The emission of Gamma Ray Bursts as a test-bed for modified gravity

S. Capozziello\textsuperscript{a,b,c}, and G. Lambiase\textsuperscript{d,e}

\textsuperscript{a}Dipartimento di Fisica, Università di Napoli "Federico II", Via Cintia, I-80126 - Napoli, Italy.

\textsuperscript{b}INFN Sez. di Napoli, Comp. Univ. di Monte S. Angelo, Edificio G, Via Cintia, I-80126 - Napoli, Italy,

\textsuperscript{c}Gran Sasso Science Institute (INFN), Via F. Crispi 7, I-67100, L’Aquila, Italy,

\textsuperscript{d}Dipartimento di Fisica "E.R. Caianiello", Università di Salerno, I-84084, Fisciano (Sa), Italy, and

\textsuperscript{e}INFN - Gruppo Collegato di Salerno, Italy.

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The extreme physical conditions of Gamma Ray Bursts can constitute a useful observational laboratory to test theories of gravity where very high curvature regimes are involved. Here we propose a sort of curvature engine capable, in principle, of explaining the huge energy emission of Gamma Ray Bursts. Specifically, we investigate the emission of radiation by charged particles non-minimally coupled to the gravitational background where higher order curvature invariants are present. The coupling gives rise to an additional force inducing a non-geodesics motion of particles. This fact allows a strong emission of radiation by gravitationally accelerated particles. As we will show with some specific model, the energy emission is of the same order of magnitude of that characterizing the Gamma Ray Burst physics. Alternatively, strong curvature regimes can be considered as a natural mechanism for the generation of highly energetic astrophysical events.

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However, recent results show that most of GRBs are narrowly beamed and the corresponding energies are $10^{51}$ ergs/s \cite{10}, making them comparable to supernovae in total energy release.

Although some features of GRBs must be still understood, there is agreement between observations and the so-called fireball model. According to the latter, GRBs are produced via a dissipation of the kinetic energy of ultra-relativistic flows. In this model, the GRB itself is produced by internal dissipation within the flow, while the afterglow\footnote{Models predict that GRBs are followed by a lower-energy afterglow and in some cases, radio afterglows have been observed several years after the bursts.}, a long-lasting emission in the x-ray, optical, and radio wavelengths, is produced via external shocks with the medium.

Owing to the several observations of GRBs and of their afterglows, it has been possible to constrain the fireball model that describes the emitting regions. There is, however, no direct evidence about the inner engine able to generate GRBs and produce the ultrarelativistic flow. For a physical characterization of these phenomena, in particular the energetic requirements and the time scales, one deduces that GRBs are correlated with the formation of black holes (via a stellar collapse) or a neutron star merger. Moreover, the requirement of the long activity of the inner engine in the fireball model (typically greater than 10 s) suggests an inner engine built on an accreting black hole. This agrees with the fact that GRBs are associated with star forming regions, indicating that GRB progenitors are massive stars. Finally, the appearance of supernova bumps in the afterglow light curve (most notably in GRB 030329 \cite{11}) suggests a correlation of GRBs with supernovae and stellar collapse.

A part the careful description of the emission in the fireball model, the very final origin of such a strong energetic mechanism is far to be fully understood at fundamental level. According to the above considerations, high curvature regimes could play an important role in the evolution of particles in the gravitational field, we shall confine ourselves to a classical description. Besides, also the back-reaction effects will be neglected in our analysis being only interested to the emission of radiation induced by the acceleration.

It is well known that particles with high acceleration generate enormous streams of photons by bremsstrahlung. The power radiated away by a particle of charge $q$ is estimated to be (for convenience, we shall use both mks and natural units) \cite{13}:

$$W = -\frac{2q^2}{3} |\dot{x}|^2,$$

where $|\dot{x}| \equiv |\mathbf{a}|$ is the modulus of the acceleration. However, in a curved spacetimes, to which we are mainly interested, the above equation generalizes to

$$W = -\frac{2q^2}{3} |D^2 x^\alpha|^2,$$

where now $D^2 x^\alpha$ represents the covariant four-acceleration of particles. Clearly for particles moving along a geodesic, the four-acceleration is zero, and no radiation can be emitted. This is not the case in models recently proposed \cite{12}. Let us assume the total interaction Lagrangian given by

$$\mathcal{L} = \mathcal{L}_{\text{grav}} + F \mathcal{L}_{\text{mat}},$$

where $\mathcal{L}_{\text{grav}}$ and $F$ can arbitrarily depend on the spacetimes metric and curvature tensors according to the effective interaction considered. The last term in \cite{4} describes theories with nonminimal coupling between matter and functions depending on curvature invariants. In general, one may consider the possibility that the coupling $F$ is a more involved function depending on nine parity-even invariants \cite{12}, so that $F = f(i_1, \ldots, i_9)$, where $i_1, \ldots, i_9$ are the scalar curvature invariants constructed by means of Ricci and Riemann tensors

$$i_1 \equiv R^2, \quad i_2 \equiv R_{\mu\nu}R^{\mu\nu}, \quad i_3 \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \quad (5)$$

$$i_4 \equiv R_{\mu\nu}^{\alpha\beta}R_{\alpha\beta}^{\sigma\rho}R_{\sigma\rho}^{\mu\nu}, \quad i_5 \equiv R_{\mu\nu}^\rho R_{\rho}^\nu R_{\rho}^\sigma, \quad i_6 \equiv R_{\mu\nu}^{\rho\sigma}R_{\rho\sigma}^{\mu\nu}R_{\alpha\beta}^{\mu\nu}, \quad i_7 \equiv R^{\mu\nu}D_{\mu\nu}, \quad i_8 \equiv D_{\mu\nu}D^{\mu\nu}, \quad i_9 \equiv D_{\mu\nu}D^{\nu\rho}R_{\rho}^\mu, \quad D_{\mu\nu} \equiv R_{\mu\nu\rho\sigma}R^{\rho\sigma} \quad (6)$$

In general, also the Gauss-Bonnet topological invariant

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$$

can be considered a term playing an important role in the matter-gravity interaction \cite{4,5}. The equations of motion for (extended) test bodies are derived from the
energy-momentum conservation law. More specifically, by using the Synge expansion technique and covariant multipolar approximation scheme, it turns out that four-acceleration is given by

\[ D^2 x^\alpha \equiv \dot{v}^\alpha = \frac{\xi}{m} (\delta^\alpha_\beta - v^\alpha v_\beta) \nabla^\beta A. \] (7)

In this equation, \( A \) is related to the function \( F \) as

\[ A = \ln F, \] (8)

\( m \) is the mass of test particle and \( v^\alpha \) its velocity, and finally, the constant \( \xi \) is a quantity that depends on the matter distribution

\[ \xi = \int_{\Sigma(s)} L_{\text{mat}} w^{x^2} d\Sigma v_x, \] (9)

where \( s \) is the proper time of the particle and the integration is over a spatial hypersurface (in general \( \xi \) does depend on gravitational background and parameters characterizing the test particle; however, in a multipolar approximation, \( \xi \) corresponds to a free particle). Eq. (7) therefore implies that a massive particle moves non-geodesically along its world-line owing to the presence of the additional force generated by the non-minimal coupling curvature function \( F \).

Consequences of (7) have been recently studied in cosmology and celestial mechanics. Stringent constraints on \( |\nabla A| \) are provided by COBE and GP-B satellites, \( |\nabla A| \approx 2 \times 10^{-4} \text{kg/sec} = 7.4 \times 10^{-5} \text{GeV}^2 \) and \( |\xi A| \approx 2 \times 10^{-10} \text{g/sec} = 7.4 \times 10^{-8} \text{GeV}^2 \).

Squaring (7) and using the normalization condition \( v^\alpha v_\alpha = -1 \), one obtains that the emitted radiation reads

\[ W = -\frac{2}{3} \frac{q^2}{m^2} K^2, \] (10)

where

\[ K^2 = |\nabla A|^2 - (v^\alpha \nabla A)^2. \] (11)

A full analysis of (10) requires to fix a form of \( A \), hence of \( F \). In the following we consider some specific case:

1. Let us first consider the case where the variation of the function \( K^2 \) is negligible over the time scale of GRBs duration. This means to consider \( K \) constant (i.e. \( A \sim C_\mu v^\mu \), where \( C_\mu \) is a constant four-vector). In order to achieve the emitted power \( 10^{51} - 10^{54} \text{erg/sec} \), the parameter \( \xi \) has to be constrained to

\[ \xi K \lesssim (10^{10} - 10^{11.5}) \text{kg/sec} = 4.1 \times (10^{12} - 10^{13.5}) \text{GeV}^2. \] (12)

2. Consider now the case where the background is described by a Schwarzschild geometry. Outside the gravitational source, the Ricci tensor vanishes, while the Riemann tensor does not. In such a case, one can assume that the coupling \( F \) is only a function of the invariant \( i_3 = 12 \frac{r^2}{r_0^6} \), where \( r_0 = 2GM \) is the Schwarzschild radius of the gravitational mass. As a more general form, we can choose

\[ F(i_3) = (\lambda^4 i_3)^\delta, \] (13)

with \( \lambda \) a constant of dimensions [length] (or [energy]^{-1}) and \( \delta \) a dimensionless constant. The functions \( F \) and \( A \) do only depend on the radial variable \( r \). For the sake of simplicity, we also assume that the motion of the particle is radial \( (v^\alpha = (v^0, v^r, 0, 0)) \). Eq. (11) gives

\[ K^2 = \frac{36 \delta^2}{(r_0^2)^2}, \]

with \( \Gamma = [1 - (v^r)^2]^{-1/2} \). Referring to an electron particle, with \( m = 0.5 \text{MeV} \) and \( q = e = 2.8 \times 10^{-1}, \) and astrophysical objects with characteristic Schwarzschild radius \( r_s = 10 \text{km} \) (the mass of black hole are typically of the order \( 3 \times 10^8 M_\odot \), where \( M_\odot \) is the solar mass), we get that the emitted power is

\[ W = 24 \frac{e^2}{m^2 \Gamma^2 r^2} (\xi \delta)^2 \]

\[ \approx 4 \times 10^{-33} \frac{\xi \delta^2}{\Gamma^2} \left( \frac{0.5 \text{MeV} \times 10 \text{km}}{r} \right)^2. \] (14)

For \( \Gamma = 10 \) one gets \( W \sim 10^{34} \text{erg/sec} \sim 4 \times 10^{30} \text{GeV} \). Eq. (10) provided \( \xi \delta \lesssim 10^{33} \text{GeV} \).

3. Finally, consider the following form of \( F \):

\[ F(i_3) = e^{(\lambda^4 i_3)^\delta}. \]

For a Schwarzschild background, one finds that the emitted power is given by

\[ W = 24 \times 10^{36} \left( \frac{0.5 \text{MeV} \times 10 \text{km}}{r_s} \right)^2 \frac{e^2}{\Gamma^2} \times \]

\[ \left( \frac{\delta \xi}{\text{GeV}} \right)^2 \left( \frac{\lambda}{r_s} \right)^{8\delta} \left( \frac{r_s}{r} \right)^{12\delta} \text{GeV}^2. \] (15)

For a characteristic length \( \lambda \) of the order of the Schwarzschild radius, \( \lambda \sim r_s \), and for distances \( r \sim 10^{10} \text{cm} = 10^9 r_s \), corresponding to the distance where the shock produces a GRB, one gets the emitted power (15) provided \( \xi \sim O(1) \text{GeV} \) and \( \delta \sim -0.62 \). This is represented in Fig. 4.

In the case where the background is described by a Kerr spacetime, the invariant \( i_3 \) reads

\[ i_3 = 12 \frac{r^2}{r_0^6} I(x), \] (16)

where

\[ I(x) \equiv \frac{(1 - x^2)[(1 - x^2)^2 - 16x^2]}{(1 + x^2)^6}, \]
with \( x = ay/r \) and \( y = \cos \theta \). The constant \( a \) is related to the angular momentum \( J \) of the gravitational source \( \sqrt{J/M} \). For \( x \ll 1 \), i.e. \( r \gg ay \), the function \( I \) approaches to 1, and one recovers the Schwarzschild results. For \( x \approx 1 \) and using (16) and (11), one gets that the emitted energy is given by

\[
W_{\text{Kerr}} = W \eta^{2\delta-1},
\]

where \( W \) is defined in (15) and \( \eta = x^2 - 1 \ll 1 \). Therefore we find that for a Kerr geometry there appears an additional factor \( \eta^{2\delta-1} \), whose effect is to amplify the emission power, i.e. \( > 1 \), provided \( \delta < 1/2 \).

In conclusion, we propose a new mechanism for the GRBs emission. The mechanism is based on the non-minimal coupling of matter with the gravitational background, which gives rise to an additional term in the equation of motion inducing a non-geodesic propagation of test particles. This aspect is very important in processes where the radiation is emitted by accelerated particles. In fact in a pure General Relativity approach, one has that \( D^2 x^\alpha = \ddot{x}^\alpha + \Gamma^\alpha_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0 \), and therefore charged particles cannot emit radiation. On the other hands, if non-minimal curvature coupling are properly taken into account, then \( D^2 x^\alpha \neq 0 \), see Eq. (7), and radiation can be emitted.

Although we have investigated some particular cases for the non-minimal coupling function \( F \), focusing on models where \( F \) does only depend on the invariant \( i_3 \), our results are more general, as follows from Eq. (10). Conversely, we can say that GRBs can be a formidable test-bed to select reliable alternative theories of gravity since, from the above mechanism, natural constraints emerge. Finally, we have to point out that here we considered only classical particles. In a more refined treatment, quantum description has to be considered.

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