Spectrum Sensing Techniques Based on Last Status Change Point Estimation for Dynamic Primary User in Additive Laplacian Noise

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Accepted: 9 August 2021 / Published online: 18 August 2021
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Abstract
A real time scenario of dynamic primary user (PU) is considered in additive Laplacian noise. Two transitions or status changes of PU in the fixed sensing time are considered. The last status change point (LSCP) is estimated with maximum likelihood estimation by using dynamic programming. We consider Cumulative Sum (CuSum) based weighted samples for detection. We consider three detection schemes such as sample mean detector, energy detection and improved absolute value cumulation detection. We derive closed form expressions of detection probability ($P_D$) and false alarm probability ($P_F$) for all the three schemes. We present our results with receiver operating characteristic (ROC) for the considered schemes. We also present simulation results, which are closely matching with their analytical counterparts. We compare the ROC of the considered system with the ROC of conventional techniques. In the conventional techniques, all the samples in the sensing time are used for detection without LSCP estimation and weight. It is found that the considered system outperforms the conventional schemes.

Keywords  Last status change point estimation · Spectrum sensing · Primary user · Laplacian noise · Receiver operating characteristic · Maximum likelihood estimation

1 Introduction
Spectrum sensing techniques in cognitive radio are used to utilize the spectrum effectively by unlicensed user or secondary users (SU) without causing interference to the primary users [1]. In spectrum sensing techniques, the PU is usually assumed to be static [2, 3]. It means PU is either present or absent during the entire sensing duration [4]. This holds good for low PU traffic. However, for medium to high traffic, PU is assumed to possess dynamic behaviour [5]. The dynamic behaviour of the PU means...
the PU randomly appears or disappears during the entire sensing duration. In [6], this randomness is modelled by Poisson process. In this case of dynamic PU, for detection, Cumulative Sum (CuSum) based test-statistic has been used in [7]. This CuSum has been adopted to various spectrum sensing schemes such as energy detection (ED) [8, 9], improved energy detection (iED) [10] and sample mean detection (SMD) [11].

The dynamic behaviour of PU is further explored at the cognitive terminal by estimating the status change points or transitions of PU in the specified sensing interval. In [11], the last status change point (LSCP) of PU was estimated and then sensing was done in the interval from LSCP to the end of the sensing time using CuSum based weighted samples in ED and cyclo-stationarity based detectors [12]. It was found that the performance of these detectors is better with LSCP estimation compared to the performance without LSCP estimation. In [13], the effect of multiple changes of PU on the detection performance was presented.

Now, all the above-mentioned papers assume thermal noise modelled by Gaussian processes. However, in a real communication system, other sources of noise such as artificial impulsive noise, co-channel interference from other PUs, emissions from microwave ovens and multiple access interference (MAI) in time hopped ultra-wideband communication system [14] are also there. The Gaussian process fails to characterize them. This non-Gaussian noise exhibits heavy tailed behaviour compared to Gaussian noise [15]. Some of the forms of non-Gaussian noise are Laplacian noise, Class A Middleton noise, Bernoulli-Gaussian noise [16]. In such non-Gaussian environment, conventional schemes such as ED, iED and eigenvalue based spectrum sensing [17] don’t perform well. The suitable probabilistic model to characterize MAI is Laplacian model compared to Gaussian or Middleton class A model [18]. Considering the additive Laplacian noise, some spectrum sensing schemes such as absolute value cumulation detection (AVCD) [19] and improved-AVCD (i-AVCD) [20] outperform ED.

In this paper, we consider a dynamic PU with a maximum of two status changes during the sensing period as of [11]. However, the considered additive noise is modelled by Laplacian instead of Gaussian. Using dynamic programming, we find MLE of the LSCP. Then, we consider the samples in the interval from the LSCP to the end of the sensing time for detection of PU. Subsequently, we use the weighted samples in CuSum based detectors. The considered detectors are sample mean detector (SMD), ED and i-AVCD. We derive closed-form expressions of probability of detection $P_D$ and probability of false alarm $P_F$. We plot the ROC using the analytical expressions and validate the same by comparing them with simulations. We found that the considered detection schemes outperform the conventional detection schemes, wherein the full sensing time is used for detection without weighted samples. Further, we have compared the ROC of the considered system with the ROC of the system by taking one status change of PU. In one status change of PU, the LSCP is estimated and sensing is done from the LSCP to the end of the sensing interval. It is found that the one status change of PU outperforms the considered system for the same LSCP in the same sensing interval. The reason is less estimation error occurred in LSCP for the case of one status change of PU.

The rest of the paper is organized as follows: Sect. 2 deals with the system model. Section 3 presents the detailed performance analysis. In this section, LSCP along with CuSum based weighing scheme is discussed in additive Laplacian noise. Section 4 presents the results with ROC followed by a brief conclusion of the work in Sect. 5.
2 System Model

The null and alternate hypothesis are denoted by $H_0$ and $H_1$ respectively. The received symbols at the cognitive terminal under the random arrival and the random departure of the PU can be expressed as

\[ H_0 : y_m = \begin{cases} w_m, & m = 1, \ldots, N_1 \\ s_m + w_m, & m = N_1 + 1, N_1 + 2, \ldots, N_2 \\ w_m, & m = N_2 + 1, N_2 + 2, \ldots, N \end{cases} \]

\[ H_1 : y_m = \begin{cases} s_m + w_m, & m = 1, \ldots, N'_1 \\ w_m, & m = N'_1 + 1, N'_1 + 2, \ldots, N'_2 \\ s_m + w_m, & m = N'_2 + 1, N'_2 + 2, \ldots, N \end{cases} \]  \hspace{1cm} (1)

where $1 < N_1 < N_2 < N$, and $1 < N'_1 < N'_2 < N$. Here, $N_1$ and $N'_1$ are first transitions of PU, and $N_2$ and $N'_2$ are the second ones. The $N$ indicates the total number of samples present during the sensing period. The $s_m$ is the unknown PU signal and $w_m$ denotes sample of Laplacian noise with mean 0 and variance $2b^2$, where $b$ is the scale parameter of Laplacian noise. The average SNR is defined as $\gamma = (1/N) \sum_{m=1}^{N} (s_m^2) / (2b^2)$. Here, $N_1$ and $N_2$ denote the first and second status change points respectively of the PU under hypotheses $H_0$.

The PDF of Laplacian noise is expressed as

\[ f_{w_m}(x) = \frac{1}{2b} \exp \left( -\frac{|x|}{b} \right). \]  \hspace{1cm} (2)

Similarly for one PU change, the system model can be expressed as

\[ H_0 : y_m = \begin{cases} s_m + w_m, & m = 1, \ldots, N_1 \\ w_m, & m = N_1 + 1, N_1 + 2, \ldots, N \end{cases} \]

\[ H_1 : y_m = \begin{cases} w_m, & m = 1, \ldots, N'_1 \\ s_m + w_m, & m = N'_1 + 1, N'_1 + 2, \ldots, N \end{cases} \]  \hspace{1cm} (3)

3 Performance Analysis

In this section, we briefly present LSCP Estimation along with CuSum based weighing scheme [11]. Then, we derive the expressions of $P_D$ and $P_F$.

The last status change point (LSCP) estimation is found out by maximum likelihood estimation (MLE). The PU ($s_m$) is assumed to be unknown constant $C$. The joint MLE of $C$, $N_1$ and $N_2$ under hypothesis $H_0$ is found out by minimizing the following cost function [11]
Similarly, under \( H_1 \), the cost function to be minimized can be expressed as

\[
J_{H_1}(C, N'_1, N'_2) = \sum_{m=1}^{N'_1} |y_m - C| + \sum_{m=N'_1+1}^{N'_2} |y_m - C| + \sum_{m=N'_2+1}^{N} |y_m|.
\]  

(5)

Here, the estimated values of \( N_2 \) and \( N'_2 \) are taken as \( \hat{N}_2 \) and \( \hat{N}'_2 \) respectively. We use dynamic programming \([11]\) for the same.

Now, with \( \hat{N}_2 \) and \( \hat{N}'_2 \), the effective hypotheses \( H'_0 \) and \( H'_1 \) can be expressed as

\[
H'_0 : y_m = w_m; \quad m = \hat{N}_2 + 1, \hat{N}_2 + 2, \ldots, N
\]

\[
H'_1 : y_m = s_m + w_m; \quad m = \hat{N}'_2 + 1, \hat{N}'_2 + 2, \ldots, N
\]

(6)

After LSCP estimation, we use CuSum based weighted samples in the following three detectors.

### 3.1 Sample Mean Detector

The test statistics can be expressed as \([11]\)

\[
Z'(y) = \frac{1}{N - L_{scp}} \left( \sum_{m=1}^{N - L_{scp}} (N - L_{scp} - m + 1)y_{N-m+1} \right).
\]  

(7)

where \( L_{scp} \) denotes the last status change point of the PU, i.e. \( L_{scp} \in \{\hat{N}_2, \hat{N}'_2\} \). Now, using the central limit theorem \([6]\), the pdf of \( Z'(y) \) can be expressed as Gaussian under both the hypotheses with the following mean and variance.

\[
E[Z'(y)|H'_0] = 0,
\]

\[
\text{var}[Z'(y)|H'_0] = \frac{2b^2}{6}\left\{ \frac{(N - \hat{N}_2 + 1)\{2(N - \hat{N}_2) + 1\}}{(N - \hat{N}_2)} \right\}
\]

(8)

\[
E[Z'(y)|H'_1] = \frac{C}{2}\left\{ N - \hat{N}'_2 - 1 \right\},
\]

\[
\text{var}[Z'(y)|H'_1] = \frac{2b^2}{6}\left\{ \frac{(N - \hat{N}'_2 + 1)\{2(N - \hat{N}'_2) + 1\}}{(N - \hat{N}'_2)} \right\}
\]

(9)
where \( E[\cdot] \) and \( \text{var}[\cdot] \) denote mean and variance, respectively. Derivation of (8) are given in appendix A and B while derivation of (9) are given in Appendix C and D. It should be noted here that similar steps given in the Appendix to calculate mean and variance have been followed in case of ED as well as i-AVCD.

### 3.2 Energy Detection

The test statistics can be expressed as

\[
Z'(y) = \frac{1}{N - L_{scp}} \left( N - L_{scp} - m + 1 \right) y_{N-m+1}^2.
\]  

(10)

Now, using the central limit theorem, the pdf of \( Z'(y) \) can be expressed as Gaussian under both the hypotheses with the following mean and variance.

\[
E[Z'(y)|H_0'] = b^2 \left( N - \hat{N}_2 + 1 \right),
\]

\[
\text{var}[Z'(y)|H_0'] = \frac{20b^4}{6} \left\{ \frac{(N - \hat{N}_2 + 1)\{2(N - \hat{N}_2) + 1\}}{(N - \hat{N}_2)} \right\}.
\]  

(11)

\[
E[Z'(y)|H_1'] = \frac{N - \hat{N}_2}{2} \left\{ 2b^2 + c^2 \right\},
\]

\[
\text{var}[Z'(y)|H_1'] = \frac{1}{6} \left\{ \frac{(N - \hat{N}_2 + 1)\{2(N - \hat{N}_2) + 1\}}{(N - \hat{N}_2)} \right\}
\]

\[
\times \left\{ 20b^4 + c^4 + 8b^2c^2 \right\}.
\]  

(12)

### 3.3 improved-Absolute Value Cumulation Detection

The test statistics can be expressed as

\[
Z'(y) = \frac{1}{N - L_{scp}} \left( N - L_{scp} - m + 1 \right) y_{N-m+1}^4.
\]  

(13)

Now, using the central limit theorem, the pdf of \( Z'(y) \) can be expressed as Gaussian under both the hypotheses with the following mean and variance.
where $\Gamma(n) = \int_0^{+\infty} e^{-t^n} dt$ is the complete Gamma function \cite{21}.

\begin{align}
E[Z'(y)|H'_o] &= \frac{b^P \Gamma(P + 1)(N - \hat{N}_2 + 1)}{2}, \\
\text{var}[Z'(y)|H'_o] &= \frac{b^{2P} \left\{ \Gamma(2P + 1) - \Gamma^2(P + 1) \right\}}{6} \\
&\quad \times \left\{ \frac{(N - \hat{N}_2 + 1) \{2(N - \hat{N}_2) + 1\}}{(N - \hat{N}_2)} \right\},
\end{align}

(14)

where $\Gamma(n) = \int_0^{+\infty} e^{-t^m} dt$ is the upper incomplete Gamma function \cite{21}.

\begin{align}
E[Z'(y)|H'_1] &= \frac{u(P) \times (N - \hat{N}_2^t + 1)}{2}, \\
\text{var}[Z'(y)|H'_1] &= \frac{v(P)}{6} \times \left\{ \frac{(N - \hat{N}_2^t + 1) \{2(N - \hat{N}_2^t) + 1\}}{(N - \hat{N}_2^t)} \right\}.
\end{align}

(15)

where

\begin{align}
u(P) &= \frac{b^P}{2} \exp \left( \frac{|C|}{\sqrt{2b^2}} \right) \Gamma \left( P + 1, \frac{|C|}{\sqrt{2b^2}} \right) \\
&\quad + \frac{b^P}{2} \exp \left( -\frac{|C|}{\sqrt{2b^2}} \right) \Gamma \left( P + 1 \right) \\
&\quad + \frac{b^P}{2} \exp \left( \frac{-|C|}{\sqrt{2b^2}} \right) M \left( P, \frac{|C|}{\sqrt{2b^2}} \right).
\end{align}

(16)

where $M(w,x) = \int_0^x e^{t^n} dt$, $\Gamma(w,x) = \int_x^{+\infty} e^{-t^m} dt$ is the upper incomplete Gamma function \cite{21}.

\begin{align}
v(P) &= u(2P) \times u^2(P).
\end{align}

(17)

The $P_F$ and the $P_D$ in each case can be expressed, by taking respective mean and variance, as

\begin{align}
P_F &= Pr\left\{ Z'(y) > \lambda | H'_o \right\} = Q \left( \frac{\lambda - E[Z'(y)|H'_o]}{\sqrt{\text{var}[Z'(y)|H'_o]}} \right),
\end{align}

(18)

Similarly,

\begin{align}
P_D &= Pr\left\{ Z'(y) > \lambda | H'_1 \right\} = Q \left( \frac{\lambda - E[Z'(y)|H'_1]}{\sqrt{\text{var}[Z'(y)|H'_1]}} \right).
\end{align}

(19)

here, $\lambda$ is the detection threshold.
4 Simulation Results

In this section, we present performance of the considered system with receiver operating characteristics (ROC). After estimating the LSCP, we apply CuSum based weighted samples for the considered detection schemes such as SMD, ED and i-AVCD. Using Neyman Pearson test, we determine threshold $\lambda$ using (18) and subsequently $P_D$ using (19). Then, we plot the ROC for the considered schemes i-AVCD for $P = 0.1$, SMD and ED. Fig. 1 shows the ROC at $\gamma = -5$ dB, $N_1 = 30$, $N_2 = 40$, $N'_1 = 10$, $N'_2 = 40$ and $N = 50$.

It can be seen that the i-AVCD outperforms the remaining two schemes. We have also presented simulation results for all the three schemes. The close matching of the simulation results with analytical counterparts, validates our analysis.

Figure 2 shows the ROC for the considered schemes at $\gamma = -10$ dB, $N_1 = 30$, $N_2 = 40$, $N'_1 = 10$, $N'_2 = 50$ and $N = 50$. We have also shown the performance of the three schemes without the LSCP and CuSum based test statistics. We refer to them as Conventional schemes. It can be seen that the considered schemes outperform the conventional scheme.

Figure 3 shows the ROC for the considered SMD scheme as ‘two PU change’ at $\gamma = -20$ dB, $N_1 = 10$, $N_2 = 30$, $N'_1 = 10$, $N'_2 = 30$ and $N = 100$. We have also presented the performance of the ‘One PU change’, in which only one transition of PU is there at $N_1$ and $N_1'$ in $H_0$ and $H_1$ respectively with total samples of $N$ in the sensing time. After applying LSCP estimation using dynamic programming, we get $\hat{N}_1$ and $\hat{N}_1'$. Then, we perform CuSum based SMD scheme using $\hat{N}_1$ and $\hat{N}_1'$. We take $N = 100$, $N_1 = 30$ and $N'_1 = 30$ and $\gamma = -20$ dB. For fair comparison between both we have kept the effective sensing time as 30 samples and total samples in sensing time as 100. It can be seen that the one PU change outperforms the two PU change. The reason behind this is less error in estimating LSCP in case of one PU change.

![Fig. 1 ROC comparison of the LSCP estimation based CuSum to SMD, ED and i-AVCD at $N = 50$ and $\gamma = -5$ dB](image-url)
In this letter, the effect of dynamic behaviour of PU was observed on the ROC in the additive Laplacian noise channel. The dynamic behaviour of the PU was assumed by two status changes of PU in the fixed sensing time. We estimated the last status change point (LSCP) by dynamic programming and then detected the PU using the samples available from the

![ROC comparison of the LSCP estimation based CuSum to SMD, ED and i-AVCD with conventional schemes at N = 50 and γ = −10 dB](image1)

Fig. 2 ROC comparison of the LSCP estimation based CuSum to SMD, ED and i-AVCD with conventional schemes at $N = 50$ and $\gamma = -10$ dB

![ROC comparison of the LSCP estimation based CuSum to SMD for one PU status change and two PU status change at $\gamma = -5$ dB and $\gamma = -20$ dB, N = 100](image2)

Fig. 3 ROC comparison of the LSCP estimation based CuSum to SMD for one PU status change and two PU status change at $\gamma = -5$ dB and $\gamma = -20$ dB, $N = 100$

5 Conclusion

In this letter, the effect of dynamic behaviour of PU was observed on the ROC in the additive Laplacian noise channel. The dynamic behaviour of the PU was assumed by two status changes of PU in the fixed sensing time. We estimated the last status change point (LSCP) by dynamic programming and then detected the PU using the samples available from the
LSCP to the end of sensing time by ignoring the samples before LSCP and boosting the samples after LSCP. Then, we used Cumulative Summation (CuSum) based weighted samples in the detection schemes such as improved absolute value cumulation detection (i-AVCD), Sample Mean Detector (SMD) and Energy Detection (ED). We derived the expressions of $P_D$ and $P_F$ for all the three schemes. We plot the ROC for all the three schemes and found that the i-AVCD outperforms the remaining two schemes. We also presented simulation results and found close matching between their analytical counterparts. We compared the considered schemes with the conventional schemes, where no LSCP or CuSum based weighted samples were used. In this case, the considered schemes outperform the conventional schemes. Finally, we compared the considered two PU status changes schemes with one PU status change. We observed that one PU status change outperforms the two PU status changes due to less estimation error in LSCP.

**Appendix**

**A. Proof of $E[Z'(y)|H'_o]$**

From (7), we have

$$Z'(y) = \frac{1}{N - L_{scp}} \sum_{m=1}^{N-L_{scp}} (N - L_{scp} - m + 1)y_{N-m+1}. \tag{20}$$

Expanding the above expression, we get

$$E[Z'(y)|H'_o] = \frac{1}{N - L_{scp}} E[(N - L_{scp})w_N + (N - L_{scp} - 1)w_{N-1} + \cdots + w_{L_{scp}+1}]. \tag{21}$$

As $w_N, w_{N-1}, w_{N-2} \cdots$ are all i.i.d, hence

$$E[w_N] = E[w_{N-1}] = \cdots E[w_{L_{scp}+1}] = 0.$$ The above expression of $E[Z'(y)|H'_o]$ can be further simplified as

$$E[Z'(y)|H'_o] = 0.$$

**B. Proof of $\text{var}[Z'(y)|H'_o]$**

We have

$$\text{var}[Z'(y)|H'_o] = \text{var} \left[ \frac{1}{N - L_{scp}} \sum_{m=1}^{N-L_{scp}} (N - L_{scp} - m + 1)y_{N-m+1} \right]$$

$$= \left( \frac{1}{N - L_{scp}} \right) \text{var} \left[ (N - L_{scp})w_N + (N - L_{scp} - 1)w_{N-1} + \cdots + w_{L_{scp}+1} \right]. \tag{22}$$
As \( w_N, w_{N-1}, w_{N-2} \ldots \) are all i.i.d.s, hence \( \text{var}[w_N] = \text{var}[w_{N-1}] = \ldots \text{var}[w_{L_{scp}+1}] = 2b^2 \). The above expression of \( \text{var}[\mathbf{Z}'(\mathbf{y})|H_o'] \) can be further simplified as

\[
\text{var}[\mathbf{Z}'(\mathbf{y})|H_o'] = \frac{2b^2}{(N - L_{scp})^2} \left[ (N - L_{scp})^2 + (N - L_{scp} - 1)^2 + \cdots + 1^2 \right]
\]

\[
= \frac{2b^2}{(N - L_{scp})^2} \left[ 1^2 + 2^2 + \cdots + (N - L_{scp} - 2)^2 + (N - L_{scp} - 1)^2 + (N - L_{scp})^2 \right]
\]

\[
= \frac{2b^2}{6} \frac{(N - L_{scp} + 1) \{2(N - L_{scp}) + 1\}}{N - L_{scp}}.
\]

As \( L_{scp} = \hat{N}_2 \), the final expression becomes

\[
\text{var}[\mathbf{Z}'(\mathbf{y})|H_o'] = \frac{2b^2}{6} \frac{(N - \hat{N}_2 + 1) \{2(N - \hat{N}_2) + 1\}}{N - \hat{N}_2}.
\]

(23)

C. Proof of \( E[\mathbf{Z}'(\mathbf{y})|H_1'] \)

We have

\[
E[\mathbf{Z}'(\mathbf{y})|H_1'] = E \left[ \frac{1}{N - L_{scp}} \left( \sum_{m=1}^{N-L_{scp}} (N - L_{scp} - m + 1) y_{N-m+1} \right) \right].
\]

(24)

Expanding the above expression, we get

\[
E[\mathbf{Z}'(\mathbf{y})|H_1'] = \frac{1}{N - L_{scp}} E \left[ (N - L_{scp}) \{w_N + C\} \right.
\]

\[
+ (N - L_{scp} - 1) \{w_{N-1} + C\} + \cdots + \{w_{L_{scp}+1} + C\}. \]

As \( w_N, w_{N-1}, w_{N-2} \ldots \) are all i.i.d.s, hence \( E[w_N] = E[w_{N-1}] = \ldots E[w_{L_{scp}+1}] = 0 \). The above expression of \( E[\mathbf{Z}'(\mathbf{y})|H_1'] \) can be further simplified as
\[ E[Z'(y)|H'_1] = \frac{1}{N - L_{scp}} \left[ C(N - L_{scp}) + C(N - L_{scp} - 1) + C(N - L_{scp} - 2) + \cdots + C \right] \]
\[ = \frac{C}{N - L_{scp}} \left[ (N - L_{scp}) + (N - L_{scp} - 1) + (N - L_{scp} - 2) + \cdots + 1 \right] \]
\[ = \frac{C}{2} \left\{ N - L_{scp} - 1 \right\}. \]

As \( L_{scp} = \hat{N}'_2 \), the final expression becomes
\[ E[Z'(y)|H'_1] = \frac{C}{2} \left\{ N - \hat{N}'_2 - 1 \right\}. \] (25)

D. Proof of var\([Z'(y)|H'_1]\)

We have
\[ \text{var}[Z'(y)|H'_1] = \text{var} \left[ \frac{1}{N - L_{scp}} \left( \sum_{m=1}^{N-L_{scp}} (N - L_{scp} - m + 1) y_{N-m+1} \right) \right] \]
\[ = \left( \frac{1}{N - L_{scp}} \right) \text{var} \left[ (N - L_{scp}) \{ w_N + C \} + (N - L_{scp} - 1) \{ w_{N-1} + C \} + \cdots + \{ w_{L_{scp}+1} + C \} \right]. \] (26)

As \( w_N, w_{N-1}, w_{N-2} \cdots \) are all i.i.d.s, hence \( \text{var}[w_N] = \text{var}[w_{N-1}] = \cdots \text{var}[w_{L_{scp}+1}] = 2b^2 \). The above expression of \( \text{var}[Z'(y)|H'_1] \) can be further simplified as
\[ \text{var}[Z'(y)|H'_1] = \frac{2b^2}{(N - L_{scp})^2} \left[ (N - L_{scp})^2 \right. \]
\[ + (N - L_{scp} - 1)^2 + \cdots + 1^2 \left. \right] \]
\[ = \frac{2b^2}{(N - L_{scp})^2} \left[ 1^2 + 2^2 + \cdots + (N - L_{scp} - 2)^2 \right] \]
\[ + (N - L_{scp} - 1)^2 + (N - L_{scp})^2 \] (27)
\[ = \frac{2b^2}{6} \left[ \frac{(N - L_{scp} + 1)(2(N - L_{scp}) + 1)}{N - L_{scp}} \right] \]
\[ = \frac{2b^2}{6} \left[ \frac{(N - \hat{N}'_2 + 1)(2(N - \hat{N}'_2) + 1)}{N - \hat{N}'_2} \right]. \]
Declarations

Conflict of Interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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