1. Introduction

Recrystallization is a process of fundamental importance in the thermomechanical processing of metals since it directly affects the microstructures in the final product. Hence, very extensive research has been conducted during the last century on the kinetics of the recrystallization process. The so-called Johnson–Mehl–Avrami–Kolmogorov equation \[ \frac{X_V}{H} = 1 - \exp\left(-kt^n\right) \] \hspace{1cm} (1) is the most widely-used theory for recrystallization and other reactions involving both nucleation and growth. This JMAK theory assumes a random distribution of nucleation sites, and then the fraction recrystallized material \( X_V \) can be derived rather simply as being

The principal parameters in the equation are the rate parameter \( k \) and the exponent \( n \), which is generally referred to as the Avrami exponent. This exponent is usually determined by plotting the equation in a double logarithmic form and taking \( n \) to be the slope of a \( \log(\log(1/(1-X_V))) \) vs. \( \log(t) \) plot. For three-dimensional growth, \( n \) is equal to 3 in the case of site saturation and a constant growth rate, 4 when both the rates of nucleation and growth are constant, and can be less than 3 if growth rate is decreasing with the proceeding recrystallization. In the case of two-dimensional or one-dimensional growth, lower values of \( n=2 \) or \( n=1 \) are predicted, respectively. Therefore, the Avrami exponent is of great importance for the understanding of recrystallization kinetics, and reveals the characteristics of nucleation and growth kinetics, although the relationship is not strictly mutual as different process conditions can lead to the same Avrami exponent.

Extensive experimental research has been focused on the kinetics of recrystallization and its interpretation in the framework of the JMAK concept, and the results can be summarized as follows:

I. The basic JMAK theory is applicable in the case of lightly deformed fine-grained metals. For example, Anderson and Mehl\(^7\) measured rates of nucleation and growth in lightly deformed fine-grained sheets of aluminum, and made a successful link between the theoretically predicted value and the experimental value for the Avrami exponent. Under similar conditions, Gordon\(^8\) studied the recrystallization of copper and obtained an Avrami exponent of approximately 4, identical to the value predicted by the JMAK model.

II. The majority of experimental studies on the recrystallization kinetics of steels, however, have shown that the Avrami exponent has a value less than 2, without any microstructural evidence that the nuclei grew in one or two dimensions.\(^1\)–\(^6\)

III. The Avrami exponent is in many cases not a constant but decreases with proceeding recrystallization.\(^1,9\)

IV. The Avrami exponent is insensitive to the deformation temperature and deformation conditions. In most studies on static recrystallization in steels the exponent is generally considered to be a constant independent of the temperature\(^2\)–\(^5\) or slightly decreasing with decreasing temperature.\(^6\) Luton \textit{et al.}\(^10\) and Ruibal \textit{et al.}\(^11\) found constant Avrami exponents smaller than 2 for a range of copper and low-alloy steels. Similarly, Laasraoui and Jonas\(^4\) have found that, over a wide range of temperatures and strain rates, the Avrami exponent is less than 1 for static recrystallization in low-

Effect of Inhomogeneous Deformation on the Recrystallization Kinetics of Deformed Metals

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In this paper, the Johnson–Mehl–Avrami–Kolmogorov equation, modified by Rollett \textit{et al.} (1989) to take the heterogeneity of recrystallization into account, was used to derive analytical equations to calculate the instantaneous and average Avrami exponent during the heterogeneous recrystallization process. The model can explain experimental observations, such as a lower exponent than predicted by the JMAK theory and a decreasing exponent with proceeding recrystallization; moreover, by building up a quantitative relationship between local strain variation and heterogeneity of recrystallization kinetics, it can also describe so-far unexplained observations, such as the range of experimentally observed exponents in recrystallization, and the very small effect of temperature and deformation conditions on the Avrami exponent.

KEY WORDS: inhomogeneous deformation; recrystallization kinetics; Avrami exponent; simulation.

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carbon steels containing various combinations of niobium, boron and copper, and concluded that this exponent shows independence of temperature and deformation. Sellars et al. also observed a similar temperature and stress rate independence for C-Mn and stainless steels.

In summary, the JMAK approach met with some difficulties to explain why the theoretically predicted Avrami exponent is generally higher than the experimentally measured one. Moreover, it cannot explain why the observed exponent is independent of deformation conditions and temperature. Humphreys et al. have concluded that the heterogeneity of recrystallization is responsible for the lower value of the Avrami exponent than that predicted by the JMAK theory.

In the present paper, the modification of the classical JMAK approach by Rollett et al. to take into account the heterogeneity of recrystallization, was used to derive analytical equations to calculate the Avrami exponent in the case of heterogeneous recrystallization. The simulations made on the basis of these equations show the origin of a smaller exponent, and can explain the independence of Avrami exponent on temperature and deformation, and even its range. Moreover, it is shown that the difference between the observed Avrami exponent and the actual local exponent is significant in the interpretation of the recrystallization kinetics.

2. Theoretical Model for the Kinetics of Heterogeneous Recrystallization

The initial microstructure at the start of the recrystallization is a deformed structure, which in most cases is not homogeneous, but varies locally in the degree of plastic deformation i.e. stored energy. This leads to the rate of recrystallization varying locally in the material. In order to take the heterogeneity of recrystallization into account, the present model starts from the idea put forward by Rollett et al., the material that is about to recrystallize can be divided into classes, with the recrystallisation kinetics in each class, labeled $i$, being described by the JMAK equation

$$X_{\text{Vex}}^i = 1 - \exp(-X_{\text{Vex}}^i) = 1 - \exp(-k_i t^i) \quad i = 1, \ldots, N$$

Where $X_{\text{Vex}}^i$ is known as the extended volume in the classical JMAK theory, i.e. the fraction of material, which would have recrystallize if the phantom nuclei were real. In the case of nucleation and growth rates both varying with time, the extended volume is:

$$X_{\text{Vex}}^i = f_i^s \left[ \int_0^t G^i(t')dt' \right]^3 \int_0^t N^i(t')dt'$$

where $f_i^s$ is a shape factor, for an example, $4/3\pi$ for a sphere; $G(t)$ and $N(t)$ are growth rate and nucleation rate respectively, they can be described by:

$$G^i(t) = G_0^i \frac{dG^i}{dt} \quad \text{and} \quad N^i(t) = N_0^i \frac{dN^i}{dt}$$

where $G_0^i$ and $N_0^i$ are coefficients of first-order differential time-dependent functions of growth and nucleation, respectively. Equation (3) incorporated with Eq. (4) can be compared with Eq. (2) as follows:

$$k_i t^i \sim f_i^s (G_0^i)^3 N_0^i \left[ \frac{dG^i(t)}{dt} \right]^3 \frac{dN^i(t)}{dt} \quad \text{..........(5)}$$

Then, it can be inferred that the rate parameter $k_i$ is determined by $f_i^s (G_0^i)^3 N_0^i$, i.e. the shape factor and the coefficients of first-order differential time-dependent functions of growth and nucleation, that is, the rate parameter $k_i$ obviously varies locally due to the variation of both nucleation and growth kinetics resulting from inhomogeneous deformation or inhomogeneous microstructure; whilst the exponent $n_i$ may be determined by $- \left[ \frac{dG^i(t)}{dt} \right]^3 \frac{dN^i(t)}{dt} / d\tau$, i.e. the first-order differential forms of the time-dependent growth and nucleation kinetics. Since we artificially divide the recrystallizing material into numerous classes, in which the classical JMAK equation can be applied, the recrystallization mechanism in each class would be same. Thus, it can be reasonably expected that the time dependence of nucleation and growth rates in each class is equal or at least similar, that is, $n_i = n$, as the nature of the recrystallisation process is the same in each class. This assumption is also in line with the experimentally observed weak dependence of $n$ on temperature and deformation conditions.

In the case of site-saturated nucleation, Eq. (3) may be changed to

$$X_{\text{Vex}}^i = f_i^s N_0^i \left[ \int_0^t G^i(t')dt' \right]^3 \quad \text{..........(6)}$$

$N_0^i$ is the number of nuclei at the beginning of recrystallization. It can be then known that the exponent $n$ is only determined by the growth kinetics whilst a heterogeneous distribution of nuclei does not affect the exponent $n$ at all. In conclusion, $n_i = n$, can be reasonably assumed even if both nucleation and growth kinetics vary locally. Then, the overall recrystallized fraction is given by

$$X_v = \frac{1}{N} \sum_{i=1}^{N} (1 - \exp(-k_i t^i)) \quad \text{..........(7)}$$

Since there is no experimental evidence available for the distribution of $k$ due to the heterogeneity of the plastic deformation, an assumption has to be made. Here, a simple assumption is taken that $k$ uniformly distributed between $k_{\text{max}}$ and $k_{\text{max}}$ and $\Delta k = (k_{\text{max}} - k_{\text{min}}) / N$, in which $k_{\text{max}}$ and $k_{\text{min}}$ should be seen as effective values representing the actual distribution of $k$, which is of course not necessarily uniform.

The right hand side of Eq. (7) can be integrated for $N$ approaching infinite, leading to

$$X_v = \frac{1}{\Delta k} \sum_{i=1}^{N} (1 - \exp(-k_i t^i)) \cdot k_{\text{max}} - k_{\text{min}}$$

$$= \int_{k_{\text{max}}}^{k_{\text{min}}} [1 - \exp(-k t^n)]dk$$

$$= 1 + \frac{\exp(-k_{\text{max}} t^n) - \exp(-k_{\text{min}} t^n)}{t^n (k_{\text{max}} - k_{\text{min}})} \quad \text{..........(8)}$$
The range of variation of local recrystallization kinetics is defined by the factor \( m = k_{\text{max}}/k_{\text{min}} \) which leads to

\[
X_v = 1 + \frac{\exp(-k_{\text{max}} t^n) - \exp(-k_{\text{min}} t^n)}{t^n (k_{\text{max}} - k_{\text{min}})} = 1 + \frac{\exp(-k_{\text{max}} t^n) - \exp(-k_{\text{max}} t^n/m)}{t^n (k_{\text{max}} - k_{\text{max}}/m)} \quad \text{.........(9)}
\]

It should be mentioned that Eq. (9) is similar to Eq. (8) in Ref. 13), derived by Rollett et al., which is given as

\[
X_v = 1 - \frac{\exp(-k_{\text{max}} t^n) - \exp(-k_{\text{min}} t^n)}{k_{\text{max}} - k_{\text{min}}} \quad \text{.........(10)}
\]

The Eqs. (9) and (10) differ in two points: (i) the factor \( t^n \) is in the numerator in Eq. (10) and in the denominator in Eq. (9), and (ii) a “−” appears in Eq. (10) where there is a “+” in Eq. (9). It is readily seen that the original equation in Ref. 13) (Eq. (10)) is affected by typing errors, since (i) the dimensions are not correct if \( t^n \) is in the numerator, and (ii) \( X_v > 1 \) results from Eq. (10) since \( k_{\text{max}} > k_{\text{min}} \).

Equation (9), however, is still not enough to calculate a value of Avrami exponent that is directly comparable to experimental observations given in the literature, since most of the experimental data on recrystallization kinetics are analyzed by the simple JMAK equation and just one value of the exponent is derived for the entire process. In order to produce a value of the Avrami exponent that can directly be compared with the extensive literature data, Eq. (9) has been differentiated to derive the transient value of the Avrami exponent during the recrystallization, \( n_v \), i.e. the momentary slope of the JMAK plot of \( \ln(\ln(1/(1-X_v))) \) vs. \( \ln(t) \), by

\[
n_v = \frac{d \ln(\ln(1/(1-X_v)))}{d \ln(t)}
\]

\[
d \ln \left\{ \frac{t^n (k_{\text{max}} - k_{\text{min}}/m)}{\exp(-k_{\text{max}} t^n) - \exp(-k_{\text{max}} t^n/m)} \right\}
\]

\[
= \frac{d \ln(t^n)}{d \ln(t)} \quad \text{.........(11)}
\]

Mathematic processing leads to the transient Avrami exponent being given by

\[
n_v = -\frac{n}{m} \left\{ m \cdot \exp(-k_{\text{max}} t^n) - m \cdot \exp(-k_{\text{max}} t^n/m) \right. \\
+ k_{\text{max}} \cdot \exp(n \cdot \ln(t) - k_{\text{max}} t^n) \cdot m \\
- k_{\text{max}} \cdot \exp \left\{ \frac{n \cdot \ln(t) - k_{\text{max}} t^n}{m} \right\} \\
\left[ \exp(-k_{\text{max}} t^n) + \exp(-k_{\text{max}} t^n/m) \right] \\
\times \ln \left\{ \frac{t^n \cdot k_{\text{max}} (1-m)}{m \cdot \exp(-k_{\text{max}} t^n) - \exp(-k_{\text{max}} t^n/m)} \right\} \quad \text{.........(12)}
\]

Note that Eq. (12) has a singular point at \( m = 1 \), for which \( n_v \) is indeed equal to \( n \).

In order to derive a single value that characterizes the entire curve, we average the value of \( n \) between \( X_v = 0.01 \) and \( X_v = 0.99 \), similar to what is common practice in using the JMAK-equation. Let the time for 1% recrystallization be \( t_1 \) and that for 99% recrystallization \( t_2 \), then the average exponent for the entire process of recrystallization is given by

\[
n_{av} = \frac{\int_{t_1}^{t_2} n_v dt}{t_2 - t_1} \quad \text{.........(13)}
\]

where \( n_v \) is a function of \( t, m, k_{\text{max}} \) (Eq. (8)), and \( t_2 \) and \( t_1 \) are functions of \( m \) and \( k_{\text{max}} \). This average value should be comparable to the values of Avrami exponent widely reported in the literature.

The kinetic parameter \( k \) in the JMAK equation, however, is still difficult to determine since it varies with chemical composition, grain size, temperature and deformation conditions. As discussed above, the heterogeneity of the recrystallization kinetics in the material is represented by the local difference of \( k \), so it is important to know the variation of \( k \) for the recrystallization. From Eq. (1), it can be derived that \( k = 0.693 t_{0.5}^{-0.6} \), in which \( t_{0.5} \) is the time for 50% accomplishment of recrystallization. As investigated by many researchers, the time parameter \( t_{0.5} \) is related to the local strain by its proportionality to \( e^{-p \cdot t} \). As an example, \( p \) varies between 3.55 and 3.81 for steels. This leads to \( m \) being given by

\[
m = \frac{k_{\text{max}}}{k_{\text{min}}} = \frac{e^{n_p}}{e^{n_{\text{min}}}} \quad \text{.........(14)}
\]

where \( e_{\text{min}} \) is the lowest strain level and \( e_{\text{max}} \) the highest strain level. With Eq. (14) it is now possible to calculate the heterogeneity of recrystallization kinetics from the magnitude of local strain variation.

3. Results and Discussion

3.1. Curves of log(ln(1/(1-X_v))) vs. log(t) and Temperature/Deformation Dependence of Avrami Exponent

Figure 1 shows typical examples of the recrystallisation kinetics as predicted for a number of relevant parameter values. The figure shows that the JMAK plots have a constant slope at the beginning of the recrystallization process, which then gradually decreases with the proceeding recrystallization. In addition, when \( m \) decreases or \( k_{\text{max}} \) or \( n \) increases, recrystallisation is getting faster. These simulation results are not only consistent with experimental observations, but also with other simulation results. For example, the Monte Carlo simulations, which have been done by Srolowitz et al., Rollett et al., and Radhakrishnan et al. respectively, also show that the slope of JMAK plot decreases with the progress of recrystallization.

The instantaneous Avrami exponent during the process of recrystallisation can be derived by Eq. (12) and is plotted against time on a logarithmic scale as shown in Figure 2(a). The arrows in the figure indicate the time ranges for \( X_v \).
being between 1 and 99% recrystallization. It is seen that the curves of $n_\text{t}$ vs. $\log(t)$ show a minimum and the region of 1–99% recrystallization is only in the decreasing part of the curve. At a given value of $m$, a higher value of $k_{\text{max}}$ just shifts the curve to longer times. The average value of the Avrami exponent is derived by integrating $n_\text{t}$ between $X_{V}/H_{11005}$ and $X_{V}/H_{11005}$, according to Eq. (13), and does not vary with $k_{\text{max}}$ at all, see Fig. 2(b). Curves of $n_{av}$ vs. $m$ calculated at the different given values of $k_{\text{max}}$ just overlap, and the average Avrami exponent $n_{av}$ changes little when $m$ is larger than 100. This suggests that $k_{\text{max}}$ has no influence on $n_{av}$.

The influence of temperature on the average exponent should be reflected by the variation in the factors $m$ and $k_{\text{max}}$ according to Eq. (12). It should be expected that deformation at higher temperature will be less heterogeneous, that is, $m$ can be expected to decrease with increasing temperature. However, when variation of $m$ induced by the temperature change is still in the stable region, for example for $m \geq 100$ in Fig. 2(b), little variation of $n_{av}$ will be observed unless $m < 100$ is reached. This strongly suggests that the observed temperature independence of the Avrami exponent may result from the fact that most of temperature changes in the experiments or practical rolling processes are not large enough to reduce the heterogeneity of the recrystallization kinetics in the materials below $m = 100$.

Since the model is based on the assumption that the strain is not uniformly distributed in the material, it is of interest to compare the required strain variation with that observed in either rolled material or material obtained by plain strain compression testing, as these conditions are the most common in recrystallisation kinetics studies. First of all it should be pointed out that the critical $m$-value of $m = 100$ corresponds to a strain variation $e_{\text{max}}/e_{\text{min}} = 1.5$ (Eq. (10), $p = 3.8$, $n = 3$). Such strain variations are easily achieved in most practical rolling processes or simulating experiments, due to the inevitable friction force introduced during the deformation and the heterogeneity of microstructures in materials, which is discussed below.

Strain distribution can be both measured experimentally and simulated by Finite Element Methods. Colas and Sellars measured the strain distribution during plane strain compression testing on austenitic stainless steel by a microgrid technique. Specimens were firstly split down the middle, and lines 0.508 mm apart were scribed by milling on the section plane. The coordinates of the mesh were photographed and measured with a traveling microscope before bolting the two halves together. After compression to a certain strain, each specimen was unbolted and the grid rephotographed. The most recent quantitative investigation of the deformation of the grid can be performed by some appropriate image analysis on a direct comparison of two SEM digital images taken at two different deformation states. Details of this technique and how to calculate the local strain can be found in Refs.16–18) With this approach, the equivalent strain contours over specimens at a certain nominal strain can be mapped. It was found that the local strain varied from 0.1 to 1.1, when nominal strain was between 0.46 and 0.73. Variation of local strain during deformation can also be calculated using Finite Element Methods. Shipway and Bhadeshia calculated the strain distribution for a cylindrical steel specimen (initial height 12 mm, radius 4 mm) compressed to a final length of 6 mm with an interfaced friction coefficient of 0.5. Their calcula-
tions showed that the longitudinal true strain was 0.69, but in regions close to the test machine platens, i.e. the dead zone, the effective strains were not greater than 0.1, whereas in regions close to the center the effective strain is close to 1.0. Moreover, the local strain variation in deformed materials, as found by Colas and Sellars, was considerable to 1.0. Moreover, the local strain variation in deformed materials, as found by Colas and Sellars, was considerable and increased with strain and strain rate. Hence an m-value greater than 100 is to be expected in almost all experiments or practical rolling processes, which leads to a deformation and temperature independence of the Avrami exponent, since m is usually located in the stable region (see Fig. 2(b)).

3.2. Range of the Observed and Actual Avrami Exponent

The transient exponent n, and the subsequent average exponent n\text{av}, can be derived only when n in Eq. (12) is known. The theoretical values for n are n=4 for the case of constant rates of nucleation and growth, and n=3 for the case of site-saturated nucleation and constant growth. Obviously, such high values of n are rarely observed in experiments because the growth rate is not necessarily constant. In the case of varying growth rate, the grain boundary migration velocity during recrystallization decreases significantly with annealing time, as revealed by Vandermeer and Gordon on aluminum, and Vandermeer and Rath on iron, and Rath et al. on titanium. Such time-dependent growth rate G is generally described by

\[ G = C \cdot r^m \] .................................(15)

where C and r are both constants, and \( r = 0.38 \) for the recrystallization of iron as investigated by Vandermeer et al. This will change the classical JMAK equation to

\[ X_V = 1 - \exp\left( -f_\gamma N_0 \int_0^t G(t') dt' \right) = 1 - \exp\left( -f_\gamma N_0 \left( \frac{C \cdot t^{1-r}}{1-r} \right)^3 \right) = 1 - \exp\left( -f_\gamma N_0 \frac{C^3}{(1-r)^3} t^{3(1-r)} \right) \] ...........(16)

Therefore, in the case of the growth rate decreasing according to Eq. (11), the Avrami exponent is given by \( n=3(1-r) \). For the recrystallization of iron in the case of varying growth rate, 1.86.

For these \( n \) values (\( n=3 \) or \( n=4 \) or \( n=1.86 \) depending on the different recrystallisation mechanisms), the range of the average Avrami exponent, which is the experimentally observed Avrami exponent, can be derived by combining Eqs. (12) with (13) and the result is shown in Fig. 3. As pointed out above, an m-value larger than 100 is very realistic for most experiments. Therefore, the predicted range of the experimentally observed exponent is between 0.77 and 2.2. When the growth rate is constant, a higher value of \( n_{av} \) (1.5–2.2) is expected, whilst a lower value, around 0.77, is expected when the growth rate significantly reduces with the annealing time. The relationship of actual local exponent n, predicted by the simple JMAK equation in each class of material, and the average exponent \( n_{av} \), calculated by the current modified JMAK equations, is shown in Fig. 4. It shows an approximately linear relation with a slope of about 1.5. Since the average exponent \( n_{av} \) is actually the experimental value of the Avrami exponent, it can be found that the predicted range of average exponent in Fig. 4, derived for the three actual values for local exponent n mentioned before, is in excellent agreement with experimental observations. For example, the experimentally observed exponent is 1 for C–Mn and Nb microalloying steels by Laasraoui and Jonas, 1.6 for a low carbon steel by Karjalainen et al. 2 for a plain carbon steel by Sun et al. in the range of 0.7–1.5 for 16 different steels as investigated by Medina and Quispe, and 0.5–1.2 for very low carbon rolled steel by Michalak and Hibbard. The model also predicts that high values for the Avrami exponent are only observed when the variation in local strain values is very small, such as in the case of lightly deformed, fine grained, texture free materials of a uniform grain size. This prediction is in excellent agreement with the experimental observations as well.

The relation between the observed Avrami exponent and the actual local Avrami exponent (Fig. 4) can now be used
to consider the mechanism behind the observed recrystallization process. The results are summarized in Table 1 for the literature data mentioned in the previous paragraph. It is clearly seen that the effect of inhomogeneous deformation does not only affect the numerical value of experimentally observed exponent $n_{av}$, but also affects the physical interpretation of the observed recrystallization kinetics. The observed Avrami exponents correspond to the local actual exponent by our modified JMAK equation, and variation of values of local exponents lead to a different interpretation on recrystallization kinetics, as illustrated in Table 1.

### 4. Conclusions

An extension to the JMAK model for the description of recrystallization kinetics, which is based on the nature of the strain heterogeneity, successfully describes the relevant experimental observations. This model can give a smaller exponent than the basic JMAK theory predicts, a larger exponent for the finer-grained materials, and a decreasing slope of JMAK plots with proceeding recrystallization. Moreover, it can quantitatively predict the range of Avrami exponents determined experimentally, and explain why the Avrami exponent is independent of temperature and deformation conditions.

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