Asymptotic Analysis of the Performance of LAS Algorithm for Large-MIMO Detection

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Abstract

In our recent work, we reported an exhaustive study on the simulated bit error rate (BER) performance of a low-complexity likelihood ascent search (LAS) algorithm for detection in large multiple-input multiple-output (MIMO) systems with large number of antennas that achieve high spectral efficiencies. Though the algorithm was shown to achieve increasingly closer to near maximum-likelihood (ML) performance through simulations, no BER analysis was reported. Here, we extend our work on LAS and report an asymptotic BER analysis of the LAS algorithm in the large system limit, where \( N_t, N_r \to \infty \) with \( N_t = N_r \), where \( N_t \) and \( N_r \) are the number of transmit and receive antennas. We prove that the error performance of the LAS detector in V-BLAST with 4-QAM in i.i.d. Rayleigh fading converges to that of the ML detector as \( N_t, N_r \to \infty \).

Keywords – High spectral efficiencies, large-MIMO detection, likelihood ascent search.

1 Introduction

Multiple-input multiple-output (MIMO) systems that employ large number of transmit and receive antennas can offer very high spectral efficiencies of the order of tens to hundreds of bps/Hz \([1,2]\). Achieving near-optimal signal detection at low complexities in such large-dimension systems has been a challenge. In our recent works, we have shown that certain algorithms from machine learning/artificial intelligence achieve near-optimal performance in large-MIMO systems that employ tens of transmit and receive antennas using V-BLAST and non-orthogonal space-time block codes (STBC) \([3]\) with tens to hundreds of dimensions in space and time, at low complexities. Such algorithms include local neighborhood search based algorithms like a likelihood ascent search (LAS) algorithm \([4,5]\) and a reactive tabu search (RTS) algorithm \([6]\), and algorithms based on probabilistic data association (PDA) \([7]\) and belief propagation (BP) \([8,9]\). Similar algorithms have been earlier reported in the context of multiuser detection \([10-14]\). In \([4-9]\), through detailed simulations, we have shown that LAS and RTS algorithms achieve increasingly closer to maximum-likelihood (ML) performance and that PDA and BP algorithms achieve near maximum a posteriori probability

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(MAP) performance for increasing number of dimensions in large-MIMO systems. For e.g., in [5], the BER performance of the basic LAS algorithm (which uses a single symbol update based neighborhood definition) and its generalized version (which uses a multiple symbol update based neighborhood definition) has been exhaustively studied through simulations. However, BER performance analysis of the LAS algorithm for large-MIMO detection has not been reported. In this correspondence, we fill some of this gap by presenting an asymptotic BER analysis of the LAS algorithm in the large system limit, where $N_t, N_r \to \infty$ with $N_t = N_r$, where $N_t$ and $N_r$ denote the number of transmit and receive antennas, respectively.

Asymptotic performance analysis of large systems in the context of multiuser detection and MIMO communication have been reported in the literature [17]-[22], using random matrix theory (e.g., [17],[18]), replica method (e.g., [19],[20],[21]), and free probability theory (e.g., [22]). We, in this correspondence, present an asymptotic BER analysis of the LAS algorithm in the large system limit. Specifically, we present an analytical proof that the error performance of the LAS detector for V-BLAST with 4-QAM in i.i.d. Rayleigh fading converges to that of the ML detector as $N_t, N_r \to \infty$ with $N_t = N_r$, which is an analytical result that has not been reported so far.

The rest of this correspondence is organized as follows. The MIMO system model and LAS detection algorithm are summarized in Section 2. The asymptotic analysis of the LAS algorithm is presented in Section 3. Lengthy proofs of lemmas and theorems are moved to the appendices. Simulation results and discussions are presented in Section 4. Conclusions are given in Section 5.

## 2 System Model

Consider a V-BLAST system with $N_t$ transmit antennas and $N_r$ receive antennas, $N_t \leq N_r$. Let $x_c \in \mathbb{C}^{N_t \times 1}$ denote the symbol vector transmitted, and $H_c \in \mathbb{C}^{N_r \times N_t}$ denote the channel matrix such that its $(i,j)$th entry $h_{i,j}$ is the complex channel gain from the $j$th transmit antenna to the $i$th receive antenna. Assuming rich scattering, we model the entries of $H_c$ as i.i.d. $\mathcal{CN}(0,1)$. Let $y_c \in \mathbb{C}^{N_r \times 1}$ and $n_c \in \mathbb{C}^{N_r \times 1}$ denote the received signal vector and the noise vector, respectively, at the receiver, where the entries of $n_c$ are modeled as i.i.d

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1Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$ and $[.]^H$ denote transpose and conjugate transpose operations, respectively. $||.||$ denotes Euclidean distance.
The received signal vector can then be written as
\[ y_c = H_c x_c + n_c. \] (1)

Let \( y_c, H_c, x_c, \) and \( n_c \) be decomposed into real and imaginary parts as follows:
\[ y_c = y_I + jy_Q, \quad x_c = x_I + jx_Q, \quad n_c = n_I + jn_Q, \quad H_c = H_I + jH_Q. \] (2)

Further, we define \( H_r \in \mathbb{R}^{2N_r \times 2N_t}, y_r \in \mathbb{R}^{2N_r \times 1}, x_r \in \mathbb{R}^{2N_t \times 1}, \) and \( n_r \in \mathbb{R}^{2N_r \times 1} \) as
\[ H_r = \begin{pmatrix} H_I - H_Q \\ H_Q & H_I \end{pmatrix}, \quad y_r = [y_I^T \ y_Q^T]^T, \quad x_r = [x_I^T \ x_Q^T]^T, \quad n_r = [n_I^T \ n_Q^T]^T. \] (3)

Now, (1) can be written as
\[ y_r = H_r x_r + n_r. \] (4)

Henceforth, we shall work with the real-valued signal model of the system in (4). For notational simplicity, we drop subscripts \( r \) in (4) and write
\[ y = H x + n, \] (5)

where \( H = H_r \in \mathbb{R}^{2N_r \times 2N_t}, y = y_r \in \mathbb{R}^{2N_r \times 1}, x = x_r \in \mathbb{R}^{2N_t \times 1}, n = n_r \in \mathbb{R}^{2N_r \times 1}. \) In this real-valued system model, the real-part of the complex data symbols will be mapped to \([x_1, \ldots, x_{2N_t}]\) and the imaginary-part of these symbols will be mapped to \([x_{N_t+1}, \ldots, x_{2N_t}]\). For \( M \)-QAM, \([x_1, \ldots, x_{N_t}]\) can be viewed to be from an underlying \( M \)-PAM signal set and so is \([x_{N_t+1}, \ldots, x_{2N_t}]\). Let \( A_i \) denote the \( M \)-PAM signal set from which \( x_i \) takes values, \( i = 1, 2, \ldots, 2N_t; \) e.g., for 4-QAM, \( A_i = \{1, -1\} \) for \( i = 1, 2, \ldots, 2N_t \). Now, define a \( 2N_t \)-dimensional signal space \( \mathbb{S} \) to be the Cartesian product of \( A_1 \) to \( A_{2N_t} \). The ML solution vector, \( d_{ML} \), is given by
\[ d_{ML} = \arg \min_{d \in \mathbb{S}} \|y - H_d\|^2 = \arg \min_{d \in \mathbb{S}} \left(d^T H^T H d - 2y^T H d\right). \] (6)

In the following subsection, we summarize the low-complexity LAS algorithm, using a neighborhood definition based on 1-symbol updates, presented in [5] for large-MIMO detection for \( M \)-QAM. The channel matrix \( H \) is assumed to be known perfectly at the receiver.

### 2.1 LAS Algorithm for Large-MIMO Detection

The LAS algorithm starts with an initial vector \( d^{(0)} \), given by \( d^{(0)} = B y \), where \( B \) is the initial solution filter, which can be a matched filter (MF) or zero-forcing (ZF) filter or MMSE
filter. The index \( m \) in \( d^{(m)} \) denotes the iteration number in a given search stage. The ML cost function after the \( k \)th iteration in a given search stage is given by

\[
C^{(k)} = d^{(k)^T} H^T H d^{(k)} - 2 y^T H d^{(k)}. \tag{7}
\]

The \( d \) vector is updated from \( k \)th to \( (k + 1) \)th iteration by updating one symbol, say, the \( p \)th symbol, as

\[
d^{(k+1)} = d^{(k)} + \lambda_p^{(k)} e_p, \tag{8}
\]

where \( e_p \) denotes the unit vector with its \( p \)th entry only as one, and all other entries as zero. Since \( d^{(k)} \) and \( d^{(k+1)} \) should belong to \( S \), \( \lambda_p^{(k)} \) can take only certain integer values. For example, for 16-QAM, \( A_p = \{-3, -1, 1, 3\} \), and \( \lambda_p^{(k)} \) can take values only from \( \{-6, -4, -2, 0, 2, 4, 6\} \). Using \( (7) \) and \( (8) \), and defining a matrix \( G \) as

\[
G \triangleq H^T H, \tag{9}
\]

we can write the cost difference \( C^{(k+1)} - C^{(k)} \) as

\[
\mathcal{F}(l_p^{(k)}) \triangleq C^{(k+1)} - C^{(k)} = l_p^{(k)} a_p - 2 l_p^{(k)} |z_p^{(k)}|, \]

where \( z_p^{(k)} \) is the \( p \)th entry of the \( z^{(k)} \) vector given by \( z^{(k)} = H^T (y - H d^{(k)}) \), \( a_p \triangleq (G)_{p,p} \) is the \((p, p)\)th entry of the \( G \) matrix, and \( l_p^{(k)} = |\lambda_p^{(k)}| \). The value of \( l_p^{(k)} \) which gives the largest descent in the cost function from the \( k \)th to the \((k + 1)\)th iteration (when symbol \( p \) is updated) is obtained as

\[
l_p^{(k)}_{p,opt} = 2 \left\lfloor \frac{|z_p^{(k)}|}{2 a_p} \right\rfloor, \tag{10}
\]

where \( \lfloor . \rfloor \) denotes the rounding operation. If \( d_p^{(k)} \) were updated using \( l_p^{(k)}_{p,opt} \), it is possible that the updated value does not belong to \( A_p \). To avoid this, we adjust \( l_p^{(k)} \) so that the updated value of \( d_p^{(k)} \) belongs to \( A_p \). Let

\[
s = \arg\min_p \mathcal{F}(l_p^{(k)}_{p,opt}). \tag{11}
\]

If \( \mathcal{F}(l_s^{(k)}_{s,opt}) < 0 \), the update for the \((k + 1)\)th iteration is

\[
d^{(k+1)} = d^{(k)} + l_s^{(k)}_{s,opt} \text{sgn}(z_s^{(k)}) e_s \tag{12}
\]

\[
z^{(k+1)} = z^{(k)} - l_s^{(k)}_{s,opt} \text{sgn}(z_s^{(k)}) g_s, \tag{13}
\]

where \( g_s \) is the \( s \)th column of \( G \). If \( \mathcal{F}(l_s^{(k)}_{s,opt}) \geq 0 \), then the search terminates, and \( d^{(k)} \) is declared as the detected data vector.
3 Asymptotic Analysis of LAS Algorithm

In this section, we prove the asymptotic convergence of the error probability of the LAS detector to that of the ML detector for $N_t, N_r \to \infty$ with $N_t = N_r$ in V-BLAST. Consider 4-QAM, i.e., $S \in \{+1, -1\}^{2N_t}$, and let $N_t = N_r$. An $n$-symbol update on a data vector $d \in S$ transforms $d$ to $(d - \Delta d_n)$ such that $(d - \Delta d_n) \in S$. Further, $(d - \Delta d_n)$ is obtained by changing $n$ symbols in $d$ at distinct indices given by the $n$-tuple $u_n \triangleq (i_1, i_2, \ldots, i_n)$, $1 \leq i_j \leq 2N_t, \forall j = 1, \ldots, n$ and $i_j \neq i_k$ for $j \neq k$. Therefore, we can write $\Delta d_n$ as

$$\Delta d_n = \sum_{k=1}^{n} 2d_{i_k}e_{i_k},$$  \hspace{1cm} (14)

where $d_{i_k}$ is the $i_k$th element of $d$. Let $\mathbb{L}_n \subseteq S$ denote the set of data vectors such that for any $d \in \mathbb{L}_n$, if a $n$-symbol update is performed on $d$ resulting in a vector $(d - \Delta d_n)$, then $||y - H(d - \Delta d_n)|| \geq ||y - Hd||$. Our main result in this section is Theorem 2. To prove Theorem 2, we need the following Lemmas 1 to 5, Slutsky’s theorem [23], and Theorem 1.

**Lemma 1** Let $d \in S$. Then, $d \in \mathbb{L}_n$ if and only if, for any $n$-update on $d$, $n \in [1, 2 \cdots, 2N_t]$, \hspace{1cm} (15)

$$n + H(x - d) + \frac{1}{2}H\Delta d_n^T(H\Delta d_n) \geq 0,$$

where $h_p$ is the $p$th column of $H$.

Proof: By definition, if $d \in \mathbb{L}_n$, then no $n$-symbol update can result in a reduction in the ML cost function. Using this, we can write

$$||y - H(d - \Delta d_n)||^2 \geq ||y - Hd||^2.$$ \hspace{1cm} (16)

Simplifying (16), we get (15). Since the choice of the indices in $u_n$ is arbitrary, the lemma holds true for all possible $n$-tuples of distinct indices. For the converse, if $d$ satisfies (15) for all possible $u_n$ for a given $n$, then, since (15) and (16) are equivalent, $d$ also satisfies (16) for all possible $u_n$. This implies that $d \in \mathbb{L}_n$. $\square$

If $d \in \mathbb{L}_1$, then using Lemma 1 and (15), we can write

$$\left(H(x - d) + hpd_p\right)^T(hpd_p) \geq 0, \forall p = 1, \cdots, 2N_t,$$

where $h_p$ is the $p$th column of $H$.

**Lemma 2** Assuming uniqueness of the ML vector $d_{ML}$ in (6), a symbol vector $d \in S$ is the ML vector if and only if the noise vector $n$ satisfies the following set of equations

$$\left(n + H(x - d) + \sum_{j=1}^{n} h_{ij}d_{ij}\right)^T\left(n + \sum_{j=1}^{n} h_{ij}d_{ij}\right) \geq 0,$$

\hspace{1cm} (18)

$\forall n = 1, \cdots, 2N_t$, and for all possible $n$-tuples $(i_1, \cdots, i_n)$ for each $n$. 


Proof: If \( d \) is the unique ML vector, then from the definition of the ML criterion in (6), it must be true that any \( n \)-update on \( d \) will not result in any decrease in the ML cost function. Therefore, \( d \in \mathbb{L}_n, \forall \ n = 1, 2 \cdots, 2N_t \). Hence, by Lemma 1 it must be true that \( d \) satisfies (15) for all \( n = 1, 2, \cdots, 2N_t \) and for all possible \( u_n \) for each \( n \). Substituting \( y = Hx + n \) in (15), we get (18). This proves the direct result. To prove the converse, let the noise vector \( n \) satisfy (18) for some vector \( d \). Since \( y = Hx + n \), the conditions in (18) imply the conditions in (15) for all \( n = 1, 2, \cdots, 2N_t \) and for all possible \( u_n \) for each \( n \). Therefore, by Lemma 1 \( d \in \mathbb{L}_n \) for all \( n = 1, 2, \cdots, 2N_t \), which then implies that \( d \) indeed is the ML vector. □

Definition: For each \( d \in \mathbb{S} \) and for each integer \( m, 1 \leq m \leq 2N_t \), we associate the set of vectors \( \mathcal{R}_{d^m} = \left\{ v | v \in \mathbb{R}^{2N} \text{ and } (v + H(x - d) + (\sum_{j=1}^{n} h_{ij} d_{ij}))^T (\sum_{j=1}^{n} h_{ij} d_{ij}) \geq 0, \forall n = 1, \cdots, m \right\} \), and define \( \mathcal{R}_d = \mathcal{R}_{d^{2N_t}} \).

Lemma 3 If the noise vector \( n \in \mathcal{R}_d \), then \( d \) is the ML vector. Let \( d_i, d_j \in \mathbb{S} \) and \( d_i \neq d_j \). Then \( \mathcal{R}_{d_i} \) and \( \mathcal{R}_{d_j} \) are disjoint.

Proof: From Lemma 2 and the definition of \( \mathcal{R}_d \), it is clear that \( d \) is the ML vector if and only if \( n \in \mathcal{R}_d \). The disjointness of \( \mathcal{R}_{d_i} \) and \( \mathcal{R}_{d_j} \), \( i \neq j \), can be shown by contradiction. If \( \mathcal{R}_{d_i} \) and \( \mathcal{R}_{d_j} \) are not disjoint, then there exists some vector \( v \) belonging to both \( \mathcal{R}_{d_i} \) and \( \mathcal{R}_{d_j} \). If \( v \) were to be the noise vector \( n \), then, \( v \) would satisfy the set of equations in (18) for both \( d = d_i \) and \( d = d_j \), since \( v \) belongs to both \( \mathcal{R}_d \) and \( \mathcal{R}_{d_j} \). This, by Lemma 2 implies that both \( d_i \) and \( d_j \) are ML vectors, which is a contradiction because of the uniqueness of the ML vector. □

Lemma 4 Let \( h \in \mathbb{R}^{2N_t} \) be a random vector with i.i.d entries distributed as \( \mathcal{N}(0, 0.5) \). Let \( \{ h_i \}, i = 1, 2, \cdots, m \) be a set of vectors, with each \( h_i \in \mathbb{R}^{2N_t} \) and having i.i.d entries distributed as \( \mathcal{N}(0, 0.5) \), \( \mathbb{E}[h_i h_j^T] = 0 \) for \( i \neq j \), and \( \mathbb{E}[hh^T] = 0 \) for \( j = 1, \cdots, m \). Then

\[
\lim_{N_t \to \infty} \frac{\sum_{k=1}^{m} h_k^T h_k}{mN_t} = 0. \tag{19}
\]

Proof: Let \( \tilde{h} \triangleq \frac{1}{\sqrt{m}} \sum_{k=1}^{m} h_k \). Then, \( \tilde{h} \sim \mathcal{N}(0, \frac{1}{2}) \). Therefore, we have

\[
\lim_{N_t \to \infty} \frac{\sum_{k=1}^{m} h_k^T h_k}{mN_t} = \lim_{N_t \to \infty} \frac{h^T \tilde{h}}{\sqrt{mN_t}}. \tag{20}
\]

We can write

\[
\lim_{N_t \to \infty} \frac{h^T \tilde{h}}{N_t} = \lim_{N_t \to \infty} \frac{\sum_{k=1}^{2N_t} h_k \tilde{h}_k}{N_t}, \quad \tag{21}
\]
where \( h_k \) and \( \tilde{h}_k \) are the \( k \)th elements of \( h \) and \( \tilde{h} \), respectively. The r.v’s \( h_k \tilde{h}_k, k = 1, \cdots, 2N_t \) are i.i.d with mean zero. From the strong law of large numbers [23], it follows that \( \lim_{N_t \to \infty} \sum_{k=1}^{2N_t} \frac{h_k \tilde{h}_k}{2N_t} = 0 \). Using this in (20) completes the proof. □

Before we present the next lemma, we present the Slutsky’s theorem on convergence of random variables, which is used to prove Lemma 5 and Theorem 1.

**Slutsky’s Theorem** [23]: Let \( \{X_m\} \) and \( \{Y_m\} \) be sequences of random variables. If \( \{X_m\} \) converges in distribution to a random variable \( X \), and \( \{Y_m\} \) converges in probability to a constant \( c \), then it is true that

1. \( \{X_m + Y_m\} \) converges in distribution to \( X + c \),
2. \( \{X_m Y_m\} \) converges in distribution to \( cX \), and
3. \( \{X_m Y_m\} \) converges in distribution to \( X c \).

**Lemma 5** For a given \( u_n \) and a given \( d \in S \), define a r.v \( z_{u_n, d} \) as

\[
    z_{u_n, d} \triangleq \sum_{k=1}^{n} \sum_{j=k+1}^{n} \frac{h_{ij}^T h_{ik} d_{ij} d_{ik}}{\sum_{j=1}^{n} \|h_{ij}\|^2}, \tag{22}
\]

where \( i_j \in u_n, j = 1, \cdots, n \). For any \( u_n \) and any \( d \in S \), \( z_{u_n, d} \) converges to zero in probability as \( N_t \to \infty \), i.e., \( z_{u_n, d} \overset{p}{\to} 0 \) as \( N_t \to \infty \), \( \forall n = 2, 3, \cdots, 2N_t \).

**Proof:** Proof of this Lemma is given in Appendix A. □

In Fig. 1, we plot the simulated pdf of \( z_{u_n, d} \) for \( n = 2N_t \) for different values of \( N_t = N_r \) for a certain \( u_n \) and \( d \) (the pdf was observed to be same for different \( u_n \) and \( d \)). We observe that with increasing \( N_t = N_r \), the pdf of \( z_{u_n, d} \) tends towards the Dirac delta function at zero. This implies that \( z_{u_n, d} \) tends to zero in distribution, and hence in probability, for large \( N_t = N_r \), which is formally proved in Lemma 5.

**Theorem 1** Let \( d \in S \) and \( n \in R_d^+ \). Then \( n \in R_d \) in probability as \( N_t \to \infty \), i.e., for any \( \delta, 0 \leq \delta \leq 1 \), there exists an integer \( N(\delta) \) such that for \( N_t > N(\delta) \), \( p(n \in R_d) > 1 - \delta \).

**Proof:** Proof of this theorem is given in Appendix B. □

**Theorem 2** The data vector/bit error probability of the LAS detector converges to that of the ML detector as \( N_t, N_r \to \infty \) with \( N_t = N_r \).

**Proof:** Let \( \mathbf{d}_{LAS} \) be the final output symbol vector of the LAS algorithm given \( \mathbf{x}, \mathbf{H} \) and \( \mathbf{n} \). The algorithm terminates if and only if no 1-update results in any further decrease of the
cost function. This implies that for the given \( x, H \) and \( n, d_{LAS} \in \mathbb{L}_1 \), and therefore it must be true that \( n \) satisfies (17) with \( d \) replaced by \( d_{LAS} \). These set of equations are the same which define the region \( R_{d_{LAS}} \). Therefore, replacing \( d \) by \( d_{LAS} \), we can equivalently claim that \( n \in R_{d_{LAS}} \). Using Theorem 1, we can further claim that asymptotically as \( N_t \to \infty \), \( n \in R_{d_{LAS}} \) in probability. From Lemma 3 we know that if \( n \in R_{d_{LAS}} \), then \( d_{LAS} \) is indeed the ML vector for the given \( x, H \) and \( n \). Therefore, we can state that asymptotically as \( N_t \to \infty \), \( d_{LAS} \) is indeed the ML vector in probability. That is, for any \( \delta, 0 \leq \delta \leq 1 \), there exists an integer \( N(\delta) \) such that for \( N_t \geq N(\delta) \)

\[
P(d_{LAS} \text{ is the ML vector}) > (1 - \delta).
\]  

(23)

Therefore, we can write that for \( N_t \geq N(\delta) \)

\[
P_{LAS}(error) = P(d_{LAS} \neq x) = P(d_{LAS} \neq x | d_{LAS} = \text{ML vector})P(d_{LAS} = \text{ML vector}) + P(d_{LAS} \neq x | d_{LAS} \neq \text{ML vector})P(d_{LAS} \neq \text{ML vector}).
\]  

(24)

From (23), we have \( P(d_{LAS} \neq \text{ML vector}) \leq \delta \). Also, \( P(d_{LAS} \neq x | d_{LAS} = \text{ML vector}) \) is the probability of error for the ML detector, which we denote by \( P_{ML}(error) \). Using these, we can bound the probability of error for the LAS detector as

\[
P_{LAS}(error) \leq P_{ML}(error) + \delta P(d_{LAS} \neq x | d_{LAS} \neq \text{ML vector}) \leq P_{ML}(error) + \delta.
\]  

(25)

Since \( \delta \) can be arbitrarily small, we can conclude from (25) that indeed as \( N_t \to \infty \), the symbol vector error probability of the LAS detector converges to that of the ML detector. This proof can be adapted to show that apart from the symbol vector error probability, the bit error probability of the LAS detector also converges to that of the ML detector. The proof for the bit error probability convergence is along the same lines as (24) and (25), except that instead of defining the error event as \( d_{LAS} \neq x \), we define error events for each bit. For example, for the \( p \)th bit, the error event is defined as \( d_{p,LAS} \neq x_p \). \( \square \)

### 4 Simulation Results and Discussions

In Fig. 2 we show the simulated BER performance of the LAS detector for V-BLAST with 4-QAM and MMSE initial vector for increasing \( N_t = N_r \). Since an analytical expression for ML performance in the large MIMO system limit is not available and simulating the ML performance for large dimensions involves prohibitively high complexity, we plot the SISO.
AWGN performance as a lower bound for comparison. It can be seen that for increasing $N_t = N_r$, the BER performance of the LAS detector approaches the SISO AWGN performance at high SNRs. Figure 3 shows the average SNR required to achieve a BER of $10^{-3}$ for increasing $N_t = N_r$ and 4-QAM. It can be seen that, for large $N_t = N_r$, the required SNR gets increasingly closer to that required in SISO AWGN for increasing $N_t = N_r$. A similar behavior can be observed in Fig. 4 for 16-QAM as well. In Figs. 3 and 4, we also see that there is an initial degradation in performance for increasing number of antennas (for $N_t < 10$). This shows that the LAS detector is suboptimal for small systems with small number of antennas and becomes optimal in the large system limit (as proved in the previous section). LAS detector achieves close to large system limit performance in systems with large number of dimensions (e.g., hundreds of dimensions in Figs. 3 and 4). Such large number of dimensions need not be realized in spatial dimension alone, as in V-BLAST. As shown in [5], exploiting time dimension in addition to space dimension, large non-orthogonal STBC MIMO systems can render large dimensions with less number of transmit antennas that can be implemented in practice. A $16 \times 16$ non-orthogonal STBC from cyclic division algebra with complex data symbols has 512 real dimensions; with 64-QAM and rate-3/4 turbo code, this STBC achieves a spectral efficiency of 72 bps/Hz. In [5], LAS algorithm has been shown to achieve near-capacity performance in $16 \times 16$ STBC MIMO systems even in the presence of spatial correlation and with estimated channel matrix. Further, considering that NTT DoCoMo has demonstrated a $12 \times 12$ V-BLAST MIMO system operating at 5 Gbps at a spectral efficiency of 50 bps/Hz at 10 Km/hr mobile speeds [24], the availability of low-complexity large-MIMO detection algorithms like the LAS algorithm analyzed in this correspondence can motivate the adoption of $16 \times 16$ and $24 \times 24$ MIMO systems operating at spectral efficiencies in excess of 50 bps/Hz in emerging wireless standards like IEEE 802.11 VHT and IEEE 802.16/LTE-A.

5 Conclusions

We conclude with the following two remarks: i) The derivation of analytical BER expressions for the ML performance in the large MIMO system limit for different signal sets is an open problem. Since large MIMO systems can be viable in practice due to the availability of low-complexity large-MIMO detection algorithms like the LAS algorithm analyzed in this correspondence can motivate the adoption of $16 \times 16$ and $24 \times 24$ MIMO systems operating at spectral efficiencies in excess of 50 bps/Hz in emerging wireless standards like IEEE 802.11 VHT and IEEE 802.16/LTE-A.

5 Conclusions

We conclude with the following two remarks: i) The derivation of analytical BER expressions for the ML performance in the large MIMO system limit for different signal sets is an open problem. Since large MIMO systems can be viable in practice due to the availability of low-complexity large-MIMO detection algorithms like the LAS algorithm analyzed in this correspondence can motivate the adoption of $16 \times 16$ and $24 \times 24$ MIMO systems operating at spectral efficiencies in excess of 50 bps/Hz in emerging wireless standards like IEEE 802.11 VHT and IEEE 802.16/LTE-A.
ity of low-complexity detectors like the LAS detector, analytical BER expressions for the ML performance in the large MIMO system limit would be quite useful as a benchmark for comparing the performance of practical detectors in large-MIMO systems. The statistical mechanics approach employed in [19] for large CDMA system BER analysis can be investigated for such an analysis. ii) While we are able to prove the asymptotic convergence of LAS performance to ML performance for 4-QAM here, our simulation results for higher order QAM (e.g., 16-QAM; see Fig. 4) show similar behavioral trend like that for 4-QAM. Consequently, we conjecture that such a convergence holds for general \( M \)-QAM and an analytical proof to show this can be attempted as an extension to this work.

Appendix A: Proof of Lemma 5

We present the proof of Lemma 5 in this appendix. The proof is by mathematical induction on \( n \).

**Base Case:** For \( n = 2 \), we have to show that

\[
d_p d_q \frac{h_p^T h_q}{\|h_p\|^2 + \|h_q\|^2} \xrightarrow{p} 0 \quad \text{as} \quad N_t \to \infty, \quad \forall \, p, q = 1, 2, \ldots, 2N_t, \ p \neq q.
\]  

(26)

We can write the random variable \( \frac{h_p^T h_q}{\|h_p\|^2 + \|h_q\|^2} \) as

\[
\frac{h_p^T h_q/(2N_t)}{(\|h_p\|^2 + \|h_q\|^2)/(2N_t)}. 
\]

(27)

As \( N_t \to \infty \), by strong law of large numbers, the denominator of (27) converges to 1 almost surely. Also, the numerator of (27) can be written as

\[
\frac{h_p^T h_q}{2N_t} = \frac{\sum_{k=1}^{2N_t} h_{p,k} h_{q,k}}{2N_t},
\]

(28)

where \( h_{p,k} \) and \( h_{q,k} \) refer to the \( k \)th entry of the vectors \( h_p \) and \( h_q \), respectively. Each \( h_{p,k} h_{q,k} \) term in the summation in (28) has the same distribution and has mean 0. Therefore, by strong law of large numbers, we can see that \( \frac{h_p^T h_q}{2N_t} \) converges to 0 almost surely. This also implies that \( \frac{h_p^T h_q}{\|h_p\|^2 + \|h_q\|^2} \) converges in distribution to the constant 0, and hence by Slutsky’s theorem, \( \frac{h_p^T h_q}{\|h_p\|^2 + \|h_q\|^2} \) converges in distribution to 0. Since, if a sequence of r.v’s converges in distribution to a constant then the sequence converges in probability to that constant, we conclude that indeed \( \frac{h_p^T h_q}{\|h_p\|^2 + \|h_q\|^2} \) converges in probability to 0. This proves the base case.

**Induction Hypothesis:** Let \( z_{u_n,d} \xrightarrow{p} 0 \) as \( N_t \to \infty, \forall \, n = 2, 3, \ldots, m. \)
Induction Step: Proof for $n = m + 1$: We have

$$z_{u(m+1),d} = \frac{\sum_{k=1}^{m+1} \sum_{j=k+1}^{m+1} h^T_{ik} h_{ik} d_j d_{ik}}{\sum_{j=1}^{m+1} \|h_{ij}\|^2}$$

(29)

$$= \frac{\sum_{k=1}^{m} \sum_{j=k+1}^{m} h^T_{ij} h_{ij} d_j d_{ik} + \sum_{k=1}^{m} h^T_{(m+1)} h_{ik} d_{(m+1)} d_{ik}}{\sum_{j=1}^{m} \|h_{ij}\|^2 + \sum_{j=1}^{m} \|h_{ij}\|^2}$$

(30)

$$= \frac{\sum_{k=1}^{m} \sum_{j=k+1}^{m} h^T_{ij} h_{ij} d_j d_{ik}}{\sum_{k=1}^{m} \sum_{j=k+1}^{m} \|h_{ij}\|^2} + \frac{\sum_{k=1}^{m} h^T_{(m+1)} h_{ik} d_{(m+1)} d_{ik}}{\sum_{j=1}^{m} \|h_{ij}\|^2}.$$  

(31)

Using Slutsky’s theorem and the strong law of large numbers, it can be shown that the denominator in (31) converges to $(1 + \frac{1}{m})$ in probability. Also, from the induction hypothesis, the term $\sum_{k=1}^{m} \sum_{j=k+1}^{m} h^T_{ij} h_{ij} d_j d_{ik} / \sum_{j=1}^{m} \|h_{ij}\|^2$ in the numerator of (31) converges in probability to 0. Therefore, the numerator in (31) converges to the same distribution that the term $\sum_{k=1}^{m} h^T_{(m+1)} h_{ik} d_{(m+1)} d_{ik} / \sum_{j=1}^{m} \|h_{ij}\|^2$ converges to. Also, the term $\sum_{j=1}^{m} \|h_{ij}\|^2 / (mN_t)$ is the same as $\frac{\sum_{k=1}^{m} h^T_{(m+1)} h_{ik} d_{(m+1)} d_{ik} / (mN_t)}{\sum_{j=1}^{m} \|h_{ij}\|^2 / (mN_t)}$. Further, from the strong law of large numbers, the term $(\sum_{j=1}^{m} \|h_{ij}\|^2) / (mN_t)$ converges almost surely to 1. Therefore, from Slutsky’s theorem, we know that $\frac{\sum_{k=1}^{m} h^T_{(m+1)} h_{ik} d_{(m+1)} d_{ik} / (mN_t)}{\sum_{j=1}^{m} \|h_{ij}\|^2 / (mN_t)}$ converges in distribution to the distribution to which the term $\frac{\sum_{k=1}^{m} h^T_{(m+1)} h_{ik} d_{(m+1)} d_{ik} / (mN_t)}{\sum_{j=1}^{m} \|h_{ij}\|^2 / (mN_t)}$ converges.

For a given vector $d$, $h_{ik} d_{ik}$ is a random vector whose distribution is the same as that of $h_{ik}$. Therefore, applying Lemma 4, we see that the term $(\sum_{k=1}^{m} h^T_{(m+1)} h_{ik} d_{(m+1)} d_{ik}) / (mN_t)$ converges almost surely to 0. Hence, the numerator in (31) converges in probability to the constant 0. Therefore, $z_{u(m+1),d} \xrightarrow{P} 0$ as $N_t \to \infty$. This proves the induction step and completes the proof of Lemma 5. □

Appendix B: Proof of Theorem 1

We present the proof of Theorem 1 in this appendix. We shall prove through induction that if $n \in \mathcal{R}_{d^1}$, then $n \in \mathcal{R}_{d^m}$ in probability, $\forall m = 2, \ldots, 2N_t$, as $N_t \to \infty$. Base Case ($m = 2$):

Let $n \in \mathcal{R}_{d^1}$. Therefore, from the definition of $\mathcal{R}_{d^m}$, $n$ satisfies (17). We show that $n \in \mathcal{R}_{d^2}$ in probability as $N_t \to \infty$. For $n$ to belong to $\mathcal{R}_{d^2}$, in addition to satisfying (17), $n$ must also satisfy the following equation $\forall p, q = 1, \ldots, 2N_t, p \neq q$:

$$\begin{align*}
(n + H(x - d) + h_p d_p + h_q d_q)^T (h_p d_p + h_q d_q) & \geq 0, \\
\end{align*}$$

(32)
which can be rewritten as

\[(n + H(x - d))^T h_p d_p + (n + H(x - d))^T h_q d_q \geq -\|h_p\|^2 - \|h_q\|^2 - 2d_p d_q h_p^T h_q. \quad (33)\]

Since \(n\) satisfies (17), it satisfies the following two equations:

\[(n + H(x - d))^T h_p d_p \geq -\|h_p\|^2, \quad \text{and} \quad (n + H(x - d))^T h_q d_q \geq -\|h_q\|^2. \quad (34)\]

Comparing (34) and (33), we notice that if \(h_p\) and \(h_q\) are orthogonal, then \(n\) trivially satisfies (33) for all \(N_t\). Therefore, when \(h_p\) and \(h_q\) are non-orthogonal, the only extra term in the RHS of (33) is \(2d_p d_q h_p^T h_q\). Applying Lemma 5 with \(n = 2\), we see that as \(N_t \to \infty\), the r.v. \(h_p^T h_q / (\|h_p\|^2 + \|h_q\|^2)\) converges to zero in probability. Then, we can write, for any \(\epsilon > 0\),

\[p \left( \frac{|h_p^T h_q|}{\|h_p\|^2 + \|h_q\|^2} > \epsilon \right) < \epsilon, \quad \forall N_t > f(\epsilon). \quad (35)\]

Now, let us analyze \(p(n \notin \mathcal{R}_{d^2})\) for the case of \(d_p d_q = +1\) (a similar analysis holds for \(d_p d_q = -1\)). Consider two disjoint events \(E_1 = \left\{ \frac{|h_p^T h_q|}{\|h_p\|^2 + \|h_q\|^2} < \epsilon \right\}\) and \(E_2 = \left\{ \frac{|h_p^T h_q|}{\|h_p\|^2 + \|h_q\|^2} > \epsilon \right\}\). Then, we can write

\[p(n \notin \mathcal{R}_{d^2}) = p(n \notin \mathcal{R}_{d^2}|E_1) p(E_1) + p(n \notin \mathcal{R}_{d^2}|E_2) p(E_2). \quad (36)\]

The event \(E_1\) can be further split into two disjoint events \(E_{11}\) and \(E_{12}\), given by \(E_{11} = \left\{ 0 < h_p^T h_q < \epsilon (\|h_p\|^2 + \|h_q\|^2) \right\}\) and \(E_{12} = \left\{ 0 > h_p^T h_q > -\epsilon (\|h_p\|^2 + \|h_q\|^2) \right\}\). Also, from (35), \(p(E_1) > 1 - \epsilon\) and \(p(E_2) < \epsilon\). Therefore, using (36), we can write

\[p(n \notin \mathcal{R}_{d^2}) < p(n \notin \mathcal{R}_{d^2}|E_{11}) p(E_{11}) + \epsilon \]

\[< p(n \notin \mathcal{R}_{d^2}|E_{11}) + p(n \notin \mathcal{R}_{d^2}|E_{12}) p(E_{12}) + \epsilon \]

\[< p(n \notin \mathcal{R}_{d^2}|E_{11}) + \epsilon. \quad (37)\]

If event \(E_{11}\) is true, then

\[- (\|h_p\|^2 + \|h_q\|^2) > - (\|h_p\|^2 + \|h_q\|^2 + 2h_p^T h_q) > - (\|h_p\|^2 + \|h_q\|^2) (1 + 2\epsilon). \quad (38)\]

Since \(n \in \mathcal{R}_{d^1}\), \(n\) satisfies (34), and hence satisfies the following equation:

\[(n + H(x - d))^T (h_p d_p + h_q d_q) \geq - (\|h_p\|^2 + \|h_q\|^2). \quad (39)\]
Using (38) and (39), we see that 

Using the above definitions, (42) can
rewritten as

\[ E \]

\[ \text{If event } E_{12} \text{ is true, then} \]

\[ - (\| h_p \|^2 + \| h_q \|^2) (1 - 2\epsilon) > - (\| h_p \|^2 + \| h_q \|^2 + 2h_p^T h_q) > - (\| h_p \|^2 + \| h_q \|^2). \] (41)

Using (33) and (41), we can write that

\[ p(n \notin \mathcal{R}_{d^2} | E_{12}) = p \left( \left[ (n + H(x - d))^T h_p d_p + (n + H(x - d))^T h_q d_q \leq - \| h_p \|^2 - \| h_q \|^2 - 2h_p^T h_q \right] | E_{12} \right) \]

\[ < p \left( - (\| h_p \|^2 + \| h_q \|^2) \leq (n + H(x - d))^T (h_p d_p + h_q d_q) \leq - (\| h_p \|^2 + \| h_q \|^2) (1 - 2\epsilon) \right) | E_{12} \}. \] (42)

Define \( \mathcal{R}_\epsilon \) to be a set of vectors in \( \mathbb{R}^{2N_i} \), as

\[ \mathcal{R}_\epsilon \triangleq \left\{ v \mid - (\| h_p \|^2 + \| h_q \|^2) \leq (v + H(x - d))^T (h_p d_p + h_q d_q) \leq - (\| h_p \|^2 + \| h_q \|^2) (1 - 2\epsilon) \right\}. \] (43)

Also, define a function \( f_2 \) as

\[ f_2(\epsilon) \triangleq p(n \in \mathcal{R}_\epsilon | E_{12}). \] (44)

Using the above definitions, (42) can rewritten as

\[ p(n \notin \mathcal{R}_{d^2} | E_{12}) < f_2(\epsilon). \] (45)

Let \( \epsilon_1, \epsilon_2 \in \mathbb{R} \), \( \epsilon_1, \epsilon_2 > 0 \), and \( \epsilon_1 > \epsilon_2 \). From the definition of \( \mathcal{R}_\epsilon \) in (43), it can be seen that \( \mathcal{R}_{\epsilon_2} \subset \mathcal{R}_{\epsilon_1} \). This implies that \( f_2(\epsilon_1) > f_2(\epsilon_2) \). Hence \( f_2 \) is a monotonically increasing function. Using (45), we can rewrite (40) as written as

\[ p(n \notin \mathcal{R}_{d^2}) < f_2(\epsilon) + \epsilon. \] (46)

Therefore,

\[ p(n \in \mathcal{R}_{d^2}) > 1 - (f_2(\epsilon) + \epsilon). \] (47)

Now define \( g_2(\epsilon) \triangleq f_2(\epsilon) + \epsilon \). So \( g_2 \) is a monotonic function and is therefore invertible. Let \( \delta = g_2(\epsilon) \). Using (35) and the above definitions, we can write that

\[ N_t > f(\epsilon) \]

\[ > f \left( g_2^{-1}(\delta) \right) \]

\[ > N_2(\delta), \] (48)
where \( N_2 \overset{\Delta}{=} f \circ g_2^{-1} \). We can then write (47) as
\[
p(n \in \mathcal{R}_{d^2}) > 1 - \delta.
\] (49)

Since \( g_2 \) is a continuous monotonic function, for any \( \delta \), \( 0 \leq \delta \leq 1 \), there exists an integer \( N_2(\delta) \) such that for \( N_t > N_2(\delta) \), \( p(n \in \mathcal{R}_{d^2}) > 1 - \delta \). Therefore, \( n \in \mathcal{R}_{d^2} \) in probability as \( N_t \to \infty \), thus proving the base case.

**Induction Hypothesis:** Let \( n \in \mathcal{R}_{d^m-1} \) in probability as \( N_t \to \infty \).

**Induction Step:** We need to prove that \( n \in \mathcal{R}_{d^m} \) in probability as \( N_t \to \infty \). For \( n \) to belong to \( \mathcal{R}_{d^m} \), \( n \) must satisfy the following equation for all possible \( m \)-tuples \((i_1, i_2, \ldots, i_m)\):
\[
(n + H(x - d) + (\sum_{j=1}^{m} h_{ij} d_{ij}))^T (\sum_{j=1}^{m} h_{ij} d_{ij}) \geq 0,
\] (50)

which can be written as
\[
(n + H(x - d))^T (\sum_{j=1}^{m-1} h_{ij} d_{ij}) + (n + H(x - d))^T h_{im} d_{im} \geq -\| \sum_{j=1}^{m-1} h_{ij} d_{ij} \|^2 - \| h_{im} \|^2 - 2\left( \sum_{j=1}^{m-1} h_{ij} d_{ij} \right)^T h_{im} d_{im}.
\] (51)

However, we know from the induction hypothesis that \((n + H(x - d))^T (\sum_{j=1}^{m-1} h_{ij} d_{ij}) \geq -\| \sum_{j=1}^{m-1} h_{ij} d_{ij} \|^2\). Also, since \( n \in \mathcal{R}_{d^1} \), we know that \((n + H(x - d))^T h_{im} d_{im} \geq -\| h_{im} \|^2\).

Therefore, if the term \( 2(\sum_{j=1}^{m-1} h_{ij} d_{ij})^T h_{im} d_{im} \) in the R.H.S of (51) were 0, then (50) would have been trivially satisfied. We now show that the contribution of the term \( 2(\sum_{j=1}^{m-1} h_{ij} d_{ij})^T h_{im} d_{im} \) when compared to the other two terms in the R.H.S (51) converges to 0 as \( N_t \to \infty \).

Define a r.v. \( v_m \overset{\Delta}{=} \frac{2(\sum_{j=1}^{m-1} h_{ij} d_{ij})^T h_{im} d_{im}}{\sum_{j=1}^{m-1} \| h_{ij} \|^2 + \| h_{im} \|^2} \). Our objective is to show that as \( N_t \to \infty \), \( v_m \to 0 \) in probability. This is equivalent to proving that \( w_m \overset{\Delta}{=} v_m + 1 = \frac{\| \sum_{j=1}^{m} h_{ij} d_{ij} \|^2}{\| h_{im} \|^2 + \| \sum_{j=1}^{m} h_{ij} d_{ij} \|^2} \) converges to one in probability as \( N_t \to \infty \). We can write \( w_m \) as
\[
w_m = \frac{\| \sum_{j=1}^{m} h_{ij} d_{ij} \|^2}{\sum_{j=1}^{m} \| h_{ij} \|^2 + \| \sum_{j=1}^{m} h_{ij} d_{ij} \|^2}.
\] (52)

From Lemma 5, we know that for any integer \( m, 1 \leq m \leq 2N_t \), it is true that \( \sum_{k=1}^{m} \sum_{j=k+1}^{m} h_{ij}^T h_{ik} d_{ij} d_{ik} \) converges to 0 in probability as \( N_t \to \infty \). By Slutsky’s theorem, this is equivalent to
\[
\frac{2 \sum_{k=1}^{m} \sum_{j=k+1}^{m} h_{ij}^T h_{ik} d_{ij} d_{ik}}{\sum_{j=1}^{m} \| h_{ij} \|^2} + 1 = \frac{\| \sum_{j=1}^{m} h_{ij} d_{ij} \|^2}{\sum_{j=1}^{m} \| h_{ij} \|^2} \to 1 - \delta
\] (53)
as $N_t \to \infty$. We shall use this result to prove the convergence of $w_m$ in (52). Using (53), it can be seen that the numerator of $w_m$ in (52) converges to 1 as $N_t \to \infty$, i.e.,

$$\frac{\| \sum_{j=1}^{m} h_{ij} d_{ij} \|^2}{\sum_{j=1}^{m} \| h_{ij} \|^2} \xrightarrow{p} 1, \text{ as } N_t \to \infty.$$  (54)

In the denominator of (52), it can be shown that the term

$$\frac{\| h_{im} \|^2}{\sum_{j=1}^{m} \| h_{ij} \|^2} \xrightarrow{p} \frac{1}{m}, \text{ as } N_t \to \infty.$$  (55)

The 2nd term in the denominator of (52) can be rewritten as

$$\frac{\| (\sum_{j=1}^{m-1} h_{ij} d_{ij}) \|^2}{\sum_{j=1}^{m} \| h_{ij} \|^2} = \frac{\| (\sum_{j=1}^{m-1} h_{ij} d_{ij}) \|^2}{\sum_{j=1}^{m-1} \| h_{ij} \|^2} \frac{\| h_{im} \|^2}{\| h_{im} \|^2 + 1}. \quad (56)$$

Similar to the derivation of (53), we can claim that the numerator in (56) converges to one in probability. From Slutsky’s theorem, it can be shown that $\frac{\| h_{im} \|^2}{\sum_{j=1}^{m-1} \| h_{ij} \|^2}$ converges to $\frac{1}{m-1}$ in probability. Using this and Slutsky’s theorem, it can be shown that (56) converges to $\frac{m-1}{m}$ in probability. Using this result along with (54), (55) and Slutsky’s theorem in (52), it can be shown that $w_m$ converges to one in probability as $N_t \to \infty$. This, therefore, implies that $v_m$ converges to zero in probability. As proved in the base case, it can be shown that for any $\delta$, $0 \leq \delta \leq 1$, there exists an integer $N_m(\delta)$ such that for $N_t > N_m(\delta)$, $p(n \in \mathcal{R}_{4^m}) > 1 - \delta$. This proves the induction step and completes the proof of Theorem 1. □

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Figure 1: Simulated pdf of $z_{u,d}$ for $n = 2N_t$ for increasing $N_t = N_r$. 4-QAM. The pdf tends towards Dirac delta function at zero.

Figure 2: Simulated BER performance of the LAS detector for V-BLAST as a function of average received SNR for increasing values of $N_t = N_r$. MMSE initial vector, 4-QAM. LAS detector achieves near SISO AWGN performance at high SNRs for large $N_t = N_r$. 

Figure 3: Average received SNR required to achieve a target BER of $10^{-3}$ in V-BLAST for increasing values of $N_t = N_r$ for 4-QAM. LAS detector with MMSE initial vector. LAS detector achieves near SISO AWGN performance for large $N_t = N_r$.

Figure 4: Average received SNR required to achieve a target BER of $10^{-4}$ in V-BLAST for increasing values of $N_t = N_r$ for 16-QAM. LAS detector with MMSE initial vector. LAS detector performance approaches SISO AWGN performance for large $N_t = N_r$. 