Investigating the magnetic inclination angle distribution of γ-ray-loud radio pulsars

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Abstract

Several studies have shown the distribution of pulsars’ magnetic inclination angles to be skewed towards low values compared with the distribution expected if the rotation and magnetic axes are placed randomly on the star. Here we focus on a sample of 28 γ-ray-detected pulsars using data taken as part of the Parkes telescope’s FERMI timing program. In doing so we find a preference in the sample for low magnetic inclination angles, $\alpha$, in stark contrast to both the expectation that the magnetic and rotation axes are orientated randomly at the birth of the pulsar and to γ-ray-emission-model-based expected biases. In this paper, after exploring potential explanations, we conclude that there are two possible causes of this preference, namely that low $\alpha$ values are intrinsic to the sample, or that the emission regions extend outside what is traditionally thought to be the open-field-line region in a way which is dependent on the magnetic inclination. Each possibility is expected to have important consequences, ranging from supernova physics to population studies of pulsars and considerations of the radio beaming fraction. We also present a simple conversion scheme between the observed and intrinsic magnetic inclinations which is valid under the assumption that the observed skew is not intrinsic and which can be applied to all existing measurements. We argue that extending the active field-line region will help to resolve the existing tension between emission geometries derived from radio polarisation measurements and those required to model γ-ray light curves.

Key words: pulsars: general – polarisation.

1 INTRODUCTION

The recent increase in the number of γ-ray pulsar detections, largely due to the FERMI satellite, allows great progress to be made in our understanding of these sources. A key question is that of the location and structure of the emission region. An important element required to test various models of this against observations is the determination of the “viewing geometry”, which describes how an observer’s line of sight samples a given pulsar’s magnetosphere. Knowledge of this for a particular pulsar allows predictions to be made corresponding to a given model of the γ-ray beam pattern, which can then be compared to observations in order to examine the models veracity. Based on radio polarisation data, viewing geometry constraints were presented by Rookyard et al. (2014) (henceforth RWJ14) for 28 pulsars which were included in the 2nd FERMI Large Area Telescope catalogue of γ-ray pulsars (Abdo et al. 2013). To this end, data taken predominantly at 1369 MHz using the Parkes radio telescope as part of the FERMI timing project (Weltevrede et al. 2010) were obtained. The aim of this timing project is to make regular monthly radio observations of a set of pulsars with a high energy loss rate $\dot{E}$ to construct timing models allowing to tag rotational phases to photons detected by the FERMI satellite. In RWJ14 these data were used to construct polarisation-calibrated averaged pulse profiles of the total, linearly polarised and circularly polarised intensity, along with profiles of the position angle (PA) of the linearly polarised component of the observed emission.

The viewing geometry is characterised by two angles, the magnetic inclination $\alpha$ (the angle between the rotation and magnetic axes) and the impact parameter $\beta$ (the angle between the line of sight and the magnetic axis at the closest approach). These parameters can be constrained by analysing the polarised radio emission and the pulse profile of the pulsar.

Fig. 1 shows the distribution of $\alpha$ values presented in Table 2 of RWJ14. For this figure and subsequent analysis $\alpha$ was mapped between 0 and 90° ($\alpha \to 180° - \alpha$ if $\alpha > 90°$) as there is no physical difference between these two regimes.

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As the pulsars in this sample are all γ-ray-detected, we would expect this selection effect to skew our observed distribution further towards higher values, by an amount which depends on the particular γ-ray model. Neither this nor the effect of the beaming fraction will be quantified further in this paper, but their effect on our results will be discussed.

An immediate question raised by the discrepancy between the observed distribution and our expectation is whether the α values should be expected to follow the birth distribution. It has been suggested that the magnetic axis should align with the rotation axis over time, meaning this assumption will only be valid for a sample of sufficiently young pulsars. However, Weltevrede & Johnston (2008) showed that for the pulsar population as a whole, if the axes are assumed to be randomly aligned at birth, the proportion of pulsars exhibiting an interpulse as a function of age is best reproduced by allowing the angle between them to decrease with a timescale $\sim 7 \times 10^3$ yr. Other estimates of this timescale are $\sim 10^5$ to $\sim 10^7$ yr (Young et al. 2010; Tauris & Manchester 1998). The highest characteristic age of any pulsar in this sample is $10^{-5.5}$ yr (PSR J1057–5226), which is significantly less than the alignment timescale. This indicates that the distribution for this sample should not have evolved significantly from the birth distribution. Further to this, it has recently been suggested (Lyne et al. 2013) that the magnetic axis of the Crab pulsar ($\tau_\gamma \approx 10^{5.1}$ yr) may be moving away from the rotation axis. If all very young pulsars undergo such a period of increasing α, this would strengthen the expectation that large α values should be favoured for the sample considered here.

In this paper we discuss various observational and other biases which could potentially cause the observed tendency towards low α values, including emission generated outside what is traditionally thought to be the open-field-line region, and find two possible explanations for the form the distribution takes. The first of these is simply that the magnetic and rotation axes are not randomly orientated at birth as expected, but instead neutron stars are more likely to be born with the axes aligned than orthogonal. Alternatively, the axes are randomly orientated and one of the assumptions used in the derivation of the α values is invalid. We argue that the most likely possibility is that the radio emission beam is larger than expected for a dipolar field by a proportion which is dependent on the magnetic inclination.

\section{Method to constrain the viewing geometry}

In the following text we give a summary of the process for constraining the viewing geometry, as applied in RWJ14 and other publications in which radio polarisation data are interpreted. This will form the mathematical basis for the discussion in this paper.

Information about the viewing geometry can be obtained by fitting the Rotating Vector Model (RVM; Radhakrishnan & Cooke 1963) to the observed PA curve of each pulsar. The RVM depends on $\alpha$ and $\beta$ and so the resulting $\chi^2$ surface in ($\alpha$, $\beta$) space constitutes an initial constraint on these parameters.

The viewing geometry can be constrained further by considering the effect of aberration and retardation (A/R;
This effect arises from relativistic motion of the emission region relative to the observer as the region corotates with the neutron star. The PA curve predicted by the RVM features a point of inflection which, neglecting the A/R effect, would be observed when the fiducial plane (the plane containing the two axes) passes the line of sight. However, the A/R effect results in a delay in pulse phase of the inflection point relative to the location of the fiducial plane inferred from the intensity profile of

\[ \Delta \phi = \frac{8 \pi h_{em}}{P_c}, \]

where \( P \) is the rotation period of the star and \( c \) is the speed of light (Blaskiewicz et al. 1991). The pulse phase at which the inflection point of the observed PA curve occurs follows from RVM fitting, and the location of the fiducial plane in terms of pulse phase can be judged somewhat subjectively using the profile morphology. The relative delay \( \Delta \phi \) then follows and so \( h_{em} \) can be determined.

With the emission height known, the half-opening-angle, \( \rho \), of the beam of radio emission can be found by assuming that the beam is bounded by tangents to the last-open-field lines of a dipolar magnetic field. Given this assumption, \( h_{em} \) yields the half-opening angle via

\[ \rho = \theta_{PC} + \arctan \left( \frac{1}{2} \tan \theta_{PC} \right), \]

where \( \theta_{PC} \), the angular radius of the open-field-line region, is given by

\[ \theta_{PC} = \arcsin \left( \sqrt{\frac{2 \pi h_{em}}{P_c}} \right) \]

(e.g., Lyne & Graham-Smith 2012). This beamwidth can finally be related to the range of rotational phase for which the line of sight samples the open-field-line region, \( W_{open} \), and the viewing geometry by

\[ \cos \rho = \cos \alpha \cos(\alpha + \beta) + \sin \alpha \sin(\alpha + \beta) \cos \left( \frac{W_{open}}{2} \right) \]

(Gil et al. 1984). The symmetry of the open-field-line region of a dipolar field means that the line of sight will sample open field lines for an equal amount of phase before and after the fiducial plane. In RWJ14 the fiducial plane was not necessarily chosen to correspond to the centre of the observed pulse, meaning that \( W_{open} \) cannot be assumed to be the same as the observed pulse width. Instead, it was taken to be twice the difference in phase between the fiducial plane position and the pulse edge furthest from it. The implications of this decision will be discussed in § 3.2. The pulse edges were in general taken to be the points at which the emission was 10% of the peak intensity. From the calculated values of \( \rho \) and \( W_{open} \), Eq. 4 can be used to constrain \( \alpha \) and \( \beta \). The best joint solution from RVM fitting and the pulse width considerations was defined as the favoured viewing geometry.

There are several possible reasons why an intrinsically sinusoidal distribution may appear skewed towards low \( \alpha \) values. These include

(i) a systematic underestimation of the phase of the PA curve inflection point;

(ii) a systematic overestimation of the phase of the fiducial plane;

(iii) a non-circularity of the emission region, and

(iv) extra-cap emission (in which the emission region is larger than expected for the open-field-line region of a simple dipolar magnetic field).

In the following subsections we consider each effect in turn and investigate whether it could sufficiently distort the \( \alpha \) distribution. We will argue that the most plausible solution which allows the \( \alpha \) distribution to be sinusoidal involves extra-cap emission for which the ratio of the beam radius to polar cap radius is \( \alpha \)-dependent.

3 CONSIDERATIONS OF POSSIBLE BIASES

3.1 Systematic underestimation of the inflection point position

The first effect we considered was a scenario in which the inflection point of the PA curve was systematically determined to be at an earlier pulse phase than its intrinsic location. Such a bias in the positioning of the inflection point could potentially be caused by scattering, which smears out the PA curve (Karastergiou 2003; Kramer & Johnston 2008). Another potential cause of a bias could be noise, which will be discussed later in this subsection. A systematic underestimation of the location of the inflection point would mean that the determined offset between the fiducial plane and the inflection point would be underestimated. Then, from Eqs. 5 and 6, the emission height and as a consequence the predicted beamwidth would be underestimated, leading to an underestimation of \( \alpha \) (Eq. 4).

To investigate the effect on the \( \alpha \) distribution the inflection point of each of the 25 pulsars which contributed to the sinusoidal distribution and the sinusoidal distribution. A lower test result indicates a greater likelihood
that the two sets of values are drawn from different parent distributions.

Fig. 2 shows the distribution obtained when an offset of $±2.2^\circ$ was applied. This was the magnitude of offset for which the distribution was found to most closely resemble a sinusoidal distribution, with a KS test result of 4.6% corresponding to a significance slightly above $2\sigma$. Such an offset is similar in magnitude to the effect of scattering observed by Kramer & Johnston (2008) in the case of PSR J0098–4913, suggesting the offset applied in the figure could potentially be explained by the effect of scattering in our sample. It can be seen that the peak at low $\alpha$ persists, accompanied by an excess of values relative to the sinusoidal distribution at $\alpha > 80^\circ$. The distribution was qualitatively similar even when offsets were considered which were too large to be explained by the amount of scattering exhibited in the profiles. Although the $\sim 2\sigma$ significance shows that the distribution is formally consistent with a sinusoidal distribution, it is somewhat marginal given that the match is not very good even after optimising the shift, which was a free parameter. Furthermore, the $\gamma$-ray selection effect and radio beaming fraction biases in our sample (see §1) are such that the observed distribution should be skewed towards high-$\alpha$ relative to the sinusoidal distribution. This cannot be reproduced by the shift. Hence we argue that, whilst scattering may play a role, it is not the main cause of the difference.

The effect of S/N on the determination of the inflection point was also investigated. The inflection point is typically at later phase than the midpoint of the profile, meaning there are more significant PA values before the inflection point. As noise contributes to the measured PA this asymmetry might, in principle, result in a bias in the determined inflection point. Polarised profiles were simulated according to the RVM using various combinations of S/N and offset between the inflection point and fiducial plane. White noise was added and the RVM was fitted to the resulting PA curve. In all cases the inflection point position was unaffected when averaged over a large number of simulations, indicating that noise does not cause a systematic bias. It is therefore apparent that if the intrinsic $\alpha$ distribution is sinusoidal the observed distribution cannot be explained by either interstellar scattering or the effects of noise on the PA swing.

### 3.2 Systematic overestimation of the fiducial plane

Another potential cause of an excess of low measured $\alpha$ values is a systematic overestimation of the phase at which the fiducial plane is located in the profile. In § 4 of RWJ14 the consequences of misplacing the fiducial plane were considered, but for single-component pulsars only; in this subsection we apply the same effect to the whole of the sample. It is important to consider this possibility as the fiducial plane of each pulsar was estimated subjectively, based on the profile shape.

There are several possible causes of such a systematic overestimation. Within the context of the core-cone model (Rankin 1993), such an effect could arise if, for example, the leading portion of the beam is less intense than the trailing portion, such that one or multiple leading components are unobserved. This could be an extremely pronounced version of the preferential illumination at later phases noted for young pulsars by Johnston & Weisberg (2006) and discussed in the context of cyclotron absorption byRussell et al. (2003). Alternatively the radio emission might not be generated in the ordered structure postulated by the core-cone model, but instead be generated in ‘patches’ distributed randomly across the open-field-line region (Lyne & Manchester 1988). It should be noted that a random distribution of patches does not explain the observed excess of low $\alpha$ values as a systematic bias is required (i.e., preferential illumination of the trailing side of the beam).

Another possible reason for an overestimation of the fiducial plane position would be that the centre of the radio beam (in either the core-cone or patchy beam model) trails the magnetic axis (that is, the fiducial plane is not at the centre of the range of phase described by $W_{\text{open}}$). However, in this situation only the relative offset of the fiducial plane and inflection point would be affected (the extent of the open-field-line region, and hence $W_{\text{open}}$, would be unaffected), making this possibility equivalent to the inflection point offsets considered in § 3.1.

The result of a systematic overestimation of the fiducial plane was first of all investigated by offsetting the fiducial plane determined in RWJ14 by a set amount of pulse phase and determining the resulting $\alpha$ distribution, using a method analogous to that described in §1. The distributions obtained using various offsets showed that as the offset becomes larger the increasing beamwidths cause the peak of the distribution to move towards higher $\alpha$. However, as the magnitude of the offset is increased further, the effect of the increasing inferred $W_{\text{open}}$ values becomes dominant and the peak of the distribution moves towards lower $\alpha$.

Fig. 3 shows the $\alpha$ distribution (solid bars) obtained with an offset of $–14^\circ$ applied to each pulsar. The data visibly match the sinusoidal distribution well, which was confirmed by a KS test result of 52%. This implies that if for every pulsar in our sample the fiducial plane is $14^\circ$ earlier than thought based on the profile morphology, this might explain the shape of the observed $\alpha$ distribution.
3.3 Emission height gradient

Gangadhara & Gupta (2001) proposed that, at a given frequency, the emission height of radiation could be greater further from the magnetic axis, and hence further from the centre of the profile. This means that the A/R effect will be most pronounced at the edges of the profile. It can be seen from Eqs. 2 and 3 that it is the emission height at the edge of the beam which leads to the overall half-opening angle of the beam. In RWJ14, however, the fiducial plane position was estimated according to the locations of component peaks, which are not at the edge of the beam and hence, according to Gangadhara & Gupta (2001), will have a lesser emission height. This will have caused an underestimation of $\Delta \phi$ and therefore a systematic bias towards late fiducial plane positions. It can be seen from Fig. 4 of Gangadhara & Gupta (2001) that for PSR B0329+54 (which has a period of 0.7 s) this effect is $\sim 4^\circ$. Considering Eq. 1 and noting that almost all the pulsars in our sample have periods less than 0.2 s, we could expect the underestimation of the fiducial plane to be greater for our sample. In addition, it is possible that high-$E$ pulsars emit over a more extended altitude range (Weltevrede & Johnston 2008), potentially increasing this effect.

The variation of the A/R effect across the profile will also move the ends of the PA curve towards later phase compared with the central region, which will distort the PA curve such that the leading half is made steeper while the trailing half is made more shallow. If not corrected for, this will cause a slight underestimation of the inflection point phase. However, as it is the ends of the PA curve at which the distortion is most pronounced, the curve will be less distorted close to the inflection point and so the observed position of the steepest gradient will change by only a small amount. The magnitude of this effect on $\phi_0$ varies between pulsars, but for this sample is expected to be typically $\sim 1^\circ$. Therefore, the effect on our fiducial plane estimate will dominate over this effect and any change to the inflection point position can be neglected.

An emission height gradient has the potential to make our observations consistent with a sinusoidal distribution. We can see from Eq. 1 that the $-14^\circ$ offset in the fiducial plane position suggested in § 3.2 could be caused by a 0.06$R_{LC}$ difference in the emission height, where $R_{LC}$ is the light cylinder radius. In other words, the points in the profiles which were used to determine the favoured position of the fiducial plane should have been emitted 6% of the light cylinder radius lower than were the edges of the pulse.

If an emission height gradient is the main cause of the observed tendency towards low $\alpha$ values, then choosing the fiducial plane position based on the pulse edges should lead to an $\alpha$ distribution which more closely resembles a sinusoid than Fig. 1. To test this, we obtained $\alpha$ values by positioning the fiducial plane at the midpoint of the observed pulse. For this we excluded pulsars for which we believed a significant fraction of the beam is not illuminated.

Despite its promise, the results of this test showed little change in the $\alpha$ values, with marginally more pulsars decreasing in $\alpha$ than increasing (as described in § 3.2 there are two competing effects acting on the $\alpha$ values). This indicates that, whilst we do not rule out an emission height gradient for these pulsars, such a gradient cannot explain the excess of low $\alpha$ values. This means that the potential $-14^\circ$ offset determined in the previous subsection would require some other physical justification.

3.4 Non-circularity of the emission region

Another possible cause of a systematic underestimation of $\alpha$ is the assumption that the beam of emission is confined to a circular open-field-line region. To investigate the effect of other beam shapes, a model was considered in which the emission region was elongated into an ellipse. The methodology to determine $\alpha$ for elliptical beams is analogous to the case of circular beams, except that the relevant half-opening angle in Eq. 1 is $\rho_{ell}$, which is a function of the magnetic longitude at which the line of sight enters and exits the beam. We define the beam such that the half-opening angle described by tangents to the last open field lines, $\rho$ (Eq. 2), forms the semi-minor axis of this ellipse. This makes $\rho_{ell}$ larger than the prediction from Eq. 2 regardless of the direction in which the beam is elongated. This systematic underestimation of the beam half opening angle as derived under the assumption of a circular beam causes a systematic underestimation of $\alpha$.

Two variants of this elliptical beam model were examined. In the first of these the major axis of the beam lay in the fiducial plane (i.e., the elongation was towards the rota-

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1 The pulsars used were PSRs J0631+1036, J0659+1414, J0729–1448, the interpulse of J0908–4913, J0940–5428, J1105–6107, J1112–6103, J1119–6127 (case (a)), J1420–6048, J1531–5610, J1648–4611, J1702–4128, J1709–4429 and J1718–3825. See RWJ14 for motivation of these choices.
be seen that the distribution tends towards higher values when the beam is elongated in the direction of rotation (e.g., the appendix in Weltevrede & Wright 2009). However, in that case the ellipticity of the beam and \( \psi \) is the magnetic longitude at which the line of sight enters and exits the beam, given by

\[
\sin(\psi_W) = \frac{\sin(\alpha + \beta) \sin(W_{\text{open}}/2)}{\sin(\rho_{\text{oil}})}
\]

(e.g., the appendix in Weltevrede & Wright 2009).

3.5 Extra-cap emission

The final considered way to make the \( \alpha \) distribution consistent with the sinusoidal distribution was “extra-cap” emission. In this situation the emission region is assumed to be circular, with a radius which exceeds the conventional polar cap radius by a factor \( s \). This is similar to the elliptical beam model in that the beam half opening angle from Eq. 2 is an underestimate which leads to an underestimation of \( \alpha \). Claims of extra-cap emission exist in the literature (Weltevrede & Wright 2009; Keith et al. 2010). In contrast, however, for PSR J0908–4913 Kramer & Johnston (2008) found \( s \leq 1 \). It therefore seems that \( s \) must differ between pulsars. Nevertheless, to compensate for the abundance of observed low \( \alpha \) values in this scenario requires \( s > 1 \) for the majority of the sample.

To test this possibility the beam half-opening angles were corrected by applying the transformation \( \theta_{PC} \rightarrow s \theta_{PC} \) to Eq. 2 New \( \alpha \) distributions were then determined using the same method detailed in § 4

Firstly \( \alpha \) distributions were obtained by applying the same \( s \) value to each pulsar. Fig. 3 shows the distribution corresponding to \( s = 1.18 \), which best fitted a sinusoidal distribution. The distributions are inconsistent, almost to the 3σ level (a KS test returns a result of 0.94%). The deficit of pulsars with 50° < \( \alpha < 70° \) cannot be compensated for in this scenario and also the optimum \( s \) value is relatively close to \( s = 1 \), indicating that the effect on the original \( \alpha \) distribution is marginal. At lower \( s \), the low \( \alpha \) peak shifted to lower values. When larger \( s \) was used the number of pulsars for which the larger beam size resulted in a constraint from the \( \alpha / \) effect which was inconsistent with the constraint from fitting of the RVM (in such cases \( \alpha \) was assumed to be 90° in line with the methodology discussed in § 4 of RWJ14). It is therefore apparent that the elliptical beam model cannot make the observed \( \alpha \) distribution consistent with a sinusoidal distribution.

Given the variation of \( s \) values reported in the literature, various \( s \) distributions were tested. For each test, \( s \) values were drawn randomly from the distribution and applied to the observations. The peak of the corresponding \( \alpha \) distributions occurred at higher \( \alpha \) when the mean \( s \) was higher and the \( \alpha \) distribution was narrower. However, even when a distribution with an unfeasibly high mean \( s = 10 \) was used, the resulting \( \alpha \) distribution retained a peak at \( \alpha = 60° \). This means that the \( \alpha \) distribution cannot be made to be consistent with the sinusoidal distribution by choosing \( s \) randomly from any distribution.

Also, consideration should be given to the \( \alpha \) distributions determined by Rankin (1990) and Gould (1993), which were analysed by Tauris & Manchester (1998). These \( \alpha \) values were calculated using empirically-determined relations between the component width and period with an assumed \( \alpha \) dependence consistent with Eq. 4 which did not rely on \( \alpha / \) effects or involve any direct assumption of \( s \). The derived empirical relations are therefore independent of \( s \), pro-
vided $s$ takes a constant value unrelated to $\alpha$. Given this, the existence of a similar skew in both Rankin’s and Gould’s $\alpha$ distributions provides more evidence that a simple scaling of $\rho$ via a parameter $s$ is not the full story. However, if $s$ is in some way related to the magnetic inclination, the $\alpha$-dependence of the component width will differ from that which was assumed by Rankin and by Gould. In this case their $\alpha$ values will have been similarly affected to the $\alpha$ values determined in RWJ14, potentially explaining the fact that the skew is apparent in all three samples. Therefore, it appears that if the observed excess of low $\alpha$ values is caused by the assumed $s$ values, then an $\alpha$-dependence of $s$ is required.

3.6 Dependence of extra-cap emission on $\alpha$

By choosing an optimum value of $s$ for each pulsar individually it is, by definition, possible to reproduce any desired $\alpha$ distribution. However, in § 3.5 we concluded that, if it is the cause of a systematic underestimation of $\alpha$, $s$ cannot be drawn randomly from a distribution and hence must either directly or indirectly depend on $\alpha$. To investigate what $\alpha$ dependence of $s$ would be able to reproduce a sinusoidal distribution, we made the assumption that the $\alpha$ values derived in RWJ14 are in the correct order. In other words, the pulsar with the lowest derived $\alpha$ was assumed to have the lowest intrinsic $\alpha$ of the sample and that there should be a monotonic relation between the intrinsic and measured $\alpha$. Keeping the pulsars in the same $\alpha$ order, an $s$ value was assigned to each pulsar such that the sinusoidal distribution was recreated. The upper panel of Fig. 6 shows the $s$ values required for this as a function of $\alpha_{\text{meas}}$, the measured $\alpha$ values as derived under the assumption that $s = 1$.

The trend in the upper panel of Fig. 6 is well fitted by an exponential function. The best exponential fit, which is also shown in the figure, is

$$s(\alpha_{\text{meas}}) = 2.0e^{-\alpha_{\text{meas}}/39^\circ} + 0.7.$$  

$$\text{(8)}$$

The errors on the fit parameters have not been quoted as the main source of error in this relation is likely to be unknown systematics. As expected, pulsars with lower $\alpha_{\text{meas}}$ values...
have, on average, larger $s$ values in order to reduce the excess of low $\alpha$ values. There are several possible reasons for the scatter of data points around this fit, such as uncertainties in the determination of $\alpha_{\text{meas}}$. Another possibility is that $s$ is not solely dependent on $\alpha$ but is also dependent on other parameters, thereby adding a random element.

It is possible to use this relationship between $\alpha_{\text{meas}}$ and $s$ to determine the intrinsic $\alpha$ for each pulsar. The $\alpha_{\text{meas}}$ value of each pulsar and Eq. [5] were used to predict $s$. This $s$ value was then applied to the pulsar to determine a “corrected” $\alpha$ value, $\alpha_{\text{intr}}$, which should represent the true value of $\alpha$. The middle panel in Fig. [6] shows $s$ plotted as a function of $\alpha_{\text{intr}}$, and the best linear fit to the data,

$$s(\alpha_{\text{intr}}) = -0.022\alpha_{\text{intr}} + 2.80,$$

where the errors have been discarded as in Eq. [5]. As noted above, any $\alpha$ distribution could be produced given the relevant set of $s$ values. However, it is encouraging (and non-trivial) that the $s$ values required to produce the sinusoidal distribution are a simple linear function of the pulsars’ intrinsic $\alpha$. In addition the half-opening angle of the beam is approximately that expected from the last open field lines of a dipolar field ($s = 1$) for orthogonal rotators.

The bottom panel in Fig. [6] contains the distribution of $\alpha_{\text{intr}}$ values, which shows a good match with the sinusoidal distribution. The match is confirmed by a KS test between these values and the sinusoidal distribution, which returns 66%. This is the highest KS test result of any scenario considered in this paper.

### 3.7 Extra-cap emission - comparison with individual pulsars

In RWJ14 three cases were found for which Eqs. [5] and [6] cannot be used as no consistent solutions were found under the assumption that $s = 1$ and hence $\alpha_{\text{meas}}$ could not be defined. This could be due to slight errors in the chosen fiducial plane or pulse edge position, or alternatively could be explained by a value $s < 1$. By calculating $\alpha_{\text{intr}}$ using the method outlined in § 2 as $s$ is varied, it is possible to plot a track for a given pulsar in $(\alpha_{\text{intr}}, s)$ space. The plotted track will only be consistent with the linear fit for a limited range of $\alpha$. Fig. [7] shows the tracks for these three pulsars, along with the best linear fit of $s(\alpha_{\text{intr}})$ and some of the data points from the middle panel of Fig. [6]. It can be seen that for these pulsars the tracks are consistent with the linear fit (within the scatter of data points) for $\alpha \gtrsim 80^\circ$. This suggests these are close to being orthogonal rotators, consistent with the conclusions of RWJ14.

A useful test of the relationship between $s$ and $\alpha$ comes from interpulse pulsars. The presence of both a main pulse and interpulse often allows a significant $s$-independent constraint on $\alpha$ from RVM fitting alone. Combining this with an estimate of $s$ allows these pulsars to be placed directly onto a plot of $s$ versus $\alpha$ and their consistency with Eq. [6] can be checked. Values of $s$ and $\alpha$ for five interpulse pulsars which are not in our sample can be found in Keith et al. (2010). These, along with PSRs J0908–4913 (Weltevrede & Johnston 2008) and the results for the two poles of PSR J1057–5226

$^2$ PSRs J0729–1448, J0742–2822 and J1105–6107.

**Figure 7.** The three curves show the relationship between $s$ and $\alpha$ for PSRs J0729–1448 (dashed curve), J0742–2822 (dotted curve) and J1105–6107 (dash-dotted curve). Data points from the middle panel of Fig. [6] have been included to indicate the magnitude of the scatter about the best linear fit (solid line).

**Figure 8.** Values of $\alpha$ and $s$ taken from the literature for PSR J0908–4913 (at $\alpha = 84^\circ$), PSR J1057–5226 from Weltevrede & Wright (2009) (at $\alpha = 75^\circ$) and five pulsars analysed by Keith et al. (2010). Main pulse and interpulse values are shown for each pulsar. The solid line is the same linear fit as shown in the middle panel of Fig. [6].
is desirable to find a relation between the measured and intrinsic values. These inferred values are those required to reproduce a sinusoidal distribution, assuming a monotonic relation with the measured inclination. An arctangential fit to the data is also shown.

4 DISCUSSION

In the previous sections we have shown that the distribution of $\alpha$ values presented in Rookyard et al. (2014) (RWJ14) differs significantly from the sinusoidal distribution one may expect for this sample of young, $\gamma$-ray-loud pulsars. We have also considered various biases which might be the cause of this difference. In this section we will first consider, in a model-independent way, the implications of a scenario in which the distribution is intrinsically sinusoidal for this sample. We devise a simple scheme to infer the intrinsic $\alpha$ value from observations which are affected by a bias which affects the measurements. We will then discuss the plausibility and implications of the two possible biases, a systematic misplacement of the fiducial plane and an $\alpha$-dependence of $s$, which have been found to be capable of reproducing an approximately sinusoidal distribution from the measured distribution. Finally we will consider the alternative scenario, in which the intrinsic $\alpha$ distribution is skewed to low $\alpha$ values compared to a sinusoidal distribution.

4.1 A model-independent relation between measured and intrinsic $\alpha$ in the case of random axis orientation

If the rotation and magnetic axes are randomly orientated, and hence the $\alpha$ distribution is intrinsically sinusoidal, it is desirable to find a relation between the measured and intrinsic $\alpha$ values. Fig. 9 shows $\alpha_{\text{intr}}$, as determined in §4.1, under the assumption that $s$ follows Eq. 9 as a function of $\alpha_{\text{meas}}$, as determined in RWJ14 under the assumption $s = 1$. However, it should be noted that any model used to recreate a sinusoidal distribution from the observed $\alpha$ values would give a similar diagram.

These data are relatively well fitted by an arctangential function. At high $\alpha$ the $\alpha_{\text{meas}}$ values are higher than the $\alpha_{\text{intr}}$ values. This is because $s$ is slightly smaller than 1 for orthogonal rotators according to Eq. 9. However, it follows from Eq. 9 that $\alpha_{\text{intr}}$ is more sensitive to $s$ (and, via Eq. 8, to errors on $\alpha_{\text{meas}}$) at values close to 90°. Therefore, accounting for the subsequent uncertainty in $\alpha_{\text{intr}}$, the data in the figure are consistent with a relation which passes through the point $\alpha_{\text{intr}} = \alpha_{\text{meas}} = 90°$. This is desirable in order to find a relation which allows for the prediction of orthogonal rotators. This was taken as a boundary condition for the fit. The best arctangential fit (shown in Fig. 9) was found to be

$$\alpha_{\text{intr}} = 1.27° \arctan\left(\frac{\alpha_{\text{meas}}}{31.25°}\right).$$

The scatter of data points demonstrates that there is significant uncertainty when applying this conversion to any particular pulsar. However, this equation can nevertheless be used as a population-averaged conversion between the measured and intrinsic $\alpha$ values. It should be noted that, although the scatter of data points can be used to estimate errors on the fit parameters, the total error is likely to be dominated by unknown systematics related to the determination of $\alpha_{\text{meas}}$. For this reason we have chosen to disregard the errors from Eq. (10).

The relation is effectively independent of the method used to determine $\alpha_{\text{intr}}$, as any correction to the measured $\alpha$ which results in a sinusoidal distribution will yield a similar set of values. The only assumption (other than that the intrinsic distribution is sinusoidal) to which the relation is sensitive is that higher measured $\alpha$ values in general correspond to higher intrinsic $\alpha$ values, or in other words that the PA curve contains at least some information about $\alpha$.

It should be noted that Eq. (10) is derived assuming an intrinsically sinusoidal $\alpha$ distribution. As discussed in §4.1, it is expected that due to the selection effects of increased $\gamma$-ray intensity modulation and radio beaming fraction when $\alpha$ is larger, this sample should contain more high-$\alpha$ values compared to a sinusoidal distribution. In this case, the relation between $\alpha_{\text{intr}}$ and $\alpha_{\text{meas}}$ would rise more steeply at low measured $\alpha$. Eq. (10) may still be used as a first order correction and the mentioned biases will make the intrinsic $\alpha$ values higher, making the correction presented here somewhat conservative.

A change in $\alpha$ will be accompanied by a change in $\beta$ in order to make the observed PA curve consistent with the RVM. However, it is possible, once $\alpha$ has been corrected, to correct $\beta$ accordingly. The two angles can be related to the maximum gradient of the measured PA curve via the equation

$$\left(\frac{d\psi}{d\phi}\right)_{\text{MAX}} = \frac{\sin \alpha}{\sin \beta}.$$  

(Komesaroff 1970). Use of this relation is justified as the gradient is, in most cases, the most constrained aspect of the RVM. The right-hand side should be identical in the cases of the measured and intrinsic viewing geometries, leading to

$$\sin \beta_{\text{intr}} = \frac{\sin \alpha_{\text{intr}} \sin \beta_{\text{meas}}}{\sin \alpha_{\text{meas}}}.$$  

As an example of this method of correction, Table 1 contains the viewing geometry constraints and favoured $\alpha$ and $\beta$ values corrected using Eqs. (10) and (12) from those given in Table 2 of RWJ14. Note that in all cases $\beta_{\text{intr}}$ is larger than $\beta_{\text{meas}}$. 

![Figure 9](image_url)
Table 1. The allowed and favoured viewing geometries for the sample after correcting the values (given in Table 2 of RWJ14) of $\alpha$ according to Eq. 10 and $\beta$ according to Eq. 12 (i.e., assuming the magnetic and rotation axes to be randomly orientated). MP and IP refer to $\alpha$ and $\beta$ values with respect to the main pulse and interpulse. In the case of PSR J1119–6127, the two cases (a) and (b) are described in RWJ14. It should be noted that values of $\alpha > 90^\circ$ have not been mapped into $0^\circ < \alpha < 90^\circ$.

| PSR          | Allowed Solutions | Favoured Solutions |
|--------------|-------------------|--------------------|
|              | $\alpha_{\text{intr}} / ^\circ$ | $\beta_{\text{intr}} / ^\circ$ | $\alpha_{\text{intr}} / ^\circ$ | $\beta_{\text{intr}} / ^\circ$ |
| J0631+1036   | 59 - 127          | -10.5 - -4         | 92    | -10       |
| J0659+1414   | 58 - 139          | -22 - -8           | 101   | -19       |
| J0729–1448   | 56 - 122          | 3 - 7              | 90    | 6         |
| J0742–2822   | 55 - 180          | -7 - 0             | 90    | -6.5      |
| J0835–4510   | 66 - 98           | -7.5 - -7          | 85    | -7.5      |
| J0908–4913 (MP) | 96 - 96.8        | -8.5 - -6.3        | 96.1  | -5.9      |
| J0908–4913 (IP) | 83.9            | 6.3                |       |           |
| J0940–5428   | 0 - 73; 102 - 180 | 0 - 21             | 117   | 18        |
| J1016–5857   | 0 - 65; 119 - 180 | -13 - 0            | 144   | -6        |
| J1048–5832   | 0 - 74            | 0 - 9              | 55    | 8         |
| J1057–5226 (MP) | 68 - 92        | 10 - 48            | 86    | 20        |
| J1057–5226 (IP) | 94               | 12                 |       |           |
| J1105–6107   | 53 - 124          | 3 - 5              | 90    | 4         |
| J1112–6103   | 0 - 180           | -5.5 - -0          | 104   | -5        |
| J1119–6127 (a) | 0 - 80; 108 - 180 | -25 - 0            | 159   | -9        |
| J1119–6127 (b) | 0 - 72; 118 - 180 | -25 - 0            | 164   | -7        |
| J1357–6429   | 0 - 77; 93 - 180  | 0 - 55             | 16    | 11        |
| J1410–6132   | 0 - 180           | 0 - 5.5            | 121   | 4.5       |
| J1420–6048   | 0 - 59            | 0 - 13             | 36    | 8         |
| J1513–5908   | 0 - 180           | 0 - 90             | 30    | 34        |
| J1531–5610   | 0 - 180           | -59 - 0            | 136   | -28       |
| J1648–4611   | 0 - 180           | -14 - 0            | 137   | -8        |
| J1702–4128   | 0 - 83; 101 - 180 | -14 - 0            | 46    | -10       |
| J1709–4429   | 27 - 72           | 12 - 24            | 57    | 21        |
| J1718–3825   | 0 - 81; 97 - 148  | 0 - 17             | 46    | 10        |
| J1730–3350   | 0 - 180           | -10 - 0            | 123   | -6        |
| J1801–2451   | 0 - 78; 107 - 180 | -14 - 0            | 121   | -11       |
| J1835–1106   | 0 - 180           | 0 - 13             | 89    | 11        |

4.2 A systematic fiducial plane offset

In §3.2 we found that a systematic offset of $\sim 14^\circ$ in the fiducial plane position could explain the difference between the measured $\alpha$ distribution and a sinusoidal distribution. The fixed offset in pulse phase implies that even in cases with high mirror symmetry in the profile (such as PSRs J1420–6048 and J1648–4611) the beam is not symmetrically illuminated.

One might expect that the offset of the fiducial plane is dependent on the pulse width. We found that the observed bias towards low values could be explained if only the trailing 45% of each beam is illuminated. In the case of PSR J1057–5226, Weltevrede & Wright (2009) argued that the main pulse represents only the trailing 50% of the beam. This suggests that at least some pulsars should be capable of having the required asymmetry in the illumination.

A problem arises when accounting for the selection effects expected due to $\gamma$-ray detectability and the radio beaming fraction (see §3). Both effects predict the intrinsic distribution to have an excess of large $\alpha$ values relative to a sinusoidal distribution. However, neither a constant offset nor an offset as a fixed proportion of the pulse width could reproduce a distribution with such an excess.

Another problem with a systematic misplacement of the fiducial plane comes from the literature. A systematic error in the fiducial plane position would affect our results as this position was used to calculate $\rho$ and $W_{\text{open}}$. However, a similar bias was found in the $\alpha$ distributions of Rankin (1990) and Gould (1994) (as shown by Tauris & Manchester 1998). As explained in §3.5 both authors used relations which are not affected by the choice of the fiducial plane position, and so the $\alpha$ values they derived should be independent of a systematic effect on the location of the fiducial plane. In light of this, the presence of a similar bias in the distributions as shown in Tauris & Manchester (1998) strongly suggests that the effectiveness of both methods of offsetting the fiducial plane in retrieving a sinusoidal distribution from our data is coincidental, and cannot explain the departure of the observed $\alpha$ distribution from a sinusoidal distribution.
4.3 Implications of an $\alpha$-dependence of $s$

An $s$ value which is dependent on $\alpha$ is a more plausible explanation for the difference between the measured and sinusoidal distributions than that discussed in the previous subsection. The additional $\alpha$ dependence which this situation would introduce would affect the $\alpha$ values as measured in RWJ14, as well as those derived by, for instance, [Rankin (1990) and Gould (1994)]. This could therefore explain the presence of a low-\(\alpha\) bias in all three samples.

Extra-cap emission has been suggested in the literature and appears to be unavoidable in some cases (e.g., the main pulse of PSR J1057–5226). Values of $s > 1$ mean either that emission can be generated on closed field lines or that some field lines traditionally thought to be closed are in fact open. The former has implications for the emission mechanism. For example, the Ruderman & Sutherland (1975) model of the emission mechanism, which draws on the Sterrock (1971) model for particle acceleration within the magnetosphere, requires the emitting particles to lie on open field lines; any emission generated on closed field lines would require a different mechanism.

The later situation has implications for the magnetic field structure. The open-field-line region we have assumed in this paper is based on a static dipolar field. We have not included higher order effects such as relativistic deformations of the field close to the light cylinder (e.g., Michel 1972), which would alter the boundary of the open-field-line region. This boundary would be a natural location for the edge of the beam, making this interpretation more plausible. However, not even conceivable correction to the field structure would yield the linear dependence of $s$ on $\alpha$ suggested by our data (§3.3). It should be noted that our results are insensitive to the shape of the beam, as our simulations of elliptical beams show (see §3.4). If the beams are elliptical, $s$ in Eq. 1 should be considered to correspond to the semi-minor axis of the beam.

Extra-cap emission also has implications for the compatibility of radio and $\gamma$-ray models. The $\gamma$-ray models with extended emission regions in the outer parts of the magnetosphere, such as the outer gap model (Cheng et al. 2000) or slot gap model (Muslimov & Harding 2004), typically require large values of $\alpha$ for pulsed emission to be observed (e.g., Watters et al. 2009). This is in stark contrast with the $\alpha$ values derived from radio polarisation measurements (Fig. 1), for which low values appear to be preferred. Making the radio beam wider will increase the radio-derived $\alpha$ values (c.f. Fig. 3), resulting in closer agreement with the expectation from $\gamma$-ray models. Note that Fig. 3 demonstrates that extra-cap emission is able to reproduce a sinusoidal $\alpha$ distribution. The fact that these pulsars are $\gamma$-ray-detected might suggest that the actual $\alpha$ distribution for this sample should be skewed to higher values, which would imply that the effect of extra-cap emission is stronger than implied by the equations in §3.3 and 3.4. A second effect of extra-cap emission is that the $\beta$ values allowed by radio modeling increase. This increase in the parameter space available to the $\gamma$-ray models will increase the potential for models to match the shape of the observed light curves.

4.4 Implications of an intrinsic low-$\alpha$ bias

Equally interesting is the possibility that the measured $\alpha$ distribution (Fig. 1) would, after correcting for the selection effects discussed in §3.1, accurately represent the birth distribution of $\alpha$ values for this sample of pulsars and hence that no additional corrections are needed. This would have implications for determined timescales for alignment of the magnetic axis. The method used by Weltevrede & Johnston (2008), for example, assumes random orientation of the axes at the birth of the neutron star; if the axes in fact already tend towards alignment at birth a longer alignment timescale than their quoted value $7 \times 10^7$ yr is required. In contrast to this, the method used by Tauris & Manchester (1998) (in which the average $\alpha$ value of a sample was analysed as a function of characteristic age) was not sensitive to the birth distribution. Young et al. (2010) calculated alignment timescales for various models with differing birth $\alpha$ distributions and showed that their value of $10^6$ yr is fairly insensitive to the initial $\alpha$ distribution.

The sample analysed in this paper consists of young $\gamma$-ray-detected pulsars. If such a birth distribution is intrinsic only to $\gamma$-ray-loud pulsars (i.e., the $\gamma$-ray-quiet population is still sinusoidal), this would be an important distinction and could allow insight into the differences between pulsars which emit $\gamma$-rays and those which do not.

Alternatively, this birth distribution may apply to the pulsar population as a whole. This would imply the cause of the bias towards low $\alpha$ is the supernova (that is, some characteristic of the supernova favours closer alignment of the magnetic and rotation axes). The presence of a similar bias in the $\alpha$ distributions of Tauris & Manchester (1998) suggest that this could be the case, as those pulsars were not selected according to detectability in $\gamma$-rays and so are a mixture of $\gamma$-ray-loud and -quiet sources.

5 CONCLUSIONS

Rookyard et al. (2014) presented the observed distribution of $\alpha$, the magnetic inclination, for a sample of young $\gamma$-ray-loud pulsars. This involved utilising several common assumptions relating to the structure and alignment of the magnetic field. This distribution is not consistent with the sinusoidal distribution expected if the magnetic and rotation axes of a neutron star are randomly orientated at birth, confirming the results of several previous studies. The observed distribution is skewed towards low values, which is opposite to the skew which would be expected from considerations of the radio beaming fraction or from the prediction by $\gamma$-ray models that pulsars are more easily detected in $\gamma$-rays when $\alpha$ is large. Assuming that the intrinsic distribution is sinusoidal we have explored a number of potential causes, including the effects of systematic underestimation of the pulse phase of the inflection point of the PA curve, systematic overestimation of the fiducial plane position, a possible emission height gradient, a possible elliptical emission beam and various distributions of the ratio $s$ between the radius of a circular emission region and that expected for the open-field-line region of a dipolar field. As a result we are left with two alternative scenarios: either the birth $\alpha$ distribution is intrinsically non-sinusoidal such that there is a significant

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excess of low-α pulsars, or the measured α is not an accurate representation of the intrinsic values. Under the assumption of an intrinsically random orientation of the rotation and magnetic axes, the measured and intrinsic α and β values may be related by Eqs. [10] and [12] respectively. However, as discussed, the intrinsic α distribution is likely to be skewed towards higher values relative to the sinusoidal distribution, in which case these relations will be a somewhat conservative first order correction.

We argue the discrepancy between the measured and intrinsic α values can be explained by a linear dependence of s on the intrinsic α (Eq. 9). This relation predicts that the emission region is larger than expected for the standard assumption of emission confined to the open-field-line region of a dipolar field when α is small, tending towards s ≈ 1 as the intrinsic α = 90°. Although there must by definition be an α-dependent s distribution which will relate any desired α distribution to the observed distribution, the fact that our result is a simple linear relation is non-trivial. It is also non-trivial that the size of the emission region tends towards that expected for a dipolar field for orthogonal rotators.

Two alternative effects were considered which could bias the α distribution: that the fiducial plane position is systematically ∼ 14° earlier in rotational phase than our expectations based on the profile morphology, and that only the trailing 45% of the beam is illuminated. These were both found to be mathematically possible but it has been argued to be implausible that either is the sole cause. We do not rule out the possibilities of elliptical beams or an emission height gradient, but find that these are not sufficient to explain the departure of the observed α distribution from a sinusoidal distribution.

Both an intrinsically non-sinusoidal birth α distribution and a discrepancy between the intrinsic and measured distributions have important implications for population studies. If the α distribution is intrinsically non-sinusoidal the population-averaged beaming fraction will be lower. In the case of an α-dependent s value the beam fraction of each pulsar will be affected differently, but the population-averaged value will be made higher. There are also implications for the timescale for possible alignment of the two axes as a neutron star ages. Both scenarios suggest this timescale to be longer than some previous estimates.

Each scenario raises important questions about neutron star physics. An α-dependence of s suggests complexities in the radio emission process or magnetic field structure. An intrinsically non-sinusoidal birth α distribution has implications for the formation of the neutron star or for the dependence of the γ-ray emission process on magnetic inclination making aligned rotators easier to detect.

Finally we note that an α-dependent s would make the radio models more compatible with the preferred γ-ray models, which place the extended emission regions far above the polar cap. This is because extra-cap emission will increase both the radio-derived α and β values. Larger α values are favoured by the γ-ray models, while the possibility of larger β values will increase the allowed parameter range available for the γ-ray models to fit the observed light curves. Extra-cap emission therefore should be seriously considered in radio beam models.

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