Effective Field Theory for a Heavy Higgs: a Manifestly Gauge Invariant Approach

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Abstract

For large values of the Higgs boson mass the low energy structure of the gauged linear sigma model in the spontaneously broken phase can adequately be described by an effective field theory. In this work we present a manifestly gauge invariant technique to explicitly evaluate the corresponding effective Lagrangian from the underlying theory. In order to demonstrate the application of this functional method, the effective field theory of the abelian Higgs model is thoroughly analyzed. We stress that this technique does not rely on any particular property of the abelian case. The application to the non-abelian theory is outlined.

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1 Introduction

The method of effective field theory has repeatedly been used in the analysis of the symmetry breaking sector of the Standard Model [1]. It provides a convenient and model independent parametrization of various scenarios discussed in the literature, such as a heavy Higgs [2] or technicolor models [3], regarding the nature of the spontaneous breaking of the electroweak symmetry. In this approach, the unknown physics is hidden in the low energy constants of an effective Lagrangian, which describes the effective field theory. Thus, in order to analyze the phenomenological implications of precision experiments with presently accessible energies on physics beyond the Standard Model, it is necessary to study the relationships between the constants of this Lagrangian and the parameters of a given underlying theory.

The physics in the low energy region of a full theory is adequately described by an effective field theory if corresponding Green’s functions in both theories have the same low energy structure. One can take this matching requirement as the definition of the effective field theory. It determines functional relationships between the low energy constants of the effective Lagrangian and the parameters of the underlying theory. Furthermore, if the coupling constants are weak in the low energy region, perturbative methods can be used to explicitly evaluate these relationships.

Functional techniques to evaluate the effective Lagrangian for a given underlying theory have recently been thoroughly discussed [4]. However, their application to gauge theories is not straightforward. The present article will continue the discussion and give a thorough account of this problem.

The special rôle of gauge theories is readily understood. The effective field theory analysis of the symmetry-breaking sector of the Standard Model should not make any particular assumptions about the underlying theory – apart from symmetry properties and the existence of a mass gap. Conceptually, this also requires parametrizing low energy phenomenology by a gauge invariant effective Lagrangian. The definition of Green’s functions, on the other hand, usually does not reflect the symmetry properties of a gauge theory. Gauge invariance is broken, and the off-shell behaviour of Green’s functions is gauge dependent.

An effective field theory defined as in the second paragraph will correctly describe low energy physics. However, if Green’s functions which enter the matching relations do not reflect the symmetry properties of the full the-
ory, it will also include the corresponding gauge artifacts. It is not clear at all, whether such an effective field theory can be described by an effective Lagrangian which is gauge invariant.

Without resolving that issue, this approach was taken in Ref. [5] in order to determine the low energy constants at order $p^4$ in the case, where the Higgs mass in the Standard Model is heavy. Under the additional assumption that the scalar coupling $\lambda$ is not too strong, perturbative methods can indeed be used to evaluate the corresponding effective Lagrangian. To pin down the low energy constants at order $p^4$, the authors matched 1PI Green’s functions of the gauge fields. The off-shell behaviour of these functions in both the full and the effective theory is gauge dependent. Thus, fine tuning of the gauge dependent structure is necessary in order to avoid inconsistencies. For the case at hand, it was sufficient to choose appropriate gauge fixing conditions in both theories. However, these Green’s functions involve gauge dependent terms of arbitrary order in the low energy expansion. Furthermore, it was shown in Ref. [6] that the 1PI Green’s functions of the gauge fields depend on the parametrization of the Goldstone boson fields. To account for this effect, the matching relations should involve all light-particle Green’s functions, including those of the Goldstone bosons. It is rather doubtful that it will be possible to maintain the off-shell matching of all these Green’s functions, yet still describe the effective field theory defined that way by a gauge invariant effective Lagrangian and a fine tuned gauge fixing term. We think that this approach will generally involve an effective Lagrangian which is not gauge invariant.

In order to avoid these problems, the authors of Ref. [6] suggested that all gauge dependence be eliminated from the matching relations. However, the approach they chose does not work in general. They projected onto the transverse components of the gauge fields in connected Green’s functions. In a non-abelian gauge theory, this does not yield gauge invariant quantities. For the case of a heavy Standard Model Higgs, it turned out that the matching relations between projected Green’s functions at order $p^4$ do not depend on the gauge fixing parameter $\xi$. However, independence of this parameter is only a necessary condition for gauge invariance. Since projecting onto the transverse components of Green’s functions does not yield gauge invariant quantities in the non-abelian case, this method is inherently gauge dependent.

Yet another approach was taken in Ref. [7], in which the background field action of both the full and the effective theory are matched. Here, too,
the off-shell behaviour of Green's functions enters the matching relation. One particular property of this action is its gauge invariance with respect to gauge transformations of the background fields, which is achieved by an appropriate choice of the gauge fixing term. This may be quite useful in certain applications, but it does not solve the problems described above. Though gauge invariant, the background field action is not gauge independent. The 1PI Green's functions explicitly depend on the gauge fixing term, which was introduced in the definition of the background field action. Thus, it is again necessary to fine tune the gauge fixing terms in both the full and the effective theory order by order in the low energy expansion. Furthermore, one still has to make sure that no gauge dependence enters the effective Lagrangian. Without further proof, independence of the gauge fixing parameter should only be considered as a necessary condition. In this work we will not distinguish between gauge invariance and gauge independence. The phrase gauge invariance will include both meanings.

This discussion indicates that any approach to determine the effective Lagrangian for a given underlying theory should match only gauge invariant quantities. Then one does not have to worry about any gauge artifact which otherwise might enter the effective field theory. Perhaps the most straightforward idea that comes to mind is to match only $S$-matrix elements. However, this approach is quite cumbersome, and one would rather like to use functional techniques like those described in Ref. [4]. The main purpose of this article is to formulate these techniques for the case of gauge theories under the condition that gauge invariance is manifest throughout the calculation. In order to avoid technical difficulties we have chosen the abelian Higgs model as a simple example to demonstrate this approach. We emphasize that our analysis does not rely on any particular property of the abelian theory. In fact, we will outline the application to the non-abelian case in the last Section of this article.

In the next Section we will briefly introduce the abelian Higgs model and discuss a manifestly gauge invariant technique to evaluate the generating functional of Green's functions of gauge invariant operators. In Section 3 the low energy constants which describe the low energy structure of the one-loop approximation in the abelian Higgs model up to order $p^6$ are determined. In Section 4 we will discuss the evaluation of 2- and higher loop corrections in our approach. Section 5 is devoted to renormalization. Finally, our results are summarized and discussed in Section 6.
2 The Abelian Higgs Model to One Loop

The Lagrangian of the abelian Higgs Model is given by

\[ \mathcal{L} = \frac{1}{2} \nabla_\mu \phi^T \nabla_\mu \phi - \frac{1}{2} m^2 \phi^T \phi + \frac{\lambda}{4} (\phi^T \phi)^2 + \frac{1}{4 g^2} F_{\mu\nu} F^{\mu\nu}, \]  

(2.1)

where \( \phi^A \) is a 2-component scalar field, coupled to the gauge field \( A_\mu \) through the covariant derivative

\[ \nabla_\mu \phi = \partial_\mu \phi + A_\mu T e \phi. \]  

(2.2)

The generator of the \( O(2) \) symmetry is of the form

\[ T_e = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \]  

(2.3)

The field strength is given by the expression \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The Lagrangian (2.1) is invariant under gauge transformations of the form

\[ \phi \to \exp (\omega T_e) \phi \]  

(2.4)

\[ A_\mu \to A_\mu - \partial_\mu \omega. \]  

(2.5)

For computational convenience we are working in euclidean space-time.

For \( m^2 > 0 \) the classical potential has its minimum at a nonzero value \( \phi^T \phi = m^2 / \lambda \) and the \( O(2) \) symmetry is spontaneously broken. Accordingly, the 2-component field \( \phi^A \) describes one massive mode, the Higgs particle, and one Goldstone boson. Thus, the spectrum of the abelian Higgs model consists of two massive states: the spin 0 Higgs particle, and the spin 1 gauge boson.

In order to have nontrivial solutions of the equations of motion, we furthermore have to couple external sources to the gauge boson and the Higgs field. The appropriate choice for these sources is crucial for an analysis which should be manifestly gauge invariant. Suppose, we couple the field \( \phi^A \) to a set of external fields with spin 0 by way of the source term

\[ f^T \phi. \]  

(2.6)
If one considers both the gauge field $A_\mu$ as well as the scalar field $f^A$ as external, one recovers exactly the situation of the ungauged linear sigma model \cite{[4]}. In that case, the generating functional was defined to be
\begin{equation}
e^{-W_{ug}^g[A_\mu,f]} = \int d\mu[\phi] \epsilon^{-\int d^dx (\mathcal{L} - f^T \phi)} .
\end{equation}
(2.7)

Derivatives of this functional with respect to the spin 0 field $f^A$ generate Green’s functions of the scalar fields $\phi^A$, while derivatives with respect to the spin 1 field $A_\mu$ generate Green’s functions of the current $(\partial_\mu \phi)^T T_\epsilon \phi$. Furthermore, the generating functional turns out to be gauge invariant under gauge transformations of the form (2.4) and (2.5), which act on the external sources. This property summarizes the Ward identities of the ungauged linear sigma model.

However, the situation is different if the gauge fields are dynamical degrees of freedom. In this case, only the field $f^A$ is an external source. The equations of motion are determined by the condition that the classical action be stationary, i.e.,
\begin{equation}
\delta \int d^dx \left( \mathcal{L} - f^T \phi \right) = 0 .
\end{equation}
(2.8)

The problem associated with this source term becomes obvious if variations of the fields corresponding to infinitesimal gauge transformations are considered. Since the Lagrangian itself is gauge invariant, we end up with the condition
\begin{equation}
f^T T_\epsilon \phi = 0 .
\end{equation}
(2.9)

Thus, the equations of motion have a solution only under the constraint that the gauge degree of freedom of the scalar field $\phi^A$ is frozen in a very particular way, depending on the direction of the external source $f^A$. If an external current $j_\mu$ is coupled to the gauge field via
\begin{equation}
j_\mu A_\mu ,
\end{equation}
(2.10)

the condition analogous to Eq. (2.9) is given by
\begin{equation}
\partial_\mu j_\mu = 0 ;
\end{equation}
(2.11)
i.e., the external current has to be conserved.

Note that the source term (2.6) explicitly breaks the symmetry in both the gauged and the ungauged linear sigma model. In the latter case this
does not hurt. The generating functional (2.7) still reflects the symmetry properties of the ungauged theory. Although fluctuations corresponding to global $O(2)$ transformations also yield a constraint, it does not affect the equations of motion. In fact, it is trivially satisfied by the classical fields.

If the symmetry is gauged, however, a symmetry breaking external source couples to the gauge degrees of freedom. This does not seem to be a suitable perturbation of the system. The action does not have a stationary point, if such a source term is present. Thus, we are naturally led to consider only external sources which couple to gauge invariant operators. Green’s functions of these operators are the proper objects to analyze in a gauge theory.

We will see below that a manifestly gauge invariant treatment is indeed possible if the analysis involves only gauge invariant sources. This is not only true at the classical level, but also if quantum corrections are taken into account.

For our analysis, a convenient choice of gauge invariant operators is the scalar density $\phi^T \phi$ and the field strength $F_{\mu\nu}$:

$$L_\sigma = \frac{1}{2} \nabla_\mu \phi^T \nabla_\mu \phi - \frac{1}{2} m^2 \phi^T \phi + \frac{\lambda}{4} (\phi^T \phi)^2 + \frac{1}{4 g^2} F_{\mu\nu} F_{\mu\nu}$$

$$- \frac{1}{2} h \phi^T \phi - \frac{1}{2} k_{\mu\nu} F_{\mu\nu} - c_{hh} h^2 - c_{mh} m^2 h .$$ (2.12)

The generating functional $W_\sigma[h,k_{\mu\nu}]$ is defined by a path integral

$$e^{-W_\sigma[h,k_{\mu\nu}]} = \int d\mu[\phi,A] e^{-\int d^4x L_\sigma} .$$ (2.13)

Derivatives of this functional with respect to the field $h$ generate Green’s functions of the scalar density $\phi^T \phi$, while derivatives with respect to the source $k_{\mu\nu}$ generate Green’s functions of the field strength $F_{\mu\nu}$. In the spontaneously broken phase, these Green’s functions have one-particle poles from both bosons. Thus, one can even extract $S$-matrix elements from the generating functional (2.13).

Green’s functions of the composite operator $\phi^T \phi$ are more singular at short distances than Green’s functions of the scalar field itself. Time ordering of these operators gives rise to ambiguities and corresponding Green’s functions are only unique up to contact terms. We will see below that renormalization of the two constants $c_{hh}$ and $c_{mh}$ in the Lagrangian (2.12) is necessary to render the generating functional (2.13) finite after the regulator is
removed. The ambiguity is then reflected by the presence of the finite parts of these two constants.

At tree-level, the generating functional is given by

\[ W_\sigma[h, k_{\mu\nu}] = \int d^d x \mathcal{L}_\sigma(\tilde{\phi}, \tilde{A}_\mu) , \]

(2.14)

where \( \tilde{\phi} \) and \( \tilde{A}_\mu \) are determined by the equations of motion

\[
\begin{align*}
(-\tilde{\nabla}_\mu \tilde{\nabla}_\mu - m^2 + \lambda \tilde{\phi}^T \tilde{\phi} - h) \tilde{\phi} &= 0 \\
(-\delta_{\mu\nu} \Box + \partial_\mu \partial_\nu) \tilde{A}_\mu + g^2 (J_\nu + \partial_\mu k_{\mu\nu}) &= 0 ,
\end{align*}
\]

(2.15)

(2.16)

with

\[
\tilde{\nabla}_\mu = \partial_\mu + T_e \tilde{A}_\mu .
\]

(2.17)

The \( O(2) \) current \( J_\mu \) is of the form

\[
J_\mu \equiv (\nabla_\mu \tilde{\phi})^T T_e \tilde{\phi} .
\]

(2.18)

It is useful to introduce the following parametrization of the classical solution \( \tilde{\phi}^A \):

\[
\tilde{\phi}^A = \frac{m}{\sqrt{\lambda}} R U^A , \quad U^T U = 1 ,
\]

(2.19)

where the radial variable \( R \) describes the massive mode, while the field \( U^A \) corresponds to the Goldstone boson. In terms of the new variables, Eqs. (2.15) and (2.16) are of the form:

\[
\begin{align*}
(-\Box + m^2 (R^2 - 1) + \tilde{\nabla}_\mu U^T \tilde{\nabla}_\mu U - h) R &= 0 \\
\partial_\mu J_\mu &= 0 \\
(-\delta_{\mu\nu} \Box + \partial_\mu \partial_\nu) \tilde{A}_\mu + g^2 (J_\nu + \partial_\mu k_{\mu\nu}) &= 0 .
\end{align*}
\]

(2.20)

(2.21)

(2.22)

Several things about these equations are worth being noticed. Gauge invariance implies that they have a whole class of solutions. Every two representatives are related to each other by a gauge transformation. Due to Eq. (2.21) and the asymmetry of the external source \( k_{\mu\nu} \), the vector field couples to a conserved current. In order to solve Eqs. (2.21) and (2.22) we write the Goldstone boson field as

\[
U = e^{w T_e} U_0 ,
\]

(2.23)
where $U_0$ is an arbitrary but constant $O(2)$ vector. The equations of motion (2.20)-(2.22) then involve the three quantities $R$, $\vec{A}_\mu^T$, and $\vec{A}_\mu^L + \partial_\mu \omega$, where $\vec{A}_\mu^L = \partial_\mu (1/\Box) \partial_\nu \vec{A}_\nu$ and $\vec{A}_\mu^T = \vec{A}_\mu - \vec{A}_\mu^L$. Neither the equations of motion nor the action depend on the vector $U_0$. The radial component $R$ describes the Higgs boson, while the transverse and the longitudinal degrees of freedom of the gauge boson are described by the two gauge invariant fields $\vec{A}_\mu^T$ and $\vec{A}_\mu^L + \partial_\mu \omega$. Picking a certain representative for our solution is equivalent to specifying how the longitudinal degree of freedom is split between the two fields $\vec{A}_\mu^L$ and $U^A$. Two extreme choices are the unitary gauge, $\omega = \text{const}$, and the condition $\partial_\mu \vec{A}_\mu = 0$.

The one-loop contribution to the generating functional can be evaluated with the saddle point method. Using the parametrization

$$\phi = \vec{\phi} + f$$

$$A_\mu = \vec{A}_\mu + g q_\mu$$

for the fluctuations around the classical fields $\vec{\phi}^A$ and $\vec{A}_\mu$, one obtains

$$e^{-W_{\alpha[h,k_{\mu\nu}]} = e^{-\int d^4x \mathcal{L}_\alpha(\vec{\phi},A_\mu) \int d\mu[A,\vec{A}] e^{-\frac{1}{2} \int d^4xy \vec{D} y} ,}$$

with

$$y = \begin{pmatrix} f \\ q_\mu \end{pmatrix} ,$$

and

$$\vec{D} = \begin{pmatrix} D^{f f} & D^{f q} \\ D^{q f} & D^{q q} \end{pmatrix} .$$

The components of the symmetric operator $\vec{D}$ are given by

$$D^{f f} = -\nabla_\mu \nabla_\mu - (m^2 - \lambda \vec{\phi}^T \vec{\phi} + h) \mathbf{1} + 2 \lambda \vec{\phi} \vec{\phi}^T$$

$$D^{f q} = -g T_e \vec{\phi} \partial_\mu - 2 g T_e (\nabla_\mu \vec{\phi})$$

$$D^{q f} = -g \vec{\phi}^T T_e \nabla_\nu + g (\nabla_\nu \vec{\phi})^T T_e$$

$$D^{q q} = -\delta_{\mu\nu} \Box + \partial_\mu \partial_\nu + \delta_{\mu\nu} g^2 \vec{\phi} \vec{\phi}^T .$$

Gauge invariance implies that the operator $\vec{D}$ has zero eigenvalues corresponding to fluctuations $y$ which are equivalent to infinitesimal gauge transformations. Indeed, if $\vec{\psi}^j = (\vec{\phi}^A, \vec{A}_\mu)$ is a solution of the equation of motion,
i.e., a stationary point of the classical action,

$$\frac{\delta S}{\delta \psi^i} \bigg|_{\psi=\bar{\psi}} = 0 \ ,$$

(2.33)

then any gauge transformation (2.4), (2.5) yields another equivalent solution. Thus, differentiating equation (2.33) with respect to the gauge parameter $\omega$ one obtains

$$\frac{\delta^2 S}{\delta \psi^i \delta \psi^j} \frac{\delta \psi^j}{\delta \omega} \bigg|_{\psi=\bar{\psi}} = 0 \ .$$

(2.34)

This can also be verified explicitly, using the following representation of the zero eigenvectors $\zeta$ in terms of scalar fields $\alpha$,

$$\zeta = P\alpha \ ,$$

(2.35)

where

$$P = \begin{pmatrix} gT_\sigma \phi^A \\ -\partial_\mu \end{pmatrix} .$$

(2.36)

Then Eq. (2.34) translates to the identity

$$P^T \bar{D} = \bar{D} P = 0 \ .$$

(2.37)

Let $\alpha_m$ be the eigenvectors of the operator $P^T P$, i.e.

$$P^T P \alpha_m = l_m \alpha_m \ .$$

(2.38)

Then, the expansion of the fluctuation $y$ in terms of eigenvectors of the operator $\bar{D}$ is given by

$$y = \sum_n a_n \zeta_n + \sum_m b_m \zeta_m \ ,$$

(2.39)

where $\zeta_m = P\alpha_m$ and $\zeta_n$ have zero and non-zero eigenvalues, respectively.

In order to evaluate the path integral (2.26), we use Polyakov’s method and equip the space of fields with a metric

$$||y||^2 = \int d^d x y^T y = \sum_n a_n^2 + \sum \ b_m^2 l_m \ .$$

(2.40, 2.41)
With our choice for the scalar fields $\alpha_m$, the metric on the kernel of the differential operator $\tilde{D}$ is indeed diagonal:

$$g_{\bar{m}m} = \int d^d x \alpha_\bar{m} P^T P \alpha_m = \delta_{\bar{m}m} l_m .$$

(2.42)

The volume element associated with this metric is then given by

$$d\mu[\phi, A] = \mathcal{N} \prod_n da_n \prod_m db_m \sqrt{\det P^T P} .$$

(2.43)

The integration over the zero-modes yields the volume factor of the gauge group, which can be absorbed by the normalization of the integral. The remaining integral over the non-zero modes is damped by the usual gaussian factor. One obtains the following result for the one-loop generating functional

$$W_\sigma[h, k_{\mu\nu}] = \int d^d x \mathcal{L}_\sigma + \frac{1}{2} \text{ln}\det' \tilde{D} - \frac{1}{2} \text{ln}\det P^T P ,$$

(2.44)

where $\det' \tilde{D}$ is defined as the product of all non-zero eigenvalues of the operator $\tilde{D}$. The evaluation of path integrals like the one in Eq. (2.26), with a semi-definite quadratic form in the exponent, is also discussed in the context of instanton calculations [9].

Since zero and non-zero eigenvectors are orthogonal to each other, implying $P^T \xi_n = 0$, one furthermore verifies the useful identity

$$\det' \tilde{D} = \frac{\det(\tilde{D} + PP^T)}{\det P^T P} .$$

(2.45)

Finally, by changing to a new basis, we separate the massless fluctuations around the classical solution $\tilde{\phi}^A$ which are bound to the sphere $U^T U = 1$, from the massive fluctuation along $U^A$:

$$f = \xi U + \eta \epsilon , \quad \epsilon = T_e U .$$

(2.46)

In order to describe the action of the differential operator $\tilde{D}$ in the new basis, we introduce the following notation:

$$D = -\Box + m^2 (R^2 - 1) + \nabla_\mu U^T \nabla_\mu U - h$$

(2.47)

$$d = D + 2m^2 R^2$$

(2.48)

$$\delta = -2f_\mu \partial_\mu - (\partial_\mu f_\mu)$$

(2.49)
\[ \delta_\mu = -2 M R f_\mu \]  
\[ \Delta_\mu = - M R \partial_\mu - 2 (\partial_\mu R) \]  
\[ f_\mu = U^T \nabla_\mu \epsilon \]  
\[ \Theta = D - \delta^T d^{-1} \delta \]  
\[ \vartheta_\mu = \Delta_\mu - \delta^T d^{-1} \delta_\mu \]  
\[ \mathcal{D}_{\mu \nu} = D^q_{\mu \nu} - \delta^T d^{-1} \delta_\nu - \vartheta_\mu \Theta^{-1} \vartheta_\nu , \]  
\[ M = m g \sqrt{\lambda} \]

is the bare mass of the gauge field. We furthermore diagonalize the quadratic form \( y^T \hat{D} y \) by the transformation
\[ \xi \to \xi - d^{-1} \delta \eta + d^{-1} \delta \Theta^{-1} \vartheta_\mu q_\mu - d^{-1} \delta_\mu q_\mu \]  
\[ \eta \to \eta - \Theta^{-1} \vartheta_\mu q_\mu . \]  

The quadratic form in Eq. (2.26) now reads:
\[ \int d^d x y^T \hat{D} y = \int d^d x (\xi d \xi + \eta \Theta \eta + q D q) . \]

The transformation of the basis also changes the metric on the space of functions. The zero-modes are now of the form
\[ \hat{P} \alpha_m \]
with
\[ \hat{P}^T = (0, 0, \partial_\mu) , \]
i.e., they correspond to longitudinal fluctuations of the gauge field. The generating functional turns out to be
\[ W_\sigma[h, k_{\mu \nu}] = \int d^d x L_\sigma + \frac{1}{2} \text{lndet} d + \frac{1}{2} \text{lndet} \Theta + \frac{1}{2} \text{lndet}' \mathcal{D} \]  
\[ - \frac{1}{2} \text{lndet} \hat{P}^T \hat{P} , \]

The last term is constant and will be absorbed in the overall normalization.
To further simplify the calculation, one can make use of the transversality of the operator $\mathcal{D}$ and write
\[
\mathcal{D}_{\mu\nu} = PT_{\mu\rho}D_{\rho\sigma}PT_{\sigma\nu},
\]
(2.62)
where
\[
PT_{\mu\nu} = \delta_{\mu\nu} - \partial_{\mu} \frac{1}{\Box} \partial_{\nu}
\]
(2.63)
projects on the non-zero modes. Eq. (2.62) implicitly involves the equations of motion and indeed simplifies the evaluation of the determinant. Using an identity similar to (2.45), one obtains
\[
\ln \det' \mathcal{D} = - \ln \det d_M + \ln \det (d_M + PT \sigma PT),
\]
(2.64)
with
\[
d_M = -\Box + M^2
\]
(2.65)
\[
\sigma_{\mu\nu} = \delta_{\mu\nu}M^2(R^2 - 1) - \delta^T_{\mu}d^{-1}\delta_{\nu} - \vartheta^T_{\mu}\Theta^{-1}\vartheta_{\nu}.
\]
(2.66)

3 The Effective Lagrangian

In this section we will discuss the effective theory of the abelian Higgs model for the case of a heavy Higgs boson mass, i.e.,
\[
p^2, M^2 \ll 2m^2.
\]
(3.1)

It is well known that the effective field theory can be described by an effective Lagrangian. In our case this Lagrangian is gauge invariant, and depends on the 2-component Goldstone boson field $U^A$, confined to the sphere $U^TU = 1$, the vector field $A_{\mu}$, and the external sources
\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}(F_{\mu\nu}, U, \nabla_{\mu}U, \nabla_{\mu}\nabla_{\nu}U, h, k_{\mu\nu}, \ldots).
\]
(3.2)

It describes the dynamics of one massive gauge boson. The corresponding generating functional is defined as a path integral,
\[
e^{-W_{\text{eff}}[h, k_{\mu\nu}]} = \int d\mu[U, A]e^{-\int d^4x \mathcal{L}_{\text{eff}}},
\]
(3.3)
The effective Lagrangian is determined by a matching relation, which requires that both the full and the effective theory yield the same Green’s functions in the low energy region:

$$W_\sigma[h, k_{\mu\nu}] = W_{\text{eff}}[h, k_{\mu\nu}] .$$  \hspace{1cm} (3.4)

At low energies, these Green’s functions have non-local contributions related to the singularities of the light vector boson. These contributions drop out of the matching relation and the remaining terms involve powers of the coupling constants, the momenta and the mass. In order to systematically evaluate the effective Lagrangian, we need to understand the counting of loops in the full theory and of the low energy expansion in the effective theory.

In the ungauged linear sigma model, the loop expansion generates a power series in the coupling constant $\lambda$, while the low energy expansion at a given loop-level produces powers of the momenta. Thus, it may seem that one has to keep track of four quantities if the theory is gauged, i.e., powers of the couplings $\lambda$ and $g$ as well as powers of the momenta and the gauge boson mass. However, such a counting scheme is ambiguous, since the gauge boson mass itself depends on the coupling constants, as shown in Eq. (2.56). This expression also indicates that it will not be very transparent to count mass factors in terms of the quantities $\lambda$ and $g^2$. The loop expansion in the full theory generates positive powers of the coupling $\lambda$, while the low energy expansion produces negative powers thereof. To simplify the bookkeeping, one may introduce a loop factor $l$, counting the number of loops in the full theory, and treat $\lambda$ and $g^2$ as order $l$. In this case $n$-loop contributions will be of order $l^{n-1}$.

Equivalently, one may discard the coupling constant $g$ from the counting scheme. This is a consequence of the definition of the vector field $A_\mu$ in Eq. (2.2), which is scaled such that the constant $g$ does not explicitly occur in the covariant derivative. As a result, this constant naturally enters all loop correction only through the gauge boson mass $M$. Regarding the one-loop contributions to the generating functional, this can readily be inferred from Eqs. (2.47)-(2.52). As we will see in the next Section, this is also true for all higher loops. Furthermore, with this bookkeeping powers of $\lambda$ count the number of loops in the full theory.

In order to evaluate the low energy expansion at a given loop-level, we treat the covariant derivative $\nabla_\mu$, the gauge boson mass $M$, and the momenta
as quantities of order $p$, while the external source $h$ is of order $p^2$. The scalar fields, the Higgs mass $\sqrt{2} m$, and the external source $k_{\mu\nu}$ are quantities of order one. In counting the mass $M$ as order $p$, the low energy expansion is carried out at a fixed ratio $p^2/M^2$, and correctly reproduces all singularities associated with the gauge boson.

The coherence of these rules requires that the coupling constant $g$ is treated as a quantity of order $p$. Note that this is different from the usual dimensional analysis: the constant $g$ has dimension (mass)$^0$, yet it counts as order $p$ in the low energy expansion. This is similar to chiral perturbation theory in QCD \cite{10}, where, for example, the light quark masses with dimension (mass)$^1$ are treated as quantities of order $p^2$.

The effective Lagrangian is a sum of terms with an increasing number of derivatives, mass factors and powers of the external fields,

$$L_{\text{eff}} = L_2 + L_4 + L_6 + \ldots ,$$

(3.5)

where $L_i$ is of order $p^i$. Furthermore, if the coupling of the scalar field is not too strong, the low energy constants admit an expansion in powers of the parameter $\lambda$,

$$d_i = \frac{1}{\lambda} d_{i}^{\text{tree}} + d_{i}^{1-\text{loop}} + \lambda d_{i}^{2-\text{loop}} + \ldots ,$$

(3.6)

corresponding to the loop expansion in the full theory. In this case the accuracy of the effective field theory description is controlled by the order of both the momentum and the coupling constant $\lambda$. For values of $\lambda$ close to the strong coupling region, one may consider higher orders in the expansion \( (3.4) \). Large values of the momentum or the gauge boson mass may require including higher orders in Eq. \((3.5)\). In the following, we will determine this Lagrangian up to order $p^6$, and the low-energy constants up to order $\lambda^0$.

In order to evaluate the low-energy constants, one can write down the most general effective Lagrangian up to order $p^6$, calculate the generating functional in both the full and the effective theory, and solve the matching relation \((3.4)\). However, we will make use of the fact that powers of the constant $\lambda$ count the number of loops. To evaluate the one-loop contribution to the generating functional of the effective theory up to order $\lambda^0$, only the parameters $d_{i}^{\text{tree}}$ in Eq. \((3.6)\) are relevant. They can be read off from the low energy expansion of the classical action of the full theory, i.e., from
\[ \int d^d x \mathcal{L}_\sigma(\phi, \bar{A}_\mu) \]

\[ = \int d^d x \left(-\frac{m^4}{4\lambda} R^4 + \frac{1}{4g^2} \bar{F}_{\mu\nu} \bar{F}_{\mu\nu} - \frac{1}{2} k_{\mu\nu} \bar{F}_{\mu\nu} - c_{hh} h^2 - c_{mh} m^2 h \right) (3.7) \]

For slowly varying external fields, the behaviour of the massive mode \( R \) is under control and the equation of motion (2.20) can be solved algebraically. The result is a series of local terms with increasing order in \( p^2 \). Since Eq. (2.20) does not involve the parameter \( \lambda \), all terms in the low energy expansion of the classical action count as order \( \lambda^{-1} \), except for the two contact terms and the source term for the field strength. Note that the kinetic term of the vector field also counts as order \( \lambda^{-1} \). In order to maintain a coherent scheme in the presence of external sources, we treat the quantities \( c_{mh}, c_{hh} \) and \( k_{\mu\nu} \) as order \( \lambda^{-1} \). One obtains

\[ \mathcal{L}^\text{tree}_2 = \frac{m^2}{2\lambda} \nabla_\mu U^T \nabla_\mu U + \frac{1}{4g^2} \bar{F}_{\mu\nu} \bar{F}_{\mu\nu} - \frac{m^2}{2\lambda} h - c_{mh} m^2 h - \frac{1}{2} k_{\mu\nu} \bar{F}_{\mu\nu} (3.8) \]

\[ \mathcal{L}^\text{tree}_4 = -\frac{1}{4\lambda} (\nabla_\mu U^T \nabla_\mu U)^2 + \frac{1}{2\lambda} h (\nabla_\mu U^T \nabla_\mu U) - \frac{1}{4\lambda} h^2 - c_{hh} h^2 \quad (3.9) \]

\[ \mathcal{L}^\text{tree}_6 = -\frac{1}{8m^2\lambda} (\nabla_\mu U^T \nabla_\mu U - h) \Box (\nabla_\nu U^T \nabla_\nu U - h) \quad . \quad (3.10) \]

Now one can evaluate the one-loop contribution to the generating functional in the effective theory, using the technique described in the previous section. At order \( \lambda^0 \), the matching relation (3.4) is of the form

\[ \int d^d x \mathcal{L}_\sigma + \frac{1}{2} \text{Indet} d + \frac{1}{2} \text{Indet} \left(1 - D^{-1} \delta^T d^{-1} \delta\right) \]

\[ + \frac{1}{2} \text{Indet} \bar{D} + \frac{1}{2} \text{Indet} (d_M + PT \bar{\sigma} PT) \]

\[ = \int d^d x (\mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6) + \frac{1}{2} \text{Indet} \bar{D} + \frac{1}{2} \text{Indet} (d_M + PT \bar{\sigma} PT) \quad . \]

The quantities on the left hand side were defined in Eqs. (2.47)-(2.55), (2.65) and (2.66), whereas those on the right hand side are given by

\[ \bar{D} = -\Box + \bar{X}_{\mu\nu} \partial_\mu \partial_\nu + (\partial_\mu \bar{X}_{\mu\nu}) \partial_\nu \quad (3.12) \]

\[ \bar{\sigma}_{\mu\nu} = -M^2 \bar{X}_{\mu\nu} - \bar{\Delta}_\mu \bar{D}^{-1} \bar{\Delta}_\nu \quad (3.13) \]
\[ \Delta_{\mu} = -M \partial_{\mu} + M \bar{X}_{\mu,\nu} \partial_{\nu} + M (\partial_{\nu} \bar{X}_{\nu\mu}) \quad (3.14) \]
\[ \bar{X}_{\mu\nu} = \frac{1}{m^2} \left( 2 f_{\mu} f_{\nu} + \delta_{\mu\nu} (\nabla_{\rho} U^T \nabla_{\rho} U - h) \right) . \quad (3.15) \]

Note that the operators on the right hand side of the matching relation (3.11) involve the solutions of the equations of motion of the effective theory, while those on the left hand side depend on the solutions of the equations of motion of the full theory. At the stationary point, however, the corresponding corrections are of second order in the shift of the fields and beyond the present accuracy. Thus, our notation will not distinguish between the two solutions.

The last two terms on both sides of Eq. (3.11) contain non-local contributions corresponding to loops which involve only the light degrees of freedom. They drop out of the matching relation. In the present case, however, the situation is even simpler. There are no non-local contributions up to order $p^6$.

This is a special property of the case $N = 2$ of the gauged linear $O(N)$ sigma model. At order $p^2$ the effective Lagrangian describes a free massive vector boson. Interaction only shows up at higher order. Furthermore, there are no loop contributions in the effective theory at order $p^4$. Thus, in our case all low-energy constants at order $p^2$ and $p^4$ are finite.

The leading contributions of both $\text{Indet} D$ and $\text{Indet} \bar{D}$ are of order $p^8$. Thus, there is only one loop contribution in the effective theory of the abelian Higgs model at order $p^6$, the tadpole graph of the gauge field:

\[ \frac{1}{2} \text{Indet} \left( 1 + d_M^{-1} PT \bar{\sigma} PT \right) = -\frac{M^2}{2} G_{\mu\nu}(0) \int d^d x \bar{X}_{\mu\nu} + \mathcal{O}(p^8) . \quad (3.16) \]

The free propagator of the vector field is of the form

\[ G_{\mu\nu}(x - y) \doteq \langle x | d_M^{-1} PT_{\mu\nu} | y \rangle . \quad (3.17) \]

For completeness sake, we list all one-loop corrections to the generating functional of the full theory which will contribute to the effective Lagrangian up to the order $p^6$. First we define

\[ d = d_m + \sigma_m \]
\[ d_m = -\Box + 2m^2 \]
\[ D = D_0 + \sigma_0 \]
\[ D_0 = -\Box . \]

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One obtains the following terms which involve only the propagator of the massive mode:

\[
\frac{1}{2} \ln \det d = \frac{1}{2} \ln \det d_m + \frac{1}{2} \text{Tr} \left( d_m^{-1} \sigma_m \right) - \frac{1}{4} \text{Tr} \left( (d_m^{-1} \sigma_m)^2 \right) + \frac{1}{6} \text{Tr} \left( (d_m^{-1} \sigma_m)^3 \right).
\]  
(3.22)

The second term, a tadpole graph, is of order \( p^2 \), whereas the third and fourth traces are of order \( p^4 \) and \( p^6 \), respectively.

Mixed loops, which contain Higgs and Goldstone boson propagators, are given by:

\[
\frac{1}{2} \ln \det \left( 1 - D^{-1} \delta^T d^{-1} \delta \right) = -\frac{1}{2} \text{Tr} \left( \delta D_0^{-1} \delta^T d^{-1} \sigma_m d^{-1} \right)
- \frac{1}{2} \text{Tr} \left( \delta D_0^{-1} \delta^T (d_m^{-1} \sigma_m)^2 d^{-1} \right) + \frac{1}{2} \text{Tr} \left( \delta D_0^{-1} \sigma_0 D_0^{-1} \delta^T d^{-1} \right)
- \frac{1}{4} \text{Tr} \left( (\delta D_0^{-1} \delta^T d^{-1})^2 \right) + \frac{1}{2} \text{Tr} \left( (\delta D_0^{-1} \delta^T d^{-1})^2 \sigma_m d^{-1} \right)
- \frac{1}{6} \text{Tr} \left( (\delta D_0^{-1} \delta^T d^{-1})^3 \right).
\]  
(3.23)

Here only the first term leads to contributions of order \( p^2 \). The second and fifth term are of order \( p^4 \), while all other terms are of order \( p^6 \).

Finally, the following terms involve the gauge boson propagator:

\[
\frac{1}{2} \ln \det \left( 1 + d_M^{-1} P T \sigma \right) = \frac{1}{2} \text{Tr} \left( d_M^{-1} P T \sigma d^{-1} \right)
- \frac{1}{2} \text{Tr} \left( d_M^{-1} P T \delta^T d^{-1} \delta \right) + \frac{1}{2} \text{Tr} \left( d_M^{-1} P T \delta^T d^{-1} \sigma_m d^{-1} \delta \right)
- \frac{1}{2} \text{Tr} \left( d_M^{-1} P T \delta^T d^{-1} \delta^T d^{-1} \delta \right).
\]  
(3.24)

The second term, involving the mixed two-point function with a propagator of the gauge field and the Higgs boson already contributes at order \( p^4 \). The tadpole graph and the three- and four-point functions contribute only at order \( p^6 \).

Techniques to evaluate the low-energy expansion of the traces in Eqs. (3.22), (3.23), and (3.24) are discussed in Ref. [4]. We obtain the following result for the effective Lagrangian of the abelian Higgs model up to order \( p^6 \) and \( \lambda^0 \):

\[
L_2 = \left( \frac{1}{4 \lambda} - 3 \Lambda_\varepsilon (2m^2) + \frac{1}{16 \pi^2} \frac{1}{4} \right) 2m^2 \nabla^\mu U^T \nabla_\mu U + \frac{1}{4g^2} F_{\mu \nu} F_{\mu \nu}
\]  
(3.25)
\[
\mathcal{L}_4 = (3\Lambda_\epsilon(2m^2) + \frac{1}{16\pi^2} \frac{1}{4}) M^2 \nabla_\mu U^T \nabla_\mu U \\
- \left( \frac{1}{6} \Lambda_\epsilon(2m^2) + \frac{1}{16\pi^2} \frac{1}{72} \right) \tilde{F}_{\mu\nu} \tilde{F}_{\mu\nu} \\
- \left( \frac{1}{4\lambda} - 5\Lambda_\epsilon(2m^2) - \frac{1}{16\pi^2} \frac{19}{12} \right) (\nabla_\mu U^T \nabla_\mu U)^2
\]  
\quad \text{(3.26)}

\[
\mathcal{L}_6 = (3\Lambda_\epsilon(2m^2) + \frac{1}{16\pi^2} \frac{1}{4}) \frac{M^4}{2m^2} \nabla_\mu U^T \nabla_\mu U \\
+ \left( \frac{1}{3} \frac{1}{16\pi^2} \right) \frac{M^2}{2m^2} \tilde{F}_{\mu\nu} \tilde{F}_{\mu\nu} \\
- \left( \frac{6\Lambda_\epsilon(2m^2) + \frac{1}{16\pi^2} \frac{9}{2} \right) \frac{M^2}{2m^2} (\nabla_\mu U^T \nabla_\mu U)^2 \\
- \frac{1}{16\pi^2} \frac{233}{72} \frac{1}{2m^2} (\nabla_\mu U^T \nabla_\mu U)^3 \\
- \left( \frac{1}{2\lambda} - 14\Lambda_\epsilon(2m^2) - \frac{1}{16\pi^2} \frac{50}{9} \right) \frac{1}{2m^2} (\nabla_\mu U^T \nabla_\mu U)(\nabla_\nu U^T \nabla_\rho U^T \nabla_\lambda U) \\
- \left( \frac{1}{2\lambda} - 14\Lambda_\epsilon(2m^2) - \frac{1}{16\pi^2} \frac{58}{9} \right) \frac{1}{2m^2} (\nabla_\mu U^T \nabla_\mu U)(\nabla_\nu U^T \nabla_\rho U^T \nabla_\lambda U) \\
- \frac{1}{16\pi^2} \frac{25}{36} \frac{1}{2m^2} (\nabla_\mu U^T \nabla_\mu U)(\nabla_\nu \nabla_\rho U^T \nabla_\lambda U) \\
- \frac{1}{16\pi^2} \frac{1}{24} \frac{1}{2m^2} (\nabla_\mu \nabla_\mu \nabla_\rho U^T \nabla_\lambda U) \\
+ \frac{1}{16\pi^2} \frac{1}{36} \frac{1}{2m^2} (\nabla_\mu \nabla_\mu \nabla_\rho U^T U)(U^T \nabla_\nu \nabla_\nu \nabla_\rho U),
\]  
\quad \text{(3.27)}

where

\[
\Lambda_\epsilon(2m^2) = \frac{1}{16\pi^2} \mu^{d-4} \left( \frac{1}{d - 4} - \frac{1}{2} \ln(4\pi + \Gamma'(1)) \right) + \frac{1}{32\pi^2} \ln(\frac{2m^2}{\mu^2}).
\]  
\quad \text{(3.28)}

Note that we have switched off the external sources \(h\) and \(k_{\mu\nu}\). An effective field theory analysis to one loop, based on this Lagrangian, will yield the low energy expansion of the one-loop approximation in the abelian Higgs model up to order \(p^6\). Two-loop corrections in the effective theory will also be of order \(p^6\). However, they are of higher order in \(\lambda\).
4 On the Loop Expansion

In this section we will briefly discuss loop contributions of arbitrary order. To go beyond the one-loop approximation, one has to include higher orders in the fluctuation $y$ around the classical solution $(\bar{\phi}^A, \bar{A}_\mu)$ in Eq. (2.26). One obtains

$$\int \! d^d x \mathcal{L}_\sigma(\phi, A) = \int \! d^d x \left( \mathcal{L}_\sigma(\bar{\phi}, \bar{A}_\mu) + \frac{1}{2} y^T \tilde{D} y + \mathcal{L}_\sigma^{[3]} + \mathcal{L}_\sigma^{[4]} \right),$$

where $y^T = (f, q_\mu)$, and

$$\mathcal{L}_\sigma^{[3]} = g q_\mu (\nabla_\mu f^T) T e f + g^2 q_\mu q_\mu \bar{\phi}^T f + \lambda f^T f \bar{\phi}^T f \quad (4.2)$$

$$\mathcal{L}_\sigma^{[4]} = \frac{g^2}{2} q_\mu q_\mu f^T f + \frac{\lambda}{4} (f^T f)^2. \quad (4.3)$$

The perturbative expansion of the generating functional to arbitrary order is then given by

$$e^{-W_{[h,k_{\mu\nu}]}} = e^{-\int \! d^d x \mathcal{L}_\sigma(\bar{\phi}, \bar{A}_\mu)} \int \! d\mu(\phi, A) e^{-\frac{1}{2} \int \! d^d x y^T \tilde{D} y} \left( 1 - \int \! d^d x \mathcal{L}_\sigma^{[4]} \right) + \frac{1}{2} \left( \int \! d^d x \mathcal{L}_\sigma^{[3]} \right)^2 + \ldots \quad (4.4)$$

Note that any 3- (4-)vertex can emit up to three (four) zero-modes. To get a first idea about how diagrams with zero-modes have to be treated, let us consider diagrams where one 3-vertex emits exactly three zero-modes. To make things well-defined, one may add an infinitesimal mass term $y^T \epsilon y$ to the quadratic form in the exponent of Eq. (4.4). Substituting

$$y = \sum_n b_n P\alpha_n \quad (4.5)$$

for every fluctuation, one obtains

$$\int \! d^d x \mathcal{L}_\sigma^{[3]} = \frac{g^3}{3} \sum_{mno} b_m b_n b_o \int \! d^d x (\partial_\mu J_\mu) \alpha_m \alpha_n \alpha_o. \quad (4.6)$$

This expression vanishes identically due to current conservation (2.21). In order to derive Eq. (4.6), it was necessary to sum over all zero-modes. Individual diagrams with zero-modes may well yield non-zero contributions to
the generating functional. However, the example shows that all diagrams where one particular 3-vertex emits three zero-modes cancel each other. The situation is more complicated if 3-vertices emit less than three zero-modes or if 4-vertices are involved. In such cases, a cancellation can only be expected between all diagrams involving the same number of zero-modes.

This complication can be traced back to our parametrization (2.24) and (2.25) of the quantum fluctuations. The Lagrangian is invariant under gauge transformations of the form (2.4) and (2.5), acting on the fields $(\phi, A_\mu)$. For fixed classical fields $(\bar{\phi}, \bar{A}_\mu)$, they imply the following transformations of the quantum fluctuations:

\begin{align}
  f' &= e^{\omega T} f + \left(e^{\omega T} - 1\right) \bar{\phi} \\
  q'_\mu &= q_\mu - \frac{1}{g} \partial_\mu \omega .
\end{align}

Only infinitesimal gauge transformations correspond to zero-modes of the operator $\hat{D}$:

\begin{equation}
y' = y + \frac{1}{g} P \omega + \mathcal{O}(\omega^2) ,
\end{equation}

where $y$ itself is also treated as order $\omega$. Thus, the quadratic form on the right hand side of Eq. (4.1) is gauge invariant only at leading order in $\omega$. Under finite gauge transformations, only the sum of the last three terms is invariant, which leads to nontrivial identities between the vertices.

In order to show that the zero-mode contributions cancel each other, it is suitable to introduce another parametrization for the quantum fluctuations:

\begin{align}
  \phi &= (1 + \tilde{f}_1) e^{\tilde{f}_2 T} \bar{\phi} \\
  A_\mu &= \bar{A}_\mu + g q_\mu .
\end{align}

With

\begin{equation}
  \tilde{f}_i = \frac{\sqrt{\lambda}}{m} \frac{\hat{f}_i}{R}
\end{equation}

one obtains

\begin{equation}
  \int d^4x \mathcal{L}_\sigma(\phi, A) = \int d^4x \left( \mathcal{L}_\sigma(\bar{\phi}, \bar{A}_\mu) + \frac{1}{2} \hat{y}^T \hat{D} \hat{y} + \hat{\mathcal{L}}^{[3]} + \hat{\mathcal{L}}^{[4]} \right) ,
\end{equation}

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where \( \hat{y}^T = (\hat{f}_1, \hat{f}_2, q_\mu) \),

\[
\hat{D} = \begin{pmatrix} d & \delta & \delta_\nu \\ \delta^T & D & \Delta_\nu \\ \delta_\mu^T & \Delta_\mu & D_{\mu\nu} \end{pmatrix}, \tag{4.14}
\]

and

\[
\hat{L}^{[3]}_\sigma = \sqrt{\lambda} m R \hat{f}_1^3 + \frac{\sqrt{\lambda}}{m} M^2 R \hat{f}_1 (q_\mu + \partial_\mu \left( \frac{\hat{f}_2}{M R} \right))^2 \\
- \frac{\sqrt{\lambda}}{m} M \hat{f}_1^2 \hat{f}_\mu (q_\mu + \partial_\mu \left( \frac{\hat{f}_2}{M R} \right)), \tag{4.15}
\]

\[
\hat{L}^{[4]}_\sigma = \frac{1}{4} \lambda \hat{f}_1^4 + \frac{1}{2} m^2 M^2 \hat{f}_1^2 (q_\mu + \partial_\mu \left( \frac{\hat{f}_2}{M R} \right))^2. \tag{4.16}
\]

The components of the operator \( \hat{D} \) are given in Eqs. (2.47)-(2.51). In the new parametrization zero-modes are of the form

\[
\hat{y}^T = (0, M R \omega, -\partial_\mu \omega), \tag{4.17}
\]

and thus correspond to gauge transformations of arbitrary size. Furthermore, the combination

\[
q_\mu + \partial_\mu \left( \frac{\hat{f}_2}{M R} \right) \tag{4.18}
\]

is gauge invariant, and vanishes identically if a zero-mode is inserted. Thus, in the new parametrization it is obvious that diagrams with zero-modes do not contribute to the generating functional. The vertices in Eqs. (4.15)-(4.16) are already grouped appropriately. If the operator \( \hat{D} \) is diagonalized by the transformation (2.57), applied to the fields \((\hat{f}_1, \hat{f}_2, q_\mu)\), the vertices in \( \hat{L}^{[3]}_\sigma \) and \( \hat{L}^{[4]}_\sigma \) change according to

\[
\hat{f}_1(x) \rightarrow \int d^d y \langle x| \left( \hat{f}_1 + v_1^{1.2} \hat{f}_2 + v_1^{1.3} q_\mu \right) |y \rangle, \tag{4.19}
\]

\[
M(q_\mu + \partial_\mu \left( \frac{\hat{f}_2}{M R} \right))(x) \rightarrow \int d^d y \langle x| \left( v_\mu^{2.2} \hat{f}_2 + v_\mu^{2.3} q_\mu \right) |y \rangle, \tag{4.20}
\]
with

\begin{align}
  v^{1,2}_\mu &= -d^{-1}\delta \\
  v^{1,3}_\mu &= -d^{-1}\delta_\mu + d^{-1}\delta\Theta^{-1}\partial_\mu \\
  v^{2,2}_\mu &= \frac{1}{MR^2}\Delta^T_\mu \\
  v^{2,3}_{\mu\nu} &= M\delta_{\mu\nu} - \frac{1}{MR^2}\Delta^T_\mu\Theta^{-1}\partial_\nu.
\end{align}

(4.21) (4.22) (4.23) (4.24)

The zero-modes are now of the form (2.59) and correspond to longitudinal fluctuations of the gauge field. They are not emitted from any vertex, since

\begin{align}
  v^{1,3}_\mu \partial_\mu \alpha_m &= v^{2,3}_{\mu\nu} \partial_\nu \alpha_m = 0.
\end{align}

(4.25)

The full gauge field propagator entering loop graphs is given by the inverse of the operator $D_{\mu\nu}$, restricted to the subspace of nonzero-modes. Thus, the calculation of loop contributions of arbitrary order is boiled down to the evaluation of Gaussian integrals. One obtains, for example,

\begin{align}
  \int d\mu[q]\langle x|D_{\mu\nu}^{-1}y\rangle e^{-\frac{1}{2}\int d^4x q\overline{D}q} = \langle x|D_{\mu\nu}^{-1}y\rangle \sqrt{\text{det}(\hat{P}T\hat{P})}/\sqrt{\text{det}(\hat{D})},
\end{align}

(4.26)

where $q^T_\mu$ is the transverse component of the fluctuation $q_\mu$, i.e., $q^T = PTq$, and

\begin{align}
  \hat{D}D^{-1}\hat{D} = D^{-1}\hat{D} = PT.
\end{align}

(4.27)

We do not need to go any further. The differential operator $\hat{D}$ as well as the vertices in the Lagrangians $\hat{L}^{[3]}_\sigma$ and $\hat{L}^{[4]}_\sigma$ depend on the coupling constant $g$ only through the gauge boson mass $M$. Furthermore, $\hat{L}^{[3]}_\sigma$ and $\hat{L}^{[4]}_\sigma$ are proportional to $\sqrt{\lambda}$ and $\lambda$, respectively. Hence, $n$-loop contributions count as order $\lambda^{n-1}$. This includes tree-level and one-loop contributions as discussed in the previous section. Tree-level corresponds to $n = 0$. The low energy expansion at a given loop level generates only powers of the momentum and the gauge boson mass $M$. Hence, it does not mix up the counting of $\lambda$. The order of $n$-loop contributions can already be inferred from Eqs. (4.2) and (4.3). The discussion in this Section shows that the proper treatment of the zero-modes does not change this result.
5 Renormalization

To render the generating functional (2.44) of the abelian Higgs model finite, one has to renormalize the bare constants $m, \lambda, g, c_{mh}, c_{hh}$, the scalar field $\phi$, and the source $h$ before the regulator can be removed. There is no wave function renormalization of the vector field on account of gauge invariance (cf. Eq. (2.2)). The ultraviolet divergences are related to the poles of the $d$-dimensional determinants which appear in the generating functional for $d = 0, 2, 4, \ldots$. For a differential operator $D$ of the form

$$D = -D_\mu D_\mu + \sigma, \quad D_\mu = \partial_\mu + \Gamma_\mu,$$

(5.1)

the pole term of the determinant at $d = 4$ is given by

$$\frac{1}{2} \text{ln} \det D = \frac{1}{d-4 \frac{1}{16\pi^2}} \int d^dx \text{tr} \left( \frac{1}{12} \Gamma_{\mu\nu} \Gamma_{\mu\nu} + \frac{1}{2} \sigma^2 \right) + O(1),$$

(5.2)

with

$$\Gamma_{\mu\nu} = [D_\mu, D_\nu].$$

(5.3)

This identity can readily be derived [4] using the heat kernel method [1]. Since the operators $P^T P$ and $\bar{D} + P P^T$ are both of the form (5.1), it is straightforward to verify that the poles in the generating functional (2.44) are removed by the following renormalization prescriptions:

$$Z_\phi = 1 - 8g_r^2 \left( \Lambda_\varepsilon(2m_r^2) + \delta z \right)$$

(5.4)

$$m^2 = m_r^2 \left( 1 - 2 \left( 4\lambda_r + g_r^2 \right) \left( \Lambda_\varepsilon(2m_r^2) + \delta m^2 \right) - (Z_\phi - 1) \right)$$

(5.5)

$$\lambda = \lambda_r \left( 1 - \left( 20\lambda_r + 4g_r^2 + 6\frac{g_{r!}^2}{\lambda_r} \right) \left( \Lambda_\varepsilon(2m_r^2) + \delta \lambda \right) - 2(Z_\phi - 1) \right)$$

(5.6)

$$g^2 = g_r^2 \left( 1 - \frac{2}{3}g_r^2 \left( \Lambda_\varepsilon(2m_r^2) + \delta g^2 \right) \right)$$

(5.7)

$$h = c_r h_r$$

(5.8)

$$c_{hh} = c_{hh,r} + \left( \Lambda_\varepsilon(2m_r^2) + \delta c_{hh} \right) - 2c_{hh,r}(c_r - 1)$$

(5.9)

$$c_{mh} = c_{mh,r} + 2 \left( \Lambda_\varepsilon(2m_r^2) + \delta c_{mh} \right) - c_{mh,r}(c_r - 1) - c_{mh,r} \left( \frac{m_r^2 - m_r^2}{m_r^2} \right)$$

(5.10)
with
\[ c_r = 1 - 2 \left( 4\lambda_r + g_r^2 \right) \left( \Lambda_\varepsilon(2m^2) + \delta c_r \right) - (Z_\phi - 1) \], (5.11)
and
\[ \phi = Z_\phi^{1/2} \phi_r \]. (5.12)

Eqs. (5.4)-(5.12) introduce the finite but otherwise completely arbitrary constants \( \lambda_r, g_r, m_r, Z_\phi, h_r, c_{hh,r}, c_{mh,r}, \delta \lambda, \delta g^2, \delta m^2, \delta z, \delta c_{hh}, \delta c_{mh}, \) and \( \delta c_r \). They are determined by the renormalization scheme. In the following we will express the effective Lagrangian (3.25)-(3.27) in terms of the physical masses \( M_H \) and \( M_W \) of the Higgs and gauge boson, as well as the parameter \( g_{\text{res}} \) defined by

\[ \langle 0 | F_{\mu\nu}(0) | k, \epsilon(\sigma) \rangle \equiv i g_{\text{res}} \left( k_\mu \epsilon_{\nu}^{(\sigma)} - k_\nu \epsilon_{\mu}^{(\sigma)} \right) . \] (5.13)

Here, \( | k, \epsilon(\sigma) \rangle \) denotes a physical state of the massive gauge boson with momentum \( k \) and polarization \( \epsilon(\sigma) \). The physical masses are determined by the pole positions of the two-point functions \( \langle 0 | T(\phi^T \phi)_{(x)}(\phi^T \phi)_{(y)} | 0 \rangle \) and \( \langle 0 | TF_{\mu\nu}(x)F_{\rho\sigma}(y) | 0 \rangle \). Furthermore, according to Eq. (5.13) the quantity \( g_{\text{res}}^2 \) is determined by the residue of the two-point function of the field strength.

All of these statements can conveniently be summarized at tree level by the following result for the generating functional \( W_\sigma[h, k_{\mu\nu}] \), expanded up to second order in the sources:

\[
W_\sigma[h, k_{\mu\nu}]_{\text{tree}} = - \left( \frac{m^2}{2\lambda} + c_{mh}m^2 \right) \int d^4x \ h - c_{hh} \int d^4x \ h^2 \\
- \left( \frac{m^2}{2\lambda} \right) \int d^4xd^4y \ h(x) \langle x | d_{m}^{-1} \rangle \ h(y) \\
- \frac{1}{2} g^2 \int d^4xd^4y \ (\partial_\mu k_{\mu\nu})(x) \langle x | d_{M}^{-1} \rangle \ (\partial_\rho k_{\rho\nu})(y) .
\] (5.14)

If the one-loop corrections in the abelian Higgs model are taken into account, one obtains

\[
M_H^2 = 2m^2 \left\{ 1 + \lambda \left[ 8 \left( 1 - \frac{3}{2} \frac{M^2}{2m^2} \right) \Lambda_\varepsilon(2m^2) \right] + \frac{1}{16\pi^2} \left( -10 + 3\sqrt{3}\pi + 2 \frac{M^2}{2m^2} \right) + \mathcal{O}(p^4) \right\} (5.15)
\]
\[ M_W^2 = M^2 \left\{ 1 + \lambda \left[ -12 \left( 1 - \frac{10}{9} \frac{M^2}{2m^2} + 2 \frac{M^4}{(2m^2)^2} \right) \Lambda_\epsilon(2m^2) \right. \right. \\
+ \left. \left. \frac{1}{16\pi^2} \left( 1 + \frac{10}{9} \frac{M^2}{2m^2} - \frac{65}{6} + 18 \ln \left( \frac{M^2}{2m^2} \right) \right) \right] + \mathcal{O}(p^6) \right\} \] \\
\tag{5.16}
\]

\[ g_{\text{res}}^2 = g^2 \left\{ 1 + g^2 \left[ \frac{2}{3} \Lambda_\epsilon(2m^2) + \frac{1}{16\pi^2} \left( \frac{1}{18} - \frac{3}{2} \frac{M^2}{2m^2} \right) + \mathcal{O}(p^4) \right] \right\}. \tag{5.17} \]

These expressions will be finite, if the renormalization prescriptions (5.4)-(5.5) are inserted on the right hand side. One obtains the same result for \( M_W \) and \( g_{\text{res}} \) in the effective field theory if all terms up to order \( p^6 \) in \( \mathcal{L}_{\text{eff}} \) are taken into account.

In terms of these physical parameters, the effective Lagrangian reads

\[ \mathcal{L}_2 = \frac{1}{2} M_W^2 \nabla_\mu U \nabla_\mu U + \frac{1}{4g_{\text{res}}^2} \bar{F}_{\mu\nu} \bar{F}_{\mu\nu} \]
\[ \mathcal{L}_4 = - \left( \frac{M_W^2}{2M_H^2 g_{\text{res}}^2} - \frac{52 - 9\sqrt{3}\pi}{192\pi^2} \right) (\nabla_\mu U \nabla_\mu U)^2 \]
\[ \mathcal{L}_6 = \left( 9\Lambda_\epsilon(M_H^2) + \frac{1}{16\pi^2} \left[ \frac{53}{24} + \frac{9}{2} \ln \left( \frac{M_W^2}{M_H^2} \right) \right] \right) \frac{M_W^4}{M_H^4} (\nabla_\mu U \nabla_\mu U)^2 \]
\[ - \frac{1}{16\pi^2} \frac{M_W^2}{M_H^4} \bar{F}_{\mu\nu} \bar{F}_{\mu\nu} - \frac{1}{16\pi^2} \frac{19}{4} \frac{M_W^4}{M_H^4} (\nabla_\mu U \nabla_\mu U)^2 \]
\[ - \frac{1}{16\pi^2} \frac{19}{72} \frac{M_W^2}{M_H^4} (\nabla_\mu U \nabla_\mu U)^3 \]
\[ - \left( \frac{M_W^2}{M_H^2 g_{\text{res}}^2} - \frac{289 - 54\sqrt{3}\pi}{288\pi^2} \right) \frac{1}{M_H^2} (\nabla_\mu U \nabla_\mu U)(\nabla_\nu U \nabla_\rho \nabla_\nu \nabla_\rho U) \]
\[ - \left( \frac{M_W^2}{M_H^2 g_{\text{res}}^2} - \frac{305 - 54\sqrt{3}\pi}{288\pi^2} \right) \frac{1}{M_H^2} (\nabla_\mu U \nabla_\mu U)(\nabla_\nu \nabla_\rho U)(\nabla_\nu \nabla_\rho U) \]
\[ - \frac{1}{16\pi^2} \frac{1}{36} \frac{M_W^2}{M_H^2} (\nabla_\mu U \nabla_\mu U)(\nabla_\nu \nabla_\rho U)(\nabla_\nu \nabla_\rho U) \]
\[ - \frac{1}{16\pi^2} \frac{1}{24} \frac{M_W^2}{M_H^2} \nabla_\mu \nabla_\mu \nabla_\rho U \nabla_\nu \nabla_\nu \nabla_\rho U \]
\[ + \frac{1}{16\pi^2} \frac{1}{36} \frac{M_W^2}{M_H^2} (\nabla_\mu \nabla_\mu \nabla_\rho U)(U)(U)(\nabla_\nu \nabla_\nu \nabla_\rho U). \] \tag{5.20}
Note, that some terms of order $p^4$ have disappeared from the effective Lagrangian. This is due to the fact that the relations (5.15) - (5.17) between bare and physical parameters are not homogeneous in the order of the momentum. Thus, renormalizing the effective Lagrangian mixes various low energy constants.

The contribution from the tadpole graph (3.16) can be accounted for by subtracting the term

$$9 \left( \Lambda_c(M_H^2) + \frac{1}{16\pi^2} \left[ \frac{1}{4} + \frac{1}{2} \ln \left( \frac{M_W^2}{M_H^2} \right) \right] \right) \frac{M_W^2}{M_H^2} \nabla_{\mu} U^T \nabla_{\mu} U$$

from the effective Lagrangian given above. This yields a finite result, since there are no other one-loop contributions in the effective theory of the abelian Higgs model up to order $p^6$.

The representation of the low energy constants in terms of the quantities $M_H$, $M_W$, and $g_{\text{res}}$ can be used to estimate the value of the Higgs boson mass where strong coupling sets in. Using Standard Model values for the gauge boson mass $M_W$ and the coupling $g_{\text{res}}$, one obtains the value $M_H \sim 4\text{TeV}$. At this point one-loop corrections to the low energy constants are of the same size as the tree level contributions. This number is larger than the corresponding estimate in the non-abelian case, since the low energy constants depend on the rank of the group $O(N)$. For larger rank this estimate is smaller.

Finally, we note that in the limit $g_{\text{res}}^2 \to 0$ and $M_W^2 \to 0$, such that

$$\frac{M_W^2}{g_{\text{res}}^2} = F^2 = \text{const},$$

the low energy constants for the ungauged $O(2)$ linear sigma model [4] are recovered.

6 Summary and Discussion

In this work we have discussed a manifestly gauge invariant\(^3\) approach to analyze the low energy structure of the gauged linear sigma model. In the

\(^3\)The literature on the background field effective action distinguishes between gauge invariance and gauge independence. In this work the phrase gauge invariant includes both meanings.
spontaneously broken phase the spectrum of this model contains one massive scalar particle, the Higgs boson. If its mass is large enough, the low energy structure of the linear sigma model can adequately be described by an effective field theory. Furthermore, if the couplings are not too strong, perturbative methods apply. Thus, there exists an intermediate range for the mass of the Higgs boson where both the effective field theory description and the perturbative treatment in the coupling constants are valid. Our analysis was concerned with this case. In particular, we discussed a functional technique to evaluate the gauge invariant effective Lagrangian which describes the effective field theory of the linear sigma model. The advantage of our approach is that it explicitly reflects the symmetry properties of the underlying theory, i.e., it is manifestly gauge invariant.

In order to avoid technical difficulties we have chosen the abelian Higgs model as a simple example to demonstrate our method. It corresponds to the case $N = 2$ of the gauged linear $O(N)$ sigma model. However, our analysis does not rely on any particular property of the abelian theory. We will see below that it can readily be extended to the non-abelian case.

The effective Lagrangian of the linear sigma model is a sum of gauge invariant terms with an increasing number of covariant derivatives and gauge boson mass factors, corresponding to an expansion in powers of the momentum and the mass. Note that the covariant derivative, the gauge boson mass $M$, and the gauge coupling $g$ all count as quantities of order $p$. Thus, the low energy expansion is carried out at a fixed ratio $p^2/M^2$, which ensures that all light-particle singularities are correctly reproduced. Furthermore, if the coupling $\lambda$ of the scalar field is small enough, the low energy constants in the effective Lagrangian admit an expansion in powers of this quantity, corresponding to the loop expansion in the full theory. $n$-loop Feynman diagrams in the abelian Higgs model yield corrections of order $\lambda^{n-1}$ to the low energy constants. Hence, in the intermediate range of the Higgs boson mass the accuracy of the effective field theory description is controlled by the order of both the momentum and the coupling constant $\lambda$. Note, however, that the correspondence between powers of the coupling $\lambda$ and the order of loops in the full theory becomes meaningless in the strong coupling limit; i.e., for very large values of the Higgs mass. In this case, the functional relationships between the low energy constants and the coupling of the scalar field cannot be evaluated with perturbative methods.

In the intermediate range for the Higgs boson mass one can make use
of this correspondence and separately specify the order of the momentum and the scalar coupling to be taken into account, depending on the energy scale and the value of the Higgs mass relevant for a given analysis. This may turn out to be useful in the effective field theory analysis of electroweak symmetry breaking, where the Standard Model with one scalar doublet serves as a reference point. For values of the Higgs mass close to the strong coupling region, one may need to go beyond the next-to-leading order approximation for the low energy constants. In this work we have evaluated the effective Lagrangian of the abelian Higgs model up to order $p^6$ and the low energy constants up to order $\lambda^0$. Thus, an effective field theory analysis based on this Lagrangian yields the low energy expansion of the corresponding one-loop approximation in the full theory up to order $p^6$. Finally, we have expressed the effective Lagrangian in terms of the physical masses of the Higgs and the gauge boson, and the gauge coupling. From this representation of the low energy constants, one can estimate the intermediate range for the mass of the Higgs boson. Using Standard Model values, this range turns out to be $0.1\text{TeV} \lesssim M_H \lesssim 4\text{TeV}$. For larger values of this mass, one enters the strong coupling regime, and perturbation theory with respect to the parameter $\lambda$ is no longer possible. For smaller values of the Higgs boson mass, the effective field theory description ceases to be valid.

It is crucial for a manifestly gauge invariant approach that the external sources respect the symmetry properties of the theory. Thus, our analysis was only concerned with Green’s functions of gauge invariant operators. In the abelian Higgs model this causes no restriction. In the spontaneously broken phase, Green’s functions of the scalar density $\phi^T \phi$ and the field strength $F_{\mu\nu}$ have poles at the masses of the Higgs and gauge boson. Thus, one can extract all $S$-matrix elements of the theory. One may even couple an external vector field to the current $(\nabla_\mu \phi)^T T e \phi$ and consider Green’s functions involving this operator as well. For large values of the Higgs mass, the low energy structure of these functions can adequately be described by an effective field theory with a gauge invariant effective Lagrangian. Thus, as far as the abelian Higgs model is concerned, things work as smoothly as in the ungauged case.

In the non-abelian theory the situation is a little bit different. The field strength and the currents are not gauge invariant. One cannot couple external sources to these operators without breaking the symmetry. Thus, in order to evaluate $S$-matrix elements in a gauge invariant framework one has to find other sources, which may either create single particles or pairs. The
latter might be more appropriate for charged particles. A particular kind of
gauge invariant source term which may help in this case is discussed in the
context of the Vilkovisky-DeWitt effective action [12].

The situation simplifies considerably if one is interested in the effective
Lagrangian. The only reason for us to couple an external source to the field
strength was to ensure that we manipulate non-vanishing classical fields dur-
ing intermediate steps of our calculation. However, this can also be achieved
without an external source simply by imposing non-trivial boundary condi-
tions on the field strength. In fact, the whole analysis in this article involved
only the solution of the equation of motion for the Higgs field. Thus, our
technique of evaluating the gauge invariant effective Lagrangian can be ap-
plied to the non-abelian case without modification.

To determine $S$-matrix elements in the effective field theory, one may
couple source terms to this Lagrangian and consider Green’s functions of
the corresponding operators. Since our renormalization of the low energy
constants in the non-abelian case will not know about these source terms,
Green’s functions will generally be well defined only in the regularized the-
ory. Thus, one has to extract $S$-matrix elements before the regulator can
be removed. Once the effective Lagrangian is given, one may even use the
Faddeev-Popov ansatz to evaluate scattering amplitudes.

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