Quantum technologies hold great promise for revolutionizing photonic applications such as cryptography, sensing and imaging. Yet their implementation in real-world scenarios is still held back, mostly due to the sensitivity of quantum states of light to scattering. The recent developments in shaping of single photons introduce new ways to control scattering of quantum light. Here we cancel scattering of entangled photons, by shaping the classical laser beam that stimulates their creation, rather than shaping them directly. We show that when the laser beam and the entangled photons pass through the same diffuser, focusing the laser using classical wavefront shaping recovers the unique correlations of entangled-photons that were scrambled by scattering. Since the shaping process is done exclusively on the classical laser beam, it does not introduce any loss to the entangled photons, and it is not limited by the low signal-to-noise ratios associated with quantum light, opening the door for efficient real-time wavefront shaping for photonic quantum applications.

Nearly a century after quantum mechanics revolutionized the way we understand nature, quantum resources such as superposition and entanglement are beginning to enter and transform technology (1, 2). Many implementations of quantum technologies such as quantum communication (3–5) and quantum imaging (6–8), are based on photonic platforms that encode quantum bits (qubits) using single photons. One of the main challenges in such applications is the low flux of single photons that can be sent per communication channel or image pixel, resulting in extremely low capacities of these systems. A promising approach for boosting the capacity of quantum systems is to encode multi-level quantum bits (coined qudits) using a single photon in a d-dimensional Hilbert space (9–11). To this end, photonic qudits have been implemented in the temporal (12), spectral (13) and spatial (14, 15) domains. The spatial domain is in particular attractive, since using spatial light modulators (SLMs) it is possible to arbitrary rotate qudits in
the d-dimensional space, simply by shaping the spatial distribution of photons (16). However, this also implies that the information carried by spatial qudits is extremely sensitive to scattering and aberrations, acting as random rotations on the qudits. For example, scattering of entangled photons that encode spatial qudits, scrambles their unique quantum correlations, resulting in a random grainy spatial correlation pattern, coined two-photon speckle (17, 18). Scattering of photonic qudits is therefore a limiting factor in implementing photonic quantum technologies in real-world applications, such as ground-satellite quantum communication or quantum imaging of biological samples. In the past few years extensive research was devoted for protecting the information carried by spatial qudits, by encoding them in spatial modes that are immune to scattering and aberrations (19–22). Nevertheless, for significant scattering all spatial modes will eventually suffer from scrambling of the information carried by the photons (23, 24).

A promising approach for cancelling scattering of classical light is wavefront shaping. Over a decade after the pioneering work of Vellekoop and Mosk (25), who focused classical light through scattering media using an SLM, a remarkable set of tools for controlling light in random media has been developed (26–28). It is therefore appealing to adopt these tools to the quantum regime. Over the past few years several important developments towards spatial control of single (29–32) and entangled photons (32) have been reported. In most demonstrations to date, photon-pairs generated by spontaneous parametric down conversion (SPDC) were sent to an SLM that directly shaped their spatial distribution before hitting a scattering sample. In a few other demonstrations, the SLM modulated the bright laser beam that stimulates the SPDC process (coined pump beam), and feedback was provided by the inherently weak quantum signal.

In this work, we take a different approach and perform the entire wavefront optimization process on the classical bright pump beam which passes through the same scattering media as the entangled photons (Fig. 1a). Remarkably, focusing the pump beam through the random media, simultaneously cancels the effect of scattering on the quantum state of the entangled photons. We explain this striking result by showing, both theoretically and experimentally, that when the pump beam scatters by the same random sample as the entangled photons, the spatial distribution of its intensity is identical to the spatial correlations of the entangled photons. Hence any manipulation of the classical pump intensity has an identical effect on the entangled photons correlations. Since wavefront shaping of a classical bright beam is much faster and more efficient than shaping weak fluxes of entangled photons, we were able, for the first time, to demonstrate real-time wavefront correction of entangled photon scattered by a dynamically moving diffuser.
Figure 1: Experimental setup. (a) Spatially entangled photons are created by pumping a non-linear crystal (PPKTP) with a $\lambda = 404 \text{nm}$ continuous-wave laser. Both the pump beam and the entangled photons pass through a diffuser, which is imaged on the crystal and SLM planes, and measured at the far-field. An optimization method is employed for compensating the scattering of the pump beam using the SLM. (b) The far field single counts, in the absence of the diffuser, is much wider than the coincidence distribution (c), indicating high spatial entanglement.

To explain why scattering of two entangled photons corresponds with scattering of a single pump photon at half the wavelength, we write the quantum state of the two photons (coined signal and idler photons), in terms of their transverse wavevector components $q_s$ and $q_i$.

$$|\psi\rangle = \int dq_s dq_i \psi(q_s, q_i) a^\dagger(q_s) a^\dagger(q_i) |0\rangle.$$  

Here $a^\dagger(q)$ is the creation operator of a photon with a transverse momentum $q$, $|0\rangle$ is the vacuum state, and we assume the signal and idler photons have the same frequency and polarization. The two-photon amplitude $\psi(q_s, q_i)$ can be expressed in terms of the angular spectrum of the pump beam $v(q)$ (33),

$$\psi(q_s, q_i) = v(q_s + q_i) \Phi(q_s - q_i),$$

where $\Phi(q) \propto \text{Sinc}(\frac{L}{4k}q^2)$ is the phase matching function of the SPDC crystal, $L$ is the crystal length and $k$ is the pump wavenumber inside the crystal. The number of inseparable modes in the superposition of the two-photon state, quantified by the Schmidt number $K$, is proportional to the ratio between the width of the phase matching function $\Phi(q)$ (determined by the crystal length $L$), and the width of the pump angular spectrum function $v(q)$ (determined by the width of the pump beam). In the so called thin-crystal regime, $K \gg 1$, the two-photon state can be approximated by $\psi(q_s, q_i) \propto v(q_s + q_i)$ (33), yielding an Einstein-Podolsky-Rosen (EPR) entangled state (34). In this regime, we can precisely control the two-photon amplitude, by tailoring the angular spectrum of the pump beam or equivalently, by controlling its spatial profile, $W(\rho)$, at the input plane of the crystal.
The effect of a thin diffuser placed right after the crystal can be modeled by a linear transformation on the creation operator $a^\dagger(q) \to \int d\rho a^\dagger(\rho) A_d(\rho) \exp(-i\rho \cdot q)$, where $A_d(\rho)$ is the amplitude transfer function of the thin diffuser and $\rho$ is the transverse spatial coordinate. We note that if the diffuser is not located right after the crystal, it can always be re-imaged on the crystal plane, as in conjugate-plane adaptive optics (35). Experimentally, the two-photon quantum state is measured using two single photon detectors placed at the far-field of the crystal. The rate of coincidence events, i.e. detection of two photons simultaneously, one photon with transverse wavevector $q_s$ and the other with transverse wavevector $q_i$, is given by $C(q_s, q_i) = \langle 0 | a(q_i) a(q_s) |\psi \rangle^2$, yielding (see Supplementary) (36),

$$C(q_s, q_i) \propto \left| \int d\rho W(\rho) A^2_d(\rho) \exp(-i\rho \cdot (q_s + q_i)) \right|^2$$

(2)

For random diffusers, the spatial distribution of the coincidence pattern $C(q_s, q_i)$ exhibits a random two-photon speckle pattern (17, 18). Surprisingly, the right hand side of Eq. (2) corresponds also to the far-field intensity profile of the pump beam (see Supplementary). Hence, by measuring the intensity profile of the pump beam, we get the spatial distribution of the two-photon state. Since this is true for any pump profile $W(\rho)$, using an SLM we can optimize the pump profile to get a focused pump spot at the far-field of the diffuser, and the two-photon spatial distribution will become localized, simultaneously. This remarkable feature of spatially entangled photons, offers orders of magnitude faster feedback than would have been possible with the inherently weak signal provided by the coincidence rate. It allows us to extend wavefront shaping to the quantum domain, by applying classical wavefront shaping to the bright pump beam.

The quantum wavefront shaping experimental setup is depicted in Fig.1a. A 2mm long PPKTP crystal is pumped by a $\lambda = 404$nm continuous-wave laser, which is shaped by a phase-only SLM imaged on the input facet of the crystal. The SPDC process in the crystal generates a continuous flux of entangled photon-pairs, with a Schmidt number of $K \approx 680$ (Fig.1b,c, Supplementary). After the crystal, both the pump beam and the entangled photons are scattered by a thin diffuser located at the image plane of the crystal, creating a fully-developed speckle pattern with no ballistic component. The pump intensity and the two-photon coincidence patterns are measured at the far-field of the crystal, after separating the pump and entangled photons using a dichroic mirror.

Figures 2a and 2b depict the measured pump intensity and two-photon coincidence rate, at the far-field of the diffuser. Even though the wavelength of the entangled photons is twice the wavelength of the pump photons, the two signals exhibit strikingly similar patterns, as predicted by Eq. (2). The wavelength difference comes into play only in the scaling of the two patterns, as the two-photon pattern is stretched by a factor of two compared to the pump pattern (see Supplementary). Since the pump and two-photon speckle patterns are remarkably similar, we can apply wavefront shaping optimization to the classical pump beam, and the quantum two-photon correlations will be optimized simultaneously. Specifically, we use the partitioning optimization algorithm to enhance the intensity of the pump beam at an arbitrary point at the far-
field of the diffuser (28) (Fig. 2d). With the exact same phase mask applied to the pump beam, the two-photon coincidence pattern is measured, showing a clear enhancement and localization of the two-photon correlations at the target area (Fig. 2e). Interestingly, the single photon counts (Fig. 2c, f) are not affected by the scattering nor by the optimization, due to the multimode nature of SPDC light in the high Schmidt number regime (18).

Figure 2: Quantum wavefront shaping. The pump beam and the entangled photons pass through the same diffuser, forming similar speckle patterns in the intensity (a) and coincidence (b) pictures, respectively. By performing classical wavefront shaping on the bright pump beam, a single speckle grain is enhanced (d), yielding a simultaneous enhancement and localization of the quantum correlations at the corresponding location (e). The single photon counts (c, f) are not affected by either the scattering or the optimization.

One of the main challenges in adopting wavefront shaping to real-world applications is scattering by a varying medium, for example in communication through turbulent atmosphere (4, 23), as it requires fast modulation rates and sufficient signal-to-noise ratios (SNR) at short integration times. The need for fast modulation rates, common for both classical and quantum wavefront shaping, can be solved by recent breakthroughs in utilizing, for example, digital mirror devices (DMDs) to obtain modulation rates of up to 350kHz (37–39). The need for high SNR at short integration times, is in particular critical for quantum light, since quantum signals are too weak for real-time optimization (40, 41). This is usually solved by performing the wavefront optimization on an auxiliary bright laser, that is carefully co-aligned with the entangled photons so that it undergoes the exact same scattering (29–32). While this approach enables fast optimization, it does not allow simultaneous transmission of the entangled photons since the auxiliary laser must have the same wavelength and polarization as the entangled photons and thus cannot be filtered out. For this reason, dynamical shaping, although crucial to many
quantum technologies, has not been achieved so far for quantum light. Since in our method the optimization is done entirely on the classical bright pump beam, the optimization rates can in principle be as fast as record-high classical wavefront shaping, making quantum wavefront shaping applicable for real-time applications. To demonstrate this feature, we emulate dynamical scattering by placing a diffuser on a moving stage, producing a time-dependent speckle pattern. The coincidence rate at the target area was measured during the movement of the diffuser, while the pump beam was simultaneously optimized to generate a focus at the far-field. Fig. 3 shows that when the optimization is turned on, both the pump (blue curve) and coincidence (red curve) signals are enhanced, even though the diffuser is constantly moving. For comparison, when the same optimization algorithm was used with the coincidence rate for the feedback, no enhancement was observed due to the poor SNR of the coincidence signal (black curve).

Figure 3: Real-time shaping. A diffuser is placed on a moving stage, creating a time dependent speckle pattern at the far-field. By using the intensity of the pump beam as feedback, real-time optimization is obtained for both the pump beam (blue) and the entangled pairs (red). When the optimization is turned off, degradation of both signals occurs due to the movement of the diffuser. Real-time optimization is not possible when the coincidence signal is used for the feedback (black), due to its inherently low signal-to-noise ratio. Here, the optimization speed is limited by the SLM response time and the feedback electronics (100 ms), yet over three orders of magnitude faster optimization rates can be achieved using deformable mirror devices and fast electronics.

Finally, we turn to discuss the expected classical and quantum enhancements of the optimization process. We first define the transmission \( t \) as the fraction of the signal arriving at some target area at the far-field. We distinguish between the effects of absorption loss and scattering loss on \( t \). In the case of absorption, if \( t_p^{(1)} \) is the transmission of the pump, then the transmission of the coincidence signal is quadratic \( t_{DC}^{(2)} = t_p^{(1)} \), as we experimentally confirm using a variable
attenuator (Fig. 4a,b). This relation results from the fact that for a coincidence event, both photons must be transmitted through the absorbing media. However, for a purely scattering sample, the situation can be quite different. To illustrate this, we measure the two-photon transmission $t_{DC}^{(2)}$ and the corresponding pump transmission $t_p^{(1)}$ at different speckle grains in the pattern formed by the diffuser (Fig. 4a). Remarkably, we get a clear linear dependency, $t_{DC}^{(2)} = t_p^{(1)}$ (Fig. 4b). The linear rather than quadratic dependence is a unique feature of entanglement in the high Schmidt number regime $K \gg 1$. To elucidate this, we numerically simulated the pump and two-photon speckle patterns formed at the far-field of a thin diffuser. The simulation was performed for two-photon states with different Schmidt numbers (corresponding to different crystal lengths), using the double-Gaussian approximation (42). In Fig. 4c, the calculated correlation coefficient between the two patterns is plotted versus the Schmidt number, $K$. At $K = 1$, there is almost no correlation between the patterns, as expected from two different wavelengths. However, as the Schmidt number increases, the patterns become increasingly correlated, yielding almost perfect correlation in our experimental conditions at $K \approx 680$. This result completes the picture given by Eq. (2) and Fig. (2)a,b, and explains the correspondence between the pump beam and entangled photon pairs in the high Schmidt number regime, that yields $t_{DC}^{(2)} = t_p^{(1)}$.

We can now easily derive the expected enhancement $\eta_{DC}^{(2)}$ of our optimization method, defined by the ratio of the coincidence rate at the target area after optimization and the average coincidence rate at the target area before optimization. As a consequence of the linear correspondence we established between the pump and two-photon signals, the coincidence enhancement $\eta_{DC}^{(2)}$ must be exactly equal to the classical enhancement of the pump beam (28),

$$\eta_{DC}^{(2)} = \eta_p^{(1)} = \frac{\pi}{4} (N - 1) + 1,$$

where $N$ is the number of degrees of freedom used in the optimization. We therefore conclude that in our method, the efficiency of the the quantum optimization is identical to the efficiency of classical wavefront shaping.
Figure 4: Quantum vs. classical transmission. (a) The transmission of the two-photon coincidence rate $t_{DC}^{(2)}$, and the transmission of the pump beam $t_p^{(1)}$, are measured in two different configurations. In the first configuration, a linear polarizer is rotated in the optical path of both beams, to induce a variable absorption loss, yielding a clear quadratic relation between $t_{DC}^{(2)}$ and $t_p^{(1)}$ (b, green points). In the second configuration, the polarizer is replaced by a diffuser, causing a speckle pattern at the far-field. Remarkably, the transmission $t_{DC}^{(2)}$, which is measured at different locations in the speckle pattern, are equal to the corresponding transmissions $t_p^{(1)}$ (b, black points), resulting in a linear dependency (black curve) instead of the classically expected quadratic one (green curve). (c) The correlation coefficient between the speckle pattern of the pump beam and the coincidence pattern is presented as a function of Schmidt number. The correlation coefficient was calculated numerically under the double Gaussian approximation. In our experiment, $K \approx 680$ (marked with a black circle), yielding a high correlation between the pump and coincidence patterns which explains the linear dependency in (b).

Our approach for shaping entangled photons, by creating them with the correct spatial correlations to compensate for the scattering, has several unique advantages over directly shaping
them after their creation. First, since it is based on shaping the classical pump beam without interacting with the entangled photons themselves, it does not introduce any loss to the quantum light. Second, since the classical pump beam is used for the optimization feedback, the optimization can be as fast as shaping of bright classical light. Moreover, since we could easily separate the pump photons from the entangled photons, we were able, for the first time, to cancel scattering of entangled-photons from a dynamically moving diffuser, without interfering their continuous transmission. This is an important step towards implementing quantum wavefront shaping in real-life scenarios, since there is no down-time for the transmission of the photons while the optimization process is performed. In this work we focused on scattering by a thin diffuser, as our method assumes different incident angles yield correlated speckle patterns. Nevertheless, it can easily be extended to thick scattering layers as well, since the so-called ’memory effect’ yields a range of incident angles, known as the ’isoplanatic patch’, for which the speckle patterns are invariant (43, 44). We therefore expect our method to find various applications in the implementation of quantum technologies in real-world scenarios, such as quantum key distribution through turbulent atmosphere (4, 23) and quantum imaging (6, 7) in living samples. It can also be used to create a two-photon source with arbitrary correlations and coherence properties by shaping both the amplitude and phase of the pump beam (45, 46), opening the door for exploiting the capacity of high dimensional qudits implemented in the spatial domain (14, 20).

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Supplementary materials

Methods

The experimental setup is presented in Fig.1a. A 2mm long type-0 PPKTP crystal is pumped by a 50mW, \(\lambda = 404nm\) continuous-wave laser. The wavefront of the pump beam is shaped by a phase-only SLM, imaged on the crystal by two lenses with focal lengths \(L_1 = 200mm\) and \(L_2 = 100mm\) respectively. Without shaping, the pump profile at the crystal plane is approximately Gaussian with a waist of 0.7mm. Both the pump beam and the entangled photons are then imaged onto a 0.25° thin diffuser by two lenses with focal lengths \(L_3 = 100mm\) and \(L_4 = 50mm\). The pump beam and the entangled photons are separated using a dichroic mirror and measured at the far-field by an CMOS camera and 100\(\mu\)m multimode fibers coupled to single photon detectors, respectively. The far-field measurements are obtained after passing through a \(L_5 = 150mm\) lens. For the coincidence measurements, 10nm interference filters around 808nm are used.

Correspondence between the pump beam and the entangled photon pairs

As discussed in the main text, in the high spatial entanglement regime (\(K \gg 1\)) there is a remarkable correspondence between the pump intensity distribution and the entangled photons coincidence pattern at the far-field. In this section, we demonstrate this correspondence in a simpler setting in which the diffuser in Fig.1a is replaced by the phase mask\(\phi(x, y) = \text{sign}(y)\frac{\pi}{2}\), for the wavelength of the SPDC entangled photons. Since the pump wavelength is half the wavelength of the entangled photons, the phase mask acts as a 2\(\pi\)-step and does not affect its wavefront (Fig.S1 insets). For the entangled pair the result is less obvious, since each individual photon will be affected by the induced \(\pi\)-step. However, when considering the pair of entangled photons, due to the high spatial entanglement, both photons hit the phase mask at the same location, thus accumulating twice the single photon phase. Therefore, although diffracting each individual photon, the two-photon wavefront is not affected by the \(\pi\)-step and the coincidence pattern does not change (Fig.S1). In the next section, this surprising correspondence is extended to a general thin diffuser, under moderate dispersion, allowing our unique use of the pump beam for the optimization feedback.
Figure S1: Pump beam and entangled photons correspondence. The diffuser in the experimental setup was replaced by an SLM. When no phase mask is applied (a), both the pump beam (inset) and coincidence pattern exhibit a peak with a width determined by the angular spectrum of the pump beam. When applying a $\pi$-step for the wavelength of SPDC light (b), both the pump beam and surprisingly the coincidence pattern remain unchanged.

Scattering of entangled pairs

In the thin crystal regime, the quantum state of the entangled photons is given by

$$|\psi\rangle = \int d\mathbf{q}_s d\mathbf{q}_i \nu(\mathbf{q}_s + \mathbf{q}_i) a^\dagger(\mathbf{q}_s) a^\dagger(\mathbf{q}_i) |0\rangle,$$

where $\nu(\mathbf{q})$ is the angular spectrum of the pump beam, $a^\dagger(\mathbf{q})$ is the creation operator of a photon with a transverse momentum $\mathbf{q}$ and $|0\rangle$ is the vacuum state. For simplicity, we assume here that the photons are created with the same frequency (corresponding to degenerate SPDC with narrow-band interference filters), and the same polarization (corresponding to type-0 SPDC process). The effect of a thin diffuser placed right after the crystal, can be modeled by a unitary transformation on the creation operator $a^\dagger(\mathbf{q}) \rightarrow \int d\rho a^\dagger(\mathbf{\rho}) A_d(\mathbf{\rho}) \exp(-i \mathbf{\rho} \cdot \mathbf{q})$, where $A_d(\mathbf{\rho})$ is the amplitude transfer function of the thin diffuser and $\mathbf{\rho}$ is the transverse spatial coordinate. By substituting the above relation into the quantum state we get that:

$$|\psi\rangle = \int d\mathbf{\rho} W(\mathbf{\rho}) A_d^2(\mathbf{\rho}) a^\dagger(\mathbf{\rho})^2 |0\rangle$$

(4)

where $W(\mathbf{\rho})$ is the spatial profile of the pump beam at the crystal plane. This representation of the quantum state stresses that, in the thin crystal regime, each pair of photons are created at the same spatial location. Substituting the quantum state into the expression for the coincidence pattern, $C(\mathbf{q}_s, \mathbf{q}_i) = |\langle 0 | a(\mathbf{q}_i) a(\mathbf{q}_s) |\psi\rangle|^2$, yields Eq. (2):

$$C(\mathbf{q}_s, \mathbf{q}_i) \propto \left| \int d\mathbf{\rho} W(\mathbf{\rho}) A_d^2(\mathbf{\rho}) \exp(-i \mathbf{\rho} \cdot (\mathbf{q}_s + \mathbf{q}_i)) \right|^2$$

(5)
Neglecting loss and material dispersion, we can model the diffuser as a random phase mask of the form \( A_d(\rho) = \exp(i\phi(\rho)) \). In this case, the scattering of the two entangled photons is determined by an effective diffuser, given by \( A_{2d}^2(\rho) = \exp(i2\phi(\rho)) \), which corresponds to the phase accumulated by the pump beam. Thus, the created two-photon speckle pattern is identical to that of the classical pump beam, up to a scaling factor of two due to the different wavelength.

**Measurement of the Schmidt number**

In order to estimate the Schmidt number \( K \) in our experimental apparatus, we use the double-Gaussian approximation for the phase matching condition, yielding a two-photon distribution of the form (42):

\[
\psi(q_s, q_i) \propto \exp \left( -\frac{(q_s + q_i)^2}{\sigma^2} \right) \exp \left( -b^2(q_s - q_i)^2 \right)
\]

where \( b^2 = \frac{L}{4\kappa} \), \( k \) is the pump wavenumber inside the crystal, and \( \sigma \) is the waist of the Gaussian pump beam.

Under the double-Gaussian approximation, the Schmidt number \( K \) could be computed exactly, and is given by (42):

\[
K = \frac{1}{4} \left( \frac{1}{b\sigma} + b\sigma \right)^2
\]

Thus, in the thin crystal or weak focusing regimes, where \( b\sigma \ll 1 \), one could approximate the Schmidt number as \( K = \frac{1}{(2b\sigma)^2} \). This expression for the Schmidt number can be understood intuitively by noticing its relation to the number of transverse spatial modes in the system (47). Therefore, the Schmidt number could be estimated experimentally by using the width of the far-field single and coincidence counts distributions, yielding \( K = 680 \pm 60 \) in our experiment, indicating high spatial entanglement.