Microscopic study of strange hadron productions at $\sqrt{s_{NN}}=2.76$ TeV, LHC energy.

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Abstract

Ratio of the yield of strange hadrons to pions is considered now as an important observable in studying the properties of the system produced in relativistic heavy ion collisions. Production of strange hadrons $K, \bar{K}, \Lambda, \Sigma, \Xi$ and $\Omega$ have been evaluated microscopically using rate equations by considering their hadronic interaction cross sections in an expanding medium. The yields obtained from rate equations are normalised with thermal pions and compared with the measurements from Pb-Pb collisions at various multiplicities at LHC energy. The calculation has been done for various initial and freeze out conditions. It is observed that each species prefers a freeze out temperature-leading to a multiple freeze out scenario of strange hadrons.

Keywords: Strangeness enhancement, Kaon, Lambda, Cascade and Omega hyperon productions, 2.76 TeV, QGP, rate equation, SPS, AGS, RHIC, LHC.

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1. Introduction

The study of strange hadrons plays a key role in extracting the properties of the medium produced in relativistic heavy ion collisions. The experiments performed at several colliding energies in several accelerator facilities like Alternating Gradient Synchrotron(AGS), Relativistic Heavy Ion Collider (RHIC), Super Proton Synchrotron(SPS) and Large Hadron Collider(LHC) provide ample of strange hadron data which help understand the QCD phase diagram. Recently, ALICE collaboration has measured the yield of strange hadrons $K, \Lambda, \Xi$ and $\Omega$ in p-p collisions at $\sqrt{s_{NN}}=7$ TeV, p-Pb collisions at $\sqrt{s_{NN}}=5.02$ TeV [1, 2] and Pb-Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV [3] in various centralities. The normalised yields of strange hadrons, $(H_s + \bar{H}_s)/(\pi^+ + \pi^-)$ are measured at various charge particle multiplicities and presented as an observable by ALICE collaboration [1]. The measured data in [1] are $(2K_s^0)/(\pi^+ + \pi^-)$, $(\Lambda + \bar{\Lambda})/(\pi^+ + \pi^-)$, $(\Xi^- + \Xi^+)/($but\), and $(\Omega^- + \Omega^+)/($but\). We call the ratio $(H_s + \bar{H}_s)/(\pi^+ + \pi^-)$ as the ‘yield-ratio’ throughout the article for the sake of convenience. The yield-ratios show a smooth increasing pattern with multiplicity and then a saturation for all strange hadrons but with a little deviation for $\Xi/\pi$ at lowest measured multiplicity and also at highest multiplicity.

These measurements are extremely important, as a smooth pattern of yield-ratio with charged multiplicity, from various colliding systems (p-p, p-Pb, Pb-Pb) at different colliding energies, would answer the question of similarity of systems with similar multiplicities produced in these collisions and a deviation may hint for new physics. The yield-ratio for $\Lambda, K, \Omega$ show a smooth increasing pattern, but as mentioned above, $\Xi$ shows a deviation. It is also observed that slopes of yield-ratio for multi strange hadrons are more compared to single strange hadrons. This may signify the enhancement of multi-strange productions compared to single-strange ones. To analyse the phenomenon, understanding of the microscopic mechanism for the production of all strange hadrons is necessary, which is the focus of this calculation.

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The study of strange hadrons was believed to be important because of their enhanced productions in heavy ion (A-A) collisions over proton-proton (p-p) collisions and was proposed long before as a good signature of quark gluon plasma (QGP) formation. Widely discussed horn like structure in the measurements of $K^+/\pi^+$ ratio with colliding energies ignited many theoretical models in the last two decades. The multi strange baryons $\Xi(uss, dss)$ and $\Omega(sss)$ also show enhancement like $K^+$ in Pb-Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV over p-p collisions. The observations of $\Xi$ & $\Omega$ yield at $\sqrt{s_{NN}}=200$ GeV, Au+Au collisions also supported the argument of strangeness enhancement in A-A collisions. Similar observations have also been made at SPS energy by WA97 collaborations while measuring $\Xi$ and $\Omega$ from Pb-Pb and p-Pb collisions at CERN. Enhancement in case of $\Xi$, $\Omega$ is a factor of 3 in Pb-Pb over p-p collisions at 158 A GeV. $\Xi/\pi$ and $\Omega/\pi$ in Pb-Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV are 1.6 and 3.3 times more compared to p-p collisions at $\sqrt{s_{NN}}=7$ TeV at LHC. In the mean time, the availability of most recent data from p-Pb collision at $\sqrt{s_{NN}}=5.02$ TeV makes it more interesting as it would help in providing a systematic study from p-p to p-A to A-A collisions.

Strangeness productions in QGP and hadronic phases have been studied by various models by several authors. The enhanced production of kaons and anti-kaons in the experiments at various colliding energy ranges such as SIS (upto 1 GeV), AGS (upto 10 A GeV) and SPS energies (11-158 A GeV) are explained by hadronic scatterings in some of the above works and also using AMPT. However, the multi strange baryon productions have not been explained satisfactorily.

Statistical Hadronisation Model also evaluated the integrated yield at those energies assuming common chemical freezeout temperature for all species including RHIC and LHC energies. However, the model could not explain the ratios of multistrange hadrons for 0-20% centrality of Pb-Pb collision at 2.76 TeV LHC energy while fitting with $p/\pi$ ratio. Similarly productions of kaons and anti-kaons at higher colliding energies such as at RHIC and LHC (also at higher SPS energies) have been explained using models with strange quark evolution assuming a QGP phase. But the multi strange productions are not explained there.

Using minimal statistical hadronization model, the authors tried to explain the momentum spectra of hyperons $\Lambda, \Xi$ measured by HADES collaboration from the collision of Ar at 1.76A GeV on fixed target KCl without considering the microscopic productions. Same authors explained the kaon productions but failed to reproduce the ratio $\Xi^-/\Lambda$ and $\Omega^-/\Xi^-$ in (NICA white paper) however got a similar trend. An attempt has also been made in to explain the multi strange productions at SPS energy using Ultra Relativistic Quantum Molecular Dynamics (UrQMD), but data were not reproduced. It was commented that the enhancement is probably due to the formation of Disoriented Chiral Condensates (DCC) from the high density matter following some topological defects in the initial stages of the collisions.

In this paper, the microscopic productions of $K, \bar{K}, \Lambda, \Sigma, \Xi$ and $\Omega$ have been discussed with their interactions in the hot-dense system along with their evolutions considering Bjorken expansion and using rate equation. We focus our calculation to analyse the yield-ratio data for all strange hadrons at different multiplicities from Pb-Pb collision at $\sqrt{s_{NN}}=2.76$ TeV.

We divide the manuscript as follows. The cross-sections of productions for Kaon, Lambda, Sigma, Cascade and Omega in a hadronic medium are discussed in section 2. The formalism for rate of production is described in Section 3. The rate equations for single- and multi-strange hadrons are discussed considering a Bjorken expansion in section 4. The evolution equations for temperature and baryon chemical potential ($\mu$) are also described here. Then the results are presented in the next section and finally, section 6 is devoted to summary and conclusion.

2. Production and interaction of strange hadrons in hadronic medium

In case of relativistic heavy ion collisions, the observed hadrons might be produced due to hadronisation of quarks when initial quark gluon state is produced or because of the nucleonic interactions of the colliding nuclei. The yield in the later case would be low. In both cases produced hadrons undergo further scatterings inside the medium till they decouple and free stream towards the detector. The dynamics of hadrons determines the properties of the system and hence the final yield. In this study, the aim is to understand the dynamics of strange degrees of freedom. While considering the production and interaction of strange
hadrons we assume the non-strange hadrons to provide thermal background. The time evolution of the hadronic system is studied with rate equation or momentum integrated Boltzmann equation along with Bjorken expansion of the system.

Various interactions involving strange and non-strange hadrons that produces $K, \Lambda, \Sigma, \Xi$ and $\Omega$ are discussed below.

### 2.1. Interaction channels and strange hadron cross sections

The production of strange mesons $K, \bar{K}$ and baryons $\Lambda, \Sigma, \Xi, \Omega$ are studied with the following hadronic interactions. They can be categorised as meson-meson(MM), meson-baryon(MB) and baryon-baryon(BB) interactions based on the hadrons in the initial channel. The reactions are, $\pi\pi \rightarrow KK, \pi N \rightarrow \Sigma K, \bar{p}p \rightarrow \Lambda \Lambda, \pi\rho \rightarrow \Sigma \pi, \rho\rho \rightarrow \bar{K} \bar{K}, \bar{K}N \rightarrow \Sigma \pi, \pi N \rightarrow \Lambda K, \rho N \rightarrow \Lambda K, \bar{p}p \rightarrow K\bar{K}^*, KN \rightarrow K\Sigma, \bar{K}\Lambda \rightarrow \pi \Xi, \bar{K} \Sigma \rightarrow \pi \Xi, \Lambda \Lambda \rightarrow N \Xi, \Lambda \Sigma \rightarrow N \Xi, \Sigma \Sigma \rightarrow N \Xi, \bar{K} \Sigma \rightarrow \pi \Xi, \Lambda K \rightarrow \Omega^- K^0, \Sigma^0 K \rightarrow \Omega^- K^0, \bar{p}p \rightarrow \Omega^- \Omega, \bar{p}p \rightarrow \Xi \Xi$. Where, $B, N$ and $Y$ represents baryons, nucleons(proton or neutron) and hyperons respectively. The production cross section for hadrons with single strangeness is described in [22, 33, 38]. Many of them are verified with experimental observations. The cross sections for inverse reactions are also taken into account using principle of detailed balance as in [39]. There are some 2 → 3 channels involving baryons in the initial channels such as $BB \rightarrow BY K$ which might be relevant for strange production at low colliding energies that is when baryon density in the system is high. However, we neglect contributions from such processes due to phase space factor.

### 2.2. Production cross sections for single-strange hadrons

Among the strange hadrons carrying single strange quantum number, Kaons ($K, \bar{K}$) are the lightest one. For Kaon production, the isospin averaged cross section ($ab \rightarrow cd$) from MM interactions ($\pi\pi \rightarrow KK$), $\rho\rho \rightarrow KK, \pi\rho \rightarrow KK^*$ and $\pi\rho \rightarrow K^* K$) is given by,

$$\sigma_{ab\rightarrow cd}(s) = \frac{1}{32\pi} \frac{P_{ab} P_{cd}}{s_{ab}} \int_{-1}^{1} dx M(s, x)$$

where, $s = (p_a + p_b)^2$ with $p_a, p_b$ being the four momenta of incoming particles $a$ and $b$; $P_{ab}$ and $P_{cd}$ are three momenta of incoming mesons and outgoing kaons in the centre-of-mass frame, $x = \cos(P_{ab}, P_{cd})$. $M$ is invariant amplitude and calculated from following interaction Langrangian densities $L_{K-K\pi} = g_{K-K\pi} K^\mu\pi^\nu K - (\partial_{\mu} K) \pi$ and $L_{\rho K-K} = g_{\rho K-K} [K^\tau(\partial_{\mu} K) - (\partial^\rho K) \tau K]^\rho$. Similar to MM interactions, MB interactions ($MB \rightarrow Y K$) also produce Kaons but strange baryons such as $\Lambda$ and $\Sigma$ are also produced along with. The dominant contributions come from $\pi N \rightarrow \Lambda K, \pi N \rightarrow \Sigma K, \rho N \rightarrow \Lambda K, \rho N \rightarrow \Sigma K$. The production cross sections are evaluated and parametrised in [33, 37]. We have calculated the cross section using the following expression from [34] considering $N_1(1650), N_2(1710), N_3(1720)$ intermediate resonant states.

$$\sigma_{MB\rightarrow Y K} = \sum_i \frac{(2J_i + 1)}{(2S_1 + 1)(2S_2 + 1)} \frac{4\pi}{k_i^2} \frac{\Gamma_i^2/4}{(s^2 - m_i)^2 + \Gamma_i^2/4} B_{i}^\text{in} B_{i}^\text{out}$$

The sum is over resonances with mass($m_i$), spin($J_i$) and decay width($\Gamma_i$). $(2S_1 + 1)$ and $(2S_2 + 1)$ are the polarisation states of the meson ($M$) and baryon($B$) in the incoming channels. $B_i$ represents the branching ratio. Other required parametres are taken from the particle data book [34].

Other channels of $\Lambda$ and $\Sigma$ productions in MB category are $KN \rightarrow \Lambda \pi$ and $KN \rightarrow \Sigma \pi$, the cross sections of which have been calculated by Ko [40] using K-matrix formalism for three coupled channels $KN, \Lambda \pi$ and $\Sigma \pi$. However, we use the experimental parametrised cross section considered in [33] which is in agreement with [40] and is as follows;

$$\sigma_{K^- p \rightarrow \Lambda \pi^0} = \begin{cases} 1.205 \ p^{-1.428} \text{ mb} & \text{if } p \geq 0.6 \text{ GeV} \\ 3.5 \ p^{0.659} \text{ mb} & \text{if } 0.6 < p \leq 1.0 \text{ GeV} \\ 3.5 \ p^{-3.97} \text{ mb} & \text{if } p > 1.0 \text{ GeV} \end{cases}$$
where $p$ in Eq.3 is the anti-Kaon momentum in the laboratory frame. We consider the isospin averaged cross section $\bar{K}N \rightarrow \Lambda \pi$. Similarly, the parametrised cross section for $\bar{K}N \rightarrow \Sigma \pi$ is as follows;

$$
\sigma_{\bar{K}N \rightarrow \Sigma \pi} = \sigma_{\bar{K}p \rightarrow \Sigma^0 \pi^0} + \sigma_{\bar{K}n \rightarrow \Sigma^0 \pi^-}
$$

where $\sigma_{\bar{K}p \rightarrow \Sigma^0 \pi^0} \approx \sigma_{\bar{K}n \rightarrow \Sigma^0 \pi^-}$ and

$$
\sigma_{\bar{K}p \rightarrow \Sigma^0 \pi^0} = \begin{cases} 
0.624 \, p^{-1.83} \text{mb} & \text{if } p \leq 0.345 \text{ GeV} \\
0.0138/[(p - 0.385)^2 + 0.0017] \text{mb} & \text{if } 0.345 < p \leq 0.425 \text{ GeV} \\
0.7 \, p^{-2.09} \text{mb} & \text{if } p > 0.425 \text{ GeV}
\end{cases}
$$

The contributions from BB category producing single strange hadrons are $pp \rightarrow K\bar{K}, pp \rightarrow \Lambda \bar{\Lambda}, pp \rightarrow \Sigma \bar{\Sigma}$. The cross-sections for $pp \rightarrow YY(\bar{MM})$ (Y is the hyperon, M is the meson, here kaon) is given below [36, 41, 42].

$$
\sigma_{pp \rightarrow YY(\bar{MM})} = \frac{C_AC_{YY(\bar{M}M)}t_0^4}{16\pi} \times \left( \frac{s - 4m_p^2}{s} \right) \times \Gamma (1 - \alpha(0))^2 \times \left( \frac{s}{s_0} \right)^{2(\alpha_0-1)} \times \frac{e^{A_1 t_{\text{min}}}}{A_1}
$$

The values of various parameters in the above expression are tabulated in Table I. The production cross sections of charged single strange hyperons are 4 times larger than the neutral hyperons from $\bar{p}p$ reactions. It has also been found that $\sigma_{pp \rightarrow \Sigma^- \Sigma^+} = 4\gamma^4 \sigma_{pp \rightarrow \Lambda \bar{\Lambda}}$ and $\sigma_{pp \rightarrow \Lambda \bar{\Lambda}} = \frac{9}{4} \sigma_{pp \rightarrow \Sigma^- \Sigma^+}$ with $\gamma^2 = 1/3$. The slopes
of the differential cross section $\Lambda$ for $\Lambda$ and $\Sigma$ are taken to be $9 \,\text{GeV}^{-2}$ and for $K$ mesons, to be $4 \,\text{GeV}^{-2}$ by fitting the data on strange hadron production from $\bar{p} - p$ collisions\cite{13,17}. The value of $g_0$ has been determined from the decay $\rho \rightarrow \pi\pi$ with $\frac{g_0}{\pi} = 2.7$.

We have considered the inverse reactions $K\bar{K} \rightarrow \pi\pi$, $K\bar{K} \rightarrow \pi\rho$, $K\bar{K} \rightarrow \rho\rho$, $K^-\bar{K}^+ \rightarrow \bar{p}\bar{p}$, $\Lambda K \rightarrow \pi N$, $\Lambda K \rightarrow \rho N$, $\Sigma K \rightarrow \pi N$, $\Lambda \pi \rightarrow \pi N$, $\Lambda \Lambda \rightarrow \pi N$ and $\Sigma^-\Sigma^+ \rightarrow \bar{p}p$, $\Sigma\pi \rightarrow K\bar{K}$ and the cross sections are calculated using principle of detailed balance as follows:

$$\sigma_{f \rightarrow i} = \frac{P_i^2}{P_f^2} \frac{g_i}{g_f} \sigma_{i \rightarrow f}$$

where $P_i, P_f$ are the centre of mass momenta and $g_i, g_f$ are the total degeneracies of the initial and final channels.

Production of single strange hadrons by other channels where multi-strange $\Xi$ and $\Omega$ are involved are described below.

2.3. Production cross sections for multi-strange hadrons: cascade(\Xi) and omega (\Omega)

The multi-strange hadrons are the baryons having strangeness more than one; $S = \pm 2, \pm 3$. Baryons like Cascade (S=-2) and Omega (S=-3) fall into this category. Due to large strangeness content, the production of multi-strange baryons from non strange hadrons is expensive and less probable. Strangeness exchange reactions become the dominant channels.

The types of reactions producing $\Xi(S = -2)$ are $K\bar{Y} \rightarrow \pi\Xi$, $Y \bar{Y} \rightarrow B\Xi$, $KB \rightarrow K\Xi$ and $B\bar{B} \rightarrow \Xi\Xi$. Here $Y$ represents $\Lambda$ or $\Sigma$. More specifically the reactions are $\Lambda\Lambda \rightarrow N\Xi$, $\Lambda\Sigma \rightarrow N\Xi$, $\Sigma\Sigma \rightarrow N\Xi$, $K\Lambda \rightarrow \pi\Xi$, $K\Sigma \rightarrow \pi\Xi$, $\bar{K}N \rightarrow K\Xi$, $\bar{p}p \rightarrow \Xi\Xi$.

Out of these above mentioned channels, the strangeness exchange reactions are $\Lambda\Lambda \rightarrow N\Xi$, $\Lambda\Sigma \rightarrow N\Xi$, $\Sigma\Sigma \rightarrow N\Xi$, $K\Lambda \rightarrow \pi\Xi$, $K\Sigma \rightarrow \pi\Xi$, $\bar{K}N \rightarrow K\Xi$. The cross sections are calculated from a SU(3) invariant Langragian density as in \cite{33,48}.

$$L = i \text{Tr} (\bar{B} \partial DB) + \text{Tr} [D_\mu P^+ D^\mu P] + g' \text{Tr} [(2\alpha - 1) \bar{B} \gamma^5 \gamma^\mu BD_\mu B \gamma^5 \gamma^\mu (D_\mu P) B]$$

(8)

$B$ and $P$ appearing in the lagrangian are the baryon and pseudo-scalar meson octets and $D_\mu = \partial_\mu - ig[V_\mu]$ is the covariant derivative, which accounts for the interaction of pseudoscalar mesons and baryons through pseudovector($V_\mu$) couplings. The octets are

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \\ -\frac{\Sigma^-}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \\ \Xi^- \\ -\Xi^- + \sqrt{\frac{2}{3}} \Lambda \end{pmatrix}$$

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{n}{\sqrt{6}} + \frac{m}{\sqrt{3}} \\ -\frac{n}{\sqrt{2}} + \frac{m}{\sqrt{6}} + \frac{\rho^-}{\sqrt{3}} \\ K^- \\ -\sqrt{\frac{2}{3}} \eta + \frac{\rho^-}{\sqrt{3}} \end{pmatrix}$$

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega^0}{\sqrt{3}} + \frac{\rho^+}{\sqrt{2}} + \frac{K^0}{\sqrt{3}} \\ \rho^- + \frac{n}{\sqrt{6}} + \frac{m}{\sqrt{3}} \\ K^- + \frac{m}{\sqrt{3}} \\ -\sqrt{\frac{2}{3}} \eta + \frac{n}{\sqrt{6}} \end{pmatrix}$$

The universal coupling constants $g$ and $g'(g' \text{ responsible for B-P interactions})$ are derived from $f_{\pi NN}$, $g_{\rho NN}$ \cite{49} and we consider the values $g=13$, $g'=14.4 \,\text{GeV}$ and parameter $\alpha=0.64 \,\text{\cite{50}}$. We also take other relevant couplings from \cite{48}. It has also been found that the contribution of $\eta$ in strangeness exchange
reactions is much less compared to the baryons [48]. Hence we don’t consider the interactions of type $\bar{K}\Lambda \to \eta\Xi$ and $K\Sigma \to \eta\Xi$.

Tensor interactions, like vector interactions of $V - B$, of D and F types have also been considered by [35, 48] and we take the SU(3) invariant Lagrangian

$$\mathcal{L}^t = \frac{g^t}{2m} \text{Tr}[(2\alpha - 1)\bar{B}\sigma^{\mu\nu}B\partial_\mu V_\nu + \bar{B}\sigma^{\mu\nu}(\partial_\mu V_\nu)B]$$

with $g^t$ obtained from rho – N tensor coupling [49] and $m$ is for the degenerate baryon mass. The feynman diagrams for some of these channels like $K\Lambda \to \pi\Xi$, $K\Sigma \to \pi\Xi$ are shown in Fig. 1. The details of the cross section for all these strangeness exchange reactions are calculated in [35]. We use the parametrised cross section [51] and evaluate the rate of production which is shown in [52].

Similarly, $\Xi$ production cross section of $BB \to \Xi\Xi$ channel, i.e., $p\bar{p} \to \Xi^-\Xi^+$ and $pp \to \Xi^0\Xi^0$ has been evaluated using quark gluon string model (QGSM) [36]. The results are also compared with experimental observation. Using this cross section the rate has been evaluated in [52].

We had already discussed that the strangeness exchange channels play crucial role in $\Xi$ productions over $BB \to \Xi\Xi$. The rates of production of $\Lambda\Lambda \to N\Xi$ or $K\Lambda \to \pi\Xi$ are $10^6$ times more compared to the channels $pp \to \Xi\Xi$ [52].

Production of $\Omega(S = -3)$ in heavy ion collisions is not well understood. However we have attempted its study of yield with the current understandings. To mention a few possible reactions for $\Omega$ productions, channels like $\Xi Y \to \Omega N$ and $K\Xi \to \Omega\pi$ seem to be important as they fall into the category of strangeness exchange reactions. But the production cross sections for these reactions are not clear by now. The authors in [53] although argue about its cross section to be similar to $K\Lambda \to \pi Y$ but the experimental coupling is not available. Other probable channels we consider are $\pi\Xi \to \Omega K$, ($\pi^0\Xi^- \to \Omega^- K^0$), $KY \to K\Omega$ ($K\Lambda \to K^0\Omega^-$), $K\Sigma^0 \to K^0\Omega^-$) and highlighted and discussed in details [52]. The other channel we have considered for $\Omega$ productions is $BB \to \Omega\Omega$ or $\bar{p}p \to \Omega\Omega$. The details of cross section and rate of production can be found in [36] and [52].

3. Rate of strange hadron production in hadronic medium

With the input of cross sections from previous section, the thermal rates of strange hadron productions in hadronic medium are evaluated considering the binary interactions in the following way. The rate, $R(T)$ at a temperature $T$ is given by [13, 54],

$$\langle \sigma v \rangle = \frac{T^4}{4m_a^2m_b^2K_2(m_a/T)K_2(m_b/T)} \int_{2m}^\infty dz \left[z^2 -(m_a/T + m_b/T)^2\right]\left[z^2 -(m_a/T - m_b/T)^2\right] \sigma K_1(z)$$
where $\sigma$ is the cross section of particular channel of interest and $v$ is the relative Moller velocity of the incoming particles of masses $m_a$ and $m_b$. $K_2$ is the modified bessel function of second kind. $z_0 = \max(m_a + m_b, m_c + m_d)/T$. The detailed derivation of rate and chemical rate equation is given in appendix A.

The rate of various channels producing single and multi-strange hadrons are discussed in the result section.

4. Yield of strange hadrons using rate equation

The number densities of $K$, $\bar{K}$, $\Lambda$, $\Sigma$, $\Xi$ and $\Omega$ are studied using following rate equations with cross sections, as described in section 2 as input. The non-strange mesons and baryons are assumed to provide thermal background to the strange hadrons which are slightly away from equilibrium. Eq[11] describes a set of coupled equations for different strange hadrons and each equation contains terms for net productions due to
binary interactions and dilution term ($n_i/t$) due to expansion of the system.

\[
\frac{dn_K}{dt} + \frac{n_K}{t} = n_\pi n_\pi \langle \sigma v \rangle_{\pi\pi} \rightarrow KK - nKNK.K.K.K - nKnK.K.K.K - nRnK.K.K.K - nKnK.K.K.K - nKnK.K.K.K - nKnK.K.K.K
\]

\[
\frac{dn_\Lambda}{dt} + \frac{n_\Lambda}{t} = n_\pi n_\pi \langle \sigma v \rangle_{\pi\pi} \rightarrow \Lambda\Lambda - nKNK.K.K.K - nKnK.K.K.K
\]

\[
\frac{dn_\Xi}{dt} + \frac{n_\Xi}{t} = n_\pi n_\pi \langle \sigma v \rangle_{\pi\pi} \rightarrow \Xi\Xi - nKNK.K.K.K
\]

We do not consider initial QGP phase in this study. The information of the strange production from QGP phase should, in principle, constrain the initial number densities ($n_i(T_i)$) of the rate equations in hadronic phase, where $T_i$ is the initial temperature. To take care of this we treat $n_i(T_i)$ as parameters here. These rate equations are numerically coded as Strange Hadron Transport in Heavy Ion Collisions (SH-THIC) to get the yield along with temperature evolution equation considering Bjorken expansion of the system. Although present study is for LHC energy, $\sqrt{s_{NN}}=2.76$ TeV Pb-Pb collisions, where the baryonic chemical potential ($\mu_b$) is very small, still we have considered the evolution of $\mu_b$ for the sake of completeness.

The evolution of the number density depends on the evolution of the temperature and chemical potential $\mu$ ($= \mu_s + \mu_b$). We consider net $\mu_s$ to be zero and $\mu = \mu_b$ is the total chemical potential. The evolution of baryonic chemical potential is obtained from the baryon number conservation equation with Bjorken expansion along $z$-direction as follows,

\[
\partial_t n_b^0 = 0
\]
where, \( n_b^\mu = n_b(\gamma, \gamma v_z) \) with \( n_b \) is the net baryon number density at \( (T, \mu) \). The above equation leads to \( n_b \tau = \text{const} \equiv k_1 \) and \( n_b = \sum_{B=N, \Lambda, \Sigma, \Xi} (n_B - n_{\bar{B}}) \) and \( \tau \) is the proper-time defined by \( \tau = \sqrt{T^2 - z^2} \). The evolution of \( \mu_b \) is obtained from the above equation. We have not considered \( \Delta \) and other massive baryons as contribution is less due to mass. Again, following Bjorken expansion \( [55] \) and energy conservation law \( \partial_\mu T^{\mu\nu} = 0 \), we get, \( \partial_\tau \left[ T^{(1+\gamma_s)} \tau \right] = 0 \) or \( T^a \tau = \text{const} = k_2 \) and \( a = \frac{4}{1+c_s^2} \) considering energy density \( \epsilon \) that goes as \( \sim T^4 \). As usual the \( T^{\mu\nu} \) represents the energy momentum tensor of the expanding fluid. \( c_s^2 \) is the square of the velocity of sound. Here \( k_1 = n_b^\mu \tau_i, k_2 = T_i^a \tau_i \), where \( n_b^\mu, \tau_i, T_i \) are the initial baryon number densities, time and temperature and are parameters. \( T_i \) is taken as the \( T_i \) from the lattice calculation.

After solving the rate equations with the evolution of temperature and chemical potential the yields have been calculated and discussed in the next section.

5. Results

Taking the cross sections from earlier section as input, the rate of production \( \langle R=\langle \sigma v \rangle \rangle_{ab \to cd} \) for strange hadrons \( K, \bar{K}, \Lambda, \Sigma, \Xi \) and \( \Omega \) have been calculated from Eq.10 The rates have been displayed in Figs.24 for the temperature ranges of our interest. Here we describe the rate of single strange hadrons \( (K, \Lambda, \Sigma) \) more explicitly as the multi strange hadron rates and yields are described in \([52] \) in detail. However the total production rates of \( \Xi \) and \( \Omega \) are discussed later.
Figure 8: Yield ratio for $K_0^0$ from 2.76 TeV Pb+Pb collisions. The solid points with error bar are the data points measured by ALICE collaboration. The solid line is the result of theoretical calculation with initial condition for various scenarios. Left panel is for scenario-I, II, III and right panel is for scenario IV, V, VI.

Figure 9: Yield ratio for $\Lambda$ from 2.76 TeV Pb+Pb collisions. The solid points with error bar are the data points measured by ALICE collaboration. The solid line is the result of theoretical calculation with initial condition for various scenarios. Left panel is for scenario-I, II, III and right panel is for scenario IV, V, VI.

The rate of Kaon ($K, \bar{K}$) productions from meson-meson (MM) interactions are shown in Fig. 2 for a temperature range of 105-170 MeV. The rate increases with temperature as expected. We have considered only binary interactions for strange hadron productions. Among these binary channels $\rho \rho \rightarrow K \bar{K}$ is the dominant one over the other channels, $\pi \pi \rightarrow K \bar{K}$ and $\pi \rho \rightarrow K \bar{K}$. The other two have similar contributions over the entire range of temperature as shown in the figure. Fig. 3 shows the rate of Kaon productions along with hyperon ($\Lambda, \Sigma, \Xi$) productions from meson-baryon (M-B) interactions. Contrary to the increase of rate with temperature, $\rho N$ channel shows a gradual decrease which is due the behaviour of cross section with centre of mass energies of the colliding $\rho$ and $N$ in the thermal system within the considered temperature range. $\rho N$ channel dominates over other channels in this (MB) category when the system is at lower temperature.

Similarly, other process producing Kaons ($K, \bar{K}$) is the interaction of $p - \bar{p}$ which is shown in the left panel of Fig. 4. Kaons are also produced from strangeness exchange reactions along with $\Xi$ and $\Omega$. Basically $\pi \Xi \rightarrow \Omega K, K \Sigma \rightarrow \Omega K, K \Lambda \rightarrow \Omega K$ are the channels, whose contributions are less to the kaon production but important for $\Omega$ productions, which are shown in Fig. 4. In the right panel of the Fig. 4, the $\Lambda$ productions are highlighted from strangeness exchange reactions with other strange hadrons like $K$ or $\Sigma$ in the outgoing channel. These channels play dominant role for the yield of hyperons $\Lambda$ and $\Sigma$. The cross sections of
NΞ → ΛΛ and NΞ → ΛΣ are most crucial for the Λ productions. The contribution from process ΩK → KΛ is less due to the massive Ω in the initial channel as shown in the right panel of Fig.4.

Rates from MB (KN, ρN, πN) and BB(pp) interactions producing K, K, and Λ are shown in the left panel of Fig.5. These are negligible contributions compared to KN → Λπ. The rates of Σ production can also be understood from Figs.5-4 and Fig.5.

As far as the production rates of Ξ are concerned we have considered the processes of YY, KY, KN and N N as discussed earlier. The contribution from YY channel is dominant and decide the cascade production. The variation of rate with temperature is slow. For details of rate of Ξ, Ω productions, one can see [52]. The total rates of K, K, Λ, Ξ and Ω are shown in Figs.5 & 7. Several initial conditions which are used to solve the rate equation to get the number densities of K, K, Λ, Σ, Ξ, Ω are tabulated in 24. The number densities are then normalised with thermal pion number density to obtain the yields.

Here we have considered 2K0 = K+ + K− and evaluated the yield ratio for Kaon and compared with the available data [1, 3] and shown in Figs.8. We have considered several scenarios with various initial conditions which are mentioned below. In scenario-I, nF for Kaon, Lambda, Sigma, Cascade and Omega are assumed to be 20% away from the equilibrium value. Tc with a value taken from lattice as mentioned above is the initial temperature for all scenarios including scenario-I. Velocity of sound for hadron phase is taken to be c2 = 1/5. The freezeout temperatures are different for different multiplicities as shown in Table-2. T is decreases with multiplicity in scenario-I and the results are shown in Fig.5 which does not explain the yield ratio data for Kaons although it explains the higher multiplicity data. This is similar to the yield ratio of Ω and unlike Ξ, where it explains the low multiplicity and disagrees with the high multiplicity data [52]. In case of Lambda it also under predicts the data which is shown in Fig.9.

In scenario-II, the system is allowed to evolve with initial density, nF to be 40% away from the equilibrium value with constant TF=144 MeV for all values. We have kept τF same for all scenarios for a particular multiplicity. τF is different for different multiplicity. Like previous scenario, Scenario-II also does not explain the kaon, lambda, cascade and omega data. With all conditions of scenario-II, but nF with 20% away from equilibrium value when considered as scenario-III, we observed kaon and cascade data are over predicted and lambda to be under predicted. However high multiplicity omega data are explained missing the lower one.

Being inspired for a TF=154 MeV for all dNch/dη at LHC energies, as predicted by statistical hadronisation model, as shown in the article by ALICE collaboration [50], we also take constant TF=154 MeV with nF 20% away from equilibrium value in scenario IV and tried to analyse the data. This is for all multiplicities. It explains most of the data points of the yield ratio for cascade and omega but does not explain for kaon and lambda. The initial conditions for scenario I-IV are tabulated in Table-2.

We tried to analyse for a scenario which could explain the data of all strange hadrons and tried to get the information of TF. That is scenario-V where nF for kaons to be 20% away from the equilibrium value as usual and with other initial conditions same. Here the freeze out temperatures that explain the yield ratio data for Kaon, Lambda, cascade and Omega data. With all conditions of scenario-V, but nF with 20% away from equilibrium value when considered as scenario-III, we observed kaon and cascade data are over predicted and lambda to be under predicted. However high multiplicity omega data are explained missing the lower one.

In scenario-VI, we tried to analyse the kaon and Lambda yield ratio considering the same freeze out temperatures which explain the Omega yield ratio. It has been observed that Kaon data at high multiplicities are explained with same TF but at lower multiplicities the data are under predicted. Here, the Lambda yield ratio does not agree with the theoretical estimation. This is depicted in the right panels of Figs.8 & 9.

6. Summary

The yield ratio of strange hadrons; (K+ + K−)/(π+ + π−), (Λ + ¯Λ)/(π+ + π−), (Σ + ¯Σ)/(π+ + π−), (Ξ− + Ξ+)/π+ + π−) and (Ω + ¯Ω)/(π+ + π−) measured from p-p, p-Pb and Pb-Pb collisions at various centralities and colliding energies are presented by ALICE collaboration as an observable in 1, 3 against the charged particle multiplicity. The smooth rise of yield ratio pose a question- does the yield depend
Table 2: Initial conditions (Freezeout temperatures, $T_F$) for various multiplicities of $K^0$, $\Lambda$, $\Xi$ and $\Omega$

| $\frac{dN_{ch}}{dy}$ | $N_{part}$ | $C_s^2$ | Scenario-I $T_f$ (in GeV) | Scenario-II $T_f$ (in GeV) | Scenario-III $T_f$ (in GeV) | Scenario-IV $T_f$ (in GeV) |
|-----------------------|------------|---------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1447.5                | 356.1      | 1/5     | 0.144                       | 0.144                       | 0.144                       | 0.154                       |
| 966                   | 260.1      | 1/5     | 0.142                       | 0.144                       | 0.144                       | 0.154                       |
| 537.5                 | 157.2      | 1/5     | 0.140                       | 0.144                       | 0.144                       | 0.154                       |
| 205                   | 68.6       | 1/5     | 0.132                       | 0.144                       | 0.144                       | 0.154                       |
| 55                    | 22.5       | 1/5     | 0.116                       | 0.144                       | 0.144                       | 0.154                       |
| 13.4                  | 4.3        | 1/5     | 0.096                       | 0.144                       | 0.144                       | 0.154                       |

Table 3: Initial conditions with freeze out temperature for scenario-V, that explains the data. The * symbol says about the unavailability of data at those multiplicities.

| $\frac{dN_{ch}}{dy}$ | $N_{part}$ | $C_s^2$ | Scenario-V $T_f$ ($K^0$) (in GeV) | Scenario-V $T_f$ ($\Lambda$) (in GeV) | Scenario-V $T_f$ ($\Xi$) (in GeV) | Scenario-V $T_f$ ($\Omega$) (in GeV) |
|-----------------------|------------|---------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| 1601                  | 383        | 1/5     | 0.145                            | 0.143                            | *                                | *                                |
| 1294                  | 330        | 1/5     | 0.144                            | 0.145                            | *                                | *                                |
| 1447.5                | 356.1      | 1/5     | *                                | *                                | 0.134                            | 0.145                            |
| 966                   | 260.1      | 1/5     | 0.143                            | 0.147                            | 0.141                            | 0.144                            |
| 537.5                 | 157.2      | 1/5     | 0.141                            | 0.152                            | 0.143                            | 0.143                            |
| 205                   | 68.6       | 1/5     | 0.138                            | 0.151                            | 0.137                            | 0.137                            |
| 55                    | 22.5       | 1/5     | 0.131                            | 0.146                            | 0.118                            | 0.118                            |
| 13.4                  | 4.3        | 1/5     | 0.122                            | 0.134                            | *                                | *                                |

explicitly on multiplicity only? Does the colliding system, whether nucleon-nucleon (p-p) or nuclei(heavy)-nuclei(heavy) not matter? Do the colliding energies, $\sqrt{s_{NN}}=2.76$ TeV or 5.02 TeV or 7 TeV matter for the yield explicitly? Answering these questions in a single step is difficult.

With an aim to answer these questions and to explain the strange hadron yields, we have made an initial framework and studied the strange hadron productions at LHC energy, $\sqrt{s_{NN}}=2.76$ TeV from Pb-Pb collisions microscopically, considering the cross sections of various interactions producing strange hadrons. We have calculated for LHC energy initially, because (i) measurements are available and (ii) the systems which are produced at various multiplicities of LHC energy have a common feature like negligible baryon chemical potential. The calculation would be extended to other colliding energies with different colliding systems.

Table 4: Chemical freezeout temperature of Scenario VI for $K^0$ and $\Lambda$

| $\frac{dN_{ch}}{dy}$ | $N_{part}$ | $C_s^2$ | Scenario-V $T_f$ ($K^0$) (in GeV) | $T_f$ ($\Lambda$) (GeV) |
|-----------------------|------------|---------|-----------------------------------|------------------------|
| 1447.5                | 356.1      | 1/5     | 0.145                            | 0.145                  |
| 966                   | 260.1      | 1/5     | 0.144                            | 0.144                  |
| 537.5                 | 157.2      | 1/5     | 0.143                            | 0.143                  |
| 205                   | 68.6       | 1/5     | 0.137                            | 0.137                  |
| 55                    | 22.5       | 1/5     | 0.118                            | 0.118                  |
| 13.4                  | 4.3        | 1/5     | 0.096                            | 0.096                  |

In this article we have calculated the rate of single and multi-strange hadron productions considering various possible hadronic interactions and their cross sections, where most of the cross sections were constrained experimentally. Then the yield of $K, \bar{K}, \Lambda, \Xi, \Omega$ are evaluated solving rate equations simultaneously.
by considering the evolution of temperature and baryonic chemical potential of the system. Considering a hadronic system at $T_c=154$ MeV we have calculated the strange hadron yield with various initial conditions and obtained the yield ratio by normalising with thermal pions and finally compared the results with experimental observations to have an information of freeze out (chemical) scenario. The best explanation of the yield ratio data (for scenario-V) suggests for (i) multiple freeze out scenario for single and multi strange hadrons, (ii) The $T_F$ increases with multiplicity and then decreases. This is for all strange hadrons. At highest multiplicity, a single freeze out scenario for $K, \bar{K}, \Lambda, \Xi, \Omega$ can be inferred but not from $\Xi$.

A smooth monotonic decrease of $T_F$ with $dN_{ch}/d\eta$ at LHC energy is expected if the yield ratio depends only on $dN_{ch}/d\eta$ or $N_{part}$. However, present calculation shows that $T_F$ increases and then decreases with $dN_{ch}/d\eta$, which suggests that energy density or finite size of the freeze out volume may be another parameter. Further improvement of calculation can be done by considering the corrections due to the volume of pion freeze out surface and considering the error bars due to the uncertainty of parameters.

Thus it would be interesting to analyse the yield ratio data for all colliding energies available with a wide range of multiplicities to have a general conclusion in future. This microscopic work set up a framework to look for a better answer in future calculation. It would help find the reason for the larger rate (slope) of multi strange productions compared to single strange hadrons at lower multiplicity when yield ratio vs $dN_{ch}/d\eta$ is considered.

We have considered $c_s^2=1/5$ while explaining the data. When $c_s^2=1/3$ is considered the theoretical estimate overestimates the experimental observations for all $dN_{ch}/d\eta$. However, parametrisation of the equation of state from lattice with $c_s^2(T)$ may improve the calculation. The yield of $\Xi, \Omega$ including single strange hadrons $K, \bar{K}, \Lambda$ are explained with this slow equation of state with $c_s^2=1/5$.

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**Appendix A. Rate Equation**

We first outline the derivation of the chemical rate equation for the evolution of number density of particle type $a$. The Boltzmann equation is given by

$$p^\mu \partial_\mu f_a = C[f_K]$$  \hspace{1cm} (A.1)

where $f_a(x, p, t)$ is the phase space density of species $a$. Assuming the phase space density to be spatially homogeneous and isotropic we have

$$E \frac{\partial f_a}{\partial t} = C[f_K]$$  \hspace{1cm} (A.2)
Let us define \( F \) where

\[
\frac{g_a}{(2\pi)^3} \int d^3p \frac{\partial f_a}{\partial t} = \frac{g_a}{(2\pi)^3} \int \frac{d^3p}{E} C[f_a]
\]

which gives

\[
\frac{dn_a}{dt} = \frac{g_a}{(2\pi)^3} \int \frac{d^3p}{E} C[f_a]
\]

where \( n_a(t) \) is given by

\[
n_a(t) = \frac{g_a}{(2\pi)^3} \int d^3p f_a(E, t)
\]

Let us define

\[
d\Pi = \frac{g}{(2\pi)^3} d^3p
\]

so that the RHS of Eq. (A.3), for some reaction \( a + b \to c + d \), may be written as (assuming classical particles)

\[
\int d\Pi_a C[f_a] = -\int d\Pi_a d\Pi_b d\Pi_c d\Pi_d (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) F_{abcd}
\]

where \( F_{abcd} \) is given by

\[
F_{abcd} = |\mathcal{M}|^2_{a+b\to c+d} f_a f_b - |\mathcal{M}|^2_{c+d\to a+b} f_c f_d
\]

and \( \mathcal{M}_{a+b\to c+d} \) denotes the amplitude for forward reaction \( a + b \to c + d \) and \( \mathcal{M}_{c+d\to a+b} \) denotes the amplitude for the reverse reaction \( c + d \to a + b \). Assuming PT invariance, we have

\[
|\mathcal{M}|^2_{a+b\to c+d} = |\mathcal{M}|^2_{c+d\to a+b} = |\mathcal{M}|^2
\]

so that we get

\[
\int d\Pi_a C[f_a] = -\int d\Pi_a d\Pi_b d\Pi_c d\Pi_d (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) |\mathcal{M}|^2 (f_a f_b - f_c f_d)
\]

The differential cross-section \[58\] for the reaction \( a + b \to c + d \) is given by

\[
d\sigma = \frac{1}{E_a E_b v_{ab}} \int d\Pi_c d\Pi_d (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) |\mathcal{M}|^2
\]

where \( v_{ab} = |v_a - v_b| \) denotes the Moller velocity (or relative velocity in loose terms) and which is given by

\[
v_{ab} = \sqrt{\frac{(p_a p_b)^2 - m_a^2 m_b^2}{E_a E_b}}
\]

The above expressions suggest the definition for the non-thermal (NTh) averaged cross section times velocity as

\[
\langle \sigma_{ab} v_{ab} \rangle_{NTh} = \frac{1}{n_a n_b} \int d\Pi_a d\Pi_b d\Pi_c d\Pi_d (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) |\mathcal{M}|^2 f_a f_b
\]

(A.4)

Hence the evolution equation for number density of particles \( a \) will be

\[
\frac{dn_a}{dt} = -n_a n_b \langle \sigma_{ab} v_{ab} \rangle_{NTh} + n_c n_d \langle \sigma_{cd} v_{cd} \rangle_{NTh}
\]

(A.5)

To make further progress we assume that the non-thermal reaction rate is approximately equal to the thermal average i.e. near chemical equilibrium and the same is also assumed for slightly away from equilibrium. \( \langle \sigma v \rangle_{NTh} \approx \langle \sigma v \rangle_{Th} \). Hence the chemical rate equation becomes

\[
\frac{dn_a}{dt} = -n_a n_b \langle \sigma_{ab} v_{ab} \rangle_{Th} + n_c n_d \langle \sigma_{cd} v_{cd} \rangle_{Th}
\]

(A.6)
From now onwards we will remove the subscripts “Th” for thermal averages. All averages that appear below should be understood as thermal averages. If the system is also expanding, then the rate equation becomes

$$\frac{dn_a}{dt} + \Gamma_{\exp} n_a = -n_a n_b \langle \sigma_{ab} v_{ab} \rangle_{Th} + n_c n_d \langle \sigma_{cd} v_{cd} \rangle_{Th}$$  \hspace{1cm} (A.7)$$

where $\Gamma_{\exp}$ is the expansion rate. For (1+1)-dimensional Bjornen expansion, $\Gamma_{\exp} = \frac{1}{T}$. Also, since we are studying the production of strange hadrons whose masses are much larger than the temperature range ($T < T_c$) in which we are interested, we can take the equilibrium distribution to be Maxwell-Boltzmann distribution. Hence the reaction rate, $R(T)$, at a temperature $T$ is given by \[13, 54,\]

$$\langle \sigma v \rangle_{ab \rightarrow cd} = \frac{\int \sigma v e^{-E_a/T} e^{-E_b/T} d^3p_a d^3p_b}{\int e^{-E_a/T} e^{-E_b/T} d^3p_a d^3p_b} = \frac{1}{16\pi^2m^2_bT^2 K_2(m_a/T)K_2(m_b/T)} \times \int \sigma v e^{-E_a/T} e^{-E_b/T} d^3p_a d^3p_b$$ \hspace{1cm} (A.8)$$

for a reaction with incoming particles $a, b$ and outgoing particles $c, d$. Here $K_2(x)$ denotes the modified bessel function of second kind. Taking a preferential direction for $p_a$ along $z$-direction and taking $\theta$ as the angle between $p_a$ and $p_b$, one gets after simplification

$$\langle \sigma v \rangle = \frac{1}{8m^2_a m^2_b T K_2(m_a/T)K_2(m_b/T)} \int_{s_0}^{\infty} ds \left[ s - (m_a + m_b)^2 \right] \left[ s - (m_a - m_b)^2 \right] \frac{1}{\sqrt{s}} \sigma K_1(\sqrt{s}/T)$$ \hspace{1cm} (A.9)$$

where $K_1(\sqrt{s}/T) = \frac{1}{T} \int dE_+ e^{-E_+/T} \sqrt{E^2_+ - s}$. With $z = \frac{\sqrt{s}}{T}$, one can write the thermal averaged reaction rate as

$$\langle \sigma v \rangle = \frac{T^4}{4m^2_a m^2_b T K_2(m_a/T)K_2(m_b/T)} \int_{z_0}^{\infty} dz \left[ z^2 - (m_a/T + m_b/T)^2 \right] \left[ z^2 - (m_a/T - m_b/T)^2 \right] \sigma K_1(z)$$ \hspace{1cm} (A.10)$$

where $z_0 = \text{max}(m_a + m_b, m_c + m_d)/T$.

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