \) to obtain the results of the antineutrino nucleus scattering. Since higher twist effects are significant for semi-inclusive reaction processes and TMD observables, e.g., twist-3 TMD PDFs often lead to azimuthal asymmetries which are different from the leading twist ones. We therefore present the complete results of the azimuthal asymmetries up to twist-3 for the neutrino induced jet production SIDIS. 6 of them are twist-3 and 18 of them are twist-3. Considering the equivalence of the transverse momenta of the quark inside the nucleon and the final jet, we calculate the intrinsic asymmetries to explore the imbalance of the transverse momentum distribution of the quark. We find that the intrinsic asymmetry decreases with respect to the energy and it is more sensitive to \( \Delta^2_t \) than \( \Delta^2_s \). For the charged current (anti)neutrino nucleus scattering process, we further define the \textit{plus and minus} cross sections to study TMD PDFs.

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