Abstract

We introduce an exact distributed algorithm to train Random Forest models as well as other decision forest models without relying on approximating best split search. We explain the proposed algorithm and compare it to related approaches for various complexity measures (time, ram, disk, and network complexity analysis). We report its running performances on artificial and real-world datasets of up to 18 billions examples. This figure is several orders of magnitude larger than datasets tackled in the existing literature. Finally, we empirically show that Random Forest benefits from being trained on more data, even in the case of already gigantic datasets. Given a dataset with 17.3B examples with 82 features (3 numerical, other categorical with high arity), our implementation trains a tree in 22h.

1. Introduction

Two families of approaches have been studied and sometimes combined to tackle the problem of training Decision Trees (DT) and Decision Forests (DF) on large datasets: (i) Approximating the building of the tree by using a subset of the dataset and/or approximating the computation of the optimal splits with a cheaper or more easily distributable computation, and (ii) using different but exact algorithms (building the same models) that allow distributing the dataset and the computation. Various works (Chan & Stolfo, 1993; Mehta et al., 1996) have shown that (i) typically leads to bigger forests and lower precision. We focus on the latter family of approaches: we propose a distributed method which is exactly equivalent to the original DT algorithm. We compare our work to the two existing methods that fall in this category: Sprint (Shafer et al., 1996) and distributed versions of Sliq (Mehta et al., 1996). Our proposed method, inspired by Sliq, aims to reach: (1) Removal of the random access memory requirement. (2) Distributed training (distribution even of a single tree). (3) Distribution of the training dataset (i.e. no worker requires access to the entire dataset). (4) Minimal number of passes in terms of reading/writing on disk and network communication. (5) Distributed computing of feature importance. While this paper mainly focuses on Random Forests, the proposed algorithm can be applied to other DF models, notably Gradient Boosted Trees (Ye et al., 2009). Our contributions in this work are as follows: (1) A distributed and exact implementation of Random Forest able to train on datasets larger than in any such past work. (2) A theoretical and numerical complexity comparison (CPU, RAM, network, disk access, disk reading, disk writing) of Sliq, Sprint, RF and our distributed version of Random Forest.

2. Distributed Random Forest

In this section, we describe the proposed Distributed Random Forest algorithm (DRF). The structure of this algorithm is different from the classical recursive Random Forest algorithm; nonetheless, as well as Sliq (Mehta et al., 1996) and Sprint (Shafer et al., 1996), the proposed algorithm is guaranteed to produce the same model as RF. DRF computation is distributed among computing units called “workers”, and coordinated by a “manager”. The manager and the workers communicate through a network. DRF is relatively insensitive to the latency of communication (Section 3). DRF also distributes the dataset between workers: each worker is assigned to a subset of columns of the dataset. Each worker only needs to read their assigned part of the dataset sequentially, i.e. no random access and no writing are needed. Workers can be configured to load the dataset in memory, or to access the dataset on drive/through network access. Finally, each worker can host a certain number of threads (details of multithreading in the supplementary material (SM)). Several types of workers are responsible for different operations. The splitter workers look for optimal candidate splits. Each splitter has access to a subset of dataset columns. The tree builder workers hold the structure of one DT being trained (one DT per tree builder) and coordinate the work of the splitters. Tree builders do not have access to the dataset. One tree builder can control several splitters, and one splitter can be controlled by several tree builders. The manager manages the tree builders.
The manager is responsible for the fully trained trees. The manager does not have access to the dataset. Like Sliq, and unlike the generic DT learning algorithm, DRF builds DTs “depth level by depth level” i.e. all the nodes at a given depth are split together. The training of a single tree is distributed among the workers. Additionally, as trees of a Random Forest are independent, DRF trains all the trees in parallel. DRF can also be used to train co-dependent sets of trees (e.g. Boosted Decision Trees). In this case, while trees cannot be trained in parallel, the training of each individual tree is still distributed.

2.1. Dataset Preparation

Consistently with existing works (Mehta et al., 1996; Shafer et al., 1996), we use presorting for numerical attributes. In the present work we do not consider other categories of attributes than categorical or numerical. The most expensive operation when preparing the dataset is the sorting of the numerical attributes. In case of large datasets, this operation is done using external sorting. In this phase, the manager distributes the dataset among the splitters. Each splitter is assigned with a subset of the dataset columns. In case several DTs are trained in parallel (e.g. RF), DRF benefits from having workers replicated i.e. several workers own the same part of the dataset and are able to perform the same computation.

2.2. Seeding

RF “bags” samples (i.e. sampling with replacement, n out of n records) used to build each tree. Instead of sending indices over the network, DRF uses a deterministic pseudorandom generator so that all workers agree on the set of bagged examples without network communication. With this method, all workers are aware of the selected samples, without the cost of transmitting or storing this information.

2.3. Mapping Sample Indices to Node Indices

At any point during training, each bagged sample is attached to a single leaf - initially the root node. When a leaf is derived into two children, each sample of this node is re-assigned to one of its child nodes according to the result of the node condition (condition = chosen split). As in Sliq (Mehta et al., 1996), DRF splitters and tree builders need to represent the mapping from a sample index to a leaf node. DRF monitors the number ℓ of active leaves (i.e. number of leaf nodes which can be further split). Therefore, \(\left\lceil \log_2 \ell \right\rceil\) bits of information are needed to index a leaf. If there is at least one non-active leaf, \(\lfloor \log_2 (\ell + 1) \rfloor\) bits are needed to encore the case of a sample being in a closed leaf. Therefore, this mapping requires \(n \left\lceil \log_2 (\ell + 1) \right\rceil\) bits of memory to store in which leaf each sample is. Depending on the size of the dataset, this mapping can either be stored entirely in memory, or the mapping can be distributed among several chunks such that only one chunk is in memory at any time. Unlike Sliq (Mehta et al., 1996), DRF does not store the label values in memory.

2.4. Finding the Best Split

During training, each splitter is searching for the optimal split among the candidate attributes it owns. The final optimal split is the best optimal split (e.g. for information gain or Gini index) among all the splitters. A split is defined as a column index \(j\) and a condition over the values of this column. For numerical columns, the condition is of the form \(x_{i,j} \leq \tau\) with \(\tau \in \mathbb{R}\). For categorical columns, the condition is of the form \(x_{i,j} \in C\) with \(C \subseteq 2^{S_j}\) and \(S_j\) the support of column \(j\). In case of attribute sampling (e.g. RF), only a random subset of attributes are considered. We call \(\text{supersplit}\) a set of splits mapped one-to-one with the open leaves at a given depth of a tree. The following subsections present how DRF computes the optimal splits for all the nodes at a given depth, i.e. the \(\text{optimal supersplit}\) at a given depth, in a single pass per feature. Computing optimal splits on categorical attributes is easily parallelized, whereas computing optimal splits in the case of numerical attributes needs presorting. Details are given in the SM for all cases, and Alg. 1 presents the algorithm for finding the optimal splits for a given numerical feature for all nodes of a given depth in one pass.

2.5. Training a Random Forest

Each decision tree is built by the tree builder with Alg. 2. To train a Random Forest, the manager queries in parallel the tree builders. This query contains the index...
of the requested tree (the tree index is used in the seeding, Section 2.2) as well as a list of splitters such that each column of the dataset is owned by at least one splitter. The answer by the tree builder is the decision tree.

3. Complexity Analysis

We present and compare the theoretical complexities (memory, parallel time, I/O and network) of generic DT, generic RF, DRF, Sprint, Sliq, Sliq/R and Sliq/D. All these algorithms operate differently, and benefit from different situations in term of time complexity: Sprint prunes records in closed leaves: a tree with a large amount of records in shallow closed leaves is fast to train. However, Sprint scans and writes continuously both the candidate and non-candidate features i.e. Sprint does not benefit from the small size of the set of candidate features. Compared to Sprint, DRF benefits from records being in closed leaves differently: records in closed leaves are not pruned, but since Sliq and DRF only scan candidate features (i.e. features randomly chosen and not closed in earlier conditions), a smaller number of records leads to a smaller number of candidate features. Although our experiments focus on the classical case of features randomly drawn at each node, we point out that Sliq and DRF benefit greatly (by a factor proportional to the number of features) from limiting the number of unique candidate features at a given depth. In particular, the trend (see Section 3.2) consisting in using the same set of features for all nodes at a given depth leads to a fast DRF with a number of machines proportional to the number of randomly drawn features instead of the total number of features. We also study the impact of equipping DRF with a mechanism to prune records similarly to Sprint: when DRF detects that this pruning becomes beneficial, the algorithm can prune the records in closed leaves. This operation is not triggered during the experimentation on the large dataset reported in Section 4.
plicate the class list in each worker. To do so, DRF requires $\log_2(1 + \text{number of open nodes})$ bits per training example. In practice this figure is significantly smaller than using a full integer (i.e. 64 bits). Sliq/D and Sprint store a class list restricted to open nodes only. This saves up space/computation for later stages of the tree building, in particular when a large part of records are already in (closed) leaves. On the other hand, the class list contains the label (and possibly the weight) for each record, which is expensive, and we need many passes of writing, namely $\Omega(\text{number of records})$ for each level of the tree. Sprint stores the class list as a distributed hashmap.

**Communications in Sprint, in Sliq and in DRF.** In Sliq/D, the class list is distributed over workers; this forces workers to query each others continuously, once per example. Sprint requests communications for updating the distributed hashmap, once per node. In DRF, for each supersplit (i.e. for each level of the tree), one bit of data is broadcasted for each training example in an open leaf. DRF and Sliq work with two passes per depth level, whereas Sprint, Sliq/D and Sliq/R work at the level of nodes. In addition, for computing a random forest (compared to only a tree), we need bagging; instead of sending massive lists of record indices over the network, DRF just sends the seed of the randomized sampling (Section 2.2).

**Computations and passes over the data in Sprint, in Sliq and in DRF.** For deciding the split, Sprint, for categorical attributes, builds count tables “attribute value $\times$ class $\rightarrow$ number of records”. For numerical attributes, incrementally compute the quality of splits (e.g. information gain), thanks to one histogram per class computed incrementally for each candidate threshold (i.e. each unique numerical attribute value) in order thanks to the presorted attributes - the histogram stores the number of individuals below the considered threshold, for each class. Sliq does the same, but for all open nodes of a given depth before broadcasting. For Sliq/D, based on shards, a large part of the cost is due to communications for combining histograms obtained on different shards. For adapting the data to the splits, Sprint splits the attribute list for the chosen feature; collects split info in a hashmap; broadcasts this hashmap. Sliq: updates the mapping row id $\rightarrow$ node. DRF can be seen as an alternate solution for distributing Sliq (compared to Sliq/D and Sliq/R); as mentioned above, thanks to the same distribution of the data as in Sliq/R (per feature), we do not have to write anything regarding features (except during the initialization); features are simply read in one pass per level of the tree (and not per node of the tree!); and we broadcast one bit per record for updating the class lists, which are themselves stored with cost logarithmic in the number of open leaves.

### 3.2. Complexity Analysis: Formalization

**Z: A critical quantity for the performance of DRF.** We distribute the $m$ features uniformly over the $w$ workers (i.e. at most $[m/w]$ features per worker) - though redundancies could be considered as detailed later. At each node, we randomly draw $m'$ features for which optimal splits are computed; the total number of drawn features is $m' \leq z m'$ with $z$ the number of independent subsets of $m'$ features drawn in a given depth. Except in the USB case detailed below, $z$ is the number of open nodes for the current depth. The computational cost associated to computing splits for a node is therefore, for a worker, proportional to the number of features which are attributed to this worker. We define $Z$ as the maximum number of features attributed to a given worker. $Z$ depends on whether we apply USB; on the number of features used in each node; on how many (and possibly which) workers have access to each feature (i.e. redundancies); and on the number of workers. Several questions naturally arise in the analysis of $Z$.

**Random forest with unique set of bagged features per depth (USB).** Importantly, the number $z$ of independently randomly drawn subsets of $m'$ features, has a big impact on $Z$ and therefore on the overall complexity - we might consider a variant of RF in which all nodes at a given depth consider a same subset of $m'$ features, in which case we can set $z = 1$ and $m'' = m'$ independently of the number of nodes. This feature, referred to as USB in the present document, was already explored by (He, 2015).

**Can we have $m''$ small at a given depth without having a narrow tree or USB?** Equivalently, we check if, without USB, features selected over the different open nodes in a single supersplit can have sufficient redundancies for reducing the total number of features to some $m'' < \min(z m', m)$. Lemmas in the SM show that there is no hope - the number of selected features, up to a constant factor, verifies $\mathbb{E}m'' = \Omega(\min(z m', m))$.

**Correctness of the approximation “nearly the same number of features are drawn on each worker”.** If we have full redundancies (all features stored on all workers and optimal allocation of tasks to workers) or if $w = m$ then we clearly have a maximum number of features to be tackled on a given worker, for a given depth level, of the form $\mathbb{E}Z = O([m''/w])$ ($\mathbb{E}Z = 1$ if $w = m$; and $\mathbb{E}Z = [m''/w]$ with full redundancies ($d = w$) and optimal allocations); is there a risk, in the case $w < m$ and no redundancies, that the number of selected features is very unbalanced, so that one worker will take much more time than others ? Essentially, Lemmas in the SM, based on VC-type inequalities for independent sampling (Lafferty et al., 2010) for the independent sampling case, and on VC-type inequalities for rejective sampling (Clemencçon et al., 2016) for the non-independent case, show that this is not the case.
and the cost will remain \( \mathbb{E}Z = O(\lceil m'/w \rceil) \) if \( m' \) increases “faster” than \( w \); and remains 
\( \mathbb{E}Z = O(\log m'/\log \log m') \)
with \( w = m' \) even without redundancies (Gonnet, 1981); details in SM.

In the case \( w = m' \), a redundant storage of features improves the complexity in particular in the USB case. \( w = m \) is for sure an ideal case (each worker deals with one and only one feature), but this might be impossible when \( m \) is large; could we save up computational resources while preserving high performance when \( m \) is large? The answer, proved under the assumption of feature sampling with replacement, is yes. Assume \( w = m' \), and let us store each feature on \( d > 1 \) workers. Then, instead of a complexity \( \mathbb{E}Z \) of order \( \log(\lceil m' \rceil)/\log \log m' \), we get \( \log \log m'/\log d \) (detailed proof in (Azar et al., 1999), more details in the SM).

This, in conjunction with USB, leads to fast computation speed (\( \mathbb{E}Z = O(1) \) if \( \log(d) = \Omega(\log \log m) \) with \( w = m' \)) instead of \( w = m \) - a significant improvement in the classical case \( m' = \lceil \sqrt{m} \rceil \).

The key advantage of DRF is the moderate number of passes over the data. In case, for some hardware, this would not matter and the pruning of Sprint might perform particularly well because of many closed leaves early in the tree, we can implement a rule for switching to Sprint’s pruning mode, and this rule detects the issue early enough for preserving the complexity of Sprint in such a case (details in SM).

4. Experiments: Artificial Datasets

In this section, we report the performance of DRF on a set of families of synthetic binary classification datasets published specifically for large scale machine learning (P. Geurts, 2018). Each family is associated with a ground truth function (e.g. XOR, Majority). The members of each family differ in the number of training samples as well as the number of informative and uninformative features. The datasets include various numbers of useless variables (UV), with no correlation with the labels. Rote learning is used as a baseline for comparison; it consists in just labelling a test sample correctly if it was in the training set, and randomly otherwise. We test DRF with hyperparameters as follows: 1, 3 or 10 trees; unbounded depth; minimum number of examples per leaf equal to 1; number of splitters equal to the number of features. Given the large number of datasets, we did not replicate any of the experiments; but each data point is obtained independently. These runs are performed with a low priority - this shows that the approach remains reliable in spite of interruptions (workers can be killed by tasks with higher priority).

Figure 1 and SM show the AUC as a function of the training set size, while Figure 2 shows the training time.

5. Experiments: Real World Dataset

The Leo dataset is a large unbalanced proprietary binary classification dataset containing 18 billions records. Each record is defined by 3 numerical features and 69 categorical features with respective arities ranging from 2 to 10'000. To put the size of this dataset into perspective, storing a single 8 byte integer for each of the samples (e.g. the index of a sample) would requires 114 gigabytes of memory. By comparison, high end consumer computer stations have between 8 and 16 gigabytes of memory. If densely represented and uncompressed (and assuming zero overhead for the dataset structure), the dataset occupies 6 terabytes of memory. As far as we know, the Leo dataset is the largest dataset ever used to train an (exact or approximate) Random Forest. We consider three versions of this dataset: Leo 1% and Leo 10%, and Leo 100%, respectively 1%, 10%, and 98% of the full dataset. We reached an UAC of .847. The best AUC on this dataset (obtained by deep learning) was 0.81. We apply the DRF algorithm on the datasets described above. The number of workers is set to 82. Small and moderate size subsets of datasets can be run with the dataset loaded entirely in memory (distributed across the workers). Reading datasets from memory is significantly faster than reading from drive. However, for the sake of the comparison, all experiments have been run with the datasets remaining on drive. In all the runs, the hyperparameters of the Random Forest have been set to some reasonable default values; the number of candidate attributes for each split is equal to the square root of the total number of attributes, the minimum number of records in each node is set to 10, 100 and 1000 respectively, and the maximum depth is set to 20. In the case of subsets of dataset, the minimum number of records has been reduced proportionally with the relative size of the subset to the original set.

Table 2 shows training time, number of nodes, node density and sample density of each tree (averaged over all the trees in the model). The node density is the ratio between the number of leaves of the tree and the number of leaf of a dense tree of similar depth (i.e. \( 2^L \) with \( D \) the depth of the tree). The sample density is the ratio of training samples that reached the bottom leaves of the tree (i.e. the leaves at depth 20). Both density measures are expected to decrease with the depth of the tree. Figure 3 shows the average training time, number of leaves, node density and sample density, when varying the maximum tree depth between 0 (i.e. a tree is a single root node attached to the majority class) and 20. Figures have been averaged across the trees of a RF. The total training time of a tree is the sum of the training times of each depth levels. As expected, the number of leaves and the training time increases with the depth of the trees as the number of leaves grows. However, while the number of leaves increases exponentially with the depth of the trees, the computation time does not. This is explained by the fact that most of computation is
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| Algorithm               | Max. memory (per worker) | Computational cost (per worker), i.e. parallel time complexity | Writing on disk & number of passes (per worker) | Network & number of passes | Reading on disk & number of passes (per worker) |
|-------------------------|--------------------------|--------------------------------------------------------------|-----------------------------------------------|----------------------------|-----------------------------------------------|
| Generic, sequential recursive tree, all in memory | $m \times n \times |value|$ | $m \times n \log(n) D$ | 0 | 0 | $(m+1) \times |value| + 1$ passes |
| Sliq (on one machine)  | $n \times |value| + |index of leaf|$ | $m' \times n D + PS$ | 0 | 0 | $(m' + 1) \times |value| + 1$ passes |
| Sprint                  | $n \times |record index|$ (nb: $|record index| \geq \log_2(n) \geq \log_2(M)$) | $KnD + PS$ | $PS + C \times K$ passes of total size $KnD$ | $n$ row indices for bagging + $Dn$ row indices in $C$ broadcasts; if we use bitmaps for saving-up communication we will pay for sorting. |
| Sliq/D                  | $\lceil n/w \rceil \times |value| + |leaf index|$ | $m' \lceil n/w \rceil D + PS$ and coordination | $PS$ | $n$ row indices for bagging and coordination and $D$ broadcasts of $Dn$ bits. |
| Sliq/R                  | $n \times |value| + |leaf index|$ | $ZnD + PS$ | $PS$ | $m' \lceil n/w \rceil D |value| + |record index|$ in $m' \times C$ passes |
| DRF                     | $n \times (1 + \log_2(M))$ | $(Z + 1)nD + PS$ | $PS$ | $Dn$ bits in $D$ allreduce. |
| DRF-USB, $w = m'$, $d = \log(m')$ | $n \times (1 + \log_2(M))$ | $nD + PS$ | $PS$ | $Dn$ bits in $D$ allreduce. |

Table 1: Complexities of some discussed algorithms in the context of bagged features (i.e. $m'$ features are randomly drawn instead of all $m$ features, with $m'$ typically scaling as $\sqrt{m}$) and bagging (bagged records). Sliq/D contains a complex expensive implementation-dependent coordination between workers (in particular for numerical features) which is not detailed here. $M$ is the maximum number of nodes per depth. We assume classification; $C$ is the number of nodes in the tree. $PS$ refers to the complexity of presorting features. $\lceil x \rceil$ refers to the size of storage for an $x$ (e.g. $\lfloor int \rfloor$ refers to the number of bits per integer and $|value|$ refers to the number of bits for storing one entry of one feature or label). $Z$ is defined in Section 3.2, and depends on $z$, $w$, $m$ and $d$; it should be averaged over the depth levels. If conditions of Section 3.2 are met, then $Z = O(\lceil \min(K, zm'/w) \rceil)$. $K$ is $\lceil m/w \rceil$.

Table 2: Average training time, number of nodes, node density and sample density for the various datasets.

| Leo | Samples | Train time (h) | Leaves | Node density | Sample density |
|-----|---------|----------------|--------|--------------|---------------|
| 1%  | $1.73 \times 10^9$ | 0.838 | 140 | $10^3$ | 0.134 | 0.766 |
| 10% | $1.73 \times 10^9$ | 3.156 | 320 | $10^3$ | 0.305 | 0.904 |
| 100%| $17.3 \times 10^9$ | 22.29 | 435 | $10^3$ | 0.415 | 0.969 |

spent on scanning the dataset, and this step does not depend on the number of leaves. A depth 20, in the case of Leo 100%, 96.9% of the training samples are still in an open leaf (i.e. a leaf that could be split if the maximum depth was increased). This indicates that the tree can still grow if the maximum depth is increased. This also shows that pruning the dataset by removing samples in closed leaves (as in Sprint) would not speed up the computation since the cost of pruning would exceed its gains (using a pruning as in Sprint would not provide any significant improvement given this 96.9%). Figure 3 also shows the AUC (Area Under the Receiver Operating Characteristic Curve; computed on a test set) of individual trees (averaged over several trees) and of the entire RF model when varying the maximum tree depth. We see that using several billions of examples is useful for improving the AUC. Non-pruned DTs are highly susceptible to overfitting. Among other causes, DT overfitting appears when the number of training samples in a node becomes too small. Then, the tree starts “learning the noise” of the training dataset, the test AUC of an individual tree decreases, and the AUC of the overall RF plateaus. The depth of a tree is the main factor leading to nodes with few training samples. Therefore, we expect for the effect described above to be correlated with the depth of the trees: in the case of Leo 1% and Leo 10%, the overfitting of individual trees starts respectively at depth 13 and 17; in the case of Leo 100%, the overfitting of individual trees has not yet started at depth 20. The overfitting of individual trees does not indicate that the overall RF is not expected to benefit from deeper training: the AUC of the corresponding RFs plateaus at depth 16 for Leo 1%, and keeps increasing after depth 20 for Leo 10% and Leo 100%. We also observe that the RF trained on more data are plateauing to greater AUCs (0.823, 0.837, and 0.847, respectively for Leo 1%, Leo 10%, and Leo 100%). This indicates that RF benefits from using large datasets and training deeper trees.

6. Conclusions

We introduced DRF, an exact distributed Random Forest (Breiman, 2001). Our method stands out from existing
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Figure 1: Impact of the number of trees and training set sizes. Area Under ROC Curve (AUC) on artificial datasets. DRF, $m' = \lceil \sqrt{m} \rceil$ randomly drawn features per node, unlimited depth, at least one record per node. Similarly to real world data in Section 5, even for gigantic datasets, increasing the training set size and/or adding trees helps - in particular with many UV (compare rows, for each column). Rote learning fails ($AUC=\frac{1}{2}$) when we have UV. Random labelling, or labelling according to majority class, leads to $AUC=\frac{1}{2}$; hence $-\log(1 - AUC) = \log(2)$. One independent run per point in the plot; hence the highly imbalanced “needle” (dashed line) leads to irregular curves, others are more stable.

It is known that more training data improves accuracy or AUC; however, it is not obvious that this is the case when increasing the number of training examples from 1 to 10 billions. This is also central to learning algorithms working on subsets of the dataset (Gehrke et al., 1999). The present results show examples of datasets (both artificial and real world) for which more training data is beneficial in case of datasets of billions of examples. Further work. The mathematical analysis has shown the importance of USB and of redundant storage of features, to be experimentally investigated; we might also switch to pure memory for nodes with small numbers of records.
Figure 2: Training time in seconds as a function of training set size. Exact random forest, \( m' = \lceil \sqrt{m} \rceil \) randomly drawn features per node, unlimited depth, at least one record per node. We have e.g. 1900s - 3000s for building one random forest tree on 3e8 examples in dimension 18. This is in an environment with preemptions; hence irregular results. The number of workers is equal to the dimension, independently of the number of trees - the different trees are built sequentially, only the presorting is amortized.

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Figure 3: Tree and RF metrics while depth-by-depth training. Shows the training time, number of open leaves, node density, sample density, individual trees' AUC and TF AUC for depth level between 0 and 20.
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