Planning with Partial Preference Models

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Abstract

Current work in planning with preferences assume that the user’s preference models are completely specified and aim to search for a single solution plan. In many real-world planning scenarios, however, the user probably cannot provide any information about her desired plans, or in some cases can only express partial preferences. In such situations, the planner has to present not only one but a set of plans to the user, with the hope that some of them are similar to the plan she prefers. We first propose the usage of different measures to capture quality of plan sets that are suitable for such scenarios: domain-independent distance measures defined based on plan elements (actions, states, causal links) if no knowledge of the user’s preferences is given, and the Integrated Convex Preference measure in case the user’s partial preference is provided. We then investigate various heuristic approaches to find set of plans according to these measures, and present empirical results demonstrating the promise of our approach.

Key words: Planning, Preferences, Heuristics, Search

1. Introduction

Most work in automated planning takes as input a complete specification of domain models and/or user preferences and the planner searches for a single solution satisfying the goals, probably optimizing some objective function. In many real world planning scenarios, however, the user’s preferences on desired plans are either unknown or at best partially specified (c.f. Kambhampati (2007)). In such cases, the planner’s job changes from finding a single optimal plan to finding a set of representative solutions (“options”) and presenting them to the user with the hope that she can find one of them desirable. As an example, in adaptive web services composition, the causal dependencies among some web services might change at the execution time.

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*This work is an extension of the work presented in Srivastava et al. (2007) and Nguyen et al. (2009).
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and as a result the web service engine wants to have a set of diverse plans/compositions such that if there is a failure while executing one composition, an alternative may be used which is less likely to be failing simultaneously (Chafle et al., 2006). However, if a user is helping in selecting the compositions, the planner could be first asked for a set of plans that may take into account the user’s trust in some particular sources and when she selects one of them, it is next asked to find plans that are similar to the selected one. The requirement of searching for a set of plans is also considered in intrusion detection (Boddy et al., 2005) where a security analysis needs to analyze a set of attack plans that might be attempted by a potential adversary, given limited (or unknown) information about the adversary’s model (e.g., his goals, capabilities, habits, ...), and the resulting analyzed information can then be used to set up defensive strategies against potential attacks in the future. Another example can be found in Memon et al. (2001) in which test cases for graphical user interfaces (GUIs) are generated as a set of distinct plans, each corresponding to a sequence of actions that a user could perform, given the user’s unknown preferences on how to interact with the GUI to achieve her goals. The capability of synthesizing multiple plans would also have potential application in case-based planning (e.g., Serina (2010)) where it is important to have a plan set satisfying a case instance. These plans can be different in terms of criteria such as resources, makespan and cost that can only be specified in the retrieval phase. In the problem of travel planning for individuals of a city in a distributed manner while also optimizing public resource (e.g., road, traffic police personnel), the availability of a number of plans for each person’s goals could make the plan merging phase easier and reduce the conflicts among individual plans.

In this work, we investigate the problem of generating a set of plans in order to deal with planning situations where the preference model is not completely specified. In particular, we consider the following scenarios:

- Even though the planner is aware that the user has some preferences on solution plans, it is not provided with any of that knowledge.

- The planner is provided with incomplete knowledge of the user’s preferences. In particular, the user is interested in some plan attributes (such as the duration and cost of a flight, or whether all packages with priority are delivered on time in a logistic domain), each with different but unknown degree of importance (represented by weight or trade-off values). Normally, it is quite hard for a user to indicate the exact trade-off values, but instead more likely to determine that one attribute is more (or less) important than some others—for instance, a businessman would consider the duration of a flight much more important than its cost. Such kind of incomplete preference specification could be modeled with a probability distribution of weights values, and is therefore assumed to be given as an input (together with the attributes) to the planner.

Even though, in principle, the user would have a better chance to find her desired plan from a larger plan set, there are two problems to consider—one computational, and other comprehensional. The computational problem is that synthesis of a single plan is often quite costly already, and therefore it is even more challenging to search for a large plan set. Coming to the second problem, it is unclear that the user will be able to inspect a large set of plans to identify the plan.

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2Even if we do not have any special knowledge about this probability distribution, we can always start by initializing it to be uniform, and gradually improve it based on interaction with the user.
she prefers. What is clearly needed, therefore, is the ability to generate a set of plans, among all sets of bounded (small) number of plans, with the highest chance of including the user’s preferred plan. An immediate challenge is formalizing what it means for a meaningful set of plans, in other words what the quality measure of plan sets should be given an incomplete preference specification.

We propose different quality measures for the two scenarios listed above. In the extreme case when the user could not provide any knowledge of her preferences, we define a spectrum of distance measures between two plans based on their syntactic features in order to define the diversity measure of plan sets. These measures can be used regardless of the user’s preference, and by maximizing the diversity of a plan set we increase the chance that the set is uniformly distributed in the unknown preference space, and therefore likely contains a plan that is close to a user’s desired one.

This measure can be further refined when some knowledge of the user’s preferences is provided. As mentioned above, we assume that the user’s preference is specified by a convex combination of plan attributes, and incomplete in the sense that the distribution of trade-off weights is given, not their exact values. The whole set of best plans (i.e. the ones with the best value function) can be pictured as the lower convex-hull of the Pareto set on the attribute space. To measure the quality of any (bounded) set of plans on the whole optimal set, we adapt the idea of Integrated Preference Function (IPF) (Carlyle et al, 2003), in particular its special case Integrated Convex Preference (ICP). This measure was developed in the Operations Research (OR) community in the context of multi-criteria scheduling, and is able to associate a robust measure of representativeness for any set of solution schedules (Fowler et al, 2005).

Armed with these quality measures, we can then formulate the problem of planning with partial preference models as finding a bounded set of plans that has the best quality value. Our next contribution therefore is to investigate effective approaches for using quality measures to bias a planner’s search to find a high quality plan set efficiently. For the first scenario when the preference specification is not provided, two representative state-of-the-art planning approaches are considered. The first, GP-CSP (Do and Kambhampati, 2001), typifies the issues involved in generating diverse plans in bounded horizon compilation approaches, while the second, LPG (Gerevini et al, 2003), typifies the issues involved in modifying the heuristic search planners. Our investigations with GP-CSP allow us to compare the relative difficulties of enforcing diversity with each of the three different distance measures (elaborated in later section). With LPG, we find that the proposed quality measure makes it more effective in generating plan set over large problem instances. For the second case when part of the user’s preferences is provided, we also present a spectrum of approaches for solving this problem efficiently. We implement these approaches on top of Metric-LPG (Gerevini et al, 2008). Our empirical evaluation compares these approaches both among themselves as well as against the methods for generating diverse plans ignoring the partial preference information, and the results demonstrate the promise of our proposed solutions.

Our work can be considered as a complement to current research in planning with preferences, as shown in Figure 1. Under the perspective of planning with preferences, most current work in planning synthesizes a single solution plan, or a single best one, in situations where user has no preferences, or a complete knowledge of preferences is given to the planner. On the other hand, we address the problem of synthesizing a set of plans when knowledge of user’s preferences is either completely unknown or partially specified.

The paper is organized as follows. Section 2 gives fundamental concepts in preferences, and formal notations. In Section 3 we formalize quality measures of plan set in the two scenarios.
Sections 4 and 5 discuss our various heuristic approaches to generate plan sets, together with the experimental results. We discuss related work in Section 6, future work and conclusion in Section 7.

2. Background and Notation

Given a planning problem with the set of solution plans $S$, a user preference model is a transitive, reflexive relation in $S \times S$, which defines an ordering between two plans $p$ and $p'$ in $S$. Intuitively, $p \preceq p'$ means that the user prefers $p$ at least as much as $p'$. Note that this ordering can be either partial (i.e. it is possible that neither $p \preceq p'$ nor $p' \preceq p$ holds—in other words, they are incomparable), or total (i.e. either $p \preceq p'$ or $p' \preceq p$ holds). A plan $p$ is considered more preferred than a plan $p'$, denoted by $p \prec p'$, if $p \preceq p'$ and $p' \not\preceq p$, and they are equally preferred if $p \preceq p'$ and $p' \preceq p$. A plan $p$ is an optimal (i.e., most preferred) plan if $p \preceq p'$ for any other plan $p'$. A plan set $\mathcal{P} \subseteq S$ is considered more preferred than $\mathcal{P}' \subseteq S$, denoted by $\mathcal{P} \prec \mathcal{P}'$, if $p \prec p'$ for any $p \in \mathcal{P}$ and $p' \in \mathcal{P}'$, and they are incomparable if there exists $p \in \mathcal{P}$ and $p' \in \mathcal{P}'$ such that $p$ and $p'$ are incomparable.

The ordering $\preceq$ implies a partition of $S$ into disjoint plan sets (or classes) $S_0, S_1, \ldots$ ($S_0 \cup S_1 \cup \ldots = S, S_i \cap S_j = \emptyset$) such that plans in the same set are equally preferred, and for any set $S_i, S_j$, either $S_i \prec S_j, S_j \prec S_i, \text{ or they are incomparable}$. The partial ordering between these sets can be represented as a Hasse diagram [Birkhoff 1948] where the sets are vertices, and there is an (upward) edge from $S_j$ to $S_i$ if $S_i \prec S_j$ and there is not any $S_k$ in the partition such that $S_i \prec S_k \prec S_j$. We denote $l(S_i)$ as the “layer” of the set $S_i$ in the diagram, assuming that the most preferred sets are placed at the layer 0, and $l(S_i) = l(S_j) + 1$ if there is an edge from $S_j$ to $S_i$. A plan in a set at a layer with the smaller value, in general, is either more preferred than...
Figure 2: The Hasse diagrams and layers of plan sets implied by two preference models. In (a), $S_1 \prec S_2 \prec S_3$, and any two plans are comparable. In (b), on the other hand, $S_1 \prec S_2 \prec S_4$, $S_1 \prec S_3$, and each plan in $S_3$ is incomparable with plans in $S_2$ and $S_4$.

or incomparable with ones at high-value layers. Figure 2 show examples of Hasse diagrams representing a total and partial preference ordering between plans.

When the preference model is explicitly specified, answering queries such as comparing two plans, finding a most preferred (optimal) plan becomes an easy task. This is possible, however, only if the set of plans is small and known upfront. Many preference languages, therefore, have been proposed to represent the relation $\preceq$ in a more compact way, and serve as starting points for algorithms to answer queries. Most preference languages fall into the following two categories:

- Quantitative languages define a value function $V: S \rightarrow R$ which assigns a real number to each plan, with a precise interpretation that $p \preceq p' \iff V(p) \leq V(p')$. Although this function is defined differently in many languages, at a high level it combines the user’s preferences on various aspects of plan that can be measured quantitatively. For instance, in the context of decision-theoretic planning (Boutilier et al., 1999), the value function of a policy is defined as the expected rewards of states that are visited when the policy executes. In partial satisfaction (over-subcription) planning (PSP) (Smith, 2004; Van Den Briel et al., 2004), the quality of plans is defined as its total rewards of soft goals achieved minus its total action costs. In PDDL2.1 (Fox and Long, 2003), the value function is an arithmetic function of numerical fluents such as plan makespans, fuel used etc., and in PDDL3 (Gerevini et al., 2009) it is enhanced with individual preference specification defined as formulae over state trajectory using linear temporal logic (LTL) (Pnueli, 1977).

- Qualitative languages provide qualitative statements that are more intuitive for lay users to specify. A commonly used language of this type is CP-networks (Boutilier et al., 2004), where the user can specify her preference statements on values of plan attributes, possibly given specification of others (for instance, “Among tickets with the same prices, I prefer airline A to airline B.”). Another example is LPP (Bienvenu et al., 2006) in which the

\[ \text{If } \preceq \text{ is a total ordering, then plans at smaller layer is more preferred than ones at higher layer.} \]
statements can be specified using LTL formulae, and possibly being aggregated in different ways.

Figure 3 shows the conceptual relation of preference models, languages and algorithms. We refer the reader to the work by Brafman and Domshlak (2009) for a more detailed discussion on this metamodel, and by Baier and McIraith (2009) for an overview of different preference languages used in planning with preferences.

From the modeling point of view, in order to design a suitable language capturing the user’s preference model, the modeler should be provided with some knowledge of the user’s interest that affects the way she evaluates plans (for instance, flight duration and ticket cost in a travel planning scenario). Such knowledge in many cases, however, cannot be completely specified. Our purpose therefore is to present a bounded set of plans to the user in the hope that it will increase the chance that she can find a desired plan. In the next section, we formalize the quality measures for plan sets in two situations where either no knowledge of the user’s preferences or only part of them is given.

3. Quality Measures for Plan Sets

3.1. Syntactic Distance Measures for Unknown Preference Cases

We first consider the situation in which the user has some preferences for solution plans, but the planner is not provided with any knowledge of such preferences. It is therefore impossible for the planner to assume any particular form of preference language representing the hidden preference model. There are two issues that need to be considered in formalizing a quality measure for plan sets:

- What are the elements of plans that can be involved in a quality measure?
- How should a quality measure be defined using those elements?

For the first question, we observe that even though users are normally interested in some high level features of plans that are relevant to them, many of those features can be considered as “functions” of base level elements of plans. For instance, the set of actions in the plan determine the makespan of a (sequential) plan, and the sequence of states when the plan executes gives the total reward of goals achieved. We consider the following three types of base level features of plans which could be used in defining quality measure, independently of the domain semantics:

- Actions that are present in plans, which define various high level features of the plans such as its makespan, execution cost etc. that are of interest to the user whose preference model could be represented with preference languages such as in PSP and PDDL2.1.
Table 1: The pros and cons of using the different base level elements of plan.

| Basis       | Pros                                           | Cons                                           |
|-------------|------------------------------------------------|------------------------------------------------|
| Actions     | Does not require problem information           | No problem information is used                 |
| States      | Not dependent on any specific plan representation | Needs an execution simulator to identify states |
| Causal links| Considers causal proximity of state transitions (action) rather than positional (physical) proximity | Requires domain theory |

- **Sequence of states that the agent goes through, which captures the behaviors resulting from the execution of plans.** In many preference languages defined using high level features of plans such as the reward of goals collected (e.g., PSP), or the whole state (e.g., MDP), or the temporal relation between propositions occur in states (e.g. PDDL3, P4P [Son and Pontelli, 2006] and LPP [Fritz and McIlraith, 2006]), the sequence of states can affect the quality of plan evaluated by the user.

- **The causal links representing how actions contribute to the goals being achieved, which measures the causal structures of plans.** These plan elements can affect the quality of plans with respect to the languages mentioned above, as the causal links capture both the actions appearing in a plan and the temporal relation between actions and variables.

A similar conceptual separation of features has also been considered recently in the context of case-based planning by Serina [2010], in which planning problems were assumed to be well classified, in terms of costs to adapt plans of one problem to solve another, in some unknown high level feature space. The similarity between problems in the space were implicitly defined using kernel functions of their domain-independent graph representations. In our situation, we aim to approximate quality of plan sets on the space of features that the user is interested in using distance between plans with respect to base level features of plans mentioned above (see below).

Table 1 gives the pros and cons of using the different base level elements of plan. We note that if actions in the plans are used in defining quality measure of plan sets, no additional problem or domain theory information is needed. If plan behaviors are used as base level elements, the representation of the plans that bring about state transition becomes irrelevant since only the actual states that an execution of the plan will take is considered. Hence, we can now compare plans of different representations, e.g., four plans where the first is a deterministic plan, the second is a contingent plan, the third is a hierarchical plan and the fourth is a policy encoding probabilistic behavior. If causal links are used, then the causal proximity among actions is now considered rather than just physical proximity in the plan.

Given those base level elements, the next question is how to define a quality measure of plan sets using them. Recall that without any knowledge about the user’s preferences, there is no way for the planner to assume any particular preference language, because of which the motivation

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4 A causal link $a_1 \rightarrow a_2$ records that a predicate is produced by $a_1$ and consumed by $a_2$. 7
behind a choice of quality measure should come from the hidden user’s preference model. Given a Hasse diagram induced from the user’s preference model, a $k$-plan set that will be presented to the user can be considered to be randomly selected from the diagram. The probability of having one plan in the set classified in a class at the optimal layer would increase when the individual plans are more likely to be at different layers, and this chance in turn will increase if they are less likely to be equally preferred by the user. On the other hand, the effect of base level elements of a plan on high level features relevant to the user suggests that plans similar with respect to base level features are more likely to be close to each other on the high level feature space determining user’s preference model.

In order to define a quality measure using base level features of plans, we proceed with the following assumption: plans that are different from each other with respect to the base level features are less likely to be equally preferred by the user, in other words they are more likely to be at different layers of the Hasse diagram. With the purpose of increasing the chance of having a plan that the user prefers, we propose the quality measure of plan sets as its diversity measure, defined using the distance between two plans in the set with respect to a base level element. More formally, the quality measure \( \zeta : 2^S \rightarrow \mathbb{R} \) of a plan set \( P \) can be defined as either the minimal, maximal, or average distance between plans:

- Minimal distance:
  \[
  \zeta_{\text{min}}(P) = \min_{p, p' \in P} \delta(p, p')
  \]  
  \( (1) \)

- Maximal distance:
  \[
  \zeta_{\text{max}}(P) = \max_{p, p' \in P} \delta(p, p')
  \]  
  \( (2) \)

- Average distance:
  \[
  \zeta_{\text{avg}}(P) = \left( \frac{|P|}{2} \right)^{-1} \times \sum_{p, p' \in P} \delta(p, p')
  \]  
  \( (3) \)

where \( \delta : S \times S \rightarrow [0, 1] \) is the distance measures between two plans.

3.1.1. Distance measures between plans

There are various choices on how to define the distance measure \( \delta(p, p') \) between two plans using plan actions, sequence of states or causal links, and each way can have different impact on the diversity of plan set on the Hasse diagram. In the following, we propose distance measures in which a plan is considered as (i) a set of actions and causal links, or (ii) sequence of states the agent goes through, which could be used independently of plan representation (e.g. total order, partial order plans).

\footnote{To see this, consider a diagram with \( S_1 = \{p_1, p_2\} \) at layer 0, \( S_2 = \{p_3\} \) and \( S_3 = \{p_4\} \) at layer 1, and \( S_4 = \{p_5\} \) at layer 2. Assuming that we randomly select a set of 2 plans. If those plans are known to be at the same layer, then the chance of having one plan at layer 0 is \( \frac{1}{2} \). However, if they are forced to be at different layers, then the probability will be \( \frac{1}{4} \).}
- **Plan as a set of actions or causal links**: given a plan $p$, let $A(p)$ and $C(p)$ be the set of actions or causal links of $p$. The distance between two plans $p$ and $p'$ can be defined as the ratio of the number of actions (causal links) that do not appear in both plans to the total number of actions (causal links) appearing in one of them:

$$\delta_A(p, p') = 1 - \frac{|A(p) \cap A(p')|}{|A(p) \cup A(p')|}$$  \hfill (4)

$$\delta_{C_L}(p, p') = 1 - \frac{|C(p) \cap C(p')|}{|C(p) \cup C(p')|}$$ \hfill (5)

- **Plan as a sequence of states**: given two sequence of states $(s_0, s_1, ..., s_k)$ and $(s'_0, s'_1, ..., s'_k)$ resulting from executing two plans $p$ and $p'$, and assume that $k' \leq k$. Since the two sequence of states may have different length, there are various options in defining distance measure between $p$ and $p'$, and we consider here two simple options. In the first one, it can be defined as the average of the distances between state pairs $(s_i, s'_i)$ $(0 \leq i \leq k')$, and each state $s_{k'+1}, ..., s_k$ is considered to contribute maximally (i.e., one unit) into the difference between two plans:

$$\delta_S(p, p') = \frac{1}{k'} \times \sum_{i=1}^{k'} \Delta(s_i, s'_i) + k - k'$$ \hfill (6)

On the other hand, we can assume that the agent continues to stay at the goal state $s'_{k'}$ in the next $(k - k')$ time steps after executing $p'$, and the measure can be defined as follows:

$$\delta_S(p, p') = \frac{1}{k'} \times \left[ \sum_{i=1}^{k'} \Delta(s_i, s'_i) + \sum_{i=k'+1}^{k} \Delta(s_i, s'_{k'}) \right]$$ \hfill (7)

The distance measure $\Delta(s, s')$ between two states $s$, $s'$ used in those two measures is defined as

$$\Delta(s, s') = 1 - \frac{s \cap s'}{s \cup s'}$$ \hfill (8)

**Example**: Figure 4 shows three plans $p_1$, $p_2$ and $p_3$ for a planning problem where the initial state is $\{r_1\}$ and the goal propositions are $\{r_3, r_4\}$. The specification of actions are shown in the table. The action sets of the first two plans ($\{a_1, a_2, a_3\}$ and $\{a_1, a_2, a_4\}$) are quite similar ($\delta_A(p_1, p_2) = 0.5$), but the causal links which involve $a_3$ (a_2 \rightarrow r_3 - a_3$, $a_3 \rightarrow r_4 - a_4$) and $a_4$ (a_1 \rightarrow r_1 - a_4, a_4 \rightarrow r_4 - a_4$) make their difference more significant with respect to causal-link based distance ($\delta_{C_L}(p_1, p_2) = \frac{3}{5}$). Two other plans $p_1$ and $p_3$, on the other hand, are very different in terms of action sets (and therefore the sets of causal links): $\delta_A(p_1, p_3) = 1$, but they are closer in term of state-based distance (16/17 as defined in the equation 5) and 0.5 if defined in the equation 7.
3.2. Integrated Preference Function (IPF) for Partial Preference Cases

We now discuss a quality measure for plan sets in the case when the user’s preference is partially expressed. In particular, we consider scenarios in which the preference model can be represented by some quantitative language with an incompletely specified value function of high level features. As an example, the quality of plans in PDDL2.1 (Fox and Long, 2003) and PDDL3 (Gerevini and Long, 2005) are represented by a metric function combining metric fluents and preference statements on state trajectory with parameters representing their relative importance.

While providing a convenient way to represent preference models, such parameterized value functions present an issue of obtaining reasonable values for the relative importance of the features. A common approach to model this type of incomplete knowledge is to consider those parameters as a vector of random variables, whose values are assumed to be drawn from a distribution. This is the representation that we will follow.

To measure the quality of plan sets, we propose the usage of Integrated Preference Function (IPF) (Carlyle et al., 2003), which has been used to measure the quality of a solution set in a wide range of multi-objective optimization problems. The IPF measure assumes that the user’s preference model can be represented by two factors: (1) a probability distribution \( h(\alpha) \) of parameter vector \( \alpha \) such that \( \int h(\alpha) \, d\alpha = 1 \) (in the absence of any special information about the distribution, \( h(\alpha) \) can be assumed to be uniform), and (2) a value function \( V(p, \alpha) : S \to \mathbb{R} \) combines different objective functions into a single real-valued quality measure for plan \( p \). This incomplete specification of the value function represents a set of candidate preference models, for each of which the user will select a different plan, the one with the best value, from a given plan set \( P \subseteq S \). The IPF value of solution set \( P \) is defined as:

\[
IPF(P) = \int_\alpha h(\alpha)V(p_\alpha, \alpha) \, d\alpha
\]
with \( p_\alpha = \text{argmin}_{p \in P} V(p, \alpha) \) is the best solution according to \( V(p, \alpha) \) for each given \( \alpha \) value. Let \( p_\alpha^{-1} \) be its inverse function specifying a range of \( \alpha \) values for which \( p \) is an optimal solution according to \( V(p, \alpha) \). As \( p_\alpha \) is piecewise constant, the IPF(\( P \)) value can be computed as:

\[
\text{IPF}(P) = \sum_{p \in P} \left[ \int_{\alpha \in p_\alpha^{-1}} h(\alpha)V(p, \alpha) \, d\alpha \right].
\]  

(10)

Let \( \mathcal{P}^* = \{ p \in \mathcal{P} : p_\alpha^{-1} \neq \emptyset \} \) then we have:

\[
\text{IPF}(P) = \text{IPF}(\mathcal{P}^*) = \sum_{p \in \mathcal{P}^*} \left[ \int_{\alpha \in p_\alpha^{-1}} h(\alpha)V(p, \alpha) \, d\alpha \right].
\]  

(11)

Since \( \mathcal{P}^* \) is the set of plans that are optimal for some specific parameter vector, IPF(\( P \)) now can be interpreted as the expected value that the user can get by selecting the best plan in \( \mathcal{P} \). Therefore, the set \( \mathcal{P}^* \) of solutions (known as lower convex hull of \( \mathcal{P} \)) with the minimal IPF value is most likely to contain the desired solutions that the user wants and in essence a good representative of the plan set \( \mathcal{P} \).

While our work is applicable to more general planning scenarios, to make our discussion on generating plan sets concrete, we will concentrate on metric temporal planning where each action \( a \in A \) has a duration \( d_a \) and execution cost \( c_a \). The planner needs to find a plan \( p = \{a_1 \ldots a_n\} \), which is a sequence of actions that is executable and achieves all goals. The two most common plan quality measures are: makespan, which is the total execution time of \( p \); and plan cost, which is the total execution cost of all actions in \( p \)—both of them are high level features that can be affected by the actions in the plan. In most real-world applications, these two criteria compete with each other: shorter plans usually have higher cost and vice versa. We use the following assumptions:

- The desired objective function involves minimizing both components: \( \text{time}(p) \) measures the makespan of the plan \( p \) and \( \text{cost}(p) \) measures its execution cost.
- The quality of a plan \( p \) is a convex combination: \( V(p, w) = w \times \text{time}(p) + (1 - w) \times \text{cost}(p) \), where weight \( w \in [0, 1] \) represents the trade-off between the two competing objective functions.
- The belief distribution of \( w \) over the range \([0, 1] \) is known. If the user does not provide any information or we have not learnt anything about the preference on the trade-off between \( \text{time} \) and \( \text{cost} \) of the plan, then the planner can assume a uniform distribution (and improve it later using techniques such as preference elicitation).

Given that the exact value of \( w \) is unknown, our purpose is to find a bounded representative set of non-dominated plans, minimizing the expected value of \( V(p, w) \) with regard to the given distribution of \( w \) over \([0, 1] \).

**Example:** Figure 5 shows our running example in which there are a total of 7 plans with their \( \text{time}(p) \) and \( \text{cost}(p) \) values as follows: \( p_1 = \{4, 25\}, p_2 = \{6, 22\}, p_3 = \{7, 15\}, p_4 = \{8, 20\}, \)

\[^6\text{A plan } p_1 \text{ is dominated by } p_2 \text{ if } \text{time}(p_1) \geq \text{time}(p_2) \text{ and } \text{cost}(p_1) \geq \text{cost}(p_2) \text{ and at least one of the inequalities is strict.}\]
$p_5 = \{10, 12\}$, $p_6 = \{11, 14\}$, and $p_7 = \{12, 5\}$. Among these 7 plans, 5 of them belong to a pareto optimal set of non-dominated plans: $P_p = \{p_1, p_2, p_3, p_5, p_7\}$. The other two plans are dominated by some plans in $P_p$: $p_3$ is dominated by $p_2$ and $p_6$ is dominated by $p_5$. Plans in $P_p$ are depicted in solid dots, and the set of plans $P^* = \{p_1, p_3, p_7\}$ that are optimal for some specific value of $w$ is highlighted by connected dots.

**IPF for Metric Temporal Planning:** The user preference model in our target domain of temporal planning is represented by a convex combination of the *time* and *cost* quality measures, and the IPF measure now is called *Integrated Convex Preference* (ICP). Given a set of plans $P^*$, let $t_p = \text{time}(p)$ and $c_p = \text{cost}(p)$ be the makespan and total execution cost of plan $p \in P^*$, the ICP value of $P^*$ with regard to the objective function $V(p, w) = w \times t_p + (1 - w) \times c_p$ and the parameter vector $\alpha = (w, 1 - w) (w \in [0, 1])$ is defined as:

$$ICP(P^*) = \sum_{i=1}^{k} \int_{w_{i-1}}^{w_i} h(w)(w \times t_{p_i} + (1 - w) \times c_{p_i})dw$$  \hspace{1cm} (12)$$

where $w_0 = 0$, $w_k = 1$ and $p_i = \arg\min_{p \in P^*} V(p, w)$ $\forall w \in [w_{i-1}, w_i]$. In other words, we divide $[0, 1]$ into non-overlapping regions such that in each region $(w_{i-1}, w_i)$ there is a single solution $p_i \in P^*$ that has better $V(p_i, w)$ value than all other solutions in $P^*$.

We select the IPF/ICP measure to evaluate our solution set due to its several nice properties:

- If $P_1, P_2 \subseteq S$ and $ICP(P_1) < ICP(P_2)$ then $P_1$ is probabilistically better than $P_2$ in the sense that for any given $w$, let $p_1 = \arg\min_{p \in P_1} V(p, w)$ and $p_2 = \arg\min_{p \in P_2} V(p, w)$, then the probability of $V(p_1, w) < V(p_2, w)$ is higher than the probability of $V(p_1, w) > V(p_2, w)$.

- If $P_1$ is obviously better than $P_2$, then the ICP measure agrees with the assertion. More formally: if $\forall p_2 \in P_2$, $\exists p_1 \in P_1$ such that $p_2$ is dominated by $p_1$, then $ICP(P_1) < ICP(P_2)$. 

Figure 5: Solid dots represents plans in the pareto set ($p_1, p_2, p_3, p_5, p_7$). Connected dots represent plans in the lower convex hull ($p_1, p_3, p_7$) giving optimal ICP value for any distribution on trade-off between *cost* and *time*.
Empirically, extensive results on scheduling problems in [Fowler et al., 2005] have shown that ICP measure "evaluates the solution quality of approximation robustly (i.e., similar to visual comparison results) while other alternative measures can misjudge the solution quality".

In the next two sections 4 and 5, we investigate the problem of generating high quality plan sets for two cases mentioned: when no knowledge about the user’s preferences is given, and when part of it is given as input to the planner.

4. Generating Diverse Plan Set in the Absence of Preference Knowledge

In this section, we describe approaches to searching for a set of diverse plans with respect to a measure defined with base level elements of plans as discussed in the previous section. In particular, we consider the quality measure of plan set as the minimal pair-wise distance between any two plans, and generate a set of plans containing \( k \) plans with the quality of at least a predefined threshold \( d \). As discussed earlier, by diversifying the set of plans on the space of base level features, it is likely that plans in the set would cover a wide range of space of unknown high level features, increasing the possibility that the user can select a plan close to the one that she prefers.

The problem is formally defined as follows:

\[
d\text{DISTANT}k\text{SET} : \text{Find } \mathcal{P} \text{ with } \mathcal{P} \subseteq \mathcal{S}, \ |\mathcal{P}\ | = k \text{ and } \zeta(\mathcal{P}) = \min_{p,q \in \mathcal{P}} \delta(p, q) \geq d
\]

where any distance measure between two plans formalized in Section 3.1.1 can be used to implement \( \delta(p, p') \).

We now consider two representative state-of-the-art planning approaches in generating diverse plan sets. The first one is GP-CSP ([Do and Kambhampati, 2001]) representing constraint-based planning approaches, and the second one is LPG ([Gerevini et al., 2003]) that uses an efficient local-search based approach. We use GP-CSP to comparing the relation between different distance measures in diversifying plan sets. On the other hand, with LPG we stick to the action-based distance measure, which is shown experimentally to be the most difficult measure to enforce diversity (see below), and investigate the scalability of heuristic approaches in generating diverse plans.

4.1. Finding Diverse Plan Set with GP-CSP

The GP-CSP planner ([Do and Kambhampati, 2001]) converts Graphplan’s planning graph into a CSP encoding, and solves it using a standard CSP solver. The solution of the encoding represents a valid plan for the original planning problem. In the encoding, the CSP variables correspond to the predicates that have to be achieved at different levels in the planning graph (different planning steps) and their possible values are the actions that can support the predicates. For each CSP variable representing a predicate \( p \), there are two special values: i) \( \perp \): indicates that a predicate is not supported by any action and is false at a particular level/planning-step; ii) “noop”: indicates that the predicate is true at a given level \( i \) because it was made true at some previous level \( j < i \) and no other action deletes \( p \) between \( j \) and \( i \). Constraints encode the relations between predicates and actions: 1) mutual exclusion relations between predicates and actions; and 2) the causal relationships between actions and their preconditions.
4.1.1. Adapting GP-CSP to Different Distance Metrics

When the above planning encoding is solved by any standard CSP solver, it will return a solution containing \( \langle \text{var}, \text{value} \rangle \) of the form \( \{ (x_1, y_1), ...(x_n, y_n) \} \). The collection of \( x_i \) where \( y_i \neq \perp \) represents the facts that are made true at different time steps (plan trajectory) and can be used as a basis for the state-based distance measure\(^7\) the set of \( (y_i \neq \perp) \land (y_i \neq \text{noop}) \) represents the set of actions in the plan and can be used for action-based distance measure; lastly, the assignments \( \langle x_i, y_i \rangle \) themselves represent the causal relations and can be used for the causal-based distance measure.

However, there are some technical difficulties we need to overcome before a specific distance measure between plans can be computed. First, the same action can be represented by different values in the domains of different variables. Consider a simple example in which there are two facts \( p \) and \( q \), both supported by two actions \( a_1 \) and \( a_2 \). When setting up the CSP encoding, we assume that the CSP variables \( x_1 \) and \( x_2 \) are used to represent \( p \) and \( q \). The domains for \( x_1 \) and \( x_2 \) are \( \{ v_{11}, v_{12} \} \) and \( \{ v_{21}, v_{22} \} \), both representing the two actions \( \{ a_1, a_2 \} \) (in that order). The assignments \( \{ \langle x_1, v_{11} \rangle, \langle x_2, v_{21} \rangle \} \) and \( \{ \langle x_1, v_{12} \rangle, \langle x_2, v_{22} \rangle \} \) have a distance of 2 in traditional CSP because different values are assigned for each variable \( x_1 \) and \( x_2 \). However, they both represent the same action set \( \{ a_1, a_2 \} \) and thus lead to the plan distance of 0 if we use the action-based distance in our plan comparison. Therefore, we first need to translate the set of values in all assignments back to the set of action instances before doing comparison using action-based distance. The second complication arises for the causal-based distance. A causal link \( a_1 \xrightarrow{p} a_2 \) between two actions \( a_1 \) and \( a_2 \) indicates that \( a_1 \) supports the precondition \( p \) of \( a_2 \). However, the CSP assignment \( (p, a_1) \) only provides the first half of each causal-link. To complete the causal-link, we need to look at the values of other CSP assignments to identify action \( a_2 \) that occurs at the later level in the planning graph and has \( p \) as its precondition. Note that there may be multiple “valid” sets of causal-links for a plan, and in the implementation we simply select causal-links based on the CSP assignments.

4.1.2. Making GP-CSP Return a Set of Plans

To make GP-CSP return a set of plans satisfying the \( d\text{DISTANT}\&\text{SET} \) constraint using one of the three distance measures, we add “global” constraints to each original encoding to enforce \( d \)-diversity between every pair of solutions. When each global constraint is called upon by the normal forward checking and arc-consistency checking procedures inside the default solver to check if the distance between two plans is over a predefined value \( d \), we first map each set of assignments to an actual set of actions (action-based), predicates that are true at different plan-steps (state-based) or causal-links (causal-based) using the method discussed in the previous section. This process is done by mapping all \( \langle \text{var}, \text{value} \rangle \) CSP assignments into action sets using a call to the planning graph, which is outside of the CSP solver, but works closely with the general purpose CSP solver in GP-CSP. The comparison is then done within the implementation of the global constraint to decide if two solutions are diverse enough.

We investigate two different ways to use the global constraints:

1. The parallel strategy to return the set of \( k \) plans all at once. In this approach, we create one encoding that contains \( k \) identical copies of each original planning encoding created using GP-CSP planner. The \( k \) copies are connected together using \( k(k-1)/2 \) pair-wise

\(^7\)We implement the state-based distance between plans as defined in equation
global constraints. Each global constraint between the \(i^{th}\) and \(j^{th}\) copies ensures that two plans represented by the solutions of those two copies will be at least \(d\) distant from each other. If each copy has \(n\) variables, then this constraint involves \(2n\) variables.

2. The greedy strategy to return plans one after another. In this approach, the \(k\) copies are not setup in parallel up-front, but sequentially. We add to the \(i^{th}\) copy one global constraint to enforce that the solution of the \(i^{th}\) copy should be \(d\)-diverse from any of the earlier \(i-1\) solutions. The advantage of the greedy approach is that each CSP encoding is significantly smaller in terms of the number of variables \((n \times k)\), smaller in terms of the number of global constraints \((1 \times k(k-1)/2)\), and each global constraint also contains lesser number of variables \((n \times 2n)\).

Thus, each encoding in the greedy approach is easier to solve. However, because each solution depends on all previously found solutions, the encoding can be unsolvable if the previously found solutions comprise a bad initial solution set.

4.1.3. Empirical Evaluation

We implemented the parallel and greedy approaches discussed earlier for the three distance measures and tested them with the benchmark set of Logistics problems provided with the Blackbox planner [Kautz and Selman 1998]. All experiments were run on a Linux Pentium 4, 3Ghz machine with 512MB RAM. For each problem, we test with different \(d\) values ranging from 0.01 (1\%) to 0.95 (95\%) and \(k\) increases from 2 to \(n\) where \(n\) is the maximum value for which GP-CSP can still find solutions within plan horizon. The horizon (parallel plan steps) limit is 30.

We found that the greedy approach outperformed the parallel approach and solved significantly higher number of problems. Therefore, we focus on the greedy approach hereafter. For each combination of \(d\), \(k\), and a given distance measure, we record the solving time and output the average/min/max pairwise distances of the solution sets.

Baseline Comparison: As a baseline comparison, we have also implemented a randomized approach. In this approach, we do not use global constraints but use random value ordering in the CSP solver to generate \(k\) different solutions without enforcing them to be pairwise \(d\)-distance apart. For each distance \(d\), we continue running the random algorithm until we find \(k_{max}\) solutions where \(k_{max}\) is the maximum value of \(k\) that we can solve for the greedy approach for that particular \(d\) value. In general, we want to compare with our approach of using global constraint to see if the random approach can effectively generate diverse set of solutions by looking at: (1) the average time to find a solution in the solution set; and (2) the maximum/average pairwise distances between \(k\geq2\) randomly generated solutions.

Table 2 shows the comparison of average solving time to find one solution in the greedy and random approaches. The results show that on an average, the random approach takes significantly more time to find a single solution, regardless of the distance measure used by the greedy approach. To assess the diversity in the solution sets, Table 3 shows the comparison of: (1) the average pairwise minimum distance between the solutions in sets returned by the random approach; and (2) the maximum \(d\) for which the greedy approach still can find a set of diverse plans. The comparisons are done for all three distance measures. For example, the first cell (0.041/0.35) in Table 3 implies that the minimum pairwise distance averaged for all solvable
\[
\begin{array}{ccccccc}
\text{Prob1} & \text{Prob2} & \text{Prob3} & \text{Prob4} & \text{Prob5} & \text{Prob6} \\
\hline
\delta_a & 0.087 & 7.648 & 1.021 & 6.144 & 8.083 & 178.633 \\
\delta_s & 0.077 & 9.354 & 1.845 & 6.312 & 8.667 & 232.475 \\
\delta_c & 0.190 & 6.542 & 1.063 & 6.314 & 8.437 & 209.287 \\
\text{Random} & 0.327 & 15.480 & 8.982 & 88.040 & 379.182 & 6105.510 \\
\end{array}
\]

Table 2: Average solving time (in seconds) to find a plan using greedy (first 3 rows) and by random (last row) approaches.

\[
\begin{array}{ccccccc}
\text{Prob1} & \text{Prob2} & \text{Prob3} & \text{Prob4} & \text{Prob5} & \text{Prob6} \\
\hline
\delta_a & 0.041/0.35 & 0.067/0.65 & 0.067/0.25 & 0.131/0.1* & 0.126/0.15 & 0.128/0.2 \\
\delta_s & 0.035/0.4 & 0.05/0.8 & 0.096/0.5 & 0.147/0.4 & 0.140/0.5 & 0.101/0.5 \\
\delta_c & 0.158/0.8 & 0.136/0.95 & 0.256/0.35 & 0.459/0.15* & 0.346/0.3* & 0.349/0.45 \\
\end{array}
\]

Table 3: Comparison of the diversity in the plan sets returned by the random and greedy approaches. Cases where random approach is better than greedy approach are marked with (*).

For \(k \geq 2\) using the random approach is \(d = 0.041\) while it is 0.35 (i.e. 8x more diverse) for the greedy approach using the \(\delta_a\) distance measure. Except for 3 cases, using global constraints to enforce minimum pairwise distance between solutions helps \(GP\text{-}CSP\) return significantly more diverse set of solutions. On average, the greedy approach returns 4.25x, 7.31x, and 2.79x more diverse solutions than the random approach for \(\delta_a\), \(\delta_s\) and \(\delta_c\), respectively.

Analysis of the different distance-bases: Overall, we were able to solve 1264 \((d, k)\) combinations for three distance measures \(\delta_a\), \(\delta_s\), and \(\delta_c\) using the greedy approach. We were particularly interested in investigating the following issues:

- **H1: Computational efficiency** - Is it easy or difficult to find a set of diverse solutions using different distance measures? Thus, (1) for the same \(d\) and \(k\) values, which distance measure is more difficult (time consuming) to solve; and (2) given an encoding horizon limit, how high is the value of \(d\) and \(k\) for which we can still find a set of solutions for a given problem using different distance measures.

- **H2: Solution diversity** - What, if any, is the correlation/sensitivity between different distance measures? Thus, how comparative diversity of solutions is when using different distance measures.

Regarding **H1**, Table 4 shows the highest solvable \(k\) value for each distance \(d\) and base \(\delta_a\), \(\delta_s\), and \(\delta_c\). For a given \((d, k)\) pair, enforcing \(\delta_a\) appears to be the most difficult, then \(\delta_s\), and \(\delta_c\) is the easiest. \(GP\text{-}CSP\) is able to solve 237, 462, and 565 combinations of \((d, k)\) respectively for \(\delta_a\), \(\delta_s\) and \(\delta_c\). \(GP\text{-}CSP\) solves \(d\)DISTANT\(k\)SET problems more easily with \(\delta_a\) and \(\delta_c\) than with \(\delta_s\) due to the fact that solutions with different action sets (diverse with regard to \(\delta_a\)) will likely cause different trajectories and causal structures (diverse with regard to \(\delta_s\) and \(\delta_c\)). Between \(\delta_s\) and \(\delta_c\), \(\delta_c\) solves more problems for easier instances (Problems 1-3) but less for the harder instances, as shown in Table 4. We conjecture that for solutions with more actions (i.e. in bigger problems) there are more causal dependencies between actions and thus it is harder to reorder actions to create a different causal-structure.

For running time comparisons, among 216 combinations of \((d, k)\) that were solved by all three distance measures, \(GP\text{-}CSP\) takes the least amount of time for \(\delta_a\) in 84 combinations, for \(\delta_s\)
in 70 combinations and in 62 for $\delta_c$. The first three lines of Table 4 show the average time to find one solution in $d$-diverse $k$-set for each problem using $\delta_a$, $\delta_s$ and $\delta_c$ (which we call $t_a$, $t_s$ and $t_c$ respectively). In general, $t_a$ is the smallest and $t_s > t_c$ in most problems. Thus, while it is harder to enforce $\delta_a$ than $\delta_s$ and $\delta_c$ (as indicated in Table 4), when the encodings for all three distances can be solved for a given $(d,k)$, then $\delta_a$ takes less time to search for one plan in the diverse plan set; this can be due to tighter constraints (more pruning power for the global constraints) and simpler global constraint setting.

To test $H2$, in Table 5, we show the cross-comparison between different distance measures $\delta_a$, $\delta_s$, and $\delta_c$. In this table, cell $(row, column) = (\delta', \delta'')$ indicates that over all combinations of $(d,k)$ solved for distance $\delta'$, the average value $d''/d'$ where $d''$ and $d'$ are distance measured according to $\delta''$ and $\delta'$ respectively ($d' \geq d$). For example, $(\delta_a, \delta_a) = 0.485$ means that over 462 combinations of $(d,k)$ solvable for $\delta_a$, for each $d$, the average distance between $k$ solutions measured by $\delta_a$ is $0.485 \times d$. The results indicate that when we enforce $d$ for $\delta_a$, we will likely find even more diverse solution sets according to $\delta_a (1.26 \times d_a)$ and $\delta_c (1.98 \times d_a)$. However, when we enforce $d$ for either $\delta_s$ or $\delta_c$, we are not likely to find a more diverse set of solutions measured by the other two distance measures. Nevertheless, enforcing $d$ using $\delta_c$ will likely give comparable diverse degree $d$ for $\delta_s (0.94 \times d_s)$ and vice versa. We also observe that $d_s$ is highly dependent on the difference between the parallel lengths of plans in the set. The distance $d_s$ seems to be the smallest (i.e. $d_s < d_a < d_c$) when all $k$ plans have the same/similar number of time steps. This is consistent with the fact that $\delta_a$ and $\delta_c$ do not depend on the steps in the plan execution trajectory while $\delta_s$ does.

### Table 4: For each given $d$ value, each cell shows the largest solvable $k$ for each of the three distance measures $\delta_a$, $\delta_s$, and $\delta_c$ (in this order). The maximum values in cells are in bold.

| $d$ | $\delta_a$ | $\delta_s$ | $\delta_c$ |
|-----|------------|------------|------------|
| 0.01 | 11.5, 28   | 9.18, 12   | 3.45, 5    |
| 0.03 | 6.3, 24    | 5.13, 9    | 2.4, 3     |
| 0.05 | 5.3, 18    | 5.10, 7    | 2.4, 3     |
| 0.07 | 2.3, 14    | 4.10, 8    | 2.4, 2     |
| 0.09 | 2.3, 14    | 6.9, 6     | 3.4, 2     |
| 0.1  | 2.3, 10    | 6.3, 6     | 2.4, 2     |
| 0.2  | 2.3, 5     | 2.9, 6     | 2.6, 6     |
| 0.3  | 2.2, 3     | 4.7, 5     | 1.4, 4     |
| 0.4  | 1.2, 3     | 3.6, 5     | 1.3, 2     |
| 0.5  | 1.1, 3     | 2.4, 5     | 1.2, 2     |
| 0.6  | 1.1, 2     | 2.3, 4     | -          |
| 0.7  | 1.1, 2     | 1.2, 2     | -          |
| 0.8  | 1.1, 2     | 1.2, 2     | -          |
| 0.9  | -          | 1.1, 2     | -          |

### Table 5: Cross-validation of distance measures $\delta_a$, $\delta_s$, and $\delta_c$.

| $\delta_a$ | $\delta_s$ | $\delta_c$ |
|------------|------------|------------|
| $\delta_a$ | -          | 1.262, 1.985 |
| $\delta_s$ | 0.485      | -          | 0.883 |
| $\delta_c$ | 0.461, 0.938 | -          |
4.2. Finding Diverse Plan Set with LPG

In this section, we consider the problem of generating diverse set of plans using another planning approach, in particular the LPG planner which is able to scale up to bigger problems, compared to GP-CSP. We focus on the action-based distance measure between plans, which has been shown in the previous section to be the most difficult to enforce diversity. LPG is a local-search-based planner, that incrementally modifies a partial plan in a search for a plan that contains no flaws (Gerevini et al., 2003). The behavior of LPG is controlled by an evaluation function that is used to select between different plan candidates in a neighborhood generated for local search. At each search step, the elements in the search neighborhood of the current partial plan \( \pi \) are the alternative possible plans repairing a selected flaw in \( \pi \). The elements of the neighborhood are evaluated according to an action evaluation function \( E \) (Gerevini et al., 2003). This function is used to estimate the cost of either adding or of removing an action node \( a \) in the partial plan \( p \) being generated.

4.2.1. Revised Evaluation Function for Diverse Plans

In order to manage \( d \)DISTANCE\(_k\)SET problems, the function \( E \) has been extended to include an additional evaluation term that has the purpose of penalizing the insertion and removal of actions that decrease the distance of the current partial plan \( p \) under adaptation from a reference plan \( p_0 \). In general, \( E \) consists of four weighted terms, evaluating four aspects of the quality of the current plan that are affected by the addition (\( E(a)^i \)) or removal (\( E(a)^r \)) of \( a \):

\[
E(a)^i = \alpha_E \cdot \text{Execution cost}(a)^i + \alpha_T \cdot \text{Temporal cost}(a)^i + \alpha_S \cdot |(p_0 - p) \cap p_R^i| + \alpha_D \cdot |(p_0 - p) \cap p_R^i|.
\]

\[
E(a)^r = \alpha_E \cdot \text{Execution cost}(a)^r + \alpha_T \cdot \text{Temporal cost}(a)^r + \alpha_S \cdot |(p_0 - p - a) \cap p_R^r| + \alpha_D \cdot |(p_0 - p - a) \cap p_R^r|.
\]

The first three terms of the two forms of \( E \) are unchanged from the standard behavior of LPG. The fourth term, used only for computing diverse plans, is the new term estimating how the proposed plan modification will affect the distance from the reference plan \( p_0 \). Each cost term in \( E \) is computed using a relaxed temporal plan \( p_R \) (Gerevini et al., 2003).

The \( p_R \) plans are computed by an algorithm, called RelaxedPlan, formally described and illustrated in (Gerevini et al., 2003). We have slightly modified this algorithm to penalize the selection of actions decreasing the plan distance from the reference plan. The specific change to RelaxedPlan for computing diverse plans is very similar to the change described in (Fox et al., 2006), and it concerns the heuristic function for selecting the actions for achieving the subgoals in the relaxed plans. In the modified function for RelaxedPlan, we have an extra 0/1 term that penalizes an action \( b \) for \( p_R \) if its addition decreases the distance of \( p + p_R \) from \( p_0 \) (in the plan repair context investigated in (Fox et al., 2006), \( b \) is penalized if its addition increases such a distance).

The last term of the modified evaluation function \( E \) is a measure of the decrease in plan distance caused by adding or removing \( a: |(p_0 - p) \cap p_R^i| \) or \( |(p_0 - p - a) \cap p_R^r| \), where \( p_R^i \) contains the new action \( a \). The \( \alpha \)-coefficients of the \( E \)-terms are used to weigh their relative importance.\[1\]

---

\[1\] These coefficients are also normalized to a value in \([0, 1]\) using the method described in (Gerevini et al., 2003).
The values of the first 3 terms are automatically derived from the expression defining the plan metric for the problem (Gerevini et al., 2003). The coefficient for the fourth new term of \( E(\alpha_D) \) is automatically set during search to a value proportional to \( d/\delta_{a}(p, p_0) \), where \( p \) is the current partial plan under construction. The general idea is to dynamically increase the value of \( \alpha_D \) according to the number of plans \( n \) that have been generated so far: if \( n \) is much higher than \( k \), the search process consists of finding many solutions with not enough diversification, and hence the importance of the last \( E \)-term should increase.

4.2.2. Making LPG Return a Set of Plans
In order to compute a set of \( k \) \( d \)-distant plans solving a \textit{distance}\_\textit{set} problem, we run the LPG search multiple times, until the problem is solved, with the following two additional changes to the standard version of LPG: (i) the preprocessing phase computing mutex relations and other reachability information exploited during the relaxed plan construction is done only once for all runs; (ii) we maintain an incremental set of valid plans, and we dynamically select one of them as the reference plan \( p_0 \) for the next search. Concerning (ii), let \( \mathcal{P} = \{ p_1, ..., p_n \} \) be the set of \( n \) valid plans that have been computed so far, and \( CPlans(p_i) \) the subset of \( \mathcal{P} \) containing all plans that have a distance greater than or equal to \( d \) from a reference plan \( p_i \in \mathcal{P} \).

The reference plan \( p_0 \) used in the modified heuristic function \( E \) is a plan \( p_{max} \in \mathcal{P} \) which has a maximal set of diverse plans in \( \mathcal{P} \), i.e.,

\[
p_{max} = \text{ARGMAX}_{\{ p_i \in \mathcal{P} \}} \{|CPlans(p_i)|\}.
\] (13)

The plan \( p_{max} \) is incrementally computed each time the local search finds a new solution. In addition to being used to identify the reference plan in \( E \), \( p_{max} \) is also used for defining the initial state (partial plan) of the search process. Specifically, we initialize the search using a (partial) plan obtained by randomly removing some actions from a (randomly selected) plan in the set \( CPlans(p_{max}) \cup \{ p_{max} \} \).

The process of generating diverse plans starting from a dynamically chosen reference plan continues until at least \( k \) plans that are all \( d \)-distant from each other have been produced. The modified version of LPG to compute diverse plans is called LPG-d.

4.2.3. Experimental Analysis with LPG-d
Recall that the distance function \( \delta_a \), using set-difference, can be written as the sum of two terms:

\[
\delta_a(p_i, p_j) = \frac{|A(p_i) - A(p_j)|}{|A(p_i) \cup A(p_j)|} + \frac{|A(p_j) - A(p_i)|}{|A(p_i) \cup A(p_j)|}
\] (14)

The first term represents the contribution of the actions in \( p_i \) to the plan difference, while the second term indicates the contribution of \( p_j \) to \( \delta_a \). We experimentally observed that in some cases the differences between two diverse plans computed using \( \delta_a \) are mostly concentrated in only one of the \( \delta_a \) components. This asymmetry means that one of the two plans can have many more actions than the other one, which could imply that the quality of one of the two plans is much worse than the quality of the other plan. In order to avoid this problem, we can parametrize \( \delta_a \) by imposing the two extra constraints

\[
\delta^A_a \geq d/\gamma \text{ and } \delta^B_a \geq d/\gamma
\]
where $\delta^A_d$ and $\delta^B_d$ are the first and second terms of $\delta_d$, respectively, and $\gamma$ is an integer parameter “balancing” the diversity of $p_i$ and $p_j$.

In this section, we analyze the performance of the modified version of LPG, called LPG-d, in three different benchmark domains from the 3rd and 5th IPCs. The main goals of the experimental evaluation were (i) showing that LPG-d can efficiently solve a large set of $(d, k)$-combinations, (ii) investigating the impact of the $\delta_d \gamma$-constraints on performance, (iii) comparing LPG-d and the standard LPG.

We tested LPG-d using both the default and parametrized versions of $\delta_d$, with $\gamma = 2$ and $\gamma = 3$. We give detailed results for $\gamma = 3$ and a more general evaluation for $\gamma = 2$ and the original $\delta_d$. We consider $d$ that varies from 0.05 to 0.95, using 0.05 increment step, and with $k = 2...5, 6, 8, 10, 12, 14, 16, 20, 24, 28, 32$ (overall, a total of 266 $(d, k)$-combinations). Since LPG-d is a stochastic planner, we use the median of the CPU times (in seconds) and the median of the average plan distances (over five runs). The average plan distance for a set of $k$ plans
solving a specific \((d, k)\)-combination \((\delta^{av})\) is the average of the plans distances between all pairs of plans in the set. The tests were performed on an AMD Athlon(tm) XP 2600+, 512 Mb RAM. The CPU-time limit was 300 seconds.

Figure 6 gives the results for the largest problem in IPC-3 DriverLog-Time (fully-automated track). LPG-d solves 109 \((d, k)\)-combinations, including combinations \(d \leq 0.4\) and \(k = 10\), and \(d = 0.95\) and \(k = 2\). The average CPU time (top plots) is 162.8 seconds. The average \(\delta^{av}\) (bottom plots) is 0.68, with \(\delta^{av}\) always greater than 0.4. With the original \(\delta_a\) function LPG-d solves 110 \((d, k)\)-combinations, the average CPU time is 160 seconds, and the average \(\delta^{av}\) is 0.68; while with \(\gamma = 2\) LPG-d solves 100 combinations, the average CPU time is 169.5 seconds, and the average \(\delta^{av}\) is 0.69.

Figure 7 shows the results for the largest problem in IPC-3 Satellite-Strips. LPG-d solves 211 \((k, d)\)-combinations; 173 of them require less than 10 seconds. The average CPU time is 12.1 seconds, and the average \(\delta^{av}\) is 0.69. We observed similar results when using the original
\[ \delta_{\omega} \text{ function or the parametrized } \delta_{\omega} \text{ with } \gamma = 2 \text{ (in the second case, LPG-d solves 198 problems, while the average CPU time and the average } \delta_{av} \text{ are nearly the same as with } \gamma = 3). \]

Figure 8 shows the results for a middle-size problem in IPC-5 Storage-Propositional. With \( \gamma = 2 \), LPG-d solves 225 \((k, d)\)-combinations, 39 of which require less than 10 seconds, while 128 of them require less than 50 seconds. The average CPU time is 64.1 seconds and the average \( \delta_{av} \) is 0.88. With the original \( \delta_{\omega} \), LPG-d solves 240 \((k, d)\)-combinations, the average CPU time is 41.8 seconds, and the average \( \delta_{av} \) is 0.87; with \( \gamma = 3 \), LPG-d solves 206 combinations, the average CPU time is 69.4 seconds and the average \( \delta_{av} \) is 0.89.

The local search in LPG is randomized by a “noise” parameter that is automatically set and updated during search (Gerevini et al., 2003). This randomization is one of the techniques used for escaping local minima, but it also can be useful for computing diverse plans: if we run the search multiple times, each search is likely to consider different portions of the search space,
which can lead to different solutions. It is then interesting to compare LPG-d and a method in which we simply run the standard LPG until \( k \)-diverse plans are generated. An experimental comparison of the two approaches show that in many cases LPG-d performs better. In particular, the new evaluation function \( E \) is especially useful for planning problems that are easy to solve for the standard LPG, and that admit many solutions. In these cases, the original \( E \) function produces many valid plans with not enough diversification. This problem is significantly alleviated by the new term in \( E \). An example of domain where we observed this behavior is logistics\(^{12}\).

5. Generating Plan Sets with Partial Preference Knowledge

In this section, we consider the problem of generating plan sets when the user’s preferences are only partially expressed. In particular, we focus on metric temporal planning where the preference model is assumed to be represented by an incomplete value function specified by a convex combination of two features: plan makespan and execution cost, with the exact trade-off value \( w \) drawn from a given distribution. The quality value of plan sets is measured by the ICP value, as formalized in Equation\(^{12}\). Our objective is to find a set of plans \( P \subseteq S \) where \(|P| \leq k\) and \( ICP(P) \) is the lowest.

Notice that we restrict the size of the solution set returned, not only for the comprehension issue discussed earlier, but also for an important property of the ICP measure: it is a monotonically non-increasing function of the solution set (specifically, given two solution sets \( P_1 \) and \( P_2 \) such that the latter is a superset of the former, it is easy to see that \( ICP(P_2) \leq ICP(P_1) \)).

5.1. Sampling Weight Values

Given that the distribution of trade-off value \( w \) is known, the straightforward way to find a set of representative solutions is to first sample a set of \( k \) values for \( w \): \( \{w_1, w_2, \ldots, w_k\} \) based on the distribution \( b(w) \). For each value \( w_i \), we can find an (optimal) plan \( p_i \) minimizing the value of the overall value function \( V(p, w_i) = w_i \times t_p + (1 - w_i) \times c_p \). The final set of solutions \( P = \{p_1, p_2, \ldots, p_k\} \) is then filtered to remove duplicates and dominated solutions, thus selecting the plans making up the lower-convex hull. The final set can then be returned to the user. While intuitive and easy to implement, this sampling-based approach has several potential flaws that can limit the quality of its resulting plan set.

First, given that \( k \) solution plans are searched sequentially and independently of each other, even if the plan \( p_i \) found for each \( w_i \) is optimal, the final solution set \( P = \{p_1, p_2, \ldots, p_k\} \) may not even be the optimal set of \( k \) solutions with regard to the ICP measure. More specifically, for a given set of solutions \( P \), some tradeoff value \( w \), and two non-dominated plans \( p, q \) such that \( V(p, w) < V(q, w) \), it is possible that \( ICP(P \cup \{p\}) > ICP(P \cup \{q\}) \). In our running example (Figure 5), let \( P = \{p_2, p_5\} \) and \( w = 0.8 \) then \( V(p_1, w) = 0.8 \times 4 + 0.2 \times 25 = 8.2 < V(p_2, w) = 0.8 \times 12 + 0.2 \times 5 = 10.6 \). Thus, the planner will select \( p_1 \) to add to \( P \) because it looks locally better given the weight \( w = 0.8 \). However, \( ICP(\{p_1, p_2, p_5\}) \approx 10.05 > ICP(\{p_2, p_5, p_7\}) \approx 7.71 \) so indeed by taking previous set into consideration then \( p_7 \) is a much better choice than \( p_1 \).

\(^{12}\)E.g., for logistics (prob3 of Table 2) LPG-d solved 128 instances, 41 of them in less than 1 CPU second and 97 of them in less than 10 CPU seconds; the average CPU time was 16.7 seconds and the average \( \delta^{av} \) was 0.38. While using the standard LPG, only 78 instances were solved, 20 of them in less than 1 CPU seconds and 53 of them in less than 10 CPU seconds; the average CPU time was 23.6 seconds and the average \( \delta^{av} \) was 0.27.
Algorithm 1: Incrementally find solution set \( \mathcal{P} \)

1. **Input:** A planning problem with a solution space \( \mathcal{S} \); maximum number of plans required \( k \); number of sampled trade-off values \( k_0 \) \((0 < k_0 < k)\); time bound \( t \);
2. **Output:** A plan set \( \mathcal{P} \) \(|\mathcal{P}| \leq k \);
3. **begin**
4. \( W \leftarrow \) sample \( k_0 \) values for \( w \);
5. \( \mathcal{P} \leftarrow \) find good quality plans in \( \mathcal{S} \) for each \( w \in W \);
6. **while** \(|\mathcal{P}| < k \) and search time \( t < \) do
7. \( \) Search for \( p \) s.t. \( ICP(\mathcal{P} \cup \{p\}) < ICP(\mathcal{P}) \)
8. \( \mathcal{P} \leftarrow \mathcal{P} \cup \{p\} \)
9. **end**
10. **Return** \( \mathcal{P} \)
11. **end**

Second, the values of the trade-off parameter \( w \) are sampled based on a given distribution, and independently of the particular planning problem being solved. As there is no relation between the sampled \( w \) values and the solution space of a given planning problem, sampling approach may return very few distinct solutions even if we sample a large number of weight values \( w \). In our example, if all \( w \) samples have values \( w \leq 0.67 \) then the optimal solution returned for any of them will always be \( p_7 \). However, we know that \( \mathcal{P}^* = \{p_1, p_3, p_7\} \) is the optimal set according to the \( ICP \) measure. Indeed, if \( w \leq 0.769 \) then the sampling approach can only find the set \( \{p_7\} \) or \( \{p_3, p_7\} \) and still not be able to find the optimal set \( \mathcal{P}^* \).

5.2. ICP Sequential Approach

Given the potential drawbacks of the sampling approach outlined above, we also pursued an alternative approach that takes into account the ICP measure more actively. Specifically, we incrementally build the solution set \( \mathcal{P} \) by finding a solution \( p \) such that \( \mathcal{P} \cup \{p\} \) has the lowest ICP value. We can start with an empty solution set \( \mathcal{P} = \emptyset \), then at each step try to find a new plan \( p \) such that \( \mathcal{P} \cup \{p\} \) has the lowest ICP value.

While this approach directly takes the ICP measure into consideration at each step of finding a new plan and avoids the drawbacks of the sampling-based approach, it also has its own share of potential flaws. Given that the set is built incrementally, the earlier steps where the first “seed” solutions are found are very important. The closer the seed solutions are to the global lower convex hull, the better the improvement in the ICP value. In our example (Figure 5), if the first plan found is \( p_2 \) then the subsequent plans found to best extend \( \{p_2\} \) can be \( p_5 \) and thus the final set does not come close to the optimal set \( \mathcal{P}^* = \{p_1, p_3, p_7\} \).

5.3. Hybrid Approach

In this approach, we aim to combine the strengths of both the sampling and ICP-sequential approaches. Specifically, we use sampling to find several plans optimizing for different weights. The plans are then used to seed the subsequent ICP-sequential runs. By seeding the hybrid approach with good quality plan set scattered across the pareto optimal set, we hope to gradually expand the initial set to a final set with a much better overall ICP value. Algorithm[1] shows the pseudo-code for the hybrid approach. We first independently sample the set of \( k_0 \) values (with
We then run a heuristic planner multiple times to find an optimal (or good quality) solution for each trade-off value \( w \) (step 5). We then collect the plans found and seed the subsequent runs when we incrementally update the initial plan set with plans that lower the overall ICP value (steps 6-8). The algorithm terminates and returns the latest plan set (step 9) if \( k \) plans are found or the time bound exceeds.

### 5.4. Making LPG Search Sensitive to ICP

Since the LPG planner used in the previous section cannot handle numeric fluents, in particular the total cost representing plan cost that we are interested in, we use a modified version of the Metric-LPG planner [Gerevini et al., 2008] in implementing our algorithms. Not only is Metric-LPG equipped with a very flexible local-search framework that has been extended to handle various objective functions, but also it can be made to search for single or multiple solutions. Specifically, for the sampling-based approach, we first sample the \( w \) values based on a given distribution. For each \( w \) value, we set the metric function in the domain file to:

\[
w \times \text{makespan} + (1 - w) \times \text{totalcost},\]

and run the original LPG in the quality mode to heuristically find the best solution within the time limit for that metric function. The final solution set is filtered to remove any duplicate solutions, and returned to the user.

For the ICP-sequential and hybrid approach, we cannot use the original LPG implementation as is and need to modify the neighborhood evaluation function in LPG to take into account the ICP measure and the current plan set \( P \). For the rest of this section, we will explain this procedure in detail.

**Background:** Metric-LPG uses local search to find plans within the space of numerical action graphs (NA-graph). This leveled graph consists of a sequence of interleaved proposition and action layers. The proposition layers consist of a set of propositional and numerical nodes, while each action layer consists of at most one action node, and a number of no-op links. An NA-graph \( G \) represents a valid plan if all actions’ preconditions are supported by some actions appearing in the earlier level in \( G \). The search neighborhood for each local-search step is defined by a set of graph modifications to fix some remaining inconsistencies (unsupported preconditions) \( p \) at a particular level \( l \). This can be done by either inserting a new action \( a \) supporting \( p \) or removing from the graph the action \( a \) that \( p \) is a precondition of (which can introduce new inconsistencies).

Each local move creates a new NA-graph \( G' \), which is evaluated as a weighted combination of two factors: \( \text{SearchCost}(G') \) and \( \text{ExecCost}(G') \). Here, \( \text{SearchCost}(G') \) is the amount of search effort to resolve inconsistencies newly introduced by inserting or removing action \( a \); it is measured by the number of actions in a relaxed plan \( R \) resolving all such inconsistencies. The total cost \( \text{ExecCost}(G') \), which is a default function to measure plan quality, is measured by the total action execution costs of all actions in \( R \). The two weight adjustment values \( \alpha \) and \( \beta \) are used to steer the search toward either finding a solution quickly (higher \( \alpha \) value) or better solution quality (higher \( \beta \) value). Metric-LPG then selects the local move leading to the smallest \( E(G') \) value.

**Adjusting the evaluation function** \( E(G') \) for finding set of plans with low ICP measure: To guide Metric-LPG towards optimizing our ICP-sensitive objective function instead of the original minimizing cost objective function, we need to replace the default plan quality measure \( \text{ExecCost}(G') \) with a new measure \( \text{ICPEst}(G') \). Specifically, we adjust the function for evaluating each new NA-graph generated by local moves at each step to be a combination of \( \text{SearchCost}(G') \) and \( \text{ICPEst}(G') \). Given the set of found plans \( P = \{p_1, p_2, ..., p_n\} \),
ICPEst\((G')\) guides Metric-LPG’s search toward a plan \(p\) generated from \(G'\) such that the resulting set \(\mathcal{P} \cup \{p\}\) has a minimum ICP value: 
\[
p = \arg\min_p \text{ICP}(\mathcal{P} \cup \{p\})
\]
Thus, ICPEst\((G')\) estimates the expected total ICP value if the best plan \(p\) found by expanding \(G'\) is added to the current found plan set \(\mathcal{P}\). Like the original Metric-LPG, \(p\) is estimated by \(p_R = G' \cup R\) where \(R\) is the relaxed plan resolving inconsistencies in \(G'\) caused by inserting or removing \(a\). The ICPEst\((G')\) for a given NA-graph \(G'\) is calculated as: 
\[
\text{ICPEst}(G') = \text{ICP}(\mathcal{P} \cup p_R)
\]
However, Equation 12 is still applicable as long as we can measure the time and cost dimensions of \(p_R\). To measure the makespan of \(p_R\), we estimate the time points at which unsupported facts in \(G'\) would be supported in \(p_R = G' \cup R\) and propagate them over actions in \(G'\) to its last level. We then take the earliest time point at which all facts at the last level appear to measure the makespan of \(p_R\). For the cost measure, we just sum the individual costs of all actions in \(p_R\).

At each step of Metric-LPG’s local search framework, combining two measures ICPEst\((G')\) and SearchCost\((G')\) gives us an evaluation function that fits right into the original Metric-LPG framework and prefers a NA-graph \(G'\) in the neighborhood of \(G\) that gives the best trade-off between the estimated effort to repair and the estimated decrease in quality of the next resulting plan set.

5.5. Experimental Results

We have implemented several approaches based on our algorithms discussed in the previous sections: Sampling (Section 5.1), ICP-sequential (Section 5.2) and Hybrid that combines both (Section 5.3) with both the uniform and triangular distributions. We consider two types of distributions in which the most probable weight for plan makespan are 0.2 and 0.8, which we will call “w02” and “w08” distributions respectively (Figure 9 shows these distributions). We test all implementations against a set of 20 problems in each of several benchmark temporal planning domains used in the previous International Planning Competitions (IPC): ZenoTravel, DriverLog, and Depots. The only modification to the original benchmark set is the added action costs. The descriptions of these domains can be found at the IPC website (ipc.icaps-conference.org). The experiments were conducted Intel Core2 Duo machine with 3.16GHz CPU and 4Gb RAM. For all approaches, we search for a maximum of \(k = 10\) plans within the 10-minute time limit for each problem (i.e., \(t = 10\) minutes), and the resulting plan set is used to compute the ICP value. In the Sampling approach, we generate ten trade-off values \(w\) between makespan and plan cost based on the distribution, and for each one we search for a plan \(p\) subject to the value function 
\[
V(p, w) = w \times t_p + (1 - w) \times c_p
\]
In the Hybrid approach, on the other hand, the first Sampling approach is used with \(k_0 = 3\) generated trade-off values to find an initial plan set, which is then improved by the ICP-Sequential runs. As Metric-LPG is a stochastic local search planner, we run it three times for each problem and average the results. In 77% and 70% of 60 problems in the three tested domains for Hybrid and Sampling approaches respectively, the standard deviation of ICP values of plan sets are at most 5% of the average values. This indicates that ICP values of plan set in different runs are quite stable. As the Hybrid approach is an improved version of ICP-sequential and gives better results in almost all tested problems, we omit the ICP-Sequential in discussions below. We now analyze the results in more detailed.

**The utility of using the partial knowledge of user’s preferences:** To evaluate the utility of taking partial preferences into account, we first compare our results against the naive approaches that
generate a plan set without explicitly taking into account the partial preference model. Specifically, we run the default LPG planner with different random seeds to find multiple non-dominated plans. The LPG planner was run with both speed setting, which finds plans quickly, and diverse setting, which takes longer time to find better set of diverse plans. Figure 10 shows the comparison between quality of plan sets returned by Sampling and those naive approaches when the distribution of the trade-off value $w$ between makespan and plan cost is assumed to be uniform. Overall, among 20 tested problems for each of the ZenoTravel, DriverLog, and Depots domains, the Sampling approach is better than LPG-speed in 19/20, 20/20 and 20/20 and is better than LPG-d in 18/20, 18/20, and 20/20 problems respectively. We observed similar results comparing the Hybrid and those two approaches: in particular, the Hybrid approach is better than LPG-speed in all 60 problems and better than LPG-d in 19/20, 18/20, and 20/20 problems respectively. These results support our intuition that taking into account the partial knowledge about user’s preferences (if it is available) increases the quality of plan set.

Comparing the Sampling and Hybrid approaches: We now compare the effectiveness of the Sampling and Hybrid approaches in terms of the quality of returned plan sets with the uniform, w02 and w08 distributions.

ICP value: We first compare the two approaches in terms of the ICP values of plan sets returned indicating their quality evaluated by the user. Table 6, 7, and 8 show the results in three domains ZenoTravel, DriverLog and Depots. In general, Hybrid tends to be better than Sampling in this criterion for most of the domains and distributions. In particular, in ZenoTravel domain it returns higher quality plan sets in 15/20 problems when the distribution is uniform, 10/20 and 13/20 problems when it is w02 and w08 respectively (both approaches return plan sets with equal ICP values for two problems with the w02 and one problem with the w08 distribution).
In the DriverLog domain, Hybrid returns better plan sets for 11/20 problems with the uniform distribution (and for other three problems the plan sets have equal ICP values), but worse with the triangular distributions: 8/20 (another 2 equals) and 9/20 (another one equals) with w02 and w08. The improvement on the quality of plan sets that Hybrid contributes is more significant in the Depots domain: it is better than Sampling in 11/20 problems with the uniform distribution (and equal in 3 problems), in 12/20 problems with the w02 and w08 distributions (with w02 both approaches return plan sets with equal ICP values for 4 problems, and for 2 problems when it is w08).

In many large problems of the ZenoTravel and DriverLog domains where Sampling performs better than Hybrid, we notice that the first phase of the Hybrid approach that searches for the first 3 initial plans normally takes most of the allocated time, and therefore there is not much time left for the second phase to improve the quality of plan set. We also observe that among the three settings of the trade-off distributions, the positive effect of the second phase in Hybrid approach (which is to improve the quality of the initial plan sets) tends to be more stable across different domains with uniform distribution, but less with the triangular, in particular Sampling wins Hybrid in DriverLog domains when the distribution is w02. Perhaps this is because with the triangular distributions, the chance that LPG planner (that is used in our Sampling approach) returns the same plans even with different trade-off values would increase, especially when the most probable value of makespan happens to be in a (wide) range of weights in which one single plan is optimal. This result agrees with the intuition that when the knowledge about user’s preferences is almost complete (i.e. the distribution of trade-off value is “peak”), then Sampling approach with smaller number of generated weight values may be good enough (assuming that a good planner optimizing a complete value function is available).

### Table 6: The ICP value of plan sets in ZenoTravel domain returned by the Sampling and Hybrid approaches with the distributions (a) uniform, (b) w02 and (c) w08. The problems where Hybrid returns plan sets with better quality than Sampling are marked with (*).

| Prob | Sampling | Hybrid | Prob | Sampling | Hybrid | Prob | Sampling | Hybrid |
|------|----------|--------|------|----------|--------|------|----------|--------|
| 1*   | 840.00   | 839.98 | 1    | 972.00   | 972.00 | 1    | 708.00   | 708.00 |
| 2*   | 2,661.43 | 2,661.25 | 2    | 3,067.20 | 3,067.20 | 2*   | 2,255.792 | 2,255.788 |
| 3*   | 1,807.84 | 1,805.95 | 3*   | 2,083.91 | 2,083.83 | 5    | 1,535.54 | 1,535.32 |
| 4*   | 3,481.31 | 3,477.49 | 4*   | 4,052.75 | 4,026.92 | 4*   | 2,960.84 | 2,947.66 |
| 5*   | 3,007.97 | 2,743.85 | 5*   | 3,171.86 | 3,171.73 | 5*   | 2,802.00 | 2,524.18 |
| 6*   | 3,447.37 | 2,755.25 | 6*   | 4,288.00 | 3,188.61 | 6*   | 3,546.95 | 3,235.63 |
| 7*   | 4,549.90 | 4,344.70 | 7*   | 5,060.81 | 5,044.43 | 7*   | 3,802.60 | 3,733.90 |
| 8*   | 6,397.32 | 5,875.13 | 9*   | 7,037.84 | 6,614.30 | 9*   | 5,469.24 | 5,040.88 |
| 10*  | 7,592.72 | 6,826.60 | 10*  | 8,515.74 | 7,472.37 | 10*  | 6,142.68 | 5,997.45 |
| 11*  | 5,307.04 | 5,050.07 | 11*  | 5,946.68 | 5,891.76 | 11*  | 4,578.09 | 4,408.36 |
| 12*  | 7,288.54 | 6,807.28 | 12*  | 7,954.74 | 7,586.28 | 12*  | 5,483.19 | 5,756.89 |
| 13*  | 10,208.11 | 9,956.94 | 13*  | 11,847.13 | 11,414.88 | 13*  | 8,515.74 | 8,479.09 |
| 14   | 11,939.22 | 11,730.87 | 14   | 14,474.00 | 15,739.19 | 14*  | 11,610.38 | 11,369.46 |
| 15   | 9,334.68 | 13,541.28 | 15   | 16,125.70 | 16,147.28 | 15*  | 11,748.45 | 11,418.59 |
| 16*  | 16,724.21 | 13,949.26 | 16   | 19,480.86 | 19,841.67 | 16*  | 14,503.78 | 15,121.77 |
| 17*  | 27,085.57 | 26,822.37 | 17   | 29,559.03 | 32,175.66 | 17*  | 21,354.78 | 22,297.65 |
| 18   | 23,610.71 | 25,089.40 | 18   | 28,520.17 | 29,020.15 | 18   | 20,107.03 | 21,727.75 |
| 19   | 29,114.30 | 29,276.09 | 19   | 34,224.02 | 36,496.40 | 19   | 23,721.90 | 25,580.24 |
| 20   | 34,939.27 | 37,166.29 | 20   | 39,443.66 | 42,790.97 | 20   | 28,178.45 | 28,961.51 |

In the DriverLog domain, Hybrid returns better plan sets for 11/20 problems with the uniform distribution (and for other three problems the plan sets have equal ICP values), but worse with the triangular distributions: 8/20 (another 2 equals) and 9/20 (another one equals) with w02 and w08. The improvement on the quality of plan sets that Hybrid contributes is more significant in the Depots domain: it is better than Sampling in 11/20 problems with the uniform distribution (and equal in 3 problems), in 12/20 problems with the w02 and w08 distributions (with w02 both approaches return plan sets with equal ICP values for 4 problems, and for 2 problems when it is w08).

In many large problems of the ZenoTravel and DriverLog domains where Sampling performs better than Hybrid, we notice that the first phase of the Hybrid approach that searches for the first 3 initial plans normally takes most of the allocated time, and therefore there is not much time left for the second phase to improve the quality of plan set. We also observe that among the three settings of the trade-off distributions, the positive effect of the second phase in Hybrid approach (which is to improve the quality of the initial plan sets) tends to be more stable across different domains with uniform distribution, but less with the triangular, in particular Sampling wins Hybrid in DriverLog domains when the distribution is w02. Perhaps this is because with the triangular distributions, the chance that LPG planner (that is used in our Sampling approach) returns the same plans even with different trade-off values would increase, especially when the most probable value of makespan happens to be in a (wide) range of weights in which one single plan is optimal. This result agrees with the intuition that when the knowledge about user’s preferences is almost complete (i.e. the distribution of trade-off value is “peak”), then Sampling approach with smaller number of generated weight values may be good enough (assuming that a good planner optimizing a complete value function is available).
| Prob | Sampling | Hybrid | Prob | Sampling | Hybrid | Prob | Sampling | Hybrid |
|------|----------|-------|------|----------|-------|------|----------|-------|
| 1    | 212.00   | 212.00| 1    | 235.99   | 236.00| 1    | 188.00   | 188.00|
| 2*   | 363.30   | 348.38| 2*   | 450.07   | 398.46| 2*   | 333.20   | 299.70|
| 3    | 176.00   | 176.00| 3    | 203.20   | 203.20| 3    | 148.80   | 148.80|
| 4*   | 282.00   | 278.45| 4*   | 336.01   | 323.79| 4*   | 238.20   | 233.20|
| 5*   | 236.83   | 236.33| 5    | 273.80   | 288.51| 5*   | 200.80   | 199.52|
| 6*   | 227.00   | 221.00| 6    | 254.80   | 254.80| 6*   | 187.47   | 187.20|
| 7    | 176.50   | 176.50| 7*   | 226.20   | 203.80| 7    | 149.20   | 149.20|
| 8*   | 338.96   | 319.43| 8    | 387.53   | 397.75| 8    | 300.54   | 323.87|
| 9*   | 369.18   | 301.72| 9*   | 420.64   | 339.05| 9*   | 316.80   | 263.92|
| 10*  | 178.38   | 170.55| 10*  | 196.44   | 195.11| 10*  | 158.18   | 146.12|
| 11*  | 289.04   | 232.65| 11*  | 334.13   | 253.09| 11*  | 245.38   | 211.60|
| 12   | 711.48   | 727.65| 12*  | 824.17   | 809.93| 12*  | 605.86   | 588.82|
| 13*  | 469.50   | 460.99| 13   | 519.92   | 521.05| 13   | 388.80   | 397.67|
| 14   | 457.04   | 512.11| 14   | 524.56   | 565.94| 14   | 409.02   | 410.53|
| 15*  | 606.81   | 591.41| 15*  | 699.49   | 643.72| 15   | 552.79   | 574.95|
| 16   | 4,432.21 | 4,490.17| 16   | 4,902.34 | 6,328.07| 16   | 3,580.32 | 4,297.47|
| 17   | 1,310.83 | 1,427.70| 17   | 1,632.86 | 1,659.46| 17   | 1,062.03 | 1,146.68|
| 18*  | 1,800.49 | 1,768.17| 18*  | 1,992.32 | 2,183.13| 18*  | 1,448.36 | 1,549.09|
| 19   | 3,941.08 | 4,278.67| 19   | 4,614.13 | 7,978.00| 19*  | 3,865.54 | 2,712.08|
| 20   | 2,225.66 | 2,397.61| 20   | 2,664.00 | 2,792.90| 20   | 1,892.28 | 1,934.11|

Table 7: The ICP value of plan sets in DriverLog domain returned by the Sampling and Hybrid approaches with the distributions (a) uniform, (b) w02 and (c) w08. The problems where Hybrid returns plan sets with better quality than Sampling are marked with (*).
Since the quality of a plan set depends on how the two features makespan and plan cost are optimized, and how the plans “span” the space of time and cost, we also compare Sampling and Hybrid approaches in terms of those two criteria. In particular, we compare plan sets returned by the two approaches in terms of (i) their median values of makespan and cost, which represent how “close” the plan sets are to the origin of the space of makespan and cost, and (ii) their standard deviation of makespan and cost values, which indicate how the sets span each feature axis.

Table 9 summarizes for each domain, distribution and feature the number of problems in which each approach (either Sampling or Hybrid) generates plan sets with better median of each feature value (makespan and plan cost) than the other. There are 60 problems across 3 different distributions, so in total, 180 cases for each feature. Sampling and Hybrid return plan sets with better makespan in 40 and 62 cases, and with better plan cost in 52 and 51 cases (respectively), which indicates that Hybrid is slightly better than Sampling on optimizing makespan but is possibly worse on optimizing plan cost. In ZenoTravel domain, for all distributions Hybrid likely returns better plan sets on the makespan than Sampling, and Sampling is better on the plan cost feature. In DriverLog domain, Sampling is better on the makespan feature with both non-uniform distributions, but worse than Hybrid with the uniform. On the plan cost feature, Hybrid returns plan sets with better median than Sampling on the uniform and w02 distribution, and both approaches perform equally well with the w08 distribution. In Depots domain, Sampling is better than Hybrid on both features with the uniform distribution, and only better than Hybrid on the makespan with the distribution w08.

In terms of spanning plan sets, Hybrid performs much better than Sampling on both features across three domains, as shown in Table 10. In particular, over 360 cases for both makespan and plan cost features, there are only 10 cases where Sampling produces plan sets with better standard deviation than Hybrid on each feature. Hybrid, on the other hand, generates plan sets with better standard deviation on makespan in 91 cases, and in 85 cases on the plan cost.

These experimental results support our arguments in Section 5.1 about the limits of sampling idea. Since one single plan could be optimal for a wide range of weight values, the search in Sampling approach with different trade-off values may focus on looking for plans only at the same region of the feature space (specified by the particular value of the weight), which can reduce the chance of having plans with better value on some particular feature. On the opposite side, the Hybrid approach tends to be better in spanning plan sets to a larger region of the space,
Table 10: The numbers of problems for each domain, distribution and feature where Sampling (Hybrid) returns plan sets with better (i.e. larger) standard deviation of feature value than that of Hybrid (Sampling), denoted in the table by $S > H$ ($H > S$, respectively). We mark bold the numbers of problems that indicate the outperformance of the corresponding approach.

| Domain    | Distribution | $S > H$ | $H > S$ | $S > H$ | $H > S$ |
|-----------|--------------|---------|---------|---------|---------|
| ZenoTravel| uniform      | 8       | 12      | 6       | 14      |
|           | w02          | 4       | 14      | 7       | 11      |
|           | w08          | 6       | 13      | 8       | 11      |
| DriverLog | uniform      | 5       | 11      | 6       | 10      |
|           | w02          | 7       | 10      | 7       | 9       |
|           | w08          | 8       | 9       | 10      | 7       |
| Depots    | uniform      | 10      | 7       | 7       | 9       |
|           | w02          | 7       | 9       | 5       | 10      |
|           | w08          | 5       | 13      | 7       | 11      |

as the set of plans that have been found is taken into account during the search.

**Contribution to the lower convex hull:** The comparison above between Sampling and Hybrid considers the two features separately. We now examine the relation between plan sets returned by those approaches on the joint space of both features, in particular taking into account the the dominance relation between plans in the two sets. In other words, we compare the relative total number of plans in the lower convex-hull (LCH) found by each approach. Given that this is the set that should be returned to the user (to select one from), the higher number tends to give her a better expected utility value. To measure the relative performance of both approaches with respect to this criterion, we first create a set $S$ combining the plans returned by them. We then compute the set $S_{LCH} \subseteq S$ of plans in the lower convex hull among all plans in $S$. Finally, we measure the percentages of plans in $S_{LCH}$ that are actually returned by each of our tested approaches. Figures 11, 12 and 13 show the contribution to the LCH of plan sets returned by Sampling and Hybrid in ZenoTravel, DriverLog and Depots domains.

In general, we observe that the plan set returned by Hybrid contributes more into the LCH than that of Sampling for most of the problems (except for some large problems) with most of the distributions and domains. Specifically, in ZenoTravel domain, Hybrid contributes more plans to the LCH than Sampling in 15/20, 13/20 (and another 2 equals), 13/20 (another 2 equals) problems for the uniform, w02 and w08 distributions respectively. In DriverLog domain, it is better than Sampling in 10/20 (another 6 equals), 10/20 (another 4 equals), 8/20 (another 5 equals) problems; and Hybrid is better in 11/20 (another 6 equals), 11/20 (another 4 equals) and 11/20 (another 4 equals) for the uniform, w02 and w08 distributions in Depots domain. Again, similar to the ICP value, the Hybrid approach is less effective on problems with large size (except with the w08 distribution in Depots domain) in which the searching time is mostly used for finding initial plan sets. We also note that a plan set with higher contribution to the LCH is not guaranteed to have better quality, except for the extreme case where one plan set contributes 100% and completely dominates the other which contributes 0% to the LCH. For example, consider the problem 14 in ZenoTravel domain: even though the plan sets returned by Hybrid contribute more than those of Sampling in all three distributions, it is only the w08 where it has a better ICP value. The reason for this is that the ICP value depends also on the range of the trade-off value (and its density) for which a plan in the LCH is optimal, whereas the LCH is constructed by simply comparing plans in terms of their makespan and cost separately (i.e. using the dominance relation), ignoring their
relative importance.

The sensitivity of plan sets to the distributions: All analysis having been done so far is to compare the effectiveness of approaches with respect to a particular distribution of the trade-off value. In this part, we examine how sensitive the plan sets are with respect to different distributions.

Optimizing high-priority feature: We first consider how plan sets are optimized on each feature (makespan and plan cost) by each approach with respect to two non-uniform distributions w02 and w08. Those are the distributions representing scenarios where the users have different priority on the features, and plan sets should be biased to optimizing the feature that has higher priority (i.e. larger value of weight). In particular, plans generated using the w08 distribution should have better (i.e. smaller) makespan values than those found with the w02 distribution (since in the makespan has higher priority in w08 than it is in w02); on the other hand, plan set returned with w02 should have better values of plan cost than those with w08.

Table 11 summarizes for each domain, approach and feature, the number of problems in which plan sets returned with one distribution (either w02 or w08) have better median value than with the other. We observe that for both features, the Sampling approach is very likely to “push” plan sets to regions of the space of makespan and cost with better value of more interested feature. On the other hand, the Hybrid approach tends to be more sensitive to the distributions on both the features in ZenoTravel domain, and is more sensitive only on the makespan feature in DriverLog and Depots domain. Those results generally show that our approaches can bias the search towards optimizing features that are more desired by the user.
Spanning plan sets on individual features: Next, we examine how plan sets span each feature, depending on the degree of incompleteness of the distributions. Specifically, we compare the standard deviation of plan sets returned using the uniform distribution with those generated using the distributions w02 and w08. Intuitively, we expect that plan sets returned with the uniform distribution would have higher standard deviation than those with the distributions w02 and w08.

Table 12 shows for each approach, domain and feature, the number of problems generated with the uniform distribution that have better standard deviation on the feature than those found with the distribution w02. We observe that with the makespan feature, both approaches return plan sets that are more “spanned” on makespan in the Depots domain, but not with ZenoTravel and DriverLog. With the plan cost feature, Hybrid shows its positive impact on all three domains, whereas Sampling shows it with the ZenoTravel and Depots domain. Similarly, Table 13 shows the results comparing the uniform and w08 distributions. This time, Sampling returns plan sets with better standard deviation on both features in the ZenoTravel and Depots domains, but not in DriverLog. Hybrid also shows this in ZenoTravel domain, but for the remaining two domains, it tends to return plan sets with expected standard deviation on the plan cost feature only. From all of these results, we observe that with the uniform distribution, both approaches likely generate plan sets that span better than with non-uniform distributions, especially on the plan cost feature.

In summary, the experimental results in this section support the following hypotheses:

- Instead of ignoring user’s preference models which are partially specified, one should take them into account during plan generation, as plan sets returned would have better quality.
Table 12: The numbers of problems for each approach, domain and feature where plan sets returned with the uniform (w02) distribution have better (i.e. higher) standard deviation of the feature value than that with w02 (uniform), denoted in the table by $U > w02$ ($w02 > U$, respectively). For each approach and feature, we mark bold the numbers for domains in which there are more problems whose plan sets returned with the uniform distribution have better standard deviation value of the feature than those with the w02 distribution.

| Approach | Domain        | $SD_{makespan}$ | $SD_{cost}$ |
|----------|---------------|-----------------|-------------|
|          | $U > w02$     | $w02 > U$       | $U > w08$   | $w08 > U$   |
| Sampling | ZenoTravel    | 9               | 10          | 10          | 7           |
|          | DriverLog     | 6               | 8           | 7           | 8           |
|          | Depots        | 9               | 6           | 8           | 7           |
| Hybrid   | ZenoTravel    | 9               | 10          | 12          | 7           |
|          | DriverLog     | 6               | 9           | 8           | 7           |
|          | Depots        | 8               | 6           | 9           | 4           |

Table 13: The numbers of problems for each approach, domain and feature where plan sets returned with the uniform (w08) distribution have better (i.e. higher) standard deviation of the feature value than that with w08 (uniform), denoted in the table by $U > w08$ ($w08 > U$, respectively). For each approach and feature, we mark bold the numbers for domains in which there are more problems whose plan sets returned with the uniform distribution have better standard deviation value of the feature than those with the w08 distribution.

| Approach | Domain        | $SD_{makespan}$ | $SD_{cost}$ |
|----------|---------------|-----------------|-------------|
|          | $U > w08$     | $w08 > U$       | $U > w08$   | $w08 > U$   |
| Sampling | ZenoTravel    | 11              | 8           | 15          | 4           |
|          | DriverLog     | 5               | 10          | 5           | 9           |
|          | Depots        | 12              | 7           | 12          | 6           |
| Hybrid   | ZenoTravel    | 10              | 9           | 15          | 4           |
|          | DriverLog     | 7               | 7           | 8           | 6           |
|          | Depots        | 5               | 8           | 11          | 4           |
In generating plan sets sequentially to cope with partial preference models, Sampling approach that searches for plans separately and independently of the solution space tends to return worse quality plan sets than Hybrid approach.

The resulting plan sets returned by Hybrid approach tend to be more sensitive to the user’s preference models than those found by Sampling approach.

6. Related Work

Currently there are very few research efforts in the planning literature that explicitly consider incompletely specified user preferences during planning. The usual approach for handling multiple objectives is to assume that a specific way of combining the objectives is available (Refanidis and Vlahavas, 2003; Do and Kambhampati, 2003), and search for one optimal plan with respect to this function. Brafman & Chernyavsky (2005) discuss a CSP-based approach to find a plan for the most preferred goal state given the qualitative preferences on goals. There is no action cost and makespan measurements such as in our problem setting. Other relevant work includes Bryce et al. (2007), in which the authors devise a variant of LAO* algorithm to search for a conditional plan with multiple execution options for each observation branch that are non-dominated with respect to objectives like probability and cost to reach the goal.

In the context of decision-theoretic planning, some work has been focused on scenarios where the value function is not completely defined, in particular due to the incompleteness in specifying the reward function. In those cases, one approach is to search for the most robust policy with different robustness criteria (e.g., Delage and Mannor, 2007; Regan and Boutilier, 2010; Nilim and Ghaoui, 2005). The idea of searching for sets of policies has also been considered recently in reinforcement learning. Specifically, in (Natarajan and Tadepalli, 2005) the reward function is incomplete with weight values changing over time, and a set of policies is searched and stored so that whenever the weights change a new best policy can be found by improving those in the set. On the other hand, Barrett and Narayanan (2008) provide Bellman equations for the Q-values using all vectors on convex hull to search for the whole pareto set.

Our work on planning with partial user’s preferences is also related to work on preference elicitation and decision making under uncertainty of preferences. For instance, Chajewska et al. (2000) consider a decision making scenario where the utility function is assumed to be drawn from a known distribution, and either a single best strategy or an elicitation question will be suggested based on the expected utility of the strategy and the value of information of the question. Boutilier et al. (2010) considers preference elicitation problem in which the user’s preference model is incomplete on both the set of features and the utility function. However, these scenarios are different from ours in two important issues: we focus on efficient approach to synthesizing plans with respect to the partial preferences, whereas the “outcomes” or “configurations” in their cases are considered given upfront (or could be obtained with low cost), and we aim to search for a set of plans based on a quality measure of plan sets (instead of a quality measure over individual outcome or configuration).

Our approach to generating diverse plan sets to cope with planning scenarios without knowledge of user’s preferences is in the same spirit as (Tate et al., 1998) and Myers (Myers, 2006; Myers and Lee, 1999), though for different purposes. Myers, in particular, presents an approach to generate diverse plans in the context of her HTN planner by requiring the metatheory of the domain to be available and using bias on the meta-theoretic elements to control search (Myers and Lee, 1999). The metatheory of the domain is defined in terms of pre-defined
attributes and their possible values covering roles, features and measures. Our work differs from hers in two respects. First, we focus on domain-independent distance measures. Second we consider the computation of diverse plans in the context of state of the art domain independent planners.

The problem of finding multiple but similar plans has been considered in the context of replanning. A recent effort in this direction is (Fox et al., 2006). Our work focuses on the problem of finding diverse plans by a variety of distance measures when the user’s preferences exist but are completely unknown.

Outside the planning literature, our closest connection is to the work by Hebrard et al. 2005, who solve the problem of finding similar/dissimilar solutions for CSPs without additional domain knowledge. It is instructive to note that unlike CSP, where the number of potential solutions is finite (albeit exponential), the number of distinct plans for a given problem can be infinite (since we can have infinitely many non-minimal versions of the same plan). Thus, effective approaches for generating diverse plans are even more critical. The challenges in finding interrelated plans also bear some tangential similarities to the work in information retrieval on finding similar or dissimilar documents (c.f. (Zhang et al., 2002)).

7. Conclusion and Future Work

In this paper, we consider the planning problem with partial user’s preference model in two scenarios where the knowledge about preference is completely unknown or only part of it is given. We propose a general approach to this problem where a set of plans is presented to the user from which she can select. For each situation of the incompleteness, we define different quality measure of plan sets and investigate approaches to generating plan set with respect to the quality measure. In the first scenario when the user is known to have preferences over plans, but the details are completely unknown, we define the quality of plansets as their diversity value, specified with syntactic features of plans (its action set, sequence of states, and set of causal links). We then consider generating diverse set of plans using two state-of-the-art planners, GP-CSP and LPG. The approaches we developed for supporting the generation of diverse plans in GP-CSP are broadly applicable to other planners based on bounded horizon compilation approaches for planning. Similarly, the techniques we developed for LPG, such as biasing the relaxed plan heuristics in terms of distance measures, could be applied to other heuristic planners. The experimental results with GP-CSP explicate the relative difficulty of enforcing the various distance measures, as well as the correlation among the individual distance measures (as assessed in terms of the sets of plans they find). The experiments with LPG demonstrate the potential of planning using heuristic local search in producing large sets of highly diverse plans.

When part of the user’s preferences is given, in particular the set of features that the user is interested in and the distribution of weights representing their relative importance, we propose the usage of Integrated Preference Function, and its special case Integrated Convex Preference function, to measure the quality of plan sets, and propose various heuristic approaches based on the Metric-LPG planner (Gerevini et al., 2008) to find a good plan set with respect to this measure. We show empirically that taking partial preferences into account does improve the quality of plan set returned to the users, and that our proposed approaches are sensitive to the degree of preference incompleteness, represented by the distribution.

While a planning agent may well start with a partial preference model, in the long run, we would like the agent to be able to improve the preference model through repeated interactions with the user. In our context, at the beginning when the degree of incompleteness of preferences
is high, the learning will involve improving the estimate of $h(\alpha)$ based on the feedback about
the specific plan that the user selects from the set returned by the system. This learning phase is
in principle well connected to the Bayesian parameter estimation approach in the sense that
the whole distribution of parameter vector, $h(\alpha)$, is updated after receiving feedback from
the user, taking into account the current distribution of all models (starting from a prior, for instance
the uniform distribution). Although such interactive learning framework has been discussed
previously, as in Chajewska et al. (2001), the set of user’s decisions in this work is assumed to
be given, whereas in planning scenarios the cost of plan synthesis should be incorporated into the
our interactive framework, and the problem of presenting plan sets to the user needs also to be
considered. Recent work by Li et al. (2009) considered learning user’s preferences in planning,
but restricting to preference models that can be represented with hierarchical task networks.

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