Analytical solution and characteristics of vertical seismic displacement of truss cable structures

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Abstract. So far, there is no analytical solution for the vertical seismic displacement of truss cable structures via modal superposition spectrum method. First, according to the first author’s previous work, the first eight symmetric and anti-symmetric frequencies and corresponding mode shapes are given. Then, the shape curves of maximum vertical seismic displacement of truss cable structure with 40m, 65m and 72m spans are presented by modal superposition spectrum method. Finally, the correctness of the above analytical solutions is verified by the results of FEM simulations. It is found that: (1) the presented analytical solutions of vertical seismic displacement of truss cable structures are of high accuracy and are simple and practical; (2) The maximum vertical seismic displacement curve of the truss cables has three shapes, which depends on the fundamental frequency and spans. It is found that for the truss cables, the maximum seismic displacement does not necessarily occur in the mid-span. This new finding should attract the attention of engineering designer when designing long-span truss cables.

1. Introduction

Truss cable structure system is a kind of reasonable structural system, and is widely applied in projects that require coverage of large areas. And the truss cable structure system studied in this paper is the lattice structure which consists of profile steel, steel rod and steel tube. It not only has the tensile capacity of flexible cable but also has the bending and shear capacity. In addition, the truss cable structure has the better stability without exerting prestress, which simplifies construction procedure and reduces loads of supporting structure\cite{1}, and it is different from the cable truss structure system. But the two types of cable structure system are all called cable-suspended structures. In recent years, many scholars have researched the static and dynamic characteristics of cable truss\cite{2-6}. However, for truss cable structure, there are not too many scholars to study its mechanical properties. Prof. Shen et al. \cite{7}, Ye and Xu \cite{8}, Liu et al. \cite{1} have studied static behavior of truss cables and acquired many important conclusions. In addition, Prof. Zhang and his teams have carried out many researches on static and dynamic behavior of truss cable structures, they have performed natural vibration analysis of truss cable with ANSYS, energy variation method and Rayleigh method\cite{9-18}. But, so far there is no research on the vertical seismic displacement analysis of truss cable structures under earthquake action. Consequently, the analytical solution of vertical seismic displacement analysis of truss cable structures will be mainly studied via modal superposition spectrum method based on the existing research findings presented by Prof. Zhang and his teams in this paper.
2. Natural vibration analysis of truss cable

2.1 Basic data

According to present code, three types of truss cable are designed in this paper. The ratio of sag to span, the height of cross section, area of upper and lower members, area of diagonal member, and mass of unit length are listed in table 1, the elastic modulus is $2.06 \times 10^{11}$ N/m$^2$ and the simplified calculation model is showed in figure 1.

Table 1. Design parameters of truss cable.

| Number | Span $l$ (m) | ratio of sag to span $f/l$ | depth of section $h$ (m) | $A_a$ | $A_b$ | $A_d$ (cm$^2$) | mass of unit length $m$ (kg/m) |
|--------|-------------|--------------------------|--------------------------|-------|-------|----------------|-----------------------------|
| DT1    | 40          | 1/12                     | 0.5                      | 20.4  | 3.57  | 728.53         |
| DT2    | 65          | 1/14                     | 0.8                      | 29.48 | 10.24 | 782.35         |
| DT3    | 72          | 1/15                     | 1.0                      | 26.56 | 8.4   | 835.26         |

Figure 1. Layout of truss cable [17].

2.2 Natural vibration of truss cable by energy variation method

According to Prof. Zhang’s paper [17], the natural vibration frequencies and corresponding mode shapes of three types of truss cables can be got by energy variation method (EVM), and the first eight symmetric and anti-symmetric frequencies and corresponding mode shapes of natural vibration of truss cables are listed in table 2 and table 3.

Table 2. First four symmetrical frequencies and corresponding mode shapes of natural vibration.

| Number | Frequency $\omega$ (1/s) | Mode shape $A_1$ | $A_3$ | $A_5$ | $A_7$ |
|--------|--------------------------|-------------------|-------|-------|-------|
| DT1    | $\omega_2$ 12.3558       | 0.6251            | -0.7803 | -0.0179 | -0.0039 |
|        | $\omega_3$ 18.0995       | -0.7784           | -0.6250 | 0.0578  | 0.0111  |
|        | $\omega_5$ 34.8799       | 0.0562            | 0.0222  | 0.9982  | -0.041  |
|        | $\omega_7$ 59.9815       | 0.0112            | 0.0040  | 0.0034  | 1.000   |
| DT2    | $\omega_1$ 8.5297        | 0.7849            | -0.6194 | -0.0176 | -0.0035 |
|        | $\omega_3$ 11.792        | -0.6188           | -0.7850 | 0.0298  | 0.0055  |
|        | $\omega_5$ 26.9054       | 0.0322            | 0.0125  | 0.9994  | -0.0020 |
|        | $\omega_7$ 49.1501       | 0.0062            | 0.0022  | 0.0018  | 1.0000  |
| DT3    | $\omega_1$ 7.1184        | -0.9155           | -0.9156 | 0.0167  | 0.0035  |
|        | $\omega_3$ 10.1570       | -0.4012           | -0.9156 | 0.0167  | 0.0032  |
|        | $\omega_5$ 24.4274       | 0.0220            | 0.0086  | 0.9997  | -0.0014 |
|        | $\omega_7$ 43.7822       | 0.0045            | 0.0016  | 0.0013  | 1.0000  |

Table 3. First four anti-symmetrical frequencies and corresponding mode shapes of natural vibration.

| Number | Frequency $\omega$ (1/s) | Mode shape $A_1$ | $A_3$ | $A_5$ | $A_7$ |
|--------|--------------------------|-------------------|-------|-------|-------|
| DT1    | $\omega_1$ 6.4109        | 1                 | 0     | 0     | 0     |
|        | $\omega_4$ 23.4956       | 0                 | 1     | 0     | 0     |
|        | $\omega_6$ 46.9559       | 0                 | 0     | 1     | 0     |
3. Analytical solution of vertical seismic displacement of truss cable structures

3.1 Analytical solution via modal superposition spectrum method

From the 5.3.4 item of code for seismic design of building (GB 50011-2010) in China [19], we know the vertical earthquake action of truss cable can be calculated by the vertical modal superposition spectrum method and the maximum value of vertical seismic influence coefficient $\alpha_{V_{\text{max}}}$ is equal to $0.65\alpha_{\text{max}}$, where $\alpha_{\text{max}}$ is the maximum value of horizontal seismic influence coefficient.

And we assume that three types of truss cable structures are all under frequent earthquake action, and the type of site is secondary site (Ⅱ), the design earthquake group is the first group, the seismic precautionary intensities is 8, the damping ratio $\xi$ of truss cable structure is 0.2. Then, according to item 5.1.4, table 5.1.4-1 and 5.1.4-2 of code for seismic design of building [19], we can get the design characteristic period of ground motion $T_g$ is 0.35s, and the maximum value of horizontal seismic influence coefficients $\alpha_{\text{max}}$ is 0.16 with 8 seismic precautionary intensity and under frequent earthquake action. Thus, according to 5.1.4 and 5.1.5 items of code for seismic design of building[19], we can get the vertical seismic influence coefficients of truss cable structures under earthquake action, which are listed in table 4, and from 5.2.2 item of code for seismic design of building[19], we can get the mode shapes participation factors of truss cable structures under earthquake action, which are listed in table 5 and the maximum vertical seismic displacement curves of truss cables with 40m, 65m and 72 spans under vertical earthquake action with 8 seismic precautionary intensity calculated by modal superposition spectrum method, which are showed in figure 3.

| Number | $\alpha_{V2}$ | $\alpha_{V3}$ | $\alpha_{V5}$ | $\alpha_{V7}$ |
|--------|---------------|---------------|---------------|---------------|
| DT1    | 0.0917        | 0.1319        | 0.1319        | 0.1319        |
| DT2    | 0.0651        | 0.0904        | 0.1319        | 0.1319        |
| DT3    | 0.0537        | 0.0758        | 0.1319        | 0.1319        |

Table 5. Mode shapes participation factors.

| Number | $\gamma_2$ | $\gamma_3$ | $\gamma_5$ | $\gamma_7$ |
|--------|-------------|-------------|-------------|-------------|
| DT1    | 0.461258    | -1.23672    | 0.328315    | 0.190219    |
| DT2    | 0.731424    | -1.1118     | 0.299273    | 0.191393    |
| DT3    | -0.99065    | -0.89073    | 0.279658    | 0.179447    |

3.2 FEM Verification via ANSYS spectrum analysis

Spectrum analysis method in ANSYS software can be used to analyze seismic response of structure. There are mainly six steps to analyze seismic response of structure by ANSYS, such as establishment of model, modal solution, spectrum solution, expansion of modes, combination of modes, observation of results[20].

From Prof. Zhang’s paper[17], we can get the maximum vertical seismic displacement curve of 40m truss cable under vertical earthquake action (with 8 seismic precautionary intensity) by spectrum analysis and show the results in figure 2. In order to verify the correctness of the modelling process and spectrum analysis of truss cables with ANSYS, the maximum vertical seismic displacement results
of the same 40m truss cable obtained from the spectrum analysis of ANSYS are also shown in figure 2.

![Figure 2. Maximum seismic displacement of 40m truss cable.](image)

Figure 2. Maximum seismic displacement of 40m truss cable.

From figure 2, it is found that the maximum vertical seismic displacement curve of the span truss cable obtained by Prof. Zhang’s paper [17] is similar to the one obtained by ANSYS in this paper. This indicates that the ANSYS modelling process and spectrum analysis of truss cable are validity.

Then, we can further study seismic response analysis of the truss cables with different spans under earthquake action by ANSYS spectrum analysis. The maximum vertical seismic displacement curves of truss cables with 40m, 65m and 72m spans obtained by spectrum analysis with ANSYS under earthquake action with 8 seismic precautionary intensity are shown in figure 3.

![Figure 3. Maximum seismic displacement along span with 8 seismic precautionary intensity.](image)

Figure 3. Maximum seismic displacement along span with 8 seismic precautionary intensity.

From figure 3, it is found that the variation curve of maximal displacement along span of truss cable obtained by modal superposition spectrum method is much more like the variation curve of maximal displacement obtained by ANSYS spectrum analysis. This shows that modal superposition spectrum method presented in this paper is correct and of high accurate. Consequently, this method is more suitable for engineers in initial design of vertical seismic displacement analysis of truss cables under earthquake action. In addition, the displacement maximum values of 72m truss cable are larger than those truss cables with 40m and 65m spans, which indicates that the displacement maximum value of truss cable under earthquake action increases with the increase of truss cable span. Furthermore, it is found that the maximum vertical seismic displacement curves of the truss cables have three shapes, which depend on the fundamental frequency and spans. The node maximum displacement of 72m truss cable occurs in the mid-span of the truss cable, as expected under earthquake action. However, the node maximum displacement of 40m truss cables appears near 1/5 span point, rather than in the mid-span of the truss cables as expected; Although the node maximum displacement of 65m truss cable occurs in the mid-span of the truss cable as expected, but the value near 1/5 span point is also very remarkably large and the resulting force should be remarkable. Therefore, these unique phenomena should attract the attention of engineering designer when designing long-span truss cables.

4. Conclusions
The following conclusions are obtained.

(1) Based on the analytical solution presented in Prof. Zhang’s paper [17], the natural vibration
frequencies and corresponding mode shapes of truss cables can be calculated easily. Then the analytical solution of the seismic displacement of truss cables can be deduced by modal superposition spectrum method under earthquake action. This new analytical solution provides a new calculated method for vertical seismic displacement analysis of truss cables under earthquake action.

(2) The node maximum displacement of 40m truss cable doesn’t always occur in the mid-span of truss cables as expected, namely, there may be different peak point in the maximum displacement curve along the span of truss cables with different spans. This new finding may help engineers to better understand the complicated characteristics of vertical seismic displacement of truss cables under earthquake action.

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