New Method for Reconstructing Effective Top Quark Spin

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Abstract

We propose a new method for reconstructing an effective spin direction of a semi-leptonically decayed top quark. The method is simple: for instance, it does not require the spin information of the antitop quark in a $t\bar{t}$ event. The reconstructed effective spin is expected to be useful in hadron collider experiments. We demonstrate its usefulness in an analysis of the top decay vertex.
After ten years since the discovery of the top quark [1], as yet only limited experimental data on its properties are available, and details of the nature of the top quark are still to be unveiled. So far, there exists no evidence for significant deviations from the Standard Model (SM) predictions concerning top quark properties. On the other hand, since the mass of the top quark approximates the electroweak symmetry-breaking scale, there are hopes that symmetry-breaking physics may manifest itself through non-standard interactions of the top quark. For detailed examinations of top quark interactions, it is expected that the top quark spin can be used as a powerful analysis tool. This is because, in the SM (and in many of its extensions), the top quark decays before it hadronizes, and the spin information of the top quark is directly reflected to the distributions of its decay products. Hence, we may utilize the top quark spin for disentangling different top quark interactions efficiently. This is in contrast to the other lighter quarks, for which hadronization effects dilute the spin information at the quark level severely.

In a future $e^+e^-$ linear collider experiment, we will be able to utilize the spin of the top quark quite efficiently. This is because produced top quarks will naturally be polarized, due to parity-violating nature of the interactions in the top quark production process. The top quark polarization can even be controlled or raised to high values by tuning the polarization of the initial electron beam.

By contrast, spin reconstruction of top quarks in hadron collider experiments is a non-trivial task. At Tevatron and LHC, top quarks are produced mainly through $t\bar{t}$ production processes, and these top quarks are known to be hardly polarized [2]. Two types of methods have been proposed and studied for utilizing the top quark spin at these colliders. One is to take advantage of the correlation between the top quark spin and antitop quark spin in the $t\bar{t}$ events. The other method is to use polarized top quarks produced through the single top production process. Unfortunately, so far, not much information has been obtained by applying these methods in analyzing real top quark data in the Tevatron experiments, due to intrinsic disadvantages of the methods. The only analysis that has been performed is a spin-correlation measurement by D0, which put a fairly loose bound on a correlation coefficient [3].

Disadvantages in using the top-antitop spin correlation in the $t\bar{t}$ events are as follows. If we analyze the $t\bar{t}$ events which decay in the dilepton channel (both $t$ and $\bar{t}$ decay semi-leptonically), data statistics is low due to the small branching fraction. Furthermore, there are two missing neutrino momenta in each event, which make reconstruction of the event topology non-trivial. Instead, if we analyze the $t\bar{t}$ events which decay in the lepton + jets channel (one of the top quarks decays semi-leptonically and the other decays hadronically), we have more statistics, but reconstruction of the spin of the hadronically-decayed top quark needs to go through complicated procedures. The reconstruction process is affected significantly by kinematical cuts, and often important information is lost by the cuts. Accurate estimation of the effects of kinematical cuts and event reconstruction is crucial as well. The complex procedures bring in sizable systematic uncertainties in the top spin reconstruction.

Disadvantages in using the single top production process are that there are huge background cross sections for $Wb\bar{b}$ and $Wb\bar{b}+jets$ processes, and that these background cross sections have not been estimated accurately. In fact, up to now the single top production process has not been observed at Tevatron, as opposed to original expectations, due to lack of data statistics and difficulty in the background estimation.

We would expect that, when a huge top quark sample is available at LHC, these difficulties
will eventually be overcome, along with reduction of statistical errors as well as better understandings of systematic uncertainties; see e.g. [4]. On the other hand, it is certainly desirable to develop another method for top spin reconstruction that can be applied to a high statistics sample, and that involves small and controlled systematic uncertainties. In this paper, we propose a new method that can meet such criteria. This method can be applied to a semi-leptonically decayed top quark, without requiring reconstruction of the spin of the antitop quark. We will demonstrate usefulness of our method in an analysis of the top-quark decay vertex. A more detailed application of our method is given in [5], where sensitivities to anomalous couplings in the top decay vertex are studied, using our method, and taking into account realistic experimental conditions expected at Tevatron and LHC. There, it is shown that our method for effective top spin reconstruction is indeed practically useful.

One may be perplexed, since there is no spin vector (polarization vector) associated with an unpolarized state; one may well argue that it is impossible to reconstruct the spin of an unpolarized top quark. While this argument is correct on its own, we can still reconstruct an effective “spin direction” of a top quark, which is practically useful in analyses of top decays.

The method we propose is simple and naive. It is based on the following two well-known facts:

(i) Within the SM, the charged lepton in the semi-leptonic decay of a 100% polarized top quark is emitted preferentially in the top quark spin direction. In fact, at tree level of the SM, the normalized angular distribution of the lepton is given by [6]

$$\frac{1}{\mathcal{N}} \frac{d\Gamma(t \rightarrow bl\nu)}{d\cos\Theta} = \frac{1 + \cos\Theta}{2}, \tag{1}$$

where $\Theta$ denotes the angle between the top spin direction and the lepton direction in the top rest frame. $\mathcal{N}$ represents the normalization constant such that the integral upon $\cos\Theta$ becomes unity. It is known that the one-loop QCD correction hardly modifies the above angular distribution [7].

(ii) If we include anomalous couplings in the top decay vertex, their effects on the lepton angular distribution enter only from quadratic dependences [8]. (All terms linear in the anomalous couplings vanish.) Namely, when the anomalous couplings are small, their effects are very suppressed.

Unpolarized top quarks can be interpreted as an admixture, where one-half of them have their spins in $+\vec{n}$ direction and the other half have their spins in $-\vec{n}$ direction, for an arbitrary chosen unit vector $\vec{n}$. The directions of the charged leptons from the top quarks with $\pm\vec{n}$ spin are emitted preferentially in the $\pm\vec{n}$ direction in the top rest frame. Hence, it seems reasonable to project the direction of the lepton $\vec{p}_l/|\vec{p}_l|$ onto the $\vec{n}$-axis and define an effective spin direction as $\text{sign}(\vec{n} \cdot \vec{p}_l) \times \vec{n}$ for each event. According to eq. (1), in this way we choose the correct direction with probability 75% on average. That we can choose any axis $\vec{n}$, and that any choice is equivalent (if we ignore experimental environment), guarantee the rotational invariance of the unpolarized state of the top quark. Due to the above property (ii), the defined direction is hardly affected by the anomalous couplings in the top decay vertex if they are small, so that it is appropriate for an effective spin direction.

Importance of this definition consists in our finding that certain angular distributions of the top decay products with respect to the effective spin direction reproduce fairly well the
corresponding angular distributions from a truly polarized top quark. This is the case even including anomalous couplings in the top decay vertex. This is the main aspect to be addressed in this article.

Provided that produced top quarks are perfectly unpolarized, and provided that we disregard effects by kinematical cuts and acceptance corrections, there is no difference on which spin axis \( \vec{n} \) we choose to project the direction of the charged lepton. In most part of the paper, we consider this ideal case. We will briefly discuss effects of incorporating realistic experimental conditions at the end.

We start by explaining our setup of the top decay vertex including form factors. We assume that deviations of the top decay form factors from the tree-level SM values are small. Then we consider only those form factors which induce deviations of the differential distributions of top decay products at the first order in the anomalous form factors. That leaves only two form factors \( f_L^1 \) and \( f_R^2 \) in the limit \( m_b \to 0 \) and for onshell \( W \), although the most general \( tbW \) vertex includes six independent form factors [10]:

\[
\Gamma_{tbW} = -\frac{g_W}{\sqrt{2}} V_{tb} \bar{u}(p_b) \left[ \gamma^\mu f_L^1 P_L - i \sigma^{\mu\nu} k_\nu f_R^2 P_R \right] u(p_t),
\]

where \( P_{L,R} = (1 \mp \gamma_5)/2 \) are the left-handed/right-handed projection operators; \( k \) is the momentum of \( W \). For simplicity, we further assume that \( f_L^1 \) and \( f_R^2 \) are real.* At tree level of the SM, \( f_L^1 = 1 \) and \( f_R^2 = 0 \). We will be concerned only with the top decay process \( t \to bW \), where the \( Q^2 \) value is fixed, therefore, we treat the form factors as constants (couplings) henceforth.

Using the above decay vertex and taking the narrow width limit for \( W \), the differential decay distribution of \( W \) and \( l \) in the semi-leptonic decay of a 100\% polarized top quark is given by [10]:

\[
\frac{d\Gamma(t \to bW \to bl\nu)}{d\cos\theta_W d\cos\theta_l d\phi_l} = A \left| (f_R^2 + f_L^1 \frac{m_t}{M_W}) \cos \frac{\theta_W}{2} \sin \theta_l + 2 \left(f_L^1 + f_R^2 \frac{m_t}{M_W}\right) e^{-i\phi_l} \sin \frac{\theta_W}{2} \sin^2 \frac{\theta_l}{2} \right|^2,
\]

with

\[
A = \frac{3G_F |V_{tb}|^2 M_W^2 (m_t^2 - M_W^2)^2}{32\sqrt{2} \pi m_t^3} \times Br(W \to l\nu).
\]

Here, \( G_F \) is the Fermi constant. \( \theta_W \) is defined as the angle between the top polarization vector and the direction of \( W \) in the top quark rest frame. \( \theta_l \) is defined as the lepton helicity angle, which is the angle of the charged lepton in the rest frame of \( W \) with respect to the original direction of the travel of \( W \). \( \phi_l \) is defined as the azimuthal angle of \( l \) around the original direction of the travel of \( W \). A schematic view of the angle definitions is shown in Fig. [11]. The above differential distribution contains fully differential information on the decay \( t \to bW \to bl\nu \).

The corresponding differential distribution with respect to the effective spin direction can be computed in the following way. An arbitrary unit vector \( \vec{n} \) is chosen as the spin axis in the

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*We note that the anomalous couplings for the right-handed bottom quark and the \( CP \)-odd anomalous couplings, which we neglect here, are severely constrained indirectly from the measurements of \( Zb\bar{b} \) vertex at LEP and of \( b \to s\gamma \) process [9]. This may provide another justification for neglecting these form factors in our simplified analysis.
top rest frame. We denote the charged lepton momentum as $p_l$ in the same frame. Then, if $\vec{n} \cdot \vec{p}_l > 0$, we define the effective spin vector to be $\vec{s}_{\text{eff}} = \vec{n}$, whereas if $\vec{n} \cdot \vec{p}_l < 0$, we define the effective spin vector to be $\vec{s}_{\text{eff}} = -\vec{n}$. The angle $\Theta_{\text{eff}}$ between $\vec{s}_{\text{eff}}$ and $\vec{p}_l$ is given by

$$\cos \Theta_{\text{eff}} \equiv \frac{\vec{s}_{\text{eff}} \cdot \vec{p}_l}{||\vec{p}_l||} = \frac{\sqrt{1 - \beta_W^2}}{1 + \beta_W \cos \theta_l} \left( \sin \theta_l \cos \phi_l \sin \theta_W + \frac{\cos \theta_l + \beta_W}{\sqrt{1 - \beta_W^2}} \cos \theta_W \right),$$

(5)

where $\beta_W = (m_t^2 - M_W^2)/(m_t^2 + M_W^2)$ denotes the velocity of $W$ in the top rest frame. $\theta_W$ and $\phi_l$ are defined as in Fig. 1 with respect to $\vec{s}_{\text{eff}}$ (instead of the top polarization vector).

Thus,

$$\frac{d\Gamma(t \rightarrow bW \rightarrow bl\nu)}{d \cos \theta_W d \cos \theta_l d \phi_l}_{\text{eff. spin}} = \left[ \frac{d\Gamma(t \rightarrow bW \rightarrow bl\nu)}{d \cos \theta_W d \cos \theta_l d \phi_l}_{\text{unpol.}} \right] \times 2 \theta(\cos \Theta_{\text{eff}}).$$

(6)

$\theta(x)$ represents the unit step function. Here, the decay distribution from an unpolarized top quark is given by

$$\left[ \frac{d\Gamma(t \rightarrow bW \rightarrow bl\nu)}{d \cos \theta_W d \cos \theta_l d \phi_l}_{\text{unpol.}} \right] = \frac{1}{4} A \left( f_1^L \frac{m_t}{M_W} + f_2^R \right)^2 \sin^2 \theta_l + 4 \left( f_1^L + f_2^R \frac{m_t}{M_W} \right)^2 \sin^4 \frac{\theta_l}{2}. \quad (7)$$

Obviously, it is independent of $\theta_W$ and $\phi_l$, since there is no reference spin vector. Therefore, the dependences on $\theta_W$ and $\phi_l$ of the differential distribution with respect to the effective spin direction enter only through the step function $\theta(\cos \Theta_{\text{eff}})$ on the right-hand-side of eq. (6).

At this fully differential level, $\left[ d\Gamma/d \cos \theta_W d \cos \theta_l d \phi_l \right]_{\text{eff. spin}}$ [eq. (6)] is only a crude approximation to $d\Gamma/d \cos \theta_W d \cos \theta_l d \phi_l$ [eq. (3)]. It can be seen, for instance, from the existence of the step function or from the factorized form of the dependences on $(f_1^L, f_2^R)$ and on $(\theta_W, \phi_l)$ in eq. (6), neither of which is in the structure of eq. (3).

Let us integrate over $\phi_l$ and compare the double angular distributions with respect to the true and effective spin directions:

\[1\text{It would be more accurate to denote these angles as } \theta_{W,\text{eff}} \text{ and } \phi_{l,\text{eff}}, \text{ but to avoid illegibility we use the same notation as in eq. (3).} \]
Figure 2: Normalized double angular distributions for \((f_1, f_2) = (1, 0)\) (a) using the true spin direction and (b) using the effective spin direction, corresponding to eqs. \(8\) and \(9\), respectively. They are normalized to unity upon integration.

\[
\frac{d\Gamma(t \to bW \to bl\nu)}{d\cos\theta_W d\cos\theta_l} = \pi A \left[ \left( f_1^L \frac{m_t}{M_W} + f_2^R \right)^2 \cos^2 \frac{\theta_W}{2} \sin^2 \theta_l \right. \\
+ 4 \left( f_1^L + f_2^R \frac{m_t}{M_W} \right)^2 \sin^2 \frac{\theta_W}{2} \sin^4 \theta_l \left. \right], \quad (8)
\]

\[
\left[ \frac{d\Gamma(t \to bW \to bl\nu)}{d\cos\theta_W d\cos\theta_l} \right]_{\text{eff. spin}} = \left[ \frac{d\Gamma(t \to bW \to bl\nu)}{d\cos\theta_W d\cos\theta_l} \right]_{\text{unpol.}} \times 2 g(y), \quad (9)
\]

where

\[
y = -\frac{\cos \theta_l + \beta_W \cot \theta_W}{\sqrt{1 - \beta_W^2}} \frac{\cot \theta_W}{\sin \theta_l}, \quad g(x) = \begin{cases} 0 & \text{if } x \geq 1 \\
\frac{2\pi}{x} & \text{if } x \leq -1 \\
\pi - 2 \arcsin x & \text{if } -1 < x < 1 \end{cases}. \quad (10)
\]

Numerically these two distributions become reasonably close to each other.\(^\dagger\) This is demonstrated in Figs. 2(a)(b), in which both double angular distributions are displayed for \((f_1^L, f_2^R) = (1, 0)\) (tree-level SM). The distributions are normalized to unity upon integration. Qualitative features of the bulk distribution shape of \(d\Gamma/d\cos\theta_W d\cos\theta_l\) are reproduced by \([d\Gamma/d\cos\theta_W d\cos\theta_l]_{\text{eff. spin}}\).

It has been known that the double angular distribution \(d\Gamma/d\cos\theta_W d\cos\theta_l\) is useful for probing the anomalous coupling \(f_2^R\).\(^\dagger\) To see sensitivities to the anomalous couplings semi-

\(^\dagger\)It is not obvious from the explicit formulas for the distributions. In particular, eq. \(9\) still has a factorized form concerning the dependences on \((f_1^L, f_2^R)\) and \(\theta_W\), which is different from eq. \(8\).
quantitatively, we divide the phase space into four regions as

\begin{align}
\text{Region } A & : \quad -1 \leq \cos \theta_W \leq 0 \quad \text{and} \quad -1 \leq \cos \theta_l \leq 0, \\
\text{Region } B & : \quad -1 \leq \cos \theta_W \leq 0 \quad \text{and} \quad 0 \leq \cos \theta_l \leq 1, \\
\text{Region } C & : \quad 0 \leq \cos \theta_W \leq 1 \quad \text{and} \quad -1 \leq \cos \theta_l \leq 0, \\
\text{Region } D & : \quad 0 \leq \cos \theta_W \leq 1 \quad \text{and} \quad 0 \leq \cos \theta_l \leq 1,
\end{align}

and define the event fraction in each region by

\[ R_i = N^{-1} \int_{\text{Region } i} d \cos \theta_W d \cos \theta_l \frac{d \Gamma(t \to bW \to bl\nu)}{d \cos \theta_W d \cos \theta_l}, \]

where

\[ N = \int_{-1}^{1} d \cos \theta_W \int_{-1}^{1} d \cos \theta_l \frac{d \Gamma(t \to bW \to bl\nu)}{d \cos \theta_W d \cos \theta_l} \]

represents the top-quark partial width to \( bl\nu \). Each \( R_i \) is a function of \( f_2^R/f_1^L \). We also define the event fractions \( R_i^{\text{eff}} \) in the same manner using the effective spin direction instead of the true spin direction.

We compare the dependences of \( R_i \) and \( R_i^{\text{eff}} \) on \( f_2^R/f_1^L \) in Figs. 3(a)(b). From the figures, we see that major features of the \( f_2^R/f_1^L \) dependences of \( R_i \) are reproduced by \( R_i^{\text{eff}} \). In fact, the dependences of \( R_i^{\text{eff}} \) are consistent with the observation that, if we use the effective spin direction, we misidentify the correct spin direction with 25% probability on average. Namely, we misidentify Region \( A \) with \( C \), and Region \( B \) with \( D \), so if we combine \( R_i \)'s in Fig. 3(a) reweighting them with this misidentification probability, we obtain the curves similar to those plotted in Fig. 3(b). Since the \( f_2^R/f_1^L \) dependence of the most sensitive event fraction \( R_i^{\text{eff}} \) is about half of that of \( R_i \), if we use the effective spin direction, we would expect a sensitivity to \( f_2^R/f_1^L \) roughly half of what would be obtained with the true spin direction. A closer examination of the sensitivities to \( f_2^R/f_1^L \) incorporating realistic experimental conditions is given in [9].

The one-loop QCD correction to \( d\Gamma/d \cos \theta_W d \cos \theta_l \) has been computed in [12]. A large part of the correction goes to a variation of the normalization of the partial decay width, which amounts to about 9%. On the other hand, the correction to the normalized double angular distribution is at the level of 1–2% or less. Although the one-loop QCD correction to \( [d\Gamma/d \cos \theta_W d \cos \theta_l]_{\text{eff. spin}} \) has not been computed yet, we expect that it would not be very different from the correction to \( d\Gamma/d \cos \theta_W d \cos \theta_l \). If this is so, we may be able to measure the QCD correction to the normalized double angular distribution at LHC, provided that a good understanding of systematic uncertainties is possible; cf. [5].

We may also compare the angular distributions \( d\Gamma/d \cos \theta_i \) and \( [d\Gamma/d \cos \theta_i]_{\text{eff. spin}} \), where \( \theta_i \) denotes the angle between the direction of particle \( i \) and the top polarization vector or the effective spin direction \( s_{\text{eff}} \) in the top rest frame. The normalized angular distributions are shown in Fig. 4 for \( i = b, W, \nu \) and \( (f_1^L, f_2^R) = (1, 0) \). The lepton angular distributions are trivial, so we do not show them here.) The angular distributions with respect to the true spin direction depend linearly on \( \cos \theta_i \). In this case, it is customary to parametrize a normalized angular distribution by \( \frac{1}{2}(1 + \alpha_i \cos \theta_i) \) and refer to \( \alpha_i \) as a correlation coefficient. Since \( b \) and \( W \) are emitted back-to-back in the top rest frame, \( \alpha_b = -\alpha_W \). The correlation coefficients \( \alpha_i \) corresponding to the true spin direction have been computed in [13]. On the other hand,
the angular distributions with respect to $\mathbf{s}_{\text{eff}}$ are not given by linear functions of $\cos \theta_i$. Their analytic expressions are complicated, which we do not present here. Numerically, the angular distributions for $b$ and $W$ are close to linear shape, while that of $\nu$ is considerably different from linear shape close to $\cos \theta_\nu = \pm 1$.

If we approximate $[d\Gamma/d\cos \theta_i]_{\text{eff, spin}}$ for $i = b, W$ by linear functions of $\cos \theta_i$, the correlation coefficients $|\alpha_b|$ and $|\alpha_W|$ for the effective spin direction are about twice larger than those for the true top spin direction. As for the angular distribution of $\nu$ with respect to $\mathbf{s}_{\text{eff}}$, on average the slope of the distribution is steeper (the correlation between the neutrino direction and $\mathbf{s}_{\text{eff}}$ is stronger) than that of $d\Gamma/d\cos \theta_\nu$. These enhancements in the angular correlations, if we use the effective spin direction instead of the true spin direction, stem from purely kinematical origins. It can be understood as follows. Consider a hypothetical case, in which no correlation between the true spin direction and direction of $W$ exists (the decay is isotropic). Even in this case, there is a positive correlation between the effective spin direction and $W$ in the top rest frame, since the charged lepton is emitted more in the direction of $W$ due to the boost by $W$. Similarly, (hypothetically) even in the absence of any correlation between the true spin direction and neutrino direction, there is a negative correlation between the lepton direction and neutrino direction in the top rest frame, since they are 100% anticorrelated (back-to-back) in the $W$ rest frame. These kinematical effects bias the angular correlations to be stronger if we use the effective spin direction.

We also examined the $f_2^R/f_1^L$ dependences of the angular distributions $d\Gamma/d\cos \theta_i$ and $[d\Gamma/d\cos \theta_i]_{\text{eff, spin}}$. The $f_2^R/f_1^L$ dependences of the latter distributions are much weaker than those of the former distributions. The $f_2^R/f_1^L$ dependences of $[d\Gamma/d\cos \theta_i]_{\text{eff, spin}}$ for $i = b, W$ are reduced as compared to the $f_2^R/f_1^L$ dependences of the double angular distribution eq. (9). This is due to cancellations of $f_2^R/f_1^L$ dependences between $R_A^{\text{eff}}$ and $R_B^{\text{eff}}$, and between $R_C^{\text{eff}}$ and $R_D^{\text{eff}}$; see Fig. 3(b). Insensitivity of $[d\Gamma/d\cos \theta_i]_{\text{eff, spin}}$ to $f_2^R/f_1^L$ stems from a strong (anti)correlation between the effective spin direction (or the lepton direction) and the $\nu$ direction. Since the $f_2^R/f_1^L$ dependences of $[d\Gamma/d\cos \theta_i]_{\text{eff, spin}}$ are weak, it is much more advantageous to use the
double angular distributions [eq. (9)] or event fractions $R^\text{eff}_i$ for gaining sensitivities to $f^R_2/f^L_1$.

Up to now, in defining the effective spin direction, we assumed that the initial top quark is completely unpolarized and neglected experimental environment. In practice, under realistic experimental conditions, different choices of spin basis (axis) $\vec{n}$ lead to different distributions of decay products. Effects of kinematical cuts are by far the largest. Based on detailed Monte Carlo simulation studies incorporating realistic experimental conditions expected at Tevatron and LHC, it is found that the top helicity axis $\vec{p}_t/|\vec{p}_t|$ defined in the $t\bar{t}$ c.m. frame (opposite of the direction of $\bar{t}$ in the top rest frame) is an optimal choice for the spin axis $\vec{n}$. Other choices, such as beamline axis and the off-diagonal spin basis [14], turn out to be inappropriate, since the original distributions [e.g. Fig. 2(b)] are strongly distorted by the effects of kinematical cuts, and also because the sensitivities to the anomalous couplings are much reduced. This can be understood as follows. If we choose the beamline axis, the small $E_T$ (transverse energy) and large $|\eta|$ (pseudorapidity) regions correspond to the regions $\cos\theta_W \simeq \pm 1$, and events that fall into these kinematical regions are rejected by cuts such as the requirements for the minimum transverse energy ($E_T$ cut) or acceptance correction ($|\eta|$ cut) for the lepton and jets. In particular, events in the kinematical regions most sensitive to a variation of $f^R_2/f^L_1$, close to $(\cos\theta_W, \cos\theta_l) = (1, 1)$ and $(-1, -1)$, are lost. At Tevatron, the status of the off-diagonal basis is similar to the beamline axis, since the off-diagonal basis is not very different from the beamline axis. (At LHC, there is no good definition of the off-diagonal basis.) On the other hand, if we choose the top helicity axis, after integrating over all top quark directions, effects of the cuts are averaged over and no significant distortion from the original distribution is found. See [5] for details.

As is clear from the above definition, our method can be applied not only to the hadron collider experiments but also to a future $e^+e^-$ collider experiment. Nevertheless, the primary motivation of our proposal is to use this method at the current Tevatron experiment and at

Figure 4: Normalized angular distributions for $b, W, \nu$ in the top rest frame for $(f^L_1, f^R_2) = (1, 0)$, using the true spin direction (solid lines) and using the effective spin directions (dashed lines).
In summary, we proposed to reconstruct an effective spin direction of a semi-leptonically decayed top quark as the projection of the lepton direction onto an arbitrary chosen axis in the top rest frame. The reconstruction method is simple so that it would be feasible in hadron collider experiments. We demonstrated that this spin direction can be used to probe anomalous couplings in the top decay vertex, through measurements of a double angular distribution or event fractions $R_{\text{eff}}^i$. Under realistic experimental conditions, the top helicity axis seems to be an optimal choice for the spin axis.

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