A remaining useful life prediction method for complex systems based on multi-index fusion with MC and HSMM

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Abstract. Remaining useful life (RUL) estimation is the core and basic of system Prognostic and Health Management (PHM) and also a challenging for complex systems. It is necessary to use performance indicators that are closely related to system status for analysis, due to the Multi-index different characterization change of system degradation, different detectability and the degree of correlation caused by system coupling. As the system status degradation shows certain scientific laws in macro, there would be certain random relationship between the system status degradation and RUL. To address these problems, the concept of imperfect condition monitoring followed by the concept of key performance indicators in order to reduce the blindness of analysis object selection. The condition degradation probability index is proposed to represent the status degradation degree of the system, whose future trend is fitted by Markov matrix obtained by the improved algorithm as a implementation of mapping condition monitoring data to CDPI. Finally, the system RUL estimation method at time $t$ combined the hidden semi-Markov model with improved forward variable is given to realize the mapping CDPI to RUL. Experiments are carried out to validate the key concepts of the developed methods, and results suggest the effectiveness.

1. Introduction

The complex systems (products) are the significant representation of a country's manufacturing level. Performance is an important technical index to evaluate the complex systems, which will degrade over time and eventually result in failure. Therefore, it is necessary to study the complex systems degradation process, and predict RUL, which is the basis of Prognostic and Health Management, and it is also a key and difficult problem in engineering practice. Meanwhile, it can carry out maintenance timely and effectively, which has positive significance[1–2].

In general, RUL prognostics techniques or approaches can be divided into three categories: 1) data-driven; 2) physics-based; 3) hybrid-based. Interesting reviews related with prognostics are given in [4,7–8]. Due to complexity of complex systems, it is impractical to establish an feasible physical model to analyze the degeneration process. Therefore, we prefer data-driven methods. The data-driven approaches mainly use information from historical data or training data to identify the characteristics of current damage status so that we can establish degradation models to predict (RUL). Classical data-driven approaches include various neural networks (NN) [9-11], hidden Markov models (HMM) [3,14], support vector machine (SVM) [15-16], Gaussian process regression [17], etc.
In contrast, large-scale CNC equipment, high-tech integrated equipment and other complex systems (i.e. Aerospace equipment, nuclear equipment, large industrial machines, etc.) have only received increasing attention recently with the rise of condition-based maintenance (CBM) and predictive maintenance (PdM), and only a few papers have considered complex systems extensively. For example, Huang [5] et al. propose a consistency test method based on multi-stage Wiener process, which detects the consistency of monitoring data before and after maintenance of aero engines, and further improved the accuracy of RUL. However, the detailed calculation is not given in the paper. Zhao et al. [6] tries to learn the regularity characteristics of aircraft engine in the progressive degradation mode using the improved neural network. The approach is verified by different datasets. Vivek [13] et al. applied Arrhenius model followed by cumulative calculation in per period with the actual life consumption of the step-up transformer in estimating the RUL. Camci et al. [15] constructed the implementation of hierarchical HMMs as dynamic Bayesian networks for health-state and remaining useful life estimation in drilling processes. Hybrid-based approaches integrate the above-mentioned methods and techniques that are applied to improve the prediction performance as details can be seen in [12].

There are several main ideas in large part of literatures for complex systems RUL prediction. The first is mapping based idea, in which the key is to construct a mapping from current monitoring data to RUL. The difficulty is to ensure the reliability of the mapping[11]. The second is the model similarity-based idea, in which the models that have existed are used to predicted the RUL[18]. The third is implicit characteristics relationship-based idea. It is assumed that the condition monitoring (CM) data and system status are related in a stochastic way, in which RUL prediction can be achieved with system state assessment and future state estimation [19,20]. The difficulty is to ensure the approximate level of system status estimation, which is also the key points to improve the accuracy of RUL prediction.

This paper is focused on a data-driven RUL prediction method for CNC machine tools. The concept of Imperfect condition monitoring is firstly proposed, upon which the key performance indicators are applied to represent the performance indicators that have a great influence on machine tools status. And then a time-varying Markov model is designed which is assumed to conform to the system state transition rules, in whose five states are given and subdivided. The condition degradation probability index is proposed to represent the status degradation degree of the system, whose future trend is fitted by Markov matrix obtained by the improved algorithm as a implementation of mapping condition monitoring data to CDPI, which ensures that the probability matrix can truly reflect the degradation law of machine tool status. The CDPI correlates the condition monitoring data achieved from condition monitoring of CNC machine tools with the health status of the system and quantify the stochastic degradation process of machine tools. Finally, the system RUL estimation method at time t combined the hidden semi-Markov model with improved forward variable is given to realize the mapping CDPI to RUL. Finally, experiments are carried out to validate the key concepts of the developed methods, and results suggest the effectiveness.

2. Key performance indicators extraction
2.1. Key performance indicators extraction
Different CM data have different influence on the system condition, so the KPI is defined as the following.

Key performance indicators (KPI) is the CM data that have a significant influence on the whole system condition in all the CM data of machine tool condition assessment system.

To get the machine tools KPI, it is necessary to sort the influence. In consideration of the deficiency in the subjective and objective weighting methods, the paper presents a method combined variation coefficient and compound correlation coefficient. Then introduce the establishment method of the influence degree correction coefficient matrix \( R_q \), which consider the correlation of qualitative and quantitative indicators and reorder the influence degree. It will make the sorting result more accurate and practical. The Pareto principle is used here to extract KPI. It is considered that the first 20%
performance indicators have the greatest influence on the system status. So they can be regarded as machine tool KPI.

2.2. Performance indicators influence analysis.

The coefficient of variation and multiple correlation coefficient will be used in this analysis and calculation process. They are described as follows.

1) Variation coefficient

In general, the degree of difference between different performance indicators can be calculated by the coefficient of variation. The calculation equation is as follows (1),

\[
\bar{x}_i = \frac{1}{m} \sum_{j=1}^{m} x_{ij}
\]

\[
s_i = \left[ \frac{1}{(m-1)} \sum_{j=1}^{m} (x_{ij} - \bar{x}_j)^2 \right]^{1/2}
\]

Where \( \bar{x}_{ij} \), \( s_{ij} \), \( \omega_{ij} \) is the mean value, standard deviation and variation coefficient of all data in the \( i \)-th performance indicators, respectively and \( j \) is the number of \( i \)-th performance indicator data.

Then the weight of the \( i \)-th performance indicator \( \omega_i \) and weight vector \( W^\top = (\omega_1, \omega_2, \ldots, \omega_n) \) can be expressed by equation (2):

\[
w_i = \frac{w_j}{\sum_{j=1}^{m} w_j}
\]

\[
W^\top = (w_1, w_2, \ldots, w_n)
\]

2) Compound correlation coefficient

Pearson correlation coefficient is easy subjected to be influenced by data transfer effect, we use multiple correlation coefficient to measure the independence of different performance indicators, as calculated by (3),

\[
R = \begin{bmatrix}
R_i & r_i \\
\bar{R}_i^T & 1
\end{bmatrix}
\]

\[
\rho = \left( \bar{R}_i^T \bar{R}_i \right)^{-1/2}
\]

Where \( R \) is the Pearson correlation coefficient matrix of all indicators, \( i \) is the number of indicators, \( \bar{R}_i \) is the correlation matrix of \( R \) removed the \( i \)-th indicator. \( r_i^T \) is Transpose matrix of \( r_i \), \( \rho_i \) is the complex criterion coefficient, that is, the multiple correlation coefficient.

Then the weight of the \( i \)-th performance indicator and weight vector can be expressed by equation (4),

\[
w_i = \rho_i^{-1} / \sum_{i=1}^{n} \rho_i^{-1}
\]

\[
W^\top = (w_1, w_2, \ldots, w_n)
\]

3) Performance indicators influence

The influence weight of machine tool performance indicators \( \omega_i \) can be calculated by equation (5),

\[
w_j^* = w_j w_j^2 / \sum_{i=1}^{m} [w_i w_i^2]
\]

\[
w_j^* = w_j \omega_i \omega_i \omega_i / W^\top W^\top
\]
Where $\omega_i^*$ is the weight considering the correlation and independence of performance indicators. Then the standard weight vector $W^*$ can be expressed as (6),

$$W^* = (w_1^*, w_2^*, \cdots, w_n^*)$$

(6)

### 2.3. Performance indicator weight correction.

The weight calculated only considers the relativity and independence of quantitative performance indicators, which is not comprehensive. In fact, qualitative indicators such as the degree of occurrence, detection degree and maintainability also influence system status.

1. Correction factor

   Occurrence degree, detection degree and maintainability, are selected as the three main types of qualitative performance indicators. Expert scoring method can be used to give the score of the correction factor in the lack of data. The corresponding scoring rules are shown in Table 1.

   **Table 1.** Correction factors and scoring principles.

| Correction factor | Scoring principle |
|-------------------|-------------------|
| Occurrence degree | According to the possibility of occurrence of performance indicators, the higher the occurrence of performance indicators, the greater the impact on the state of machine tools. |
| Detection degree | According to the different degree of performance index detection, the lower the degree of performance index detection, the smaller the impact on machine status. |
| Maintainability | The maintainability is evaluated according to the difficulty degree of performance index repair. The lower the maintainability of performance indicator, the greater the impact on the machine status. |

The fuzzy description, definition and scoring rules of each correction factor in Table 1 can be seen in Table 2 to 4.

**Table 2.** Description, definition and occurrence scoring.

| Description    | Definition                                                                 | Scoring |
|----------------|---------------------------------------------------------------------------|---------|
| Inevitable     | It happens at least once a day, or almost every day.                      | 10      |
| Very high      | Happens almost weekly.                                                    | 9       |
| High           | Occurs at least once a month.                                             | 8       |
| Moderate       | It happens every four months.                                            | 6, 7    |
| Low            | Occurs rarely or every six months.                                       | 4, 5    |
| Very low       | Occurs rarely or once a year.                                            | 2, 3    |
| Impossible     | There have been few failures and no records of failures                  | 1       |

**Table 3.** Description, definition and detection degree scoring.

| Description        | Definition                                                                 | Scoring |
|--------------------|---------------------------------------------------------------------------|---------|
| Extremely low      | Existing methods cannot detect failures.                                  | 10      |
| Very low           | Fault detection is difficult, and the detection process is not easy to implement. | 8, 9    |
| Low                | Lack of suitable testing equipment can only rely on manual testing, manual testing, and luck testing. | 6, 7    |
| Common             | Failure requires multiple inspections, and the inspection process needs to be strictly controlled to get a correct conclusion. | 5       |
| High               | The correct conclusion can be drawn only by detecting the fault once, but the detection process is not automatic. | 3, 4    |
| Very high          | The fault only needs to be detected once to obtain the fault.             | 2       |
| Extremely high     | The machine is equipped with an automatic stop or limit program to avoid malfunctions. | 1       |
Table 4. Description, definition and maintainability scoring.

| Description     | Definition                                                                 | Scoring |
|-----------------|-----------------------------------------------------------------------------|---------|
| Extremely High  | Performance indicators are very easy to maintain, and the system can be     | 1       |
|                 | restored to the previous working state after repair.                       |         |
| Very high       | The performance indicators are easy to maintain, and the restored          | 2,3     |
|                 | performance indicators restore the system to the previous working state.    |         |
| High            | Performance indicators are difficult to maintain, but the repaired          | 4,5     |
|                 | performance indicators can restore the system to the previous working state.|         |
| Common          | Performance indicators are more difficult to maintain, but the repaired     | 6,7     |
|                 | performance indicators can restore the system to the previous working state.|         |
| Low             | Performance indicators are more difficult to maintain, and it is more       | 8       |
|                 | difficult to restore the system to the previous working state after repair. |         |
| Very low        | Performance indicators are difficult to maintain, and it is difficult to     | 9       |
|                 | return to the previous working state after repair.                         |         |
| Extremely low   | The performance index is very difficult to maintain, even if it is repaired,| 10      |
|                 | it cannot restore the working state of the previous moment.                |         |

(2) Influence correction coefficient matrix

The influence of performance indicators will be different as a result of qualitative indicators influencing quantitative indicators. To solve the problem, the grey matter-element method (GME) is adopted to construct the influence correction coefficient matrix to correct the influence weight of the performance indicators. The GME is described as follows:

A. GME matrix and optimal GME matrix

GME is an ordered triple \( R = (N, C, V) \) to describe the basic element of things, which includes the name of the thing \( N \), the characteristic \( C \), and the gray value \( V \). It is assumed that the machine tool monitoring items, performance indicators, and monitoring data in different time periods are treated as the names, features, and gray values in the triplet, the GME matrix \( R_{mn} = (N_m, C_m, V) \) can be expressed by equation (7),

\[
R_{mn} = (N_m, C_m, V)
\]  

(7)

Where \( N_i (i = 1, 2, \ldots, m) \) is the \( i \)-th machine tool monitoring item, \( C_j (j = 1, 2, \ldots, n) \) is the \( j \)-th performance indicator, and \( V_{ij} \) is the gray value of each performance indicator. Then the optimal gray matter-element matrix \( R_o \) is determined with the relative optimization criterion to compare the different performance indexes of the machine tool, as expressed by equation (8).

\[
R_o = (N_o, C_o, V_o)
\]  

(8)

B. Association analysis

In order to eliminate analysis error, standard quantification is made by the relative optimization criterion, as shown in Table 5. for details.

Table 5. Relative optimization criteria.

| The bigger the better | The smaller the better | Moderate |
|-----------------------|------------------------|----------|
| \( V'_{ij} = \frac{V_{ij} - \min V_{ij}}{\max V_{ij} - \min V_{ij}} \) | \( V'_{ij} = \frac{\max V_{ij} - V_{ij}}{\max V_{ij} - \min V_{ij}} \) | \( V'_{ij} = \frac{\min (V_{ij}, u_{ij})}{\max (V_{ij}, u_{ij})} \) |

C. Influence degree correction coefficient matrix

The influence degree correction coefficient matrix \( R_{\lambda} \) can be expressed by equation (9),

\[
R_{\lambda} = (N_m, C_n, \lambda_{\text{norm}})
\]  

(9)
Where $\lambda_{ij}$ is Correlation coefficient gray value after the performance indicator is quantized.

The gray value of the correlation coefficient $\lambda_{ij}$ can be calculated by equation (10),

$$\begin{align*}
\Delta_{ij} &= |\otimes V_{ij} - \otimes V_{ij}| \\
\lambda_{ij} &= \frac{\Delta_{\text{max}} + \zeta \Delta_{\text{max}}}{\Delta_{ij} + \zeta \Delta_{\text{max}}} 
\end{align*}$$  

(10)

Where: $\otimes V_{ij}$ is the optimal quantized gray value, $\zeta$ is the resolution coefficient.

In general, resolution coefficient has influence on the results. But it can be estimated according to equation (11) and experience, as shown in Table 6.

$$\begin{align*}
\Delta_{ij} &= \frac{1}{m n} \sum_{j=1}^{m} \sum_{i=1}^{n} \Delta_{ij} \\
\varepsilon &= \frac{\Delta_{\text{max}}}{\Delta_{ij}} 
\end{align*}$$  

(11)

Where $\Delta_{ij}$ is the average of range sequence, and $\varepsilon$ is a judgment factor.

### Table 6. Selection principle of $\zeta$.

| Resolution coefficient | Reference for X-ray spectral data analysis | Description | Value range |
|------------------------|------------------------------------------|-------------|-------------|
| $\zeta$                |                                         | Stable sequence ($1 \leq \varepsilon \leq 6$) | $\zeta \geq 0.5$ |
|                        |                                         | Fluctuate sequence ($\varepsilon > 6$) | $0 < \zeta < 0.5$ |
|                        |                                         | Special sequence ($\varepsilon > 10$) | $\zeta < 0.1$ |

(3) Key performance indicator extraction

The weight of $i$-th performance indicator can be corrected by influence degree correction coefficient matrix $R_{ij}$, as shown in equation (12).

$$\begin{align*}
W &= (w_1, w_2, \ldots, w_n) = R_{ij} W^T \\
w_i &= \lambda_{ij} w_i^* + \lambda_{ij} w_2^* + \ldots + \lambda_{ij} w_n^* 
\end{align*}$$  

(12)

The standardized weight vector $W$ and correction weight $w_i$ can be expressed as below (13), the corrected weights are used to re-order influence. Then the KPI can be extracted with extraction criteria.

$$\begin{align*}
W &= (w_1, w_2, \ldots, w_n) \\
w_i &= \frac{w_i}{\sum_{j=1}^{m} w_j} 
\end{align*}$$  

(13)

### 3. Proposed CDPI-based RUL prediction

The proposed scheme is shown in Figure 1. In applying this scheme, there are three requisite questions that need to be addressed: 1) how to define the CDPI; 2) How to obtain CDPI from KPI data, and 3) how to map CDPI to the RUL prediction. They will be addressed below.
3.1. Condition deterioration probability indicator (CDPI)
CDPI is used to represent the overall degradation degree of a machine tool. Without loss of generality, CDPI is set to have the range of $[0, 1]$, where 0 means the machine tool is new, while 1 means the failure (i.e., soft failure). The degradation can be assumed to be either linear or nonlinear. Figure 3 shows the linear growth (see "CDPI-linear") and nonlinearly growth (see "CDPI-nonlinear").

Intuitively speaking, nonlinear CPDI seems to be more realistic than linear CPDI. However, without sufficient prior knowledge, it is difficult for us to determine RUL with the Mathematical expression form of nonlinear CPDI. Therefore, we need to find a suitable RUL expression with linear CDPI.

3.2. Markov algorithm model
We would like to highlight that both linear and nonlinear CPDI describe the relationship of CDPI to the time $t$, but not to the data $X$ observed at time $t$. That is, the growth, decline or shock in the trend of CDPI is caused by the uncertainty and stochasticity of data. Therefore, it is necessary to find an algorithm model for CDPI fitting. To solve this problem, the Markov algorithm model with good prediction of uncertainty and stochasticity is used to fit CDPI trend.

3.2.1. Markov assumption.
We then use an Markov model for fitting CDPI by giving the following assumptions.
Assumption 1. The status stochastic degradation can be considered as multiple degradation processes, each of which is regarded as a status, and all status constitute Markov status space.
Assumption 2. Only consider the situation that the machine tools work in normal conditions instead of the special situations such as shutdown, serious uncontrollable faults.
Assumption 3. The status at time $T+1$ is only related to time $T$, and has nothing to do with the time before time $T$.
Assumption 4. We assume that the sub-status degradation will not affect system status. In other words, the CM data is only related to system status, and will not be affected by the degradation of sub-status.

3.2.2. Markov model.
The degradation process are divided into five status corresponding to five intervals in linear CDPI in Figure.2 following Markov hypothesis. The Markov model topology diagram is shown in Figure.3.

Figure 1. CDPI-based RUL prediction scheme.

Figure 2. Example of linear and nonlinear degradation of CDPI.

Figure 3. Markov model topology diagram.

Explanations need for Figure.3 are that the status degradation is a irreversible process, that is, reverse jump transition is impossible (e.g., the third status transfers to the first). However, due to maintenance, the degradation status will change, which is generally very small. We assume that it
can restore to the previous status before a series of maintenance except for failure (i.e. the transition from the second status to the first). Based on the second hypothesis proposed, the status can move across one status to another.

### 3.3.3. Data processing.
To better reflect the characteristics of CDPI scientifically and accurately with CM data, we assume that CM data and CDPI satisfy a certain functional relationship, as expressed in (14),

\[
CDPI(t) = \rho \left[ \delta_1(t), \delta_2(t), \ldots, \delta_n(t) \right]
\]  (14)

Where \( \delta_i(t) \) is KPI of extracted in Section 2.3, \( \rho \) represents the relationship function. We use a linear weighted mathematical model to calculate CDPI, which is expressed as (15)

\[
CDPI(t) = \rho_1 \delta_1(t) + \rho_2 \delta_2(t) + \cdots + \rho_n \delta_n(t)
\]  (15)

Where \( \rho_i \) represents the weight of KPI, which can be obtained by weight reassignment. Before the fitting, all the CM data are first normalized so that the CM data can map to five status interval of CDPI 0-1. The \( |Z| \)-score approach was selected for the normalization, as calculated by (16), where \( \mu \) and \( \sigma \) are

\[
|Z| = \frac{|x - \mu|}{\sigma}
\]  (16)

the mean and standard deviation of a feature in the CM data, \( x \) is the original data before normalization, and \( |Z| \) is the normalized data of the CM data.

### 3.3.4. Improved algorithm and CDPI fitting.
The key to Markov model is the construction of status transition probability matrix. Due to the state transition probability is fixed, which can not track the status degradation very well and lack of real-time, the result has a relatively large error with practice and does not have a good robustness. Therefore, we must optimize with a algorithm shown in Figure 4. The key part is described as follows:

1. **Initial state transition probability matrix** \( P_{t_0} \)
   - The initial state transition probability matrix \( P_{t_0} \) can be constructed as equation (17)
   \[
P_{t_0} = P_{t_0} \left( T_a \right)
   \]  (17)

2. **Fitting error**
   - The fitting error indicates the degree of similarity of the system state \( s_i \) at time \( T \), as indicated in equation (18),
   \[
e_i(T) = p_i(T) - \sum_{j=1}^{n} P_{ij} p_j(T-1)
   \]  (18)

   The degree of similarity is determined by a likelihood function, which is used to determine whether the fitting error is within the allowable range.

3. **Status fitting degree**
   - The fitting degree \( Q \) is called the sum of all states of the \( Q \), as indicated by equation (19),
   \[
Q = \sum_{i=1}^{n} \sum_{T=1}^{T_a} \left[ e_i(T) \right]^2
\]  (19)

4. **Objective function**
   - It transforms the solution of the state transition probability matrix into a nonlinear optimization of the
Figure 4. The algorithm flowchart.

objective function, that is, when the fitness $Q$ reaches the minimum which is given, we can obtain the state transition probability matrix that we need. The Objective function is established as followed:

$$\begin{align*}
\min Q &= \sum_{i=1}^{n} \sum_{j=1}^{T} \left[ p_i(T) - \sum_{j=1}^{n} P_{ji}(T-1) \right] \\
\sum_{j=1}^{n} P_{ji} &= 1, i = 1, 2, \cdots, n \\
0 &\leq P_{ij} \leq 1, i, j = 1, 2, \cdots, n
\end{align*}$$

(5) probability matrix column transformation and row transformation

The matrix row transformation in the algorithm is shown in the equation (21),

$$\begin{align*}
P_{(T_e+1)ij} &= \frac{P_{T_eij} \cdot Q_{T_ej}}{Q_{(T_e+1)i}} \\
Q_{(T_e+1)i} &= \sum_{j=1}^{n} P_{(T_e+1)ij} \\
Q_{(T_e+1)j} &= Q_{T_ej}
\end{align*}$$

The matrix column transformation in the algorithm is shown in the equation (22),
3.4. RUL prediction based HSMM

We assume the full life cycle \( T \) is used to represent the full length of a brand new device, and \( D(s_i) \) represents the duration time of state \( s_i \), the n-th status represents the failure, then \( T \) can be expressed as (23):

\[
T = \sum_{i=0}^{n-1} D(s_i)
\]

Then, the concept of state transition point is given to represent the time point of system transition. Markov model can well determine the state transition probability, however, it cannot obtain the status duration time. If the system starts to predict RUL at time \( T_0 \), it is necessary to determine the remaining time in this status. Therefore, HSMM is used to calculate the status duration time based on Markov model. The parameter mapping relationship of the HSMM is shown in Figure 5. The system state transition. The specific form and expression of parameters are listed in Table 7.

![Figure 5. The parameter mapping relationship of the HSMM.](image)

**Table 7. Description, expression and Symbol of parameters in HSMM.**

| Description     | Expression | Symbol |
|-----------------|------------|--------|
| system status   | \( S_i \)  | \( S_n \) |
| I sub-status    | \( S_{r_1} \) | \( S_{r_n} \) |
Duration time

\[ D_1 = T_1 \]
\[ D_2 = T_2 - T_1 \]
\[ \cdots \]
\[ D_{n-1} = T_{n-1} - T_{n-2} \]
\[ D_n \]

Time unit

\[ (1, \ldots, T_1) \]
\[ (T_1 + 1, \ldots, T_2) \]
\[ \cdots \]
\[ (T_{n-2} + 1, \ldots, T_{n-1}) \]
\[ T_n \]

II sub-status

\[ (s_1, \ldots, s_{T_1}) \]
\[ (s_{T_1+1}, \ldots, s_{T_2}) \]
\[ \cdots \]
\[ (s_{T_{n-1}+1}, \ldots, s_{T_n}) \]

Data

\[ (x_1, \ldots, x_{T_1}) \]
\[ (x_{T_1+1}, \ldots, x_{T_2}) \]
\[ \cdots \]
\[ (x_{T_{n-1}+1}, \ldots, x_{T_n}) \]

However, the transition between two sub states \( s_{i-1} \rightarrow s_i \) do not satisfy Markov process. Then, we can combine Markov model and Hidden semi-Markov model as the implementation of RUL estimation. The RUL network of state transition prediction based on MC and HSMM is shown in Figure 6.

If the starting point is randomly given for predicting RUL (i.e. \( T_i + k \) in Figure 7.), then the predicted RUL expression can be expressed as (25),

\[ RUL = d_k (s_{T_i}) + \sum_{j=i+1}^{n-1} D(s_{T_j}) \]  \hspace{1cm} (25)

Where \( d_k (s_{T_i}) \) represents the remaining stay time in this state, and \( k \) represents the accumulated time length. We use improved forward backward algorithm to estimate the remaining stay time. We improve the forward variables to represent the probability under the condition that we know the observation sequence, in which the status of time \( T_i + k \) is \( s_{T_i} \) and the remaining stay time is \( \tau \), as shown in equation (26),

\[ \alpha_{T_i+k} (i, d) = P \left( s_{T_i+k} = s_{T_i}, \tau = d \mid x^{T_i+k-1} \right) \]  \hspace{1cm} (26)

The recursive equation of the forward algorithm is expressed by (27),

\[ \begin{cases} 
\alpha_{T_i+k+1} (i, d) = S_{T_i+k} (i) p_i (d) + q_i (x^{T_i+k}) \alpha_{T_i+k} (i, d+1) \\
p_i (d) = P_{d-1} (1 - P_d) 
\end{cases} \]  \hspace{1cm} (27)

Then, the remaining stay time \( d_k (s_{T_i}) \) of the current state at time \( T_i + k \) is expressed in equation(28),

\[ d_k (s_{T_i}) = \sum_{d=1}^{D} \alpha_{T_i+k+1} (i, d) \cdot (d + 1) \]  \hspace{1cm} (28)

The duration of each I sub-status can be solved by modeling the state duration density function \( P(D_n | s_{T_i}) \) with a single Gaussian distribution estimation, and finally the mean and variance of the data are used to obtain the state duration time, as indicated in equation (29),

\[\text{Figure 6. The RUL network of state transition prediction based on MC and HSMM.}\]
\[
\begin{align*}
D_n(s_r) &= \mu(s_r) + \rho \sigma^2(s_r) \\
\rho &= \left( T - \sum_{j=0}^{n-1} \mu(s_r) \right) / \sum_{j=0}^{n-1} \sigma^2(s_r)
\end{align*}
\]

(29)

4. Case studies
We selected 25 kinds of condition monitoring data from the monitoring data management system as the machine tool performance indicators, and calculated their weights to find KPI. Weight value for each performance indicator is given in Table 8.

**Table 8. Weight value for each performance indicator.**

| Weight | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|        | 0.01| 0.0239 | 0.0147 | 0.088 | 0.0057 | 0.19 | 0.009 | 0.021 | 0.017 |
| 10     | 0.02 | 0.018 | 0.201 | 0.152 | 0.0154 | 0.0114 | 0.0164 | 0.002 | 0.007 |
| 19     | 0.026 | 0.024 | 0.015 | 0.109 | 0.001 |       |       |       |       |

The key performance indicators are extracted with the extraction method and redistribute the weight. The results can be seen in Table 9.

**Table 9. Weight for key performance indicators (KPI).**

| KPI | 1   | 2   | 3   | 4   | 5   |
|-----|-----|-----|-----|-----|-----|
|     | 0.272 | 0.257 | 0.205 | 0.147 | 0.119 |

Then we use equation (14) and equation (15) to fuse KPI into CDPI. We use improved Markov matrix to fit CDPI. The results are shown in Figure 7 and Figure 8. Figure 7 shows the results with fixed state transition probability matrix. It can be found it can obtain better fitting results in early stage, but poor fit since 120 (cycle), so it will make a big error in RUL prediction. Figure 8 shows the results with improved algorithm is significantly better than Figure 7, which provides a mathematical basis for RUL prediction. The state transition probability obtained by algorithm is given in Table 10.

**Figure 7. CPDI fitting with constant status transition probability matrix**

**Figure 8. CPDI fitting with improved status transition probability matrix**

**Table 10. Transition probability between five system status.**

| Status  | Normal | Better | Medium | Bad | Very poor |
|---------|--------|--------|--------|-----|-----------|
| Normal  | 0.8913 | 0.0453 | 0.0621 | 0.0000 | 0.0000    |
| Better  | 0.2125 | 0.6901 | 0.0444 | 0.0530 | 0.0000    |
| Medium  | 0.0000 | 0.1735 | 0.7562 | 0.0691 | 0.0012    |
| Bad     | 0.0000 | 0.0000 | 0.1374 | 0.8503 | 0.0123    |
| Very poor | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000    |

Finally, using CDPI data for RUL prediction, HSMM model training is used to obtain the average value and variance of each working condition duration. The results are compared with the traditional
RUL calculation method. The results are shown in Table 11. From the historical data, it can be seen that the historical average life cycle of the machine tool is 180 units. From table 11, we can see that under the condition that the variance is approximately close to the confidence interval, the error between the average life of the whole life cycle calculated by the proposed method and the historical average life time is only 0.0214, which is less than the error of 0.0394 calculated by the traditional method, which is closer to the machining machine. The actual average life of the bed shows that the method based on MC and HSMM is effective and can be applied to the practical application of large data sets.

Table 11. Life calculation results of different methods.

| Status   | Life cycle |
|----------|------------|
|          | Normal     | Better | Medium | Low |          |
| Proposed method | 176.14   | 73.56  | 52.43  | 30.23 | 20.25  |
| Traditional method | 187.1    | 78.14  | 56.25  | 31.52 | 21.46  |

5. Conclusion
1) Combination weighting methods are applied to establish analysis mathematical model for performance indicators. The intrinsic link between system status and CM data is mathematically analyzed to further obtain machine tool KPI.
2) The Markov matrix obtained by improved algorithm is applied to fit the CDPI, which has solved the problem of large fitting error of original matrix and provide a mathematical basis for RUL prediction.
3) To accurately estimate the status duration time and remaining stay time, the explicit expression of HSMM is used to train and obtain the mean RUL.
4) Problems that need to be further solved: First, how to prove accuracy of KPI and the comparative tests need to be implemented. Second, to approach the machine tool actual status, errors caused by sub-status change should be considered.

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