CAUSALITY AND SUPERLUMINAL FIELDS

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The expression of causality depends on an underlying choice of chronology. Since a chronology is provided by any Lorentzian metric in relativistic theories, there are as many expressions of causality as there are non-conformally related metrics over spacetime. Although tempting, a definitive choice of a preferred metric to which one may refer to is not satisfying. It would indeed be in great conflict with the spirit of general covariance. Moreover, a theory which appear to be non causal with respect to (hereafter, w.r.t) this metric, may well be causal w.r.t another metric. In a theory involving fields that propagate at different speeds (e.g. due to some spontaneous breaking of Lorentz invariance), spacetime is endowed with such a finite set of non-conformally related metrics. In that case one must look for a new notion of causality, such that 1. no particular metric is favored and 2. there is an unique answer to the question: “is the theory causal?”. This new causality is unique and defined w.r.t the metric drawing the wider cone in the tangent space of a given point of the manifold. Moreover, which metric defines the wider cone may depend on the location on spacetime. In that sense, superluminal fields are generically causal, provided that some other basic requirements are met.

1. Introduction

Many authors argue that superluminal fields are not causal\(^1\)–\(^4\) (but see Refs.\(^5\)). This is not true, unless one refers to an indefensible notion of causality. Indeed, as the notion of causality evolves from Newtonian dynamics to Special Relativity (SR), one must as well reconsider the notion of causality from Special or General Relativity (GR), in which spacetime is only endowed with the flat (resp. gravitational) metric, to the case where it is endowed with a finite set of Lorentzian metrics (notably then, if there are superluminal fields).

In this short communication based on the more detailed paper,\(^6\) we thus look for an expression of causality in such a multi-metric framework. The gravitational metric field is denoted by \(g\), and \(\mathcal{M}\) is a four-dimensional differentiable manifold.

2. Causality and chronology in field theories

The analysis of the notion of causality leads in particular to the following:

**Observation 1**: Since causes must precede effects, causally connected events must be time-ordered. Causality thus needs a notion of chronology to be expressed.

**Observation 2**: Any Lorentzian metric over \(\mathcal{M}\) defines a local chronology (in the tangent space), through the special relativistic notions of absolute future and past.

Gluing these two points together, we get the following

**Main point**: In relativistic field theories, there are *as many* notions of causality as there are non-conformally related metrics over \(\mathcal{M}\). These metrics \(h_i\) are the one along which the various fields \(\psi_i\) propagate, with velocities \(c_i \neq c_j, \forall i \neq j\).
This plurality of the notion of causality is the crucial feature of multi-metric theories. Indeed, it does not make any sense to assert that a given theory is -or not- causal, if one does not define to which metric (i.e. to which chronology) he refers to. A theory which appear to be non causal w.r.t some metric may be causal w.r.t another metric.

To face this issue, one may be tempted to assume that there exists a preferred metric field over $\mathcal{M}$. In other words, one may fix a preferred chronology and its associated causal structure. Most of the literature on causality and superluminal fields is based -often implicitly- on this first approach. In their famous textbook, Hawking and Ellis recognize explicitly that their notion of causality is defined w.r.t the gravitational metric. This constitutes a “postulate which sets the metric $g$ apart from the other fields on $\mathcal{M}$ and gives it its distinctive geometrical character” (p.60). As a consequence, fields that propagate faster than gravitons are not causal. Thus, “the null cones of the matter equations [must] coincide or lie within the null cone of the spacetime metric $g$” (p.255).

Although such an attitude does not pose any problem when spacetime is endowed with only one metric, as is the case of GR plus matter fields that couple to $g$, it becomes highly problematic in the multi-metric case. First, indeed, there is no way to find which metric should be favored, and which should not. Thus, by invoking causality, different authors may find opposite requirements on the theory.

Second, let us consider two fields $\psi_i$ propagating along the metrics $h_i$ ($i = 1, 2$), such that $\psi_2$ travels faster than $\psi_1$. Following the above reasoning, we can define causality w.r.t the metric $h_1$. Then two observers that are spacelike related w.r.t $h_1$ (and hence, non time-ordered) but timelike related w.r.t $h_2$ must be considered as causally disconnected, whereas they can interact thanks to the field $\psi_2$. The only way to avoid so an absurd conclusion is to define causality w.r.t to the metric that defines the wider cone in the tangent space (see below).

Third, any choice of a preferred metric is equivalent to a choice of preferred coordinates which, locally, diagonalize it. But the existence of preferred coordinates, or equivalently, of preferred rods and clocks, is in great conflict with the whole spirit of GR, namely diffeomorphism invariance; coordinates are meaningless in GR.

The above attitude is thus irrelevant in the multi-metric case. As an application, one should not invoke such a notion of causality to put constraints on the theory (notably in order to fix various signs), contrary to what is done in the literature.

3. An extended notion of causality and superluminal behaviors

There is only one relevant notion of (extended) chronology that does not refer to a given metric. This consists in defining the extended future of a point $P$ as the union of the futures of $P$ defined by each metric $h_i$. The corresponding (extended) notion of causality is thus in accordance with the notion of interaction. It is very
permissive in the sense that, by construction, any field theory is a priori causal provided that the various fields propagate along Lorentzian metrics, so that the (extended) spacelike region is never empty. Moreover, interactions cannot threaten this causal behavior, since, by construction, the extended future and past are defined at each point of $\mathcal{M}$. Which metric defines the wider cone may thus depend on the location on spacetime. In particular, superluminal fields are a priori causal.

Of course, this construction is not sufficient. Causality also requires, first, that the whole theory has an initial value formulation. This is generically the case if the field equations form a quasilinear, diagonal and second order hyperbolic system.\textsuperscript{6,7} Beware however that initial data must be assigned on hypersurfaces that are spacelike in the extended sense, that is spacelike w.r.t to all metrics $\mathbf{h}_i$. All the difficulties in the Cauchy problem of superluminal fields found in the literature arise from an irrelevant choice of initial data surfaces.\textsuperscript{1,2} Second, a local chronology is not enough. We must have at hand a global chronology over spacetime, in order to prevent, e.g. the existence of closed timelike curves. In the multi-metric case, we shall also require that our extended chronology is a global one, that is that no closed extended-timelike curves exist.

It has been shown\textsuperscript{2} that a particular superluminal scalar field may suffer from such a global pathology. This is however not enough to kill this theory, for the very reason that GR itself may suffer from such causal anomalies. Therefore, difficulties at a global level do not signal an intrinsic disease of superluminal fields. Rather, they originate from the fact that the global topology of the Universe is not imposed by local field equations. It is therefore necessary to assume that spacetime does not involve any closed (extended) timelike curves to ensure causality.

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