GUT, Neutrinos, and Baryogenesis

Hitoshi Murayama\textsuperscript{a,b}\textsuperscript{*}

\textsuperscript{a}Department of Physics, University of California
Berkeley, CA 94720

\textsuperscript{b}Theoretical Physics Group, Lawrence Berkeley National Laboratory
Berkeley, CA 94720

It is an exciting time for flavor physics. In this talk, I discuss recent topics in baryogenesis and leptogenesis in light of new data, and implications in $B$ and neutrino physics. I also discuss current situation of grand unified theories concerning coupling unification, proton decay, and indirect consequences in lepton flavor violation and $B$ physics. I explain attempts to understand the origin of flavor based on flavor symmetry, in particular “anarchy” in neutrinos.

1. Introduction

Flavor physics is going through a big revolution. Neutrino oscillation has been a big discovery, and new data on quark sector are pouring in. Given the excitement in flavor physics, I will discuss recent topics on baryogenesis and grand unification, and their connection to flavor physics in this talk.

2. Baryogenesis

Why there is only matter in Universe but no anti-matter is one of the big questions in cosmology and particle physics. Because of its fundamental importance, it is often quoted as one of the primary reasons to study CP violation in flavor physics.

First of all, the amount of baryon we have in our Universe had been quite well determined by Big-Bang Nucleosynthesis. The only free parameter in this theory is the amount of baryons that determines the rate of nuclear fusion process in early Universe. People often quote the baryon-to-photon ratio $\eta = n_B/n_\gamma$, namely the ratio of the baryon number over the number of photons in a fixed volume, because this quantity does not change as the Universe expands. There was a period when different determinations of baryon-to-photon ratio did not agree with each other, and it was said to be a crisis. The problem has largely disappeared recently, and there emerged a consensus on the baryon-to-photon ratio. Let me quote two quantitative analyses:

\begin{equation}
\eta = \frac{n_B}{n_\gamma} = \left\{ \begin{array}{ll}
(4.7^{+1.0}_{-0.8}) \times 10^{-10} & [1] \\
(5.0 \pm 0.5) \times 10^{-10} & [2]
\end{array} \right.
\end{equation}

This determination of the baryon-to-photon ratio is also consistent with the analysis that combines power spectrum in cosmic microwave anisotropy, cluster mass, and large scale structure. Therefore, we know the baryon-to-photon ratio of the Universe with a good confidence.

When I mention the word “anti-matter” in a public talk, it causes a certain level of fear among the audience. It sounds something scary to them. And they are right. When the Universe was hot, at a temperature of 1 GeV, there were practically equal amount of matter and anti-matter. As the temperature decreased, anti-matter has annihilated with matter, leaving only radiation. However, at the level of one out of ten billions or so, there was an excess in the amount of matter.

\textsuperscript{*}This work was supported in part by the DOE Contract DE-AC03-76SF00098 and in part by the NSF grant PHY-0098840.

\textsuperscript{2}When a particle species freezes out, however, the ratio does change. The baryon-to-entropy ratio is constant across the thresholds.
over anti-matter, and this small excess is \textit{us}. We have survived \textit{The Great Annihilation}. This realization immediately leads to a question: “What caused a small excess in the amount of matter over anti-matter?” This excess is called the baryon asymmetry of Universe.

Sakharov pointed out that the small baryon asymmetry may be understood as a consequence of microphysics from a Universe with no asymmetry only if three conditions are satisfied:

1. Existence of process that violates the baryon number.
2. CP Violation.
3. Departure from thermal equilibrium.

The first requirement is obvious. If the Universe had no asymmetry as its initial condition, generation of baryon asymmetry is possible only if the baryon number can change. Then there may be a finite rate of a process that increases the baryon number \( \Gamma(\Delta B > 0) \neq 0 \). If, however, CP were an exact symmetry, a process and its CP conjugate process would have the same rate. Because the baryon number is CP-odd, it would imply that \( \Gamma(\Delta B > 0) = \Gamma(\Delta B < 0) \), and no baryon asymmetry can be generated. To make these rates different, CP violation is mandatory. Even so, thermal equilibrium, by definition, has the same rates for a process and its inverse process. Similarly to the CP conservation, it would also imply \( \Gamma(\Delta B > 0) = \Gamma(\Delta B < 0) \), and no baryon asymmetry would be generated. Therefore, departure from thermal equilibrium is necessary to generate the baryon asymmetry.

It was once hoped that grand unified theories (GUTs) would provide the mechanism of generating baryon asymmetry, a.k.a. baryogenesis \footnote{\textcopyright 2023}. GUTs indeed necessarily break baryon number. If a heavy particle from GUTs remain in the early Universe after the temperature drops below its production threshold, the leftover amount exhibits the departure from thermal equilibrium. Then their decay, if CP is violated, may preferentially produce baryons over anti-baryons, thereby generating baryon asymmetry. It is encouraging that such a decay asymmetry had been established, namely \( \varepsilon' \neq 0 \) or equivalently \( \Gamma(K^0 \rightarrow \pi^+\pi^-) \neq \Gamma(K^0 \rightarrow \pi^-\pi^+) \).

However, the effect of electroweak anomaly changed the picture quite dramatically. The Standard Model actually violates \( B \) \footnote{\textcopyright 2023}. In the Early Universe when the temperature was above 250 GeV, there was no Higgs boson condensate and \( W \) and \( Z \) bosons were massless (so where all quarks and leptons). Therefore \( W \) and \( Z \) fields were just like electromagnetic field in the hot plasma and were fluctuating thermally. The quarks and leptons move around under the fluctuating \( W \)-field background. To see what they do, we solve the Dirac equation for fermions coupled to \( W \). There are positive energy states that are filled in the “vacuum” configuration.

Figure 1. The energy levels of the Dirac equation in the presence of fluctuating \( W \)-field move up and down. All negative energy states are occupied while the positive energy states vacant in the “vacuum” configuration.
Figure 2. Once in a while, the fluctuation in the $W$-field becomes so large that the energy levels of the Dirac equation in the presence of fluctuating $W$-field shift all the way by one unit. Then a positive energy state is occupied and a particle is created.

Note that $\Delta(B-L) = 0$; the electroweak anomaly preserves $B-L$.

Because of this process, the pre-existing $B$ and $L$ are converted to each other to find the chemical equilibrium at $B \sim 0.35(B-L)$, $L \sim -0.65(B-L)$ \(\square\). In particular, even if there was both $B$ and $L$, both of them get washed out if $B-L$ was zero.

Given this problem, there are now two major directions in the baryogenesis. One is the electroweak baryogenesis \(\square\), where you try to generate $B = L$ at the time of the electroweak phase transition so that they do not get washed out further by the electroweak anomaly. The other is the leptogenesis \(\square\), where you try to generate $L \neq 0$ but no $B$ from neutrino physics well before the electroweak phase transition, and $L$ gets partially converted to $B$ due to the electroweak anomaly.

2.1. Electroweak Baryogenesis

It appears at the first sight that the baryogenesis is possible in the Minimal Standard Model. First, the baryon number is violated as we discussed above. Second, CP is also violated in the Standard Model. Third, if the phase transition of electroweak symmetry breaking is first order, the coexistence of broken and unbroken phases at the phase transition is a departure from equilibrium. Then all three conditions by Sakharov are satisfied. The question is if enough baryon asymmetry can be generated quantitatively \(\square\).

There are at least two big problems in the Standard Model. The first problem is the order of phase transition. The first order phase transition is possible only if the Higgs boson is relatively light, $m_H \lesssim 60$ GeV. Above this mass, the phase transition becomes second order, and there is no departure from equilibrium. The LEP bound on the Higgs boson has excluded this possibility. The other problem is the size of CP violation. In the Standard Model, any CP violating effects must be proportional to the so-called Jarlskog parameter, $J = \Im(\det[M^u_d M^d_u, M^l_d M^d_l])$. At the phase transition temperature $T_{EW} \simeq v$, the dimensionless quantity that characterizes the size of CP violation is $J/v^{12} \sim 10^{-20} \ll 10^{-10}$. Unless there is a mechanism of tremendous enhancement by ten orders of magnitude, the resulting baryon asymmetry would be too small.

The Minimal Supersymmetric Standard Model (MSSM) can go around both problems. The first order phase transition becomes a possibility again if one of the scalar top quark is very light, $m_{t_R} \lesssim 160$ GeV, despite the LEP bound. There is also a new CP violating phase $3(\mu^* M_2)$ in the chargino sector which could in principle be order one. See Fig. 3 and the caption for the mechanism of the baryogenesis.

However, the model is getting cornered; the available parameter space is becoming increasingly limited due to the LEP constraints on chargino, scalar top quark and Higgs boson \(\square\). This is because the LEP bound on the lightest Higgs boson requires a large radiative correction, and hence a large scalar top quark mass. Because we need a light right-handed scalar top quark to achieve the first order phase transition, only the left-handed scalar top quark can be raised above TeV for this purpose. $\tan \beta$ also needs to be large to raise the Higgs mass. However, the CP violation is a relative phase between $\mu$ and $M_2$ in the chargino mass matrix

$$
\begin{bmatrix}
M_2 & \sqrt{2}m_W \cos \beta \\
\sqrt{2}m_W \sin \beta & \mu
\end{bmatrix},
$$

and the phase becomes unphysical as $\cos \beta \to 0$ ($\tan \beta \to \infty$). Therefore, $\tan \beta$ cannot be large
Figure 3. The mechanism of baryogenesis in the MSSM. The bubbles of the true vacuum with broken electroweak symmetry $v \neq 0$ forms and expands into the false vacuum with unbroken electroweak symmetry. The charginos bounce off the expanding bubble walls. Because of the CP violation in the chargino mass matrix, the reflection probabilities are different for different charginos. The interaction of light scalar top quark with the charginos convert the difference in the charginos to the asymmetry between left-handed and right-handed top quarks. At this point, there is no overall top quark asymmetry and no baryon number. Then the asymmetry in the left-handed top quark is partially converted to the lepton asymmetry due to the electroweak anomaly. The reduced asymmetry in the left-handed top quark and the remaining (unaffected) asymmetry in the right-handed top quark no longer cancel and there is net baryon asymmetry.

To retain enough CP violation, causing a tension with the requirement of heavy enough Higgs mass. Moreover, the constraints from electric dipole moments are quite severe if the relative phase between $\mu$ and $M_2$ is order unity. What it means is that we are supposed to find a right-handed scalar top quark, charginos “soon” with a large CP violation in the mass matrix.

It is important to ask if there is any interesting consequence of this scenario in CP violation on B systems. It turns out, however, that there is no new CP violation in B systems from the relevant phase for the electroweak baryogenesis in the MSSM. The most important effect is the contribution to $B_d,s-\bar{B}_{d,s}$ mixing. Surprisingly, the new CP violating phase does not appear in the diagram, and hence the mixing amplitude has the same phase as in the Standard Model. There is, however 20–30% enhancement in the mixing amplitude. Such an enhancement cannot be established now because of the theoretical uncertainty in the $B$-parameter. However lattice calculations are expected to reduce the uncertainty down to 5% level in the near future, and this enhancement may be seen experimentally. It would require a complicated analysis. In the case of $B_s$ mixing, the Standard Model prediction must be improved with a better determination of $V_{cb}$ (and hence $V_{ts}$ through unitarity) and a better calculation of the $B$-parameter. In the case of $B_d$ mixing, we used to determine $V_{td}$ from the mixing. For the purpose of extracting a new contribution in the mixing, we have to determine $V_{td}$ by alternative method. By an improved determina-

3 After my talk, the constraints from electric dipole moments had been shown to be even more severe.

4 It is quite possible that other models of electroweak baryogenesis lead to new CP violation in $B$-physics.
The most important contribution to $B$-physics from particles relevant for the electroweak baryogenesis in the MSSM. The phase of the amplitude is exactly the same as that of the Standard Model diagram. The same argument applies to $B_s$ mixing.

2.2. Leptogenesis

In leptogenesis, you generate $L \neq 0$ first. Then $L$ gets partially converted to $B$ by the electroweak anomaly. The question then is how you generate $L \neq 0$. In the original proposal \[3\], it was done by the decay of a right-handed neutrino (say $N_1$), present in the seesaw mechanism, with a direct CP violation. At the tree-level, a right-handed neutrino decays equally into $lH$ and $\bar{\nu}_lH^*$. At the one-loop level, however, the interference between diagrams shown in Fig. 6 cause a difference in the decay rates of a right-handed neutrino into leptons and anti-leptons as

$$\frac{\Gamma(N_1 \to lH) - \Gamma(N_1 \to \bar{l}_iH)}{\Gamma(N_1 \to l_iH) + \Gamma(N_1 \to \bar{l}_iH)} \approx \frac{1}{8\pi} \Re(h_{ij}h_{ik}h_{kj}^*h_{ij}^*) \frac{M_1}{|h_{1i}|^2 M_3}. \tag{3}$$

Once the right-handed neutrinos are produced in early Universe, their long lifetime would allow them to decay out of equilibrium, thereby generating an asymmetry between leptons and anti-leptons. The lepton asymmetry then is partially converted to baryon asymmetry thanks to electroweak anomaly. Much more details had been worked out in the light of recent neutrino oscillation data and it had been shown that a right-handed neutrino of about $10^{10}$ GeV can well account for the cosmic baryon asymmetry from its out-of-equilibrium decay \[4\].

There is some tension in the supersymmetric version because of cosmological problems caused by the gravitino, which prefers a low reheating temperature after inflation. But a supersymmetric problem has a supersymmetric solution. It can be circumvented using the superpartner of right-handed neutrino that can have a coherent oscillation after the inflation \[15\]. Leptogenesis can work.

Can we prove leptogenesis experimentally? Lay Nam Chang, John Ellis, Belen Gavela, Boris Kayser, and myself got together at Snowmass 2001 and discussed this question. The short answer is unfortunately no. There are additional CP violating phases in the heavy right-handed neutrino sector that cannot be seen by studying the light left-handed neutrinos\[4\]. For example, even

\[5\text{If you believe in a certain scenario of supersymmetry breaking, low-energy lepton-flavor violation can carry information about CP violation in the right-handed neutrino sector \[4\]. However, such connection depends on the as-}
two-generation seesaw mechanism is enough to have CP violation that can potentially produce lepton asymmetry, unlike the minimum of three-generations for CP violation in neutrino oscillation. However, we decided that if we will see (1) electroweak baryogenesis ruled out, (2) lepton-number violation e.g. in neutrinoless double beta decay, and (3) CP violation in the neutrino sector e.g., in very long-baseline neutrino oscillation experiment, we will probably believe it based on these “archaeological” evidences.

3. Grand Unified Theories

As we have discussed, baryogenesis may not be a good motivation for GUTs any more. However, there are still many reasons to consider GUTs seriously to answer some of the big questions in the Standard Model. For example, electric charges are quantized in the unit of $e/3$ among quarks and leptons, while the electromagnetic $U(1)$ gauge invariance does not require such quantization. Similarly, the non-trivial anomaly cancellation and seemingly random hypercharge assignments hint at deeper organizing principles behind the quantum numbers of quarks and leptons. Three forces are seemingly unrelated, but they are all based on the gauge principle. Also philosophically, unified description of all forces has been a dream since Einstein. GUTs address these questions beautifully.

The quarks and leptons are unified in GUT multiplets. In the case of $SU(5)$ group, $5^*$ multiplet includes a lepton doublet and a right-handed down quark of three colors, and $10$ multiplet includes a quark doublet and a right-handed up quark of three colors, and right-handed charged lepton.

Phenomenologically, the original non-supersymmetric $SU(5)$ GUT no longer works because the gauge coupling constants measured precisely at LEP and SLC extrapolated to higher energies do not meet at a point. However with the MSSM particle content, running of the gauge couplings changes and they do meet within a percent level accuracy at $2 \times 10^{16}$ GeV. Therefore, the Minimal supersymmetric $SU(5)$ GUT appears a success phenomenologically.

On the other hand, another prediction of GUTs than the gauge coupling unification, namely the proton decay, has not been observed. In fact, the limit from SuperKamiokande $\tau(p \rightarrow \bar{\nu}K^+) > 6.7 \times 10^{32}$ yr implies the mass of the color-triplet $SU(5)$-partner of the Higgs boson to be heavier than $7.6 \times 10^{16}$ GeV, while the coupling unification requires it to be well below $10^{16}$ GeV (Fig. 8). Even with the so-called “decoupling” limit where first- and second-generation squarks are assumed to be heavy, the lower bound comes down only

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{The gauge coupling constants extrapolated to higher energies using the particle contents of the Minimal Standard Model and the Minimal Supersymmetric Standard Model.}
\end{figure}
to $5.7 \times 10^{16}$ GeV. Therefore, the Minimal supersymmetric $SU(5)$ GUT is now excluded \cite{18}. 

Unfortunately, the prediction of the proton lifetime is sensitive to the ugly part of GUT model building, namely how to accommodate observed quark and lepton masses, which the minimal model gets wrong by a factor of three, and how to keep only the doublet Higgs light while making color-triplet partner heavy ("triplet-doublet splitting problem"). Expanded particle content at the GUT scale also makes the prediction more uncertain \cite{18}. Some fine-tuning can suppress the decay rate \cite{14}. A recent proposal of breaking $SU(5)$ using extra dimension on a one-dimensional orbifold $S^1/Z_2$ eliminates this mode of proton decay \cite{21}. Therefore, one cannot say that supersymmetric GUTs are excluded. In other words, proton decay may be just around the corner. It should be remembered that proton decay is a truly unique window to physics at $>10^{15}$ GeV scale, and it is worth pursuing anyway. Some other supersymmetric GUT models predict $p \rightarrow e^+\pi^0$ mode close to the experimental limit, including flipped $SU(5)$ model \cite{18} and orbifold GUT \cite{20}.

In the absence of direct signal (proton decay), it is natural to look for other indirect effects of GUT. It has been well studied that quark-lepton unification causes flavor-changing effects among leptons through top quark Yukawa coupling, giving rare phenomena such as $\mu \rightarrow e\gamma$, $\mu \rightarrow e$ conversion at experimentally accessible rates \cite{21}. Recent discovery of $\nu_\mu \rightarrow \nu_\tau$ atmospheric neutrino oscillation with large mixing angle may also give observable $\tau \rightarrow \mu\gamma$ etc at near future experiments \cite{22}.

I would like to advertise yet another interesting signal of GUT in $B$-physics \cite{23}. Take the large mixing between $\nu_\mu$ and $\nu_\tau$. In $SU(5)$ GUT, they are in the same multiplets as $s_R$ and $b_R$, respectively. Therefore, it is natural to expect a large mixing between $s_R$ and $b_R$. Such mixing drops out completely from CKM matrix because it keeps track of mixing only among left-handed quarks that participate in the charged-current weak interaction. However, in presence of supersymmetry, a large mixing between $\tilde{s}_R$ and $\tilde{b}_R$ would lead to observable effects. For example, supersymmetric contribution to $B_s$ mixing may well be comparable to the standard model contribution even with squarks at 1 TeV. Supersymmetric penguin contribution to $b \rightarrow s\bar{s}s$ allows di-

---

**Figure 8.** The GUT-scale mass parameters $M_{H^C}$ (mass of color-triplet Higgs that mediates the proton decay) and $M_{GUT} = (M_X^2 M_Z^2)^{1/3}$ ($M_X$ is the mass of the $X$ boson, and $M_Z$ that of the adjoint Higgs) from the requirement that coupling constants unify.

**Figure 9.** The diagrams that can contribute to direct CP violation in $B_d \rightarrow \phi K_s$ and $B_s$ mixing.
rect CP violation and may give different \(\sin 2\beta\) in \(B_d \to \phi K_s\) from that in \(B_d \to J/\psi K_s\).\(^7\)

4. Models of Flavor

The most pressing question on the origin of flavor is what distinguishes flavor. Three generations of quarks and leptons have exactly the same quantum numbers. But then, how come that they have so different masses and small mixings? When we learn quantum mechanics, we see that states with similar quantum numbers have similar energies, and they mix greatly. Due to some reason, three generations do not follow this common wisdom. Why?

One way to answer this question, developed very actively in the past few years, is the concept of explicitly broken flavor symmetry. The idea is simple: there must a new (but hidden) quantum number that distinguishes different flavors. Three generations have very different masses and mix little. According to Nöther’s theorem, a quantum number means symmetry, and hence flavor symmetry. The flavor symmetry must allow top quark Yukawa coupling because it is a coupling of order unity and hence flavor symmetry. The MNS matrix of explicitly broken flavor symmetry. The idea has been a big surprise to all of us.

Models of Flavor

The most pressing question on the origin of flavor is what distinguishes flavor. Three generations of quarks and leptons have exactly the same quantum numbers. But then, how come that they have so different masses and small mixings? When we learn quantum mechanics, we see that states with similar quantum numbers have similar energies, and they mix greatly. Due to some reason, three generations do not follow this common wisdom. Why?

One way to answer this question, developed very actively in the past few years, is the concept of explicitly broken flavor symmetry. The idea is simple: there must a new (but hidden) quantum number that distinguishes different flavors. Three generations have very different masses and mix little. According to Nöther’s theorem, a quantum number means symmetry, and hence flavor symmetry. The flavor symmetry must allow top quark Yukawa coupling because it is a coupling of order unity and hence flavor symmetry.

To make the discussion more concrete, let us study the following SU(5)-like flavor quantum number assignment. I choose a simple \(U(1)\) charge, and I assign \(^2\)

\[
\begin{align*}
\text{10}_1 (+2), & \quad \text{10}_2 (+1), & \quad \text{10}_3 (0), \\
\text{5}_1 (+1), & \quad \text{5}_2 (+1), & \quad \text{5}_3 (+1).
\end{align*}
\]

The subscripts refer to the generations, and the charges shown in parentheses. With this assignment, the only allowed Yukawa couplings are the ones for the top quark. Other Yukawa couplings are forbidden. Next I assume that the \(U(1)\) symmetry is broken by a small parameter \(\epsilon \sim 0.05\) that carries the charge \(-1\). Then it becomes possible to fill in all elements of the mass matrices,

\[
M_u \sim \begin{pmatrix} e^4 & e^3 & e^2 \\ e^3 & e^2 & e \\ e^2 & e & 1 \end{pmatrix},
\]

\[
M_d \sim \begin{pmatrix} e^3 & e^2 & e^2 \\ e^2 & e & e \\ e & e & e \end{pmatrix},
\]

\[
M_l \sim \begin{pmatrix} e^3 & e^2 & e \\ e^2 & e & e \\ e & e & e \end{pmatrix}.
\]

The symbol \(\sim\) emphasizes that we cannot predict precise numbers based on this simple idea, but only the order of magnitudes suppressed by powers of \(\epsilon\). A simple approximate prediction out of this charge assignment is that

\[
m_u : m_c : m_t \sim m_d^2 : m_s^2 : m_b^2 \\
\sim m_e^2 : m_\mu : m_\tau \sim \epsilon^4 : \epsilon^2 : 1,
\]

which is phenomenologically acceptable. Note that up quarks are doubly hierarchical compared to the down quarks and charged leptons. Mixing angles are also predicted, \(V_{cb} \sim V_{ts} \sim \epsilon, V_{ub} \sim V_{td} \sim \epsilon^2\), which work quite well, and \(V_{us} \sim V_{cd} \sim \epsilon\) is a little bit too small but not crazy.

What about neutrinos? Indeed, recent data on neutrino oscillations have shed considerable insight into the flavor symmetry. The MNS matrix suggested by atmospheric, solar (LMA solution), and reactor data has the form

\[
(e \mu \tau) \begin{pmatrix} \text{large} & \text{large} & \text{smallish} \\ \text{large} & \text{large} & \text{large} \\ \text{large} & \text{large} & \text{large} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},
\]

\[
\frac{\Delta m^2_{21}}{\Delta m^2_{23}} = 0.01 - 0.2.
\]

The mass hierarchy is not very large, especially after taking square roots. All angles are large except \(|U_{e3}| \lesssim 0.15\), but even this constraint is not particularly strong compared to \(U_{e2} \sim 0.4\), \(U_{\mu 3} \sim 0.7\). It has been a big surprise to all of us.
that the pattern is so different from quarks and charged leptons.

I actually find the neutrino masses and mixings very natural. In view of the question I posed earlier, if three quantum mechanical states share the same quantum numbers, their energies (masses) are expected to be similar, and their mixings unsuppressed. We have been so much used to hierarchical masses and small mixings over many decades, but what is surprising is not the neutrinos but other quarks and leptons rather! I view the observed pattern of neutrino masses and mixings a great confirmation of our naive intuition.

In terms of flavor quantum numbers, all we need to do then is to assign the same quantum numbers to three generations of neutrinos. Indeed, the charge assignment was motivated by this requirement to obtain

$$M_\nu \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$  \hspace{1cm} (8)

Here, the overall suppression of $\epsilon^2$ is dropped because the overall mass scale of neutrino masses is probably determined by other physics such as seesaw mechanism.

But you may wonder if such an argument may only produce large mixing angles, but never dramatically maximal mixing as in atmospheric neutrino oscillations. It turns out that a maximal mixing is in a sense the most natural angle. Using a simple Monte Carlo of random neutrino mass matrices, you can see that there is a peak in the distribution at $\sin^2 2\theta_{23} = 1$. Actually, you can understand this distribution based on purely group-theoretic consideration. The unique invariant measure (Haar measure) of SU(3) MNS matrix gives this distribution.

We called this simple fact “anarchy” \cite{24,25}. Because there is no quantum number to distinguish three generations of neutrinos, neutrino mass matrix lacks any particular structure. That alone explains large mixing angles and small hierarchy. Even though $\sin^2 2\theta_{13}$ appears indeed small compared to the obtained distribution, if you get three quantities $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$, and $\Delta m^2_{12}/\Delta m^2_{23}$ right with one outlier, I find it perfectly reasonable. The anarchy then predicts the LMA solution, testable at KamLAND, and $\theta_{13}$ just below the bound. CP violation is also $O(1)$. This is the perfect scenario for long-baseline neutrino oscillation experiments.

5. Conclusion

Flavor physics is going through an amazing period. Just to name a few important points I covered in this talk, (1) Electroweak baryogenesis is getting cornered, (2) Leptogenesis is gaining momentum, and (3) Neutrinos provide new insight into the origin of flavor. The good news is that we will obtain more data, including rare decays and possible deviations from the Standard Model, to hopefully pin down the flavor symmetry and eventually its dynamical origin.

Acknowledgements

I thank the organizers for inviting me to the exciting workshop and also for the patience waiting...
for my manuscript. This work was supported in part by the DOE Contract DE-AC03-76SF00098 and in part by the NSF grant PHY-0098840.

REFERENCES

1. T. X. Thuan and Y. I. Izotov, arXiv:astro-ph/0101282.
2. S. Burles, K. M. Nollett and M. S. Turner, Astrophys. J. 552, L1 (2001) arXiv:astro-ph/0010173.
3. See, e.g., M. Tegmark, M. Zaldarriaga and A. J. Hamilton, Phys. Rev. D 63, 043007 (2001) arXiv:astro-ph/0008167.
4. M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978) [Erratum-ibid. 42, 746 (1978)]; A. Y. Ignatev, N. V. Krasnikov, V. A. Kuzmin and A. N. Tavkhelidze, Phys. Lett. B 76, 436 (1978).
5. G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976).
6. V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155, 36 (1985).
7. S. Y. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B308 (1988) 885; J. A. Harvey and M. S. Turner, Number Violation,” Phys. Rev. D 42 (1990) 3344.
8. M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).
9. See a review and references therein: A. G. Cohen, D. B. Kaplan and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 43, 27 (1993) arXiv:hep-ph/9302210.
10. M. Carena, J. M. Moreno, M. Quiros, M. Seco and C. E. Wagner, Nucl. Phys. B 599, 158 (2001) arXiv:hep-ph/0011053.
11. J. M. Cline, M. Joyce and K. Kainulainen, JHEP 0007, 018 (2000) arXiv:hep-ph/0006119; Erratum, arXiv:hep-ph/0110031.
12. H. Murayama and A. Pierce, arXiv:hep-ph/0201261.
13. D. Chang, W. F. Chang and W. Y. Keung, arXiv:hep-ph/0205084; A. Pilaftsis, arXiv:hep-ph/0207277.
14. See, e.g., W. Buchmüller, Presented at 8th International Symposium on Particle Strings and Cosmology (PASCOS 2001), Chapel Hill, North Carolina, 10-15 Apr 2001, arXiv:hep-ph/0107153.
15. H. Murayama and T. Yanagida, Phys. Lett. B 322, 349 (1994) arXiv:hep-ph/9310297; K. Hamaguchi, H. Murayama and T. Yanagida, arXiv:hep-ph/0109030.
16. J. R. Ellis, J. Hisano, S. Lola and M. Raidal, arXiv:hep-ph/0109125.
17. H. Murayama and A. Pierce, arXiv:hep-ph/0206177.
18. H. Murayama and A. Pierce, Phys. Rev. D 65, 055009 (2002) arXiv:hep-ph/0108104.
19. K. S. Babu and S. M. Barr, Phys. Rev. D 48, 5354 (1993) arXiv:hep-ph/9306242.
20. Y. Kawamura, Prog. Theor. Phys. 105, 999 (2001) arXiv:hep-ph/0101212; L. J. Hall and Y. Nomura, Phys. Rev. D 64, 055003 (2001) arXiv:hep-ph/0103123; arXiv:hep-ph/0205055.
21. R. Barbieri and L. J. Hall, Phys. Lett. B 338, 212 (1994) arXiv:hep-ph/9408406; R. Barbieri, L. J. Hall and A. Strumia, Nucl. Phys. B 445, 219 (1995) arXiv:hep-ph/9501334; R. Barbieri, L. J. Hall and A. Strumia, Nucl. Phys. B 449, 437 (1995) arXiv:hep-ph/9504373.
22. See, e.g., J. Hisano and D. Nomura, Phys. Rev. D 59, 116005 (1999) arXiv:hep-ph/9810479.
23. D. Chang, A. Masiero and H. Murayama, arXiv:hep-ph/0205111.
24. B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0207070; T. Auzhev (Belle collaboration), talk presented at ICHEP 2002, Amsterdam, July 26, 2002; Doug Wright (BABAR collaboration), talk presented at ICHEP 2002, Amsterdam, July 26, 2002.
25. N. Haba and H. Murayama, Phys. Rev. D 63, 053010 (2001) arXiv:hep-ph/0009174.
26. L. J. Hall, H. Murayama and N. Werner, Phys. Rev. Lett. 84, 2572 (2000) arXiv:hep-ph/9911341.