Abstract

The Q-analysis governance approach and the use of simplicial complexes—type of hypergraph—allow to introduce the formal concepts of dimension and conjugacy between the network of entities involved in governance (typically organizations) and the networks of those attributes taken into account (e.g. their competences), which offer a specific angle of analysis. The different sources of existing data (e.g. textual corpora) to feed the analysis of governance—environmental in particular—are mentioned, their reliability is briefly discussed and the required pre-processing steps are identified in the perspective of evidence-based analyses. Various indices are constructed and evaluated to characterize the context of governance as a whole, at mesoscale, or locally, i.e. at the level of each of the entities and each of the attributes considered. The analysis of ideal-type stylizing boundary cases provides useful references to the analysis of concrete systems of governance and to the interpretation of their empirically observed properties. The use of this governance modeling approach is illustrated by the analysis of a health-environment governance system in Southeast Asia, in the context of a One Health approach.

Keywords: governance, modeling, simplicial complex, evidence-based analysis, topology, One Health, ideal type, indices

1. Introduction

In April 2010, in the Gulf of Mexico, started the BP Deepwater Horizon oil spill, considered one of the largest marine oil spills in the history of the petroleum industry (estimated to over 600,000 tons of oil released in Gulf of Mexico over 3 months) killing 11 workers and leading to a major environmental disaster. It raised a number of legal issues involving a variety of actors, various levels of decision-making and regulation (from international to local). Presented as “an important example of multidimensional governance in action” by Osofsky [1], it led to an attempt by the same author to provide a conceptual model for understanding complex regulatory
problems. If the multidimensional aspect of governance is effectively considered as the central challenge in this complex socio-environmental tragedy and has been debated as such as it will be later on, in the case of climate change litigation [2], it is not at all addressed empirically but stay at a very descriptive level. Furthermore, some descriptions of multidimensionality through legal lenses are in contradiction with the mathematical notion of dimension (cf. the paragraph on multidimensionality in Ref. [3]).

Nevertheless, the notion of dimension is fundamental in mathematics and physics and therefore in disciplines using their formal representations (e.g. in ecology or epidemiology modeling). It is declined in various ways, depending on whether it attempts to characterize the space in which interactions are deployed (embedding dimension, local dimension), the geometry of an object (e.g. fractals) or the development of instabilities that work on the evolution of the state of a system (e.g. Lyapunov dimension) [4, 5]. The analysis of governance by political scientists or international relations scholars has made only an extremely limited use of this notion, which, however, is adaptable to the needs of this field of research and is likely to consolidate an empirical, evidence-based approach of governance.

The situation is similar concerning the notion of conjugacy: if a group of organizations is involved in the management of a set of environmental issues, the symmetrical point of view considers that these issues solicit organizations, thus offering another perspective on governance. This kind of duality of approaches is shown in a conjugate relation between two expressions of a formal entity, in this case a simplicial complex, a particular type of hypergraph [9, 10]. Continuing our approach of providing the network governance study with formal tools and the concepts that they provide [11, 12], we apply in this paper the notions of dimension and conjugacy as used in a discrete modeling of governance based on Q-analysis.

This approach proposed in the 1970s by the mathematician Ron Atkin [13, 14] has been used to formalize various problems in social sciences [15–19]. It is now developed in the context of the application of hypergraphs to the analysis of various complex systems (e.g. [19]). The formalism of the simplicial complexes intervenes in a very wide range of applications (for a brief overview, see e.g. [20–22]). A more general survey of data processing using topology methods is found in [23, 24].

The main notions of Q analysis are presented in Section 2. This study aims at illustrating their use by analyzing a network of health-environment institutions and themes in Southeast Asia are presented in Section 2. The concepts of dimension and conjugacy are presented in Section 3. As soon as the dimension of simplices or of paths in the structure is higher than 3, their representation is not readable. For the analysis, we rely on indicators defined in Section 4. Models corresponding to classical general ideal type of governance are then presented in Section 5 and are used as references to analyze empirical governance systems. In Section 6, we discuss the role of generalist organizations (organizations with a large and diverse portfolio of competences) as seen as high-dimensional simplices in a governance system.

1The abundant indexes of the subjects of three relatively recent synthesis books—the Oxford Handbook of Governance [6], the Oxford Handbook of Political Methodology [7] and the Oxford Handbook of International Relations [8]—do not contain the dimension, conjugacy or duality entries.
2. Actors and competencies

We consider a set of organizations with expertise on themes emerging from the analysis of the emergence or re-emergence of infectious diseases in Southeast Asia in a context of environmental change. Epidemiology shows that human health is likely to be affected by a wide variety of pathogens, themselves dependent on their vectors and hosts and on environmental (precipitation and ambient temperature climatology, surface hydrological regime) or socio-ecological dynamics (land cover and land use, biodiversity state and uses, economic exchanges, migration) [25]. In response to the risks of pandemics, the One Health approach [26, 27] promotes simultaneous consideration of the determinants of human health, animal health (domestic animals and wildlife) and environmental health.

This posture leads to considering both public health and environmental themes—such as climate change [28, 29] or the loss of biological diversity [30]—linked by epidemiological dynamics [31], as well as organizations operating from international to regional or local levels in these areas. Health governance in Southeast Asia, a hot spot for the emergence or re-emergence of infectious diseases and biodiversity [32], is also based on political or legal texts (e.g. international conventions [33]), which are themselves integral parts of governance systems [34]. In the One Health perspective, the following health and environmental themes are identified: human health (HH label), animal health (AH), ecosystem health (EH), climate change (CC), land use and land cover (LU), water resources (WR) and risk assessment or risk analysis (RA). The organizations we consider are listed in Table 1 with the themes for which they display competencies.

Types are indicated by combining the following initials: I = international; R = regional; O = organization; Ob = observation; N = network; Po = policy; Pr = project or initiative; NG = non-governmental and PF = platform. The labels of themes read as follows: HH = human health; AH = animal health; EH = ecosystem health; CC = climate change; BD = biodiversity; FS = food security; LU = land use and land cover; WR = water resources and RA = risk assessment or risk analysis.

Under the generic term organization, we target organizations as such (FAO, WHO), networks (e.g. TROPMED, APEIR, GEOBON) or network of networks of organizations (CORDS), consortia (MBDS), information systems (ARAHIS), fora (FREH) or technical or cooperation platforms (ARAHIS, EVIPNeT). ASEAN2025 [35] outlines the ASEAN policy strategy for collaboration and development in member countries, and as such participates in regional governance, particularly on health-environment issues. All these entities have an institutionalized existence. Regional health-environment governance involves in fact the diversity of

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2The criteria and methodology used for this choice of organization are described and discussed in Ref. [11].
| L   | T   | Websites of organizations | Acronym = Subset of Themes |
|-----|-----|---------------------------|---------------------------|
| 01  | IO  | http://www.fao.org/home/en/ | FAO = {HH,AH,EH,CC,BD,FS,LU,WR,RA} |
| 02  | IO  | www.oie.int/               | OIE = {AH,RA}            |
| 03  | IO  | www.who.int/               | WHO = {HH,CC,RA}        |
| 04  | IObN| http://geobon.org/         | GEOBON = {BD}           |
| 05  | IO  | www.cordsnetwork.org/      | CORDS = {HH,AH,EH,RA}   |
| 06  | RPo | See reference [35]         | ASEAN2025 = {HH,CC,BD,FS,LU,WR,RA} |
| 07  | RO  | https://www.aseanbiodiversity.org/ | ACB = {BD,RA} |
| 08  | RPr | http://environment.asean.org/ | ACEenv = {HH,CC,BD,WR}  |
| 09  | RN  | www.asfnsec.org/           | ASFN = {CC,BD,FS,RA}   |
| 10  | RO  | http://aichr.org/          | AICHR = {HH,CC}         |
| 11  | NGO | https://www.aseanlawassociation.org/ | ALAWASS = {RA} |
| 12  | RPF | http://www.rr-asia.oie.int/ | ARAHIS = {AH}          |
| 13  | RO  | www.seameo.org/            | SEAMEO = {HH}          |
| 14  | RO  | https://www.biota.org/     | BIOTROP = {HH,AH,EH,BD,WR} |
| 15  | RO  | www.seameo-recfon.org/     | RECFON = {HH,FS,RA}    |
| 16  | RO  | http://www.search.org/     | SEARCA = {CC,BD,FS,LU,WR} |
| 17  | RN  | http://seameotropmednetwork.org/ | TROPMED = {HH,RA} |
| 18  | RN  | www.mbdsnet.org/          | MBDS = {HH,RA}        |
| 19  | RPr | http://www.cdcseomoh.gov.kh/25-cdc2-project/ | CDC-ADB = {HH,AH,CC,RA} |
| 20  | RO  | http://www.mrcmekong.org/  | MRC = {HH,AH,EH,CC,BD,FS,LU,WR,RA} |
| 21  | NGO | www.mekonglawcenter.org/  | MRLC = {BD}            |
| 22  | RN  | http://cansea.org.vn/      | CANSEA = {CC,BD,FS,LU,WR} |
| 23  | RPr | https://www.adb.org/projects/40253-012/main | BCI/CoP = {HH,EH,CC,BD,FS,LU,WR,RA} |
| 24  | RPr | http://lowermekong.org/    | LMI = {HH,BD,FS,WR,RA} |
| 25  | RON | http://www.esabii.biodic.go.jp/ap-bon/ | APBON = {BD} |
| 26  | RPF | http://www.aehin.org/      | AeHIN = {HH,RA}        |
| 27  | RPF | http://www.who.int/evidence/en/ | EVIPNetA = {HH} |
| 28  | RO  | http://aepir.net/          | APEIR = {HH,AH,EH,RA}  |
| 29  | RO  | www.pemsea.org/            | PEMSEA = {EH,BD,FS,WR}  |
| 30  | RO  | www.asiadhrhra.org/       | ADHRRA = {CC,BD,FS}   |
| 31  | RPF | http://www.esabii.biodic.go.jp/ | ESABII = {BD} |
| 32  | RPr | http://www.wpro.who.int/riish/en/ | RFEH = {HH,EH,CC,RA} |
| 33  | RPr | www.cobsea.org/            | COBSEA = {EH,CC,BD,FS,LU,WR} |
| 34  | RN  | www.aecen.org/             | AECEN = {HH,CC,BD,RA}  |

Table 1. List of organizations (level L and type T in columns 2 and 3) as simplices over the health-environment-related themes.
organizations and political and legal mechanisms that must be taken into account in an empirical approach.

3. Governance structure: dimensions and conjugacy

To present the concepts we are interested in, we work out a small-size case and introduce some notations. We consider a set $X$ of $M = 4$ organizations (with acronyms WHO, SEARCA, LMI, APEIR), a set $Y$ of $N = 6$ themes or issues (with labels HH, AH, EH, CC, LU, RA) and relation $R$ so that $x_j \mathbin{R} y_k$ means that organization $x_j \in X$ has competency on theme $y_k \in Y$ as indicated by the checked cells of the table in Plate 1A. This information is coded in the incidence matrix $R$ with a 1 at the intersection of $j$th line and $k$th column (zero value otherwise; see Plate 1B).

Now consider each organization $x_j$ as the set of themes with which it is related, say $\overline{x_j} = \{y_k \text{ such as } x_j \mathbin{R} y_k\}$. For example, we have $\overline{\text{APEIR}} = \{HH, AH, EH, RA\}$. The organization APEIR can be represented as a regular polyhedron of four linked vertices (the related themes), say as a tetrahedron or 3-simplex (which is 3-dimensional). In the same way, $\overline{\text{WHO}} = \{HH, CC, RA\}$ is a 2-simplex (three vertices, triangle, 2-dimensional). $\overline{\text{SEARCA}} = \{CC, LU\}$ and $\overline{\text{LMI}} = \{HH, RA\}$ are two disjoint 1-simplices (2 linked vertices, line segment, 1-dimensional). Altogether these simplices form the simplicial complex $K^{\text{ex}}_{\text{X}}(Y, R)$, the subscript X indicating that the simplices represent organizations (elements of $X$) and the superscript “ex” standing for “example.” Figure 1A shows that LMI is a 1-common face (line segment) of both the APEIR tetrahedron and WHO triangle. WHO and SEARCA share a 0-face (with a single vertex $\{CC\}$).

In a symmetrical or conjugated way, we can consider each theme as the set of organizations with which it is bound by the inverse relation $R^{-1}$: $\overline{y_k} = \{x_j \text{ such as } y_k \mathbin{R^{-1}} x_j\}$. The conjugate simplicial complex $K^{\text{ex}}_{\text{Y}}(X, R^{-1})$ is represented in Figure 1B. $\overline{\text{LU}} = \{\text{SEARCA}\}$, $\overline{\text{AH}} = \{\text{APEIR}\}$ and $\overline{\text{RH}} = \{\text{APEIR}\}$ are 0-simplices, the last two not being distinguishable in this specific context. $\overline{CC}$ is a 1-simplex; $\overline{HH}$ and $\overline{RA}$ are undistinguishable 2-simplices (same triangle). The $M \times M$ symmetric matrix $RR^T$ (with elements $a_{jj}$; $R^T$ is the transposed matrix of $R$) convey information on $K^{\text{ex}}_{\text{X}}(Y, R)$: $a_{jj}$ is the number of vertices forming the simplex $\overline{x_j}$; $a_{jk}$ is the number

| $R$ | HH | AH | EH | CC | LU | RA |
|-----|----|----|----|----|----|----|
| WHO | ×  | ×  | ×  | ×  |    |    |
| SEARCA | ×  | ×  | ×  | ×  |    |    |
| LMI   | ×  | ×  | ×  | ×  |    |    |
| APEIR | ×  | ×  | ×  | ×  |    |    |

Plate 1. (Left) Table of the relation between organizations (lines) and themes (columns) and (right) corresponding incidence matrix.
of vertices shared by simplices $x_j$ and $x_k$. In the same way, matrix $R^T R$ encodes information on $K_{YX}^q; R/C_0 ...$ the vertices are themes. (B) 2D simplicial complex $K_{YX}^q$. Each simplex is a theme, and the vertices are organizations.

As for graphs, it is possible to define paths in a simplicial complex, but of various dimensions. The intersection between two simplices—for example $x_j$ and $x_{j+1}$—is either empty or is a set of vertices that form a simplex $x_j \cap x_{j+1}$ of $K_{YX}^q$. Two simplices $x_I$ and $x_n$ are connected by a path of length $(m - 1)$ if the sequence of simplices $x_1, x_2, \ldots, x_m$ satisfies $x_j \cap x_{j+1} \neq \emptyset$ for every $j \in \{1, 2, \ldots, m - 1\}$. It is also a $q$-path if:

$$\min_{j=1 \ldots (m-1)} \{\dim(x_j \cap x_{j+1})\} = q$$

That is to say each pair of consecutive simplices of the sequence shares at least $(q + 1)$ vertices. $x_1$ and $x_n$ are then $q$-connected. Any path of minimum length between two simplices is called a geodesic. The relation $x_I R_q x_k$ if $x_j$ and $x_k$ are $q$-connected is an equivalence relation on $K_{YX}^q$. The equivalence classes of $R_q$ are called the $q$-connected components of $K_{YX}^q$. We denote by $Q_q$ their number. The graphical representation of simplicial complexes is not readable as soon as the dimension of the simplices is greater than 3 or when the complex is composed of numerous simplices with common faces. This limitation is bypassed by the use of indicators.

4. Governance complex: global to local indexes

We define three types of indexes to characterize a simplicial complex $K$ defined from a relation involving a space $X$ of cardinal $N$. A global index characterizes a simplicial complex in its entirety. A mesoindex takes into account the positioning of each simplex in the whole structure of the simplicial complex. A local index is attached to each simplex and allows to evaluate the configuration of their local insertion, with their immediate neighbors, in the complex. The first global index associated to a simplicial complex $K$ is its dimension $\dim K$: it is the dimension of
its higher dimensional simplex. In our example of Figure 1, we have $\text{dim}K_X = 3$ and $\text{dim}K_Y = 2$. The structure vector $Q(K)$ is formed from the number of q-connected components of $K$, for $q$ varying from 0 to $\text{dim}K$:

$$Q(K) = [Q_0, Q_1, Q_2, \ldots, Q_{\text{dim}K}]$$ (2)

A global size index is evaluated according to the formula:

$$G_{SI}(K) = \frac{2}{(\text{dim}K + 1)(\text{dim}K + 2)} \sum_{q=0}^{\text{dim}K} (q + 1)Q_q$$ (3)

If $K$ is complete, then $G_{SI} = 1$. If none of the $N$ vertices of $K$ is connected to another vertex, then $G_{SI} = N$. In order to compare the percentage of dispersion of vertices between complexes that do not have the same number of vertices, one also defines a normalized size index:

$$\overline{G_{SI}}(K) = 100 \times |G_{SI}(K) - 1|/(N - 1)$$ (4)

$\overline{G_{SI}}$ can vary from 0% for a clique to 100% for isolated vertices (i.e. for a stable set in the terminology of graph theory). Meso-indexes take into account the insertion of each simplex in the network. For each simplex $x_j$, one defines a size index by:

$$M_{SI}(x_j) = \frac{2}{(N - 1)(\text{dim}K + 1)(\text{dim}K + 2)} \sum_{q=0}^{\text{dim}K} (\text{dim}K + 1 - q) \times n_q(x_j)$$ (5)

where $n_q(x_j)$ is the number of simplices $y_j$, with $y \neq x_j$, connected to $x_j$ by a q-path. $M_{SI}(x_j)$ is 0 when $x_j$ is isolated (not connected to any other simplex $y_j$, with $y \neq x_j$). It increases in particular when the dimension of the complex is high and the simplex has connections with many other simplices along low-dimensional q-paths. A path index $P_q(x_j)$ is also defined for each simplex, which also depends on a threshold dimension $q$ (with $q$ varying from 0 to $\text{dim}K$):

$$P_q(x_j) = \frac{2}{(N_q - 1)(g_q + 1)(g_q + 2)} \sum_{k=1}^{g_q + 1} (g_q + 2 - k) \times m_{q,k}(x_j)$$ (6)

where $m_{q,k}(x_j)$ is the number of simplices $y_j$, with $y$ different from $x_j$ and connected by a q-path of length at most $k$. $g_q$ is the maximum length of q-geodesics and $N_q$ is the number of simplices $x$ with dimension greater or equal to $q$. $P_q(x_j)$ varies from 0 when $x_j$ is isolated (no access to this simplex) to 1 when $x_j$ includes all other simplices (as faces: immediate access). Eccentricity is a local index attached at each simplex. Considering a simplex $x_j$ of dimension $\text{dim}(x_j)$ and which higher q-connectivity is of degree $\text{dim}^\prime(x_j)$, we define the eccentricity of $x_j$ by:

$$\eta(x_j) = \frac{\text{dim}(x_j) - \text{dim}^\prime(x_j)}{\text{dim}^\prime(x_j) + 1}$$ (7)
The eccentricity of $x_j$ is maximal if it is only connected to the other simplices by a 0-path: its value is then $\eta(x_j) = \dim(x_j)$. $\eta(x_j) = 0$ if $x_j$ is a sub-simplex (say if there is a $x_k$ such that $x_j \subset x_k$). By convention, we set $\eta(x_j) = -1$ if $x_j$ is an isolated simplex.

5. Governance ideal types versus empirical types

To better understand the specificities of the system we are studying, we propose four models of comparisons corresponding to limiting types of organization of governance, say of ideal types. In the following examples, we assume the same number $N = 8$ or organizations and competences. The global indexes of the corresponding complexes are summarized in Table 2. Note that for the no-dependency, full dependency and cyclic ideal types, the incidence matrices are symmetric so that the properties of the simplicial complex $K_X$ and of its conjugate $K_Y$ are the same.

5.1. Ideal type 1: no dependency $K^{nodep}$

In this model, each of the $N$ organizations has a single competence (works on a single theme) and there is no overlap in the areas of competence of the organizations. This governance structure induces a unitary diagonal $[N \times N]$ square matrix (identity matrix). Each organization is a 0-simplex (a single vertex) with eccentricity $-1$ since there is no path between organizations (each organization is isolated). The simplicial complex $K^{nodep}$ is of zero dimension $\dim K^{nodep} = 0$. The vector of structure $Q$ also includes only one component equal to the

| $K^{nodep}_{X or Y}$ | $dimK$ | $G_{SI}$ | $G_{SI}$ | $Q(K)$ | $\eta(x_j)$ | $M_{SI}(x_j)$ | $P_q(x_j)$ |
|-----------------------|---------|---------|---------|--------|------------|-------------|----------|
| $K^{nodep}_{X}$       | 0       | 8       | 100     | [8]    | -1         | 0           | 0        |
| $K^{fulldep}_{X or Y}$| 7       | 1       | 0       | [1,1,1,1,1,1,1,1] | 0 | 1 | 1 |
| $K^{vertical}_{X}$    | 7       | 1       | 0       | [1,1,1,1,1,1,1,1] | $\{j = 1\}:7$ | $\{j = 1\}:0.22$ | $\{j = 1, q = 0\}:1$ | $\{j \neq 1, q = 0\}:0.43$ |
| $K^{vertical}_{Y}$    | 1       | 5       | 57.1    | [1,7]  | $\{j = 1\}:0$ | $\{j = 1\}:0.62$ | $\{q = 0\}:1$ | $\{q = 1\}:0$ |
| $K^{vertical2}_{X or Y}$ | 1       | 5.7     | 66.7    | [1,8]  | 1 | 0.67 | $\{q = 0\}:0.56$ | $\{q = 1\}:0$ |
| $K^{vertical2}_{X or Y}$ | 2       | 4.5     | 50.0    | [1,8]  | 1/2 | 0.83 | $\{q = 0\}:0.71$ | $\{q = 1\}:0.56$ | $\{q = 2\}:0$ |
| $K^{vertical3}_{X or Y}$ | 3       | 3.8     | 40.0    | [1,8]  | 1/3 | 0.90 | $\{q = 0\}:0.90$ | $\{q = 1\}:0.71$ | $\{q = 2\}:0.56$ | $\{q = 3\}:0$ |

The values of $j$ and $q$ are specified (between braces) only if the value of the index considered is related to them. In the $K^{vertical}_{X}$ complex, $j = 1$ corresponds to the generalist organization $x_{VI}$, which has all the competences.

Table 2. Indices and structure vectors of simplicial complexes corresponding to main ideal type of governance (assuming $8 \times 8$ incidence matrices)—see text.
global size index $G_{SI}$ of the complex. This index is equal to the number of independent organizations considered $G_{SI} = N$ (and thus $Q = [N]$) and the maximum dispersion $\overline{G_{SI}} = 100\%$.

5.2. Ideal type 2: full interdependency $K_{fulldep}$

Here, on the contrary, the organizations all work on all the themes and are thus fully interdependent, with no structural leadership. The incidence matrix $[N \times N]$ is full of 1. The dimension of the complex of the organization is determined by the number of themes, $\dim K_{X}^{fulldep} = (N - 1)$. The complex has only one component, all the organizations being connected by an $(N-1)$ path. Each simplex has dimension $(N - 1)$ and zero eccentricity: indeed, each simplex coincides with each of the other simplices. The vector of structure $Q(K_{X}^{fulldep})$ is an all-one vector of $N$ components. However, the amalgam of the organizations in a compact structure is expressed by the value of the size index $G_{SI} = 1$ (and dispersion index $\overline{G_{SI}} = 0$), and therefore does not depend on the number $N$ of themes. By symmetry, the conjugate complex $K_{Y}^{fulldep}$ has similar properties.

5.3. Ideal type 3: vertical integration $K_{vertical}$

For comparison with the other models, in this ideal type, we also consider that the number of organizations is equal to the number of competences $N$. In the vertical integration model, one of the organizations $x_{VI}$, integrates all the skills the other organizations having only one of these skills, each time different from the skill of the other organizations. The corresponding incidence matrix is an identity $[N \times N]$ matrix with the addition of the first line (corresponding to the integrative organization) composed of unit elements. The dimension of the simplicial complex $K_{X}^{vertical}$ of the organizations is given by the dimension of the organization $x_{VI}$ which integrates all the competences ($[N - 1]$-simplex), say $\dim K_{X}^{vertical} = (N - 1)$. All other organizations are 0-simplices attached to $x_{VI}$ by the competence that each one shares with it: there is only one 0-path that binds all organizations. They all have zero eccentricity, being a face (of dimension zero) of the simplex $x_{VI}$ with eccentricity $\eta(x_{VI}) = \dim(x_{VI})$. The structure vector $Q_{X}$ has $N$ unit components and the global size index is $G_{SI} = 1$ (and dispersion $\overline{G_{SI}} = 0$).

While the diagram associated with $K_{X}^{vertical}$ consists mainly of the simplex $x_{VI}$ of the integrative organization (the other organizations coinciding with its vertices), the diagram of $K_{Y}^{vertical}$ is a star diagram (a single connected component). The matrix of incidence of the relation $R^{-1}$ is an identity matrix with the first column filled with 1s. The theme addressed only by $x_{VI}$ is a 0-simplex and the others are all 1-simplices (addressed by $x_{VI}$ and one and only one other organization), thus $\dim K_{Y}^{vertical} = 1$. Its structure vector has two components $Q_{Y} = [1, N - 1]$ (a 0-simplex and $N - 1$ 1-simplices). $K_{Y}^{vertical}$ has only one connected component (star diagram, no isolated vertex). The eccentricity values do not depend on the size of the network: the eccentricity is zero for the theme taken into account only by the organization $x_{VI}$, and 1 for the themes covered by two organizations ($x_{VI}$ and one and only one other organization). The
global size index depends on the number of organizations with \( G_{SL} = 2(N - 1)/3 \). Finally, it should be noted that in this model, horizontal integration (a competence shared by all organizations, other competences being held by only one organization) is obtained by simply transposing the incidence matrix associated with vertical integration (with simplicial complex \( K_{X_{vertical}}^{\text{vertical}} \)).

5.4. Ideal type 4: cyclic integration \( K_{\text{cycle}}^{\theta} \)

Let us suppose that we have \( N \) themes \( \{y_1, y_2, \ldots, y_N\} \) and that each organization has competencies on the same number \( k < N \) of themes but so that the first organization covers the themes \( \{y_1, y_2, \ldots, y_k\} \), the second one the themes \( \{y_2, y_3, \ldots, y_{k+1}\} \) and so on till the last organization with competences on \( \{y_N, y_1, \ldots, y_{k-1}\} \). The corresponding complex is formed from \( N \) simplices of dimension \( (k - 1) \) connected by two along a \( (k - 2) \)-path forming a cycle. We shall say that this cycle has a thickness \( \theta = (k - 1) \). Two organizations opposite to each other on the cycle have no common focus (theme and competence). However, they are connected by the \( (k - 1) \)-path, but separated by a hole. They may be led to dialog, but through other neighboring organizations (with whom they share themes of interest), with some themes being shared between contiguous organizations along this path. As R. Atkin notes, the hole in the middle of the cycle is not just the absence of common competences between opposite organizations on the cycle: it is a real obstacle to cooperation (viewed from the sharing of competences). Table 2 presents the values of the indexes for three cyclic models with respective thickness \( \theta = 1, 2 \) and 3 (assuming again \( 8 \times 8 \) incidence matrices). The dimension of the cycle complexes is given by \( \dim K_{X_{\text{cycle}}^{\theta}}^{\text{cycle}} \). The \( \theta \) first values of the structure vector are 1 s, and the last \( (\theta + 1) \)th component equals \( N \). All simplices have the same eccentricity \( \eta = \theta^{-1} \). For a given value of \( \theta \), all the simplices have the same meso-index of size \( M_{SI} \). The path index \( P_q(\overline{X}) \) takes quantized values and follows a pattern when changing \( \theta \) and \( q \) as can be seen in Table 2.

6. The role of generalist organizations

Consider now the complex \( K_{X_{\text{all}}}^{\text{all}} \) and its conjugate \( K_{Y_{\text{all}}}^{\text{all}} \) representing the organizations involved in Southeast Asia with the distribution of their competences in environment and health as given in Table 1. With expertise in each of the areas we are interested in, we consider FAO (Food and Agriculture Organization of the United Nations) and MRC (Mekong River Commission) in this context as generalist organizations. The fact that they cover all the competences has several consequences: (a) whatever the dimension threshold considered, the complex has only one connected component (the vector of structure is a all-one vector; it is also the case with \( K_{X_{\text{global}}}^{\text{global}} \) and \( K_{X_{Mekong}}^{\text{Mekong}} \) for the same reason; Table 3); (b) all other organizations are faces, of the FAO-MRC simplex with no dispersion of the organizations \( G_{SI} = 0 \); (c) the eccentricity of all organizations of \( K_{X_{\text{all}}}^{\text{all}} \) is zero; (d) the dimension values of the simplices and their meso-index of size \( M_{SI} \) are congruent (provide the same information) as seen in Figure 2. Overall, the \( K_{X_{\text{all}}}^{\text{all}} \) simplicial complex is very similar to the ideal type of vertical integration \( K_{X_{\text{vertical}}}^{\text{vertical}} \).
No competence to solicit all organizations, the structure of the conjugate complex $K^{all}_Y$ is less homogeneous. Two factors contribute to a high value of the mesoindex of size: a high number of vertices connected by q-geodesics of maximum length and in addition that this degree q is low—see Eq. (5). This is the case of simplices $\mathcal{H}$ (human health), $\mathcal{R}$ (risk assessment) and $\mathcal{B}$ (biodiversity) (Figure 3). The eccentricity varies according to the competence considered. In this context of governance, the skill regarding biodiversity is more eccentric, less integrated to the set of other competences. Indeed $\mathcal{R}$ and $\mathcal{B}$ are of the same dimension (18) but the degrees of q-connectivity are $q = 14$ for $\mathcal{R}$ and $q = 10$ for $\mathcal{B}$; $\mathcal{H}$ is of dimension 19 and of higher q-connectivity $q = 14$.

The structure vector $Q^{all}_Y$ also contains very useful information. It indicates the number of co-existing cliques when only the simplices of a dimension greater or equal to a threshold dimension are maintained. In the case of $K^{all}_Y$, the cliques are represented in the Q-analysis tree in Figure 4. The lower dimensional simplices (disappearing first from the tree of cliques) are land use ($\mathcal{L}$), then animal health ($\mathcal{A}$) and ecosystem health ($\mathcal{E}$). $\mathcal{A}$ and $\mathcal{E}$ are also the first simplices to dissociate from the main clique. These properties show that animal health and

![Table 3](image)

Table 3. Global indices and structure vectors of various complexes corresponding to empirical types of health-environment governance.

![Figure 2](image)

Figure 2. Values of the dimension + 1 (diamonds) and meso-index of size $M_{31}$ (squares) for the simplices (organizations) of $K^{all}_Y$. The values of the meso-index of size obtained by considering each organization group separately (global, ASEAN, SEAMEO, organizations of the Mekong Basin, Asia-Pacific organizations—see Table 1) are also represented (triangles).
ecosystem health skills are the least well integrated in this context of regional governance, while their integration with human health is central to the One Health approach. Similarly, land use skills are very important—especially if they are linked to epidemiological competences—the life cycle of several vectors and pathogens being influenced by land use and land cover changes [36]. Finally, the clique of competences that we can classify under the label environmental changes (climate change, biodiversity, water resources, food security) also disassociates quite quickly (in dimension 10), revealing an institutional gap between these competencies (in this context again).

The Q-analysis can be done by considering in turn each subgroup of organizations—global organizations, ASEAN, SEAMEO, Mekong Basin and Asia-Pacific organizations (see Table 1). The trees showing the fragmentation of the competence cliques with the increase of the

![Figure 3. Number of vertices (diamonds), meso-size index $M_{3l} \times 100$ (squares) and eccentricity $\eta \times 100$ (triangles) of the complex of competences $K^{all}_Y$.](image)

![Figure 4. Q-analysis tree of $K^{all}_Y$: clique of competences (with labels given in Table 1) as a function of the threshold dimension $q$, with structure vector $Q(K^{all}_Y) = [1, 1, 1, 1, 1, 2, 3, 2, 1, 2, 4, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 1]$. The threshold dimension $q$ is indicated in the bottom boxes.](image)
threshold dimension are very different from each other (Figure 5) and do not make it possible to infer a priori that which results from their association in Figure 4. At the beginning, each group presents all the competences distributed among its member organizations (except ASEAN without competence in environmental health). But according to the organizations involved, each competence is more or less shared in the group. The main ones (at the top of the trees) will tend to promote the associated theme as one that federates the activities of the organization group: risk assessment for global organizations, human health for Asia-Pacific and SEAMEO, the importance of climate change for ASEAN, etc. Groups with the most member organizations tend to have higher competence trees (5 for Asia-Pacific). It is also observed that although the Global and Mekong groups have each a generalist organization (FAO and MRC respectively), the competence cliques are not comparable.

Of course, the association of all these groups produces a higher clique tree ($q = 19$, Figure 4), with a more robust network of competences with respect to a change of skill, or even the discontinuance of an organization. The integration of groups in the regional governance system has differentiated effects for each organization. In Figure 2, it is observed, for example, that the meso-index of size $MS_I$ decreases for the MRC generalist organization, whereas it increases for the FAO. GEOBON’s relative size decreases with this integration, whereas that of APBON (both dedicated to the management of biodiversity observations) remains unchanged. The competence portfolio (and hence the number of vertices) remains unchanged by the integration of organizations, thus any change in the size meso-index reflects the

![Q-analysis tree of cliques of competences considered separately for each group of organizations (labels in the rectangular boxes at the bottom). The threshold dimension $q$ is indicated for each level.](image)

**Figure 5.** Q-analysis tree of cliques of competences considered separately for each group of organizations (labels in the rectangular boxes at the bottom). The threshold dimension $q$ is indicated for each level.
modifications of the q-paths, the low-dimensional ones being weighted more in this index (cf. Eq. (5)).

Without going into details, the tree of the cliques of organizations obtained according to the threshold dimension is less interesting in this context than that of competences presented in Figure 5. Indeed, the tree associated with each organization group resembles more or less that corresponding to another ideal type. This one, which we call *pyramidal* ideal type (inverted), is composed of \( n = 1 \ldots N \) organizations, the \( n \)th having \( (N - n + 1) \) competencies. At the top there is a generalist organization and at the bottom an organization presenting only one competence. In this situation, the tree of the cliques has only one trunk that loses an organization with each unit increment of the threshold dimension. The path indexes (Eq. (6)) of almost all organizations change with their integration in the larger “all” governance system as can be seen for the FAO and COBSEA (Coordinating Body on the Seas of East Asia) organizations and for the strategic policy program ASEAN2025 [35] in Figure 6.

The change in the \( P_q \) path indexes expresses the fact that in general the integration of an organization in a large governance systems multiplies the q-paths and the opportunities to find some potential partners with similar competences and interest in common themes. Of course, a generalist organization like FAO takes maximum advantage of such integration. But it is also interesting to see that a political strategy as expressed by the ASEAN countries in their ASEAN 2025 policy [35] offers new perspectives and new connectivity, when considered in a broader governance context. Similarly, an organization like COBSEA, focused on issues related

![Figure 6](image-url)
to the management of regional seas and marine resources and environments, is to be reconsidered in the larger governance system, as it is true that the relative position that each occupies depends closely on the context taken into account. All the information produced by Q-analysis is not exploited here, but position and importance of attributes of each entity—actors (e.g. organizations) and framework for action (policy strategy, legal instruments, etc.)—can be analyzed according to different governance contexts where it integrates or wishes to integrate.

7. Discussion

The mathematical concepts used in this article remain elementary, but it is important to note that the two conjugate complexes associated with the same relation, even though they have generally very different combinatorial appearances, share strong topological properties. From the mathematical point of view, this is reflected in the identity of their homology groups and their homotopy groups [37, 38]. This goes well beyond the elementary considerations to which we limit ourselves here in our modeling but the identity of these topological characteristics reinforces the importance of the principle of conjugacy between the two simplicial complexes naturally associated with a given relation.

For governance studies, the interest of such an approach is that it allows understanding very different contexts of governance by describing the actors and organizations already into action and the way they connect to each other. Ultimately, it also makes possible to delineate the institutions and issues at stake and to highlight the different levels of decision-making and thus of regulations involved. It can apply in various settings. For instance, one of the issues underlined by Osofsky [1] in the case of the environmental disaster resulting from the BP Deepwater Horizon oil spill is the need for integration across scales. The spill stretched over the shoreline of five states of the United States, and due to the multiplicity of decision levels (local and federal governments) and the variety of institutions involved (such as the Department of Agriculture, Department of Defense, Department of Energy, Department of Homeland Security, Department of Justice, Department of the Interior, Department of Labor, Environmental Public Agency, Health and Human Services or National Aeronautics and Space Administration ...), one of the legal difficulties was to disentangle the overlaps of regulations or on the contrary the gaps resulting from the legal fragmentation.

The approach can thus be used in this kind of context or either to determine in a specific area, like the Southeast Asian region, how the health and environmental governance works to identify the missing linkages or the possibilities for synergies. It is a flexible approach and the results and their interpretations are depending on the context chosen as well as on the organizations, networks and themes considered in the research scope. This flexibility can be seen not only as a limit of the approach but also as an advantage as it allows to change the analysis framework: in a first phase, we could choose to consider a specific type of organizations (in a predetermined typology) and thus extend the research to other types of organizations. It is
particularly relevant when it comes to describing and interpreting multidimensional and multilevel interactions.

The modeling approach is also very useful when governance systems are composed of hundreds of organizations and tens of attributes or when the ambition is to simulate the impact of changes of governance structure through scenarios. System wide indexes (global indexes), local indexes attached to each organization or attribute and meso-indexes assessing how they are inserted are exploited not only to construct global governance diagnoses but also to follow each entity in the evolving governance architecture.

On a semantic point of view, the use of the term *model* itself in the legal or political arena is different than in mathematics, physics or computer sciences. This can have methodological repercussions, as the term “model” can be used to define a descriptive approach closer to an enumeration of facts than to a systemic approach. Indeed, when speaking about models of governance, legal scholars usually refers to an analytical or normative framework rather than to a model integrating interactions and showing a dynamic expressed through mathematical properties translating types of behaviors or linkages. Nevertheless, this type of formal model opens the perspective of many analyzes of real systems of governance seen from new and diversified angles.

### 8. Conclusion

We have enriched our analytical tools with another approach to modeling systemic governance based on Q-analysis and using the simplicial complexes as a mathematical object (type of hypergraph or hyper-network). The model allows taking into account a variety of entities as elements of governance, say organizations, networks (of networks) of organizations, technical platforms, but also legal instruments (e.g. norms, agreements) and public policies. Since these entities can be characterized in different ways, modeling leads us to consider governance under as many different angles as there are types of attributes associated with entities.

The simplicial complexes introduce formal concepts of dimension and conjugacy between the hyper-network of entities (e.g. organizations) and the hyper-network formed by a choice of attributes, the two simplicial complexes being bound by topological properties. Several indicators are evaluated to characterize the global (overall), mesoscale and local (at the scale of each organization or attribute) properties of each of the two conjugated complexes associated with a given context of governance. Moreover, these indicators also make it possible to compare distinct systems of governance. Thus, we have also established the indices associated with several ideal type of governance that stylizes limit situations between organizations (or other entities): complete independence, full interdependence, vertical integration and horizontal integration and cyclic governance. The flexibility of the analytical tool makes it suitable for exploring a wide variety of governance systems, the case discussed in more detail here considering groups of organizations involved in South-east Asia on health-environment issues.
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Author details

Pierre Mazzega¹*, Claire Lajaunie² and Etienne Fieux³

*Address all correspondence to: pierre.mazzegaciamp@get.omp.eu

1 Geosciences Environment Toulouse UMR5563, CNRS, University of Toulouse, Toulouse, France

2 INSERM DICE-Ceric UMR7318—International, Comparative and European Law, Aix-Marseille University, Aix en Provence, France

3 Institute of Mathematics of Toulouse UMR 5219, University of Toulouse, Toulouse, France

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