Condensates near the Argyres-Douglas point in SU(2) gauge theory with broken $\mathcal{N}=2$ supersymmetry

A. Gorsky$^a$

$^a$Institute of Experimental and Theoretical Physics, Moscow 117259,

Abstract

The behaviour of the chiral condensates in the SU(2) gauge theory with broken $\mathcal{N}=2$ supersymmetry is reviewed. The calculation of monopole, dyon, and charge condensates is described. It is shown that the monopole and charge condensates vanish at the Argyres-Douglas point where the monopole and charge vacua collide. This phenomenon is interpreted as a deconfinement of electric and magnetic charges at the Argyres-Douglas point.

Talk given at ”Quarks-2000”, Pushkino, May 2000
1 Introduction

This talk is based on the paper [?] where the behaviour of the supersymmetric gauge theories near the Argyres-Douglas point was considered. The main question discussed concerned the nature of the hypothetical phase transition occured at the Argyres-Douglas point. It is widely believed that the theory near this point flows into the superconformal point in the infrared however the physics of this critical point was unclear. Since the results presented below are based on the exact statements concerning $\mathcal{N}=1$ and $\mathcal{N}=2$ supersymmetric theories the identification of the phase transition at the Argyres-Douglas point as a deconfinement phase transition is rigorous.

The derivation of exact results in $\mathcal{N}=1$ supersymmetric gauge theories based on low energy effective superpotentials and holomorphy was pioneered in [2, 3] and then strongly developed, mostly by Seiberg, see [4] for review. An extra input was provided by the Seiberg-Witten solution of $\mathcal{N}=2$ supersymmetric gauge theories with and without matter [5]. It was also clarified that Seiberg-Witten solution amounts the set of vacua in the corresponding $\mathcal{N}=1$ theory [6, 7, 8, 11, 12]. Different vacua are distinguished by values of chiral condensates, such as gluino condensate $\langle \text{Tr} \lambda \lambda \rangle$ and the condensate of the fundamental matter $\langle \bar{Q}Q \rangle$. Recently some points concerning the formation of the condensate and the identification of the relevant field configurations were clarified in [12, 13, 14, 15].

We compare then the condensate of the adjoint matter with the discriminant locus defined by Seiberg-Witten solution in $\mathcal{N}=2$ theory and find a complete matching. Our results for matter and gaugino condensates are consistent with those obtained by ‘integrating in’ method [16, 17, 18] and can be viewed as an independent confirmation of the method. What is specific for our approach is that we start from weak coupling regime where notion of effective Lagrangian is well defined and then use holomorphy to extend results for chiral condensates into strong coupling.

Then we determine monopole, dyon and charge condensates following to the Seiberg-Witten approach, i.e. considering effective superpotentials near singularities on the Coulomb branch in $\mathcal{N}=2$ theory. Again, holomorphicity allows us to extend results to the domain of strong $\mathcal{N}=2$ breaking.

Our next step is study of chiral condensates in the Argyres-Douglas (AD) points. These points were originally introduced in moduli/parameter space of $\mathcal{N}=2$ theories as points where two singularities on the Coulomb branch collide [18, 19, 20]. It is believed that the theory at the AD point flows in infrared to a nontrivial superconformal theory. The notion of AD point continue to make sense even when the $\mathcal{N}=2$ theory is broken to $\mathcal{N}=1$ by nonzero $\mu$, in the $\mathcal{N}=1$ theory it is the point in parameter space where two vacua collide.

Particularly, we consider collision of monopole and charge vacua at certain value of the mass of the fundamental flavor. Our key result is that both monopole and charge condensates vanish at the AD point. We interpret this as deconfinement of both electric...
and magnetic charges at the AD point.

Let us remind that the condensation of monopoles ensures confinement of quarks in the monopole vacuum \( [5] \), while the condensation of charges provides confinement of monopoles in the charge vacuum. As it was shown by ’t Hooft \([21]\) it is impossible for these two phenomena to coexist. This leads to a paradoxical situation in the AD point where the monopole and charge vacua collide. Our result resolves this paradox.

This paradox is a part of more general problem: whether there is a uniquely defined theory in the AD point. Indeed, when two vacua collide the Witten index of the emerging theory is 2, i.e. there are two bosonic vacuum states. The question is if there is any physical quantity which could serve as an order parameter differentiating these two vacua. The continuity of chiral condensates in the AD point we found shows that these condensates are not playing this role. The same continuity leads also to vanishing of tension of domain walls interpolating between colliding vacua when we approach the AD point.

## 2 Matter and gaugino condensates

Let us consider \( \mathcal{N} = 1 \) theory with SU(2) gauge group where the matter sector consists of the adjoint field \( \Phi^a_\alpha = \Phi^{\alpha (\tau^a)/2}_\beta \) (\( \alpha, \beta = 1, 2; a = 1, 2, 3 \)) and two fundamental fields \( Q^f_\alpha \) (\( f = 1, 2 \)) describing one flavor. The most general renormalizable superpotential for this theory has the form,

\[
W = \mu \text{Tr} \Phi^2 + \frac{m}{2} Q^f_\alpha Q^f_\alpha + \frac{1}{\sqrt{2}} h^{fg}_{\beta} Q^f_\alpha \Phi^\beta_\gamma Q^\gamma_\delta.
\]  

(1)

Here parameters \( \mu \) and \( m \) are related to masses of the adjoint and fundamental fields, \( m_\Phi = \mu/Z_\Phi, m_Q = m/Z_Q \), by corresponding \( Z \) factors in kinetic terms. Having in mind normalization to the \( \mathcal{N} = 2 \) case we choose for bare parameters \( Z_\Phi^0 = 1/g_\Phi^2, Z_Q^0 = 1 \). The matrix of Yukawa couplings \( h^{fg}_{\beta} \) is the symmetric, summation over color indices \( \alpha, \beta = 1, 2 \) is explicit. Unbroken \( \mathcal{N} = 2 \) SUSY appears when \( \mu = 0 \) and \( \det h = -1 \).

To get an effective theory similar to SQCD we integrate out the adjoint field \( \Phi \) implying that \( m_\Phi \gg m_Q \). In classical approximation this integration reduces to the substitution

\[
\Phi^\beta_\gamma = -\frac{1}{2\sqrt{2} \mu} h^{fg}_{\beta} \left( Q^f_\alpha Q^\alpha_\gamma - \frac{1}{2} \delta^\beta_\gamma Q^f_\alpha Q^\alpha_\gamma \right),
\]  

(2)

which follows from \( \partial W/\partial \Phi = 0 \). It is well known from the study of SQCD that perturbative loops do not contribute and nonperturbative effects are exhausted by the Affleck-Dine-Seiberg (ADS) superpotential generated by one instanton \([3]\). The effective superpotential then is

\[
W_{\text{eff}} = m V - \frac{(-\det h)}{4\mu} V^2 + \frac{\mu^2 \Lambda^3}{4V},
\]  

(3)
where the gauge and subflavor invariant chiral field \( V \) is defined as
\[
V = \frac{1}{2} Q^\alpha_f Q^f_{\alpha}.
\]

The third nonperturbative term in Eq. (3) is the ADS superpotential. The coefficient \( \mu^2 \Lambda^3_1/4 \) in the ADS superpotential is an equivalent of \( \Lambda^5_{SQCD} \) in SQCD. The factor \( \mu^2 \) in the coefficient reflects four zero modes of the adjoint field, see e.g. Ref. [22, 14] for details.

When \( \det h \) is nonvanishing we have three vacua, marked by vevs of the lowest component of \( V \),
\[
v = \langle V \rangle.
\]
These vevs are roots of the algebraic equation \( dW_{\text{eff}}/dv = 0 \) which looks as
\[
m - \frac{(- \det h)}{2} v \frac{\Lambda^3}{\mu} - \frac{\Lambda^3}{4} \left( \frac{\mu}{v} \right)^2 = 0.
\]
This equation shows, in particular, that although the second term in the superpotential (3) looks as suppressed at large \( \mu \) it is of the same order as the ADS term. From Eq. (3) it is also clear that the dependence on \( \mu \) is given by scaling \( v \propto \mu \).

To see dependence on other parameters let us substitute \( v \) by the dimensionless variable \( \kappa \) as
\[
v = \mu \sqrt[4]{\frac{\Lambda^3}{4m}} \kappa.
\]
Then Eq. (3) in terms of \( \kappa \)
\[
1 - \sigma \kappa - \frac{1}{\kappa^2} = 0
\]
is governed by the dimensionless parameter \( \sigma \),
\[
\sigma = \frac{(- \det h)}{4} \left( \frac{\Lambda_1}{m} \right)^{3/2}.
\]

To verify this interesting mapping we need to find out vevs for
\[
u = \langle U \rangle = \langle \text{Tr} \Phi^2 \rangle.
\]
This can be done using set of Konishi anomalies. Generic equation for arbitrary matter field \( Q \) looks as follows (we are using notations of the review [11]):
\[
\frac{1}{4} \bar{D}^2 J_Q = Q \frac{\partial W}{\partial Q} + T(R) \frac{\text{Tr} W^2}{8\pi^2},
\]
where \( T(R) \) is the Casimir in the matter representation. The left hand side is the total derivative in superspace so its average over supersymmetric vacuum vanishes. In our
case it results in two relations for condensates,

\[
\left\langle \frac{m}{2} Q^a_i Q^i_a + \frac{1}{\sqrt{2}} h^{fg} Q_{\alpha f} \Phi^\beta_g Q^\beta_g + \frac{1}{2} \frac{\text{Tr} W^2}{8\pi^2} \right\rangle = 0
\]

\[
\left\langle 2 \mu \text{Tr} \Phi^2 + \frac{1}{\sqrt{2}} h^{fg} Q_{\alpha f} \Phi^\beta_g Q^\beta_g + 2 \frac{\text{Tr} W^2}{8\pi^2} \right\rangle = 0
\]  

(12)

From the first relation after substitution (2) and comparison with Eq. (6) we find the expression for gluino condensate

\[
s = \frac{\langle \text{Tr} \lambda^2 \rangle}{16\pi^2} = -\frac{\langle \text{Tr} W^2 \rangle}{16\pi^2} = \frac{\mu^2 \Lambda_1^3}{4 v}.
\]  

(13)

This is consistent with the general expression \([T_G - \sum T(R)]\langle \text{Tr}\lambda^2 \rangle/16\pi^2\) for the non-perturbative ADS piece of the superpotential (3) (24). Combining then two relations (12) we express the condensate value of u via v,

\[
u = \frac{1}{2\mu} (mv + 3s) = \frac{1}{2\mu} \left( mv + \frac{3}{4} \frac{\mu^2 \Lambda_1^3}{v} \right) = \frac{\sqrt{m\Lambda_1^3}}{4} \left( \kappa + \frac{3}{\kappa} \right).
\]  

(14)

Now we see that at the limit of large m two vacua \(\kappa = \pm 1\) are in perfect correspondence with \(u = \pm \Lambda_0^3\) for the monopole and dyon vacua of SYM. Indeed, \(\Lambda_0^3 = m\Lambda_1^3\) is a correct relation between scale parameters of the theories.

For the third vacuum at large m the value \(u = m^2/(-\det h)\) corresponds on the Coulomb branch to the so called charge vacuum, where some fundamental fields become massless. Moreover, the correspondence with \(\mathcal{N}=2\) results can be demonstrated for three vacua at any value of m. To this end we use the relation (14) and Eq. (8) to derive the following equation for u,

\[(-\det h) u^3 - m^2 u^2 - \frac{9}{8} (-\det h) m\Lambda_1^3 u + m^3\Lambda_1^3 + \frac{27}{2^8 (-\det h)^2 \Lambda_1^3} = 0.\]  

(15)

Three roots of this equation are vevs of Tr \(\Phi^2\) in the corresponding vacua.

How does it look from \(\mathcal{N}=2\) side? The Riemann surface governing the Seiberg-Witten solution is given by the curve

\[y^2 = x^3 - u x^2 + \frac{1}{4} \Lambda_1^3 m x - \frac{1}{64} \Lambda_1^6.\]  

(16)

Singularities of the metric, i.e. the discriminant locus of the curve, is defined by two equations, \(y^2 = 0\) and dy^2/dx = 0,

\[x^3 - u x^2 + \frac{1}{4} \Lambda_1^3 m x - \frac{1}{64} \Lambda_1^6 = 0, \quad 3x^2 - 2u x + \frac{1}{4} \Lambda_1^3 m = 0,\]  

(17)

which lead to

\[u^3 - m^2 u^2 - \frac{9}{8} m\Lambda_1^3 u + m^3\Lambda_1^3 + \frac{27}{2^8 \Lambda_1^3} = 0.\]  

(18)
We see that this is a particular case of the $\mathcal{N}=1$ equation (13) at $\det h = -1$.

The point in the parameter manifold where two vacua coincide is the AD point [18]. In $SU(2)$ theory these points were studied in [19]. Mutually non-local states, say charges and monopoles becomes massless at these points. On the Coulomb branch of $\mathcal{N}=2$ theory these points correspond to non-trivial conformal field theory [19]. Here we study the $\mathcal{N}=1$ SUSY theory, where $\mathcal{N}=2$ is broken down by the mass term for the adjoint matter as well as by the difference of the Yukawa coupling from its $\mathcal{N}=2$ value.

But collisions of two vacua still occur in the theory. In this subsection we find the values of $m$ at which AD points appear and calculate values of condensates at this point. In the next section we study what happen to the confinement of charges in the monopole point at non-zero $\mu$ once we approach AD point.

First let us work out the AD values of $m$, generalizing the consideration [19]. Collision of two roots for $v$ means that together with Eq. (6) the derivative of its left-hand-side should also vanish,

$$m - \frac{(- \det h)}{2} \frac{v}{\mu} - \frac{\Lambda_1^3}{4} \left( \frac{\mu}{v} \right)^2 = 0, \quad -(- \det h) + \Lambda_1^3 \left( \frac{\mu}{v} \right)^3 = 0. \tag{19}$$

This system is consistent only at three values of $m = m_{AD}$,

$$m_{AD} = \frac{3}{4} \omega \Lambda_1 (- \det h)^{2/3}, \quad \omega = e^{2\pi in/3} (n = 0, \pm 1), \tag{20}$$

related by $Z_3$ symmetry. The condensates at the AD vacuum are

$$v_{AD} = \omega \frac{\mu \Lambda_1}{(-\det h)^{1/3}},$$

$$u_{AD} = \omega^{-1} \frac{3}{4} \Lambda_1^2 (- \det h)^{1/3},$$

$$s_{AD} = \omega^{-1} \frac{1}{4} \mu \Lambda_1^2 (- \det h)^{1/3}. \tag{21}$$

### 3 Dyon condensates

In this section we calculate various dyon condensates at three vacua of the theory. As it was discussed above holomorphicity allows us to find these condensates starting from consideration on the Coulomb branch in $\mathcal{N}=2$ near the singularities associated with given massless dyon. Namely, we calculate the monopole condensate near the monopole point, the charge condensate near the charge point and the dyon $(n_m, n_e) = (1, 1)$ condensate near the point where this dyon is light. Although we start with small value of adjoint mass parameter $\mu$, our results for condensates are exact for any $\mu$. 


3.1 Monopole condensate.

Let us start with calculation of the monopole condensate near the monopole point. Near this point the effective low energy description of our theory can be given in terms of $\mathcal{N}=2$ dual QED $^3$. It includes light monopole hypermultiplet interacting with vector (dual) photon multiplet in the same way as electric charges interact with ordinary photons. Following Seiberg and Witten $^5$ we write down the effective superpotential in the following form.

$$W = \sqrt{2} \tilde{M} M A_D + \mu U,$$

(22)

where $A_D$ is a chiral neutral field (it is a part of $\mathcal{N}=2$ dual photon multiplet in $\mathcal{N}=2$ theory) and $U = \text{Tr} \Phi^2$. The second term breaks $\mathcal{N}=2$ supersymmetry down to $\mathcal{N}=1$.

Variating this superpotential with respect to $A_D$, $M$ and $\tilde{M}$ we find that $A_D = 0$, i.e. the monopole mass vanishes, and

$$\langle \tilde{M} M \rangle = -\frac{\mu}{\sqrt{2}} \frac{du}{dA_D}. \quad (23)$$

The condition $A_D = 0$ means that the Coulomb branch near the monopole point, where the monopole mass vanishes, shrinks to the single vacuum state at the singularity while Eq. (23) together with D flatness condition (up to gauge transformation) $\tilde{M} = M$ determines the value of monopole condensate.

The non-zero value of monopole condensate ensures the U(1) confinement for charges via the formation of Abrikosov-Nielsen-Olesen vortices. Let us work out the r.h.s. of Eq. (23) to determine the $\mu$ and $m$ dependence of the monopole condensate. From exact Seiberg-Witten solution $^3$ we have

$$\frac{dA_D}{du} = \frac{\sqrt{2}}{8\pi} \int y(x). \quad (24)$$

Here for $y(x)$ given by Eq. (16) we use the form

$$y^2 = (x - e_0)(x - e_-)(x - e_+). \quad (25)$$

We get finally

$$\langle \tilde{M} M \rangle = 2i\mu \left( u_M^2 - \frac{3}{4} m \Lambda_1^3 \right)^{1/4}. \quad (26)$$

Now let us address the question: what happens with the monopole condensate when we reduce $m$ and approach the AD point. The AD point corresponds to particular value of $m$ which ensures colliding of monopole and charge singularities in the $u$ plane. Near the monopole point we have condensation of monopoles and confinement of charges while near the charge point we have condensation of charges and confinement of monopoles. As it was shown by 't Hooft these two phenomena cannot happen.
The question is: what happens when monopole and charge points collide in the \( u \) plane?

The monopole condensate at the AD point is given by Eq. (26) when \( m_{AD} \) and \( u_{AD} \) from Eqs. (20) and (21) are substituted,

\[
\langle \tilde{M}M \rangle_{AD} = 0.
\]  

We see that monopole condensate goes to zero at the AD point. Our derivation above makes clear why it happens. At the AD point all three roots of \( y^2 \) become degenerate, \( e_+ = e_- = e_0 \), so the monopole condensate which is proportional to \( \sqrt{e - e_0} \) naturally vanishes.

In the next subsection we calculate the charge condensate in the charge point and show that it also goes to zero as \( m \) approaches its AD value (20). Thus we interpret the AD point as a deconfinement point for both monopoles and charges.

### 3.2 Charge and dyon condensates

In this subsection we use the same method to calculate values of charge and dyon condensate near charge and dyon points respectively. We first consider \( m \) above AD value (20) and then continue our results to values of \( m \) below \( m_{AD} \). In particular in the limit \( m = 0 \) we recover \( Z_3 \) symmetry.

Let us start with the charge condensate. At \( \mu = 0 \), \( \det h = -1 \) and large \( m \) the effective theory near the charge point

\[
a = -\sqrt{2} m
\]  

on the Coulomb branch is \( \mathcal{N}=2 \) QED. The half of degrees of freedom in color doublets becomes massless whereas the other half acquire large mass \( 2m \). These massless fields form one hypermultiplet \( \tilde{Q}_+, Q_+ \) of charge particle in the effective electrodynamics. Once we add the mass term for the adjoint matter the effective superpotential near the charge point becomes

\[
W = \frac{1}{\sqrt{2}} \tilde{Q}_+ Q_+ A + m \tilde{Q}_+ Q_+ + \mu U
\]  

Minimizing this superpotential we get condition (28) as well as

\[
\langle \tilde{Q}_+ Q_+ \rangle = -\sqrt{2} \mu \frac{du}{da}.
\]  

Now following the same steps which led us from (23) to (26) we get

\[
\langle \tilde{Q}_+ Q_+ \rangle = 2 \mu (u_C^2 - \frac{3}{4} m \Lambda_1^3)^{1/4}
\]  

8
Here $u_C$ is the position of charge point in the $u$ plane, $u_C = m^2$ at large $m$, see Eq. (??). Thus, at large $m$

$$
\langle \tilde{Q}_+ Q_+ \rangle = 2 \mu m .
$$

(32)

Holomorphicity allows us to extend the result (??) to arbitrary $m$ and $\det h$. So we can use Eq. (??) to find the charge condensate at the AD point. Using Eqs. (20) and (21) we see that the charge condensates vanishes in the AD point the same way the monopole one does. As it was mentioned we interpret this as deconfinement for both charges and monopoles.

Similarly to the monopole condensate we can relate the charge condensate with the quark one $v$

$$
\langle \tilde{Q}_+ Q_+ \rangle^2 = v^2 - \frac{\mu^3 \Lambda^3}{v} = v^2 - 4\mu s ,
$$

(33)

This expression differs from the one for the monopole condensate only by sign. The coincidence of the charge condensate with the quark one at large $v$, i.e. at weak coupling is natural. The difference is due to nonperturbative effects and similar to the difference between $a^2/2$ and $u$ on the Coulomb branch of the $\mathcal{N}=2$ theory. In strong coupling the difference is not small, in particular, the charge condensate vanishes in the AD point while the quark condensate remains finite.

Note that near the AD point we can consider an effective superpotential which includes both light monopole and charge fields simultaneously. Such consideration leads to the same results for condensates.

Now let us work out the dyon condensate. More generally let us introduce the dyon field $D_i$, $i = 1, 2, 3$, which stands for charge, monopole and $(1,1)$ dyon, $D_i = (Q_+, M, D)$. The arguments of the previous subsection which led us to the result (26) for monopole condensate gives for $\langle \tilde{D}_i D_i \rangle$

$$
\langle \tilde{D}_i D_i \rangle = 2i \zeta_i \mu \left( u_i^2 - \frac{3}{4} m \Lambda_1^3 \right)^{1/4} ,
$$

(34)

where $u_i$ is the position of the $i$-th point in the $u$ plane and $\zeta_i$ are phase factors.

For the monopole condensate at real values of $m$ larger than $m_{AD} = (3/4) \Lambda_1 (\det h)^{2/3}$ Eq. (26) gives

$$
\zeta_M = 1,
$$

(35)

while for charge from Eq. (??)

$$
\zeta_C = -i .
$$

(36)

In fact one can fix the phase factor for charge imposing the condition that the charge condensate should approach the value $2m\mu$ in the large $m$ limit. For dyon the phase factor is

$$
\zeta_D = i .
$$

(37)

At the AD point monopole and charge condensates go to zero, while the dyon one remains non-zero, see (34). Below the AD point condensates are given by the same
Eq. (34), but the phase factors for charge and monopole can change its values. The dyon phase factor (37) is not changing when we move through the AD point because the dyon condensate does not vanish at this point.

4 The Argyres-Douglas point: how well the theory is defined

As we discussed in Introduction in the AD point we encounter the problem of not uniquely defined vacuum state. Indeed, when the mass parameter $m$ approaches its AD value $m_{AD}$ we deal with two vacuum states which can be distinguished by values of chiral condensates. It is unlikely that the number of states with zero energy will change when we reach the AD point, it is very much similar to Witten index. However, the continuity of chiral condensates we obtained above shows that they are no longer parameters which differentiate two states once we reach the AD point.

A natural possibility to consider is domain walls interpolating between colliding vacua. In case of BPS domain walls their tension is given by central charges,

$$T_{ab} = 2 |W_{\text{eff}}(v_a) - W_{\text{eff}}(v_b)|$$

where $a,b$ label colliding vacua. The central charge here is expressed via values of exact superpotential (3) in corresponding vacua. The continuity of the condensate $v$ shows that the domain wall becomes tensionless in the AD point. If such domain wall were observable it could serve as a signal of two vacua.

Let us note one more interesting question. Namely the BPS tension should obey the Picard-Fuchs equation providing the dependence on the quark mass. The mass corresponding to the position of the Argyres-Douglas point plays the role of the ”strong coupling singularity” like the monopole singularity in the Seiberg-Witten solution. The generic structure of the monodromies at the complex m plane should provide the unique solution for the multiplet of tensions. It would be very interesting to compare the solutions of Picard-Fuchs equations with the tensions followed from the exact superpotentials.

5 Conclusions

We analyze monopole, charge and dyon condensates departing from the Coulomb branch of the $\mathcal{N}=2$ theory. It results in the explicit relations between these condensates and those of the fundamental matter. The most interesting phenomenon occurs in the AD point: when the monopole and charge vacua collide both the monopole

*Note that quantum numbers of “charge” and “monopole” are also changed, see [25]
and charge condensates vanish. We interpret this as a deconfinement of electric and magnetic charges in the AD point.

Let us mention a relation to finite-dimensional integrable systems. It was recognized that $\mathcal{N}=2$ theories are governed by finite-dimensional integrable systems. The integrable system responsible for $\mathcal{N}=2$ SQCD was identified with the nonhomogenous XXX spin chain [27]. After perturbation to the $\mathcal{N}=1$ theory the Hamiltonian of the integrable system is expected to coincide with the superpotential of corresponding $\mathcal{N}=1$ theory. This has been confirmed by direct calculation in the pure $\mathcal{N}=2$ gauge theory [28] as well in the theory with massive adjoint multiplet [29]. It would be very interesting to find a similar connection between spin chain Hamiltonians and superpotentials in the $\mathcal{N}=1$ SQCD. One more point to be clarified is a meaning of the AD point within approach based on integrability. Since the quark mass is identified as a value of spin [27] one could expect that at particular values of spins corresponding to the AD mass XXX spin chain has additional symmetries similar to superconformal ones.

I am grateful to A. Vainshtein and A. Yung for the collaboration on this issue. The work was partially supported by the grant INTAS-99-1705 and CRDF-RP1-2108.

References

[1] A. Gorsky, A. Vainshtein and A. Yung, Nucl. Phys. B584 (2000) 197; [hepth/0004087]

[2] I. Affleck, M. Dine and N. Seiberg, Phys. Lett. B137,187 (1984).

[3] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov Nucl. Phys. B229, 407 (1983) [Reprinted in Supersymmetry, Ed. S. Ferrara (North Holland/World Scientific, Amsterdam – Singapore, 1987), Vol. 1, page 606]; M. Shifman and A. Vainshtein, Nucl. Phys. B296, 445 (1988);

[4] K. Intriligator and N. Seiberg, Nucl. Phys. Proc. Suppl. 45BC, 1 (1996) [hep-th/9509060].

[5] N. Seiberg and E. Witten, Nucl. Phys. B426, 19 (1994); (E) B430, 485 (1994) [hep-th/9407087]; B431, 484 (1994) [hep-th/9408099].

[6] D. Kutasov, A. Schwimmer and N. Seiberg, Nucl. Phys. B459, 455 (1996).

[7] K. Intriligator and N. Seiberg, Nucl. Phys. B431, 551 (1994) [hep-th/9408153].

[8] S. Elitzur, A. Forge, A. Giveon, and E. Rabinovici, Phys. Lett. B353, 79 (1995) [hep-th/9504080]; Nucl. Phys. B459,160 (1996) [hep-th/9509130];

S. Elitzur, A. Forge, A. Giveon, K. Intriligator, and E. Rabinovici, Phys. Lett. B379, 121 (1996) [hep-th/9603051].
[9] S. Terashima and S. Yang, Phys. Lett. B391, 107 (1997) hep-th/9607151.

[10] K. Konishi and H. Terao, Nucl. Phys. B511 (1998) 264 hep-th/9707003.

[11] M. Shifman and A. Vainshtein, In *M.A. Shifman: ITEP lectures on particle physics and field theory,* vol. 2, page 485, World Scientific, Singapore, 1999 hep-th/9902018.

[12] N. M. Davies, T. J. Hollowood, V. V. Khoze and M. P. Mattis, Nucl. Phys. B559, 123 (1999) hep-th/9905015.

[13] N. M. Davies and V. V. Khoze, JHEP 0001, 015 (2000) hep-th/9911112.

[14] A. Ritz and A. Vainshtein, Nucl. Phys. B566, 311 (2000) hep-th/9909073.

[15] G. Carlino, K. Konishi and H. Murayama, JHEP 0002, 004 (2000) hep-th/0001036.

[16] K. Intriligator, R. G. Leigh and N. Seiberg, Phys. Rev. D50, 1092 (1994) hep-th/9403198.

[17] K. Intriligator, Phys. Lett. B336, 409 (1994) hep-th/9407106.

[18] P. C. Argyres and M. R. Douglas, Nucl. Phys. B448, 93 (1995) hep-th/950506.

[19] P. C. Argyres, M. R. Plesser, N. Seiberg and E. Witten, Nucl. Phys. B461, 71 (1996) hep-th/9511154.

[20] T. Eguchi, K. Hori, K. Ito and S. Yang, Nucl. Phys. B471, 430 (1996) hep-th/9603002.

[21] G. ’t Hooft, Nucl. Phys. B138, 1 (1978); Nucl. Phys. B153, 141 (1979).

[22] A. Yung, Nucl. Phys. B485, 38 (1997) hep-th/9604096.

[23] N. Seiberg, Phys. Rev. D49, 6857 (1994) hep-th/9402043.

[24] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. 260B, 157 (1985).

[25] A. Bilal and F. Ferrari, Nucl. Phys. B516, 175 (1998) hep-th/9706145.

[26] A. Hanany, M. Strassler and A. Zaffaroni, Nucl. Phys. B513, 87 (1998) hep-th/9707244.

[27] A. Gorsky, A. Marshakov, A. Mironov and A. Morozov, Phys. Lett. B380, 75 (1996) hep-th/9603140.

A. Gorsky, S. Gukov and A. Mironov, Nucl. Phys. B517, 409 (1998) hep-th/9707120.

A. Gorsky and A. Mironov, Nucl. Phys. B550, 513 (1999) hep-th/9902030.
[28] S. Katz and C. Vafa, Nucl. Phys. B\textbf{497}, 196 (1997) [hep-th/9611090].

[29] N. Dorey, JHEP \textbf{9907}, 021 (1999) [hep-th/9906011].