Compressed Sensing Radar Detectors Under the Row-Orthogonal Design Model: A Statistical Mechanics Perspective

Siqi Na, Tianyao Huang, Yimin Liu, Takashi Takahashi, Yoshiyuki Kabashima, and Xiqin Wang

Abstract—Compressed sensing (CS) model of complex-valued data can represent the signal recovery process of many types of radar systems, especially when the measurement matrix is row-orthogonal. Based on debiased least absolute shrinkage and selection operator (LASSO), the detection problem under the Gaussian random design model, i.e., the elements of the measurement matrix are drawn from a Gaussian distribution, is studied by literature. However, these approaches are unsuitable for the row-orthogonal measurement matrices, which are of more practical relevance. In view of statistical mechanics approaches, we provide derivations of more accurate test statistics and thresholds (or p-values) under the row-orthogonal design model and theoretically analyze the detection performance of the present detector. Such a detector can analytically provide the threshold according to a given false alarm rate, which is not possible with the conventional CS detector, and the detection performance is proved to be better than that of the traditional LASSO detector. Compared with other debiased LASSO based detectors, simulation results indicate that the proposed approach can achieve a more accurate probability of false alarm when the measurement matrix is row-orthogonal, leading to better detection performance under the Neyman-Pearson principle.

Index Terms—Compressed sensing, radar detection, LASSO, row-orthogonal matrix, replica method, statistical mechanics.

I. INTRODUCTION

CS model of complex-valued data assumes a scenario of recovering an \( N \)-dimensional vector \( x_0 \in \mathbb{C}^N \) from an \( M \)-dimensional vector \( y \in \mathbb{C}^M \), given by

\[
y = A x_0 + \xi,
\]

where \( A \in \mathbb{C}^{M \times N} \) is the measurement matrix (or the sensing matrix) and \( \xi \in \mathbb{C}^M \) is the complex additive white Gaussian noise (AWGN) with i.i.d. components \( \xi_i \sim \mathcal{CN}(0, \sigma^2) \). Basically, for the CS model, \( M \) is less than \( N \), and we regard \( \gamma = M/N \) as compression rate. The original signal \( x_0 \) is a sparse vector containing only \( k = \rho N \) non-zero entries, where \( \rho(0 \leq \rho \leq 1) \) is referred to as the signal density.

In many practical applications, the signal processing can be modeled as above, while the sensing matrix has a specific structure. Particularly, in this paper, we focus on radar applications, in which \( y, x_0, \) and \( A \) refer to the sampled received signal, radar observation scene, and observation (or steering) matrix, respectively. Generally, in the radar application scenarios, the number of non-zero entries in \( x_0 \), which represents the intensity of scattering points, is small enough to satisfy \( k \ll N \). The indices of the non-zero entries indicate the “position” of the scattering points (or the targets), such as range, azimuth, radial velocity, etc.

The observation matrix \( A \) incorporates the observation scene’s geometry and the transmitting waveform’s design. In the present paper, we are concerned with the case where \( A \) is a row-orthogonal matrix. That is, \( A_i, A_i^H = 0 \) for \( i \neq j \) and \( A_i, A_i^H = 1 \), in which \( A_i \) represents the \( i \)-th row of matrix \( A \). Many types of radar-transmitting waveform utilize a row-orthogonal steering matrix, and below we list some of them for illustration:

1) The partial observation problem of a pulse Doppler radar system, such as those working in complex electromagnetic environments [1], leads to a result of a partial Fourier steering matrix, which is apparently row-orthogonal.

2) The steering matrix of the frequency-agile radar system is similarly row-orthogonal [2], [3], which possesses a variety of merits such as good electronic countermeasures (ECCM) performance, low hardware system cost, and convenience of spectrum sharing.

3) Sub-Nyquist radar systems [4], [5] realize the observation and compression on several dimensions, such as the temporal domain, spatial domain, and spectral domain. The steering matrix of sub-Nyquist radar, which is proved to be the Kronecker product of several partial Fourier matrices (depending on the number of dimensions of the observation scene) in [5], satisfies the row-orthogonal property.

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For modern radar systems, it is crucial to detect targets element-wisely. As previously mentioned, the indices of the non-zero entries in $x_0$ represent the information about the targets, and judging whether each entry in $x_0$ is non-zero can inform us about the existence of the targets and their location. We summarize such requirements as solving the following hypothesis testing problems:

$$
\begin{align*}
\mathcal{H}_{0,i} : & x_{0,i} = 0, \\
\mathcal{H}_{1,i} : & x_{0,i} \neq 0, 
\end{align*}
$$

for $i = 1, 2, \ldots, N$ and designing thresholds for these tests. Note that it is different from the task to detect whether there exists any target in the whole observed scene. The latter is generally simpler, which can be expressed as

$$
\begin{align*}
\mathcal{H}_0 : & y = \xi, \\
\mathcal{H}_1 : & y = Ax_0 + \xi,
\end{align*}
$$

and can be easily solved by conventional approaches such as the generalized likelihood ratio test (GLRT) or the Wald test.

However, since the linear model (1) is underdetermined, the maximum likelihood estimation (MLE) of $x_{0,i}$ is unavailable. Consequently, one cannot directly apply conventional detectors to solve detection problem (2). Therefore, it is natural that we estimate $x_0$ by CS approaches and complete the tests on the basis of such estimation, in which LASSO [6] is a frequently used technique. The complex-valued LASSO estimator $\hat{x}_{\text{LASSO}}$ is given by

$$
\hat{x}_{\text{LASSO}} = \arg \min_{x} \left\{ \frac{1}{2} \| y - Ax \|_2^2 + \lambda \| x \|_1 \right\}, \tag{4}
$$

where

$$
\| x \|_1 = \sum_{i=1}^{N} \sqrt{(\Re(x_i))^2 + (\Im(x_i))^2}. \tag{5}
$$

As a convex optimization problem, LASSO can be solved by standard techniques such as interior point methods [7], [8]. There are also plenty of iterative thresholding algorithms such as approximate message passing (AMP), iterative shrinkage-thresholding algorithm (ISTA), etc. Besides the computationally feasible nature of $\hat{x}_{\text{LASSO}}$ has been studied in a large literature, which mainly focuses on the prediction error $\| A(\hat{x}_{\text{LASSO}} - x_0) \|_q / M$ [9], the estimation error $\| \hat{x}_{\text{LASSO}} - x_0 \|_q$ with $q \in \{1, 2\}$ [10], [11], [12], and variable selection (or support estimation) of $x_0$ [13], [14], [15], denoted by the support set $S_0 = \{ i \leq N : x_{0,i} \neq 0 \}$ and the estimation of support set $\hat{S}$ such that $\mathbb{P}(\hat{S} = S_0)$ is bounded. However, the above study is not enough to solve detection problems (2): we still cannot get their p-values (or the threshold with a given false alarm rate, which is extremely important in radar applications).

Naturally, one would wonder about the distribution of $\hat{x}_{\text{LASSO}}$ for the purpose of designing the detector.

A certain linear transformation of the LASSO estimator, called debiased LASSO [16], which is also known as desparsified LASSO [17] in the context of statistics, is given by

$$
\hat{x}^d = \hat{x}_{\text{LASSO}} + \frac{1}{\lambda} A^H (y - Ax_{\text{LASSO}}), \tag{6}
$$

where $\lambda > 0$ is the debiased coefficient computed from known variables and contains information about the structure of the sensing matrix $A$. Asymptotic analysis of the LASSO solution based on AMP algorithm was developed in [18] and [19], which proves that the empirical distribution of the difference

$$
w = \hat{x}^d - x_0, \tag{7}
$$

converges weakly to a Gaussian distribution under the real-valued Gaussian random design model (the entries of the observation matrix $A$ are i.i.d. drawn from a Gaussian distribution). This conclusion derives two studies for the design of detectors for solving the detection problem (2) based on debiased LASSO: [16] for the real-valued CS model and [20] for the complex-valued one. While all the studies above restrict the measurement matrix to be Gaussian, work in [21] based on statistical mechanics methods as well provides different ways for obtaining $\hat{x}^d$. The methodologies used allow the asymptotic analysis of LASSO solutions and the construction of debiased LASSO estimator for multiple real-valued observation matrix ensembles such as Gaussian, row-orthogonal, and random discrete cosine transformation (DCT). We aim to derive debiased LASSO in complex-valued form based on [21], especially under the row-orthogonal design model, and analyze the detection performance of the resulting detector.

The other difference between this paper with other works on detection based on LASSO estimators is the definition of the detection problem and consequently the false alarm rate. As shown in Equation (2), the detection problem we aim to deal with is to see whether each component of $x_0$ is zero. Similar research [22] provides an approach to approximately calculate the threshold of a given false alarm rate. However, considering the hypothesis testing problem (2), the false alarm will only happen in the noise-only case, which means $x_0 = 0$. This is quite different from the standard definition of false alarm rate in (2), in which false alarm will also happen when $x_0 \neq 0$. Again, there are some papers related to the LASSO estimator [23], [24] that consider the control of the family-wise error rate (FWER) [25], [26]. The definition of FWER, which is the probability of at least one Type I error, is obviously different from the one we are concerned with in this paper. We believe that controlling the false alarm rate of problem (2) is more difficult due to the challenges of obtaining the distribution of $\hat{x}^d_{\text{LASSO}}$ when $x_0 \neq 0$.

The main contributions of this paper are twofold. First, we present a general debiased LASSO detector framework and analyze its detection performance. We summarize the existing research on detection problems, including [16] and complex approximate message passing (CAMP) based [20], and find that they all used the debiased LASSO estimator to formulate the test statistics. Therefore, we construct a general detector framework based on debiased LASSO and analyze its detection performance. We prove that the detection performance of the debiased LASSO detector is better than that of the traditional LASSO detector under the Neyman-Pearson principle, and its detection threshold can be analytically calculated by a given false alarm rate, thus the detection rate can be further quantified. In contrast, most CS methods do not have closed-form solutions, and the distribution is not
available. Consequently, the threshold cannot be given by the false alarm rate either. Therefore, we believe that the debiased LASSO detector can completely replace the traditional CS detector when solving the detection problem (2).

Second, we extend the results of [21] to the complex domain to enhance their applicability to engineering applications. Some methods in statistical mechanics are used in reference [21], which sacrifice part of the mathematical rigor to obtain more attractive results: the debiased LASSO estimator for some non-Gaussian sensing matrices can be derived, and a more accurate estimation method for the variance $\sigma_w^2$ of $w$ (as defined in (7)) can be provided. These results make it possible to implement debiased LASSO detector under the row-orthogonal design model. Therefore, for engineering applications, we extend these derivation processes to the complex domain. We find that the debiased coefficient under the row-orthogonal assumption is different from the results in [16] and [20], and the correctness of the results obtained by our method is verified by simulation experiments. Numerical results also verify that our method can estimate the variance $\sigma_w^2$ of $w$ more accurately than [16] and [20]. This leads to a more accurate threshold for the designed debiased LASSO detector and therefore to a better reaching of a given false alarm rate. Such a conclusion is also verified by simulation results.

The recovery problem considered restricts the condition that the signal is affected by white Gaussian noise. However, in many radar scenarios, the noise is colored with an unknown covariance matrix and may be non-Gaussian. The targets may also be disturbed by clutter. In such cases, the proposed detector may not work properly. With appropriate pre-processing or the use of the prior information about $x_0$ and the noise, these problems may be solved, and we treat them as future work.

Furthermore, the validity of the proposed method depends on the selection of the regularization parameter $\lambda$. The results in our paper hold with regulation parameters $\lambda \in [\lambda_{\min}, \lambda_{\max}]$ for some $0 < \lambda_{\min} < \lambda_{\max} < \infty$ [16], [19], [27], [28]. Though the analysis in these researches is based on the Gaussian sensing matrices, we believe that similar properties also hold for row orthogonal matrices due to their deterministic singular values. It is obvious that the variance $\sigma_w^2$ of $w$ relies on how $\lambda$ is chosen. Our future work also includes the optimization of $\lambda$ by minimizing the variance $\sigma_w^2$, see [29].

The organization of the present paper is as follows. In Section II, we provide our results on the design of the detector for the compressed sensing radar system and the analysis of its detection performance. The derivation of the test statistic and the threshold of the presented detector is elaborated in Section III. Section IV furnishes some numerical validation of the obtained asymptotic analysis. We conclude the paper in Section V.

Throughout the paper, we use $a$, $a$, and $A$ as a number, a vector, and a matrix, respectively. For a set $S$, $\#S$ denotes its cardinality. Denote by $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ for the real and imaginary parts of a complex-valued component, respectively. The function $\delta(\cdot)$ is Dirac’s delta function and $\Theta(x)$ is Heaviside’s step function. The operators $()^T$, $()^*$, and $()^H$ represent the transpose, conjugate, and conjugate transpose of a matrix or a vector, respectively. Denote by $\Phi(x) = \int_{-\infty}^{\infty} e^{-u^2/2} / \sqrt{2\pi} du$ the standard Gaussian distribution function.

![Fig. 1. Frameworks of two detectors for detection problem (2): (a) LASSO detector; (b) Debiased LASSO detector.](image)

II. DESIGN AND ANALYSIS OF THE DEBIASED LASSO DETECTOR

In this section, we first introduce the debiased LASSO detector. The advantage of the debiased LASSO detector is elaborated mainly by comparing it with the LASSO detector in Section II-A. While both the proposed detector and the ones in [16], [20], [21] are debiased LASSO detectors, the difference is explained in Section II-B. Particularly, this paper inherits some derivation from [21], and we will briefly describe the contribution of this work over [21] in Section II-B.

A. Debiased LASSO Detector

Concerning the detection problem (2), we here define the following performance metrics. Denote by $\varphi_i = \begin{cases} 1, & \text{if detector reject } H_{0,i}; \\ 0, & \text{otherwise}. \end{cases}$

Define the false alarm probability $P_{fa}$ as

$$P_{fa} = \lim_{N \to \infty} \frac{1}{N - k} \sum_{i \in S^c} \varphi_i,$$

and the detection probability $P_d$ as

$$P_d = \lim_{N \to \infty} \frac{1}{k} \sum_{i \in S} \varphi_i,$$

where $S = \{i : x_{0,i} \neq 0\}$ is the support set of $x_0$ with $\#S = k = \rho N$ and $S^c = \{1, \ldots, N\} \setminus S$. The LASSO detector, which is a natural idea, possesses the structure shown in Fig. 1(a), in which the test statistic is provided by LASSO estimator. Indeed, other traditional CS radar detectors have a form similar to the LASSO detector, which contains the compressed sensing module and a test of judging whether the amplitude is greater than a threshold.

On the other hand, the debiased LASSO detector for compressed sensing radar has the structure shown in Fig. 1(b). The test statistic is given by the debiased estimator $x_d$ and the threshold $\kappa_d$ can fix the probability of false alarm. In the present paper, we propose the debiased LASSO detector for the complex-valued row-orthogonal observation matrix called Complex Row-Orthogonal Debiased detector (CROD), whose procedure is shown in Algorithm 1. The detailed derivation of the involved parameters will be presented later in Section III.

We declare here that the benefits of the debiased LASSO detector are mainly twofold. First, the relationship between the threshold $\kappa_d$ and the false alarm rate $P_{fa}$ can be computed...
Algorithm 1 CROD

Input: $y$, $A$, regularization parameter $\lambda$, probability of false alarm $P_{fa}$, variance of noise $\sigma^2$

Output: debiased LASSO estimator $\hat{x}^d$, threshold $\kappa_d$

1: Let

$$\hat{x}^{LASSO}_{\lambda} = \arg\min_x \left\{ \frac{1}{2} \| y - Ax \|_2^2 + \lambda \|x\|_1 \right\}. $$

2: The debiased LASSO estimator is obtained from

$$\hat{x}^d = \hat{x}^{LASSO}_{\lambda} + \frac{1}{\Lambda_{CROM}} A^H(y - Ax^{LASSO}_{\lambda}),$$

with $\Lambda_{CROM}$ and $\rho_{CA}$ are solved from the following fixed point equation

$$\Lambda_{CROM} = \frac{\gamma - \rho_{CA}}{1 - \rho_{CA}},$$

$$\rho_{CA} = \frac{1}{2N} \sum_{i=1}^{N} \left[\left(2 - \frac{\lambda}{\Lambda_{CROM} \|x^{LASSO}_{\lambda}\|_2} + \lambda\right) \cdot \Theta\left(\|x^{LASSO}_{\lambda}\|_2\right)\right].$$

3: The threshold $\kappa_d$ is given by

$$\kappa_d = -\hat{\sigma}^2_w \ln P_{fa},$$

where

$$\hat{\sigma}^2_w = \frac{\gamma(1-\gamma)}{(\gamma-\rho_{CA})^2} \text{RSS} + \sigma^2,$$

$$\text{RSS} = \frac{1}{M} \|y - Ax^{LASSO}_{\lambda}\|_2^2.$$

analytically, as will be detailed later in Section II-A1. This is usually not possible for traditional CS detectors, because the distribution of the solution obtained by CS methods is unavailable. Second, its detection performance is better than that of the LASSO detector, which will be proved in Section II-A2.

1) Analytical Expression of False Alarm Probability: The most significant advantage of the debiased LASSO detector over the conventional CS detector is that the analytical relationship between the threshold $\kappa_d$ and the false alarm rate $P_{fa}$ can be obtained. Controlling the false alarm rate is particularly important in many radar applications due to resource allocation and other reasons. Asymptotic analysis shows that as $N \to \infty$, if $\Lambda$ is suitably chosen, $\hat{x}^d$, the $i$-th entry of $\hat{x}^d$, approximately follows a Gaussian distribution with mean $x_{0,i}$, where $x_{0,i}$ is the $i$-th entry of $x_0$. For the convenience of readers, we here restate the definition and conclusion in [18] (which is similar to that in [19]) to describe this result more precisely.

**Definition II.1:** ([18]). For a given $(\gamma, \rho) \in [0, 1]^2$, a sequence of instances $\{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}}$ indexed by $N$ is said to be a converging sequence of the Gaussian random design model if the empirical distribution of the entries $x_0(N) \in \mathbb{R}^N$ converges weakly to a probability measure $p_X$ with a bounded second moment, the empirical distribution of the entries $\xi(N) \in \mathbb{R}^M (M = \gamma N)$ converges weakly to a probability measure $p_{\xi}$ with a bounded second moment and the elements of $A(N) \in \mathbb{R}^{M \times N}$ are i.i.d. drawn from a Gaussian distribution.

**Lemma II.2:** ([18]). Let $\{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}}$ be a converging sequence of the Gaussian random design model. The empirical law of $w(N) = \hat{x}^d(N) - x_0(N)$ converges to a zero-mean Gaussian distribution almost surely as $N \to \infty$ for a specific $\Lambda$.

The previous lemma shows the existence of the debiased coefficient $\Lambda$ for the random Gaussian random design model. The uniqueness of $\Lambda$ can also be inferred.

**Remark II.3:** Let $\{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}}$ be a converging sequence of the Gaussian random design model. If there exists $\Lambda > 0$ and let $\hat{x}^d = \hat{x}^{LASSO}_{\lambda} + \frac{1}{\Lambda} A^H(y - Ax^{LASSO}_{\lambda})$, such that the empirical law of $w(N) = \hat{x}^d(N) - x_0(N)$ converges to a zero-mean Gaussian distribution almost surely as $N \to \infty$. Then for all $\Lambda' \neq \Lambda$ and $\hat{x} = \hat{x}^{LASSO}_{\lambda' + 1} A^H(y - Ax^{LASSO}_{\lambda'})$, the empirical law of $w'(N) = \hat{x}(N) - x_0(N)$ does not converge to a Gaussian distribution as $N \to \infty$.

Thanks to the decoupling principle by Guo and Verdu [30], whose validity is rigorously proved by [18], and the conclusion of Claim III.1, the LASSO solution converges in law as

$$\hat{x}^{LASSO}_{\lambda} = \frac{h_i}{|h_i| - \lambda} \Theta\left(|h_i| - \lambda\right),$$

where

$$h_i = \Lambda x_{0,i} + \sqrt{2\chi} z_i,$$

in which $z_i \sim \mathcal{CN}(0, 1)$ are i.i.d. standard complex Gaussian random variables and $\chi$ is a positive real number. The debiased estimator is given by

$$\hat{x}^d = \hat{x}^{LASSO}_{\lambda} + \frac{1}{\Lambda} a_{i}^H(y - Ax^{LASSO}_{\lambda}) = x_{0,i} + \frac{\sqrt{2\chi} z_i}{\Lambda},$$

where $a_i$ is the $i$-th column of $A$. This yields

$$a_i^H(y - Ax^{LASSO}_{\lambda}) = \Lambda (x_{0,i} - \hat{x}^{LASSO}_{\lambda}) + \sqrt{2\chi} z_i.$$

Therefore,

$$w'_i = \hat{x}_i - x_{0,i} = \hat{x}^{LASSO}_{\lambda} + \frac{1}{\Lambda} a_{i}^H(y - Ax^{LASSO}_{\lambda}) - x_{0,i} = \hat{x}^{LASSO}_{\lambda} + \frac{1}{\Lambda} \left(\Lambda (x_{0,i} - \hat{x}^{LASSO}_{\lambda}) + \sqrt{2\chi} z_i\right) - x_{0,i} = \left(1 - \frac{1}{\Lambda}\right) \hat{x}^{LASSO}_{\lambda} - x_{0,i} + \frac{\sqrt{2\chi} z_i}{\Lambda}.$$
\(R^M (M = \gamma N)\) converges weakly to a probability measure with a bounded second moment and the singular values of \(A \in R^{M \times N}\) are all 1 with the right singular basis of \(A\) is a random sample from the Haar measure of \(N \times N\) orthogonal matrices.

**Remark II.5:** Let \(\{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}}\) be a converging sequence of the row-orthogonal design model. If there exists \(\Lambda > 0\) and let \(\hat{x}' = \hat{x}_{\text{LASSO}} + \frac{1}{\Lambda} A^H (y - A \hat{x}_{\text{LASSO}})\), such that the empirical law of \(w(N) = \hat{x}'(N) - x_0(N)\) converges to a zero-mean Gaussian distribution almost surely as \(N \to \infty\). Then for all \(\Lambda' \neq \Lambda\) and \(\hat{x} = \hat{x}_{\text{LASSO}} + \frac{1}{\Lambda'} A^H (y - A \hat{x}_{\text{LASSO}})\), the empirical law of \(w'(N) = \hat{x}(N) - x_0(N)\) does not converge to a Gaussian distribution as \(N \to \infty\).

This conclusion shares the same derivation process as Remark II.3.

Denote the sample variance of \(w(N)\) by \(\sigma^2_w\), it is natural to get the following analytical relationship between the probability of false alarm and the threshold \(\kappa_d\) of the detector.

**Theorem II.6:** Let \(\{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}}\) be a converging sequence of the row-orthogonal design model. If there exists \(\Lambda > 0\) and let \(\hat{x}' = \hat{x}_{\text{LASSO}} + \frac{1}{\Lambda} A^H (y - A \hat{x}_{\text{LASSO}})\), such that the empirical law of \(w(N) = \hat{x}'(N) - x_0(N)\) converges to a zero-mean Gaussian distribution almost surely as \(N \to \infty\). Then the false alarm probability \(P_{fa}\) of the debiased LASSO detector satisfies:

\[
\kappa_d = -\sigma^2_w \ln P_{fa},
\]

where the test is

\[
\varphi_i = \begin{cases} 1, & |\hat{x}'_i|^2 > \kappa_d; \\ 0, & \text{otherwise}. \end{cases}
\]

**Proof:** For \(i \in S^c\), which means that \(x_{0,i} = 0\), the empirical law of \(\{\hat{x}'_i\}\) converges to \(CN(0, \sigma^2_w)\) as \(N \to \infty\). Therefore, the empirical distribution of \(\{|\hat{x}'_i|^2\}\) converges to an exponential distribution with rate parameter \(1/\sigma^2_w\), leading to:

\[
P_{fa} = \lim_{N \to \infty} \frac{1}{N - k} \sum_{i \in S^c} \varphi_i = \exp \left( -\frac{\kappa_d}{\sigma^2_w} \right),
\]

which proves (20).

From Theorem II.6, we conclude that the threshold \(\kappa_d\) of the debiased LASSO detector can be analytically calculated by the false alarm probability. Such a mission cannot be achieved by traditional CS detector for the reason that almost all of the solutions obtained by CS methods do not have a closed form, nor the distribution of the solutions. In the present paper, we will provide an asymptotic analysis from a statistical mechanics perspective and the process of calculating the coefficients \(\Lambda\) with variance \(\sigma^2_w\) in Section III, which can be adapted to the row-orthogonal matrix design of \(A\) mentioned for radar applications.

2) Better Detection Performance: At first glance, the debiased LASSO estimator destroys the sparsity brought by the LASSO results, but we will next prove theoretically that treating \(|\hat{x}'_i|^2\) as a test statistic compared to \(|\hat{x}_i^2|\) does not worsen the detection performance of the detector.

**Theorem II.7:** Let \(x_0 \in \mathbb{C}^N\), \(\xi \in \mathbb{C}^M (M = \gamma N)\), and \(A \in \mathbb{C}^{M \times N}\). For the same false alarm probability \(P_{fa}\), the detection probability of the debiased LASSO detector \(P_{fa}^{\text{d}}\) is not less than that of LASSO detector \(P_{fa}^{\text{LASSO}}\).

**Proof:** See Appendix A (available online) for the proof.

Theorem II.7 suggests that applying such a non-sparse solution in the detector instead leads to better detection performance. This conclusion does not depend on \(N \to \infty\), nor does it require any assumptions about the distribution or structure of \(\{x_0, \xi, A\}\).

### B. Comparison With Existing Debiased LASSO Detectors

In this subsection, we compare the proposed detector CROD with SDL-test [16], CAMP [20], and the Row-Orthogonal Debiased detector (ROD) constructed by the conclusions obtained from [21]. Due to the fact that the frameworks of all the debiased LASSO detectors are the same, we mainly list the differences in the debiased coefficient \(\Lambda\) and the approaches to estimate the variance \(\sigma^2_w\).

1. **SDL-test [16]** suggests that

\[
\Lambda_G = \gamma - \rho_a,
\]

where \(\rho_a = \# \{i \mid \hat{x}_i^2 \neq 0\} / N\) denotes the active component density and

\[
\hat{\sigma}_w = \frac{\sqrt{\gamma}}{\Phi^{-1}(0.75) (\gamma - \rho_a)} \text{median} \left( |y - A \hat{x}_{\text{LASSO}}| \right),
\]

for the real-valued Gaussian random sensing matrix, where \(|\cdot|\) is applied to each component of a vector and \(\text{median}(\cdot)\) is the median of the \(N\) entries.

2. **CAMP [20]** provides

\[
\Lambda_{CG} = \gamma - \rho_{CA},
\]

where \(\rho_{CA}\) given by (II-A) denotes the complex active component density and

\[
\hat{\sigma}_w = \frac{1}{\sqrt{\ln 2}} \text{median} \left( |\hat{x}'| \right)
\]

for the complex-valued Gaussian random sensing matrix.

3. **ROD from [21]** claims that

\[
\Lambda_{ROM} = \frac{\gamma - \rho_a}{1 - \rho_a}
\]

for the real-valued row-orthogonal sensing matrix, and \(\hat{\sigma}_w^2\) is given by replacing \(\rho_{CA}\) in (13) to \(\rho_a\).

4. The proposed CROD claims that

\[
\Lambda_{CROD} = \frac{\gamma - \rho_{CA}}{1 - \rho_{CA}},
\]

and \(\hat{\sigma}_w^2\) is given by (13) and (14) for the complex-valued row-orthogonal sensing matrix.

Moreover, the methodologies in [21] can obtain the same result as (23) under the Gaussian random matrix design. Our work also gives the same debiased coefficient as (25) for the complex-valued Gaussian sensing matrix in Corollary III.4. Based on the derivation in Section III, we believe that the debiased coefficient \(\Lambda\) is related to the asymptotic eigenvalue distribution \(\rho_J(s)\) of
$J = A^T A$ (or $J = A^H A$), which is referred to as Claim III.2. For a Gaussian matrix, $\rho_J(s)$ can be calculated by (37), which yields (25). For other sensing matrices, the proposed method is also applicable if $\rho_J(s)$ is obtainable. We summarize the results of the comparison in Table I.

Recall that we have proved the uniqueness of the debiased coefficient $\Lambda$. When the steering matrix $A$ is row-orthogonal, the correctness of the debiased coefficient $\hat{A}_{\text{CROM}}$ given in the present paper will be verified by the numerical result in Section IV. Besides the construction of the debiased LASSO estimator, the approach to estimating $\sigma_0^2$ in the present paper is different from CAMP and SDL-test. There is also a numerical result of the accuracy of the three estimation methods presented in Section IV.

The derivation, including the construction of the debiased LASSO estimator and the estimation of the variance, in Section III is primarily based on [21]. However, since the signal model in radar systems is composed of complex vectors or matrices, our next derivations are in complex form, which differs from both the process and the results of [21]. We will verify in our simulation experiments that the direct use of the real-valued version results is incorrect. This is because the complex LASSO differs from the real one with regard to the computation of the $\ell_1$-norm of $x$ in the regularization term.

### III. DERIVATION OF THE DEBIASED LASSO ESTIMATOR AND THE ESTIMATION OF ITS VARIANCE

In Algorithm 1, we simply present the results of the debiased LASSO estimator and estimation of its variance $\sigma_0^2$. In this section, we will provide the detailed derivation of these results, based on some statistical mechanics approaches. Led by [31], more and more work applies statistical mechanics approaches for theoretical analysis in information theory and communications theory. Although a mathematically rigorous justification of the replica method is still in progress, it has been proven extensively successful in very difficult problems and applied to derive a number of captivating results, see [30], [32], [33], [34].

While with respect to the replica method for LASSO, a recent work [35] guarantees its mathematical correctness, and we prefer to leave the complex-valued version for future investigation. In addition, we refer to the results in this section as claims.

Particularly, the asymptotic analysis of the distribution of LASSO solutions will first be introduced, which tells us that a “hidden” random variable obeying a Gaussian distribution with a mean of $x_0$ exists. Then, we aim to get the closed-form expression of such a “hidden” variable, which will be regarded as the debiased LASSO estimator. At the end of this section, the procedure for estimating the variance of the debiased estimator will be presented.

### A. Main Results

Our first result provides the asymptotic distribution of the LASSO solution $\hat{x}_{\text{LASSO}}$.

**Claim III.1:** Let $\{\nu_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}}$ be a converging sequence of the row-orthogonal design model. The asymptotic distribution of the LASSO solution $\hat{x}_{\text{LASSO}}$ can be inferred as follows:

$$
\hat{x}_{\text{LASSO}} = ST_{\lambda, Q}(h_i) = \frac{h_i}{|h_i|} \left( |h_i| - \Theta(|h_i| - \lambda) \right),
$$

where $h_i = \bar{m} x_{0,i} + \sqrt{2} \bar{x} z_i$,

in which $z_i \sim \mathcal{CN}(0, 1)$ are i.i.d. standard complex Gaussian random variables and $\bar{m}$, $\bar{Q}$, $\bar{x}$ are positive real numbers.

Such a conclusion reminds us that the debiased LASSO estimator can be easily obtained if $h_i$ is available. We next introduce the construction of debiased LASSO $\hat{x}^d$ for certain matrix ensembles through deriving $h_i$.

**Claim III.2:** Suppose that the right singular basis of the sensing matrix $A$ is a random sample from the Haar measure of $N \times N$ orthogonal matrices. Suppose the Gram matrix $J = A^H A$ has deterministic asymptotic eigenvalue distribution $\rho_J(s)$ and denotes the debiased LASSO estimator by

$$
\hat{x}^d = \hat{x}_{\text{LASSO}} + \frac{1}{\Lambda} A^H (y - A \hat{x}_{\text{LASSO}}),
$$

then the debiased coefficient is

$$
\Lambda = \frac{t \cdot \rho_{\text{CA}}}{\rho_{\text{CA}} - 1},
$$

in which $t$ is the solution of

$$
\int \frac{\rho_J(s)}{t - s} ds = \frac{1 - \rho_{\text{CA}}}{t},
$$

and $\rho_{\text{CA}}$ is the complex active component density of the LASSO solution defined in (II-A).

Setting the sensing matrix ensemble to be row-orthogonal, the asymptotic eigenvalue distribution of $J$ is given by

$$
\rho_J(s) = (1 - \gamma) \delta(s) + \gamma \delta(s - 1).
$$

Therefore, one can obtain the following corollary.
Corollary III.3: Let \{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}} be a converging sequence of the row-orthogonal design model. Denote the debiased LASSO estimator \( \hat{x}^d \) by
\[ \hat{x}^d = \hat{x}^{LASSO} + \frac{1}{\Lambda_{\text{CHROM}}} A^H (y - A\hat{x}^{LASSO}), \]
then the debiased coefficient is
\[ \Lambda_{\text{CHROM}} = \frac{\gamma - \rho_{\text{PCA}}}{1 - \rho_{\text{PCA}}}, \]
where \( \rho_{\text{PCA}} \) is the complex active component density of the LASSO solution defined in (II-A).

According to [36], when the entries of \( A \) are all i.i.d. Gaussian ensembles with mean 0 and variance \( 1/N \), for \( \gamma \leq 1 \), the asymptotic eigenvalue distribution is given by
\[ \rho_I(s) = (1 - \gamma) \delta(s) + \frac{1}{2\pi} \sqrt{(\lambda_+ - s)(s - \lambda_-)} \mathbb{I}_{[\lambda_-, \lambda_+]}(s), \]
\[ \lambda_{\pm} = (1 \pm \sqrt{\gamma})^2, \]
\[ \mathbb{I}_S(x) = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{otherwise} \end{cases}, \]

Then comes the following corollary.

Corollary III.4: Let \{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}} be a converging sequence of the Gaussian random design model in which all entries of \( A \) are i.i.d. complex Gaussian variables with mean 0 and variance \( 1/N \). Denote the debiased LASSO estimator \( \hat{x}_d \) by
\[ \hat{x}_d = \hat{x}^{LASSO} + \frac{1}{\Lambda_{\text{CG}}} A^H (y - A\hat{x}^{LASSO}), \]
then the debiased coefficient is \( \Lambda_{\text{CG}} = \gamma - \rho_{\text{PCA}} \), where \( \rho_{\text{PCA}} \) is the complex active component density of the LASSO solution defined in (II-A).

In addition, we find that the debiased coefficient \( \Lambda_{\text{CG}} \) is the same as the result in [20] in the case of the complex Gaussian design model.

The third result presents a method for estimating the variance \( \sigma_w^2 \).

Claim III.5: Let \{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}} be a converging sequence of the row-orthogonal design model. The sample variance \( \sigma_w^2 \) of \( w = \hat{x}_d - x_0 \) converges to
\[ \hat{Q} = G'(-\chi; J), \]
and
\[ \hat{Q} = \frac{\gamma G''(-\chi; J) - 2G''(-\chi; J)\chi}{2G''(-\chi; J) - 2G''(-\chi; J)\chi + \frac{G''(-\chi; J)\gamma + (G'(-\chi; J))^2}{2G''(-\chi; J) - 2G''(-\chi; J)\chi} \sigma_w^2}. \]

Here,
\[ \text{RSS} = \lim_{N \to \infty} \frac{1}{N^2} \left\| y - A\hat{x}^{LASSO} \right\|_2^2 \]
\[ G'\left( x; J \right) \]
\[ G\left( x; J \right) \]
which \( \rho_J(s) \) is the asymptotic eigenvalue distribution of \( J \). The derivative of the function \( G(x; J) \) has the following form:
\[ G'(x; J) = t(x) - \frac{1}{x}, \]
where \( t(x) \) is implicitly determined by the extreme value condition of (44):
\[ x = \int \frac{\rho_J(s)}{t(x) - s} ds. \]
Thus,
\[ G'(-\chi; J) = t(-\chi) + \frac{1}{\chi}, \]
\[ G''(-\chi; J) = t'(-\chi) + \frac{1}{\chi^2}. \]

The definition of the residual sum of squares \( \text{RSS} \) is given in (88), while it is unrealistic to calculate in practice. In a single hypothesis testing, we estimate the value of \( \text{RSS} \) in this way:
\[ \text{RSS} = \frac{1}{M} \left\| y - A\hat{x}^{LASSO} \right\|_2^2, \]
which gives a reasonable estimate in large system sizes because of the self-averaging property of \( \text{RSS} \).

B. Proof of Claim III.1
We evaluate the free energy density corresponding to the LASSO Hamiltonian \( H(x) = \| y - Ax \|_2^2 / 2 + \lambda \|x\|_1 \) at a zero-temperature limit:
\[ f(\lambda) = -\lim_{\beta \to \infty} N^{-\infty} \frac{1}{\beta N} \mathbb{E}_{A, \xi} [\ln Z(y, A; \lambda, \beta)], \]
where \( \beta \) is the inverse temperature and \( Z \) is the partition function:
\[ Z(y, A; \lambda, \beta) = \int \exp \left( -\frac{\beta}{2} \| y - Ax \|_2^2 - \beta \lambda \|x\|_1 \right) dx. \]
In the zero-temperature limit \( \beta \to \infty \), the Boltzmann distribution \( e^{-\beta H(x)}/Z \) is dominated by the configurations of the LASSO solution. Hence, one can evaluate how the LASSO estimator depends on \( x_0 \), \( A \), \( \xi \) by analyzing the macroscopic behavior of the typical free energy density (53) using statistical mechanics.

Based on the replica method and the replica symmetric (RS) ansatz, the following is claimed.

Claim III.6: Let \{x_0(N), \xi(N), A(N)\}_{N \in \mathbb{N}} be a converging sequence of the row-orthogonal design model. The free energy density (53) can be evaluated by (55) at the top of the next page,
\[
f = \text{extr}_{Q, \chi, \hat{m}, \hat{\chi}} \left\{ G'(-\chi; J) \left( Q - 2m + \frac{\rho}{2} - \frac{\chi^2}{2} + \frac{\gamma}{2} \sigma^2 - \hat{\chi} \hat{\sigma} + 2\hat{m}m \right) + \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \min_{x_i} \left[ \frac{Q}{2} |x_i|^2 - \text{Re} \left( (\hat{m} x_{0,i} + \sqrt{2} \chi z_i)^* x_i \right) + \lambda |x_i| \right] Dz_i \right\},
\]

where \( \text{extr}_X \{ F(X) \} \) denotes the extremization of a function \( F(X) \) with respect to \( X \). We define \( \int (\ldots) Dz \) and \( J \) as follows:

\[
\int (\ldots) Dz = \int (\ldots) \frac{\exp(-|z|^2/\pi)}{\pi} dz,
\]

\[
J = A^H A,
\]

where \( z \) is a complex number and the function \( G(x; J) \) and its derivative \( G'(x; J) \) are defined in (44) and (45).

Proof: The proof is given in Appendix B (available online).

We can then obtain the possible distribution of LASSO solution by the free energy density.

On one hand, from the extreme value condition of the free energy density (55) of variable \( \hat{m} \), that is \( \frac{\partial}{\partial \hat{m}} = 0 \), we have

\[
m = \lim_{N \to \infty} \frac{1}{2N} \sum_{i=1}^{N} \text{Re} \left( x_{0,i}^* \hat{x}_i \right) Dz_i,
\]

where \( \hat{x}_i \) satisfies

\[
\hat{x}_i = \arg \min_{x_i} \left[ \frac{Q}{2} |x_i|^2 - \text{Re} \left( (\hat{m} x_{0,i} + \sqrt{2} \chi z_i)^* x_i \right) + \lambda |x_i| \right].
\]

The above optimization problem has its analytical solution, given by

\[
\hat{x}_i = \text{ST}_h, \hat{\chi} (h_i),
\]

where \( h_i \) is given in (30).

On the other hand, from the definition of the macroscopic physical observable \( m \), that is \( m = \frac{1}{2N} \text{Re} \left( x_0^H x_a \right) \), we have

\[
m = \lim_{N \to \infty} \frac{1}{2N} E_{A, \xi} \left[ \text{Re} \left( x_0^H \hat{x}^{\text{LASSO}} \right) \right].
\]

Comparing (58) with (61), the distribution of the LASSO solution \( \hat{x}^{\text{LASSO}} \) can be inferred as follows:

\[
\hat{x}^{\text{LASSO}} = \hat{x}_i = \text{ST}_h, \hat{\chi} (h_i).
\]

C. Proof of Claim III.2

Denote by \( h = [h_1, h_2, \ldots, h_N]^T \), where \( h_i \) is defined in (30). In the cavity approach, \( h \), called the local field, is assumed to follow a Gaussian distribution by invoking the central limit theorem as \( N \to \infty \) [37]. Denote by \( \langle x \rangle \) the average of \( x \) taken by the Boltzmann distribution \( e^{-\beta H(x)}/Z \), which is given by

\[
\langle x \rangle, \left\langle |x|^2 \right\rangle / (2N) = \arg \min_{m, q} \Gamma(m, Q).
\]

We will evaluate it by expectation consistent approximate inference [38].

Claim III.7: The alternative Gibbs free energy (64) possesses the expression of (66) at the top of the next page under the limit \( \beta \to \infty \), where \( \chi = \beta (Q - q) \) and \( q = \sum m_i^2 / (2N) \).

Proof: The expectation consistent inference provides the following approximation:

\[
\Gamma(m, Q) \approx \phi_{\text{ada}}(m, Q) = \phi(m, Q; l = 0) + \phi^G(m, Q; l = 1) - \phi^G(m, Q; l = 0),
\]

where

\[
\phi(m, Q; l) = \text{extr}_{h, \Lambda} \left\{ \text{Re} \left( h^H m \right) - N\Lambda Q - \frac{1}{\beta} \ln \int e^{-\beta \left[ \|y - Ax\|^2 + \beta \text{Re}(h^H x) - \frac{\beta}{2} \Lambda \|x\|^2 - \beta \lambda \|x\|_1 \right] } dx \right\},
\]

and

\[
\phi^G(m, Q; l) = \text{extr}_{h, \Lambda} \left\{ \text{Re} \left( h^H m \right) - N\Lambda Q - \frac{1}{\beta} \ln \int e^{-\beta \left[ \|y - Ax\|^2 + \beta \text{Re}(h^H x) - \frac{\beta}{2} \Lambda \|x\|^2 \right] } dx \right\}.
\]

With such approximation, \( \phi(m, Q; l = 1), \phi^G(m, Q; l = 1), \) and \( \phi^G(m, Q; l = 0) \) can be easily calculated as shown at the
\begin{equation}
\hat{\phi}(m, Q; l = 0) = \text{extr}_{h, \lambda} \left\{ \text{Re} \left( h^H m \right) - N \Lambda Q - \sum_{i=1}^{N} \frac{|h_i| - \lambda|^2}{2\lambda} \cdot \Theta(|h_i| - \lambda) \right\},
\end{equation}

\begin{equation}
\phi^G(m, Q; l = 1) = -\frac{N}{\beta} G(-\chi; J) + \frac{1}{2} \| y - Am \|^2 + \frac{N}{\beta} \ln \frac{\beta}{2\pi} - \frac{N}{\beta} \ln |\chi| - \frac{N}{\beta},
\end{equation}

\begin{equation}
\phi^G(m, Q; l = 0) = \frac{N}{\beta} \ln \frac{\beta}{2\pi} - \frac{N}{\beta} \ln |\chi| - \frac{N}{\beta},
\end{equation}

(70)–(72), where (70) is calculated under the limit \( \beta \to \infty \). Therefore, \( \phi_{ade}(m, Q) \) possesses the expression of (66).

Taking the limit \( \beta \to \infty \) and \( N \to \infty \), we can get the mean field equation by the extreme value condition on \( h, \Lambda \), and \( Q \) of (66) and linear response argument [31], [37], given by

\begin{equation}
h = \Lambda m + A^H (y - Am),
\end{equation}

\begin{equation}
m_i = \frac{h_i}{|h_i|} \cdot \frac{|h_i| - \lambda}{\Lambda} \cdot \Theta(|h_i| - \lambda),
\end{equation}

\begin{equation}
\Lambda = G'(\chi; J),
\end{equation}

\begin{equation}
\chi = \frac{1}{2\Lambda N} \sum_{i=1}^{N} \left[ \left( 2 - \frac{\lambda}{|h_i|} \right) \cdot \Theta(|h_i| - \lambda) \right],
\end{equation}

(73)–(75), will be derived later, is obtained by linear response argument. Denote by \( \rho_{CA} \), the complex active component density of the LASSO solution as follows:

\begin{equation}
\rho_{CA} = \frac{1}{2\Lambda N} \sum_{i=1}^{N} \left[ \left( 2 - \frac{\lambda}{|h_i|} \right) \cdot \Theta(|h_i| - \lambda) \right],
\end{equation}

which is the same as (II-A). According to (73), one can obtain the debiased estimator \( \hat{\chi} \) as follows:

\begin{equation}
\hat{\chi} = \frac{\rho_{CA} \chi}{\chi} = \frac{G'(\chi; J)},
\end{equation}

(76) then we get the following equations according to the extreme condition in function \( G'(\chi; J) \) (46) and (79):

\begin{equation}
\Lambda = G'(\chi; J) = \frac{\rho_{CA} \chi}{\chi},
\end{equation}

\begin{equation}
-\chi = \int \frac{\rho_{\hat{\chi}}(s)}{t(-\chi) - s} ds,
\end{equation}

where \( \rho_{\hat{\chi}}(s) = t(-\chi) - s \).

(80)–(81)

(82)

which lead to (32) and (33).

We here provide the derivation of \( \chi \) through the linear response argument. Denote by

\begin{equation}
\Gamma_0(m, h, Q, \Lambda) = \text{Re} \left( h^H m \right) - N \Lambda Q
\end{equation}

\begin{equation}
-\frac{1}{\beta} \ln \int e^{-\frac{1}{2} \| y - Ax \|^2 + \beta \text{Re}(h^H x)} \frac{1}{(2\pi)^{\frac{N}{2}}} \Omega(||x||^2 - \beta \lambda ||x||) d\lambda,
\end{equation}

By recalling the definition of the Boltzmann average \( \langle x \rangle \), one can find that

\begin{equation}
\frac{\partial \Gamma_0}{\partial \text{Re}(h_i)} \bigg|_{h_i = h_{0,i}} = \text{Re} \left( m_i \right) - \text{Re} \left( \langle x_i \rangle \right),
\end{equation}

(83)–(85)

(88)

and

\begin{equation}
\frac{\partial^2 \Gamma_0}{\partial \text{Re}(h_i)^2} \bigg|_{h_i = h_{0,i}} + \frac{\partial^2 \Gamma_0}{\partial \text{Im}(h_i)^2} \bigg|_{h_i = h_{0,i}} = \beta \left( \langle |x_i|^2 \rangle - \langle |\chi|^2 \rangle \right),
\end{equation}

We compute \( \chi \) with the following approximation:

\begin{equation}
\chi = \frac{1}{2N} \sum_{i=1}^{N} \left( \frac{\partial^2 \Gamma_0}{\partial \text{Re}(h_i)^2} \bigg|_{h_i = h_{1,i}} + \frac{\partial^2 \Gamma_0}{\partial \text{Im}(h_i)^2} \bigg|_{h_i = h_{1,i}} \right),
\end{equation}

(84)–(86)

D. Proof of Claim III.5

We first define some macroscopic observables.

Claim III.8: The following relationships between the free energy density, regularization term, and residual sum of squares hold:

\begin{equation}
f = \frac{\gamma}{2} \text{RSS} + \bar{r},
\end{equation}

\begin{equation}
\bar{r} = \text{E}_A \xi \left[ \langle \sum_{i=1}^{N} |x_i| \rangle \right] = \bar{\chi} \lambda + 2 \bar{m} m - 2 \bar{Q} Q,
\end{equation}

\begin{equation}
\text{RSS} = \text{E}_A \xi \left[ \text{RSS} \right] = \text{E}_A \xi \left[ \langle \frac{1}{M} \| y - Ax \|^2 \rangle \right],
\end{equation}

\begin{equation}
= \frac{2}{\gamma} \left[ G'(\chi; J) \left( Q - 2m + \frac{\rho}{2} - \frac{\chi}{2} \sigma^2 \right) + \frac{\gamma}{2} \sigma^2 - \bar{\chi} \chi \right],
\end{equation}

(87)–(89)

where \( \bar{r} \) and \( \text{RSS} \) represent the per-element average of the regulation term and residual sum of squares, respectively.
Proof: Consider the extreme value condition of the free energy density \( f \) (55) on \( \hat{Q}, \hat{m}, \) and \( \hat{\chi} \), given by

\[
\frac{\partial f}{\partial Q} = -Q + \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} |\hat{x}_i|^2 D_z = 0, \\
\frac{\partial f}{\partial \hat{m}} = 2\hat{m} - \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \text{Re} \left( \hat{x}_i^* \hat{x}_i \right) D_z = 0, \\
\frac{\partial f}{\partial \hat{\chi}} = -G'(-\chi; J) \left( Q - 2\hat{m} + \frac{\rho}{2} - \frac{\chi^2}{2} \right) - \frac{\sigma^2}{2} G'(-\chi; J) + \hat{\chi} = 0.
\]

(90), (91), and (92) follow the derivation of (45). Derivatives act on (46) leads to the expressions of \( t(-\chi) \) and \( t'(-\chi) \).

IV. NUMERICAL EXPERIMENTS

In this section, we mainly provide numerical simulations to examine the following capabilities.

1. Gaussianity of \( w \) in the case of the row-orthogonal matrix design, where \( \bar{w} = \bar{x}^d - x_0 \) denotes the difference between the debiased LASSO estimator and the original signal \( x_0 \).
2. Accuracy of estimating the variance \( \sigma_w^2 \) in both the cases of the row-orthogonal and the Gaussian design model.
3. Detection performance of debiased LASSO detector.

A. Settings

In all the numerical experiments, we artificially generate the original signal \( x_0 \), the observation matrix \( A \), and the noise \( \xi \). The original signal \( x_0 \) is generated from the Bernoulli-Gaussian distribution: \( p_x = (1 - \rho)\delta(x) + \frac{\rho}{\sqrt{2\pi}\sigma_x} e^{-x^2/2\sigma_x^2} \). For the observation matrix \( A \), we separately consider the Gaussian design and the row-orthogonal design. The former is achieved by setting all the entries of \( A \) i.i.d. complex Gaussian variables: \( A_{ij} \sim CN(0, 1/N) \), and the latter is achieved by randomly selecting \( M \) rows from a randomly generated \( N \times N \) orthogonal matrix. The entries of the noise \( \xi \) are i.i.d. complex Gaussian variables: \( \xi_i \sim CN(0, \sigma_x^2) \). We consider the matched filtering (MF) definition of the signal-to-noise ratio (SNR) [5], that is

\[
\text{SNR} = \frac{\gamma^2 \sigma_x^2}{\sigma^2}.
\]

(101)

B. Gaussianity of \( w \) in the Case of the Row-Orthogonal Matrix Design

We investigate the Gaussianity of \( w \) in the case of the row-orthogonal matrix design by comparing the empirical cumulative distribution function (ECDF) of its real and imaginary parts with a Gaussian distribution. The comparisons are performed for the complex row-orthogonal debiased estimator \( \hat{x}^{d,CROM} \) (with \( \Lambda_{CROM} \) given by (28)) and the complex Gaussian debiased estimator \( \hat{x}^{d,CG} \) (with \( \Lambda_{CG} \) given by (25)). Denote by \( \bar{w}^{CROM} = \hat{x}^{d,CROM} - x_0 \) and \( \bar{w}^{CG} = \hat{x}^{d,CG} - x_0 \). Let \( \bar{w}^{CROM} = \sqrt{2}u^{CROM}/\sigma_{u,CROM}^2 \), \( \bar{w}^{CG} = \sqrt{2}u^{CG}/\sigma_{u,CG}^2 \). Consequently, the ECDF of the real and imaginary part of these two vectors, denoted by \( F_{CROM,i}(x) \), \( F_{CROM,i}(x) \), \( F_{CG,i}(x) \), and \( F_{CG,i}(x) \) respectively, are considered to converge to the standard Gaussian distribution \( \Phi(x) \) weakly. We divide \( \bar{w}^{CROM} \) and \( \bar{w}^{CG} \) into four parts: real and imaginary parts of non-zero entries and zero entries, and verify their Gaussianity by demonstrating the differences such as \( \Phi(x) - F_{CROM,i}(x) \), \( F_{CROM,i}(x) \), \( F_{CG,i}(x) \), and \( F_{CG,i}(x) \) respectively.

The observation matrix \( A \) is a partial Fourier matrix with a size of \( M = 768 \) and \( N = 1024 \). The variance \( \sigma_x^2 \) of the non-zero entries in \( x_0 \) is set to be 1 and the regularization parameter \( \lambda \) of LASSO is 0.1. We set the signal density to be \( \rho = 0.1 \) and SNR to be 5dB. The empirical laws are obtained by 10^5 Monte-Carlo trials.
### Table II

| p-values | Re ($\mathbf{w}_{\text{CROM}}^{\text{H}_1}$) | Im ($\mathbf{w}_{\text{CROM}}^{\text{H}_1}$) | Re ($\mathbf{w}_{\text{CROM}}^{\text{H}_0}$) | Im ($\mathbf{w}_{\text{CROM}}^{\text{H}_0}$) | Re ($\mathbf{w}_{\text{CG}}^{\text{H}_1}$) | Im ($\mathbf{w}_{\text{CG}}^{\text{H}_1}$) | Re ($\mathbf{w}_{\text{CG}}^{\text{H}_0}$) | Im ($\mathbf{w}_{\text{CG}}^{\text{H}_0}$) |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $N = 64$ | 0.0077 | 0.3456 | 0 | 0 | 0 | 0 | 0 | 0 |
| $N = 128$ | 0.0318 | 0.3262 | 0.0002 | 0.0054 | 0 | 0 | 0 | 0 |
| $N = 256$ | 0.1414 | 0.8750 | 0.2601 | 0.0779 | 0 | 0 | 0 | 0 |
| $N = 512$ | 0.8232 | 0.6075 | 0.3676 | 0.2627 | 0 | 0 | 0 | 0 |
| $N = 1024$ | 0.7563 | 0.8259 | 0.6702 | 0.6090 | 0 | 0 | 0 | 0 |

Fig. 2. The difference between the Gaussian distribution and the empirical CDF of the real and imaginary parts of non-zero entries and zero entries of $\mathbf{w}_{\text{CROM}}$ and $\mathbf{w}_{\text{CG}}$.

Fig. 3. REE of CROD, CAMP and SDL-test under the Gaussian and partial Fourier design model.

Fig. 4. REE of CROD, CAMP and SDL-test under the Gaussian and partial Fourier design model.

Fig. 5. REE of CROD, CAMP and SDL-test under the Gaussian and partial Fourier design model.

Fig. 2 shows the difference between the standard Gaussian distribution $\Phi(x)$ and the ECDF of each part of $\mathbf{w}_{\text{CROM}}$ and $\mathbf{w}_{\text{CG}}$. The results show that in the case of partial Fourier matrix design, the ECDF of $\mathbf{w}_{\text{CROM}}$ is very close to a Gaussian distribution, while the ECDF of $\mathbf{w}_{\text{CG}}$ has a significant difference from a Gaussian distribution. We also employ Kolmogorov-Smirnov (KS) test [39] on them, with the p-values shown in Table II. The KS test compares the ECDF of the input samples with a certain distribution (here we set it to be a Gaussian distribution) and the larger of p-values means the higher probability that the samples come from the given distribution.

It is obvious that for each part of $\mathbf{w}_{\text{CROM}}$, KS test verifies that its ECDF well meets the expected theoretical distribution while the other one indicates the opposite result. We remark that although the derivations were made in the asymptotic limit, the Gaussianity holds even if the value of $N$ is only hundreds. Our verification of the Gaussianity of $\mathbf{w}_{\text{CROM}}$ in turn verifies the correctness of the debiased coefficient $\Lambda_{\text{CROM}}$ in the case of the row-orthogonal matrix design.
C. Accuracy of Estimating $\sigma_w^2$

In this section, we investigate the accuracy of the estimation of $\sigma_w^2$ by comparing the relative estimation error of different approaches, which is defined as follows

$$\text{REE} = \frac{|\hat{\sigma}_w - \sigma_w|}{\sigma_w},$$

where $\hat{\sigma}_w$ is the estimated result, and the ground truth $\sigma_w$ is obtained by

$$\sigma_w = \sqrt{\frac{1}{N} \sum_{i=1}^{N} |w_i|^2}.$$  \hspace{1cm} (103)

We compare REE of CROD, CAMP, and SDL-test in the case of the Gaussian matrix design and the row-orthogonal matrix design, respectively. In the following experiments, we set the length of $x_0$ to $N = 256$ and $\sigma_x^2$ to 1. The variance of the noise is $\sigma^2 = 0.05$, corresponding to a SNR of 13dB. Noting that the estimation approach of SDL-test in [16] is not suitable for complex-valued data, we have modified it to the following form:

$$\hat{\sigma}_w = \frac{\sqrt{N}}{\sqrt{\ln 2(\gamma - \rho_{CA})}} \text{median} \left( |y - Ax^{\text{LASSO}}| \right).$$ \hspace{1cm} (104)

Fig. 6. (a) Probability of false alarm and (b) detection of different detectors vary with SNR. Here $\rho = 0.1$ and $\gamma = 0.5$.

D. Detection Performance of Debiased LASSO Detector

We apply several detectors for solving the sub-Nyquist radar detection problem, in which the observation matrix $A$ is partial Fourier, and examine their detection performance. Debiased detectors listed in Table I, including CROD, CAMP, SDL-test and ROD, are compared. Their thresholds $\kappa_d$ are all derived by (20). In the following experiments, we set the length of $x_0$ to $N = 256$ and $\sigma_x^2$ to 1. The probability of false alarm is set to 0.01 and all the results are obtained by $10^5$ Monte-Carlo trials. In Figs. 6–8, we demonstrate how the detection performance of these detectors varies with SNR, signal density, and compression rate, respectively. The results suggest that CROD has the best ability to maintain the false alarm rate under multiple...
parameter variations, i.e., it can most accurately calculate the threshold (or p-value) for a given false alarm rate.

Next, we present a practical experiment. The radar system we used is AWR1843 of Texas Instrument (TI), with a carrier frequency of 77GHz and a bandwidth of 3.36GHz. It transmits 16 pulses in a coherent processing interval (CPI), whose carrier frequencies cover 128 frequency points, resulting in a length of $x_0$ of $N = 16 \times 128 = 2048$, i.e. the radar observation scene contains 2048 range-Doppler bins. We set the radar on a walkway and its observation scene is shown in Fig. 9 and the signal density $\rho$ is about 0.02. The echo has an original length of $N = 2048$, and the compression is achieved by randomly selecting $M = \gamma N$ entries of it. In the following experiments, the false alarm rate is set to 0.01 and all the results are obtained by $10^3$ Monte-Carlo trials. Figs. 10 and 11 show the detection performance of these detectors varies with SNR and compression rate. The experimental results also demonstrate that the proposed detector has a better ability to control the false alarm rate.

V. CONCLUSION

In this paper, we provide the design of the debiased LASSO detector solving the detection problem of compressed sensing radar under the row-orthogonal design model. To this aim, we generalize the derivation of [21] to a complex-valued version. The detection performance of the present detector is theoretically analyzed and proved to be better than that of the LASSO detector. We also compare the proposed approach with other debiased LASSO detectors, simulation results indicate that our approach can provide test statistics and threshold (or p-value) more accurately. Such merit allows precise control of the false alarm rate, resulting in a higher detection rate than other CS detectors.

Fig. 8. (a) Probability of false alarm and (b) detection of different detectors vary with compression rate. Here $\rho = 0.1$ and SNR = 13dB.

Fig. 9. Observation scene of Texas Instruments AWR1843.

Fig. 10. (a) False alarm rate and (b) detection probability of different detectors vary with SNR. Here $\gamma = 1/4$.

Fig. 11. (a) False alarm rate and (b) detection probability of different detectors vary with SNR. Here $\gamma = 1/4$.
Fig. 11. (a) False alarm rate and (b) detection probability of different detectors vary with compression rate. Here SNR = 13dB.

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