TECHNICOLOR 2000
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ABSTRACT
This review is based on lectures on technicolor and extended technicolor presented at the Frascati Spring School in May 2000. I summarize the motivation and structure of this theory of dynamical breaking of electroweak and flavor symmetries. Particular attention is paid to the main phenomenological obstacles to this picture—flavor-changing neutral currents, precision electroweak measurements, and the large top–quark mass—and their proposed resolutions—walking technicolor and topcolor–assisted technicolor. I then discuss the signatures for technicolor and the existing and upcoming searches for them at LEP, the Tevatron Collider, and the Large Hadron Collider. The final section lists some outstanding theoretical questions.
1. The Motivation for Technicolor and Extended Technicolor

The elements of the standard model of elementary particles have been in place for more than 25 years now. These include the $SU(3) \otimes SU(2) \otimes U(1)$ gauge model of strong and electroweak interactions [1, 2]. And, they include the Higgs mechanism used to break spontaneously electroweak $SU(2) \otimes U(1)$ down to the $U(1)$ of electromagnetism [3]. In the standard model, couplings of the elementary Higgs scalar bosons also break explicitly quark and lepton chiral–flavor symmetries, giving them hard (Lagrangian) masses. In this quarter century, the standard model has stood up to the most stringent experimental tests [4, 5]. The only indications we have of physics beyond this framework are the existence of neutrino mixing and, presumably, masses (though some would say this physics is accommodated within the standard model); the enormous range of masses, about $10^{12}$, between the neutrinos and the top quark; the need for a new source of CP–violation to account for the baryon asymmetry of the universe; the likely presence of cold dark matter; and, possibly, the cosmological constant. These hints are powerful. But they are also obscure, and they do not point unambiguously to any particular extension of the standard model.

In addition to these experimental facts, considerable theoretical discomfort and dissatisfaction with the standard model have dogged it from the beginning. All of it concerns the elementary Higgs boson picture of electroweak and flavor symmetry breaking—the cornerstone of the standard model. In particular:

1. Elementary Higgs models provide no dynamical explanation for electroweak symmetry breaking.

2. Elementary Higgs models are unnatural, requiring fine tuning of parameters to enormous precision.

3. Elementary Higgs models with grand unification have a “hierarchy” problem of widely different energy scales.

4. Elementary Higgs models are “trivial”.

5. Elementary Higgs models provide no insight to flavor physics.

In nonsupersymmetric Higgs models, there is no explanation why electroweak symmetry breaking occurs and why it has the energy scale of 1 TeV. The Higgs doublet self–interaction potential is $V(\phi) = \lambda(\phi^\dagger \phi - v^2)^2$, where $v$ is the
vacuum expectation of the Higgs field $\phi$ when $v^2 \geq 0$. Its experimental value is $v = 2^{-1/4}G_F^{-1/2} = 246 \text{ GeV}$. But what dynamics makes $v^2 > 0$? What dynamics sets its magnitude? In supersymmetric Higgs models, the large top–quark Yukawa coupling drives $v^2$ positive, but this just replaces one problem with another or, to be generous, replaces two with one.

Elementary Higgs boson models are unnatural. The Higgs boson’s mass, $M_H^2 = 2\lambda v^2$ is quadratically unstable against radiative corrections [6]. Thus, there is no natural reason why $M_H$ and $v$ should be much less than the energy scale at which the essential physics of the model changes, e.g., a unification scale or the Planck scale of $10^{16} \text{ TeV}$. To make $M_H$ very much less that $M_P$, say 1 TeV, the bare Higgs mass must be balanced against its radiative corrections to the fantastic precision of a part in $M_P^2/M_H^2 \sim 10^{32}$.

In grand–unified Higgs boson models, supersymmetric or not, there are two very different scales of gauge symmetry breaking, the GUT scale of about $10^{16} \text{ GeV}$ and the electroweak scale of a few 100 GeV. This hierarchy is put in by hand, and must be maintained by unnaturally–fine tuning in ordinary Higgs models, or by the “set it and forget it” feature of supersymmetry.

Taken at face value, elementary Higgs boson models are free field theories [7]. To a good approximation, the self–coupling $\lambda(\mu)$ of the minimal one–doublet Higgs boson at an energy scale $\mu$ is given by

$$\lambda(\mu) \approx \frac{\lambda(\Lambda)}{1 + (24/16\pi^2) \lambda(\Lambda) \log(\Lambda/\mu)}.$$  \hspace{1cm} (1)

This coupling vanishes for all $\mu$ as the cutoff $\Lambda$ is taken to infinity, hence the description “trivial”. This feature persists in a general class of two–Higgs doublet models [8] and it is probably true of all Higgs models. Triviality really means that elementary–Higgs Lagrangians are meaningful only for scales $\mu$ below some cutoff $\Lambda_\infty$ at which new physics sets in. The larger the Higgs couplings are, the lower the scale $\Lambda_\infty$. This relationship translates into the so–called triviality bounds on Higgs masses. For the minimal model, the connection between $M_H$ and $\Lambda_\infty$ is

$$M_H(\Lambda_\infty) \approx \sqrt{2\lambda(M_H)} v = \frac{2\pi v}{\sqrt{3} \log(\Lambda_\infty/M_H)}.$$  \hspace{1cm} (2)

Clearly, the cutoff has to be greater than the Higgs mass for the effective theory to have some range of validity. From lattice–based arguments [7], $\Lambda_\infty \gtrsim 2\pi M_H$. Since $v$ is fixed at 246 GeV in the minimal model, this implies the triviality bound
If the standard Higgs boson were to be found with a mass this large or larger, we would know for sure that additional new physics is lurking in the range of a few TeV. If the Higgs boson is light, less than 200–300 GeV, as it is expected to be in supersymmetric models, this transition to a more fundamental theory may be postponed until very high energy, but what lies up there worries us nonetheless.

Finally, in all elementary Higgs models, supersymmetric or not, every aspect of flavor is completely mysterious, from the primordial symmetry defining the number of quark and lepton generations to the bewildering patterns of flavor breaking. The presence of Higgs bosons has no connection to the existence of multiple identical fermion generations. The flavor–symmetry breaking Yukawa couplings of Higgs bosons to fermions are arbitrary free parameters, put in by hand. As far as we know, it is a logically consistent state of affairs that we may not understand flavor until we understand the physics of the Planck scale. I do not believe this. And, I cannot see how this problem, more pressing and immediate than any other save electroweak symmetry break itself, can be so cavalierly set aside by those pursuing the “theory of everything”.

The dynamical approach to electroweak and flavor symmetry breaking known as technicolor (TC) \cite{10, 11, 12} and extended technicolor (ETC), \cite{13, 14} emerged in the late 1970s in response to these shortcomings of the standard model. This picture was motivated first of all by the premise that every fundamental energy scale should have a dynamical origin. Thus, the weak scale embodied in the Higgs vacuum expectation value $v = 246$ GeV should reflect the characteristic energy of a new strong interaction—technicolor—just as the pion decay constant $f_\pi = 93$ MeV reflects QCD’s scale $\Lambda_{QCD} \sim 200$ MeV. For this reason, I write

$$F_\pi = 2^{-1/4}G_F^{1/2} = 246 \text{ GeV}$$

to emphasize that this quantity has a dynamical origin.

Technicolor, a gauge theory of fermions with no elementary scalars, is modeled on the precedent of QCD: The electroweak assignments of quarks to left–handed doublets and right–handed singlets prevent their bare mass terms. Thus, if there are no elementary Higgses to couple to, quarks have a large chiral symmetry,

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1. Precision electroweak measurements suggesting that $M_H < 200$ GeV do not take into account additional interactions that occur if the Higgs is heavy and the scale $\Lambda$ relatively low. Chivukula and Evans have argued that these interactions allow $M_H = 400$–500 GeV to be consistent with the precision measurements \cite{3}.

2. This is not quite fair. In the early days of the second string revolution, in the mid 1980s, there was a great deal of hope and even expectation that string theory would provide the spectrum—quantum numbers and masses—of the quarks and leptons. Those string pioneers and their descendants have learned how hard the flavor problem is.
$SU(6)_L \otimes SU(6)_R$ for three generations. This symmetry is spontaneously broken to the diagonal (vectorial) $SU(6)$ subgroup when the QCD gauge coupling grows strong near $\Lambda_{QCD}$. This produces 35 massless Goldstone bosons, the “pions”. According to the Higgs mechanism—whose operation requires no elementary scalar bosons [4]—this yields weak boson masses of $M_W = M_Z \cos \theta_W = \frac{1}{2} \sqrt{3} g f_\pi \simeq 50$ MeV [10]. These masses are 1600 times too small, but they do have the right ratio. Suppose, then, that there are technifermions belonging to a complex representation of a technicolor gauge group (taken to be $SU(N_{TC})$) whose coupling $\alpha_{TC}$ becomes strong at $\Lambda_{TC} = 100$s of GeV. If, like quarks, technifermions form left–handed doublets and right–handed singlets under $SU(2) \otimes U(1)$, then they have no bare masses. When $\alpha_{TC}$ becomes strong, the technifermions’ chiral symmetry is spontaneously broken, Goldstone bosons appear, three of them become the longitudinal components of $W^\pm$ and $Z^0$, and the masses become $M_W = M_Z \cos \theta_W = \frac{1}{2} g F_\pi$. Here, $F_\pi \sim \Lambda_{TC}$ is the decay constant of the linear combination of the absorbed “technipions”. Thus, technicolor provides a dynamical basis for electroweak symmetry breaking, one that is based on the familiar and well–understood precedent of QCD.

Technicolor, like QCD, is asymptotically free. This solves in one stroke the naturalness, hierarchy, and triviality problems. The mass of all ground–state technihadrons, including Higgs–like scalars (though that language is neither accurate nor useful in technicolor) is of order $\Lambda_{TC}$ or less. There are no large renormalizations of bound state masses, hence no fine–tuning of parameters. If the technicolor gauge symmetry is embedded at a very high energy $\Lambda$ in some grand unified gauge group with a relatively weak coupling, then the characteristic energy scale $\Lambda_{TC}$—where the coupling $\alpha_{TC}$ becomes strong enough to trigger chiral symmetry breaking—is naturally exponentially smaller than $\Lambda$. Finally, asymptotically free field theories are nontrivial. A minus sign in the denominator of the analog of Eq. (1) for $\alpha_{TC}(\mu)$ prevents one from concluding that it tends to zero for all $\mu$ as the cutoff is taken to infinity. No other scenario for the physics of the TeV scale solves these problems so neatly. Period.

Technicolor alone does not address the flavor problem. It does not tell us why there are multiple generations and it does not provide explicit breaking of quark and lepton chiral symmetries. Something must play the role of Higgs bosons to communicate electroweak symmetry breaking to quarks and leptons. Furthermore, in all but the minimal TC model with just one doublet of technifermions, there are Goldstone bosons, technipions $\pi_T$, in addition to $W_L^\pm$ and $Z_L^0$. These must be given mass and their masses must be more than 50–100 GeV for them to have escaped detection. Extended technicolor (ETC) was invented to address all these aspects of
It was also motivated by the desire to make flavor understandable at energies well below the GUT scale solely in terms of gauge dynamics of the kind that worked so neatly for electroweak symmetry breaking, namely, technicolor. Let me repeat: the ETC approach is based on the gauge dynamics of fermions only. There can be no elementary scalar fields to lead us into the difficulties technicolor itself was invented to escape.

2. Dynamical Basics

In extended technicolor, ordinary $SU(3)$ color, $SU(N_{TC})$ technicolor, and flavor symmetries are unified into the ETC gauge group, $G_{ETC}$. Thus, we understand flavor, color, and technicolor as subsets of the quantum numbers of extended technicolor. Technicolor and color are exact gauge symmetries. Flavor gauge symmetries are broken at a one or more high energy scales $\Lambda_{ETC} \approx M_{ETC}/g_{ETC}$ where $M_{ETC}$ is a typical flavor gauge boson mass.

In these lectures, I assume that $G_{ETC}$ commutes with electroweak $SU(2)$. In this case, it must not commute with electroweak $U(1)$, i.e., some part of that $U(1)$ must be contained in $G_{ETC}$. Otherwise, there will be very light pseudoGoldstone bosons which behave like classical axions and are ruled out experimentally [13, 16].

More generally, all fermions—technifermions, quarks, and leptons—must form no more than four irreducible ETC representations: two equivalent ones for left–handed up and down–type fermions and two inequivalent ones for right–handed up and down fermions (so that up and down mass matrices are not identical). In other words, ETC interactions explicitly break all global flavor symmetries so that there are no very light pseudoGoldstone bosons or fermions.

The energy scale of ETC gauge symmetry breaking is high, well above the TC scale of 0.1–1.0 TeV, into $SU(3) \otimes SU(N_{TC})$. The broken gauge interactions, mediated by massive ETC boson exchange, give mass to quarks and leptons by connecting them to technifermions (Fig. 1a). They give mass to technipions by connecting technifermions to each other (Fig. 1b).

The graphs in Figs. 1 are convergent: The changes in chirality imply insertions on the technifermion lines of the momentum–dependent dynamical mass, $\Sigma(p)$. This function falls off as $1/p^2 (\log(p/\Lambda_{TC}))^c$ in an asymptotically free theory at weak coupling and, in any case, at least as fast as $1/p$ [17, 18]. For such a power law, the dominant momentum running around the loop is $M_{ETC}$. Then, the opera-

[3] I leave neutrinos out of this discussion. Their very light masses are not yet understood in the ETC framework.
tor product expansion tells us that the generic quark or lepton mass and technipion mass are given by the expressions

\[ m_q(M_{ETC}) \simeq m_\ell(M_{ETC}) \simeq \frac{g^2_{ETC}}{M^2_{ETC}} \langle T_L T_R \rangle_{ETC}; \quad (3) \]

\[ F_T^2 M^2_{\pi T} \simeq 2 \frac{g^2_{ETC}}{M^2_{ETC}} \langle \bar{T}_L T_R \bar{T}_R T_L \rangle_{ETC}. \quad (4) \]

Here, \( m_q(M_{ETC}) \) is the quark mass renormalized at \( M_{ETC} \). It is a hard mass in that it scales like one for energies below \( M_{ETC} \). Above that, it falls off more rapidly, like \( \Sigma(p) \). The technipion decay constant \( F_T = F_\pi/\sqrt{N} \) in TC models containing \( N \) identical electroweak doublets of color–singlet technifermions. The vacuum expectation values \( \langle \bar{T}_L T_R \rangle_{ETC} \) and \( \langle \bar{T}_L T_R \bar{T}_R T_L \rangle_{ETC} \) are the bilinear and quadrilinear technifermion condensates renormalized at \( M_{ETC} \). The bilinear condensate is related to the one renormalized at \( \Lambda_{TC} \), expected by scaling from QCD to be

\[ \langle T_L T_R \rangle_{TC} = \frac{1}{2} \langle \bar{T}T \rangle_{TC} \simeq 2\pi F_T^3, \quad (5) \]

by the equation

\[ \langle \bar{T}T \rangle_{ETC} = \langle \bar{T}T \rangle_{TC} \exp \left( \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma_m(\mu) \right). \quad (6) \]

The anomalous dimension \( \gamma_m \) of the operator \( \bar{T}T \) is given in perturbation theory by

\[ \gamma_m(\mu) = \frac{3 C_2(R)}{2\pi} \alpha_{TC}(\mu) + O(\alpha^2_{TC}), \quad (7) \]

where \( C_2(R) \) is the quadratic Casimir of the technifermion \( SU(N_{TC}) \)–representation \( R \). For the fundamental representation of \( SU(N_{TC}) \), \( C_2(N_{TC}) = (N_{TC}^2 - 1)/2N_{TC} \).
Finally, in the large–$N_{TC}$ approximation (which will be questionable in the walking technicolor theories we discuss later, but which we adopt anyway for rough estimates)

$$\langle \bar{T}_LT_R\bar{T}_RT_L \rangle_{ETC} \simeq \langle \bar{T}_LT_R \rangle_{ETC} \langle \bar{T}_RT_L \rangle_{ETC} = \frac{1}{4} \langle \bar{T}T \rangle_{ETC}^2. \quad (8)$$

We can obtain an estimate of $M_{ETC}$ if we assume that technicolor is QCD–like. In that case, its asymptotic freedom sets in quickly (or “precosously”) at energies above $\Lambda_{TC}$ and $\gamma_m(\mu) \ll 1$ for $\mu$ greater than a few times $\Lambda_{TC}$. Then Eq. (5) applies to $\langle \bar{T}T \rangle_{ETC}$. For $N$ technidoublets, the ETC scale required to generate $m_q(M_{ETC}) \simeq 1$ GeV is

$$\Lambda_{ETC} \equiv \frac{M_{ETC}}{g_{ETC}} \simeq \sqrt{\frac{2\pi F_\pi^3}{m_qN^{3/2}}} \simeq \frac{10 \text{ TeV}}{N^{3/4}}. \quad (9)$$

This is pretty low, but the estimate is rough. The typical technipion mass implied by this ETC scale is

$$M_{\pi_T} \simeq \frac{\langle TT \rangle_{TC}}{\sqrt{2\Lambda_{ETC}F_T}} \simeq \frac{55 \text{ GeV}}{N^{1/4}}. \quad (10)$$

Finally, some phenomenological basics: In any model of technicolor, one expects bound technihadrons with a spectrum of mesons paralleling what we see in QCD. The principal targets of collider experiments are the spin–zero technipions and spin–one isovector technirhos and isoscalar techniomegas. In the minimal one–technidoublet model ($T = (T_U, T_D)$), the three technipions are the longitudinal components $W_L$ of the massive weak gauge bosons. Susskind [10] pointed out that the analog of the QCD decay $\rho \rightarrow \pi\pi$ is $\rho_T \rightarrow W_LW_L$. In the limit that $M_{\rho_T} \gg M_{W,Z}$, the equivalence theorem states that the amplitude for $\rho_T \rightarrow W_LW_L$ has the same form as the one for $\rho \rightarrow \pi\pi$. If we scale technicolor from QCD and use large–$N_{TC}$ arguments, it is easy to estimate the strength of this amplitude and the $\rho_T$ mass and decay rate [19]:

$$M_{\rho_T} = \sqrt{\frac{3}{N_{TC}}} \frac{F_\pi}{f_\pi} M_\rho \simeq 2\sqrt{\frac{3}{N_{TC}}} \text{ TeV},$$

$$\Gamma(\rho_T \rightarrow W_LW_L) = \frac{2\alpha_{\rho_T}p_T^3}{3M_{\rho_T}^2} \simeq 500 \left(\frac{3}{N_{TC}}\right)^{3/2} \text{ GeV}. \quad (11)$$

Here, the naive scaling argument gives $\alpha_{\rho_T} = (3/N_{TC})\alpha_\rho$ where $\alpha_\rho = 2.91$.

In the minimal model, a very high energy collider, such as the ill–fated Superconducting Super Collider (SSC) or a 2 TeV linear collider, is needed to discover
the lightest technihadrons. In nonminimal models, where $N \geq 2$, the signatures of technicolor ought to be accessible at the Large Hadron Collider (LHC) and at a comparable lepton collider. We shall argue later that technicolor signatures are even likely to be within reach of the Tevatron Collider in Run II! Before we can do that, however, we must face the obstacles to technicolor dynamics and see how they are overcome.

3. Dynamical Perils

Technicolor and extended technicolor are challenged by a number of phenomenological hurdles, but the most widely cited causes of the “death of technicolor” are flavor–changing neutral current interactions (FCNC) [13, 22], precision measurements of electroweak quantities (STU) [23], and the large mass of the top quark. We discuss these in turn.

3.1 Flavor–Changing Neutral Currents

Extended technicolor interactions are expected to have flavor–changing neutral currents involving quarks and leptons. The reason for this is simple: Realistic quark mass matrices require ETC transitions between different flavors: $q \rightarrow T \rightarrow q'$. Thus, there must be ETC currents of the form $\bar{q}_L,R^\prime \gamma_\mu T_{L,R}$ and $\bar{T}_{L,R} \gamma_\mu q_{L,R}$; their commutator algebra includes the ETC currents $\bar{q}_L,R^\prime \gamma_\mu q_{L,R}$, and ETC interactions necessarily produce $\bar{q}q\bar{q}q$ operators at low energy. Similarly, there will be $\bar{q}q\ell\ell$ and $\bar{\ell}\ell\ell\ell$ operators. Even if these interactions are electroweak–eigenstate conserving (or generation–conserving), they will induce FCNC four–fermion operators after diag-

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4It is possible that, like the attention paid to discovering the minimal standard model Higgs boson, this emphasis on the $W_LW_L$ decay mode of the $\rho_T$ is somewhat misguided [11]. Since the minimal $\rho_T$ is so much heavier than $2M_W$, this mode may be suppressed by the high $W$–momentum in its decay form factor. Then, $\rho_T$ decays to four or more weak bosons may be competitive or even dominate. This means that the minimal $\rho_T$ may be wider than indicated in Eq. (11) and, in any case, that its decays are much more complicated than previously thought. Furthermore, walking technicolor [21], discussed below, implies that the spectrum of technihadrons cannot be exactly QCD–like. Rather, there must be something like a tower of technirhos extending almost up to $M_{ETC} \gtrsim$ several 100 TeV. Whether or not these would appear as discernible resonances is an open question [21]. All these remarks apply as well to the isoscalar $\omega_T$ and its excitations.

5Run II of the Tevatron Collider begins in Spring 2001. The first stage, Run IIa, is intended to collect 2 fb$^{-1}$ of data with significantly enhanced CDF and DØ detectors featuring new silicon tracking systems. It is planned that, after a brief shutdown to replace damaged silicon, Run IIb will bring the total data sets for each detector to 15 fb$^{-1}$ or more before the LHC is in full swing in 2006 or so.

6Much of the discussion here on FCNC and STU is a slightly updated version of that appearing in my 1993 TASI lectures [11].
onalization of mass matrices and transformation to the mass–eigenstate basis. No satisfactory GIM mechanism has ever been found that eliminates these FCNC interactions [24].

The most stringent constraint on ETC comes from $|\Delta S| = 2$ interactions. Such an interaction has the generic form

$$
\mathcal{H}'_{|\Delta S|=2} = \frac{g_{ETC}^2 V_{ds}^2}{M_{ETC}^2} \bar{d}\gamma^\mu s \bar{d}\Gamma^\prime_\mu s + \text{h.c.} \tag{12}
$$

Here, $V_{ds}$ is a mixing–angle factor; it may be complex and seems unlikely to be much smaller in magnitude than the Cabibbo angle, say $0.1 \lesssim |V_{ds}| \lesssim 1$. The matrices $\Gamma_\mu$ and $\Gamma^\prime_\mu$ are left– and/or right–chirality Dirac matrices. I shall put $\Gamma_\mu, \Gamma^\prime_\mu = \frac{1}{2} \gamma^\mu (1 - \gamma^5)$ and count the interaction twice to allow for different chirality terms in $\mathcal{H}'_{|\Delta S|=2}$. The contribution of this interaction to the $K_L - K_S$ mass difference is then estimated to be

$$
(\Delta M_K)_{ETC} \equiv 2\text{Re}(M_{12})_{ETC} = \frac{4g_{ETC}^2 \text{Re}(V_{ds}^2)}{8M_K M_{ETC}^2} \langle K^0 | \bar{d}\gamma^\mu (1 - \gamma^5) s \bar{d}\gamma^\mu (1 - \gamma^5) s | \bar{K}^0 \rangle
$$

$$
\simeq \frac{g_{ETC}^2 \text{Re}(V_{ds}^2)}{M_{ETC}^2} f_K^2 M_K, \tag{13}
$$

where I used the vacuum insertion approximation with $\langle \Omega | \bar{d}\gamma^\mu \gamma^5 s | \bar{K}^0(p) \rangle = i \sqrt{2} f_K p_\mu$ with $f_K \simeq 110$ MeV. This ETC contribution must be less than the measured mass difference, $\Delta M_K = 3.5 \times 10^{-18}$ TeV. This gives the limit

$$
\frac{M_{ETC}}{g_{ETC} \sqrt{\text{Re}(V_{ds}^2)}} \gtrsim 1300 \text{ TeV}. \tag{14}
$$

If $V_{ds}$ is complex, $\mathcal{H}'_{|\Delta S|=2}$ contributes to the imaginary part of the $K^0 - \bar{K}^0$ mass matrix. Using $\text{Im}(M_{12}) = \sqrt{2} \Delta M_K |\epsilon| \simeq 1.15 \times 10^{-20}$ TeV, the limit is

$$
\frac{M_{ETC}}{g_{ETC} \sqrt{\text{Im}(V_{ds}^2)}} \gtrsim 16000 \text{ TeV}. \tag{15}
$$

If we use these large ETC masses and scale the technifermion condensates in Eqs. (3,4) from QCD, i.e., assume the anomalous dimension $\gamma_m$ is small so that $\langle \bar{T}T\bar{T}T \rangle_{ETC} \simeq \langle \bar{T}T \rangle_{ETC}^2 \simeq \langle \bar{T}T \rangle_{TC}^2 \simeq (4\pi F_T^3)^2$, we obtain quark and lepton and technipion masses that are 10–1000 times too small, depending on the size of $V_{ds}$. This is the FCNC problem. It is remedied by the non–QCD–like dynamics of technicolor with a slowly running gauge coupling, walking technicolor, which will be described in the next section.
3.2 Precision Electroweak Measurements

Precision electroweak measurements actually challenge technicolor, not extended technicolor. The basic parameters of the standard $SU(2) \otimes U(1)$ model—$\alpha(M_Z)$, $M_Z$, $\sin^2 \theta_W$—are measured so precisely that they may be used to limit new physics at energy scales above 100 GeV [23]. The quantities most sensitive to new physics are defined in terms of correlation functions of the electroweak currents:

$$\int d^4x e^{-iq \cdot x} \langle \Omega| T \left( j_\mu^i(x) j^\nu_j(0) \right) |\Omega \rangle = ig^{\mu\nu} \Pi_{ij}(q^2) + q^\mu q^\nu \text{ terms}.$$  \hspace{1cm} (16)

Once one has accounted for the contributions from standard model physics, including a single Higgs boson (whose mass $M_H$ must be assumed), new high–mass physics affects the $\Pi_{ij}$ functions. Assuming that the scale of this physics is well above $M_W, M_Z$, it enters the “oblique” correction factors $S, T, U$ defined by

$$S = 16\pi \frac{d}{dq^2} \left[ \Pi_{33}(q^2) - \Pi_{3Q}(q^2) \right]_{q^2=0} \equiv 16\pi \left[ \Pi'_{33}(0) - \Pi'_{3Q}(0) \right],$$

$$T = \frac{4\pi}{M_Z^2 \cos^2 \theta_W \sin^2 \theta_W} \left[ \Pi_{11}(0) - \Pi_{33}(0) \right],$$

$$U = 16\pi \left[ \Pi'_{11}(0) - \Pi'_{33}(0) \right].$$  \hspace{1cm} (17)

The parameter $S$ is a measure of the splitting between $M_W$ and $M_Z$ induced by weak–isospin conserving effects; the $\rho$–parameter is given by $\rho \equiv M_W^2/M_Z^2 \cos^2 \theta_W = 1 + \alpha T$; the $U$–parameter measures weak–isospin breaking in the $W$ and $Z$ mass splitting. The experimental limits on $S, T, U$ are [4]

$$S = -0.07 \pm 0.11 \ ( -0.09),$$

$$T = -0.10 \pm 0.14 \ ( +0.09),$$

$$U = +0.11 \pm 0.15 \ ( +0.01).$$  \hspace{1cm} (18)

The central values assume $M_H = 100$ GeV, and the parentheses contain the change for $M_H = 300$ GeV. The $S$ and $T$–parameters and $M_H$ cannot be obtained simultaneously from data because the Higgs loops resemble oblique effects.

The $S$–parameter is the one most touted as a show–stopper for technicolor [23, 25]. The value obtained in technicolor by scaling from QCD is $O(1)$. For example, for $N$ color–singlet technidoublets, Peskin and Takeuchi found the positive result

$$S = 4\pi \left( 1 + \frac{M_{\rho T}^2}{M_{\rho T}^2 + M_{\pi T}^2} \right) \frac{F_T^2}{M_{\rho T}^2} \simeq 0.25 N \frac{N_{TC}}{3}.$$  \hspace{1cm} (19)

The resolution to this problem may also be found in walking technicolor. One thing is sure: naive scaling of $S$ from QCD is unjustified and probably incorrect in walking
gauge theories. No reliable estimate exists because no data on walking gauge theories are available to put into the calculation of $S$.

### 3.3 The Top Quark Mass

The ETC scale required to produce $m_t = 175$ GeV in Eq. (3) is $0.75$ TeV/$N^{3/4}$ for $N$ technidoublets. This is uncomfortably close to the TC scale itself. In effect, TC becomes strong and ETC is broken at the same energy; the representation of broken ETC interactions as contact operators is wrong; and all our mass estimates are questionable. It is possible to raise the ETC scale so that it is considerably greater than $m_t$, but then one runs into the problem of fine-tuning the ETC coupling $g_{ETC}$ (just as in the Nambu–Jona-Lasinio model, where requiring the dynamical fermion mass to be much less than the four-fermion mass scale $\Lambda$ requires fine-tuning the NJL coupling very close to $4\pi$) \[26\]. This flouts our cherished principle of naturalness, and we reject it. Another, more direct, problem with ETC generation of the top mass is that there must be large weak isospin violation to raise it so high above the bottom mass. This adversely affects the $\rho$ parameter \[27\]. The large effective ETC coupling to top quarks also makes a large, unwanted contribution to the $Z \to b\bar{b}$ decay rate, in conflict with experiment \[28\].

In the end, there is no plausible way to understand the top quark’s large mass from ETC. Something more is needed. The best idea so far is topcolor-assisted technicolor \[29\], in which a new gauge interaction, topcolor \[30\], becomes strong near 1 TeV and generates a large $\bar{t}t$ condensate and top mass. This, too, will be described in the next section.

### 4. Dynamical Rescues

The FCNC and STU difficulties of technicolor have a common cause: the assumption that technicolor is a just a scaled-up version of QCD. Let us focus on Eqs.(3,4,6), the key equations of extended technicolor. In a QCD–like technicolor theory, asymptotic freedom sets in quickly above $\Lambda_{TC}$, the anomalous dimension $\gamma_m \ll 1$, and $\langle \bar{T}T \rangle_{ETC} \simeq \langle \bar{T}T \rangle_{TC}$. The conclusion that fermion and technipion masses are one or more orders of magnitude too small then followed from the FCNC requirement in Eqs. \[14,13\] that $M_{ETC}/g_{ETC}|V_{ds}| \gtrsim 1000$ TeV. Scaling from QCD also means that the technihadron spectrum is just a magnified image of the QCD–hadron spectrum, hence that $S$ is too large for all technicolor models except, possibly, the minimal one-doublet model with $N_{TC} \lesssim 4$.

The solution to these difficulties lies in technicolor gauge dynamics that
are distinctly not QCD–like. The only plausible example is one in which the gauge coupling $\alpha_{TC}(\mu)$ evolves slowly, or “walks”, over the large range of energy $\Lambda_{TC} \lesssim \mu \lesssim M_{ETC}$ \[20\]. In the extreme walking limit in which $\alpha_{TC}(\mu)$ is constant, it is possible to obtain an approximate nonperturbative formula for the $\bar{T}T$ anomalous dimension $\gamma_m$, namely,

$$\gamma_m(\mu) = 1 - \sqrt{1 - \alpha_{TC}(\mu)/\alpha_C} \quad \text{where} \quad \alpha_C = \frac{\pi}{3C_2(R)}. \quad (20)$$

This reduces to the expression in Eq. (7) for small $\alpha_{TC}$. It has been argued that $\gamma_m = 1$ is the signal for spontaneous chiral symmetry breaking \[18\], and, so, $\alpha_C$ is called the critical coupling for $\chi_{SB}$, with $\pi/3C_2(R)$ its approximate value. If we identify $\Lambda_{TC}$ with the scale at which technifermions in the $SU(N_{TC})$ fundamental representation condense, then $\alpha_{TC}(\Lambda_{TC}) = \alpha_C$.

In walking technicolor, $\alpha_{TC}(\mu)$ is presumed to remain close to its critical value from $\Lambda_{TC}$ almost up to $M_{ETC}$. This implies $\gamma_m(\mu) \simeq 1$, and by Eq. (20), the condensate $\langle \bar{T}T \rangle_{ETC}$ is enhanced by a factor of 100 or more. This yields quark masses up to a few GeV and reasonably large technipion masses despite the very large ETC mass scale. This is still not enough to account for the top mass; more on that soon.

Another consequence of the walking $\alpha_{TC}$ is that the spectrum of technihadrons, especially the $I = 0, 1$ vector and axial vector mesons, $\rho_T, \omega_T, a_{1T}$ and $f_{1T}$, cannot be QCD–like \[11, 32, 33\]. In QCD, the lowest lying isovector $\rho$ and $a_1$ saturate the spectral functions appearing in Weinberg’s sum rules \[14\]. Then, the relevant combination $\rho_V - \rho_A$ of spectral functions falls off like $1/p^6$ for $p > M_{\rho,a_1} \sim \Lambda_{QCD}$, and the spectral integrals converge very rapidly. This “vector meson dominance” of the spectral integrals is related to the precocious onset of asymptotic freedom in QCD. The $1/p^6$ momentum dependence is just what one would deduce from a naive, lowest–order calculation of $\rho_V - \rho_A$ using the asymptotic $1/p^2$ behavior of the quark dynamical mass $\Sigma(p)$ \[17\]. In walking technicolor, the technifermion’s $\Sigma(p)$ falls only like $1/p^{2-\gamma_m} \sim 1/p$ for $\Lambda_{TC} \lesssim M_{ETC}$, so that $\rho_V - \rho_A \sim 1/p^4$ up to very high energies. To account for this in terms of spin–one technihadrons, there must be something like a tower of $\rho_T$ and $\omega_T$ extending up to $M_{ETC}$. Their mass spectrum, widths, and couplings to currents cannot be predicted. Thus, without experimental knowledge of these states, it is impossible to estimate $S$ reliably, any more than it would have been in QCD before the $\rho$ and $a_1$ were discovered and measured.

Another issue that may affect $S$ is that it is usually defined assuming that the new physics appears at energies well above $M_{W,Z}$. We shall see below that, on
the contrary, walking technicolor suggests that there are $\pi_T$ and $\rho_T$ starting near or not far above $100 \text{ GeV}$.

We have seen that extended technicolor cannot explain the top quark’s large mass. An alternative approach was developed in the early 90s based on a new interaction of the third generation quarks. This interaction, called topcolor, was invented as a minimal dynamical scheme to reproduce the simplicity of the one–doublet Higgs model and explain a very large top–quark mass [30]. Here, a large top–quark condensate, $\langle \bar{t}t \rangle$, is formed by strong interactions at the energy scale, $\Lambda_t$ [35]. To preserve electroweak $SU(2)$, topcolor must treat $t_L$ and $b_L$ the same. To prevent a large $b$–condensate and mass, it must violate weak isospin and treat $t_R$ and $b_R$ differently. In order that the resulting low–energy theory simulate the standard model, particularly its small violation of weak isospin, the topcolor scale must be very high—$\Lambda_t \sim 10^{15} \text{ GeV} \gg m_t$. Therefore, this original topcolor scenario is highly unnatural, requiring a fine–tuning of couplings of order one part in $\Lambda_t^2/m_t^2 \simeq 10^{25}$ (remember Nambu–Jona-Lasinio!).

Technicolor is still the most natural mechanism for electroweak symmetry breaking, while topcolor dynamics most aptly explains the top mass. Hill proposed to combine the two into what he called topcolor–assisted technicolor (TC2) [29]. In TC2, electroweak symmetry breaking is driven mainly by technicolor interactions strong near $1 \text{ TeV}$. Light quark, lepton, and technipion masses are still generated by ETC. The topcolor interaction, whose scale is also near $1 \text{ TeV}$, generate $\langle \bar{t}t \rangle$ and the large top–quark mass. The scale of ETC interactions still must be at least several $100 \text{ TeV}$ to suppress flavor-changing neutral currents and, so, the technicolor coupling still must walk. Their marriage neatly removes the objections that topcolor is unnatural and that technicolor cannot generate a large top mass. In this scenario, the nonabelian part of topcolor is an ordinary asymptotically free gauge theory.

Hill’s original TC2 scheme assumes separate color $SU(3)$ and weak hypercharge $U(1)$ gauge interactions for the third and for the first two generations of quarks and leptons. In the simplest example, the (electroweak eigenstate) third generation $(t, b)_{L,R}$ transform with the usual quantum numbers under the top-color gauge group $SU(3)_1 \otimes U(1)_1$ while $(u, d), (c, s)$ transform under a separate group $SU(3)_2 \otimes U(1)_2$. Leptons of the third and the first two generations transform in the obvious way to cancel gauge anomalies. At a scale of order $1 \text{ TeV}$, $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2$ is dynamically broken to the diagonal subgroup

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$^8$Three massless Goldstone “top–pions” arise from top–quark condensation. Thus, ETC interactions must contribute a few GeV to $m_t$ to give the top–pions a mass large enough that $t \rightarrow b\pi^+_t$ is not a major decay mode.
of ordinary color and weak hypercharge, $SU(3)_C \otimes U(1)_Y$. At this energy, the $SU(3)_1 \otimes U(1)_1$ couplings are strong while the $SU(3)_2 \otimes U(1)_2$ couplings are weak. This breaking gives rise to massive gauge bosons—a color octet of “colorons” $V_8$ and a color singlet $Z'$.

Top, but not bottom, condensation is driven by the fact that the $SU(3)_1 \otimes U(1)_1$ interactions are supercritical for top quarks, but subcritical for bottom. The difference between top and bottom is caused by the $SU(3)$$_1$$\otimes U(1)_1$ couplings of $t_R$ and $b_R$. If this TC2 scenario is to be natural, i.e., there is no fine–tuning of the $SU(3)_1$, the $U(1)_1$ couplings cannot be weak. To avoid large violations of weak isospin in this and all other TC2 models [36], right as well as left–handed members of individual technifermion doublets $T_{L,R} = (T_U, T_D)_{L,R}$ must carry the same $U(1)_1$ quantum numbers, $Y_{1L}$ and $Y_{1R}$, respectively [37].

Hill’s simplest TC2 model does not how explain how topcolor is broken. Since natural topcolor requires it to occur near 1 TeV, the most likely cause is technifermion condensation. In Ref. [38], it was argued that this can be done for $SU(3)_1 \otimes SU(3)_2 \rightarrow SU(3)_C$ by arranging that technifermion doublets $T_1$ and $T_2$ transforming under $SU(N_{TC}) \otimes SU(3)_1 \otimes SU(3)_2$ as $(N_{TC}, 3, 1)$ and $(N_{TC}, 1, 3)$ condense with each other as well as themselves, i.e.,

$$\langle \bar{T}_{iL}T_{jR} \rangle = -U_{ij}\Delta_T \ (i, j = 1, 2),$$

where $U$ is a nondiagonal unitary matrix and $\Delta_T$ the technifermion condensate of $O(\Lambda_{TC}^2)$. The strongly coupled $U(1)_1$ plays a critical role in tilting $U$ away from the identity, which is the form of the condensate preferred by the color interactions.

The breaking $U(1)_1 \otimes U(1)_2 \rightarrow U(1)_Y$ is trickier. In order that there is a well–defined $U(1)_Y$ boson with standard couplings to all quarks and leptons, this must occur at a somewhat higher scale, several TeV. Thus, the $Z'$ boson from this breaking has a mass of several TeV and is strongly coupled to technifermions, at least. To employ technicolor in this $U(1)$ breaking too, technifermions $\psi_{L,R}$ belonging to a higher–dimensional $SU(N_{TC})$ representation are introduced. They condense at higher energy than the fundamentals $T_{iL,R}$ [41]. The critical reader will

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[9] A large bottom condensate is not generated by $SU(3)_1$ because it is broken and its coupling does not grow stronger as one descends to lower energies.

[10] In Ref. [38] the fermions of the first two generations also need to couple to $U(1)_1$. The limits on these strong couplings and $M_{Z'}$ from precision electroweak measurements were studied by Chivukula and Terning [38]. Another variant of TC2 has all three generations transforming in the same way under topcolor [40]. This “flavor–universal topcolor” has certain phenomenological advantages (see the second paper of Ref. [38]), but the problems of the strong $U(1)_1$ coupling afflict it too.
Table 1: Relative vector and axial vector amplitudes for $V_T \to G \pi_T$ with $V_T = \rho_T, \omega_T$ and $G$ a transverse electroweak boson, $\gamma, Z^0, W^\pm$; from Ref. [46].

| $V_T$ Decay Mode | $V(V_T \to G \pi_T) \times M_V/e$ | $A(V_T \to G \pi_T) \times M_A/e$ |
|------------------|---------------------------------|---------------------------------|
| $\omega_T \to \gamma \pi_T^0$ | $(Q_U + Q_D) \cos \chi$ | 0 |
| $\to \gamma \pi_T^0$ | $(Q_U + Q_D) \cos \chi'$ | 0 |
| $\to Z^0 \pi_T^0$ | $\cos \chi \cot 2\theta_W$ | 0 |
| $\to Z^0 \pi_T^0$ | $-(Q_U + Q_D) \cos \chi' \tan \theta_W$ | 0 |
| $\to W^\pm \pi_T^\pm$ | $\cos \chi' / (2 \sin \theta_W)$ | 0 |

| $\rho_T^0 \to \gamma \pi_T^0$ | $(Q_U + Q_D) \cos \chi$ | 0 |
| $\to \gamma \pi_T^0$ | $(Q_U + Q_D) \cos \chi'$ | 0 |
| $\to Z^0 \pi_T^0$ | $-(Q_U + Q_D) \cos \chi \tan \theta_W$ | 0 |
| $\to Z^0 \pi_T^0$ | $\cos \chi' \cot 2\theta_W$ | 0 |
| $\to W^\pm \pi_T^\pm$ | 0 | $-\cos \chi / (2 \sin \theta_W)$ |

| $\rho_T^\pm \to \gamma \pi_T^\pm$ | $(Q_U + Q_D) \cos \chi$ | 0 |
| $\to \gamma \pi_T^\pm$ | $(Q_U + Q_D) \cos \chi'$ | $\cos \chi / \sin 2\theta_W$ |
| $\to Z^0 \pi_T^0$ | $-(Q_U + Q_D) \cos \chi \tan \theta_W$ | $\cos \chi / (2 \sin \theta_W)$ |
| $\to W^\pm \pi_T^0$ | 0 | 0 |

| $\rho_T^\pm \to \gamma \pi_T^\pm$ | $(Q_U + Q_D) \cos \chi$ | 0 |
| $\to \gamma \pi_T^\pm$ | $(Q_U + Q_D) \cos \chi'$ | $\cos \chi / \sin 2\theta_W$ |
| $\to Z^0 \pi_T^0$ | $-(Q_U + Q_D) \cos \chi \tan \theta_W$ | $\cos \chi / (2 \sin \theta_W)$ |
| $\to W^\pm \pi_T^0$ | 0 | 0 |

Table 1: Relative vector and axial vector amplitudes for $V_T \to G \pi_T$ with $V_T = \rho_T, \omega_T$ and $G$ a transverse electroweak boson, $\gamma, Z^0, W^\pm$; from Ref. [46].

note that this scenario also flirts with unnatural fine tuning because the multi–TeV $Z'$ plays a critical role in top and bottom quark condensation. Another pitfall is that the strong $U(1)_1$ coupling may blow up at a Landau singularity at a relatively low energy [38, 42]. To avoid this, unification of $U(1)_1$ with the nonabelian $G_{ETC}$ must occur at a lower energy still. This is not a very satisfactory state of affairs, but that is how things stand for now with TC2. There are many opportunities for improvement.

A variant of topcolor models is called the “top seesaw” mechanism [43]. Its motivation is to realize the original, supposedly more economical, top–condensate idea of the Higgs boson as a fermion–antifermion bound state [35]. Apart from its fine tuning problem, that way failed because it implied a top mass of about 250 GeV. In top seesaw models, an electroweak singlet fermion $F$ acquires a dynamical mass of several TeV. Through mixing of $F$ with the top quark, it gives the latter a much smaller mass (the seesaw) and the scalar $\bar{F}F$ bound state acquires a component with an electroweak symmetry breaking vacuum expectation value. The latest twist on this variant is called the “topcolor jungle gym” [44]. We’ll say no more about these approaches here as they are off our main line of technicolor and extended technicolor. The interested reader should consult the literature.
Figure 2: Technivector meson total decay rates versus $M_V = M_A$ for $\rho^0_T$ (solid curve) and $\rho^\pm_T$ (long-dashed) with $M_{\rho_T} = 210\,\text{GeV}$, and $\omega_T$ with $M_{\omega_T} = 200$ (lower dotted), 210 (lower short-dashed), and 220 GeV (lower medium-dashed); $Q_U + Q_D = 5/3$ and $M_{\pi_T} = 110\,\text{GeV}$; from Ref. [46].

5. Technicolor Phenomenology

The coupling $\alpha_{TC}$ in walking technicolor decreases slowly if the beta–function $\beta(\alpha_{TC}) = \mu d\alpha_{TC}/d\mu$ is negative and near zero for a large range of energy $\mu$ above $\Lambda_{TC}$. This small beta–function may be achieved by having many technifermions in the fundamental representation of $SU(N_{TC})$, or a few in higher-dimensional representations, or both [41]. For different reasons, models of topcolor–assisted technicolor also seem to require many technifermions [37, 38]. The technidoublets include $\sim 5$ that are color singlets as well as the color triplets $T_1 \in (N_{TC}, 3, 1)$ and $T_2 \in (N_{TC}, 1, 3)$ mentioned above. The color singlets insure that all quarks and leptons get the appropriate ETC mass and that there is sufficient mixing between the third generation quarks and the two light ones (so that weak decays of the $b$–quark are allowed).

These requirements suggest that the technicolor scale is much lower than
previously thought. If the number $N$ of technidoublets is $\mathcal{O}(10)$ (including 3 for each color triplet), then $\Lambda_{TC} \simeq F_T = F_{\pi}/\sqrt{N} \lesssim 100$ GeV. This sets the mass scale for the lightest color–singlet technivector mesons, $M_{\rho^T} \simeq M_{\omega^T} \simeq 2\Lambda_{TC} \lesssim 200$ GeV. These states are produced in hadron and lepton colliders. The mechanism is good old–fashioned vector meson dominance of the $s$–channel production of $\gamma$, $Z^0$, and $W^\pm$. The lightest color–octet $\rho^{T8}$, bound states of the color–triplet technifermions of TC2, will be heavier, starting, perhaps, at 400–500 GeV. Hadron colliders are needed to produce these states. Isosinglet $\rho^{T8}$ bosons are produced by their couplings to the QCD gluon and the $V_8^0$ colorons.

In the limit that color interactions are weak compared to technicolor, the chiral symmetry of $N$ technidoublets is $SU(2N)_L \otimes SU(2N)_R$. When it is spontaneously broken, there result $4N^2 - 4$ technipions in addition to $W_E^\pm$ and $Z_2^0$, a large number of states if $N$ is large. In QCD–like technicolor, these technipions would be very light and the $\rho_T$ and $\omega_T$ would decay to two or more technipions, with $\rho^{T8}$ decaying to color–octet and color–triplet (leptoquark) pairs. Walking
Figure 4: Production rates in $\bar{p}p$ collisions at $\sqrt{s} = 2$ TeV for the sum of $\omega_T$, $\rho_T^0$, $\rho_T^\pm \rightarrow \gamma \pi_T$ versus $M_V$, for $M_{\rho_T} = 210$ GeV and $M_{\omega_T} = 200$ (dotted curve), 210 (solid), and 220 GeV (short-dashed); $Q_U + Q_D = 5/3$, and $M_{\pi_T} = 110$ GeV; from Ref. [46].

Technicolor dramatically changes this expectation. In the extreme walking limit, $\langle \bar{T}T \rangle_{ETC} \simeq (M_{ETC}/\Lambda_{TC}) \langle \bar{T}T \rangle_{TC}$, so that technipions masses are of order $\Lambda_{TC}$, and they are not pseudoGoldstone bosons at all. Though this extreme limit is theoretically problematic because it is exactly scale–invariant, it is clear that walking technicolor enhances $\pi_T$ masses significantly more than it does the $\rho_T$ and $\omega_T$ masses. Thus, it is likely that $M_{\pi_T} \gtrsim \frac{1}{2} M_{\rho_T,\omega_T}$ and, so, the nominal isospin–conserving decay channels $\rho_T \rightarrow \pi_T \pi_T$ and $\omega_T \rightarrow \pi_T \pi_T \pi_T$ are closed [11]. If the color–singlet $\rho_T$ start near 200 GeV, we expect $M_{\pi_T} \gtrsim 100$ GeV. Of course, an explicit ETC model will be needed to make firm mass estimates.

This “low–scale technicolor” may be within the reach of CDF and DØ in Run II of the Tevatron Collider [45, 46, 47]. It certainly will be accessible at

\footnote{Many of these signatures are now encoded in \textsc{Pythia} [48].}
Figure 5: Production rates in $\bar{p}p$ collisions at $\sqrt{s} = 2$ TeV for $\omega_T$, $\rho_T^0$, $\rho_T^\pm \rightarrow W\pi_T$ (upper curves) and $Z\pi_T$ (lower curves) versus $M_V$, for $M_{\rho_T} = 210$ GeV and $M_{\omega_T} = 200$ (dotted curve), 210 (solid), and 220 GeV (short-dashed); $Q_U + Q_D = 5/3$ and $M_{\pi_T} = 110$ GeV. Also shown is $\sigma(\rho_T \rightarrow \pi_T\pi_T)$ (lowest dashed curve); from Ref. [46].

the LHC. Color–singlet $\rho_T$ and $\omega_T$ may even be detected at LEP200. If a lepton collider with $\sqrt{s} \lesssim 500$ GeV is built, it will be able carry out precision studies of color–singlet technihadrons. The Very Large Hadron Collider or a multi–TeV lepton collider will be needed to explore more fully the strongly coupled region of walking technicolor.

In the rest of this section, we describe a simple model, suitable for experimental studies, of our expectations for the low–lying states of low–scale technicolor—first for the the color–singlet sector, then for color–nonsinglets.
Figure 6: Invariant mass distributions in $\bar{p}p$ collisions at $\sqrt{s} = 2$ TeV for $\omega_T$, $\rho_T^0 \rightarrow e^+e^-$ for $M_{\rho_T} = 210$ GeV and $M_{\omega_T} = 200$ (short-dashed curve), 210 (solid), and 220 GeV (long-dashed); $M_V = 100$ GeV. The standard model background is the sloping dotted line. $Q_U + Q_D = 5/3$ and $M_{\pi_T} = 110$ GeV; from Ref. [46].

5.1 Theory and Experiment for Color–Singlet Technihadrons

The flavor problem is hard whether it is attacked with extended technicolor or from any other direction. We theorists need experimental guidance. Experimentalists, in turn, need input from theorists to help design useful searches. Supersymmetry has its MSSM. What follows is a description of the corresponding thing for technicolor, in the sense that it defines a set of incisive experimental tests in terms of a limited number of adjustable parameters. I call this the “Technicolor Straw Man” model (or TCSM).

In the TCSM, we assume that we can consider in isolation the lowest-lying bound states of the lightest technifermion doublet, $(T_U, T_D)$. If these technifermions belong to the fundamental representation of $SU(N_{TC})$, they probably are color singlets. In walking technicolor, ordinary color interactions contribute significantly to
the hard mass of $SU(3)$ triplets \[\Box]. The lightest technidoublet’s electric charges are unknown; we denote them by $Q_U$ and $Q_D = Q_U - 1$.

The bound states in question are vector and pseudoscalar mesons. The vectors include a spin–one triplet $\rho_T^{\pm,0}$ and a singlet $\omega_T$. In topcolor–assisted technicolor, there is no need to invoke large isospin–violating extended technicolor interactions to explain the top–bottom splitting. Techni–isospin can be, and likely must be, a good approximate symmetry. Then, $\rho_T$ and $\omega_T$ will be mostly isovector and isoscalar, respectively, and they will be nearly degenerate. Their production in $\bar{q}q$ and $e^+e^-$ annihilation is described using vector meson dominance, with propagator matrices that mix them with $W^\pm$ and $\gamma, Z^0$. The details are given in Ref. \[\Box\], called TCSM–1 below. I reiterate, mixing of these $\rho_T$ and $\omega_T$ with their excitations is ignored in the TCSM as is the production of the axial vector $a_{1T}$ and the like.

The lightest pseudoscalar $\bar{T}T$ bound states, the technipions, also comprise an isotriplet $\Pi_T^{\pm,0}$ and an isosinglet $\Pi_T^0$. However, these are not mass eigenstates; all
color–singlet isovector technipions have a $W_L$ component. To limit the number of parameters in the TCSM, we make the simplifying assumption that the isotriplets are simple two–state mixtures of the $W_L^\pm$, $Z_L^0$ and the lightest mass eigenstate pseudo–Goldstone technipions $\pi_T^\pm$, $\pi_T^{0'}$:

$$|\Pi_T\rangle = \sin \chi |W_L\rangle + \cos \chi |\pi_T\rangle .$$  \hspace{1cm} (22)$$

Here, $\sin \chi = F_T/F_\pi = 1/\sqrt{N} \ll 1$ is an adjustable parameter. The isosinglet is also an admixture, $|\Pi_T^{0'}\rangle = \cos \chi' |\pi_T^{0'}\rangle + \cdots$, where $\chi'$ is another adjustable mixing angle and the ellipsis refers to other technipions needed to eliminate the two-technigluon anomaly from the $\Pi_T^{0'}$ chiral current.

It is unclear whether, like $\rho_T^0$ and $\omega_T$, the neutral technipions $\pi_T^0$ and $\pi_T^{0'}$ will be degenerate as we have previously supposed \[45\]. On one hand, they both contain the lightest $TT$ as constituents. On the other, $\pi_T^{0'}$ must contain other, presumably
heavier, technifermions as a consequence of anomaly cancellation. The calculations and searches presented here assume that $\pi_T^0$ and $\pi_T^{0'}$ are nearly degenerate. If this is true, and if their widths are roughly equal, there will be appreciable $\pi_T^0 - \pi_T^{0'}$ mixing. Then, the lightest neutral technipions will be ideally-mixed $\bar{T}_U T_U$ and $\bar{T}_D T_D$ bound states.

In any case, these technipions are expected to couple most strongly to the heaviest fermion pairs that they can. The reason for this is that $\pi_T$ couple to ordinary fermions via extended technicolor, $\pi_T \rightarrow \bar{T}T \rightarrow \bar{f}f$. Figure 1 suggests that this coupling is proportional to $m_f$ (more precisely, the ETC contribution to $m_f$). In our studies we assume technipions to be lighter than $m_t + m_b$. Then, we expect them to decay as follows: $\pi_T^+ \rightarrow c\bar{b}$ or $c\bar{s}$ or even $\tau^+\nu_\tau$; $\pi_T^0 \rightarrow b\bar{b}$ and, perhaps $c\bar{c}$, $\tau^+\tau^-$; and $\pi_T^{0'} \rightarrow gg, b\bar{b}, c\bar{c}, \tau^+\tau^-$. This puts a premium on heavy-flavor identification.

12See Ref. [40] for a discussion and estimate of $\pi_T$ decay rates.
in collider experiments. However, this is only an educated guess. The reader is cautioned that the mass–eigenstate neutral $\pi_T$ may have a sizable branching ratio to gluon (or even light–quark) pairs.

For vanishing electroweak couplings $g$ and $g'$, the $\rho_T$ and $\omega_T$ decay as

$$
\begin{align*}
\rho_T & \to \Pi_T\Pi_T = \cos^2 \chi (\pi_T \pi_T) + 2 \sin \chi \cos \chi (W_L \pi_T) + \sin^2 \chi (W_L W_L); \\
\omega_T & \to \Pi_T\Pi_T\Pi_T = \cos^3 \chi (\pi_T \pi_T \pi_T) + \cdots .
\end{align*}
\tag{23}
$$

As noted above however, the all–$\pi_T$ modes are likely to be closed. Thus, major decay modes of the $\rho_T$ will be $W_L \pi_T$ or, if $M_{\rho_T} \lesssim 180$ GeV (a possibility we regard as unlikely, if not already eliminated by LEP data), $W_L W_L$. The $W^\pm \pi_T^{\mp}$ and $Z^0 \pi_T^{\pm}$ decays of $\rho_T$ have striking signatures in any collider. Only at LEP is it now possible to detect $\rho_T^0 \to W^+ W^-$ above the standard model background. If $M_{\omega_T} < 250$ GeV, all the $\omega_T \to \Pi_T\Pi_T\Pi_T$ modes are closed. In all cases, the $\rho_T$ and $\omega_T$ are very narrow, $\Gamma(\omega_T) \lesssim \Gamma(\rho_T) \lesssim 1$ GeV, because of the smallness of $\sin \chi$ and the limited phase
Figure 11: (a) The distribution of $M_{jj}$ vs. $M_{jj\gamma} - M_{jj}$ for events with a photon, b-tagged jet and a second jet. (b) Projection of this data in $M_{jj\gamma} - M_{jj}$; from Ref. [53].

space. Therefore, we must consider other decay modes. These are electroweak, suppressed by powers of $\alpha$, but not by phase space.

The decays $\rho_T, \omega_T \rightarrow G\pi_T$, where $G$ is a transversely polarized electroweak gauge boson, and $\rho_T, \omega_T \rightarrow \bar{f} f$ were calculated in TCSM–1. The $G\pi_T$ modes have rates of $\mathcal{O}(\alpha)$, while the fermion mode $\bar{f} f$ rates are $\mathcal{O}(\alpha^2)$. The $\Gamma(\rho_T, \omega_T \rightarrow G\pi_T)$ are suppressed by $1/M_V^2$ or $1/M_A^2$, depending on whether the vector or axial vector part of the electroweak current is involved in the decay. Here, $M_{V,A}$ are masses of order $\Lambda_{TC}$ occurring in the dimension–5 operators for these decays. We usually take them equal and vary them from 100 to 400 GeV. For the smaller values of $M_{V,A}$, these modes, especially $\rho_T, \omega_T \rightarrow \gamma \pi_T$, are as important as the $W_L\pi_T$ modes. For larger $M_{V,A}$ and $|Q_U + Q_D| \gtrsim 1$, the $\bar{f} f$ decay modes may become competitive. As an illustration, Table 1 lists the relative strengths of the decay amplitudes for the $\rho_T, \omega_T \rightarrow G\pi_T$ processes. Figure 2 gives a sense of the $M_{V,A}$ dependence of the total decay rates of $\rho_T$ and $\omega_T$ for $M_{\rho_T} = 210$ GeV, $M_{\omega_T} = 200–220$ GeV, $M_{\pi_T} = 110$ GeV,
Figure 12: The 90%, 95% and 99% CL exclusion regions for the CDF search for $\omega_T \to \gamma \pi_T$ in Ref. [53]. The inset shows the limit on $\sigma_B$ for $M_{\pi_T} = 120$ GeV. The circles represent the limit and the solid line the prediction from the second paper in Ref. [45].

and $Q_U = Q_D + 1 = 4/3$. Figure 3 shows the decay rates for $Q_U = -Q_D = 1/2$. Note how narrow the $\rho_T$ and $\omega_T$ are. These and all subsequent calculations assume that $N_{TC} = 4$ and $\sin \chi = \sin \chi' = 1/3$. Experimental analyses quoted below use the same defaults and (usually) $Q_U = Q_D + 1 = 4/3$.

Figures 4 and 5 show the cross sections in $\bar{p}p$ collisions at $\sqrt{s} = 2$ TeV for production of $\gamma \pi_T$ and for $W \pi_T$, $Z \pi_T$ and $\pi_T \pi_T$ as a function of $M_V = M_A$. Figure 6 shows the $e^+e^-$ rate for $M_V = 100$ GeV. The production rates in these figures, all in the picobarn range, are typical for the Tevatron for $M_{\rho_T,\omega_T} \lesssim 250$ GeV and $M_{\pi_T} \lesssim 150$ GeV. That is why we believe Run II will probe a significant portion of the parameter space of low–scale technicolor.

Let us turn to the recent searches for color–singlet technihadrons. We begin with analyses by the L3 [50] and DELPHI [51] collaborations at LEP. Note
that the LEP experiments can be sensitive to $\rho_T$ and $\omega_T$ masses significantly above the $e^+e^-$ c.m. energy, $\sqrt{s}$. This is because the $e^+e^-$ cross section on resonance is very large for the narrow $\rho_T$. Furthermore, masses below the nominal c.m. energy are scanned by the process of radiative return, $e^+e^- \rightarrow \rho_T^0/\omega_T + n \gamma$.

The L3 search is based on 176 $pb^{-1}$ of data taken at an average energy of 189 GeV. The analysis used TCSM–1 for the channels $e^+e^- \rightarrow \rho_T^0 \rightarrow W^+W^-$; $W_T^\pm \pi_T^\mp \rightarrow \ell \nu \ell bc$; $\pi_T^+\pi_T^- \rightarrow t\bar{c}b\bar{c}$; and $\gamma\pi_T^0 \rightarrow \gamma b\bar{b}$. The TC–scale masses were fixed at $M_V = M_A = 200$ GeV and the technifermion charges ranged over $Q_U + Q_D = 5/3, 0, -1$. The resulting 95% confidence limits in the $M_{\rho_T} - M_{\pi_T}$ plane are shown in Fig. 7.

The DELPHI collaboration searched for $\rho_T^0 \rightarrow W^+W^-; W^\pm \pi_T^\mp; \pi_T^+\pi_T^-$; and $\mu^+\mu^-$. Data was taken over a range of $\sqrt{s}$ between 161 and 202 GeV with a variety of integrated luminosities. The modes $\rho_T^0, \omega_T \rightarrow \pi_T^0\gamma, \pi_T^0\gamma$ were neglected. This is not a good assumption if $M_V \lesssim 200$ GeV. The DELPHI exclusion plot is shown in
Since these LEP analyses were done, I have realized that the cross section formulae stated in TCSM–1 are inappropriate for $\sqrt{s}$ well below $M_{\rho_T}$. This is unimportant for the Tevatron and LHC, where the production rate comes mainly from integrating parton distributions over the resonance pole. However, it may have a significant effect on limits derived from $e^+e^-$ annihilation. This is especially true for the $W^+W^-$ channel, which has a large standard model amplitude interfering with the TCSM one.\footnote{I thank F. Richard for drawing my attention to this shortcoming of TCSM–1. A correction will be issued soon.} Another feature of these analyses not evident in the exclusion plots is that limits on $M_{\pi_T}$ approaching $\sqrt{s}/2$ should be derivable from $e^+e^- \rightarrow \pi_T^+\pi_T^-$. We look forward to the new LEP limits that will be announced in the summer of 2000.

Fig. 8.

Both Tevatron collider collaborations have searched for signals of low–scale technicolor. In Run I, only CDF had a vertex detector to find the detached vertices of $b$–quark decays. The collaboration used this capability to search for processes
Figure 15: Reach of the DØ detector in Tevatron Run IIa for $\rho_T^0, \omega_T \rightarrow e^+e^-$; from Ref. [56].

signalled by a $W$ or photon plus two jets, one of which is $b$-tagged:

$$\bar{q}q \rightarrow W^\pm, \gamma, Z^0 \rightarrow \rho_T^{\pm,0} \rightarrow W_L^\pm \pi_T \rightarrow \ell^\pm \nu_\ell b + \text{jet}$$

$$\rightarrow \rho_T^{\pm,0}, \omega_T \rightarrow \gamma \pi_T \rightarrow \gamma b + \text{jet}. \quad (24)$$

These analyses were carried out before the publication of TCSM–1, so they do not include the $G\pi_T$ and $\bar{f}f$ processes and corresponding branching ratios. They will be included in Run II data analyses. Figure 9 shows data for the $W\pi_T$ search on top of a background and signal expected for default parameters with $M_{\rho_T} = 180$ GeV and $M_{\pi_T} = 90$ GeV. The topological cuts leading to the lower figure are described in the second paper of Ref. [45]. The region excluded at 95% confidence level is shown in Fig. 10 [52].

Figure 11 shows the invariant mass of the tagged and untagged jets and the invariant mass difference $M(\gamma+b+\text{jet}) - M(b+\text{jet})$ in a search for $\omega_T, \rho_T \rightarrow \gamma \pi_T$ [53]. The good resolution in this mass difference is controlled mainly by that of the electromagnetic energy. The exclusion plot is shown in Fig. 12. It is amusing that
the $\sim 2\sigma$ excesses in Figs. 9 and 12 are both consistent with expectations for a signal with $M_{\rho T,\omega_T} \approx 200$ GeV and $M_{\pi_T} \approx 100$ GeV.

The expected reach of CDF in Run IIa for the $\rho_T \rightarrow W^\pm \pi_T \rightarrow \ell^\pm \nu_\ell b$ jet processes is shown in Fig. 13 for $M_V = M_A = 200$ GeV [54]. This study uses all the processes of TCSM–1. It also assumes the same selections and systematic uncertainty as in the published Run I data [52], but double the signal efficiency (1.38% vs. 0.69%). The 5$\sigma$ discovery reach goes up to $M_{\rho_T} = 210$ GeV and $M_{\pi_T} = 110$ GeV, larger than the 95% excluded region in Run I. The region that can be excluded in Run IIa extends up to $M_{\rho_T} = 250$ GeV and $M_{\pi_T} = 145$ GeV. When $M_V = 400$ GeV, the 5$\sigma$ discovery and 95% exclusion regions are only slightly larger

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Figure 16: Simulated event and background rates in the ATLAS detector for $\rho_T^\pm \rightarrow W^\pm Z \rightarrow \ell^\pm \nu_\ell \ell^+\ell^-$ for various $M_{\rho_T}$ and $M_{\pi_T}$; from Ref. [57].
The Run I DØ detector had superior calorimetry and hermiticity. The collaboration studied its Drell–Yan data to search for $\rho_T^-,\omega_T^+\rightarrow e^+e^-$. The data and the excluded region are shown in Fig. 14 for $Q_U = Q_D + 1 = 4/3$, $M_V = 100–400$ GeV and $M_{\rho_T} - M_{\pi_T} = 100$ GeV. Increasing $M_V$ and decreasing $M_{\rho_T} - M_{\pi_T}$ both increase the branching ratio for the $e^+e^-$ channel. For the parameters considered here, $M_{\rho_T} = M_{\omega_T} < 150–200$ GeV is excluded at the 95% CL. The expected reach of DØ in Run IIa for $\rho_T^-,\omega_T^+\rightarrow e^+e^-$ with $M_V = 100$ and 200 GeV and other TCSM parameters (see above) is shown in Fig. 15 [56]. As long as $Q_U + Q_D = \mathcal{O}(1)$, masses $M_{\rho_T}, M_{\omega_T}$ up to 450–500 GeV should be accessible in the $e^+e^-$ channel.

The ATLAS collaboration has studied its reach for $\rho_T^+\rightarrow W^\pm Z, W^\pm\pi_T, Z\pi_T$ and for $\omega_T^+\rightarrow \gamma\pi_T$ [57]. Figure 16 shows $\rho_T^+\rightarrow W^\pm Z \rightarrow \ell^\pm\nu_\ell\ell^\mp\nu_\ell$ for several $\rho_T$ and $\pi_T$ masses and a luminosity of 10 fb$^{-1}$. Detailed studies have not been published in which all the TCSM processes have been included and the parameters varied over a wide range. Still, it is clear from Fig. 16 that the higher energy and luminosity of the LHC ought to make it possible to completely exclude, or discover, low–scale technicolor for any reasonable parameterization.

5.2 Color–Nonsinglet Technihadrons

The experimental searches so far in the color–nonsinglet sector of low–scale technicolor have been inspired by the phenomenology of a pre–TCSM, one–family TC model. This model contains a single doublet each of color–triplet techniquarks $Q = (U,D)$ and of color–singlet technileptons $L = (N,E)$ [58, 19, 49]. We consider these searches first, commenting on the status of the TCSM for color nonsinglets at the end.

Assuming that techni–isospin is conserved, production of color–nonsinglet states is assumed to proceed through the lightest isoscalar color–octet technirhino, $\rho_{TS}$:

$$\bar{q}q, gg \rightarrow g \rightarrow \rho_{TS} \rightarrow \pi_{TS}\pi_{TS}, \pi_{LQ}\pi_{QL} \rightarrow \bar{q}q, gg$$

(25)

Here, $\pi_{TS} = \pi_{TS}^\pm, \pi_{TS}^0, \pi_{TS}^0 \equiv \eta_T$ are four color–octet technipions that are expected to decay to heavy $\bar{q}q$ pairs; $\pi_{LQ}$ are four color–triplet leptoquarks expected to decay to heavy $\ell_\ell$ with the corresponding charges. If TC2 is invoked, the neutral $\pi_{TS}$ decay to $\bar{b}b$ and, possibly, $gg$ as readily as to $\bar{t}t$.  

\[^{14}\text{The ATLAS collaboration has studied } gg \rightarrow \eta_T \rightarrow \bar{t}t \text{ [57]. Even if this is not the dominant}\]
Figure 17: The 95% CL exclusion regions for various $M_{\pi_{LQ}} - M_{\pi_{\bar{L}Q}}$ from a CDF search for $\rho_{T8} \rightarrow \pi_{LQ} \pi_{\bar{Q}L} \rightarrow \tau^+ \tau^- + jet$ jet; from Ref. [59].

The two $\rho_{T8} \rightarrow \pi_{T} \pi_{T}$ searches are by CDF for leptoquark technipions: $\pi_{ED} \rightarrow \tau^+ b$ where the $b$ is not tagged [59] and $\pi_{SD} \rightarrow \nu b, \nu c$ with one or two tagged jets [60]. These are based on 110 pb$^{-1}$ and 88 pb$^{-1}$ of Run I data, respectively. The exclusion plot for the $\tau^+ \tau^- +$dijet signal is shown in Fig. 17 as a function of the $\pi_{TS} - \pi_{LQ}$ mass difference. The theoretically likely case is that this mass difference is about 50 GeV, implying a 95% excluded region extending over $200 \lesssim M_{\rho_{TS}} \simeq 2 M_{\pi_{LQ}} \lesssim 500$ GeV. Figure 18 shows the reach for $\rho_{T8}^0 \rightarrow b\bar{b} \nu \bar{\nu}$ with at least one $b$-jet tagged. Here the 95% limits extends over $300 \lesssim M_{\rho_{TS}} \simeq 2 M_{\pi_{LQ}} \lesssim 600$ GeV. The search for $\pi_{LQ} \rightarrow c \nu$ excludes a similar range. These limits are quite impressive. However, it is not clear how they will be affected by the complications of topcolor-assisted technicolor.

Given the walking technicolor enhancement of $\pi_{T}$ masses, it is likely that decay mode of an $\eta_{T}$ in a TC2 model, other bosons, such as the colorons $V_{8}$, will have a sizable $t\bar{t}$ branching ratio and the ATLAS study serves as a promising prototype of a search for this process.
the $\rho_{TS} \rightarrow \pi_T \pi_T$ channels are closed. In that case, one looks for $\rho_{TS}$ as a dijet resonance: $\bar{q}q, gg \rightarrow g \rightarrow \rho_{TS} \rightarrow g \rightarrow \bar{q}q, gg$. Searches have been made by CDF for both untagged \[^{15}\] and $b$–tagged \[^{16}\] dijet resonances. The latter mode has a better signal–to–background ratio, but the rates and $b$–identification efficiencies in Run I were not high enough to make this advantage significant; the best limits come from untagged–dijets. The results of such a search are shown in Fig. 19. The region $260 < M_{\rho_{TS}} < 460$ GeV is excluded at the 95\% confidence level. This is a stringent constraint, but its applicability to TC2 models is uncertain.

Finally, and very briefly, we turn to the effect of topcolor–assisted technicolor on experimental studies of the color–nonsinglet sector \[^{17}\]. As I mentioned, the simplest implementation of TC2 models requires two color $SU(3)$ groups, one

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\[^{15}\]The decays $\rho_{TS} \rightarrow g\pi_{TS}$ and $g\pi_{T1}$ may occur and deplete the $\rho_{TS} \rightarrow$ dijets rate. These modes were not expected to be important in Ref. \[^{19}\] and they were not taken into account in the CDF analyses. They are included in the studies in Ref. \[^{17}\] (TCSM–2) discussed below.
strongly–coupled at 1 TeV for the third generation quarks (t, b) and one weakly–coupled for the two light generations. These two color groups must be broken down to the diagonal SU(3) near 1 TeV, and this remaining symmetry is identified with ordinary color. The most economical way I know to achieve this is to have the two technifermion doublets $T_1 = (U_1, D_1) \in (N_{TC}, 3, 1)$ and $T_2 = (U_2, D_2) \in (N_{TC}, 1, 3)$ condense with each other to achieve the desired breaking to SU(3)$_C$.

The main phenomenological consequence of this scenario is that the SU(3) gluons mix with the $SU(3)_1$ octet of massive colorons, $V_8$, and with four color–octet technirhos, $\rho_{ij} \sim T_i \lambda A T_j$ ($i, j = 1, 2$). The colorons decay strongly to top and bottom quarks and weakly to the light quarks. Alternatively, in the flavor–universal variant of TC2, the colorons decay with equal strength to all quark flavors. In Ref. [17], we assume for simplicity that all $\rho_{ij}$ are too light to decay to pairs of technipions. Then, they decay (via gluon and coloron dominance) into $\bar{q}q$.

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The colored technipion sector of a TC2 model is bound to be very rich. Thus, it is not clear how the limits on leptoquarks discussed above are to be interpreted. This is work for the future.
and $gg$ dijets and into $g\pi T_8$ and $g\pi T_1$.

Even this simplified minimal TC2 version of the TCSM has a much richer set of dijet spectra and other hadron collider signals than the one–family model discussed above [49, 61]. We are just beginning to study it. Some preliminary examples of dijet production based on the assumptions of TCSM–2 are shown in Figs. 20 and 21 for $\sqrt{s} = 2$ TeV at the Tevatron. In both figures the coloron mass is 1.2 TeV while the input $\rho_{T8}$ masses range from 350 to 500 GeV. Figure 20 shows $b\bar{b}$ production with a strong resonance at 300 GeV (i,e., below $t\bar{t}$ threshold). Figure 21 shows $t\bar{t}$ production with roughly a factor of two enhancement of the total cross section over that predicted in the standard model. Both signals are ruled out by Run I measurements of the $b\bar{b}$ and $t\bar{t}$ cross sections [62, 64].

Many more studies of both the color–singlet and nonsinglet sectors of the TCSM need to be carried out. The Fermilab Workshop on Strong Dynamics at Run II will begin these studies this autumn, in time to be of use when the run starts.

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17 The pole masses are shifted somewhat from these input values by mixing effects.
Figure 21: Production of $\bar{t}t$ in $\bar{p}p$ collisions at $\sqrt{s} = 2$ TeV according to the TCSM model of Ref. [47].

in Spring 2001. The CDF and DØ collaborations will carry out detector-specific simulations in the next year or two. More detailed and more incisive $e^+e^- \rightarrow \rho_T, \omega_T$ studies will come from the LEP experiments this year. The ATLAS and CMS collaborations likewise ought to study a broad range of signals for strong dynamics before they begin their runs later in the decade.

6. Open Problems

My main goal in these lectures has been to attract some bright young people to the dynamical approach to electroweak and flavor symmetry breaking. Many difficult problems remain open for study there. These lectures provide a basis for starting to tackle them. All that’s needed now are new ideas, new data, and good luck. Here are the problems that vex me:

1. First and foremost, we need a reasonably realistic model of extended technicolor, or *any other* natural, dynamical description of flavor. To repeat: This is
the hardest problem we face in particle physics. It deserves much more effort. I think the difficulty of this problem and the lack of a “standard model” of flavor are what have led to ETC’s being in such disfavor. Experiments will be of great help, possibly inspiring the right new ideas. Certainly, experiments that will be done in this decade will rule out, or vindicate, the ideas outlined in these lectures. That is an exciting prospect!

2. More tractable is the problem of constructing a dynamical theory of the top–quark mass that is natural, i.e., requires no fine–tuning of parameters, and has no nearby Landau pole. Like topcolor–assisted technicolor and top–seesaw models, such a theory is bound to have testable consequences below 2–3 TeV. So hurry—before the experiments get done!

3. Neutrino masses are at least as difficult a problem as the top mass. In particular, it is a great puzzle how ETC interactions could produce $m_\nu \lesssim 10^{-7} m_e$. It seems unnatural to have to assume an extra large ETC mass scale just for the neutrinos. Practically no thought has been has been given to this problem. Is there some simple way to tinker with the basic ETC mass–generating mechanism, some way to implement a seesaw mechanism, or must the whole ETC idea be scrapped? The area is wide open.

4. My favorite problem is “vacuum alignment” and CP violation [33]. The basic idea is this: Spontaneous chiral symmetry breaking implies the existence of infinitely many degenerate ground states. These are manifested by the presence of massless Goldstone bosons (technipions). The “correct” ground state, i.e., the one on which consistent chiral perturbation theory for the technipions is to be carried out, is the one which minimizes the vacuum expectation value of the explicit chiral symmetry breaking Hamiltonian $\mathcal{H}'$ generated by ETC. As Dashen discovered, it is possible that an $\mathcal{H}'$ that appears to conserve CP actually violates it in the correct ground state. This provides a beautiful dynamical mechanism for the CP violation we observe. Or it could lead to disaster—strong CP violation, with a neutron electric dipole moment ten orders of magnitude larger than its upper limit. This field of research is just beginning in earnest. If the strong–CP problem can be controlled (there is reason to hope that it can be!), there are bound to be new sources of CP violation that are accessible to experiment.
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