Truncation effect on estimation of transport parameters for slug-injection tracer tests

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Abstract

For slug-injection tracer tests, tracer concentrations below the detection limit of the measurement instrument can cause truncation of the observed data. This study investigated the truncation effect on the estimation error of parameters based on analytical solutions and the results of laboratory-scale experiments. Spatial moment analysis was performed to estimate the measured total mass and transport parameters, including the pore velocity and the longitudinal and transverse dispersivities. Increasing the travel distance and detection limit caused the measured mass and dispersivities to be underestimated regardless of the dimensionality because hydrodynamic dispersion occurs with increasing travel distance, which smoothens the concentration front. The one- and two-dimensional cases showed that the truncation effect on the measured mass and longitudinal dispersivity depended on dimensionality. In contrast, the pore velocity showed no such dependence; the center of mass did not change as the unmeasured portion due to truncation was increased because the plume, which exhibited a Gaussian distribution, was truncated symmetrically. In the experiment, the measured mass and dispersivities likewise depended on the travel distance and detection limit, but there were large differences in the detection limit at which the dimensionless parameter reached a value of zero between the experimental results and analytical solution. This is because the initial plume in the experiment was of a finite size. Thus, experimental design factors such as the scale, device, and dimensionality should be considered to minimize the estimation error of transport parameters, excluding the pore velocity.

Keywords Slug-injection tracer test · Truncation · Analytical solution · Laboratory-scale experiment · Transport parameter · Spatial moment analysis

Introduction

The advection–dispersion equation (ADE) includes constant transport parameters for the pore velocity and dispersivity. Despite its known limitations, it is the most commonly used tool for characterizing the behavior of solute contaminants in natural soils and aquifers because of its simplicity (Levy and Berkowitz 2003; Liang et al. 2018). The slug-injection tracer test is widely used to estimate these parameters in the laboratory (Chao et al. 2000; Greiner et al. 1997; Levy and Berkowitz 2003; Liang et al. 2018; McNeil et al. 2006; Kurasawa et al. 2020; Kurotori et al. 2019; Qian et al. 2015; Zhang et al. 2019) and field (Adams and Gelhar 1992; Butters et al. 1989; Frank et al. 2020; Freyberg 1986; LeBlanc et al. 1991). In a typical test, a prepared tracer solution is instantaneously injected as a slug in a test well, and the evolution of the tracer plume is observed at downstream sampling locations. If detailed spatial information on a tracer plume is detected at several times, the parameters are commonly estimated based on plume spatial moments that represent global features of the transport process (Adams and Gelhar 1992; Fernández-Garcia et al. 2005; Tompson and Gelhar 1990).

In practice, unfortunately, observed concentration data are often truncated because of the detection limit constraint, i.e., the tracer concentration is below the detection limit of the measurement instrument (Drummond et al. 2012). In this case, only tracer concentrations above a certain value are detected. Estimating transport parameters from the spatial moments of truncated concentration data may yield errors. In porous media such as natural soils and aquifers, hydrodynamic dispersion
occurs because of the combined action of molecular diffusion resulting from concentration gradients and advection resulting from velocity variations at the pore scale. Hydrodynamic dispersion reduces the plume concentration and increases the plume entropy with increasing travel distance (i.e., travel time) of the solute (Bear 1972; Xu et al. 2019). With increasing travel distance, the concentration front of the solute is smoothed out, which can make the truncation effect on transport parameters more pronounced. The truncation effect may also depend on dimensionality because the dilution of the solute plume (i.e., reduction in concentration and increase in entropy) by hydrodynamic dispersion is stronger in higher-dimensional systems (Ye et al. 2015). Thus, the scale, detection device, and dimensionality of the experimental design for a slug-injection tracer test may need to be considered to minimize the estimation errors of parameters.

A few studies have addressed the truncation effect on the estimation of transport parameters. For reactive transport, Heyse et al. (1997) and Young and Ball (2000) noted that truncation of the tailing edge of breakthrough curves at fixed locations significantly influences the estimation of retardation factors with the method of moments. Skopp (1984) conducted a similar study with a nonreactive tracer. With regard to computation of an analytical solution for one-dimensional nonreactive solute transport, Kurasawa et al. (2020a) found that the longitudinal dispersivity estimated from a spatial moment of the concentration distribution exhibits a dependence on time (i.e., scale), whereas the pore velocity does not. However, an evaluation of multidimensional problems and experimental proof of the truncation effect are still lacking.

In this study, the aim was to investigate the truncation effect on the estimation error of parameters in a slug-injection tracer test. Simulations were performed to develop analytical solutions for two- and one-dimensional transport problems, which were used to obtain spatial concentration distributions. These distributions were then artificially truncated, and the parameters were estimated from the spatial moments of the truncated distributions. Finally, relationships were identified between the estimated transport parameters and experimental design factors (i.e., scale, detection limit, and dimensionality). A two-dimensional laboratory-scale slug-injection tracer tests were conducted in homogeneous porous medium to provide experimental evidence of the truncation effect. An optical tracer was employed to observe concentration distribution at a high spatial resolution.

**Evaluation by analytical solution**

**One-dimensional analytical solution and truncation**

Figure 1 illustrates a solute tracer slug that is instantaneously injected (i.e., the Dirac input) into a system governed by a one-dimensional uniform flow along the x-axis. The boundaries are at $x = \pm \infty$ and do not affect the tracer test results. An analytical solution for a slug of conservative tracer instantaneously injected at $x = 0$ can be written as (Bear 1972; Liang et al. 2018; Zheng and Bennett 2002)

$$
\frac{C(x,t)}{C_0} = \frac{1}{\sqrt{4D_L \pi t}} \exp \left( - \frac{(x - \bar{x})^2}{4D_L t} \right),
$$

where $C$ is the solute concentration (M/L$^3$), $t$ is the time (T), $C_0$ is the initial solute concentration (M/L$^3$), $D_L = \alpha_L |v| + D_0$ is the longitudinal dispersion coefficient (L$^2$/T), $\alpha_L$ is the longitudinal dispersivity (L), $v$ is the pore velocity (L/T) along the x-axis, $D_0$ is the effective molecular diffusion coefficient in porous media (L$^2$/T), and $\bar{x} = vt$ is the average travel distance (L). Unless the pore velocity is extremely low, the effective molecular diffusion coefficient is negligible. In this study, $D_0 = 0$ was assumed, which usually holds for most practical problems (Liang et al. 2018).

Equation (1) was used to compute the concentration distributions of the solute at various travel distances. The input parameters were set to $\alpha_L = 1.0$ and $v = 1.0$. Figure 2 shows the evolution of the solute plume along the x-axis.
The maximum tracer concentration was at the centroid position and decayed exponentially as the average travel distance $\bar{x}$ increased.

The detection limit of the measurement instrument was assumed as follows to truncate the concentration distributions of the solute:

$$
C_{tr}(\mathbf{x}, t) = \begin{cases} 
C(\mathbf{x}, t) & C(\mathbf{x}, t) \geq C_{lim} \\
0 & \text{otherwise}
\end{cases},
$$

(2)

where $\mathbf{x}$ is the position vector, $C_{tr}$ is the truncated concentration distribution (M/L³), and $C_{lim}$ is the detection limit (M/L³). According to Eq. (2), only tracer concentrations above the detection limit $C_{lim}$ (M/L³) can be detected. As an example, Fig. 3 shows the truncated concentration distributions for $C_{lim}/C_0 = 2.0 \times 10^{-2}$. Note that the unmeasured portion of the solute plume increases with the average travel distance. This shows that the truncation effect on parameter estimation may be more pronounced at large travel distances.

Two-dimensional analytical solution and truncation

A two-dimensional transport problem associated with a uniform flow was considered. The molecular diffusion was assumed negligible (i.e., $D_y = 0$). The analytical solution for a slug of conservative tracer instantaneously injected at $\mathbf{x} = (x_1, x_2) = (0, 0)$ can be written as follows (Bear 1972; Liang et al. 2018):

$$
\frac{C(\mathbf{x}, t)}{C_0} = \frac{1}{4\pi D_T t} \exp \left(-\frac{(x_1 - x_1^c)^2}{4D_T t} - \frac{x_2^2}{4D_T t}\right),
$$

(3)

where $D_T = \alpha_T v + D_0$ is the transverse dispersion coefficient (L²/T) and $\alpha_T$ is the transverse dispersivity (L). Here, $x_1$ and $x_2$ denote the longitudinal (main flow) and transverse (orthogonal to main flow) directions, respectively. Equation (3) was used to compute two-dimensional concentration maps at several travel distances, as shown in Fig. 4. The input parameters were set to $\alpha_L = 1.0$, $\alpha_T = 0.1$, and $v = 1.0$. Similar to the results shown in Fig. 2, the maximum tracer concentration was at the centroid position, and the plume entropy increased with the travel distance. Equation (2) was used to truncate the two-dimensional concentration maps in Fig. 4. Figure 5 shows the truncated concentration maps for $C_{lim}/C_0 = 2.0 \times 10^{-3}$. Similar to the one-dimensional transport, the unmeasured mass of the solute plume increased with the average travel distance.

Parameter estimation by spatial moment analysis

The spatial moments of the concentration distributions provide an appropriate means of evaluating the transport characteristics of tracer plumes. They have been widely used to estimate the solute transport parameters in experiments (Freyberg 1986; Kurasawa et al. 2020; Sudicky 1986; Tompson and Gelhar 1990) and numerical simulations (Fernández-García et al. 2005; Fernández-García and Gómez-Hernández 2007). The measured total mass $M_{tr}$ of the truncated distribution was obtained by computing the zeroth spatial moment:

$$
M_{tr}(t) = \int C_{tr}(\mathbf{x}, t)d\mathbf{x}.
$$

(4)
The subscript \( tr \) indicates that a value was estimated from the truncated distribution. The centroid position of the plume was calculated as follows:

\[
X_{G,\text{tr}}(t) = \frac{1}{M_{\text{tr}}(t)} \int x_i C_{\text{tr}}(\mathbf{x}, t) dx_i
\]

where \( X_{G,\text{tr}}(t) \) is the centroid position along the \( x_i \) direction as estimated from the truncated distribution. The longitudinal centroid position \( X_{G,\text{tr},1} \) was used to calculate the pore velocity \( v_{tr} \) as follows:

\[
v_{tr}(t) = \frac{X_{G,\text{tr},1}(t)}{t}.
\]

The longitudinal and transverse dispersivities \( \alpha_{L,\text{tr}} \) and \( \alpha_{T,\text{tr}} \), respectively, were calculated as follows:

\[
\alpha_{L,\text{tr}}(t) = \frac{1}{M_{\text{tr}}(t)} \int x_1^2 C_{\text{tr}}(\mathbf{x}, t) dx_1 - (X_{G,\text{tr},1}(t))^2 \frac{2X_{G,\text{tr},1}(t)}{2X_{G,\text{tr},1}(t)}
\]

\[
\alpha_{T,\text{tr}}(t) = \frac{1}{M_{\text{tr}}(t)} \int x_1^2 C_{\text{tr}}(\mathbf{x}, t) dx_1 - (X_{G,\text{tr},2}(t))^2 \frac{2X_{G,\text{tr},1}(t)}{2X_{G,\text{tr},1}(t)}
\]

Results and discussion for the analytical solutions

For the one-dimensional problem, the measured total mass \( M_{\text{tr}} \) and parameters \( (\alpha_{L,\text{tr}}, v_{\text{tr}}) \) were calculated at several travel distances \( (\bar{x} = 10^0, 10^1, 10^2) \) from the spatial moment of the truncated distribution. Figure 6 shows the trends as a function of the dimensionless detection limit \( C_{\text{lim}}/C_0 \). The same input parameters \( (\bar{x} = 1.0, v = 1.0) \) were used as for the results shown in Figs. 2 and 3. While the study conditions represent only one case of many different possible site or sample characteristics (i.e., different parameter sets), several useful observations and conclusions can be made from the present results. In Fig. 6a, the vertical axis shows the measured total mass \( M_{\text{tr}} \) divided by \( M_0 \), which was estimated from the complete distribution (i.e., \( C_{\text{lim}}/C_0 = 0 \)). The vertical axes in Fig. 6b and c represent the estimated parameters \( (\alpha_{L,\text{tr}} \) and \( v_{\text{tr}}) \) divided by the input parameters \( (\bar{x} \) and \( v \), respectively). Values of > 1 and < 1 on the vertical axis indicate over- and underestimation, respectively. As shown in Fig. 6a, truncation did not affect the measured total mass when the detection limit was low, but \( M_{\text{tr}}/M_0 \) was greatly underestimated when the detection limit was high. \( M_{\text{tr}}/M_0 \) also decreased more rapidly to zero when the travel distance was large rather than small. More specifically, \( M_{\text{tr}}/M_0 = 0 \) was obtained at about \( C_{\text{lim}}/C_0 = 3.0 \times 10^{-1} \) for \( \bar{x} = 10^0 \) (red line) but at about \( C_{\text{lim}}/C_0 = 3.0 \times 10^{-2} \) for \( \bar{x} = 10^2 \) (blue line). This dependence of the measured total mass on scale was represented by the increase in the unmeasured mass with travel distance, as shown in Fig. 3.

As shown in Fig. 6b, the estimation of the dimensionless longitudinal dispersivity was likewise dependent on the average travel distance and detection limit, which indicates that the increasing unmeasured mass due to truncation leads to underestimation of the solute spreading around the center of mass. In contrast, the dimensionless pore velocity \( v_{\text{tr}}/v \) remained constant at 1.0. Equation (1) and Fig. 2 indicate that the solute plume exhibited a Gaussian distribution that was characterized by the input parameters of the pore velocity \( v \) and longitudinal dispersivity \( \alpha_L \). This caused the tracer plume to be truncated symmetrically, so the center of mass did not change as the unmeasured mass due to truncation increased. Therefore, the pore velocity estimated by Eq. (6) does not depend on the travel distance or detection limit.

To reflect the truncation effect, Fig. 7 shows the measured total mass and parameters estimated from two-dimensional concentration maps of the slug as functions of the detection limit for several travel distances \( (\bar{x} = 10^0, 10^1, 10^2) \). The input parameters are the same \( (\alpha_L = 1.0, v = 1.0) \) as those used for the results shown in Figs. 4 and 5, except for the transverse dispersivity \( \alpha_T \). The transverse dispersivity was varied to consider two dispersivity ratio cases \( (\alpha_T/\alpha_L = 0.1, \)
$\alpha_T/\alpha_L = 0.1$ and $\alpha_T/\alpha_L = 0.5$. In particular, the truncation effect was more pronounced at large travel distances. This is because two-dimensional cases have a stronger dilution effect as spreading occurs in two directions ($x_1$ and $x_2$) compared with one-dimensional cases. Thus, the measured total mass and longitudinal dispersivity estimated from the truncated distribution showed a dependence on the dimensionality. The estimated transverse dispersivity (Fig. 7c) showed the same trend as the longitudinal dispersivity. This indicates that the dispersivity from the truncated distribution does not depend on direction. The measured total mass and dispersivities decreased more rapidly for $\alpha_T/\alpha_L = 0.5$ than for $\alpha_T/\alpha_L = 0.1$ as the detection limit was increased. Specifically, $a_{t,tr}/a_l$ reached zero at $C_{lim}/C_0 = 2.0 - 3.0 \times 10^{-3}$ when $\bar{x} = 10^2$ for $\alpha_T/\alpha_L = 0.1$ (blue line on the left-hand side of Fig. 7b). In contrast, it reached zero at about $C_{lim}/C_0 = 1.0 \times 10^{-3}$ for $\alpha_T/\alpha_L = 0.5$ (blue line on the right-hand side of Fig. 7b). The transverse dispersivity of the input parameters was set to 0.1 and 0.5 for the cases of $\alpha_T/\alpha_L = 0.1$ and 0.5, respectively. Therefore, the case of $\alpha_T/\alpha_L = 0.5$ had a relatively strong dilution effect, which led to a relatively strong truncation effect. Figure 7d also shows that, similar to the one-dimensional results, the dimensionless pore velocity $v_{tr}/v$ remained constant at 1.0 for the two-dimensional problem. This indicates that the pore velocity does not depend on the dimensionality in addition to the distance and detection limit.

The dispersivities estimated from truncated distributions were found to significantly depend on the detection limit, travel distance, and dimensionality. To further evaluate this dependence, the relative error $\epsilon$ was calculated between the longitudinal dispersivity $a_{l,tr}$ estimated from truncated distributions and the input parameter $a_L$ as follows:

$$\epsilon = \frac{|a_{l,tr} - a_L|}{a_L}.$$  \hspace{1cm} (9)

Figure 8 maps the relative error $\epsilon$ as a function of the average travel distance and detection limit for the one- and two-dimensional cases. Because the estimated longitudinal and transverse dispersivities exhibited the same dependence on the experimental design factors, only maps of the estimated longitudinal dispersivity are presented here. In Fig. 8, the darker regions represent a smaller relative error. Under the smaller $C_{lim}/C_0$ and $\bar{x}_1$ conditions, $\epsilon$ was relatively low. Figure 8a–c highlight that the estimation error for the dispersivity due to the truncated distribution largely depends on the dimensionality. These results demonstrate the importance of considering experimental design factors including the scale, detection device, and dimensionality for slug-injection tracer tests.
Evaluation with a laboratory-scale experiment

Experimental device and porous media

Using a quasi-two-dimensional laboratory sandbox as shown in Fig. 9, a total of three slug-injecting tracer tests were performed to provide experimental evidence of the truncation effect on parameter estimation. These experiments were carried out using three different injection ports (labeled a, b and c). The sandbox was made of acrylic, glass, and stainless steel with internal dimensions of \(200 \text{ cm} \times 3 \text{ cm} \times 80 \text{ cm}\). The front side had a window with a 3 cm thick glass pane for visual observation of the dye tracer.
plume within the sandbox. Constant head spill reservoirs were connected with tubes (internal diameter of 1 cm) to 10 ports at the inlet and outlet. All connections to the sandbox and the inlet and outlet ports were covered with a steel mesh to prevent sand from flowing out of the system. The rear side had an acrylic plate with 10 pressure measurement ports and a solute injection port (black dot).

For the slug-injection tracer tests, silica sand was used with a relatively uniform grain size. Table 1 presents the properties of the silica sand. The hydraulic conductivity was determined as follows. First, a one-dimensional column was packed with sand. The flow rate and hydraulic gradient were measured under the steady-state condition, and Darcy’s law was used to calculate the hydraulic conductivity.

For the two-dimensional sandbox shown in Fig. 9, sand was packed under the saturated condition to prevent air bubbles from being trapped in the porous medium, which can clog pores and change the hydraulic conductivity. The packing was done layer by layer to remove air. The porosity was estimated gravimetrically as 0.437. After packing, the flow rate \( Q \) (about 160 cm\(^3\)/min) through the sandbox was adjusted by controlling the water level of the constant head spill reservoirs. The measured flow rate was used to estimate the pore velocity \( v_{sb} \) through the sandbox as follows:

\[
v_{sb} = \frac{Q}{A n_e},
\]

where \( A \) is the area of the cross section that is perpendicular to the flow direction (L\(^2\)) and \( n_e \) is the effective porosity (–). While the top of the sandbox had an unsaturated zone, this zone was relatively small, so the area of the cross section was assumed equal to the internal area of the sandbox \( (A = 240 \text{ cm}^2) \). The effective porosity was assumed as equal to 0.437, which was the value estimated gravimetrically.

### Table 1 Properties of the silica sand

| Property                  | Value (unit) |
|--------------------------|--------------|
| Average grain size       | 0.13 (cm)    |
| Coefficient of uniformity| 1.99 (–)     |
| Porosity                 | 0.437 (–)    |
| Hydraulic conductivity   | 0.367 (cm/s) |

![Fig. 8](image)

**Fig. 8** Relative error \( \epsilon \) of the longitudinal dispersivity as a function of the average travel distance and detection limit for the one- and two-dimensional cases: **a** one-dimensional case, **b** two-dimensional case \( (\alpha_T/\alpha_L = 0.1) \), and **c** two-dimensional case \( (\alpha_T/\alpha_L = 0.5) \)

![Fig. 9](image)

**Fig. 9** Schematic for the experimental setup of the sandbox: **a** front view and **b** top view

![Table 1](image)
Thus, the pore velocity through the sandbox was estimated as about \( v_{sb} = 1.52 \text{ cm/s} \).

**Experimental procedure**

To visualize the solute transport, dye (Brilliant Blue FCF) was used as the solute tracer at a concentration of \( C_0 = 0.2 \text{ mg/cm}^3 \). This dye is readily visible in soils, is moderately mobile, and has low toxicity (Flury and Flühler 1994); it has been used successfully in previous studies (Forrer et al. 1999; Inoue et al. 2016; Laine-Kaulio et al. 2015; Yang et al. 2014; Zinn et al. 2004). Once a steady-state flow was reached, the dye tracer was instantaneously injected into the two-dimensional sandbox at a specific injection port, which is shown in Fig. 9 as a black dot. Two light sources and a digital camera were placed approximately 100 cm from the sandbox. The digital camera was used to continuously record color images of the slug-injection tracer test. Figure 10 displays an image of the initial plume from injection port b, which exhibited a nearly circular shape with a radius of about 2 cm.

Each pixel of the images displayed a color combining red, green, and blue. The three color channels (i.e., red (R), green (G), and blue (B)) can be divided to transform each picture into three different grayscale images. Each pixel can be assigned an integer value from 0 (black) to 255 (white), and the pixel intensity of the whole image can then be measured for each channel. In this study, only the red color intensity was considered. The dye concentration was correlated with the red color intensity by using the following calibration procedure: (1) inject a solute plume at a known concentration in the porous medium; (2) acquire the RGB image; (3) restart from step (1) with a new solution concentration. This procedure was performed for different concentrations to obtain an adequate calibration curve. Figure 11 shows the calibration curve of the dye concentration against the red color intensity. This calibration curve was used to obtain two-dimensional concentration maps of the solute plume at several times (i.e., several travel distances) and the maps were artificially truncated with Eq. (2). Here, “real” truncation was expected to occur because of the sensitivity limit of the digital camera. By using a relatively large value for \( C_{\text{lim}} \), the effect of “real” truncation can be made negligible. Equations (4)–(8) were used to estimate the measured total mass \( M_{\text{tr}} \) and the parameters \( \alpha_{L, tr} \), \( \alpha_{T, tr} \), and \( v_{tr} \). The average parameter values resulting from estimation for the complete distribution (i.e., \( C_{\text{lim}}/C_0 = 0 \)) of the experiments were \( \alpha_{L, tr} = 0.40 \text{ cm}, \alpha_{T, tr} = 0.0095 \text{ cm} \) and \( v_{tr} = 1.4 \text{ cm/s} \). These dispersivity values are reasonably close to those estimated by Citarella et al. (2015) (\( \alpha_L = 0.106 \text{ cm}, \alpha_T = 0.045 \text{ cm} \)) for a porous medium comprising glass beads with a slightly smaller grain size than in the experiment. In addition, the experimental value of \( v_{tr} = 1.4 \text{ cm/s} \) is close to \( v_{sb} = 1.52 \text{ cm/s} \) calculated from Eq. (10).

**Experimental results and discussion**

Figure 12a and b show concentration maps at several times \( t = 10, 40, 70 \text{ min} \) for \( C_{\text{lim}}/C_0 = 0 \) and \( C_{\text{lim}}/C_0 = 1.5 \times 10^{-1} \), respectively. When \( C_{\text{lim}}/C_0 = 0 \), the shape of the solute plume was weakly irregular but similar to that of concentration maps computed from the analytical solution (see Fig. 4). The irregular behavior of the plume may be attributed to the difficulty with achieving a truly homogeneous porous media, as reported by Huang et al. (1995) and Irwin et al. (1996). Also, the distribution turned gradually Gaussian shape at later times as can be seen in Figure 12a, implying the emergence of the transition to Fickian dispersion. After solute plume have access to the full spectrum of velocity fluctuation for pore structures, asymptotic dispersion (i.e., Fickian dispersion) behavior appears. Previous studies have shown that non-Fickian (i.e., pre-asymptotic) behavior occurs during early times of transport in both heterogeneous and homogeneous porous media (Aris 1956; Dentz et al. 2018; Hinton and Woods 2020; Zhang

![Image of the initial solute plume that exhibited a nearly circular shape with a radius of about 2 cm](image-url)
et al. 2019). This is also one of the causes of the irregular shape for the solute plume. When \( C_{\text{lim}}/C_0 = 1.5 \times 10^{-1} \), the unmeasured mass of the solute plume due to truncation increased with time (i.e., travel distance). This experimental observation is consistent with the computational results (see Fig. 5).

Figure 13 depicts the measured total mass and parameters estimated at travel distances \( \bar{x} = 56, 98 \) cm as a function of the dimensionless detection limit. Here, values obtained at each travel distance below the detection limit \( C_{\text{lim}}/C_0 = 0 \) were regarded as true values (i.e., \( M_0 \), \( \alpha_L \), \( \alpha_T \), and \( v ) \), and the dimensionless parameters (values of vertical axis) of each experiment were calculated. The results computed from the analytical solution are also shown. The average estimated values of the parameters (\( \alpha_{L,T} = 0.40 \) cm, \( \alpha_{T,pr} = 0.0095 \) cm, \( v_{pr} = 1.4 \) cm/s) for the complete distribution (i.e., \( C_{\text{lim}}/C_0 = 0 \)) from the experiments were used as the input parameters for the simulation. For both the measured total mass and dispersivities, the estimated values from the analytical solution and experiment showed similar decreasing trends as the detection limit was increased. However, there were large differences of approximately one order of magnitude in the dimensionless parameter reached a value of zero. For the analytical solution to Eq. (3), the tracer injection was assumed to be a Dirac input, in which a tracer slug instantaneously enters the flow system. However, such input is generally difficult in practice. In fact, the solute plume had a finite initial size with a radius of about 2 cm in the experiment, as shown in Fig. 10. As a result, the plume diluted more slowly in the experiments, and the truncation effect on the estimated values of the parameters was smaller than that on the analytical solution. Several previous studies (Young and Ball 2000; Yu et al. 1999) reported the dependence of truncation error of transport parameters from moment method on initial condition of solute input. For instance, Young and Ball (2000) showed that step input can give more accurate transport parameters (such as retardation factors and mass transfer rate between mobile and immobile regions) than pulse input.

While use of a large plume may have the advantage of minimizing the estimation error due to truncation, the data of our study are not sufficient to accurately evaluate such effects. Thus, future work is required to estimate the impact of initial solute distribution on truncation error of transport parameters. In contrast to the measured total mass and dispersivities, the pore velocity in the experiment remained constant at about 1.0, similar to the analytical solution. This demonstrates that the tracer plume was truncated symmetrically regardless of the initial plume dimensions.

**Conclusions**

This study investigated the truncation effect on the estimated values of transport parameters based on analytical solutions and laboratory-scale experiments. Analytical solutions were calculated for one- and two-dimensional slugs of the solute to obtain spatial concentration distributions at several travel distances, which were truncated. Then, the measured total mass and transport parameters were estimated from spatial moments of the truncated distributions. The results showed that, for both the one- and two-dimensional problems, increasing the travel distance and detection limit caused the measured total mass to be underestimated. This dependence was attributed to the occurrence of hydrodynamic dispersion with increasing travel distance, which smooths the concentration front. In the two-dimensional case, the dimensionless longitudinal and transverse dispersivities
Similarly decreased with increasing travel distance, which was attributed to the truncation effect causing underestimation of the solute spreading around the center of mass. In contrast, the dimensionless pore velocity was estimated to remain constant at 1.0, and it showed no dependence on the travel distance or detection limit. This was attributed to the Gaussian distribution of the solute plume, so it was truncated symmetrically. The measured total mass and longitudinal dispersivity estimated from the truncated distribution also showed a dependence on dimensionality. In the two-dimensional experiments, the estimated values of the measured total mass and dispersivities also showed a dependence on the travel distance and detection limit. This was attributed to the Gaussian distribution of the solute plume, so it was truncated symmetrically. The measured total mass and longitudinal dispersivity estimated from the truncated distribution also showed a dependence on dimensionality. In the two-dimensional experiments, the estimated values of the measured total mass and dispersivities also showed a dependence on the travel distance and detection limit. However, there were large differences (approximately one order of magnitude) in the detection limits at which a parameter reached a value of zero between the experimental results and analytical solution. This is because the initial solute plume in the experiment had a finite initial size. In contrast, the estimated pore velocity in the experiments showed no dependence on the travel distance or detection limit. This indicates that the tracer concentration distribution was truncated symmetrically regardless of the initial plume dimension. These results emphasize the need for considering experimental design factors including the scale, detection device, and dimensionality for slug-injection tracer tests. However, such considerations are not needed for the pore velocity.

Slug-injection tracer tests can be used to study a wide range of engineering problems including storage of carbon dioxide, underground radioactive repository as well as groundwater remediation. In general, field-scale tracer tests require relatively long travel distance, and thus significant truncation of the plume may occur. It is important to understand the effects of truncation on parameter estimation for site specific conditions (i.e., the ratio of detection limit to initial concentration of the plume, travel distance, solute transport characteristics and dimensionality).

As mentioned above, it was suggested that the discrepancy between the estimated values (except for the pore velocity) from the analytical solution and experiment is due to the difference in the initial solute distributions. However,
the data of this study are insufficient to accurately evaluate the impact of initial solute distribution on truncation error of parameter estimation. In future work such effect needs to be examined.

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**Declarations**

**Conflict of interest**  The authors have not disclosed any competing interests.

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