Pair reversal in homogeneous isotropic turbulence

BJ Devenish and DJ Thomson
Met Office, Fitzroy Road, Exeter, EX1 3PB, UK
E-mail: ben.devenish@metoffice.gov.uk, david.thomson@metoffice.gov.uk

Abstract. We show that the separation of particle pairs in the inertial subrange of homogeneous isotropic turbulence is strongly influenced by the pairs that separate quasi-diffusively. We quantify the influence of the diffusive separators by considering the probability that a pair will ‘reverse’ direction across a given separation i.e. its separation will decrease (before eventually increasing) and derive an analytical expression for the expected number of reversals across this separation for a quasi-one-dimensional model of relative dispersion in the inertial subrange with Gaussian turbulence. We compare this theoretical result with three different Lagrangian stochastic models in which the influence of the diffusive and ballistic separators (the latter dominated by velocity memory) can be varied by means of the value of \( C_0 \), the constant of proportionality in the Lagrangian velocity structure function, which appears explicitly in Lagrangian stochastic models. We also compare these results with data from a direct numerical simulation of turbulence. The results indicate the importance of the transverse relative velocity component (i.e. the ability of pairs to rotate), which is absent in Q1D models, in determining the correct quantitative relative dispersion statistics.

1. Introduction

It is well known that in the inertial subrange of homogeneous isotropic turbulence the mean square separation of a pair of particles, \( \langle r^2 \rangle \), is predicted to grow like \( \varepsilon t^3 \), where \( \varepsilon \) is the mean rate of kinetic energy dissipation and \( t \) is time, once the initial separation, \( r_0 \), of the pair is forgotten. The difficulty in measuring this result directly, either experimentally or in numerical simulations, has prompted the analysis of particle-pair statistics in terms of exit times which are defined to be the time taken for the separation, \( r \), to change from, say, \( R/\rho \) to \( R \) (see e.g. Boffetta & Sokolov (2002); Biferale et al. (2005); Devenish & Thomson (2011)). When \( \rho \gg 1 \) the physics of the separation process is primarily diffusive whereas for \( \rho - 1 \ll 1 \) the physics is primarily ballistic (here we use ballistic to mean that the relative velocity of the pairs is only slowly changing rather than strictly constant, that is, the separation process is dominated by the velocity memory). In a recent paper, Devenish & Thomson (2011) argued that the mean exit time must satisfy a consistency condition between small and large \( \rho \)-values. They also showed that the distribution of exit times for \( \rho - 1 \ll 1 \) has a long tail of slow separators which will in general move inward to separations much less than \( R/\rho \) before separating to \( R \). In effect, these slow separators (which tend to separate diffusively) constrain the mean exit time.

In this paper we attempt to quantify the influence of the slow (diffusive) separators. We use Lagrangian stochastic models (LSM) to illustrate our arguments as here the relative importance of the ballistic and diffusive separators can be varied in a way that is not possible in real turbulence. As the constant of proportionality in the Lagrangian velocity structure function,
$C_0$, appears explicitly in LSM, it is possible to illustrate the effects of velocity memory by varying $C_0$. Since $C_0$ is inversely proportional to the magnitude of the Lagrangian time scale (see e.g. Pope (2000), p.486), as $C_0$ increases the relative velocity of the pairs decorrelates increasingly rapidly and particle pairs are more likely to separate diffusively. Conversely, as $C_0$ decreases, the relative velocity decorrelates more slowly and the pairs are more likely to separate ballistically.

Since the relative velocity of the diffusively separating pairs decorrelates more rapidly compared with the ballistic separators, the diffusively separating pairs are more likely to change direction. When pairs reverse direction, their separation converges, to possibly a value smaller than $R$, and it can take a long time for their separation to grow beyond $R$. The expected number of reversals across $R$, $E_R$, varies with $C_0$ and increases with increasing $C_0$. An analytical form for $E_R$ can be derived for a (transformed) quasi-one dimensional (Q1D) model of relative dispersion in the inertial subrange with Gaussian turbulence and takes the form

$$E_R = \frac{1}{\sqrt{2\pi}} \frac{3C_0}{7^{3/2}} \exp\left(-\frac{49C_0^3}{18C_0^2}\right) - \frac{1}{2} \text{erfc}\left(\frac{7C_0^{3/2}}{3\sqrt{2}C_0}\right)$$

where $C$ is the constant of proportionality in Kolmogorov’s two-thirds law (typically $C \approx 2$). The variation of (1) with $C_0$ is shown in figure 1: note the rapid decrease as $C_0$ decreases from $C_0 \approx 5$. The expected number of reversals can be regarded as a measure of the decorrelation associated with diffusive motion. We compare (1) with three different LSM and data from a direct numerical simulation (DNS) of turbulence.

![Figure 1](image.png)

**Figure 1.** The expected number of reversals as a function of $C_0$ according to (1) (solid line). The dashed and dotted lines are respectively the small and large $C_0$ asymptotes of (1).

### 2. Pair reversal in LSM and DNS

In the inertial subrange of turbulence the Q1D models of Kurbannuradov (1997) (shown in figure 2) and Borgas & Yeung (2004) (shown in figure 2), both formulated with non-Gaussian velocity statistics, produce values of $E_R$ which are close to the theoretical values even for $C_0 \gg 1$. The three-dimensional (3-D) LSM of Thomson (1990) with Gaussian turbulence, on the other hand, leads to values of $E_R$ which are significantly larger than (1) for $C_0 \gg 1$ (see figure 2) and which are closer to the value calculated from DNS (shown in figure 2). Only for large values of $C_0$, when the statistics are dominated by the diffusive separators, does this model agree with (1). (Note that the increase in $E_R$ for large values of $r$ (except for $C_0 = 100$) is due to the effects of the integral scale, $L$. The final decrease in $E_R$ (for all values of $C_0$) results from ‘losing’ pairs i.e. pairs that do not reach the largest values of $r$ before the end of the simulation.)

2
Figure 2. The expected number of reversals for the Q1D model of Kurbanmuradov (1997) for $C_0 = 5$ (dashed), $C_0 = 10$ (dot-dashed) and $C_0 = 100$ (dotted). The horizontal lines represent the theoretical result (1) for the same values of $C_0$.

Figure 3. The expected number of reversals for the Q1D model of Borgas & Yeung (2004) showing the variation with Reynolds number for $C_0 = 5$: $Re\lambda = 10^4$ (red), $Re\lambda = 10^5$ (blue) and $Re\lambda = 284$ (cyan). The solid lines represent pairs with $r_0 = 0.25\eta$, the dashed lines $r_0 = 2\eta$ and the dotted lines $r_0 = 20\eta$. The horizontal black line represents the theoretical result (1) for $C_0 = 5$.

Compared with the Q1D model and the 3-D LSM, DNS is, of course, complicated by the presence of a dissipation range and limited by a relatively short inertial subrange. Thus, here one might expect to see more variation of $E_R$ with $r$ which is indeed shown in figure 2. The DNS data is taken from Biferale et al. (2005) for which $C_0 = 5.2$ and the Taylor scale Reynolds number, $Re\lambda = 284$. We speculate that the initial decrease in $E_R$ may be due to the effects of $r_0$ and that the peak that occurs for small $r_0$ may be due to intermittency effects. For $r$ in the range $30\eta \lesssim r \lesssim 200\eta$ (where $\eta$ is the Kolmogorov scale), which is the approximate extent of the inertial subrange (Biferale et al., 2005), the curves corresponding to different values of $r_0$ show an approximate collapse. The increase (followed by a decrease) of $E_R$ for values of $r \sim 100\eta$ is consistent with the 3-D LSM shown in figure 2 and occurs for the same reasons.

As the model of Borgas & Yeung (2004) has an explicit Reynolds number dependence, includes intermittency effects and is formulated to model the transition from the dissipation range to the inertial subrange, we can use this model to gain some insight into the behaviour of $E_R$ calculated from the DNS data. Figure 2 shows the expected number of reversals for the Q1D model of Borgas & Yeung (2004) for $C_0 = 5$ (which is comparable with the DNS value) and three values...
Figure 4. The expected number of reversals for the 3-D LSM of Thomson (1990) for \( C_0 = 5 \) (dashed), \( C_0 = 10 \) (dot-dashed) and \( C_0 = 100 \) (dotted). The horizontal line represents the theoretical result (1) for \( C_0 = 100 \).

Figure 5. The expected number of reversals for the DNS data: \( r_0 = 1.23\eta \) (solid), \( r_0 = 2.45\eta \) (dashed), \( r_0 = 9.82\eta \) (dot-dashed) and \( r_0 = 19.64\eta \) (dotted).

of \( Re_\lambda \) including one that is comparable with the DNS data. The behaviour of \( E_R \) shows some qualitative agreement with the DNS results in figure 2 notably a local peak for \( r \gg \eta \). As stated above, for \( r \)-values in the inertial subrange, \( E_R \) is much smaller than the DNS and 3-D LSM values and is closer to the value of (1) for \( C_0 = 5 \). Furthermore, the value of \( E_R \) in the inertial subrange is almost insensitive to changes in \( Re_\lambda \). We will use these results to argue that the transverse relative velocity component (i.e. the ability of pairs to rotate), which is absent in Q1D models, plays an important role in determining the correct quantitative relative dispersion statistics.

References

Boffetta, G. & Sokolov, I. M. 2002 Statistics of two-particle dispersion in two-dimensional turbulence. Phys. Fluids 14, 3224-3232.

Biferale, L., Boffetta, G., Celani, A., Devenish, B.J., Lanotte, A. & Toschi, F. 2005 Lagrangian statistics of particle pairs in homogeneous isotropic turbulence. Phys. Fluids 17, 115101.

Devenish, B.J. & Thomson, D.J. 2011 Quantifying turbulent dispersion by means of exit times. Submitted to Phys. Fluids
POPE, S.B. 2000 Turbulent Flows. Cambridge University Press.
KURBANMURADOV, O.A. 1997 Stochastic Lagrangian models for two-particle relative dispersion in high-Reynolds number turbulence. Monte Carlo Meth. Appl. 3, 37-52.
THOMSON, D.J. 1990 A stochastic model for the motion of particle pairs in isotropic high-Reynolds-number turbulence, and its application to the problem of concentration variance. J. Fluid Mech. 210, 113-153.
BORGAS, M.S. & YEUNG, P.K. 2004 Relative dispersion in isotropic turbulence: Part 2. A new stochastic model with Reynolds number dependence. J. Fluid Mech. 503, 125-160.