Nuclear Transparency and Effective NN Cross Section in Heavy Ion Collisions at 14.6 GeV/nucleon

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Abstract

The probability of a projectile nucleon to traverse a target nucleus without interaction is calculated for central Si-Cu and Si-Pb collisions. Special attention is given to the impact parameter range which contributes to events with large transverse energy. A fit to the data from E814 requires an effective NN cross section of $\sigma_{\text{eff}} = 54.2 \pm 5.0$ mb (compared to the free space value $\sigma_{\text{im}}^{NN} = 30$ mb) and is interpreted as one related to wounded target nucleons.

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The dynamics of a heavy ion collision is a complicated many-body problem for various reasons. It is the task of appropriately designed experiments to isolate one particular aspect of the dynamics and elucidate its physics. Wounded nucleons are one of the open problems. In a heavy ion collision a nucleon may undergo a sequence of collisions, which follow so rapidly one after another, that the nucleon is no more in its ground state, even not necessarily in any definite excited baryonic resonance (like $\Delta$ or $N^*$). We will speak of a wounded...
nucleon. In a next encounter with another nucleon this wounded nucleon will not interact with the free space NN cross section $\sigma_{NN}^{NN}$ but with an effective one $\sigma_{eff}$. Is it possible to determine $\sigma_{eff}$ from experiment?

The E814 collaboration has designed an experiment to answer this question [1]. At the energy of 14.6 GeV/nucleon, the projectile $^{28}Si$ collides with $Al$, $Cu$ and $Pb$ and the beam rapidity nucleons are studied as a function of the centrality of the reaction (controlled by a measurement of the transverse energy). The beam-rapidity nucleons belong to the projectile and have not lost any energy in the reaction. Of course, in a peripheral reaction one always has the so called spectator nucleons, which pass the target nucleus without interaction. They are not interesting for our purpose. However, in a central event, e.g., for $Si$ on $Pb$, one still sees beam rapidity nucleons. The target nucleus is transparent for these projectile nucleons. We expect the transparency of a heavy nucleus like $Pb$ to be small. Indeed, the “survival probability” $S$ for a projectile nucleon to pass through the target without any inelastic interaction has been measured to $S_{exp} = 3.5 \cdot 10^{-3}$ for central $Si-Pb$ collisions [1]. Does this result contain information about wounded nucleons and effective NN cross sections?

In a heavy ion collision those projectile nucleons which arrive first at the target nucleus, interact with ground state nucleons of the target, but may transform them into wounded ones. The next wave of projectile nucleons already finds wounded target nucleons and interacts with them via a modified, probably larger cross section and transforms them to a higher degree of woundedness. This procedure repeats itself for the third and following waves. A complicated situation like this may best be simulated by a cascade code. However, present day computer codes are not yet in a position to handle off-energy-shell situations. Instead they use on-shell baryons (in their ground and excited states) together with mesons as their basic degrees of freedom. A code like this, ARC, has been applied to the data from E814 by Schlagel et al. [3] and has lead to agreement with the data without wounded nucleons and effective cross section, though with large error bars because of the low statistics. As we will argue on the basis of time scales, the basic assumptions of the cascade code may not be fulfilled and another approach may be better justified. It will be presented in this paper.

The survival probability $S$ for a projectile nucleon to pass through the target without inelastic interaction is calculated in a Glauber type approximation (straight lines, frozen
nucleons) as

\[
S(b, \sigma_{\text{eff}}) = \int d^2 s T_p(\vec{b} - \vec{s}) e^{-\sigma_{\text{eff}} A_t T_t(\vec{s})},
\]

(1)

where \(\vec{b}\) is the impact parameter of the nucleus-nucleus collision, \(T_p(b)\) and \(T_t(b)\) are the thickness functions \((T(b) = \int dz \rho(b, z), \int d^3 x \rho(x) = 1)\) of projectile and target, respectively. The straight line geometry is certainly justified for the through-going nucleons. The use of “frozen” nucleons and the neglect of any other degrees of freedom, like mesons, needs justification. We consider a target nucleon and estimate the time \(\Delta t\) in its rest system which has elapsed between the arrival times of the first and the last projectile nucleons:

\[
\Delta t \leq 2R_p/\gamma_p
\]

where \(2R_p \approx 7\text{ fm}\) is the diameter of the \(Si\) projectile and \(\gamma_p \approx 15.6\) is the Lorenz factor for the experiment under consideration. The time \(\Delta t \approx 0.5\text{ fm/c}\) is rather short: (i) The target nucleon has not moved significantly in space (frozen approximation is good). (ii) According to the uncertainty principle the intrinsic excitation energy is uncertain by \(\Delta E \geq (\Delta t)^{-1} \approx 0.4\text{ GeV}\) and does not allow the definition of a definite excited state. (iii) Secondary hadrons are not yet formed since typical formation times are of order 1 fm/c.

The nature of a wounded nucleon is not clear. We parametrise any modification into an effective cross section \(\sigma_{\text{eff}}\) between a ground-state nucleon (of the projectile) and a wounded nucleon (of the target), and determine it from experiment. Using Saxon-Woods parametrisations for the densities \(\rho_t\) and \(\rho_p\) with the surface thickness \(a = 0.52\text{ fm}\) for all nuclei and the half-density radius \(r_A\) such that the root-mean-square radius of the nucleus equals the charge radius, the survival probability \(S(b, \sigma_{\text{eff}})\) is calculated for two values \(\sigma_{\text{eff}} = 30\) and 50mb, and the results are displayed in Fig.1. The experimental point, measured at transverse energy \(E_t^c = 15.5\text{ GeV}\) is also shown in the figure. It is obtained from the measured mean multiplicity \(< M_c >\) of beam rapidity protons by

\[
S(E_t) = < M_c > (E_t)/Z,
\]

(2)

where \(Z = 14\) is the number of protons in \(Si\). We have assumed - as the authors of the experiment do - that the high value \(E_t^c\) corresponds to a central collision which is assigned a value \(b = 0\). Then the experimental value is close to the curve \(\sigma_{\text{eff}} = 30\text{mb}\). In fact
the equation \( S(b = 0, \sigma_{\text{eff}}) = S_{\text{exp}}(E_t^c = 15.5 \text{GeV}) \) leads to a value \( \sigma_{\text{eff}} = 31.1 \pm 0.7 \) mb. This result reproduces a similar calculation using uniform density distributions by the E814 collaboration [1], who have concluded that \( \sigma_{\text{eff}} = \sigma_{\text{NN}} \) within error bars and no anomaly being visible.

The crucial step in the argument is the assignment of \( b = 0 \) to the central value of transverse energy \( E_t \). Indeed, a given value of \( E_t \) determines an impact parameter \( b_m(E_t) \) only within a band \( \Delta b(E_t) \). And \( \Delta b(E_t) \) may be very large! We study the relation between \( E_t, b_m \) and \( \Delta b \) in the form of a probability distribution \( P(E_t, b) \), which gives the probability that values of \( b \) contribute to events with a given value of \( E_t \). We normalize it as \( \int dE_t P(E_t, b) = 1 \). With this function and the differential inelastic heavy ion cross section \( d\sigma_{\text{in}}/d^2b \), one can obtain the dependence of the survival function \( S(E_t, \sigma_{\text{eff}}) \) as function of transverse energy

\[
S(E_t, \sigma_{\text{eff}}) = \int d^2b S(b, \sigma_{\text{eff}}) P(E_t, b) d\sigma_{\text{in}}/d^2b.
\]

where

\[
\frac{d\sigma_{\text{in}}}{dE_t} = \int d^2b P(E_t, b) \frac{d\sigma_{\text{in}}}{d^2b}.
\]

We will use Eq.\([2]\) to determine \( P(E_t, b) \) from a comparison with the measured transverse energy distribution \( d\sigma_{\text{in}}/dE_t \). Only then \( S(E_t, \sigma_{\text{eff}}) \) can be calculated from Eqs.\([1]\) and \([3]\) without ambiguity. We discuss our parametrisations.

The inelastic cross section \( d\sigma_{\text{in}}/d^2b \) is taken from the folding model

\[
\frac{d\sigma_{\text{in}}}{d^2b}(b) = 1 - \exp[-\sigma_{\text{in}}^{NN} A_p A_t \int d^2s T_p(b - \vec{s})T_t(s)].
\]

For central collisions (small value of \( b \)), the exponential is practically zero and any uncertainties, e.g., the choice of \( \sigma_{\text{in}}^{NN} \), are unimportant. The parametrisation of \( P(E_t, b) \) is more model dependent. We choose a Gaussian parametrisation

\[
P(E_t, b) = \frac{1}{\sqrt{2\pi\sigma_t^2(b)}} \exp\{-\frac{[E_t - E_t(b)]^2}{2\sigma_t^2(b)}\},
\]

which satisfies the normalization condition. We make the usual assumptions\([4]\)\([5]\) for the functions \( E_t(b) \) and \( \sigma_t(b) \).
\[ E_t(b) = N(b)\epsilon_0, \quad (7) \]

\[ \sigma_t^2(b) = N(b)\epsilon^2_0\omega. \quad (8) \]

Here \( N(b) \) is calculated either in the “collision model” \( N_c(b) \) or in the “wounded nucleon model” \( N_w(b) \).

\[ N_c(b) = \sigma_{NN}^{in} A_t A_p \int d^2b T_p(\vec{s}) T_t(\vec{b} - \vec{s}), \quad (9) \]

\[ N_w(b) = A_t \int d^2b T_p(\vec{s}) \exp[-\sigma_{NN}^{in} T_t(\vec{b} - \vec{s})] + (t \leftrightarrow p). \quad (10) \]

\( N_c(b) \) equals the mean number of NN collisions in a projectile-target interaction with impact parameter \( b \), and \( N_w(b) \) equals the number of nucleons in the overlap volume of projectile and target. The proportionality constant \( \epsilon_0 \) between the observed transverse energy \( E_t \) and \( N(b) \) depends on the dynamics of hadron production, but also on the experimental set up (chosen rapidity interval and acceptance of counter). It will be a fit parameter. In the collision model, which we will use, only the product \( \epsilon_0\sigma_{NN}^{in} \) enters and the physics of Eq.(6) is completely independent of the choice of \( \sigma_{NN}^{in} \). For the wounded nucleon model this is only approximately true. The ansatz \( \sigma_t^2 \propto N(b) \) corresponds to the hypothesis that we deal with statistical fluctuations in \( N(b) \). It is known, however, that the proportionality constant \( \omega \) in Eq.(8) depends strongly on the rapidity interval of the accepted particles. The reason is not clear. We take \( \omega \) as a free parameter.

Using expressions (5) to (8) we have calculated \( d\sigma_{in}/dE_t \) and have varied the parameters \( \epsilon_0 \) and \( \omega \) until the data for Si-Pb are fitted. The result is shown in Fig.2 with \( \sigma_{NN}^{in} = 30mb, \epsilon_0 = 0.067GeV, \omega = 7.8 \) for the collision model and \( \epsilon_0 = 0.1GeV, \omega = 6. \) for the wounded nucleon model. Both models for the calculation of \( N(b) \), describe the data Si-Pb equally well. But when the same parameters are used to describe \( d\sigma_{in}/dE_t \) for Si-Al and Si-Cu collisions, the collision model gives better fits and therefore we discard the other model.

According to Eq.(3), \( P(E_t, b) \) determines the integration region in impact parameter \( b \), which contributes to the integral for a given value of \( E_t \). As an example, Fig.1 shows
the weight function $P(E_t^c, b)$. Its width is unexpectedly large. In particular the survival probability $S(b, \sigma_{eff})$ changes considerably in the $b$-region, where $P(E_t^c, b)$ is large. This invalidates the assumption to put the experimental point at $b = 0$.

Since $P(E_t, b)$ plays such important role in the understanding of the experiment, we have studied its properties in more detail. For the functions $P(E_t, b)$ used to fit $d\sigma / dE_t$ we have calculated the value $b_m(E_t)$ of the impact parameter which contributes maximally to a given $E_t$ and $\Delta b(E_t)$ which is the width of the band of impact parameters around $b_m(E_t)$. We define these two quantities by

$$\frac{d}{db} P(E_t, b) \bigg|_{b=b_m(E_t)} = 0,$$

and

$$[\Delta b(E_t)]^2 = \int d^2 b (b - b_m)^2 P(E_t, b) / \int d^2 b P(E_t, b).$$

For the Gaussian parametrisation Eq. (6) one has approximately the relations

$$E_t[b_m(E_t)] = E_t,$$

$$[\Delta b(E_t)]^2 = \omega \frac{N(b_m)}{[N'(b_m)]^2},$$

where $N'(b) = dN/db$. Note that the width of the band in impact parameter, which contributes to a given $E_t$ is proportional to the width $\omega$ of the Gaussian of $P(E_t, b)$.

Fig. 3 shows the position $b_m(E_t)$ of the maximum of $P(E_t, b)$ and the width $\Delta b(E_t)$ as a function of $E_t$ for Si-Pb collisions. As expected the position $b_m$ goes to zero and then stays zero for large values of $E_t$. However, the width $\Delta b$ shows a break at $E_t^0$ where $b_m(E_t)$ becomes zero. This is related to a sudden change in the shape of $P(E_t, b)$ at $E_t^0$. For $E_t > E_t^0$ the width $\Delta b$ decreases only very slowly.

Fig. 1 shows as example the distribution $P(E_t, b)$ for a central collision ( $E_t^c = 15.5$ GeV ). We have compared this form with the corresponding one $P_U(E_t^c, b)$ calculated with the help of the code UrQMD [9]. The shape of $P_U$ is similar to $P$, but narrower. (The value of $< b >$ is smaller by about 25%). However, also $d\sigma / dE_t$ calculated from UrQMD falls off faster than the experimental data. Using the functions $S(b, \sigma_{eff})$ and
as shown in Fig.1, the calculated value of \( S(E^c_t, \sigma_{eff}) \) gets contributions from a considerable range of values of \( b \). We find

\[
S(E^c_t, \sigma_{eff})/S(b = 0, \sigma_{eff}) = 2.86
\]

for \( \sigma_{eff} = 30 \text{mb} \). Since \( S(b = 0, \sigma_{eff} = 30 \text{mb}) \) corresponded essentially to the experimental value, \( S(E^c_t, \sigma_{eff} = 30 \text{mb}) \) does not. A solution of the equation for \( \sigma_{eff} \)

\[
S(E^c_t, \sigma_{eff}) = S_{exp}(E^c_t)
\]

leads to

\[
\sigma_{eff} = 54.2 \pm 5.0 \pm 6.4 \pm 5.6 \text{mb}.
\]

The meanings of the errors are explained below and in the caption of Table 1. The same equation for central events ( \( E^c_t = 9.5 \text{ GeV} \)) in Si-Cu collisions leads to \( \sigma_{eff} = 50.3 \pm 15.0 \pm 4.6 \pm 6.4 \text{mb} \), in agreement with the value from Si-Pb. Since Al is a smaller nucleus than Si, the beam rapidity nucleons from Si-Al will always be contaminated by spectator nucleons and not very sensitive to effects of the nuclear transparency. The results of the values for \( \sigma_{eff} \) are compiled in Table 1. We have also calculated \( \sigma_{eff} \) by using \( P_U(E^c_t, b) \) from UrQMD and have obtained \( \sigma_{eff} = 40.3 \pm 3.0 \text{mb} \), where the error reflects only the statistical fluctuations in \( P_U \).

The result for \( \sigma_{eff} \) from our analysis is at variance with the conclusion by the authors of the E814 experiment [1] and of the cascade calculation of Ref. [2]. While we are unable to understand the difference to the cascade calculation[3], the difference to the argument of the E814 collaboration is clear: They assume that central collisions means \( b = 0 \) with very small width, while our analysis shows that even for central collisions a fairly wide band of impact parameters contributes. The width of this band depends on the parameters \( \epsilon_0 \) and \( \omega \) in the function \( P(E_t, b) \). How sensitive are the data to these parameters? Table 1 gives the uncertainties in the extracted values of \( \sigma_{eff} \) if one assumes uncertainties of 5% in \( \epsilon_0 \) and 10% in \( \omega \), respectively. A reduction by 25% in \( \omega \) would reproduce the results of UrQMD for \( d\sigma/dE_t \) and \( \sigma_{eff} \) but would be in contradiction to the data for \( d\sigma/dE_t \) and therefore the extracted value \( \sigma_{eff} \approx 40 \text{mb} \) is doubtful. According to the results presented in Table 1 and from the cascade calculations the extracted effective cross section seems to
be definitely larger than the inelastic cross section \( \sigma_{\text{in}}^{NN} \) for a NN collision in vacuum.

The difference may be interpreted in terms of wounded nucleons. We try to estimate the effect by the following model\[10\] which a wounded nucleon \( N^+ \) consists of a nucleon plus a semi-hard prompt gluon, which is radiated as bremsstrahlung in quark-quark interactions during an inelastic NN collision\[11\]. We furthermore use the additive quark model (a nucleon consists of 3 quarks) to relate the \( NN \) cross section to the nucleon-quark (\( Nq \)) one via \( \sigma^{NN} = 3\sigma^{Nq} \). Then

\[
\sigma^{NN^+} = 3\sigma^{Nq} + \sigma^{Ng} = \sigma^{NN}(1 + \frac{\sigma^{Ng}}{3\sigma^{Nq}}).
\]  \hspace{1cm} (17)

If in addition one assumes \( \sigma^{Ng}/\sigma^{Nq} \approx 9/4 \), the color factor, one obtains \( \sigma_{\text{in}}^{NN^+} \approx 50 \text{ mb} \). In view of all the uncertainties in the definition of \( \sigma_{\text{eff}} \), in the model for \( N^+ \) and in the additive quark model, the close agreement between experiment and the theoretical estimate must be considered fortuitous, but it may show that the order of magnitude of the extracted value for the effective cross section is not unreasonable.

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Captions:

Fig. 1 Survival probabilities \( S(b, \sigma_{\text{eff}}) \) of a beam rapidity nucleon calculated for \( \sigma_{\text{eff}} = 30 \) mb and \( \sigma_{\text{eff}} = 50 \) mb (solid lines) and the correlation \( P(E_T^c = 15.5\text{GeV}, b) \) between transverse energy and impact parameter (dashed line) for \( \text{Si-Pb} \) collisions as a function of \( b \). The data point is obtained from the measured multiplicity \( < M_c > = 0.049 \pm 0.005 \) of beam rapidity protons at \( E_T^c \).

Fig. 2 Transverse energy distributions in the rapidity range \(-0.5 < \eta < 0.8\) for a \( \text{Si} \) beam at 14.6GeV/nucleon on different targets. Data are taken from [7]. The calculations refer to the collision model (solid lines) and the wounded nucleon model (dashed curves). For each model one set of parameters \( \epsilon_0, \omega \) has been determined by a fit to the \( \text{Pb} \) data and then applied to the other data.

Fig. 3 The position \( b_m(E_t) \) of the maximum of \( P(E_t, b) \) in impact parameter \( b \) (solid line) and the width \( \Delta b(E_t) \) around it (dashed line) as a function of \( E_t \) for \( \text{Si-Pb} \) collisions.

Table 1 The fitted effective NN cross sections from the experimental value of the multiplicity \( < M_c > \) of beam rapidity nucleons in collisions of \( \text{Si-Pb} \) and \( \text{Si-Cu} \) at AGS energy of 14.6GeV/nucleon. If one assumes that the \( E_T^c \) corresponds to \( b = 0 \) and uses distributions of uniform density (u.d.) or Saxon-Woods (S.W.) one finds the values in columns 4 [1] and 5, respectively. The last column corresponds to our result. The first error corresponds to the uncertainty in \( < M_c > \), while the second and third errors correspond to uncertainties of 5% in \( \epsilon_0 \) and 10% in \( \omega \), respectively.
$$S(b, \sigma_{\text{eff}}), P(E_T^c, b)$$

$S(b, 30\text{mb})$

$S(b, 50\text{mb})$

Exp. $P(15.5\text{GeV}, b)$

Fig. 1
Fig. 2

Graph showing the relationship between $d\sigma/dE_T$ (mb/GeV) and $E_T$ (GeV) for different systems: Si+Al, Si+Cu, Si+Pb.

- $E^0_T = 15.5$ GeV

Note: The graph shows a decrease in $d\sigma/dE_T$ as $E_T$ increases.
$b_m, \Delta b (\text{fm})$

$E_T (\text{GeV})$

$E_T^0 = 15.5 \text{GeV}$

Fig. 3
| $E_T^p$ (GeV) | $< M_c >$ | $\sigma_{eff}$ (mb) |
|-------------|---------|------------------|
|             |         | $b=0$ (u.d.)     | $b=0$ (S.W.) | $E_i = E_T^p$ |
| Pb          | 15.5    | 28.8±0.5         | 31.1±0.7     | 54.2±5.0±6.4±5.6 |
| Cu          | 9.5     | 28.8±1.8         | 35.8±2.5     | 50.3±15.0±4.6±6.4 |

Table 1