Semiotic Analysis of Mathematics Problems-Solving: Configure Mathematical Objects Viewed from High Mathematical Disposition

L F Claudia¹,², T A Kusmayadi², and L Fitriana¹
¹ Department of Mathematics Education, Faculty of Teacher Training and Education, Sebelas Maret University
² Department of Mathematics, Faculty of Mathematics and Natural Sciences, Sebelas Maret University
Jalan Ir. Sutami 36A Kentingan Jebres, Surakarta, Indonesia 57126
Email: *lidya.fransisca23@gmail.com

ABSTRACT. The main purpose of this study was to describe the semiotic of students in solving mathematical problems with analysis using mathematical objects. In learning, mathematics was inseparable from signs and symbols. Mathematics in general was a subject that contains symbols, numbers, graphs, tables, diagrams. We used semiotic to analyze the configuration of mathematical objects (language, problems, concepts, procedures, propositions, and arguments). Subject of high school students in the City of Kediri using a technique is called qualitative descriptive and the subject has been selected according to a purposive sampling technique, and then fourteen students has been chosen who represent the category of high mathematical disposition. The results of this study describe semiotic students, who are students with high mathematical disposition obtaining all indicators of semiotic.

1. Introduction
The development of the age of education also develops in Indonesia, such as the industrial era 4.0 which requires students to be able to develop more academically and non-academically. Despite the changes in the teaching and learning process, mathematics is still a subject that is considered difficult by students. Basically, mathematics is a basic tool for teaching other subjects [1]. Mathematics in general is a subject that contains symbols, numbers, graphs, tables, diagrams, and so forth. Because in this student-centered research, students use many symbols and representations in mathematical practice, therefore they use them to communicate, solve, explain, and argue about a problem. At this time, many symbols represent mathematical objects and are the focus of research in the field of mathematics education [2].

The low understanding of symbols is also caused by a lack of understanding of mathematical language. Mathematics as a language certainly has the same function as a language in general, like a naming (labeling). Students understand variables or letters as symbolic replacements for certain numbers. As a result, assumptions of student by applying on the 2 equations "\( x + y = 4 \)" or the equation "\( x + y + z \)" will never have the same value as the equation "\( x + p + z \)" [3]. Symbols, variables and signs, mathematics is a communication tool or as a language. In mathematics everything related to signs, symbols and relations between symbols is called semiotic. Semiotic includes all signs that are visual and verbal.
The opinion of Saussure that semiotic is a semiology that is expected if human actions and behavior can carry meaning and the meaning is generated from the sign system used by some people [4]. Semiotic which emphasizes the relationship between the definition of meaning and understanding [5]. There is an approach developed by Godino in 2007, the Onto-Semiotic Approach (OSA). The Onto-Semiotic approach is a theoretical framework that combines semiotics and anthropology about mathematics, and social constructivist principles and relations for the study of the learning process [6]. OSA can be used to solve problems regarding mathematical understanding, which is an ontology and semiotic analysis that uses semiotic theoretical constructs as the ontology of mathematical objects [7]. Taking the relationship between ontology with the sign that underlies a sign at certain limits, this ontology, and semiotic approach are used for computing which is characterized by understanding and using semiotic especially in sign studies [8]. Ontology is a condition for, and a result of, semiotic processes [9]. We can say if semiotic is part of the onto-semiotic approach, using mathematical objects contained in the onto-semiotic approach can be used as a tool to analyze students' semiotic abilities.

The system of semiotic is composed of a series of signs, some rules that produce signs and a set of relationships between signs and the meanings contained. Signs that can be written, pronounced, moved, or attached to an object with the aim of achieving a goal are Radford's semiotic ideas [10]. Semiotic is the first reason in the chosen field of knowledge to study the signs and their use [11]. Semiotic has an important role in mathematics learning and has gained the attention of researchers who are interested in advancing the understanding of the processes involved [12]. Semiotic is a methodology, not a philosophical doctrine in which all thoughts are formed with a sign that can give meaning that represents the sign and that is not far from representation [13]. The symbols used are usually in the form of signs or images that are concrete and represent an idea [14]. Semiotic is used to differentiate the use of signs in the form of index, icon, and symbol but also signs in the form of verbal language [15]. Signs in mathematics have two processes: the development of concepts and interactions between signs and their agreed-upon meaning [16]. Generally, semiotic is the science used to study the signs in mathematics learning and where the signs have their own recognized meanings.

Semiotic is part of an onto-semiotic approach that pays attention to the meaning of mathematical objects. Three aspects of mathematics, namely: problem-solving activities, symbolic language, logical, and organized conceptual systems. Mathematical objects have six main entities in the semiotic approach; there are (1) Language, (2) Situation, (3) Subject's actions, (4) concepts, (5) properties or attributes, and (6) arguments [17]. Mathematical objects that will be used as a standard for analysis: (1) Language that includes terms, symbols, tables in the problem and used in solving mathematical problems, (2) Problems that include tasks or exercises that used, (3) Definition /concept which includes the definition or description of a concept used in solving mathematical problems, (4) Procedures that include strategies or steps used to solve mathematical problems, (5) Propositions which include the properties or principles consisting of several facts in a mathematical concept, and (6) Arguments that include statements used to justify answers in solving mathematical problems [18]. Regarding mathematics, representation in semiotic is what enables conceptualization, reasoning, and problem solving [19]. That semiotic is suitable for analysis tools of students' ability to solve mathematical problems because they have elements of representation, conceptualization, reasoning, and problem-solving. One of the routine activities in learning mathematics is problem solving, where students can apply the information received from the teacher and students can explore the cognitive domains.

Students in learning mathematics must also balance between the cognitive and affective domains. Some of the factors that make mathematics a subject that is considered difficult are interest and motivation to like mathematics. Students have an interest in something, will be easy to accept something of it. One of the affective domains that can foster self-confidence, interest, and perseverance in positively accepting mathematics learning is mathematical disposition. Mathematical disposition is a student's belief in the value and usefulness of mathematics as well as beliefs, attitudes, self-abilities, and self-identity towards mathematics learning [20]. Mathematical disposition is a tendency to: (1) view mathematics as something that can be understood, (2) feel mathematics as something useful and useful, (3) believe that a diligent and tenacious effort in learning mathematics will produce results, (4) perform
actions as active learners and mathematical workers [21]. Students who have self-confidence, interested, diligent and flexible thinking to find several solutions in a mathematical problem can be said that these students have an affective-domains called a mathematical disposition [22].

In general, these students fail to enroll in advanced mathematics classes. After all, their mathematical disposition is negative because they do not have innate abilities. The existence of a mathematical disposition observer whose aim is to find out how students view mathematics which can later be influential in solving mathematical problems [23]. Similar to the research conducted by Lestari et al. (2019) with the categorization of a high, medium, and low mathematical disposition is very influential on how students' abilities [24]. Several indicators of mathematical disposition: (1) The nature of self-confidence and perseverance in doing mathematical tasks, solving problems, communicating mathematically, and in giving mathematical reasons; (2) The nature of flexibility in investigating, and trying to find alternatives in solving problems; (3) shows interest, and curiosity, the nature of wanting to monitor and reflect the way they think; (4) trying to apply mathematics to other situations; (5) appreciates the role of mathematics in culture, values, and mathematics as tools and language [25]. The existence of a mathematical disposition can determine the positive attitude of students in accepting mathematics learning, students with good semiotic abilities if they have a positive outlook in accepting mathematics learning.

2. Methods

Research activities in this study, the researchers will express and describe a way of thinking, ideas, opinions, attitudes and can also be the behavior of the observed subject using qualitative descriptive research. Qualitative research in significant scientific fields develops gradually and is often descriptive and leads to hypotheses [26]. Qualitative research can emphasize aspects other than quantitative projects in their protocol. The main advantage of qualitative research is that it can collect as much data as possible with descriptions and examples, and the language and interests of the most important participants [27]. Such qualitative use is very useful for researchers who want to know who, what, and where the event took place [28]. This study uses subjects from 11th-grade students at 8 High School in Kediri City.

Subject selection used a purposive sampling technique that has the provisions of students who have received a linear program. The instruments in this study used two instruments, there are a mathematical disposition questionnaire and a description of the problem, as well as an interview that aims to dig deeper into students’ semiotic. The stages used in this study are: (1) The selection of classes to be used in research and based on the recommendations of the teacher, (2) Giving a mathematical disposition questionnaire, (3) The researcher describes the problem with a linear program, (4) Categorizing students into 3 categories: high, medium, and low mathematical dispositions, (5) Three students selected representing each category, (6) Analysis of student responses to semiotic abilities and conducting interviews. As students’ categories are presented in Table 1 below.

| Mathematical disposition | Category        |
|--------------------------|-----------------|
| \( Score > \bar{X} + 0.5s \) | High            |
| \( \bar{X} - 0.5s \leq Score \leq \bar{X} + 0.5s \) | Medium          |
| \( Score < \bar{X} - 0.5s \) | Low             |

Source: adopted from Ramadhani, et al. [29]
3. Results and Discussion

The results of this study are presented following the elaboration of the research findings that describe the students' semiotic. The first instrument used was a questionnaire about the mathematical disposition that has the aim to categorize students into three categories. The first category is a high mathematical disposition, the second is a medium mathematical disposition and the third is a low mathematical disposition. The mathematical disposition questionnaire was given to 36 students in one class, and then selected using purposive sampling. This study uses three subjects where each subject represents each category of mathematical disposition. The following is a categorization of students' mathematical dispositions shown in Table 2.

| Mathematical disposition | Category    | Total students |
|--------------------------|-------------|----------------|
| Score > 136              | High        | 14             |
| 110 ≤ Score ≤ 136        | Medium      | 9              |
| Score < 110              | Low         | 13             |

Table 1 shows the number of students who have a high mathematical disposition are 14 students, the numbers of the students who have a medium mathematical disposition are 9 students, and the numbers of students who have a low mathematical disposition are 13 students. Then from the data, selected 14 students who are included in the category of high mathematical disposition. Here is a semiotic analysis of students in solving mathematical problems in linear program material in terms of mathematical disposition.

In general, seeing the response given by students to a mathematical disposition can be seen as how students respond and accept mathematics as a subject that is useful and useful for solving mathematical problems and the role of mathematics in everyday life. Subjects who have a high mathematical disposition, which can be expected that the subject can solve mathematical problems very well. Reference analysis that is used to describe students' semiotic is using mathematical objects. The mathematical object is an expression that can be used as content: language, problems, concepts, procedures, propositions, and arguments [6]. In this study, mathematical object analysis can be used as follows: language and problems, concepts and propositions, procedures, and arguments.

3.1. Mathematical object: language and problems

The first step in solving problems is to understand the information contained in the problem then change the problem story into making symbols or examples with mathematical models. The first mathematical objects are language and problems, an analysis of aspects of language that includes symbols and tables contained in the problem. That language includes terms, expressions, notations, graphics, while problems include assignments, exercises, and examples [18]. Language includes terms, expressions, notations, and graphics, while problems include situations, problems, and extra applications [30]. The following is students' answers to mathematical language objects and problems.
Based on the results of interviews between researchers and subjects, it can be said that the subject understands what information is contained in the problem. Subject explains the use of variable $x$ for machine I and $y$ for machine II. Subject also understands the use of symbols for the inequality "$\geq"$ as a sign more than equal to, where the problem has a meaning that can produce more than specified. Subject understands what the problem is but does not write back what is asked. Therefore, from the results of written tests and interview results it was concluded that the subject is able to meet the indicators of mathematical objects in language and problems.

**Figure 1.** Written test results for language and problems

|                          | Big Vase | Medium Vase | Small Vase | Operating Costs |
|--------------------------|----------|-------------|------------|-----------------|
| Machine I                | 1 kg     | 3 kg        | 5 kg       | 700,000 days    |
| Machine II               | 4 kg     | 4 kg        | 4 kg       | 1,200,000 days  |
| Total                    | 100 kg   | 180 kg      | 220 kg     |                 |

Mathematical modeling:

a. $x \geq 0$

b. $y \geq 0$

c. $x + 4y \geq 100$

d. $3x + 4y \geq 180$

e. $5x + 4y \geq 40$

f. $f(x, y) = 700,000x + 1,200,000y$

3.2. Mathematical object: concepts and propositions

The next mathematical object is a concept which includes a description of a concept used in solving mathematical problems and a proposition that includes several facts in a mathematical concept. Definitions / concepts are introduced through definitions or descriptions, explicit or vice versa (straight lines, points, numbers, functions, averages), while propositions include statements about concepts [18]. Definitions or descriptions of mathematical ideas (numbers, points, straight lines, averages) while propositions are usually given as statements [30] that propositions can be used to check a concept [31].

This stage analyzes how subjects understand the concepts of linear programming material and true or false statements about a concept called a proposition. Researchers analyzed using written test results and supported by interviews between researchers and subjects. The following is explained one by one proposition about the concepts contained in the linear program material using data from the results of written tests and the results of interviews between researchers and subjects.

First there is the use of symbols, variables and tables in rewriting information contained in the problem. Here are the results of the written test and the results of the interview quote showing the first proposition in Figure 3.
Figure 3. Written test results for the first proposition

| Big Vase | Medium Vase | Small Vase | Operating Costs |
|----------|-------------|------------|-----------------|
| Machine I | 1 kg        | 3 kg       | 5 kg            | 700,000 days   |
| Machine II | 4 kg       | 4 kg       | 4 kg            | 1,200,000 days |
| Total     | 100 kg      | 180 kg     | 220 kg          |                |

Mathematical modeling:

a. \( x \geq 0 \)

b. \( y \geq 0 \)

c. \( x + 4y \geq 100 \)

d. \( 3x + 4y \geq 180 \)

e. \( 5x + 4y \geq 40 \)

f. \( f(x, y) = 700,000x + 1,200,000y \)

Based on written test data results and interview results can be known for the first proposition about the use of symbols, variables, and tables. Subject shows it by writing the information that is known to the problem with the table and using variable \( x \) as machine I and variable \( y \) as machine II and using symbols to be more than equal to \( \geq \). Therefore, subjects satisfy the first proposition.

Second, there is a mathematical model of an inequality. Following are the results of the written test and the results of the interview quote showing the second proposition in Figure 5.

Figure 4. Subject interview for the first proposition

Figure 5. Written test results for the second proposition
P : What is the mathematical model? Anything?
S : For the first and second because of this declared goods, then \( x \geq 0 \) and \( y \geq 0 \). Then the third is to produce large vases, \( x + 4y \geq 100 \)

P : Why use more than equal sign (\( \geq \))?
S : Because of that, it can produce more than 100.

P : What can you model the mathematics using? Variable right? What do you mean by that?
S : To make it easier, Ma'am in counting.

P : What variables do you use?
S : I use variables \( x \) and \( y \) where \( x \) is machine I and \( y \) is machine II.

**Figure 6.** Subject interview for the second proposition

Based on data from written test results and interview results can be known for the second proposition that is about the existence of a mathematical model of an inequality. Subject can write mathematical models correctly and correctly and the use of the hyphen symbol “ \( \geq \) ” which means that the goods produced can exceed the target, for example in a large vase means it can be more than 100 kg. Subject also writes \( x \geq 0 \) and \( y \geq 0 \), Subject has an argument for stating goods. Therefore, the subject satisfies the second proposition.

Third, there is the goal problem which is usually called the objective function. Following are the results of the written test and the results of the interview quote showing the third proposition in Figure 7.

\[
 f(x, y) = 700,000x + 1,200,000y
\]

**Figure 7.** Written test results for the third proposition

**Figure 8.** Subject interview for the third proposition

Based on written test data and interview results can be known for the third proposition that is the purpose of the problem that is commonly called the objective function. Subject writes down and can argue that \( f(x, y) \) is an objective function. The existence of the objective function can be used to determine the optimum value (maximum or minimum). Therefore, subject satisfies the third proposition. Fourth, there is a graph to make it easier to find the settlement set area.
Figure 9. Written test results for the fourth proposition

| Elimination: $3x + 4y = 180$ | Elimination: $5x + 4y = 220$ |
|--------------------------------|--------------------------------|
| $x + 4y = 100$                | $3x + 4y = 180$                |
| $2x = 80$                     | $2x = 40$                      |
| $x = 40$                      | $x = 20$                       |
| For $x = 40$, then $x + 4y = 100$ | For $x = 20$, then $3x + 4y = 180$ |
| $40 + 4y = 100$               | $60 + 4y = 180$               |
| $4y = 60$                     | $4y = 120$                    |
| $y = 15$                      | $y = 30$                      |
| So, $y = 15$ and then $x = 40 \rightarrow (40;15)$ | So, $y = 30$ and then $x = 20 \rightarrow (20;30)$ |

Figure 10. Subject interview for the fourth proposition

Based on written test data and interview results can be known for the fourth proposition, namely the existence of a graph to facilitate the search set of settlement areas. The written test results of the subject have drawn a graph, utilizes the graph to find the set of settlement areas. According to subject the area which is the settlement set area is the shaded area so can find out which points can be used. Subject looks for points on the graph using the elimination method. Subject gets the first 2 cut points $(40,15)$ and the second one $(20,30)$. Overall subject can understand the meaning of the use of graphs and the elements contained in the graph. Therefore, subject satisfies the fourth proposition.

The fifth is the optimum (maximum or minimum) value associated with the existence of the settlement area.

Figure 11. Written test results for the fifth proposition

| $f(x,y)$ | $700,000x + 1,200,000y$ | $\varepsilon$ |
|----------|------------------------|---------------|
| $(0;55)$ | $0 + 66,000,000$       | $66,000,000$  |
| $(20;30)$| $14,000,000 + 36,000,000$ | $50,000,000$  |
| $(40;15)$| $28,000,000 + 18,000,000$ | $46,000,000 \rightarrow \text{Min}$ |
| $(100;0)$| $70,000,000 + 0$       | $70,000,000$  |

So, the lowest operating costs is 46,000,000

Figure 12. Subject interview for the fifth proposition

| P : After knowing the dots, your next step? |
|--------------------------------------------|
| S : We enter the destination function for each point. There could be 66,000,000,50,000,000,46,000,000 and 70,000,000. |
| P : Then what was asked about the matter? |
| S : Minimal operational costs. |
| P : Okay. Which means? |
| S : Which is 46,000,000 with its point $(40,15)$. |
| P : Okay. Means that by looking at the results of your work, does your answer already represent what is asked? |
| S : Yes, ma'am. |

8
Based on the data of the written test results and the interview results can be known for the fifth proposition that is the existence of the optimum value (maximum or minimum) associated with the existence of the settlement area. The problem found in the problem is to find how many (days) machine I and machine II work with minimal operational costs, it can be interpreted to find the smallest or maximum value. Subject looks for the smallest value by using the objective function which then selects the points that are in the settlement set area and substitutes these points in the objective function equation in the form of \( f(x, y) = 700,000x + 1,200,000y \). Subject write the points that are in the region of the set of settlements, there are \((0.55), (20.30), (40.15), and (100.0)\). Subject gets the smallest value which is \(46,000,000\) at point \((40,15)\). Therefore, the subject satisfies the fifth proposition.

3.3. Mathematical object: procedures

The mathematical objects are procedures, procedures include algorithms, operations, and calculation techniques [30]. Procedures or actions of subjects when completing mathematical tasks in the form of operations, algorithms, techniques and procedures [30]. This stage analyzes how subjects perform procedures in solving mathematical problems in linear program material. The following is a description of the procedure or steps taken by subject in solving problems. The step chosen by the subject is to rewrite the information obtained from the question using the table. Subject writes down the information obtained by modeling the mathematics. Apply symbols and use variables. Subject writes the objective function and writes the terms \(x \geq 0\) and \(y \geq 0\) and gives an argument writing the condition with the reason for stating the goods. After knowing all the points, the subject draws a graph to find the area of the set of solutions. Subject looks for cut points using the elimination method and gets the first cut point \((40.15)\) and the second cut point \((20,30)\). Subject states that the settlement area is the shaded area. This argument is reinforced by interviews between researchers and subjects.

The next step, the subject knows which points are in the settlement set, which are then substituted into the objective function. The objective function is \( f(x, y) = 700,000x + 1,200,000y \). Subject writes the dots \((0.55), (20.30), (40.15),\) and \((100.0)\). Subject substitutes the points to the objective function to find out the minimum operational costs, which are as follows. For point \((0.55)\) gets a value of \(66,000,000\), for point \((20.30)\) gets a value of \(50,000,000\), for point \((40.15)\) gets a value of \(46,000,000\), and for point \((100.0)\) gets a value of \(70,000,000\). Subject gets the value for minimal operational costs at point \((40.15)\) with \(46,000,000\). Subject writes the conclusion that machine I worked for 40 days and machine II worked for 15 days.

Based on the description of the problem-solving procedure in the linear program. The subject as a whole can already understand the procedure chosen to find the answer to the question. In concluding subjects can represent or answer the problem contained in the problem there are subject draw conclusions by stating how many (days) machine I and machine II work.

3.4. Mathematical object: arguments

The indicator for the sixth mathematical object is the argument, arguments include discourse to validate and explain propositions [18]. What arguments are used to validate and explain propositions or to compare (justify or refute) the actions of the subject [30]. This stage emphasizes more on how subjects corrected whether the answers obtained were correct or not by explaining again using their language so that students could be sure whether or not the work was done for the question. The researcher conducted interviews with subjects to find out how to understand the subject in solving problems. Quotes of interviews between researchers and subject are as follows.
Okay. Now try using your own language from beginning to end, how you solve problem number two!

Okay, just go ahead for the mathematical model, which is because it states the goods, then for equations one and two are

\[ x \geq 0 \] and \[ y \geq 0. \]

And then for large vases \[ x + 4y \geq 100, \]

for medium vases \[ 3x + 4y \geq 180, \]

and for small vases \[ 5x + 4y \geq 220. \]

Why is it more than the same as?

And the objective function \( f(x, y) = 700,000x + 1,200,000y. \)

And after the mathematical model, we put it on the graph and then find the set of solutions that is shaded. And there are points where there are two intersection points, are searched using the elimination method to find points \((40, 15)\) and \((20, 30)\).

After we know the points, we put them in the destination function, and those that have a minimum value of 46,000,000, and for the duration of work, are for machine I for 40 days and machine II for 15 days.

Okay. Is your answer correct?

Now I ask, why do you use the symbol \( x \) as machine I and \( y \) as machine II?

Yes because it is for convenience. And as the teacher teaches.

**Figure 13.** Subject interview for arguments

Based on quotes from interviews conducted between researchers and subjects, it is known that subjects can re-express the steps from the start of the work up to the final results and subject realizes that the answers obtained are the right answers because subject realizes that for a long time it works is a machine I for 40 days and machine II 15 days. Therefore, the subject can provide arguments about the right or wrong in solving linear program problems.

In the research findings, we can discuss students’ semiotic in terms of mathematical disposition. Students in the category of high mathematical disposition can be described that the student can meet all mathematical objects that are used as reference semiotic analysis. Semiotic students for mathematical objects, there are: (1) Language where students can convert information into mathematical symbols by making an example using variables and students can draw tables. (2) The problem where students can understand the problem asked for the problem, it’s just that students do not write in writing and only understand verbally. (3) The concept where students can understand the concept of a linear program that is writing information using symbols and variables then writing also with tables, applying variables as examples of information that is then formed into a mathematical model, understanding the problems contained in the problem, using procedures correctly and understanding the optimum point. (4) Procedure where students can understand the procedures used in a linear program in a row by the problems contained in the problem. (5) Proposition where students can explain a statement of right or wrong about the concept of linear program material. (6) Arguments, where students can provide an argument in correcting whether the answers obtained, are correct or not by explaining again using their language so that students can be sure whether or not the work is done.

4. Conclusion

There are six mathematical objects in semiotic: language, problems, concepts, procedures, propositions, and arguments. The mathematical object can be used as an analysis of student semiotic. In connection with the results obtained in this study implies this research is the importance of instilling basic concepts in students. The most basic thing in studying mathematics and other fields of science is an understanding of the concept. The existence of a mathematical disposition that can be used as a reference in knowing students' interest in learning mathematics where there is a categorization of mathematical dispositions in which the teacher can see the enthusiasm, self-confidence, and interest of students in learning mathematics which can result in the success of learning achievement, which must begin with interest and love math lessons. Semiotic can also help teachers in knowing the ability of students in the use of symbols, variables, signs as language and problems, concepts, procedures, propositions and arguments.
References

[1] Naval Education and Training (NAVEDTRA) 2003 Mathematics Basic Math and Algebra (United States: Naval Education and Training Professional Development and Technology Center)

[2] Stender P and Keiser G 2015 ZDM-Mathematics Education 47 (7) 1255-1267

[3] Booth L 1988 Children’s Difficulties in Beginning Algebra in a Coxford and a Shulte (eds.) The Ideas of Algebra K–12 (Reston (VA): NCTM)

[4] Krampen M 1987 Classics of Semiotics (West Germany: University of the Arts Berlin)

[5] Sierpinska A 1994 Understanding in Mathematics (London: The Falmer Press)

[6] Godino J D, Batanero C and Font V 2007 ZDM- The International Journal on Mathematics Education 39 (1) 127-135

[7] Gusmao T, Santana E, Cazorla I and Cajaraville J 2010 A Semiotic Analysis of “Mônica’s Random Walk: Activity to Teach Basic Concepts of Probability Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8) (Netherlands: International Statistical Institute)

[8] Cabitza F and Mattozzi A 2017 Journal of Visual Language and Computing 40 65-90

[9] Kockelman, P 2013 Agent, Person, Subject, Self (United States of America: Oxford University Press)

[10] Weinberg A, Dresen J and Slater T 2016 Journal of Mathematical Behavior 43 70–88

[11] Duval R 2017 Understanding the Mathematical Way of Thinking – the Registers of Semiotic Representations (France: Springer International Publishing)

[12] Presmeg N, Radford L, Roth W M and Kadunz G 2016 Semiotics in Mathematics Education (Switzerland: Springer International Publishing)

[13] Otte M 2018 Chapter 9: Semiotics, Epistemology, and Mathematics. N. Presmeg et al. (eds.), Signs of Signification, ICME-13 Monographs (Switzerland: Springer International Publishing)

[14] Semetsky I 2007 Studies in Philosophy Education 26 (3) 179–183

[15] Seel N M 2012 Semiotics and Learning. In: Seel N.M. (eds) (Boston: Springer International Publishing)

[16] Gutiérrez Á, Gilah C Leder G C and Boero P 2016 The Second Handbook of Research on the Psychology of Mathematics Education (Netherlands: Sense Publishers)

[17] Font V, Godino J D and D’Amore B 2007 An Onto-Semiotic Approach to Representation in Mathematics Education For the Learning of Mathematics 27 (2) 2–7

[18] Borji V, Font V, Alamolodei H and Sanchez A 2018 EURASIA Journal of Mathematics, Science and Technology Education 14 (6) 2301-2315

[19] Salazar J V F 2018 Chapter 12: Semiotic Representations: A Study of Dynamic Figural Register. N. Presmeg et al. (eds.) Signs of Signification, ICME-13 Monographs (Switzerland: Springer International Publishing)

[20] An S A, Zhang M, Flores M, Tillman D A and Serna L 2015 Journal of Mathematics Education 8 (2) 39-55

[21] Kilpatrick J Swafford J and Findell B 2001 Adding It Up: Helping Children Learn Mathematics (Washington D.C.: National Academy Press)

[22] Katz L G1993 Disposition as Educational Goals (United States: ERIC Custom Transformations Team)

[23] Cai J, Robinson V, Moyer J, Wang J and Nie B 2012 Mathematical Dispositions and Student Learning: A Metaphorical Analysis Mathematics, Statistics and Computer Science (United States: Department of Faculty Research and Publications Marquette University)

[24] Lestari S D, Kartono and Mulyono 2019 Unnes Journal of Mathematics Education Research 8 (2) 157-164
[25] Polking J 1998 *Response to NCTM’s Round 4 Questions* (Washington D.C.: American Mathematical Society)

[26] Arnesen H, Bechensteen A G, Jacobsen A F and Omenaas E 2017 *The Research Handbook: From Idea to Publication* 7th edition (Oslo: Oslo University Hospital)

[27] Leavy P 2017 *Research Design: Quantitative, Qualitative, Mixed Methods, Arts-Based, and Community-Based Participatory Research Approaches* (New York: A Division of Guilford Publications Inc.)

[28] Sandelowski M 2000 Focus on Research Methods: Whatever Happened to Qualitative Description? *Research in Nursing & Health*, John Wiley & Sons Inc. 23 334–340

[29] Ramadhani M R, Usodo B and Subanti S 2017 *Journal of Physics: Conf. Series* 1157 042101

[30] Montiel M, Wilhelmi M R, Vidakovic D and Elstak I 2012 Vectors, Change of Basis and Matrix Representation: Onto-Semiotic Approach in the Analysis of Creating Meaning *International Journal of Mathematical Education in Science and Technology* 43 (1) 11-32

[31] Afifah D S N, Nafi’an M I, Juniati D and Siswono T Y E 2016 *International Conference on Mathematics, Science, and Education 2016 (ICMSE 2016)* 3 (1) 110-113