Diffusion-thermo and thermal-diffusion effects with inclined magnetic field on unsteady MHD slip flow over a permeable vertical plate

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Abstract

In this study, various fluid physical quantities effects such as diffusion-thermo, thermal-diffusion, thermal radiation, viscous dissipation, inclined magnetic field on unsteady MHD slip flow over a permeable vertical plate are considered. The coupled and nonlinear partial differential governing equations consisting of momentum, energy and species equations are reduced to ordinary differential equations using perturbation technique. The resulted coupled, nonlinear ordinary differential equations are solved by using collocation method with the aid of assumed Legendre polynomial. The impacts of different physical parameters on fluid properties are discussed and presented both graphically and tabularly. Both Dufour and Soret have the tendency of enhancing velocity profiles.

Keywords: Inclined magnetic field, Diffusion-thermo, Thermal-diffusion, MHD, Unsteady, Porous medium

Mathematics Subject Classification: 65L60, 76A05, 76M55, 76Sxx, 76W05

Introduction

In heat and mass simultaneous occurrence, it is evident that intricacy resulted due to fluxes and driving potentials relationship. Energy flux effect due to concentration gradient is referred to as diffusion-thermo (Dufour), and mass flux effect as a result of temperature gradient is known as thermal-diffusion (Soret). Most often, Dufour and Soret effects are neglected in heat and mass transfer analysis based on the assumption that they are of smaller magnitude compared to other effects as depicted by Fick’s and Fourier’s law. However, with reference to its applications in area like petrology, geosciences, Solar collectors, hydrology, Combustion flames, and building energy conservation, the significance of these effects become unavoidable. On this note, many researchers like Reddy et al. [1] examined Soret and Dufour effects on MHD flow via exponentially stretching sheet in the presence of viscous dissipation and thermal radiation. The result revealed that both the fluid temperature and concentration increased with a slowdown in Soret and speed-up in Dufour. Studies on effects of Soret and Dufour on Micropolar fluid were considered by Babu et al. [2]. The effects of radiation and magnetic field were
also examined. It was reported that temperature profiles declined with the increase in
Soret. 
Furthermore, Sekhar and Manjula [3] studied Casson fluid inclined permeable plate in
the presence of Soret and Dufour with slip condition. Runge–Kutta fourth-order method
was used to solve resulting ordinary differential equations. The result revealed that the
increase in the angle of inclination accelerates velocity profiles. Effects of Dufour and
Soret on Casson fluid were presented by Reddy and Janardhan [4]. In the study, radia-
tion and chemical reaction effects were not given recognition. It was reported that
concentration declined with the increase in Soret number. The impacts of Soret and
Dufour on MHD flow with mass and heat transfer in a wavy channel were examined by
Gbadeyan et al. [5]. The result showed that velocity declined with a rise in Soret, while
a reverse trend is experienced for Dufour. Parandhama et al. [6] investigated effects of
Soret on MHD Casson fluid via a vertical plate. The result showed that the increase in
Soret decelerated temperature profiles. Salawu and Dada [7] considered pressure-driven
inclined magnetic fluid flow through a Darcy Forchheimer medium in the presence of
Soret and Dufour. It was reported that a rise in Soret and Dufour increases the skin fric-
tion. Iftikhar et al. [8] studied mixed convection MHD Jeffery fluid flow in the presence
of Soret, Dufour and thermophoresis. The resulted partial differential equations are
solved using Optimal Homotopy Analysis Method (OHAM). Convective unsteady MHD
fluid flow via a permeable vertical plate with Soret and Dufour effects was carried out
by Sarada and Shanker [9]. It was noticed that velocity speeds up with a rise in Dufour
parameter. None of the above studies considered variable suction.

Boundary layer fluid flows in the presence of heat and mass transfer with effects of
thermal radiation are applicable in numerous engineering and industrial processes such
as extraction and manufacture of polymer. In addition, radiative effect has its applica-
tions in areas like cooling processes of electronic devices, nuclear reactors and technolo-
gical processes involving high temperature. Navier–Stokes equations theory is basically
on no-slip for the boundary conditions of the flow. But practically, it is not always appli-
cable, especially for non-Newtonian fluid. Problems involving slip conditions and ther-
mal radiation for various fluids have attracted the attention of researchers. Kumar [10]
investigated the flow of fluid over stretched variable thickness surface of natural convec-
tive MHD Casson fluid in the presence of thermal radiation. The problem was solved
numerically, and it was observed that temperature profiles are enhanced by radiation
parameter. The impacts of slip conditions and radiation on stagnation point MHD flow
via a stretching sheet were examined by Sumalatha et al. [11]. According to their results,
the slip parameter has the tendency of declining the velocity of the fluid. Rajakumar et al.
[12] presented the effects of diffusion-thermo, radiation and viscous dissipation on natu-
ral convective flow of Casson fluid over an oscillatory permeable vertical plate. Ion-slip
current was put into consideration. Sreenivasulu et al. [13] considered impacts of radia-
tion on boundary layer MHD slip flow through an exponential porous stretching sheet
with dissipation and joule heating. The resulted differential equations are solved numeri-
cally with Runge–Kutta fourth-order method. The study concluded that temperature of
the fluid is improved by dissipation. Agunbiade and Dada [14] discussed dissipation and
chemical reaction effects on rotatory Rivlin–Ericksen fluid flow via a vertical permeable
plate in the presence of thermal radiation.
In addition, Reddy and Reddy [15] examined slip boundary layer flow of convective MHD through an inclined porous surface with thermal radiation and chemical reaction. It was observed that fluid velocity tend to be higher when the angle of inclination is set to zero, while it declined with a rise in the angle of inclination. Eyring–Powell Unsteady Nanofluid flow of Hydromagnetic through a stretched inclined porous sheet in the presence of radiation and joule heating was reported by Kumar and Srinivas [16]. It is noticed that thermophoresis parameters speed up the fluid velocity. Ragavan et al. [17] examined inclined magnetic field for Walter’s Liquid B fluid with entropy generation via a stretching sheet. It was reported that magnetic field strengthens with the increase in the angle of inclination. Reddy [18] discussed MHD Casson fluid flow through an inclined permeable exponentially stretching surface in the presence of chemical reaction and thermal radiation. The results revealed that inclination parameter has the tendency of enhancing velocity profiles.

This study is an extension of Sharma and Choudhary [19], and in view of the above studies, combined effects of diffusion-thermo, thermal-diffusion, viscous dissipation and thermal radiation on unsteady MHD slip flow with inclined magnetic field over a permeable vertical plate have not being given adequate attention. Practically, combined effects of these parameters are significantly important in numerous engineering and industrial processes. Therefore, this study is motivated to consider diffusion-thermo and thermal-diffusion effects on unsteady MHD slip flow with inclined magnetic field, thermal radiation, viscous dissipation and variable suction. The results of the coupled nonlinear differential equations governing the flow were obtained using collocation method with the aid of assumed Legendre polynomial.

**Main text**

Consider unsteady free convective MHD laminar incompressible, electrically conducting fluid flow via a permeable medium with slip condition in the presence of inclined magnetic field, diffusion-thermo and thermal-diffusion. The plate is considered to be porous and infinite, x-axis is considered in the vertical direction of the plate, while η-axis is perpendicular to the plate. There is application of inclined magnetic field in η-axis direction. Soret and Dufour are not negligible because it is assumed to be of substantial magnitude. Since application of voltage externally is absent, electric field is not considered. The flow configuration is shown below (Fig. 1).

Based on Boussinesq approximation and the above assumptions, the equations governing the flow can be expressed as

\[
\frac{\partial v^*}{\partial \eta^*} = 0
\]

\[
\frac{\partial h^*}{\partial t^*} - V^* \frac{\partial h^*}{\partial \eta^*} = \frac{\partial H^*_\infty}{\partial t^*} + g \beta \left( \tilde{\theta}^* - \tilde{\theta}_\infty \right) - g \beta \left( \tilde{\zeta}^* - \tilde{\zeta}_\infty \right) \frac{\nu}{K^*} \frac{\partial h^*}{\partial \eta^*} + \frac{\nu}{K^*} + B \sin^2 \psi \left( h^* - H^*_\infty \right)
\]

\[
\frac{\partial \tilde{\theta}^*}{\partial t^*} - V^* \frac{\partial \tilde{\phi}^*}{\partial \eta^*} = \kappa \rho C_p \left( \frac{\partial^2 \tilde{\theta}^*}{\partial \eta^*^2} - \frac{1}{K^*} \frac{\partial q^*_\infty}{\partial \eta^*} \right) + \mu \rho C_p \left( \frac{\partial h^*}{\partial \eta^*} \right)^2 + DK_p \frac{\partial^2 \tilde{\zeta}^*}{\partial \eta^*^2}
\]
where $h^*$ and $v^*$ are dimensional velocity components in $x^*$ and $\eta^*$ directions, respectively, $t^*$ is the dimensional time, $V^*$ is the constant suction velocity, $H^*_\infty$ is the free stream velocity, $g$ is the acceleration as a result of gravitational force, $\beta$ is the thermal expansion coefficient, $\beta^*$ concentration expansion coefficient, $\vartheta^*$ is the dimensional temperature, $\vartheta^*_\infty$ is the temperature of the fluid at the free stream, $\zeta^*$ is the boundary layer species concentration, $\zeta^*_\infty$ is the species concentration at the free stream, $\nu$ is the kinematic viscosity, $K^*$ is the permeability of porous medium, $B$ is the magnetic field intensity, $\psi$ is the angle of inclination of magnetic field, $k$ is the thermal conductivity, $C_p$ is the specific heat at constant pressure, $q^*_r$ is the radiation heat flux, $\rho$ is the fluid density, $\mu$ is the viscosity coefficient, $D$ is the mass diffusivity, $K_\vartheta$ is the thermal diffusion ratio, $C_s$ concentration susceptibility, $k_1^*$ is the chemical reaction coefficient, and $\vartheta_m$ is the mean fluid temperature.

The boundary conditions are:

$$
\frac{\partial \zeta^*}{\partial t^*} - V^* \frac{\partial \zeta^*}{\partial \eta^*} = D \frac{\partial^2 \zeta^*}{\partial \eta^*^2} - k_1^* (\zeta^* - \zeta^*_\infty) + \frac{DK_\vartheta}{\vartheta_m} \frac{\partial^2 \vartheta^*}{\partial \eta^*^2}
$$

(4)

where $h^*$ and $v^*$ are dimensional velocity components in $x^*$ and $\eta^*$ directions, respectively, $t^*$ is the dimensional time, $V^*$ is the constant suction velocity, $H^*_\infty$ is the free stream velocity, $g$ is the acceleration as a result of gravitational force, $\beta$ is the thermal expansion coefficient, $\beta^*$ concentration expansion coefficient, $\vartheta^*$ is the dimensional temperature, $\vartheta^*_\infty$ is the temperature of the fluid at the free stream, $\zeta^*$ is the boundary layer species concentration, $\zeta^*_\infty$ is the species concentration at the free stream, $v$ is the kinematic viscosity, $K^*$ is the permeability of porous medium, $B$ is the magnetic field intensity, $\psi$ is the angle of inclination of magnetic field, $k$ is the thermal conductivity, $C_p$ is the specific heat at constant pressure, $q^*_r$ is the radiation heat flux, $\rho$ is the fluid density, $\mu$ is the viscosity coefficient, $D$ is the mass diffusivity, $K_\vartheta$ is the thermal diffusion ratio, $C_s$ concentration susceptibility, $k_1^*$ is the chemical reaction coefficient, and $\vartheta_m$ is the mean fluid temperature.

The boundary conditions are:

$$
\begin{align*}
\frac{\partial h^*}{\partial \eta^*} &= S^* \left( \frac{\partial \vartheta^*}{\partial \eta^*} \right), \quad \vartheta^* = \vartheta_w + \varepsilon (\vartheta_w - \vartheta^*_\infty) e^{i \omega^* t^*}, \quad \zeta^* = \zeta_w + \varepsilon (\zeta_w - \zeta^*_\infty) e^{i \omega^* t^*} \quad \text{at } \eta^* = 0; \\
h^* &\to H^*_\infty(t^*) = H_0 \left( 1 + \varepsilon A e^{i \omega^* t^*} \right), \quad \vartheta^* \to \vartheta^*_\infty, \quad \zeta^* \to \zeta^*_\infty \quad \text{at } \eta^* = \infty
\end{align*}
$$

(5)

Here $\varepsilon$ is the scalar constant, $A$ is the non-dimensional suction velocity parameter, $\omega^*$ is the dimensional oscillation parameter, $S^*$ is the slip parameter, $\vartheta_w$ and $\zeta_w$ are the temperature and species concentration at the plate, respectively, $H_\infty$ is the free stream velocity, and $H_0$ is a constant. The suction velocity is normal to the plate and is expressed as a function of time in the form

$$
V^* = -V_0 \left( 1 + \varepsilon A e^{i \omega^* t^*} \right)
$$

(6)

Radiative heat flux is given as

$$
q_r^* = -\frac{4 \sigma_s}{3k_e} \frac{\partial \vartheta^*}{\partial \eta^*}
$$

(7)
Expanding $\varphi^{*4}$ using Taylor series expansion and ignoring higher order terms from second order gives

$$\varphi^{*4} = 4\varphi^{*3}\varphi^{*} - 3\varphi^{*4}$$  \hspace{1cm} (8)

### Method of solution

The following non-dimensional quantities

$$h = \frac{h_*}{V_0}, \eta = \frac{\nu_0\varphi^{*} - \varphi^{*} \xi}{\varphi^{*} - \varphi^{*}}, \xi = \frac{\varphi^{*} - \varphi^{*}}{\varphi^{*} - \varphi^{*}}, \zeta = \frac{\varphi^{*} - \varphi^{*}}{\varphi^{*} - \varphi^{*}}, t = \frac{t_* V_0^2}{\nu_0}, \omega = \frac{4\varphi^{*}}{\nu_0}, D_u = \frac{D K (\varphi^{*} - \varphi^{*})}{c_0 c_0 (\varphi^{*} - \varphi^{*})},$$

$$S_t = \frac{D K (\varphi^{*} - \varphi^{*})}{v_0 \varphi_0 (\varphi^{*} - \varphi^{*})}, H_g = \frac{v_0 g (\varphi^{*} - \varphi^{*})}{v_0}, M_s = \frac{v_0 g (\varphi^{*} - \varphi^{*})}{v_0}, M = \frac{v_0 g (\varphi^{*} - \varphi^{*})}{v_0}, S = \frac{v_0 g (\varphi^{*} - \varphi^{*})}{v_0}, k_1 = \frac{v_0^2}{v_0}, v = \frac{v_0}{v_0}$$  \hspace{1cm} (9)

Equations (10)–(13) are obtained by introducing Eqs. (6)–(9) to Eqs. (2)–(5)

$$\frac{1}{4}\frac{\partial h}{\partial t} - \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial h}{\partial \eta} = \frac{1}{4}\frac{\partial H}{\partial \eta} + \frac{\partial^2 \xi}{\partial \eta^2} + H_g \xi + M_s \xi - \left(M \sin^2 \psi + 1 \right) (h - H)$$  \hspace{1cm} (10)

$$\frac{1}{4}\frac{\partial \xi}{\partial t} - \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial \xi}{\partial \eta} = \frac{1}{P_f} \left(1 + \frac{4}{3 R_d} \right) \frac{\partial^2 \xi}{\partial \eta^2} + E_c \left(\frac{\partial h}{\partial \eta}\right)^2 + D_u \frac{\partial^2 \xi}{\partial \eta^2}$$  \hspace{1cm} (11)

$$\frac{1}{4}\frac{\partial \xi}{\partial t} - \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial \xi}{\partial \eta} = \frac{1}{S_c \partial \eta^2} - k_1 \xi + S_c \frac{\partial^2 \xi}{\partial \eta^2}$$  \hspace{1cm} (12)

The boundary in non-dimensional form is

$$h = \left\{\begin{array}{ll}
S \left(\frac{\partial h}{\partial \eta}\right), & \xi = 1 + \varepsilon e^{i\omega t}, \zeta = 1 + \varepsilon e^{i\omega t} \quad \text{at} \eta = 0; \\
\lambda (1 + \varepsilon e^{i\omega t}), & \xi \rightarrow 0, \xi \rightarrow 0 \quad \text{at} \eta \rightarrow \infty
\end{array}\right\}$$  \hspace{1cm} (13)

where $S$ is the slip parameter, $\lambda = \frac{h_0}{V_0}$, $h$ is the non-dimensional velocity along $x$-axis, $\xi$ is the non-dimensional fluid temperature, $\zeta$ is the non-dimensional concentration, $D_u$ is the Dufour parameter, $S_t$ is the non-dimensional fluid temperature, respectively, $H_g$ and $M_s$ are the Grashof number for heat and mass transfer, respectively, $M$ is the Hartmann number, $P_f$ is the Prandtl number, $K$ is the permeability parameter, $R_d$ is the radiation parameter, $E_c$ is the Eckert number, $S_c$ is the Schmidt number, $H$ is the free stream velocity, $k_1$ is the chemical reaction parameter, $\sigma$ is the fluid electrical conductivity, and $k_e$ is the mean absorption coefficient.

Considering the associated boundary conditions, the assumed solutions can be expressed as

$$h(\eta, t) = h_0(\eta) + \varepsilon e^{i\omega t} h_1(\eta)$$
$$\xi(\eta, t) = \xi_0(\eta) + \varepsilon e^{i\omega t} \xi_1(\eta)$$
$$\zeta(\eta, t) = \zeta_0(\eta) + \varepsilon e^{i\omega t} \zeta_1(\eta)$$  \hspace{1cm} (14)

Applying assumed solutions (14) to Eqs. (10)–(13) and equating the terms, harmonic and non-harmonic, gives
Now, the boundary conditions reduced to

\[ \xi''_0 + P_r \left(1 + \frac{4}{3R_d}\right)^{-1} \xi'_0 = E_c P_r \left(1 + \frac{4}{3R_d}\right)^{-1} \xi_0'' - D_a \xi''_0 \]  \hspace{1cm} (17)

\[ \xi''_1 + P_r \left(1 + \frac{4}{3R_d}\right)^{-1} \xi'_1 - \frac{i \omega P_r}{4} \left(1 + \frac{4}{3R_d}\right)^{-1} \xi_1 = -A P_r \left(1 + \frac{4}{3R_d}\right)^{-1} \xi_1' - D_a \xi''_1 \]  \hspace{1cm} (18)

\[ \zeta''_0 + S_c \zeta'_0 - k_1 S_c \zeta_0 = -S_t \zeta''_0 \]  \hspace{1cm} (19)

\[ \zeta''_1 + S_c \zeta'_1 - S_c \left(\frac{i \omega}{4} + k_1\right) \zeta_1 = -S_c A \zeta''_0 - S_t \zeta''_1 \]  \hspace{1cm} (20)

Now, the boundary conditions reduced to

\[ h_0 = S \left(\frac{\partial h_0}{\partial \eta}\right), h_1 = S \left(\frac{\partial h_1}{\partial \eta}\right), \xi_0 = 1, \xi_1 = 1, \zeta_0 = 1, \zeta_1 = 1 \text{ at } \eta = 0; \]
\[ h_0 \to \lambda, h_1 \to A \lambda, \xi_0 \to 0, \xi_1 \to 0, \zeta_0 \to 0, \zeta_1 \to 0 \text{ at } \eta \to \infty \]  \hspace{1cm} (21)

Legendre collocation method
The solutions to the coupled, nonlinear ordinary differential Eqs. (15)–(20) with the boundary conditions (21) are obtained using collocation method with assumed Legendre polynomial. Applying the domain truncation method, the interval \([0, \infty)\) is transformed to \([0, L]\). The Legendre polynomial is of interval \([-1, 1]\) which is transformed to \([0, L]\) using the transformation

\[ y = \frac{2 \eta}{L} - 1. \]  \hspace{1cm} (22)

Hence, the boundary value problem is solved within the region \([0, L]\) instead of \([0, \infty)\), where \(L\) (scaling parameter) is taken to be sufficiently large enough to take care of the thickness of the boundary layer (Olagunju et al. [20] and Aysun and Salih [21]). Therefore, the Legendre polynomial is expressed as

\[ h_0 = \sum_{j=0}^{N} a_j P_j(y) \text{ for } j = 0, 1, 2, 3, 4, \ldots, N \]  \hspace{1cm} (23)

Hence,
Substituting Eqs. (22) and (25) in Eq. (24) gives

\[ h_0 = a_0 P_0(y) + a_1 P_1(y) + a_2 P_2(y) + \cdots \]  

where

\[ P_0(y) = 1, \quad P_1(y) = y, \quad P_2(y) = \frac{1}{2} \left( 3y^2 - 1 \right) \]  

Substituting Eqs. (22) and (25) in Eq. (24) gives

\[ h_0 = a_0 + a_1 \left( \frac{2\eta}{L} - 1 \right) + \frac{a_2}{2} \left[ 3 \left( \frac{2\eta}{L} - 1 \right)^2 - 1 \right] + \cdots \]  

For \( L = 20 \) and \( N = 20 \), Eq. (26) becomes

\[ h_0 = a_0 + \frac{1}{10}(-10 + \eta) a_1 + \frac{1}{200} \left( 200 - 60\eta + 3\eta^2 \right) a_2 + \cdots \]  

Similarly,

\[ h_1 = b_0 + \frac{1}{10}(-10 + \eta) b_1 + \frac{1}{200} \left( 200 - 60\eta + 3\eta^2 \right) b_2 + \cdots \]  

\[ \xi_0 = c_0 + \frac{1}{10}(-10 + \eta) c_1 + \frac{1}{200} \left( 200 - 60\eta + 3\eta^2 \right) c_2 + \cdots \]  

\[ \xi_1 = d_0 + \frac{1}{10}(-10 + \eta) d_1 + \frac{1}{200} \left( 200 - 60\eta + 3\eta^2 \right) d_2 + \cdots \]  

\[ \zeta_0 = e_0 + \frac{1}{10}(-10 + \eta) e_1 + \frac{1}{200} \left( 200 - 60\eta + 3\eta^2 \right) e_2 + \cdots \]  

\[ \zeta_1 = f_0 + \frac{1}{10}(-10 + \eta) f_1 + \frac{1}{200} \left( 200 - 60\eta + 3\eta^2 \right) f_2 + \cdots \]  

\( \{a_0, a_1, \ldots, a_N\}, \{b_0, b_1, \ldots, b_N\}, \{c_0, c_1, \ldots, c_N\}, \{d_0, d_1, \ldots, d_N\}, \{e_0, e, \ldots, e_N\} \) and \( \{f_0, f, \ldots, f_N\} \) are unknown coefficients which can be obtained using the coupled, nonlinear differential Eqs. (15)–(20) with the boundary conditions (21). Hence, the approximate solutions of the truncated series (27)–(32) can be obtained.

\[ \text{Collpoints} = \text{NSolve}[\text{Expand}[\text{LegendreP}[N - 1, 2\frac{\eta}{L} - 1]] = 0, \eta] \]  

Equation (33) is a MATHEMATICA Software Language used to generate the following collocation points for values of \( \eta \):

\[
0.0759316, 0.397918, 0.968441, 1.77285, 2.79034, 3.99455, \\
5.35429, 6.83436, 8.39641, 10.0000, 11.6036, 13.1657, \\
14.646, 16.0051, 17.2097, 18.2273, 19.0319, 19.6002, 19.9241
\]  

Equations (27)–(32) are substituted in Eqs. (15)–(20), with the default values for the fluid parameters; \( H_g = 5, M_g = 5, P_r = 0.71, E_c = 0.01, \psi = \frac{\pi}{2}, k_1 = 0.1, S_c = 0.22, \)
\(R_d = 1.6, K = 0.1, D_a = 0.1, S_t = 0.2, M = 1, t = 0.1, A = 0.5, S = 0.2, \lambda = 0.5, \omega = 1,\) and \(\varepsilon = 0.1,\) to give six residual equations. By imposing the boundary conditions (21) on Eqs. (27)–(32), twelve equations are derived. Each residual equation is collocated at the above collocation points to yield one hundred and fourteen collocation equations. Consequently, there are a total number of one hundred and twenty-six equations with one hundred and twenty-six unknown coefficients. These equations are solved using MATHEMATICA 11.0 software. The numerical values obtained for the unknown coefficients are then substituted back into Eqs. (27)–(32). Hence, Eq. (14) becomes

\[
h = 0.631525 + 5.71453 \times 10^{-25} i - \left(0.0248978 + 3.12151 \times 10^{-25} i\right) \left(200 - 60\eta + 3\eta^2\right)
- \left(0.000480709 - 8.45787 \times 10^{-27} i\right) \left(-400 + 240\eta - 30\eta^2 + \eta^3\right) - \ldots
\]

\[
\xi = 0.140729 + 1.49935 \times 10^{-24} i - \left(0.030269 + 7.79491 \times 10^{-25} i\right) - 10 + \eta
+ (0.00136063 + 1.65512 \times 10^{-26} i)(200 - 60\eta + 3\eta^2) - \ldots
\]

\[
\zeta = 0.194627 - 2.1262 \times 10^{-25} i - \left(0.00373656 - 8.6365 \times 10^{-26} i\right) (-10 + \eta)
+ (0.00134708 - 1.58859 \times 10^{-27} i)(200 - 60\eta + 3\eta^2) - \ldots
\]

Skin-friction:
The coefficient of skin-friction in non-dimensional form is expressed as

\[
C_f = \left(\frac{\partial}{\partial \eta} \left(h_0(\eta) + \varepsilon e^{i\omega t} h_1(\eta)\right)\right)_{\eta=0}
\]  

\[
(38)
\]

Nusselt Number
The rate of heat transfer at the plate is expressed as

\[
\mathrm{Nu} = - \left(\frac{\partial}{\partial \eta} \left(\xi_0(\eta) + \varepsilon e^{i\omega t} \xi_1(\eta)\right)\right)_{\eta=0}
\]  

\[
(39)
\]

Sherwood Number
The rate of mass transfer in non-dimensional form is given as

\[
\mathrm{Sh} = - \left(\frac{\partial}{\partial \eta} \left(\zeta_0(\eta) + \varepsilon e^{i\omega t} \zeta_1(\eta)\right)\right)_{\eta=0}
\]  

\[
(40)
\]

Results and discussion
Ordinary differential Eqs. (15)–(20) with the boundary conditions (21) are solved using collocation method with the aid of assumed Legendre polynomial. On velocity, temperature and concentration profiles, the impacts of various parameters are considered and
the results are presented both in graphical and tabular forms. For the computations, the default values are: \(H_g = 5\), \(M_g = 5\), \(P_r = 0.71\), \(E_c = 0.01\), \(\psi = \frac{\pi}{2}\), \(k_1 = 0.1\), \(S_c = 0.22\), \(R_d = 1.6\), \(K = 0.1\), \(D_u = 0.1\), \(S_t = 0.2\), \(M = 1\), \(t = 0.1\), \(A = 0.5\), \(S = 0.2\), \(\lambda = 0.5\), \(\omega = 1\), and \(\varepsilon = 0.1\). The comparison of this work with the work of Sharma and Choudhary [19] is presented in Table 1, by setting Dufour and Soret parameters to zero, an excellent agreement is observed. In order to further validate the results, the comparison of Legendre collocation method with fourth-order Runge–Kutta method for skin friction, Nusselt number and Sherwood Number is displayed in Table 2 where \(d\) is the difference between the collocation method and fourth-order R-K.

Figures 2 and 3 display the variation of thermal \(H_g\) and solutal \(M_g\) Grashof number, respectively, on velocity profiles. Physically, increase in thermal Grashof number will make the buoyancy force to rise, which in turn accelerates within the channel the viscous hydrodynamics. Solutal Grashof number can be expressed as ratio of concentration buoyancy force to viscous hydrodynamic force. From these figures, it is obvious that a rise in \(H_g\) enhanced velocity profiles. The same trend is evident in Fig. 3, a hike in \(M_g\) speed-up velocity profiles.

Effects of angle of inclination parameter \(\psi\) is depicted in Fig. 4. Increase in \(\psi\) retards the velocity distribution. This is as a result of magnetic field being strengthen with a rise in \(\psi\). Magnetic field exhibits a resistance force known as Lorentz force. This force resists the fluid motion. Hence, in Fig. 5 it is evident that Hartmann number \(M\) has the tendency of retarding the fluid velocity.

The comparison of the present work with Sharma and Choudhary [19] is presented in Table 1, by setting Dufour and Soret parameters to zero, an excellent agreement is observed. In order to further validate the results, the comparison of Legendre collocation method with fourth-order Runge–Kutta method for skin friction, Nusselt number and Sherwood Number is displayed in Table 2 where \(d\) is the difference between the collocation method and fourth-order R-K.

### Table 1 Comparison of the present work with Sharma and Choudhary [19]

| \(R\) | \(k_1\) | Sharma and Choudhary | Present work |
|-------|--------|-----------------------|--------------|
|       |        | Temp. (\(\xi\)) | Conc. (\(\zeta\)) | Temp. (\(\xi\)) | Conc. (\(\zeta\)) |
| 1.0   | 0.1    | 0.3062 | 0.2978          | 0.2999 | 0.2988 |
| 1.6   | 0.4    | 0.3895 | 0.2978          | 0.3812 | 0.2988 |

### Table 2 Comparison of the collocation method with fourth-order Runge–Kutta method

| \(D_u\) | \(S_t\) | \(E_c\) | \(k_1\) | \(R_d\) | Collocation method | 4th order R-K | \(|d|\) |
|--------|--------|--------|--------|--------|-------------------|---------------|--------|
| 0.3    |        |        |        |        | 5.10354          | 5.10354       | 0.00000 |
| 0.6    |        |        |        |        | 5.13080          | 5.13080       | 0.00000 |
| 0.2    | 0.05   | 5.08704 | 5.08704 | 0.00000 |
| 0.6    | 0.07   | 5.14038 | 5.14038 | 0.00000 |
| 0.3    | 0.05   | 5.09275 | 5.09275 | 0.00000 |
| 0.7    | 0.07   | 5.09561 | 5.09561 | 0.00000 |
| 0.3    | 0.05   | 5.04433 | 5.04432 | 0.00001 |
| 0.7    | 0.07   | 4.98716 | 4.98715 | 0.00001 |
| 1.7    | 0.07   | 5.13774 | 5.13773 | 0.00001 |

The comparison of this work with the work of Sharma and Choudhary [19] is presented in Table 1, by setting Dufour and Soret parameters to zero, an excellent agreement is observed. In order to further validate the results, the comparison of Legendre collocation method with fourth-order Runge–Kutta method for skin friction, Nusselt number and Sherwood Number is displayed in Table 2 where \(d\) is the difference between the collocation method and fourth-order R-K.

Figures 2 and 3 display the variation of thermal \(H_g\) and solutal \(M_g\) Grashof number, respectively, on velocity profiles. Physically, increase in thermal Grashof number will make the buoyancy force to rise, which in turn accelerates within the channel the viscous hydrodynamics. Solutal Grashof number can be expressed as ratio of concentration buoyancy force to viscous hydrodynamic force. From these figures, it is obvious that a rise in \(H_g\) enhanced velocity profiles. The same trend is evident in Fig. 3, a hike in \(M_g\) speed-up velocity profiles.

Effects of angle of inclination parameter \(\psi\) is depicted in Fig. 4. Increase in \(\psi\) retards the velocity distribution. This is as a result of magnetic field being strengthen with a rise in \(\psi\). Magnetic field exhibits a resistance force known as Lorentz force. This force resists the fluid motion. Hence, in Fig. 5 it is evident that Hartmann number \(M\) has the tendency of retarding the fluid velocity.
Figure 6 portrays effects of permeability of the porous medium parameter ($K$) on velocity profiles. Momentum boundary layer thickness is boosted with higher values of $K$. Physically, this result can be justified by ignoring the permeability holes.
Variation of chemical reaction parameter ($k_1$) on concentration and velocity distribution is detected in Figs. 7 and 8. It is observed in Fig. 7 that a hike in the value of $k_1$ slowed down the concentration profiles. This is as a result of reduction in solutal boundary layer thickness and mass transfer increases due to destructive chemical.
The same trend is apparent in Fig. 8; $k_1$ has the tendency of retarding the velocity of the fluid.

The effects of Prandtl number ($P_r$) on temperature and velocity profile are elucidated in Figs. 9 and 10. Prandtl can be expressed as relativity of momentum diffusivity to thermal diffusivity; therefore, high thermal conductivity reduced the velocity of the fluid. High values of $P_r$ imply high thermal conductivity; hence from the heated surface, heat diffuses away more rapidly. Consequently, the thermal boundary layer is lessened with hike in $P_r$, as displayed in Fig. 10.

Figures 11 and 12 show the variation of thermal radiation ($R_d$) on temperature and velocity profiles. Dominance of conduction over $R_d$ accelerated with a rise in thermal radiation. As a result, both buoyancy force and thermal boundary temperature are slowed down. Generally, though trivial fact, there is inverse proportionality between thermal radiation and temperature. From Fig. 12, it is noticed that the increase in $R_d$ decelerated the velocity distributions.

It is shown in Figs. 13 and 14 the effects of Schmidt number ($S_c$) on concentration and velocity distribution. Both concentration and velocity are retarded with the increase in $S_c$. Physically, a boost in $S_c$ implies a reverse trend in molecular diffusion.
Figures 15, 16 and 17 reveal the effects of thermal-diffusion \((S_t)\) on concentration, temperature and velocity distributions. A careful examination of these figures shows that concentration and velocity improved for higher values of \(S_t\). Soret \((S_t)\) resulted
from mass flux; hence, there is tendency for concentration to increase. In Fig. 16, the effect of $S_t$ on the temperature of the fluid is not noticeable. Figures 18, 19 and 20 depict the variation of diffusion-thermo ($D_u$) on concentration, temperature and
velocity. Both temperature and velocity accelerated as the values of $D_u$ speed up. The enhancement of temperature due to a rise in $D_u$ is as a result of energy flux being
generated. Consequently, there is increase in heat of the fluid flow. It is apparent in Fig. 18 that concentration profiles are indifference to the increase in $D_u$.

Tables 3, 4 and 5 are the numerical presentation of Figs. 15, 16 and 17, respectively. In Table 4, there is evidence of slight impact of Soret on the fluid temperature, though this effect is not noticeable on the graph. It is obvious in Table 4 that from $\eta = 0$ to
\( \eta = 4 \), temperature tends to decline, while a different trend is observed from \( \eta = 6 \) to \( \eta = 20 \) where Soret (\( S_t \)) shows the tendency of enhancing the fluid temperature.

Table 4 displays the effects of \( E_c \), \( R \), \( k_1 \), \( \psi \), \( S_t \) and \( D_u \) on Skin-friction \( C_f \), Nusselt number \( Nu \) and Sherwood Number \( Sh \). It is discovered that \( E_c \), \( S_t \) and \( D_u \) have the tendency of enhancing \( C_f \), while it is slowed down by \( R \), \( k_1 \) and \( \psi \). The heat transfer is improved with the increase in \( R \), \( \psi \) and \( S_t \). A reverse trend is noticed for a rise in \( E_c \), \( k_1 \) and \( D_u \). Finally, mass transfer accelerated as a result of a hike in \( E_c \), \( k_1 \) and \( D_u \). On the other hand, \( Sh \) is lessened with the increase in \( R \), \( \psi \) and \( S_t \).

### Conclusion

This study is carried out to examine the combined effects of diffusion-thermo, thermal-diffusion, viscous dissipation and thermal radiation on unsteady MHD slip flow with inclined magnetic field over a permeable vertical plate. The nonlinear coupled differential equations are solved using collocation method with the aid of assumed Legendre polynomial. The results are graphically and tabularly presented. From the study, though Dufour and Soret has the tendency of improving the fluid velocity but mass flux as a result of temperature gradient has insignificant impact on temperature of the fluid, likewise energy flux on concentration. The result revealed that Soret and Dufour effect is relevant in mixture of gases of light molecular weight and is applicable in different areas.

It is detected that:

(i) Velocity of the fluid is retarded with a rise in \( \psi \), \( M \), \( k_1 \), \( P_t \), \( R_d \) and \( S_c \). On the other hand, the fluid velocity improved for higher values of \( H_g \), \( M_g \), \( K \), \( S_t \) and \( D_u \).

(ii) Increase in \( P_t \) and \( R_d \) reduced the thermal boundary layer.
(iii) The concentration profiles are improved for higher values of $S_t$, while they decelerated with hike in $k_1$.

### Table 6 Values of skin-friction, Nusselt number and Sherwood Number for different parameters

| $R_d$ | $E_c$ | $k_1$ | $\psi$ | $S_t$ | $D_u$ | $C_f$ | $Nu$ | $Sh$ |
|-------|-------|-------|--------|-------|-------|-------|------|------|
| 0.5   | 5.58856 | 0.19238 | 0.30835 |
| 1.0   | 5.53619 | 0.31281 | 0.28514 |
| 1.5   | 5.50341 | 0.39306 | 0.26950 |
| 0.02  | 5.49998 | 0.39145 | 0.26987 |
| 0.03  | 5.50165 | 0.37699 | 0.27276 |
| 0.04  | 5.50332 | 0.36251 | 0.27565 |
| 0.2   | 5.46935 | 0.39956 | 0.33086 |
| 0.4   | 5.42733 | 0.39012 | 0.42709 |
| 0.6   | 5.49998 | 0.39145 | 0.26987 |
| $\pi$ | 5.56836 | 0.40512 | 0.26714 |
| $\pi$ | 5.55877 | 0.40523 | 0.26712 |
| $\pi$ | 5.54367 | 0.40539 | 0.26709 |
| 0.1   | 5.48278 | 0.40254 | 0.30093 |
| 0.4   | 5.53015 | 0.41279 | 0.19703 |
| 0.7   | 5.57999 | 0.42367 | 0.08659 |
| 0.2   | 5.50922 | 0.37886 | 0.27225 |
| 0.5   | 5.54415 | 0.29078 | 0.28942 |
| 0.8   | 5.58280 | 0.19053 | 0.30902 |

**Abbreviation**

MHD: Magneto-hydrodynamic.

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**Authors’ contributions**

TL formulated the problem of this research work and interpreted and discussed the results. SA computed the results and did the major work in the writing of the manuscript. Both authors read and approved the final manuscript.

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