Integrability and coherence of hopping between 1D correlated electrons systems∗

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Abstract

We present numerical evidence that the hopping of electrons between chains described by the $t-J$ model is coherent in the integrable cases ($J = 0$ and $J = 2$) and essentially incoherent otherwise. This effect is not related to the value of the exponent $\alpha$, (which is restricted to the interval $[0,1/8]$ when $0 \leq J \leq 2$), and we propose that enhanced coherence is characteristic of integrable systems.

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After the proposal by Anderson that two-dimensional models of correlated electrons might behave as Luttinger liquids \[1\], i.e. as their one-dimensional analogs, a lot of work has been devoted to studying the effect of a small transverse hopping $t_\perp$ between two chains. The first conclusion that has been obtained by several authors \[2\] is that a small transverse hopping is a relevant perturbation as long as the exponent $\alpha$ describing the power-law singularity of the momentum distribution in the isolated chains is smaller than 1, suggesting that a 2D system of coupled Hubbard chains flows toward a Fermi liquid fixed point, $\alpha$ being at most $1/8$ in that case. More recently \[3\], another point of view has been emphasized according to which what really matters is not whether $t_\perp$ is relevant, but whether it induces a coherent hopping between the chains. A study of the short time dynamics of two coupled Luttinger liquids by Clarke, Strong and Anderson \[4\] suggests that coherence can be destroyed even if $t_\perp$ is relevant. Within their approximation, the hopping between chains becomes incoherent for $\alpha = 1/2$, whereas $t_\perp$ remains relevant until $\alpha = 1$. While this result opens new perspectives in the problem of coupled chains, this analysis suggests that the key parameter is still $\alpha$, which somehow reflects the strength of the correlations within the chains. In a very recent paper about a related problem in other systems, Chakravarty and Rudnick \[5\] suggest that incoherence is actually a generic feature, and that coherence is limited to a very narrow range of parameters. The origin of this coherence is left as an open issue.

In this Letter, we identify integrability as a very important factor of coherence between chains. More specifically, we show that the short time dynamics of the hopping of electrons between two chains is coherent if the model describing each chain is integrable, that this coherence is destroyed if one goes away from integrable points, and that this effect cannot be understood in terms of the exponent $\alpha$. The system we have studied consists of two chains (so called $2 \times L$ ladder) described by the $t - J$ model and coupled by a perpendicular hopping $t_\perp$. The Hamiltonian can be written

\[
H = \sum_{\alpha=1,2} \left( -t \sum_{i,\sigma} P^\alpha G (c_{i\sigma} c_{i+1\sigma} + h.c.) P^\alpha G + J \sum_i \vec{S}_i^\alpha \cdot \vec{S}_{i+1}^\alpha \right) - t_\perp \sum_i (c_{i\sigma} c_{i+1\sigma} + h.c.) \tag{1}
\]
where $P^G_\alpha$ is the Gutzwiller projection operator that excludes double occupancy on chain $\alpha$. For small $J/t$ ratios (say $J/t < 0.5$) an additional small transverse exchange coupling $J_\perp = J(\frac{t_\perp}{t})^2$ has also been included in order to improve the equivalence with the large-$U$ Hubbard ladder in this parameter regime. In the isotropic regime ($t_\perp = t$) transverse coherence can be established [6] but hereafter we restrict ourselves to the small $t_\perp$ regime. Following Clarke et al, our analysis of coherence is based on the probability $P(\tau)$ that a system comes back after some time $\tau$ to its initial state if one initially puts more particles on one chain than on the other. More precisely, $P(\tau)$ is defined by

$$P(\tau) = |A(\tau)|^2$$

$$A(\tau) = \langle \psi_0 | e^{i(H-E_0)\tau} | \psi_0 \rangle$$

where $|\psi_0\rangle$ is the lowest energy $(E_0)$ eigenstate of the system with $t_\perp$ set to zero and with $\Delta N_e$ more particles on one chain than on the other, while $H$ is the full Hamiltonian. There are actually two such states because the excess particles could be on either chain. In the following, we have taken the symmetric combination of these states for numerical convenience [7].

In the case of non-interacting particles (referred to hereafter as $U = 0$) $P(\tau)$ exhibits an oscillatory behavior characteristic of coherent transverse motion. More precisely, $P(\tau) = \left| \cos^{\Delta N_e}(t_\perp \tau) + i \Delta N_e \sin^{\Delta N_e}(t_\perp \tau) \right|^2$ showing a characteristic period $\pi/(4t_\perp)$ for all the excess particles to exactly move in phase from one chain to the next [7]. In contrast, when interaction between particles is switched on we expect that $P(\tau)$ never reaches 1 for finite non-zero $\tau$ although an oscillatory behavior can still occur in the case of coherent transverse hopping.

Since this issue cannot be adressed by perturbative methods exact diagonalizations of small ladder clusters have been performed. Using Lanczos technique, it is easier to calculate first the Fourier transform of $A(T)$ defined by

$$A(E) = -\frac{1}{\pi} \text{Im} \langle \psi_0 | \frac{1}{E - H + E_0 + i\epsilon} | \psi_0 \rangle$$
which can be obtained through a continued fraction expansion. This function is itself quite useful because coherence shows up as symmetric peaks around $E = 0$, while an incoherent system is expected to have a broad peak centered around 0. We have calculated these quantities for a system of 2 chains with $L = 16$ sites each, and for a total number of particles $N_e = 8$. For clarity, most of the results presented below correspond to $\Delta N_e = 8$, i.e. to an initial state having all the particles on the same chain, but the main conclusions of this work do not depend dramatically on the value of $\Delta N_e$ as discussed later. Note that antiperiodic boundary conditions have been used in the chains direction to ensure closed shell fillings of the corresponding non-interacting systems. Numerical data obtained for $t_{\perp}/t = 0.2, \Delta N_e = 8$ and for various values of $J/t$ ranging from 0 to 2.4 are presented in Figures 1, 2 and 3.

The first important result is that the behaviour depends dramatically on the value of $J/t$. Two typical cases have emerged:

i) $A(E)$ is a smooth distribution centered around $E = 0$, and $P(\tau)$ decreases monotonically with $t$. This is for instance what we have obtained for $J/t = 0.25$ (see Figs. 1a and 2b). In this case, the probability that the system goes back to its initial state is continuously decreasing, which corresponds to a totally incoherent dynamics.

ii) $A(E)$ has three narrow peaks, one at $E = 0$, the other two symmetric around $E = 0$, and $P(\tau)$ exhibits damped oscillations. This situation is best exemplified by $J/t = 2$ (see Figs. 1b and 3). In that case, the probability for the system to go back to its initial state reaches again substantial values after going down to almost 0, at least for moderate values of $T$, and the short time dynamics is coherent. Very regular oscillations have also been observed for $J/t = 0$, with a smaller amplitude than for $J/t = 2$ though (see Fig. 2b).

The basic properties of a single chain described by the $t-J$ model are known from the work of Ogata et al. Since we have 8 particles on $N = 2 \times 16$ sites, the relevant band-filling is $n=1/8$ (4 particles on each chain). Then, going for $J/t = 0$ to $J/t = 2$, the exponent $\alpha$ decreases monotonically from $1/8$ to 0. Beyond that point, a gap opens in the spin sector, although the exact location of the critical value is not known. Finally, a
phase separation occurs at $J/t \simeq 2.7$. Comparing this behavior with our results (coherence for $J/t = 0$ and 2, incoherence for $J/t = 0.25$), we conclude that the coherence we have detected cannot be related in any simple way to the exponent $\alpha$.

We finish the discussion of the data by a comment on the actual role of the parameter $\Delta N_e$. So far we have considered the case $\Delta N_e/N_e = 1$ where the initial state at $\tau = 0$ corresponds somehow to a macroscopic perturbation of the system (with full Hamiltonian $H$) far from its equilibrium state. Fig. 4 shows a comparative study of $P(\tau)$ at $J/t = 0.25$ for $\Delta N_e = 8$ and $\Delta N_e = 2$. Although $\Delta N_e = 2$ corresponds to a much smaller deviation from the absolute ground state (with an excitation energy of only $\sim 0.7t$ compared to $\sim 4.3t$ for $\Delta N_e = 8$) it nevertheless gives rise to a qualitatively similar behavior for $P(\tau)$ showing again no coherence in the transverse motion.

A natural question which arises then is what causes the special behavior observed only for $J/t = 0$ and $J/t = 2$. The answer we propose is that these are the only two points for which the $t-J$ model is integrable. For $J/t = 0$, the model is equivalent to the finite energy sector of the infinite $U$ Hubbard model, which is known to be soluble by Bethe ansatz since the work of Lieb and Wu [10], while for $J/t = 2$ an additional supersymmetry again makes the model integrable by Bethe ansatz [11]. The remarkable differences in Fig. 3 between the results for $J/t = 2$ and those obtained for $J/t = 1.6$ and $J/t = 2.4$ support this point of view. For both non integrable cases, the oscillations are much smaller in amplitude and involve several frequencies, a sign of a much less coherent dynamics. Coherence might be completely lost for arbitrary deviation from the supersymmetric case provided that the system size is large enough.

So the main conclusion of this work is that the short-time dynamics of hopping of electrons between chains is remarkably coherent if the model describing the chains is integrable and more or less incoherent otherwise.

Let us now compare our results with those obtained previously by other authors. The main conclusion of Clarke et al [4], namely that incoherence can be achieved for $\alpha < 1$, is confirmed by our results for $J/t = 0.25$, which corresponds to $\alpha \simeq 0.1$. Note that the
necessary condition \( t_\perp \gg 2\pi \frac{\Delta N}{\mathcal{L}} (v_c - v_s) \) for the observation of coherence proposed by Clarke et al is clearly satisfied for \( J/t = 0.25 \) because the charge and spin velocities \( v_c \) and \( v_s \) are nearly equal for that particular value of \( J/t \). However, as already been pointed out, our results show that coherence does not seem to be linked with the value of \( \alpha \). More importantly it seems also that the analysis of the \( \tau^2 \) term in the expansion of \( P(\tau) \) for short times is not sufficient to study coherence. In Fig. 2a, we have depicted the short time behavior of \( P(\tau) \) for 3 extreme cases: \( U = 0 \) (totally coherent), \( J/t = 0 \) (partially coherent) and \( J/t = 0.25 \) (totally incoherent). The curvatures at \( \tau = 0 \) are indistinguishable.

The main conclusion of Chakravarty and Rudnick \[5\], namely that incoherence is the generic behavior, and coherence is somehow accidental, is also in agreement with our results. To go beyond this general statement, one should understand what integrability means in terms of the models they have studied, and this is not clear yet.

Finally the reason why integrability and coherence are related remains an open issue. Integrability is already known to have dramatic consequences on the level statistics \[12\], the distribution being Poisson instead of Wigner in that case, and on the finite temperature conductivity \[13\]. While the first effect can be qualitatively understood in terms of level repulsion, a good explanation of the second one is also missing. More work is clearly needed to understand the influence of integrability on the dynamical properties of correlated electrons.

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FIGURES

FIG. 1. Spectral function $A(E)$ for a $2 \times 16$ $t$-$J$ ladder with $N_e = \Delta N_e = 8$. Energies and parameters are measured in unit of $t$ (i.e. $t' = t_{\perp}/t$). (a) Non-integrable case $J = 0.25$ (and $J_{\perp} = 0.01$); (b) Supersymmetric case $J = 2$.

FIG. 2. Probability $P(\tau)$ vs time $\tau$ calculated at $J/t = 0$ and $J/t = 0.25$ for $\Delta N_e = 8$ and a transverse coupling $t' = t_{\perp}/t = 0.2$. The non-interacting case is also shown (thin dashed line) for comparison. Time is measured in unit of the inverse hopping integral $1/t$. (a) Small time region; (b) Enlarged time interval.

FIG. 3. Probability $P(\tau)$ vs time $\tau$ calculated at the supersymmetric point $J/t = 2$ and in its vicinity for $\Delta N_e = 8$. The non-interacting case is also shown and time is measured in unit of $1/t$.

FIG. 4. Probability $P(\tau)$ vs time $\tau$ calculated at $J/t = 0.25$ and for both $\Delta N_e = 2$ (full line) and $\Delta N_e = 8$ (dashed line).
Energy \[ t \]

\[ J=0.25 \]

\[ t'=0.2 \]
Energy $t$

$A(E) = J=2$

t' = 0.2
The diagram shows the relationship between time and a variable labeled P. The graph plots time on the x-axis and P on the y-axis. Three lines are depicted, each with a different symbol or style:

1. A dashed line labeled \( J = 0.25 \)
2. A solid line labeled \( J = 0 \)
3. A dashed-dotted line labeled \( U = 0 \)

In addition, the following annotations are present:

- \( t' = 0.2 \)
- \( N = 2 \times 16 \)

The figure is labeled as (a).
\(N = 2 \times 16\)
\(t' = 0.2\)

- Dashed line: \(N1 - N2 = 8\)
- Solid line: \(N1 - N2 = 2\)