Limitations of model-fitting methods for lensing shear estimation

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ABSTRACT

Gravitational lensing shear has the potential to be the most powerful tool for constraining the nature of dark energy. However, accurate measurement of galaxy shear is crucial and has been shown to be non-trivial by the Shear TEsting Programme. Here, we demonstrate a fundamental limit to the accuracy achievable by model-fitting techniques, if oversimplistic models are used. We show that even if galaxies have elliptical isophotes, model-fitting methods which assume elliptical isophotes can have significant biases if they use the wrong profile. We use noise-free simulations to show that on allowing sufficient flexibility in the profile the biases can be made negligible. This is no longer the case if elliptical isophote models are used to fit galaxies made up of a bulge plus a disc, if these two components have different ellipticities. The limiting accuracy is dependent on the galaxy shape, but we find the most significant biases (∼1 per cent of the shear) for simple spiral-like galaxies. The implications for a given cosmic shear survey will depend on the actual distribution of galaxy morphologies in the Universe, taking into account the survey selection function and the point spread function. However, our results suggest that the impact on cosmic shear results from current and near future surveys may be negligible. Meanwhile, these results should encourage the development of existing approaches which are less sensitive to morphology, as well as methods which use priors on galaxy shapes learnt from deep surveys.

Key words: gravitational lensing – methods: data analysis – cosmology: observations.

1 INTRODUCTION

Dark energy dominates the mass energy of the Universe and the goal to discover the nature of dark energy, or even whether it truly exists, is of paramount importance in cosmology. Cosmic shear provides one of the most promising methods for constraining the nature of dark energy (Albrecht et al. 2006; Peacock & Schneider 2006). Cosmic shear is the mild distortion of distant galaxy images due to the bending of light by intervening matter. Typically galaxy images are stretched by only a few per cent, for example an intrinsically circular galaxy image would become an ellipse with major-to-minor axis ratio of about 1.06. The clumpier the intervening dark matter, the greater the distortions. Dark energy affects the rate of gravitational collapse, therefore it can be investigated by measuring cosmic shear at different times in the history of the Universe.

A number of observational surveys are planned to capitalize on this, including ground-based projects KIlo-Degree Survey (KIDS), the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS),1 the Dark Energy Survey (DES)2 and the Large Synoptic Survey Telescope (LSST),3 and space missions the International Dark Energy Cosmology Survey or Euclid and/or the Joint Dark Energy Mission (JDEM). If we are to fully utilize the potential of these future cosmology surveys then the potential systematics associated with measuring cosmic lensing must be understood and controlled. The main areas for work are (i) measurement and calibration of galaxy redshifts (ii) measurement and subtraction of galaxy intrinsic alignments and (iii) accurate shear measurement from images. In this paper, we focus on the last of these.

Shear measurement is difficult because (i) images are convolved with a kernel due to the atmosphere, telescope optics and measurement devices, (ii) they are then pixelized and (iii) they are noisy mainly due to the finite number of photons collected. The convolution kernel [usually referred to as the point spread function (PSF)] is typically a similar size to the unconvolved galaxy image and is generally not circular. It must be accurately measured either from a detailed model of the telescope or, more usually, from stars in the image, which can be treated as point objects before the convolution. Many works, including this paper, focus on the case where the PSF is perfectly known. However, the shear measurement problem is still very difficult due to the high noise levels in the images and the

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very small signals that need to be measured. The signal-to-noise ratio on shear measurement from any single galaxy image is typically about 0.1, and the signal from many millions of galaxies must be combined to make useful measurements of cosmology.

The Shear TEsting Programme (STEP) (Heymans et al. 2004; Massey et al. 2007) is a collaborative effort to quantify the biases associated with current shear measurement methods. Crucially, the programme has validated the implementation of the Kaiser, Squires & Broadhurst (1995) (KSB) method by several groups to obtain shears from real data. In brief, the method measures the quadrupole moments of the image which are combined to estimate the ellipticity of the galaxy. The presence of noise in the images requires the addition of a weighting factor. It is now widely believed however that KSB methods will not be sufficiently accurate to obtain shears from future surveys observing billions of galaxies.

Several groups are working on model-fitting methods to obtain shear, using either Gaussian-weighted Hermite polynomials (‘shapelets’) to model the galaxy (Bernstein & Jarvis 2002; Nakajima & Bernstein 2007) or elliptical profiles (Kuijken 1999; Bridle et al. 2002; Kuijken 2006; Irwin, Shmakova & Anderson 2007; Miller et al. 2007; Kitching et al. 2008). Alternatively, statistics from shapelets can also be considered as shear estimators that generalize and improve on weighted quadrupole moments (Refregier 2003; Refregier & Bacon 2003; Massey & Refregier 2005). The GRavitational Lensing Accuracy Testing 2008 (GREAT08) Challenge (Bridle et al. 2009) has recently been run to draw expertise from researchers in statistical inference, inverse problems and computational learning.

There is a large variety of galaxy morphologies, whereas the amount of information in any single typical galaxy image is extremely small. Model-fitting methods must therefore make some assumptions. Lewis (2009) has shown that both the PSF and the galaxy shape must be accurately modelled to remove biases; in particular the paper proves that this is a direct result of the symmetries broken by the PSF. In this paper, we concentrate solely on the galaxy model, quantifying the bias on the shear for models using elliptical profiles and assume the PSF is known precisely. In addition we assume infinite signal-to-noise ratio. We first consider the case where the simulated galaxy also has elliptical isophotes, adopting the widely used de Vaucouleurs and exponential profiles. We also consider more realistic simulated galaxies with non-elliptical isophotes, in particular two-component systems representing early (elliptical) and late-type (spiral or disc-dominated) galaxies in which each component has a different profile shape and ellipticity.

The paper is organized as follows. In Section 2, we summarize the equations governing gravitational shear and describe the method used to quantify the accuracy of the shear measurement method. In addition, we discuss the requirements on the accuracy for future DESs. In Section 3, we describe the simulations used to test the method and in Section 4 we describe the shape measurement method. We then present results for different galaxy shapes in Sections 5 and 6. Finally we discuss the implications of these results on the development of future methods in Section 7.

2 SHEAR ESTIMATION

2.1 Gravitational shear

Light from a source passing a thin lens at position \( \theta \) in the lens plane suffers a deflection through an angle \( \alpha \) given by

\[
\alpha = \nabla \psi (\theta),
\]

where \( \psi \) is the dimensionless gravitational potential of the lens projected along the line of sight. If \( \beta \) is the true position of the source then the observed position \( \theta \) is related to \( \alpha \) through the lens equation

\[
\alpha (\theta) = \theta - \beta.
\]

where we have assumed the source is at infinite distance from the observer. The gravitational potential of the lens at \( \theta \) is related to its surface mass density \( \Sigma (\theta) \) via the Poisson equation

\[
\nabla^2 \psi (\theta) = \frac{\Sigma (\theta)}{\Sigma_{\text{crit}}} = 2 \kappa (\theta),
\]

where \( \kappa (\theta) \) is the convergence and the critical surface density is

\[
\Sigma_{\text{crit}} = \frac{c^2}{4 \pi G D_s D_{ls}},
\]

where \( D_s \), \( D_l \) and \( D_{ls} \) are the angular diameter distances between the observer and the source, the observer and the lens and the lens and the source, respectively.

Differentiating equations (1) and (2) with respect to \( \theta \), we obtain the Jacobian or magnification matrix, relating the apparent position \( \theta \) to the unlensed position \( \beta \) in terms of the gradients of the gravitational potential

\[
M = \frac{\partial \beta}{\partial \theta} = \left( \begin{array}{cc} 1 - \psi_{11} & -\psi_{12} \\
-\psi_{21} & 1 - \psi_{22} \end{array} \right),
\]

Defining the complex gravitational shear as

\[
\gamma = \gamma_1 + i \gamma_2,
\]

with

\[
\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22}), \quad \gamma_2 = \psi_{12} = \psi_{21},
\]

the magnification matrix becomes

\[
M = \left( \begin{array}{cc} 1 - \kappa & -\gamma_2 \\
\gamma_2 & 1 - \kappa + \gamma_1 \end{array} \right).
\]

Under this transformation, an object with intrinsic complex ellipticity given by

\[
e^i = \frac{a - b}{a + b} e^{i \phi},
\]

where \( a \) and \( b \) are the major and minor axes and \( \phi \) is the orientation of the major axis from the \( x \)-axis, is sheared to an object with observed complex ellipticity, \( e^o \), given by

\[
e^o = e^i + g \frac{e^i + g}{1 + g^2 e^i},
\]

(Seitz & Schneider 1997), where \( g = \gamma/(1 - \kappa) \) is the reduced shear and we have assumed \( |g| < 1 \). The reduced shear is equal to the shear \( \gamma \) when \( \kappa = 0 \), which we assume throughout this paper.

2.2 Quantifying the bias on the shear estimator

Shape noise is the statistical noise arising from the random distribution of galaxy shapes. We quantify the bias on the shear measured for different galaxy shapes in the absence of shape noise (i.e. in the limit of an infinite number of galaxy orientations). To achieve this, we follow Nakajima & Bernstein (2007) by performing a ‘ring test’, whereby the same galaxy is rotated around a ring prior to shearing. The mean ellipticity over the ring provides a shear estimate which, as explained below, is free from shape noise to first order. Our shear estimator, \( \tilde{\gamma} \), is the measured galaxy ellipticity, \( e^m \). For a perfect
shear measurement method, the measured ellipticity is equal to the true observed ellipticity, given in equation (10). Even in this case, averaging over galaxy orientations $i$ gives the following expression for the mean true observed ellipticity
\[ \langle e'_{ij} \rangle = y_i + \langle e_i \rangle + \langle (\epsilon_i + \gamma_i) - y_i \epsilon_i + O(\gamma_i) \rangle \]
where $\gamma_i$ is the true input shear. The term $\langle e_i \rangle$ is zero for a pair of identical galaxies rotated by 90° from each other. Measuring biases for galaxy pairs was suggested by Nakajima & Bernstein (2007) and adopted in the STEP2 simulations (Massey et al. 2007) as a useful method for reducing the intrinsic shape noise. We find that using three linearly spaced pairs of galaxies in the ring test is enough to reduce the total contribution to the shape noise (i.e. including higher order terms in the sum in equation 11) to a negligibly small value. To test the effects of PSF convolution and pixelization on the accuracy of our (non-perfect) shear measurement method (i.e. in which $e^\text{m} \neq e^\text{s}$), we use a set of galaxies rotated by 10°, 20°, and 30° from each other. Measuring biases on the shear measured do not change if we double the number of angles used.

We quantify the bias on the shear estimator in terms of multiplicative and additive errors, $m_i$ and $c_i$ respectively, following Heymans (2006), such that
\[ \hat{y}_i = (1 + m_i) y_i + c_i, \]
where we assume there is no cross contamination of, e.g., $\hat{y}_i$ depending on the value of $y_i$ or vice versa. We measure $m_i (m_2)$ by shearing along (at 45°) the $x$-axis with a magnitude of 0.03, i.e. we measure $m_1$ by shearing using $y_1 = 0.03$, $y_2 = 0$ and $m_2$ by shearing using $y_1 = 0$, $y_2 = 0.03$.

### 2.3 Bias requirements for future surveys

Amara & Réfrégier (2008) derived requirements on $m_i$ and $c_i$ for general current and future surveys covering $A \text{deg}^2$ of sky, with $n_{\text{gal}}$ galaxies per arcmin$^2$ and with a median redshift $z_0$ (their equations 21 and 22). They consider general functional forms for the redshift evolution of these parameters and the systematic biases from shear calibration to be less than the random uncertainties for a two-parameter dark matter bias.

We consider three sets of survey parameters ($A, n_{\text{gal}}, z_0$): (170, 12, 0.8), (5000, 12, 0.8) and (2 $\times$ 10$^4$, 35, 0.9). These parameter sets are chosen to represent the Canada–France–Hawaii Telescope Legacy Survey (CFHTLS), the DES and Euclid. We assume the limit on the additive error $c_i$ is equal to the limit on $\sigma_{\text{sys}}$ in their equation (21); therefore, this gives the limits given in Table 1 for each of the three fiducial surveys.

The GREAT08 Challenge (Bridle et al. 2009) has set a target accuracy level, described by the quality factor $Q$. The quality factor can be related to the $m$ and $c$ values via the equation
\[ Q = \frac{10^{-4}}{m^2 c^4 + c^2_i}, \]
where $i$ refers to the two shear components, and we have written $\sigma_y$ as the residual shear used in the simulation (technically, this is the reduced shear rather than the shear) and assumed that $m_i$ and $c_i$ are the same for all data. We further assumed that the mean true shear in the simulation is zero. Typically $\sigma_y \sim 0.03$ for cosmic shear. The GREAT08 Challenge has set a target accuracy level of $Q = 1000$. Therefore, if $m_i = 0$ then this corresponds to the Euclid requirement on $c_i$.

### 3 SIMULATIONS

We next describe the simulations we have used to investigate biases in shear measurement. In Section 5, we investigate shear measurement from simulated de Vaucouleurs and exponential profiles and in Section 6 two-component galaxies in which each component has a different Sérsic index and ellipticity. Therefore, here we discuss the two different galaxy profiles considered, the method used for convolution and the two-component models.

#### 3.1 Galaxy profiles

Galaxies are broadly classified in the literature as ellipticals, pure spheroids or spheroid (bulge) plus disc systems. The de Vaucouleurs profile has long been used to model the light from elliptical galaxies (de Vaucouleurs 1948) and the exponential profile provides a good description of disc galaxies both in the local Universe (Freeman 1970; Kormendy 1977; de Jong 1996; MacArthur, Courtney & Holtzman 2003) and at high redshift (Elmegreen et al. 2005). Historically, pure spheroids and bulge components have also been modelled using a de Vaucouleurs profile, though recent studies have revealed a range of profile shapes (Graham & Worley 2008).

Both the de Vaucouleurs and exponential profiles belong to a family of functions known as the Sérsic profiles (Sérsic 1968). The Sérsic intensity at position $x$ is given by
\[ I(x) = A e^{-b(x-x_0)^2} C(x - x_0)^{1/2n}, \]
where $x_0$ is the centre, $A$ is the peak intensity, $n$ is the Sérsic index and $C$ (proportional to the inverse covariance matrix if $n = 0.5$) has elements
\[ C_{11} = \frac{\cos^2 \phi + \sin^2 \phi}{a^2 - b^2} \]
\[ C_{12} = \frac{1}{2} \left( \frac{1 - \frac{1}{a^2}}{\frac{1}{b^2}} \right) \sin(2\phi) \]
\[ C_{22} = \frac{\sin^2 \phi + \cos^2 \phi}{a^2 + b^2} \]
where $\phi$ is the angle (measured anticlockwise) between the $x$-axis and the major axis of the ellipse and the minor-to-major axis ratio is $b/a$. The Sérsic index defines the profile ‘type’, with $n = 0.5$, 1 and 4 for Gaussian, exponential and de Vaucouleurs profiles, respectively. If $k$ is defined as $k = 1.9992n - 0.3271$ then for a circular profile $r_c = a = b$, referred to as the ‘effective radius’ or ‘half-light radius’, is the radius enclosing half the total flux. (Note that for a Gaussian profile $a^2$ and $b^2$ are the two-dimensional variances if $k = 0.5$; for the exponential profile $h = a = b$ is known as the ‘scalelength’.
461

we define pixels in size. The PSF FWHM is \(4.14/\Gamma(1)\) pixels.

We find that the results are largely insensitive to changes in the centroid position within the central pixel.

**3.2 Shear and convolution**

We model the PSF as a single Gaussian aligned along the x-axis with ellipticity \(e_p = 0.05\) and FWHM of 2.85 pixels. We define the FWHM of an elliptical object such that the area of the ellipse is equal to the area of a circle with the same FWHM. The default value used for the galaxy ellipticity is \(e = 0.2\). The galaxy size is chosen such that the FWHM of the PSF-convolved image is \(\sim 1.5\) times that of the PSF.\(^4\) Galaxies smaller than this are generally cut from catalogues used in weak lensing analyses.

The size, defined here as the product of the major and minor axis, and ellipticity, \(|e^2|\), of the unlensed galaxy is the same at each point in the ring. We use equation (10) to calculate the post-shear complex ellipticity, \(e_s\), of each galaxy used in the ring test. The post-shear size is equal to the unlensed size divided by a correction factor equal to \(1 - |e^2|\). For the two-component galaxies, we compute the size and complex ellipticity of the bulge and disc separately.

Fig. 1 shows the relationship between the galaxy effective radius and the FWHM of the PSF-convolved image for Gaussian, de Vaucouleurs and exponential profiles. The horizontal dashed line shows the value used in this study. Fig. 2 shows cross-sections through the galaxy and PSF-convolved galaxy profiles for the chosen galaxy parameters, compared with a Gaussian galaxy image. We see that the de Vaucouleurs has an extremely sharp galaxy profile before the PSF convolution and larger wings after convolution.

By default the galaxy is convolved numerically with the PSF on a large, fine grid (25 \(\times\) 45) pixels in size. The PSF FWHM is sampled by (2.85 \(\times\) 45) pixels. Following the convolution the grid is binned up by a factor of 45 to obtain a square image 25 pixels across in which the FWHM of the PSF is 2.85 pixels. Finally, we cut the grid down to obtain a postage stamp 15 pixels across. We try increasing the resolution used for the convolution such that the PSF FWHM is sampled by (2.85 \(\times\) 55) pixels. The grid is (25 \(\times\) 55) pixels in size and, following the convolution, is binned up by a factor of 55. We also try increasing the size of the grid used for the convolution to (31 \(\times\) 45) pixels, keeping the PSF FWHM at the default value and binning up by a factor of 45. In both cases, it is the central 15 pixels which are analysed. We find that the results do not change when we increase either the resolution or the grid size used for the convolution.

The true galaxy centroid is at the centre of the postage stamp. We find the results are largely insensitive to changes in the centroid position within the central pixel.

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\(\Gamma(1)\) is given by pixels which are analysed. We find that the results are largely insensitive to changes in the centroid position within the central pixel.

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\(^4\) Specifically, we first compute the FWHM of the galaxy for the case where both the PSF and the galaxy are circular and the FWHM of the PSF-convolved image is 1.5 times that of the PSF. We then adjust the FWHM of the galaxy to keep the area of the ellipse constant as the ellipticity is increased.
3.3 Two-component models

As discussed above, the de Vaucouleurs and exponential profiles are widely used to describe the light distribution in elliptical and disc galaxies. However, real galaxies do not have constant ellipticity isophotes. Therefore, in this paper, we also explore galaxies with both a bulge and a disc component and, crucially, with non-constant ellipticity isophotes, since we allow the bulge and disc to have different ellipticities.

We consider two different two-component systems: one which closely models realistic spiral (disc-dominated) galaxies and one which represents ellipticals with a small disc (exponential) component. For the spiral galaxies, the bulge is modelled as a Sérsic profile with index $n_b = 1.5$. While for many years it was believed that bulges were universally described by the $r^{1/4}$ model (de Vaucouleurs 1948; de Vaucouleurs 1958; de Vaucouleurs & Pence 1978), it is now generally accepted that most bulges have Sérsic indices $n < 4$ (Graham 2001; Balcells et al. 2003; MacArthur et al. 2003; Laurikainen et al. 2006; Graham & Worley 2008) and typically between $1 \sim 2$ for a range of Hubble types [Graham & Worley 2008, see their fig. 3]. Studies also suggest that the bulge-to-disc size ratio is reasonably independent of Hubble type, with Graham & Worley (2008) finding a median value for $r_d/h$ equal to 0.22. We adopt a similar size ratio, with $r_d/r_b$ equal to 6.5, where $r_d$ and $r_b$ are the disc and bulge effective radii, respectively. Our second model is chosen to represent ellipticals, which are well described by de Vaucouleurs profiles. We therefore set $n_b = 4$ and add a small exponential component ($n_g = 1$) with $r_g/r_b = 0.5$. For both models, the bulge and disc ellipticities are set equal to $e_b = 0.05$ and $e_d = 0.2$, respectively. The bias on the shear is measured for a range of bulge-to-total ($B/T$) flux ratios between 0 and 1. At each $B/T$ value, the ratio $r_d/r_b$ is held constant and the bulge and disc effective radii computed for circular PSF and galaxy profiles such that the FWHM of the PSF-convolved image is 1.5 times the FWHM of the PSF. The total flux in each galaxy component is calculated by integrating the flux from $r = 0$ to infinity, as given in equation (19). The bulge and disc effective radii are then adjusted to keep the area of each component constant as the ellipticity is increased from zero.

4 MODEL FITTING USING SUMS OF GAUSSIANS

In this paper, we model galaxies as a sum of co-elliptical (homeoidal) Gaussians of varying size and amplitude. This model was first suggested by Kuijken (1999) and developed by Bridle et al. (2002) into a publicly available code (IM2SHAPES) which has been used to measure cluster masses (e.g. Cypriano et al. 2004) and tested in the STEP1 simulations (Heymans 2006). We stress that the results found in this paper are general for all models adopting elliptical isophotes since any such model can be completely described in terms of a sum of Gaussians. Adopting a sum of Gaussians to model the galaxy has the particular advantage that the convolution with the PSF can be carried out analytically (assuming the PSF is also modelled as a sum of Gaussians).

If the PSF and galaxy intensity profiles are of the form

$$ I_p(x) = \frac{k}{\pi} C_p |C_p|^{-1/2} e^{-|x-x_0|^2/C_p(x-x_0)} $$

and

$$ I_g(x) = A_g e^{-|x-x_0|^2/C_g(x-x_0)} $$

respectively, then the PSF-convolved intensity for a sum of $n_g$ Gaussians is given by

$$ I_g(x) = \sum_{i=1}^{n_g} A_{g,i} \frac{C_{g,i}}{|C_{g,i}|^{1/2}} e^{-|x-x_0|^2/C_{g,i}(x-x_0)} $$

where

$$ C_{g,i} = \frac{1}{|C_p| + C_{g,i}^2} |C_p| C_{g,i} + C_{g,i}^2 |C_p| $$

The centre, ellipticity and orientation of each Gaussian used to model the galaxy are tied. Thus the number of free parameters in the fit is 4 ($x_0, e, \phi$) plus 2$n_g$ ($A_i, n_g, a_i$). The best-fitting parameters are found using $\chi^2$ minimization. We speed up the calculation by computing the normalizations of the Gaussians analytically. This is possible because the model is linear in these parameters.

Images are generated on a grid $15^2$ pixels in size. The intensity in each pixel is the sum of the intensity computed at the centres of $n_g$ sub-pixels, where we refer to $n_p$ as the pixel integration level. The default pixel integration level used in the simulated galaxies is $n_p = 45$ (see Section 3.2).

5 RESULTS FOR GALAXIES WITH ELLIPTICAL ISOPHOTES

In this section, we simulate galaxies with elliptical isophotes and fit them with different elliptical isophote models. First we try using a single Gaussian when fitting an exponential or de Vaucouleurs profile. We explain our results qualitatively using a one-dimensional toy model. Then we use multiple Gaussians to allow a more accurate fit to the simulations.

5.1 Using the wrong elliptical isophote model

We first ask whether model fitting using a single Gaussian provides an unbiased shear estimate for a galaxy with elliptical isophotes. We use two different profiles to simulate the true galaxy shape: a de Vaucouleurs and an exponential. The default model for the PSF is a single Gaussian aligned along the x-axis with perfectly known ellipticity and size. We first investigate how shear measurement biases vary with the size of the pixels used for the observation when the wrong elliptical isophote model is used. We calculate the biases both with the default PSF model and with the PSF set to a delta function. The default value used in this paper for the PSF FWHM is 2.85 pixels, but in Fig. 3 we vary the resolution from 1 to 15 pixels per PSF FWHM, while keeping the relative size of the galaxy and PSF the same. For the case where the PSF is a delta function, the galaxy size is set equal to that computed for the default PSF model (thus the galaxy size is the same at each point on the x-axis in Fig. 3). We ensure that the resolution is the only quantity which changes as the PSF FWHM is increased. This is achieved by convolving the galaxy with the PSF on a large, fine grid and then binning the pixels to obtain images with decreasing resolution. The convolution is carried out as described in Section 3.2, on a grid $(25 \times 45)^2$ pixels in size, except here the PSF FWHM is sampled by 45 pixels instead of $(2.85 \times 45)$ pixels. The grid is then binned by a factor of $3 (5, 9, 15, 45)$ to obtain an image in which the PSF FWHM is 15 (9, 5, 3, 1) pixels. The pixel integration level used in each pixel in the galaxy and PSF images prior to the convolution is 1, thus each binned PSF-convolved image has a pixel integration level equal to the binning factor. At each PSF resolution we use the same pixel integration level in the galaxy model as used in the simulated galaxy image.
We explain this result qualitatively using Fig. 4, which shows results from a toy problem using a one-dimensional image of infinite resolution. We simulate a galaxy with a one-dimensional exponential profile and convolve it with a Gaussian PSF. The convolved image is then fitted with a Sérsic profile convolved with the correct PSF. The galaxy size (scale radius) is varied to find the best fit, and this is compared to the best fit in the absence of a PSF. The best-fitting size of the galaxy is either overestimated or underestimated, depending on the value of the Sérsic index. The amount by which it is overestimated or underestimated increases as the PSF size increases relative to the galaxy. Fitting a Gaussian galaxy profile (Sérsic index = 0.5) causes the fitted galaxy radius to be more overestimated as the PSF to galaxy size ratio increases.

Consider now a two-dimensional image of a galaxy with elliptical isophotes aligned along the x-axis. Very roughly we can consider biases in the measured ellipticity by considering a one-dimensional slice along the x-axis, where the galaxy radius is at its largest relative to the PSF, and then a one-dimensional slice along the y-axis where the galaxy radius is at its smallest. For an elliptical galaxy, therefore, we expect that if we use a Gaussian to model the galaxy, the size of the major axis will be overestimated less than the minor axis. This will result in a more circular best-fitting object, and the shear will be biased low. By contrast, if instead we fit the exponential galaxy using a de Vaucouleurs profile then, using similar arguments, the estimated shear will be biased high.

This conclusion can also be seen qualitatively by considering the two-dimensional image that is being fit. Without the PSF, each point between the PSF and the galaxy is the same (except for the galaxy pair at 0° and 90°).

The biases for the de Vaucouleurs profile (right-hand panel) are larger than for the exponential profile, which is not surprising considering that it is even further from the single Gaussian used in the fit. Inserting the bias values into equation (13) for the exponential galaxy simulation (left-hand panel) gives $Q \sim 30$, and for the de Vaucouleurs galaxy gives $Q \sim 6$.

We discuss the dotted and dash–dotted curves in Section 6.
around an elliptical isophote has equal weight in the $\chi^2$, but when the PSF is added, different parts of the galaxy profile are weighted differently.

In summary, the presence of a convolution causes a bias in the measured shear of an elliptical object, if the wrong profile is assumed.

5.3 Allowing the right elliptical isophote model

We have found that to obtain an unbiased estimate of the galaxy ellipticity, even when the PSF is known and the pixels are small, the galaxy must be modelled well. Next we improve our model by increasing the number of Gaussians used in the sum. An infinite number of homeoidal Gaussians would allow perfect reconstruction of any elliptical isophote galaxy. In Fig. 5, we show the biases as a function of the number of Gaussians used. We see that the biases reduce to below far-future requirements for both galaxy profiles when four Gaussians are used. For galaxies with an exponential profile only three Gaussians are required in the sum. Note that we do not tune practical computational parameters (especially number of sub-pixels used for pixel integration) for points which already lie well below the requirements for future surveys (darkest shaded area).

In Fig. 6, we plot the biases as a function of the number of sub-pixels used in the pixel integration. The x-axis shows the number of sub-pixels $n_p$ in one direction, so the pixel integration sums over values in $n_p^2$ sub-pixels. Recall that the default value used e.g. in Fig. 5 was $n_p = 13$. Specifically, the biases flatten when limited by the number of Gaussians and decrease when limited by the pixel integration level. If a small number of sub-pixels are used in the fit then the galaxy is more elliptical than in the unpixellated case. This results in an estimated ellipticity which is rounder than the true ellipticity. This effect however cancels out in the ring test, and the decrease in bias with increasing pixel integration is entirely a result of the improvement in the pixel model. We see that $n_p \sim 10$ is more than sufficient for foreseeable future surveys, and $n_p \sim 5$ is sufficient for mid-term surveys. However, $n_p \sim 1$ is insufficient even for current surveys.

Figure 5. Multiplicative (top) and additive (bottom) biases for exponential (left) and de Vaucouleurs (right) profiles as a function of the number of Gaussians used in the fit. The PSF is included. The pixel integration level $n_p$ is 13. Open squares (crosses) show $m_1$, $c_1(m_2$, $c_2)$. The $c_2$ values are smaller than the minimum on the y-axis. Shaded regions as in Fig. 3.

Figure 6. Multiplicative (top) and additive (bottom) biases for exponential (left) and de Vaucouleurs (right) profiles as a function of the pixel integration level, $n_p$. Blue dot–dashed, green dashed, cyan dotted and red solid curves show the biases when two, three, four and five Gaussians are included in the model, respectively. Results for one Gaussian are larger than the maximum value on the y-axis. The PSF is included. Squares (crosses) show $m_1$, $c_1(m_2$, $c_2)$. Shaded regions as in Fig. 3.

6 RESULTS FOR BULGE PLUS DISC GALAXIES

So far all our simulated galaxies have had elliptical isophotes. However, this is not the case in the Universe, and the simple deviation we consider in this paper is a two-component bulge plus disc model. In Section 3.3, we described two fiducial two-component models, one to model a spiral galaxy with a bulge ($r_d/r_b = 7.5$, $n_b = 1.5$, $n_d = 1$), and one to model an elliptical galaxy with a small disc ($r_d/r_b = 0.5$, $n_b = 4$, $n_d = 1$). We repeat the previous shear measurement bias analysis, always using an elliptical isophote model in the fit, despite the non-elliptical isophotes of the simulated images. The purpose is to see whether elliptical isophote models can be used for shear measurement from non-elliptical isophote galaxies.

High-resolution images of the two-component galaxies with $B/T$ flux ratios giving rise to the largest biases on the shear (see Fig. 9) are shown in Fig. 7 before and after convolution with the PSF. The $B/T$ flux ratios for the elliptical- and spiral-like galaxies are $B/T = 0.7$ and $B/T = 0.2$, respectively. Constant intensity contours are shown at radii enclosing 0.1, 10, 50 and 70 per cent of the total flux. The bulges of the elliptical- and spiral-like galaxies are similar in size, with $\sim 10$ per cent of the flux contained within the FWHM of the PSF-convolved image. The elliptical-like galaxy is more compact, with 50 per cent of the flux contained within three times the FWHM. The spiral-like galaxy has an extended disc. For both galaxies, the ellipticities of the intensity isophotes increase with radius.

In Fig. 8, we plot the biases for both of the two-component models described above as a function of the number of (co-elliptical) Gaussians used in the fit. For reference, we also show the results when the bulge ellipticity is equal to the disc ellipticity ($\epsilon_b = \epsilon_d = 0.2$), i.e. the simulated galaxy has elliptical isophotes. When the bulge and disc ellipticities are the same, the biases decrease as the number of Gaussians used in the fit increases. This type of behaviour was already seen in Fig. 5, and the results are slightly different now due to the different galaxy profile arising from the sum of exponential and de Vaucouleurs components.
the failure of the model to take account of galaxies with varying ellipticity isophotes.

For comparison, we also indicate in Fig. 8 the additive biases when the PSF ellipticity is doubled from the fiducial value to $e_p = 0.1$ (green dotted) and halved to $e_p = 0.025$ (green dash-dotted). As expected, the additive biases increase (decrease) when the PSF ellipticity increases (decreases). The largest change in the bias is for the spiral-like galaxy, with a factor of $\sim 7$ decrease in $e_p$ when the PSF ellipticity is halved. The multiplicative biases are not affected by the above changes in the PSF ellipticity from the fiducial value.

The dotted and dash–dotted curves in Fig. 3 show the biases on the shear for the bulge plus disc galaxies described above (with $e_b = 0.05$ and $e_d = 0.2$) as a function of pixel resolution when the galaxy is modelled as a single Gaussian ($n_g = 1$). The results are shown with the default PSF model (red dotted) and with the PSF set to a delta function (blue dash–dotted). The plot shows the relative effects of convolution with the PSF and pixelization. When there is no PSF, the biases decrease as the pixel size decreases. When the PSF is included the biases are large, independent of the pixel size. As discussed earlier, this is consistent with the result found by Lewis (2009) that the shear is unbiased even if a complex galaxy shape is modelled with an elliptical profile if there is no PSF and in the limit of infinite image resolution. Conversely, if the galaxy image is either convolved with a PSF or pixellated or both then the bias on the shear will be zero only if we use the right galaxy model (for infinite signal-to-noise ratio). We note that the biases are larger when the simulated galaxy has radially varying ellipticity isophotes (the dotted and dash–dotted lines are higher than the solid and dashed lines).

The purpose of this paper is to quantify the level of the bias for simple two-component galaxies when the galaxy model is intrinsically limited in its ability to describe the true galaxy shape; in particular when the galaxy model has non-varying ellipticity isophotes. We note that, for realistic pixel sizes ($\sim 2$–3 pixels inside the FWHM of the PSF), the biases resulting from pixelization and convolution with the PSF are similar when we model a two-component galaxy with a single Gaussian.

We next investigate how the size of the bias depends on the amount of flux in each component. In Fig. 9, we plot the biases as

When the bulge and disc have different ellipticities, however, the bias is not reduced by increasing the number of Gaussians beyond $n_g \sim 3$. We have checked that this bias is not due to the finite resolution used for the pixel integration. We conclude that it is

FIGURE 7. Constant intensity contours (black solid) showing the radius enclosing 0.1, 10, 50 and 70 per cent of the total flux overlaid on the galaxy (upper panels) and PSF-convolved galaxy (lower panels) images for two-component elliptical-like ($B/T = 0.7$; left) and spiral-like ($B/T = 0.2$; right) models. The blue dashed contours in the lower panels show the FWHM of the PSF-convolved images (equal to 1.5 times the PSF FWHM). The 70 per cent contour lies outside the image region shown for the spiral-like galaxy. For the default pixel size used in this paper, the image regions shown are $\sim 22$ pixels across.

FIGURE 8. Multiplicative (top) and additive (bottom) biases for two-component elliptical-like ($B/T = 0.7$; left) and spiral-like ($B/T = 0.2$; right) galaxies as a function of the number of Gaussians used in the fit. Blue dashed and red solid lines show results for the fiducial PSF ellipticity ($e_p = 0.05$) for the case where the bulge and the disc have the same ellipticity ($e_b = e_d = 0.2$) and different ellipticities ($e_b = 0.05$, $e_d = 0.2$), respectively. The green dotted and dash–dotted curves show the additive biases for PSF ellipticities of 0.1 and 0.025, respectively, with $e_b = 0.05$ and $e_d = 0.2$. Shaded regions as in Fig. 3.

FIGURE 9. Multiplicative (top) and additive (bottom) biases for elliptical (left) and spiral (right) two-component galaxies as a function of the $B/T$ flux ratio. Red solid and blue dashed curves as in Fig. 8. Shaded regions as in Fig. 3.

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a function of the $B/T$ flux ratio for the spiral and elliptical galaxy models for $n_T = 5$. Again, we include a reference curve for the case where the bulge and disc ellipticities are equal. As expected, the biases fall to the residual level as $B/T$ approaches zero or unity. The biases differ slightly from the reference curve for $B/T = 1$ because the bulge ellipticity is 0.05 for the solid curve but 0.2 for the dashed (reference) curve. The elliptical-like galaxy (left-hand panel) has negligible additive biases and has multiplicative biases below the requirements of upcoming mid-term surveys at all $B/T$ ratios. The behaviour at $B/T = 0.1$ is due to a change in sign of $m_i$ from negative at $B/T = 0$ to positive at higher $B/T$ values.

For the spiral galaxy model, both additive and multiplicative biases peak at $B/T \sim 0.2$. The multiplicative bias at this $B/T$ is small enough for current surveys, but worse than the requirements for upcoming surveys. The additive bias is slightly above the requirements for far-future surveys. Most disc galaxies have $B/T < 1/3$ (Kormendy 2008), with a median value of 0.24 for early-type spiral galaxies (Sa–Sb) and 0.04 for late-type spiral galaxies (Scd–Sm) (Graham & Worley 2008). It is likely that on averaging over all galaxy types, the biases are lower than the requirements for upcoming surveys. However, the exact bias for any particular survey will need to be calculated incorporating the galaxy selection criteria and PSF.

7 DISCUSSION

To fully capitalize on the potential of gravitational lensing as a cosmological probe biases on galaxy shear estimates must be reduced to the sub-per cent level. In this paper, we have shown that the effects of convolution with the PSF make this a non-trivial problem. In particular, the unlensed galaxy must be very accurately modelled even if the PSF is known precisely and the pixels are small. We have isolated this effect by restricting our investigation to noise-free images.

We have illustrated that fitting a single elliptical Gaussian to an elliptical exponential or de Vaucouleurs profile causes no bias on the measured shear, in the unrealistic case where the pixels are infinitesimally small and there is no PSF. For the fiducial galaxy size we chose, application of a realistic PSF causes a significant shear measurement bias, too large even to use single-Gaussian fitting for current cosmic shear data. This illustrates the general point that even if galaxies have elliptical isophotes, a model-fitting method must use a realistic galaxy profile. We explained this qualitatively by considering a one-dimensional toy model. Lewis (2009) proved that the presence of a PSF will result in biased shear estimates when the wrong galaxy model is used. In this paper, we have quantified the level of the bias when the wrong model was used. There is a sum of co-elliptical Gaussians, but stress that our results are general for any model-fitting method using elliptical profiles. We find that if galaxies have elliptical isophotes then a sum of four Gaussians is sufficient for future surveys. For bulge plus disc galaxies increasing the number of Gaussians in the model beyond $\sim 3$ does not significantly reduce the biases.

Earlier versions of LensFit (Miller et al. 2007; Kitching et al. 2008) used a de Vaucouleurs profile to fit galaxies of all types, including exponentials. Thus, this is expected to lead to a small residual bias. We found that using an overly flat profile (Gaussian), the shears were biased low relative to the truth. Our toy model predicts that fitting an overly peaky profile (e.g. a de Vaucouleurs to an exponential) will overestimate the shears.

\textit{im2shape} (Bridle et al. 2002) fits a sum of co-elliptical Gaussians, however there is usually no strong prior on the relative sizes and amplitudes of the components. Therefore, when applied to noisy data it is possible that they might not sum to make a particularly peaky profile, and may produce results closer to those expected from fitting a single Gaussian. This could be rectified by applying priors to the relative sizes and amplitudes of the Gaussians, however, for best results these priors should be tuned to the expected profiles in the data.

This result may also be relevant for shapelet methods, which are based on a Gaussian. If only a low-order shapelet expansion is used then the profile will be less centrally peaked, and have smaller wings, than an exponential or de Vaucouleurs. A similar expansion based on the sech function has been proposed to address these problems (van Uitert & Kuijken, in preparation).

Model-fitting techniques adopting co-elliptical profiles (Bridle et al. 2002; Kuijken 2006; Miller et al. 2007; Kitching et al. 2008) cannot, by definition, provide an exact fit to multicomponent galaxies with varying ellipticity isophotes. We find that this introduces a fundamental limit to the accuracy of these methods which can produce biases on shear measurements from individual galaxies which are too large for future surveys. The size of the bias depends on the true galaxy morphology, and we investigate just two example morphologies over a range in bulge-to-disc flux ratios. The bias is largest for spiral-like galaxies with about 20 per cent of the flux in a bulge component. The precise impact on future surveys would require a detailed modelling of galaxy properties and the survey selection function, and is beyond the scope of this work. Further, it may be possible to use fudge parameters which correct for the biases resulting from model-fitting with elliptical profiles. It is unclear at this stage how well this would work given the wide range of underlying galaxy morphologies.

The galaxies simulated in GREAT08 are comparable to the model used in this paper to represent ellipticals. In addition, the PSF model we use (a single Gaussian) has a similar shape to a Moffat profile with $\beta = 3$, used in GREAT08. Further, we adopt the same PSF FWHM (in pixels), and the galaxy to PSF size ratio is close to the central value in the GREAT08 simulations. From the left-hand panel of Fig. 9, we find $m \sim 3.5 \times 10^{-3}, c \sim 3.5 \times 10^{-5}$ which would give a GREAT08 $Q$ of $\sim 8000$. This is indicative of an upper limit to the GREAT08 $Q$ obtainable by shape measurement techniques using model-fitting with elliptical isophotes.

We note that although the simple bulge plus disc galaxies considered here are only an approximation to real systems which often contain more than two structural components, such as nuclear sources, bars, spiral arms and H\textsc{ii} regions, the results we obtain provide an illustration of the level of bias that may be incurred and show that more detailed simulations would be required to test elliptical isophote model-fitting methods for future surveys. In addition, further investigation is required to quantify the bias level for various (survey-dependent) PSF models (e.g. models including extended wings, dipole moments, etc.). Simulations incorporating complex galaxy and PSF models are anticipated for some GREAT Challenges in the future.

Stacking many galaxies in a similar region of sky should circumvent the dependence on individual galaxy properties, as suggested by Kuijken (1999) and Lewis (2009). If we are interested only in some average shear for these galaxies then this may be measured from the stacked image, from which detailed galaxy substructure will have been washed out to leave an elliptical object with an

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6The latest LensFit version fits a co-elliptical bulge plus disc model. See http://www.physics.ox.ac.uk/lensfit/
ellipticity corresponding to the average shear (in the limit of an infinite number of averaged galaxies). This approach now requires more detailed study to determine its practical feasibility.

We have shown that the underlying galaxy shape must be accurately modelled to obtain unbiased shear estimates. However, considerable information about the galaxy shape is lost when images are pixellated and noise added. The optimal freedom in the model may be determined by a balance which allows the model to account for the wide range of galaxy morphologies while restricting it from fitting to noise spikes. Future shape measurement methods should capitalize on the wealth of knowledge gathered in the field of galaxy shape classification. Information about, for example, the narrow range in bulge-to-disc size ratios observed in spirals (Graham & Worley 2008) could be fed into shape measurement methods using a Bayesian approach. Such methods would need to be fine-tuned for different surveys.

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