Setting and solving the game confrontation problem of the hardware-redundant dynamic system with an attacking enemy operating under incomplete information in the conflict process

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Abstract. The game task of confrontation of the attacked hardware-redundant dynamic system with an attacking enemy operating in conditions of incomplete information about the behavior of the attacked enemy in the process of conflict was posed and solved numerically and analytically. The attacking party aspires to increase the intensity of the failures of the components of the attacked system at the expense of its attack resources, up to its total failure. The attacked party, due to the corresponding strategy of redistribution of reserve blocks of the hardware-redundant dynamic system between the failed main blocks at the appropriate instants of time, strives to maximize the probability of failure-free operation of the attacked system at the end of the confrontation (game) with the attacking enemy. Behavior of the system under attack in the process of conflict is approximated by the Markov process, and the number of operable states is equal to the number of failed functional blocks that do not exceed the number of standby blocks. As a payment function in the game in question, the probability of failure-free operation of the attacked system is used by the time the game ends. The solution of the game is the vector of the system setup moments after the corresponding failures of the functional blocks and the set of reservation vectors corresponding to the instantaneous settings of the system being attacked, which maximizes the probability of system failure during the conflict. The differential game model is reduced to a multi-step matrix model with given probabilities of the states of the attacking enemy. Numerical algorithms for calculating the vector of reservation for the attacked system that maximizes the probability of its trouble-free operation by the end of the game and for solving the game problem in question are presented in a form convenient for implementation on a personal computer.

1. Introduction
Research in the field of setting and solving confrontation problems in conflict situations of dynamic systems of various nature, including problem-oriented game problems, is the subject of many scientific works, the closest of which in content to the problems considered in this article are the following works [1–12]. In the above works, it was directly or indirectly assumed that the conflicting parties in the conflict process have full information about the enemy’s behavior and the results of their actions, as well as about the state of the game in the previous steps. However, in practice this is not the case. In real conflict situations taking place in the economy, business, military, social affairs and other areas, for objective reasons [1], the conflicting parties participating in the conflict situation often
receive only probabilistic information about the results of their actions and the actions of the adversary, the rules of the game and the behavioral strategies of the players or their winning functions. In other words, in real conditions, one often encounters game tasks when one or both opposing sides act in conditions of incomplete information during a conflict. There are many models of such game tasks, among which, from the point of view of the author of this work, the most important are practical models related to the study of behavioral strategies and optimizing the reliability of the defending side in the conflict situation from enemy attacks. This article is devoted to the formulation and solution of one of these game problems with incomplete information related to the choice of the reserve strategy of a hardware-redundant dynamic system involved in a conflict that maximizes its probability of failure-free operation by the end of the confrontation (game) with the attacking opponent.

2. Formulation of the problem
We will assume that the hardware-redundant dynamic system \( S_A(n,m,s) \) controlled by player \( A \) consists of \( n, n = n_1 + n_2 + \ldots + n_q \) main functionalities and \( m, m = s_1 + s_2 + \ldots + s_q \) reserve blocks divided into \( q \) respective groups in each of which it is possible to replace any failed main functional block with reserve \( s = s_1 + s_2 + \ldots + s_q \) is integer reservation vector. In this case, we assume that the reserve blocks, after appropriate adjustment by reprogramming, can be arbitrarily redistributed by player \( A \) commands between \( q \) groups of the system \( S_A(n,m,s) \) to replace the failed main function blocks in the corresponding group. In the future, to simplify the model, we will neglect the time for setting up and redistributing reserve blocks instead of the failed main ones during the conflict, as well as the ultimate reliability of the system for monitoring the operation of main blocks, setting up and redistributing reserve blocks.

If necessary, the considered model of the hardware-redundant dynamic system \( S_A(n,m,s) \) can be complicated by introducing additional constraints considered in [13], which naturally leads to complication of all computational procedures when solving the considered problem. In the following, we will assume that each of the \( q \) main blocks \( n_1, n_2, \ldots, n_q \) of the system \( S_A(n,m,s) \) corresponds to the failure rates \( \lambda_1(t), \lambda_2(t), \ldots, \lambda_q(t) \), but not included to the reserve blocks \( s_1, s_2, \ldots, s_q \) corresponds to the failure rate \( \lambda_0(t) \), and \( \lambda_0(t) \leq [\lambda_1(t), \lambda_2(t), \ldots, \lambda_3(t)] \). After connecting the reserve block instead of the failed main in the \( i \)-th group, it starts working in the same mode as the main block, i.e. with a failure rate of \( \lambda_i(t), 1 \leq i \leq q \). In the process of conflict, the enemy, at the expense of his own means of attack, seeks to increase the failure rate of the components of the system \( S_A(n,m,s) \), therefore the functions \( \lambda_i(t) \) are increasing.

Let the attacked side \( A \) have the above described system \( S_A(n,m,s) \). On the time interval \( [0,t_f] \), where \( t_f \) is the end time of the game, we introduce the vector \( \tau = \{\tau_0, \tau_1, \tau_2, \ldots, \tau_L\}, \tau_0 = 0, \tau_L < t_f \), whose elements correspond to the moments redistribution of reserve elements of the hardware-redundant system between \( q \) groups. In the following, the vector \( \tau \) will be called the setting vector of the system \( S_A(n,m,s) \). To each time \( \tau_k, 0 \leq k \leq L \), we assign the distribution vector of the reserve blocks \( s(\tau_k) = \{s_1(\tau_k), s_2(\tau_k), \ldots, s_q(\tau_k)\} \). Thus, player \( A \) has a set of strategies \( W^A = \{\tau, s(\tau)\} \), satisfying the following constraints

\[
\min_{0 \leq k \leq L - 1} (\tau_{k+1} - \tau_k) \geq \alpha \tag{1}
\]

\[
\sum_{i=1}^{q} s_i(\tau_k) = m - \varphi(\tau_k) \tag{2}
\]
\[ \varphi(\tau_k) = \sum_{l=0}^{m} p_l(\tau_k) \]  

where \( \varphi(\tau_k) \) is the mathematical expectation of the number of elements of the system \( S_A(n,m,s) \) that have failed by the time \( \tau_k \). \( p_l(\tau_k) \) is the probability of finding the considered system in states with \( l \) refusals.

The physical meaning of the constraint (1) is that the side \( A \) is not allowed to make two consecutive settings too quickly, i.e. \( \alpha \) is the minimum time between two adjacent settings. The meaning of the constraint (2) is that by the time of setting \( \tau_k \), due to the replacement of the failed main function blocks of the system with the reserve, the number of the latter will decrease by the value \( \varphi(\tau_k) \).

Let side \( B \) attacking in the process of conflict on the system \( S_A(n,m,s) \) can be in one of the states \( B_1, B_2, \ldots, B_N \), characterized by the corresponding result of the attack on this system as a vector of failure rates of the system \( S_A(n,m,s) \) \( \lambda(t) = \{\lambda_i(t)\} \), each element of which represents a set of failure rates in \( q \) groups of main functional blocks and reserve blocks not included in the work \( \lambda_i(t) = \{\lambda_{i1}(t), \lambda_{i2}(t), \ldots, \lambda_{iq}(t)\} \), \( i = 1, 2, \ldots, N \). If for each state of the attacking side \( B_i \) are given the probabilities of its being in these states \( Q(t) = \{Q_i(t)\} \), then the set of player strategies from the side \( B \) can be defined as \( W^B = \{Q(t), \lambda(t)\} \). Thus, the actions of player \( B \) consist in the random selection of one of \( N \) states, which correspond to the intensity of non-stationary Poisson flows of failures of the main and reserve blocks of the system \( S_A(n,m,s) \).

Strategies of player \( B \) have the following constraints

\[ \sum_{i=1}^{N} Q_i(t) = 1 \]  

(4)

\[ \sum_{i=0}^{\infty} \int_0^{\infty} \lambda^j_i(t) dt \leq \Lambda^j, \quad j = 1, 2, \ldots, N \]  

(5)

the meaning of the first of which is obvious, and the physical meaning of the second is that for each state of the attacking side \( B \) the total attack on the main and reserve elements of the system \( S_A(n,m,s) \) is limited.

As a payment function in this game we will use the probability of failure-free operation of the system \( S_A(n,m,s) \) by the time the game ends \( P(t_f) \). Then the decision of the game will be the moment vector of settings \( \tau \) of the system \( S_A(n,m,s) \), and the set of reservation vectors \( \{s(\tau_k)\}, 0 \leq k \leq L \), corresponding to the moments of the settings \( \tau_k \), maximizing the probability of failure-free operation \( P(t_f) \) of the system under attack.

Consequently, the attacked side (player \( A \)) in the process of confrontation with the attacking side (player \( B \)) to defend against the enemy’s attacks is to redistribute the reserve blocks between the failed cores at the \( \tau_k \) setting points, so that by the end of the game the probability of the attacked system is maximum, i.e.

\[ P(t_f) = \max P(t_f, s(\tau_k)) \]  

(6)

To solve this problem, we use the method described in [10].

3. Solution of the problem of optimizing the probability of failure-free system \( S_A(n,m,s) \)
We assume that the behavior of the attacked system \( S_A(n,m,s) \) in the process of conflict is approximated by a Markov process, and the number of working states \( E_k, 0 \leq k \leq m \) is equal to the
number of failed main functional blocks not exceeding the number of reserve ones. It is obvious that the state \( E_{k+1} \) is the state of complete failure of the system \( S_A(n,m,s) \), that is, the absorbing state. Denote \( A_k, 1 \leq k \leq m \) is the intensity of transitions of the system from the state \( E_{k-1} \) to the state \( E_k \); \( B_k, 1 \leq k \leq m+1 \) is intensity of system transitions from the state \( E_{k-1} \) to the state of complete failure. Then the system of Kolmogorov differential equations describing the behavior of the system \( S_A(n,m,s) \) in the process of conflict will have the following form

\[
\begin{aligned}
    p_0(t) &= D_0 p_0(t) \\
    p_k(t) &= A_k p_{k-1}(t) - D_{k+1} p_k(t), k = 1,2,\ldots,m \\
    p_{m+1}(t) &= \sum_{k=1}^{m+1} B_k \lambda(t) p_{k-1}(t)
\end{aligned}
\]  

with initial conditions

\[
\begin{aligned}
    p_0(0) &= 1 \\
    p_i(0) &= 0, 1 \leq i \leq m
\end{aligned}
\]

Wherein

\[
\begin{aligned}
    D_k &= A_k + B_k, 1 \leq k \leq m \\
    D_{m+1} &= B_{m+1}
\end{aligned}
\]

The coefficients of the system of equations (7) are calculated in accordance with the expressions

\[
\begin{aligned}
    A_k &= \sum_{i=0}^{q} \alpha_i(k) \lambda_i(t), k = 1,2,\ldots,m \\
    D_k &= \sum_{i=0}^{q} \beta_i(k) \lambda_i(t), k = 1,2,\ldots,m+1
\end{aligned}
\]

where at \( 0 \leq k \leq m \)

\[
\begin{aligned}
    \alpha_i(k) &= \begin{cases} (m-k+1)R_k, & i = 0 \\ \delta_i n_i R_k, & 1 \leq i \leq q \end{cases} \\
    \beta_i(k) &= \begin{cases} (m-k+1)R_k, & i = 0 \\ \delta_i n_i R_k + n_i \Theta_i(k), & 1 \leq i \leq q \end{cases}
\end{aligned}
\]

The coefficients \( \delta_i \) and \( \Theta_i(k) \), which are elements of vectors \( \delta = (\delta_1,\delta_2,\ldots,\delta_q) \) and \( \Theta(k) = (\Theta_1(k),\Theta_2(k),\ldots,\Theta_q(k)) \) are defined as follows

\[
\begin{aligned}
    \delta_i &= \begin{cases} 0, & s_i = 0 \\ 1, & s_i \geq 1 \end{cases} \\
    \Theta_i(k) &= \begin{cases} 0, & k \leq s_i \\ 1, & k \geq s_i + 1 \end{cases}
\end{aligned}
\]

It's obvious that \( \Theta_i(1) = 1 - \delta_i \)

The variable \( R_k \) determines the number of possible hits of the system \( S_A(n,m,s) \) in the state \( E_k \) and is calculated using the formula

\[
R_k = \sum_{v \in \Omega(k,\bar{s})} \prod_{i=1}^{q} \left( n_i + s_i \right)
\]

where

\[
\Omega(k,\bar{s}) = \{ v \mid v_1 + v_2 + \ldots + v_q = k : \forall i, 0 \leq v_i \leq s_i \}
\]
\(\mathbf{v} = (v_1, v_2, \ldots, v_q)\) is an integer vector representing the sum of integer vectors \(\mathbf{v} = \mathbf{x} + \mathbf{z}, \mathbf{x} = (x_1, x_2, \ldots, x_q)\) and \(\mathbf{z} = (z_1, z_2, \ldots, z_q)\).

Expression (17) was obtained under the assumption that \(k\) failures in the system were distributed as follows: in the \(i\)-th group of the main blocks \(x_i\) of failures, in the \(i\)-th group of the reserve blocks \(z_i\) of failures for \(1 \leq i \leq q\). If \(x_i = 0\) or \(z_i = 0\), then there was no failure in the corresponding group.

The task is as follows. For a given time \(t_f > 0\), find the vector \(\mathbf{v}\) that maximizes the probability of failure-free operation \(P(t_f, s)\) of the technical system \(S(n, m, s)\) described by equations (7), given the constraints on the system parameters.

It is not possible to obtain an exact solution to this problem, since the differential equations in system (7) have variable coefficients. Therefore, we use the discretization method to obtain an approximate solution.

To do this, we calculate the minimum positive integer \(r\) satisfying the conditions

\[
\max_{0 \leq i \leq q} \max_{0 \leq s \leq r} \left| \lambda_i(t) - \lambda_{iv} \right| \leq \epsilon \quad (19)
\]

where

\[
\Delta_v = [t_{v-1}, t_v], t_v = v\Delta t, \Delta t = \frac{t_f}{r} \quad (20)
\]

\[
\lambda_{iv} = \frac{1}{2} \left[ \lambda_i(t_{v-1}) + \lambda_i(t_v) \right] \quad (21)
\]

\(\epsilon\) is a given positive number that represents the largest allowable deviation of the functions \(\lambda_i(t)\) from the constants \(\lambda_{iv}\) on the sampling intervals \(\Delta_v\) for all \(1 \leq v \leq r\).

Obviously, \(t_0 = 0, t_r = t_f\).

Then the system of equations (7) splits into \(r\) systems with constant coefficients for \(t \in \Delta_v\)

\[
\begin{cases}
p_{0,v}(t) = -D_{0,v} p_{0,v}(t) \\
p_{k,v}(t) = A_{k,v} p_{k-1,v}(t) - D_{k+1,v} p_{k,v}(t), k = 1, 2, \ldots, m
\end{cases} \quad (22)
\]

with initial conditions

\[
p_{k,v}(t_{v-1}) = \begin{cases} p_k(0), v = 1 \\ p_{k,v-1}(t_{v-1}), 2 \leq v \leq r \end{cases} \quad (23)
\]

The coefficients of the system of differential equations (22) are as follows

\[
A_{k,v} = \sum_{i=0}^{q} \alpha_i(k) \lambda_{iv} \quad (24)
\]

\[
D_{k,v} = \sum_{i=0}^{q} \beta_i(k) \lambda_{iv} \quad (25)
\]

The solution of the system of equations (22) for \(t \in \Delta_v\) can be written in the following form

\[
\begin{cases}
p_{0,v}(t) = p_{0,v-1}(t_{v-1}) \exp(-D_{0,v} t) \\
p_{k,v}(t) = \sum_{j=0}^{k} p_{j,v-1}(t_{v-1}) \sum_{i=j+1}^{k+1} A_{i,v} \sum_{i=j+1}^{k} \frac{\exp(-D_{i,v} t)}{D_{i,v} - D_{i,v}} p_{j,v-1}(t_{v-1}) - D_{k+1,v} p_{k,v}(t), k = 1, 2, \ldots, m
\end{cases} \quad (26)
\]

Denote by \(S(m)\) the set \(S(m) = \{s \mid q_1 + q_2 + \ldots + q_q = m, \forall i q_i \geq 0\}\).
Now the problem of calculating the vector \( s = (s_1, s_2, \ldots, s_q) \), which maximizes the probability of failure-free operation of the \( P(t_f, s) \) of the system \( S(n, m, s) \) can be solved using the following algorithm.

Algorithm 1

Begin.
1. Set natural numbers \( m, q \), array \( \{n_1, n_2, \ldots, n_q\} \), functions \( \lambda_i(t), 0 \leq i \leq q \), array \( \{1, 0, 0, \ldots, 0\} \) of initial values, number \( t_f > 0 \), number \( \varepsilon > 0 \).
2. Suppose \( r = 2 \).
3. Calculate number \( \Delta t = \frac{t_f}{r} \) and array \( \{t_0, t_1, \ldots, t_r\} \).
4. Suppose \( i = 0 \).
5. Suppose \( v = 1 \).
6. Calculate \( \lambda_{i,v} = \frac{1}{2} \left[ \lambda_i(t_{v-1}) + \lambda_i(t_v) \right] \).
7. Calculate number \( \phi_{i,v}^r = \max \lambda_i(t) - \lambda_{i,v} = \frac{1}{2} \left[ \lambda_i(t_v) - \lambda_i(t_{v-1}) \right] \).
8. Suppose \( v = v + 1 \).
9. If \( v \leq r \), go to step 6.
10. Calculate number \( \phi_{i}^r = \max_{1 \leq v \leq r} \left\{ \phi_{i,v}^r \right\} \).
11. Suppose \( i = i + 1 \).
12. If \( i \leq m \), go to step 5.
13. Calculate number \( \phi^r = \max_{0 \leq i \leq m} \left\{ \phi_{i}^r \right\} \).
14. If \( \phi^r \leq \varepsilon \), go to step 17.
15. Suppose \( r = r + 1 \).
16. Go to step 3.
17. Set integer vector \( \bar{s} \in S(m) \).
18. Suppose \( v = 1 \).
19. Calculate \( p_{k,v}(t_v), 0 \leq k \leq m \), by the formula (27).
20. Suppose \( v = v + 1 \).
21. If \( v \leq r \), go to step 19.
22. Calculate \( P(t_f, s) = \sum_{k=0}^{m} p_{k,v}(t_f) \).
23. Perform steps 17 – 22 for all \( s \in S(m) \).
24. Calculate vector \( s(t_k) \), for which \( P(t_f, s(t_k)) = \max_{s \in S(m)} P(t_f, s) \).

End. \((s(t_k)) \) is the desired reservation vector.

4. Solution of the game problem of confrontation

Consider the game \( G_1 \) with \( Q_i(t) = \text{const} \) with the payment function \( P(t_f) \), where \( t_f \) is the end time of the game. Side \( A \) has a system \( S_A(n, m, s) \) and a set of strategies \( W^A \), side \( B \) (opponent) can be in \( N \) states and has a set of strategies \( W^B \). Let the time points \( \{t_0, t_1, t_2, \ldots, t_z\}, t_0 = 0, t_z < t_f \) be given on the
interval \([0,t_f]\). To each fixed moment in time \(t_i\), we assign the probability vector of finding the attacking enemy in a given state \(Q(t_i)\) and will assume that the probabilities of the enemy state do not change on the interval \([t_i,t_{i+1}]\), \(t_{i+1} = t_f\). Let for each interval \(T_{i+1} = [t_i, t_{i+1}]\) set the number of system \(S_A(n,m,s)\) settings \(L_{T_{i+1}}\). Then the considered game can be represented as a set of \(Z\) games, each of which has a payment function \(P(t_{i+1})\). Successive solutions of \(Z\) games with appropriate initial conditions give a solution to the game \(G_i\).

Consider the solution of one of \(Z\) games on the interval \([t_i,t_{i+1}]\) and determine the initial conditions for the next \(Z\)-game.

Let in the considered time interval the probabilities of the enemy being in the states \(B_1, B_2, \ldots, B_N\), characterized by the failure rates vectors of the main and reserve blocks of the system \(S_A(n,m,s)\)

\[
\lambda' (t) = \{ \lambda_1'(t), \lambda_2'(t), \ldots, \lambda_N'(t) \}, i = 1, 2, \ldots, N,
\]

are equal to \(Q_1, Q_2, \ldots, Q_N\). We introduce the set

\[
\chi = \{ t_{i+1} - \alpha, t_{i+1} - 2\alpha, \ldots, t_{i+1} - (\omega - 1)\alpha \}, \quad \alpha \text{ is the minimum time between two adjacent settings of the attacker systems, and } \omega \text{ is the integer part } \frac{t_{i+1}}{\alpha}.
\]

Obviously, \(\tau_0 = t_i\). Sequence of moment of settings \(\{\tau_1, \tau_2, \ldots, \tau_L\}\) on points of the set \(\chi\) can be distributed in \(C^L_{\omega-1}\) ways. Since for each moment \(\tau_k\) the total number of settings is defined as the number of integer non-negative roots of equation (2), the number of strategies of player \(A\) can be calculated by the formula

\[
M = \left( q + m - \varphi(t_i) - 1 \right) + \left( \frac{\omega - 1}{L_{T_{i+1}}} \right) \prod_{i=1}^{L_{T_{i+1}}} \left( q + m - \varphi(t_k) - 1 \right)
\]

where \(\varphi(t)\) is defined by expression (3) and has the same meaning. Obviously, the number of strategies of player \(B\) is equal to the number of enemy states \(-N\).

We form a payment matrix \(A = [a_{ij}]\) of dimension \(M \times N\), in which at the intersection of the \(i\)-th row and \(j\)-th column we write the payment function \(P(t_{i+1})\), calculated under the assumption of the \(j\)-th state of the enemy with the corresponding failure rates of the components of the attacked system over the entire time interval \(T_{i+1} = [t_i, t_{i+1}]\). The payment function is determined by Algorithm 1 by successively integrating the system of differential equations (7), describing the probabilities of finding the system \(S_A(n,m,s)\) in states with \(l, 0 \leq l \leq m\) failures, on time intervals determined by dividing the interval \(T_{i+1} = [t_i, t_{i+1}]\) moments of settings \(\tau_k\). At the same time, the initial conditions on the interval \(T_k = [t_0, t_1]\) at the time \(\tau_k = 0\) have the form \(p_0(0) = 1, p_1(0) = p_2(0) = \ldots = p_m(0) = 0\), and each the subsequent time \(\tau_k\) is defined as the probabilities of finding the system \(S_A(n,m,s)\) in states with \(l, 0 \leq l \leq m\) failures by this time.

By the initial time instant \(t_i\) of the next \(Z\)-game, the initial conditions are determined through probabilities that the attacked system is in states with \(l, 0 \leq l \leq m\) failures by this moment, calculated under the assumption \(j\)-th state enemy, using the formula

\[
p_l(t_i) = \sum_{j=1}^{N} p_{lj}(t_i)Q_j
\]

Strategy of player \(A\) in each \(Z\)-game is defined as a line of the payment matrix, for which the mathematical expectation of winning, considering the probabilities of all possible states of the enemy, is maximized.
\[
\alpha_i = \sum_{j=1}^{N} Q_{ij} \alpha_j \rightarrow \max
\]  
\hspace{1cm} (30)

It is obvious that such a strategy is optimal (close to optimal) not in each individual case, but on average. The solution of the game should be sought in pure strategies, since for any mixed strategy, the weighted average of the winnings corresponding to pure strategies cannot exceed the maximum of them. The solution of the considered game problem is possible using the following algorithm.

Algorithm 2

Begin.
1. Set \( N, t_j, \alpha, Z \).
2. For \( z = 0,1, \ldots, Z \) set \( \lbrace t_z \rbrace, \lbrace L_{t_{z+1}} \rbrace, Q(T_{z+1}), \lambda(T_{z+1}) \).
3. Suppose \( z = 0 \).
4. Calculate \( \alpha = \left[ \frac{t_{z+1}}{\alpha} \right] \), where \([x]\) is the whole part of \( x \).
5. Form set \( \chi = \lbrace t_{z+1} - \alpha, t_{z+1} - 2\alpha, \ldots, t_{z+1} - (\alpha - 1)\alpha, t_z \rbrace \).
6. Set all possible moment vectors of system \( S_A(n,m,s) \) settings \( \bar{\tau}_{z+1} = \lbrace \tau_0, \tau_1, \ldots, \tau_{L_{t_{z+1}}} \rbrace \), where \( \tau_k \in \chi, 0 \leq k \leq L_{t_{z+1}}, \tau_0 = t_z \), and the corresponding reservation vectors \( s_{z+1}(\tau_k) \) are defined as whole non-negative solutions of equation (2).
7. Form a payment matrix \( A^z = \|a_{ij}\|_{M \times N} \), where \( a_{ij} = P(t_{z+1}, \tau_i, \sigma_j) \) payment calculated by Algorithm 1.
8. Determine \( i \) for which \( \alpha_i = \sum_{j=1}^{N} Q_{ij} \alpha_j \rightarrow \max \).
9. Suppose \( \tau(z+1) = \tau^t_{z+1}, s(z+1) = s^t \).
10. Suppose \( z = z + 1 \).
11. Calculate the initial conditions for integrating the system of differential equations (7) on the interval \( [t_z, t_{z+1}] \) by the formula \( p_i(t_z) = \sum_{j=1}^{N} p_{ij}(t_z)Q(T_z), i = 0,1, \ldots, m \).
12. Perform steps 4 – 11 for all \( z, 0 \leq z \leq Z \), assuming \( (Z+1) = t_f \)
End. (Vectors \( \tau \) and \( s \) are the desired strategies of player \( A \).)

5. Conclusion

The solution of the game problem considered above was obtained under the assumption of the known probabilities of the states of the attacking enemy, that is, the problem of choosing a solution under uncertainty, meaning not complete information, is reduced to the problem of choosing a solution under certainty conditions so that the solution obtained is not optimal in every single case, but on average. If the probabilities of the states of the attacking enemy cannot be estimated or calculated, then the decision under uncertainty can be made based on the pessimism criterion [14].

6. Reference

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