Effects of impurity on fidelity of quantum state transfer via spin channels

Wen-Wen Zhang\(^1\), Ming-Liang Hu\(^1,\)*, Dong-Ping Tian\(^{1,2}\)

\(^1\)School of Science, Xi’an University of Posts and Telecommunications, Xi’an 710061, China
\(^2\)Xi’an University of Architecture and Technology, Xi’an 710055, China

By adopting the concept of fidelity, we investigated efficiency of quantum state transfer with the XX chain as the quantum channel. Different from the previous works, we concentrated on effects of spin and magnetic impurity on fidelity of quantum state transfer. Our results revealed that the spin impurity cannot prevent one from implementing perfect transfer of an arbitrary one-qubit pure state across the spin channel, however, the presence of magnetic impurity or both spin and magnetic impurities may destroy the otherwise perfect spin channels.

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I. INTRODUCTION

Since the seminal work of Bose [1], quantum state transfer along linear arrays of interacting qubits or spins [2, 3], which is closely associated with its potential applications in quantum communications [4, 7], has been discussed by a number of authors [5–21]. In Bose’s scheme [1], the state to be transmitted is initially encoded at one end of an unmodulated XXX spin chain by the sender Alice, then time evolution of the system, and after certain intervals of time, the state will be received by the receiver Bob at another end of the chain with some fidelity. Since then, many schemes focused on the implementation of perfect quantum state transfer (i.e., transferring a quantum state with fidelity equals to unity) by adopting pre-engineered spin chains as quantum channels have been proposed [11–20]. In particular, by using the identity of the otherwise perfect quantum spin channel will be enhanced amount of entanglement [32–34]. This fact naturally arises the following question: how the otherwise perfect quantum spin channel works if impurities are introduced? The purpose of this paper is to address this issue by investigating average fidelity of quantum state transfer across a spin chain with the presence of spin impurity, magnetic impurity [29] as well as both spin and magnetic impurities. Our results revealed that the presence of spin impurity cannot rule out the possibility of perfect state transfer, while the presence of magnetic impurity or both spin and magnetic impurities may destroy the otherwise perfect spin channels.

The structure of this paper is arranged as follows. In Section II, we examined effects of spin impurity on average fidelity of quantum state transfer by using the XX chain as the quantum channel, and gave the corresponding methods to maximize the average fidelity to its maximum value 1. Then in Sections III and IV, the calculation in the preceding section is repeated by changing the spin impurity to magnetic impurity as well as both spin and magnetic impurities, respectively, through which we show that the otherwise perfect quantum spin channel will be destroyed for these two cases. We also demonstrated how to minimize the detrimental effects introduced by magnetic impurity as well as both spin and magnetic impurities by performing local unitary operations in these two cases.
sections. Finally, we concluded this paper in Section V.

II. THE QUANTUM CHANNEL WITH SPIN IMPURITY

In this section, we examine effects of spin impurity (denoted by a spin-1 particle) on state transfer in an XX spin chain. We assume the quantum state to be transmitted is encoded at the first site as $|\varphi_{in}\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$ (with $|0\rangle$ and $|1\rangle$ represent the state of spin up and down, respectively), and all the other spins in the chain are initialized to the ground state $|0\rangle$, thus the initial state of the whole system at time $t = 0$ becomes

$$|\psi(0)\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle,$$

where $|0\rangle = |0_1,0_2,\ldots,0_N\rangle$ and $|1\rangle = |1_1,0_2,\ldots,0_N\rangle$, with $\theta \in [0,\pi]$ and $\phi \in [0,2\pi]$ being the polar and the azimuthal angles, respectively.

We first consider efficiency of quantum state transfer by using the two-spin XX chain as the quantum channel, with the impurity spin located at the first site. Then the Hamiltonian of the system can be expressed as

$$\hat{H} = J(S^x_1 s^x_2 + S^y_1 s^y_2) + B(S^z_1 + s^z_2),$$

where $S^x_\alpha$ and $S^y_\alpha$ ($\alpha = x,y,z$) denote the spin-1/2 and spin-1 operators (in units of $\hbar$) at the $i$th site (same notations are used throughout this paper). $J$ and $B$ are the coupling strength between the two neighboring spins and the intensity of the external magnetic fields applied to the two-spin system.

For the initial state $|\psi(0)\rangle$, the state at a given time, say $t$, is represented by $|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$. From the explicit form of the system Hamiltonian $\hat{H}$, one can show that its dynamics is completely determined by the time evolution in the zero and single excitation subspace $\mathcal{H}_{\rho \neq 0}$, thus it suffices to restrict our attention to the dynamics of $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ in this 3-dimensional subspace spanned by $\{|0\rangle,|1\rangle,|2\rangle\}$, which yields

$$|\psi(t)\rangle = \cos\frac{\theta}{2} f_0 |0\rangle + e^{i\phi}\sin\frac{\theta}{2}\sum_{n=1}^{2} f_n |n\rangle,$$

where $|n\rangle = |0_1,0_2,\ldots,0_{n-1},1_n,0_{n+1},\ldots,0_N\rangle$ denotes the site basis ($N$ is the number of sites in the chain, here $N = 2$), and the other three parameters $f_n$ ($n = 0,1,2$) are given by

$$f_0 = \langle 0|e^{-i\hat{H}t}|0\rangle, \quad f_n = \langle n|e^{-i\hat{H}t}|1\rangle.$$

In the present paper, we adopt the concept of average fidelity (the fidelity $F = \langle \varphi_{in}|\rho_2(t)|\varphi_{in}\rangle$ averaged over all pure input states in the Bloch sphere) $\bar{F} = \frac{1}{4\pi} \int dF\int d\Omega f F d\Omega = \frac{1}{2\pi} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta F \sin \theta$ as an estimation of the quality of state transfer from the sender Alice to the receiver Bob.

[1] For state $|\psi(t)\rangle$, the reduced density matrix $\rho_2(t)$ can be obtained by tracing qutrit 1 from $\rho(t)$ as

$$\rho_2(t) = \left( \begin{array}{ccc} 1 - \sin^2\frac{\theta}{2}|f|^2 & \frac{1}{2} \sin \theta e^{-i\phi} f_0 \gamma^* & 0 \\ \frac{1}{2} \sin \theta e^{i\phi} f_0 \gamma & 1 - \sin^2\frac{\theta}{2}|f|^2 & 0 \\ 0 & 0 & \sin^2\frac{\theta}{2}|f|^2 \end{array} \right).$$

From Eqs. (2) and (4) one can show that $|f_0| = 1$, thus if we define $f = f_0^* f_2$ (when $f_0 = 1$, $f$ is just the transfer fidelity of an excitation), then the reduced density matrix can be rewritten as

$$\rho_2(t) = \left( \begin{array}{ccc} 1 - \sin^2\frac{\theta}{2}|f|^2 & \frac{1}{2} \sin \theta e^{-i\phi} f_0 \gamma^* & 0 \\ \frac{1}{2} \sin \theta e^{i\phi} f_0 \gamma & 1 - \sin^2\frac{\theta}{2}|f|^2 & 0 \\ 0 & 0 & \sin^2\frac{\theta}{2}|f|^2 \end{array} \right).$$

Eq. (6) enables one to compute the state transfer fidelity $F = \langle \varphi_{in}|\rho_2(t)|\varphi_{in}\rangle$ as

$$F = \cos \frac{\theta}{2} \left( 1 - |f|^2 \sin^2\frac{\theta}{2} + 2 |f| \sin\frac{\theta}{2} \cos \gamma \right) + |f|^2 \sin^2\frac{\theta}{2},$$

which yields

$$F = \frac{1}{2} + \frac{|f| \cos \gamma}{3} + \frac{|f|^2}{6},$$

where $\gamma = \arg(f)$ denotes the argument of the complex number $f$.

From Eq. (8) it is easy to conclude that if we want to attain perfect state transfer for all kinds of initial pure states (i.e., $F_{\text{max}} = 1$), we demand that $|f(t_c)| = 1$ and $\gamma(t_c) = 2k\pi$ ($k \in \mathbb{Z}$), or equivalently, $\text{Re}\{f(t_c)\} = 1$ and $\text{Im}\{f(t_c)\} = 0$, where $\text{Re}\{f(t_c)\}$ and $\text{Im}\{f(t_c)\}$ represent the real and imaginary part of $f$, respectively. So based on this consideration, we only need to discuss effects of impurity on dynamics of $f = f_0 f_2$ in the following.

To obtain the explicit expressions of $f_0 = \langle 0|e^{-i\hat{H}t}|0\rangle$ and $f_2 = \langle 2|e^{-i\hat{H}t}|1\rangle$, one needs to obtain the eigenvalues as well as the eigenvectors of the Hamiltonian $\hat{H}$. Since for the initial state $|\psi(0)\rangle$ expressed in Eq. (1), its dynamics is completely determined by the time evolution in the subspace spanned by the site basis $\{|0\rangle,|1\rangle,|2\rangle\}$, one can calculate the eigenvalues of the Hamiltonian $\hat{H}$ in this subspace, which is $\epsilon_0 = \frac{3}{2} B$ and $\epsilon_{1,2} = \frac{1}{2}(B \pm \sqrt{2}J)$, with the corresponding eigenstates given by

$$|\varphi_0\rangle = |00\rangle, \quad |\varphi_{1,2}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).$$

From the explicit expressions of the eigenstates given in Eq. (9), one can obtain directly that $|00\rangle = |\varphi_0\rangle$ and $|10\rangle = \frac{1}{\sqrt{2}}(|\varphi_1\rangle - |\varphi_2\rangle)$. Substituting these results into Eq. (4), one can obtain

$$f_0 = e^{-i3Bt/2}, \quad f_2 = -ie^{-iBt/2} \sin(\sqrt{2}Jt/2),$$

which yields

$$f = -ie^{iBt} \sin(\sqrt{2}Jt/2).$$
From the above analysis, one can see that the presence of spin impurity does not prevent one from implementing perfect transfer of an arbitrary one-qubit state. Here we address the question of how the average fidelity of quantum state transfer depends on various parameters. We first discuss quantum state transfer in the presence of a spin XX chain in the presence of magnetic impurity, with the Hamiltonian given by

\[ H = J \sum_{i} S_i^x S_{i+1}^x + J \sum_{i} S_i^z S_{i+1}^z + B S_i^z. \]

From the above expressions, one can obtain directly the explicit expression of the average fidelity \( F \) for this system has the same form as given in Eq. (8), with the corresponding eigenstates given by

\[ |0\rangle = |000\rangle, \quad |1\rangle = \frac{1}{\sqrt{2}} (|000\rangle + i|100\rangle), \quad |2\rangle = \frac{1}{\sqrt{2}} (|000\rangle - i|100\rangle). \]

For this system, its eigenvalues can be obtained explicitly as \( \epsilon_0 = 2B, \epsilon_1 = B + J \) in the subspace \( |0\rangle \), \( |1\rangle \), \( |2\rangle \), and \( \epsilon_2 = B - J \) in the subspace \( |000\rangle \) of initial states \( |0\rangle \). In the following we investigate the efficiency of quantum state transfer through a spin XX chain, and discuss dynamics of the average fidelity of quantum state transfer.

### III. The Quantum Channel with Magnetic Impurity

By using the same method, one can show that the expression of the average fidelity \( F \) for this system has the same form as given in Eq. (8), with the corresponding eigenstates given by

\[ \rho = |000\rangle \langle 000| + \sqrt{2} |100\rangle \langle 100| + \frac{1}{2} (|000\rangle \langle 000| - 2 |100\rangle \langle 100|). \]

When the impurity spin locating at the edge site of the three-spin XX chain, one can show that perfect transfer of the XX chain is possible by performing the local unitary operation \( \mathbf{U} = |0\rangle \langle 0| + \frac{1}{\sqrt{2}} (|1\rangle \langle 1| - |2\rangle \langle 2|) \) (or \( \mathbf{U} = |0\rangle \langle 0| - \frac{1}{\sqrt{2}} (|1\rangle \langle 1| + |2\rangle \langle 2|) \)).

When the impurity spin is located at the critical time \( t = (2k + 1)\pi / 2J t \), with \( k = 0, 1, 2, \ldots \) and the corresponding time \( t = \max \), \( \gamma = \pi/4 \) when \( k = 0 \). This is because for the XX chain, one can also show that perfect transfer of the spin state is also possible by performing the local unitary operation \( \mathbf{U} = |0\rangle \langle 0| + \frac{1}{\sqrt{2}} (|1\rangle \langle 1| - |2\rangle \langle 2|) \) (or \( \mathbf{U} = |0\rangle \langle 0| - \frac{1}{\sqrt{2}} (|1\rangle \langle 1| + |2\rangle \langle 2|) \)).

When the initial state \( |000\rangle \) is the initial state of the spin chain, with \( \gamma = \pi/4 \) when \( k = 0 \).
with $\mu = \sqrt{B^2 + J^2}$, and the corresponding eigenstates are given by

$$|\varphi_0\rangle = |00\rangle,$$

$$|\varphi_{1,2}\rangle = \frac{1}{\sqrt{2\mu(\mu \pm B)}}[(B \pm \mu)|01\rangle + J|10\rangle].$$

(17)

For this kind of imperfect quantum spin channel, it can be shown that the expression of the average fidelity $F$ has the same form as that expressed in Eq. (8). Moreover, from Eq. (17) one can obtain directly that $|00\rangle = |\varphi_0\rangle$, $|10\rangle = J|\varphi_1\rangle/\sqrt{2\mu(\mu + B)} + J|\varphi_2\rangle/\sqrt{2\mu(\mu - B)}$, which yields

$$f_0 = e^{-iBt/2}, \quad f_2 = -i\frac{J}{\mu} \sin \frac{\mu t}{2},$$

(18)

and

$$f = -ie^{iBt/2} \frac{J}{\mu} \sin \frac{\mu t}{2}.\quad \quad \quad (19)$$

Since $J < \mu$ when $B \neq 0$, we have $|f| < 1$ at any instant of time, which implies that one cannot implement perfect transfer of an arbitrary purely input state $|\varphi_{in}\rangle$ in the presence of one magnetic impurity. This is different from that of the spin impurity, which does not exclude the possibility of perfect state transfer of $|\varphi_{in}\rangle$. Theoretically, for given $\mu$, one can modulate the coupling strength $J$ of the neighboring spins so that $J \gg B$, for which the parameter $f$ can be approximated by $f \simeq -ie^{iBt/2} \sin \frac{\mu t}{2}$, thus when $t_c = (4k + 1)\pi/J, \quad B_c = (4l + 1)\pi/t_c$ or $t_c = (4k + 3)\pi/J, \quad B_c = (4l + 3)\pi/t_c (k, l = 0, 1, 2, \ldots)$, one can obtain the maximum average fidelity $F_{\text{max}} \simeq 1$. However, the realization of $J$ large enough might be a difficult experimental task.

Now we discuss average fidelity of quantum state transfer via the three-spin channel, with the magnetic impurity locating at the central site. The Hamiltonian of the system is given by

$$\hat{H} = J(s_z^1s_z^2 + s_z^1s_z^3 + s_z^2s_z^3 + s_x^1s_y^2) + Bs_z^2. \quad \quad \quad (20)$$

In the subspace spanned by $\{|00\rangle, |11\rangle, |22\rangle, |33\rangle\}$, the eigenvalues of the system can be obtained as $\epsilon_{0,1} = \frac{1}{2}B$ and $\epsilon_{2,3} = \pm \frac{1}{2} \sqrt{2B^2 + 4J^2}$ and the corresponding eigenstates given by

$$|\varphi_0\rangle = |00\rangle, \quad |\varphi_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle),$$

$$|\varphi_{2,3}\rangle = \frac{1}{\sqrt{2\nu(\nu \mp B)}}[J|00\rangle - (B \mp \nu)|01\rangle + J|10\rangle],$$

(21)

from which one can obtain $|00\rangle$ and $|10\rangle$ in terms of the eigenstates as $|00\rangle = |\varphi_0\rangle$, and $|10\rangle = J|\varphi_2\rangle/\sqrt{2\nu(\nu - B)} + J|\varphi_3\rangle/\sqrt{2\nu(\nu + B)} - |\varphi_1\rangle/\sqrt{2}$. Thus the parameters $f_0, f_3$ and $f = f_0^{*}f_3$ can be obtained straightforwardly as

$$f_0 = e^{-iBt/2},$$

$$f_3 = \frac{J^2e^{-ivt/2}}{2\nu(\nu - B)} + \frac{J^2e^{ivt/2}}{2\nu(\nu + B)} - \frac{1}{2}e^{-iBt/2}, \quad \quad \quad (22)$$

and

$$f = \frac{J^2e^{i(\nu - v)t/2}}{2\nu(\nu - B)} + \frac{J^2e^{i(\nu + v)t/2}}{2\nu(\nu + B)} - \frac{1}{2}. \quad \quad \quad (23)$$

Similar to the two-spin channel, one still cannot obtain the maximum average fidelity $F_{\text{max}} = 1$ since $|f| < 1$ and $\gamma \neq 2k\pi (k \in \mathbb{Z})$, i.e., the three-spin quantum channel is also destroyed by the presence of the magnetic impurity. Even when the coupling strength $J$ is strong enough, the average fidelity attainable still cannot reach its maximum value 1 since $f \simeq \frac{1}{2}e^{iBt/2} \cos(Jt/\sqrt{2} - 1)$ under the condition of $J \gg B$.

If the receiver Bob can perform a local unitary operation $U$ to the spin at his hands, the average fidelity may be maximized to a certain maximum but not unity. Since this requires $f(t_c) = 1$ and $\gamma(t_c) = 2k\pi (k \in \mathbb{Z})$, the unitary operation $U$ must satisfying the following relations $U|0\rangle = |0\rangle$ and $U|1\rangle = |\varphi_1\rangle$, from which one can obtain $U = \text{diag}[1, e^{-i\vartheta}, e^{-i\vartheta}], \quad \vartheta = \tan^{-1} [\text{Im}(f)/\text{Re}(f)]$. Using this method, the average fidelity $F$ can be greatly maximized. For example, when $J = 2\sqrt{2}B/3$, we have $F_{\text{max}} \simeq 0.9678$, which is very close to its maximum value unity.

**IV. THE QUANTUM CHANNEL WITH BOTH SPIN AND MAGNETIC IMPURITIES**

Now we investigate efficiency of quantum channel with both spin and magnetic impurities. From the above two sections one can see that the presence of spin impurity does not rule out the possibility of perfect state transfer through an XX chain, while the magnetic impurity may destroy the quantum channel and induce unavoidable loss of quantum information during the transmission process, thus it is natural to conjecture that under the influence of both spin and magnetic impurities, the quantum channel may also be destroyed. To show this is true, we consider the three-spin XX chain with both spin and magnetic impurities locating at the central site, then the Hamiltonian can be written as

$$\hat{H} = J(s_z^1S_z^2 + s_z^1S_z^3 + s_z^2S_z^3 + S_z^2S_z^3) + BS_z^2. \quad \quad \quad (24)$$

It can be shown that the explicit expression of the average fidelity $F$ has the same form as that expressed in Eq. (8), with however, $f = f_0f_3$. Moreover, the eigenvalues of the system can be calculated as $\epsilon_{0,1} = B$ and $\epsilon_{2,3} = \frac{1}{2}(B \pm \xi)$ in the subspace spanned by the site basis $\{|00\rangle, |11\rangle, |22\rangle, |33\rangle\}$, with $\xi = \sqrt{B^2 + 4J^2}$, and the eigenvectors are given by

$$|\varphi_0\rangle = |00\rangle, \quad |\varphi_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle),$$

$$|\varphi_{2,3}\rangle = \frac{1}{\sqrt{\xi(\xi + B)}}[J|00\rangle - B\frac{\pm \xi}{2}(|01\rangle + |10\rangle) + J|10\rangle]. \quad \quad \quad (25)$$

Thus $|00\rangle$ and $|10\rangle$ can be expressed in terms of the eigenstates $|\varphi_i\rangle (i = 0, 1, 2, 3)$ as $|00\rangle = |\varphi_0\rangle$, and
\[ |100\rangle = J|\varphi_2\rangle / \sqrt{\xi(\xi - B)} + J|\varphi_3\rangle / \sqrt{\xi(\xi + B)} - |\varphi_1\rangle / \sqrt{2}, \]
which yields
\[ f_0 = e^{-iBt}, \]
\[ f_3 = \frac{J^2e^{-i(\beta + \xi) t/2}}{\xi(\xi - B)} + \frac{J^2e^{-i(\beta - \xi) t/2}}{\xi(\xi + B)} - \frac{1}{2} e^{-iBt}, \tag{26} \]
and
\[ f = \frac{J^2e^{i(\beta - \xi) t/2}}{\xi(\xi - B)} + \frac{J^2e^{i(\beta + \xi) t/2}}{\xi(\xi + B)} - \frac{1}{2}. \tag{27} \]

Clearly, perfect transfer of all the purely input states \(|\varphi_{in}\rangle\) is also impossible since \(|f| < 1\) and \(\gamma \neq 2k\pi (k \in \mathbb{Z})\).
However, the average fidelity \(F\) can also be maximized to a certain maximum value if Bob performs a local unitary operation \(U = diag\{1, e^{-i\delta}\}\) to the spin at his hands, with \(\delta = \tan^{-1}[\text{Im}(f)/\text{Re}(f)]\). For example, when \(J = 2B/3\), the average fidelity \(F\) can be adjusted to a certain maximum value of about 0.9678 when the \(U\) operation is performed.

\section{Conclusion}
In conclusion, in this paper we have investigated effects of spin and magnetic impurity on average fidelity of quantum state transfer by using the XX spin chain as quantum channels. Our results revealed that even in the presence of spin impurity, one can still implement perfect transfer of an arbitrary one-qubit pure state by tuning the strength of the external magnetic filed according to the instant of time the receiver Bob decodes the information. One can also maximize the average fidelity by performing relevant local unitary operations at the spin belonging to Bob. When the magnetic impurity or both spin and magnetic impurities are present, however, the quantum channel is destroyed and thus one cannot obtain the maximum average fidelity \(F_{\text{max}} = 1\), which implies that some information is lost during the transmission process of the quantum states. Though for some special cases (e.g., the three-spin quantum channel), the average fidelity can be maximized to a certain maximum value which is very close to unity by performing a proper local unitary operation at the spin belonging to Bob, however, this procedure does not work out for a general case.

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