Binary Dynamics from Worldline QFT for Scalar-QED

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We investigate the worldline quantum field theory (WQFT) formalism for scalar-QED and observe that a generating function emerges from WQFT, from which the scattering angle ensues. This generating function bears important similarities with the radial action in that it requires no consideration of exponentiation of lower-order contributions. We demonstrate the computations of this generating function and the resulting scattering angle of a binary system coupled to electromagnetic field up to the third order in the Post-Minkowskian expansion (3PM).

Recent developments in gravitational-wave physics [1–5] call for innovations of theoretical framework that facilitate both numerical [6–8] and analytical [9–36] high-precision computations of the dynamics of binary black hole or neutron star mergers.

It has proven fruitful to extract classical observables from scattering amplitudes in perturbative quantum field theories [37–42], thanks to modern tools based on on-shell techniques [43–52] and effective field theory [38, 53]. However, to expose the classical quantity, amplitudes-based approaches often require a delicate analysis which removes quantum and superclassical contributions alike [39, 40, 54, 55]. Alternative methods that capture classical observables more directly are therefore in demand and several explorations in this direction [56–59] have been shown to be beneficial.

It is in this light that the worldline quantum field theory (WQFT) [60], in which worldline degrees of freedom are quantised, is formulated, providing a formal link between black hole observables extracted from scattering amplitudes and time-ordered correlators in WQFT. WQFT Feynman rules circumvent the need for the effective potential in traditional worldline EFT methods [10, 61, 62] and streamline loop calculations encountered in amplitudes-based approaches to summing over diagrams of tree topologies only, yielding classical observables directly. Recent applications of WQFT involve a series of work on spinning black holes [63–65] and the state-of-the-arts derivations of the conservative momentum impulse and the spin kick up to the third order in Post-Minkowskian (3PM) expansion and quadratic order in spin have been obtained from WQFT [66].

As established in amplitudes-based approaches, conservative and radiative dynamics in classical relativistic scattering can be extracted from the eikonal phase [55, 67–70]. Inspired by the eikonal approximation, an amplitude-action relation has been revealed [55] and the radial action [71–74] serves as another generating function for the scattering angle. Another closely related generating function is defined in the heavy-particle EFT [59] which agrees with the radial action in their real parts, but differs in the imaginary part. One crucial difference between these functions and the standard eikonal exponentiation is that iterations from lower orders can be discarded for the former.

WQFT is expected to have the potential of capturing such generating functions too. The classical eikonal phase can be obtained from WQFT in various contexts up to 2PM/next-to-leading order (NLO) [60, 65, 75, 76]. However, the calculation of the eikonal phase at 3PM and beyond in WQFT remains somewhat ambiguous in the $\imath\epsilon$-prescription of the worldline propagator. On the other hand, it is conceivable that WQFT speaks more directly to a generating function whose classical part is readily isolated than the eikonal. In this letter, we seek to explore the construction of such a generating function from WQFT.

In this letter, we consider the WQFT counterpart of scalar-QED as a toy model, which is shown to be a useful playground for higher PM gravitational computations [74, 77, 78]. We illustrate that a generating function emerges from WQFT in a highly streamlined fashion, which reproduces both the conservative and radiative contributions of the scattering angle. This generating function bears similarities with the radial action and the eikonal exponentiation. The WQFT integrands can be made to match with those in the heavy-mass limit of scalar-QED in the comparable-masses sector and in those diagrams responsible for the radiation reaction. We expect these observations to carry over straightforwardly to WQFT in gravitational background.

WQFT Formalism for Scalar-QED The worldline action describing a charged massive non-spinning point-particle in an electromagnetic background reads [79, 80]

\begin{equation}
  S_i = -m_i \int d\sigma \left[ \frac{1}{2} \left( \eta^{-1} \dot{x}_i^2 + \eta \right) + \imath \epsilon \frac{q_i}{m_i} A_\mu \dot{x}_i^\mu \right],
\end{equation}

where the worldline coordinate \(x^\mu\) is parameterised by \(\sigma\) and \(\dot{x}^\mu = dx^\mu/d\sigma\). \(q_i\) and \(m_i\) denote the charge and mass of the scalar \(i = 1, 2\). The worldline is coupled to the electromagnetic (EM) field \(A_\mu\) and the bulk theory is simply given by the usual EM action. For convenience, we set the einbein \(\eta(\sigma) = 1\). As shown in [60], specialising the

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photon to plane waves of fixed momenta and polarisations, the photon-dressed Feynman-Schwinger propagator [81] can be identified with the path integral for the WQFT correlator, with external legs amputated through the LSZ reduction.

Expanding the worldline around straightline trajectories $x_i^\mu = b_i^\mu + u_i^\mu \sigma + z_i^\mu(\sigma)$, the WQFT Feynman rules are readily expressed in frequency/momentum space: $z_i^\mu(\sigma) = \int \mathcal{D}z \ e^{-i\sigma \oint \mathcal{A}_\mu},$ where we have used the shorthand notations $\int \mathcal{D}z / \mathcal{D}k$ as introduced in [60]. The explicit expressions of WQFT Feynman rules for worldline-photon interactions are given in Appendix A.

Inspired by the eikonal exponentiation [60], we consider the phase identified with the WQFT path integral in the classical limit,

$$e^{i\delta} = \mathcal{Z}_{\text{WQFT}} = \int \mathcal{D}[A] \prod_{j=1}^{\mathcal{G}} \mathcal{D}[z_j] e^{i(S_{\text{EM}} + \sum_{j=1}^{\mathcal{G}} S_j)}, \quad (2)$$

where $S_{\text{EM}}$ denotes the standard action for the electromagnetic field in the bulk. We note that the identification above is designed to hold in the classical limit. Hence the phase $\delta$ is a purely classical quantity. That is, $\delta$ is uniform in $h$ and only admits an expansion in the coupling constant $e^2$. Taking logarithm on both sides, we identify $\delta$ at each order of $e^2$ with the sum of connected WQFT diagrams, without iteration corrections from lower orders, which sets apart from the eikonal approach proposed in [60]. It may be tempting to identify it with the radial action due to the similar definitions; but preliminary evidences suggest that differences occur in their respective imaginary parts. Similar to the HEFT phase [59], we restrict ourselves to the real parts of this generating function and the resulting scattering angle. The imaginary part is beyond the scope of this letter.

The evaluation of WQFT path integrals is normally sensitive to the $ie$-prescription of the worldline propagator. We observe that only the principal-value part of the time-symmetric propagator [60] is relevant for the construction of this generating function. Hence we propose the principal-value prescription for the worldline propagator and the propagator simply reads

$$\omega \quad z^\mu \quad z' \quad = -\frac{i \eta^{\mu\nu}}{m} \frac{\omega^2}{\omega^2}. \quad (3)$$

This treatment is reminiscent of [59, 73, 75, 82]. The Feynman rules are given in terms of kinematic variables $b^\mu$ and $u^\mu$, the interpretation of which depends on the worldline trajectory they describe [60]. The kinematics of the $2 \rightarrow 2$ scattering is given by the momenta:

$$p_1 = \bar{p}_1 + q/2, \quad p_2 = \bar{p}_2 - q/2,$$
$$p'_1 = \bar{p}_1 - q/2, \quad p'_2 = \bar{p}_2 + q/2,$$

with $p_1^2 = \bar{p}_1^2 = m_1^2$ and $p'_2^2 = \bar{p}'_2^2 = m_2^2$. The initial trajectory $(\sigma = -\infty)$ corresponds to $p_i^\mu = m_i u_i^\mu$ and the initial impact parameter is given by $b^\mu = b_1^\mu - b_2^\mu$. The in-between trajectory $(\sigma = 0)$ corresponds to the “barred variable” $\bar{p}_i^\mu = \bar{m}_i \bar{u}_i^\mu$ and $\bar{b}^\mu$. The differences between the two sets of variables come at $\mathcal{O}(q^2)$. Similar to the observations in [59], the phase $\delta$ is free from iterations and hence the barred variables can be traded with the unbarred ones at no cost.

1PM & 2PM At the leading (1PM) and subleading (2PM) orders, the phase $\delta$ is given by

$$\delta^{(0)} + \delta^{(1)} = \ldots + \ldots + \ldots$$

$$= \int_{\mathcal{D}} e^{i\bar{b} q} \sigma \delta(q \cdot u_1) \delta(q \cdot u_2) \left[ -ie^{q_1 q_2}\gamma \right] + \ldots$$

$$+ ie^{q_1 q_2} \left[ (2D - 7)\gamma^2 - 1 \right] (m_1 + m_2) \Gamma_i(1) \right], \quad (4)$$

where we have adopted the notations in [60] for the integration measure and $\delta(x) := 2\pi \delta(x), \gamma = u_1 \cdot u_2$ and $D$ denotes the spacetime dimension. The integral $\Gamma_i(1)$ in $D = 4 - 2\epsilon$ reads

$$\Gamma_i(1) \int_{\mathcal{D}} \frac{\delta(q \cdot u_1)}{\epsilon} \left[ \frac{4\pi^{\epsilon} - \frac{3}{2} \Gamma(\frac{1}{2} - \epsilon) \Gamma(\frac{3}{2} + \epsilon)}{(-q^2)^{\frac{D}{2} + \epsilon} \Gamma(1 - \epsilon)}. \quad (5)$$

Note that both the impact parameter $b^\mu$ and the total momentum transfer $q^\mu$ are spacelike and the Fourier transform is performed in $(D - 2)$ dimensions due to the two $\delta$-functions as follows,

$$\int_{\mathcal{D}} e^{i\bar{b} q} \left[ -b^{a - 1 + \epsilon} \Gamma(1 - \alpha - \epsilon) \right], \quad \frac{4b^a - 1}{\Gamma(\alpha)} \Gamma(1 - \epsilon). \quad (6)$$

Fourier transforming to the impact parameter space, we obtain

$$\delta^{(0)} = \alpha q_1 q_2 \gamma \frac{\Gamma(1) - (-\epsilon)}{\sqrt{1 - \epsilon}} (b^2)^\epsilon, \quad (7)$$
$$\delta^{(1)} = -\alpha q_1 q_2^2 \frac{\Gamma(1 - \epsilon)}{2m_1 m_2 \gamma^2 - 1} b^2, \quad (8)$$

where $b = |b| = \sqrt{b^2}$ and $\alpha \equiv e^2/(4\pi)$.

Moving on to the next-to-next-to-leading order (3PM), we consider the comparable-masses sector ($m_1 \sim m_2$) and the probe-limit sector ($m_1 \ll m_2$ or $m_1 \gg m_2$). The former contributes to both the conservative and the radiative parts of the scattering angle whereas the latter contributes only to the conservative part. In addition, the scattering angle begins to receive the so-called radiation reactions at 3PM [74, 83–88] and we shall consider them separately.

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1 We multiply a factor of $\frac{4\pi^{\epsilon}}{(4\pi)^2 \epsilon} (4\pi e^{-\gamma E})^\epsilon$ per loop in the end to restore the proper normalization [66, 68].
3PM Comparable Masses  The conservative contribution from this sector is computed by the following diagrams,

\[ i\delta^{(2)} \bigg|_{m_1 m_2} = \frac{1}{2} \left( \sum \text{ diagrams} + \sum \text{ diagrams} \right) \]

\[ = \frac{ie^6 q_1^2 q_2^2}{2m_1 m_2} \int \frac{d^4q}{q} \prod_{i=1}^{2} \delta(q \cdot u_i) \int \frac{d^4 \ell_1 d^4 \ell_2 (q - \ell_1 - \ell_2)}{\ell_1^2 \ell_2^2 (q - \ell_1 - \ell_2)^2} \left[ \gamma (q - \ell_1)^2 + \gamma (q - \ell_2)^2 \right] \]

\[ = -\frac{\gamma^2 q^2}{(\ell_1 \cdot u_1) (\ell_2 \cdot u_2)} \cdot \frac{2(\ell_1 \cdot u_1)}{2(\ell_2 \cdot u_2)} \left[ \frac{\gamma (q - \ell_1)^2}{(\ell_2 \cdot u_2)^2} + \frac{\gamma (q - \ell_2)^2}{(\ell_1 \cdot u_1)^2} \right] \]

\[ \left( \delta (\ell_1 \cdot u_2) \delta (\ell_2 \cdot u_1) \right) \]

\[ \rho_1 = \ell_1 \cdot u_1, \quad \rho_2 = \ell_2 \cdot u_2, \quad \rho_3 = \ell_2^2, \quad \rho_4 = \ell_2^2, \quad \rho_5 = (q - \ell_1 - \ell_2)^2, \quad \rho_6 = (q - \ell_1)^2, \quad \rho_7 = (q - \ell_2)^2. \]

We list the coefficients in \( D = 4 - 2\epsilon \) below:

\[ a_1 = -\frac{2\epsilon (\gamma^4 (4\epsilon^2 - 2\epsilon - 1) + \gamma^2 (2\epsilon + 3) - 3)}{\gamma (\gamma^2 - 1)(1 - 2\epsilon)}, \]

\[ a_2 = \frac{(2\epsilon + 1) (\gamma^4 (2\epsilon (6\epsilon - 5) - 1) + \gamma^2 (6\epsilon + 3) - 3) q^2}{3\gamma (\gamma^2 - 1)^2 (1 - 2\epsilon)\epsilon}, \]

\[ a_3 = -\frac{\gamma^2 (2\epsilon^2 \epsilon + 3) q^2}{3\gamma (\gamma^2 - 1)^2 (1 - 2\epsilon)} \]

These integrals are extensively studied in literature [59, 66, 67, 86, 90] using differential equations [91–95]. We display the explicit expressions for the real parts of the master integrals present in (10) in Appendix B and (10) is readily evaluated. Fourier transforming to the impact parameter space and taking \( \epsilon \to 0 \), we obtain

\[ \text{Re} \left( \delta^{(2)} \right) \bigg|_{m_1 m_2} = -\frac{2(\alpha q_f)^2 (\gamma^4 - 3\gamma^2 + 3)}{3m_1 m_2 b^2 (\gamma^2 - 1)^{1/2}} \]

\[ + \frac{2(\alpha q_f)^2 \gamma^2 (\gamma \sqrt{\gamma^2 - 1} - \arccosh \gamma)}{m_1 m_2 b^2 (\gamma^2 - 1)^{1/2}}. \]

The two terms are identified with the conservative and radiative contributions, because they result from boundary values computed in different regions. The first term comes purely from the potential region identified in [59] while the second from the radiative region. As will be demonstrated shortly, they reproduce the conservative and radiative parts of the scattering angle respectively.

3PM Probe Limit  Similarly in the probe limit we consider the following diagrams,

\[ i\delta^{(2)} \bigg|_{m_2} = \frac{1}{3} \left( \sum \text{ diagrams} + \sum \text{ diagrams} + \sum \text{ diagrams} \right) \]

\[ = \frac{ie^6 q_1^2 q_2^2}{12m_1^4} \int \frac{d^4q}{q} \prod_{i=1}^{2} \delta(q \cdot u_i) \int \frac{d^4 \ell_1 d^4 \ell_2 (q - \ell_1 - \ell_2)}{\ell_1^2 \ell_2^2 (q - \ell_1 - \ell_2)^2} \left[ \gamma ((q - \ell_1)^2 - q^2) \right] \]

\[ \left\{ \frac{\gamma (q - \ell_1)^2}{(\ell_1 \cdot u_1)^2} + \frac{2\gamma ((q - \ell_2)^2 - q^2)}{(\ell_2 \cdot u_2)^2} + \frac{\gamma^3 q^2 (q - \ell_2)}{(\ell_1 \cdot u_1)^2 (\ell_2 \cdot u_1)^2} \right\} \]

\[ + \frac{\gamma^3 (q - \ell_2)^4}{(\ell_1 \cdot u_1)^2 (\ell_2 \cdot u_1)^2} + \frac{\gamma^3 (q - \ell_2)^2 ((q - \ell_2)^2 + (q - \ell_1)^2 - q^2)}{(\ell_1 \cdot u_1)^2 (\ell_2 \cdot u_1)^2}. \]

Here we have symmetrized the diagrams by labelling the momenta universally in all three diagrams. This symmetrization helps to reproduce all the pole structures expected explicitly in the classical limit of the corresponding Feynman diagrams in this sector. The two probe limit sectors are simply related by relabelling \( m_1 \leftrightarrow m_2 \).

After IBP reduction using LiteRed, the integrand in
is simplified to one single master integral
\[
\frac{i e^2 q_1^2 q_2^2 (6 \epsilon - 1) \gamma (\gamma^2 (6 \epsilon - 2) + 3)}{6 m_1^2 (\gamma^2 - 1)^2} G_2^{(2)},
\]
where the master integral in \( D = 4 - 2 \epsilon \) reads
\[
G_2^{(2)} = \int_\epsilon \frac{\delta (\ell_1 \cdot u_1) - \delta (\ell_2 \cdot u_2)}{e^{2 \ell_1 \cdot q} (q - \ell_1 - \ell_2)^2}
= - \frac{(4 \pi)^{-3 + 2 \epsilon} \Gamma \left( \frac{1}{2} - \epsilon \right)^3 \Gamma (2 \epsilon)}{(q^2)^{2 \epsilon}}.
\]
That only one master integral contributes is also observed in the context of gravity [59, 73]. We note that the matching with the heavy-mass limit of scalar-QED in the probe limit is less manifest. The two integrands can be shown to be equal after IBP reduction. Fourier transforming to impact parameter space, we have
\[
\text{Re} \, \delta^{(2)} \bigg|_{m_2^2} = \frac{(\alpha q_1 q_2)^2 \gamma (2 \gamma^2 - 3)}{m_1^2 b^2 (\gamma^2 - 1)^{3/2}}.
\]
We will see shortly this reproduces the conservative part of the scattering angle in the probe limit.

**3PM Radiation Reaction** The radiation reaction is accounted for by the following diagrams,
\[
i \delta^{(2)} \bigg|_{\text{r.r.}} = \left( \begin{array}{c}
\delta^{(2)} \bigg|_{\text{r.r.}} \\
\delta^{(2)} \bigg|_{\text{r.r.}} \\
\delta^{(2)} \bigg|_{\text{r.r.}}
\end{array} \right) \]
\[
= \frac{i e^2 q_1 q_2^2}{2 m_1^2} \int_q e^{i b q} \prod_{i=1}^{2} \delta (q \cdot u_i) \int_{\ell_1, \ell_2} \frac{\delta (\ell_1 \cdot u_2) \delta (\ell_2 \cdot u_1)}{e^{2 \ell_1 \cdot q} (q - \ell_1 - \ell_2)^2 (q - \ell_1 - \ell_2)^2}
\left[ 1 + \frac{(\ell_2 \cdot u_2)^2}{(\ell_1 \cdot u_1)^2} \right]
= \frac{\gamma^2 q^2}{2 (\ell_1 \cdot u_1)^2} + \frac{\gamma q_1 q_2}{(\ell_1 \cdot u_1)^2} \left\{ \{ q_1, m_1 \} \leftrightarrow \{ q_2, m_2 \} \right\}.
\]
We again apply IBP reductions to the expression above, which leads to one single master integral \( G_{0,0,1,0,1,1,0} \) as defined in (11). Hence the radiation reaction contribution reads
\[
\text{Re} \, \delta^{(2)} \bigg|_{\text{r.r.}} = \frac{-2 (\alpha q_1 q_2)^2 \gamma^2}{3 m_1 m_2 b^2 (\gamma^2 - 1)} \left[ \frac{q_1/m_1}{q_2/m_2} + \frac{q_2/m_2}{q_1/m_1} \right].
\]

**Scattering Angle** It is straightforward to compute the scattering angle in the center of mass frame via
\[
\chi = - \frac{\partial \delta}{\partial J},
\]
where \( J \) denotes the total angular momentum and we have \( J = p b \) with
\[
p = \frac{m_1 m_2 \sqrt{\gamma^2 - 1}}{E}, \quad E = \sqrt{m_1^2 + m_2^2 + 2 m_1 m_2 \gamma}.
\]
Plugging in (7) and (8), we obtain the scattering angle at 1PM and 2PM
\[
\chi^{(0)} = \frac{2 \alpha q_1 q_2 E \gamma}{m_1 m_2 b (\gamma^2 - 1)},
\]
\[
\chi^{(1)} = \frac{- (\alpha q_1 q_2)^2 \pi E (m_1 + m_2)}{2 m_1^2 m_2 b^2 (\gamma^2 - 1)}.
\]

The conservative part of the 3PM scattering angle follows from the first term of the comparable-mass (17) and the two probe-limit sectors (21),
\[
\chi^{(2)}_{\text{con}} = - \frac{4 (\alpha q_1 q_2)^3 E (\gamma^4 - 3 \gamma^2 + 3)}{3 m_1^2 m_2 b^4 (\gamma^2 - 1)^3} + \frac{2 (\alpha q_1 q_2)^3 E (m_1^2 + m_2^2) (2 \gamma^2 - 3)}{3 m_1^2 m_2 b^4 (\gamma^2 - 1)^3}.
\]
Likewise, the radiative part at 3PM follows from the second line of (17) and the radiation reaction term (23),
\[
\chi^{(2)}_{\text{rad}} = \frac{4 (\alpha q_1 q_2)^3 E \gamma^2}{m_1^2 m_2 b^3} \left[ \frac{\gamma}{(\gamma^2 - 1)^{1/2}} \left( \frac{\gamma}{(\gamma^2 - 1)^{1/2}} \right) - \frac{1}{3 m_1^2 m_2^2 (\gamma^2 - 1)^{3/2}} \left( \frac{q_1/m_1}{q_2/m_2} + \frac{q_2/m_2}{q_1/m_1} \right) \right].
\]
For (28) and (29) we find agreement with known results in literature [74, 88].

**Discussions** We have demonstrated a highly streamlined method for obtaining both the conservative and radiative contributions of the scattering angle in the WQFT formalism for scalar-QED. The scattering angle is computed from a generating function that naturally arises from the WQFT path integral. This generating function is constructed to be purely classical by virtue of WQFT and coincides with the recently proposed “HEFT phase”, although the precise connection between the two remains to be clarified. Its real part also agrees with the radial action, while the differences between their respective imaginary parts are yet to be investigated. These observations are expected to hold in other WQFTs, especially those in a gravitational background, which we leave to future work. It is also interesting to further clarify the relation between this generating function and the eikonal phase in the context of WQFT. Another immediate followup is to study higher PM orders. In particular, the probe limit involves only one diagram (up to symmetrization) at any order, for which the vertices are known on closed forms. In this limit, it is promising to obtain all-loop results from WQFT.

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Appendix A: WQFT Feynman Rules for Scalar-QED

The WQFT Feynman rules can be read off from the action with the trajectory \( x^\mu(\sigma) = b^\mu + u^\sigma + z^\mu(\sigma) \) and the plane wave \( A_\mu(x) = \sum_{n=1}^\infty \epsilon_{\mu
u} e^{ik_{\nu}x} \) plugged in. The interaction term of the worldline coupled to one photon then reads

\[
S_{\text{int}} = \sum_{n=0}^{\infty} \frac{i^n e q_0}{n!} \int \delta(k \cdot u + \sum_i \omega_i) \prod_i z^{\rho_i} \left\{ \prod_{i} k_{\rho_i} u^\mu + \sum_i \omega_i \delta^\mu_{\rho_i} \prod_{j \neq i} k_{\rho_j} \right\}.
\]

Hence the momentum-space Feynman rule for the worldline-photon interaction at the zeroth order of \( z^\rho \) is given by

\[
A_\mu(k) = -ie q_0 \exp (ik \cdot b) \delta (k \cdot u) u^\mu.
\]  

At the linear order of \( z^\rho \), we have

\[
A_\mu(k) = e q_0 e^{ik \cdot b} \delta (k \cdot u + \omega) \left( k_{\rho} u^\mu + \omega_{\rho} \delta^\mu_{\rho} \right).
\]

Finally, at the quadratic order, the worldline vertex reads

\[
A_\mu \left( \begin{array}{c} z^{\rho_1} \\ z^{\rho_2} \end{array} \right) = i e q_0 e^{ik \cdot b} \delta (k \cdot u + \omega_1 + \omega_2)
\left( k_{\rho_1} k_{\rho_2} u^\mu + \omega_1 \delta^\mu_{\rho_1} k_{\rho_2} + \omega_2 \delta^\mu_{\rho_2} k_{\rho_1} \right).
\]

This completes all the worldline interactions needed for the computation in the main text.

In general, for a worldline vertex emitting a photon with \( n \) deflections, the Feynman rule is at the \( n \)-th order of \( z^\rho \) and satisfies the same recursion relation as in \([60]\)

\[
\frac{\partial}{\partial b^\nu} \nabla^{\mu_1 \ldots \mu_n} (k, \omega_1, \ldots, \omega_n) = \nabla^{\mu_1 \ldots \mu_{n+1}} (k, \omega_1, \ldots, \omega_n, 0) \bigg|_{\rho_{n+1} = \nu}.
\]

Appendix B: Master Integrals

Here we list the explicit expressions for the real part of the master integrals in \( D = 4 - 2\epsilon \) used in the main text, which can be found in \([59, 66, 67]\). For detailed discussions on these integrals including the imaginary part, see \([59, 71, 73]\). For convenience, we adopt the basis below

\[
I_1 = (-g^2)^{1+2\epsilon} G_{0,0,0,0,1,1,1}, \quad (B1)
I_2 = (-g^2)^{1+2\epsilon} G_{0,0,0,0,2,1,1}, \quad (B2)
I_3 = (-g^2)^{1+2\epsilon} G_{0,0,0,0,1,2,1}, \quad (B3)
I_4 = (-g^2)^{1+2\epsilon} G_{1,1,0,0,1,1,1}, \quad (B4)
I_5 = (-g^2)^{2\epsilon} G_{0,0,1,0,1,1,0}. \quad (B5)
\]

Then the real parts of the integrals above read

\[
I_1 = -\left( \frac{4\pi^2}{\sqrt{\gamma^2 - 1}\epsilon} \right) \arccosh \gamma,
I_2 = -\left( \frac{4\pi^2}{\sqrt{\gamma^2 - 1}\epsilon} \right) \frac{1}{1 + 2\epsilon} \left( 6\sqrt{\gamma^2 - 1}\epsilon \arccosh \gamma + \frac{2\gamma}{\sqrt{\gamma^2 - 1}} \right),
I_3 = -\left( \frac{4\pi^2}{\sqrt{\gamma^2 - 1}\epsilon} \right) \frac{1}{1 + 2\epsilon} \left( 1 + 2\epsilon \right),
I_4 = -\left( \frac{4\pi^2}{\sqrt{\gamma^2 - 1}\epsilon} \right) \frac{2\arccosh \gamma}{(\gamma^2 - 1)\epsilon},
I_5 = -\left( \frac{4\pi^2}{\sqrt{\gamma^2 - 1}\epsilon} \right) \frac{8\epsilon^2 - 6\epsilon + 1}{1 + 2\epsilon}. \quad (B10)
\]

For each integral, the contribution from the potential region is colored black while that from the radiative region is colored gray. The conservative/radiative part computed in the main text can be obtained by restricting the integrals above to their respective potential/radiative regions.

Appendix C: Comparison With HEFT

Here we demonstrate the matching between WQFT graphs and HEFT ones. On the WQFT side, consider a single photon emitted from a worldline and we have

\[
\times ie q_1 (\varepsilon_1 \cdot u_1) = \frac{1}{m_1} A_{s\text{QED}}^3 (\varepsilon_1, u_1), \quad (C1)
\varepsilon_1^\mu (\ell_1)
\]

where we have omitted the factor \( e^{i\varepsilon_1 \cdot b_1} \) and the \( \delta \)-function in the WQFT Feynman rule (A2), which will be restored in order to assemble the relevant integrand. \( A_{s\text{QED}}^3 (2, u_1) \) denotes the 3-point HEFT amplitude in scalar-QED and we follow the notation in \([59]\). Similarly,
it is straightforward to check that the following WQFT diagram is proportional to the 4-point HEFT amplitude up to a trivial overall factor $e^{i(\ell_1 - \ell_2) \cdot b_1}$ and the $\delta$-function in the WQFT Feynman rule:

\[
\begin{align*}
\frac{i e^2 q_1^2 (\ell_1 \cdot \ell_2)(\varepsilon_1 \cdot u_1)(\varepsilon_2 \cdot u_1)}{m_1 (\ell_1 \cdot u_1)^2} + \frac{i e^2 q_1^2 (\ell_1 \cdot \varepsilon_2)(\varepsilon_1 \cdot u_1)}{m_1 (\ell_1 \cdot u_1)} & = \frac{1}{m_1} A_{4\text{QED}}^{\varepsilon_1, \varepsilon_2, u_1}. \\
A_{4\text{QED}}^{h_1, h_2}(\ell_1, \ell_2, u_1) A_{3}^{-h_1}(\ell_1, u_2) A_{3}^{-h_2}(\ell_2, u_2) & = \frac{1}{m_1^2 \ell_1^2 \ell_2^2} \int \frac{d^4q}{(2\pi)^4} \delta(q \cdot u_1) \delta(q \cdot u_2) \int \delta(\ell_1 \cdot u_2) \\
& \times \sum_{h_1, h_2} \int \frac{d^4q}{(2\pi)^4} \delta(q \cdot u_1) \delta(q \cdot u_2) \int \delta(\ell_1 \cdot u_2)
\end{align*}
\]

At 1PM and 2PM the WQFT graphs in Eq. (4) in the main text are now readily identified with the HEFT integrand. For instance,

\[
\begin{align*}
\int \frac{d^4q}{(2\pi)^4} \delta(q \cdot u_1) \delta(q \cdot u_2) \int \delta(\ell_1 \cdot u_2) & = \int f^{ib}\delta(q \cdot u_1) \delta(q \cdot u_2) \int \delta(\ell_1 \cdot u_2)
\end{align*}
\]

where $\ell_1 + \ell_2 = q$ and $h_1, h_2$ denote the helicities of the photons. The first line in gray comes from the factors we have ignored above. The $\delta$-function is precisely the massive “cut-propagator” in the HEFT computation. At 3PM, a graph-to-graph matching in the same fashion can be easily seen in the comparable-mass sector. In the probe-limit sector, the integrands computed from WQFT and HEFT can be shown to be equivalent up to IBP relations.

[1] B. P. Abbott et al. (LIGO Scientific, Virgo), “Observation of Gravitational Waves from a Binary Black Hole Merger,” Phys. Rev. Lett. 116, 061102 (2016), arXiv:1602.03837 [gr-qc].
[2] B. P. Abbott et al. (LIGO Scientific, Virgo), “GW170817: Observation of Gravitational Waves from a Binary Neutron Star Insipiral,” Phys. Rev. Lett. 119, 161101 (2017), arXiv:1710.05832 [gr-qc].
[3] B. P. Abbott et al. (LIGO Scientific, Virgo), “GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs,” Phys. Rev. X 9, 031040 (2019), arXiv:1811.12907 [astro-ph.HE].
[4] R. Abbott et al. (LIGO Scientific, Virgo), “GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run,” Phys. Rev. X 11, 021053 (2021), arXiv:2010.14527 [gr-qc].
[5] R. Abbott et al. (LIGO Scientific, VIRGO), “GWTC-2.1: Deep Extended Catalog of Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run,” (2021), arXiv:2108.01045 [gr-qc].
[6] Frans Pretorius, “Evolution of binary black hole spacetimes,” Phys. Rev. Lett. 95, 121101 (2005), arXiv:gr-qc/0507014.
[7] Manuela Campanelli, C. O. Lousto, P. Marronetti, and Y. Zlochower, “Accurate evolutions of orbiting black-hole binaries without excision,” Phys. Rev. Lett. 96, 111101 (2006), arXiv:gr-qc/0511048.
[8] John G. Baker, Joan Centrella, Dae-II Choi, Michael Koppitz, and James van Meter, “Gravitational wave extraction from an inspiraling configuration of merging black holes,” Phys. Rev. Lett. 96, 111102 (2006), arXiv:gr-qc/0511103.
[9] A. Buonanno and T. Damour, “Effective one-body approach to general relativistic two-body dynamics,” Phys. Rev. D 59, 084006 (1999), arXiv:gr-qc/9811091.
[10] Walter D. Goldberger and Ira Z. Rothstein, “An Effective field theory of gravity for extended objects,” Phys. Rev. D 73, 104029 (2006), arXiv:hep-th/0409156.
[11] Barak Kol and Michael Smolkin, “Classical Effective Field Theory and Caged Black Holes,” Phys. Rev. D 77, 064033 (2008), arXiv:0712.2822 [hep-th].
[12] James B. Gilmore and Andreas Ross, “Effective field theory calculation of second post-Newtonian binary dynamics,” Phys. Rev. D 78, 124021 (2008), arXiv:0810.1328 [gr-qc].
[13] Stefano Foffa and Riccardo Sturani, “Effective field theory calculation of conservotive binary dynamics at third post-Newtonian order,” Phys. Rev. D 84, 044031 (2011), arXiv:1104.1122 [gr-qc].
[14] Stefano Foffa, Pierpaolo Mastrolia, Riccardo Sturani, and Christian Sturm, “Effective field theory approach to the gravitational two-body dynamics, at fourth post-Newtonian order and quintic in the Newton constant,” Phys. Rev. D 95, 104009 (2017), arXiv:1612.00482 [gr-qc].
[15] Rafael A. Porto and Ira Z. Rothstein, “Apparent ambiguities in the post-Newtonian expansion for binary systems,” Phys. Rev. D 96, 024062 (2017), arXiv:1703.06433 [gr-qc].
[16] J. Blümlein, A. Maier, and P. Marquard, “Five-Loop Static Contribution to the Gravitational Interaction Potential of Two Point Masses,” Phys. Lett. B 800, 135100 (2020), arXiv:1902.11180 [gr-qc].
[17] Stefano Foffa and Riccardo Sturani, “Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach I: Regularized Lagrangian,” Phys. Rev. D 100, 024047 (2019).
Stefano Foffa, Rafael A. Porto, Ira Rothstein, and Riccardo Sturani, “Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach II: Renormalized Lagrangian,” Phys. Rev. D 100, 024048 (2019), arXiv:1903.05118 [gr-qc].

J. Blümlein, A. Maier, P. Marquard, and G. Schäfer, “Fourth post-Newtonian Hamiltonian dynamics of two-body systems from an effective field theory approach,” Nucl. Phys. B 955, 115041 (2020), arXiv:2003.01692 [gr-qc].

J. Blümlein, A. Maier, P. Marquard, and G. Schäfer, “Testing binary dynamics in gravity at the sixth post-Newtonian level,” Phys. Lett. B 807, 135496 (2020), arXiv:2003.07145 [gr-qc].

Donato Bini, Thibault Damour, and Andrea Geralico, “Sixth post-Newtonian local-in-time dynamics of binary systems,” Phys. Rev. D 102, 024061 (2020), arXiv:2004.05407 [gr-qc].

Donato Bini, Thibault Damour, and Andrea Geralico, “Sixth post-Newtonian nonlocal-in-time dynamics of binary systems,” Phys. Rev. D 102, 084047 (2020), arXiv:2007.11239 [gr-qc].

J. Blümlein, A. Maier, P. Marquard, and G. Schäfer, “The fifth-order post-Newtonian Hamiltonian dynamics of two-body systems from an effective field theory approach,” Nucl. Phys. B 983, 115900 (2022), arXiv:2110.13822 [gr-qc].

J. Blümlein, A. Maier, P. Marquard, and G. Schäfer, “The fifth-order post-Newtonian Hamiltonian dynamics of two-body systems from an effective field theory approach: potential contributions,” Nucl. Phys. B 965, 115352 (2021), arXiv:2010.13672 [gr-qc].

Stefano Foffa, Riccardo Sturani, and William J. Torres Bobadilla, “Efficient resummation of high post-Newtonian contributions to the binding energy,” JHEP 02, 165 (2021), arXiv:2010.13730 [gr-qc].

J. Blümlein, A. Maier, P. Marquard, and G. Schäfer, “The 6th post-Newtonian potential terms at $O(\alpha_s^4)$,” Phys. Lett. B 816, 136260 (2020), arXiv:2101.08630 [gr-qc].

Gregor Källin, Zhengwen Liu, and Rafael A. Porto, “Conservative Dynamics of Binary Systems to Third Post-Minkowskian Order from the Effective Field Theory Approach,” Phys. Rev. Lett. 125, 261103 (2020), arXiv:2007.04977 [hep-th].

Gregor Källin, Zhengwen Liu, and Rafael A. Porto, “Conservative Tidal Effects in Compact Binary Systems to Next-to-Leading Post-Minkowskian Order,” Phys. Rev. D 102, 124025 (2020), arXiv:2008.06047 [hep-th].

Stavros Mougiakakos, Massimiliano Maria Riva, and Filippo Vernizzi, “Gravitational Bremsstrahlung in the post-Minkowskian effective field theory,” Phys. Rev. D 104, 024041 (2021), arXiv:2102.08339 [gr-qc].

Massimiliano Maria Riva and Filippo Vernizzi, “Radiated momentum in the post-Minkowskian worldline approach via reverse unitarity,” JHEP 11, 228 (2021), arXiv:2110.10140 [hep-th].

Christoph Diapa, Gregor Källin, Zhengwen Liu, and Rafael A. Porto, “Dynamics of binary systems to fourth Post-Minkowskian order from the effective field theory approach,” Phys. Lett. B 831, 137203 (2022), arXiv:2106.08276 [hep-th].

Christoph Diapa, Gregor Källin, Zhengwen Liu, and Rafael A. Porto, “Conservative Dynamics of Binary Systems at Fourth Post-Minkowskian Order in the Large-Eccentricity Expansion,” Phys. Rev. Lett. 128, 161104 (2022), arXiv:2112.11296 [hep-th].

Walter D. Goldberger, Jingping Li, and Ira Z. Rothstein, “Non-conservative effects on spinning black holes from world-line effective field theory,” JHEP 06, 053 (2021), arXiv:2012.14869 [hep-th].

Jung-Wook Kim, Michèle Levi, and Zhewei Yin, “Quadratic-in-spin interactions at fifth post-Newtonian order probe new physics,” Phys. Lett. B 834, 137410 (2022), arXiv:2112.01509 [hep-th].

Gihyuk Cho, Rafael A. Porto, and Zixin Yang, “Gravitational radiation from inspiralling compact objects: Spin effects to fourth Post-Newtonian order,” (2022), arXiv:2201.05138 [gr-qc].

Gregor Källin and Rafael A. Porto, “Post-Minkowskian Effective Field Theory for Conservative Binary Dynamics,” JHEP 11, 106 (2020), arXiv:2006.01184 [hep-th].

N. E. J. Bjerrum-Bohr, Poul H. Damgaard, Guido Festuccia, Ludovic Plante, and Pierre Vanhove, “General Relativity from Scattering Amplitudes,” Phys. Rev. Lett. 121, 171601 (2018), arXiv:1806.04920 [hep-th].

Clifford Cheung, Ira Z. Rothstein, and Mikhail P. Solon, “From Scattering Amplitudes to Classical Potentials in the Post-Minkowskian Expansion,” Phys. Rev. Lett. 121, 251101 (2018), arXiv:1808.02489 [hep-th].

Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, and Mao Zeng, “Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order,” Phys. Rev. Lett. 122, 201603 (2019), arXiv:1901.04424 [hep-th].

Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, and Mao Zeng, “Black Hole Binary Dynamics from the Double Copy and Effective Theory,” JHEP 10, 206 (2019), arXiv:1908.01493 [hep-th].

Duff Neill and Ira Z. Rothstein, “Classical Space-Times from the S Matrix,” Nucl. Phys. B 877, 177–189 (2013), arXiv:1304.7263 [hep-th].

Andrea Cristofoli, Riccardo Gonzo, David A. Kosower, and Donal O’Connell, “Waveforms from Amplitudes,” (2021), arXiv:2107.10193 [hep-th].

Zvi Bern, Lancer J. Dixon, David C. Dunbar, and David A. Kosower, “One loop n point gauge theory amplitudes, unitarity and collinear limits,” Nucl. Phys. B 425, 217–260 (1994), arXiv:hep-ph/9403226.

Zvi Bern, Lancer J. Dixon, David C. Dunbar, and David A. Kosower, “Fusing gauge theory tree amplitudes into loop amplitudes,” Nucl. Phys. B 435, 59–101 (1995), arXiv:hep-ph/9409265.

Ruth Britto, Freddy Cachazo, and Bo Feng, “Generalized unitarity and one-loop amplitudes in N=4 super-Yang-Mills,” Nucl. Phys. B 725, 275–305 (2005),
arXiv:hep-th/0412103.

[66] N. E. J. Bjerrum-Bohr, John F. Donoghue, and Pierre Vanhove, “On-shell Techniques and Universal Results in Quantum Gravity,” JHEP 02, 111 (2014), arXiv:1309.0804 [hep-th].

[67] Andreas Luna, Isobel Nicholson, Donal O’Connell, and Chris D. White, “Inelastic Black Hole Scattering from Charged Scalar Amplitudes,” JHEP 03, 044 (2018), arXiv:1711.03901 [hep-th].

[68] H. Kawai, D. C. Lewellen, and S. H. H. Tye, “A Relation Between Tree Amplitudes of Closed and Open Strings,” Nucl. Phys. B 269, 1–23 (1986).

[69] Z. Bern, J. J. M. Carrasco, and Henrik Johansson, “New Relations for Gauge-Theory Amplitudes,” Phys. Rev. D 78, 085011 (2008), arXiv:0805.3993 [hep-ph].

[70] Zvi Bern, John Joseph M. Carrasco, and Henrik Johansson, “Perturbative Quantum Gravity as a Double Copy of Gauge Theory,” Phys. Rev. Lett. 105, 061602 (2010), arXiv:1004.0476 [hep-th].

[71] Z. Bern, J. J. M. Carrasco, L. J. Dixon, H. Johansson, and R. Roiban, “Simplifying Multiloop Integrands and Ultraviolet Divergences of Gauge Theory and Gravity Amplitudes,” Phys. Rev. D 85, 105014 (2012), arXiv:1201.5366 [hep-th].

[72] Zvi Bern, John Joseph Carrasco, Marco Chiodaroli, Henrik Johansson, and Radu Roiban, “The Duality Between Color and Kinematics and its Applications,” (2019), arXiv:1909.01358 [hep-th].

[73] N. E. J Bjerrum-Bohr, John F. Donoghue, and Barry R. Holstein, “Quantum gravitational corrections to the nonrelativistic scattering potential of two masses,” Phys. Rev. D 67, 084033 (2003). [Erratum: Phys.Rev.D 71, 069903 (2005)], arXiv:hep-th/0211072.

[74] Zvi Bern, Andres Luna, Radu Roiban, Chia-Hsien Shen, and Mao Zeng, “Spinning black hole binary dynamics, scattering amplitudes, and effective field theory,” Phys. Rev. D 104, 065014 (2021), arXiv:2005.03071 [hep-th].

[75] Zvi Bern, Julio Parra-Martinez, Radu Roiban, Michael S. Ruf, Chia-Hsien Shen, Mikhail P. Solon, and Mao Zeng, “Scattering Amplitudes and Conservative Binary Dynamics at O(G4),” Phys. Rev. Lett. 126, 171601 (2021), arXiv:2101.07254 [hep-th].

[76] David A. Kosower, Ben Maybee, and Donal O’Connell, “Amplitudes, Observables, and Classical Scattering,” JHEP 02, 137 (2019), arXiv:1811.10950 [hep-th].

[77] Ben Maybee, Donal O’Connell, and Justin Vines, “Observables and amplitudes for spinning particles and black holes,” JHEP 12, 156 (2019), arXiv:1906.09260 [hep-th].

[78] Andreas Brandhuber, Gang Chen, Gabriele Travaglini, and Congkao Wen, “A new gauge-invariant double copy for heavy-mass effective theory,” JHEP 07, 047 (2021), arXiv:2104.11206 [hep-th].

[79] Andreas Brandhuber, Gang Chen, Gabriele Travaglini, and Congkao Wen, “Classical gravitational scattering from a gauge-invariant double copy,” JHEP 10, 118 (2021), arXiv:2108.04216 [hep-th].

[80] Gustav Mogull, Jan Plefka, and Jan Steinhoff, “Classical black hole scattering from a worldline quantum field theory,” JHEP 02, 048 (2021), arXiv:2010.02865 [hep-th].

[81] Walter D. Goldberger and Ira Z. Rothstein, “Towers of Gravitational Theories,” Gen. Rel. Grav. 38, 1537–1546 (2006), arXiv:hep-th/0605238.

[82] Barak Kol and Michael Smolkin, “Non-Relativistic Gravitation: From Newton to Einstein and Back,” Class. Quant. Grav. 25, 145011 (2008), arXiv:0712.4116 [hep-th].

[83] Gustav Uhre Jakobsen, Gustav Mogull, Jan Plefka, and Jan Steinhoff, “Classical Gravitational Bremsstrahlung from a Worldline Quantum Field Theory,” Phys. Rev. Lett. 126, 201103 (2021), arXiv:2101.12688 [gr-qc].

[84] Gustav Uhre Jakobsen, Gustav Mogull, Jan Plefka, and Jan Steinhoff, “Gravitational Bremsstrahlung and Hidden Supersymmetry of Spinning Bodies,” Phys. Rev. Lett. 128, 011101 (2022), arXiv:2106.10256 [hep-th].

[85] Gustav Uhre Jakobsen, Gustav Mogull, Jan Plefka, and Jan Steinhoff, “SUSY in the sky with gravitons,” JHEP 01, 027 (2022), arXiv:2109.04465 [hep-th].

[86] Gustav Uhre Jakobsen and Gustav Mogull, “Conservative and Radiative Dynamics of Spinning Bodies at Third Post-Minkowskian Order Using Worldline Quantum Field Theory,” Phys. Rev. Lett. 128, 141102 (2022), arXiv:2201.07778 [hep-th].

[87] Julio Parra-Martinez, Michael S. Ruf, and Mao Zeng, “Extremal black hole scattering at O(G4): graviton dominance, eikonal exponentiation, and differential equations,” JHEP 11, 023 (2020), arXiv:2005.04336 [hep-th].

[88] Paolo Di Vecchia, Carlo Heissenberg, Rodolfo Russo, and Gabriele Veneziano, “The eikonal approach to gravitational scattering and radiation at O(G4),” JHEP 07, 169 (2021), arXiv:2104.03256 [hep-th].

[89] Carlo Heissenberg, “Infrared divergences and the eikonal exponentiation,” Phys. Rev. D 104, 046016 (2021), arXiv:2105.04594 [hep-th].

[90] Paolo Di Vecchia, Carlo Heissenberg, Rodolfo Russo, and Gabriele Veneziano, “The eikonal operator at arbitrary velocities I: the soft-radiation limit,” (2022), arXiv:2204.02378 [hep-th].

[91] Pou H. Damgaard, Ludovic Plante, and Pierre Vanhove, “On an Exponential Representation of the Gravitational S-Matrix,” (2021), arXiv:2107.12891 [hep-th].

[92] Uri Kol, Donal O’Connell, and Ofri Telem, “The radial action from probe amplitudes to all orders,” JHEP 03, 141 (2022), arXiv:2109.12092 [hep-th].

[93] N. Emil J. Bjerrum-Bohr, Ludovic Plante, and Pierre Vanhove, “Post-Minkowskian radial action from soft limits and velocity cuts,” JHEP 03, 071 (2022), arXiv:2111.02976 [hep-th].

[94] Zvi Bern, Juan Pablo Gatica, Enrico Herrmann, Andres Luna, and Mao Zeng, “Scalar QED as a toy model for higher-order effects in classical gravitational scattering,” (2021), arXiv:2112.12243 [hep-th].

[95] Canxin Shi and Jan Plefka, “Classical double copy of worldline quantum field theory,” Phys. Rev. D 105, 026007 (2022), arXiv:2109.10345 [hep-th].

[96] Fiorenzo Bastianelli, Francesco Comberiati, and Leonardo de la Cruz, “Light bending from eikonal in worldline quantum field theory,” JHEP 02, 209 (2022), arXiv:2112.05013 [hep-th].

[97] Konrad Westphal, “High-Speed Scattering of Charged and Uncharged Particles in General Relativity,” Fortsch. Phys. 33, 417–493 (1985).
