RELATIVELY MOVING SYSTEMS IN “TRUE TRANSFORMATIONS RELATIVITY”

Tomislav Ivezić

Rudjer Bošković Institute, P.O.B. 180, 10002 Zagreb, Croatia
ivezic@sirb.hr

In this paper the physical systems consisting of relatively moving subsystems are considered in the “true transformations relativity”. It is found in a manifestly covariant way that there is a second-order electric field outside stationary current-carrying conductor. It is also found that there are opposite charges on opposite sides of a square loop with current and these charges are invariant charges.

Key words: covariant length, current, electric field and charge.

Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows and only a kind of union of the two will preserve an independent reality. - H. Minkowski

1. INTRODUCTION

In the recent paper [1] I have shown that due to the fundamental difference between the true transformations (TT) and the apparent transformations (AT) (see [1] and [2]) one can speak about two forms of relativity: the “TT relativity” and the “AT relativity.” The “TT relativity,” which is a covariant formulation of relativity, is based on the TT of physical quantities as 4-dimensional (4D) spacetime tensors, i.e., on the covariant definition of the spacetime length, and the covariant electrodynamics with 4-vectors $E^\alpha$ and $B^\alpha$, see [1] and [3]. This formulation of electrodynamics is equivalent to the usual covariant electrodynamics with the electromagnetic field tensor $F^{\alpha\beta}$, as shown in [1]. The TT are the transformations of 4D spacetime tensors referring to the same quantity (in 4D spacetime) considered in different inertial frames of reference (IFRs), or in different coordinatizations of some IFR. The TT do conform with the special relativity as the theory of 4D spacetime with pseudo-Euclidean geometry, i.e., they leave the interval $ds$ and thus the geometry of spacetime unchanged. An example of the TT are the Lorentz transformations (LT) of 4D tensor quantities. The “AT relativity” is the conventional special relativity based on Einstein’s relativity of simultaneity and on the synchronous definition of the spatial length, i.e., on the AT of the spatial length (the Lorentz contraction, see [1,2,3,4]) and the time distance (the conventional dilatation of time), and, as shown in [1] (see also [3]), on the AT of the electric and magnetic three-vectors (3-vectors) $\mathbf{E}$ and
B (the conventional transformations of \( E \) and \( B \)). The AT are not the transformations of 4D spacetime tensors and they do not refer to the same quantity (in 4D spacetime), but, e.g., they refer to the same measurement in different IFRs.

In this paper we investigate physical systems consisting of relatively moving subsystems, as it is a current-carrying conductor (CCC), using a covariant formulation of physical quantities and physical phenomena, i.e., the “TT relativity”. First we examine the covariant definition of length when defined in geometrical terms and in different coordinatizations of an IFR. We also report an expression for the Lorentz transformations, which is independent of the chosen synchronization, i.e., coordinatization of an IFR. Further, the AT of the spatial length - the Lorentz contraction - is examined in detail. Then the covariant definition of length in Einstein’s coordinatization is applied to the consideration of the well-known “relativistic” paradox “Car and garage paradox.” It is found that in the “TT relativity” and, if one wants to retain the connection with the prerelativistic physics in which one deals with the “spatial length”, then only the rest length (volume) of the object is well defined quantity.

From this result and the covariant definition of charge we also find that in the “TT relativity” the charge density as the three-dimensional (3D) quantity has definite physical meaning only for charges at rest. In order to avoid from the beginning the misunderstanding of the “TT relativity” and of our choice of the rest frame of the object, as the starting frame for the definitions of 4D quantities, we emphasize that the “TT relativity” is covariant in the usual sense. In the “TT relativity” one can define 4D physical quantities and investigate physical laws connecting such 4D quantities in any IFR, not only in the rest frame of the object. The LT will correctly connect the results of measurements of the same 4D quantity in two, arbitrary, relatively moving IFRs. Thus, the “TT relativity” does not use a preferred reference frame. Our choice of the rest frame of the object does not mean in any way that this frame is a preferred IFR. The rest frame is, in fact, the most convenient for the purpose of comparison with the prerelativistic physics, in which one does not deal with 4D quantities but with “3+1” quantities (the quantities defined in “3+1” space and time), and with the “AT relativity,” in which one works in 4D spacetime but with quantities, e.g., the spatial length, the time distance, the 3-vectors \( E \) and \( B \), etc., that are not 4D tensor quantities. Taking this into account we show that the current density 4-vector \( j^\mu \) for a CCC in an arbitrary IFR is determined as the sum \( j_+^\mu + j_-^\mu \), where the current density 4-vectors \( j_+^\mu \) and \( j_-^\mu \) for positive and negative charges, respectively, have to be found in their own rest frames, and then transformed by the Lorentz transformation to the considered IFR. Then in Sec.3.2 we quote the covariant Maxwell equations when written by the electromagnetic field tensor \( F^{\alpha\beta} \) and by the 4-vectors \( E^\alpha \) and \( B^\alpha \), (both forms were already found in [1]), and also we report a new form - the covariant Majorana form of Maxwell’s equations. Then the 4-vectors \( E^\alpha \) and \( B^\alpha \) are determined for a CCC (instead of the usual 3-vectors \( E \) and \( B \)) and it is obtained in such a covariant way, i.e., in the “TT relativity,” that, for the observers at rest in the rest frame of that CCC, there is a second-order electric field outside stationary conductor with steady current. Such fields are already theoretically predicted on different
grounds in [5], see also [6]. In contrast to previous works we also find in such a
covariant manner that there are opposite charges on opposite sides of a square
loop with current and these charges are Lorentz invariant charges. In the usual
approach, i.e., in the “AT relativity,” it is found that there is an electric moment
\( P \) for a moving loop with a current. However we find that such loop, regarding
the electric effects, _always_, i.e., for a stationary loop as well, behaves at long
distances as an electric dipole (4-vector).

2. COVARIANT AND SYNCHRONOUS DEFINITIONS OF LENGTH

As discussed in [1], (and [3]) according to the “modern” point of view the
special relativity is the theory of 4D spacetime with pseudo-Euclidean geometry. Quantities of physical interest, both local and nonlocal, are represented in the
special relativity by spacetime tensors, i.e., as covariant quantities, and the
laws of physics are written in a manifestly covariant way as tensorial equations.
The geometry of the spacetime is generally defined by the invariant infinitesimal
spacetime distance \( ds \) of two neighboring points, \( ds^2 = dx^a g_{ab} dx^b \). I adopt the
following convention with regard to indices. Repeated indices imply summation. Latin indices \( a, b, c, d, \ldots \) are to be read according to the abstract index notation, see [7], Sec.2.4.. They designate geometric objects in 4D spacetime. Thus \( dx^a, b \)
and \( g_{ab} \), and of course \( ds \), are defined independently of any coordinate system,
e.g., \( g_{ab} \) is a second-rank covariant tensor (whose Riemann curvature tensor \( R_{abcd} \)
is everywhere vanishing; the spacetime of special relativity is a flat spacetime,
and this definition includes not only the IFRs but also the accelerated frames
of reference). Greek indices run from 0 to 3, while latin indices \( i, j, k, l, \ldots \) run
from 1 to 3, and they both designate the components of some geometric object
in some coordinate chart, e.g., \( x^\mu(x^0, x^i) \) and \( x^\nu(x^0, x^i) \) are two coordinate
representations of the position 4-vector \( x^a \) in two different inertial coordinate
systems \( S \) and \( S' \), and \( g_{\mu\nu} \) is the \( 4 \times 4 \) matrix of components of \( g_{ab} \) in some
coordinate chart.

2.1. The Spacetime or the TT Length

In general, in 4D spacetime of special relativity it is not possible to separate the
spatial and temporal parts of \( ds \), or according to Minkowski’s words, quoted here
as a motto, _the spatial and temporal parts taken separately loose their physical
meaning_. Therefore, only the invariant spacetime length (the Lorentz scalar)
between two points (events) in 4D spacetime does have definite physical meaning
in the “TT relativity” and it is defined as

\[
l = (l^a g_{ab} l^b)^{1/2},
\]

where \( l^a(l^b) \) is the distance 4-vector between two events \( A \) and \( B \), \( l^a = x^a_B - x^a_A \),
$x^A_{a,b}$ are the position 4-vectors, and $g_{ab}$ is the metric tensor.

Using different coordinatizations of a given reference frame, which can be realized, for example, by means of different synchronizations, we find different expressions, i.e., different representations of the spacetime length $l$, Eq. (1). Obviously the coordinates $x^\mu$ of an event, when written in some coordinatization of an IFR, do not have an intrinsic meaning in 4D spacetime. However the spacetime length $l$ (1) does have the same value for all relatively moving inertial coordinate systems and it represents an intrinsic feature of the spacetime.

Different synchronizations are determined by the parameter $\varepsilon$ in the relation

$$t_2 = t_1 + \varepsilon(t_3 - t_1),$$

where $t_1$ and $t_3$ are the times of departure and arrival, respectively, of the light signal, read by the clock at $A$, and $t_2$ is the time of reflection at $B$, read by the clock at $B$, that has to be synchronized with the clock at $A$. Usually physicists prefer Einstein’s synchronization convention with $\varepsilon = 1/2$ in which the measured coordinate velocity of light (the one-way speed of light) is constant and isotropic. A nice example of a non-standard synchronization is “everyday” clock synchronization [8] in which $\varepsilon = 0$ and there is an absolute simultaneity; see also [9] for an absolute simultaneity in the special relativity, and for the review on synchronisation and test theories see the recent article [10]. As explained in [8]: “For if we turn on the radio and set our clock by the standard announcement "...at the sound of the last tone, it will be 12 o’clock,” then we have synchronized our clock with the studio clock in a manner that corresponds to taking $\varepsilon = 0$ in $t_2 = t_1 + \varepsilon(t_3 - t_1).”

When Einstein’s synchronization of distant clocks and cartesian space coordinates $x^i_0$ are used in an IFR $S$ (this coordinatization will be named Einstein’s or “e” coordinatization) then, e.g., the geometric object $g_{ab}$ is represented by the $4 \times 4$ matrix of components of $g_{ab}$ in that coordinate chart, i.e., it is the Minkowski metric tensor $g_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, where “e” stands for Einstein’s coordinatization. With such $g_{\mu\nu}$ the space $x^i_0$ and time $t_0$ ($x^0_0 = c t_0$) components of $x^i_0$ do have their usual meaning. Then $ds^2$ can be written with the separated spatial and temporal parts, $ds^2 = (dx^i_0 dx_{i\nu}) - (dx^0_0)^2$, and the same happens with the spacetime length $l$ (1), $l^2 = (l^i l^0_0) - (l^0)^2$. Such separation remains valid in other inertial coordinate systems with the Minkowski metric tensor, and in $S’$ one finds $l'^2 = (l'^i l'^0_0) - (l'^0)^2$, where $l'^i_0$ in $S’$ is connected with $l^i_0$ in $S$ by the LT.

In the usual form the LT connect two coordinate representations (in the “e” coordinatization) $x^\mu_i$, $x'^\mu_i$ of a given event. $x^\mu_i$, $x'^\mu_i$ refer to two relatively moving IFRs (with the Minkowski metric tensor) $S$ and $S'$,

$$x^\mu_i = L^\mu_{i,\nu} x^\nu_{\nu}, \quad L^0_{0,0} = \gamma, \quad L^0_{i,0} = L^1_{i,0} = -\gamma V^1_0, \quad \delta^i_j + (\gamma - 1) V^i_0 V^j_0 V^2_0;$$

where $V^i_0 = dx^i_0/dt_0$ are the components of the ordinary velocity 3-vector, and $\gamma = (1 - V^2_0/c^2)^{1/2}$. As explained in [11], when such usual representations of pure Lorentz transformations are applied to covariant expressions they destroy the covariant form: “because they employ three-vector notation, because they treat the spatial and temporal components separately, and because they are parametrized by the ordinary velocity three-vector $V.$” In order to obtain a
covariant expression for $L^\mu_{\nu,e}$ the ordinary velocity is replaced in [11] by the proper velocity 4-vector $v^\mu_e \equiv dx^\mu_e/d\tau = (\gamma_e c, \gamma_e v^0_e)$, $d\tau \equiv dt_e/\gamma_e$ is the scalar proper-time, the unit vector $n^\mu_e \equiv (1, 0, 0, 0)$ along the temporal axis is introduced, and $\delta^\mu_j$ is replaced with the Minkowski metric tensor $g^\mu_{\nu,e}$. This shows that the cartesian space coordinates $x^i_e$ and Einstein’s synchronization of distant clocks are explicitly chosen in [11]. In such a way the covariant expression for $L^\mu_{\nu,e}$ in the “e” coordinatization is found in [11], Eq.(5),

$$L^\mu_{\nu,e} \equiv L^\mu_{\nu,e}(v) = g^\mu_{\nu,e} - \frac{2n^\mu_e v^\nu_e}{c} + \frac{(n^\mu_e + v^\mu_e/c)(n_\nu e + v^\nu e/c)}{1 - n_e \cdot v/c}. \tag{1}$$

Since we want to use the LT in different coordinatizations we generalize the expression for $L^\mu_{\nu,e}$ from [11] and find

$$L^a_b \equiv L^a_b(v) = g^a_b - \frac{2n^a e v^b e}{c} + \frac{(n^a + v^a /c)(n^b + v^b /c)}{1 - n \cdot v /c}. \tag{2}$$

Such form (2) of the LT can be applied to an arbitrary inertial coordinate system in which the metric tensor can be different than the Minkowski metric tensor, and thus the form of the covariant 4D Lorentz transformations (2) is independent of the chosen synchronization, i.e., coordinatization of reference frames. But we have to note that $n^a$ in (2) is a specific quantity. Namely it always has to be taken as the unit vector along the temporal axis in the chosen IFR and the chosen coordinatization. Nevertheless $L^a_b$ correctly transforms some 4D tensor quantity from an IFR to another relatively moving IFR. For example, when $L^a_b$ is applied to the position 4-vector $x^a$ one finds (in the abstract index notation)

$$x'^a = x^a + \frac{(n \cdot x - (2\gamma + 1) v \cdot x/c)n^a + (n \cdot x + v \cdot x/c)v^a/c}{1 - n \cdot v /c}. \tag{3}$$

Let us examine the relations (2) and (3) in two different coordinatizations. First in the “e” coordinatization, in which the Minkowski metric tensor is used, $n^a$ becomes $n^\mu_e \equiv (1, 0, 0, 0)$, $v^\nu_e = (\gamma_e c, \gamma_e v^0_e)$, and $\gamma_e = -n^\mu_e v_\mu e/c$, as in [11]. From the general relation $v^\nu v_\nu = -c^2$ one finds, in the “e” coordinatization, that $v^\nu_e = (c^2 + v^0_e v^0_e)^{1/2}$, which shows that the expression for $L^\mu_{\nu,e}$ is parametrized essentially by the three spatial components $v^i_e$ of the proper velocity 4-vector $v^\nu_e$. Then, using the above expressions for $n^\nu_e$, $v^\nu_e$, and $\gamma_e$ one finds from (2) and (3) the usual expressions for pure LT, as in [11], i.e., the above mentioned $L^\mu_{\nu,e}$ and $x'^a$, but with $v^\nu_e$ replacing the components of the ordinary velocity 3-vector $V$. Also, we find the above mentioned usual expressions in the “e” coordinatization for $ds^2 = ds_e^2 = (dx^i_e dx^i_e) - (dx^0_e)^2$ and $l^2_e = l^2_e = (l^i_e l^i_e) - (l^0_e)^2$, with the separated spatial and temporal parts.

In the similar way we use the relations (2) and (3) to write the corresponding expressions in another coordinatization, “r” coordinatization, of an IFR, which is found in [8], where “everyday” or “radio” synchronization of distant clocks is used. For simplicity we consider 2D spacetime as in [8]. Then the metric tensor $g_{ab}$ becomes $g_{\mu\nu r} = \begin{pmatrix} -1 & -1 \\ -1 & 0 \end{pmatrix}$, where “r” stands for “radio” (it differs from
that one in [8] since the Minkowski tensors are different). The LT $L^\mu\nu,r$ in the “r” coordinatization can be easily found from (2), from the known $g_{\mu\nu,r}$, with $n_1^e = (1,0)$ and $\gamma_r = -n_1^e\gamma/e = \gamma_r$. These relations can be found as in [8], or by means of the matrix $T^\mu\nu$, which is given below. Thus the pure Lorentz transformation matrix $L_a^b$ (2) becomes in the “r” coordinatization

$$L^\mu\nu,r = \begin{pmatrix} K & 0 \\ -\beta_r/K & 1/K \end{pmatrix},$$

Also we find the “r” representation $x^\mu_r$ of $x^a$ (3),

$$x^0_r = Kx^0_e, \quad x^1_r = (1/K)(-\beta_rx^0_e + x^1_e),$$

where $K = (1 + 2\beta_r)^{1/2}$ and $\beta_r = dx^1_r/dx^0_r$ is the velocity of the frame $S'$ as measured by the frame $S$. Further $ds^2 = dx^a g_{ab} dx^b$ becomes in the “r” coordinatization $ds^2 = ds^2 = -\left[(dx^0_r)^2 + 2dx^0_r dx^1_r\right]$. We see that in the “r” coordinatization the spatial and temporal parts of $ds^2$ are not separated, that is different than in the coordinatization with the Minkowski metric tensor. The same holds for the spacetime length $l$, which is in the “r” coordinatization determined as $l^2 = l^2_r = -\left[(l^0_r)^2 + 2l^0_r l^1_r\right]$. Expressing $dx^\mu_e$, or $l^\mu_e$, in terms of $dx^\mu_r$, or $l^\mu_r$ (the transformation matrix between “r” and “e” coordinatizations is

$$T^\mu\nu = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix},$$

whence, e.g., $x^0_r = x^0_e - x^1_e$, $x^1_r = x^1_e$, and $\beta_r = \beta_e/(1 - \beta_e)$, see [11]) one finds that $ds^2_e = ds^2_r$, and also, $l^2_e = l^2_r$, as it must be.

The whole preceding discussion about the geometric quantities $x^a$, $l^a$, $ds$, $l$, .. and their different representations can be illustrated in a way which better clarifies the difference between two sorts of quantities. Again we consider the TT length (1) (we use the words - the TT length, the spacetime length, and the covariantly defined length as synonyms) in two relatively moving IFRs $S$ and $S'$ and in two coordinatizations “e” and “r” in these IFRs. Now, let the spacetime be endowed with base vectors, the temporal and the spatial base vectors. The bases $\{e_\mu\}$, with the base vectors $\{e_0, e_1\}$, and $\{r_\mu\}$, with the base vectors $\{r_0, r_1\}$, are associated with “e” and “r” coordinatizations, respectively, of a given IFR. The temporal base vector $e_0$ is the unit vector directed along the world line of the clock at the origin. The spatial base vector by definition connects simultaneous events, the event “clock at rest at the origin reads 0 time” with the event “clock at rest at unit distance from the origin reads 0 time”, and thus it is synchronization-dependent. The spatial base vector $e_1$ connects two above mentioned simultaneous events when Einstein’s synchronization ($\varepsilon = 1/2$) of distant clocks is used. The temporal base vector $r_0$ is the same as $e_0$. The spatial base vector $r_1$ connects two above mentioned simultaneous events when “everyday” clock synchronization ($\varepsilon = 0$) of distant clocks is used. All the spatial base vectors $r_1, r'_1, ..$ are parallel and directed along an (observer-independent) light line. Hence, two events that are everyday (“r”) simultaneous
in S are also “r” simultaneous for all other IFRs. The connection between the bases \( \{e_\mu\} \) and \( \{r_\mu\} \) is \( r_0 = e_0, r_1 = e_0 + e_1 \), see [8]. Then the geometrical quantity, e.g., the distance 4-vector \( l_{AB}^a \) between two events A and B, will be represented by the vector in 2D spacetime, which have different decompositions, representations, with respect to \( \{e_\mu\} \), \( \{e'_\mu\} \) and \( \{r_\mu\} \), \( \{r'_\mu\} \) bases. Note that in the “TT relativity” the same distance 4-vector \( l^a \) is decomposed with respect to

\[
S \{ \}
\]

and not with 4D quantities. \( A \) and \( B \) are decomposed with respect to \( \{e_\mu\} \) base as \( x^a_A = x^0_A e_0 + x^1_A e_1 = 0e_0 + 0e_1 \), and \( x^a_B = x^0_B e_0 + x^1_B e_1 = 0e_0 + l_0 e_1 \), and the distance 4-vector \( l_{AB}^a = x^a_B - x^a_A \) is decomposed as

\[
l_{AB}^a = l^0 e_0 + l^1 e_1 = 0e_0 + l_0 e_1.
\]

Thus in S the position 4-vectors \( x^a_{A,B} \) are determined simultaneously, \( x^0_{Be} - x^0_{Ae} = l^0_e = 0 \), i.e., the temporal part of \( l_{AB}^a \) is zero. The spacetime length \( l \) is written in the \( \{e_\mu\} \) base as

\[
l = l_e = (l^0_e l_{ee})^{1/2} = (l^1_e l_{ee})^{1/2} = l_0,
\]

as in the prerelativistic physics; it is in that case a measure of the spatial distance, i.e., of the rest spatial length of the rod. The observers in all other IFRs will look at the same events but associating with them different coordinates; it is the essence of the covariant description. They all obtain the same value \( l \) for the spacetime length. It has to be pointed out that in the “TT relativity” it is not necessary to start in this example with the rest frame of the object and to choose the events A and B to be simultaneous in that frame. The whole consideration can be done in the same covariant manner for other choices of IFRs and of the events A and B in the chosen IFR. For any starting choice the covariant LT (2) will correctly connect the results of measurements of the same 4D quantity in two relatively moving IFRs. The rest frame of the object and the simultaneity of the events A and B in it are chosen only to have the connection with the prerelativistic physics, which deals with “3+1” quantities and not with 4D quantities.

Let us then consider the same 4-vector \( l_{AB}^a \) in \( S' \), (where in “3+1” picture the rod is moving). The position 4-vectors \( x^a_A \) and \( x^a_B \) of the events A and B respectively are decomposed with respect to \( \{e'_\mu\} \) base as \( x^a_A = x^0_A e'_0 + x^1_A e'_1 = 0e'_0 + 0e'_1 \), and \( x^a_B = x^0_B e'_0 + x^1_B e'_1 = -\beta e'_0 l_0 e'_0 + \gamma e'_0 l_0 e'_1 \), and the distance 4-vector is decomposed as

\[
l_{AB}^a = x^a_B - x^a_A = l^0_e l_{ee}' + l^1_e l_{ee}' = -\beta e'_0 l_0 e'_0 + \gamma e'_0 l_0 e'_1.
\]

Note that in the “e” coordinatization, commonly used in the “AT relativity,” there is a dilatation of the spatial part \( l^1_e = \gamma l_0 \) with respect to \( l^1_e = l_0 \) and
not the Lorentz contraction as predicted in the “AT relativity.” However it is clear from the above discussion that comparison of only spatial parts of the components of the distance 4-vector \( l^A_{AB} \) in \( S \) and \( S' \) is physically meaningless in the “TT relativity.” All components of the distance 4-vector \( l^A_{AB} \) are transformed by the LT from \( S \) to \( S' \). \( l^e_x \) and \( l^e_y \) are different representations of the same physical quantity \( l^a_{AB} \) measured in two relatively moving IFRs \( S \) and \( S' \). The invariant spacetime length of that object in \( S' \) is

\[
l = l'_e = (l'_e, l'_e, l'_e, l'_e)^{1/2} = l_0.
\]

Note that if \( l'^0_e = 0 \) then \( l'_e \) in any other IFR \( S' \) will contain the time component \( l'^0_e \). We conclude from the above discussion that if one wants in the “TT relativity” to compare in a physically meaningful sense the “lengths” of two different objects than it is possible only by comparing their invariant spacetime lengths.

In the “r” coordinatization the position 4-vectors of the events \( A \) and \( B \), \( x^A_x \) and \( x^A_y \) in \( S \) are decomposed with respect to \( \{ r_\mu \} \) base as \( x^A_x = x^A_0 r_0 + x^A_1 r_1 = 0 r_0 + 0 r_1 \), and \( x^A_y = x^A_0 r_0 + x^A_1 r_1 = -l_0 r_0 + l_0 r_1 \), and the distance 4-vector \( l^A_{AB} = x^A_B - x^A_A \) is decomposed as

\[
l^A_{AB} = l^0_{r,r} r_0 + l^1_{r,r} r_1 = -l_0 r_0 + l_0 r_1,
\]

and the TT length \( l \) is

\[
l = l_r = (l^0_r, l^1_r)^{1/2} = l_e = l_0
\]
as it must be.

In \( S' \) and in the \( \{ r'_\mu \} \) base the position 4-vectors of the events \( A \) and \( B \) are \( x'^0_A = 0 r'_0 + 0 r'_1 \) and \( x'^0_B = x'^0_0 r'_0 + x'^0_1 r'_1 = -K l_0 r'_0 + (1 + \beta_r)(1/K) l_0 r'_1 \), and the components \( l'^0_\mu \) of the distance 4-vector \( l'^A_{AB} \) are equal to the components \( x'^0_B - x'^0_A \), i.e., \( l'^0_r = x'^0_B - x'^0_A \). Thus \( l'^A_{AB} \) is decomposed as

\[
l'^A_{AB} = l'^0_{r,r} r'_0 + l'^1_{r,r} r'_1 = -K l_0 r'_0 + (1 + \beta_r)(1/K) l_0 r'_1.
\]

If only spatial parts of \( l^e_r \) and \( l'^e_r \) are compared than one finds that \( \infty \approx l'^e_r \geq l_0 \) for \(-1/2 < \beta_r \leq 0 \) and \( l_0 \leq l'^e_r \lesssim \infty \) for \( 0 \leq \beta_r < \infty \), which once again shows that such comparison is physically meaningless in the “TT relativity.” However the invariant spacetime length always takes the same value

\[
l = l'_e = (l'^e, l'^e, l'^e, l'^e)^{1/2} = l_0,
\]

and as already said, it can be compared in a physically meaningful sense in the “TT relativity.” One concludes from this discussion that, e.g., our particular 4-vector \( l^A_{AB} \) (a geometrical quantity) is represented in different bases \( \{ e_\mu \}, \{ e'_\mu \}, \{ r_\mu \} \) and \( \{ r'_\mu \} \) by its coordinate representations \( l^e_\mu, l'^e_\mu, l^r_\mu \) and \( l'^r_\mu \), respectively.

We see that in the “TT relativity” the geometrical quantities, e.g., the 4-vectors \( x^e, l^e, \ldots \), have different representations depending on the chosen IFR and the chosen coordinatization in that IFR, e.g., \( x^e, l^e, \ldots \). Although
the Einstein coordinatization is preferred by physicists due to its simplicity and symmetry it is nothing more “physical” than others, e.g., the “r” coordinatization. The coordinate dependent quantities have not an intrinsic physical meaning. The spacetime length \( l \) is an example of a well defined quantity that is independent of the chosen IFR and also of the coordinatization taken in that IFR; it is an intrinsic property of the spacetime. From this consideration an important conclusion emerges; the usual 3D length of a moving object cannot be defined in the 4D spacetime of the TT relativity in an adequate way, since it is only the spatial length and not a 4D tensor quantity.

2.2. The AT of Length

In contrast to the covariant definition of the spacetime length and the TT of the spacetime tensors considered in the “TT relativity” the synchronous definition of spatial length, introduced by Einstein [12] defines length as the spatial distance between two spatial points on the (moving) object measured by simultaneity in the rest frame of the observer. To see the difference with respect to the “TT relativity” we determine the spatial length of the rod considered in the previous section. As shown above in the “TT relativity,” in contrast to the “AT relativity,” one cannot speak about the spatial distance, as a correctly defined physical quantity, but only about 4D tensor quantities, the geometrical quantities - the position 4-vectors \( x^a_{A,B} \), the distance 4-vector \( l^a_{AB} \), the spacetime length \( l \), etc., and their 4D representations, \( x^\mu_{A,B,..,e,r,..} \), \( l^\mu_{ABe,..,r,..} \), \( l^\mu_{e,..,r,..} \).

Instead of to work with geometrical quantities \( x^a_{A,B}, l^a_{AB} \) and \( l \) one deals, in the “AT relativity,” only with the spatial, or temporal, parts of their coordinate representations \( x^\mu_{Ae,r}, x^\mu_{Be,r} \) and \( l^\mu_{e,r} \). First the “e” coordinatization, which is almost always used in the “AT relativity,” is considered. According to Einstein’s definition [12] of the spatial length the spatial ends of the rod must be taken simultaneously in the chosen coordinatization. In 4D (at us 2D) spacetime and in the “e” coordinatization the simultaneous events \( A \) and \( B \) (whose spatial parts correspond to the spatial ends of the rod) are the intersections of \( x^e_1 \) axis (that is along the spatial base vector \( e_1 \)) and the world lines of the spatial ends of the rod that is at rest in \( S \) and situated along the \( x^e_1 \) axis. The position 4-vectors (in the “e” base) \( x^\mu_{Ae} \) and \( x^\mu_{Be} \) of the simultaneous (at \( t_e = a = 0 \)) events \( A \) and \( B \) in \( S \) are \( x^\mu_{Ae} = 0e_0 + 0e_1 \), or, in short, \( x^\mu_{Ae} = (0, 0) \), and \( x^\mu_{Be} = (0, l_0) \), and the distance 4-vector (in the “e” base) \( l^\mu_{ABe} = x^\mu_{Be} - x^\mu_{Ae} = (0, l_0) \). We emphasize that it is necessary in the “AT relativity” to take the end points of the spatial length of the rod to be simultaneous, whereas in the “TT relativity” the events \( A \) and \( B \) can be, in principle, taken at arbitrary \( x^0_{Ae} \neq x^0_{Be} \). Then in \( S \), the rest frame of the object, the spatial part \( l^1_{ABe} = l_0 \) of \( l^\mu_{ABe} \) is considered to define the rest spatial length (the temporal part of \( l^\mu_{ABe} \) is taken to be zero). Further one uses the inverse Lorentz transformations to express \( x^\mu_{Ae}, x^\mu_{Be}, \) and \( l^\mu_{ABe} \) in \( S \) in terms of the corresponding quantities in \( S' \), in which the rod is moving. This procedure yields \( x^\alpha_{A,Be} = ct_{A,Be} = \gamma_c (ct_{A,Be} + \beta_c x^\alpha_{A,Be}) \), and
\[ x_{A,Bc} = \gamma_c(\beta_e c t'_{A,Bc} + x'_{A,Bc}) \]

whence

\[ l_{0,ABc} = c t_{Be} - c t_{Ac} = \gamma_c(c t'_{Be} - c t'_{Ac}) + \gamma_c \beta_e(x'_{Be} - x'_{Ac}) = \gamma_c t_{0,ABc} + \gamma_c \beta_e l_{0,ABc} \]  

(4)

and

\[ l_{1,ABc} = x'_{Be} - x'_{Ac} = \gamma_c(x'_{Be} - x'_{Ac}) + \gamma_c \beta_e(c t'_{Be} - c t'_{Ac}) = \gamma_c l_{1,ABc} + \gamma_c \beta_e l_{0,ABc}. \]  

(5)

Now comes the main difference between the two forms of relativity. Instead of to work with 4D tensor quantities and their LT (as in the “TT relativity”) in the “AT relativity” one forgets about the transformation of the temporal part \( l_{0,ABc} \), Eq. (4), and considers only the transformation of the spatial part \( l_{1,ABc} \), Eq. (5). Further, in that relation for \( l_{1,ABc} \) one assumes that \( t'_{Be} = t'_{Ac} = t'_e = b \), i.e., that \( x'_{Be} \) and \( x'_{Ac} \) are simultaneously determined at some arbitrary \( t'_e = b \) in \( S' \). However, in 4D (at us 2D) spacetime such an assumption means that in \( S' \) one actually does not consider the same events \( A \) and \( B \) as in \( S \) but some other two events \( C \) and \( D \), whence \( t'_{Be} = t'_{Ac} \) has to be replaced with \( t'_{De} = t'_{Ce} = b \). The events \( C \) and \( D \) are the intersections of the line (the hypersurface \( t'_e = b \) with arbitrary \( b \)) parallel to the spatial axis \( x^0 \) (which is along the spatial base vector \( e_1' \)) and of the above mentioned world lines of the spatial end points of the rod. Then in the above transformation for \( l_{1,ABc} \) one has to write \( x'_{Ce} - x'_{De} = l_{1,CDc} \) instead of \( x'_{Be} - x'_{Ac} = l_{1,ABc} \). The spatial parts \( l_{1,ABc} \) and \( l_{1,CDc} \) are the spatial distances between the events \( A, B \) and \( C, D \), respectively. The spatial distance \( l_{1,ABc} = x'_{Be} - x'_{Ac} \) defines in the “AT relativity,” and in the “e” base, the spatial length of the rod at rest in \( S \), while \( l_{1,CDc} = x'_{De} - x'_{Ce} \) is considered in the “AT relativity,” and in the “e” base, to define the spatial length of the moving rod in \( S' \). With these definitions we find from the equation for \( l_{1,ABc} \) the relation between \( l_{1} = l_{1,CDc} \) and \( l_{1} = l_{1,ABc} = l_0 \) as the famous formulae for the Lorentz contraction of the moving rod

\[ l_1 = x'_{De} - x'_{Ce} = l_0 / \gamma_e = (x'_{Be} - x'_{Ac})(1 - \beta_e^2)^{1/2}, \]  

(6)

with \( t'_{Ce} = t'_{De} \), and \( t_{Be} = t_{Ac} \), where \( \beta_e = V_e / c \), \( V_e \) is the relative velocity of \( S \) and \( S' \). Note that the spatial lengths \( l_0 \) and \( l_1 \) refer not to the same 4D tensor quantity, as in the “TT relativity,” but to two different quantities in 4D spacetime. These quantities are obtained by the same measurements in \( S \) and \( S' \); the spatial ends of the rod are measured simultaneously at some \( t_e = a \) in \( S \) and also at some \( t'_e = b \) in \( S' \), and \( a \) in \( S \) and \( b \) in \( S' \) are not related by the LT or any other coordinate transformation. While in the “TT relativity” one deals with events as correctly defined quantities in 4D spacetime in Einstein’s approach [12] the spatial and temporal parts of events are treated separately, and moreover the time component is not transformed in the Lorentz contraction.

The LT (2) is the transformation in 4D spacetime and it transforms some 4D tensor quantity \( Q_{\mu,..}^{\nu,..}(x^0, x^1, \ldots) \) from \( S \) to \( Q_{\mu,..}^{\nu,..}(x^0, x^1, \ldots) \) in \( S' \), (all parts of the quantity are transformed), which means that in 4D spacetime it is not possible to neglect the transformation of \( l_0 \) as a part of \( l_0 \), as done in the derivation of the Lorentz contraction (6). However, if one does not forget the transformation
of the temporal part $t'_{A'B'}$, Eq. (4), and takes in it that $t'_{B'C'} = t'_{A'C'}$, $t_{B'C'} = t_{A'C'}$ (as in the derivation of the Lorentz contraction), then one finds from (4) that $x'_{B'C'} = x'_{A'C'}$, which is in the obvious contrast with the formulae for the Lorentz contraction.

Let us also see does the Lorentz contraction, as the coordinate transformation, change the interval $ds$, which defines the geometry of the spacetime. In $S$ and in the “$e$” base the interval $ds$ is $ds^2 = ds'^2 = (dx^1)^2 - (c^2 dt')^2$, and with $dt = 0$, as assumed in the derivation of the Lorentz contraction, it becomes, in $S'$, where it is assumed that $dt' = 0$, and with the relation for the Lorentz contraction (6), $dx^1 = dx^1_0 / \gamma$, the infinitesimal spacetime distance $ds'$ becomes, in $S'$ $ds'^2 = (dx^1_0)^2 / \gamma^2$, and thus $ds' \neq ds$.

Let us now consider the Lorentz “contraction” in the “$r$” coordinatization. According to Einstein’s definition [12] of the spatial length the spatial ends of the rod must be taken simultaneously in the chosen coordinatization. In 4D (at us 2D) spacetime and in the “$r$” base the spatial ends of the considered rod, that is at rest in $S$, must lie on the light line, i.e., on the $x^1_0$ axis (that is along the spatial base vector $r_1$). Hence the simultaneous events $E$ and $F$ (whose spatial parts correspond to the spatial ends of the rod) are the intersections of $x^1_0$ axis and the world lines of the spatial ends of the rod. Note that in our 2D spacetime the events $E$ and $F$ are not the same events as the events $A$ and $B$, considered in the “$e$” base for the same rod at rest in $S$, since the simultaneity of the events is defined in different ways. The $\{r_\mu\}$ representations of the position 4-vectors $x^0_{E}$ and $x^0_{F}$ of the events $E$ and $F$ in $S$ are $x^\mu_{E} = (0, 0)$ and $x^\mu_{F} = (0, l_0)$, and of the distance 4-vector $l^0_{EF}$ is $l^\mu_{EF} = x^\mu_{F} - x^\mu_{E} = (0, l_1) = (0, l_0)$. However, as noticed above, in 4D spacetime the spatial length in the “$r$” base $l^0_r = l_0$ (with $x^0_{Br} = x^0_{Ar}$), since the simultaneity is defined in a different way. Applying the same procedure as in the case of the derivation of the Lorentz contraction in the “$e$” base we find the relations for $l^0_r$ and $l^1_r$ corresponding to (4) and (5), respectively,

\begin{equation}
I^0_{EFr} = x^0_{Fr} - x^0_{Er} = (1/K)(x^0_{Fr} - x^0_{Er}) = (1/K)I^0_{EFr},
\end{equation}

\begin{equation}
I^1_{EFr} = x^1_{Fr} - x^1_{Er} = (\beta_r/K)(x^0_{Fr} - x^0_{Er}) + K(x^1_{Fr} - x^1_{Er}) = (\beta_r/K)l^0_{EFr} + Kl^1_{EFr},
\end{equation}

$K = (1 + 2\beta_r)1/2$. Further, in the “$r$” base, one again forgets the transformation of the temporal part $t^0_{EFr}$ of $t^\mu_{EFr}$ and assumes that in the relation for $I^1_{EFr}$ $x^0_{Fr}$ and $x^0_{Er}$ are simultaneously determined at some $x^0_{Fr} = x^0_{Er} = b$ in $S'$. However, in the same way as in the “$e$” base, in 4D (at us 2D) spacetime such an assumption means that in $S'$ one actually \textit{does not consider the same events} $E$ and $F$ as in $S$ but some other two events $G$ and $H$, and that the equality $x^0_{Fr} = x^0_{Er} = b$ has to be replaced by $x^0_{Gr} = x^0_{Br} = b$. The events $G$ and $H$ are the intersections of the line (the hypersurface $x^0_{Gr} = x^0_{Br} = 0$ with arbitrary $b$) parallel to the spatial axis $x^1_0$ (which is along the spatial base vector $r_1$) and of the above mentioned world lines of the spatial end points of the rod. Then in the above transformation for $I^1_{EFr}$ $l^0_{GFr} = 0$, and $l^0_{EFr} = x^1_{Fr} - x^1_{Er}$
is replaced by $l_{GHr}^1 = x_{GHr}^1 - x_{Gr}^1$. Now, in the “r” base, the spatial distance $l_{EFr}^1 = x_{Fr}^1 - x_{Er}^1$ defines in the “AT relativity” the spatial length of the rod at rest in $S$, while $l_{GHr}^1 = x_{GHr}^1 - x_{Gr}^1$ defines the spatial length of the moving rod in $S'$. Then, from the equation for $l_{EFr}^1$, and with these definitions, we find the relation between $l_{EFr}^1 = l_{GHr}^1$ and $l_{EFr}^1 = l_0$ as the Lorentz “contraction” of the moving rod in the “r” base,

$$l_{EFr}^1 = x_{Fr}^1 - x_{Er}^1 = l_0/K = (1/K)(x_{Fr}^1 - x_{Er}^1),$$

(9)

with $x_{Fr}^0 = x_{Gr}^0$ and $x_{Fr}^0 = x_{Er}^0$. In contrast to the “e” coordinatization we find that in the “r” base there is a length dilatation $\infty \succ l_{EFr}^1 \succ l_0$ for $-1/2 < \beta_r < 0$ and the standard “length contraction” $l_0 \succ l_{EFr}^1 \succ 0$ for positive $\beta_r$, which clearly shows that the “Lorentz contraction” is not physically correctly defined transformation.

We see from the preceding discussion that - the Lorentz contraction is the transformation connecting different quantities (in 4D spacetime) in different IFRs and different coordinatizations, and also it changes the infinitesimal spacetime distance $dS$ and consequently the pseudo-Euclidean geometry of the 4D spacetime. Such characteristics of the Lorentz contraction as the coordinate transformation clearly show that the Lorentz contraction belongs to - the AT. In the same way one can show that the usual “time dilatation” does have the same characteristics as the Lorentz contraction, i.e., that it is also - an AT, but this will not be done here.

Although the Lorentz contraction is an AT it is still widely used in numerous textbooks and papers as an “important relativistic effect.” Thus, for example, it is almost generally accepted in ultra-relativistic nuclear collisions, see, e.g., [13]: “that in the center-of-mass frame two highly Lorentz contracted nuclei pass through each other ....” In the recent paper [14] it is supposed that: “... Čerenkov radiation of the charged two-particle system involves the Lorentz contraction of their rest distance.” An experiment, based on this idea, is suggested in [14] for the verification of the Lorentz contraction. Moreover it is argued in [15] that the authors have experimentally succeeded to observe the Lorentz contraction of magnetic flux quanta (vortices) in Josephson tunnel junction. In all these examples it is understood that a Lorentz boost transforms the rest length to the contracted length. But, as it is already explained above, a Lorentz boost is a TT transforming from an IFR $S$ to another IFR $S'$, e.g., all four coordinates as a 4-vector; the same events are considered in $S$ and $S'$. Also, a Lorentz boost transforms a physical quantity represented by a 4D spacetime tensor, e.g., $Q(x)$ in $S$, to $Q'(x')$ in $S'$, thus again considering the same quantity in $S$ and $S'$. On the contrary, as already said, in the Lorentz contraction, Eqs. (10) and (11), the time component is not transformed and the Lorentz contraction is an AT from the relativity viewpoint, which has nothing in common with a Lorentz boost as a TT. In the “TT relativity” one cannot say that the nucleus must contract (as argued in literature on ultra-relativistic nuclear collisions), or that the rest distance between two charged particles undergoes the Lorentz contraction (as considered in [14]), or that the vortices contract when moving (as argued to
be proved in experiments [15]), since the Lorentz contraction is certainly not a relativistic relation, i.e., the relation belonging to the “TT relativity,” and cannot be used either to illustrate or to test any part of the “TT relativity.”

The above discussion reveals the main differences between the spacetime length considered in the “TT relativity” and the spatial length considered in the “AT relativity.” The same example (a rod at rest in \(S\)) is investigated in the “TT relativity,” Sec. 2.1, and in the “AT relativity,” this section. The “TT relativity” deals with 4D quantities in 4D spacetime. We associate with the mentioned rod a 4D quantity - a distance 4-vector \(l^{\mu}_{AB}\), and consider this quantity \(l^{\mu}_{AB}\) in two IFRs \(S\) and \(S'\) (in which the rod is moving) and in two coordinatizations, “e” and “r”. Four different decompositions, representations, are found for the same \(l^{\mu}_{AB}\). Different representations of \(l^{\mu}_{AB}\) in \(S\) and \(S'\) are connected by the TT - the LT. In terms of \(l^{\mu}_{AB}\) the spacetime length \(l^{1}\) is constructed and it does have the same value for all four representations of \(l^{\mu}_{AB}\). An essentially different treatment of that rod is performed in the “AT relativity.” This form of relativity does not deal with 4D quantities in 4D spacetime. In the “AT relativity” we associate with that rod four different spatial lengths, i.e., four different 3D quantities in 4D spacetime; in the “e” base they are \(l^{\mu}_{ABe}\) in \(S\) and \(l^{\mu}_{CDe}\) in \(S'\), and in the “r” base they are \(l^{\mu}_{EFr}\) in \(S\) and \(l^{\mu}_{GHR}\) in \(S'\). The quantities in the same base but in different IFRs are connected by the AT - the Lorentz “contraction” \(l^{\mu}_{ABe}\) and \(l^{\mu}_{CDe}\) with (6), and \(l^{\mu}_{EFr}\) and \(l^{\mu}_{GHR}\) with (9). None of these quantities is well defined in 4D spacetime. We conclude from the whole previous consideration that when the 4D structure of our spacetime is correctly taken into account then there is no place for the Lorentz contraction formulae, and only the spacetime length and the spacetime quantities are well defined quantities.

### 2.3. “Car and Garage Paradox”

In the previous sections we have examined the main characteristics of both forms of relativity. Now we want to show the difference between the treatments of relatively moving systems in the “AT relativity” and the “TT relativity.” Usually such systems are treated in a noncovariant manner, i.e., in the “AT relativity,” but here we shall present the treatment of such systems in a manifestly covariant manner, i.e., in the “TT relativity.” In order to see the differences between both treatments we do not need to work completely in geometrical terms, but we can choose some specific coordinatization, e.g., the simplest one, the ”e” coordinatization. Therefore, in the following, we restrict ourselves to the “e” coordinatization and, for simplicity in notation, we omit the subscript “e” in all quantities. However, we shall often write the important relations in geometrical terms, and also we shall explain which results and conclusions are independent of the chosen coordinatization.

As already discussed, if in an IFR \(S'\) in which the time component of the distance 4-vector \(l^{\mu}_{AB}\), i.e., \(l^{0}_{AB}\), is zero (simultaneously determined events \(A\) and \(B\)), then \(l^{\mu}_{AB}\) comprises only spatial components. Hence, in \(S'\) the invariant spacetime length \(l'\) is given as the usual 3D distance between \(A\) and \(B\). But,
in such a case, $l_{AB}^\mu$ in the rest frame $S$ of the object does have $l_{AB}^\mu \neq 0$, and the spacetime length $l$ in $S$ (it is $= l'$) could take different values depending on the chosen IFR $S'$. Such an arbitrariness in $l$, although not forbidden by any physical law, would complicate both theory and experiment. Furthermore, one wants to retain the connection with the prerelativistic concept of the spatial length. Therefore, the most convenient choice for the frame in which the time component $l_{AB}^0$ of $l_{AB}^\mu$ is zero is the $S$ frame. Thus in $S$ the position 4-vectors $x_{AB}^\mu$ are determined simultaneously, $x_{B}^0 - x_{A}^0 = l_{AB}^0 = 0$, and the spacetime length $l$, with the rest frame length $l_0$, i.e., $l = (l_{AB}^\mu l_{AB}^\mu)^{1/2} = l_0$, as in the prerelativistic physics. The observers in all other IFRs will look at the same events but associating with them different coordinates; they all find (measure) the same value $l = l_0$ for the spacetime length. Note, as we have mentioned, that if $l_{AB}^0 = 0$ then $l_{AB}^\mu$ in any other IFR $S'$ will contain the time component $l_{AB}^0 \neq 0$. We once again emphasize that the choice of the rest frame of the object as the starting frame for the consideration is not dictated by physical requirements. This choice is, in fact, determined only by our desire to have a quantity that corresponds in the 4D spacetime to the prerelativistic spatial length. As already said at the end of Sec. 2.1 the usual 3D length of a moving object cannot be defined in the 4D spacetime, i.e., in the “TT relativity”, in an adequate way. Only the spacetime length $l$ does have a definite theoretical and experimental meaning and it is an invariant quantity. This holds for all possible synchronizations. With our choice of the rest frame of the object as the starting frame, i.e., with $l = l_0$, the spatial rest length determined simultaneously in $S$ obtains the properties of the spacetime length $l$. Then, the coordinate measurements of $x_{AB}^\mu$ in an IFR $S'$ in which an object is moving are not of interest in their own right but they have to enable one to find the rest spatial length $l_0$. In the prerelativistic “3+1” picture, and in the “AT relativity,” one can compare the spatial lengths of two relatively moving objects. But in the “TT relativity” the spacetime lengths (that contain both spatial and temporal parts) are well defined quantities in 4D spacetime and they, or the rest spatial lengths, can be compared in a physically meaningful way.

Let us illustrate the preceding discussion considering the well-known “Car and garage paradox” (see, e.g., [7], p.9). The common assertion about this “paradox” is that it comes out due to, [7]: ”The lack of a notion of absolute simultaneity in special relativity ...,” and consequently due to the relativity of the Lorentz contraction. However, as discussed above, the relativity of simultaneity is a coordinate dependent effect and, for example, for $\varepsilon = 0$, [8], the absolute simultaneity is preserved. Also, the Lorentz contraction is an AT and it has nothing to do with the 4D pseudo-Euclidean geometry of the special relativity. Therefore, we discuss this “paradox” using covariant 4D quantities, i.e., in the “TT relativity”. But for our purposes, as it is already explained, there is no need to discuss the “paradox” in geometrical terms, than it can be considered in the inertial coordinate systems with the Minkowski metric tensors, that is in the “e” coordinatization. The frame in which a garage is at rest is denoted by $S$ while that one in which a car is at rest by $S'$. The unprimed quantities are in $S$ and the primed ones in $S'$. Instead of 4D spacetime we work here with 2D
3. CURRENT-CARRYING CONDUCTOR AND EXTERNAL ELECTRIC FIELDS IN THE “TT RELATIVITY”

Let us now apply these ideas to the consideration of a CCC in the “TT relativity”. An infinite straight wire with a steady current is situated along the \(x^1\) axis. A current is flowing in \(-x^1\) direction and accordingly electrons move in \(+x^1\) direction. We suppose that positive and negative charge densities are of equal magnitude when both subsystems are relatively at rest, i.e., before a current is established in the wire. In a CCC the wire (i.e., the ions) is supposed to be at rest in \(S\), while the electrons are at rest in \(S'\).

Before determining the current density 4-vectors \(j^\mu\) in \(S\) and \(S'\) we give the
manifestly covariant definition of a charge within a boundary \( \delta H \) of an arbitrary hypersurface \( H \) (see, e.g., [16, 17]),

\[
Q_{\delta H} = \int_{H} j^{\mu} d\sigma_{\mu},
\]

(10)

where \( d\sigma^{\mu} \) is the 4-vector of an element of the hypersurface \( H \). (This expression can be written in a more general form, i.e., in geometrical terms, replacing Greek index \( \mu \) by the abstract index \( a \).) The invariance of charge defined by (10) is proved in [16] for a linear CCC, and also for the general case of 4D spacetime in [17]. If the hypersurface \( H \) is chosen in the rest frame of charges in such a way that it is the space-like plane \( t = \text{const.} \), then the charge \( dQ \) is given as in the prerelativistic physics \( dQ = \rho dV \): the 4-vector \( j^{\mu} \) has only the time component \( j^{0} = c\rho \), since we are in the rest frame of charges, and \( d\sigma_{\mu} \) also has only time component which is the synchronously defined rest volume \( dV \). Consequently, the charge density \( \rho \) is defined as the ratio of \( dQ \) given by (10), but taken simultaneously in the rest frame of the charges, and the synchronously defined rest volume \( dV \), \( \rho = dQ/dV \), and it is well defined quantity from the “TT relativity” viewpoint. Obviously the charge density has the common prerelativistic meaning only in the rest frame of the charges. The charge density of moving charges is not a well defined quantity from the “TT relativity” point of view in the same way as the spatial length or the volume of a moving object are not correctly defined quantities in the “TT relativity”. This is in contrast with the “AT relativity” and the Lorentz contraction, where the charge density of the moving charges is defined; it is enhanced by \( \gamma = (1 - \beta^{2})^{1/2} \) relative to the proper charge density due to the Lorentz contraction of the moving volume. Thereby, when determining the current density 4-vector \( j^{\mu} \) in some IFR in which the charges are moving one first has to find that vector in the rest frame of the charges, where the space component \( j^{0} = 0 \) and \( \gamma = 1 \), and then to transform by the LT so determined \( j^{\mu} \) to the considered IFR. According to this consideration the simplest and the correct way, from the “TT relativity” viewpoint, to determine the current density 4-vector \( j^{\mu} \) in some IFR for a CCC is the following: The current density 4-vectors \( j^{\mu}_{+} \) and \( j^{\mu}_{-} \) for positive and negative charges, respectively, have to be determined in their rest frames and then transformed by the LT to the given IFR. It has to be noted that in the “TT relativity” it is not necessary to determine \( j^{\mu} \) for a CCC, in an arbitrary IFR, in the mentioned way. In that frame we could start in (10) with an arbitrary space-like hypersurface \( H \) and determine \( j^{a} \) and \( d\sigma^{a} \) in some coordinatization that is different than the “e” coordinatization. But then we loose the connection with the prerelativistic notions, the charge density \( \rho \), the current density \( j \) (3-vector), the spatial length and volume, etc., and with the prerelativistic relation \( dQ = \rho dV \).

3.1. The Current Density \( j^{\mu} \) in the Ions’ Rest Frame S

Hence, the current density 4-vector in \( S \), for the considered wire with cur-
rent, is \( j^\mu = j^\mu_+ + j^\mu_- \), where \( j^\mu_+ = (c \rho_0, 0) \). The positive charge density \( \rho_+ \) is \( \rho_0 \), where \( \rho_0 \) is the positive charge density for the wire at rest but without a current. To find \( j^\mu_- \) in \( S \) one has, as already said, to find the electrons’ charge density \( \rho'_- \), and the current density 4-vector of the electrons \( j'^\mu_- \) in their rest frame \( S' \), where \( \rho'_- \) is well defined quantity, and then to transform them to the ions’ rest frame \( S \). In the rest wire, but without a current, the charge density of the electrons, which are at rest there, is \( -\rho_0 \). Then, it follows from the previous consideration that in \( S' \), where the electrons in that wire, but with a current, are at rest, the proper charge density \( \rho'_- \) of the electrons must again be equal to \( -\rho_0 \), i.e.,

\[
\rho'_- = -\rho_0, \quad j'^\mu_- = (-c \rho_0, 0).
\]  

(11)

By means of (11) and the LT we find the current densities in \( S \) as

\[
j^\mu_- = (-c \gamma \rho_0, -c \gamma \beta \rho_0), \quad j^\mu_+ = (c (1 - \gamma) \rho_0, -c \gamma \beta \rho_0).
\]  

(12)

Eqs. (11) and (12) are in contrast to all previous works from the time of Clausius, Clausius hypothesis, see [18], until today. In the Clausius hypothesis it is simply supposed that in the ions’ rest frame \( S \) the charge density of the moving electrons \( \rho_- = -\rho_0 \). However the same equations were already obtained in [5], where the “AT relativity” with the Lorentz contraction is used. This may seem surprising that the same equations exist in [5] (with the “AT relativity”) and here, where the “TT relativity” is considered and thus only the covariant quantities are used. But, we must note that the results obtained in [5] are not actually based on the AT, i.e., on the Lorentz contraction, than on the assumption that in the electrons’ rest frame \( S' \) the electrons’ charge density \( \rho'_- \) is \( -\rho_0 \). In a covariant approach, i.e., in the “TT relativity,” Eq. (11) is neither hypothesis (as in the traditional approach) nor the assumption (as in [5]), but it is a consequence of the covariant definition of an invariant charge (10) and the invariance of the rest length, i.e., it resulted from the use of correctly defined covariant quantities.

3.2. The \( F^{\alpha \beta} \) and the \( E^\alpha, B^\alpha \) Formulations of Electrodynamics

Having determined the sources \( j^\mu \) we find the electric and magnetic fields for that infinite wire with current. One way is to start with the covariant Maxwell equations with \( F^{\alpha \beta} \) and its dual \( *F^{\alpha \beta} \)

\[
\partial_\alpha F^{\alpha \beta} = -j^\beta / \varepsilon_0 c, \quad \partial_\alpha *F^{\alpha \beta} = 0
\]  

(13)

where \( *F^{\alpha \beta} = -(1/2) \varepsilon^{\alpha \beta \gamma \delta} F_{\gamma \delta} \) and \( \varepsilon^{\alpha \beta \gamma \delta} \) is the totally skew-symmetric Levi-Civita pseudotensor. In such a covariant formulation \( F^{\alpha \beta} \) is the primary quantity; it is the solution of (13), or the corresponding wave equation

\[
\partial^\alpha \partial_\alpha F_{\alpha \beta} - (1/\varepsilon_0 c) (\partial_\beta j^\alpha - \partial_\alpha j^\beta) = 0,
\]  

(14)

and it conveys all the information about the electromagnetic field. There is no need to introduce either the intermediate electromagnetic 4-potential \( A^\mu \) or the
connection of the components of $F^{\alpha\beta}$ with the usual 3-vectors $E$ and $B$. The general solution in the retarded representation of (14) or (13) is

$$F^{\alpha\beta}(x^\mu) = (2k/i\pi c) \int \left\{ \frac{[j^\alpha(x^\mu)(x - x')^\beta - j^\beta(x'^\mu)(x - x')^\alpha]}{[(x - x')^\sigma(x - x')^\sigma]^2} \right\} d^4x',$$  

where $x^\alpha$, $x'^\alpha$ are the position 4-vectors of the field point and the source point respectively, $k = 1/4\pi\varepsilon_0$. After transforming by the LT (13) to the $S'$ frame one finds the same equations with primed quantities replacing the unprimed ones, since the transformations of all quantities in (13) are – the TT.

Instead of to work with $F^{\alpha\beta}$ formulation one can equivalently use the $E^\alpha$, $B^\alpha$- formulation, which is presented in [1] and [3]. It is shown there that the “TT relativity” one has to use the 4-vectors $E^\alpha$ and $B^\alpha$ instead of the usual 3-vectors $E$ and $B$. The usual transformations of $E$ and $B$, obtained by the identification of the components of $E$ and $B$ with the components of $F^{\alpha\beta}$ ($E_i = F^{0i}$ and $B_i = \ast F^{0i}$), are shown to be the AT referring to the same measurements in different IFRs and not to the same quantity, see [1] and [3]. In that way it is found in [1] (and [3]) that contrary to the common belief the usual noncovariant formulation with the 3-vectors $E$ and $B$ is not equivalent to the covariant formulations. $E^\alpha$ and $B^\alpha$ are determined by the covariant Maxwell equations derived in [1] (and [3]),

$$\partial_\alpha(\delta^{\alpha\beta}_\mu v^\mu E^\nu) + c\partial_\alpha(\varepsilon^{\alpha\beta\mu\nu} B^\mu v^\nu) = -j^\beta/\varepsilon_0,$$

$$\partial_\alpha(\delta^{\alpha\beta}_\mu v^\mu B^\nu) + (1/c)\partial_\alpha(\varepsilon^{\alpha\beta\mu\nu} v_\mu E^\nu) = 0,$$  

where $E$ and $B$ are the electric and magnetic field 4-vectors measured by a family of observers moving with 4-velocity $v^\mu$, and $\delta^{\alpha\beta}_\mu = \delta^{\alpha}_\mu \delta^\beta_\nu - \delta^{\alpha}_\nu \delta^\beta_\mu$. For the given sources $j^\mu$ one could solve these equations and find the general solutions for $E^\alpha$ and $B^\alpha$.

We note that it is possible to write Eqs. (16) in a somewhat simpler form, the covariant Majorana form, introducing $\Psi^\alpha = E^\alpha - icB^\alpha$. Then the covariant Majorana form of Maxwell’s equations becomes

$$(\gamma^\mu)^\beta_\alpha \partial_\mu \Psi^\alpha = -j^\beta/\varepsilon_0,$$  

where the $\gamma$-matrices are

$$(\gamma^\mu)^\beta_\alpha = \delta^\mu_\rho \gamma^\rho g^\alpha_\beta + i\varepsilon^{\mu\alpha\beta\gamma} v^\gamma.$$  

In the case that $j^\mu = 0$ Eq. (17) becomes Dirac-like relativistic wave equation for free photons

$$(\gamma^\mu)^\beta_\alpha \partial_\mu \Psi^\alpha = 0.$$  

We shall not further discuss the covariant Majorana formulation since it will be reported elsewhere.
3.3. \(E^\alpha\) for a CCC in the “TT Relativity”

Instead of solving (16) or (17) to find \(E^\alpha\) and \(B^\alpha\) for a CCC we first find \(F^\alpha\beta\) from (15) inserting into it \(j^\mu\) from (12) and performing the integration. Then we use the relations given in [1] (and [3]), which connect \(F^\alpha\beta\) and \(E^\alpha\), \(B^\alpha\)-covariant formulations,

\[
E^\alpha = \frac{1}{c} F^\alpha\beta v_\beta, \quad B^\alpha = \frac{1}{c^2} \ast F^\alpha\beta v_\beta. \tag{20}
\]

The inverse relations connecting the \(E^\alpha\), \(B^\alpha\) and \(F^\alpha\beta\)-covariant formulations are also given in [1] (and [3]) and they are

\[
F^\alpha\beta = \frac{1}{c} \delta^\alpha\beta \mu \nu E^\nu + \varepsilon^{\alpha\beta\mu\nu} B^\mu v_\nu, \quad \ast F^\alpha\beta = \delta^\alpha\beta \mu \nu B^\nu + \frac{1}{c} \varepsilon^{\alpha\beta\mu\nu} v_\mu E^\nu. \tag{21}
\]

Taking that the family of observers who measures \(E^\alpha\) is at rest in the \(S\) frame, i.e., that \(v_\mu = (-c, 0)\) one finds from (20) that \(E^0 = 0\), \(E^i = F^{0i}\), whence

\[
E^1 = 0, \quad E^2 = 2k(1 - \gamma) \rho_0 y (y^2 + z^2)^{-1}, \quad E^3 = 2k(1 - \gamma) \rho_0 z (y^2 + z^2)^{-1}. \tag{22}
\]

The equation (22) shows that the observer who is at rest relative to a wire with steady current will see, i.e., measure, the second order electric field outside such a CCC. Thus the result which is for such fields predicted on different grounds in [5] is proved to be correct in the “TT relativity” too, i.e., when all quantities are treated in a covariant manner, see the discussion at the end of Sec.3.1. We thus find that these electric fields naturally come out in the “TT relativity” treatment of physical systems consisting of relatively moving subsystems. Such fields were not searched for and, it seems, were not observed earlier due to their extreme smallness. (To be more precise, the similar second-order electric fields \((\propto v^2/c^2)\) have been detected in [18] and [19], but it is not sure that they are caused by the effect predicted in [5] and here.) However, I suppose that such fields must play an important role in many physical phenomena with steady currents, particularly in tokamaks and astrophysics, where high currents exist, and in superconductors, where the electric fields of zeroth order outside CCCs are absent, (see [20]).

Similarly, the magnetic field 4-vector \(B^\mu\) can be also obtained from (20) and the expression for \(F^\alpha\beta\) (15). For the observers with \(v_\mu = (-c, 0)\) one finds \(B^0 = 0\) and \(B^i = (-1/c) \ast F^{0i}\). In terms of the known \(\ast F^{0i}\) we find for \(B^i\) the usual expression for the magnetic field of an infinite straight wire with current, (only the current density is \(\gamma\) times bigger).

4. CHARGES ON A CURRENT LOOP IN THE “TT RELATIVITY”

In this section we discuss the macroscopic charge of a square loop with a steady current in the “TT relativity”. This is already discussed in numerous previous works but from the “AT relativity” viewpoint by using the synchronous definition of length and the Lorentz contraction. Let a square loop is at rest in the
x, y plane in an IFR S. The spatial x, y coordinates of the corners are: A(0,0), B(1,0), E(1,1), F(0,1). The electrons move from A to B in the $AB$ side. We first consider the charge in the $AB$ side in two frames; in $S$, the ions’ rest frame, and $S_{AB}'$, the electrons’ rest frame. The evaluation of the charge in the $AB$ side in $S$ and $S_{AB}'$ using (10) is already correctly performed in [16]. However there is an important difference between the calculation of $Q_{AB}$ in [16] and in this paper. In [16] the charge density of the electrons in $S_{AB}'$ is not specified but simply taken to have some undetermined value $-\lambda$ and its value in $S$ is found by means of the LT. The covariance of the definition (10) (the Lorentz scalar) will yield that $Q_{AB}$ in $S$ is equal to $Q_{AB}'$ in $S_{AB}'$ for any choice of $-\lambda$. Our discussion of an infinite wire with a current reveals that $\rho_\perp$ and $j_\mu$ in $S_{AB}'$ are determined by (11). Then $j_\mu'$ in $S$ is obtained by the LT, and it is given by (12). Hence, we find from (10) that

$$Q_{AB} = (1/c) \int_0^t \left( j_+^0 + j_-^0 \right) dx = (1 - \gamma) \rho_0 l \quad (23)$$

This result is already found in [5]. Similarly, to find $Q_{AB}'$ in $S_{AB}'$ one first determines $j_+^0$ in $S$, $j_+^0 = (c\rho_0, 0)$, and then by the LT one obtains $j_-^\mu = (c\gamma \rho_0, -c\gamma \beta \rho_0)$ in $S_{AB}'$. $Q_{AB}'$, for the moving loop in $S_{AB}'$, is found from (10) in the same way as in [16] and it is, of course, equal to $Q_{AB}$. The evaluation of $Q_{EF}$ in the $EF$ side, which is parallel to the $AB$ side, proceeds in the same way as for $Q_{AB}$. However the relative velocity of the $S_{EF}'$ frame, the rest frame for the electrons in the $EF$ side, and the $S$ frame is now $v^\mu = (\gamma c, -\gamma v)$. Hence, $j_\mu'$ is now $j_-^0 = (-c\gamma \rho_0, c\gamma \beta \rho_0)$ and

$$Q_{EF} = (1/c) \int_0^t \left( j_+^0 + j_-^0 \right) dx = -Q_{AB}. \quad (24)$$

Thus we find that there are charges $Q_{AB}$ and $-Q_{AB}$ on the sides $AB$ and $EF$, respectively, in the $S$ frame in which the loop is at rest. According to (10) $Q_{EF}$ for the moving loop is $= Q_{EF} = -Q_{AB}$. Moreover, it can be immediately concluded that the charge $Q_{BE}$ in $S$, in the vertical side $BE$, must be the same as $Q_{AB}$; it is the simple change of the spatial axes $x$ and $y$. Thus $Q_{BE} = Q_{AB}$. Similarly the charge $Q_{FA}$, in $S$, in the vertical side $FA$ is $Q_{FA} = -Q_{AB}$. The total charge in $S$, the rest frame of the loop with current, is zero $Q = Q_{AB} + Q_{BE} + Q_{EF} + Q_{FA} = 0$, as it must be. It remains zero in every IFR since the charges in all sides are invariant charges according to (10), which means that they are the same for both, moving and stationary current loop. Thus, we find the same behaviour for a moving loop with current and for the same loop but at rest in a given IFR.

In the traditional “AT relativity” approach with the synchronous definition of length and the Lorentz contraction (see, e.g., [21]) the charges on all sides are supposed to be zero in the rest frame of a loop with current. In the IFR in which the loop with current is moving it is found that $Q_{AB}' \neq Q_{AB}$, due to the Lorentz contraction, and that the charge on the $EF$ side is ”n” of that one on the $AB$ side. Furthermore, it is obtained that the charges on the vertical

20
sides of a loop with current are zero for both the loop at rest and moving loop, since for vertical sides there is no Lorentz contraction. Such results obtained in the common approach led the physics community (I am not aware of any exception) to conclude that there is an electric moment \( P \) for a moving loop with current, (see [21], Eq.(18-58)). The appearance of this “relativistic” effect and its consequences are discussed in numerous papers and books. Contrary to all these works we find in the “TT relativity” that at points far from that current loop such a distribution of charges always (in any IFR) behaves like an electric dipole, but as a 4D geometric quantity.

5. CONCLUSIONS

In this paper the “TT relativity” is consistently applied to the investigation of relatively moving systems. We presented the expressions for the covariantly defined spacetime length (1) and for covariant 4D Lorentz transformations (2) when both are written in geometrical terms, and in the “e” and “r” coordinations. It is also shown that it is not the spatial length but the spacetime length (1) which is well defined quantity from the “TT relativity” viewpoint; in the rest frame of the object and when the temporal part of \( l_{AB} \) is zero the spacetime length \( l \) becomes the rest spatial length of the object. Using this result and the covariant definition of charge (11) the expressions (12) for the current density 4-vectors of a CCC were found in the ions’ rest frame. Then the 4-vectors \( E^\alpha \) and \( B^\alpha \) for a CCC are determined by means of the known \( F^\alpha \beta \) (15) and the relations (20) and (21), which connect the \( F^\alpha \beta \) and \( E^\alpha , B^\alpha \) formulations of electrodynamics. This yields Eq. (22), which is one of the main results found in this paper. It shows that for the observers at rest in the ions’ rest frame the spatial components \( E^i \) of \( E^\alpha \) are different from zero outside a CCC. The second important result, which is also found in a completely covariant manner, i.e., in the “TT relativity” treatment, is the existence of invariant charges on a loop with current. There are opposite charges on opposite sides of a square loop with current, but the total charge of that loop is zero. These results are different from those found in all previous works in which mainly the “AT relativity” is used.

Acknowledgements. I am indebted to Prof. F. Rohrlich for reading the first version of this paper and for useful suggestions and encouragement, and also to an anonymous referee for useful comments.

REFERENCES

1. T. Ivezic, Found. Phys. Lett. 12, 105 (1999).
2. F. Rohrlich, Nuovo Cimento B 45, 76 (1966).
3. T Ivezic, preprint SCAN-9802018, on the CERN server.
4. A. Gamba, Am. J. Phys. 35, 83 (1967).
5. T. Ivezić, *Phys. Lett. A* **144**, 427 (1990).
6. T. Ivezić, preprint SCAN-9802017 (on the CERN server).
7. R.M. Wald, *General relativity* (The University of Chicago Press, Chicago, 1984).
8. C. Leubner, K. Aufinger and P. Krumm, *Eur. J. Phys.* **13**, 170 (1992).
9. G. Cavalleri and C. Bernasconi, *Nuovo Cimento B* **104**, 545 (1989).
10. R. Anderson, I. Vetharaniam, G.E. Stedman, *Phys. Rep.* **295**, 93 (1998).
11. D.E. Fahnline, *Am. J. Phys.* **50**, 818 (1982).
12. A. Einstein, *Ann. Physik* **17**, 891 (1905), tr. by W. Perrett and G.B. Jeffery, in *The principle of relativity* (Dover, New York).
13. K. Geiger, *Phys. Rep.* **258**, 240 (1995).
14. M. Pardy, *Phys. Rev. A* **55**, 1647 (1997).
15. A. Laub, T. Doderer, S.G. Lachenmann and R.P. Huebener, *Phys. Rev. Lett.* **75**, 1372 (1995).
16. N. Bilić, *Phys. Lett. A* **162**, 87 (1992).
17. L. Baroni, E. Montanari and A.D. Pesci, *Nuovo Cimento B* **109**, 1275 (1994).
18. W.F. Edwards, C.S. Kenyon and D.K. Lemon, *Phys. Rev. D* **14**, 922 (1976).
19. D.K. Lemon, W.F. Edwards and C.S. Kenyon, *Phys. Lett. A* **62**, 105 (1992).
20. T. Ivezić, *Phys. Rev. A* **44**, 2682 (1991).
21. W.K.H. Panofsky and M. Phillips, *Classical electricity and magnetism*, 2nd edn. (Addison-Wesley, Reading, Mass., 1962).