3-&4-Body Final States in $B$, $D$ & $\tau$ Decays about Features of New Dynamics with CPT Invariance

or

”Achaeans outside Troy”

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Abstract

The ‘landscape’ of fundamental dynamics has changed even for the ‘known’ matter. The Standard Model has produced at least the leading source of CP violation in $B$ decays; the data have not shown CP asymmetries in $D$ transitions. It needs more data and better technologies to understand the underlying forces. Probing three- and four-body final states in $B$ & $D$ & $\tau$ decays with better accuracy is crucial about the existence and the features of New Dynamics. Theoretical tools produced about MEP will show even more about HEP in the future. We have to work on the correlations between different final states on several CKM levels and the connection between known matter and Dark Matter in indirect ways. CPT invariance is usable in $D$ and $\tau$ decays.

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1 Prologue – Landscapes of CP Asymmetries

Reminding Gary Larson’s ‘Far Side’ cartoon when cowboys were surrounded by indians. The defenders of the Standard Model (SM) tell us about signs of New Dynamics (ND): SM can do it and who cares of the ‘footstool’: neutrino oscillations, huge asymmetry in matter vs. anti-matter, ‘Dark’ whatever it is?

SM gives

- at least the leading source of measured CP violation (CPV) in $B$ decays;
- no CPV in $\tau$ decays beyond measured $K^0 - \bar{K}^0$ oscillations;
- small CPV in singly Cabibbo suppressed (SCS) in $D(s)$ decays;
- close to zero in doubly Cabibbo suppressed (DCS) charm ones.

Penguin quark diagrams deal with inclusive decays. The concept of ‘duality’ connect inclusive final states (FS) measured with hadrons and those with quarks (& gluons) that can be calculated. There are real challenges, namely to understand the underlying dynamics that give the measured rates.

The main point is: we have to probe FS about the existence and features of ND with both the best theoretical and experimental tools. One should remember that CP asymmetries need only one SM amplitude and one ND amplitudes – i.e., it gives much higher reach about ND; of course interference has to happen.
We enter a new era:

- The goal is to go from ‘accuracy’ to ‘precision’.

- CPT invariance can be used for CP asymmetries in $D$ & $\tau$ decays or gives the direction for $B$ ones.

The outline: Sect.2 about 3-&4-body FS of CP asymmetries in $D$, $\tau$ & $B$ decays; Sect.3 about the impact of CPT invariance; Sect.4 about parameterization of CKM matrix through higher order; Sect.5 about theoretical tools for treating FS interacting (FSI); Sect.6 gives comments about $B_d \to K\pi$ vs. $B_s \to K\pi$ and Sect.7 about some lessons learnt in the FPCP2013 conference; finally resume in Sect.8 and Epilogue in Sect.9.

2 3-&4-Body FS about CPV & Impact of ‘Penguins’

Probing FS with two hadrons (including narrow resonances) is not trivial to measure CPV; on the other hand one gets ‘just’ numbers. However 3-body FS are described by 2-dimensional plots. For 4-body FS one has even more dimensional landscape. There is a price: it needs much more work for experimenters to produce such data with more accuracy, and for theorists to understand the information given by the data. Yet there is a prize – namely finding existence of ND and its (or their) features.

There are several subtle points:

- Penguin diagrams were introducted for $K$ decays to understand the forces needed for $\Delta I = 1/2 \gg \Delta I = 3/2$ amplitudes and later about the ratio $\epsilon'/\epsilon$. They are given by local operators due to $M_K < m_c \ll m_t$; their impacts are greatly enhanced for two pseudo-scalars FS. There is hardly any difference between inclusive and exclusive transitions.

- The landscapes for $B$ transitions are more complex. Quark diagrams including penguin ones lead to local operators and can deal with inclusive rates based on quark-hadron duality. It needs work, but it can be done and did it in many cases for inclusive CPV with ‘hard’ FSI [1, 2, 3].

Allow me to show simple example, namely $B^- = [b\bar{u}] \to d\bar{u}u\bar{u}$ in the theorists’ world; in the real world of hadrons they show up with measured FS separately of $2\pi$, $4\pi$ etc. and $3\pi$, $5\pi$ etc. due to $G$ parity.

One can calculate exclusive FS including penguin diagrams. They give the directions of ‘soft’ FSI with hadrons at best in a semi-quantitative way; i.e., one cannot deal with local operators. Often such contributions are called ‘effective penguins’; that are fine for leading amplitudes.

When we want to probe for non-leading sources, we have to use other theoretical technologies, namely to think about correlations with other FS based on global symmetries like chiral, isospin, $SU(3)_{fl}$ etc. Their violations in exclusive FS are
sizable larger than inclusive ones. *Exclusive* rates depend on \( M_\pi \ll M_K \). *Inclusive* ones depend on \( m_u, m_d \ll m_s \) – but more importantly on \( m_s \ll \bar{\Lambda} \) which control the impact of QCD less flavour dependant.

- The situations are even more complex for \( D \) decays, while on the other hand easier. If one has \( c \rightarrow u \) penguin diagram with internal beauty quarks, it leads to local operator, and one can calculate it – but it gives only insignificant impact. However internal \( d \) and \( s \) with \( m_d, m_s \ll m_c \) lines are mostly given by non-perturbative QCD – in particular about exclusive FS.

However CPT invariances are ‘usable’ in \( D \) and \( \tau \) decays, namely about correlations between different FS like \( D^+ \rightarrow 3\pi \) vs. \( D^+ \rightarrow \pi K\bar{K} \) or \( \tau^- \rightarrow \nu\bar{K}^0\pi^0 \) vs. \( \tau^- \rightarrow \nu\bar{K}^0\pi^- \) (or \( \tau^- \rightarrow \nu K_S\pi^- \)).

- Long time before QCD was found as ‘the’ theory about strong forces, one gave predictions based on global symmetries like I-, U- and V-spin as parts of SU(3)fl. It was known that I-spin violation are much smaller than U- and V-spin ones and also somewhat smaller than total SU(3)fl. Once the technologies of QCD were applied in many situations, it was clear that violation of U- and V-spin symmetries are usually larger in exclusive decays than inclusive ones.

- Measured rates depend on the areas of Dalitz plots and the production of the decaying state \( P \). However the ‘local’ ratios of Dalitz plots of \( P \) vs. \( \bar{P} \) do not depend on the production of \( P \) vs. \( \bar{P} \) (in principle).

- Measuring ‘local’ CP asymmetries needs much more data than ‘averaged’ one, but gives much more information about the underlying dynamics in time.

- Surprising sign of direct CPV in \( \tau \) decays was given:

\[
A_{\text{CP}}(\tau^+ \rightarrow \bar{\nu}K_S\pi^+)_{\text{SM}} = +(0.36 \pm 0.01)\% \quad (1)
\]

\[
A_{\text{CP}}(\tau^+ \rightarrow \bar{\nu}K_S\pi^+ [+\pi^0's])_{\text{BaBar2012}} = -(0.36 \pm 0.23 \pm 0.11)\% \quad (2)
\]

- Since we have no infinite data (and no infinite time), we have to think and discuss which ways are the best depending on the features of ND.

- Collaboration of Hadronic Dynamics/MEP and HEP physicists is very important.

Indirect and direct CPV has been established in 2-body FS of \( B_d \); we need more precision and probe 3- & 4-body FS with accuracy.

SM expects small indirect CPV about \( B_s \) transitions. It was said a decade ago (look at the history given in Ref.[3]) even ND cannot produce large CPV; one needs more accuracy there. It is a very good achievement by LHCb collab. to establish the first CP asymmetry in \( B_s \) decays:

\[
A_{\text{CP}}(B_s \rightarrow K^-\pi^+) = 0.27 \pm 0.04 \pm 0.01 \quad (3)
\]
Yet it is not surprising to find large direct CPV from the theoretical side in a qualitative way; yet it is surprising to me that it is so large; I will come back to it in Sect.\ref{sect:6}.

We need precision and probe 3- and 4-body FS with accuracy – including the topologies of the asymmetries. Furthermore you have to measure correlations between different FS and understand of which reasons they are based.

There is no evidence for indirect or direct CPV in 2-body FS in $D$ decays. The data are very consistent with SM predictions which give small and less CP asymmetries. ND cannot give large CPV, however sizable ones. We need precision there and probe 3- & 4-body FS.

There is evidence for direct CPV in $\tau^+ \to \bar{\nu} K_S \pi^+$, see Eqs.\ref{eq:tempscras},\ref{eq:tempscras}. We need precision there and probe hadronic 2- & 3-body FS.

We need accuracy on different CKM levels and correlations.

\section{Impact of CPT Invariance}

CPT symmetry gives equalities for the masses and widths of particles $P$ vs. anti-particles $\bar{P}$. However that invariance tell us much more about the underlying dynamics, namely equalities of different classes of FS due to ‘mixing’/‘re-scattering’\footnote{\textsuperscript{1}Mostly the words ‘mixing’ and ‘oscillation’ are seen as equivalent, however they are not\footnote{\textsuperscript{3}}.} in the amplitudes \ref{eq:tempscras},\ref{eq:tempscras}:

\begin{align}
T(P \to a) &= \exp(i\delta_a) \left[ T_a + i \sum_{aj \neq a} T_{aj} T_{aj,a}^{\text{resc}} \right] \\
T(\bar{P} \to \bar{a}) &= \exp(i\delta_a) \left[ T_a^{\ast} + i \sum_{aj \neq a} T_{aj}^{\ast} T_{aj,a}^{\text{resc}} \right]
\end{align}

Direct CPV is measured with

$$\Delta \Gamma(a) = |T(\bar{P} \to \bar{a})|^2 - |T(P \to a)|^2 = 4 \sum_{aj \neq a} T_{aj,a}^{\text{resc}} \text{Im}T_a^{\ast}T_{aj}$$

CPV has to vanish upon summing over all mixed states $a$ due to CPT invariance, since $T_{aj,a}^{\text{resc}}$ is symmetric and $\text{Im}T_a^{\ast}T_{aj}$ anti-symmetric: $\sum_a \Delta \Gamma(a) = 4 \sum_a \sum_{aj \neq a} T_{aj,a}^{\text{resc}} \text{Im}T_a^{\ast}T_{aj} = 0$. We do not know how to calculate strong FSI: $\Delta \Gamma(a)$ cannot predict direct CPV quantitatively even if only SM gives weak phases.

CPT symmetry gives relations between CP asymmetries in different channels. Finding CP asymmetry in one channel one infers which channel(s) have to compensate asymmetries based on CPT invariance. Finally analyzing those decays teach us important lessons about the inner working of QCD. CPT invariance in $D$ and $\tau$ decays is ‘practical’, since a ‘few’ channels can be combined.

Landscapes are different between $D$ & $\tau$ decays on one side and $B$ ones on the other side. Furthermore one has to deal with different experimental and theoretical challenges:
• To find non-leading sources for ND one has to deal with large ‘background’ for SM about CP asymmetries in $B$ decays; the impacts of penguin diagrams are subtle and CPT symmetry gives only ‘directions’, but not more: ‘price’ vs. ‘prize’.

• SM gives small ‘background’ about CPV in SCS and near zero in DCS $D$ decays; the impact of penguin diagrams are subtle, but different reasons: ‘prize’ vs. ‘price’.

• SM gives near zero ‘background’ in $\tau$ transition, and the correlations with the forces producing neutrino oscillations; on the other hand there could be correlations with CP asymmetries in $D$ decays: ‘prize’ vs. ‘price’.

It was first suggested to use penguins diagrams about FSI [7]. However the situations are more subtle and complex as discussed in Ref.[1].

4 Parameterization of CKM Matrix through $\mathcal{O}(\lambda^6)$

PDG and HFAG show also the ‘exact’ CKM matrix with three families of quarks. However experimenters and theorists do not use exact CKM matrix as you can see in their papers and talks. The pattern can be much more obvious in parameterization, tell us when we need more data where and the existence of ND and its features. Now we need precision.

In Wolfenstein parameterization one gets six triangles that are combined into three classes with four parameters $\lambda$, $A$, $\bar{\eta}$ and $\bar{\rho}$ with $\lambda \simeq 0.223$. Those are probed and measured in $K$, $B$, $B_s$ and $D$ transitions: $A \sim 1$, but the two ones are not of $\mathcal{O}(1)$: $\bar{\eta} \simeq 0.34$ and $\bar{\rho} \simeq 0.13$. It is assumed – usually without mentioning – that one applies them with no expansion of $\bar{\eta}$ and $\bar{\rho}$. Obviously it is a ‘smart’ parameterization with a clear hierarchy.

When we need a parameterization of the CKM matrix with more precision for non-leading sources in $B$ decays and very small CP asymmetries in $D$ decays with little ‘background’ from SM. Several ‘technologies’ was given like in Ref.[8] with $\lambda$ as before, but $f \sim 0.75$, $\bar{h} \sim 1.35$ and $\delta_{\text{QM}} \sim 90^\circ$. Now we get somewhat different six classes, and it is more subtle for CP violation:

\[
\begin{pmatrix}
1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} - \frac{\lambda^6}{16}, & \lambda, & \bar{h}\lambda^4 e^{-i\delta_{\text{QM}}}, \\
-\lambda + \frac{\lambda^5}{2} f^2, & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{16} (1 + 4 f^2) - f \bar{h}\lambda^5 e^{-i\delta_{\text{QM}}} + \frac{\lambda}{16} (4 f^2 - 4 \bar{h}^2 - 1), & f \lambda^3, \\
- f \lambda^2 - \bar{h}\lambda^4 e^{i\delta_{\text{QM}}} + \frac{\lambda^6}{2} f, & 1 - \frac{\lambda^4}{16} f^2 - f \bar{h}\lambda^5 e^{-i\delta_{\text{QM}}}, & -\frac{\lambda}{2} \bar{h}^2
\end{pmatrix} + \mathcal{O}(\lambda^7)
\]

Class I.1 : $V_{ud} V_{us}^*$ \ $[\mathcal{O}(\lambda)] + V_{cd} V_{cs}^*$ \ $[\mathcal{O}(\lambda)] + V_{td} V_{ts}^*$ \ $[\mathcal{O}(\lambda^{5k6})] = 0$ \ (8)
Class I.2 : $V_{ud} V_{cd}^*$ \ $[\mathcal{O}(\lambda)] + V_{us} V_{cs}^*$ \ $[\mathcal{O}(\lambda)] + V_{ub} V_{cb}^*$ \ $[\mathcal{O}(\lambda^{6k7})] = 0$ \ (9)
Class II.1 : $V_{us} V_{ub}^*$ \ $[\mathcal{O}(\lambda^5)] + V_{cs} V_{cb}^*$ \ $[\mathcal{O}(\lambda^{2k3})] + V_{ts} V_{tb}^*$ \ $[\mathcal{O}(\lambda^2)] = 0$ \ (10)
Class II.2 : $V_{cd} V_{d}^*$ \ $[\mathcal{O}(\lambda^3)] + V_{cs} V_{s}^*$ \ $[\mathcal{O}(\lambda^{2k3})] + V_{cb} V_{b}^*$ \ $[\mathcal{O}(\lambda^{2k3})] = 0$ \ (11)
Class III.1 : $V_{ud} V_{ub}^*$ \ $[\mathcal{O}(\lambda^4)] + V_{cd} V_{cb}^*$ \ $[\mathcal{O}(\lambda^{3k4})] + V_{td} V_{tb}^*$ \ $[\mathcal{O}(\lambda^3)] = 0$ \ (12)
Class III.2 : $V_{ud} V_{td}^*$ \ $[\mathcal{O}(\lambda^3)] + V_{us} V_{s} \ [\mathcal{O}(\lambda^{3k4})] + V_{ub} V_{b}^*$ \ $[\mathcal{O}(\lambda^4)] = 0$ \ (13)
One finds the same pattern as from Wolfenstein parametrization, namely ‘large’ CP asymmetries in Class III.1, sizable ones in Class II.1 and ‘small’ one in Class I.1. However, the pattern is not so obvious, and it is similar in a semi-quantitative way:

- CP asymmetries in $B_d \rightarrow \psi K_S$ and $B^+ \rightarrow D_\pi K^+$ control Class III.1 triangle. Due to interference between the two contributions one gets from CKM dynamics:

$$S(B_d \rightarrow \psi K_S) = \sin 2\phi_1 \approx 0.62 - 0.68 \text{ for } \delta_{QM} \approx 75^\circ - 90^\circ \quad (14)$$

$$S(B_d \rightarrow \psi K_S) = \sin 2\phi_1 \sim 0.72 \text{ for } \delta_{QM} \approx 100^\circ - 120^\circ ; \quad (15)$$

i.e., CKM dynamics produce $S(B_d \rightarrow \psi K_S) \sim 0.72$ as largest value for CP asymmetry with $\delta_{QM} \approx 100^\circ - 120^\circ$ to compare with the measured

$$S(B_d \rightarrow \psi K_S) \sim 0.676 \pm 0.021 . \quad (16)$$

Therefore it seems at first sight that CKM dynamics give very close to ‘maximal’ value possible there, but not close to 100 %. However the situation is more subtle as mentioned next.

- We are searching for non-leading source of CP violation in $B$ transitions, in particular in $B^0 - \bar{B}^0$ oscillations. ND’s impact could ‘hide’ there in ”SM predicted” CP asymmetries. ‘Data’ given by HFAG, for example, are averaged over values of $|V_{ub}/V_{cb}|$ from $B \rightarrow l\nu\pi$ and $B \rightarrow l\nu X_c$; actually the ‘central’ value is closer to $|V_{ub}|_{excl}$ rather than the larger $|V_{ub}|_{incl}$. It is quite possible that the theoretical uncertainties about extracting of $|V_{cb}|$, $|V_{ub}|$ and $|V_{ub}/V_{cb}|$ from $B \rightarrow l\nu\pi$ vs. $B \rightarrow l\nu D^*$ are sizably larger than claimed; some details are told about it in Ref.[9].

- The information from the data now and in the future about ND has to be based on accuracies and its correlations with different FS in several $B$, $D$ and $K$ transitions and rare decays.

- It gives more deeper insight into flavour dynamics and QCD’s impact, but also about inner structures for non-perturbative forces.

5 Theoretical Tools for dealing with FSI

The goal is to find its (or theirs) existence and its nature. When the impact of ND has been established, one wants to find its features due to interferences between scalar & pseudoscalar, vectors & axial-vectors etc. etc.

- It is non-perturbative QCD that mostly controls FSI.

- One has to probe CPV in $K$, $D$, $B$ and $\tau$ decays.

- There is experience from Hadronic Dynamics(HD)/MEP.
5.1 ‘Catholic’ Road to ND – 3-Body FS

For $D/B \to P_1P_2P_3$ or $\tau \to \nu P_1P_2$ decays there is a single path to ‘heaven’, namely asymmetries in the Dalitz plots. One can rely on relative rather than absolute CPV; it is much less dependent on production asymmetries. However one needs a lot of statistics – and robust pattern recognition\footnote{You might remember the known history.}

5.1.1 CP Asymmetries in $B^\pm$ Decays

One such procedure have given and simulated about 3-body FS in $B^\pm$ decays, namely Refs.[10] [11]; another one can be found in Ref.[12].

Early data from LHCb have found CPV averaged in $B^\pm$ decays to 3-body FS [13]:

\[
A_{CP}(B^\pm \to \pi^\pm \pi^+\pi^-) = +0.120 \pm 0.020\text{(stat)} \pm 0.019\text{(syst)} \pm 0.007(J/\psi K^\pm) (17)
\]
\[
A_{CP}(B^\pm \to \pi^\pm K^+K^-) = -0.153 \pm 0.046\text{(stat)} \pm 0.019\text{(syst)} \pm 0.007(J/\psi K^\pm) (18)
\]

It is interesting – but not more (yet) – that these CP asymmetries come with opposite signs. It makes it ‘easier’ to think about the impact of CPT invariance.

Very recent data from LHCb show very sizable averaged CP asymmetries with more accuracy, correlations and isospin symmetry[14]:

\[
A_{CP}(B^\pm \to K^\pm \pi^+\pi^-) = +0.034 \pm 0.009\text{(stat)} \pm 0.004\text{(syst)} \pm 0.007(J/\psi K^\pm) (19)
\]
\[
A_{CP}(B^\pm \to K^\pm K^+K^-) = -0.046 \pm 0.009\text{(stat)} \pm 0.005\text{(syst)} \pm 0.007(J/\psi K^\pm) (20)
\]

The data of CKM suppressed $B^+$ decays to charged three-body FS

\[
\begin{align*}
\text{BR}(B^+ \to K^+\pi^-\pi^+) & = (5.10 \pm 0.29) \cdot 10^{-5} \quad (21) \\
\text{BR}(B^+ \to K^+K^-K^+) & = (3.37 \pm 0.22) \cdot 10^{-5} \quad (22) \\
\text{BR}(B^+ \to \pi^+\pi^-\pi^+) & = (1.52 \pm 0.14) \cdot 10^{-5} \quad (23) \\
\text{BR}(B^+ \to \pi^+K^-K^+) & = (0.52 \pm 0.07) \cdot 10^{-5} \quad (24)
\end{align*}
\]

show the impact of penguins/re-scattering diagrams, since the FS with $\Delta S \neq 0$ are larger than with $\Delta S = 0$. However one can remember that penguins operators show only hard re-scattering and focus on inclusive decays.

It is important to measure the averaged CP asymmetries, but also probe the Dalitz plots ‘locally’ and probe the correlations with different FS as shown above.

5.1.2 CP Asymmetries in $D^\pm_{(s)}$ Decays

$D^\pm$ has two all charged 3-body FS on the SCS level – namely $D^\pm \to \pi^\pm \pi^+\pi^-$ and $D^\pm \to \pi^\pm K^+K^-$ [15] – and also on the DCS one – $D^\pm \to K^\pm \pi^+\pi^-$ and $D^\pm \to K^\pm K^+K^-$. $D^\pm_s$ has two ones on the SCS level – $D^\pm_s \to K^\pm \pi^+\pi^-$ and $D^\pm_s \to K^\pm K^+K^-$ – however only one for DCS level – $D^\pm_s \to K^\pm K^+\pi^\pm$. 

As stated above, for SCS FS SM gives small ‘background’ for CPV and close to zero about DCS. However data give limits about CPV that are somewhat small or happen in rare FS. We have to use good experimental and theoretical technologies to get the information about the underlying dynamics; we have to probe FS with broad resonances – in particular scalar ones like $\sigma$ and $\kappa$ – and their interferences.

5.1.3 CP Asymmetries in $\tau^\pm$ Decays and Correlations with $D(s)$ Decays

There may be a sign - may be – of ND in $\tau$ decays, see Eqs. (12) about averaged CP asymmetries. It is crucial to probe CP ‘locally’. Furthermore one has to measure correlations with $D(s)$ decays [16].

One should focus on SCS decays with the impact with two hadrons in the FS, namely $\tau^- \rightarrow \nu K^+\pi^0$ and $\tau^+ \rightarrow \nu K_S\pi^+$ and FS with more hadrons.

5.2 ‘Protestant’ Road to ND – 4-Body FS

There are several ways to probe CPV in 4-body FS and to differential the impact of SM vs. ND, since the landscapes are more complex. One can compare $T^\text{odd}$ moments or correlations in $D$ vs. $\bar{D}$. For example one has to measure the angle $\phi$ between the planes of $\pi^+ - \pi^-$ and $K - \bar{K}$ and described its dependence [3, 16]:

$$\frac{d\Gamma}{d\phi}(D \rightarrow K\bar{K}\pi^+\pi^-) = \Gamma_1\cos^2\phi + \Gamma_2\sin^2\phi + \Gamma_3\cos\phi\sin\phi$$ (25)

$$\frac{d\Gamma}{d\phi}(\bar{D} \rightarrow K\bar{K}\pi^+\pi^-) = \bar{\Gamma}_1\cos^2\phi + \bar{\Gamma}_2\sin^2\phi - \bar{\Gamma}_3\cos\phi\sin\phi$$ (26)

The partial width for $D[\bar{D}] \rightarrow K\bar{K}\pi^+\pi^-$ is given by $\Gamma_{1,2}[\bar{\Gamma}_{1,2}]$: $\Gamma_1 \neq \bar{\Gamma}_1$ and/or $\Gamma_2 \neq \bar{\Gamma}_2$ represents direct CPV in the partial width.

$\Gamma_3$ and $\bar{\Gamma}_3$ represent $T^\text{odd}$ correlations; by themselves they do not necessarily indicate CPV, since they can be induced by strong FSI; however [17, 18, 3]:

$$\Gamma_3 \neq \bar{\Gamma}_3 \rightarrow \text{CPV}$$ (27)

Integrated rates give $\Gamma_1 + \Gamma_2$ vs. $\bar{\Gamma}_1 + \bar{\Gamma}_2$; integrated forward-backward asymmetry

$$\langle A \rangle = \frac{\Gamma_3 - \bar{\Gamma}_3}{\pi/2(\Gamma_1 + \Gamma_2 + \bar{\Gamma}_1 + \bar{\Gamma}_2)}$$ (28)

gives full information about CPV. One could disentangle $\Gamma_1$ vs. $\bar{\Gamma}_1$ and $\Gamma_2$ vs. $\bar{\Gamma}_2$ by tracking the distribution in $\phi$.

5.3 Theoretical Tools for treating FSI

Tools about FSI in 3- and 4-body FS have been produced after the last 10 - 15 years mostly based on dispersion relations. See a list of papers [19]. There are some points: chiral symmetry is a good tool for probing FS with just pions, but not about $D$ and $\tau$ decays with kaons. However the connection of CPT and chiral symmetries is subtle.
6 CP Asymmetry in $B_s \rightarrow K^\pi$ vs. $B_d \rightarrow K^\pi$

It is an important achievement that LHCb has found the first CP asymmetry in $B_s$ decays, and it is large: $A_{CP}(B_s \rightarrow K^\pi) = 0.27 \pm 0.04 \pm 0.01$. There is an obvious reason to compare it with $A_{CP}(B_d \rightarrow K^\pi) = -0.080 \pm 0.007 \pm 0.003$. The correlation of those CP asymmetries come with opposite signs is not surprising. Furthermore it gives with an amazing experimental certainty $\Delta_{LHCb} = -0.02 \pm 0.05 \pm 0.04$.[20]. There is a statement just before the Eq.(11): "These results allow a stringent test of the validity of the relation between $A_{CP}(B^0 \rightarrow K^\pi)$ and $A_{CP}(B^0_s \rightarrow K^\pi)$ in the SM given in Ref.[21]. However in the 2005 paper Lipkin gave a theoretical uncertainty of ‘... the order of 10 - 20 per cent ...’ on p. 6 of arXiv paper. It is not enough to just read the ‘Abstract’ of Lipkin’s papers, but more. In this paper the prediction was based on a model about CPT invariance that is stated in the body of that paper. CPT symmetry is ‘practical’ about charm and $\tau$ decays, but for beauty decays only about ‘directions’. Furthermore measured FS in $B$ decays are based on hadrons, not quarks. Theory tells about underlying forces for inclusive transitions in a quantitative way including ‘hard’ re-scattering due to the total QCD. The challenge for exclusive ones is greater: the impact of strong FSI is important for correlations between members of the same class defined by symmetries, but not by the numbers of hadrons; however we cannot calculate them[1 2 3]. Therefore it has to probe $B$ decays with three- and four-body FS in the future with ‘local’ CP asymmetries – in particular about regions where ND can have more impact like from exchanges with charged Higgs etc. Furthermore SM gives at least the leading source of beauty CPV.

One more comment: for a long time it was stated that Cabibbo flavoured quark penguin loop diagrams can compete with Cabibbo suppressed quark tree diagrams on a similar level – even more – and produre interferences leading to direct CP violation on the scale of around 10% – as shown for $B_d \rightarrow K^\pi$ as suggested in Ref.[7 1]. However the situation is quite different for $B_s \rightarrow K^\pi$, where one have to compare Cabibbo favoured quark tree with Cabibbo suppressed quark penguins loop diagrams. As stated before, quark diagrams deal with inclusive decays: $B_d \rightarrow KX_{S=0}$ vs. $B_s \rightarrow KX_{S=0}$. The impact of strong forces due to re-scattering is important or even crucial for exclusive hadronic FS. The data tell us that the rates of $B_s \rightarrow K^\pi$ are smaller than for $B_d \rightarrow K^\pi$, but not more than a factor of two or three. It seems to me that might be a sign of ND’s impact there, unless there is an impact of a resonance. Obviously I need more thinking about it based on the concept of ‘duality’ in subtle ways[22].

7 Lessons from FPCP2013 Conference in ‘Person’

Talking in person is much more important than connection by internet about fundamental physics. It helps to understand items covered in talks at conferences where one can think about and discuss them not only with the speaker, but with other attendees. One example: BaBar/Belle data show that $B$ decays have probability with baryon-antibaryon FS with $(6.8 \pm 0.6)\%$. Based on a parton model a 1981 prediction gave $(5-10)\%$
in the range for these *inclusive* FS [23]. It is surprising about these data show that known exclusive FS give only 10% of these inclusive ones. It tells us that our control of non-perturbative dynamics is quite limited. We have to think about the impact of resonances (in particular broad one), threshold enhancement etc. Of course, there is no other candidate about strong forces different from QCD. It shows that we have to think more about impact of ND in hadrons decays. It would not be surprising that *semi-leptonic* FS with baryon-antibaryon are less complex than those non-leptonic ones.

Finally the central point is how important meetings, conferences and workshop are even in the internet era.

8 Summary of Searching for ND in 3- & 4-Body FS

The goal of flavor dynamics is to find the *existence* and *features* of ND. The SM with $SU(3)_{\text{QCD}} \times SU(2)_L \times U(1)$ is not complete even beyond thinking about symmetries – namely the structure of our Universe, neutrino oscillations, asymmetry in matter vs. antimatter. It has a good chance to show the correlation between known matter vs. dark matter in heavy flavor transitions. Therefore we have to probe 3- & 4-body FS of $D$, $B$ & $\tau$ decays with correlations. We need detailed analyses of 3- & 4-body FS including CPV despite the large start-up work. CPT invariance does not have just an academic reason, but also a practical one at least in $D$ and $\tau$ decays. We should have real collaborations between theorists from HD/MEP & HEP and experimentalists from HEP. It is important whether penguin quark diagrams lead to a local operator or not. We have to remember that U- and V-spin violations enter different landscapes in exclusive vs. inclusive decays.

Most physicists start with minimal version of ND for practical reasons; however the real world does not care about convenience for our powers of calculations. The best example is SUSY: there are several causes for the existence of SUSY in our world – however those do not give us reasons for minimal version of SUSY or close to it.

A few words after this Conference: A very, very recent paper from LHCb collab. is dealing with $B_{(s)}^0 \rightarrow K_S h^+ h^-$ with $h = \pi, K$ [24]. It needs more thinking about the informations that the data give us about CKM suppressed $B_{(s)}^0$ rates including re-scattering – and about CP asymmetries, signs of ND existence and its features. This landscape of three-body FS seems to be more ‘complex’ based on quark diagrams.

9 Epilogue: ‘Achaeans outside Troy’

It was said for a very, very long time that one could find the ‘Devil’ at least in one paintings produced by Giotto in the Basilica San Francesco in Assisi in Italy in the 14th century, see Fig.1. A few years ago it was found in a subtle locality. Obviously it took many efforts to find ‘him’ even with his horn, beard and strong nose. Now you can see him
in the cloud \[3\], see Fig.2. There is a much longer history that painting, namely the Greek history about taking Troy. We hope that the features of ND will be seen and measured in the next ten years with the modern analogy, see the Fig.3.

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\[3\] Have you heard the word ‘Cloud’ more recently in another, but still ‘sacred’ situation?
Figure 2: Finding the Devil in the Basilica San Francesco in Assisi in Italy

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Figure 3: Achaeans needed force, but also ‘cunning’ to win – and Odysseus produced both for taking Troy!

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