Extended Chiral Transformations Including Diquark Fields As Parameters

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Abstract

We introduce extended chiral transformation, which depends both on pseudoscalar and diquark fields as parameters and determine its group structure. Assuming soft symmetry breaking in diquark sector, bosonisation of a quasi-Goldstone $ud$-diquark is performed. In the chiral limit the $ud$-diquark mass is defined by the gluon condensate, $m_{ud} \approx 300 MeV$. The diquark charge radius is $\langle r_{ud}^2 \rangle^{1/2} \approx 0.5 fm$.

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1 Introduction.

Diquarks, which were introduced almost three decades ago [1], became now efficient tool for studying various processes in hadron physics (see e.g. [2,3] and reviews [4]). The diquark model was analysed from various points of view. However, the complete picture is still lacking.

It was suggested by Dosch et al [3] that wave function of pion and $ud$-diquark are the same at the origin. It was also shown [3] that the diquark decay constant following from this suggestion is very close to that estimated from the QCD sum rules.

In this paper we propose to go a bit further: we suppose that the similarity of wave functions of pion and diquark is due to common origin as parameters of a certain anomalous transformation which does not preserve the measure of the quark path integral. While at the classical level the chiral symmetry is broken by quark mass, the extended chiral ($E\chi$) symmetry is broken by quark mass and gluon fields. $E\chi$-group is $U(2N)$ for $N$ internal degrees of freedom, $N = N_c N_f$. Non-anomalous (measure preserving) generators span the Lie algebra of $O(2N)$, anomalous generators belong to the coset $U(2N)/O(2N)$. Anomalous generators describe chiral rotations and transformations with diquark variables (“diquark” rotations), non-anomalous part consists out of gauge transformations and combined chiral “diquark” rotations.

We assume that $E\chi$-symmetry breaking due to quark masses and gluon fields is soft in the sense that the action for bosonised diquark fields can be obtained by integrating corresponding $E\chi$-anomaly. Colorless chiral fields after bosonisation give rise to Goldstone particles – pseudoscalar mesons. We suggest that at low energies bosonised diquark parameters of $E\chi$-transformations with quantum numbers of lightest $J^P = 0^+$ $ud$-diquark can be treated as a Goldstone-like particle. Therefore, in bosonisation we restrict ourselves to the case of $E\chi$-transformations with $ud$-diquark fields. The $E\chi$-group in this case is $SU(4)$, non-anomalous transformations are just gauge transformations $SU(3) \times U(1)$ and the diquark Goldstone degrees of freedom belongs to $CP^3 = SU(4)/SU(3) \times U(1)$.

Our analysis shows that the $ud$-diquark introduced a la Goldstone becomes massless in the limit of vanishing gluon condensate and current quark masses. Furthermore, we calculate the diquark mass and charge radius for the actual value of gluon condensate. The obtained values fall into the region allowed in other models [4]. Note, that our approach is a direct generalization of chiral bosonisation scheme [5] for the case of new anomalous transformation. We introduce no new parameters.

2 Group structure of $E\chi$-transformations.

In our previous paper [6] we demonstrated that in order to consider quark-antiquark and quark-quark composites on equal footings one should introduce eight-component spinors $\Psi$ constructed from ordinary Dirac spinors $\psi$

$$\Psi = \begin{pmatrix} \psi \\ \psi^T \end{pmatrix}$$  \hspace{1cm} (1)
The quark lagrangian can be rewritten in the form
\[ \mathcal{L} = \frac{1}{2} \Psi^T \hat{F} \Psi, \quad \hat{F} = \begin{pmatrix} C \Phi & -D^T \\ D & \Phi C \end{pmatrix} \quad F = -F^T \] (2)
where \( D \) is the Dirac operator \( D = i \gamma^\mu (\partial_\mu + v_\mu + \gamma_5 a_\mu) \), \( ^t \rightarrow \) means transposition and \( \Phi = \gamma^\mu (\phi_{5\mu} + \gamma_5 \phi_\mu) \). We have introduced various external fields \( v_\mu, a_\mu, \phi_\mu \) and \( \phi_{5\mu} \) generating both \( \bar{\psi} \psi \) and \( \psi \psi \) composites. \( C \) is charge conjugation matrix. The quark path integral becomes
\[ Z_\psi = \int \mathcal{D} \Psi \exp i \int d^4x \mathcal{L} = (\text{det} \hat{G})^{1/2} \]
\[ \hat{G} = \begin{pmatrix} D & \Phi \\ \Phi & D_c \end{pmatrix}, \quad D_c = C^{-1} D^T C, \]
\[ \hat{F} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & C^{-1} \end{pmatrix} \hat{G} \begin{pmatrix} 1 & 0 \\ 0 & C \end{pmatrix} \] (3)
where the operator \( \hat{G} \) is \( \gamma_0 \)-hermitian. A similar construction was considered by Ball [7] for Majorana spinors.

The lagrangian (4) is invariant under the following transformations
\[ \delta \Psi = -\Omega \Psi, \quad \Omega = \begin{pmatrix} \alpha + \gamma_5 \chi & (\xi + \gamma_5 \omega) C \\ (-\xi^* + \gamma_5 \omega^*) C & \alpha^* - \gamma_5 \chi^* \end{pmatrix} \] (4)
provided external fields in the operators \( \hat{G} \) and \( \hat{F} \) transform according to the rules
\[ \hat{F} \rightarrow \hat{F}' = \exp \Omega^T \hat{F} \exp \Omega \]
\[ \hat{G} \rightarrow \hat{G}' = \exp(-\Xi + \gamma_5 \Theta) \hat{G} \exp(\Xi + \gamma_5 \Theta) \]
\[ \Xi = \begin{pmatrix} \alpha & \xi \\ \xi^* & \alpha^* \end{pmatrix} \quad \Theta = \begin{pmatrix} \chi & \omega \\ -\omega^* & -\chi^* \end{pmatrix}. \] (5)
The matrices \( \alpha \) and \( \chi \) are antihermitian, \( \xi \) is antisymmetric and \( \omega \) is symmetric in internal indices. The transformations (6) do not destroy the structure (3) of the eight-component spinor \( \Psi \). These transformations can be absorbed in transformations of background fields.

Due to the noninvariance of the measure, only part of the transformations (4) do not change the path integral (3). These are the transformations generated by \( \Xi \). The generators \( \Theta \) lead to quantum anomalies. The operators \( \alpha \) generate gauge transformations, \( \chi \) describe chiral rotations, the anomalous transformations \( \omega \) include fields with diquark quantum numbers, the generators \( \xi \) are needed for closure of the algebra.

The matrix commutator
\[ [\Xi(\alpha_1, \chi_1) + \gamma_5 \Theta(\xi_1, \omega_1), \Xi(\alpha_2, \chi_2) + \gamma_5 \Theta(\xi_2, \omega_2)] = \Xi(\alpha_3, \chi_3) + \gamma_5 \Theta(\xi_3, \omega_3) \] (6)
induces the following Lie structure
\[ \begin{align*}
\alpha_3 &= [\alpha_1, \alpha_2] + [\chi_1, \chi_2] + \xi_1 \xi_2^* - \xi_2 \xi_1^* - \omega_1 \omega_2^* + \omega_2 \omega_1^*, \\
\chi_3 &= [\alpha_1, \chi_2] - [\alpha_2, \chi_1] - \xi_1 \omega_2^* - \omega_2 \xi_1^* + \xi_2 \omega_1^* + \omega_1 \xi_2^*, \\
\xi_3 &= \alpha_1 \xi_2 + \xi_1 \alpha_2^* - \alpha_2 \xi_1 - \xi_2 \alpha_1^* + \chi_1 \omega_2 - \omega_1 \chi_2^* - \chi_2 \omega_1 + \omega_2 \chi_1^*, \\
\omega_3 &= \alpha_1 \omega_2 + \chi_1 \xi_2 - \xi_1 \chi_2^* + \omega_1 \alpha_2^* - \alpha_2 \omega_1 - \chi_2 \xi_1 + \xi_2 \omega_1^* - \omega_2 \xi_1^*. \end{align*} \] (7)
One can verify that the composition laws (7) are induced also by the matrix commutator without $\gamma_5$

$$[\Xi(\alpha_1,\chi_1) + \Theta(\xi_1,\omega_1), \Xi(\alpha_2,\chi_2) + \Theta(\xi_2,\omega_2)] = \Xi(\alpha_3,\chi_3) + \Theta(\xi_3,\omega_3)$$

(8)

This means that the Lie algebras (6) and (8) are isomorphic. For the case of $N$ internal degrees of freedom, $N = N_c N_f$, and maximally extended algebra (i.e. when $\alpha, \chi, \xi$ and $\omega$ are all matrixes satisfying the above mentioned hermiticity and symmetry properties), the $\Xi + \Omega$ form the space of hermitian matrices $2N \times 2N$. Hence the algebra (8) is $U(2N)$. The generators $\Xi$ preserve symmetric non-degenerate bilinear form $O$

$$O = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Xi O + O \Xi^T = 0.$$ 

(9)

Consequently the non-anomalous generators $\Xi$ span the Lie algebra of $O(2N)$ and the anomalous generators belong to the coset $U(2N)/O(2N)$. The generators $\alpha$ and $\omega$ preserve symplectic form

$$\Sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(10)

and thus span the Lie algebra of the subgroup $Sp(N)$. The generators obviously form $U(N)$.

In principle, any transformation of $\Theta$ could be related to a Goldstone particle, whose dynamics is governed by quantum anomalies. However in realistic models most of the symmetries (6) are broken already at classical level by the presence of quark masses and gluon fields, and the vector field prescribed by the transformation rules (4). Only colorless chiral fields $\chi$ are definitely interpreted as pseudoscalar mesons. Some other states were also considered in literature [8]. We suggest, that at certain energy scale the fields $\omega$ with quantum numbers of lightest $J^P = 0^+$ $ud$-diquarks can also generate Goldstone-like particles. In what follows we shall restrict ourselves to the transformations

$$\omega = (1/F_\omega)\omega_c(i\sigma_2)_{jk}\epsilon_{abc}$$

(11)

corresponding to $0^+$ $ud$-diquarks where j,k are flavor and a,b,c are color indices.

In this special case transformations close in a smaller group. To see this one should exclude the $i\sigma_2$ in the same way, as it was done previously with $\gamma_5$, and use the commutation relations (6). One can obtain that after removing $i\sigma_2$ and $\gamma_5$ the algebra becomes formally equivalent to that generated by the $\alpha$ and $\xi$ operators in the case $N = 3$. Hence, the complete group is $O(6) \sim SU(4)$, and the non-anomalous transformations, that are now represented by $\alpha$ and $\xi$ operators, belong to $U(3) \sim SU(3) \times U(1)$. The anomalous (Goldstone) diquark degrees of freedom belong to the complex projective space $CP^3 = SU(4)/SU(3) \times U(1)$. The same result could be obtained in a straightforward but tedious way by computing matrix commutators in an appropriate basis.

As a consistency check we shall demonstrate that the diquark mass vanishes for zero gluon condensate and zero current quark masses. We shall also compute the diquark mass and charge radius for actual value of gluon condensate.
3 The diquark bosonisation. Gluon condensate as a source of diquark mass.

To define the diquark parameters we should regularize the quark path integral. We also need a method of extracting a non-invariant part of the path integral corresponding to anomalous transformations.

To reduce possible regularization dependence [9] we shall use exactly the same scheme [5] which was developed for chiral bosonisation and generalized [6] for the presence of diquark variables. Since the parameters of this scheme were defined through chiral dynamics, we will be able to compare our results for diquark with pion physics directly.

The basic object is the quark path integral over low scale region

\[ Z^L_L \psi = \left( \det \{ \hat{G} \theta (1 - (\hat{G} - M)^2 / \Lambda^2) \} \right)^{1/2} \]

\[ \theta(x) = \int_{-\infty}^{\infty} dt \frac{\exp(itx)}{2\pi i(t-i0)}. \]  

(12)

The parameters \( \Lambda \) and \( M \) are defined below. The functional (12) can be represented in the form

\[ Z^L_L \psi = (Z^L_L \psi Z^{-1}_{inv})Z_{inv}^{-1} \]

\[ Z_{inv}^{-1} = \int D\Theta(Z^L_L \psi(\Theta))^{-1} \]  

(13)

where we integrate over anomalous transformations \( \Theta \), \( D\Theta \) is invariant measure on the corresponding coset space and \( (Z^L_L \psi(\Theta)) \) is the path integral (12) with background fields transformed as in eq.(3). The \( Z_{inv} \) does not depend on degrees of freedom described by \( \Theta \). Hence all information over \( \Theta \)-noninvariant processes is contained in \( (Z^L_L \psi Z^{-1}_{inv}) \) and the effective action for \( \Theta \) can be defined as

\[ (Z^L_L \psi Z^{-1}_{inv}) = \int D\Theta \exp(iW_{eff}(\Theta)) \]  

(14)

The effective action is obtained by integration of the corresponding anomaly \( \mathcal{A}(x) \)

\[ \mathcal{A}(x; \Theta) = \frac{1}{i} \frac{\delta \ln Z^L_L \psi(\Theta)}{\delta \Theta} \]

\[ W_{eff}(\Theta) = -\int d^4x \int_0^1 ds \mathcal{A}(x; s\Theta)\Theta(x) \]  

(15)

Previously [5] this method was applied to \( \pi \)-mesons. It was found that the parameters \( \Lambda \) and \( M \) are related to the pion decay constant

\[ F_\pi^2 = \frac{N_c}{4\pi^2}(\Lambda^2 - M^2) \]  

(16)

We will not report here details of computations of \( W_{eff}(\omega) \) for the case \( \Theta = \omega \), where \( \omega \) is given by (11). They can be performed in same manner as in the papers [5,6]. Neglecting all external fields except vector gauge fields

\[ v_\mu = -iQA_\mu + \frac{\lambda^a}{2i}G^a_\mu, \quad Q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} \]  

(17)
where $A_\mu$ is electromagnetic field, $G^a_\mu$ are gluons and taking zero current quark masses we obtain in quadratic order of $\omega$

$$W_{eff}(\omega) = \frac{1}{96\pi^2 F_\omega^2} \text{tr}_{(c,f)} \left\{ 6(A^2 - M^2)[D_\mu, \omega^*][D^\mu, \omega] + [D_\mu, [D^\mu, \omega^*]][D^\nu, [D^\nu, \omega]] + 2[D_\mu, F_{\mu\nu}^a](\omega[D^\nu, \omega^*] + [D^\nu, \omega] \omega^*) + (F_{\mu\nu}^a \omega^* \omega^* - F_{\mu\nu}^a \omega^* F^T_{\mu\nu} \omega^*) \right\}$$

(18)

where $[D_\mu, \omega] = (\partial_\mu \omega) + v_\mu \omega + \omega v^T_\mu$, $[D_\mu, \omega^*] = (\partial_\mu \omega^*) - v^T_\mu \omega^* - \omega^* v_\mu$ and $F_{\mu\nu} = (\partial_\mu v_\nu) - (\partial_\nu v_\mu) + [v_\mu, v_\nu]$. From (18) we see that the mass of $ud$-diquark $\omega$ is defined by the gluon condensate $\langle G^2_\mu \rangle$ ($N_c = 3$)

$$M^2_{\omega} = -4\pi^2 F_\pi^2 + \sqrt{16\pi^4 F_\pi^4 + \frac{\langle G^2_\mu \rangle}{12}}$$

(19)

and vanishes when $\langle G^2_\mu \rangle \to 0$. For derivation of (19) we used

$$\langle G^a_\mu G^{b\nu}_\mu \rangle = \frac{1}{8} \delta^{ab} \langle G^2_\mu \rangle$$

(20)

For $\langle G^2_\mu \rangle = (365MeV)^4$ we get $M_\omega \approx 300MeV$. The correction of this evaluation due to quark masses is provided by $M^2_{\omega}(m_q \neq 0) = M^2_{\omega}(m_q = 0) + m^2_\pi$. This gives $M^2_{\omega}(m_q \neq 0) \approx 340MeV$, which falls into the region allowed in the other models [4], though lies close to the lower boundary. $F_\omega$ is defined by requirement that the residue of the diquark propagator at $k^2 = M^2_{\omega}$ is unity,

$$F^2_{\omega} = \frac{\Lambda^2 - M^2}{4\pi^2} + \frac{1}{12\pi^2} M^2_{\omega}$$

(21)

The coefficient before the term $\partial^2 A_\mu(\omega^*(\partial_\mu \omega) - (\partial_\mu \omega^*) \omega)$ allows us to evaluate the mean square radius of diquark charge distribution

$$\langle r^2 \rangle^{1/2} \approx 0.5 fm$$

(22)

This value is also compatible with other data [4] for diquark effective radius.

Our desire to describe diquarks as a quasi-particle similar to $\pi$ meson has more than aesthetic grounds. This model allows to explain relatively low mass of the scalar diquark and include diquark variables in framework of current algebra and chiral perturbative theory. As far as we were able to verify, this suggestion does not lead to any contradictions. We obtained quite sensible results for diquark mass and charge radius. The model has no free parameters. All this indicates that broken $E\chi$-symmetry deserves further investigations.

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