Percolation and Galam Theory of Minority Opinion Spreading

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Abstract: The way in which an opinion rejecting reform can finally become the consensus of everyone was studied by Galam (2002) in a probabilistic model. We now replace his clusters by those formed via random percolation by letting particles diffuse on a lattice. Galam's rejection of reform is reproduced below the percolation threshold, whereas at and above the threshold, also approval of a reform becomes possible.

Keywords: Monte Carlo, fixed point, cluster, majority rule, $d$ dimensions

In principle, the laws of a society should be based on the opinion of the majority. However, to protect minorities or to ensure stability, constitutions and other basic laws allow some changes only with a majority far above 50 percent, and pose other hindrances. For example, in the USA the equal-rights amendment to the constitution failed because not enough states did ratify it in the allotted time. Galam [1,2], however, pointed out that even for normal reforms which require only a majority of 50.01 percent for approval, this reform, due to the step-wise aspects of some opinion-forming and legislative processes, is always blocked even though initially half of the people or more may be in favour. This blocking of change comes from the widespread simple rule that a tie vote means a “no”. We confirm it here in a more realistic computer simulation.

If in a democracy based on majority vote, a tie between yes and now means the continuation of the status quo and the rejection of the reform, then in some cases according to Galam [1,2] reform can become impossible even if initially about half of the voters are in favour of reform. This happens [2] if the vote proceeds in a hierarchical form, in which each group selects one representative who represents at the next-higher level the majority opinion of the group, and who votes for status quo if the represented group was evenly divided. In a less artificial way, recently Galam [2] divided the population of $L$ individuals into groups groups of various sizes $i$, with an arbitrary probability distribution function for $i$. Starting from a random distribution of opinions, with 50 percent probability in favour, each group determines its majority (biased by giving “no” in the case of a tie vote) and gives one common vote
in an overall referendum. The resulting fraction of group yes votes is the 
input for the probability of individuals to vote yes at the next time step. This procedure is iterated for several time steps until a consensus is reached, and this consensus is always a “no”. (See refs.3 for recent papers citing other consensus models, and refs.4 for other examples of sociophysics.)

Now we replace the general and fixed group size distribution by the 
dynamical one resulting from $x L^d$ individuals diffusing randomly on a $d$-
dimensional hypercubic lattice of $L^d$ sites), and we take as groups the clusters 
of nearest neighbours forming in this way. Thus instead of a whole group size 
distribution we have just one concentration $x$ which determines the initial 
cluster size distribution as in percolation theory. Initially, the individuals 
favor “yes” with probability $p$ and no with probability $1 - p$. Thereafter, 
the diffusion process changes these clusters; in every time step, on average 
every particle tries to move once to an empty neighbour site, and then the 
biased majority vote is taken. All individuals of each cluster adopt the ma-
jority opinion. This adoption finishes this time step; afterwards the next 
time step starts with the diffusion of individuals (each individual keeps its 
opinion when diffusing).

The bias against reform is strongest for small clusters where ties are 
frequent. Large clusters favor yes and no with nearly equal probability. Thus 
we first take the concentration $x$ such that the number of isolated pairs 
is maximal: $x = 1/2d$. Then on the square lattice $(d = 2, 23 \leq L \leq 
3001, p = 1/2)$ we found in all 20 samples that after about $10^2$ time steps 
everybody voted no. Fig.1 shows the average time to reach consensus to 
increase as $\log(L)$, perhaps due to the fact that the largest cluster size below 
the percolation threshold increases as $\log(L)$.

With instead $x$ equal to the percolation threshold 0.59, the results were 
mixed about half and half; the border between blocking of all reforms and 
equal chance for blocking or success seems to be near $x = 0.53$ (smaller for 
small lattices). At $x = 1/4$, $L = 1001$ the initial opinion must be larger than 
0.9999 in favor of reform to give it a chance. Also in higher dimensions at 
$x = 1/2d$ with $L = 23$, $d = 3, 4, 5$, reform was always blocked.

In summary, we confirmed qualitatively the conclusion of Galam that 
reforms can be blocked by biased majority voting; but quantitatively our 
times to reach this consensus are much longer than the examples given by 
Galam, and increase logarithmically with system size.

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Figure 1: Logarithmic increase of the time needed to reach the negative consensus, versus lattice size in two dimensions.

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