I. INTRODUCTION

An understanding of the low energy properties in the theory of strong interactions described by quantum chromodynamics (QCD) may be based on the complementarity between a Higgs- and a confinement description of the vacuum. Complementarity has been exploited already for the high temperature and high density phases of gauge theories. In the electroweak standard model the Higgs phase at low temperature and the high temperature confined phase were shown to be continuously connected. In QCD it has been pointed out that the high baryon number density phase admits a Higgs description by the phenomenon of color-flavor locking. Recently, it has been proposed that the vacuum of QCD can be characterized by a color octet quark-antiquark condensate involving three quark flavors. As a consequence color is broken completely and a successful phenomenology emerges. A dynamical mechanism for the formation of the octet condensate is provided by the anomalous multiquark interaction induced by instantons.

In this Letter we propose a Higgs description for the vacuum of QCD in the limit of two quark flavors. The Higgs picture gives a simple relation between the microscopic (quark–gluon) and the macroscopic (hadron) degrees of freedom, which overcomes the classic qualitative problems of an analytic approach to low-energy QCD. Both spontaneous chiral symmetry breaking and confinement are associated to the condensation of an adjoint quark-antiquark and an antitriplet quark-quark pair. The condensates leave a global vector–like Goldstone bosons associated to the spontaneous breaking of chiral symmetry can be identified with the pions. In the presence of both condensates all elementary excitations acquire integral charges. Quarks can be identified with baryons and gluons with vector mesons. In contrast to diquark condensation yields a modified conserved baryon number. As a consequence the dressed quarks acquire integer baryon number, two of them carrying the quantum numbers of the proton and the neutron. The quark degrees of freedom and the baryon degrees of freedom can thus be described by the same fields. Since color is broken completely all gluons acquire a mass by the Higgs phenomenon and four of them carry the quantum numbers of the $\rho$– and $\omega$–mesons. The pseudo-Goldstone bosons associated to the spontaneous breaking of chiral symmetry can be identified with the pions.

In addition to the expected nucleons and mesons the remaining quarks and gluons correspond to soliton-type fermionic excitations which carry zero baryon number and bosonic excitations with baryon number one. Excitations with similar quantum numbers are known to occur in two-flavor QCD at sufficiently high baryon density. In the vacuum these excitations comprise half–integer isospin representations of bosons and integer isospin representations of fermions. In the non-relativistic quark model for two-flavor QCD these states could only be constructed from an infinite number of quarks and antiquarks. We emphasize that within our approach the appearance of stable nonlinear excitations follows only from group-theoretical arguments independent of the detailed dynamics. This provides an important test for the idea of color symmetry breaking in the vacuum. Even though two-flavor QCD is an idealization and not accessible to direct experimental observation it can be simulated by lattice QCD.

Consequences of a possible condensation of an adjoint quark-antiquark and a quark-quark pair have been recently discussed at high baryon number density. It was found that simultaneous condensation can be observed in the high density phase for sufficiently strong interactions in the respective channels. In this case vacuum and nuclear matter in the non-superfluid phase may be continuously connected.

II. COLOR-ISOSPIN LOCKING

Color octet, antitriplet and singlet condensates. In the proposed Higgs description the vacuum can be characterized by the condensation of a color-octet quark-antiquark pair.
\[ \chi \sim \left\langle \bar{\psi}_a \sum_{s=1}^3 (\tau_3)_{ab} (\lambda_s)^i \psi_b \right\rangle \] (1)

and a color-antitriplet diquark
\[ \Delta \sim \left\langle \bar{\psi}_a^i (\tau_2)_{ab} (\lambda_2)^i \psi_b^i \right\rangle \] (2)

where \((\tau_s)_{ab}\) \((a,b = 1, 2)\) are the Pauli matrices in flavor space, and \((\lambda_s)^ij\) \((i,j = 1, 2, 3)\) denote the Gell-Mann matrices in color space. Here we have suppressed the Dirac structure.

Since the adjoint quark-antiquark condensate \(\chi\) belongs to a flavor triplet it breaks both color and flavor symmetries separately. It is invariant under the global vector-like \(SU(2)_{\text{color}} + V\) subgroup which applies color and flavor transformations simultaneously. The condensate locks the flavor generators \(\tau_s\) to the transposed color generators \(\lambda_s\) of a \(SU(2)_{\text{color}}\) subgroup and extends the notion of color-flavor locking to a two-flavor theory. Condensation of the form \((\chi)\) leaves a local \(U(1)_8\) subgroup of color unbroken. It gives a mass to seven of the eight gluons by the Higgs mechanism.

The color-antitriplet diquark condensate \(\Delta\) given in \((\Delta)\) is a flavor singlet and does not break chiral symmetry (see also \((\chi)\)). Condensation of this form leaves an \(SU(2)\) subgroup of \(SU(3)_{\text{color}}\) unbroken. Consequently three gluons remain massless if only \(\Delta\) condenses.

Neither the adjoint chiral diquark nor the antitriplet diquark condensate alone break color completely. Only both condensates together give a mass to all gluons. Chiral symmetry is spontaneously broken by \(\chi \neq 0\) and four quarks get a mass even in the chiral limit of a vanishing current quark mass \(m_q\). The remaining two quarks become massive for a nonvanishing singlet condensate
\[ \phi \sim \left\langle \bar{\psi}_a^i \psi_b^i \right\rangle. \] (3)

Further condensates consistent with the symmetries include \((\phi)\) a flavor quark-antiquark singlet bilinear involving the color \(\lambda_q\) generator, or a pure glue condensate of the form \((F_{ik}^\mu F_{kj}^{\mu \nu} - \frac{1}{3} F_{ik}^{\mu \nu} F_{kl}^{\mu \nu} \delta_{ij}) \sim (\lambda_8)_{ij}\). These condensates affect some of the quantitative aspects but do not change the picture as far as symmetries are concerned.

### III. QUARK-GLUON DESCRIPTION OF HADRONS

**Integer charges and modified baryon number.** The electromagnetic gauge symmetry with fractional quark charges is broken in presence of the condensates \((\chi)\) or \((\phi)\). The vacuum is invariant under a combination of quark-electric charge \(Q_q\) and abelian color charge \(Q_c = \frac{1}{2} \lambda_3 + \frac{\gamma}{\sqrt{2}} \lambda_8\) of the quarks. As a consequence, the physical electric charge contains a color component
\[ Q = Q_q - \frac{1}{2} \lambda_3 - \frac{1}{2\sqrt{3}} \lambda_8 \] (4)

where \(Q_q = 2/3\) for up and \(Q_q = -1/3\) for down quarks. One observes that all fields carry integral electric charge \(Q\).

The diquark condensate \((\phi)\) breaks baryon number. However, there is a conserved modified baryon number
\[ B' = B_\eta - \frac{\lambda_8}{\sqrt{3}} \] (5)

with \(B_\eta = 1/3\) for both quarks. The condensates \(\Delta\) and \(\chi\) are neutral under \(B'\). Electric charge is related to \(B'\) and the third component of isospin, \(I_3\), by \(Q = I_3 + B'/2\).

**Quarks.** We show the quark-electric charge \(Q_q\), abelian color charge \(Q_c\), and the quantum numbers \(Q, I_3\) and \(B'\) of the six quarks in table 1. There are only two degrees of freedom, corresponding to the quarks of the third color, which carry baryon number \(B' = 1\). The dressed up quark of the third color carries electric charge \(Q = +1\) and the corresponding down quark is electrically neutral. They carry the quantum numbers of the proton and the neutron, respectively. In presence of the condensates \((\chi)\), \((\phi)\) and \((\Delta)\) the nucleons acquire a mass
\[ M^2_q = M^2_{\text{n}} = (m_q + \phi)^2 \] (6)

independent of \(\chi\) and \(\Delta\). The mass matrix for the remaining four quark fields contains Majorana-type entries similar to the case of neutrinos. Indeed, no quantum number forbids a mixing of the quark fields with their antiparticles. There is an isospin triplet \((S^0, S^+, S^-)\) with charge \(Q = (0, 1, -1)\) and a neutral singlet \((L^0)\) which get contributions from all three condensates
\[ M^2_q = (m_q + \phi - \chi)^2 + \Delta^2, \quad M^2_{L^0} = (m_q + \phi + 3\chi)^2 + \Delta^2. \] (7)

| Quarks | \(Q_q\) | \(Q_c\) | \(Q\) | \(I_3\) | \(B'\) |
|--------|--------|--------|------|------|------|
| \(u_3\) | \(2/3\) | \(-1/3\) | \(1\) | \(1/2\) | \(1\) |
| \(d_3\) | \(-1/3\) | \(-1/3\) | \(0\) | \(-1/2\) | \(1\) |
| \(u_1\) | \(2/3, -1/3\) | \(2/3, -1/3\) | \(0\) | \(0\) | \(S^0\) |
| \(u_2\) | \(2/3\) | \(-1/3\) | \(1\) | \(1\) | \(0\) |
| \(d_1\) | \(-1/3\) | \(2/3\) | \(-1\) | \(-1\) | \(S^-\) |
| \(u_1 + d_2\) | \(2/3, -1/3\) | \(2/3, -1/3\) | \(0\) | \(0\) | \(L^0\) |

Table 1: Charges of the up and down quarks.

**Vector mesons.** All eight gluons carry integer electric charge. Three gluons carry the quantum numbers of the isospin triplet \(\rho\)-mesons. The isospin singlet corresponding to the eighth gluon can be associated with the neutral \(\omega\)-meson. The adjoint chiral condensate \((\phi)\) contributes to the \(\rho\)-meson mass while the diquark condensate \((\phi)\) is responsible for a nonvanishing mass of the \(\omega\)-meson
\[ M^2_\rho = c_\chi g^2 \chi^2, \quad M^2_{\omega} = c_\Delta g^2 \Delta^2. \] (8)
The proportionality factors $c_\chi$ and $c_\Delta$ reflect the relative strength of the coupling of the condensates to quarks and gluons. The vector meson masses are proportional to the strong gauge coupling $g$. The mass of the two isospin doublets $(\kappa^+, \kappa^0)$ and $(\kappa^-, \kappa^0)$ get contributions from both colored condensates,

$$M_\kappa = \frac{3}{8} c_\chi g^2 \lambda^2 + \frac{3}{4} c_\Delta g^2 \Delta^2.$$  \hspace{1cm} (9)

| Gluons  | $Q_c$ | $Q$ | $I_3$ | $B'$ |
|--------|------|----|------|------|
| $A_3$  | 0    | 0  | 0    | $\rho^0$ |
| $A_1+iA_2$ | -1  | 1  | 1    | $\rho^+$ |
| $A_1-iA_2$ | 1   | -1 | -1   | $\rho^-$ |
| $A_0$  | 0    | 0  | 0    | $\omega$ |
| $A_1+A_4$ | -1  | 1  | 1/2  | $\kappa^+$ |
| $A_6+iA_7$ | 0   | 0  | -1/2 | $\kappa^0$ |
| $A_4-iA_5$ | 1   | -1 | -1/2 | $\kappa^-$ |
| $A_6-iA_7$ | 0   | 0  | -1/2 | $\kappa^0$ |

Table 2: Charges of the gluons.

Pseudo-Goldstone bosons. There is also a triplet of collective modes associated with chiral symmetry breaking by the $\chi$ or $\phi$ condensates. The broken symmetry generators are given by the axial charges and the bosons with mass squared $m_\chi^2 \sim m_\phi = m_q$ match the quantum numbers of the three pions. The interactions of the pions are dictated by spontaneously broken chiral symmetry. No Goldstone bosons are generated by $\Delta \neq 0$.

IV. "Strange" Particles

Soliton-type excitations. As a consequence of the complete breaking of color we find, apart from the expected nucleons and mesons, additional excitations $S, L^0$ and $\kappa$ with unusual properties. These states carry integral charges and form part of the spectrum. They can be expected to be relatively heavy since their masses get contributions from all condensates $\chi, \Delta$ and $\phi$ (cf. eqs. [6], [8]). From table 1 one observes that there are integer isospin representations of fermions, a triplet $(S, S^+, S^-)$ and a singlet $(L)$, which carry zero baryon number. Table 2 shows half-integer isospin representations of bosons, two doublets $(\kappa^+, \kappa^0)$ and $(\kappa^-, \kappa^0)$, with baryon number one. Their presence could be tested in lattice Monte Carlo simulations of two-flavor QCD. Such a test is not completely straightforward since integer isospin fermions and half-integer isospin bosons do not couple in the channels of standard operators representing a finite number of quarks. Nontrivial operators in the gluon sector are needed.

For an understanding of this it is instructive to consider the nonrelativistic quark model. In the quark model for two-flavor QCD the building blocks of hadrons are up and down quarks belonging to isospin doublets. No half-integer isospin representations can occur as $q\bar{q}$-states or any bosonic state involving a finite number of quarks $q$ and antiquarks $\bar{q}$. In fact, there is a simple selection rule that the product of a finite even number of half-integer isospin representations has integer isospin and an odd number has half-integer isospin. Bosons build from a finite number of fermions must involve an even number and therefore carry integer isospin. We conclude that the bosonic $\kappa$-states can arise in the two-flavor quark model only as soliton-type excitations involving infinitely many quarks and antiquarks. Similarly, the fermions with integral isospin $(S, S^+, S^-$ and $L)$ cannot be represented by a finite odd number of quarks or antiquarks in the quark model.

The $\kappa$-states carry integer $B'$. They can therefore not decay into $\rho$-mesons and pions (or photons) which all carry $B' = 0$. The decay into protons and neutrons involves an even number of fermions with $B' = \pm 1$. It would therefore lead to a final state with $B'$ even and is again forbidden by $B'$ conservation. The $\kappa$-states are stable if their mass is below the masses of $S$ and $L^0$. The (lightest of the) latter ones are stable since they carry $B' = 0$ and cannot decay into protons and neutrons. Fermion number is only conserved modulo two, as is manifest already from the Majorana-type entries in the mass matrix for the $S$– and $L$–states.

"Strangeness". The quantum numbers of the additional particles become more familiar if seen from a three-flavor perspective. In terms of standard three-flavor hadronic quantum numbers one may write $B' = "B" + "S"$ for "baryon number $B"$ and "strangeness $S"$. One observes from table 1 that with the association "$B" = 1, \ "S" = -1 for the triplet of $S$–states there is a precise correspondence to the quantum numbers of the hyperon $\Sigma$-triplet. Similarly, $L^0$ can be associated with the singlet $A^0$. From table 2 one finds that the $\kappa$-states with "$B" = 0 can be mapped to the $K^*$-mesons. In this language the appearance of “strange” particles is a consequence of the fact that color is broken and the definition of “$S$" contains a color component. Despite the apparent correspondence we emphasize that there is no additional $U(1)$ symmetry associated to strangeness present in the two-flavor theory.

V. High Density or Temperature

High density and Higgs description of nuclear matter. The appearance of “strange” states in the proposed vacuum of two-flavor QCD follows only from the symmetry breaking pattern and is independent of the detailed dynamics. It is interesting to note that fermionic states carrying zero baryon number are known from two-flavor QCD at high baryon number density where diquark condensation of the form $\bar{q}q$ occurs [3,4]. The high density
ground state is neutral under $B'$ given in eq. (5) and exhibits four fermionic excitations with $B' = 0$. A major difference to the vacuum is that at asymptotically high density no adjoint chiral condensate is present and, in particular, all fermionic excitations carry half-integer isospin.

At nuclear matter densities, however, a nonvanishing adjoint chiral condensate cannot be excluded. A Higgs description of nuclear matter with simultaneous condensation of quark-quark and adjoint quark-antiquark pairs can be achieved at moderate densities if sufficiently strong interactions in the respective channels are present. This opens the possibility that nuclear matter in the non-superfluid phase can be continuously connected to the vacuum. At zero temperature nuclear matter is expected to be in a superfluid phase due to nucleon pairing breaking isospin. In the Higgs description the quarks of the third color carry the same quantum numbers as the proton and the neutron. The possible channels for condensation correspond precisely to the known possibilities for pairing in nuclear matter (see also [4]). If isospin is broken there is one exactly massless boson associated to superfluidity with the quantum numbers of the $a_0$-meson. We note that two light scalars correspond to the breaking of the generators $I^\pm$. The masses of the corresponding $a^+ - a^-$ and $a^0$-mesons vanish in the limit of equal up and down quark masses.

**High temperature.** Lattice simulations for two-flavor QCD at high temperature and zero density find that chiral symmetry restoration and deconfinement occur approximately at the same temperature $T_c$ [1]. Sufficiently below $T_c$ the equation of state is rather well approximated by a gas of hadrons. Above $T_c$ the dominant thermodynamic degrees of freedom are gluons and quarks. In the present Higgs description both spontaneous chiral symmetry breaking and confinement are associated to the condensation of quark-antiquark and quark-quark pairs. The transition to the high temperature state proceeds by the melting of these condensates.

Qualitatively different aspects of the transition are described by the differing condensates. The adjoint chiral condensate breaks both chiral symmetry and color up to an abelian $U(1)$ gauge symmetry. If $\chi$ becomes zero at some critical temperature $T_c$ then chiral symmetry is restored and at the same temperature part of the gluons become massless. This provides a simple mechanism for the connection between chiral symmetry restoration and deconfinement in QCD at high temperature.

For $\Delta = 0$ all quarks carry fractional baryon number $B_q = 1/3$. A vanishing diquark condensate signals the hadron-quark transition. For $\chi \neq 0$ the absence of the diquark condensate has a comparably small effect on the gluon sector. Only the $\omega$-meson becomes massless.

Whether or not the hadron–quark transition happens at the temperature when all gluons become massless, i.e. if $\chi$ and $\Delta$ vanish at the same $T_c$, is a question of the detailed dynamics. Also the singlet chiral condensate $\phi$ will play a role for the quantitative aspects of chiral symmetry restoration.

Two flavor QCD is an idealization which is expected to resemble in many aspects realistic QCD. The proposed Higgs description, in particular, predicts intriguing features as the existence of soliton-type excitations. Lattice Monte Carlo simulations can constitute important tests of this picture.

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