Effects of radiative energy losses on the structure of stellar wind interaction with the interstellar medium

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Abstract. We present two-dimensional axisymmetric gas dynamic simulations of the hypersonic stellar wind interaction with the surrounding interstellar medium moving with supersonic speed in the stellar rest frame. Our model takes into account radiative losses that appear in optically thin plasma. We aim to explore the influence of radiative cooling on the size and global shapes of astrospheres. It is demonstrated in the paper that solution depends on five dimensionless parameters: \( \gamma \) (the adiabatic index), \( M_\infty \) (the Mach number in the undisturbed interstellar medium), \( \chi \) (the ratio between terminal speed and interstellar medium speed), as well as \( \alpha \) and \( \theta \) which are related to the energy losses; \( \alpha \) is responsible for radiative cooling power, \( \theta \) is the interstellar medium temperature divided by \( 10^4 \) K. Results of the simulations demonstrate that radiative losses have a significant influence on the flow pattern. The process of radiative cooling leads to an increase of density and pressure which results in compression of the interaction region and changing the position of the astropause. We also explore the growth of Kelvin-Helmholtz instability appearing at tangential discontinuities with increasing cooling power \( \alpha \).

1. Introduction

The stellar wind blows into the surrounding interstellar medium forming a structure called astrosphere. The shapes of such structures depend on the parameters of both stellar wind and interstellar medium. Analysis of the properties of the astrospheres plays an important part in the exploration of stellar evolution.

The pioneer research in this field has been made by Parker [1]. He developed the first stationary gas dynamic models of the stellar wind extension into interstellar space. Parker considers the flow from the star to be radial with spherical symmetry and having a supersonic velocity. Due to the lack of observational information, he makes some assumptions about the properties of the interstellar medium and proposes three types of solutions. In one of them, the interstellar medium is supposed to be a steady subsonic parallel flow without magnetic field. In this case, the flow pattern is divided into three parts: supersonic stellar wind, shocked stellar wind which goes through a shock transition, and interstellar gas flow.

Baranov et al [2] considered the interaction of the stellar wind with the supersonic interstellar flow. In this case, the flow pattern consists of four areas: supersonic stellar wind, shocked stellar wind (called inner heliosheath for the Sun), shocked interstellar flow (called outer heliosheath),
and supersonic interstellar gas flow (Fig. 1). Numerical research [3] demonstrates that in the
tail or downwind part of the interaction region a complex structure with secondary shock waves
and tangential discontinuities occurs.

![Figure 1. Left panel: a qualitative model of interaction between the stellar wind and the
interstellar medium. ISM – undisturbed interstellar medium, BS – bow shock, HP – tangential
discontinuity, TS – terminal shock, dashed lines are for secondary shock waves and tangential
discontinuities. Right panel: Structure of interaction region with radiative losses taken into
account; 1 – undisturbed stellar wind, 2 – shocked stellar wind, 3 – a layer of cold dense interstellar
medium, 4 – a layer of hot shocked interstellar medium, 5 – undisturbed interstellar flow.

Some physical effects can have a significant influence on the structure of the astrospheres.
One of such effects is radiative cooling. Weaver et al. [4] present the detailed quantitative model
of the structure of the interstellar bubbles created by early-type stars to explain the presence of
$O_{VI}$ in their spectrum. The interstellar medium is considered to be at rest. Firstly, they show
the idealized adiabatic solution. Secondly, the effects of thermal conduction and radiative losses
are included to modify the solution. Density and temperature distributions demonstrate that a
cool dense layer appears near the bow shock; moreover, a conduction front forms between this
cold layer and hot stellar wind.

Modern studies continue to take into account thermal conduction and radiative losses.
Comerón and Kaper [5] provide semi-analytical approximate and numerical solutions of the
problem of the massive runaway OB stars motion in the diffuse medium. Such stars move
supersonically, therefore, the flow pattern resembles the one described in Fig. 1 but a cool dense
layer and a conduction front appear in the area of the shocked interstellar medium. Consequently,
there are six different regions: 1) a zone of free stellar wind, 2) a hot shocked stellar wind with
a broad conduction zone of decreasing temperature and increasing density, 3) a cool dense layer
of the interstellar wind, 4) a thick diffuse layer of the shocked interstellar wind, 5) undisturbed
interstellar gas. It is also shown that the structure becomes unstable when the difference between
the surface density of the cool dense layer and the overlying layer of cooling shocked interstellar
gas is high. Overall, these results suggest that there is a variety of shapes and structures of
astrospheres. Meyer et al. [6] present the numerical results for massive runaway MS and RSG
stars. The authors found that the effects of thermal conduction lead to an expansion of the
shocked interstellar medium region and the entire structure of the astrosphere as a whole. In
addition, the luminosity of the astrospheres was calculated. The interaction region of RSG stars
has a high luminosity in the infrared range, the brightest part is the shocked interstellar medium. The studies mentioned above have been modeled specific objects; such a method does not allow obtaining an understanding of the influence of individual processes on the interaction region structure. This paper aims to explore the influence of radiative losses on the astrospheric shape. The quantitative model of the interaction between the stellar wind and the supersonic interstellar medium is presented. We give the dimensionless mathematical formulation of the problem establishing dimensionless parameters and perform the parametric study.

The rest of this paper is organized as follows. Section 2 presents the mathematical formulation of the problem and its dimensionless form. We also discuss the physics of radiative losses in Subsection 2.1. Section 3 gives an overview of the numerical methods used. Results of the numerical calculations are presented and discussed in Section 4. The conclusions are given in Section 5.

2. Model
In order to describe stellar and interstellar winds several assumptions are made:

- Both winds are supposed to be fully ionized gases and may be described as ideal gases with \( \gamma = \frac{5}{3} \). The pressure and the temperature can be linked as \( p = 2n_T kT \). The influence of the magnetic fields is neglected (the ideal MHD model is presented, for example, in [7]).
- The stellar wind is assumed to be a supersonic spherically symmetric point source flow with constant mass loss rate \( \dot{M}_\odot \) and terminal velocity \( v_0 \).
- The interstellar wind is a supersonic parallel flow with given values of pressure \( p_\infty \), density \( \rho_\infty \), and velocity \( v_\infty \).
- The chosen radiative cooling function is constructed for optically thin plasma. We assume the cooling rates for the undisturbed stellar and interstellar winds to be zero.

We aim to explore stationary solutions for the wind interaction problem, therefore, the governing Euler equations with the radiative cooling term in the right part can be written as:

\[
\begin{align*}
\nabla \cdot (\rho \mathbf{v}) &= 0, \\
\nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} + \mathbf{p} \mathbf{I} \right] &= 0, \\
\nabla \cdot [(e + p) \mathbf{v}] &= q, \quad q = -\frac{\rho^2}{m_p^2} \Lambda(T).
\end{align*}
\] (1)

where \( e \) is the total energy density: \( e = p/(\gamma - 1) + \rho v^2/2 \). The cooling function \( \Lambda(T) \) is described in Subsection 2.1.

In this paper, the problem is two-dimensional and axisymmetric. The outer boundary conditions are set at \( x \to \infty \) with the parameters of the undisturbed interstellar medium. The point source (described by \( \dot{M}_\odot \) and \( v_0 \)) is situated at \( x = 0 \). Consequently, the solution depends on six dimensional parameters: \( p_\infty, \rho_\infty, v_\infty = -v_\infty \mathbf{e}_x, \dot{M}_\odot, v_0, \gamma \).

2.1. Radiative losses
In the absorbing medium the photon beam loses its intensity as follows: \( dI_\nu = -\alpha_\nu I_\nu ds \), where \( \alpha_\nu \) is the attenuation coefficient. This coefficient can also be written as \( \alpha_\nu = \sigma_\nu n \) where \( \sigma_\nu \) is the attenuation cross section. The integral density decay is:

\[
I_\nu(s) = I_\nu(0) \exp \left( -\int_{s_0}^s \alpha_\nu ds' \right).
\] (2)
Then we can make a substitution \( d\tau_\nu = \alpha_\nu ds \) called optical depth. The integral form can be written as follows:

\[
\tau_\nu = \int_{s_0}^s \alpha(s') \, ds'
\]  

(3)

Optical depth defines attenuation along the propagating light beam. When \( \tau \ll 1 \) the medium is considered to be optically thin. In this case, photons can leave the plasma freely. Thus, the main cause for energy losses in optically thin plasma is photon emission.

Several processes can contribute to radiative cooling: collisional excitation, collisional ionization, recombination, and bremsstrahlung. In the range of \( 10^4 \) to \( 10^7 \) K collisional excitation dominates, and a major contribution to the cooling process is made by metal line transitions [8]. For high temperatures the leading process is bremsstrahlung.

Collisions between particles and photons are very rare, consequently, the effects of photoionization are neglected. It is also assumed that all excited ions return to the ground state in times short in comparison with the time between collisions. Therefore, every ion is in the ground state when a collision occurs. In that case, the mentioned cooling processes have similar density dependencies, and the emissivity \( \Lambda \) depends only on temperature and metallicity. Therefore, the cooling rate can be described as \( -n^2 \Lambda(T) \). In our case, we use a simple approximation of cooling function for solar metallicity [9]:

\[
\Lambda(T) = \begin{cases} 
0, & T < 10^4 \, \text{K} \\
10^{-24}T^{0.55}, & 10^4 \, \text{K} < T < 10^5 \, \text{K} \\
6.2 \cdot 10^{-19} \cdot T^{-3/5}, & 10^5 \, \text{K} < T < 4 \cdot 10^7 \, \text{K} \\
2.5 \cdot 10^{-27} \cdot T^{0.5}, & T > 4 \cdot 10^7 \, \text{K}.
\end{cases}
\]

(4)

### 2.2. Dimensionless parameters

The solution depends on five parameters: \( \rho_\infty, p_\infty, v_\infty = -v_\infty e_x, \dot{M}_\odot, v_0 \). The chosen independent basic parameters are \( \frac{v_0 M_\odot}{4\pi}, \rho_\infty, v_\infty \). Consequently, the dimensionless boundary conditions are:

\[
\dot{M}_\odot = 4\pi/\chi, \quad \dot{v}_0 = \chi, \quad \dot{\rho}_\infty = 1, \quad \rho_\infty = 1/(\gamma M_\odot^2), \quad \bar{v}_\infty = 1.
\]

The characteristic distance is \( R_* = \left( \frac{v_0 M_\odot}{4\pi \rho_\infty v_\infty^2} \right)^{1/2} \). The dimensionless Euler equations can be written as:

\[
\begin{align*}
\hat{\nabla} : (\hat{\rho}\hat{\nabla}) &= 0, \\
\hat{\nabla} : \left[ \hat{\rho} \hat{\nabla} \hat{\nu} + \hat{p} I \right] &= 0, \\
\hat{\nabla} : [(\hat{\epsilon} + \hat{\rho}) \hat{\nabla}] &= -\alpha \rho^2 \hat{\Lambda}(\gamma M_\odot^2 \theta \hat{\rho})
\end{align*}
\]

where \( \epsilon = p/(\gamma - 1) + \rho v^2/2 \),

\[
\begin{align*}
\theta &= T_\infty/T_0 \\
\alpha &= \frac{R_* \rho_\infty^2 \bar{v}_\infty^3}{m_p} \Lambda(T_0), \\
\hat{\Lambda} &= \begin{cases} 
0, & \hat{T} < 1 \\
\hat{T}^{0.55}, & 1 < \hat{T} < 10 \\
15.62 \cdot \hat{T}^{-3/5}, & 10 < \hat{T} < 4 \cdot 10^3, \\
1.58 \cdot 10^{-3} \cdot \hat{T}^{0.5}, & \hat{T} > 4 \cdot 10^3
\end{cases}
\end{align*}
\]
where \( \Lambda(T_0) \approx 1.58 \cdot 10^{-22}\text{erg} \cdot \text{cm}^3 \cdot \text{s}^{-1} \), \( T_0 = 10^4\text{K} \), \( T_\infty \) is interstellar medium temperature.

Although the stationary solution for the stellar wind in the hypersonic approximation does not depend on the distance at which the boundary conditions are set, in numerical simulations, we must determine the inner boundary \( \hat{R}_\text{in} \). In dimensionless form, the boundary conditions on \( \hat{R}_\text{in} \) are defined as follows:

\[
V_\text{in} = \frac{V_0}{v_\infty} = \chi, \quad \rho_\text{in} = \frac{1}{\chi^2 \hat{R}_\text{in}^2}.
\]

Therefore, the solution now depends on these dimensionless parameters: \( \chi = v_0/v_\infty, M_\infty, \alpha, \theta, \gamma; \alpha \) shows radiative cooling power, \( \theta \) is the interstellar medium temperature divided by \( 10^4 \text{K} \).

In the case of ideal gas without radiative losses, the geometrical pattern of the stationary solution does not depend on \( \chi \). If there is a solution \( \rho_1(r), V_1(r), p_1(r) \) for a set of parameters \( (\chi_1, M_\infty) \) in the free stellar wind region, a solution for a new set \( (\chi_2, M_\infty) \) can be constructed (the solution in the interstellar medium region remains the same):

\[
\rho_2(r) = \left( \frac{\chi_1}{\chi_2} \right)^2 \rho_1(r), \quad V_2(r) = \frac{\chi_2}{\chi_1} V_1(r), \quad p_2(r) = p_1(r).
\]

These functions satisfy the equations of ideal gas dynamics, boundary conditions at the tangential discontinuity, and Rankine–Hugoniot conditions. Consequently, the gas dynamic functions can be recalculated.

In the model with radiative losses, we assume that in the undisturbed stellar wind \( \hat{\Lambda} \approx 0 \), because its density and temperature at large distances from the star are small. Then, there is no dependence on the \( \chi \) parameter either. Thus, it will be enough to carry out calculations for only three parameters: \( M_\infty, \alpha, \theta \).

3. Numerical approach

The calculations were performed using a finite volume Godunov-type method where non-stationary equations are solved with formulated above boundary conditions. Stationary solutions can be obtained at \( t \to \infty \). We use a 2D cylindrical coordinate system due to the axial symmetry of the problem. Therefore, the following non-stationary equations are:

\[
\sigma = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad a = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ \rho uv \\ (e + p)u \end{bmatrix}, \quad b = \begin{bmatrix} \rho v \\ p + \rho v^2 \\ \rho uv \\ (e + p)v \end{bmatrix}, \quad f_0 = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 \\ (e + p)v \end{bmatrix}, \quad q = \begin{bmatrix} 0 \\ 0 \\ 0 \\ q \end{bmatrix}.
\]

The evolution of the matter, energy, and momentum densities in the cell \( (j - 1/2, k - 1/2) \) can be written as:

\[
\sigma^{j-1/2,k-1/2} = \sigma^{j-1/2,k-1/2} - \frac{\tau}{\Delta x} \left( A_{i,k-1/2} - A_{j-1,k-1/2} \right) - \frac{\tau}{\Delta y} \left( B_{i-1/2,k} - B_{j-1/2,k} \right) - \frac{\tau}{\Delta y_{j-1/2}} f_{0,j-1/2,k-1/2} + \frac{\tau}{\Delta x_{i-1/2}} q_{j-1/2,k-1/2},
\]

where \( \Delta x \) and \( \Delta y \) are the constant steps, \( \tau \) is the time interval. \( A \) and \( B \) are found by solving Riemann problems at each inter-cell boundary. We use several methods to solve the Riemann problem: the approximate solvers (HLL, HLLC [10]) and exact solutions like in the classical Godunov method. The approximate methods work faster, but their numerical viscosity is large; thus, the tangential discontinuity can be smeared out. The minmod flux limiter is used that makes the scheme second-order accurate in space.
4. Results

In this section, the solutions for different values of parameters α and θ are presented; Mach number \( M_\infty \) and \( \chi \) are the same for all calculations (\( M_\infty = 3, \chi = 1.628 \)). The order of numerical scheme, type of solver, and grid resolution are given for each of the presented calculations in the figure captions.

Figure 2 shows how the flow pattern changes with the increase of α parameter. It can be seen that the interaction region becomes thinner and more compressed. The triple point (where the secondary tangential discontinuity and reflected shock wave emanate from) moves out, the bow shock, the terminal shock, and the tangential discontinuity move closer to the star. It is shown that in the case of ideal non-heat conducting gas without radiative cooling, Kelvin-Helmholtz (K-H) instability appears at primary and secondary tangential discontinuities [11]. In order to observe K-H instability, one needs to perform computations with higher spatial resolution and using the second-order methods. When \( \alpha = 0 \) the tangential discontinuity is unstable only in the tail part (Figure 2.a). Figures 2.b and 2.c demonstrate that with the increase of \( \alpha \) the instability grows and spreads to the upwind part. The parameter \( \theta \), which refers to the interstellar medium temperature, also has an influence on the pattern (Figure 3): its decrease reduces the effects of radiative losses.

![Figure 2](image)

**Figure 2.** Isolines of Mach number for various α parameters, second-order HLLC, number of grid-cells: 3584 × 2200.

Figure 4 presents a more detailed gas dynamic parameters distribution. It can be seen in this figure that even at small values of the parameter \( \alpha = 0.01 \), the gas density in the region of the disturbed interstellar medium increases by more than 4 times in comparison with the case of \( \alpha = 0 \). The outer shock wave and the tangential discontinuity are located closer to the star,
while the position of the inner shock wave almost does not change. Moreover, not only the area of the outer shock layer becomes thinner, but also the inner one. When $\alpha = 0.1$ the bow shock and the tangential discontinuity continue to move closer to the star; the terminal shock also changes its position.

Figure 3. The influence of $\theta$ on the flow pattern. Isolines of Mach number for $\alpha=0.1$, first-order HLL, number of grid-cells: $1792 \times 1280$.

Figure 4. Dimensionless gas dynamic parameters at $y=0$ in the upwind part

The gas cooling leads to a smaller outflow of gas from the point of interaction. If the gas
has enough time to cool down significantly before it moves further downstream towards the tail, then a cold dense zone may form near the tangential discontinuity. With an increase of radiative losses parameters, and, consequently, the rate of cooling, this zone grows. The density rise causes the increase in pressure and the compression of the interaction region.

Figure 5. Isolines of the Mach number and streamlines obtained with various numerical schemes., $\alpha = 0.1$, $\theta = 1.0$, $3584 \times 2200$, $\chi = 1.628$.

Figure 6. Isolines of the Mach number for various resolutions of numerical grid, HLLC+tvd, $\alpha = 0.01$, $\theta = 1.0$, $\chi = 1.628$.

The first-order computational methods can have large numerical viscosity that stabilizes the flow. Figure 5 shows the isolines of Mach number calculated for the same parameters $\alpha$ and $\theta$ by various methods. It can be seen that the first-order HLL method (Fig. 5.a) gives a more stable solution due to the strong smearing of tangential discontinuities. Figure 5.b shows
the solution obtained using the first order HLLC method. In this case, the instability of the secondary tangential discontinuity becomes more noticeable. All second-order methods show approximately the same results: all figures show strong instabilities of primary and secondary tangential discontinuities. Thus, we can say that for almost all methods (except the first-order HLL), the formation of the Kelvin-Helmholtz instability is observed.

Numerical viscosity also depends on the numerical grid resolution. That can be seen in Figure 6: large cell size leads to smearing of the Kelvin-Helmholtz instability. With higher grid resolutions the instability becomes visible.

5. Conclusions

The aim of this study is to investigate the effects of radiative losses on the structure of the interaction region. In dimensionless form, the solution depends on five parameters: $\gamma$, $M_\infty$, $\chi$, $\alpha$, $\theta$. In the stationary case, the geometry of the pattern does not depend on $\chi$. Therefore, we present the numerical solutions for various $\alpha$ and $\theta$ parameters. The calculations are done using various methods with first and second order in space. Results show that radiative cooling causes the compression of the interaction region. The shock waves and the tangential discontinuity in the upwind part move closer to the star. The cause of such changes is a significant increase in density and pressure in the interaction region. The density difference can also lead to the growth of Kelvin-Helmholtz instability. It can be seen that the instability grows and moves to the upwind part with the increase of $\alpha$.

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