DIMENSIONAL DEPENDENCE OF THE HYDRODYNAMICS OF CORE-COLLAPSE SUPERNOVAE

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ABSTRACT

A major goal over the last decade has been understanding which multidimensional effects are crucial in facilitating core-collapse supernova (CCSN) explosions. Unfortunately, much of this work has necessarily assumed axisymmetry. In this work, we present analyses of simplified two-dimensional (2D) and three-dimensional (3D) CCSN models with the goal of comparing the hydrodynamics in setups that differ only in dimension. Not surprisingly, we find many differences between 2D and 3D models. While some differences are subtle and perhaps not crucial, others are dramatic and make interpreting 2D models problematic. In particular, axisymmetric models produce excess power at the largest spatial scales, power that has been deemed critical in previous explosion models. Nevertheless, our 3D models, which have an order of magnitude less power than 2D models on large scales, explode earlier. Since explosions occur earlier in 3D than in 2D, the vigorous large-scale sloshing is either not critical in any dimension or the explosion mechanism operates differently in 2D and 3D. On the other hand, we find that the average parcel of matter in the gain region has been exposed to net heating for up to 30% longer in 3D than in 2D, an effect we attribute to the differing characters of turbulence in 2D and 3D. We suggest that this effect plays a prominent role in producing earlier explosions in 3D. Finally, we discuss a simple model for the runaway growth of buoyant bubbles that is able to quantitatively account for the growth of the shock radius and predicts a critical luminosity relation.

Key words: hydrodynamics – neutrinos – stars: interiors – supernovae: general

Online-only material: color figures

1. INTRODUCTION

Multidimensional hydrodynamic instabilities play a central role in the explosion mechanism of most core-collapse supernovae (CCSNe). Though this has been repeatedly demonstrated in a variety of contexts (Herant et al. 1992; Burrows & Fryxell 1993; Herant et al. 1994; Burrows et al. 1995; Janka & Mueller 1996; Marek & Janka 2009), there is as yet no definitive understanding of the role of multidimensional effects in facilitating explosions. Important effects may include increased dwell times in the gain region (Murphy & Burrows 2008), expansion of the shock due to turbulent pressure support from neutrino-driven convection (Herant et al. 1994; Burrows et al. 1995; Murphy et al. 2012) and/or the standing accretion shock instability (SASI; e.g., Scheck et al. 2008), simultaneous accretion and explosion (Burrows et al. 1995), suppression of cooling beneath the gain region (Pejcha & Thompson 2012), and other still unidentified processes.

With the exception of a few preliminary results in three dimensions (3D; Bruenn et al. 2009; Kuroda et al. 2012; Takiwaki et al. 2012), the enormous computational expense has limited the most sophisticated supernova models, including the multi-species, multi-group neutrino transport, to two-dimensional (2D) axisymmetric simulations (Ott et al. 2008; Müller et al. 2012). Unfortunately, the fundamentally 3D hydrodynamics in the post-shock turbulent flow is qualitatively different if axisymmetry is imposed, as we discuss in detail below. Understanding the differences between 2D and 3D behavior and how dimension effects the mechanism of explosion is critical in the interpretation of realistic 2D simulations and ultimately in elucidating how real stars explode. If we find explosions in axisymmetry, then should we expect them in 3D? If so, then are the conditions identified in 2D as crucial to producing explosions manifest in 3D? How are explosions triggered in 3D?

In this work, we do not perform sophisticated radiation hydrodynamic simulations of realistic CCSNe. Rather, we perform a series of simplified numerical experiments designed to clarify how dimension effects the hydrodynamics leading to explosions (see also Murphy & Burrows 2008; Nordhaus et al. 2010; Hanke et al. 2012). We find that there are many differences between 2D and 3D models, some quite dramatic, and that conditions identified as important in 2D may not remain so in more realistic 3D models. Nevertheless, our 3D models explode whenever the 2D models explode and, moreover, 3D models explode earlier.

We begin in Section 2 with a discussion of our numerical setup and solution technique. Section 3 gives an overview of the basic results of our simulations and discusses qualitative differences in the structures of 2D and 3D models. Section 4 begins the quantitative analyses of the simulations, showing how the global structures of the flows are different between 2D and 3D models. Section 5 compares various measures of the 2D and 3D turbulence in the post-shock flows. Section 6 discusses popular explosion metrics and introduces a simple model for explosions based on the runaway growth of bubbles. Finally, Section 7 discusses our results and conclusions.

2. NUMERICAL SETUP

Our setup is the same as that presented in Burrows et al. (2012) and Murphy et al. (2012). We use the CASTRO adaptive mesh refinement hydrodynamics code (Almgren et al. 2010) to evolve 2D (axisymmetric) and 3D models of the collapse, bounce, and subsequent evolution of the $15 M_\odot$ non-rotating solar-metallicity progenitor of Woosley & Weaver (1995). The
adapted mesh uses six levels of factor two refinement with \( \approx 0.5 \text{ km} \) resolution in the inner 50 km and 2 km or better resolution everywhere behind and including the shock during the stalled pre-explosion phase. We ensure that both 2D and 3D simulations utilize the same refinement criteria to minimize any differences that might arise from different grid structures. Self-gravity is included with the monopole approximation. The domains include the inner 5000 km in radius. We note that a zero-gradient outer boundary condition has been adopted. Though Couch (2012) points out that this may lead to accretion rates that are larger than they should be at late times, possibly suppressing marginal explosions, this should have little effect on the comparison between 2D and 3D models presented in this work.

As in previous works, we adopt the “light bulb” prescription for neutrino heating and cooling (Murphy & Burrows 2008; Nordhaus et al. 2010; Hanke et al. 2012; Burrows et al. 2012; Murphy et al. 2012; Couch 2012). In this prescription, neutrino heating is parameterized by a constant driving electron neutrino luminosity \( L_{\nu e} \), (the electron neutrino and antineutrino luminosities are assumed to be equal), and we present results for three luminosities: \( 2.1 \times 10^{52} \text{ erg s}^{-1} \), \( 2.2 \times 10^{52} \text{ erg s}^{-1} \), and \( 2.3 \times 10^{52} \text{ erg s}^{-1} \). Neutrino cooling occurs at a rate \( \propto T^9 \) (Bethe 1990). Since we do not explicitly treat the neutrino transport, the electron fraction \( Y_e \) is evolved according to the prescription given by Liebendörfer et al. (2005). Finally, the equation of state is based on the relativistic mean-field theory of Shen et al. (1998a, 1998b) and we assume nuclear statistical equilibrium.

3. OVERVIEW OF SIMULATION RESULTS

We find that the structure and evolution of the post-bounce hydrodynamics can be clearly distinguished between simulations that differ only in dimension. In this section, we develop a qualitative view of the structure and evolution of 2D and 3D models.

Figure 1 shows the development of the average shock radius for all three driving neutrino luminosities in both 2D and 3D. For the first 100 ms after bounce, the models are nearly indistinguishable, but diverge thereafter. The stalled shock radii in 3D are generally larger than in 2D and are less variable in both angle and time. The models with driving luminosities of \( 2.2 \times 10^{52} \text{ erg s}^{-1} \) and \( 2.3 \times 10^{52} \text{ erg s}^{-1} \) have sufficient neutrino heating to explode within the simulated time in both 2D and 3D. The explosions, however, occur earlier in 3D than in 2D, in qualitative agreement with Nordhaus et al. (2010) despite deficiencies in that work (Burrows et al. 2012). Moreover, once explosions set in, the shock radii grow monotonically in 3D, whereas they show oscillatory behavior in 2D, at least early in the explosion phase.

To develop a qualitative sense for some of the underlying hydrodynamic differences, we show snapshots of various quantities at select times in the evolution. Figure 2 shows slices of the entropy, radial velocity, magnitude of the vorticity \( |\mathbf{\nabla} \times \mathbf{v}| \), and velocity divergence \( \mathbf{V} \cdot \nabla \) at 250 ms post-bounce for the \( L_{\nu e} = 2.2 \times 10^{52} \text{ erg s}^{-1} \) models in 2D and 3D. Figure 3 shows the same quantities at 500 ms post-bounce. Perhaps the most obvious difference between 2D and 3D is the clear presence of a preferred axis, leading to a distinctly prolate distortion of the post-shock flow. This is a generic result seen in all of the 2D simulations, even those that include sophisticated neutrino transport and relativistic effects (Ott et al. 2008; Marek & Janka 2009; Müller et al. 2012). Importantly, this may be an artifact of assuming axisymmetry, the consequences of which are difficult to clarify. Another important difference, identified previously, is the existence of more small-scale structure in the flow in 3D (Hanke et al. 2012), as can be seen in the entropy, radial velocity, vorticity, and velocity divergence. Interestingly, when comparing the entropy and radial velocity maps in both the 2D and 3D models, there is a strong correspondence between large-scale structures; high-entropy plumes are associated with outflow, whereas low-entropy regions are associated with inflow, a natural consequence of buoyancy-driven convection, which it has been argued dominates the flow in the stalled accretion shock phase (Burrows et al. 2012; Murphy et al. 2012). In 3D, these rising plumes also have lower vorticities and larger velocity divergences than the surrounding flow, suggesting that these structures are relatively coherent and expanding. In 2D, these associations are more difficult to make by eye.

The 2D and 3D post-shock flows have qualitative differences that are easily identified. The geometries of the flows are different and the characteristics of the turbulent, convective, post-shock flows are different. One consequence of these differences is a larger averaged stalled shock radius in 3D relative to 2D. In the remainder of this work, we undertake detailed analyses of the post-shock hydrodynamics to better understand how these qualitative differences manifest quantitatively in quantities that have been suggested to be of importance.

4. GLOBAL STRUCTURES

4.1. Integral Quantities

While unable to capture the full scope of the differences between 2D and 3D flows, integral quantities offer the
Figure 2. Snapshots of the entropy, radial velocity, magnitude of the vorticity (|∇ × v|), and velocity divergence (∇·v) at 250 ms post-bounce for the 2D (left) and 3D (slice, right) $L_{\nu} = 2.2 \times 10^{52}$ erg s$^{-1}$ models. As evident in all four quantities and in contrast with the 2D models, 3D models show significantly more small scale structure, have no preferred axis, and tend to be more spherical in the early stalled accretion shock phase.

(A color version of this figure is available in the online journal.)
Figure 3. Same as Figure 2, but at 500 ms post-bounce. As compared with the snapshot at 250 ms, the 3D model is developing significant asymmetry as it evolves toward explosion. The 2D model is qualitatively similar at 500 ms post-bounce to 250 ms post-bounce, with its distinctive prolate distortion and characteristically larger structures compared with the 3D model.

(A color version of this figure is available in the online journal.)
advantages of simplicity and widespread usage. In the context of the neutrino mechanism, the integrated heating and cooling in the post-shock flow are naturally important quantities. Figures 4, 5, and 6 show the total integrated heating, cooling, and net heating minus cooling rates, respectively, in the region between the neutrinosphere (where the optical depth to neutrinos is approximately unity) and shock. While there tends to be marginally more heating in the 3D models, they have significantly more cooling and, therefore, less net heating than their corresponding 2D models. If we focus on the gain region (i.e., only the region with net neutrino heating), then we see in Figure 7 that the 2D models have significantly more heating than their 3D counterparts. This arises, in part, because, prior to explosion, there is more mass in the gain region in 2D than in 3D, as shown in Figure 8. Finally, in spite of the higher heating rates in 2D, the average specific entropy in the gain region,

\[
\langle s \rangle_{\text{gain}} = \frac{1}{M_{\text{gain}}} \int_{V_{\text{gain}}} \rho s dV,
\]

is larger in 3D than in 2D, as shown in Figure 9 and suggested by Nordhaus et al. (2010). This is consistent with the results shown in Hanke et al. (2012), who found somewhat larger average entropies in 3D than in 2D. We note, however, that a higher average entropy is not directly related to earlier explosions, as is clearly demonstrated by the 3D \( L_{\nu_e} = 2.1 \times 10^{52} \text{ erg s}^{-1} \) model, which has the highest average entropy of any model shown but does not explode within the simulated time. In any case, since entropy depends on the integrated heating, not the heating rate,
this suggests that perhaps some material is exposed longer to heating in 3D than in 2D, which we return to in Section 5.1.

With the exception of the average entropies, the integral quantities just shown seem to indicate that 2D models might have more favorable conditions for explosion. Since our 3D models explode earlier, as shown in Figure 1, important differences between 2D and 3D must remain which these integral quantities have yet to elucidate.

### 4.2. Radial Profiles

Moving beyond integral quantities, we can look at radial profiles, averaged over solid angle, to distinguish the global structures of 2D and 3D models. In order to highlight the trends with luminosity and dimension, we compute two sets of profiles, one time-averaged from 200 ms to 300 ms post-bounce and another time-averaged from 450 ms to 550 ms post-bounce. The $L_{\nu_e} = 2.3 \times 10^{52}$ erg s$^{-1}$ models are excluded from the latter set of plots because they are already well into explosion. Time averaging is essential for the 2D profiles to minimize the large fluctuations that obscure their underlying quasi-steady structure.

Figure 10 shows the time- and spherically-averaged entropy profiles. Consistent with Figure 1, the entropy profiles clearly show that the shock radii are systematically larger in 3D than in 2D prior to explosion. The 3D models also have higher peak entropies than their corresponding 2D models, consistent with Figure 9. Interestingly, the peak entropies vary little over the luminosity range considered for models with a given number of dimensions, but there is a clear distinction between the 2D and 3D models. On the other hand, the entropy profiles between ~50 km and ~100 km are remarkably similar between all models.

Figure 11 shows the profiles of radial velocity. As with the entropy profiles, the radial velocities in the post-shock region vary more with dimension than over the range of luminosities of models with a given number of dimensions. The magnitudes of the radial velocities in the post-shock flow are generally larger in 2D than in 3D. This remains true also when doing mass-weighted spherical averages, though the profiles are somewhat closer. The larger post-shock radial velocities in 2D are associated with lower densities and, therefore, lower heating rates for radii beyond ~120–150 km as compared with 3D. This has the effect of lowering the temperatures, which suppresses cooling and moves the gain radius inward (relative to 3D) toward higher densities and neutrino fluxes. Figures 12 and 13 show the radial profiles of the net neutrino heating rate and bear out these arguments. In the end, the smaller gain radius in 2D leads to a larger integrated net heating rate in the gain region in 2D as shown in Figure 7 and a larger mass in the gain region as shown in Figure 8, even though the densities and net heating rates are larger in 3D at radii beyond ~120 km and ~150 km, respectively.

Finally, Figure 14 shows the radial profiles of the transverse kinetic energy. From 200 ms and 300 ms and between ~100 km and the shock, the 2D models have nearly twice as much transverse kinetic energy as the 3D models. Again, the profiles vary much more with dimension than between the different luminosity models of the same dimension. At this stage in the evolution, the turbulence in the post-shock flow is artificially vigorous in 2D axisymmetric models, consistent with multidimensional simulations of stellar convection (Meakin & Arnett 2007). At later times, the transverse kinetic energies in the 3D models have grown to be roughly comparable with the 2D models, though the $L_{\nu_e} = 2.2 \times 10^{52}$ erg s$^{-1}$ models still differ by ~50%.

The radial structures of 2D and 3D models show interesting systematic differences, though none are obviously implicated in causing the discrepant explosion times. For example, Figure 13 shows that beyond 150 km, the 3D models have higher net specific heating rates. At the same time, Figures 6 and 7 show that the 2D models have higher integrated heating rates. Which is more important in producing explosions is not a priori clear. The 2D models tend to have larger transverse kinetic energies, which has been claimed to be conducive to explosion (Hanke et al. 2012), but our results seem at odds with this claim. Finally, consistent with Figure 9, the entropy in the gain region is systematically larger in 3D than in 2D, suggesting that there may be interesting differences in the temporally integrated heating, as discussed below in Section 5.1.
Figure 10. Time- and spherically-averaged entropy profiles in the inner 500 km for 2D (dashed) and 3D (solid) models. The left panel shows the profiles time-averaged from 200 to 300 ms post-bounce, while the right panel shows the profiles time-averaged from 450 to 500 ms post-bounce. The $L_{\nu_e} = 2.3 \times 10^{52}$ erg s$^{-1}$ models explode before 500 ms and are therefore not included in the latter. The 3D models generically have higher averaged entropies between $\sim 100$ km (near the gain radius) and the shock.

(A color version of this figure is available in the online journal.)

Figure 11. The same as Figure 10, but for the time- and spherically-averaged radial velocity. The 3D models (solid) tend to have smaller radial velocity magnitudes between $\sim 100$ km and the shock than the 2D models (dashed).

(A color version of this figure is available in the online journal.)

5. TURBULENCE DIAGNOSTICS

5.1. Dwell-time Distributions

During the stalled accretion shock phase, fluid elements advect through the gain region and eventually settle onto the proto-neutron star. In spherical symmetry, the time to advect through the gain region (the integrated dwell time) is short and shared by all of the fluid elements belonging to the same mass shell. In multiple dimensions, the aspherical shock structure and post-shock turbulence lead to a distribution of dwell times, with some fraction of the mass being exposed longer to net neutrino heating (Murphy & Burrows 2008). This effect can increase the neutrino heating efficiency and has naturally been suggested to be one key element of the multidimensional picture (Burrows et al. 1995; Murphy & Burrows 2008). Here, we focus on the $L_{\nu_e} = 2.1 \times 10^{52}$ erg s$^{-1}$ models since the dwell time distribution is difficult to interpret once fluid elements begin to participate in explosion. We consider two related, but distinct, definitions of the dwell time distribution. The first is

the standard integrated dwell time distribution introduced by Murphy & Burrows (2008), which answers the question, for a particular shell of accreting matter, what is the distribution of total integrated times spent in the region of net neutrino heating? Next, we introduce the instantaneous dwell time distribution. At any particular instant, the instantaneous dwell time distribution tells us the distribution of times all of the parcels of matter in the gain region have thus far been exposed to net heating. We suggest that the instantaneous dwell time distribution is of more direct relevance in producing explosions.

We measure the dwell-time distributions by following the trajectories of Lagrangian tracer particles which move according to the time-dependent velocity field. We initialize the particles at a radius of 400 km at 250 ms$^5$ post-bounce and begin integrating their dwell times once they pass through the shock. In 3D, we use approximately $2^{18}$ particles distributed quasi-uniformly on

$^5$ We also looked at results based on particles injected at 500 ms post-bounce and the basic conclusions remain unchanged.
Figure 12. The same as Figure 10, but for the time- and spherically averaged net heating rate. Overall, the 2D (dashed) and 3D (solid) models have very similar net heating profiles. Note that the “gain radius” is found at $\sim 100$ km where the net heating changes sign.

(A color version of this figure is available in the online journal.)

Figure 13. The same as Figure 12, but zoomed into the region of significant net neutrino heating. The 2D models (dashed) tend to have more heating at smaller radii (near the gain radius where the net heating changes sign), but less heating at larger radii, compared to the 3D models (solid).

(A color version of this figure is available in the online journal.)

Figure 14. The same as Figure 10, but for the time- and spherically-averaged transverse kinetic energy per unit mass. In the early stalled accretion shock phase (200–300 ms post-bounce), the 2D models (dashed) have $\sim 50\%$–$100\%$ more specific transverse kinetic energy than the 3D models (solid). This early turbulent vigor seems to be an artifact of assuming axisymmetry. At later times, the 2D/3D difference is reduced, but is still significant ($\sim 30\%$).

(A color version of this figure is available in the online journal.)
the mean integrated dwell time, use 16 shells of approximately 212 particles injected over 2 ms mass shell, even at the same resolution. To combat this, we are fewer independent trajectories for fluid elements of a given sphere. In 2D, the fewer degrees of freedom means that there is the data beyond ∼400 ms for the 2D model is dominated by shot noise associated with the finite number of tracer particles.

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\[ \langle \tau \rangle , \text{ is about 5}\% \text{ longer in 2D.} \]

In a steady state, the integrated dwell time distribution simply by \( M_{\text{gain}} = M(\tau) \) and so a longer mean dwell time in 2D is consistent with the larger gain mass seen previously in Figure 8. Second, the integrated dwell time distribution in 3D has a longer, shallower tail than the 2D distribution. Integrating over the distributions, we find that about 25% of the accreted material spends more time in the gain region in 3D than in 2D. While the converse is also true—about 75% of the accreted material has a longer dwell time in 2D—much of this material simply settles onto the proto-neutron star without having had much neutrino heating or participated in the turbulent flow in the gain region. By recording the peak entropy reached by each tracer particle, we have found that there is a strong correlation between dwell time and peak entropy. Thus, the more prominent tail at long dwell times translates into having relatively more material in 3D exposed to heating for long times and, consequently, reaching high entropies. Which effect is more important—a longer mean integrated dwell time (2D) or a more prominent tail at long dwell times (3D)—is not immediately clear.

The integrated dwell time distribution does not tell us directly about the instantaneous state of material in the gain region, as might be expected to bear on a model’s ability to transition to explosion. A more relevant measure is the distribution of instantaneous dwell times for all of the parcels of matter in the gain region. This distribution tells us, at any particular instant, how long parcels of matter in the gain region have thus far been exposed to net heating. In a steady state, the instantaneous dwell time distribution \( (F(\tau)) \) is related to the standard time-integrated dwell time distribution \( (f(\tau)) \) discussed above by

\[ F(\tau) = N \int_{\tau}^{\infty} f(\tau')d\tau', \]

where \( N \) is a normalization factor that assures \( F(\tau) \) integrates to unity. For CCSNe, the turbulence is established at some time \( t_0 \), so \( F(\tau) = 0 \) for \( \tau > t - t_0 \), with the above definition of \( F(\tau) \) being correct only asymptotically. Aside from the normalization, \( F(\tau) \) is just one minus the cumulative time-integrated distribution function. Figure 16 shows the instantaneous dwell time distribution, computed according to Equation (2). Apart from the first 50–100 ms, the distributions are very nearly exponential \( (\propto \exp(-\tau/t_0)) \), with a time constant of 55 ms in 3D and 43 ms in 2D. By computing the mean of the distributions, we find that at late times, the average parcel of matter in the gain region has been exposed to net heating for about 30% longer in 3D than in 2D, a much larger difference than the tail of the integrated dwell time distribution becomes longer than in 2D as the tail of the integrated dwell time distribution becomes increasingly relevant. Thereafter, the fractional difference between the 3D and 2D mean instantaneous dwell times grows approximately as \( 0.3[1 - \exp(-t/(100\text{ms})] \). Thus, soon after turbulence develops, a typical fluid element in the gain region has been exposed to net heating for longer in 3D than in 2D. We note that this is consistent with explosion times in 2D and 3D approaching one another as the delay to explosion becomes shorter, as we have found and as previously reported (Nordhaus et al. 2010; Hanke et al. 2012). This is also likely the source of the larger entropies in 3D shown in Figures 9 and 10 and in...
other works (Nordhaus et al. 2010; Hanke et al. 2012). Importantly, this may also be one of the main reasons for the earlier explosions in 3D.

5.2. Turbulent Energy Spectra

Perhaps the single most distinguishing characteristic between the 2D and 3D turbulent post-shock flows is found in their energy spectra. As shown by Kraichnan (1967) and later confirmed experimentally in various contexts (see, e.g., Boffetta & Ecke 2012), turbulent cascades are different in 2D and 3D. In 3D turbulence, energy is the only constant of the motion and this leads to a single turbulent cascade that transfers energy from some driving wavenumber \( k_\text{d} \) toward larger \( k \) (smaller scales). In 2D, both energy and squared vorticity are constants of the motion which lead to cascades of energy and enstrophy (proportional to the mean squared vorticity). The enstrophy cascade transports enstrophy in \( k \)-space from the driving wavenumber \( k_\text{d} \) toward larger \( k \) (smaller scales). This leads to a characteristic \( k^{-3} \) scaling of the velocity energy spectrum for \( k > k_\text{d} \) (Kraichnan 1967). The energy cascade, by contrast, transports energy from \( k_\text{d} \) to smaller \( k \) (larger scales) and leads to a \( k^{-5/3} \) scaling of the velocity energy spectrum for \( k < k_\text{d} \), in direct analogy with the Kolmogorov theory of turbulence. This is the inverse energy cascade of 2D turbulence, which tends to exaggerate motions on the largest scales of the flow. These turbulence theories were developed within highly idealized setups, assuming, for example, steady isotropic turbulence, and do not necessarily apply directly to the turbulence seen in the core-collapse context. Nevertheless, we find, as did Hanke et al. (2012), that the basic predictions of these theories—the predominance of energy at the largest scales in 2D, the excess energy at the largest scales in 2D relative to 3D, and the shallower slope of the velocity energy spectrum for \( k > k_\text{d} \) (and therefore more energy for large \( k \)) in 3D relative to 2D—are all confirmed in our simulations.

The most natural basis to represent the matter fields in the quasi-spherical post-shock flow is the basis of real spherical harmonics. We decompose the arbitrary scalar quantity \( Q \) into spherical harmonics with time- and radially-dependent coefficients

\[
a_{\ell m}(t, r, \rho) = \oint Q(t, r, \rho, \theta, \phi) Y_{\ell m}^*(\theta, \phi) \, d\Omega,
\]

where

\[
Y_{\ell m}^*(\theta, \phi) = \begin{cases} \sqrt{2} N_{\ell m}^m P_{\ell m}^m (\cos \theta) \cos m \phi & m > 0, \\ N_{\ell m}^0 P_{\ell m}^0 (\cos \theta) & m = 0, \\ \sqrt{2} N_{\ell m}^{-m} P_{\ell m}^{-m} (\cos \theta) \sin |m| \phi & m < 0 \end{cases}
\]

and

\[
N_{\ell m}^m = \sqrt{\frac{2\ell + 1}{4\pi} \frac{(\ell - m)!}{(\ell + m)!}}.
\]

In 2D, axisymmetry implies that all coefficients with \( m \neq 0 \) are identically zero. Here, we consider \( Q = \{ \rho, P, s, \sqrt{\rho} v_r \} \), where \( v_r \) represents the spherical velocity components \( \{ v_r, v_\theta, v_\phi \} \) in 2D and \( \{ v_r, v_\theta, v_\phi \} \) in 3D. We compute the discrete energy spectrum as a function of spherical harmonic degree \( \ell \) as

\[
E(\ell) = \sum_{m=-\ell}^{\ell} a_{\ell m}^2,
\]

where it should be understood that \( a_{\ell m} \) and, therefore, \( E(\ell) \), depend on time and radius (Burrows et al. 2012).

Figure 17 shows the energy spectra of \( \sqrt{\rho} v_\theta \) for the \( L_{\nu_\text{e}} = 2.1 \times 10^{52} \text{ erg s}^{-1} \) models at a radius of 150 km and time-averaged between 450 and 500 ms after bounce. We also reproduce the results of Hanke et al. (2012) for comparison. While these curves change in detail for different quantities, over time, and at different radii, the qualitative trends and relation between 2D and 3D are quite robust. At \( \ell = 1, 2 \) has an order of magnitude more energy in all quantities \( Q = \{ \rho, P, s, \sqrt{\rho} v_r \} \),

![Figure 17](image-url)
consistent with the idea of an inverse energy cascade that pumps energy into the largest scales. The results of Hanke et al. (2012) are even more extreme, with a factor ≈50 more energy in the \( l = 1 \) mode in 2D relative to 3D. In real-space, this excess energy at the largest scales manifests as the characteristic “sloshing” always found in 2D simulations (including those with coupled neutrino transport). This sloshing is typically associated with the development of the SASI (Blondin et al. 2003; Scheck et al. 2008; Foglizzo et al. 2007), but the inverse energy cascade will always produce excess energy at \( l = 1 \) in 2D, even if the turbulence is driven by, for example, neutrino-driven convection. The SASI may be capable, in principle, of producing significant energy in \( l = 1 \), but disentangling its effects from the inevitable \( l = 1 \) energy associated with the inverse cascade would seem to require 3D simulations. Whether the low-\( l \) energy is a result of the SASI or not, all of the 3D supernova simulations presented in the literature thus far show muted or nonexistent sloshing (Iwakami et al. 2008; Hanke et al. 2012; Burrows et al. 2012), though there has yet to be a fully self-consistent 3D radiation hydrodynamic simulation. Therefore, the results seen thus far suggest that these violent sloshing motions are an artifact of assuming axisymmetry, not a feature that must be incorporated into 3D models as suggested by Hanke et al. (2012). Importantly, this artifact is not a small effect; most of the energy in the flow in 2D simulations is at low-\( l \). This artifact, in fact, dominates the flow and has poorly understood consequences on other coupled aspects of the problem, including the neutrino transport.

At intermediate \( l \), the spectra are significantly steeper in 2D (\( \sim l^{-2.0} \)) than in 3D (\( \sim l^{-1} \)). That these power-laws differ from those naively expected from simple theoretical arguments should not be surprising given that the turbulence analyzed here is, among other things, not steady-state or isotropic. Qualitatively, however, the expectation of a steeper slope in 2D relative to 3D is confirmed and we find our results to be generally consistent with those of Hanke et al. (2012). The transition to these slopes occurs around \( l \sim 10 \) in 2D and \( l \sim 4 \) in 3D, which each reflect the corresponding characteristic driving scales for convection. The lower \( l \) driving in 3D is likely the result of the larger radial extent of the gain region owing to the larger shock radius (Chandrasekhar 1961). The 3D model has significantly more energy at small-scales than the 2D model. At \( l \gtrsim 40 \) (spatial scales \( \sim 10 \) km at a radius of \( 150 \) km), we begin to see the effects of grid-scale (2 km at this radius) dissipation associated with numerical diffusion.

Hanke et al. (2012) showed that their results, especially in 3D, were sensitive to resolution. The comparison of energy spectra in Figure 17 affords us the opportunity to directly compare the effective resolutions of our independent calculations by comparing the scales at which dissipation begins to set in. As noted above, our 3D results begin to deviate from the \( l^{-1} \) power-law at \( l \approx 40 \), while the results in Hanke et al. (2012) begin to deviate at \( l \approx 25 \), confirming our expectation that our models have nearly double the effective resolution. Hanke et al. (2012) argue that 3D models become less prone to explosion as resolution is increased, yet our higher resolution 3D models explode earlier than the 2D models. We also note that we carried out a 2D \( L_{\nu_e} = 2.2 \times 10^{52} \) erg s\(^{-1}\) simulation with double the resolution of the models presented in this work and found that it exploded marginally later (\( \sim 10 \) ms) than the model discussed herein, both countering the results of Hanke et al. (2012) and suggesting that our runs (in 2D, at least) are reasonably converged. The source of these apparent discrepancies is unclear.

6 MULTIDIMENSIONAL EXPLOSIONS

6.1. Explosion Conditions

A number of quantities have been proposed in the literature that are meant to distinguish exploding from non-exploding models and, furthermore, to define in a systematic way the time at which a model transitions into the exploding phase. The most widely used condition is based on the idea of a critical ratio of the advection to heating timescales (Janka & Keil 1998; Thompson 2000; Thompson et al. 2003). There are numerous ways of defining these timescales. Here, we define the advection time as

\[
\tau_{\text{adv}} = \int_{R_{\text{esc}}}^{R_{\text{gain}}} \frac{dr}{\left\langle v_r \right\rangle},
\]

where \( \left\langle v_r \right\rangle \) is the spherically averaged radial velocity. In multidimensional models, the shock radius \( R_s \) and the gain radius \( R_{\text{gain}} \) are not uniquely defined bounds.7 We appeal to the radial profiles shown previously and define the shock radius as the outermost zero in the radial velocity gradient and the gain radius as the first zero crossing in the net heating rate interior to the shock (always around \( \sim 100 \) km). Furthermore, to minimize the large fluctuations in \( \tau_{\text{adv}} \) that appear in 2D due to transient and localized fluctuations in \( v_r \), we time-average the velocity profiles over a \( \pm 15 \) ms window. We define the heating timescale as

\[
\tau_{\text{heat}} = \frac{\int_{R_{\text{esc}}}^{R_{\text{gain}}} \rho v_{\text{esc}}^2 (\langle \rho \rangle - \langle \rho \rangle_0) (\langle Y_e \rangle_0) 4\pi r^2 dr}{\int_{R_{\text{esc}}}^{R_{\text{gain}}} \rho (\langle H - C \rangle) 4\pi r^2 dr},
\]

where angle brackets indicate solid-angle averaging, \( \varepsilon \) is the specific internal energy, \( \varepsilon_0 \) is the zero-point energy of the EOS, and \( H - C \) is the net heating rate per unit mass. The results are shown in Figure 18. After an initial transient phase, the models settle on an approximately (model-dependent) constant value. Rather than there being a particular critical value for the ratio of advection to heating timescales, explosions seem to be robustly connected to rapid growth from these model-dependent quasi-steady values. Nonetheless, we define an explosion time in a systematic manner by measuring the time at which this ratio exceeds 0.5 without later dropping below this value.

An alternative explosion condition (the “antesonic” condition) was suggested by Pejcha & Thompson (2012). Based on parameterized 1D steady-state models, they suggested that explosions occur when \( \max(c_{\text{esc}}^2/v_{\text{esc}}^2) \) reaches a critical value \( \approx 0.2 \). Murphy & Meakin (2011) tested this condition with their parameterized 2D simulations and found it to be consistent with their results. Müller et al. (2012) on the other hand, using 2D radiation hydrodynamic simulations, assessed the validity of the antesonic condition and suggest that it may not be a robust indicator of explosion, but, in any case, the critical value should at least be somewhat larger (\( \sim 0.35 \)). Like Müller et al. (2012), we find that a larger value for the critical condition is required and we adopt 0.3 as the critical ratio. Unfortunately, \( \max(c_{\text{esc}}^2/v_{\text{esc}}^2) \) is a noisy quantity, with brief spikes as large as \( \approx 0.5 \) that then return well below the critical value. The smoothed curves, however, seem to reliably indicate when explosions set in, but these

6 Hanke et al. (2012) report scalings of \( l^{-3} \) and \( l^{-5/3} \) in 2D and 3D, respectively, but Figure 17 shows that their results are consistent with the shallower slopes reported here.

7 Indeed, the gain region need not even be bounded from below by a single closed surface.
smoothed curves are necessarily produced ex post facto. In other words, the antesonic condition is not a reliable indicator of explosion given the instantaneous value of the ratio. The evolutions for the 2D and 3D models, smoothed for clarity, are shown in Figure 19. As above, we identify the time of explosion as the time when the smoothed version of \(
abla c_r^2/v_{esc}^2\) exceeds 0.3 without later dropping below this value.

A final, phenomenological, explosion condition is simply when the average shock radius exceeds 400 km without later receding below this value, as used in Nordhaus et al. (2010). In this case, explosion times can be read directly from the shock radius evolution curves in Figure 1.

Table 1 shows the explosion times and corresponding mass accretion rates as determined by the three conditions above for the \(L_{\nu_e} = 2.2 \times 10^{52}\) erg s\(^{-1}\) and \(L_{\nu_e} = 2.3 \times 10^{52}\) erg s\(^{-1}\) models. All three conditions give comparable numbers, but the antesonic condition tends to give the earliest indication of explosion. We note, however, that this conclusion may be somewhat sensitive to the particular critical values adopted.

That the three conditions give comparable explosion times is not surprising; the ratio of advection to heating times, the ratio of sound to escape speed, and the average shock radius all grow as models transition into explosion. None of these conditions is able to robustly predict when or even if an explosion will occur before the explosion begins. This suggests that while these conditions may be indicative of explosion, they are by no means the full story.

With the results shown in Table 1 in hand, we can quantify the delay between the 2D explosions and the earlier 3D explosions. Taking the minimum difference found between the explosion conditions, we find that the \(L_{\nu_e} = 2.3 \times 10^{52}\) erg s\(^{-1}\) 3D model explodes at least 87 ms earlier than the corresponding 2D model, while the \(L_{\nu_e} = 2.2 \times 10^{52}\) erg s\(^{-1}\) 3D model explodes at least 259 ms earlier than the corresponding 2D model. These delays may translate into appreciable differences in the explosion energy at infinity (Yamamoto et al. 2012), with earlier explosions (3D) being more energetic.

6.2. Explosion Trigger in 3D

What conditions trigger explosions in 3D? The multidimensional nature of the hydrodynamics is generally agreed to be important in producing such conditions. Turbulence in the post-shock flow leads to a distribution of dwell times for accreting parcels of matter, increasing the mass in the gain region and, therefore, the efficiency of neutrino heating. In this way, turbulence plays a crucial, but secondary, role as an aid to neutrino heating. Murphy & Meakin (2011) argue that convection modifies the quasi-steady global structure of the flow by introducing turbulent fluxes of, for example, enthalpy and entropy. Their model focuses on how turbulent convection modifies averaged radial profiles, rather than on the effects of particular turbulent fluctuations, and was able to account for the differences in, for example, the radial profiles of entropy between 1D and 2D models. While these are important roles for the post-shock turbulent motions, here we suggest that the turbulent fluctuations themselves, driven by neutrino heating, may be instrumental in triggering explosions.

The dominant hydrodynamic instability, aside from the explosion itself, in our parameterized 3D models is neutrino-heating-driven convection (Burrows et al. 2012; Figure 18. Ratio of advection to heating timescales (defined in the text) for the 3D (solid) and 2D (dashed) models. Explosions are associated with a rapid growth in this ratio, though it is difficult to identify a particular critical value for the onset of explosions. With a critical value of 0.5, the \(L_{\nu_e} = 2.3 \times 10^{52}\) erg s\(^{-1}\) model explodes 159 ms earlier in 3D relative to 2D, while the \(L_{\nu_e} = 2.2 \times 10^{52}\) erg s\(^{-1}\) model explodes 333 ms earlier in 3D.

(A color version of this figure is available in the online journal.)

Figure 19. Maximum of the ratio of the square of the local sound speed to escape speed for the 3D (solid) and 2D (dashed) models, smoothed for clarity, as discussed by Pejcha & Thompson (2012) in their antesonic condition. In agreement with Müller et al. (2012), we find that the critical value of 0.2 suggested by Pejcha & Thompson (2012) is too low, but using a larger value (≈0.3) may be a useful diagnostic of explosion. With a critical value of 0.3, the \(L_{\nu_e} = 2.3 \times 10^{52}\) erg s\(^{-1}\) model explodes 87 ms earlier in 3D relative to 2D, while the \(L_{\nu_e} = 2.2 \times 10^{52}\) erg s\(^{-1}\) model explodes 309 ms earlier in 3D.

(A color version of this figure is available in the online journal.)
Figure 20. Sequence of volume renderings of the specific entropy for the 3D $L_\nu = 2.2 \times 10^{52}$ erg s$^{-1}$ model. As seen at the bottom left of each image, a high entropy plume forms around $\sim250$–$300$ ms and persists for hundreds of milliseconds, eventually pushing the shock out far enough to seemingly trigger the global explosion. A similar structure appears around $\sim450$ ms at the top right of each image, which leads to similar local shock expansion thereafter. (A color version of this figure is available in the online journal.)

Murphy et al. (2012). In the nonlinear phase, the post-shock turbulent flow involves a complex interaction between buoyantly rising, neutrino-heating-driven plumes, negatively buoyant accretion streams, turbulent entrainment resulting from secondary Kelvin–Helmholtz instabilities, and dissipative interactions between rising plumes and the bounding shock. The evolution of a buoyant “bubble” depends on parameters including the heating rate, bubble size, and background inflow velocity. Small bubbles tend to be shredded by turbulent entrainment, while large bubbles can rise all the way to the shock and locally push it outward. Our simulations suggest that some bubbles can continue rising, locally pushing the shock to ever larger radii until the global explosion is triggered. Figure 20 shows a sequence of volume renderings illustrating that large buoyant features form in the flow and push the shock to larger radii. Figure 21 shows the evolution of the shock surface where the same local growth of the shock radius can be clearly seen. Interestingly, the growth of these features occurs not on a dynamical time, but appears to proceed quasi-statically. The same basic picture holds in all of our 3D simulations, suggesting that this may be a generic feature of the transition to explosion.

We can try to understand the conditions for the runaway growth of bubbles (and the associated triggered explosions) by considering the highly simplified toy model presented in the Appendix. In this model, the evolution of a bubble’s outer radius ($R_b$) is determined by the competition between neutrino driving power ($L_\nu \tau$, $L_\nu$ is the neutrino driving luminosity and $\tau$ is the effective optical depth of the bubble) and accretion power ($\alpha G M M / R_b$, $\alpha$ is a constant defined in the Appendix) associated with the ram pressure of material immediately behind the shock. The evolution follows

$$\frac{dR_b}{dt} = \frac{\Omega_0}{4\pi} \frac{R_b^2}{G M M_b} \left( L_\nu \tau - \alpha \frac{G M M}{R_b} \right),$$

which has the solution

$$R_b(t) = \frac{\alpha G M M}{L_\nu \tau + e^{\alpha G M M / R_b - L_\nu \tau}},$$

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where

\[ \lambda = \alpha \frac{\Omega_0}{4\pi} \frac{\dot{M}}{M_b} \]  

Here, \( \Omega_0 \) is the bubble’s constant solid angle and \( M_b \) is the bubble’s fixed mass. Qualitatively, \( R_b(t) \) has two types of behavior. When the neutrino driving term exceeds the ram pressure term, \( R_b(t) \) grows without bound, presumably triggering the global explosion (Thompson 2000). When the ram pressure term dominates, the bubble recedes, perhaps allowing another bubble to take its place.

This simple model makes two predictions. First, the model predicts a characteristic growth of the maximum shock radius, at least in the quasi-static growth phase. Figure 22 shows the maximum shock radii (\( R_{\max} \)) from the \( L_{\nu_e} = 2.2 \times 10^{52} \text{ erg s}^{-1} \) and \( L_{\nu_e} = 2.3 \times 10^{52} \text{ erg s}^{-1} \) 3D models along with fitted model solutions. To compute the fits, we fix \( \dot{M} = 1.6 M_\odot \), \( \dot{M} = 0.25 M_\odot \text{ s}^{-1} \), and \( M_b/\Omega_0 = 2.5 \times 10^{30} \text{ g} \) (chosen so that \( 4\pi M_b/\Omega_0 \lesssim M_{\text{gain}} \)) and fit the model solution to the data between 250 ms post-bounce and the time when \( R_{\max} = 1000 \text{ km} \), leaving \( R_0 \), \( \tau \), and \( \alpha \) as free parameters. Both fits give \( \tau \approx 0.04 \) (about 20% smaller than the average \( \tau \) for the gain region) and \( \alpha \approx 0.25 \). We note that \( \alpha \approx 0.25 \) gives an “effective drag coefficient” \( C_d \approx 12.5 \), which seems quite large, but it is nontrivial to estimate reliably what value \( C_d \) should take in this context, especially considering the crude dimensional analysis on which the equations are based. Given the simplicity of the model, the agreement between the hydrodynamical simulation results and the model predictions is quite surprising and encouraging.

Second, there is a critical luminosity for a given mass accretion rate and shock radius above which a bubble will runaway. If the stalled shock radius scales as \( L^\beta \), then the critical luminosity for runaway bubble growth is proportional to \( \dot{M}^{1/(1+\beta)} \).
Empirically, $\beta \sim 3$, and if we adopt the parameters used above we find

$$L_{\text{crit}} \approx 2.2 \left( \frac{M}{1.6 M_\odot} \right)^{1/4} \left( \frac{M}{0.25 M_\odot \text{s}^{-1}} \right)^{1/4} \times \left( \frac{\tau}{0.04} \right)^{-1/4} \times 10^{52} \text{erg s}^{-1},$$

which seems consistent with the critical explosion curves from parameterized multidimensional models shown in other works (Murphy & Burrows 2008; Nordhaus et al. 2010; Hanke et al. 2012; Couch 2012). Inasmuch as runway bubble growth is associated with explosions, this may be viewed as an alternative, albeit crude, derivation of the critical luminosity curve of Burrows & Goshy (1993). This may suggest that the reduction in the critical luminosity in going from 1D to 2D and 3D models might arise, in part, from the emergence of bubbles and their runaway growth. The crudeness of the model precludes us from distinguishing between 2D and 3D, but we suggest that important differences may include the different geometries of the bubbles (rings in 2D versus quasi-spherical in 3D) and the efficiency of turbulent entrainment.

7. DISCUSSION AND CONCLUSIONS

We have presented analyses of parameterized 2D axisymmetric and 3D CCSN models. Our basic conclusion is not surprising or controversial—the hydrodynamics of CCSNe are different between 2D axisymmetric and 3D models. These differences are many and, while some are subtle and perhaps not crucial to understanding the mechanism, some are quite dramatic and make interpreting 2D supernova models problematic.

Our parameterized models indicate that the global structures of the flows are different between 2D and 3D. We see this reflected in integral measures like the mass and average entropy in the gain region and in the radial profiles of various quantities. In spite of identical heating and cooling prescriptions in 2D and 3D, our analyses show how the different global structures effect the heating and cooling rates. We find that the 2D models have significantly higher integrated net heating rates than their corresponding 3D models and attribute this to the smaller gain radius (and more mass in the gain region) in 2D, which may in turn be related to the larger radial velocity magnitudes in 2D. On the other hand, the 3D models tend to have higher densities at large radii and larger average shock radii.

The dwell time distributions for accreted parcels of matter offer another vantage point from which to distinguish 2D and 3D models. In both cases, the post-shock turbulence leads to a broad distribution of dwell times. In our simulations, the mean integrated dwell time is about 5% longer in 2D than in 3D, consistent with the larger mass in the gain region in 2D than in 3D, but the 3D distribution has a relatively prominent tail toward long dwell times. About 25% of the accreted material spends more time in the gain region in 3D than in 2D, being exposed to more integrated heating and reaching higher peak entropies. We also introduce the instantaneous dwell time distribution, which reflects, at any instant, the distribution of current dwell times for all parcels of matter in the gain region. We argue that this distribution bears more directly on a model’s propensity to explode. After turbulence is established, the mean instantaneous dwell time is first up to 10% longer in 2D, but soon after becomes longer in 3D, asymptoting to about 30% longer in the few hundred milliseconds following the development of turbulence. We suggest that this longer mean instantaneous dwell time, a direct result of the relatively shallow tail toward long integrated dwell times, plays a prominent role in yielding earlier explosions in 3D than in 2D.

Ultimately, many of the differences we see are plausibly associated with the character of the post-shock turbulent flow. In 2D, turbulent energy is pumped into the largest scales of the flow, which inevitably gives rise to the sloshing behavior manifest in all modern 2D CCSN simulations. This sloshing behavior has been identified as a crucial ingredient in nearly all of the successful neutrino-driven explosion models to date (Buras et al. 2006; Marek & Janka 2009; Müller et al. 2012). In 3D, this sloshing motion is muted or absent, at least in part, because the turbulent energy transport is predominantly toward small scales. Since we see explosions earlier in 3D than in 2D, vigorous sloshing is either not critical in any dimension or the explosion mechanism operates differently in 2D and 3D. On the other hand, the dwell time distributions must be intimately connected to the character of the turbulence. It may be that the relative prominence of small scale structures in 3D leads to the shallower tail in the integrated dwell time distributions, and consequently to the longer mean instantaneous dwell time for all fluid elements in the gain region in 3D compared to 2D.

Finally, we present a toy model that describes the evolution of buoyant bubbles driven by the competition between neutrino heating power and accretion power. The simple model is able to account for the quasi-static growth of the maximum shock radius in the exploding 3D models with neutrino luminosities of $L_{\nu e} = 2.2 \times 10^{52} \text{erg s}^{-1}$ and $L_{\nu e} = 2.3 \times 10^{52} \text{erg s}^{-1}$. It also predicts the existence of a critical luminosity for a given mass accretion rate and shock radius beyond which a bubble will explode. We speculate that this runaway growth triggers the global explosion of the star, and therefore that the critical luminosities for runaway bubble growth and explosion are effectively the same.

The mean background and fluctuating turbulent components of the post-shock flow in 2D axisymmetric models are different from 3D models and, more importantly, likely not representative of the hydrodynamics of supernova cores in nature. While the models presented in this work are incomplete, lacking adequate
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APPENDIX

TOY MODEL OF A NEUTRINO-DRIVEN BUBBLE

Consider a bubble of fixed solid angle \(\Omega_0\) pinned beneath the shock at radius \(R_b\). The energy injection rate due to neutrino heating is \(\sim(\Omega_0/4\pi)L_\nu \tau\). For a quasi-steady system, this energy injection rate should be balanced by the rate of energy loss. The interaction between the bubble and the surrounding flow will remove energy from the bubble at a rate \(\sim C_d \rho v^2 \Omega_0 R_b^2\), where \(C_d\) is an effective drag coefficient that encapsulates information about the geometry and character of the flow, including effects like turbulent entrainment. For a bubble pinned beneath the shock, the density and velocity are given by the shock jump conditions, so this loss term can be rewritten as \(\sim C_d(\Omega_0/4\pi) f^2/R^2 (2GM/M_b)\), where \(f\) is the pre-shock fraction of the free-fall velocity \((\approx 1/\sqrt{2})\) and \(R\) is the shock compression ratio. Also, lifting the bubble out of the gravitational potential requires a power \(\sim dR_b/dt G M M_b/R_b^2\).

Equating the driving neutrino power to the sum of the ram and gravitational powers, we find

\[
\frac{dR_b}{dt} = \frac{\Omega_0}{4\pi} \frac{R_b^2}{GM_b} \left( L_\nu \tau - \alpha \frac{GM}{R_b} \right),
\]

where \(\alpha = 2C_d f^2/R^2\) is a dimensionless constant of the order of unity.

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