Effect of Relative Permittivity with Strain in Dielectric Elastomer Peristaltic Actuator

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Abstract. This article, exemplify the deformation behaviour of dielectric elastomer peristaltic actuator with variation in relative permittivity as a function of voltage induced strain. Modified model is proposed to predict the behaviour of peristaltic actuators, best suited for biomedical applications to transport incompressible fluids. Transportation is accomplished by a series of radially expanding and axially inextensible tubular modules, made up of dielectric elastomer. Consecutive equations are derived using the hypothesis of linear elasticity. The static electromechanical actuation performance is estimated and compared well with existing experimental as well as modelling data. Results indicate an efficient model is attained by considering crucial material parameters that significantly influence electromechanical performance. In the highlight, many other behaviours like viscoelasticity, dielectric loss, breakdown strength, etc. need attention to realize a reliable model for dielectric elastomer actuator towards commercial applications.

Keywords: Biomedical; Electromechanical actuation; peristaltic actuator; Dielectric elastomers; Strain; Permittivity

1. Introduction

Dielectric elastomers (DEs) illustrate significant change in shape and size under the imposition of high voltage electric field. Research on this smart electromechanical deformation progressed tremendously over the past decade and varieties of DE configured devices are proposed such as actuators, force sensors, energy harvesters, artificial muscle, etc. [1]. DE-based peristaltic actuator is similar and useful to transport incompressible fluid in biomedical systems, infusion pumps, microfluidic devices, locomotion in crawling robots, etc. [2][3][4].

The peristaltic actuation at present is undertaken by the complex mechanism of electromagnetic motors, electrostatic actuators, pneumatic mechanisms and similar [5]–[8]. The materials explored towards the development of such devices counts; piezoelectric, shape memory alloys, magnetic fluids, polymer-metal composite, etc. [9][10][11]. However, DE ensures the feasibility for soft application owing to the features of incompressible softness, ease in scaling, high; power-to-weight ratio, mechanical compliance, flexibility, less structural complexity, good response-time, etc.

In general, the actuation response of DE actuator works on the principle of Maxwell stress. Which reveals the actuation strain is due to electrostatic pressure generated by oppositely charge electrode, along with the thickness of the elastomer. The actuation performance again relies on system parameters like dielectric permittivity and elastic modulus properties, applied voltage, as represented in Eq. (1) and (2).

\[ p = \varepsilon_0 E^2 = \varepsilon_0 \left( \frac{V}{d} \right)^2 \]  

(1)
These electromechanical properties of dielectric elastomer are recognized to vary with pre-strain, which is applied to prevent electromechanical pull-in instability and to decrease elastic modulus owing to reduced thickness. And it simultaneously decreases dielectric permittivity of elastomer in a significant manner, despite which the overall performance of DE actuator improves with pre-strain.

The concept of DE peristaltic actuator is earlier proposed by Capri et al. [2] whose schematic illustration is as shown in fig. 1(a).

The system peristaltic actuator comprises serially arranged DE tubular modules that electromechanically expands in radial and subsequent contract. Longitudinal or the actuation in the axial direction is bounded by inextensible wires. Further, as shown in fig. 1(b), in a single direction flow, two adjacent tubular modules expand radially so as to intake the fluid inside, the deactivation of the first stage and the activation of the second is repeated in the entire length of the tube step by step manner to maintain the fluid flow.

The contraction and expansion of the actuator can be explained by Maxwell principle. On removal of the voltage from one stage of the tube, the closed valve leads to constant volume condition. And the flowing fluid exert pressure on tube wall to reduce its radius which will create a pressure difference between the inside ($P_{in}$) and the outside $P_{out}$ (i.e. $P_{in} - P_{out} > 0$). Simultaneously, the pressure difference is generated to open the valves and allow the fluid to flow to the next stage of the tubes. Therefore, this actuated deformation in the tube is expected to decrease the dielectric permittivity as discussed. However, it is assumed to be constant in the existing model of peristaltic actuator [2].

In this article, change in relative permittivity with strain is considered to improve the effectiveness of the existing model for a peristaltic actuator. Axial deformation is presumed to be fixed by flexible but inextensible wire. Consecutive equations are derived through the hypothesis of linear elasticity which is also used to obtain prediction from the existing model. A flow chart presenting the systematic view to steps followed in modelling is shown in fig. 2. The results are found comparatively closer experimental data. Thus, consideration of varying permittivity is recognized to be of utmost importance to present a realistic model for the practical application.
2. Analytical Modelling

As shown in fig. 1(b), a single module of the tube actuator is considered in the static condition. Analysis of electrical and mechanical deformation induced by the applied voltage in terms of inside and outside pressure difference is studied separately.

\[ P_A = \frac{\varepsilon V^2}{A} \Gamma \]  

(3)

\[ P_B = \frac{\varepsilon V^2}{B} \Gamma \]  

(4)

Where \( \varepsilon = \varepsilon_0 \cdot \varepsilon_r \), \( \varepsilon_0 \) is the dielectric permittivity of the vacuum and \( \varepsilon_r \) is relative permittivity.

\[ \Gamma = \frac{1}{2AB(A^2 - B^2)\ln^2(B/A)} \sqrt{A^6 + B^6 - A^2B^2 - A^2B^2 + 8A^2B^2(B^2 - A^2)\ln(B/A)} \]  

(5)

\( \Gamma \) is the geometrical parameter.

Hence, the forces on the lateral surfaces of the tubes are equal i.e. \( F_A = F_B \). And the calculation for pressure induced radial deformation is as in the following section.

2.2. Active radial deformations
Stresses and strain in the cylindrical module due to pressure \( P_{\theta} \) and \( P_{z} \) is estimated by a simplified quasi-static analytical model. The elastomer is assumed to be linearly elastic, isotropic and homogeneous. Navier’s equation for displacement is used to relate displacement in terms of pressure-induced strains in the actuator.

In a cylindrical coordinate \( r, \theta \) and \( z \) in absence of body forces and under the static condition equations are:

\[
\left( \lambda + \mu \right) \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \right) + \mu \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right) = 0
\]

Similarly for the other two perpendicular \( \theta - direction \) and \( z - direction \).

Where \( \lambda \) and \( \mu \) are Lamé’s constant. \( u_r, u_\theta \) and \( u_z \) are the deformation along \( r - direction \), \( \theta - direction \) and \( z - direction \) respectively. Lamé’s constants are defined as:

\[
\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}
\]

\[
\mu = \frac{E}{2(1+\nu)}
\]

Where \( E \) : Young’s modulus \( \mu \) : Poisson’s ratio.

In the absence of torsion, the movement along the tangential direction \( \theta \) is assumed to be null. Further, the displacement of the tube in the axial direction is also absent due to isometric condition, the deformation along the radial direction only depends on the radial coordinate \( r \) we neglect the edge effects. These assumptions can be listed as follows:

\[
u u_\theta = 0, u_z = 0, u_r = u_r \left( r \right)
\]

On substituting assumptions and on rewriting., Naiver equation in \( \theta - direction \) and \( z - direction \) becomes null and equations (4) becomes

\[
\frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} (ru_r) \right) = 0
\]

The generalized solution for the above equation (8) given by:

\[
u u_r = c_1 \frac{r}{2} + c_2 \frac{1}{r}
\]

Where \( c_1 \) and \( c_2 \) are the arbitrary constants can be obtained by imposing the boundary conditions.

The strains ( \( \varepsilon_{rr}, \varepsilon_{r\theta} \) and \( \varepsilon_{zz} \) ) and displacements (7) are related as:

\[
\varepsilon_{rr} = \frac{du_r}{dr}, \varepsilon_{r\theta} = \frac{u_r}{r} + \frac{1}{r} \frac{du_\theta}{d\theta}, \varepsilon_{zz} = \frac{du_z}{dz}
\]

By using equation (9) in equation (10) strains become

\[
\varepsilon_{rr} = \frac{c_1}{2} - \frac{c_2}{r^2}
\]

\[
\varepsilon_{r\theta} = \frac{c_1}{2} + \frac{c_2}{r^2}
\]

\[
\varepsilon_{zz} = 0
\]
Stresses ($\sigma_{rr}$, $\sigma_{\theta\theta}$, and $\sigma_{zz}$) and strains ($\varepsilon_{rr}$, $\varepsilon_{\theta\theta}$ and $\varepsilon_{zz}$) are related by material constitutive equations, represented by the Hooke’s law and expressed in terms of the Lame’s constant as:

$$
\sigma_{rr} = \frac{1}{2}(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu \varepsilon_{rr}
$$
(16)

$$
\sigma_{\theta\theta} = \frac{1}{2}(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu \varepsilon_{\theta\theta}
$$
(17)

$$
\sigma_{zz} = \frac{1}{2}(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu \varepsilon_{zz}
$$
(18)

Further, the boundary conditions to find the value of arbitrary constants $c_1$ and $c_2$ are:

$$
\sigma_{rr} \big|_{r=A} = -P_A
$$
(19)

$$
\sigma_{rr} \big|_{r=B} = -P_B
$$
(20)

Here negative sign is taken by assuming stresses compressive. On solving these equations for unknown $c_1$ & $c_2$ and substituting in stress equations (14), (15), (16)

$$
\sigma_{rr} = \frac{P_B B^2 - P_A A^2}{A^2 - B^2} - \frac{(P_B - P_A) A^2 B^2}{(A^2 - B^2)} \frac{1}{r^2}
$$
(21)

$$
\sigma_{\theta\theta} = \frac{P_B B^2 - P_A A^2}{A^2 - B^2} + \frac{(P_B - P_A) A^2 B^2}{(A^2 - B^2)} \frac{1}{r^2}
$$
(22)

$$
\sigma_{zz} = \frac{\lambda}{\lambda + \mu} \left(\frac{P_B - P_A}{A^2 - B^2}\right)
$$
(23)

For the squeezing action of the tube along the radial and longitudinal directions stresses $\sigma_{rr} < 0$ and $\sigma_{zz} < 0$, while $\sigma_{\theta\theta} \big|_{r=A} > 0$ and $\sigma_{\theta\theta} \big|_{r=B} < 0$.

By using the following Hooke’s law, written in terms of $E$ and $\nu$

$$
\varepsilon_{rr} = \frac{1}{E} \left[ \sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz}) \right]
$$
(24)

$$
\varepsilon_{\theta\theta} = \frac{1}{E} \left[ \sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz}) \right]
$$
(25)

$$
\varepsilon_{zz} = \frac{1}{E} \left[ \sigma_{zz} - \nu(\sigma_{rr} + \sigma_{\theta\theta}) \right]
$$
(26)

From equation (5) and (6) it can be deduced that $\frac{\lambda}{\lambda + \mu} = 2\nu$

On substituting the values in equation (22), (23), (24) and rewriting:

$$
\varepsilon_{rr} = \frac{1 + \nu}{E(A^2 - B^2)} \left[ (1 - 2\nu) \left(\frac{P_B B^2 - P_A A^2}{A^2 - B^2} - A^2 B^2 \frac{1}{r^2}\right) \right]
$$
(27)

$$
\varepsilon_{\theta\theta} = \frac{1 + \nu}{E(A^2 - B^2)} \left[ (1 - 2\nu) \left(\frac{P_B B^2 - P_A A^2}{A^2 - B^2} + A^2 B^2 \frac{1}{r^2}\right) \right]
$$
(28)

$$
\varepsilon_{zz} = 0
$$
(29)
To obtain two quantities of interest for the description of the actuator performance: the engineering strains of the inner and outer radii are defined as the percentage variation in the respective dimension. It can be calculated as

$$\Delta A |_{\text{active}} = \frac{1}{2\pi} \int_0^{2\pi} \varepsilon_{\theta \theta} (A) d\theta$$

$$\frac{\Delta A}{A} |_{\text{active}} = \frac{(1+\nu)}{E(A^2 - B^2)} \left[ P_a (2B^2 - 2\nu B^2) - P_a \left( A^2 + B^2 - 2\nu A^2 \right) \right]$$

$$\frac{\Delta B}{B} |_{\text{active}} = \frac{1}{2\pi} \int_0^{2\pi} \varepsilon_{\theta \theta} (B) d\theta$$

$$\Delta B |_{\text{active}} = \frac{(1+\nu)}{E(A^2 - B^2)} \left[ P_a (A^2 + B^2 - 2\nu B^2) - P_a \left( 2A^2 - 2\nu A^2 \right) \right]$$

Substituting (1) and (2) into the above equations (31) and (32) provides the relation between the active relative radial deformations exhibited by the actuator. It is worth reminding that the pressures $P_a$ and $P_b$ are assumed to be constants. For a constant applied voltage, the pressure will change due to modification in the thickness of the elastomer. Such an effect through iterative procedures is not considered in this linear model. However, presented analytical model matches well with the experimental results.

Substituting the values of $P_a$ and $P_b$ in the equation we get:

$$\Delta A |_{\text{active}} = \frac{(1+\nu) \delta (B-A)^2}{A^2 E} \left( \frac{V}{B-A} \right)^2$$

$$\Delta B |_{\text{active}} = \frac{(1+\nu) \delta (B-A)^2}{B^2 E} \left( \frac{V}{B-A} \right)^2$$

Where $\gamma = \Gamma / (A + B)$.

2.3. Passive radial deformation

If the pressure inside the tube by the flowing fluid be $P_m$ and that of the external environment be $P_{ext}$, then pressure difference of $P_m - P_{ext}$ will induce deformation in the radial direction. These pressure are not affected by the electrical action on the tubes so named as passive action. Deformation caused by passive action can be calculated by following the same procedure in the previous section (2.2).

$$\Delta A |_{\text{passive}} = \frac{(1+\nu)}{E(A^2 - B^2)} \left[ P_{ext} (2B^2 - 2\nu B^2) - P_m \left( A^2 + B^2 - 2\nu A^2 \right) \right]$$

$$\Delta B |_{\text{passive}} = \frac{(1+\nu)}{E(A^2 - B^2)} \left[ P_{ext} (A^2 + B^2 - 2\nu B^2) - P_m \left( 2A^2 - 2\nu A^2 \right) \right]$$

These can be used to quantify the effects of variable amounts of fluid contained inside each module.

2.4. Total radial deformation

The combined effect of the active i.e. electrical effect and the passive i.e. the effect of pressure difference can be quantified by superimposing the effects separately, which can be given by:
\[
\frac{\Delta A}{A_{\text{Total}}} = \frac{\Delta A}{A_{\text{active}}} + \frac{\Delta A}{A_{\text{passive}}} \\
\Delta A = (1+\nu)\delta E\left(\frac{B-A}{A^2E}\right)^2 + \frac{1+\nu}{E(A^2-B^2)}\left[P_{\text{ext}}(2B^2-2\nu B^2) - P_{\text{in}}(A^2 + B^2 - 2\nu A^2)\right] \\
\frac{\Delta B}{B_{\text{Total}}} = \frac{\Delta B}{B_{\text{active}}} + \frac{\Delta B}{B_{\text{passive}}} \\
\Delta B = (1+\nu)\delta E\left(\frac{B-A}{B^2E}\right)^2 + \frac{1+\nu}{E(A^2-B^2)}\left[P_{\text{ext}}(A^2 + B^2 - 2\nu B^2) - P_{\text{in}}(2A^2 - 2\nu A^2)\right]
\]

The values of change in the relative permittivity with a change in the uniaxial pre-strain at different frequencies is referred to fig. 2 [12].

![Figure 3. Relative permittivity as a function of strain on a different frequency](image)

At a frequency of 100Hz the variation of the relative permittivity as a function of strain of the material is found to be:

\[
\varepsilon_r = e^{(1.52847 - 0.01467x - 0.00117x^2)}
\]

Where \(x\) represents the strain introduced to the thin film and \(\varepsilon_r\) is relative permittivity corresponding to a particular strain due to an applied voltage.

### 3. Result and Discussion

For a micro-pump module by keeping the internal radius of 4.1 mm and the external radius of 5 mm, with Young’s modulus (E) at 0 % strain as 100 kPa. The module is assumed to be a constant volume (i.e. incompressible, \(\nu = 0.499\)). Module is assumed to be linear elastic for small deformation. The axial deformation of a single module is constrained with the help of externally inextensible but flexible wires. The consecutive equation is driven to deduce the radial deformation of a single module of the peristaltic pump. MATLAB platform is used to calculate the radial deformation of a module with the help of all consecutive equations after incorporating variable relative permittivity [12] eq. (42). The step size in the increase of the voltage for analytical modelling is 1. Other parameters of the module are listed in Table 2.
Table-1 Comparison of radial deformation in the existing model and modified model

| Voltage per unit thickness (1×10^6 V/m) | Relative radial deformation (%) | Existing Model | Modified model |
|----------------------------------------|---------------------------------|----------------|----------------|
| 0                                      | 0                               | 0              | 0              |
| 1.1111                                 | 0.0475                          | 0.0438         |
| 2.2222                                 | 0.1901                          | 0.1753         |
| 3.3333                                 | 0.4276                          | 0.3944         |
| 4.4444                                 | 0.7602                          | 0.7011         |
| 5.5555                                 | 1.1879                          | 1.0954         |
| 6.6666                                 | 1.7105                          | 1.5772         |
| 7.7777                                 | 2.3289                          | 2.1466         |
| 8.8889                                 | 3.0409                          | 2.8035         |

Figure 4. relative radial deformation with voltage per unit thickness

Table-2 Parameter of the micro pump module

| Parameter                | Value        |
|--------------------------|--------------|
| Electrode length         | L = 163.00 mm|
| Actuator length          | L_{tot} = 174.0 mm|
| Internal radius          | A = 4.1 mm   |
| External radius          | B = 5.0 mm   |
| Wall thickness           | B - A = 0.9 mm|
| Poisson’s ratio          | ν = 0.499    |

The result produced by the modified analytic modelling, suited well with the experimental data as shown in fig. 3. Incorporation of relative varying permittivity as a function of strain shows a significant difference as compared to existing Carpi model [2] as represented in Table 1. The possible
reason for the reduced deformation with an increase in the voltage per unit thickness is due to polar segments of a dielectric elastomer get less movement space under stretched condition. With the strain, the affinity of alignment of polar molecule reduces relative permittivity, which in turn decrease the relative radial deformation of the micro pump module.

4. Conclusions
The effect of a change in relative permittivity with the voltage induced strain is considered to incorporate in existing model for DE-based peristaltic actuator. The attained results are found closer to experimental data for actuation performance. Hence, the existing model is modified to be a more efficient model is presented in comparison to existing one. For more accuracy in the estimation of performance, the peristaltic pump should be analysed under dynamic and nonlinear condition. Further, the effects of constraint in axial direction and consideration of other crucial parameters indeed for designing practical peristaltic actuator.

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