Can Kozai-Lidov cycles explain Kepler-78b?

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ABSTRACT

Kepler-78b is one of a growing sample of planets similar, in composition and size, to the Earth. It was first detected with NASA’s Kepler spacecraft and then characterised in more detail using radial velocity follow-up observations. Not only is its size similar to that of the Earth (1.2 \(R_\oplus\)), it also has a very similar density (5.6 g cm\(^{-3}\)). What makes this planet particularly interesting is that it orbits its host star every 8.5 hours, giving it an orbital distance of only 0.0089 au. What we investigate here is whether or not such a planet could have been perturbed into this orbit by an outer companion on an inclined orbit. In this scenario, the outer perturber causes the inner orbit to undergo Kozai-Lidov cycles which, if the periapse comes sufficiently close to the host star, can then lead to the planet being tidally circularised into a close orbit. We find that this process can indeed produce such very-close-in planets within the age of the host star (\(\sim 600 - 900\) Myr), but it is more likely to find such ultra-short-period planets around slightly older stars (\(\geq 1\) Gyr). However, given the size of the Kepler sample and the likely binarity, our results suggest that Kepler-78b may indeed have been perturbed into its current orbit by an outer stellar companion. The likelihood of this happening, however, is low enough that other processes – such as planet-planet scattering – could also be responsible.

Key words: planets and satellites : formation - planets and satellites : general - planets and satellites - terrestrial planets - planet-star interactions

1 INTRODUCTION

Analysis of data from NASA’s Kepler spacecraft (Borucki et al. 2010; Batalha et al. 2013) indicates that planets with radii similar to that of the Earth are common (Petigura, Marcy & Howard 2013; Dressing & Charbonneau 2013). Recently it was announced that one of the Kepler targets (Kepler-78) showed a 0.02% decline in brightness that was associated with a planet with a radius of only 1.16 ± 0.19\(R_\oplus\) (Sanchis-Ojeda et al. 2013). Follow-up observations, using HARPS-N (Cosentino et al. 2012) and the High Resolution Echelle Spectrometer (HIRES) (Vogt et al. 1994), confirmed that this is indeed a planet with a mass of about 1.86\(M_\oplus\) and a density of about 5.6 g cm\(^{-3}\) (Pepe et al. 2013; Howard et al. 2013).

Of course it is fascinating that we are now detecting planets with sizes and densities similar to that of the Earth, but what makes this planet particularly interesting is that it has an orbital period of only 8.5 hours, meaning that it is orbiting at a distance of only 0.0089 au from its parent star. Quite how such a planet can end up in such an orbit is very uncertain. It almost certainly could not have formed where it now resides, as the temperature in the disc that region would have been too high even for dust grains to condense (Bell et al. 1997). It could potentially have migrated inwards through disc migration. However, such low-mass planets would migrate in the gapless, Type I regime (Ward 1997) which is typically thought to be so fast (Tanaka, Takeuchi & Ward 2002; Kley & Crida 2008) that it would seem unlikely that such objects could be left stranded so close to their parent stars. Population synthesis models (Ida & Lin 2008; Mordasini, Alibert & Benz 2009) typically assume a reduced Type I migration rate.

Alternatively, such close-in planets could be scattered onto eccentric orbits (Ford & Rasio 2006) that are then circularised through tidal interactions with the parent star (Rasio et al. 1996). It has indeed been suggested that if such a process were to occur, we should be seeing some very short period hot super-Earths (Schlaufman, Lin & Ida 2010), so this could be an explanation for the origin of Kepler-78b. However, even this study suggested that typical orbital periods would be greater than the 8.5 hour orbital period of Kepler-78b.

Another mechanism for forming close-in planets, related to dynamical interactions in multi-planet systems, is for the planet to undergo Kozai-Lidov cycles driven by a stellar companion on a highly inclined orbit (Kozai 1962; Lidov 1962). If the eccentricity is sufficiently large, so that the periastron becomes very small, the planet’s orbit may be circularised through tidal interactions with its host star (Wu & Murray 2003; Fabrycky & Tremaine 2007). What we want to investigate in this paper is whether or not this process could indeed explain the origin of Kepler-78b. Given that binarity amongst solar-like stars is quite high (Duquennoy & Mayor 1991; Abi & Willmart 2006) it seems likely that this could play a role in producing close-in, Earth-sized planets.
In this paper we present results from a series of Monte Carlo simulations in which we consider how a planet with a mass and density the same as that of Kepler-78b, but initially orbiting between 0.5 and 2 au, is influenced by perturbations from a binary stellar companion. We also include the influence of tides, which would allow the orbit to circularise if the eccentricity becomes sufficiently large, and the influence of general relativistic and apsidal precession. The paper is organised as follows: in Section 2 we present equations of motion, in Section 3 we describe the basic setup of the problem, in Section 4 we discuss the results, and in Section 5 we discuss the results and draw some conclusions.

## 2 Equations of motion

The goal is to evolve an inner binary (planet and star) under the influence of tidal interactions, perturbing accelerations from stellar and planetary distortions due to tides and rotation, perturbing accelerations from a third body, and general relativistic apsidal precession. To quadrupole order, these equations were first presented by (Eggleton & Kiseleva 2001) and can also be found in Wu & Murray (2003) and Fabrycky & Tremaine (2007). Rather than using the equations in Eggleton & Kiseleva (2001), we’ve implemented those from Barker & Ogilvie (2009) and Barker (2011), which are regular at $e = 0$. We want to evolve an inner system (planet + host star) where the bodies have masses $M_s$ and $M_p$, radii $R_s$ and $R_p$, and in which the orbit has an eccentricity $e$, semi-major axis $a$, and orbital angular frequency $n = \sqrt{G(M_s + M_p)/a^3}$. The vector quantities that we want to evolve are, therefore, the spin of the parent star $\Omega_s$, the spin of the planet $\Omega_p$, the eccentricity of the inner orbit $e$, and angular momentum vector of the inner orbit $\mathbf{h} = r \times \mathbf{r} = na^2 \sqrt{1 - e^2} \mathbf{h}$. We’ve built our model by considering, initially, only the equations that evolve these quantities through tidal dissipation and a stellar wind. From Barker & Ogilvie (2009) we have

\[
\frac{d\Omega_s}{dt} = \frac{\mu}{I_s} \left( \frac{dh}{dt} \right)_s + \dot{\Omega}_{\text{wind}}
\]

\[
\frac{d\Omega_p}{dt} = \frac{\mu}{I_p} \left( \frac{dh}{dt} \right)_p,
\]

where $I_s$ and $I_p$ are the moments of inertia of the star and planet, $\dot{\Omega}_{\text{wind}}$ represents the stellar wind, and $\mu = M_s M_p / (M_s + M_p)$ is the reduced mass of the inner system. We also need to define the tidal friction timescales for the star and planet ($t_{f_s}$ and $t_{f_p}$), which depend on the star and planet’s tidal quality factors ($Q'_s$ and $Q'_p$), and the functions of the eccentricity.

\[
\frac{dt_{f_s}}{dt} = \left( \frac{9n}{2Q'_s} \right) \left( \frac{M_p}{M_s} \right) \left( \frac{R_s}{a} \right)^5
\]

\[
\frac{dt_{f_p}}{dt} = \left( \frac{9n}{2Q'_p} \right) \left( \frac{M_s}{M_p} \right) \left( \frac{R_p}{a} \right)^5
\]
We also include the contributions due to an additional outer body from Barker (2011) apsidal precession (outer body’s orbit (not to be confused with quadrupolar distortions of the inner star and planet due to tidal and rotational bulges (qs and qp), and we include general relativistic apsidal precession (GR). The equations, shown below, are taken from Barker (2011)

\[
\begin{align*}
 f_1(e^2) &= 1 + \frac{25}{8} e^2 + \frac{45}{8} e^4 + \frac{5}{4} e^6 \quad (1 - e^2) \frac{d}{dt}
 f_2(e^2) &= 1 + \frac{2}{3} e^2 + \frac{5}{4} e^4 \quad (1 - e^2)^2 \frac{d}{dt}
 f_3(e^2) &= 1 + \frac{25}{8} e^2 + \frac{5}{4} e^4 \quad (1 - e^2) \frac{d}{dt}
 f_4(e^2) &= 1 + \frac{15}{4} e^2 + \frac{45}{8} e^4 + \frac{5}{4} e^6 \quad (1 - e^2) \frac{d}{dt}
 f_5(e^2) &= 3 + \frac{1}{2} e^2 \quad (1 - e^2)^2 \frac{d}{dt}
 f_6(e^2) &= 1 + \frac{25}{8} e^2 + \frac{25}{8} e^4 + \frac{5}{4} e^6 + \frac{25}{8} e^8 \quad (1 - e^2)^8 \frac{d}{dt}
\end{align*}
\]

We also include the contributions due to an additional outer body (b) of mass \( M_o \), orbital angular frequency \( \nu_o \), semi-major axis \( a_o \), and eccentricity \( e_o \). Additionally, we add contributions from quadrupolar distortions of the inner star and planet due to tidal and rotational bulges (qs and qp), and we include general relativistic apsidal precession (GR). The equations, shown below, are taken from Barker (2011)

\[
\begin{align*}
 (\frac{dh}{dt})_{qs} &= -\frac{\alpha_s}{h(1-e^2)^2}(\Omega_s \cdot h)(\Omega_s \times h)
 (\frac{dh}{dt})_{qp} &= -\frac{\alpha_p}{h^2(1-e^2)^2}(\Omega_p \cdot h)(\Omega_p \times h)
 h (\frac{de}{dt})_{qs} &= 3\alpha_h(1-e^2)[2(h \times e) - (n \cdot h)(n \times e) + 5(n \cdot e)(n \times e)]
 h (\frac{de}{dt})_{qp} &= \frac{\alpha_s}{(1-e^2)^2}\left[\frac{3}{8}(\Omega_s \cdot h)^2 - \Omega_s^2\right] \\
 &+ 15G M_p a^3 f_2(e^2)(1-e^2)^2(h \times e)
 &+ \frac{\alpha_s}{h^2(1-e^2)^2}(\Omega_s \cdot h)(\Omega_s \cdot h \times e) h
 h (\frac{de}{dt})_{GR} &= 3G(M_s + M_p)n \quad ac^2(1-e^2)^2(h \times e),
\end{align*}
\]

\[
\begin{align*}
 \alpha_s &= \frac{R_o^3 k_s M_p}{2\mu_{na} a^5}
 \alpha_p &= \frac{R_o^3 k_p M_s}{2\mu_{na} a^5}
 C_b &= \frac{M_o}{M_s + M_p + M_o} \frac{n^2}{n(1-e^2)^{1/2}(1-e^2)^{3/2}}
\end{align*}
\]

In Equation (18), \( k_s \) and \( k_p \) are the inner star and planet’s tidal love numbers.

**2.1 Octupole terms**

It now appears that expanding the equations only to quadrupole order may not be appropriate for many systems (Naoz et al. 2011; Naoz, Farr & Rasio 2013), so we’ve also included the octupole terms. This allows us to consider situations in which the outer body’s mass is comparable to that of the inner planet, and to consider situations in which the outer orbit is eccentric.

We’re unable to write the octupole terms in a way that is regular at \( e = 0 \), so have implemented the form in (Mardling & Lith 2002). The octupole contributions are

\[
\begin{align*}
 (\frac{dh}{dt})_{oct} &= \frac{G(M_s + M_p)}{a} \left( \frac{M_o}{M_s + M_p} \right) \left( \frac{M_s - M_p}{M_s + M_p} \right) \\
 &\times \left( \frac{a}{R} \right)^4 \frac{15 e}{16} \left\{ 10(1-e^2)\dot{R}_1 \dot{R}_2 \dot{R}_3 \dot{e} \\
 &+ [(4 + 3e^2)\dot{R}_3 - 5(3 + 4e^2)]\dot{R}_1^2 \dot{R}_3 \\
 &- 5(1-e^2)\dot{R}_2^2 \dot{R}_3 \dot{q} \\
 &- [(4 + 3e^2)\dot{R}_2 - 5(1 + 6e^2)]\dot{R}_1^2 \dot{R}_2 \\
 &- 5(1-e^2)\dot{R}_2^3 \dot{h} \right\} \\
 (\frac{de}{dt})_{oct} &= -\frac{G(M_s + M_p)}{a} \left( \frac{M_o}{M_s + M_p} \right) \left( \frac{M_s - M_p}{M_s + M_p} \right) \\
 &\times \left( \frac{a}{R} \right)^4 \frac{15 e}{16} \left\{ -(4 + 3e^2)\dot{R}_2^2 \\
 &+ (5 + 6e^2)\dot{R}_1^2 \dot{R}_2 + 5(1-e^2)\dot{R}_3^3 \dot{e} \\
 &+ [(4 + 3e^2)\dot{R}_1 - 5(1 - 3e^2)]\dot{R}_1 \dot{R}_2^2 \\
 &- 5(1+ 4e^2)\dot{R}_3^3 \dot{q} \\
 &+ 10e^2\dot{R}_1 \dot{R}_2 \dot{R}_3 \dot{h} \right\},
\end{align*}
\]

where \( R \) is the co-ordinate of the outer body, and the co-ordinate frame is defined by the basis vectors \( (\hat{e}, \hat{q}, \hat{h}) \), with \( \hat{q} = \hat{h} \times \hat{e} \). The other unit vectors above are \( \hat{R}_1 = \dot{R} \cdot \hat{e}, \hat{R}_2 = \dot{R} \cdot \hat{q}, \hat{R}_3 = \dot{R} \cdot \hat{h} \).
2.2 Integrating the outer orbit

The octupole terms described above need the co-ordinate of the outer body, which we determine by solving for the eccentric anomaly, $E$. This can be done by iterating the following equations until $dE$ is below a threshold (we use $10^{-12}$)

$$dE = \frac{-(E - e_o \sin E - l)}{1 - e_o \cos E}$$

$$E = E + dE,$$

where $l$ is the mean anomaly $[l = n_o (t - P)]$, with $P$ the orbital period and $t$ the time since the completion of the last full orbit of the outer body. In all of our simulations, we fix the outer body to lie in the $xy$ plane and so its co-ordinates are then

$$R_x = a_o (\cos E - e_o)$$
$$R_y = a_o \sqrt{1 - e_o^2} \sin E$$
$$R_z = 0,$$

where $a_o$ and $e_o$ are the semi-major axis and eccentricity of the outer orbit. In this work, we neglect perturbations on the outer orbit.

2.3 Stellar wind

Without a stellar wind, or with a very weak stellar wind, it is possible that tidal interactions between the star and planet can result in the planet being trapped in a close orbit (Dobbs-Dixon, Lin & Mardling 2004). Most stars, however, have winds that continue to remove angular momentum, and so once tidal interactions become significant, we would typically expect the planet to continue spiralling in towards the central star. We implement here a very simple magnetic braking form for the stellar wind (Weber & Davis 1967; Kawaler 1988; Collier Cameron & Jianke 1994) so that in the unsaturated regime, the stellar wind term is

$$\dot{\Omega}_{\text{wind}} = -\kappa_w \Omega^3,$$

where $\kappa_w$ is the braking efficiency coefficient. In the saturated regime (when $\Omega_s > \Omega$) this becomes

$$\dot{\Omega}_{\text{wind}} = -\kappa_w \Omega^2 \dot{\Omega}.$$  

The braking efficiency coefficient, $\kappa_w$, is set so that the stellar rotation period matches that expected for the star being considered, and $\dot{\Omega}$ is set to be $14 \Omega_{\odot}$. The vector associated with the stellar wind is always set so as to point in to opposite direction to that of the spin of the planet host star.

2.4 Putting it together

Ultimately we want to evolve the angular momentum, $h$, and eccentricity, $e$, of the inner orbit, and the spins of the planet and its host star, $\Omega_D$ and $\Omega_s$. The evolution of the stellar spin is determined by combining the stellar wind equations [Equations (24) and (25)] with Equation (3). The evolution of the spins of the star and planet [Equation (4)], both depend on the tidal evolution of the orbital angular momentum [Equation (1)].

To evolve the angular momentum of the inner orbit, we need to add the contributions from tides [Equation (1)], perturbations from an outer body expanded to quadrapole and octupole order [Equations (15), and (21)], perturbations from distortions of the inner star and planet [Equation (16)] and general relativistic apsidal precession [Equation (17)].

2.5 Some basic tests

Since this is a new code, we ran a few comparison tests to check that it was working properly. The first was that introduced by Wu & Murray (2003). It comprises a $1.1M_\odot$ star with a $7.8M_{\text{Jup}}$ planetary companion, the star having a radius of $1R_\odot$ and the planet having a radius the same as that of Jupiter. The initial stellar and planetary spin periods are, respectively, 20 days and 10 hours. The inner system’s orbit has a semimajor axis of $a = 5.0$ au, and eccentricity $e = 0.1$, the tidal love numbers are $k_{2s} = 0.028$ and $k_{3s} = 0.51$, and the tidal dissipation quality factors are $Q'_{e} = 5.35 \times 10^7$ and $Q'_{\theta} = 5.88 \times 10^5$. The system also has a $1.1M_\odot$ companion with $a_p = 1000$ au, $e_p = 0.5$ and with an orbital plane inclined at 85.6° to that of the plane of the inner orbit.

Figure 1 shows the time evolution of the semi-major axis (dashed line) and periastr (solid line) of the system described above, and appears the same as that in Fabrycky & Tremaine (2007), who also performed this test. It’s not quite the same as in Wu & Murray (2003), but we can match their results if we remove the apsidal precession term and match our results if we ignore the term representing the apsidal precession due to the spin and tidal bulges of the planet.
The system we want to consider specifically is Kepler-78 (Sanchis-Ojeda et al. 2013). The companion planet, with a mass of 0.1 $M_\oplus$ and $M_p = 1M_\oplus$, a semi-major axis chosen randomly in log $a$, between $a_o = 40$ au and $a_o = 20000$ au, and a randomly chosen eccentricity between $e_o = 0$ and $e_o = 1$. We then fix the outer companion’s orbit to be in the $xy$ plane and randomly orientate the inner orbit so that the mutual inclination, $i$, is isotropic (Wu, Murray & Ramsaha 2007). We also randomly orientate the longitude of the planet’s ascending node. By choosing such a high-mass companion, we’re essentially in the test particle regime (Lithwick & Naoz 2011). Such companions will also produce a large maximum eccentricity (for the inner orbit) than lower mass companions (Teyssandier et al. 2013). As such, we might expect a reasonably large number of tidal disruption events (Naoz, Farr & Rasio 2012; Li et al. 2014; Petrovich 2015). As such, our results only apply to a situation where the companion is of stellar mass. We also impose stability criteria (Lithwick & Naoz 2011; Naoz et al. 2013; Mardling & Aarseth 2000) and insist that
\[
\frac{a_o}{a} \left(1 - \frac{e_o}{2} \right) < 0.1, \tag{26}
\]
and that
\[
\frac{a_o}{a} > 2.8 \left(1 + \frac{M_o}{M_s + M_p}\right)^{2/5} \left(\frac{1 + e_o}{1 - e_o}\right)^{2/5} \left(1 - \frac{0.3i}{180^\circ}\right). \tag{27}
\]
Equation (26) ensures that we are in the regime where the quadrupole and octupole terms dominate, while Equation (27), in which $i$ is the mutual inclination of the two orbits, ensures that the triple system is long-term stable (Mardling & Aarseth 2001). Equation (27) is almost always satisfied for the initial conditions used here.

4.1 Initial results

The tidal quality factor for a terrestrial planet is thought to lie between $Q_p = 10$ and $Q_p = 500$ (Goldreich & Soter 1966). Since we’re considering a young system in which the planet likely retains a lot of its initial internal heat, we assume a value at the top of this range ($Q_p = 500$), and also a more extreme case where $Q_p = 5000$ (Henning, O’Connell & Sassellar 2005). For the star, we assume tidal quality factors of $Q_* = 5 \times 10^5$ and $Q_* = 5 \times 10^6$, within the range expected for exoplanet host stars (Baraffe, Chabrier & Barman 2010; Brown et al. 2011). For each simulation we select the initial conditions as described above and evolve the system until $t = 800$ Myr, using a fourth-order Runge-Kutta integrator. We repeat this 10000 times for each set of parameters, and the basic result is shown in Figure 3. The top panel is for $Q_p = 500$ and the bottom for $Q_p = 5000$. The solid line in each figure is for $Q_* = 5 \times 10^5$, the dashed line is for $Q_* = 5 \times 10^6$, and the vertical dash-dot line indicates the current semimajor axis of Kepler-78b. In each case, the number of planets still located between 0.5 and 2 is very large and their distribution extends well above the limits shown on the y-axis.

From Figure 3 it seems clear that it is possible for a planet to be perturbed into an orbit inside $a = 0.01$ au within 800 Myr. However, the numbers are typically small. For $Q_p = 500$ it is 10 ($Q_* = 5 \times 10^5$) and 4 ($Q_* = 5 \times 10^6$), while for $Q_p = 5000$ it is

![Figure 2](image-url)
and 7 and 4 respectively. Even though the number of planets surviving inside $a = 0.01$ au is small, in most cases, a much larger number reach their Roche limit $[a = R_p/(2M*2M_p)^{1/3} = 0.0056$ au] (Faber, Rasio & Willems 2005) and are assumed to be tidally disrupted and destroyed. With the exception of the $Q_p' = 500$, $Q_s' = 5 \times 10^5$ simulation (in which the numbers were small), in excess of 100 - out of a sample of 10000 - reached the Roche limit.

Given that a large numbers of planets do become tidally destroyed, it is useful to know for how long a planet might exist inside $a = 0.01$ au. Figure 3 shows single planet simulations, one for each combination of $Q_p'$ and $Q_s'$, each of which is run until the planet reaches its Roche limit. The amount of time such a planet spends inside $a = 0.01$ au depends, primarily, on the star’s tidal quality factor. For $Q_p' = 5 \times 10^5$, the planet reaches the Roche limit in 480 Myr, while for $Q_s' = 5 \times 10^5$ it takes 48 Myr. Therefore, it would seem that for reasonable estimates of the star’s tidal quality factor, a planet such as Kepler-78b will only be detectable inside $a = 0.01$ au for a few hundred Myrs at most.

Our initial results would therefore seem to suggest that it is possible for an outer companion to perturb a planet like Kepler-78b into a very close orbit ($a < 0.01$ au) within the age of the system ($\sim 800$ Myr). However, the numbers are small, with at most 10 out of 10000 surviving inside $a = 0.01$ au at $t = 800$ Myr.

### 4.2 Age of the system

The previous simulations only considered the likelihood of a system with an age similar to that of Kepler-78, having a planetary companion with orbital properties similar to the of Kepler-78b. The results suggest it is possible, but probably rare. Additionally, the age distribution of a sample of 950 Kepler object of interest host stars (Walkowicz & Basri 2013) suggests that about 10% have ages less than 1 Gyr. This is also probably biased towards younger stars because of the way in which the sample was selected. Given that about 40% of these would have stellar companions (Raghavan et al. 2010) would further reduce the likelihood of actually observing a Kepler-78b type system.

To investigate how our results might depend on the age of the system, we reran our simulations with the same set of parameters as described above, but allowed the age of the star to vary, uniformly, from 500 Myr, to 2 Gyr. The resulting histograms are shown in Figure 4 and are very similar to those in Figure 3. There is a slight increase in the number surviving inside $a = 0.01$ au for $Q_p' = 5 \times 10^5$. In these runs there were 17 and 16 for $Q_p' = 500$ and $Q_p' = 5000$ respectively, compared to 10 and 7 when the age of the system was fixed at $t = 800$ Myr. For $Q_p' = 5 \times 10^5$, the numbers are similar to the runs with the age fixed at $t = 800$ Myr.

To further see the influence of the age of the system, we plot in Figure 5 the final semimajor axis of the planet against age of the system, for all those systems in which planets end up inside 0.05 au. We only show, however, the results for $Q_p' = 5000$, $Q_s' = 5 \times 10^5$ as that produced the largest number of surviving planets inside $a = 0.01$ au. The first thing to note is that it appears more likely to detect such a planet for systems older than 1 Gyr (Kepler-78 has an age of between 600 and 900 Myr). However, Figure 5 does show 4 systems with an age $< 1$ Gyr, and with a planet inside $a = 0.01$ au.
Walker & Bass (2013) also suggest that maybe 20% of the Kepler targets have ages less than 2 Gyr. Kepler observed about 150,000 stars (Borucki et al. 2010), which suggests maybe as many as 30,000 could have ages less than 2 Gyr. Candidates as small as Kepler-78b, however, are typically found around quieter - and therefore older - stars. Kepler is therefore incomplete for stars with high Combined Differential Photometric Precision (Batalha et al. 2013; Christiansen et al. 2013) and so the number of such planets is likely an underestimate. In the scenario shown in Figure 6 - out of 10,000 - survive inside \( a = 0.01 \) au. If we assume that 40% of those stars have stellar companions (Raghavan et al. 2010), and that all of those stars could host terrestrial planets (Cassan et al. 2012; Greaves & Rice 2011), then we might expect as many as 20 of the 30,000 Kepler targets, with ages below 2 Gyr, to host such a ultra-close-in planet. Of course, our other simulations suggest that the number surviving could be as low as 2 (depending on the tidal properties of the star and planet), but given that the chance of such a system transiting is actually quite high (46%), observing such a system is still quite likely.

Similarly, if we consider only those systems with ages below 1 Gyr, Figure 6 suggests that maybe as many as 4 out of 10,000 could survive inside \( a = 0.01 \) au. Repeating the calculation above suggests that maybe 4 - 5 stars with ages similar to that of Kepler-78 could host such a close-in planet. Again, given the high transit probability for such a close-in system, detecting a planet such as Kepler-78b becomes possible. Our results therefore suggest that it is possible for this process to have produced a planet like Kepler-78b. Of course, if Kepler-78 is closer in age to 600 Myr, than to 900 Myr, Figure 6 suggests that it would become less likely.

### 4.3 Perturber properties

The results above suggest that it is possible for an outer perturber to drive a Kepler-78b-like planet into a close-in orbit within the age of Kepler-78. To see how the properties of the outer body influences the inner planet, we show - in Figure 7 - how the final semimajor axis of the planet depends on the mass of the outer body. Again, we only shows results from the simulation with \( Q_p = 5000 \) and \( Q_s = 5 \times 10^6 \). Figure 7 suggests that there isn’t a particularly strong mass dependence, consistent with our simulations essentially being in the test particle regime (Lithwick & Naoz 2011). However, Kepler-78, which has an apparent magnitude of \( m_v = 12 \), is not known to host a stellar companion. Since Kepler is sensitive down to an apparent magnitude of \( m_v = 14 \), that would suggest that if there is an undetected companion it would need to have a mass less that about 0.5 M_\odot.

Figure 8 shows how the final semimajor axis of the planet, \( a \), depends on the orbital properties of the outer body. The top panel shows that it is more likely that the planet will end up close to the parent star, if the outer body is in a relatively close orbit (\( a_o \approx 100 \) au). Kepler’s has a 4” pixel size and so Figure 8 does suggest that a sufficiently faint, non-variable companion - that could have perturbed a planet into a Kepler-78b-like orbit - could indeed have avoided detection. The bottom panel of Figure 8 shows how the final semimajor axis of the planet depends on the on the eccentricity (\( e_o \)) of the outer body. It indicates that close-in orbits are a little more likely when the companion has a high eccentricity (\( e > 0.4 \)) but are still possible for those with smaller eccentricities.
Figure 7. Figure showing the planet’s final semi-major axis plotted against the mass of the outer companion. There appears to be little dependence on companion mass. *Kepler* would probably have detected a companion with a mass in excess of \( \sim 0.5 \, M_\odot \), but this figure does show that a low-mass companion \((M < 0.5 \, M_\odot)\) could indeed have produced a system like Kepler-78b system.

Figure 8. Figure showing how the orbital properties of the outer companion influences the final semimajor axis of the inner planet. The top panel shows that outer companions with smaller semi-major axes \((a_0)\) are more likely to drive the planet to within \( a = 0.01 \, \text{au} \). The bottom panel shows that outer companions with large eccentricities are more likely to perturb inner planets into very-close-in orbits, but that it is still possible for outer perturbers with low eccentricities.

Figure 9. Figure showing the obliquity of the inner system. The inner system starts with the orbital angular momentum aligned with the spins of the central star and planet. The perturbation from the outer companion can, however, cause the inclination of the inner orbit to oscillate and those systems that are tidally circularised can end up with a range of obliquities.

4.4 The pile-up inside 0.1 au

Figures 3 and 5 show a pile-up of planets inside \( a = 0.1 \, \text{au} \), peaking at \( \sim 0.02 \, \text{au} \). In our simulations, between 350 and 850 (between 3.5% and 8.5% of the full sample of 10000) had final semi-major axes inside \( a = 0.1 \, \text{au} \) (and had not reached their Roche limit). If we assume that 40% of the *Kepler* sample could have a binary companion (either primordial or through an exchange interaction) - and that most Sun-like stars have terrestrial-mass, planetary companions \((\text{Cassan et al. 2012; Greaves & Rice 2011})\) - then our results suggest that as much as 3% of the *Kepler* sample might have planets that have been perturbed into close-in orbits, with a distribution that peaks at about 0.02 au. This is intriguingly similar to the suggestion in \( \text{Sanchis-Ojeda et al. 2014} \) that about 1 in 200 *Kepler* stars hosts a planet with a period of 1 day or less.

4.5 Obliquity

An interesting aspect of the Kozai-Lidov process is that it can perturb a planet into an orbit that is inclined with respect to its initial plane and, hence, inclined with respect to the spin of the host star \((\text{Wu, Murray & Ramsahai 2003; Fabrycky & Tremaine 2007})\). We now have a number of close-in, ‘hot’ Jupiters that are on orbits inclined with respect to the spin of the host star \((\text{Hebrard et al. 2008; Triaud et al. 2010})\) and these are thought to be a consequence of Kozai-Lidov cycles. Figure 9 shows the final angle (obliquity) between the angular momentum vector of the inner planet’s orbit and the spin of the central star, and shows that a wide range of obliquities is possible. All the systems initially have obliquities of zero (the angular momentum of the inner orbit is aligned with the spins of the parent star and planet) and Figure 9 shows that those that are perturbed into an inner orbit can then be tidally circularised with a large range of obliquities, consistent with other similar studies \((\text{Fabrycky & Tremaine 2007; Naoz, Farr & Rasio 2012})\). Using the Rossiter-McLaughlin method to determine such a mis-alignment \((\text{e.g. Queloz et al. 2001})\) is probably not possible for such a low-mass planet, but it may be possible to do so using spot-crossing \((\text{Desert et al. 2011})\) or astro-seismology \((\text{Chaplin et al. 2013})\).
5 DISCUSSION AND CONCLUSIONS

We have considered, here, if systems like Kepler-78b (an Earth-like exoplanet with a very close-in orbit) could be due to a perturbation from an outer companion on an, initially, inclined orbit. To do this, we consider a system in which the star and planet have the same masses and radii as in the Kepler-78 system (Sanchis-Ojeda et al. 2013, Pepe et al. 2013, Howard et al. 2013), but in which the planet initially has an almost circular orbit with a semi-major axis between 0.5 and 2 au. The system is also assumed to have an outer companion with a semimajor axis between 40 and 20000 au, with an orbital eccentricity that can be as high as \( e = 1 \) (but constrained by stability criteria) and that may be inclined with respect to the plane of the inner orbit.

We ran a suite of Monte Carlo simulations in which we randomly select the inner and outer systems semi-major axes, the eccentricity of the outer system, the mass of the outer companion and the mutual inclination of the two orbits. We ran two sets of simulations, one where each system was evolved for \( t = 800 \) Myr, similar to the expected age of the Kepler-78 system, and the other where the age of the system was randomly selected to be between 500 Myr and 2 Gyr. Our basic results are that:

- it is possible for a planet to be perturbed into an orbit similar to that of Kepler-78b around a star with an age \((600 - 900 \) Myr) similar to that of Kepler-78. Out of a sample of 10000, between 4 and 10 survive inside \( a = 0.01 \) au.
- if we consider a broader age range, the likely binarity of the Kepler sample, and the size of the Kepler sample, our results suggest that as many as 20 of the Kepler targets with ages less than 2 Gyr could host a Kepler-78b-like planet. Additionally, we find that a system with an age similar to that of Kepler-78 could indeed have been found to host a Kepler-78b-like planet.
- a planet such as Kepler-78b will, quite quickly, reach its Roche limit and be tidally destroyed. Our results suggest that such a planet would only survive inside \( a = 0.01 \) au for a few hundred Myrs, at most. In the simulations here, typically in excess of 100, but no more than 300, (out of 10000) reached their Roche limit and were assumed to be destroyed. This appears consistent with other work that has also suggested that this process could lead to the tidal destruction of perturbed planets (Naoz, Farr & Rasio 2013).
- given Kepler’s 4" pixel size and magnitude limit, it is possible that a faint, non-variable companion that could drive Kozai-Lidov cycles may have gone undetected. That the companion appears to need to be inside 100 au, means that it may be possible to detect the resulting radial velocity drift.
- If a planet such as Kepler-78b were perturbed into its current orbit through Kozai-Lidov cycles, we might expect the star’s rotation axis to be misaligned with respect to the planet’s orbit. Measuring the star’s obliquity is quite difficult, but could be possible using spot-crossing (Desert et al. 2011) or using astro-seismology (Chaplin et al. 2013).
- even though it appears possible that a system such as Kepler-78b could form in this way, it appears to be more likely for system older than 1 Gyr, than for systems younger than 1 Gyr.

Our basic results, therefore, suggest that such a process could operate, but there are some caveats. Although it is possible for a system with an age similar to that of Kepler-78 to host a planet like Kepler-78b, the numbers are small (we may expect the Kepler sample to host only a few such planets). Additionally it seems that it would have been more likely to have found such a planet in a slightly older system. These results, therefore, suggest that Kozai-Lidov cycles could have played a role in the evolution of Kepler-78b, but don’t rule out that there could be an alternative explanation, such as planet–planet scattering (Rasio et al. 1996). This, however, may suffer from the similar issues, since the dominant constraint - given the relatively low age of Kepler-78 - is the tidal evolution timescale. This constraint would probably also apply to the tidal downsizing hypothesis (Navakshin 2010), in which a massive gas-giant planet formed in the outer parts of the system, migrates rapidly inwards and loses masses via tidal stripping. That, of course, leaves the possibility that disc migration (Ward 1997) moved this planet into a very close orbit which has since evolved, through tidal interactions with the host star, into the orbit it inhabits today. Again, this would also involve tidal evolution once the disc has dispersed and so may also have a similar timescale issue, unless disc migration can place the planet sufficiently close to the parent star so that it can then tidally evolve to where it is today.

Recent work by Sanchis-Ojeda et al. (2014) suggest that about 1 out of every 200 stars hosts an ultra short period (USP) planet (period of 1 day or less). Although we’ve focussed on Kepler-78b here and found that few of our simulated systems have final periods as short as Kepler-78b (8.5 hours), many more have periods of 1 day or less. The exact number depends on the chosen parameters, but it varies from \( \sim 100 \) to just over \( 300 \) (from a total sample of 10000). Given that not all stars are binaries, this is intriguingly similar to the result in Sanchis-Ojeda et al. (2014). Similarly, our results suggest that such a process should lead to a pile-up of planets with a peak at about 0.02 au, again consistent with Sanchis-Ojeda et al. (2014) who find that the occurrence rate rises with period from 0.2 to 1 day. It may, however, be difficult to distinguish a pile-up due to Kozai-Lidov cycles from what is expected from scattering in multi-planet systems (Schlaufman, Lin & Ida 2011). Sanchis-Ojeda et al. (2014) do, however, suggest that almost all USPs have companion planets with period \( P < 50 \) days, which may provide a constraint on the formation process for these USPs.

We should also acknowledge the possibility that our assumptions do not properly represent the possible initial conditions such a system could have. The initial distribution of the planet in semi-major axis space may be different to what we’ve assumed and the orbital properties of the outer perturber may also be different. Similarly, the tidal properties of the parent star and planet may differ from what we’ve assumed. However, we should at least acknowledge that even though our results suggest that Kozai-Lidov cycles will rarely produce a planet with properties similar to that of Kepler-78b, Kepler-78b is itself rare. In that sense our results could be seen as somewhat consistent with our knowledge of such planets, but that - alone - doesn’t allow us to determine if it is likely that such a process did indeed play a role in the evolution of Kepler-78b.

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