Analysis of eigenmodes in a swirling jet and flame: 3D PIV and PLIF study

S Abdurakipov\textsuperscript{1,2,*}, A Lobasov\textsuperscript{1,2}, L Chikishev\textsuperscript{1,2} and V Dulin\textsuperscript{1,2}

\textsuperscript{1} Kutateladze Institute of Thermophysics SB RAS, Novosibirsk, Russia
\textsuperscript{2} Novosibirsk State University, Novosibirsk, Russia

*E-mail: s.s.abdurakipov@gmail.com

Abstract. The existence of spiral structures was directly confirmed experimentally on the basis of 3D Particle Image Velocimetry (PIV) and planar laser-induced fluorescence (PLIF) measurements both in a strongly swirling non-reacting jet and in a fuel-lean premixed methane-air flame. Flows were characterized by the presence of vortex breakdown and precession of the vortex core. The differences in magnitude and spatial distribution of flow eigenmodes with distance from the nozzle are analyzed by using Proper Orthogonal Decomposition (POD) based on Spatial Fourier Transform over azimuthal coordinate. The analysis of flow eigenmodes reveal that for a high-swirl isothermal jet, the vortex core co-existed with the pair of counter-rotating helical vortices, which were located in outer shear layer and inside the recirculation zone. This double-vortex helical structure was also detected in a swirling flame, but its magnitude was suppressed compared with isothermal flow. It was determined that the change in the shape of the chemical reaction area was associated with two types of large-scale coherent structures: nearly axisymmetric mode $m = 0$ of the flame front deformation, presumably due to the effect of buoyancy forces on the combustion products, and the quasi-solid rotation of the asymmetric global mode $|m| = 1$ in the form of double-vortex structure due to precession of the swirling flow. It was shown that the energy of the axisymmetric mode $m = 0$ in a reacting jet increased downstream by almost 10 $\%$, unlike other modes, whose energy was only diminished.

1. Introduction

In modern burners and combustion chambers lean premixed combustion regimes are used to reduce the level of emissions of nitrogen oxides NOx and carbon monoxide CO [1]. However, lean premixed flames are known to be sensitive to various disturbances, for example, to slight changes in the excess fuel ratio, hydrodynamic and thermoacoustic instabilities. This can lead to undesirable burning conditions in technical devices. Therefore, the study of sensitivity and stability, turbulent structure and dynamics of lean premixed flames at the moment is a very significant task. Its solution is necessary to understand the operating conditions and improve the efficiency of modern burners [2].

The study of the structure of turbulent flows with swirl and combustion attracts great attention in the literature [2, 3]. Swirling flow is often used in burner devices to stabilize the flame, especially under adverse conditions, for example, with super-lean combustion conditions close to the extinction limit. For a swirling jet, when the critical value of the swirl parameter is reached, the vortex core
breakdown with formation of central recirculation zone takes place. Strongly swirling flows with vortex breakdown and precession of the vortex core (PVC) are often used to stabilize the premixed flame, since the intense turbulent fluctuations caused by large-scale spiral vortices provide intense heat and mass transfer between the fresh and burnt mixture in the region of reverse flow [3]. The presence of combustion makes the structure of the swirling flow even more complicated due to the sharp expansion of the jet, the effects associated with density variation. In particular, combustion can influence the onset of disintegration of the vortex core. Mourtazin and Cohen [4] have shown that PVC can be suppressed or enhanced depending on the negative or positive temperature difference between the jet core and the surrounding fluid. The PVC in swirling reactive jets was analyzed in review paper [3]. Syred [3] indicates that the occurrence and amplitude of PVC in reacting jets largely depends on many parameters, for example, the composition and stoichiometry of the mixture, nozzle geometry, size of the combustion chamber, etc. Oberleithner etc. [5] based on velocity measurements and density estimates, as well as stability analysis, have shown that the appearance of a global mode in the form of PVC and two secondary helices is substantially dependent on the shape of the flame. It was found that the appearance of a global mode for all the cases considered depended on two factors: intensity of the recirculation zone and level of stratification. The smaller the ratio of the density in the jet core to the density in the free stream, the greater the intensity of the reverse flow is required to become globally unstable to the first azimuthal mode |m| = 1. This work [5] confirms that the suppression of PVC is caused by an inhomogeneous density distribution induced by the flame. The authors determined that the characteristics of PVC depended to a greater extent on the radial density gradient and to a lesser extent on the change in the shape of the mean velocity profile.

The development of modern optical techniques, such as tomographic PIV [6], planar laser-induced fluorescence (PLIF) [7, 8] can provide a deeper understanding of this problem. In recent works [7-14], a study of large-scale vortex structures in a swirling jet has been carried out using modern optical diagnostics and statistical methods [15, 16]. However, the role of large-scale vortex structures in occurrence of non-stationary combustion regimes in swirling jet has not been sufficiently studied [2].

The aim of the present study is to investigate 3D vortex structures in a swirling turbulent jet and in fuel-lean premixed methane-air flame by using tomographic PIV and PLIF techniques. The focus is placed on statistical analysis of eigenmodes associated with coherent structures in the high-swirl flows.

2. Experimental setup and data processing

Experimental studies were conducted on an open model burner [8, 12]. Isothermal and reactive jet flows were organized by a Vitoshinsky nozzle with an outlet diameter d = 15 mm. The bulk velocity was $U_0 = 5$ m/s. Swirl of the flow was organized by using swirler mounted to the nozzle, and the degree of swirl was determined by the swirler geometry according to the definition of [3] and was equal to $S = 1.0$. The degree of swirl is considered to be high because the swirl forms a jet flow with pronounced vortex breakdown and central recirculation zone (in the case of a non-reactive flow). The experiments with reacting jets were carried out at the values of the fuel excess ratio of methane-air mixture $\Phi = 0.7$. The Reynolds number, constructed from the bulk velocity $U_0$ and air viscosity, was $\text{Re} = 5000$.

The tomographic measurement system [12] consisted of 8 cameras: 4 ImperX IGV-B2020 cameras and 4 ImperX IGV-B2020 cameras with Sigma AF # 50 lenses and band-pass optical filters (full width 10 nm and half maximum) from Edmund Optics with 60% transmittance at 532 nm; Nd: YAG laser (Quantel EverGreen 200) with an energy of 200 mJ per pulse; and two mirrors that were used to organize a multi-pass scheme for the laser beam. Flow was seeded with Al$_2$O$_3$ particles 4 μm in size. The depth of the illuminated volume was 45 mm. To calibrate the optical system, a calibration target (Edmund Optics) was installed above the nozzle. A self-calibration procedure similar to [17] was used. The 3D images were reconstructed and processed using MLOS-SMART (15 iterations) and MTE algorithms. An iterative cross-correlation procedure with continuous displacement and volume deformation was applied to estimate the three-dimensional velocity fields. The final size of the
measurement volume was 403 voxels with an overlap of 75%. The experimental setup and experiment details are described in more detail in [8, 12].

The PLIF measurements were carried out in the flow cross-sections at various distances from the nozzle exit. The distance from the nozzle to the measuring plane was varied using a motorized coordinate device that moved the nozzle along the vertical axis. Positioning accuracy was 100 microns. To excite HCHO fluorescence, third-harmonic radiation (355 nm) of a pulsed Nd: YAG laser (Quantel Brilliant B with an energy of 45 mJ of each pulse) was used. A transition in the A – X band was excited. The standard deviation of the laser energy did not exceed 5%. In order to create a laser sheet with a thickness of less than 0.8 mm, collimating optics were used in the measurement area. The HCHO fluorescence signal was recorded by an sCMOS camera (LaVision Imager Pro X, 16-bit dynamic range) equipped with a brightness amplifier based on an electron-optical converter (LaVision IRO), quartz lens (f # 2.8) and optical HCHO PLIF filter (LaVision). The laser flash duration was approximately 10 ns. The image recording frequency was 10 Hz, the exposure time of each frame was 200 ns. Using the FlameMaster software, the background signal was subtracted from the obtained images with correction of the irregularity in laser sheet intensity and the changes in laser energy from pulse to pulse, as well as inhomogeneous spatial sensitivity of the camera sensor. Details of experiment can be found in [8].

![Figure 1](image)

**Figure 1.** (a) Photograph of a strongly swirling flame of a methane-air mixture (S=1.0, Φ=0.7). (b) An example of instantaneous 2D distribution of PLIF signal pulsations in a flow cross-section (y/d = 1.5). (c) Example of instantaneous 3D velocity distribution in a swirling flame (for clarity, two flow cross-sections are presented). Gray and red isosurfaces of the Q-criterion visualize vortex structures in the outer mixing layer and inside the recirculation zone, respectively.

In figure 1, for example, a photograph of a lean premixed methane-air flame, 2D PLIF realization in cross-section y/d = 1.5, and instantaneous 3D velocity field of a strongly swirling methane-air flame (S = 1.0 and Φ = 0.7) are shown. For clarity, two cross sections of y/d = 0.2 and 1.5 are depicted. Gray and red isosurfaces of the Q-criterion $Q = \Omega_{ij} \Omega_{ij} - S_{ij} S_{ij}$ ($\Omega_{ij}$ and $S_{ij}$ are the antisymmetric and symmetric part of the velocity gradient tensor) visualize vortex structures in the outer mixing layer and inside the recirculation zone, respectively. The obtained 150 three-dimensional velocity fields and 1000 PLIF realizations were then investigated using Fourier analysis and Proper Orthogonal Decomposition method.
3. Analysis of 3D PIV measurements

To reveal the coherent modes, present in the set of velocity fields, we performed the Fourier transform procedure of the three-dimensional instantaneous velocity fields using azimuth angle θ. Thus, each 3D velocity field \( \mathbf{u}(r, \theta, y, t_k) \) was represented as the complex Fourier amplitudes \( \hat{\mathbf{u}}^m(r, y, t_k) \), according to the formula:

\[
\hat{\mathbf{u}}^m(r, y, t_k) = \frac{1}{2\pi} \int_0^{2\pi} \mathbf{u}(r, \theta, y, t_k) e^{-im\theta} d\theta 
\approx \frac{1}{N_\theta} \sum_{l=1}^{N_\theta} \mathbf{u}(r, \theta_l, y, t_k) e^{-im\theta_l} \quad (1)
\]

The Fourier transform procedure requires interpolating data from a Cartesian coordinate system onto a cylindrical grid. This is done with a constant grid spacing in each direction using the method of weighted nearest neighbors, where \( r \) is the radial coordinate, defined as \( r^2 = x^2 + z^2 \). The 3D velocity fields decompose to maximum wave number \( |m| = M = 30 \).

![Figure 2](image-url)

**Figure 2.** The distribution of the total turbulent kinetic energy (TKE) \( k \) of velocity fluctuations for different azimuthal modes \( m \) for the case of (a) a strongly swirling isothermal jet (\( S = 1.0, \Phi = 0 \)) and (b) a strongly swirling methane-air flame (\( S = 1.0, \Phi = 0.7 \)).

Figure 2 shows the total turbulent kinetic energy (TKE) distribution \( k = < u'u' + v'v' + w'w' > /2 \), averaged over time and azimuthal angle, for an isothermal and reacting jets, respectively. Figures 2b and 3b show the spatial distributions of the kinetic energy of pulsations obtained after the spatial
Fourier analysis for the first four azimuthal modes ($|m|$ from 0 to 3). As you can see, the azimuthal mode $|m| = 1$ in an isothermal jet contains the greatest amount of kinetic energy of fluctuations compared with other modes. For strongly swirling isothermal jet, the mode $|m| = 1$ have the strongest amplitude and are associated with intensive velocity fluctuations in the central recirculation zone. The axisymmetric mode $m = 0$ is mainly produced by velocity fluctuations around the jet axis. According to contribution of different velocity components to the amplitude of azimuthal modes in the strongly swirling jet, the axisymmetric mode $m = 0$ is related to the axial velocity fluctuations mainly along the jet axis, for $|m| = 1$, all velocity components have comparable contributions. The azimuthal and radial components are the most intensive at the jet axis, near $y/d = 0.2$. Strong velocity fluctuations in this region are concluded to be produced by the vortex core precession just downstream from the nozzle exit. According to the spatial stability analysis [13], the maximum growth rate of eigenmode $m = 1$ is observed in this region. Precession of the swirling jet core near the bottom stagnation point produces roll-up of the large-scale vortices in the inner shear layer.

For each azimuth mode $m$, the obtained sequence of complex Fourier amplitudes $\hat{u}(r, \theta, y, t_k)$ was analyzed by the POD method [14, 19] to extract the most energetic coherent perturbations:

$$
\textbf{u}(r, \theta, y, t_k) = \sum_{m=-M}^{+M} \hat{u}^m(r, y, t_k) e^{im\theta} = \sum_{m=-M}^{+M} \sum_{q=1}^{N_t} a_q^m(t_k) \lambda_q^m \Phi_q^m(r, y) e^{im\theta} \quad (2)
$$

where

$$
\int_{B_{r-y}} \Phi_i^m \Phi_j^m r dr dy = \delta_{ij}, \quad \frac{1}{N} \sum_{k=1}^{N_t} a_i^m(t_k) a_j^m(t_k) = \delta_{ij} \quad (3)
$$

The POD method is based on representing each $k$-th instantaneous complex Fourier coefficient $\hat{u}^m(r, y, t_k)$ of the corresponding velocity field $\textbf{u}(r, \theta, y, t_k)$ as a finite series (2) of the products of complex-valued orthonormal spatial basic functions $\Phi_q^m$ and time coefficients $a_q^m$ satisfying equality (3), and real eigenvalues $\lambda_q^m$ characterizing the amplitude of each POD mode in the data sequence. $N_t$ corresponds to the number of measured velocity fields, i.e., $N_t = 150$. Eigenmodes and POD coefficients for wavenumbers $m$ with the opposite sign are complex conjugate.

![Figure 3](image-url)

Figure 3. The eigenvalue spectra of the POD decomposition of the velocity fluctuations for (a) strongly swirling isothermal jet and (b) strongly swirling methane-air flame for different azimuthal modes $m$. The dependence of the real and imaginary parts of the correlation coefficients for the mode $|m| = 1, q = 1$ in the case of (c) strongly swirling isothermal jet ($S = 1.0, \Phi = 0$) and (d) strongly swirling methane-air flame ($S = 1.0, \Phi = 0.7$).

To reduce the computational needs of the POD algorithm, we used the singular value decomposition (SVD) (see [9, 16]) to calculate spatial orthonormal basis functions, eigenvalues, and
temporal amplitudes. This procedure was previously tested and used in [9, 14, 18, 19]. The spatial structure of the dominant azimuthal modes was analyzed as a superposition of the $U_{\text{mean}}$ average velocity field and the most energetic POD modes (i.e., for $q = 1$) using the formula:

$$u_{LO}^{ml}(r, \theta, y, t) = U_{\text{mean}}(r, \theta, y) + u_{m}^{l}(r, \theta, y, t) = U_{\text{mean}} + a_{q}^{m}(t)\lambda_{q}^{m}\phi_{q}^{m}(r, y)e^{im\theta} + a_{q}^{m}(t)\lambda_{q}^{m}\phi_{q}^{m}(r, y)e^{-im\theta} = U_{\text{mean}} + 2\text{Re}\left[a_{q}^{m}(t)\lambda_{q}^{m}\phi_{q}^{m}(r, y)e^{im\theta}\right]$$  \hspace{1cm} (4)

Figure 4. The spatial distributions of the real and imaginary parts of the most energetic POD mode $|m| = 1, q = 1$ for the case of (a) a strongly swirling isothermal jet and (b) a strongly swirling methane-air flame. Reconstruction of the phase-averaged flow structure based on the average 3D velocity field and POD mode $|m| = 1, q = 1$ for the case of (c) a strongly swirling isothermal jet and (d) a strongly swirling flame. Gray isosurfaces of the Q-criterion visualize large-scale helical vortex structures in the outer mixing layer and inside the recirculation zone.

Figure 3 and 4 show POD results for azimuthal modes for a strongly swirling jet and flame. The spectra (see Figure 3) demonstrate that the most energetic POD mode $|m| = 1, q = 1$, containing approximately 10% of the total TKE for the case of non-reacting jet and 7% for reacting jet. According to the reconstruction of phase-averaged velocity field (4), eigenmode $|m| = 1$ is associated with a double helix structure consisting of a spiral vortex core and another spiral vortex in the outer layer. Based on the visualization, it is concluded that these helical structures are associated with counter-winding eigenmode $m = +1$. This double-vortex helical structure was also detected in a swirling flame, but magnitude of PVC and helical vortices was suppressed compared with isothermal flow. As Figure 3 c and d show, real and imaginary parts of correlation coefficients of the most energetic POD mode $|m| = 1, q = 1$ of velocity fluctuations in the high-swirl jet and flame are distributed around a circle Lissajous figure, which indicates the quasi-periodic dynamics of the dominant mode, as previously shown in [7, 9, 14].
4. Analysis of PLIF measurements

The planar laser-induced fluorescence of formaldehyde was used to study the shape of the chemical reaction region in a methane-air mixture in a turbulent flow of a swirling jet with vortex breakdown and precession of the vortex core. To determine the coherent structures arising from deformation of the flame front, the measured sets of “instantaneous” realizations were decomposed into the main flow eigenmodes. The azimuthal modes in PLIF realizations in the swirling jet and flame were analyzed in a manner similar to the 2D approach used in [18]. This 2D analysis differs from the 3D detection of helical modes in velocity fields, which is described above. The focus was on the differences in magnitude and spatial distribution of flow eigenmodes with a distance from the nozzle y/d.

The averaged squared fluctuations $\langle l_m'(r)l_m'(r) \rangle$ of the azimuthal modes $m$ averaged over all measurements depending on the radial coordinate $r/d$:

(a) for the axial position $y/d = 0.2$ and 0.5, solid and dashed curves, respectively;
(b) for the axial position, $y/d = 1.0$ and 1.5.

Figure 5. The quadratic fluctuations of PLIF signal (strongly swirling methane-air flame) for different azimuthal modes $m$ averaged over all measurements depending on the radial coordinate $r/d$:
(a) for the axial position $y/d = 0.2$ and 0.5, solid and dashed curves, respectively;
(b) for the axial position, $y/d = 1.0$ and 1.5.

The averaged squared fluctuations $\langle l_m'(r)l_m'(r) \rangle$ of the azimuthal modes $m$ from 0 to 4 in the high-swirl flame are presented in figure 5. The curves correspond to the distribution in the radial direction. Figure 5 a show a comparison of distributions at positions $y/d = 0.2$ (solid lines) and $y/d = 0.5$ (dashed lines). Figure 5 b shows positions $y/d = 1.0$ and $y/d = 1.5$. At a small distance from the nozzle, a sharp increase in the amplitude of all azimuthal modes is observed. Axisymmetric mode $|m| = 0$ has the greatest amplitude. The maximum is in a mixing layer of reacting jet and shifts to the right with $y/d$, which is related to the flow expansion. As the distance from the nozzle increases, the amplitude of pulsations of all azimuthal modes begins to decrease, except for the axisymmetric mode $|m| = 0$. A similar picture can be observed when analyzing the first two POD eigenvalues $q = 1, 2$ for various azimuthal modes (see figure 6). For the first azimuthal mode $|m| = 1$, the squared fluctuations begin growing to a distance $y/d = 0.5$, and then decrease in downstream direction, at the same time, axisymmetric mode $|m| = 0$ is permanently amplified downstream (“energy” proportion rises from 5 to about 15%) presumably due to the effect of buoyancy forces on the combustion products. The presence of low-frequency oscillations of the global mode associated with the action of buoyancy forces was previously documented in [11].
Figure 6. The dependence of (a) the first $q = 1$ and (b) the second $q = 2$ eigenvalue (according to magnitude) of POD decomposition of PLIF fluctuations (strongly swirling methane-air flame) for different azimuthal modes $m$ from the distance to the nozzle $y/d$.

Four POD modes for “instantaneous” PLIF realizations ($|m|$ from 0 to 3) in different flow cross-sections are visualized in figure 7 and 8 based on 2D form of equation (4). Axisymmetric POD modes for $|m| = 0$ correspond to the oscillations in the jet core and in the outer region due to buoyancy effect, what is most pronounced after $y/d = 0.5$. The first POD of the azimuthal mode $|m| = 1$ is related to fluctuations in the inner and outer mixing layer, presumably during quasi-solid rotation of the asymmetric global self-oscillating mode in the form of double-vortex structure due to the precession of swirling flow. The maximum amplitude of POD modes for $|m| = 1$ is reached around $y/d = 0.5$ (see figure 7). The amplitudes of $|m| = 2$ and $|m| = 3$ decrease downstream. The proportion of quadratic fluctuations drops from 7 to 3% (see figure 8).

Figure 7. Dependence of maximum POD eigenvalue ($q = 1$) of PLIF fluctuations (strongly swirling methane-air flame) for two azimuthal modes $|m| = 0$ and 1 from the nozzle distance $y/d$. The color shows the spatial distribution of fluctuations of the most energetic first POD mode $|m| = 0$, $q = 1$ and $|m| = 1$, $q = 1$ varying on the distance to the nozzle.
Figure 8. Dependence of maximum POD eigenvalue ($q = 1$) of PLIF fluctuations (strongly swirling methane-air flame) for two azimuthal modes $|m| = 2$ and $3$ from the nozzle distance $y/d$. The color shows the spatial distribution of the fluctuations of the most energetic first POD mode $|m| = 2$, $q = 1$ (top row) and $|m| = 3$, $q = 1$ (bottom row) varying on the distance to the nozzle.

Conclusions
Isothermal swirling air jet flow with vortex breakdown and fuel-lean methane-air flame were investigated by using a tomographic PIV and planar laser-induced fluorescence method. The existence of spiral structures was directly confirmed on the basis of 3D PIV and PLIF measurements both in a strongly swirling jet and in a flame. The spatial shape and dynamics of the vortex core and spiral coherent structures are identified by using modal decomposition technique Proper Orthogonal Decomposition (POD) based on spatial Fourier transform over azimuthal coordinate. The POD method allowed to obtain the spatial phase-averaged structure of the most energetic flow eigenmodes from the ensemble of PIV and PLIF realizations. For PLIF measurements, the analysis was performed in cross-sections at different distances from the nozzle. The differences in squared fluctuations and shape of eigenmodes with distance from the nozzle are analyzed. Vortex structures in instantaneous velocity fields are visualized using vortex identification criteria based on the analysis of the local velocity gradient tensor and its invariants. The analysis of flow eigenmodes reveal that for a high-swirl isothermal jet, the vortex core co-existed with the pair of co-rotating counter-winding helical vortices, which were located in the outer shear layer and inside the recirculation zone. It was determined that each vortex of the pair, and the whole double-helix structure winded in the direction, opposite to the flow swirl. The vortices in the shear layers were convected by the flow, and the double-helix structure rotated in the same direction as jet swirling. This double-vortex structure was also detected in the swirling flame, but its amplitude (together with PVC) was suppressed as compared with the isothermal flow. From PLIF measurements, it is shown that the change in the shape of the chemical reaction area is associated with two types of large-scale coherent structures: nearly axisymmetric mode $m = 0$ of the flame front deformation, presumably due to the effect of Archimedes forces on the combustion products, and rotation of the asymmetric global mode $|m| = 1$ in the form of double-vortex structure due to precession of the swirling flow. The energy of the axisymmetric mode in the reacting jet increased downstream by almost 10%, unlike other azimuthal modes, whose energy was only diminished.

Acknowledgements
This work was financially supported by the Russian Science Foundation, grant number 19-79-30075.
References

[1] Dunn-Rankin D 2008 Lean Combustion: Technology and Control (Amsterdam: Elsevier)
[2] Lieuwen T 2012 Unsteady combuster physics (Cambridge University Press)
[3] Syred N 2006 Progress in Energy and Combustion Science 32 (2) 93–161
[4] Mourtazin D, Cohen J 2007 Journal of Fluid Mechanics 571 177–89
[5] Terhaar S, Oberleithner K, Paschereit C 2015 Proceedings of the Combustion Institute 35 3347–54
[6] Scarano F 2013 Measurement Science and Technology 24 012001
[7] Stohr M, Sadanandan R, Meier W 2011 Experiments in Fluids 51 (4) 1153–67
[8] Lobasov A, Abdurakipov S, Chikishev L, Dulin V, Markovich D 2018 Explosion and Shock Waves 54 (6) 642–8
[9] Markovich D, Abdurakipov S, Chikishev L, Dulin V, Hanjalic K 2014 Physics of Fluids 26 (6) 065109
[10] Sharaborin D, Abdurakipov S, Dulin V 2016 Journal of Physics: Conference Series 754 (7) 072003
[11] Abdurakipov S, Dulin V, Markovich D, Hanjalić K 2013 Technical Physics Letters 39 (3) 308–11
[12] Dulin V, Chikishev L, Tokarev M, Markovich D 2014 Proceedings of 17th International Symposium on Applications of Laser Techniques to Fluid Mechanics 7–10
[13] Abdurakipov S, Dulin V, Markovich D 2018 Thermophysics and Aeromechanics 25 (3) 379–86
[14] Markovich D, Dulin V, Abdurakipov S, Kozinkin L, Tokarev M, Hanjalić K 2016 Journal of Turbulence 17 (7) 678–98
[15] Holmes P, Lumley J, Berkooz G 1998 Turbulence coherent structures dynamical systems and symmetry (Cambridge university press)
[16] Taira K, Brunton S, Dawson S, Rowley C, Colonius T, McKeon B, Ukeiley L 2017 AIAA Journal 4013–41
[17] Wienke B 2008 Experiments in Fluids 45 549–56
[18] Alekseenko S, Abdurakipov S, Hrebtov M, Tokarev M, Dulin V, Markovich D 2018 International Journal of Heat and Fluid Flow 70 363–79
[19] Mullyadzhanov R, Sandberg R, Abdurakipov S, George W, Hanjalić K 2018 Physical Review Fluids 3 062601