Three-dimensional simulation of bubble dynamics in a narrow pipe using lattice Boltzmann method

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Abstract. In the paper, a three-dimensional model of a gravity-driven bubble rising in a narrow pipe filled with viscous liquid is built using the lattice Boltzmann method. On the Cartesian grid, the free-energy multiphase lattice Boltzmann model and the no-slip bounce-back scheme are combined together to implement the bubble interface and the solid boundary treatment, respectively. To start with, the Laplace law for bubble interface is verified with the newly built model in this paper. Then the cases where the pipe with the radius 1.2 to 2.5 times the bubble radius are carried out to investigate the effects of pipe dimension on the bubble motion, including rising velocity, deformation and jet formation. Moreover, the asymmetric characteristics of bubble biases the centre axle are explored further. The results show that the boundary condition effect consisting of the pipe dimension and the offset of bubble biasing the centre axle is of great significance to the bubble dynamics in a narrow pipe. The former factor mostly affect the velocity characteristics of the bubble, while the latter one mostly focuses on the bubble deformation and trajectory.

1. Introduction

The motion of bubble in a pipe is a normal phenomenon that widely exists in engineering fields, such as hydraulic system, pipeline valve and heat collector, etc. Due to the difficulty in dealing with the multiphase interface and the complex fluid properties, quite a lot of researchers have investigated the process for a long time by using many numerical methods [1-3]. Still, many unknown phenomenon need people to uncover.

In researching bubble dynamics, quite a lot of work has been carried out in terms of experiments, theory, and numerical simulation. A common view is that solid walls have a significant effect on bubble dynamics. Zhang A M and Ni B Y et al. studied the bubble dynamics in the vortex field induced by solid boundaries [4], Popinet S et al. researched the effects of viscosity on bubble collapse near a solid wall [5], Zhang A M studied the characteristics of jet and impact combing the BEM and SPH [6]. Therefore, the wall effect has become an important branch in the research area. For the bubble dynamics in a narrow pipe, the wall effect is more obvious.

Generally, for a traditional method, the question can be described by solving a macroscopic control equation, such as the Navier-Stokes equation. However, the lattice Boltzmann method (LBM) based on the molecular theory can recover the macroscopic equation by adding up the particles distribution on the mesoscopic level. Due to its natural microcosmic property, it can easily deal with complex solid
boundary conditions and the microphysical quantity acting on the interface. Till now, a series of models treating gas-liquid interface, such as colour model, phase field model, and the free-energy model exist.

Combining the free-energy multiphase lattice Boltzmann model with the no-slip solid boundary condition, a buoyancy-driven bubble rising in narrow pipe is simulated. Both the relative radius size and the offset of bubble biasing the centre axle effecting the bubble motion and topological deformation are researched.

2. Numerical method and implementation

2.1. Lattice Boltzmann equations

Based on the free-energy model, Zheng H C et al. [7] introduced another distribution function $g$, to describe the multiphase flow interface dynamics which can be written as

$$g_i(x + \Delta x, t + \Delta t) - g_i(x, t) = -\frac{1}{\tau_g}(g_i(x, t) - g_i^e(x, t)) + Q,$$  \hspace{1cm} (1)

where $i$ is the discrete velocity direction. In the D3Q19 model, $i=0 \sim 18$, and in the D3Q7 model, $i=0 \sim 6$. For the distribution function $g$, the D3Q7 model is used. $\tau_g$ is the relaxation parameter, and $\tau_g = 0.6$ in this paper. $g_i^e(x, t)$ is the equilibrium distribution function satisfying the Maxwell distribution [7,8]. The last term on the right hand of equation (1) is a modified term related to the distribution condition of particles at the two adjoining node positions. Combining the step by step operation [9] and introducing the controls on transfer efficiency between lattice nodes [10], equation (1) can be divided into two steps. The first step is

$$g_i(x, t) = g_i(x, t) + \frac{1}{\tau_g}(g_i^e(x, t) - g_i(x, t)),$$  \hspace{1cm} (2)

and the second step is

$$g_i(x + \Delta x, t + \Delta t) = g_i(x + \Delta x, t) + q(g_i(x, t) - g_i(x + \Delta x, t)).$$  \hspace{1cm} (3)

The first step only occurs on the local node position, so it can be easily implemented parallel process in three dimensional calculations. The second step is the streaming operation where $q$ is the controlled parameter, $q = \frac{2}{1 + 2 \tau_g}$. Using the Chapman-Enskog expansion, equation (1) can recover the Cahn-Hilliard equation which is widely used to trace interface movement with the second-order accuracy. The Cahn-Hilliard equation can be written as

$$\partial \phi / \partial t + \nabla \cdot (\phi \mathbf{u}) = \sigma \nabla^2 \phi,$$  \hspace{1cm} (4)

where $\phi$ is the order parameter, $\phi = \sum g_i$. $\sigma$ is the mobility and $\theta$ is the chemical potential parameter. The evolution of the original distribution function which can recover the Navier-Stokes equations by Chapman-Enskog expansion is

$$f_i(x + \Delta x, t + \Delta t) - f_i(x, t) = -\frac{1}{\tau_f}(f_i(x, t) - f_i^e(x, t)) + F_i,$$  \hspace{1cm} (5)

where $\tau_f$ is the relaxation parameter and $f_i^e(x, t)$ is the equilibrium distribution function [11] corresponding to the distribution function $f_i$. In this paper, $\tau_f = 0.75$. $F_i$ is discrete body force based on the lattice structure which plays an important role in the numerical simulation.
2.2. Body force implementation
Different from the traditional discrete operation of a body force, the discrete lattice effects must be considered into the lattice Boltzmann model especially the multiphase flow model. In 2002, Guo Z L et al. [12] examined several popular existing force treatments, and proposed an effective representation of the forcing term in which the discrete lattice effects are introduced. The representation can be written as

\[ F_i = 3w_i (1 - \frac{1}{2\tau_i})[(e_i - u) + 3(e_i \cdot u) \cdot e_i] \cdot F. \]  

(6)

where \( w_i \) is the weighting factor, for the D3Q19 model, \( w_0 = 1/3, w_{1-6} = 1/18, w_{7-18} = 1/36 \). \( e_i \) is the discrete velocity whose value is determined by the lattice structure. For a single-phase flow simulation, the body force \( F \) is traditionally composed of the gravity, however the interface force must be considered in a multiphase flow. \( F \) can be written as \( F = \theta \vec{\nabla} \phi + G \), where \( G \) is the gravity acceleration. \( \theta \) and \( \phi \) are same with those defined in the previous article.

2.3. Solid boundary implementation
Another important issue that needs much more attention is the solid boundary implement. In the model, a no-slip bounce-back scheme is applied to deal with the curved walls. For a fluid node near to the solid wall, the distribution function coming from the solid node after the stream process can be calculated as [13]

\[ f_i(x_i, t + \Delta t) = f_i^{\text{eq}}(x_i, t) - 6w_i \rho (e_i \cdot u). \]

(7)

\( \vec{i} \) is the negative direction with the discrete direction \( i \). \( x_i \) stands for the fluid node position adjacent to the solid wall. \( u' \) is the intermediate velocity value on the virtual boundary composed of the midpoints between the solid node adjacent to the solid wall with \( x_i \). \( u \) can be calculated through interpolation and extrapolation [13].

3. Results and discussion

3.1. Verification
In the simulation, twenty thousand iterations are used at the beginning to eliminate the spurious velocity caused by the initial conditions. Related parameters are defined as \( \tau_f = 0.6, \tau_g = 0.75, \sigma = 2.0 \). The convergence is detected according to the Laplace Law, \( \Delta P = 2\sigma / R \). The results of bubbles with different scales are shown in figure 1.

![Figure 1. Duration curve of pressure difference inside and outside bubble.](image-url)
Figure 1 shows the pressure difference $P_{err}$, which is defined as

$$P_{err} = (2\sigma / R_g - \Delta P) \cdot R_g / \sigma ,$$

with $R_p / R_g = 1.2 \sim 2.5$. $R_p$ stands for the inner radius of the pipe, and $R_g$ stands for the bubble radius. The curves show that the results converge well and match well with the theoretical value.

### 3.2. Effects of size ratio

Keeping the pipe radius stable, several cases where the radius of bubble is different are carried out. The detailed information is show in Table 1.

| Pipe Radius($R_p$) | Relative Size($R_p / R_g$) | Bubble Radius($R_g$) |
|--------------------|-----------------------------|-----------------------|
| 50                 | 1.2                         | 41.67                 |
|                    | 1.5                         | 33.33                 |
|                    | 2.0                         | 25                    |
|                    | 2.5                         | 20                    |

Figures 2-3 show the rising process for bubbles with radius different times the pipe radius, $R_p / R_g = 1.2$ in figure 2, and $R_p / R_g = 2.5$ in figure 3. In each figure, time increases from left to right, and bubble terminal shape gradually formed. The bubbles tend to have a quite different deformation behaviour as the size ratio changes. This result indicates that the solid wall plays an important role in the evolution of the bubble shape as it rises under gravity. For the size ratio $R_p / R_g = 1.2$, the bubble experiences a large amount of deformation. By the joint action of fluid viscosity and solid wall effect, the interface adjacent to the wall moves slower than the central part, and a bubble with long strip shape is formed in the end. The phenomenon is consistent with the summary provided by Lee T [14] that if the size ratio is smaller than 1.25, the bubble deformation will not be as consistent. For the case, $R_p / R_g = 2.5$, the bubble rises as an isolated bubble in a free field. The results show that when the size ratio is larger than 1.5, bubbles can turn to be an approximately oblate spheroid as the wall effect decreases with large size ratio.

**Figure 2.** Bubble rising process with the size ratio $R_p / R_g = 1.2$.

**Figure 3.** Bubble rising process with the size ratio $R_p / R_g = 2.5$.  


Figure 4 shows the vertical position of the bubble with different size ratio, the vertical position $H$ is defined as the distance between the top node position of the bubble and the pipe bottom divided by $R_p$. The initial position of the four lines is determined by the bubble position and radius size. It can be found that the bubble rises faster as the size ratio increases which means that the wall can restrain bubble moving. Whatever, the bubble will come to a stable rising velocity just as it will reach a terminal shape. The results also show that the jet deformation becomes inapparent, as the size ratio decreases, and a negative jet velocity appears when $R_p / R_b = 1.2$.

![Figure 4](image)

**Figure 4.** Vertical position of bubble coaxial with the pipe centre axle defined by the distance between its top node and the pipe bottom divided by $R_p$.

3.3. Effects of offsets

In the earlier paragraphs, bubble dynamics in a narrow pipe where the bubble is coaxial with the pipe centre axle is studied. Next, effect of bubbles biasing the centre axle on bubble motion is researched. Keeping the pipe and bubble radius stable, offset values range from 10 to 25 as shown in Table 2.

| Pipe Radius | Bubble Radius | Relative Size | Offset value ($\Delta S$) |
|-------------|---------------|---------------|--------------------------|
| 50          | 20            | 2.5           | 10                       |
|             |               |               | 15                       |
|             |               |               | 20                       |
|             |               |               | 25                       |

**Table 2.** Cases of simulating bubble rising in a pipe with different offsets biasing the centre axle

![Figure 5](image)

**Figure 5.** Bubble rising process with the offset $\Delta S = 25$.

Figure 5 shows the bubble rising process in a pipe where the bubble biases the pipe centre axle with the offset $\Delta S = 25$. Due to the phenomenon becomes apparent, as the offset increases, the case with the offset $\Delta S = 25$ is chosen to be shown in Figure 5. Three main phenomena can be found. Firstly, under the condition of solid wall, a jet towards the wall is induced. Further, one can see the researches by Zhang A M [6] and Popinet S [5] to know solid walls affecting bubble jet motion. Secondly, the wall will attract the bubble at the initial stage, and then repel it. The bubble will produce an oscillate-rising trajectory. The phenomenon is consistent the results presented by Samaroo R [15]. The third one is the bubble will approach to an equilibrium position coaxial with the pipe.
Figures 6-7 indicate the position information of the bubble. Figure 6 show the vertical position of bubble defined as the distance between its top node and the pipe bottom divided by $R_p$. Figure 7 show the horizontal position of bubble defined as the distance between its rightmost node and the pipe left side divided by $R_p$. The figures show that the offset has worse effects on the horizontal position of bubble than on the vertical position. Comparing with figure 4, the results also indicate that size ratio mostly effects on the velocity characteristics of the bubble, while offset focuses on the bubble deformation and trajectory when the size ratio is not too small. It can also be seen in figure 7 that bubbles with different offsets biasing the pipe centre axe will finally come close to the equilibrium position on the pipe centre axe.

**Figure 6.** Vertical position of bubble biasing the pipe centre axe defined as the distance between its top node and the pipe bottom divided by $R_p$.

**Figure 7.** Horizontal position of bubble biasing the pipe centre axe defined as the distance between its right node and the pipe left side divided by $R_p$.

4. Conclusion

By using the lattice Boltzmann method, a three dimensional numerical model of bubbles moving in a narrow pipe is established. Effects of size ratio ($R_p / R_b$) and offsets that bubbles bias the pipe centre axe ($\Delta S$) on bubble dynamics are researched. The results can be summarized as:

1) Relative to size ratio which is not too small, offsets can induce a sensible horizontal movement and asymmetrical jet towards the solid wall which mainly influence the bubble deformation and trajectory.

2) When the size ratio is small, for example, smaller than 1.5, the wall effect becomes significant for coaxial cases. Bubble rising velocity goes down, and the interface adjacent to the wall moves slower than the central part, and a bubble with long strip shape is formed in the end.

3) Bubbles with a offset biasing the pipe axe will produce an oscillate-rising trajectory, and approach to an equilibrium position coaxial with the pipe.

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