INTRODUCTION

With the gradual scarcity of terrestrial resources, mankind's demand for green energy is becoming increasingly strong. As one of the most promising renewable green energy sources in the world, wind energy has huge reserves and is widely distributed. At present, the most widely used wind energy equipment in the world is the wind turbine generator set (WTGS), which converts wind energy into mechanical energy and then into electrical energy for production and living. The WTGS is a complex mechanical equipment system, and its design, manufacture, operation, and maintenance have an important effect on the utilization rate of wind energy and power generation. With the continuous increase in installed...
capacity, many potential problems need to be solved, such as fatigue, failure, operation, and maintenance. Moreover, wind farms are usually located in remote areas with special terrain. The harsh environment and high cost have resulted in high maintenance and repair costs for the WTGS. Therefore, in the early stage of production design, the reliability allocation of the WTGS is particularly important. It can not only reduce the design difficulty and manufacturing cost of each component of the WTGS but also provide a reference for operation and maintenance during the life of the WTGS.

Reliability allocation is a top-down process, which means that the specified reliability indexes are reasonably allocated to each subsystem to make the reliability of each subsystem meet requirements. In the early stage of product design, reliability allocation methods that have frequently been used include Aeronautical Radio Inc (ARINC) allocation method, Advisory Group of Reliability of Electronic Equipment (AGREE) allocation method, feasibility of objectives (FOO) allocation method, and so on. Although these classical allocation methods can consider different influencing factors according to the actual situation to determine the allocation weight, they still have many limitations when solving the reliability problems of large and complex mechanical equipment. Therefore, many scholars have studied and improved the reliability allocation technology of WTGSs. For example, Li and Sheng statistically analyzed the reliability analysis method of the WTGS and the fault modes of each component, which provided a reference for the reliability analysis and research of the WTGS. Chen et al. took the mean time between failure (MTBF), mean time to repair (MTTR), annual times of shutdown, power curve compliance, and availability as reliability evaluation indexes in consideration of the actual operation of wind turbines. Combined with fuzzy set theory, the technique for order preference by similarity to an ideal solution (TOPSIS) method and membership function were used to evaluate the system. Through the combination of the expert experience method and entropy weight method, the composite weight of five evaluation indexes was calculated objectively and fairly, and the reliability evaluation of each unit was realized. Zhou et al. based on the analysis of the historical fault maintenance data of 1.5 MW wind turbine, took the number of faults and the average maintenance shutdown time of each subsystem as the influencing factors of reliability allocation and realized the overall reliability index to be allocated to each subsystem from top to bottom. Rajeevan et al. combined the Markov chain with condition monitoring with the analysis of the failure rate and repair rate of each component of the WTGS system and established the reliability allocation model of the wind energy system, thereby improving the reliability. Guo used a weighted comprehensive scoring method based on factors such as fault data, technical level, complexity, and importance to allocate reliability to the entire wind turbine generator set. In this paper, via failure mode effects and criticality analysis (FMECA), reliability allocation based on the failure rate is realized. Zhao et al. adopted a method combining an improved fuzzy analytic hierarchy process and entropy weight method to realize reliability allocation for the WTGS under comprehensive consideration of the working environment, task situation, importance, cost, technical level, and complexity. The above scholars’ research on the reliability allocation method of the WTGS and key components considers increasingly comprehensive factors and is closer to the actual situation. The WTGS is a complex system that includes electronic and machinery and other parts. In addition, the above studies are based on the interdependence of subsystems; that is, failures between subsystems will not affect each other. According to the actual operation of the WTGS, this assumption is not sufficiently accurate, which not only affects the reliability allocation results but also increases unnecessary costs. The failure of the subsystems is related not only to the fatigue caused by long-term operation but also to the failure of other systems, which is the so-called failure correlation. Therefore, the failure correlation should be fully considered in the reliability allocation of the WTGS.

In recent years, scholars have considered the failure correlation of the WTGS and its subsystems and applied it to reliability allocation. For example, Tang proposed a reliability allocation method based on the multivariate copula function, which provides a new idea and new method for mechanical system reliability allocation considering failure correlation. Zhou established a gear transmission system dynamic reliability model considering failure correlation in the WTGS and analyzed and evaluated the component reliability from the perspective of stress intensity. Han et al. introduced the copula function to allocate system reliability to key components considering the correlation and described in detail the specific analysis method. The above scholars have proposed a reliability allocation method for the WTGS that considers the correlation of component failures in combination with the copula function model but only considers the failure and maintenance conditions and does not comprehensively consider the influence of internal and external factors on the reliability of the WTGS.

Based on the above research, vine copula correlation theory was adopted in this paper to establish the reliability allocation model of the WTGS considering different failure correlations. Through in-depth analysis and research on the inside and outside factors of the WTGS, comprehensive consideration of the complexity of the subsystems and the harm degree of each failure mode, including environmental conditions, technical level, the number of components, degree of severity and occurrence of each failure mode and so on, the allocation weight of each subsystem is determined. Then the multivariate Copula function is used to establish the reliability model of the WTGS considering failure correlation , and by introducing the Vine Copula model structure, the above model is simplified, and then the reliability allocation of the
WTGS is completed, which makes the analysis result more realistic and provides some theoretical reference and support for the reliability design and analysis of the WTGS.

2 VINE COPULA MODEL FUNCTIONS

The copula is a function proposed by Sklar to describe the correlation among variables. By Sklar’s theorem17: For the joint distribution function \( F(x_1, x_2, \ldots, x_n) \) of \( n \) dimensional random variables \( X = (X_1, X_2, \ldots, X_n) \), the marginal probability distribution function is \( F_i(x_i) \). For any \( x \in [0, 1]^n \), there exists a copula function \( C(\cdot) \) such that:

\[
F(x_1, x_2, \ldots, x_n) = P(X_1 \leq x_1, X_2 \leq x_2, \ldots, X_n \leq x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))
\] (1)

If \( F_i(x_i) \), \( i = 1, 2, \ldots, n \) is continuous, the copula function \( C(\cdot) \) is uniquely determined. Its joint distribution probability density function is:

\[
f(x_1, x_2, \ldots, x_n) = c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \prod_{i=1}^{n} f_i(x_i)
\] (2)

where \( f_i(x_i) \) is the edge probability density function and \( C(\cdot) \) is the density function of the copula function:

\[
c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) = \frac{\partial^n C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))}{\partial F_1(x_1) \cdot \partial F_2(x_2) \cdots \partial F_n(x_n)}
\] (3)

In recent years, the copula function has been widely used in many fields, such as finance, machinery, and electronics. There are many types of copula functions, and the Archimedean copula function is a common type.

The Archimedes copula function is a particularly important type of function that describes the correlation of variables. The Archimedes copula function form is as follows:

\[
C(u_1, u_2, \ldots, u_n) = \phi^{-1}\{\phi(u_1) + \phi(u_2) + \cdots + \phi(u_n)\}
\] (4)

where \( u_i \in [0, 1] \), and the generating function \( \phi: [0, 1] \to [0, \infty) \) is a strictly decreasing function, satisfying \( \phi(1) = 0 \) and \( x \to \lim_{x \to 0} \phi(x) = 0 \).

According to the different forms of the generating function \( \phi(\cdot) \), the Archimedes copula function can be divided into the forms of Frank copula, Clayton copula, and Gumbel copula.18 The expressions, parameter forms, and tail correlation of several functions are shown in Table 1.

As you can see from Table 1, different copula functions have different tail correlations. The Gumbel copula can describe the asymmetrical correlation between the upper tail height and lower tail height and is very insensitive to changes in the upper tail. The Clayton copula can describe the asymmetrical correlation between the lower tail height and upper tail low, and it is sensitive to changes in the lower tail. The Frank copula can describe the symmetrical tail structure and is insensitive to the change in the correlation between the upper and lower tails. It can not only describe the symmetric correlation between variables but also describe the negative correlation between variables.

In mechanical systems, the Gumbel copula function is commonly used to express failure correlation, which is suitable for exponential and Weibull distributions.19 The parameter \( \theta \) value can be obtained by Kendall correlation coefficient theory.20 Given that the random variables \( X \) and \( Y \) are continuous variables of the copula function \( C(\cdot) \), its Kendall correlation coefficient \( \tau_{ij} \) can be expressed as

\[
\tau_{ij} = 4 \int\int_{[U]} C(u, v) dC(u, v) - 1
\]

where \( u = F_X(\cdot) \) and \( v = F_Y(\cdot) \) are the marginal distribution functions of \( X \) and \( Y \), respectively.

Substituting the copula function expression into the above formula, the calculation formula of the correlation parameter \( \theta_{ij} \) can be obtained as follows:

\[
\theta_{ij} = \frac{1}{1 - \tau_{ij}}
\]

The vine copula function expresses the correlation of multiple variables. It measures and analyses the correlation between multiple variables from a new perspective. The vine copula function is not so much a function as it is a graphical modeling tool that expresses the correlation between multiple variables in the form of vines. The core idea is to decompose the multivariate joint distribution function into the product of the marginal distribution function and multiple binary copula functions.21

## Table 1 Expression and tail correlation of the common Archimedes copula functions

| Copula types | \( C(u, v|\theta) \) | \( \theta \in \Omega \) | Tail correlation \( \lambda \) |
|-------------|-----------------|-----------------|-----------------|
| Gumbel      | \( \exp\left(-\left[\frac{1}{\theta} (\ln u)^2 + (\ln v)^2\right]\right) \) | \( (0, 1) \) | \( \lambda_L = 0; \lambda_U = 2 - 2^{\theta} \) |
| Clayton     | \( (u^{-\theta} + v^{-\theta} - 1)^{\frac{1}{\theta}} \) | \( (0, +\infty) \) | \( \lambda_L = 2^{-1/\theta}; \lambda_U = 0 \) |
| Frank       | \( -\frac{2}{\theta} \ln\left(1 + \frac{u^{1-\theta} - 1}{\theta v^{1-\theta} - 1}\right) \) | \( (-\infty, 0) \cup (0, +\infty) \) | \( \lambda_L = \lambda_U = 0 \) |
There are \( n(n-1)/2 \) ways to decompose the density function of the \( n \) element distribution by using pair copula.\(^{22}\) The two most common types of decomposition are C-vine and D-vine. Both forms have root nodes and \( n-1 \) tree connections to express the failure relationship between multiple variables. Then, the structure of the C-vine and D-vine models of \( n \) dimensional random variables is as follows\(^{23}\):

C-vine model probability density function:

\[ f_C(x_1, x_2, \ldots, x_n) = \prod_{d=1}^{n} f_d(x_d) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{ij+[i+1] \ldots [j-1]} \]

\[ F_{\beta_1,\ldots,\beta_{n-1}}(x_1, \ldots, x_{n-1}, x) = F_{\beta_1}(x_1) \ldots F_{\beta_{n-1}}(x_{n-1}, x) \quad (7) \]

D-vine model probability density function:

\[ f_D(x_1, x_2, \ldots, x_n) = \prod_{d=1}^{n} f_d(x_d) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i+j+j+1 \ldots i+j-1} \]

\[ F_{\alpha_{i+j+1},\ldots,\alpha_{i+j-1}}(x_i, x_{i+j+1}, \ldots, x_{i+j-1}) = F_{\alpha_{i+j+1}}(x_i) \ldots F_{\alpha_{i+j-1}}(x_{i+j+1}, x_{i+j-1}) \quad (8) \]

where \( f_d(x_d) \), \( d = 1, 2, \ldots, n \) is the marginal distribution density function of the variable; \( c_i(\cdot) \) describes the correlation between variables \( i \) and \( j \) and is the density function of the copula function; \( i \) is the edge of each tree; and \( j \) is the level of the tree.

For the conditional distribution function, Joe\(^{21}\) expressed the conditional distribution function as

\[ F_{x|v}(x|v) = \frac{\partial C_{x,v}(F(x), F(v), F(v|v_j))}{\partial F(v_j)} \quad (9) \]

where \( v \) is the vector composed of any part of the elements in the vector \( v \), and \( v_j \) is the vector composed of the remaining elements after removing \( v_j \).

When \( v \) is a single variable, the function can be expressed as

\[ F_{x|v}(x|v) = \frac{\partial C_{x,v}(F(x), F(v))}{\partial F(v)} \quad (10) \]

The failure correlation of the WTGS system is variable because of the internal structure and task conditions of the system or the external technical level and working environment. Therefore, the vine copula function can be used to solve this problem, and it can decompose the multidimensional failure correlation into that between the two systems. The failure correlation between the two systems can be considered separately to make the reliability analysis of the WTGS more precise and reasonable.

However, regarding the above optimization scheme of the WTGS reliability analysis, there are two key problems that must be solved: how to select the appropriate vine copula model tree structure and how to determine the appropriate binary copula function. First, the C-vine model tree structure expresses the strong dependence between variables, and there is a central node in each layer of the tree; each layer of the D-vine model structure is linear, which is generally used to express the relationship between variables without obvious correlation.\(^{22}\) Based on the analysis of the structure and failure mechanism of the WTGS, the failure dependency among subsystems cannot be accurately determined. Therefore, the D-vine model structure is adopted in this paper to establish the failure dependence among the subsystems. Second, for the selection of the optimal vine copula model tree structure and the optimal binary copula function, this paper adopts the Akaike information criterion (AIC) to select the copula function with a good fitting degree on the basis of the data set samples. Meanwhile, the AIC values of different vine copula models are obtained to verify the rationality of the structure selection of the D-vine model in this paper.

AIC is a standard to measure the goodness of fit of statistical models, and its basic model is \( \eta_{AIC} = -2 \ln(L) + 2k \); in this paper, the AIC model is as follows:

\[ \eta_{AIC} = -2 \sum_{i=1}^{n} \ln(L_{i1}, L_{i2}) + 2k \quad (11) \]

where \( k \) is the number of parameters; \( L_{i1}(x_i) \) is the marginal distribution function of the \( i \) th sample of variable \( X_i \); and \( n \) is the sum of the sample size. For the same sample data, the smaller the calculation results of \( \eta_{AIC} \) are, the better the fitting degree.

### 3 WIND TURBINE GENERATOR SET RELIABILITY MODEL BASED ON THE VINE COPULA FUNCTION

As a complex equipment system, the WTGS consists of different subsystems. According to its working principle, the entire system is primarily divided into blades and hub systems, transmission systems, generators, braking systems, yaw systems, towers, and control systems. Their working principle diagram is shown in Figure 1. Through the failure mode, effect, and criticality analysis of the key components of WTGS,\(^{24}\) once one of the key subsystems fails, it will cause the entire machine to be shut down for maintenance and even overall failure, so it is suitable to be expressed in a series logic relationship. The failure logic relationship between the various subsystems of the WTGS is shown in Figure 2.

It is assumed that the life of several subsystems of the WTGS is expressed by a variable as \( X = (X_1, X_2, \ldots X_n) \), where \( n \) is the number of subsystems. Then, the reliability is as follows:

\[ R_i(t) = P(X_i > t) = 1 - F_i(t) \quad (12) \]

where \( F_i(t) \) is the failure rate of the subsystem \( i \).
According to the characteristics of the series system, the reliability of the WTGS is expressed as follows:

\[
R_s(t) = 1 - P\left( \bigcup_{i=1}^{n} X_i \leq x_i \right) \\
= 1 - \sum_{i=1}^{n} P\left( X_i \leq x_i \right) - \sum_{1 \leq i < j \leq n} P\left( X_i \leq x_i, X_j \leq x_j \right) + \cdots \\
+ (-1)^k \sum_{1 \leq i < j < k \leq n} P\left( X_i \leq x_i, X_j \leq x_j, X_k \leq x_k \right) + \cdots \\
+ (-1)^{n-1} P\left( X_1 \leq x_1, X_2 \leq x_2, \ldots, X_n \leq x_n \right) 
\]

(13)

FIGURE 1  Cooperative working principle of the various subsystems of the WTGS

FIGURE 2  Reliability block diagram of the WTGS

Considering the failure correlation between systems, the above formula for the WTGS reliability is combined with the copula function. The joint life distribution function of the \( n \) subsystems with failure correlation is as follows:

\[
F(x_1, x_2, \ldots, x_n) = P(X_1 \leq x_1, X_2 \leq x_2, \ldots, X_n \leq x_n) 
\]

(14)

Because the marginal distribution function \( F_i(x_i) \) is continuous, according to Sklar’s theorem, the expression for the reliability of the WTGS is as follows:

\[
R_s(t) = 1 - \sum_{i=1}^{n} F_i(x_i) - 1ijn \leq \sum_{i=1}^{n} C_{ij}(F_i(x_i), F_j(x_j)) + \cdots \\
+ (-1)^k 1ij\cdots kn \leq \sum_{i=1}^{n} C_{ik}(1, 1, \ldots, F_i(x_i), 1, \ldots, F_j(x_j), 1, \ldots, F_k(x_k), 1, \ldots, 1) \\
+ \cdots + (-1)^{n-1} C_n(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) 
\]

(15)

The above multidimensional copula function can be directly expressed by the multivariate Gumbel copula function, but it is necessary to assume that multiple systems have the same degree of failure, that is, the parameter \( \theta \) values are uniform. Assuming that the target reliability of the system composed of four subsystems is 0.95, the system reliability is calculated using the multivariate Gumbel copula function calculation model. The system reliability variation under different \( \theta \) values is shown in Figure 3. As the \( \theta \) value increases, the system reliability also increases.

To accurately express the failure correlation between subsystems, this paper proposes the decomposition calculation formula of equation (8) using the structure of vines. If the D-vine model structure is adopted to decompose the multivariate copula function in this paper, the specific decomposition model structure diagram is shown in Figure 4.
FIGURE 4 Subsystem failure correlation structure diagram based on the D-vine

Using formula (2) and formula (8), combined with the failure correlation model diagram of each subsystem of the WTGS, the multidimensional copula function is converted into the expression of multiple two-dimensional copula functions, as follows:

\[
c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) = \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,j+i+1,\ldots,i+j-1}(F_{i|i+1,\ldots,i+j-1}(x_i | x_{i+1}, \ldots x_{i+j-1})),
\]

\[
F_{i+j|i+1,\ldots,i+j-1}(x_{i+1}, \ldots x_{i+j-1})
\]

(16)

Combining the above formulas, the reliability model of the WTGS is as follows:

\[
R_s(t) = 1 - \left[ \sum_{i=1}^{n} F_i(x_i) - \sum_{1 \leq i \leq n} C_2(F_i(x_i), F_j(x_j)) \ldots \right.
\]

\[
+(-1)^{k-j} \sum_{i=1}^{n-j} \prod_{k \leq i \leq l} c_{i+j+1,\ldots,i+j-1}(F_{i+j+1,\ldots,i+j-1}(x_i | x_{i+1}, \ldots x_{i+j-1})),
\]

\[
F_{i+j|i+1,\ldots,i+j-1}(x_{i+1}, \ldots x_{i+j-1})dF_1(x_1)dF_2(x_2)\ldots dF_n(x_n)
\]

\[
+(-1)^{n-j-1} \left\{ \prod_{i=1}^{n-j} \prod_{j=1}^{n} c_{i+j+1,\ldots,i+j-1}(F_{i+j+1,\ldots,i+j-1}(x_i | x_{i+1}, \ldots x_{i+j-1})),
\]

\[
F_{i+j|i+1,\ldots,i+j-1}(x_{i+1}, \ldots x_{i+j-1})dF_1(x_1)dF_2(x_2)\ldots dF_n(x_n) \right\]

(17)

The parameter expression is omitted in the above formula, but when calculating the reliability, the failure correlation degree parameter \( \theta_{i+j|i+1,\ldots,i+j-1} \) between different systems needs to be calculated according to the actual situation to distinguish the difference.

4 | RELIABILITY ALLOCATION METHOD OF A WIND TURBINE GENERATOR SET CONSIDERING FAILURE CORRELATION

4.1 | Failure mode and critical analysis of the WTGS

Xie\textsuperscript{25} and Wang et al\textsuperscript{26} proposed that the complexity of the system is determined by the number of system components, operating environment, and technical level factors. Therefore, the complexity of the WTGS system is expressed as

\[
I_i = \frac{h(n_i, T_i, E_i)}{\sum_{j=1}^{n} h(n_j, T_j, E_j)}
\]

(18)

where \( E_i \) represents the working environment factor of the WTGS, the value range is \( E_i \in (0, 1] \), and the value increases with the increase in the harsh environment. \( T_i \) is used to indicate the technical level of the turbines, and the value range is \( T_i \in (0, 1] \); the higher the technical level is, the more concentrated the dispersion of its product performance will be. In addition, in the above formula, function \( h(n_i, T_i, E_i) = n_i^{1+(T_i/E_i)} \).

Importance refers to the level of influence on the reliability of the entire system when the WTGS subsystem fails. Researchers have proposed that, on the basis of the risk priority number (RPN) method,\textsuperscript{27} it is more reasonable to convert the severity \( S_u \) and occurrence \( O_p \) of each failure mode before calculating the criticality of the subsystem.\textsuperscript{28}

The severity calculation formula is as follows:

\[
\begin{align*}
S_u &= \exp(\alpha S_u) \\
O_p &= \max(S_{11}, S_{12}, \ldots S_{MN})
\end{align*}
\]

(19)

where \( \alpha \) is an undetermined coefficient, and the larger its value is, the more obvious the gap between different score levels will be after conversion. This paper takes \( \alpha = 0.6 \) from the literature.\textsuperscript{29}

Because of the consideration of the influence on the failure correlation between the various subsystems, this article converts the occurrence degree according to the actual operating environment and technical level of the WTGS. The formula is as follows:

\[
\bar{O}_{ij} = \exp \left( \beta \left( \frac{T_i}{E_i + \frac{T_i}{I_i}} \right) O_{ij} + \gamma \right)
\]

\[
\bar{O} = \sum_{j=1}^{N_i} \bar{O}_{ij}
\]

(20)

where \( \beta \) and \( \gamma \) can be determined based on existing similar products, and their values can be determined under the
assumption that the failures between subsystems are completely independent. This paper takes $\beta = 0.8$ and $\gamma = -10$ from the literature.\(^{29}\)

4.2 WTGS reliability allocation considering the failure correlation of the subsystems

In reliability engineering, the analysis of system failure modes is a prerequisite for improving system reliability and reducing maintenance costs. Therefore, experts proposed the relationship between the failure rate and the cost of reducing the failure rate\(^{28}\):

$$P_i = -\frac{\ln O_i}{r}$$ \hspace{1cm} (21)

where $r$ is an undetermined coefficient related to the cost resources, such as manpower and materials consumed in product improvement. This paper takes $r = 100$.

Normalize $O_i$ and $P_i$, and the criticality of the subsystems $C_i$ is

$$C_i = \frac{S_i/P_i}{\sum_{i=1}^{n} S_i/P_i}$$ \hspace{1cm} (22)

The calculation formula for the weight $\omega_i$ of reliability allocation among the various subsystems of the WTGS is as follows:

$$\omega_i = \frac{I_i/C_i}{\sum_{i=1}^{n} I_i/C_i}, \quad i = 1, 2, \ldots, n$$ \hspace{1cm} (23)

According to the allocation weight, the allocation ratio is determined as follows:

$$\lambda_i = w_i, \quad i = 1, 2, \ldots, n$$ \hspace{1cm} (24)

The failure time of the WTGS usually obeys three distribution forms: the Weibull distribution, gamma distribution, and lognormal distribution.\(^{4}\) Here, the two-parameter Weibull distribution is taken as an example to discuss reliability allocation:

$$R_i(t) = e^{-\left(\frac{t}{\beta}\right)^{\gamma}}$$ \hspace{1cm} (25)

where $t \geq 0$ and $\beta$ and $k_i$ are the scale parameter and shape parameter of the Weibull distribution, respectively.

Therefore, the reliability function under consideration of various influencing factors is as follows:

$$R_i(t) = \lambda_i \cdot \lambda_0 \cdot \left[e^{-\left(\frac{t}{\beta}\right)^{\gamma}}\right]$$ \hspace{1cm} (26)

where $\lambda_0$ is the benchmark allocated to the reliability of the subsystems.

5 | CASE ANALYSIS AND VERIFICATION

5.1 Wind turbine reliability allocation considering failure correlation

The reliability calculation model based on the vine copula function in Section 3 is used to allocate the reliability of seven key subsystems of the WTGS. It is assumed that the system reliability is 0.95 after one year. Taking the historical failure data statistics of a horizontal axis wind turbine on wind farm\(^{30}\) as an example, the failure mode analysis of seven key subsystems is performed, and the values of the influencing factors and the allocated weights are shown in Table 2, namely, the Con—control system, BH—blades and hub, Tra—transmission system, Gen—generator, Bra—brake system, Yaw—yaw system, and Tow—tower.

| Subsystems | $E_i$ | $T_i$ | $S_i$ | $O_i$ | $P_i$ | $I_i$ | $C_i$ | $\omega_i$ |
|------------|------|------|------|------|------|------|------|----------|
| Con        | 0.55 | 0.45 | 64.68 | $1 \times 10^{-3}$ | 0.0690 | 0.1035 | 0.1022 | 0.0983   |
| BH         | 0.49 | 0.39 | 110.51 | $6.38 \times 10^{-4}$ | 0.0736 | 0.0938 | 0.1746 | 0.1271   |
| Tra        | 0.44 | 0.62 | 66.69 | $5.44 \times 10^{-4}$ | 0.0750 | 0.1423 | 0.0940 | 0.1508   |
| Gen        | 0.58 | 0.48 | 20.09 | $2.24 \times 10^{-4}$ | 0.0840 | 0.1553 | 0.0253 | 0.1377   |
| Bra        | 0.67 | 0.31 | 36.60 | $7.53 \times 10^{-5}$ | 0.0949 | 0.0859 | 0.0407 | 0.1633   |
| Yaw        | 0.32 | 0.55 | 121.51 | $8.82 \times 10^{-4}$ | 0.0703 | 0.2767 | 0.1826 | 0.1581   |
| Tow        | 0.81 | 0.89 | 221.41 | $8.53 \times 10^{-4}$ | 0.0615 | 0.1425 | 0.3806 | 0.1647   |
Combining the fault information of each subsystem with formula (5), the Kendall rank correlation coefficient $\tau$ between each subsystem is calculated. The matrix element values show that the correlation between each subsystem is variable, there are large and small values, and some subsystems are not even related.

According to the above research on the method of determining the tree structure of the vine copula model and the binary copula function, the AIC analysis method was adopted to analyze the fault information of each subsystem based on the C-vine and D-vine model structure and different copula function models. AIC values are shown in Table 3.

It can be seen from Table 3 that compared with the two vine model trees, the value of the D-vine model structure is the smallest; therefore, it is reasonable to use the D-vine model structure to express the correlation between subsystems in the paper. At the same time, compared with the AIC values of different copula functions, the Gumbel copula function has the best fitting degree. Therefore, the reliability allocation of seven key subsystems of the WTGS is carried out through the following three calculation models: the allocation model based on the assumption of independent subsystems, the allocation model based on the multivariate Gumbel copula function (taking the average of $\theta = 1.66$ of all correlation parameters), and the allocation model based on the vine copula calculation model.

Assuming that the subsystems are independent, the reliability of the subsystems can be calculated from the following equation:

$$ R_s = \prod_{i=1}^{7} \lambda_i \cdot \lambda_0 \left[ e^{-\left( \frac{1}{\theta} \right)} \right] = 0.95 \quad (27) $$

where the reliability value of each subsystem is obtained by substituting the obtained value $\lambda_0$ into formula (26).

By substituting the copula function in Equation (15) with the expression of the Gumbel copula function and combining Equations (10) and (17), the reliability of each subsystem is solved. Table 4 shows the reliability values of each subsystem under the three models.

### Result analysis

1. According to the weakest link theory, the reliability of the WTGS system should meet:

$$ \prod_{i=1}^{n} R_i(t) \leq R_s(t) \leq \min(R_1(t), R_2(t), \ldots, R_n(t)) \quad (28) $$

where $R_i(t)$ is the reliability of the $i$th subsystem, $i = 1, 2, \ldots, 7$, and $R_s(t)$ is the reliability of the WTGS. Substitute the reliability value of the subsystem in the third row of Table 3 into the above formula: $0.9462 < R_s < 0.9899$, which satisfies the reliability allocation criterion. Therefore, the reliability allocation method of wind turbines in this paper is effective.

2. The reliability of each subsystem in the third column of Table 3 shows that the reliability of the control system is the lowest, the tower is the highest, and the blades and hub system, the transmission system, the generator,

| Model category | C-vine | D-vine | Gumbel | Clayton | Frank |
|----------------|--------|--------|--------|---------|-------|
| AIC            | −421.3016 | −532.6795 | −84.2603 | −74.7334 | −70.4959 |

### TABLE 3 AIC values of different vine model tree structures and copula function models

| Models                              | Results | Con  | BH   | Tra  | Gen  | Bra  | Yaw  | Tow  |
|-------------------------------------|---------|------|------|------|------|------|------|------|
| The independent model               | 0.9919  | 0.9942 | 0.9954 | 0.9948 | 0.9959 | 0.9957 | 0.9962 |
| The multivariate Gumbel copula function model | 0.9902  | 0.9930 | 0.9940 | 0.9936 | 0.9945 | 0.9944 | 0.9950 |
| The vine copula calculation model   | 0.9899  | 0.9903 | 0.9916 | 0.9915 | 0.9920 | 0.9936 | 0.9960 |

### TABLE 4 Reliability allocation results under three methods
the braking system, and the yaw system are in the middle. The cumulative failure rate shows the opposite behavior; it is highest for the control system, lowest for the tower, and intermediate for the other systems. This result is consistent with the actual operation of each subsystem. Figure 5 shows that the abovementioned reliability allocation result is basically consistent with the average value of the cumulative failure rate of each key subsystem, which verifies the rationality of the proposed method.

As shown in Figure 6, the allocation results of the three methods are basically identical. The results are lower when considering the failure correlation than when assuming independence between subsystems, and compared with the reliability solution method based on the multivariate copula function, the allocated results based on the vine copula function are reduced. At the same time, the reliability of the seven subsystems in this paper is allocated using the reliability allocation method of Zhou’s literature, which only considers the fault maintenance data. Compared with the proposed allocation method that comprehensively considers internal and external factors, the reliability results allocated to each subsystem have the same trend, but the overall trend is higher. This result verifies the effectiveness and rationality of the proposed method.

The allocation results show that the proposed reliability allocation method based on the vine copula function has greatly increased the failure rate assigned to each subsystem compared with those of the other two methods. The increase percentage is shown in Table 5.

Table 5 shows that when the failure rate of the WTGS is maintained at 0.05, that of each subsystem is increased by more than 50% compared with the two allocation methods considering failure correlation and assuming independence. For the two allocation methods that consider failure correlation, the allocation method based on the vine copula increases the failure rate of each subsystem by more than 20% compared with that of the multivariate copula function.

6 | CONCLUSIONS

In this paper, combined with the structural and functional characteristics of the WTGS system and the actual operating conditions, the reliability allocation calculation method of the seven key subsystems is explored and researched. The article comprehensively considers the internal and external factors that affect the reliability of the WTGS, including complexity, failure hazard, technical level, working environment, and cost. Three methods of assumption independence, a multivariate copula function calculation model, and a vine copula function calculation model are adopted to allocate reliability to the key problems. Compared with the method in Zhou’s literature, the allocation results in this article are basically consistent and have been improved. In this paper, the vine copula function calculation model is used instead of the multivariate copula function model to allocate the reliability of the WTGS considering the failure correlation. Not only the difference in the degree of failure correlation between
the subsystems can be considered, but it can also be closer to the operating conditions of the WTGS when performing reliability allocation, thus making the allocation result more reasonable and effective.

Through comparative analysis of the allocation results of the three methods, the allocation method based on the vine copula function has been allocated a lower reliability to each subsystem, that is, the failure rate is higher. In this way, not only is the allocated reliability more reasonable but also the design, manufacturing, and maintenance costs of the WTGS can be reduced, thereby improving their overall economic benefits. However, the exploration of the calculation method for the reliability allocation of the WTGS proposed in this paper is only on the logical basis of series connection, and the target reliability is set after one year. The actual WTGS system is more complicated, and it is difficult to accurately express its failure logic only with a simple series system; its life span is generally approximately 20 years. For specific applications in engineering, it must be deduced according to specific conditions.

ACKNOWLEDGMENTS
This work was supported by the National Natural Science Foundation of China (Grant No.51565055).

ORCID
Yuanyuan Wu https://orcid.org/0000-0003-0048-0019

REFERENCES
1. Pérez J, Márquez F, Tobias A, Papaelias M. Wind turbine reliability analysis. Renew Sustain Energy Rev. 2013;23:463-472.
2. Fang M. Research on Reliability Allocation Method of Mechanical System in Conceptual Design Stage. Tianjin: Tianjin University; 2013.
3. Forcina A, Silvestri L, Bona DG, Silvestri A. Reliability allocation methods: a systematic literature review. Qual Reliab Engng Int. 2020;36(6):1-23.
4. Li Y, Zhu C. Research and development of the wind turbine reliability. Int J Mech Eng Technol. 2018;6(2):35-45. http://dx.doi.org/10.11648/j.ijmea.20180602.14
5. Sheng S, O’Connor R. Reliability of wind turbines. Wind Energy. 2017;299-327.
6. Chen H, Pang Z, Xu Z, Ouyang C. Research on reliability of wind turbine based on fuzzy evaluation of operation data. Automat Instrum. 2019;6:87-90.
7. Zhou X, Li Z. Reliability allocation of MW scale permanent magnet direct-drive wind power generator based on failure maintenance date. Coal Mine Machinery. 2016;37(9):84-87.
8. Rajeevan AK, Shouri PV, Nair U. Markov modeling and reliability allocation in wind turbine for availability enhancement. Life Cycle Reliab Safe Eng. 2018;7(3):147-157.
9. Guo X. Study on Reliability on and its Growth Technology of Wind Power Turbines. Jilin: Northeast Dianli University; 2016.
10. Zhao L, Sun W, Zhang S. Research on the method of wind power generator reliability allocation based on combination weights. Renewable Energy Resources. 2018;36(9):1369-1374.
11. Sun Y, Ma L, Mathew J, Zhang S. An analytical model for interactive failures. Reliab Eng Syst Safe. 2006;91(5):495-504.
12. Cheng Y, Du X. System reliability analysis with dependent component failures during early design stage - a feasibility study. J Mech Des. 2016;138(5):51405.
13. Chen Y, Yang L, Ye C, Kang R. Failure mechanism dependence and reliability evaluation of non-repairable system. Reliab Eng Syst Safe. 2015;138(6):273-283.
14. Tang J, Zhao Y, He P, Song D. Copulas new theory for reliability calculation involving correlation in mechanical systems. Mech Sci Technol Aerospace Eng. 2009;28(4):532-535.
15. Zhou Z, Qin D, Yang J, Chen H. Time-dependent reliability analysis of gear transmission system of wind turbine considering dependent failure. Acta Energiae Solar Sin. 2013;34(7):1212-1219.
16. Han S, Dong H. System reliability analysis model of wind turbine based on Copula function. J Lanzhou Jiaotong Univ. 2016;35(6):90-94.
17. Sklar A. Fonctions de répartition à n dimensions et leurs marges. Publ Inst Stat Univ Paris. 1959;8:229-231.
18. You Y, Fang R, Li X. Allocating active redundancies to k-out-of-n reliability systems with permutation monotone component lifetimes. Appl Stochastic Model Business Indust. 2016;32(5):607-620.
19. Huang M, Wang Q, Li Y, Liang A. An approach for improvement of avionics reliability assessment based on copula theory. In: The Proceedings of 2011 9th International Conference on Reliability, Maintainability and Safety, IEEE. 2011; 179-183.
20. Kendall M, Stuart A, Ord JK. Kendall’s Advanced Theory of Statistics, vol. 1. New York, NY: Oxford University Press; 1987.

---

| Symbol of subsystems | Compared with independent method/% | Compared with Gumbel copula method/% |
|----------------------|-----------------------------------|-------------------------------------|
| Con                  | 24.6                              | 3.06                                |
| BH                   | 67.24                             | 38.57                               |
| Tra                  | 82.61                             | 40                                  |
| Gen                  | 63.46                             | 32.82                               |
| Bra                  | 95.13                             | 45.45                               |
| Yaw                  | 48.84                             | 14.29                               |
| Tow                  | 5.26                              | -20                                 |
| Average/%            | 55.31                             | 22.03                               |

**TABLE 5** Percentage increase in the failure rate of each subsystem
21. Dorota K, Joe H. Dependence modeling: vine copula handbook. World Scientific. 2010;6-33.
22. Aas K, Czado C, Frigessi A, Frigessi A, Henrik B. Pair-copula constructions of multiple dependence. Insurance Math Econ. 2006;44(2):182-198.
23. Bedford T, Cooke RM. Probability density decomposition for conditionally dependent random variables modeled by vines. Ann Math Artific Intell. 2001;32(1):245-268.
24. Magomedov IA, Magomadov VS, Rahimov AA, Alikhadzhiev SK, Gudaev MA. FMMA and FMECA for analysis of reliability of a wind turbine. J Phys Conf Ser. 2019;1399:55074.
25. Xie L, Wang Z, Zhou J. Basic Theories and Methods of Mechanical Reliability (2nd ed.). Beijing: Science Press; 2012:156-157.
26. Wang Y, Yam R, Zuo M, Tse P. A comprehensive reliability allocation method for design of CNC lathes. Reliab Eng Syst Saf. 2001;72(3):247-252.
27. Kang R, Zheng T. Fuzzy mathematics method in criticality analysis. J Beijing Univ Aeronaut Astronaut. 1995;21(4):60-65.
28. Yadav OP, Zhuang X. A practical reliability allocation method considering modified criticality factors. Reliab Eng Syst Saf. 2014;129(14):57-65.
29. Zhang Y, Yu T, Song B. A reliability allocation method of mechanism considering system performance reliability. Qual Reliab Eng Int. 2019;35:1-21.
30. Yang M. Falut Mode Statistic & Analysis and Failure Diagnosis of Large-scale Wind Turbines. Beijing: North China Electric Power University; 2009.

How to cite this article: Wu Y, Sun W. Research on the reliability allocation calculation method of a wind turbine generator set based on a vine copula correlation model. Energy Sci Eng. 2021;00:1–11. https://doi.org/10.1002/ese3.927