Advantages of NOMA for Multi-User BackCom Networks

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Abstract—Ambient backscatter communication (BackCom) is faced with the challenge that a single BackCom device can occupy multiple orthogonal resource blocks unintentionally. As a result, in order to avoid co-channel interference, a conventional approach is to serve multiple BackCom devices in different time slots, which reduces both spectral efficiency and connectivity. This letter demonstrates that the use of non-orthogonal multiple access (NOMA) can efficiently improve the system throughput and support massive connectivity in ambient BackCom networks. In particular, two transceiver design approaches are developed in the letter to realize different tradeoffs between system performance and complexity.

Index Terms—Non-orthogonal multiple access, backscatter communications, space division multiple access, orthogonal frequency-division multiple access.

I. INTRODUCTION

Recently, various novel backscatter communication (BackCom) approaches have been developed to support the envisioned ultra-massive machine type communications (umMTC) [1–5]. The key idea of these BackCom approaches is to use the signals sent by users in a legacy system for exciting the circuits of BackCom devices. This type of BackCom is featured by the challenge that a single BackCom device can occupy multiple orthogonal resource blocks unintentionally. For example, if the legacy system is based on orthogonal frequency-division multiple access (OFDMA), a BackCom device needs to reflect all the legacy signals sent at different subcarriers, which means that a signal sent by the device appears at multiple subcarriers. A similar challenge also presents if space division multiple access (SDMA) is used in the legacy system. In order to avoid the co-channel interference between BackCom devices, orthogonal multiple access (OMA) approaches have been conventionally used, e.g., BackCom devices are served in different time slots [2, 3], which reduces both spectral efficiency and connectivity.

This letter is to demonstrate the advantage of using non-orthogonal multiple access (NOMA) to improve the system throughput and connectivity in BackCom networks. In particular, this letter considers a full-duplex (FD) network, where downlink users are viewed as legacy users and multiple uplink BackCom devices are served concurrently in the downlink users’ channels by using BackCom assisted NOMA (BAC-NOMA) [5]. Unlike [5], multiple legacy users are considered in this letter, where the use of both SDMA and OFDMA in the legacy system is investigated. Two transceiver design approaches are proposed in the letter, where one approach can realize the sum capacity of the multiple access channel (MAC) and the other one can be implemented in a low complexity manner. The provided simulation results demonstrate that BAC-NOMA realizes a larger throughput and support more devices than OMA, even if random resource allocation is used.

II. SYSTEM MODEL

Consider an ambient BackCom network, where SDMA is used in the legacy system. In particular, in the legacy system, a base station serves users, denoted by $U_k$, via spatial beamforming vectors, denoted by $w_k$, 1 ≤ $k$ ≤ $K$. The application of BAC-NOMA can ensure that additional $M$ uplink BackCom devices, denoted by $BD_m$, 1 ≤ $m$ ≤ $M$, are admitted to the bandwidth used by the legacy system [5]. It is assumed that each user/device is equipped with a single antenna, the base station has $N$ antennas with the full-duplex capability, and each uplink device is equipped with a BackCom circuit. We note that the proposed BAC-NOMA scheme can also be applied to the case with OFDMA based legacy systems, as shown in Section IV.

Denote $x_k$ by the signal sent by downlink user $U_k$. Based on the principle of SDMA, the base station broadcasts the following superimposed signals: $s_0 = \sqrt{P_0} \sum_{k=1}^{K} w_k x_k$, where $P_0$ denotes the transmit power of the base station [6]. Optimizing the transmission strategy of the legacy system, i.e., optimizing $w_k$ and $P_0$, is beyond the scope of this letter, since it should happen before the uplink BackCom devices are admitted via BAC-NOMA.

Each downlink user receives the following:

$$y_k = \mathbf{g}_k^T s_0 + \sum_{m=1}^{M} \sqrt{\eta_m |g_{m,k}|^2} \mathbf{h}_{m}^T s_m + n_k,$$  \hspace{1cm} (1)

where $\mathbf{g}_k$ denotes the channel vector between the base station and downlink user $U_k$, $\mathbf{h}_{m}$ denotes the channel vector between the base station and uplink device $BD_m$, $g_{m,k}$ denotes the channel gain between $U_k$ and $BD_m$, $\eta_m$ denotes the BackCom reflection coefficient of $BD_m$ [7], $s_m$ denotes the signal sent by $BD_m$, and $n_k$ denotes the receiver noise. For notational convenience, it is assumed that the noise terms at different receivers have the same power, denoted by $\sigma^2$.

Because $s_m$ is unknown to the downlink users, the term, $I \triangleq \sum_{m=1}^{M} \sqrt{\eta_m |g_{m,k}|^2} \mathbf{h}_{m}^T W_{m}^T$, can be treated as interference, whose power is given by $E_{x_k,s_m} \{ I^* \} = P_0 \sum_{m=1}^{M} \eta_m |g_{m,k}|^2 |h_{m,k}|^2 W_{m}^T$, where $E\{ \cdot \}$ denotes the expectation operation, and $W = [w_1 \cdots w_K]$. Therefore, the data rate of downlink user $U_k$ is given by

$$R_k^D = \log \left( 1 + \frac{P_0 |g_{k}^T w_k|^2}{P_0 \sum_{i=1, i \neq k}^{K} |g_{k}^T w_i|^2 + \epsilon_{x_k,s_m} \{ I^* \} + \sigma^2} \right).$$

The base station receives the following observation:

$$y_{BS} = \sum_{m=1}^{M} \sqrt{\eta_m |h_{m}|^2} \mathbf{W}_{m}^T s_m + n_{BS},$$  \hspace{1cm} (2)

where $n_{BS}$ is assumed to be complex Gaussian distributed, i.e., $n_{BS} \sim CN(0, \alpha P_0 C_{BS})$, $C_{BS}$ denotes the covariance matrix of...
the self-interference channels and $\alpha$, $0 \leq \alpha \leq 1$, indicates the amount of the residual self-interference [8].

Depending on how successive interference cancellation (SIC) is carried out, different sum rates can be realized for the BackCom devices, as shown in the following section.

III. TWO APPROACHES WITH DIFFERENT TRADEOFFS BETWEEN PERFORMANCE AND COMPLEXITY

A. Approach I – A Sum-Capacity Approaching Design

By applying the noise pre-whitening process [9], i.e., applying a detection matrix $(\sigma^2I_N + \alpha P_0 C_S)^{-\frac{1}{2}}$ to $y_{BS}$, the model in (2) can be rewritten as follows:

$$\tilde{y}_{BS} = \sum_{m=1}^{M} \sqrt{\eta_m} h_m^T s_0 \tilde{h}_m s_m + \tilde{n}_{BS} = \tilde{H} D s + \tilde{n}_{BS}$$

(3)

where $\tilde{y}_{BS} = (\sigma^2 I_N + \alpha P_0 C_S)^{-\frac{1}{2}} y_{BS}$, $\tilde{h}_m = (\sigma^2 I_N + \alpha P_0 C_S)^{-\frac{1}{2}} h_m$, $\tilde{H} = [\tilde{h}_1 \cdots \tilde{h}_M]$, $\tilde{n}_{BS} = (\sigma^2 I_N + \alpha P_0 C_S)^{-\frac{1}{2}} n_{BS}$. $D$ is a diagonal matrix with the elements on its main diagonal as $D_{m,m} = \sqrt{\eta_m} h_m^T s_0$, and $s = [s_1 \cdots s_M]^T$.

By treating $D$ as a power allocation matrix, the system model in (3) can be viewed as a special case of a conventional MAC, whose sum capacity can be realized as follows [9]. Without loss of generality, assume that BD$_m$, signal is decoded at the $m$-th stage of SIC, where it straightforward to show that different SIC decoding orders lead to the same sum capacity. Further assume that prior to decoding BD$_m$, signal, the signals from BD$_i$, $1 \leq i \leq m - 1$, have been decoded correctly. By applying $\sqrt{\eta_m} h_m^T s_0 h_m^H$ as a detection matrix, the following data rate is achievable to BD$_m$:

$$R_{m}^{MAC} = \log \det \left( I_N + \left( I_N + \sum_{i=m+1}^{M} \eta_i h_i^T s_0^2 h_i^H \right) \right)^{-1} \times \eta_m h_m^T s_0^2 h_m^H.$$ (4)

for $1 \leq m \leq M - 1$, and $R_{MAC}^{sum} = \log \det \left( I_N + \eta_m h_m^T s_0^2 h_m^H \right)$.

Therefore, the sum capacity of the uplink BackCom devices achieved by this type of SIC is given by

$$R_{MAC}^{sum} = \log \det \left( I_N + \sum_{m=1}^{M} \eta_m h_m^T s_0^2 h_m^H \right).$$ (5)

This letter considers a problem of throughputs maximization for the BackCom devices, which can be formulated as follows:

$$\max_{\eta_m} R_{MAC}^{sum} \quad \text{(P1a)}$$

subject to

$$\sum_{m=1}^{M} \eta_m \left| g_{m,k} \right|^2 \left| h_m^T w \right|^2 \leq \tau_k, \quad 1 \leq k \leq K \quad \text{(P1b)}$$

$$0 \leq \eta_m \leq 1, \quad 1 \leq m \leq M, \quad \text{(P1c)}$$

where $\tau_k$ indicates the tolerable interference experienced by downlink user $U_k$. If $U_k$ has a target data rate, denoted by $R_k$, one choice of $\tau_k$ is given by $\tau_k = \frac{\ln \left| \sum_{m=1}^{K} \eta_m h_m^T w_k^2 \right|^2 \left| h_m^T w_k^2 \right|^2}{\eta_k^2}$ and $\eta_k = 2^R_k - 1$.

Remark 1: Problem $[P1]$ is a concave optimization problem since its objective function is in a log-det form and its constraints are affine [10]. Various optimization solvers, such as Matlab fmincon, can be straightforwardly applied to find the optimal solution of Problem $[P1]$.

Remark 2: The solution of Problem $[P1]$ is based on an instantaneous realization of $x_k$. Therefore, significant system overhead can be consumed for the base station to inform the BackCom devices about the optimal choices of $\eta_m$. A low-complexity alternative is to use random choices of $\eta_m$ which satisfy $[P2a]$ and $[P2c]$. The simulation results provided in Section V show that Approach I with random $\eta_m$ can still significantly outperform OMA.

B. Approach II - A Low-complexity QR Based Design

Assume that the composite channel matrix $\tilde{H}$ in (3) can be decomposed via QR decomposition as follows: $\tilde{H} = QR$, where $Q$ is an $N \times N$ unitary matrix, and $R$ is an $N \times M$ upper triangular matrix [11]. The base station can use $Q^H$ as a detection matrix, which simplifies the system model in (3) as follows:

$$Q^H \tilde{y}_{BS} = RD s + Q \tilde{n}_{BS}. \quad \text{(6)}$$

By using the upper triangular structure of $R$, SIC can be implemented in a low-complexity manner [11]. In particular, during the $(M - m + 1)$-th step, the signal from BD$_m$ can be decoded with the following data rate:

$$R_m = \log \left( 1 + R_{m,m}^2 \eta_m \left| h_m^T s_0 \right|^2 \right), \quad \text{(7)}$$

where $R_{m,m}$ is defined similar to $D_{m,m}$. To further reduce system overhead, it is ideal to formulate the resource allocation problem based on the following average sum rate:

$$R_{sum} = \sum_{m=1}^{M} E_k \left\{ \log \left( 1 + R_{m,m}^2 \eta_m \left| h_m^T s_0 \right|^2 \right) \right\}. \quad \text{(8)}$$

Therefore, the considered long-term throughput maximization problem can be formulated as follows:

$$\max_{\eta_m} \bar{R}_{sum} \quad \text{s.t.} \quad [P2a], \quad [P2b]. \quad \text{(P2a)}$$

We note that there is an analogy between Problem $[P2]$ and the one developed for cognitive MAC [12]; however, the explicit expression of $R_{sum}$ can be obtained, as shown in the following. First define $\bar{R}_m$ as follows:

$$\bar{R}_m = E_k \left\{ \log \left( 1 + R_{m,m}^2 \eta_m \left| h_m^T s_0 \right|^2 \right) \right\}. \quad \text{(9)}$$
Recall that $s_0 = \sqrt{R_0^{\sum_{k=1}^{K} W_k x_k}}$, which means that $R_m$ can be rewritten as follows:

$$ R_m = \mathcal{E}_{x_k} \left\{ \log \left( 1 + P_0 R_{m,m}^{\eta_m} x^H W^H h_m^T W x \right) \right\}. $$

(10)

A closed-form expression of $R_m$ can be found by using the method developed in [13]:

$$ R_m = \log(e) \mathcal{E}_{x_k} \left\{ \int_0^\infty \left( e^{-t} \frac{t}{1+t \left( P_0 R_{m,m}^{\eta_m} x^H W^H h_m^T W x \right)} \right) dt \right\}, $$

(11)

which can be simplified as follows:

$$ R_m = \log(e) \int_0^\infty \left( e^{-t} \frac{t}{1+t \left( P_0 R_{m,m}^{\eta_m} \right) \left| h_m^T W x \right|^2} \right) dt. $$

(12)

Assuming that $x_k$’s are independent and identically distributed (i.i.d.) complex Gaussian variables with mean zero and unit variance, $R_m$ can be evaluated as follows:

$$ R_m = \log(e) \int_0^\infty \left( e^{-t} \frac{t}{1+t \left( P_0 R_{m,m}^{\eta_m} \right) \left| h_m^T W x \right|^2} \right) dt = \log(e) f \left( P_0 R_{m,m}^{\eta_m} \left| h_m^T W x \right|^2 \right), $$

(13)

where $f(x) \triangleq - e^x E_1 (\frac{x}{e})$ and the last step follows from [14, 3.352.6] and $E_1(\cdot)$ denotes the exponential integral function. We note that an alternative way to obtain (13) is to treat $h_m^T W x$ in (10) as a complex Gaussian variable with mean zero and variance $\left| h_m^T W x \right|^2$.

So Problem [P2] can be recast in the following equivalent form:

$$ \max_{\eta_m} \sum_{m=1}^{M} - e^{-P_0 R_{m,m}^{\eta_m} \left| h_m^T W x \right|^2} E_1 \left( - \frac{1}{P_0 R_{m,m}^{\eta_m} \left| h_m^T W x \right|^2} \right) $$

(13a)

s.t. \hspace{0.5cm} (P1b), (P1b).

Although Problem [P3] contains the exponential integral function, it is still concave as shown in the following lemma.

**Lemma 1.** Problem [P3] is a concave optimization problem.

**Proof.** See Appendix A

Remark 4: Because Problem [P3] is concave, it can be straightforwardly solved by using various optimization solvers. We note that the computation of the function $f(x)$ in [13] can be difficult even for a moderately small $x$. For example, for $x = 0.0013$, $\frac{1}{e} = 750$, and Matlab returns $f(750) = \text{Inf}$. To overcome this computational issue, the following approximation of $f(x)$ is used for small $x$. Recall that $-E_1 (-\frac{1}{x}) = \Gamma (0, \frac{1}{x})$, where $\Gamma (\beta, y)$ denotes the incomplete gamma function and can be approximated as follows [14]:

$$ \Gamma (\beta, y) \approx y^{\beta-1} e^{-y} \sum_{m=0}^{L-1} \frac{(-1)^m \Gamma (1-\beta+m) y^m \Gamma (1-\beta)}{m!}, $$

(14)

for $y \to \infty$. By letting $L = 1$ and $\beta = 0$, $f(x)$ can be approximated as follows:

$$ f(x) = - e^x \Gamma \left( 0, \frac{1}{x} \right) \approx x, \hspace{0.2cm} x \to 0, $$

(15)

which can be used to approximate [P3a].

Remark 5: Compared to Approach I, the QR based design can be implemented with low computational complexity, as explained in the following. First, there is no need to calculate the inverse of a matrix with size of $N$ at each SIC step. Second, the resource allocation solution is not based on the instantaneous realizations of $x_k$, which reduces the system complexity. However, it is worth to point out that, unlike Approach I, Approach II cannot achieve the sum capacity of MAC, or support the overloading case, i.e., $M > N$.

IV. EXTENSION TO OFDMA-BASED LEGACY SYSTEMS

The aforementioned BAC-NOMA scheme can also be applied to the case, where OFDMA is used in the legacy system. In particular, in the considered legacy system, a base station serves $K$ downlink users, each denoted by $U_k$, $1 \leq k \leq K$, via $K$ orthogonal OFDMA subcarriers. If OMA is used, all the $K$ subcarriers will be occupied by a single BackCom device, since a signal reflected by one BackCom device can block all subcarriers. The application of BAC-NOMA can ensure that $M$ uplink BackCom devices are simultaneously admitted to share the subcarriers. For the purpose of illustration, it is assumed that each node is equipped with a single antenna.

Without loss of generality, assume that downlink user $U_k$ is served at the $k$-th subcarrier. Following the ambient BackCom model in [2], [3], at subcarrier $k$, the frequency-domain baseband signal received by downlink user $U_k$ is given by

$$ y_k^D = \sqrt{P_0 G_k x_k} + \sqrt{P_0 \sum_{m=1}^{M} \sqrt{\eta_m} G_{m,k} H_{m,k} x_k s_m + n_k^D}, $$

(16)

where $G_k$ denotes the channel gain between the base station and $U_k$ at subcarrier $k$, $H_{m,k}$ denotes the forward channel gain from the base station to uplink device $BD_m$ at subcarrier $k$, $G_{m,k}$ denotes the channel gain between $U_k$ and $BD_m$ at subcarrier $k$, and $n_k^D$ denotes the receiver noise.

Because $s_m$ is unknown to the downlink users, the term, $I_0 \triangleq \sqrt{P_0 \sum_{m=1}^{M} \sqrt{\eta_m} G_{m,k} H_{m,k} x_k s_m}$, is again treated as interference. The power of this interference term is given by $\mathcal{E}_{x_k,s_m} \{ I_0^2 \} = P_0 \sum_{m=1}^{M} \eta_m |G_{m,k}|^2 |H_{m,k}|^2$. Therefore, the data rate of downlink user $U_k$ is given by

$$ R_k^D = \log \left( 1 + \frac{P_0 |G_k|^2}{P_0 \sum_{m=1}^{M} \eta_m |G_{m,k}|^2 |H_{m,k}|^2 + \sigma^2} \right). $$

(16)

At the base station, the frequency-domain baseband signal at the $k$-th subcarrier is given by

$$ y_k^{BS} = \sqrt{P_0} \sum_{m=1}^{M} \sqrt{\eta_m} F_{m,k} H_{m,k} x_k s_m + s_k^{SI} + n_k^{BS}, $$

(17)

where $F_{m,k}$ denotes BD$_m$’s backward channel gain at subcarrier $k$, $s_k^{SI}$ denotes the self-interference and $n_k^{BS}$ denote the noise. As in the previous section, it is assumed that $s_k^{SI}$ ~
\(CN(0, \alpha P_0 |h_{SI}|^2)\), where \(h_{SI}\) denotes the self-interference channel. Furthermore, it is assumed that self-interference at different subcarriers is independent.

By applying the pre-whitening process, the system model at the base station can be expressed as follows:

\[
g_{k}^{\text{BS}} = \sqrt{P_0} x_k h_k^H \eta^2 s + n_{k}^{\text{BS}},
\]

where \(g_{k}^{\text{BS}} = (\alpha P_0 |h_k|^2 + \sigma^2)^{-\frac{1}{2}} g_k^{\text{BS}}, \quad \hat{n}_k = (\alpha P_0 |h_k|^2 + \sigma^2)^{-\frac{1}{2}} \left[ F_1 \bar{H}_{1,k} \cdots \cdot F_M \bar{H}_{M,k} \right]^H, \quad s = [s_1 \cdots s_M]^T, \quad \eta = \text{diag}\{\eta_1, \cdots, \eta_M\}, \quad \text{and} \quad \bar{n}_k^{\text{BS}}\text{ is a complex Gaussian white noise with mean zero and unit variance.}

Stacking the \(K\) observations in one vector, the system model at the base station can be rewritten as follows:

\[
y^{\text{BS}} = \sqrt{P_0} D x H^H \eta^2 s + \bar{n}^{\text{BS}},
\]

where \(D\) is a \(K \times K\) diagonal matrix, i.e., \(D = \text{diag}\{x_1, \cdots, x_K\}\), \(H = [\bar{h}_1 \cdots \bar{h}_K]^T\), \(y^{\text{BS}} = [g_1^{\text{BS}} \cdots g_K^{\text{BS}}]^T\), and \(\bar{n}^{\text{BS}}\) is constructed similarly to \(\bar{y}^{\text{BS}}\).

By defining \(H = \sqrt{P_0} D x H^H\) and treating \(\eta^2\) as a power allocation matrix, one can view the system model in (19) as a special case of conventional MAC, whose sum capacity can be realized as follows. Without loss of generality, assume that BDm\(_m\)' signal is decoded at the \(m\)-th stage of SIC, where it is noted that different SIC decoding orders lead to the same sum capacity. Further assume that prior to decoding BDm\(_m\)' signal, the signals from BD\(_i\), \(1 \leq i \leq m - 1\), have been decoded correctly. By applying \(\bar{h}_i^H (I_K + \sum_{i=m+1}^{M} \eta_i \bar{h}_i \bar{h}_i^H)^{-1}\) as the detector, the following data rate is achievable to BD\(_m\):

\[
R_{m}^{\text{MAC}} = \log \det \left( I_K + \sum_{i=m+1}^{M} \eta_i \bar{h}_i \bar{h}_i^H \right)^{-1} \eta_m \bar{h}_m \bar{h}_m^H,
\]

for \(1 \leq m \leq M - 1\), and \(R_{M}^{\text{MAC}} = \log \det (I_K + \eta_M \bar{h}_M \bar{h}_M^H)\), where \(\bar{h}_i\) denotes the \(i\)-th column of \(H\). Therefore, the sum capacity of the uplink BackCom devices is given by

\[
R_{\text{sum}}^{\text{MAC}} = \log \det (I_K + \sum_{m=1}^{M} \eta_m \bar{h}_m \bar{h}_m^H),
\]

which can be used to formulate the following throughput maximization problem:

\[
\max_{\eta_m} \quad R_{\text{sum}}^{\text{MAC}} \quad \text{(P4a)}
\]

\[
\text{s.t.} \quad \sum_{m=1}^{M} \eta_m |G_{m,k}|^2 |H_{m,k}|^2 \leq \tau_k, \quad 1 \leq k \leq K \quad \text{(P4b)}
\]

Similar to (P4a), (P4b) is also in the concave log-det form. Therefore, Problem (P4) is also concave and hence can be straightforwardly solved.
transmission improves the throughput, which are consistent to the observations made in Figs. 1 and 2.

VI. CONCLUSIONS

This letter has demonstrated the advantages of NOMA for ambient BackCom networks with OFDMA and SDMA used in legacy systems. By using BAC-NOMA, multiple BackCom devices can be served simultaneously, instead of being served in different time slots as in OMA. Two resource allocation approaches have been proposed in order to realize different tradeoffs between system performance and complexity.

APPENDIX A

PROOF FOR LEMMA I

Recall that all the constraints of Problem 23 are affine, and therefore the lemma can be proved by showing that the objective function of Problem 23 is concave. Recall that \( f(x) = -e^{x} x^{-1} \), \( x \geq 0 \). The concavity of the objective function in 23 can be proved by showing that \( f(x) \) is a concave function. Recall that the first order derivative of \( f(x) \) has been obtained in [5] as follows:

\[
    f'(x) = e^{x} x^{-1} x^{-2} E_i((-1)^{2}) + x^{-1}.
\]

By using (21) and with some algebraic manipulations, the second order derivative \( f''(x) \) can be expressed as follows:

\[
    f''(x) = e^{x} x^{-1} x^{-4} \times (-E_i((-1)^{2}) - x e^{-x} - 2 x E_i((-1)^{2}) - x^{2} e^{-x}).
\]

Define \( u(x) = -E_i((-1)^{2}) - x e^{-x} - 2 x E_i((-1)^{2}) - x^{2} e^{-x} \). In order to show \( f''(x) \leq 0 \), it is sufficient to show that \( u(x) \leq 0 \), since \( x \geq 0 \). The first order derivative of \( u(x) \) is given by

\[
    u'(x) = e^{x} x^{-1} - e^{x} x^{-1} - e^{x} x^{-1} - 2 E_i((-1)^{2}) + 2 x e^{-x} x^{-1} - 2 x e^{-x} x^{-1} - 2 e^{-x} x^{-1}
\]

\[= -2 E_i((-1)^{2}) - 2 x e^{-x},\]

where the first step follows from the fact that \( \frac{dE_i((-1)^{2})}{dx} = -x^{-2} \).

Define \( g(x) \triangleq E_i((-1)^{2}) + x e^{-x} \), and \( u'(x) \) can be shown as a function of \( g(x) \) as follows:

\[
u'(x) = -2 \left( E_i((-1)^{2}) + x e^{-x} \right) = -2 g(x).
\]