EXTENDED LIE DERIVATIVES AND A NEW FORMULATION OF 
D=11 SUPERGRAVITY

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Abstract

Introducing an extended Lie derivative along the dual of $A$, the three form field of $d=11$ supergravity, the full diffeomorphism algebra of $d=11$ supergravity is presented. This algebra suggests a new formulation of the theory, where the three-form field $A$ is replaced by bivector $B^{ab}$, bispinor $B^{\alpha\beta}$, and spinor-vector $\eta^{a\beta}$ one-forms. Only the bivector one-form $B^{ab}$ is propagating, and carries the same degrees of freedom of the three-form in the usual formulation, its curl $D_{[\mu}B^{\nu]}$ being related to the $F_{\mu\nu\delta}$ curl of the three-form. The other one-forms are auxiliary, and the transformation rules on all the fields close on the equations of motion of $d=11$ supergravity.

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1 Introduction

Supergravity in eleven dimensions \[1\] (a particular limit of M-theory, for a review see for ex. \[2\]), can be formulated within the framework of free differential algebras (FDA’s) \[3, 4, 5, 6, 8, 9\], a generalization of Lie algebras which include \( p \)-form potentials. In this framework the 3-form of \( d = 11 \) supergravity acquires an algebraic interpretation.

In preceding papers \[9, 10\] it was shown how FDA’s can be related to ordinary Lie algebras via extended Lie derivatives: the Lie algebra underlying the FDA of \( d = 11 \) supergravity was identified in \[10\], and found to coincide with an algebra discussed in 1989 \[11\] in the context of supermembranes.

In Section 2 we recall briefly the FDA formulation of \( d = 11 \) supergravity of \[4\]. In Section 3 we present the full diffeomorphism algebra of \( d = 11 \) supergravity on the FDA “manifold”, which encodes all the symmetries of the theory. In Section 4 a new formulation is proposed, where the 3-form \( A \) is replaced by 1-form potentials with (couples of) Lorentz vector and spinor indices.

For a resumé on FDA’s and the notion of extended Lie derivative along duals of \( p \)-forms we refer to \[10\]. Here the general theory is applied to the FDA of \( d = 11 \) supergravity.

2 The FDA of \( d=11 \) supergravity, Bianchi identities, field equations and transformation rules

The FDA structure \[4\] is contained in the following curvature definitions:

\[
\begin{align*}
R^{ab} &= d\omega^{ab} - \omega^{ac}\omega^{cb} \\
R^{a} &= dV^{a} - \omega^{ab}V^{b} - \frac{i}{2}\bar{\psi}\Gamma^{a}\psi \equiv \mathcal{D}V^{a} - \frac{i}{2}\bar{\psi}\Gamma^{a}\psi \\
\rho &= d\psi - \frac{1}{4}\omega^{ab}\Gamma^{ab}\psi \equiv \mathcal{D}\psi \\
R(A) &= dA - \frac{1}{2}\bar{\psi}\Gamma^{ab}\psi V^{a}V^{b}
\end{align*}
\]

(2.1)

The Bianchi identities are obtained by taking the exterior derivative of (2.1):

\[
\begin{align*}
dR^{ab} + R^{ac}\omega^{cb} - \omega^{ac}R^{cb} &\equiv \mathcal{D}R^{ab} = 0 \\
\mathcal{D}R^{a} + R^{ab}V^{b} - i\bar{\psi}\Gamma^{a}\rho &\equiv 0 \\
\mathcal{D}\rho + \frac{1}{4}R^{ab}\Gamma^{ab}\psi &\equiv 0 \\
dR(A) - \bar{\psi}\Gamma^{ab}\rho V^{a}V^{b} + \bar{\psi}\Gamma^{ab}\psi R^{a}V^{b} &\equiv 0
\end{align*}
\]

(2.2)

The superPoincaré curvatures \( R^{ab}, R^{a}, \rho \) (respectively the Lorentz curvature, the torsion and the gravitino curvature) are 2-forms, and \( R(A) \) is a 4-form. These can be expanded on a superspace basis spanned by the vielbein \( V^{a} \) and the gravitino \( \psi \).
The group-geometric method of \[7, 6, 8\] requires the Ansatz that all “exterior” components of the curvatures be expressed in terms of the “spacetime” ones, spacetime meaning along the \(V^a\) vielbeins only.

The Bianchi identities then become equations for the curvatures, whose solution \[4\] is:

\[
R^{ab} = R^{ab}_{\ cd} V^c V^d + i (2 \bar{\rho}_{[a} \Gamma_{b]} - \bar{\rho}_{ab} \Gamma_c) \psi V^c + F^{abcd} \bar{\psi} \Gamma^{cd} \psi + \frac{1}{24} F_{c_1 c_2 c_3 c_4} \bar{\psi} \Gamma^{a b c_1 c_2 c_3 c_4} \psi \tag{2.3}
\]

\[
R^a = 0 \tag{2.4}
\]

\[
\rho = \rho_{ab} V^a V^b + \frac{i}{3} (F^{ab_1 b_2 b_3} \Gamma^{b_1 b_2 b_3} - \frac{1}{8} F^{b_1 b_2 b_3 b_4} \Gamma^{a b_1 b_2 b_3 b_4}) \psi V^a \tag{2.5}
\]

\[
R(A) = F^{a_1 \ldots a_4} V^a_1 V^{a_2} V^{a_3} V^{a_4} \tag{2.6}
\]

where the spacetime components \(R^{ab}_{cd}, \rho_{ab}, F^{a_1 \ldots a_4}\) satisfy the well known propagation equations (Einstein, gravitino and Maxwell equations):

\[
R^{ac}_{\ bc} - \frac{1}{2} \delta^a_b R = 3 F^{ac_1 c_2 c_3} F^{b c_1 c_2 c_3} - \frac{3}{8} \delta^a_b F^{c_1 \ldots c_4} F^{c_1 \ldots c_4} \tag{2.7}
\]

\[
\Gamma^{a b}_{c} \rho_{b c} = 0 \tag{2.8}
\]

\[
D_a F^{ab_1 b_2 b_3} = -\frac{1}{2 \cdot 4! \cdot 7!} \bar{\epsilon}^{b_1 b_2 b_3 a_1 \ldots a_8} F^{a_1 \ldots a_4} F^{a_5 \ldots a_8} = 0 \tag{2.9}
\]

In the group geometric formulation the symmetries gauged by the superPoincaré fields \(V^a, \omega^{ab}\) and \(\psi\) are seen as diffeomorphisms on the “FDA manifold”, generated by the Lie derivative along the tangent vectors \(t_a, t_{ab}, \tau\) dual to these one-form fields. Thus, setting \(\epsilon = \epsilon^a t_a + \epsilon^{ab} t_{ab} + \epsilon \tau\), the transformation rules under local supertranslations and Lorentz rotations are generated by the Lie derivative

\[
\ell_\epsilon \equiv d \ i_\epsilon + i_\epsilon \ d \tag{2.10}
\]

Explicitly

\[
\delta V^a = \ell_\epsilon V^a = D \epsilon^a + \epsilon^{ab} V^b + i \bar{\epsilon} \Gamma^a \psi \tag{2.11}
\]

\[
\delta \omega^{ab} = \ell_\epsilon \omega^{ab} = D \epsilon^{ab} + 2 R^{ab}_{\ cd} \epsilon^c V^d + i (2 \bar{\rho}_{[a} \Gamma_{b]} - \bar{\rho}_{ab} \Gamma_c) (\epsilon V^c - \psi \epsilon^c) - 2 F^{abcd} \bar{\psi} \Gamma^{cd} \epsilon - \frac{1}{12} F_{c_1 c_2 c_3 c_4} \bar{\psi} \Gamma^{a b c_1 c_2 c_3 c_4} \epsilon \tag{2.12}
\]

\[
\delta \psi = \ell_\epsilon \psi = D \epsilon + i \frac{1}{4} \epsilon^{ab} \Gamma_{ab} \psi + 2 \bar{\rho}_{ab} \epsilon^a V^b + i \frac{1}{3} (F^{ab_1 b_2 b_3} \Gamma^{b_1 b_2 b_3} - \frac{1}{8} F^{b_1 b_2 b_3 b_4} \Gamma^{a b_1 b_2 b_3 b_4}) (\epsilon V^a - \psi \epsilon^a) \tag{2.13}
\]

\[
\delta A = \ell_\epsilon A = -\bar{\psi} \Gamma^{ab} \epsilon V^a V^b + \bar{\psi} \Gamma^{ab} \psi \epsilon^a V^b + 4 F^{a_1 \ldots a_4} \epsilon^{a_1} V^{a_2} V^{a_3} V^{a_4} \tag{2.14}
\]

where the exterior derivatives on the fields have been expressed in terms of the curvatures \((2.1), \) and the solutions \((2.3)-(2.6)\) have been used. The closure of these transformations is then equivalent to the propagation equations \((2.7)-(2.9),\) as is usual in locally supersymmetric theories.
3 The algebra of diffeomorphisms on the d=11 supergravity FDA “manifold”

On a soft group manifold, i.e. a manifold whose vielbeins $\mu^A$ have in general nonvanishing curvatures

$$R^A = d\mu^a + \frac{1}{2} C^C_{AB} \mu^A \mu^B,$$  \hspace{1cm} (3.1)

the algebra of diffeomorphisms is given by the commutators of Lie derivatives:

$$[\ell_{\varepsilon^A t_A}, \ell_{\varepsilon^B t_B}] = \ell_{[\varepsilon^A t_A, \varepsilon^B t_B]} t_C$$  \hspace{1cm} (3.2)

where $t_A$ are the tangent vectors dual to the one-forms $\mu^A$, and

$$\mathcal{R}^C_{AB} \equiv R^C_{AB} - \frac{1}{2} C^C_{AB}$$  \hspace{1cm} (3.3)

involves the curvature components on the vielbein basis and the group structure constants. The closure of the algebra requires the Bianchi identities

$$\partial_B \mathcal{R}^A_{CD} + 2 \mathcal{R}^A_{E[B} \mathcal{R}^E_{CD]} = 0$$  \hspace{1cm} (3.4)

On a soft “FDA manifold”, the algebra of diffeomorphisms includes also the diffeomorphisms in the $p$-form directions, generated by an extended Lie derivative $\ell_{\varepsilon t}$, where $t$ is a “tangent vector” dual to the $p$-form, and $\varepsilon$ is a $p-1$ form parameter $[9] [10]$. 

In the case of $d = 11$ supergravity all the local symmetries of the theory are given by the following FDA diffeomorphism algebra:

$$[\ell_{\varepsilon^A t_A}, \ell_{\varepsilon^B t_B}] = \ell_{[\varepsilon^A t_A, \varepsilon^B t_B]} t_C +$$

$$+ \ell(-\varepsilon^A \Gamma^B_{CD} V^C \varepsilon^B \varepsilon^A V^C - \varepsilon^A \varepsilon^B \varepsilon^C \varepsilon^C V^A + 2 \varepsilon^A \varepsilon^B \Gamma^C_{AB} \varepsilon^C V^A - 2 \varepsilon^A \varepsilon^B \varepsilon^C \varepsilon^C V^A) t +$$

$$[\ell_{\varepsilon^A t_A}, \ell_{\varepsilon^B t_B}] = \ell_{\varepsilon t}$$  \hspace{1cm} (3.5)

where the indices $A,B,C,...$ run on the Lie algebra directions (corresponding to the vielbein, gravitino and spin connection one-forms), i.e. $A = a, \alpha, ab$(Lorentz). The quantities $\mathcal{R}^C_{AB}$ (cf. [31]) are given by the solutions for $R^a, R^\alpha, \rho$ in (2.3)-(2.5) and by the superPoincaré structure constants encoded in the first three lines of (2.1). The two-form parameters $\varepsilon$ and $\zeta$ in (3.6) are

$$\varepsilon \equiv \varepsilon_{ab} V^a V^b + 2 \varepsilon_{a\alpha} V^a \psi^\beta + \varepsilon_{a\beta} \psi^a \psi^\beta$$

$$\zeta \equiv \varepsilon^A (D_A \varepsilon_{cd}) V^c V^d - 2 \varepsilon^c (D_c \varepsilon_{ab}) V^d + 2 i \varepsilon_{cd} \varepsilon^d \Gamma^c \rho^d V^d + i \varepsilon_{cd} \varepsilon^d \Gamma^c \psi +$$

$$+ 2 \varepsilon^A (D_A \varepsilon_{ab}) V^c \psi^a - 2 \varepsilon^c (D_c \varepsilon_{ab}) \psi^a - 2 \varepsilon^a (D_a \varepsilon_{cd}) V^c + 2 i \varepsilon_{cd} \varepsilon^d \Gamma^c \psi +$$

$$+ i \varepsilon_{a\alpha} \varepsilon_{a\beta} \psi^a \psi^\beta - 2 \varepsilon_{a\alpha} (D_a \varepsilon_{ab}) \psi^b + 2 \varepsilon_{a\gamma} (\rho_{ab} \varepsilon^A V^b + \rho_{a\beta} \varepsilon_{a\beta} \psi^\beta) \psi^\gamma -$$

$$- 2 \varepsilon_{a\beta} \varepsilon_{a\alpha} \psi^a \psi^\beta$$  \hspace{1cm} (3.8)
The Lie derivative along $t$, the dual of the three form $A$, is a particular case of the extended Lie derivatives along $p$-forms $B^i$ ($i$ being a $G$-representation index) introduced in $[9, 10]$, the fields in this general setting being the $G$ Lie algebra one-forms $\mu^A$ supplemented by the $p$-form $B^i$. The extended Lie derivative is given by

$$\ell_{\varepsilon t} \equiv i_{\varepsilon t} d + d i_{\varepsilon t} \quad (3.10)$$

the contraction operator $i_{\varepsilon t}$ being defined by its action on a generic form $\omega = \omega_{i_1...i_nA_1...A_m} B^{i_1} \wedge ... B^{i_n} \wedge \mu^{A_1} \wedge ... \mu^{A_m}$ as

$$i_{\varepsilon t} \omega = n \varepsilon^j \omega_{ji_2...i_nA_1...A_m} B^{i_2} \wedge ... B^{i_n} \wedge \mu^{A_1} \wedge ... \mu^{A_m} \quad (3.11)$$

where $\varepsilon^j$ is a $(p-1)$-form. Thus the contraction operator still maps $p$-forms into $(p-1)$-forms. Note that i) $i_{\varepsilon t}$ vanishes on forms that do not contain at least one factor $B^i$; ii) the extended Lie derivative commutes with $d$ and satisfies the Leibnitz rule.

Returning to the FDA of $d = 11$ supergravity, since $A$ is a three-form in the identity representation of the superPoincaré Lie algebra, parameters in the extended Lie derivative (along the dual $t$ of $A$) are 2-forms carrying no representation index, and are explicitly given for the algebra of FDA diffeomorphisms in (3.9) and (3.8).

The action of the extended Lie derivative on the basic fields is simply:

$$\ell_{\varepsilon t} \mu^A = 0, \quad \ell_{\varepsilon t} A = d \varepsilon \quad (3.12)$$

with $\mu^A = V^a, \omega^{ab}, \psi$. Using these rules together with the variations (2.1)-(2.4) (generated by the usual Lie derivative) leads to the diffeomorphism algebra given in eqs (3.5)-(3.7). As discussed in ref. [10] for the general case, the algebra of FDA diffeomorphisms closes provided the FDA Bianchi identities hold. Therefore, if we use in (3.5) and (3.6) the solutions (2.3)-(2.5) for the curvatures, the algebra (3.5)-(3.7) closes on the $d = 11$ field equations (2.7) - (2.9).

Note that the commutator of two ordinary Lie derivatives, computed on the 3-form $A$, does not close on the usual (3.2) diffeomorphism algebra, but develops an extra piece, i.e. the second line in the “extended” diffeomorphism algebra of (3.5), containing the extended Lie derivative.

4 A new formulation of D=11 supergravity

The idea is to reinterpret the extended Lie derivative $\ell_{\varepsilon t}$ of the $d = 11$ FDA in terms of ordinary Lie derivatives along new tangent vectors $t_{ab}, t_{a\beta}, t_{\alpha\beta}$, via the following identification:

$$\ell_{\varepsilon t} = \ell_{\varepsilon_{ab} V^a V^b t + \varepsilon_{a\beta} V^a \psi^\beta t + \varepsilon_{\alpha\beta} \psi^\alpha \psi^\beta t} \equiv \ell_{\varepsilon_{ab} t_{ab} + \varepsilon_{a\beta} t_{a\beta} + \varepsilon_{\alpha\beta} t_{\alpha\beta}} \quad (4.1)$$

The 0-forms $\varepsilon_{ab}, \varepsilon_{a\beta}, \varepsilon_{\alpha\beta}$, i.e. the coefficients of the expansion on the superspace basis of the 2-form parameter $\varepsilon$ in the extended Lie derivative, are reinterpreted as
parameters of ordinary Lie derivatives along the new tangent vectors \( t_{ab} = V^a \psi^b t \), \( t_{a\beta} = V^a \psi^\beta t \), \( t_{a\beta} = \psi^a \psi^\beta t \).

This is possible when the set
\[
\ell_{\varepsilon A}, \ell_{\varepsilon a b}, \ell_{\varepsilon a \beta}, \ell_{\varepsilon a \beta}, \ell_{\varepsilon a \beta}
\]
closes on a diffeomorphism algebra similar to the one in (3.2), i.e. a diffeomorphism algebra of an ordinary group manifold. If this is the case the new operators can be seen as \textit{bona fide} Lie derivatives, generating ordinary diffeomorphisms along new directions.

Now (3.5) indeed is of the form (3.2), and the extra piece on the right hand side simply defines new curvatures and structure constants in \( R^C_{AB} \) of (3.3). However the other commutations (3.6) contain terms with exterior (covariant) derivatives of the parameters \( \varepsilon^{ab}, \varepsilon^{a\beta}, \varepsilon^{a\beta} \), not amenable to the form of the derivative terms in (3.2). These parameters (associated with the new directions) will therefore be taken to be covariantly constant in the arguments that follow. This is the price to pay if we want to interpret the \( d = 11 \) diffeomorphism algebra (3.5)-(3.7) as an algebra of ordinary Lie derivatives.

In other words: the algebra (3.5)-(3.7) with \( D\varepsilon_{cd} = D\varepsilon_{ca} = D\varepsilon_{a\beta} = 0 \) can be considered the diffeomorphism algebra of a manifold, whose vielbeins are \( V^a, \omega^{ab}, \psi, B^{ab}, B^{a\beta}, \) and \( \eta^{a\beta} \) (the last three being the vielbeins dual to the tangent vectors \( t_{ab}, t_{a\beta}, t_{a\beta} \)).

Comparing (3.5)-(3.7) with the general form (3.2) we deduce the new curvature components that satisfy the Bianchi identities (3.4) implied by (3.2):
\[
R^{ab}, T^{ab}, T^{\alpha\beta}, \Sigma^{a\beta}
\]
remain unchanged as given in eq.s (2.3)-(2.5), whereas the solutions for the curvatures \( T^{ab}, T^{\alpha\beta}, \Sigma^{a\beta} \) of the new potentials \( B^{ab}, B^{a\beta}, \eta^{a\beta} \) are
\[
T^{ab} = 24 F^{abcd}_{cd} V^c V^d - \frac{3}{4} \rho^b_{[a\beta]} V^a \delta V^b - \frac{1}{4} \rho^a_{[a[b]} \eta_{b]} \delta \psi^a - \frac{1}{2} \rho^a_{ab} \psi^b \gamma \delta (4.3)
\]
\[
T^{\alpha\beta} = \frac{1}{4} \rho^a_{a[a \beta]} V^a (4.4)
\]
\[
\Sigma^{b\beta} = -i \rho^a_{ab} B^{a\beta} \delta V^a - \frac{i}{2} \rho^a_{a[b]} \gamma \delta V^a - \frac{i}{2} \rho^a_{b[a \beta]} \gamma \psi^a (4.5)
\]
(all contractions Lorentz invariant, position of indices not relevant).

Using the standard formula for the variation of group manifold vielbeins \( \mu^A \) under diffeomorphisms:
\[
\delta \mu^A = d\varepsilon^A - 2 R^A_{BC} \mu^B \varepsilon^C (4.6)
\]
we find the variations of the potentials \( B^{ab}, B^{a\beta}, \) and \( \eta^{a\beta} \):
\[
\delta B^{ab} = \tilde{\psi} \delta^{a b} - 48 F^{abcd}_{cd} V^c V^d - \frac{3}{4} \rho^b_{[a\beta]} V^a \delta V^b + \frac{1}{2} \rho^a_{[a[b]} \eta_{b]} \delta \psi^a + \rho^a_{ab} \psi^b \gamma \delta (4.7)
\]
\[
\delta B^{a\beta} = 2(C T_{ab})^{a\beta} V^a \varepsilon^b - \frac{3}{2} (CT_{c})_{a\beta} \varepsilon^c (4.8)
\]
\[
\delta \eta^{a\beta} = \psi \varepsilon^a \psi^b \delta V^a + \frac{1}{2} \rho^a_{a[a \beta]} \gamma \delta V^a - \frac{i}{2} \rho^a_{b[a \beta]} \gamma \psi^a (4.9)
\]
\[ + i (C_{\Gamma})^{\alpha\beta}(\varepsilon^d B^{cd} - \varepsilon^{ad} V^d) - \frac{1}{2} \rho^\gamma_{(\alpha}(\varepsilon^\gamma_{\beta)} V^a - \varepsilon^a B^{\gamma}_{\beta}) \]  

\[ \delta \eta^{\alpha\beta} = 2(C_{\Gamma})_{\delta\beta}(V^a \varepsilon^\delta - \psi^\delta \varepsilon^a) + 2i(C_{\Gamma})_{\alpha\beta}(\psi^\alpha \varepsilon_{bc} - B^{bc} \varepsilon^\alpha) + 
+ 2i \rho^\gamma_{\alpha}(\varepsilon^\gamma_{\beta}) V^a - \varepsilon^a \eta^\gamma_{\beta}) \]  

\[ + i \rho^\gamma_{\alpha}(\varepsilon^\gamma_{\beta}) \psi^\alpha - \varepsilon^a B_{\beta\gamma} \) \] (4.9)

having used \( D_{\varepsilon^{ab}} = D_{\varepsilon^{\alpha\beta}} = D_{\varepsilon^{\beta\alpha}} = 0 \) in (4.3). The variations of \( V^a, \omega^{ab}, \psi \) are unchanged and given in (2.11)-(2.13).

In summary: the new formulation of \( d = 11 \) supergravity proposed here contains the fields:

\[ V^a, \omega^{ab}, \psi, B^{ab}, B^{\alpha\beta}, \eta^{\alpha\beta} \] (4.10)

The transformation rules of these fields, given in (2.11)-(2.13) and (4.7)-(4.9) close under the same conditions necessary for the closure of the algebra (3.5)-(3.7), since it is just a reformulation of this algebra. The only "spurious" element in this reformulation is the fact that some of the parameters (i.e. \( \varepsilon^{ab}, \varepsilon^{\alpha\beta}, \varepsilon^{\beta\alpha} \)) must be taken to be covariantly constant. Even in this case the algebra (3.5)-(3.7) closes only provided the Bianchi identities (2.2) hold, which implies the \( d = 11 \) field equations (2.7)-(2.9). Thus the closure of the transformation rules on the fields (4.10) requires the \( d = 11 \) field equations, a situation analogous to the one in type IIB supergravity [12].

Finally, we can relate the covariant curl \( D_{[\mu} B_{\nu]}^{ab} \) to the curl of the three-form \( F_{\mu\nuab} \): indeed the curvature of \( B^{ab} \), according to the definition (3.1), reads

\[ T^{ab} = D B^{ab} = \frac{1}{2} \bar{\psi} \Gamma^{ab} \psi \] (4.11)

(see also [10]) where we have used the structure constants deduced by recasting the diffeomorphism algebra (3.5)-(3.7) in the form (3.2). Comparing the \( V^c V^d \) components of the definition of \( T^{ab} \) (4.11) and its solution (4.13) yields

\[ (DB^{ab})_{cd} = 24 F^{ab}_{cd} \] (4.12)

\( (DB^{ab})_{cd} \) being the \( V^c V^d \) components of the two-form \( DB^{ab} \). The other fields \( B^{\alpha\beta}, \eta^{\alpha\beta} \) are auxiliary: their curvature solutions, given respectively in (4.4) and (4.5), have no spacetime \( (VV) \)-components, and the external components only contain the gravitino curvature.

In conclusion, we have found a set of transformation rules on the dynamical fields \( V^a, \omega^{ab}, \psi, B^{ab} \) and auxiliary fields \( B^{\alpha\beta}, \eta^{\alpha\beta} \) that close on the (usual) field equations of \( d = 11 \) supergravity, \( F^{abcd} \) being now related to the curl of \( B^{ab} \).

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References

[1] E. Cremmer and B. Julia, *Supergravity theory in eleven dimensions*, Phys. Lett. **B76** (1978) 409; *The SO(8) supergravity*, Nucl. Phys. **B159** (1979) 141.

[2] P.K. Townsend, *Four lectures on M theory*, Proceedings of “High energy physics and cosmology”, 385-438, Trieste 1996, [hep-th/9612121]; *M theory from its superalgebra*, Proceedings of “Strings, branes and dualities”, 141-177, Cargese 1997, [hep-th/9712004].

[3] D. Sullivan, *Infinitesimal computations in topology*, Bull. de L’ Institut des Hautes Etudes Scientifiques, Publ. Math. **47** (1977).

[4] R. D’Auria and P. Fré, *Geometric supergravity in D=11 and its hidden supergroup*, Nucl. Phys. **B201** (1982) 101;

[5] L. Castellani, P. Fré, F. Giani, K. Pilch and P. van Nieuwenhuizen, *Gauging of D=11 supergravity?* Ann. Phys. **146** (1983) 35; P. van Nieuwenhuizen, *Free graded differential algebras* in: Group theoretical methods in physics, Lect. Notes in Phys. 180 (Springer, Berlin, 1983).

[6] L. Castellani, R. D’Auria and P. Fré, *Supergravity and superstrings: a geometric perspective*, World Scientific, Singapore 1991.

[7] Y. Ne’eman and T. Regge, Phys. Lett. **B74** (1978) 31; Riv. Nuovo Cimento **1** (1978) 5; A. D’Adda, R. D’Auria, P. Fré and T. Regge, Riv. Nuovo Cimento **3** (1980) 6; R. D’Auria, P. Fré and T. Regge, Riv. Nuovo Cimento **3** (1980) 12.

[8] L. Castellani, *Group geometric methods in supergravity and superstring theories*, Int. J. Mod. Phys. **A7** (1992) 1583.

[9] L. Castellani and A. Perotto, *Free differential algebras, their use in field theory and dual formulation*, Lett. Math. Phys. **38** (1996) 321, [hep-th/9509031].

[10] L. Castellani, *Lie derivatives along antisymmetric tensors, and the M-theory superalgebra*, [hep-th/0508213].

[11] E. Bergshoeff, E. Sezgin, *New spacetime superalgebras and their Kac-Moody extension*, Phys. Lett. **B232** (1989) 96.

[12] M.B. Green and J.H Schwarz, Phys. Lett. **109B** (1982) 444; **122B** (1983) 43.