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On maximal product sets of random sets. (English) [Zbl 1472.11059]
J. Number Theory 224, 13–40 (2021).

Summary: For every positive integer $N$ and every $\alpha \in [0, 1)$, let $B(N, \alpha)$ denote
the probabilistic model in which a random set $A \subset \{1, \ldots, N\}$ is constructed by choosing
independently every element of $\{1, \ldots, N\}$ with probability $\alpha$. We prove that, as $N \to +\infty$,
for every $A$ in $B(N, \alpha)$ we have $|AA| \sim |A|^2/2$ with probability $1 - o(1)$, if and only if
$$\frac{\log(\alpha^2 \log N^{\log 4 - 1})}{\sqrt{\log \log N}} \to -\infty.$$ This improves on a theorem of Cilleruelo, Ramana and Ramaré [J. Cilleruelo et al., Proc. Steklov Inst.
Math. 296, 52–64 (2017; Zbl 1371.11023); translation in Tr. Mat. Inst. Steklova 296, 58–71 (2017)],
who proved the above asymptotic between $|AA|$ and $|A|^2/2$ when $\alpha = o(1/\sqrt{\log N})$, and
supplies a complete characterization of maximal product sets of random sets.

MSC:
11B30 Arithmetic combinatorics; higher degree uniformity

Keywords:
product sets; random models; localised divisor functions; distribution of the number of prime factors

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