Photons mass from inflation

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We consider vacuum polarization from massless scalar electrodynamics in de Sitter inflation. The
theory exhibits a 3+1 dimensional analogue of the Schwinger mechanism in which a photon mass
is dynamically generated. The mechanism is generic for light scalar fields that couple minimally to
gravity. The non-vanishing of the photon mass during inflation may result in magnetic fields on
cosmological scales.

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1. Introduction. The mass of the photon has been
under scrutiny from the early days of quantum mechanics [1], and this has resulted in stringent limits. The
best laboratory bounds of \( m_\gamma \lesssim 10^{-14} \text{eV} \approx 10^{-14} \text{eV} \) are derived from measurements of potential deviations from
the Coulomb law [2]. The most precise direct bounds
are based on measurements of Earth’s magnetic field [3]
and the Pioneer-10 measurements of Jupiter’s magnetic field [4] and yield \( m_\gamma \lesssim 10^{-15} \text{eV} \). For a review of other
methods and limits, see [5].

Although there is little direct evidence about the photon
mass before the time of matter-radiation decoupling, it is usually assumed to have been equally small on the
basis of current (approximately flat space) data, the
conformal invariance of classical electromagnetism, and the
deduction that the geometry of the early universe was
conformally flat to a high degree. It is well known,
however, that quantum electrodynamic (QED) corrections
break conformal invariance in curved space [6, 7].
This may induce important effects in the early universe [8, 9, 10].

The problem of full nonlocal vacuum polarization in-
duced by matter loops in curved spacetimes has so far
not been considered. That is precisely the subject of this work. We show that, in a locally-de-Sitter inflationary
spacetime and in the presence of a light, minimally cou-
pled, charged scalar field, the polarization of the vacuum induces a photon mass at the one-loop level. The effect is
caused by the coupling of the gauge field to infrared scalar
modes that generically undergo superadiabatic amplifi-
cation on superhorizon scales. This represents a novel
mechanism by which gauge fields can become massive; it
is analogous to the Schwinger mechanism [1], according
to which the photon of 1+1 dimensional QED acquires
a mass \( m_\gamma = e^2/\sqrt{\pi} \). The photon vacuum polarization

\[
\frac{1}{4} \sqrt{-g} \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu} F_{\rho \sigma} - \sqrt{-g} g^{\mu \nu} (D_\mu \phi)^\dagger \partial_\nu \phi, \quad (1)
\]

where \( D_\mu = \partial_\mu + ieA_\mu \) is the covariant derivative, \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\mu A_\nu \) is the gauge field strength, \( g_{\mu \nu} = a^2 \eta_{\mu \nu} \) is

Our perturbative result is in agreement with that of the
authors of Refs. [12, 13], where it was argued that a dynam-
ically generated gauge-field mass in inflation may result
in the generation of large-scale magnetic fields, which
could seed the galactic dynamo and thus offer an ex-
planation for the micro-Gauss-strength magnetic fields observed today [14]. An analogous effect arises in
a more conventional Higgs mechanism realised in inflation [16].

The resulting magnetic field spectrum is of the
form \( B_\ell \propto \ell^{-1} \), where \( \ell \) is the correlation length. This
can be sufficiently strong to seed the galactic dynamo mechanism [16] in flat universes with a dark-energy com-
ponent [17]. For reviews of other mechanisms that may
generate large-scale magnetic fields, see [8, 9, 14, 18].

The authors of Refs. [12, 13] have used a mean-field
approximation to model the backreaction of superhorizon
calar fields on gauge fields. Their analysis indicates that
the photon acquires a mass in inflation. In this Letter, we
calculate the gauge-invariant photon self-energy at the
one-loop level, from which we obtain the photon mass.
a conformally flat metric and \( g = \text{det}[g_{\mu\nu}] = -a^{2D} \). Our calculation was performed in \( D \) spacetime dimensions using dimensional regularization \cite{19}. However, with only two minor modifications it can be understood by working in \( D = 4 \).

We require that the scalar field be light in comparison to the Hubble parameter \( H, m_{\phi} \ll H \sim 10^{13} \text{ GeV} \), so that scalar-field perturbations may grow during inflation. The current experimental bounds on the mass of a charged scalar particle \( m_{\phi} \gtrsim M_{\text{P}} \sim 10^2 \text{ GeV} \) can be amply satisfied. The obvious candidates for \( \phi \) are the charged Higgs particles and the supersymmetric partners of the Standard-Model leptons and quarks.

The scalar propagator \( i\Delta(x, x') = \langle \Omega | \hat{\phi}(x)\phi(x')|\Omega \rangle \), where \( |\Omega \rangle \) denotes the Bunch-Davies vacuum, satisfies for \( D = 4 \) the equation

\[
\partial^\mu \left( a^2 \partial_\mu i\Delta(x, x') \right) = i\delta^4(x - x'),
\]

where the raising of indices is from now on defined as \( \partial^\mu \equiv \eta^{\mu\nu}\partial_\nu \). In the de Sitter spacetime, where the scale factor is given by \( a = -1/H\eta \), \( H \) denotes the Hubble parameter and \( \eta \) denotes conformal time, one can show that the solution of (2) reads \cite{21}

\[
i\Delta(x, x') = \frac{H^2}{4\pi^2} \left( \frac{\eta\eta'}{\Delta x^2} - \frac{1}{2} \ln(H^2\Delta x^2) \right),
\]

where \( \Delta x^2 = -(\eta - \eta') - i\epsilon^2 + |\vec{x} - \vec{x}'|^2 \). Our metric convention is \( \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \) and \( \alpha^\mu = (x^0, \vec{x}) \).

On the other hand, the propagation of free photons in de Sitter inflation is governed, on the classical level, by the flat-space Maxwell equations, \( \partial^\mu F_{\mu\nu} = 0 \).

3. Photon self-energy. Consider now the photon self-energy, which acquires one-loop level contributions from the diagrams shown in Fig. 1 and can be written as

\[
i[\Pi^\nu](x, x') = -2ie^2 a^i\Delta(x, x') \eta^{\mu\nu} \delta^4(x - x') + 2e^2\eta^{\rho\sigma} \eta^{\nu\sigma} \left( \partial_\rho \left[ a^i \Delta(x, x) \right] \partial_\sigma \left[ a^i \Delta(x, x') \right] \right) + [a^i \Delta(x, x')] \partial_\sigma \partial_\sigma \left[ a^i \Delta(x, x') \right],
\]

where \( \partial_\sigma \equiv \partial/\partial x^\sigma \) and \( a = a(\eta) \), \( a' = a'(\eta') \), we used the symmetry \( i\Delta(x, x') = i\Delta(x', x) \) of (2) and neglected for the moment the contribution from the counterterm in Fig. 1.

After some algebra, the one-loop self-energy \( [\Pi^\nu] \) can be recast in the form

\[
i[\Pi^\nu](x, x') = \eta^{\mu\nu} \eta^{\rho\sigma} \left( \eta_{\rho\sigma} \partial' - \partial' \partial_\sigma \right) i\Pi^{(1)}(x, x') + \eta^{\mu\nu} \eta^{\rho\sigma} \left( \eta_{\rho\sigma} \partial' - \partial' \partial_\sigma \right) i\Pi^{(2)}(x, x'),
\]

where

\[
i\Pi^{(1)}(x, x') = \frac{e^2}{8\pi^4} \left[ \frac{1}{6\Delta x^4} - \frac{1}{\eta\eta'} \left( \frac{1}{2\Delta x^2} + \frac{\ln(H^2\Delta x^2)}{2\Delta x^2} \right) \right] \]

\[
i\Pi^{(2)}(x, x') = \frac{e^2}{8\pi^4} \left[ \frac{1}{\eta\eta'} \left( \frac{1}{2} \ln(H^2\Delta x^2) + \frac{1}{8} \ln(H^2\Delta x^2) \right) \right].
\]

The first term in (3) is the standard Minkowski vacuum contribution. This term is singular in the (ultraviolet) coincident limit \( x \to x' \), while the other terms originate from the nonconformal coupling of scalar fields to gravity in the de Sitter background and are completely integrable. The ultraviolet problems are resolved by using dimensional regularization, that is by calculating in \( D \) spacetime dimensions and, subsequently, by renormalizing the self-energy. The result of this rather technical analysis, which we present in \cite{19}, is that \( i\Pi^{(1)} \to i\Pi^{(1)}_{\text{ren}} \), where

\[
i\Pi^{(1)}_{\text{ren}}(x, x') = i \delta \Pi_{\text{anom}}(x, x') + \frac{e^2}{8\pi^4} \left[ \partial' \cdot \partial \ln(\mu^2\Delta x^2) + \frac{1}{16} \ln(H^2\Delta x^2) \right],
\]

and \( i\Pi^{(2)}_{\text{ren}} = i\Pi^{(2)} \) remains unchanged. Here, \( \mu \) is the renormalization scale and

\[
i \delta \Pi_{\text{anom}}(x, x') = -\frac{\alpha_e}{6\pi} \ln(a) \delta^4(x - x')
\]

where \( \alpha_e = e^2/4\pi \) is a local, anomalous contribution resulting from an imperfect cancellation in expanding backgrounds between the local term and counterterm in Fig. 1.

Upon combining the classical action \( S_0 \) with the anomaly contribution \( \delta S_{\text{anom}} \), we get

\[
S_0 + \delta S_{\text{anom}} = -\frac{1}{2} \int d^4xd^4x' A_\mu(x) \left( \eta^{\mu\nu} \partial' \cdot \partial - \partial' \partial_\nu \partial' \partial_\nu \right) \times \left[ 1 + \frac{\alpha_e}{6\pi} \ln\left( \frac{a}{a_0} \right) \right] \delta^4(x - x').
\]

This is the scalar electrodynamics equivalent of the Dolgov anomaly \cite{22}. Since the anomalous contribution to the effective action is proportional to \( \ln a \), we infer that the anomaly affects the photon dynamics quite
mildly \[2\] when compared with the effect of the photon mass, which contributes as \(\propto a^2\) and hence is parametrically much larger.

The transverse structure of the vacuum polarization [8] implies that the Ward identities \(\partial_\mu [\Pi^\nu] = 0 = \partial_\nu [\Pi^\mu]\) are obeyed, so that gauge invariance remains unbroken. The structure of our result [8]-[10] is very similar to that of thermal QED \[22\], which may have something to do with regarding inflationary particle production as Hawking radiation. The spacetime generalization of the standard thermal transverse and ‘longitudinal’ projectors are

\[
P_T^{\mu\nu} = \eta^{\mu i} \eta^{\nu j} \left( \delta_{ij} - \frac{\partial_i}{\partial x} \frac{\partial_j}{\partial x} \right), \quad P_L^{\mu\nu} = \eta^{\mu i} \frac{\partial^j}{\partial x} \left( \delta_{ij} - \frac{\partial_i}{\partial x} \frac{\partial_j}{\partial x} \right) - P_T^{\mu\nu},
\]

while the transverse and ‘longitudinal’ polarizations are

\[
\Pi_t(x, x') = \partial' \cdot \partial \Pi_{\text{ren}}^{(1)} (x, x') + \nabla' \cdot \nabla \Pi_{\text{ren}}^{(2)} (x, x'),
\]

\[
\Pi_l(x, x') = \partial' \cdot \partial \Pi_{\text{ren}}^{(1)} (x, x').
\]

However, this analogy has its limitations. The absence of time-translation invariance in our case makes it non-trivial to extract local physical quantities such as the photon mass or dissipative rate. This is nevertheless possible. Below, we perform a perturbative analysis and show how one can extract a local photon mass from the self-energy (5)-(8).

4. Photon mass. In order to study the effects of the photon self-energy [8]-[10] on the photon propagation, we make use of the Schwinger-Keldysh formalism [23, 24] and write the photon field equation of motion as follows:

\[
\partial_\mu F^{\mu\nu} + \int d^4x' [\eta^{\mu\nu}]_{\text{ren}} (x, x') A_\nu (x') = 0,
\]

where \([\eta^{\mu\nu}]_{\text{ren}} (x, x') = \left[ \Pi_{\text{ren}}^{\mu\nu} (x, x') + \Pi_{\text{ren}}^{\nu\mu} (x, x') \right]\) defines the retarded photon self-energy in terms of the Feynman \([\Pi_{\text{Feyn}}^{\mu\nu}]\) and Wightman \([\Pi_{\text{Wight}}^{\mu\nu}]\) self-energies so that the photon propagation is manifestly causal. The tensors \([\Pi_{\text{Feyn}}^{\mu\nu}]\) and \([\Pi_{\text{Wight}}^{\mu\nu}]\) are obtained from (8) by making the replacements \(\Delta x^2 \rightarrow -\Delta x_+^2 = - (\eta - \eta') i e^2 + |\vec{x} - \vec{x}'|^2\) and \(\Delta x^2 \rightarrow \Delta x_{-}^2 = -(\eta - \eta' + i e^2) + |\vec{x} - \vec{x}'|^2\), respectively [19] [24].

Since the vacuum polarization is only known to one loop order we solve Eq. (13) perturbatively, expanding the photon wave function as

\[
A_\mu = A_\mu^{(0)} + A_\mu^{(1)} + \ldots .
\]

Here \(A_\mu^{(1)} = \mathcal{O}(e^2)\) is the one-loop amplitude, \(A_\mu^{(0)} = \varepsilon_\mu e^{ik \cdot x}\) (with \(k_0 = \pm |k|\)) is the plane-wave solution to the free Maxwell equation, and \(\varepsilon_\mu\) is the (transverse) photon polarization vector, which in Lorentz gauge satisfies \(\varepsilon_0 = 0, \varepsilon \cdot \vec{k} = 0\). The one-loop contribution to Eq. (13) then reads

\[
\left( \eta^{\mu \nu} \partial_\nu - \partial^\nu \partial_\nu \right) A_\mu^{(1)} (x) + \int d^4x' [\eta^{\mu \nu}]_{\text{ren}} (x, x') A_\nu^{(0)} (x') = 0.
\]

We are primarily interested in photons that are subhorizon (\(k \gg H\)) at the initial time \(\eta_0 = -H^{-1}\), and then become superhorizon at some later time \(\eta, k_{\text{phys}} = k/\alpha(\eta) \ll H\), as illustrated in Fig. 2. Upon inserting equations (7), (9) and (10) into (15), we obtain the following approximate equation for the gauge field

\[
\left( \eta^{\mu \nu} \partial_\nu - \partial^\nu \partial_\nu \right) A_\mu^{(1)} (x) - a^2 m_\gamma^2 \eta^{\mu \nu} A_\nu^{(0)} (x') = 0,
\]

When evaluated at the leading logarithmic order in \(k/H\) and \(H a/k\), the photon mass-squared is

\[
m_\gamma^2 = \frac{e^2 H^2}{2\pi^2} \ln \frac{k}{H}.
\]

![FIG. 2: Evolution of the physical scales in de Sitter inflation.](image)

In what follows we shall discuss the origin and the physical relevance of this result. Note first that we have calculated only the leading logarithmic contribution to the photon mass. This will be a good approximation for modes that satisfy \(\ln(k/H) \gg 1\). The mass \(m_\gamma^2\) corresponds to that of space-like transverse excitations, so it is associated with the transverse polarization \(\Pi_t\) in (12).

Even though the scalar excitations produced by an inflationary universe are not thermal, one commonly defines the ‘Hawking temperature’ \(T_H = H/2\pi\). In terms of this temperature the photon mass-squared \(m_\gamma^2\) is

\[
m_\gamma^2 = 2e^2 T_H^2 \ln(k/H).
\]

The logarithmic enhancement is a consequence of the nonthermal nature of the spectrum of charged scalar excitations.

The mathematics of our photon mass generation mechanism bears an interesting resemblance to that of the Schwinger model [11] in which the photon acquires a mass \(m_\gamma^2 = -\epsilon^2/\pi\) in flat, two dimensional, scalar QED the charged field propagators are logarithmic, which results in the vacuum polarization failing to vanish on shell. The scalar propagator goes like \(1/\Delta x^2\) in 3+1 dimensional flat space, and the photon stays massless. In de Sitter background the scalar propagator has a logarithmic tail which is responsible for our mass generation effect. The two extra spatial dimensions are compensated, in the integration, by two factors of \(1/\eta\), and the net result is quite similar to Schwinger’s.

We now relate the photon mass \(m_\gamma^2\) to the Hartree-approximation result \(m_{\text{Hartree}}^2 = 2e^2 \langle \Omega | \phi^\dagger \phi | \Omega \rangle\) considered in [13]. The Hartree mass arises from the local
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