Moduli Stabilisation in Heterotic Models with Standard Embedding

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Abstract

In this note we analyse the issue of moduli stabilisation in 4d models obtained from heterotic string compactifications on manifolds with $SU(3)$ structure with standard embedding. In order to deal with tractable models we first integrate out the massive fields. We argue that one can not only integrate out the moduli fields, but along the way one has to truncate also the corresponding matter fields. We show that the effective models obtained in this way do not have satisfactory solutions. We also look for stabilised vacua which take into account the presence of the matter fields. We argue that this also fails due to a no-go theorem for Minkowski vacua in the moduli sector which we prove in the end. The main ingredient for this no-go theorem is the constraint on the fluxes which comes from the Bianchi identity.
1 Introduction

In recent years there have been considerable progress in the field of moduli stabilisation in string compactifications especially due to the idea of flux compactification (for reviews see [1, 2]). However, moduli stabilisation in realistic models is far from being well understood. This is an important issue in trying to construct viable models from string theory as, stabilising moduli by using fluxes may influence the matter sector in a significant way. It is not really clear in general whether the moduli stabilisation sector is independent of the matter sector and if this is not the case, what is the relation between these sectors. The best one can achieve so far is to analyse this kind of questions on a case by case basis.

Moduli stabilisation in realistic models[1] was studied mostly in type II or F-theory models, mainly due to the better control on the fluxes in these theories [3, 4, 5, 6]. As it is well known, the lack of ordinary fluxes in heterotic string is a big challenge for the moduli stabilisation problem. The presence of fluxes inevitably deforms the compactification geometry away from Calabi–Yau manifolds [7]–[19] and one has to deal with non-Kähler manifolds. Moreover, the simple solutions with fluxes in type IIB are mapped via duality to torsional backgrounds in heterotic theories [9, 20]. Therefore, one has to understand the theories which result from compactifications on backgrounds with torsion. Using a supergravity approach, the effective theory which comes from heterotic string compactification on certain manifolds with $SU(3)$ structure was derived in [21, 22]. The background consists of half-flat manifolds, which have been encountered in the context of mirror symmetry [23, 24], and generalisations thereof [25, 26, 27, 28]. In [29] we started to consider the moduli stabilisation problem in the models of [22] and here we shall continue along the same route in that we make a supergravity analysis of the moduli stabilisation problem in models which contain matter fields. The particular setup we shall consider is that of the standard embedding. To be more precise, we have in mind as a starting point, heterotic string compactifications on Calabi–Yau manifolds with standard embedding. The actual background is obtained by deforming the Calabi–Yau manifold into a manifold with $SU(3)$ structure as prescribed in [23] so that we still retain control on the compactification procedure and we are able to write down an effective action in four dimensions[2].

The good part of this approach is that the calculation holds for any Calabi–Yau manifold and its corresponding deformation to a manifold with $SU(3)$ structure. This is mainly because of the standard embedding we use for solving the Bianchi identity. In particular it can be applied to the interesting models with three generations recently found in [31]. On the other hand, as we shall see later in the paper, the format of the theory is fairly rigid and the connection between the matter and moduli sector (which is again due to the standard embedding procedure) makes it difficult – if not impossible – to find good models with stabilised moduli.

Finding solutions in supergravity coupled to a few chiral fields is already a challenging task. For the case at hand we would have to deal with large number of fields, both neutral

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1By realistic here we mean models which include a matter sector.
2For another view on the standard embedding in heterotic string compactifications on manifolds with $SU(3)$ structure see [30].
and charged and therefore a complete classification of the solutions is far outside our reach. Instead, we shall find effective models – which are easier to solve – by integrating out certain massive fields. It turns out that the masses for the matter fields and for the moduli are of the same order, and therefore, if we want to integrate our certain moduli fields we should also integrate out the corresponding matter fields. This will essentially leave us with very simple models which have very small flexibility of finding good solutions.

One important aspect which we will have in mind is that of the stabilisation of the dilaton. The superpotentials we consider do not depend on the dilaton and therefore the only possibility which is left, is to consider its stabilisation via non-perturbative effects such as gaugino condensate in the hidden sector. For a realistic stabilisation we would need to obtain a small value for the superpotential after stabilising all other fields. This is actually the main challenge in our analysis as it was realised long ago [32, 33]. Finding small values for the flux superpotential directly seems difficult because of the fact that the effective models we work with have a too simple superpotential which can not be made small and at the same time ensure large values for the moduli fields which are required by the consistency of the compactification. Another possibility we consider is to find small superpotentials which are generated from the matter fields. However this requires a vanishing flux-superpotential which we show it is impossible to obtain in the context of the standard embedding.

The organization of the paper is as follows. In section 2 we shall review the model under consideration. This was originally obtained in [22]. In section 3 we shall analyse the issue of the masses of the moduli and matter fields. We shall make the remark that the masses for matter and moduli fields are related and therefore if we wish the moduli to get large masses, the same will happen with certain matter fields. Therefore when integrating out massive moduli we would have to integrate out at the same time the massive matter fields as well. In section 4 we study two types of effective models, the first one containing only Kähler moduli and the second containing only complex structure moduli. We shall argue that none of these models has satisfactory solutions. Finally in section 5 we shall analyse the possibility of obtaining a small superpotential only using the matter sector. For this we will need to find solutions with vanishing flux superpotential, but as we shall claim in this section this turns out to be impossible provided one satisfies the Bianchi identity. In section 6 we present our conclusions.

2 The general model

As explained in the Introduction we shall stay in the realm of heterotic string compactifications on manifolds with $SU(3)$ structure with standard embedding. To be more precise, we consider Calabi–Yau compactifications of the $E_8 \times E_8$ heterotic string with standard embedding and deform the compactification manifold to one with $SU(3)$ structure [22]. The effective four-dimensional theory consists of $N = 1$ supergravity coupled to a $E_8$ su-
per Yang-Mills theory\(^3\) and to a certain number of chiral (super)fields. The chiral fields – which we shall be concentrating on in the following – come in two categories:

- neutral fields: the axio-dialton, \( S \), the Kähler moduli, \( T^i \) and the complex structure moduli \( Z^a \).
- charged (matter) fields: transforming in the representation \( 27, D^a \) and in \( \overline{27}, C^i \), of \( E_6 \) (we have suppressed the \( E_6 \) index)

On top of these fields one also has to consider the so-called bundle moduli which parameterise the possible deformations of the gauge bundle. However, dealing with these fields is still more difficult and we shall not consider them in the following.

The Kähler moduli, as well as the matter fields in \( 27 \) are in one to one correspondence with the \((1,1)\) forms on the compactification manifold and therefore they carry an index \( i = 1, \ldots, h^{1,1} \) while the complex structure moduli and the matter fields in \( 27 \) are in one to one correspondence with the \((2,1)\) forms and therefore carry an index \( a = 1, \ldots, h^{2,1} \), where \( h^{1,1} \) and \( h^{2,1} \) are dimensions of the corresponding cohomology groups. The kinetic terms for the chiral fields are given in terms of the Kähler potential

\[
K(S, T, Z, C, D) = K_0(S, T, Z) + \alpha' K_1(T, Z, C, D) ,
\]

where

\[
K_0 = -\log (S + \bar{S}) - \log \frac{1}{6}[K_{ijk}(T^i + \bar{T}^i)(T^j + \bar{T}^j)(T^k + \bar{T}^k)] - \log \frac{1}{6}[	ilde{K}_{abc}(Z^a + \bar{Z}^a)(Z^b + \bar{Z}^b)(Z^c + \bar{Z}^c)]
\]

\[
K_1 = 4e^{(K_{Cas} - K_{K})/3} g_{ij} C^{aP} \tilde{C}^{jP} + e^{(K_{K} - K_{Cas})/3} g_{ab} D^{aP} \bar{D}^{bP}_P - 2 \left( K_i K_a C^i_P D^{aP} + \text{c.c.} \right)
\]

So far, this is the usual result of Calabi-Yau compactifications of the heterotic string with standard embedding. The fact that we consider a manifold with \( SU(3) \) structure and background fluxes shows up in the superpotential which reads

\[
W(T, Z, C, D) = W_0(T, Z) + \alpha' W_1(Z, C, D) ,
\]

where

\[
W_0 = i \left( \xi + i e_i T^i \right) + \left( \epsilon_a + i p_{ia} T^i \right) Z^a + \frac{i}{2} \left( \mu^a + i q^a_i T^i \right) \tilde{K}_{abc} Z^b Z^c + \frac{1}{6} \left( \rho + i r_i T^i \right) \tilde{K}_{abc} Z^a Z^b Z^c
\]

\[
W_1 = 2 \left[ i p_{ia} + \left( \frac{1}{2} r_i Z^a - q^a_i \right) \tilde{K}_{abc} Z^b \right] C^i D^c
\]

\[
- \frac{1}{3} \left[ K_{ijk} j_{PRS} C^{aP} C^{jR} C^{kS} + \tilde{K}_{abc} j_{PRS} D^{aP} D^{bR} D^{cS} \right].
\]

In the above we have used the following conventions and notation. \( K_{ijk} \) and \( \tilde{K}_{abc} \) denote the triple intersections on the manifold with \( SU(3) \) structure we use in order to compactify

\(^3\)There is also a hidden sector consisting of the second \( E_6 \) factor of the ten-dimensional gauge group, but this will be irrelevant for most of the issues discussed here.
the heterotic string and on its mirror respectively. By $K_{\epsilon a}$ and $K_K$ we have denoted the parts in the zeroth order Kähler potential, $K_0$, which depend on the complex structure and Kähler moduli respectively. Finally, the symbols $\xi$, $e_i$, $\epsilon_a$, $p_{\alpha a}$, $\mu^i$, $q_i^\alpha$, $\rho$ and $r_i$ are the flux parameters. The Latin letters denote the fluxes due to the $SU(3)$ structure while the Greek letters denote the usual $H$-fluxes.

The setup above is subjected to the following constraints

$$
\begin{align*}
p_{\alpha a}q_i^\alpha - p_j^a q_i^a - e_i r_j + e_j r_i &= 0, \\
\xi r_i - \epsilon_a q_i^a + \mu_{\alpha a} p_{\alpha a} - \rho e_i &= 0,
\end{align*}
$$

(2.7)

where the first of the constraints comes from the consistency conditions on the manifold with $SU(3)$ structure while the second comes from the Bianchi identity $dH = 0$ which has to be satisfied in the standard embedding case. The model above was derived in [22] in a supergravity approximation and in order to insure its validity one has to make sure that the moduli stay in the large volume and large complex structure regime. Mathematically this translates into the conditions

$$
T^i + \bar{T}^i \gg 1; \quad Z^a + \bar{Z}^a \gg 1.
$$

(2.8)

On the other hand, the action for the charged fields is obtained perturbative and this means it is valid only for matter fields which are small fluctuations around the zero value

$$
|C^i| \ll 1; \quad |D^a| \ll 1.
$$

(2.9)

Clearly, in general, the setup above is quite complicated as it contains a large number of fields and parameters, and therefore it is tedious to analyse. In order to make any progress we shall consider that pairs of fields get large masses (due to the fluxes) and we can integrate them out. We shall make this more precise in the next section where we shall argue that the masses for (some of) the gauge fields and those for the moduli are related and we can not integrate out only moduli or matter fields.

### 3 Masses for moduli and matter fields

In this section we study the masses for the moduli and matter fields in the model presented previously. The motivation behind is that if we find that certain fields acquire large masses, we can try to implement a two step procedure in finding the vacuum, where in the first instance we integrate out the heavy fields and then, in the second step we find the vacua of the remaining theory. Therefore, if we have fluxes which are generic enough, we can hope that the effective model we have to analyse is sufficiently simple so that we can analyse it analytically.

Let us assume that in the first step we are able to find a supersymmetric solution which preserves the full $E_6$ gauge group. It is only a matter of algebra to compute the mass matrix. Starting from the supergravity expression of the scalar potential

$$
V = e^K \left( D_i W D_j W g^{ij} - 3 |W|^2 \right)
$$

(3.1)
one computes the second derivative of the potential as

\[ \partial_i \partial_j V = -e^K \partial_i \partial_j W \bar{W} - e^K \partial_i \partial_j K |W|^2 + e^K \partial_i K \partial_j K |W|^2 ; \]
\[ \partial_i \partial_j V = -2e^K g_{ij} |W|^2 + e^K g^{kl} \partial_k D_k \bar{W} \partial_l D_l W , \]

(3.2)

Where we have made use of the the fact that in a supersymmetric vacuum

\[ D_i W = 0 . \]

(3.3)

The indices \( i, j, \ldots \) label all the chiral fields present (including the charged matter fields) and should not be confused with the indices \( i, j, \ldots = 1, \ldots h^{1,1} \) which were used in the previous section to label the Kähler moduli and the charged fields in \( \mathbb{C}^7 \). In fact, in the following we shall come back to the notation in the previous section where the indices \( i, j, \ldots \) label the Kähler moduli.

The condition that the gauge group is not broken ensures that the mass matrix splits into one for the matter fields and one for the moduli. Moreover, in the matter field mass matrix, only the gauge invariant terms survive. Splitting the indices into \( T, Z \) for the moduli (Kähler and complex structure respectively) and \( C, D \) for the matter fields (in \( \mathbb{C}^7 \) and \( \mathbb{C}^7 \) respectively) and suppressing extra indices on these fields in order to avoid clutter, we find that the only non-vanishing terms of the matter field mass matrix are \( \partial_C \partial_D V, \partial_C \partial_C V \) and \( \partial_D \partial_D V \). Computing these terms at the first order in \( \alpha' \) we obtain

\[ \partial_C \partial_D V = e^{-K} (\partial_C \partial_D W_1 \bar{W}_0 + \partial_C \partial_D K |W_0|^2) \alpha' + O(\alpha'^2) ; \]
\[ \partial_C \partial_C V = \left( -2e^K g_{CC} |W_0|^2 + e^K g_{DD} \partial_C D_D \bar{W}_1 \partial_C D_D W_1 \right) \alpha' + O(\alpha'^2) ; \]
\[ \partial_D \partial_D V = \left( -2e^K g_{DD} |W_0|^2 + e^K g^{CC} \partial_D D_C \bar{W}_1 \partial_D D_C W_1 \right) \alpha' + O(\alpha'^2) . \]

(3.4)

Similar formulae can be written for the moduli fields

\[ \partial_T \partial_T V = -e^K \partial_T \partial_T W \bar{W} - e^K \partial_T \partial_T K |W|^2 + e^K \partial_T K \partial_T K |W|^2 ; \]
\[ \partial_T \partial_Z V = -e^K \partial_T \partial_Z W \bar{W} - e^K \partial_T \partial_Z K |W|^2 + e^K \partial_T K \partial_Z K |W|^2 ; \]
\[ \partial_Z \partial_Z V = -e^K \partial_Z \partial_Z W \bar{W} - e^K \partial_Z \partial_Z K |W|^2 + e^K \partial_Z K \partial_Z K |W|^2 ; \]
\[ \partial_T \partial_T V = -2e^K g_{TT} |W|^2 + e^K g^{TT} \partial_T D_T W \partial_T D_T W + e^K g^{ZZ} \partial_T D_Z W \partial_T D_Z W ; \]
\[ \partial_T \partial_Z = e^K g^{TT} \partial_Z D_T W \partial_T D_T W + e^K g^{ZZ} \partial_Z D_Z W \partial_T D_Z W ; \]
\[ \partial_Z \partial_Z = -2e^K g_{ZZ} |W|^2 + e^K g^{TT} \partial_Z D_T W \partial_Z D_T W + e^K g^{ZZ} \partial_Z D_Z W \partial_Z D_Z W ; \]

(3.5)

Note that the masses for the matter fields come at order \( \alpha' \) while the masses for the moduli are only at zeroth order as usually the \( \alpha' \) corrections to the moduli are given by terms including matter fields, which are taken to be zero in the background.

So far the formulae written above are fairly general and do not make use of the specific form of the superpotential or the Kähler potential, apart from the fact that the Kähler
potentialAction is, at the zeroth order in α’, a sum of the Kähler potentials for the Kähler moduli and for the complex structure moduli. In the following we shall also make use of some specific properties of these quantities defined in (2.2), (2.3), (2.5) and (2.6). First note that ∂_T ∂_T W = 0 as the flux superpotential comes linear in T. Then, for E_6 preserving solutions ∂_T ∂_Z K = 0 as such terms appear multiplied by matter fields which have to vanish in the background. Finally, it is easy to see that

\begin{align}
\partial_C \partial_D W |_{C=D=0} &= 2 \partial_T \partial_Z W_0 ; \\
\partial_C \partial_D K_1 |_{C=D=0} &= -2 \partial_T K_0 \partial_Z K_0 ; \\
\partial_C \partial_C K_1 |_{C=D=0} &= 4 e^{(K_X - K_K)/3} g_{TT} ; \\
\partial_D \partial_D K_1 |_{C=D=0} &= e^{(K_K - K_{cs})/3} g_{ZZ} ,
\end{align}

which allow us to write all the masses only in terms of derivatives of the zeroth order quantities K_0 and W_0.

There is one more thing we should note related to the formulae for the mass matrix elements. The superpotential appears quadratically in each term with or without derivatives. Since first order derivatives of the superpotential can be replaced from (3.3) there will only be terms involving two derivatives of the superpotential or plain superpotential terms without any derivative. Moreover it is easy to see that each plain superpotential factor comes together with a derivative of the Kähler potential with respect to the moduli, K_X, which, in the supergravity limit we are using, behaves like

\begin{equation}
K_X \sim \frac{1}{X + X} \ll 1 .
\end{equation}

Therefore, if we are looking for a regime where the superpotential is small, the leading terms in the mass matrix elements are the ones which are quadratic in the second derivative of the superpotential. Writing now for clarity the indices which label the various fields – but still ignoring the E_6 indices which are all contracted as the results should be gauge invariant – the relevant elements of the mass matrix take the form

\begin{align}
e^{-K} \partial_C, \partial_D V &= 4 \alpha' e^{(K_X - K_K)/3} g^{ba} \partial_j \partial_b W \partial_a \partial_a W + \mathcal{O}(W) ; \\
e^{-K} \partial_D^a, \partial_D^b V &= \alpha' e^{(K_X - K_K)/3} g^{ab} \partial_i \partial_j W \partial_a \partial_i W + \mathcal{O}(W) ; \\
e^{-K} \partial_T, \partial_Z^a V &= g^{ba} \partial_a \partial_a W + \mathcal{O}(W) ; \\
e^{-K} \partial_T, \partial_F^a V &= g^{ba} \partial_a \partial_a W + \mathcal{O}(W) ; \\
e^{-K} \partial_Z^a, \partial_Z^b V &= g^{cd} \partial_d \partial_d W + e^K g^{ba} \partial_b \partial_j W \partial_a \partial_i W + \mathcal{O}(W) ,
\end{align}

where the rest of the elements are order \( \mathcal{O}(W) \) or higher. It is now clear that the masses for the matter fields and those for the moduli are related. In particular it can be seen that the masses are mainly controlled by the mixed derivatives of the superpotential \( \partial_T \partial_Z W \). If this has zero eigenvalues, meaning that the charged fields are massless, also the mass
matrix for the $T$-moduli will have zero eigenvalues. Therefore, massless matter fields are only possible if some of the moduli remain flat directions.

One can explicitly check for a toy model which contains only one $T$ and one $Z$ modulus that this is indeed the case. For certain values of the flux parameters which give rise to a small $W$, $[37]$, the masses for the matter fields turn out to be suppressed only by the fact that they come in a higher order in the $\alpha'$ expansion. Given that in the setup we consider the $\alpha'$ scale is not much below the Planck scale, any field which is massive can be safely removed form the spectrum.

It should be clear that the above pattern can be used only for pairs of fields $(T, Z)$ and $(C, D)$. The fields which remain “outside” such pairs are bound to have lower masses, of order $|W|$. This should be clear for the unpaired matter fields, because by gauge invariance and holomorphy of the superpotential the only possible second derivatives of the superpotential involve both fields in $27$ and fields in $\bar{27}$. For the moduli fields this may not be totally correct because of the existence of second derivatives of the superpotential of the form $\partial_Z \partial_Z W$ which may give large masses to the unpaired $Z$-fields. Nonetheless, the $T$-moduli are light.

Before we go further let us summarise the results of this section. We found that in general the masses for the moduli and matter fields are related and even though the latter may be lower than the former they are still large and, in a low energy approximation, one should truncate both the moduli and the corresponding matter fields. This means that we can effectively remove from the spectrum $h = \min(h^{1,1}, h^{2,1})$ pairs of moduli $(T, Z)$ and the corresponding pairs of matter fields $(C, D)$. This will be our strategy in the following in order to obtain simpler models which can be more easily analysed.

4 Effective low energy models

In the previous section we studied the masses for the matter fields and for the moduli in a supersymmetric, $E_6$ preserving vacuum. Even though in the general case it is hard to perform explicit calculations, we argued that the masses for the matter fields and for the moduli are related. We now apply these findings and truncate from the spectrum $h = \min(h^{1,1}, h^{2,1})$ pairs of moduli, $T, Z$ and the corresponding matter fields $C, D$. Note that we are not doing a proper integration out of the heavy fields, but rather freeze them at the value they have at the supersymmetric solutions analysed in the previous section. The conditions under which such a procedure is consistent were analysed in $[38, 39]$. As for the case at hand we do not know the solution explicitly these conditions are hard to analyse. We shall nevertheless assume that freezing the heavy fields at the supersymmetric value is consistent.

In the following, w+e shall distinguish two cases

1. $h^{1,1} > h^{2,1}$,
2. $h^{2,1} > h^{1,1}$.
In the first case we assume that \( h^{2,1} \) pairs of moduli \((T, Z)\) get large masses and can be removed from the spectrum. The same will happen with \( h^{2,1} \) pairs of matter fields \((C, D)\) and we are effectively left with a model containing only Kähler moduli and the corresponding charged fields \( C \) in the \( \overline{27} \) of \( E_6 \). In the second case the situation will be opposite and we will be left with an effective model with only complex structure moduli and the corresponding matter fields, \( D \), in \( 27 \) of \( E_6 \).

Before we discuss each case let us comment on the constraints one would have to satisfy in these effective models. In general we need to choose the flux parameters such that the constraints (2.7) are satisfied. However, this would be relevant if we solved the equations of motion in full generality. What we want to do instead is to see the effect of the constraints (2.7) on the effective models we want to analyse.

The important point to note is that these constraints are non-trivial only if both electric and magnetic type fluxes are present. For example if the superpotential (2.5) contained only up to linear terms in the complex structure moduli, the constraints (2.7) would be trivial. Also, if the superpotential did not depend on the Kähler, or on the complex structure moduli, the constraints (2.7) would be again identically satisfied. Therefore it is likely that these constraints do not have a descendant in the effective models we have listed above as they only contain one type of moduli fields (either Kähler or complex structure moduli).

We can nevertheless expect that the constraints (2.7) play a role in the process of integrating out the massive fields. One thing which may happen is that the constraints enforce that the masses take particular values which may not be consistent with the integrating out procedure we want to implement. In particular it would be disturbing if the constraints imply that the masses for certain fields we wanted to integrate out, vanish. As we have seen in the previous section the masses for the fields which pair up are controlled by the matrix \( \partial_T \partial_Z W \), which in such a case would have zero eigenvalues. By looking at a system which contains only one pair of moduli, \( T, Z \), and one pair of matter fields \( C, D \), this does not seem to be the case so in the following we will assume that the constraints do not play a role in the values of the masses for the paired up fields.

One other obstruction may be that the constraints (2.7) do not allow values for the moduli within the region where the approximations we use are valid. Some simple computer scan over the parameter space shows that actually this is not the case. Moreover, in section 5 we shall see that there is enough freedom in the parameters to fix the values of the moduli in the region of validity for our approximations. As we shall learn in this section this can be done at the expense of not having the value of the superpotential as a free parameter anymore. In particular we shall learn that the superpotential can not be made zero while keeping the moduli fields in a region where the approximations we use still hold.

In conclusion, we assume that we can fix the moduli at values consistent with all the approximations we use and that we can give them large enough masses so that they are effectively removed from the spectrum. Moreover, we assume that the effective models listed above do not have to satisfy any constraint like (2.7). The only restriction will be on the value of the superpotential which will actually appear as a constant in the
effective superpotential obtained after integrating out the massive fields. This term will be in general large (at least of order one) and since we can not control it very well we should avoid moduli stabilisation mechanisms which make use of such a term.

4.1 \( h^{1,1} > h^{2,1} \)

In this case we will be effectively left over with \( h^{1,1} - h^{2,1} \) Kähler moduli and the corresponding matter fields in $27$. The superpotential has the form

\[ W = w_0 + eT + C^3, \tag{4.1} \]

and it was analysed in [29]. In the case that \( w_0 = 0 \) the system was shown to have no proper solution with fixed moduli. In the case \( w_0 \neq 0 \) one can find an \( E_6 \) solution and the value of the Kähler modulus will directly depend on the (complex) parameter \( w_0 \). Unless one does a proper integration over the massive fields which were removed form the spectrum, the constant \( w_0 \) is unknown, and it is not clear what would be the values for this constant which are compatible with all the constraints on the theory. Finally, the value for the superpotential in such cases is typically large and incompatible with the stabilisation of the dilaton via a gaugino condensate in the hidden sector.

4.2 \( h^{2,1} > h^{1,1} \)

This case is similar to the previous one, with the difference that now we are dealing with complex structure moduli rather than Kähler moduli and accordingly, the matter fields transform in the $27$ of $E_6$. Compared to the previous case, the superpotential for the complex structure moduli is more complicated and allows for more flexibility in fixing the moduli

\[ W = w_0 + eZ + \frac{\mu}{2} Z^2 + \frac{\rho}{3} Z^3 + D^3. \tag{4.2} \]

As we said before, we are looking for solutions which have a small value for the superpotential at the critical point so that in the end we can use the gaugino condensate in order to stabilise the dilaton. To gain a qualitative picture, note that such solutions are given in a first approximation by solving the global supersymmetry equations, \( \partial_Z W = 0 \) (see [37] for details). For the case at hand, this equation can be easily solved and one can also compute the value of the superpotential at this point. With enough flexibility in tuning the flux parameters, one can achieve a small \( \text{Re} W \), but it can be easily seen that the imaginary part is proportional to \( z^3 \). The consistency of the model requires that \( z \gg 1 \) and therefore it is impossible to achieve a small value for the superpotential in this regime. In conclusion, even though the starting superpotential (2.5) looks complex enough so that one may naïvely think that it is easier to tune the parameters such that a small value for \( W \) is achieved, in practice, we have argued that this does not happen and we encounter the old problem that we have to balance \( W_{\text{flux}} \), which is of order one or greater due to flux quantization, against the non-perturbative small effect of gaugino condensation [32, 33, 34, 35].
5 Breaking $E_6$

The strategy we followed so far was to integrate out the massive fields and analyse the simpler models obtained in this way. We have seen in the previous section that there may be solutions to these simple models, but in general the value for the superpotential is large and incompatible with the stabilisation of the dilaton via gaugino condensation in the hidden sector. Small values for $W$ are also needed in order to be sure that the truncation of the massive modes, we have done in the first step, is indeed consistent. In this section we shall investigate another possibility of obtaining small values for the superpotential.

Since the fluxes are integers, the solutions for the moduli fields will generically be rational numbers. Therefore the superpotential will be a polynomial with integer coefficients of these rational numbers. It is more or less clear that unless one chooses a big hierarchy between the fluxes, arbitrarily small, but nonvanishing values for the superpotential are not possible. However, from rational and integer numbers it is much easier to construct a vanishing quantity. Still, we need a non-vanishing superpotential in order to be able to fix the dilaton via gaugino condensation in the hidden sector. This can be achieved in principle by vevs of the matter fields. If the values for the charged fields are small – which is in fact required by the consistency of the derivation of the effective action in the presence of charged fields – the superpotential will naturally be a small quantity. We shall see in this section that actually such solutions are not possible as they violate the constraint on the fluxes coming from the Bianchi identity.

Note that this idea is not entirely new, but it appeared before in the literature in various forms [32, 34, 36]. In these references it was suggested that the small quantity which should balance the gaugino condensate comes from the Chern-Simons correction to the field strength $H$ of the antisymmetric tensor field. Recall that for the case at hand, the superpotential is generated from the formula [21]

$$W = \int H \wedge \Omega,$$

(5.1)

where $\Omega$ is the $(3,0)$ form which one can define on a manifold with $SU(3)$ structure. Moreover, the matter superpotential comes entirely from the the Chern-Simons correction to $H$ which shows the relation with the references above. The difference now is that for the case at hand we have a very specific model where moduli stabilisation can be discussed explicitly and we can check whether such a mechanism works or not.

5.1 Preliminaries

For the model described in section 2 suppose we are given a supersymmetric solution in flat Minkowski space at zeroth order in $\alpha'$. The purpose is to find a solution at order $\alpha'$ which has non-vanishing superpotential which can be used later to fix the dilaton via gaugino condensation in the hidden sector. At first order in $\alpha'$ the equations for supersymmetric solutions read

$$\partial C W_1 = \partial D W_1 = 0.$$

(5.2)
In the above equations, the term proportional to the superpotential is absent as $W_0$ vanishes by assumption while the term with $W_1$ is higher order in $\alpha'$. A non-vanishing $W_1$ can only be obtained if the charged fields have a nonzero vev which in turn implies that the $E_6$ gauge group is broken. The equations above can be written schematically as

$$jC^2 + D = 0, \quad jD^2 + C = 0,$$

where $j$ denotes the totally symmetric cubic invariant of $E_6$. Let us make a few comments here. First of all, note that the presence of the mass term $CD$ in the superpotential is crucial as, in its absence, only $E_6$ preserving solutions can be found. Moreover, the range of the gauge groups which are preserved by the solutions to the above equations is limited and can not be any subgroup of $E_6$. For example let us assume that we are looking for a solution which preserves an $SO(10)$ gauge group. The 27 of $E_6$ decomposes under $SO(10)$ as

$$27 = 1 \oplus 10 \oplus 16,$$

and therefore this breaking would be triggered by a non-vanishing vev of the singlet field in the above decomposition. It is easy to see that in this case the first term in (5.3) does not contribute because of the coefficient of the cubic coupling of the singlet field vanishes, ie $j_{111} = 0$. This means that the only solution will be the trivial one which actually preserves the full $E_6$ group and not only a $SO(10)$ subgroup.

Even if, as noticed above we do not have full flexibility in choosing a solution to the above equations, reasonable solutions with $C, D \ll 1$ to these equations may be found depending on the combinatorial and group theoretical factors. We do not insist any further on this aspect as the purpose of this preliminary section was only to show that apriori the line of reasoning we have chosen can indeed work. We still need to check that the initial assumption of the existence of a Minkowski ground state at the zeroth order in $\alpha'$. We shall actually prove in the following that this is not possible in the setup we consider.

### 5.2 No-go theorem for Minkowski solutions

Now let us look for Minkowski solutions in the low energy theories obtained from heterotic string compactifications on manifolds with $SU(3)$ structure with standard embedding. A computer scan of the flux parameter space shows that at zeroth order in $\alpha'$, many solutions with vanishing superpotential exist with moderate values for the fluxes. However none of these solutions satisfy the constraint on the fluxes imposed by the Bianchi identity. It is important to stress that this constraint has to be imposed as we have already chosen, by embedding the spin connection into the gauge group, that the right hand side of the Bianchi identity

$$dH = tr(F \wedge F - R \wedge R),$$

identically vanishes. Therefore we have to make sure that for the solutions we find, $H$ satisfies $dH = 0$. 

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In order to be able to manage the calculations let us choose a setup which has one Kähler and one complex structure modulus. Together with these fields we have to consider the corresponding charged fields which we have briefly analysed in the previous subsection. As we mentioned there, the term \(CD\) in the superpotential is crucial for finding solutions, and this is why we need both matter fields in \(27\) and in the \(\overline{27}\) of \(E_6\). The zeroth order (in \(\alpha'\)) superpotential for this model is given by

\[
W = i(\xi + ieT) + (\epsilon + ipT)Z + \frac{i}{2}(\mu + iqT)Z^2 + \frac{1}{6}(\rho + irT)Z^3 ,
\]

where \(T\) and \(Z\) are the Kähler and respectively complex structure moduli fields and \(\xi, \epsilon, \mu, \rho, e, p, q, r\) are the flux parameters which satisfy the constraint [21]

\[
\xi r - \epsilon q + \mu p - \rho e = 0 .
\]

The strategy is fairly clear: the equation (5.8) is cubic and can be solved for \(Z\). Then one replaces this solution into (5.9) in order to determine \(T\). Finally, we have to impose the constraints (5.7) and \(W = 0\) which will restrict the flux parameter space. Apriori one should think that there are enough flux parameters which we can tune such that we get the desired solution, but as we shall see in the following, this intuition is not correct.

It is useful to remark the following trick which sometimes eases the calculations. Let us denote

\[
T = t + i\tau , \quad Z = z + i\zeta ,
\]

where \(t\) and \(z\) are the true moduli fields, ie the moduli which govern the actual size of the corresponding cycles, while \(\tau\) and \(\zeta\) are their axionic superpartners. With this notation, it is a matter of algebra to derive

\[
Re \left( t\partial_T W + z\partial_Z W - W \right) = z^2(-qt + \frac{1}{3}\rho z - rt\zeta - \frac{1}{3}rz\tau) .
\]

We are looking for a solution for which each of the terms on the left hand side vanishes and since the approximations we are using require that \(z \gg 1\) (which in particular means that \(z \neq 0\)) we find that

\[
-qt + \frac{1}{3}\rho z - rt\zeta - \frac{1}{3}rz\tau = 0 ,
\]

which is a condition much easier to analyse than the generic \(Re W = 0\).

In the following we shall distinguish two cases which are intrinsically different: \(r \neq 0\) and \(r = 0\).
5.2.1 $r \neq 0$ case

Let us start with some simple observation. Let us shift the field $Z$ by some purely imaginary constant

$$Z = Z' + ia, \quad a \in \mathbb{R}.$$  

(5.13)

Defining new flux parameters by

$$\begin{pmatrix} e' \\ p' \\ q' \\ r' \end{pmatrix} = M \begin{pmatrix} e \\ p \\ q \\ r \end{pmatrix}$$

and

$$\begin{pmatrix} \xi' \\ \epsilon' \\ \mu' \\ \rho' \end{pmatrix} = M \begin{pmatrix} \xi \\ \epsilon \\ \mu \\ \rho \end{pmatrix},$$

(5.14)

where the matrix $M$ is defined as

$$M = \begin{pmatrix} 1 & a & -\frac{1}{2}a^2 & -\frac{1}{6}a^3 \\ 0 & 1 & -a & -\frac{1}{2}a^2 \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

(5.15)

the superpotential (5.6) has precisely the same form, but with the primed quantities. In matrix notation the constraint reads

$$(e, p, q, r)L \begin{pmatrix} \xi \\ \epsilon \\ \mu \\ \rho \end{pmatrix} = 0,$$

(5.16)

where $L$ is the simplectic form

$$L = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}.$$  

(5.17)

It is easy to verify that $M^TLM = L$ which means that the constraint also has the same form in the primed parameters.

The next thing to notice is that the equation which determines $Z$, (5.8), always has (at least) one purely imaginary solution which we denote by $Z_0$. To see this note that making the change $Z \to iZ$ changes the equation in a cubic equation with real coefficients which always admits a real solution.

Now let us combine the above remarks and make a shift in $Z$ by the purely imaginary solution of (5.8)\footnote{Note that we are interested in setups which give consistent values for the moduli, i.e. $ReZ \neq 0$. This means that equation (5.8) should have precisely one purely imaginary solution which makes the above shift unambiguous.}. In the notation above it means that $a = -iZ_0$. The advantage is that in the new (primed) variables the flux parameter $e' = e - ipZ_0 + \frac{1}{2}qZ_0^2 - i\frac{1}{6}rZ_0^3 \equiv 0$. 

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By the above tricks we have ended with precisely the same theory we started from, but with one vanishing flux parameter. Note that the shift in $Z$ is not of any physical significance, as the imaginary part of $Z$ was an axion before turning on fluxes, and its vev does not appear in the calculation of any physical quantities. If, on the other hand, the same trick had required a shift in the real part of $Z$ instead, we would have had to be more careful as the value of this field is important in the consistency of the setup.

Finally, we should stress that the key ingredient here is the fact that the fluxes are real parameters. This is unlike type IIB theory where the flux superpotential contains complexifications of the flux parameters and so the above argument can not hold.

With this comments we shall start analysing the solutions of the system defined by the equations (5.8) and (5.9) subject to the constraints $Re W = 0$ (which is equivalent to (5.12)), $Im W = 0$ and (5.7). We shall consider that $e = 0$ which we can do, as explained above, without loss of generality.

Equation (5.8) can now be solved easily. The solution $Z = 0$ is not physical as we need for consistency that $z \equiv Re Z \gg 1$. We find that

$$\zeta = -\frac{3q}{2r} \quad \text{and} \quad z^2 = -\frac{9q^2}{4r^2} - \frac{6p}{r}.$$  (5.18)

Note that this solution is valid only if the expression defining $z^2$ is positive. Therefore we necessarily have $p \neq 0$.

To solve for $T$ is is easier to use the constraint (5.12). From this we can express $\tau$ as a function of $t$, $z$ and $\zeta$ and replace the result into $Im \partial_Z W$. After a little algebra, making use of the solution for $Z$ we find

$$\frac{t}{z} = \frac{\mu - \rho q}{2rp}.$$  (5.19)

At this stage we have found a solution for $T$ and $Z$ and we have to impose the constraints coming from $Re \partial_Z W = 0$, $Im W = 0$ and (5.7). The strategy will be to express $\epsilon$ and $\xi$ from the first and last equations and replace these results together with the solution for $T$ and $Z$ in $Im W$. After straightforward algebraic calculations one finds

$$Im W = \frac{4}{3}ptz.$$  (5.20)

The consistency of the setup requires that $t$ and $z$ be non-zero. Moreover, we have explained above that a non-vanishing solution for $z$ is possible only for $p \neq 0$ and therefore the right hand side of the above equation can not vanish. This ends our proof that for $r \neq 0$ the setup we have considered has no supersymmetric Minkowski solutions which satisfy the constraint (5.7).

5.2.2 $r = 0$ case

As we have explained at the beginning of this section, the computations above apply only to the case $r \neq 0$ and for vanishing $r$ we have to redo all the calculations. First of
all note that if $r = 0$, the equation (5.8) no longer has a purely imaginary solution by which we can shift $Z$. Therefore our argument that we can shift $Z$ in order to make $e = 0$ no longer works. In fact, in order to have a non-vanishing solution for $z$ we need that $p^2 + 2qe < 0$ which actually requires that both $q$ and $e$ are non-zero. Then the solution to (5.8) reads

$$\zeta = \frac{p}{q} \quad \text{and} \quad z^2 = -\frac{p^2}{q^2} - 2\frac{q}{e}.$$  \hspace{1cm} (5.21)

The equation for $T$, (5.9), can easily be solved to yield

$$\tau = \frac{\rho p + \mu q}{q^2} \quad \text{and} \quad t = \frac{1}{qz} \left( \epsilon - \frac{\mu p}{q} - \frac{\rho e}{q} - \frac{p^2 \rho}{q^2} \right).$$  \hspace{1cm} (5.22)

The first two terms in the bracket can be replaced from the constraint (5.7) and one easily finds

$$\frac{t}{z} = \frac{\rho}{q}.$$  \hspace{1cm} (5.23)

Looking at the constraint coming from $Re \, W = 0$, (5.12), for the case $r = 0$ we see that the two equations are not compatible with each other, and so, also for the case $r = 0$ there is no Minkowski solution which satisfies (5.7). In the above it seems that we have not even made use of the constraint coming from $Im \, W = 0$. This is indeed true, but this constraint can always be satisfied by choosing the flux parameter $\xi$ accordingly. Moreover, this flux parameter does not modify the constraint (5.7) because $r = 0$. The only problem may come from the fact that fluxes are quantised, but by running a simple computer code to scan over the flux parameter space it can be seen that there exist several solutions which have $\xi$ integer.

This ends our proof that in heterotic models compactified on manifolds with $SU(3)$ structure with standard embedding there are no supersymmetric Minkowski solutions with all moduli fixed. Therefore the trial to find solutions which have small $W$ which is generated entirely from the matter sector fails.

We should close this section by one remark. The proof we have presented above is entirely at the zeroth order in $\alpha'$ and is independent of the presence of matter fields. Therefore one would have expected that such an analysis had appeared before in the literature. Indeed, this problem was studied before in [37]. There solutions with small $W$ were found by perturbing global supersymmetric solutions (which are the same as Minkowski solutions). The reason such solutions were found was precisely because the Bianchi identity was not imposed. The consistency there was argued via the possibility of adding NS5 branes in order to satisfy the Bianchi identity. For the case at hand this is not a possible solution anymore because in order to deal with matter fields we have to specify our solution to the Bianchi identity in the first place (ie by standard embedding in this case) and the matter field spectrum only follows after that. We have chosen the standard embedding because we know how to treat the matter fields and this means that the Bianchi identity can not be modified any further by the inclusion of NS5 branes.
6 Conclusions and outlook

In this paper we have analysed the issue of moduli stabilisation in models derived from heterotic strings compactified on manifolds with SU(3) structure at first order in \( \alpha' \). The setup used was the one derived in [22] where the solution to the Bianchi identity was obtained via the standard embedding. However, this setup suffers from various problems, some of which can be traced back to the original discussions about moduli stabilisation in heterotic compactifications [32, 33, 34, 35]. One of the problems which has been observed in this paper is that the moduli and matter field sectors are not independent, but the masses in these sectors are the same in many cases of interest. This can be directly related to the fact that the moduli and matter fields arise from expansions in the same set of forms on the internal manifold in the standard embedding case. Therefore, giving masses to some of the moduli by making some of the expansion forms non-harmonic, immediately implies that the corresponding charged fields are also massive removing them from the spectrum. Thus the first question to ask is how one can be left with massless matter fields in the 4d theory and at the same time give masses to all the moduli. One answer is that gauge symmetry prevents the matter fields which transform in complex representations of the gauge group to acquire masses unless pair anti-matter fields are present. Such cases were discussed in section 4 and the main conclusion is that the remaining moduli superpotential is too simple in order to have a satisfactory picture of moduli stabilisation. In particular, in the case where there are more Kähler than complex structure moduli, one can argue along the lines of [29] that there is no solutions with all moduli fixed, while in the other case, where there are more complex structure than Kähler moduli, the superpotential is too large in order to have the stabilisation of the dilaton via gaugino condensate in the hidden sector.

Finally we have analysed one possibility of obtaining a small superpotential at the minimum by finding a Minkowski flux solution (ie vanishing superpotential at order \( \alpha'^0 \)) and generating a small superpotential by giving small vevs to the matter fields and thus breaking the original \( E_6 \) gauge group. Even if apriori it is not clear why such a setup can not exist, we have been able to prove analytically that Minkowski solutions to the flux superpotential at zeroth order in \( \alpha' \) which obey the standard embedding Bianchi identity do not exist. A similar analysis of moduli stabilisation was performed also in [37] where a solution was found precisely by relaxing the Bianchi identity constraint which can be achieved by introducing NS5 branes. In order to deal with the matter fields, however, one has to specify an explicit solution to the Bianchi identity, which can no longer be modified at will when dealing with matter fields as the way the matter fields are defined intrinsically depends on the solution to the Bianchi identity. Therefore, if we want to modify the constraint which comes from the Bianchi identity by adding branes, we would have to modify the matter sector accordingly and so, the model from which we start would be different from the one we have considered in this paper. There exist another logical possibility of obtaining a consistent solution in the framework of section 5. There we have considered non-trivial vevs for the matter fields and wanted to use them in order to obtain a small superpotential. The vacuum expectation values for the matter fields modify the anomaly cancellation condition. Therefore the constraint [5.7] would strictly speaking have some non-vanishing correction on the right hand side.
This correction is however small because we know how to treat only the matter fields which are small fluctuations. This correction would show up as a correction to (5.20), but this can not compensate for the large contribution on the RHS of (5.20) because, as explained before, $p$ is quantised and therefore can not be made arbitrarily small, while for consistency we need that $t, z \gg 1$.

It looks like all the problems encountered in this communication point towards the fact that in heterotic string compactifications with standard embedding moduli can not be stabilised in a satisfactory way. One possible direction is to study heterotic string compactifications with fluxes outside the standard embedding scenario. Another possibility is to consider generalisations which include non-geometric backgrounds, but both these directions have been unexplored so far.

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