A simple protocol for secure decoy-state quantum key distribution with a loosely controlled source

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Abstract

The method of decoy-state quantum key distribution (QKD) requests different intensities of light pulses. Existing theory has assumed exact control of intensities. Here we propose a simple protocol which is secure and efficient even there are errors in intensity control. In our protocol, decoy pulses and signal pulses are generated from the same father pulses with a two-value attenuation. Given the upper bound of fluctuation of the father pulses, our protocol is secure provided that the two-value attenuation is done exactly. We propose to use unbalanced beam-splitters for a stable attenuation. Given that the intensity error is bounded by ±5%, with the same key rate, our method can achieve a secure distance only 1 km shorter than that of an ideal protocol with exactly controlled source.

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Recently, some methods have been proposed for secure quantum key distribution (QKD) with coherent states where Alice randomly changes the intensity of her pulses among a few values (e.g., 3 values: 0, $\mu$ and $\mu'$) or infinite values and then she can verify the fraction of single-photon counts in the raw key. With this information, a secure final key can be distilled by using the separate theoretical results. So far a number of experiments on decoy-state QKD have been done, in optical fiber or in free space, in polarization space or with phase-coding. However, the existing theory of decoy-state method assumes the exact control of pulse intensities. A problem met in practice is how to carry out the decoy-state method efficiently given the inexact control of pulse intensity. As we have shown, actually, one can verify the single-photon counts rather efficiently with simple tomography even though the intensity fluctuations of each light pulses are large. However, doing it in that way Alice needs additional operation of tomography. Here we present a simpler protocol for decoy-state method QKD.

Here we shall consider the effects of inexact control of $\mu, \mu'$ in a decoy-state protocol requesting 3 intensities, $\{0, \mu, \mu'\}$. (Actually, all decoy-state protocols with a few intensities, e.g., 2 or 4 intensities have the same basic problem of inexact control of $\mu, \mu'$.) For clarity, let’s first consider an ideal protocol with exact intensity control, as shown in Fig. 1:a). At any time $t$, if Alice wants to send a pulse of intensity $\mu$ or $\mu'$, she first produces a father pulse of intensity $\Omega$. After that she attenuates the pulse by $A(t) = \mu/\Omega$ or $A(t) = \mu'/\Omega$ randomly and a pulse of intensity $\mu$ or $\mu'$ is produced randomly and sent out to Bob. In this ideal protocol, Both $\Omega$ and $A(t)$ are controlled exactly.

For practical use, we propose a similar protocol as shown in Fig. 1:b). Alice wants to produce an intensity $\Omega$ for the father pulse. She then takes the same random attenuations as that in the ideal protocol. Here we assume that Alice can control the attenuation factors of $A(t)$ exactly (either $\mu/\Omega$ or $\mu'/\Omega$) but she can not control $\Omega$ exactly. (To control the instantaneous attenuation $A(t)$ exactly we can use unbalanced beam-splitters as we are going to show.) At each time, she has actually produced intensities of $\{\Omega_t\}$ for the father pulses. Although we can never control the intensity exactly, by our currently existing technology, we can definitely control the intensity in a small range, say, e.g., controlling the fluctuation within $\pm5\%$ of $\Omega$. That is to say, Alice knows the upper-bound of $\{\Omega_t\}$. We denote such an upper bound value as $\Omega_M$. 


Given this upper-bound value $\Omega_M$, the set-up in Fig. 1:b) is equivalent to a virtual set-up as shown in Fig. 2:a). In the virtual protocol shown in Fig. 2:a), every time a father pulse of constant intensity $\Omega_M$ is first produced and then the pulse is attenuated randomly, with attenuation factors of $A'(t) = \frac{\Omega}{\Omega_M}$. Alice cannot control this $A'(t)$. After this attenuation, a pulse of intensity $\Omega_t$ is produced. Note that $A'(t)$ is independent of $\mu, \mu'$, since we can imagine that Alice decides to use $\mu$ or $\mu'$ after the attenuation $A'(t)$. Since all attenuators are inside Alice’s Lab., it makes no difference if Alice exchanges the order of attenuators $A(t)$ and $A'(t)$. The physical meaning of order exchange is that Alice first decides to use $\mu$ or $\mu'$ and then arrange the attenuation $A'(t)$ which is independent of Alice’s decision of using $\mu$ or $\mu'$. This is just the virtual protocol in Fig. 2:b). In Fig. 2:b), after the pulse passes $A(t)$ but before passes $A'(t)$, the intensity should be either $\tilde{\mu} = \frac{\mu}{\Omega_M} \Omega$ or $\tilde{\mu}' = \frac{\mu'}{\Omega_M} \Omega_M$ exactly. That is to say, during the virtual stage between $A(t)$ and $A'(t)$, the light intensities of each pulses are either exactly $\tilde{\mu}$ or exactly $\tilde{\mu}'$. But after a pulse passes through $A'(t)$, the intensity is changed to inexact values of $\mu_i$ or $\mu'_i$. For the security proof of the real set-up in Fig. 1:b), we show the following lemma first. **Lemma:** The set-up in Fig. 2:b) is unconditionally secure if Alice regards it as a 3-intensity decoy-state protocol with each light intensities being randomly chosen from $\{0, \tilde{\mu}, \tilde{\mu}'\}$.

**Proof:** First we suppose Eve controls $A'(t)$. The dashed square can be regarded as an exact source for a decoy-state protocol using intensities $0, \tilde{\mu}, \tilde{\mu}'$. As it has been known already, decoy-state method with exact intensity control is secure given whatever channel. Here what Eve can do is first using $A'(t)$ for attenuation and then do whatever. This is only a type of specific channel therefore cannot be used to cheat Alice and Bob. In the set-up of Fig. 2:b), actually the attenuator $A'(t)$ is not controlled by Eve, definitely the set-up is secure because Eve cannot attack the protocol better with her power being reduced. Alternatively, we imagine that $A'(t)$ is controlled by Alice’s friend, Clare. If Alice choose to disregard Clare’s existence then to Alice this is just a decoy-state protocol with exactly controlled intensities of $\tilde{\mu}, \tilde{\mu}'$.

Given this lemma, we immediately have the **Theorem:** The set-up shown in Fig. 1:b) is unconditionally secure even though there are intensity fluctuations in values $\mu, \mu'$ if: 1) the values of $\{\Omega_t\}$ are upper-bounded by $\Omega_M$; 2) attenuation $A(t)$ is exactly controlled; 3) Alice assumes that she had used exact intensities of $\{0, \mu \Omega_M, \mu' \Omega_M\}$ in calculating the fraction of single-photon counts and distilling the final key.
The proof is simply that the final light pulses produced in Fig. 1:b) and final light pulses produced in Fig. 2:b) are identical. While the scheme in Fig. 2:b) has been proven to be secure by our Lemma.

The question remaining is then how to make stable attenuations $A(t)$ used above. We can realize $A(t)$ by beam-splitters as shown in Fig.(3). First, we attenuate each father pulses $(\Omega t)$ by a fixed attenuation factor of $A_0 = \frac{\mu + \mu'}{2}$. ($A_0$ can be realized by an unbalanced beam-splitter.) Second, after this $A_0$ attenuation, we split the beam by a $\mu : \mu'$ beam-splitter and then either the transmitted beam $b$ or the reflected beam $b'$ will be blocked and the other one will be guided to the optical fiber and sent to Bob.

Suppose the ideal protocol with parameters $\mu_e, \mu'_e$ would produce a good key rate in a certain experimental condition. Alice can try to use any intensities around $\mu_e, \mu'_e$ and then assumes that she had used larger intensities for security. One good choice is that Alice tries to produce \{0, $\mu_e/\lambda, \mu'_e/\lambda$\} with $\lambda = \Omega_M / \Omega$ for quantum communication with Bob and then assumes to have used \{0, $\mu_e, \mu'_e$\}. For simplicity, we shall only consider this option hereafter.

The actual efficiency of the protocol can be concluded by a real experiment. But we can still roughly estimate the efficiency theoretically on what should be found if we did the experiment. One can calculate the final key rate\[5, 16\] if he knows fraction of single-photon counts and the quantum bit error rate (QBER).

We consider the normal case where there is no Eve. and the channel transmittance is linear. We shall first compare the key rate of the following two protocols: our protocol with channel transmittance $\eta$ (protocol $P(\eta)$) and the ideal protocol with channel transmittance $\eta/\lambda$ (protocol $P_0(\eta/\lambda)$). In both protocols they will assume exact values 0, $\mu_e, \mu'_e$ in the calculation of single-photon counts and final key distillation. However, in protocol $P(\eta)$, Alice had done her best to produce 0, $\mu_e/\lambda, \mu'_e/\lambda$ for quantum communication. In both protocols, Alice and Bob can find the values of $S_{\mu_e} = 1 - e^{-\eta \mu_e/\lambda} + d_B$, $S_{\mu'_e} = 1 - e^{-\eta \mu'_e/\lambda} + d_B$ as the counting rates (yields) of pulses of intensities (or assumed intensities) of $\mu_e, \mu'_e$, respectively, where $d_B$ is the dark count rate of Bob’s detector. We have the following joint equations[2] to calculate the single-photon transmittance for both protocols:

\[
\begin{align*}
e^{-\mu_e} s_0 + \mu_e e^{-\mu_e} s_1 + c s_c &= S_{\mu_e}, \\
e^{-\mu'_e} s'_0 + \mu'_e e^{-\mu'_e} s'_1 + \left(\frac{\mu'_e}{\mu_e}\right)^2 e^{-\mu'_e} s'_c &\leq S_{\mu'_e}
\end{align*}
\]

Here $c = 1 - e^{-\mu_e} - \mu_e e^{-\mu_e}$. Parameters of $s_x$ are counting rates for states $|x\rangle\langle x|$ from $x$.
pulses \((x = 0, 1)\), \(s_c\) is the counting rates of state \(\rho_c\) (state of those multi-photon pulses) from \(\mu_e\) pulses. Parameters \(s'_c\) are counting rates of the same state as defined for \(s_x\), but they are for those states from \(\mu'_e\) pulses only. The values of \(s_0, s'_0\) can be deduced from the observed counting rate of those vacuum pulses. (For the two-intensity protocol \([5]\), one can assume \(s_0 = s'_0 = 0\) for the minimum key rate.) Asymptotically, \(s_x = s'_x, s_c = s'_c\). Given a finite number of pulses, \(s_x\) and \(s'_x\) can be a bit different but their possible range of difference can be bounded by classical statistics\([2, 3, 5]\) with exponential certainty therefore minimum values of \(s_1, s'_1\) can be calculated numerically. Since \(s_1, s'_1\) values for both protocols are calculated from the same equations with same parameters, the value of verified single-photon transmittance \((s_1, s'_1)\) of our protocol \(P(\eta)\) is equal to that of the ideal protocol \(P_0(\eta/\lambda)\). This indicates that the two protocols should have the same fraction of single-photon counts for each intensity.

We use notation \(E, E'\) for the observed error rates of pulses of intensity \(\mu_e, \mu'_e\), respectively. If they are determined by dark counts, alignment error and transmission error, these values and the deduced value of single-photon QBER in our protocol should be the same with (or a bit less than) those of the ideal protocol \(P_0(\eta/\lambda)\).

Given all these values requested for the final key distillation are the same for the two protocols, we conclude that that the key rate of our protocol \(P(\eta)\) is the same with that of the ideal protocol \(P_0(\eta/\lambda)\). In the case that the intensity fluctuation is bounded by 5% in our protocol (i.e., \(\lambda = 1.05\)) and the light intensity decreases by a half for every 15 kms in both protocols, the QKD distance of our protocol is only shorter than that of the ideal protocol by less than 1.06 km if we request the same key rate for two protocols. Similar calculation shows that even the maximum fluctuation is 20%, the shortened distance is less than 4 kms. This shows that the secure distance of currently existing experiments\([18, 19]\) would keep on exceeding 100 kms if they had used the proposed method here. Since currently existing experimental results\([18, 19]\) have not adopted our protocol (producing different intensities from the same laser device with attenuation), it should be interesting to redo the decoy-state QKD experiment using our protocol for a securer result.

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FIG. 1: a) The ideal protocol that Alice can produce constant intensity Ω for the father pulse therefore intensity μ, μ' are controlled exactly. b) The real protocol used in practice. At each time, Alice wants to produce intensity Ω for the father pulse, however, she actually produces {Ωₜ} at each time t. Consequently, the intensities of output pulses are {μᵢ}, {μᵢ'}. We assume that Alice can control the attenuator A(t) exactly in a real protocol. After a father pulse is produced, Alice randomly choose the attenuation factor by A(t) = μ/Ω or A(t) = μ'/Ω. Here the subscript t is from 1 to N + N', subscript i for {μᵢ} is from 1 to N, subscript i for {μᵢ'} is from 1 to N'.

FIG. 2: Equivalent virtual protocols. Our real protocol in Fig. 1:b) is equivalent to a virtual protocol as shown in part a) of this figure. Here Alice first produces a constant intensity Ωₘ for each father pulses and then attenuates each of them by attenuator A'(t). After A'(t), the pulse intensity is Ωₜ. It makes no difference to the output light if we exchange the order of A(t) and A'(t), therefore a) is equivalent to b). In part b), we can also regard the dashed square as our source and A'(t) as part of the channel.
FIG. 3: Realizing $A(t)$ by unbalanced beam-splitters. $A_0$ : a constant attenuator with attenuation factor $\mu + \mu'$, this can be realized by an unbalanced beam-splitter. BS: a $\mu : \mu'$ beam-splitter. OS: optical switcher. OF: optical fiber.