Estimating the evolution of gas in the Fornax dwarf spheroidal galaxy from its star formation history: an illustrative example

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ABSTRACT

We propose that detailed data on the star formation history of a dwarf spheroidal galaxy (dSph) may be used to estimate the evolution of the total mass $M_{\text{g}}(t)$ for cold gas in its star-forming disk. Using Fornax as an illustrative example, we estimate its $M_{\text{g}}(t)$ and the corresponding net gas flow rate $\Delta F(t)$ assuming a global star formation rate $\psi(t) = \lambda_s(t)[M_{\text{g}}(t)/M_\odot]^{\alpha}$ consistent with observations of nearby galaxies. We identify the onset of the transition in $\Delta F(t)$ from a net inflow to a net outflow at the time $t_{\text{sat}}$ at which the Fornax halo became a Milky Way satellite and estimate the evolution of its total mass $M_{\text{h}}(t)$ at $t < t_{\text{sat}}$ using the median halo growth history in the $\Lambda$CDM cosmology and its present mass within the half-light radius derived from observations. We examine three different cases of $\alpha = 1, 1.5, \text{and} 2$, and justify the corresponding $\lambda_s(t)$ by comparing the gas mass fraction $f_g(t) = M_g(t)/M_h(t)$ at $t < t_{\text{sat}}$ with results from simulations of gas accretion by halos in a reionized universe. We find that the Fornax halo grew to $M_h(t_{\text{sat}}) \approx 2 \times 10^9 M_\odot$ at $t_{\text{sat}} \sim 5$ or $8$ Gyr, in broad agreement with previous studies using data on its stellar kinematics and its orbital motion. We describe qualitatively the evolution of Fornax as a satellite and discuss potential extension of our approach to other dSphs.

Key words: galaxies: dwarf — galaxies: evolution — galaxies: formation — galaxies: individual (Fornax dwarf galaxy) — Local Group.

1 INTRODUCTION

As a result of hierarchical structure formation, the cold dark matter (CDM) halo associated with a main galaxy has many subhalos. In principle, this can account for the dwarf spheroidal galaxies (dSphs) orbiting the Milky Way (MW). However, due to the complicated gas dynamics involved in star formation and the resulting feedback, it is far from straightforward to associate the subhalos found in CDM simulations with the actual dSphs possessing stellar populations. A prominent example is the so-called too-big-to-fail (TBTF) problem: the largest subhalos of an MW-like halo appear unsuitable for hosting the brightest MW dSphs because of incompatible mass distributions (e.g., [Boylan-Kolchin, Bullock & Kaplinghat 2011]). Proposed solutions to this problem mostly call for either modifying the nature of DM (e.g., [Herpich et al. 2014] or including baryons and the associated gas dynamics in CDM simulations (see [Pontzen & Governato 2014] for a review and [Sawala et al. 2014] for recent simulation results on the Local Group galaxies). It was also shown that there might not be a TBTF problem if the CDM halo of the MW has a relatively low mass of $\approx 10^{12} M_\odot$ (e.g., [Wang et al. 2012]). In any case, gas dynamics is the crucial bridge linking DM halos with observed galaxies.

In this paper we propose an empirical approach to study halo evolution and global gas dynamics of dSphs. We use Fornax, the brightest MW dSph, as an illustrative example. Our approach is in the same spirit as that of [Qian & Wasserburg 2012], who used the data on metallicity distributions of MW dSphs to infer the generic evolution of their gas. We show that the global star formation rate (SFR) $\psi(t)$ of Fornax derived from observations by de Boer et al. (2012b) may be used to estimate its total gas mass $M_{\text{g}}(t)$ and the corresponding net gas flow rate $\Delta F(t)$ as functions of time $t$. We
consider two regimes of gas evolution separated by the time \( t_{\text{sat}} \), at which Fornax ceased evolving independently and became a MW satellite. We assume \( \psi(t) = \lambda(t)[M_g(t)/M_\odot]^\alpha \) with \( \lambda(t) = \lambda(t_{\text{sat}}) \) for \( t > t_{\text{sat}} \). We consider that the accretion of Fornax by the MW caused the former’s global gas flow to change rapidly from a net inflow to a net outflow and identify the onset of this transition in \( \Delta F(t) \) as \( t_{\text{sat}} \). We assume that the total mass \( M_g(t) \) of the Fornax halo at \( t < t_{\text{sat}} \) follows the median halo growth history in the \( \Lambda \)CDM cosmology and determine \( M_h(t_{\text{sat}}) \) by requiring that the corresponding density profile give the mass enclosed within the half-light radius as derived from observations.

Considering the central role of the assumed global star formation law (SFL) \( \psi(t) = \lambda(t)[M_g(t)/M_\odot]^\alpha \) in our approach, we examine three different cases of \( \alpha = 1, 1.5, \) and 2, and justify the corresponding \( \lambda(t) \) by comparing the gas mass fraction \( f_g(t) = M_g(t)/M_h(t) \) at \( t < t_{\text{sat}} \) with the results from cosmological simulations of gas accretion by halos in a reionized universe. For each case we also perform statistical realizations of the data on \( \psi(t) \) to gauge how uncertainties in \( \psi(t) \) affect our results, especially \( \Delta F(t) \). Using representative results for the baseline case of \( \alpha = 1.5 \), we give a qualitative description of Fornax’s overall gas evolution, especially its gas loss through a combination of ram-pressure stripping and tidal interaction with the MW at \( t > t_{\text{sat}} \).

This paper is organized as follows. In §2 we discuss theoretical and empirical SFLs and motivate our assumed SFL for dSphs. In §3 we use Fornax as an illustrative example to estimate \( M_h(t), \Delta F(t), t_{\text{sat}} \), and \( M_h(t_{\text{sat}}) \) from its star formation history (SFH), approximately taking into account uncertainties in the SFH. We also compare our estimates of \( t_{\text{sat}} \) and \( M_h(t_{\text{sat}}) \) with relevant results from previous studies, justify choices of \( \lambda(t) \) by considering gas evolution at \( t < t_{\text{sat}} \), and examine how our results are affected by our assumed SFLs. In §4 we qualitatively relate \( \Delta F(t) \) at \( t > t_{\text{sat}} \) to the orbital motion of Fornax as an MW satellite, and discuss how it lost all of its gas through a combination of ram-pressure stripping and tidal interaction with the MW. We summarize our results and discuss potential extension of our approach to other dSphs in §5.

2 SFLS AND STAR FORMATION IN A DSPH

An in-depth study of the halo evolution and the associated gas dynamics of a dSph would require simulations that incorporate hierarchical structure formation in the \( \Lambda \)CDM cosmology and treat a wide range of physical processes, such as gas accretion by the hosting halo before and after reionization of the universe, cooling and condensation of gas to form stars, conversion of cold into hot gas by radiation and supernova (SN) explosion, and gas expulsion from the halo (see e.g., Somerville & Davé 2015 for a review). A representative example of this approach is the recent work of Shen et al. (2014), who simulated the formation and evolution of seven field dwarf galaxies with present-day virial masses of \( M_{\text{vir}} \approx 4.4 \times 10^8 - 3.6 \times 10^{10} M_\odot \). Our approach here is empirical and phenomenological, especially regarding star formation and gas inflows and outflows. Central to our approach is the SFL relating the SFR to gas properties. Therefore, we start with a brief discussion of theoretical and empirical SFLs (see e.g., Kennicutt & Evans 2012 Krumholz 2014 for comprehensive reviews).

2.1 Theoretical and empirical SFLs

Star formation is most directly associated with molecular gas (see e.g., Gao & Solomon 2004 and Krumholz & Thompson 2007 for discussion of observations using different molecular tracers). For a cloud with a total mass \( M_{\text{mol}} \) of molecular gas (predominantly consisting of \( H_2 \) molecules and the associated He atoms), a theoretical estimate of the SFR (e.g., Krumholz & Tan 2007) is

\[
dM_g/\text{dt} = \epsilon_{\text{ff}} M_{\text{mol}}/t_{\text{ff}},
\]

where \( M_* \) is the mass of stars formed,

\[
t_{\text{ff}} \equiv \sqrt{\frac{3\pi}{32G\rho_{\text{mol}}}}
\]

is the free-fall time at the density \( \rho_{\text{mol}} \) of the molecular gas, \( G \) is the gravitational constant, and \( \epsilon_{\text{ff}} \) is the fraction of the molecular gas turned into stars during one free-fall time. Krumholz & Tan (2007) obtained \( \epsilon_{\text{ff}} \sim 10^{-2} \) from observations covering a wide range of \( M_{\text{mol}} \equiv \rho_{\text{mol}}/M_{\text{p}} \sim 10^{-2} - 10^{-5} \text{ cm}^{-3} \), where \( M_p \) is the proton mass. An alternative form of Eq. (1) is

\[
\Sigma_{\text{SFR}} = \epsilon_{\text{ff}} f_{H_2} \Sigma_g/t_{\text{ff}},
\]

where \( \Sigma_{\text{SFR}} \) is the SFR per unit area, \( \Sigma_g \) is the surface mass density of gas in all forms, and \( f_{H_2} \) is the fraction of \( H \) mass in \( H_2 \) molecules. As star formation typically occurs in gaseous disks, Eq. (3) is more convenient to use, especially for comparison with observations. Both Eqs. (1) and (3) can be applied to individual star-forming regions and in the disk-averaged form, to individual galaxies (see review by Krumholz 2014). For illustration, we compare them with two observational results below.

From observations of nearby galaxies on sub-kpc scales, Bigiel et al. (2008) obtained an SFL that applies to local regions within a galaxy:

\[
\frac{\Sigma_{\text{SFR}}}{M_\odot \text{ yr}^{-1} \text{ kpc}^{-2}} = 10^{-2.1 \pm 0.2} \left( \frac{\Sigma_{H_2}}{10 M_\odot \text{ pc}^{-2}} \right)^{1.0 \pm 0.2},
\]

where \( \Sigma_{H_2} \) is the surface mass density of \( H_2 \) molecules in a star-forming region. The above SFL covers \( \Sigma_{H_2} \approx 3 - 50 M_\odot \text{ pc}^{-2} \). Taking the central values of the fitting parameters, we can rewrite it as \( dM_g/\text{dt} \sim M_{\text{mol}}/t_{\text{depl}} \sim 1.4M_{H_2}/t_{\text{depl}} \), where \( M_{H_2} \) is the total mass of \( H_2 \) molecules and \( t_{\text{depl}} \sim 2 \text{ Gyr} \) is the time needed for star formation to deplete all the molecular gas. This depletion time can be understood from \( t_{\text{depl}} \sim t_{\text{ff}}/\epsilon_{\text{ff}} \) [see Eq. (1)]. The star-forming regions studied by Bigiel et al. (2008) were resolved to 750 pc. Assuming a reasonable scale-height \( H \sim 100 \text{ pc} \) for such regions with a typical \( \Sigma_{H_2} \sim 10 M_\odot \text{ pc}^{-2} \), we obtain \( \rho_{\text{mol}} \sim 1.4\Sigma_{H_2}/H \sim 0.14 M_\odot \text{pc}^{-3} \rho_{\text{mol}} \sim 5.7 \text{ cm}^{-3} \), which corresponds to \( t_{\text{depl}} \sim 22 \text{ Myr} \) [see Eq. (2)] or \( t_{\text{depl}} \sim 2.2 \text{ Gyr} \) for \( \epsilon_{\text{ff}} \sim 10^{-2} \).

Based on observations of 61 normal spiral galaxies and 36 starburst galaxies, Kennicutt (1998) found a tight correlation between \( \Sigma_{\text{SFR}} \) and \( \Sigma_g \), both of which are averaged over the disk of an individual galaxy. This global SFL is
\[ \frac{\Sigma_{\text{SFR}}}{M_\odot \text{ yr}^{-1} \text{kpc}^{-2}} = (2.5 \pm 0.7) \times 10^{-4} \left( \frac{\Sigma_g}{M_\odot \text{ pc}^{-2}} \right)^{1.4 \pm 0.15}, \]  

where \( \Sigma_g \) ranges from \( \sim 10 \) to \( 10^4 M_\odot \text{ pc}^{-2} \) for the galaxies used. The power-law form \( \Sigma_{\text{SFR}} \propto \Sigma_g^a \) was first suggested by [Schmidt, 1955], and Eq. (5) is commonly referred to as the Kennicutt-Schmidt law. [Kennicutt, 1998] also found an alternative form of this SFL:

\[ \frac{\Sigma_{\text{SFR}}}{M_\odot \text{ yr}^{-1} \text{kpc}^{-2}} = 1.7 \times 10^{-2} \left( \frac{\Sigma_g}{M_\odot \text{ pc}^{-2}} \right) \left( \frac{\text{yr}}{\tau_{\text{syn}}} \right), \]

where \( \tau_{\text{syn}} \) is the dynamical timescale taken to be the orbit time at half of the outer radius of the star-forming disk and ranges from \( \sim 10^5 \) to \( 10^8 \) yr. Detailed models to explain Eqs. (5) and (6) were given by [Krumholz & McKee, 2005]. Here we consider the limit of \( \Sigma_g \gtrsim 10^7 M_\odot \text{ pc}^{-2} \), for which gas predominantly consists of H\(_2\) molecules [Krumholz & McKee, 2005] and Eq. (3) can be applied to give \( \Sigma_{\text{SFR}} \sim \epsilon_H \Sigma_g/\tau_{\text{ff}} \). Evaluating \( \tau_{\text{ff}} \) at the gas density \( \rho_g / M_\odot \text{ pc}^{-2} \), we obtain

\[ \frac{\Sigma_{\text{SFR}}}{M_\odot \text{ yr}^{-1} \text{kpc}^{-2}} \sim 1.2 \times 10^{-4} \left( \frac{\epsilon_H}{10^{-2}} \right) \left( \frac{100 \text{ pc}}{\mathcal{H}} \right)^{0.5} \times \left( \frac{\Sigma_g}{M_\odot \text{ pc}^{-2}} \right)^{1.5}, \]

which is in good agreement with Eq. (5) for \( \epsilon_H \sim 10^{-2} \) and disk scale-heights of \( \mathcal{H} \sim 100 \text{ pc} \). A similar argument was used by [Kennicutt, 1998] to explain qualitatively the power-law form of Eq. (5).

### 2.2 Examples of treating star formation in galaxy simulations

One way to treat star formation in modeling galaxies is to use Eq. (1) or its equivalent, Eq. (3). In either case, estimates of \( f_{\text{H}_2} \) are required. For systems with sufficiently high SFRs, \( f_{\text{H}_2} \) depends on \( \Sigma_g \), the metallicity, and the clumpiness of gas [Krumholz, McKee & Tumlinson, 2009]. For systems with very low SFRs, \( f_{\text{H}_2} \) also depends on \( \Sigma_{\text{SFR}} \) [Krumholz, 2013]. Incorporating all the above dependences of \( f_{\text{H}_2} \) [Krumholz, 2013] showed that Eq. (3) provides a good description of the data on star formation for conditions ranging from poor to rich in molecular gas.

However, many simulations of galaxy formation do not evaluate \( f_{\text{H}_2} \) in implementing Eq. (1) or (3) for star formation (see e.g., the review by Somerville & Davé, 2015). Instead, stars are assumed to form when gas has cooled below some temperature \( T_{\text{max}} \) and condensed to some threshold number density of atoms \( n_{\text{min}} \). For example, [Stinson et al., 2007] chose \( T_{\text{max}} = 1.5 \times 10^4 \text{ K} \) and \( n_{\text{min}} = 0.1 \text{ cm}^{-3} \). For a region with a total mass of \( M_g \) of the gas that satisfies the conditions and has a dynamic timescale \( \tau_{\text{dyn}} \propto (G\rho_g)^{-1/2} \), stars were assumed to form stochastically at a rate

\[ \frac{dM_\ast}{dt} = 0.05 \frac{M_g}{\rho_g \tau_{\text{dyn}}}, \]

In comparing the above equation with Eq. (1), \( M_g \) is a proxy for \( M_{\text{mol}} \) while the numerical coefficient and \( \tau_{\text{dyn}} \) play the roles of \( \epsilon_H \) and \( \tau_{\text{ff}} \), more. More recent simulations of [Shen et al., 2014] adopted \( T_{\text{max}} = 10^4 \text{ K} \), \( n_{\text{min}} = 10^2 \text{ cm}^{-3} \), and an SFR per unit volume

\[ \frac{d\rho_\ast}{dt} = 0.1 \frac{\rho_\ast}{\tau_{\text{dyn}}}, \]

where \( \rho_\ast \) is the density of stars formed. The above SFL is qualitatively the same as Eq. (5).

### 2.3 Assumed dSph SFL based on gas mass

As our goal is to gain some understanding of the overall picture for halo evolution and gas dynamics of dSphs in general and Fornax in particular, we are interested in the relation of a dSph’s global SFR to its properties on the galactic scale. While we recognize the important role of molecular gas in the making of stars, our simple approach here will focus on the relative amount of cold gas in atomic and molecular forms, which dominates the total gas mass in the star-forming disk. Based on observations of seven nearby spiral galaxies resolved to 750 pc, [Bigiel et al., 2008] found that for the range of SFR \( \sim 1-10^4 M_\odot \text{ pc}^{-2} \) for the net surface mass density of H atoms and H\(_2\) molecules in star-forming regions, \( \Sigma_{\text{SFR}} \) can be described by

\[ \frac{\Sigma_{\text{SFR}}}{M_\odot \text{ yr}^{-1} \text{kpc}^{-2}} = 10^{-2.39 \pm 0.08} \left( \frac{\Sigma_g}{10 M_\odot \text{ pc}^{-2}} \right)^{1.85 \pm 0.70}. \]

More recently, [Roychowdhury et al., 2015] carried out a spatially-resolved study of star-forming regions dominated by H atoms in nearby massive spiral and faint dwarf irregular galaxies. They found a much tighter relation

\[ \frac{\Sigma_{\text{SFR}}}{M_\odot \text{ yr}^{-1} \text{kpc}^{-2}} \sim 10^{-3} \left( \frac{\Sigma_g}{10 M_\odot \text{ pc}^{-2}} \right)^{1.5}, \]

over the range of \( \Sigma_g \sim 0.3-30 M_\odot \text{ pc}^{-2} \). In addition, the Kennicutt-Schmidt law in Eq. (5) describes the global \( \Sigma_{\text{SFR}} \) for individual galaxies with \( \Sigma_g \sim 10^{-10} M_\odot \text{ pc}^{-2} \). The above three empirical results on SFLs have overlapping ranges of \( \Sigma_g \) and are in broad agreement over these ranges within their uncertainties. Guided by these results, we assume that the global SFR \( \psi(t) \) in a dSph is related to the total mass \( M_\ast(t) \) of gas in its star-forming disk as

\[ \psi(t) = \frac{dM_\ast}{dt} = \lambda_\ast(t) \left[ \frac{M_g(t)}{M_\odot} \right]^\alpha, \]

where \( \lambda_\ast(t) \) is a rate function.

We can use the effective area \( A_{\text{disk}}(t) \) of the star-forming disk to rewrite Eq. (12) as

\[ \frac{\Sigma_{\text{SFR}}(t)}{M_\odot \text{ yr}^{-1} \text{kpc}^{-2}} = \lambda_\ast(t) \frac{10^{10 \cdot s_{\text{SFR}}} M_\odot \text{ yr}^{-1}}{10 \text{kpc}^2} \left[ \frac{A_{\text{disk}}(t)}{10 \text{ kpc}^2} \right]^{\alpha-1} \times \left[ \frac{\Sigma_g(t)}{10 M_\odot \text{ pc}^{-2}} \right]^\alpha. \]

For the above equation to agree with the empirical SFLs in Eqs. (6), (10), and (11), we take

\[ \lambda_\ast(t) \sim 10^{-2-8_{\text{SFR}}} \left[ \frac{10 \text{kpc}^2}{A_{\text{disk}}(t)} \right]^{\alpha-1} \frac{M_\odot \text{ yr}^{-1}}{A_{\text{disk}}(t)} \]

and examine cases of \( \alpha = 1, 1.5, \) and 2. We assume that \( A_{\text{disk}}(t) \) grew with time until a dSph became a satellite at \( t = t_{\text{sat}} \) and remained fixed at \( A_{\text{disk}}(t_{\text{sat}}) \) during the subsequent SFH. As the majority of stars in the present-day Fornax are distributed within a region of \( r_s \sim 2 \text{ kpc} \) in...
radius (de Boer et al. 2012b), we take $A_{\text{disc}}(t_{\text{sat}}) \sim \pi r_s^2 \sim 10 \text{kpc}^2$ and estimate $\lambda_\ast(t_{\text{sat}}) \sim 10^{-2.8} M_\odot \text{yr}^{-1}$. We will provide another justification for the choice of $\lambda_\ast(t_{\text{sat}})$ in $§ 4$. Note that the case of $\alpha = 1$ is special because the corresponding $\lambda_\ast(t)$ is independent of time [see Eq. (14)]. In this case, $\lambda_\ast \sim 10^{-10} M_\odot \text{yr}^{-1}$ is simply the fixed rate of gas consumption by star formation.

3 GAS MASS, NET GAS FLOW, AND HALO EVOLUTION FOR FORNAX

The data of de Boer et al. (2012b) on Fornax’s $\psi(t)$ are binned across 13.75 Gyr and give the average SFR $\bar{\psi}$ for each time bin. As discussed in Appendix $A$, we obtain a smooth $\psi(t)$ using a quadratic-spline fit to the mean of $\bar{\psi}$ in each bin. For each accepted realization, we obtain a quadratic-spline fit to $\psi(t)$ and use it to calculate $M_\psi(t)$ and $\Delta F(t)$ from Eqs. (15) and (16), respectively.

3.1 Baseline case

We first discuss our baseline case of $\alpha = 1.5$ and $\lambda_\ast(t_{\text{sat}}) = 10^{-14} M_\odot \text{yr}^{-1}$. The $M_\psi(t)$ calculated from the smooth $\psi(t)$ shown in Fig. 1 assuming that $\lambda_\ast(t) = \lambda_\ast(t_{\text{sat}})$ is shown as the top curve in Fig. 2. The corresponding $\Delta F(t)$ is shown as the curve in Fig. 3. The shaded region in this figure indicates the 68% confidence interval for $\Delta F(t)$ estimated from the Monte Carlo simulations constrained by the total mass of stars formed.

We take $t_{\text{sat}}$ to coincide with the onset of suppression of the net gas inflow. Based on the $\Delta F(t)$ shown in Fig. 3, we consider two possibilities of $t_{\text{sat}} \approx 4.8$ and 7.8 Gyr, respectively. Using the Via Lactea II cosmological simulations,

**Figure 1.** Data (de Boer et al. 2012b) filled circles with error bars) on Fornax’s global star formation rate $\psi$ as a function of time $t$ since the big bang. The data are binned according to time (converted from stellar age) and the horizontal error bar represents the size of each time bin. The filled circles give the mean for the average star formation rate $\bar{\psi}$ in each bin and the vertical error bar represents the 1σ uncertainty in $\psi$. A quadratic-spline fit (see Appendix $A$) corresponding to the filled circles is shown as the solid curve. We take the centroid of the time bin at $t \approx 0.29 \text{Gyr}$ as the onset and $t \approx 13.6 \text{Gyr}$ given by the fit as the end of star formation in Fornax.

**Figure 2.** Total mass $M_\psi$ for cold gas in Fornax’s star-forming disk as a function of time $t$ derived from Eq. (15) and the smooth $\psi(t)$ shown in Fig. 1 for the baseline case of $\alpha = 1.5$ and $\lambda_\ast(t_{\text{sat}}) = 10^{-14} M_\odot \text{yr}^{-1}$. The top curve assumes a disk of fixed size with $\lambda_\ast(t) = \lambda_\ast(t_{\text{sat}})$. The middle (bottom) curve is for a disk growing until $t_{\text{sat}} \approx 4.8$ (7.8) Gyr. The middle and top (bottom) curves have the same $M_\psi(t)$ for $t > 4.8$ (7.8) Gyr. Note that all curves reach $M_\psi = 0$ at $t \approx 13.6 \text{Gyr}$, the end of star formation in Fornax.
Figure 3. Rate $\Delta F$ of net gas flow to or from Fornax’s star-forming disk as a function of time $t$ estimated from Eqs. (15) and (16) and the data shown in Fig. 1 for the baseline case of $\alpha = 1.5$ and $\lambda_s(t_{sat}) = 10^{-14} M_{\odot} \text{ yr}^{-1}$. A net inflow or outflow corresponds to $\Delta F > 0$ and $< 0$, respectively. In each panel, the curve is calculated from the smooth $\psi(t)$ shown in Fig. 1 and the shaded region indicates the 68% confidence interval. Panel (a) assumes a disk of fixed size with $\lambda_s(t) = \lambda_s(t_{sat})$. Panels (b) and (c) are for a disk growing until $t_{sat} \approx 4.8$ and 7.8 Gyr, respectively. We attribute the onset of suppression of the net gas inflow at $t \approx 4.8$ or 7.8 Gyr to effects from Fornax’s becoming an MW satellite. See text for details.

Rocha et al. (2012) found that the infall times $t_{infall}$ for the MW satellites can be constrained by their present-day kinematics (radial velocities, proper motions, and Galactocentric positions). They found that $t_{infall} \approx 4.8-5.8$ or 7.8–9.8 Gyr for Fornax, with the former range being more probable. It is intriguing that our estimates of $t_{sat}$ based on Fornax’s SFH are consistent with theirs for $t_{infall}$ based on Fornax’s present-day kinematics.

For each choice of $t_{sat}$, we assume that the Fornax halo stopped growing at $t = t_{sat}$ and that at $t < t_{sat}$, its total mass $M_h(t)$ followed the median halo growth history in the $\Lambda$CDM cosmology. As discussed in Appendix B, we use the model of Zhao et al. (2009) to estimate the dependence of the CDM density profile on the halo mass and collapse time, and select the appropriate $M_h(t_{sat})$ to obtain a mass $M(< r_{1/2}) = 7.39^{+0.14}_{-0.36} \times 10^7 M_{\odot}$ enclosed within the half-light radius $r_{1/2} = 944 \pm 53$ pc as derived from observations (Wolf et al., 2010). We find that $M_h(t_{sat}) \approx 1.8 \times 10^9 M_{\odot}$ and $2.4 \times 10^9 M_{\odot}$ for $t_{sat} \approx 4.8$ and 7.8 Gyr, respectively. Using CDM mass distributions in subhalos from high-resolution simulations of the Aquarius Project, Strigari et al. (2010) found a present-day halo mass of $7 \times 10^8 M_{\odot}$ for Fornax based on its stellar kinematics. Considering the approximate nature of our approach and that the Fornax halo likely lost CDM at $t > t_{sat}$ due to tidal interaction with the MW (see §4), we regard our estimates of $M_h(t)$, especially the lower one for $t_{sat} \approx 4.8$ Gyr, as consistent with this more rigorous result on Fornax’s present-day halo mass.

Using the model of Zhao et al. (2009), we show $M_h(t)$ at $t < t_{sat}$ for $t_{sat} \approx 4.8$ and 7.8 Gyr as the solid and dashed curves, respectively, in Fig. 3. We now make better estimates of $M_h(t)$ for $t < t_{sat}$ by considering a growing star-forming disk. In accordance with the model of Mo, Mao & White (1998), we assume that the effective disk radius grew in proportion to the virial radius $r_{vir}$ of the halo (see Appendix C) until $t = t_{sat}$ and then remained fixed during the subsequent SFH. Thus we obtain

$$\lambda_s(t) = \begin{cases} \lambda_s(t_{sat})[r_{vir}(t_{sat})/r_{vir}(t)]^{2(\alpha-1)}, & t < t_{sat}, \\ \lambda_s(t_{sat}), & t \geq t_{sat}. \end{cases}$$

For the baseline case of $\alpha = 1.5$ and $\lambda_s(t_{sat}) = 10^{-14} M_{\odot} \text{ yr}^{-1}$, we show the evolution of $r_{vir}$ at $t < t_{sat}$ for the two estimates of $t_{sat}$ in Fig. 3. The corresponding $M_h(t)$ calculated from the smooth $\psi(t)$ in Fig. 1 is shown as the solid and dashed curves in Fig. 2. The corresponding $\Delta F(t)$ is shown in Figs. 3a and 3b. Note that a growing disk does not change $\Delta F(t)$ at $t < t_{sat}$ qualitatively, and therefore, does not affect our above estimates of $t_{sat}$.

The choice of $\lambda_s(t)$ can now be justified by comparing our results on $M_h(t)$ and $M_h(t)$ at $t < t_{sat}$ with simulations of gas accretion by halos after reionization of the intergalactic medium (IGM) at time $t_{reion}$. Okamoto, Gao & Theuns (2008) showed that at $t > t_{reion}$, the baryonic mass fractions for halos of mass $M$ are distributed around the mean value

$$\bar{f}_b(M, t) = \langle f_b \rangle \left[ 1 + (2^{2/3} - 1) \left( M_h(t)/M \right)^2 \right]^{-3/2},$$

where $M_h(t)$ is a characteristic halo mass determined from...
Figure 5. Virial temperature $T_{\text{vir}}$ (solid curve) and radius $r_{\text{vir}}$ (dashed curve) for the Fornax halo as functions of time $t$ before it became an MW satellite at $t = t_{\text{sat}}$. Panels (a) and (b) are for $t_{\text{sat}} \approx 4.8$ and $7.8$ Gyr, respectively. The horizontal part of the solid curve corresponds to the transition from a neutral to a fully ionized primordial gas at $T_{\text{vir}} = 1.5 \times 10^4$ K. See Appendix C for details.

Figure 6. Fornax’s gas mass fraction $f_g = M_g/M_h$ as a function of time $t$ (solid and dashed curves) before it became an MW satellite at $t = t_{\text{sat}}$ for the baseline case of $\alpha = 1.5$ and $\lambda = (t_{\text{sat}}) = 10^{-14} M_\odot$ yr$^{-1}$. Panels (a) and (b) are for $t_{\text{sat}} \approx 4.8$ and $7.8$ Gyr, respectively. In each panel, the shaded region indicates the 68% confidence interval due to uncertainties in $M_h(t)$ only. The dotted curve shows the mean baryon mass fraction $f_b(t)$ calculated from Eq. (18) for Fornax-like halos in a universe reionized at $z = 9$ ($t \approx 0.55$ Gyr). The horizontal dotted line indicates the cosmic mean mass fraction of baryons $\langle f_b \rangle$. We estimate $t_{\text{reion}} \approx 0.52$ and $0.49$ Gyr corresponding to $z_{\text{reion}} \approx 9.4$ and $9.8$ for panels (a) and (b), respectively, and set $f_g(t) = \langle f_b \rangle$ for $t < t_{\text{reion}}$. The agreement between $f_g$ and $\langle f_b \rangle$ justifies the choice of $\lambda(t)$.

In comparing the $f_g(t)$ and $\langle f_b \rangle(t)$ shown in Fig. 6, we note that Okamoto et al. (2008) assumed a reionization redshift of $z = 9$ corresponding to $t = 0.55$ Gyr. We now estimate the time $t_{\text{reion}}$ for reionization of the IGM surrounding Fornax. The oldest stellar age bin for the data on Fornax’s SFH is $13-14$ Gyr. In view of the substantial uncertainties of $\sim 2.5$ Gyr for these ages (de Boer et al. 2012b), we take the centroid of this age bin at $13.5$ Gyr as a reasonable age estimate for Fornax’s oldest stars. With an adopted age of $13.79$ Gyr for the universe, we estimate that star formation started in Fornax at $t_{\text{SP}} \approx 0.29$ Gyr (redshift $z_{\text{SP}} \approx 14.5$), when the halo mass reached $M_h(t_{\text{SP}}) \approx 1.3 \times 10^6 M_\odot$ or $1.1 \times 10^6 M_\odot$ for $t_{\text{sat}} \approx 4.8$ or $7.8$ Gyr, respectively (see Fig. 4). This is reasonable because the corresponding virial temperature $T_{\text{vir}}(t_{\text{SP}}) \approx 1.8 \times 10^3$ or $1.6 \times 10^3$ K (see Appendix C and Fig. 5) could readily enable cooling by H$_2$ molecules as required for formation of the first stars (see e.g., Bromm & Larson 2004 for a review). With $t_{\text{SP}} \approx 0.29$ Gyr, star formation most likely started in Fornax at $t < t_{\text{reion}}$. For $t_{\text{SP}} < t < t_{\text{reion}}$, accretion of gas by the Fornax halo should be proportional to that of CDM and $f_g(t) = M_g(t)/M_h(t)$ should be close to the cosmic mean mass fraction of baryons $\langle f_b \rangle \approx 0.16$. The $f_g(t)$ shown in Fig. 6(b) (c) reaches $\langle f_b \rangle$ at $t \approx 0.52$ (0.49) Gyr and exceeds this value at earlier times (not shown). Our assumed SFL most likely does not apply to the early epoch near the onset of star formation. So we could take $t_{\text{reion}} \approx 0.52$ (0.49) Gyr corresponding to $z_{\text{reion}} \approx 9.4$ (9.8) and make a revised estimate of $M_h(t) \approx (f_b)M_h(t)$ for $t_{\text{SP}} < t < t_{\text{reion}}$. The above estimates of $z_{\text{reion}}$ are in good agreement with $z = 8^{+2}_{-2}$ found for reionization of the IGM surrounding MW dwarf galaxies in simulations by Busha et al. (2010).

With $z_{\text{reion}} \approx 9.4$ (9.8) being close to the value assumed by Okamoto et al. (2008), our estimated $f_g$ can be directly compared with $\langle f_b \rangle$ and the agreement is rather good.

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as shown in Fig. 8. We note that this agreement is not affected by adding the contribution of stars to $f_g$ as the mass of stars is always significantly smaller than that of gas for $t < t_{\text{sat}}$. Therefore, we consider that our choice of $\lambda_*(t)$ with $\lambda_*(t_{\text{sat}}) = 10^{-14} M_\odot \text{yr}^{-1}$ for the baseline case is reasonable.

3.2 Other cases

The case of $\alpha = 1$ is special because $\lambda_*(t) = \lambda_*(t_{\text{sat}})$ is independent of time (see 2.3). We choose $\lambda_*(t_{\text{sat}})$ for this case to obtain the same $M_g$ at $t \approx 4.8$ Gyr as the top curve in Fig. 2 for the baseline case. This gives $\lambda_*(t_{\text{sat}}) = 7 \times 10^{-11} M_\odot \text{yr}^{-1}$, which is consistent with the estimate based on empirical SFLs as discussed in 2.3. The corresponding $\Delta F(t)$ is shown in Fig. 7 where the same onset of suppression of the net gas inflow at $t_{\text{sat}} \approx 4.8$ or 7.8 Gyr as for the baseline case can be seen. In Fig. 8 we take $t_{\text{sat}} \approx 4.8$ Gyr and show the corresponding $f_g(t)$, which gives $t_{\text{reion}} \approx 0.88$ Gyr ($z_{\text{reion}} \approx 6.3$). As $f_b$ assumes $z = 9$ for reionization, straightforward comparison between $f_g$ and $f_b$ can be made only for $t > 0.88$ Gyr. The agreement is rather good for $t \gtrsim 1.2$ Gyr.

For the case of $\alpha = 2$, we first consider $\lambda_*(t) = \lambda_*(t_{\text{sat}})$ and choose $\lambda_*(t_{\text{sat}})$ to obtain the same $M_g$ at $t \approx 4.8$ Gyr as the top curve in Fig. 2 for the baseline case. This gives $\lambda_*(t_{\text{sat}}) = 1.4 \times 10^{-13} M_\odot \text{yr}^{-1}$ in accord with the estimate based on empirical SFLs (see 2.3). The corresponding $\Delta F(t)$ shown in Fig. 9a again suggests $t_{\text{sat}} \approx 4.8$ or 7.8 Gyr. The $\Delta F(t)$ for a disk growing until $t \approx 4.8$ Gyr is shown in Fig. 9b and the corresponding $f_g(t)$ in Fig. 10. An estimate of $t_{\text{reion}} \approx 0.36$ Gyr ($z_{\text{reion}} \approx 12.3$) can be made and there is good agreement between $f_g$ and $f_b$ at $t > 0.55$ Gyr, i.e., after the time of reionization assumed for the latter.

Based on the above results, we consider that our results on $t_{\text{sat}}$, and hence $M_b(t_{\text{sat}})$, are insensitive to the exact form of our assumed SFL so long as it is consistent with the empirical SFLs discussed in 2.3.

4 GLOBAL GAS DYNAMICS AND EVOLUTION OF FORNAX AS AN MW SATELLITE

Based on the discussions in 3, the baseline case appears to give representative results on the general evolution of Fornax. Below we focus on this case with $t_{\text{sat}} \approx 4.8$ Gyr to discuss the global gas dynamics and evolution of Fornax as an MW Satellite. Considering that our discussion is mostly qualitative, we use the solid curve for $\Delta F(t)$ at $t > 4.8$ Gyr.
1. Same as Fig. but for the case of $\alpha = 2$ and $\lambda (t_{\text{sat}}) = 1.4 \times 10^{-18} M_\odot \text{yr}^{-1}$.

in Fig. 3 as a guide. This is reproduced in Figure 11, which shows that $\Delta F$ dropped to zero at $t \approx 6$ Gyr and the first net outflow ($\Delta F < 0$) occurred for $t \approx 6-7$ Gyr. This was followed by the final round of net inflow ($\Delta F > 0$) for $t \approx 7-8.6$ Gyr, and then by several episodes of net outflow until star formation ended in Fornax. As we discuss below, the episodic gas flows at $t > 4.8$ Gyr shown in Fig. 11 were driven by Fornax’s orbital motion and its tidal interaction with the MW.

4.1 Orbital motion and tidal interaction

Observations show that Fornax currently moves in an elliptical orbit with a period of $\sim 3.2$ Gyr. Its distance from the MW center is $R_p \sim 118$ kpc at the perigalacticon and $R_a \sim 152$ kpc at the apogalacticon (Piatek et al. 2007). Using the median halo growth history and the corresponding halo structure given by the model of Zhao et al. (2009), we find that for a present-day MW halo mass in the range of $(1-2) \times 10^{12} M_\odot$, the MW mass enclosed by Fornax’s present-day orbit is $\sim 1.5$ times the MW halo mass at $t_{\text{sat}} \approx 4.8$ Gyr when Fornax became a satellite. In view of this moderate change, we assume that Fornax’s orbit for $t \gtrsim 4.8$ Gyr can be approximately described by the present-day parameters. For estimates, we take the MW mass enclosed within Fornax’s orbit to be $M_{\text{MW}}^{\text{incl}} \sim 7.5 \times 10^{11} M_\odot$ on average. This gives an average orbital velocity of $v_{\text{tor}} \sim (G M_{\text{MW}}^{\text{incl}}/a)^{1/2} \sim 150$ km/s for Fornax, where $a = (R_p + R_a)/2 \sim 135$ kpc. For comparison, the circular velocity characterizing matter motion inside the Fornax halo at $t_{\text{sat}} \approx 4.8$ Gyr is $v_{\text{circ}} = (G M_h/r_{\text{vir}})^{1/2} \approx 21$ km/s with $M_h \approx 1.8 \times 10^9 M_\odot$ and $r_{\text{vir}} \approx 17$ kpc (see Fig. 3).

The effects of tidal interaction between the MW and Fornax can be estimated from the radii $r_{\text{RS},p}$ and $r_{\text{RS},a}$ of the instantaneous Roche spheres (King 1962) at the perigalacticon and apogalacticon, respectively. Ignoring the small eccentricity of $\sim 0.13$ for Fornax’s orbit (Piatek et al. 2007), we have

$$\frac{r_{\text{RS},p}}{R_p} \approx \frac{r_{\text{RS},a}}{R_a} \approx \left( \frac{M_{\text{Forn}}}{3 M_{\text{MW}}^{\text{incl}}} \right)^{1/3},$$

where $M_{\text{Forn}}$ is the total mass of Fornax and generally differs from $M_h$ at $t_{\text{sat}} \approx 4.8$ Gyr due to tidal interaction. The CDM and gas outside the Roche sphere would be lost from Fornax. However, because the Roche sphere would expand as Fornax moved from the perigalacticon to the apogalacticon, some of the CDM and gas lost earlier could be reaccreted. Guided by the simulations of Han et al. (2012) and our derived $\Delta F$, we assume that reaccretion occurred only once. Subsequently, $M_{\text{Forn}}$ settled down to a value set by the mass enclosed within $r_{\text{RS},p}$. For estimates, we use the density profile given by the model of Zhao et al. (2009) for a halo with $M_h \approx 1.8 \times 10^9 M_\odot$ at $t_{\text{sat}} \approx 4.8$ Gyr. Assuming that Fornax was at the apogalacticon at $t_{\text{sat}} \approx 4.8$ Gyr with $M_{\text{Forn}} \approx 1.8 \times 10^9 M_\odot$, we estimate that its total mass was quickly reduced to $\sim 1.6 \times 10^9 M_\odot$, which corresponds to the mass enclosed within the Roche sphere at this time. The mass of the Fornax halo was further reduced to $\sim 1.3 \times 10^9 M_\odot$ at $t \sim 6.4$ Gyr when it was at the perigalacticon with a smaller Roche sphere. It then reaccreted some of the CDM (and gas) lost earlier to obtain $M_{\text{Forn}} \approx 1.5 \times 10^9 M_\odot$ at $t \sim 8$ Gyr when it was at the apogalacticon again. After that $M_{\text{Forn}}$ settled down to $\sim 1.2 \times 10^9 M_\odot$, which is enclosed within $r_{\text{RS},p}$ estimated from Eq. (19) for this value of $M_{\text{Forn}}$. During the above evolution
of the Fornax halo, \( r_{\text{RS},a} \) started at \( \sim 14 \) kpc and settled down to \( \sim 12 \) kpc while \( r_{\text{RS},p} \) decreased from \( \sim 11 \) kpc to \( \sim 10 \) kpc. The evolution of \( M_{\text{for}, a} \), \( r_{\text{RS},a} \) and \( r_{\text{RS},p} \) is sketched in Fig. 11.

Based on the above discussion, we estimate that Fornax lost \( \sim 1/3 \) of its CDM through tidal interaction. However, this loss occurred in the outermost region of the halo and had little effect on the structure of its interior. For example, because \( r_{1/2} \approx 944 \) pc was well within the Roche sphere at all times, the mass \( M(<r_{1/2}) \) enclosed within \( r_{1/2} \), which is used to determine \( M_h \) at \( t_{\text{sat}} \approx 4.8 \) Gyr (see Appendix B), would not have been affected by tidal interaction. Similarly, with a radius of \( \sim 2 \) kpc, the star-forming disk would have remained largely intact as Fornax orbited the MW so long as there was a sufficient supply of cold gas.

4.2 Gas inflow and outflow

Over Fornax’s SFH, the star-forming disk was surrounded by hot gas either accreted from the IGM or expelled from the disk due to feedback from star formation, which includes SN explosions and heating by stellar radiation. Cooling of this hot gas gave rise to an inflow onto the disk while stellar feedback produced an outflow from it. The net gas flow rate \( \Delta N \) represents the difference between the two. At \( t < 4.8 \) Gyr, accretion of the IGM by the growing Fornax halo dominated its global gas dynamics and supplied hot gas that cooled to provide a net inflow. In general, reducing the amount of hot gas would suppress the inflow, thereby causing \( \Delta N \) to decrease. This accounts for the decline of \( \Delta N \) for \( t \approx 4.8-6 \) Gyr (see Fig. 11) as accretion of the IGM was disrupted and the initial tidal interaction between Fornax and the MW resulted in loss of hot gas along with CDM. This gas loss was also aided by ram-pressure stripping as discussed in 4.3. The corresponding suppression of the inflow allowed the brief first occurrence of a net outflow for \( t \approx 6-7 \) Gyr as hot gas escaped from the star-forming disk. This outflow carried only \( \sim 2 \times 10^8 M_\odot \) and was soon quenched when \( \sim 10^7 M_\odot \) of the hot gas lost earlier was re-accreted and cooled to produce an inflow for \( t \approx 7-8.6 \) Gyr (see Fig. 11).

As mentioned in 4.1, some of the gas and CDM lost earlier was re-accreted because the Roche sphere expanded at the apogalacticon (see Fig. 11). This re-accretion was discussed for gas by [Nichols, Lin & Bland-Hawthorn (2012)] and seen for CDM in simulations by [Han et al. (2012)]. However, our derived \( \Delta N \) indicates that there was only one episode of significant re-accretion for Fornax, which might represent a variation from the scenario of episodic starbursts with repeated gas expulsion and re-accretion as discussed by [Nichols et al. (2012)]. In the absence of further re-accretion, several episodes of net outflow occurred in Fornax for \( t \approx 8.6-13.6 \) Gyr (see Fig. 11). We consider that the episodic nature of these outflows is mainly due to Fornax’s orbital motion and is very different from the episodic behavior purely driven by stellar feedback as discussed for example, by [Stinson et al. (2007)]. Specifically, we argue that although stellar feedback initiated the escape of hot gas from the star-forming disk, the loss of such gas from the Fornax halo at \( t \approx 8.6-13.6 \) Gyr was driven by tidal interaction and ram-pressure stripping.

4.3 Ram-pressure stripping

We consider that the gas subject to ram-pressure stripping was dispersed outside Fornax’s star-forming disk but within its Roche sphere of \( \sim 10 \) kpc in radius. From the \( \Delta F \) shown in Fig 11, we estimate that \( \sim 10^7 M_\odot \) of gas were stripped for each episode. So this gas had a density of \( n \sim 10^{-4} \) cm\(^{-3}\) while inside Fornax. Its internal pressure was \( P \sim nkT \sim n m_p k T \), with \( T \) being its temperature, \( v_{th} \sim (kT/m_p)^{1/2} \sim 29(T/10^5 \text{ K})^{1/2} \) km/s its thermal velocity, and \( k \) the Boltzmann constant. The ram pressure can be estimated as

\[
P_{\text{ram}} \sim n_p n_{\text{med}} v_{\text{For}}^2,
\]

where \( n_{\text{med}} \) is the density of the gaseous medium through which Fornax moved. Ram-pressure stripping occurred when \( P_{\text{ram}} > P \), i.e.,

\[
n_{\text{med}} v_{\text{For}}^2 > n_{\text{th}} v_{\text{th}}^2.
\]

For the hot gas accreted from the reionized IGM right before Fornax became an MW satellite, we estimate \( v_{\text{th}} \sim 2 \times 10^{-6} \) cm\(^{-3}\).

The majority of the gas stripped from Fornax first escaped from its star-forming disk due to stellar feedback. For a crude estimate, we assume that the gas in this disk consisted of three components ([McKee & Ostriker 1977]): cold gas with a density of \( n_{\text{cold}} \sim 40 \) cm\(^{-3}\) and a temperature of \( T_{\text{cold}} \sim 80 \) K, warm partially-ionized gas with \( n_{\text{warm}} \sim 0.3 \) cm\(^{-3}\) and \( T_{\text{warm}} \sim 8000 \) K, and hot ionized gas with \( n_{\text{hot}} \sim 4 \times 10^{-3} \) cm\(^{-3}\) and \( T_{\text{hot}} \sim 5 \times 10^5 \) K. The total gas mass was dominated by the cold component that fed star formation. Cold gas was also converted by stellar feedback to maintain the hot component as hot gas escaped from the disk in an outflow. Assuming adiabatic cooling during its escape, we estimate an upper limit of \( T \sim T_{\text{hot}}(n_{\text{hot}}/n_{\text{med}})^{2/3} \sim 4 \times 10^4 \) K corresponding to \( v_{\text{th}} \sim 18 \) km/s for the hot gas when it was dispersed inside the Roche sphere. Its stripping requires \( n_{\text{med}} \gtrsim 10^{-6} \) cm\(^{-3}\).

Based on the above discussion, with \( n_{\text{med}} \gtrsim 2 \times 10^{-6} \) cm\(^{-3}\) for the gaseous medium around Fornax’s orbit at \( \sim 118-152 \) kpc from the MW center, ram-pressure stripping would have occurred to both accreted hot gas for \( t \approx 4.8-6 \) Gyr and gas heated by stellar feedback for \( t \approx 8.6-13.6 \) Gyr in Fornax. The gas density in the present MW halo can only be estimated by indirect means and its time evolution over the MW’s history can only be estimated by simulations of galaxy formation. Simulations of [Kaufmann et al. (2009)] give \( n_{\text{med}} \sim 10^{-5}-10^{-4} \) cm\(^{-3}\) at \( \sim 100-200 \) kpc from the galactic center with no significant changes over a period of 10 Gyr. These results are consistent with estimates based on observations of MW halo clouds (e.g., [Hsu et al. (2011)]). However, [Murail (2000)] derived a present limit of \( n_{\text{med}}<10^{-5} \) cm\(^{-3}\) at 50 kpc from the MW center by considering effects on the Magellanic Stream. For comparison, the cosmic mean baryon density at \( t_{\text{sat}} \approx 4.8 \) Gyr (\( z_{\text{sat}} \approx 1.33 \)) was \( n_b \approx 3 \times 10^{-6} \) cm\(^{-3}\). We consider \( n_{\text{med}} \approx 3 \times 10^{-6}-10^{-5} \) cm\(^{-3}\) as reasonable around Fornax’s orbit for \( t \gtrsim 4.8 \) Gyr. Therefore, ram-pressure stripping would have been effective in removing gas from Fornax.

For the three gas components in Fornax’s star-forming disk with densities and temperatures assumed above, they
were in pressure equilibrium and their internal pressure was \( \gtrsim 10^3 \) times the ram pressure. Consequently, for those conditions the ram pressure would have had little impact on the disk. However, when a sufficient amount of gas had been removed, the three-component structure would be disrupted and gas density in the disk would drop, eventually allowing gas to be ram-pressure stripped directly from the disk. This might account for the sharp increase in the net outflow rate at the end \((t \sim 12.8-13.6 \text{ Gyr, see Fig. 11})\).

### 4.4 Gas loss from Fornax

In summary, we propose the following outline for gas dynamics in Fornax as it orbited the MW. Tidal interaction was responsible for removing \( \sim 1/3 \) of its original CDM. Because collisionless CDM does not suffer ram pressure, the CDM lost during Fornax’s first orbital period appeared to have stayed close to the orbit and kept the simultaneously-lost gas from dispersing. This was crucial to the reaccretion of some of the lost CDM and gas as the Roche sphere expanded near the end of this period. Subsequently, there was no significant clustering of CDM near the orbit and once ram-pressure-stripped gas moved outside the largest Roche sphere of \( \sim 12 \) kpc in radius, it was lost from Fornax. We note that although the change in the Roche-sphere radius from \( \sim 10 \) kpc at the perigalacticon to \( \sim 12 \) kpc at the apogalacticon appeared small, the corresponding change in the Roche-sphere volume was by a significant factor of \( \sim 1.7 \).

The non-monotonic evolution of net gas outflow for \( t \approx 8.6-13.6 \) Gyr might have resulted because an expanding Roche sphere would oppose while a shrinking one would enhance the effect of ram pressure. This would explain the coincidence of \( \Delta F \approx 0 \) at \( t \sim 11.2 \) Gyr with the transition from an expanding Roche sphere to a shrinking one (Fig. 11). We also note that subsequently the net outflow rate \( |\Delta F| \) increased as Fornax approached its perigalacticon. However, the net outflow rate started to decrease at \( t \sim 12.3 \) Gyr and dropped to \( \sim 0 \) when Fornax reached its perigalacticon at \( t \sim 12.8 \) Gyr. This is in contradiction to the expected effect of the smallest Roche sphere at the perigalacticon.

Interestingly, several observations (e.g., Coleman et al. [2004], de Boer et al. [2013]) found evidence that Fornax might have accreted a smaller satellite or reaccreted some of the lost gas \( \sim 1.5-2 \) Gyr ago \((t \approx 11.8-12.3 \text{ Gyr})\). This would have explained the puzzling drop of the net outflow rate described above. In any case, it appears that Fornax lost all of its gas at \( t \approx 13.6 \) Gyr before returning to its apogalacticon (estimated to occur at \( t \approx 14.4 \text{ Gyr or } \sim 0.6 \text{ Gyr from now} \)).

Gas loss from satellites through ram-pressure stripping and tidal interaction was studied in detail by simulations of Mayer et al. [2006], who emphasized the importance of the UV background in keeping the gas widely distributed inside the satellites. Gas depletion in Local Group dwarfs was studied analytically by Nichols & Bland-Hawthorn [2011], who emphasized heating of gas by stellar feedback. In general accord with these detailed studies, we propose that gas in the star-forming disk was first heated by SNe to \( T_{\text{hot}} \sim 5 \times 10^5 \) K, and then escaped to be dispersed outside the disk before getting removed from Fornax. It is also possible that heating by both SNe and the UV background was needed, but we focus on SN heating below.

Using the same initial mass function as adopted by de Boer et al. [2012b] in deriving the SFR and assuming that stars of \( 8-120 M_\odot \) result in SNe, we obtain an SN rate of \( R_{\text{SN}}(t) \approx 10^{-2} \psi(t)/M_\odot \). For SNe to heat a total mass \( \Delta M_{\text{hot}} \) of gas that escaped during an episode of outflow, the required efficiency per SN is

\[
\epsilon_{\text{SN}} \sim \frac{(\Delta M_{\text{hot}}/m_\odot)T_{\text{hot}}}{\Delta N_{\text{SN}}E_{\text{expl}}} \\
\approx 0.083 \left(\frac{\Delta M_{\text{hot}}}{10^3 M_\odot}\right) \left(\frac{5 \times 10^5 K}{T_{\text{hot}}}ight) \left(\frac{10^4}{\Delta N_{\text{SN}}}\right) \\
\times \left(\frac{10^{51} \text{ ergs}}{E_{\text{expl}}}\right),
\]

where \( \Delta N_{\text{SN}} \) is the total number of SNe occurring in this episode and \( E_{\text{expl}} \) is the average explosion energy per SN. From the \( \psi \) and \( \Delta F \) shown in Figs. 1 and 11 respectively, we estimate that \( \Delta M_{\text{hot}} \sim 1.2 \times 10^2 M_\odot \) (2.2 \times 10^2 M_\odot) and \( \Delta N_{\text{SN}} \approx 6 \times 10^4 (1.4 \times 10^5) \) for the episode of \( t \approx 8.6-11.2 \) Gyr (11.2-12.8 Gyr), which correspond to \( \epsilon_{\text{SN}} \sim 1.7 \% \) (13\%) for \( T_{\text{hot}} \approx 5 \times 10^5 \) K and \( E_{\text{expl}} \approx 10^{51} \) ergs. For the last episode of \( t \approx 12.8-13.6 \) Gyr, there was a sharp increase in the net outflow rate and \( \Delta M_{\text{hot}} \approx 8 \times 10^3 M_\odot \), which would have required \( \epsilon_{\text{SN}} \sim 51\% \) with \( \Delta N_{\text{SN}} \approx 1.3 \times 10^5 \). As noted in [13], for this terminal episode, gas could have been ram-pressure stripped directly from the star-forming disk due to the decrease in density. So a lower SN-heating efficiency could have been sufficient. Heating by Type Ia SNe, which is ignored in the above discussion, could also reduce the required \( \epsilon_{\text{SN}} \). Therefore, it appears that SN-heating coupled with ram-pressure stripping and tidal interaction for a few orbital periods was responsible for removing gas and terminating star formation in Fornax.

### 5 DISCUSSION AND CONCLUSIONS

We have presented an empirical model for the halo evolution and global gas dynamics of Fornax. Guided by data on star formation, especially those reported by Bigiel et al. [2008] and Roychowdhury et al. [2015] for nearby galaxies, we have assumed a global SFR \( \psi(t) = \lambda_\odot(t) M_\odot(t)/M_\odot \) that is dependent on the total mass \( M_\odot(t) \) for cold gas in the star-forming disk. We have examined three different cases of \( \alpha = 1, 1.5, \) and 2, and chosen the corresponding \( \lambda_\odot(t) \) in agreement with the empirical SFRs. For each case, we have used the data on Fornax’s \( \psi(t) \) provided by de Boer et al. [2012b] to derive \( M_\odot(t) \) and estimate the rate of net gas flow \( \Delta F(t) \) to or from Fornax’s star-forming disk. We have identified the onset of the transition in \( \Delta F(t) \) from a net inflow to a net outflow as the time \( t_{\text{sat}} \) at which Fornax ceased evolving independently and became an MW satellite.

We have determined the mass \( M_\odot(t_{\text{sat}}) \) of the Fornax halo at this time by using the median halo growth history in the model of Zhao et al. [2009] and requiring that the corresponding density profile give the mass enclosed within the half-light radius as derived from observations. We have further justified our chosen \( \lambda_\odot(t) \) by comparing the gas mass fraction \( f_g(t) = M_g(t)/M_\odot(t) \) at \( t < t_{\text{sat}} \) with the results from cosmological simulations of Okamoto et al. [2008] on gas accretion by halos in a reionized universe.

Our main results for Fornax are the evolution of its total gas mass \( M_g(t) \) and net gas flow rate \( \Delta F(t) \), the time \( t_{\text{sat}} \),...
when it became an MW satellite, and its halo mass $M_h(t_{\text{sat}})$ at this time. For our baseline case of $\alpha = 1.5$ and $\lambda_*(t_{\text{sat}}) = 10^{-14} M_\odot \, \text{yr}^{-1}$, we have obtained $t_{\text{sat}} \approx 4.8$ (7.8) Gyr and $M_h(t_{\text{sat}}) \approx 1.8 \times 10^8 M_\odot (2.4 \times 10^8 M_\odot)$. These results are in broad agreement with previous studies based on Fornax’s orbital motion (Rocha et al. 2012) and stellar kinematics (Strigari et al. 2010). We also found that the evolution of the gas mass fraction $f_g(t) = M_g(t)/M_h(t)$ for $t < t_{\text{sat}}$ is consistent with cosmological simulations of Okamoto et al. (2008). We have checked that these results, especially $t_{\text{sat}}$ and $M_h(t_{\text{sat}})$, are not sensitive to different choices of $\alpha = 1$ and 2.

Using the results for the baseline case with $t_{\text{sat}} \approx 4.8$ Gyr and the present orbital parameters of Fornax, we have related $\Delta F(t)$ at $t > 4.8$ Gyr to its orbital motion as an MW satellite and estimated the effects of ram pressure and tidal interaction with the MW. The Fornax halo lost $\sim 1/3$ of its CDM through tidal interaction but reaccreted some of the lost CDM near the end of its first orbital period. This reaccretion was responsible for the last episode of significant net gas inflow. Otherwise, gas was removed from the star-forming disk as hot gas created by SN heating escaped and was then lost from Fornax through ram-pressure stripping and tidal interaction. This lasted a few orbit periods and eventually terminated star formation in Fornax.

Our assumed global SFL most likely does not apply to the epoch near the onset of star formation and prior to reionization. By using the median halo growth history $M_h(t)$ prior to the time $t_{\text{reion}}$ for reionization and assuming the corresponding gas mass fraction $f_g(t) = M_g(t)/M_h(t)$ to be the cosmic mean mass fraction of baryons ($f_b$), we have estimated $t_{\text{reion}}$ as the time at which the $f_g(t)$ derived from the assumed SFL becomes equal to $f_b$. We have obtained $t_{\text{reion}} \sim 0.52$ (0.49) Gyr corresponding to $z_{\text{reion}} \sim 9.4$ (9.8) for the baseline case with $t_{\text{sat}} \approx 4.8$ (7.8) Gyr. $t_{\text{reion}} \sim 0.88$ Gyr ($z_{\text{reion}} \sim 6.3$) for the case of $\alpha = 1$ and $t_{\text{reion}} \sim 0.36$ Gyr ($z_{\text{reion}} \sim 12.3$) for the case of $\alpha = 2$. These estimates of $z_{\text{reion}}$ are close to $z = 8^{+2}_{-1}$ found for reionization of the IGM surrounding MW dwarf galaxies in simulations by Busha et al. (2010). Our estimate of the reionization time can be improved significantly by using detailed CDM simulations to estimate halo evolution instead of the model of Zhao et al. (2009), which only gives the median halo growth history. We note that data on SFHs are available for other dwarf galaxies of the Local Group (e.g., de Boer et al. 2012a, 2012b; Weisz et al. 2014). Combining CDM simulations with our approach to study these systems may provide an interesting probe of reionization.

In conclusion, we have provided a reasonable picture for the halo evolution and global gas dynamics of Fornax based on empirical data. The input to our model includes the SFH, the mass enclosed within the half-light radius, and the present-day orbit of Fornax. Essential to our approach is the SFH and we have given only an approximate treatment of the correlated errors in each time bin to obtain a smooth SFH. We emphasize that a rigorous treatment should be carried out to make our approach more quantitative. Our approach can be extended to other dSphs for which the data listed above are available, e.g., Sculptor and Carina. Our results on $t_{\text{sat}}$ and $M_h(t_{\text{sat}})$ may help identify the halos that are most likely to host the observed dSphs in CDM simulations of the formation of an MW-like galaxy. Our results on $M_h(t)$ and $\Delta F(t)$ may be used along with the data on SFHs to develop models of chemical evolution for dSphs (e.g., Qian & Wasserburg 2012 and references therein). Our empirical approach also serves to illustrate how different sets of observational data may be integrated to provide a coherent description of the evolution of individual dSphs. We hope that this would help motivate dedicated cosmological simulations with gas physics to reproduce the observed dSphs with detailed star formation and chemical evolution histories, as well as present-day mass distributions and orbits. Such simulations are the best way to resolve the TBTF problem.

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APPENDIX A: FITTING FORNAX’S SFR TO DATA

The data of de Boer et al. (2012b) on Fornax’s SFH were binned according to the age $t_*$ of stars. We label the bin boundaries as $t_0 < t_1 < \cdots < t_n$ in terms of the time of star formation $t = t_n - t_0$, where $t_n$ is the age of the universe and $t = 0$ corresponds to the big bang. We obtain a smooth SFR $\psi(t)$ from a quadrature-spline fit to the data that conserves the total number of stars formed in each bin and guarantees the continuity of $\psi(t)$ and $\psi'(t) \equiv d\psi/dt$ (see e.g., C. Chan et al., in preparation). Specifically, for the $i$th bin $[t_{i-1}, t_i]$ with an average SFR $\bar{\psi}_i$ given by the data, we seek

$$\psi(t) = \psi(t_{i-1}) + p_i(t - t_{i-1}) + q_i(t - t_{i-1})^2,$$  \hspace{1cm} (A1)

where $p_i$ and $q_i$ are coefficients to be determined. The conservation of the total number of stars formed in this bin requires

$$\psi(t_{i-1}) = \bar{\psi}_i - \frac{1}{2} p_i \Delta_i - \frac{1}{3} q_i \Delta_i^2,$$  \hspace{1cm} (A2)

where $\Delta_i \equiv t_i - t_{i-1}$. In addition, Eq. (A1) gives

$$\psi'(t_{i-1}) = p_i,$$  \hspace{1cm} (A3)

$$\psi'(t_i) = p_i + 2q_i \Delta_i.$$  \hspace{1cm} (A4)

Inspection of Eqs. (A1)–(A4) shows that $\psi(t)$ for all $t$ in the $i$th bin can be obtained once $\psi'(t_{i-1})$ and $\psi'(t_i)$ are determined.

Equation (A1) gives

$$\psi(t_i) = \psi(t_{i-1}) + p_i \Delta_i + q_i \Delta_i^2.$$  \hspace{1cm} (A5)

Substituting $\psi(t_{i-1})$ from Eq. (A2) into the above equation, we obtain

$$\psi(t_i) = \bar{\psi}_i + \frac{1}{2} p_i \Delta_i + \frac{2}{3} q_i \Delta_i^2.$$  \hspace{1cm} (A6)

All of the equations for the $i$th bin can be generalized to other bins. For example, Eq. (A2) can be rewritten as

$$\psi(t_i) = \bar{\psi}_{i+1} - \frac{1}{2} p_{i+1} \Delta_{i+1} + \frac{1}{3} q_{i+1} \Delta_{i+1}^2.$$  \hspace{1cm} (A7)

Combining Eqs. (A6) and (A7) gives

$$\frac{1}{2} p_i \Delta_i + \frac{2}{3} q_i \Delta_i^2 + \frac{1}{2} p_{i+1} \Delta_{i+1} + \frac{1}{3} q_{i+1} \Delta_{i+1}^2 = \bar{\psi}_{i+1} - \bar{\psi}_i,$$  \hspace{1cm} (A8)

Using Eqs. (A3) and (A4) to eliminate $p_i$, $q_i$, $p_{i+1}$, and $q_{i+1}$ from the above equation, we obtain

$$\psi'(t_i) + \frac{\Delta_i + \Delta_{i+1}}{3} \bar{\psi}'(t_i) + \frac{\Delta_{i+1}}{6} \psi'(t_{i+1}) = \bar{\psi}_{i+1} - \bar{\psi}_i,$$  \hspace{1cm} (A9)

which can be rewritten as a matrix equation for $\left\{\psi'(t_i), 0 \leq i \leq n\right\}$. The unknown $\left\{\psi'(t_i), 0 < i < n\right\}$ can be solved from this equation by specifying $\psi'(t_0)$ and $\psi'(t_n)$.

We take $\psi'(t_0) = \psi'(t_n) = 0$ to obtain the smooth $\psi(t)$ that is shown in Fig. 1 along with the data. We use $t_0 = 13.79$ Gyr for the adopted cosmology to convert the stellar age $t_*$ into the time $t$ of star formation. The youngest stars covered by the data have $t_* = 0.25 - 0.5$ Gyr corresponding to $t = 13.29 - 13.54$ Gyr. Considering that there is no current star formation in Fornax, we have included $13.54 \leq t \leq 13.79$ Gyr in the fit as the $n$th bin with $\bar{\psi}_n = 0$. To ensure that no net stars were formed in this bin, the corresponding $\psi(t)$ must drop from positive to negative values. We take the part of the fit with $\psi(t) \geq 0$ and ignore the unphysical part with $\psi(t) < 0$. This gives an estimate of $t \approx 13.6$ Gyr for the end of star formation in Fornax. The oldest stars covered by the data have $t_* = 13 - 14$ Gyr. So formally our first bin corresponds to $-0.21 \leq t \leq 0.79$ Gyr. However, the stellar ages in this bin have uncertainties of $\sim 2.5$ Gyr (de Boer et al. 2012b). Therefore, we take the centroid of this bin at $t = 0.29$ Gyr as approximately the onset of star formation in Fornax and ignore the part of the fit for $t < 0.29$ Gyr. The uncertainties associated with the first and last bins for our fit represent end effects that are mostly restricted to the corresponding periods. Our main results are based on the fitted $\psi(t)$ outside these periods.
APPENDIX B: DETERMINATION OF FORNAX’S HALO MASS

In discussing halo evolution, it is convenient to use redshift $z$. For a flat $\Lambda$CDM cosmology, we have

$$\frac{dz}{dt} = -(1+z)\sqrt{\frac{8\pi G \rho_c(z)}{3}}, \tag{B1}$$

where

$$\rho_c(z) \equiv \rho_c(0) \left[ \Omega_m(1+z)^3 + \Omega_\Lambda \right] \tag{B2}$$

is the critical density at redshift $z$, $\rho_c(0) \equiv 3H_0^2/(8\pi G)$ is the critical density at the present time, $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$ is the Hubble parameter, and $\Omega_m = \Omega_{\text{CDM}} + \Omega_\Lambda$ and $\Omega_\Lambda$ are the fractional contributions to $\rho_c(0)$ from non-relativistic matter (CDM plus baryons) and the cosmological constant, respectively. Throughout this paper, we adopt $h = 0.69$, $\Omega_m = 0.29$ ($\Omega_{\text{CDM}} = 0.243$, $\Omega_\Lambda = 0.047$), and $\Omega_\Lambda = 0.71$. In calculating the growth of a halo, we use a primordial power spectrum with a spectral index $n_s = 0.96$ and the transfer function of Eisenstein & Hu (1998) with a present temperature of 2.726 K for the cosmic microwave background to obtain the linear power spectrum, the amplitude of which is fixed by $\sigma_8 = 0.82$. The above cosmological parameters are consistent with the final analysis of the WMAP experiment (Hinshaw et al. 2013).

In addition to the total mass of a halo, another important parameter is its virial radius. For a halo of mass $M_h$ collapsing at redshift $z$, its virial radius $r_{\text{vir}}$ (Bryan & Norman 1998) is defined through

$$M_h = \frac{4\pi}{3}r_{\text{vir}}^3 \rho_c(z) \Delta_c, \tag{B3}$$

where

$$\Delta_c = [18\pi^2 + 82(\Omega_\Lambda - 1) - 39(\Omega_\Lambda - 1)^2], \tag{B4}$$

and

$$\Omega_\Lambda \equiv \frac{\Omega_m(1+z)^3 + \Omega_\Lambda}{\Omega_m(1+z)^3 + \Omega_\Lambda}. \tag{B5}$$

The mass distribution of a halo can be approximated by the Navarro-Frenk-White (NFW) density profile

$$\rho(r) = \rho_0 \left( \frac{r}{r_s} \right)^{2} \left( 1 + \frac{r}{r_s} \right)^{-3}, \tag{B6}$$

where $\rho_0$ and $r_s$ are two parameters characteristic of the halo. Using the above profile, we obtain

$$M_h \equiv \int_0^{r_{\text{vir}}} 4\pi r^2 \rho(r) dr = 4\pi \rho_0 r_s^3 \left[ \ln(1 + c) + \frac{c}{1 + c} \right], \tag{B7}$$

where $c \equiv r_{\text{vir}}/r_s$ is referred to as the concentration parameter. For a halo of mass $M_h$ collapsing at redshift $z$, the concentration parameter $c(M_h, z)$ can be estimated from the model of Zhao et al. (2009).

We assume that the density profile of the Fornax halo was approximately fixed at $z_{\text{sat}} \approx 0.77$ when the halo mass reached $M_h(z_{\text{sat}})$. As discussed in Bryan & Norman (1998), tidal interaction with the MW would not have affected the density profile within the half-light radius $r_{1/2}$ for the present-day Fornax. Using the NFW profile and Eq. (B7), we obtain the mass $M(< r_{1/2})$ enclosed within $r_{1/2}$ as

$$M(< r_{1/2}) = M_h(z_{\text{sat}}) \frac{f(cr_{1/2}/r_{\text{vir}})}{f(c)}, \tag{B8}$$

where

$$f(x) = \ln(1 + x) + \frac{x}{1 + x}, \tag{B9}$$

and both $c$ and $r_{\text{vir}}$ are evaluated for $M_h(z_{\text{sat}})$. Estimating $c(M_h, z_{\text{sat}})$ with the model of Zhao et al. (2009), we solve Eq. (B8) by iteration to obtain $M_h(z_{\text{sat}}) \approx 1.8 \times 10^9 M_\odot$ for $M(< r_{1/2}) = 7.39 \times 10^7 M_\odot$ and $r_{1/2} = 944 \pm 53$ pc (Wolf et al. 2010).

APPENDIX C: CHARACTERISTIC PARAMETERS OF A HALO

We define some useful quantities (e.g., Barkana & Loeb 2001) for discussing the evolution of a halo. For a halo of total mass $M_h$ collapsing at redshift $z$, its virial radius $r_{\text{vir}}$ is defined by Eq. (B3) and can be evaluated as

$$r_{\text{vir}} = \frac{7.85}{1 + z} \left( \frac{M_h}{10^8 M_\odot} \right)^{1/3} \left( \frac{18\pi^2}{\Delta_c} \right)^{1/3} \left( \frac{\Omega_\Lambda}{\Omega_m h^2} \right)^{1/3} \text{kpc}. \tag{C1}$$

Its circular velocity is

$$v_{\text{circ}} = \sqrt{\frac{GM_h}{r_{\text{vir}}}} = 20.7 \left( \frac{M_h}{10^8 M_\odot} \right)^{1/2} \left( \frac{\text{kpc}}{r_{\text{vir}}} \right)^{1/2} \text{ km s}^{-1}, \tag{C2}$$

and its virial temperature is

$$T_{\text{vir}} = \frac{\mu m_p v_{\text{circ}}^2}{2k},$$

where

$$\mu = \frac{M_p}{M_e},$$

and $M_p$ is the mean molecular weight for those electrons, nuclei, and atoms that contribute to the gas pressure, and $k$ is the Boltzmann constant. We take the primordial mass fractions of protons and $^4$He nuclei to be 0.75 and 0.25, respectively, and use $\mu = 1.23$ and 0.59 for neutral and fully ionized gas, respectively. Following Busha et al. (2010), we take $\mu = 1.23$ for $T_{\text{vir}} < 1.5 \times 10^5$ K and $\mu = 0.59$ for $T_{\text{vir}} > 1.5 \times 10^5$ K. The transition between these two regimes is assumed to occur at a fixed $T_{\text{vir}} = 1.5 \times 10^5$ K. Using $M_h(t)$ for the Fornax halo shown in Fig. 3, we show the corresponding $r_{\text{vir}}(t)$ and $T_{\text{vir}}(t)$ in Fig. 4.