On Dark Energy and Accelerated Reference Frames

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Received XXXX, revised XXXX, accepted XXXX  
Published online XXXX

Key words Accelerated observer; Dark energy; Torsion Tensor.

The paper is devoted to an explanation of the accelerated rate of expansion of the Universe. Usually the acceleration of the Universe, which is described by FRW metric, is due to dark energy. It is shown that this effect may be considered as a manifestation of torsion tensor for a flat Universe in the realm of Teleparallel gravity. An observer with radial field velocity obey Hubble’s Law. As a consequence it is established that this is radial acceleration in a flat Universe. In Eq. (35) the acceleration is written in terms of the deceleration parameter, the Hubbles constant and the proper distance. This may be interpreted as an acceleration of the Universe.

1 Introduction

An accelerated expansion of the Universe has been verified by recent observational data [1, 2]. The existence of dark energy is considered as an explanation of this fact [3, 4, 5, 6]. Even though the nature of such an energy remains obscure. The main feature of this energy, that is almost 70% of the content of the Universe, is that it has a repulsive gravitational force.

There are several ansatz about the nature of the dark energy. One of them associates cosmological constant to dark energy [8, 9, 10]. This explanation is popular since it behaves as a cosmological fluid with small constant energy density and negative pressure. In this model the equation of state is \( \omega = \frac{p}{\rho} = -1 \), where \( p \) is the pressure and \( \rho \) is the fluid density. There is no time evolution in such a model for \( \omega \). Other models of dark energy do have some time dependence.

A scalar field has been considered as the source of such an energy [11, 12, 13, 14, 15]. This includes the well-known quintessence model [16]. In such models the negative pressure arises when the potential energy becomes dominant in the temporal evolution. Such a behavior is obtained if the kinetic term is proportional to the derivative of the potential energy. A third option is the so called f(R) theories. The idea is to modify the Einstein-Hilbert lagrangian density adding functions of Ricci scalar which become important at late times and for small scalar curvature [17, 18, 19]. This procedure modifies the Friedmann’s equation with an extra acceleration term.

Recently in the realm of Einstein-Cartan theory, it is suggested that dark energy could arise from the coupling between torsion tensor and fermions. A cosmological BCS mechanism is used to derive the effective equations of motion which leads to an acceleration [20]. Another approach, in the same context, is given in [21], where an effective lagrangian density is used to derive an effective cosmological constant.

In this paper a possible explanation is given for the accelerated expansion of the Universe based on the teleparallel gravity. The main advantage of this method is that it leads to an expression for the acceleration naturally in terms of the torsion tensor. It does not need ad-hoc modifications of the Einstein-Hilbert equations in order to accommodate the concept of dark energy. In fact Schucking showed that the curvature does not describe acceleration but simply its gradient [22].
The paper is organized as follows. In section 2, the teleparallel gravity is introduced and an accelerated reference frame in spacetime is defined. In section 3 the FRW metric is used to describe the Universe. The acceleration of an observer in a galaxy in a radial velocity field is calculated, considering the Earth as stationary. In particular the flat Universe is analyzed. Section 4 has some concluding remarks.

Notation: space-time indices $\mu, \nu, \ldots$ and SO(3,1) indices $a, b, \ldots$ run from 0 to 3. Time and space indices are indicated according to $\mu = 0$, $i, a \equiv (0), (i)$. The flat, Minkowski space-time metric tensor raises and lowers tetrad indices and is fixed by $e_\alpha^\mu e_\beta^\nu g_{\mu\nu} = (- + + +)$. The determinant of the tetrad field is represented by $e = \det(e^a_\mu)$. It is used $G = c = 1$.

2 Accelerated Reference Frames in Teleparallel Gravity

In General Relativity the metric tensor $g_{\mu\nu}$ plays the role of a dynamical variable. This leads to the curvature tensor in terms of the Christoffel symbols $\Gamma^\lambda_{\mu\nu}$ and the corresponding torsion tensor vanishes. However, the field equations in teleparallel gravity are constructed using the torsion tensor of the Weitzenböck space-time [23], defined as

$$T^\lambda_{\mu\nu}(e) = e_a^\lambda (\partial_\mu e^a_\nu - \partial_\nu e^a_\mu).$$

The dynamical variable in the teleparallel gravity is the tetrad field $e^a_\nu$. The curvature tensor vanishes identically. It should be noted that $T^\lambda_{\mu\nu}$ is also the object of anholonomy.

Let $0\omega_{\mu ab}$ represent the torsion-free Levi-Civita connection,

$$0\omega_{\mu ab} = -\frac{1}{2} e_\mu (\Omega_{abc} - \Omega_{bac} - \Omega_{cab}),$$

with

$$\Omega_{abc} = e_{ab}(e_c^\mu \partial_\mu e_a^c - e_c^\mu \partial_\mu e_b^c),$$

The Levi-Civita connection is related to $0\Gamma^\lambda_{\mu\nu}$ as

$$0\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + e^a_\mu e^b_\nu (T_{\lambda\mu\nu} + T_{\nu\lambda\mu} + T_{\mu\lambda\nu}),$$

where $\Gamma^\lambda_{\mu\nu}$, the Weitzenböck connection.

The following identity is relevant in the construction of the teleparallel gravity,

$$0\omega_{\mu ab} = -K_{\mu ab},$$

where $K_{\mu ab} = \frac{1}{4} e_\lambda e^a_\nu (T_{\lambda\mu\nu} + T_{\nu\lambda\mu} + T_{\mu\lambda\nu})$ is the contorsion tensor of the Weitzenböck connection.

Then the curvature scalar is given by

$$eR(0, \omega) \equiv -e \left(\frac{1}{4} T^{abc} T_{abc} + \frac{3}{2} T^{abc} T_{bac} - T^a T_a\right) + 2\partial_\mu (e T^\mu).$$

with $T^a = T_b^a$. Both sides of (6) are invariant under Lorentz transformations. Dropping the divergence term the Lagrangian density is
\[ \mathcal{L}(e_{a\mu}) = -k e \left( \frac{1}{4} T^{abc}T_{abc} + \frac{1}{2} T^{abc}T_{bac} - T^{a}T_{a} \right) - \mathcal{L}_M \]

\[ \equiv -k e \Sigma^{abc}T_{abc} - \mathcal{L}_M, \tag{7} \]

where \( k = 1/(16\pi) \), \( \mathcal{L}_M \) is the Lagrangian density for the matter fields and \( \Sigma^{abc} \) is defined by

\[ \Sigma^{abc} = \frac{1}{4}(T^{abc} + T^{bac} - T^{cab}) + \frac{1}{2}(\eta^{ac}T_{b} - \eta^{ab}T_{c}). \tag{8} \]

The field equation is obtained using a variational derivative with respect to \( e_{a\lambda} \)

\[ \partial_{\nu}(e \Sigma^{a\lambda\nu}) = \frac{1}{4k} e e_{a\mu}(t^{\lambda\mu} + T^{\lambda\mu}), \tag{9} \]

where

\[ t^{\lambda\mu} = \frac{k}{4}(4\Sigma^{bc\lambda}T_{bc}^{\mu} - \eta^{\lambda\mu}\Sigma^{bcd}T_{bcd}), \tag{10} \]

and \( e_{a\mu}T^{\lambda\mu} = \frac{\delta e_{a\mu}}{\delta e_{a\mu}} \) defines the energy-momentum tensor of the matter field. It is possible to show that Eq. (9) is equivalent to the Einstein field equations.

Tetrad fields may be interpreted as reference frames [24,25] adapted to a class of observers in spacetime. If the world line \( C \) of an observer in spacetime is given by \( x^\mu(\tau) \) and its velocity by \( u^\mu(\tau) = dx^\mu/d\tau \) where \( \tau \) is the proper time, then the observer’s velocity is identified with \( e^{(0)}_{a\mu} \). Thus \( u^\mu(\tau) = e^{(0)}_{a\mu} \) along \( C \). The acceleration \( a^\mu \) is given by the derivative of \( u^\mu \) along \( C \) [26,27], i.e.

\[ a^\mu = \frac{Du^\mu}{d\tau} = \frac{De^{(0)}_{a\mu}}{d\tau} = u^\alpha \nabla_\alpha e^{(0)}_{a\mu}, \tag{11} \]

where the covariant derivative is calculated using Christoffel symbols which implies that the manifold preserves parallel transport, with Weitzenböck connection \( a^\mu \) will vanish.

Following [26,27], the absolute derivative of \( e_{a\mu} \) is given by

\[ \frac{De_{a\mu}}{d\tau} = \phi_{ab} e_{b\mu}, \tag{12} \]

where \( \phi_{ab} \) is the acceleration tensor. The acceleration tensor divides the acceleration of the whole frame along \( C \). In analogy with the Faraday tensor the identification \( \phi_{ab} \rightarrow (a, \Omega) \), where \( a \) is the translational acceleration (\( \phi_{(0)(i)} = a_{(i)} \)) and \( \Omega \) is the frequency of rotation of the local spatial frame with respect to a nonrotating (Fermi-Walker transported) frame, proves to be plausible [24,25,28].

In view of the orthogonality between tetrads it is possible to isolate \( \phi_{ab} \) in relation (12), i.e.

\[ \phi_{ab} = e^b_{\mu} \frac{De_{a\mu}}{d\tau} = e^b_{\mu} u^\lambda \nabla_\lambda e_{a\mu}, \tag{13} \]

where \( \nabla_\lambda e^b_{\mu} = \partial_\lambda e^b_{\mu} + \Sigma^{\lambda\mu\sigma}T_{\sigma}e^b_{\mu} \). The meaning of such tensor may be made clearer when the acceleration vector is written in terms of the acceleration tensor. The projection of \( a^\mu \) in (11) on a frame yields

\[ a^b = e^b_{\mu} a^\mu = e^b_{\mu} u^\alpha \nabla_\alpha e^{(0)}_{a\mu} = \phi_{(0)}^{ab}. \tag{14} \]
Therefore $a^\mu$ and $\phi_{(0)\beta}$ are not different accelerations of the frame.

The expression of $a^\mu$ given by Eq. (11) may be rewritten as

$$a^\mu = u^\alpha \nabla_\alpha e^\mu(0) = \frac{dx^\alpha}{d\tau} \left( \frac{\partial u^\mu}{\partial x^\alpha} + \Gamma^\mu_{\alpha\beta} u^\beta \right),$$

\hspace{5cm} (15)

If $u^\mu = e^\mu(0)$ represents a geodesic trajectory, then the frame is in free fall with $a^\mu = 0$ which means $\phi_{(0)(i)} = 0$. Therefore the nonvanishing values of $\phi_{(0)(i)}$ do represent non-geodesic accelerations of the frame.

Due to the orthogonality of the tetrads, eq. (13) is written as

$$\phi_{a\beta} = -u^\lambda e_a^{\mu} \nabla_\lambda e^\beta_{\mu},$$

\hspace{5cm} (16)

Combining with Eq. (4), the components $\phi_{a\beta}$ may be expressed as

$$\phi_{a\beta} = e^\mu(0) \left( 0_{0\omega_{a\beta}} \right).$$

Taking into account Eq. (5), we get

$$\phi_{ab} = \frac{1}{2} [T_{(a)\beta} + T_{a(0)\beta} - T_{b(0)a}].$$

(17)

This is the acceleration tensor [29, 30] expressed in terms of components of the torsion tensor of the Weitzenböck space-time [25]. The symmetric part of acceleration tensor vanishes.

The acceleration tensor is invariant under coordinate transformations. However it is not invariant under SO(3,1) which means it is frame-dependent. Therefore given a set of tetrad field the translational acceleration of the frame along $C$ follows from $\phi_{(0)(i)}$ and the angular velocity from $\phi_{(i)(j)}$. Consequently the acceleration tensor is suitable to describe geometrically an observer in space-time. It does not contain any dynamical feature which depends on field equations.

3 The Isotropic and Homogeneous Universe

The cosmological principle asserts that the large-scale structure of the Universe reveals homogeneity and isotropy [31]. The general form of the line element preserving such features is the Friedman-Robertson-Walker (FRW) line element

$$ds^2 = -dt^2 + \frac{R^2(t)}{1 + k^\prime r^2} [dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)],$$

\hspace{5cm} (18)

where $R(t)$ is the scale factor and $k^\prime$ is the normalized curvature of the Universe. It assumes the values -1, 0 or 1. The above line element is an ansatz based on cosmological postulates, therefore the temporal dependence of scale factor will be given by Einstein’s equations. There are numerous ideas to determine the scale factor, scalar field and vanishing curvature. In all such cases the expansion of the Universe has to depend on the scale factor [32]. From field equations we find out that the scale factor obeys Friedmann’s equation which reads

$$3 \left( \frac{\dot{R}^2 + k^\prime}{R^2} \right) - \Lambda = 8\pi \rho,$$

$$2R\ddot{R} + \dot{R}^2 + k^\prime \frac{\dot{R}}{R} - \Lambda = 8\pi \rho,$$

\hspace{5cm} (19)
where $\rho$ is the mean density of the Universe, $p$ is the pressure and $\Lambda$ is the cosmological constant.

The characterization of the field velocity $U^\mu$ of an observer is given by the component $a = (0)$ of the tetrad $e_a^\mu$ by the identification $U^\mu = e_{(0)}^\mu$. It is chosen using the following tetrad field

$$e_a^\mu = \begin{pmatrix} A & 0 & 0 & 0 \\ B \sin \theta \cos \phi & D \sin \theta \cos \phi & E r \cos \theta \cos \phi & -F r \sin \theta \sin \phi \\ B \sin \theta \sin \phi & D \sin \theta \sin \phi & E r \cos \theta \sin \phi & F r \sin \theta \cos \phi \\ B \cos \theta & D \cos \theta & -F r \sin \theta & 0 \end{pmatrix} ,$$

(20)

where $A, B, C, D, E$ and $F$ are functions of $r$ and $t$, only. They are given by

$$E^2 = F^2 = \frac{R^2(t)}{(1 + \frac{4}{3} r^2)^2} ,$$
$$B^2 - A^2 = -1 ,$$
$$D^2 - C^2 = \frac{R^2(t)}{(1 + \frac{4}{3} r^2)^2} ,$$
$$AC = BD .$$

(21)

The metric tensor is given as $g_{\mu\nu} = e_a^\mu e_a^\nu$. The tetrad field is adapted to a reference frame, which will be designated by $K'$, with a radial field velocity in relation to a reference frame at rest, $K$, with $U^\mu = e_{(0)}^\mu = (A, -g^{11}C, 0, 0)$. Indeed there is a freedom in the choice of the function $C$, corresponding to the radial component of velocity for different observers.

The acceleration tensor $\phi_{ab}$ reveals some fundamental features and details of the observer with radial velocity. The components of $\phi_{(0)(i)}$ are

$$\phi_{(0)(i)} = \frac{1}{2} g^{00} g^{11} (e_{(0)0} e_{(i)1} - e_{(0)1} e_{(i)0}) (g^{00} e_{(0)0} T_{001} + g^{11} e_{(0)1} T_{101}) .$$

(22)

There are only two relevant components of the torsion tensor in above expression, i. e.

$$T_{001} = -A \partial_0 C + B \partial_0 D ,$$
$$T_{101} = \frac{1}{2} \partial_0 (g_{11}) + C \partial_1 A - D \partial_1 B .$$

(23)

The we have

$$\phi_{(0)(1)} = \frac{1}{2} (AD - BC) g^{00} g^{11} X \sin \theta \cos \phi ,$$
$$\phi_{(0)(2)} = \frac{1}{2} (AD - BC) g^{00} g^{11} X \sin \theta \sin \phi ,$$
$$\phi_{(0)(3)} = \frac{1}{2} (AD - BC) g^{00} g^{11} X \cos \theta ,$$

(24)

where $X$ is defined by the expression

$$X = \frac{B}{C} (g_{11} \partial_0 B - \partial_1 D) + \frac{1}{2} g^{11} (C \partial_0 g_{11} + A \partial_1 g_{11}) .$$

(25)
The acceleration is given as

\[ a = \phi(0)(1) \hat{x} + \phi(0)(2) \hat{y} + \phi(0)(3) \hat{z}, \]  

leading to

\[ a = \frac{1}{2} (AD - BC) g^{00} g^{11} X \hat{r}. \]  

The frame treated here has no angular velocity which is a desirable feature since it is supposed to be adapted to the movement of galaxies. It is clear that a non-rotating frame has to be described by a tetrad field with vanishing \( \phi(i)(j) \).

Then \( \phi(i)(j) \) reduces to

\[ \phi(i)(j) = \left[ g^{00} g^{11} (e_{(i)0} e_{(j)1} - e_{(i)1} e_{(j)0}) (g^{00} e_{(0)0} T_{001} + g^{11} e_{(0)1} T_{101}) \right], \]

that leads to

\[ \phi(i)(j) = 0, \]  

once \( (e_{(i)0} e_{(j)1} - e_{(i)1} e_{(j)0}) \) always vanish. It is to be noted that an observer with radial field velocity is radially accelerated as well, which means the expansion between \( K' \) and \( K \) is accelerated.

### 3.1 The Flat Universe Case

The flat Universe is set up once the parameter \( \kappa \) is equal to zero. It worths to analyze such an Universe since it appears that the curvature of our Universe is approximately zero \[33,36\]. Besides it may provide new insights into the nature of the dark energy. The Hubble’s law implies that the velocity of recession of galaxies is roughly proportional to the mean separation between them \[37,38\].

A co-moving observer with a galaxy has the field velocity

\[ U^\mu = e_{(0)}^\mu = (U^0, U^1, 0, 0), \]  

in the radial direction. The function \( A \) is identified with \( U^0 \) and \( C \) with \( -g_{11} U^1 \). The radial component of field velocity is fixed by

\[ U^1 = \frac{\dot{R}}{R} r, \]  

where \( \dot{R} \) is the time derivative of scale factor (\( R(t) \)) and \( r \) is the comoving distance from the origin. This is the Hubble’s Law with Hubble’s constant as \( H = \frac{\dot{R}}{R} \) \[37,39\]. Such a movement is not geodesic since it deals with relative velocities between \( K' \) and \( K \). The Earth may be taken as \( K \), which is assumed to be stationary, and some arbitrary galaxy as \( K' \). In this context what is important is the relative acceleration between galaxies. Thus a non-geodesic acceleration could be established simulating effects of dark energy.

With this choice, the functions appearing in the tetrad field are...
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\[ E = F = R, \]
\[ C = -R\ddot{R}r, \]
\[ D = R(1 + R^2 r^2)^{1/2}, \]
\[ B = R^{-1}C, \]
\[ A = R^{-1}D. \]  

(32)

Substituting this in (25), we have

\[ X = -\dot{R}^2 r - R\ddot{R}r - \frac{\dot{R}^2 r}{[1 + (R\ddot{R}r)^2]^{1/2}}. \]  

(33)

Then using (27) the non-geodesic acceleration is

\[ a = \frac{1}{2R} \left( \dot{R}^2 r + R\ddot{R}r + \frac{\dot{R}^2 r}{[1 + (R\ddot{R}r)^2]^{1/2}} \right). \]  

(34)

In order to compare this to observational data, it should be noted that the first two terms of the above expression assume arbitrary values when \( r \) tend to infinity while the last term remains constant. Thus for large values of \( r \) it is possible to drop the last term, therefore an accelerated rate of expansion between \( K \) and \( K' \) approximately is

\[ a \approx \frac{1}{2}(1 - q)H^2 d_M, \]  

(35)

where \( q = -\frac{\ddot{R}}{R^2} \) is the deceleration parameter and \( d_M = Rr \) is the proper distance which is related to the luminosity distance by \( d_l = d_M(1 + z) \).

The quantity (35) represents the acceleration of an observer whose field velocity obeys the Hubble’s Law for big separation between \( K \) and \( K' \). Since the galaxies approximatively expand from each other in a similar way, it is interpreted as the acceleration of the Universe. Old measurements point to the current value of \( q \) as being 1 which means an approximately non-expanding Universe [37]. However new data aims to a negative value of \( q \), this is taken as an indication of the existence of dark energy [40–42]. Let’s see the predictions of Eq. (35) for different values of \( q \).

Cosmography is an important tool to map the expansion of the Universe. In reference [43] the deceleration parameter is reconstructed using a combination of SN Ia, BAO and CMB data. Thus for redshift around \( z \approx 1.2 \), we see \( q \approx 0.2 \), then using \( H \approx 71 \text{ K}m \text{s}^{-1} \text{Mpc}^{-1} \), the non-geodesic acceleration is

\[ a \approx 6.9 \text{ K}m^2 \text{s}^{-2} \text{pc}^{-1} \]

where the luminosity distance is approximately given by \( d_l = (c z/H)(1 + (1 - q)z/2) \) and it is used \( c = 3.10^8 \text{ m/s} \) rather than the unity. For \( z \approx 0.2 \) we see \( q \approx -0.5 \) and \( a \approx 3.1 \text{ K}m^2 \text{s}^{-2} \text{pc}^{-1} \). Now it is due to experimentalists to confirm (or not) the above prediction for the acceleration of the Universe.

The acceleration in (35) was obtained from the acceleration tensor given by Eq. (17), and the latter is constructed out of the torsion tensor, which is ultimately responsible for the non-geodesic acceleration of the Universe. It is important to note that expression (35) is always positive even when \( 0 < q < 1 \) and dark energy could be associated to the non-geodesic acceleration of the expanding Universe composed of ordinary matter. In this case the dark energy may be an effect due to the torsion tensor. In a general case, maybe in an Universe with \( k \neq 0 \), an acceleration could exist due to torsion and curvature, however in such a case the effect of accelerated reference frames has to be taken into account together with the presence of energy-momentum source in order to explain the observational data. The exclusion of such a possibility will lead to an erroneous analysis since it is impossible to express Eq. (17) in terms of the curvature.

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4 Conclusion

In this paper an observer moving along the same geodesic of a galaxy which obeys approximately the Hubble’s Law has a radial acceleration, therefore it is interpreted as the acceleration of the Universe. The Hubble’s Law gives the movement of galaxies only approximately, thus one can see that this acceleration, in fact, does not represent the real picture in the Universe. However a realistic field velocity can be chosen by an appropriate choice of the function $C$ (a null radial velocity implies $C = 0$).

The case of a flat Universe ($k = 0$) is analyzed, leading to a finite and positive radial acceleration even if $0 < q < 1$ (ordinary matter). This seems to be in agreement with what our Universe looks like since recent observations lead to an expanding Universe approximately flat [34]. As pointed by [44] all indications for the existence of dark energy comes from distance measurements for a given redshift, thus for an Universe composed by ordinary matter the relation between the luminosity distance and redshift could be affected by the presence of the non-geodesic acceleration similarly to what happens with dark energy. Hence it is concluded that the observed acceleration could be an effect of the torsion tensor. In a general case when the curvature is different from zero such an analysis plays an important role in determining the nature of possible dark energy.

Acknowledgements I would like to thank Prof. Maluf (Universidade de Brasilia) for helpful discussions, Prof. Santana (Universidade de Brasilia) and Prof. Khanna (University of Alberta) for an English revision of the manuscript. I also would like to thank the anonymous referee for contributing by bringing to my attention so important points.

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