Incomplete noise-induced synchronization of spatially extended systems

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Abstract

A new type of noise-induced synchronous behavior is described. This phenomenon, called incomplete noise-induced synchronization, arises for one-dimensional Ginzburg-Landau equations driven by common noise. The mechanisms resulting in the incomplete noise-induced synchronization in the spatially extended systems are revealed analytically. The different model noise are considered. A very good agreement between the theoretical results and the numerically calculated data is shown.

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Introduction

Noise-induced synchronization \cite{1, 2, 3} is an ubiquitous phenomenon in nonlinear science. It arises as the interplay between determined and random dynamics \cite{4}, with both the synchronization and noise influence being recently the subjects of considerable interest of scientific community. Indeed, on the one hand the synchronous behavior of nonlinear systems has attracted great attention of researchers for a long time \cite{5, 6, 7, 8}. On the other hand discovering the fact that fluctuations can actually induce some degree of order in a large variety of nonlinear systems is one of the most surprising results of the last decades in the field of stochastic processes \cite{9, 10, 11}. Moreover, both these phenomena are relevant for physical, chemical, biological and other systems described in terms of nonlinear dynamics (see, e.g., \cite{12, 13, 14, 15}).

Noise-induced synchronization (NIS) means that the random signal influencing two identical uncoupled dynamical chaotic systems $u(t)$ and $v(t)$ (starting from the different initial conditions $u(t_0)$ and $v(t_0)$, $u(t_0) \neq v(t_0)$) results in their synchronous (i.e., identical) behavior $u(t) = v(t)$ after transient finished.

Noise-induced synchronization can be detected by means of direct comparison of the states of two chaotic systems, $u(t)$, and, $v(t)$, being under the influence of noise. The other method of diagnostics of NIS is calculating the largest Lyapounov exponent (LE) of dynamical system that measures the stability of the motion. Indeed, in driven chaotic system the largest Lyapunov exponent may become negative, that results in synchronization: both systems forced by the same noise “forget” their initial conditions and evolve to identical state \cite{16}. If the noise influence is infinitely small the largest Lyapunov exponent is positive for such a system.

In all cases of the noise-induced synchronization being known hitherto the boundary of the noise-induced synchronization regime is associated with the point on the parameter axis where the largest Lyapunov exponent of the system under study crosses the zero value when its sign is changed from “plus” to “minus”. In this paper we report for the first time that the noise-induced synchronization regime of two spatially extended uncoupled identical systems driven by common noise may be preceded by a new type of behavior, when the largest Lyapunov exponent remains zero in a finite range of parameter values. This kind of behavior called “incomplete noise induced synchronization” (INIS) demonstrates the features
of the synchronous motion of two uncoupled identical systems driven by common noise: although the states of the system differ, trajectories can be transformed into each other by an appropriate spatial shift.

The structure of the paper is the following. In Sec. I we describe the system under study being the complex Ginzburg–Landau equations driven by common noise and the new type of their behavior (incomplete noise induced synchronization) observed for the certain set of the values of the control parameters. In Sec. II we consider the mechanisms being responsible for the INIS regime. Sec. III deals with the different models of noise with the distinct probability densities. The final conclusions are given in Sec. IV.

I. SYSTEM UNDER STUDY AND INCOMPLETE NOISE-INDUCED SYNCHRONIZATION

The system under study is represented by a pair of uncoupled complex Ginzburg–Landau equations (CGLEs) driven by common noise:

\begin{align}
\frac{du}{dt} &= u - (1 - i\beta)|u|^2u + (1 + i\alpha)u_{xx} + D\zeta(x, t), \\
\frac{dv}{dt} &= v - (1 - i\beta)|v|^2v + (1 + i\alpha)v_{xx} + D\zeta(x, t),
\end{align}

where \( u(x, t) \), \( v(x, t) \) are complex states of the considered systems, \( \alpha \) and \( \beta = 4 \) are the control parameters, \( D \) defines the intensity of a noise term \( \zeta = \zeta_1 + i\zeta_2 \). We have used model noise with the asymmetrical probability distribution of the real \( \zeta_1 \) and imaginary \( \zeta_2 \) parts of the random variable

\[
p(\zeta_{1,2}) = \begin{cases} 
2\zeta_{1,2}, & \text{if } 0 \leq \zeta_{1,2} \leq 1, \\
0, & \text{otherwise}
\end{cases}
\]

(2)
on the unit interval \( \zeta_{1,2} \in [0; 1] \). The simulation of the random variables \( \zeta_{1,2} \) with required probability distribution \( p(\zeta_{1,2}) \) was carried out in the same way as it was described in [17] for the exponential stagger distribution. Equation (1) was solved with periodic boundary conditions

\[
u(x, t) = u(x + L, t) \quad \text{and} \quad v(x, t) = v(x + L, t),
\]

(3)
with all numerical calculations being performed for a fixed system length \( L = 40\pi \) and random initial conditions. To evaluate (1) the standard numerical scheme has been used [18], the value of the grid spacing is \( \Delta x = L/1024 \), the time step of the scheme \( \Delta t = 2.0 \times 10^{-4} \).
FIG. 1: The evolution of the difference of the system states $|u(x, t) - v(x, t)|$ described by complex Ginzburg-Landau equations (1) (a) without noise and (b) with noise with the intensity $D = 3$. In the second case the difference of the states of both systems in every point of space tends to be zero (after transient), which means the presence of the noise-induced synchronization regime. The control parameter values are $\alpha = 2$, $\beta = 4$.

If the noise intensity is equal to zero ($D = 0$) and initial conditions $u(x, 0)$ and $v(x, 0)$ are not identical, both systems demonstrate the complex chaotic behavior (both in time and in space), with the system states being different, i.e., $u(x, t) \neq v(x, t)$ (Fig. 1a). Alternatively, if the noise intensity $D$ is large enough the states of both systems coincide with each other (Fig. 1b), that is the evidence of the noise-induced synchronization.

To detect the presence of the noise–induced synchronization regime the averaged difference

$$\varepsilon = \frac{1}{TL} \int_0^\tau \int_0^L |u(x, t) - v(x, t)| \, dx dt,$$

between the spatio-temporal states of two CGLEs driven by common noise was calculated. The averaging process starts after a long-time transient with duration $\tau = 200$.

In the NIS regime the relation $\varepsilon = 0$ takes place, since in this case the difference between the states of two identical spatially extended systems (1) in every point of space tends to zero. We have also calculated the largest Lyapunov exponent $\lambda$ for one of the systems (1).
FIG. 2: The dependencies of (a) the averaged difference (4) and (b) the largest Lyapounov exponent of the CGLE on the noise intensity $D$ for the different values of the control parameter $\alpha$. Curves 1 correspond to the case of $\alpha = 1$, curves 2 were calculated for $\alpha = 2$. The values of noise intensity corresponding to the onset of noise-induced synchronization are shown by arrows with labels $D_{\text{NIS}}^1$ and $D_{\text{NIS}}^2$ for the curves 1 and 2, respectively. The boundary of the incomplete noise-induced synchronization is also shown by arrow marked as $D_{\text{INIS}}$.

As it was mentioned above in the NIS regime the largest Lyapunov exponent $\lambda$ should be negative.

The dependencies of the largest Lyapunov exponent $\lambda(D)$ and the averaged difference $\varepsilon(D)$ on the noise intensity $D$ are shown in Fig. 2 for two different values of the control parameter $\alpha$. It is easy to see that for the control parameter $\alpha = 1$ (curves 1 in Fig. 2a,b) the value of the noise intensity $D$ for which the largest Lyapunov exponent $\lambda$ crosses zero value and becomes negative coincides with the point where the averaged difference (4) starts being vanishingly small. So, in this case the noise-induced synchronization boundary is $D_{\text{NIS}} \approx 1.5$ (see arrows in Fig. 2a,b) and we deal with the occurrence of the noise-induced synchronization regime being typical and well-known.

Alternatively, a different scenario is observed in the same system if the control parameter value $\alpha = 2$ is considered (see curves 2 in Fig. 2a,b). For such a choice of $\alpha$-parameter...
value the largest Lyapunov exponent becomes equal to zero for the large enough intensity of noise $D_{INIS} \approx 1.53$ whereas the averaged difference $\varepsilon$ between the spatio-temporal states of two CGLEs driven by common noise exceeds zero value sufficiently (Fig. 2, a,b). With further increase of the noise intensity $D$ (when $D$ is equal to $D_{NIS} \approx 2.5$) the value of $\varepsilon$ becomes equal to zero (see Fig. 2 a) and the largest Lyapunov exponent starts to be negative which is the evidence of the presence of the noise-induced synchronization regime.

In other words, there is the finite interval of the noise intensity values $(D_{INIS}; D_{NIS})$ for which the noise-induced synchronization is not observed, and the largest Lyapunov exponent $\lambda$ is equal to zero. To prove this fact we have calculated the largest Lyapunov exponent of the complex Ginzburg-Landau equation for different values of the spatial grid spacing. The range of the noise intensities corresponding to the plateau where $\lambda = 0$ is shown in Fig. 3. One can see that the largest Lyapunov exponent calculations with the different values of the spatial grid step give the similar results. Based on these calculations we come to conclusion that the largest Lyapunov exponent is actually equal to zero in the finite range of the noise intensity.

Despite the fact that the noise-induced synchronization is not observed in the region where $\lambda = 0$ (see Fig. 2), this range of the noise intensities corresponds to the behavior showing the features of synchronous dynamics. The manifestation of synchronism can be observed if one of the complex media described by the Ginzburg-Landau equation starts to shift along
the second one with the spatial shift $\delta$. In other words, if one uses the shifted state of one of the system $v = v(x + \delta, t)$ in Eq. (1) the averaged difference $\varepsilon$ changes depending on this shift $\delta$. This movement of one of the systems supposed to be very slow for the transient to be completed. In this case such a spatial shift $\delta_0$ may be found that both Ginzburg-Landau equations start to behave identically, with the largest Lyapunov exponent being equal to zero. Therefore, we have called this regime “incomplete noise-induced synchronization” (INIS).

This statement is illustrated in Fig. 4 where the dependence of the difference $\varepsilon$ on the space shift $\delta$ is shown. One can see that there is value $\delta_0$ of shift $\delta$ for which the averaged difference $\varepsilon$ becomes equal to zero. Therefore, for this space shift $\delta_0$ both systems demonstrate identical behavior and the noise-induced synchronization is observed. This shift $\delta_0$ depends on the initial conditions. For the other values of the spatial shift $\delta$ the system states (both in space and time) are different, but the largest Lyapunov exponent is always equal to zero for the considered set of the control parameter values.

II. MECHANISMS RESULTING IN THE INCOMPLETE NOISE-INDUCED SYNCHRONIZATION REGIME

Let us discuss the mechanisms resulting in the occurrence of the incomplete noise-induced synchronization regime. In work [4] it has been shown, that for dynamical systems with small number of degrees of freedom the mechanisms of the onset of noise-induced synchronization
and generalized synchronization are equivalent. The mechanism of the generalized synchronization occurrence can be considered with the help of the modified system approach as it was done in Ref. [19] for the chaotic systems with the small number of degrees of freedom and in Ref. [20] for the spatially extended system. We interpret the mechanism of the onset of incomplete noise-induced synchronization in a similar way. Therefore, following Ref. [4, 19, 20] we consider the dynamics of the modified spatially extended system with the additional term determined by the mean value of noise.

The deterministic modified Ginzburg-Landau equation with the additional term, determined by the mean value \( \langle D\zeta \rangle \) of the noise term \( \zeta \) in stochastic equation (1) can be written as

\[
\frac{\partial u_m}{\partial t} = u_m - (1-i\beta)|u_m|^2 u_m + (1+i\alpha)\frac{\partial^2 u_m}{\partial x^2} + \langle D\zeta \rangle. \tag{5}
\]

For the selected kind of noise with the probability distribution (2) \( \langle D\zeta \rangle = \frac{2D}{3} \). Equation (5) is forced CGLE, widely studied and well documented in the literature (see, e.g. [21, 22, 23]). It is well-known, that different types of the spatio-temporal patterns may be observed depending on the domain of the control parameter values. If the value \( D \) is large enough, the homogeneous stationary state \( u_0 = u_0(x,t) = \text{const} \) is observed in the system (5). In this case the largest Lyapunov exponent is negative, with the stationary state regime in the system (5) corresponding to the noise-induced synchronization in the system (1). With decrease of the noise intensity \( D \) the stationary state \( u_0 \) loses its stability that corresponds to the boundary of the noise-induced synchronization of the initial Ginsburg-Landau equations (1) driven by noise.

At the same time the loss of the stability of the homogeneous stationary state occurs in different ways depending on the control parameter values of the modified Ginzburg-Landau equation (5).

Indeed, the homogeneous stationary state \( u_0 \) can be obtained numerically from equation

\[
u_0 - (1-i\beta)|u_0|^2 u_0 + 2D/3 = 0, \tag{6}\]

e.g., by a Newton method [24]. To analyze the stability of Eq. (6) we have to consider the linearization of the modified Ginzburg-Landau equation in the vicinity of the stationary solution \( u_0 \). Let \( \tilde{u} = \tilde{u}_r + i\tilde{u}_i \) be a small perturbation of the homogenous stationary state \( u_0 = u_r + iu_i, \) i.e., \( u_m = u_0 + \tilde{u} \). Having linearized equation (5) and assuming that
FIG. 5: The dependencies of the real part of the eigenvalues $\Lambda$ on the wave number $k$ for the different values of $D$-parameter when the control parameter $\alpha$ has been fixed as (a) $\alpha = 1$ and (b) $\alpha = 2$

$\tilde{u}_r(x, t) = \hat{u}_r(k) \exp(\Lambda t + ikx)$, $\tilde{u}_i(x, t) = \hat{u}_i(k) \exp(\Lambda t + ikx)$ we obtain the dispersion relation

$$
\begin{vmatrix}
1 - u_i^2 - 3u_r^2 - (\beta u_i^2 + 3\beta u_r^2 + 2\beta u_i u_r - k^2 - \Lambda) & 2u_r u_i - \alpha k^2 \\
2\beta u_i u_r - k^2 - \Lambda & 2u_r u_i - \alpha k^2 \\
\beta u_i^2 - 2u_i u_r + 2\beta u_r u_i - u_r^2 & 3\beta u_r^2 - \alpha k^2 \\
3\beta u_i^2 - \alpha k^2 & 3u_i^2 + 1 - k^2 - \Lambda
\end{vmatrix} = 0. \quad (7)
$$

determining the stability of the homogenous stationary state $u_0$. The homogenous stationary state $u_0$ is stable if the condition $\text{Re} \, \Lambda(k) < 0$, $\forall k$ is satisfied.

The evolution of $\text{Re} \, \Lambda(k)$ with the decrease of $D$-value for $\alpha = 1$ and $\alpha = 2$ is shown in Fig. 5a and Fig. 5b, respectively. One can see, that for $\alpha = 1$ the homogenous stationary state $u_0$ loses its stability when $D \approx 1.5$. In this case the spatial perturbation with the wave number $k = 0$ starts growing exponentially. As a result, the stationary state $u_0$ becomes unstable, the spatio-temporal chaos taking place in system (5). With the largest Lyapunov
FIG. 6: The profiles of the spatial stationary states $|u_0|^2$ and $|u_k(x)|^2$ observed in [5] for the different values of the noise intensity $D$ and $\alpha = 2$. Dashed lines correspond to the unstable state $|z_0|^2$.

exponent becoming positive both in the modified [5] and original [1] Ginzburg-Landau equation, the noise-induced synchronization regime in Eq. (1) is destroyed.

For the value of the control parameter $\alpha = 2$ the homogenous stationary state $u_0$ loses its stability for $D \approx 2.5$ and the spatial mode with the wave number $k = \pm 0.5$ becomes unstable in contrast to the case of $\alpha = 1$ considered before (see Fig. [5] b). Therefore, for $\alpha = 2$ the periodic spatial state $u_k(x) = u_k(x + l)$ (where $l$ is close to $2\pi/k$ due to periodical boundary conditions), which is stationary in time replaces the homogenous state $u_0$ in the modified Ginzburg-Landau equation. The example of the profiles of such stationary in time but periodical in space states is shown in Fig. [6]. Obviously, for such stationary states the largest Lyapunov exponent is equal to zero. Evidently, in the initial Ginzburg-Landau equation driven by noise, $D\zeta(x, t)$, with the mean value $\langle D\zeta \rangle$ the stationary in time and periodical in space structure $u_k(x)$ is perturbed by the fluctuations. Therefore, the spatio-temporal dynamics of $u_k(x)$ looks like aperiodic motion, with the largest Lyapunov exponent being also equal to zero. Since two identical media, $u(x, t)$, and, $v(x, t)$, driven by common noise start with different initial conditions $u(x, 0)$ and $v(x, 0)$, the spatially periodical structures do not coincide with each other, i.e., $u_k(x) \neq v_k(x)$, but there is such a shift in space $\delta_0$ depending on the initial conditions $u(x, 0)$ and $v(x, 0)$ where $u_k(x) = v_k(x + \delta_0)$. Therefore, for $D_{INIS} < D < D_{NIS}$ Ginzburg-Landau equations (1) driven by common noise are characterized by zero largest Lyapunov exponent and their states are not identical. If the first of the systems is shifted along the second one with a certain shift $\delta_0$ that, depending on
initial conditions, satisfies the requirement \( u_k(x) = v_k(x + \delta_0) \), the identical behavior of both considered systems is observed.

Note, also, a very good agreement between the values of the noise intensity \( D \) corresponding to the loss of the stability of the homogenous stationary state \( u_0 \) (see Fig. 5(b)) and to the point where the largest Lyapunov exponent becomes equal to zero (see Fig. 2(b)).

It should be noted that the INIS phenomenon considered above is determined by the peculiarity of the periodic boundary conditions. Due to the usage of such a kind of the boundary conditions, any spatially periodic solution can be moved arbitrarily along the spatial coordinate, and, therefore, in this case there is additional translational degree of freedom. Evidently, zero Lyapunov exponent corresponds to this translational degree of freedom, whereas all other Lyapunov exponents are negative. Instantaneous states evolved from the different initial conditions under the influence of the given realization of noise can be transformed into each other by means of the corresponding spatial shift. Taking this aspect into consideration the INIS regime does not seem to be observed in the absence of the translational invariance, i.e., for the other kind of the boundary conditions.

To illustrate this statement we have considered the complex dynamics of Ginsburg–Landau equation \((11)\) with the same set of the control parameter values but with the alternative types of the boundary conditions eliminating the translational invariance in the system under study. We have calculated both the largest Lyapunov exponent and \( \lambda \) and the averaged difference \( \varepsilon \) for two types of the boundary conditions:

\[
\frac{\partial u}{\partial x} \bigg|_{x=0} = \frac{\partial u}{\partial x} \bigg|_{x=L} = 0 \tag{8}
\]

and

\[
\frac{\partial v}{\partial x} \bigg|_{x=0} = \frac{\partial v}{\partial x} \bigg|_{x=L} = 0
\]

\[
\text{Re } u(0, t) = \text{Re } u(L, t) = 1, \\
\text{Im } u(0, t) = \text{Im } u(L, t) = 1, \\
\text{Re } v(0, t) = \text{Re } v(L, t) = 1, \\
\text{Im } v(0, t) = \text{Im } v(L, t) = 1. \tag{9}
\]

The obtained curves \( \lambda(D) \) and \( \varepsilon(D) \) are shown in Fig. 7.

Evidently, in both cases there is no translation invariance in the system, and, as a result, the largest Lyapunov exponent is close to zero, but negative, in the range of \( D \)-values
FIG. 7: (Color online) The dependencies of (a) the largest Lyapounov exponent of the CGLE and (b) the averaged difference $\bar{\varepsilon}$ on the noise intensity $D$ for control parameter values $\alpha = 2$ and $\beta = 4$. Curves 1 (♦) correspond to the boundary conditions (8) and curves 2 (□) — to the boundary conditions (9), respectively.

where for the periodical boundary conditions (3) the INIS regime is observed. The averaged difference $\bar{\varepsilon}$, in turn, is equal to zero in this range of the noise intensity values, that shows the impossibility of the INIS regime existence if there is no translation invariance in the system.

III. THE INCOMPLETE NOISE-INDUCED SYNCHRONIZATION AND THE NOISE CHARACTERISTICS

To illustrate that the onset of the incomplete noise-induced synchronization regime is caused by the mean value of noise only, we examine how the different model noise influence the considered media described by CGLEs (1). One of the typical probability density is the Gaussian one, therefore, it seems to be reasonable to consider the dynamics of the complex Ginzburg-Landau equations (1) driven by common noise with the Gaussian probability distribution of the real and imaginary parts of the random variable

$$p(\xi) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\xi - a)^2}{2\sigma^2}\right).$$  \hspace{1cm} (10)
FIG. 8: (Color online) The dependencies of the largest Lyapunov exponent $\lambda$ on the value $a$ calculated for the noise with Gaussian distribution (10) for the different values of the variance: $\text{◊} - \sigma = 0.1$, $+$ $\sigma = 0.2$, $\square$ $\sigma = 0.5$. The plateau where $\lambda = 0$ is observed. The control parameter values of complex Ginzburg-Landau equation are $\alpha = 2$, $\beta = 4$

FIG. 9: (Color online) The dependencies of the largest Lyapunov exponent $\lambda$ on the value $a$ calculated for the uniform distribution of the random variable with the different values of the variance: $\text{◊} - \sigma = 0.5$, $+$ $\sigma = 1.0$, $\square$ $\sigma = 1.5$, $\times$ $\sigma = 2.0$. The plateau where $\lambda = 0$ is observed. The control parameter values of complex Ginzburg-Landau equation are $\alpha = 2$, $\beta = 4$

Since in Sec. I and Sec. II we were not able to separate the influence of the mean value and variance from each other, we can do it easily for Gaussian probability density. To do that, the value of the noise intensity $D = 1$ has been fixed. In this case the mean value of the random process is governed by the choice of $a$-parameter, while the intensity of noise is determined by the variance $\sigma$. The random variable with the required probability density has been generated following Ref. [24].

We have calculated the dependence of the largest Lyapunov exponent $\lambda$ on the mean
value $a$ of the probability density (10) for different values of the variance $\sigma$. The results of these calculations are given in Fig. 8 where the curves $\lambda$ vs. $a$-value are shown. As it follows from the numerical calculations the incomplete noise-induced synchronization is observed in the range $a \in [1.0; 1.7]$ (as well as in the case considered in Sec. II) for all values of the variance $\sigma$. Therefore, the value of the variance $\sigma$ (i.e., the noise intensity) does not seem to play the key role in the occurrence of the INIS regime.

Having examined the spatio-temporal behavior of two complex Ginzburg-Landau equations (11) driven by common noise with the probability density (10) we have found for the range $a \in [1.0; 1.7]$ that the states of them do not coincide with each other therefore there is no noise-induced synchronization as expected. Nevertheless, there is the spatial shift $\delta$ for which the dynamics of system $u(x, t)$ and $v(x + \delta, t)$ is identical, which is also the evidence of the presence of the incomplete noise-induced synchronization regime.

To compare the results described in Sec. II and Sec. III with each other we have to take into account that the value $2D/3$ has been substituted in Sec. II for the mean value of noise (see the explanation given below Eq. (5)). Obviously, if we substitute the same value $2D/3$ for the mean value $a$ (i.e., denoting $a = 2D/3$) in the case of noise with the Gaussian probability distribution, we obtain, that the range of the INIS-regime is $D \in [1.5; 2.5]$ that agrees very well with the results of the analytical consideration given in Sec. II.

The very same results have been obtained for the uniform distribution of the random variables with the mean value $a$ and variance $\sigma$ (Fig. 9): there is the range of the values of $a$-parameter where the largest Lyapunov exponent is equal to zero and the incomplete noise-induced synchronization regime takes place.

Based on this consideration we come to conclusion that the occurrence of the incomplete noise-induced synchronization regime is determined by the mean value of noise, whereas the variation of it practically does not play any role.

IV. CONCLUSION

In conclusion, we have reported for the first time a new type of noise-induced synchronous behavior occurring in the spatially extended systems. Such a type of incomplete noise-induced synchronization differs remarkably from all other types of synchronous behavior known so far. It may be observed in a certain range of the noise intensity values, where
the largest Lyapunov exponent is equal to zero and the states of two identical spatially extended systems driven by common noise are different, although there is an indication of the synchronism: if one of the systems is shifted along the second one on the certain shift the identical behavior of the considered systems is observed. The theoretical equations allowing to explain the mechanism resulting in such a type of behavior have also been given, and they are in perfect agreement with the numerically obtained data. The influence of a different model noise on the occurrence of the noise-induced synchronization regime is considered. Though the INIS regime has been observed here in the complex Ginzburg-Landau equations driven by common noise with non-zero mean value, we expect that the very same type of behavior can be observed in many other relevant circumstances. Since the noise influence may result in a pattern formation (see, e.g., [25]) we suppose that incomplete noise-induced synchronization can be also observed for noise with the zero mean value, with the other types of the spatio-temporal patterns (e.g., traveling waves) being observed.

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