Dynamic Event—Triggered Discrete—Time Linear Time—Varying System with Privacy—Preservation

Xuefeng Yang, Li Liu, Member, IEEE, Wenju Zhou, Member, IEEE, Jing Shi, Yinggang Zhang, Xin Hu and Huiyu Zhou

Abstract—This paper focuses on discrete-time wireless sensor networks with privacy-preservation. In practical applications, information exchange between sensors is subject to attacks. For the information leakage caused by the attack during the information transmission process, privacy-preservation is introduced for system states. To make communication resources more effectively utilized, a dynamic event-triggered set-membership estimator is designed. Moreover, the privacy of the system is analyzed to ensure the security of the real data. As a result, the set-membership estimator with differential privacy is analyzed using recursive convex optimization. Then the steady-state performance of the system is studied. Finally, one example is presented to demonstrate the feasibility of the proposed distributed filter containing privacy-preserving analysis.

Index Terms—Set-membership estimation, wireless sensor networks, privacy-preservation, event-triggered scheme.

I. INTRODUCTION

DISTRIBUTED computing is the sharing of information among multiple pieces of software, which can run on a single machine or connecting by multiple computers over a network. Distributed computing applications are decomposed into multiple small parts and distributed to multiple computers for processing, which allows for reducing running time and sharing resource. Therefore, state estimation based on distributed computing has become a popular research topic [1–3].

Wireless sensor networks (WSNs) are mainly multi-hop self-organized distributed sensing networks, which are formed by large amount sensor nodes on the basis of wireless communication technology. Due to the advantage of the unrestricted formation, the uncertain network structure and the decentralized control among sensor nodes, WSNs are widely used in military [4], industrial [5] and commercial fields [6]. However, performing efficient distributed processing is an extremely challenging topic, a great deal of studying on this topic are conducted, such as Kalman filter [7], [8] and $H\infty$ filter [9], [10]. Kalman filter is mainly applied to systems with deterministic noise or models [11], [12]. Considering the different applications in practice, to improve the performance of the traditional Kalman filter, novel methods are proposed, such as the unscented Kalman filter, the extended Kalman filter and the cubature Kalman filter [13–15]. When the noise or system model is uncertain, $H\infty$ filter is applied to obtaining more accurate estimates and ensures the robustness of the system [16–19]. However, in practical engineering applications, due to the noise complexity and modeling imprecision, the inaccurate values or even very terrible results are caused during the estimation process. The set-membership estimator (SME) is more suitable for solving such problems under unknown but bounded (UBB) noise [20]. In recent years, the ellipsoid algorithm in the SME algorithm has been extensively studied [21–23]. The ellipsoid algorithm mainly restricts the bounded noise to a set of ellipsoids. However, The ellipsoid algorithm is first transformed into a recursive convex optimization problem, which is then solved using interior point polynomials.

Information exchange between sensors is required in most estimation algorithms, which means that sensors need to broadcast information to their neighbors within a specific sampling period. However, continuous or periodic information exchange between sensors consumes a lot of communication resources, which can even lead to network congestion or packet loss [24], [25]. Therefore, designing a feasible algorithm to decrease the frequency of data transmission between sensors is important for the sustainable use of communication resources. In practical research, event-triggered schemes (ETS) are divided into static event-triggered schemes (SETS) [26] and dynamic event-triggered schemes (DETS) [27–29]. The threshold parameter of SETS is a fixed scalar, while DETS introduces an auxiliary parameter for the threshold. Auxiliary dynamic variables and dynamic threshold parameters are two typical algorithms for DETS. Since the DETS has higher resource utilization than SETS, DETS is more widely used.

Meanwhile, the advent of information era has infringed the network information security. During the information exchange, information tampering and leakage, the transmission procedure is confronted with the main threats to network communication [30–33]. Similarly, the openness of interactive channels inevitably brings threats to information security. Therefore, it is necessary to protect the privacy of information. There are two main privacy-preserving methods: specially designed random noise and differential privacy methods. However, differential privacy methods have been widely studied due to the rigorous formulation derivation in the application and the proven security [34], [35]. Differential privacy methods protect the privacy of information using random states. It is very difficult for an attacker to deduce the real information. To prevent the data from being tampered and leaked during the exchange of sensor information, the filter estimation that incorporates privacy-preserving still needs to be further explored.

However, in reality, the existing models are unable to address several challenges at the same time. Firstly, the complexity of the noise leads to inaccurate models, despite simple
assumptions for noise. It becomes a challenge to build more accurate models. Secondly, data transmission when sensors exchange information consumes too many resources, but it is difficult to reduce the frequency of data transmission. Thirdly, the open nature of the information exchange channel poses a threat to data security. It is a challenge to ensure data security.

In summary, this paper studies a dynamic event-triggered discrete-time linear time-varying system (DTLVS) containing privacy-preservation. Considering the accuracy of the calculation and alleviating the complexity of the system, this paper uses an optimal bounding ellipsoid algorithm. The primary work is summarized as follows.

First, privacy-preserving noise is used to protect the initial state information from being stolen and leaked, and then analyze the privacy of the system. Section IV simulates the proposed model, where each element $a_{ij}$ represents the weight of edge between nodes is denoted by $N$. When data transfer is possible between two adjacent nodes. When data transfer is possible between two neighboring nodes, $a_{ij} = 0$, otherwise, $a_{ii} = 0$. The set of all neighbors for the node $i$ is denoted by $N_i = \{j \in V : (i,j) \in \varepsilon \}$. The directed graph consisting of $N$ nodes is denoted by $A = (V,\varepsilon, A)$.

Considering the noise from the external environment is described as UBB, and the ellipsoidal ensemble form is described as UBB, and the ellipsoidal representation can be written as $Z \triangleq \{a : (a-b)^TP^{-1}(a-b) \leq 1\}$, where $P = EE^T$ according to the cholesky decomposition.

### A. System Model

In the field of ship navigation, some noise is inevitably generated because of unknown environmental changes during sailing. At the same time, the resistance generates changes so that the speed of the sailing ship is affected. Consequently, the data of the sailing speed and resistance, which are sensed or transmitted by the sensor, are containing noise. The disturbed data is processed by the sensor to obtain the original data. As the sensors send and receive signals in a distributed manner, the open nature of their transmission channels may allow attacks to be made in the process causing distortion of the data. The security of the data is not guaranteed. Erroneous data is transmitted and sensed by neighboring sensors resulting in inaccurate data being obtained by the final estimator. It is necessary to ensure the security of its initial state. Consequently, introducing privacy noise protects the data security with mixing noise for the state value of the system state.

The model of the ship navigation is established as follows.

\[
\begin{aligned}
\dot{z}_k &= x_k + \eta_k \\
x_{k+1} &= C_k z_k + F_k w_k,
\end{aligned}
\]

where $x_k \in \mathbb{R}^n_k$ describes the state variable. The wind and waves, the ship sways. The ship swaying changes periodically due to the wave activities. $C_k$ and $F_k$ are defined as the time-varying periodic matrices, and $z_k \in \mathbb{R}^n$ describes the state after privacy-preserving, $\eta_k \in \mathbb{R}^n$ describes the random privacy noise obeying the Laplace distribution. When the state information is leaked or stolen, the stealer obtains...
the information contains privacy noise, and the real state information cannot be obtained. This noise is represented as

\[
\begin{cases}
\eta_k \sim \text{Lap}(b) \\
b = cq^k
\end{cases},
\]

where \(c\) and \(q\) satisfy the following conditions

\[
\begin{align*}
&c > 0, \\
&q \in (0, 1).
\end{align*}
\]

\(w_k \in \mathbb{R}^{n_w}\) in Eq. (1) expresses the process noise limited within a certain ellipsoid

\[
W_k \triangleq \left\{ w_k : w_k^T (R_k)^{-1} w_k \leq 1 \right\},
\]

where \(R_k = (R_k)^T > 0\) expresses the real-valued matrix.

**B. Measurement Output Model**

The output measurement value of sensor \(i\) at moment \(k\) is expressed as follows

\[
y_k^i = H_k^i x_k + D_k^i v_k^i,
\]

where \(y_k^i \in \mathbb{R}^{n_y}\) expresses the output measurement of sensor \(i\), \(H_k^i\) and \(D_k^i\) represent the time-varying coefficient matrices, and \(v_k^i \in \mathbb{R}^{n_y}\) is measurement noise limited within a certain ellipsoid

\[
V_k^i \triangleq \left\{ v_k^i : (v_k^i)^T (Q_k^i)^{-1} v_k^i \leq 1 \right\},
\]

where \((Q_k^i)^T > 0\) is the real-valued matrix.

Remark 1: For a known \(\varsigma > 0\), when there exists

\[
|x_{i0}^{\varsigma, 2} - x_{i0}^{\varsigma, 1}| \leq \left\{ \begin{array}{ll}
\varsigma, & i = i_0 \\
0, & i \neq i_0,
\end{array} \right.
\]

the two sets of initial states \(x_{i0}^{\varsigma, 2}\) and \(x_{i0}^{\varsigma, 1}\) of system \(i\) can be named \(\varsigma > 0\)-adjacent [35].

Remark 2: Assuming that \(\Pr [A \log(x^1) \in \Xi] \leq e^{\varsigma} \Pr [A \log(x^2) \in \Xi]\) holds, adjacent initial states \(x_{i0}^{\varsigma, 2}\) and \(x_{i0}^{\varsigma, 1}\) can be obtained such that the algorithm meets \(\epsilon\)-differential privacy performance, i.e., \(\epsilon_i = \frac{\varsigma q}{c_i (q - 1)}\).

**C. Dynamic Event-Triggered Scheme**

To decrease the frequency of data transmission between sensors and improve the sustainable use of resources, a DETS [37] is presented.

The event-triggered moment of the sensor \(i\) is described as follows

\[
t_{k+1}^i = \inf_{k \in \mathbb{N}} \left\{ k > t_k^i \mid \theta_i^k > \delta_k^i \right\},
\]

where

\[
\begin{align*}
\eta_k &\sim \text{Lap}(b) \\
b &\equiv cq^k
\end{align*}
\]

\[
\left\{ l_k = (h_k^i)^T \psi_k^i \bar{h}_k^i - \sigma_i (\bar{y}_k^i)^T \psi_k^i \bar{y}_k^i, \\
l_k^i &\equiv \bar{y}_k^i - \bar{y}_k^i, \\
\bar{y}_k^i &\equiv y_k^i - C_k^i x_k^i.
\right.
\]

Here \(\sigma_i \in (0, 1)\) denotes the specified threshold parameter, \(\psi_k^i \equiv (\psi_k^i)^T > 0\) denotes the sequence of weighting matrices to be determined, and \(\bar{y}_k^i\) denotes the measured residuals at moment \(k\). \(\delta_k^i\) is the key parameter and represents the auxiliary offset parameter, which satisfies Eq. (10)

\[
\delta_k^i = \rho_i \delta_k^i - t_k^i.
\]

\(\delta_k^i\) is the initial value of the auxiliary system. In Eqs. (8) and (10), \(\rho_i\) and \(\theta_i\) satisfy

\[
\left\{ \begin{array}{ll}
0 < \rho_i < 1 \\
\theta_i \geq 1/\rho_i.
\end{array} \right.
\]

At the moment \(k\), the packet scheduler \(i\) is used to check the current packet \((k, \bar{y}_k^i)\). When the event-triggered condition under Eq. (8) is satisfied, the release moment \(t_{k+1}^i\) is calculated, at the same time, the packet at this moment is sent to the estimator \(i\) and transmitted to its neighbors. Otherwise, this packet is discarded and the last reserved transmission packet \((t_k^i, \bar{y}_k^i)\) is used. The auxiliary offset variable in Eq. (10) is the key parameter in DETS, and then it can dynamically adjust the interval between two consecutive sampling moments, i.e., \((t_{k+1}^i - t_k^i)\).

**D. Event-Triggered Set-Membership Estimator**

State estimation needs to achieve the full-scale confidence level. Therefore, this section aims to design an event-triggered set-membership estimator, and the one-step prediction for estimator \(i\) is formulated as follows

\[
\hat{x}_{k+1}^i = \hat{A}_k^i \hat{x}_k^i + \hat{B}_k^i \sum_{j \in N_i} a_{ij} \tilde{y}_j^i,
\]

where \(\tilde{y}_j^i \equiv \arg \min_{\tilde{y}_j^i} \left\{ k - t_k^j \mid k > t_k^j, \tilde{y}_j^i \in N_i \right\}, \hat{A}_k^i\) and \(\hat{B}_k^i\) are the time-varying estimated gain matrices, while the initial state estimated value \(\hat{x}_0^i\) satisfies

\[
X_0^i \triangleq \left\{ x_0: (x_0 - \hat{x}_0^i)^T (U_0^i)^{-1} (x_0 - \hat{x}_0^i) \leq \beta_i \right\},
\]

where \(U_0^i = (U_0^i)^T > 0\) and \(\beta_i > 0\) represent the time-varying real-valued matrix and the parameter variables of the ellipsoid, respectively.

In a system containing privacy noise \(\eta_k\), UBB processes \(w_k\) and measurement noise \(v_k\), a confidence interval containing all estimates can be obtained by the set-membership estimator

\[
X_{k+1}^i \triangleq \left\{ x_{k+1}: (c_{k+1}^i)^T (U_{k+1}^i)^{-1} c_{k+1}^i \leq \beta_i \right\},
\]

where \(c_{k+1}^i = x_{k+1} - \hat{x}_{k+1}^i\) denotes the estimation error and \(U_{k+1}^i = (U_{k+1}^i)^T > 0\) is the time-varying real-valued matrix.

To satisfy the set-membership estimator with privacy-preserving, for a given sequence of scalars \(\sigma_i \in [0, 1], \beta_i > 0, \rho_i\) and \(\theta_i\) from Eq. (11), set \(\eta_k \in \mathbb{R}^{n_y}, w_k \in \mathbb{R}^{n_w}\) and \(v_k \in \mathbb{R}^{n_y}, i \in v\). Assuming that \(U_{k+1}^i > 0, \psi_k^i > 0, \bar{A}_k^i\) and \(\bar{B}_k^i\) exist, the one-step prediction state \(x_{k+1}\) can be resolved within the ellipsoid \(X_{k+1}^i\) of the estimated state.
III. Model Analysis

The distributed system consists of \( N \) subsystems, and the subsystems need to interact over communication. Therefore, to analyze the model, we restructure the parameters.

\[
\hat{e}_k = \text{col}_N \{ e_k^i \}, \quad \hat{x}_k = \text{col}_N \{ x_k \}, \quad \hat{v}_k = \text{col}_N \{ v_k^i \}, \\
\hat{\psi}_k = \text{diag}_N \{ \psi_k^i \}, \quad \hat{\Sigma} = \text{col}_N \{ \sigma_i \}, \\
\alpha = \text{col}_N \{ \alpha_i \}, \quad \beta = \text{diag}_N \{ (\beta_i)^2 \}, \quad \hat{U}_k = \text{diag}_N \{ U_k^i \}. 
\]

The following analysis focuses on the conditions guaranteeing steady-state performance, which satisfy the set-membership estimation after adding differential privacy.

A. Analyzing Set-Membership Estimation with Differential Privacy

The proposed set-membership estimator is designed to achieve DETS and privacy preservation, meanwhile it satisfies the one-step prediction state of \( x_{k+1} \) being within estimation ellipsoid \( X_{k+1} \).

Theorem 1: For a given scalar \( \sigma_i \in (0, 1) \), \( \beta_i > 0 \), \( \rho_i \) and \( \theta_i \) satisfying Eq. (11), set \( \eta_k \in \mathbb{R}^{n_u} \), \( w_k \in \mathbb{R}^{n_w} \) and \( v_k^i \in \mathbb{R}^{n_v}, \) \( i \in i \). Suppose that \( U_k^i > 0 \), \( \psi_k^i > 0 \), \( A_k^i \), \( B_k^i \) and a sequence of scalar \( e_k^m > 0 \), \( m = 1, 2, 3, 4 \), there exists

\[
\begin{align*}
\left( I - \tilde{U}_{k+1} \right) \Phi_k \Lambda_k &\leq 0, \forall k \in N, \\
\Lambda_k &\in [\Lambda_k^q]_{6 \times 6} \\
\end{align*}
\]

where

\[
\Phi_k = \left[ (\tilde{C}_k - \hat{A}_k) \hat{e}_k, \beta (\tilde{C}_k - \hat{B}_k A \hat{H}_k) L_k, F_k, \hat{C}_k, \hat{B}_k A \hat{D}_k, \hat{B}_k A \right],
\]

and \( \Lambda_k = [\Lambda_k^q]_{6 \times 6} \) are real-valued matrices. The non-zero terms contained in \( \Lambda_k = [\Lambda_k^q]_{6 \times 6} \) are

\[
\begin{align*}
\Lambda_k^{1,1} &= -N \beta_i + e_k^i N + e_k^i N + e_k^i N + \sum_{i=1}^{N} \delta_i^i, \\
\Lambda_k^{2,2} &= -e_k^i I + (\tilde{L}_k)^T (\tilde{H}_k)^T \beta \theta \tilde{\Sigma} \hat{\psi}_k \beta \tilde{H}_k \tilde{L}_k, \\
\Lambda_k^{2,5} &= (\tilde{L}_k)^T (\tilde{H}_k)^T \beta \theta \tilde{\Sigma} \hat{\psi}_k \beta \tilde{L}_k, \\
\Lambda_k^{2,6} &= - (\tilde{L}_k)^T (\tilde{H}_k)^T \beta \theta \tilde{\Sigma} \hat{\psi}_k \beta \tilde{L}_k, \\
\Lambda_k^{3,3} &= -e_k^i (\tilde{R}_k)^{-1}, \quad \Lambda_k^{4,4} = e_k^i (\tilde{M}_k)^{-1}, \\
\Lambda_k^{5,5} &= -e_k^i (\tilde{Q}_k)^{-1}, \quad \Lambda_k^{6,6} = e_k^i \tilde{\Sigma} (\tilde{\Sigma} - I) \hat{\psi}_k.
\end{align*}
\]

Proof: First, to simplify Eq. (13), \( (e_k^0)^T (U_k^0)^{-1} e_k^0 \leq \beta_i \) is obtained. If there exists \( x_k \in X_k^i \) satisfying

\[
(e_k^i)^T (U_k^i)^{-1} e_k^i \leq \beta_i,
\]

it is sufficient to prove that \( x_{k+1} \in X_{k+1} \) exists and \( (e_{k+1}^i)^T (U_{k+1}^i)^{-1} e_{k+1}^i \leq \beta_i \).

An alternative formulation of the ellipsoid, where the estimation error can be obtained from \( e_{k+1}^i = x_{k+1} - \hat{x}_{k+1} \) at time \( k+1 \), expressed as \( (x_k - \hat{x}_k)^T (U_k^{-1}) (x_k - \hat{x}_k) \leq \beta_i \).

To simplify the set of ellipsoids where the estimated errors are located \( \beta_i = (e_{k+1}^i)^T (U_{k+1}^i)^{-1} e_{k+1}^i \), which may be transmitted as \( \beta_i^{-1} e_{k}^i (e_{k}^i)^T \leq U_{k}^i \) under the Schur complementary. Then, we decompose \( U_k^i \) using the cholesky factorization, i.e. \( U_k^i = L_k^i (L_k^i)^T \), where \( L_k^i \) is a lower triangular matrix satisfying all elements on the diagonal are positive. Therefore, \( (\beta_i)^{-1} e_{k}^i (e_{k}^i)^T \leq L_k^i (L_k^i)^T \) is redefined. Let \( \alpha_i = (\beta_i)^{-1} \), then

\[
\alpha_i^T \alpha_i = (\beta_i)^{-1} (x_k - \hat{x}_k)^T (U_k^{-1}) (x_k - \hat{x}_k) \leq 1,
\]

i.e. meeting \( \| \alpha_i \| \leq 1 \).

From Eq. (13), the state is rewritten as

\[
x_k = (\beta_i)^{\frac{1}{2}} L_k^i \alpha_i + \hat{x}_k.
\]
Note that,
\[
\begin{align*}
\delta_k^i - \theta_i \left( (h_k^i)^T \psi_k^i \hat{h}_k^i - \sigma_i (\hat{y}_k^i - h_k^i)^T \psi_k^i (\hat{y}_k^i - h_k^i) \right) \\
= \delta_k^i + \theta_i \sigma_i (h_k^i)^T \psi_k^i \hat{h}_k^i - \theta_i (h_k^i)^T \psi_k^i h_k^i \\
+ \theta_i \sigma_i \alpha_i^T (L_k^i)^T (\beta_k^{1/2})^T (H_k^i)^T \psi_k^i h_k^i \beta_k^{1/2} L_k^i \alpha_i \\
+ \theta_i \sigma_i \alpha_i^T (L_k^i)^T (\beta_k^{1/2})^T (H_k^i)^T \psi_k^i D_k^i v_k \\
+ \theta_i \sigma_i (v_k^i)^T (D_k^i)^T \psi_k^i h_k^i \beta_k^{1/2} L_k^i \alpha_i \\
+ \theta_i \sigma_i (v_k^i)^T (D_k^i)^T \psi_k^i D_k^i v_k - \theta_i \sigma_i (h_k^i)^T \psi_k^i h_k^i - \theta_i \sigma_i (v_k^i)^T (D_k^i)^T \psi_k^i h_k^i \\
- \theta_i \sigma_i \alpha_i^T (L_k^i)^T (\beta_k^{1/2})^T (H_k^i)^T \psi_k^i h_k^i.
\end{align*}
\]

The nonzero terms of matrix \( \Xi_k = [\Xi_{k}^{0 \cdot q}]_{6 \times 6} \) from Eq. (23) are defined as
\[
\Xi_{k}^{1,1} = \sum_{i=1}^{N} \delta_k^i, \quad \Xi_{k}^{2,2} = \Lambda_k^{2,2} + \epsilon_k^1 T, \quad \Xi_{k}^{3,3} = \Lambda_k^{3,3} + \epsilon_k^1 \hat{Q}_k, \quad \Xi_{k}^{4,4} = \Lambda_k^{4,4}
\]
and \( \Xi_{k}^{6,6} = \Lambda_k^{6,6} \).

According to the S-procedure, assuming that a sequence of scalars \( \epsilon_k^m > 0 \), i.e. \( m = 1, 2, 3, 4 \) exists and all of them are positive, inequality Eq. (21) can be rewritten as
\[
\Phi_k^T \tilde{U}_{k+1} \Phi_k + \Theta + \epsilon_k^1 \Gamma_k^1 + \epsilon_k^2 \Gamma_k^2 + \epsilon_k^3 \Gamma_k^3 + \epsilon_k^4 \Gamma_k^4 + \Xi_k \leq 0. \tag{24}
\]
The inequality Eq. (24) is simplified using the Schur complement to obtain Eq. (16).

**B. Analysis of System Stability**

The defined \( g_k = E \left[ x_k x_k^T \right] \) satisfies the following equation
\[
g_{k+1} = C_0 g_k C_0^T + C_0 M_k C_0^T + F_0 R_k F_k^T. \tag{25}
\]

Theorem 2: For the system in Eqs. (1) and (5), the solution \( g_k \) with any initial value \( g_0 \geq 0 \) converges to a unique semi-positive definite solution \( g \) of the Lyapunov equation. Note that the matrix \( C \) is stable.
\[
g = C_0 g_k C_0^T + C_0 M_k C_0^T + C_0 R_k C_0^T.
\]

In addition, \( \lim_{k \to \infty} g_k = g \), \( \lim_{k \to \infty} R_k = R \), \( \lim_{k \to \infty} Q_k = Q \), and \( \lim_{k \to \infty} M_k = M \) are held.

Proof: Since matrix \( C \) is stable, the spectral radius \( \rho(C) < 1 \) of matrix \( C \) is obtained. In this context, \( g = \lim_{k \to \infty} g_k \). In addition, \( \lim_{k \to \infty} R_k = R \), \( \lim_{k \to \infty} Q_k = Q \), \( \lim_{k \to \infty} M_k = M \) are held. So when \( g_k \) converges to the unique semi-positive definite solution \( g \) of the Lyapunov equation, the system has the steady-state performance.

**IV. NUMERICAL SIMULATION**

This section applies the model to a ship navigation system, which has the similar description in [39], where a DTLVS can be modeled from all external factors
\[
\begin{bmatrix}
x_{k+1}^1 \\
x_{k+1}^2
\end{bmatrix} =
\begin{bmatrix}
0.9653 & -0.0021 \\
-0.054 & 0.7654 + 0.2 \times \sin(k)
\end{bmatrix}
\begin{bmatrix}
x_k^1 + \eta_k^1 \\
x_k^2 + \eta_k^2
\end{bmatrix},
\tag{27}
\]
where \( x_k^1 \) and \( x_k^2 \) denote the speed and resistance in ship navigation, respectively. In the actual ship navigation, the noise

is mainly from the disturbance of the external environment, such as unpredictable weather, waves, etc. Note that the privacy noise \( \eta_k \) is random to protect the system safety. Therefore, the time-varying matrix is defined as
\[
C_k =
\begin{bmatrix}
0.9653 & -0.0021 \\
-0.054 & 0.7654 + 0.2 \times \sin(k)
\end{bmatrix},
\tag{28}
\]
\[
F_k =
\begin{bmatrix}
0.22 + 0.22 \times \sin(k) \\
0.22
\end{bmatrix}.
\tag{29}
\]

To provide more accurate and convincing data in the simulation, a network topology with five sensor nodes is designed, as shown in Fig. 2. Each sensor receives and transmits data only to its neighboring sensors. Sensor nodes \( i \) and \( j \) exchange information to receive or transmit data when the element \( a_{ij} \) in the adjacency matrix is set as 1. The sensor is affected by the measurement noise \( v_k \) while transmitting data, where the time-varying coefficient matrices are
\[
H_k =
\begin{bmatrix}
0 & 1.1 + 0.11^* (i + 1) - 0.11^* \sin(k) \\
0 & 1.1 + 0.11^* (i + 1) - 0.11^* \sin(k)
\end{bmatrix}
\]
and \( D_k = 1/(i + 1) \).

In this section, let the initial speed and initial resistance of the ship sailing be 1.7m/s and 3.7kgf. Set the estimated values of the initial moment as \( \dot{x}_0^1 = [1.8 \ 3.7] \), \( \dot{x}_0^2 = [1.6 \ 3.8] \), \( \dot{x}_0^3 = [1.8 \ 3.55] \), \( \dot{x}_0^4 = [1.35 \ 3.3] \) and \( \dot{x}_0^5 = [1.5 \ 3.9] \). Let \( U_0 = \text{diag}_2 \{40 \ 40\} \), \( \beta_i = 1 \), \( R_k = 0.4 \), \( \hat{Q}_k = 0.2 \) and \( b = 1 \).

In the DTS model, set the values of its corresponding parameters as follows: \( \sigma_i = \{0.98, 0.9, 0.8, 0.85, 0.93\} \), \( \rho_i = 0.7 \), \( \theta_i = 30 \) and \( \delta_i^0 = \{0.25, 0.2, 0.15, 0.1, 0.05\} \), where \( i = 1, 2, 3, 4, 5 \).

In Figs. 3 and 4, \( x_k^1 \), \( x_k^2 \) denote the sailing speed and resistance parameters in the state variables, and \( x_k^{1,1} \) and \( x_k^{1,2} \) denote the estimation value of the sailing speed and resistance. The designed set-membership estimation model with privacy-preserving is simulated and compared with the model in [39].

The actual value is always in the estimation interval, and the estimated value obtained by the estimator is close to the actual value given in Fig. 3. From Fig. 4 when the other models are applied to the ship navigation system, there is a certain disparity between the estimated value and the actual value. And there is a certain difference in the numerical fluctuation of the speed and resistance of the ship. The actual state value of the system cannot be judged by the estimator. Therefore, the estimator designed in this paper can be well applied in the ship navigation system.
Adding DETS to the ship application, the release time interval of specific events is shown in Fig. 5. It shows that the dynamic event triggering scheme is applied, the triggering time is discontinuous, and the number of event-triggered events is significantly decrease. The DETS is able to better decrease the frequency of data transmission between sensors and their neighbors, and then, improve the sustainable use of resources.

From Fig. 6, the estimation error values of each sensor can be fluctuated within a certain range, and the fluctuation range is very small. It may be obtained that the model designed can be efficiently applied in the ship navigation system, and optimal estimation values can be obtained at each moment.

V. CONCLUSION

This paper addressed information privacy-preservation for DTLVS. The initial state of the system was protected using
differential privacy. To improve the system security, an event-triggered set-membership estimation method with privacy-preserving was proposed. Meanwhile, the system stability was analyzed. Finally, it was indicated through simulation experiments that the estimator could be successfully applied, and the introduction of DETS improves the sustainable utilization of resources. This could be followed by research in the incorporation of cyber attacks.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China under Grant 61903172, Grant 61877065 and Outstanding Youth Innovation Team Project of Shandong Higher Education Institution under Grant 2021KJ042.

REFERENCES

[1] M. N. Kurt, Y. Yılmaz, and X. D. Wang, “Secure distributed dynamic state estimation in wide-area smart grids,” IEEE Transactions on Information Forensics and Security, vol.15, pp. 800–815, 2020.
[2] J. X. Wang, and T. Li, “Distributed multi-area state estimation for power systems with switching communication Graphs,” IEEE Transactions on Smart Grid, vol.12, no. 1, pp. 787–797, 2021.
[3] M. Z. Mohamed, S. Saxena, and H. E. Farag, “Optimal design of islanded microgrids considering distributed dynamic state estimation,” IEEE Transactions on Industrial Informatics, vol. 17, no. 3, pp. 1–1, 2020.
[4] X. H. Liu, Z. H. Xu, L. S. Wang, W. Dong, and S. P. Xiao, “Cognitive dwell time allocation for distributed radar sensor networks tracking via cone programming,” IEEE Sensors Journal, vol.20, no. 10, pp. 5092–5101, 2020.
[5] J. M. Liu, Z. Y. Zhao, J. Ji, and M. L. Hu, “Research and application of wireless sensor network technology in power transmission and distribution system,” Intelligent and Converged Networks, vol.1, no. 2, pp. 199–220, 2020.
[6] W. Youn, H. Lim, H. S. Choi, M. B. Rhudy, H. Ryu, S. Kim, et al., “State estimation for HALE UAVs with deep-learning-aided virtual AOA/SSA sensors for analytical redundancy,” IEEE Robotics and Automation Letters, vol.6, no. 3, pp. 5276–5283, 2021.
[7] J. He, C. Sun, B. Zhang and P. Wang, “Adaptive Error-State Kalman Filter for Attitude Determining on a Moving Platform,” IEEE Transactions on Instrumentation and Measurement, vol.70, pp. 1–10, 2021.
[8] D. R. Ding, Q. L. Han, Z. D. Wang, and X. H. Ge, “A survey on model-based distributed control and filtering for industrial cyber-physical systems,” IEEE Transactions on Industrial Informatics, vol.15, no. 5, pp. 2483–2499, 2019.
[9] Y. Chen, Z. D. Wang, Y. Yuan, and P. Date, “Distributed $H\infty$ filtering for switched stochastic delayed systems over sensor networks with fading measurements,” IEEE Transactions on Cybernetics, vol.50, no. 1, pp. 2–14, 2020.
[10] Y. Q. Luo, Z. D. Wang, Y. Chen, and X. J. Yi, “$H\infty$ state estimation for coupled stochastic complex networks with periodical communication protocol and intermittent nonlinearity switching,” IEEE Transactions on Network Science and Engineering, vol.8, no. 2, pp. 1414–1425, 2021.
[11] L. Liu, A. Yang, X. Tu, M. Fei, and W. Naeem, “Distributed weighted fusion estimation for uncertain networked systems with transmission time-delay and cross-correlated noises,” Neurocomputing, vol.270, pp. 54–65, 2017.
[12] J. K. Lee, “A Parallel Attitude-Heading Kalman Filter Without State-Augmentation of Model-Based Disturbance Components,” IEEE Transactions on Instrumentation and Measurement, vol.67, no. 7, pp. 2668–2670, 2019.
[13] G. Battistelli, and L. Chisci, “Stability of consensus extended Kalman filter for distributed state estimation,” Automatica, vol.68, pp. 169–178, 2016.
[14] W. Muhammad, and A. Ahsan, “Airship aerodynamic model estimation using unscented Kalman filter,” Journal of Systems Engineering and Electronics, vol.31, pp. 1318–1329, 2020.
[15] W. L. Wang, C. K. Tse, and S. Y. Wang, “Dynamic state estimation of power systems by p-norm nonlinear Kalman filter,” IEEE Transactions on Circuits and Systems I: Regular Papers, vol.67, no. 5 pp. 1715–1728, 2020.
[16] J. B. Zhao, and L. Mili, “A decentralized $H\infty$ unscented Kalman filter for dynamic state estimation against uncertainties,” IEEE Transactions on Smart Grid, vol.10, pp. 4870–4880, 2019.
[17] L. Liu, W. J. Zhou, M. R. Fei, Z. L. Yang, H. Y. Yang, and H. Y. Zhou, “Distributed fusion estimation for stochastic uncertain systems with network-induced complexity and multiple noise,” IEEE Transactions on Cybernetics, vol.52, no. 9, pp. 2168–2207, 2022.
[18] H. H. Gong, Y. Yu, L. N. Zheng, B. L. Wang, L. Zhen, T. Fernando, et al., “Nonlinear $H\infty$ filtering based on tensor product model transformation,” IEEE Transactions on Circuits and Systems II: Express Briefs, vol.67, no. 6, pp. 1074–1078, 2020.
[19] M. G. Hua, D. D. Zheng, F. Q. Deng, J. T. Fei, P. Cheng, and X. S. Dai, “$H\infty$ Filtering for nonhomogeneous markovian jump repeated scalar nonlinear systems with multiplicative noises and partially mode-dependent characterization,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol.51, no. 5, pp. 3180–3192, 2021.
[20] Y. L. Zhang, N. Xia, Q. L. Han, and F. W. Yang, “Set-membership global estimation of networked systems,” IEEE Transactions Cybern, vol.52, no. 3, pp. 1454–1464, 2022.
[21] D. Bhattacharjee, and K. Subbarao, “Set-membership filter for discrete-time nonlinear systems using state-dependent coefficient parameterization,” IEEE Transactions on Automatic Control, vol.67, no. 2, pp. 894–901, 2022.
[22] E. MousaviNajaf, X. H. Ge, Q. L. Han, T. J. Lim, and L. Vlasic, “An ellipsoidal set-membership approach to distributed joint state and sensor fault estimation of autonomous ground vehicles,” IEEE/CAA Journal of Automatica Sinica, vol.8, no. 6, pp. 1107–1118, 2021.
[23] D. R. Ding, Z. D. Wang, and Q. L. Han, “A set-membership approach to event-triggered filtering for general nonlinear systems over sensor networks,” IEEE Transactions on Automatic Control, vol.65, no. 4, pp. 1792–1799, 2020.
X. M. Zhang, Q. L. Han, X. H. Ge, D. R. Ding, L. Ding, D. Yue, et al., “Networked control systems: a survey of trends and techniques,” *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 1, pp. 1–17, 2019.

C. Peng, Q. L. Han, and D. Yue, “To transmit or not to transmit: a discrete event-triggered communication scheme for networked Takagi–Sugeno fuzzy systems,” *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 1, pp. 164–170, 2013.

H. Yang, Z. D. Wang, Y. X. Shen, and F. E. Alsaadi, “Self-triggered filter design for a class of nonlinear stochastic systems with Markovian jumping parameters,” *Nonlinear Analysis: Hybrid Systems*, vol. 40, 2021.

D. R. Ding, Z. D. Wang, and Q. L. Han, “A scalable algorithm for event-triggered state estimation with unknown parameters and switching topologies over sensor networks,” *IEEE Transactions on Cybern.*, vol. 50, no. 9, pp. 4087–4097, 2020.

X. H. Ge, Q. L. Han, L. Ding, Y. L. Wang, and X. M. Zhang, “Dynamic event-triggered distributed coordination control and its applications: a survey of trends and techniques,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, pp. 3112–3125, 2020.

X. H. Ge, Q. L. Han, X. M. Zhang, L. Ding, and F. Yang, “Distributed event-triggered estimation over sensor networks: a survey,” *IEEE Transactions on Cybern.*, vol. 50, no. 9, pp. 1306–1320, 2020.

K. D. Lu and Z. G. Wu, “Genetic Algorithm-Based Cumulative Sum Method for Jamming Attack Detection of Cyber-Physical Power Systems,” *IEEE Transactions on Instrumentation and Measurement*, vol. 71, no. 9004810, pp. 1–10, 2022.

D. Yue and Q. L. Han, “Guest editorial special issue on new trends in energy internet: artificial intelligence-based control, network security, and management,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 8, pp. 1551–1553, 2019.

H. Long, Z. Wu, C. Fang, W. Gu, X. C. Wei, and H. Y. Zhan, “Cyber-attack detection strategy based on distribution system state estimation,” *Journal of Modern Power Systems and Clean Energy*, vol. 8, no. 4, pp. 669–678, 2020.

D. J. Du, L. Wu, C. D. Zhang, Z. X. Fei, L. S. Yang, M. R. Fei, and H. Y. Zhou, “Co-Design Secure Control Based on Image Attack Detection and Data Compensation for Networked Visual Control Systems,” *IEEE Transactions on Instrumentation and Measurement*, vol. 71, pp. 1–14, 2022.

C. Y. Yin, J. W. Xi, R. X. Sun, and J. Wang, “Location privacy protection based on differential privacy strategy for big data in industrial internet of things,” *IEEE Transactions on Industrial Informatics*, vol. 14, no. 8, pp. 3628–3636, 2018.

H. P. Huang, D. J. Zhang, F. Xiao, K. Wang, J. T. Gu, and R. C. Wang, “Privacy-preserving approach PBCN in social network with differential privacy,” *IEEE Transactions on Network and Service Management*, vol. 17, no. 2, pp. 931–945, 2020.

A. J. Wang, X. F. Liao, and H. B. He, “Event-triggered differentially private average consensus for multi-agent network,” *IEEE/CAA Journal of Automatica Sinica*, vol. 6, no. 1, pp. 75–83, 2019.

A. Girard, “Dynamic triggering mechanisms for event-triggered control,” *IEEE Transactions on Automatic Control*, vol. 60, pp. 1992–1997, 2015.

P. Han, C. Ting, and L. Xi, “De-correlated unbiased sequential filtering based on best unbiased linear estimation for target tracking in Doppler radar,” *Journal of Systems Engineering and Electronics*, vol. 31, no. 6, pp. 1167–1177, 2020.

X. H. Ge, Q. L. Han, and Z. D. Wang, “A dynamic event-triggered transmission scheme for distributed set-membership estimation over wireless sensor networks,” *IEEE Transactions on Cybern.*, vol. 49, no. 1, pp. 171–183, 2019.
This figure "fig1.png" is available in "png" format from:

http://arxiv.org/ps/2210.15875v1