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Does the shape of economic recovery matter? An alternative unit root test with new smooth transition model

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ABSTRACT

The subject of economic recovery after the Coronavirus pandemic has received much attention in the media and by academics in recent years. Pandemic experience creates a new transition between the pre-pandemic era trend and the post-pandemic era trend related to the major economic indicators’ time series path. This paper offers a new smooth transition model and a unit root testing procedure to test null of non-stationary against the alternatives of stationary that allow for a pit shape smooth transition from the pre-pandemic trend to post-pandemic trend. The properties of the test statistics are investigated with several simulation studies. Also, the new model and unit root testing procedure are applied to industrial production index, consumer price index and the unemployment rates of Global 8 countries and results state the usefulness of these new tests.

1. Introduction

Recent years have witnessed a growing academic interest in impacts of the Coronavirus pandemic on economics and the main question was: how will the pandemic shape the economy? The term of shape indicates the trend of economic time series and a structural change in related trend that Coronavirus health crisis caused. It is clear today that the negative economic impact of the pandemic will be temporary for advanced economies. Hence, economists have already started to discuss the shape of economic recovery (Sharma et al., 2021; Spelta et al., 2020; Wen & Arbogast, 2020). The discussions generally focus on “V” and/or “U” shapes due to the time series path of several macroeconomic indicators such as industrial production, employment, and gross domestic product. To the best of our knowledge, there is only one study that proposes a new statistical method to analyse non-linear trend patterns of the Coronavirus time series. Kumar et al. (2021) introduce a Bayesian spline-based time series model to study the trend pattern of Coronavirus infection data. However, non-linear statistical methods have not yet been used often enough to understand the impact of the pandemic on economic time series. Therefore, a new non-linear time series modelling technique is needed to fit the observed “pit” shaped structural changes in the trend of the series. This study aims to contribute to the growing topic of Coronavirus research by offering a new non-linear transition model and a unit root testing process. To achieve that, the Leybourne et al. (1998) (LNV) approach is expanded upon with a new time varying exponential function. By adopting this, it will be able to test whether the pandemic has a significant statistical impact on the stationary condition of the economic time series. Thus, due to stationary being a vital statistical property of an economic variable, this new test offers a useful insight to researchers before designing an empirical study that contains Coronavirus periods.

The overall structure of the study takes the form of five chapters. The following chapter defines the new smooth transition approach and the unit root testing procedure. The third chapter is about experimental designs and results of the Monte-Carlo simulation studies.
for the new unit root test statistics. An empirical study, which investigates the Coronavirus’ impact on time series trends of selected Global 8 (G8) countries’ several macroeconomic indicators, is reported in Chapter 4. Finally, the fifth chapter summarizes the study.

2. A new approach for smooth transition model and unit root tests

The LNV method introduces three models that allow one to estimate two linear trends around one specific time point of structural change. This structural change is represented with a logistic smooth transition function and the purpose of this function is to provide a proper estimation of the “S” shaped transition between two linear trends. On the other hand, the LNV approach also represents a very useful tool to develop various type of non-linear unit root testing processes. The smooth transition is a difficult concept to perceive and is confused by many researchers because there are many studies on it. Nonlinear dynamics in a time series can be considered in two frameworks. The first of these is the structural change (break) dynamic, which determines the nonlinear trend over time. The other is the regime change dynamic, which considers the change in the trend path according to the past values of the time series of interest. Considering this basic distinction in nonlinear time series, unit root tests on the concept of smooth transition can be categorised as in Table 1.

The LNV approach offers the following three exponential smooth transition regression models, which could be also considered as alternatives against the null of difference stationary, are employed:

Model A:  
\[ y_t = a_0 + \alpha_1 S_1(y, t_0) + \varepsilon_t \]  
(1)

Model B:  
\[ y_t = a_0 + \phi_1 t + \alpha_2 S_2(y, t_0) + \varepsilon_t \]  
(2)

Model C:  
\[ y_t = a_0 + \phi_1 t + \alpha_3 S_3(y, t_0) + \phi_2 t S_1(y, t_0) + \varepsilon_t \]  
(3)

where \( \varepsilon_t \) is a zero mean stationary error, \( t \) is a trend variable, and \( S_i(y, t_0) \) is the time varying exponential function that could be written as:

\[ S_i(y, t_0) = 1 + \exp \{- \gamma |t - t_0| \}^{-1} \]  
(4)

In equation (4), the \( t_0 \) parameter indicates midpoint time of smooth transition and the \( t \) parameter controls the speed of transition. The process starts with Equations (1)–(4) defining a smooth structural change in the trend of a time series. Moreover, it is important to remember that smooth transition is controlled by the variable \( t \) and the parameter of \( t_0 \) in (4), not by the lags of the time series \( y \). This is why it is a time varying structural change process, and it could not be categorised as a regime switching process like Smooth Transition Autoregressive Model (STAR), Logistic STAR (LSTAR) and Exponential STAR (ESTAR) models. After the LNV’s path breaking introduction of the time varying smooth structural change modelling approach, Sollis et al. (1999) offer more advanced time driven exponential function instead of (4) and could be shown as:

\[ G_i(y, t_0, \theta) = 1 + \exp \{- \gamma (t - t_0) / \theta \}^{-\theta} \quad 0 < \theta \leq 1 \]  
(5)

The roles of \( \gamma \) and \( t_0 \) are the same as with (4). The key parameter of (5) is \( \theta \). \( \theta \) controls the asymmetry of the smooth transition and when it is equal to 1 (\( \theta = 1 \) ) \( G_i \) function turns into a symmetric logistic function as introduced by the LNV approach.

\[ G_i(y, t_0, 1) = S_i(y, t_0) \]  
(6)

The logistic functions of (4) and (5) are proposed to fit the “S” shape nonlinear trend of a time series. Therefore, these functions are not capable of estimating the impact of the coronavirus pandemic on economic time series. As mentioned before, the pandemic mostly created “V” or “U” shaped nonlinear trends in economic indicators because of the recovery measures that economic authorities applied. It is clear that some modifications are needed for them to fit within the LNV models to the pandemic related nonlinear trend of time series. Firstly, the models of (1), (2) and (3) are expanded with a binary variable that could be defined as:

\[ \delta_i = \begin{cases} 
0 & \text{if } t_i \leq t_0 \ 
1 & \text{if } t_i > t_0 \end{cases} \]  
(7)

where \( T \) is the size of the sample. The exponential smooth transition regression models could be written as:

Table 1  
Categorization of the smooth transition unit root tests according to non-linear dynamics.

| Structural Change | Regime Switching | Hybrid* |
|-------------------|------------------|---------|
| Leybourne et al. (1998) | Sollis et al. (2002) | Sollis (2004) |
| Sollis et al. (1999) | Kapetanios et al. (2003) | Ozcan and Yurdakul (2022) |
| Reuse (2011) | Hepsag (2021) | |

* These studies consider both structural change and regime switching dynamics together.
Model A: \[ y_t = \alpha_0 + \alpha_1 P_t(\gamma, \tau_0) + \alpha_2 \delta_t P_t(\gamma, \tau_0) + \varepsilon_t \]  
(8)

Model B: \[ y_t = \alpha_0 + \varphi_1 t + \alpha_1 P_t(\gamma, \tau_0) + \alpha_2 \delta_t P_t(\gamma, \tau_0) + \varepsilon_t \]  
(9)

Model C: \[ y_t = \alpha_0 + \varphi_1 t + \alpha_1 P_t(\gamma, \tau_0) + \alpha_2 \delta_t P_t(\gamma, \tau_0) + \varphi_2 t \delta_t P_t(\gamma, \tau_0) + \varepsilon_t \]  
(10)

where \( \varepsilon_t \) is a zero mean stationary error and \( t \) is a trend variable. In the same perspective, the unit root testing procedure is introduced with the three exponential smooth transition regression models above, which could be also considered as alternatives against the null of difference stationary. The second modification is the new function, \( P_t(\gamma, \tau_0) \) is the time varying exponential function that could be written as:
\[ P_t(\gamma, \tau_0) = 1 - \exp\{- \gamma [t - \tau_0 T]^2 \} \]  
(11)

where \( \gamma \) controls width of pit shape or in other words, the speed of reaching the bottom point of the transition and should be greater than zero (\( \gamma > 0 \)), \( \tau_0 \) indicates the mid-point of the transition (\( 0 < \tau_0 < 1 \)), \( \alpha_2 \) and \( \alpha_1 \) determines asymmetry. If \( \alpha_1 + \alpha_2 > \alpha_1 \), this means that \( \tau_0 \) is greater than the mean before \( \tau_0 \). Conversely, when \( \alpha_1 + \alpha_2 < \alpha_1 \), this means that \( \tau_0 \) is smaller than the mean before \( \tau_0 \). Additionally, \( \alpha_2 = 0 \) indicates symmetry. Before defining unit root test statistics that consider stationary around a special form of smooth transition between two linear trends, the null hypothesis of non-stationary could be stated as follows:
\[ y_t = \zeta_1, \quad \zeta_1 = \zeta_{1-1} + \alpha_0, \quad \zeta_0 = 0 \]  
(12)

where, as in common practice, \( \alpha_0 \) is a zero mean and unit variance stationary process. Moreover, the LNV method proposes the usage of proper test statistics to test null of non-stationary in a two-step procedure. Following the same process, firstly, the smooth transition models (8), (9) and (10) are estimated with the nonlinear least square (NLS) estimation method. By doing this, the NLS residuals can be estimated. In this study, sequential quadratic programming algorithm is used to get NLS residuals from the smooth transition models.

\[ \hat{\varepsilon}_t = y_t - (\hat{\alpha}_0 + \hat{\alpha}_1 P_t(\hat{\gamma}, \hat{\tau}_0) + \hat{\alpha}_2 \hat{\delta}_t P_t(\hat{\gamma}, \hat{\tau}_0)) \]  
(13)

\[ \hat{\varepsilon}_t = y_t - (\hat{\alpha}_0 + \hat{\varphi}_1 t + \hat{\alpha}_1 P_t(\hat{\gamma}, \hat{\tau}_0) + \hat{\alpha}_2 \hat{\delta}_t P_t(\hat{\gamma}, \hat{\tau}_0)) \]  
(14)

\[ \hat{\varepsilon}_t = y_t - (\hat{\alpha}_0 + \hat{\varphi}_1 t + \hat{\alpha}_1 P_t(\hat{\gamma}, \hat{\tau}_0) + \hat{\alpha}_2 \hat{\delta}_t P_t(\hat{\gamma}, \hat{\tau}_0) + \hat{\varphi}_2 t \hat{\delta}_t P_t(\hat{\gamma}, \hat{\tau}_0)) \]  
(15)

Secondly, test statistics are estimated for residuals (\( \hat{\varepsilon}_t \)) from (13), (14) and (15) with a Dickey Fuller (1981) (DF) type autoregressive model
\[ \Delta \hat{\varepsilon}_t = \rho \hat{\varepsilon}_{t-1} + \sum_{i=1}^{k} \delta_i \Delta \hat{\varepsilon}_{t-i} + \varphi_t \]  
(16)

where \( \varphi_t \) is stationary error with the zero mean for an optimal lag length of \( k \). Three test statistics \( p_{\alpha}, p_{\alpha|\beta} \) and \( p_{\alpha|\varphi} \), which are connected with models (13), (14), and (15) respectively, could be calculated from the DF \( t \) ratio associated with \( \hat{\rho} \) in (16):
\[ p_{\alpha, \alpha|\beta, \alpha|\varphi} = \hat{\rho}_{p_{\alpha, \alpha|\beta, \alpha|\varphi}} / \hat{\sigma} (p_{\alpha, \alpha|\beta, \alpha|\varphi}) \]  
(17)

3. Simulation studies

This section contains Monte-Carlo simulation studies that are performed to obtain critical values, the finite sample size, and empirical power values of the test statistics \( p_{\alpha}, p_{\alpha|\beta}, \) and \( p_{\alpha|\varphi} \) under different data generation processes (DGP).

| \( T \) | \( p_{\alpha} \) | \( p_{\alpha|\beta} \) | \( p_{\alpha|\varphi} \) | \( p_{\alpha} \) | \( p_{\alpha|\beta} \) | \( p_{\alpha|\varphi} \) |
|---|---|---|---|---|---|---|
| 50 | -4.653 | -5.033 | -5.785 | -5.133 | -5.508 | -6.272 | -5.603 | -5.969 | -6.720 |
| 100 | -4.246 | -4.582 | -5.206 | -4.736 | -5.055 | -5.757 | -5.078 | -5.431 | -6.060 |
| 200 | -3.939 | -4.258 | -4.835 | -4.459 | -4.779 | -5.393 | -4.801 | -5.114 | -5.729 |
| 300 | -3.829 | -4.135 | -4.743 | -4.358 | -4.681 | -5.294 | -4.691 | -4.989 | -5.600 |
| 400 | -3.753 | -4.059 | -4.652 | -4.288 | -4.588 | -5.202 | -4.618 | -4.912 | -5.504 |
| 500 | -3.717 | -4.026 | -4.562 | -4.254 | -4.537 | -5.119 | -4.567 | -4.867 | -5.436 |
3.1. Critical values

Calculated with 10,000 replications under the null hypothesis of non-stationary, the critical values of \( p_{\alpha} \), \( p_{\alpha(t)} \) and \( p_{\alpha(g)} \) statistics for \( T = \{50, 100, 200, 300, 400, 500\} \) are reported in Table 2. The nominal significance levels are selected as 1%, 5% and 10% respectively.

3.2. Finite sample size

To observe specific size distortions under various conditions, a Monte-Carlo experiment is designed with the following DGP:

\[
\Delta y_t = \varphi \Delta y_{t-1} + \eta_t \quad y_0 = 0 \quad \text{and} \quad \eta_t \sim N(0, 1)
\]  

(18)

\( \varphi \) varies among \(-0.5, 0, 0.5\) and autoregressive lag \( k \) varies among \(0, 1, 4\). Simulation results are reported in Table 3. It could be seen that, for \( \varphi = 0 \) choosing \( k = 0 \) causes serious size distortions. Also, when \( k = 0 \), \( \varphi < 0 \) leads large empirical sizes and \( \varphi > 0 \) creates small empirical sizes. Lastly, increasing autoregressive lag produces empirical sizes that are below the nominal level. These findings are consistent with the reported results found in Leybourne et al. (1998).

3.3. Empirical power

Another important simulation study for the test statistics is needed to be done to investigate power values. For simplicity, the powers of \( p_{\alpha} \) and DF \( \tau_2 \) are calculated. In order to obtain the stationary series under various conditions, a new DGP is offered as:

\[
y_t = 1.0 + 1.0 P_i(y, t_b) + \alpha_t P_t(y, t_b) + \omega_t
\]  

(19)

\[
\omega_t = 0.5 \omega_{t-1} + \epsilon_t \quad \omega_0 = 0 \quad \epsilon_t \sim N(0, 1)
\]  

(20)

In DGP (19), asymmetry parameter \( \alpha_t \) varies among 0 (symmetry case), 5 and 10 (asymmetry case), the mid-point of transition \( t_b \) is set 0.5 and 0.8 and the speed of transition \( \gamma \) is chosen as 0.01 and 0.1. The computed power values are reported in Table 4. The findings indicate that over-fitting lag leads to significant power declines and, as expected, \( p_{\alpha} \) statistics has more power than \( \tau_2 \) with increasing asymmetry \( (\alpha_t) \). However, changing of the mid-point of transition \( (t_b) \) and the speed of transition \( (\gamma) \) do not cause noticeable power differentiations.

4. Empirical study

As discussed in Islam et al. (2020) and Al-Thaqeb et al. (2020), the macroeconomic indicators that the pandemic most affected are total production, employment and general price level. It is expected that the negative effects of the pandemic will decrease production and increase prices and unemployment. In order to get consistent results from nonlinear unit root tests, high frequency monthly industrial production index (IPI), consumer price index (CPI) and unemployment rates of Global 8 (G8) countries are chosen to apply the new method that is proposed in this paper. The data were observed monthly between September 2013 and December 2021. Also, natural logarithms of the IPI and the CPI are used, and series were collected from OECD Short-Term Economic Indicators database.

In all the countries examined, the industrial production index was adversely affected by the pandemic, and a temporary decline was observed. The findings show that the proposed smooth transition regression model could estimate the pit shape impact of the Coronavirus era can be seen in Fig. 1, Fig. 2 and Fig. 3 respectively.

In the all countries examined, the industrial production index was adversely affected by the pandemic, and a temporary decline was observed. The findings show that the proposed smooth transition regression model could estimate the pit shape impact of the Coronavirus and the trend lines with high precision for the IPI series. On the contrary, the time series path of consumer prices in the G8

| \( \varphi \) | \( k \) | \( T = 100 \) | \( T = 200 \) |
|---|---|---|---|
| \( p_{\alpha} \) | \( p_{\alpha(t)} \) | \( p_{\alpha(g)} \) | \( p_{\alpha} \) | \( p_{\alpha(t)} \) | \( p_{\alpha(g)} \) |
| 0 | 0 | 4.79 | 5.04 | 5.08 | 4.93 | 5.18 | 4.89 |
| 0 | 1 | 4.38 | 4.44 | 4.40 | 4.64 | 4.89 | 4.53 |
| 0 | 4 | 2.38 | 2.12 | 1.8 | 3.28 | 3.00 | 2.81 |
| -0.5 | 0 | 84.14 | 95.00 | 95.51 | 87.14 | 97.34 | 98.21 |
| -0.5 | 1 | 34.03 | 40.58 | 38.00 | 39.85 | 50.5 | 53.18 |
| -0.5 | 4 | 3.97 | 3.45 | 2.42 | 5.69 | 5.63 | 5.12 |
| 0.5 | 0 | 0.29 | 0.14 | 0.03 | 0.36 | 0.10 | 0.09 |
| 0.5 | 1 | 11.20 | 13.42 | 13.79 | 12.13 | 13.97 | 15.08 |
| 0.5 | 4 | 1.93 | 1.51 | 1.26 | 2.53 | 2.27 | 2.18 |

The nominal size is 5%. The results are based on 10000 replications.
countries are different from the industrial production index series, and the pit shape could not be observed in all countries. Canada, Germany, Italy and the United States experienced a temporary decrease in 2020 caused by the pandemic, and this situation could be captured by the proposed smooth transition model. However, the estimated non-linear trends show that there have been more serious structural changes in consumer prices in France, Japan, the United Kingdom and Russia at different dates than that which was caused by the pandemic. Finally, the sudden and temporary increase in unemployment rates observed during quarantine periods could be successfully modelled with the method proposed in this study. Among the G8 countries, the fluctuation in France’s unemployment rates due to the pandemic has not been as severe as expected and it is also a significant surprise that the unemployment rate in Italy has experienced a sharp and temporary decline contrary to expectations; both of these structural changes have been successfully modelled.

Another important point to be considered here is that the smooth transition trend model revealed in this study successfully captures the inverse “U” and/or “V” shaped structural breaks observed in the macroeconomic indicators. Considering all these structural breaks and non-linear trends revealed in the empirical study, the unit root test results in Table 1 can be interpreted. The unit root null could be

Table 4
Empirical powers of the tests $p_\alpha$ and ADF ($\tau_t$)

| $a_2$ | $h_\tau$ | $\gamma$ | $k = 0$ | $p_\alpha$ | $\tau_t$ | $k = 4$ | $p_\alpha$ | $\tau_t$ |
|-------|----------|---------|--------|----------|--------|--------|----------|--------|
| 0     | 0.5      | 0.01    | 99.84  | 99.99    | 24.47  | 57.61  |
| 0     | 0.5      | 0.1     | 99.89  | 100.00   | 28.17  | 73.41  |
| 0     | 0.8      | 0.01    | 99.81  | 99.99    | 25.86  | 64.18  |
| 0     | 0.8      | 0.1     | 99.90  | 100.00   | 28.62  | 74.5   |
| 5     | 0.5      | 0.01    | 98.89  | 65.30    | 19.29  | 0.32   |
| 5     | 0.5      | 0.1     | 99.51  | 70.25    | 23.61  | 2.49   |
| 5     | 0.8      | 0.01    | 98.80  | 44.06    | 17.58  | 0.08   |
| 5     | 0.8      | 0.1     | 99.49  | 33.49    | 22.49  | 0.28   |
| 10    | 0.5      | 0.01    | 98.68  | 0.01     | 16.79  | 0.00   |
| 10    | 0.5      | 0.1     | 99.45  | 0.01     | 22.29  | 0.00   |
| 10    | 0.8      | 0.01    | 98.19  | 0.00     | 13.02  | 0.00   |
| 10    | 0.8      | 0.1     | 99.40  | 0.00     | 21.35  | 0.00   |

The nominal size is 5% and $T$ is 100. The results are based on 10000 replications.

Table 5
Unit root test results of G8 industrial production indexes data.

| Country      | Series | $k$ for $\tau_t$ | $\tau_t$ | $k$ for $p_\alpha$ | $p_\alpha$ | $\tau_t$ | Mid-point date |
|--------------|--------|------------------|---------|---------------------|-----------|---------|---------------|
| Canada       | $ipi_t$ | 1                | −3.217* | 1                   | −3.236    | 0.803   | 04-2020       |
|              | $unp_t$ | 2                | −2.674  | 3                   | −2.365    | 0.806   | 05-2020       |
|              | $cpi_t$ | 1                | −2.887  | 1                   | −4.556    | 0.825   | 06-2020       |
| Germany      | $ipi_t$ | 2                | −2.512  | 1                   | −3.053    | 0.802   | 04-2020       |
|              | $unp_t$ | 3                | −2.315  | 1                   | −3.855    | 0.798   | 03-2020       |
|              | $cpi_t$ | 3                | −3.029  | 3                   | −4.175    | 0.876   | 12-2020       |
| France       | $ipi_t$ | 2                | −3.213* | 2                   | −4.200    | 0.800   | 04-2020       |
|              | $unp_t$ | 1                | −4.545**| 1                   | −6.011**  | 0.366   | 09-2016       |
|              | $cpi_t$ | 1                | −1.829  | 1                   | −3.388    | 0.468   | 07-2017       |
| Italy        | $ipi_t$ | 1                | −4.988***| 2                   | −3.084    | 0.798   | 04-2020       |
|              | $unp_t$ | 2                | −4.246***| 1                   | −4.151    | 0.799   | 04-2020       |
|              | $cpi_t$ | 2                | −1.010  | 2                   | −2.565    | 0.833   | 07-2020       |
| Japan        | $ipi_t$ | 1                | −3.158* | 1                   | −4.421    | 0.813   | 05-2020       |
|              | $unp_t$ | 2                | −1.260  | 1                   | −6.407***| 0.749   | 11-2019       |
|              | $cpi_t$ | 7                | −2.320  | 1                   | −2.981    | 0.084   | 04-2014       |
| Russia       | $ipi_t$ | 4                | −2.804  | 4                   | −4.064    | 0.811   | 05-2020       |
|              | $cpi_t$ | 2                | −2.409  | 1                   | −2.477    | 0.199   | 04-2015       |
| United Kingdom | $ipi_t$ | 1                | −4.545***| 2                   | −3.768    | 0.804   | 04-2020       |
|              | $unp_t$ | 4                | −2.287  | 3                   | −4.958    | 0.686   | 05-2019       |
|              | $cpi_t$ | 6                | −1.723  | 6                   | −2.742    | 0.397   | 12-2016       |
| United States | $ipi_t$ | 1                | −3.727**| 1                   | −1.732    | 0.804   | 04-2020       |
|              | $unp_t$ | 1                | −3.362* | 1                   | −4.538    | 0.803   | 04-2020       |
|              | $cpi_t$ | 2                | −0.547  | 2                   | −2.554    | 0.879   | 12-2020       |

*, ** and *** represent rejection of the null unit root hypothesis at the 10%, 5% and 1% significance levels respectively. $k$ indicates estimated autoregressive lag.

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rejected for Canada, France, Japan United Kingdom and the United States’ IPI series with linear \( \tau \) statistics at different significance levels. Similar results could be found for the unemployment series of France and United States. However, according to \( p_{null} \) statistics the null hypothesis of non-stationary could not be rejected for only the unemployment series of France and Japan. Except for those related to France’s unemployment rates, all other results clearly show that the temporal impact of the Coronavirus on economic indicators could change stationary conditions of related time series. Therefore, ignoring the effects of the Coronavirus era in economics may lead
to serious econometric problems in applied studies.

5. Conclusion

The main question of this paper was: does the shape of economic recovery after the Coronavirus pandemic matter? To answer this

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Fig. 2. Fitted non-linear trends for CPI series with new smooth transition model.
question from an econometric perspective, a new smooth transition model and unit root testing process are offered in this study. Although it is not a direct alternative of linear DF test statistics, the new test statistics that are introduced in this paper have more power than linear DF if the DGP is a stationary asymmetric smooth transition model. The monthly industrial production index, the consumer price index and the unemployment rate series, which belong to G8 countries, are chosen for empirical study and (most of) their time series path clearly show a pit or inverse pit shape recovery pattern. The empirical findings thus indicate that the shape of

Fig. 3. Fitted non-linear trends for unemployment series with new smooth transition model.
economic recovery which could be observed in industrial production, consumer prices and unemployment, econometrically matters for G8 countries. The motivation of this study was born from pandemic experience. However, the proposed method of this study could be used for any specific recession-recovery cases like the Coronavirus pandemic causes of 2020.

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CRediT authorship contribution statement

Mehmet Özcan: Conceptualization, Methodology, Software, Data curation, Writing – original draft, Visualization, Investigation, Software, Validation, Writing – review & editing.

Declaration of competing interest

The author has no relevant financial or non-financial interests to disclose.

Data availability

The dataset sample files and the R Programming Language codes are available at the following link: https://github.com/mehmet-ozcan/Shape_of_economic_recovery/releases/tag/published.

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