Abstract— Distribution systems are experiencing increasing penetration of Distributed Generation (DG). One attractive option is to use the available DG capacity during utility outages by forming planned micro-grids. Load sharing among different DG, during micro-grid operation, could be accomplished by equipping each DG with a P-f and Q-V droop characteristic. In this paper, we analyze the impact of the loads’ voltage and frequency dependence on the micro-grid’s frequency and voltage deviation during a planned islanding condition. The system was modeled and analyzed using PSCAD/EMTDC. The simulation results and mathematical analysis show that the load’s voltage and frequency dependence is an important factor when choosing the DG droop characteristic. The results also show that some loading conditions could lead to an unstable micro-grid operation.

Index Terms— Distributed Generation, Inverter, Micro-grid, Static Load Models.

I. INTRODUCTION

The Micro-grid could be viewed as a group of DG that could operate either connected to the grid or intentionally islanded. Although both the IEEE Std. 929 and UL 1741 standard do not allow islanded operation; the IEEE 1547 standard presents it as one of the topics that needs to be addressed in the future[1]-[3]. Micro-grids can offer many advantages which include increase system reliability, ability to isolate during utility outage, and reduce need for central dispatch [4].

Distributed Generation are designed to operate close to unity power factor as per the IEEE Std. 1547 [3]. It would not be possible to operate a micro-grid with such control [4]. The operation of a micro-grid requires a robust control strategy that would facilitate load sharing between different DG units. Two main strategies have been proposed in literature for controlling DG on a micro-grid which include the Master/Slave approach and the droop control approach. In the master/slave approach, one DG (master) operates as a voltage source where it is responsible for controlling the micro-grid voltage while the other DG (slaves) operate as a current source [5]-[6]. The DG designed to operate as slaves are responsible for supplying active power to meet the load demand. The distributed generators communicate with each other for load sharing. In the droop control approach, the DG are equipped with a P-f and Q-V droop characteristic [7]-[9]. The amount of load supplied by each DG depends on the DG droop. The main advantage of this technique is that it does not require any communication links. In [6] and [9], the master slave and droop approach were both implemented where some DG were designed to supply a predefined active and reactive power and others were equipped with a droop characteristic.

In this paper, we focus on the micro-grid load and its impact on the voltage and frequency deviation during micro-grid operation for inverter based DG equipped with droop controls. The load is modeled as a static load with both voltage and frequency dependence. The static load model parameters are varied and both the frequency and voltage waveforms are examined.

The paper in organized as follows: Section II provides a brief description of the load model and DG droop characteristic. Section III presents the micro-grid system and DG interface model under study. Section IV highlights the simulation results for the different load model parameters. Lastly, conclusions are drawn in Section V.

II. MICRO-GRID LOAD AND INVERTER CHARACTERISTICS

The frequency and voltage at which the micro-grid will stabilize depends on both the load and DG characteristic. Both the active and reactive power of the load are expressed as functions of voltage and frequency [10]. The load’s voltage dependence could be modeled through either a polynomial or exponential relationship. The load’s frequency dependence is represented by multiplying either the polynomial or exponential load model by a factor. An exponential load model with both voltage and frequency dependence is given below in (1) and (2)

\[ P = P_0 \left( \frac{V}{V_0} \right)^{NP} \left( 1 + k_f \left( f - f_0 \right) \right) \]  

(1)
where $V_o$ represents the initial operating voltage (rated voltage) and $P_o$ and $Q_o$ represent the active and reactive power corresponding to the initial operating voltage. The parameter $f$ and $f_o$ represent the operating and rated frequency respectively. The parameter $V$ represents the operating voltage. $NP$, $NQ$, $k_p$, and $k_q$ correspond to the parameters of the load model [10].

As mentioned earlier, the DG is equipped with a $P-f$ and $Q-V$ droop characteristic as shown in Fig. 1 and Fig. 2. The $P-f$ characteristic is designed such that the DG delivers its maximum power at rated frequency. In a similar manner, the $Q-V$ characteristic is designed such that the DG supplies zero reactive power at rated voltage. Since both the voltage and frequency are controlled by the utility during parallel operation, the DG would operate at unity power factor.

During micro-grid operation, the amount of voltage and frequency deviation will depend on the amount of active and reactive power mismatch, the load characteristic and the DG droop model. The $P-f$ and $Q-V$ droop characteristic could be mathematically represented as follows

$$P_{DG} = a_1 (f - f_o) + b_1$$

$$Q_{DG} = a_2 (V - V_o) + b_2$$

where $P_{DG}$ and $Q_{DG}$ are the DG active and reactive power output. Parameters $a_1$, $a_2$, $b_1$, and $b_2$ represent the droop characteristic parameters. The frequency and voltage at which the micro-grid will stabilize could be found by equating (1) and (2) to (3) and (4) respectively as follows

$$P_{DG} = a_1 (f - f_o) + b_1$$

$$Q_{DG} = a_2 (V - V_o) + b_2$$

It can be seen from (5) and (6) that deviations in voltage are dependent on deviations in frequency and vice versa. For constant power loads ($NP=NQ=0$) with no frequency dependence ($k_p = k_q = 0$), the voltage deviation and frequency deviation are decoupled.

As a whole, it can be concluded that the DGs are equipped with a $P-f$ and $Q-V$ droop characteristic and a maximum power control. The frequency and voltage at which the DGs will operate can be determined by equating (1) and (2) to (3) and (4) respectively. The parameters $a_1$, $a_2$, $b_1$, and $b_2$ correspond to the parameters of the load model [10].

### III. SYSTEM UNDER STUDY

The system under study consists of a load represented by a static load model and two inverter based DG as shown in Fig. 3. The distribution system was modeled by an equivalent source behind an impedance. Each DG is equipped with the interface control presented in Fig. 4. The frequency and voltage at the Point of Common Coupling (PCC) are measured and through the droop characteristic the active and reactive power reference values ($P_{ref}$ and $Q_{ref}$) are determined. The power reference values are then compared with the DG output active and reactive power ($P_{DG}$ and $Q_{DG}$) and the error is passed to a Proportional Integral (PI) controller to generate the current $d$-axis and $q$-axis reference values ($I_{dref}$ and $I_{qref}$). Through Park’s transformation, the DG a-b-c current are transformed into the $d$-$q$ axis frame and are compared with the current $d$-$q$ axis reference values. The error is passed through a second PI controller to determine the inverter switching signals. Table I presents the system, DG and droop characteristic parameters.
TABLE I
SYSTEM, INVERTER AND DROOP DATA

| Grid Parameters          | Value       |
|--------------------------|-------------|
| Voltage (line to line)   | 480V        |
| Frequency                | 60Hz        |
| Grid Resistance          | 0.02Ω       |
| Grid Inductance          | 0.3 mH      |

| DG Inverter Controller Parameters | Value       |
|-----------------------------------|-------------|
| $k_p' = 5$                        | $k_q' = 0.07$|
| $k_p = 3$                         | $k_q = 0.08$|
| $a_1 = -0.1$                      | $b_1 = 0.1$ |
| $a_2 = -0.5$                      | $b_2 = 0$   |

The load on the micro-grid was modeled using (1) and (2). In order to examine the effect of the load model on the micro-grid frequency and voltage deviations, the different load parameters were varied individually while fixing the remaining load parameters.

IV. SIMULATION RESULTS WITH DIFFERENT LOAD MODELS

The system under study is implemented on the PSCAD/EMTDC simulator. A micro-grid case is simulated by opening the utility breaker at the Point of Common Coupling (PCC) at $t = 5$ seconds. Both the PCC voltage and frequency are monitored to determine the effect of load parameters on the micro-grid voltage and frequency deviations. The next subsections highlight the simulation results for each case study.

A. Load Sharing with Droop Control

The DG interface control is tested by opening the switch at the micro-grid isolating point (refer to Fig. 3). The load is modeled as a constant power load with no frequency dependence ($NP=0$ and $k_p=k_q=0$) and micro-grid operation is initiated at $t = 5$ seconds. The load’s active and reactive power are 200 kW and 90 kVAR respectively. Figure 5 and Fig. 6 present the DG active power, reactive power, PCC frequency and voltage. Since the two DG presented in this work are identical with the same droop characteristic, the DG share the load equally. During grid connected operation both DG supply a 100 kW of active power. Since the PCC voltage is less than 1 p.u during grid connected operation, each DG will supply 15 kVAR of reactive power which is dictated by the droop characteristic. From Fig. 5, it can be seen that the frequency will stabilize at a value of 60 Hz and the voltage will stabilize at approximately 0.91 per unit. This result coincides with the mathematical results which are presented in Table II in the Appendix.

B. Effect of Voltage Dependence

In this scenario, the effect of the load’s voltage dependence is analyzed by varying the values of NP and NQ. Similarly,
the load’s active and reactive power are adjusted to 200 kW and 90 kVAR respectively. A micro-grid is initiated at $t = 5$ seconds. The load’s frequency dependence ($k_{pf}=k_{qf}=0$) is set to zero. Figure 7 presents the frequency and voltage waveforms for $NP=1$ (constant impedance load), $NP=Q=1$ (constant current load) and for $NP=Q=0$ (constant power load). The frequency deviations decrease as the value of $NP$ and $Q$ decreases. On the contrary, the voltage deviations increase as the value of $NP$ and $Q$ decrease for the presented cases. The results coincide with the mathematical results presented in the Appendix.

C. Effect of Frequency Dependence

This scenario focuses on the effect of the load’s frequency dependence on voltage and frequency deviations during micro-grid operation. The micro-grid operation is analyzed for different values of $k_{pf}$ and $k_{qf}$ while fixing $NP=Q=2$. Figure 8 presents the voltage and frequency waveforms for various values of $k_{pf}$ and $k_{qf}$. Similarly, the load was adjusted to consume 200 kW and 90 kVAR at rated conditions. From Fig. 8, the amount of frequency deviation will decrease as the value of $k_{pf}$ and $k_{qf}$ increase. On the other hand, the amount of voltage deviations will increase with the increase in $k_{pf}$ and $k_{qf}$ (refer to Appendix).

A second case was simulated where the load is adjusted to 176 kW and 90 kVAR at rated conditions. The load’s voltage dependence parameter is set equal to zero ($NP=Q=0$). Figure 9 presents the voltage and frequency deviations for various values of $k_{pf}$ and $k_{qf}$. Similarly the frequency deviations decrease while voltage deviations increase with the increase in $k_{pf}$ and $k_{qf}$. Frequency and voltage deviations are dependent on the load parameters as well as the amount of active and reactive power mismatch. For constant power loads with no frequency dependence (C1 and C7 in Appendix), the voltage deviation is independent on the amount of frequency deviation.

Fig. 7. Frequency and voltage at the PCC for different values of $NP$ and $Q$.

Fig. 8. Frequency and voltage at the PCC for different values of $k_{pf}$ and $k_{qf}$ with $NP=Q=2$.

Fig. 9. Frequency and voltage at the PCC for different values of $k_{pf}$ and $k_{qf}$ with $NP=Q=0$. 
D. Unstable Operation

In this case, the load was adjusted to consume 176 kW and 90 kVAR at rated conditions. Similarly, the micro-grid operation was initiated at t = 5 seconds. The load's voltage dependence parameters was set fixed (NP=NQ=3). Figure 10 presents the frequency and voltage waveforms for different values of \( k_{pf} \) and \( k_{qf} \). For the cases where \( k_{pf}=k_{qf}=0 \) and \( k_{pf}=k_{qf}=5 \), the micro-grid was stable and the frequency and voltage stabilize at values that coincide with those given in Table II. For the case where \( k_{pf} \) and \( k_{qf} \) are set equal to -5, there is no stable micro-grid operating point. The frequency drifts until it reaches the interface controller limits. Although case C10, C11, and C12 (refer to Table II) have the same loading conditions, some case might result in stable operation while others might result in an unstable operating condition. Table II shows that, mathematically, there is no solution for either the frequency or voltage for this specific loading condition.

The frequency and voltage at which the micro-grid will stabilize could be determined from (5) and (6). Table II presents the values for the frequency and voltage during micro-grid operation for the loading conditions under study in this paper.

### Table II

| Case | \( P \) (kW) | \( Q \) (kVAR) | NP | NQ | \( k_{pf} \) | \( k_{qf} \) | \( f \) (Hz) | \( V \) (p.u) |
|------|--------------|---------------|----|----|----------|----------|---------|----------|
| C1   | 200          | 90            | 0  | 0  | 0        | 0        | 60      | 0.91     |
| C2   | 200          | 90            | 1  | 1  | 0        | 0        | 60.08   | 0.917    |
| C3   | 200          | 90            | 2  | 2  | 0        | 0        | 60.147  | 0.923    |
| C4   | 200          | 90            | 2  | 2  | 1        | 1        | 60.085  | 0.9177   |
| C5   | 200          | 90            | 2  | 2  | 3        | 3        | 60.06   | 0.915    |
| C6   | 200          | 90            | 2  | 2  | 5        | 5        | 60.03   | 0.929    |
| C7   | 176          | 90            | 0  | 0  | 0        | 0        | 60.12   | 0.91     |
| C8   | 176          | 90            | 0  | 0  | 2        | 2        | 60.04   | 0.902    |
| C9   | 176          | 90            | 0  | 0  | 5        | 5        | 60.022  | 0.9      |
| C10  | 120          | 90            | 3  | 3  | 0        | 0        | 60.52   | 0.928    |
| C11  | 120          | 90            | 3  | 3  | 5        | 5        | 60.19   | 0.879    |
| C12  | 120          | 90            | 3  | 3  | -5       | -5       | No sol.  | No sol.   |

V. CONCLUSIONS

This paper examines the effect of the load's voltage and frequency dependence on micro-grid operation for inverter based distributed generation. An exponential load model with both voltage and frequency dependence was modeled. The DG interface control was equipped with a \( P-f \) and \( Q-V \) droop characteristic. The results and analysis show that the frequency deviations and voltage deviation during micro-grid operation are dependent on each other. Although the micro-grid load could be within the available DG capacity, unstable operating conditions could occur as a result of the load's voltage and frequency dependence. It is important when designing the DG droop characteristic to take into account different possible loading conditions and models. The mathematical analysis coincides with the simulation results.

VI. APPENDIX

The frequency and voltage at which the micro-grid will stabilize could be determined from (5) and (6). Table II presents the values for the frequency and voltage during micro-grid operation for the loading conditions under study in this paper.

### Table II

| Case | \( P \) (kW) | \( Q \) (kVAR) | NP | NQ | \( k_{pf} \) | \( k_{qf} \) | \( f \) (Hz) | \( V \) (p.u) |
|------|--------------|---------------|----|----|----------|----------|---------|----------|
| C1   | 200          | 90            | 0  | 0  | 0        | 0        | 60      | 0.91     |
| C2   | 200          | 90            | 1  | 1  | 0        | 0        | 60.08   | 0.917    |
| C3   | 200          | 90            | 2  | 2  | 0        | 0        | 60.147  | 0.923    |
| C4   | 200          | 90            | 2  | 2  | 1        | 1        | 60.085  | 0.9177   |
| C5   | 200          | 90            | 2  | 2  | 3        | 3        | 60.06   | 0.915    |
| C6   | 200          | 90            | 2  | 2  | 5        | 5        | 60.03   | 0.929    |
| C7   | 176          | 90            | 0  | 0  | 0        | 0        | 60.12   | 0.91     |
| C8   | 176          | 90            | 0  | 0  | 2        | 2        | 60.04   | 0.902    |
| C9   | 176          | 90            | 0  | 0  | 5        | 5        | 60.022  | 0.9      |
| C10  | 120          | 90            | 3  | 3  | 0        | 0        | 60.52   | 0.928    |
| C11  | 120          | 90            | 3  | 3  | 5        | 5        | 60.19   | 0.879    |
| C12  | 120          | 90            | 3  | 3  | -5       | -5       | No sol.  | No sol.   |

VII. REFERENCES

[1] IEEE Std. 929-2000, “IEEE recommended practice for Utility Interface of Photovoltaic (PV) Systems”, April 2000.
[2] UL 1741, Static inverter and charge controllers for use in photovoltaic systems, Underwriters Laboratories Inc., Northbrook, IL.
[3] IEEE Std. 1547-2003, “IEEE Standard for Interconnecting Distributed Resources with Electric Power Systems”, July 2003.
[4] R. H. Lasseter, and P. Paigi, “Microgrid: A conceptual solution”, IEEE 35th Power Electronics Specialists Conference, vol. 6, pp. 4285 – 4290, June 2004.
[5] J. Liang, T. C. Green, G. Weiss, and Q. Zhong, “Hybrid control of multiple inverters in an island-node distribution system”, 2003 IEEE 34th Annual Power Electronics Specialist Conference, 2003, PESC ’03, Vol. 1, pp. 61 – 66, June 2003.
[6] J. A. Lopes, C. Moreira, and A. Madureira, “Defining Control Strategies for MicroGrids Islanded Operation”, IEEE Transactions on Power Systems, vol. 21, No.2, pp. 916-924, May 2006.
[7] S. Barsali, M. Ceroaldo, P. Pelacchi, and D. Poli, “Control techniques of Dispersed Generators to improve the continuity of electricity supply”, IEEE Power Engineering Society Winter Meeting, Vol. 2, pp. 789 - 794, 2002.
[8] F. Katiraei, and M. R. Iravani, “Power Managemnet Strategies for a Microgrid With Multiple Distributed Generation Units”, IEEE Transactions on Power Systems, vol. 21, no.4, pp. 1916-1924, Nov.2008.
[9] C. Moreira, F. Resende, and J. Lopes, “ Using Low Voltage MicroGrids for Service Restoration”, IEEE Transactions on Power Systems, vol. 22, no.1, pp. 395-403, Feb. 2007.
[10] IEEE Task Force on Load Representation for Dynamic Performance, ‘Load representation for dynamic performance analysis’, IEEE Transactions on Power Systems, Vol.8, No.2, pp. 472-482, May 1993.

VIII. BIOGRAPHIES

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