ANISOTROPIC NULL STRING COSMOLOGIES

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Abstract

We study string propagation in an anisotropic, cosmological background. We solve the equations of motion and the constraints by performing a perturbative expansion of the string coordinates in powers of $c^2$, the world-sheet speed of light. To zeroth order the string is approximated by a tensionless string (since $c$ is proportional to the string tension $T$). We obtain exact, analytical expressions for the zeroth and the first order solutions and we discuss some cosmological implications.

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1. Introduction.

Strings in curved spacetimes provide the best framework for studying gravity in the context of string theory. The action for a bosonic string propagating in a $D$-dimensional Riemannian manifold is given by

$$S = -\frac{T}{2} \int d\tau d\sigma \sqrt{-h} h^{\alpha\beta}(\tau, \sigma) G_{\mu\nu}(X) \partial_{\alpha} X^\mu \partial_{\beta} X^\nu$$

where $\mu, \nu = 0, 1, \ldots (D - 1)$ are spacetime indices, $\alpha, \beta = 0, 1$ are worldsheet indices and $T = (2\pi \alpha')^{-1}$ is the string tension. The metric $h^{\alpha\beta}(\tau, \sigma)$ describes the intrinsic geometry of the two-dimensional worldsheet while $G_{\mu\nu}(X)$ describes the geometry of the $D$-dimensional target space. This particular action does not permit us to discuss the limit $T \to 0$. Following the analogous massless particle case [1] we are led to a reformulated action which in the conformal gauge ($h^{\alpha\beta}(\tau, \sigma) = \exp[\phi(\tau, \sigma)] \eta^{\alpha\beta}$), where $\phi(\tau, \sigma)$ is an arbitrary function and $\eta^{\alpha\beta} = \text{diag}(-1, +1)$, leads to the classical equations of motion

$$\ddot{X}^\mu - c^2 (T) (X^\mu)'' + \Gamma^\mu_{\nu\rho}(X) [\dot{X}^\nu \dot{X}^\rho - c^2 (T) X'^\nu X'^\rho] = 0$$

with $c = 2\lambda T$ ($\lambda$ is a Lagrange multiplier) and $\Gamma^\mu_{\nu\rho}(X)$ are the Christoffel symbols associated to the metric $G_{\mu\nu}(X)$. Variation of this action with respect to the two-dimensional metric $h^{\alpha\beta}(\tau, \sigma)$ also yields two constraints

$$G_{\mu\nu}(X) [\dot{X}^\mu \dot{X}^\nu + c^2 (T) X'^\mu X'^\nu] = 0, \quad G_{\mu\nu}(X) \dot{X}^\mu X'^\nu = 0.$$  

Dot and prime stand for differentiation with respect to $\tau$ and $\sigma$ respectively. Different perturbative approaches were introduced in order to solve the EOM and the constraints of strings propagating in curved backgrounds: the center of mass expansion [2], the $\tau$-expansion [3], the null string expansion [4] while in [5] inverse scattering methods and soliton techniques were introduced in order to obtain exact solutions. Solutions in closed form have also been found for some relevant gravitational backgrounds in [6]. The null string method consists of inserting into Eqns. (2) and (3) the expansion

$$X^\mu(\tau, \sigma) = A^\mu(\tau, \sigma) + c^2 B^\mu(\tau, \sigma) + O(c^4), \quad |B^\mu| \ll |A^\mu|.$$  

To zeroth order in $c^2$ we find

$$\ddot{A}^\rho + \Gamma^\rho_{\kappa\lambda} \dot{A}^\kappa \dot{A}^\lambda = 0, \quad \dot{A}^\mu \dot{A}^\nu G_{\mu\nu} = 0, \quad \dot{A}^\mu A'^\nu G_{\mu\nu} = 0.$$  

We observe that the coordinates $A^\mu(\tau, \sigma)$ describe a null string [7], a collection of points moving independently along null geodesics. The second constraint ensures that each point of the string propagates in a direction perpendicular to the string. The next order correction $B^\mu(\tau, \sigma)$ (first order in $c^2$) obeys the EOM

$$\ddot{B}^\mu + \Gamma^\mu_{\alpha\beta} (\dot{A}^\alpha \dot{B}^\beta + \dot{B}^\alpha \dot{A}^\beta) + \Gamma^\mu_{\alpha\beta,\nu} \dot{A}^\alpha \dot{A}^\beta B^\nu = A''^\mu + \Gamma^\mu_{\kappa\lambda} A'^\kappa A'^\lambda.$$  

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supplemented with the constraints

\[ 2 \dot{A}^\mu \dot{B}^\nu G_{\mu\nu} + \dot{A}^\mu \dot{A}^\nu B^\rho G_{\mu\nu,\rho} + \dot{A}^\mu A^\nu \dot{G}_{\mu\nu} = 0 \]
\[ (\dot{A}^\mu B^\nu + \dot{B}^\mu A^\nu) G_{\mu\nu} + \dot{A}^\mu A^\nu B^\rho G_{\mu\nu,\rho} = 0 \]  

(7)

where \( G_{\mu\nu,\rho} \) and \( \Gamma^\mu_{\alpha\beta,\rho} \) indicate differentiation with respect to \( A^\rho \). We note that in this approximation scheme we expand around the null string, an extended object as opposed to the center of mass expansion in which the zeroth order solution corresponds to the center of mass of the string, a pointlike structure.

2. Null Strings in Milne Spacetimes.

In this section we will apply the null string approach to solve the EOM and the constraints of a string propagating in a Milne cosmological background. Null strings propagating in various spacetime geometries including anisotropic models, were previously considered in [8]. The line element of the Milne spacetime is defined as

\[ ds^2 = -(dX^0)^2 + (dX^1)^2 + (dX^2)^2 + (X^0)^2 (dX^3)^2. \]

(8)

Although Milne spacetimes are merely unconventional coordinatizations of flat geometries analogous to the Rindler metrics, they lead to non-trivial effects. The equations of motion and the constraints for \( A^\mu \), Eqn. (5) become

\[ \ddot{A}^0 + (A^0)(\dddot{A}^3)^2 = 0, \quad \ddot{A}^1 = \dddot{A}^2 = 0, \quad \ddot{A}^3 + \frac{2\dot{A}^0\dddot{A}^3}{A^0} = 0 \]
\[ (\dddot{A}^0)^2 = (\dddot{A}^1)^2 + (\dddot{A}^2)^2 + (A^0)^2 (\dddot{A}^3)^2, \quad (\dddot{A}^0 A^0) = (\dddot{A}^1 A^1) + (\dddot{A}^2 A^2) + (A^0)^2 (\dddot{A}^3 A^3). \]  

(9)

There are two classes of solutions:

\[ (A^0)^2(\tau, \sigma) = ((P^1)^2 + (P^2)^2)(\sigma)^\tau + 2P^0(\sigma)\tau + I^0(\sigma), \quad A^{1,2}(\tau, \sigma) = P^{1,2}(\sigma)\tau + I^{1,2}(\sigma) \]
\[ A^3(\tau, \sigma) = \frac{1}{2} \ln \left( \frac{(P^0 + P^3)\tau + I^0}{(P^0 - P^3)\tau + I^0} \right) + I^3(\sigma) \]  

(10)

provided that \( \Delta = I^0((P^1)^2 + (P^2)^2) - P^{02} < 0 \) and \( P^\mu, I^\mu \) are arbitrary functions of \( \sigma \) that obey the constraints

\[ (P^0)^2 = I^0((P^1)^2 + (P^2)^2) + (P^3)^2, \quad P^1(I^1)^\prime + P^2(I^2)^\prime + P^3(I^3)^\prime = \frac{P^0 I^0^\prime}{2I^0}. \]  

(11)

These constraints tie together the initial momentum \( P^\mu(\sigma) \) and shape of the string \( I^\mu(\sigma) \). When the worldsheet timelike variable vanishes (\( \tau = 0 \)), then the cosmic time variable (\( A^0 \))
Anisotropic Null String Cosmologies... assumes finite value \( \sqrt{I_0} \) and the position of the string is specified by \((I^1(\sigma), I^2(\sigma), I^3(\sigma))\). As the string evolves \((\tau \to +\infty)\), and \((A^0 \to +\infty)\), we notice that \(A^3 \to A_{max}^3\)

\[
A_{max}^3 = \frac{1}{2} \ln \left( \frac{P_0^0 + P_3^3}{P_0^0 - P_3^3} \right) = \ln \left( \rho + \sqrt{1 + \rho^2} \right)
\]  

(12)

with \(\rho = \frac{P_3^3}{\sqrt{((P_1^1)^2 + (P_2^2)^2)I_0}}\). The string bounces at a finite distance from the origin as it moves towards the direction of the anisotropy. One needs increasingly higher values of the momentum \(P_3^3\) in order to penetrate into higher values of the direction of anisotropy.

The second class of solutions corresponds to \(\Delta = 0(P_3^3 = 0)\) and is given by

\[
(A^0)^2(\tau, \sigma) = ((P_1^1)^2 + (P_2^2)^2)(\sigma)^2 + 2P_0^0(\sigma)\tau + I_0^0(\sigma), \quad A^{1,2}(\tau, \sigma) = P^{1,2}(\sigma)\tau + I^{1,2}(\sigma) \\
A^3(\tau, \sigma) = I^3(\sigma)
\]

(13)

while the constraints read

\[
(P_0^0)^2 = I_0^0((P_1^1)^2 + (P_2^2)^2), \quad P^1(1) + P^2(2) = \frac{P_0^0 I_0^0}{2I_0^0}.
\]

(14)

The form of this solution suggests that string propagation has been restricted to two space dimensions. The energy-momentum tensor for the null string is obtained by varying the action with respect to the metric \(G_{\mu\nu}(X)\) at the spacetime point \(X\)

\[
\sqrt{-G}T_{\mu\nu}(A) = \frac{1}{\lambda} \int d\tau d\sigma \dot{A}^\mu \dot{A}^\nu \delta^{(4)}(A - A(\tau, \sigma))
\]

(15)

where \(A^\mu(\tau, \sigma)\) are the string coordinates. Furthermore we integrate \(T_{\mu\nu}(A)\) over a spatial volume thus enclosing the string at fixed time \(A^0\) as it was proposed in [9],

\[
I_{\mu\nu}(A^0) = \int \sqrt{-G}T_{\mu\nu}(A)d^3\vec{A}.
\]

(16)

If we insert Eqn. (10) into Eqn. (16) we find the following expressions for the energy and the momentum of the null string

\[
I^{00} = \frac{1}{\lambda} \int d\sigma \sqrt{\frac{(P_0^0)^2 - (P_3^3)^2}{I_0} + \frac{(P_3^3)^2}{(A^0)^2}}, \quad I^{0i} = \frac{1}{\lambda} \int d\sigma P^i(\sigma), \quad I^{03} = \frac{1}{\lambda} \int d\sigma \frac{P^3(\sigma)}{(A^0)^2}
\]

(17)

with \(i = 1, 2\). Similarly we find for the components of the pressure

\[
I^{ii} = \frac{1}{\lambda} \int d\sigma \sqrt{\frac{(P_i^i)^2}{I_0} - \frac{(P_3^3)^2}{(A^0)^2}}, \quad I^{33} = \frac{1}{\lambda} \int d\sigma \frac{(P_3^3)^2}{(A^0)^4} \sqrt{\frac{(P_0^0)^2 - (P_3^3)^2}{I_0} + \frac{(P_3^3)^2}{(A^0)^2}}
\]

(18)
while the components $I^{i3}(i=1,2)$ and $I^{12}$

$$I^{i3} = \frac{1}{\lambda} \int d\sigma \frac{(P^i P^3)}{A^0 \sqrt{\frac{(P^0)^2 - (P^3)^2 (A^0)^2}{(P^0)^2} + \frac{(P^3)^2}{(A^0)^2}}}$$

$$I^{12} = \frac{1}{\lambda} \int d\sigma \frac{(P^1 P^2)}{\sqrt{\frac{(P^0)^2 - (P^3)^2}{P^0} + \frac{(P^3)^2}{(A^0)^2}}} \quad (19)$$

represent stresses which act on the string. We observe that the isotropic pressure components $I^{ii}(i=1,2)$ attain constant values as $A^0 \mapsto \infty$ while the anisotropic component $I^{33}$ tends to zero. If we now let a sequence of strings to move towards the anisotropy with $\left(\frac{dN}{dA^0}\right) = N_0$ for a constant number of strings that move from the origin ($A^3 = 0$) towards the anisotropy we have

$$\frac{dW}{dA^0} = N_0 \frac{dW}{dN} = N_0 I^{00}. \quad (20)$$

Integrating from the initial cosmic time ($A^0 = \sqrt{\sigma}$) to $(A^0 = \bar{A}^0)$ one obtains an expression for the energy of $N$ strings

$$W = \frac{N_0}{2\lambda} \int d\sigma \left[ \sqrt{(P^3)^2 + ((P^1)^2 + (P^2)^2)(A^0)^2} - P^3 \ln \left( \frac{P^3 + \sqrt{(P^3)^2 + ((P^1)^2 + (P^2)^2)(A^0)^2}}{\sqrt{((P^1)^2 + (P^2)^2)A^0}} \right) \right]_{\sqrt{\sigma}}^{\bar{A}^0} \quad (21)$$

The distribution of strings thus gains energy that grows linearly with respect to the cosmic time ($A^0$). On the contrary if we constrain the strings to move only along the anisotropic direction the energy grows as $W \sim \ln A^0$. Therefore in the case in which one would allow for the backreaction, the strings would gain energy from the gravitational field, thereby altering the anisotropy.

### 3. First Order Corrections.

Let’s now discuss the first order corrections $B^\mu(\tau, \sigma)$. The equations of motion become

$$\ddot{B}^0 + 2A^0(\dot{A}^3 \dot{B}^3) + (\dot{A}^3)^2 B^0 = (A^0)'' + A^0(\dot{A}^3)^2, \quad \ddot{B}^{1,2} = (A^{1,2})''$$

$$\ddot{B}^3 + \frac{2}{A^0}(\dot{A}^3 \dot{B}^3 + \dot{A}^3 \dot{B}^3) - \frac{2\dot{A}^0 \dot{A}^3 B^0}{(A^0)^2} = (A^3)'' + \frac{2}{A^0}(A^0)'(A^3)' \quad (22)$$

and the constraints

$$- (\dot{B}^0 A^0 + \dot{A}^0 B^0) + (\dot{B}^1 A^1 + \dot{A}^1 B^1) + (\dot{B}^2 A^2 + \dot{A}^2 B^2)$$

$$+ (A^0)^2(\dot{B}^3 A^3 + \dot{A}^3 B^3) + 2A^0 A^3 A^3 B^0 = 0$$

$$- 2\dot{A}^0 \dot{B}^0 + 2\dot{A}^1 \dot{B}^1 + 2\dot{A}^2 \dot{B}^2 + 2(A^0)^2 \dot{A}^3 \dot{B}^3 + 2A^0 (\dot{A}^3)^2 B^0$$

$$- (A^0)^2 + (A^1)^2 + (A^2)^2 + (A^0)^2 (A^3)^2 = 0. \quad (23)$$
Finally by substituting $B$ into Eqn. (26) and integrating we find an expression for $B^0(\tau, \sigma)$

$$B^0(\tau, \sigma) = \int^{\tau} d\tau' \frac{1}{A^0(\tau', \sigma)^3} \int^{\tau'} d\tilde{\tau} h(\tilde{\tau}, \sigma) + \int^{\tau} d\tau' \frac{g(\sigma)}{A^0(\tau', \sigma)^3} + l(\sigma)$$

(29)

Finally by substituting $B^0$ into Eqn. (26) and integrating we find an expression for $B^3(\tau, \sigma)$

$$B^3(\tau, \sigma) = \int^{\tau} d\tilde{\tau} \frac{f'(\tilde{\tau}, \sigma)}{A^0(\tilde{\tau}, \sigma)^2} - 2 \int^{\tau} d\tilde{\tau} \frac{P^3(\sigma)}{A^0(\tilde{\tau}, \sigma)} + m(\sigma)$$

(30)

The general solution $B^\mu(\tau, \sigma)$ is complicated and depends on arbitrary functions of $\sigma$ which determine the initial shape and momentum of the string. In order to simplify the calculation we will consider some special cases. Initially we will assume that the string at $\tau = 0$ forms a circle of radius $R$ in the $X^1 - X^3$ plane and furthermore that all the points of the string move with the same momentum $P^1$ parallel to the $X^1$ axis, more specifically

$$I^1(\sigma) = R \sin \sigma, \quad I^3(\sigma) = R \cos \sigma, \quad P^1 \neq 0, \quad P^2 = P^3 = 0.$$  

(31)
From the constraints Eqn. (11) we find that \((P^0)^2 = I^0(P^1)^2 \) and \(I^0 = R^2 \sin^2 \sigma \). The overall solution for the \(A\)'s becomes

\[ A^0(\tau, \sigma) = (P^1 \tau + R \sin \sigma), \quad A^1(\tau, \sigma) = P^1 \tau + R \sin \sigma, \quad A^2(\tau, \sigma) = 0, \quad A^3(\tau, \sigma) = R \cos \sigma. \] (32)

It is also interesting to calculate the distance of the string between \((\sigma, \tau)\) and \((\sigma + d\sigma, \tau)\).

We find that for the solution (32)

\[
ds^2 = (P^1 R \tau \sin \sigma + R^2 \sin^2 \sigma)^2 d\sigma^2 = (A^0)^2 R^2 \sin^2 \sigma d\sigma^2 \] (33)

We observe that the string is expanding at the same rate as the spacetime. The solution then to the Eqn. (22), using Eqn. (32) is

\[
B^0(\tau, \sigma) = -R \sin \sigma \frac{\tau^2}{2} + P^1 R^2 \sin^2 \sigma \frac{\tau^3}{6} + R^3 \sin^3 \sigma \frac{\tau^2}{2} + H^0(\sigma) \tau + Q^0(\sigma) \\
B^1(\tau, \sigma) = -R \sin \sigma \frac{\tau^2}{2} + H^1(\sigma) \tau + Q^1(\sigma) \\
B^2(\tau, \sigma) = H^2(\sigma) \tau + Q^2(\sigma) \\
B^3(\tau, \sigma) = -R \cos \sigma \frac{\tau^2}{6} - \frac{4 R^2}{3 P^1} (\cos \sigma \sin \sigma) \tau - \frac{4 R^4 \cos \sigma \sin^3 \sigma}{3 (P^1)^2} + \frac{H^3(\sigma)}{(P^1)} \big( (P^1 \tau + R \sin \sigma)^{-1} + Q^3(\sigma) \big)
\] (34)

while the constraints Eqn. (23) imply that the functions \(H^\mu(\sigma)\) and \(Q^\mu(\sigma)\) satisfy the following relations

\[ 2P^1(H^0 - H^1) = R^4 \sin^4 \sigma, \quad P^1(Q^{0'} - Q^{1'}) + R \cos \sigma (H^0 - H^1) + R \sin \sigma = 0 \] (35)

Similarly we can assume that the string at \(\tau = 0\) forms a circle of radius \(R\) in the \(X^2 - X^3\) plane and furthermore that all the points of the string move with the same momentum \(P^3\) parallel to the \(X^3\) axis, the anisotropy direction, more specifically

\[ I^1 = 0, \quad I^2(\sigma) = R \cos \sigma, \quad I^3(\sigma) = R \sin \sigma, \quad P^3 \neq 0, \quad P^1 = P^2 = 0. \] (36)

The constraints then imply that \((P^0)^2 = (P^3)^2\) and \(I^0(\sigma) = e^{2R \sin \sigma}\) while the overall solution reads

\[
(A^0)^2(\tau, \sigma) = 2P^3 \tau + e^{2R \sin \sigma}, \quad A^1(\tau, \sigma) = 0, \quad A^2(\tau, \sigma) = R \cos \sigma \\
A^3(\tau, \sigma) = \frac{1}{2} \ln \left( 2P^3 \tau + e^{2R \sin \sigma} \right)
\] (37)

For this particular solution we find that the distance of the string between \((\sigma, \tau)\) and \((\sigma + d\sigma, \tau)\) is

\[
ds^2 = R^2 \sin^2 \sigma d\sigma^2 \] (38)
which implies that the cosmological expansion balances exactly the string contraction. The first order corrections $B^0(\tau, \sigma)$ and $B^3(\tau, \sigma)$ become

$$B^0(\tau, \sigma) = C^0(\sigma)(2P^3\tau + e^{2R\sin\sigma})\frac{\tau}{2} + D^0(\sigma)(2P^3\tau + e^{2R\sin\sigma})\frac{\tau}{2}
+ H^0(\sigma)(2P^3\tau + e^{2R\sin\sigma}) - \frac{\tau}{2} + Q^0(\sigma)$$

$$B^3(\tau, \sigma) = C^3(\sigma)(2P^3\tau + e^{2R\sin\sigma}) + D^3(\sigma)\ln(2P^3\tau + e^{2R\sin\sigma})
+ H^3(\sigma)(2P^3\tau + e^{2R\sin\sigma}) - \frac{\tau}{2} + Q^3(\sigma)$$

where $C(\sigma)$ and $D(\sigma)$ are known functions of $e^{2R\sin\sigma}$ and $P^3$ and $H(\sigma), Q(\sigma)$ are arbitrary functions of $\sigma$. The other components $B^{1,2}(\tau, \sigma)$ retain their previous form. The form of the solution $X^\mu = A^\mu + e^2 B^\mu$ suggests that in the null string expansion the overall motion of the string is treated exactly while the internal motion is approximated by a $\tau$-expansion.

Finally we would like to comment on the algebra of the constraints and the quantization of the null string. The constraints

$$G(\sigma) = (P^0)^2 - I^0((P^1)^2 + (P^2)^2) - (P^3)^2, \quad F(\sigma) = P^1(I^1)' + P^2(I^2)' + P^3(I^3)' - \frac{P^0 I^0'}{2I^0}$$

generate reparametrizations of the form $\tau = f(\tilde{\tau}, \tilde{\sigma}), \sigma = h(\tilde{\sigma})$ and obey the following Poisson brackets

$$\{F(\sigma), F(\sigma')\} = 2F(\sigma')\delta'(\sigma - \sigma') - F'(\sigma')\delta(\sigma - \sigma'), \quad \{G(\sigma), G(\sigma')\} = 0,$$

$$\{G(\sigma), F(\sigma')\} = 2G(\sigma')\delta'(\sigma - \sigma') - G'(\sigma')\delta(\sigma - \sigma').$$

This is the classical algebra for the constraints of the null string [10] and it arises as the Inonu-Wigner contraction ($T \rightarrow 0$) [11] of the algebra of the constraints of the tensile string. Quantum mechanically the Poisson brackets will be replaced by commutators and this offers the possibility for the emergence of central extension in the algebra [12]. We hope to report our results towards this direction in a future communication.

4. Conclusions.

In this section we will briefly recapitulate what we have done in this paper. We studied string propagation in an anisotropic Milne background. The equations of motion and the constraints were solved by means of the null string expansion. In this scheme the string equations of motion and the constraints are systematically expanded in powers of the world-sheet speed of light $c^2$ which is proportional to the string tension $T$. To zeroth order the string is described by a null (tensionless) string—an extended object. The points of the null string interact only with the gravitational background. The first order correction introduces the string tension $T$ and thus the self-interaction among the points of the string. We derived exact analytical expressions for the zeroth and first order solutions. We also
calculated the stress tensor for the null string and examined some of its cosmological implications.

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