Paritonic calculation of $\gamma Z$ Exchange Corrections to Parity-Violating Elastic Electron-Proton Scattering

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We calculate $\gamma Z$ exchange corrections to the parity-violating asymmetry of the elastic electron-proton scattering in a parton model using the formalism of generalized parton distributions (GPDs). We also examine the validity of the zero-momentum-transfer approximation adopted in the literature and find that it overestimates the $\gamma Z$ exchange effect significantly at the forward scattering angles.

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The parity asymmetry $A_{PV} = \sigma_R - \sigma_L$ in scattering of longitudinally polarized electrons elastically from unpolarized protons is an important observable because it provides crucial information of the strangeness content of the proton. At the tree level, the parity asymmetry $A_{PV}$ arises from the interference of diagrams with one-photon-exchange (OPE) and $Z$-boson exchange shown in Fig. 1(a) and (b), respectively. The interactions between photon, $Z$-boson and proton are described by the form factors of the proton defined as

$\langle p' | J_Z^p | p \rangle = \bar{u}(p') \left( F_1^{zp}\gamma_\mu + F_2^{zp} i\gamma_\mu q'^\nu + G_A^{zp} \gamma_\mu \gamma_5 \right) u(p),$

$\langle p' | J_\mu^p | p \rangle = \bar{u}(p') \left( F_1^{\mu p}\gamma_\mu + F_2^{\mu p} i\gamma_\mu q'^\nu + G_M^{\mu p} \gamma_\mu \right) u(p),$

where $q = q' - p$ and $M$ is the mass of the proton. $F_1^{zp}$, $F_2^{zp}$ and $G_A^{zp}$ are the form factors of the proton electromagnetic and neutral weak current, respectively. The parity asymmetry $A_{PV}$ can be expressed by the form factors defined above as

$A_{PV}(1 + Z) = \frac{G_E Q^2}{4\pi \alpha\sqrt{2}} \frac{A_E + A_M + A_A}{[\epsilon(G_E^2) + \tau(G_M^2)]^2},$ (1)

where $M$ is the proton mass and $G_E$ is Fermi constant and $Q^2 = -q^2$, $\tau = Q^2/(4M^2)$, and $\epsilon = (1 + 2(1 + \tau) \tan^2 \frac{\theta_W}{2})^{-1}$. $A_E$, $A_M$ and $A_A$ are defined as

$A_E = \epsilon G_{E}^{zp} G_{E}^{\gamma}\gamma, A_M = \tau G_{M}^{zp} G_{M}^{\gamma}\gamma, A_A = (-1 + 4 \sin^2 \theta_W) \sqrt{\tau(1 + \tau)(1 - \epsilon^2)} G_{E}^{zp} G_{M}^{\gamma}\gamma$,$$ (2)

where $\theta_W$ is the weak mixing (Weinberg) angle and $G_{E}^{zp} = F_1^{zp} - \tau F_2^{zp}$, $G_{M}^{zp} = F_1^{zp} + F_2^{zp}$. When combined with proton and neutron electromagnetic form factors and with the use of charge symmetry, one obtains the following relation [1]:

$G_{E,M}^{zp} = (1 - 4 \sin^2 \theta_W) G_{E,M}^{\gamma}\gamma - G_{E,M}^{n}\gamma - G_{E,M}^{s}\gamma$. (3)

Consequently Eq. (1) becomes

$A_{PV}(1 + Z) = A_1 + A_2 + A_3$.

$A_1 = -a \left[ 1 - 4 \sin^2 \theta_W - \frac{\epsilon G_{E}^{zp} G_{E}^{\gamma}\gamma + \tau G_{M}^{zp} G_{M}^{\gamma}\gamma}{\epsilon(G_E^2) + \tau(G_M^2)} \right],$

$A_2 = \frac{\epsilon G_{E}^{zp} G_{E}^{\gamma}\gamma + \tau G_{M}^{zp} G_{M}^{\gamma}\gamma}{\epsilon(G_E^2) + \tau(G_M^2)},$

$A_3 = a(1 - 4 \sin^2 \theta_W) \sqrt{\tau(1 + \tau)(1 - \epsilon^2)} G_{E}^{zp} G_{M}^{\gamma}\gamma,$ (4)

where $a = \frac{G_E Q^2}{4\pi \alpha\sqrt{2}}$. The strange form factors $G_{E,M}^{s}\gamma$ can be determined through Eq. (3). Four experimental programs SAMPLE [2], HAPPEX [3], A4 [4], and G0 [5] have been designed to measure $A_{PV}$, which is small and ranges from 0.1 to 100 ppm. Their results demonstrate that the sum of $A_1$ and $A_2$ is much larger than $A_3$ and the theoretical uncertainty is comparable in size to $A_2$. This calls for greater efforts to reduce the theoretical uncertainty in order to arrive at a more precise determination of the strange form factors.

The main theoretical uncertainty is from higher-order electroweak radiative corrections which have been care-
fully studied in [6, 7]. Those corrections are from the interference of the electroweak one-loop diagrams with the tree level diagrams. Among those one-loop diagrams, the two-boson-exchange box diagrams shown in Figs. 1(c) and 1(d) are particularly intricate because they both depend on the nucleon structure.

Recently the Two-Photon-Exchange (TPE) diagram (Fig 1(c)) has been evaluated in [8] in a parton model using GPDs where the handbag approximation is used, namely the two photons are coupled directly to the same point-like quark. The contribution of the interference of the two-photon-exchange process of Fig. 1(c) with the one-boson-exchange diagram of Fig. 1(d) are particularly intricate because they both depend on the nucleon structure.

In this Letter, we report on the result of the partonic calculation of the parity-violating exchange effects. They have however been computed under a certain approximation where the momenta carried by the two bosons were assumed to be equal, corresponding with the forward scattering case. Furthermore the electron momenta were assumed to be zero, which is relevant kinematical limit to calculate parity violation in atoms, considered in [6].

To reduce the theoretical uncertainty at finite $Q^2$, one must go beyond this particular approximation in the calculation of the $\gamma Z$ exchange effects. In the same parton model developed in [8]. We first calculate the subprocess on a quark $e(k) + q(P_q) \rightarrow e(k') + q(P_{q'})$ in denoted by the scattering amplitude $\hat{H}$ in Fig. 2. Subsequently we embed the quarks in the proton as described through the nucleon’s generalized parton distributions (GPDs). Here only GPDs of $u$ and $d$ quarks are included because the strange quark contribution to the box diagrams is expected to be very small.

In the standard model the electromagnetic current and the neutral weak current of quarks are $J_{\mu}^{em} = \sum_q Q_q \bar{q} \gamma_\mu q$ and $J_{\mu}^{\nu} = \sum_q g(q) \bar{q} \gamma_\mu (1 + \gamma_5)q$. Here we follow the notation of [9] and have $g_v^q = 1 - \frac{2}{3} \sin^2 \theta_W$, $g_A^u = -1$ and $g_A^d = 1 + \frac{2}{3} \sin^2 \theta_W$, $g_A^A = +1$. The quark-level parity-violating amplitudes of the $\gamma Z$ exchange diagrams are

$$\begin{align*}
\mathcal{M}_{\gamma Z, Parton}^{PV} & = -i \frac{G_F}{2 \sqrt{2}} \hat{u}_e \gamma_\mu u_e \hat{u}_N \Gamma_{1}\gamma_5 u_N \\
& \quad + \hat{u}_e \gamma_\mu \gamma_5 u_e \hat{u}_N \Gamma_2 \gamma_5 u_N |_{\frac{P_{\mu}}{M}} u_N, \quad (7)
\end{align*}$$

Here $\hat{u}_e = (P_q + k)^2, \hat{u} = (P_q - k)^2$ and $\lambda$ is the infrared cut-off input by infinitesimal photon mass. The amplitudes are separated into the soft and hard parts, the soft part corresponds with the situation where the photon carries zero four momentum and one obtains $c_1^{soft} = \frac{-e^2}{16 \pi^2} \ln \left( \frac{\lambda^2}{Q^2} \right) + \frac{3 e^2}{16 \pi^2} \ln \left( \frac{\hat{u}}{M_Z^2} \right)$ and $c_2^{soft} = -7 + 3 \ln \left( \frac{\hat{u}}{M_Z^2} \right) + 3 \ln \left( \frac{\hat{u}}{M_Z^2} \right)$.

$\Gamma_1, \Gamma_2$ and $\Gamma_3$ are defined as

$$\begin{align*}
\Gamma_1 & = \frac{1 + \epsilon}{2 \epsilon} F - \frac{1 + \epsilon}{2 \epsilon} \frac{Q^2}{s - u} D, \\
\Gamma_2 & = \frac{1 + \epsilon}{2 \epsilon} D - \frac{1 + \epsilon}{2 \epsilon} \frac{Q^2 + 4M^2}{s - u} F, \\
\Gamma_3 & = \frac{1}{1 + \tau} \left[ \Gamma_2 - \frac{1 + \epsilon}{2 \epsilon} E \right], \quad (8)
\end{align*}$$

where $D, E$ and $F$ are defined as

$$\begin{align*}
D & = \int_{-1}^{1} \sum_{q = u, d} \frac{dx}{x} \frac{Q^2 t_q^2 + (\hat{u} - \hat{u}) t_q^2}{s - u} (H^q + E^q), \\
E & = \int_{-1}^{1} \sum_{q = u, d} \frac{dx}{x} \frac{Q^2 t_q^2 + (\hat{u} - \hat{u}) t_q^2}{s - u} (H^q - \tau E^q), \\
F & = \int_{-1}^{1} \sum_{q = u, d} \frac{dx}{x} \frac{Q^2 t_q^2 + (\hat{u} - \hat{u}) t_q^2}{s - u} \sgn(x) \tilde{H}^q, \quad (9)
\end{align*}$$

where $s = (p_1 + p_3)^2, \hat{u} = (p_2 - p_3)^2$ and $H^q, E^q$ and $\tilde{H}^q$ are the GPDs for a quark in the nucleon. To estimate the amplitudes of Eq. (9) one needs to specify a model for
tering, corresponding with the blob denoted by $H$ in Fig. 2.

$\begin{array}{cccc}
\gamma & k' & k & k \\
\gamma & k & k' & k \\
\gamma & k & k' & k \\
\end{array}$

$\begin{array}{cccc}
P_q & P_{q'} & P_q & P_{q'} \\
P_q & P_{q'} & P_q & P_{q'} \\
P_q & P_{q'} & P_q & P_{q'} \\
\end{array}$

FIG. 3: The direct and crossed box diagrams to describe the TPE and $\gamma Z$ exchange contribution to the lepton-quark scattering, corresponding with the blob denoted by $H$ in Fig. 2.

\[ \begin{array}{ll}
2.5 \text{ GeV}^2 & 4 \text{ GeV}^2 \\
0.5 & 0.5 \\
\end{array} \]

\[ \begin{array}{ll}
2.5 \text{ GeV}^2 & 4 \text{ GeV}^2 \\
0.5 & 0.5 \\
\end{array} \]

FIG. 4: TPE and $\gamma Z$ exchange corrections to parity-violating asymmetry as functions of $\epsilon$. The left of upper panel for $Q^2=1$ GeV$^2$, the right for $Q^2=2$ GeV$^2$. The left of lower panel for $Q^2=5$ GeV$^2$, the right for $Q^2=9$ GeV$^2$. The various contributions are $\gamma(\gamma\gamma)$ (dashed line), $Z(\gamma\gamma)$ (dotted lines), TPE total contribution $\delta_{2\gamma}$ (dash-dotted lines) and $\delta_{Z\gamma}$ (solid lines).

The GPDs. Again we follow [8] to adopt an unfactorized model of GPDs in terms of a forward parton distributions and a gaussian factor in $x$ and $-Q^2$. For the details we refer the readers to [8]. In Fig. 4 we show the TPE and $\gamma Z$ exchange corrections to $A_{PV}$ by plotting $\delta$, defined by

\[ A_{PV}(1+Z+2\gamma+\gamma Z) = A_{PV}(1+Z)(1+\delta_{2\gamma}+\delta_{Z\gamma}), \]

at four different values of $Q^2=1, 2, 5$ and 9 GeV$^2$. $A_{PV}(1+Z+2\gamma+\gamma Z)$ includes the effects of TPE and $\gamma Z$ exchange. The interferences between OPE and $\gamma Z$ exchange denoted by $\delta_{Z\gamma}$ are represented by solid lines. The effects of the parity conserving interference between OPE and TPE ($\gamma(\gamma\gamma)$), entering the denominator of the asymmetry, are represented by dashed lines. The interference between $Z$-exchange and TPE ($Z(\gamma\gamma)$) are represented by dotted lines, with their sum ($\delta_{2\gamma}$) denoted by dot-dashed lines. Our result of $\gamma(\gamma\gamma)$ and $Z(\gamma\gamma)$ is in agreement with [9].

Our result shows that TBE effects are about few percent as expected. However $\delta_{Z\gamma}$ and $\delta_{2\gamma}$ behave quite differently. In handbag calculation, $\delta_{Z\gamma}$ is sensitive to $Q^2$. For example, its value is tripled when $Q^2$ decreases from 5 GeV$^2$ to 1 GeV$^2$. On the other hand, the value of $\delta_{2\gamma}$ is between $1 \sim 2\%$ at all $Q^2$ values. Furthermore the value of $\delta_{2\gamma}$ at backward angles is in general larger than the one at forward angles but $\delta_{Z\gamma}$ is insensitive to $\epsilon$. On the contrary, the result of the hadronic model [10] shows that both of $\delta_{Z\gamma}$ and $\delta_{2\gamma}$, have very strong $\epsilon$ dependencies and always decrease into zero when $\epsilon$ approaches one. Such a difference may be able to be explained by the following: In the hadronic model the loop momentum integration is mostly from the low loop momenta because the form factors are inserted as regulators. However, in the partonic calculation the loop momentum integration is dominated by the high loop momentum. Naively one should add the results of the two calculation together. How to combine the results of these two calculations remains an open issue. Another important issue to address is the validity of the previous estimate of $\gamma Z$ exchange effects made by Marciano and Sirlin [4] because their estimate is widely used. They evaluated the $\gamma Z$ box diagrams in Fig. 3 in the limit of $k=k'=0$ and $P_q=P_{q'}$. Their result shows that

\[ \mathcal{M}_{\gamma Z}^{PV, MS}(eq \rightarrow eq) = \frac{-iG_F}{\sqrt{2}} \sum_{q=u,d} |C_{1q}| \]

\[ (\bar{u}_e \gamma_\mu \gamma_5 u_e)(\bar{q}\gamma^\mu q) + C_{2q}(\bar{u}_e \gamma_\mu u_e)(\bar{q}\gamma^\mu \gamma_5 q), \]

\[ (10) \]

here $C_{1q}$ and $C_{2q}$ are

\[ C_{1u} = \frac{\alpha}{2\pi} (1 - 4\sin^2 \theta_W) C, \quad C_{1d} = \frac{\alpha}{4\pi} (1 - 4\sin^2 \theta_W) C, \]

\[ C_{2u} = \frac{\alpha}{2\pi} \left(1 - \frac{8}{3}\sin^2 \theta_W\right) C, \quad C_{2d} = \frac{\alpha}{4\pi} \left(1 - \frac{4}{3}\sin^2 \theta_W\right) C, \]
here $C = \ln \frac{M^2}{\Lambda^2} + \frac{3}{4}$. $M$ is a hadronic mass scale associated with the asymptotic behavior and its value is set to be 1 GeV $[6]$. Because $C_{1q}$ and $C_{2q}$ are constants, the amplitude of Fig 1(d) can be written as

$$P_{\gamma} = \frac{iG_F}{\sqrt{2}} \bar{u}_c \gamma_\mu \gamma_5 u_e \sum_{q=u,d} C_{1q} \bar{u}_N [\bar{q} \gamma_\mu q] u_N,$$

$$P_{\gamma} = \frac{iG_F}{\sqrt{2}} \bar{u}_c \gamma_\mu u_e \sum_{q=u,d} C_{2q} \bar{u}_N [\bar{q} \gamma_\mu \gamma_5 q] u_N.$$

According to charge symmetry, the form factors associated with the matrix elements of quark vector current can be written as

$$G_{E,M}^{p/p} = 2G_{\gamma p}^{\gamma p} + G_{\gamma M}^{\gamma n} + G_{E,M}^n,$$

$$G_{E,M}^{p/p} = G_{\gamma M}^{\gamma p} + 2G_{\gamma M}^{\gamma p} + G_{E,M}^p.$$

(11)

If we follow notation from [6]:

$$C_{1u} = \frac{1}{2} \Delta \rho \left[ 1 - \frac{8}{3} \Delta \kappa \sin^2 \theta_W \right],$$

$$C_{1d} = -\frac{1}{2} \Delta \rho \left[ 1 - \frac{4}{3} \Delta \kappa \sin^2 \theta_W \right],$$

(12)

then one obtains the following widely used formula,

$$A_{PV}^{MS}(\gamma Z) = 2 Re[M_{1\gamma}^{\dagger} M_{\gamma Z}^{PV,MS}] = \hat{A}_1 + \hat{A}_2 + \hat{A}_3,$$

$$\hat{A}_1 = -a \Delta \rho \left[ 1 - 4 \Delta \kappa \sin^2 \theta_W - \frac{e G_{\gamma M}^{\gamma p} G_{\gamma M}^{\gamma p} + \tau G_{\gamma M}^{\gamma p} G_{\gamma M}^{\gamma p}}{\epsilon (G_{\gamma M}^{\gamma p})^2 + \tau (G_{\gamma M}^{\gamma p})^2} \right],$$

$$\hat{A}_2 = a \Delta \rho \frac{e G_{\gamma M}^{\gamma p} G_{\gamma M}^{\gamma p} + \tau G_{\gamma M}^{\gamma p} G_{\gamma M}^{\gamma p}}{\epsilon (G_{\gamma M}^{\gamma p})^2 + \tau (G_{\gamma M}^{\gamma p})^2},$$

$$\hat{A}_3 = a (1 - 4 \sin^2 \theta_W) \sqrt{1 + \tau} \left[ 1 - (1 - \epsilon^2) \right] \frac{G_{\gamma M} \bar{G}_A}{\epsilon (G_{\gamma M}^{\gamma p})^2 + \tau (G_{\gamma M}^{\gamma p})^2},$$

(13)

here $\bar{G}_A$ absorbs the contribution from $M_{\gamma Z}^{PV,MS}$ which is suppressed at the forward scattering angles.

We therefore define the following quantity to characterize the difference between our partonic calculation and the previous estimate taken at the limit $k = k' = 0$ and $P_q = P_{\gamma}$:

$$\Delta_{MS} = \frac{A_{PV}^{Parton}(\gamma Z)}{A_{PV}^{MS}(\gamma Z)} = \frac{Re[M_{1\gamma}^{\dagger} M_{\gamma Z}^{PV,Parton}]}{Re[M_{1\gamma}^{\dagger} M_{\gamma Z}^{PV,MS}]}.$$

If MS’s approximation overestimates the $\gamma Z$ exchange effect then $\Delta_{MS}$ will be smaller than one. $\Delta_{MS}$ is a function of $\epsilon$ and $Q^2$. Here we are mostly concerned about the case of low $Q^2$ and the forward angles. In Fig. 5 we choose $\epsilon = 0.99$ (dash-dotted line), 0.97(dotted line) and 0.95(dashed line) and 0.03 GeV$^2 \leq Q^2 \leq 1$ GeV$^2$.

It shows that MS’s approximation always overestimates the $\gamma Z$ exchange effect and the extent of overestimate grows as $Q^2$ and $\epsilon$ increase. Note that $\Delta_{MS}$ grows fast when $Q^2 \leq 0.1$ GeV$^2$ which means $A_{PV}^{Parton}(\gamma Z)$ grows fast there. It is interesting because similar feature also appears in the result of hadronic model [10]. However even when $Q^2=0.03$ GeV$^2$ and $\epsilon=0.99$, $\Delta_{MS} \approx 0.55$ which means MS’s approximation still overestimates $\gamma Z$ exchange effect up to 45%. It is obvious that $A_{PV}^{MS}(\gamma Z)$ is significantly larger than $A_{PV}^{Parton}(\gamma Z)$. Hence we corroborate the claim made by [10] that the $\gamma Z$ exchange effect is indeed overestimated by the zero-momentum-transfer approximation adopted by the previous estimate.

We conclude that the TPE and $\gamma Z$ exchange effects contribute to the parity asymmetry at a level of a few percent according to our partonic calculation. In particular, the $\gamma Z$ exchange effect is sensitive to $Q^2$ and its magnitude increases when $Q^2$ decreases but it is insensitive to $\epsilon$. We also compare the $A_{PV}(\gamma Z)$ under the approximation relevant for the calculation of atomic parity violation with $A_{PV}(\gamma Z)$ based on our partonic calculation. We find that $A_{PV}^{MS}(\gamma Z)$ is significantly larger than $A_{PV}^{Parton}(\gamma Z)$. Therefore we corroborate the claim made by [10] that $\gamma Z$ exchange effect is indeed overestimated by the zero momentum-transfer approximation.

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