Abstract

Background/Objectives: Aluminum dross as waste is always a problematic issue. The aim of this study is to apply a systematic method for the Aluminum waste management system selection problem in environment management.

Methods/Statistical Analysis: In this study, a methodology extended fuzzy TOPSIS and based on LCA is applied by using the nearest weighted interval approximations. The functional unit includes aluminum dross and aluminum scrap, which are defined as 1000 kg. The model is confirmed in the case of aluminum waste management in the city of Arak. The scenarios are ranked based on their closeness coefficient to the ideal solution.

Results: Five scenarios, in a step-wise manner were surveyed. Based on study results and particularly the closeness coefficients, scenario S_4 was assigned as the most preferred choice with a weight of 0.723514. Also the scenario S_1 with a value of 0.448137, scenario S_5 with a value of 0.354226, scenario S_2 with a value of 0.314215 and scenario S_3 with a value of 0.204909 were ranked from second to fifth respectively. The results of the present study illustrated that the procedure is simple in calculations and sets priorities. It is very appropriate for solving MCDM problems.

Conclusion/Application: From the application perspective, this research will provide a valuable insight for managers to attempt to improve the environmental, social and economic condition all together at the same time.

Keywords: Aluminum Waste, Dross, LCA, TOPSIS

1. Introduction

In aluminum industries, the secondary aluminum production grows rapidly due to pertaining to the environment considerations and growing of consumption demands. The production of this material will reach 2.60 × 10^7 t in 2015^1. Authors have shown that aluminum waste recycling is an economic and environmentfriendly^2. Complex decision-making situations relates to aluminum waste management system which needs the understanding of different sectors within the industry and society.

In this paper, integrated fuzzy TOPSIS based LCA results methodology have been considered to evaluate the environmental, economic, and social impacts of aluminum waste. The selection aluminum waste management system using environmental attributes together with social and economic criteria based on decision maker opinions become a suitable solution where conflicting objectives exist. Before, a LCA was initially conducted for aluminum producing and recycling processes. Details of the LCA results are reported in other papers. To improve the MCDM method and facilitate aluminum waste management system selection process, the paper will use the
proposed method in a fuzzy environment. In addition, life cycle assessment results evaluate by experts by using fuzzy linguistic terms. This approach allowed us to mathematically represent the uncertainty and vagueness and reflect the decision maker's perception to decision making process.

The rest of this study is so: In section 2, we review some literatures about proposed problem. In section 3, as methodology, we review some basic concept about LCA, Fuzzy Numbers (FNs), the Nearest Weighted Interval Approximation (NWIA), fuzzy TOPSIS method and finally proposed algorithm explained. In section 4, we present a case study related to aluminum waste management. The study ends with discussion and conclusion.

2. Literature Review

Nowadays, aluminum is used in industry, transportation, construction and packaging increasingly. The production of aluminum from bauxite needs much more energy than other metals and lead to large quantities of greenhouse gas emissions. Aluminum production is responsible for 1.1% of the annual greenhouse gas emissions. It is about 75% of all the aluminum produced since the 1880s is still in generative use. Recycling of scrap demands about 20 times less energy than primary aluminum. Among the aluminum LCA, usual, primary aluminum analysis is used for analyzing the environmental impacts of products or system. Several applications of LCA are available in the literature to aluminum, for example. Despite of the fact that previous studies have successfully quantified the environmental impacts of aluminum life cycle several steps in terms of emissions and natural resource consumption, until now, little attention has been given to environmental aspects of processing output of aluminum dross and aluminum scrap as aluminum waste in the LCA studies or other methodologies. LCA was also proven to be a time-consuming and tedious process. In this situation, the main solution is to incorporate social and economic issues into LCA, resulting in an integrated Fuzzy MCDM along with apply LCA results. In this case, using two tools cooperatively can remove or reduce the weakness and provide the condition for using strength points.

The other point is that the FNs is based on the theory of fuzzy sets which presented by. Fuzzy set idea was used by to manage uncertainty in inventory data. In proposed a two-stage methodology, partial indicator obtain in fuzzy space, for LCIA appraisal phase. In modified and compared the fuzzy approach presented in to the “traditional” valuation method of LCA. In made simplified LCA process by fuzzy numbers for representing of the emissions magnitude. Thus the applications of fuzzy logic in LCA are to manage uncertainty, to make less complex LCA process by using fuzzy to appraise the characterization results so that show the significance of impact attributes. TOPSIS method is presented in Chen and Hwang. This method is one of the classic methods for resolving the decision making problem. A more detailed description about the TOPSIS can be found in. In real-world condition, the data are often not so crisp and they commonly are ambiguity. Therefore, some authors employ TOPSIS method for ambiguity data. But, it is limitation for TOPSIS method. In order to overwhelm this limitation, the fuzzy set theory can be used with the traditional TOPSIS approach to allow decision-makers to incorporate unquantifiable information, incomplete information, non-obtainable information, and partially ignorant facts into the decision model. Hence, fuzzy TOPSIS can be successfully used in the various application areas of MCDM problems.

3. Methodology

The aim of this work is to apply a systematic method for the management system selection problem in environment management. We apply a methodology extended TOPSIS in fuzzy environmental numbers and based on LCA by using NWIAa FN.

3.1 Life Cycle Assessment (LCA)

LCA is a systematic method that addresses the environmental aspects and potential environmental impacts throughout a product’s life cycle from raw material acquisition through production, use, end of life treatment, recycling and final disposal (i.e., cradle to grave). But in practice few reviewed studies achieve this due to time, data, and knowledge limits. On the other hand LCA is one of several environmental management methods and might not be the most appropriate method to apply in all condition. LCA typically does not address the economic or social aspects of a product, but the life cycle approach described in International Standard ISO 14040-2006 and ISO 14044-2006 can be used to these other aspects.
3.2 Fuzzy Number (FN)

A FN $A = (a, b, c, d)$ is called a trapezoidal FN if its membership function $A(x)$ has the following form:

$$A(x) = \begin{cases} 
\frac{x-a}{b-a}, & x \in [a,b] \\
1, & x \in [b,c] \\
\frac{d-x}{d-c}, & x \in [c,d] \\
0, & \text{otherwise.}
\end{cases} \quad (1)$$

If $b = c$, then $A = (a, b, d)$ is a triangular FN. (see Figure 1).

Linguistic terms corresponding to FNs in Table 1 were employed for assessment of the criteria. The importance of criteria is identified by decision makers as shown Table 2.

### Table 2. Linguistic variable for scenarios assessment

| Linguistic term       | Membership Function |
|-----------------------|---------------------|
| Very Poor (VP)        | (0,0,1)             |
| Poor (P)              | (1,2,2)             |
| Moderate-Poor (MP)    | (2,3,4)             |
| Moderate (M)          | (3,4,5)             |
| Moderate-Good (MG)    | (5,6,7)             |
| Good (G)              | (7,8,8)             |
| Very Good (VG)        | (9,9,10)            |

3.3 Decision Maker Group

Decision makers are five experts from the production managers of the proposed industries and academics in the environment and Management field who contribute to the decision-making process. The relative importance of each attribute and the preferences of the decision makers are transformed to triangular fuzzy numbers. We use 0–1 and 0–10 scale to express their opinions independently on the rating of the criteria and scenarios with respect to the criteria.

3.4 Decision Matrix in Fuzzy Environment

We consider a group of $k$ decision matrix $(D_1, D_2, \ldots, D_k)$ including $m$ scenarios $S_1, S_2, \ldots, S_m$ and $n$ criteria $F_1, F_2, \ldots, F_n$ for a MCDM problem which is clearly expressed in a matrix format:

$$D = [\bar{x}_{ij}] = \begin{bmatrix} 
\bar{x}_{11} & \bar{x}_{12} & \cdots & \bar{x}_{1m} \\
\bar{x}_{21} & \bar{x}_{22} & \cdots & \bar{x}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\bar{x}_{n1} & \bar{x}_{n2} & \cdots & \bar{x}_{nm} 
\end{bmatrix}, \quad i=1,2,\ldots,m; \quad j=1,2,\ldots,n; \quad k=1,2,\ldots,k. \quad (2)$$

Also, suppose $W = [w_1, w_2, \ldots, w_n]$, be a weighting vector, where $w_i$ is the weight of criterion $F_i$.

3.5 The Nearest Weighted Interval Approximations

Let a weighting function be a function as:

$$f = (f, \bar{f}): ([0,1], [0,1]) \rightarrow (\mathbb{R}, \mathbb{R})$$

so that the functions $f, \bar{f}$ be not negative, regularly increasing and satiates the following normalization condition:

$$\int_0^1 f(a)da = \int_0^1 \bar{f}(a)da = 1 \quad (3)$$

So, let $\tilde{A}$ Be a FN with $A_{\tilde{A}} = [\varphi(\alpha), \bar{\varphi}(\alpha)]$ and $f(\alpha) = (f(\alpha), \bar{f}(\alpha))$ being a weighted function. Then the interval

$$NWIAf(A) = \left[ \int_0^1 f(a)\varphi(\alpha)da, \int_0^1 \bar{f}(a)\varphi(\alpha)da \right] \quad (4)$$

is NWIA to FN $\tilde{A}$.  

![Figure 1. Typical triangular FNs.](image-url)
If \( A \in F \) be a \( FN \) with \( A_a = \left[ q(a), \bar{a}(a) \right] \) and 
\[
 f(a) = \left( \bar{f}(a), \bar{f}(a) \right)
\]
be weighting function, then, the nearest fuzzy-weighted interval approximation (NWIAf) of \( A \) is defined as bellows:

\[
 NWIAf(A) = \left[ C^L_f, C^U_f \right] = \left[ \int_{a_{\bar{L}}}^{1} f(a) \bar{a}(a) da, \int_{0}^{1} f(a) \bar{a}(a) da \right]
\]  
(5)

Where \( C^L_f \) is the nearest lower fuzzy weighted pointed approximation (NLWPAf(A)) and \( C^U_f \) is the nearest upper fuzzy-weighted pointed approximation (NLWPAf(A)).

In practice, the function \( f(a) \) can be chosen according to the actual situation.

If \( A = (a, b, c) \) be a triangular \( FN \) and 
\[
 f(a) = (na^{-2}, na^{-1})
\]
be a weighting function. Then:

\[
 NWIAf(A) = \left[ \frac{a + nb}{n + 1}, \frac{nb + c}{n + 1} \right].
\]  
(6)

Example 1: Let \( \tilde{A} = (3, 4, 7) \) be a triangular \( FN \) and also let 
\[
 f_1(a) = (2a, 2a) \quad \text{and} \quad f_2(a) = (4a, 4a)
\]
be two weighting functions. Then the nearest weighted interval to \( \tilde{A} \) is as follows:

\[
 NWIAf(\tilde{A}) = \left[ \frac{11}{3}, \frac{5}{3} \right], NWIAf_2 = \left[ \frac{19}{5}, \frac{23}{5} \right]
\]  
(7)

3.6 Normalization through Jahanshahloo Method\(^{33}\)

Propose \( X = (x_{ij})_{m \times n} \) be an interval decision matrix which 
\[
 x_{ij} = [x_{ij}^L, x_{ij}^U],
\]
then the normalized interval decision matrix can be stated as \( N = (n_{ij})_{m \times n} \), where \( n_{ij} = [n_{ij}^L, n_{ij}^U] \) and

\[
 n_{ij}^L = \frac{x_{ij}^L}{\sqrt{\sum_{i=1}^{m} (x_{ij}^L)^2 + (x_{ij}^U)^2}}, \quad j = 1, \ldots, n, \quad i = 1, \ldots, m \quad (8)
\]

\[
 n_{ij}^U = \frac{x_{ij}^U}{\sqrt{\sum_{i=1}^{m} (x_{ij}^L)^2 + (x_{ij}^U)^2}}, \quad j = 1, \ldots, n, \quad i = 1, \ldots, m \quad (9)
\]

Now, by regarding the different importance of criteria, we can construct the weighted normalized interval decision matrix 
\( V = (v_{ij})_{m \times n} = \left[ (v_{ij}^L, v_{ij}^U) \right]_{m \times n} \) as:

\[
 v_{ij}^L = w_i^j n_{ij}^L, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n,
\]

\[
 v_{ij}^U = w_i^j n_{ij}^U, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \quad (10)
\]

3.7 Fuzzy TOPSIS

As pointed before, TOPSIS is a multiple criteria method to identify solutions from a set of scenarios and used for ranking problem in real time situations. The basic principle is that the chosen alternative should have the shortest distance from the negative ideal solution. Fuzzy TOPSIS can be successfully used in the various application areas of MCDM problems.

3.8 Algorithm

An algorithm to select the most preferable alternative among all possible scenarios, with extended fuzzy TOPSIS approach based on LCA is given as following steps:

**Step1:** Establish the expert group

In order to assess scenarios, an expert group composed of scientists and managers should be formed.

**Step 2:** Identify criteria and generate the scenarios

For this propose, it is necessary for the decision makers determine criteria and generate the scenarios based on proposed the main target.

**Step 3:** Identify the fuzzy weight of criteria

In this section the fuzzy weight of criteria determine by each decision maker.

**Step 4:** Build the decision matrix and analysis the scenarios

Each decision maker evaluates the scenarios with to respect to the values of the criterion functions which are \( FNs \). (see Equation (2)).

**Step 5:** Aggregate the opinion of decision makers

In this step, the decision makers’ aggregate evaluations are performed. We aggregate fuzzy matrices constructed by decision makers by using geometric mean method and convert them to unit fuzzy matrix as:

\[
 (a^L_{ij}, a^M_{ij}, a^U_{ij}), \quad i, j = 1, 2, \ldots, n \quad (11)
\]

where 
\[
 a^L_{ij} = \left( \prod_{k=1}^{5} a^L_{ik} \right)^{\frac{1}{5}}, a^M_{ij} = \left( \prod_{k=1}^{5} a^M_{ik} \right)^{\frac{1}{5}}, a^U_{ij} = \left( \prod_{k=1}^{5} a^U_{ik} \right)^{\frac{1}{5}}
\]

and 
\[
 (a^L_{ik}, a^M_{ik}, a^U_{ik}), \quad k = 1, 2, \ldots, 5
\]

is the importance opinion of \( K^{th} \) decision maker.
Step 6: Compute \( NWIA \) of each \( FN \ x_{ij} \)

We apply an interval operator and a fuzzy weighted distance quantity in environment fuzzy. Then, we obtain the interval approximations for a \( FN \) (see Equation (6)).

Suppose \( NWIA \) of \( x_{ij} \) is as \([x_{ij}^L, x_{ij}^U]\), then we have an interval decision matrix.

Step 7: Calculate the normalized interval decision matrix. Until now, each \( FN \ x_{ij} \) is transformed to an interval. By (Equation (8) and (9)) this interval transforms into the normalized interval. So, interval \([n_{ij}^L, n_{ij}^U]\) is normalized of interval \([x_{ij}^L, x_{ij}^U]\).

Step 8: Calculate \( NWIA \) to each importance weight \( w_j \) by (Equation (5)).

Suppose \( NWIA \) to \( w_j \) is as: \([w_j^L, w_j^U]\), then we have the vector of interval importance weights.

Step 9: Construct the weighted normalized interval decision matrix

So, we have converted the fuzzy decision matrix into interval decision matrix. (See Equation (10)). Hence, we apply extended fuzzy TOPSIS by the obtained interval decision matrix for rating of the scenarios.

Step 10: Identify positive ideal solution (\( A^+ \)) and negative ideal solution (\( A^- \))

For this work, we employ method of jahanshahloo. At first, we consider the weighted normalized interval ideal solution. Then, we should determine positive ideal solution and negative ideal solution as follows:

\[
A^+ = v^1_i, ..., v^m_i = \left( \frac{\max v^L_{ij} / J \in I, \min v^U_{ij} / J \in I}{} \right) \tag{12}
\]

Where \( I \) is related to benefit criteria, and \( J \) is related to cost criteria.

\[
A^- = v^-1_i, ..., v^-m_i = \left( \frac{\min v^L_{ij} / J \in I, \max v^U_{ij} / J \in I}{} \right) \tag{13}
\]

Where \( I \) is related to benefit criteria, and \( J \) is related to cost criteria.

Step 11: Obtain positive ideal solution \( (d^+ \) and negative ideal solution \( (d^- \)

We calculate the separation of each scenario from \( (d^+ \) and \( (d^- \). The separation of each scenario \( i \), \( i = 1, ..., m \) from the positive ideal solution, using the n-dimensional Euclidean distance, can be usually formulated as:

\[
d^+ = \left\{ \sqrt{\sum_{j \in I} \left( v^L_{ij} - v^+ j \right)^2 + \sum_{j \in I} \left( v^U_{ij} - v^+ j \right)^2} \right\}^{\frac{1}{2}} \tag{14}
\]

Step 12: Calculate the relative closeness of each alternative to positive ideal solution \( (R) \)

A closeness coefficient is defined to determine the ranking order of all scenarios once the \( d^+ \) and \( d^- \) each scenario \( S_i \) has been calculated. The relative closeness of the scenario \( S_i \) with respect to \( A^+ \) defined as:

\[
R_i = \frac{d^-}{(d^+ + d^-)}, \ i = 1, ..., m. \tag{16}
\]

Step 13: Rank the priority order of all scenarios with respect to the closeness coefficient.

The scenarios are ranked based on their closeness coefficient to the ideal solution. A scenario will be the best scenario only with the largest relative closeness to the positive ideal solution.

4. Case Study

A case study on aluminum industries and particularly aluminum remelting plants in the Arak industrial area in the central of the Islamic Republic of Iran is presented to illustrate how the proposed fuzzy TOPSIS methodology according to above described algorithm can be applied to support decision making for environmentally friendly aluminum waste system with regard social and economic aspects. Twenty-nine re-melting facilities were incorporated in this research.

Secondary aluminum re-melting considered as unit function. All the results are based on the reference flow of 1 ton of aluminum bath include aluminum new and old scraps and aluminum dross. The operations associated with this unit process include:

- Recovery of internal process scrap.
- Metal treatment and casting operations.
- Upkeep and repair of plant and equipment.

Materials are provided to this life-cycle stage from both primary and secondary aluminum processing. In this study, crucible furnace for secondary aluminum re-melting is used. As pointed before, a LCA was initially conducted for aluminum producing and recycling processes. Details of the LCA results are reported in
other papers. The criteria used for the aluminum waste management system selection is identified through literatures. We developed the questionnaire to achieve the weight preference for the criteria and to rank the scenarios related to the criteria. After this, a questionnaire was send to the academic group for content evaluate. Then, decision makers, who similar with the context of aluminum waste management systems are invented to evaluate the scenarios of improved questionnaire. Decision makers were five experts from the production managers of the proposed industries and academics in the environment and Management field who contribute to the decision-making process.

We were, finally, selected criteria as: Global Warming (GW), Human Toxicity (HT), Land Use (LU), Health and Safety at work (H&S), Regulation (Reg), turnover and gain.

Also, five management scenarios for aluminum waste management in the city of Arak presented:

- Scenario S₁ is defined as “positive transactor system”: Aluminum crucible for re-melting, aluminum batch include primary aluminum ingot 99.5–20% and secondary aluminum –aluminum scrap 98–80%, beneficiation activities related to input aluminum scrap such as washing, separating and sorting, export aluminum dross to other place and duplicate recycling, landfill.

- Scenario S₂ is defined as “current system”: Aluminum crucible for re-melting, aluminum batch include primary aluminum ingot 99.5–20% and secondary aluminum–aluminum scrap 98–80%, re-melting without beneficiation activities, export remain aluminum dross to other place and duplicate recycling, release in environment.

- Scenario S₃ is defined as “trafficker system”: Aluminum crucible for re-melting, aluminum batch include primary aluminum ingot 99.5–20% and secondary aluminum–aluminum scrap (in form billets from Iraq country) 96–80%, re-melting without beneficiation activities, export remain aluminum dross to other place and duplicate recycling, release in environment.

- Scenario S₄ is defined as “proposed system” aluminum crucible for re-melting, aluminum batch include primary aluminum ingot 99.5–20% and secondary aluminum–aluminum scrap 98–80%, beneficiation activities related to input aluminum scrap such as washing, separating and sorting, duplicate aluminum dross recycling in plant, release in environment.

- Scenario S₅ is defined as “business system”: Aluminum crucible for re-melting, aluminum batch include primary aluminum ingot 99.5–20% and secondary aluminum –aluminum scrap 98–80%, beneficiation activities related to input aluminum scrap such as washing, separating and sorting, duplicate aluminum dross recycling in plant, release in environment.

Then, details of five different waste management scenarios were presented to the experts before the evaluation. Once the responses were received, the questionnaire results were checked and interviews were conducted to ensure the data validity. The five decision makers express their opinions on the ratings of each alternative with respect to the seven criteria separately. The opinions of various experts were integrated to create one matrix though the geometric mean method. The computed fuzzy aggregated criteria weights matrix respect to scenarios shown in Table 5. Also Table 4 represents aggregated Fuzzy relative importance and the nearest weighted interval approximation of criteria by the decision makers.

| Criteria | F₁ | F₂ | F₃ | F₄ | F₅ | F₆ | F₇ |
|----------|----|----|----|----|----|----|----|
| D₁ | H | VH | H | MH | MH | M | M |
| D₂ | H | H | VH | MH | MH | M | MH |
| D₃ | H | VH | VH | H | MH | M | MH |
| D₄ | H | H | MH | MH | MH | MH | MH |
| D₅ | MH | H | MH | MH | H | M | MH |

Table 3. Linguistic assessments criteria by the decision makers
and social aspects are more important. Scenario S₄ represents 'waste export' where combines the concept of recycling and waste export, where approximately 70% of aluminum waste that is normally land filled, are exported to other place for duplicate recycling and finally, is land filled. Scenario S₅ is defined as the environment friendly aluminum waste management system.

After that each FNₓᵢ was transformed to an interval, the obtained fuzzy interval matrix is normalized by following Equations (10) (See Table 7).

The interval weights of the criteria were product in the normalized matrix to form an interval weights normalized fuzzy decision matrix (See Table 8).

After obtaining the weighted normalized fuzzy decision matrix, the interval fuzzy positive ideal solution (IFPIS) and the Interval Fuzzy Negative Ideal Solution (IFNIS) are determined applying Equations (11) and (13) (Table 9). Then, the distance of the scenarios from the FPIS and FNIS is computed applying Equations (14) and (15) (Tables 10 and 11).

The final results obtained from the proposed fuzzy TOPSIS approach for the MCDM problem of the aluminum waste management systems required for the aluminum companies are shown in the Table 12. (Equation (16)).

As shown in Table 12, scenario S₄ is assigned as the most preferred choice with a weight of 0.723514, followed by scenario S₁ with a value of 0.448137, scenario S₅ with a value of 0.354226, scenario S₂ with a value of 0.314215, scenario and S₃ with a value of 0.204909.
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### Table 7. Normalized fuzzy-decision matrix for the proposed scenarios

| D_i | F_1      | F_2      | F_3      | F_4      | F_5      | F_6      | F_7      |
|-----|----------|----------|----------|----------|----------|----------|----------|
| S_1 | [0.35,0.36] | [0.3,0.32] | [0.3,0.32] | [0.43,0.46] | [0,0.03] | [0.0,14.16] | [0.41,0.43] |
| S_2 | [0.35,0.36] | [0.3,0.32] | [0.3,0.32] | [0.16,0.19] | [0,0.03] | [0.3,0.32] | [0.42,0.44] |
| S_3 | [0.3,0.31]  | [0.28,0.3] | [0.32,0.34] | [0,0.02]  | [0,0.02] | [0.29,0.31] | [0.28,0.31] |
| S_4 | [0.15,0.16] | [0.27,0.29] | [0.3,0.32] | [0.44,0.47] | [0.64,0.67] | [0.38,0.39] | [0.15,0.17] |
| S_5 | [0.36,0.37] | [0.37,0.38] | [0.29,0.31] | [0.23,0.26] | [0.23,0.26] | [0.36,0.37] | [0.12,0.14] |

### Table 8. Weighted normalized fuzzy-decision matrix for the proposed scenarios

| D_i | F_1      | F_2      | F_3      | F_4      | F_5      | F_6      | F_7      |
|-----|----------|----------|----------|----------|----------|----------|----------|
| S_1 | [0.26,0.29] | [0.25,0.28] | [0.23,0.27] | [0.28,33] | [0,0.02] | [0.0,07.09] | [0.23,0.27] |
| S_2 | [0.26,0.29] | [0.25,0.28] | [0.23,0.27] | [0,1.13]  | [0,0.02] | [0.14,0.17] | [0.16,0.19] |
| S_3 | [0.22,0.25] | [0.24,0.27] | [0.24,0.28] | [0,0.01]  | [0,0.02] | [0.15,0.18] | [0.23,0.27] |
| S_4 | [0.11,0.13] | [0.23,0.26] | [0.23,0.27] | [0.28,0.33] | [0.42,0.47] | [0.19,0.22] | [0.08,0.11] |
| S_5 | [0.26,0.29] | [0.31,0.34] | [0.23,0.26] | [0.15,0.18] | [0.15,0.18] | [0.18,0.21] | [0.07,0.08] |

### Table 9. Interval FPIS (A⁺) and interval FNIS (A⁻) for each criterion

| Criterion | F_1  | F_2  | F_3  | F_4  | F_5  | F_6  | F_7  |
|-----------|------|------|------|------|------|------|------|
| A⁺        | [0.109303,0.109303] | [0.232170,0.232170] | [0.228908,0.228908] | [0.333660,0.333660] | [0.479906,0.479906] | [0.071944,0.071944] | [0.27187,0.27187] |
| A⁻        | [0.293564,0.293564] | [0.341007,0.341007] | [0.282193,0.282193] | [0,0]  | [0,0]  | [0,219311,0.219311] | [0.068484,0.068484] |

### Table 10. d⁺ Values for various scenarios in FTOPSIS method

| d⁺ value | d⁺ value |
|----------|----------|
| d1⁺      | 0.507585 |
| d2⁺      | 0.559379 |
| d3⁺      | 0.610619 |
| d4⁺      | 0.234619 |
| d5⁺      | 0.476333 |

### Table 11. d⁻ Values for various scenarios in FTOPSIS method

| d⁻ value | d⁻ value |
|----------|----------|
| d1⁻      | 0.412182 |
| d2⁻      | 0.256298 |
| d3⁻      | 0.157368 |
| d4⁻      | 0.613958 |
| d5⁻      | 0.261282 |

### Table 12. Final ranks of the aluminum waste management system scenarios

| Scenario | Closeness Coefficient | Rank |
|----------|-----------------------|------|
| Scenario 1 | 0.448137               | 2    |
| Scenario 2 | 0.314215               | 4    |
| Scenario 3 | 0.204909               | 5    |
| Scenario 4 | 0.723514               | 1    |
| Scenario 5 | 0.354226               | 3    |

5. Discussion
As previously noted, the global secondary aluminum production grows rapidly because of environmental considerations and continuous growing of consumption demands. In this case, over 200kg of aluminum black dross as waste are produced for each ton of secondary aluminum black dross, is either duplicate recovered as by-products...
or landfilled. Releasing this quantity in environment can produce considerable consequences from air, water and soil pollution point of view. The presented model for waste management scenarios implemented in the case of aluminum waste systems in the industrial city of Arak. The life-cycle thinking is a unique way of addressing environmental problems from a system or holistic perspective. In this way of thinking, a product or system can be with a goal of reducing environmental impacts over its whole life cycle. On the basis of such consideration, the application of LCA to aluminum waste management system will be a feasible way. However, LCA studies have generally an intrinsic uncertainty related to various categories. In addition, no single solution is available as each industry in each country has different characteristics in terms of geographical environmental as well as social and economic aspects. Several management decisions are required to provide efficient aluminum waste management systems. The aim of this paper is to propose an evaluation model combining LCA and fuzzy TOPSIS in aluminum waste management system. This paper has presented a model, based on the fuzzy logic, to evaluate aluminum waste management systems. It can systematically evaluate and contains interdependency relationship among criteria under uncertainty. The results of the present study illustrated that the procedure is simple in calculations and set priorities. On the other hand, it is very appropriate for solving MCDM problems. At this study we applied environmental, social and economic criteria for evaluation and support decision-making within the aluminum industry as a MCDM problem. The result of study shown that fuzzy TOPSIS can be used not only as a way to handle the inner dependences within a set of aspects and criteria, but also as a way of producing more valuable information for decision making. This model, also have disadvantages. For example fuzzy TOPSIS model uses aggregated categories data in which several subcategories are evaluated under the same main category. This will increase the uncertainty in fuzzy TOPSIS-based in LCA results that can be solved by using more specific life cycle data for several steps of aluminum waste. It is important to note that the weights of criteria obtained from expert judgment are also subject to uncertainties. With changing weights, a Fuzzy MCDM decision making method might give different results for the ranking of aluminum waste management scenarios. It is important to note that this research contributed to the emerging field of life cycle assessment results with an integrated methodology that includes expert judgment and a fuzzy MCDM. Presented criteria are broken-down into Seven categories, namely: GW, HT, LU, H&S, regulation, turnover and gain. First, decision makers evaluated each waste management scenario for selected criteria. Second, we obtained these evaluation results taking into account the weight of each criterion in a fuzzy environment. In this paper the nearest weighted interval approximation (NWIA) are used in order to reduce the ambiguity of the linguistic assessments. Hence we transformed collected data into interval fuzzy version. After finally, we applied real MCDM problem for proposed decision making method which aims to rank scenarios. Therefore, integrated fuzzy TOPSIS is used to model the aluminum waste management options selection for aluminum industries.

Based on study results, and particularly the closeness coefficients, the ranks obtained using extended fuzzy TOPSIS method for the available five aluminum waste management systems are as follows:

- Proposed System.
- Positive Transactor System.
- Business System.
- Current System.
- Trafficker System.

From the obtained results, we can conclude that scenario $S_5$ with a weight of 0.723514 has the highest ranking compared to other scenarios. It was first preference to be selected for the aluminum waste management system. Also the scenario $S_5$ and scenario $S_4$ were ranked second and third, respectively. Apart from these three, other scenarios of aluminum waste management system as scenario $S_2$ and scenario $S_3$, were ranked fourth and fifth, respectively. Each scenario presents a solution for the aluminum waste management system with a certain degree of trade-off between benefit and its consequences related to environmental, social and economic aspects. For example, despite selection choice of scenario $S_3$ and scenario $S_4$ could be increased by increasing amount of aluminum scraps related to primary aluminum production process. Also scenarios $S_1$, $S_2$ and $S_3$ represent the export of aluminum waste to other places in the form of black dross or aluminum dross with less metallurgical aluminum. From the application perspective, this research will provide a valuable insight for managers to attempt to improve the environmental, social and economic condition all together at the same time.
6. Conclusions

Until now, little attention has been given to environmental aspects of processing output of aluminum dross and aluminum scrap as aluminum waste. The aim of this study is model combining LCA and fuzzy to rank and select the most appropriate aluminum waste management system based on decision makers’ judgments. All the information collected was related to LCA result, written documents and findings from interviews. For the proposed integrated fuzzy TOPSIS model, the five scenarios are investigated. In this study we applied the NWIA-a-FNs to convert each fuzzy element of the matrix to its NWIA. The results showed that scenario S_5 was assigned as the best and scenario S_4 was assigned as the worst preferred choice with weights of 0.723514 and 0.204909, respectively. At the scenario S_5, aluminum batch includes primary aluminum ingot 99.5-20% and secondary aluminum-aluminum scrap 98-80%, beneficiation activities related to input aluminum scrap such as washing, separating and sorting, duplicate aluminum dross recycling in plant and landfill. While in scenario S_4, aluminum batch include primary aluminum ingot 99.5-20% and secondary aluminum-aluminum scrap (in form billets from Iraq country) 96–80%, re-melting without beneficiation activities, export remain aluminum dross to other place and duplicate recycling and release in environment.

According above consideration, uncertainty in the LCA results and limitations of the current method should be taken into account by decision makers. First of all, the proposed methodology can be utilized for similar problems where multiple criteria are present. This study will provide a valuable insight for managers to attempt to improve the environmental, social and economic condition all together at the same time. In future research, current fuzzy approach can be developed and apply for different MCDM problems in industry where conflicting criteria exist.

7. References

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