Comment on Neutrino Masses and Oscillations
in an SU(3)$_L$ × U(1)$_N$ Model
with Radiative Mechanism

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(March, 2001)

We discussed how neutrino masses and oscillations are radiatively generated in an SU(3)$_L$ × U(1)$_N$ gauge model with a symmetry based on $L_e - L_μ - L_τ$ ($\equiv L'$). The model is characterized by lepton triplets $ψ^i = (ν^i, ℓ^i, E^{-i})$, where $E^{-i}$ are negatively charged heavy leptons, an SU(3)$_L$ triplet Higgs scalar $ξ$ and a singlet Higgs scalar $κ_+$. These Higgs scalars can be interpreted as a Zee’s and Zee-Babu’s scalar for radiative mechanisms. We demonstrated that the mass hierarchy of $Δm^2_{atm} \gg Δm^2_⊙$ arises as a consequence of the dynamical hierarchy between $L'$-conserving one-loop effects and $L'$-violating two-loop effects, and our model is relevant to yield quasivacuum solution for solar neutrino problem.

PACS: 12.60.-i, 13.15.+g, 14.60.Pq, 14.60.St

Keywords: neutrino mass, neutrino oscillation, radiative mechanism, lepton triplet

There is definitive evidence for neutrino oscillations from atmospheric and solar neutrino observations. For the atmospheric neutrino oscillations, the recent SuperKamiokande (SK) data indicates that the observed deficit of $ν_μ$ is due to the $ν_μ \leftrightarrow ν_e$ oscillation [1,2], while for the solar neutrino oscillations [3], the SK, Homestake [4], SAGE [5], GALLEX [6] and GNO [7] data indicate the $ν_e \leftrightarrow ν_μ, ν_τ$ oscillation. The existence of these neutrino oscillations implies the neutrinos are massive particles [8]. The mass squared differences for atmospheric oscillations $Δm^2_{atm}$ is measured as $Δm^2_{atm} \sim 3 \times 10^{-3}$ eV$^2$ [9]. On the other hand, there are some solutions to explain the observed solar neutrino oscillation data as (1) $Δm^2_{atm} \sim 10^{-5}$ eV$^2$ for the LMA solution, (2) $Δm^2_{atm} \sim 10^{-6}$ eV$^2$ for the SMA solution, (3) $Δm^2_{atm} \sim 10^{-7}$ eV$^2$ for the LOW solution, (4) $Δm^2_{atm} \sim 10^{-10}$ eV$^2$ for the VO solution and recently proposed (5) $Δm^2_{atm} \sim 10^{-9}$ eV$^2$ for the quasi-VO (QVO) solution [10]. To sum up, we have $Δm^2_{atm} \sim 10^{-3}$ eV$^2$ and $Δm^2_⊙ \lesssim 10^{-5}$ eV$^2$ indicating the hierarchy of $Δm^2_{atm} \gg Δm^2_⊙$ exist. In the theoretical view, this mass hierarchy suggests that the neutrino mass matrix has bimaximal structure [11-13].

Recently, radiative mechanisms to generate tiny neutrino masses and oscillations in SU(3)$_L$ × U(1)$_N$ gauge models [14-18] with the $L'$ symmetry have been extensively studied [16-18]. Here $L' = L_e - L_μ - L_τ$ is a new lepton number and the conservation of this quantum number is one of the possibilities of the origin of the bimaximal structure [16-18].

Three SU(3)$_L$ × U(1)$_N$ gauge models are used to accomodate such radiative mechanisms. Each of the SU(3)$_L$ × U(1)$_N$ models can be distinguished by the lepton triplets $ψ^i$ ($i = 1, 2, 3$) in the models: (a) $ψ^i = (ν^i, ℓ^i, ω^{0i})$ model [16], (b) $ψ^i = (ν^i, ℓ^i, κ^{+i})$ model [17] and (c) $ψ^i = (ν^i, ℓ^i, E^{-i})$ model [18], where $ω^{0i}$, $κ^{+i}$ and $E^{-i}$ are denoted by electrically neutral heavy leptons, positively charged heavy leptons and negatively charged heavy leptons, respectively. In the model (a) and the model (b), the atmospheric neutrino oscillations are generated by a one-loop radiative mechanism with $L'$-conserving interactions [21], and the solar neutrino oscillations are induced from a two-loop radiative mechanism with $L'$-violating interactions [22]. Consequently, the mass hierarchy of $Δm^2_{atm} \gg Δm^2_⊙$ is explained as a result of the smallness of the two-loop effects compared with one-loop effects [21-22]. On the other hand, in the model (c), there is no one-loop interaction and both of the atmospheric and solar neutrino oscillations come from two-loop radiative effects [21-22]. The mass hierarchy of $Δm^2_{atm} \gg Δm^2_⊙$ is related to the dynamical hierarchy of the $L'$-conserving and $L'$-violating two-loop interaction effects.

In this article, we show that it is possible to construct the other SU(3)$_L$ × U(1)$_N$ model with lepton triplets $ψ^i = (ν^i, ℓ^i, E^{-i})$ [23]. The model has similar particle content to the model (c): however, one-loop interactions also exist and neutrino masses are induced by the $L'$-conserving one-loop radiative mechanism as well as the $L'$-violating two-loop radiative mechanism.

The particle content in our SU(3)$_L$ × U(1)$_N$ gauge model is summarized as follows:

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1The model with $ψ^i = (ν^i, ℓ^i, κ^{+i})$ has been lately examined to yield tiny neutrino masses and observed neutrino oscillations [15].
\begin{equation}
\psi_{L}^{i=1,2,3} = (\nu^{i}, \ell^{i}, E^{i})_{L}^{T}: (3,-2/3), \quad \ell_{R}^{i,2,3}: (1,-1), \quad E_{R}^{i,2,3}: (1,-1), \tag{1}
\end{equation}

in the lepton sector, where we have denoted $E^{-i}$ by $E^{i}$,

\begin{equation}
Q_{L}^{i} = (u^{i}, d^{i}, d^{i})_{L}^{T}: (3,0), \quad Q_{L}^{i=2,3} = (d^{i}, -u^{i}, u^{i})_{L}^{T}: (3^{*},1/3), \quad u_{R}^{i,2,3}: (1,2/3), \quad \ell_{R}^{i,2,3}: (1,-1/3), \quad u_{R}^{i,2,3}: (1,2/3), \quad d_{R}^{i}: (1,-1/3), \tag{2}
\end{equation}

in the quark sector, and

\begin{equation}
\eta = (\eta^{0}, \eta^{-}, \eta^{+})^{T}: (3,-2/3), \quad \rho = (\rho^{0}, \rho^{0}, \rho^{0})^{T}: (3,1/3), \quad \chi = (\chi^{+}, \chi^{0}, \chi^{0})^{T}: (3,1/3), \quad \xi = (\xi^{+}, \xi^{+}, \xi^{+})^{T}: (3,4/3), \quad k^{++}: (1,2), \tag{3}
\end{equation}

in the Higgs sector, where the quantum numbers are specified in the parentheses by $(SU(3)_{L}, U(1)_{Y})$. Let $N/2$ be the $U(1)_{Y}$ quantum number, then the hypercharge $(Y)$ and electric charge $(Q_{e})$ are given by $Y = \lambda^{8}/\sqrt{3} + N$ and $Q_{e} = (\lambda^{3} + Y)/2$, respectively, where $\lambda^{8}$ is the $SU(3)$ generator with $\text{Tr}(\lambda^{a}\lambda^{b}) = 2\delta^{ab}$ $(a,b = 1, \ldots, 8)$. Three Higgs triplets $\eta, \rho$ and $\chi$ are the minimal set to generate masses of quarks and leptons in $SU(3) \times U(1)_{Y}$ models. An additional Higgs triplet $\xi$ is introduced as a triplet version of the Zee scalar to realize the one-loop radiative mechanism and an additional Higgs singlet $k^{++}$ is introduced to realize the two-loop radiative mechanism.

Here, we introduce two constraints to obtain the relevant Yukawa interactions. The first is the $L' \equiv L_{e} - L_{\mu} - L_{\tau}$ conservation imposed on our interactions to reproduce the observed atmospheric neutrino oscillations as mentioned. The $L'$ assignment is shown in Table 1. The second is the discrete symmetry based on $Z_{2}$ to suppress unwanted flavor-changing-neutral-currents (FCNC) interactions in the quark sector and the lepton sector. In the quark sector, there are quarks with the same charge, thus, quark mass terms can be generated by $\rho$ and $\chi$ between $Q_{L}^{i}$ and down-type quarks and by $\rho^{0}$ and $\chi^{0}$ between $Q_{L}^{i=2,3}$ and up-type quarks. FCNC is induced from these interactions. Also, in the lepton sector, $\ell^{i}$ and $E^{i}$ $(i = 1,2,3)$ has the same charge and the similar FCNC problem can occur. To avoid such interactions, Yukawa interactions must be constrained such that a quark (lepton) flavor gains a mass from only one Higgs scalar.

These situations can be realized by introducing the following $Z_{2}$ symmetry into the model:

\begin{equation}
\begin{aligned}
\psi_{L}^{i,2,3} &\rightarrow i\psi_{L}^{i,2,3}, \quad \ell_{R}^{i,2,3} \rightarrow \ell_{R}^{i,2,3}, \quad E_{R}^{i,2,3} \rightarrow -iE_{R}^{i,2,3}, \quad Q_{L}^{i} \rightarrow iQ_{L}, \quad Q_{L}^{2,3} \rightarrow -iQ_{L}^{2,3}, \quad u_{L}^{i,2,3} \rightarrow u_{L}^{i,2,3}, \quad d_{L}^{i,2,3} \rightarrow d_{L}^{i,2,3}, \quad u_{R}^{i,2,3} \rightarrow iu_{R}^{i,2,3}, \quad d_{R}^{i,2,3} \rightarrow -id_{R}^{i,2,3}, \quad \eta \rightarrow i\eta, \quad \rho \rightarrow i\rho, \quad \chi \rightarrow -\chi, \quad \xi \rightarrow -\xi, \quad \text{and} \quad k^{++} \rightarrow ik^{++}.
\end{aligned}
\end{equation}

With these constraints, the Yukawa interactions are given by

\begin{equation}
\begin{aligned}
-L_{Y} &= \epsilon^{\alpha\beta\gamma} \sum_{i=2,3} f_{i}[\psi_{L}^{i} \psi_{L}^{i} \xi_{\gamma} + \sum_{i=1,2,3} \psi_{L}^{i} (\bar{\rho}_{L}^{i} \ell_{L}^{i} + f_{E} E_{L}^{i})] \\
&+ \sum_{i,j=2,3} f_{ij}(\bar{\ell}_{L}^{i} \rho_{L}^{j} + \bar{\ell}_{L}^{j} \rho_{L}^{i} + \bar{Q}_{L}^{i} \eta_{R}^{j} + \rho_{D_{R}}^{i} + \chi_{D_{R}}^{j} + \xi_{U_{R}}^{j} + \rho_{E} E_{R}^{j}) \\
&+ \sum_{i=2,3} \bar{Q}_{L}^{i} (\eta^{i} D_{R}^{i} + \rho^{i} U_{R}^{i} + \chi^{i} U_{R}^{i}) + (h.c.),
\end{aligned}
\end{equation}

where $f$'s are Yukawa couplings with the relation $f_{i}[j] = -f_{[j]}$ by the Fermi statics, and right-handed quarks are denoted by $U_{R} = \sum_{i=1}^{3} f_{w_{u}} u_{R}^{i}, \quad D_{R} = \sum_{i=1}^{3} f_{w_{d}} d_{R}^{i}, \quad U_{R}^{i} = f_{w_{2}} u_{R}^{i} u_{R}^{i}, \quad d_{R}^{i} = f_{d_{1}} d_{R}^{i} d_{R}^{i}$. For simplicity, we have assumed diagonal mass terms for the leptons. Note that there is no term which can induce FCNC interactions such as $Q_{L}^{i} \chi_{D_{R}}, \quad Q_{L}^{i} \rho_{D_{R}}, \quad Q_{L}^{2,3} \chi_{U_{R}^{2,3}}, \quad Q_{L}^{2,3} \rho_{U_{R}^{2}}, \quad \psi_{L}^{i} \ell_{R}^{j}$ and $\psi_{L}^{i} \rho E_{R}^{j}$.

The Higgs interactions are given by self-Hermitian terms of $\phi_{\alpha}^{i} \phi_{\beta}^{i} (\phi = \rho, \eta, \chi, \xi, k^{++})$, and two types of non-self-Hermitian Higgs potentials:

\begin{equation}
\begin{aligned}
V_{0} &= \lambda_{0} \epsilon^{\alpha\beta\gamma} \eta_{\alpha} \rho_{\beta} \chi_{\gamma} + \lambda_{1} (\eta^{i} \rho)(\xi^{i} \chi) + \lambda_{2} (\eta^{i} \chi)(\xi^{i} \rho) + (h.c.),
V_{b} &= \mu_{b} \epsilon^{\xi^{i} k^{++}} + (h.c.),
\end{aligned}
\end{equation}

where $\lambda_{0,1,2}$ stands for $L'$-conserving coupling constants and $\mu_{b}$ denotes the $L'$-violating mass scale. The interaction of the $\eta \rho \chi$ type in Eq. (5) is a guarantee of the orthogonal choice of vacuum expectation values (VEVs) for three Higgs scalars, $\eta, \rho$ and $\chi$ as $(0)\eta(0) = (\eta_{0}, 0, 0)^{T}$, $(0)\rho(0) = (0, \rho_{0}, 0)^{T}$, and $(0)\chi(0) = (0, 0, \chi_{0})^{T}$, respectively.

We note that there are two main differences between the model (c) discussed in Ref. 13 and the model in this article (current model). The first is the absence of an $SU(3)_{L}$ singlet Higgs scalar $k^{++}$ in the current model. The model (c)
has two Zee-Babu type Higgs scalars called \( k^{++} \) and \( k'^{++} \), which are needed to realize \( L' \)-conserving and \( L' \)-violating two-loop interactions. However, in the current model, only one Higgs \( k^{++} \) is introduced and no additional singlet Higgs is needed because \( L' \)-conserving one-loop effects will serve as the \( L' \)-violating two-loop effects in the model (c). The second is the implementation of the discrete symmetry into the models. The discrete symmetry based on \( Z_2 \) is required in model (c) to avoid the FCNC interactions and to prevent the realization of the one-loop effects. Meanwhile, in the current model, the discrete symmetry based on \( Z_4 \) is introduced and the one-loop effects are allowed.

Now, let us demonstrate how radiative corrections induce neutrino masses in our model. The Yukawa interaction denoted by \( \mathcal{L}_Y \) and \( L' \)-conserving Higgs potential \( V_0 \) work together to generate one-loop interactions as shown in Fig. 1, also \( \mathcal{L}_Y \) and \( L' \)-violating Higgs potential \( V_6 \) yield two-loop interactions as shown in Fig. 2. From the one-loop diagrams, we obtained the following Majorana neutrino masses:

\[
m^{(1)}_{11} = f_{[1]} \left[ \lambda_1 \frac{m^2 F \left( m^2_{\nu}, m^2_{\nu}, m^2_{\nu} \right)}{v^2} - m^2 F \left( m^2_{\nu}, m^2_{\nu}, m^2_{\nu} \right) \right] + \lambda_2 \frac{m^2 E \left( m^2_{\nu}, m^2_{\nu}, m^2_{\nu} \right)}{v^2} v_\nu v_\nu v_\nu, \tag{6}
\]

where

\[
F(x, y, z) = \frac{1}{16\pi^2} \left[ \frac{x \ln x}{(x - y)(x - z)} + \frac{y \ln y}{(y - x)(y - z)} + \frac{z \ln z}{(z - y)(z - x)} \right], \tag{7}
\]

and, from the two-loop diagrams, we obtain:

\[
m^{(2)}_{11} = -2 \sum_{i,j=2,3} \lambda_2 f_{[1]} f_{[2]} f^{ij} \mu_i \mu_j m_{E} v_\nu v_\nu \chi I^{(2)} \tag{8}
\]

with

\[
f(2) = \frac{G(m^2_{\nu}, m^2_{\nu})}{m^2_{\nu}} \left( G(m^2_{\nu}, m^2_{\nu}) - G(m^2_{\nu}, m^2_{\nu}) \right),
\]

\[
G(m^2_{\nu}, m^2_{\nu}) = \frac{1}{16\pi^2} \frac{m^2 \ln(m^2_{\nu}/m^2_{\nu}) - m^2 \ln(m^2_{\nu}/m^2_{\nu})}{m^2 - m^2_{\nu}}, \tag{9}
\]

where the relation of \( m_k \gg \) (masses of other particle) has been used. The outline of the derivation of the two-loop integral, Eq. (6), is shown in the Appendix of Ref. [14].

The neutrino mass matrix is composed of these Majorana masses as

\[
M_\nu = \begin{pmatrix}
m^{(1)}_{12} & m^{(1)}_{11} & m^{(1)}_{13} \\
m^{(1)}_{12} & 0 & 0 \\
m^{(1)}_{13} & 0 & 0
\end{pmatrix}, \tag{10}
\]

from which we find the following relations for the neutrino oscillations in our \( SU(3)_L \times U(1)_N \) model

\[
\Delta m^2_{atm} = m^{(1)}_{12}^2 + m^{(1)}_{13}^2 (\equiv m^2_\nu), \quad \Delta m^2_{\beta} = 2 m_\nu |m^{(2)}_{11}|. \tag{11}
\]

The bimaximal structure of \( M_\nu \) is realized by requiring that \( |m^{(1)}_{12}| \sim |m^{(1)}_{13}| \), thereby, leading to \( m_{E_1} \sim m_{E_3} \) or \( m_{E_2,E_3} \ll m_{E_1} \) because the charged lepton contributions are to be neglected in our case. We assume that \( m_{E_2} \sim m_{E_3} \) in our analysis.

In order to see that our result, Eq. (11), really regenerates the observed neutrino oscillations, we make the following assumptions on relevant free parameters in the same way as those in Ref. [13]: (1) \( \eta = \frac{v_w}{20}, v_\nu = v_{\nu_e}, v_\tau = v_{\nu_\tau} \) and \( v_\chi = 10 v_{\nu_e} \), where \( v_w = (2\sqrt{2}G F)^{-1/2} \approx 174 \) GeV, (2) \( m_\tau \sim m_\nu = v_\nu, m_{\chi,k} = 10 v_{\nu_e}, m_{E_2,E_3} = e v_\chi \) to enhance the bimaximal mixing and \( m_{E_1} = 0.9 m_{E_2,E_3} \) to contribute to \( \Delta m^2_{atm} \), where \( e \) stands for the electromagnetic coupling, (3) \( f_{[1]} \sim 10^{-7}, \lambda_1 = \lambda_2 = f^{ij}_{[2]} = 1 \) and \( \mu_0 = e v_\chi \), where \( f_{[1]} \) is determined by \( \Delta m^2_{atm} = m^{(1)}_{12}^2 + m^{(1)}_{13}^2 = 3.0 \times 10^{-3} \) eV^2.
From numerical calculations of Eq. (11), we find $f_{\nu i} = 0.93 \times 10^{-7} \text{ eV}^2$, which reproduce $\Delta m^2_{atm} = 3.0 \times 10^{-3} \text{ eV}^2$ and $\Delta m^2_{\odot} = 0.91 \times 10^{-9} \text{ eV}^2$. As a result, the mixing angle $\theta$ for atmospheric neutrinos defined by $\cos \theta = m^2_{12}/m^2_{\nu}$ is computed to yield $\sin^2 2\theta = 0.93$, where the charged lepton contributions to $\Delta m^2_{atm}$ give the deviation form $\sin^2 2\theta = 1$ for the bimaximal mixing case. The estimated $\Delta m^2_{\odot}$ lies in the allowed region of the QVO solution to the solar neutrino problem.

Summarizing our discussion, we have constructed an $SU(3)_L \times U(1)_N$ gauge model with lepton triplets $\psi^i = (\nu^i, \ell^i, E^{-i})$, where $E^{-1}$ are negatively charged heavy leptons. This model has a triplet version of the Zee scalar $\xi$ and a singlet as the Zee-Babu scalar $k^{++}$. Owing to the existence of these scalars, our $SU(3)_L \times U(1)_N$ model is capable of generating tiny neutrino masses by the radiative mechanism. The atmospheric neutrino oscillation is related to $L'$-conserving one-loop interactions, while the solar neutrino oscillation is related to $L'$-violating two-loop interactions, where $L' \equiv L_e - L_{\mu} - L_{\tau}$. As a result, the bimaximal structure of the neutrino mass matrix is enhanced by the approximate degeneracy between masses of heavy leptons of $E^2$ and $E^3$. The observed mass hierarchy of $\Delta m^2_{atm} \gg \Delta m^2_{\odot}$ is explained by the difference between one-loop and two-loop effects. From our numerical estimate, our model reproduces the observed neutrino oscillation data $\Delta m^2_{atm} = 3.0 \times 10^{-3} \text{ eV}^2$ with $\sin^2 2\theta = 0.93$ and $\Delta m^2_{\odot} = 0.91 \times 10^{-9} \text{ eV}^2$. Our model is, thus, relevant to yield quasi-vacuum solution for solar neutrino problem.

The author would like to thank Prof. M. Yasuè for many helpful suggestions, useful comments and a careful reading of this article.

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Table Captions

TABLE I. $L$ and $L'$ quantum number.

| Fields   | $\eta, \rho, \chi, \xi$ | $\psi^L_1, \psi_1^R, E_1^R$ | $\psi^{L', E}_2 \psi^{L', E}_3, E_2^{L', E}_3$ | $k^{L', E}_1$ |
|----------|------------------------|-----------------------------|-----------------------------------------------|----------------|
| $L$      | 0                      | 1                           | 1                                             | -2             |
| $L'$     | 0                      | 1                           | -1                                           | -2             |

Figure Captions

FIG. 1. $L'$-conserving one-loop diagrams.

FIG. 2. $L'$-violating two-loop diagrams.
FIG. 1 L'-conserving two-loop diagrams

FIG. 2 L'-violating two-loop diagrams