Modeling the Plasma Frequency for F2-Region Using Modified Chapman Function and NeQuick2 Model over Different Geographical Locations and Months

Abstract- This study aims to modeling the plasma frequency profile of the F2 region as a function of geographical location and month of the year. The most important model and function used are Chapman function and NeQuick2 model which have been defined both by exponential function. These models need some ionospheric parameters such as the critical frequency of F2 layer (foF2), maximum peak height (hmF2), semi thickness (ymF2), and the M factor (M (3000) F2). The results of these models are compared with the results of the International Reference Ionosphere (IRI) model. For north hemisphere, the results of Chapman function has great fit with the results of IRI2012 model for low and high latitudes. For southern hemisphere the MAPE has greater values at high latitudes and drops to low latitudes. For NeQuick model, MAPE has a periodic behavior with latitudes. The monthly mean of the MAPE of the results obtained by modeling the plasma frequency profile using Chapman function and NeQuick2 model equal 0.466 and 0.259. The analysis of the MAPE for ten months gives a best correlation between the MAPE and foF2.

Keywords- Chapman function, Electron Density, Empirical Model, F2- region., Ionosphere, NeQuick Model

1. Introduction

The modeling of the electron density for F2-region is challenging due to both limitations availability of measured data through all the world, and to the different specified functions that show these behaviors. Over the past decades, a range of approaches to F2-region ionospheric modeling and representation of the (Ne) variation over different locations and altitudes have been developed [1]. The electron density modeling at different locations, time and altitudes gives an important reliable HF radio propagation prediction. The solution of Boltzmann, continuity, energy, and momentum equations for electrons and ions lying on the theoretical modeling of the ionosphere [2]. The Parameterized ionospheric model (PIM) is a global model of the theoretical climatology of the ionosphere, which is a parameterization of the output from a combination of theoretical ionospheric models, including model for high latitudes with model for low and mid latitudes, enhanced by the empirical plasmasphere model [3]. Empirical modeling means the use of the real data obtained from different stations over the world wild and times, also, it is unable to predict the storm dynamics and abnormal variability.

The ionosphere can be divided into two divisions, bottomside ionosphere lies below the maximum peak height (hmF2) while the topside extending from hmF2 to an important height called upper transition height (UTH). This description is shown on figure 1 [1].

Figure 1 an illustration of the different regions into which the ionosphere can be divided [1]

The scientists, engineers, and educators need to an empirical model which uses a reliable data for a...
specific parameter which depends on the Sun-Earth relation [4].

The NeQuick is an ionospheric electron density model advanced at the Abdus Salam International Centre for Theoretical Physics (ICTP), and at the Institute for Geophysics, Astrophysics and Meteorology (IGAM) of the Uni. of Graz, Austria [5]. It is founded on the DGR “profiler” suggested by Di Giovanni and Radicella [6], thereafter adjusted by Radicella and Zhang [7] and is a quick run model for trans-ionospheric propagation applications. Moreover amortiation have been performed by Radicella and Leitinger [8]. A modified bottomside has been introduced by Leitinger, Zhang, and Radicella [9]. A modified topside has been suggested by Coïsson, Radicella, Leitinger and Nava [10]. All these efforts aimed to the developments of a new version of the model, have drove to the enforcement of the NeQuick2 [11]. A specific version of NeQuick has been adopted as Galileo Single-Frequency Ionospheric Correction algorithm and its the performance have been recently confirmed during In-Orbit Validation [12-14].

The analytic Chapman function is simpler to fit the electron density profiles [15, 16]. Therefore this function is used to represent the topside electron density using a various scale height that continuously with height [17, 18]. The dependence of a Chapman \( q \) or Chapman \( \beta \) layer on the varying scale height can closely match the observed topside distribution [16, 19, 20, 21]. Ezquier et al. [22, 23] used the Chapman function with atomic oxygen scale height to estimate the total electron content (TEC). Reinisch and Huang [24] step inside a Chapman function with a constant scale height obtained from the digital ionosonde to clarify the topside electron density profile. This method can also obtain sensible TEC assess up to 1000 km [25, 27]. Luan et al. [28] and Liu et al. [29] suggested a Chapman function with the scale height of the neutral atmosphere at the F2 peak height to calculate the topside electron density content in tantamount winds calculation. The Chapman \( q \) layer and the \( \beta \) layer have a 0.5 and 1.0 factor, so the supposition of \( q \) or \( \beta \) chapman layer with a same constant scale height may drive to considerable difference in producing the topside profile [30].

2. Mathematical Models

I. Base Point Model (BPM)

The base point model assumes two modified Chapman profile functions for modeling the whole ionosphere, bottomside and topside. The Chapman requires a specified points called base points. With this function the four parameters \((N_m, h_m, H, c)\) and the following modified Chapman expression are required [4, 31].

\[
N(h) = N_m F_2 \* \exp \left( c \left( 1 - z \* \exp \left( -z \right) \right) \right) 
\]

Where

- \( c \): The type coefficient (unitless)
- \( N_m F_2 \): The maximum electron density \( (m^3) \)
- \( N(z) \): The electron density at \( z \) \( (m^3) \)
- \( h_m \): The altitude where \( N(z) = N_m \), \( (Km) \)
- \( z \): The reduced height, \( (Km) \)

\[
z = \frac{h-h_m}{H}
\]

For the bottomside the following expression is assumed according to the Chiu model [6] with \( c=1 \) by:

\[
N(h)=N_m F_2 \* \exp(1-z-\exp(-z)) \quad (3)
\]

So, in eq. (3) only three parameters \((N_m, h_m \) and \( H)) \) are required. The parameters that largely determines the shape of the bottomside profile is the scale height \([1]\). This parameter can be calculated using a simplified empirical formula using a Chiu model [32] as shown below:

\[
H=0.2*h+40 \quad h \leq h_m F_2 
\]

The total electron content (TEC) of ionosphere which is obtained from the global positioning system (GPS) is used to calculate the vertical scale height [33].

II. The NeQuick2 Model

The NeQuick2 model is an empirical model of ionosphere that generates profile of electron density based on parameters extracted from ionograms [34]. Historically the NeQuick has to be considered as an evolution of the DGR profiler proposed by Di Giovanni and Radicella [6], and subsequently modified by Radicella and Zhang [7].

The F2-layer of the bottom side of the NeQuick2 can be expressed as [34]:

\[
N_{F2} = \frac{4+N_m F_2}{1+\exp \left( \frac{h-h_m F_2}{B2} \right)} \* \exp \left( \frac{h-h_m F_2}{B2} \right) 
\]

Where \( B2 \) is the thickness parameter and is given by:

\[
B2 = \frac{0.385 \times N_m F_2}{\left( \frac{dN}{dh} \right)_{max}} 
\]

\[
\left( \frac{dN}{dh} \right)_{max} : \text{The maximum value of the electron density derivative with respect to height (10}^9 \text{m}^{-3} \text{km}^{-1}).
\]

\( M \) (3000) \( F2 \): M-factor at 3000 (km). \( \text{foF2} \): Critical frequency of F2 layer (MHz) This maximum is computed from \( \text{foF2} \) and \( M \) (3000).
F2 values, using the empirical relation [35]:

\[ \ln \left( \frac{dn}{dn} \right)_{\text{max}} = -3.467 + 1.714 \times \ln(\text{foF2}) + 2.02 \times \ln(M(3000)F2) \]  

(7)

The plasma frequency is related to the electron density by the following equation [36]:

\[ f_N = 9 \times \sqrt{N_e} \text{ (m}^{-3}\text{)} \text{ (Hz)} \]  

(8)

3. Method of Calculation

The evaluation of the electron density profile of the F2 region using analytic functions shown above will be implemented firstly for nine different locations over the world in universal time, secondly for Jeju city (33.43° N and 126.3° E) over twelve months of 2015. The analytical functions need more than ionospheric parameters such as the critical frequency of the F2 region, foF2, the maximum peak height of F2 region, hmF2 and the M(3000)F2. These ionospheric parameters have been obtained using the real data of Global Ionospheric Observatory (GIRO) (http://giro.uml.edu/ionogrammovies/) which provides accurate specification of electron density in the earth's ionosphere at more than 60 locations in the world. Table 1 shows the geographical coordinates for study area and the main ionospheric parameters shown above associated with it taken for 15/2/2016. Table 2 shows the main ionospheric parameters for twelve months of 2015 taken from GIRO for Jeju city at midday hour.

4. Results and Discussions

Figures 2-6 show the plasma frequency height profiles of F2 region for different ten locations for midday hour using Eqs. (3, 5, 8) and data shown in Table 1 compared with data of IRI2012. It can be seen that the plasma frequency height profile calculated using the modified chapman function is more closely to the data by IRI2012, especially at the mid-latitudes, while the results of the NeQuick2 model is more slightly different from the IRI2012.

![Figure 2](image1.png)

(a) (b)

Figure 2 the plasma frequency height profile for (a) Port Stanley (b) Grahamstown city.

![Figure 3](image2.png)

(a) (b)

Figure 3 the plasma frequency height profile for (a) Boavista city (b) Guam city.
Figure 4 the plasma frequency height profile for (a) Melrose city (b) Beijing city

Figure 5 the plasma frequency height profile for (a) Jeju city (b) Chilton city

Figure 6 the plasma frequency height profile for (a) Moscow city (b) Tromso city
Table 1: the main ionospheric parameters for different locations for February 2016

| City       | Latitude | Longitude | Ionospheric Parameters |
|------------|----------|-----------|------------------------|
|            |          |           | foF2 MHz   | hmF2 Km | M(3000) |
| Port Stanley | -51.6   | 302.1     | 6.5        | 213     | 3.67    |
| Grahamstown | -33.3   | 26.5      | 9.8        | 301.9   | 2.88    |
| Boavista   | 2.8      | 299.3     | 7.45       | 237.2   | 3.55    |
| Guam       | 13.62    | 144.86    | 9.2        | 306     | 3.07    |
| Melrose    | 29.71    | 278       | 4.5        | 247.7   | 3.4     |
| Jeju       | 33.43    | 126.3     | 4.82       | 301.3   | 3.14    |
| Beijing    | 40.3     | 116.2     | 5.35       | 269.1   | 3.35    |
| Chilton    | 51.5     | 359.4     | 10.17      | 237.4   | 3.46    |
| Moscow     | 55.47    | 37.3      | 8.25       | 213.3   | 3.52    |
| Tromso     | 69.6     | 19.2      | 6.425      | 241.7   | 3.4     |

Table 2: the main ionospheric parameters for twelve months 2015 for Jeju city

| Month  | Ionospheric Parameters |
|--------|------------------------|
|        | foF2 MHz   | hmF2 Km | M(3000) | ymF2 Km |
| January| 4.325       | 262.6   | 3.39    | 42.2    |
| February| 5.325     | 326.4   | 3.00    | 82.9    |
| March  | 5.8        | 310.9   | 2.98    | N/A     |
| April  | N/A        | N/A     | N/A     | N/A     |
| May    | 8.45       | 377.1   | 2.68    | 119.2   |
| June   | 5.3        | N/A     | N/A     | N/A     |
| July   | 7.8        | 325     | 2.92    | 95.6    |
| August | 5.725      | 413.5   | 2.51    | 92.4    |
| September| 5.3       | 316.3   | 2.97    | 87.6    |
| October| 3.85       | 323.8   | 2.96    | 71      |
| November| 3.875     | 313.2   | 3.11    | 63.7    |
| December| 3.95       | 315.6   | 3.04    | 90      |

Figures 7-11 show the plasma frequency height profiles for Jeju city for ten months at midday hour using modified Chapman function and NeQuick2 model and data of Table 2 shown above compared with the results of the IRI2012 model.

It can be seen that the plasma frequency profile using the modified Chapman function for January, October, November and December has best results than for results of NeQuick2 model, which has more closely results with IRI2012 for February, March, May, July, August and September.

(a)                                                                                           (b)

Figure 7 the plasma frequency height profile for (a) January (b) February
To investigate the accuracy of modified Chapman function and NeQuick2 model, it is desirable to use an important statistical parameter called the mean absolute percent error (MAPE) which is given by [37]: Table 3 shows that the MAPE has greater values for the NeQuick2 model than for modified Chapman function, with mean values of MAPE equal 0.267 and 0.124 respectively. For north hemisphere, the Chapman function has great fit with the results of IRI2012 model for low and high latitudes while has great value of MAPE for mid latitude with 0.358. For southern hemisphere the MAPE has greater values at high latitudes and drops at low latitudes. For NeQuick2 model, MAPE has a periodic behavior with latitudes with minimum value at 40.3° N equals 0.187 and maximum value equals 0.379 at 2.8°N.

High MAPE = M \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Ne(R)-Ne (analytical)}{Ne(R)} \right| \quad 9

The results of equation (9) accomplished for the two cases shown above have been presented in Tables 3-4.
Table 3 shows that the MAPE has greater values for the NeQuick2 model than for modified Chapman function, with mean values of MAPE for NeQuick2 model results from the fact that its modeling must take into account the three regions not a separate regions. While the difference of modified Chapman function from the results of IRI2012 model came from the calculation of the scale height which control the bottomside shape of the plasma frequency profile as well as the selection of the input ionospheric parameters.

From Table 4 it is observed that the behavior of the MAPE is roughly identical with the behavior of the critical frequency for F2 region with positive correlation as shown in figures 12-13.

The MAPE has greatest value at May with 3.146 and 1.702 for both modified Chapman function and NeQuick2 model respectively, with critical frequency equal 8.45 MHz, where the minimum value of MAPE for Chapman and NeQuick2 models are 0.02 and 0.129, respectively. The mean value of MAPE for Chapman function is 0.735 while equals 0.394 for NeQuick2 model.
The monthly mean of the MAPE of the results obtained by modeling the plasma frequency profile using modified Chapman function and NeQuick2 model equal 0.466 and 0.259. The analysis of the MAPE for ten months gives a best correlation between the MAPE and foF2 as seen from figure 12.

5. Conclusions
The results of modeling the plasma frequency profile as a function of geographical location using the modified Chapman function are more accurate than results of the NeQuick2 model when comparing both results with the results of the IRI2012 model. For north hemisphere, the results of modified Chapman function has great fit with the results of IRI2012 model for low and high latitudes. For southern hemisphere the MAPE has greater values at high latitudes and drops at low latitudes. The MAPE of the results of NeQuick2 model has a periodic behavior with latitudes. Also, it is observed that the behavior of the MAPE is roughly has positive correlation with the behavior of the critical frequency for F2 region. For high ionosphere (denser ionosphere), both functions have a results more slightly from the real data.

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