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A note on the stability characteristics of the respiratory events

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ABSTRACT

The present outbreak enables the researchers from fluid mechanics to widen the understanding of expelling respiratory liquids from a unique perspective to diminish the persistence of COVID-19. This article focuses on uncovering the instability mechanism responsible for forming droplets and aerosols during respiratory events such as breathing, talking, coughing and sneezing. We illustrate a mathematical framework by revisiting the model (Vadivukkarasan and Panchagnula, 2017) and show the associated instabilities during respiratory events. We envisage the combined Rayleigh–Taylor–Kelvin–Helmholtz (R–T–K–H) model as a robust tool for respiratory events. This study highlights the distinct possibility of respiratory droplet formation over multiple instabilities and provides a fundamental understanding. We present the different dominant modes through a ternary phase diagram for three-dimensional numbers (Bond number and Weber numbers). Furthermore, this model can be extended phenomenologically to viscous fluids to satisfy mucus and saliva in the respiratory liquids.

The adverse effects of the rapid worldwide surge of severe acute respiratory syndrome coronavirus (SARS-CoV-2) pose a monumental challenge to researchers in multidisciplinary fields [1,2]. One of the prominent fields is fluid mechanics [3], and its role is crucial in understanding the formation [4], dispersion [5] of droplet-containing viruses [9] of respiratory jets [10]. The impact of respiratory liquids on the surfaces [11,12] at different ambient conditions [13, 14] and in closed environments [15,16] is still in its infant stage. Several fluid-mechanical methods, such as high-speed imaging [17,18], particle image velocimetry [19], schlieren imaging [20], and a few other in silico approaches using computational modeling [21] elucidate certain features in the recent past. The researchers hitherto have advanced the understanding of the peculiar dynamics of respiratory liquids to an extent, which are multi-physics complex systems in nature. Despite recent advances, knowledge concerning the instability mechanism responsible for the bulk respiratory fluid that transforms into tiny droplets during a respiratory event would add tremendous value [3,22].

Three hydrodynamic instability mechanism [3,22] that are responsible for respiratory events are (i) Rayleigh–Taylor (R–T) instability, (ii) Kelvin–Helmholtz (K–H) instability and (iii) Plateau–Rayleigh (P–R) instability. The distinction between the R–T, K–H, and P–R instability is well documented in the literature [23,24]. The R–T instability on the interface is likely to occur when a lighter fluid is accelerated into the heavier liquid under an unfavorable stratification. The K–H instability is caused at the interface between fluids due to the existence of a shear caused due to the velocity difference. The P–R instability refers to the liquid thread breaking up into the droplets due to the dominance of surface tension force. Respiratory fluids ultimately lead to the formation of droplets (> 5 µm) and aerosols (< 5 µm) resulting from the above-mentioned simultaneous hydrodynamic instabilities [25]. Thus, a qualitative attempt to understand the implications of such combined mechanisms of instability and its intricate nature enables effective control over the mitigation of the transmission of diseases.

A human expels the respiratory liquids into the ambient through one of the following events: (i) breathing [29], (ii) talking [30], (iii) coughing [31] and (iv) sneezing [32]. Liquid expelled from the human mouth during natural respiratory events is either shaped into a cylindrical jet or a sheet before it emerges as a droplet. These events generate aerosols and droplets of varying sizes [4,3,32] are similar to the process of liquid atomization [25] and are the most common mode of disease transmission. While breathing, the tiny droplets in the size of microns are exhaled, which can be referred to as aerosols. During talking [30], a human ejects a respiratory liquid as a cylindrical jet and transforms into tiny droplets. This event is analogous to the breakup of liquid thread that breaks up due to P–R instability. In parallel, other respiratory events such as coughing and sneezing are the spasmodic and complex event [3,22,26,27] that triggers the involuntary discharge of fluid from the lungs [1,33]. A cough is a spontaneous and persistent reflex that effectively clears the large breathing passages of fluids and other foreign particles. In
The respiratory liquid sheet destabilizes due to the competition between the inertial and capillary forces as shown in Fig. 1. The magnitude of the inertial force of the expelled liquid sheet of each respiratory event determine the primary instability mechanism responsible for breaking up. It could be either due to independent (K–H alone) or combination (R–T–K–H) of instabilities. The conservation laws that account for the competing forces between the inertial (axial and radial) and capillary forces determine the shape of the expelling fluid. The inertial forces comprise fluid that moves axially (K–H instability) and radially expands (R–T instability), and capillary forces indicate the surface tension of the cylindrical interface during coughing and sneezing.

In parallel, numerous studies have identified the typical velocity of liquid expulsion during respiratory events. The breathing rate [29,34] ranges 0.5 to 2 m/s, while the sneeze [35] rate varies from 2 to 5 m/s. Similarly, the cough [31,36–39] velocity ranges from 5 to 25 m/s. Likewise, the sneeze [26,27,40–43] varies from 25 to 50 m/s. The velocities of the each event sets up the magnitude of the inertial forces. It is worth mentioning that radial acceleration during respiratory events occurring at a higher velocity while coughing and sneezing is not quantified in the existing literature.

The density of the respiratory fluids ranges from approximately from 800 to 1200 kg/m³ whereas the surface tension varies from 0.072 to 0.1 N/m. These values gives us the indication on the magnitude of inertial forces. In a real situation, the liquid properties, the thickness of the liquid, and the disjunct forces into axial and radial forces vary from human to human [32].

Vadivukkarasan and Panchagnula [44] developed the combined effect of three-dimensional R–T and K–H instabilities in the presence of surface tension to understand the behavior of gas turbine atomizers. They observed the instabilities characteristics to be primarily governed by dimensionless quantities such as (i) Bond number (Bo) signifying the effect of R–T instability and (ii) Weber numbers at the two interfaces based on the relative velocities (We and Weo) signifying the effect of the K–H instabilities. They classified three instability modes namely (a) Taylor mode, (b) flute mode and (c) helical mode. Here, the helical mode is the three-dimensional whereas the Taylor and flute modes are two-dimensional.

Here we revisit a theoretical model [44] that describes the combined behavior of the R–T and K–H instabilities of a cylindrical interface [45–47]. Consider an infinite annular cylindrical interface as shown in Fig. 2 with an internal radius (R(t)) and external radius (R0) of varying densities (ρO, ρ1 and ρ2) and interfacial tensions (σ1 and σ2). The interfaces are subjected to axial and radial motion. It is presumed that the motion in the axial direction (Wx, Wt and Wz) is uniform. The motion in the radial direction includes radial velocity (R) as well as radial acceleration (R).

The governing equations for this case are continuity and Euler equations. Among the boundary conditions, (i) the set of kinematical boundary conditions relates the movement of the interface in the radial direction and (ii) the dynamic boundary condition ensures that the jump in the normal stress balances the capillary (interfacial tension) forces. We consider the three-dimensional infinitesimal disturbances. The linearized governing equation along with boundary conditions are non-dimensionalized by the following scales: length by mean radius of the interface Rm, mass by mRm3 and time by √mRm2/σm. Here ρm = max(ρ1, ρO, ρ2), Rm = R0 + R/2 and surface tension ratio, σm = (σ1 + σ2)/2. Evaluating the equations will lead to the dispersion relation between ω, k and m along with other flow parameters. Here ω is the growth rate, k and m represents the wavenumbers in the axial and azimuthal directions. The detailed derivation and the closed form of the dispersion relation can be found in Vadivukkarasan and Panchagnula [44]. The dimensionless dispersion
Fig. 2. Schematic of the physical problem. An infinite cylindrical liquid sheet in \((r, \theta, z)\) coordinates with an internal radius \((R_i)\) and outer radius \((R_o)\) of varying densities \((\rho_i, \rho_l\) and \(\rho_m\)) and axial velocities \((W_i, W_l\) and \(W_m\)). \(\tilde{R}\) indicates the radial motion (acceleration).

relation of the combined Rayleigh–Taylor and Kelvin–Helmholtz mechanisms of expelling liquid respiratory sheets can be written as,

\[
\mathcal{R}(\omega, k, m) := \mathcal{F}_d\omega^4 + \mathcal{F}_2\omega^3 + \mathcal{F}_1\omega^2 + \mathcal{F}_0 = 0
\]

where \(\omega = \sqrt{\mathcal{A}/\mathcal{B}_m}\), \(k = \mathcal{K}_m\) and \(\mathcal{F}_n\), \((n = 0...4)\) are functions of the following dimensionless numbers, (i) Bond number \((B_o)\) representing the ratio of radial acceleration force to the surface tension force, (ii) Weber numbers \((W_o, W_e)\) representing the aerodynamic force due to the relative velocity to the surface tension force at each interface and (iii) density ratios between the two pairs of fluids \((Q)\) where \(j = i, o\), indicates the inner and outer interfaces. These dimensionless numbers can be written mathematically as,

\[
B_o = \frac{\rho_i (Q_i - 1) \tilde{R} R_m^2}{\sigma_m^i}; \quad W_j = \frac{Q_j \rho_j (W_j - W_m)^2 R_m}{\sigma_m^j}; \quad Q_j = \frac{\rho_j}{\rho_i};
\]

Note that the \(B_o\) and \(W_j\) accounts the density ratio, surface tension ratio and influence of the mean radius. This helps in analyzing only three dimensional numbers. This analysis attempts to find the temporal instability characteristics from the above dispersion relation for a set of dimensionless numbers. The wavenumbers that manifest the dominant instability growth rate \((\omega^*)\) are referred to as the most unstable axial ad circumferential wavenumbers \((k^*\) and \(m^*)\) respectively. Depending upon the values of \(k^*\) and \(m^*\) for a set of flow conditions, three distinct destabilization modes are possible \([44]\). They are (i) Taylor mode \((k^* > 0, m^* = 0)\), (ii) Flute mode \((k^* = 0, m^* > 0)\) and (iii) Helical mode \((k^* > 0, m^* > 0)\). While the interface destabilizes through Taylor mode, one can expect the liquid sheet in the form of rings whereas flute mode yields the ligaments. The shape of the helical mode is arbitrary and difficult to predict. To rationalize the instability modes, we introduce a characteristic length scale \((L^*)\) and investigate it. \(L^*\) is defined as \(2\pi/\epsilon\) for Taylor mode, \(2\pi/\epsilon m^*\) for the flute mode and \(2\pi/\epsilon m^*\) for the helical mode.

Recall the respiratory droplet size depends upon the radial acceleration field (first term of \(\epsilon\)) and in the form of relative kinetic energy in the two streams (second and third terms of \(\epsilon\)) of fluid. Consequently, \(R_m^2/\epsilon = B_o + W_i + W_o = \xi\). Here, \(W_i\) indicates the interaction between the inner airflow and respiratory liquid sheet and \(W_o\) indicates the interaction between the respiratory liquid sheet and ambient \([4]\). It is worth recalling that the typical (mean) axial velocity of the respiratory events ranges from 0.5 to 50 m/s and there exists no study that has quantified any data to date concerning the radial motion to the best of our knowledge. On the other hand, researchers elucidated that both the axial (K–H instability) and radial (R–T instability) forces need to be accounted for in the respiratory events’ stability characteristics \([3,22,25]\). Henceforth, the present work conceptualizes the mean velocity to be attributed to radial velocity and axial relative shear velocities. In other words, \(\xi \sim \tilde{R}^2 + (W_i - W_j)^2\) is to be considered to understand the stability characteristics. For instance, if one considers the axial forces alone and disregard the radial forces, the typical \(\xi\) ranges from 0 to 900 for the respiratory events \([25,27,29,31,34–43]\) being \(B_o = 0\). However, in a real physical problem, \(\xi\) should be ascertained as the summation of all the forces and investigated. This study accounts \(B_o \neq 0\) and addresses the unique parameter, \(\xi\). Note that \(\xi\) has multiple combinations. A range of parametric conditions \((B_o, W_i, W_o)\), it is possible to manifest only one or three dimensional modes. Also, notice that this study does not prescribe any specific value to the radial acceleration or axial velocities. Nevertheless, the investigation possessing unprecedented theoretical simplicity with the help of a few dimensionless numbers \((B_o, W_i, W_o)\) to understand the destabilization characteristics are discussed here, and it may add value to future numerical modeling.

Intuitively, one can consider the range of \(\xi\) and its predominant forces for the respiratory events from the existing studies \([17,27,29,31,34–43]\). \(\xi \lesssim 5\) and \(5 \lesssim \xi \lesssim 20\) applies for breathing and sneezing. In such a situation, axial forces are higher than the radial forces and one could expect the thin cylindrical sheet is ejecting out. Therefore, it suffices to analyze the Weber numbers alone and setting \(B_o = 0\). Likewise, there is a presence of the radial motion along with the axial motion for a cough, and one may notice an expanding cylindrical jet or sheet ejecting out. Henceforth, \(B_o\) should be considered as a non-zero and \(\xi \lesssim 200\). Sneezing arises while \(\xi \lesssim 200\), the radial force can be in the same order or more than the axial forces and the respiratory sheet ejects out radially. However, the quantification of axial and radial forces remains challenging. Moreover, the distribution of forces can be in any combination depending upon the physical characteristics of a human \([32]\).

Under this condition, it would be interesting to analyze the effect of \(\xi\) in different combinations using a ternary plot. Since \(\xi < 100\) yields only Taylor (two dimensional) mode, we present here
Fig. 3. A ternary phase diagram showing contours of constant $L^*$ as a function of $(Bo, We_i, We_o)$. (a) $\zeta = Bo + We_i + We_o = 100$, (b) $\zeta = Bo + We_i + We_o = 200$, (c) $\zeta = Bo + We_i + We_o = 500$ and (d) $\zeta = Bo + We_i + We_o = 800$. The three sides of the triangle of the regime map correspond to the Bond number ($Bo$), inner Weber number ($We_i$) and outer Weber number ($We_o$). $\zeta = Bo + We_i + We_o$ for all points in this figure. The solid lines indicate contours of constant values of $L^*$. A dashed line separates the Taylor mode regime and the helical mode.

starting from $\zeta = 100$. Taylor (helical) mode can be hypothesized to be the representation of droplets (aerosols).

Figs. 3 and 4 shows such a phase diagram. Every point inside the ternary diagram is represented by the three co-ordinates, $Bo$, $We_i$ and $We_o$ such that $Bo + We_i + We_o = \zeta$ in these figures. A dashed line is indicated in this figure separating regions where Taylor mode is manifest from regions where helical and flute modes dominate. As can be seen from all these Figs. 3 and 4, higher values of $Bo$ tend to give rise to helical modes, which may lead to aerosols. It signifies the accounting the radial (acceleration) with the axial motions yields us a smaller droplets than without it. These plots also shows contours of $L^*$ in the three parameter space.

Firstly, it can be observed that for low values of $Bo \ (< 400)$ (see Figs. 3(a) and (b)), these lines of constant $L^*$ are parallel to the $We_o$ axis, which are lines of constant $Bo$. This implies that for $Bo < 400$, $L^*$ is only a function of $Bo$ and is relatively insensitive to both $We_i$ and $We_o$. In addition, as $Bo$ increases, $L^*$ monotonically decreases. It can be observed from Fig. 3(a) that the Taylor modes are alone manifested for any combination of $\zeta$. In contrast, as can be seen from the Fig. 3, the onset of helical modes occurs in between $(Bo, We_i, We_o) = (120, 0, 80)$ and $(120, 80, 0)$. Note that the value of $L^*$ is order less in the region of helical mode.

Recall that $\zeta \lesssim 200$ represents the range of coughing and we will discuss the range of sneezing where $\zeta \gtrsim 200$. The boundaries of coughing and sneezing cannot be accurately demarcated. With the higher radial motion ($Bo$), there is a possibility of generation of aerosols in place of droplets. Notice that $We_o$ is insensitive in determining $L^*$, but it can assist the drops/aerosols to transport to larger distances [4,31,48–50].

The ternary phase diagram for $Bo + We_i + We_o = \zeta = 500$ and $Bo + We_i + We_o = 800$ can be seen from Fig. 3(c) and (d). As $Bo$ increases further (i.e. $Bo > 400$) or $Bo + We_i + We_o = \zeta < 400$, the lines of constant $L^*$ are no longer parallel to lines of constant $Bo$, indicating the effect of combining R–T and K–H mechanisms as shown in Fig. 3(c) and (d) for $Bo + We_i + We_o = \zeta = 500$ and $Bo + We_i + We_o = \zeta = 800$, respectively. Note the bottom right corner of the Figs. 3(c) and (c), 4, the lines of constant $L^*$ show significant non-monotonic behavior. It can be observed the value of overall $L^*$ decreases as increase in the value of $\zeta$ as shown in Fig. 3.

To explain it clearly, one can assume $Bo + We_i + We_o = \zeta = 900$ as shown in Fig. 4. For $\zeta = 900$, the global minimum value of $L^*(\approx 0.14)$ occurs at $(Bo, We_i, We_o) \approx (900, 0, 0)$. This point is indicated by the circle in Fig. 4. Several local minima are also indicated in the figure by other symbols to represent the
existence of helical mode. It implies that if the 90% of inertial forces of the respiratory liquid sheet is disjunct into induce (axial) radial motion, it yields a (maximum) minimum value of $L^*$ and obtain (droplets) aerosols.

We show here that the role of radial acceleration induced R–T instability along with the axial motion-induced K–H instability on the respiratory events plausibly plays as predominant instability mechanisms in determining the size of the droplets or aerosols. But it is still required to confirm either by experimental or computational effort to either substantiate or contradict that theory. The rheological properties (stretching nature of the fluid) of the mucosalivary fluid that contain a disproportionate amount of glucose, mucus, electrolytes and lipids [3,30,48] and could alter the instability characteristics. There is little information on the breakup behavior of viscoelastic fluids. From a practical point of view, one needs to include viscosity to capture the real dynamics of respiratory events. With a close look, in the case of R–T instability, the viscous effects help in destabilization [51,52], whereas, in K–H, it opposes destabilization [24]. Furthermore, the thickness of the expelling respiratory liquid sheet is non-uniform, and it tends to decrease along the axial direction while acting against the surface tension force, which would alter instability characteristics. Henceforth, investigating the viscous effects in the combined R–T–K–H will be interesting. In addition, a complex relationship also exists between the size of the ligaments, the rate of evaporation, and the effect of environmental factors on the overall transmission of the disease. Finally, this novice attempt to delineate the instability modes and their behavior serves as a theoretical framework for extending it to a holistic model with other factors. These findings may be of interest to future investigations.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Availability of data**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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