Addendum

Optically trapped atom interferometry using the clock transition of large $^87$Rb Bose–Einstein condensates

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Abstract. In our original paper (Altin et al 2011 New J. Phys. 13 065020), we presented the results from a Ramsey atom interferometer operating with an optically trapped sample of up to $10^6$ Bose-condensed $^87$Rb atoms in the $m_F = 0$ clock states. We were unable to observe projection noise fluctuations on the interferometer output, which we attribute to the stability of our microwave oscillator and background magnetic field. Numerical simulations of the Gross–Pitaevskii equations for our system show that dephasing due to spatial dynamics driven by interparticle interactions accounts for much of the observed decay in fringe visibility at long interrogation times. The simulations show good agreement with the experimental data when additional technical decoherence is accounted for, and suggest that the clock states are indeed immiscible. With smaller samples of $5 \times 10^4$ atoms, we observe a coherence time of $\tau = 1.0^{+0.5}_{-0.3}$ s.

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1. Interferometer stability

As noted in our original paper (section 3.2.2 of [1]), fluctuations in the frequency difference $\Delta$ between the microwave reference oscillator and the atomic resonance during the beamsplitter pulses affect the transition probability $p$ after a full interferometer sequence to first order. In addition, fluctuations in $\Delta$ during the interrogation time $T$ inevitably translate into an uncertainty in $p$, as this is precisely what the interferometer is designed to measure. The latter dominates when the beamsplitter pulses are short compared with the interrogation time, as in our system. Assuming ideal resonant $\pi/2$ pulses, the transition probability after a complete Ramsey sequence is

$$ p = \frac{1}{2}[1 + \cos(T\Delta)], $$

where $\Delta$ represents the detuning during the interrogation time. The uncertainty in $p$ caused by detuning fluctuations $\delta\Delta$ is therefore $\delta p = |\partial p/\partial \Delta| \delta \Delta = T/2 |\sin(T\Delta)| \delta \Delta$. At the midpoint of a Ramsey fringe, $T\Delta = (2n + 1)\pi/2$ for integer $n$, so $|\sin(T\Delta)| = 1$ and $\delta p = T \delta \Delta/2$. (In fact, this is an underestimate, as it neglects the effect of fluctuations during the pulses.) Comparing this with the projection noise limit $\sigma_p = 1/\sqrt{4N}$, we can place an upper bound on $\delta\Delta$:

$$ \delta\Delta \leq \frac{1}{T \sqrt{N}}. \quad (2) $$

Fluctuations in detuning larger than this will preclude projection-noise-limited performance in a Ramsey interferometer with $N$ atoms and an interrogation time $T$. With $T = 5$ ms and $N = 10^6$, the allowable fluctuation is $\delta\Delta \leq 0.03$ Hz. This requires a relative frequency stability of $4 \times 10^{-12}$ in the microwave source, and magnetic field variations below 7 $\mu$G at a bias field of 4 G.

Operating our interferometer at mid-fringe with $N = 5 \times 10^5$ atoms and an interrogation time of $T = 5$ ms, we measured run-to-run fluctuations in the transition probability of $\sigma_p = 0.011$ after a complete $\pi/2-\pi/2$ Ramsey sequence. This is a factor of 16 higher than the projection noise limit for this atom number. In light of the above discussion, we conclude that the performance is limited by the stability of the microwave source (specified at $2 \times 10^{-11}$ over 1 s) and the magnetic bias field. Remarkably, the measured fluctuations in $p$ are below what would be expected given our measured background field variations of 4 mG, which should cause $\delta\Delta = 20$ Hz and thus a substantially larger $\delta p = 0.3$. This can perhaps be attributed
to averaging of fast fluctuations over the interrogation time. Nonetheless, it is evident that achieving projection-noise-limited performance with large atom numbers will necessitate a high degree of stability in the magnetic environment, with low-noise current supplies and effective magnetic shielding. It will also demand a highly stable reference oscillator to effect coupling between the states.

2. Fringe visibility

In the original paper, we speculated that the observed decay of fringe visibility with interrogation time was due to a combination of inelastic loss, interaction-induced dephasing and technical noise. Here we show that while spatial dynamics driven by the interparticle interactions can account for much of the visibility reduction, it is unlikely that inelastic losses have a significant effect. We also quantify the decoherence due to technical noise sources, both with and without spin-echo pulses.

2.1. Differential loss

In a simple two-mode model, the only agent that can cause a reduction in fringe visibility (aside from imperfect coupling pulses) is differential loss from the interferometer states. If one of the internal states experiences greater loss than the other, the superposition prepared by the first $\pi/2$ pulse—which initially has equal amplitudes in both states—will evolve to an unequal superposition by the end of the interrogation time. The interference at the final beamsplitter will then be incomplete, yielding fringes with decreased amplitude.

Suppose that the losses are such that, at the end of the interrogation time, a fraction $k_i$ of the atoms in state $|i\rangle$ remain in the system. The character and time-dependence of these losses are immaterial; they may arise from two- or three-body processes, background gas collisions or scattering by photons from the dipole trapping laser. The state vector before the final beamsplitter is then

$$\Psi_{\pi/2-T} = \frac{1}{\sqrt{2}} \left( \sqrt{k_1} \right. - i \sqrt{k_2} e^{i\phi} \left. \right),$$

and the fraction of atoms found in state $|2\rangle$ at the output of the interferometer, normalized to the total number $p_1 + p_2 = \frac{1}{2}(k_1 + k_2)$, is

$$p = \frac{1}{2} \left[ 1 + \frac{2\sqrt{k_1k_2}}{k_1 + k_2} \cos \phi \right],$$

giving a fringe visibility of

$$V = \frac{2\sqrt{k_1k_2}}{k_1 + k_2}.$$
Thus, having twice as many atoms remaining in one state as that in the other state reduces the fringe visibility by only 6%—even with 90% loss from one state (and none from the other) the fringe visibility would still be 57\%.

In our system, the interferometer states \(|1\rangle\) and \(|2\rangle\) do suffer from differential loss, primarily due to inelastic spin-exchange collisions which populate the \(|F = 1, m_F = \pm 1\rangle\), \(|F = 2, m_F = \pm 1\rangle\) states as demonstrated in the original paper (figure 3 of [1]). However, the above analysis demonstrates that this should have a minimal effect on the fringe visibility—even after 1 s of evolution, when \(k_1 \simeq 0.45\) and \(k_2 \simeq 0.18\), the visibility should remain above 90\%. Over a typical interrogation time of tens of milliseconds, the contribution of the measured differential loss to decreased fringe visibility should be negligible (<0.1\%); to explain the visibility \(V \simeq 0.1\) observed at \(T = 30\) ms would require a loss rate 400 times larger in one state than the other\(^7\). We conclude that differential loss does not contribute significantly to fringe visibility reduction in our system.

2.2. Spatial dynamics

The external state of the atoms participating in the interferometer—and in particular its associated phase—can also affect the output. The external state of a two-component Bose–Einstein condensate may be written in spinor notation as

\[
\Psi(r, t) = \begin{pmatrix} \psi_1(r, t) \\ \psi_2(r, t) \end{pmatrix},
\]

where \(\psi_1(r, t)\) represent the spatial wavefunctions of the two interferometer states, which obey a pair of coupled Gross–Pitaevskii equations (GPEs) and are normalized such that \(|\psi_1|^2 + |\psi_2|^2 = N\). These can be expressed in terms of the particle density \(n_i(r, t) = |\psi_i|^2\) and a phase \(\phi_i(r, t)\) as \(\psi_i(r, t) = \sqrt{n_i(r, t)} e^{i\phi_i(r, t)}\). The phase difference \(\phi_2(r, t) - \phi_1(r, t)\) between the spatial wavefunctions, which can vary in space as well as with time, cannot be distinguished from that accrued during the interrogation time due to the energy difference of the internal states. As noted in the original paper (section 4.2 of [1]), inhomogeneous broadening mechanisms such as the position-dependent ac-Stark shift due to the dipole trapping potential or local magnetic field inhomogeneities cause the phase evolution during the interrogation time to proceed at different rates across the trapped cloud, reducing the fringe visibility. Those that are constant in time can be reversed using a spin–echo pulse.

Even without an externally imposed phase gradient, however, inhomogeneous phase evolution (with accompanying spatial dynamics) can be elicited by interparticle interactions. To investigate the impact of spatial dynamics on fringe visibility, we simulate the evolution of the two-component system in the mean-field approximation. For this purpose, we assume ideal \(\pi/2\) pulses that perform the mappings \(\psi_1 \rightarrow (\psi_1 - i\psi_2)/\sqrt{2}, \psi_2 \rightarrow (\psi_2 - i\psi_1)/\sqrt{2}\). The initial

\(^6\) This weak scaling of the fringe visibility is entirely analogous to the situation in optical heterodyning: here, the weaker ‘signal’ state is amplified by mixing with a stronger ‘local oscillator’. Of course, while losses affect the fringe visibility only inasmuch as they differ between the two states, an overall decrease in atom number raises the quantum projection noise limit and can therefore also decrease the interferometer’s sensitivity.

\(^7\) It should be noted that this analysis neglects spatial dynamics stimulated by the loss of atoms from each state; in particular, in the case of density-dependent two- and three-body loss, the removal of atoms changes the density distribution and can excite oscillations. However, this should represent only a small perturbation in the limit that the loss rate is small compared with the trap frequencies, as is the case here (\(\tau_{\text{loss}}^{-1} \ll \bar{\omega}/10\)).
Figure 1. The impact of spatial dynamics on fringe visibility in a Ramsey interferometer with $N = 10^6$ atoms. From top to bottom: two-dimensional density distributions, radial density profiles, relative phase profiles and Ramsey fringes obtained from a simulation of two-component condensate evolution using the GPE. The spatial structure oscillates around the two-component ground state—a ball-and-shell arrangement due to the large intraspecies repulsion. When the relative velocity of the two components is highest, the relative phase exhibits a large variation across the cloud, reducing the fringe visibility.

state is taken to be a pure $|1\rangle$ condensate occupying the ground state of the external potential; to make the computation tractable we assume cylindrical symmetry and trap frequencies $\omega_{z,\rho} = 2\pi \times \{30, 55\}$ Hz, such that $\bar{\omega}$ matches that of our crossed dipole trap. After an initial $\pi/2$ pulse, the evolution of the two components is determined by numerically integrating the following coupled GPEs:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V + g_{11} |\psi_1|^2 + g_{12} |\psi_2|^2 \right] \psi_1,$$  

$$ (7)$$

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Figure 2. Visibility of Ramsey fringes from the in-trap interferometer with (filled circles) and without (open circles) a ‘spin-echo’ $\pi$ pulse. Note that the data for the spin-echo interferometers represent the inferred visibility, which gives a lower bound on the actual fringe visibility. The lines show the fringe visibility determined from GP simulations of the spatial dynamics as described in the text; solid—no spin-echo pulse; dotted—with spin-echo pulse. The results of the spin-echo simulation agree with the experimental data when an additional decoherence $e^{-T/\tau}$ is added (dashed line).

\[ i\hbar \frac{\partial \psi_2}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V + g_{22} |\psi_2|^2 + g_{12} |\psi_1|^2 \right] \psi_2, \]  

where the nonlinear terms proportional to $|\psi_i|^2$ with $g_{ij} = 4\pi \hbar^2 a_{ij}/m$ describe the effect of interparticle interactions, characterized by the scattering lengths $a_{11} = 100.9 a_0$, $a_{12} = 98.9 a_0$ and $a_{22} = 94.9 a_0$. After evolution of the states for a time $T$, another $\pi/2$ pulse mapping is performed with phase $\phi_{\pi/2}$ relative to the first, and the number of atoms in each state determined from $\psi_1$ and $\psi_2$. By scanning the phase $\phi_{\pi/2}$ of the final beamsplitter, we can simulate Ramsey fringes and determine their visibility.

Figure 1 shows the results of this simulation for an interferometer with $N = 10^6$ atoms and interrogation times up to $T = 40$ ms. For the scattering lengths given above, the miscibility parameter $\mu = a_{11}(a)_{22}/a_{12}^2 \simeq 0.98 < 1$ predicts that the states will be immiscible, since the cross-species repulsion exceeds the self-repulsion. Indeed, it can be seen from the simulated density profiles that the two states spatially separate, developing a ball-and-shell structure over several tens of milliseconds. As they are initially overlapped and, thus, far from the two-component ground state, oscillations ensue. At the times when the relative velocity of the two components is greatest, a large gradient manifests in the relative phase and the visibility of the Ramsey fringes is reduced.

The simulated fringe visibility is plotted as a function of interrogation time in figure 2 (solid line), together with the experimental data from our original paper (figure 2(b) of [1]). A faster
Figure 3. Spatial dynamics during a Ramsey interferometer with a spin-echo $\pi$ pulse. The pulse is applied after 10 ms of interrogation time, exchanging the density distributions of the two states (upper plots) and inverting their relative phase profile (lower plots). During the second half of the interrogation time, there is a partial rephasing of the wavefunctions. The dashed lines in the lower plots show the relative phase evolution in the absence of a spin-echo pulse, which by the end of the interrogation time spans most of $2\pi$ across the cloud.

reduction in visibility (well described by an exponential decay with time constant $\tau = 14(1)$ ms) is observed in the experiment, which we ascribe to inhomogeneous broadening effects such as the ac-Stark shift of the trapping laser and technical noise. The simulated fringe visibility also exhibits a different shape compared to the exponential decay seen in the experimental data, and we do not observe the predicted revival in fringe visibility after $T \simeq 20$ ms. Qualitative disagreement between the experiment and simulation could arise from the condensate not being in its ground state before the first pulse, which would inevitably affect the subsequent dynamics. Perhaps more importantly, the simulation is cylindrical symmetric; the asymmetry of our crossed dipole trap may tend to wash out the periodic rephasing of the spatial dynamics.$^8$

We can also use the simulation to examine the effect of a spin-echo pulse on the spatial dynamics. To do this, we assume an ideal $\pi$ pulse that performs the transformation $\psi_1 \rightarrow -i\psi_2$, $\psi_2 \rightarrow -i\psi_1$ at time $t = T/2$, and again simulate fringes by scanning the phase of the final beamsplitter. The results for a $T = 20$ ms interferometer sequence with a spin-echo pulse are shown in figure 3; in this case the predicted fringe visibility is increased from 20 to 75% (cf figure 1). The pulse clearly does not reverse the nonlinear spatial dynamics, but partially mitigates the phase gradient that develops during the first half of the interrogation time, resulting in higher visibility fringes. The cancellation is not perfect: the altered density distribution causes the phase evolution due to mean-field interaction to be different during the second half of the interrogation time, preventing complete rephasing. Nonetheless, the pulse does

$^8$ Although the fringes are simulated by scanning the phase of the final beamsplitter rather than the detuning, we do not expect this (or the assumption of ideal and instantaneous $\pi/2$ pulses) to significantly affect the outcome since the interrogation times are much longer than the duration of the beamsplitter pulses. This is verified by comparing the simulated fringe visibility with that obtained by solving the full GP equations including coupling terms and varying $\Delta$—the results agree within 2%.
Figure 4. (a) Ramsey fringes from an interferometer with \( N = 5 \times 10^4 \) and varying interrogation time. Data points represent the measured fractional population in state \( |2\rangle \), \( p = N_2 / N \), and the solid lines are sinusoidal fits. At long interrogation times, run-to-run resonance fluctuations randomize the phase, thus increasing the noise on the fringes. Where sinusoidal fitting fails, we take the fringe visibility to be the range of the scattered data (dashed lines). (b) Fringe visibility as a function of interrogation time. The dotted line represents the visibility obtained from a GP simulation of the spatial dynamics. The solid line is a fit of the form \( e^{-T/\tau} \), from which we extract a coherence time of \( \tau = (300 \pm 50) \text{ ms} \).

significantly increase the coherence time of the interferometer. The fringe visibility obtained from the simulation is overlaid with the experimental spin-echo data in figure 2 (dotted line). The simulated spin-echo visibility qualitatively resembles the experimental data much more strongly than without the \( \pi \) pulse, suggesting that reversible inhomogeneous broadening does indeed contribute significantly to the decay in visibility in that case. By assuming an additional decoherence not remedied by the spin-echo pulse, \( V(t) = V(e^{-T/\tau}) \), with time constant \( \tau = (120 \pm 20) \text{ ms} \), we find excellent quantitative agreement with our experimental data (dashed line).

2.3. Small atom numbers

One would expect irreversible inhomogeneous broadening (such as due to technical noise) to cause a decay in fringe visibility independent of atom number\(^9\), while interaction-induced dephasing should affect large condensates more due to their higher density and correspondingly higher interaction energy. This hypothesis is substantiated by GP simulations which show that, although smaller two-component condensates still separate spatially, the timescale of

\(^9\) Aside from a small effect due to the increased spatial extent of larger condensates; in the Thomas–Fermi limit, the condensate radius scales weakly with atom number as \( r \propto N^{2/5} \).
the dynamics is much longer, resulting in lower relative velocities and thus reduced phase gradients. As a final test of the impact of interaction-induced spatial dynamics, therefore, we run the interferometer with a much smaller sample of $N = 5 \times 10^4$ atoms. The resulting fringes are presented in figure 4(a), displaying high visibility $V \simeq 90\%$ for interrogation times up to $T = 52.5$ ms. We are unable to observe fringes at larger $T$ due to crippling phase noise, which scatters the data randomly as seen at $T = 102.5$ ms. Nonetheless, this is not indicative of decoherence, as the output does not converge to $p = 0.5$ as with the larger condensates. For the purposes of understanding decoherence, we can take the range of the scattered data as an estimate of the fringe visibility. The visibility is plotted as a function of interrogation time in figure 4(b). Again, the visibility decays faster than predicted by the simulation. An exponential fit of the form $e^{-T/\tau}$ yields a coherence time of $\tau = 0.30(5)$ s, a factor of 20 longer than for $N = 10^6$ atoms.

Adding a spin-echo pulse to this interferometer allows us to estimate the true coherence limit of our system in a regime where interaction-induced dephasing is minimal. Figure 5 plots the fraction of atoms measured in state $|1\rangle$ at the output of the interferometer after a $\pi/2-\pi-\pi/2$ sequence as a function of interrogation time. Unexpectedly, this fraction drops below $p = 0.5$ after 100 ms—most of the atoms are observed in state $|2\rangle$ at $T = 200$ ms. This cannot be explained by a phase shift that is constant in time, since this would be cancelled by the spin-echo pulse. A likely candidate for this effect is a deterministic drift in the magnetic bias field. The difference in phase accumulated before and after the spin-echo pulse is $\delta \phi = (\Delta_2 - \Delta_1)T/2$, where $\Delta_1$ and $\Delta_2$ represent the average detuning during each half of the interrogation time. If $\Delta$ drifts at a constant rate $\nu$, then $\delta \phi = \nu T^2/4$, resulting in fringes whose spacing decreases with $T$. The solid line in figure 5 is a fit of this form, including an exponential decay to account

**Figure 5.** Fraction of atoms measured in state $|1\rangle$ after a $\pi/2-\pi-\pi/2$ pulse sequence with $N = 5 \times 10^4$ atoms. The population oscillates due to a systematic phase shift that is not cancelled by the spin-echo pulse, most likely due to a drift in the bias magnetic field changing the detuning during the interrogation time. An exponential fit to the envelope (dashed line) yields a coherence time of $\tau = 1.0^{+0.3}_{-0.3}$ s.
Figure 6. The effect of interspecies interactions on fringe visibility in an in-trap Ramsey interferometer with \( N = 10^6 \) atoms, calculated from numerical GP simulations. The solid lines correspond to scattering lengths \( a_{12} \) where the two states are miscible; the dashed line shows the visibility for the background scattering length \( a_{12} = 98.9 a_0 \) at which the states are immiscible. A reduction in \( a_{12} \) of only 10\% is sufficient to maintain fringe contrast above 90\% over 100 ms.

for decoherence. From the fit, we calculate a drift rate of approximately \( \nu = 2\pi \times 50 \text{ Hz} \cdot \text{s}^{-1} \), which at a bias field of 4 G requires a drift in magnetic field of roughly 10 mG \( \text{s}^{-1} \). This is consistent with the field drift measured in our system, which we believe to be caused by warming of our bias field coils. Assuming that the measured points represent lower bounds on the fringe visibility at a given interrogation time, the reduction in visibility can be described by an exponential decay \( e^{-T/\tau} \) with a coherence time of \( \tau = 1.0^{+0.5}_{-0.3} \text{ s} \).

2.4. Miscibility

In our original paper, we reported that no spatial structure was observed in the two clouds at the output of the interferometer, and contemplated the possibility that the clock states were in fact miscible, in contrast to the prediction of the calculated scattering lengths. In light of the agreement between the experimental data and the simulations presented above, we believe it is likely that the states are indeed immiscible as predicted by coupled-channel calculations. The expansion of the condensate during time-of-flight, the Stern–Gerlach separation pulse and the repumping pulse applied before the measurement, along with the line-of-sight integration inherent in the absorption imaging itself, could all contribute to smoothing spatial inhomogeneities, concealing the structure developed during the interrogation time. As noted, states which are miscible are more favourable for interferometry, as only minimal evolution of the spatial distributions occurs due to low-level breathing-mode oscillations excited by the first \( \pi/2 \) pulse. Figure 6 shows the simulated fringe visibility for an \( N = 10^6 \) atom interferometer with various values of the interspecies scattering length \( a_{12} \). With \( a_{12} \) adjusted by only 10\%, the fringe visibility does not drop below 90\% over 100 ms of interrogation time. This suggests that
an interspecies Feshbach resonance could be exploited to obviate interaction-induced dephasing and prevent the consequent decrease in interferometric sensitivity.

References

[1] Altin P A, McDonald G, Döring D, Debs J E, Barter T H, Close J D, Robins N P, Haine S A, Hanna T M and Anderson R P 2011 Optically trapped atom interferometry using the clock transition of large $^{87}\text{Rb}$ Bose–Einstein condensates *New J. Phys.* **13** 065020