ACCRETION ONTO BLACK HOLES FROM LARGE SCALES REGULATED BY RADIATIVE FEEDBACK. III. ENHANCED LUMINOSITY OF INTERMEDIATE-MASS BLACK HOLES MOVING AT SUPersonic SPEEDS

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ABSTRACT

In this third paper of a series, we study the growth and luminosity of black holes (BHs) in motion with respect to their surrounding medium. We run a large set of two-dimensional axisymmetric simulations to explore a large parameter space of initial conditions and formulate an analytical model for the accretion. Contrary to the case without radiation feedback, the accretion rate increases with increasing BH velocity reaching a maximum value at \( v_{\text{BH}} = 2c_{s,\infty} \sim 50 \text{ km s}^{-1} \), where \( c_{s,\infty} \) is the sound speed inside the “cometary-shaped” H II region around the BH, before decreasing as \( v_{\text{BH}}^3 \). When the BH becomes gravitationally bound to the BH and is accreted from the downstream direction. The generalized Bondi–Hoyle–Lyttleton formula approaches the classical Bondi–Hoyle–Lyttleton solution. The increase of the accretion rate with \( v_{\text{BH}} \) is produced by the formation of a D-type (dense) I-front preceded by a standing bow shock that reduces the downstream gas velocity to transonic values. There is a range of densities and velocities where the dense shell is unstable producing periodic accretion rate peaks which can significantly increase the detectability of intermediate-mass BHs. We find that the mean accretion rate for a moving BH is larger than that of a stationary BH of the same mass if the medium temperature is \( T_{\infty} < 10^4 \text{ K} \). This result could be important for the growth of seed BHs in the multi-phase medium of the first galaxies and for building an early X-ray background that may affect the formation of the first galaxies and the reionization process.

Key words: accretion, accretion disks – dark ages, reionization, first stars – hydrodynamics – methods: numerical – radiative transfer

Online-only material: color figures

1. INTRODUCTION

Gravitationally driven gas inflow onto moving point masses such as black holes (BHs) or neutron stars has been described analytically in the 1940s by Bondi–Hoyle–Lyttleton (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Bondi 1952). The generalized formula for the accretion onto a point mass moving with velocity \( v_{\text{BH}} \) is obtained from the Bondi formula by replacing the gas sound speed, \( c_{s,\infty} \), with an effective speed \( v_{\text{eff}} = (c_{s,\infty}^2 + v_{\text{BH}}^2)^{1/2} \).

The accretion rate \( M \propto \rho v_{\text{eff}} \sigma_{\text{eff}} \) is roughly estimated as the gas flux through the effective cross section \( \sigma_{\text{eff}} = \pi r_{\text{eff}}^2 \), where \( r_{\text{eff}} = GM_{\text{BH}}/v_{\text{eff}}^2 \) is the distance at which the escape velocity of the gas equals \( v_{\text{eff}} \).

Despite the similarities between the Bondi and Lyttleton formulae, the second case the accretion onto the BH is not spherically symmetric: most of the gas streams past the BH and is gravitationally focused on the axis of symmetry of the problem. The component of the gas velocity perpendicular to the direction of the BH motion is converted into thermal energy and is mostly dissipated, thus a fraction of the gas becomes gravitationally bound to the BH and is accreted from the downstream direction. The generalized Bondi–Hoyle–Lyttleton formula is

\[
\dot{M}_{\text{BHL}} = \frac{\dot{M}_B}{(1 + v_{\text{BH}}^2/c_{s,\infty}^2)^{3/2}},
\]

where \( \dot{M}_B \) is the Bondi accretion rate onto non-moving BHs \( \dot{M}_B = \pi v^2 \rho \gamma G^2 M_{\text{BH}}^2 c_{s,\infty} \) (assuming isothermal equation of state: \( \gamma = 1 \)). Here, \( M_{\text{BH}} \) is the BH mass, and \( \rho_{\infty} \) is the density of the ambient gas. The term in the denominator of Equation (1) is the only term that accounts for the motion of the point mass, thus for supersonic BH motion the accretion rate decreases with increasing velocity as \( v_{\text{BH}}^3 \).

Numerical simulations of accretion onto moving BHs have confirmed the validity of the Lyttleton equation (Shima et al. 1985; Ruffert & Arnett 1994; Ruffert 1996), even in the presence of non-axisymmetric flows that necessarily arise in three-dimensional (3D) simulations due to hydrodynamical instabilities (Cowie 1977; Matsumi et al. 1987; Fryxell & Taam 1988; Taam & Fryxell 1988; Soker 1990; Koide et al. 1991; Livio et al. 1991; Foglizzo & Ruffert 1997, 1999; Foglizzo et al. 2005).

For the case of accretion onto stationary BHs, the radiation emitted near the gravitational radius of the BH typically reduces the rate of gas supply to the BH from large scales well below the value given by the Bondi formula. This is because X-ray and UV heating increases the sound speed of the gas in the proximity of the BH (Ostriker et al. 1976, 2010; Cowie et al. 1978; Binsoniyi–Kogan & Blinnikov 1980; Krolik & London 1983; Vitello 1984; Wandelt et al. 1984), and because radiation pressure accelerates the gas and dust away from the BH (Shapiro 1973; Ostriker et al. 1976; Begelman 1985; Ricotti et al. 2008). Indeed, the maximum accretion rate that can be achieved in most cases is the Eddington rate. In this limit, the outward acceleration of the gas due to Compton scattering of radiation with free electrons equals the inward gravitational acceleration. For supermassive BHs (SMBHs), the picture is far more complex because the hot ionized region and the Bondi radius extend to galactic scales. Thus, it is necessary to model the evolution of the host galaxy, with its stellar populations and the interstellar medium (ISM) in addition to the accreting SMBH. Despite the complexity of this problem, a significant amount of work exists in the...
literature on the self-regulation mechanisms for the growth of SMBHs at the centers of elliptical galaxies (Ciotti & Ostriker 2001, 2007; Sazonov et al. 2005; Ciotti et al. 2009; Lusso & Ciotti 2011; Novak et al. 2011, 2012), for radiation-driven axisymmetric outflows in active galactic nuclei (Proga 2007; Proga et al. 2008; Kurosawa et al. 2009; Kurosawa & Proga 2009a, 2009b) and more generally on the co-evolution of galaxies and their SMBHs. The study of the cosmological origin and growth of SMBHs is a multi-scale problem often approached using large-scale simulations. Typically, the limited resolution of these cosmological simulations precludes resolving the SMBH Bondi radius, hence, the accretion rate is modeled using sub-grid recipes largely based on the Eddington-limited Bondi formula (Volonteri & Rees 2005; Pelupessy et al. 2007; Greif et al. 2008; Alvarez et al. 2009; Kim et al. 2011; Blecha et al. 2011, 2013), sometimes with the introduction of a normalization parameter for the accretion rate to match observations (Di Matteo et al. 2005, 2008; Springel et al. 2005).

Most interestingly, we have discovered that there are two distinct modes of oscillations with very different duty cycles (6% and 50%), governed by different depletion processes of the gas inside the ionized bubble (see Papers I and II).

This paper is the continuation of our previous work on characterizing how the Bondi–Lyttleton problem is modified by radiation feedback, but here we focus on the growth rate and luminosity of BHs in motion with respect to their surrounding medium: i.e., Lyttleton accretion modified by a radiative feedback loop. Considering the motion of the BH is important for the study of stellar BHs (e.g., Wheeler & Johnson 2011) and IMBHs accreting from the ISM, both in the early universe and in the local galaxies as candidates for ULXs (Krolik et al. 1981; Krolik & Kallman 1984; Krolik 2004; Ricotti & Ostriker 2004; Ricotti et al. 2005; Ricotti 2007; Strohmayer & Mushotzky 2009), and SMBHs in merging galaxies. Both radiation feedback and BH motion are expected to reduce the accretion rate with respect to the Bondi rate, but it is unknown what is the combined effect of these two processes. To the best of our knowledge, the present study is the first to consider radiation feedback effects on moving BHs.

This paper is organized as follows. In Section 2, we introduce basic definitions used throughout this series of papers and describe the numerical simulations. In Sections 3 and 4, we show our simulation results and the analytical model describing the set of simulations, respectively. Finally, we summarize and discuss the implications of our work in Section 5.

2. BASIC DEFINITIONS AND RADIATION-HYDRODYNAMIC SIMULATIONS

For the sake of consistency with the previous papers of this series, we define the dimensionless accretion rate $\lambda_{\text{rad}} \equiv M/M_B$, where $M_B$ is the Bondi accretion rate for isothermal gas ($\gamma = 1$). In Paper II, we found that the mean accretion rate for stationary BHs regulated by radiation feedback is

$$\langle \dot{M} \rangle = \min \{1\% T_{\infty,5}^{5/2} M_B, \eta^{-1} M_{\text{Edd}} \},$$

where we have defined $T_{\infty,5} \equiv T_{\infty}/(10^5 \text{ K})$. Equation (2) is valid for density $n_{\text{H},5} \gtrsim 10^5 M_2^{-1} \text{ cm}^{-3}$ where a similar definition $M_{\text{MB}} \equiv M_5^{\text{MB}}/(10^5 M_5)$ is made. Instead, if $n_{\text{H},5} \lesssim 10^5 M_2^{-1} \text{ cm}^{-3}$, the dimensionless accretion rate in the sub-Eddington regime shows a dependency with the density $(\lambda_{\text{rad}}) \approx 1\% T_{\infty,5}^{5/2} n_{\text{H},5}^{1/2}$, where again we define $n_m \equiv n_{\text{H},5}/10^5 \text{ cm}^{-3}$. The Eddington luminosity and accretion rates are defined as $L_{\text{Edd}} = 4\pi GM_5^2 m_p c \sigma_T = (3.3 \times 10^6 L_5) M_5$, and $M_{\text{Edd}} \equiv L_{\text{Edd}}/c^2$, respectively. Note that we have adopted a definition of $M_{\text{Edd}}$ independent of the radiative efficiency $\eta$, thus for Eddington limited accretion the accretion rate in units of $M_{\text{Edd}}$ is $\dot{m} = m/1/\eta > 1$.

The numerical method used in this paper is the same as in Papers I and II. We run a set of 2D radiation-hydrodynamic simulations using a modified parallel version of the non-relativistic hydrodynamics code ZEUS-MP (Stone & Norman 1992; Hayes et al. 2006) with our photon-conserving UV and X-ray 1D radiative transfer equation solver (Ricotti et al. 2001; Whalen & Norman 2006). Photoheating, photoionization, and chemistry of H/He in multi-frequency are manifested in the code to see how high-energy UV and X-ray photons regulate gas accretion onto moving BHs. Radiation pressure both on electrons and H I is calculated to simulate the effect of momentum transfer from ionizing photons to gas. See Paper I and Paper II for detailed description.
At the inner boundary, in the radial direction, we apply flow-out boundary conditions for the second half ($0$ $\leq$ $\theta$ $\leq$ $\pi$) and gas densities $n_{H,\infty}$ $\approx$ $10^5$ cm$^{-3}$, and $T_{\infty}$ $=$ $10^4$ K. The time duration of the transient phase is proportional to the cross timescale $\tau_*$, and thus inversely proportional to the Mach number. The mean dimensionless accretion rate $\langle \lambda_{\text{rad}} \rangle$ (in units of the Bondi rate) is measured when the accretion reaches a steady state while for the non-steady cases, such as $M$ $=$ $4$, we calculate the time-averaged accretion rate. As found in Paper I, at low gas densities, i.e., $n_{H,\infty}$ $\lesssim$ $10^5$ cm$^{-3}$ for simulations with $M_{\text{bh}}$ $=$ $100$ $M_\odot$, $\langle \lambda_{\text{rad}} \rangle$ is proportional to square root of density ($\langle \lambda_{\text{rad}} \rangle$ $\propto$ $n_{H,\infty}^{1/2}$). Figure 2 shows that after correcting ($\lambda_{\text{rad}}$) for the aforementioned density dependence also found for stationary BH, the same functional form for the accretion rate fits all simulations with different gas densities (large symbols for $\eta$ $=$ $0.1$ and gas densities $n_{H,\infty}$ $=$ $10^2$–$10^5$ cm$^{-3}$) and radiative efficiencies (small pentagons for a simulation with $\eta$ $=$ $0.01$ and $n_{H,\infty}$ $=$ $10^5$ cm$^{-3}$). Open symbols indicate simulations that show non-steady accretion rate due to instabilities of the dense post-shock layer forming in the upstream direction (see Section 4.3). The quasi-periodic oscillations of the accretion rate found in the simulation with $M$ $=$ $4$ have ($\lambda_{\text{rad}}$) divided by the aforementioned density dependence for $\langle \lambda_{\text{rad}} \rangle$ $\propto$ $n_{H,\infty}^{1/2}$.

Figure 1. Accretion rate in units of the Bondi rate as a function of time for simulations with $M_{\text{bh}}$ $=$ $100$ $M_\odot$, $n_{H,\infty}$ $=$ $10^5$ cm$^{-3}$, and $T_{\infty}$ $=$ $10^4$ K. The lines show simulations for BH moving at Mach numbers $M$ $=$ $1$, $2$, $4$, $8$, and $10$ (see labels under each line). The early phase in all the simulations shows oscillatory behavior of the accretion rate due to non-equilibrium initial conditions. However, in most simulations the accretion rate quickly reaches a steady state. The average accretion rate increases with increasing Mach number for Mach numbers from $1$ to $4$ and decreases for larger Mach numbers. Quasi-periodic bursts of accretion are seen in the simulation with $M$ $=$ $4$. (A color version of this figure is available in the online journal.)

3. NUMERICAL RESULTS

3.1. Accretion Rate as a Function of Mach Number

The classical Bondi–Hoyle–Lyttleton accretion predicts a monotonic decrease of accretion rate with increasing velocity of the BH with respect to the ambient gas as in Equation (1). However, our simulations of moving BHs with radiative feedback show a very different dependence of accretion rate as a function of the Mach number.

Figure 1 shows the time evolution of accretion rate for different Mach numbers $M$ $=$ $1$, $2$, $4$, $8$, and $10$ for gas density $n_{H,\infty}$ $=$ $10^5$ cm$^{-3}$. Most simulations, except the one with $M$ $=$ $4$, show steady accretion rates after an initial transient phase that is the result of out-of-equilibrium initial conditions. The time duration of the transient phase is proportional to the cross timescale $\tau_*$, and thus inversely proportional to the Mach number. The mean dimensionless accretion rate $\langle \lambda_{\text{rad}} \rangle$ (in units of the Bondi rate) is measured when the accretion reaches a steady state while for the non-steady cases, such as $M$ $=$ $4$, we calculate the time-averaged accretion rate. As found in Paper I, at low gas densities, i.e., $n_{H,\infty}$ $\lesssim$ $10^5$ cm$^{-3}$ for simulations with $M_{\text{bh}}$ $=$ $100$ $M_\odot$, $\langle \lambda_{\text{rad}} \rangle$ is proportional to square root of density ($\langle \lambda_{\text{rad}} \rangle$ $\propto$ $n_{H,\infty}^{1/2}$). Figure 2 shows that after correcting ($\lambda_{\text{rad}}$) for the aforementioned density dependence also found for stationary BH, the same functional form for the accretion rate fits all simulations with different gas densities (large symbols for $\eta$ $=$ $0.1$ and gas densities $n_{H,\infty}$ $=$ $10^2$–$10^5$ cm$^{-3}$) and radiative efficiencies (small pentagons for a simulation with $\eta$ $=$ $0.01$ and $n_{H,\infty}$ $=$ $10^5$ cm$^{-3}$). Open symbols indicate simulations that show non-steady accretion rate due to instabilities of the dense post-shock layer forming in the upstream direction (see Section 4.3). The quasi-periodic oscillations of the accretion rate found in Paper I and Paper II for stationary BHs are still observed in simulations with low Mach numbers $M$ $\lesssim$ $0.5$, which maintain the main characteristics of spherically symmetric accretion
discussed in Paper I and Paper II. This implies that introducing small systematic subsonic velocity to spherically symmetric accretion does not significantly alter the oscillatory behavior of the accretion. However, the average accretion rate \( \langle \lambda_{\text{rad}} \rangle \) decreases steeply as a function of Mach number in this Mach number range, and \( \langle \lambda_{\text{rad}} \rangle \) at \( M \sim 1 \) is roughly one order of magnitude smaller than for non-moving BHs (including radiation feedback), and three orders of magnitude smaller compared to stationary BHs with no radiative feedback, when all the other parameters are held constant. This decrease of the accretion rate with increasing velocity found for subsonic BH velocities is only qualitatively similar to Bondi–Hoyle–Lyttleton accretion, but does not have the same scaling with BH velocity.

The spherically symmetric accretion model fails for supersonic BH motion (\( M \gtrsim 1 \)). The shape of the H\( \text{\textsc{ii}} \) region makes a transition to a well-defined axis-symmetric geometry, elongated along the direction of the gas flow in the downstream direction, while a bow-shaped dense shell develops in front of the H\( \text{\textsc{ii}} \) region in the upstream direction, significantly affecting the velocity field of the gas inflow. In most simulations, steady-state accretion is achieved for supersonic BH motion, since gas is continuously supplied to the BH without interruption.

Interestingly, as the BH motion becomes supersonic and a bow-shock and dense shell form, \( \langle \lambda_{\text{rad}} \rangle \) increases as a function of Mach number. This is clearly at odds with the results expected from the classical Bondi–Hoyle–Lyttleton model. A Mach number \( M = M_R \) (\( M_R \sim 4 \) for \( T_\infty = 10^5 \) K), at which \( \langle \lambda_{\text{rad}} \rangle \) reaches a maximum value, before starting to decrease with increasing BH velocity. An instability of the dense shell that leads to bursts of accretion rate is observed in some simulations in this Mach number range. This result will be discussed in Section 4.3. At higher Mach numbers (\( M > M_R \)), a steady-state solution is achieved once again since the dense shell does not form due to the high velocity of the gas inflow (as shown in Section 4.1, the I-front transitions from D-type to R-type). For \( M > M_R \), \( \langle \lambda_{\text{rad}} \rangle \) decreases monotonically as a function of Mach number and converges to the Bondi–Hoyle–Lyttleton solution (with no radiative feedback) shown as a dashed line in Figure 2. In this high velocity regime, the I-front becomes R-type and the gas flow is weakly affected by radiative feedback.

### 3.2. Structure of the Gas Flow and H\( \text{\textsc{ii}} \) Region

For Mach numbers \( 1 < M < M_R \), a dense bow shock forms in front of the H\( \text{\textsc{ii}} \) region in the upstream direction, followed by a D-type I-front (see Figure 3). Most of the gas inflow propagates through the bow shock without changing direction, while a small fraction of the gas inflow in the outer parts of the bow shock is re-directed farther from the axis of symmetry. The formation of a bow shock in the upstream direction changes the gas density and velocity behind the shock, while the gas temperature remains relatively unaffected (isothermal shock). Note that the H\( \text{\textsc{ii}} \) region has a cometary shape, with overall length increasing linearly with increasing Mach number. The size of the H\( \text{\textsc{ii}} \) region in the upstream direction is not sensitive to
the Mach number, while in the downstream direction the length of the ionized tail shows a linear relationship with the Mach number as shown in Figure 3. The upper panels in Figure 3 show the changes in the density structure and the H II region shape for $M = 1, 2,$ and 4, respectively. The lower panels show the vector fields over the gas density for each simulation. In the bottom panels, we use a logarithmic scale for the radial direction to better show the motion of gas in the vicinity of the BH. The Bondi radius calculated for the gas temperature inside the H II region is close to, but within the location of the I-front. We will see in the next section that this will allow us to derive simple analytical formulae for the accretion rate using the Bondi formula for the gas inside the H II region and a model for the I-front.

For $M = 1$, the size of the H II region in the downstream direction is ~4 times its length in the upstream direction. The density structure in the downstream direction is very complex as shown in Figure 3. The re-directed gas streams form high-density regions and shocks. However, since most of the gas downstream of the BH is not accreted onto the BH, we will focus on understanding the upstream structures. The size of the H II region in the upstream direction will be discussed in greater detail in Section 4.4.

4. ANALYTIC MODELING

In this section, we show that properties of the gas flow around the moving BH can be understood using a simple 1D model for a standing I-front in the frame of reference moving with the BH. There are two regimes for the I-fronts: an R-type front and D-type front, determined by the BH velocity and density of the ambient medium. As in the previous papers we find that the accretion rate onto the BH can be estimated using the Bondi–Lyttleton formula for the gas inside the I-front, since the effective Bondi radius lays within the ionized region.

4.1. Transition from R-type to D-type I-front

In most simulations, we found that the gas flow reaches a steady-state solution. In these cases, the position of the I-front with respect to the BH is stationary, hence the I-front propagates with respect to the gas upstream with constant velocity $v_{bh}$. Although the geometry of the H II region and the bow shock are clearly not planar, we can approximate the flow as one dimension (parallel to the direction of motion of the BH) near the I-front location in the region around the axis of symmetry of the problem. But we will show that accounting for deviations from the planar symmetry is necessary to understand the simulation results.

Let us start by writing down the equations for the propagation of 1D I-front through a homogeneous medium with density $\rho_\infty$. The density ratio across the I-front can be estimated by solving the mass and momentum conservation conditions $\rho_{\infty}v_{\infty} = \rho_{\infty}v_{bh} = J\mu$, where $J$ is radiation flux and $\mu = 1.27m_H$, assuming the helium becomes singly ionized in the front (Spitzer 1981):

$$\Delta \rho \equiv \frac{\rho_{\infty}}{\rho_{\infty}} = \frac{(1 + M^2) \pm \sqrt{(1 + M^2)^2 - 4M^2\Delta T}}{2\Delta T}.$$  (3)

Here, we have defined $\Delta T \equiv T_{\infty}/T_\infty \geq 1$. Due to the condition for the density ratio in Equation (3) to have real positive values, the Mach number must be $M \leq M_D$, where $D$ stands for dense gas, or $M \geq M_R$ where $R$ refers to rarefied gas. D- and R-critical Mach numbers are, respectively,

$$M_D = \sqrt{\Delta T(1 - \sqrt{1 - 1/\Delta T})} \frac{1}{\Delta T^{\frac{1}{2}}},$$  (4)

$$M_R = \sqrt{\Delta T(1 + \sqrt{1 - 1/\Delta T})} \frac{1}{\Delta T^{\frac{1}{2}}}.$$  (5)

Also, $M_D M_R = 1$, thus $M_D \equiv M_R^{-1}$. In terms of the BH velocity the critical velocities are

$$v_D = c_{s,\infty}(1 - \sqrt{1 - 1/\Delta T}) \frac{c_{s,\infty}}{2c_{s,\infty}},$$  (6)

$$v_R = c_{s,\infty}(1 + \sqrt{1 - 1/\Delta T}) \frac{c_{s,\infty}}{2c_{s,\infty}}.$$  (7)

Only for the special case in which $\Delta T \equiv 1$, we have $M_D \approx M_R \approx 1$ and a solution exists for any BH velocity. However, for all physically motivated cases with $\Delta T > 1$ a solution is not possible for BH velocities $v_D < v_{bh} < v_R$. In our simulations if $v_{bh} > v_R \sim 2c_{s,\infty}$, the analytical solution that reproduces the data is the one with the negative sign in Equation (3). This solution describes a weak R-type I-front that has $\rho_{\infty} \sim \rho_{\infty}$ and supersonic motions of the gas both ahead and behind the I-front (in the frame of reference comoving with the BH).

If $v_D < c_{s,\infty}/2 < v_{bh} < v_R \sim 2c_{s,\infty}$ a solution is not possible. A shock front must precede the I-front, increasing the density and reducing the gas velocity below $v_D$ (D-type solution). This is indeed observed in the simulations. For a D-type front, the density near the front is always lower than the density upstream. Figure 4 shows that the post-shock density and velocity are a function of the Mach number, while the temperature of the shell is always $T_{sh} \approx T_\infty$, i.e., the shock is isothermal ($\gamma = 1$). The ratio between the densities at infinity and behind the isothermal shock is

$$\frac{\rho_{sh}}{\rho_{\infty}} \approx M^2,$$  (8)

which is in good agreement with the simulation results (see Figure 5, left panel). However, for the calculation of the gas velocity in the shell our simplifying assumption of planar geometry fails. Assuming planar geometry, mass flux conservation in the direction of the BH motion implies $\rho_{sh}v_{sh} = \rho_{\infty}v_{bh}$. Thus, the Mach number in the shell $M_{sh} = v_{sh}/c_{s,sh}$ is $M_{sh} = M^{-1} < 1$. However, this assumption fails to reproduce the simulation results and is inconsistent with obtaining a D-type solution for the I-front. Hence, the model must assume (as verified in the simulations) that the gas velocity inside the shell has a non-zero tangential component. It follows that the component of the velocity parallel to the bow shock. As illustrated in the left panel in Figure 5, the simulations do not indicate a strong trend of $M_{sh}$ with $M$:  

$$M_{sh} \sim A M_D,$$  (9)

implying that $v_{sh}\rho_{sh}/(\rho_{sh}v_{bh}) = A(M/M_R) < 1$. Note that in order to be consistent with our model, the data points for $M_{sh}$ are calculated from the simulation data by enforcing mass conservation across the I-front: $M_{sh} = \rho_{sh}v_{sh}/(\rho_{sh}c_{s,sh})$. This ensures that the jump conditions across the I-front ignore the component of the velocity parallel to the bow shock.
As discussed above, across the I-front, the density ratio between the gas in the shell and in the H\textsc{ii} region can be estimated by solving the mass and momentum conservation conditions:

\[
\frac{\rho_m}{\rho_{sh}} = \frac{(1 + M_{sh}^2) \pm \sqrt{(1 + M_{sh}^2)^2 - 4M_{sh}^2\Delta T}}{2\Delta T}
\]

\[
\sim 2M_D^2, \quad \text{for } A \sim 1.
\]

For Equation (10) to have real positive values, we must have \(M_{sh} \leq M_D = M_R^{-1}\) (D-type solution). Clearly, if \(A \leq 1\) (i.e., \(M_{sh} \leq M_D\)) a D-type solution is possible for any Mach number \(M < M_R\). Note that a D-type solution would not be possible if we had assumed 1D flow geometry for which \(M_{sh} = M^{-1}\). A 1D flow assumption would give a D-type solution only for \(M > M_R\), which is the regime in which we expect an R-type front, and is thus ruled out. In general, for \(A^2 < 1\) the D-type solution consistent with the simulation
results is the one with the negative sign in Equation \((10)\), which has the larger relative decrease in density across the front: \(\rho_{\infty}/\rho_{bh} \sim A^2 M_{sh}^2\). This solution describes a strong D-type front. By combining Equations \((8)\)–\((10)\), we obtain

\[
\Delta_p = \frac{\rho_{\infty} - \rho_{in}}{\rho_{\infty}} \approx \frac{\rho_{in} \rho_{bh}}{\rho_{\infty}} \rho_{bh} \rho_{\infty}
\]

\[
\approx \frac{M^2}{2A_T} \approx 2 \left(\frac{M}{M_R}\right)^2 \quad \text{for } A \sim 1,
\]

or \(\Delta_p = A^2(M/M_R)^2 \) for \(A^2 < 1\). The velocity ratio between the gas inside the \textsc{Hii} region and the BH velocity is \(v_{in}/v_{bh} = (\rho_{bh}/\rho_{in})(M_{sh}/M) \approx M_{sh}/M\) for \(A \sim 1\) or \(v_{in}/v_{bh} = M_{sh}/A M_{s}^2 \) for \(A^2 < 1\). In this regime (D-type I-front), the model predicts that the Mach number of the gas inside the \textsc{Hii} region is a constant close to unity, as is indeed observed in all the simulations: \(M_{in} \approx (v_{in}/v_{bh})(\Delta_p A_T^2) \approx 1\) for \(A \sim 1\) (or \(M_{in} = 2/A \) for \(A^2 < 1\)).

The symbols in the upper left panel in Figure 5 show the ratio between the gas density at infinity and inside the dense shell as a function of \(M\) for a set of simulations with different parameters (i.e., gas density and radiative efficiency as shown in the figure’s legend). Our model is shown as a solid line. The lower left panel shows the Mach number in the shell \(M_{sh}\) in units of \(M_D\), as a function of \(M\) for the same simulations. The right panels in Figure 5 show the density and velocity ratios between the gas at infinity and inside the \textsc{Hii} region as a function of Mach number. We measure the gas density and velocity within the \textsc{Hii} region where the density profiles have a minimum behind the I-front. As discussed above, for a plane parallel I-front and shock the mass flux of the gas is conserved: \(\rho_{\infty}/\rho_{in} = \rho_{\infty}/\rho_{bh} = 1\). However, due to the formation of a bow shock when the I-front becomes D-type, the gas flow has a velocity component perpendicular to the direction of the BH motion and \(\rho_{in}/\rho_{bh} \approx M_{sh}/M_R < 1\), in agreement with the simulation results shown in the right bottom panel of Figure 5. In our fiducial simulations with \(T_{\infty} = 10^4\) K, we have \(M_{R} \sim 4.7\) \((A_T \sim 6\) and \(T_{sh} = 6 \times 10^4\) K). The simulation results confirm that a D-type I-front forms for \(M < M_{R}\), while a transition to R-type occurs at \(M \sim M_{R}\).

4.2. Accretion Rate

In this section, we describe a model that fits the gas accretion rates measured at the inner boundary (i.e., near the BH) in all our simulations. Since the effective Bondi radius calculated using the velocity and the sound speed inside the \textsc{Hii} region is smaller than the radius of the I-front in the upstream direction, we treat this problem as a “mini” Bondi accretion within the \textsc{Hii} region. The mean accretion rate can be estimated using the Bondi–Hoyle–Lyttleton formula for gas inside the \textsc{Hii} region: \(M \propto M_{bh}^{2} \rho_{s,in}^{-3}(1 + M_{in}^2)^{-3/2}\). As in previous papers of this series, we define the dimensionless accretion rate in units of the Bondi rate \(M_B \propto M_{bh}^{2} \rho_{s,in}^{-3} \rho_{\infty}^{-3}\).

\[
\langle \lambda_{rad} \rangle \equiv \frac{\dot{M}}{M_B} = \frac{\rho_{in}}{\rho_{\infty}} \left(\frac{c_s^{3}}{c_{s,in}^{3}}\right) \frac{1}{1 + M_{in}^{2}}^{3/2}
\]

\[
= \frac{\Delta_{p}}{A_T^{3/2}} \frac{1}{1 + M_{in}^{2}}^{3/2}.
\]

We have identified two regimes corresponding to the D-type and R-type I-front solutions. If \(1 \leq M < M_R\), in the previous section we found \(\Delta_p \approx 2(M/M_{R})^2 = 2(v_{bh}/v_{in})^2\), and \(M_{in} \approx 1\). This means that inside the \textsc{Hii} region the density increases as the BH moves faster but the Mach number remains constant (approximately transonic). This model explains in simple terms the increase in the accretion rate with increasing BH velocity observed in the simulations in this regime. If \(M > M_R\), the shock front does not form and \(\Delta_p \rightarrow 1\) (if \(M > M_R\)), while the Mach number is \(M_{in} \approx M/M_{p} \Delta_T^{1/2}\). Hence, in dimensionless units:

\[
\langle \lambda_{rad} \rangle \approx \frac{(2\Delta_T)^{-3/2} M^2 \approx \frac{3/2}{M_{R}^2}}{\frac{M_{R}^2}{M_{B}} \left(\frac{M_{R}^2}{M_{R}^2} + M^2 \right)^{-3/2}} \quad \text{if } M < M_R.
\]

\[
\Delta_{p}^{2}(\Delta_{p} A_M + M_{in})^{-3/2} \frac{M_{sh}}{M_{R}} \left(\frac{M_{R}^2}{M_{R}^2} + M^2 \right)^{-3/2} \quad \text{if } M > M_R.
\]

The solid line in Figure 2 shows our model for the accretion rate onto moving BHs. The model generally is a good fit to the simulation results, except the model slightly overestimates the accretion rate around the critical Mach number \(M_R\).

We run a complementary set of simulations to study more precisely the changes of the physical properties as a function of Mach number, since the simulations with constant velocities have a coarse sampling in velocity space and show an intrinsic scatter which is probably the result of out-of-equilibrium initial conditions. We start the simulation assuming sonic motion of the BH \((M \sim 1)\) and increase gradually the velocity of the gas inflow at the boundary. This type of “wind-tunnel” numerical experiments is useful to focus on the changes of physical properties as a function of velocity, while holding the other parameters fixed. The critical Mach number \(M_R\) and the peak luminosity depend on the temperature ratio \(\Delta_T\) as in Equation \((5)\). Figure 6 shows the accretion rate as a function of Mach number for different gas temperatures at infinity \(T_{\infty} = 7 \times 10^3\) K and \(M_{bh} = 100 M_{\odot}\), \(n_{H_{\infty}} = 10^8\) cm\(^{-3}\), and \(\eta = 0.1\). Our model is in good agreement with the simulation results as shown in Figure 6. The peak accretion rates and the critical Mach numbers in these cases are very close to the model predictions. However, the caveat is that the dense shell which initially forms at the beginning of the simulations does not change its location as the velocity of gas increases as observed in the simulations in which \(v_{bh}\) was held constant.

If we express the accretion rate as a function of the BH velocity in physical units, we find that the accretion rate is independent of the temperature of the ambient medium and peaks at about twice the sound speed inside the \textsc{Hii} region: \(v_{b}^{max} = v_{bh} \lesssim 2 c_{s,in}\) (see Figure 7). This is contrary to the case of a static BH (or moving at subsonic speed) for which \(\langle \lambda_{rad} \rangle \propto T_{\infty}^{3/2}\) and hence the accretion rate \(M \propto T_{\infty}\) is proportional to the temperature of the ambient gas. The gas accretion rate for a moving BH is

\[
\dot{M} \approx \frac{\rho_{\infty}(GM_{bh})^2}{c_{s,in}^3} \times \left(\frac{v_{bh}}{c_{s,in}}\right)^2 \left[1 + (v_{bh}/c_{s,in})^2\right]^{-3/2} \quad \text{if } v_{bh} \gg 2 c_{s,in}.
\]

The velocity for peak accretion depends only on the sound speed inside the \textsc{Hii} region and is \(v_{bh} = 50\) km s\(^{-1}\) for \(T_{\infty} = 6 \times 10^4\) K \((c_{s,in} = 25\) km s\(^{-1}\)). We find a mild dependence of \(T_{sh}\) on the density of the ambient medium. As explored in more detail in
Our model has a peak accretion rate of about 70% of the Bondi rate in a gas with temperature $T_\infty$ and density $\rho_\infty$, in good agreement with our "wind-tunnel" type simulations (however, as noted before, the simulations with constant BH Mach number produce smaller accretion rates at peak accretion: about a factor of five smaller than the model prediction for the peak value). Interestingly, in our model and simulations a BH moving at 20–50 km s$^{-1}$ with respect to a gas with temperature $T_\infty \lesssim 10^4$ K has a faster growth rate and accretion luminosity than if it was at rest (or moving at subsonic speed). Figure 7 is an extrapolation of our results to lower temperature regime showing accretion rates as a function of BH velocity $v_{bh}$ for gas temperatures $T_\infty = 10^3$, $3 \times 10^3$, and $10^4$ K. In all cases, if $T_\infty > 10^4$ K, we find that the peak accretion rate at $v_{bh} \sim 2c_{s,in}$ is larger than the corresponding accretion rate onto a stationary BH. Clearly, this is an important result because significant BH accretion is only possible when the BH is in a dense medium, for instance, a molecular cloud or the cold neutral medium in a galaxy that generally have low temperatures. In addition, the colder the gas, the smaller the BH velocity needs to be to achieve the supersonic speed that leads to significant increase of the BH accretion rate.

In the next section, we will discuss another important result: in the regime when the I-front is $D$-type, instabilities of the dense and thin shell behind the bow shock may lead to its fragmentation and hence produce periodic oscillations of the accretion rate and luminosity of the BH. This effect can further increase the peak accretion luminosity of the BH by roughly a factor $M^2$ with respect to the mean values estimated above in Equation (16). For instance, the simulation with $M \sim 3$, $n_{H,\infty} = 10^4$ cm$^{-3}$, and $T_\infty = 10^4$ K shows this instability and the peak accretion rate is roughly a factor of 10 larger than the mean.

4.3. Stability of Bow Shock and Periodic Oscillations of the Luminosity

As discussed in the previous section, the average accretion rate ($\dot{\lambda}_{rad}$) increases with increasing Mach number if $1 < M < M_R$. For the lower values in this Mach number range, all simulations approach a steady-state accretion rate. Interestingly, as the Mach number approaches $M_R$, the bow shock in simulations with ambient gas densities in the range $n_{H,\infty} = 10^3$–$10^4$ cm$^{-3}$ becomes unstable producing intermittent bursts of accretion due to a cyclic formation/destruction of a dense shell in the upstream direction (see Whalen & Norman 2011). Figure 8 shows time evolution of a simulation with $M_{bh} = 100 M_\odot$, gas density $n_{H,\infty} = 10^4$ cm$^{-3}$, temperature $T_\infty = 10^4$ K, and Mach number $M = 2.7$. As seen in Figure 3, the ionizing radiation creates a "cometary-shaped" H$\textsc{ii}$ region around the BH. In the early stages of the simulation, the gas flow remains relatively steady. However, with time some instabilities start growing leading to the fragmentation of the shell. As a result, the ionizing radiation and the hot gas inside the H$\textsc{ii}$ region are no longer contained by the dense shell downstream of the bow shock and an explosion takes place. Fragments from the broken dense shell fall onto the BH significantly increasing the accretion rate, thus creating more ionizing photons that blow out further the thinner parts of the shell. However, after a time delay from the burst roughly estimated as the H$\textsc{ii}$ region sound crossing time, the dense shell re-forms, resetting the initial conditions for the next burst cycle.
Since the dense shell is located outside the inner Bondi radius, the gravitational acceleration on the shell is relatively small and hence the timescale for Rayleigh–Taylor instability is long. In addition, the radiation has a stabilizing effect as it tends to smooth out the growth of linear perturbations on small scales that have the faster grow rate. However, when $v_{bh} \gtrsim c_{s,\text{in}}$ (i.e., $\mathcal{M} \sim \mathcal{M}_b/2$) the thermal pressure becomes comparable to the thermal pressure inside the H ii region and the I-front is pushed closer to the BH. In this case, the increased gravitational acceleration on the shell and the increased sharpness of the pressure gradient seem to trigger the growth of instabilities. Also the column density of dense shell is roughly constant as the flow through the I-front of neutral gas of density $n$ and velocity $v_{bh}$:

$$N_{\text{ion}} = \frac{4\pi}{3} \langle R_s \rangle^3 \alpha_{\text{rec}} n_e^2 + 4\pi \langle R_s \rangle^3 n_e v_{bh} \cos(\theta),$$  \hspace{1cm} (17)$$

where $N_{\text{ion}}$ is the number of emitted ionizing photons, being directly related to the luminosity of the BHs (which is a function of Mach number). When the magnitudes of the two terms on the right side of Equation (17) are compared, at $\theta = 0$ the first term is dominant over the second term due to the BH motion. The electron number density inside the H ii region for $1 < \mathcal{M} < \mathcal{M}_b$ is

$$n_e \sim x_en_{\text{H,\text{in}}} = \frac{\mathcal{M}^2}{(2\Delta T)^{3/2}} n_{\text{H,\infty}}.$$  \hspace{1cm} (18)$$

Since $\langle \lambda_{\text{rad}} \rangle \propto n_{\text{H,\infty}}^{1/2}$ (for $n_{\text{H,\infty}} \lesssim 10^5$ cm$^{-3}$ and $M_{bh} = 100 \, M_\odot$), the average size of the H ii region in the upstream direction is

$$\langle R_s \rangle_{\theta=0} \propto n_{\text{H,\infty}}^{-1/6}.$$  \hspace{1cm} (19)$$

Figure 10 shows the size of the H ii region at $\theta = 0$ as a function of Mach number for simulations with various densities ($n_{\text{H,\infty}} = 10^2$–$10^3$ cm$^{-3}$) and radiative efficiencies ($\eta = 0.1$, 0.01). The model is a good fit to the simulation results.
We model the size of the H\textsc{ii} region in the downstream direction in a similar manner. In the downstream direction $\theta = \pi$ the second term on the right-hand side of Equation (17) dominates over the first term:

$$N_{\text{ion}} \simeq 4\pi \langle R_s \rangle_{\theta=\pi}^2 n_{\text{H,ion}} v_{\text{bh}},$$  \hspace{1cm} (20)

where $n_{\text{H,ion}}$ can be calculated simply using pressure equilibrium condition $n_{\text{H,ion}} \propto n_{\text{H,\infty}}^\eta$. The size of the H\textsc{ii} region in the downstream direction for $1 < M < M_R$ is

$$\langle R_s \rangle_{\theta=\pi} \propto \eta^{1/2} n_{\text{H,\infty}}^{1/4} (1 + M^2)^{1/2},$$  \hspace{1cm} (21)

where $\langle R_s \rangle_{\theta=\pi}$ is approximately proportional to the Mach number. Figure 11 shows that the model reproduces the length of the tail of the H\textsc{ii} region in the downstream direction in simulations for $n_{\text{H,\infty}} = 10^3$ cm$^{-3}$. The length of the H\textsc{ii} region at $\theta = \pi$ is shorter than the model prediction because of the gravitational focusing of the gas by the BH that increases the gas density on the axis of symmetry behind the BH.

5. SUMMARY AND DISCUSSION

In this third paper of a series on radiation-regulated accretion onto BHs, we have focused on the effect of the BH motion relative to the surrounding gas, i.e., the Hoyle–Lyttleton problem modified by the effects that photoheating and radiation pressure from the radiation emitted near the BH have on the accretion flow. As in previous papers of this series, we have used radiation-hydrodynamic simulations to explore a large parameter space of initial conditions to inform us on how to formulate an analytical model that reproduces the simulation results. The following are our key findings:

1. The quasi-periodic oscillation of the accretion rate observed in simulations of non-moving BHs is only observed for subsonic motions of the BH, while the accretion rate becomes steady at supersonic velocities (in most cases).

2. In the supersonic regime, we observe an axis-symmetric gas flow and a “cometary-shaped” H\textsc{ii} region with tail length proportional to the BH velocity $v_{\text{bh}}$. For BH velocities $c_s, \infty < v_{\text{bh}} < v_R \approx 50$ km s$^{-1}$ the ionization front becomes
For subsonic motion of the BH, we find that the accretion rate onto the BH decreases with increasing BH velocity. However, contrary to naive expectations, the accretion rate increases with increasing BH velocity in the regime \( c_{\infty, s} < v_{\text{bh}} < 50 \text{ km s}^{-1} \), when the I-front is D-type. The accretion rate peaks at BH velocity \( v_{\text{bh}} \approx 50 \text{ km s}^{-1} \) before it starts decreasing with increasing BH velocity, converging to the well-known Hoyle–Lyttleton solution without radiation feedback.

4. Based on the simulation results, we formulate a simple analytical model of the problem based on modeling the jump conditions across the I-front. The transition of the I-front from R-type to D-type happens at \( v_{\text{bh}} = 2c_{\text{s}, \infty} \), where \( c_{\text{s}, \infty} \) is the sound speed inside the H\( \text{II} \) region. Isothermal jump conditions for the bow shock reproduce fairly well the gas flow properties in simulations with ambient gas density \[ n_{\text{HII}} > 10^5 \text{ cm}^{-3} \]. Because the inner Bondi radius (for the gas inside the H\( \text{II} \) region) is comparable to but smaller than the I-front radius, the accretion rate is well reproduced in our analytical model assuming Bondi-type accretion from the ionized gas downstream of the bow shock. The BH moves subsonically or transonically with respect to the gas downstream of the bow shock. In this regime, the accretion rate increases with BH velocity because the density of the ionized gas inside the H\( \text{II} \) region increases with increasing velocity reaching \( n_{\text{HII}} \rightarrow n_{\text{HII}, \infty} \) at \( v_{\text{bh}} \approx 2c_{\text{s}, \infty} \).

5. Simulations of BHs accreting from a high-density medium \( (n_{\text{HII}} = 10^5 \text{ cm}^{-3}) \) show steady accretion rate for all Mach numbers. However, at intermediate densities \( (n_{\text{HII}} = 10^3-10^4 \text{ cm}^{-3}) \) we find intermittent bursts of accretion rate in the Mach number range \( 2.5 < M < 3 \). The oscillatory behavior of accretion rate is due to the development of instabilities in the thin shell in the upstream direction that cause its cyclic fragmentation and re-formation. In the lower density regime, \( n_{\text{HII}} < 10^2 \text{ cm}^{-3} \), the post-shock density is lower than expected assuming isothermal shock jump conditions; as a result the dense shell is thicker and less prone to instabilities. For \( v_{\text{bh}} > v_R \) the dense shell never forms, providing steady accretion rates.

6. Contrary to the case of radiation-regulated accretion onto non-moving BHs in which \( M \propto T_\infty^{3/2} M_R \propto T_\infty \) (see Papers I and II), the accretion rate onto supersonic BHs is independent of the temperature of the ambient medium (see Equation (16)). It follows that if \( T_\infty < 10^4 \text{ K} \) the accretion rate onto a BH moving with velocity \( v_{\text{bh}} \approx 50 \text{ km s}^{-1} \) is about \( 5(10^4 \text{ K}/T_\infty) \) times larger than the accretion rate at \( v_{\text{bh}} = 0 \). Hence, the growth rate and the mean accretion luminosity of BHs moving supersonically can be significantly larger than those of non-moving BHs with the same mass and accreting from the same medium.

Our simulations are 2D, having assumed rotational symmetry around the axis defined by the BH motion and a spherically symmetric radiation field emitted near the BH. Below we discuss how our results and conclusions might be sensitive to the inclusion of the extra degree of freedom in more realistic 3D simulations and different assumptions on the feedback mechanism.

It is well known that Hoyle–Lyttleton accretion without radiation feedback is prone to side-to-side “flip-flop” instability (e.g., Matsuda et al. 1987; Blondin & Pope 2009; Blondin & Raymer 2012) that cannot be captured in our axis-symmetric simulations. This instability breaks the axial symmetry of the flow downstream of the BH where the gas, gravitationally focused by the BHs, is shocked and accreted. The resulting accretion of angular momentum produces the formation of temporary accretion disks spinning in alternating directions, with a burst of mass accretion when the direction of rotation is flipped. This non-axisymmetric instability has been seen in numerous hydrodynamic simulations of 2D planar accretion (Matsuda et al. 1991; Zarinelli et al. 1995; Benensohn et al. 1997; Shima et al. 1998; Pogorelov et al. 2000) but it has not been unambiguously identified in 3D simulations (Ruffert 1999), possibly due to insufficient spatial resolution. In our simulations, radiation feedback may stabilize this instability for the cases, shown in Figure 3, when a stable bow-shock forms upstream of the BH. In this regime, the post-shock gas is transonic and the flow is better described by Bondi-type accretion than Hoyle–Lyttleton. For this reason, and because of the stabilizing effect of radiative heating (Blondin et al. 1990), the flip-flop instability may not be important in Hoyle–Lyttleton accretion when radiation feedback is included.

An extra spatial degree of freedom is likely to play a role in determining the stability of the thin cooling layer that forms behind the bow shock. As seen in Figure 8, probably because of the axis-symmetric assumption in our simulations, the fastest growing mode in the unstable shock is a “dimple” forming along the axis of symmetry. Both the instability growth rate and the shock breakup conditions may change given the additional degrees of freedom in 3D simulations. For instance, the spherical accretion shock instability found in supernova simulations (e.g., Blondin & Shaw 2007) is less important in 3D simulations than in 2D ones.

In this paper, as in the previous papers in this series, we assume that the radiation is emitted in a spherically symmetric manner near the BH. However, it is possible that the formation of optically thick structures near the BHs, such as an accretion disk, which are not resolved in our simulations, may change the angular dependence of the emitted radiation. For instance, Proga et al. (1998) have studied extensively the case of accretion on non-moving SMBHs in which UV photons are emitted preferentially in the direction perpendicular to the accretion disk. Here, we have not explored how such an assumption would affect our results for a moving BH. Neither have we explored the feedback effect from a collimated wind or jet that may also develop in some phases of the BH accretion cycle.

Despite the aforementioned simplifying assumptions in our simulations, our results are a substantial improvement with respect to the Hoyle–Lyttleton model. The dependence of the growth rate and luminosity of BHs on their velocity with respect to the ambient gas is qualitatively modified by radiation feedback. The results reported in this paper may have significant repercussions in modeling and understanding the growth of stellar BHs in the early universe, the buildup of an early X-ray background and provide important clues for explaining ULXs (Krolik et al. 1981; Krolik & Kallman 1984; Krolik 2004; Ricotti & Ostriker 2004; Ricotti et al. 2005; Ricotti 2007; Strohmayer & Mushotzky 2009; Volonteri & Bellovary 2012). We plan to address some of these issues in forthcoming papers.

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