Cosmological abundances of right-handed Dirac neutrinos

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The equilibration of the right-helicity states $\nu_+$ of light Dirac-neutrinos is discussed. I point out that the $\nu_+$ production rate is enhanced by weak gauge boson pole effects so that the right-helicity component of $\nu_\tau$ is brought into equilibrium at $T \approx 10$ GeV independently of the initial abundance, provided $m_{\nu_\tau} \gtrsim 10$ keV. Neutrino spin flip in primordial magnetic fields and the resulting bound on $\mu_\nu$ is also discussed.

1 Relic abundances of right-helicity neutrinos

Primordial nucleosynthesis is a remarkable probe of neutrino properties. To some extent primordial nucleosynthesis could be sensitive even to the Dirac vs. Majorana nature of neutrinos, because in the Dirac case the small relic abundance of the inert right-handed component of Dirac neutrino would also contribute at nucleosynthesis. Of course, presently one cannot hope to differentiate between the Dirac and Majorana nature of neutrinos on cosmological grounds, but in principle this is an interesting problem. The right-helicity states of Dirac neutrinos can be produced (and destroyed) in spin-flip transitions induced by the Dirac mass or the neutrino magnetic moment. Spin-flip transitions may also be induced by primordial magnetic fields, if such exist.

The actual cosmological density of the right-helicity neutrinos depends not only on the production rate near the QCD phase transition, but also on whether the right-helicity neutrinos had a chance to equilibrate at some point during the course of the evolution of the universe. This depends on the thermal scattering rates of neutrinos. An important source of $\nu_+$'s are also the non-equilibrium neutrino scatterings and decays of pions. Such processes give rise to the bound $m_{\nu_\tau} \lesssim 130$ keV and $m_{\nu_\tau} \lesssim 150$ keV, using $T_{\text{QCD}} = 100$ MeV and assuming that nucleosynthesis allows less than 0.3 extra neutrino families. Let us note that there seems to be no window of opportunity for a sufficiently stable ($\tau_\nu \gtrsim 10^2$ sec) tau neutrino in the MeV region because of the production of non-equilibrium electron neutrinos in $\nu_\tau\bar{\nu}_\tau$ annihilations, which would disrupt the successful nucleosynthesis predictions.

In the Standard Model there are 68 purely fermionic $2 \rightarrow 2$ processes in which a right-helicity muon or tau neutrino can be produced. In addition, a right-helicity tau neutrino can also be produced in 11 lepton and quark three-body decays, and the muon neutrino in another set of 11 three-body decays. In principle, one has also to consider processes involving $W^\pm$, $Z$ and $H$. There are 16 such processes. Finally,
there are 3 two-body decays of $W^\pm$, $Z$ and $H$ bosons which are capable producing right-helicity muon and tau neutrinos.

The thermally averaged production rate in $2 \to 2$ scattering $a + b \to \nu_+ + d$ per one right-helicity neutrino $\nu_+$ is

$$\Gamma_+ = \frac{1}{n_{\nu_+}^{FD}} \int d\Pi_a d\Pi_b d\Pi_+ d\Pi_d (2\pi)^4 \delta^4 (p_a + p_b - p_+ - p_d) S |\mathcal{M}_{ab \to +d}|^2 \times f_a^{FD} f_b^{FD} (1 - f_+^{FD}) (1 - f_d^{FD}) ,$$

(1)

where $n_{\nu_+}^{FD}$ is the the equilibrium number density of the right-handed neutrinos, $d\Pi_i \equiv d^3 p_i / ((2\pi)^3 2 E_i)$, $S$ is the symmetry factor taking into account identical particles in the initial and/or final states, and $f_i^{FD}$ are Fermi-Dirac distribution functions. The fermionic processes exhibit an enhancement of the $\nu_+$ production rate which is due to gauge boson pole effects\(^6\). This is demonstrated in Fig. 1, where the enhancement is apparent in the s-channel process $u \overline{d} \to \nu_+ \tau^+$, where $\nu_+$ is a right helicity tau neutrino, as compared with the crossed t-channel process. It turns out that in the temperature of interest, the processes involving gauge or Higgs bosons can be neglected in comparison with the purely fermionic processes. This is demonstrated in Fig. 1, where the thermally averaged rate for the bosonic process $\tau^- \gamma \to \nu_+ W^-$ is shown.

Let us note that at high $T$ the rate Eq. (1) is infrared sensitive to the thermal corrections in the propagators. In most cases it is an excellent approximation just to modify the propagators by introducing a Debye mass $M_i^2 (T) = \Pi_i (\omega, \mathbf{k} = 0)$, which may be approximated by $M_i^2 (T) \simeq M_i^2 + 0.1 T^2$ ($i = W, Z$).

The relic density of the right-helicity tau neutrinos can be found by solving the
Boltzmann equation
\[
\left( \frac{\partial}{\partial t} - H|p_+| \frac{\partial}{\partial |p_+|} \right) f_+ = \left( \frac{\partial f_+}{\partial t} \right)_{\text{coll}}.
\] (2)

Considering only purely fermionic $2 \rightarrow 2$ and $1 \rightarrow 3$ processes, we can write the collision term in the form
\[
\left( \frac{\partial f_+}{\partial R} \right)_{\text{coll}} = \left( C_{2 \rightarrow 2} + C_{1 \rightarrow 3} \right) (1 - f_+) - \left( C'_{2 \rightarrow 2} + C'_{1 \rightarrow 3} \right) f_+ ,
\] (3)

where the coefficients $C_I$ represent production and $C'_I$ destruction of $\nu_+$, with $I = 2 \rightarrow 2, 1 \rightarrow 3$. The explicit expressions for the quantities $C_{2 \rightarrow 2}$ and $C_{1 \rightarrow 3}$ are
\[
C_{2 \rightarrow 2}(|p_+|, R) = \sum_{\text{scatt}} \frac{1}{2E_+} \int d\Pi_a d\Pi_b d\Pi_d (2\pi)^4 \delta^{(4)}(p_a + p_b - p_+ - p_d) \\
\times |M_{a b e + d}|^2 f_a^{FD} f_b^{FD} (1 - f_d^{FD}) ,
\]
\[
C_{1 \rightarrow 3}(|p_+|, R) = \sum_{\text{dec}} \frac{1}{2E_+} \int d\Pi_f d\Pi_g d\Pi_h (2\pi)^4 \delta^{(4)}(p_f - p_g - p_+ - p_h) \\
\times |M_{f e + g + h}|^2 f_f^{FD} (1 - f_g^{FD})(1 - f_h^{FD}) .
\] (4)

The Boltzmann equation Eq. (2) can be solved numerically (by using e.g. 30 momentum bins). One can consider two extreme initial conditions for $\nu_+$ at $T \approx 100$ GeV: (i) full equilibrium with $f_+ = f_{eq}$ and (ii) complete decoupling with $f_+ = 0$. The evolution of the $\nu_+$ energy density $\rho_+$ for different masses and for the two different initial conditions is shown in Fig. 2. One observes that right-helicity tau neutrinos equilibrate independently of the initial condition provided $m_{\nu_\tau} > \sim 10$ keV. The relic density, in units of extra neutrino species $\Delta N_\nu$, is also shown in Fig. 2. For $m_{\nu_\tau} < \sim 10$ keV the relic density is very close to the expected value $\rho_+ / \rho_L = [g_*(T)/g_*(T_i)]^{1/3} \rho_+(T_i)/\rho_L(T_i)$, where $\rho_L$ is the density of left-handed neutrinos and $T_i$ is the initial temperature. For $m_{\nu_\tau} \gtrsim 10$ keV one would obtain a small extra effect at nucleosynthesis, independently of the initial abundance. The results are for $\nu_\tau$, but for $\nu_\mu$ the graphs would look very similar.

2 Effects of magnetic fields

The abundance of the right-helicity states is also sensitive to primordial magnetic fields. Large magnetic fields could be created by cosmological phase transitions at microscopic length scales. There are indications that the small scale field cascades into larger length scales because of hydromagnetic turbulence. Electrical conductivity in the early universe is high so that the field is frozen in the plasma. Typically, such a primordial magnetic field is random so that $\langle B \rangle = 0$. When a Dirac neutrino propagates through such magnetic field, its spin will precess. At the same time, the neutrino is subject to scattering and thermal corrections. Neutrino dispersion relations
Figure 2: (a) The evolution of the right-helicity tau neutrino relic density $\rho_+\rho_{FD}^{-1}$, in units of the equilibrium density $\rho_{FD}$; (b) the relic density of the right-helicity tau neutrino in units of extra neutrino species; the upper set of curves is for right-helicity neutrinos in equilibrium at $T \simeq 100$ GeV, the lower set corresponds to zero initial density.

In magnetized plasma have recently been carefully studied. The actual spin evolution is best described by a relativistic kinetic equation, and the result depends very much on whether the coherence length of $B$ is larger or smaller than the neutrino scattering length. Another unknown is how to average over the randomly varying magnetic field. In the case of neutrino propagation, the appropriate statistical procedure might be the line average. Purely phenomenologically, one can write

$$B_{\text{rms}} = B_0 \left( \frac{T}{T_0} \right)^2 \left( \frac{d}{L_0} \right)^p,$$

where $L_0$ is the coherence length of the magnetic field, $d$ is the distance scale and $p$ is unknown; for a line average, $p = 1/2$.

In the case of small-scale random magnetic field, neutrino forward scattering tends to depolarize the spin. If the field is completely uncorrelated at distances $d \gg L_0$ so that

$$\langle B(t)B(t_1) \rangle = B_{\text{rms}}^2 L_0 \delta(t - t_1),$$

one finds that the spin flip probability reads

$$P_{\nu_L \rightarrow \nu_R} = \frac{1}{2} \left( 1 - \exp(-\Gamma t) \right),$$

where the damping parameter $\Gamma$ is given by

$$\Gamma = \frac{8}{3} \mu^2 B_{\text{rms}}^2 L_0^2.$$
Setting $t = H^{-1}$ and requiring that $\Gamma < H$ at $T = T_{\text{QCD}}$ so that the right-helicity states are not in equilibrium below QCD phase transition, and in particular during nucleosynthesis, results in the bound

$$\mu_\nu B_{\text{rms}}(T_{\text{QCD}}, H^{-1}) \lesssim 6.7 \times 10^{-3} \mu_B G \left( \frac{L_W}{L_0} \right)^{1/2}.$$  \hspace{1cm} (9)

In order to translate this to a bound on $\mu_\nu$ one needs to assume something about the magnitude of $B_{\text{rms}}$ at the horizon scale at $T = T_{\text{QCD}}$. No reliable estimate exists at present time. However, the bound Eq. (9) could in principle be as restrictive as $\mu_\nu \lesssim 10^{-20} \mu_B$.

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