Testing Dark Matter with the Anomalous Magnetic Moment in a Dark Matter Quantum Electrodynamics Model

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We consider a model of dark quantum electrodynamics which is coupled to a visible photon through a kinetic mixing term. We compute the $g_{\chi} - 2$ for the dark fermion, where $g_{\chi}$ is its gyromagnetic factor. We show that the $g_{\chi} - 2$ of the dark fermion is related to the $g_{\gamma} - 2$ of (visible) quantum electrodynamics through a constant which depends on the kinetic mixing factor. We determine $g_{\chi} - 2$ as a function of the mass ratio $\kappa = m_B/m_{\chi}$ where $m_B$ and $m_{\chi}$ denote the masses of the dark photon and the dark fermion respectively and we show how $g_{\chi} - 2$ become very different for light and heavy fermions around $m_B \leq 10^{-4}$ eV.

I. INTRODUCTION

The search for dark matter is one of the important challenges in physics today because its existence may explain some of the puzzles of conventional physics, such as the rotation curves of the galaxies, the new phenomenon of emission of light from the center of galaxies. It can also provide new insights into old problems such as matter-antimatter asymmetry, primordial magnetic fields and so on. Dark matter can also open up new areas of research in particle physics, astrophysics and cosmology [1–6].

The dark matter interacts very weakly with conventional matter and the effects produced by either light or heavy dark fermions are small in general. On the other hand, some properties, such as the magnetic dipole moment for electron/muon like dark fermions may depend dramatically on the mass of the dark fermion as well as the way they couple to the visible sector. (The Pauli coupling, for example, depends on the fermion mass with the mass in the denominator.) The possibility of a mass dependence for such effects makes the heavy and the light fermion regimes very different since they correspond to different parameter spaces. Precisely for this reason, the distinction between weakly interacting slightly particle (WISP) and weakly interacting massive particle (WIMP) is introduced in these studies.

Although the observational and theoretical reasons for the existence of dark matter are many (for a review see [7, 8]), there are issues that still need to be addressed. For example, for models mirroring the visible sector, such as the one considered in the present work, it would be interesting to find the effects of such dark systems on some of the high-precision measurements in particle physics [9, 10].

The electron gyromagnetic factor is one of the greatest triumphs of quantum field theory (quantum electrodynamics) and, therefore, the study of the magnetic moment of a dark Dirac fermion would be a very relevant and important issue in this context. However contrary of the conventional standard model calculations, the calculation for the magnetic moment of dark fermions must take into account the range of masses in which the theory is considered. For the case of WISP the typical fermion masses are $m_{\text{wisp}} \sim 10^{-3}$ eV or less, while in the case of WIMP the fermion masses are typically $m_{\text{wimp}} \sim 10^{2}$ GeV or more and, therefore the magnetic moment of a WISP and a WIMP can be very different.

In this paper we would like to concentrate mainly on the sector of WISP where the space of parameters can be tested in a family of experiments running at present [11, 12]. The masses of dark light fermions are bounded typically as $m_\chi < 10^{-3}$ eV which can be easily studied in these experiments.

In the low energy sector, which is the region to be discussed in this paper, there are still a number of important phenomena that require careful study such as the distortion of cosmic microwave radiation [13], deviations from Coulomb’s law [14, 15], the distortion of planetary magnetic fields [16], the level shifts in atomic physics [17] and so on. The ranges of parameters that may be relevant in these studies are $m_\chi \sim 10^{-8} – 10^6$ eV for dark fermion masses.
and $\xi \sim 10^{-14} - 10^{-4}$ for the kinetic mixing parameter (to be introduced in the next section) and, as we will see in the following, this can be perfectly tested with the current experiments mentioned above.

The calculation of the magnetic moment for the light dark fermions [13, 22] requires particular attention because of the smallness of the mass of the fermion. In this case, the loop corrections of the standard model (e.g. from $Z$ boson loops) are not relevant. In the opposite case of heavy dark fermions, on the other hand, the calculation of the magnetic moment is similar to that of the muon.

The paper is organized as follows. In section II, we set up the problem and present the model to be studied. Details of the calculations are given in section III where we also discuss the implications of the results. In section IV we summarize and discuss some open problems and in the appendix we provide some details of the diagonalisation of the mass matrix.

II. DARK MATTER AND KINETIC MIXING

In order to understand this problem systematically, let us build our model of dark QED. First, we consider a dark fermion $\chi$ with mass $m_\chi$ coupled to a hidden (dark) photon $B_\mu$. We choose the coupling constant (for minimal coupling) of the dark fermion to the hidden photon to be unity $e_h = 1$ for simplicity so that the covariant derivative in the dark sector can be written as $D_\mu[B] = \partial_\mu + iB_\mu$. We note that since we are interested in dipole interactions, we have to couple the dark fermion non-minimally and we choose the Lagrangian density for the dark fermion to be given by

$$\mathcal{L}_\chi = \bar{\chi} \left( iD_\mu[B] + \frac{g_h}{\Lambda} \sigma_{\mu\nu} F^{\mu\nu}(B) - m_\chi \right) \chi. \quad (1)$$

The Pauli coupling term in the Lagrangian density may arise either from radiative corrections or can be included at the tree level Lagrangian density in an effective theory. Here $\Lambda$ is a mass scale which, in the case of (visible) QED, is proportional to the electron mass $m$. Therefore, by analogy, $\Lambda$ can be chosen in the present case to be proportional to the mass of the dark fermion ($m_\chi$). The constant $g_h$ can be thought of as the gyromagnetic factor for the dark fermion in analogy with standard QED. We point out that the $(4 \times 4)$ matrix $\sigma_{\mu\nu} F^{\mu\nu}$ in the Pauli term can be written explicitly in terms of electric and magnetic fields as

$$\frac{1}{2} \sigma_{\mu\nu} F^{\mu\nu} = -\text{diag}\{\sigma \cdot (B_h + iE_h), \sigma \cdot (B_h - iE_h)\}, \quad (2)$$

where $E_h, B_h$ correspond respectively to the dark electric and magnetic fields associated with $B_\mu$. Therefore the Dirac fermion, coupled non-minimally to the gauge field $B_\mu$, contains both an electric dipole term $\sigma \cdot E_h$ and a magnetic dipole term $\sigma \cdot B_h$. We note here that in spite of the eventual importance of magnetic moment in the search for dark matter, at present there is no discussion of this in the literature beyond the tree level (except for [19], [26] within the context of technicolor dark matter and an interesting recent reference [27] within the context of early universe). In this work, we carry out the analysis of the magnetic moment at the one loop level.

Next, we choose the dynamics for the hidden photon to be given by the Lagrangian density

$$\mathcal{L}_B = -\frac{1}{4} F_{\mu\nu}(B)F^{\mu\nu}(B) + \frac{m_B^2}{2} B_\mu B^\mu. \quad (3)$$

Here $m_B$ is the mass of the dark photon which can arise, for example, through the Higgs mechanism or the St"{u}ckelberg formalism in the hidden sector. Similarly, in the visible sector, the dynamics of the photon can be given by the standard Maxwell term. We note that although the mass of the hidden (dark) photon $m_B$ may be very small, we find from our analysis that the relevant parameter in the study of the magnetic moment is the ratio of the masses of the dark photon and the dark fermion (which we denote later as $\kappa = m_B/m_\chi$). This ratio can be extracted appropriately from the experimental data and, indeed, can give nontrivial effects even when the individual masses are small (see Figures 2 and 3 later).

We will assume the kinetic mixing model of [28] which allows for a mixing between the gauge bosons in the visible and the hidden sectors through the Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}(A)F^{\mu\nu}(A) + \frac{m_A^2}{2} A_\mu A^\mu + \frac{\xi}{2} F_{\mu\nu}(A)F^{\mu\nu}(B) + \mathcal{L}_B. \quad (4)$$

Here $\xi$ is a dimensionless mixing parameter assumed to be small and we can also add a term which represents the interaction of the photon with the visible charged current [29]. However, we neglect this term for simplicity since it is not relevant for our analysis. In the present model we have admitted a mass $m_A$ for the photon of the visible sector.
This mass is presently constrained by the fact that there exist galactic magnetic fields coherent on the galactic size of a few kpc \[23\] \[24\] leading to

\[ m_A < 1 \times 10^{-27} \text{eV}, \quad \text{or,} \quad \lambda_\gamma > 10^{22} \text{cm}, \]

where \( \lambda_\gamma \) denotes the Compton wave length of the photon.

The simultaneous diagonalisation of the kinetic mixing and the mass terms is achieved in two steps (for details see the appendix) allowing us to write the (diagonalized) Lagrangian density

\[
\mathcal{L} = -\frac{1}{4} F^2(A'') - \frac{1}{4} F^2(B'') + \frac{m_B^2}{2} (B'')^2 + \frac{m_A^2}{2} (A'')^2 + \bar{\chi} \left( i \partial_\mu \chi B'' + \xi_A A'' - m_\chi \right) \chi + \mathcal{L}_P, \tag{5}
\]

where \( \xi_A \approx -\frac{m_A^2}{m_B^2} \xi \) and \( \xi_B \approx 1 \) for \( \xi \ll 1 \), which is the case we study in our model. With these redefinitions, the Pauli term \( \mathcal{L}_P \) takes the form

\[
\mathcal{L}_P = \frac{g_\chi}{\Lambda} \bar{\chi} \left( \sigma_{\mu\nu} F^{\mu\nu}(B'') + \xi_A \sigma_{\mu\nu} F^{\mu\nu}(A'') \right) \chi. \tag{6}
\]

We see that the diagonalization introduces a coupling of the dark fermion to the visible photon both in the minimal as well as in the nonminimal terms.

For \( \Lambda = 8 m_\chi \), we see from \[6\] that \( g_\chi \) in the Pauli interaction can be thought of as the gyromagnetic factor of the dark fermion whose magnetic moment is given by \( \mu = \frac{1}{2} \sigma \) (\( \sigma \) denotes the spin of the fermion)

\[
\mu = -\frac{g_\chi}{2m_\chi} S,
\]

in complete analogy with standard QED. However, the second term in \[6\]

\[
\frac{g_\chi \xi_A}{8m_\chi} \bar{\chi} \sigma_{\mu\nu} F^{\mu\nu}(A'') \chi,
\]

represents an interaction of the dark fermion with the visible photon. In the static limit of a constant (visible) magnetic field \( B \) (\( B \) is the magnetic field associated with the visible photon \( A''_\mu \) and is not the space component of the dark photon \( B'_\mu \)) this term leads to

\[
\frac{g_\chi \xi_A}{8m_\chi} \bar{\chi} \sigma_{\mu\nu} F^{\mu\nu}(A'') \chi \xrightarrow{\text{static} A''} \bar{\chi} \left[ -\frac{g_\chi}{2m_\chi} S \cdot B \right] \chi.
\]

Therefore, the coefficient \( g'_\chi \equiv g_\chi \xi_A \) plays the role of the gyromagnetic factor for the interaction of the dark fermion with the visible photon. This is interesting because although \( g_\chi \) is an unknown parameter, as it originates in processes in the dark sector of the theory (namely, the first term in the Pauli coupling in \[6\]), the second term involves the interaction with the visible photon opening up new possibilities for its measurement.

The one loop quantum vertex corrections arising from both the minimal coupling terms in \[6\] involve the two diagrams shown in Figure \ref{fig:diagram}. Fig. \ref{fig:diagram}(a) denotes the vertex diagram contributing to the gyromagnetic factor in the dark sector, while Fig. \ref{fig:diagram}(b) contributes to the gyromagnetic factor for the interaction of the dark fermion with the photon in the visible sector. The detailed calculations for these are described in the following section.

III. DETAILS OF THE CALCULATION

We have already seen that in our model we can identify two gyromagnetic factors. One of them (\( g_\chi \)) is a genuine correction to the magnetic moment of the dark fermion obtained from diagram Fig. \ref{fig:diagram}(a) while the other comes from the interaction with the visible photon, as is shown in Fig. \ref{fig:diagram}(b). The integrands for the two diagrams are the same (except for a multiplicative constant) and, therefore, the calculations can be done in the conventional manner \[31\]. Even though this calculation is well known, we will describe this in some detail in order to clarify differences with the standard case.

The effect of loop contributions to the vertex is equivalent to modifying the vertex \( \gamma^\mu \rightarrow \Gamma^\mu \) where the new vertex function can be parameterized as

\[
\Gamma^\mu = \gamma^\mu F_1 \left( \frac{p^2}{m_\chi^2} \right) + \frac{\sigma_{\mu\nu}}{2m_\chi} p_\nu F_2 \left( \frac{p^2}{m_\chi^2} \right). \tag{7}
\]
Here, $F_1$ and $F_2$ are two form factors which depend on the Lorentz invariant combination $(p^2/m^2_{\chi})$ of the transferred momentum $p_\mu$ and the mass scale of the fermion $m_{\chi}$. Therefore, the diagram in Fig. 1(a) can be thought of as giving rise to the amplitude

$$iM^\mu = -i\bar{u}(q_2)\Gamma^\mu u(q_1) = -i\bar{u}(q_2)\left[F_1\left(\frac{p^2}{m^2_{\chi}}\right)\gamma^\mu + \frac{\sigma^{\mu\nu}}{2m_{\chi}}p_\nu F_2\left(\frac{p^2}{m^2_{\chi}}\right)\right]u(q_1),$$

where $q_1^\mu, q_2^\mu$ are the four momenta of the incoming and the outgoing fermions respectively. The four momentum of the external photon defines the momentum transfer $p^\mu$. Diagram in Fig. 1(b) gives rise to a similar contribution except that the coupling of the dark fermion with visible photon has an extra factor of $\xi_A$. Diagram (b), on the other hand, corresponds to the contribution coming from the coupling of the dark fermion to the visible photon. Both diagrams differ by a factor of $\xi_A$.

Form factors can be calculated from the diagrams as follows. The amplitude for the diagram in Fig. 1(a) is given by

$$iM^\mu = -i\bar{u}(q_2)\int \frac{d^4k}{(2\pi)^4} \frac{\eta_{\alpha\beta}(p + k + m_{\chi})\gamma^\alpha(k + m_{\chi})\gamma^\beta}{((k + q_1)^2 - m^2_{\chi} + i\varepsilon)((p + k)^2 - m^2_{\chi} + i\varepsilon)[k^2 - m^2_{\chi} + i\varepsilon]}u(q_1).$$

After some algebra (long but straightforward) one can identify contributions proportional to the matrix $\sigma^{\mu\nu}p_\nu$ which we are interested in (see [3]), to correspond to $(x, y, z)$ denote Feynman combination parameters

$$F_2^{(a)}(p^2) = -i 8 m^2_{\chi} \int_0^1 dx dy dz \delta(x + y + z - 1) \int \frac{d^4k}{(2\pi)^4} \frac{z(1 - z)}{(k^2 - \Delta + i\varepsilon)^3 + \cdots}$$

where $\Delta = -xy + (1 - z)^2 m^2_{\chi} + m^2_B z$.

The integral over $k$ can be calculated in the standard manner [31] and leads to

$$F_2^{(a)}(p^2) = \frac{1}{8\pi^2} \int_0^1 dx dy dz \delta(x + y + z - 1) \frac{z(1 - z)}{(1 - z)^2 + \kappa^2 z - x y m^2_{\chi}}.$$  

with $\kappa^2 = m^2_B / m^2_{\chi}$. This is the standard result of QED if the photon is massive which is the case for our hidden photon.

In the limit $p_\mu \to 0$, the form factor turns out to be

$$F_2^{(a)}(0) = \frac{1}{8\pi^2} \int_0^1 dx dy dz \delta(x + y + z - 1) \frac{z(1 - z)}{(1 - z)^2 + \kappa^2 z}.$$
Furthermore, in the limit $\kappa \to 0$ (massless dark photon), the value of the integral is $1/2$ and, in this case, $2F_2(0) = \alpha/2\pi$ (if we reinsert the factors of charge $e^2$ which we have set to unity in the hidden sector) which coincides with Schwinger’s result for the gyromagnetic factor $g - 2$ for QED \cite{9}. Furthermore, for diagram 1(b), which gives rise to the magnetic moment of the dark fermion through coupling to the visible photon, the calculation is completely parallel and we can write the results for the contributions from the two diagrams (to the magnetic moment) as

$$ F_2^{(a)}(0) = \frac{1}{8\pi^2} f(\kappa), \quad F_2^{(b)}(0) = \frac{\xi_A}{8\pi^2} f(\kappa). \quad (14) $$

The function $f(\kappa)$ can be obtained from (13) and a direct calculation gives

$$ f(\kappa) = -\frac{\kappa}{\sqrt{4 - \kappa^2}} \tan^{-1} \left( \frac{\sqrt{4 - \kappa^2}}{\kappa} \right) + \frac{1}{2} \left[ 1 - 2\kappa^2 + 2\kappa^2(\kappa^2 - 2) \ln \kappa \right]. \quad (15) $$

It may appear that this function is defined only for $0 < \kappa < 2$. However, this result is, in fact, valid for all values of $\kappa > 0$. This can be seen directly by noting that the imaginary parts arising from $\tan^{-1}$ (arctan) cancel with those coming from the square root in the denominator. Figure 2(a) illustrates this (reality) behavior.

The gyromagnetic factors coming from the two diagrams (a) and (b) ($g_\chi$ and $g'_\chi$) turn out to be (see (9) and (14))

$$ g_\chi - 2 = 2 F_2^{(a)} = \frac{1}{4\pi^2} f(\kappa), \quad (16) $$

$$ g'_\chi - 2 = 2 F_2^{(b)} = \frac{\xi_A}{4\pi^2} f(\kappa), \quad (17) $$

implying the identity

$$ g'_\chi - 2 = \xi_A(g_\chi - 2). \quad (18) $$

This relates the gyromagnetic factor in the dark sector to that for interaction with the visible photon.

A final comment is in order here. A different diagram, which can also contribute to the gyromagnetic factor of the dark fermion, can be obtained by replacing the internal dark photon propagator by that of the visible photon. Although this diagram is allowed from the interactions in (5), it is suppressed by a factor of $\xi_A^2$ (each visible photon vertex would have an extra factor of $\xi_A$) which is very small \cite{33} and we are discarding terms of this order in this calculation.

Possible values of $g_\chi - 2$ factor are shown in Figure 3 for different values of the fermion masses as a function of the gauge boson mass. We note from the shape of $f(\kappa)$ in Figure 2 (a) that, as the mass of the dark fermion becomes larger (for a fixed value of $m_B$), the value of the gyromagnetic factor approaches the value of the gyromagnetic factor of the electron only if the electric charge of the hidden fermion is equal to the electric charge of the electron. This behavior is expected because for heavy dark fermions the mass of the hidden photon can be (effectively) neglected which is like the visible sector (zero mass) photon. In this case, one obtains the same gyromagnetic factor provided the hidden electric charge is equal to the electron charge. It is also interesting to note that for low mass dark fermions,
VI. FINAL COMMENTS AND CONCLUSIONS

In this paper we have calculated the magnetic moment of the dark fermion in a model of hidden quantum electrodynamics coupled to visible photons through a kinetic mixing term. The kinetic mixing term couples the dark and the visible sectors. The dark fermion has a $g_\chi$ (gyromagnetic) factor associated with its interaction with the dark photon as well as a $g'_\chi$ factor because of its interaction with the visible photon. An explicit relation between $g_\chi$ and $g'_\chi$ has been obtained in [18]. From the analysis of [18] along with comparison with the data of measurements, we conclude that $g_\chi$ changes drastically from the conventional value if the dark fermions are light, otherwise the same behavior, as in standard (visible) quantum electrodynamics is obtained if the electric charge of the dark fermion is the same as the electron charge. It is quite possible that in other applications of quantum electrodynamics, such as atomic physics tests and pair production, one may find interesting results.
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V. APPENDIX: ON THE DIAGONALIZATION PROCEDURE

In this appendix we would like to give some details related to the diagonalization of the Lagrangian with the kinetic mixing, followed by the mass matrix diagonalization. Let us start with the Lagrangian density describing only the hidden ($B$) and the visible ($A$) gauge fields

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}(A) F^{\mu\nu}(A) - \frac{1}{4} F_{\mu\nu}(B) F^{\mu\nu}(B) + \frac{\xi}{2} F_{\mu\nu}(B) F^{\mu\nu}(A) + \frac{1}{2} m_B^2 B_{\mu} B^{\mu} + \frac{1}{2} m_A^2 A_{\mu} A^{\mu}. \]

\[ \equiv \mathcal{L}_g + \mathcal{L}_{\text{mass}}, \]

where $\mathcal{L}_g$ describes the massless part of $\mathcal{L}$.

In order to diagonalize the kinetic mixing term in [19], consider the following linear transformation of fields

\[ A_\mu = a A'_\mu + b B'_\mu, \quad B_\mu = c A'_\mu + d B'_\mu \]

with $a, b, c, d \in \mathbb{R}$. The massless part of $\mathcal{L}$ reads now

\[ \mathcal{L}_g = -\frac{1}{4} F^2(A') \left[ a^2 + c^2 - 2\xi ac \right] - \frac{1}{4} F^2(B') \left[ b^2 + d^2 - 2\xi bd \right] + \frac{1}{2} F(A') F(B') \left[ \xi (ac + bd) - ab - cd \right] \]

so that the condition for diagonalization becomes

\[ \xi = \frac{ab + cd}{ad + bc} \]

This condition reduces the number of parameters in the transformation (20) to three. There exist also one-parameter choices of coefficients $a, b, ...$ satisfying (22) which will be discussed at the end of the present section.

We redefine fields by

\[ \tilde{A}_\mu = A'_\mu \sqrt{a^2 + c^2 - 2\xi ac}, \quad \tilde{B}_\mu = B'_\mu \sqrt{b^2 + d^2 - 2\xi bd}, \]

and then the Lagrangian density $\mathcal{L}_g$ turns out to be

\[ \mathcal{L}_g = -\frac{1}{4} F^2(\tilde{A}) - \frac{1}{4} F^2(\tilde{B}). \]

The massive part of $\mathcal{L}$ in [19] now reads as

\[ \mathcal{L}_{\text{mass}} = \frac{m_B^2}{2} \left[ \tilde{a}^2(\tilde{A})^2 + \tilde{b}^2(\tilde{B})^2 + 2(\tilde{A}) \cdot (\tilde{B}) \tilde{c} \tilde{d} \right] + \frac{m_A^2}{2} \left[ \tilde{a}'^2(\tilde{A})^2 + \tilde{b}'^2(\tilde{B})^2 + 2(\tilde{A}) \cdot (\tilde{B}) \tilde{a} \tilde{b} \right], \]

with the definitions

\[ \tilde{a} = \sqrt{a^2 + c^2 - 2\xi ac}, \quad \tilde{c} = \frac{c}{\sqrt{a^2 + c^2 - 2\xi ac}}, \]

\[ \tilde{b} = \frac{b}{\sqrt{b^2 + d^2 - 2\xi bd}}, \quad \tilde{d} = \frac{d}{\sqrt{b^2 + d^2 - 2\xi bd}}. \]

Following [34], we rotate fields in order to diagonalize the new mass matrix. This rotation does not change the kinetic terms and it is given by

\[ \tilde{A}_\mu = \cos \theta A''_\mu + \sin \theta B''_\mu, \]

\[ \tilde{B}_\mu = -\sin \theta A''_\mu + \cos \theta B''_\mu. \]
The mass term in the Lagrangian can be written now

$$\mathcal{L}|_{\text{mass}} = \left( A'' \ B'' \right)_{\mu} \mathbb{M} \left( A'' \ B'' \right)^{\mu}.$$  \hfill (29)

with the mass matrix $\mathbb{M}$ given by

$$\mathbb{M}_{11} = (\hat{c}^2 m_B^2 + \hat{a}^2 m_A^2) \cos^2 \theta + (\hat{d}^2 m_B^2 + \hat{b}^2 m_A^2) \sin^2 \theta - (\hat{e} \hat{d} m_B^2 + \hat{a} \hat{b} m_A^2) \sin(2\theta)$$ \hfill (30)

$$\mathbb{M}_{22} = (\hat{d}^2 m_B^2 + \hat{b}^2 m_A^2) \cos^2 \theta + (\hat{c}^2 m_B^2 + \hat{a}^2 m_A^2) \sin^2 \theta + (\hat{e} \hat{d} m_B^2 + \hat{a} \hat{b} m_A^2) \sin(2\theta)$$ \hfill (31)

$$\mathbb{M}_{12} = (\hat{e} \hat{d} m_B^2 + \hat{a} \hat{b} m_A^2) \cos(2\theta) - \frac{1}{2} \left( (\hat{d}^2 - \hat{c}^2)m_B^2 + (\hat{b}^2 - \hat{a}^2)m_A^2 \right) \sin(2\theta)$$ \hfill (32)

$$= \mathbb{M}_{21}$$ \hfill (33)

The condition for diagonal matrix can be read from here to be

$$\tan(2\theta) = \frac{2(\hat{e} \hat{d} m_B^2 + \hat{a} \hat{b} m_A^2)}{(\hat{d}^2 - \hat{c}^2)m_B^2 + (\hat{b}^2 - \hat{a}^2)m_A^2}.$$ \hfill (34)

Finally, we impose the condition for non kinetic mixing \cite{22} through the following parametrization

$$a = \delta = \cosh \tau, \quad b = c = \sinh \tau,$$ \hfill (35)

with the condition $\tanh(2\tau) = \xi$. By replacing parameters according to \cite{23}, we find in the limit $\xi \ll 1$

$$\mathbb{M}_{11} = m_A^2 - \frac{m_A^4}{m_B^2 - m_A^2} \xi^2 + \mathcal{O}(\xi^4),$$ \hfill (36)

$$\mathbb{M}_{22} = m_B^2 + \frac{m_B^2}{m_B^2 - m_A^2} \xi^2 + \mathcal{O}(\xi^4).$$ \hfill (37)

Finally, we observe that the vertices in the coupling of hidden fermions with the $B$ field are modified since

$$B_{\mu} = \hat{c} \hat{A}_{\mu} + \hat{d} \hat{B}_{\mu},$$

$$= (\hat{c} \cos \theta - \hat{d} \sin \theta) A''_{\mu} + (\hat{c} \sin \theta + \hat{d} \cos \theta) B''_{\mu},$$

$$\equiv \xi_A A''_{\mu} + \xi_B B''_{\mu},$$ \hfill (38)

where the last line is a redefinition of the coupling constant. These coupling constant are related through

$$\xi_A^2 + \xi_B^2 = \hat{c}^2 + \hat{d}^2 = \frac{1}{1 - \xi^2},$$ \hfill (39)

which is independent of the parametrization of the restriction.

The coupling constants modifying the vertices in the limit $\xi \ll 1$ turn out to be

$$\xi_A = -\frac{m_A^2}{m_B^2 - m_A^2} \xi + \mathcal{O}(\xi^3),$$ \hfill (40)

$$\xi_B = \left( 1 + \frac{m_B^2 - 2m_B^2 m_A^2}{2(m_B^2 - m_A^2)^2} \xi^2 \right) + \mathcal{O}(\xi^3).$$ \hfill (41)

For $m_B \gg m_A$ we take, for linear corrections in $\xi$

$$\xi_A \approx -\frac{m_A^2}{m_B^2} \xi,$$ \hfill (42)

$$\xi_B \approx 1 + \xi^2 \approx 1.$$ \hfill (43)

\[1\] The influential and important work of Y. Zeldovich and collaborators is summarized in A. D. Dolgov and Y. B. Zeldovich, Rev. Mod. Phys. 53, 1 (1981).
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