Universally Leptophilic Dark Matter
From Non-Abelian Discrete Symmetry

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Abstract

The positron anomaly recently reported by the cosmic-ray measurements can be explained by the decaying dark matter scenario, where it decays mainly into leptons with the lifetime of $O(10^{26})$ second. When the dark matter is a fermionic particle, the lifetime of this order is known to be obtained by a dimension 6 operator suppressed by the unification scale ($\sim 10^{16}$ GeV), while such decay operators do not necessarily involve only leptons. In addition, the scenario would be spoiled if there exist lower-dimensional operators inducing the dark matter decay. We show in this letter that a single non-Abelian discrete symmetry such as $A_4$ is possible to prohibit all such harmful (non-leptonically coupled and lower-dimensional) operators. Moreover, the dark matter decays into charged leptons in a flavor-blind fashion due to the non-Abelian flavor symmetry, which results in perfect agreements not only with the PAMELA data but also with the latest Fermi-LAT data reported very recently. We also discuss some relevance between the discrete symmetry and neutrino physics.
1 Introduction

The existence of non-baryonic dark matter, which accounts for about 23% of the energy density in the present universe, has been established thanks to the recent cosmological observations such as the WMAP (Wilkinson Microwave Anisotropy Probe) experiment \cite{1}. The detailed nature of the dark matter is, however, still unrevealed and is a great mystery not only in astrophysics and cosmology but also in particle physics. In order to detect and study the dark matter, various theoretical and experimental efforts have been devoted, and possible signals for the dark matter have recently been reported from the indirect detection measurements at the PAMELA (a Payload for Antimatter Matter Exploration and Light-nuclei Astrophysics) \cite{2} and Fermi-LAT (The Fermi Large Area Telescope) \cite{3,4} experiments, where anomalous excesses of cosmic-ray positrons (electrons) have been found. Though it is under debate whether these anomalies are interpreted as dark matter signals, they have motivated many theoretical study to explore the nature of the dark matter.

There are several types of scenarios to explain the cosmic-ray anomalies. Among these, we focus on the decaying dark matter scenario \cite{5,6,7} where the dark matter is assumed to be unstable with the lifetime much longer than the age of the universe and its decay in the halo of our galaxy explains the anomalies. The observational data of positron (electron) excesses as well as the non-observation of anti-proton excesses in the cosmic ray \cite{8} suggest that the dark matter mass should be on the TeV scale and it decays mainly into leptons with the lifetime of $\mathcal{O}(10^{26})$ sec. An important question for this scenario is why the lifetime is so long, in other words, what is the origin of meta-stability of the dark matter. An attractive answer is that the meta-stability is derived from very high-energy physics such as Grand Unified Theory (GUT). When the dark matter is a TeV-scale fermionic particle, it could decay through a four-Fermi operator suppressed by the GUT scale $\Lambda \sim 10^{16}$ GeV and the width is estimated as $\Gamma \sim (\text{TeV})^{5}/\Lambda^{4} \sim 10^{-26}/\text{sec}$. With this interesting relation between $\Lambda$ and $\Gamma$, various explicit studies have been performed so far \cite{9}.

In the context of decaying dark matter, it seems however difficult to realize the main decay mode contains only leptons, not hadrons. In addition, there generally exists lower-dimensional operators inducing the dark matter decay, and then the estimation of $\Gamma$ may be disturbed. A reasonable solution to these problems is to implement appropriate symmetry which forbids the rapid and/or non-leptonic decay of the dark matter. In this letter, we point out that the leptonically-decaying dark
matter is guaranteed with use of non-Abelian discrete symmetry acting on the generation space. We focus on the $A_4$ flavor symmetry \cite{9} as the simplest example. The dark matter is assumed to be a Majorana fermion that is singlet under $A_4$ and the standard-model gauge groups. Identifying the effective decay operators and evaluating the positron (electron) flux from the decay, we show that the $A_4$ invariance leads to a novel flavor pattern of the dark matter decay which well describes the cosmic-ray anomaly reported by the PAMELA collaboration. It also turns out that the total electron and positron flux is in perfect agreement with the latest Fermi-LAT data reported very recently \cite{4}. We also discuss some relevance of discrete symmetry on the neutrino physics, i.e. the masses and generation mixing of neutrinos.

2 Decaying dark matter and discrete symmetry

In addition to the standard-model fields, a gauge-singlet fermion $X$ is introduced as the dark matter (DM) particle. We assume that the baryon number is preserved at least at perturbative level. It then turns out that there exist various gauge-invariant operators up to dimension 6 \cite{10} which induce the DM decay:

| Dimensions | DM decay operators |
|------------|--------------------|
| 4          | $\tilde{L}H^c X$   |
| 5          | –                  |
| 6          | $\tilde{L}E\tilde{L}X$, $H^c \tilde{L}H^c X$, $(H^c)^\dagger D_\mu H^c \tilde{E}_\gamma^\mu X$, $\tilde{Q}D\tilde{L}X$, $\tilde{U}Q\tilde{L}X$, $\tilde{L}D\tilde{Q}X$, $\tilde{U}\gamma_\mu D\tilde{E}_\gamma^\mu X$, $D_\mu H^c D_\mu \tilde{L}X$, $D^\mu D_\mu H^c \tilde{L}X$, $B_{\mu \nu} \tilde{L} \sigma^{\mu \nu} H^c X$, $W^a_{\mu \nu} \tilde{L} \sigma^{\mu \nu \tau a} H^c X$ |

Table 1: The decay operators of the gauge-singlet fermionic dark matter $X$ up to dimension 6. Here, $L$, $E$, $Q$, $U$, $D$, and $H$ denote left-handed leptons, right-handed charged leptons, left-handed quarks, right-handed up-type quarks, right-handed down-type quarks, and higgs field, respectively ($H^c = eH^*$). On the other hand, $B_{\mu \nu}$, $W^a_{\mu \nu}$, and $D_\mu$ are the field strength tensor of hypercharge gauge boson, that of weak gauge boson, and the electroweak covariant derivative.

This general operator analysis shows that the dark matter $X$ can decay into not only leptons but also quarks, higgs, and gauge bosons at similar rates. Furthermore, a
quick decay of DM is induced if the dimension 4 Yukawa operator $\bar{L}H^cX$ is allowed. One may try to impose an Abelian (continuous or discrete) symmetry to prohibit unwanted decay operators, but it does not work. The reason is the following: the abelian charges of $L$, $E$, and $H$ are assigned to be $q_L$, $q_E$, and $q_H$, respectively. The operator $\bar{L}HE$ should be invariant under the symmetry in order to have the masses of charged leptons, and the relation $q_L = q_E + q_H$ is hold. The invariance of $\bar{LE}\bar{L}X$ is also needed because this is the unique operator in Table 1 for the leptonic decay of dark matter, and leads to $2q_L = q_E + q_X$. These charge relations turn out to imply that $q_L + q_H = q_X$ and the operator $\bar{L}H^cX$ necessarily becomes symmetry invariant. The discussion is unchanged even when the charged-lepton Yukawa coupling is generated from higher-dimensional effective operators, in which case, an unfavorable decay via $\bar{L}H^cX$ is found to be suppressed by at most (electron mass)/(electroweak scale) and still leads to a short lifetime. Further, in Table 1 there are other leptonic decay operators such as $H^\dagger H\bar{L}H^cX$ which do not contain hadrons. However they have the same property as $\bar{L}H^cX$ with respect to the Abelian charge.

In the following, we show that the desirable DM decay is guaranteed with use of non-Abelian discrete symmetry. Namely, non-Abelian symmetry allows us to prohibit the dangerous dimension 4 operator as well as other operators leading to non-leptonic DM decay, while keeping the operator $\bar{L}E\bar{L}X$ invariant. In this letter, we present a model with the discrete symmetry $A_4$, though it is possible to construct different models with similar DM decay using other discrete symmetry. The $A_4$ group has one real triplet $3$ and three independent singlet representations $1$, $1'$, $1''$ [11]. The multiplication rules of these representations are as follows;

$$
3 \otimes 3 = 3 \oplus 3 \oplus 1 \oplus 1' \oplus 1'', \quad 3 \otimes 1' = 3 \otimes 1'' = 3, \\
1' \otimes 1' = 1'', \quad 1'' \otimes 1'' = 1', \quad 1' \otimes 1'' = 1.
$$

One notice is that the multiplication of two $3$’s contains both $3$ and real singlet $1$, and hence any products of more than two $3$’s can be invariant under the $A_4$ transformation.

With this property of the $A_4$ symmetry, we consider the $A_4$ charge assignment given in Table [2]. Remarkably, all the decay operators in Table [1] except $\bar{L}E\bar{L}X$ are forbidden due to this single symmetry, and the dark matter mainly decays into leptons. With the notation $L_i = (\nu_e, e_L), (\nu_\mu, \mu_L), (\nu_\tau, \tau_L)$ and $E_i = (e_R, \mu_R, \tau_R)$,
### Table 2: The $A_4$ charge assignment of the SM fields and the dark matter $X$

| SU(2) × U(1) | $Q$    | $U$   | $D$   | $L$   | $E$   | $H$   | $X$   |
|--------------|--------|-------|-------|-------|-------|-------|-------|
| $A_4$        | $2_{1/6}$ | $1_{2/3}$ | $1_{-1/3}$ | $2_{-1/2}$ | $1_{-1}$ | $2_{1/2}$ | $1_0$ |
| singlets     | singlets | singlets | 3     | 3     | $(1,1',1'')$ | 1     |

The four-Fermi decay interaction is explicitly written as

$$
\mathcal{L}_{\text{decay}} = \frac{\lambda^+}{\Lambda^2} (\bar{L}E) \bar{L}X + \frac{\lambda^-}{\Lambda^2} (\bar{L}E)' \bar{L}X + \text{h.c.} \quad (2.2)
$$

$$
= \sum_{\pm} \frac{\lambda_\pm}{\Lambda^2} \left[ (\tau_\tau \mu_R \pm \nu_\tau \nu_R) \bar{\nu}_L X - (\tau_L \mu_R \pm \mu_L \tau_R) \bar{\nu}_e X \\
+ (\nu_\tau \nu_R \pm \nu_\tau \nu_R) \bar{\nu}_L X - (\nu_L \nu_R \pm \nu_L \nu_R) \bar{\nu}_e X \\
+ (\nu_\mu \nu_R \pm \nu_\mu \nu_R) \bar{\nu}_L X - (\nu_L \nu_R \pm \nu_L \nu_R) \bar{\nu}_e X \right] + \text{h.c.} \quad (2.3)
$$

There are two types of operators, which we have denoted with the coefficients $\lambda_\pm$, corresponding to the fact that there are two ways to construct the $A_4$ triplet representation from two $3$’s. It should be noted that, due to the non-Abelian $A_4$ symmetry, the decay vertices have specific structures of chirality and generations.

We have introduced three higgs doublets $H_{1,1',1''}$ to have the masses of charged leptons (the details of lepton masses and mixing will be discussed in later section). It was shown [12] that the introduction of multi higgs doublets in this manner does not lead to dangerous flavor-changing processes.

## 3 Cosmic-ray anomaly

In this section, we show by calculating the positron (electron) flux that the scenario given above, which has a special generation structure of DM decay vertices, is possible to excellently describe the cosmic-ray anomalies reported by the PAMELA and Fermi-LAT experiments.

### 3.1 Positron production from DM decay

First, we consider the branching fraction of the DM decay through the $A_4$-invariant operator $\bar{L}E \bar{L}X$. Due to the typical generation structure given in (2.3), the dark matter $X$ decays into several tri-leptons final state with the equal rate, where each final states include all three flavors:

$$
\text{Br}(X \to e^\mp \mu^\mp \nu_\tau) = \text{Br}(X \to \tau^\pm e^\mp \nu_\mu) = \text{Br}(X \to \mu^\pm \tau^\mp \nu_e) = \frac{1}{6}. \quad (3.1)
$$
Here we have omitted the masses of charged leptons in the final states. The branching fractions indicate that the spectrum of positrons (electrons) in cosmic rays is uniquely determined in the present framework with $A_4$ symmetry, which allows us to predict the spectrum of cosmic-ray anomalies. The total decay width of DM turns out to be

$$\Gamma = \frac{m_X^5}{512\pi^3A_4^4}(|\lambda_+|^2 + 3|\lambda_-|^2), \quad (3.2)$$

where $m_X$ is the DM mass.

Given the decay width and the branching fractions, the positron (electron) production rate (per unit volume and unit time) at the position $\vec{x}$ of the halo associated with our galaxy is evaluated as

$$Q(E, \vec{x}) = n_X(\vec{x}) \Gamma \sum_f Br(X \to f) \left[ \frac{dN_{e^\pm}}{dE} \right]_f, \quad (3.3)$$

where $[dN_{e^\pm}/dE]_f$ is the energetic distribution of positrons (electrons) from the decay of single DM with the final state `$f$'. We use the PYTHIA code [13] to evaluate the distribution $[dN_{e^\pm}/dE]_f$. The DM number density $n_X(\vec{x})$ is obtained by the profile $\rho(\vec{x})$, the DM mass distribution in our galaxy, through the relation $\rho(\vec{x}) = m_X n_X(\vec{x})$.

In this work we adopt the Navarro-Frank-White profile [14],

$$\rho_{\text{NFW}}(\vec{x}) = \rho_\odot \frac{r_\odot(r_\odot + r_c)^2}{r(r + r_c)^2}, \quad (3.4)$$

where $\rho_\odot \simeq 0.30$ GeV/cm$^3$ is the local halo density around the solar system, $r$ is the distance from the galactic center whose special values $r_\odot \simeq 8.5$ kpc and $r_c \simeq 20$ kpc are the distance to the solar system and the core radius of the profile, respectively.

In the present model, the dark matter decays into not only $e^\pm$ and $\mu^\pm$ which result in pure leptonic decays, but also $\tau^\pm$ leading to hadronic decays, and antiprotons may also be produced in the halo of our galaxy. It is however obvious that the dominant decay channels are leptonic and the branching fractions of hadronic decay are made tiny by the electroweak coupling and the phase space factor. The suppression of hadronic decays is consistent with the $\bar{p}$ data obtained in the PAMELA experiment [8]. On the other hand, the injections of high-energy positrons (electrons) in the halo give rise to gamma rays through the bremsstrahlung and inverse Compton scattering processes. Comprehensive analyses of cosmic-ray fluxes [5] show that the gamma-ray flux from leptonically decaying DM is also consistent with the Fermi-LAT data [15]. As a result, we concentrate on the calculation of positron (electron) flux in what follows.
3.2 Diffusion model

Next, we consider the propagation of positrons (electrons) produced by the DM decay in our galaxy. The charged particles \(e^\pm\) suffer from the influence of tangled magnetic fields in the galaxy before arriving at the solar system. The physics of the propagation can be described by the diffusion equation [16, 17],

\[
K_{e^\pm}(E) \nabla^2 f_{e^\pm}(E, \vec{x}) + \frac{\partial}{\partial E} \left[ b(E) f_{e^\pm}(E, \vec{x}) \right] + Q(E, \vec{x}) = 0. \tag{3.5}
\]

The number density of \(e^\pm\) per unit energy, \(f_{e^\pm}\), satisfies the condition \(f_{e^\pm} = 0\) at the boundary of the diffusion zone. The diffusion zone is approximated to be a cylinder with the half-height of 4 kpc and the radius of 20 kpc. The diffusion coefficient \(K_{e^\pm}(E)\) and the energy-loss rate \(b(E)\) are set to be

\[
K_{e^\pm}(E) = 1.12 \times 10^{-2} \text{[kpc}^2/\text{Myr}] \times E_{\text{GeV}}^{0.70}, \tag{3.6}
\]

\[
b(E) = 1.00 \times 10^{-16} \text{[GeV/sec]} \times E_{\text{GeV}}^2, \tag{3.7}
\]

where \(E_{\text{GeV}} = E/(1 \text{GeV})\). To fix these parameters, we have used the MED set for the propagation model of \(e^\pm\) [18], which gives the best fit value in the boron-to-carbon ratio (B/C) analysis as well as in the diffused gamma-ray background. Once \(f_{e^\pm}\) is determined by solving the above equation, the \(e^\pm\) fluxes are given by

\[
[\Phi_{e^\pm}(E)]_{\text{DM}} = \frac{c}{4\pi} f_{e^\pm}(E, \vec{x}_\odot), \tag{3.8}
\]

where \(\vec{x}_\odot\) is the location of the solar system, and \(c\) is the speed of light. For the total fluxes of \(e^\pm\), we have to estimate the background fluxes produced by collisions between primary protons and interstellar medium in our galaxy. In the analysis, the following fluxes for cosmic-ray electrons and positrons [17] are adopted:

\[
[\Phi_{e^-}]_{\text{prim}} = \frac{0.16 E_{\text{GeV}}^{-1.1}}{1 + 11 E_{\text{GeV}}^{0.9} + 3.2 E_{\text{GeV}}^{2.15}}, \tag{3.9}
\]

\[
[\Phi_{e^-}]_{\text{sec}} = \frac{0.70 E_{\text{GeV}}^{0.7}}{1 + 110 E_{\text{GeV}}^{1.5} + 600 E_{\text{GeV}}^{2.9} + 580 E_{\text{GeV}}^{4.2}}, \tag{3.10}
\]

\[
[\Phi_{e^+}]_{\text{sec}} = \frac{4.5 E_{\text{GeV}}^{0.7}}{1 + 650 E_{\text{GeV}}^{2.3} + 1500 E_{\text{GeV}}^{4.2}}, \tag{3.11}
\]

in unit of \((\text{GeV cm}^2/\text{sec str})^{-1}\). With these backgrounds, the total fluxes and the positron fraction \(R_{e^+}\), which is measured by the PAMELA experiment, are found to
Figure 1: The positron fraction and the total $e^+ + e^-$ flux predicted in the leptonically-decaying DM scenario with $A_4$ symmetry. The DM mass is fixed to 1, 1.5, and 2 TeV. As for the DM decay width used in the fit, see the text.

be

$$[\Phi_{e^+}]_{\text{total}} = [\Phi_{e^+}]_{\text{DM}} + [\Phi_{e^+}]_{\text{sec}},$$

$$[\Phi_{e^-}]_{\text{total}} = [\Phi_{e^-}]_{\text{DM}} + a[\Phi_{e^-}]_{\text{prim}} + [\Phi_{e^-}]_{\text{sec}},$$

$$R_{e^+} = [\Phi_{e^+}]_{\text{total}} / ([\Phi_{e^+}]_{\text{total}} + [\Phi_{e^-}]_{\text{total}}).$$

Note that the primary flux for electrons measured by Fermi-LAT should be multiplied by the normalization factor $a = 0.7$ so that our evaluation is consistent with the experimental data in the low-energy range [19].

3.3 Results for PAMELA and Fermi-LAT

The positron fraction and the total flux $[\Phi_{e^-}]_{\text{total}} + [\Phi_{e^+}]_{\text{total}}$ are depicted in Figure 1 for the scenario of the leptonically decaying DM with $A_4$ symmetry. For the DM mass $m_X = 1$, 1.5, and 2 TeV, the results are shown with the experimental data of PAMELA and Fermi-LAT. The total decay width $\Gamma$ is fixed for each value of DM mass so that the best fit value explains the experimental data. With a simple $\chi^2$ analysis, we obtain $\Gamma^{-1} = 1.7 \times 10^{26}$, $1.2 \times 10^{26}$, and $9.5 \times 10^{25}$ sec for $m_X = 1$, 1.5, and 2 TeV, respectively. It can be seen from the figure that the PAMELA anomaly is well explained in the decaying DM scenario with $A_4$ symmetry. Furthermore, the latest Fermi-LAT data is perfectly fitted in this scenario if the DM mass is around 2 TeV.
4 Lepton masses and mixing

So far, the dark matter property, especially the leptonic decay, has been analyzed for the gauge-singlet fermion $X$. In this section, we discuss the lepton masses and mixing in the same setup as Table 2 and also in two types of its extensions.

For the matter content and the $A_4$ assignment given in Table 2, the charged-lepton and neutrino masses come from the symmetry-invariant operators

$$\mathcal{L} = - \sum_{i=1,1',1''} (y_e)_i H_i \bar{L} E + \text{h.c.} + \sum_{i,j=1,1',1''} (y_\nu)_{ij} \bar{L} H_i^a H_j^a L.$$

(4.1)

The subscripts $i$ mean the singlet representations of $A_4$ symmetry, $i = 1, 1', 1''$. The higgs fields are assumed to develop vacuum expectation values $\langle H_i \rangle = (0, v_i/\sqrt{2})^t$. The lepton mass matrices turn out to take the forms

$$M_e = \begin{pmatrix} m_e & m_{\mu} & m_{\tau} \end{pmatrix}, \quad M_\nu = \begin{pmatrix} m_1 & m_2 & m_3 \end{pmatrix},$$

(4.2)

$$m_e = f(v_1, v_{1'}, v_{1''}), \quad m_1 = g(v_1, v_{1'}, v_{1''}),$$

$$m_\mu = f(v_1, \omega v_{1'}, \omega^2 v_{1''}), \quad m_2 = g(v_1, \omega^2 v_{1'}, \omega v_{1''}),$$

$$m_\tau = f(v_1, \omega^2 v_{1'}, \omega v_{1''}), \quad m_3 = g(v_1, \omega v_{1'}, \omega^2 v_{1''}),$$

(4.3)

where $\omega = e^{2\pi i/3}$, and the functions $f$ and $g$ are given by

$$f(v_1, v_{1'}, v_{1''}) = \frac{1}{\sqrt{2}} \sum_i (y_e)_i v_i, \quad g(v_1, v_{1'}, v_{1''}) = \frac{1}{2} \sum_{ij} (y_\nu)_{ij} v_i v_j.$$  

(4.4)

With suitable values of the coupling constants, the experimentally-observed masses (differences) are able to be reproduced. The generation mixing is, however, absent unless some ingredient is added. In the following, we will present two possible examples to remedy this problem without causing a rapid decay of the dark matter.

The first example is to introduce extra higgs doublets which induce Majorana neutrino mass, i.e., additional dimension 5 operator like (4.1). The extra higgses $H'$ belong to the triplet representation of $A_4$ symmetry in order for non-trivial flavor mixing to be generated. Further, $H'$ should be charged under some symmetry not

\footnote{When $v_i$ are the electroweak scale, the neutrino mass $M_\nu \sim 10^{-(1-2)}$eV seems to imply that the effective scale of $LLHH$ operator, $y_{\nu}^{-1} \sim \Lambda'$, is somewhat below the unification scale, namely, the lepton number symmetry is valid above $\Lambda'$ in low-energy effective theory.}
to have the interactions (the effective operators listed in Table I) which cause the DM decay and disturb the previous result. To satisfy this requirement, we consider a simple example with $Z_2$ parity under which only $H'$ is negative. As a result, the decay operators involving $H'$ with dimensions less than 7 are not permitted, except for dimension 6 operators $H'^\dagger H' \bar{L}H'X$ and $H'D_{\mu}H'\bar{X}\gamma^\mu E$. It is found that they cannot be forbidden by any Abelian (discrete) symmetry while other necessary terms remain intact. Therefore, if one assumes that the DM decay from these operators is sub-dominant, the expectation values of $H'$ should be suppressed.

The remaining is the decay operator of dark matter $\bar{L}E\bar{L}X$ and the additional source of neutrino masses $y'_{\nu}L\bar{E}LH'H'$. The charged-lepton masses are unchanged and the neutrino mass matrix turns out to be

$$M'_{\nu} = \begin{pmatrix}
    m'_1 & y'_{\nu}v'_2v'_3 & y'_{\nu}v'_1v'_3 \\
    y'_{\nu}v'_2v'_3 & m'_2 & y'_{\nu}v'_1v'_2 \\
    y'_{\nu}v'_1v'_3 & y'_{\nu}v'_1v'_2 & m'_3
\end{pmatrix},$$

(4.5)

where $v'_i$ are the expectation values of $H'_i$. The diagonal elements $m'_i$ are shifted by $O(y'_{\nu}v'^2_i)$ from $m_i$ due to the new interaction, and their exact forms are determined by the $A_4$ invariance. The additional 3 degrees of freedom (the off-diagonal matrix elements) can fit the experimental values of neutrino mixing.

Another way to have non-vanishing generation mixing is to consider a different type of neutrino mass operator than (4.1) with use of the SU(2)-triplet scalar $\Delta$. The simplest tree-level term for neutrino mass is constructed with the scalar $\Delta$:

$$L_{\Delta} = y_{\Delta}\bar{L}\ell\Delta L.$$

(4.6)

Similar to the first example, $\Delta$ should belong to the triplet representation of $A_4$ symmetry for non-trivial generation mixing of neutrinos. The electroweak gauge invariance implies that the above term only induces off-diagonal elements in the neutrino mass matrix. Assuming nonzero expectation values $v_{\Delta i}$ for the neutral components of $\Delta_i$ ($i = 1, 2, 3$), we obtain the neutrino mass matrix

$$M^\Delta_{\nu} = \begin{pmatrix}
    m_1 & y_{\Delta}v_{\Delta3} & y_{\Delta}v_{\Delta2} \\
    y_{\Delta}v_{\Delta3} & m_2 & y_{\Delta}v_{\Delta1} \\
    y_{\Delta}v_{\Delta2} & y_{\Delta}v_{\Delta1} & m_3
\end{pmatrix}.$$

(4.7)

The phenomenological analysis based on this type of Majorana mass matrix has been performed in Ref. [20], where the solar and atmospheric neutrino anomalies and the neutrino-less double beta decay have been studied.
It is noticed that the triplet scalar $\Delta$ gives rise to new decay interactions of dark matter. For operators with dimensions more than 5, their contributions to the decay amplitude are suppressed when $\Delta$ is heavier than the dark matter and $v_\Delta$ is much smaller than $v_i$ to satisfy the electroweak precision (the $\rho$ parameter constraint). The gauge and flavor invariance then leave a single dimension 5 operator

$$\lambda_\Delta H\Delta^\dagger \bar{L}X. \quad (4.8)$$

It is easily found that this operator cannot be forbidden by imposing any symmetry, if one allows the necessary operators for the lepton masses and the DM decay through $\bar{L}E\bar{L}X$. To avoid a rapid DM decay via the operator (4.8), $v_\Delta$ should be smaller than $(\text{TeV})^2/\Lambda \sim \text{eV}$. Then the coupling $y_\Delta$ in (4.6) is $\mathcal{O}(1)$ for non-negligible neutrino mixing. Integrating out the heavy scalar $\Delta$ with its mass $m_\Delta$, we have an effective operator

$$\frac{y_\Delta \lambda_\Delta}{\Lambda m_\Delta^2} H \bar{L}^c L \bar{L}X. \quad (4.9)$$

Since the coupling $y_\Delta$ is $\mathcal{O}(1)$, this dimension 7 operator might give a sizable effect on the $X$ decay. In other words, if one requires that the dominant decay vertex is the four-Fermi operator $\bar{L}E\bar{L}X$, the triplet scalar should be heavier than the intermediate scale: $m_\Delta \gtrsim \sqrt{|\lambda_\Delta|} v_\Delta$. Such an SU(2)-triplet scalar with an intermediate mass and a tiny expectation value might be incorporated in SO(10) unified theory with the intermediate Pati-Salam group, where the potential analysis is slightly shifted by the electroweak scale. We finally mention that a tiny value of $\lambda_\Delta$ ($\lesssim \text{TeV}/\Lambda$) might also be a solution with low-mass $\Delta$. That however means the effective theory description is invalid and the model should be improved.

5 Conclusion

We have considered the decay of gauge-singlet dark matter for the cosmic-ray anomalies reported by the PAMELA and Fermi-LAT experiments. The decaying dark matter recently attracts much attention because, if it is a TeV-scale fermionic particle, the suggested order of meta-stability is just derived from a four-Fermi interaction suppressed by the GUT scale. It is also noted that the cosmic-ray anomalies are explained by the DM decay, while the relic abundance may be determined by DM annihilation process, e.g. mediated by a light singlet scalar.
The scenario is however spoiled due to the existence of other operators which force a rapid DM decay and/or induce non-leptonic DM decay. In this letter, we have pointed out that such harmful decay vertices are prohibited by implementing a single non-Abelian flavor symmetry such as $A_4$. Any Abelian symmetry cannot play the same role. We have also shown that the $A_4$ invariance leads to the flavor-universal decay channels of DM, with which the cosmic-ray anomalies are captured very well with the DM mass around 2 TeV. Further we have discussed the relevance of discrete flavor symmetry on neutrino phenomenology and offered two independent mechanisms to generate lepton masses and mixing without disturbing the successful decaying DM scenario. It would be therefore interesting to construct a high-energy completion, i.e. a concrete GUT model involving both the dark matter candidate and mechanism to generate neutrino masses with a non-Abelian discrete symmetry.

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