Constraints on $Wtb$ anomalous coupling with $B \rightarrow X_s \gamma$ and $B_s \rightarrow \mu^+ \mu^*$

Zhao-Hua Xiong$^1$, Lian Zhou$^1$

$^1$ Institute of Theoretical Physics, Beijing University of Technology, Beijing 100124, China

Abstract

We calculate the amplitudes of $b \rightarrow s$ transition in extension of the Standard Model with $Wtb$ anomalous couplings. We found that i) there exist the Ward identity violating terms in effective vertex of $b \rightarrow s \gamma$. The terms, which come from the tensor parts of $Wtb$ anomalies, and can be canceled exactly by introducing corresponding $Wtb \gamma$ interactions; ii) $Br(B_s \rightarrow \mu^+ \mu^-)$ provides unique information on $\delta v_L$ which is set to zero in top decay experiments, and stringent bounds on $v_R, g_L$ by $Br(B \rightarrow X_s \gamma)$ are obtained.

Keywords: $Wtb$ Anomalous Coupling; Wilson Coefficients; Rare B Processes

PACS: 12.60.Fr, 14.40.Nd, 14.65.Ha

1 Introduction

The standard model (SM) is a very successful model in describing physics below the Fermi scale and is in good agreement with the most experiment data, especially the last particle predicted in the SM has been discovered at LHC[1, 2]. And yet, the completion of the SM particle spectrum does not mean the completion of the SM itself until all the SM interaction forces could be firmly measured. It is universally accepted that there must be something more than what has been so brilliantly conceived and verified.

One important issue is searching the new physics (NP) signatures directly, for example, probing $Wtb$ couplings in top quark decay. With its mass around the electroweak symmetry breaking (EWSB) scale, the top quark is believed to play an important role to connect the SM and new physics. A first glance at the top decay has been given through the study of $W$ helicity fractions and related observables[3]. Production cross sections have been measured both for $tt$ pairs[4, 5] and for single top quarks[6]. The lastest analysis for $Wtb$ anomalous couplings by top decay at LHC has presented in [7]. Another important issue is searching the new physics signatures indirectly, i.e., probing $Wtb$ anomalous couplings at some of which derive from flavour changing neutral-current processes (FCNC), such as rare B processes. The new B factories, BELLE and LHCb, are providing us now with more and more high statistics data, which will carry FCNC tests to the next precision level. Measurements of rare B meson decays such as $B \rightarrow X_s \gamma, B_s \rightarrow \mu^+ \mu^-$[8], are likely to provide sensitive tests of the SM [9, 10]. These rare B decays have been studied extremely at the leading logarithm order[11], high order in the SM[12] and various new physics models [13, 14, 15].

Currently, most works on anomalous $Wtb$ couplings are probing NP directly in processes of top quark production/decay[9, 10], only a few authors have studied virtual effect of the anomalous $Wtb$ couplings on some rare decays through loop-level[10, 17]. In Ref.[10], the authors calculated contribution of the anomalous couplings to $b \rightarrow s \gamma$ and presented some constraints on four parameters, but they did not presented a proof that the effective vertex of $b \rightarrow s \gamma$ satisfies the Ward identity. Ref.[17] only considered the $V + A$ contributions of $Wts$ to $B_s - \bar{B}_s$ mixing. The contribution is just proportional to the anomalous coupling squared, which is hard to present a strong constraints on the parameters.

In this paper, we will evolute the effect of the $Wtb$ anomalous couplings on the rare B decays. We will present a detailed one-loop correction for the $b \rightarrow s \gamma$, including contributions from all anomalous couplings related to the third generation quarks such as $tt \gamma, \bar{b}b \gamma$, as well as $Wtb \gamma$. We will prove that with effect of new $Wtb \gamma$ anomalous

*Supported by National Natural Science Foundation of China (NFSC No. 11775012)
couplings taken into account, the Ward identity violating terms in the effective vertex with $Wtb$ anomalies cancel exactly, and this is independent of the unitarity of the quark mixing matrix. Then we obtain the Wilson coefficients $C_i$ ($i = 7$ to 10) for branching ratios of $B \rightarrow X_s \gamma$ and $B_s \rightarrow \mu^+ \mu^-$. We also present some stringent constraints on the four anomalous couplings.

The work is organized as follows. In Section 2, we will render a detailed one loop calculation for $b \rightarrow s \gamma$, proving the effective vertex satisfies the Ward identity. And then we obtain the Wilson coefficients from the effective vertices of $b \rightarrow s \gamma$ and $b \rightarrow s \ell^+ \ell^-$. In Section 3, we express the branching ratios of $B \rightarrow X_s \gamma$ and $B_s \rightarrow \mu^+ \mu^-$ in terms of the $Wtb$ anomalous couplings, and present some numerical bounds for the anomalous couplings. Some inputs are presented in the Appendix.

## 2 Amplitudes of $b \rightarrow s$ transition

### 2.1 Effective vertices of $b \rightarrow s \gamma$

Before going to detailed calculation, let us start with the most general $Wtb$ vertex:

$$L_{Wtb} = -\frac{g V_{tb}^*}{\sqrt{2}} [\gamma \mu (v_L L + v_R R) + ig_{\mu\nu} k^\nu (g_L L + g_R R)] \frac{i}{m_W} W^- \mu \tag{1}\]$$

where $v_L = 1 + \delta v_L$. In the SM, $\delta v_L, v_R, g_L, g_R$ are zero. $L, R = (1 \pm \gamma_5)/2$, and $k$ is the W boson momentum. In the phenomenology, new physics effects can be parameterized in terms of an effective field theory approach. Then the effective vertex $\Gamma_\mu$ in amplitude $A(b \rightarrow s \gamma)$ has form as:

$$\Gamma_\mu = e \frac{g^2}{32 \pi^2} V_{tb} V_{ts}^* [\gamma \mu LF_0 + m_b^2 \gamma \mu LF_1 + \frac{g^2}{m_W^2} \gamma_\mu q_{\mu} LF_2 + \frac{im_b \sigma_{\mu\nu} q^{\nu}}{m_W^2} LF_3] \tag{2}\]$$

where $q$ is the photon momentum. It is clear that the first two terms donate the Ward identity violating terms. Note differing from the top decay which occurs at tree level, $b \rightarrow s$ processes occur at loop level. In model we research the couplings of three up-type quark with $W$ boson are not universal, only top quark in loop contribution to factors $F_3$. Thus, the constants and divergent terms in $F_j$, which will disappear due to CKM unitarity in the SM, may remain. The divergence will be assumed canceled by heavy particles in the model. Furthermore, whether the Ward identity for $b \rightarrow s V$ ($V = g, \gamma$) vertex are guaranteed needs to be checked.

| $v_L$ | $i = 1$ | $-\frac{1-\delta}{2} + 2B_0^0 - 3B_1^0 - \frac{1}{m_W^2} B_2^0 + \frac{B_3^0}{m_W^2}$ | $+ A(m_t)$ | $\frac{3}{2} - \frac{1-\delta}{2} (1 + C_{24}^0) + 2m_t^2 C_{0}^0 + \frac{\bar{B}_{0}^{2}}{m_W^2}$ | $\frac{3 - \delta}{2} - \frac{3}{2}(2 - \delta_1) \bar{C}_{24}^{0}$ |
|---|---|---|---|---|---|
| $g_R$ | $1$ | $-1 + 3B_1^0$ | $2$ | $1 - \frac{1}{2} C_{24}^{0}$ | $3 - \frac{1-\delta}{2} - \frac{3}{2} \bar{C}_{24}^{0} + \frac{3 \bar{B}_{0}^{0}}{4 m_W^{2}}$ | $3 - \delta_1 + 3 \bar{B}_{0}^{0} - \frac{3 \bar{B}_{0}^{2}}{4 m_W^{2}}$ |

It is well known, at matching scale $\mu_W$ the factors $F_j$ are functions of $\delta_t = m_t^2/m_W^2$ and can be written as follows:

$$F_j = v_L F_j^{v_L} + v_R \frac{m_t}{m_b} F_j^{v_R} + g_L \frac{m_W}{m_b} F_j^{g_L} + g_R \frac{m_t}{m_W} F_j^{g_R} \tag{3}\]$$

The task left to us is deriving these factors, then obtain the Wilson coefficients. We use unitary gauge and take naive dimension renormalization (NDR) scheme in calculation. The factors $F_j$ receive contributions from Feynman
In B physics the loop functions can be expanded order by order as diagrams of 1) self-energy; 2) vertex with one W in loop; 3) vertex with two W bosons; 4) vertex with Wtb coupling and 5) vertex with ttγ anomalies, as displayed in Fig. 1. We express each type Feynman diagram contributions to $F_j$, denoted by $F_j^i$, in terms of loop functions $A$, $B$, $C$ and list them in Table 1 to Table 3. Then $F_j$ are obtained as

$$F_j = e_d F_j^1 + e_u F_j^2 + F_j^3 + F_j^4 + F_j^5,$$

(4)

where $e_u = 2/3$, $e_d = -1/3$ stand for charge numbers of quarks. To trace our calculation easily, we present defination for the loop functions as follows:

$$B_\mu(p; m_1, m_2) = p_\mu B_1,$$

$$C_\mu(p, p'; m_1, m_2, m_3) = C_{11} p_\mu + C_{12} p'_\mu,$$

$$C_{\mu\nu}(p, p'; m_1, m_2, m_3) = C_{21} p_\mu p_\nu + C_{22} p'_\mu p'_\nu + C_{23}(p_\mu p'_\nu + p_\nu p'_\mu) + \frac{g_{\mu\nu}}{4} C_{24}$$

(5)

In B physics the loop functions can be expanded order by order as

$$B(p; m_1, m_2) = B^0 + \frac{p^2}{m_W^2} B^1 + \cdots,$$

$$C(p_1, p_2; m_1, m_2, m_3) = C^0 + \frac{p^2}{m_W^2} C^{1.1} + \frac{2p_1 p_2}{m_W^2} C^{1.2} + \frac{p^2}{m_W^2} C^{1.3} + \cdots,$$

(6)

with functions $B^\alpha$, $C^{\mu, i}$ being independent of momenta. The definitions and corresponding expansions can be found in Ref. [18].

Table 2: Factors $F_1^i$ for $b \to s\gamma$

| $v_L$ | $i$ | $F_1^i$ |
|------|-----|---------|
| $v_L$ | $i = 1$ | $-\frac{1}{6} + 2B_1^0 - 3B_1^1 + B_2^0 - \frac{1}{3} B_2^1$ |
| $v_L$ | 2 | $\tilde{B}_1^0 - \tilde{B}_1^0 - \frac{3}{2}(\tilde{C}_{11}^{1.1} + \tilde{C}_{12}^{1.2}) + 2m_1^2 (\tilde{C}_{0}^{1.1} + \tilde{C}_{0}^{1.2}) + m_2^2 (3\tilde{C}_{23}^{0} - 2\tilde{C}_{11}^{0})$ |
| $v_L$ | 3 | $\tilde{B}_0^0 - m_1^2 [(4 - 3\delta_1)\tilde{C}_{11}^{1.1} - (12 - 3\delta_1)\tilde{C}_{23}^{0}] - \frac{1}{3} \tilde{C}_{0}^{24} + \frac{3(2 - \delta_1)}{2} (\tilde{C}_{24}^{1.1} + \tilde{C}_{24}^{1.2})$ |
| $v_R$ | 1 | $-\frac{1}{6} - B_1^0 + \frac{B_1^1 + B_2^1}{m_W^2}$ |
| $v_R$ | 2 | $-\frac{1}{2} - \tilde{B}_2^0 + \tilde{C}_{24}^0 + (6\tilde{C}_{11}^0 - 4\tilde{C}_{0}^0) m_2^2$ |
| $v_R$ | 3 | $-4\tilde{C}_{24}^0 + 4\tilde{C}_{0}^0 - (8 - 2\delta_1)\tilde{C}_{11}^0$ |
| $g_L$ | 1 | $-3 (B_1^0 - B_2^0) - \frac{1}{2} + \frac{3m_2^2}{2m_W^2}$ |
| $g_L$ | 2 | $- \frac{m_1^2}{2m_W^2} [\tilde{C}_{11}^0 - 2(1 + 2\delta_1)\tilde{C}_{0}^1] + \frac{3}{2} \tilde{C}_{24}^0$ |
| $g_L$ | 3 | $\frac{1}{2} \frac{B_1^1 - B_2^1}{2} - \tilde{C}_{24}^0 + m_2^2 (6\tilde{C}_{11}^0 - 3\tilde{C}_{0}^0)$ |
| $g_R$ | 1 | $3B_1^1$ |
| $g_R$ | 2 | $3m_2^2 (\tilde{C}_{11}^0 - 3\tilde{C}_{23}^0 - \frac{3}{2}(\tilde{C}_{24}^{1.1} + \tilde{C}_{24}^{1.2})$ |
| $g_R$ | 3 | $-3m_2^2 (\tilde{C}_{0}^0 - 3\tilde{C}_{11}^0 + 3\tilde{C}_{23}^0) - \frac{3}{2}(\tilde{C}_{24}^{1.1} + \tilde{C}_{24}^{1.2})$ |

It is time to check our results by Ward identity of $b \to s g$ and $b \to s\gamma$, which implies $F_0$ and $F_1$ in (2) should be zero. The decay $b \to s g$ is governed by operator $O_8$ in next Subsection and also contribution to $b \to s\gamma$ as QCD correction included. Considering the strong interactions with quarks are independent of quark charges, using the analytic formulae presented in Table 1 to Table 3 we find $F_0^1 + F_0^2 = F_1^1 + F_1^2 = 0$ as expected. However, this is not the case in $b \to s\gamma$. The effective vertex of $b \to s\gamma$ receives contributions not only from diagrams to $b \to s g$ with $g$ replaced by $\gamma$, also diagram with two W bosons in loop. We find that

$$e_d F_0^1 + e_u F_0^2 + F_0^3 = 0$$

(7)
is not satisfied for $g_R$ term, and

$$e_dF^1_d + e_uF^2_1 + F^3_1 = 0$$  \hspace{1cm} (8)$$
is not satisfied for the $g_L$ term. This implies that without the new anomalous interactions introduced as extracting $Wtb$ anomalous couplings in top decay, the effective vertex of $b \rightarrow s\gamma$ does not satisfy the Ward identity. It is obvious, the source of the new anomalous couplings should the same as $g_L$ and $g_R$. We expect that new introduced contributions can cancel part contributions to $g_L, g_R$ terms.

In this work, we follow Ref.[10] where $Wtb\gamma$ anomalous couplings are always along with the tensor parts of $Wtb$ interactions. They have the origin from dimension-six operators:

$$Q_{LRt} = \bar{q}_L\sigma^{\mu\nu}\tau^a t_R\tilde{\phi}W^a_{\mu\nu} + h.c.,$$  \hspace{1cm} (9a)$$
$$Q_{LRb} = \bar{q}_L'\sigma^{\mu\nu}\tau^a b_R\phi W^a_{\mu\nu} + h.c.$$  \hspace{1cm} (9b)$$
Here $\phi$ denotes the Higgs doublet, $\tilde{\phi} = i\tau^2\phi^*$, and $W^a_{\mu\nu}$ is the field strength and $q_L = (t_L, V^*_tb_L)^T, q'_L = (V^*_tb_L, b_L)^T$.

Note that for consistency, in this work we do not consider $Wts, Wcb, Wub$ anomalies, but take $tt\gamma$ and $bb\gamma$ anomalies into account. The additional anomalous interactions are given by

$$\delta\mathcal{L} = -\frac{ge}{\sqrt{2m_W}}iV^*_tb\sigma^{\mu\nu}(g_RL + g_LR)bW^+_{\nu}\epsilon_\mu + \frac{1}{2}i\frac{e}{m_W}\bar{g}_Rt\epsilon_\mu - \frac{1}{2}i\frac{e}{m_W}\bar{g}_Lb\epsilon_\mu,$$  \hspace{1cm} (10)$$
where $\epsilon$ is the photon polarization vector. Our treatment is some different from Ref.[10]. We will discuss more in next subsection.

Figure 1: The Feynman diagrams of $b \rightarrow s$ transition. (a) Self-energy; (b) Vertices with one loop with one and two $W$ bosons; (c) Vertex with $Wtb\gamma$ anomalous couplings; (d) Vertex with $tt\gamma$ anomalous couplings and Box diagram

With the Feynman rules given in above, we find the $bb\gamma$ anomalous has no contribution to $b \rightarrow s\gamma$. The Feynman diagrams of $Wtb\gamma$ and $tt\gamma$ contribution to $b \rightarrow s\gamma$ are plotted in Fig. 1. The corresponding factors $F_j$ are also listed in Table H to Table L. We find $F^3_0 = F^4_1 = 0$. Thus, the Ward identity in $b \rightarrow s\gamma$ requires that the tensor parts of $Wtb$ anomalies should be accompanied with the $Wtb\gamma$ anomalies. Although the $tt\gamma$ anomalous couplings are not related to the Ward identity, for consistency its contribution to $g_L$ part of $b \rightarrow s\gamma$ also should be included.

2.2 Effective vertex of $b \rightarrow s\ell^+\ell^-$

In this subsection, we calculate the effective vertex of $b \rightarrow s\ell^+\ell^-$. The vertex receives contribute for $F_3$ which is related to the operator $O_7$, $F_2$ which is related to the operator $O_3$ in vertex of $b \rightarrow s\gamma$, i.e., off-shell $\gamma$-penguin, and the $Z$-penguin, $F^Z$, which can be obtained by replacement $\gamma$ with $Z$ boson in $b \rightarrow s\gamma$ diagrams. Using the relation $F^1_d = -F^2_u = F^3_1 + F^4_1$ we obtain

$$F^{Z,\nu\ell} = \overline{C}^0_0 m_t^2 - \frac{3\delta_1}{4}(\overline{C}^0_24 - 1),$$  \hspace{1cm} (11)$$

$$F^{Z,\gamma\ell} = -\frac{3}{4}\overline{C}^0_24^2 + \frac{5}{4}.$$  \hspace{1cm} (11)$$
The effective vertex of $b \to s \ell^+ \ell^-$ also receives contribution from box diagrams, as displayed in Fig. 1. The contributions denoted by $F^B$ read:

$$F^{B,s_L} = m^2_W c^0_0 - \frac{7}{4} c^0_{24} + \frac{\bar{B}^0_{22}}{4m^2_W} + 1,$$

$$F^{B,s_R} = \frac{3}{2} \bar{c}^0_{24} - 1. \quad (12)$$

At the end of this Section, we obtain the Wilson coefficients of operators at matching scale by comparing the amplitudes of $b \to s \gamma$ and $b \to s \ell^+ \ell^-$ with the effective Hamiltonian:

$$\mathcal{H}^{eff}(b \to s) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^{\ast} \sum_{i=1}^{10} C_i(\mu) O_i. \quad (13)$$

The operators in (13) related to the factors $F_j$ directly are defined as follows:

$$O_7 = \frac{e}{16\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} Rb_i F_{\mu\nu},$$

$$O_8 = \frac{g_3}{16\pi^2} \bar{s}_i \sigma^{\mu\nu} R T^a_{ij} b_j G_{\mu\nu}^a,$$

$$O_9 = \frac{e}{16\pi^2} (\bar{s}_i \gamma^\mu Lb_i)(\bar{\ell}_i \gamma_\mu \ell),$$

$$O_{10} = \frac{e}{16\pi^2} (\bar{s}_i \gamma^\mu Lb_i)(\bar{\ell}_i \gamma_5 \gamma_\mu \ell). \quad (14)$$

where $F_{\mu\nu}$ and $G_{\mu\nu}$ are the electromagnetic tensor and strong tensor, respectively, and $T^a$ stand for the $SU(3)$ generators. Note coefficients of the four quark operators $O_i$ (i=1 to 6) are the the same as those in the SM, other Wilson coefficients at matching scale are written in terms of factor $F_j$ as:

$$C_7(\mu_W) = -\frac{1}{2} F_3,$$

$$C_8(\mu_W) = -\frac{1}{2} F^2_3,$$

$$C_9(\mu_W) = \frac{F^Z + F^B}{4 \sin^2 \theta_W} - F^Z + F_2,$$

$$C_{10}(\mu_W) = \frac{F^B + F^Z}{4 \sin^2 \theta_W} \quad (15)$$

Note in the SM, $C_9(\mu_W)$ has a constant 4/9 which is come from the correction of the four-quark operators [11].
we obtained by using unitary gauge, the formulae for $B$ are not the same as those calculated in Feynman gauge even the constants deleted by the CKM unitarity. In this gauge except for constants and divergent terms. They are presented as follows:

For comparison with the SM results, we first present the Wilson coefficients related to $v_L$ at matching scale for $b \to s\gamma$:

$$C^\nu_L(\mu_W) = \frac{3\delta^3_1 - 2\delta^2_1}{4(\delta_1 - 1)^4} \ln \delta_1 + \frac{22\delta^3_1 - 153\delta^2_1 + 159\delta_1 - 46}{72(\delta_1 - 1)^3}$$

$$C^\nu_S(\mu_W) = -\frac{3\delta^2_1}{4(\delta_1 - 1)^4} \ln \delta_1 + \frac{5\delta^3_1 - 9\delta^2_1 + 30\delta_1 - 8}{24(\delta_1 - 1)^3}$$

(16)

It is clear that

$$C^S_7 = C^S_8 = C^\nu_L - \frac{23}{36}$$

where the constants in $C^\nu_L$ in the SM disappear due to the CKM unitarity. Consider the $C^S_7(m_W) = -0.19$, one can see the constant is very important in phenomenological analysis.

When $C_9$ and $C_{10}$ related to $v_L$ are compared with the SM values for $b \to s\ell^+\ell^-$, we stress that the factors $F_j$ we obtained by using unitary gauge, the formulae for $B = -F^B/4$, $C = F^Z/4$, $D = -F_2$ used in expressing $C^S_{9,10}$ are not the same as those calculated in Feynman gauge even the constants deleted by the CKM unitarity. In this sense, comparing our calculation with $B^{SM}$, $C^{SM}$, $D^{SM}$ separately has no meaning since they are gauge dependent. However, from Eq.(15) one can see that $C - B = (F^B + F^Z)/4$ and $4C + D = F^Z - F_2$ should be independent of gauge expect for constants and divergent terms. They are presented as follows:

$$\frac{1}{4}(F^B, v_L + F^Z, v_L) = -\frac{3}{4} \log(\mu_W/m_W) - \frac{1 - \delta_1 + 5\delta^2_1}{8(1 - \delta_1)^2} + \frac{3\delta^2_1 \ln \delta_1}{8(1 - \delta_1)^2}$$

$$= (C - B)^{SM} - \frac{3}{4} \log(\mu_W/m_W) - \frac{1}{8},$$

$$F^Z, v_L - F^v_L = \frac{1}{3} \log(\mu_W/m_W) + \frac{26 - 53\delta_1 - 363\delta^2_1 + 379\delta^3_1 - 54\delta^4_1}{108(1 - \delta_1)^3} + \frac{8 - 74\delta_1 + 159\delta^2_1 - 90\delta^3_1}{18(1 - \delta_1)^4} \ln \delta_1$$

$$= (4C + D)^{SM} - \frac{8}{3} \log(\mu_W/m_W) + \frac{61}{54}. (17)$$
Other Wilson coefficients read:

\[ C_{7i}^{gr} = -\frac{3\delta_7^2 - 2\delta_t}{2(\delta_t - 1)^3} \ln \delta_t + \frac{-5\delta_t^2 + 31\delta_t - 20}{12(\delta_t - 1)^2} \]  \hspace{1cm} (18)

\[ C_{8}^{gr} = \frac{3\delta_t}{2(\delta_t - 1)^3} \ln \delta_t - \frac{\delta_t^2 + \delta_t + 4}{4(\delta_t - 1)^2} \]  \hspace{1cm} (19)

\[ C_{7i}^{gL} = (\delta_i - \frac{7}{3}) \log(\mu_W/m_W) + \frac{8 - 27\delta_t + 7\delta_i^2 + 6\delta_i^3}{12(1 - \delta_i)^2} - \frac{\delta_i(2 + 3\delta_t + \delta_i^2 - 3\delta_i)}{6(1 - \delta_i)^3} \ln \delta_t \]  \hspace{1cm} (20)

\[ C_{8}^{gL} = -2 \log(\mu_W/m_W) + \frac{2 - 9\delta_t + \delta_i^2}{4(1 - \delta_i)^2} - \frac{\delta_t(1 - 6\delta_t + 2\delta_i^2)}{2(1 - \delta_i)^3} \ln \delta_t \]  \hspace{1cm} (21)

\[ C_{7i}^{gr} = \frac{1}{4} \ln(\mu_W/m_W) + \frac{17 - 31\delta_t + 5\delta_i^2 - 3\delta_i^3}{48(1 - \delta_i)^3} + \frac{2 - 2\delta_t - 3\delta_i^2 + 2\delta_i^3 - \delta_i^4}{8(1 - \delta_i)^4} \]  \hspace{1cm} (22)

\[ C_{8}^{gr} = \frac{2 + 5\delta_t - \delta_i^2}{8(1 - \delta_i)^3} + \frac{3\delta_t}{4(1 - \delta_i)^4} \ln \delta_t \]  \hspace{1cm} (23)

\[ \frac{1}{4}(F_{B,gr} + F_{Z,gr}) = \frac{3}{8} \log(\mu_W/m_W) - \frac{17\delta_t + 1}{32(1 - \delta_i)} - \frac{3\delta_t(\delta_t + 2) \ln \delta_t}{16(1 - \delta_i)^2} \]  \hspace{1cm} (24)

From Eqs. (16) and (17), it is clear that \( C_{7,8}^{gr} \) and \( C_{7,8}^{gL} \) originate from the ultraviolet-finite diagrams and depend on \( \delta_t \) only. However, divergent terms appear in other Wilson coefficients. Consequently, \( \ln(\mu_W/m_W) \) are left in these functions. We also compare our formulae of \( C_{7,8} \) with Ref. [10], find that there is a 1/2 factor in our calculation of \( C_{7,8}^{gL} \), and \( C_{8}^{gL} \) has different expression. And there are no constants and divergences in \( C_{7,8}^{gr} \) in Ref. [10] since the authors included \( Wtb \), \( Wab \) anomalous couplings contributions, as mentioned in last subsection. Further, the non-log term in \( C_{8}^{gr} \) has opposite sign with that in Ref. [10]. We confirm our calculation.

### 3 Constraints on Wtb anomalous couplings with rare decays

With all Wilson coefficients at \( \mu_W \) scale ready, we can express the branching ratios in terms of the Wilson coefficient, further, of the anomalous couplings. Because the SM contributions to \( C_i(\mu_b) \) and the corresponding operator matrix elements are mostly real, the linear terms in \( \delta C_i \), which stem from SM-NP interference contributions contribute mostly as \( Re[\delta C_i] \), we neglect the small contributions of \( Im[\delta C_i] \) [19] and set \( \mu_W = m_W \). The \( B \rightarrow X_s \gamma \) branching ratio can be expressed as follows [20]:

\[ Br(B \rightarrow X_s \gamma) = [3.15 \pm 0.23 - 14.81\delta C_7(\mu_b) + 16.68(\delta C_7(\mu_b))^2] \times 10^{-4} \]  \hspace{1cm} (25)

Using \( \delta C_7(\mu_b) = 0.627\delta C_7(m_W) \), \( \delta C_8(\mu_b) = 0.747\delta C_8(m_W) \) and \( \delta C_7^{eff} = \delta C_7 + 0.24\delta C_8 \), as well as the Wilson coefficients in Eqs (16) and (17) we rewritten the branching as

\[ Br(B \rightarrow X_s \gamma) = [3.15 \pm 0.23 - 3.40\delta v_L + 339.09v_R - 78.49g_L - 1.12g_R \]  
\[ + (0.94\delta v_L - 93.51v_R + 21.65g_L + 0.31g_R)^2] \times 10^{-4}. \]  \hspace{1cm} (26)

The branching ratio for \( B_s \rightarrow \mu^+\mu^- \) is given by [19]

\[ Br(B_s \rightarrow \mu^+\mu^-) = 1.8525 \times 10^{-10} \left[ | - 4.3085 + \delta C_{10}|^2 \pm 1.7274 \right] \]  
\[ = 1.8525 \times 10^{-10} \left[ | - 4.3085 - 29.17\delta v_L - 7.11g_R|^2 \pm 1.7274 \right] \]  \hspace{1cm} (27)

From Eqs (26) and (27), we can see that \( Br(B_s \rightarrow \mu^+\mu^-) \) depends only parameters \( \delta v_L \) and \( g_R \) and is more sensitive to \( \delta v_L \) which is set to zero in top decay research [17]. And \( Br(B \rightarrow X_s \gamma) \) depend all four parameters and will exert more stronger limits on \( v_R \) and \( g_L \) than in top decay since enhanced factors \( m_t/m_b, m_W/m_b \), as pointed out in Ref. [10].
couplings are small, the coefficients of

Figure 3: The scatter plot of $\delta v_L$ as a function of $g_R$ constrained by $Br(B_s \to \mu^+\mu^-)$ and top decay where $\delta v_L$ is set to zero. The solid lines stand for 2$\sigma$ bounds for $Br(B_s \to \mu^+\mu^-)$, while the dashed lines from top decay.

We first constrain parameters $\delta v_L$ and $g_R$ by $Br(B_s \to \mu^+\mu^-)$. Using the branching ratio by experimental measurements and the SM prediction, as well as constraints from lastest top decay experiment in Appendix, we plot $\delta v_L$ as a function of $g_R$ in Fig. 2. We find that $B_s \to \mu^+\mu^-$ exerts bounds more tighter restrictions on $\delta v_L$ but slight weaker on $g_R$ than those at LHC via top decays with $\delta v_L = 0$.

Under the limits by $Br(B_s \to \mu^+\mu^-)$ and lastest top decay experiment at LHC [7], we plot $Br(B \to X_s\gamma)$ as a function of $Br(B_s \to \mu^+\mu^-)$ in Fig. 3. Since $Br(B_s \to X_s\gamma)$ depends on all four parameters and are more sensitive to $g_L$, $g_R$ which are less constrained by top decay, leading to very large branching ratio, we also use the bounds from $Br(B \to X_s\gamma)$ presented in Ref. [10] for comparison. Note at one loop level, $Br(B \to X_s\gamma)$ is proportional to $|C_7^{[1]}|^2$, the NP contributions to the branching ratio may as large as the SM value. Although the anomalous couplings are small, the coefficients of $g_R$, $g_L$ have large enhancements, as mentioned in Subsection 1. Thus, we keep up to the anomalous couplings squared in expression of $Br(B \to X_s\gamma)$. Besides, some Wilson coefficients are also different from those in Ref. [10]. From Fig. 3 we obtain more stringent constraints on $g_L$, $g_R$ than in top decay and slightly tighter constraints than previous work [10].

Figure 3: The scatter plot of $Br(B \to X_s\gamma)$ (in unit $10^{-4}$) as function of $Br(B_s \to \mu^+\mu^-)$ (in unit $10^{-9}$). The parameters $\delta v_L$, $g_R$ are restrictioned by $B_s\mu^+\mu^-$, and allow region of $g_L$, $g_R$ are obtained by top decay (denoted by triangles) and $Br(B \to X_s\gamma)$ in Ref. [10] (denoted by crosses). The 2$\sigma$ bounds for $Br(B_s \to \mu^+\mu^-)$ and $Br(B \to X_s\gamma)$ are also shown as solid and dashed lines, respectively.

Now we would like to summarize our work. We calculated the amplitudes of $b \to s$ transition in extension of the Standard Model with $Wtb$ anomalous couplings in unitary gauge. We presented all anomalous couplings related to the third generation quarks including anomalous $Wtb$, $Wtb\gamma$ and $tt\gamma$ couplings needed in calculation of rare B decays, and a detailed one-loop correction calculation for the $b \to s\gamma^{(*)}$. We found that effective vertex of $b \to s\gamma$ does not satisfy the Ward identity without $Wtb\gamma$ vertex contribution taken into account. This is independent of the CKM matrix unitarity even the photon is off-shell. We express branching ratios $B \to X_s\gamma$ and $B_s \to \mu^+\mu^-$ in terms of the Wilson coefficients and the anomalous couplings. The numerical results show that unlike to the inclusive decays, $Br(B_s \to \mu^+\mu^-)$ depends only on two anomalous parameters, it exerts unique constraints on $\delta v_L$.
since $\delta v_L$ is set to zero in top decay at LHC. Under the constraints, we obtain stringent limits on other parameters by $B \to X_s\gamma$ than obtained at LHC via $t \to W^+b$ decay and previous $B \to X_s\gamma$ analysis. Our work will be useful in experimental measurements for $Wtb$ anomalous couplings with more data at high energy colliders and B factories.

**Appendix**

In this Appendix, we present some inputs for numerical analysis.

1. **Experimental measurements and SM predictions of the Branching ratios**

| Process | $B_{\text{ex}}$ | $B_{\text{SM}}$ |
|---------|----------------|-----------------|
| $B \to X_s\gamma$ | $(3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$ | $(3.15 \pm 0.23) \times 10^{-4}$ |
| $B_s \to \mu^+\mu^-$ | $(3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$ | $(3.42 \pm 0.54) \times 10^{-9}$ |

2. **Bounds on $Wtb$ anomalous couplings by top decays and $Br(B \to X_s\gamma)$**

Table 6: 95% C.L. limits on the real components of the anomalous couplings. These limits were extracted from the combination of $W$-boson helicities and single top quark production cross section measurements at LHC at 14 TeV with high luminosity[7] and $Br(B \to X_s\gamma)$ [10]

| $\delta v_L$ | $g_R$ | $g_L$ | $\nu_R$ |
|--------------|-------|-------|--------|
| Allowed regions | 0 | $[-0.07, 0.07]$ | $[-0.16, 0.17]$ | $[-0.25, 0.34]$ |
| Allowed regions | $[-0.13, 0.03]$ | $[-0.0007, 0.0025]$ | $[-0.0013, 0.0004]$ | $[-0.15, 0.57]$ |

**References**

[1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B716: 1 (2012), [arXiv:1207.7214][hep-ph]
[2] P. W. Higgs, Phys. Rev. Lett. 12: 132(1964)
[3] G. Aad *et al.* [ATLAS Collaboration], JHEP 06: 088 (2012)
[4] V. Khachatryan *et al.* [CMS Collaboration], Phys. Lett. B 695: 424 (2011)
[5] G. Aad *et al.* [Atlas Collaboration], Eur. Phys. J. C71: 1577 (2011)
[6] CMS Collaboration, CMS-PAS-TOP-10-008; G. Aad *et al.* [Atlas Collaboration], ATLAS-CONF-2011-027
[7] J.L Birman *et al.*, [arXiv:1605.02673][hep-ph]
[8] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. 118: 191801(2017)
[9] C. Bernardo et.al., Phys. Rev. D 90: 113007 (2014). J. A. Aguilar-Saavedra, N. F. Castro, A. Onofre, Phys. Rev. D83:117301(2011)[arXiv:1105.0117][hep-ph]
[10] Bohdan Grzadkowski, Mikolaj Misiak, Phys. Rev. D78: 077501(2008)[arXiv:0802.1413][hep-ph]
[11] A. J. Buras, M. Misiak, M. Münz and S. Pokorski, Nucl. Phys. B 424: 374 (1994)
[12] M. Misiak et al., Phys. Rev. Lett. 98: 022002 (2007)[hep-ph/0609232]. T. Hurth, Rev. Mod. Phys. 75: 1159 (2003)[hep-ph/0212304]. C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, JHEP 0404: 071 (2004)[hep-ph/0312090]. A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, Nucl. Phys. B 685: 351 (2004)[hep-ph/0312128]. H. H. Asatryan, H. M. Asatrian, C. Greub and M. Walker, Phys. Rev. D 65: 074004 (2002)[hep-ph/0109140].

[13] Z. Heng et al., Phys. Rev. D77: 095012(2008); G. Lu, Z. Xiong, Y. Cao, Nucl. Phys. B487: 43(1997); Z. H. Xiong, J. M. Yang, Nucl. Phys. B602: 289(2001) and 628: 193(2002); G. Lu, et al. Phys. Rev. D54: 5647(1996); C. D. Lü, D. X. Zhang, Phys. Lett. B397: 279(1997); Y. Xu, R. Wang, Y. D. Yang, Phys. Rev. D74: 114019(2006); W. J. Li, Y. B. Dai, C. S. Huang, Eur. Phys. J. C40: 565(2005); C. S. Huang, X. H. Wu, Nucl. Phys. B657: 304(2003); C. S. Huang, W. Liao, Q. S. Yan, S. H. Zhu, Eur. Phys. J. C25: 103(2002); Phys. Rev. D63: 114021(2001)

[14] T. M. Aliev, E. O. Iltan, Phys. Rev. D58: 095014(1998); C. S. Kim, Y. G. Kim, C. D. Lü, Phys. Rev. D64: 094014(2001); A. Ali, G. Hiller, Phys. Rev. D60: 034017(1999); J. X. Chen, Z. Y. Hou, C. D. Lü, Commun. Theor. Phys.47: 299(2007); Z. J. Xiao, C. D. Lü, W. J. Huo, Phys. Rev. D67: 094021(2003); W. J. Zou, Z. J. Xiao, Phys. Rev. D72: 094026(2005)

[15] C. S. Huang, W. Liao, Phys. Lett. B525: 107(2002); C. S. Huang, S. H. Zhu, Eur. Phys. Rev. D61: 015001(2001); Z. J. Xiao, L. X. Lü, Phys. Rev. D74: 034016(2006); W. Liu, C. X. Yue, H. D. Yang, arXiv:0901.3463; J. J. Cao, G. U. Lu, Z. J. Xiao, Phys. Rev. D64: 014012(2001)

[16] Q. H. Cao, B. Yan, J. H. Yu, C. Zhang, arXiv:1408.7063 [hep-ph]; S. F. Taghavi, M. Mohammadi Najafabadi, Int.J.Theor.Phys. 53: 4326(2014)

[17] J. P. Lee and K. Y. Lee, Phys. Rev. D78: 056004(2008)

[18] Gunion J F, Haber H E. Nucl. Phys. B278, : 449(1986)

[19] S. Descotes-Genon, D. Ghosh, J. Matiasc, M. Ramon, JHEP 1106: 099 (2011). arXiv:1104.3342 [hep-ph]

[20] Jure Drobnak, Svjetlana Fajfer, and Jernej F. Kamenik, arXiv:1109.2357 [hep-ph]

[21] C. Patrignani et al. (Particle Data Group), 2017 Review of Particle Physics, Chin. Phys. C, 40: 100001 (2016)