11 Correlations in Minimal $U(2)^3$ models and an $SO(10)$ SUSY GUT model facing new data

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Abstract Models with an approximate $U(2)^3$ flavour symmetry represent simple non-MFV extensions of the SM. We compare correlations of $\Delta F = 2$ observables in CMFV and in a minimal version of $U(2)^3$ models, $MU(2)^3$. Due to the different treatment of the third generation $MU(2)^3$ models avoid the $M_{d,s} - |\epsilon_K|$ correlation of CMFV which precludes to solve the $S_{\psi K_S} - |\epsilon_K|$ tension present in the flavour data. While the flavour structure in $K$ system is the same for CMFV and $MU(2)^3$ models, CP violation in $B_{d,s}$ system can deviate in $MU(2)^3$ models from CMFV. We point out a triple correlation between $S_{\psi \phi}$, $S_{\psi K_S}$ and $|V_{ub}|$ that can provide a distinction between different $MU(2)^3$ models.

GUTs open the possibility to transfer the neutrino mixing matrix $U_{PMNS}$ to the quark sector which leads to correlations between leptonic and hadronic observables. This is accomplished in a controlled way in an $SO(10)$ SUSY GUT model proposed by Chang, Masiero and Murayama (CMM model) whose flavour structure differ significantly from the CMSSM. We present a summary of a global analysis of several flavour processes containing $B_{d,s}$ mixing, $b \to s \gamma$ and $\tau \to \mu \gamma$. Furthermore we comment on the implications on the model due to the latest data of $S_{\psi \phi}$, $\theta_{13}$ and the Higgs mass.

11.1 Current situation of the flavour data

With the start of the LHCb experiment a new era in precision measurements in flavour physics started. The present 95% C.L. upper bound $B(B_s \to \mu^+\mu^-) \leq 4.5 \cdot 10^{-9}$ [1] is already close to the SM prediction $B(B_s \to \mu^+\mu^-)^{SM} = (3.1 \pm 0.2) \cdot 10^{-9}$ [2,3]. New data on mixing induced CP violation in $B_s - \bar{B}_s$ mixing measured by $S_{\psi \phi} = 0.002 \pm 0.0087$ [7] is consistent with the SM prediction of $S_{\psi \phi}^{SM} = 0.0035 \pm 0.002$ and excludes ranges from CDF and DØ with large $S_{\psi \phi}$. Thus there is not much room left for new physics (NP).

However a slight tension in the flavour data concerns $|\epsilon_K|$, $B^+ \to \tau^+ \nu$ and $S_{\psi K_S}$ which can be related with the so-called $|V_{ub}|$-problem. Both $|\epsilon_K| \propto \sin 2\beta |V_{cb}|^4$ and $S_{\psi K_S}$ can be used to determine $\sin 2\beta$. In Fig.11.1 (left) one can see that the $\sin 2\beta$ derived from the experimental value of $S_{\psi K_S}$ is much smaller that the one derived from $|\epsilon_K|$. This issue was discussed in

\footnote{In [4] the “non-radiative” branching ratio that corresponds to the branching ratio fully inclusive of bremsstrahlung radiation was calculated to $B(B_s \to \mu^+\mu^-) = (3.23 \pm 0.27) \cdot 10^{-9}$. When the corrections from $\Delta \Gamma$ pointed out in [5, 6] are taken into account the experimental upper bound is reduced to $4.1 \cdot 10^{-9}$.}
The “true” value of $\beta$ depends on the value of $|V_{ub}|$ and $\gamma$. However there is a tension between the exclusive and inclusive determinations of $|V_{ub}|$ [10]:

$$|V_{ub}^{\text{incl.}}| = (4.27 \pm 0.38) \cdot 10^{-3}, \quad |V_{ub}^{\text{excl.}}| = (3.38 \pm 0.36) \cdot 10^{-3}. \quad (11.1)$$

Now one can distinguish between these two benchmark scenarios: If one uses the exclusive value of $|V_{ub}|$ to derive $\beta_{\text{true}}$ and then calculates $S_{\psi K_S}^{\text{SM}} = \sin 2\beta_{\text{true}}$ one finds agreement with the data whereas $|\epsilon_K|$ stays below the data. Using the inclusive $|V_{ub}|$ as input for $\beta_{\text{true}}$, $S_{\psi K_S}$ is above the measurements while $|\epsilon_K|$ is in agreement with the data. However in such considerations one has to keep in mind the error on $|\epsilon_K|$ coming dominantly from the error of $|V_{cb}|$ and the error of the QCD factor $\eta_1$ [11].

The branching ratio $B(B^+ \rightarrow \tau^+ \nu)$ can also be used to measure $|V_{ub}|$. The SM prediction $B(B^+ \rightarrow \tau^+ \nu)_{\text{SM}} = (0.80 \pm 0.12) \cdot 10^{-4}$ as calculated in [12] where one eliminates the uncertainties of $F_{B^+}$ and $|V_{ub}|$ by using $\Delta M_d$, $\Delta M_d/\Delta M_s$ and $S_{\psi K_S}$ is about a factor 2 below the experimental world average based on results by BaBar [13] and Belle [14]: $B(B^+ \rightarrow \tau^+ \nu)_{\text{exp}} = (1.67 \pm 0.30) \cdot 10^{-4}$ [15]. Consequently this favors a large $|V_{ub}|$ and leads to a $S_{\psi K_S} - B(B^+ \rightarrow \tau^+ \nu)$ tension discussed for example in [16]. Recently new results have been provided by BaBar $B(B^+ \rightarrow \tau^+ \nu)_{\text{exp}} = (1.79 \pm 0.48) \cdot 10^{-4}$ [17] and by Belle $B(B^+ \rightarrow \tau^+ \nu)_{\text{exp}} = (0.72 \pm 0.27 \pm 0.46) \cdot 10^{-4}$ [18] where the latter value went down and is consistent with the SM prediction.

It is now interesting to see if a certain new physics model can solve these problems and if yes, which $|V_{ub}|$ scenario is chosen. In the following we will confront constraint minimal flavour violation (CMFV) and models with a global $U(2)^3$ symmetry to this tension. At the end we discuss the CMM model as an alternative to MFV.

### 11.2 Correlations of $\Delta F = 2$ observables: CMFV vs. $MU(2)^3$

A very simple extension of the SM is CMFV, where the CKM matrix is the only source of flavour and CP violation and only SM operators are relevant below the electroweak scale.
These properties lead to the following equations describing \( \Delta F = 2 \) observables where only three new parameters appear

\[
S_{\psi K_S} = \sin(2\beta + 2\varphi_{\text{new}}), \quad S_{\psi \phi} = \sin(2|\beta_s| - 2\varphi_{\text{new}}), \\
\Delta M_{s,d} = \Delta M_{s,d}^{\text{SM}} r_B, \quad \epsilon_K = r_K \epsilon_K^{\text{SM,tt}} + \epsilon_K^{\text{SM,cc+ct}}.
\]

Phenomenological consequences of CMFV concerning \( \Delta F = 2 \) observables are the following:

First, since there are no new CP violating phases the mixing induced CP asymmetries stay as in the SM: \( S_{\psi K_S} = \sin 2\beta, S_{\psi \phi} = \sin 2|\beta_s| \). Second, \( \Delta M_{s,d} \) and \( |\epsilon_K| \) can only be enhanced simultaneously relative to the SM [18–20]. Third, CMFV chooses exclusive \( S_{\psi K_S} \) stays as in the SM and \( |\epsilon_K| \) can be enhanced. But if one wants to solve the \( |\epsilon_K| - S_{\psi K_S} \) tension one gets a problem with \( \Delta M_{s,d} \). This \( \Delta M_{s,d} - |\epsilon_K| \) tension is shown in Fig. 11.1 (right).

Models with a global \( U(2)^3 \) flavour symmetry represent simple non-MFV extensions of the SM and can help avoiding this \(\Delta M_{s,d} - |\epsilon_K| \) tension. The \( U(2)^3 \) symmetry was first studied in [22, 23] and then in [24–30] where a detailed description of the model can be found (see also talk by F. Sala during this workshop). A nice feature of \( U(2)^3 \) is that one can easily embed SUSY with heavy 1st/2nd sfermion generation and a light 3rd generation which is still consistent with current collider bounds on sparticle masses. In a minimal version of this model, called \( MU(2)^3 \), the symmetry is broken minimally by three spurions and only SM operators are relevant. General consequences of \( MU(2)^3 \) concerning \( \Delta F = 2 \) observables are the following:

- The flavour structure in the \( K \)-meson system is governed by MFV (no new phase \( \varphi_K \)).
- Corrections in \( B_{d,s} \) system are proportional to the SM CKM structure and universal.
- There exists one new universal phase that only appears in \( B_{d,s} \) system: \( \varphi_{\text{new}} \).

These properties lead to the following equations describing \( \Delta F = 2 \) observables where only three new parameters appear

\[
S_{\psi K_S} = \sin(2\beta + 2\varphi_{\text{new}}), \quad S_{\psi \phi} = \sin(2|\beta_s| - 2\varphi_{\text{new}}), \\
\Delta M_{s,d} = \Delta M_{s,d}^{\text{SM}} r_B, \quad \epsilon_K = r_K \epsilon_K^{\text{SM,tt}} + \epsilon_K^{\text{SM,cc+ct}}.
\]
The parameters \( r_K, B \) are real and positive definite and further \( r_K \geq 1 \). In contrast to CMFV \( r_B \) and \( r_K \) are in principle unrelated. However in concrete realizations of the model, e.g. SUSY they both depend on SUSY masses. In [21] we point out a triple \( S_{\psi K_5} - S_{\psi \phi} - |V_{ub}| \) correlation which will provide a crucial test of the \( MU(2)^3 \) scenario once the three observables will be precisely known. This is shown in Fig. 11.2 (left) for fixed \( \gamma = 68^{\circ} \). Negative \( S_{\psi \phi} \) is only possible for small \( |V_{ub}| \) in the ballpark of the exclusive value. For inclusive \( |V_{ub}|, S_{\psi \phi} \) is always larger than the SM prediction. \( MU(2)^3 \) models that are consistent with this correlation should also describe the data for \( |\Delta m_{ee}| \) and \( \Delta M_{d,s} \). For example for \( S_{\psi \phi} < 0 \) the particular \( MU(2)^3 \) model must provide a 25\% enhancement of \( |\Delta m_{ee}| \) (see Fig. 11.2 right plot). Moreover, if this \( MU(2)^3 \) flavour symmetry turns out to be true one can determine \( |V_{ub}| \) by means of precise measurements of \( S_{\psi K_5} \) and \( S_{\psi \phi} \) with small hadronic uncertainties. The dependence of \( |\Delta m_{ee}| \) (only central values) on \( |V_{ub}| \) for different values of \( r_K \) is shown in the right plot of Fig. 11.2. Fixing \( S_{\psi K_5} = 0.679 \) to its central experimental value we can use the triple correlation to get the connection between \( |\Delta m_{ee}| \) and \( S_{\psi \phi} \) (see Fig. 4 in [21]). Thus we see that even in \( MU(2)^3 \) models correlations between \( B- \) and \( K- \) physics are possible.

### 11.3 \( SO(10) \) SUSY GUT: CMM model

In an \( SO(10) \) SUSY GUT model proposed by Chang, Masiero and Murayama [31,32] the neutrino mixing matrix \( U_{PMNS} \) is transferred to the right-handed down quark and charged lepton sector. In [33] we have performed a global analysis in the CMM model including an extensive renormalization group (RG) analysis to connect Planck-scale and low-energy parameters. A short summary can be found in [2,16,34]. In view of the new knowledge about the Higgs mass and the latest measurements of the reactor neutrino mixing angle \( \theta_{13} \) an updated analysis of this model would be desirable.

The basic ingredient of the flavour structure is that not only the neutrinos are rotated with \( U_{PMNS} \) but the whole \( 5 \)-plets of SU(5) \( 5_i = (d_R^c, \ell_R, -\nu_L)^T \). Including SUSY the atmospheric neutrino mixing angle \( \theta_{23} \approx 45^\circ \) is responsible for large \( \tilde{b}_R - \tilde{s}_R \) - and \( \tilde{\tau}_L - \tilde{\mu}_L \)-mixing which can then induce \( b \to s \) and \( \tau \to \mu \) transitions via SUSY loops. For a more detailed derivation starting from an \( SO(10) \) superpotential see [33]. From the superpotential and the requirement of perturbative couplings up to the Planck scale one can derive a range for \( \tan \beta \): 2.7 \( \lesssim \tan \beta \lesssim 10 \). Rotating from flavour to mass eigenstate basis the right-handed down squark mass matrix at \( M_Z \) reads

\[
m^2_D = U_D m^2_d U^*_D \approx m^2_{\tilde{d}_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \frac{1}{2} \Delta_{\tilde{d}} & -\frac{1}{2} \Delta_{\tilde{d}} e^{i\xi} \\ 0 & -\frac{1}{2} \Delta_{\tilde{d}} e^{-i\xi} & 1 - \frac{1}{2} \Delta_{\tilde{d}} \end{pmatrix},
\]

(11.4)

where the neutrino mixing enters through \( U_D = U^*_{PMNS} \text{diag}(1, e^{i\xi}, 1) \) and \( \Delta_{\tilde{d}} \in [0,1] \) defines the relative mass splitting between the 1\(^{st}\)/2\(^{nd}\) and 3\(^{rd}\) down-squark generation. It is generated by RG effects of the top Yukawa coupling and can reach 0.4. Thus the CMM model shares the feature of \( U(2)^3 \) models of heavy 1\(^{st}\)/2\(^{nd}\) squark generations but a light 3\(^{rd}\) generation. The 23-entry \( \alpha \Delta_{\tilde{d}} \) is responsible for \( \tilde{b}_R - \tilde{s}_R \)-mixing and a new CP violating

\(^2\)Varying \( \gamma \) between 63\(^\circ\) and 73\(^\circ\) does not change the result significantly.
phase $\xi$ enters that affects $B_S - \bar{B}_S$-mixing. The \(\approx\) sign in (11.4) gets a \(\approx\) if one usestribimaximal mixing in $U_{PMNS}$. However, the latest data show that $\theta_{13}$ is non-zero [35–37]. Including $\theta_{13} \neq 0$ the 12- and 13-entry in (11.4) are no longer zero, but still much smaller than the 23-entry. This gives small corrections to $K - \bar{K}$- and $B_d - \bar{B}_d$-mixing.

Flavour processes where we expect large CMM contributions are $B_S - \bar{B}_S$ mixing, $b \to s\gamma$ and $\tau \to \mu\gamma$ since here the angle $\theta_{23} \approx 45^\circ$ enters. CMM effects in $B(\bar{B}_S \to \mu^+\mu^-)$ are small and compatible with the LHCb bound because at the electroweak scale the CMM model is a special version of the MSSM with small $\tan\beta$. Due to the structure of (11.4) the contributions to $K - \bar{K}$ mixing, $B_d - \bar{B}_d$ mixing and $\mu \to e\gamma$ are absent. However there are two sources of small corrections: a non-vanishing $\theta_{13}$ and corrections due to dimension-5-Yukawa terms that are needed to fix $Y_{d} = Y_{l}^T$ for the 1st/2nd generation. The latter point was worked out in [38] where it was also shown that the $|\epsilon_K| - S_{\psi}\kappa_S$ tension can be removed with the help of higher-dimensional Yukawa couplings.

Results from our global analysis are the following: $\tau \to \mu\gamma$ constrains the sfermion masses of the first two generations to lie above 1 TeV while the third generation can be much lighter ($\tau \to \mu\gamma$ gives stronger bounds than $b \to s\gamma$). Concerning $B_S - \bar{B}_S$ mixing the situation changed after the LHCb data for $S_{\psi}\phi$. Due to the free phase $\xi$ it is possible to get large CP violation in the $B_S$ system in the CMM model while at the same time $\Delta M_S$ stays within its experimental range. In view of the data from CDF and DØ on $S_{\psi}\phi$, this property was very welcomed in 2010. The new data on $S_{\psi}\phi$ implies new constraints on the model parameters, especially on $\xi$ and on the ratio of gluino and squark masses $m_{\tilde{g}}/M_{\tilde{q}}$ which must now be smaller than before. This was exemplarily shown in [2].

Another observable that needs further investigation is the Higgs mass. In the CMM model
the mass of the lightest neutral Higgs is very sensitive to $\tan \beta^3$. In [33] we pointed out that $\tan \beta = 3$ is excluded due to the LEP bound. For $\tan \beta = 6$ the Higgs mass can be up to 120 GeV in the parameter range consistent with flavour observables. Consequently one has to increase $\tan \beta$ further to accommodate a Higgs mass of 125 GeV.

11.4 Summary

In the first part we studied and compared correlations of $\Delta F = 2$ observables in CMFV and in a minimal version of models with an approximate global $U(2)^3$ flavour symmetry. These $MU(2)^3$ models are very simple non-MFV extensions of the SM that avoid the $\Delta m_{s,d}$ tension present in CMFV. We pointed out a triple correlation between $S_{\psi\phi}$, $S_{\psi K_S}$ and $|V_{ub}|$ that constitutes an important test for $MU(2)^3$ models. In the last part an $SO(10)$ SUSY GUT model, the CMM model was under consideration where a summary can be found in Fig. [11.3]

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$^3$Decreasing $\tan \beta$ also decreases the Higgs mass because a larger $y_t$ increases the mass splitting $\Delta_3$ in the RG running which leads to smaller stop masses.
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