On Non-Chiral Extension of Kerr/CFT

Noriaki OGAWA
(IPMU)

[arXiv:1010.4291] & [arXiv:1102:3423]

Collaboration with:
T. Azeyanagi (RIKEN) & S. Terashima (YITP)

String Seminar @Nagoya Univ.
May 30, 2011
Today's theme: Extension of Kerr/CFT

Kerr/CFT = AdS/CFT in extremal BH, without excitations above it.

Where is non-extremal excitations ??

Toward holography in realistic BH's!
Strategy: Zero-entropy extremal BH and AdS$_3$/CFT$_2$

Extremal BH

Near Horizon Geometry

Local AdS$_3$ (BTZ) geometry

AdS$_3$/CFT$_2$ (Future extension...)

$C_R = C_L$
Thank you for your patience!
Appendix
AdS/CFT in Extremal Black Holes (Kerr/CFT)

[Guica-Hartman-Song-Strominger, 0809.4266]

Gaze into the near-horizon

Extremal Kerr Black Hole

\[ \text{AdS}_2 \times S^1 \]

\((t, r) \times (\phi)\)

Asymptotic symmetry

Virasoro alg. x 1

\[ \text{AdS/CFT} \quad (\text{Symmetry Correspondence}) \]

2d chiral CFT!
Boundary condition and asymptotic symmetry

Near Hrizon Extremal Kerr(NHEK) metric

\[ ds^2 = 2G_4 J \Omega(\theta)^2 \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + \Lambda(\theta)^2 (d\phi - r dt)^2 + d\theta^2 \right] \]

\[ \text{\(\theta\)-dep \(S^1\)-fibrated AdS}_2 \text{ isometry} = \text{SL}(2, \mathbb{R}) \times \text{U}(1)_\phi \]

Bekenstein-Hawking entropy

\[ S_{BH} = 2\pi J \]

Boundary condition

\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \sim \begin{pmatrix} r^2 & r^{-2} & r^{-1} & 1 \\ r^{-3} & r^{-2} & r^{-1} & 1 \\ r^{-1} & r^{-1} & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} t \\ r \\ \theta \\ \phi \end{pmatrix} \]

\( \leftrightarrow \) choice of Hilbert space

Asymptotic symmetry generator

\[ \xi_n^L = -e^{-in\phi}(inr \partial_r + \partial_\phi) \quad \xi_0^R = -\partial_t \]

\[ [\xi_m^L, \xi_n^L]_{Lie} = -i(m - n)\xi_{m+n}^L \]

Virasoro algebra

\( = \) conformal sym of 2d (chiral) CFT!
Asymptotic charges and AdS/CFT

Conserved charges in the dual theory (ex. Virasoro charges)
= asymptotic charges in the gravity side

\[ Q_\xi = \int_{\partial \Sigma} k_\xi[h; \bar{g}] \]

\[ k_\xi[h; \bar{g}] = -\frac{\sqrt{-g}}{16\pi} \left( \bar{D}^{[\nu} (h\xi^{\mu]}) + \bar{D}_\sigma (h^{[\mu\sigma} \xi^{\nu]) + \bar{D}^{[\mu} (h^{\nu\sigma} \xi_{\sigma}) + \frac{3}{2} h \bar{D}^{[\mu} \xi^{\nu]} + \frac{3}{2} h^{[\nu} \bar{D}^{\mu] \xi_{\sigma} + \frac{3}{2} h^{[\nu\sigma} \bar{D}_{\sigma} \xi^{\mu]} \right) (d^{D-2}x)_{\mu\nu} \]

(In the case of Einstein gravity, [Barnich-Brandt, hep-th/0111246])

For Virasoro charges, \( \tilde{L}_n \simeq Q_{\xi} L_n \).

Commutators ↔ Poisson brackets

\[ \{ Q_\xi, Q_\eta \}_{PB} = Q_{[\xi,\eta]}_{Lie} + \int_{\partial \Sigma} k_\eta [\mathcal{L} \xi \bar{g}, \bar{g}] \]

Central extension

\[ c = 12j \]
Non-chiral (→ Non-extremal) Kerr/CFT ?

There have been some suggestions that Kerr/CFT is extended to non-extremal BH, where the dual theory is non-chiral CFT$^2$.

[Bredberg-Hartman-Song-Strominger (2009)], [Castro-Larsen (2009)], [Castro-Maloney-Strominger (2010)], ...

Can we derive it from asymptotic symmetry ?
Attempts to non-chiral Kerr/CFT

Several attempts have been done, to enhance U(1)\_t to right-handed Virasoro...

A slightly modified b.c. for NHEK yields the ASG

\[ \xi_n^L = -e^{-in\phi} (inr \partial_r + \partial_\phi) \quad \xi_n^R = -e^{-int/\beta} (inr \partial_r + \beta \partial_t) \]

Two Virasoro algebras !

[Matsuo-Tsukioka-Yoo, 0907.4272]

However....

- The right-handed Virasoro charges are all divergent.
- The right-handed central charge = 0.
Non-chiral Kerr/CFT in extremal BTZ

Near-horizon extremal BTZ

\[ ds^2 = \frac{L^2}{4} \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + (d\phi - r dt)^2 \right] \]

\( (t, \phi) \sim (t, \phi + 2\pi \ell) \)

Boundary cond.

\[ h_{\mu\nu} \sim \begin{pmatrix} 1 & r\ell^{-1} & 1 \\ r^{-3} & r^{-1} & 1 \\ 1 & 1 & 1 \end{pmatrix} \]

Asymptotic sym.

\[ \xi^L_n = -e^{-in\phi/\ell} \left( inr \partial_r + \ell \partial_\phi \right) \]

\[ \xi^R_n = -e^{-in\ell/\beta} \left( inr \partial_r + \beta \partial_t \right) \]

Two Virasoros as asymptotic symmetry.

All asymptotic charges are finite.

Central charges

\[ c_L = \frac{3L}{2G_3}, \quad c_R = 0 \]

Vanishing central charge again...
Regularization of Time-slice

Look at the geometry again....

\[ ds^2 = \frac{L^2}{4} \left[ \frac{dr^2}{r^2} - 2r dt d\phi + d\phi^2 \right] \]

Time-slice is light-like at the boundary! (null-orbifold)

It can make the asymptotic charge ill-defined...

We make the time-slice space-like, by a regularization

\[ t' \equiv t + \alpha \phi, \quad \phi' \equiv \phi, \quad (t', \phi') \sim (t', \phi' + 2\pi \ell) \]

(we changed the orbifold.)

(We take \( \alpha \to 0 \) limit at last.)
Regularization of Time-slice (2)

\[ ds^2 = \frac{L^2}{4} \left[ \frac{dr^2}{r^2} - 2rdt'd\phi' + (1+2\alpha r)d\phi'^2 \right] \]

\[ t = t' - \alpha \phi \quad \rightarrow \quad t \text{ becomes periodic automatically.} \]

Explicit calculation yields

\[ c_R = c_L = \frac{3L}{2G_3} \]

Equal & finite values !  

(also they agrees with Brown-Henneaux)
Origin of the regularization

\[ ds^2 = \frac{L^2}{4} \left[ \frac{dr^2}{r^2} - 2r dt'd\phi' + (1 + 2\alpha r) d\phi'^2 \right] \]

BTZ metric:

\[ ds^2 = L^2 \left[ -\frac{\rho^4}{\rho^2 + r_+^2} d\tau^2 + \frac{d\rho^2}{\rho^2} + (\rho^2 + r_+^2) \left( d\psi - \frac{r_+^2}{\rho^2 + r_+^2} d\tau \right)^2 \right] \]

Near horizon transformation:

\[ \rho^2 = \frac{\lambda r}{2}, \quad \tau = -\frac{r_+}{\lambda} t, \quad \psi = \frac{\phi}{2r_+} - \frac{r_+}{\lambda} t \]

Resulting metric before the limit:

\[ ds^2 = \frac{L^2}{4} \left[ \frac{dr^2}{r^2} - 2r dt'd\phi' + \left(1 + \frac{\lambda}{2r_+^2} r\right) d\phi'^2 \right] \]
Zero entropy limit for extremal BH

Near horizon geometry (4D):

\[ ds^2 = A(\theta)^2 \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + B(\theta)^2 (d\phi - k r dt)^2 \right] + F(\theta)^2 d\theta^2 \]

\( \theta \)-dep \( S^1 \)-fibrated \( AdS_2 \)

isometry = \( SL(2, \mathbb{R}) \times U(1)_\phi \)

Bekenstein-Hawking entropy

\[ S_{BH} = \frac{\pi}{2G_4} \int d\theta A(\theta) B(\theta) F(\theta) \]

We take zero-entropy limit for this geometry.

In this limit,
if \( AdS_2 \) unbroken & existence of dual theory
→ no singular behaviors expected.
→ The geometry is expected to be regular.
BTZ-structure Emergence

\[ ds^2 = A(\theta)^2 \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + B(\theta)^2 (d\phi - k r dt)^2 \right] + F(\theta)^2 d\theta^2 \]

Scalings of the parameters:

\[ B(\theta) = \epsilon B'(\theta), \quad k = \frac{k'}{\epsilon}, \quad B(\theta)k = B'(\theta)k' = 1 + \epsilon^2 b(\theta) \]

At the same time, we rescale the angular coordinate:

\[ \phi' = \frac{\phi}{k} \quad (\phi' \sim \phi' + \frac{2\pi}{k}) \]

In the \( \epsilon \to 0 \) limit,

\[ ds^2 = A(\theta)^2 \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + (d\phi' - r dt)^2 \right] + F(\theta)^2 d\theta^2 \]

Structure of (massless) extremal BTZ emarges.

(Many concrete examples are known: [Guica-Strominger], etc...)
Non-chiral Kerr/CFT for zero-entropy extremal BH

\[ ds^2 = A(\theta)^2 \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + (d\phi - r dt)^2 \right] + F(\theta)^2 d\theta^2 \]

(t, \phi) \sim (t, \phi + 2\pi \delta)

\[ ds^2 = A(\theta)^2 \left[ \frac{dr^2}{r^2} - 2rdt'd\phi' + (1+2\alpha r)d\phi'^2 \right] + F(\theta)^2 d\theta^2 \]

Boundary condition

\[ h_{\mu\nu} \sim \begin{pmatrix} 1 & r^{-1} & 1 & 1 \\ r^{-3} & r^{-2} & r^{-1} & 1 \\ r^{-1} & 1 & 1 & 1 \end{pmatrix} \]

Asymptotic symmetry

\[ \xi_n^L = -e^{-in\phi/\delta}(inr\partial_r + \delta \partial_\phi) \]

\[ \xi_n^R = -e^{-int/\alpha \delta}(inr\partial_r + \alpha \delta \partial_t) \]

\[ c_R = c_L = \frac{3}{G_4} \int d\theta \ A(\theta) F(\theta) \]
Origin of our regularization (2)

When we take the limits of zero-entropy and near-horizon at the same time,

\[
\begin{align*}
    ds^2 &= A(\theta)^2 \left[ \frac{dr^2}{r^2} - 2r dt' d\phi' + (1+Cr) d\phi'^2 \right] + F(\theta)^2 d\theta^2 \\
    C &\equiv \lim \frac{\lambda}{\epsilon} \\
    \lambda &\text{ : scaling parameter of near-horizon} \\
    \epsilon &\text{ : scaling parameter of zero-entropy}
\end{align*}
\]

We can identify \( \alpha = C/2 \)

Our regularization is automatically introduced.
Summary

• Kerr/CFT is extended to a non-chiral form, for extremal BTZ and zero-entropy BH.

• Naive prescription leads to $c_R = 0$
  $\rightarrow$ A regularization yields $c_R = c_L$.

• The regularization appears automatically and naturally, as a remnant of the near horizon parameter.

• Generalization to nonzero-entropy BH would be challenging....
Copyrights

Pictures of cats (🐱🐱🐱)

(©ねこのおしごと  http://members.jcom.home.ne.jp/0412269401/)