PLANE SPDC-QUANTUM MIRROR

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Abstract

In this paper the kinematical correlations from the phase conjugated optics (equivalently with crossing symmetric spontaneous parametric down conversion (SPDC) phenomena) in the nonlinear crystals are used for the description of a new kind of optical device called SPDC-quantum mirrors. Then, some important laws of the plane SPDC-quantum mirrors combined with usual mirrors or lens are proved only by using geometric optics concepts. In particular, these results allow us to obtain a new interpretation of the recent experiments on the two-photon geometric optics.

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1 Introduction

The spontaneous parametric down conversion (SPDC) is a nonlinear optical process [1] in which a laser pump (p) beam incident on a nonlinear crystal leads to the emission of a correlated pair of photons called signal (s) and idler (i). If the S-matrix crossing symmetry [2] of the electromagnetic interaction in the spontaneous parametric down conversion (SPDC) crystals is taken into account, then the existence of the direct SPDC process

\[ p \rightarrow s + i \]  \hspace{1cm} (1)

will imply the existence of the following crossing symmetric processes [3]

\[ p + \bar{s} \rightarrow i \]  \hspace{1cm} (2)
as real processes which can be described by the same transition amplitude. Here, by $\bar{s}$ and $\bar{i}$ we denoted the time reversed photons (or antiphotons in sense introduced in Ref. [4]) relative to the original photons $s$ and $i$, respectively. In fact the SPDC effects (1)-(3) can be identified as being directly connected with the $\chi^{(2)}$-second-order nonlinear effects called in general three wave mixing (see Ref.[5]). So, the process (1) is just the inverse of second-harmonic generation, while, the effects (2)-(3) can be interpreted just as emission of optical phase conjugated replicas in the presence of pump laser via three wave mixing.

In this paper a new kind of geometric optics called quantum SPDC-geometric optics is systematically developed by using kinematical correlations of the pump, signal and idler photons from the SPDC processes. Here we discuss only the plane quantum mirror. Other kind of the SPDC-quantum mirrors, such as spherical SPDC quantum mirrors, parabolic quantum mirrors, etc., will be discussed in a future paper.

## 2 Quantum kinematical correlations

In the SPDC processes (1) the energy and momentum of photons are conserved:

$$\omega_p = \omega_s + \omega_i, \quad k_p = k_s + k_i$$

(4)

Moreover, if the crossing SPDC-processes (2)-(3) are interpreted just as emission of optical phase conjugated replicas in the presence of input pump laser then Eqs. (4) can be identified as being the phase matching conditions in the three wave mixing (see again Ref. [5]). Indeed, this scheme exploits the second order optical nonlinearity in a crystal lacking inversion symmetry. In such crystals, the presence of the input pump $(E_p)$ and of the signal $(E_s)$ fields induces in the medium a nonlinear optical polarization (see Eqs. (26)-(27) in Pepper and Yariv Ref.[5]) which is: $P_{NL}^i = \chi_{ijk}^{(2)} E_p j(\omega_p) E^*_k(\omega_s) \exp\{i[(\omega_p - \omega_s)t - (k_p - k_s) \cdot r] \} + c.c.,$ where $\chi_{ijk}^{(2)}$ is the susceptibility of rank two tensor components of the crystal. Consequently, such polarization, acting as a source in the wave equation will radiate a new wave $E_i$ at frequency $\omega_i = \omega_p - \omega_i$, with an amplitude proportional to $E^*_i(\omega_i)$, i.e., to the complex conjugate of the spatial amplitude of the low-frequency
probe wave at $\omega_s$. Then, it is easy to show that a necessary condition for a phase-coherent cumulative buildup of conjugate-field radiation at $\omega_i = \omega_p - \omega_s$ is that the wave vector $k_i$ at this new frequency must be equal to $k_i = k_p - k_s$, i.e., we have the phase matching conditions (4). Hence, the optical phase conjugation by three-wave mixing help us to obtain a complete proof of the existence of the crossing reactions (2)-(3) as real processes which take place in the nonlinear crystals when the energy-momentum (or phase matching) conditions (4) are fulfilled.

Now, it is important to introduce the momentum projections, parallel and orthogonal to the pump momentum, and to write the momentum conservation law from (4) as follows

$$k_p = k_s \cos \theta_{ps} + k_i \cos \theta_{pi}$$  \hspace{1cm} (5)

$$k_s \sin \theta_{ps} = k_i \sin \theta_{pi}$$  \hspace{1cm} (6)

where the angles $\theta_{pj}, j = s, i$, are the angles (in crystal) between momenta of the pump (p)≡($\omega_p, k_p, e_p, \mu_p$), signal (s)≡($\omega_s, k_s, e_s, \mu_s$) and idler (i)≡($\omega_i, k_i, e_i, \mu_i$) photons. By $e_j$ and $\mu_j, j \equiv p, s, i$, we denoted the photon polarizations and photon helicities, respectively. Now, let $\beta_{ps}$ and $\beta_{pi}$ be the corresponding exit angles of the signal and idler photons from crystal. Then from (6) in conjunction with Snellius law, we have

$$\sin \beta_{ps} = n_s \sin \theta_{ps}, \quad \sin \beta_{pi} = n_i \sin \theta_{pi}$$  \hspace{1cm} (7)

$$\omega_s \sin \beta_{ps} = \omega_i \sin \beta_{pi}$$  \hspace{1cm} (8)

3 Quantum mirrors via SPDC phenomena

(D.1) Quantum Mirror (QM). By definition a quantum mirror (QM) is a combination of standard devices (e.g., usual lenses, usual mirrors, lasers, etc.) with a nonlinear crystal by which one involves the use of a variety of quantum phenomena to exactly transform not only the direction of propagation of a light beam but also their polarization characteristics.

(D.2) SPDC-Quantum Mirror (SPDC-QM). A quantum mirrors is called SPDC-QM if is based on the quantum SPDC phenomena (1)-(3) in order to transform signal photons characterized by ($\omega_s, k_s, e_s, \mu_s$) into idler photons with ($\omega_p - \omega_s, k_p - k_s, e_s^*, -\mu_s$)≡($\omega_i, k_i, e_i, \mu_i$).

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Now, since the crossing symmetric SPDC effects (2)-(3) can be interpreted just as emission of optical phase conjugated replicas in the presence of pump laser via three wave mixing, the high quality of the SPDC-QM will be given by the following peculiar characteristics: (i) Coherence: The SPDC-QM preserves high coherence between s-photons and i-photons; (ii) Distortion undoing: The SPDC-QM corrects all the aberrations which occur in signal or idler beam path; (iii) Amplification: A SPDC-QM amplifies the conjugated wave if some conditions are fulfilled.

3.1. Plane SPDC-quantum mirrors. The quantum mirrors can be plane quantum mirrors (P-QM) (see Fig.1), spherical quantum mirrors (S-QM), hyperbolic quantum mirror (H-QM), parabolic quantum mirrors (PB-QM), etc., according with the character of incoming laser wave fronts (plane waves, spherical waves, etc.). Here we discuss only the plane SPDC-quantum mirror. Other kind of the SPDC-quantum mirrors, such as spherical SPDC quantum mirrors, parabolic quantum mirrors, etc., will be discussed in a future paper.

In order to avoid many complications, in the following we will work only in the thin crystal approximation. Moreover, we do not consider here the so called optical aberrations.

(L.1) Law of thin plane SPDC-quantum mirror: Let BBO be a SPDC crystal illuminated uniform by a high quality laser pump. Let \( Z_s \) and \( Z_i \) be the distances shown in Fig.1 (from the object point \( P \) to crystal (point A) and from crystal (point A) to image point \( I \). Then, the system behaves as a plane mirror but satisfying the following important laws:

\[
\frac{Z_i}{Z_s} = \frac{\omega_i}{\omega_s} = \frac{\sin \beta_{ps}}{\sin \beta_{pi}} = \frac{n_s \sin \theta_{ps}}{n_i \sin \theta_{pi}}, \quad M = \frac{\omega_s Z_i}{\omega_i Z_s} = 1
\]

where \( M \) is the linear magnification of the plane SPDC-quantum mirror.

3.2. Plane SPDC-QM combined with thin lens. The basic optical geometric configurations of a plane SPDC-QM combined with thin lens is presented in Figs. 2a and 2b. The system in this case behaves as in usual geometric optics but with some modifications in the non degenerate case introduced by the presence of the plane SPDC-quantum mirror. The remarkable law in this case is as follows.

(L.2) Law of the thin lens combined with a plane SPDC-QM: The distances \( S \) (lens-object), \( S' \) (lens-crystal-image plane), \( D_{CI} \) (crystal-image plane) and \( f \) (focal distance of lens), satisfy the following thin lens equation

\[
\frac{1}{S} + \frac{1}{S'} + \left( \frac{\omega_s}{\omega_i} - 1 \right) \frac{1}{D_{CI}} = \frac{1}{f}
\]
The SPDC-QM system in this case has the magnification $M$ given by

$$M = \frac{S'}{S} + \left(\frac{\omega_s}{\omega_i} - 1\right) \frac{D_{CI}}{S} = M_0 + \left(\frac{\omega_s}{\omega_i} - 1\right) \frac{D_{CI}}{S}$$

(11)

In degenerate case ($\omega_s = \omega_i = \omega_p/2$) we obtain the usual Gauss law for thin lens with the magnification $M_0 = S'/S$.

**Proof:** The proof of the predictions (10)-(11) can be obtained by using the basic geometric optical configuration presented in Fig. 2a. Hence, the image of the object P in the thin lens placed between the crystal and object is located according to the Gauss law

$$\frac{1}{S} + \frac{1}{S_1} = \frac{1}{f}$$

(12)

where $S_1$ is the distance from lens to image $I_1$. Now the final image $I$ of the image $I_1$ in the plane SPDC-QM is located according to the law (9). Consequently, if $d$ is the lens-crystal distance then we have

$$S_1 = S' + (Z_s - Z_i) = S' + \left(\frac{\omega_s}{\omega_i} - 1\right) D_{CI}$$

(13)

since $S_1 = d + Z_s$, $S'=d+Z_i$,and $D_{CI}$ is the crystal-image distance. A proof of the magnification factor can be obtained on the basis of geometric optical configuration from Fig. 2b. Hence, the magnification factor is

$$M = \frac{y_I}{y_o} = \frac{y_I}{y_{I1}} = \frac{y_{I1}}{y_o}$$

(14)

since the plane SPDC-QM has the magnification $\frac{y_{I1}}{y_o} = 1$. Obviously, from $\Delta PP'V \sim \Delta I_1 I_{11}V$, we get $y_{I1}/y_o = S_1/S$ and then with (13) we obtain the magnification (11).

(L.3) Law of thin lens + plane SPDC-QM with the null crystal-lens distance

$$\frac{1}{S} + \frac{1}{\omega_i S'} = \frac{1}{f}, \quad M = \frac{\omega_s}{\omega_i} \frac{S'}{S}$$

(15)

**Proof:** Here we note that (L.4) is the particular case of (L.3) with $d=0$ for which we get $S_1 = Z_s$, and $S' = Z_i$. Then from (9) and (12) we obtain (15).

3.3. Thin lens combined with plane SPDC-QM and classical mirror.
Law of thin lens + plane SPDC-QM + classical mirror (see the basic geometric optical configuration presented in Fig. 3). The distances S (lens-object), S'_1 (lens-crystal-first image plane I_1), S'_2 (lens-crystal-second image plane I_2), D_{CI_1} (crystal-first image plane), D_{CI_2} (crystal-second image plane) and f (focal distance of lens), must satisfy the following law:

$$\frac{1}{S} + \frac{1}{S'_{1} + (\frac{\omega_{s}}{\omega_{i}} - 1) D_{CI_{1}}} = \frac{1}{f} \quad (16)$$

and the magnification M_1 given by

$$M_1 = \frac{S'_{1} + (\frac{\omega_{s}}{\omega_{i}} - 1) D_{CI_{1}}}{S} \quad (17)$$

and

$$\frac{1}{S + 2D_{OM}} + \frac{1}{S'_{2} + (\frac{\omega_{s}}{\omega_{i}} - 1) D_{CI_{2}}} = \frac{1}{f} \quad (18)$$

the magnification M_2 given by

$$M_2 = \frac{S'_{2} + (\frac{\omega_{s}}{\omega_{i}} - 1) D_{CI_{2}}}{S} \quad (19)$$

where \(D_{OM}\) is the distance from object to the classical mirror M (see Fig. 3). The proof of (L.4) is similar to that of (L.3) and here will be omitted.

4 Experimental tests for the geometric SPDC-quantum optics

For an experimental test of the Gauss like law of the thin lens combined with a plane SPDC-QM we propose an experiment based on a detailed setup presented in Fig. 4 and in the optical geometric configuration shown in Fig. 2b. Then, we predict that the image I of the object P (illuminated by a high quality signal laser SL with \(s(\omega_s, k_s, e_s, \mu_s)\)) will be observed in the idler beam, \(i(\omega_i, k_i, e_i, \mu_i) \equiv i(\omega_p-\omega_s, k_p-k_s, e^*_s, -\mu_s)\), when distances lens-object (S), lens-crystal-image plane (S'), crystal-image plane (D_{CI}) and focal distance f of lens, satisfy thin lens+QM law (10). Moreover, if thin lens+QM law (10) is satisfied, the image I of that object P can be observed even when instead of the signal source SL we put a detector D_s. This last statement is clearly confirmed recently, in the degenerate case \(\omega_s = \omega_i = \omega_p/2\), by
a remarkable two-photon imaging experiment [8]. Indeed, in these recent experiments, inspired by the papers of Klyshko et al (see refs. quoted in [9]), was demonstrated some unusual two-photon effects, which looks very strange from classical point of view. So, in these experiments, an argon ion laser is used to pump a nonlinear BBO crystal ($\beta$-BaB$_2$O$_4$) to produce pairs of orthogonally polarized photons (see Fig. 1 in ref. [8] for detailed experimental setup). After the separation of the signal and idler beams, an aperture (mask) placed in front of one of the detectors ($D_s$) is illuminated by the signal beam through a convex lens. The surprising result consists from the fact that an image of this aperture is observed in coincidence counting rate by scanning the other detector ($D_i$) in the transverse plane of the idler beam, even though both detectors single counting rates remain constants. For understanding the physics involved in their experiment they presented an ”equivalent “ scheme ( in Fig. 3 in ref. [8]) of the experimental setup. By comparison of their ”scheme” with our optical configuration from Fig. 2b we can identify that the observed validity of the two-photon Gaussian thin-lens equation

$$\frac{1}{f} = \frac{1}{S} + \frac{1}{S'}$$

as well as of the linear magnification

$$M_0 = \frac{S'}{S} = 2$$

can be just explained by our results on the two-photon geometric law (10)-(11) of the thin lens combined with a plane SPDC-QM for the degenerate case $\omega_s = \omega_i = \omega_p/2$. Therefore, the general tests of the predictions (10)-(11) using a setup described in Fig. 4, are of great importance not only in measurements in presence of the signal laser LS (with and without coincidences between LS and idler detector $D_i$), but also in the measurements in which instead of the laser LS we put the a signal detector $D_s$ in coincidence with $D_i$.

5 Conclusions

In this paper the class of the SPDC-phenomena (1) is enriched by the introducing the crossing symmetric SPDC-processes (2)-(3) satisfying the same energy-momentum conservation law (4). Consequently, the kinematical correlations (4)-(8) in conjunction with the Snellius relations (7) allow us to
introduce a new kind of optical devices called *quantum mirrors*. Then, some laws of the *quantum mirrors*, such as: law (9) of *thin plane SPDC-quantum mirror*, the law (10)-(11) of the *thin lens combined with a plane SPDC-QM*, as well as, the laws (16)-(19), are proved. These results are natural steps towards a *new geometric optics* which can be constructed for the kinematical correlated SPDC-photons. In particular, the results obtained here are found in a very good agreement with the recent results [8] on *two-photon imaging experiment*. Moreover, we recall that all the results obtained in the *two-photon ghost interference-diffraction* experiment [6] was recently explained by using the concept of *quantum mirrors* (see Ref. [3]).

Finally, we note that all these results can be extended to the case of the *spherical quantum mirrors*. Such results, which are found in excellent agreement to the recent experimental results [7] on *two-photon geometric optics*, will be presented in a future publication. (This paper was published in Romanian Journal of Physics, Vol.45, P. 15, Bucharest 2000)

**References**

[1] A. Yariv, *Quantum Electronics*, Wiley, New York, 1989.

[2] See e. g., A. D. Martin and T. D. Spearman, *Elementary Particle Theory*, Nord Holland Publishing Co., Amsterdam, 1970.

[3] D. B. Ion and P. Constantin, *A New Interpretation of two-photon entangled Experiments, NIPNE-1996 Scientific Report*, National Institute for Physics and Nuclear Engineering Horia Hulubei, Bucharest, Romania, p. 139; D. B. Ion, P. Constantin and M. L. D. Ion, Rom. J. Phys. 43 (1998) 3.

[4] M. W. Evans, in *Modern Nonlinear Optics, Vol. 2*, (M. W. Evans and S. Kielich (Eds)), John Wiley&Sons, Inc.,1993, pp.249.

[5] For a review see for example: D. M. Pepper and A. Yariv, in R. A. Fischer (Ed.), *Optical Phase Conjugation*, Academic Press, Inc., 1983, p 23; See also: H. Jagannath et al., *Modern Nonlinear Optics, Vol 1*, (M. Evans and S .Kielich (Eds)) John Wiley&Sons, Inc.,1993, pp.1.

[6] D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, Phys. Rev. Lett. 74 (1995) 3600.
[7] T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, Phys. Rev. A 52 (1995) R3429.

[8] T. B. Pittman, D. V. Strekalov, D. N. Klyshko, M. H. Rubin, A. V. Sergienko, and Y. H. Shih, Phys. Rev. A 53 (1996) 2804.

[9] A. V. Belinski and D. N. Klyshko, Sov. JETP 78 (1994) 259.
Figures

Fig. 1: The basic optical configuration of a plane SPDC-quantum mirror.

Fig. 2a: The basic optical configuration for usual lens combined with a plane SPDC-quantum mirror.
Fig. 2b: The basic optical configuration for a proof of magnification factor for a usual lens combined with a plane SPDC-quantum mirror.

Fig. 3: The basic optical configuration for usual lens combined with a plane SPDC-quantum mirror and with a classical mirror.
Fig. 4: The scheme of the experimental setup for a test of the geometric optics of correlated photons. The QM indicates the SPDC-quantum mirror, PBS is a polarization beam splitter, SL is a signal laser, P is an object, L a convergent lens, Dₗ is an idler detector and CC is the coincidence circuit.