Goldstone Superfield Actions in AdS\textsubscript{5} backgrounds

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Abstract

Nonlinear realizations superfield techniques, pertinent to the description of partial breaking of global $N = 2$ supersymmetry in a flat $d = 4$ super Minkowski background, are generalized to the case of partially broken $N = 1$ AdS\textsubscript{5} supersymmetry $SU(2,2|1)$. We present, in an explicit form, off-shell manifestly $N = 1, d = 4$ supersymmetric minimal Goldstone superfield actions for two patterns of partial breaking of $SU(2,2|1)$ supersymmetry. They correspond to two different nonlinear realizations of the latter, in the supercosets with the AdS\textsubscript{5} and AdS\textsubscript{5} $\times S^1$ bosonic parts. The relevant worldvolume Goldstone supermultiplets are accommodated, respectively, by improved tensor and chiral $N = 1, d = 4$ superfields. The second action is obtained from the first one by dualizing the improved tensor Goldstone multiplet into a chiral Goldstone one. In the bosonic sectors, the first and second actions yield static-gauge Nambu-Goto actions for a L3-brane on AdS\textsubscript{5} and a scalar 3-brane on AdS\textsubscript{5} $\times S^1$.

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1 Introduction

The description of superbranes in terms of worldvolume Goldstone superfields based on the concept of partial spontaneous breaking of global supersymmetry (PBGS) \([1, 2]\) is advantageous in many respects. Its main attractive feature is that the corresponding invariant actions reveal manifest off-shell linearly realized worldvolume supersymmetry. The second half of the full supersymmetry is non-linearly realized \([3]-[11]\) (see \([12, 13]\) for a review and further references).

A group-theoretical basis of the PBGS theories is provided by the appropriate versions of the general nonlinear realizations method \([14]\). The PBGS approach has many potential capabilities and implications in string theory, e.g. for constructing non-abelian Born-Infeld actions, as well as their supersymmetric extensions, for describing different possibilities of a non-standard partial supersymmetry breaking (such as \(1/4, 3/4, \ldots\)) and studying the Hamiltonian and quantum structure of the relevant models, etc (see e.g. \([15]-[18]\)).

Most PBGS theories constructed so far correspond to superbranes on flat super Minkowski backgrounds. On the other hand, in the framework of the AdS/CFT correspondence \([19]\), of primary interest are the AdS\(\times S^n\) and pp-wave type \([20]\) superbackgrounds, with \(S^n\) being some curved Riemannian manifold, e.g. a sphere \(S^n\). While Green-Schwarz type actions for branes on such backgrounds are known (see e.g. \([21, 22]\)), not too many explicit examples of the worldvolume superfield PBGS actions were presented. Until now such actions were given only for \(N = 1\) supermembrane in AdS\(_4\) \([23]\), \(N = 1\) superstring in AdS\(_3\) \([24]\) and \(N = 2\) superparticle in AdS\(_2\) \([18]\) (the two latter examples are dimensional reductions of the first one).

The corresponding groups of superisometries coincide with superconformal groups in dimensions lower by 1 (in \(d = 3, d = 2\) and \(d = 1\)), so the construction of these PBGS systems amounts to setting up appropriate non-linear realizations of superconformal symmetries.

It is tempting to construct PBGS versions of superstring and D3-brane on the AdS\(_5\times S^5\) superbackground which is the basic ingredient of the original AdS/CFT conjecture. They should reappear in this context as theories of partial breaking of the \(N = 4\) superconformal group \(SU(2, 2|4)\). It is natural to begin with some their truncations, corresponding to non-linear realizations of the simpler cases of \(N = 1\) and \(N = 2, d = 4\) superconformal groups \(SU(2, 2|1)\) and \(SU(2, 2|2)\). In \([25]\) an attempt to construct a PBGS model for \(SU(2, 2|1)\) was undertaken. This model generalizes that of \([4]\) and should reduce to it in the limit of infinite AdS\(_5\) radius \(R\). Similarly to the model of \([4]\), also its generalization employs a \(N = 1\) chiral superfield as the basic Goldstone one and is expected to describe a scalar 3-brane on AdS\(_5\times S^1\). However, the proper minimal Goldstone superfield action was not constructed in \([23]\), though it was suggested that such an action could be regained like this has been done in the flat case in \([4, 5, 6]\). There, an alternative PBGS action with a linear (tensor) \(N = 1\) Goldstone multiplet was firstly constructed and then it was dualized into an action of \(N = 1\) chiral Goldstone supermultiplet.

The basic aim of the present paper is to carry out such a construction in the case of AdS\(_5\) background.\(^1\) Instead of dealing with a non-linear realization of \(SU(2, 2|1)\) in the standard approach \([14]\), we follow the line of refs. \([4, 5, 6, 23, 9]\). We firstly define a non-standard \(N = 2, d = 4\) superspace of \(SU(2, 2|1)\) which contains Grassmann coordinates associated with both Poincaré and conformal supersymmetry generators. Then we seek for a representation of \(SU(2, 2|1)\) on the chiral and improved tensor \(N = 1\) superfields defined as \(N = 1\) components of a suitable ‘\(N = 2, d = 4\) tensor multiplet’ with a Goldstone-type transformation law. We find that requiring the closure of \(SU(2, 2|1)\) on this set of \(N = 1\) superfields necessarily implies

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\(^1\) Its sketchy exposition was given in \([26]\).
the constraints, which are a generalization of those used in [1] [2] [3]. They allow one to trade the chiral superfield for the improved tensor one and to construct a $SU(2,2|1)$ invariant action of the tensor $N = 1$ Goldstone superfield. The action describes a AdS$_5$ L3-superbrane$^2$ in a static gauge and goes into that of refs. [1] [2] [3], upon taking the $R = \infty$ limit. Then we dualize the Goldstone tensor multiplet into a chiral one and thus obtain the Goldstone superfield action with the AdS$_5 \times S^1$ 3-brane bosonic part, i.e. just that corresponding to the PBGS option analyzed in ref. [25].

This paper is organized as follows. In Section 2 we recall the basic properties and existing superfield formulations of the $N = 2,d = 4$ tensor multiplet which plays a central role in our construction. In Section 3 we repeat, in a somewhat different setting, the construction of the PBGS superfield action for the $N = 1$ Goldstone tensor multiplet in a flat $d = 4$ super Minkowski background. We demonstrate that the requirement of covariance under the nonlinearly realized $d = 5$ Lorentz $SO(1,4)$ symmetry provides an alternative way of deducing the constraints which should be imposed [4] on the linear tensor constraints which properly generalize those of the flat background. The resulting nonlinear realization is the genuine generalization of the flat case one, and the $SU(2,2|1)$ invariant action of the Goldstone improved tensor $N = 1$ superfield can be constructed in a similar fashion. In Section 5 we perform a duality transformation of the improved tensor Goldstone $N = 1$ superfield action we have constructed and arrive at the invariant action of a chiral Goldstone $N = 1$ superfield. The latter properly generalizes the minimal Goldstone superfield action of refs. [3] [4]. An outline of some open problems is the content of the concluding Section.

2 $N=2$ tensor multiplet

Since the central role in our study is played by a proper generalization of the $N = 2$ tensor multiplet, we start with a brief recapitulation of the basic properties of the multiplet and its different formulations.

In the standard $N = 2,d = 4$ superspace $z = (x^{\alpha \dot{\alpha}}, \theta^i \bar{\theta}^i)$ the $N = 2$ tensor multiplet is described by an isotriplet of scalar fields $L^{ij}(z)$ satisfying the constraints [28]

$$D^i_\alpha L^{jk} = 0, \quad \bar{D}^{(i}L^{jk)} = 0, \quad (L^{ij})^\dagger = -\epsilon_{ik}\epsilon_{jl}L^{kl}, \quad (2.1)$$

where$^3$

$$D^i_\alpha = \frac{\partial}{\partial \theta^i_\alpha} + i\bar{\theta}^{\dot{\alpha}}\partial_{\alpha \dot{\alpha}}, \quad \bar{D}_{\dot{\alpha}i} = -\frac{\partial}{\partial \bar{\theta}^i_\alpha} - i\theta^{\alpha}_i\partial_{\dot{\alpha} \alpha}, \quad (\theta^i_\alpha)^\dagger = \bar{\theta}^{\dot{\alpha} i}. \quad (2.2)$$

It is convenient to introduce two sets of Grassmann coordinates and covariant derivatives

$$(\theta_{\alpha 1}, \bar{\theta}_{\dot{\alpha} 1}) \equiv (\theta_{\alpha}, \bar{\theta}_{\dot{\alpha}}), \quad (D^1_\alpha, \bar{D}_{\dot{\alpha} 1}) \equiv (D_\alpha, \bar{D}_{\dot{\alpha}}),$$

$$(\theta_{\alpha 2}, \bar{\theta}_{\dot{\alpha} 2}) \equiv (\xi_{\alpha}, \bar{\xi}_{\dot{\alpha}}), \quad (D^2_\alpha, \bar{D}_{\dot{alpha}}) \equiv (\nabla_\alpha, \nabla_{\dot{\alpha}}). \quad (2.3)$$

$^2$See [27] for the relevant nomenclature.

$^3$We mostly use the conventions of [29].
and rewrite (2.1) as follows:

\[
\begin{align*}
D_\alpha F &= 0, \\
D_\alpha L - \nabla_\alpha F &= 0, \\
\nabla_\alpha L + D_\alpha \bar{F} &= 0, \\
\nabla_\alpha F &= 0,
\end{align*}
\]

where

\[
L \equiv 2L^{12} = \bar{L}, \quad F \equiv L^{11}, \quad \bar{F} \equiv -L^{22}.
\]

One observes that the covariant derivatives of all superfields \(\{L, F, \bar{F}\}\) with respect to \(\xi, \bar{\xi}\) are expressed from (2.4) as covariant derivatives with respect to \(\theta, \bar{\theta}\). Therefore, the only independent \(N = 1\) superfield components of these \(N = 2\) superfields are the \(\xi = \bar{\xi} = 0\) components of the latter subjected to the constraints (which also follow from (2.4))

\[
D_\bar{\alpha} F = 0, \quad D_\alpha F = 0, \quad D^2 L = D^2 \bar{L} = 0.
\]

Hence, they are the \(N = 1\) chiral \((F, \bar{F})\) and tensor \((L)\) superfields.

Two other ways to describe the \(N = 2\) tensor multiplet are the formulations in \(N = 2\) harmonic \([30, 29]\) and projective \([31]\) superspaces. Such formulations are basically equivalent, as shown in \([32]\). However, the projective superspace formulation is more suitable for our eventual purpose of constructing PBGS \(N = 1\) Goldstone superfield actions, because it allows one to pass easily to \(N = 1\) superfield formulations.

Projective superspace includes one additional complex bosonic \(\mathbb{C}P^1\) coordinate \(\omega\). One defines

\[
D_\omega^\alpha = \omega D_\alpha - \nabla_\alpha, \quad \bar{D}_\omega^\bar{\alpha} = \bar{D}_{\bar{\alpha}} + \omega \nabla_{\bar{\alpha}},
\]

\[
\{D_\omega^\alpha, D_\beta^\omega\} = 0, \quad \{D_\alpha^\omega, D_\beta^\omega\} = 0, \quad \{D_\omega^\alpha, D_\beta^\omega\} = 0,
\]

and rewrites the original constraints (2.1) as

\[
D_\omega^\alpha \mathcal{L}^\omega = 0, \quad \bar{D}_\omega^\bar{\alpha} \mathcal{L}^\omega = 0,
\]

where

\[
\mathcal{L}^\omega = \omega F + L - \frac{\bar{F}}{\omega}.
\]

Thus, the components of \(\mathcal{L}^\omega\) are effectively \(N = 1\) superfields and one can construct \(N = 2\) invariants which look much like \(N = 1\) ones and contain integration only over \(N = 1\) superspace.

### 3 Goldstone \(N=1\) tensor multiplet in a flat background

The idea to utilize the \(N = 1\) tensor multiplet as the Goldstone one for describing the partial breaking of global \(N = 2, d = 4\) Poincaré supersymmetry down to \(N = 1\) has been worked out in \([4, 5, 6]\). The basic ingredients of this construction are, first of all, an appropriate modification of the linear \(N = 2\) tensor multiplet and, secondly, the covariant constraints imposed on the latter, in order to end up with a nonlinear realization of the \(N = 2 \rightarrow N = 1\) PBGS pattern in terms of a single Goldstone \(N = 1\) tensor multiplet. The corresponding Goldstone superfield action is a manifestly \(N = 1\) supersymmetric worldvolume form of the action of \(N = 1, d = 5\).
L3-brane in a flat background. Here we reproduce this construction in a different setting, which admits a direct generalization to the case we are mainly interested in, i.e. the partial breaking of the $SU(2,2|1)$ supersymmetry down to the $N = 1, d = 4$ Poincaré supersymmetry. In passing, we show that the constraints of refs. [4, 5, 6] naturally arise from the requirement of 5-dimensional Lorentz $SO(1, 4)$ covariance.

We basically follow the line of refs. [3, 4]. As a first step, we should define a ‘linear’ version of $N = 2 \rightarrow N = 1$ PBGS in $N = 2, d = 4$ superspace with $N = 2$ tensor multiplet as the relevant Goldstone one. We start with $N = 2, d = 4$ Poincaré superalgebra extended by a real central charge $D$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2P_{a\dot{a}}, \{S_\alpha, \bar{S}_{\dot{\alpha}}\} = 2P_{a\dot{a}}, \{Q_\alpha, S_\beta\} = -\varepsilon_{\alpha\beta}D, \{\bar{Q}_{\dot{\alpha}}, \bar{S}_{\dot{\beta}}\} = -\varepsilon_{\dot{\alpha}\dot{\beta}}D. \quad (3.1)$$

Here $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ and $S_\alpha, \bar{S}_{\dot{\alpha}}$ are generators of unbroken and broken $N = 1$ supersymmetries, respectively. The latter generators and the 4-translation generator $P_{a\dot{a}}$ possess standard commutation relations with the Lorentz $so(1, 3)$ generators $(M_{\alpha\beta}, \dot{M}_{\dot{\alpha}\dot{\beta}})$

$$i[M_{\alpha\beta}, M_{\rho\sigma}] = \varepsilon_{\alpha\rho}M_{\beta\sigma} + \varepsilon_{\alpha\sigma}M_{\beta\rho} + \varepsilon_{\beta\rho}M_{\alpha\sigma} + \varepsilon_{\beta\sigma}M_{\alpha\rho} \equiv (M)_{\alpha\beta, \rho\sigma},$$

$$i[M_{\dot{\alpha}\dot{\beta}}, \dot{M}_{\dot{\rho}\dot{\sigma}}] = (\dot{M})_{\dot{\alpha}\dot{\beta}, \dot{\rho}\dot{\sigma}}, \quad i[M_{\alpha\beta}, P_{\rho}] = \varepsilon_{\alpha\rho}P_{\beta} + \varepsilon_{\beta\rho}P_{\alpha},$$

$$i[M_{\dot{\alpha}\dot{\beta}}, P_{\dot{\rho}}] = \varepsilon_{\dot{\rho}\dot{\beta}}P_{\dot{\rho}} + \varepsilon_{\dot{\rho}\dot{\alpha}}P_{\dot{\rho}}, \quad i[M_{\alpha\beta}, Q_{\gamma}] = \varepsilon_{\alpha\gamma}Q_{\beta} + \varepsilon_{\beta\gamma}Q_{\alpha} \equiv (Q)_{\alpha\beta, \gamma},$$

$$i[M_{\dot{\alpha}\dot{\beta}}, \dot{S}_{\dot{\gamma}}] = (\dot{S})_{\dot{\alpha}\dot{\beta}, \dot{\gamma}}, \quad i[\dot{M}_{\dot{\alpha}\dot{\beta}}, S_{\gamma}] = (S)_{\dot{\alpha}\dot{\beta}, \gamma}. \quad (3.2)$$

Our basic superfield is $\mathcal{L}^\omega$, eq. (2.10). We associate it with the generator $D$ as the relevant coset parameter, i.e. as a Goldstone $N = 2$ superfield. Together with the $N = 2$ superspace coordinates $\{x^{a\dot{\alpha}}, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}, \xi^\alpha, \bar{\xi}_{\dot{\alpha}}\}$, the latter parameterizes the coset space of $N = 2$ Poincaré supergroup over its $d = 4$ Lorentz subgroup $SO(1, 3)$

$$g = e^{-ix^{a\dot{\alpha}}P_{a\dot{\alpha}} + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}_{\dot{\alpha}}} e^{i\xi^\alpha S_\alpha + i\bar{\xi}_{\dot{\alpha}}\dot{S}_{\dot{\alpha}}} e^{i\mathcal{L}^\omega D}. \quad (3.3)$$

The $\mathbb{C}P^1$ co-ordinate $\omega$ on which $\mathcal{L}^\omega$ depends can be regarded, like harmonic coordinates, to parameterize a coset of the $R$-symmetry group $SU(2)$. However, this internal symmetry is explicitly broken in the $N = 2 \rightarrow N = 1$ PBGS actions [4]. For this reason in what follows we shall not be interested in it. We treat $\omega$ as an external parameter, which allows one to incorporate all superfield components of the $N = 2$ tensor multiplet into the coset geometry. Thus, in the present case, we are dealing with a special coset realization of $N = 2$ supersymmetry. We call it ‘linear’ because, though the $S$-supersymmetry and $D$-transformations of the Goldstone superfield $\mathcal{L}^\omega$ are inhomogeneous, they do not exhibit any nonlinearity.

The full set of $N = 2$ super Poincaré transformations of the coset parameters in (3.3) can be found by acting on (3.3) from the left by various group elements. Most relevant are the $Q$ and $S$ supersymmetry transformations.

Unbroken $Q$ supersymmetry $(g_0 = e^{i\eta^\alpha Q_\alpha + i\bar{\eta}_{\dot{\alpha}}\bar{Q}_{\dot{\alpha}}})$ reads:

$$\delta x^{a\dot{\alpha}} = -i \left( \eta^{\alpha} \bar{\theta}_{\dot{\alpha}} + \bar{\eta}^{\dot{\alpha}} \theta^\alpha \right), \quad \delta \theta^\alpha = \epsilon^\alpha, \quad \delta \bar{\theta}_{\dot{\alpha}} = \bar{\epsilon}_{\dot{\alpha}}. \quad (3.4)$$

Broken $S$ supersymmetry $(g_0 = e^{i\eta^\alpha S_\alpha + i\bar{\eta}_{\dot{\alpha}}\dot{S}_{\dot{\alpha}}})$ reads:

$$\delta x^{a\dot{\alpha}} = -i \left( \eta^{\alpha} \bar{\xi}_{\dot{\alpha}} + \bar{\eta}^{\dot{\alpha}} \xi^\alpha \right), \quad \delta \xi^\alpha = \eta^\alpha, \quad \delta \bar{\xi}_{\dot{\alpha}} = \bar{\eta}^{\dot{\alpha}}, \quad \delta \mathcal{L}^\omega = -i \left( \theta \cdot \eta - \bar{\theta} \cdot \bar{\eta} \right). \quad (3.5)$$
From (3.4) and (3.5) one observes that the ‘active’ form of the transformations of our basic superfield $L^\omega$ is standard with respect to the $Q$ supersymmetry

$$\delta_Q^* L^\omega = \epsilon^\alpha Q_\alpha L^\omega + \bar{\epsilon}_a \bar{Q}^\alpha L^\omega$$

(3.6)

and is modified by a $\theta$-dependent shift under $S$ supersymmetry

$$\delta_S L^\omega = -i \left( \theta \cdot \eta - \bar{\theta} \cdot \bar{\eta} \right) + \eta^\alpha S_\alpha L^\omega + \bar{\eta}_\dot{a} \bar{S}^\dot{a} L^\omega.$$  

(3.7)

Here

$$Q_\alpha = -\frac{\partial}{\partial \theta^\alpha} + i\bar{\theta}^\dot{a} \partial_{a\dot{a}} , \quad \bar{Q}_\dot{a} = \frac{\partial}{\partial \bar{\theta}^\dot{a}} - i\theta^a \partial_{a\dot{a}} , \quad S_\alpha = -\frac{\partial}{\partial \bar{\theta}^\dot{a}} + i\bar{\theta}^\dot{a} \partial_{a\dot{a}} , \quad \bar{Q}_\dot{a} = \frac{\partial}{\partial \theta^\alpha} - i\bar{\theta}^\dot{a} \partial_{a\dot{a}} .$$

(3.8)

This modification, being independent of $\omega$, affects only the transformation law of $L$, leaving that of $F, \bar{F}$ in its standard form. Nevertheless, this modification is crucial. Firstly, it implies that the spinor derivatives of $L$ with respect to $\theta, \bar{\theta}$, i.e. $D_\alpha L$ and $\bar{D}_\dot{a} L$, are shifted by Grassmann parameters $\eta_\alpha, \bar{\eta}_\dot{a}$ under $S$ supersymmetry and so are Goldstone fermions for the considered linear realization of the $N = 2 \rightarrow N = 1$ PBGS (the first component of $L$ is the Goldstone field for the broken 5th translation with the generator $D$). Secondly, due to this modification, the basic constraints of $N = 2$ tensor multiplet (2.4) cease to be covariant and should also be properly modified.

In order to find a proper deformation of (2.4), one should construct the covariant derivatives of the Goldstone superfield $L^\omega$ by the standard methods of nonlinear realizations [14]. First, one calculates the Cartan forms

$$g^{-1} dg = i\omega^{\alpha\dot{a}} P_{\alpha\dot{a}} + i\mu \cdot Q + i\bar{\mu} \cdot \bar{Q} + i\nu \cdot S + i\bar{\nu} \cdot \bar{S} + i\omega_D D + i\omega_M M + i\bar{\omega}_M \bar{M} ,$$

(3.9)

$$\omega^{\alpha\dot{a}} = -dx^{\alpha\dot{a}} + i d\bar{\theta}^{\dot{a}} \theta^\alpha + i d\theta^\alpha \bar{\theta}^{\dot{a}} + i d\xi^{\alpha\dot{a}} \xi^\alpha + i d\bar{\xi}^{\dot{a}\alpha} \bar{\xi}^\dot{a} , \quad \nu^\alpha = d\xi^\alpha , \quad \bar{\nu}^{\dot{a}} = d\bar{\xi}^{\dot{a}} ,$$

$$\mu^\alpha = d\theta^\alpha + \bar{\xi}_\dot{a} dx^{\alpha\dot{a}} , \quad \bar{\mu}^{\dot{a}} = d\bar{\theta}^{\dot{a}} + \xi^\alpha dx^{\alpha\dot{a}} , \quad \omega_M^\alpha = \bar{\omega}_M^{\dot{a}} = 0 ,$$

$$\omega_D = dL^\omega + i \left( \xi \cdot d\theta - \bar{\xi} \cdot d\bar{\theta} \right).$$

(3.10)

Next one defines the covariant derivatives of $L^\omega$ as

$$dL^\omega = -\omega^\alpha_{\alpha\dot{a}} \partial_{a\dot{a}} L^\omega + \mu^\alpha D^\alpha L^\omega - \bar{\mu}^{\dot{a}} \bar{D}_\dot{a} L^\omega + \nu^\alpha D^\alpha L^\omega - \bar{\nu}^{\dot{a}} \bar{D}_\dot{a} L^\omega ,$$

(3.11)

where

$$D^\alpha L^\omega = D_\alpha L^\omega + i\xi^\alpha , \quad \bar{D}^\dot{a} L^\omega = \bar{D}_\dot{a} L^\omega - i\bar{\xi}_\dot{a} , \quad D^\xi L^\omega = \nabla_\alpha L^\omega , \quad \bar{D}^\bar{\xi} L^\omega = \bar{\nabla}_\dot{a} L^\omega .$$

(3.12)

Thus, only the spinor derivatives with respect to $\theta, \bar{\theta}$ get modified. Once again, this modification affects only the superfield $L$ in $L^\omega$, while the covariant derivatives of $F, \bar{F}$ retain their previous form.

Now one can write the covariant constraints defining the $N = 2$ tensor multiplet with the modified transformations properties (3.6), (3.7) by substituting the covariant derivatives (3.12) in (2.9) or (2.4) for the previous ones. This gives

$$\begin{cases}
D_\alpha F = 0 , \\
D_\alpha L + i\xi_\alpha - \nabla_\alpha F = 0 , \\
\nabla_\alpha L + D_\alpha \bar{F} = 0 , \\
D_\alpha F = 0 , \\
\bar{\nabla}_\dot{a} F = 0 , \\
\bar{\nabla}_\dot{a} L + \bar{D}_\dot{a} F = 0 , \\
D_\alpha L - i\bar{\xi}_\dot{a} - \nabla_\alpha \bar{F} = 0 , \\
\bar{D}_\dot{a} F = 0 .
\end{cases}$$

(3.13)
The $S$-supersymmetry transformations (3.5) of the $N = 2$ tensor multiplet $\mathcal{L}^\omega$ induce the following transformations for the $N = 1$ superfield components $L, F, \bar{F}$ (with the constraints (3.13) taken into account)

$$\delta L = -i \left( \eta^\alpha \theta_\alpha - \bar{\eta}^\dot{\alpha} \bar{\theta}^{\dot{\alpha}} \right) + \eta^\alpha D_\alpha \bar{F} - \bar{\eta}^\dot{\alpha} \bar{D}_{\dot{\alpha}} F ,$$

$$\delta F = -\eta^\alpha D_\alpha L , \quad \delta \bar{F} = \bar{\eta}^\dot{\alpha} \bar{D}_{\dot{\alpha}} L .$$

(3.14)

They of course coincide with those given in [4]. The superfields $L$ and $F, \bar{F}$ are subjected to the same $N = 1$ constraints (2.6), which remain covariant with respect to the modified transformations (3.14). The entire set of Goldstone fields (the goldstino for the $N = 2 \to N = 1$ breaking and the Goldstone field for the broken $D$ translations) is now accommodated by the Goldstone $N = 1$ superfield $L$.

It is evident from (3.14) that one can construct an invariant ‘action’ as follows

$$S = \frac{1}{4} \int d^4x d^2\theta \bar{F}^2 + \frac{1}{4} \int d^4x d^2\bar{\theta} F^2 .$$

(3.15)

In order to make it meaningful, one should express the chiral supermultiplet $F, \bar{F}$ in terms of the Goldstone tensor multiplet $L$ by imposing proper covariant constraints. These additional constraints were simply guessed in [4] and later re-derived in [5] from the nilpotency conditions imposed on the appropriate $N = 2$ superfield. They read

$$F = -\frac{D^\alpha L D_\alpha L}{2 - D^2 F} , \quad \bar{F} = -\frac{\bar{D}_{\dot{\alpha}} L D^{\dot{\alpha}} L}{2 - \bar{D}^2 \bar{F}}$$

(3.16)

and can be easily solved [4, 5]

$$F = -\psi^2 + \frac{1}{2} D^2 \left[ \frac{\psi^2 \bar{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}} \right] ,$$

(3.17)

where

$$\psi_\alpha \equiv D_\alpha L , \quad \bar{\psi}_{\dot{\alpha}} \equiv \bar{D}_{\dot{\alpha}} L ,$$

$$A = \frac{1}{2} \left( D^2 \bar{\psi}^2 + \bar{D}^2 \psi^2 \right) , \quad B = \frac{1}{2} \left( D^2 \bar{\psi}^2 - \bar{D}^2 \psi^2 \right) .$$

(3.18)

Finally, the action (3.15) becomes

$$S = -\frac{1}{4} \int d^4x d^2\theta \psi^2 - \frac{1}{4} \int d^4x d^2\bar{\theta} \bar{\psi}^2 + \frac{1}{4} \int d^4x d^4\theta \frac{\psi^2 \bar{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}} .$$

(3.19)

It is a nonlinear extension of the standard $N = 1$ tensor multiplet action. In the bosonic sector it gives rise to the static-gauge Nambu-Goto action for a L3-brane in $d = 5$ Minkowski space, with one physical scalar of $L$ being the transverse brane coordinate and another one being represented by the notoph field strength. After dualizing $L$ into a pair of conjugated chiral and antichiral $N = 1$ superfields (the notoph strength is dualized into a scalar field), the PBGS form of the static-gauge action of super 3-brane in $d = 6$ is reproduced [4].

Now we would like to demonstrate that the constraints (3.16) which play a central role in deriving the action (3.15) are intimately related to the 5-dimensional nature of the brane under consideration. They can be derived from the requirement of 5-dimensional Lorentz covariance.
Indeed, in order to find a proper place for the basic superfield $L^\omega$ in the coset space, the $N = 2, d = 4$ Poincaré superalgebra has been extended by a central charge generator $D$, eqs. (3.1). This generator can be treated as the generator of translations in the 5th direction and the full automorphism algebra of (3.1) can be checked to be $so(1,4)$ (we ignore the $R$-symmetry $SU(2)$ automorphisms). The $d = 5$ Lorentz algebra $so(1,4)$ includes, besides the $d = 4$ Lorentz generators $M_{\alpha \beta}, M_{\alpha \dot{\beta}}$, an additional $d = 4$ vector $K_{\alpha \dot{\alpha}}$ belonging to the coset $SO(1,4)/SO(1,3)$. The full set of the additional commutation relations reads as follows:

\begin{align*}
i \left[ M_{\alpha \beta}, K_{\rho \dot{\beta}} \right] &= \varepsilon_{\alpha \rho} K_{\beta \dot{\beta}} + \varepsilon_{\beta \rho} K_{\alpha \dot{\beta}} , \\
i \left[ M_{\alpha \dot{\alpha}}, K_{\rho \dot{\beta}} \right] &= -\varepsilon_{\alpha \rho} M_{\dot{\alpha} \dot{\beta}} - \varepsilon_{\dot{\alpha} \rho} M_{\alpha \dot{\beta}} , \\
i [ D, K_{\alpha \dot{\alpha}} ] &= 2 P_{\alpha \dot{\alpha}} , \\
i [ P_{\alpha \dot{\alpha}}, K_{\beta \dot{\beta}} ] &= \varepsilon_{\alpha \beta} \varepsilon_{\dot{\alpha} \dot{\beta}} D , \\
i [ K_{\alpha \dot{\alpha}}, Q_{\beta \dot{\beta}} ] &= -\varepsilon_{\alpha \beta} \bar{S}_{\dot{\alpha}} , \\
i [ K_{\alpha \dot{\alpha}}, S_{\beta \dot{\beta}} ] &= \varepsilon_{\alpha \beta} \bar{Q}_{\dot{\alpha}} ,
\end{align*}

(3.20)

One can ask whether the linear realization of the $N = 2 \rightarrow N = 1$ PBGS with the Goldstone superfield $L^\omega$ constructed above is compatible with this extra $SO(1,4)$ covariance. The $SO(1,4)/SO(1,3)$ transformations of the superspace coordinates and superfield $L^\omega$ can be easily found from the left action of the group element $g_0 = e^{iaaK_{\alpha \dot{\alpha}}}$ on the coset element (3.3):

\begin{align*}
\delta x^{a\dot{\alpha}} &= 2a^{a\dot{\alpha}} L^\omega + ia^{a\dot{\alpha}} \left( \theta_{\beta \dot{\alpha}} \xi^\alpha + \theta^\alpha \xi_{\beta \dot{\alpha}} \right) + ia^{a\dot{\alpha}} \left( \bar{\theta}_{\dot{\beta} \alpha} \bar{\xi}^{\dot{\alpha}} + \bar{\theta}^{\dot{\alpha}} \bar{\xi}_{\alpha \dot{\beta}} \right) , \\
\delta \theta^\alpha &= -ia^{a\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} , \\
\delta \bar{\theta}_{\dot{\alpha}} &= -ia^{a\dot{\alpha}} \xi^\alpha , \\
\delta \xi^\alpha &= -ia^{a\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} , \\
\delta \bar{\xi}^{\dot{\alpha}} &= -ia^{a\dot{\alpha}} \theta^\alpha .
\end{align*}

(3.21)

Now one should verify whether the defining constraints (3.13) are consistent with the transformations (3.21). An equivalent but simpler check consists in passing to the $N = 1$ superfield components $L, F$ and $\bar{F}$ and examining whether their $SO(1,4)$ transformations are consistent with the constraints (2.6). The active form of $SO(1,4)$ transformations of these $N = 1$ superfields can be found from (3.21) with taking into account the $\omega$ dependence of $L^\omega$ in (2.10):

\begin{align*}
\delta^* L &= a_{a\dot{a}} x^{a\dot{\alpha}} - a^{a\dot{\alpha}} \partial_{a\dot{a}} (L^2 - 2FF) + ia^{a\dot{\alpha}} \theta_{a} D_{\dot{\alpha}} F - ia^{a\dot{\alpha}} \bar{\theta}_{\dot{a}} D_{a} F , \\
\delta^* F &= -2a^{a\dot{\alpha}} \partial_{a\dot{a}} (FL) + ia^{a\dot{\alpha}} \bar{\theta}_{\dot{a}} D_{a} L , \\
\delta^* \bar{F} &= -2a_{a\dot{a}} \partial_{a\dot{a}} (\bar{F}L) - ia^{a\dot{\alpha}} \theta_{a} D_{\dot{\alpha}} L .
\end{align*}

(3.22)

Note that these transformations are already essentially nonlinear, compared with the $S$- and $D$ transformations. Then the chirality of $F, \bar{F}$ implies

\begin{align*}
D_{\alpha} \bar{F} = \bar{D}_{\dot{\alpha}} F &= 0 \quad \Rightarrow \quad \partial_{a\dot{\alpha}} \left( FD_{\beta} L \right) = \partial_{a\dot{\alpha}} \left( \bar{F}D_{\dot{\beta}} L \right) = 0 ,
\end{align*}

(3.23)

while the $N = 1$ tensor multiplet constraint yields one more condition

\begin{align*}
D^2 L &= 0 \quad \Rightarrow \quad a^{a\dot{\alpha}} \partial_{a\dot{\alpha}} \left[ D^2 \left( L^2 - 2FF \right) + 4F \right] = 0 .
\end{align*}

(3.24)

These conditions obviously cannot be satisfied with independent $N = 1$ superfields $L$ and $F, \bar{F}$. At the same time, it is straightforward to see that the constraints (3.16) solve both (3.20) and (3.24). Thus the implementation of the $d = 5$ $SO(1,4)$ Lorentz covariance can be achieved only provided we impose the constraint (3.16) on our $N = 2$ tensor multiplet, i.e. within the nonlinear realization framework. This observation is crucial for the AdS case, where all generators of the automorphism group appear in the anticommutators of $Q$ and $S$ supersymmetries. This case is the subject of the next Section.
adS\textsubscript{5} background: Goldstone N=1 improved tensor multiplet

In the previous Section we have shown that the implementation of the automorphism SO(1, 4) symmetry in the framework of a ‘linear’ realization of spontaneously broken N = 2 supersymmetry puts additional strong constraints on the Goldstone N = 2 tensor multiplet, giving rise to the genuine nonlinear realization in terms of the Goldstone N = 1 tensor multiplet pioneered in \cite{4}. Here we exploit this observation, in order to construct a nonlinear realization describing the partial 1/2 breaking of the simplest adS\textsubscript{5} supersymmetry SU(2, 2|1), with a suitable generalization of the N = 1 tensor multiplet as a Goldstone one.

The superalgebra su(2, 2|1) contains a so(2, 4) × u(1) bosonic subalgebra with generators \{P_{a\dot{a}}, M_{\alpha\beta}, M_{\dot{\alpha}\dot{\beta}}, K_{a\dot{a}}, D\} and \{J\} and eight supercharges \{Q_\alpha, \bar{Q}_{\dot{\alpha}}, S_\alpha, \bar{S}_{\dot{\alpha}}\}. It can be considered either as a N = 1 superconformal algebra in d = 4 or as the supersymmetry of superspaces with the adS\textsubscript{5} or adS\textsubscript{5} × S\textsuperscript{1} bosonic bodies (depending on whether the \gamma\textsubscript{5} generator J is placed in the stability subgroup or in the coset). We choose the basis in such a way, that the generators K\textsubscript{a\dot{a}} form a so(1, 4) subalgebra together with the d = 4 Lorentz generators \{M_{\alpha\beta}, M_{\dot{\alpha}\dot{\beta}}\}, as in the first two lines of (3.20). The remaining non-trivial commutators read:

\[
i [M_{\alpha\beta}, P_{\rho\dot{\rho}}] = \varepsilon_{\alpha\rho}P_{\beta\dot{\rho}} + \varepsilon_{\beta\rho}P_{\alpha\dot{\rho}},
\]
\[
i [D, P_{a\dot{a}}] = m P_{a\dot{a}}, \ i [D, K_{a\dot{a}}] = 2 P_{a\dot{a}} - m K_{a\dot{a}},
\]
\[
i [P_{a\dot{a}}, K_{\beta\dot{\beta}}] = \varepsilon_{\alpha\beta}\varepsilon_{\dot{a}\dot{\beta}}D - \frac{m}{2} (\varepsilon_{\alpha\beta} M_{\dot{a}\dot{\beta}} + \varepsilon_{\dot{a}\dot{\beta}} M_{\alpha\beta}).
\]
\[
\{Q_\alpha, Q_\beta\} = -\varepsilon_{\alpha\beta}(D + imJ) + m M_{\alpha\beta}, \ \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = -\varepsilon_{\dot{\alpha}\dot{\beta}}(D - imJ) + m M_{\dot{\alpha}\dot{\beta}},
\]
\[
\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2P_{a\dot{a}}, \ \{S_\alpha, \bar{S}_{\dot{\alpha}}\} = 2P_{a\dot{a}} - 2m K_{a\dot{a}},
\]
\[
i [M_{\alpha\beta}, Q_\gamma] = \varepsilon_{\alpha\gamma} Q_\beta + \varepsilon_{\beta\gamma} Q_\alpha \equiv (Q)_{\alpha\gamma\beta}, \ i [M_{\alpha\beta}, S_\gamma] = (S)_{\alpha\gamma\beta},
\]
\[
i [D, Q_\alpha] = \frac{m}{2} Q_\alpha, \ i [D, \bar{Q}_{\dot{\alpha}}] = \frac{m}{2} \bar{Q}_{\dot{\alpha}}, \ i [D, S_\alpha] = \frac{-m}{2} S_\alpha, \ i [D, \bar{S}_{\dot{\alpha}}] = \frac{-m}{2} \bar{S}_{\dot{\alpha}},
\]
\[
[J, Q_\alpha] = \frac{3}{2} Q_\alpha, \ [J, \bar{Q}_{\dot{\alpha}}] = \frac{-3}{2} \bar{Q}_{\dot{\alpha}}, \ [J, S_\alpha] = \frac{3}{2} S_\alpha, \ [J, \bar{S}_{\dot{\alpha}}] = \frac{3}{2} \bar{S}_{\dot{\alpha}},
\]
\[
i [K_{a\dot{a}}, Q_\beta] = -\varepsilon_{\alpha\beta} \bar{S}_{\dot{\alpha}}, \ i [K_{a\dot{a}}, \bar{Q}_{\dot{\beta}}] = -\varepsilon_{\dot{\alpha}\dot{\beta}} S_\alpha, \ i [K_{a\dot{a}}, S_\beta] = \varepsilon_{\alpha\beta} Q_\alpha,
\]
\[
i [K_{a\dot{a}}, \bar{S}_{\dot{\beta}}] = \varepsilon_{\dot{\alpha}\dot{\beta}} Q_\alpha, \ i [P_{a\dot{a}}, S_\beta] = m \varepsilon_{\alpha\beta} \bar{Q}_{\dot{\alpha}}, \ i [P_{a\dot{a}}, \bar{S}_{\dot{\beta}}] = m \varepsilon_{\dot{\alpha}\dot{\beta}} Q_\alpha.
\]

This provides an example of the ‘AdS basis’ of conformal superalgebras \cite{33, 34, 23, 17, 18} which perfectly suits their interpretation as the superisometry groups of the appropriate AdS superspaces. Indeed, the generators \{P_{a\dot{a}}, D, J\} form a maximal solvable bosonic subgroup in su(2, 2|1) and span the coset SO(2, 4)/SO(1, 4) × U(1) ∼ adS\textsubscript{5} × S\textsuperscript{1}. The parameter m has the meaning of the inverse AdS\textsubscript{5} radius, m = R\textsuperscript{-1}. In the limit m = 0 (R = ∞) \cite{11} goes into the N = 1, d = 5 Poincaré superalgebra considered in the previous Section, with D becoming the generator of translations along the 5th dimension. The generators J and K\textsubscript{a\dot{a}}, M\textsubscript{\alpha\beta}, M\textsubscript{\dot{\alpha}\dot{\beta}} decouple and generate outer u(1) ⊕ so(1, 4) automorphisms.

Our goal is to construct an adS\textsubscript{5} version of the Goldstone N = 2 tensor supermultiplet and then properly generalize the constraints \cite{33, 10}. In terms of N = 1 superfields this version is expected to involve some modification of the N = 1 tensor multiplet L and, as before, a pair of mutually conjugated N = 1 chiral and anti-chiral superfields F, \bar{F}. On these N = 1 superfields we wish to realize an additional supersymmetry, such that it forms, together with the manifest N = 1 supersymmetry, just the adS\textsubscript{5} SU(2, 2|1) supersymmetry. Besides, in a close analogy
with the flat case, we wish that the following ‘action’:

\[ S = \frac{1}{4} \int d^4 xd^2 \theta F + \frac{1}{4} \int d^4 xd^2 \bar{\theta} F \]  \hspace{1cm} (4.2)\\

be an invariant of the AdS supersymmetry. Since the right-chiral integration measure \( d^4 xd^2 \theta \) has a \( D \) weight \(-3m\) and, with our normalization of \( J \), a \( U(1) \) charge \(-3\), the superfield \( F \) should carry \( D \) and \( J \) weights equal to \( 3m \) and \( 3 \) (\( \bar{F} \) has the same \( D \) weight and a \( J \) charge equal to \(-3\)). Such an assignment will be a useful hint in finding out the \( SU(2,2|1) \) transformation laws of \( L, \bar{F} \) and the appropriate constraints.

Our further strategy is similar, in its basic points, to what we did in the flat case. The latter should be recovered as the \( m = 0 \) limit of the \( SU(2,2|1) \) construction, what provides us with one more hint. We first define a non-standard coset realization of \( SU(2,2|1) \) in a \( N = 2, d = 4 \) superspace by choosing \( SO(1,3) \times U(1) \) as the stability subgroup and associating two sets of Grassmann coordinates \((\theta^\alpha, \bar{\theta}^\dagger)\) and \((\xi^\alpha, \bar{\xi}^\dagger)\) with the generators \((Q_\alpha, \bar{Q}_\dagger)\) and \((S_\alpha, \bar{S}_\dagger)\). The coset parameters corresponding to the \( SO(2,4)/SO(1,4) \) generators \( P_{\alpha \dagger} \) and \( D \) are the 4-coordinate \( x^{\alpha \dagger} \) and the Goldstone \( N = 2 \) superfield \( \mathcal{L}^\omega(x, \omega, \theta, \xi) \). We need not specify how the latter depends on \( \omega \). The only assertion to be exploited is that an analog of the inhomogeneously transforming \( N = 2 \) superfield \( L \) of the flat case is still given by the \( \omega \)-independent part of \( \mathcal{L}^\omega \) (cf. \( (2.10) \))

\[ \mathcal{L}^\omega = L(x, \theta, \xi) + \mathcal{F}(\omega, \frac{1}{\omega}) \]  \hspace{1cm} (4.3)\\

Thus, the supercoset element we start with is basically of the same form \((3.3)\) as in the flat case.

The coset space techniques allow us to find the transformation properties of the \( N = 1 \) superspace coordinates \((x^{\alpha \dagger}, \theta^\alpha, \bar{\theta}^\dagger)\) and the remaining coset parameters \((\xi^\alpha, \bar{\xi}^\dagger, \mathcal{L}^\omega)\) under \( S \)-supersymmetry. For our purposes, it is enough to know them to zeroth order in \((\xi^\alpha, \bar{\xi}^\dagger)\)

\[ \delta x^{\alpha \dagger} = im \left( \eta_\beta x^{\delta \alpha} \theta^\alpha + \bar{\eta}_\beta x^{\alpha \delta} \bar{\theta}^\dagger \right) + \frac{m}{2} \left( \bar{\theta}^2 \theta^\alpha \bar{\eta}^\dagger + \theta^2 \eta^\alpha \bar{\theta}^\dagger \right) , \]
\[ \delta \theta^\alpha = -m \bar{\eta}_\alpha x^{\alpha \dagger} - im \left( \theta^2 \eta^\alpha - \bar{\theta}^\dagger \bar{\eta}^\alpha \bar{\theta}^\dagger \right) , \]
\[ \delta \bar{\theta}^\dagger = -m \eta_\alpha x^{\alpha \dagger} + im \left( \bar{\theta}^2 \bar{\eta}^\alpha - \theta^\alpha \eta_\alpha \bar{\theta}^\dagger \right) , \]
\[ \delta \xi^\alpha = \eta^\alpha , \quad \delta \bar{\xi}^\dagger = \bar{\eta}^\dagger , \]
\[ \delta L = -i \left( \theta^\alpha \eta_\alpha - \bar{\theta}^\dagger \bar{\eta}^\dagger \right) . \]  \hspace{1cm} (4.4)\hspace{1cm} (4.5)\\

Thus, the active form of the \( S \) transformation of the superfield \( L \) (to zeroth order in \((\xi, \bar{\xi})\)) reads:

\[ \delta^* L = -i \left( \theta^\alpha \eta_\alpha - \bar{\theta}^\dagger \bar{\eta}^\dagger \right) - \Delta x^{\alpha \dagger} \partial_{\alpha \dagger} L + \Delta \theta^\alpha D_\alpha L - \Delta \bar{\theta}^\dagger D_{\bar{\theta}^\dagger} L - \eta^\alpha \frac{\partial}{\partial \xi^\alpha} L - \bar{\eta}^\dagger \frac{\partial}{\partial \bar{\xi}^\dagger} L , \]  \hspace{1cm} (4.6)\\

where

\[ \Delta x^{\alpha \dagger} = 2im \left( \eta_\beta x^{\delta \alpha} \theta^\alpha + \bar{\eta}_\beta x^{\alpha \delta} \bar{\theta}^\dagger \right) - m \left( \theta^2 \eta^\alpha \bar{\eta}^\dagger + \bar{\theta}^2 \bar{\eta}^\dagger \theta^\alpha \right) , \]
\[ \Delta \theta^\alpha = m \bar{\eta}_\alpha x^{\alpha \dagger} + im \left( \theta^2 \eta^\alpha - \bar{\theta}^\dagger \bar{\eta}^\alpha \bar{\theta}^\dagger \right) , \]
\[ \Delta \bar{\theta}^\dagger = m \eta_\alpha x^{\alpha \dagger} - im \left( \bar{\theta}^2 \bar{\eta}^\dagger - \theta^\alpha \eta_\alpha \bar{\theta}^\dagger \right) , \]  \hspace{1cm} (4.7)\\

and we should put \( \xi^\alpha = \bar{\xi}^\dagger = 0 \) in both sides of \((4.6)\).
From the transformation law (4.6) one can guess the constraint which replaces (2.6) in the AdS case
\[
\frac{1}{m} D^2 e^{-2mL} = \frac{1}{m} D^2 e^{-2mL} = 0 . \tag{4.8}
\]
Indeed, in the limit \( m \to 0 \), (4.8) reduces to (2.6). If we neglect the last two terms in (4.6), for the \( N = 1 \) superfield \( e^{-2mL} \) we just recover the standard superconformal transformation law of the improved \( N = 1 \) tensor multiplet \([35]\), with (4.8) being covariant under this realization. However, since we wish to gain a generalization of the \( N = 2 \) tensor multiplet transformation law \([31, 14]\), we cannot suppress such terms in (4.6). Similar terms should also modify the standard superconformal transformation laws of the chiral \( N = 1 \) superfields \( F, \bar{F} \). In principle, we could fix the \( \xi, \bar{\xi} \) dependence of the \( N = 2 \) superfields \( L(x, \theta, \xi), F(x, \theta, \xi), \bar{F}(x, \bar{\theta}, \bar{\xi}) \) entering \( \mathcal{L}_\omega \), by finding an analog of the covariant constraints (3.13) for the present case. However, in view of the highly non-trivial structure of the superalgebra (4.1), such a method would entail complicated technicalities. In fact, since we need a realization of \( SU(2, 2|1) \) only on the \( N = 1 \) superfield components \( L(x, \theta), F(x, \theta), \bar{F}(x, \bar{\theta}) \), it is simpler to guess the precise form of \( (\partial L/\partial \xi^a)|, (\partial L/\partial \bar{\xi}^\dot{a})| \) and similar contributions to the transformation laws of \( F, \bar{F} \) (here \( | \) denotes retaining \( \xi, \bar{\xi} \) independent parts only). It turns out that the reasoning based on the \( D \) and \( J \) weights, together with the requirement of compatibility with the modified constraints (4.8) and the chiral ones for \( F, \bar{F} \), fix these terms, up to a numerical coefficient. The latter is then determined by the requirement that, in the flat \( m = 0 \) limit, the transformations (3.14) are reproduced. One should take into account that \( L \) is shifted by a constant parameter under the action of the generator \( D \), so \( e^{-mL} \) has a \( D \) weight \( m \) (spinor derivatives have a weight \( m/2 \).

In this way, we come to the following modified realization of conformal \( S \) supersymmetry on the \( N = 1 \) superfields \( L, F, \bar{F} \):
\[
\begin{align*}
\delta^* \bar{F} &= 6im\theta^a \eta_\alpha \bar{F} - \Delta x^{\alpha \dot{a}} \partial_{\alpha \dot{a}} \bar{F} + \Delta \theta^a D_\alpha \bar{F} + ie^{-2mL} \bar{\eta}^{\dot{a}} \bar{D}_\dot{a} L , \\
\delta^* F &= -6im\bar{\theta}_{\dot{a}} \eta^\dot{a} F - \Delta x^{\alpha \dot{a}} \partial_{\alpha \dot{a}} F - \Delta \bar{\theta}^{\dot{a}} \bar{D}_{\dot{a}} F + ie^{-2mL} \eta^a D_a L , \\
\delta^* L &= -i(\theta^a \eta_\alpha - \bar{\theta}_{\dot{a}} \bar{\eta}^{\dot{a}}) - \Delta x^{\alpha \dot{a}} \partial_{\alpha \dot{a}} L + \Delta \theta^a D_\alpha L - \Delta \bar{\theta}^{\dot{a}} \bar{D}_{\dot{a}} L \\
&\quad -ie^{-2mL} \left[ \eta^a D_a \left( e^{2mL} F \right) + \bar{\eta}^{\dot{a}} \bar{D}_{\dot{a}} \left( e^{2mL} \bar{F} \right) \right] . \tag{4.9}
\end{align*}
\]
In the limit \( m = 0 \), the transformations (4.9) go into (3.14). The compatibility of the \( F, \bar{F} \) transformation laws with the chirality conditions is just a consequence of the constraint (4.8). The appearance of the weight pieces in the transformations can be traced to the transformation properties of the chiral integration measures in (4.12), and is required for the invariance of (4.2). It is easy to find, e.g.
\[
\delta (d^4 x R d^2 \bar{\theta}) = \left( \frac{\partial \delta x^{\alpha \dot{a}}}{\partial x^{R \alpha \dot{a}}} - \frac{\partial \delta \bar{\theta}^{\dot{a}}}{\partial \bar{\theta}^{\dot{a}}} \right) d^4 x, d^2 \bar{\theta} = 6im \langle \bar{\theta} \cdot \bar{\eta} \rangle d^4 x_R d^2 \bar{\theta} , \tag{4.10}
\]
where
\[
x^{\alpha \dot{a}}_R = x^{\alpha \dot{a}} - i\theta^a \bar{\theta}^{\dot{a}} , \quad \delta x^{\alpha \dot{a}}_R = 2im \bar{\eta}_{\bar{\alpha}} x^{\alpha \dot{a}}_R \bar{\theta}^{\dot{a}} , \quad \delta \bar{\theta}^{\dot{a}} = -m \eta_\alpha x^{\alpha \dot{a}}_R + im \bar{\eta}^{\dot{a}} \bar{\theta}^2 . \tag{4.11}
\]
Taking the transformations of \( F, \bar{F} \) in a passive form, when only the first and last terms in (4.9) are retained, we see that the weight pieces are necessary for the invariance of (4.2).

The standard part of the transformation of \( L \) in (4.9) is compatible with the constraints (4.8), while in the additional piece (the last line of (4.9)) only the first and second terms
manifestly obey the first and second constraints in (4.8), respectively. Their sum, with \( L, F, \bar{F} \) being independent \( N = 1 \) superfields, is clearly inconsistent with (4.8). However, recall that the \( SO(1,4) \) covariance in the flat case can be implemented, only provided the constraints (3.16) are imposed. In our case, the \( SO(1,4) \) transformations appear in the closure of \( Q \) and \( S \) supersymmetries, so it is natural to expect that it is necessary to take into account similar constraints already at the level of the \( S \) transformations (4.9) for self-consistency of the latter. It turns out that this is indeed the case. We checked that the variations (4.9) properly reproduce, in their closure with both themselves and manifest \( N = 1 \) supersymmetry, the remaining transformations of the \( AdS_5 \) superalgebra (4.1), only if \( F, \bar{F} \) are subjected to the following nonlinear constraints:

\[
F = -e^{-2mL} D^\alpha L D_\alpha L, \quad \bar{F} = -\frac{e^{-2mL} \bar{D}_\dot{\alpha} L \bar{D}^\dot{\alpha} L}{2 - e^{4mL} D^2 F}.
\]  

(4.12)

The latter constraints are compatible with both (4.8) and the chirality properties of \( F, \bar{F} \). They go into (3.16) at \( m = 0 \). Once again, this modification of (3.16) can be easily guessed from the condition that \( F \) and \( \bar{F} \) possess \( D \) weight 3 and \( J \) charges \( \pm 3 \). Now, it is easy to check that the full transformation of \( L \) in (4.9) is compatible with the constraint (4.8) on the surface of (4.12). It is also a matter of a straightforward computation, to check that (4.12) by themselves are covariant under the transformations (4.9). Thus, we see that, in the \( AdS_5 \) case, there are no direct analogs of either the linear PBGS realization of the flat case, or the Goldstone tensor \( N = 2 \) superfield. An analog of the latter can be consistently defined only on the surface of the nonlinear constraints (4.12). Hence we are led, at once, to deal with a nonlinear realization of \( SU(2,2|1) \) in terms of the improved tensor \( N = 1 \) superfield \( L(x, \theta) \).

Similarly to their flat counterparts, the constraint (4.12) can be easily solved

\[
F = -e^{-2mL} \psi^2 + \frac{1}{2} D^2 \left[ \frac{\psi^2 \bar{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}} \right],
\]

(4.13)

where

\[
\psi_\alpha \equiv D_\alpha L, \quad \bar{\psi}_\dot{\alpha} \equiv \bar{D}_\dot{\alpha} L,
\]

\[
A = \frac{1}{2} e^{2mL} \left( D^2 \psi^2 + \bar{D}^2 \bar{\psi}^2 \right), \quad B = \frac{1}{2} e^{2mL} \left( D^2 \psi^2 - \bar{D}^2 \bar{\psi}^2 \right).
\]

(4.14)

Finally, the action (4.12) can be written in the form

\[
S = -\frac{1}{4} \int d^4x d^2\theta e^{-2mL} \bar{\psi}^2 - \frac{1}{4} \int d^4x d^2\bar{\theta} e^{-2mL} \bar{\psi}^2 + \frac{1}{4} \int d^4x d^4\theta \frac{\psi^2 \bar{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}},
\]

(4.15)

or, equivalently, as

\[
S = \frac{1}{4} \int d^4x d^4\theta \left[ \frac{1}{m} Le^{-2mL} + \frac{\psi^2 \bar{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}} \right].
\]

(4.16)

The first term in (4.16) is recognized as the action of the improved tensor \( N = 1 \) superfield
To see which sort of supersymmetric extended object the action (4.16) describes let us examine its bosonic core. Defining the bosonic components as follows:

\[
\phi = L|_{\theta=0}, \quad [D_\alpha, \bar{D}_\dot{\alpha}] e^{-2mL}|_{\theta=0} = -2mV^{\alpha\dot{\alpha}},
\]

where, in virtue of (4.12),

\[
\partial_{\alpha\dot{\alpha}} V^{\alpha\dot{\alpha}} = 0,
\]

the bosonic part of (4.15) proves to be

\[
S_B = \int d^4x e^{-4m\phi} \left[ 1 - \sqrt{1 + \frac{1}{2} e^{6m\phi} V^2 - 2e^{2m\phi}(\partial\phi)^2 - e^{8m\phi}(V^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \phi)^2} \right].
\]

The latter is a conformally-invariant extension of the static gauge Nambu-Goto action for a L3-brane in \(d = 5\): the dilaton \(\phi\) can be interpreted as a radial brane coordinate, while \(V^{\alpha\dot{\alpha}}\) is the field strength of the notoph, and it contributes one more scalar degree of freedom on shell. In the limit \(V^{\alpha\dot{\alpha}} = 0\), the action (4.19) becomes just the static-gauge form of the Nambu-Goto action for 3-brane on AdS_5 in the ‘solvable-subgroup’ parametrization [33, 23].

Thus we conclude that the superfield action (4.16) describes the static-gauge AdS_5 super L3-brane which can be defined as a superconformally invariant generalization of the flat superspace L3-brane of refs. [4]-[6]. To our knowledge, such an action was never given before. On the other hand, the flat superspace L3-brane action beyond the static gauge was deduced in ref. [27] in the framework of superembedding approach. It would be tempting to recover the action (4.16) as a gauge-fixed form of some appropriate worldvolume action in this approach.

We wish to point out that the Goldstone superfield action (4.16) corresponds to the one-half partial breaking of SU(2|1) down to its \(N = 1, d = 4\) super Poincaré subgroup which, together with the \(R\)-symmetry (or \(\gamma_5\)) subgroup generated by \(J\), are the only linearly realized symmetries of this PBGS pattern. Likewise, the only linearly realized symmetry of the bosonic action (4.19) is the 4-dimensional worldvolume Poincaré symmetry.

The \(U(1)\) \(R\)-symmetry generated by \(J\) gets nonlinearly realized after performing a duality transformation. As well known, the notoph field strength \(V^{\alpha\dot{\alpha}}\) can be dualized into an off-shell scalar, by first introducing the constraint (4.18) into the action with a Lagrange scalar multiplier and then eliminating \(V^{\alpha\dot{\alpha}}\), using its algebraic equation of motion, in terms of the additional scalar field. Extending (4.19) as follows:

\[
S_B \Rightarrow S^\text{dual}_B = S_B + \int d^4x \lambda \partial_{\alpha\dot{\alpha}} V^{\alpha\dot{\alpha}}
\]

and eliminating \(V^{\alpha\dot{\alpha}}\), after some algebra we obtain

\[
S^\text{dual}_B = \int d^4x e^{-4m\phi} \left\{ 1 - \sqrt{1 - 2e^{2m\phi}[(\partial\phi)^2 + (\partial\lambda)^2] + 4e^{4m\phi}[(\partial\phi)^2(\partial\lambda)^2 - (\partial\phi\partial\lambda)^2]} \right\}.
\]

After passing to Cartesian \(\mathbb{R}^2\) coordinates

\[
Z^1 = r \cos \vartheta, \quad Z^2 = r \sin \vartheta, \quad r \equiv e^{-m\phi}, \quad \vartheta \equiv m \lambda,
\]

one can rewrite (4.21) in a nice form

\[
S^\text{dual}_B = \int d^4x |Z|^4 \left[ 1 - \sqrt{-\det \left( \eta_{\mu\nu} - \frac{2}{m^2} \partial_{\mu}Z^n \partial_{\nu}Z^n \right)} \right].
\]
which is just the $S^5 \to S^1$ reduction of the scalar part of the D3-brane action on $AdS_5 \times S^5$ [19, 22], i.e. the static-gauge Nambu-Goto action of a scalar 3-brane on $AdS_5 \times S^1$ (with $AdS_5$ and $S^1$ having equal radii $\sim m^{-1}$). In the next Section we shall perform this duality transformation at the full superfield level and obtain a $SU(2,2|1)$ invariant action of the Goldstone chiral $N = 1$ superfield which yields in its bosonic sector just (4.21) or (4.23). It will be argued there that $\lambda(x)$ is the coset parameter associated with the $U(1)$ generator $J$, which supports its interpretation as an angular variable in (4.22).

5 Dual Goldstone superfield action on $AdS_5 \times S^1$

In order to carry out a duality transformation at the full superfield level, we begin with the superfield action (4.16) and relax the basic constraints (4.8) by adding a Lagrange multiplier

$$S_{\text{dual}} = \frac{1}{4} \int d^4 x d^2 \theta d^2 \bar{\theta} \left[ -\frac{1}{2m^2} Y (\ln Y - 1) + \frac{Y^{-4}}{(2m)^4} (DY)^2 (DY)^2 f + \frac{Y}{2m} (\varphi + \bar{\varphi}) \right] , \quad (5.1)$$

where

$$Y \equiv e^{-2mL} , \quad \bar{D}_\alpha \varphi = D_\alpha \bar{\varphi} = 0 . \quad (5.2)$$

Next, we vary the action (5.1) with respect to $Y$, in order to obtain an algebraic equation that would allow us to trade $Y$ for $\varphi, \bar{\varphi}$

$$\frac{1}{2m^2} \ln Y - \frac{1}{2m} (\varphi + \bar{\varphi}) = a_1 (DY)^2 + a_2 (\bar{D}Y)^2 + c_{\alpha\dot{\alpha}} D^\alpha Y \bar{D}^{\dot{\alpha}} Y + a_3^2 D_\alpha Y (\bar{D}Y)^2 + a_4^2 \bar{D}_\dot{\alpha} Y (DY)^2 + a_5 (DY)^2 (\bar{D}Y)^2 , \quad (5.3)$$

where all terms in the r.h.s come from the variation of the four-fermion term in (5.1). The coefficients $a_\alpha$ and $c_{\alpha\dot{\alpha}}$ are functions of $Y$ and its derivatives.

Plugging (5.3) back into the action (5.1) gives us the following expression:

$$S_{\text{dual}} = \frac{1}{4} \int d^4 x d^2 \theta d^2 \bar{\theta} e^{m(\varphi + \bar{\varphi})} \left[ \frac{1}{2m^2} + \left( \frac{Y^{-4}}{(2m)^4} f - 2m^2 a_1 a_2 - \frac{m^2}{4} c^2 \right) (DY)^2 (\bar{D}Y)^2 \right] . \quad (5.4)$$

Thus, we need to solve the rather complicated equation (5.3), which expresses $Y$ in terms of $\varphi, \bar{\varphi}$ only up to the second order in the fermions. Moreover, the functions $a_1$ and $a_2$ are proportional to $\bar{D}^2 Y$ and $D^2 Y$, respectively. It was explicitly shown in [6] that such terms can be reabsorbed into a redefinition of a chiral Lagrangian multiplier. Therefore we can discard such terms in the action.

To summarize, the equation we have to solve reads:

$$-\frac{1}{2m^2} \ln Y + \frac{1}{2m} (\varphi + \bar{\varphi}) + c_{\alpha\dot{\alpha}} D^\alpha Y \bar{D}^{\dot{\alpha}} Y = 0 , \quad (5.5)$$

where

$$c_{\alpha\dot{\alpha}} = \frac{Y^{-4}}{8m^4} \left[ H G_{\alpha\dot{\alpha}} - \bar{H} G_{\alpha\dot{\alpha}} \right] . \quad (5.6)$$

Here,

$$H = 2f + (f_A + f_B) (A + B) , \quad \bar{H} = 2f + (f_A - f_B) (A - B) , \quad (5.7)$$

Some appropriate periodicity conditions should be imposed on $\lambda(x)$ to make such an interpretation correct.
and

\[ G_{\alpha \dot{\alpha}} \equiv D_{\alpha} D_{\dot{\alpha}} Y \, , \quad G_{\alpha \dot{\alpha}} \equiv D_{\dot{\alpha}} D_{\alpha} Y . \] (5.8)

Acting by two spinor derivatives on eq. (5.3) and omitting terms with fermions we obtain the following nonlinear equations:

\[ -2i \partial_{\alpha \dot{\alpha}} \varphi = \frac{Y^{-1}}{m} \left[ \left(1 - \frac{1}{4} (A + B) \bar{H} \right) G_{\alpha \dot{\alpha}} + \frac{1}{4} (A - B) H \bar{G}_{\alpha \dot{\alpha}} \right] , \]

\[ -2i \partial_{\alpha \dot{\alpha}} \bar{\varphi} = \frac{Y^{-1}}{m} \left[ \left(1 - \frac{1}{4} (A - B) H \right) G_{\alpha \dot{\alpha}} + \frac{1}{4} (A + B) \bar{H} \bar{G}_{\alpha \dot{\alpha}} \right] . \] (5.9)

Squaring these equations and taking their cross-product we find the relations between \( A, B \) and \( P \) defined as

\[ P \equiv \frac{Y^{-3}}{2m^2} G^{\alpha \dot{\alpha}} \bar{G}_{\alpha \dot{\alpha}} , \] (5.10)
on the one hand, and the corresponding duals

\[ a = -e^{-m(\varphi + \bar{\varphi})} \left[ (\partial \varphi)^2 + (\partial \bar{\varphi})^2 \right] , \quad b = -e^{-m(\varphi + \bar{\varphi})} \left[ (\partial \varphi)^2 - (\partial \bar{\varphi})^2 \right] , \]

\[ p = -e^{-m(\varphi + \bar{\varphi})} \partial \varphi \partial \bar{\varphi} \] (5.11)
on the other. The relations

\[ a + b = (A - B) \left[ \left(1 - \frac{1}{4} \bar{H} (A + B) \right)^2 + \frac{1}{2} \bar{H} P \left(1 - \frac{1}{4} \bar{H} (A + B) \right) + \frac{1}{16} \bar{H}^2 (A^2 - B^2) \right] , \]

\[ a - b = (A + B) \left[ \left(1 - \frac{1}{4} H (A - B) \right)^2 + \frac{1}{2} H P \left(1 - \frac{1}{4} H (A - B) \right) + \frac{1}{16} H^2 (A^2 - B^2) \right] , \]

\[ a + p = A + P \] (5.12)
can be exactly solved, yielding the following expressions:

\[ A = \frac{2(a^2 - b^2 + a(2 + p))}{b^2 - a^2 + (p + 2)^2} , \quad B = -\frac{2b}{\sqrt{b^2 - a^2 + (p + 2)^2}} , \quad P = a + p - A . \] (5.13)

As a last step, we should express \((DG)^2(\bar{D}G)^2\) in terms of spinor derivatives of \(\varphi, \bar{\varphi}\). Once again, acting on the basic equation (5.3) with the derivatives \(D_{\alpha}\) and \(\bar{D}_{\dot{\alpha}}\) and keeping only terms linear in the fermions, we find

\[ D_{\alpha} \varphi = \frac{1}{m} \bar{D}_{\dot{\alpha}} \ln Y + 2mc_{\beta \dot{\beta}} G^\alpha_{\beta} D_{\beta} Y \, , \quad \bar{D}_{\dot{\alpha}} \bar{\varphi} = \frac{1}{m} D_{\alpha} \ln Y - 2mc_{\beta \dot{\beta}} G^\alpha_{\dot{\alpha}} \bar{D}_{\dot{\beta}} Y . \] (5.14)

From (5.14) one can deduce

\[ (DG)^2(\bar{D}G)^2 = \frac{m^4 Y^4 (D\varphi)^2 (\bar{D}\bar{\varphi})^2}{(1 + 2m^2 Y c_{\alpha \dot{\alpha}} G^\alpha_{\alpha} + m^4 Y^2 c^2 G^2) (1 - 2m^2 Y c_{\alpha \dot{\alpha}} G^\alpha_{\dot{\alpha}} + m^4 Y^2 c^2 G^2)} . \] (5.15)

Finally, plugging all this in the actions (5.4), we find the dual action in a rather simple form

\[ S_{\text{dual}} = \frac{1}{8} \int d^4 x d^4 \theta \left( \frac{1}{m^2} e^{m(\varphi + \bar{\varphi})} + \frac{1}{8} (D\varphi)^2 (\bar{D}\bar{\varphi})^2 \right) . \] (5.16)
This action goes into the flat $N = 2 \to N = 1$ chiral Goldstone superfield action of \cite{5, 6} in the limit $m = 0$ and is obviously $SU(2, 2|1)$ invariant, as it was obtained by dualizing the $SU(2, 2|1)$ invariant action \cite{11, 12}.

In principle, one can find the precise form of the $SU(2, 2|1)$ transformations of the chiral Goldstone superfields $\varphi$, $\bar{\varphi}$, but we do not give them here, because they look not too illuminating. We only note that the standard $U(1)$ isometry associated with the duality transformation, viz. $\delta \varphi = i \alpha$, $\delta \bar{\varphi} = -i \alpha$, now appears in the closure of $Q$ and $S$ transformations of the Goldstone superfields, with the imaginary part of $\varphi$ as the corresponding Goldstone field. Hence, it is just the $J$ (or $\gamma_5$) symmetry of $SU(2, 2|1)$. In other words, performing a duality transformation brings this symmetry from the stability subgroup into the coset. Actually, this can be seen already at the level of dualization of the standard improved $N = 1$ tensor supermultiplet, which corresponds to the approximation of neglecting the last line in the transformation law of $L$ in \cite{9} and keeping only the first term in the action \cite{11, 12}. In this case, it is easy to find the precise expression of $Y$ in terms of $\varphi, \bar{\varphi}$

\[ Y = e^{m(\varphi + \bar{\varphi})}. \] (5.17)

Then, the standard $S$ transformation law of $Y$ is reproduced by the following transformations of chiral superfields

\[ \delta^s \varphi = 2i\theta^\alpha \eta_\alpha - \Delta x^{\dot{\alpha}} \partial_{\alpha \dot{\alpha}} \varphi + \Delta \theta^\alpha D_\alpha \varphi, \quad \delta^s \bar{\varphi} = -2i\bar{\theta}_{\dot{\alpha}} \bar{\eta}^{\dot{\alpha}} - \Delta x^{\alpha \dot{\alpha}} \partial_{\alpha \dot{\alpha}} \bar{\varphi} - \Delta \bar{\theta}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} \bar{\varphi}. \] (5.18)

It is easy to see that, in the closure of this transformation with the standard $N = 1$ Poincaré supersymmetry, there appears a complex bracket parameter. Its real and imaginary parts are just the dilatonic and $\gamma_5$ weight transformations. This property persists in the complete nonlinear version of \cite{5}. A similar ‘jumping’ of the $R$-symmetry generator from the stability subgroup to the coset after a duality transformation was observed in \cite{36}, in the study of the relation between real and complex forms of $N = 2$ superconformal mechanics associated with the nonlinear realization of the $N = 2, d = 1$ superconformal group $SU(1, 1|1) \sim OSP(2|2)$.

The bosonic part of the action \cite{11, 12} reads:

\[ S_B^{\text{dual}} = \int d^4 x e^{2m(\varphi + \bar{\varphi})} \left[ 1 + \sqrt{(1 - e^{-m(\varphi + \bar{\varphi})} \partial \varphi \partial \bar{\varphi})^2 - e^{-2m(\varphi + \bar{\varphi})}(\partial \varphi)^2(\partial \bar{\varphi})^2} \right] \] (5.19)

and it coincides with \cite{12}, after the following identifications:

\[ \phi = -\frac{1}{2} (\varphi + \bar{\varphi}), \quad \lambda = \frac{i}{2} (\varphi - \bar{\varphi}). \] (5.20)

Thus, we conclude that the Goldstone superfield action \cite{5, 10} describes a situation where $SU(2, 2|1)$ is nonlinearly realized in its coset over the subgroup $SO(1, 3)$, with a $N = 1, d = 4$ Poincaré supersymmetry realized in the standard linear way on $N = 1$ superspace coordinates and Goldstone superfields. The $S$ supersymmetry is broken, along with the $D$, $J$ and $SO(1, 4)/SO(1, 3)$ $K_{a \dot{a}}$ transformations (the ‘Goldstone field’ for the latter is basically the $x$-derivative of the dilaton). The independent bosonic Goldstone fields and $x^{a \dot{a}}$ parameterize the coset manifold $AdS_5 \times S^1 \propto \{ x^{a \dot{a}}, \phi \} \otimes \{ \lambda \}$. The bosonic part of the action \cite{5, 10} is just the static-gauge Nambu-Goto action of 3-brane on the latter manifold.

This solves the problem of constructing a minimal Goldstone superfield action for the PBGS option considered.
6 Discussion

In this paper we have constructed new nonlinear realizations of the simplest AdS$_5$ superisometry group (viz. $N = 1, d = 4$ superconformal group) $SU(2, 2|1)$, in terms of a $N = 1, d = 4$ improved tensor and chiral Goldstone superfields. We have set up the minimal Goldstone superfield action for the first option, by generalizing the approach applied earlier to the case of flat Minkowski background. This generalization is not straightforward, and it essentially relies on a novel interpretation of the basic constraints of refs. [4, 5, 6], as a guarantee of hidden $d = 5$ $SO(1, 4)$ covariance. The minimal Goldstone superfield action for the second PBGS option has been obtained by dualizing the action for the first one. The actions constructed contain no free parameters (as opposed to the candidate Goldstone superfield action of ref. [25]). They provide a manifestly $N = 1$ supersymmetric off-shell superfield form of the worldvolume actions for a L3-superbrane on AdS$_5$ and a scalar super 3-brane on AdS$_5 \times S^1$, respectively. The latter is a truncation of the action for a super AdS$_5 \times S^5$ D3-brane. In the limit of infinite AdS$_5$ radius, the new actions go into their flat superspace counterparts which describe the partial breaking of $N = 2, d = 4$ supersymmetry down to $N = 1$ supersymmetry [4, 5, 6, 3]. Similarly to the flat superspace Goldstone superfield Lagrangians, their AdS$_5$ analogs do not behave under $SU(2, 2|1)$ supersymmetry transformations as tensors. Rather, they are shifted by a total derivative. In this respect, they are reminiscent of the WZW or CS Lagrangians.

The study in this paper, together with the results of [23], can be regarded as first steps in a program of constructing off-shell Goldstone superfield actions for various patterns of partial breaking of AdS×S supersymmetries and their non-trivial contractions corresponding to pp-wave type backgrounds. One of the obvious related tasks (as already mentioned in [25]) is the quest for an action corresponding to the half-breaking of the $N = 2$ AdS$_5$ supergroup $SU(2, 2|2)$, in a supercoset with the AdS$_5 \times S^1$ bosonic part. In this case, the basic Goldstone superfield that we expect to deal with should be the appropriate generalization of the $N = 2$ Maxwell superfield strength. The relevant minimal action should be a superconformally invariant version of the Dirac-Born-Infeld action describing $N = 4 \rightarrow N = 2$ partial breaking in flat superspace [37, 38]. Note that in the flat case there exists one more $N = 2 \rightarrow N = 1$ PBGS option associated with the choice of a vector $N = 1, d = 4$ multiplet as a Goldstone one and corresponding to a space-filling $N = 1$ D3-brane [39]. No AdS$_5$ analog of this realization exists. The reason is that the $SU(2, 2|1)$ invariance requires the presence of a dilaton field in the relevant $N = 1$ Goldstone supermultiplet. In the Goldstone $N = 2$ vector supermultiplet there are two scalar fields and, therefore, the above objection can be circumvented.

Another interesting problem is to extend the ‘holographic map’ of ref. [17] to the superconformal PBGS cases, including those studied in the present paper. We expect the existence of a nonlinear change of Goldstone superfields and $N = 1$ superspace coordinates which maps the nonlinear realization [19] (and its counterpart for the chiral Goldstone superfields) onto the standard nonlinear realization of $SU(2, 2|1)$ regarded as a $N = 1, d = 4$ superconformal group [40]. The minimal Goldstone superfield actions constructed above are expected to be mapped onto some non-linear higher-derivative extensions of the standard $N = 1, d = 4$ superconformal actions of the improved tensor and chiral $N = 1$ superfields used as the Goldstone ones for the standard nonlinear realizations of $SU(2, 2|1)$, similarly to what takes place for bosonic AdS actions [17].
Acknowledgements

This work was partially supported by INTAS grant No 00-0254 and the Iniziativa Specifica MI12 of the INFN Commissione Nazionale IV. The research of S.B. is supported in part by European Community’s Human Potential Programme contract HPRN-CT-2000-00131 and NATO Collaborative Linkage Grant PST.CLG.979389. The research of E.I. and S.K. is supported in part by grants DFG No.436 RUS 113/669, RFBR-DFG 02-02-04002, RFBR-CNRS 01-02-22005 and a grant of the Heisenberg-Landau program. E.I. and S.K. thank INFN-LNF for warm hospitality during the course of this work.

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