Testing super-deterministic hidden variables theories

Abstract We propose to experimentally test non-deterministic time evolution in quantum mechanics by consecutive measurements of non-commuting observables on the same prepared state. While in the standard theory the measurement outcomes are uncorrelated, in a super-deterministic hidden variables theory the measurements would be correlated. We estimate that for macroscopic experiments the correlation time is too short to have been noticed yet, but that it may be possible with a suitably designed microscopic experiment to reach a parameter range where one would expect a super-deterministic modification of quantum mechanics to become relevant.

Keywords Hidden Variables · Determinism

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1 Introduction

Quantum mechanics in the standard Copenhagen interpretation is fundamentally non-deterministic. The collapse of the wave function cannot be predicted, and outcomes of measurements are merely probabilistic. Because of its implications for the nature of true coincidence this issue has caused plenty discussions. The free will, even consciousness itself, has been placed within this randomness, for within a truly deterministic world the future might be unknown to us, but is in principle entirely determined by the past and unchangeable through human action – it is the world of Laplace’s demon.

Despite the philosophical and interpretational upheaval, physicists have come to live with the quantum mechanical non-determinism in a very pragmatic style. It is without doubt that quantum mechanics and its younger
cousin quantum field theory are experimentally extremely well confirmed. But the aim of our scientific endeavors is pushing beyond what we already know, and the axioms of quantum mechanics left us with the question whether this non-determinism is really fundamental, or whether it may arise from an underlying deterministic theory. If it was, our inability to predict the outcome of a measurement could be due to an incomplete knowledge of the initial state, the complete knowledge existent but unavailable to us, encoded in what is commonly named ‘hidden variables.’

For a locally causal theory, an explanation of quantum effects through hidden variables can be shown to be in disagreement with experiment via Bell’s theorem [1], or its generalization respectively [2]. Many tests [3, 4, 5, 6, 7, 8, 9] have meanwhile been performed and found that the hidden variables theories for which Bell’s theorem applies are not realized in nature. There are various loopholes in the conclusions that can be drawn from the set-up of these experiments and not all of these loopholes have yet been satisfactorily closed (for a recent summary see e.g. [10]). But even so, local hidden variables are strongly disfavored. However, Bell’s theorem relies on the assumption that one has the freedom to choose the detector’s settings without modifying the prepared state that one wishes to measure. In case this freedom is not given, the conclusion of Bell’s theorem does not apply. Theories that violate this assumption have become known as ‘super-deterministic,’ and usually do not find much of an appreciation because they seem to rely on a ‘conspiracy’ that is an inability of the experimentalist to do as he wishes with his detector.

However, despite our unease with super-determinism it might be the way nature works. We will consider here such super-determinism from a very general point of view and propose a possibility to experimentally test and constrain such theories. This is an investigation of interest not only with regard to the foundations of quantum mechanics, but also for our ongoing search for a theory of quantum gravity: Possibly our inability to unify quantum theory with classical general relativity is due to our incomplete understanding of quantum mechanics.

2 Super-determinism

Let us start with clarifying which type of theory we will be examining. It is not our aim to construct a detailed model here. Instead, we want to test super-deterministic hidden variable theories (SDHVT) in a way as model-independent as possible, much like Bell’s theorem tests for hidden variables by only making a few assumptions rather than using a specific model.

We will consider our space-time as a differentiable 4-manifold with a 3+1 slicing into space-like hypersurfaces $\Sigma_t$. An initial state $\psi(t, x)$ on $\Sigma_t$ can be evolved forward and backward by help of the evolution equation, or the Hamiltonian respectively, which is an axiom of the theory. This evolution is deterministic also in quantum mechanics. The non-deterministic ingredient of the Copenhagen interpretation is the probabilistic computation of measurement outcomes from the initial state, given by Born’s rule: The outcome of a measurement of an observable described by an operator $\hat{O}$ is an eigenvalue
of that operator $\hat{O}\psi = O\psi$, where $\psi$ is the corresponding eigenvector. The probability $P(\psi, O)$ of measuring a state $\Psi(t, x)$ at value $O$ is then given by the square of the projection of the state on the eigenvector

$$P(\psi, O) = |\langle \psi | \psi_O \rangle|^2,$$

and the expectation value of the measurement is

$$E(\psi, O) = \langle \psi | \hat{O} | \psi \rangle.$$

In a SDHVT, the initial state depends on additional parameters $\lambda_i$, and the outcome of the measurement is a function of the hidden variables $\lambda_i$. If all $\lambda_i$ were known, the outcome of the measurement could be predicted in principle. We will assume that there are finitely many such hidden variables, $i \in \{1...N\}$. To good precision such a SDHVT has to reproduce Born’s rule when the hidden variables are unknown and are considered to be stochastically distributed over typical values describing the setting.

The wave-function contains both the state that one wants to measure and the detector on the slice $\Sigma_t$ — in fact a separation might not be uniquely possible. Normally both parts of the wave-function are considered as separate entities. Super-determinism comes in through the impossibility to change the part of the wave-function describing the detector independently from the one describing the prepared state. This violates a central assumption of Bell’s theorem.

A simple example to understand such a feature is to consider a wave-function $\psi(t, x, \lambda_i)$ that is analytic on the whole slice $\Sigma_t$. It is then not possible to prepare any wave-function on compact support, and to add non-overlapping wave-functions. Instead, a different setting of the detector necessarily implies a different prepared state, even though both are spatially separated. This change of initial conditions in the detector affecting the initial conditions of the prepared state might be small, but it affects the hidden variables.

One can now see why such super-deterministic theories are often dubbed ‘conspiracy theories.’ In a certain sense, the detector already ‘knows’ it will be detecting the state that is being prepared. Not because there is backward causation though — causality is maintained in its usual form — but because it is impossible to prepare the state independently of the detector to begin with. If this dependence goes unnoticed, it results in a seemingly non-deterministic measurement outcome. The impossibility to prepare state and detector independently is not specifically due to an inability of the conscious beings conducting the experiments, but it is a consequence of fundamental determinism.

At first sight, such super-determinism might seem nonlocal. To clarify the notion of local causality, let us examine whether it is locally causal according to Bell’s definition:

“A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region
Fig. 1 Local causality

2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3...

(See Figure 1 and [12]). Thinking of our example with the analytic function, super-determinism can thus be locally causal according to the second part of the definition (“what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3”) but not according to the first part (“the probabilities attached to values of local beables in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region 2.”) The violation of the second part of the definition is exactly what makes it impossible to choose the detector settings without altering the state that one wants to observe. It does however a priori not necessitate superluminal exchange of information or action at a distance.

In the following we will assume that the hidden variables in our theory are environmentally induced by which we mean their origin is in the impossibility to treat the prepared state as independent from the rest of the experiment. The environment is thus constituted of what is necessary to completely specify the experiment, including the detector and any apparatus necessary to prepare the state and to set up the experiment. It does not include further non-necessary additions, such as for example an actual observer reading out the measurement outcome. This is a minimalistic assumption about the origin of the hidden variables, and a crucial one to specify the sort of theory we are interested in on which we will comment in the discussion. Note that this notion of environment becomes ambiguous in ordinary quantum mechanics. There, it would be possible to not only change the settings of the detector as one wishes, but it would in principle be possible to change the whole detector after one had set up the experiment. Then one would be faced with the question if one had to consider all possible detectors to be part of the possible environment. Luckily, in the SDHVT, we do not have this complication, since the detector cannot be altered independently of the prepared state without changing the whole experiment.

3 Testing Super-determinism

We can now make some general considerations about these super-deterministic hidden variables theories. Based on this, we can then propose experimental tests.

The decisive feature of a deterministic hidden variable theory is that knowing the variables allows one to predict the measurement outcome. To
investigate this feature, what we will be interested in here is the correlation between measurements on identically prepared states for which also the hidden variables are identical. In ordinary quantum mechanics, the outcome of measurements on identically prepared states will be statistically distributed according to Born’s rule, and entirely uncorrelated. In a hidden variables theory, the outcomes of these measurements may be unknown if the hidden variables are unknown, but the measurement outcomes will be the same.

The difference between the both cases can be illustrated with a simple example. Consider you are pregnant with twins (‘identically prepared’) and are given a risk estimate of \( r \) (probability of measurement), based on your and your partner’s family history, for a genetic disease of your offspring. If the twins are fraternal (corresponding to usual quantum mechanics), the risk that both children will have the disease is \( r^2 \). In this analogy, quantum mechanics would imply there is no way to make states more ‘identical’ than having the same parents – there are no hidden variables. If the twins are identical (corresponding to the hidden variables theory) the risk that both twins have the disease is \( r \). On the risk of overstretched this example, if you were given data of a large group of women with either all identical or all fraternal twins, you could find out which case you are dealing with by studying the correlation of diseases among the twins. That, in essence, it what we will do.

Let us denote with \( \kappa \in \{1...n\} \) a sequence of \( n \) identically prepared states \( \phi_{\kappa} \) and \( P(\phi_{\kappa}, O) \) the probability to measure value \( O \) for observable \( \hat{O} \). For simplicity we assume that the expectation value \( E(\phi_{\kappa}, O) = 0 \). Then, in ordinary quantum mechanics the joint probability for two subsequent measurements of the same value \( O \) is

\[
P(\phi_{\kappa}, O \land \phi_{\kappa+1}, O) = P(\phi_{\kappa}, O)^2 \quad ,
\]

whereas in the SDHVT

\[
P(\phi_{\kappa}, O \land \phi_{\kappa+1}, O) = P(\phi_{\kappa}, O) \quad .
\]

The state \( \phi_{\kappa} \) carries all the available information about the system, i.e. in Eq. 3 it is the usual wave-function, whereas in Eq. 4 it is a function also of the hidden variables. The above means in other words that the correlation

\[
\text{Corr}(\nu, \kappa) = \frac{E(\phi_{\nu}, O \land \phi_{\kappa}, O)}{E(\phi_{\nu}, O^2)}
\]

does in the SDHVT not vanish for \( \nu \neq \kappa \), whereas in standard quantum mechanics it does. We will denote with \( \text{Corr}_{\kappa} = \text{Corr}(0, \kappa) \) the comparison to the first state in the sequence.

The problem with detecting this modification of quantum mechanics is that to produce an identically prepared initial state in the hidden variables theory, one would have to make sure the hidden variables have identical values too, which, given that they are hidden, is difficult to achieve. In a non-ideal setup, which one generally will be faced with, the correlation \( \text{Corr}_{\kappa} \) lies between 0 and 1, tending towards the usual quantum mechanical value of zero the less reliably one can expect the states to be identical. The important
point is that this correlation time is measurable, and any value larger than zero is in contradiction with standard quantum mechanics.

The number of hidden variables, \( N \), that describe the setting of prepared state and detector will increase the more degrees of freedom the system under consideration has which means in most cases, the larger it is. This is the reason why it is plausible to assume the hidden variables are statistically distributed, resulting in a seemingly random measurement outcome. The more variables we have to take into account, the more difficult it will be to measure any correlation.

The hidden variables come in through both the preparation of the state and the detector, from which the preparation of the state is the more problematic part because it usually involves an even larger system. Luckily, this latter problem can be circumvented by doing repeated measurements on the same state. This requires to chose a setting in which the state (at least with some probability) is returned into the initial state. In this case we are then interested in the correlation of measurements of the same observable \( \hat{O} \) of the same state \( \phi \) at time steps labeled by \( \kappa \). With a slight abuse of notation, we are thus interested in the autocorrelation of the time series:

\[
\text{Corr}_\kappa = \frac{E(\phi, O, t = 0 \land \phi, O, t = \kappa)}{E(\phi, O^2)}.
\]  

(6)

It then remains the task of choosing repeatable measurements in between which the detector itself undergoes as few changes as possible.

For the autocorrelation we make the usual ansatz of exponential decay

\[
\text{Corr}_\kappa = \exp(-\kappa/\tau),
\]  

(7)

where \( \tau \) is some timescale, the autocorrelation time, encoding the rate of change of the experimental setup and thus the environmentally induced hidden variables. For this ansatz we have assumed that the divergence of the system from a perfect repetition happens by incremental disturbances that are described by a homogeneous Poisson-process, i.e. the changes that lead to the decay of the autocorrelation are statistically independent and follow a probability distribution which is uniform in time. The larger the setting and the shorter the typical timescales on which its degrees of freedom change, the smaller \( \tau \) and the faster the correlation will decrease. In the limit \( \tau \to 0 \), one reproduces standard quantum mechanics.

With these considerations, we can now identify the following criteria as useful for the purpose of constraining SDHVTs:

1. Instead of measuring a sequence of individually prepared states, chose a setting in which the state (at least with some probability) is returned into the initial state and repeated measurements on the same state can be performed.
2. The experimental setup itself and the detector should be as small as possible to minimize the number of hidden variables (i.e. \( N \) should be small).
3. The repetition of measurements should be as fast as possible so any changes to the hidden variables of the detector in between measurements are minimized (i.e. \( \kappa < \tau \)).
4 Proposed Experiment

The quantity Corr$_x$ will be a function of those hidden variables that change during the time interval $\kappa$. Since we assumed these variables to be induced by the environment that, in the super-deterministic theory, can no longer be treated as independent from the prepared state, the correlation time will depend on the experimental setup. We will here propose a general outline for an experiment that should be doable with technology available today, and allow a measurement that constrains the SDHVT.

The setup of a possible experiment is shown in Figure 2. First, one needs a source for single particles, most conveniently electrons or photons. These particles enter through a one-way mirror a system consisting of two detectors, A and B, behind which there is another mirror. The particle then, ideally, enters a loop. The two detectors measure two non-commuting variables, $A$ and $B$, with $[A, B] \neq 0$, in a way that, according to standard quantum mechanics, measurement of one variable should destroy information obtained in the previous measurement of the other variable. Since actual one-way mirrors do not exist, one should understand this as a material with a small chance of the particle trespassing, say $p = 1\%$. Then, only one out of 100 particles will on the average enter the system, but once it has done so, it has a 99% probability of staying for the next loop.

To see whether such an experiment is possible in an interesting parameter range, let us estimate what is the typical decay time to expect. As an example, consider a photodetector constituted of $N$ atoms that measure an infalling photon by excitation of an electron into the conduction band. The energy gap of the material be $\Delta E$ and the temperature be $T$. At that temperature, one single atom has a probability of $\exp(-\Delta E/T)$ to become excited by thermal motion, and it remains so for the average electron-hole recombination time.
The time $\tilde{\tau}$ for $N$ atoms to undergo a statistical change is thus

$$\tilde{\tau} \approx \exp(\Delta E/T) \tau_r. \quad (8)$$

There are many other sorts of noise, but this is the one with the highest frequency, and thus the one relevant for the decay time.

We set this time in relation to the decay time of the correlation by $\tau = \alpha \tilde{\tau}$. Here, the dimensionless parameter $\alpha$ is the parameter we want to constrain by experiment. For a SDHVT one would expect it to be of order one, whereas for ordinary quantum mechanics it is zero.

If one inserts typical values of $\Delta E \sim 5$eV, $T \sim 300$K, $\tau_r \sim 1$ ns, and $N \approx 10^{20}$ one finds $\tilde{\tau} \sim 10^{-20}$s and, for $\alpha \approx 1$, a hopelessly small correlation time. One might think about cooling the whole system down. However, this would practically necessitate immersion in some liquid, thereby increasing $N$ and the difficulty to perform an experiment to begin with. However, if we consider instead a microscopically small detector, maybe with an extension of some $\mu$m, we could get down to $N \approx 10^{15}$. Let us further take a semiconductor with a fairly large band gap of $\Delta E \approx 1$ eV. Then we get down to $\tilde{\tau} \approx 10^{-6}$s. This now has to be compared with the typical time in which measurements can be repeated.

In $10^{-6}$sec a photon travels a distance of about 100m. The experiment cannot be arbitrarily small because it still should be larger than the wavelength of the photon, but a size of a mm is, with visible light, still large enough. In this case, the time for a photon to get from one end of the experiment to the other is below $10^{-11}$sec, and is as desired much shorter than the decay time, leaving space for non-ideal material and other sources of experimental uncertainty.

For the example of the photon, the detectors A and B could measure different polarization states in a photonic version of the Stern-Gerlach experiment, when such devices can be made sufficiently small. Alternatively, one can use electrons and spin-filters [13]. While the research in this area is not yet sufficiently advanced, it may soon be. To return the electron into the initial state, the function of a mirror could for example be mimicked by Bragg-reflection on sufficient layers of suitably spaced atoms. Electrons bring the advantage that the wavelength is smaller and thus the experiment can be made smaller. It has the disadvantage that it is more difficult to handle charged particles.

In either case, the detector would let the particle (photon or electron) trespass straight for one particular combination of measurement outcomes (polarization or spin), while the particle would be deviated to leave the loop if it does not have this particular combination. The quantity of interest is then time it takes for the particle to leave the loop at either detector A or B. The average value of these time measurements is the correlation time.

Let us denote the measurement outcomes the particle needs to have to stay in the loop with $a$ and $b$, corresponding to the two detectors (for example spin up and left). The probabilities for these measurement outcomes be $p_a$ and $p_b$, respectively. In ordinary quantum mechanics the probability for a particle that has entered the system to be deviated out of the system at
detector A or B before it returns to the point of entry is \((1-p_a)^2(1-p_b)\). Then, it has a probability of \(p\) that it leaves through the imperfect one-way mirror, and the probability to make it into the next loop is \((1-p)(1-p_a)^2(1-p_b)\). The probability that the particle is still in the system after the \(m\)th reflection at the one-way mirror is \((1-p)^m(1-p_a)^{m+1}(1-p_b)^m\). In the SDHVT however, we know that if the particle has fulfilled the conditions to make the loop once it will continue to do so and, on a duration shorter than the autocorrelation-time, it should return to the mirror on the second screen with probability \((1-p)^m(1-p_a)(1-p_b)\).

With \(p \ll 1\) and \(p_a, p_b\) of order one, in ordinary quantum mechanics the probability is thus very high that the photon will be detected within only a few rounds. In the SDHVT, detection should occur either in the first round, or with a delay relative to the standard case. There is not much use in making the propagation time of the particle very short by making the experiment very small, since the time still has to be measurable. However, with present-day technology, it seems feasible to measure a correlation time of \(10^{-6}\) sec and distinguish it from no correlation.

In addition to the above considerations about measurability of the autocorrelation, there is of course experimental error that needs to be taken into account. Most crucially, we have so far assumed that the mirrors are perfectly even. In reality, they will of course have a finite surface roughness and the electron or photon will only return to its initial state with some limited precision, which then alters the probability for it to make the same loop again. We can estimate this effect by noting it will become relevant if the unwanted dislocation of the particle per loop, \(\Delta z_m\), through a non-perfect reflection makes a non-negligible change to its relative location to the degrees of freedom of the environment. In the case considered here, the relevant distance would be of about one atomic diameter, or \(10^{-10}m\). If we denote this distance with \(z_N\), then we have an additional contribution to the autocorrelation time that increases with \(m\),

\[
Corr_m = \exp \left( -\frac{\kappa}{\alpha^2} \right) + \exp \left( -m \frac{\Delta z_m}{z_N} \right) .
\]

5 Discussion

The here proposed test aims directly at falsifying the non-deterministic nature of quantum mechanics. Since we know that local theories with hidden variables are strongly disfavored by experiment already, the type of model that could replace standard quantum mechanics considered here is non-local in the specific sense of being super-deterministic, i.e. the prepared state is correlated with the detector.

Of course any proposed test is only as good as the assumptions going into deriving the constraints, so let us summarize them. Most crucially, we have made the minimal assumption that the hidden variables stem from the correlation with the detector and possibly other parts of the experimental setup. This is a restriction to a subclass of SDHVTs since in principle the remainder of the universe might contain additional variables relevant to the
evolution of the subsystem we are considering. While this is possible, this would be a type of SDHVT that is in practice not testable anyway, so we have focused here on a version of SDHVT that, if realized in nature, could falsify non-determinism in quantum measurement. (This is not to say that the type of theory considered in this paper is the only type of SDHVT which is testable.) We also have not addressed here the possibility of a combination of a super-deterministic evolution with a limited version of free will, a possibility that merits mentioning since it has recently been shown that allowing for a small initial correlation between measurement device and measured system is sufficient to reproduce nonlocal correlations [13].

Further, we have implicitly made the assumption that variables that are not relevant for the description of the experiment in the standard theory are not among the hidden variables either. This includes for example that the space-time the prepared state and the detector are embedded in does not carry additional variables which would have to be taken into account for the measurement outcome, and neither does the description of the detector’s constituents on scales smaller than relevant for the experiment add to the hidden variables. Another assumption we have made is that we explicitly used there are finitely many hidden variables. We also used an ansatz according to which the correlation decays exponentially in time.

It should also be noted that the assumption of a super-deterministic correlation between prepared state and detector is not in conflict with a local time evolution assuming that their past lightcones intersect. However, this assumption will always be fulfilled for Earth based experiments.

Finally, it should be noted that we have assumed that the commutators between the measured observables remain the same as in standard quantum mechanics.

6 Closing Remark

During the work on this manuscript, it was brought to my attention footnote 1 of E. P. Wigner’s paper [15]:

“Von Neumann often discussed the measurement of the spin component of a spin-$\frac{1}{2}$ particle in various directions. Clearly, the possibilities for the two possible outcomes of a single such measurement can be easily accounted for by hidden variables [...] However, Von Neumann felt that this is not the case for many consecutive measurements of the spin component in various different directions. The outcome of the first such measurement restricts the range of values which the hidden parameters must have had before that first measurement was undertaken. The restriction will be present also after the measurement so that the probability distribution of the hidden variables characterizing the spin will be different for particles for which the measurement gave a positive result from that of the particles for which the measurement gave a negative result. The range of the hidden variables will be further restricted in the particles for which a second measurement of the spin component, in a different direction, also gave a positive result.”
great number of consecutive measurements will select particles the hidden variables of which are all so closely alike that the spin component has, with a high probability, a definite sign in all directions. However, according to quantum mechanical theory, no such state is possible. Schrödinger raised the objection against this argument that the measurement of a spin component in one direction, while possibly specifying some hidden variables, may restore a random distribution of some other hidden variables. It is this writer’s impression that Von Neumann did not accept Schrödinger’s objection. His point was that the objection presupposed hidden variables in the apparatus used for the measurement. Von Neumann’s argument needs to assume only two apparatus, with perpendicular magnetic fields, and a succession of measurements alternating between the two apparatus. Eventually, even the hidden variables of both apparatus will be fixed by the outcomes of many subsequent measurements of the spin component in their respective directions so that the whole system’s hidden variables will be fixed. Von Neumann did not publish this apparent refutation of Schrödinger’s objection.”

In a later volume of the same journal J. F. Clauser commented on Wigner’s recollection of Von Neumann’s and Schrödinger’s discussion and provided a particular example for hidden variables which are able to reproduce the predictions of quantum mechanics for repeated measurements of polarization directions [16]. In his reply to Clauser, Wigner (ibid) pointed out that Clauser’s example comes at the cost sacrificing two-way causality, i.e. there is no one-to-one correspondence between states at two instants of time and no time-reversal invariance. To this Clauser replied (ibid) that time-reversal invariance is maintained if one takes into account the history of the measurements. Clauser’s example, though admittedly contrived and applicable only for a special case, thus represents a case of hidden variables not of the type discussed here, one in which the hidden variables have nothing to do with the correlation between prepared state and detector, and the correlation time is zero.

The here proposed test would be a realization of Von Neumann’s thought experiment, taking into account that the alternating measurements can in practice not be done on exactly the same state. Thus, instead of entirely fixing the outcome of measurements, the effect of the hidden variables is a non-vanishing correlation time between measurement outcomes.

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