Broken Symmetry and Nonequilibrium Superfluid $^3$He

Circular Birefringence of Propagating Transverse Currents

J. A. Sauls
Northwestern University, Evanston, Illinois 60208, USA

Abstract. The superfluid phases of $^3$He provide a unique physical system to study the dynamical effects of spontaneous symmetry breaking in condensed matter. The theory of superfluid $^3$He is grounded in two of the most successful theories of strongly interacting matter, Landau’s Fermi-liquid theory and the BCS pairing theory of superconductivity. These two theories were placed into a common theoretical framework in the late 60’s. I discuss applications of this theory to nonequilibrium dynamics of superfluid $^3$He. In 1957 Landau predicted that liquid $^3$He would support propagating shear waves at low temperatures, i.e. a transverse sound mode. Such waves have recently been observed at low temperatures in the superfluid B-phase of liquid $^3$He. These observations provide a beautiful example of the effect of spontaneous symmetry breaking in condensed matter. I discuss the theory of transverse wave propagation in $^3$He and the recent detection of these waves by magneto-acoustic rotation of the polarization in a magnetic field.

1. Introduction

The physics of superfluid $^3$He is sufficiently rich that it can provide interesting analogues of theoretical models in astrophysics, high-energy physics and cosmology (Volovik, 1999), and more recently in the field of ultra-low-temperature atomic gases (Ho and Shenoy, 1996). But the significance of liquid $^3$He to theoretical physics may well be that it provides the model system for developing and extending one of the most successful theories of strongly interacting matter - the Fermi-liquid theory of superconductivity.¹ These lecture notes provide an introduction to non-equilibrium superfluid $^3$He, for a detailed description see (Serene and Rainer, 1983).

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$^3\text{He}$, with applications to high-frequency excitations, including aspects of symmetry breaking in $^3\text{He}$ and its effects on collective mode dynamics.

The density and current modes of liquid $^3\text{He}$ are governed by conservation laws for mass (or particle number) and momentum,

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{J} = 0, \quad \frac{\partial J_i}{\partial t} + \frac{1}{m} \frac{\partial \Pi_{ij}}{\partial x_j} = 0,$$

(1)

where $n(\mathbf{R}, t)$ is the particle density, $\mathbf{J}(\mathbf{R}, t)$ is the particle current density, $m$ is the atomic mass of $^3\text{He}$ and $\Pi_{ij}(\mathbf{R}, t)$ is the momentum stress tensor of the liquid. The conservation laws are supplemented by equations relating the density, current and stress tensor. In the hydrodynamic limit these constitutive equations are obtained from local thermodynamics. The assumption of hydrodynamics is that the characteristic timescale for a disturbance from equilibrium is long compared with the timescale for the restoration of equilibrium locally on the scale of a typical wavelength of the disturbance. The equations of hydrodynamics provide an accurate description of the long wavelength, low frequency ($\omega \ll 1/\tau$) dynamics of $^3\text{He}$. However, this description breaks down at sufficiently high frequencies or low temperatures. The regime of applicability of hydrodynamics to liquid $^3\text{He}$ at low temperatures is severely restricted by Fermi statistics. The time-scale for the restoration of local equilibrium increases rapidly below $100 \text{mK}$, $\tau(T) \simeq 1 \mu\text{sec}\cdot\text{mK}^2/T^2$. Thus, for excitation frequencies of order $\omega/2\pi \simeq 20 \text{MHz}$ the hydrodynamic regime is restricted to temperatures above $T \simeq 15 - 20 \text{mK}$. The physics of $^3\text{He}$ at lower temperatures, or higher frequencies, departs radically from the predictions of hydrodynamics.

Liquid $^3\text{He}$ at these excitation energies is governed by coupled dynamical equations for quasiparticle (distribution function) and Cooper pair (pair correlation amplitude) excitations.

**Transport Theory**

In his seminal papers on Fermi liquid theory (Landau, 1956; Landau, 1957) Landau explained how a system of strongly interacting Fermions could exhibit both a spectrum of low-lying Fermionic excitations with a well-defined Fermi surface, as well as low lying Bosonic modes associated with deformations of the Fermi surface. The stability of the Fermi surface requires that interactions between the Fermionic excitations ("quasiparticles") act to restore the Fermi surface to its equilibrium configuration. Under these conditions the "Fermi sea" behaves as an elastic medium in which restoring forces lead to natural oscillations, or free vibrations of the Fermi surface, called "zero sound modes". The dynamical variables describing these modes are related to the deformation of the Fermi surface, which
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is defined in terms of the deviation of the quasiparticle distribution function from its equilibrium form,

$$
\phi(p, R; t) = \int d\varepsilon \Phi(p, R; \varepsilon, t), \tag{2}
$$

where $\Phi(p, R; \varepsilon, t)$ is the distribution function for quasiparticles with momentum near the Fermi surface, $p = p_f \hat{p}$, and excitation energies, $\varepsilon$, that are small compared with the Fermi energy, $|\varepsilon| \ll E_f$. BCS pairing in $^3\text{He}$ is an instability of the Fermi surface; thus, one should expect that a description of collisionless modes in the superfluid phases requires new dynamical variables describing the condensed phase, which are coupled to the dynamics of the distribution function.

The transport equation for the distribution function is the central equation of Landau’s theory of normal $^3\text{He}$,

$$
\left( \frac{\partial}{\partial t} + v_f \cdot \nabla R \right) \Phi(p, R; \varepsilon, t) - \left( \frac{\partial \Phi_0}{\partial \varepsilon} \right) \frac{\partial}{\partial t} U_{\text{tot}}(p, R; t) = I[\Phi]. \tag{3}
$$

The first two terms describe the ballistic propagation of quasiparticles with a group velocity given by the Fermi velocity, $v_f = v_f \hat{p}$. The third term represents the action of external and internal forces acting on the quasiparticles; $U_{\text{tot}}(p, R; t) = u_{\text{ext}}(R; t) + E(p, R; t)$ is the sum of the external potential energy ($u_{\text{ext}}$) and the interaction energy ($E$) of a quasiparticle with momentum $p$ on the Fermi surface with the distribution of other nonequilibrium quasiparticles. The derivative of the equilibrium distribution, $\partial \Phi_0(\varepsilon)/\partial \varepsilon$, restricts the dynamics to low-energy excitations near the Fermi level. Both external and internal forces accelerate the quasiparticles leading to smooth changes in the distribution function in space and time. The right side of the transport equation determines the rate of change of the distribution function from quasiparticle collisions. This term leads to irreversibility and relaxation of the distribution function on the timescale of the mean time between collisions, i.e. $I[\Phi] \sim -1/\tau \delta \Phi$. The collision terms are generally small for the typical frequencies of interest, i.e. $\omega \gg 1/\tau$. Thus, we can often neglect collisional effects, except for collisional broadening of otherwise sharp collective modes.

The distribution function determines the density and momentum fluctuations associated with nonequilibrium states of $^3\text{He}$,

$$
n(R, t) = \frac{p_f^3}{3\pi^2 \hbar^3} + \frac{N_f}{1 + F_0^2} \int d\Omega \int d\varepsilon \Phi(p, R; \varepsilon, t), \tag{4}
$$

$^2$In the microscopic formulation of Fermi liquid theory the distribution function is defined in terms of a correlation function in the limit, $\hbar q \ll p_f$, $\hbar \omega \ll E_f$ (Serene and Rainer, 1983).
\[
\mathbf{J} = N_f \int \frac{d\Omega}{4\pi} \mathbf{v}_f \int d\varepsilon \Phi(\hat{\mathbf{p}}, \mathbf{R}; \varepsilon, t),
\]

(5)

where \(N_f\) is the density of states at the Fermi level and \(F_0^s\) is the \(\ell = 0\) Landau interaction parameter (defined below). The conservation laws for mass and momentum follow from the transport equation and conservation of fermion number and momentum in collision processes: \(\int d\Omega_p \int d\varepsilon I[\Phi] = 0\) and \(\int d\Omega_p \mathbf{p} \int d\varepsilon I[\Phi] = 0\). The continuity equation for \(\mathbf{J}\) determines the stress tensor,

\[
\Pi_{ij} = N_f \int \frac{d\Omega}{4\pi} [\mathbf{v}_f]_i [\mathbf{p}_f]_j \int d\varepsilon \Phi(\hat{\mathbf{p}}, \mathbf{R}; \varepsilon, t).
\]

(6)

The result for \(\Pi_{ij}\) follows from the microscopic theory of a Fermi liquid and is valid above and below the superfluid transition, as well as in the nonlinear response regime, so long as \(u_{\text{ext}} \ll E_f\) (McKenzie and Sauls, 1990).

Normal \(^3\)He is isotropic so the eigenmodes of the Fermi surface are given by the amplitudes, \(\phi_{\ell,m}\), of the spherical harmonic expansion of the distribution function. For example, \(\phi_{0,0}\) is the amplitude for an isotropic expansion or contraction of the Fermi surface, and determines the density fluctuation,

\[
\delta n(\mathbf{R}, t) = \frac{N_f}{1 + F_0^s} \phi_{0,0},
\]

(7)

while the \(\ell = 1\) modes determine the current density,

\[
\mathbf{J}(\mathbf{R}, t) = \frac{1}{3} N_f v_f \sum_{m=0,\pm 1} \phi_{1,m} \mathbf{e}^{(m)},
\]

(8)

where \(\mathbf{e}^{(m)}\) are unit vectors defining the three linearly independent current modes: the longitudinal current, \(\mathbf{J}_l \sim \mathbf{e}^{(0)} = \hat{\mathbf{q}}\), and the two circularly polarized transverse modes, \(\mathbf{J}_\pm \sim \mathbf{e}^{(\pm)}\). The circularly polarized basis vectors are defined by, \(\mathbf{e}^{(0)} = \mathbf{e}_3\) and \(\mathbf{e}^{(\pm)} = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2}\), where \(\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}\) is a Cartesian triad with \(\mathbf{e}_3 = \hat{\mathbf{q}}\). The \(\ell = 2\) amplitudes are related to the five traceless and symmetric components of the stress tensor,

\[
\pi_{ij} = \Pi_{ij} - \frac{1}{3} \text{Tr}[\Pi] \delta_{ij} = \frac{2}{15} N_f p_f v_f \sum_{m=0,\pm 1,\pm 2} \phi_{2,m} \mathbf{t}^{(2,m)}_{ij},
\]

(9)

where \(\mathbf{t}^{(2,m)}_{ij}\) are the \(\ell = 2\) spherical tensors with the quantization axis chosen as \(\mathbf{e}^{(0)} = \hat{\mathbf{q}}\) (see Eqs. (18) below). The stress tensor plays a central role in the nonequilibrium response of liquid \(^3\)He. The solution of the transport equation leads to constitutive equations relating the stress tensor, the density and current response, and in the superfluid phase to additional variables representing the dynamics of the condensate.
In the collisionless limit restoring forces for spin-independent deformations of the Fermi surface originate from

\[ \mathcal{E}(\hat{p}, R; t) = \int \frac{d\Omega p'}{4\pi} A^s(\hat{p}, \hat{p}') \phi(\hat{p}', R; t). \]  

(10)

This term represents the interaction of a quasiparticle of momentum \( p_f \hat{p} \) with the distribution of quasiparticles that represent the deformation of the Fermi surface. The function \( A^s(\hat{p} \cdot \hat{p}') \) is the spin-independent amplitude for forward scattering of two quasiparticles with momenta \( p \) and \( p' \) on the Fermi surface, and is related to the spin-independent interaction energy between two quasiparticles, \( F^s(\hat{p}, \hat{p}') \), by (Landau, 1957)

\[ A^s(\hat{p} \cdot \hat{p}') = F^s(\hat{p} \cdot \hat{p}') + \int \frac{d\Omega p''}{4\pi} F^s(\hat{p} \cdot \hat{p}'') A^s(\hat{p}'' \cdot \hat{p}'). \]  

(11)

The underlying force between \(^3\)He atoms is rotationally invariant. Thus, \( A^s \) is a function only of the relative orientation of \( p \) and \( p' \), which allows us to parametrize \( F^s \) (or \( A^s \)) by a set of interactions for quasiparticles in relative angular momentum channels, i.e. \( F^s = \sum \ell F^s_{\ell} P_{\ell}(\hat{p} \cdot \hat{p}') \), where the \( F^s_{\ell} \) are the dimensionless Landau parameters. These interaction parameters determine many of the physical properties of the Fermi liquid, e.g. the change in the interaction energy induced by an isotropic deformation of the Fermi surface, a dilatation, is determined by the \( \ell = 0 \) interaction, \( F^s_0 \). Similarly, the interaction parameter in the \( \ell = 1 \) channel represents the current-current interaction between quasiparticles. Galilean invariance of the interactions in liquid \(^3\)He provides a relation connecting the effective mass of a quasiparticle to the \( \ell = 1 \) interaction, \( m^s/m = (1/F^s_1 + F^s_0/3) \). Since the effective mass can be obtained from the low-temperature heat capacity we obtain from fundamental measurements the quasiparticle interaction in the current-current channel. Both interactions determine the compressibility and hydrodynamic sound velocity of \(^3\)He. Strong repulsive interactions in liquid \(^3\)He lead to a sound velocity that is significantly larger than the velocity of quasiparticle excitations, \( c_1/v_f = \sqrt{\frac{1}{3}(1 + F^s_0)(1 + F^s_1)} \). Measurements of the heat capacity and sound velocity yield values of \( F^s_0 \simeq 10 \) (\( F^s_1 \simeq 6 \)) at \( p = 1 \) bar to \( F^s_0 \simeq 100 \) (\( F^s_1 \simeq 15 \)) at \( p = 34 \) atm, corresponding to interaction energies \( \sim 10-100 \) times the Fermi energy. \(^3\)Thus, although quasiparticles are long-lived single-particle excitations at low energies and low temperatures, liquid \(^3\)He can never be described as a “gas” of weakly interacting quasiparticles.

\(^3\)The spin-dependent interaction energy originates from exchange interactions which are described by the spin-dependent Landau parameters, \( \{F^a_{\ell}\} \). These interactions are important for \(^3\)He in a magnetic field; they produce exchange enhancement of an applied magnetic field.
An important consequence of strong interactions is that zero sound modes exist at low temperatures in the collisionless regime, \( \omega \tau(T) \gg 1 \). The restoring forces are provided by strong repulsive quasiparticle interactions. The first observation of longitudinal zero sound was reported in Ref. (Abel et al., 1966), and has been studied extensively (Halperin and Varoquaux, 1990). The signatures of the cross-over from hydrodynamic to collisionless sound are a small change in sound velocity in the collisionless regime, \( (c_0 - c_1)/c_1 \approx \frac{2}{3}(1 + \frac{1}{2}F_s^2)/(1 + F_s^2) \sim 10^{-2} \), and a dramatic change in the temperature dependence of the attenuation, \( \alpha_1 \sim \omega^2/T^2 \) for \( \omega \tau < 1 \) compared with \( \alpha_0 \sim T^2 \) for \( \omega \tau > 1 \) (Abel et al., 1966).

In addition to longitudinal zero sound, Landau predicted a transverse mode of the Fermi surface in the collisionless regime (Landau, 1957). This mode should be observable as a propagating shear wave, or transverse zero sound (TZS). The velocity of TZS in normal \(^3\text{He}\) is expected to be close to the Fermi velocity, \( c_t \approx v_f \) (Fomin, 1968), and the collisional damping is predicted to be serveral orders of magnitude larger than the damping of the longitudinal zero sound mode (Corruccini et al., 1969). In contrast to the detection of longitudinal zero sound, the search for TZS has been more difficult. Early experimental efforts to observe this mode were inconclusive (Roach and Ketterson, 1976; Flowers et al., 1976). The recent observation of a propagating transverse current mode in superfluid \(^3\text{He-B}\) (Lee et al., 1999) provides new insight into the dynamics of normal and superfluid \(^3\text{He}\), as well as new information on many-body correlation effects. Before discussing the theory of TZS and order parameter dynamics, I review some basic facts about superfluid \(^3\text{He}\).

2. Symmetry Breaking and the Order Parameter

The Hamiltonian that determines the properties of \(^3\text{He}\) has a high degree of symmetry. The ground state electronic configuration of the \(^3\text{He}\) atom is a filled 1s shell, so the atom is isotropic and the interaction energy between two \(^3\text{He}\) atoms is dominated by a central potential, \( v(r) \), which is also isotropic and spin-independent. This interaction energy is of order \( v(\bar{r}) \approx -10 \text{ K} \) at the minimum of the \(^3\text{He-}\(^3\text{He}\) potential, and has a hard repulsive core at \( \bar{r} \approx 2.5 \text{ Å} \). By comparison, the nuclear dipolar energy at these densities is of order \( v_d = (\gamma \hbar)^2/\bar{r}^3 \approx 10^{-7} \text{ K} \). The dipolar energy is tiny even in comparison to the energy scale of the superfluid transition, \( T_c \approx 10^{-3} \text{ K} \). Thus, to an excellent approximation the equilibrium phase of \(^3\text{He}\) above the superfluid transition is separately invariant under rotations in orbital space and in spin space.

Above \( T_c \) \(^3\text{He}\) also possesses discrete symmetries under space-inversion (\( \mathcal{P} \)), time-inversion (\( \mathcal{T} \)), and an approximate symmetry under particle to
hole conversion ($C$), i.e. the transformation of a quasiparticle with energy $\varepsilon = \xi_p (> 0)$ and spin projection $\uparrow$ into a quasihole with the same excitation energy ($|\xi_p| = \xi_p$) and spin projection $\downarrow$. The transformation is represented by a unitary operator, $C^\dagger a_{\alpha}^\dagger p_{\beta} C = [i\sigma_y]_{\alpha\beta} a_{\beta}^\dagger p_{\alpha}$, where $a_{\alpha}^\dagger p_{\beta}$ creates a quasiparticle (quasihole) with momentum $p$ and spin projection $\alpha$. Under the particle-hole transformation the low-energy effective Hamiltonian for quasiparticles is approximately invariant. Particle-hole symmetry in a Fermi liquid is analogous to charge conjugation. The order of magnitude of the particle-hole asymmetry is given by

$$\zeta = \frac{N'(E_f)\varepsilon}{N(E_f)} \sim \frac{\varepsilon}{E_f}.$$  

Particle-hole symmetry becomes exact for $\varepsilon \to 0$; at $T \approx T_c$, particle-hole asymmetry is $\zeta \sim 10^{-2}$. The normal state of liquid $^3$He is also invariant under global gauge transformations, $\psi(r) \to \psi'(r) = \psi(r) e^{i\chi}$. When combined with the discrete symmetries under $P$, $T$ and $C$, the symmetry group of liquid $^3$He below $T \sim 100$ mK and above $T_c \sim 1$ mK is

$$G = SO(3)_S \times SO(3)_L \times U(1)_\chi \times P \times T \times C.$$  

(12)

This high degree of symmetry of liquid $^3$He is spontaneously broken at the superfluid transition. The residual symmetry of the superfluid phase(s) is reflected in the invariant subgroup of $G$ defined by the order parameter; the latter is a particular realization of an orbital $p$-wave, spin-triplet amplitude for Cooper pairs. The order parameter is defined in terms of a superposition of quasiparticle pair states with zero total momentum in the low-energy shell ($|\xi_p| \leq \varepsilon_c \ll E_f$) about the Fermi surface,

$$\Delta_{\alpha\beta}(\hat{p}, \hat{R}; t) = \int \frac{d\Omega_{\hat{p}'}}{4\pi} V_{\chi}(\hat{p} \cdot \hat{p}') \int_{-\varepsilon_c}^{\varepsilon_c} \frac{d\varepsilon}{2\pi i} f_{\alpha\beta}(\hat{p}', \hat{R}; \varepsilon, t),$$  

(13)

where $f_{\alpha\beta} \sim \langle a_{p\alpha} a_{-\hat{p}\beta}^\dagger \rangle$ is the amplitude for Cooper pairs in the spin state $|\alpha\beta\rangle$, and $V_{\chi} = 3V_1 \hat{p} \cdot \hat{p}'$ is the pairing interaction in the spin-triplet, $p$-wave channel. The matrix structure in spin-space represents the three spin-triplet amplitudes, each of which is a function of the orbital momentum of the pairs. The order parameter can be represented as (Balian and Werthamer, 1963)

$$\Delta_{\alpha\beta}(\hat{p}, \hat{R}; t) = \sum_{\mu} \sum_{i} (i\sigma_\mu \sigma_y)_{\alpha\beta} d_{\mu i}(\hat{R}, t) \hat{p}_i,$$  

(14)

where the $3 \times 3$ complex matrix, $d_{\mu i}$, transforms as a vector (with respect to $\mu$) under rotations in spin space and, independently, as a vector (with respect to $i$) under rotations in orbital space.

$^4$The quasiparticle operators are defined on a restricted Hilbert space of states in the low-energy band $|\xi_p| < \varepsilon_c$ about the Fermi surface, and are constructed to generate the low-energy quasiparticle Green’s function (Nozières, 1964; Serene, 1983).
The equilibrium B-phase of $^3$He is identified (Leggett, 1975) with an order parameter belonging to the Balian-Werthamer (BW) class of p-wave states (Balian and Werthamer, 1963). The simplest BW state, $d_{\mu i} \sim \delta_{\mu i}$, corresponds to pairing with quantum numbers $S = 1$, $L = 1$ and $J = 0$, where $J = L + S$ is the total angular momentum. The $J = 0$ state belongs to a continuous manifold of degenerate ground states; i.e., any order parameter obtained from the $J = 0$ state by a relative rotation of the spin and orbital coordinates and a uniform gauge transformation minimizes the free energy. Thus, a general BW state is,

$$d_{\mu i} = \frac{\Delta}{\sqrt{3}} e^{i\chi} \mathcal{R}_{\mu i} [\hat{n}, \vartheta],$$

where $\mathcal{R}[\hat{n}, \vartheta]$ is an orthogonal matrix representing a relative rotation of the spin and orbital coordinates about the axis $\hat{n}$ by the angle $\vartheta$. Note that the relative spin-orbit symmetry is spontaneously broken below $T_c$. Since parity and gauge symmetry are also broken below $T_c$ the residual symmetry group for $^3$He-B includes the group of combined spin and orbital rotations and time-inversion,

$$G_{\text{resid}} = SO(3)_{L+S} \times \mathcal{T} \times C,$$

with the generator for the rotation group given by $J = L + \mathcal{R}^{-1} \cdot S$.

The dynamics of the order parameter is governed by the time-dependent ‘gap equation’, Eq. (13), where the pairing amplitude and quasiparticle distribution function obey coupled quasiclassical transport equations. The coupling of the quasiparticle and condensate dynamics is rooted in particle-hole coherence, and is responsible for most of the novel physics associated with nonequilibrium superfluidity (Kurkijärvi and Rainer, 1990), including the coupling between the mass current and the collective modes of the order parameter. The rotational symmetry of the ground state of $^3$He-B allows us to classify the order parameter excitations, which are Bosonic modes, in terms of the eigenvalues of $J^2$ and $J_3$ (Maki, 1974). Furthermore, there is a doubling of the spectrum for each $(J, M)$ labeled by $C$ parity, $\zeta = \pm$. Within the spin-triplet, p-wave subspace we can expand the order parameter fluctuations in the eigenfunctions of $(J^2, J_3, C)$,

$$\delta d_{\mu i} = \sum_{J, M, \zeta = \pm} D^{(C)}_{J, M}(\zeta) t^{(J, M)}_{\mu i},$$

5The degeneracy of the homogeneous BW state is partially resolved by weak perturbations, e.g. the nuclear dipolar energy or the magnetic Zeeman energy.
TABLE 1. The order parameter collective mode spectrum for p-wave, spin-triplet pairing fluctuations in $^3$He-B. There are four Goldstone modes associated with the spontaneous breaking of gauge and relative spin-orbit rotational symmetry. All other modes are ‘massive’ and correspond to fluctuations of the order parameter that are not related by a long wavelength rotation or gauge transformation of the ground state order parameter.

| $D^{(J)}_{1,M}$ | Mode          | Frequency | Degeneracy | Coupling to | π |
|-----------------|---------------|-----------|------------|-------------|---|
| $D_{0,0}$       | phase         | $\omega = \frac{1}{\sqrt{2}}v_f q$ | 1          | 1           | (L) |
| $D_{0,0}$       | amplitude     | $\omega = 2\Delta$ | 1          | $\zeta \left( \frac{q v_f}{\Delta} \right)$ | (L) |
| $D_{1,M}$       | amplitude     | $\omega = 2\Delta$ | 3          | -           | -   |
| $D_{1,M}$       | spin-waves    | $\omega = \frac{1}{\sqrt{2}}v_f q$ | 3          | -           | -   |
| $D_{2,0}$       | Im squashing  | $\omega = \sqrt{\frac{3}{2}} \Delta$ | 5          | $\left( \frac{\omega}{\Delta} \right)$ | (L,T) |
| $D_{2,0}$       | Re squashing  | $\omega = \sqrt{\frac{3}{2}} \Delta$ | 5          | $\zeta \left( \frac{q v_f}{\Delta} \right)$ | (L,T) |

where $t^{(J,M)}_{\mu \nu}$ are spherical tensors that transform according to the $J^{th}$ irreducible representation of the rotation group,

\[
\begin{align*}
t^{(0,0)}_{ij} &= \frac{1}{\sqrt{3}} \delta_{ij} \\
t^{(1,M)}_{ij} &= \frac{1}{\sqrt{2}} \varepsilon_{ijk} e_k^{(M)} \quad t^{(2,\pm 1)}_{ij} &= \frac{1}{\sqrt{2}} \left( e_i^{(0)} e_j^{(\pm)} + e_i^{(\pm)} e_j^{(0)} \right) \\
t^{(2,\pm 2)}_{ij} &= e_i^{(\pm)} e_j^{(\pm)}
\end{align*}
\]

The tensors are defined in terms of the circularly polarized basis vectors, $e^{(0)} = e_3$ and $e^{(\pm)} = (e_1 \pm ie_2)/\sqrt{2}$, where $\{e_1, e_2, e_3\}$ is a Cartesian triad of unit vectors defining the coordinate system for the Cooper pairs.

There are 2 (gauge) $\times$ 3 (spin) $\times$ 3 (orbital) = 18 p-wave, spin-triplet Bosonic modes of $^3$He-B, which are listed in Table 2 and labeled by the quantum numbers, $(J^{C}, M)$. The eigenfrequencies are given for the limit $q \to 0$ neglecting interactions. There are four Goldstone modes associated with the spontaneous breaking of gauge and relative spin-orbit rotational symmetry. These modes are observable because they couple to the mass and spin currents. The $J = 0^−$ mode is the phase mode (Anderson, 1958; Bogoliubov et al., 1958), which is essential for understanding the propagation of longitudinal sound in superfluid $^3$He. The Goldstone modes with $J = 1^+$ correspond to long wavelength fluctuations in the axis of rotation, $n$, and the angle of rotation, $\vartheta$, and are related to the NMR and spin dynamics in $^3$He-B (Leggett, 1975). All other modes have an excitation energy
of order $\Delta$ and correspond to deformations of the order parameter that are unrelated by a rotation or gauge transformation to the ground state order parameter.

Figure 1 shows the spectrum of uncoupled acoustic and $J = 2$ order parameter collective modes of superfluid $^3$He-B. The pairbreaking continuum onsets at $\omega = 2\Delta(T)$. Below the pairbreaking edge the zero sound mode crosses the dispersion curves for the $J = 2^{\pm}$ order parameter collective modes. Resonant absorption and anomalous dispersion of longitudinal sound result from the coupling of these modes to density, current and stress fluctuations (Wölfle, 1973; Maki, 1974). Quasiparticle interactions lead to renormalization of the mode frequencies (Sauls and Serene, 1981), as well as phenomena that cannot be anticipated from a theory based on uncoupled bosonic modes and weakly interacting quasiparticles (McKenzie and Sauls, 1990).

\[ H = 0 \]

\[ g \gamma H_{\text{eff}} \]

\[ \hbar \omega \]

\[ 2\Delta \]

\[ \Omega_2^- \]

\[ \Omega_2^+ \]

\[ \frac{1}{\xi_0} \]

\[ q \]

\[ \text{LZS} \]

\[ \text{Broken Pair Continuum} \]

\[ \text{H} \neq 0 \]

\[ \text{g} \gamma H_{\text{eff}} \]

\[ \text{Sound propagates in normal liquid } ^3\text{He because quasiparticle interactions and collisions provide restoring forces against density fluctuations. The restoring forces enter through the stress tensor. In the collisionless regime the main contributions to the induced stress fluctuations in normal } ^3\text{He come from the interaction of a quasiparticle with the density and current fluctuations. Both channels contribute to the longitudinal zero sound velocity; however, only the current fluctuations contribute to the restoring force for transverse zero sound. In the superfluid phases new physics enters because dynamical order parameter fluctuations also contribute to the stress tensor, and therefore couple to density and current fluctuations.} \]
The general form of the stress tensor follows from symmetry considerations. In $^3$He-B the stress fluctuations are described by a symmetric second-rank tensor under the group of joint rotations, $SO(3)_{L+S}$,

$$\delta \Pi_{ij} \simeq \Pi_n \delta n_{ij} + \Pi_J (J_i q_j + J_j q_i) + \Pi_{d+} (\delta d_{ij}^{(+)} + \delta d_{ji}^{(+)}) + \Pi_{d-} (\delta d_{ij}^{(-)} + \delta d_{ji}^{(-)}) ,$$

(19)

where $\delta d_{ij}^{(\pm)}$ represents order parameter fluctuations which are even (+) and odd (−) under particle-hole conversion, i.e. $\delta d_{ij}^{(\pm)} \xi \rightarrow \delta d_{ij}^{(\pm)}$. In the limit of exact particle-hole symmetry the density, current and stress fluctuations are odd under $C$: $\delta n \rightarrow -\delta n$, $J \rightarrow -J$ and $\delta \Pi_{ij} \rightarrow -\delta \Pi_{ij}$. This implies that coupling of the stress to the order parameter fluctuations, $\delta d_{ij}^{(+)})$, is non-vanishing only because of particle-hole asymmetry (Koch and Wölfle, 1981; Serene, 1983). Thus, the relative magnitude of the $J = 2^{\pm}$ coupling is $\Pi_{d+} \sim (\Delta/E_f) \Pi_{d-}$. The contributions of these dynamical fluctuations to the stress tensor are obtained by solving the transport equations for the quasiparticle distribution function and pairing amplitude.

For the acoustic modes, the analysis of the transport equations is simplified by expressing the conservation laws for particle number and momentum in terms of the amplitudes of the distribution function, $\phi_{lm}$, using Eqs. (7)-(8). The generalization of the amplitudes, $\phi_{lm}$, to the superfluid phases is discussed for example in (McKenzie and Sauls, 1990).

The continuity equation for the number density is equivalent to,

$$\omega \phi_{0,0} - \frac{1}{3} (1 + F_0^s) q v_f \phi_{1,0} = 0 ,$$

(20)

while the equations for momentum conservation can be expressed as,

$$\omega \phi_{1,0} - \left(1 + \frac{1}{3} F_1^s\right) q v_f \left(\phi_{0,0} + \frac{2}{5} \sqrt{\frac{2}{3}} \phi_{2,0}\right) = 0 ,$$

(21)

for the longitudinal current, and

$$\omega \phi_{1,\pm 1} - \left(1 + \frac{1}{3} F_1^s\right) q v_f \frac{2}{5} \sqrt{\frac{2}{3}} \phi_{2,\pm 1} = 0 ,$$

(22)

for the transverse current.

3. Longitudinal Modes

The dispersion relation for longitudinal sound is obtained by combining Eqs. (20) and (21),

$$\omega^2 - c_1^2 q^2 \left\{1 + \frac{2}{5} \sqrt{\frac{2}{3}} \xi(\omega, q)\right\} = 0 ,$$

(23)
where \( c_1 \) is the hydrodynamic sound velocity. The effects of mode coupling are contained in the response function,

\[
\xi(\omega, q) = \left[ \frac{\delta \phi_{2,0}}{\delta \phi_{0,0}} \right]_{\text{tot}} = \left( \frac{\delta \phi_{2,0}}{\delta \phi_{1,0}} \right) \left( \frac{\delta \phi_{1,0}}{\delta \phi_{0,0}} \right) + ..., \tag{24}
\]

which represents the total stress induced by a density fluctuation. To calculate the response function we solve the coupled dynamical equations for the quasiparticle distribution function, time-dependent order parameter, and the Landau interactions self-consistently, \textit{c.f.} \citep{McKenzie90}. The solution for the longitudinal component of the induced stress is

\[
\phi_{2,0} = A_0 \alpha_0(q, \omega) \phi_{0,0} + \frac{1}{3} A_1 \alpha_1(q, \omega) \phi_{1,0} + \left( \frac{\omega}{2 \Delta} \right) \left[ \beta_0(q, \omega) D_{0,0}^{(-)} + \beta_2(q, \omega) D_{2,0}^{(-)} \right] + \left( \frac{qv_f}{2 \Delta} \right) \left[ \zeta_0(q, \omega) D_{0,0}^{(+)} + \zeta_2(q, \omega) D_{2,0}^{(+)} \right], \tag{25}
\]

where \( \alpha_0 = \xi_0, \alpha_1 = (qv_f/\omega)[\sqrt{2} \xi_2 + \xi_1], \beta_0 = \frac{1}{3 \sqrt{2}}[\lambda_1 - \frac{1}{3} \lambda_0], \beta_2 = \sqrt{2}[\lambda_2 - \frac{2}{3} \lambda_1 + \frac{1}{3} \lambda_0], \) and the functions, \( \xi_0, \xi_1, \lambda_0, \lambda_1 \) and \( \lambda_2 \) are given in Ref. \citep{Moores93}. In addition to the density, \( \phi_{0,0} \), and longitudinal current fluctuation, \( \phi_{1,0} \), the order parameter modes with \( J = 0, 2 \) and \( M = 0 \) contribute to the longitudinal stress. The stress induced by the \( J = 0^+ \) and \( J = 2^+ \) modes is small compared with the \( J = 0^- \) and \( J = 2^- \) modes by the particle-hole asymmetry parameter, \( \zeta \). For the particle-hole asymmetric couplings, see \citep{Koch81}. Unless stated otherwise, I will omit the contributions to the stress tensor coming from the Fermi-liquid interactions with \( \ell \geq 2 \) as well as higher angular momentum pairing channels.

The equations for the order parameter modes, \( D_{0,0}^{(-)} \) and \( D_{2,0}^{(-)} \), obtained from solutions of the time-dependent gap equation, reduce to,

\[
\left[ \omega^2 - \frac{1}{3} (qv_f)^2 \right] D_{0,0}^{(-)} = 2 \sqrt{3} (\omega \Delta) \phi_{0,0} + \frac{2 \sqrt{2}}{15} (qv_f)^2 D_{2,0}^{(-)} \tag{26}
\]

\[
\left[ (\omega + i \Gamma)^2 - \Omega_2^2 - \frac{7}{5} q^2 v_f^2 \right] D_{2,0}^{(-)} = \frac{2}{15} (qv_f)^2 D_{0,0}^{(-)} + \frac{A_1}{3} \left( \frac{qv_f}{2 \Delta} \right) \left( \frac{8}{5} \Delta^2 \right) \sqrt{\frac{2}{3}} \phi_{1,0}. \tag{27}
\]
Equation (26) is the wave equation for the Anderson-Bogoliubov phase mode, $D_{0,0}^{(-)}$ (Anderson, 1958; Bogoliubov et al., 1958). For $q \to 0$ the driving term on the right side of Eq.(26) is proportional to the density fluctuation, and in this limit the equation for $D_{0,0}^{(-)}$ is equivalent to the Josephson equation, $i\hbar \partial_t \chi = \delta \mu$ (McKenzie and Sauls, 1990), for the order parameter phase, $\chi$, where $\delta \mu$ is the change in the chemical potential. For shorter wavelengths and higher frequencies, the phase fluctuations are also coupled to the high-frequency modes. Equation (27) describes driven oscillations of the $J = 2^-, M = 0$ order parameter mode by the change in the Landau interaction energy induced by a current fluctuation; note the Landau parameter $A_{11}$. The driving term includes the longitudinal current, $\phi_{1,0}$, and the phase fluctuation mode, $D_{0,0}^{(-)}$. The stress induced by fluctuations of the $J = 2^+ \pm$ mode is much weaker than that of the $J = 2^-$ mode because of approximate particle-hole symmetry. The $J = 2^+ \pm$ mode is excited by density and current fluctuations, but the coupling is again small by a factor of $\zeta$.

\[
\left[ (\omega + i\Gamma)^2 - \Omega_{2^+}^2 - \frac{7}{5} q^2 v_f^2 \right] D_{2,0}^{(+)} = 2\zeta \omega \Delta D_{2,0}^{(-)}. 
\]  

(28)

Combining Eqs. (24)-(28) we obtain the dispersion relation for longitudinal sound (Wölfe, 1973; Koch and Wölfe, 1981) with

\[
\xi(q, \omega) = \frac{2}{5} \left( \frac{c_1 q}{\omega} \right)^2 \left( \frac{1}{1 + F_{0}^s} \right) 
\times \left\{ \rho_s(\omega) + \frac{3}{5} \rho_n(\omega) \left[ \frac{\omega^2}{(\omega + i\Gamma)^2 - \Omega_{2^-}^2 - \frac{7}{5} q^2 v_f^2} \right] \right. 
\left. + \zeta^2 \rho_s(\omega) \left[ \frac{\omega^2}{(\omega + i\Gamma)^2 - \Omega_{2^+}^2 - \frac{7}{5} q^2 v_f^2} \right] \right\}. 
\]  

(29)

The response function exhibits the contribution from the quasiparticle restoring force $\sim \rho_n(\omega)$, as well as the $J = 2^\pm, M = 0$, order parameter modes, $\sim \rho_s(\omega)$. Note that $\rho_s$ and $\rho_n$ represent condensate and non-condensate response functions with $\rho_s + \rho_n = 1$; these functions reduce to the superfluid and normal fluid densities in the limit $\omega = 0$.\(^6\) The contribution from the $J = 2^+$ mode is also proportional to the particle-hole asymmetry factor, $\zeta^2$. The mode couplings are enhanced near the resonance frequency of the modes. The resonance linewidth is determined by quasiparticle scattering rate which becomes exceedingly small at low temperatures, $\Gamma(T) \simeq \Gamma_n(T_c/T)^\frac{3}{2} e^{-\Delta/T} \ll \Omega_{2^\pm} \sim \Delta$, with $\Gamma_n \simeq 1/\tau(T_c) \sim 0.1$ MHz.

\(^6\)The condensate response function $\rho_s(\omega)$ is equivalent to the Tsuneto function $\lambda(\omega)$ in (Moores and Sauls, 1993).
(Halperin, 1982). Thus, even weakly coupled modes are observable near resonance as strong absorption features in the attenuation spectrum of longitudinal sound. In addition to the resonant contributions to the absorption of longitudinal sound, there is an absorption band for frequencies above the pair-breaking threshold, $\omega \geq 2\Delta(T)$. These features are shown in the calculated attenuation spectrum of Fig. 2 as a function of temperature for longitudinal sound at $P \simeq 0$ bar ($T_c = 0.93\text{ mK}$) and a frequency of $35.8\text{ MHz}$. The calculation assumes $\zeta = 10^{-2}$. The peaks correspond to resonant absorption by the $J = 2^\pm$ modes. The relative weight of the resonance peaks reflects the weak coupling of the $J = 2^+$ mode due to approximate particle-hole symmetry.

In the calculated attenuation spectrum of Fig. 2 as a function of temperature for longitudinal sound at $P \simeq 0$ bar ($T_c = 0.93\text{ mK}$) and frequency of $\omega/2\pi = 35.8\text{ MHz}$. The sharp resonance peak at the lowest temperature is the $J = 2^+$ resonance, while the huge absorption peak at a higher temperature is $J = 2^-$ resonance. Pair-breaking gives rise to the broad absorption band onsetting at $T_c$. The $J = 2^+$ mode resonance was observed in sound absorption by (Giannetta et al., 1980) and (Mast et al., 1980). The identification of this resonance with the $J = 2^+$ mode was made by (Avenel et al., 1980) who observed the Zeeman splitting of the absorption line in a magnetic field.

**Zeeman Splitting of the $J = 2^\pm$ Modes**

The $J = 2$ modes are five-fold degenerate in zero field; however, the
nuclear magnetic moment of $^3$He leads to a Zeeman coupling in a magnetic field that lifts the degeneracy (Tewordt and Schopohl, 1979), producing five Zeeman levels in a field. For weak magnetic fields the level splitting is linear in the field and given by

$$\Omega_{2\pm,M} = \Omega_{2\pm} + M g_{2\pm} \omega_{\text{eff}},$$

where $g_{2\pm}$ is the g-factor for the $J = 2\pm$ modes (Tewordt and Schopohl, 1979; Sauls and Serene, 1982) and $\omega_{\text{eff}}$ is the effective Larmor frequency of the excited pairs,

$$\omega_{\text{eff}} = \gamma H \frac{(1 + \frac{1}{5} F_2^{\alpha})}{1 + F_0^{\alpha} \left( \frac{2}{3} + \frac{1}{5} Y(T) \right) + \frac{1}{5} F_2^{\alpha} \left[ \frac{1}{3} + \left( \frac{2}{3} + F_0^{\alpha}\right) Y(T) \right]},$$

where $Y(T)$ is the Yosida function, and $\gamma$ is the gyromagnetic ratio for the $^3$He nucleus. Equation (31) includes the exchange field enhancement of the applied field, $H$, in the superfluid state. The g-factors for the excited pair states, including Fermi liquid effects and higher angular momentum pairing channels, were calculated by (Sauls and Serene, 1982).

Measurements of the Zeeman splitting provide an experimental determination of the Lande g-factor. The g-factor is sensitive to exchange and pairing interactions, and is an excellent parameter to study many-body correlations and strong-coupling effects in superfluid $^3$He (Sauls and Serene, 1982). Measurements of the g-factor from the Zeeman splitting of the absorption spectrum of LZS for the $J = 2^+$ modes are in reasonable agreement with theoretical calculations. However, precision measurements for the $J = 2^-$ modes have not previously been possible because the coupling of the $J = 2^-$ modes is so strong that high magnetic fields are required to resolve different Zeeman levels. At high magnetic fields the $J = 2$ level shifts evolve nonlinearly with $H$ as a result of gap distortion by the magnetic field (Shivaram et al., 1983). For a discussion of these effects see (Schopohl et al., 1983; Fishman and Sauls, 1986; Halperin and Varoquaux, 1990). Nevertheless, (Movshovich et al., 1988) obtained a value of $g_{2^-} \simeq 0.04$ at $P = 19$ bar from their measurements at fields above 1kG, in reasonable agreement with theoretical expectations. However, as I show below the g-factor for the $J = 2^-$ modes can be determined with high accuracy from an analysis of transverse sound propagation in a magnetic field. But, first I describe the mechanisms that lead to a propagating transverse current mode.

4. Transverse Modes

The dispersion relation for a transverse current excitation is given by the momentum conservation equation, Eq. (22), and the response function for
the stress induced by a transverse current fluctuation,

$$\left(\frac{\omega}{qv_f}\right)_{\pm} = \frac{2}{5} \left(1 + F_1^s/3\right) \frac{1}{\sqrt{2}} \left[\frac{\delta \phi_{2,\pm1}}{\delta \phi_{1,\pm1}}\right]_{\text{tot}}$$

where the subscripts $\pm$ refer to the two independent circular polarizations of the transverse current. In addition to the direct contribution to $\phi_{2,\pm1}$ coming from the current fluctuations, order parameter modes with $J = 2$ and $M = \pm 1$ also induce transverse stress fluctuations and therefore couple to the transverse current. The result for the transverse stress is (Moores and Sauls, 1993),

$$\phi_{2,\pm1} = \frac{1}{3} A_1^s \left(\frac{qv_f}{\omega}\right) \xi_1(q,\omega) \frac{1}{\sqrt{2}} \phi_{1,\pm1} + \left(\frac{\omega}{2\Delta}\right) A_1(q,\omega) D_{2,\pm}(-),$$

where the term $\sim A_1^s \phi_{1,\pm1}$ represents the quasiparticle contribution to the transverse restoring force, while the condensate contribution, which also enters through the Landau interaction, is proportional to the amplitude of the $J = 2^-$, $M = \pm 1$ order parameter modes, $D_{2,\pm}(-)$. I omit the $J = 2^+$, $M = \pm 1$ modes because the coupling of these modes is smaller by a factor of the particle-hole asymmetry ratio, $\zeta$. Unlike the case of LZS, the $J = 2^+$ modes cannot be easily detected in the TZS spectrum because the TZS mode is suppressed for frequencies below the $J = 2^-$ mode, which lies above the $J = 2^+$ mode. The equations of motion for the $J = 2^-$, $M = \pm 1$ order parameter modes are given by

$$\left[(\omega + i\Gamma)^2 - \Omega_{2^-,-1}^2 - \frac{2}{5} q^2 v_f^2\right] D_{2,\pm} = \left(\frac{8\Delta^2}{2\Delta}\right) \left(\frac{qv_f}{2\Delta}\right) A_1^s \frac{1}{3} \frac{1}{\sqrt{2}} \phi_{1,\pm1}.$$ (34)

In zero field the dispersion relation for transverse current excitations is independent of the polarization,

$$\left(\frac{\omega}{qv_f}\right)^2 = \frac{F_1^s}{15} \rho_n(\omega) + \frac{2F_1^s}{78} \rho_s(\omega) \left\{\frac{\omega^2}{(\omega + i\Gamma)^2 - \Omega_{2^-}^2 - \frac{2}{5} (qv_f^2)\right\},$$

which displays contributions from the quasiparticle restoring force ($\sim \rho_n$), and from the condensate ($\sim \rho_s$) (Maki and Ebisawa, 1977; Moores and Sauls, 1993). Equation (35) is based on a long-wavelength expansion, i.e. $qv_f \ll \omega$, which is valid in the B-phase for frequencies in the range, $\Gamma \ll |\omega - \Omega_{2^-}| \ll \omega$.

\footnote{Results for shorter wavelengths and frequencies far off resonance can be found in (Moores and Sauls, 1993).}
In normal $^3$He transverse sound propagates with a phase velocity $c_t \geq v_f$ provided the $\ell = 1$ and $\ell = 2$ Landau parameters satisfy, $F_1^s/3 + F_2^s/(1 + F_1^s/3) > 2$ (Fomin, 1968); in the limit $F_1^s \gg 1$, $c_t \simeq \sqrt{F_1^s/15} v_f$, which is the normal-state result in the limit $qv_f/\omega \ll 1$. The condition for a propagating TZS mode is obeyed for pressures above approximately 1 bar; however, the predicted velocity is close to the Fermi velocity at all pressures. As a result it is difficult to differentiate transverse oscillations of the Fermi surface from an incoherent current of quasiparticles; and early attempts to observe transverse sound in the normal phase of $^3$He were inconclusive, (Roach and Ketterson, 1976; Flowers et al., 1976).

The onset of superfluidity in $^3$He has dramatic effects on the collisionless transverse modes in $^3$He. The opening of a gap in the excitation spectrum suppresses the quasiparticle contribution to the restoring force. This suppression comes from the factor of $\rho_n$ in Eq. (35), which decreases rapidly below $T_c$. For this reason early theoretical investigations emphasized that the transverse zero sound mode, even if observable in the normal state, would disappear below $T_c$ (Maki, 1974; Combescot and Combescot, 1976; Maki and Ebisawa, 1977). The condensate also contributes to the dispersion relation through the coupling to the $J = 2^-$, $M = \pm 1$ order parameter modes. The condensate response suppresses TZS at low frequencies, as can be seen from Eq. (35). However, at intermediate frequencies, $\Omega_2^- < \omega < 2\Delta$, the condensate provides a strong restoring force through the Landau interaction energy, leading to a propagating transverse current mode at low temperatures. The calculated temperature dependences of the phase velocity and attenuation of a transverse current excitation in $^3$He-B are shown in Fig. 3 taken from Ref. (Moores and Sauls, 1993). At low frequency, $\omega \ll \Omega_2^-(0)$, the transverse current mode is highly attenuated over the full temperature range below $T_c$. At high frequencies, $\omega = 1.9\Delta_0 > \Omega_2^-(0)$, a propagating transverse current mode develops with a large phase velocity and low absorption that decreases exponentially at low temperature. For intermediate frequency, $\omega = 1.54\Delta_0$, TZS develops for $T < T_{pb}$; the attenuation drops until $T = T_*$, at which point TZS is extinguished by resonant absorption from the $J = 2^-$ modes.

**Circular Birefringence of Transverse Waves**

The B-phase of $^3$He is symmetric under time-inversion, and consequently is non-birefringent for transverse wave propagation, i.e. right- and left-circularly polarized waves propagate at the same velocity. However, *circular birefringence*, i.e. $C_+ \neq C_-$, can be induced by a magnetic field. If the field is applied parallel to the direction of propagation of the waves, $\mathbf{H}||\mathbf{q}$, then axial symmetry is preserved and the eigenmodes for the transverse current...
are the right- and left-circular polarization states. However, the eigenfrequencies of these modes, or equivalently the complex phase velocities, are no longer degenerate. This leads to the acoustic analog of (circular) optical birefringence ($\text{Re} C_+ \neq \text{Re} C_-$) and dichroism ($\text{Im} C_+ \neq \text{Im} C_-$) (Moore and Sauls, 1993). At low temperatures and high frequencies, $\omega > \Omega_{2^-, \pm 1}$, circular birefringence gives rise to the acoustic Faraday effect in which the direction of a linearly polarized transverse wave rotates along the direction of propagation.

The dispersion relations for RCP (+) and LCP (−) transverse current modes in a magnetic field, $\mathbf{H} || \mathbf{q}$, are given by

$$
\left( \frac{\omega}{q v_f} \right)^2 \pm = \Lambda_n + \Lambda_s \left( \frac{\omega^2}{(\omega + i \Gamma)^2 - \Omega_{2^-, \pm 1}^2(T, \omega, H)} \right) 
$$

where $\Lambda_n = (F_s^0 / 15) \rho_n(\omega, T)$ and $\Lambda_s = (2F_s^0 / 75) \rho_s(\omega, T)$ in the long-wavelength limit. The condensate term dominates at low temperature ($\Lambda_s \gg \Lambda_n$) and is anomalously large when the sound frequency is nearly resonant with the $J = 2^-, M = \pm 1$ modes,

$$
D_{\pm}^2(\omega, H, T) = (\omega + i \Gamma)^2 - \Omega_{2^-, \pm 1}^2(\omega, H, T) \approx 0 .
$$

The frequencies of the $M = \pm 1$ modes include the Zeeman shifts,

$$
\Omega_{2^-, \pm 1}^2 = \Omega_{2^-}^2(T) \pm 2 g_2^-(T) \omega \omega_{\text{eff}}(H, T) .
$$
where $\Omega_{2-}(T)$ is the frequency of the $J = 2^-$ modes in zero field.

Consider a transverse current excitation of frequency $\omega$ at $z = 0$, linearly polarized along $x$ and propagating in the $z$-direction, $J(\omega, z = 0) = J(\omega) \hat{x}$. The RCP and LCP modes propagate with different phase velocities; thus the current evolves as,

$$J(\omega, z) = \frac{\mathcal{J}}{\sqrt{2}} e^{i q_+(\omega) z} \hat{e}_+ + \frac{\mathcal{J}}{\sqrt{2}} e^{i q_- (\omega) z} \hat{e}_- ,$$

(39)

where $\hat{e}_\pm = (\hat{x} \pm i \hat{y})/\sqrt{2}$ are the polarization vectors for RCP and LCP current modes. This response corresponds to a pure Faraday rotation of the polarization if the phase velocities are real. This is the case for low temperatures, $T \ll T_c$, and frequencies above the collective mode resonances, $\omega > \Omega_{2-\pm 1}(T)$. The spatial period of the rotation of the polarization is identified by writing

$$J(\omega, z) = J e^{iq(\omega) z} (\cos(\delta q z) \hat{x} - \sin(\delta q z) \hat{y}) ,$$

(40)

where the sum and difference wavevectors are $q = (q_+ + q_-)/2$ and $\delta q = (q_+ - q_-)/2$. The linearly polarized transverse wave propagates with a mean phase velocity, $\bar{c}_t = \omega/q$, and the polarization rotates with the spatial period,

$$\Lambda(\omega, H, T) = \frac{2\pi}{|\delta q|} = \frac{4\pi}{q_- - q_+} .$$

(41)

Near the $M = +1$ mode crossing the wavenumber for the RCP wave vanishes; $q_+ (T_+ ) = 0$, since $D_+(\omega, T_+ ) = 0$ and $\Lambda_n \ll \Lambda_s$. The temperature dependence of the Faraday rotation period is then determined by the response of this mode off resonance,

$$q_+ (T) \simeq q_f \frac{D_+ (T)}{\sqrt{\Lambda_s \omega}} = q_f \sqrt{s(T/T_+ - 1)} ,$$

(42)

where $q_f = \omega/v_f$,

$$s \equiv \frac{2|\Omega_+'|T_+}{\Lambda_s \Omega_+} ,$$

(43)

$\Omega_+ = \Omega_{2-}(T_+) = \omega$, and $\Omega_+' = d\Omega_{2-}(T)/dT|_{T_+}$ are evaluated at the mode resonance, $T = T_+$. The field dependence originates from the linear Zeeman splitting of the $M = \pm 1$ modes, and enters via the wavenumber of the $M = -1$ mode,

$$q_- \simeq q_f \sqrt{\frac{2|\Omega_+'| (T - T_+) + 4g_2 - \omega_{\text{eff}}}{\Lambda_s \Omega_+}} ,$$

(44)
for $T_+ \lesssim T \ll T_c$. Thus, the Faraday rotation period varies with field on the scale,

$$B_+ \equiv \frac{\Lambda_x \Omega_+}{4 \gamma_{\text{eff}} g_2} ,$$

(45)

where $\gamma_{\text{eff}} \equiv \omega_{\text{eff}}/H$ is the effective gyromagnetic ratio at the mode crossing. I can express the temperature and field dependence of $\Lambda$ near the $J = 2^-$, $M = +1$ mode crossing in terms of scaled temperature and field variables, $\alpha = s (T/T_+ - 1)$ and $\beta = H/B_+ ,$

$$\frac{\Lambda}{\lambda_f} = \frac{2}{\sqrt{\alpha + \beta - \sqrt{\alpha}}} = \left\{ \begin{array}{ll}
4 \sqrt{\alpha/\beta} , & \beta \ll \alpha \\
2 \left(1 + \sqrt{\alpha/\beta}\right)/\sqrt{\beta} , & \beta \gg \alpha ,
\end{array} \right.$$

(46)

where $\lambda_f = 2 \pi v_f / \Omega_+$. For $T_+/T_c \simeq 0.4$, $\omega/2\pi = 80$ MHz and $F_{s} \simeq 15$ the field scale is $B_+ \simeq 1$ Tesla. Thus, except for a narrow temperature range near resonance, i.e. $|T - T_+| < 10^{-2}T_+$ at $H = 100$ G, the Faraday rotation period varies as

$$\Lambda \simeq \frac{4 B_+}{H} \sqrt{s(T - T_+)} \simeq \frac{\xi_{\Omega} \pi v_f}{\gamma_{\text{eff}} g_2} \sqrt{\frac{2 F_{s} \rho_s \sqrt{T - T_+}}{H}} ,$$

(47)

with $\xi_{\Omega} = 8 |\Omega_+| T_+/\Omega_+ \simeq 1$ at $T \simeq 0.44T_c$. In magneto-optics the field-independent prefactor, $V = \Lambda H/2\pi$, is called the Verdet constant.

**Broken Spin-Orbit Symmetry**

The acoustic Faraday effect provides a clear demonstration of *spontaneously broken spin-orbit symmetry*. The direction of mass flow is altered by the application of a magnetic field along the propagation direction. The role of broken spin-orbit symmetry is evident in the contribution to the stress fluctuations from the order parameter,

$$\delta \pi_{ij} \sim \sum_M D_{2,M}^{(-)} t_{ij}^{2,M} .$$

(48)

Inspection of Eqs. (18) shows that the transverse stress couples the transverse current fluctuations only to the $M = \pm 1$ modes with total angular momentum $J = 2 ,$

$$t_{ij}^{(2,\pm)} \sim (\hat{q}_i \hat{e}_j^{\pm} + \hat{e}_i^{\pm} \hat{q}_j) .$$

(49)

Broken time-reversal symmetry is also important for producing circular birefringence. The Zeeman splitting of the $M = \pm 1$ modes in a field aligned along $\mathbf{q}$ produces a difference in the stress induced by the $M = +1$ and $M = -1$ modes, and thus, different propagation velocities.
These general symmetry considerations also imply that superfluid $^3$He-A, which spontaneously breaks time-reversal symmetry as well as relative spin-orbit rotation symmetry, should exhibit spontaneous acoustic circular birefringence or dichroism for transverse excitations propagating along the $\ell$ vector. In fact observation of this effect for a uniform texture would provide a direct evidence of broken time-reversal symmetry in $^3$He-A (Yip and Sauls, 1992).

5. Observation of Transverse Current Waves in $^3$He-B

Propagating transverse current waves were recently observed in $^3$He-B at low pressure (Lee et al., 1999). The excitation and detection of these waves was made in an acoustic cell with a transducer operating at a frequency of $\omega/2\pi = 82.26$ MHz that generates and detects transverse currents with a fixed linear polarization. In a high attenuation regime the acoustic response measures the transverse acoustic impedance of bulk $^3$He. However, if a propagating mode develops with low attenuation then current waves propagate across the cell, are reflected from the boundary opposite the transducer and are detected by the same transducer. In this regime standing waves develop and are observable in the acoustic impedance because of interference between the source and reflected wave at the detector. Oscillations of the impedance occur as the phase velocity and attenuation change with temperature, pressure or field. A linearly polarized wave with $\mathbf{q} \parallel \mathbf{H}$ undergoes Faraday rotation of its polarization as it propagates. Upon reflection from the opposite wall of the cavity the reflected wave propagates with its polarization rotating with the same handedness relative to the direction of the field, i.e. the rotation of the polarization accumulates after reflection from a surface. The rotation of the polarization produces an oscillatory modulation of the impedance as a function of magnetic field, with the period of the impedance oscillations being determined by, $\Lambda/2 \propto 1/H$.

The transverse acoustic response measured by (Lee et al., 1999) is shown for $^3$He at a pressure of 4.31 bar in Fig. 4. The impedance varies smoothly as the temperature drops below $T_c$ and shows a peak at the pairbreaking (PB) edge, $\omega = 2\Delta(T_{pb})$. Oscillations develop below a temperature of $T \simeq 0.65T_c$, indicating the presence of a propagating mode. The amplitude of the impedance oscillations increases as the temperature decreases indicating that the attenuation of the propagating mode decreases with decreasing temperature. The oscillations disappear at the temperature $T_\ast \simeq 0.42 T_c$, corresponding to the resonance condition, $\omega = \Omega_{2-}(T_\ast)$; i.e. transverse waves are extinguished below this temperature. All of these features are in agreement with theoretical predictions (Moores and Sauls, 1993).
Figure 4. Transverse impedance oscillations in $^3$He-B at $\omega/2\pi = 82.26$ MHz, $P = 4.31$ bar and zero field. The oscillations develop in the intermediate frequency range $\Omega_2^- < \omega < 2\Delta(T)$, between the pairbreaking edge, labeled by PB ($\omega = 2\Delta(T_{PB})$), and the extinction point determined by the resonance frequency of the $J = 2^-, M = \pm 1$ modes, i.e. $\omega = \Omega_2^-(T_s)$. The pair-breaking and resonance conditions for fixed frequency are shown in the inset.

Figure 5 (reproduced from (Lee et al., 1999)) shows the impedance oscillations at a pressure of 4.42 bar for magnetic fields of 52 G, 101 G and 152 G. The magnetic field modulates the zero field impedance oscillations. The transducer detects only transverse waves having the same linear polarization as the source wave. A field of 52 G suppresses the impedance oscillations near $T = 0.465 T_c$ that were present in zero field, corresponding to a $90^\circ$ rotation of the polarization of the reflected wave; i.e. the polarization of the reflected wave is orthogonal to the detection direction. Doubling the field restores the impedance oscillations near $T = 0.465 T_c$. The oscillations are suppressed again at a field to 152 G. Near the $90^\circ$ and $270^\circ$ points, there are small amplitude impedance oscillations, with shorter period than the primary oscillations. These oscillations are due to interference of the source and waves that traverse the cavity twice. This interpretation was verified by comparing the impedance oscillations with the calculated cavity response, shown in the right panels of Fig. 5. The calculation is based on Eqs. (40) and (47) and takes as input the measured attenuation and phase velocity in zero field. The Verdet constant is obtained from the measurement at 52 G. The simulation reproduces the observed features of the impedance.
as a function of temperature including the maximum in the modulation at $T/T_c = 0.465$, $H = 101$ G and the minimum at $T/T_c = 0.465$, $H = 152$ G, as well as the fine structure oscillations in the impedance near the points labeled $90^\circ$ and $270^\circ$. The fine structure is observed when the polarization rotates by an odd multiple of $90^\circ$ upon a single round trip in the cell. Waves that traverse the cell twice are then $180^\circ$ out of phase relative to the source wave, and consequently the period of the impedance oscillations is halved. The amplitude of the oscillations is reduced because of attenuation over the longer pathlength. These observations of the rotation of the polarization of the current by a magnetic field prove that the impedance oscillations result from the interference of propagating transverse current waves (Lee et al., 1999).

The g-factor for the $J = 2^-$ Modes

The temperature and field variations of the impedance oscillations provide a strong test of the theoretical predictions for transverse wave propagation in $^3$He-B. Here I compare the theoretical result for $\Lambda$ in Eq. (46) with the experimental data for magneto-acoustic rotation period for low fields, $H \ll 1$ kG, and temperatures above, but near, the extinction point $T_\ast$. Note that the temperature, $T_\ast$, is the extinction point in a field is slightly higher.

Figure 5. Left panels: Magnetic field dependence of the acoustic cavity response measured at 4.42 bar. The angles indicate the rotation of the polarization after one round-trip. Right panels: Simulation of the acoustic impedance for the same path length, temperature, pressure and magnetic fields.
than the zero-field extinction point, $T_\ast$; at $H = 100$ G, $T_+ - T_\ast \approx 1 \mu K$. The magnitude of the Faraday rotation period depends on well known properties of $^3$He-B, except for the Landé g-factor, $g_{2-}$. For the calculations shown in Fig. 6 I used the measured Fermi-liquid data tabulated for $T_c$, $v_f$, $F_1$, $F_0$, and $F_2$ as a function of pressure in (Halperin and Varoquaux, 1990). The mode data are $T_+/T_c$ and $\Omega_+(T_+) = \omega$, and $\gamma = 3.2434$ MHz/kG. The only undetermined parameter that enters Eq. (46) for $\Lambda$ is the Landé g-factor, which scales the magnitude of the Faraday rotation angle. The data exhibit the temperature dependence and pressure dependence predicted by the theory and provide a new determination of the Landé g-factor the $J = 2^-$ modes at low fields. The calculated results are in excellent agreement with the experimental data at three nearby pressures for $g_{2-} = 0.02 \pm 0.002$.

![Figure 6](image_url)

Figure 6. Comparison between the theoretical and experimental results for the temperature dependence of the Faraday rotation period for a field of $H = 100$ Gauss at pressures, $p = 4.31$ bar (red), $p = 4.42$ bar (blue), and $p = 4.52$ bar (green). The data for the corresponding pressures are shown as the solid squares with error bars (Y. Lee, et al. (1999)).

This result differs from the value of $g_{2-} \simeq 0.042$ reported by (Movshovich et al., 1988) for $p = 19$ bar, which was obtained from the splitting of the $J = 2^-$ multiplet in the absorption spectrum of longitudinal sound for fields above 2 kG. At these high fields the non-linear field contribution to the level splittings is comparable to the linear Zeeman splitting, which makes and accurate determination of the Landé g-factor from the absorption spectrum difficult. This complication is not present in the analysis of the Faraday rotation period at $H \approx 100$ G.
Earlier theoretical calculations for the order parameter collective mode frequencies showed that that the g-factors for these excitations are sensitive to Fermi liquid interaction effects and higher angular momentum pairing correlations (Sauls and Serene, 1982). Figure 7 shows the g-factor for the $J = 2^-$ modes as a function of temperature for several values of the f-wave pairing interaction, expressed in terms of the f-wave and p-wave transition temperatures, $x_3 = \ln(T_{f\text{-wave}}/T_c)$. Negative values correspond an attractive f-wave pairing interaction. The g-factor, $g_{2^-}$, also depends on the Landau interaction parameter, $F_s^2$, which is known from measurements of the normal-state zero sound velocity. The calculated values of $g_{2^-}$ are shown for a pressure of 4.31 bar, corresponding to $F_s^2 \simeq -0.05$. The calculated g-factor is strongly reduced by attractive f-wave correlations.

![Figure 7](image)

*Figure 7.* Comparison between the theoretical and experimental results for the Landé g-factor of the $J = 2^-$ modes. The theoretical curves are include the effects of f-wave pairing correlations. The experimental result for $g_{2^-}$ at $T/T_c = 0.46$ and $P = 4.32$ bar is fit with an attractive f-wave interaction, $x_3 \simeq -0.33$.

The analysis of the Faraday rotation period yields a measurement of $g_{2^-}$. The fact that the $J = 2^-$ g-factor is intrinsically small (in contrast to the $J = 2^+$ g-factor), makes this parameter a sensitive test of many-body correlation effects in superfluid $^3$He. The implications of this result are that either substantial attractive f-wave pairing correlations or strong-coupling effects reduce the g-factor by a factor of two compared to the weak-coupling result. The value of $g_{2^-}$ obtained from Faraday rotation measurements suggests that f-wave pairing correlations at 4 bar are attractive with $T_{f\text{-wave}} \simeq 0.07 T_c$. 
If the reduction is due to f-wave pairing correlations then there should be a collective mode with total angular momentum $J = 4$ (Sauls and Serene, 1981) lying just below the pair breaking edge which might be observable in the dispersion of TZS at high frequencies near, but below the pair breaking edge. In summary, the discovery of propagating transverse currents and magneto-acoustic rotation of the polarization of mass currents in superfluid $^3$He-B opens a new spectroscopy of collective excitations in superfluid $^3$He-B. The precision of this spectroscopy offers the possibility of measuring strong coupling effects on the collective mode frequencies and Zeeman levels, as well as effects of higher angular momentum pairing correlations at ultra-low temperatures.

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