Transverse momentum decorrelation of the flow vector in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

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The individual studies of the anisotropic flow vector, flow angle and flow magnitude fluctuations with multi-particle correlations provide insight into the initial conditions and properties of the quark-gluon plasma (QGP) created in high-energy heavy-ion collisions. Recent measurements of these fluctuations have been available and the comparison to hydrodynamic models shows unique sensitivities to the initial conditions of the system, but also a puzzling dependence on the specific shear viscosity $\eta/s$. In this paper, a systematic study with A Multi-Phase Transport (AMPT) model using different tunings of the initial conditions, partonic cross section and hadronic interactions investigates the $p_T$-dependent flow vector, flow angle and flow magnitude fluctuations. It is found that the transport model reasonably describes the flow vector, flow angle and flow magnitude fluctuations observed in data and that the fluctuations are driven by fluctuations in the initial state. The comparison of data and model presented in this paper enables further constraints on the initial conditions of the heavy-ion collisions.

I. INTRODUCTION

The primary purpose of ultra-relativistic heavy-ion collisions, such as those at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC), is to create the strongly-coupled Quark-Gluon Plasma (QGP) and measure its properties [1,5]. The azimuthal anisotropy of produced particles is a key observable in probing the QGP [6, 7], and it is typically characterized by a Fourier expansion of the azimuthal angle distribution [8]

$$P(\varphi) = \sum_{n=-\infty}^{\infty} \sqrt{n} e^{-in\varphi},$$  \hspace{1cm} (1)

where $\varphi$ is the azimuthal angle of the produced particles and $\sqrt{n} = v_n e^{in\Psi_n}$ is the n-th order flow vector with magnitude $v_n$ and orientation $\Psi_n$. The magnitude $v_n$ is the n-th order flow coefficient, and the orientation $\Psi_n$ is the symmetry plane angle. The anisotropic flow has been studied extensively at RHIC [9–12] and the LHC [13–23]. Comparison to theoretical model calculations have shown that the QGP behaves like an almost perfect fluid [24, 27] with very low specific shear viscosity, $\eta/s$. A good theoretical model description on the experimental data have a significant impact on the understanding of the QGP. The current state-of-the-art understanding of the QGP comes from models based on Bayesian inference [28, 30], which use multiple experimental measurements to simultaneously constrain the parameters of the initial conditions and the hydrodynamical model. The extracted information can be better constrained by including higher-order flow and correlations of different order flow harmonics in the experimental observables [31]. Additional constraints can be obtained by including the $p_T$-differential studies of the flow coefficient $v_n(p_T)$ and its fluctuations, as they contain additional information about the dynamics of the expansion and the initial conditions of the heavy-ion collisions [30]. Fluctuations of the flow vector with the transverse momentum, $p_T$, are predicted in hydrodynamical simulations [32, 33] and have been measured at the LHC [34–37]. The event-by-event fluctuations of the flow coefficient $v_n$ are well known [7, 24, 38]; however, the decorrelation of the flow coefficient in $p_T$ has not been studied separately from the flow angle fluctuations [39]. Additionally, the fluctuations of the flow angle $\Psi_n$ has proved hard to measure experimentally [33, 39–44], and the dependence of the flow angle on the transverse momentum [35, 44–47] results in a decorrelation between the integrated symmetry plane, $\Psi_n$, and the $p_T$-differential plane, $\Psi_n(p_T)$. Such fluctuations potentially affect measurements that rely on the assumption of a common symmetry plane. The overall flow vector fluctuation gives insight into the event-by-event fluctuating initial conditions and separating the fluctuations into the flow angle and flow magnitude fluctuations enables the individual study of the sources of the fluctuations in heavy-ion collisions. Measurements of $p_T$-dependent flow vector fluctuations have traditionally been measured with two-particle correlations [29, 39]. Recently, we proposed new four-particle correlations to measure the flow angle and flow magnitude fluctuations separately [41, 48]. Additional hydrodynamical model calculations are compared to the ALICE data in [50], which were successful in capturing the trend of the data. However, the model calculations of the flow magnitude fluctuations showed a puzzling trend when using large values of $\eta/s$. At low $p_T$, where the expectation is to see little

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II. THE MODEL

In this paper, AMPT [51,52] is used to study the transverse momentum decorrelation of the flow vector, flow angle and flow magnitude fluctuations in Pb-Pb collisions at 5.02 TeV. The model consists of four stages: initial conditions, partonic scattering, hadronization, and hadronic rescattering. The initial conditions are generated with HIJING [57], which includes the spatial and momentum distributions of mini-jet partons and soft string excitations. Hadrons produced from string fragmentation are then converted with string melting into their valence quarks and anti-quarks. The interaction of partons is treated with ZDC [55], which only includes two-body scatterings. The cross sections are obtained from pQCD with screening masses

\[ \sigma = \frac{9\pi\alpha_s^2}{2\mu^2}, \]

where \( \alpha_s \) is the strong coupling and \( \mu \) is the Debye screening mass. At LHC energies, a strong coupling constant of \( \alpha_s = 0.33 \) is used [59]. Partons are converted into hadrons with a quark coalescence model, combining two quarks into mesons and three quarks into baryons [60]. Finally, the hadronic rescattering is handled by ART [61,62], where the dynamics of the hadronic matter are described by a hadron cascade. In order to study the effects of the initial state on the flow vector fluctuations, the Lund string parameters a and b, which are related to the string tension, are varied. The default setting is \( a = 0.3 \) and \( b = 0.15 \) [60] [59], and the variation is \( a = 0.5 \) and \( b = 0.9 \), which are the default HIJING values [63,64]. The effect of the QGP properties is studied by varying the partonic cross section using different values of the screening mass \( \mu \). The values \( \mu = 3.2042 \text{ fm}^{-1} \) and \( \mu = 2.2814 \text{ fm}^{-1} \) correspond to cross sections of \( \sigma \approx 1.5 \) mb and \( \sigma \approx 3 \) mb, respectively. A smaller cross section in AMPT corresponds to a larger viscosity in viscous hydrodynamics [65,66]. The specific shear viscosity can be approximated by

\[ \eta/s \approx \frac{3\pi}{40\alpha_s^2} \left( 9 + \frac{2\pi^2}{\alpha_s^2} \right) \ln \left( \frac{18\pi^2/\alpha_s^2}{T^2 - 2} \right) - 81, \]

where \( T \) is the initial temperature. At the LHC, the initial temperature is approximately 468 MeV [55], which leads to values of specific shear viscosity around 0.18 for 3 mb and 0.28 for 1.5 mb. Additionally, the effects of the hadronic rescatterings are explored by changing the rescattering time in ART between 0.6 fm/c (ART effectively OFF) and 30 fm/c (ART ON). In total, five different sets of parameters are used for AMPT in this study, as seen in table I. The results in this paper are presented in different centrality intervals. The centrality is determined by the impact parameter \( b \) as in [64]. Around 320K events are generated in the 0-10% central regions, and around 300K events are generated for each 10% interval of the other centralities.

| Partonic cross section | a | b | ART |
|-----------------------|---|---|-----|
| Par1                  | 1.5 mb | 0.3 | 0.15 | ON |
| Par2                  | 1.5 mb | 0.3 | 0.15 | OFF|
| Par3                  | 1.5 mb | 0.9 | 0.5  | OFF|
| Par4                  | 3.0 mb | 0.3 | 0.15 | ON |
| Par5                  | 3.0 mb | 0.3 | 0.15 | OFF|

TABLE I. Table of the different sets of parameters used for AMPT in this study. The cross section is determined from the screening mass \( \mu \) with Eq. (2).

III. OBSERVABLES

The \( p_T \)-dependent flow vector fluctuations are studied with \( p_T \)-differential flow coefficients. The regular \( p_T \)-differential flow coefficient is defined as

\[ v_n\{2\} = \frac{\langle \cos n(\varphi_{POI} - \varphi_2) \rangle}{\sqrt{\langle \cos n(\varphi_1 - \varphi_2) \rangle}} = \frac{\langle v_n(p_T) v_n \cos n[\Psi_{n}(p_T) - \Psi_n] \rangle}{\sqrt{\langle v_n^2 \rangle}}, \]

where \( \varphi_{POI} \) is the azimuthal angle of a particle of interest (POI) chosen from a narrow \( p_T \) range and \( \varphi \) is the azimuthal angle of a reference particle (RP) chosen from a wide \( p_T \) range. The single bracket denotes an average over all events, and double brackets indicate an average over all particles and all events. The \( v_2\{2\} \) probes the \( p_T \)-differential flow and is sensitive to fluctuations of
both the flow magnitude and flow angle. Another $p_T$-differential flow observable was proposed in [32], which is unaffected by the flow vector fluctuations

$$v_n[2] = \sqrt{\langle \langle \cos n(\varphi_{\text{POI}} - \varphi_2) \rangle \rangle} = \sqrt{v_n^2(p_T)}.$$  

(5)

Since $v_n[2]$ is not affected by the flow angle and flow magnitude fluctuations, the $p_T$-dependent flow vector fluctuations can be probed by taking the ratio of $v_n\{2\}$ and $v_n[2]:$

$$v_n\{2\} = \frac{v_n(p_T)v_n\cos n[\Psi_n(p_T) - \Psi_n]}{\sqrt{\langle v_n^2(p_T) \rangle}}.$$  

(6)

A value of $v_n\{2\}/v_n[2] < 1$ indicates the presence of $p_T$-dependent flow vector fluctuations. Similarly, the double-differential factorization ratio [33], which tests the factorization of the two-particle correlation into the product of single-particle flow coefficients, is a measure of the flow vector fluctuations. It provides more detailed information about the correlation structure

$$r_n = \frac{\langle \langle \cos n(\varphi_n^a - \varphi_n^p) \rangle \rangle}{\sqrt{\langle \langle \cos n(\varphi_n^a - \varphi_n^p) \rangle \rangle \langle \langle \cos n(\varphi_n^a - \varphi_n^p) \rangle \rangle}} = \frac{\langle v_n(p_T^a)v_n\cos n[\Psi_n(p_T^a) - \Psi_n] \rangle}{\sqrt{\langle v_n^2(p_T^a) \rangle \langle v_n^2(p_T^p) \rangle}}.$$  

(7)

Here, $\varphi_n^a$ is the azimuthal angle of the associate particle, and $\varphi_n^p$ is the azimuthal angle of the trigger particle with transverse momentum $p_T^a$ and $p_T^p$, respectively. The $p_T$-range is fixed for all values of $p_T$. Again, a value of $r_n < 1$ indicates that factorization is broken and that $p_T$-dependent flow vector fluctuations are present. Both Eqs. (6) and (7) are based on two-particle correlations and may be affected by fluctuations in both the flow magnitude and flow angle. Previously we proposed to use four-particle correlations to separate the two types of fluctuations [48] so that they can be measured experimentally. The flow angle fluctuations are given by

$$A_n^T = \frac{\langle \langle \cos n(\varphi_n^a + \varphi_n^2 - \varphi_n^3 - \varphi_n) \rangle \rangle}{\langle \langle \cos n(\varphi_n^a + \varphi_n^2) \rangle \rangle \langle \langle \cos n(\varphi_n^3 - \varphi_n) \rangle \rangle} = \frac{\langle v_n^2(p_T)^2 \rangle - 2n[\langle v_n(p_T) \rangle - \Psi_n] \rangle}{\langle v_n^2(p_T) \rangle^2}.$$  

(8)

where the last equality holds if the non-flow is approximately the same in numerator and denominator. Then $A_n^T < 1$ suggests $p_T$-dependent flow angle fluctuations. The flow magnitude is given by the double ratio of the $p_T$-differential flow fluctuations against the reference flow baseline fluctuations

$$M_n^T = \frac{\langle \langle \cos n[\varphi_n^a + \varphi_2 - \varphi_3 - \varphi_4] \rangle \rangle}{\langle \langle \cos n[\varphi_n^a - \varphi_3] \rangle \rangle} / \frac{\langle \langle \cos n[\varphi_2 - \varphi_4] \rangle \rangle}{\langle \langle \cos n(\varphi_2 - \varphi_4) \rangle \rangle} = \frac{\langle v_n^2(p_T) \rangle^2 / \langle v_n^2(p_T) \rangle}{\langle v_n^2(p_T) \rangle^2}.$$  

(9)

Any deviation from unity indicates the presence of $p_T$-dependent flow magnitude fluctuations.

IV. METHOD

The anisotropic flow observables are calculated using the two- and multi-particle correlation method [67, 68]. This method has been widely used at RHIC [69] and the LHC [13, 14] in the study of anisotropic flow. In this study, the Generic Framework [70, 71] is used, which enables a fast and exact calculation of the multi-particle correlations. However, additional correlators had to be implemented in the Generic Framework to separate the flow angle and flow magnitude fluctuations as in Eqs. (8) and (9). While the method has been used for the measurements in [60], a detailed explanation of the method has not been available before this paper.

A. Standard multi-particle correlations

The anisotropic flow correlations are expressed in terms of the Q-vector

$$Q_{n,p} = \sum_{k=1}^{M} w_k e^{in\varphi_k},$$  

(10)

where $M$ is the multiplicity of the selected particles in a given kinematic range, $\varphi$ is the azimuthal angle of the particles, and $w$ is a particle weight that corrects for detector inefficiencies in experimental measurements. In general, the single-event m-particle correlation can be calculated with

$$\langle m \rangle_{n_1,...,n_m} = \sum_{k_1 \neq \ldots \neq k_m} \sum_{k_1 \neq \ldots \neq k_m} \frac{w_{k_1} \ldots w_{k_m} e^{in_1\varphi_1 + \ldots + n_m\varphi_m}}{w_{k_1} \ldots w_{k_m}}.$$  

(11)

The numerator and denominator in Eq. (11) are trivially related

$$N\langle m \rangle_{n_1,...,n_m} = \sum_{k_1 \neq \ldots \neq k_m} w_{k_1} \ldots w_{k_m} e^{in_1\varphi_1 + \ldots + n_m\varphi_m}$$  

(12)

$$D\langle m \rangle_{n_1,...,n_m} = \sum_{k_1 \neq \ldots \neq k_m} w_{k_1} \ldots w_{k_m}.$$  

(13)

Due to the trivial nature of the denominators, they will be omitted from the following part unless necessary. The exact equations for the two- and four-particle azimuthal
single-event correlations \(N(2)_{n_1,n_2}\) and \(N(4)_{n_1,n_2,n_3,n_4}\) calculated with the Generic Framework are given by Eqs. (19)-(22) in [70]. The event-averaged two- and four-particle correlations can then be obtained with

\[
\langle \langle 2 \rangle \rangle = \frac{\sum_{\text{events}} N(2)_{n_1,n_2}}{\sum_{\text{events}} D(2)_{n_1,n_2}},
\]

\[
\langle \langle 4 \rangle \rangle = \frac{\sum_{\text{events}} N(4)_{n_1,n_2,n_3,n_4}}{\sum_{\text{events}} D(4)_{n_1,n_2,n_3,n_4}},
\]

where the double brackets refer to an average over all particles and events. \(D(m)_{n_1,n_2,...,n_m}\) is used as an event weight in order to reduce the effect of the varying multiplicity [70].

For \(p_T\)-differential anisotropic flow, one or more of the particles are selected from a narrow \(p_T\) range by constructing a \(p\)-vector analogous to the \(Q\)-vector consisting only of particles of interest.

\[
p_{n,p} = \sum_{k=1}^{M_{POI}} w_k^{n_p} e^{i\phi_k},
\]

where \(M_{POI}\) is the multiplicity of the POIs in the selected kinematic region. The \(p\)-vector enables the calculation of various \(p_T\)-differential anisotropic flow coefficients, such as \(v_n\{2\}\) by selecting one POI and one RP or \(v_n\{2\}\) by selecting two POIs, in the calculation of the two-particle correlations. Additionally, the double-differential two-particle correlations are calculated with one \emph{associate} particle and one \emph{trigger} particle from different narrow \(p_T\)-ranges. The \(p_T\)-differential two-particle correlations are then

\[
N(2')_{n_1,n_2} = p_{n_1,p_{n_2,1}} - q_{n_1+n_2,2},
\]

\[
N(2'')_{n_1,n_2} = p_{n_1,p_{n_2,1}} - p_{n_1+n_2,2},
\]

\[
N(4',a,t)_{n_1,n_2} = p_{n_1,p_{n_2,1}},
\]

where \(q_{n,p}\) is the overlap-vector of particles that are both POIs and RPs, and the superscript \(a\) refers to \emph{associate} particles, and \(t\) refers to \emph{trigger} particles. Eq. (17) takes one POI and correlates it with the reference particles, whereas Eq. (18) correlates two POIs from the same \(p_T\)-range. Eq. (19) is a special case of Eq. (18) when the two POIs are taken from different \(p_T\)-ranges. The event-averaged \(p_T\)-differential correlations then follow similarly to Eqs. (14) and (15)

\[
\langle \langle 2' \rangle \rangle = \frac{\sum_{\text{events}} N(2')_{n_1,n_2}}{\sum_{\text{events}} D(2')_{n_1,n_2}},
\]

\[
\langle \langle 2'' \rangle \rangle = \frac{\sum_{\text{events}} N(2'')_{n_1,n_2}}{\sum_{\text{events}} D(2'')_{n_1,n_2}},
\]

\[
\langle \langle 2',a,t \rangle \rangle = \frac{\sum_{\text{events}} N(2',a,t)_{n_1,n_2}}{\sum_{\text{events}} D(2',a,t)_{n_1,n_2}},
\]

**B. Subevent method**

Non-flow effects are correlations not due to collective effects, such as resonance decays and jets. They can be suppressed by applying a gap in pseudorapidity, \(|\Delta \eta|\), between two subevents, A and B, from which the particles are selected.

\[
N(2)_{n_1,n_2} |\Delta \eta| = Q_{n_1,1}^A \cdot Q_{n_2,1}^B,
\]

\[
N(4)_{n_1,n_2,n_3,n_4} |\Delta \eta| = N(2)_{n_1,n_2} |\Delta \eta| \cdot N(2)_{n_3,n_4}.
\]

For \(p_T\)-differential correlators the subevents are implemented analogous to Eq. (23)

\[
N(2')_{n_1,n_2} |\Delta \eta| = p_{n_1,1}^A O_{n_2,1}^B,
\]

\[
N(2'')_{n_1,n_2} |\Delta \eta| = p_{n_1,1}^A P_{n_2,1}^B,
\]

\[
N(4',a,t)_{n_1,n_2} |\Delta \eta| = P_{n_1,1}^A P_{n_2,1}^B.
\]

The four-particle correlations, which are needed to calculate Eqs. (8) and (9), are obtained with the subevent method to reduce the possible non-flow contamination. For four-particle correlations with a pseudorapidity gap,
there is a choice of how to group the POIs and RPs. Either the POIs are chosen from the same subevents (SS), or one POI is chosen from opposite subevents (OS), as shown in fig. [1]

\[
N(4')_{SS} = (p_{n1,1}p_{n2,1} - p_{n1+n2,2})^A \cdot (Q_{n3,1}Q_{n4,1} - Q_{n3+n4,2})^B, \tag{28}
\]

\[
N(4')_{OS} = (p_{n1,1}Q_{n2,1} - q_{n1+n2,2})^A \cdot (p_{n3,1}Q_{n4,1} - p_{n3+n4,2})^B. \tag{29}
\]

The event average of the correlators described in Eqs. [17], [29] are obtained by replacing the correlators in Eqs. [14] and [15]. The four-particle correlations are, by construction, less sensitive to non-flow correlations, as the latter typically involves fewer particles. By also applying a pseudorapidity gap, it ensures that the non-flow contamination is minimal. The non-flow also applying a pseudorapidity gap, it ensures that as the latter typically involves fewer particles. By construction, less sensitive to non-flow correlations, (17)-(29) are obtained by replacing the correlators in Eqs. (30)-(32) are obtained by replacing the correlators in Eqs. (33)-(34) are obtained by replacing the correlators in Eqs. (30)-(34). The event average of the correlators described in Eqs.

\[
N_{\text{flow}} = \langle\langle 4'\rangle\rangle_{\text{flow}}, \tag{30}
\]

and the alternative \(p_T\)-differential flow coefficient \(v_n[2]\) becomes

\[
v_n[2] = \sqrt{\langle\langle 2'\rangle\rangle}, \tag{31}
\]

The factorization ratio \(r_n\) is given by

\[
r_n = \frac{\langle\langle 2'\rangle\rangle_{a,t}}{\langle\langle 2'\rangle\rangle_{a,t}}, \tag{32}
\]

Finally, the observables based on four-particle correlations, \(A_n^f\) and \(M_n^f\), which describe the flow angle and flow magnitude fluctuations, respectively, are given by

\[
A_n^f = \frac{\langle\langle 4'\rangle\rangle_{SS}}{\langle\langle 4'\rangle\rangle_{OS}}, \tag{33}
\]

\[
M_n^f = \frac{\langle\langle 4'\rangle\rangle_{OS}/\langle\langle (2')\rangle\rangle/\langle\langle (2)\rangle\rangle^2}{\langle\langle 4\rangle\rangle/\langle\langle (2)\rangle\rangle^2}. \tag{34}
\]

As the non-flow contribution is small for multi-particle cumulants [23], a smaller \(\eta\)-gap is used for the four-particle correlations compared to the two-particle correlations to increase the event sample size. No particle weights are used to correct detector inefficiencies and acceptance for the generated AMPT events.

V. RESULTS AND DISCUSSION

The \(p_T\)-dependence of the \(v_2[2]/v_2[2]\) ratio calculated from the different configurations of the AMPT model is shown in fig. [2] for centrality ranges 0-5% to 40-50%. The deviation of \(v_2[2]/v_2[2]\) from unity is largest in the 0-5% central collisions and then becomes progressively smaller as the centrality increases. The effect of changing the initial conditions via the Lund string parameters, \(a\) and \(b\), can be seen by comparing the results from Par3 and Par2. The effect depends on the transverse momentum with an increasing effect up to 4 GeV/c, after which it decreases slightly. The string parameters affect the ratio \(v_2[2]/v_2[2]\) by up to 35% in central collisions and up to 30% in non-central collisions. The \(v_2[2]/v_2[2]\) shows a weak dependence on the partonic cross section: comparing the different cross sections with ART ON shows at most a 10% difference (Par1 vs Par4). Without ART, the effect of changing the cross section is negligible except in the 0-5% central collisions (Par2 vs Par5). The comparison with ART OFF may better reflect the true sensitivity to the QGP properties as the final state hadronic interactions may differ significantly when ART is enabled (Par1 vs Par2, Par4 vs Par5). A smaller partonic cross section in AMPT corresponds to a larger specific viscosity in hydrodynamics. The expectation is therefore that the calculation of \(v_2[2]/v_2[2]\) with a smaller partonic cross section should show less deviation from unity as a larger specific shear viscosity tends to dampen fluctuations from the initial state [24]. This expectation is consistent with what is observed in fig. [2] the Par1 calculation with smaller partonic cross section deviates less from unity than the Par4 calculations. However, the difference is small compared to the sensitivity to the initial state. The effect of the hadronic rescattering on the ratio \(v_2[2]/v_2[2]\) increases with \(p_T\) and is at most 10% in the presented \(p_T\)-range across all centralities for the Par5 vs Par4 calculations. The same is true for the Par2 vs Par1 results except in the 0-5% most central collisions, where a 20% difference is seen in the highest \(p_T\) bin. The above comparisons suggest that the flow vector fluctuations are highly sensitive to the initial state of heavy-ion collision and, unlike the \(v_2\) flow coefficient itself, are less sensitive to the transport properties of the created matter. The effect of the hadronic rescattering (ART ON vs ART OFF) is constant across the non-central collisions. Measurements of \(v_2[2]/v_2[2]\) from the ALICE experiment in Pb–Pb collisions at \(\sqrt{s_{NN}} = 5.02\) TeV are shown for comparison. All the configurations of the AMPT model overestimate the deviation from unity, especially in non-central collisions, suggesting that the model needs further tuning to the data, particularly of the initial conditions.
The calculation with 1.5 mb cross section, $a = 0.3$, $b = 0.15$ and ART ON (Par1) is closest in describing the data and only overestimates the data in the highest $p_T$ data point.

The factorization ratio of the second-order flow harmonic, $v_2$, is shown in fig. 3 as a function of associate particle transverse momentum, $p_T$, in centrality ranges 0-5%, 10-20% and 30-40% and different trigger particle transverse momentum, $p_T^t$, ranges. The factorization ratio in centrality ranges 5-10%, 20-30% and 40-50% is shown in Appendix B. The deviation of the factorization ratio from unity is largest in the 0-5% central collisions and increases with the difference $|p_T^a - p_T^t|$, which has also been observed in both hydrodynamics $^{33}$ and data $^{30, 75}$. The different configurations of AMPT all predict factorization breaking, with the largest deviation from unity observed for the Par3 model, which has different parameters for the initial conditions. The effect of the initial stages on the factorization ratio by changing the Lund string parameters from $a = 0.3$, $b = 0.15$ to $a = 0.5$, $b = 0.9$ is up to 40% in central collisions and increases with $|p_T^a - p_T^t|$. However, for the lower $p_T^t$-ranges, $p_T^t < 1.5$ GeV/$c$, the effect does not increase for $p_T > 3$ GeV/$c$ for central collisions and $p_T > 4$ GeV/$c$ for non-central collisions. In non-central collisions, the effect is up to 30% for large values of $|p_T^a - p_T^t|$. Changing the cross section from 1.5 mb to 3.0 mb affects $r_2$ by up to 10% at high $|p_T^a - p_T^t|$ with ART OFF (Par5 vs Par2). This is consistent with $v_2^a/v_2^t$, as expected since $r_2$ is basically the double-differential $v_2^a$/$v_2^t$. The hadronic phase also shows an effect of around 10% at high $|p_T^a - p_T^t|$ across all centralities for 1.5 mb (Par2 vs Par1) and 3.0 mb (Par5 vs Par4). The ALICE measurements favour the model with a 1.5 mb cross section, $a = 0.3$, $b = 0.15$ and ART ON (Par1), although the model overestimates the breaking of factorization as $|p_T^a - p_T^t|$ increases.

In fig. 4 the flow angle fluctuations $A^f$ are shown as a function of transverse momentum in centrality ranges 0-5% to 40-50%. A deviation of $A^f$ from unity is observed for all the model calculations, with the majority showing deviations for $p_T > 2.5$ GeV/$c$ and the Par3 calculation with $a = 0.5$, $b = 0.9$ showing deviation from unity for $p_T > 1.5$ GeV/$c$. The deviation from unity indicates that the $p_T$-dependent flow angle, $\Psi_a(p_T)$ fluctuates around the $p_T$-integrated flow angle $\Psi_a$ in the AMPT model, and the fluctuations are strongest in the central collisions, where the initial state density fluctuations dominate. However, significant fluctuations of up to 50% for Par3 and 30-40% for the other configurations can be seen.
FIG. 3. The $p_T$-dependence of the factorization ratio $r_2$ in different centrality and trigger particle transverse momentum ranges from the AMPT model with different configurations: $\sigma = 1.5$ mb and ART ON (Par1), $\sigma = 1.5$ mb and ART OFF (Par2), $\sigma = 1.5$ mb, ART ON, $a = 0.5$ and $b = 0.9$ (Par3), $\sigma = 3.0$ mb and ART ON (Par4), and $\sigma = 3.0$ mb and ART OFF (Par5). ALICE data points are shown as black circles for comparison.
from unity at measurements, only slightly overestimating the deviation (Par1) shows the most accurate description of the ALICE central collisions and at shown in bottom left (Par3 vs Par2). The large uncertainties whether ART is ON (Par4 vs Par1) or not (Par5 vs Par2). No significant effect of the hadronic rescattering is observed at high $p_T$ due to the large uncertainties. At lower $p_T$, the effect is ∼20% for the 1.5 mb calculations (Par2 vs Par1) and even less for the 3.0 mb calculations (Par5 vs Par4). Therefore, the fluctuating flow angles are most likely driven mainly by the event-by-event fluctuations in the initial state. The AMPT model with 1.5 mb cross section, Lund string parameters $a = 0.3$ and $b = 0.15$, and with hadronic interactions (Par1) shows the most accurate description of the ALICE measurements, only slightly overestimating the deviation from unity at $p_T > 3.0$ GeV/$c$ in the 0-5% and 5-10% central collisions and at $p_T > 2.0$ GeV/$c$ in centralities 30-40% and 40-50%. The flow angle fluctuations, which have been observed in data and hydrodynamic models, are also reproduced in the AMPT model.

The flow magnitude fluctuations $M^f_2$ are shown in fig. 4 as a function of transverse momentum in centrality ranges 0-5% to 40-50%. As $M^f_2$ is normalized to the baseline $p_T$-integrated flow fluctuations $\langle v_2^4 \rangle / \langle v_2^2 \rangle$, it does not depend on the magnitude of the flow coefficient itself. The flow magnitude fluctuations are strongest in the 0-5% central collisions and then decrease as the centrality increases. The largest deviation from unity is observed for the Par3 calculation, which also deviates from unity at lower values of $p_T$ compared to the other calculations. The large deviation from unity suggests a strong sensitivity of the flow magnitude fluctuations to the initial conditions by changing the Lund string parameters with an effect of up to 20% when comparing Par3 vs Par1. The flow magnitude fluctuations $M^f_2$ show no sensitivity to the partonic cross section within uncertainties, regardless of whether ART is enabled (Par4 vs Par1) or not (Par5 vs Par2). The puzzling $\eta/s$-dependence shown in based on the models from [49] is not observed in the low $p_T$ region even though the change in $\eta/s$ is larger in the AMPT calculations. The AMPT calculations show $M^f_2 = 1$ at low $p_T$ and the results are compatible for different $\sigma$, as expected. The hadronic rescattering plays a small role in the flow magnitude fluctuations with ∼10% difference between the calculations with and without ART in the second-highest $p_T$ bin, which has smaller uncertainties. Both the 1.5 mb (Par4 vs Par1) and 3.0 mb (Par5 vs Par2) calculations are equally affected by the hadronic

FIG. 4. The $p_T$-dependence of the flow angle fluctuations, $A^f_2 \approx \langle \cos 2n(\Psi_n(p_T) - \Psi_n) \rangle$, in different centrality ranges from the AMPT model with different configurations: $\sigma = 1.5$ mb and ART ON (Par1), $\sigma = 1.5$ mb and ART OFF (Par2), $\sigma = 1.5$ mb, ART ON, $a = 0.5$ and $b = 0.9$ (Par3), $\sigma = 3.0$ mb and ART ON (Par4), and $\sigma = 3.0$ mb and ART OFF (Par5). ALICE data points are shown as black circles for comparison [50].

Across the centralities the flow angle fluctuations are highly sensitive to the initial conditions. Changing the Lund string parameters for the initial conditions affects the $A^f_2$ by up to 70% in the 20-30% centrality range as shown in bottom left (Par3 vs Par2). The large uncertainties at high $p_T$ in central collisions do not allow for a quantitative statement about the effect, but at lower $p_T$, the effect is up to 50%. The flow angle fluctuations show no sensitivity to the partonic cross section within the uncertainties whether ART is ON (Par4 vs Par1) or OFF (Par5 vs Par2). No significant effect of the hadronic rescattering is observed at high $p_T$ due to the large uncertainties. At lower $p_T$, the effect is ∼20% for the 1.5 mb calculations (Par2 vs Par1) and even less for the 3.0 mb calculations (Par5 vs Par4). Therefore, the fluctuating flow angles are most likely driven mainly by the event-by-event fluctuations in the initial state. The AMPT model with 1.5 mb cross section, Lund string parameters $a = 0.3$ and $b = 0.15$, and with hadronic interactions (Par1) shows the most accurate description of the ALICE measurements, only slightly overestimating the deviation from unity at $p_T > 3.0$ GeV/$c$ in the 0-5% and 5-10% central collisions and at $p_T > 2.0$ GeV/$c$ in centralities 30-40% and 40-50%. The flow angle fluctuations, which have been observed in data and hydrodynamic models, are also reproduced in the AMPT model.
\[ \sigma = 1.5 \text{ mb}, a = 0.3, b = 0.15 \text{ and ART ON describes the ALICE data reasonably well in all centralities. The flow magnitude fluctuations are reproduced in the transport model, and the data vs model comparison enable a further opportunity to tune the initial conditions in AMPT and, in general, the use of ALICE data to constrain the initial conditions of the heavy-ion collisions.} \]

**VI. SUMMARY**

In this paper, the AMPT calculations of \( p_T \)-dependent flow vector, flow angle and flow magnitude fluctuations in Pb–Pb collisions at \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \) are presented. The different configurations of the AMPT model probe the sensitivity of the fluctuations to 1) the initial conditions, 2) the partonic cross section, and 3) the hadronic rescattering to pinpoint the source of these fluctuations. It is found that the observables are highly sensitive to the changes in the initial conditions. The observables show weak to no dependence on changes in the partonic cross section and the hadronic rescattering. The high sensitivity to changes in the initial state suggests that the fluctuations mainly originate from the event-by-event fluctuations of the initial state and are only slightly dependent on the transport properties of the QGP. The presented studies on the \( p_T \)-dependent flow vector fluctuations and comparison between models and data can help constrain the initial stage of the QGP formation.

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**Appendix A: Model ratios**

In order to compare the different configurations against each other the ratios of the model calculations are presented here. Fig. 6 shows the comparison for \( v_2(2)/v_2[2] \). The \( r_2 \) comparison is shown in fig. 7. Finally, the ratios for the flow angle fluctuations \( A_2 \) and the flow magnitude fluctuations \( M_2 \) are shown in figures 8 and 9, respectively.
Appendix B: Factorization ratio

The factorization ratio of the second-order flow harmonic, $r_2$, is shown in Fig. 10 as a function of associate particle transverse momentum, $p_T^A$, in centrality ranges 5-10%, 20-30% and 40-50% and different trigger particle transverse momentum, $p_T^P$, ranges. These are the centrality ranges not presented in the main body of this paper.

[1] I. Arsene et al. (BRAHMS), Quark gluon plasma and color glass condensate at RHIC: The Perspective from the BRAHMS experiment, Nucl. Phys. A 757, 1 (2005). arXiv:nucl-ex/0410020

[2] J. Adams et al. (STAR), Experimental and theoretical challenges in the search for the quark gluon plasma: The STAR Collaboration’s critical assessment of the evidence from RHIC collisions, Nucl. Phys. A 757, 102 (2005). arXiv:nucl-ex/0501009

[3] K. Adcox et al. (PHENIX), Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: Experimental evaluation by the PHENIX collaboration, Nucl. Phys. A 757, 184 (2005). arXiv:nucl-ex/0410003

[4] B. B. Back et al. (PHOBOS), The PHOBOS perspective on discoveries at RHIC, Nucl. Phys. A 757, 28 (2005). arXiv:nucl-ex/0410022

[5] B. Muller, J. Schukraft, and B. Wyslouch, First Results from Pb+Pb collisions at the LHC, Ann. Rev. Nucl. Part. Sci. 62, 361 (2012) arXiv:1202.3233 [hep-ex]

[6] J. Y. Ollitrault, Anisotropy as a signature of transverse collective flow, Phys. Rev. D46, 229 (1992)

[7] S. A. Voloshin, A. M. Poskanzer, and R. Snellings, Collective phenomena in non-central nuclear collisions, Landolt-Bornstein 23, 293 (2010) arXiv:0809.2949 [nucl-ex]

[8] S. Voloshin and Y. Zhang, Flow study in relativistic nuclear collisions by Fourier expansion of Azimuthal particle distributions, Z. Phys. C70, 665 (1996) arXiv:hep-ph/9407282 [hep-ph]

[9] K. H. Ackermann et al. (STAR), Elliptic flow in Au+Au collisions at (S(NN))**(1/2) = 130 GeV, Phys. Rev. Lett. 86, 402 (2001) arXiv:nucl-ex/0009011

[10] S. S. Adler et al. (PHENIX), Elliptic flow of identified hadrons in Au+Au collisions at (S(NN))**(1/2) = 200 GeV, Phys. Rev. Lett. 91, 182301 (2003) arXiv:nucl-ex/0305013

[11] L. Adamczyk et al. (STAR), Third Harmonic Flow of Charged Particles in Au+Au Collisions at sqrt(sNN) = 200 GeV, Phys. Rev. C 88, 014904 (2013) arXiv:1301.2187 [nucl-ex]

[12] A. Adare et al. (PHENIX), Measurements of elliptic and triangular flow in high-multiplicity He+Au collisions at sqrt(sNN) = 200 GeV, Phys. Rev. Lett. 115, 142301 (2015) arXiv:1507.06273 [nucl-ex]

[13] K. Aamodt et al. (ALICE), Elliptic flow of charged particles in Pb-Pb collisions at 2.76 TeV, Phys. Rev. Lett. 105, 252302 (2010) arXiv:1011.3014 [nucl-ex]

[14] K. Aamodt et al. (ALICE), Higher harmonic anisotropic flow measurements of charged particles in Pb-Pb collisions at sqrt(sNN) = 2.76 TeV, Phys. Rev. Lett. 107, 032301 (2011) arXiv:1105.3865 [nucl-ex]

[15] B. Abelev et al. (ALICE), Elliptic flow of identified hadrons in Pb-Pb collisions at sqrt(sNN) = 2.76 TeV, JHEP 06, 190 arXiv:1405.4632 [nucl-ex]

[16] J. Adam et al. (ALICE), Anisotropic flow of charged particles in Pb-Pb collisions at sqrt(sNN) = 5.02 TeV, Phys. Rev. Lett. 116, 132302 (2016) arXiv:1602.01119 [nucl-ex]

[17] S. Acharya et al. (ALICE), Linear and non-linear flow modes in Pb-Pb collisions at sqrt(sNN) = 2.76 TeV, Phys. Lett. B773, 68 (2017) arXiv:1705.04377 [nucl-ex]

[18] G. Aad et al. (ATLAS), Measurement of the azimuthal anisotropy for charged particle production in sqrt(sNN) = 2.76 TeV lead-lead collisions with the ATLAS detector, Phys. Rev. C86, 014907 (2012) arXiv:1203.3087 [hep-ex]

[19] G. Aad et al. (ATLAS), Measurement of the pseudorapidity and transverse momentum dependence of the elliptic flow of charged particles in lead-lead collisions at sqrt(sNN) = 2.76 TeV with the ATLAS detector, Phys. Lett. B707, 330 (2012) arXiv:1108.6018 [hep-ex]

[20] G. Aad et al. (ATLAS), Measurement of the distributions of event-by-event flow harmonics in lead-lead collisions at sqrt(sNN) = 2.76 TeV with the ATLAS detector at the LHC, JHEP 11, 183 arXiv:1305.2942 [hep-ex]

[21] S. Chatrchyan et al. (CMS), Centrality dependence of di-hadron correlations and azimuthal anisotropy harmonics in PbPb collisions at sqrt(sNN) = 2.76 TeV, Eur. Phys. J. C72, 2012 (2012) arXiv:1201.3158 [nucl-ex]

[22] S. Chatrchyan et al. (CMS), Measurement of the elliptic anisotropy of charged particles produced in PbPb collisions at sqrt(sNN)=2.76 TeV, Phys. Rev. C87, 014902 (2013) arXiv:1204.1409 [nucl-ex]

[23] S. Chatrchyan et al. (CMS), Azimuthal anisotropy of charged particles at high transverse momenta in PbPb collisions at sqrt(sNN) = 2.76 TeV, Phys. Rev. Lett. 109, 022301 (2012) arXiv:1204.1850 [nucl-ex]

[24] U. Heinz and R. Snellings, Collective flow and viscosity in relativistic heavy-ion collisions, Ann. Rev. Nucl. Part. Sci. 63, 123 (2013) arXiv:1301.2826 [nucl-th]

[25] M. Luzum and H. Petersen, Initial State Fluctuations and Final State Correlations in Relativistic Heavy-Ion Collisions, J. Phys. G41, 063102 (2014) arXiv:1312.5503 [nucl-th]

[26] E. Shiroyuk, Strongly coupled quark-gluon plasma in heavy ion collisions, Rev. Mod. Phys. 89, 035001 (2017) arXiv:1412.8393 [hep-ph]

[27] H. Song, Y. Zhou, and K. Gajdosova, Collective flow and hydrodynamics in large and small systems at the LHC, Nucl. Sci. Tech. 28, 99 (2017) arXiv:1703.00670 [nucl-th]

[28] J. E. Bernhard, J. S. Moreland, and S. A. Bass, Bayesian estimation of the specific shear and bulk viscosity of quark-gluon plasma, Nature Phys. 15, 1113 (2019)

[29] D. Everett et al. (JETSCAPE), Multisystem Bayesian constraints on the transport coefficients of QCD matter, Phys. Rev. C 103, 054904 (2021) arXiv:2011.01430 [hep-ph]
[30] G. Nijs, W. van der Schee, U. Gürtsoy, and R. Snellings, Bayesian analysis of heavy ion collisions with the heavy ion computational framework Trajectum, Phys. Rev. C 103, 054909 (2021) [arXiv:2010.15134 [nucl-th]]

[31] J. E. Parkkila, A. Omerstad, S. F. Taghavi, C. Mordasini, A. Bilandzic, and D. J. Kim, New constraints for QCD matter from improved Bayesian parameter estimation in heavy-ion collisions at LHC, (2021), arXiv:2111.08145 [hep-ph]

[32] U. Heinz, Z. Qiu, and C. Shen, Fluctuating flow anisotropies in heavy ion collisions, Phys. Rev. C 87, 054913 (2013) [arXiv:1302.3535 [nucl-th]]

[33] P. F. Gardim, F. Grassi, M. Luzum, and J.-Y. Ollitrault, Breaking of factorization of two-particle correlations in relativistic heavy ion collisions, Eur. Phys. J. C 87, 031901 (2013) [arXiv:1211.0989 [nucl-th]]

[34] S. Acharya et al. (ALICE), Searches for transverse momentum dependent flow vector fluctuations in Pb-Pb and p-Pb collisions at the LHC, JHEP 99, 032 (2012) [arXiv:1707.05690 [nucl-ex]]

[35] S. Chatrchyan et al. (CMS), Studies of azimuthal di-hadron correlations in ultra-central PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, JHEP 02, 088 (2012) [arXiv:1312.1845 [nucl-ex]]

[36] V. Khachatryan et al. (CMS), Evidence for transverse momentum and pseudorapidity dependent event plane fluctuations in PbPb and pPb collisions, Phys. Rev. C 92, 034911 (2015) [arXiv:1503.01692 [nucl-ex]]

[37] K. Aamodt et al. (ALICE), Harmonic decomposition of two-particle angular correlations in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, Phys. Lett. B708, 249 (2012) [arXiv:1109.2501 [nucl-ex]]

[38] S. Acharya et al. (ALICE), Anisotropic flow and flow fluctuations of identified hadrons in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, (2022) [arXiv:2206.04587 [nucl-ex]]

[39] Z. Qiu and U. W. Heinz, Event-by-event shape and flow fluctuations of relativistic heavy-ion collision fireballs, Phys. Rev. C 84, 024911 (2011) [arXiv:1104.0650 [nucl-th]]

[40] D. Teaney and L. Yan, Triangularity and Dipole Asymmetry in Heavy Ion Collisions, Phys. Rev. C 83, 064904 (2011) [arXiv:1010.1876 [nucl-th]]

[41] J. Jia and S. Mohapatra, A Method for studying initial geometry fluctuations via event plane correlations in heavy ion collisions, Eur. Phys. J. C 73, 2510 (2013) [arXiv:1203.5905 [nucl-th]]

[42] G. Aad et al. (ATLAS), Measurement of event-plane correlations in $\sqrt{s_{NN}} = 2.76$ TeV lead-lead collisions with the ATLAS detector, Phys. Rev. C 90, 024905 (2014) [arXiv:1403.0489 [hep-ex]]

[43] Z. Qiu and U. Heinz, Hydrodynamic event-plane correlations in Pb+Pb collisions at $\sqrt{s} = 2.76$A TeV, Phys. Lett. B717, 261 (2012) [arXiv:1208.1200 [nucl-th]]

[44] P. Bozek, Angle and magnitude decorrelation in the factorization breaking of collective flow, Phys. Rev. C 98, 064906 (2018) [arXiv:1808.04248 [nucl-th]]

[45] F. G. Gardim, F. Grassi, P. Ishida, M. Luzum, P. S. Magalhaes, and J. Noronha-Hostler, Sensitivity of observables to coarse-graining size in heavy-ion collisions, Phys. Rev. C 97, 064919 (2018) [arXiv:1712.03912 [nucl-th]]

[46] W. Zhao, H.-J. Xu, and H. Song, Collective flow in 2.76 A TeV and 5.02 A TeV Pb+Pb collisions, Eur. Phys. J. C77, 645 (2017) [arXiv:1703.10792 [nucl-th]]

[47] L. Barbosa, F. G. Gardim, F. Grassi, P. Ishida, M. Luzum, M. V. Machado, and J. Noronha-Hostler, Predictions for flow harmonic distributions and flow factorization ratios at RHIC, (2021) [arXiv:2105.12792 [nucl-th]]

[48] E. G. Nielsen (for the ALICE collaboration), Fluctuations and correlations of flow in heavy-ion collisions measured by alice (2021), The 13th International Conference on the Initial Stages of High-Energy Nuclear Collisions, https://indico.cern.ch/event/854124/contributions/4134638/

[49] P. Bozek and R. Samanta, Factorization breaking for higher moments of harmonic flow, Phys. Rev. C 105, 034904 (2022) [arXiv:2109.07781 [nucl-th]]

[50] S. Acharya et al. (ALICE), Observation of flow angle and flow magnitude fluctuations in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV at the LHC, (2022) [arXiv:2206.04574 [nucl-ex]]

[51] Z.-W. Lin, C. M. Ko, B.-A. Li, B. Zhang, and S. Pal, A Multi-phase transport model for relativistic heavy ion collisions, Phys. Rev. C 72, 064901 (2005) [arXiv:nucl-th/0411110 [nucl-th]]

[52] Z.-W. Lin and L. Zheng, Further developments of a multi-phase transport model for relativistic nuclear collisions, Nucl. Sci. Tech. 32, 113 (2021) [arXiv:2110.02989 [nucl-th]]

[53] Z.-W. Lin and C. M. Ko, Partonic effects on the elliptic flow at RHIC, Phys. Rev. C 65, 034904 (2002) [arXiv:nucl-th/0108039]

[54] J. Xu and C. M. Ko, Triangular flow in heavy ion collisions in a multiphase transport model, Phys. Rev. C 84, 014903 (2011) [arXiv:1105.5187 [nucl-th]]

[55] J. Xu and C. M. Ko, Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in a multiphase transport model, Phys. Rev. C 83, 034904 (2011) [arXiv:1101.2231 [nucl-th]]

[56] Z. Feng, G.-M. Huang, and F. Liu, Anisotropic flow of Pb+Pb $\sqrt{s_{NN}} = 5.02$ TeV from a Multi-Phase Transport Model, Chin. Phys. C 41, 024001 (2017) [arXiv:1606.02416 [nucl-ex]]

[57] X.-N. Wang and M. Gyulassy, hijing: A parton cascade for ultrarelativistic heavy ion collisions, Comput. Phys. Commun. 119, 193 (1998) [arXiv:nucl-th/9709009]

[58] Z.-W. Lin, Evolution of transverse flow and effective temperatures in the parton phase from a multi-phase transport model, Phys. Rev. C 90, 014904 (2014) [arXiv:1403.6321 [nucl-th]]

[59] L.-W. Chen and C. M. Ko, System size dependence of elliptic flows in relativistic heavy-ion collisions, Phys. Lett. B634, 205 (2006) [arXiv:nucl-th/0505044 [nucl-th]]

[60] B.-A. Li and C. M. Ko, Formation of superdense hadronic matter in high-energy heavy ion collisions, Phys. Rev. C52, 2037 (1995) [arXiv:nucl-th/9505016 [nucl-th]]

[61] B. Li, A. T. Sustich, B. Zhang, and C. M. Ko, Studies of superdense hadronic matter in a relativistic transport model, Int. J. Mod. Phys. E 10, 267 (2001)

[62] X.-N. Wang and M. Gyulassy, HIJING: A Monte Carlo model for multiple jet production in pp, pA, and AA collisions, Phys. Rev. D44, 3501 (1991)

[63] B. Zhang, ZPC I 0.1: A Parton cascade for ultrarelativistic heavy ion collisions, Comput. Phys. Commun. 109, 193 (1998) [arXiv:nucl-th/9709009]

[64] B. Li, A. T. Sustich, B. Zhang, and C. M. Ko, Studies of superdense hadronic matter in a relativistic transport model, Phys. Rev. C52, 2037 (1995) [arXiv:nucl-th/9505016 [nucl-th]]

[65] B. Li, A. T. Sustich, B. Zhang, and C. M. Ko, Studies of superdense hadronic matter in a relativistic transport model, Int. J. Mod. Phys. E 10, 267 (2001)

[66] M. Gyulassy and X.-N. Wang, HIJING 1.0: A Monte Carlo program for parton and particle production in high-energy hadronic and nuclear collisions, Comput. Phys. Commun. 83, 307 (1994) [arXiv:nucl-th/9502021]
[65] M. Gyulassy, Y. Pang, and B. Zhang, Transverse energy evolution as a test of parton cascade models, Nucl. Phys. A 626, 999 (1997) [arXiv:nucl-th/9709025].

[66] B. Zhang, M. Gyulassy, and C. M. Ko, Elliptic flow from a parton cascade, Phys. Lett. B 455, 45 (1999) [arXiv:nucl-th/9902016].

[67] N. Borghini, P. M. Dinh, and J.-Y. Ollitrault, Flow analysis from multiparticle azimuthal correlations, Phys. Rev. C 64, 054901 (2001) [arXiv:nucl-th/0105040].

[68] A. Bilandzic, R. Snellings, and S. Voloshin, Flow analysis with cumulants: Direct calculations, Phys. Rev. C 83, 044913 (2011) [arXiv:1010.0233 [nucl-ex]].

[69] G. Agakishiev et al. (STAR), Energy and system-size dependence of two- and four-particle $v_2$ measurements in heavy-ion collisions at RHIC and their implications on flow fluctuations and nonflow, Phys. Rev. C 86, 014904 (2012) [arXiv:1111.5637 [nucl-ex]].

[70] A. Bilandzic, C. H. Christensen, K. Gulbrandsen, A. Hansen, and Y. Zhou, Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations, Phys. Rev. C 89, 064904 (2014) [arXiv:1312.3572 [nucl-ex]].

[71] P. Huo, K. Gajdosov, J. Jia, and Y. Zhou, Importance of non-flow in mixed-harmonic multi-particle correlations in small collision systems, Phys. Lett. B 777, 201 (2018) [arXiv:1710.07567 [nucl-ex]].

[72] N. Magdy, Measuring differential flow angle fluctuations in relativistic nuclear collisions, Phys. Rev. C 106, 044911 (2022) [arXiv:2207.04530 [nucl-th]].

[73] S. Acharya et al. (ALICE), Investigations of Anisotropic Flow Using Multiparticle Azimuthal Correlations in pp, p-Pb, Xe-Xe, and Pb-Pb Collisions at the LHC, Phys. Rev. Lett. 123, 142301 (2019) [arXiv:1903.01790 [nucl-ex]].

[74] B. Schenke, S. Jeon, and C. Gale, Elliptic and triangular flow in event-by-event (3+1)D viscous hydrodynamics, Phys. Rev. Lett. 106, 042301 (2011) [arXiv:1009.3243 [hep-ph]].

[75] S. Acharya et al. (ALICE), Searches for transverse momentum dependent flow vector fluctuations in Pb-Pb and p-Pb collisions at the LHC, JHEP 09, 032 (2017) [arXiv:1707.05690 [nucl-ex]].
FIG. 6. The ratio of the AMPT calculations of $v_2^2/v_2[^2]_0$ with different configurations: ART OF vs ART ON with 1.5 mb (Par2/Par1), ART OF vs ART ON with 3.0 mb (Par5/Par3), 3.0 mb vs 1.5 mb with ART OFF (Par5/Par2), 3.0 mb vs 1.5 mb with ART ON (Par4/Par1), and $a = 0.5$, $b = 0.9$ vs $a = 0.3$, $b = 0.15$ with 1.5 mb and ART OFF (Par3/Par2).
FIG. 7. The ratio of the AMPT calculations of $r_2$ with different configurations: ART OF vs ART ON with 1.5 mb (Par2/Par1), ART OF vs ART ON with 3.0 mb (Par5/Par3), 3.0 mb vs 1.5 mb with ART OFF (Par5/Par2), 3.0 mb vs 1.5 mb with ART ON (Par4/Par1), and $a = 0.5, b = 0.9$ vs $a = 0.3, b = 0.15$ with 1.5 mb and ART OFF (Par3/Par2).
FIG. 8. The ratio of the AMPT calculations of $A_f^2$ with different configurations; ART OF vs ART ON with 1.5 mb (Par2/Par1), ART OF vs ART ON with 3.0 mb (Par5/Par3), 3.0 mb vs 1.5 mb with ART OFF (Par5/Par2), 3.0 mb vs 1.5 mb with ART ON (Par4/Par1), and $a = 0.5, b = 0.9$ vs $a = 0.3, b = 0.15$ with 1.5 mb and ART OFF (Par3/Par2).

FIG. 9. The ratio of the AMPT calculations of $M_f^2$ with different configurations; ART OF vs ART ON with 1.5 mb (Par2/Par1), ART OF vs ART ON with 3.0 mb (Par5/Par3), 3.0 mb vs 1.5 mb with ART OFF (Par5/Par2), 3.0 mb vs 1.5 mb with ART ON (Par4/Par1), and $a = 0.5, b = 0.9$ vs $a = 0.3, b = 0.15$ with 1.5 mb and ART OFF (Par3/Par2).
FIG. 10. The $p_T$-dependence of the factorization ratio $r_2$ in centralities 5-10%, 20-30% and 40-50% with different trigger particle transverse momentum ranges from the AMPT model with different partonic cross sections, different Lund string parameters and with and without a hadronic rescattering phase. ALICE data points are shown as black circles for comparison.