Abstract

I propose a resolution of the strong CP problem of quarks based on vacuum alignment in theories of dynamically broken electroweak and flavor symmetries. Mixing in the neutral $B$ and $K$–meson systems is used to restrict the form of quark mass and mixing matrices and to constrain the masses of the topcolor gauge bosons $V_8$ and $Z'$ to be greater than 5 TeV. The $K^0$ mixing parameter $\epsilon$ is calculated for models whose quark mass matrix leads to weak, but not strong, CP violation. It is easy to accommodate its value in some models while, in others, no reasonable combination of extended technicolor and topcolor masses can account for $\epsilon$. 

$K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ CONSTRAINTS ON TECHNICOLOR

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1. Outline

In this talk I discuss the dynamical approach to CP violation in technicolor theories and a few of its consequences. Special attention will be paid to constraints from the $K^0$ and $B^0_d$ systems. I will cover the following topics:

1. Vacuum alignment in technicolor theories and the rational–phase solutions.
2. A proposal to solve the strong CP problem without an axion or a massless up quark.
3. The structure of quark mass and mixing matrices in extended technicolor (ETC) theories with topcolor–assisted technicolor (TC2). In particular, realistic Cabbibo–Kobayashi–Maskawa (CKM) matrices are easily generated.
4. Flavor–changing neutral current interactions from extended technicolor and topcolor.
5. Strong constraints on TC2 gauge boson masses from $B_d$–$\bar{B}_d$ mixing.
6. New results on the $K^0$–$\bar{K}^0$ CP–violating parameter $\epsilon$ and constraints on ETC masses.

2. Vacuum Alignment in the TechniFermion Sector

In 1971, Dashen stressed the importance of matching the ground state $|\Omega\rangle$ of a theory containing spontaneously broken chiral symmetries with the perturbing Hamiltonian $\mathcal{H}'$ that explicitly breaks those symmetries. He also showed that this process, known as vacuum alignment, can lead to a spontaneous breakdown of CP invariance: it may happen that the CP symmetry of $|\Omega\rangle$ is not the same as that of the the aligned $\mathcal{H}'$. This idea found its natural home in dynamical theories of electroweak symmetry breaking—technicolor—because they have large groups of flavor/chiral symmetries that are spontaneously broken by strong dynamics and explicitly broken by extended technicolor (ETC). Furthermore, the perturbation $\mathcal{H}'$ generated by exchange of ETC gauge bosons is naively CP–conserving if CP is unbroken above the technicolor energy scale. Thus, in 1979, Eichten, Preskill and I proposed that CP violation occurs spontaneously in theories of dynamical electroweak symmetry

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1Invited talk at Les Rencontres de Physique de la Valle d’Aoste, La Thuile, Italy, March 4-10, 2001. Most of the work reported here was done in collaboration with Gustavo Burdman, Estia Eichten and Tunguç Rador.
breaking [3]. Our goal, unrealized at the time, was to solve the strong CP problem of QCD without invoking a Peccei–Quinn symmetry or a massless up quark [6].

This problem was taken up again a few years ago with Eichten and Rado [8]. We studied the first important step in reaching this goal: vacuum alignment in the technifermion sector. We considered models in which a single kind of technifermion interacts with quarks via ETC interactions. There are \( N \) technifermion doublets \( T_{L,R,I} = (U_{L,R,I}, D_{L,R,I}) \), all transforming according to the fundamental representation of the technicolor gauge group \( SU(N_{TC}) \). There are 3 generations of \( SU(3)_C \) triplet quarks \( q_{L,R,i} = (u_{L,R,i}, d_{L,R,i}) \), \( i = 1, 2, 3 \). The left-handed fermions are electroweak \( SU(2) \) doublets and the right-handed ones are singlets.

Here and below, we exhibit only flavor, not technicolor nor color, indices. The technifermions are assumed for simplicity to be ordinary color–singlets, so the chiral flavor group of our model is \( G_f = [SU(2N)_L \otimes SU(2N)_R] \otimes [SU(6)_L \otimes SU(6)_R] \).

When the TC and QCD couplings reach their required critical values, these symmetries are spontaneously broken to \( S_f = SU(2N)_V \otimes SU(6)_V \), with fermion bilinear condensates given by

\[
\langle \Omega | \bar{U}_{L,I} U_{R,J} | \Omega \rangle = \langle \Omega | \bar{D}_{L,I} D_{R,J} | \Omega \rangle = -\delta_{IJ} \Delta_T \\
\langle \Omega | \bar{u}_{L,i} u_{R,j} | \Omega \rangle = \langle \Omega | \bar{d}_{L,i} d_{R,j} | \Omega \rangle = -\delta_{ij} \Delta_q .
\]

(1)

Here, \( \Delta_T \approx N_{TC} \Lambda_{TC}^3 \) and \( \Delta_q \approx N_C \Lambda_{QCD}^3 \) when they are renormalized at their respective strong interaction scales. Of course, \( N_C = 3 \).

All of the \( G_f \) symmetries except for the gauged electroweak \( SU(2) \otimes U(1) \) must be explicitly broken by extended technicolor interactions [3]. In the absence of a concrete ETC model, we write the interactions broken at the scale \( M_{ETC}/g_{ETC} \sim 10^2–10^4 \text{TeV} \) in the phenomenological four-fermion form (sum over repeated indices) [3]

\[
\mathcal{H}' \equiv \mathcal{H}'_{TT} + \mathcal{H}'_{Tq} + \mathcal{H}'_{qq} \equiv \Lambda_{TT}^{T} \bar{T}_{I,J} \gamma^\mu T_{J,K} \bar{T}_{K,L} \gamma^\mu T_{L,I} + \Lambda_{T}^{T} \bar{T}_{I,J} \gamma^\mu q_{L,I} \bar{q}_{R,J} \gamma^\mu T_{L,R} + h.c. \\
+ \Lambda_{q}^{q} \bar{q}_{L,i} \gamma^\mu q_{L,j} \bar{q}_{R,k} \gamma^\mu q_{R,l} .
\]

(2)

\footnote{The fact that heavy quark chiral symmetries cannot be treated by chiral perturbative methods will be addressed below. We have excluded anomalous \( U_A(1)'s \) strongly broken by TC and color instanton effects.}

\footnote{See the Appendix for estimates of \( M_{ETC}/g_{ETC} \).}

\footnote{We assume that ETC interactions commute with electroweak \( SU(2) \), though not with \( U(1) \) nor color \( SU(3) \). All fields in Eq. (2) are electroweak, not mass, eigenstates.
where \( T_{L,R} \) and \( q_{L,R} \) stand for all 2N technifermions and 6 quarks, respectively. Here, \( M_{ETC} \) is a typical ETC gauge boson mass and the \( \Lambda \) coefficients are \( \mathcal{O}(g_{ETC}^2/M_{ETC}^2) \) times mixing factors for these bosons and group theoretical factors. The \( \Lambda \)'s may have either sign. In all calculations, we must choose the \( \Lambda \)'s to avoid unwanted Goldstone bosons. Hermiticity of \( \mathcal{H}' \) requires

\[
(\Lambda_{IJKL}^{TT})^* = \Lambda_{JILK}^{TT}, \quad (\Lambda_{Iijj}^{Tq})^* = \Lambda_{iijj}^{Tq}, \quad (\Lambda_{ijkl}^{qq})^* = \Lambda_{jilk}^{qq}.
\]  

(3)

The assumption of time-reversal invariance for this theory before any potential breaking via vacuum alignment means that the instanton angles \( \theta_{TC} = \theta_{QCD} = 0 \) (at tree level) and that all \( \Lambda \)'s are real. Thus, e.g., \( \Lambda_{IJKL}^{TT} = \Lambda_{JILK}^{TT} \).

Having chosen a standard chiral-perturbative ground state, \( |\Omega\rangle \), vacuum alignment proceeds by minimizing the expectation value of the rotated Hamiltonian. This is obtained by making the \( G_f \) transformation \( T_{L,R} \rightarrow W_{L,R} T_{L,R} \) and \( q_{L,R} \rightarrow Q_{L,R} q_{L,R} \), where \( W_{L,R} \in SU(2N)_{L,R} \) and \( Q_{L,R} \in SU(6)_{L,R} \):

\[
\mathcal{H}'(W,Q) = \mathcal{H}_{TT}^{TT}(W,L,R) + \mathcal{H}_{Tq}^{Tq}(W,Q) + \mathcal{H}_{qq}^{qq}(Q_L,Q_R) \]

\[
= \Lambda_{IJKL}^{TT} \bar{T}_{L'I'} W_{L'I'}^{\dagger} \gamma^\mu W_{L'J'} T_{L'J'} \bar{T}_{R'K'} W_{R'K'}^{\dagger} \gamma^\mu W_{R'L'} T_{R'L'} + \cdots.
\]  

(4)

Since \( T \) and \( q \) transform according to complex representations of their respective color groups, the four-fermion condensates in the \( S_f \)-invariant \( |\Omega\rangle \) have the form

\[
\langle \Omega | \bar{T}_{L'I'} \gamma^\mu T_{LJ} \bar{T}_{R'K'} \gamma_\mu T_{RL} | \Omega \rangle = -\Delta_{TT} \delta_{LJ} \delta_{JK}, \\
\langle \Omega | \bar{T}_{L'I'} \gamma^\mu q_{L'I'} \bar{q}_{R'L'} \gamma_\mu T_{RL} | \Omega \rangle = -\Delta_{Tq} \delta_{LJ} \delta_{ij}, \\
\langle \Omega | \bar{q}_{L'I'} \gamma^\mu T_{LJ} \bar{q}_{R'L'} \gamma_\mu q_{RL} | \Omega \rangle = -\Delta_{qq} \delta_{LJ} \delta_{jk}.
\]  

(5)

The condensates are positive, renormalized at \( M_{ETC} \) and, in the large-\( N_{TC} \) and \( N_C \) limits, they are given by \( \Delta_{TT} \simeq (\Delta_T(M_{ETC}))^2 \), \( \Delta_{Tq} \simeq \Delta_T(M_{ETC}) \Delta_q(M_{ETC}) \), and \( \Delta_{qq} \simeq (\Delta_q(M_{ETC}))^2 \). In walking technicolor \cite{ref}, \( \Delta_T(M_{ETC}) \simeq (M_{ETC}/\Lambda_{TC}) \Delta_T(\Lambda_{TC}) = 10^2-10^4 \times \Delta_T(\Lambda_{TC}) \). In QCD, however, \( \Delta_q(M_{ETC}) \simeq (\log(M_{ETC}/\Lambda_{QCD}))^{\gamma_m} \Delta_q(\Lambda_{QCD}) \simeq \Delta_q(\Lambda_{QCD}) \), where \( \gamma_m \simeq 2\alpha_C/\pi \) for \( SU(3)_C \). Thus, the ratio

\[
r = \frac{\Delta_{Tq}(M_{ETC})}{\Delta_{TT}(M_{ETC})} \simeq \frac{\Delta_{qq}(M_{ETC})}{\Delta_{Tq}(M_{ETC})}.
\]  

(6)

\[\text{So long as vacuum alignment preserves electric charge conservation, the alignment matrices will be block-diagonal:}
\]

\[
W_{L,R} = \begin{pmatrix} W^U & 0 \\ 0 & W^D \end{pmatrix}_{L,R}, \quad Q_{L,R} = \begin{pmatrix} U & 0 \\ 0 & D \end{pmatrix}_{L,R}.
\]
is at most $10^{-10}$. This is $10$ to $10^4$ times smaller than in a technicolor theory in which the coupling does not walk.

With these condensates, the vacuum energy is a function only of $W = W_L W_R^\dagger$ and $Q = Q_L Q_R^\dagger$, elements of the coset space $G_f / S_f$:

$$E(W, Q) = E_{TT}(W) + E_{Tq}(W, Q) + E_{qq}(Q)$$

$$= -\Lambda_{TT}^{IJKL} W_{JK} W^\dagger_{LI} \Delta_{TT} - \left(\Lambda_{Tq}^{ij} Q_{ij} W^\dagger_{JI} + c.c.\right) \Delta_{Tq} - \Lambda_{qq}^{ijkl} Q_{jk} Q^\dagger_{li} \Delta_{qq}$$

$$= -\Lambda_{TT}^{IJKL} W_{JK} W^\dagger_{LI} \Delta_{TT} + \mathcal{O}(10^{-10}).$$

Note that time–reversal invariance of the unrotated Hamiltonian $\mathcal{H}'$ implies that $E(W, Q) = E(W^*, Q^*)$. Hence, spontaneous CP violation occurs if the solutions $W_0, Q_0$ to the minimization problem are complex.

The last line of Eq. (7) makes clear that we should first minimize the technifermion sector energy $E_{TT}$. This determines $W_0$ up to corrections of $\mathcal{O}(10^{-10})$. This result is then fed into $E_{Tq}$ to determine $Q_0$—and the nature of quark CP violation—up to corrections which are also $\mathcal{O}(10^{-10})$.

In Ref. [8] it was shown that just three possibilities naturally occur for the phases in $W$. (We drop the subscript “0” from now on.) Let us write $W_{IJ} = |W_{IJ}| \exp(i\phi_{IJ}).$ Consider an individual term $-\Lambda_{TT}^{IJKL} W_{JK} W^\dagger_{LI} \Delta_{TT}$ in the vacuum energy. If $\Lambda_{TT}^{IJKL} > 0$, this term is least if $\phi_{IL} = \phi_{JK}$; if $\Lambda_{TT}^{IJKL} < 0$, it is least if $\phi_{IL} = \phi_{JK} \pm \pi$. We say that $\Lambda_{TT}^{IJKL} \neq 0$ links $\phi_{IL}$ and $\phi_{JK}$, and tends to align (or antialign) them. Of course, the constraints of unitarity may partially or wholly frustrate this alignment. The three possibilities for the phases are:

1. The phases are all unequal, irrational multiples of $\pi$ that are random except for the constraints of unitarity and unimodularity.

2. All of the phases may be equal to the same integer multiple of $2\pi / N$ (mod $\pi$). This occurs when all phases are linked and aligned, and the value $2\pi / N$ is a consequence of unimodularity. In this case we say that the phases are “rational”.

3. Several groups of phases may be linked among themselves and the phases only partially aligned. In this case, their values are various rational multiples of $\pi / N'$ for one or more integers $N'$ from 1 to $N$.  

\footnote{Two sorts of corrections to this statement are under study. The first are higher–order ETC and electroweak corrections to $E_{TT}$. The second are due to $T\bar{t}T$ terms in $E_{Tq}$ which are important if the top condensate is large. I thank J. Donoghue and S. L. Glashow for emphasizing the importance of these corrections.}

\footnote{Because $W$ is block diagonal, $E_{TT}$ factorizes into two pieces, $E_{UU} + E_{DD}$, in which $W_U$ and $W_D$ may each be taken unimodular. Thus, totally aligned phases are multiples of $2\pi / N$, not $\pi / N$.}
We stress that, as far as we know, rational phases occur naturally only in ETC theories. They are a consequence of $E_{TT}$ being quadratic, not linear, in $W$. With these three outcomes in hand, we proceed to investigate the strong CP violation problem of quarks.

3. A Dynamical Solution to the Strong CP Problem

There are two kinds of CP violation in the quark sector. Weak CP violation enters the standard weak interactions through the CKM phase $\delta_{13}$ and, for us, in the ETC and TC2 [10] interactions through phases in the quark alignment alignment matrices $U_{L,R}$ and $D_{L,R}$ discussed in Section 4. Strong CP violation, which can produce electric dipole moments $10^{16}$ times larger than in the standard model, is a consequence of instantons [7]. No discussion of the origin of CP violation is complete which does not eliminate strong CP violation. Resolving the strong CP problem amounts to making $\bar{\theta}_q = \arg\det(M_q) \lesssim 10^{-10}$ (in a basis with instanton angle $\theta_{QCD} = 0$), so that the neutron electric dipole moment is below its experimental bound of $0.63 \times 10^{-25} \text{e--cm}$ [11]. Here, $M_q$ is the hard or current algebra mass matrix of the quarks. It includes both the TC2 and ETC–generated parts of the top quark’s mass.

The “primordial” quark mass matrix, the coefficient of the bilinear $\bar{q} \gamma^\mu q$ of quark electroweak eigenstates, is generated by ETC interactions and is given by

$$ (M_q)_{ij} = \sum_{I,J} \Lambda^{Tq}_{iIjJ} W_{JI}^\dagger \Delta_T(M_{ETC}) \quad (q, T = u, U \text{ or } d, D) \quad (8) $$

The $\Lambda^{Tq}_{iIjJ}$ are real ETC couplings of order $(10^{2}–10^{4} \text{TeV})^{-2}$ (see the Appendix). Furthermore, the quark alignment matrices $Q_{L,R}$ which diagonalize $M_q$ to $M_q$ are unimodular. Thus, $\arg\det(M_q) = \arg\det(M_q) = \arg\det(M_u) + \arg\det(M_d)$, and the question of strong CP violation is determined entirely by the character of vacuum alignment in the technifermion sector, i.e., by the phases $\phi_{IJ}$ of $W$, and by how the ETC factors $\Lambda^{Tq}_{iIjJ}$ map these phases into the $(M_q)_{ij}$.

If the $\phi_{IJ}$ are random irrational phases, $\bar{\theta}_q$ could vanish only by the most contrived, unnatural adjustment of the $\Lambda^{Tq}$. If all $\phi_{IJ} = 2m\pi/N (\text{mod } \pi)$, then all elements of $M_u$ have the same phase, as do all elements of $M_d$. Then, $U_{L,R}$ and $D_{L,R}$

\footnote{The matrix element $M_{tt}$ arises almost entirely from the TC2–induced condensation of top quarks. We assume that $\langle \bar{t} t \rangle$ and $M_{tt}$ are real in the basis with $\theta_{QCD} = 0$. Since technicolor, color, and topcolor groups are embedded in ETC, all CP–conserving condensates are real in this basis.}
$D_{L,R}$ will be real orthogonal matrices, up to an overall phase. There may be strong CP violation, but there will no weak CP violation in any interaction.

There remains the possibility, which we assume henceforth, that the $\phi_{I,J}$ are different rational multiples of $\pi$. Then, strong CP violation will be absent if the $A^T$ map these phases onto the primordial mass matrix so that each element $(M_q)_{ij}$ has a rational phase and these add to zero in $\arg \det(M_q)$. In the absence of an explicit ETC model, we are not certain this can happen, but we see no reason that it cannot. For example, there may be just one nonzero $A^T_{I,J}$ for each pair $(ij)$ and $(IJ)$. An ETC model which achieves such a phase mapping will solve the strong CP problem, i.e., $\bar{\theta}_q < \sim 10^{-10}$, without an axion and without a massless up quark. This is, in effect, a “natural fine-tuning” of phases in the quark mass matrix.\footnote{9\Thank C. Sommerfield for this description.} There is, of course, no reason weak CP violation will not occur in this model. We shall illustrate this with some examples in Sections 4 and 6.

Determining the quark alignment matrices $Q_{L,R}$ begins with minimizing the vacuum energy

$$E_{Tq}(Q) \cong -\frac{1}{2} \text{Tr} \left( M_q Q + \text{h.c.} \right) \Delta_q(M_{ETC})$$

(9)

to find $Q = Q_L Q_R^\dagger$. Whether or not $\bar{\theta}_q = 0$, the matrix $Q^\dagger M_q$ is hermitian up to the identity matrix \footnote{10\Since quark vacuum alignment is based on first order chiral perturbation theory, it is inapplicable to the heavy quarks $c,b,t$. When $\bar{\theta}_q = 0$, Dashen’s procedure is equivalent to making the mass matrix diagonal, real, and positive. Thus, it correctly determines the quark unitary matrices $U_{L,R}$ and $D_{L,R}$ and the magnitude of strong and weak CP violation.},

$$M_q Q - Q^\dagger M_q^\dagger = i\nu_q 1,$$

(10)

where $\nu_q$ is the Lagrange multiplier associated with the unimodularity constraint on $Q$, and $\nu_q$ vanishes if $\bar{\theta}_q$ does. Thus, $Q^\dagger M_q$ may be diagonalized by the single unitary transformation $Q_R$ and so

$$M_q \equiv \begin{pmatrix} M_u & 0 \\ 0 & M_d \end{pmatrix} = Q_R^\dagger M_q Q Q_R = Q_R^\dagger M_q Q_L.$$  

(11)

4. Quark Mass and Mixing Matrices in ETC/TC2

4.1 General Considerations

If $\bar{\theta}_q = 0$, the matrix $M_q$ is brought to real, positive, diagonal form by the block-diagonal $SU(6)$ matrices $Q_{L,R}$. From these, one constructs the CKM matrix $V = U_L^\dagger D_L$. Carrying out the vectorial phase changes on the $q_{L,R}$ required to put $V$
in the standard Harari–Leurer form with the single CP–violating phase $\delta_{13}$, one obtains \[12, 11\]

$$
V \equiv \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
$$

\begin{align*}
= & \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{12} c_{13} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{13} e^{-i\delta_{13}} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13}
\end{pmatrix}.
\end{align*}

Here, $s_{ij} = \sin \theta_{ij}$, and the angles $\theta_{12}, \theta_{23}, \theta_{13}$ lie in the first quadrant. Additional CP–violating phases appear in $U_{L,R}$ and $D_{L,R}$ and they are rendered observable by ETC and TC2 interactions. We will study their contribution to $\epsilon$ in Section 6. Before that, we need to discuss the constraints on $M_{u,d}$ and $U_{L,R}, D_{L,R}$ imposed by ETC and TC2.

First, limits on flavor–changing neutral current (FCNC) interactions, especially those mediating $|\Delta S| = 2$, require that ETC bosons coupling to the two light generations have masses $M_{ETC} \gtrsim 1000$ TeV \[3, 4\]. These can produce quark masses less than about $m_s(M_{ETC}) \simeq 100$ MeV in a walking technicolor theory (see the Appendix). Extended technicolor bosons as light as 50–100 TeV are needed to generate $m_b(M_{ETC}) \simeq 3.5$ GeV. Flavor–changing neutral current interactions mediated by such light ETC bosons must be suppressed by small mixing angles between the third and the first two generations.

The most important feature of $M_u$ is that the TC2 component of $M_{tt}$, $(m_t)_{TC2} \simeq 160$ GeV, is much larger than all its other elements, all of which are generated by ETC exchange. In particular, off-diagonal elements in the third row and column of $M_u$ are expected to be no larger than the 0.01–1.0 GeV associated with $m_u$ and $m_c$. Thus, $M_u$ is very nearly block–diagonal and, so, $|U_{L,R u_{ti}}| \simeq |U_{L,R u_{ti}}| \approx \delta_{tu_i}$.

The matrix $M_d$ has a triangular or nearly triangular structure. One reason for this is the need to suppress $\bar{B}_d – B_d$ mixing induced by the exchange of “bottom pions” of mass $M_{n_b} \sim 300$ GeV \[13, 14\]. Furthermore, since $U_L$ is block–diagonal, the observed intergenerational mixing in the CKM matrix must come from the down sector. These requirements are met when the $d_R, s_R \leftrightarrow b_L$ elements of $M_d$ are much smaller than the $d_L, s_L \leftrightarrow b_R$ elements. In Ref. \[15\], the strong topcolor $U(1)$ charges were chosen to exclude ETC interactions that induce $M_{db}$ and $M_{sb}$. This makes $D_R$, like $U_{L,R}$, nearly $2 \times 2$ times $1 \times 1$ block–diagonal.
From these considerations and $V_{tb} \cong 1$, we have

$$V_{td_i} \cong V_{tb}^* V_{td} \cong U_{L_{tt}} D_{L_{tb}} U_{L_{bt}}^* D_{L_{bd}} \cong D_{L_{tb}} D_{L_{bd}}.$$  

(13)

This relation, which is good to 10% (see Section 4.2 for examples), will be used in Section 6 to put strong limits on the TC2 $V_{8}$ and $Z'$ masses from $\bar{B}_d - B_d$ mixing [16].

One more interesting property of the quark alignment matrices is this: The vacuum energy $E_{Tq}$ is minimized when the elements of $U$ and $D$ have almost the same rational phases as $M_u$ and $M_d$ do. In particular, all the large diagonal elements of $U, D$ have rational phases (see Section 4.2). This is generally not true of $U_{L,R}$ and $D_{L,R}$ individually. However, since $Q_{ii} = \sum_j Q_{Lij} Q_{Rij}^*$ ($Q = U, D$) has a rational phase, $E_{Tq}$ is likely to be minimized when each term in the sum has the same rational phase. Thus, like DNA, in which the patterns of the two strands are linked,

$$\arg Q_{Lij} - \arg Q_{Lik} = \arg Q_{Rij} - \arg Q_{Rik} \pmod{\pi} \text{ for } i, j, k = u, c, t \text{ or } d, s, b.$$  

(14)

In particular, $\arg V_{td_i} \cong \arg D_{L_{bd_i}} - \arg D_{L_{bb}} \cong \arg D_{R_{bd_i}} - \arg D_{R_{bb}} \pmod{\pi}$ for $d_i = d, s, b$.

4.2 Examples

Our proposal for solving the strong CP problem in technicolor theories rests on the fact that phases in the technifermion alignment matrices $W = (W_U, W_D)$ can be different rational multiples of $\pi$, and on the conjecture that these phases may be mapped by ETC onto the primordial mass matrix $(\mathcal{M}_q)_{ij} = \Lambda_{ij}^T \mathcal{T} q W_{ij}^* \Delta_T$ so that $\bar{\theta}_q = \arg \det(\mathcal{M}_q) = 0$. Corrections to $\bar{\theta}_q$ are expected to be at most $O(10^{-10})$. In this section we present two examples of quark mass matrices for which we have engineered $\bar{\theta}_q = 0$. They lead to similar alignment and CKM matrices, except that one example has $\delta_{13} = 0$. Nevertheless, as we see in Section 6, both sections lead to successful calculations of CP-violating parameter $\epsilon$.

**Model 1:**

In this model, $\delta_{13} = 0$, but CP violation will arise from phases in $U_{L,R}$ and $D_{L,R}$. The primordial quark mass matrices renormalized at $M_{ETC}$ are taken to be of seesaw form with phases that are multiples of $\pi/3$:

$$\mathcal{M}_u = \begin{pmatrix} (0, 0) & (200, 1/3) & (0, 0) \\ (15.6, -1/3) & (900, 1) & (0, 0) \\ (0, 0) & (0, 0) & (162620, 0) \end{pmatrix}$$
\[ \mathcal{M}_d = \begin{pmatrix} (0, 0) & (23.3, 0) & (0, 0) \\ (21.7, 0) & (102, 1/3) & (0, 0) \\ (17.0, 1/3) & (144, 2/3) & (3505, 0) \end{pmatrix}. \]

The notation is \(|(\mathcal{M}_q)_{ij}|, \arg((\mathcal{M}_q)_{ij})/\pi\). Here, we have made \(\arg \det(\mathcal{M}_u) = \arg \det(\mathcal{M}_d) = \pi\). We imposed the same kind of structure on \(\mathcal{M}_u\) as \(\bar{B}_d-B_d\) mixing requires of \(\mathcal{M}_d\). The quark mass eigenvalues may be extracted from \(\mathcal{M}_q\). Their values at \(M_{E_{TC}} \sim 10^3\) TeV are (in MeV):

\[
\begin{align*}
m_u &= 3.35, \quad m_c = 924, \quad m_t = 162620 \\
m_d &= 4.74, \quad m_s = 106, \quad m_b = 3508
\end{align*}
\]

The alignment matrices \(U = U_L^\dagger U_R\) and \(D = D_L^\dagger D_R\) obtained by minimizing \(E_{Tq}\) are

\[
U = \begin{pmatrix} (0.973, 0) & (0.232, 1/3) & (0, 0) \\ (0.232, -1/3) & (0.973, 1) & (0, 0) \\ (0, 0) & (0, 0) & (1, 0) \end{pmatrix}
\]

\[
D = \begin{pmatrix} (0.915, -2/3) & (0.404, 0) & (0.0046, -1/3) \\ (0.404, 0) & (0.914, -1/3) & (0.0400, -2/3) \\ (0.0119, -1/3) & (0.0384, -2/3) & (0.999, 0) \end{pmatrix}
\]

The cloning of the \(\mathcal{M}_{u,d}\) phases onto \(U, D\) is apparent. Diagonalizing the aligned quark mass matrices yields \(Q_{L,R}\):

\[
U_L = \begin{pmatrix} (0.9999, -0.859) & (0.0164, 0.141) & (0, 0) \\ (0.164, -1.193) & (0.9999, -1.193) & (0, 0) \\ (0, 0) & (0, 0) & (1, -0.526) \end{pmatrix}
\]

\[
D_L = \begin{pmatrix} (0.980, 1.141) & (0.199, 1.141) & (0.00485, 1.141) \\ (0.199, -0.192) & (0.979, 0.808) & (0.0412, 0.808) \\ (0.00344, -0.526) & (0.0413, 0.474) & (0.999, -0.526) \end{pmatrix}
\]

\[
U_R = \begin{pmatrix} (0.976, -0.859) & (0.216, -0.859) & (0, 0) \\ (0.216, -1.193) & (0.976, -0.192) & (0, 0) \\ (0, 0) & (0, 0) & (1, -0.526) \end{pmatrix}
\]

\[
D_R = \begin{pmatrix} (0.977, -0.192) & (0.214, 0.808) & (0.000273, 0.808) \\ (0.214, 1.141) & (0.977, 1.141) & (0.00122, 1.141) \\ (5 \times 10^{-6}, -0.526) & (0.00125, 0.474) & (1, -0.526) \end{pmatrix}
\]
As required, all the mixing in $U_{L,R}$ and $D_R$ is between the first two generations; mixing of these two with the third generation comes entirely from $D_L$. A perusal of the phases will reveal differences which are multiples of $\pi/3$. Finally, the CKM matrix is

$$V = \begin{pmatrix}
(0.977, 0) & (0.215, 0) & (0.00552, 0) \\
(0.215, 1) & (0.976, 0) & (0.0411, 0) \\
(0.00344, 0) & (0.0413, 1) & (0.999, 0)
\end{pmatrix}. \quad (19)$$

Note its similarity to $D_L$ (including phase differences). This corresponds to the angles $\theta_{12} = 0.217, \theta_{23} = 0.0411, \theta_{13} = 0.00552, \delta_{13} = 0. \quad (20)$

The angles $\theta_{ij}$ are in good agreement with those in the Particle Data Group’s book [11]. We will see in Section 6 that, even though $\delta_{13} = 0$, the CP–violating angles in $D_{L,R}$ can easily account for the measured value of $\epsilon$.

Model 2:

The second model is based on a $W$–matrix whose phases are multiples of $\pi/5$. The primordial quark mass matrices renormalized at $M_{ETC}$ are again taken to be of seesaw form, but we allow off–diagonal terms $|M_{ij}| \sim \sqrt{(|M_{ii}|M_{jj})}$ (all masses refer to the ETC contribution only):

$$M_u = \begin{pmatrix}
(7, 0.2) & (2, -0.4) & (0, 0) \\
(100, 0.4) & (890, -0.2) & (0, 0) \\
(50, -0.4) & (500, 0.2) & (160000, 0)
\end{pmatrix}.$$ \quad (21)

$$M_d = \begin{pmatrix}
(8, 0) & (1, -0.2) & (0, 0) \\
(25, -0.2) & (100, -0.4) & (0, 0) \\
(10, 0) & (140, -0.4) & (3500, 0.4)
\end{pmatrix}.$$ \quad (21)

Here, we have made $\text{arg det}(M_u) = \text{arg det}(M_d) = 0$. We again imposed the same kind of structure on $M_u$ as $\bar{B}_d$–$B_d$ mixing requires of $M_d$. The quark mass eigenvalues are (in MeV):

$$m_u = 6.84, \quad m_c = 896, \quad m_t = 160000$$

$$m_d = 7.52, \quad m_s = 103, \quad m_b = 3503 \quad (22)$$

The alignment matrices $U = U_L^T U_R$ and $D = D_L^T D_R$ obtained by minimizing $E_{Tq}$ are

$$U = \begin{pmatrix}
(0.994, -0.2) & (0.110, -0.4) & (0.0031, 0.4) \\
(0.110, -0.6) & (0.994, 0.2) & (0.00311, -0.2) \\
(0.00062, 0.505) & (0.00306, -0.6) & (1, 0)
\end{pmatrix}.$$
The FCNC effects that concern us arise from four-quark interactions induced by the exchange of heavy ETC gauge bosons and of TC2 color-octet “colorons” $V_8$ and color-singlet $Z'$. Lepton interactions are not dealt with here.

At low energies and to lowest order in $\alpha_{ETC}$, the ETC interaction involves products of chiral currents. Still assuming that the ETC gauge group commutes

\[ D = \begin{pmatrix} (0.976, 0) & (0.217, 0.2) & (0.00265, -0.0178) \\ (0.217, -0.8) & (0.975, 0.4) & (0.0389, 0.4) \\ (0.00664, -0.679) & (0.0384, 0.603) & (0.999, -0.4) \end{pmatrix}. \]

Again, the cloning of the $M_{u,d}$ phases onto the large elements of $U, D$ is apparent. The $Q_{L,R}$ are:

\[ U_L = \begin{pmatrix} (0.994, 0.873) & (0.112, 0.336) & (0.00031, 0.535) \\ (0.112, 0.472) & (0.994, 0.936) & (0.00313, -0.0652) \\ (0.0063, -0.422) & (0.00308, 0.138) & (1, 0.135) \end{pmatrix} \]

\[ U_R = \begin{pmatrix} (1, 1.073) & (0.00198, 0.534) & (0, 0) \\ (0.00198, 0.274) & (1, 0.736) & (1.8 \times 10^{-5}, -0.322) \\ (0, 0) & (1.8 \times 10^{-5}, 0.192) & (1, 0.135) \end{pmatrix} \]

\[ D_L = \begin{pmatrix} (0.970, 0.881) & (0.245, 0.727) & (0.00286, 0.535) \\ (0.245, 0.0810) & (0.969, 0.927) & (0.0400, 0.936) \\ (0.00771, 0.213) & (0.0394, 1.131) & (0.999, 0.135) \end{pmatrix} \]

\[ D_R = \begin{pmatrix} (1, 0.881) & (0.0284, 0.727) & (1.7 \times 10^{-5}, 0.661) \\ (0.0284, -0.319) & (1, 0.527) & (0.00116, 0.531) \\ (1.7 \times 10^{-5}, 0.611) & (0.00116, -0.470) & (1, 0.535) \end{pmatrix}. \]

The CKM matrix is (compare it to $D_L$)

\[ V = \begin{pmatrix} (0.972, 0) & (0.234, 0) & (0.00315, 0.305) \\ (0.233, 0.9999) & (0.971, 8.6 \times 10^{-6}) & (0.0431, 0) \\ (0.00867, 0.0930) & (0.0423, 0.995) & (0.999, 0) \end{pmatrix}. \]

This corresponds to the angles

\[ \theta_{12} = 0.236, \quad \theta_{23} = 0.0431, \quad \theta_{13} = 0.00315, \quad \delta_{13} = -0.957. \]

Again, the angles $\theta_{ij}$ are in reasonable agreement with those in the Particle Data Group’s book. In this model, $\delta_{13}$ is large.

5. ETC and TC2 Four–Fermion Interactions

The FCNC effects that concern us arise from four-quark interactions induced by the exchange of heavy ETC gauge bosons and of TC2 color-octet “colorons” $V_8$ and color-singlet $Z'$. Lepton interactions are not dealt with here.

At low energies and to lowest order in $\alpha_{ETC}$, the ETC interaction involves products of chiral currents. Still assuming that the ETC gauge group commutes
with electroweak $SU(2)$, it has the form

$$
\mathcal{H}_{ETC} = \Lambda_{ijkl}^{LL} \left( \bar{u}'_{Li} \gamma^\mu u'_{Lj} + \bar{d}'_{Li} \gamma^\mu d'_{Lj} \right) \left( \bar{u}'_{Lk} \gamma^\mu u'_{Ll} + \bar{d}'_{Lk} \gamma^\mu d'_{Ll} \right) 
+ \left( \bar{u}'_{Li} \gamma^\mu u'_{Lj} + \bar{d}'_{Li} \gamma^\mu d'_{Lj} \right) \left( \Lambda_{ijkl}^{uLR} \bar{u}'_{Rk} \gamma^\mu u'_{Rl} + \Lambda_{ijkl}^{dLR} \bar{d}'_{Rk} \gamma^\mu d'_{Rl} \right) 
+ \sum_{i,j} \Lambda_{ijkl}^{uR} \bar{u}'_{Ri} \gamma^\mu u'_{Rj} + \Lambda_{ijkl}^{dR} \bar{d}'_{Ri} \gamma^\mu d'_{Rj} 
+ \Lambda_{ijkl}^{uR} \bar{u}'_{Ri} \gamma^\mu u'_{Rj} \bar{d}'_{Rk} \gamma^\mu d'_{Rl},
$$

(27)

where primed fields are electroweak eigenstates. The ETC gauge group contains technicolor, color and topcolor singlets. The $\Lambda$’s in $\mathcal{H}_{ETC}$ are of order $g_{ETC}^2 M_{ETC}^2$, whose magnitude is discussed below, and the operators are renormalized at $M_{ETC}$. Hermiticity of $\mathcal{H}_{ETC}$ implies that $\Lambda_{ijkl} = \Lambda_{jikl}^\dagger$. We assume that this primordial ETC interaction conserves CP, i.e., that all the $\Lambda$’s are real. When written in terms of mass eigenstate fields $q_{L,R;i} = \sum_j (Q_{L,R;j}^i \bar{q}_{L,R})$ with $Q = U, D$, an individual four–quark term in $\mathcal{H}_{ETC}$ has the form

$$
\left( \sum_{i,j,k,l} \Lambda_{ijkl}^{q_1q_2q_3q_4} Q_{\lambda_1}^{i} Q_{\lambda_2}^{j} Q_{\lambda_2}^{k} \bar{q}_{\lambda_2}^{l} \bar{q}_{\lambda_1}^{l} \right) \bar{q}_{\lambda_1}^i \gamma^\mu q_{\lambda_2}^j \bar{q}_{\lambda_2}^k \gamma^\mu q_{\lambda_2}^l.
$$

(28)

A reasonable and time–honored guess for the magnitude of the $\Lambda_{ijkl}$ is that they are comparable to the ETC masses that generate the quark mass matrix $M_q$. We elevate this to a rule: The ETC scale $M_{ETC}/g_{ETC}$ in a term involving weak eigenstates of the form $\bar{q}_i' q_j' q'_k q'_l$ or $\bar{q}_i' q'_k q'_j q'_l$ (for $q_i' = u_i'$ or $d_i'$) is approximately the same as the scale that generates the $\bar{q}_R' q_L'$ mass term, $(M_q)_{ij}$. A plausible, but approximate, scheme for correlating a quark mass $m_q(M_{ETC})$ with $M_{ETC}/g_{ETC}$ is presented in the Appendix. The results are shown in Fig. 1. There, $\kappa > 1$ parameterizes the departure from the strict walking technicolor limit; i.e., $\alpha_{TC} = \text{constant}$ and the anomalous dimension $\gamma_m$ of $\bar{T}T$ equals one up to the highest ETC mass scale divided by $\kappa$, and $\gamma_m = 0$ beyond that. The ETC masses run from $M_{ETC}/g_{ETC} = 46 \text{ TeV}$ for $m_q = 5 \text{ GeV}$ to $2.34/\kappa \times 10^4 \text{ TeV}$ for $m_q = 10 \text{ MeV}$. We rely on Fig. 1 for estimating the $\Lambda$’s in $\mathcal{H}_{ETC}$.

Extended technicolor masses, $M_{ETC}/g_{ETC} \gtrsim 1000 \text{ TeV}$, are necessary, but not sufficient, to suppress FCNC interactions of light quarks to an acceptable level. This is especially true for $\Delta M^0_K$ and $\epsilon$ [3, 4]. Thus, we assume that $\mathcal{H}_{ETC}$ is electroweak generation conserving, i.e.,

$$
\Lambda_{ijkl}^{q_1q_2q_3q_4} = \delta_{il} \delta_{jk} \Lambda_{ij}^{q_1q_2q_3q_4} + \delta_{ij} \delta_{kl} \Lambda_{ik}^{q_1q_2q_3q_4}.
$$

(29)

Considerable FCNC suppression then comes from off–diagonal elements in the alignment matrices $Q_{L,R}$. 
Figure 1: Extended technicolor scale $M_{ETC}/g_{ETC}$ as a function of quark mass $m_q$ renormalized at $M_{ETC}$ for $\kappa = 1$ (solid curve), $\sqrt{10}$ (dashed), and $10$ (solid); see the Appendix for details.

In all TC2 models, color $SU(3)_C$ and weak hypercharge $U(1)_Y$ arise from the breakdown of the topcolor groups $SU(3)_1 \otimes SU(3)_2$ and $U(1)_1 \otimes U(1)_2$ to their diagonal subgroups. Here, $SU(3)_1$ and $U(1)_1$ are strongly coupled, $SU(3)_2$ and $U(1)_2$ are weakly coupled, with the color and weak hypercharge couplings given by

$$g_C = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \equiv \frac{g_1 g_2}{g_V} \equiv g_2 \cos \theta_C \simeq g_2;$$

$$g_Y = \frac{g'_1 g'_2}{\sqrt{g'_1^2 + g'_2^2}} \equiv \frac{g'_1 g'_2}{g_Y} \equiv g'_2 \cos \theta_Y \simeq g'_2. \quad (30)$$

Top and bottom quarks are $SU(3)_1$ triplets. The broken topcolor interactions are mediated by a color octet of colorons, $V_8$, and a color singlet $Z'$ boson, respectively. By virtue of the different $U(1)_1$ couplings of $t_R$ and $b_R$, exchange of $V_8$ and $Z'$ between third generation quarks generates a large contribution, $(m_t)_{TC2} \simeq 160 \text{ GeV}$, to the top mass, but none to the bottom mass.

There are two variants of TC2: The "standard" version [10], in which only the third generation quarks are $SU(3)_1$ triplets, and the "flavor–universal"
version [17], in which all quarks are $SU(3)_1$ triplets. In standard TC2, $V_8$ and $Z'$ exchange gives rise to FCNC that mediate $|\Delta S| = 2$ and $|\Delta B| = 2$. In flavor-universal TC2, only $Z'$ exchange generates such FCNC. We shall write the four-quark interaction for standard TC2, but our results apply to $Z'$ exchange interactions in flavor-universal TC2 as well.

The TC2 interaction at energies well below $M_{V_8}$ and $M_{Z'}$ is

$$\mathcal{H}_{TC2} = \frac{g_{V_8}^2}{2M_{V_8}^2} \sum_{A=1}^{8} J^A_{\mu} J^A_{\mu} + \frac{g_{Z'}^2}{2M_{Z'}^2} J_{Z'}^{\mu} J_{Z'}^{\mu}. \quad (31)$$

The coloron and $Z'$ currents written in terms of electroweak eigenstate fields are given by (color indices are suppressed)

$$J^A_{\mu} = \cos^2 \theta_C \sum_{i=t,b} \bar{q}_i \gamma_{\mu} \frac{\lambda_A}{2} q'_i - \sin^2 \theta_C \sum_{i=u,d,c,s} \bar{q}_i \gamma_{\mu} \frac{\lambda_A}{2} q'_i;$$

$$J_{Z'}^{\mu} = \cos^2 \theta_Y J_{1\mu} - \sin^2 \theta_Y J_{2\mu} \equiv \sum_{\lambda=L,R} \sum_i \left( \cos^2 \theta_Y Y_{1\lambda i} - \sin^2 \theta_Y Y_{2\lambda i} \right) \bar{q}_{\lambda i} \gamma_{\mu} q'_{\lambda i}. \quad (32)$$

The $U(1)_1$ and $U(1)_2$ hypercharges satisfy $Y_{1\lambda i} + Y_{2\lambda i} = Y_{\lambda i} = 1/6, Q_{EM}$ for $\lambda = L, R$. Consistency with $SU(2)$ symmetry requires $Y_{Lt} = Y_{Lb}$, etc. The suppression of light quark FCNC requires $Y_{1Li} \equiv Y_{1i}$ for $i = u, d, c, s$ and $Y_{1Ru} = Y_{1Rc}, Y_{1Rd} = Y_{1Rs}$. Remaining FCNC are suppressed by small mixing angles.

6. TC2 Constraints from $B_d - \bar{B}_d$ Mixing

If topcolor is to provide a natural explanation of $(m_t)_{TC2}$, the $V_8$ and $Z'$ masses ought to be $O(1 \text{ TeV})$. In the Nambu–Jona-Lasinio (NJL) approximation, the degree to which this naturalness criterion is met is quantified by the ratio [18]

$$\frac{\alpha(V_8) + \alpha(Z') - (\alpha^*(V_8) + \alpha^*(Z'))}{\alpha^*(V_8) + \alpha^*(Z')} = \frac{\alpha(V_8) r_{V_8} + \alpha(Z') r_{Z'}}{\alpha(V_8)(1 - r_{V_8}) + \alpha(Z')(1 - r_{Z'})}. \quad (33)$$

Here,

$$\alpha(V_8) = \frac{4\alpha_1 \cos^4 \theta_C}{3\pi} = \frac{4\alpha_C \cot^2 \theta_C}{3\pi},$$

$$\alpha(Z') = \frac{\alpha_Z Y_{Ll} Y_{Rb} \cos^4 \theta_Y}{\pi} = \frac{\alpha_Y Y_{Ll} Y_{Rb} \cot^2 \theta_Y}{\pi}; \quad (34)$$

$$\tan \theta_C = \frac{g_2}{g_1}, \quad \tan \theta_Y = \frac{g_2'}{g_1'}, \quad r_i = \frac{(m_i^2)_{TC2}}{M_i^2} \ln \left( \frac{M_i^2}{(m_i^2)_{TC2}} \right), \quad (i = V_8, Z').$$
and $Y_{t_{L,R}}$ are the $U(1)_1$ charges of $t_{L,R}$. The NJL condition on the critical couplings for top condensation is $\alpha^*(V_8) + \alpha^*(Z') = 1$.

In Ref. [14] we showed that, for such large couplings, TC2 is tightly constrained by the magnitude of $B_d - B_s$ mixing. The relevant observable is the $B^0_L - B^0_S$ mass difference. Since $\Gamma_{12} \ll M_{12}$, it is given by $\Delta M_{B_d} = 2|\Delta M_{12}|$. (See Ref. [19] for the standard model description of meson mixing and definitions of the constants and the functions $S_0$ used below.) The dominant contributions to $M_{12}$ come from $\bar{b}'b'$ terms in Eq. (31). After Fierzing the product of color–octet currents into a product of singlets, they give

$$2(M_{12})_{TC2} = \frac{4\pi}{3} \left[ \frac{\alpha C \cot^2 \theta_C}{3M^2_{V_8}} + \frac{\alpha Y \cot^2 \theta_Y (\Delta Y_L)^2}{M^2_{Z'}} \right] \eta_B M_{B_d} f^2_{B_d} B_{B_d} (D^*_{Lbb} D_{Lbd})^2.$$

(35)

Here, $\Delta Y_L = Y_{bl} - Y_{dl} = Y_{bl} - Y_{sl}$ is a difference of strong $U(1)_1$ hypercharges. We take $\alpha C \cot^2 \theta_C = \alpha Y (\Delta Y_L)^2 \cot^2 \theta_Y = 3\pi/8$ so that their sum is NJL–critical if $(\Delta Y_L)^2 \simeq Y_{tL}Y_{tR}$. The QCD radiative correction factor for the LL product of color–singlet currents is $\eta_B = 0.55 \pm 0.01$. We use $f_{B_d} \sqrt{B_{B_d}} = (200 \pm 40)$ MeV, where $f_{B_d}$ and $B_{B_d}$ are, respectively, the $B_d$–meson decay constant and bag parameter. This TC2 contribution is to be added to the standard model one,

$$2(M_{12})_{SM} = \frac{G^2_F}{6\pi^2} \eta_B M_{B_d} f^2_{B_d} B_{B_d} M^2_W S_0(x_t) (V^*_{td} V_{td})^2,$$

(36)

where the top–quark loop function $S_0(x_t) \simeq 2.46$ for $x_t = m_t^2(m_t)/M^2_W$ and $m_t(m_t) = 170$ GeV.

We argued in Section 4 that the quark–mixing factors in these two contributions are nearly the same in ETC/TC2 models; see Eq. (13). Thus, the two add without ambiguity and can be used to restrict TC2 parameters. So long as ETC and TC2 do not contribute significantly to the direct CP–violating parameter $\epsilon'$ [4], we can use its measurement to limit $|V_{td}|$. Using $\Delta M_{B_d} = (3.11 \pm 0.11) \times 10^{-13}$ GeV [11], we found the limits

$$M_{V_8} \simeq M'_{Z} \gtrsim 5 \text{ TeV}.$$  \hspace{1cm} (37)

From Eq. (33), these limits imply that the topcolor coupling $\alpha(V_8) + \alpha(Z')$ must be within less than 1% of its critical value. We regard this tuning to be unnaturally fine. \footnote{11} One way to eliminate this fine–tuning problem is to invoke the “top seesaw” mechanism in which the topcolor interactions operate on a quark whose mass is several TeV, and the top’s mass comes to it by a seesaw mechanism [21].

\footnote{11}We are studying this presumption and, tentatively, believe it is correct.

\footnote{12}Other limits on $M_{V_8}$ were obtained in Refs. [20].
7. ETC and TC2 Contributions to the CP–Violating Parameter $\epsilon$

The CP–violating parameter $\epsilon$ is defined by

$$\epsilon \equiv \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = \frac{e^{i\pi/4} \text{Im} M_{12}}{\sqrt{2} \Delta M_K},$$

(38)

where $2M_K M_{12} = \langle K^0|\mathcal{H}|\Delta S=2|K^0 \rangle$ and we use the phase convention that $A_0 = \langle (\pi\pi)_{I=0}|\mathcal{H}|\Delta S=1|K^0 \rangle$ is real. Experimentally, $\epsilon = (2.271\pm0.017) \times 10^{-3} \exp(i\pi/4)$ [1].

The standard model contribution to $\epsilon$ is

$$\epsilon_{SM} = \frac{e^{i\pi/4} G_F^2 M_W^2 f_K^2 \hat{B}_K M_K}{3\sqrt{2}\pi^2 \Delta M_K} \text{Im} \left[ \lambda^*_c \eta_1 S_0(x_c) + \lambda^*_t \eta_2 S_0(x_t) + 2\lambda^*_c \lambda^*_t \eta_3 S_0(x_c, x_t) \right],$$

(39)

where $f_K = 112$ MeV is the kaon decay constant, $\hat{B}_K = 0.80 \pm 0.15$ is the kaon bag parameter, $\lambda_{i=c,t} = V_{id}^* V_{is}^*$, $S_0(x_t) \simeq 2.46$, $S_0(x_c) \simeq 1.55 \times 10^{-4}$, and $S_0(x_c, x_t) = 1.40 \times 10^{-3}$ [13].

Despite the large ETC gauge boson masses of several 1000 TeV and the stringent $B_d$–$B_d$ mixing constraint leading to TC2 gauge masses of at least 5 TeV, both interactions can contribute significantly to $\epsilon$. The main ETC contribution comes from $\bar{s'}s'\bar{s}''s''$ interactions and is given by

$$\epsilon_{ETC} \simeq \frac{e^{i\pi/4} f_K^2 M_K \hat{B}_K}{3\sqrt{2} \Delta M_K} \left\{- \left[ \frac{M_K}{m_s + m_u} \right]^2 + \frac{3}{2} \right\} \Lambda_{ss}^{LR} \text{Im} (D_{Lss}^* D_{Lss} D_{Rss}^* D_{Rss})$$

$$+ 2 \left[ \Lambda_{ss}^{LL} \text{Im} (D_{Lss}^2 D_{Lss}^*) + \Lambda_{ss}^{RR} \text{Im} (D_{Rss}^2 D_{Rss}^*) \right].$$

(40)

Note the suppression of $O((\theta_{12})^2)$ from mixing angle factors. This $\bar{s'}s'\bar{s}''s''$ contribution as well as those from the standard model and TC2 vanish for Model 1.

For that model, $\text{Im}(M_{12})_{ETC}$ comes from $\bar{s}'d\bar{d}'s'$ terms and has a form similar to Eq. (10).

The dominant TC2 contribution comes from $\bar{b}'_L b'_L \bar{b}'_L b'_L$ interactions; terms involving $b'_R$ are suppressed by the very small $D_{Rbd}$ and $D_{Rbs}$:

$$\epsilon_{TC2} \simeq \frac{e^{i\pi/4} 4\pi f_K^2 M_K \hat{B}_K}{3\sqrt{2} \Delta M_K} \left[ \frac{\alpha_C \cot^2 \theta_C}{M_{V_{ts}}^2} + \frac{\alpha_Y (\Delta Y_L)^2 \cot^2 \theta_Y}{M_{Z'}^2} \right] \text{Im} (D_{Lbs}^2 D_{Lbs}^*).$$

(41)

The couplings and mixing angles were defined in Eq. (31); we again take $\alpha_C \cot^2 \theta_C = \alpha_Y (\Delta Y_L)^2 \cot^2 \theta_Y = 3\pi/8$ [15].

The various contributions to $\epsilon$ for several different “models” of the primordial quark mass matrix $M_q$ are given in Table 1. Models 1 and 2 are present as are
Table 1: Contributions to $\epsilon e^{-i\pi/4} \times 10^3$ for various “models” of $\mathcal{M}_q$ with $\bar{\theta}_q = 0$. Unless otherwise indicated, all $\Lambda_{ss} = (2000 \text{ TeV})^{-2}$ and $\Lambda_{qs} = (2000 \text{ TeV})^{-2}$.

| Model | SM | (ETC)$_{\text{LR}}$ | (ETC)$_{\text{LL}}$ | (ETC)$_{\text{RR}}$ | (TC2)$_{\text{LL}}$ | Comments |
|-------|----|---------------------|---------------------|---------------------|---------------------|----------|
| 1     | 0  | 2.38                | 0                   | 0                   | 0                   | Fit for $\Lambda_{sd} = (4000 \text{ TeV})^{-2}$ |
| 1’    | 2.28 | 9.61               | 0.88                | 1.02                | 8.34                | Fit for $M_{\text{ETC}} \to \infty$, $M_{V_{q,z'}} \to \infty$ |
| 2     | -1.98 | 9.44              | -7.68               | -0.11               | -4.57               | $\epsilon_{\text{ETC+TC2}} = 4.22$ for $\Lambda_{ss} = (1250 \text{ TeV})^{-2}$, $M_{V_{q,z'}} \to \infty$ |
| 2’    | 1.97 | -6.30              | 7.76                | 0.05                | 4.52                | Approximate fit for $M_{\text{ETC}} \to \infty$, $M_{V_{q,z'}} \to \infty$ |
| 2”    | -2.02 | 31.25             | -7.92               | -1.21               | -4.68               | $\epsilon_{\text{ETC+TC2}} = 4.33$ for $\Lambda_{ss} = (2500 \text{ TeV})^{-2}$, $M_{V_{q,z'}} = 6.9 \text{ TeV}$ |
| 3     | 2.18 | -8.94              | -0.97               | -0.80               | 8.20                | $\epsilon_{\text{ETC+TC2}} = 0.10$ for $M_{V_{q,z'}} = 8.7 \text{ TeV}$ |

some related ones (e.g., model 2’ is similar to model 2, but the complex conjugate input $\mathcal{M}_q$ is used; differences apart from signs are due to computer round–off error). As indicated, typical ETC masses from Fig. 1 are used for $\Lambda_{ss}$ and $\Lambda_{sd}$ (the latter for model 1 only). Note that model 1 accounts very well for $\epsilon$ from ETC interactions alone. ETC interactions that lead to models 1’ and 2’ are ruled out. In the other models, $\epsilon$ is easily accounted for because large cancellations occur among the ETC contributions or between ETC and TC2 contributions. These would be disturbing if we had not already seen them in a standard context: the large cancellations between QCD and electroweak penguin terms in the calculation of $\epsilon'/\epsilon$ [19].

8. Summary and Conclusions

We have presented a dynamical picture of CP nonconservation arising from vacuum alignment in extended technicolor theories. This picture leads naturally to a mechanism for evading strong CP violation without an axion or a massless up quark. We derived complex quark mixing matrices from ETC/TC2–based constraints on the primordial mass matrices $\mathcal{M}_u$ and $\mathcal{M}_d$. These led to very realistic–looking CKM matrices. We categorized 4–quark contact interactions arising from ETC and TC2...
and proposed a scheme for estimating the strengths of these interactions. Putting this together with the quark mixing matrices, we showed that TC2 couplings must be tuned to within 1% of their NJL critical values to avoid conflict with $B_d - \bar{B}_d$ mixing. We also calculated contributions to the CP–violating parameter $\epsilon$, obtaining quite good (or powerfully constraining) results for a variety of “models” of $M_q$. Future work will include calculating $\epsilon'/\epsilon$ and $\sin(2\beta)$ in these models.

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**Appendix. ETC Gauge Boson Mass Scales**

To set the ETC mass scales that enter $H_{ETC}$ in Eq. (27), we assume a model containing $N$ identical electroweak doublets of technifermions. The technipion decay constant (which helps set the technicolor energy scale) is then $F_T = F_{\pi}/\sqrt{N}$, where $F_{\pi} = 246$ GeV is the fundamental weak scale. We estimate the ETC masses in $H_{ETC}$ by the rule stated in Section 5: The ETC scale $M_{ETC}/g_{ETC}$ in a term involving weak eigenstates of the form $\bar{q}_i'q_j'\bar{q}_i'q_j'$ (for $q_i' = u_i'$ or $d_i'$) is approximately the same as the scale that generates the $\bar{q}_R^i q_L^j$ mass term, $(M_q)_{ij}$.

The ETC gauge boson mass $M_{ETC}(q)$ giving rise to a quark mass $m_q(M_{ETC})$—an element or eigenvalue of $M_q$—is defined by

$$m_q(M_{ETC}) \simeq \frac{g_{ETC}^2}{M_{ETC}^2(q)} \langle \bar{T}T \rangle_{ETC}.$$  \hspace{1cm} (42)

Here, the quark mass and the technifermion bilinear condensate, $\langle \bar{T}T \rangle_{ETC}$, are renormalized at the scale $M_{ETC}(q)$. The condensate is related to the one renormalized at the technicolor scale $\Lambda_{TC} \simeq F_T$ by the equation

$$\langle \bar{T}T \rangle_{ETC} = \langle \bar{T}T \rangle_{TC} \exp \left( \int_{\Lambda_{TC}}^{M_{ETC}(q)} \frac{d\mu}{\mu} \gamma_c(\mu) \right).$$  \hspace{1cm} (43)

Scaling from QCD, we expect

$$\langle \bar{T}T \rangle_{TC} \equiv \Delta_T \simeq 4\pi F_{\pi}^3 = 4\pi F_{\pi}^3 / N^{3/2}.$$  \hspace{1cm} (44)
The anomalous dimension $\gamma_m$ of the operator $\bar{T}T$ is given in perturbation theory by

$$\gamma_m(\mu) = \frac{3C_2(R)}{2\pi} \alpha_{TC}(\mu) + O(\alpha_{TC}^3), \tag{45}$$

where $C_2(R)$ is the quadratic Casimir of the technifermion $SU(N_{TC})$ representation $R$. For the fundamental representation of $SU(N_{TC})$ to which we assume our technifermions $T$ belong, it is $C_2(N_{TC}) = (N_{TC}^2 - 1)/2N_{TC}$. In a walking technicolor theory, however, the coupling $\alpha_{TC}(\mu)$ decreases very slowly from its critical chiral symmetry breaking value at $\Lambda_{TC}$, and $\gamma_m(\mu) \simeq 1$ for $\Lambda_{TC} < \mu < M_{ETC}$.

An accurate evaluation of the condensate enhancement integral in Eq. (43) requires detailed specification of the technicolor model and knowledge of the $\beta(\alpha_{TC})$--function for large coupling. Lacking this, we estimate the enhancement by assuming that

$$\gamma_m(\mu) = \begin{cases} 1 & \text{for } \Lambda_{TC} < \mu < M_{ETC}/\kappa^2 \\ 0 & \text{for } \mu > M_{ETC}/\kappa^2 \end{cases} \tag{46}$$

Here, $M_{ETC}$ is the largest ETC scale, i.e., the one generating the smallest term in the quark mass matrix for $\kappa = 1$. The number $\kappa > 1$ parameterizes the departure from the strict walking limit (i.e., $\gamma_m = 1$ constant all the way up to $M_{ETC}/\kappa^2$). Then, using Eqs. (42,43), we obtain

$$\frac{M_{ETC}(q)}{g_{ETC}} = \begin{cases} \sqrt{\frac{64\pi^3\alpha_{ETC} F_q^2}{Nm_q}} & \text{if } M_{ETC}(q) < M_{ETC}/\kappa^2 \\ \sqrt{\frac{4\pi M_{ETC}F_q^2}{\kappa^2 Nm_q}} & \text{if } M_{ETC}(q) > M_{ETC}/\kappa^2 \end{cases} \tag{47}$$

To evaluate this, we take $\alpha_{ETC} = 3/4$, a moderately strong value as would be expected in walking technicolor [22], and $N = 10$, a typical number of doublets in TC2 models with topcolor breaking [13]. Then, taking the smallest quark mass at the ETC scale to be 10 MeV, we find $M_{ETC} = 7.17 \times 10^4$ TeV. The resulting estimates of $M_{ETC}/g_{ETC}$ were plotted in Fig. 1 for $\kappa = 1, \sqrt{10},$ and 10. They run from $M_{ETC}/g_{ETC} = 46$ TeV for $m_q = 10$ GeV to $2.34/\kappa \times 10^4$ TeV/ for $m_q = 10$ MeV. Very similar results are obtained for $\alpha_{ETC} = 1/2$ and $N = 8$.

References

1. R. F. Dashen, Phys. Rev. D3, 1879 (1971); J. Nuyts, Phys. Rev. Lett. 26, 1604 (1971).

13See Ref. [22] for an attempt to calculate this integral in a walking technicolor model.
2. S. Weinberg, Phys. Rev. D19, 1277 (1979);  
   L. Susskind, Phys. Rev. D20, 2619 (1979).

3. E. Eichten and K. Lane, Phys. Lett. B90, 125 (1980).

4. For a review of modern technicolor, see K. Lane, *Technicolor 2000*, Lectures at the LNF Spring School in Nuclear, Subnuclear and Astroparticle Physics, Frascati (Rome), Italy, May 15–20, 2000; [hep-ph/0007304](https://arxiv.org/abs/hep-ph/0007304).

5. R. S. Chivukula, *Models of Electroweak Symmetry Breaking*, NATO Advanced Study Institute on Quantum Field Theory Perspective and Prospective, Les Houches, France, 16–26 June 1998, [hep-ph/9803219](https://arxiv.org/abs/hep-ph/9803219).

6. E. Eichten, K. Lane and J. Preskill, Phys. Rev. Lett. 45, 225 (1980);  
   K. Lane, Physica Scripta 23, 1005 (1981);  
   Also see J. Preskill, Nucl. Phys. B177, 21 (1981);  
   M. E. Peskin, Nucl. Phys. B175, 197 (1980).

7. For a review of the strong CP problem and the status of axion searches, see R. D. Peccei, *QCD, Strong CP and Axions*, [hep-ph/9606475](https://arxiv.org/abs/hep-ph/9606475).

8. K. Lane, T. Rador, and E. Eichten, Phys. Rev. D62, 015005 (2000); [hep-ph/0001056](https://arxiv.org/abs/hep-ph/0001056).

9. B. Holdom, Phys. Rev. D24, 1441 (1981); Phys. Lett. B150, 301 (1985);  
   T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. 57, 957 (1986);  
   T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D36, 568 (1987);  
   K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. 56, 1335 (1986);  
   T. Akiba and T. Yanagida, Phys. Lett. B169, 432 (1986).

10. C. T. Hill, Phys. Lett. 345B, 483 (1995).

11. Review of Particle Properties, D. E. Groom, *et al.*, Eur. Phys. J. C15, 1 (2000).

12. H. Harari and M. Leurer, Phys. Lett. B181, 123 (1986).

13. D. Kominis, Phys. Lett. B358, 312 (1995), [hep-ph/9506305](https://arxiv.org/abs/hep-ph/9506305).

14. G. Buchalla, G. Burdman, C. T. Hill, and D. Kominis, Phys. Rev. D53, 5185 (1996), [hep-ph/9510370](https://arxiv.org/abs/hep-ph/9510370).
15. K. Lane and E. Eichten, Phys. Lett. B352, 382 (1995);
   K. Lane, Phys. Rev. D54, 2204 (1996);
   K. Lane, Phys. Lett. B433, 96 (1998).

16. G. Burdman, K. Lane, and T. Rador, \( \bar{B} - B \) Mixing Constrains Topcolor–Assisted Technicolor, to be published in Physics Letters, hep-ph/0012073.

17. R. S. Chivukula, A. G. Cohen and E. H. Simmons, Phys. Lett. B380, 92 (1996),
    hep-ph/9603311;
    M. Popovic and E. H. Simmons, Phys. Rev. D58, 095007 (1998), hep-ph/9806287.

18. R. S. Chivukula, B. A. Dobrescu and J. Terning, Phys. Lett. B353, 289 (1995),
    hep-ph/9503203.

19. A. J. Buras and R. Fleischer, Heavy Flavors II, World Scientific, A. J. Buras and
   M. Lindner, eds. (1997), hep-ph/9704376.

20. Earlier, somewhat lower limits on \( M_{V_s} \) and \( M_{Z'} \) were obtained in Ref. [18], in
    R. S. Chivukula and J. Terning, Phys. Lett. B385, 209 (1996), hep-ph/9606233,
    and in I. Bertram and E. H. Simmons, Phys. Lett. B443, 347 (1998), hep-ph/9809472. The first contains a model–independent bound from the \( t-b \) loop contributions to the \( \rho \)–parameter. The latter two are based on fits of standard
    TC2 to precision electroweak data and of flavor–universal TC2 to Tevatron jet
    data, respectively.

21. B. A. Dobrescu and C. T. Hill, Phs. Rev. Lett. 81, 2634 (1998), hep-ph/9712319;
    R. S. Chivukula, B. A. Dobrescu, H. M. Georgi, and C. T. Hill, Phys. Rev. D59,
    075003 (1999), hep-ph/9809470.
    G. Burdman and N. Evans, Phys. Rev. D59, 115005 (1999), hep-ph/9811357.

22. K. Lane and M. V. Ramana, Phys. Rev. D44, 2678 (1991).