Realization of Cyons and Anyons by Atoms

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Received: 11 March 2020 / Accepted: 4 July 2020 / Published online: 14 July 2020
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Abstract
We propose theoretical schemes to realize the cyon and anyon by atoms which possess non-vanishing electric dipole moments. To realize a cyon, besides the atom, we need a magnetic field produced by a long magnetic-charged filament. To realize an anyon, however, apart from these we need a harmonic potential and an additional magnetic field produced by a uniformly distributed magnetic charges. We find that the atom will be an anyon when cooled down to the negligibly small kinetic energy limit. The relationship between our results and the previous ones is investigated from the electromagnetic duality.

Keywords Cyon and anyon · Electric dipole moments · Canonical angular momentum

1 Introduction

Anyons are quasi-particles with fractional spins. They only exist in the two-dimensional space since the rotation group in two-dimensional space is Abelian which cannot impose any constraint on the eigenvalues of the canonical angular momentum [1, 2]. Anyons play important roles in understanding some two-dimensional phenomena [3, 4]. It was shown that after coupling charged particles to the pure Chern-Simons gauge field on a plane, the eigenvalues of the canonical angular momentum of both non-relativistic and relativistic charged particle are fractional. This means that anyons can be realized by coupling charged particles or charged fields to the pure Chern-Simons gauge field [5–8]. Recently, it was predicted theoretically that anyons may exist in the quantum Hall states [9–11].
In Ref. [12], the author proposed a different approach to realize anyons, i.e., a trapped ion was coupled to two types of magnetic potentials instead of coupling to Chern-Simons gauge field. One is the dynamical magnetic potential whose magnetic field does not vanish in the area where the ion moves, the other is the Aharonov-Bohm type which is produced by an infinitely long-thin solenoid. As expected, the eigenvalues of the canonical angular momentum of the ion take quantized numbers in the unit of \( h \). However, the author found that the rotation property of the reduced model, which was obtained by cooling down the kinetic energy of the ion to the lowest level, behaved interestingly since the eigenvalues of the canonical angular momentum of this reduced model were fractional. The fractional part was proportional to the magnetic flux inside the solenoid. It is interesting to show that both of these two potentials played important roles in producing fractional angular momentum although it seems that only one of them appeared in the final result.

As prototype of anyons, cyons also attracted much attention. A cyon is a compound system which was originally realized by a planar charged particle and a perpendicular infinitely long-thin solenoid [13]. As expected, eigenvalues of the canonical angular momentum take integers. However, it is shown that the integer eigenvalues are divided into two parts: one is localized at the charged particle which generally is fractional and is related to the spin of the cyon, the other is also fractional which is located at the spatial infinity. It was pointed out that the part located at spatial infinity was irrelevant to phenomena in a finite length scale and thus only the part attached to the charged particle was relevant since we are only interested in the behaviors of the charged particle [14]. Therefore, the compound system has a fractional statistic property if two identical compound systems are interchanged. Even though both cyons and anyons were mostly realized by charged particles before, one may wonder whether it is possible to realize cyons and anyons by neutral particles. In this paper, we show that it is indeed possible.

We organize our paper as follows. In Section 2, we first review the original approach of realizing a cyon by a charged planar particle and then propose a scheme to realize it by using an atom, which has a non-vanishing electric dipole moment and a magnetic field produced by an infinitely long magnetic-charged filament. Then, in Section 3, we propose a scheme to realize anyons. Compared with the case of cyon, we need one more magnetic field and a harmonic potential used to trap the atom. The differences between cyon and anyon are also analyzed from the point of the conversation. The relationship between our results and previous ones is investigated from the electromagnetic duality in the last section.

## 2 Realize a Cyon by an Atom

Traditionally, a cyon is realized by a charged planar particle and a perpendicular infinitely long-thin solenoid. The action which describes dynamics of this charged particle is (Latin indices run from 1 to 2. To be specific, \( x_1 = x \), \( x_2 = y \), and the summation convention is used throughout this paper)

\[
S = \int dt \ L = \int dt \left( \frac{1}{2} m x_i^2 - \frac{q}{c} A_i \dot{x}_i \right),
\]

(1)

where \( q \) is the charge that the particle carries, \( c \) is speed of light in vacuum and \( A_i \) is the magnetic potential produced by the infinitely long-thin solenoid in the region where the magnetic field is non-zero.
charged particle moves. In fact, it is the Aharonov-Bohm type magnetic potential. We choose symmetric gauge and write the magnetic potential as

\[ A_i = -\frac{\Phi}{2\pi} \epsilon_{ij} x_j \frac{1}{r^2}, \tag{2} \]

where \( \Phi \) is the magnetic flux inside the solenoid, \( \epsilon_{ij} \) is the Levi-Civita symbol in two-dimensional space and \( r = (x_i x_i)^{1/2} = \sqrt{x^2 + y^2} \) is the distance between the charged particle and the solenoid. The magnetic potential can also be written in the vector form as

\[ \mathbf{A} = \frac{\Phi}{2\pi} \frac{n \times \mathbf{r}}{r^2} \]

where \( n \) is the unit vector normal to the plane on which the charged particle moves. It can be checked that the magnetic field corresponding to the above magnetic potential is \( \mathbf{B} = \nabla \times \mathbf{A} = \epsilon_{ij} \partial_i A_j = \Phi \delta^2(\mathbf{r}) \). It ensures the area where the charged particle moves is magnetic-field-free.

Introduce canonical momenta with respect to \( x_i \), we get

\[ p_i = \frac{\delta S}{\delta \dot{x}_i} = m \dot{x}_i - \frac{q}{c} A_i. \tag{3} \]

Classical Poisson brackets among canonical variables \( (x_i, p_i) \) are

\[ \{x_i, x_j\} = \{p_i, p_j\} = 0, \quad \{x_i, p_j\} = \delta_{ij}, \tag{4} \]

which will be replaced by quantum commutators, i.e., \( \{\ , \ \} \rightarrow \frac{1}{i\hbar} [\ , \ , ] \) when the canonical quantization is performed.

Considering canonical momenta (3), we write the canonical angular momentum \( J_c = \epsilon_{ij} x_i p_j \) as

\[ J_c = J_k - \frac{q}{c} \epsilon_{ij} x_i A_j \tag{5} \]

where \( J_k = m \epsilon_{ij} x_i \dot{x}_j \) is the kinetic angular momentum. On the other hand, the canonical angular momentum (5) can also be written as \( J_c = -i \hbar \frac{\partial}{\partial \varphi} \) with \( \varphi \) being the polar angle.

The single-valuedness of the wavefunctions requires the eigenvalues of the canonical angular momentum should be quantized, i.e., \( J_{cn} = n \hbar, \ n = 0, \pm 1, \pm 2, \ldots \). Substituting the explicit expression \( A_i (2) \) into the canonical angular momentum, we find that

\[ J_c = J_k - \frac{q \Phi}{2\pi c} \tag{6} \]

The canonical angular momentum differs from the kinetic one only by a constant, so they commute each other. As a result, they have common eigenstates. We label eigenvalues of canonical and kinetic angular momenta by \( J_{cn} \) and \( J_{kn} \) respectively. Acting (5) on their common eigenstates, we find that eigenvalues of the canonical angular momentum (6) are split into two parts: one is the fractional kinetic angular momentum \( J_{kn} = n \hbar + \frac{q \Phi}{2\pi c} \), \( n = 0, \pm 1, \pm 2, \ldots \) which is localized at the charged particle. The other part \(-\frac{q \Phi}{2\pi c}\) is also fractional. The authors of Ref. [14] pointed out that this part was located at the spatial infinity and was irrelevant to the studies for phenomena on a finite length. Thus, only the kinetic angular momentum is relevant to the local physical phenomena. The kinetic angular momentum with the quantum number \( n = 0 \) divided by \( \hbar \) is defined as the spin, i.e.,

\[ s = \frac{J_{k0}}{\hbar}. \]

Therefore, the spin of a cyon is fractional, i.e. \( s = \frac{q \Phi}{2\pi\hbar c} \) [15, 16].

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We shall show that a cyon can also be realized by an atom which possesses a non-vanishing electric dipole moment in the background of a specific magnetic field. The magnetic field is produced by a long magnetic-charged filament which is parallel to the electric dipole moment. Both the long filament and the electric dipole moment are perpendicular to the plane where the atom moves. This configuration is identical to that of the He-Mckellar-Wilkens (HMW) effect, which predicted that a neutral particle with a non-vanishing electric dipole moment will acquire a topological phase if it circles around a long magnetic-charged filament with its electric dipole moment parallel to the filament [17, 18].

The magnetic field produced by this long magnetic-charged filament is

\[ B_i^{(1)} = \frac{\lambda_m x_i}{2\pi r^2} \quad \text{or} \quad B^{(1)} = \frac{\lambda_m r}{2\pi r^2} \] (7)

where \( \lambda_m \) is magnetic charges per unit length on the long filament, and \( r \) is the radius vector on the plane.

The Hamiltonian which describes dynamics of this atom is given by [30]

\[ H = \frac{1}{2m} \left( p_i - \frac{d}{c^2} \epsilon_{ij} B_j^{(1)} \right)^2 \] (8)

where \( d \) is the magnitude of the electric dipole moment. Hamiltonian (8) is the non-relativistic limit of a relativistic spin-half particle which possesses a non-vanishing electric dipole moment in the background of a magnetic field [17, 18]. The action corresponding to this Hamiltonian is

\[ S = \int dt \, L = \int dt \left( \frac{1}{2} m \dot{x}_i^2 + \frac{d}{c^2} \epsilon_{ij} \dot{x}_i B_j^{(1)} \right) \] (9)

Introduce canonical momenta with respect to \( x_i \)

\[ p_i = \frac{\delta S}{\delta \dot{x}_i} = m \dot{x}_i + \frac{d}{c^2} \epsilon_{ij} \dot{x}_i B_j^{(1)}. \]

The canonical angular momentum is

\[ J_c = \epsilon_{ij} x_i p_j = m \epsilon_{ij} x_i \dot{x}_j - \frac{d}{c^2} x_i B_i^{(1)}. \] (10)

As usual, the eigenvalues of the canonical angular momentum are quantized, i.e., \( J_c = n \hbar, n = 0, \pm 1, \pm 2, \ldots \).

The canonical angular momentum (10) can also be written as a summation of two parts,

\[ J_c = J_k + J_s \] (11)

where \( J_k \) is the kinetic angular momentum and \( J_s = -\frac{d}{c^2} x_i B_i^{(1)}. \) Using two dimensional Dirac delta function, \( \delta_i (\frac{x_i}{r^2}) = 2\pi \delta^{(2)}(r) \), we can write \( J_s \) as a surface term, i.e.,

\[ J_s = -\frac{d}{2\pi c^2} \int d^2 x \, \partial_i \left( \frac{x_i x_j B_j^{(1)}}{r^2} \right) \] (12)

Therefore, the difference between \( J_c \) and \( J_k \) in the model (9) is only a surface term, which takes values at the spatial infinity. However, contrary to common cases, the contribution of this surface term does not vanish. This indicates that the eigenvalues of canonical angular momentum are split into two fractional part, one is localized at the atom, the other is at the
spatial infinity which is irrelevant to the finite length phenomena. Finally, we get the spin of the atom,

$$s = \frac{\lambda_m d}{2\pi \hbar c^2},$$

(13)

which is proportional to the magnetic charges per unit along the filament. Therefore, a cyon can also be realized by a neutral particle which possesses a non-vanishing electric dipole moment and a specific magnetic field.

### 3 The Realization of Anyons

In this section, we shall propose a scheme to realize an anyon. To do that, besides the atom which possesses a non-vanishing electric dipole moment and the magnetic field (7), we need an extra magnetic field

$$B_i^{(2)} = \frac{\rho_m x_i}{2} \quad \text{or} \quad B_i^{(2)} = \frac{\rho_m r}{2}$$

(14)

and a harmonic potential to trap it. The corresponding action is

$$S = \int dt \left( \frac{1}{2} m \dot{x}_i^2 + \frac{d}{c^2} \epsilon_{ij} \dot{x}_i B_j^T \right) - \frac{1}{2} K x_i^2$$

(15)

where $B_i^T = B_i^{(1)} + B_i^{(2)}$ is the total magnetic field, $K$ is a constant which describes the intensity of the harmonic potential. To study the rotation properties of the model (15), we quantize this model canonically. The canonical momenta are defined as

$$p_i = \frac{\delta S}{\delta \dot{x}_i} = m \dot{x}_i + \frac{d}{c^2} \epsilon_{ij} B_j^T.$$  

(16)

After quantization, canonical variables $x_i, p_i$ satisfy the standard Heisenberg algebra,

$$[x_i, x_j] = [p_i, p_j] = 0, \quad [x_i, p_j] = i\hbar \delta_{ij}. \quad (17)$$

Starting from the action (15), one can get the Hamiltonian via the Legendre transformation as

$$H = \frac{1}{2m} \left( p_i - \frac{d}{c^2} \epsilon_{ij} B_j^T \right)^2 + \frac{K}{2} x_i^2,$$

(18)

which is analogous to a planar charged harmonic oscillator interacting with a magnetic field. Similarities between neutral particles which have electric or magnetic dipole moments in the background of electromagnetic field and charged particles interacting with magnetic field was first reported in Ref. [19], in which the authors found that the Landau levels, which are spectra of a charged planar particle in the background of a uniform perpendicular magnetic field, can be simulated by a neutral particle which possesses a permanent magnetic dipole moment interacting with a specific electric field. Since then, the similarity between Landau levels and eigenvalues of neutral particles interacting with electromagnetic fields in various backgrounds has attracted much attention [20–27].

Using the basic commutators (17), it is easy to prove that the canonical angular momentum

$$J_c = \epsilon_{ij} x_i p_j$$

(19)

is not only conserved $[J, H] = 0$, but also is the generator of rotation transformation, $[J, x_i] = i\hbar \epsilon_{ij} x_j, \quad [J, p_i] = i\hbar \epsilon_{ij} p_j$. Obviously, the eigenvalues of the canonical angular momentum can only be integer.

Now, we investigate rotation properties of the reduced model which is the limit of cooling down the atom to the negligibly small kinetic energy. This kind of limit was considered in
the Chern-Simons quantum mechanics [28]. Mathematically, this limit amounts to set the kinetic energy term in the action (15) to zero. The action of the reduced model can be easily obtained from the full one (15) by neglecting kinetic energy as

\[ S_r = \int dt \left( \frac{d}{c^2} \epsilon_{ij} \dot{x}_i B_j^T - \frac{1}{2} K x_i^2 \right). \]  

(20)

Introduce canonical momenta with respect to \( x_i \)

\[ p_i = \delta S \delta \dot{x}_i = \frac{d}{c^2} \epsilon_{ij} B_j^T. \]  

(21)

This results in primary constraints,

\[ \phi_i = p_i - \frac{d}{c^2} \epsilon_{ij} B_j^T \approx 0. \]  

(22)

In terminology of Dirac, the symbol ‘\( \approx \)’ is weak equivalence which means equivalent only on the constraint hypersurface. The existence of constraints indicates that there are redundant degrees of freedom. The Poisson brackets between primary constraints are

\[ \{ \phi_i, \phi_j \} = -\frac{d}{c^2} \epsilon_{ij}. \]  

(23)

According to the classification of constraints, primary constraints (22) belong to second class and can be used to get rid of redundant degrees of freedom.

The canonical angular momentum of the reduced model (20) is also defined as \( J = \epsilon_{ij} x_i p_j \). However, since there are redundant degrees of freedom, \( x_i \) and \( p_i \) are not independent. The redundant degrees of freedom can be eliminated by substituting constraints (22). In terms of independent variables, we write the canonical angular momentum as

\[ J_r = \epsilon_{ij} x_i p_j = -\frac{d}{2c^2} \left( \rho_m x_i^2 + \frac{\lambda_m}{\pi} \right). \]  

(24)

Because of the primary constraints (22), the standard Poisson brackets (4) among canonical variables \((x_i, p_i)\) should be replaced by Dirac brackets

\[ \{x_i, x_j\}_D = \{x_i, x_j\} - \{x_i, \phi_m\} \{\phi_m, \phi_n\}^{-1} \{\phi_n, x_j\} \]  

(25)

After a direct calculation, we get \( \{x_i, x_j\}_D = \frac{d^2}{dp_m} \epsilon_{ij} \), from which we have

\[ [x_i, x_j] = \frac{i \hbar c^2}{d \rho_m} \epsilon_{ij}. \]  

(26)

It is easy to find that the canonical angular momentum (24) is analogous to a one-dimensional harmonic oscillator. We can get eigenvalues of \( J_r \) directly

\[ J_{rn} = \left( n + \frac{1}{2} \right) \hbar - \frac{\lambda_m d}{2\pi c^2}, \quad n = 0, 1, 2, \cdots. \]  

(27)

Obviously, apart from a ‘normal’ part, the eigenvalues of the canonical angular momentum contain a fractional part which is equivalent to the one we studied in previous section. It is showed that eigenvalues of the canonical angular momentum can take fractional values.

It is also interesting to observe that there is an extraordinary term \( \frac{\hbar}{2} \) in the ‘normal’ part in the eigenvalues of angular momentum (27). It is one of the characteristics of Chern-Simons quantum mechanics [5]. In fact, the model (15) has the same structure as Chern-Simons quantum mechanics if we turn off the magnetic field \( B^{(1)} \). By neglecting the kinetic energy term in the Chern-Simons quantum mechanics, one finds that a extraordinary term \( \frac{\hbar}{2} \) will appear in the eigenvalues of canonical angular momentum.
There is a great difference between results (13) and (27). The canonical angular momenta (19) and (24) are always conserved. However, the kinetic angular momentum is not. It will be more transparent if we consider the case of varied line density of the magnetic charges, i.e., $\lambda_m = \lambda_m(t)$. Both (19) and (24) are still conserved since they are all Noether charges of the planar rotation transformation ($\delta \varphi$ is an infinitely small angle)

$$x_i \rightarrow x'_i = x_i + \delta \varphi \epsilon_{ij} x_j$$

regardless the line density $\lambda_m$ is time-dependent or not. It means that $\dot{J}_c = 0$. As a result, we get $\dot{s} = \frac{d \lambda_m(t)}{2 \pi \hbar \epsilon_0^2} \neq 0$. Therefore, the kinetic angular momentum and the spin of a cyon are not conserved in the case of varied line density of the magnetic charges.

### 4 Conclusions and Further Discussions

In this paper, we propose to realize cyons and anyons by using atoms which have non-vanishing electric dipole moments. The results presented here can also be understood from the electromagnetic duality point of view.

Historically, after the discovery of AB effect [29], Aharonov and Casher predicted that a neutral particle with a non-vanishing magnetic dipole moment would acquire a topological phase if it moves around a uniformly charged infinitely long filament with its direction paralleling to the filament. It is the Aharonov-Casher (AC) effect [30–34]. It is generally accepted that the AB effect is dual to the AC effect in the sense that the solenoid in the AB effect can be thought of a linear array of magnetic dipoles and exchanges the magnetic dipoles with the electric charges [30, 35]. Therefore, for the AC effect one has a line of electric charges and a particle with a magnetic dipole moment moving around this line.

The Hamiltonian describing the interaction between a neutral particle with a non-vanishing magnetic dipole moment and electric fields is

$$H = \frac{1}{2m} \left( p_i + \frac{\mu}{c^2} \epsilon_{ij} E_j \right)^2$$

where $\mu$ is the magnitude of the magnetic dipole moment. It is the non-relativistic limit of a relativistic spin half neutral particle with a non-vanishing magnetic dipole moment interacting with electric fields [30].

In [36], the authors realized the fractional angular momentum from the above Hamiltonian by applying two electric fields $E_i^T = E_i^{(1)} + E_i^{(2)}$ and confining it by a harmonic trap potential. Two electric fields are

$$E_i^{(1)} = \frac{\lambda_e x_i}{2 \pi \epsilon_0 r^2} \text{ or } E^{(1)} = \frac{\lambda_e r}{2 \pi \epsilon_0 r^2},$$

$$E_i^{(2)} = \frac{\rho_e x_i}{2 \epsilon_0} \text{ or } E^{(2)} = \frac{\rho_e r}{2 \epsilon_0}$$

where $\lambda_e$ and $\rho_e$ are the charges per unit length on the long filament and the charge density respectively, $\epsilon_0$ is the dielectric constant. After taking the limit of cooling down the kinetic energy of the atom to its lowest level, one gets the canonical angular momentum as [36]

$$J = \frac{\mu}{2c^2 \epsilon_0} \left( \rho_e x_i^2 + \frac{\lambda_e}{\pi} \right)$$
where $x_i$ are coordinates of the atom on the plane which is perpendicular to the magnetic dipole moment. They satisfy the commutation relations

\[ [x_i, x_j] = -i\hbar \epsilon_0 e^2 \mu \rho_e. \] (33)

Obviously, the eigenvalues of the canonical angular momentum can be read directly from (32) and commutation relation (33) as [36]

\[ J_n = \left(n + \frac{1}{2}\right)\hbar + \frac{\mu \lambda_e}{2\pi c^2 \epsilon_0}. \] (34)

However, it is argued that it is the HMW effect [17] rather than the AB effect that is dual to the AC effect. Because there is a natural correspondence between the AC and HMW effects: the magnetic dipole moment in the AC effect corresponds to the electric dipole moment in the HMW effect, the electric charges in the AC effect corresponds to the magnetic charges in the HMW effect.

The duality between the AC effect and the HMW effect can be expressed exactly as

\[ \{ E \}_{d} \rightarrow \frac{1}{\sqrt{\epsilon_0 \mu_0}} \{ B \}_\mu, \quad \{ \lambda_e \}_{\rho_e} \rightarrow \sqrt{\frac{\epsilon_0}{\mu_0}} \{ \lambda_m \}_{\rho_m} \] (35)

and

\[ \{ B \}_\mu \rightarrow -\sqrt{\epsilon_0 \mu_0} \{ E \}_{d}, \quad \{ \lambda_m \}_{\rho_m} \rightarrow -\sqrt{\frac{\mu_0}{\epsilon_0}} \{ \lambda_e \}_{\rho_e} \] (36)

where the minus arises from the asymmetric nature of the electromagnetic duality.

It is easy to check that the AC effect is dual to the HMW effect through (35) and the HMW effect is dual to the AC effect through (36). Furthermore, the Maxwell equations are also invariant under the dualities (35), (36) and

\[ J_e \rightarrow \sqrt{\frac{\epsilon_0}{\mu_0}} J_m, \quad J_m \rightarrow -\sqrt{\frac{\mu_0}{\epsilon_0}} J_e \] (37)

with $J_e$ ($J_m$) being the electric (magnetic) current density provided the magnetic charges are presented [37]. Interestingly, the authors of [38] predict that there is a new topological effect which is dual to the AB effect according to the duality relations (35) and (36).

These duality relations are also applied in [39], in which the authors find that energy spectra of an electric dipole moment in the background of the magnetic field (14) are dual to spectra of a magnetic dipole moment interacting with the electric field (31) through (35, 36) [19].

The dualities (35) and (36) not only hold in the Hamiltonians (18) and (29), but also in the canonical angular momentum (24) and (32) as well as the commutation relations between $x_i$ in the reduced models (26) and (33). Therefore, the studies of the present paper can be regarded as the electromagnetic duality of the [36] from the point of view (35) and (36). It may be curious that eigenvalues of the angular momentum (27) take negative values. The minus sign in eigenvalues of the angular momentum (27) originates from (24), which is the direct result of the electromagnetic dualities (35) and (36). It can also be understood classically since the atom can only stay in the stable equilibrium provided the direction of the classical angular momentum is inverse to the electric dipole moment.

Finally, it is very interesting to wonder whether we can realize cyons or anyons by using neutral particles which possess non-vanishing electric or magnetic quadruple moments. Such a study will be considered in near future.
Acknowledgements  The authors appreciate referees’ many invaluable comments which are greatly helpful for us to improve our manuscript. This work is supported by NSFC with Grant No. 11465006 and partially supported by 20200981-SIP-IPN and the CONACyT under grant No. 288856-CB-2016.

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