THREE DISK OSCILLATION MODES OF ROTATING MAGNETIZED NEUTRON STARS

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ABSTRACT

We discuss three specific modes of accretion disks around rotating magnetized neutron stars which may explain the separations of the kilohertz quasi-periodic oscillations (QPOs) seen in low-mass X-ray binaries. The existence of these modes requires that there be a maximum in the angular velocity of the accreting material, and that the fluid be in stable, nearly circular motion near this maximum rather than moving rapidly toward the star or out of the disk plane into funnel flows. It is currently not known whether these conditions occur, but we are exploring this with 3D magnetohydrodynamic simulations and will report the results elsewhere. The first mode is a corotation mode which is radially trapped in the vicinity of the maximum of the disk rotation rate and is unstable. The second mode, relevant to relatively slowly rotating stars, is a magnetically driven eccentric (\( m = 1 \)) oscillation of the disk excited at a Lindblad radius in the vicinity of the maximum of the disk rotation. The third mode, relevant to rapidly rotating stars, is a magnetically coupled eccentric (\( m = 1 \)) and an axisymmetric (\( m = 0 \)) radial disk perturbation which has an inner Lindblad radius also in the vicinity of the maximum of the disk rotation. We suggest that the first mode is associated with the upper QPO frequency, the second with the lower QPO frequency \( \nu_i = \nu_u - \nu_s \), and the third with the lower QPO frequency \( \nu_i = \nu_u - \nu/2 \), where \( \nu_s \) is the star’s rotation rate.

Subject headings: accretion, accretion disks — MHD — stars: neutron — X-rays: binaries

1. INTRODUCTION

Low-mass X-ray binaries often display twin kilohertz quasi-periodic oscillations (QPOs) in their X-ray emissions (van der Klis 2006; Zhang et al. 2006). A wide variety of different models have been proposed to explain the origin and correlations of the different QPOs. These include the beat frequency model (Miller et al. 1998; Lamb & Miller 2001; Lamb & Miller 2003), the relativistic precession model (Stella & Vietri 1999), the Alfven wave model (Zhang 2004), and warped disk models (Shirakawa & Lai 2002; Kato 2004).

A puzzling aspect of the some of the twin QPO sources considered in this work is that the difference between the upper \( \nu_u \) and lower \( \nu_l \) QPO frequencies is roughly either the spin frequency of the star \( \nu_s \) (3 cases where \( \nu_s = 270, 300, \) and 363 Hz) or one-half this frequency, \( \nu_s/2 \) (4 cases where \( \nu_s = 401, 524, 581, \) and 619 Hz), for the cases where \( \nu_s \) is known, even though \( \nu_s \) and \( \nu_s/2 \) vary significantly (see, e.g., Zhang et al. 2006). A further type of behavior appears in the source Cir X-1 (Boutloukos et al. 2006), but this is not considered here. The cases where \( \nu_i - \nu_u \approx \nu_s \) may be explained by the beat frequency model (Miller et al. 1998), but the explanation of the cases where \( \nu_i - \nu_u \approx \nu_s/2 \) is obscure.

Section 2.1 discusses the corotation instability, \( \Sigma \ 2.2 \) the eccentric \( (m = 1) \) mode of the disk driven by the star’s rotating magnetic field, and \( \Sigma \ 2.3 \) the coupled eccentric plus axisymmetric mode \( (m = 0 \) and 1) also due to the star’s rotating magnetic field. Section 3 gives the conclusions.

2. THREE MODES

2.1. Corotation Instability

We assume a pseudo-Newtonian potential \( \Phi_s = -GM_s/(r - r_s) \), where \( M_s \) is the star’s mass and \( r_s \equiv 2GM_s/c^2 \). In the absence of the star the angular velocity of disk matter is \( \Omega_{\phi} = (GM_s/[r(r - r_s)])^{1/2} \) for \( r \geq 3r_s \). Near the star the disk’s angular rotation rate in the equatorial plane is modeled as

\[
\Omega_{\phi}(r) = \frac{\Omega_f(r)}{1 + f(r)} + \frac{\Omega_{\phi,0}(r)}{1 + f(r)},
\]

where \( f(r) = \exp\left[-(r - r_0)/\Delta r\right] \) with \( r_0 \) the standoff distance of the boundary layer and \( \Delta \) its thickness, which are expected to depend on the accretion rate and the star’s magnetic field.

The radial force equilibrium in the midplane of an axisymmetric disk is \( \rho r (\Omega_{\phi}^2 - \Omega_\phi^2,0) = d\rho B^2/8\pi dr \), where \( \rho \) is the midplane density and the midplane field \( B = B(r)\). Figure 1 shows the equilibrium quantities for an illustrative case. Clearly, \( \Omega_{\phi}(r) \) has a maximum value outside of the star at a distance denoted \( r_0 \sim r_s \). The importance of this maximum for models of QPOs was discussed earlier by Alpar & Psaltis (2005). Three-dimensional magnetohydrodynamic (MHD) simulations of disk accretion to rotating magnetized stars can in principle be used to determine \( \Omega_{\phi}(r) \) for different conditions (Romanova et al. 2007). However, for the present purposes the dependence of equation (1) is used. It is needed only for distances \( r \geq r_s \) as discussed below. In this region the radial epicyclic frequency is \( \Omega_k(r) = [r^2d(r^4\Omega_{\phi}^2,0)/dr]^1/2 \). The maximum value of \( \Omega_k(r) \) has the approximate dependence max \( \Omega_k(r) \approx 2040 (3r_s/r_0)^{7/4} \) Hz for \( 3 < r_s/r_0 < 5 \), \( \Delta r/\rho = 0.05 \), and \( M_*=1.4 M_\odot \). For this range of \( r_0 \), max \( \Omega_k(r) \) changes by a factor of 2.4.

We consider a WKB treatment of the corotation (\( \Omega \approx M_\Omega_r \)) or Rossby type wave of the disk with pressure perturbation

\[ \delta p \sim \exp\left(i \int dr k + im\phi - i\omega t\right), \]

where \( k \) is the radial wavenumber, \( m = 1, 2, \ldots, \) and \( \omega = \omega_0 + i\omega_1 \), with \( \omega_0 \), the angular frequency of the perturbation and \( \omega_1 \), the growth rate. For the conditions of Figure 1, the Alfven
speed $c_\lambda = B/(4\pi \rho)^{1/2}$ is much larger than the sound speed $c_s$. Also, we assume and verify later that $|\omega - m \Omega_\odot|^2 \ll \Omega_\odot^2$. Under these conditions,

$$k^2(r) = -\left(\frac{m^2}{r}\right)^2 - \left(\frac{\Omega_\odot}{c_\lambda}\right)^2 - \frac{1}{L^2_d} \frac{d}{dr} \left(\frac{r F}{\Omega_\odot L^2_d}\right)$$

$$- \frac{2m}{r} \left(\frac{2}{L^2_d} \frac{d}{dr} \frac{F}{\Delta \omega^2}\right) \Re\left(\frac{\Omega_\odot}{\Delta \omega}\right) - \frac{m^2}{L^2_d} \frac{2}{\Omega_\odot} \Re\left(\frac{1}{\Delta \omega^2}\right),$$

(2)

where $\Delta \omega = \omega - m \Omega_\odot$, $L^2_d \equiv d \ln (B/\rho) / d r$, $L^2_i \equiv d \ln (B/\rho) / d r$, $F = \rho \Omega_\odot / \Omega_\odot^2$, which all depend on $r$, and $\Re(\cdots)$ denotes the real part (R. V. E. Lovelace et al. 2007, in preparation). Equation (2) generalizes the calculation of Lovelace et al. (1999) to include the influence of the magnetic field perturbation $\delta B = \delta B(r, \phi, t) \xi$.

Figure 2 shows the radial dependence of $k^2$ for a representative case. For the chosen value $\omega / 2\pi = 1100$ Hz, which is somewhat less than the maximum value of $\Omega_\odot$, and $m = 1$, it is seen that $k^2(r) \geq 0$ in a finite radial interval in the vicinity of the maximum of $\Omega_\odot$. Thus the wave is radially trapped in the vicinity of the maximum of $\Omega_\odot$. Analogous radially trapped modes were analyzed earlier by Lovelace et al. (1999) and Li et al. (2000) and verified in two-dimensional hydrodynamic simulations (Li et al. 2001). The Bohr-Sommerfeld quantization condition $|\omega(I)| dr k = (n + 1/2)\pi$, $n = 0, 1, \ldots$ allows the determination of the growth rate $\omega$. For the case shown, $\omega / 2\pi = \nu \approx 55$ Hz for $n = 0$, which gives the largest growth rate, and $r(I/n)/r_\odot = 4.08, r(out)/r_\odot = 4.48$. The growth rate increases as $\Delta$ decreases. Similar values of the growth rates are found for $\omega$, somewhat less than $m \max(\Omega_\odot)$ for $m = 2, 3, \ldots$ The nonlinear saturation of the growth of the modes can in principle be found by MHD simulations (Koldoba et al. 2002; Romanova et al. 2007).

From Figure 2 we see that the validity of equation (1) is needed only from the vicinity of the maximum of $\Omega_\odot$, that is, from $r/r_\odot = 4.08$, where $\max(\Omega_\odot) = 0.94$ (the inner turning point), and outward (including the outer turning point at $r/r_\odot = 4.48$).
and super-Alfvénic relative to , and excitation of the disk
regions and are forbidden. The existence of
the product of the two perturbations
magnetic perturbation of the disk is in general nonlinear. The
result is a leading spiral wave with .
Consequently the perturbation is a leading spiral wave with .
Disk velocity is supersonic and super-Alfvénic relative to
region of the mentioned corotation instability, and
same conditions as Figure 2. For higher , all values of
at the outer resonance. However, the solution of equation
at the inner Lindblad resonance is expected to be stronger than
rC which turn gives rise to the
induced magnetic field of the disk B. The radial force includes
a coherent component B' which can be written as \( \delta F = C_i \exp (i \phi - i \Omega_t t) \), where \( C_i \) is the average
over \( \phi \). Considering only the coherent component, equation
(4) is then a driven oscillator. With \( \omega \sim \exp (i \phi - i \Omega_t t) \), it becomes
\[
\left[ \frac{d^2}{dr^2} - \frac{D(r)}{r} \right] \mathcal{E} = \frac{C_i}{\epsilon_f^2}, \tag{5}
\]
with \( D = \{ \Omega^2 - (\Omega - \Omega_0)^2 \} \epsilon_f^2 \). Depending mainly on the
value of \( \Omega_0 \), there may be Lindblad resonances with inner and
outer Lindblad radii \( r_{i} \) where \( D(r_{i}) = 0 \). The region be-
tween the two radii is permitted in the sense that \( D \leq 0 \) while
the regions \( r > r_i \) and \( r < r_i \) are forbidden. The existence of
the Lindblad resonances means that a weak magnetic dis-
trance can give rise to a strong disk response in the vicinity of
\( r_i \) proportional to \( C_i(r_i) \) (Goldreich & Tremaine 1979). Be-
cause of the rapid decrease of the magnetic field the response
at the inner Lindblad resonance is expected to be stronger than
that at the outer resonance. However, the solution of equation
(5) is beyond the scope of this work. We find that there are
Lindblad resonances only for \( \nu_i = \Omega_i/2\pi < 380 \) Hz for the
same conditions as Figure 2. For higher \( \nu_i \), all values of \( r \) are
forbidden. For \( \nu_i = 300 \) Hz, \( r_i/r_0 \approx 4.34 \), which is within the
region of the mentioned corotation instability, and \( r_i/r_0 \approx 4.83 \), which is outside the region of corotation instability. The
disk velocity \( \Omega_r \) is supersonic and super-Alfvénic relative
to the velocity of the perturbation \( \Omega, r \) for \( r \) larger than \( r_i \). Con-
sequently the perturbation is a leading spiral wave with \( k < 0 \).
At the outer Lindblad radius the disk velocity is also supersonic
and super-Alfvénic relative to \( \Omega, r \), and excitation of the disk
at this radius gives a trailing spiral, \( k > 0 \) for \( r < r_i \).

The interaction between the corotation perturbation and the
magnetic perturbation of the disk is in general nonlinear. The
perturbed disk surface temperature can be represented as a
product of the two perturbations \( \tilde{T}(r, \phi, t) \sim [1 + \epsilon_m \tilde{E}_r \exp (-i \phi - i \Omega_t t)] [T_0 + \epsilon_0 \tilde{E}_r \exp (-i \phi - i \omega t) + \cdots] = T_0 + \epsilon_m \tilde{E}_r \tilde{E}_t \exp [-i \omega t - \Omega_0 t] + \cdots \) where \( \epsilon_m \ll 1 \). Conse-
quently, there is a contribution to the source flux \( \delta \mathcal{L} \sim \left| \frac{d}{dr} \right| \mathcal{E} \times \exp (-i \phi) \tilde{S}(r, \phi, t) \sim 4 \left| \frac{d}{dr} \right| T_0 \epsilon_m \tilde{E}_r \tilde{E}_t \exp [-i \omega t - \Omega_0 t] \), where \( T \sim T_0 \). For \( \nu_i \approx 380 \) Hz, we interpret \( \omega_i - \Omega_0/2 \), as the lower frequency component of twin QPOs.

2.3. Magnetically Coupled \( m = 0 \) and 1 Mode

Consider now higher rotation frequencies \( \nu_i \). Note that the
radial force perturbation includes a contribution of the form
\( \delta F \sim \Omega_0^{-1} \tilde{S}(\dot{\epsilon}_c \tilde{B}_s) \), which again has a coherent term
proportional to \( B' \). We consider only the coherent term which
\( \Omega_0 \) \( \approx \), so that the right-hand side of the second
\( \Omega_0 \) \( \approx \Omega_0 \). Evidently the equation for \( \mathcal{E} \) corresponds to free
oscillations. It has Lindblad radii approximately where
\( \Omega_0 \approx \Omega_0 \). Consequently the perturbation is a leading spiral wave
with . The mode may be driven at \( r_i \), by noise or fluctuations in the disk at this radius, and it is a leading spiral wave. The mode amplitude can in principle be found
using three-dimensional MHD simulations (Koldoba et al.
2002; Romanova et al. 2007).

The influence of the magnetically coupled modes on the
flux follows the discussion of § 2.3. We find \( \delta \mathcal{L} \sim \left| \frac{d}{dr} \right| \mathcal{E} \times \exp (-i \phi) \tilde{S}(r, \phi, t) \sim 4 \left| \frac{d}{dr} \right| T_0 \epsilon_m \tilde{E}_r \tilde{E}_t \exp [-i \omega t - \Omega_0 t] \). For \( \nu_i \approx 380 \) Hz, we interpret \( \omega_i - \Omega_0/2 \) as the lower frequency component of twin QPOs.

3. CONCLUSIONS

We discuss three modes of accretion disks around rotating
magnetized neutron stars which may explain the frequency
separations of the twin kilohertz QPOs seen in accreting X-ray
binaries. The existence of these modes requires that there be
a maximum in the angular velocity of the accreting material
and that the fluid be in stable, nearly circular motion near this
maximum rather than moving rapidly toward the star or out of
the disk plane into funnel flows. It is currently not known
whether these conditions occur, but we are exploring this with
3D magnetohydrodynamic simulations and will report the
results elsewhere. The first mode is a corotation mode which is
radially trapped in the vicinity of the maximum of the disk
rotation rate and is unstable. A simple dependence is assumed
for the angular rotation rate of the disk \( \Omega(r) \) which has a maximum
at a radius \( r_o \) outside the neutron star. The unstable mode
has a frequency \( \omega_o \), somewhat less than \( \Omega(r_o) \) and
this can vary by a significant factor depending on the state of
the disk (e.g., the accretion rate). We suggest that this mode is
associated with the upper QPO frequency \( \nu_i = \omega_o/2\pi \). The
second mode is a magnetically driven eccentric \( (m = 1) \) os-
cillation of the disk excited at the inner Lindblad radius which
is in the vicinity of the maximum of the disk rotation. The
star’s magnetic field is assumed to be the form discussed by
Ruderman (2006). The Lindblad radii occur only for relatively
slowly rotating stars, \( \nu_i \approx 380 \) Hz. For these stars we suggest
that the lower QPO frequency is \( \nu_i = \nu_i - \nu_o \). The third mode,
relevant to more rapidly rotating stars, is a magnetically coupled
eccentric \( (m = 1) \) and an axisymmetric \( (m = 0) \) radial disk
perturbation. It has an inner Lindblad radius also in the vicinity
of a large distance ( ). This mode may be driven at by
noise or fluctuations in the disk at this radius, and it is a leading spiral wave.
of the maximum of the disk rotation. For these stars the lower QPO frequency is \( \nu_l = \nu_H - \nu_r/2 \). A problem remaining for future work is the determination of the saturation amplitudes of the different modes.

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