Deeply Virtual Compton Scattering: How to Test Handbag Dominance?

T. Gousset\textsuperscript{a,b}, M. Diehl\textsuperscript{c,d}, B. Pire\textsuperscript{c} and J.P. Ralston\textsuperscript{e}

\textit{a)} NIKHEF, P. O. Box 41882, 1009 DB Amsterdam, The Netherlands.
\textit{b)} SUBATECH, B. P. 20722, 44307 Nantes CEDEX 3, France.
\textit{c)} CPhT, Ecole Polytechnique, 91128 Palaiseau, France.
\textit{d)} DAPNIA/SPhN, C. E. Saclay, 91191 Gif sur Yvette, France.
\textit{e)} Dept. of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, USA.

Abstract

We propose detailed tests of the handbag approximation in exclusive deeply virtual Compton scattering. Those tests make no use of any prejudice about parton correlations in the proton which are basically unknown objects and beyond the scope of perturbative QCD. Since important information on the proton sub-structure can be gained in the regime of light cone dominance we consider that such a class of tests is of special relevance.

1 Why deeply virtual Compton scattering...

There has recently \cite{1, 2, 3} been considerable interest in exclusive virtual Compton scattering (VCS) and the proposal that off-diagonal matrix elements of quark operators in proton states might be measurable there. The idea is that the process

\[ \gamma^*(q) + p(p) \rightarrow \gamma(q') + p(p') \]  

proceeds via the short-distance Compton scattering on a single quark (cf. Fig. 1) in the Bjorken limit of large \( Q^2 = -q^2 \) at fixed \( x_B = Q^2/(2p \cdot q) \), and for a limited range of the momentum transfer \( t = (p - p')^2 \), or in other words of the transverse momentum \( \Delta_T \) of the scattered proton (in the c.m. of the collision), which should be of the order of a hadronic scale. (Through a hard quark loop the two photons can also couple to two gluons.) Following the old terminology of deep inelastic scattering those diagrams where two quark/gluon fields are connected to the proton states are referred to as \textit{handbag} diagrams. The short-distance processes can be computed in perturbation theory and the VCS amplitude is a sum of convolutions of those perturbative amplitudes with the Fourier transforms of off-diagonal matrix elements of quark or gluon fields such as (in the \( A^+ = 0 \) gauge)

\[ \langle p'|\bar{\psi}(0)\gamma^+\psi(z)|p \rangle, \quad \langle p'|G^{+i}(0)G^{+,i}(z)|p \rangle, \]  

(2)
quantities that have been introduced in the literature some time ago (for references cf. e.g [4]). We note that handbag dynamics singles out both specific Dirac/Lorentz components of the quark/gluon fields and light-like separations between them. At large $Q^2$ other contributions including correlations of three or more fields are power suppressed. We note that handbag dynamics singles out both specific Dirac/Lorentz components of the quark/gluon fields and light-like separations between them. At large $Q^2$ other contributions including correlations of three or more fields are power suppressed.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{handbag_diagrams.png}
\caption{The handbag Born diagrams for virtual Compton scattering.}
\end{figure}

From this line of reasoning it is clear that the process shares similarities with inclusive deep inelastic scattering, whose cross section is directly proportional to the imaginary part of the forward Compton amplitude. There one also selects in the Bjorken limit products of two quark or two gluon fields with light-like separations. The Compton scattering reaction is however richer because of the difference between the initial and final state protons. The light-like separation being probed in a different manner may help us to extend our understanding of proton structure at the quark-gluon level.

A very interesting feature discovered by Ji [1] is that sum rules for the off-diagonal matrix elements carry information on the orbital angular momentum of quarks and gluons in the proton. From this point of view their study naturally extends that of proton spin structure, which has been an object of intense research in the last decade.

2 ... and how?

There is now some theoretical evidence for the separation of long and short distance dynamics in the framework of perturbative QCD. An important question is then to ask how to test this prediction experimentally. We have in mind tests which rely on general principles and avoid theoretical prejudices or model assumptions about the unknown off-diagonal matrix elements.

To formulate tests of the dominance of the handbag diagrams we make use of their spin selection rules for the $\gamma^* p \rightarrow \gamma p$ process. For this purpose we consider the perturbative Compton scattering of a virtual photon on a free quark target in the high energy limit. In a frame where all particles are fast and in the region of nearly collinear kinematics we are considering, the process has the remarkable property of helicity conservation for the photon. This follows from the fact that a massless quark cannot change its helicity. (QCD loop corrections to order $\alpha_S$ may violate the photon helicity conservation rule by two units, but interestingly not by one [4].) At $\Delta_T = 0$ the proton helicity can then not be flipped either because of angular momentum conservation.
For finite small $\Delta_T$, however, the handbag does give proton helicity flip amplitudes at leading twist, with an overall factor of $\Delta_T R$, where $R$ is of the order of the proton radius.

The reaction where one can study virtual Compton scattering is the electroproduction process

$$e(k) + p(p) \rightarrow e(k') + p(p') + \gamma(q') .$$

A great opportunity is offered by the interference of VCS with the Bethe-Heitler (BH) amplitude, where the photon is radiated off the electron. In addition to the kinematical invariants introduced to describe VCS we use the standard variable $y = (q \cdot p)/(k \cdot p)$ and the angle $\varphi$ between the leptonic and hadronic scattering planes in the c.m. of the scattered photon and scattered proton. The $\varphi$-dependence of the $ep$ cross section contains a wealth of information, as we shall see.

With appropriate phase conventions for the external particles the contribution of VCS to the amplitude of (3) for a given helicity $\lambda$ of the intermediate photon $\gamma^*(q)$ depends on $\varphi$ as

$$\mathcal{T}_{VCS} \propto \exp(-i\lambda \varphi) .$$

The BH amplitude has in general a rather complicated $\varphi$-dependence, but at leading order in $1/Q$ it simplifies to

$$\mathcal{T}_{BH} \propto \exp(-2i\lambda' \varphi) + O\left(\frac{1}{Q}\right)$$

for a scattered photon of helicity $\lambda'$. With this one can analyze the $\varphi$-dependence of the $ep$ cross section $\mathcal{B}$. It displays a hierarchy in powers of $1/Q$, with $|\mathcal{T}_{BH}|^2$, BH–VCS interference and $|\mathcal{T}_{VCS}|^2$ respectively going like 1, $1/Q$ and $1/Q^2$ in the handbag model, accompanied with a $\varphi$-distribution going like $\cos(n\varphi)$ with $n = 0, 1, 2$ and 3. A powerful consequence is that extracting the angular distribution does not require any binning of data. Using the weighted cross sections

$$\langle \langle f(\varphi) \rangle \rangle = \frac{Q^4}{x_B y^2} \int d\varphi \frac{d\sigma}{d\varphi dt dQ^2 dx_B} \cdot f(\varphi)$$

with $f(\varphi) = 1, \cos \varphi, \cos 2\varphi, \cos 3\varphi$, one directly projects out the coefficients of $\cos(n\varphi)$ in $|\mathcal{T}_{VCS} + \mathcal{T}_{BH}|^2$ and can formulate a number of tests for the predictions of the handbag approximation. We restrict ourselves here to the case where $y$ is not too small; then the BH–VCS interference dominates over the squared VCS amplitude.

We can test the scaling properties of the handbag. Let us define helicity amplitudes $M_{h,h'}^{\lambda,\lambda'}$ for $\gamma^* p \rightarrow \gamma p$, where $\lambda$ ($\lambda'$) is the helicity of the initial (final) state photon and $h$ ($h'$) that of the initial (final) state proton. To order $1/Q$ the $\cos \varphi$-term in the BH–VCS interference is multiplied by the photon helicity conserving amplitudes $M_{h,h'}^{1,1}$ which the handbag approximation predicts to be constant in $Q$. There is also a $\cos \varphi$-term in the square of the BH amplitude, which has the same global power $1/Q$ and thus must be subtracted if we want to investigate VCS. Note that the BH process including its QED radiative corrections can be calculated, and that the elastic proton form factors are well...
parameterized in the region of small $t$ where they are needed. We then have as a test for the scaling properties in the handbag that at fixed $\Delta_T$, $x_B$ and $y$

$$\langle\langle \cos \varphi \rangle\rangle - \langle\langle \cos \varphi \rangle\rangle_{BH\text{ only}} \sim 1/Q \ ,$$  \hspace{1cm} (7)

where $\langle\langle \cos \varphi \rangle\rangle_{BH\text{ only}}$ denotes the contribution of the squared BH amplitude. Of course this scaling behavior is to be understood as up to logarithms due to QCD radiative corrections in the handbag.

To test photon helicity conservation we can use $\langle\langle \cos 2\varphi \rangle\rangle$ and $\langle\langle \cos 3\varphi \rangle\rangle$. To order $1/Q$ they receive contributions from the BH–VCS interference proportional to $M^{0,1}_{h,h'}$ and $M^{-1,1}_{h,h'}$, respectively, which are zero in the handbag approximation and thus should be power suppressed. $|T_{BH}|^2$ does not contain any $\cos 2\varphi$ or $\cos 3\varphi$ up to order $1/Q$ so that, without needing to subtract this contribution, we have as a test for the handbag that

$$\langle\langle \cos 2\varphi \rangle\rangle , \langle\langle \cos 3\varphi \rangle\rangle \sim 1/Q^n \ , \ n \geq 2 \ .$$  \hspace{1cm} (8)

From (4) and (8) one has of course that $\langle\langle \cos 2\varphi \rangle\rangle$ and $\langle\langle \cos 3\varphi \rangle\rangle$ should be small compared with $\langle\langle \cos \varphi \rangle\rangle$, a test that can even be done without much lever arm in $Q^2$.

For lack of space we shall not discuss here the kinematic regime where $y$ is close to zero and where the VCS contribution to the amplitude dominates over the contribution from BH. Additional and complementary information on VCS can be gained using lepton beams with longitudinal polarization, while still averaging over the proton spin; the interested reader is referred to Ref. [4].

**Acknowledgments**

This work is partially funded through the European TMR Contract No. FMRX-CT96-0008 and through DOE grant No. 85ER40214. T. G. was carrying out his work under the European Contract No. ERBFMBICT950411. CPhT is Unité propre 14 du Centre National de la Recherche Scientifique.

**References**

[1] X. Ji, Phys. Rev. Lett. **78** 610 (1997); Phys. Rev. **D55** 7114 (1997).

[2] A.V. Radyushkin, Phys. Lett. **B380** 417 (1996); Phys. Rev. **D56** 5524 (1997).

[3] C. Hyde-Wright, Nucl. Phys. **A622** 268c (1997).

[4] M. Diehl, T. Gousset, B. Pire and J.P. Ralston, Phys. Lett. **B411** 193 (1997).