ISOLATED STATES AND THE CLASSICAL PHASE SPACE

OF 2-D STRING THEORY

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Abstract

We investigate the classical phase space of 2-d string theory. We derive the linearised covariant equations for the spacetime fields by considering the most general deformation of the energy-momentum tensor which describes \( c = 1 \) matter system coupled to 2-d gravity and by demanding that it respect conformal invariance. We derive the gauge invariances of the theory, and so investigate the classical phase space, defined as the space of all solutions to the equations of motion modulo gauge transformations. We thus clarify the origins of two classes of isolated states.

1. Introduction

One interesting property of string theory is that, in addition to the usual physical fields, it possesses so-called isolated states. Unlike the usual particle-like excitations, these states exist only at particular momenta, and may [1] be relics of a conjectured topological phase [2] in which the full symmetry of the theory is unbroken (an argument is given in reference [3] as to why topological world-sheet field theories correspond to such a symmetric phase).

Isolated states were first identified in the \( c = 1 \) matrix model [4], and were then sought and found in a continuum analysis of \( c = 1 \) matter coupled to two dimensional gravity [1]. A careful analysis of the BRST cohomology [5] identified these isolated physical states, as well as relatives at non-zero ghost number. More recently, Witten [6] has argued that much of the physics of the isolated states is captured by a ring of dimension-zero operators, while Klebanov and Polyakov [7] and Gross and Danielson [8] have studied their interactions.

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The continuum world-sheet version of this model consists of a single scalar field coupled to two-dimensional gravity. Naively, this should correspond to a string embedded in a one-dimensional target space. However, as is by now well known [9] [10], the conformal degree of freedom of the world-sheet metric does not decouple and gives rise to a second dimension for the target space-time.

In two dimensions there are no transverse spatial directions, and so we would naturally expect that the only degree of freedom of the theory would be a massless, “tachyon”. However, it turns out that the higher excitations of the theory cannot quite be gauged away and the isolated states are the remnants. At mass-level one, the usual continuum argument for this [1] starts from the physical state conditions

\[ p^\mu (p_\mu + 2Q_\mu)\epsilon^{\lambda\nu} = 0 \]  
\[ (p_\mu + 2Q_\mu)\epsilon^{\mu\nu} = 0 \]

where \( p_\mu \) is the momentum of the state, \( Q_\mu \) is the “background charge,” proportional to the gradient of the dilaton, and \( \epsilon^{\mu\nu} \) is the polarisation of the state. For generic momenta \( p_\mu \), equations (1) and (2) are conditions on the polarisation \( \epsilon^{\mu\nu} \), but for the particular momentum \( p_\mu + 2Q_\mu = 0 \), both conditions are satisfied for arbitrary polarisation. It is in this way that isolated states are usually identified.

While this argument is undoubtedly correct (it yields new BRST cohomology classes [5] and singularities in matrix model amplitudes [4]) it also has some puzzling features. Equation (1) is a mass-shell condition, but equation (2) is a space-time gauge condition. Selecting the gauge should be a free choice of the physicist and devoid of physical content, yet it is the degeneration of this gauge condition which relaxes the requirement of transversality, and so yields new physical states. For example, imposing instead the Hilbert gauge condition

\[ p^\mu \epsilon^{\mu\nu} + \frac{1}{2}p_\nu \epsilon^{\mu\nu} = 0 \]

would not indicate anything of interest at \( p_\mu = -2Q_\mu \). Furthermore, the physical state conditions (1) and (2) are not sufficient by themselves to eliminate all degrees of freedom, even at generic momenta: equation (1) is a mass-shell condition, while equation (2) enforces transversality in the two-dimensional space-time. Obviously, \( \epsilon = 0 \) is not the only solution to equation (2). To eliminate these degrees of freedom it is necessary to invoke additional gauge invariances not manifest in either (1) or (2). The origin of the states is thus rather unclear from (1) or (2) alone.

Isolated states appear in another context in [11]. The present authors took critical string theory, and considered the most general infinitesimal deformation of naive dimension two that preserves conformal invariance. It was found that there is a distinct such deformation for every solution to a set of linearized, gauge covariant space-time equations of motion for the massless states of the theory. However, over and above this there remains a finite-dimensional space of additional deformations, which, it was argued, correspond to adding to the world-sheet action terms proportional to instanton topological charges. If these deformations are integrable (can be made finite rather than just infinitesimal—a non trivial question) they can be thought of as free parameters of string theory itself (rather than just moduli of the solution space). However, they may equally well be thought of as corresponding to isolated states, a fact which may be seen as follows. To each conformal field theory there corresponds a nilpotent BRST operator \( Q_B \), and a conformal deformation therefore corresponds to a deformation of \( Q_B \) which preserves nilpotency, \( Q_B^2 = (Q_B + \delta Q_B)^2 = 0 \). Thus \( \{Q_B, \delta Q_B\} = 0 \), which implies that \( \delta Q_B \) acting on one physical state yields another. Note that this argument holds irrespective of the integrability of the conformal deformation.
These isolated states (in the critical case, at zero momentum, as they must be since Lorentz invariance is unbroken) are thus directly associated with global aspects of the embedding of the string in space-time, once again suggesting a connection with a topological phase of the theory.

There are certain similarities between the two types of isolated state; both involve additions to the world-sheet energy-momentum tensor which are total derivatives, and so, “topological” in nature. It is therefore desirable to have a formalism in which both types of isolated states may be examined, and which clarifies the origin of the first class without appealing to the degeneration of a gauge condition. We shall exhibit such a formalism by using the method of [11] applied to the case of two-dimensional non-critical string theory. By considering infinitesimal conformal deformations of the world-sheet energy-momentum tensor we shall obtain the fully gauge covariant linearised equations of motion for the space-time fields, and by discussing the appropriate inner automorphisms we shall derive the gauge invariances of the interacting theory.

Together, these two pieces of information are sufficient for us to construct (locally) the classical phase space of the theory in an elementary way; we simply fix the gauge of the solutions until there is no gauge freedom left. The classical phase space is then isomorphic to this gauge-fixed solution-space. In this way, both classes of isolated state can be seen straightforwardly, and we do not have to appeal to the degeneration of one particular gauge condition to see them.

2. Conformal Deformations and Equations of Motion

In the continuum language, the two-dimensional string is usually (and most simply) described by the world-sheet energy-momentum tensor

\[
T(\sigma) = \frac{1}{2} \eta_{\mu \nu} \partial X_\mu \partial X_\nu(\sigma) + Q_\mu \partial^2 X^\mu(\sigma)
\]

\[
\overline{T}(\sigma) = \frac{1}{2} \eta_{\mu \nu} \overline{\partial} X_\mu \overline{\partial} X_\nu(\sigma) + Q_\mu \overline{\partial}^2 X^\mu(\sigma)
\]

where \(Q_\mu = (Q, 0)\) and \(X^\mu\) stands for the two space-time coordinates \((\phi, X)\).

Since moments of this choice of \(T\) and \(\overline{T}\) give two mutually commuting copies of the Virasoro algebra, they define a conformal field theory [12], and the central charge is

\[
c = 2 + 12Q^2.
\]

Thus a sensible string theory requires that \(Q = \sqrt{2}\) [13].

Conformal field theories (with the correct central charge) are solutions to the classical equations of motion of string theory [14]. Thus we may obtain the linearised space-time equations of motion by considering infinitesimal conformal deformations of \(T\) and \(\overline{T}\) that preserve the two mutually commuting Virasoro algebras [3], [15] (see also [16]). The variations in \(T\) and \(\overline{T}\) must therefore satisfy the deformation equations

\[
[\delta T(\sigma), T(\sigma')] + [T(\sigma), \delta T(\sigma')] = 2\delta T(\sigma') \delta (\sigma - \sigma') - \delta T'(\sigma') \delta (\sigma - \sigma')
\]

\[
[\delta \overline{T}(\sigma), \overline{T}(\sigma')] + [\overline{T}(\sigma), \delta \overline{T}(\sigma')] = -2\delta \overline{T}(\sigma') \delta (\sigma - \sigma') + \delta \overline{T}'(\sigma') \delta (\sigma - \sigma')
\]

\[
[\delta T(\sigma), \overline{T}(\sigma')] + [T(\sigma), \delta \overline{T}(\sigma')] = 0
\]

The usual physical state conditions (equations (1) and (2)) may be obtained by considering canonical deformations [3], [15], [17], which are defined by \(\delta T(\sigma) = \delta \overline{T}(\sigma) = \Phi_{(1,1)}(\sigma)\), a primary field of dimension \((1,1)\). For a deformation corresponding to a state at the first mass-level, such a deformation would be of the form

\[
\delta T(\sigma) = \delta \overline{T}(\sigma) = h^{\mu \nu}(X) \partial X_\mu \overline{\partial} X_\nu
\]
where the right hand side of this equation is a (1,1) primary field only if $h^{\rho\sigma}$ obeys

$$(\Box + 2Q_\rho \partial^\rho)h^{\mu\nu}(X) = 0$$

$$(\partial_\mu + 2Q_\mu)h^{\mu\nu}(X) = 0$$

which are obviously the same as equations (1) and (2).

However, as was shown in [11], canonical deformations are not the most general. We may find deformations that replace the equation of motion and gauge condition of equations (6) by a gauge-covariant equation of motion alone. The derivation is very similar to that in [11], so we shall only outline it here, and refer the interested reader to [11] for more details.

We shall consider the most general local deformation which only contains terms of naive dimension two for the graviton, dilaton and antisymmetric tensor fields and a term of naive dimension zero for the tachyon field, and write it as

$$\delta T = H^{\nu\lambda}(X)\partial X_\nu \overline{\partial} X_\lambda + A^{\nu\lambda}(X)\partial X_\nu \partial X_\lambda$$

$$+ B^{\nu\lambda}(X)\overline{\partial} X_\nu \overline{\partial} X_\lambda + C^{\nu\lambda}(X)\overline{\partial} X_\nu + D^{\nu\lambda}(X)\overline{\partial}^2 X_\lambda + T(X)$$

$$\delta \overline{T} = H^{\nu\lambda}(X)\partial X_\nu \overline{\partial} X_\lambda + A^{\nu\lambda}(X)\partial X_\nu \partial X_\lambda$$

$$+ B^{\nu\lambda}(X)\partial X_\nu \partial X_\lambda + C^{\nu\lambda}(X)\partial^2 X_\lambda + D^{\nu\lambda}(X)\overline{\partial}^2 X_\lambda + \overline{T}(X)$$

The tensors $H^{\nu\lambda} \ldots \overline{T}$ are initially taken to be completely independent. This ansatz is then substituted into the deformation equations (4) yielding fifty-four distinct equations for the ten tensors $H^{\nu\lambda} \ldots \overline{D}^{\lambda}$. After some calculation these equations reduce to the following

$$\delta T(\sigma) = K^{\nu\lambda}\partial X_\nu \overline{\partial} X_\lambda + (\partial - \overline{\partial}) [C^{\nu\lambda}\partial X_\nu - D^{\nu\lambda}\overline{\partial} X_\lambda] + T(X)$$

$$\delta \overline{T}(\sigma) = K^{\nu\lambda}\partial X_\nu \overline{\partial} X_\lambda - (\partial - \overline{\partial}) [C^{\nu\lambda}\overline{\partial} X_\nu - D^{\nu\lambda}\overline{\partial} X_\lambda] + \overline{T}(X)$$

where we have introduced a new field $K^{\nu\lambda}$, defined by

$$K^{\nu\lambda} = H^{\nu\lambda} + \partial^\nu C^{\nu\lambda} + \partial^\nu D^{\nu\lambda}$$

The quantities $C^\nu$, $\overline{C}^\nu$, $D^\mu$, $\overline{D}^\mu$ are given in terms of $K^{\mu\nu}$ by

$$\partial_\nu C^\nu + 2Q_\nu C^\nu = 0$$

$$D^\lambda = -\frac{1}{2}\partial_\mu K^{\mu\lambda} - Q_\mu K^{\mu\lambda}$$

$$\overline{D}^\lambda = -\frac{1}{2}\partial_\mu K^{\mu\lambda} - Q_\mu K^{\mu\lambda}$$

$$\partial^{\nu\lambda} C^{\nu\lambda} = \frac{1}{2} \Box K^{\nu\lambda} + Q_\mu \partial^\nu K^{\nu\lambda} - \frac{1}{2} \partial^\nu \partial_\mu K^{\mu\lambda} - Q_\mu \partial^\nu K^{\mu\lambda}$$

$$\partial^{\nu\lambda} \overline{C}^{\nu\lambda} = \frac{1}{2} \Box K^{\nu\lambda} + Q_\mu \partial^\nu K^{\nu\lambda} - \frac{1}{2} \partial^\nu \partial_\mu K^{\mu\lambda} - Q_\mu \partial^\nu K^{\mu\lambda}$$

while the equation of motion for the tachyon field is given by

$$\Box T + 2Q_\mu \partial^\mu T + 2T = 0$$

At first sight there is no equation of motion for the physical field $K^{\mu\nu}$. However, just as in the critical case [11], the last two equations cannot be solved for $C^\nu$ and $\overline{C}^\nu$ for arbitrary $K^{\mu\nu}$. There is an integrability condition which $K^{\mu\nu}$ must satisfy which turns out to yield an equation of motion

$$\Box h^{\nu\lambda} + 2Q_\nu \partial^\nu h^{\nu\lambda} - \partial^\nu \partial_\mu h^{\mu\lambda} - 2Q_\mu \partial^\nu h^{\mu\lambda} - \partial^\lambda \partial_\mu h^{\mu\nu} - 2Q_\mu \partial^\lambda h^{\mu\nu} + \partial^\nu \partial^\lambda h = \partial^\nu \partial^\lambda \phi$$

(12)
for some scalar function $\phi(X)$ which we identify as the dilaton. Here $h^{\mu\nu}$, which we identify as the graviton, is the symmetric part of $K^{\mu\nu}$.

For the two-form field $b_{\mu\nu}$, the anti-symmetric part of $K_{\mu\nu}$, the fact that the space-time is two-dimensional makes for some significant differences from the critical case. As in the critical case, the general solution to the integrability condition involves a constant two-form, $\alpha_{\mu\nu}$, which makes its appearance in the equation of motion for $b_{\mu\nu}$:

$$\partial^\mu H_{\mu\nu\lambda} + 2Q^\mu H_{\mu\nu\lambda} = \alpha_{\nu\lambda}$$

where $H_{\mu\nu\lambda} = \partial_{[\mu}b_{\nu\lambda]}$ and square brackets imply anti-symmetrisation. However, in two dimensions $H$ vanishes identically, so that we have to take $\alpha = 0$, and $b_{\nu\lambda}$ is not constrained by any equation of motion.

Superficially, this might suggest that the isolated states of the type found in [11] do not occur in two dimensions (in the critical case they were associated with the constants $\alpha_{\nu\lambda}$). However, in section 4 we shall see that the difference is small, and that the same isolated states occur in both theories.

It only remains to derive the equation of motion for the dilaton. The integrability condition for $K^{\mu\nu}$ provides an expression for the $C^\nu$ field, which is

$$2C^\nu = \partial^\nu(\phi - K) + \partial_\mu K^{\nu\mu} + 2Q_\mu K^{\nu\mu}$$

By substituting this expression into the equation (10a) and using the equation of motion for the $h^{\mu\nu}$ field we derive the following equation of motion for the dilaton field,

$$\Box \phi + 4Q_\mu \partial^\mu \phi = 2Q_\mu \partial^\mu h - 4Q_\mu \partial_\nu h^{\mu\nu} - 8Q_\mu Q_\nu h^{\mu\nu}$$

We have thus achieved our goal of finding deformations which are associated with covariant equations of motion and no gauge conditions. This will enable us to investigate the isolated states after we have understood the gauge symmetries of the theory. It is to this question that we now turn.

### 3. Symmetries

In this section we shall use the method developed in [3], [11], [15] to derive the gauge invariances of string theory around flat two-dimensional space-time in the presence of a dilaton background linear in $\phi$. The basic idea is that a symmetry transforms one solution of the classical equations of motion to another, without changing the physics. In the case of string theory, we are therefore interested in physically indistinguishable conformal field theories.

Given a conformal field theory, one simple way to get another which is physically indistinguishable is to take the operators of the given theory and perform the same similarity transformation on them. This gives a new conformal field theory which is physically identical to the old one, since none of the algebraic properties are changed by a similarity transformation. However, in general the energy-momentum tensors are different fields, and if that change can be interpreted as a change in the space-time fields, then that change is a symmetry transformation.

If we restrict ourselves to infinitesimal transformations, then $\Phi \to \Phi + \delta \Phi$ is a symmetry if there exists an operator $h$ such that

$$i[h, T_\Phi(\sigma)] = T_{\Phi + \delta \Phi}(\sigma) - T_{\Phi}(\sigma)$$

where $T_\Phi(\sigma)$ is the energy-momentum tensor of the field $\Phi$. The operator $h$ is called the generator of the symmetry transformation.
Which operators $h$ satisfy equation (15), when $T_Φ$ is the energy-momentum tensor that describes the $c = 1$ matter system coupled to two-dimensional gravity? Because the commutator preserves naive dimension [11], the generators of interest are

$$h = \int d\sigma (\xi^\mu(X) \partial X_\mu + \zeta^\mu(X) \bar{\partial} X_\mu)$$

(16)

Now since

$$\delta T(\sigma) = -i[h, T(\sigma)]$$

$$\delta T(\sigma) = -i[h, T(\sigma)]$$

(17)

is an automorphism, it automatically satisfies the deformation equations and so must correspond to a solution of the type specified in equations (8) and (10). Let us therefore compute the right hand side of equation (17), with $h$ given by (16). The result provides us with the transformation properties of the physical fields under coordinate and two-form gauge transformations

$$K^{\nu\lambda} = H^{\nu\lambda} + \partial^\nu C^\lambda + \partial^\nu D^\lambda = \partial^\nu \xi^\lambda + \partial^\nu \zeta^\lambda$$

(18a)

$$\partial^\nu \phi = -2Q_\mu \partial^\nu \xi^\mu - 2Q_\mu \partial^\nu \zeta^\mu$$

(18b)

These are the canonical general-coordinate and two-form gauge transformations when the background consists of a flat space-time and a non-constant dilaton. Thus, having obtained the linearised equations of motion and gauge symmetries, we now proceed to the construction of the classical phase space.

4. Classical Phase Space

We proceed now to investigate the classical phase space of our theory. It is defined as the space of all solutions of the classical equations of motion modulo gauge transformations.

As we saw in the last section, the equations of motion for the physical fields are invariant under two-dimensional coordinate transformations

$$h^{\nu\lambda} \rightarrow h^{\nu\lambda} + \frac{1}{2} \partial^\nu u^\lambda + \frac{1}{2} \partial^\lambda u^\nu$$

(19)

where $u^\lambda = \xi^\lambda + \zeta^\lambda$, and two-form gauge transformations

$$b^{\nu\lambda} \rightarrow b^{\nu\lambda} + \frac{1}{2} \partial^\nu w^\lambda - \frac{1}{2} \partial^\lambda w^\nu$$

(20)

where $w^\lambda = \xi^\lambda - \zeta^\lambda$, while the dilaton transforms in a non-trivial manner

$$\partial^\nu \phi \rightarrow \partial^\nu \phi - 2Q_\mu \partial^\nu u^\mu$$

(21)

Let us begin with the graviton and dilaton. We find it convenient to impose first the Lorentz gauge condition

$$Q_\mu h^{\mu\nu} = 0$$

(22)

where $Q_\mu = (Q, 0)$. Thus equation (22) is equivalent to the two conditions $h^{00} = 0$ and $h^{01} = 0$, leaving $h^{11}$ as the sole non-zero component. It is straightforward to see that we may impose this condition, starting from an arbitrary $h^{\mu\nu}$; we need to find $u^\mu$ which satisfies

$$\partial_0 u_0 = -h_{00}$$

$$\partial_0 u_1 = -2h_{01} - \partial_1 u_0$$

(23)

(24)
Equations (23) and (24), thought of as equations for $u_0$ and $u_1$ respectively, obviously have solutions for arbitrary right-hand sides.

In this gauge the equations for the graviton-dilaton system reduce to the following set of equations

$$\partial_0 \partial^0 (\phi - h) = 0, \quad h = h^{11}$$  \hspace{1cm} (25)
$$\partial^0 (\partial_0 + 2Q_0) h = \partial_1 \partial^1 \phi$$  \hspace{1cm} (26)
$$\partial^0 \partial_1 \phi = 0$$  \hspace{1cm} (27)
$$\partial^0 \partial_0 \phi + \partial^1 \partial_1 \phi + 4Q_0 \partial^0 \phi = 2Q_0 \partial^0 h$$  \hspace{1cm} (28)

Although we have imposed a gauge condition there is still residual coordinate invariance, namely coordinate transformations which respect this particular choice. These are generated by those vector fields $u_\mu = (u_0, u_1)$ which obey the following relations as can be seen by equation (19)

$$\partial_0 u_0 = 0, \quad \partial_1 u_0 = \partial_0 u_1$$  \hspace{1cm} (29)

with general solution

$$u_0 = u_0(x^1), \quad u_1(x^0, x^1) = (\partial_1 u_0) x^0 + f(x^1)$$  \hspace{1cm} (30)

where $f = f(x^1)$ is an arbitrary function of $x^1$.

To fix the gauge further, we must make use of the equations of motion. The most general solution to the equation (27) is

$$\partial_1 \phi = y(x^1)$$  \hspace{1cm} (31)

where $y = y(x^1)$ is an arbitrary function of $x^1$. According to (21) $\partial_1 \phi$ transforms under a coordinate transformation as

$$\partial_1 \phi \rightarrow \partial_1 \phi - 2Q_0 \partial_1 u_0$$

We choose $u_0(x^1)$ so that $2Q_0 \partial_1 u_0 = y(x^1)$ and so set $\partial_1 \phi$ equal to zero. Thus the gauge freedom associated with $u_0$ has been exhausted. At our disposal we still have the transformations which are generated by $u_1 = u_1(x^1)$.

Using $\partial_1 \phi = 0$ and equations (25), (26) and (28) it is straightforward to show that

$$\phi(x^0) - h(x^0, x^1) = \rho(x^1)$$  \hspace{1cm} (32)

where $\rho(x^1)$ is an arbitrary function of $x^1$ only. Using coordinate transformations which are generated by $u_1 = u_1(x^1)$ we can set $\rho(x^1)$ equal to zero, and all gauge freedom is now used up. We are thus reduced to a single degree of freedom depending on $x^0$ alone: $\phi(x^0) = h(x^0)$. Substituting this degree of freedom into the equations of motion, we find the following general solution:

$$h(x^0) = \phi(x^0) = -\frac{c_1}{2Q_0} \exp(-2Q_0 x^0) + c_2$$  \hspace{1cm} (33)

where $c_1$, $c_2$ are arbitrary integration constants.

These, then, are the first class of isolated states. Recalling that these linearised solutions are the wave functions for the graviton and dilaton vertex operators, these new factors might have been anticipated since the effective string coupling constant is $\exp(\phi) = \exp(-2Q_0 x^0)$. 

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Higher massive fields are expected to contribute in an analogous manner to the phase space of the theory, namely states which correspond to discrete values of momenta. Consequently our phase space consists of one field, the tachyon, and a tower of states which are remnants of the higher massive modes of the string.

Finally, we turn to the two-form field, $b_{\mu\nu}$. Recall that this field is not constrained by any equation of motion, and is invariant under the gauge transformation, equation (20). Thus, in two dimensions the two-form is always locally pure gauge (recall that there are no three-forms, so every two-form is closed): we do not even have to use an equation of motion to eliminate all propagating degrees of freedom. However, depending on the global properties of space-time, there may be cohomology. For example, if space-time is compact, $b_2 = 1$ and there is a one dimensional space of gauge-inequivalent solutions.

Although the derivation is superficially different, we wish to argue that these states are the two-dimensional analogues of the second class of isolated states found in [11]. The reason is that non-trivial cohomology is associated with non-trivial homotopy [18], which in turn implies the existence of instantons. Indeed, the two-form $b$ is just proportional to the instanton topological charge density. It was argued in [11] that the isolated states found there were associated with instanton topological terms in the action, of just this type, which correspond to two-forms representing non-trivial elements of $H^{(2)}$. Thus the absence of the parameters $\alpha_{\mu\nu}$ in two dimensions is not due to the absence of the isolated states, but rather to the absence of a propagating two-form, so that there is no equation of motion to be modified.

We have thus succeeded in exhibiting both classes of isolated states.

6. Conclusions

In summary, we have used the techniques of [11] to derive isolated states of string theory in a very physical way; we obtained linearised equations of motion and the gauge symmetries, and then derived the phase space as the set of all solutions, modulo gauge transformations.

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