Accuracy Degradation Model and Residual Accuracy Life Prediction of CNC Machine Tools Based on Wiener Process

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Abstract. In order to predict the accuracy residual life of CNC machine tools, an accuracy degradation model and accuracy residual life prediction method of CNC machine tools based on accuracy degradation data are proposed. Considering the non-linear and uncertainties in the process of accuracy degradation, an accuracy degradation model of CNC machine tools was established based on the nonlinear Wiener process; The accuracy failure is judged when the measured accuracy value exceeds the failure threshold for the first time. An accuracy residual life prediction model based on Inverse Gaussian distribution is established, and the accuracy residual life is calculated as a probability density function. To solve the problem of lack of historical data and prior information for a single CNC machine tool, particle filter is used to recursively update the posterior values of the model parameters based on the current observation data to achieve continuous optimization of the residual life prediction results; Finally, the correctness and effectiveness of the accuracy degradation model and the prediction method of the accuracy residual life are verified through the example analysis of the accuracy degradation experimental data of CNC machine tool.

Index terms. CNC machine tools, accuracy retentivity, residual life prediction, Wiener process, particle filter.

1. Introduction
Accuracy retentivity is one of the important indexes to evaluate the performance of CNC machine tools, and it is also the main bottleneck to affect the performance of domestic CNC machine tools.
Accuracy retentivity is the ability that the accuracy characteristics of machine tools can be kept within the required range for a long time under normal use conditions. Accuracy retentivity has a crucial impact on the machining quality and efficiency of CNC machine tools [1].

The accuracy retentivity of CNC machine tools mainly depends on the design, manufacturing and assembly. However, the ability to maintain the accuracy of CNC machine tools is difficult to be quantified in these links and factory acceptance, it can only be reflected in the use process. And in the process of using the machine tool, users often lack the effective prediction of the accuracy index of the machine tool. Only when there is a large degree of accuracy degradation and product quality problems, the manufacturer can be invited to carry out maintenance, this kind of passive maintenance makes the long maintenance time and the high maintenance cost [2]. Therefore, it is necessary to analyze the degradation process of CNC machine tools and predict the accuracy residual life, for realizing the active maintenance of the accuracy of the CNC machine tools and reduce the maintenance cost.

Considering that CNC machine tools is a large and complex product, it not only has high technology content and complex structure, but also has various failure modes, and the performance of each functional component affects each other [3]. At the same time, the factors that affect the accuracy of each component are more complicated. At present, the research on the prediction of accuracy residual life mainly focuses on several key components that constitute the CNC machine tool, such as feeding system [4-6], spindle [7-8], CNC turntable [9]. However, there are few reports on residual life prediction of complex mechanical equipment such as CNC machine tools. Due to the lack of comprehensive consideration of various factors affecting the accuracy degradation, it is difficult to effectively apply the research on the accuracy retention of key components to the accuracy retentivity of the whole CNC machine tools. Therefore, it is urgent to establish a reasonable accuracy degradation model to accurately predict the accuracy residual life of CNC machine tools.

At present, the residual life prediction methods mainly include physical model methods and data-driven methods [10]. Based on the mechanism of accuracy degradation, a physical model is established, which can be used to predict the residual life without analyzing a lot of data. Fan et al. [5] proposed a mathematical model that can calculate the geometric error caused by the wear of the guide rail according to the given parameters, and then can predict the positioning error of the guide rail after a period of work. Cheng et al. [6] established a dynamic model for the ball screw of the machine tool feed system to describe the sliding and rolling between the ball and the raceway, and thus realized the wear prediction of the ball screw. Tan Yanqing et al. [11] based on the wear characteristics of sliding guide rails of machine tools, established an accuracy retentivity model of sliding guide rails based on the quantitative relationship between the accuracy of the guide rail and the working condition parameters, operating parameters, material properties and wear status of the guide rail. Physical models usually need to fully study the deep-seated failure mechanism, but for complex mechanical and electrical equipment such as CNC machine tools, it is very difficult to establish a model that can accurately and comprehensively reflect the degradation of the accuracy of CNC machine tools.

The data-driven methods predict the residual life directly from the condition monitoring data by machine learning, probability and statistics [12]. Machine learning uses neural network, support vector
machine, hidden Markov model and other methods to fit historical data for residual life prediction. Deng Chao et al. [4] established a mapping model of positioning error of machine tool feed system based on BP neural network, and then realized the prediction of positioning accuracy. Wang Gang et al. [9] proposed using hidden Markov model and particle filter algorithm trained by vibration signal as prediction model to calculate the accuracy residual life of CNC turntable. Wang et al. [13] Proposed a state prediction method of CNC machine tools based on support vector machine, using the vibration signal sequence data collected in the processing process to predict the performance state trend of CNC machine tools. The model based on machine learning has a high fitting accuracy to the existing data, but the long-term prediction effect is often poor. When building the model, a lot of training is often needed, which requires a lot of data and time. At the same time, the physical interpretation ability of the degradation process is poor. The prediction methods of probability and statistics are based on the theory of random process and statistical inference to establish the prediction model of performance degradation, and the prediction results are provided by the probability density function of residual life. Guo et al. [14] proposed a performance evaluation method for CNC machine tools based on degradation analysis, which regards performance degradation as an independent incremental process, using gamma process to describe the performance degradation of CNC machine tools. Deng Chao et al. [3] used Wiener process to model the performance degradation of CNC machine tools, and proposed a residual life prediction method based on threshold distribution.

2. Establishment of accuracy degradation model

The accuracy degradation process of the machine tool is the result of the wear of the moving parts and the residual stress release of the bed, the preload loss of the bolts, and the assembly stress [15]. The degradation process caused by wear has certain directionality and determinacy, while the degradation process caused by residual stress release and bolt preload loss has greater randomness. Therefore, the process of accuracy degradation is a combination of regular degradation and uncertainty degradation, it is not a completely deterministic and monotonic degradation process. At the same time, in the actual measurement process of machine tool accuracy, due to the influence of temperature, air pressure, relative humidity, accuracy of measuring instruments and other factors, measurement errors are inevitable, which cannot accurately measure the state of degradation of accuracy, resulting in measurement uncertainty.

The degradation model established based on the random process can well describe the uncertainty existing in the accuracy degradation process of machine tools. At the same time, it can make a reasonable explanation for the degradation mechanism of accuracy. Therefore, compared with physical model and machine learning, the statistical prediction method based on stochastic process is more suitable for residual accuracy life prediction of CNC machine tools. Common random processes include Gamma process, Wiener process and Markov chain. Since the Gamma process is a monotonous random process, it is only suitable for degradation modeling of measurement data with strict monotonic degradation characteristics. At the same time, the increment of the Gamma process conforms to the Gamma distribution. The expression of this distribution is more complicated and difficult to base on the latest degradation data. Update the model; due to the memorylessness of the Markov chain, the accuracy life prediction result based on this model is only determined by the accuracy state of the current machine tool, and the measurement data of the entire degradation process.
cannot be fully utilized, and it is difficult to guarantee the accuracy of the prediction result. At the same time, the discrete characteristic of Markov chain makes it difficult to model a machine tool with a continuous degradation process. Wiener process is a random process with independent increments, also known as Brownian motion process. It is a mathematical model of Brownian motion. Brownian motion is a Gaussian process with zero mean and time-dependent variance. The randomness of Brownian motion can be better. The ground describes the uncertain degradation process. At the same time, due to the good mathematical properties of the Wiener process, the analytical formula of the probability distribution of the remaining life can be directly obtained.

Therefore, this paper uses the Wiener process to model the accuracy degradation process of the machine tool, and its expression is:

$$X(t) = \lambda t + \sigma_B B(t)$$

(1)

Where: $\lambda$ is the drift coefficient; $\sigma_B$ is the diffusion coefficient; $B(t)$ is standard Brownian motion, which has the following three properties:

1. $B(0) = 0, \quad B(t) \in (-\infty, +\infty)$;
2. $B(t) - B(t - \Delta t) \sim N(0, \Delta t)$;
3. $B(t) \sim N(0, t)$.

According to the actual degradation of the accuracy of CNC machine tools, a nonlinear Wiener process is used to model the degradation process of accuracy in this paper. The expression of the machine tool accuracy state $x(t)$ at time $t$ is:

$$X(t) = x_0 + \lambda A(t; \theta) + \sigma_B B(t)$$

(2)

Where: $x_0$ is starting point of degradation process, represents the initial accuracy state of CNC machine tools; $\lambda$ is the degradation rate of the regular degradation of machine tool accuracy; $A(t; \theta)$ is nonlinear function of time $t$, characterizing the trend of machine tool accuracy degradation; if $A(t; \theta) = t$, then the model is transformed into a linear model of formula (1); $\sigma_B B(t)$ is used to describe the uncertainty degradation in the process of accuracy degradation of CNC machine tools.

At the same time, in the actual measurement process of machine tool accuracy, due to the influence of instrument accuracy, working environment and other factors. The measured degradation data cannot fully show the real degradation state of the machine tool. To describe the uncertainty of real state mapping to observation data, the observation equation can be expressed as:

$$Y(t) = X(t) + \varepsilon$$

(3)

Where: $\varepsilon$ is the measurement error, $\varepsilon \sim N(0, \sigma^2)$. Considering that the measurement of the accuracy of CNC machine tools is discrete, for a certain accuracy index of CNC machine tools. If the accuracy degradation data monitored at the time $t_1 < t_2 < \cdots < t_n$ is $Y_{1n} = (y_1, y_2, \cdots, y_n)$, the corresponding real accuracy state is $X_{1n} = (x_1, x_2, \cdots, x_n)$. After obtaining the interval measurement data of the accuracy of CNC machine tools, a discrete state transition model can be established. Because of the Markov property of Brownian motion, the process of state transition obeys the first-order Markov model. That is, the state $x_n$ of the current time is only related to the state $x_{n-1}$ of the previous time. And if the observations are independent of each other,
that is to say, the observation value $y_n$ is only related to the real state $x_n$ at time $t_n$.

The state space equation of accuracy degradation is:

$$x_n = x_{n-1} + \lambda \left[ A(t_n; \theta) - A(t_{n-1}; \theta) \right] + \sigma_B \left( B_n - B_{n-1} \right)$$

$$y_n = x_n + \epsilon$$

(4)

Where: $t_n$ is the corresponding measurement time; $\epsilon$ is the measurement error, $\epsilon \sim N\left(0, \sigma^2_{\epsilon}\right)$.

3. Prediction of accuracy residual life

The failure of accuracy of CNC machine tools generally refers to the degradation of accuracy to a certain extent. Once the accuracy value $X(t)$ of CNC machine tools degenerate to the failure threshold $\omega$ of accuracy, the accuracy at this time cannot meet the accuracy requirements of parts processing, and CNC machine tools are regarded as the failure of accuracy failure. The accuracy life of CNC machine tools is the time that the accuracy of CNC machine tools goes through from the beginning of operation to reaching the failure threshold. The failure threshold $\omega$ is generally set according to the requirements of users and machining accuracy. The accuracy life $T$ of CNC machine tools can be expressed as:

$$T = \inf \left\{ t; x(t) \geq \omega, t > 0 \right\}$$

(5)

For the Wiener process with (1) linear drift, the time $T$ when the degradation process reaches the failure threshold for the first time obeys the inverse Gaussian distribution after the values of parameters $\lambda$ and $\sigma_B$ are given [16]. The probability density function (PDF) of accuracy life $T$ is

$$f\left(t \mid \lambda, \sigma_B\right) = \frac{\omega}{\sqrt{2\pi \sigma_B^3}} \exp\left\{ -\frac{(\omega - \lambda t)^2}{2\sigma_B^3} \right\}$$

(6)

For the nonlinear Wiener process in the accuracy degradation model (2) established in this paper, the probability density function of the corresponding accuracy life $t$ can be obtained after the values of parameters $\lambda$, $\theta$ and $\sigma_B$ are given. Taking $A(t; \theta) = t^\theta$ obtained from the data of later examples as an example, the probability density function of the accuracy life is:

$$f\left(t \mid \lambda, \theta, \sigma_B\right) \approx \frac{\omega + (\theta - 1)\lambda t^\theta}{\sqrt{2\pi \sigma_B^3 t^3}} \exp\left\{ -\frac{(\omega - x_0 - \lambda t^\theta)^2}{2\sigma_B^3 t} \right\}$$

(7)

The accuracy residual life of the CNC machine tools refers to the time after the CNC machine tools run for a period $\tau$, from the accuracy value of the current time $\tau$ to the failure threshold. According to the relationship between life and residual life, the accuracy residual life $L_\tau$ of CNC machine tools can be expressed as:

$$L_\tau = \inf \left\{ t; x(t + \tau) \geq \omega, t > 0 \right\} = T - \tau$$

(8)

The probability density function of the accuracy residual life $L_\tau$ of CNC machine tools is:
It can be seen from equation (9) that in order to obtain the distribution of the residual life of CNC machine tools at time $t_n$, it is necessary to estimate the reasonable parameter value of $\lambda$, $\theta$, $\sigma_B$ based on the historical data $Y_{1:n}$ of the previous $n$ times of accuracy degradation, so as to obtain the probability density function of the accuracy residual life $L$ of the CNC machine tool.

4. Model parameter estimation based on particle filter

Due to the Markov property of accuracy degradation process, that is, given the current degradation state and the historical degradation state, the future degradation process is only related to the current state. Therefore, if we want to accurately predict the accuracy residual life of machine tools, it is particularly important to accurately judge the current running state of machine tools, that is, to obtain the reasonable degradation model parameters under the current state. At the same time, CNC machine tools, which are large and complex mechanical products, are very expensive, so it is impossible to use a large number of CNC machine tools for testing to evaluate their reliability, resulting in the limited degradation data that can be used. In order to solve the problem of limited available degraded data, a series of historical degraded data can be used as prior information by using Bayesian method.

When the current degraded data is obtained, the prior information can be modified by using Bayesian method. Updating the parameters of the accuracy degradation model to achieve more accurate prediction of the accuracy residual life. The parameter vector of the degenerate model is defined as $\Theta_n = (\lambda_n, \theta_n, \sigma_{Bn})$ after the parameter is updated at time $t_n$, and the parameter set $\Theta_{1:n}$ is considered to be a random parameter subject to a certain distribution, so as to reflect the current accuracy degradation of CNC machine tools.

Bayesian method transforms the problem of parameter state estimation into solving the posterior probability density of parameters by Bayesian formula, and expresses the estimation value of random parameters in the form of probability distribution. Recursively calculate the credibility of the current state based on the previous series of existing data, first predict the future parameter state through the existing prior information to obtain the prior probability density $p(\Theta_n | Y_{1:n-1})$ of the parameter state, and then use the latest accuracy measurement value to modify the prior probability density to obtain the posterior probability density:

$$p(\Theta_n | Y_{1:n}) = \frac{p(y_n | \Theta_n, Y_{1:n-1}) p(\Theta_n | Y_{1:n-1})}{p(y_n | Y_{1:n-1})}$$

(10)

In the nonlinear system of this paper, it is very difficult to solve the analytical solution of the posterior distribution of parameters. Monte Carlo method approximates the probability density function of random parameters through the random sampling of the posterior distribution, and replaces the integral calculation of the probability density function with the mean value of the sample to obtain the estimated value of parameters.

Particle filter is a kind of approximate Bayesian filter based on Monte Carlo method. It is very
convenient to solve the time series problem in this paper by combining the sequential analysis method with Monte Carlo method. Recursively estimate the posterior probability density of the parameter set based on the degraded sequence data $Y_{ln}$, and finally obtain the updated posterior estimate of the parameter set $\Theta_n$. Because the random parameters of the conjugate prior distribution have good statistical and computational characteristics in Bayesian statistical inference, and at the same time, when sufficient information of the random parameter prior distribution cannot be obtained, it can be assumed that the parameters are independent of each other and obey Gaussian distributed. Note that $\Theta_0$ is the prior distribution of the parameter set, where the parameters obey the conjugate prior Gaussian distribution $\mathcal{N}(\lambda_0, \sigma_{\lambda})$, $\mathcal{N}(\theta_0, \sigma_{\theta})$ and $\mathcal{N}(\sigma_{B0}, \sigma_{B})$ respectively. In order to update the parameters’ posterior estimates according to the degraded data $Y_{ln}$, it is necessary to establish the parameter state transition equation at the adjacent measurement time points. The parameter state transition process based on the accuracy degradation model also needs to obey the first-order Markov model. The parameter state $\Theta_n$ of the current time is only related to the parameter state $\Theta_{n-1}$ of the previous time and the current accuracy degradation $y_n$. Since the prior distribution of each parameter is Gaussian distribution, the state transition equation of each parameter is defined as:

$$
\lambda_n = \lambda_{n-1} + y_{\lambda} \sim \mathcal{N}(0, \sigma_{\lambda})
$$

$$
\theta_n = \theta_{n-1} + y_{\theta} \sim \mathcal{N}(0, \sigma_{\theta})
$$

$$
\sigma_{Bn} = \sigma_{B_{n-1}} + y_{\sigma_B} \sim \mathcal{N}(0, \sigma_{\sigma_B})
$$

Through the parameter state transition equation, the prior distribution $p(\Theta_n | \Theta_{n-1})$ of the next parameter state $\Theta_n$ can be established according to the previous parameter estimate $\Theta_{n-1}$ and the variance $\sigma_{\lambda}$, $\sigma_{\theta}$ and $\sigma_{\sigma_B}$ of the random parameter, and the recursive update of the parameter posterior estimate can be realized.

Monte Carlo method is used to express the probability density by random sampling from the posterior distribution of parameters, the samples are called particles. The integral operation of the posterior probability density is replaced by the mean of samples to estimate the posterior value of parameters. Therefore, the posterior probability density $\Theta_n$ of parameter set $p(\Theta_n | \Theta_{n-1}, Y_{ln})$ should be calculated. According to the properties of the first-order Markov process, the parameter state $\Theta_n$ at the current time is only related to the parameter state $\Theta_{n-1}$ at the previous time and the current accuracy degradation $y_n$, the posterior probability density of parameter set can be expressed as $p(\Theta_n | \Theta_{n-1}, Y_n)$. However, such a posterior probability density usually has no analytical solution and cannot be obtained directly. Therefore, importance sampling method is used to sample from a reference distribution $q(\Theta_n | \Theta_{n-1}, Y_n)$ which is similar to the posterior probability density function and easy to sample. Then, the credibility of each particle is tested according to the accuracy observation data as a posteriori information, and each particle is given the corresponding importance weight to express its credibility, to select the particles that can represent the posteriori distribution of parameters. Then, the sample particles are weighted and approximately summed to obtain $p(\Theta_n | \Theta_{n-1}, Y_n)$. The common reference distribution is the prior probability density of parameter set $p(\Theta_n | \Theta_{n-1}, Y_n)$:
\[ q(\Theta_n | \Theta_{n-1}, Y_n) = p(\Theta_n | \Theta_{n-1}) \]  

(12)

The importance weight of each particle is:

\[ w^{(i)}_n = \frac{p(Y_n | \Theta^{(i)}_n) p(\Theta_n | \Theta_{n-1})}{q(\Theta_n | \Theta_{n-1}, Y_n)} = w^{(i)}_{n-1} p(Y_n | \Theta^{(i)}_n) \]  

(13)

Since resampling is required after calculating the particle weight, and there is \( w^{(i)}_{n-1} = 1/N \) after resampling, formula (13) can be further simplified as:

\[ w^{(i)}_n = p(Y_n | \Theta^{(i)}_n) \]  

(14)

The reference distribution is randomly sampled to obtain the particle set \( \{\Theta^{(i)}_n, i = 1, 2 \cdots M\} \), where \( \Theta^{(i)}_n = (\lambda^{(i)}_n, \theta^{(i)}_n, \sigma_{Bn}^{(i)}) \), and \( M \) is the number of particles. The generation of each particle obeys the state transfer equation of each parameter in formula (11):

\[ \lambda^{(i)}_n \sim N(\lambda_{n-1}, \sigma_{\lambda}) \]
\[ \theta^{(i)}_n \sim N(\theta_{n-1}, \sigma_{\theta}) \]
\[ \sigma_{Bn}^{(i)} \sim N(\sigma_{Bn-1}, \sigma_{\sigma}) \]  

(15)

Substitute the parameters in the generated particles into the \( (2) \) type accuracy degradation model, and according to the particles, the predicted value of the corresponding accuracy observation value of each particle can be calculated.

\[ y^{(i)}_n = x_t + \lambda^{(i)}_n A(t; \theta^{(i)}_n) + \sigma_{Bn}^{(i)} B(t) \]  

(16)

Through the difference between the predicted value \( y^{(i)}_n \) of the accuracy observation value and the actual observation value \( y_n \), we can judge the credibility of the current particle \( \Theta^{(i)}_n \), and then give each particle the corresponding importance weight. The higher the credibility of the particle, the greater the weight is given. Since the measurement noise is Gaussian distribution, the weight of the \( i \)-th particle is:

\[ w^{(i)}_n = p(y_n | \Theta^{(i)}_n) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(y_n - \lambda^{(i)}_n A(t; \theta^{(i)}_n) + \sigma_{Bn}^{(i)} B(t))^2}{2\sigma^2} \right\} \]  

(17)

The weights of all particles are normalized by the following formula:

\[ w^{(i)}_n = \frac{w^{(i)}_n}{\sum_{i=1}^{M} w^{(i)}_n} \]  

(18)

In order to avoid particle degradation, the systematic resampling algorithm is used to resample the particles, that is, the particles with significant weight are copied many times, and the particles with low weight are eliminated. A more reasonable parameter set is selected from the particle set, and the optimization of parameter set is realized. For the ordered pair of particles set and weight \( \left\{\Theta^{(i)}_n, w^{(i)}_n\right\}_{i=1}^{M} \),
it becomes \( \{ \Theta^{(i)}_n \cdot 1/M \}_{i=1}^M \) after resampling, the total number of particles remains the same. According to the idea of Monte Carlo method, the posterior probability density of parameter set is expressed in the form of mean value of particle set:

\[
\tilde{\Theta}_n = \frac{1}{M} \sum_{i=1}^M \Theta^{(i)}_n
\]  \hspace{1cm} (19)

After substituting the parameters in the mean value \( \tilde{\Theta}_n \) of the particle set into equation (9), the probability density function \( f \left( \ell_r | \lambda, \theta, \sigma_B \right) \) of the accuracy residual life \( L_n \) of the CNC machine tool can be obtained.

5. An illustrative example

In order to verify the validity of the accuracy residual life prediction method proposed in this paper, it is verified by the degradation data generated from the test run of a horizontal machining center, under the simulated working condition before leaving the factory in the major science and technology project of "high-grade CNC machine tools and basic manufacturing equipment". Taking the degraded data of X-axis positioning accuracy of a horizontal machining center as an example to predict the residual life, the failure threshold of X-axis positioning accuracy is 80 \( \mu m \). The positioning accuracy was measured by a grating ruler with a time span of 2000 hours, and the interval of the measurement time was 20 hours, thereby generating 100 sets of accuracy degradation data.

**Table 1.** Measurement values of positioning accuracy at different measurement time.

| Time/h | Value/\( \mu m \) | Time/h | Value/\( \mu m \) | Time/h | Value/\( \mu m \) | Time/h | Value/\( \mu m \) |
|--------|-----------------|--------|-----------------|--------|-----------------|--------|-----------------|
| 20     | 1.811           | 520    | 35.732          | 1020   | 59.571          | 1520   | 69.796          |
| 40     | 3.632           | 540    | 36.042          | 1040   | 59.994          | 1540   | 69.909          |
| 60     | 4.487           | 560    | 36.164          | 1060   | 60.812          | 1560   | 71.902          |
| 80     | 7.291           | 580    | 36.336          | 1080   | 60.995          | 1580   | 72.725          |
| 100    | 9.506           | 600    | 36.836          | 1100   | 61.432          | 1600   | 73.638          |
| 120    | 11.022          | 620    | 37.732          | 1120   | 61.532          | 1620   | 74.538          |
| 140    | 12.938          | 640    | 38.328          | 1140   | 61.942          | 1640   | 74.912          |
| 160    | 14.554          | 660    | 39.524          | 1160   | 62.436          | 1660   | 75.321          |
| 180    | 15.837          | 680    | 40.424          | 1180   | 63.032          | 1680   | 76.193          |
| 200    | 18.186          | 700    | 41.316          | 1200   | 63.658          | 1700   | 76.356          |
| 220    | 19.702          | 720    | 42.212          | 1220   | 63.715          | 1720   | 76.429          |
| 240    | 21.818          | 740    | 43.662          | 1240   | 63.942          | 1740   | 77.012          |
| 260    | 23.634          | 760    | 45.455          | 1260   | 64.258          | 1760   | 78.201          |
| 280    | 24.255          | 780    | 47.276          | 1280   | 64.523          | 1780   | 78.371          |
| 300    | 25.452          | 800    | 49.096          | 1300   | 65.128          | 1800   | 78.956          |
| 320    | 26.367          | 820    | 49.285          | 1320   | 65.312          | 1820   | 79.159          |
| 340    | 27.277          | 840    | 50.904          | 1340   | 65.448          | 1840   | 80.374          |
| 360    | 29.059          | 860    | 51.263          | 1360   | 65.992          | 1860   | 80.526          |
| 380    | 30.722          | 880    | 52.064          | 1380   | 66.163          | 1880   | 81.635          |
According to the above accuracy degradation data, we can determine the degradation path of the accuracy degradation process, which is basically a power function of nonlinear form. Therefore, we assume that \( A(t; \theta) = t^\theta \) in the accuracy degradation model. After 1840 hours, the positioning accuracy exceeded the accuracy failure threshold for the first time, reaching 80.374 \( \mu \)m. Therefore, this time is regarded as the failure time of accuracy, the accuracy life of this accuracy index is 1840 hours.

In this paper, the data series in different time are used to predict the residual life of the accuracy index, that is, the degradation data in 0 ~ 1400 hours, 0 ~ 1500 hours, 0 ~ 1600 hours, 0 ~ 1700 hours and 0 ~ 1800 hours are used to predict the residual life of the accuracy index. Bayesian method is used to update the posterior estimates of the parameter set of the degradation model to test the prediction ability of the prediction method.

Firstly, the data series \( Y_{1:90}, Y_{1:85}, Y_{1:80}, Y_{1:75} \) and \( Y_{1:70} \) are used as the prior data to initialize the parameter estimation, and the initial value of the estimated parameter is used as the prior distribution \( \Theta_0 \) of the parameter set. For the estimation of initial values \( \lambda_0 \) and \( \theta_0 \), the least square method can be used. Firstly, the nonlinear accuracy degraded data \( Y_{1:n} \) and the corresponding time \( t_{1:n} \) are transformed into linear data by logarithmic transformation. The linear equation after logarithmic transformation is obtained by using the least square method, and finally the linear equation is reduced to the curve equation. The curve fitting of the accuracy degradation data \( Y_{1:n} \) can be realized, and the corresponding initial parameters \( \lambda_0 \) and \( \theta_0 \) can be obtained, and then the \( \sigma_{B0} \) under this parameter state can be calculated.

According to equation (11), the probability density function of accuracy life predicted by each measuring time point can be calculated. In order to express the calculation results of accuracy life more intuitively, the time corresponding to the maximum probability in the probability density function is taken as the time point of accuracy failure. The estimated value of initial parameter value and the corresponding accuracy life prediction results are shown in Table 2.

| Operation hours | \( \lambda \) | \( \theta \) | \( \sigma \) | Accuracy life /h |
|----------------|-------------|-------------|-------------|-----------------|
| 1800           | 0.280       | 0.765       | 0.105       | 1623.19         |
| 1700           | 0.270       | 0.772       | 0.104       | 1591.17         |
| 1600           | 0.260       | 0.778       | 0.103       | 1577.10         |
| 1500           | 0.248       | 0.787       | 0.101       | 1540.27         |
| 1400           | 0.236       | 0.796       | 0.105       | 1508.74         |

The results of life prediction with initial parameter values reflect the predictive ability of life prediction with traditional parameter estimation methods such as maximum likelihood estimation and
least squares. The life prediction result in the table deviates greatly from the actual life (1840 hours), and the minimum prediction deviation also exceeds 200 hours. This is because it is often necessary to use the whole data for curve fitting when using the idea of traditional parameter estimation method to estimate parameters, the obtained parameters reflect the fitting results of the whole data, but cannot reflect the current running status of the machine tool. In this case, even after obtaining the latest data, because the number of new data accounts for a small proportion in the overall data, it is difficult to affect the result of parameter estimation, and it is impossible to update the parameter value with the new data, or update the prediction result.

In this paper, particle filter combined with real-time measured degradation data is used to adjust the degradation model by updating the posterior value of parameters, and then more accurate degradation model and prediction results of residual life are obtained. Taking 0-1600 hours precision degraded data as an example, the posterior estimation of parameter set is updated by particle filter algorithm. Set the super parameters \( \sigma_\lambda = 0.005 \), \( \sigma_\sigma = 0.005 \) and \( \sigma_\theta = 0.005 \) in the prior distribution of each parameter according to the influence degree of each parameter value on the accuracy residual life in Table 2. The prior distribution of the parameter set is composed of the initial values of the parameters obtained above. The number of sampling particles is updated according to the data sequence through equations (11) to (19), the update process of parameter posterior value is shown in Figure 1.

![Figure 1. Update process of parameter posterior estimates.](image)

After the posterior estimation of the parameters of the accuracy degradation model is obtained, the probability density function of the accuracy residual life at different measuring time points can be calculated by equation (9), and get the corresponding maximum probability accuracy residual life.
prediction value. Figure 2 shows the probability density curve of residual life estimation corresponding to 1600h operation time.

**Figure 2.** Probability density curve of residual life at 1600h.

Due to the randomness of the mechanism of particles generated by Monte Carlo method, therefore, the posteriori estimated value of the parameter calculated by each filter has certain randomness. In order to reduce the chance in the process of updating the posteriori estimated value of the parameter, the average value of the parameter posterior value $\Theta_{n-5n}$ of the last five outputs of the parameter posterior estimate update is taken as the parameter value of the current state. In addition, the results of the five filtering calculations are compared. The parameter posterior estimates and accuracy residual life prediction values after filtering are shown in Table 3.

**Table 3.** Parameter posterior estimation value and residual life prediction result at 1600h.

| $\lambda$ | $\theta$ | $\sigma$ | RL predictive result | Actual RL |
|----------|----------|----------|----------------------|-----------|
| 0.2538   | 0.7669   | 0.0684   | 211.55               | 240.00    |
| 0.2508   | 0.7692   | 0.0754   | 198.96               | 240.00    |
| 0.2829   | 0.7529   | 0.0613   | 203.10               | 240.00    |
| 0.2262   | 0.7829   | 0.0637   | 200.24               | 240.00    |
| 0.2488   | 0.7700   | 0.0698   | 203.66               | 240.00    |

In order to evaluate the prediction ability of the prediction model quantitatively, root mean square error (RMSE) of the prediction model is used as the absolute error index. According to this evaluation model, the smaller the RMSE is, the higher the prediction accuracy is. The RMSE at the defined time $t_n$ is:

$$\text{RMSE} = \sqrt{\frac{1}{k} \sum_{i=1}^{k} (L(t_n) - \hat{L}(t_n, \Theta_n))^2}$$

(20)

Where: $L(t_n)$ is the actual accuracy residual life at time $t_n$; $\hat{L}(t_n, \Theta_n)$ is the accuracy residual life predicted by parameter set $\Theta_n$ at time $t_n$; $k$ is the number of predictions.

In Table 3, the RMSE of the accuracy residual life prediction results is 36.76 hours, that is, the
average deviation of the accuracy residual life prediction is about 36.76 hours. The accuracy residual life of the rest time is predicted in the same steps, and the probability distribution of residual life prediction at different measuring time points is shown in Figure 3.

**Figure 3.** Probability distribution of residual life prediction at different time.

Table 4 shows a set of parameters with the accuracy residual life in the median value at each time and the prediction results of the accuracy residual life.

| Operation hours | λ     | θ     | σ     | RL prediction result | Actual RL | RMSE  |
|-----------------|-------|-------|-------|-----------------------|-----------|-------|
| 1800            | 0.2319| 0.7783| 0.0633| 22.56                 | 40.00     | 16.74 |
| 1700            | 0.3182| 0.7378| 0.0645| 92.43                 | 140.00    | 46.14 |
| 1600            | 0.2829| 0.7529| 0.0613| 203.10                | 240.00    | 36.76 |
| 1500            | 0.2209| 0.7882| 0.0642| 264.02                | 340.00    | 82.12 |
| 1400            | 0.2516| 0.7728| 0.0348| 330.09                | 440.00    | 112.75|

The prediction results show that the parameters estimated by particle filter have a significant improvement compared with the initial parameters. When the time span is small, the accuracy of residual life prediction can achieve a good prediction effect, with a deviation of about 40 hours. When the time span of prediction becomes larger, the accuracy of prediction decreases, with a deviation of 120 hours in 1400 hours. This is because when predicting the accuracy residual life with the parameter estimation value of a certain measurement time point, it is assumed that the parameter state will not change from the current time until the accuracy residual life is reached. But in fact, because of the uncertainty of the accuracy degradation process, the parameter state is constantly changing from beginning to end, and the parameters can only be updated by the latest degradation data. Therefore, it can gradually approach the parameter state when the degradation process is close to the accuracy life, so that the closer the prediction time point is to the accuracy life, the closer the prediction result is to the real accuracy residual life. When the prediction time span is large and the deviation from the real residual life is large, it does not mean that the prediction result is inaccurate. But based on the known degradation trend and degradation data, the prediction results that are most consistent with the current situation can be made. After that, the parameter changes of the degradation model are uncertain, so the prediction results with large deviation appear when the time span is long.

6.Conclusion
In view of the uncertainty in the process of accuracy degradation of CNC machine tools, this paper presents a nonlinear method of accuracy degradation modeling. Based on the accuracy degradation model, the accuracy residual life prediction model is established, and the uncertainty in the degradation process is incorporated into the predicted residual life distribution. The particle filter is used to integrate the accuracy degradation data of CNC machine tools, which overcomes the problem that a single CNC machine tool lacks historical data and prior information, and significantly improves the accuracy of residual life prediction. At the same time, the prediction results also have strong individual pertinence, finally, the accuracy degradation measurement data of CNC machine tools verify the correctness and effectiveness of the proposed method.
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