Determining Changes in the Engagement of Screw Rotors Due to Transmission Error

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Abstract. As a rule, the structural analysis of screw compressors during their design comes down to determining the geometric dimensions of the screws, the position of their window edges, as well as the gas forces acting on the compressor rotors. The latter determine the loads on the compressor bearings and affect vibration of the rotors and the compressor as a whole. Their effect plays the dominant role but not the only one. To create a mathematical model describing the vibration state of rotors, determining all additional forces acting on the compressor rotors that arise in the course of its operation under normal operating conditions is also required. These include the manufacturing inaccuracies causing skewing and bending of the rotor axles, as well as contact forces caused by the interaction of rotors due to their deviation from the nominal position. The latter are the least studied and, as a rule, are typical only for oil-flooded screw compressors with no rotor synchronisation mechanism. This article is devoted to the determination of rotor dynamics and the forces caused by rotor teeth engagement. In future, its results will serve as a basis for developing a rotor vibration state model. Meanwhile, the analysis of results obtained already at the current research stage allows to decrease their impact and effect.

1. Introduction

One of the defining factors directly influencing the performance indicators of screw compressors is their vibration level. It is this in particular that ultimately determines the reliability of compressor systems, their service life, and can indicate the presence or emergence of faults or damage to the internal compressor assemblies. Currently compressor systems are designed mainly on the basis of the experience gained from designing similar previous-generation systems, leading to the need for upgrading operations, which in turn increases the net cost of the R&D.

At present, diagnostics during operation of compressor equipment using its vibration and acoustic characteristics is very limited and requires upgrading the compressor unit configuration with systems such as Bentley Nevada [1]. It, in turn, significantly increases both the cost of the compressor unit itself and the operating costs. The economic expediency limits the adoption of such systems.

Based on the foregoing, creating a mathematical model describing the vibration state of compressor systems with screw compressors is highly relevant. Afterwards, this will be used as a basis for developing specialised systems adapted to particular screw compressor systems. The following tasks need to be accomplished to achieve this goal:

- development of a mathematical description of the rotor dynamics and compressor vibration in the general formulation;
• specification of variables in the mathematical description that are dependent upon the operating process parameters and the structural features of screw compressor working elements;
• refinement of the screw compressor operating process mathematical model in order to more accurately calculate the parameters with the largest effect on the compressor vibration;
• specification of variables in the mathematical description that depend upon the internal compressor assemblies (in particular, upon their current condition);
• experimental validation of the mathematical description for the dynamics of rotors and vibration with its subsequent refinement;
• conducting a comprehensive numerical analysis of various factors influencing the level of compressor unit vibration;
• development of recommendations for predicting the vibration state of newly designed equipment and development of diagnostic procedures for operating equipment.

Accomplishing these tasks requires, first of all, an in-depth analysis of the already available approaches, their re-thinking and adaptation to the solution of the stated problems.

This article analyses the factors in the mathematical description, variables dependant upon the operating process parameters and the structural features of screw compressor working elements, as well as the methods for their calculation in the first-order approximation based on the already available methods.

2. Mathematical Description

It can easily be shown that any screw compressor rotor can be presented in the computational model as a rigid rotor, which is asymmetric with respect to elastic supports. In such case, for a rotor free of supports and external loads [2], the line passing through the geometrical cross-sectional centers of gravity determines its stiffness axis. The line passing through the mass centers of gravity determines its axis of inertia. Imbalance is defined as a deviation of the axis of inertia from the stiffness axis in each cross-section and is characterised in each rotor cross-section by linear variables $e_1$, $e_2$ and angular variables $\gamma_f$ and $\gamma_t$ (figure 1), which characterise the position of the axes of inertia relative to the stiffness axes. In an isotropic rotor, which includes screw compressor rotors, all the cross-sectional axes of inertia are the main axes. The rotor has variable longitudinal flexural rigidity $EJ(x)$, distributed mass per unit length $m(x)$, identical with respect to the $y$ and $z$ axes longitudinally distributed axial moment of inertia $i(x)$ and polar moment of inertia $i_p(x)$ with respect to the $x$ axis. In case of translational displacements of the rotor cross-sections in the $y$ and $z$ directions, distributed reactive power loads arise with the coefficients of proportionality $c_{yy}$, $c_{yz}$, $c_{zy}$, $c_{zz}$ at the corresponding displacements, and $k_{yy}$, $k_{yz}$, $k_{zy}$, $k_{zz}$ at the corresponding velocities. Similarly, in case of angular rotations of cross-sections around the $y$ and $z$ axes, distributed moment loads arise with the coefficients $s_{yy}$, $s_{yz}$, $s_{zy}$ at angular displacements, and $r_{yy}$, $r_{yz}$, $r_{zy}$, $r_{zz}$ at angular velocities. The forces of internal friction in the material distributed over the rotor length are characterised by the coefficient of proportionality $f(x)$. The rotor is loaded by the longitudinally distributed vector power load $p(x, t)$ with the components $p_y$ and $p_z$, and the moment load $l(x, t)$ with the components $p_y$ and $p_z$.

The deformed state in each cross-section is characterised by the deformation parameters including the displacement vector $u(x, t)$ with components $u_y$ and $u_z$, and the rotation angle vector $\phi(x, t)$ with components $\phi_y$ and $\phi_z$, and the force parameters — transverse force vector $Q(x, t)$ with components $Q_y$ and $Q_z$, and the bending moment vector $M(x, t)$ with components $M_y$ and $M_z$. The differential equations for the rotor motion (not including the transverse forces whose effect on the rotor deformation is not substantial in most cases) have the following form:

\[
\begin{cases}
\frac{\partial^2 u_y^0}{\partial t^2} = -\frac{\partial Q_y}{\partial x} - c_{yy}u_y - c_{yz}u_z - k_{yy}\frac{\partial u_y}{\partial t} - k_{yz}\frac{\partial u_z}{\partial t} + p_y; \\
\frac{\partial^2 u_z^0}{\partial t^2} = -\frac{\partial Q_z}{\partial x} - c_{zy}u_y - c_{zz}u_z - k_{zy}\frac{\partial u_y}{\partial t} - k_{zz}\frac{\partial u_z}{\partial t} + p_z.
\end{cases}
\]
For angular displacements:

\[
\left\{
\begin{array}{l}
 i(x) \frac{\partial^2 \theta_x}{\partial t^2} + i_y(x) \frac{\partial \theta_x}{\partial t} = -\frac{\partial M_x}{\partial x} + Q_x - s_y \theta_y - s_x \theta_x - r_y \frac{\partial \theta_y}{\partial t} - r_x \frac{\partial \theta_x}{\partial t} + I_y; \\
 i(x) \frac{\partial^2 \theta_y}{\partial t^2} - i_y(x) \frac{\partial \theta_y}{\partial t} = -\frac{\partial M_y}{\partial x} - Q_y - s_x \theta_x - s_y \theta_y - r_x \frac{\partial \theta_x}{\partial t} - r_y \frac{\partial \theta_y}{\partial t} + I_y.
\end{array}
\right.
\]

(2)

The following relationships should be added to equations (1) and (2):

**Figure 1.** The computational model for determining displacements
In equations (1) and (2) the superscript "0", as above, corresponds to the linear and angular displacements of the rotor axis of inertia; while the point displacements of the axis of inertia and the stiffness axis are connected by the following formulas:

\[
\begin{align*}
\theta^0_x &= \theta_x + \gamma_1 \cos \theta - \gamma_2 \sin \theta; \\
\theta^0_z &= \theta_z + \gamma_1 \sin \theta + \gamma_2 \cos \theta.
\end{align*}
\]

The main difficulty in solving the presented equations is determining the relationship of the external forces \( p(x, \tau) \) included in the presented equations. Whereas its components caused by the gas forces, the forces of friction against the gas-oil mixture and the reactions in supports have been studied extensively and the procedure for their determination is presented in [3,4], the rotor engagement forces have not received extensive attention, and in fact are not considered in the screw compressor design. Their magnitude is, as expected, less than the rest, and on the whole will not exert a significant effect on the reactions of the supports; however, they depend upon the compressor operation mode and upon the transmission error caused by a manufacturing inaccuracy of its rotors, and can cause additional vibration harmonics. The rest of the article is devoted to the development of a method for their approximate determination.

Taking into account the complexity of the rotor interaction system in the general formulation, as well as the displacement limitations imposed by the bearing support, the rotor interaction forces can approximately be determined on the basis of the rotational motion only. We will limit ourselves to considering the approach as applied to a free (female) rotor, assuming that the male rotor rotates at a constant angular speed. This assumption is justified by a large moment of inertia of the male rotor, in particular, due to the elements fitted thereon (mechanical seal and couplings), as well as due to a relatively rigid coupling with the drive.

The approach to the determination of \( \frac{d^2\theta}{d\tau^2} \) is considered in [5, 6, 7], and some extracts from that article are provided below. The dynamics equation can be written as follows:

\[
\frac{d^2\theta}{d\tau^2} = \frac{M_{GF} + M_{AF} - M_R - M_{F1} - M_{F2} - M_{MEC}}{J_2}
\]

where \( J_2 \) is the moment of inertia of the free rotor; \( M_{GF} \) is the moment created by the gas forces that it is determined on the basis of the (P-V) indicator chart according to the "subtending chord" method [4, 5, 6]; \( M_{MEC} \) is the moment of resistance of the compressor mechanical assemblies such as seals, balance pistons and bearings (its magnitude is mostly determined by the construction of the rotor mechanical assemblies); \( M_R, M_{F1}, M_{F2} \) are the moments of forces of friction against a compressible medium in the radial and end clearances respectively; \( M_{AF} \) is the moment created by adhesion forces. It can easily be shown that in this case the concept of "adhesion" can be replaced with the closely related concept of "cohesion". Then this moment will be proportional to the surface tension coefficient, the contact spot area and the contact angle; it should also be noted that this moment will be significantly less than the remaining moments of forces, which means that it can be disregarded in further calculations: \( M_{AF} \approx 0 \).

The moments of forces of friction against the compressible medium in the end clearances depend, according to the computational model, on the operation mode [4, 5, 6]:

\[
\begin{align*}
\theta^0_x &= \theta_x + \gamma_1 \sin \theta - \gamma_2 \cos \theta; \\
\theta^0_z &= \theta_z + \gamma_1 \cos \theta + \gamma_2 \sin \theta.
\end{align*}
\]
\[
M_{F1,2} = \begin{cases} 
\frac{\pi \cdot \omega_2 \cdot \eta_{\text{mix}} \cdot R_{\text{eq}}^4 \left(1 - \frac{r_k^4}{R_{\text{eq}}^4}\right)}{2 \cdot \delta_{F1,2}}, & \text{if } \Re_{F1,2} < 10^4 \\
\frac{\rho_{\text{mix}} \cdot \omega_3^3}{4} \left(c_{\text{M1,2},R}^3 \cdot R_{\text{eq}}^3 - c_{\text{M1,2},z}^3 \cdot R_{\text{eq}}^3\right), & \text{if } 10^6 \geq \Re_{F1,2} \geq 10^4 
\end{cases}
\]  

(6)

where \(c_{\text{M1,2},R} = 0.0277 \cdot \Re_F^{-0.2} \cdot \left(\frac{\delta_{F1,2}}{R}\right)^{-0.2}\) is the coefficient of friction, to determine \(c^*_M\), \(r_k (R = r_k)\) is used as the defining size, \(R_{\text{eq}} (R = R_{\text{eq}})\) is used in all remaining cases; \(\Re_F = \frac{R^2 \cdot \omega_3 \cdot \rho_{\text{mix}}}{\eta_{\text{mix}}}\) is the Reynolds number; \(\omega_2 = \omega_1 \cdot \frac{z_1}{z_2}\) is the nominal angular velocity of the free rotor; \(\rho_{\text{mix}}, \eta_{\text{mix}}\) are the density and the kinematic viscosity of the gas-oil mixture in the working cavity of the compressor, respectively; \(\delta_{F1}, \delta_{F2}\) are the end clearances between the rotor and the casing on the suction and discharge sides respectively. The equivalent radius is determined as 
\[
R_{\text{eq}} = \sqrt[3]{\frac{1}{2 \cdot \pi} \int_0^1 \left(r(t);0 \right) \cdot dr},
\]

where \(r(t);0\) is the relationship describing the rotor profile.

Similarly, the moments of forces of friction against the compressible medium in the radial clearance can be written as follows [5, 6, 7, 8, 9]:
\[
M_r = \begin{cases} 
\frac{2 \cdot \pi \cdot \xi \cdot \omega_2 \cdot \eta_{\text{mix}} \cdot R_{\text{z2}}^3 \cdot z_2 \cdot b_k}{(1 + \xi) \cdot \delta_{r}}, & \text{if } \Re_r < 2500 \\
\pi \cdot c_r \cdot \rho_{\text{mix}} \cdot \omega_3^2 \cdot z_2 \cdot b_k \cdot R_{\text{z2}}^3, & \text{if } 10^5 \geq \Re_r \geq 2500 
\end{cases}
\]  

(7)

where \(c_r \approx 0.0076 \cdot \Re_r^{-0.25}\) is the coefficient of friction [4, 5, 6], \(\Re_r = \frac{\omega_3 \cdot R_{\text{z2}} \cdot \delta_{r} \cdot \rho_{\text{mix}}}{\eta_{\text{mix}}}\) is the Reynolds number, \(b_k\) is the rotor tooth width in the the Z axis direction, \(R_{\text{z2}}\) is the outer rotor diameter, \(\delta_{r}\) is the radial clearance between the compressor rotor and casing, \(\xi = \frac{m_{\text{gas}}}{m_{\text{oil}}}\) is the gas-oil ratio.

The thermodynamic parameters of the mixture are determined according to the following relationships [5, 6, 7, 8, 9]:
\[
\eta_{\text{mix}} = \left(\frac{1}{1 + \xi}\right) \cdot \eta_{\text{gas}} + \left(\frac{\xi}{1 + \xi}\right) \cdot \eta_{\text{oil}} 
\]  

(8)

\[
\rho_{\text{mix}} = (1 + \xi) \cdot \rho_{\text{gas}} 
\]  

(9)

In [5, 6, 7, 8, 9] it was shown that equation (5) can also serve as the criterion of rotor disengagement:
\[
\frac{d^2 \theta_2}{d \tau^2} \geq \frac{d^2 \beta_2}{d \theta_1^2} \cdot \alpha_3^2 
\]  

(10)

where \(\beta_2\) is the angle characterising the freeplay in engagement in the direction of rotor rotation with respect to the estimated position, ultimately it will characterise the transmission error. The connection between the integration time \(\tau\) and the rotation angle of the male rotor can be determined by the following expression:
\[
\omega_1 = \frac{d \theta_1}{d \tau} 
\]  

(11)
At the moment when the inequality (11) is no longer valid, the position of the female rotor should be determined by integration of differential equation (5) with the following boundary conditions, taking the moment of rotor zero position as the time reference point [10]:

\[
\begin{align*}
\theta_2(\tau_1) &= \theta_1 \frac{z_1}{z_2} + \beta_2 \\
\frac{d\theta_2}{d\tau}(\tau_1) &= \omega_1 \frac{z_1}{z_2}.
\end{align*}
\] (12)

At the moment of the rotor contact, their linkage separation occurs; the integration should be restarted from that time point with the following initial conditions:

\[
\begin{align*}
\theta_2(\tau_2) &= \theta_1 \frac{z_1}{z_2} + \beta_2 \\
\frac{d\theta_2}{d\tau}(\tau_2) &= \omega_1 \frac{z_1}{z_2} - (\omega_2 - \omega_1) \cdot k.
\end{align*}
\] (13)

where \( k \) is the recovery coefficient.

3. Calculation results

We have selected the standard screw compressor developed at JSC NII turbokompressor named after V. B. Shnepp (the successor of SKBK) as the object of study, with the rotor diameter of 250 mm, length of the rotor profile of 270 mm, screw pitch of 480 mm and transfer ratio of 4/6. The improved SKBK profile has been selected as the profile surface with the standard downward bias described in [4]; its essence is the following: the male rotor profile remains theoretical, whereas the female rotor profile has an equidistant downward bias with respect to the theoretical one. The magnitude of equidistant downward bias of the female rotor is variable over the length and varies from the maximum radial downward bias value (0.13 mm) for points with the minimum radial coordinate, with a smooth (linear) decrease to the circumferential (end face) downward bias (0.07 mm). The following values have been adopted for the end clearances between the rotor and the casing: on the suction side – 0.6 mm, on the discharge side – 0.07 mm. The value of 0.2 mm is adopted as the radial clearance. The compressor operation mode using air with the compression from 1 atm (abs.) to 6 atm (abs.) (pressure ratio \( \Pi = 6 \)), suction temperature of 25 °C, discharge temperature of 100 °C, temperature of injected oil of 40 °C, the geometric degree of compression of 4.5, and the rotational speed of the male rotor of 3,000 was selected as the operating mode. The gas-oil ratio was calculated at a constant discharge temperature.

The calculation results are presented in figure 2.

As can be seen from the calculations, the rotor disengagement leads to a loss of contact along a lengthy segment, with subsequent impact entry into engagement. The recovery coefficient (coefficient characterising the linkage separation) is the least studied. The latter has been taken as 0.3 based on the presence of an oil layer serving as a damper. This choice corresponds to the coefficient of linkage separation of the compressor valve plates whose operation mode is similar.

Thus, it has been demonstrated that there are two points of engagement which should be reflected in the vibration characteristics of rotors.

Similar results were obtained by analyzing of other compressor working conditions and by analyzing of compressors with different size of the rotors. Those factors mainly influence on the period duration when the inequality (10) is broken. The female rotor behavior changing can be explained by the changing of the relation between forces inducted by the gas pressure and by the friction on rotors’ gaps, which are changed depending on the working conditions and rotors’ sizes. Even though of the conditions, when the inequality (10) is broken only on the moment of rotors’ tooth disengagement, the rotors’ contact are lost for a quite long period. It also inducts a shock contact and an additional vibration.
4. Conclusions

The research performed has allowed to evaluate, in the first-order approximation, the changes in the engagement of screw rotors due to transmission errors which lead to the emergence of additional forces causes by impact entry into engagement. In addition, we have analysed the engagement from the point of view of the contact. The time of teeth entry into contact and, consequently, the points of contact have been obtained. The presented method also allows to quantitatively evaluate the magnitudes of the contact entry forces. They can be calculated taking into account the variation of the rotor momentum, as well as by specifying the rotor contact duration at the level of $10^{-5} \ldots 10^{-4}$ s.

The obtained results based on the determination of the female rotor correction angle have a good agreement with experimental measurement values presented at [11] and have a good correlation with experimental data presented at [12].

At the same time, it should be noted that the conducted research leads to the formulation of the following additional problems: specification of the values of the linkage separation coefficients during interaction of rotors; narrowing the range and elaboration of recommendations for selecting the contact time, and experimental testing of the approach developed herein. This will be done in future.

References

[1] Bently Nevada website :http://www.ge-mcs.com/en/bently-nevada.htm
[2] Chelomey V N 1980 Vibratsiya v tekhnike: Spravochnik v 6 tomakh (t.3) (Vibration of equipments: handbook included 6 volume (v.3)) (Moscow: Mashinostroenie)
[3] Stosic N, Smith I K, Kovacevic A, 2005 Screw Compressors Mathematical Modelling and Performance Calculation (Berlin Heidelberg New York: Springer)
[4] Khisameev I G and Maksimov V A 2000 Dvukhrotornye vintovye I pryamozubye kompressory. Teoriya, raschetiproektirovanie (Twin rotors screw and spur compressors. Theory, calculation and design (in Russian)) (Kazan: Fen)
[5] Yakupov R R, Mustafin T N, Nalimov V N, Khamidullin M S and Khisameev I G 2013 Discussion of actual profile clearances’ calculation method in rotary compressors in the absence of rotor timing units 8th Int. Conf. on Compressors and their Systems (London: City University) 209
[6] Yakupov R R, Mustafin T N, Nalimov V N, Khamidullin M S and Khisameev I G 2014 Analysis of transmission error depending on compressor working conditions J of Proc. Mechanical
Engineering (Proc. of the Institution of Mechanical Engineers) Part E (London: SAGE Publications)

[7] Adams G P and Soedel W 1994 Dynamic simulation of rotor Contact Forces in Twin Screw Compressors Proc. Int. Compressor Engineering Conference, (West Lafayette: Purdue University)

[8] Mustafin T N and et. 2015 Analysis of the screw compressor rotors’ non-uniform thermal field effect on transmission error IOP Conference Series: Materials Science and Engineering (Vol. 90, No. 1) (IOP Publishing) p 012004

[9] Mustafin T N and et.:2016 Analysis of influence of screw compressor construction parameters and working condition on rotor temperature fields Procedia Engineering 152 pp 423-433.

[10] Mustafin T N and et. 2017 Determining of actual profile clearances and screw compressor rotor positions depending on working conditions. IOP Conference Series: Materials Science and Engineering (Vol. 232, No. 1) (IOP Publishing) p 012023

[11] Mustafin T N and et. 2018 Calculation and experimental analysis of profile clearance values in screw compressor rotors AIP Conference Proceedings Volume 2007, 23 August 2018

[12] Holmes, C. S., Williamson, T., 2001, The manufacture of hardened screw compressor rotors, International Conference on Compressors and their Systems, City University London.