Research Article

Theoretical and Experimental Study considering the Influence of T-Stress on the Fracture Behavior of Compression-Shear Crack

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The influence of the nonsingular stress term (T-stress), existing in the Williams expansion of the crack tip stress field, on the fracture behavior of the compressive-shear crack is comprehensively considered in this paper. According to the stress boundary conditions on the crack surfaces, the theoretical solution of the stress field around the closed crack tip is established using the complex potential theory, which includes not only the singular term containing the stress intensity factor $K_{II}$ but also three nonsingular terms ($T_x$, $T_y$, and $T_{xy}$) without $K_{II}$. Then, the differences between tangential stresses and shear stresses at the crack tip obtained in the cases with and without T-stress are compared, respectively. Outcome indicates that there are tangential tensile stress and compressive stress zones at the crack tip when considering T-stress, while only the former exists when ignoring T-stress. In the case of considering T-stress, an improved crack initiation criterion is proposed, where the maximum tangential tensile stress and maximum shear stress are utilized simultaneously to predict the crack initiation and failure mode under the multiaxial compression stress state. Besides, the triaxial compression tests were carried out on the prismatic rock-like material samples with a central crack, and the effectiveness and validity of the proposed criterion are verified by comparing the experimental results and theoretical predictions. It reveals that the proposed criterion can reliably predict the crack initiation angles and failure modes with higher accuracy than traditional fracture criterion.

1. Introduction

Rock material, well recognized for its excellent mechanical behavior and economic efficiency, is widely used as a manufactured material in modern engineering construction, such as buildings, dams, nuclear power plants, tunnels, and subways. Indeed, it is inevitable that the defects similar to faults or microcracks are pervasive in the process of material forming or manufacturing [1–4]. However, the cracks and faults form the boundaries or discontinuities of the rock materials that may cause many fracture disasters and related phenomena in terms of mudslides, earthquakes, landslides, collapses, and gas explosions—in response to the initiation, propagation, interaction, and interconnection of those cracks. As a result, there exists a practical implication for in-depth investigating fractures such as cracks and faults for the sake of modern engineering structures in such fields as structural engineering, geotechnical engineering, and underground engineering [5, 6].

With respect to the crack problem, the stress field based on fracture mechanics can better describe the stress variation law around the crack tip. The stress field proposed by Williams [7] is one of the most classical theories, and its expansion includes not only the singular stress term containing $r^{1/2}$, but also the nonsingular stress term and higher-order term $O(r^{1/2})$. Most researchers [8, 9] over the past decades only utilized the singular stress term in the expansion when studying the stress field at the crack tip, ignoring the nonsingular and higher-order term, although the higher-order term $O(r^{1/2})$ tends to 0 when $r$ is close to 0.
However, the nonsingular term is constant and does not change with \( r \), this term is generally called \( T \)-stress. The influence of \( T \)-stress on the tip stress field has been revealed by many studies [10–12], which can be summarized as follows: when \( r \) tends to 0, the singular stress term in the expansion plays a major role in controlling the tip stress field. With the gradual increase of \( r \), the singular stress term decreases rapidly, while the proportion of nonsingular stress term increases gradually; in this case, the nonsingular stress term cannot be ignored. This demonstrates a practical demand for considering the nonsingular stress term in the expansion.

Building on stress field around the crack tip, the evaluation of crack initiation and propagation is the key to solve rock engineering fracture problems, and the direction of cracking angle is also an important parameter for exploring crack propagation behavior. For the mixed cracking problem with mode I cracks, the direction of crack initiation can be predicted by the existing mature theories, including the maximum tangential stress criterion [13], the minimum strain energy density criterion [14], and the maximum energy release rate criterion [15]. Based on the maximum tangential stress criterion, the mixed mode crack begins to initiate from the tip along the direction of the maximum tangential stress. Numerous experiments and theoretical foundations have made it mature to consider the effect of \( T \)-stress on crack initiation and growth under tensile conditions [16, 17], yet the influence of \( T \)-stress on the stress field and initiation angle of crack tip under compression-shear stress has not been comprehensively and systematically investigated so far.

In reality, rock materials in nature or engineering are usually subjected to the stress state of compression-shear [18, 19]. Due to the closure and friction of the two surfaces of crack, the complex nonlinear interaction of relatively dislocation or sliding exists between both. In some cases, these interactions may be beneficial as preventing cracks from initiating, and they make the stress field of tip not only have nonsingular stress term along the crack direction, but also nonsingular stress terms perpendicular to the crack surface and shear direction, which is different from the nonsingular stress term only exists in the crack direction under tensile load. Therefore, the evaluation of fracture behavior, initiation, and propagation path of crack under compression-shear is more complicated than that of mode I crack problems. Some researchers conducted uniaxial compression fracture tests using prismatic samples with inclined precast cracks [20, 21]. Others used the center cracked disk, usually called the Brazilian disk (BD), to conduct uniaxial compression tests to study the cracking angle of compression-shear cracks in rock materials [22–25]. The initiation and propagation of cracks under uniaxial stress can be well described and predicted in these studies from the theoretical level and the experimental level.

However, most existing fracture criteria are established on the premise that infinite plates are subjected to remote multiaxial stress. It is far from enough to verify the correctness of the fracture criteria by only using the cracking angle from uniaxial fracture tests. Although some scholars [26–28] conducted multiaxial fracture tests on cylindrical samples, the crack initiation in cylindrical samples cannot be simply regarded as a plane problem, and the applicability of fracture model remains to be testified. According to the conclusions of Melin [29], with the increase of lateral confining pressure, the tensile stress at the crack tip gradually disappears, while the shear stress still existed. Therefore, the failure mode of brittle rock materials is usually oblique shear failure under confining pressure [30, 31]. The existing fracture mechanics theories rarely explore the mechanism of shear failure from this perspective.

Based on the previous studies, there exists a practical demand to explore the effects of slip and friction at the crack surface on the stress field at the tip of the compression-shear crack. In this study, the stress field expression of the center crack tip under the remote compressive load was established based on the complex potential theory, considering the influence of slip and friction between the surfaces of the crack. Besides, the differences of tangential stress, shear stress, and initiation angle at crack tip with and without \( T \)-stress are rationally compared. An improved crack initiation criterion is then proposed, where the maximum tangential stress and the maximum shear stress are simultaneously employed for predicting the crack initiation and failure mode. Finally, the correctness and validity of the above criterion are verified through multiaxial compression test of rock-like materials containing preexisting crack with two different inclination angles. The overall framework of this research is illustrated in Figure 1, including theoretical and experimental modules. In theoretical study, the theoretical inclination angles are obtained by the proposed criterion, and the actual initiation angles as well as failure modes of the crack are verified through experimental study. The research results provide a theoretical guidance for accurately and quantitatively analyzing the damage tolerance of structures made from rock materials.

2. Analytical Solution of Stress Field at the Tip of Crack

2.1. Complex Stress Function of Crack under Far-Field Compressive Stress. Consider an elastic infinite plate containing one center through-thickness closed crack of length \( 2a \) with two crack tips as depicted in Figure 2. The plate is subjected to two remote and mutually perpendicular loads \( \sigma_1 \) and \( \sigma_3 \). The angle between the crack and the maximum principal stress \( \sigma_1 \) is \( \alpha \), and the angle between the crack and the minimum principal stress \( \sigma_3 \) is \( \beta \). Due to the closure of crack under compressive load, the mechanical interaction between the upper and lower surfaces of the crack exists. A crack therefore can be regarded as a pair of planes in frictional contact, and the frictional resistance \( \tau_f \) of the surfaces is

\[
\tau_f = \mu \sigma_n = \frac{1}{2} \mu (\sigma_1 + \sigma_3) + (\sigma_1 - \sigma_3) \cos 2 \beta,
\]

where \( \mu \) represents the friction coefficient of the surfaces on both sides of the crack.
The compressive normal stress $\sigma_n$ and the shear normal stress $\tau_s$ on the surfaces of crack can be expressed as follows:

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\beta,$$

$$\tau_s = \frac{\sigma_1 - \sigma_3}{2} \sin 2\beta.$$  

Whether the frictional sliding occurs on the crack surfaces depends on the relationship between the magnitude of shear normal stress and frictional resistance. The criterion of sliding is clearly defined as the following equation pair:

$$\tau_s \geq \tau_f: \text{Frictional sliding},$$

$$\tau_s < \tau_f: \text{No slip.}$$  

In general, the boundary condition can be written in the following form:

Upper surface: $\sigma_y^+ = \sigma_{n_1}, \tau_{xy}^+ = \tau_f,$  

Lower surface: $\sigma_y^- = \sigma_{n_2}, \tau_{xy}^- = \tau_f,$

where $\sigma_y^+, \tau_{xy}^+$, and $\sigma_y^-, \tau_{xy}^-$ refer to the stress boundary values of the upper and lower surfaces of the crack, respectively. For the plane problem, according to the Muskhelishvili complex potential theory [32], the stress field at the crack tip can be expressed in two complex variable functions $\Phi(z)$ and $\Omega(z)$:

$$\sigma_x + \sigma_y = 4\text{Re}\{\Phi(z)\},$$

$$\sigma_y - i\tau_{xy} = \Phi(z) + \Omega(\bar{z}) + (z - \bar{z})\Phi'(z),$$

where $z = x + iy$ and $\bar{z} = x - iy$. Incorporating equations (4) and (5) into equation (7), the boundary conditions of the crack surface thus can be written in the following form:

$$\Phi^+(t) + \Omega^-(t) = \sigma_y^+ - i\tau_{xy}^+,$$

$$\Phi^-(t) + \Omega^+(t) = \sigma_y^- - i\tau_{xy}^-.$$  

Figure 1: Flowchart for analyzing the fracture behavior of compression-shear crack.

Figure 2: Stress status of an inclined crack.
Adding and subtracting, one obtains
\[ [\Phi(t + \Omega(t))^+ + \Phi(t + \Omega(t))^+] = 2P = 2\left(\sigma_y - i\tau_{xy}\right) \]
\[ = 2\left(\sigma_n - i\tau_f\right), \quad t \in L, \]
(9)
\[ [\Phi(t - \Omega(t))^+ + \Phi(t - \Omega(t))^+] = 2Q = 0, \quad t \in L, \]
(10)
where \( L \) is the crack surface and \( P \) and \( Q \) are known functions on \( L \).

This is a typical Riemann–Hilbert problem, and its general form is as follows:
\[ F^+(t) = gF^-(t) + f. \]
(11)
Assuming \( F(t) = \Phi(t) + \Omega(t) \), when \( g = -1, \ f = 2(\sigma_y - i\tau_{xy}) \), (9) can be obtained.
Assuming \( F(t) = \Phi(t) - \Omega(t) \), when \( g = -1, \ f = 0 \), (10) can be obtained.

Therefore, the two general solutions to this problem can be obtained as follows:
\[ \Phi(z) + \Omega(z) = \frac{1}{mX(z)} \int_X \frac{X(t)Pdt}{L - t - z} + \frac{2P}{X(z)}, \]
\[ \Phi(z) - \Omega(z) = \frac{1}{mX(z)} \int_X \frac{X(t)Qdt}{L - t - z} - \Gamma', \]
(12)
Further, the formula of \( \Phi(z) \) and \( \Omega(z) \) can be written as
\[ \Phi(z) = \frac{1}{2mX(z)} \int_X \frac{X(t)Pdt}{L - t - z} + \frac{P(z)}{X(z)} - \frac{1}{2} \Gamma', \]
\[ \Omega(z) = \frac{1}{2mX(z)} \int_X \frac{X(t)Qdt}{L - t - z} + \frac{P(z)}{X(z)} + \frac{1}{2} \Gamma', \]
(13)
where \( X(z) = \sqrt{z^2 - a^2} \) and \( P(z) = C_0z; \) one obtains
\[ \Phi(z) = \frac{\left(C_0 - (1/2)P\right)z}{\sqrt{z^2 - a^2}} + \frac{1}{2} P - \frac{1}{2} \Gamma', \]
\[ \Omega(z) = \frac{\left(C_0 - (1/2)P\right)z}{\sqrt{z^2 - a^2}} + \frac{1}{2} P + \frac{1}{2} \Gamma', \]
(14)
(15)
Assume that tension stress is positive and compression is negative, where
\[ P = -\frac{1}{2}(1 - i\mu)[\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3)\cos 2\beta], \]
\[ \Gamma' = \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta - i\sin 2\beta, \]
\[ C_0 = \Gamma + \frac{1}{2}\Gamma' = -\frac{1}{4}(\sigma_1 + \sigma_3) - \frac{1}{4}(\sigma_1 - \sigma_3)\cos 2\beta - i\sin 2\beta. \]
(16)

2.2. Solution of Stress Intensity Factor. The stress intensity factors \( K_I \) and \( K_{II} \) of inclined cracks in an infinite plane can be expressed as [33, 34]
\[ K_I - iK_{II} = \lim_{z \to a} 2\sqrt{2\pi(z - a)}\Phi(z). \]
(17)
Thus,
\[ K_I - iK_{II} = -\frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta \]
\[ -\mu[\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3)\cos 2\beta]. \]
(18)

The expressions of \( K_I \) and \( K_{II} \) can be obtained as
\[ K_I = 0, \]
\[ K_{II} = -\frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta \]
\[ -\mu[\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3)\cos 2\beta]. \]
(19)
(20)
It can be seen from equation (19) that for the inclined crack under compression, the mode I stress intensity factor at the crack tip is always 0, indicating that the singular term of mode I crack disappears. It is also consistent with the previous research results [35, 36].

2.3. Stress Field at the Tip of a Closed Crack. For the crack subjected to compression and shear stress, the stress field near the tip of the crack can be calculated through equations (6) and (7), where the complex stress functions \( \Phi(z) \) and \( \Omega(z) \) can be obtained from the calculation of equations (14) and (15). The variables \( z = x + iy \) in the stress functions \( \Phi(z) \) and \( \Omega(z) \) can be expressed by a complex plane coordinate system with the crack centroid as the coordinate origin, as shown in Figure 3.

According to the position of crack in the coordinate system, the relevant variables are defined in the polar form as follows:
\[ r = r_1e^{i\theta}; \]
\[ z - a = r e^{i\theta}; \]
\[ z + a = r_2 e^{i\theta_2}, \]
\[ \sqrt{z^2 - a^2} = \sqrt{(z + a)(z - a)} = \sqrt{r e^{i\theta/2}} \sqrt{r_2 e^{i\theta_2/2}}. \]
(21)
Thus,
\[ \Phi(z) = \frac{-iK_{II}r_1}{2\sqrt{marr_2}} \left[ \cos \left(\theta_1 - \frac{\theta + \theta_2}{2}\right) + i \sin \left(\theta_1 - \frac{\theta + \theta_2}{2}\right) \right] + \frac{p}{2} - \frac{1}{2} \Gamma' \]
(22)
when \( r \) is close to 0, \( \theta_1 = \theta_2 = 0 \), and \( r_1 = a, r_2 = 2a \). A simplified form of the above equation can be derived as follows:
Φ(\(z\)) = \(\frac{K_{II}}{2\sqrt{\pi r}}\left(\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}\right) + \frac{P}{2} - \frac{1}{2}T'\). \hspace{1cm} (23)

Similarly,

\(\Omega(\bar{z}) = \frac{iK_{II}r_1}{2\sqrt{\pi r_2}}e^{-i(\theta_1 - (\theta + \theta_2/2))} + \frac{1}{2}P + \frac{1}{2}T' = \frac{K_{II}}{2\sqrt{2\pi r}}\left(-\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}\right) + \frac{P}{2} + \frac{1}{2}T'\),

\(\Phi'(z) = -a^2\left(C_0 - (P/2)\right)\left(z^2 - a^2\right)^{3/2} - a^2\left(C_0 - (P/2)\right)e^{-i(3/2)(\theta + \theta_2)} = \frac{K_{II}}{4r\sqrt{2\pi r}}\left(\sin \frac{3\theta}{2} + i \cos \frac{3\theta}{2}\right). \hspace{1cm} (24)\)

\(\Phi'(\bar{z}) = \frac{K_{II}}{4r\sqrt{2\pi r}}\left(\sin \frac{3\theta}{2} - i \cos \frac{3\theta}{2}\right).\)

Substituting the complex stress functions \(\Phi(z)\), \(\Omega(\bar{z})\), and \(\Phi'(z)\) into (6) and (7), the combination of stress fields near the tip of crack thus becomes

\(\sigma_x + \sigma_y = 4\text{Re}[\Phi(z)] = 4\text{Re}\left[\frac{-K_{II}}{2\sqrt{2\pi r}}\left(\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}\right) + \frac{P}{2} - \frac{1}{2}T'\right] = -\frac{2K_{II}}{\sqrt{2\pi r}}\sin \frac{\theta}{2} - (\sigma_1 + \sigma_3), \hspace{1cm} (25)\)

\(\sigma_y - i\tau_{xy} = \Phi(z) + \Omega(\bar{z}) + (z - \bar{z})\Phi'(z) = -\frac{K_{II}}{2\sqrt{2\pi r}}\left(\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}\right) + P - \frac{K_{II}}{2\sqrt{2\pi r}}\left(-\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}\right)\)

\(+ (z - \bar{z})\frac{K_{II}}{4r\sqrt{2\pi r}}\left(\sin \frac{3\theta}{2} - i \cos \frac{3\theta}{2}\right). \hspace{1cm} (26)\)

in which \(z - \bar{z} = z - a + a - \bar{z} = re^{i\theta} - re^{-i\theta} = 2ir \sin \theta = 4ir \sin(\theta/2)\cos(\theta/2);\) hence,
\[\sigma_y - i\tau_{xy} = \Phi(z + \Omega(z)) - (z - \overline{z})\Phi'(z)\]

\[= \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} - \frac{1}{2} \left[\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3) \cos 2\beta\right] - \frac{iK_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} + \frac{iK_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} + \frac{i\mu}{2} \left[\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3) \cos 2\beta\right].\]

The solutions of the simultaneous of equations (25) and (27) are as follows:

\[\sigma_x = -\frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2}\right) + T_x + 2\cos^2 \theta,\]

\[\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + T_y,\]

\[\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) + T_{xy} + \frac{1}{2} \sigma_1 \sin \theta + \frac{1}{2} \sigma_3 \sin \theta - \frac{1}{2} T_x \sin \theta + \frac{1}{2} T_y \sin \theta + \frac{1}{2} T_{xy} \sin \theta.\]

The above equations are the expression of the progressive stress field at the crack tip, which can also be simplified as

\[\sigma_r = \frac{K_{II}}{2\sqrt{2\pi r}} \sin \frac{\theta}{2} (3 \cos \theta - 1) + T_x \cos^2 \theta + T_y \sin^2 \theta + T_{xy} \sin 2\theta,\]

\[\sigma_\theta = -\frac{3K_{II}}{2\sqrt{2\pi r}} \sin \theta \cos \frac{\theta}{2} + T_x \sin^2 \theta + T_y \cos^2 \theta - T_{xy} \sin 2\theta,\]

\[\tau_{r\theta} = \frac{K_{II}}{2\sqrt{2\pi r}} \cos \frac{\theta}{2} (3 \cos \theta - 1) + \frac{1}{2} (T_y - T_x) \sin 2\theta + T_{xy} \cos 2\theta,\]

where \(\sigma_r\) is radial normal stress, \(\sigma_\theta\) is tangential normal stress, and \(\tau_{r\theta}\) is shear normal stress. In the new equations of the stress field in polar coordinates, the stress term in each direction consists of singular stress term containing \(K_{II}\) and nonsingular stress term containing \(T\)-stress.

### 3. Effect of \(T\)-Stress on the Tangential Stress and Shear Stress at Crack Tip

#### 3.1. Prediction of Tangential Stress with and without \(T\)-Stress

Equation (31) about the tangential stress can be derived as

\[\sigma_\theta = -\frac{3K_{II}}{2\sqrt{2\pi r}} \sin \theta \cos \frac{\theta}{2} + T_x \sin^2 \theta + T_y \cos^2 \theta - T_{xy} \sin 2\theta,\]

in which \(y = \sqrt{2r_1/a}\), \(r_1\) represents the critical radius of crack zone or fracture process zone of the material and can be regarded as the characteristic material parameters \([37, 38]\). For the same materials, \(r_1\) is a constant and considered to be the radius of a circular zone surrounding the crack tip \([39]\). Ayatollahi and Aliha \([40]\) pointed out that the values of \(r_1\) for marble, limestone, and granite are 0.6, 5.2, and 0.8 mm, respectively. It is assumed that the value of \(r_1\) is 0.8 for granite according to \([40]\) to investigate the tangential stress around crack tip.

Dividing both sides of (33) by \(\sigma_1\), the following equation can be obtained:

\[\frac{\sigma_\theta}{\sigma_1} = -\frac{3K_{II}}{2\sqrt{2\pi a}} \sin \theta \cos \frac{\theta}{2} + \frac{T_x}{\sigma_1} \sin^2 \theta + \frac{T_y}{\sigma_1} \cos^2 \theta - \frac{T_{xy}}{\sigma_1} \sin 2\theta.\]

In order to investigate the influence of the cases with and without \(T\)-stress on the tangential stress, Figure 4 plots the variation regularities of the ratio of the tangential stress to
the axial stress $\sigma_\theta/\sigma_1$ when the confining pressure coefficient is 0, 0.15, 0.3, and 0.5, respectively, under the condition of $a = 10\, \text{mm}$, $\beta = 30^\circ$, and $\mu = 0.15$. It is worth noting that both tensile tangential stresses and compressive tangential stresses at the crack tip are considered in this paper.

In general, the tensile tangential stresses of the cases with and without $T$-stress both decrease with the increasing of confining pressure coefficient $\lambda$, while the compressive tangential stresses increase with $\lambda$. As can be seen from Figure 4, only tensile tangential stress exists at the crack tip for the case without $T$-stress, which is obviously inconsistent with the actual stress situation and significantly different from the simultaneous existence of tensile and compressive tangential stress for the case with $T$-stress. Additionally, Figure 4(d) shows that tensile tangential stress disappears at $\lambda = 0.5$ for the case with $T$-stress. This phenomenon can be attributed that the confining pressure $\sigma_3$ not only increases the pressure between the crack surfaces, but also limits the relative sliding between them. The angles corresponding to the maximum tensile tangential stress are approximately $70^\circ$ and $-70^\circ$ ($290^\circ$) for the case without $T$-stress, whilst only $70^\circ$ for the case with $T$-stress. Under the same confining pressure coefficient, the range of tensile tangential stress for the case without $T$-stress is larger than that for the case with $T$-stress. This has also demonstrated that the compressive $T$-stress on the crack surface has an inhibitory effect on the crack initiation.

3.2. Prediction of Shear Stress with and without $T$-Stress. Dividing both sides of the (32) by the major principal stress $\sigma_1$, the following equation can be derived:
\[
\frac{T_{r\theta}}{\sigma_1} = \frac{1}{2y} \frac{K_{II}}{\sqrt{\pi a}} \cos \theta \left(3 \cos \theta - 1\right)
\]
\[
+ \frac{1}{2} \frac{T_y - T_x}{\sigma_1} \sin 2 \theta + \frac{T_{xy}}{\sigma_1} \cos 2 \theta. \tag{35}
\]

Similarly, it is assumed that the half-length of the crack \(a = 10\) mm, the inclination angle \(\beta = 30^\circ\), and the friction coefficient \(\mu = 0.15\), the variation of the ratio of shear stress to the major principal stress \(T_{r\theta}/\sigma_1\) with angle \(\theta\) is drawn when considering the confining pressure coefficient \(\lambda = 0, 0.15, 0.3, \) and 0.5, respectively, as depicted in Figure 5.

As illustrated in Figure 5, for the cases with and without \(T\)-stress, both positive shear stress and negative shear stress exist simultaneously at the crack tip, and the area of shear stress zone decreases for the two cases as the increasing of confining pressure coefficient. Under the same confining pressure coefficient, the absolute values of the maximum shear stress and the minimum shear stress for the case with \(T\)-stress are larger than those without \(T\)-stress, and the angles corresponding to the maximum shear stress and the minimum shear stress are also different between the two cases. It can be concluded that the angle corresponding to the maximum absolute value of shear stress without \(T\)-stress is always 0, which is inconsistent with the existing test results [26–28, 30, 31, 41].

4. Criterion of Crack Initiation considering \(T\)-Stress

4.1. Basis and Premise of the Criterion. The crack initiation of brittle rock-like materials subjected to compression is explored in numerous experimental studies, and the failure mode of crack initiation can be broadly classified into three types: (i) tensile failure; (ii) shear failure; and (iii) tensile and shear failure occurring simultaneously. Tensile failure, as the main failure mode, has been confirmed by many scholars in theory and experiment. However, the research on generation conditions of shear failure is relatively limited, although many experiments have revealed that the shear failure mode often appears in the failure process of sample with preexisting crack. Therefore, it is worthwhile attempting to study the crack initiation under far-field compression considering both tensile and shear failure criteria.

As described in Section 3.1, the stress field at the crack tip is controlled by tensile stress in most of the angular ranges, while being controlled by compressive stress in a smaller range. It can be seen from Figures 4 and 5 that when the confining pressure coefficient is large to a certain extent, the tensile tangential stress at the crack tip disappears, whilst the shear stress at the tip still exists. In this case, the crack initiation cannot be studied according to the maximum tensile tangential stress criterion. Previously, scholars mainly studied the effect of tensile tangential stress on the crack initiation, while the effect of shear stress was ignored. The shear stress occurs when the shear stress reaches the ultimate shear strength of the material. Therefore, the crack initiation can be predicted by judging whether the shear stress reaches its ultimate shear strength.

4.2. Criterion of Crack Initiation under Far-Field Compression-Compression Combination. Building on these views and prior work, both tangential stress \(\sigma_{r\theta}\) and shear stress \(T_{r\theta}\) are attempted to be considered simultaneously as the conditions for judging the initiation crack in the case with \(T\)-stress. The loading path is shown in Figure 6. It is assumed that the infinite plate is subjected to the same far-field confining pressure \(\sigma_1 = \sigma_3\) in the initial state; then \(\sigma_2\) keeps constant and \(\sigma_1\) gradually increases until the crack begins to initiate.

The variation of the confining pressure coefficient \(\lambda\) gradually approaches from 1 in the initial state to 0 in the final state. The maximum tangential stress and shear stress under different confining pressure coefficient need to be calculated in order to construct the predication lines such as that given in Figure 7. It is worth noting that, at the beginning of loading, the shear normal stress on the crack surface is less than the frictional resistance \(T_x < T_y\) due to the large confining pressure coefficient, and the shear force cannot drive the relative sliding of the crack surfaces. In this case, there is no stress concentration at the crack tip, and the stress concentration occurs at the crack tip only after the confining pressure coefficient is less than a certain value; the stress field at the crack tip can be calculated through equations (30)–(32).

In the case of constant friction coefficient, it is known from the above description that when the confining pressure coefficient exceeds a certain value, the tensile tangential stress at the crack tip will disappear, and only the compressive tangential stress exists. Therefore, the tangential stress is compressive at the beginning of the loading and gradually changes from compressive state to tensile state during loading. For the shear stress, both maximum shear stress and minimum shear stress exist in the loading process. The maximum shear stress is positive and gradually increases with the loading progresses; that is, the confining pressure coefficient is gradually reduced; the minimum shear stress is negative and gradually decreases with the loading progresses, as shown in Figure 7.

In Figure 7, \(\sigma_{\text{max}}\) is the maximum tangential stress, \(\tau_{r\theta\text{max}}\) the maximum shear stress, \(\tau_{r\theta\text{min}}\) the minimum shear stress, \(\sigma_{r\theta}\) the ultimate tensile strength, and \(\tau_{r\theta}\) the ultimate shear strength. \(\lambda_\theta\) is the confining pressure coefficient corresponding to the maximum tangential stress reaching the ultimate tensile strength, and \(\lambda_{r\theta}\) is the confining pressure coefficient corresponding to the maximum or minimum shear stress reaching the ultimate shear strength. It can be seen from Figure 7 that the failure mode of crack initiation depends on the magnitude between \(\lambda_\theta\) and \(\lambda_{r\theta}\) as the loading progresses.

When \(\lambda_\theta > \lambda_{r\theta}\), the maximum tangential stress reaches the ultimate tensile strength earlier, and the tensile failure thus occurs at crack tip under \(\lambda = \lambda_\theta\). In this condition, the maximum tangential stress criterion should be utilized to predict the initiation angle of crack, and the crack initiation angle \(\theta\) of tensile crack should simultaneously satisfy the following equation:
Figure 5: The ratio $\tau_{r\theta}/\sigma_1$ at crack tip under different confining pressure coefficients. (a) $\mu = 0.15$, $\lambda = 0$. (b) $\mu = 0.15$, $\lambda = 0.15$. (c) $\mu = 0.15$, $\lambda = 0.3$. (d) $\mu = 0.15$, $\lambda = 0.5$.

Figure 6: Variation of confining pressure coefficient with loading process.
\[ \frac{\partial \sigma_{\theta}}{\partial \theta} = 0, \]
\[ \frac{\partial^2 \sigma_{\theta}}{\partial \theta^2} < 0, \]
\[ \sigma_{\theta}^{\text{max}} \geq \sigma_{\theta}^{\text{max}}. \]

When \( \lambda_{\theta} < \lambda_{\theta} \), the shear stress reaches the ultimate shear strength earlier, and the shear failure thus occurs at crack tip under \( \lambda = \lambda_{\theta} \). The maximum shear failure stress criterion should be adopted to predict the initiation angle of crack. The assumptions of shear stress failure criterion are as follows: (i) the positive and negative shear stress \( \tau_{\theta} \) only represent the difference in the direction of stress; (ii) the shear failure starts when the absolute value of the shear stress \( |\tau_{\theta}| \) reaches the ultimate shear strength \( \tau_{\theta}^\text{u} \). According to the maximum shear stress failure criterion, the crack initiation angle \( \theta \) of shear crack should simultaneously satisfy the following equation:

\[ \frac{\partial \tau_{\theta}}{\partial \theta} = 0, \]
\[ \tau_{\theta}^{\text{max}} \geq \tau_{\theta}^{\text{u}}, \]
\[ \tau_{\theta}^{\text{max}} \geq - \tau_{\theta}^{\text{u}}. \]

When \( \partial^2 \tau_{\theta}/\partial \theta^2 > 0 \), the maximum value \( \tau_{\theta}^{\text{max}} \) can be obtained; when \( \partial^2 \tau_{\theta}/\partial \theta^2 > 0 \), the minimum value \( \tau_{\theta}^{\text{min}} \) can be derived.

In particular, when \( \lambda_{\theta} = \lambda_{\theta} \), the maximum tangential stress and shear stress reach their respective ultimate strengths, and both tensile failure and shear failure thus occur simultaneously at the crack tip under \( \lambda = \lambda_{\theta} = \lambda_{\theta} \). In this case, both the maximum tangential stress failure criterion and maximum shear stress failure criterion are applicable to predict the initiation angle \( \theta \) of crack.

5. Compression Tests of Rock-Like Material with Preexisting Crack

5.1. Comparison of Mortar and Rock. In order to evaluate the fracture behavior of rock-like materials with preexisting crack for the case with \( T \)-stress, the mix proportion of mortar was designed in accordance with the basic mechanical properties of granite. The mortar could be deemed as a rock replica only when the basic mechanical properties of mortar are comparable to the granite rock [42]. Through various repeated tests, analyses, and improvements, the mix proportion of the mortar was determined, as listed in Table 1. The basic mechanical properties of the mortar are shown in Table 2.

An extensive literature review [45–62] is conducted in this paper to investigate the basic mechanical parameters of granite so as to compare the difference of mechanical properties between mortar samples and granite. The deformation index proposed by Deere and Miller [43] and the brittleness index specified by Tatone [44] are used to evaluate the similarity of the basic mechanical properties between the prepared mortar samples and granite. Figures 8 and 9 show the comparison between deformation index and brittleness index from the statistical data of granite and experimental data of the rock replica in this paper, respectively.

It can be seen from Figures 8 and 9 that the experimental data of the mortar all fall in the region where the statistical data of granite are intensive. Additionally, the brittleness index and the deformation index are also adopted to evaluate the similarity, and the difference between mortar and granite in terms of the Poisson’s ratio, dry density, and friction angle between mortar and granite are also compared in Figure 10. It shows that the dotted line representing the three parameters of mortar is in the middle of the maximum and minimum values of the three parameters of granite. According to the data information in Figures 8–10, it can be concluded that the basic mechanical properties of the prepared mortar sample are basically similar to those of granite,
so it is reasonable to simulate the granite rock by using the mortar.

### 5.2. Test Method and Equipment

Two kinds of prismatic samples were produced according to the mix proportion in Table 1, with the dimensions of 175 mm × 70.7 mm × 41 mm, the inclination angles $\beta = 30^\circ$ and $\beta = 60^\circ$ of preexisting crack. The crack was prefabricated by the sharp steel blade with the width of 10 mm, and the thickness of steel blade is only 0.4 mm so that the surfaces of crack can be fully contacted and closed during loading process. Similar to intact samples, triaxial compression tests were carried out on the samples with preexisting crack under confining pressures of 5, 20, and 40 MPa, respectively. Before the test, three layers of PTFE sheets with a thickness of 0.1 mm were placed between the indenter and the upper as well as lower ends of the sample, and lubricant was also added into each layer of PTFE sheets so as to reduce the friction between the indenter and the end of sample. Besides, one layer of thermoplastic sleeve is wrapped around the sample to prevent the hydraulic oil from contacting with the sample. The test device and the geometric dimensions of sample were shown in Figures 11 and 12, respectively.

By plotting the Mohr circles of the intact samples under various confining pressures, the failure envelope line of the sample can be obtained, and the ultimate shear strength of the material under each confining pressure can be determined from the tangent points between the envelope and the Mohr circles, as shown in Figure 13. The ultimate shear strength values of the mortar under confining pressure at 5, 20, and 40 MPa are 62.87, 95.26, and 120.32 MPa, respectively. The friction coefficient of the crack surface can be taken as $\mu = 0.4$ as specified in [63], and the critical radius of granite can be taken as $r_c = 0.8$ mm according to Ayatollahi and Aliha [40].

### 5.3. Comparison between Experimental and Theoretical Results

As described in Section 4, the maximum tangential stress and maximum shear stress can be calculated through equations (31) and (32), and the crack is assumed closed during loading process. Figure 14 shows the variation of the maximum circumferential stress and the maximum shear stress of the samples with the inclination angle $\beta = 30^\circ$ and $60^\circ$ under different confining pressure coefficients. Three corresponding confining pressure coefficients and inclination angle were marked in figure when the maximum tangential stress and shear stress reach their respective ultimate strengths. The three confining pressure coefficients at the crack initiation are listed in order from large to small, corresponding to three different prediction results 1, 2, and 3 in the figure, and the order represents the probability magnitude of the initiation form at the crack tip.

Figure 14 illustrates that when the confining pressure is 5, 20, and 40 MPa, the confining pressure coefficient $\lambda_{r\theta}$ corresponding to the shear stress reaching its ultimate strength is greater than the confining pressure coefficient $\lambda_{\theta}$ corresponding to the tangential stress reaching its ultimate strength during the loading process of this rock-like material, demonstrating that the crack initiation at the tip is in the form of shear. It is also shown that the absolute values of the positive shear stress and the negative shear stress are almost equal under the same confining pressure coefficient, and the corresponding confining pressure coefficients when the positive shear stress reach the ultimate strength are largely the same as those when the negative shear stress reaches the ultimate strength.

Figure 15 shows the directions of inclination angles after the test (indicated by the white dashed line) and theoretically predicted directions of inclination angles (indicated by the red dashed line) at crack tips, where $\theta_e$ and $\theta_p$ represent the experimental and predictive value of inclination angle at

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**Table 1**: Mix proportion of the mortar.

| Materials                     | Value |  
|-------------------------------|-------|
| Portland cement P-O52.5R     | 1     |
| Quartz sand (0.16–0.63 mm)   | 1     |
| Water                         | 0.2   |
| Quartz powder (<80 μm)       | 0.27  |
| Silica fume (0.1 μm)         | 0.18  |
| FAQ defoamer                  | 0.001 |
| Water reducer                 | 0.018 |

**Table 2**: Mechanical properties of the mortar.

| Properties                          | Mean value |
|-------------------------------------|------------|
| Uniaxial compressive strength ($\sigma_{uc}$) | 143.52 MPa |
| Triaxial compressive strength ($\sigma_t$)   |            |
| $\sigma_1 = 5$ MPa                   | 180.41 MPa |
| $\sigma_2 = 20$ MPa                  | 252.18 MPa |
| $\sigma_3 = 40$ MPa                  | 322.90 MPa |
| Indirect tensile strength ($\sigma_i$)    | 9.53 MPa   |
| Elastic modulus ($E$)                  | 35.32 GPa  |
| Poisson’s ratio ($\nu$)                | 0.18       |
| Dry density ($\rho$)                   | 2.45 g/cm$^3$ |
| Friction angle ($\phi$)                | 37.87°     |

![Figure 8: Comparison between the mortar and granite based on deformation index $E/\sigma_{uc}$](image)}
crack tip, respectively. The test results of inclination angles at the two tips are, respectively, marked in the figure, since the stress fields of the two tips are symmetrical about the crack centroid center, three theoretical prediction results of crack initiation angles are marked only on tip 1.

It can be seen from Figure 15 that the failure modes of the two types of samples are shear failure under the action of three confining pressures. By comparing the confining pressure coefficients when the crack initiating, it can also be determined that the initiation form of crack tip is shear mode.

Figure 16 shows the comparison between the test results and the theoretical predictions considering the $T$-stress. The dashed line represents the theoretical value of initiation angle obtained by using the maximum tangential stress without considering the $T$-stress. For the samples with inclination angle $\beta = 30^\circ$, the theoretical prediction results 1 of crack initiation angles at the two tips are all well in agreement with the test results. For the samples with inclination angle $\beta = 60^\circ$, the theoretical prediction results 1 of crack initiation angles are only in good agreement with the
**Figure 12:** Geometric dimensions of rock-like material samples with preexisting crack.

**Figure 13:** Mohr circles and failure envelopes of intact samples under different confining pressures.

**Figure 14:** Continued.
Figure 14: The changes of $\sigma_{\theta}^{max}$, $\tau_{\theta\theta}^{min}$, and $\tau_{\theta\theta}^{max}$ of two different samples under different confining pressures. (1) $\beta = 30^\circ$, (2) $\beta = 60^\circ$ (a) $\sigma_\lambda = 5$ MPa (1) $\beta = 30^\circ$, (2) $\beta = 60^\circ$ (b) $\sigma_\lambda = 20$ MPa (1) $\beta = 30^\circ$, (2) $\beta = 60^\circ$ (c) $\sigma_\lambda = 40$ MPa.
Test results at the confining pressure of 5 and 20 MPa. As for the confining pressure of 40 MPa, reverse oblique shear cracks exhibit at the tips, and the initiation angle is in better accordance with the results of the theoretical results of 2. Since the confining pressure coefficients corresponding to the prediction results 1 and 2 are almost same, the possibility

![Graphs and images showing test results and theoretical predictions.](image-url)

Figure 16: Comparison between the test results and theoretical predictions of initiation angle. (a) \( \beta = 30^\circ \). (b) \( \beta = 60^\circ \).
of the occurrence of 2 cannot be excluded; this may be related to the uneven force on the sample under high confining pressure.

The failure modes of the crack tip in the test of this paper are almost similar to the results of previous researches [26–28]. However, it is worth noting that due to the stochastic mechanical properties of rock-like materials, other failure modes that are different from the test results in this paper cannot be excluded.

6. Conclusions

The fracture behavior of compression-shear crack by theoretical and experimental studies is presented in this paper. In the case of considering T-stress, an improved crack initiation criterion proposed can accurately predict the initiation angles and failure modes of compressive-shear crack. The research results provide a theoretical and experimental guidance to analyze the damage tolerance of structure made of rock materials accurately and quantitatively. The following conclusions can be drawn.

(1) The expression of the stress field at the crack tip is established by using the complex potential theory based on the stress boundary conditions on the surfaces of the compression-shear crack. Compared with the traditional fracture theory which fails to consider the nonsingular stress term (T-stress) at the crack tip, proposed stress field expression includes both the singular term containing stress intensity factor \( K_\text{II} \) and the nonsingular stress terms containing \( T_x \), \( T_y \), and \( T_{xy} \).

(2) Only tensile stress zone exists in the circumferential direction near the crack tip for the case without T-stress, while both compressive and tensile stress zones exist in the circumferential direction when considering the T-stress. As the confining pressure coefficient increases, the compressive stress zone gradually increases, whilst the tensile stress zone gradually decreases or even disappears. Similarly, the shear stress zone gradually decreases with the increase of confining pressure coefficient. The maximum shear stresses and the corresponding angles under different confining pressure coefficients for the case with T-stress are significantly different from those when ignoring T-stress.

(3) Under the consideration of T-stress, a crack initiation criterion considering the maximum tangential stress and the maximum shear stress simultaneously is proposed. By comparing the theoretical predictions of the criterion with the triaxial test results of the rock-like material with preexisting crack, the crack initiation criterion can predict the crack initiation angles and failure modes of the rock-like material with different inclination angles more accurately than the conventional maximum tangential stress criterion.

Data Availability

No data were used to support this study

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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