All-optical non-demolition measurement of single-hole spin in a quantum-dot molecule

F. Troiani, I. Wilson-Rae, and C. Tejedor

Departamento de Física Teórica de la Materia Condensada,
Universidad Autónoma de Madrid, 28049 Madrid, Spain
(Dated: March 23, 2022)

We propose an all-optical scheme to perform a non-demolition measurement of a single hole spin localized in a quantum-dot molecule. The latter is embedded in a microcavity and driven by two lasers. This allows to induce Raman transitions which entangle the spin state with the polarization of the emitted photons. We find that the measurement can be completed with high fidelity on a timescale $T \sim 10^2$ ps, shorter than the typical $T_2$. Furthermore, we show that the scheme can be used to induce and observe spin oscillations without the need of time-dependent magnetic fields.

PACS numbers: 03.67.-a, 42.50.Ct, 42.50.Ar

The capability of encoding and manipulating information at the single level represents a key challenge for semiconductor-based spintronics and quantum-information [1, 2]. A reliable read-out of an individual-spin state is likely to require the measurement to be repeatable. This calls for it to be non-destructive and carried out on timescales shorter than those characterizing spin decoherence [3, 4]. In this respect, optical manipulation of single carriers in quantum dots (QDs) is specially attractive due to the orders of magnitude separation between optical timescales and those associated to the intrinsic spin dynamics [5]. While most of the attention has been centered in the past on electron spin, it can be argued that hole spin could offer novel alternatives. Along these lines, it has been noted that the decoherence due to hyperfine interactions is suppressed compared to that affecting the electron [6].

Here we propose a novel technique to perform a fast and robust non-demolition measurement of single hole spin in a QD-microcavity (MC) system. To illustrate its merit, we also discuss how it could be used to study the spin decoherence with a photon-correlation experiment. The basic idea is to exploit virtual Raman transitions that entangle the spin ($|\uparrow\rangle$ or $|\downarrow\rangle$) with the polarization ($\sigma_+$ or $\sigma_-$) of the photons emitted into the cavity. The semiconductor heterostructure we consider consists of two self-assembled QDs, coherently coupled with each other and embedded in a high-Q optical MC. The quantum-dot molecule (QDM) is doped with an excess hole, and its lowest-energy trion transition is strongly coupled to a pair of degenerate cavity modes with frequency $\omega_c$, damping constant $\kappa$, and polarizations $\sigma_\pm$ [7]. In the absence of a magnetic field, the ground state of the hole is doubly degenerate and each of the two eigenstates of its spin along the optical axis ($\hat{z}$ in Fig. 1(a)) couples to a different set of trion states. The system’s dynamics is driven by two linearly polarized lasers (1 and 2) with frequencies $\omega_1$ and $\omega_2$. The photons emitted by the cavity are sorted out with a $\lambda/4$ phase shifter followed by a polarizing beam splitter. The photons with polarization $\sigma_+$ ($\sigma_-$) are finally sent to the right (left) detector where photocounts $N_+$ ($N_-$) are recorded. The outcome of the spin ($J_z$) measurement is decided based on whether $N_+ \geq N_-$. This scheme corresponds to each of the two subspaces, “+” and “−”. Optical transitions are induced by two lasers (blue arrows) with frequencies $\omega_1$ and $\omega_2$ linearly polarized along $\hat{x}$, and two degenerate cavity modes (red) with frequency $\omega_c$. The scheme relies on Raman transitions between the states $|1\pm\rangle$ and $|2\pm\rangle$, that involve $\sigma_\pm$ radiation (solid arrows). All deleterious virtual processes that involve the emission of $\sigma_\mp$ photons (an example of which is given by the dotted arrows) are very off-resonant.

A typical QDM is formed by two vertically-stacked QDs. There, the combined effect of strain and effective-mass asymmetry strongly suppresses the hole-state hybridization, while allowing the formation of molecular-like bonding and antibonding states for electrons [8]. Thus, we assume that the heavy hole ($J_z = \pm 3/2$) re-
mains localized either in the larger dot (L) or in the smaller one (S), while the electron (S_e = ±1/2) bonding state (B) is significantly delocalized over the two. At low temperatures (T < 5K) and for near resonant interaction with the laser and cavity fields, we can restrict ourselves to a ground-state manifold: \(|\pm\rangle = d_{1/L}^\dagger|0\rangle\), \(|\pm\rangle = d_{1/S}^\dagger|0\rangle\), and an excited-state (trion) manifold: \(|\mp\rangle = e_{1/L}^\dagger B d_{1/L}^\dagger|0\rangle\), \(|\mp\rangle = e_{1/S}^\dagger B d_{1/S}^\dagger|0\rangle\), and \(|\mp\rangle = e_{1/L}^\dagger B d_{1/S}^\dagger|0\rangle\). Each of these states is polarized and can be measured into a many photon state.

The interaction of both lasers is equivalent to a cycling transition \(\sigma_+\) \(\sigma_-\). The cornerstone of the scheme allows to amplify the single states of the QDM, \(|\pm\rangle\), \(|\mp\rangle\), and \(|\mp\rangle\). The lasers are switched between \(|\pm\rangle\) \(|\mp\rangle\), \(|\mp\rangle\) while \(|\pm\rangle\) - \(|\mp\rangle\) - \(|\mp\rangle\) - \(|\pm\rangle\) - \(|\mp\rangle\) - \(|\pm\rangle\) - \(|\mp\rangle\).

In order to study its interaction with the radiation field, we treat the laser driven QDM coupled to the MC as an open quantum system. We apply a time-dependent canonical transformation \(O \to e^{iH_0 t} O e^{-iH_0 t}\) defined by

\[
s(t) = -i \sum_{\zeta = \pm; n = 1, 2} \left[ \hat{\omega}_n t - \frac{(1)^n}{2} \phi(t) \right] \sigma_{nm}^{(C)} - \frac{\hat{\omega}_n t}{2} a_\zeta^\dagger a_{\zeta},
\]

where: \(\sigma_{nm}^{(C)} = |m\rangle\langle n|\), \(a_\pm\) is the annihilation operator for the \(\sigma_{\pm}\) polarized cavity mode, \(\hat{\omega}_n \equiv (3\omega_{1/2} + \omega_{2/1})/4\), and \(\phi(t) \equiv \sum_m \int d\tau N_{m}^{(C)}(\tau)/(-\omega_{2}\delta_T + (3 - 4\delta_{n,m})\Delta)\). Here we also introduce \(\delta_T \equiv (\omega_1 - \omega_{2/2})/2\), \(\omega_1 \equiv \omega_1 + \omega_{2} + \Delta - 2\omega_T\), and the Rabi frequencies \(\Omega_{1/2}(t)\), \(\Omega_{1/2}(t)\) for the transitions between \(|1/2\rangle\) and \(|1/2\rangle\) and the trion states induced by laser 1 (2). (In this representation the system’s Hamiltonian is given by \(H(t) = H_0(t) + V(t) + H_T\), with \(h \equiv 1\):

V(t) = \(\sum_{\zeta = \pm} \sum_{n = 1, 2} g_n e^{i\phi(t)/2} [\sigma_{m}^{(C)} - \sigma_{m}^{(C)} - \delta_\zeta a_\zeta^\dagger a_\zeta, \)

and \(H_T = -\delta_T / 2 \sum_{\zeta = \pm} \sum_{n = 1, 2} \sum_{n = 1, 2} \sum_{\zeta = \pm} \sum_{n = 1, 2} \left[ \sigma_{n}^{(C)} - \sigma_{n}^{(C)} + \sigma_{n}^{(C)} - \delta_\zeta a_\zeta^\dagger a_\zeta \right] + \text{H.c.,}

1. The above treatment is valid provided \(|\Omega_{1/2}(t)/|\Delta| - |\delta_T|\|^2\) and \(\kappa, \Gamma, |\delta_{c/s}|, |\Omega_{1/2}(t)/|\Omega_{1/2}(t)|\) \(\ll |\Delta| - |\delta_T|\) are satisfied for \(n = 1, 3\); where we assume \(g_2 < g_1 < |\Omega_{1/2}(t)/|\Delta| - |\delta_T|\) and that the typical cavity occupations are at most of order unity. We take laser and cavity frequencies so that \(\delta_0 = 0\), \(\delta_\zeta = \Delta_{c/\delta}, \Delta = 0\). The asymmetry of the molecule ensures \(\delta_\zeta = \Delta_{c/\delta} \sim \Delta\) (\(n = 1, 3\)). The relative intensities of the two lasers are chosen so that the coefficients of \(\delta_\zeta\) and \(\delta_\zeta\) are equal. This yields \(H_{\text{eff}} = -\tilde{\Omega}(t)/2 \sum_{\zeta} \sigma_{n}^{(C)} (a_\zeta^\dagger + a_\zeta)\) with \(\tilde{\Omega}(t) = |\Omega_{1/2}(t)/|g_2^2 - g_2^2|\Delta_2\). The lasers are switched
on at $t = 0$ so that for negative times $\tilde{\Omega}(t) = 0$ with the cavity modes in the vacuum.

As will be borne out below, to analyze the measurement process it is useful to consider the time evolution conditioned upon having no photocounts detected: $\mu = \sum_{\xi} \mu_{\xi} = \sum_{\xi} -i[H_{\text{eff}}, \mu] + \mathcal{L}_{\xi}^{(C)}(\eta)\mu$ with $\mathcal{L}_{\xi}^{(C)}(\eta)\mu = (\kappa/2)[2(1 - \eta)\alpha_{\xi}\alpha_{\xi} - \{\alpha_{\xi}^{\dagger}\alpha_{\xi}, \mu\}]$. Here $\eta$ is the collection efficiency times the efficiency of the detectors and for $\eta = 0$ one recovers the standard time evolution. This equation for $\mu(t)$ has the following solution:

$$\mu(t) = \sum_{\xi = \pm} e^{-\epsilon\cdot\xi P(t)} e^{-i\epsilon t} \sigma^{(C)}_{\xi}(\alpha_{\xi}^{\dagger} + \alpha_{\xi}) |0\rangle_+ \langle 0|_+ \prod |0\rangle_0 \langle 0|_0 \cdot \frac{\text{Tr}[\sigma^{(C)}_{\xi}(\mu)]}{2} \sigma^{(C)}_{\xi} e^{i\epsilon t} \sigma^{(C)}_{\xi}(\alpha_{\xi}^{\dagger} + \alpha_{\xi})$$

(2)

where $\xi_{1/2} = 1, \xi_{3/2} = 2 - \eta$, and $P(t) = \int_0^t d\tau p(\tau)^2$ with $p(t)$ satisfying: $2\dot{p} = -\tilde{\Omega}(t) - \kappa p, p(0) = 0$. Here $|0\rangle_0 \langle 0|$ are the vacuum states for the cavity modes, $\sigma^{(C)}_{\xi} \equiv \sigma^{(C)}_{\eta} + \sigma^{(C)}_{\eta}$, and $\sigma^{(C)}_{\eta}$ with $\nu > 0$ correspond to the Pauli matrices.

The profiles of the laser pulses are chosen so that $\tilde{\Omega}(t) = \tilde{\Omega}_0[e^{-8(t/t_s - 1)^2}\Theta(t_s - t) + \Theta(t - t_s)]$, where $t_s$ is a switch-on time. We choose $1/\Delta \ll t_s < 1/\tilde{\Omega}_0$ so that $t_s\tilde{\Omega}_0$ is a small parameter and one can keep only the zeroth order. This corresponds to $H_P(t) = \kappa P(t) e^{2\eta/(t/t_s - 1)}$ where $\int[d\tau p(\tau)^2]$ from Eq. 2, which specifies $\mu(t)$.

If we now consider in Eq. 2 $\eta = 0$, $t \to \infty$ we obtain the steady state to which the system converges starting from a given initial condition for the hole: $\rho_h$.

$$\mu_{\text{ss}} = \sum_{\xi = \pm} |\xi\rangle_+ \langle \xi|_{\rho_h} \langle \xi|_+ \langle \xi| \cdot \langle \xi|_\eta \langle \xi|_\eta\rangle_0 \langle 0|_0 \langle 0|_0 \cdot \frac{\kappa P(t)}{2} \sigma^{(C)}_{\xi} e^{i\epsilon t} \sigma^{(C)}_{\xi}(\alpha_{\xi}^{\dagger} + \alpha_{\xi})$$

(3)

Here we have defined $|\xi\rangle_\pm = (|\pm\rangle \pm |\pm\rangle)/\sqrt{2}$ and introduced the cavity coherent states $|\lambda\rangle_\pm$. We note the perfect correlation between the initial state of the spin and the polarization of the cavity mode that does not remain in the vacuum, and its independence from the initial orbital state of the carrier. In addition we find that the eigenstates of the orbital pseudospin $\sigma_z$ become correlated with the phase quadratures of the cavity fields. Thus if homodyne detection is performed for both circular polarizations one can also measure the orbital state of the hole in the $|\pm\rangle \pm |\pm\rangle$ basis.

To assess the performance of the proposed measurement strategy [Fig. 1(a)] one can take the signal output density matrix for the spin $\rho_o = \sum_{\xi} \langle \xi|_o \langle \xi|_o\rangle_{\text{Tr}}\text{orb}$, where the orthogonal projectors $\hat{P}_\xi(T) = \hat{S}(T)\hat{S}(T) = 1/2$ correspond to the events $N_+(T) \geq N_-(T)$. Here $\hat{S}(T) \equiv \text{sgn}[N_+(T) - N_-(T)]$ and $T$ is the total time over which the photocounts are integrated. The signal input density matrix can be defined as $\rho_s = \text{Tr}\text{orb}\{\sum_{\xi} \sigma_{\eta}^{(C)} \rho_{\xi} \sigma_{\eta}^{(C)}\}$. Then, the quality of the measurement can be characterized by $F_{\text{Min}}(T) = \text{Min}(\text{Tr}[\rho_{\text{O}(T)}\rho_1])$, where $F$ is the square of the standard fidelity [2]. The probability of measurement “failure” ($N_+ = N_-$) is given by $(1 - S(T)^2)$.

FIG. 2: (Color online). Simulated time evolution of the unconditional (a) and conditional (b) density matrices under the effect of two consecutive laser pulses, with frequencies $\omega_1$ (light gray area) and $\omega_2$ (dark gray). The curves show the occupations of: state $|1+\rangle$ (red line), state $|2+\rangle$ (blue), the trion manifold $|n > 2 \pm\rangle$ (black dotted), and the “...” subspace (black dashed). The solid black line in panel (b) corresponds to Tr$(\rho(t))$. Figure inset: photon occupations $\langle a^\dagger(t) a(t) \rangle$ for $\zeta = +$ (red) and $\zeta = -$ (blue, x5), in the conditional (solid lines) and unconditional (dotted) cases. The values of the parameters are: $g_{1,2} = 0.2, 0.15\text{meV}$, $\Omega^{(1)}(t_s) = 0.4\text{meV}$, $\Omega^{(2)}(t_s) = 0.3\text{meV}$, $\kappa = 0.1\text{ps}^{-1}$, $\Gamma_s = 1\text{ns}^{-1}$, $\Gamma = 2\text{ns}^{-1}$; $t_s = 7.5\text{fs}$.
on the simultaneous switch-on (SSO) of the two lasers. An alternative approach consists in applying an alternating sequence of non-overlapping pulses with frequencies $\omega_1, \omega_2, \omega_1, \omega_2, \ldots$ (see Fig. 2). In this case, each pulse triggers the emission of a single photon. We find that an analysis based on $H_{\text{eff}}$, analogous to the one performed for the SSO strategy, allows to establish $\kappa_{\text{opt}} \approx 2 \hat{\Omega}_{\text{max}}$, and that $T_{\text{min}} \approx 1/\eta \hat{\Omega}_{\text{max}}$ for the complete pulse sequence. Thus, though the SSO strategy is preferable for low $\eta$, in the range we will focus on ($\eta \gtrsim 0.7$) the timescales $T$ for the two approaches are comparable. On the other hand, the “pulsed” scenario directly relates to photon-correlation experiments, that allow to probe the spin’s “intrinsinc” dynamics.

In the following, we numerically solve the complete conditional master equation for the QDM-MC system. The unitary part of the evolution is induced by the Hamiltonian $H(t)$ (see Eq. 1). The dissipative contribution, instead, is given by $\mathcal{L}_{\sigma}^{(\uparrow \downarrow)}(\eta) + \mathcal{L}_{\sigma}^{(\downarrow \uparrow)}(\eta) + \mathcal{L}_B(t) + \mathcal{L}_S$, where the collapse operators of the Liouvillian $\mathcal{L}_S$, accounting for the spin-flip process, are $\sqrt{\Gamma_S} \sigma_\uparrow$ and $\sqrt{\Gamma_S} \sigma_\downarrow$, with $\sigma_\uparrow \equiv \sum_{n=1}^{\infty} |n+\rangle \langle n-|$. In Fig. 4 we plot the system’s time evolution under the effect of two consecutive laser pulses (shaded gray areas) for both the unconditional [$\eta = 0$, panel (a)] and the conditional case [$\eta \neq 0$, panel (b)]. The first pulse essentially induces a population transfer from the initial state $|1+\rangle$ (blue line) to $|2+\rangle$ (red). The second Raman transition, due to the following laser pulse, drives the hole back to dot $L$. While the overall occupation of the excitonic manifold (black dotted lines) is kept negligible throughout the process, $\rho$ suffers a population leakage to the subspace “$-$” (dashed lines), which is responsible for the finite probability of emitting a $\sigma_-$ photon (blue lines in the inset). We note that these numerical simulations clearly support the approximations underpinning the effective Hamiltonian $H_{\text{eff}}$. The merits of the measure ultimately depend on the occupations of the cavity mode. In particular, we find that the final probability of having recorded no photocounts $(\text{Tr}\{\rho\}|_{\eta \neq 0})$ falls below 0.1, while $(\bar{P}_+) = 0.71$ (0.87) and $(\bar{P}_-) = 0.027$ (0.048) after one pulse (two pulses), yielding $\tilde{T}^{\text{min}} = 0.963$ (0.947). The repetition of the measurement while decreasing $\tilde{T}^{\text{min}}$, slightly worsens the fidelity. This is because the repetition time is not sufficiently short compared to $1/\Gamma_S$.

The non-destructive nature of our measurement scheme turns it into an ideal means to probe the spin’s dynamics. In particular, the polarization correlations between two photons detected at times $t$ and $t + \tau$ can be used to investigate the evolution that the spin undergoes in such time interval. As above, the system’s state is driven by a sequence of two non-overlapping laser pulses, with frequencies $\omega_1$ and $\omega_2$ [Fig. 5(a)]. The first pulse (light gray area) induces a Raman transition which displaces the hole from dot $L$ to dot $S$. The cases where a $\sigma_+$ photon is detected at a given time $t$ (vertical black line) are post-selected: this first measurement (approximately) projects $\rho$ onto the $|2+\rangle$ state, thus initializing the spin to a pure state. A tunable time-interval $\Delta T$ follows, during which the spin freely evolves under the effect of the spin-flip process, and eventually of an applied magnetic field $B$. The corresponding time-evolution, conditioned upon having detected a photon at time $t$, is given by the second-order correlation functions $G_{\zeta,\alpha}(t, t + \tau) = \langle a^\dagger_\zeta(t) a_\alpha(t + \tau) \rangle$, with $\xi, \beta = \pm$. Finally, the second laser pulse (dark gray) probes the spin state, while displacing the carrier back to dot $L$. If the magnetic field is applied in the $z$ direction, the polarization correlations between the first and the second detected photons, given by $G_{\zeta,\alpha}(t, t + \tau) = \langle a^\dagger_\zeta(t) a^\dagger_\beta(t + \tau) a_\beta(t + \tau) a_\alpha(t) \rangle$, only reflect the effect of $\mathcal{L}_S$, allowing to infer the value of $T_3 = 1/\Gamma_S$. If $B \parallel \hat{x}$ instead [12], $J_z$ is no longer a constant of motion of $H(t)$, and its time-evolution will consist of damped oscillations between the states $|2+\rangle$ and $|2-\rangle$ (solid and dotted curves, respectively). Due to the high fidelity of the initializing measurement, the initial conditions do not play here a crucial role. As the energy splitting induced by $B$ is small compared to $k_B T$, we take $\rho_0 = \langle |1+\rangle \langle 1+| + |1-\rangle \langle 1-| \rangle / 2$. In Fig. 5(c) we...
plot $G^{(2)}_{\zeta\beta}(t, t+\tau)$, for $\zeta = +$ and $\beta = \pm$ (upper and lower panels, respectively), and for different values of $\Delta T$. The time integrals of these functions $I_{\zeta\beta} = \int G^{(2)}_{\zeta\beta}(t, t+\tau)\,d\tau$ clearly show an oscillatory behavior as a function of $\Delta T$ [Fig. 3(b)]. The free damped oscillations we observe reflect the decay of the initial coherence between the eigenstates of $J_x$, and thus would allow to infer the $T_2$ for a transverse field.

In conclusion, we have proposed an all-optical robust scheme to perform a QND measurement of a single hole spin in sub-nanosecond timescales. Furthermore, we have pointed out how in the presence of a static magnetic field photon correlation experiments would allow to study the spin decoherence. Beyond measurement, the entanglement between the carrier and the photon could enable generation of EPR pairs. In the case of correlations with the phase quadratures [Eq. (3)] one could also envisage the generation of Schrödinger cat states of the emitted light. Finally, the same system could be operated in a continuous measurement regime with the spin-flip processes inducing quantum jumps in the output.

We thank J.C. Cuevas and J. Eschner for discussions. Work supported by the Spanish MEC under the contracts MAT2005-01388 and NAN2004-09109-C04-4, by CAM under Contract S-0505/ESP-0200, and by the EU within the RTN’s COLLECT and CLERMONT2.

*filippo.troiani@uam.es

[1] I. Žutić, et al., Rev. Mod. Phys., 76, 323 (2004).
[2] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, England, 2000).
[3] T. Meunier, et al., cond-mat/0603794.
[4] R.-B. Liu, et al., Phys. Rev. B. 72, 81306(R) (2005).
[5] F. Troiani, et al., Phys. Rev. Lett., 90, 206802 (2003).
[6] D. V. Bulaev and D. Loss, Phys. Rev. Lett. 95, 76805 (2005).
[7] E. A. Stinaff, et al., Science, 311, 636 (2006).
[8] A micropillar structure affords a physical realization. See, e.g., J.P. Reithermaier et al., Nature, 432, 197 (2004).
[9] G. Bester, et al., Phys. Rev. Lett. 93, 47401 (2004).
[10] C. W. Gardiner and P. Zoller, Quantum Noise (Springer-Verlag, Berlin, 2004).
[11] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, Atom-Photon Interactions (Wiley, US, 1992).
[12] For the effective spin Hamiltonian $H_B$ and typical parameters, see: H. W. van Kesteren, et al., Phys. Rev. B. 41, 5283 (1990); M. Bayer, et al., Phys. Rev. B. 61, 7273 (2000). Here, we also include the dark states $|6\pm\rangle \equiv c^{\dagger}_{\uparrow/\downarrow}d^{\dagger}_{\uparrow}d^{\dagger}_{\downarrow}|0\rangle$.
[13] T. C. Ralph, et al., Phys. Rev. A 73, 12113 (2006).