Nonlinear Dynamic Bending and Domain Wall Motion in Functionally Graded Piezoelectric Actuators under AC Electric Fields: Simulation and Experiment*

Yasuhide SHINDO**, Fumio NARITA**, Masaru MIKAMI** and Fumitoshi SAITO**

This paper describes the results of our numerical and experimental studies of the nonlinear bending behavior due to domain wall motion in functionally graded piezoelectric actuator under alternating current electric fields. A nonlinear three-dimensional finite element method is employed to simulate the dynamic response of cantilever functionally graded piezoelectric actuator. A phenomenological model of domain wall motion is used in computation, and the effects of ac electric field amplitude and frequency, number of layers, and property gradation on the deflection and internal stresses of the functionally graded bimorphs are examined. It is shown that the predicted deflection results, obtained from the numerical model, agree well with the corresponding experimental results.

Key Words: Elasticity, Finite Element Method, Material Testing, Functionally Graded Piezoelectric Materials, Dynamic Bending, Domain Wall Motion, Smart Structures

1. Introduction

Functionally graded piezoelectric materials (FGPMs) have recently been introduced and applied in the development of electronic components such as sensors, actuators and transducers. FGPMs possess continuously varying microstructure and electromechanical properties(1)–(3). In contrast to the sharp bimaterial interfaces that commonly arise in traditional bimorph or unimorph actuators, the gradual change in electromechanical properties throughout a FGPM seems to improve their resistance to interfacial delamination. In some actuator and transducer applications, fairly large electric fields are applied to these materials at relatively low frequency. One of the limitations for practical use of these piezoelectric materials and products is their nonlinear behavior, which occurs due to polarization switching(4)–(6) and/or domain wall motion(7),(8) at high driving levels. Experimental studies on soft lead zirconate titanate (PZT) have shown that the dielectric and piezoelectric coefficients increase with electric field due to the extrinsic contribution at room temperature(9)–(11). In order to optimize the performance of the piezoelectric devices with functionally graded microstructure under these conditions, it is essential to understand the nonlinear behavior in FGPMs.

In this work, we investigate both analytically and experimentally the nonlinear bending behavior of functionally graded piezoelectric actuators under alternating current (ac) electric fields. The functionally graded piezoelectric actuators consist of many thin soft PZT layers. A simple phenomenological model of a vibrating domain wall in ac electric fields is used, and the contribution of domain wall motion to the elastic compliance, piezoelectric coefficient and dielectric permittivity in PZT layers is evaluated. A nonlinear finite element analysis is then employed to examine the effects of the amplitude and frequency of electric fields, number of layers, and property gradation on the deflection and internal stresses for cantilever actuators. Experimental results are also presented to validate the predictions using functionally graded bimorph actuators. Calculations show reasonable agreement with the experimental data, and suggest that domain wall motion plays an important role in the nonlinear material response of the functionally graded piezoelectric actuators under ac electric fields.
2. Analysis

2.1 Basic equations

Consider the orthogonal coordinate system with axes \( x_1, x_2, \) and \( x_3 \). The basic equations for piezoelectric materials are

\[
\sigma_{ij} = \rho \ddot{u}_i, \quad D_i = 0
\]

(1)

\[
\varepsilon_{ij} = s_{ijkl} \sigma_{kl} + d_{klj} E_k
\]

(2)

\[
D_i = d_{ijkl} \sigma_{kl} + \epsilon_{il}^T E_k
\]

(3)

\[
\varepsilon'_{ij} = \frac{1}{2} (u_{,ij} + u_{,ji})
\]

(4)

\[
E_i = -\phi_j
\]

(5)

where \( \sigma_{ij}, D_i, \varepsilon_{ij} \) and \( E_i \) are the stress, electric displacement, strain and electric field intensity, \( u_i \) and \( \phi \) are the displacement and electric potential, \( \rho \) is the mass density, and \( s_{ijkl}, d_{klj} \) and \( \epsilon^T_{il} \) are the elastic compliances, piezoelectric coefficient and dielectric permittivity at constant stress, respectively. A comma followed by an index denotes partial differentiation with respect to the space coordinate \( x_i \) or the time \( t \). We have introduced the summation convention for repeated tensor indices. Valid symmetry conditions for the material constants are

\[
s_{ijkl} = s_{jikl} = s_{ijk} = s_{klij}, \quad d_{klj} = d_{klij}, \quad \epsilon^T_{il} = \epsilon^T_{i}\]

(7)

The constitutive equations (3) and (4) for PZT poled in the 1, 2 direction of the basic unit of a crystallite with a displaceable domain wall are found in Appendix A.

2.2 Model of domain wall motion

Figure 1(a) shows the domain structure in piezoelectric materials. The piezoelectric effect consists of two parts, the intrinsic and the extrinsic effects. The intrinsic effect results from the response of a single domain crystal under the application of an electric field, and the extrinsic effect represents the elastic deformation caused by the motions of domain walls. For simplicity here, the direction of the applied ac electric field \( E_0 \exp(\i\omega t) \) is parallel to the direction of spontaneous polarization \( P^s \) in one of the domains as shown in Fig. 1(b); \( E_0 \) is the ac electric field amplitude and \( \omega \) is the input frequency. A domain wall displacement \( \Delta l \) gives rise to the following changes of the strain \( \Delta \varepsilon_{ij} \) and electric dipole moment \( \Delta P_i \) of this basic unit:

\[
\Delta \varepsilon_{11} = -\frac{\Delta l}{l} \gamma^s, \quad \Delta \varepsilon_{22} = 0, \quad \Delta \varepsilon_{33} = \frac{\Delta l}{l} \gamma^s, \quad \Delta \varepsilon_{12} = 0, \quad \Delta \varepsilon_{23} = 0, \quad \Delta \varepsilon_{31} = 0
\]

(8)

\[
\Delta P_1 = \frac{\Delta l}{l} P^s, \quad \Delta P_2 = 0, \quad \Delta P_3 = \frac{\Delta l}{l} P^s
\]

(9)

where \( l \) is the domain width and \( \gamma^s \) is the spontaneous strain. The equation of motion of the domain wall may be written as

\[
m_{D} \ddot{\Delta l} + \beta \Delta l + f_D \Delta l = \frac{\partial W}{\partial \Delta l}
\]

(10)

where \( m_{D} \) is the effective mass per unit area of the wall, \( \beta \) is the damping constant of the wall motion, \( f_D \) represents the force constant for the domain wall motion process, and \( W = -\sigma_{ij} \Delta \varepsilon_{ij} + E_0 \Delta P_1 / 2 \) is the induced energy. The frequency \( \omega \) of the applied field is usually much smaller than the resonance frequency of the domain wall. Damping may be occasioned by coupling with lattice vibrations and other causes, but for the present we set \( \beta = 0 \) purely for convenience. Setting \( \Delta l = \Delta l_0 \exp(\i\omega t) \), from Eqs. (8) – (10), we have

\[
\Delta l_0 = \frac{1}{2 f_D} [\gamma^s (\sigma_{33} - \sigma_{11}) + P^s (E_3 - E_1)] \exp(-\i\omega t)
\]

(11)

The configuration of the piezoelectric actuator under consideration is shown in Fig. 2. The actuator is driven by
applying an ac voltage \( V_0 \exp(i\omega t) \); \( V_0 \) is the ac voltage amplitude. The induced strain \( \Delta \epsilon_{31} \) and polarization \( \Delta P_3 \) by the domain wall motion of the actuator according to Fig. 2 are given by \(^{(14)}\)

\[
\Delta \epsilon_{11} = \Delta \epsilon_{31}(\sigma_{11} + \Delta d_{31}) E_3 \tag{12}
\]

\[
\Delta P_3 = \Delta d_{31}(\sigma_{11} + \Delta \epsilon_{33}^T) E_3 \tag{13}
\]

where

\[
\Delta \epsilon_{11} = \gamma^2 \frac{E_0}{2I_D} \tag{14}
\]

\[
\Delta d_{31} = -\gamma^2 P^s \frac{E_0}{2I_D} \tag{15}
\]

\[
\Delta \epsilon_{33}^T = \frac{p^2}{2I_D} \tag{16}
\]

The strain \( \epsilon_{11} \) and electric displacement \( D_3 \) become

\[
\epsilon_{11} = s_{11}^{**} \sigma_{11} + s_{12}^{**} \sigma_{22} + s_{13}^{**} \sigma_{33} + d_{31}^{**} E_3 \tag{17}
\]

\[
D_3 = d_{31}^{**} \sigma_{11} + d_{31}^{**} \sigma_{22} + d_{31}^{**} \sigma_{33} + \epsilon_{33}^T E_3 \tag{18}
\]

where

\[
s_{11}^{**} = s_{11} + \Delta s_{11} \tag{19}
\]

\[
d_{31}^{**} = d_{31} + \Delta d_{31} \tag{20}
\]

\[
\epsilon_{33}^T = \epsilon_{33} + \Delta \epsilon_{33} \tag{21}
\]

All terms with \( \Delta \) are contributions from the domain wall motion.

Experimental studies on PZTs have shown that as much as 45–70\% of dielectric and piezoelectric moduli values may originate from the extrinsic contributions \(^{(9,10)}\). The extrinsic dielectric constant \( \Delta \epsilon_{33}^T \) was approximately estimated by Li et al. \(^{(15)}\) as the two thirds of the measured value. The following equation is utilized to describe \( \Delta \epsilon_{33}^T \) in terms of ac electric field amplitude \( E_0 \) and coercive electric field \( E_c \):

\[
\Delta \epsilon_{33}^T = \frac{\epsilon_{33}^T}{\epsilon_{33}^{**}} \frac{2E_0}{3E_c} \tag{22}
\]

2.3 Model

A three-dimensional piezoelectric plate model is illustrated in Fig. 3. Let the coordinate axes \( x = x_1 \) and \( y = x_2 \) be chosen such that they coincide with the middle plane of the hybrid laminate and the \( z = x_3 \) axis is perpendicular to this plane. Dimensions of the functionally graded piezoelectric plate are: \( a \) the length, \( b \) the width, \( h \) the thickness.

The piezoelectric material is graded through the thickness only, from pure PZT-A at the surface \( z = -h/2 \) to pure PZT-B at \( z = h/2 \). The plate is discretized into \( N \) layers which are assumed to have constant material properties. The thickness of the \( k \)th layer is \( h_k \). PZT of each layer is poled in the \( z \) direction, and the properties do not vary with \( z \) within each layer.

In the numerical examples, the functionally graded model concerning the material inhomogeneity is employed. The material constants of the \( k \)th layer for \( N \) layered functionally graded piezoelectric actuator are

\[
\begin{align*}
(s_{11})_k &= s_{11}^A V_k + s_{11}^B (1 - V_k) \\
(s_{33})_k &= s_{33}^A V_k + s_{33}^B (1 - V_k) \\
(s_{12})_k &= s_{12}^A V_k + s_{12}^B (1 - V_k) \\
(s_{13})_k &= s_{13}^A V_k + s_{13}^B (1 - V_k) \\
(d_{31})_k &= d_{31}^A V_k + d_{31}^B (1 - V_k) \\
(d_{32})_k &= d_{32}^A V_k + d_{32}^B (1 - V_k) \\
(d_{33})_k &= d_{33}^A V_k + d_{33}^B (1 - V_k) \\
(d_{15})_k &= d_{15}^A V_k + d_{15}^B (1 - V_k) \\
(\epsilon_{33}^T)_k &= \epsilon_{33}^T V_k + \epsilon_{33}^B (1 - V_k) \\
(\epsilon_{33}^{**})_k &= \epsilon_{33}^{**} V_k + \epsilon_{33}^{**} (1 - V_k)
\end{align*}
\]

where the superscripts \( A \) and \( B \) represent the materials PZT-A and PZT-B, respectively, and

\[
V_k = \left(1 - \frac{k}{N} \right)^{1/m}
\]

In Eq. (26), \( m \) is the functionally graded material volume fraction exponent, and governs the distribution pattern of the electroelastic properties across the thickness of the FGPMs. The material constants \( \Delta s_{11}, \Delta d_{31} \) and \( \Delta \epsilon_{33}^T \) of the \( k \)th layer are described by

\[
\begin{align*}
(\Delta s_{11})_k &= s_{11}^A V_k + (\Delta s_{11})_k (1 - V_k) \\
(\Delta d_{31})_k &= d_{31}^A V_k + (\Delta d_{31})_k (1 - V_k) \\
(\Delta \epsilon_{33}^T)_k &= \epsilon_{33}^T V_k + (\Delta \epsilon_{33}^T)_k (1 - V_k)
\end{align*}
\]

2.4 Finite element method

We use a commercial finite element package ANSYS to perform the analysis. The finite element model uses three-dimensional eight-noded elements. From Eqs. (14)–(16) and (22), the elastic compliance \( (s_{11})_k = (s_{11}) + (\Delta s_{11})_k \), piezoelectric coefficient \( (d_{31})_k = (d_{31}) + (\Delta d_{31})_k \) and dielectric permittivity \( (\epsilon_{33}^T)_k = (\epsilon_{33}^T) + (\Delta \epsilon_{33}^T)_k \) vary with domain wall motion. \( P^s = 0.3 \text{C/m}^2 \) and \( \gamma^2 = 0.004 \) are used to get \( \Delta s_{11}, \Delta d_{31} \) and \( \Delta \epsilon_{33}^T \). Making use of electric field dependent piezoelectric material properties, the model calculated the nonlinear dynamic bending behavior.

3. Experimental Procedure

Cantilever functionally graded piezoelectric bimorph actuator was prepared using soft PZTs C-91 and C-6 (Fuji...
Table 1. Material properties of soft PZTs

| Property | C-91 | C-6 |
|----------|------|-----|
| $\varepsilon_{11}$ | $17.1$ | $16.6$ |
| $\varepsilon_{33}$ | $18.6$ | $20.5$ |
| $\varepsilon_{15}$ | $4.1 \times 10^{-12}$ | $53.2 \times 10^{-12}$ |
| $\varepsilon_{13}$ | $-6.3$ | $-5.1$ |
| $\varepsilon_{35}$ | $-7.3$ | $-8.2$ |
| $d_{31}$ | $-340$ | $-224$ |
| $d_{33}$ | $645$ | $471$ |
| $d_{15}$ | $836$ | $771$ |
| $\rho$ | $395$ | $203$ |
| $\rho$ | $490$ | $180$ |
| $\rho$ | $7800$ | $7400$ |

Ceramics Co., Ltd., Japan). 3-layered FGPMs of thickness $h_1 + h_2 + h_3 = 0.3$ mm and $h_4 + h_5 + h_6 = 0.3$ mm, created by extrusion, are respectively added to the upper and lower surfaces of an electrode film to make the 6-layered inward series bimorph. The thickness of each layer (54 mm long, 20 mm wide) is about $h_k = 0.1$ mm ($k = 1, \ldots, 6$). The specimen has dimensions of approximately $54$ mm $\times$ $20$ mm $\times$ $0.6$ mm.

The properties are graded according to Eqs. (23)–(29) from the PZT C-6 at the inner layers ($k = 3, 4$) to the PZT C-91 at the outer layers ($k = 1, 6$), and $m$ is taken to be 1. The material properties of the PZT samples C-91 and C-6 are listed in Table 1. The coercive electric fields of C-91 and C-6 are about 0.35 and 0.45 MV/m, respectively.

The free length was set to 40 mm. The center electrode was grounded, while an ac voltage was applied to the surface electrode of the lower element. Using a microscope, the amplitude of the dynamic displacement of the cantilever bimorph was measured at different ac voltages.

4. Results and Discussion

Tip deflection and internal stresses of the cantilever functionally graded piezoelectric actuators are obtained. All actuators are inward series bimorphs and have the same geometric dimensions, i.e. free length of 40 mm, width of 20 mm and total thickness of 0.6 mm. Consider two types of grading through the thickness as shown in Fig. 4, type I where the piezoelectric properties increase toward the mid-plane, and type II where the piezoelectric properties decrease toward the mid-plane. The PZT C-91 is characterized by high piezoelectric constants and low coercive electric field.

We first present analytical and experimental results for cantilever 6-layered inward series bimorph ($m = 1$) with a symmetric construction of type II [C-91/.../C-6]$_s$, where [ ]$_s$ designates symmetry about the middle surface. Figure 5 shows the amplitude of the tip deflection $w_{tip}$ as a function of ac voltage $V_0$ at frequency $f = \omega/2\pi = 60$ Hz. The dashed line represents the values of deflection predicted by the finite element analysis (FEA) without extrinsic effect, the solid line represents the deflection after the extrinsic effect has been applied, and the open circle denotes the experimental data. It may be seen that the differences between the predicted tip deflections with and without the extrinsic effect increase as ac voltage increases. This is an anticipated result based on the enhanced extrinsic contribution (domain wall motion) to electromechanical response. The calculation results with the extrinsic effect are in good agreement with experimental measurements.

Next, the tip deflection and internal stresses of the actuators obtained from the FEA with the extrinsic effect are discussed in detail. Figure 6 shows the amplitude of the tip deflection $w_{tip}$ for two types of 8- and 6-layered bimorphs ($m = 1$) as a function of the ac voltage $V_0$ at 60 Hz. For comparison, the results for traditional C-91 and C-6 bimorphs without shim (2-layered actuators) are included. In considering the tip deflection of the type I [C-6/.../C-91]$_s$ bimorph, the largest deflection is observed for the
Fig. 6 Dynamic deflections versus voltage ($m = 1$)

Fig. 7 Distributions of normal stress $\sigma_{xx}$ along the thickness direction ([C-6]/.../[C-91]$_s$, $m = 1$)

Fig. 8 Distributions of shear stress $\sigma_{zx}$ along the thickness direction ([C-6]/.../[C-91]$_s$, $m = 1$)

Fig. 9 Distributions of normal stress $\sigma_{xx}$ along the thickness direction ([C-91]/.../[C-6]$_s$, $m = 1$)

The effect of piezoelectric property distributions on the tip deflection versus ac voltage is examined in Fig. 11 for the 8-layered type I [C-6]/.../[C-91]$_s$ bimorphs at 60 Hz. For an equivalent voltage (for example, 60 V), the actuator with volume fraction exponent $m = 1.4$ is predicted to have a tip deflection that is greater than the actuator fabricated with $m = 1$. The deflection increases as $m$ increases and this trend may be more clearly observed in Fig. 12. In Fig. 13, we have plotted the through-the-thickness variation of the normal stress $\sigma_{xx}$ at $x = 20$ mm and $y = 0$ mm of the 8- and 6-layered type II [C-91]/.../[C-6]$_s$ bimorphs with different volume fraction exponents under $w_{tip} = 200 \mu$m at 60 Hz. The driving voltages of the actuators with $m = 1.8$, exhibits the through-the-thickness variation of the normal stress $\sigma_{xx}$ at $x = 20$ mm and $y = 0$ mm of the 8- and 6-layered type II [C-91]/.../[C-6]$_s$ bimorphs ($m = 1$) under $w_{tip} = 200 \mu$m at 60 Hz. Also shown are the results of the traditional C-6 bimorph. The driving voltages of the actuator under $w_{tip} = 200 \mu$m are about 84.7 V for the 8-layered [C-91]/.../[C-6]$_s$ bimorph, 86.6 V for the 6-layered [C-91]/.../[C-6]$_s$ bimorph, and 89.2 V for the traditional C-6 bimorph, respectively. Figure 10 shows the similar results for the shear stress $\sigma_{zx}$.

The traditional C-91 bimorph and increasing $V_0$, in general, $w_{tip}$ increases. The functionally graded actuators can result in smaller deflections. In our results for the type II [C-91]/.../[C-6]$_s$ bimorph, larger deflection can be archived with the functionally graded actuators than with the traditional C-6 bimorph. The deflection of 8-layered type II [C-91]/.../[C-6]$_s$ bimorph is larger than that of 6-layered type II [C-91]/.../[C-6]$_s$ bimorph. The variation of normal stress $\sigma_{xx}$ along the thickness direction are calculated for the 8- and 6-layered type I [C-6]/.../[C-91]$_s$ bimorphs ($m = 1$) at a chosen point ($x = 20$ mm and $y = 0$ mm here) and the results are shown in Fig. 7. The result for the traditional C-91 bimorph is shown for comparison purposes. All calculations were done at a fixed deflection of 200$\mu$m and frequency of 60 Hz. The driving voltages of the actuator under $w_{tip} = 200 \mu$m are about 62.5 V for the 8-layered [C-6]/.../[C-91]$_s$ bimorph, 63.0 V for the 6-layered [C-6]/.../[C-91]$_s$ bimorph, and 53.7 V for the traditional C-91 bimorph, respectively. The magnitudes of the normal stresses increase toward the mid-plane as is expected. The highest normal stress is noted for the traditional C-91 bimorph. In other words, the functionally graded piezoelectric actuators give lower normal stress. Similar results for the shear stress $\sigma_{zx}$ are shown in Fig. 8. Lower shear stress is also found in the functionally graded actuators. Figure 9.

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Fig. 10 Distributions of shear stress $\sigma_{zx}$ along the thickness direction ([C-91/$\ldots$/C-6]$_s$, $m = 1$)

Fig. 11 Dynamic deflections versus voltage ([C-6/$\ldots$/C-91]$_s$, 8-layered)

Fig. 12 Dynamic deflection versus material volume fraction exponent

1, 0.6 under $u_{tp} = 200 \mu m$ are about 59.8, 62.5, and 65.2 V, respectively. The volume fraction exponent has very little effect on the normal stress. Also, shear stress is not influenced by the volume fraction exponent (see Fig. 14). At higher volume fraction exponents, a 200 $\mu m$ deflection is produced at lower voltages, but the internal stresses are almost the same.

Fig. 13 Distributions of normal stress $\sigma_{xx}$ along the thickness direction ([C-6/$\ldots$/C-91]$_s$, 8-layered)

Fig. 14 Distributions of shear stress $\sigma_{zx}$ along the thickness direction ([C-6/$\ldots$/C-91]$_s$, 8-layered)

5. Conclusions

A phenomenological model of domain wall motion has been presented for the dynamic finite element analysis of functionally graded piezoelectric actuators. Validation of the present calculation was obtained by comparing, with excellent agreement, the measured tip deflection of a cantilever functionally graded bimorph under ac voltage. For a given ac voltage, the variation pattern of the electroelastic properties are shown to govern the tip deflection of the actuator. When the piezoelectric properties increase toward the mid-plane of the bimorph actuators, functional grading of materials could effectively reduce the magnitude of internal stresses or obtain an optimal pattern of normal and shear stresses in actuators for a given design application. The results of this study are expected to encourage further numerical and experimental research on the manufacture, design, and characterization of FGPMs that could function safely under ac voltage.

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For piezoelectric materials which exhibit symmetry of a hexagonal crystal of class 6 mm with respect to principal $x_1$, $x_2$, and $x_3$ axes, the constitutive relations can be written in the following form:

$$
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6 \\
\end{bmatrix} =
\begin{bmatrix}
s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\
s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\
s_{13} & s_{13} & s_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & s_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & s_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & s_{66} \\
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 \\
\end{bmatrix}
$$

$$
\begin{bmatrix}
E_1 \\
E_2 \\
E_3 \\
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & d_{31} \\
0 & 0 & d_{31} \\
d_{15} & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
$$

$$
\begin{bmatrix}
D_1 \\
D_2 \\
D_3 \\
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & d_{15} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 \\
\end{bmatrix}
$$

$$
\begin{bmatrix}
\varepsilon_1^T \\
\varepsilon_2^T \\
\varepsilon_3^T \\
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & e_{33}^T \\
0 & 0 & 0 \\
\end{bmatrix}
$$

where

$$
\begin{align}
\sigma_1 &= \sigma_{11}, \quad \sigma_2 = \sigma_{22}, \quad \sigma_3 = \sigma_{33} \\
\sigma_4 &= \sigma_{23} = \sigma_{32}, \quad \sigma_5 = \sigma_{31} = \sigma_{13}, \\
\sigma_6 &= \sigma_{12} = \sigma_{21} \\
\end{align}
$$

$$
\begin{align}
\varepsilon_1 &= \varepsilon_{11}, \quad \varepsilon_2 = \varepsilon_{22}, \quad \varepsilon_3 = \varepsilon_{33}, \\
\varepsilon_4 &= 2\varepsilon_{23} = 2\varepsilon_{32}, \quad \varepsilon_5 = 2\varepsilon_{31} = 2\varepsilon_{13}, \\
\varepsilon_6 &= 2\varepsilon_{12} = 2\varepsilon_{21} \\
\end{align}
$$

$$
\begin{align}
s_{11} &= s_{1111}, \quad s_{12} = s_{1122}, \quad s_{13} = s_{1133}, \quad s_{1233} = s_{3333} \\
s_{33} &= s_{2222} + 2s_{1111}, \quad s_{12} = s_{1122}, \quad s_{13} = s_{1133}, \quad s_{1233} = s_{3333} \\
s_{44} &= s_{3322} + s_{3311}, \quad s_{55} = s_{3312} = 2(s_{11} - s_{12}) \\
d_{15} &= 2d_{31} = 2d_{31}, \quad d_{33} = d_{33} = d_{33} \\
\end{align}
$$

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