A Dam Performance Model Based on Rehabilitation of the Kariba Dam

Junxiao Zhou*, Junwei Cheng, Qian Li
College of Systems Engineering National University of Defense Technology, Changsha, Hunan province, China.

*myaxiao@126.com

Abstract. Because of the damage in its major parts such as the plunge pool and sluice gates, the Kariba Dam is now in a dangerous state that needs urgent actions of rehabilitation. In this paper, we use the multiple dam system in Kariba to verify the dam performance models we propose. First, a dam performance model is built with 3 parts which are focused on hydropower generation, water storage capacity, and the costs of a single dam respectively. Next, we simulated the geometrical characteristic of the dams based on the terrain of Zambezi river basin. The number and placements of the multiple dams system are determined to satisfy the generation and water capacity of the existing dam. Discussion of the costs-benefits difference between multiple dams system and other 2 solutions are made, the result shows that multiple dam system is the optimal solution, and the model we proposed were proved to be reliable.

Keyword: Dam Performance, Dam Rehabilitation, Multiple Dams System, Kariba Dam

1. Introduction
The Kariba Dam is a double curvature concrete arch dam situated in the Kariba Gorge between Zambia and Zimbabwe. It is considered to be one of the largest reservoirs in the world. The Kariba Dam is the main source of hydropower for Zambia and Zimbabwe, each country has its own generation station on the north and south bank of the dam separately. Opened in 1959, the Kariba dam’s bedrock has been badly eroded, causing the dam’s plunge pool’s instability. The Kariba Dam is in a dangerous state now and possible catastrophic dam failure may happen in the future. If such failure were to occur, would put approximately 3.5 million people’s lives in danger and knock out 40% of southern Africa’s hydroelectric capability. Therefore, urgent rehabilitation project is required to prevent further safety issues and damage.

In recent global conventions on dams, issues about performance improvement and rehabilitation of aging dams have been actively discussed. Choi et al. suggested an approach to dam rehabilitation assessment to deal with complex dam safety and environmental issues arising from climate changes, which used a survey-based Delphi–analytic hierarchy process (AHP) method in accordance with the expertise of an expert panel[1]. Talon et al. developed a performance assessment application of dams based on a four-phase method[2]. Anjasmoro et al. analyzed the priority of small dams’ construction under a limited budget by using Cluster Analysis, AHP, and Weighted Average Method [3]. Ronco et al. developed a simplified numerical model to analyze the present and future effects of the presence of the Kariba and Cahora Bassa dams on the morphology of lower Zambezi River[4]. Ji et al. discussed the
value and specific process of design and construction of the dismantlement of intake cofferdam in the extension project of the north shore hydropower station in Kariba[5].

The Zambezi River Authority (ZRA), a corporation jointly and equally owned by the governments of Zambia and Zimbabwe, is the project proponent for the proposed Kariba Dam Rehabilitation Works. The ZRA proposed to repair the Kariba dam with classical measures of rehabilitating a large-scale dam: enhance the stability of the plunge pool and six sluice gates of the dam’s spillway. The rehabilitation of the Kariba dam, clearly, is an issue that involves large capital investment and takes many years to accomplish. From the perspective of long-term development, the Kariba dam needs not only to be repaired or rebuilt but upgraded. Meanwhile, based on a long-term study of the Zambezi river, ZRA also gives a series of development plans to build new dams at potential dam sites, which is considered to be a better choice to fix the Kariba dam problem.

Therefore, based on the situation of Kariba dam, we built a dam performance model that can evaluate the construction of the dam through its main parameter and local terrain. The plan of developing new dams through Zambezi river is called a multiple dams system(also named multi-dam system).

2. The Models

2.1. Water Characteristics

2.1.1. The Characteristic of Water Levels
Since the precipitation in Zimbabwe has uneven spatiotemporal distribution, the water level would be different during the dry season and rainy season, accordingly the hydropower generation would be different. The local reservoir operation is an annual work, which determines the time of opening a water outlet according to the annual precipitation to regulate the runoff. Based on the characteristic[6-8] of water levels, we divide the reservoir into 5 levels as shown in Fig.1 below.

![Fig. 1 Characteristic of water levels](image)

2.1.2. Hydropower Generation in Different Seasons
During the rainy season, if the water level reaches to flood control level before releasing water, then the water of overlap reservoir capacity and utilizable capacity can be used for hydropower generation[9-11].
Rainy season hydropower generation:

\[ Q_{\text{rain}} = \frac{n_{\text{rain}}q_s g (H^2 - H_0^2)}{2} \]  

while \( n_{\text{rain}} \) indicates the times of generating.

Dry season hydropower generation:

\[ Q_{\text{drought}} = \frac{n_{\text{drought}}q_s g (H^2 - H_0^2)}{2} \]  

while \( n_{\text{drought}} \) indicates the times of generating.

2.1.3. Annual Generation

We search the monthly precipitation of catchment area[10,11], which would become water flow in Kariba reservoir. We assume the precipitation in the catchment area would flow into the reservoir in a certain ratio. The times of generating during the rainy season

\[ n_{\text{rain}} = \frac{N}{S(H-H_0)} \]  

While \( N \) indicates the amount of water that can be used for generating; \( S \) indicates the area of the reservoir.

2.2. Water Storage Capacity Model

We approximate the dam’s water capacity as

\[ v_i = s_i h_i \]

While \( s_i \) is the surface area of the dam; \( h_i \) is the water level.

As for the dams newly built in downstream, they have a basic altitude of water level. After storing water the level would rise \( \Delta h \) and the width of the water surface would also increase from \( w_{\text{former}} \) to \( w_{\text{now}} \). So the added water capacity \( v_i \) is

\[ v_i = \frac{(w_{\text{former}} + w_{\text{now}}) + \sqrt{w_{\text{former}} w_{\text{now}}} \Delta h}{3} \]  

By simulating the shape of the reservoir, which is shown in Fig.2, we can calculate its approximate width, length, and water capacity.
2.3. Single Dam Costs Model
We consider the costs of building one single dam. The costs of building a dam can be divided into building cost and maintenance model. As for a newly built dam, the maintenance cost takes little proportion; while for an old dam like Kariba Dam, the maintenance cost needs to be considered separately.

2.3.1. Building Cost
The construction cost mainly includes the costs on aspects like project design, building materials, and construction. Through studying and comparing the dams of Zambezi river basin, we find that the dams are mostly arch dam[12-14], thus we approximate the dam to cuboid whose crest length is $a$, thickness is $b$ and height is $h$.

The construction of the dam should meet a definite aspect ratio and thickness-height ratio.
(a) Aspect ratio: the ratio of river valley width $L$ and dam’s height $H$
(b) Thickness-height ratio: the ratio of dam’s thickness $T$ and height $H$

If $L/H < 1.5$, $T/H < 0.2$, the dam is a thin arch dam;
If $L/H = 1.5\sim3.0$, $T/H = 0.2\sim0.35$, the dam is a normal dam;
If $L/H = 3.0\sim4.5$, $T/H = 0.35\sim0.60$, the dam is a gravity arch dam.

The volume $V$ of the dam is

$$V = a \cdot b \cdot h$$  \hspace{1cm} (6)

Because of the particularity of the Kariba dam structure, the ratio of its top thickness to its maximum bottom thickness makes the sides of the dam closer to a trapezoid. Take $a$ as the dam’s length, $b$ as the top thickness, $c$ as the bottom thickness, and $h$ as the dam’s height, and modify its volume to

$$V = \frac{(b+c)ah}{2}$$  \hspace{1cm} (7)

If the average cost of $1m^3$ building materials is $m_1$, the total cost of building materials would be $m_1V$. The price of the generator is $m_2$, and the dam needs $n$ generators, so the cost of generators
would be $m_2$. The cost of project design is $m_3$, and the construction cost per stere is $m_4$, so the total cost can be indicated as

$$\text{cost} = V(m_1 + m_4) + nm_2 + m_3$$ \hspace{1cm} (8)

2.3.2. Profits Sources
The dam’s profits mainly come from hydropower generation. Hydropower would also improve the development of industry and agriculture. Considering that the feedback on electricity price would directly influence the profits of hydropower station, so we would not count the profits on industry and agriculture repeatedly.

If the water level before releasing water is $H$, while the water level after releasing water is $H_0$. According to the law of energy conversion, the hydropower can be indicated as

$$\Delta Q = \alpha \rho S \Delta h (H - H_0)$$ \hspace{1cm} (9)

While $\Delta Q$ is the hydropower generated when the water depth is $\Delta H$ and the altitude difference of water level is $H - H_0$. 
$\alpha$ indicates the efficiency of energy conversion; 
$S$ indicates the surface area of the reservoir; 
$\rho$ is the density; 
$g$ is the local acceleration of gravity; 
Integral the formulate above, we get

$$Q = \frac{\alpha \rho S g (H^2 - H_0^2)}{2}$$ \hspace{1cm} (10)

When discussing the profit of the dam, we also consider the loss caused by the inability to obtain hydropower generation revenue during the dam project.

Therefore, we build the profits model of the dam as:

$$\text{profit} = \begin{cases} 
  m_e Q - \text{cost} - m_e D & \text{new dam} \\
  m_e Q - m_{re} & \text{old dam}
\end{cases}$$ \hspace{1cm} (11)

while $m_e$ indicates electricity price; 
$m_{re}$ indicates the maintenance cost of the old dam; 
$\text{cost}$ indicates the building cost of a new dam. 
$D$ indicates the loss of hydropower generation

3. MultipleDams System
3.1. Location Choice

Through searching for information, we found that after the Southern African Power Pool (SAPP) based on field surveys and hydrological surveys, 9 dam sites in the multi-dam system have been selected. The selection of SAPP is based on the long-term research and observation of the Zambezi River Basin, and it is also consistent with the aspects we should consider when constructing dams. Therefore, we have reason to think that the dam sites of SAPP are scientific. In this paper, these 9 dam sites are selected to form a multi-dam system to verify our model. The name and geographic coordinates of those dams are shown in Tab.1. To make it clear, the dams’ location on the map is shown in Fig.3.

| Number | Name                          | Geographic coordinates |
|--------|-------------------------------|------------------------|
| 0      | Kariba dam                    | 16°31'S, 28°45'E       |
| 1      | Katombora dam                 | 17°50'S, 25°24'E       |
| 2      | Victoria Falls Southern Bank dam | 17°49'S, 25°42'E   |
| 3      | Batoka Gorge dam              | 17°54'S, 26°09'E       |
| 4      | Devil’s Gorge dam             | 17°59'S, 26°52'E       |
| 5      | Mupata Gorge dam              | 15°37'S, 30°04'E       |
| 6      | Cabora Bassa dam              | 15°35'S, 32°42'E       |
| 7      | Mphanada Nkuwa dam            | 15°48'S, 33°13'E       |
| 8      | Boroma dam                    | 16°07'S, 33°30'E       |
| 9      | Lupata dam                    | 16°34'S, 33°59'E       |

**Tab. 1** Number and name and geographic coordinates of chosen dams and Karibadam

**Fig. 3** Location choices
We need to determine the height, water storage capacity, and hydropower generation so that we can obtain costs and benefits. According to the available data [15-17], the investment costs of hydropower stations numbered 2, 3, 5, and 7 can be found, so the known costs are given directly in the table. Based on the model we constructed, the main parameters of these dams can be obtained. Among them, the Katombora Reservoir does not calculate its hydropower generation because its installed capacity is 0.

After determining other smaller dams, there would still be a small Kariba Dam that needs to take a significant duty of hydropower generation and water storage. Since Zimbabwe has planned to send eight generators from Kariba to the national grid, the new Kariba dam still retains four generators. The number of the new small Kariba dam is 0, and the dam parameters of the complete multi-dam system are shown in Tab.2.

| Number | Water storage capacity/ km³ | Height/m | Cost/billion dollars | Hydropower generation/ TJ |
|--------|----------------------------|----------|----------------------|---------------------------|
| 0      | 35.58                      | 30       | 2.05                 | 9351.9                    |
| 1      | 68.24                      | 30       | 2.50                 | 5526.14                   |
| 2      | 0.02                       | 30       | 3.24                 | 22671.36                  |
| 3      | 2.86                       | 181      | 25                   | 17003.52                  |
| 4      | 1.68                       | 140      | 14.79                | 17003.52                  |
| 5      | 0.95                       | 90       | 15.16                | 17003.52                  |
| 6      | 66.13                      | 163.5    | 59.85                | 58803.84                  |
| 7      | 2.76                       | 86       | 23.85                | 37195.2                   |
| 8      | 0.82                       | 30       | 3.68                 | 6291.3                    |
| 9      | 0.96                       | 30       | 5.43                 | 9266.91                   |

Tab. 2 The main parameters of the multi-dam system

Total cost: 155.45 billion dollars
Hydropower generation: 183113.7TJ
Water storage capacity: 180km³

4. Discussion
This paper calculates the performance of multiple dam system with the model we give. To proved the reliability of our model, 2 options that ZRA proposed before are chosen as comparisons. The total solutions are shown below:
Solution1: Rebuilding the existing Kariba dam;
Solution 2: Repairing the existing Kariba dam;
Solution 3: Removing the Kariba dam and replacing it with a series of 10 to 20 smaller dams along the Zambezi river.

Since building a dam is a long-term project and normally the dam would have a risk problem in about 50 years, we decide the assess time limit as 50 years. Here we assume all the dams are built with the same type of materials and electric generators.

When adopting solution (1), the potential costs include the maintenance cost and daily management cost; when adopting solution (2), the potential costs include removing cost, rebuilding cost, and daily management cost; when adopting solution (3), the potential costs include removing cost, cost of building smaller dams and daily management cost. And no matter what solution we choose, the benefits all come from hydropower generation, improving industry and agriculture, and development of tourism.
Through searching for former studies, we learn that the Kariba Dam was completed in 1977 with a total cost of $480,000,000,000. According to the data from the world bank, the cost of Kariba Dam rehabilitation is 2.94 billion dollars. By contrast, the daily management cost is so little that we would not take it into consideration. For the Kariba Dam, which has been built and used for more than 50 years, even repairing can improve its situation, the maintenance cost would constantly increase in the next 50 years, and equal-scale maintenance will be required every 10 years to ensure the normal operation of the dam.

Based on the scale of Kariba Dam, we can calculate the cost of rebuilding the dam with our single dam cost model, including 150 million dollars of removing the dam [18]. We take 10 years as the construction period of the multi-dam system as a whole. To make the comparison result clearer, we give the benefit-input ratio of 3 solutions respectively. The following table shows a comparison of the costs and benefits of the three solutions in 50 years.

| Solution                        | Costs/billion dollars | Benefits/million dollars | Benefit-input ratio |
|---------------------------------|-----------------------|--------------------------|---------------------|
| Repairing Kariba Dam            | 11.76                 | 64                       | 5.44                |
| Rebuilding Kariba Dam           | 25.39                 | 55.04                    | 2.17                |
| Replacing Kariba Dam with dam system | 156.95             | 1004.49                  | 6.40                |

**Tab. 3 Solution’s costs and benefits.**

From Tab.3 we can conclude that repairing the dam earns the least benefits while replacing the dam with smaller dams earns the most benefits. Although replacing the dam with a dam system would take a long time to accomplish and generates less power in short term, in the future the Kariba Dam would need more maintenance cost while the power generation capability cannot compare with the new dams. Therefore, considering in the long term, the benefits difference between repairing the dam and building new dams would keep increasing. Unfortunately, we do not have enough data to calculate how much benefit the dam construction will bring to the local tourism, agriculture, and industry in the future, otherwise, our calculation results will be more comprehensive and closer to the real situation.

Furthermore, adopting multiple dam system can share the risk of disaster, which improves safety and makes the maintenance work more easily. Therefore, replacing the dam with a multiple dam system is considered to be the optimal choice.

5. Conclusions
Due to the instability of the Kariba dam’s plunge pool and sluice gates, the rehabilitation project has become increasingly significant to save the dam from its unstable state. This study proposed a dam performance model and analyzed the performance of a multiple dams system on Zambezi river to replace the existing Kariba dam.

From the perspective of dam performance, we built 3 mathematical models, which focused on generation, water storage, and costs respectively. After choosing the location of the multiple dam system given by SAPP, we focused on the water capabilities to see if the system would have the same or even better performance than the existing dam. We measure the water capabilities by the Hydropower generation and water storage. We also discuss the water management of Kariba dam and multiple dam systems. Finally, the costs and benefits of 3 proposed solutions in 50 years were assessed and the multiple dams system shows out to be the optimal choice, which again proves that our model is promising and can be used in solving real-life dam problems.
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