D-term Inflation and M-theory

Christopher Kolda† and David H. Lyth‡

† Theoretical Physics Group, Lawrence Berkeley National Laboratory, University of California, Berkeley, CA 94720, USA
‡ Department of Physics, Lancaster University, Lancaster, LA1 4YB, UK

Abstract
Models of supersymmetric D-term inflation require a new mass scale near $10^{15−16}$ GeV in order to match the density perturbation spectrum observed by COBE. Attempts to obtain such a scale from the anomalous U(1) of string theories fail in most string models. However there is hope that models based on non-standard embeddings in M-theory can solve the discrepancy. We will show that such models still suffer from the other drawback of D-inflationary models, namely that Planckian field values are required to drive inflation. Thus it is hard to understand why the inflaton potential remains so flat without imposing stringent symmetries on the superpotential. We also examine a fascinating quasi-fixed point behavior for the gauge coupling of the anomalous U(1) in these extra-dimensional models, and show that either the presence of large numbers of fields in the 5-dimensional bulk or strongly suppressed U(1) gauge couplings is required in order to restore naturality to the inflationary potential.
1 Introduction

Particle physics models of inflation must incorporate two key elements: a potential for the inflaton that is sufficiently flat to allow a long period of inflation (\(\gtrsim 40\) to 60 e-folds) and a mechanism by which that inflation is ended. Hybrid, or "two-field," inflationary models\(^{1,2}\) are probably the simplest and most attractive solution to both of these problems: while one field (the inflaton) slowly rolls, inflation is driven by the false vacuum energy of the second field which is prevented from dropping to its true minimum by its coupling to the inflaton. Eventually, the inflaton vacuum expectation value falls below a critical value at which it can no longer support the second field in its false vacuum. At that time, the second field drops smoothly to its true (zero energy) minimum and inflation ends.

Within the context of supersymmetry\((\text{SUSY})\), a large number of hybrid inflationary models have been built which, at lowest order, provide all of the elements necessary for successful inflation. These SUSY models fall into two classes, because (in the language of global SUSY) vacuum energy is always either the result of non-zero \(F\)-terms or \(D\)-terms. The models of \(F\)-term inflation are known to suffer from a serious problem. If \(\psi\) is the field whose \(F\)-vev drives inflation (\(|F_\psi|^2 = 3M_P^2H^2\) where \(H\) is the Hubble parameter) and \(\phi\) is the inflaton which is slowly rolling down its potential, then at the non-renormalizable level, the Kähler potential generically contains terms of the form:

\[
K = \frac{1}{M_P^2} \phi^\dagger \phi \psi^\dagger \psi
\]

where \(M_P = (8\pi G_N)^{-1/2}\). In a full supergravity scenario, terms such as this one, along with others that arise in the scalar potential, produce a mass term for the inflaton \(m_\phi^2 = a|F_\psi|^2/M_P^2 = aH^2\) with \(a \sim \mathcal{O}(1)\). The resulting equation of motion for \(\phi\) is not slow-rolling, but near critical damping:

\[
\ddot{\phi} + 3H \dot{\phi} + 2aH^2 \phi = 0
\]

This feedback of the SUSY-breaking \(F\)-type vacuum energy into the inflaton potential is very general\(^3\), though ways of avoiding this problem have been discussed in Ref.\(^2\).

The alternative class of models, those in which \(D\)-terms dominate the vacuum energy\(^4\), do not suffer from this same malady since terms corresponding to Eq.\(^1\) do not exist unless the inflaton carries a gauge charge, which one would already assume not to be the case. Models of \(D\)-term inflation make use of a U(1) gauge symmetry for which one can write an explicit energy density which is both SUSY- and gauge-invariant: the Fayet-Iliopoulos term, \(g_\xi \int d^4 \theta V\). In the superpotential, \(\phi\) couples to two fields, \(\psi\) and \(\bar{\psi}\), which carry U(1) charges \(\pm 1\) respectively: \(W = \lambda \phi \bar{\psi} \psi\). The resulting potential is:

\[
V = |\lambda \phi|^2 \left( |\psi|^2 + |\bar{\psi}|^2 \right) + |\lambda \bar{\psi} \psi|^2 + g^2 \left( |\psi|^2 - |\bar{\psi}|^2 + \xi \right)^2
\]

(We will define \(\xi > 0\) for convenience.) For large initial values of \(\phi\), the \(\bar{\psi}\) and \(\psi\) fields receive large masses, forcing them to \(\bar{\psi} = \psi = 0\). The resulting vacuum energy is then \(V_0 = g^2 \xi^2/2\), driving a period of inflation and breaking SUSY. With SUSY broken, a potential for \(\phi\) is generated at one-loop:

\[
V(\phi; \mu) = \frac{1}{2} g^2 \xi^2 \left[ 1 + C \frac{g^2}{8\pi^2} \log \left( \frac{|\lambda \phi|}{\mu} \right) \right],
\]
where \( C \geq 1 \) is the number of pairs of fields to which \( \phi \) couples, and \( V \) is to be evaluated at the scale \( \mu \). Ideally, \( \phi \) rolls slowly in its logarithmic potential until it reaches its critical value, \( \phi_c = g\sqrt{\xi}/\lambda \), at which time it falls quickly to \( \sqrt{\xi} \), SUSY is restored and inflation ends.

This model has two problems, the first well-known but model-dependent, the second less-known but generic. The first problem stems from a calculation of the density perturbations resulting from the above potential. COBE imposes the normalization (see, e.g., Ref. [2]):

\[
(V_0/\epsilon)^{1/4} = 6.7 \times 10^{16} \text{ GeV}
\] (4)

where \( \epsilon \) is one of the slow-roll parameters: \( \epsilon \equiv 1/2 M_P^2 (V'/V)^2 \). In principle, this expression should be evaluated \( N \) e-folds before the end of inflation, where \( N \) is the number of e-folds remaining after the scales probed by COBE left the horizon \( (40 \lesssim N \lesssim 60) \). Plugging into these the potential \( V(\phi; \mu = \phi) \) from above (the result is actually almost independent of the details of \( V(\phi) \)), one finds

\[
\sqrt{\xi} = 8.5 \times 10^{15} \text{ GeV} \times \left( \frac{50C}{N} \right)^{1/4}.
\] (5)

In and of itself, this is not a problem, since we have not specified a source for \( \xi \) in the toy model above. However, the most attractive mechanism for generating \( \xi \) of this order is via the pseudo-anomalous \( U(1) \) of string theory. In many string theories, one finds a \( U(1) \) gauge group, which at the level of the massless fermion fields, appears to be anomalous (it has both \( [U(1)]^3 \) and \( G \times [U(1)]^2 \) anomalies for each group \( G \) in the four-dimensional theory). The anomaly is fictitious, however, as it is eliminated by the Green-Schwarz mechanism: the model-independent axion transforms under the \( U(1) \), cancelling the \( F \tilde{F}\)-terms generated by the anomaly in the fermion sector so that the action remains invariant. However, the transformation of the axion field generates a Fayet-Iliopoulos term for the \( U(1) \) which can be calculated in any string model. In heterotic models, the result was first obtained in Refs. [3]:

\[
\xi = \frac{1}{192\pi^2} \langle \text{Tr} Q \rangle M_s^2
\] (6)

where \( \text{Tr} Q \) sums the \( U(1) \) charges of all massless states, and \( M_s \) is the string scale, equal to \( gM_P \) at weak coupling. For phenomenological values of \( g \approx 0.7 \), \( \xi \) is always larger than \( (2 \times 10^{16} \text{ GeV})^2 \); in realistic string models, \( \text{Tr} Q \sim 100 \) usually, so that typical \( \xi \) are approximately \( (\text{few} \times 10^{17} \text{ GeV})^2 \). In either case, the natural scale for \( \xi \) greatly exceeds the bound imposed by COBE.

We now know that Eq. (6), with \( M_s = gM_P \), holds more generally in string theory [7], as long as the axion whose transformation cancels the anomaly is the partner of the perturbative dilaton. Recently, however, string theories have been discussed in which the anomalous \( U(1) \) and its Green-Schwarz axion arise non-perturbatively. Such models would avoid the constraint of Eq. (6). In this paper we will examine extensions of one particular string model, the so-called Hořava-Witten (HW) model [8]. The minimal HW model itself contains no anomalous \( U(1) \), but its generalizations can [9, 10].

The second problem faced by models of \( D \)-term inflation is more generic. Up until now, we have assumed that inflation ended when \( \phi = \phi_c \). However, inflation also requires
slow-roll, so that $\epsilon$ defined above must be $\ll 1$. Likewise, inflation also requires that the second slow-roll parameter, $\eta \equiv M_P^2 |V''/V|$, be $\ll 1$, since $\eta$ measures the consistency of the slow-roll approximation. Thus inflation could end for $\phi \gg \phi_c$ if either $\epsilon \sim 1$ or $\eta \sim 1$ first. This is in fact what happens \cite{11}. Using the potential in Eq. (3),

$$\eta(\phi) = \sqrt{\frac{C\alpha}{2\pi} \frac{M_P}{\phi}}$$

(7)

where $\alpha = g^2/4\pi$ as usual. Inflation will end when $\eta \sim 1$, which happens when $\phi = \phi_f \simeq \sqrt{C\alpha/2\pi M_P}$. For the phenomenological value of $\alpha \simeq 1/25$ and $C = 1$, inflation ends for $\phi_f \simeq 0.1 M_P$, which is much larger than $\phi_c$ given the COBE normalization of $\xi$ and $\lambda \sim \mathcal{O}(1)$.

This value in and of itself is not overly large, but we must remember $\phi_f$ represents the end of inflation. To calculate the initial value of $\phi$ needed for $N$ e-folds, we use the relation $dN = -(V/V') M_P^2 d\phi$. Solving, one finds

$$\phi_i \simeq \sqrt{\frac{C\alpha N}{\pi} M_P}$$

(8)

which means $\phi_i \simeq 0.8 M_P$ for $N = 50$. As in models of chaotic inflation, field values so close to the Planck scale are not necessarily unreasonable. However, they do mean that the field theory may not be under control. In particular, corrections to the superpotential of the form $\phi^n/M_P^{n-3}$ are not particularly suppressed, so that the slope of $V(\phi)$ is naturally $\mathcal{O}(1)$. It is possible to eliminate the higher-order terms with $R$-symmetries, for example, though this may have a strong impact on the post-inflationary reheating \cite{11}. In any case, one is still left with the conclusion that the inflationary dynamics is occurring at scales where string theory or quantum gravity replace field theory. Thus the model may not be internally consistent.

In this paper, we examine the solution to the first problem (COBE normalization) which is provided by the extended HW models and show that the second problem (Planck-scale inflation) is still present. Thus we will show that models of the HW-type do not solve all the problems faced by $D$-term inflation.

2 Extra Dimensions and Anomalous U(1)’s

The Hořava-Witten action \cite{8} represents one particular limit of M-theory in which 11-dimensional supergravity is compactified in two stages: first, dimensions 6...11 are compactified on a 6-dimensional Calabi-Yau manifold at the string scale as is customary; but the fifth dimension is compactified on a line segment ($S^1/\mathbb{Z}_2$) whose length, $\pi R$, is much larger than the inverse string scale. After the compactification of the fifth dimension, the theory resembles the usual $E_8 \times E_8$ heterotic string but with the surviving subgroups of each $E_8$ (one $E_8$ for the visible sector and one for the hidden sector) on two different 4-dimensional “walls,” a distance $\pi R$ apart. Standard Model gauge interactions and charged matter are confined to one or the other wall and so the Standard Model gauge theory appears 4-dimensional even at distances shorter than $R$. Gravity, however, knows about the additional dimension immediately at $R$. 

3
Once gravitational interactions become 5-dimensional, the gravitational potential falls off more quickly. Matching the 4- and 5-dimensional theories at \( R \), one finds:

\[
M_P^2 = (2\pi R)M_5^3 = (2\pi R)V_{CY}M_5^{11}
\] (9)

where \( M_P \) is the usual 4-dimensional (reduced) Planck mass, \( M_5(M_{11}) \) is its equivalent in the 5(11)-dimensional theory, and \( V_{CY} \) is the volume of the 6-dimensional Calabi-Yau manifold on which the full 11-dimensional theory is compactified down to \( d = 5 \); we identify the string scale by \( M_s = V_{CY}^{-1/6} \). Witten proposed that this matching could explain why the gauge couplings of the MSSM do not meet at the usual weak-coupling string scale [12] but rather at \( 3 \times 10^{16} \) GeV. In this theory, the running of the 4-dimensional gauge couplings is left unchanged, but for \( M_R \equiv 1/\pi R \simeq 5 \times 10^{15} \) GeV, one finds that the “true” 11-dimensional Planck mass, \( M_{11} \), moves down to roughly \( 6 \times 10^{16} \) GeV and the corresponding string scale, \( M_s \), to \( 3 \times 10^{16} \) GeV [12, 13, 10]. Notice that both \( M_P \) and \( M_5 \) are larger than the true \( d = 11 \) Planck scale.

The minimal HW model contains no anomalous U(1) gauge groups. Such a group would have to interact simultaneously with fields on both walls and must then live in the intermediate “bulk,” the region between the two 4-dimensional walls. The action, however, contains no fields which can play the role of the new photon. Thus, this model appears to solve neither of the two problems present in D-term inflation which we discussed earlier.

Nonetheless, the minimal HW action can be extended in a way which is interesting to us. Non-perturbative effects (represented perhaps by adding additional D-branes to the model) can give rise to gauge fields and charged matter in the bulk [9, 10]. One of these groups could be an anomalous U(1), for which a Green-Schwarz mechanism would still operate. Assuming that such a U(1) does arise, the resulting Fayet-Iliopoulos term can be calculated [9]. The result reproduces exactly Eq. (6), where now the trace \( \text{Tr} Q \) is over all matter in the bulk and on either wall and \( M_s \) is henceforth the true string scale, i.e., the unification scale of the MSSM gauge couplings; that is, the apparent 4-dimensional string scale has been replaced with the true 11-dimensional one. Given \( M_s \simeq 3 \times 10^{16} \) GeV, it appears that deriving values of \( \xi \) consistent with the COBE bound is no longer a problem, as was noticed in Ref. [9]. This then would appear to be a great success for both M-theory and D-term inflation.

3 Slow-Roll and the End of Inflation

Before examining D-term inflation in the context of these generalized HW models, we need to verify that the usual inflation model-building will still work. In that vein, we would like to demand that \( H < M_R \) during inflation (i.e., the Hubble radius is larger than \( R \) so that on cosmological scales the universe can be described by a \( d = 4 \) metric) and that inflation is driven by \( d = 4 \) particle dynamics, so that the usual calculation of density perturbations holds [14].

We can actually check to see if \( H < M_R \) self-consistently in these schemes. Given the COBE normalization in Eq. (4) (with \( \epsilon < 1 \)):

\[
H = \left( \frac{V_0}{3M_P^2} \right)^{1/2} < 1.1 \times 10^{15}
\] (10)
which is obviously smaller than $M_R \simeq 5 \times 10^{15}$ GeV. (This result makes more precise a result first presented in Ref. [3].) Thus we can safely do cosmology in 4 dimensions regardless of the existence of the bulk.

Given $H < M_R$, the inflationary potential is unchanged from Eqs. (5) and (6), though now $\xi \sim 10^{15-16}$ GeV, consistent with COBE. The slow-roll parameter $\eta$ is still given by Eq. (1) and the arguments proceed as before. In particular, the requirement of $N \epsilon$-folds of inflation gives $\phi_i > \sqrt{C \alpha N / \pi M_P}$ again. But there is a further requirement which must hold in order for the theory to remain in the field-theoretic regime, and for the non-renormalizable contributions to $W$ to be naturally suppressed, namely $\phi \ll M_{11}$ (recall that the field theory cut-off is no longer $M_P$, but is instead $M_{11}$, and so all non-renormalizable operators are suppressed only by $M_{11}$). Combining these two requirements yields:

$$M_{11} \gg \sqrt{\frac{C \alpha N}{\pi}} M_P \simeq 4 \sqrt{\frac{C \alpha N}{50}} M_P .$$

(11)

For $M_{11} \approx 6 \times 10^{16}$ GeV, the above constraint requires a very small value for $\alpha$, namely $\alpha \lesssim 10^{-5}$.

We turn now to the question of the natural size of $\alpha$, the gauge coupling of the anomalous $U(1)$, evaluated at the string scale. Even without specifying a particular realization of the anomalous $U(1)$ in the bulk, we can still broadly consider the expectations for $\alpha(M_s)$. If $\alpha$ is roughly the same size as the unified coupling on the Standard Model wall, then $\alpha \sim 1/25$, clearly too large. It has also been found [10] in particular embeddings that $\alpha$ is intermediate to the gauge couplings on the two walls; under the usual assumption that the theory on "our" wall is the weaker, then again $\alpha$ is again too large. We cannot rule out the other possibility, that our wall is the strongly-coupled theory, which means that $\alpha$ could be very small indeed (though we know of no suggestions to make it as small as $10^{-5}$).

Of course, the value of $\alpha$ being discussed is its value as measured on the 4-dimensional wall of the Standard Model at the energy scale of the inflationary potential, $\xi$. This value can be calculated from its value at the 5-dimensional string scale via the usual renormalization group procedure. How $\alpha$ will run depends intimately on whether or not there exist any charged fields living in the bulk. If there are no charged fields in the bulk, then $\alpha$ as measured on the wall containing the inflationary potential will run only according to the usual 4-dimensional $\beta$-function:

$$\alpha^{-1}(\mu) = \alpha^{-1}(M_s) + \frac{1}{2\pi}(\text{Tr} Q^2) \log \left( \frac{M_s}{\mu} \right)$$

(12)

at any scale $\mu < M_s$. Here $\text{Tr} Q^2$ sums over all the fields with mass $m < \mu$. The resulting change in $\alpha$ is running down from the string scale is therefore insignificant and does not rescue inflation.

However, in the more likely case that there are charged fields in the bulk, the result changes significantly. Now, the gauge coupling of the $U(1)$ runs not in 4 dimensions but in 5. Or equivalently, the gauge coupling runs in 4 dimensions, but with the contributions of the bulk Kaluza-Klein (KK) excitations included. The renormalization group equation is simplest to derive from this latter point of view [13]. To do so, imagine running $\alpha$ from $M_R$ to some arbitrary scale $\mu$ such that $M_R \leq \mu \leq M_s$. Each state in the “massless” spectrum
(by which we means states with mass \( \ll M_R \), either on the walls or in the bulk) contributes to the \( \beta \)-function a factor \( Q^2/2\pi \). But for each massless state in the bulk, there is a tower of KK excitations with masses \( m_n^2 = n^2 M_R^2 \) where \( n = 1 \cdots \infty \). At the scale \( \mu \) there are \( \mu/M_R \) states with masses \( m < \mu \). Unless \( \mu \) is very close to \( M_R \), these KK contributions will overwhelm the contributions from the massless states in the bulk and on the walls, and so we can drop these latter contributions. The \( \beta \)-function then has the \( \mu \)-dependence one would expect in a 5-dimensional theory:

\[
\frac{d\alpha^{-1}}{d \log \mu} = -\frac{1}{2\pi} \frac{\mu}{M_R} \text{Tr}' Q^2,
\]

where \( \text{Tr}' \) represents a sum over only the massless states in the bulk but not their KK excitations. (The states confined to the walls are also not included in the \( \text{Tr}' Q^2 \) as they contribute only to the logarithmic running which we are dropping.) Running this down from \( M_s \), one finds:

\[
\alpha^{-1}(\mu) = \alpha^{-1}(M_s) + \frac{1}{2\pi} \frac{(M_s - \mu)}{M_R} \text{Tr}' Q^2
\]

for \( M_R \leq \mu \leq M_s \). This running is linear in \( \mu \), not logarithmic, and so \( \alpha \) falls rapidly. At scales \( \mu \ll M_s \), Eq. (14) reduces to:

\[
\alpha(\mu) \simeq \left( \frac{2\pi}{\text{Tr}' Q^2} \right) \frac{M_R}{M_s}.
\]

independent of \( \mu \); even for \( \text{Tr}' Q^2 \) as small as 1, \( \alpha(\mu) \) is suppressed far below its string-scale value. In fact, Eq. (14) can be thought of as a quasi-fixed point solution to the renormalization group equation (14). For almost any initial value of \( \alpha(M_s) \), the value at \( \mu \ll M_s \) will be given by Eq. (15). The only exception would be if \( \alpha(M_s) \lesssim (2\pi M_R)/(\text{Tr}' Q^2 M_s) \).

In the case at hand, with \( M_s/M_R \approx 10 \), the validity and utility of this quasi-fixed point depends strongly on the value of \( \text{Tr}' Q^2 \).

We see that the strong running of \( \alpha(\mu) \), caused by the extra dimension, can sharply reduce its value during inflation. This strong running is however not experienced by \( V_0 = \frac{1}{2} g^2 \xi^2 \). By imposing the requirement \( dV/d\mu = 0 \) at the scale \( \mu = \phi \), one finds the renormalization group equation

\[
\frac{d \log V_0}{d \log \mu} = \frac{C \alpha(\mu)}{2\pi}.
\]

Taking \( \alpha(\mu) \) from Eq. (15), one sees that the fractional change in \( V_0 \) between \( \mu = M_s \) and \( \mu = M_R \) is small. In contrast with the situation for \( \alpha \), it can be neglected just as is done for the case of \( D \)-term inflation without large extra dimensions.\(^1\)

The strong damping of \( \alpha \) due to the extra KK modes in the bulk (for large \( \text{Tr}' Q^2 \)) gives us hope of naturally suppressing \( \epsilon \) and \( \eta \) so that inflation can begin at \( \phi_i \ll M_{11} \). Recall the

---

\(^1\)As is usual, we are taking \( \mu \) to have a fixed value, chosen to be within the regime of \( \phi \) where inflation takes place so that two-loop and higher corrections are negligible. An alternative procedure is to set \( \mu = \phi \) and ignore all loop corrections, giving what is called the renormalization group improved tree-level potential. The two procedures are equivalent provided that the variation of \( \phi \) during inflation is not too big, which is the case for the present model. Indeed, one finds in that case that the second procedure gives a potential that is practically linear in \( \log \phi \), in agreement with the first procedure.
self-consistency condition we derived in Eq. (11). Plugging in the quasi-fixed point result for \( \alpha \) produces:

\[
M_{11}^2 \gg \frac{C\alpha N}{\pi} M_{P}^2 \simeq \frac{2CN}{\text{Tr}^' Q^2} \frac{M_{R} M_{P}^2}{M_{s}^2} \simeq \frac{C N}{\text{Tr}^' Q^2} (10^{18} \text{GeV})^2
\]

(17)

which can only hold if

\[
\frac{C N}{\text{Tr}^' Q^2} \ll 2 \times 10^{-3}
\]

(18)

which in turn requires \( \text{Tr}^' Q^2 \gg 10^{44} \). An equivalent statement is that although the small value of \( \alpha \) did indeed lower \( \phi_i \), the extra dimension also lowered the cut-off scale for the theory, by roughly the same amount. Thus, the HW-type models with one bulk dimension cannot solve the slow-roll problem of 4-dimensional D-term inflation, unless the bulk is full of new fields carrying the U(1) charge, enough to get \( \text{Tr}^' Q^2 \gg 10^{4} \) (or, alternatively, \( \alpha(M_{s}) \) is far below its expected value).

The preceding analysis seems to give hope that in string theories with more than one “large” dimension the fast running of \( \alpha \) (which goes as a power of the number of bulk dimensions) could solve the problem of Planckian field values. Consider what would happen if a string theory in \( d \) dimensions compactified down to \( 4+\delta \) dimensions at some string scale which, for simplicity, we will also identify with the true \( d \)-dimensional Planck scale, \( M_{d} \). Then at some second scale \( M_{R} \) the remaining \( \delta \) dimensions compactify down to give the usual 4-dimensional world. Then we have the relation \( M_{d}^{\delta+2} = M_{R}^2 M_{P}^2 \). The generalization of Eq. (14) requires us to count all the KK modes associated with the \( \delta \)-dimensions. In this case, the KK states are labelled by a \( \delta \)-dimensional set of integers: \( m^2 = \sum_{i=1}^{\delta} n_i^2 M_{R}^2 \).

However, now the number of states with mass \( m < \mu \) is given by \( X_{\delta}^{' \delta} (\mu/M_{R})^{\delta} \) where \( X_{\delta} \) is the volume inside a \( \delta \)-dimensional unit sphere. The renormalization group equation is then:

\[
\frac{d\alpha^{-1}}{d \log \mu} = -\frac{1}{2\pi} X_{\delta}^{' \delta} (\text{Tr}^' Q^2) \left( \frac{\mu}{M_{R}} \right)^{\delta}
\]

(19)

for \( M_{R} < \mu < M_{d} \). This too has a quasi-fixed point solution for \( \mu \ll M_{d} \), namely:

\[
\alpha(\mu) \simeq \left( \frac{2\pi}{X_{\delta}^{' \delta} \text{Tr}^' Q^2} \right) \left( \frac{M_{R}}{M_{d}} \right)^{\delta}.
\]

(20)

Eqs. (11) and (17) generalize to

\[
M_{d}^2 \gg \frac{2CN}{X_{\delta}^{' \delta} \text{Tr}^' Q^2} \left( \frac{M_{R}}{M_{d}} \right)^{\delta} M_{P}^2 = \frac{2CN}{X_{\delta}^{' \delta} \text{Tr}^' Q^2} M_{d}^2
\]

(21)

for

\[
\frac{2CN}{X_{\delta}^{' \delta} \text{Tr}^' Q^2} \ll 1.
\]

(22)

So we find that although the presence of extra dimensions could result in a strong suppression of \( \alpha \) as measured at the scale of the vacuum energy, the corresponding decrease in the \( d \)-dimensional Planck scale cancels the effect completely, so there is nothing to be gained by increasing the number of “large” dimensions.
4 Conclusions

Inflationary models driven by non-zero Fayet-Iliopoulos terms, $\xi$, suffer from a slow-roll problem which requires inflation to begin when the inflaton field value is Planckian. Furthermore, attempts to connect the source of the $\xi$-term to the anomalous $U(1)$ of string theory are inconsistent with the COBE data unless the underlying string theory has some new scales present. The most promising class of such models are the HW models which can be shown to produce $\xi$-terms consistent with COBE. One might also expect these models to solve the slow-roll problem (by suppressing $\alpha$) since the $U(1)$ gauge coupling has a strongly attractive quasi-fixed point at very weak coupling if there is charged matter in the bulk. However, the slow-roll problem remains even in these new models due to a cancellation between the suppression of the gauge coupling and the suppression of the $d$-dimensional Planck scale. Only in models with very large amounts of matter in the bulk or strongly suppressed values of $\alpha(M_s)$ can the field theory be made self-consistent without imposing stringent symmetries on the superpotential.

Acknowledgments

We would like to thank Tony Gherghetta for discussions at the beginning of this project. CK would also like to thank the Department of Physics at Lancaster University for their hospitality while this work was completed. This work was supported in part by NATO grant CRG–970214. CK is also supported by US Department of Energy contract DE–AC03–76SF00098.

References

[1] A. Linde, Phys. Rev. D49, 748 (1994).
[2] D.H. Lyth and A. Riotto, hep-ph/9807278, to appear in Physics Reports.
[3] E. Copeland et al., Phys. Rev. D49, 6410 (1994);
   M. Dine, L. Randall and S. Thomas, Nucl. Phys. B458, 291 (1996).
[4] E. Stewart, Phys. Rev. D51, 6847 (1995);
   E. Halyo, Phys. Lett. B387, 43 (1996);
   P. Binetruy and G. Dvali, Phys. Lett. B388, 241 (1996).
[5] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B28, 589 (1987);
   J. Atick, L. Dixon and A. Sen, Nucl. Phys. B292, 109 (1987).
[6] D.H. Lyth and A. Riotto, Phys. Lett. B412, 28 (1997).
[7] J. March-Russell, Phys. Lett. B437, 318 (1998).
[8] P. Hořava and E. Witten, Nucl. Phys. B460, 506 (1996); Nucl. Phys. B475, 94 (1996).
[9] P. Binetruy, C. Deffayet, E. Dudas and P. Ramond, hep-th/9807079.
[10] Z. Lalak, S. Pokorski and S. Thomas, hep-ph/9807503.

[11] C. Kolda and J. March-Russell, hep-ph/9802358.

[12] E. Witten, Nucl. Phys. B471, 135 (1996).

[13] T. Banks and M. Dine, Nucl. Phys. B479, 173 (1996).

[14] D.H. Lyth, hep-ph/9810320.

[15] K. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436, 55 (1998); hep-ph/9806292.