SHATTERING FLARES DURING CLOSE ENCOUNTERS OF NEUTRON STARS

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ABSTRACT

We demonstrate that resonant shattering flares can occur during close passages of neutron stars in eccentric or hyperbolic encounters. We provide updated estimates for the rate of close encounters of compact objects in dense stellar environments, which we find are substantially lower than given in previous works. While such occurrences are rare, we show that shattering flares can provide a strong electromagnetic counterpart to the gravitational wave bursts expected from such encounters, allowing triggered searches for these events to occur.

Key words: galaxies: star clusters: general – gravitational waves – stars: kinematics and dynamics – stars: neutron

Online-only material: color figures

1. INTRODUCTION

The major expected source of gravitational waves (GWs) for the advanced LIGO (Harry et al. 2010) class of GW detectors is compact binary systems. The long inspiral signals from such binaries will be detected by matched filtering with theoretical templates, which allows signal-to-noise (S/N) to be built up over many orbits (Cutler & Flanagan 1994).

GWs are also emitted as broad band bursts when compact objects undergo close passages, either during single parabolic or hyperbolic encounters, or during repeated eccentric encounters (Kocsis et al. 2006). Such events occur rarely, but are more likely in dense stellar environments, such as globular clusters or galactic nuclear clusters. The brief duration of such bursts does not allow a large integrated buildup of S/N, and they may be difficult to detect without some electromagnetic trigger.

Recently, Tsang et al. (2012) showed that during binary inspiral of neutron stars (NSs), resonant tidal excitation of the interface mode—a natural mode of a NS peaked at the crust–core boundary—could result in an isotropic resonant shattering flare, and that these were consistent with short gamma-ray burst (sGRB) precursors observed seconds before some sGRBs (Troja et al. 2010). Coincident timing of such precursor flares and the GW inspiral signal can be used to provide strong constraints on the NS equation of state (Tsang et al. 2012).

In this paper, we show that resonant shattering flares can also occur during close passages of other compact objects, such as another NS or a black hole (BH), and that such flares could serve as electromagnetic counterparts to GW bursts, allowing triggered searches for these bursts.

2. TIDAL ENERGY TRANSFER DURING PARABOLIC AND ECCENTRIC ENCOUNTERS

Tidal energy transfer during close encounters can be determined in a Newtonian approximation through the procedure outlined in Press & Teukolsky (1977). While a fully relativistic formulation would be preferable, the Newtonian formulation is sufficiently accurate for a periapse distance much larger than the NS radius; relativistic effects would only slightly modify the frequencies and increase the strength of the interaction.

In general, the energy transfer rate to a star is given by

$$\frac{dE}{dt} = \int d^3x \rho v \cdot \nabla U,$$

where the fluid velocity $v \equiv \partial \xi / \partial t$ is the time derivative of the Lagrangian displacement $\xi$ and $U$ is the gravitational potential.

To examine the response of a NS with mass $M_1$, it is convenient to decompose the potential due to a star with mass $M_2$ into spherical harmonics $Y_{lm}(\theta, \phi)$,

$$U(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} U_{lm} Y^{*}_{lm}(\theta, \phi)$$

$$U_{lm} = W_{lm} \frac{G M_2 r^l}{R(y)^{l+1}} e^{im\phi},$$

where $\Phi$ is the true anomaly of the system, $^*$ denotes the complex conjugate, $R(t)$ is the distance between the stars, and $(r, \theta, \phi)$ is the comoving coordinate system centered with $M_1$. Note that for the purposes of mode excitation, we are only concerned with the tidal ($l > 2$) component of the potential. Assuming the normalization for the spherical harmonics given in Jackson (1999), the constants $W_{lm}$ are

$$W_{lm} = (-1)^{(l+m)/2} \left[ \frac{4\pi}{2l+1} \right] \binom{l-m}{l+m}! \left[ \binom{l+m}{l-m} \right]^{-1},$$

where $(-)^k$ is defined to be zero when $k$ is a non-integer.

The energy transferred to a particular mode (assuming no nonlinear effects, such as crust fracture) during a periapse passage for a parabolic or highly eccentric encounter can be estimated by Equation (40) of Press & Teukolsky (1977),

$$\Delta E_{nlm} = 2\pi^2 \frac{G M_1^2}{R_1} \left( \frac{M_2}{M_1} \right)^2 \left( \frac{R_1}{R_{\text{min}}} \right)^{2l+2} |Q_{nl}|^2 |K_{nlm}|^2,$$

where $R_{\text{min}}$ is the separation of the stellar centers of mass at the periapse, $R_1$ is the radius of the NS, $Q_{nl}$ is the overlap integral for the NS displacement eigenmode $\xi_{nlm}$, with radial mode number $n$, and $K_{nlm}$ is given by

$$K_{nlm} = \frac{W_{lm}}{2\pi} \int d^3x \rho \xi_{nlm}^{*} \cdot \nabla [\rho^2 Y_{lm}(\theta, \phi)],$$

and

$$Q_{nl} = \frac{1}{MR^2} \int d^3x \rho \xi_{nlm}^{*} \cdot \nabla [\rho^2 Y_{lm}(\theta, \phi)].$$
and summarized in Table 1. If for various equations of state given by Tsang et al. (2012) a tidal disruption may occur. The periapse distance is too small, then the stars may collide, or with the magnetic field result in strong transverse electric fields, which can accelerate particles to high energy, sparking a pair-photon fireball. The luminosities of resonant shattering flares with photon fireball. The luminosities of resonant shattering flares can accelerate particles to high energy, sparking a pair-photon fireball.

\[
I_{\text{in}}(\omega_{\text{nlm}}) \equiv \int_0^{\infty} (1 + x^2)^{-1} \cos(2^{1/2} \omega_{\text{nlm}}(x + x^3/3) + 2m \tan^{-1} x) \, dx. \tag{8}
\]

Here, \( \tilde{\omega} \equiv [M_1/(M_1 + M_2)]^{1/2}(R_{\text{min}}/R_1)^{3/2} \) and \( \omega_{\text{nlm}} \equiv \omega_{\text{nlm}}(M_1 + M_2)^{-1/2} R_{\text{min}}^{3/2} \) are the Keplerian frequency at the NS surface and the mode frequency, respectively, both scaled by the Keplerian frequency at periapse.

Modes with frequency much higher than periapse Keplerian frequency (\( \omega_{\text{nlm}} \gg 1 \)) cannot be strongly excited. In contrast, if the periapse distance is too small, then the stars may collide, or a tidal disruption may occur.

We can calculate the energy transfer to the interface mode \( \Delta E_i \) by utilizing the i-mode frequencies and overlap integrals for various equations of state given by Tsang et al. (2012) and summarized in Table 1. If \( \Delta E_i > E_b \), the mode energy required for the crust to reach the breaking strain, then a shattering flare can occur. \( \Delta E_i/E_b \) for various equations of state are shown as a function of periapse distance \( R_{\text{min}} \) for parabolic encounters between a 1.4 \( M_\odot \) NS (Figure 1) and a 10 \( M_\odot \) BH and a 1.4 \( M_\odot \) NS (Figure 2).

### 3. Resonant Shattering

The process that produces a resonant shattering flare is outlined in Figure 3. During a close encounter (or at orbital resonance), energy is extracted from the kinetic energy of the orbit through resonant tidal coupling. The interface mode is excited strongly, which drives the mode to an amplitude at which the breaking strain of the crust is exceeded.

The crust fractures, depositing \( \sim \epsilon_b \mu \Delta \rho ^3 \sim 10^{43} \) erg of seismic energy into the crust, where \( \epsilon_b \sim 0.1 \) is the breaking strain (Horowitz & Kadu 2009), \( \mu \sim 10^{30} \) erg cm\(^{-3}\) is the shear modulus, and \( \Delta \rho \sim 10^5 \) cm is roughly the thickness of the crust. These broad spectrum seismic waves are peaked at characteristic frequency \( \sim (\mu/\rho)^{1/2} / (2\pi \Delta \rho) \sim 200 \) Hz, where \( \rho \) is the density of the crust. Low-frequency seismic waves cannot couple efficiently to the magnetic field (Blaes et al. 1989), and the energy builds up in the NS crust as the interface mode is driven further and more fractures occur. This seismic energy builds until the crust reaches the elastic limit \( E_{\text{elastic}} = \int dV \epsilon_b \mu \sim 10^{46} \) erg when it shatters, scattering the mode and seismic energy to high-frequency oscillations which can then couple to the magnetic field. Strong perturbations of the magnetic field result in strong transverse electric fields, which can accelerate particles to high energy, sparking a pair-photon fireball. The luminosities of resonant shattering flares

### Table 1

| EOS       | \( R_{1.4} \) (km) | \( f_i \) (Hz) | \( Q_i \) | \( E_b \) (erg) |
|-----------|------------------|----------------|---------|--------------|
| SLy4      | 11.7             | 188            | 0.041   | 5 \times 10^{46} |
| Sk6       | 12.5             | 67.3           | 0.017   | 3 \times 10^{45} |
| Rs        | 13.0             | 32.0           | 0.059   | 1 \times 10^{46} |

**Notes.** \( R_{1.4} \) is the radius of a 1.4 \( M_\odot \) NS, \( f_i \) is the i-mode frequency, \( Q_i \) is the overlap integral for the i-mode and the tidal field, while \( E_b \) is the mode energy required to reach the breaking strain in the crust.

**Figure 1.** Top: ratio of the maximum energy transfer through tidal resonance to the interface mode, \( \Delta E_i/E_b \), to the mode energy required to reach the breaking strain, \( E_b \), for a parabolic or highly eccentric 10 \( M_\odot \) BH–1.4 \( M_\odot \) NS encounter as a function of periapse distance \( R_{\text{min}} \) for various equations of state. Below a critical periapse distance, the mode energy exceeds the breaking energy \( \Delta E_i/E_b > 1 \) and a resonant shattering flare could occur. The upper axis shows the Keplerian orbital frequency at the periapse, \( f_K(R_{\text{min}}) = [G(M_1 + M_2)/R_{\text{min}}^3]^{1/2}/2\pi \). Bottom: sky and observer inclination averaged signal-to-noise ratio \( (S/N) \) for a single advanced LIGO gravitational wave detector for a gravitational wave burst at 50 Mpc from a parabolic 10–1.4 \( M_\odot \) close encounter with periapse \( R_{\text{min}} \).

(A color version of this figure is available in the online journal.)

**Figure 2.** Top: ratio of the maximum energy transfer through tidal resonance to the interface mode, \( \Delta E_i/E_b \), to the mode energy required to reach the breaking strain, \( E_b \), for a parabolic or highly eccentric 1.4 \( M_\odot \) NS–1.4 \( M_\odot \) NS encounter as a function of periapse distance \( R_{\text{min}} \) for various equations of state. Below a critical periapse distance, the mode energy exceeds the breaking energy \( \Delta E_i/E_b > 1 \) and a resonant shattering flare could occur. The upper axis shows the Keplerian orbital frequency at the periapse, \( f_K(R_{\text{min}}) = [G(M_1 + M_2)/R_{\text{min}}^3]^{1/2}/2\pi \). Bottom: sky and observer inclination averaged signal-to-noise ratio \( (S/N) \) for a single advanced LIGO gravitational wave detector for a gravitational wave burst at 50 Mpc from a parabolic 1.4–1.4 \( M_\odot \) close encounter with periapse \( R_{\text{min}} \).

(A color version of this figure is available in the online journal.)
When the total seismic energy in the NS crust exceeds the elastic limit of the fracture, more energy is deposited into seismic energy in the crust. Fracture occurs, releasing elastic energy to high-frequency oscillations. High-frequency oscillations couple strongly to the magnetic field (Blaes et al. 1989; Thompson & Blaes 1998) by strongly vibrating their footprints. Strong perturbations of the magnetic field at the neutron star surface drive strong electric fields, which can accelerate charged particles, triggering pair production and a relativistic fireball with luminosity $10^{57}$–$10^{48}$ erg s$^{-1}$. A color version of this figure is available in the online journal.

are expected to be up to $10^{47}$–$10^{48}$ erg s$^{-1}$ (Tsang et al. 2012) if the precursor flare timescales are assumed.

Troja et al. (2010) found precursors occurring in 3 of the 49 soft gamma repeaters analyzed, implying that not every binary merger should result in a detectable shattering flare. We note that the extraction of seismic energy from the crust by the magnetic field is limited by the strength of the magnetic field at the surface of the NS. The maximum luminosity that can be extracted from the crust by the magnetic field can be estimated by

$$L_{\text{max}} = \int_{\text{surf}} (\mathbf{v} \times \mathbf{B}) \times \mathbf{B} \cdot dA \sim 10^{47} \text{erg s}^{-1}(v/c)(B_{\text{surf}}/10^{13} \text{G})^2(R/10 \text{ km})^2,$$  \hspace{1cm} (9)

where $v$ is the maximum velocity of the perturbation to the field line, $R$ is the NS radius, and $B_{\text{surf}}$ is the local surface field strength, which can be significantly higher than the large scale dipole field. Thus, only shattering flares from NSs with sufficiently strong surface fields can be detected.

4. ELECTROMAGNETIC COUNTERPARTS TO GRAVITATIONAL WAVE BURSTS

To calculate the expected GW S/N due to parabolic encounters, we follow the procedure outlined in Kocsis et al. (2006). The strain caused by a GW burst due to a parabolic encounter is given by (Flanagan & Hughes 1998)

$$h(f) = \frac{\sqrt{3}}{2\pi} \frac{G^{1/2}}{c^{3/2}} \frac{1+z}{d_L} \left[ \int_0^\infty \frac{dE}{df} [(1+z)f] \right].$$  \hspace{1cm} (10)

where $z$ is the redshift, $d_L$ is the luminosity distance, and $dE/df$ is the total GW energy emitted by encounter per unit frequency, which is given for a parabolic ($e = 1$) encounter in the non-relativistic limit by Equation (46) from Turner (1977). The S/N for a sky- and orientation-averaged signal on a single detector is given by (Dalal et al. 2006; Nissanke et al. 2010)

$$S/N = \frac{8}{5} \left( \int_0^{\infty} \frac{|h(f)|^2}{S_n(f)^2} df, \right)$$  \hspace{1cm} (11)

where $S_n(f)$ is the spectral noise density for a given detector. In Figures 1 and 2, the S/N is shown for the NS–NS and BH–NS encounters assuming a single encounter at 50 Mpc ($z \sim 0.011$) for advanced LIGO, with spectral noise density given by Harry et al. (2010).

Blind detection ($S/N \gtrsim 6$ coincident at each detector; see, e.g., Aasi et al. 2013) of a single GW burst from a NS close encounter would be extremely challenging at reasonable distances, with fairly low S/N even for close passages, in particular for NS–NS encounters. Using X-ray or gamma-ray detections of resonant shattering flares as electromagnetic counterparts, triggered GW searches could be performed, significantly lowering the S/N threshold for GW burst detection (Koehanek & Piran 1993; Nissanke et al. 2010; Kelley et al. 2013; Dietz et al. 2013). Networks of detectors can also be used to enhance burst detection through coincident and coherent methods (Schutz 2011; Nissanke et al. 2013; Aasi et al. 2013).

Kocsis & Levin (2012) also show that repeated GW bursts from eccentric captures can be combined with the final chirp to boost the integrated S/N by roughly an order of magnitude, and would optimistically allow detection of bursts from BH–NS eccentric captures out to $\sim 300$ Mpc, and NS–NS encounters to $\sim 150$ Mpc. The pattern of these repeated bursts can be modeled for given orbital parameters. Resonant shattering flares can be seen significantly farther than the GW bursts. If they occur for a given system, then they will happen for sufficiently close passages, which are also those that contribute the largest component of the GW burst signal. If repeated flares are seen, then these could also be used to characterize the orbit and target a burst search to accumulate S/N over multiple passages. However, significant changes to the current GW templates may be necessary to detect eccentric captures and mergers (East et al. 2013; Huerta & Brown 2013).

5. EVENT RATES

Close encounters of NSs with other compact objects are much more likely to occur in dense stellar environments, such as globular clusters and galactic nuclei. While it is beyond the scope of this paper to perform an extremely detailed evaluation of the event rates for close encounters of compact objects, we will briefly discuss the event rates for such encounters in both of these environments and provide updated estimates for some of the rates in the literature.

5.1. Globular Clusters

Kocsis et al. (2006) calculated the parabolic encounter rate for compact objects in globular clusters using simplified globular cluster models, predicting a rate of $\gtrsim 1$ detection per year for advanced LIGO in optimistic scenarios. However, their detection rates are dominated by rare distant events involving close encounters of $\gtrsim 20$ $M_{\odot}$ BHs.
Lee et al. (2010) examine various dynamical pathways to sGRBs in globular clusters, including binary interactions and tidal capture. They calculate a high rate of close encounters for two NSs, $\Gamma_{\text{NS-NS}}^{\text{GC}} \sim 55 \text{ yr}^{-1} \text{ Gpc}^{-3}$, using as a calibration the estimate of $\sim 10^4$ NSs in the collapsed core of M15, from the Fokker–Planck calculations of Dull et al. (1997). This would require an extremely high NS retention fraction. Subsequent more careful calculations by Murphy et al. (2011) have determined the number of NSs in the core of M15 to be closer to $\sim 10^3$, which is consistent with $\sim 1\%–10\%$ retention fraction estimates from pulsar kick velocities (Drukier 1996; Hansen & Phinney 1997; Davies & Hansen 1998). These reduces the estimates of Lee et al. (2010) to $\Gamma_{\text{NS-NS}} \sim 0.5 \text{ yr}^{-1} \text{ Gpc}^{-3}$, however, Samsing et al. (2013) have recently showed that considering chaotic resonances in binary–single interactions can significantly increase the expected rate of eccentric binaries.

5.2. Galactic Nuclei

While there are many globular clusters per galaxy, high kick velocities at NS birth significantly lower their retention fraction. The deeper potentials of galactic nuclei may provide dense stellar environments where the NS retention fraction is higher and close encounters are more likely to occur. O’Leary et al. (2009) and Kocsis & Levin (2012) both provide estimates of $\sim 1–100$ BH close passages per year within a few Gpc detectable by advanced LIGO, with GW-detectable NS–BH encounters estimated to be $\sim 1\%$ of this. However, as we discuss in Appendix A below, they scale by a factor $\xi$, representing the contribution due to the variance of the nuclear cluster density across galaxies. They take this factor to be $\xi \gtrsim 30–100$, but we find that $\xi$ is more correctly evaluated to be significantly lower, even with the most optimistic assumptions.

In Appendix A, we have re-evaluated the rates for single–single eccentric captures of compact objects in nuclear star clusters containing massive central BHs, assuming a simplified isothermal density distribution, as in Kocsis & Levin (2012). We find that the $10 M_{\odot}$ BH–BH eccentric capture rate is $\Gamma_{\text{BH–BH}} \sim 0.02 \text{ yr}^{-1} \text{ Gpc}^{-3}$, which is significantly lower than the previously estimated values. However, a more top-heavy initial mass function (IMF; Bartko et al. 2010), along with enhanced segregation and spatial flattening for heavier BHs, may help to increase this value.

The NS–NS and NS–BH (10 $M_{\odot}$) rates can be estimated in a similar fashion for an isothermal $r^{-2}$ density distribution to be $\Gamma_{\text{NS–NS}}^{\text{EC}} \sim 0.04–6 \text{ yr}^{-1} \text{ Gpc}^{-3}$, and $\Gamma_{\text{NS–BH}}^{\text{EC}} \sim 0.05–0.6 \text{ yr}^{-1} \text{ Gpc}^{-3}$ with the range mainly due to uncertainty in the IMF and mass loss models for NS progenitors (O’Connor & Ott 2011). However, there is reason to suspect that the slope of the density distribution is somewhat flattened due to interaction with segregated BHs (see, e.g., O’Leary et al. 2009 and references therein). Taking a NS density distribution $\propto r^{-3/2}$ as a lower bound for our rates, we find in this case $\Gamma_{\text{NS–NS}}^{\text{EC}} \simeq 0.003–0.3 \text{ yr}^{-1} \text{ Gpc}^{-3}$.

Note that in evaluating the above rates, we have made very optimistic assumptions about systematic versus intrinsic variation of the relevant observations, as in O’Leary et al. (2009). Possibly more realistic estimates for this intrinsic scatter reduce these rates by a factor of $\sim 4$.

In the high density, high relative-velocity region near the center of nuclear star clusters, there may be a significant rate of hyperbolic passages where the periapse is sufficiently close to trigger a shattering flare, but insufficient to result in eccentric capture by GW emission. In Appendix B, we have calculated the rate of encounters that result in a shattering flare during the first close passage, and find this to be higher than the eccentric capture rate for the fiducial model used. We find, for the optimistic assumptions about intrinsic variation used above, and assuming our canonical isothermal model, $\Gamma_{\text{NS–NS}}^{\text{SP}} \simeq 0.2–60 \text{ yr}^{-1} \text{ Gpc}^{-3}$. For a more flattened density profile $\propto r^{-3/2}$, we have $\Gamma_{\text{NS–NS}} \simeq 0.005–0.5 \text{ yr}^{-1}$. Taking our less generous estimates for the intrinsic scatter across galaxies significantly reduces these rates by a factor of $\sim 6$.

5.3. Other Possible Event Rate Contributions

In the above discussion, we have primarily considered single–single interactions in determining the event rates in dense clusters. Binary–single or binary–binary interactions have larger cross sections and could increase the rates for such events (see, e.g., Miller & Lauburg 2009). Recently, Katz & Dong (2012) and Kushnir et al. (2013) demonstrated that Kozai–Lidov type interactions can drive the inner binaries of hierarchical triple systems toward extreme eccentricity, with collisions occurring when the periapse is driven below the stellar radius. They claim that the rates for such Kozai-oscillation driven collisions between white dwarfs in field triple star systems can be comparable to the SN Type Ia rate. Similar interactions could potentially drive close encounters of NSs or BHs in triple systems. However, the periapse for shattering flares is two orders of magnitude smaller than those considered by Katz & Dong (2012), and NSs and BHs are substantially more rare than white dwarfs, particularly outside of dense clusters. Within clusters, such triple systems would need to reach these extreme eccentricities quickly, before other encounters ionize away the softer less-bound outer companion.

6. DISCUSSION

We have calculated the energy transfer to the interface mode through tidal interaction for NSs during close encounters with other compact objects, and we have shown that resonant shattering flares can occur during parabolic or eccentric encounters if the periapse is sufficiently close and the local surface field of the NS is sufficiently high. Such flares are similar to resonant shattering flares during binary inspirals, and should have luminosity $\sim 10^{37}–10^{38}$ erg s$^{-1}$ (Tsang et al. 2012).

Broad band GW bursts are also generated by such encounters. While they are rare, there is intense interest in such GW burst events which are detectable by the next generation of GW detectors. GW bursts with high S/N are also those for which shattering flares may occur and act as an electromagnetic counterpart to trigger burst searches. Highly eccentric captures of NSs in dense stellar environments are expected to result in repeated GW bursts (Kocsis & Levin 2012), but may also lead to repeated shattering flares at each periapse passage. Radioactive emission from ejecta (so-called kilonova; Metzger et al. 2010) and radio flares is also expected to be stronger for eccentric mergers than for their circularized counterparts (East & Pretorius 2012), which may allow easier identification and localization of a shattering flare through coincident detections.

We have also reviewed and updated the rates for compact object encounters presented in the literature for dense stellar environments, and find that these rates should be revised significantly downward. While these estimated rates for close encounters involving NSs within the horizon of advanced LIGO are low, shattering flares from such encounters can be detected
significantly farther, and may occur at rates of up to \( \Gamma_{\text{NS–NS}}^{(\text{GN, SF})} \approx 0.2–60 \text{ yr}^{-1} \text{ Gpc}^{-3} \), subject to large model uncertainties. More conservative assumptions substantially lower this rate.

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APPENDIX A

ECCENTRIC CAPTURE RATES IN NUCLEAR STAR CLUSTERS

Here, we will carefully estimate the eccentric capture rate of compact objects in nuclear star clusters. We begin by following the general procedure outlined in Appendix C of Kocsis & Levin (2012), and calculate the eccentric capture rate for a single fiducial galaxy.

A.1. The Rate for a Single Galaxy

The cross section for eccentric capture is basically the cross section for which the energy emitted by GW (or lost due to tidal interactions) during a close encounter exceeds the kinetic energy of the objects at infinity. The maximum periapsis for capture is given by Quinlan & Shapiro (1989) as

\[
r_{p, \text{max}} = \left[ \frac{85 \pi \sqrt{2} G^7 m_i m_j (m_i + m_j)^{3/2}}{12 c^8 |v_i - v_j|^2} \right]^{2/7},
\]

\[
= 190 \text{ km} \left( \frac{\eta}{0.25} \right)^{2/7} \left( \frac{m_{\text{tot}}}{2.8 M_\odot} \right) \left( \frac{v_{\text{rel}}}{10^3 \text{ km s}^{-1}} \right)^{-4/7},
\]

(A1)

where \( \eta \) is the symmetric mass ratio, \( m_{\text{tot}} \) is the total mass of the two objects, and \( v_{\text{rel}} \) is the relative speed of the two objects at infinity. We only consider GW capture here since, for compact objects, the tidal capture cross section is much smaller than the GW capture cross section.

We can then calculate the cross section, using the standard formula for gravitational focusing

\[
\sigma_{\text{cs}} = \pi r_{p, \text{max}}^2 \left[ 1 + \frac{2 G m_{\text{tot}}}{r_{p, \text{max}} v_{\text{rel}}^2} \right] \approx \frac{2 \pi G m_{\text{tot}} r_{p, \text{max}} v_{\text{rel}}}{v_{\text{rel}}^2}
\]

\[
= 1.3 \times 10^{23} \text{ cm}^2 \left( \eta/0.25 \right)^{2/7} (m_{\text{tot}}/20 M_\odot)^2 \times (v_{\text{rel}}/84 \text{ km s}^{-1})^{-11/7},
\]

(A2)

where we have scaled this to fiducial values that will be used later. The rate of eccentric captures for a single galaxy with a central supermassive black hole (SMBH) of mass \( M_\bullet \) and velocity dispersion \( \sigma_{\text{disp}}(M_\bullet) \) is then

\[
\Gamma_{\text{gal}}(M_\bullet) \approx \int_{r_{\text{min}}}^{r_{\text{max}}} \! d4\pi r^2 n_1(r, M_\bullet) n_2(r, M_\bullet) \sigma_{\text{cs}} v_{\text{rel}},
\]

(A3)

where \( n_1 \) and \( n_2 \) are the number densities of each type of object as a function of radius, \( r_i \equiv GM_\bullet/\sigma_{\text{disp}}^2 \) is the radius of influence of the BH, and \( r_{\text{min}} \) is approximately the radius inside which there is only a single object.

We take the density of objects to be the same as in an isothermal distribution, and the velocity distribution to be Maxwellian at each radius with relative speed \( v_{\text{rel}}^2 = 2 v_{\text{circ}}^2 \equiv 2GM_\bullet/r = 2v_{\text{disp}}^2(r/r_i)^{-1} \), which captures the behavior well near the central BH, where the contribution to the rate is the largest. The number densities are then given by

\[
n_i(r) = \frac{N_i}{4\pi r_i^3} \left( \frac{r}{r_{\text{dyn}}} \right)^{-2},
\]

(A4)

where \( r_{\text{dyn}} \) defines the dynamical radius inside which twice the mass of the central BH is contained such that \( N_i \equiv 2 \kappa_i M_\bullet/m_i \) and \( \kappa_i \) is the number of objects and total mass fraction of type \( i = 1, 2 \) within \( r_{\text{dyn}} \), respectively. We take these scalings of \( n(r) \) and \( v_{\text{rel}}(r) \) for simplicity to highlight the sources of uncertainty and provide a rate estimate for the simplest case. For how different and more realistic scalings may alter the basic single galaxy rate, see the detailed discussion in O’Leary et al. (2009) and Appendix C of Kocsis & Levin (2012).

It is also convenient to define the geometric means of the number \( \tilde{N} \) and mass fraction \( \kappa \), such that

\[
\tilde{N}^2 \equiv N_1 N_2 = \frac{4\tilde{k}^2 M_\bullet^2}{\eta m_{\text{tot}}^2}, \quad \tilde{k}^2 \equiv \kappa_1 \kappa_2,
\]

(A5)

as well as the fiducial scalings \( \sigma_{\text{disp}} \equiv \sigma_{\text{disp}}/(84 \text{ km s}^{-1}) \), \( \eta_0.25 \equiv \eta/0.25 \), and \( m_{20} \equiv m_{\text{tot}}/(20 M_\odot) \). Approximating \( r_{\text{min}} \approx \tilde{N}^{-1} r_{\text{dyn}} \), we find

\[
\Gamma_{\text{gal}}(M_\bullet) \approx \int_{r_{\text{min}}}^{r_{\text{max}}} \! dr 4\pi r^2 \frac{\tilde{N}^2}{(4\pi r_{\text{dyn}}^3)^2} \left( \frac{r}{r_{\text{dyn}}} \right)^{-4} \sigma_{\text{cs}} v_{\text{rel}}
\]

\[
\simeq 1.3 \times 10^{30} \text{ cm}^2 \text{ s}^{-1} \left( \eta_0.25 \right)^{2/7} \left( m_{20} \right)^{-11/7} \left( \frac{r_i}{r_{\text{dyn}}} \right)^{-11/14}
\]

\[
\times \frac{\tilde{N}^2}{4\pi r_{\text{dyn}}^3} \int_{r_{\text{min}}/r_{\text{dyn}}}^{r_{\text{max}}/r_{\text{dyn}}} \! d xx^{-2+11/14},
\]

(A7)

where we have averaged over the Maxwellian distribution, \( v_{\text{circ}}(r_i) = 1.15 \sigma_{\text{disp}}^{-1/7} \), and changed integration variables to \( x \equiv r/r_{\text{dyn}} \). Assuming \( (\tilde{N} r_i/r_{\text{dyn}})^{3/14} \gg 1 \), we have

\[
\Gamma_{\text{gal}}(M_\bullet) \approx 1.2 \times 10^{-10} \text{ yr}^{-1} (r_i/r_{\text{dyn}})^{3/14}
\]

\[
\times \left( \eta_0.25 \right)^{-23/28} \left( m_{20} \right)^{3/14} M_{4\odot}^{-11/14} \sigma_{\odot}^{31/14},
\]

(A8)

where \( M_{4\odot} \equiv M_\bullet/(4 \times 10^6 M_\odot) \) is scaled to the Milky Way, and \( \kappa_{2.5} \equiv \kappa/(2.5\%) \) as in Kocsis & Levin (2012). Applying the \( M–\sigma \) relation, \( M_{4\odot} = \sigma_{\odot}^{3} \) (Tremaine et al. 2002), we finally have

\[
\Gamma_{\text{gal}}(M_\bullet) \approx 1.2 \times 10^{-10} \text{ yr}^{-1} (r_i/r_{\text{dyn}})^{3/14}
\]

\[
\times \left( \eta_0.25 \right)^{-23/28} \left( m_{20} \right)^{3/14} M_{4\odot}^{-28/28}. \]

(A9)

Thus far, this agrees relatively well with Kocsis & Levin (2012) and O’Leary et al. (2009).
where the scatter in the inferred nuclear star cluster relaxation time $T_r$ for the low $\sigma_{\text{disp}}$ galaxies given in Figure 1 of Merritt et al. (2007), O’Leary et al. (2009) and Kocsis & Levin (2012) claim that the variance of the central number density scales the average rate per galaxy by a factor $\xi = \frac{n^2}{\bar{n}^2} \gtrsim 30$, increasing their total rate substantially.

Here, in calculating the average rate over many galaxies, we will carefully consider the effect of variation in both the $M-\sigma$ relation and the scaling of the central density implied by Merritt et al. (2007)—related to the parameter $r_1/r_{\text{dyn}}$—and show that such a substantial increase in the inferred rate is not warranted.

**Variation in the $M-\sigma$ relation.** We begin by taking the generous assumption that the intrinsic scatter in the $M$—$\sigma$ relation is $\sim 0.5$ dex in $M_5$ (Tremaine et al. 2002). We take the distribution of $M_5$ for a fixed $\sigma_{\text{disp}}$ to be log-normal such that $M_{5e} = C_{M_5} \times \sigma_{\text{disp}}$, where $C_{M_5}$ is a random variable with log-normal probability distribution with geometric mean $(C_{M_5}) = 1$ and scale factor $\sigma_{M_5} = \ln \sqrt{10}$.

Our single galaxy rate (Equation (A8)) has scaling such that

$$\xi_{M_5} \equiv \frac{C_{M_5}}{(C_{M_5})^{-31/28}} \equiv \exp \left[ \frac{1}{2} \left( \frac{31}{28} \right)^2 \frac{\sigma_{M_5}^2}{(C_{M_5})^{-31/28}} \right] \simeq 2.25, \quad \text{(A10)}$$

where $\xi_{M_5}$ denotes averaging of $\sigma_{M_5}$ over the distribution.

**Variation in $r_1/r_{\text{dyn}}$.** The ratio of the radius of influence, $r_1$, to the dynamical radius, $r_{\text{dyn}}$, determines the relative density of the nuclear cluster, and varies for different $N$-body models from $0.1$ to $1$. (Binney & Tremaine 2008). For simplicity we will take this ratio to be a log-normal distributed random variable independent of $C_{M_5}$.

Figure 1 of Merritt et al. (2007) showed an estimate of the relaxation time at the dynamical radius as a function of the velocity dispersion $\sigma_{\text{disp}}$ for nuclei of early-type galaxies in the ACS Virgo Cluster Survey (Côté et al. 2004). While the majority of the scatter in this distribution is for low-luminosity unresolved nuclear star clusters, an indication that much of this scatter may be due to observational uncertainty, for the purposes of this discussion we will assume, as in O’Leary et al. (2009), that this scatter is intrinsic. We will again assume, for simplicity, that the relaxation time $T_r$ for fixed $\sigma_{\text{disp}}$ is distributed log-normally, with a generous estimate of $1.5$ dex standard deviation for the scatter in log $T_r$, such that the scale factor is $\delta_{T_r} \simeq 1.5 \ln 10$.

The nuclear relaxation time is given by

$$T_r(r_{\text{dyn}}) \simeq \frac{0.34 r_{\text{dyn}}^3}{G^2 n(r_{\text{dyn}}) \ln \Lambda} = \frac{0.34 r_{\text{dyn}}^3}{G^2 n \ln \Lambda} \frac{4\pi r_{\text{dyn}}^3}{2 \kappa M_S} \quad \text{(A11)}$$

(Spitzer 1987), where $\ln \Lambda$ is the Coulomb logarithm. For fixed $\sigma_{\text{disp}}$ we can rewrite the $T_r$ in terms of the random variables $C_{M_5}$ and $r_1/r_{\text{dyn}}$:

$$T_r(r_{\text{dyn}}) \simeq \frac{0.68 \pi G M_S^2}{\ln \Lambda \kappa \sigma_{\text{disp}}} \left( \frac{r_1}{r_{\text{dyn}}} \right)^{-3} \sim \sigma_{\text{disp}}^2 C_{M_5}^2 \left( \frac{r_1}{r_{\text{dyn}}} \right)^{-3}. \quad \text{(A12)}$$

For independent log-normal random variables $C_{M_5}$ and $r_1/r_{\text{dyn}}$,

$$\delta_{T_r}^2 = (2\delta_{M_5})^2 + (3\delta_{r_1/r_{\text{dyn}}})^2, \quad \text{(A13)}$$

where $\delta_{\text{disp}}$ is the standard deviation of $\ln n_0/r_{\text{dyn}}$. This then gives the scaling due to variation in $r_1/r_{\text{dyn}}$ to the average rate of

$$\xi_{r_1/r_{\text{dyn}}} = \frac{(r_1/r_{\text{dyn}})^{31/14}}{(r_1/r_{\text{dyn}})^{31/14}} = \exp \left[ \frac{1}{2} \left( \frac{31}{14} \right)^2 \delta_{r_1/r_{\text{dyn}}}^2 \right] \simeq 6.1. \quad \text{(A14)}$$

**Final rate.** The central BH mass function can be estimated by

$$\Phi(M) \equiv \frac{dn_{\text{gal}}}{d\ln M} \simeq 0.0077 \text{Mpc}^{-3} \times \left( \frac{M}{M_\odot} \right)^{\alpha+1} \exp \left[ -\left( \frac{M}{M_\odot} \right)^{-\beta} \right] \quad \text{(A15)}$$

(Shankar et al. 2004), where $\alpha \simeq -1.11$, $\beta \simeq 0.5$, and $M_\odot \simeq 6.4 \times 10^3 M_\odot$. Assuming the local Hubble constant $H_0 = 70 \text{km s}^{-1} \text{Mpc}^{-1}$.

Integrating our mass dependence $M_{4\text{e}6}^{0.28}$ over the mass function, we can obtain the effective density

$$n_{\text{gal}, \text{eff}} = \int M_{\text{gal}, \text{min}} M_{\text{gal}, \text{max}}^{0.28} \Phi(M_\text{gal}) \frac{dM_\text{gal}}{M_\text{gal}} \quad \text{(A16)}$$

with which to multiply our single galaxy rate evaluated at $M_{4\text{e}6} = 1$. The mass function (Equation (A15)) is only constructed to be valid between $10^6 M_\odot \lesssim M_\odot \lesssim 5 \times 10^8 M_\odot$, however, we expect significant contribution from smaller galaxies. The halo mass function increases for lower mass, however, for dwarf galaxies the stellar mass (and therefore SMBH mass) to halo mass ratio drops significantly and the expected number density at that mass should also fall. If we integrate down to only $M_{\text{gal}, \text{min}} \approx 10^6 M_\odot$, then this gives us an effective density $n_{\text{gal}, \text{eff}} \simeq 0.043 \text{Mpc}^{-3}$, while extending this mass function down to a cutoff of $M_{\text{gal}, \text{min}} \approx 10^4 M_\odot$ yields $n_{\text{gal}, \text{eff}} \simeq 0.067 \text{Mpc}^{-3}$. With this in mind, we take the fiducial value of the effective density to be $n_{\text{gal}, \text{eff}} \approx n_{\text{gal}, \text{eff}} = 0.05 \times 0.05 \text{Mpc}^{-3}$.

Scaling to the Milky Way where O’Leary et al. (2009) assume a fiducial value of $r_1/r_{\text{dyn}} \approx 0.5$, we have our final rate of eccentric captures in galactic nuclei,

$$\Gamma_{\text{hot}}^{(\text{GN, EC})} = \frac{4}{3} \pi d^3 n_{\text{gal}, \text{eff}} \int r_{\text{gal}}(M_{4\text{e}6} = 1) \quad \text{(A17)}$$

$$\simeq 0.6 \text{yr}^{-1} \left( \frac{\xi_{M_5}}{2.25} \right) \left( \frac{\xi_{\text{disp}}}{6.1} \right) \left( \frac{\langle r_1/r_{\text{dyn}} \rangle^{31/14}}{0.5} \right) \left( \frac{H_0}{r} \right)^{-2}$$

$$\times \eta_0^{25/28} \left( \frac{k_{\text{disp}}}{m_{\text{20}}} \right)^{21/14} n_{\text{gal}, \text{disp}} d_2 \text{Gpc}. \quad \text{(A18)}$$

within $d_2 \text{Gpc} \times 2 \text{Gpc}$, where we take $2 \text{Gpc}$ for the fiducial value as it is roughly the advanced LIGO horizon distance for $10 M_\odot$ BH–BH eccentric captures. Here, we have also included an additional factor $(H/r)^{-2}$, which may increase the density for nuclear clusters where significant flattening has occurred (B. Kocsis 2013, private communication). Assuming the generous fiducial values for $\xi_{M_5}$ and $\xi_{\text{disp}}$ and no significant flattening, the rate for eccentric capture of $10 M_\odot$ BH–BH encounters is

$$\Gamma_{\text{BH–BH}}^{(\text{GN, EC})} \simeq 0.02 \text{yr}^{-1} \text{Gpc}^{-3}. \quad \text{(A19)}$$

Less generous estimates for the intrinsic scatter, $\delta_{M_5} \simeq 0.3 \ln 10$ (Tremaine et al. 2002) and $\delta_{r_1} \simeq 10$ give $\xi_{M_5} \simeq 1.33$ and $\xi_{\text{disp}} = 2.52$, which reduces the above rate by a factor of $\sim 4$. 

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**A.2. Averaging Over Many Galaxies**

From the scatter in the inferred nuclear star cluster relaxation time $T_r$ for the low $\sigma_{\text{disp}}$ galaxies given in Figure 1 of Merritt et al. (2007), O’Leary et al. (2009) and Kocsis & Levin (2012) claim that the variance of the central number density scales the average rate per galaxy by a factor $\xi = \frac{n^2}{\bar{n}^2} \gtrsim 30$, increasing their total rate substantially.
A.3. Neutron Star Rates

We are now (finally) ready to estimate the rates for NS eccentric captures in galactic nuclei. There is large uncertainty in the NS production rate in nuclear star clusters, mostly due to two factors. First, the IMF is unknown and could range from the standard Salpeter IMF to an extremely top-heavy IMF (e.g., Bartko et al. 2010). Second, there is great uncertainty in the effects of mass-loss for determining the fraction of stars with $M_{ZAMS} \gtrsim 8 M_\odot$ that will become NSs, and the fraction that will collapse to form BHs (see, e.g., O’Connor & Ott 2011 for discussion).

With these considerations, we will assume that between $\sim 1%$ and 10% of the stars in a nuclear star cluster will become NSs. The low end of this range is for a standard IMF, with low mass loss and metallicity, while the high end roughly corresponds to a top-heavy IMF with more mass loss in the NS progenitors. Assuming an average stellar mass in nuclear star clusters of $\sim 0.5 M_\odot$, this gives $\kappa_{NS} \sim 0.03–0.3$.

NSs have only had time to segregate in galaxies for which $\sigma_{\text{disp}} \lesssim 50$ km s$^{-1}$ (Miller & Lauburg 2009). Unfortunately, these low mass galaxies also have escape velocities $\sim 2 \sigma_{\text{disp}}$, and thus NS kick velocities of $\sim 100$ km s$^{-1}$ will significantly reduce the retention fraction after NS formation, much like in globular clusters. So we will assume the scaling above with no significant enhancements due to segregation or flattening.

Substituting $m_{\text{tot}} = 2.8 M_\odot$, $\eta = 0.25$, and $\kappa_{NS} = 0.03–0.3$ into (A18) and continuing to use the very generous assumptions above for $\xi_{M\sigma} \sim 2.25$ and $\xi_{\text{dyn}} \sim 6.1$, we get a rate for eccentric capture of

$$\Gamma_{\text{NS–NS}}^{(\text{GN,EC})} \sim 0.04–6 \text{ yr}^{-1} \text{ Gpc}^{-3}. \quad (A20)$$

For 10 $M_\odot$ BH–1.4 $M_\odot$ NS encounters we take $m_{\text{tot}} = 11.4 M_\odot$, $\eta \approx 0.11$, $\kappa_{\text{BH}} \approx 0.25$, and $\kappa_{NS} \approx 0.03–0.3$, and get a rate

$$\Gamma_{\text{BH–NS}}^{(\text{GN,EC})} \sim 0.05–0.6 \text{ yr}^{-1} \text{ Gpc}^{-3}. \quad (A21)$$

However, in systems where the BHs dominate the core, a cusp of more massive objects tends to flatten out the distribution of lighter objects compared to an isothermal density profile. O’Leary et al. (2009) find that the distribution can approach a power law index of 1.5 for the lighter objects in their Fokker–Planck calculation. Repeating the above calculations for $n_{NS} \propto r^{-3/2}$ as a lower bound, we find

$$\Gamma_{\text{NS–NS}}^{(\text{GN,EC})} \sim 0.003–0.3 \text{ yr}^{-1} \text{ Gpc}^{-3}, \quad (A22)$$

noting that the rate in a single galaxy is no longer dominated by the contribution due to the innermost objects.

APPENDIX B

RATES FOR SHATTERING DURING FIRST PASSAGE

Eccentric captures, described above, provide multiple close passages, however, the cross section for eccentric capture only exceeds that for shattering flares from the first passage below $v_{\text{rel}} \sim 1000$ km s$^{-1}$. Near the center of nuclear star clusters—where the density and relative velocity dispersion are the highest—the rate for hyperbolic passages that result in shattering flares but are not bound through GW emission can be significant. In this Appendix, we calculate this rate for shattering flares during first passage.

We begin by assuming that the speed at infinite separation is small compared to the speed at periapse $v_{\text{rel}} \ll \sqrt{2 G m_{\text{tot}} / r_{p,s}}$. The maximum periapse distance for shattering is then only a function of the stellar masses, and the equation of state

$$r_{p,s} = r_{p,s}(\eta, m_{\text{tot}}, \text{EOS}), \quad (B1)$$

which can be determined through setting $\Delta E_i = E_i$ as in Section 2 of the main text above. We can then calculate the single galaxy rate as for eccentric captures, giving

$$\Gamma_{\text{gal}}^{(\text{GN,SF})} = \int_{r_{\text{min}}}^{r_{\text{max}}} 4 \pi r^2 \frac{\dot{N}^2}{(\Delta r)^2} \left(\frac{r}{r_{\text{dyn}}}\right)^{-4} \frac{2 \pi G m_{\text{tot}} r_{p,s}}{v_{\text{rel}}} dr \quad (B2)$$

$$\approx 1.3 \times 10^{-10} \text{ yr}^{-1} \left(\frac{r_{p,s}}{200 \text{ km}}\right) \left(\frac{\kappa}{0.03}\right)^{5/2} \times \left(\frac{r_i / r_{\text{dyn}}}{0.5}\right)^{5/2} \left(\frac{\eta_{0.25} m_{2.8}}{C_{M\sigma} M_{4e6}}\right)^{3/4}, \quad (B3)$$

where we have used the $M – \sigma$ relation $M_{4e6} = C_{M\sigma} \sigma_{4e6}^3$ and the fact that $(v_{\text{rel}})^{-1} \sim 1.38 \times (v_{\text{rel}})^{-1}$ over a Maxwellian distribution.

Again, for the same extremely generous assumptions for the intrinsic variation in $M – \sigma$ and $T_{\sigma}(\sigma)$ as we did above, we can evaluate

$$\xi(SF)_{M\sigma} = \frac{C_{M\sigma}^4}{(C_{M\sigma})^{3/4}} \sim 2.82, \quad \xi(SF)_{\text{dyn}} = \frac{(r_i / r_{\text{dyn}})^{5/2}}{(\dot{r}_i / r_{\text{dyn}})^{5/2}} \approx 9.99. \quad (B4)$$

To calculate the effective density, we take

$$n_{\text{gal,eff}} = \int M_{\text{max}}^{3/4} M_{4e6}^{1/4} \Phi(M_S) \frac{dM_S}{M_S} \sim 0.1 \text{ Mpc}^{-3}, \quad (B5)$$

which gives us our rate for shattering flare encounters of

$$\Gamma_{\text{tot}}^{(\text{GN,SF})} \sim 0.5 \text{ yr}^{-1} \xi_{M\sigma} \xi_{\text{dyn}} \left(\frac{r_{p,s}}{200 \text{ km}}\right) \left(\frac{\kappa}{0.03}\right)^{5/2} \times \left(\frac{r_i / r_{\text{dyn}}}{0.5}\right)^{5/2} \left(\frac{\eta_{0.25} m_{2.8}}{C_{M\sigma} M_{4e6}}\right)^{3/4} \frac{1}{d_{2}^{3} \text{ Gpc}}. \quad (B6)$$

For $\kappa_{NS} \approx 0.03–0.3$ we then have the range

$$\Gamma_{\text{NS–NS}}^{(\text{GN,EC})} \sim 0.2–60 \text{ yr}^{-1} \text{ Gpc}^{-3}. \quad (B7)$$

for the fiducial values above, including our optimistic estimates for $\xi_{M\sigma} \approx 2.82$ and $\xi_{\text{dyn}} \approx 9.99$.

Similar to the case for eccentric captures, less generous estimates for the intrinsic scatter significantly reduce these rates. Taking $\delta_{M\sigma} \approx 0.3 \ln 10$ and $\delta_{T_{\sigma}} \approx 0.1 \ln 10$, we have $\xi_{M\sigma} \approx 1.45$ and $\xi_{\text{dyn}} \approx 3.24$, which reduces the above rates by a factor of $\sim 6$.

If we repeat the above calculations for the flattened distribution $n_{NS} \propto r^{-3/2}$, then we obtain a reduced rate estimate of

$$\Gamma_{\text{NS–NS}}^{(\text{GN,EC})} \sim 0.005–0.5 \text{ yr}^{-1} \text{ Gpc}^{-3}. \quad (B8)$$

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