Submillimeter Radiation as the Thermal Component of the Neupert Effect

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Abstract The Neupert effect is the empirical observation that the temporal evolution of non-thermal emission (e.g. hard X-rays) is frequently proportional to the temporal derivative of the thermal emission flux (soft X-rays), or vice versa, that time-integrated non-thermal flux is proportional to thermal flux. We analyzed the GOES M2.2 event SOL2011-02-14T17:25, and we found that the 212 GHz emission plays quite well the role of the thermal component of the Neupert effect. We show that the maximum of the hard X-ray flux for energies above 50 keV is coincident in time with the temporal derivative of the 212 GHz flux, within the uncertainties. The microwave flux density at 15.4 GHz, produced by optically thin gyrosynchrotron mechanism, and hard-X rays above 25 keV mark the typical impulsive phase, and they have similar temporal evolution. On the other hand, the 212 GHz emission is delayed by about 25 seconds with respect to the microwave and hard X-ray peak. We argue that this delay cannot be explained by magnetic trapping of non-thermal electrons. With all of the observational evidence, we suggest that the 212 GHz emission is produced by thermal bremsstrahlung, initially in the chromosphere, and shifting to optically thin emission from the hot coronal loops at the end of the gradual phase.

Keywords Flares, dynamics · Flares, X-rays · Flares submillimeter radiation · Chromospheric evaporation

1. Introduction

The different temporal evolution of hard X-rays (HXR) and soft X-rays (SXR) during impulsive bursts is known since the Orbiting Solar Observatory-1 (OSO-1) observations...
(White, 1964). Moreover, OSO-1 data also showed that microwaves (MW) and HXR are temporally coincident during the impulsive bursts (Frost, 1964), lending support to their close origin. It was Neupert (1968) who noted for the first time that the SXR flux is better correlated with the time-integrated flux density (fluence) at the MW frequency $\nu = 2.695$ GHz, i.e.

$$ F_{\text{SXR}}(t) \propto \int_{t_0}^{t} F_{\text{MW}}(t') \, dt', $$

(1)

where $F_{\text{SXR}}$ is the SXR flux and $F_{\text{MW}}$ is the MW flux density. This relation holds until $F_{\text{SXR}}$ reaches its maximum flux, which should be coincident with the end of the MW emission. A similar relation is observed between HXR and SXR (Hudson, 1991). This observational fact was interpreted as the atmospheric response to the heating produced by the energetic particles when they precipitate into the lower and denser layers: energetic electrons spiraling within the magnetic fields produce synchrotron radiation observed at MW, while the HXR is non-thermal bremsstrahlung produced by Coulomb collisions, transferring energy to the plasma that expands and emits thermal bremsstrahlung observed at SXR: a phenomenon also known as chromospheric evaporation (Neupert, 1968; Hudson and Ohki, 1972; Antonucci, Gabriel, and Dennis, 1984). Conversely, Equation 1 can be written in terms of the SXR temporal derivative (Hudson, 1991):

$$ \frac{dF_{\text{SXR}}(t)}{dt} \propto F_{\text{MW}}(t), \quad \text{or} \quad \frac{dF_{\text{SXR}}(t)}{dt} \propto F_{\text{HXR}}(t). $$

(2)

Equations 1 and 2 are the mathematical representation of the Neupert effect. Statistical analysis of SXR and HXR data show that in around half of the bursts the effect is present, which means that for around 50% of the cases, there is evidence for a more complex heating mechanism than only electron-beam-driven (Veronig et al., 2002a). Furthermore, McAteer and Bloomfield (2013) analyzed the energy-band pairs for which the Neupert effect is better observed, concluding that the best agreement is between the bands 12–25 keV (SXR) and 100–300 keV (HXR).

All of the work mentioned is concentrated on the SXR wavelength domain to observe the thermal emission. Trottet et al. (2000) observed that H$\alpha$ has a slow (accumulative) and fast (direct) relationship with HXR: either chromospheric evaporation or continuous coronal heat flux may be responsible for the slow response. In another statistical work, Veronig et al. (2002b) analyzed the timing between SXR, HXR, and H$\alpha$, showing that in 90% of all cases, SXR starts before HXR while H$\alpha$ starts simultaneously with HXR. Moreover, SXR and H$\alpha$ maximum fluxes are temporally coincident with the end of HXR.

Solar-flare observations at submillimeter frequencies (here considered to be frequencies > 100 GHz) are relatively new. For this reason the physical origin of the emission is still a subject of debate. Thermal bremsstrahlung and synchrotron radiation, or a combination of both, are the main candidate sources of radiation at high frequencies (see, e.g., Bastian, Benz, and Gary, 1998; Pick and Vilmer, 2008). Other mechanisms have been suggested (Kaufmann and Raulin, 2006; Fleishman and Kontar, 2010; Krucker et al., 2013), but the lack of a better spectral coverage toward the THz range does not allow us to draw definitive conclusions. In the work of Trottet et al. (2002, 2011), Lüthi, Magun, and Miller (2004), and Lüthi, Lüdi, and Magun (2004) the submillimeter radiation during the time-extended or gradual phase of the solar flares was compatible with thermal bremsstrahlung, while during the impulsive phase it was considered to be of synchrotron origin. Moreover Tsap et al. (2016) have shown a particular flare whose emission between 93 and 140 GHz increases and can be attributed solely to thermal bremsstrahlung.
We present in this work a peculiar event where submillimeter emission can be interpreted as the thermal component of the Neupert effect. We show that the 212 GHz temporal profile is very different from the temporal profiles at HXR and MW, that it does not show the typical impulsive phase (Krucker et al., 2013), and that its temporal evolution is in agreement with both Equations 1 and 2. In contrast to other events (Lüthi, Magun, and Miller, 2004; Lüthi, Lüdi, and Magun, 2004; Trottet et al., 2011, 2015) the submillimeter time-extended temporal evolution is not well correlated with SXR.

2. Observations and Data Analysis

The event SOL2011-02-14T17:25 (hereafter SOL2011-02-14 for simplicity) is associated with a GOES M2.2-class SXR flare in the Active Region 11158. On 14 February 2011, at 0 UT, the region was located at Heliographic Latitude S20 and Longitude W04, it was 10° wide in longitude, and within it occurred several C-class events before the M2.2, subject of the present analysis. The Solar Submillimeter Telescope (SST: Kaufmann et al., 2008) tracked AR11158 since the day before the event. On 14 February, the atmospheric conditions were not favorable for millimeter observations: at 212 GHz the zenith optical depth was 0.8, and we can only estimate the lower limit for the optical depth at 405 GHz to be \( \approx 4.5 \). Therefore, at the antenna elevations during the event, the signal was attenuated 57% and \( > 99 \% \) at 212 and 405 GHz, respectively. In addition to this correction, the antenna temperature of Beam 1 was subtracted from the antenna temperatures of the multi-beam array (Beams 2, 3, and 4) which are 7' away (see Trottet et al., 2011, for a detailed explanation). While in this way we remove most of the atmospheric fluctuations, we note that, since Beam 1 is off the active region, a variation in opacity is less amplified by the cool background than the same variation observed by Beams 2, 3, and 4 which are over the hot active region. However, at 212 GHz active regions are at most 20% hotter than quiet Sun (Silva et al., 2005), and therefore this is a limited effect within the quoted uncertainty.

The SST observed the flare at 212 GHz with the three beams that compose its multi-beam system providing the instantaneous emitting centroid and its flux density every 0.04 seconds using the technique described in Section 2.3. The flux density reaches its maximum between 17:25:45 and 17:25:52 UT (see Figure 1), with mean \( \langle F_{x_{212}} \rangle_{\text{max}} = 220 \pm 30 \text{ SFU} \). The peak is relatively smooth, temporal fluctuations around the maximum have a standard deviation of about 4% of \( \langle F_{x_{212}} \rangle_{\text{max}} \).

After the peak, the flux density decreases slowly; the event overall duration is \( \approx \) nine minutes. At 405 GHz no significant signal excess was detected, a logical consequence of the high optical depth. A rough estimation of the detectable source flux threshold was obtained from the measured antenna-temperature fluctuations, \( \delta T_{405} \approx 15 \text{ K} \), corrected for optical depth and for the beam offset (which we can infer from the multi-beam solution) and converted to flux. The result yields \( F_{x_{405}} \gtrsim 10^4 \text{ SFU} \). This number should be considered our uncertainty in the 405 GHz flux.

The flare has been detected in HXR by the Gamma-Ray Burst Monitor (GBM) onboard Fermi (Meegan et al., 2009). GBM is composed of 12 sodium-iodide (NaI) and two bismuth-germanium-oxide (BGO) detectors. In our analysis we used the NaI 128 energy channels in the range from 4 keV to 2000 keV and 1.024 second temporal resolution. Unfortunately there are no Ramaty High Energy Solar Spectroscopic Imager (RHESSI) data during the impulsive phase of the event. We complement our analysis with MW from the United States Air Force (USAF) Radio Solar Telescope Network (RSTN: Guidice et al., 1981) at 1.415, 2.695, 4.995, 8.8, and 15.4 GHz with one-second temporal resolution, and GOES 1 – 8 Å (1.5 – 12 keV) with two-second temporal resolution.
Figure 1 Temporal evolution at selected frequencies/energies. From top to bottom: GOES 1–8 Å (1.5–12 keV); Fermi/GBM NaI 12–25 keV, spectral index $\alpha$ obtained between 15.4 and 212 GHz (see Section 2.2), SST 212 GHz, RSTN 15.4 GHz, and Fermi/GBM NaI 100–300 keV HXR.

2.1. Temporal Evolution

Figure 1 presents a selection of the different frequency/energy temporal profiles. By using GOES and Fermi low-energy channels, we cover the whole SXR domain. At the lowest energies, GOES 1.5–12 keV, the emission has a smooth evolution starting at 17:23 UT, with a simple structure peaking at 17:26 UT, and returning to pre-flare level not before 18:40 UT. The Fermi 12–25 keV band starts and ends with GOES 1–8 Å, but it shows two peaks: at 17:25:20 UT and at around 17:26:40 UT. On the other hand the impulsive phase at MW and HXR have a common shape, with delays of less than one second between the strongest structures. Emission starts at around 17:24 UT and ends by 17:27 UT.
A closer look at the temporal evolution at different frequencies and energy bands can be seen in Figure 2, where the fluxes are normalized to facilitate the comparison. In the left panel we present the normalized flux at 212 GHz (shaded dark gray), 15.4 GHz (shaded light gray) and 100–300 keV HXR (black curve). We observe that during the impulsive phase, defined by HXR, there is an excellent match between features observed at HXR and 15.4 GHz, with no detectable delay. This implies that 15.4 GHz has an optical depth $\tau \lesssim 1$, i.e. is near the peak of the spectrum. In contrast, the submillimeter emission is, as noted before, smoother, almost featureless, and its peak is delayed by $> 20$ seconds with respect to HXR. The cross correlation between HXR and 15.4 GHz (HXR and 212 GHz), graphically exposed in the inset with a continuous line (dashed line) are a quantitative way to remark the coincidence (lack of coincidence) of the temporal evolution at different frequencies. The cross correlation between HXR and 15.4 GHz is maximum for a lag $= 0$ seconds, while between HXR and 212 GHz is maximum for a lag $= 25$ seconds. We have also compared the temporal evolution at the three HXR energy bands (right panel in Figure 2). The shaded light-gray curve represents the lowest energy (25–50 keV), the dark-gray curve the middle energy (50–100 keV), and the black curve the highest detected energy (100–300 keV). All of the peaks match well; therefore we do not observe any delay within the data’s temporal resolution.

### 2.2. Spectra

The Fermi/GBM HXR fitted photon spectrum during the peak time four-second interval (17:25:24–17:25:28 UT) is shown in Figure 3 (left panel). The spectrum is best fitted by a thermal component (dot–dashed curve) with temperature $T = 13$ MK and emission measure $EM = 56 \times 10^{69}$ cm$^{-3}$, plus a thick-target component from a power–law distribution of electrons, with an electron rate of $F = 2.6 \times 10^{35}$ s$^{-1}$, spectral index $\delta = 4.6$; and low-energy cutoff $E_c = 16$ keV (continuous gray curve). From the figure it can be seen that for
energies $>20$ keV the emission can be considered as non-thermal bremsstrahlung, and that there is no detected emission above $\approx 200$ keV.

The radio spectra at different temporal intervals of one-second duration are shown in Figure 3 (right panel). No firm conclusion can be drawn about the submillimeter emission origin during the maximum of the microwave emission, intervals b to d, because of the gap between 15.4 and 212 GHz. For the same reason we cannot determine the peak frequency, but, as noted before, the emission at 15.4 GHz has an optical depth $\tau_{15.4} \lesssim 1$. After the maximum, intervals e and f, flux density at 8.8, 15.4 and 212 GHz are very similar, which can be considered an indication of pure thermal emission. Moreover, while the 15.4 GHz flux varies by a factor of around 15 between interval c through f, the submillimeter emission just halves its flux during the same period. As an illustration of this behavior we show in Figure 1 the spectral index between 212 and 15.4 GHz $\alpha = -\log(F_{212}/F_{15.4})/\log(212/15.4)$ as a function of time. We remark that $|\alpha| \leq 0.5$ which is a rather hard index. Indeed, applying the Dulk (1985) semi-empirical formulation for gyrosynchrotron emission, we get an electron index $\delta \leq 1.9$. Moreover, we note that during interval a, before the peak, $\alpha < 0.5$ (and $\delta < 2$), indicating again the thermal origin of the 212 GHz emission.

### 2.3. Submillimeter Position

In order to obtain position and flux for our submillimeter observations, we used the iterative multi-beam technique first introduced by Herrmann et al. (1992) for observations made with the 13.7-m antenna of the Itapetinga Radio Observatory and later applied to SST observations by Cristiani et al. (2007). The original method considered point-like sources, and
therefore it needed at least three independent observations. Lüthi, Lüdi, and Magun (2004) expanded this method by introducing extended sources, for which they obtained position, flux, and an effective area using four independent observations in order to get a unique solution. In our case we used a matrix representation of the beams obtained after the deconvolution of solar maps observed in 2006 following the method developed by Costa et al. (2002).

We also considered Gaussian extended sources defined by a four-tuple \((F_s, x, y, \sigma_s)\) with \(F_s\) the maximum of the source flux density, \(x, y\) its position, and \(\sigma_s\) its Gaussian standard deviation.

The iterative method compares a combination of measured and model-calculated relative fluxes of the three different receivers:

\[
Q(t_i, x, y, \sigma_s) = \sum_k \left| \frac{F_{\text{meas}}^k(t_i)}{F_{\text{meas}}^{p\neq k}(t_i)} - \frac{F_{\text{cal}}^k(t_i, x, y, \sigma_s)}{F_{\text{cal}}^{p\neq k}(t_i, x, y, \sigma_s)} \right|
\]

\[
F_{\text{meas}}^k(t_i) = 2 \frac{k_B T_k(t_i)}{A_e},
\]

with \(T_k(t_i)\) the antenna excess temperature of beam \(k\) at instant \(t_i\); \(k_B, A_e\) the Boltzmann constant and the antenna effective area, respectively, and \(F_{\text{cal}}^k(t_i, x, y, \sigma_s)\) the corresponding calculated flux obtained after convolving the source with the beam. We then looked for the location \((x_o, y_o)\) that overall minimizes \(Q\) matrices throughout the whole event for a fixed source size. Namely

\[
Q(x, y, \sigma_s) = \sum_i Q(t_i, x, y, \sigma_s), \quad \frac{\partial^2 Q(x, y, \sigma_s)}{\partial x \partial y} \bigg|_{\sigma_s} = 0.
\]

We prepared low-resolution profiles taking values every 15 seconds between 17:24:00 UT and 17:30:00 UT, as is shown in Figure 4. A grid of \(350'' \times 350''\) with \(10''\) spacing between points is used to calculate \(Q(t_i, x, y, \sigma_s)\). The resulting \(Q(x_o, y_o, \sigma_s)\) values, normalized to 100, as a function of the source angular radius \(\varphi = \sigma_s \sqrt{\ln(4)}\) are shown in Figure 4 (circles).

**Figure 4** Left: Calibrated low-resolution flux-density temporal profiles used to determine the source flare position. The dataset was built picking values every 15 seconds between 17:24:00 and 17:30:00 UT for every receiver of the 212 GHz multi-beam array. Normalized light curves from GOES 1 – 8 Å and RSTN 15.4 GHz are shown for comparison. Right: \(Q(x_o, y_o, \sigma_s)\) normalized to 100 as a function of \(\varphi = \sigma_s \sqrt{\ln(4)}\) (filled circles), the dashed curve shows the global trend. The diamonds represent the distance between the source positions determined with the multi-beam technique \((x_o, y_o)\) as a function of \(\varphi\).
Figure 5  UV image at 1700 Å taken by SDO/AIA at 17:28:31 UT, to avoid the saturated pixels. The red cross is the solution of the iterative multi-beam technique, the dashed-red circle is its absolute uncertainty. The small-green cross shows the center of the brightest and biggest UV source. The dashed-black curves are the three 212 GHz beams used in the multi-beam solution represented at 50% and 75% levels; black crosses are the beam centers.

We note that $Q$ decreases when the source increases in size up to $25''$, then stabilizes, i.e. the method becomes insensitive to changes in size. The same figure also shows the distance from $(x_0, y_0)$ to the center of the biggest and brightest UV source (diamonds). We observe a similar behavior, when the source achieves a size of $\approx 30''$, the distance stabilizes around $10''$. We conclude that $\varphi \geq 25''$, which corresponds to a source area $A_{MB} \geq 3.8 \times 10^{17}$ cm$^2$. We remark that, since we do not have four independent observations, we cannot provide a unique solution such as those obtained by Lüthi, Lüdi, and Magun (2004), or even by Giménez de Castro et al. (1999) who worked with analytical expressions and nominal beams.

In Figure 5 the submillimeter centroid position for 17:24:00 and 17:30:00 UT period is shown over a 1700 Å UV image as a red cross; the dashed-red circle represents its absolute uncertainty. The small-green cross labeled UV marks the centroid position of the brightest and biggest UV source, and it is the reference for the distance to the submillimeter solutions. The excess flux of the source was obtained after its position and size were determined.

3. Origin of the Submillimeter Emission

The most common hypothesis for the millimeter and submillimeter emission during the impulsive phase of solar flares is gyrosynchrotron from non-thermal electrons. The same electrons should produce HXR, implying similar HXR and radio light curves; when delays between them appear, some trapping is considered responsible. However, in this particular case we do not find evidence of gyrosynchrotron and trapping in the light curves. Our arguments supporting this conclusion are:

i) The temporal evolution at 15.4 GHz greatly differs from that at 212 GHz, which can be hardly attributed to a $\approx 25$ second electron trapping in the magnetic loop. Had the emission at both frequencies been produced by gyrosynchrotron, they would have presented similar curves, with or without trapping effects.
ii) There is no indication of > one-second trapping when comparing the HXR curves at several energy bands and 15.4 GHz as is illustrated in Figure 2.

iii) The similarities of the high-energy HXR and 15.4 GHz curves, i.e. presence of a number of peaks and lack of time delays, indicate that the electrons producing the microwave emission are not affected by magnetic trapping. It also suggests that the 15.4 GHz emission is mostly optically thin, and therefore, it is, at least, very close to the peak of the gyrosynchrotron spectrum. As noted before, the 212 GHz curve is smooth, and it does not follow the 15.4 GHz.

iv) A 25-second trapping is a rather extreme condition. A long trapping time has a strong impact in other source parameters, as the magnetic-field intensity. It has been shown by Giménez de Castro et al. (2009) that the greater the trapping time, for a given HXR flux, the smaller the magnetic field. A 25 second trapping would imply a magnetic field of a few Gauss, which would be too low to produce any significant emission at 212 GHz.

v) If the emission had been produced in a homogeneous source, the spectral index \( \alpha \) between 15.4 and 212 GHz (Figure 1) would give us information on the electron distribution. However, during the event is always \( \alpha < 0.5 \), which would suggest a very hard, and unlikely, electron distribution with \( \delta < 1.9 \) (Dulk, 1985). Therefore, such a hard electron distribution is inconsistent with the value \( \delta = 4.6 \) derived from the HXR observed spectra. Moreover, a circumstantial indication supporting our conclusion is the absence of observational evidence for \( > 1 \) MeV electrons in the HXR data: the highest HXR energy detected significantly is \( \approx 200 \) keV. It is well known that relativistic electrons are the main source of the 212 GHz synchrotron (e.g. Ramaty et al., 1994; Trottet et al., 2015).

### 3.1. The Impulsive Phase

Taking into account all of the arguments presented above, we conclude that it is very unlikely that 212 GHz emission is due to gyrosynchrotron. This leaves thermal bremsstrahlung as the most likely mechanism to produce the observed 212 GHz radiation. The emission at these frequencies, during the gradual phase of flares, has been interpreted as due to thermal bremsstrahlung (Trottet et al., 2002, 2011; Lüthi, Magun, and Miller, 2004; Lüthi, Lüdi, and Magun, 2004; Tsap et al., 2016). Our observations and analysis put thermal bremsstrahlung as the dominating mechanism during the impulsive phase of SOL2011-02-14 as well.

It has been shown that ionized plasmas at \( T < 1 \) MK in the chromosphere are efficient mm-wave sources (Kašparová et al., 2009; Heinzel and Avrett, 2012; Simões et al., 2017). In particular, Simões et al. (2017) have shown that once the energy deposition in the chromosphere stops, the free electrons quickly recombine with ions, thus reducing the main source of free–free emission after the impulsive phase. This decrease of the chromospheric emission could allow the optically thin free–free emission from the coronal plasma to dominate late in the gradual phase and produce the microwave spectrum observed at the time intervals e and f in Figure 3.

Moreover, Trottet et al. (2015) interpreted that most of the 30 THz radiation observed during the flare SOL2012-03-13 is thermal bremsstrahlung of an optically thin source located above the temperature minimum in a \( T \approx 8000 \) K plasma heated by the energy deposited by precipitated particles (electrons, protons and \( \alpha \)). Simões et al. (2017) have reached similar conclusions, using their results from numerical modeling to interpret the mid-infrared flare reported by Penn et al. (2016). Extending their calculations into the sub-THz range, they show that, during the impulsive phase, the sub-THz emission would be associated with the upper chromosphere with temperature around \( T \approx 10^{4.6} \) K. Therefore, sub-THz as the thermal counterpart of the Neupert effect is certainly possible.
We present simple calculations to show that the observed flux density at 212 GHz can be easily explained by optically thick free–free emission originating in the upper chromosphere. The observed flare excess $\Delta S_f$ is simply the difference between the total flux during the flare $S_f$ and pre-flare $S_b$, as observed by the SST 212 GHz beam:

$$\Delta S_f = S_f - S_b.$$  

(5)

Using the Rayleigh–Jeans law, this becomes

$$\Delta S_f = \frac{2k_b \nu^2}{c^2} \left( \frac{T_f A}{D^2} - T_\odot \Omega_b \right)$$

(6)

where $c$ is the speed of light, $T_\odot$ is the brightness temperature of the quiet Sun at $\nu = 212$ GHz, $\Omega_b = 1.06 \times 10^{-6}$ str is the solid angle of the $4'$ beam angular diameter, and finally $D$ is the Sun–Earth distance: one astronomical unit (AU). With numerical values, $\Delta S_f = 220$ sfu, $T_f = 5500$ K (Figure 3 of Selhorst, Silva, and Costa, 2005), and rearranging, Equation 6 becomes

$$A_{20} = \frac{1.66}{T_4},$$

(7)

where $A_{20}$ is the flare area in $10^{20}$ cm$^2$ and $T_4$ is the brightness temperature at 212 GHz in $10^4$ K.

Following the procedure introduced by Simões et al. (2017), we calculated the brightness temperature [$T_b$] at 212 GHz for two models of the F-CHROMA flare model database (www.fchroma.org/?page_id=24). The database contains more than 90 flare models and it was generated using the code RADYN (Carlsson and Stein, 1995; Allred, Kowalski, and Carlsson, 2015), starting from a quiet-Sun atmospheric model based on VAL-C (Vernazza, Avrett, and Loeser, 1981). RADYN solves the coupled, non-linear, equations of hydrodynamics, atomic level populations, radiative transfer in a 1D atmosphere subject to energy input by a beam of accelerated electrons. The electron transport and energy deposition is treated by solving the Fokker–Planck equation for an initial electron power-law distribution with spectral index $\delta$, low-energy cutoff $E_c$, and with an energy flux $F$. We only present a brief description of the RADYN code here and note that Allred, Kowalski, and Carlsson (2015) should be consulted for more details.

We selected the models with electron-beam parameters closer to the ones estimated from the HXR spectral analysis, namely, Model 21 ($\delta = 5$, $E_c = 15$ keV, $F = 10^{10}$ erg s$^{-1}$ cm$^{-2}$) and Model 27 ($\delta = 5$, $E_c = 15$ keV, $F = 3 \times 10^{10}$ erg s$^{-1}$ cm$^{-2}$). The maximum $T_b$-values found are $\approx 6.7 \times 10^4$ K and $\approx 10 \times 10^4$ K, for Models 21 and 27, respectively. Using these values for $T_b$ in Equation 7 gives an estimate of the necessary emitting areas to produce the observed flare emission of 220 sfu: $A = 2.5 \times 10^{15}$ and $A = 1.6 \times 10^{19}$ cm$^2$, for Models 21 and 27, respectively.

The resulting contribution function [CF] and optical depth [$\tau$] for 212 GHz are shown in Figure 6. CF indicates the formation height of the radiation (e.g. Carlsson, 1998; Simões et al., 2017), which originates near the base of the photosphere in the quiet Sun but shifts to the upper chromosphere during a flare. The emission is optically thick in both cases as indicated by the large optical depth $\tau > 1$.

The area of the flaring chromosphere can be estimated from AIA images. The best Atmospheric Imaging Assembly (AIA) band for this purpose is 1700 Å, since, unfortunately, most AIA bands saturate heavily during this event, especially during the impulsive phase. According to Simões et al. (2019), the flare-excess emission captured by the AIA 1700 Å
band originates in the chromosphere. To estimate the flaring area, we constructed histograms of AIA 1700 Å images and subtracted the average histogram of pre-flare images from the flare histograms (Figure 7). This resulted in the total number of pixels with values enhanced by the flare. The flaring area is then simply obtained by adding all of the histogram bins and multiplying the result by the area relative to the AIA pixel (0.6 × 0.6 arcsec² corresponding to ≈1.9 × 10¹⁵ cm²). Ignoring the saturated images, we found an average area of A ≈ 2.6 × 10¹⁹ cm², which is sufficient to produce the maximum observed flux density (≈220 sfu), with the assumed T₄-values above; and also much bigger than the lower limit value A₉Β obtained from the multi-beam technique.

These calculations are not an attempt to model this specific event. Our goal is to show that typical flare characteristics are sufficient to generate an observable flare signature at sub-mm from optically thick free–free from the upper chromosphere.

3.2. The Gradual Phase

After the end of the HXR emission (around 17:26), the radio spectrum from microwaves to the submillimeter is consistent with thermal bremsstrahlung. Even if we lack intermediate frequencies between 15.4 and 212 GHz to better characterize the spectrum, it is likely that for ν ≥ 15 GHz the emission is optically thin with a flux Fgradual ≈ 40–50 SFU. From GOES data we obtain E_MGOES ≈ 10⁹⁹ cm⁻³ and T.GOES ≈ 16 MK. If the radio source were coronal, it would produce a flux density at 212 GHz, which is a tenth of the observed one. This problem was observed already in other works (e.g., Lüthi, Magun, and Miller, 2004; Cristiani et al., 2007). Trottet et al. (2011) addressed the same question for SOL2003-10-27T12:30 and solved the problem using a multi-thermal coronal source, where the lower layers are cooler. In this way the relatively high submillimeter flux density was explained. We note that SOL2003-10-27T12:30, an M6.7 GOES class event, has a gradual phase with a similar flux density at submillimeter frequencies as SOL2011-02-14, and that the GOES emission measure (≈10⁹⁹ cm⁻³) and temperature (≈12 MK) are also quite similar. Therefore, conclusions from Trottet et al. (2011) can also be applied to this work.

3.3. Neupert Effect

We binned the Fermi NaI 128 energy channels in three HXR bands: 25−50 keV, 50−100 keV, and 100−300 keV. These bands were chosen in order to avoid the contribution from the thermal emission below 20 keV and the noise above 300 keV. We numerically integrated the HXR counts between t₀ = 17:23 UT and 17:32 UT. The resulting curves were compared with the 212 GHz flux-density temporal evolution (Figure 8, top-left) and with
Figure 7 Histograms of pixel values \([\text{DN s}^{-1}]\) of AIA 1700 Å images at pre-flare (black) and flare times (blue). We used their difference (orange) to estimate the number of flaring pixels and hence the flaring area.

the GOES SXR 1.5 – 12 keV and Fermi 12 – 25 KeV (Figure 8, bottom left). We note a coincidence between the maximum of the fluence 50 – 100 keV and the peak at 212 GHz. For the 25 – 50 keV band the fluence only reaches its maximum at the end of the time interval. Nonetheless it is worth noting that it follows the initial 212 GHz emission curve between 17:24 UT and 17:25 UT remarkably well. The association with SXR, however, is not so good. We observe that the SXR emission starts about 30 seconds before the HXR fluence. GOES 1.5 – 12 keV peak occurs simultaneously with the HXR fluence, meanwhile the Fermi 12 – 25 keV first peak is 30 seconds behind, only the second, less intense peak is coincident with the fluence maximum.

We computed the temporal derivative (Equation 2) of the 212 GHz flux density using a three-point quadratic Lagrangian interpolation for unevenly spaced data and plotted its positive values along with the 50 – 100 keV count rate (Figure 8, top-right panel). We have smoothed the 212 GHz data, using a 12-second running mean, to reduce the noise from the derivative procedure. Applying the same procedure to the SXR reveals a different behavior (Figure 8 bottom-right).

4. Conclusions

We present in this work a peculiar event observed at 212 GHz whose emission can be attributed to thermal bremsstrahlung during the impulsive and gradual phases. During the impulsive phase the 212 GHz emission comes from a thermal source at chromospheric heights. Moreover, its temporal derivative mimics the HXR flux (Figure 8, top-right), conforming with the thermal counterpart of the Neupert effect. During the extended phase, the sub-THz emission might be characterized as optically thin thermal bremsstrahlung, from a coronal multi-thermal source as described by Trottet et al. (2011).

Differently from the sub-THz, in the SXR domain, we observe that during the event the emission starts well in advance of the HXR energy accumulation (Figure 8 bottom-left). Moreover, its temporal derivatives are significantly different from the HXR temporal evolution. This is not completely unexpected, since statistics show that 50% of all events
observed at SXR do not follow the Neupert-effect hypothesis (Veronig et al., 2002a). In our case, SXR starts before the HXR fluence, which can be interpreted as a pre-heating of the plasma (Veronig et al., 2005). That the temporal evolution of the sub-THz bremsstrahlung and SXR are not always coincident during a flare is already known (e.g. Trottet et al., 2002; Tsap et al., 2016), which explains why the Neupert effect is observed at 212 GHz and not at SXR.

We note that this is the first time that the iterative multi-beam technique has been used to deduce position, flux, and effective area with the SST. Cristiani et al. (2007) used the SST iterative method for point-like sources, and the derived positions were compatible with the magnetic structures that gave rise to the flare. In the present case, we found that in order to
minimize the difference between expected and observed fluxes, the matrices \(Q(t_i, x, y, \sigma_s)\), we need an extended source with \(\varphi \geq 25''\). At the same time, and reinforcing the result, this solution is the closest to the main UV emitter (Figures 4 and 5). As we pointed out above, we do not find a unique solution here because we do not have four independent observations; we got a lower limit bound instead. The fact that this solution stabilizes above a threshold \(\varphi \geq 25''\) is an indication of an extended source (Giménez de Castro et al., 1999).

As a final remark, we stress the importance of the submillimeter and THz observing range for energy-transport diagnostics in the solar atmosphere. Observations with new instrumentation, at submillimeter frequencies (e.g. ALMA: Wedemeyer et al., 2016), THz/infrared (Kaufmann et al., 2013, 2016; Penn et al., 2016), and/or near-infrared (Kleint et al., 2016) should help the theoretical work to produce more refined flare models helping to increase our knowledge about the flaring solar atmosphere.

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References

Allred, J.C., Kowalski, A.F., Carlsson, M.: 2015, A unified computational model for solar and stellar flares. Astrophys. J. 809, 104. DOI. ADS.

Antonucci, E., Gabriel, A.H., Dennis, B.R.: 1984, The energetics of chromospheric evaporation in solar flares. Astrophys. J. 287, 917. DOI. ADS.

Bastian, T.S., Benz, A.O., Gary, D.E.: 1998, Radio emission from solar flares. Annu. Rev. Astron. Astrophys. 36, 131. ADS.

Carlsson, M.: 1998, Radiative transfer and radiation hydrodynamics. In: Vial, J.C., Bocchialini, K., Boumier, P. (eds.) Space Solar Physics: Theoretical and Observational Issues in the Context of the SOHO Mission; Proc. of a Summer School, Lecture Notes in Physics 507, Springer, New York, 163. DOI. ADS.

Carlsson, M., Stein, R.F.: 1995, Does a nonmagnetic solar chromosphere exist? Astrophys. J. Lett. 440, L29. DOI. ADS.

Costa, J.E.R., Silva, A.V.R., Lüdi, A., Magun, A.: 2002, Beam profile determination by tomography of solar scans. Astron. Astrophys. 387, 1153. ADS.

Cristiani, G., Martinez, G., Mandrini, C.H., Giménez de Castro, C.G., da Silva, C.W., Rovira, M.G., Kaufmann, P.: 2007, Spatial characterization of a flare using radio observations and magnetic field topology. Solar Phys. 240, 271. DOI.

Dulk, G.A.: 1985, Radio emission from the Sun and stars. Annu. Rev. Astron. Astrophys. 23, 169. ADS.

Dulk, G.A., Kontar, E.P.: 2010, Sub-Thz radiation mechanisms in solar flares. Astrophys. J. Lett. 709, L127. DOI. ADS.

Fleishman, G.V., Kontar, E.P.: 2010, Sub-Thz radiation mechanisms in solar flares. Astrophys. J. Lett. 709, L127. DOI. ADS.

Frost, K.J.: 1964, Comments on high energy X-ray bursts observed by OSO I. NASA SP-50, 139. ADS.

Giménez de Castro, C.G., Raulin, J.-P., Makramut, V.S., Kaufmann, P., Costa, J.E.R.: 1999, Instantaneous positions of microwave solar bursts: Properties and validity of the multiple beam observations. Astron. Astrophys. Suppl. Ser. 140, 373. ADS.
Giménez de Castro, C.G., Trottet, G., Silva-Valio, A., Krucker, S., Costa, J.E.R., Kaufmann, P., Correia, E., Levato, H.: 2009, Submillimeter and X-ray observations of an X class flare. *Astron. Astrophys.* **507**, 433. [DOI, ADS](https://doi.org/10.1051/0004-6361:200812819)

Guidice, D.A., Cliver, E.W., Barron, W.R., Kahler, S.: 1981, The air force RSTN system. *Bull. Am. Astron. Soc.* **13**, 553. [ADS](https://ui.adsabs.harvard.edu/abs/1981BAAS...13..553G)

Heinzel, P., Avrett, E.H.: 2012, Optical-to-radio continua in solar flares. *Solar Phys.* **277**, 31. [DOI, ADS](https://doi.org/10.1007/s11207-011-9818-3)

Herrmann, R., Magun, A., Costa, J.E.R., Correia, E., Kaufmann, P.: 1992, A multibeam antenna for solar mm-wave burst observations with high spatial and temporal resolution. *Solar Phys.* **142**, 157. [ADS](https://ui.adsabs.harvard.edu/abs/1992SoPh..142..157H)

Hudson, H.S.: 1991, Differential emission-measure variations and the “Neupert effect”. *Bull. Am. Astron. Soc.* **23**, 1064. [ADS](https://ui.adsabs.harvard.edu/abs/1991BAAS...23.1064H)

Hudson, H.S., Ohki, K.: 1972, Soft X-ray and microwave observations of hot regions in solar flares. *Solar Phys.* **23**, 155. [DOI, ADS](https://doi.org/10.1007/BF00160699)

Kašparová, J., Heinzel, P., Karlický, M., Moravec, Z., Varady, M.: 2009, Far-IR and radio thermal continua in solar flares. *Cent. Eur. Astrophys. Bull.* **33**, 309. [ADS](https://ui.adsabs.harvard.edu/abs/2009CEB....33..309K)

Kaufmann, P., Raunig, J.-P.: 2006, Can microbunch instability on solar flare accelerated electron beams account for bright broadband coherent synchrotron microwaves? *Phys. Plasmas* **13**, 701. [DOI, ADS](https://doi.org/10.1063/1.2151357)

Kaufmann, P., Levato, H., Cassiano, M.M., Correia, E., Costa, J.E.R., Giménez de Castro, C.G., Godoy, R., Kingsley, R.K., Kingsley, J.S., Kudaka, A.S., Marcon, R., Martin, R., Marun, A., Melo, A.M., Pereyra, P., Raunig, J.-P., Rose, T., Silva Valio, A., Walber, A., Wallace, P., Yakubovich, A., Zakia, M.B.: 2008, New telescopes for ground-based solar observations at submillimeter and mid-infrared. In: *Soc. Photo-Opt. Instrum. Eng. (SPIE)* **CS-7012**, [DOI](https://doi.org/10.1117/12.791959)

Kaufmann, P., White, S.M., Freeland, S.L., Marcon, R., Fernandes, L.O.T., Kudaka, A.S., de Souza, R.V., Aballay, J.I., Fernandez, G., Godoy, R., Marun, A., Valio, A., Raunig, J.-P., Giménez de Castro, C.G.: 2013, A bright impulsive solar burst detected at 30 THz. *Astrophys. J.* **768**, 134. [DOI, ADS](https://doi.org/10.1088/0004-637X/768/2/134)

Kaufmann, P., Abrantes, A., Bortolucci, E., Caspi, A., Fernandes, L.O.T., Kropotov, G., Kudaka, A., Laurent, G.T., Machado, N., Marcon, R., Marun, A., Nicolaev, V., Hidalgo Ramirez, R.F., Raunig, J.-P., Saint-Hilaire, P., Shihi, A., Silva, C., Timofeevsky, A.: 2016, Solar observations at THz frequencies on board of a trans-Antartic stratospheric balloon flight. In: *AAS/Solar Phys. Div. Meet.* **47**, 6.11. [ADS](https://ui.adsabs.harvard.edu/abs/2016ASD...47...61K)

Kleint, L., Heinzel, P., Judge, P., Krucker, S.: 2016, Continuum enhancements in the ultraviolet, the visible and the infrared during the X1 flare on 2014 March 29. *Astrophys. J.* **816**, 88. [DOI, ADS](https://doi.org/10.3847/0004-637X/816/2/88)

Krucker, S., Giménez de Castro, C.G., Hudson, H.S., Trottet, G., Bastian, T.S., Hales, A.S., Kašparová, J., Klein, K.-L., Kretzschmar, M., Lüthi, T., Mackinnon, A., Pohjolainen, S., White, S.M.: 2013, Solar flares at submillimeter wavelengths. *Astron. Astrophys. Rev.* **21**, 58. [DOI, ADS](https://doi.org/10.1007/s00159-013-0058-8)

Lüthi, T., Lüdi, A., Magun, A.: 2004, Determination of the location and effective angular size of solar flares with a 210 GHz multibeam radiometer. *Astron. Astrophys.* **420**, 361. [ADS](https://ui.adsabs.harvard.edu/abs/2004A&A...420..361L)

Lüthi, T., Magun, A., Miller, M.: 2004, First observation of a solar X-class flare in the submillimeter range with KOSMA. *Astron. Astrophys.* **415**, 1123. [ADS](https://ui.adsabs.harvard.edu/abs/2004A&A...415.1123L)

McAteer, R.T.J., Bloomfield, D.S.: 2013, The bursty nature of solar flare X-ray emission. II. The Neupert effect. *Astrophys. J.* **776**, 66. [DOI, ADS](https://doi.org/10.1088/0004-637X/776/1/66)

Meegan, C., Lichti, G., Bhat, P.N., Bissaldi, E., Briggs, M.S., Connaughton, V., Diehl, R., Fishman, G., Greiner, J., Hoover, A.S., van der Horst, A.J., von Kienlin, A., Kippen, R.M., Kouveliotou, C., McBreen, S., Paciesas, W.S., Preece, R., Steinle, H., Wallace, M.S., Wilson, R.B., Wilson-Hodge, C.: 2009, The *Fermi* Gamma Burst Monitor. *Astrophys. J.* **702**, 791. [DOI, ADS](https://doi.org/10.1088/0004-637X/702/1/791)

Neupert, W.M.: 1968, Comparison of solar X-ray line emission with microwave emission during flares. *Astrophys. J.* **Lett.** **153**, L59. [DOI, ADS](https://doi.org/10.1086/180830)

Penn, M., Krucker, S., Hudson, H., Iabvala, M., Jennings, D., Lunsford, A., Kaufmann, P.: 2016, Spectral and imaging observations of a white-light solar flare in the mid-infrared. *Astrophys. J. Lett.* **819**, L30. [DOI, ADS](https://doi.org/10.3847/2041-8213/819/2/L30)

Pick, M., Vilmer, N.: 2008, Sixty-five years of solar radioastronomy: Flares, coronal mass ejections and Sun Earth connection. *Astrophys. Rev.* **16**, 6. [DOI, ADS](https://doi.org/10.1103/RevModPhys.80.16)

Ramaty, R., Schwartz, R.A., Enome, S., Nakajima, H.: 1994, Gamma-ray and millimeter-wave emissions from the 1991 June X-class solar flares. *Astrophys. J.* **436**, 941. [ADS](https://ui.adsabs.harvard.edu/abs/1994ApJ...436..941R)

Selhorst, C.L., Silva, A.V.R., Costa, J.E.R.: 2005, Solar atmospheric model with spicules applied to radio observation. *Astron. Astrophys.* **433**, 365. [DOI, ADS](https://doi.org/10.1051/0004-6361:20041382)

Silva, A.V.R., Laganá, T.F., Gimenez Castro, C.G., Kaufmann, P., Costa, J.E.R., Levato, H., Rovira, M.: 2005, Diffuse component spectra of solar active regions at submillimeter wavelengths. *Solar Phys.* **227**, 265. [DOI, ADS](https://doi.org/10.1007/s11207-005-4662-8)

Simões, P.J.A., Kerr, G.S., Fletcher, L., Hudson, H.S., Giménez de Castro, C.G., Penn, M.: 2017, Formation of the thermal infrared continuum in solar flares. *Astron. Astrophys.* **605**, A125. [DOI, ADS](https://doi.org/10.1051/0004-6361/201629229)

Simões, P.J.A., Reid, H.A.S., Milligan, R.O., Fletcher, L.: 2019, The spectral content of SDO/AIA 1600 and 1700 Å filters from flare and plage observations. *Astrophys. J.* **870**(2), 114. [DOI, ADS](https://doi.org/10.3847/1538-4357/870/2/114)
Trottet, G., Rolli, E., Magun, A., Barat, C., Kuznetsoy, A., Sunyaev, R., Terekhov, O.: 2000, The fast and slow Hα chromospheric responses to non-thermal particles produced during the 1991 March 13 hard X-ray/gamma-ray flare at ~08 UTC. *Astron. Astrophys.* **356**, 1067. ADS.

Trottet, G., Raulin, J.-P., Kaufmann, P., Siarkowski, M., Klein, K.-L., Gary, D.E.: 2002, First detection of the impulsive and extended phases of a solar radio burst above 200 GHz. *Astron. Astrophys.* **381**, 694. ADS.

Trottet, G., Raulin, J.-P., Giménez de Castro, C.G., Lüthi, T., Caspi, A., Mandrini, C., Luoni, M.L., Kaufmann, P.: 2011, Origin of the submillimeter radio emission during the time-extended phase of a solar flare. *Solar Phys.* **273**(2), 340. DOI.

Trottet, G., Raulin, J.-P., Mackinnon, A., Giménez de Castro, G., Simões, P.J.A., Cabezas, D., de La Luz, V., Luoni, M., Kaufmann, P.: 2015, Origin of the 30 THz emission detected during the solar flare on 2012 March 13 at 17:20 UT. *Solar Phys.* **290**, 2809. DOI. ADS.

Tsap, Y.T., Smirnova, V.V., Morgachev, A.S., Motorina, G.G., Kontar, E.P., Nagnibeda, V.G., Strekalova, P.V.: 2016, On the origin of 140 GHz emission from the 4 July 2012 solar flare. *Adv. Space Res.* **57**, 1449. DOI. ADS.

Veronig, A., Vršnak, B., Dennis, B.R., Temmer, M., Hanslmeier, A., Magdalenić, J.: 2002a, Investigation of the Neupert effect in solar flares. I. Statistical properties and the evaporation model. *Astron. Astrophys.* **392**, 699. DOI. ADS.

Veronig, A., Vršnak, B., Temmer, M., Hanslmeier, A.: 2002b, Relative timing of solar flares observed at different wavelengths. *Solar Phys.* **208**, 297. DOI. ADS.

Veronig, A.M., Brown, J.C., Dennis, B.R., Schwartz, R.A., Sui, L., Tolbert, A.K.: 2005, Physics of the Neupert effect: Estimates of the effects of source energy, mass transport, and geometry using RHESSI and GOES data. *Astrophys. J.* **621**, 482. DOI. ADS.

Wedemeyer, S., Bastian, T., Brajša, R., Hudson, H., Fleishman, G., Loukitcheva, M., Fleck, B., Kontar, E.P., De Pontieu, B., Yagoubov, P., Tiwari, S.K., Soler, R., Black, J.H., Antolin, P., Scullion, E., Gunár, S., Labrosse, N., Ludwig, H.-G., Benz, A.O., White, S.M., Hauschildt, P., Doyle, J.G., Nakariakov, V.M., Ayres, T., Heinzel, P., Karlicky, M., Van Doorsselaere, T., Gary, D., Alissandrakis, C.E., Nindos, A., Solanki, S.K., Rouppe van der Voort, L., Shimojo, M., Kato, Y., Zaqarashvili, T., Perez, E., Selhorst, C.L., Barta, M.: 2016, Solar science with the Atacama Large Millimeter/Submillimeter Array – A new view of our Sun. *Space Sci. Rev.* **200**, 1. DOI. ADS.

White, W.A.: 1964, Solar X rays: A comparison with microwave radiation. *NASA SP-50*, 131. ADS.