Preparing, linking, and unlinking cluster-type polarization-entangled states by integrating modules

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Integrating the modules of the controlled phase-flip gate, the distributed entanglement gate, and the simplified entanglement gate, we propose schemes for preparing, linking, and unlinking standard cluster-type polarization-entangled states, which are achieved by performing different operations for cluster-type states consisting of even and odd photons, in company with Hadamard gate operations and other single-photon operations. Mediated by weak cross-Kerr nonlinear interactions, two individual photons interact with each other depending on the assistance of the coherent state in Kerr media. Exploiting homodyne measurement and polarization beam splitters, the photon paths take on a certain ambiguity and photons entangle together in the polarization mode. Assisted by a classical feed-forward, these aforementioned modules can be realized with an efficiency approaching unity. Employing the module of the controlled phase-flip gate, standard even- and odd-photon cluster-type states can be prepared, which are distinguished by the different photons performed by Hadamard gate operations. Linking two or more standard cluster-type states, a new standard cluster-type state can be produced after combination of the module of the controlled phase-flip gate and Hadamard gate operations. Dividing, truncating, or pinching a multi-photon cluster-type state can be accomplished by applying the module of the controlled phase-flip gate and single-photon operations including Hadamard gate operations.

Subject Index A61

1. Introduction

One-way quantum computation or measurement-based quantum computation [1–3] is a highly promising candidate to fulfill the tasks of quantum information processing, differentiating from traditional circuit-based quantum computation implemented by single-qubit and multiqubit gates. Following it, some ideas and approaches on relevant applications have been researched, such as many fundamental issues of quantum mechanics on cluster-type states [4], the implementation of Glover’s search algorithm based on cluster-type states [5], and the realization of Deutsch’s algorithm on a four-qubit cluster-type state [6]. Moreover, quantum computing with only linear optics [7] has been constructed, and applications on the scale of several qubits have been demonstrated [8].

Due to these important potential applications, numerous researchers have considered the generation of cluster-type states in various physics systems such as photons and continuous variables [6–15], trapped ions [16,17], cavity QED setups [18–25], and superconducting quantum circuits [26–28].
The interaction strength between photons and the environment is weak enough to guarantee a good coherence time. Moreover, good progress has been made in the relevant optical technique, which is maturing as a result of the outstanding work done by many researchers. By including the feature of maximal transmission velocity, photonic cluster-type states and quantum computing have become a promising candidate for quantum information processing, attracting considerable interest. However, due to its natural properties, the success probability of linear-optics-based cluster-type states is much less than unity; it therefore needs quantum memory to assist it, which is not easy to realize using current technologies.

In this context, an optical hybrid system consisting of photons and coherent states provides an opportunity for generating cluster-type states and fulfilling other tasks of quantum information processing. Depending on homodyne measurements, discrete photons are considered to generate cluster-type states with the help of continuous variable coherent states in Kerr media [12,13].

Employing entanglement gates, Lin and He [29] conceived the production scheme of multi-photon cluster-type states. Combining interlocking bricks constructed with a single and reused auxiliary coherent state, Horsman et al. [30] proposed an optimally efficient method for generating cluster-type states. For saving operations, Brown et al. [31] considered three cases to generate cluster-type states depending on the ability of manipulators to destroy previously created controlled-phase links between qubits, i.e., without the destroying ability and optimal savings, having it but being more complicated, and a halfway scheme.

In this paper, we consider how to prepare a multi-photon standard cluster-type state from single-photon polarization superposition states, link EPR (Einstein–Podolsky–Rosen) photon pairs or other standard cluster-type states, and unlink a standard cluster-type state, by adopting a series of modules of a controlled phase-flip gate and entanglement gates, and performing single-photon operations such as Hadamard gate operations. In Sect. 2, by letting the photons pass through the successive modules of the controlled phase-flip gate and Hadamard gate, the standard multi-photon cluster-type state is prepared. Applying the EPR pairs previously generated from the single-photon states depending on the entanglement gates or by other methods, the standard even- or odd-photon cluster-type states are linked with the controlled phase-flip gate modules in Sect. 3. In Sect. 4, on the basis of the combination of single-photon operations such as Hadamard gate operations and the modules of controlled phase-flip gates, we propose a scheme for unlinking a standard even- or odd-photon cluster-type state to several standard cluster-type states. Finally, some discussions about the practical implementations and a summary are presented in Sect. 5.

2. Preparation of a standard \( N \)-photon cluster-type polarization-entangled state with controlled phase-flip gate and Hadamard gate modules

To begin with, we introduce the formula of an \( N \)-photon cluster-type polarization-entangled state, which can be denoted as [2]

\[
| \text{Cluster}_N \rangle = \frac{1}{2^{N/2}} \bigotimes_{i=1}^{N} (|H \rangle_i \sigma_z^{(i+1)} + |V \rangle_i),
\]

(1)

where the subscript ‘\( i \)’ labels the position of the photons, \( \sigma_z = |H \rangle \langle H| - |V \rangle \langle V| \), and by convention \( \sigma_z^{(N+1)} \equiv I \).

According to the definition of cluster-type states, a module functioning as a controlled phase-flip gate [32] is necessary. Depending on weak cross-Kerr nonlinear interaction rather than only linear optics, two-photon logical quantum gates such as a controlled-NOT gate or controlled arbitrary phase gate [33–35] can be constructed.
Between the interaction of the individual photons, the Kerr media play an important bridge role. Let a signal state denoted as

\[ |\phi\rangle_s = a |0\rangle_s + b |1\rangle_s, \]  

where \(|0\rangle_s\) and \(|1\rangle_s\) denote the vacuum state and the single-photon state, and a probe coherent state denoted as

\[ |\alpha\rangle_p = \exp \left(-\frac{1}{2} |\alpha|^2 \right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \]  

where \(|n\rangle\) represents the state consisting of \(n\) photons, pass through Kerr media, and the hybrid system state is evolved as

\[ U_{ck} (|\phi\rangle_s |\alpha\rangle_p) = e^{iH_{ck}t/\hbar} \left([a|0\rangle_s + b|1\rangle_s]|\alpha\rangle_p\right) = a|0\rangle_s |\alpha\rangle_p + b|1\rangle_s |ae^{i\theta}\rangle_p. \]  

Based on the feature of the phase shift on the coherent state, the presence and absence of signal photons can be confirmed.

The homodyne measurement (e.g. \(\hat{X}\) quadrature measurement) [33,34,36] disentangles the entanglement between the probe coherent state and the signal photons, and generates entanglement of the signal photons. By performing the \(\hat{X}\) measurement, photon paths are partially confirmed, and photons are entangled together due to the property of ambiguity. To decrease the error probability and facilitate scalable applications, the displacement measurement [37–40] may be adopted, which can displace the phase shift on the coherent state to its amplitude, by which the error probability can be reduced from \(\frac{1}{2}\text{erfc}(|\alpha\theta|^2/2\sqrt{2})\) (\(\hat{X}\) measurement) to \(\frac{1}{2}\text{erfc}(|\alpha\theta|/2\sqrt{2})\) (displacement measurement). The displacement operation can be achieved by inputting the probe coherent state into a beam splitter with higher reflectivity [37] or higher transmissivity [41] than 50 : 50, in company with a high-intensity coherent state. After that, number-resolving detectors are applied to obtain the photon number of the probe coherent state. Here we use the displacement measurement in our schemes. As Fig. 1 illustrates, the photon paths \((a, e)\) and \((b, f)\) cannot be confirmed because the displacement measurement has no way to detect the difference between the phase shift \(\theta\) and \(-\theta\) of the probe coherent state. Moreover, the experimenter is ignorant of the discrimination of the photon paths \((a, f)\) and \((b, e)\), in that their overall phase shifts equal zero \((-\theta + \theta = 0\). Referring to the measurement outcomes, the classical feed-forward is exploited to modulate the relative phase difference and flip the unwanted qubit to the correct one, and fulfill the task entangling two photons into an EPR pair on demand.

Therefore, the tasks of two-qubit entanglement gates [33,36,42–44] and quantum logical gates [33–35,45] or other quantum information processing can be accomplished. Many properties and applications of the cross-Kerr nonlinear interaction have been discussed and analyzed by many researchers [8,33–36,38,44–58].

After revising the controlled arbitrary phase gate [45], we can obtain the module of the controlled phase-flip gate, also called a CZ gate, which can realize the evolution from the initial state

\[ |\Psi\rangle_i = \alpha |H\rangle |H\rangle + \beta |H\rangle |V\rangle + \gamma |V\rangle |H\rangle + \delta |V\rangle |V\rangle, \]  

to the final state

\[ |\Psi\rangle_f = e^{i\xi} (\alpha |H\rangle |H\rangle - \beta |H\rangle |V\rangle + \gamma |V\rangle |H\rangle + \delta |V\rangle |V\rangle), \]  

where the former and latter photons function as the control and target qubits respectively, and \(e^{i\xi}\) denotes the overall phase shift on two photons, which can be neglected.
In Fig. 1, dotted red lines and red symbols (phase shifter (PS) \(\pi\) and PS \(2\phi\)) denote the feed-forward based on the displacement measurement outcome, and the dashed green line and green element (PS \(\pi\)) denotes the feed-forward according to the \(\hat{Y}\) measurement outcome. Moreover, exploiting additional phase-locking lasers, the method actively stabilizing the interferometers [59] is adopted to guarantee long-term stability. They propagate along the reverse direction of photons 1 and 2 (polarization beam splitter, PBS\(_2\) \(\rightarrow\) PBS\(_1\), PBS\(_4\) \(\rightarrow\) PBS\(_3\), PBS\(_6\) \(\rightarrow\) PBS\(_5\), beam splitter, BS\(_2\) \(\rightarrow\) BS\(_1\)). At the output ports, feedback signals are sent to modulate the difference based on the measurement results, which are not plotted in Fig. 1.

Exploiting the modules of the controlled phase-flip gate, we consider the generation of a standard \(N\)-photon cluster-type polarization-entangled state.

As preliminary resources, all the single photons are prepared in the superposition of the polarization state, \(\frac{1}{\sqrt{2}} (|H\rangle + |V\rangle)\). In order to prepare the \(N\)-photon cluster-type polarization-entangled state denoted as Eq. (1), the module of the controlled phase-flip gate can be utilized as the fundamental assembly, which is operated \(N - 1\) times.

Coming out from the output ports, \(N/2\) (\(N\) is even) photons or \((N + 1)/2\) (\(N\) is odd) photons should undergo Hadamard gate operations to obtain a standard even- or odd-photon cluster-type state, which is a simpler presentation compared with Eq. (1). As shown in Fig. 2, the half wave plate (HWP) 22.5° outside of the shadow rectangle box are put into lines before the output ports. That is to say, Hadamard gate operations should be done on the photons after they depart from the modules of the controlled phase-flip gate.

Moreover, there exists a difference between the generation processes of the standard even- and odd-photon cluster-type polarization-entangled states. Explicitly, for generating a standard odd-photon cluster-type state \(N (\geq 3)\), the HWP 22.5° (\(H\) gate operations) need to be performed on the photons labeled \(N\), \(N - 2i\), \(i = 0, 1, 2, \ldots, (N - 1)/2\), i.e., only the odd-number photons need to undergo \(H\) gate operations to achieve cluster-type states in a standard presentation. Therefore, there exists only one type for a standard odd-photon cluster-type state. As an example, a standard five-photon cluster-type state (5-S) can be denoted as

\[
5\text{-S} : \frac{1}{2} [ |HHH\rangle_{12} (|VHV\rangle - |HVH\rangle)]_{345} - |VVV\rangle_{12} (|HHV\rangle - |VVH\rangle)]_{345}. \tag{7}
\]

With regard to the other scenarios, for preparing a standard even-photon cluster-type state with simple presentation on the basis of \(|H\rangle, |V\rangle\), \(H\) gate operations need to be performed on the photons.
Fig. 2. A plot to illustrate a generation scheme for even-photon (II)-type and standard odd-photon cluster-type polarization-entangled states. There exist two types of standard even-photon cluster-type polarization-entangled states distinguished by the different photons undergoing Hadamard gate operations. Explicitly, the yellow symbols (HWP 22.5°) with black dotted border and the grey symbols with black solid border are used to generate I-type states; the red symbols (HWP 22.5°) with black dotted border and the grey symbols with black solid border are used to obtain II-type states. As for odd-photon cluster-type polarization-entangled states, there is only one type, i.e., the yellow symbols (HWP 22.5°) with black dotted border and the grey symbols with black solid border are used to produce the states.

ordered by $N, \ldots, N - 2i, \ldots, 1$ (or 2) where $i = 0, 1, 2, \ldots, (N - 1)/2$ (or $(N - 2)/2$). So there are two types of standard even-photon cluster-type states, called I-type (II-type) states, which can be transformed into each other by performing two Hadamard gate operations on photons 1 and 2.

Here, a standard four-photon cluster-type state is adopted to illustrate these two types of states. Explicitly, the difference between two types of standard four-photon cluster-type states is that photons 1 and 4 undergo Hadamard gate operations to generate the 4-I state denoted as Eq. (8), and photons 2 and 4 undergo Hadamard gate operations to generate the 4-II state denoted as Eq. (9):

$$4\text{-I}: \frac{1}{2} \left[ |HH\rangle_{12}(|HV\rangle - |VH\rangle)_{34} - |VV\rangle_{12}(|HV\rangle + |VH\rangle)_{34} \right],$$  
(8)

$$4\text{-II}: \frac{1}{2} \left[ |H\rangle_{1}(|VHV\rangle - |HVH\rangle)_{234} + |V\rangle_{1}(|HHV\rangle - |VVH\rangle)_{234} \right].$$  
(9)

To facilitate later description, we label single-photon polarization states and EPR states with the following symbols:

$$1\text{-I}: \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle),$$

$$1\text{-II}: \frac{1}{\sqrt{2}} (|H\rangle - |V\rangle),$$

$$2\text{-I}: |\Psi^+\rangle = \frac{1}{\sqrt{2}} (|HV\rangle + |VH\rangle),$$

$$2\text{-II}: |\Phi^-\rangle = \frac{1}{\sqrt{2}} (|HH\rangle - |VV\rangle).$$

(10)
Fig. 3. A plot to illustrate the random preparation of a 2-I(II) state with the distributed entanglement gate. When the displacement measurement outcome indicates that the phase shift of the probe coherent state is not equal to zero, a 2-I state ($|\Psi^+\rangle$) can be obtained with the feed-forward (phase shifter $\pi$ and $2\phi$) denoted by blue dotted lines. In the other case, a 2-II state ($|\Phi^-\rangle$) can be generated.

Fig. 4. A plot to illustrate the random preparation of a 2-I(II) state with the simplified entanglement gate. The two different displacement measurement outcomes correspond to the generation of two states, i.e. PS = 0: 2-II state ($|\Phi^-\rangle$), PS $\neq$ 0: 2-I state ($|\Psi^+\rangle$). To obtain a 2-I state, the feed-forward (phase shifter $2\phi$ and the module of Z gate) denoted by blue dotted lines should be in operation.

Besides the aforementioned method with the single-photon polarization-entangled state and the modules of the controlled phase-flip gate, the manipulator may employ the controlled phase-flip gate modules to link several EPR pairs and a superposition of a single-photon polarization state to generate standard even- or odd-photon cluster-type states, $|\text{Cluster}_N\rangle$, which will be considered in the next section.

3. Linking standard multi-photon cluster-type polarization-entangled states

Employing the modules of the controlled phase-flip gate and Hadamard gate operations, we consider how to obtain standard $N$-photon polarization-entangled cluster-type states by linking cluster-type states. At first, we consider how to link several EPR pairs and a single-photon polarization state to prepare a standard polarization-entangled cluster-type state. We assume that a good number of 1-I states ($1/\sqrt{2}(|H\rangle + |V\rangle)$), 2-I states ($|\Psi^+\rangle$), and 2-II states ($|\Phi^-\rangle$) have been prepared.

A 2-I (2-II) state can be obtained from two 1-I states with a distributed entanglement gate or simplified entanglement gate. The function of the distributed entanglement gate [33] can be fulfilled if four polarization beam splitters and Kerr media are correctly placed, illustrated in Fig. 3. The two beam splitters in Fig. 1 have been replaced with two polarization beam splitters and the other irrelevant elements have been discarded, and the distributed entanglement gate is obtained as a consequence.

Releasing the two polarization beam splitters in the distributed entanglement gate, Guo et al. [44] present a simplified entanglement gate confined in the local nodes. As another feature, the simplified
entanglement gate increases the success probability of the entanglement gate by the appropriate circuit doubling the phase shift of the affected probe coherent state, which can be seen in Fig. 4.

Exploiting the distributed entanglement gate, a 2-I state or a 2-II state can be obtained at random, illustrated by Fig. 3. When the displacement measurement outcome indicates a nonzero phase shift \( \phi \neq 0 \) on the affected probe coherent state \( |\alpha\rangle \), the classical feed-forward (phase shifter \( 2\phi, \pi \)), denoted by the blue dotted lines, is in operation. As a consequence, a 2-I state emerges on the output ports. If a displacement measurement outcome of zero phase shift \( \phi = 0 \) is presented, a 2-II state will be generated on the terminals \( O_1, O_2 \).

Moreover, the aforementioned states can be prepared by exploiting the simplified entanglement gate, shown in Fig. 4. The two different displacement measurement outcomes reflect the generation of a 2-I state and a 2-II state. That is, the zero phase shift of the affected probe coherent state \( \phi = 0 \) corresponds to the preparation of a 2-II state, and the nonzero phase shift of the affected probe coherent state \( \phi \neq 0 \) corresponds to a 2-I state in the output ports. To obtain the 2-I state, the feed-forward \( \phi \) and the optical element module of \( \sigma_2 \) denoted by blue dotted lines should be in operation on the photons passing through path \( b \).

There are different methods for preparing the standard \( N \)-photon cluster-type states consisting of even and odd photons. In the following, we present preparation schemes for even-photon I-type cluster-type states (representing standard even-photon cluster-type states) and standard odd-photon cluster-type states.

(1) The preparation of even-photon I-type cluster-type states

Here we present a scheme for preparing an even-photon I-type cluster-type state with the combination of EPR pairs (2-I states, 2-II states) and the modules of the controlled phase-flip gate and Hadamard gate operations. For preparing an even-photon \( (2N \)-photon) I-type cluster-type state, the manipulator needs to do the following work beforehand. In general, he selects the left end of the multiphoton cluster-type state to be prepared as its head and the right end as its tail, and labels the photons in the state to be generated 1–\( N \) from its head to its tail. In our construction scheme, one 2-I state is utilized as the tail of the state (the extreme right end), and one 2-II state is selected as the head of the state (the extreme left end). The body of the state can be composed of two kinds of EPR states. If a 2-I state is exploited to compose one part of the body of the state to be produced, the right photon in the 2-I state needs to undergo a Hadamard gate operation before entering into the module of the controlled phase-flip gate. For another selection, if a 2-II state is applied to form one part of the body of the state to be prepared, the photon on the left end needs to undergo a Hadamard gate operation.

After the above preparatory work on 2-I(II) states, \( N - 1 \) modules of the controlled phase-flip gate are adopted to link these EPR states, where the photon on the right end of each EPR state is the Control qubit and the photon on the left end of the right adjacent EPR state is the Target qubit, i.e., the tail photon \( (C) \) of an EPR state at the left side controls the head photon \( (T) \) of the EPR state at its right side to flip the phase of the state \( |HV\rangle_{CT} \).

Passing through the modules of the controlled phase-flip gate, the even-numbered photons should undergo Hadamard gate operations to obtain the standard presentation, except for the second head photon (No. 2) and the tail photon (No. \( N \)). As a consequence, no Hadamard gate operation is necessary in the generation of a four-photon I-type cluster-type polarization-entangled state \( (4-I) \) from the composition of the left 2-II state and the right 2-I state.

As an example, we present a scheme for generating an eight-photon I-type cluster-type entangled state \( (8-I) \) with a composition of 2-I states and 2-II states. We numbered the photons 1–8 from head
to tail of the state to be prepared. In preparation, we select the $|\Phi^-\rangle_{12}, |\Phi^-\rangle_{34}, |\Psi^+\rangle_{56}, \text{ and } |\Psi^+\rangle_{78}$ states to generate the 8-I state. The explicit generation process can be illustrated as follows:

$$|\psi\rangle_I = |\Phi^-\rangle_{12}|\Phi^-\rangle_{34}|\Psi^+\rangle_{56}|\Psi^+\rangle_{78}$$

$$\xrightarrow{H_6} \frac{1}{2}|\Phi^-\rangle_{12}|\Phi^-\rangle_{34}(|HH\rangle - |HV\rangle + |VH\rangle + |VV\rangle)|\Psi^+\rangle_{56}|\Psi^+\rangle_{78}$$

$$\xrightarrow{\text{CPF}_{67}} \frac{1}{2\sqrt{2}}|\Phi^-\rangle_{12}|\Phi^-\rangle_{34}(|HH\rangle_{56}|\Psi^-\rangle_{78} - |HV\rangle_{56}|\Psi^+\rangle_{78}$$

$$+ |VH\rangle_{56}|\Psi^+\rangle_{78} + |VV\rangle_{56}|\Psi^+\rangle_{78})$$

$$\xrightarrow{H_6} \frac{1}{2}|\Phi^-\rangle_{12}|\Phi^-\rangle_{34}(|HV\rangle_{56}|HV\rangle_{78} + |VH\rangle_{56}|HV\rangle_{78} - |HH\rangle_{56}|VH\rangle_{78} - |VV\rangle_{56}|VH\rangle_{78})$$

$$\xrightarrow{\text{CPF}_{45}} \frac{1}{2}|\Phi^-\rangle_{12}(|\Phi^-\rangle_{34}|HV\rangle_{56}|HV\rangle_{78} - |\Phi^+\rangle_{34}|VH\rangle_{56}|HV\rangle_{78}$$

$$- |\Phi^-\rangle_{34}|HH\rangle_{56}|VH\rangle_{78} + |\Phi^+\rangle_{34}|VV\rangle_{56}|VH\rangle_{78})$$

$$\xrightarrow{H_3} \frac{1}{4}|\Phi^-\rangle_{12}[(|HH\rangle + |VV\rangle)_{34}(|VVH\rangle - |VHV\rangle)_{5678}$$

$$+ (|HV\rangle - |VH\rangle)_{34}(|VHV\rangle - |HVH\rangle)_{5678} + |HHV\rangle + |VVH\rangle)_{5678}]$$

$$\xrightarrow{\text{CPF}_{23}} \frac{1}{4\sqrt{2}}[(|HH\rangle + |VV\rangle)_{12}(|HH\rangle - |VV\rangle)_{34}(|VVH\rangle - |VHV\rangle)_{5678}$$

$$+ (|HV\rangle + |VH\rangle)_{34}(|VVH\rangle - |HVH\rangle)_{5678}]$$

$$\xrightarrow{H_4} \frac{1}{2\sqrt{2}}[|\Phi^-\rangle_{12}|VV\rangle_{34}(|VVH\rangle - |HVH\rangle)_{5678}$$

$$+ |HH\rangle_{34}(|VVH\rangle - |VHV\rangle)_{5678}]$$

$$\xrightarrow{H_4} \frac{1}{2\sqrt{2}}[|\Phi^+\rangle_{12}|VH\rangle_{34}(|VHV\rangle - |HHV\rangle)_{5678}$$

$$- |VV\rangle_{34}(|VHH\rangle - |VHV\rangle)_{5678}] \quad (11)$$

From Eq. (11), it can be seen that the 8-I state can be obtained by performing the operations of $H_6, \text{CPF}_{67}, H_6, \text{CPF}_{45}, H_3, \text{CPF}_{23}, H_4$, where the external orders ($\cdots$; $\cdots$; $\cdots$) of the three operation combinations ($H_6, H_3$; $\text{CPF}_{67}, \text{CPF}_{45}, \text{CPF}_{23}$; $H_6, H_4$) cannot be changed, while their internal orders ($\cdots, \cdots, \cdots$) can be exchanged.

(2) The preparation of standard odd-photon cluster-type states

If the aforementioned even-photon I-type cluster-type state has been prepared, a standard odd-photon cluster-type state can be generated by attaching one single photon in the 1-I state before the head of the prepared even-photon cluster-type state. After performing a Hadamard gate operation on the head photon of the prepared even-photon I-type cluster-type state, the single-photon (the Control qubit) and the head photon of the prepared state (the Target qubit) are sent into the module of the controlled phase-flip gate. After that, two Hadamard gate operations should be performed on the attached single photon and the second head photon (No. 2) of the original even-photon I-type cluster-type state to obtain the standard presentation of the newly prepared odd-photon cluster-type state on the basis of $\{|H\rangle, |V\rangle\}$.

For the other case, an even-photon II-type cluster-type state is assumed to have been generated. With the single photon in the 1-I state and the head photon (No. 1) of the prepared even-photon II-type cluster-type state entering the module of the controlled phase-flip gate with 1-I state photon
as the Control qubit, and the head photon (No. 1) of the prepared state as the Target qubit, a new odd-photon cluster-type state will be prepared. Finally, one Hadamard gate operation should be performed on the attached single photon to obtain the standard odd-photon cluster-type state.

Next, we consider the generation of a multi-photon cluster-type state by combining several cluster-type states. Exploiting \( m \) groups of \( n_i (\geq 2) \)-photon standard cluster-type states (even-photon I-type states and the standard odd-photon states) prepared beforehand, where the total number of photons is equal to \( N (\Sigma_{i=1}^{m} n_i = N) \), and \( m - 1 \) modules of the controlled phase-flip gate and the appropriate Hadamard gate operations, the required \( N \)-photon cluster-type state can be prepared.

After Hadamard gate operations on the photons at both sides of the junction, they are sent into the modules of the controlled phase-flip gate, where the photon at the left side of the junction is utilized as the Control qubit, and the photon at the right side of the junction is utilized as the Target qubit. Finally, the appropriate Hadamard gate operations should be performed to generate the standard cluster-type state. For obtaining the new odd-photon cluster-type state with the standard presentation, Hadamard gate operations should be executed on the odd-numbered photons, except for the photons on the two endpoints (head and tail).

In the generation of standard even-photon cluster-type states, the photons undergoing Hadamard gate operations are different. If an even-photon I-type cluster-type state is required, the Hadamard gate operations should be operated on the even-numbered photons, excluding the second head (No. 2) photon and the tail (No. \( N \)) photon in the last step. For generating the even-photon II-type cluster-type state, the Hadamard gate operations should be performed on the head photon (No. 1) and the even-numbered photons, other than the tail photon (No. \( N \)).

Excluding the photons at both sides of the junction, it is necessary to note that, in the generation of the new standard odd-photon cluster-type state, if the odd-numbered photons in the new cluster-type state have undergone Hadamard gate operations in the original standard cluster-type state, no Hadamard gate operation is required. Conversely, if the even-numbered photons in the new cluster-type state have undergone a Hadamard gate operation in the generation of the original state, Hadamard gate operations should be performed on these photons to restore them to the status without having undergone the Hadamard gate operations. After these operations, a new standard odd-photon cluster state has been constructed. For the generation of standard even-photon cluster-type states, the above rules of Hadamard gate operations on odd- and even-numbered photons should be reversed.

4. **Unlinking standard multi-photon cluster-type polarization-entangled states**

Suppose that a multi-photon standard cluster-type state is provided, and we can divide it into several cluster-type states on demand according to the following rules:

**Dividing body:** Here, we consider how to divide a standard cluster-type state into two new parts, the rules of which are also appropriate for a partition into many parts. If the photon sitting at either side of the unlinking point has undergone Hadamard gate operation in the generation process of the original standard cluster-type state, Hadamard gate operations should be performed on it at first. If any photon at the two sides of the unlinking point has not undergone a Hadamard gate operation, no Hadamard gate operation needs to be performed and the scheme skips to next step, which only happens on the photons (Nos. 2 and 3) of even-photon I-type cluster-type states.

Subsequently, two photons sitting at each side of the unlinking point enter into the module of the controlled phase-flip gate, where the photon at the left side of the unlinking point functions as the Control qubit and the photon at the right side is the Target qubit.
In the last step, some photons need to undergo single-photon local transformation operations such as Hadamard gate operations to obtain two new standard cluster-type states.

For example, unlinking the entanglement between photon 4 and the adjacent photons (3 or 5) of a six-photon I-type state (6-I), i.e., entanglement between photons (3,4) or photons (4,5), Hadamard gate operations should be performed on photon 4 rather than photon 3 or photon 5 at first. Then photons (3,4) or photons (4,5) should be transmitted through the module of the controlled phase-flip gate, i.e., CPF_{34} or CPF_{45}. Finally, an $H_4 \otimes H_3$ or $H_4$ operation should be performed to obtain two three-photon GHZ states (3-S) or one 4-I state and one 2-I state. The explicit steps can be also seen in Table 2.

**Truncating tail:** Unlike the above method separating the original cluster-type state into two parts, if we disentangle the tail (No. $N$) photon (the second tail (No. $N-1$) photon) from the standard $N$-photon cluster-type state, the photon to be disentangled should undergo $H$ gate operation first. After that, photon $N-1$ and photon $N$ are transmitted to the module of the controlled phase-flip gate; the former (latter) photon is the Control qubit and the latter (former) one is the Target qubit, respectively. As a consequence, photon No. $N$ (photon No. $N-1$) is separated from the original cluster-type state. Finally, some single-photon operations should be performed to obtain the standard cluster-type state consisting of $N-1$ photons.

**Pinching head:** As for the separation of the head (No. 1) photon (photon (No. 2) in the second head place) in the even-photon I-type cluster-type states or the standard odd-photon cluster-type states, there is a small difference in the explicit process. Photon No. 1 (photon No. 2) should undergo a Hadamard gate operation at first. Subsequently, photon No. 1 and photon No. 2 are sent into the module of the controlled phase-flip gate to perform the operation of $CPF_{12}$ ($CPF_{21}$) to unlink photon No. 1 (photon No. 2) from the original standard cluster-type state. In the last step, appropriate Hadamard gate operations need to be performed on photons to obtain the standard cluster-type state. If an even-photon II-type cluster-type state is required to pinch photon No. 1 (photon No. 2), a Hadamard gate operation should be performed on photon No. 2 (photon No. 1) in the first step rather than photon No. 1 (photon No. 2), while the operations in the latter two steps are the same as those of even-photon I-type cluster-type states.

In the following, we unlink two states, a 5-S state and a 6-I state, to illustrate the unlinking rules of the standard odd-photon and even-photon I-type cluster-type states.

**Example:** (1) Unlinking a 5-S state.
First, we unlink a 5-S state, which represents the standard odd-photon cluster-type states. The explicit unlinking steps can be seen in Table 1. For a standard even-photon cluster-type state, the unlinking operations are a little more complicated than those of standard odd-photon cluster-type states. Without loss of generality, a 6-I state is utilized as an example to describe the unlinking rule of standard even-photon cluster-type states as follows.

**Example:** (2) Unlinking a 6-I state.
A 6-I state can be denoted as
\[
6 - 1 : \frac{1}{2\sqrt{2}}[(|HH\rangle - |VV\rangle)_{12}|H\rangle_3(|VHV\rangle - |HVH\rangle)_{456} - (|HH\rangle + |VV\rangle)_{12}|V\rangle_3(|HHV\rangle - |VVH\rangle)_{456}].
\] (12) The detailed steps for unlinking a 6-I state can be seen in Table 2. From the above tables (Tables 1 and 2), it can be seen that a 5-I state and 6-I state can be unlinked by combining the modules of the controlled phase-flip gate and single-qubit operations. As a rational
Table 1. Unlinking a 5-S state. TPP denotes the two parts of photons to be unlinked; EOS denotes the explicit operating steps: \( H \) denotes Hadamard gate operation, CPF denotes the module of the controlled phase-flip gate, and the corresponding subscripts denote the photons undergoing the operations; RS denotes the resulting states.

| TPP   | EOS             | RS                                                                 |
|-------|-----------------|-------------------------------------------------------------------|
| 123,45| 3-S: \( \frac{1}{\sqrt{2}}(\langle HHV \rangle - \langle VVH \rangle)_{123}; \) |
|       | 2-I: \( \frac{1}{\sqrt{2}}(\langle VHV \rangle + \langle VHY \rangle)_{123}; \) |
| 12,345| 2-II: \( \frac{1}{\sqrt{2}}(\langle HHV \rangle - \langle VHV \rangle)_{12}; \) |
|       | 3-S: \( \frac{1}{\sqrt{2}}(\langle HHY \rangle - \langle VYV \rangle)_{345}; \) |
| 1,2345| 1-I: \( \frac{1}{\sqrt{2}}(\langle H \rangle + \langle V \rangle)_3; \) |
|       | 4-II: \( \frac{1}{\sqrt{2}}(\langle H \rangle_2(\langle VH \rangle - \langle VY \rangle)_{12} + \langle V \rangle_2(\langle HHV \rangle - \langle VYH \rangle)_{345}; \) |
| 2,1345| 1-I: \( \frac{1}{\sqrt{2}}(\langle H \rangle + \langle V \rangle)_3; \) |
|       | 4-II: \( \frac{1}{\sqrt{2}}(\langle H \rangle_1(\langle VH \rangle - \langle VY \rangle)_{345} + \langle V \rangle_1(\langle HHV \rangle - \langle VYH \rangle)_{345}; \) |
| 4,1235| 1-I: \( \frac{1}{\sqrt{2}}(\langle H \rangle + \langle V \rangle)_4; \) |
|       | 4-I: \( \frac{1}{\sqrt{2}}(\langle HH \rangle_{12}(\langle VH \rangle - \langle VH \rangle)_{345} + \langle V \rangle_{12}(\langle HH \rangle + \langle V \rangle)_{35}; \) |
| 5,1234| 1-I: \( \frac{1}{\sqrt{2}}(\langle H \rangle + \langle V \rangle)_5; \) |
|       | 4-I: \( \frac{1}{\sqrt{2}}(\langle HH \rangle_{12}(\langle VH \rangle - \langle VH \rangle)_{34} - \langle V \rangle_{12}(\langle HH \rangle + \langle V \rangle)_{34}; \) |

Table 2. Unlinking a 6-I state. The symbol conventions are the same as those in Table 1.

| TPP   | EOS             | RS                                                                 |
|-------|-----------------|-------------------------------------------------------------------|
| 1234,56| 4-I: \( \frac{1}{\sqrt{2}}(\langle HH \rangle_{12}(\langle VH \rangle - \langle VH \rangle)_{34} - \langle V \rangle_{12}(\langle HH \rangle + \langle V \rangle)_{34}; \) |
|       | 2-I: \( \frac{1}{\sqrt{2}}(\langle VHV \rangle + \langle VHY \rangle)_{356}; \) |
| 12,3456| 2-II: \( \frac{1}{\sqrt{2}}(\langle HH \rangle - \langle VY \rangle)_{12}; \) |
|       | 4-II: \( \frac{1}{\sqrt{2}}(\langle HH \rangle_{35}(\langle VH \rangle - \langle VY \rangle)_{345} + \langle V \rangle_{3}(\langle HH \rangle - \langle VY \rangle)_{345}; \) |
| 12,3456| 3-S: \( \frac{1}{\sqrt{2}}(\langle HH \rangle - \langle VHV \rangle)_{123}; \) |
|       | 3-S: \( \frac{1}{\sqrt{2}}(\langle HH \rangle - \langle VHV \rangle)_{123}; \) |
| 1,23456| 1-I: \( \frac{1}{\sqrt{2}}(\langle H \rangle + \langle V \rangle)_3; \) |
|       | 5-S: \( \frac{1}{\sqrt{2}}(\langle HH \rangle_{23}(\langle VH \rangle - \langle VH \rangle)_{345} - \langle VV \rangle_{23}(\langle HH \rangle - \langle VY \rangle)_{345}; \) |
| 2,13456| 1-I: \( \frac{1}{\sqrt{2}}(\langle H \rangle + \langle V \rangle)_3; \) |
|       | 5-S: \( \frac{1}{\sqrt{2}}(\langle HH \rangle_{13}(\langle VH \rangle - \langle VH \rangle)_{345} - \langle VV \rangle_{13}(\langle HH \rangle - \langle VY \rangle)_{345}; \) |
| 12345,6| 5-S: \( \frac{1}{\sqrt{2}}(\langle HH \rangle_{12}(\langle VH \rangle - \langle VH \rangle)_{345} - \langle VV \rangle_{12}(\langle HH \rangle - \langle VY \rangle)_{345}; \) |
|       | H_3, H_4, H_5. |
| 12346,5| 1-I: \( \frac{1}{\sqrt{2}}(\langle H \rangle + \langle V \rangle)_5; \) |
|       | 5-S: \( \frac{1}{\sqrt{2}}(\langle HH \rangle_{12}(\langle VH \rangle - \langle VH \rangle)_{346} - \langle VV \rangle_{12}(\langle HH \rangle - \langle VY \rangle)_{346}; \) |

5. Discussion and summary

Exploiting a combination of the presented modules of the controlled phase-flip gate, the distributed entanglement gate, and the simplified entanglement gate, the lengths of cluster-type states can be unlinked using similar methods.
polarization-entangled states can be extended. Moreover, the modules can be applied to disentangle the cluster-type state accompanying Hadamard gate operations and other single-photon operations.

In the practical implementation, the efficiencies of the practical detectors and the effect of the dissipated probe coherent states should be taken into account. They increase the difficulty of distinguishing the photon paths \(a, f; b, e\) and \(a, e; b, f\) illustrated in Fig. 1, \(a, d; b, c\) and \(a, c; b, d\) illustrated in Fig. 3, and \(a, b; b, a\) and \(a, a; b, b\) illustrated in Fig. 4. Explicitly, in the propagation process, photons of the probe coherent states may be lost due to interaction with the environment, and this effect can be viewed as the amplitude attenuation of the probe coherent states \([37–39,60]\).

We utilize the parameter \(\alpha\sqrt{\eta_d} (\sqrt{\eta_l} < 1)\) to represent the influence on the probe coherent state \(|\alpha\rangle\). For practical detectors, in the sense of error probability, their effect is equal to reducing the intensity of the probe coherent state owing to their imperfect detection efficiency, so this can be denoted as the amplitude \(\alpha\) multiplied by the parameter \(\sqrt{\eta_d}(<1)\). Based on the above analyses, the error probability of the controlled phase-flip gate and the entanglement gates can be denoted as

\[
P_{\text{err}} = \frac{1}{2}\text{erfc} \left( |\alpha\theta\sqrt{\eta_d}\eta_l| / 2\sqrt{2} \right).\tag{13}
\]

From Eq. (13), it can be seen that the dissipated coherent state and the imperfect detectors increase the error probability, from \(\frac{1}{2}\text{erfc}(|\alpha|/\sqrt{2})\) to \(\frac{1}{2}\text{erfc}(|\alpha\theta\sqrt{\eta_d}\eta_l|/2\sqrt{2})\). However, the effect of \(\sqrt{\eta_d}\eta_l\) can be compensated by increasing the intensity \(\alpha\) of the probe coherent state and/or the strength \(\theta\) of cross-Kerr nonlinearity, including the interaction time and length \([37–40]\).

Turchette et al. \([61]\) proposed the first pioneering experimental work on the conditional phase gate with cross-Kerr nonlinearity in cavity quantum electrodynamics, which indicates that natural Kerr nonlinearity is too weak to provide a sufficiently large phase shift. Furthermore, phase noise \([41]\) hinders it from fulfilling the tasks of quantum information processing with cross-Kerr nonlinearity.

To obtain a large phase shift, there exist two methods that can be adopted. One is to generate an effective nonlinear interaction strength by coupling light to a physics system possessing certain energy levels, such as a Rydberg electromagnetically induced transparencies (EIT) system \([47,62–65]\) and cavity EIT system \([48,66–68]\), or several methods including the double EIT effect \([69]\) and weak measurements \([51]\). Recently, under EIT conditions, a large cross-Kerr nonlinearity has been shown to be achievable by exploiting negative index metamaterials \([70]\). In particular, atomic systems working under EIT conditions \([62]\) become promising candidates for applying cross-Kerr nonlinearity. Under this condition, the phase noise model \([41]\) may not be considered. Moreover, only weak cross-Kerr nonlinear interaction is required, and the potential photon loss is negligible in an EIT medium where the loss of the signal photons and the coherent beam is intrinsically lowered \([48,71]\). The other method is to prolong the interaction time or length \([52,72–75]\). Interaction between two stopped light pulses has been studied and demonstrated by Chen et al. \([76]\). With double slow light pulses, Lin et al. \([77]\) demonstrated an all-optical switching. Applying a general theory of interaction between continuous-mode photonic pulses, He et al. \([78]\) studied the interaction between a single photon and a coherent state. They showed that the simultaneous requirements of high fidelities, nonzero conditional phases, and high photon numbers can be achieved under conditions where the photonic pulses fully pass through each other and the unwanted transverse-mode effects are suppressed.

Moreover, the performance of the detectors is being improved step by step. Recently, Pernice et al. \([79]\) demonstrated a high-speed traveling wave single-photon detector possessing a high probability approaching 91%, which is embedded in nanophotonic circuits.

The synchronization between the coherent states and the local oscillators applied in logical gates is necessary for the present schemes. Exploiting temperature-insensitive time-delay elements and
balanced photodetection to suppress thermal drift and amplitude noise, Kim et al. [80,81] proposed an optical–optical synchronization scheme associated with single-crystal balanced optical cross-correlation.

To sum up, we have proposed schemes for preparing, linking, and unlinking cluster-type polarization-entangled states assisted by weak cross-Kerr nonlinear interaction, which can be considered to be possible in other physics systems. The modules of the distributed entanglement gate, the simplified entanglement gate, and the controlled phase-flip gate are integrated to accomplish the tasks for preparing, linking, and unlinking cluster-type states, together with Hadamard gate operations and single-photon operations. The presented schemes require resources such as single-photon polarization states, currently feasible techniques including classical feed-forward, and some available optical elements including beam splitters, polarization beam splitters, reflection mirrors, half-wave plates functioning as Hadamard gate operations, and phase shifters to delay the phases on demand, etc.

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References
[1] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).
[2] H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. 86, 910 (2001).
[3] L.-M. Duan and R. Raussendorf, Phys. Rev. Lett. 95, 080503 (2005).
[4] V. Scarani, A. Ac, E. Schenck, and M. Aspelmeyer, Phys. Rev. A 71, 042325 (2005).
[5] P. Walther, K. J. Resch, T. Rudolph, E. Schenck, H. Weinfurter, V. Vedral, M. Aspelmeyer, and A. Zeilinger, Nature 434, 169 (2005).
[6] M. S. Tame, R. Prevedel, M. Paternostro, P. Bö, M. S. Kim, and A. Zeilinger, Phys. Rev. Lett. 98, 140501 (2007).
[7] E. Knill, R. Laflamme, and G. J. Milburn, Nature 409, 46 (2001).
[8] P. Kok, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, Rev. Mod. Phys. 79, 135 (2007).
[9] R. Prevedel, P. Walther, F. Tiefenbacher, P. Bö, R. Kaltenbaek, T. Jennewein, and A. Zeilinger, Nature 445, 65 (2007).
[10] T. Bodiya and L.-M. Duan, Phys. Rev. Lett. 97, 143601 (2006).
[11] N. Lindner and T. Rudolph, Phys. Rev. Lett. 103, 113602 (2009).
[12] S. Louis, K. Nemoto, W. Munro, and T. Spiller, Phys. Rev. A 75, 042323 (2007).
[13] S. G. R. Louis, K. Nemoto, W. J. Munro, and T. P. Spiller, New J. Phys. 9, 193 (2007).
[14] H. F. Wang and S. Zhang, Eur. Phys. J. D 53, 359 (2009).
[15] J.-W. Pan, Z.-B. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Zukowski, Rev. Mod. Phys. 84, 777 (2012).
[16] S.-B. Zheng, Phys. Rev. A 73, 065802 (2006).
[17] L.-M. Duan, Rev. Mod. Phys. 82, 1209 (2010).
[18] J. Cho and H.-W. Lee, Phys. Rev. Lett. 95, 160501 (2005).
[19] X. Zou and W. Mathis, Phys. Rev. A 72, 013809 (2005).
[20] P. Dong, Z.-Y. Xue, M. Yang, and Z.-L. Cao, Phys. Rev. A 73, 33818 (2006).
[21] X. Zhang, K. Gao, and M. Feng, Phys. Rev. A 75, 034308 (2007).
[22] P. P. Munhoz, F. L. Seminati, A. Vidiella-Barranco, and J. A. Roversi, Phys. Lett. A 372, 3580 (2008).
[23] E. M. Becerra-Castro, W. B. Cardoso, A. T. Avelar, and B. Baseia, J. Phys. B: At. Mol. Opt. Phys. 41, 085505 (2008).
[24] H.-F. Wang, X.-Q. Shao, Y.-F. Zhao, S. Zhang, and K.-H. Yeon, J. Phys. B: At. Mol. Opt. Phys. 42, 175506 (2009).
[25] D. Gonta, T. Raitke, and S. Fritzsche, Phys. Rev. A 79, 062319 (2009).
[26] T. Tamamoto, Y.-x. Liu, S. Fujita, X. Hu, and F. Nori, Phys. Rev. Lett. 97, 230501 (2006).
[27] X. L. Zhang, K. L. Gao, and M. Feng, Phys. Rev. A 74, 024303 (2006).
[28] G.-P. Guo, H. Zhang, T. Tu, and G.-C. Guo, Phys. Rev. A 75, 050301(R) (2007).
[29] Q. Lin and B. He, Phys. Rev. A 82, 022331 (2010).
[30] C. Horsman, K. Brown, W. Munro, and V. Kendon, Phys. Rev. A 83, 042327 (2011).
[31] K. Brown, C. Horsman, V. Kendon, and W. Munro, Phys. Rev. A 85, 052305 (2012).
[32] L.-M. Duan and H. Kimble, Phys. Rev. Lett. 92, 127902 (2004).
[33] K. Nemoto and W. J. Munro, Phys. Rev. Lett. 93, 250502 (2004).
[34] Q. Lin and J. Li, Phys. Rev. A 82, 022331 (2010).
[35] C. Horsman, K. Brown, W. Munro, and V. Kendon, Phys. Rev. A 83, 042327 (2011).
[36] K. Brown, C. Horsman, V. Kendon, and W. Munro, Phys. Rev. A 85, 052305 (2012).
[37] L.-M. Duan and H. Kimble, Phys. Rev. Lett. 92, 127902 (2004).
[38] K. Nemoto and W. J. Munro, Phys. Rev. Lett. 93, 250502 (2004).
[39] Q. Lin and B. He, Phys. Rev. A 80, 042310 (2009).
[40] A. Feizpour, X. Xing, and A. M. Steinberg, Phys. Rev. Lett. 107, 133603 (2011).
[41] B. He, A. MacRae, Y. Han, A. Lvovsky, and C. Simon, Phys. Rev. A 83, 033806 (2011).
[42] Y.-F. Chen, C.-Y. Wang, S.-H. Wang, and I. Yu, Phys. Rev. Lett. 96, 043603 (2006).
[43] B. He, A. MacRae, Y. Han, A. Lvovsky, and C. Simon, Phys. Rev. A 83, 033806 (2011).
[44] Y. Zhu, Opt. Lett. 35, 303 (2010).
[45] M. Mü, E. Figueroa, J. Bochmann, C. Hahn, K. Murr, S. Ritter, C. J. Villas-Boas, and G. Rempe, Nature 465, 755 (2010).
[46] T. Kampschulte, W. Alt, S. Brakhane, M. Eckstein, R. Reimann, A. Widera, and D. Meschede, Phys. Rev. Lett. 105, 153603 (2010).
[47] Z.-B. Wang, K.-P. Marzlin, and B. C. Sanders, Phys. Rev. Lett. 97, 063901 (2006).
[48] M. Siomau, A. A. Kamli, S. A. Moiseev, and B. C. Sanders, Phys. Rev. A 85, 050303(R) (2012).
[49] B. He, M. Nadeem, and J. Bergou, Phys. Rev. A 79, 035802 (2009).
[50] S. Harris and L. Hau, Phys. Rev. Lett. 82, 4611 (1999).
[51] M. D. Lukin and A. Imamoglu, Phys. Rev. Lett. 84, 1419 (2000).
[52] M. Bajcsy, A. S. Zibrov, and M. D. Lukin, Nature 426, 638 (2003).
[53] H.-Y. Lo, Y.-C. Chen, P.-C. Su, H.-C. Chen, J.-X. Chen, Y.-C. Chen, I. A. Yu, and Y.-F. Chen, Phys. Rev. A 83, 041804(R) (2011).
[54] Y.-H. Chen, M.-J. Lee, W. Hung, Y.-C. Chen, Y.-F. Chen, and I. A. Yu, Phys. Rev. Lett. 108, 173603 (2012).
[77] C.-C. Lin, M.-C. Wu, B.-W. Shiau, Y.-H. Chen, I. A. Yu, Y.-F. Chen, and Y.-C. Chen, Phys. Rev. A 86, 063836 (2012).
[78] B. He, Q. Lin, and C. Simon, Phys. Rev. A 83, 053826 (2011).
[79] W. H. P. Pernice, C. Schuck, O. Minaeva, M. Li, G. N. Goltsman, A. V. Sergienko, and H. X. Tang, Nat. Commun. 3, 1325 (2012).
[80] J. Kim, J. Chen, Z. Zhang, F. N. C. Wong, F. X. Kärtner, F. Loehl, and H. Schlarb, Opt. Lett. 32, 1044 (2007).
[81] J. Kim, J. A. Cox, J. Chen, and F. X. Kärtner, Nat. Photonics 2, 733 (2008).