Tortuosity and Percolation Probability on 3 Dimensional Rock Models with Different Model Sizes

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Abstract. Tortuosity, percolation probability and porosity are important rock parameters because all those things affect the fluid flow in rocks. In this study, 3-dimensional cube-shaped rock models were randomly generated with \(N \times N \times N\) size containing matrices and pores. Rock models were made with porosities ranging from 0.1 to 1 with increasing of value 0.1. The selected \(N\) is \(N = 5, 6, 7, 8, 9, 10, 25, 125\) and for every \(N\) there are 10,000 random configurations. Then, tortuosity was calculated using a nearest-neighbor sites method with 6 neighboring sides. In addition, with the site percolation model, percolation probability was calculated by determining the comparison between the number of percolate configurations and the total number of configurations created. The results show that in large-size rock models, the average tortuosity will increase. Further, the percolation probability is zero at a small porosity and rises to 1 as the porosity increases. Moreover, the slope of the percolation probability curve will increase for the larger model. In other words, the larger the size of the model, the percolation threshold will become larger or more difficult to percolate for small porosity.

1. Introduction
Tortuosity is an important physical quantity because it can characterize a porous medium such as rock. Tortuosity is a representation level of complexity of the pore arrangement of the rocks when certain fluids are passed. The high or low tortuosity affects how easily the fluid can pass through the rock. There are several types of porous rocks, but not all of those have pores that are connected to each other. The rock whose pore is not connected between one side and the other side is called non-percolation, and vice versa. The size of the rock model is also interesting to study because more configurations can be made. Therefore, it is important to determine the effect of the model size on tortuosity and percolation.

In a porous medium, there is a quantity that characterizes the number of pores in the medium, namely porosity \([1]\). Mathematically, porosity is defined as in equation (1).

\[
\phi = \frac{V_p}{V_T}
\]  

(1)

Tortuosity was first examined by Carman [2]. To date, there are several definitions of tortuosity depending on the perspective of experts in their scientific fields such as geology, engineering and chemistry. Palaciauskas defines tortuosity (\(\tau\)) as a ratio of the fluid flow path \(L_e\) to the shortest distance of the flow \(L\). Mathematically, tortuosity is defined as follows [3].

\[
\tau = \frac{L_e}{L}
\]  

(2)
Percolation theory was first implicitly used by Flory (1941) [4] and Stockmayer (1943) [5] to describe polymerization, which is a phenomenon when monomers and small branched molecules react and form large macromolecules. Percolation \((p)\) comes from the Latin language \((\text{percōlāre})\) which refers to the movement and filtering of fluids through pivoting material. The state of the rock that has two opposite sides connected to each other is called a percolate and vice versa is non-percolate. Comparison of the number of rock configurations that are percolate and non-percolate is called percolation probability. The percolation probability is in the range \(p = [0,1]\). A configuration is called percolated when the percolation probability \((p)\) is greater than 0 [6]. There is a term called percolation threshold \((p_c)\) which is the limit on a configuration when it reaches the percolate. There are two types of percolation models namely bond percolation and site percolation as shown in the following figure [7]. In this paper, we use site percolation as the model.

![Figure 1. Type of percolation model.](image)

If we use an \(N \times N \times N\) model and \(N_\phi\) pore cells, there will be \(W_\phi\) configurations and fulfill the following equation [8].

\[
W_\phi = \frac{N^3!}{N_\phi! (N^3 - N_\phi)!}
\]  

(3)

2. Method

In this study, 3D cube rock models made with \(N \times N \times N\) size. In one configuration model, porosity \((\phi)\) in equation (1) is calculated by comparing the number of pore cells \((N_\phi)\) and total cells \((N)\). The configuration of pore and matrix cells is created randomly as has been researched by T. Ariwibowo [9]. Furthermore, porosity varied from 0.1 to 1 with an increasing value is 0.1. Then, the selected \(N\) is \(N = 5, 6, 7, 8, 9, 10, 25, 125\) and for every \(N\), there are 10,000 random configurations.

Tortuosity, \(\tau\), is calculated using the 6 nearest-neighbor sites method [7]. This means that fluid flow can only pass through the 6 closest sites. In-depth tortuosity calculation is carried out using algorithms in previous studies [8]. In addition, with the site percolation model as in Figure 1, percolation probability is calculated by determining the comparison between the number of percolate configurations and the total number of configurations created which is 10,000.

3. Result and discussion

Tortuosity for each porosity on various model sizes has been calculated. The results show that for small size models \((N \leq 25)\), the tortuosity will increase first and then decrease as porosity increases (see Figure 2 and Figure 3). This happens because, in small rock models, the percolation probability \((p)\) for 10,000 random configurations is large enough. In certain configurations, there is a model that has a number of pore cells \((N_\phi)\) close to the length of the model side \((N)\). If the configuration is percolate (both sides of the model are connected), then the tortuosity \((\tau)\) will approach to 1. For example, if a model with size \(N = 5\) which has a number of pore cell \(N_\phi = 5\) is percolate, then the tortuosity is \(\tau = 1\). Therefore, the model is simpler instead of more complex. This is consistent with the results in Figure 2, a model with size \(N = 5\), which shows the tortuosity \(\tau = 1\) when \(p = 0.0005\) (see Table 1). The \(p = 0.0005\) means that for 10,000 configurations, there are 5 configurations that have tortuosity \(\tau = 1\). The
tortuosity $\tau = 1$ in low porosity also appears in models with size $N = 6$, but with $p = 0.0001$ (see Table 1). Furthermore, the tortuosity will increase to a certain point because of the appearance of complex model configurations is more frequent. Finally, tortuosity decreases due to the reduced model complexity in high porosity.

In the other hand, for large rock models ($N > 25$), the tendency of tortuosity decreases immediately. In large size models, the tortuosity in low porosity is high and then decreases with increasing porosity. This is because, at low porosity, the model will be more complex than in high porosity. The fluid will flow through a more winding path because of the many pathways blocked by the rock matrix. For model size $N = 125$, there is no tortuosity for $\phi = 0.1, 0.2, 0.3$. This happens because there is no percolate configuration that appears on that porosity.

![Tortuosity for N = 5, 6, 7, 8 models.](image)

**Figure 2.** Tortuosity for $N = 5, 6, 7, 8$ models.
Figure 3. Tortuosity for \( N = 9, 10, 25, 125 \) models.

Percolation probability increases starting at certain porosity and then forms an S curve with a range of \( p = [0.1] \) (see Figure 4). The increasing percolation probability is caused by increasing porosity so that the possibility of connecting both sides of the model will be higher. The S curve will be increasingly sharp with increasing model size (\( N \)) (see Figure 4). This is because the percolation probability in porosity will decrease as the size of the rock model increases. For example, see the percolation probability at \( \phi = 0.3 \) for each \( N \) (Table 1). It is clear that the percolation probability decreases with increasing \( N \). The decrease in percolation probability exists because when the size of the rock model increases, the number of configurations \( (W_\phi) \) for particular porosity will increase. However, the number of configurations is only 10,000. Of course, there will be more and more percolate configurations that cannot be covered by just making 10,000 configurations. Therefore, a percolation probability will decrease further. For example, suppose we take \( \phi = 0.3 \) with \( N = 5, N = 25 \) and \( N = 125 \). The number of configurations that can be formed for \( N = 5, N = 25 \) and \( N = 125 \) are \( W_\phi \approx \)
\(3.06 \times 10^{31}, W_{\phi} \approx 3.35 \times 10^{41^{42}}\) and \(W_{\phi} \approx 3.39 \times 10^{51^{150}}\). If we calculate the difference in the configuration that has not been made \((W_{\phi} - 10,000)\), then more and more configurations have not been covered as the \(N\) increases.

Figure 4. Percolation probability for each \(N \times N \times N\) models and percolation threshold.

Percolation threshold, which states the minimum porosity to produce a percolate configuration, increases as the size of the rock model increases (see Figure 4). That means, compared to large size models, small size models will be easier to get percolate configurations with small porosity. For larger models, greater porosity is needed so that the percolate configuration can be formed. As shown in Table 1, the percolation threshold for \(N = 5\) is \(p_c = 0.1\) while the percolation threshold for \(N = 125\) is \(p_c = 0.4\). That happens for the same reason as the S curve gets sharper. With the increase in model size \((N)\) while the configuration is only 10,000, more and more percolate configurations not covered. Therefore, the percolate configuration in small porosity will be more difficult to form, causing an increase in the minimum porosity to produce a percolate configuration which is the percolation threshold.

Table 1. Percolation probability for each model size.

| \(\phi\) | \(N = 5\) | \(N = 6\) | \(N = 7\) | \(N = 8\) | \(N = 9\) | \(N = 10\) | \(N = 25\) | \(N = 125\) |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.1    | 0.0005  | 0.0001  | 0       | 0       | 0       | 0       | 0       | 0       |
| 0.2    | 0.0268  | 0.0147  | 0.008   | 0.0045  | 0.002   | 0.0011  | 0       | 0       |
| 0.3    | 0.2173  | 0.2025  | 0.2017  | 0.199   | 0.1648  | 0.1702  | 0.039   | 0       |
| 0.4    | 0.6259  | 0.6994  | 0.7987  | 0.849   | 0.8751  | 0.9309  | 1       | 1       |
| 0.5    | 0.9263  | 0.973   | 0.9948  | 0.9984  | 0.9994  | 1       | 1       | 1       |
| 0.6    | 0.9958  | 0.9991  | 1       | 1       | 1       | 1       | 1       | 1       |
| 0.7    | 0.9999  | 1       | 1       | 1       | 1       | 1       | 1       | 1       |
| 0.8    | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       |
| 0.9    | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       |
4. Conclusion
Tortuosity for small size models ($N \leq 25$) will increase first and then decrease with increasing porosity. This happens because there is a model that has a pore cell number ($N_p$) approaching the length of the model ($N$). If the configuration is percolate (both sides of the model are connected), then the tortuosity ($\tau$) will approach to 1. Furthermore, the tortuosity will increase to a certain point due to the high complexity of the model before finally decreasing for high porosity. This does not occur in large size models. For model size $N = 125$, there is no tortuosity in $\phi = 0.1, 0.2, 0.3$. This happens because there is no percolate configuration that appears on that porosity.

Percolation probability for certain porosity will decrease with increasing rock size. This happens because the number of configurations in porosity will increase if the model size ($N$) is enlarged. On the other hand, the configuration for one porosity is made in 10,000 on all model sizes ($N$). Therefore, the more configurations that have not been covered as model size increases ($N$). In addition, percolate configuration in small porosity will be more difficult to form, resulting in an increased percolation threshold.

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