Heavy to Light Semileptonic Transitions in the Heavy Quark Effective Theory

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ABSTRACT

The scaling behavior of semileptonic form-factors in Heavy to Light transitions is studied in the Heavy Quark Effective Theory. In the case of $H \to \pi e\nu$ it is shown that the same scaling violations affecting the heavy meson decay constant will be present in the semileptonic form-factors.


1 Introduction

The Heavy Quark Limit (HQL), in which the mass of the heavy quark $m_Q$ is much larger than the typical scale of hadronic interactions, has been the object of intense study [1]. The symmetries arising in this limit, $SU(2)_{\text{flavor}} \times SU(2)_{\text{spin}}$, have been successfully applied to the matrix elements involved in transitions between heavy mesons, reducing the number of form-factors to one universal function that contains all the long distance effects. In the Heavy-to-Light transitions, although the number of form-factors cannot be reduced by symmetry arguments, the HQL can be used to relate $D \to L$ to $B \to L$ decays, where $L = \pi, \rho, \omega, \ldots$. This is potentially useful to extract the CKM matrix element $|V_{ub}|$ by using $D$ decay data plus the well defined scaling behavior of the form-factors with the heavy mass in the HQL [2]. Several issues should be addressed in the implementation of this program. First, the $D$ decay data will not cover the full kinematic range in the corresponding $B$ decay, as a consequence of which a large portion of the $B$ data will be excluded. Second, the presence of poles near the physical region, particularly in the $\pi$ modes, introduces a different scaling behavior and, because they enter in $D$ and $B$ decays with different weights, they should be taken into account. However, the most pressing issue concerning the extraction of $|V_{ub}|$ by using HQL arguments is the size of the corrections to the symmetry limit. In general, $1/m_Q$ corrections will result in scaling violations in the semileptonic form-factors. These corrections are thought to be important in heavy meson decay constants, a result suggested by lattice calculations [3]. It is then imperative to confront this issue in semileptonic form-factors for $H \to L$ transitions, for the existence of potentially large scaling violations could introduce an uncertainty in the scaling procedure that would seriously undermine our ability to extract $|V_{ub}|$ within the framework of a controlled approximation.

The Heavy Quark Effective Theory (HQET) [4] is a convenient framework to systematically take into account the $1/m_Q$ corrections [5, 7, 9]. In this letter we will show the structure of the $1/m_Q$ corrections to semileptonic form-factors in $H \to L$ transitions. We will focus on the case $L = \pi$, where the formulation of its chiral couplings to heavy mesons [12, 13] gives a relation that can be used to estimate the size of the scaling violation effects in terms of the heavy quark symmetry breaking in the heavy meson decay constant.

2 Semileptonic form-factors in the HQET

In the HQL, form-factors entering in the semileptonic decays of heavy mesons to light mesons have a definite scaling with the heavy mass. The matrix element for the decay to a pion can be written as
\[ < \pi(p)|\bar{q}\gamma_{\mu}Q|M(P) > = f_+(q^2)(P+p)_{\mu} + f_-(q^2)(P-p)_{\mu} \]  

(1)

with the normalization of states given by

\[ < M(P)|M(P') > = 2m_M\delta^3(P-P') \]  

(2)

In the HQL the left hand-side of (1) has to be independent of the heavy mass, after properly dividing by the factor \( \sqrt{2m_M} \) from the normalization of the heavy meson. This automatically implies

\[ f_+ + f_- \sim m_M^{-1/2} \]  

(3)

\[ f_+ - f_- \sim m_M^{1/2} \]

This simple scaling behavior is, however, affected by corrections that are multiplied by inverse powers of the heavy mass (as well as by QCD corrections). The HQET provides a systematic approach to these corrections. The heavy quark field in the effective theory is defined, in terms of the quark field in the full theory \( Q(x) \), as

\[ h_Q(v,x) = \exp^{imQ\not{v}\cdot x}Q(x) \]  

(4)

with \( v \) the velocity of the heavy quark. In this way, derivatives on \( h_Q(x) \) are ‘small’ or independent of the heavy mass

\[ i\partial h_Q(x) = (\not{P} - m_Q k)h_Q(v,x) = kh_Q(v,x) \]  

(5)

where \( k \) is the off-shellness of the heavy quark, and it is of the order of the \( \Lambda_{QCD} \) scale for soft processes. In principle, perturbative hard gluon exchange could upset this description in terms of velocities. However, in practice, perturbative off-shell effects seem small throughout the kinematic range relevant for \( D \) and \( B \) decays.

The effective Lagrangian is then written in terms of \( h_Q \) as an expansion in \( 1/m_Q \). Neglecting terms involving operators of dimension six or higher

\[ \mathcal{L}_{eff} = \bar{h}_Q i\gamma_\mu D_{\mu}h_Q + \frac{1}{2m_Q}\bar{h}_Q[D^2 + \frac{1}{2}gs\sigma^{\mu\nu}G_{\mu\nu}]h_Q \]  

(6)

where \( D_{\mu} = \partial_{\mu} - ig_sA_{\mu} \) is the usual covariant derivative and \( G_{\mu\nu} = [D_{\mu}, D_{\nu}]/ig_s \). The last two terms in (6) break the \( SU(2)_{flavor} \) symmetry due to the explicit presence of \( m_Q \), whereas only the last term breaks \( SU(2)_{spin} \). Additional \( m_Q \) dependence will appear when QCD corrections are taken into account.
The current operators will also have an expansion in $1/m_Q$. For the Heavy-Light current, at tree level, we have

$$\bar{q}\Gamma Q = \bar{q}\Gamma h_Q + \frac{1}{2m_Q}\bar{q}\Gamma i D h_Q$$

where contributions from operators of dimension five or higher are neglected.

To obtain the form-factors in the HQET we need the matrix elements of the operators in the right-hand side of (7). The leading contribution corresponds to the operator $Q_0 = \bar{q}\Gamma h_Q$. Using the trace formalism this can be written as

$$<\pi(p)|\bar{q}\Gamma h_Q| M(v) > = Tr[\gamma_5 (A \not{\!v} + B \not{\!p}) \Gamma M(v)]$$

Here the heavy meson with velocity $v$ is represented by

$$M(v) = \sqrt{m_M} \frac{(1 + \not{\!v})}{2} \gamma_5$$

It satisfies the conditions $\frac{(1 + \not{\!v})}{2} M(v) = M(v)$ and $\frac{(1 - \not{\!v})}{2} M(v) = 0$. The explicit mass dependence in (9) is in correspondence with the normalization (2). The form-factors $A$ and $B$ are independent of $m_M$. They are functions of $v.p$, the light hadron energy in the heavy meson rest frame. The expression (8) is completely general and independent of the Dirac structure. In particular, for the vector current it will give the form-factors defined in (1) to leading order in the HQET. The naive scaling of (3) is then recovered in the form:

$$f_+ + f_- = 2A m_M^{-1/2}$$

$$f_+ - f_- = 2B m_M^{1/2}$$

At tree level, the $1/m_Q$ corrections to (11) come exclusively from the second term in (7), the operator $Q_1 = \bar{q}\Gamma i D h_Q$. Dimension four operators can be written in a general form by absorbing the Dirac matrix contracting the covariant derivative into a general Dirac structure $\Gamma$. Their matrix elements can then be expressed as

$$<\pi(p)|\bar{q}\Gamma i D_\alpha h_Q| M(v) > = Tr[(F_1 v_\alpha + F_2 \gamma_\alpha + F_3 p_\alpha) \gamma_5 \Gamma M(v)]$$

where the $F_i$'s are new long distance parameters, functions of $v.p$ but independent of the heavy mass. In what follows we will apply the methods that are used in Ref.3 for the case of the decay constant in order to eliminate these new parameters in favor of $A$ and $B$. 

3
First, the equation of motion for the heavy quark, to leading order, is $iv.Dh_Q = 0$. When this is applied to (11) we obtain the constraint

$$F_1 - F_2 + v.pF_3 = 0$$

(12)

The corrections to (12) are not relevant to the order we are working.

Furthermore, the $F_i$’s can be related to the leading form factors $A$ and $B$ by making use of the fact that both equations (8) and (11) are independent of the Dirac structure of the currents. If we write

$$\bar{q}\Gamma iD_\alpha h_Q = i\partial_\alpha (\bar{q}\Gamma h_Q) - (-iD_\alpha)\bar{q}\Gamma h_Q$$

(13)

and then set $\Gamma = \gamma^o \hat{\Gamma}$, the last term in (13) will vanish in the limit $m_q = 0$ due to the equation of motion for the light quark. In this way we can write

$$<\pi(p)|\bar{q}\gamma^\alpha \hat{\Gamma} iD_\alpha h_Q|\pi_M(v)> = i\partial_\alpha <\pi(p)|\bar{q}\gamma^\alpha \hat{\Gamma} h_Q|\pi_M(v)>$$

(14)

$$= \overline{\Lambda} \nu_\alpha <\pi(p)|\bar{q}\gamma^\alpha \hat{\Gamma} h_Q|\pi_M(v)>$$

(15)

Here $\overline{\Lambda} = m_M - m_Q$ is a low energy parameter that expresses the fact that in the effective theory the current only carries a small momentum.

Using (8), (11) and (15), we obtain two equations relating the $F_i$’s with the leading form-factors. Together with (12) they give

$$F_1 = \overline{\Lambda} \left(\frac{A}{3} + v.pB\right)$$

$$F_2 = \overline{\Lambda} \left(\frac{A}{3} + 2v.pB\right)$$

$$F_3 = \overline{\Lambda}B$$

(16)

We can now proceed to write down the semileptonic form-factors at tree level and to order $1/m_Q$ in the heavy quark expansion. Using (1), (7), (8), (11) and (16) we have:

$$f_+ + f_- = 2A m_M^{-1/2} [1 - \frac{\overline{\Lambda}}{2m_M} (1 + 4v.p B) + \ldots]$$

(17)

$$f_+ - f_- = 2B m_M^{1/2} [1 - \frac{\overline{\Lambda}}{2m_M} + \ldots]$$

(18)

It is interesting to analyze (17) and (18) in the chiral limit, $v.p \rightarrow 0$. First, we observe that in this limit the same scaling violation occurs in both the non-leading (17)
and the leading (18) combinations. Moreover, the chiral couplings of heavy mesons to the pseudo-goldstone bosons implies \[ f_+ + f_- = \frac{f_M}{f_\pi} \] \[ (19) \]

We then conclude that in the chiral limit, the symmetry breaking terms affecting (17) are the same as the ones affecting the heavy meson decay constant \( f_M \). Then, whatever the size of the symmetry breaking effects in \( f_M \), equations (17), (18) and (19) tell us how they contaminate the scaling procedure to extract \(|V_{ub}|\) from \( D \to \pi \) and \( B \to \pi \) data. The kinematic range shared by the two processes corresponds to \( v.p < 1\text{GeV} \), where (19) is expected to be valid.

Large scaling violation effects in the heavy meson decay constant are suggested by lattice calculations as well as by QCD sum rules results. Thus, before applying heavy quark symmetry arguments to phenomenology, these calculations should be fully understood and under control.

Equations (17) and (18) were derived at tree level. However the essential result, which relates the symmetry breaking in the decay constant \( f_M \) to the semileptonic form-factors, will not change when going beyond tree level.

### 3 Beyond Tree Level

Beyond tree level there will be additional operators contributing to the \( 1/m_Q \) expansion of the current. There will be an additional dimension three operator, \( Q'_0 = \bar{q}v_\mu h_Q \). Also \( Q_1 \) will mix with other dimension four operators. The vector current including \( 1/m_Q \) corrections and QCD corrections can be written as

\[
\bar{q}\gamma_\mu Q = C_0(\mu)Q_0 + C_1(\mu)Q'_0 + \frac{1}{m_Q} \sum_{i=1}^6 B_i(\mu)Q_i + \ldots
\]

(20)

The \( C_i \)'s and \( B_i \)'s are the short distance coefficients of the dimension three and dimension four operators respectively and they introduce additional dependence on the heavy mass. Following Ref.\[9\], we choose the basis for the dimension four operators to be
\[ Q_1 = \bar{q} \gamma_\mu i D h_Q \quad Q_4 = (iv.D) \bar{q} \gamma_\mu h_Q \]
\[ Q_2 = \bar{q} v_\mu i D h_Q \quad Q_5 = (iv.D) \bar{q} v_\mu h_Q \]
\[ Q_3 = \bar{q} i D h_Q \quad Q_6 = (iD_\mu) \bar{q} h_Q \quad (21) \]

The short distance coefficients are calculated in [5, 7]. The matrix elements of the dimension four operators in (20) will account for the subleading contributions coming from the expansion of the current in the HQET.

Additional \(1/m_Q\) corrections come from corrections to the heavy quark propagator. These can be taken into account, when calculating the matrix element (1) in the HQET, as insertions of the dimension four operators in the effective Lagrangian (6) into the leading order current operator \(Q_0\) [9].

\[
< \pi(p)|i \int d^4 x T[(\bar{q} \Gamma h_Q)_0, (\bar{h}_Q(iD)^2 h_Q)_x]|M(v) > = G_1(\mu) Tr[\gamma_5 (A(\mu) \not{p} + B(\mu) \not{\bar{p}}) \Gamma M(v)] \quad (22)
\]

\[
< \pi(p)|i \int d^4 x T[(\bar{q} \Gamma h_Q)_0, (\bar{h}_Q \sigma^{\mu \nu} G_{\mu \nu} h_Q)_x]|M(v) > =
Tr[W^{\mu \nu} \gamma_5 (A(\mu) \not{p} + B(\mu) \not{\bar{p}}) \Gamma(\frac{1+ \not{p}}{2} \sigma^{\mu \nu} M(v)] \quad (23)
\]

where \(T\) denotes time-ordered product and \(W^{\mu \nu}\) is an antisymmetric tensor that can be written as

\[ W^{\mu \nu} = G_2(\mu) \sigma^{\mu \nu} + G_3(\mu)(\gamma^\mu p^\nu - \gamma^\nu p^\mu) \quad (24) \]

\(G_1(\mu)\) and \(G_2(\mu)\) are new long-distance parameters evaluated at the energy scale \(\mu\). The matrix elements of \(Q_2\) and \(Q_3\) can be written as in (11) and then their computation is similar to \(Q_1\) with different Dirac structure. The matrix elements of the operators \(Q_4\), \(Q_5\) and \(Q_6\) are related to those by using (13). The result, to all orders in \(\alpha_s\) and to order \(1/m_Q\) in the HQET is

\[
f_++f_- = 2A(\mu)m_M^{-1/2}(C_0(\mu) + C_1(\mu))[1 - \frac{\Lambda}{2m_M} B_+(\mu) + \frac{G_+(\mu)}{m_M}] \quad (25)\]

\[
f_+-f_- = 2B(\mu)m_M^{1/2}C_0(\mu)[1 - \frac{\Lambda}{2m_M} B_-(\mu) + \frac{G_-(\mu)}{m_M}] \quad (26)\]
Here we defined

\[
B_+ = B_1(1 + 4v.p \frac{B}{A}) + B_2(1 + v.p \frac{B}{A}) + B_3v.p \frac{B}{A} \\
+ 2B_4(1 + 2v.p \frac{B}{A}) + B_5(1 + \frac{5}{2}v.p \frac{B}{A}) + B_6(1 + v.p \frac{B}{A})
\] (27)

\[
B_- = B_1 + B_3 + 2B_4 + B_6
\] (28)

\[
G_+ = G_1 + 16G_2 + 4G_3m_x^2
\] (29)

\[
G_- = G_1 + 16G_2 - 8G_3v.p
\] (30)

The situation looks, in principle, more complicated than at tree level. When taking the chiral limit to make a connection with the heavy meson decay constant via (19), we observe that the contributions from the \(1/m_Q\) corrections to the hadronic wave function -\(G_1(\mu)\) and \(G_2(\mu)\)- as well as the contributions from \(Q_1, Q_4\) and \(Q_6\) are the same in (25) and (26). However the symmetry breaking induced by \(Q_2, Q_3\) and \(Q_5\) differs. Also the short distance coefficient \(C_1(\mu)\), corresponding to \(Q'_0\), only affects \(f_+ + f_-\).

Nevertheless, the essence of the result from the previous section is still present: in the chiral limit the symmetry breaking terms in (25) are the same as the ones in the heavy meson decay constant \(f_M\). The scaling violating short and long distance parameters entering in (25) and (26) are the same (and, to a very good approximation, enter in the same way in both). Then, any calculation addressing the scaling violations in \(f_M\) is at the same time a calculation of the effect in the semileptonic form-factors. For instance, to make contact with the work on \(f_M\) in Ref.[9] we can define \(A(\mu) = F(\mu)/2f_\pi\), with \(F(\mu)\) defined in [9]. Thus the QCD sum rules results for \(F(\mu), G_1(\mu)\) and \(G_2(\mu)\) directly apply to the semileptonic form-factors in (25) and (26).

### 4 Conclusions

We have calculated the semileptonic form-factors entering the decay \(H \rightarrow \pi e\nu\) to order \(1/m_Q\) in the HQET, both at tree level and to all orders in QCD. By using equation (19), a result of the unified heavy quark and chiral symmetric formulation, we have shown that the same symmetry breaking terms affecting the heavy meson decay constant \(f_M\) enter in both the non-leading and the leading combinations of semileptonic form-factors.

Equations (25) and (26) help identify the origin of the most important scaling violations. The long distance parameters should be computed on the lattice or by QCD sum rules, as applied to \(f_M\). Only when they are known with confidence in these or any other calculational scheme, will the extraction of \(|V_{ub}|\) from \(D, B \rightarrow \pi e\nu\) plus mass scaling be a phenomenologically sensible procedure.
The extension of the calculation to include final states with light vector mesons ($\rho, K^*$, etc) is straightforward. Here, however, there does not seem to be an equivalent to equation (19) relating $f_M$ with the semileptonic form-factors. The $1/m_Q$ corrections arising from the operators in (21) are still the same (they only depend on the long distance parameter $\Lambda$); but the ones induced by insertions like those in (22,23) need not be related, in principle, to the equivalent symmetry breaking terms in $f_M$. The formulation of the chiral couplings of heavy mesons to light vector mesons [14] does not seem to provide, by itself, such a connection.

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