CP Violation and the Muon Anomaly in $N = 1$ Supergravity

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Abstract

The one loop supersymmetric electro-weak correction to the anomalous magnetic moment of the muon is derived in the minimal $N = 1$ supergravity unification with two CP violating phases. A numerical analysis of the CP violating effects on $g_{\mu} - 2$ is carried out with the cancellation mechanism to guarantee the satisfaction of the experimental limits on the electric dipole moments of the electron and on the neutron. It is found that the effects of the CP phases can significantly affect the supersymmetric electro-weak correction to $g_{\mu} - 2$, and that the numerical size of such a correction can be as large or larger than the Standard Model electro-weak correction to $g_{\mu} - 2$. These results are of import for the new Brookhaven experiment which is expected to increase the sensitivity of the $g_{\mu} - 2$ measurements by a factor of 20 and allow for a test of the electro-weak correction in the near future.
1 Introduction

Supersymmetric theories contain a large number of CP violating phases which arise from the soft supersymmetry (SUSY) breaking sector of the theory and contribute to the electric dipole moment (EDM) of the quarks and the leptons. Currently there exist stringent limits on the neutron[1] and on the electron[2] EDM. Thus CP violations in supersymmetric theories is severely constrained by experiment. To satisfy these constraints it has generally been assumed that the CP violating phases are small[3, 4]. However, small phases constitute a fine tuning and an alternative possibility suggested is that that the CP violating phases can be large O(1) and the EDM constraints could be satisfied by the choice of a heavy spectrum[5]. However, for CP phases O(1) the satisfaction of the EDM constraints may require the SUSY spectrum to lie in the several TeV region thus putting the spectrum even beyond the reach of the Large Hadron Collider (LHC). More recently a third possibility has been proposed, and that is of internal cancellations among various contributions to the electron and the neutron EDMs[6], and there have been further developments of this idea[7, 8, 9, 10, 11]. Since the cancellation mechanism allows for the possibility of large CP violating phases, it is of considerable interest to explore the effects of such large phases on low energy physics and several studies exploring the effects of large phases have recently been reported. These include the effects of large CP phases on dark matter[12, 13], on low energy phenomena[14, 15, 16, 17], as well as other SUSY phenomena[18, 19, 20, 21, 22, 23].

In this paper we investigate the effects of CP violation on the supersymmetric electro-weak contributions to $g_\mu - 2$. This analysis extends the previous analyses of supersymmetric electro-weak contributions without the inclusion of the CP violating effects[24, 25]. This investigation is timely since the Brookhaven experiment E821 has started collecting data and in the near future will improve the sensitivity of the $g_\mu - 2$ measurements to allow a test of the Standard Model electro-weak contribution[26]. It is already known that the supersymmetric electro-weak contributions to $g_\mu - 2$ can be as large or larger[23, 24, 25] than the Standard Model electro-weak contribution[26] and it is thus of interest to investigate the effects of large CP violating phases on the supersymmetric muon anomaly.

We begin by exhibiting the SUSY breaking sector of the CP violating phases relevant for our case. It is given by

$$V_{SB} = m_1^2|H_1|^2 + m_2^2|H_2|^2 - [B\mu\epsilon_{ij}\bar{H}_1^i\bar{H}_2^j + H.c.] + m_{\tilde{L}}^2[\bar{\tilde{\nu}}_{\mu}\tilde{\nu}_\mu + \bar{\tilde{\nu}}_{\mu}\tilde{\nu}_\mu] + m_{\tilde{R}}^2[\bar{\tilde{\nu}}_{\mu}\tilde{\nu}_\mu + \bar{\tilde{\nu}}_{\mu}\tilde{\nu}_\mu]$$
\[
+ \frac{g m_0}{\sqrt{2} m_W} \epsilon_{ij} \left[ \frac{m_{\mu}}{\cos \beta} H_1^i \tilde{\mu}^*_R + H.c. \right] + \frac{1}{2} [\tilde{m}_2 \tilde{W}^a \tilde{W}^a + \tilde{m}_1 \tilde{B}\tilde{B}] + \Delta V_{SB} \quad (1)
\]

where \( \tilde{l}_L \) is the \( SU(2)_L \) smuon doublet, \( tan \beta = |< H_2 > / < H_1 > | \) where \( H_1 \) gives mass to the muon. The quantities \( A_{\mu}, \mu, \) and \( B \) are in general complex.

In this analysis we shall limit ourselves to the framework of the minimal supergravity model [30]. In the minimal supergravity framework (mSUGRA) the soft SUSY breaking is characterized by the parameters \( m_0, m_1^2, A_0, tan \beta, \theta_{\mu0} \) and \( \alpha_{A0} \), where \( m_0 \) is the universal scalar mass at the GUT scale, \( m_1^2 \) is the universal gauginos mass at the GUT scale, \( A_0 \) is the universal trilinear coupling at the GUT scale, \( \theta_{\mu0} \) is the phase of \( \mu_0 \) at the GUT scale, and \( \alpha_{A0} \) is the phase of \( A_0 \). In the analysis we use one-loop renormalization group equations (RGEs) for the evolution of the soft SUSY breaking parameters and for the parameter \( \mu \), and two-loop RGEs for the gauge and Yukawa couplings. The phase of \( \mu \) does not run because it cancels out of the one loop renormalization group equation of \( \mu \). However the magnitude and the phase of \( A_\mu \) do evolve. Thus while the phase of \( A_\mu \) is modified from \( \alpha_{A_\mu0} \) at the GUT scale to its value \( \alpha_{A_\mu} \) at the electro-weak scale, the phase of \( \mu \) is unaffected at the one loop level, i.e., \( \theta_\mu = \theta_{\mu0} \).

The outline of the rest of the paper is as follows: In Sec.2 we derive a general formula for the contribution to \( a_f = g_f - \frac{2}{2} \) in the presence of CP violating phases. In Sec.3 we compute the supersymmetric electro-weak corrections to \( a_\mu \) from the chargino exchange and in Sec. 4 we compute the supersymmetric electro-weak corrections to \( a_\mu \) from the neutralino exchange. A discussion of these results is given in Sec. 5 and a numerical analysis of the effects of CP violating phases is given in Sec. 6. We summarize our results in Sec. 7.

## 2 g-2 Calculation with CP Violation in SUSY

In this section we derive a general formula for the contribution to \( a_\mu \) for an interaction with CP violating phases which would be typical of the interactions that we will encounter in Secs. 3 and 4. For a theory of a fermion \( \psi_f \) of mass \( m_f \) interacting with other heavy fermions \( \psi_i \)'s and heavy scalars \( \phi_k \)'s with masses \( m_i \) and \( m_k \), the interaction that contains CP violation is in general given by

\[
- \mathcal{L}_{int} = \sum_{ik} \tilde{\psi}_f (K_{ik} \frac{1 - \gamma_5}{2} + L_{ik} \frac{1 + \gamma_5}{2}) \psi_i \phi_k + H.c. \quad (2)
\]
Here $\mathcal{L}$ violates CP invariance iff $Im(K_{ik}L_{ik}^*)$ is different from zero. The one loop contribution to $a_f$ is given by

$$a_f = a_f^1 + a_f^2$$

(3)

where $a_f^1$ and $a_f^2$ are coming from Fig. 1(a) and Fig. 1(b) respectively. $a_f^1$ is a sum of two terms: $a_f^1 = a_{f1}^1 + a_{f2}^1$ where

$$a_{f1}^1 = \sum_{ik} \frac{m_f}{16\pi^2 m_i} Re(K_{ik}L_{ik}^*) F_1 \left( \frac{m_k^2}{m_i^2} \right)$$

(4)

and

$$F_1(x) = \frac{1}{(x-1)^3} (1 - x^2 + 2lnx)$$

(5)

and where

$$a_{f2}^1 = \sum_{ik} \frac{m_f}{96\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) F_2 \left( \frac{m_k^2}{m_i^2} \right)$$

(6)

and

$$F_2(x) = \frac{1}{(x-1)^4} (-x^3 + 6x^2 - 3x - 2 - 6lnx).$$

(7)

Similarly, $a_f^2$ also consists of two terms: $a_f^2 = a_{f1}^2 + a_{f2}^2$ where

$$a_{f1}^2 = -\sum_{ik} \frac{m_f}{16\pi^2 m_i} Re(K_{ik}L_{ik}^*) F_3 \left( \frac{m_k^2}{m_i^2} \right)$$

(8)

and

$$F_3(x) = \frac{1}{(x-1)^3} (3x^2 - 4x + 1 - 2x^2lnx)$$

(9)

and where

$$a_{f2}^2 = \sum_{ik} \frac{m_f}{96\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) F_4 \left( \frac{m_k^2}{m_i^2} \right)$$

(10)

and

$$F_4(x) = \frac{1}{(x-1)^4} (2x^3 + 3x^2 - 6x + 1 - 6x^2lnx).$$

(11)

3 Chargino Contributions with CP Violating Phases

The chargino matrix with CP violating phases is given by

$$M_C = \begin{pmatrix} \tilde{m}_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & |\mu| e^{i\theta_\mu} \end{pmatrix}$$

(12)

This matrix can be diagonalized by a biunitary transformation $U^* M_C V^{-1} = diag(\tilde{m}_{\chi_1^+}, \tilde{m}_{\chi_2^+})$. By looking at the muon-sneutrino-chargino interaction:

$$- \mathcal{L}_{\mu-\tilde{\nu}-\tilde{\chi}_+} = g\tilde{\mu} [V_{11} P_R - \kappa_\mu U_{12}^* P_L] \tilde{\chi}_1^+ \tilde{\nu}$$

$$+ g\tilde{\mu} [V_{21} P_R - \kappa_\mu U_{22}^* P_L] \tilde{\chi}_2^+ \tilde{\nu} + H.c.,$$

(13)
where $\kappa = \frac{m_\mu}{\sqrt{2}M_W \cos \beta}$, we find that the chargino exchange to $a_\mu$ is given by

$$a^{\chi^+}_\mu = a^{21}_\mu + a^{22}_\mu$$  

(14)

where

$$a^{21}_\mu = \frac{m_\mu \alpha_{EM}}{4\pi \sin^2 \theta_W \sqrt{2} M_W \cos \beta} \sum_{i=1}^2 \frac{1}{M_{\chi_i^+}} \text{Re}(U_{i2}^* V_{i1}^*) F_3\left(\frac{M^2_\mu}{M_{\chi_i^+}^2}\right)$$  

(15)

and

$$a^{22}_\mu = \frac{m_\mu^2 \alpha_{EM}}{24\pi \sin^2 \theta_W} \sum_{i=1}^2 \frac{1}{M_{\chi_i^+}^2} \left(\frac{m_\mu^2}{2m_W^2 \cos^2 \beta} |U_{i2}|^2 + |V_{i1}|^2\right) F_4\left(\frac{M^2_\mu}{M_{\chi_i^+}^2}\right).$$  

(16)

The phase which enters here is $\theta_\mu$ through the matrix elements of $U$ and $V$.

4 Neutralino Contributions with CP Violating Phases

The neutralino mass matrix $M_{\chi^0}$ is a complex symmetric matrix and is given by

$$
\begin{pmatrix}
\tilde{m}_1 & 0 & -M_z \sin \theta_W \cos \beta & M_z \sin \theta_W \sin \beta \\
0 & \tilde{m}_2 & M_z \cos \theta_W \cos \beta & -M_z \cos \theta_W \sin \beta \\
-M_z \sin \theta_W \cos \beta & M_z \cos \theta_W \cos \beta & 0 & -|\mu| e^{i\theta_\mu} \\
M_z \sin \theta_W \sin \beta & -M_z \cos \theta_W \sin \beta & -|\mu| e^{i\theta_\mu} & 0
\end{pmatrix}
$$

(17)

The matrix $M_{\chi^0}$ can be diagonalized by the unitary transformation

$$X^T M_{\chi^0} X = \text{diag} (\tilde{m}_{\chi_1^0}, \tilde{m}_{\chi_2^0}, \tilde{m}_{\chi_3^0}, \tilde{m}_{\chi_4^0}).$$

(18)

The smuon $(mass)^2$ matrix is given by

$$M^2_{\tilde{\mu}} = \begin{pmatrix}
M^2_L + m_\mu^2 - M^2_z (\frac{1}{2} - \sin^2 \theta_W) \cos 2\beta & m_\mu (A^*_\mu m_0 - \mu \tan \beta) \\
m_\mu (A_\mu m_0 - \mu^* \tan \beta) & M^2_R + m_\mu^2 - M^2_Z \sin^2 \theta_W \cos 2\beta
\end{pmatrix}$$

(19)

This matrix is hermitian and can be diagonalized by the unitary transformation

$$D^\dagger_{\tilde{\mu}} M^2_{\tilde{\mu}} D_{\tilde{\mu}} = \text{diag}(M^2_{\tilde{\mu}_1}, M^2_{\tilde{\mu}_2})$$

(20)

The muon-smuon-neutralino interaction in the mass diagonal basis is defined by

$$-\mathcal{L}_{\mu-\tilde{\mu}-\tilde{\chi}^0} = \sum_{j=1}^4 \sqrt{2} \tilde{\mu}_j (\alpha_{\mu j} D_{\mu 11} - \gamma_{\mu j} D_{\mu 21}) P_L$$

$$+ (\beta_{\mu j} D_{\mu 11} - \delta_{\mu j} D_{\mu 21}) P_R]_{\chi_j^0} \tilde{\mu}_1 + \sqrt{2} \tilde{\mu}_j (\alpha_{\mu j} D_{\mu 12} - \gamma_{\mu j} D_{\mu 22}) P_L$$

$$+ (\beta_{\mu j} D_{\mu 12} - \delta_{\mu j} D_{\mu 22}) P_R]_{\chi_j^0} \tilde{\mu}_2 + \text{H.c.}$$  

(21)
where $\alpha$, $\beta$, $\gamma$ and $\delta$ are given by

$$\alpha_{\mu j} = \frac{g m_\mu X_{3j}}{2m_W \cos \beta}$$  \hspace{1cm} (22)

$$\beta_{\mu j} = e Q_\mu X_{1j}^* + \frac{g}{\cos \theta_W} X_{2j}' (T_{3\mu} - Q_\mu \sin^2 \theta_W)$$  \hspace{1cm} (23)

$$\gamma_{\mu j} = e Q_\mu X_{1j}' - \frac{g Q_\mu \sin^2 \theta_W}{\cos \theta_W} X_{2j}'$$  \hspace{1cm} (24)

$$\delta_{\mu j} = -\frac{g m_\mu X_{3j}^*}{2m_W \cos \beta}$$  \hspace{1cm} (25)

and where

$$X_{1j}' = X_{1j} \cos \theta_W + X_{2j} \sin \theta_W$$  \hspace{1cm} (26)

$$X_{2j}' = -X_{1j} \sin \theta_W + X_{2j} \cos \theta_W,$$  \hspace{1cm} (27)

The neutralino exchange contribution to $a_\mu$ is given by

$$a_\mu^{\chi_0} = a_{\mu}^{11} + a_{\mu}^{12}$$  \hspace{1cm} (28)

where

$$a_{\mu}^{11} = \frac{m_\mu \alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{j=1}^{4} \sum_{k=1}^{2} \frac{1}{M_{\chi_j}^2} \eta_{\mu j}^k F_1 \left( \frac{M_{\mu j}^2}{M_{\chi_j}^2} \right)$$  \hspace{1cm} (29)

and

$$a_{\mu}^{12} = \frac{m_\mu^2 \alpha_{EM}}{24\pi \sin^2 \theta_W} \sum_{j=1}^{4} \sum_{k=1}^{2} \frac{1}{M_{\chi_j}^2} X_{\mu j}^k F_2 \left( \frac{M_{\mu j}^2}{M_{\chi_j}^2} \right)$$  \hspace{1cm} (30)

Here

$$\eta_{\mu j}^k = -\tan^2 \theta_W \Re(X_{1j}^2 D_{1k}^* D_{2k}) - \tan \theta_W \Re(X_{2j} X_{1j} D_{1k}^* D_{2k})$$

$$+ \frac{m_\mu \tan \theta_W}{M_W \cos \beta} |D_{2k}|^2 \Re(X_{3j} X_{1j}) - \frac{m_\mu \tan \theta_W}{2M_W \cos \beta} |D_{1k}|^2 \Re(X_{3j} X_{1j})$$

$$- \frac{m_\mu}{2M_W \cos \beta} |D_{1k}|^2 \Re(X_{3j} X_{2j}) + \frac{m_\mu^2}{2M_W^2 \cos^2 \beta} \Re(X_{3j}^2 D_{2k}^* D_{1k})$$  \hspace{1cm} (31)

and

$$X_{\mu j}^k = \frac{m_\mu^2}{2M_W^2 \cos^2 \beta} |X_{3j}|^2$$

$$+ \frac{1}{2} \tan^2 \theta_W |X_{1j}|^2 (|D_{1k}|^2 + 4|D_{2k}|^2) + \frac{1}{2} |X_{2j}|^2 |D_{1k}|^2$$

$$+ \tan \theta_W |D_{1k}|^2 \Re(X_{1j} X_{2j}^*)$$

$$+ \frac{m_\mu \tan \theta_W}{M_W \cos \beta} \Re(X_{3j} X_{1j} D_{1k} D_{2k}^*) - \frac{m_\mu}{M_W \cos \beta} \Re(X_{3j} X_{2j} D_{1k} D_{2k}^*)$$  \hspace{1cm} (32)

The matrix elements of $X$ carry the phase of $\mu$ and the matrix elements of $D$ carry both the phase of $\mu$ and the phase of the trilinear parameter $A_\mu$ where $A_\mu$ is the renormalization group evolved value of $A_{\mu_0}$ at the Z-scale.
5 Discussion of Results

It is interesting to consider the supersymmetric limit of our results when the soft susy breaking terms vanish. In this limit Eq.(14) which arises from the chargino exchange gives a contribution which is equal in magnitude and opposite in sign to the contribution from the W exchange. Thus we find that in the supersymmetric limit

\[ a_W^\mu + a_{\chi^+}^\mu = 0 \] (33)

Similarly taking the supersymmetric limit of Eq.(28) we find that the massive modes neutralino exchange contribution is equal in magnitude and opposite in sign to the Z boson exchange contribution so that

\[ a_Z^\mu + a_{\chi^0}^\mu (\text{massive modes}) = 0 \] (34)

This is what one expects on general grounds\[31, 32\] and our explicit evaluations satisfy Eqs.(33) and (34). The proof of Eqs.(33) and (34) is given in Appendix A. A result similar to Eq.(34) holds for the massless modes but its proof requires extension of the results of Sec.2 to include \( m_f \) corrections in the loop integrals. This extension will be discussed elsewhere.

Next we discuss the limit of vanishing CP violating phases. In this limit the unitary matrices \( U \) and \( V \) becomes orthogonal matrices. Using the notation

\[ V^{-1} \rightarrow O_1, U^* \rightarrow O_2^T \] (35)

where \( O_1 \) and \( O_2 \) are orthogonal matrices, the chargino contributions take on the form

\[ a_{\chi^+}^{21} = \frac{m_\mu^{\alpha EM}}{4\pi \sin^2 \theta_W} \frac{m_\mu}{\sqrt{2} m_W \cos \beta} \sum_{i=1}^2 \frac{1}{M_{\chi_i^+}} O_{22i} O_{1i1}^T F_3(\xi_{\nu_i}) \] (36)

and

\[ a_{\chi^+}^{22} = \frac{m_\mu^{2\alpha EM}}{24\pi \sin^2 \theta_W} \sum_{i=1}^2 \frac{1}{M_{\chi_i^+}^2} \left( \frac{m_\mu^2}{2 m_W^2 \cos^2 \beta} (O_{22i})^2 + (O_{1i1}^T)^2 \right) F_4(\xi_{\nu_i}) \] (37)

where \( \xi_{\nu_i} = M_{\mu}^2 / M_{\chi_i^+}^2 \). The neutralino exchange contributions in the CP violating limit can similarly be gotten from Eqs.(28-32) by the replacement

\[ X \rightarrow O, \quad D \rightarrow S \] (38)

where \( O \) and \( S \) are orthogonal matrices. Our results for the chargino and neutralino contributions go to the result of the previous works\[25\] in the vanishing CP phase limit considered above.
6 Analysis of CP Violating Effects

Before discussing the effects of CP violating phases on the supersymmetric contributions to $a_{\mu}$ we summarize briefly the current experimental and theoretical situation regarding $a_{\mu}$. The most accurate determination of $a_{\mu}$ is from the CERN experiment[33] which gives a value of $a_{\mu}^{\exp} = 11659230(84) \times 10^{-10}$ while the Standard Model determination including $\alpha^5$ QED contributions[34], hadronic vacuum polarization[35] and light by light hadronic contributions[36], and the complete two loop standard Model electro-weak contributions[37] is $a_{\mu}^{SM} = 11659162(6.5) \times 10^{-10}$. Here essentially the entire error shown in parenthesis comes from the hadronic sector. It is expected that the new Brookhaven $g_{\mu}$ experiment[26, 38] will improve by a factor of 20 the determination of $a_{\mu}$ over the previous $a_{\mu}$ measurement[33], i.e., the error in the experimental determination of $a_{\mu}$ is expected to go down to $4 \times 10^{-10}$. This means that even with no further reduction in the hadronic error the new $g_{\mu}$ experiment will be able to test the Standard Model electro-weak corrections which contribute an amount[37] $a_{\mu}^{EW}(SM) = 15.1(0.4) \times 10^{-10}$. However, it has been pointed out that the supersymmetric electro-weak corrections can be as large or larger than the Standard Model electro-weak corrections, and thus the new $g_{\mu}$ experiment will also probe supersymmetry[25, 27, 28]. In this context it is important to know how large the CP violating effects are on the supersymmetric electro-weak anomaly.

Previous analyses of $g_{\mu}$ in supersymmetry did not consider the effects of CP violating phases because the effects of such phases were expected to be generally small due to the electric dipole moment (EDM) constraints. As mentioned in Sec.1 in the conventional scenarios the current experimental constraints on the electron EDM ($d_e$) and on the neutron EDM ($d_n$) are satisfied either by the choice of small CP violating phases[3, 4] or by the choice of a heavy mass spectrum of supersymmetric particles[5]. For the first case, the CP violating effects are small because of the smallness of the CP violating phases, while for the second the supersymmetric contribution to $g_{\mu} - 2$ will itself be small compared to the Standard Model result to be of relevance. However, as also pointed out in Sec. 1 with the cancellation mechanism[6] one can satisfy the EDM constraints with large CP violating phases and not too massive a SUSY spectrum and thus it is of relevance to examine the effects of CP violating phases on $a_{\mu}$.

For the case of the electron EDM the cancellations occur between the chargino and the neutralino exchange contributions while for the case of the neutron EDM
the cancellations can occur in a two step process. Thus for the neutron case, the
EDM receives contributions from the electric dipole, the chromo-electric dipole
and the purely gluonic dimension six operators. For the electric and the chromo-
electric dipole operators cancellations can occur between the chargino, the gluino
and the neutralino exchange contributions. There is, however, the possibility of
a further cancellation, and that is among the electric dipole, the chromo-electric
dipole and the purely gluonic contributions. Recently, it has been pointed out that
in addition to the above contributions certain two loop graphs may also contribute
significantly in some regions of the parameter space\[39\]. In our analysis we have
included the effects of these contributions as well. However, we find that the effect
of these terms is relatively small compared to the other contributions. The regions
of interest in the parameter space are those where the cancellations among different
components happen simultaneously for the case of the electron EDM and of the
neutron EDM so as to satisfy the experimental lower limits, which for the neutron
is\[1]\)

\[ |d_n| < 6.3 \times 10^{-26} \] (39)

and for the electron is\[2]\)

\[ |d_e| < 4.3 \times 10^{-27} \] (40)

We discuss now the size of the CP violating effects on \(a^\mu_{SSY}\). In Fig.2 we
exhibit the effect of the variation of the CP violating phase \(\theta_{\mu_0}\) on \(a^\mu_{SSY}\), without
the imposition of the EDM constraint, as a function of \(\theta_{\mu_0}\). The values of the
other parameters (\(m_0, m_\tilde{g}, \tan \beta, \alpha_{A_0}\)) for the curves (1)-(5) can be read off from
Table 1 for the cases (1) - (5). We find that the effect of the CP violating phase
is very substantial. A similar analysis of the effects of the variation of the CP
violating phase \(\alpha_{A_0}\) on \(a^\mu_{SSY}\), also without the imposition of the EDM constraint,
is given in Fig.3. Again the value of the parameters other than \(\alpha_{A_0}\) for the curves
labeled (1)-(5) can be read off from Table 1. Here again the effects of the CP
violating phase \(\alpha_{A_0}\) are found to be quite substantial although not as large as
those from \(\theta_{\mu_0}\). The reason for this discrepancy is easily understood. In the
region of the parameter space considered the chargino contribution is large and
this contribution is independent of \(\alpha_{A_0}\) since \(\alpha_{A_0}\) does not enter in the chargino
mass matrix. Thus the \(\alpha_{A_0}\) dependence enters only via the smuon mass matrix,
while \(\theta_{\mu_0}\) enters via all mass matrices.

Inclusion of the EDM constraint puts stringent constraints on the parameter
space of mSUGRA. As an illustration we give in Fig. 4 the plot of the EDM of the electron and for the neutron as a function of $\alpha_{A_0}$ for curves (1) and (3) of Fig. 3. The figure illustrates the simultaneous cancellation occurring for the electron and the neutron EDM in narrow regions of $\alpha_{A_0}$ and in these regions the experimental EDM constraints can be satisfied. We note the appearance of two cancellation minima in the cases considered. These double minima reveal the strong dependence on the phases of the various terms that contribute to the EDMs. The effects of CP violating phases on $a_\mu$ can be significant in these domains. In Table 1 we give a set of illustrative points where a simultaneous cancellation in the electron and the neutron EDM occurs. The size of the effects of CP violating phases on $a_\mu$ can be seen from the values of $a_\mu$ at these points and the corresponding four CP conserving cases in Table 2. A comparison of the results of Tables 1 and 2 with those of Figs. 2 and 3 shows that with the inclusion of the EDM constraints the CP violating effects are much reduced for the points chosen here. However, even with the inclusion of the EDM constraints the CP effects on $a_\mu$ can still be quite substantial as a comparison of Tables 1 and 2 exhibits. Inclusion of more than two phases makes the satisfaction of the EDM constraints much easier and detailed analyses show that there are significant regions of the parameter space where the CP violating phases are large and cancellations occur to render the electron and the neutron EDM in conformity with experiment[7, 9]. Such regions are of considerable interest in the investigations of SUSY phenomena at low energy. The effects of CP violating phases in these regions could be substantial. However, a quantitative discussion of these effects requires inclusion of nonuniversal effects which are outside the framework of mSUGRA model discussed here.

| $\theta_{\mu_0}$ | $\alpha_{A_0}$ | $d_\mu(10^{-26}\text{ecm})$ | $d_e(10^{-27}\text{ecm})$ | $a_\mu(\theta_{\mu_0},\alpha_{A})(10^{-9})$ |
|-----------------|-----------------|------------------------|------------------------|----------------------------------|
| (1) 3.108       | -0.2            | 5.4                    | -2.7                   | -3.6                             |
| (2) 3.08        | -0.45           | 4.86                   | -4.26                  | -2.5                             |
| (3) 3.02        | -1.0            | -3.6                   | -3.1                   | -1.7                             |
| (4) 3.1         | -0.2            | -4.9                   | -0.93                  | -1.1                             |
| (5) 3.02        | -1.0            | -5.0                   | 1.1                    | -0.78                            |

Table caption: The other parameters corresponding to the cases (1)-(5) are: (1) $m_0=60, m_{1/2}=123, \tan\beta=3.5, |A_0|=5.45$, (2) $m_0=65, m_{1/2}=119, \tan\beta=2.6, |A_0|=2.93$, (3) $m_0=80, m_{1/2}=147, \tan\beta=2.6, |A_0|=2.93$, (4) $m_0=120, m_{1/2}=228, \tan\beta=3.5, |A_0|=5.47$, (5) $m_0=120, m_{1/2}=220, \tan\beta=2.6, |A_0|=2.93$, where all masses are in GeV units.
Table 2:

|   | $a_\mu(0,0)(10^{-9})$ | $a_\mu(0,\pi)(10^{-9})$ | $a_\mu(\pi,0)(10^{-9})$ | $a_\mu(\pi,\pi)(10^{-9})$ |
|---|------------------------|--------------------------|--------------------------|--------------------------|
| (1) | 3.25                   | 4.18                     | -3.5                     | -2.6                     |
| (2) | 2.49                   | 3.1                      | -2.6                     | -1.98                    |
| (3) | 1.5                    | 1.86                     | -1.9                     | -1.34                    |
| (4) | 0.75                   | 1.12                     | -1.13                    | -0.75                    |
| (5) | 0.62                   | 0.82                     | -0.89                    | -0.6                     |

Table caption: The values of $a_\mu$ for four CP conserving cases with all other parameters the same as in the corresponding cases in Table 1.

7 Conclusions

In this paper we have derived the general one loop formula for the effects of CP violating phases on the anomalous magnetic moment of a fermion. We then specialized our analysis to the case of the calculation of CP violating effects on the supersymmetric muon anomaly. Here the contributions arise from the one loop chargino and neutralino exchange diagrams. The numerical analysis of the CP violating effects is strongly constrained by the experimental EDM constraints on the electron[2] and on the neutron[1]. Our analysis including these constraints shows that the size of the CP violating effects is strongly dependent on the region of the parameter space one is in and that the CP violating phases can produce substantial affects on the supersymmetric electro-weak contribution. We also find that the supersymmetric contribution to the muon anomaly in the presence of large CP violating phases consistent with the EDM constraints can be as large or larger than the Standard Model electro-weak contribution. These results are of interest in view of the new BNL muon g-2 experiment which will improve the accuracy of the muon g-2 measurement by a factor of 20 and test the electro-weak correction to $g_\mu - 2$.

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Appendix A: The Supersymmetric Limit

In this appendix we exhibit the supersymmetric limit of the chargino exchange contribution. The supersymmetric limit corresponds to $M_\tilde{\nu} = 0$, $\tilde{m}_i = 0$ (1,2),
$\tan \beta = 1$ and $\mu = 0$. In this limit $F_3(0) = -1$, $F_4(0) = 1$, and the unitary matrices $U$ and $V$ take the values

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

(41)

where $U^* M_C V^{-1} = \text{diag}(M_W, M_W)$. In this limit $a_{\mu}^{21}$, $a_{\mu}^{22}$ and the total chargino contribution $a_{\mu}^{\chi^+}$ are given by

$$a_{\mu}^{21} = -\frac{\alpha_{EM}}{4\pi \sin^2 \theta_W} \frac{m_{\mu}^2}{M_W^2}, \quad a_{\mu}^{22} = -\frac{\alpha_{EM}}{24\pi \sin^2 \theta_W} \frac{m_{\mu}^2}{M_W^2}$$

(42)

$$a_{\mu}^{\chi^+} = -\frac{5\alpha_{EM}}{24\pi \sin^2 \theta_W} \frac{m_{\mu}^2}{M_W^2}$$

(43)

The result of Eq.(43) is to be compared with the contribution arising from the exchange of the W boson$[29]$

$$a_{\mu}^{W} = \frac{5m_{\mu}^2G_F}{12\pi^2\sqrt{2}}$$

(44)

Using $G_F = \pi \alpha_{em}/(M_W^2\sqrt{2}\sin^2 \theta_W)$ we then find that the sum of the chargino and the W exchange contributions vanish in the supersymmetric limit.

Next we consider the neutralino exchange contribution to $a_{\mu}^{\chi^0}$ in the supersymmetric limit. In this limit two of the eigen-values of the neutralino mass matrix are zero and the other two are $\pm M_Z$. However, we choose the unitary transformation $X$ so that the non-vanishing eigen-values are all positive definite, i.e.,

$$X^T M_{\chi^0} X = \text{diag}(0,0,M_Z,M_Z)$$

(45)

In this case the unitary matrix $X$ takes on the form

$$
\begin{pmatrix}
\alpha & \beta \\
\alpha \tan \theta_W & \beta \tan \theta_W \\
\alpha & \frac{1}{2}\beta \sec^2 \theta_W \\
\alpha & \frac{1}{2}\beta \sec^2 \theta_W \\
\end{pmatrix}
\begin{pmatrix}
\frac{\sin \theta_W}{\sqrt{2}} \\
\frac{\sin \theta_W}{\sqrt{2}} \\
\frac{-\cos \theta_W}{\sqrt{2}} \\
\frac{-\cos \theta_W}{\sqrt{2}} \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta_W \\
\sin \theta_W \\
\cos \theta_W \\
\cos \theta_W \\
\end{pmatrix}
- \frac{i\sin \theta_W}{\sqrt{2}}
\cos \theta_W \\
\cos \theta_W \\
\cos \theta_W \\
\cos \theta_W \\
\end{pmatrix}
$$

(46)

where

$$\alpha = \frac{1}{\sqrt{3 + \tan^2 \theta_W}}, \quad \beta = \frac{1}{\sqrt{1 + \tan^2 \theta_W + \frac{1}{2}\sec^4 \theta_W}}$$

(47)

The appearance of $i(=\sqrt{-1})$ in the last column in $X$ is to guarantee that the eigenvalues are all positive definite. In the supersymmetric limit $\eta_{\mu j}^k$ take on the following form

$$\eta_{\mu j}^1 = -\frac{m_{\mu}}{\sqrt{2}M_W} \text{Re}(X_{3j}X_{2j}) - \frac{m_{\mu}}{\sqrt{2}M_W} \tan \theta_W \text{Re}(X_{3j}X_{1j})$$

(48)
and
\[ \eta_{\mu ij}^2 = \frac{\sqrt{2} m_\mu}{M_W} \tan \theta_W \text{Re}(X_{3j}X_{1j}) \]  

(49)

while \( X_{\mu ij}^k \) take on the form
\[ X_{\mu ij}^1 = \frac{m_\mu^2}{M_W^2} |X_{3j}|^2 + \frac{1}{2} \tan^2 \theta_W |X_{1j}|^2 + \frac{1}{2} |X_{2j}|^2 + \tan \theta_W \text{Re}(X_{1j}X_{2j}^*) \]  

(50)

\[ X_{\mu ij}^2 = \frac{m_\mu^2}{M_W^2} |X_{3j}|^2 + 2 \tan^2 \theta_W |X_{1j}|^2 \]  

(51)

Using the above and the limit \( F_1(0) = -1, F_2(0) = -2 \) and by ignoring the terms of higher order of \( m_\mu \), one finds that \( a_{\mu 11} \) and \( a_{\mu 12} \) simplify as follows:

\[ a_{\mu 11} = \sum_{j=3}^{4} \frac{m_\mu^2 \alpha_{EM}}{4 \sqrt{2} \pi \sin^2 \theta_W M_W M_Z} (\text{Re}(X_{3j}X_{2j})+\tan \theta_W \text{Re}(X_{3j}X_{1j})-2 \tan \theta_W \text{Re}(X_{3j}X_{1j})) \]  

(52)

\[ a_{\mu 12} = -2 \sum_{j=3}^{4} \frac{m_\mu^2 \alpha_{EM}}{48 \pi \sin^2 \theta_W M_Z^2} (5 \tan \theta_W |X_{1j}|^2 + |X_{2j}|^2 + 2 \tan \theta_W \text{Re}(X_{1j}X_{2j}^*)) \]  

(53)

Substitution of the explicit form of X from Eq.(46) in Eqs.(52) and (53) gives

\[ a_{\mu 11} = \frac{m_\mu^2 G_F}{2 \sqrt{2} \pi^2} \left( \frac{1}{2} \right) \]  

(54)

\[ a_{\mu 12} = -\frac{m_\mu^2 G_F}{2 \sqrt{2} \pi^2} \left( \frac{4}{3} \sin^4 \theta_W - \frac{2}{3} \sin^2 \theta_W + \frac{1}{6} \right) \]  

(55)

and

\[ a_{\mu 0} = -\frac{m_\mu^2 G_F}{2 \sqrt{2} \pi^2} \left( \frac{4}{3} \sin^4 \theta_W - \frac{2}{3} \sin^2 \theta_W - \frac{1}{3} \right) \]  

(56)

The result of Eq.(56) is to be compared to the Standard Model Z exchange contribution

\[ a_{\mu}^Z = \frac{m_\mu^2 G_F}{2 \sqrt{2} \pi^2} \left( -\frac{5}{12} + \frac{4}{3} \sin^2 \theta_W - \frac{1}{4} \right)^2 \]  

(57)

Thus one finds that in the supersymmetric limit the sum of the neutralino and the Z boson exchange contributions vanish.
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Figure 1: The one loop contribution to $g_\mu - 2$ from (a) neutralino exchange, and (b) chargino exchange diagrams.

Figure 2: Plot of $a_{\mu}^{SUSY}$ as a function of the CP violating phase $\theta_{\mu_0}$. The values of the other parameters for the curves (1)-(5) correspond to the cases (1)-(5) in Table 1.
Figure 3: Plot of $a^{SUSY}_{\mu}$ as a function of the CP violating phase $\alpha_{A_0}$. The values of the other parameters for the curves (1)-(5) correspond to the cases (1)-(5) in Table 1.

Figure 4: Exhibition of the dependence of the $|EDM|$ of the electron (solid) and the neutron (dashed) and the cancellation as a function of $\alpha_{A_0}$. The curves with minima to the extreme left and the extreme right have other parameters corresponding to case (1) of Table 1, while the curves with two minima in the middle have other parameters corresponding to case (3) of Table 1.