Leptogenesis and $\mu - \tau$ symmetry

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If an exact $\mu \leftrightarrow \tau$ symmetry is the explanation of the maximal atmospheric neutrino mixing angle, it has interesting implications for the origin of matter via leptogenesis in models where small neutrino masses arise via the seesaw mechanism. For seesaw models with two right handed neutrinos ($N_\nu, N_\tau$), lepton asymmetry vanishes in the exact $\mu \leftrightarrow \tau$ symmetric limit, even though there are nonvanishing Majorana phases in the neutrino mixing matrix. On the other hand, for three right handed neutrino models, lepton asymmetry is nonzero and is given directly by the solar mass difference square. We also find an upper bound on the lightest neutrino mass.

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INTRODUCTION

One of the most puzzling aspects of neutrino mixings observed in various oscillation experiments is the near maximal value of the $\nu_\mu - \nu_\tau$ mixing angle (i.e. $\theta_{23} \approx \pi/4$). This was needed to explain the original atmospheric neutrino data and is now supported by data from the K2K experiment that uses accelerator neutrinos. The corresponding parameter in the quark sector is very small (about 4%) and is believed to be connected to the mass hierarchy among quarks. The large value of $\theta_{23}$ may therefore be telling us about some new symmetries of Nature.

In the basis where charged leptons are mass eigenstates, a symmetry that has proved useful in understanding maximal atmospheric neutrino mixing is $\mu \leftrightarrow \tau$ interchange symmetry. The mass difference between the muon and the tau lepton of course breaks this symmetry. Thus we expect this symmetry to be an approximate one. It may however happen that the symmetry is truly exact at a very high scale; but at low mass scales, the effective theory only has the $\mu - \tau$ symmetry in the neutrino couplings but not in the charged lepton sector so that we have $m_\tau \gg m_\mu$. We will consider this class of theories in this note. For this case, a convenient parameterization of the neutrino mass matrix is (assuming the neutrinos to be Majorana fermions):

$$
\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_{\odot}}}{2} \begin{pmatrix}
ce^n & dc & de \\
dc & 1 + \epsilon & -1 \\
de & -1 & 1 + \epsilon
\end{pmatrix}
$$

where $n \geq 1$. An immediate prediction of this mass matrix is that $\theta_{23} = \pi/4$ and $\theta_{13} = 0$; we also get $\epsilon \sim \sqrt{\Delta m^2_{\odot}/\Delta m^2_{\odot}}$.

We can now use $\theta_{13}$ as a probe of how leptonic $\mu \leftrightarrow \tau$ symmetry is broken in Nature and through that one may hope for an understanding of the origin of the near maximal (maximal?) $\theta_{23}$, as has been emphasized in ref. (1) (and also perhaps the $\mu - \tau$ mass difference). In particular, different ways of breaking $\mu \leftrightarrow \tau$ symmetry will lead to $\theta_{13} \sim \sqrt{\Delta m^2_{\odot}/\Delta m^2_{\odot}}$ or $\theta_{13} \sim \Delta m^2_{\odot}/\Delta m^2_{\odot}$. These predictions are clearly timely and interesting in view of many proposals to measure the parameter $\theta_{13}$.

1 The $\mu - \tau$ symmetry in supersymmetric seesaw models also leads to other phenomenological predictions such as the $B(\mu \rightarrow e + \gamma)/B(\mu \rightarrow e\nu\bar{\nu}) = B(\tau \rightarrow e + \gamma)/B(\tau \rightarrow e\nu\bar{\nu})$. 

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In this paper, we discuss implications of exact $\mu \rightarrow \tau$ symmetry for the origin of matter via leptogenesis and find several new results: (i) we find that if there are only two right handed neutrinos ($N_\mu, N_\tau$) that via seesaw mechanism lead to neutrino masses, then primordial lepton asymmetry arising from right handed neutrino decay vanishes in the $\mu - \tau$ symmetric limit even though in the low energy neutrino mass matrix may have Majorana phases; (ii) secondly, for the case of three right handed neutrinos, the primordial lepton asymmetry is directly proportional to the solar mass difference square. These predictions are very different from the generic three neutrino case. In both these case we assume that neutrino masses arise via the type I seesaw formula. These results are independent of any detailed model.

**PRIMORDIAL LEPTON ASYMMETRY WITH TWO RIGHTHANDED NEUTRINOS**

We start with the neutrino part of the superpotential:

$$W = e^T Y_\ell L d + N^c T Y_\nu L h_u + \frac{1}{2} M_R N^c N^c$$

where we assume that $N^c \equiv (N^c_\mu, N^c_\tau)$. As noted earlier, we work in a basis where $Y_\ell$ is diagonal. While naively, one may think that in such models $m_\mu = m_\tau$, there are models where one can split the muon and tau masses consistent with this symmetry in the neutrino sector.

The basic assumption of this work is that we have models where $Y_\nu$ and $M_R$ obey $\mu \leftrightarrow \tau$ symmetry under which $(N_\mu \leftrightarrow N_\tau)$ and $L_\mu \leftrightarrow L_\tau$ whereas the $m_\mu \neq m_\tau$. The general structure of $Y_\nu$ and $M_R$ are then given by:

$$M_R = \begin{pmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{pmatrix}$$

$$Y_\nu = \begin{pmatrix} h_{11} & h_{22} & h_{23} \\ h_{11} & h_{23} & h_{22} \end{pmatrix}$$

The seesaw formula in our notation is

$$M_\nu = -Y_\nu^T M_R^{-1} Y_\nu v_{u,w}^2$$

and the formula for primordial lepton asymmetry in this case, caused by right handed neutrino decay is

$$\epsilon_1 = \frac{1}{4\pi} \sum_j \left| \frac{\text{Im}[\tilde{Y}_\nu \tilde{Y}_\nu^T]_{12}^2}{\tilde{Y}_\nu \tilde{Y}_\nu^T_{11}} \right| F(x)$$

where $\tilde{Y}_\nu$ is defined in a basis where righthanded neutrinos are mass eigenstates and $F(x) \simeq -\frac{x}{2}$ for small $x$ which follows from our assumption that the right handed neutrino masses are hierarchical. In order to use this formula, we must diagonalize the righthanded neutrino mass matrix and change the $Y_\nu$ to $\tilde{Y}_\nu$. Since $M_R$ is a symmetric complex $2 \times 2$ matrix, it can be diagonalized by a transformation matrix $U(\pi/4) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ i.e. $U(\pi/4) M_R U^T(\pi/4) = \text{diag}(M_1, M_2)$ where $M_{1,2}$ are complex numbers. In this basis we have

$$\tilde{Y}_\nu = U(\pi/4)Y_\nu.$$ We can therefore rewrite the formula for $n_\ell$ as

$$\epsilon_1 \propto \sum_j \text{Im}[U(\pi/4) Y_\nu Y_\nu^T(\pi/4)]_{12}^2 F(M_1/M_2)$$

Now note that $Y_\nu Y_\nu^T$ has the form $\begin{pmatrix} A & B \\ B & A \end{pmatrix}$ which can be diagonalized by the matrix $U(\pi/4)$. Therefore it follows that $n_\ell = 0$.

An interesting feature of this model is that one can determine the neutrino masses and mixings explicitly in terms of the parameters of the model. We find a hierarchical mass pattern i.e. $m_1 \ll m_2 \ll m_3$ with the lightest neutrino being massless i.e.

$$m_1 = 0; m_2 = \frac{2}{M_+} (h_2^2 + 2h_1^2); m_3 = \frac{2}{M_-} (h_2^2)$$

where $M_\pm$ are the masses of the two right handed neutrinos with $M_- \ll M_+$ and $h_\pm = (h_2^2 + h_3^2)$.

Even though there is no lepton asymmetry in the model, there are Majorana CP phases in the light neutrino mixing which we denote by $K = (e^{i\alpha_1}, e^{-i\alpha_2})$. It is easy to see the origin of the phases: by appropriate choice of the phases of the fields one can show that $M_R$ has only one phase and $Y_\nu$ also has only one phase. After using the seesaw formula, one gets the light neutrino mass matrix which therefore has only one phase after redefinition of the light neutrino fields.

**$\mu - \tau$ SYMMETRY BREAKING WITH TWO RIGHT HANDED NEUTRINOS**

From the above discussion, it is natural to expect the model to have nonzero lepton asymmetry once $\mu - \tau$ symmetry is broken, as well as also a nonvanishing $\theta_{13}$. One may then expect that $\epsilon_1 \propto \theta_{13}$. The details however depend on how the symmetry is broken. As an example we note that when the symmetry is broken only by the masses of the RH neutrinos i.e. a RH neutrino mass matrix of the form $M_R = \text{diag}(M_1, M_2)$ and no off diagonal terms, since $Y_\nu Y_\nu^T$ is a real matrix, $\epsilon_1 = \text{Im}[Y_\nu Y_\nu^T]_{12} = 0$ despite the $\mu - \tau$ symmetry breaking. It is easy to check that $\theta_{13} \simeq \frac{\epsilon_1}{\epsilon_1} \propto (M_1 - M_2) \neq 0$.

One may however break $\mu \leftrightarrow \tau$ symmetry in the Dirac mass terms for the neutrinos i.e. in $Y_\nu$. This can be done in many ways e.g. by choosing $Y_\nu = \begin{pmatrix} h_{11} & h_{22} & h_{23} \\ h_{12} & h_{23} & h_{22} \end{pmatrix}$ or $Y_\nu = \begin{pmatrix} h_{11} & h_{22} & h_{23} \\ h_{11} & h_{23} & h_{33} \end{pmatrix}$ etc. In all these cases, one
gets $\epsilon_1 \neq 0$ and also $\theta_{13} \neq 0$ and $\theta_A \neq \pi/4$. One lesson one can draw from this observation is that, if leptogenesis is the true mechanism for the origin of matter, then the limit on $\theta_{13}$ going down by an order of magnitude could teach us about the nature of right handed neutrino spectrum. For instance, a very small $\theta_{13}$ (i.e. $\theta_{13} \leq \frac{\Delta m^2_{31}}{2M_\nu}$) would indicate a nearly exact $\mu - \tau$ symmetry and therefore sufficient leptogenesis would then require the existence of three right handed neutrinos or some complicated way of breaking $\mu - \tau$ symmetry.

THE CASE OF THREE RIGHT HANDED NEUTRINOS

In this case, the right handed neutrino mass matrix $M_R$ and the Dirac Yukawa coupling $Y_\nu$ can be written respectively as:

$$M_R = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{22} \end{pmatrix} \quad (9)$$

$$Y_\nu = \begin{pmatrix} h_{11} & h_{12} & h_{12} \\ h_{21} & h_{22} & h_{23} \\ h_{21} & h_{23} & h_{22} \end{pmatrix}$$

where $M_{ij}$ and $h_{ij}$ are all complex. An important property of these two matrices is that they can be cast into a block diagonal form by the transformation matrix $U_{23}(\pi/4) \equiv \begin{pmatrix} 1 & 0 \\ 0 & U(\pi/4) \end{pmatrix}$ and then be subsequently diagonalized by the most general $2 \times 2$ unitary matrix as follows:

$$V^T(2 \times 2) U_{23}^T(\pi/4) M_R U_{23}(\pi/4) V(2 \times 2) = M_R^d \quad (10)$$

where $V(2 \times 2) = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix}$ where $V$ is the most general $2 \times 2$ unitary matrix given by $V = e^{i\alpha} P(\beta) R(\theta) P(\gamma)$ with $P(\beta) = \text{diag}(e^{i\beta}, e^{-i\beta})$; $R(\theta) = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$; $(c, s)$ being cosine and sine of $\theta$ respectively. We will denote $V(2 \times 2)$ simply by $V_{LR}$ depending on whether it acts on left handed or the RH neutrinos.

We now change to the basis where the right handed neutrino mass matrix is diagonal (Eq.(10)). The Dirac Yukawa coupling in this basis has the form

$$\tilde{Y}_\nu = V^T(2 \times 2) U_{23}^T(\pi/4) Y_\nu \quad (11)$$

Due to the special form of $Y_\nu$ dictated by $\mu \leftrightarrow \tau$ symmetry, it is easy to see that

$$\tilde{Y}_\nu = V^T(2 \times 2) Y_\nu' U_{23}^T(\pi/4) \quad (12)$$

where $Y_\nu'$ is in block diagonal form. An important point to realize at this stage is that the the $3 \times 3$ matrix problem has reduced to a $2 \times 2$ problem. So all the matrices from now on will be $2 \times 2$ and the third neutrino (the heaviest of the light neutrinos) has completely “decoupled” from the considerations below of both seesaw formula for neutrino masses as well as lepton asymmetry. This is a direct consequence of $\mu - \tau$ symmetry and of course considerably simplifies the discussions.

Restricting to the $2 \times 2$ case, we can use the seesaw formula to write down the left handed neutrino mass matrix as follows in units of $-\nu_{atk}^2$:

$$M_\nu = -\tilde{Y}_\nu^T M_R^{d - 1} \tilde{Y}_\nu \quad (13)$$

Next, we go to a basis where $M_\nu$ (the upper $2 \times 2$ block of it) is diagonalized by a matrix $V_L$ i.e. $V_L^T M_\nu V_L = M^d_\nu$. In this basis, the Dirac Yukawa coupling $Y_\nu$ becomes $V_L^T Y'_\nu \equiv Y'_\nu$. Let us write $Y'_\nu$, which is a $2 \times 2$ matrix as

$$Y'_\nu = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \quad (14)$$

The $Z_{ij}$ obey the constraints: $Z_{12} = -Z_{21} \frac{M_{22}}{M_{11} M_{12}}$ and the neutrino masses are given by

$$m_1 = \frac{Z_{11}^2}{M_1} \rho \text{e}^{i\eta} \quad (15)$$

$$m_2 = \frac{Z_{22}^2}{M_2} \rho \text{e}^{i\eta}$$

where $\rho \text{e}^{i\eta} = \left(1 + \frac{M_{12}^2}{M_{11} M_{12}}\right)$.

Let us now calculate the out of equilibrium for the decay of the lightest right handed neutrino, which we assume to be the lighter of the two mass eigenstates of the $2 \times 2$ right handed neutrino mass matrix considered above. It is given by:

$$\Gamma_1 = \frac{1}{8\pi} (Y'_\nu Y'^{-1}_\nu)_{11} M_1$$

$$= \frac{M_1 (|Z_{11}|^2 + |Z_{12}|^2)}{8\pi} \leq 14 \frac{M_L^2}{M_{PL}} \quad (16)$$

where $M_{PL}$ appears in the right hand side from the Hubble expansion formula $H^2 \simeq \sqrt{g_T} T^2/M_{PL}$ in a radiation dominated Universe. Using Eq.(15), which gives $(|Z_{11}|^2 + |Z_{12}|^2) \simeq \frac{M_1}{\rho \text{e}^{i\eta} - 1} [m_1 + |\rho \text{e}^{i\eta} - 1||m_2|]$, we can rewrite this inequality as a constraint on the following combination of the masses of the two lightest neutrino eigenstates:

$$\frac{|m_1| + |\rho \text{e}^{i\eta} - 1||m_2|}{\rho} \leq 10^{-3} \text{ eV} \quad (17)$$

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2 After this paper was posted, it was brought to our attention that leptogenesis for a $\mu - \tau$ symmetric model with the specific restriction that $Y_\nu = \text{diag}(a, b, b)$ was considered in Ref. [11]. Our consideration is more general.
For hierarchical right handed neutrino mass spectrum (i.e. $M_2 > M_1$), $\rho \sim 1$ and we get

$$|m_1 + 2|m_2| |\sin \eta/2| \leq 10^{-3} \text{ eV}$$  \hspace{1cm} (18)

This puts a limit on the two lightest neutrino masses. For instance, it implies that the lightest neutrino mass $m_1 \leq 10^{-3} \text{ eV}$. The solar neutrino oscillation would require $|\sin \eta/2| \sim 0.07$ so that $m_2$ will match the central value required by data.

We now proceed to calculate the primordial lepton asymmetry $\epsilon_1$ in this model. It turns out that $\epsilon_1$ is directly proportional to the solar mass difference square as we show below. We start with the expression for $\epsilon_1$,

$$\epsilon_1 \approx \frac{3}{8 \pi} \frac{M_1}{M_2} \frac{\Delta m_{21}^2 \sin \eta}{|m_1 + |\rho e^{\phi m} - 1||m_2|}$$  \hspace{1cm} (19)

Using the constraints on $Z_{ij}$ discussed in Eq. (15) and the relation just prior to it, we get,

$$|Z_{ij}|^4 Im \left( \frac{Z_{21}^{2}}{Z_{11}^{2}} \right) + |Z_{21}|^2 \frac{M_1}{M_2} Im \left( \frac{Z_{21}^{2}}{Z_{11}^{2}} \right) \sin \eta$$

Plugging this expression into Eq. (19), we can express the primordial lepton asymmetry $\epsilon_1$ in terms of neutrino masses $m_{1,2}$ and the parameters $\rho$ and $\eta$ as follows:

$$\epsilon_1 \approx \frac{3}{8 \pi} \frac{M_1}{v_{ek}^2} \frac{\Delta m_{21}^2 \sin \eta}{|m_1 + |\rho e^{\phi m} - 1||m_2|} \times \frac{10^{-3}}{|m_1 + |\rho e^{\phi m} - 1||m_2|}$$  \hspace{1cm} (20)

We see that the origin of matter in this model is predicted primarily in terms of the solar mass difference square and the unknown phase $\eta$ whose value is already determined by Eq. (18). Thus given a value for the lightest right handed neutrino mass, the model predicts the value of primordial lepton asymmetry $\epsilon_1$. In Eq. (20), we have assumed $M_1 \approx 10^{10} \text{ GeV}$. Note that our result is based on only three assumptions: (i) type I seesaw formula for neutrino masses and (ii) hierarchy among right handed neutrinos. This is very different from generic seesaw models without $\mu \leftrightarrow \tau$ where the dominant contribution to $\epsilon_1$ comes from the atmospheric neutrino mass difference square and depends on unknown parameters related to the Dirac neutrino Yukawa coupling. It is also interesting that origin of matter is tied to the existence of solar neutrino oscillation and it is the LMA solution to the solar neutrino problem that reproduces the correct order of magnitude for the lepton asymmetry which after taking into the dilution factor and sphaleron effects, can give rise to the magnitude for the observed baryon to photon ratio. The value of $10^{10} \text{ GeV}$ for the mass of the lightest right handed neutrino is chosen to show that the model when embedded into an extension of MSSM can avoid the reheat temperature constraint coming from gravitino production. Finally it is important to stress that this result is valid for both normal and inverted mass hierarchy among light neutrinos.

In conclusion, we have discussed the consequences of the hypothesis that the large atmospheric neutrino mixing angle arises from an intrinsic $\mu \leftrightarrow \tau$ symmetry for leptons for origin of matter via leptogenesis. We point out that if there are two right handed neutrinos obeying $\mu \leftrightarrow \tau$ interchange symmetry, then lepton asymmetry vanishes whereas for three right handed neutrinos, it is given directly the solar mass difference square provided one assumes type I seesaw formula for neutrino masses. We also obtain an upper limit on the lightest neutrino mass of a milli-eV under these assumptions.

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