Brane inflation and Swampland Criteria

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Abstract

We consider inflation in a five -dimensional space time with the the inflaton field confined to live on a brane word. In this scenario, we study type potentials for the inflaton, discuss their observational consequences, and compare with data. We find that some class of potentials are in good agreement with observation and that the value of the inflaton field can be sub-planckian. Moreover, we investigate the recent proposed swampland criteria in this scenario and determine the type of inflaton potential that satisfy such criteria.

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I. INTRODUCTION

The inflationary scenario is known as one of the best candidate for describing the very early universe which has been strongly supported by the observational data [1–3]. Since the first proposal of the scenario [4–8] many inflationary model have been introduced such as non-canonical inflation [9–16], tachyon inflation [17–20], DBI inflation [21–26], G-inflation [27–30], warm inflation [31–38], in which the most common picture is that inflation is drives by a scalar field which slowly rolls down to minimum of its potential [39–42].

The standard model of inflation has been generalized in different ways which one of them is the inflationary scenario in modified gravity models where the brane gravity model is known as one of the interesting generalized theory of gravity. The brane theory of gravity is a higher dimensional model of gravity which has been inspired from M-theory. The first model of brane world was introduced by Randall and Sundrum (RS) in 1999 where the main motivation of the model was to find a solution for the Hierarchy problem between electroweak scale and Planck scale [43, 44]. The general picture is that all standard particles are confined to a four-dimensional space-time (brane) and only gravity could propagates in higher dimension. In another words, our universe is a three brane embedded in five-dimensional space-time which is called bulk. The model introduces an interesting and novel feature in the evolution equation. The Friedmann equation in brane world gravity includes both quadratic and linear terms of the energy density while in four-dimensional cosmology there is only linear term. The quadratic term of the energy density dominates over the linear term in the high energy regime (where energy density is larger than the brane tension, i.e. $\rho \gg \lambda$). Consequently, the Hubble parameter in this regime is proportional to the energy density, $H \propto \rho$ and it is no longer proportional to $H \propto \sqrt{\rho}$ [45–47].

A theoretical constraint on the inflationary models has been recently proposed which is known as the swampland criteria [48, 49]. The origin of these criteria stands in string theory where they are realized as a measure to recognize the consistence low-energy effective field theory (EFT) from the inconsistence ones. It includes two conjectures: I) There is an upper bound on the field range, i.e. $\Delta \phi / M_p < c_1$ where $c_1$ is of order of unity, which rise from this belief that the effective Lagrangian in the EFT is valid only for a finite radius; II) putting an upper bound on the gradient of the potential of the field of any EFT, i.e. $M_p |V'|/V \geq c_2$ or the refined version of this conjecture, given by
\( M_p \frac{V''}{V} \geq c_2 \) where the most recent studies determines that \( c \) could be even of order of \( \mathcal{O}(0.1) \) \cite{51}. In the first look, the second criterion is in direct tension with the slow-roll inflation where the slow-roll parameter \( \epsilon_\phi = M_p^2 \left( \frac{V'}{V} \right)^2 \) must be smaller than one. In general, these two criteria rule out some of the inflationary models, however, the recent studies \cite{51,58} have determined that some non-standard models of inflation might still survive these two criteria, in which the brane inflation could be one of them.

The main reasons that motivates us to consider the inflationary scenario in the frame of RSII brane gravity model are two folds. First, due to interesting feature of the Friedmann equation in the brane world model which is expected to lead to some novel conclusions. The scenario is studied for different well-known potentials, and the free parameters of the model are determined by comparing the results with observational data. In this regard, our method is different from the previous studies where instead of testing the results of the model for two or three sets of the constant parameters of the model, we find a parameter space in which every point is consistent with the data. The observational data, during the past years is getting better and there are chance that some of the potential be throwing out due to their inconsistency with data. Considering the consistency with the swampland criteria is another motivation for the present work. There is a growing interest to find the inflationary models which simultaneously agree with observational data and swampland criteria. Then, after constraining the free parameters of the model for every potential, we are going to find whether the model could satisfy the swampland criteria.

The paper is organized as follows: After the introduction, the main dynamical equations of the model are presented in Sec.II. In Sec.III, the slow-roll parameters are introduced for a general form of the potential, and the perturbations parameters are described in terms of the potential. Next, in Sec.IV we are going to consider the consistency of the model with data for different well-known types of the potential, then try to find out the consistency of the result with the swampland criteria. The results will be summarize and discussed in Sec.V.
II. THE MODEL

Our study will be limited to Randall-Sundrum II brane gravity model, with the following action

\[ S_5 = \int d^5x \sqrt{-g} \left( \frac{M_5^3}{2} \mathcal{R} - \Lambda_5 \right) + \int d^4x \sqrt{-q} \left( L_b - \lambda \right), \tag{1} \]

where \( \mathcal{R} \) is the Ricci scalar, \( \Lambda_5 \) the five-dimensional cosmological constant, \( M_5 \) stands for five-dimensional Planck mass, and \( q_{\mu\nu} \) the induced metric on the brane which is related to the five-dimensional metric \( g_{AB} \) by the relation \( g_{AB} = q_{AB} + n_A n_B \), where \( n^A \) is a unit normal vector. \( L_b \) indicates the Lagrangian of matter that has confined on the brane and \( \lambda \) is the brane tension. By taking variation of the above action with the metric we obtain the field equation of motion

\[ G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \left( \frac{8\pi}{M_4^2} \right) \tau_{\mu\nu} + \left( \frac{8\pi}{M_5^3} \right)^2 \Pi_{\mu\nu} - E_{\mu\nu}, \tag{2} \]

with

\[ \Lambda_4 = \frac{4\pi}{M_5^3} \left( \Lambda_5 + \frac{4\pi}{3M_5^3} \lambda^2 \right), \]
\[ M_4^2 = \frac{3}{4\pi} \frac{M_5^6}{\lambda}, \]
\[ E_{\mu\nu} = C_{MRNS} n^M n^N q^R_{\mu} q^S_{\nu}, \]
\[ \tau_{\mu\nu} = -2 \frac{\delta L_b}{\delta g^{\mu\nu}} + g_{\mu\nu} L_b, \]
\[ \Pi_{\mu\nu} = -\frac{1}{4} \tau_{\mu\alpha} \tau_{\nu}^{\alpha} + \frac{1}{12} \tau \tau_{\mu\nu} + \frac{q_{\mu\nu}}{8} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{q_{\mu\nu} \tau^2}{24}. \]

Here \( M_4 \) is the effective four-dimensional Planck mass, \( \Lambda_4 \) the cosmological constant on the brane is defined by \( \Lambda_4 \) which is a combination of the five-dimensional cosmological constant and the tension of the brane, \( E_{\mu\nu} \) is the projection of the five-dimensional Weyl tensor \( C_{MRNS} \) on the brane, and \( \tau_{\mu\nu} \) is the brane energy momentum tensor. Note that both the linear and quadratic terms contribute to the effective four-dimensional energy-momentum tensor.

Assuming the homogeneity and isotropy of the universe and a spatially flat five-dimensional Friedmann-Lemaître-Robertson-Walker (FLRW) metric, defined as

\[ ds^2_5 = -dt^2 + a^2 \delta_{ij} dx^i dx^j + dy^2, \tag{3} \]
where $\delta_{ij}$ is a maximally symmetric three-dimensional metric and $y$ denotes the fifth coordinate, the corresponding Friedmann equation reads

$$H^2 = \frac{\Lambda_4}{3} + \left( \frac{8\pi}{3M_4^2} \right) \rho + \left( \frac{4\pi}{3M_5^2} \right)^2 \rho^2 + \frac{C}{a^4}. \quad (4)$$

The last term on the right hand side of the above equation arises from the term $E_{\mu\nu}$, which describes the influence of the bulk graviton on the brane evolution, and is known as the dark radiation. Because it scales as $a^{-4}$, the dark radiation gets rapidly diluted during inflationary phase, and hence can be neglected. Also, we will set $\Lambda_4 = 0$ as in the original RS model. Therefore, the Friedmann equation is rewritten as

$$H^2 = \frac{8\pi}{3M_4^2} \rho \left( 1 + \frac{\rho}{2\lambda} \right), \quad (4)$$

In the high energy region, there the contribution of the term quadratic in the energy density is dominant in the expression of the Hubble parameter, where as in the regime where $\rho \ll \lambda$, the Friedmann equation the usual form of standard cosmology. Since standard cosmology is very successful in describing the evolution of the universe from the time of nucleosynthesis, it requires the brane tension as $\lambda \geq 1\text{MeV}^4$, leading to the five-dimensional Planck mass $M_5 \geq 10\text{TeV}$ \cite{59, 60}. Moreover, the Newtonian law of gravity receives a correction of order $M_5^6/\lambda^2 r^2$, which should be small on scales larger than $r \geq 1\text{mm}$, and consequently yields to the stronger constraint $M_5 \geq 10^5\text{TeV}$ \cite{60}. There are also various astrophysical implications which set strong limit on the brane tension $\lambda \geq 5 \times 10^8\text{MeV}^4$ (see \cite{60}).

The matter confined to the brane satisfy the same energy conservation equation as in standard cosmology, i.e

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (5)$$

Using this equation and taking the time derivative of Eq.(4), we obtain the second Friedmann equation

$$\dot{H} = -\frac{4\pi}{M_4^2} \left( 1 + \frac{\rho}{\lambda} \right) (\rho + p). \quad (6)$$

III. BRANE INFLATION

We assume the inflaton is scalar field living on the brane and has the energy density and pressure $\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$ and $p = \frac{\dot{\phi}^2}{2} - V(\phi)$, respectively, which is governed by the equation
of motion
\[ \ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0. \]  
(7)
where \( V(\phi) \) is the potential of the inflaton. The common picture for the universe is that, the scalar field slowly rolls down toward the minimum of its potential. During this slow-rolling phase, the scalar field yields very small kinetic energy which can be neglected compared to its potential energy. Also, it is assumed that the term \( \ddot{\phi} \) is much smaller than the friction term \( H \dot{\phi} \) and the slope of the potential \( V' \). These assumptions are known as the slow-roll conditions and are described by the smallness of the slow-roll parameters:
\[ \epsilon = \frac{-\dot{H}}{H^2}, \quad \eta = \frac{-\ddot{\phi}}{H \dot{\phi}} \]  
(8)
With these parameters, the dynamical equations of the model could be rewritten as
\[ H = \frac{8\pi}{3M^2_\text{Pl}} V(\phi) \left( 1 + \frac{V(\phi)}{2\lambda} \right), \]  
(9)
\[ \dot{H} = \frac{-4\pi}{M^2_\text{Pl}} \left( 1 + \frac{V(\phi)}{\lambda} \right) \dot{\phi}^2, \]  
(10)
\[ 3H \dot{\phi} = -V'(\phi). \]  
(11)
Using these equations, we can express the slow-roll parameters in terms of the potential and its derivatives as
\[ \epsilon = \frac{M^2_\text{Pl}}{16\pi} \left( \frac{V''(\phi)}{V(\phi)} \right)^2 \frac{4\lambda(\lambda + V(\phi))}{(2\lambda + V(\phi))^2}, \]  
(12)
\[ \eta = \frac{M^2_\text{Pl}}{8\pi} \frac{V''(\phi)}{V(\phi)} \frac{2\lambda}{2\lambda + V(\phi)}. \]  
(13)
Compared to the standard cosmology, here we have a generalized Friedmann equation with some modified terms. These It is important to note that in the high energy limit, i.e. \( \rho \gg \lambda \), The quadratic term of the energy density dominates over the linear term and the Hubble parameter is proportional to the potential, in contrast to the standard cosmology where \( H \propto V^{1/2}(\phi) \). For the rest of the work, we will assume that inflation occurs in the high energy limit, in which case the slow-roll parameters get the simpler form
\[ \epsilon = \frac{1}{3} \left( \frac{3M^3_\text{Pl}}{4\pi} \right)^2 \frac{V''(\phi)}{V^2(\phi)} , \quad \eta = \frac{1}{3} \left( \frac{3M^3_\text{Pl}}{4\pi} \right)^2 \frac{V''(\phi)}{V^2(\phi)} \]  
(14)
The expansion of the universe during inflation is quantified by the number of e-fold which describes how long this exponential phase should last, and is defined as
\[ N = \int_{t_i}^{\epsilon} H dt = -3 \left( \frac{4\pi}{3M^2_\text{Pl}} \right)^2 \int_{\phi_i}^{\phi} \frac{V^2(\phi)}{V'(\phi)} d\phi \]  
(15)
where in obtaining the second equality Eqs. (9) and (11) have been used.

A. Cosmological perturbations

Quantum perturbations in inflationary scenario are of three types: as scalar, vector, and tensor, in which the scalar perturbations are the seeds for large scale structure of the universe and tensor perturbations are known as the primordial gravitational waves. The vector perturbations are less important since they behave as the inverse of the scale factor and rapidly diluted during inflation.

Let us consider an arbitrary scalar perturbation to the background FLRW metric

\[ ds^2 = -(1 + 2A)dt^2 - 2a^2(t)\nabla_i Bdx^i dt + a^2(t) \left[ (1 - 2\psi)\delta_{ij} + 2\nabla_i \nabla_j E \right] dx^i dx^j. \]  

(16)

where \( \delta_{ij} \) is the spatial metric of the background and \( \nabla_i \) stands for covariant derivative with respect to the metric. The quantity \( \psi \) is called the curvature perturbations due to the fact the intrinsic curvature of the spatial hypersurface is directly related to the this parameter as \( ^3R = 4\nabla^2 \psi / a^2 \). The curvature perturbation is gauge dependent and changes under arbitrary coordinate transformation. However, the curvature perturbations in the uniform density hypersurface, given by \( \zeta = \psi + \frac{H\delta\rho}{\rho} \) is a gauge invariant perturbation parameter. For the single scalar field inflationary models, where the perturbations can be assumed to be adiabatic, the curvature perturbation \( \zeta \) is conserved and remains almost constant at the superhorizon scale [45, 61]. This is the most important feature of the parameter. On the spatially flat hypersurface, \( \psi = 0 \), using the scalar field energy density, the gauge-invariant curvature perturbation \( \zeta \) is obtained as

\[ \zeta = \frac{H}{\phi} \delta\phi, \]  

(17)

where \( \delta\phi = H/2\pi \). Following the notation of [45, 61], the amplitude of the scalar perturbation is defined as \( P_s = 4\langle \zeta^2 \rangle / 25 \) [45, 62], and making use of the slow-roll approximations we have

\[ P_s = \frac{9}{25\pi^2} \left( \frac{4\pi}{3M_5^3} \right)^6 \left( \frac{V^6(\phi)}{V'^2(\phi)} \right) \]  

(18)
Using above relation, we obtain the scalar spectral index

\[ n_s - 1 = \frac{d \ln(P_s)}{d \ln(k)} = -6\epsilon + 2\eta. \] (19)

The derivation of the amplitude of the tensor perturbations for this model is a little trickier than in the standard four-dimensional cosmology since here the graviton can propagate along the fifth dimension as well. It is given by [63–65]

\[ P_g = \frac{16\pi}{25\pi M_p^2} \left( \frac{H}{2\pi} \right)^2 F^2(x) \] (20)

where

\[ F^2(x) = \left[ \sqrt{1 + x^2} - x^2 \sinh^{-1} \left( \frac{1}{x} \right) \right]^{-1/2}, \quad x \equiv \sqrt{\frac{3}{4\pi\lambda}} M_p H \] (21)

Using Eq.(9) and considering the high energy limit, the perturbations reads

\[ P_g = \frac{9}{50\pi^2} \left( \frac{4\pi}{3M_5^2} \right)^4 V^3(\phi) \] (22)

The tensor perturbation is measured indirectly through the tensor-to-scalar ratio

\[ r = \frac{3 \epsilon}{2}. \] (23)

Thus far there is no evidence for such contribution which yields an upper limit \( r < 0.064 \) from the Planck data combined with the BICEP2/Keck Array BK14 data [3].

**IV. CONSISTENCY WITH OBSERVATION AND SWAMPLAND CRITERIA**

In this section, we consider in details different types of inflaton potentials and for each we determine the model parameter space that is consistent with the latest observational data.

**A. Power-law Potential**

For our first case, we study the power-law potential

\[ V(\phi) = V_0 \phi^n, \] (24)

where \( V_0 \) and \( n \) are constant. Substituting this potential into Eq.(14) yields the slow-roll parameters

\[ \epsilon = \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{n^2}{3V_0} \frac{1}{\phi^{n+2}}, \quad \eta = \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{n(n-1)}{3V_0} \frac{1}{\phi^{n+2}} \] (25)
By setting $\epsilon = 1$, we can infer the value of the scalar field at the end of inflation as

$$\phi_{\epsilon}^{n+2} = \left(\frac{3M_5^3}{4\pi}\right)^2 \frac{n^2}{3V_0}. \quad (26)$$

Applying this result to Eq.(15) yields the scalar field at the horizon crossing

$$\phi_{\star}^{n+2} = \left(\frac{3M_5^3}{4\pi}\right)^2 \frac{n^2}{3V_0} \left[1 + \left(\frac{n+2}{n}\right)^N\right]. \quad (27)$$

Substituting the above relation into Eqs.(25), (19), and (23), we get

$$\epsilon(n, N) = \left(1 + \frac{(n+2)N}{n}\right)^{-1}, \quad \eta(n, N) = \frac{(n-1)}{n} \left(1 + \frac{(n+2)N}{n}\right)^{-1} \quad (28)$$

which is a function of only the power of the potential and the number of e-folds. Consequently, the scalar spectral index and the tensor-to-scalar depend on just on the two parameters $n$ and $N$. Using the Planck $r - n_s$ diagram, we show in Fig.1 the parameter space for $(n, N)$ for which the model predictions are in agreement with the Planck data.

![FIG. 1: The parameter space $(n, N)$ for the power law potential that yield values $(r, n_s)$ allowed by the Planck data at the 95% CL (light blue) and the 68% CL (dark blue).](image)

Also, from the expression of the amplitude of the scalar perturbations, Eq.(18), the constant $V_0$ is determined as

$$V_0^\frac{6n+2}{n+2} = \frac{25\pi^2 n^2 P_s}{9} \left(\frac{3M_5^3}{4\pi}\right)^6 \left[\frac{3n^2}{16\pi^2} M_5^5 \left(1 + \frac{(n+2)N}{n}\right)\right]^{-\frac{2(2n+1)}{n+2}} \quad (29)$$

and depends on the values of $n$, $N$, and the five-dimensional Planck mass. Setting $M_5 = 2 \times 10^{14}$ GeV, in Table.II we present the values of $V_0$ and the energy scale $V^\star$ for different values of $n$ and $N$ taken from Fig.1. Considering the constraint on the brane tension $\lambda$, stated in Sec.II, and for the same chosen value of $M_5$, we find that $\rho/\lambda \sim \mathcal{O}(10^{11} - 10^{18})$,
TABLE I: The constant $V_0$ and the energy scale of the inflation for different values of $(n, N)$ taken from Fig.1 and $M_5 = 2 \times 10^{14}$ GeV

which is much larger than unity, and hence in consistent with our assumption that inflation occurred in the high energy regime.

Now that we determined the model parameter space that are consistent with observation, the next step is to use these values and determine whether the model satisfy the swampland criteria. For that, in Figs.2 we display the behavior of $\Delta \phi/M_p$ and $M_p|V'/V|$ for different values of the $n$ and the number of e-folds. Fig.2(a) and Fig.2(b) respectively show that $\Delta \phi/M_p < 1$ and $M_p|V'/V| > 1$ at the horizon crossing time for allowed values of $n$ (based on Fig.1). From Fig.2(c), we see that for all the chosen values of $n$, we have $\Delta \phi/M_p < 1$ during the whole time of inflation, and it gets even smaller as $n$ decreases. On the other hand, Fig.2(d) shows that $M_p|V'/V| > 1$ for all the chosen values of $n$, and gets even larger for smaller $n$. We therefore conclude that for the all values of $n$ and $N$ presented in Fig.1 both swampland criteria are satisfied during the whole time of inflation. We also note that smaller values of the five dimensional Planck mass support the swampland criteria, namely by reducing the value of $M_5$, $\Delta \phi/M_p$ decreases and $M_p|V'/V|$ increases, respectively.

B. Axion-like Potential

The second type of potential we consider is a periodic potential of the form

$$V(\phi) = V_0 \left( 1 - \cos \left( \frac{\phi}{f} \right) \right),$$

(30)

here $V_0$ and $f$ are constant parameters. In this case, the slow-roll parameters are given by

$$\epsilon = \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{1}{3f^2V_0} \frac{(1 + \cos(\Phi))}{(1 - \cos(\Phi))^2}, \quad \eta = \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{1}{3f^2V_0} \frac{\cos(\Phi)}{(1 - \cos(\Phi))^2}.$$

(31)
where $\Phi \equiv \phi / f$. At the end of inflation, we have

$$\cos(\Phi_e) = \frac{1}{2} \left[ (\gamma + 2) \pm \sqrt{(\gamma + 2)^2 - 4(\gamma - 1)} \right], \quad \gamma \equiv \left( \frac{3M_p^2}{4\pi} \right)^2 \frac{1}{3f^2V_0}, \quad (32)$$

and after inserting it into Eq.(15), the field during at horizon crossing reads

$$\cos(\Phi) = -1 - 2W \left[ -\frac{1}{2} \exp \left( -\frac{1}{2} - \zeta \right) \right] \quad (33)$$

where

$$\zeta \equiv \gamma \cdot N + \cos(\Phi_e) - 2 \ln (1 + \cos(\Phi_e)) \,.$$

and $W[x]$ is the Lambert function. Substituting Eq.(33) in Eq.(31), one find that the scalar spectral index and tensor-to-scalar ratio are only a function of the constant $\gamma$ and the number of e-folds $N$. Then, we can extract the allowed values of the model parameters $(\gamma, N)$ that yield values of $(r, n_s)$ in agreement with Planck data, as shown in Fig.3. On the other hand,
FIG. 3: The parameter space $(\gamma, N)$ for the axion-like potential that yield values $(r, n_s)$ allowed by the Planck data at the 95% CL (light blue) and the 68% CL (dark blue).

| $n$  | $N$  | $f$ (GeV)       | $V_0$ (GeV$^4$) | $V^*$ (GeV$^4$) |
|------|------|-----------------|-----------------|-----------------|
| 0.055| 55   | $3.66 \times 10^{17}$ | $4.01 \times 10^{58}$ | $7.32 \times 10^{58}$ |
| 0.060| 60   | $3.72 \times 10^{17}$ | $3.56 \times 10^{58}$ | $6.68 \times 10^{58}$ |
| 0.065| 65   | $3.79 \times 10^{17}$ | $3.17 \times 10^{58}$ | $6.06 \times 10^{58}$ |
| 0.070| 65   | $3.73 \times 10^{17}$ | $3.04 \times 10^{58}$ | $5.84 \times 10^{58}$ |
| 0.075| 70   | $3.82 \times 10^{17}$ | $2.70 \times 10^{58}$ | $5.25 \times 10^{58}$ |

TABLE II: The constant $V_0$ and the energy scale of the inflation for different values of $(n, N)$ taken from Fig.3 and $M_5 = 5 \times 10^{15}$ GeV.

After some algebra, the amplitude of the scalar perturbation can be expressed as

$$P_s = \left( \frac{V_0}{75\pi^2\gamma^3 f^4} \right) \frac{(1 - \cos(\Phi_s))^5}{(1 + \cos(\Phi_s))}$$  \hspace{1cm} (34)

Using the observational data for $P_s$, the expression of the scalar field in Eq. (33), and the values of $\gamma$ and $N$ from Fig.3, we determine the possible values of the other constants of the model as presented in Table II. To see if the swampland criteria is met in this type models, we depict in Fig.4 the quantities $\Delta \phi/M_p$ and $M_p|V'/V|$ for different values of $\gamma$ and the number of e-fold $N$. For instance, Figs. 4(a) and 4(b) determine $\Delta \phi/M_p$ and $M_p|V'/V|$ at a specific time during inflation (horizon crossing time) for different values of $\gamma$. We note that when $\gamma$ decreases, both $\Delta \phi/M_p$ and $M_p|V'/V|$ decreases, however, $\Delta \phi/M_p$ remains smaller than unity and $M_p|V'/V|$ is still bigger than one. On the other hand, Figs. 4(c) and 4(d) display the behavior of these quantities from the start to the end for different values of $\gamma$, and as inflation approaches the end, $\Delta \phi/M_p$ decreases, while $M_p|V'/V|$ increases. Thus, in
the brane gravity the axion-like potential satisfy both swampland criteria.

FIG. 4: The figures show the behavior of: a) $M_p \Delta \phi \equiv \Delta \phi$ and b) $M_p V/V' \equiv \Delta V$ versus the number of e-fold for different values of $n$ taken from Fig 3.

C. Exponential Potential

The exponential potential is another case which we are about to consider in this part. The potential is given as

$$V(\phi) = V_0 \exp (\alpha \phi),$$

where $V_0$ and $\alpha$ are two constants of the model. Substituting this potential in Eq. (14), the slow-roll parameters are found as

$$\epsilon = \left( \frac{3M_5^2}{4\pi} \right)^2 \frac{\alpha^2}{3V_0} \exp (-\alpha \phi), \quad \eta = \epsilon.$$
Finding the scalar field at the end of inflation by solving the relation \( \epsilon = 1 \), and using that in Eq. (15), the scalar field during inflation is obtained in terms of the number of e-fold as

\[
\exp (\alpha \phi) = \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{\alpha^2}{3V_0} (1 + N)
\]

Then, the slow-roll parameters are given as

\[
\epsilon(N) = \eta(N) = (1 + N)^{-1},
\]

and from Eqs. (19) and (23), the scalar spectral index and tensor-to-scalar ratio are obtained only as a function of the number of e-fold. Fig. 5 illustrates the behavior of the tensor-to-scalar versus the scalar spectral index in terms of the number of e-fold. The curve cross the region of \( r - n_s \) only for the number of e-fold \( N > 90 \).

![Graph showing the tensor-to-scalar ratio versus the scalar spectral index where the variable is the number of e-fold. The small and big point on the curve are respectively for \( N = 80 \) and \( N = 100 \).](image)

D. T-mode Potential

This is a hyperbolic function of the field:

\[
V(\phi) = V_0 \tanh^2 \left( \frac{\phi}{\sqrt{6\alpha}} \right).
\]

where \( V_0 \) and \( \alpha \) are free constant parameters. The slow-roll parameters are

\[
\epsilon = \frac{2\gamma (1 - \tanh^2 \left( \frac{\phi}{\sqrt{6\alpha}} \right))}{\tanh^4 \left( \frac{\phi}{\sqrt{6\alpha}} \right)}, \quad \eta = \frac{\gamma (1 - \tanh^2 \left( \frac{\phi}{\sqrt{6\alpha}} \right)) (1 - 3 \tanh^2 \left( \frac{\phi}{\sqrt{6\alpha}} \right))}{\tanh^4 \left( \frac{\phi}{\sqrt{6\alpha}} \right)},
\]

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with the parameter $\gamma$ given by

$$\gamma \equiv \left(\frac{3M_3^3}{4\pi}\right)^2 \frac{1}{9V_0\alpha}$$

The scalar field at the horizon crossing time is

$$\cosh^2\left(\frac{\phi_e}{\sqrt{6\alpha}}\right) - \ln\left(\cosh^2\left(\frac{\phi_e}{\sqrt{6\alpha}}\right)\right) = \cosh^2\left(\frac{\phi_e}{\sqrt{6\alpha}}\right) - \ln\left(\cosh^2\left(\frac{\phi_e}{\sqrt{6\alpha}}\right)\right) + 2\gamma N, \quad (41)$$

Here $\phi_e$ is the value of the field at the end of inflation, given by

$$\cosh^2\left(\frac{\phi_e}{\sqrt{6\alpha}}\right) = 1 + \sqrt{2}\gamma$$

Comparing the model predictions for $n_s$ and $r$ with the Planck data, we present in Fig.6 the corresponding allowed range of the constants for $\gamma$ and $N$ at the 68% CL (in dark blue) and 95% CL (in light blue).

![Graph showing allowed parameter space (\gamma, N) for the T-mode potential that yield values of (r, n_s) that are in agreement with observation at the 68% CL (dark blue) and 95% CL (light blue).](image)

**FIG. 6:** The allowed parameter space ($\gamma, N$) for the T-mode potential that yield values of ($r, n_s$) that are in agreement with observation at the 68% CL (dark blue) and 95% CL (light blue).

Next, the amplitude of scalar perturbations at the crossing horizon time can be show to be expressed as

$$\alpha^3 = \frac{1}{(150\pi^2 \times 81) \gamma^4 P_s} \frac{\left(\cosh^2\left(\frac{\phi_e}{\sqrt{6\alpha}}\right) - 1\right)^5}{\cosh^6\left(\frac{\phi_e}{\sqrt{6\alpha}}\right)} \quad (42)$$

Then, by choosing specific values of $\gamma$ from the Fig.6, the allowed potential parameters $\alpha$ and $V_0$ can be determined and are shown in Table III. We also show in the values of the last two columns of the table the values of $\Delta\phi/M_p$ and $M_p|V''/V|$ where we see that they satisfy the swampland criteria [67]. Therefore, the T-mode potential can be a viable model for inflation that satisfies the swampland criteria.
\[ 1.5 \times 10^{-5} \quad 66 \quad 4.01 \times 10^{36} \quad 1.64 \times 10^{60} \quad 9.95 \times 10^{58} \quad 0.406 \quad 3.33 \\
3 \times 10^{-5} \quad 67 \quad 2.84 \times 10^{36} \quad 1.15 \times 10^{60} \quad 9.83 \times 10^{58} \quad 0.409 \quad 3.25 \\
5 \times 10^{-5} \quad 69 \quad 3.65 \times 10^{36} \quad 8.86 \times 10^{59} \quad 9.68 \times 10^{58} \quad 0.416 \quad 3.15 \\
6.5 \times 10^{-5} \quad 71 \quad 1.98 \times 10^{36} \quad 7.65 \times 10^{59} \quad 9.55 \times 10^{58} \quad 0.423 \quad 3.07 \\
8 \times 10^{-5} \quad 73 \quad 1.82 \times 10^{36} \quad 6.78 \times 10^{59} \quad 9.42 \times 10^{58} \quad 0.430 \quad 2.49 \\

TABLE III: The constants \(\alpha\), \(V_0\) and the energy scale of the inflation for different values of \((\gamma, N)\) taken from Fig.6 and \(M_5 = 5 \times 10^{15}\) GeV. Also, the last two columns of the table determine the \(\Delta \phi/M_p\) and \(M_p |V'/V|\) and give some insight about the swampland criteria.

E. Generalized T-mode Potential

Here we consider a slightly modified T-mode potential

\[ V(\phi) = V_0 \left(1 - \tanh^2(\alpha \phi)\right) \tag{43} \]

Following similar steps as we did with the previous type of potentials, we obtain the slow-roll parameters at the crossing time

\[ \epsilon = \frac{\gamma \exp(-\gamma N)}{(1 + \gamma) - \exp(-\gamma N)}, \quad \eta = -\frac{\gamma}{2} \left(1 + \gamma - 3 \exp(-\gamma N) \right) \tag{44} \]

where the defined constant \(\gamma\) here is given by

\[ \gamma \equiv \left(\frac{3M_5^3}{4\pi} \right)^2 \frac{4\alpha^2}{3V_0}. \]

By comparing the model predictions for \(n_s\) and \(r\) with the Planck \(r - n_s\) diagram, we find that only for small range of the parameters \(\gamma\) and \(N\) the model is in agreement with the observational data, as depicted in Fig.7.

For the amplitude of the scalar perturbations at the horizon crossing time, we find

\[ V_0 \alpha^4 = \frac{75\pi^2 P_s}{16\gamma^3} \frac{(1 + \gamma)^3 \exp(-\gamma N)}{((1 + \gamma) - \exp(-\gamma N))^4} \tag{45} \]

Thus, for a given point of the allowed region in Fig.7 we can use of the data for the amplitude of the scalar perturbations to determine the parameters of the potential. In Table IV we give the values of the constants \(\alpha\) and \(V_0\) for a chosen set of points from Fig.7.
FIG. 7: The allowed parameter space \((\gamma, N)\) for the generalized T-mode potential that yield values of \((r, n_s)\) that are in agreement with observation at the 68% CL (dark blue) and 95% CL (light blue).

| \(\gamma\) | \(N\) | \(\alpha\) (GeV\(^{-1}\)) | \(V_0\) (GeV\(^4\)) | \(V^*\) (GeV\(^4\)) |
|---|---|---|---|---|
| 0.010 | 89 | \(2.94 \times 10^{-16}\) | \(4.21 \times 10^{61}\) | \(2.50 \times 10^{61}\) |
| 0.015 | 84 | \(2.12 \times 10^{-16}\) | \(1.46 \times 10^{61}\) | \(1.05 \times 10^{61}\) |
| 0.020 | 76 | \(1.74 \times 10^{-16}\) | \(7.38 \times 10^{60}\) | \(7.38 \times 10^{60}\) |
| 0.025 | 80 | \(1.39 \times 10^{-16}\) | \(3.79 \times 10^{60}\) | \(3.29 \times 10^{60}\) |
| 0.030 | 85 | \(1.15 \times 10^{-16}\) | \(2.13 \times 10^{60}\) | \(1.97 \times 10^{60}\) |

TABLE IV: The constants \(\alpha\), \(V_0\) and the energy scale of the inflation for different values of \((\gamma, N)\) taken from Fig.7 and \(M_5 = 2 \times 10^{14}\) GeV

To examine the swampland criteria, in Fig.8 we plot the quantities \(\Delta \phi/M_p\) and \(M_p|V'/V|\) for different values of \(\gamma\) and \(N\). In Figs.8(a) and 8(b) we note that as \(\gamma\) increases, \(\Delta \phi/M_p\) and \(M_p|V'/V|\) respectively increases and decreases. Figs.8(c) and 8(d) which represent the behavior of these quantities during inflation (versus the number of e-fold) for different values of the constant \(\gamma\), and as the inflaton approaches the end of inflation, \(\Delta \phi/M_p\) and \(M_p|V'/V|\) respectively decreases (as was expected) and increases, and hence during the whole period of inflation the swampland criteria are satisfied. Therefore, the potential of the form (43) can be in consistent with the Planck data and satisfy the swampland criteria.
FIG. 8: The figures show the behavior of: a) $M_p \Delta \phi \equiv \Delta \phi$ and b) $M_p V/V' \equiv \Delta V$ versus the number of e-fold for different values of $n$ taken from Fig.1

V. CONCLUSION

We studied the inflationary scenario in the framework of brane gravity, where all standard particle live on a four-dimensional space-time embedded in five-dimensional space-time. In particular, the inflaton is confined on the brane and its energy density dominates the universe. Unlike in the standard cosmology, the Friedmann equation contains a term quadratic in the energy density which affects the dynamics in the high energy regime. After deriving the general expressions of the slow-roll parameters and the density perturbations generated during inflation, we investigated in details some well known class of inflaton potentials. We determined the allowed range of the potential parameters that yield values of the spectral index and the tensor-to-scalar ratio that are in agreement with the Planck data. We also showed that these type of potentials satisfy the swampland criteria.
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Note that different values of $N$ correspond to different horizon crossing time.
Note that since the scalar field approaches $\phi_e$ as time passes, the quantity $\Delta\phi/M_p$ gets smaller and smaller.