Neutrino Radiation Challenges and Proposed Solutions for Many-TeV Muon Colliders

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Abstract. Neutrino radiation is expected to impose major design and siting constraints on many-TeV muon colliders. Previous predictions for radiation doses at TeV energy scales are briefly reviewed and then modified for extension to the many-TeV energy regime. The energy-cubed dependence of lower energy colliders is found to soften to an increase of slightly less than quadratic when averaged over the plane of the collider ring and slightly less than linear for the radiation hot spots downstream from straight sections in the collider ring. Despite this, the numerical values are judged to be sufficiently high that any many-TeV muon colliders will likely be constructed on large isolated sites specifically chosen to minimize or eliminate human exposure to the neutrino radiation. It is pointed out that such sites would be of an appropriate size scale to also house future proton-proton and electron-positron colliders at the high energy frontier, which naturally leads to conjecture on the possibilities for a new world laboratory for high energy physics. Radiation dose predictions are also presented for the speculative possibility of linear muon colliders. These have greatly reduced radiation constraints relative to circular muon colliders because radiation is only emitted in two pencil beams directed along the axes of the opposing linacs.

I INTRODUCTION

Neutrinos interact so rarely that, only 50 years ago, even their detection was not considered to be feasible. It is therefore quite surprising that the design of future muon colliders will usually be constrained by the need to limit hazards from neutrino radiation (1; 2). Neutrinos are produced copiously at muon colliders from the decays of the large currents of muons circulating in the collider ring:

1) To appear in Proc. HEMC’99 Workshop – Studies on Colliders and Collider Physics at the Highest Energies: Muon Colliders at 10 TeV to 100 TeV; Montauk, NY, September 27-October 1, 1999, web page http://pubweb.bnl.gov/people/bking/heshop. This work was performed under the auspices of the U.S. Department of Energy under contract no. DE-AC02-98CH10886.
The decays of muons in a muon collider will produce a neutrino radiation disk emanating out tangentially from the collider ring. Radiation hot spots in the disk will occur directly downstream from straight sections in the collider ring.

\[
\begin{align*}
\mu^- & \rightarrow \nu_\mu + \bar{\nu}_e + e^- , \\
\mu^+ & \rightarrow \nu_\mu + \nu_e + e^+ .
\end{align*}
\]  

(1)

The neutrino direction is tightly collimated to within a characteristic angle, \( \theta_\nu \), of the decaying muon’s direction, where:

\[
\theta_\nu = 1/\gamma_\mu = \frac{m_\mu c^2}{E_\mu} \approx \frac{10^{-4}}{E_\mu [\text{TeV}]} ,
\]

(2)

for \( \gamma_\mu \) the relativistic boost factor of the muon, \( m_\mu \) the muon rest mass, \( c \) the speed of light and \( E_\mu \) the muon energy. (Units are given in square brackets in the equations throughout this paper.)

The combined effect of all the muon decays will be a disk of neutrinos emanating out in the plane of the collider ring (2), as shown in figure 1. Straight sections in the ring will cause radiation hot spots in the disk (1) where all of the decays in the straight section line up into a pencil beam that is superimposed on the disk, again with a characteristic opening half-angle for the cone of \( 1/\gamma_\mu \). As a notable contrast to all other radiation hazards, the neutrino attenuation length is too long for the beam to be appreciably attenuated by any practical amount of shielding material, including even the expanse of ground between the collider ring and where the radiation disk breaks ground.

Example parameter sets for muon colliders that illustrate the radiation hazard are given in table 1. The entries in the table will be referred to and explained throughout this paper. For now, we note that the radiation doses may be compared with the U.S. Federal off-site limit of 1 mSv/year or, in alternative units, 100 mrem/year. (The limit is comparable to typical background radiation levels of 0.4 to 4 mSv/year (3).) The radiation hazard is seen to rise sharply with muon collider energy, increasing from a tiny fraction of the legal limit for the lower energy colliders to well above the limit for the collider scenarios at 10 TeV and 100 TeV.
This behavior will be quantified in the two sections that follow; the next section will characterize the radiation dose for colliders up to the TeV energy scale and the section after that will discuss some mitigating factors that come into play for the many-TeV muon colliders that are the subject of this workshop. Possible solutions for the hazard at many-TeV energies will be discussed in the next-to-last section of the paper before rounding out with a summary section.

**TABLE 1.** Straw-man muon collider specifications and the corresponding neutrino radiation parameters for muon colliders at 0.1, 4, 100 and 100 TeV. The first two muon collider scenarios were taken from references (4) and (5), respectively, while the final two scenarios were straw-man parameter sets for this workshop (6). The calculation of the neutrino radiation parameters is discussed in the text.

| center of mass energy, $E_{\text{CoM}}$ | 0.1 | 4 TeV | 10 TeV | 100 TeV |
|----------------------------------------|-----|-------|--------|--------|
| additional description                  |     |       |        |        |
| collider luminosity, $L$ [cm$^{-2}$s$^{-1}$] |     |       |        |        |
| collider int. lum., $\int L$ [fb$^{-1}$/yr] |     |       |        |        |
| muon beam energy, $E_{\mu}$ [TeV]      | 0.05| 2     | 5      | 50     |
| muon decays/yr, $N_{\mu}^+$ [10$^{20}$] |     |       |        |        |
| collider reference depth, $d$ [m]      | 10  | 300   | 100    | 100    |
| $\nu$ beam distance to surface, $L$ [km] | 11  | 62    | 36     | 36     |
| $\nu$ beam radius at surface [m]       | 24  | 3.3   | 0.8    | 0.08   |
| ave. rad. dose in plane [mSv/yr]       | $2 \times 10^{-5}$ | $5 \times 10^{-4}$ | 2.3 | 10 |
| str. sec. len. for 10x ave. rad. [m]   | 1.9 | 1.1   | 1.0    | 4.2    |

II THE NEUTRINO RADIATION HAZARD FOR MUON COLLIDERS UP TO TEV ENERGY SCALES

This section reviews reference (2) in characterizing the potential neutrino radiation hazard and giving numerical estimates for the radiation dose in the so-called equilibrium situation that pertains to the beams from muon colliders at the TeV energy scale and below.

Neutrinos at energies beyond a few GeV interact predominantly through deep inelastic scattering off nucleons. The radiation hazard arises from the showers of penetrating charged particles produced through neutrino interactions with any material bathed by the neutrino radiation disk, as is indicated in figure 2. Starting from the initial interaction products, an avalanche effect of secondary, tertiary, etc. interactions, produces the vast majority of the charged particles. For TeV-scale neutrinos, neutrino interactions in people themselves may therefore only account for as little as 0.1% of their radiation dose (7) because the primary hadrons from the interaction will typically exit the person before interacting to commence the shower of charged particles.

The development of particle showers makes the dose very dependent on the local surroundings. Radiation hazard calculations must conservatively consider the
worst case configuration where a person is (i) completely bathed in the radiation disk or the pencil beam downstream from a straight section and (ii) is surrounded by material that will initiate showers from neutrino interactions. A further requirement for the validity of these calculations is that the characteristic density of the surrounding material must be sufficient to contain the showers within the pencil beam or radiation disk. Such geometries can be contrasted with the more common situation where the showers will spread out transversely beyond the beam and, hence, dilute the dose received by someone within the disk. Instead, this containment criterion will be satisfied only by materials where the nuclear interaction length that characterizes shower development is shorter than the beam radius, as will generally be the case for solids or liquids but not for air or other gaseous media. As examples (3), water and quartz have interaction lengths of 85 cm and 43 cm, respectively, while the interaction length of air, 700 meters, is much larger than the typical few-meter beam radii expected at TeV-scale muon colliders (see table 1).

The worst-case situation can be conveniently modeled for numerical calculations (2) by considering a person completely enclosed within a “tissue equivalent medium” i.e. a medium with the approximate density of water. (The “scuba diver” configuration.) This geometry is illustrated in figure 3. For this simplified model and rather artificial geometry, it is clear that the energy–per–unit–mass absorbed by a person is constrained simply by conservation of energy to be approximately equal to the summed energy of the neutrino interactions in the person. This applies even though most of the deposited energy is from shower products of interactions upstream from the person. This approximate equality is referred to as the equilibrium approximation. For more realistic and general geometries with inhomogeneous distributions of mass, it can be argued (2) that the equilibrium approximation is either valid or, alternatively, conservatively overestimates the radiation dose.

The calculation of radiation doses is straightforward (2) within this equilibrium assumption because neutrino interaction cross sections are well known and the approximate neutrino flux within the pencil beams can be simply predicted from the known decay kinematics and relativistic kinematics. Here we merely reproduce, in a slightly reorganized form, the formula of reference (2) for the average whole-body radiation dose, $D^{\text{ave}}$, in the plane of the collider ring:
FIGURE 3. Conceptual illustration of the “equilibrium approximation” geometry used for quantitative worst-case radiation calculations. The entire person is bathed by the neutrino radiation disk in the plane of the collider ring and perhaps additionally by the radiation cone downstream from a straight section in the collider ring. The person is also enclosed in a “tissue equivalent medium” that has sufficient density to localize the charged particle showers from neutrino interactions so that little of the showers’ energy spills out beyond the radiation disk.

\[
D^{\text{ave}}[mSv] \simeq 3.7 \times N_\mu^+ [10^{20}] \times \frac{(E_\mu[TeV])^3}{(L[km])^2}.
\]

(3)

\(N_\mu^+\) is the number of muons decaying in the collider ring, per year and per charge sign and given in appropriate units of \(10^{20}\) decays. \(L\) is the tangential distance from the ring to where the dose is measured – typically where the radiation disk exits the ground.

The additional radiation hot-spot from a straight section of length \(l^{ss}\) is given by (2):

\[
D^{ss}[mSv] = 1.1 \times 10^5 \times N_\mu[10^{20}] \times f^{ss} \times \frac{(E_\mu[TeV])^4}{(L[km])^2},
\]

(4)

where the fraction, \(f^{ss}\), of the ring circumference, \(C\), corresponds to \(l^{ss}\) through:

\[
f^{ss} = \frac{l^{ss}}{C}.
\]

(5)

Equation 4 can be rewritten in terms of \(l^{ss}\) and the average bending magnetic field in the collider ring, \(B^{\text{ave}}\), as

\[
D^{ss}[mSv] \simeq 5.3 \times N_\mu^+ [10^{20}] \times l^{ss}[m] \times B^{\text{ave}}[T] \times \frac{(E_\mu[TeV])^3}{(L[km])^2},
\]

(6)

by making use of the relation

\[
C[km] = \frac{2\pi \cdot E_\mu[TeV]}{0.3 \cdot B^{\text{ave}}[T]}.
\]

(7)
Equations 3, 4 and 6 are not claimed to be accurate at much better than order-of-magnitude level and detailed follow-up Monte Carlo simulations have confirmed their predictions to this level of accuracy (8).

The ratio of equations 3 and 6 immediately gives the length of straight section, $l_{\text{equiv}}$, that approximately doubles the in-plane average radiation dose:

$$l_{\text{equiv}} [\text{meters}] \simeq \frac{0.7}{B_{\text{ave}}[T]}.$$ (8)

This is only of order 10 cm for the typical average bending fields expected in muon collider storage rings.

The energy-cubed dependences of equations 3 and 6 deserve further comment. The dependence in equation 3 is the product of three linear factors: 1) the inverse width of the disk, which goes as $1/\theta_\nu$, 2) the neutrino cross-section, $\sigma_{\nu N}$, and 3) the average neutrino energy, $<E_\nu>$, that is deposited per interaction:

$$\text{disk average dose}, \ D_{\text{ave}} \sim \frac{1}{\theta_\nu} \cdot \sigma_{\nu N} \cdot <E_\nu> \propto E_\mu \cdot E_\mu \cdot E_\mu = E_\mu^3.$$ (9)

For the straight sections, the inverse disk width is replaced by the inverse cross-sectional area of the pencil beam, which goes as $1/\theta_\nu^2$. Also, the proportionality of equation 4 on $f^{ss}$ brings in a factor of $1/E_\mu$ for a given value of $l^{ss}$ in equation 6, using equations 5 and 7. The energy scaling of equation 6 can therefore be broken down into:

$$\text{str. section dose}, \ D^{ss} \sim \left(\frac{1}{\theta_\nu}\right)^2 \cdot \sigma_{\nu N} \cdot <E_\nu> \cdot f^{ss} \propto E_\mu^2 \cdot E_\mu \cdot E_\mu/E_\mu = E_\mu^3.$$ (10)

Equations 3 and 6 predict the numerical radiation doses in table 1 for the 0.1 TeV and 4 TeV collider examples. The values used for $L$ assume a collider located at a specified depth, $d$, under a site with a smooth surface having the average curvature of the Earth, so that

$$L_{\text{exit}} = \left(2 \times d \times R_E\right)^{1/2},$$ (11)

where the Earth’s radius has the value $R_E = 6.4 \times 10^6$ m and the equation very reasonably assumes that $d \ll R_E$.

The preceding discussion in this section has provided the background information for the numerical estimates of table 1 for the 0.1 TeV and 4 TeV parameter sets. The tabulated radiation predictions for the 10 TeV and 100 TeV collider parameters anticipate some mitigating factors at multi-TeV energies that will now be discussed.

### III MITIGATING FACTORS AT MANY-TEV ENERGIES

Equations 3 and 6 are too pessimistic at the many-TeV energies addressed at this workshop, for two reasons. The smaller of the two effects is a partial leveling
FIGURE 4. The worst-case geometry for radiation exposure at many-TeV muon colliders, to be contrasted with the “equilibrium approximation” geometry of the preceding figure.

TABLE 2. Parameterization of the cross section factor $X(E_{\mu})$ that describes the fall-off from a linear cross section rise with energy. By definition, $\alpha \equiv \log_{10}(E_{\mu} [\text{TeV}])$. The parameterizations are simple logarithmic energy interpolations between the cross sections given in reference (9). The numerical factors in the expressions, 1.453, 1.323, 1.029, 0.512 and 0.175 are the total summed neutrino-nucleon and antineutrino-nucleon cross-sections-divided-by-energy (9) at the neutrino energies of 100 GeV, 1 TeV, 10 TeV, 100 TeV and 1 PeV, respectively, given in units of $10^{-38}$ cm$^2$/GeV. As an adequate approximation to avoid convolutions with neutrino energy spectra, the muon energies in the table have been set equal to the corresponding neutrino energies. By definition, $X(E_{\mu}) \equiv 1$ for $E_{\mu} = 100$ GeV, which is the reference energy used for the radiation dose parameterizations of equations 3 and 6.

| muon energy range | expression for $X(E_{\mu})$ |
|-------------------|-------------------------------|
| $E_{\mu} < 1$ TeV | $(-1.453 \times \alpha + 1.323 \times (\alpha + 1))/1.453$ |
| 1 TeV < $E_{\mu} < 10$ TeV | $(1.323 \times (1 - \alpha) + 1.029 \times \alpha)/1.453$ |
| 10 TeV < $E_{\mu} < 100$ TeV | $(1.029 \times (2 - \alpha) + 0.512 \times (\alpha - 1))/1.453$ |
| 100 TeV < $E_{\mu} < 1000$ TeV | $(0.512 \times (3 - \alpha) + 0.175 \times (\alpha - 2))/1.453$ |
| 1000 TeV < $E_{\mu}$ | $(0.175/1.453) \times 3^{3-\alpha}$ |

off in the neutrino cross section. Rather than a continuation of the linear rise up to TeV energy scales, it is predicted that (9), for example, the neutrino cross section at 100 TeV is only 33 times that at 1 TeV, instead of a 100-fold increase.

More significantly, the beam radius, $\theta_{\nu} L$, ceases to be large compared to the size of a person or to the width of the shower it produces. This prevents the possibility of a person ever experiencing the geometry of figure 3 that is needed for the equilibrium approximation to apply. Instead, a modified worst-case geometry that often applies for many-TeV muon colliders is illustrated in figure 4.

The formulae 3 and 6 can be modified to apply roughly to the case of a narrow, many-TeV neutrino radiation disk or pencil beam by considering how the equilib-
rium approximation breaks down for a beam radius that, due to either increasing \( E_\mu \) or decreasing \( L \), becomes comparable to or smaller than a hadron shower radius. Instead of depositing the dose over a progressively smaller vertical band (in the case of a radiation disk), or a spot (in the case of the pencil beam), the shower radius imposes a limiting transverse size scale. For definiteness, the calculations for table 1 spread the disk-average dose out evenly over a half-height of 0.5 meters – which is comparable to the 0.43 m interaction length of granite that was mentioned previously – and spread the hot-spot dose over a circle of the same radius.

The explicit many-TeV equations corresponding to the lower energy equations 3 and 6 become, respectively:

\[
D_{\text{ave many-TeV}}^{\text{ave}} [mSv] \simeq 3.7 \times N_\mu^+ [10^{20}] \times \frac{(E_\mu [TeV])^3}{(L[km])^2} \times X(E_\mu) \times F(E_\mu, L) \tag{12}
\]

and

\[
D_{\text{ss many-TeV}}^{\text{ss}} [mSv] \simeq 5.3 \times N_\mu^+ [10^{20}] \times l^{ss}[m] \times B_\text{ave}[T] \times \frac{(E_\mu [TeV])^3}{(L[km])^2} \times X(E_\mu) \times (F(E_\mu, L))^2, \tag{13}
\]

where \( X(E_\mu) \) and \( F(E_\mu, L) \) are the high energy suppression factors for the cross section leveling and small spot size, respectively.

The cross-section suppression factor, \( X(E_\mu) \), has, by definition, the value unity for \( E_\mu = 100 \) GeV, which was the energy chosen to calculate the numerical coefficients for equations 3 and 6. Its slow decrease with increasing energy has been approximated by simple logarithmic energy interpolations between the cross sections given in reference (9) and the form and coefficients for the interpolations are given in table 2.

The appropriate form of \( F(E_\mu, L) \), incorporating the assumed minimum transverse shower dimension of 0.5 m that was discussed above, is:

\[
F(E_\mu, L) = \min \left( 1, \frac{\theta_\mu [\text{rad}] L[km]}{5 \times 10^{-4}} \right) = \min \left( 1, \frac{L[km]}{5 \times E_\mu [TeV]} \right), \tag{14}
\]

where equation 2 has also been used to obtain the second form of the expression.

Substituting in the explicit form of equation 14 for the case where \( L[km] < 5 \times E_\mu [TeV] \) returns the narrow-beam form of equations 12 and 13:

\[
D_{\text{ave many-TeV}}^{\text{ave}} [mSv] \rightarrow 0.74 \times N_\mu^+ [10^{20}] \times \frac{(E_\mu [TeV])^2}{L[km]} \times X(E_\mu) \tag{15}
\]

and

\[
D_{\text{ss many-TeV}}^{\text{ss}} [mSv] \rightarrow 0.21 \times N_\mu^+ [10^{20}] \times l^{ss}[m] \times B_\text{ave}[T] \times E_\mu [TeV] \times X(E_\mu). \tag{16}
\]

The second of these equations is independent of the distance \( L \) in this limit, as it intuitively must be: the beam is now narrow enough that a person will intercept
essentially the entire beam, so the dose should become independent of distance in this limit.

We can now review the energy scaling of the radiation dose at many-TeV energies, to compare the power law dependences with those for the lower energy colliders that are given in equations 9 and 10. The 0.5 m fixed half-height removes the dependence of equation 9 on the disk height, $\theta \nu L$. Combined with the partial leveling of the cross-section, this softens the radiation rise with energy relative to slightly less than quadratic in energy:

\[ D_{\text{many-TeV}} \sim \sigma_{\nu N} \cdot E_{\nu} \cdot \langle E_{\nu} \rangle \propto E_{\mu}^{<1} \cdot E_{\mu} = E_{\mu}^{<2}. \]

Similarly, the many-TeV dose from straight sections loses both powers of $E_{\mu}$ that came from the $1/(\gamma_{\mu} L)^2$ factor in equation 10, giving:

\[ D_{\text{many-TeV str. sec.}} \sim \sigma_{\nu N} \cdot E_{\nu} \cdot f_{ss} \propto E_{\mu}^{<1} \cdot E_{\mu} / E_{\mu} = E_{\mu}^{<1}, \]

i.e. a less-than-linear rise with energy for radiation hot spots from a straight section of fixed length $f_{ss}$.

The assumed shower radius of 0.5 meters is clearly a somewhat arbitrary choice, so it should be borne in mind that equations 12 and 13 will be even more approximate than the lower energy predictions of equations 3 and 6. These predictions at many-TeV energies should also be interpreted even more as worst case scenarios than those at lower energies, since the material surrounding the person is now required to have a density comparable to granite in order to confine the hadron showers to the assumed transverse dimensions of 0.5 meters.

No detailed follow-up Monte Carlo simulations for various material geometries have yet been performed for the Many-TeV scenarios discussed in this section. Such simulations would be very valuable in confirming and refining the rough numerical estimates obtained from applying the above high-energy modifications to equations 3 and 6.

**IV PROPOSED SOLUTIONS**

Several means to reduce the radiation hazard have been proposed previously in reference (2):

1. minimize straight sections in the collider ring, e.g. by superimposing some bending field on all focusing magnets
2. improve the luminosity per unit current, from better beam cooling etc.
3. use fenced-off radiation enclosures downstream from the largest straight sections
4. bury the muon collider ring deeper underground to increase the distance before the neutrino disk exits the ground. Optionally, the ring can also be tilted and oriented to take advantage of natural geological features.

5. choose to build the muon collider on a site where human exposure to the radiation disk will be minimized or, ideally, nobody at all will be exposed to the neutrino radiation.

The orders-of-magnitude reductions desired for many-TeV colliders appear difficult to achieve through any combination of items 1 through 4 alone so the final option – a specially chosen site – may well be unavoidable for muon colliders at the highest energies.

Two classes of siting options are shown in figure 5. The first is an isolated site, where nobody is exposed to the radiation disk before it exits into the atmosphere due to the local curvature of the Earth. The height above ground, $h$, at distance $L$ from a collider ring close to the surface of a spherical Earth is given by rearranging equation 11 to:
FIGURE 6. The ultimate high energy physics laboratory? See further discussion in reference (10). Figure reproduced from reference (10).

\[ h = \frac{L^2}{2R_E}. \]  \hspace{1cm} (19)

FIGURE 7. An example to illustrate the size of a HEP laboratory with a 400 km site boundary circumference. A circle of this diameter has been drawn in the Great Victoria Desert (just above the “A” in the label “South Australia”), showing that the outline of a laboratory of this size would even be visible on a map of Australia. A 1000 km long strip of land for the 1 PeV linear collider would perhaps not be included inside the original laboratory boundary. The choice of country and positioning of the site are for illustration only. Figure reproduced from reference (10).
As an example that is discussed further below, a distance to the site boundary of \( L = 64 \text{ km} \) corresponds to \( h = 320 \text{ m} \), which might be considered a very comfortable clearance height at the site boundary for an isolated region. Some zoning restrictions could also be placed on tall structures near the laboratory site if this was additionally required.

The second option in figure 5, siting the laboratory on elevated land, would take advantage of the local topology to provide either a smaller site or higher clearances for the radiation disk at the site boundary. In practice, both siting options would likely be combined by choosing the most elevated site in an isolated region.

A third siting option, placing the collider in a valley or other depressed region to extend the distance before the disk exits the ground, is discussed elsewhere in these proceedings (11). This can be considered to be a variation on item 4 of the above list. As speculation, this siting option might well be practical up to perhaps the 10 TeV energy scale but would likely give an inadequate dose reduction for muon colliders at the highest energies and luminosities.

The transient radiation doses to people in planes and birds flying through the disk would clearly be negligible for either of the siting options of figure 5 since the doses in table 1 must be accumulated over a full accelerator year of \( 10^7 \text{ seconds} \) and also, as explained previously, the conservatively calculated doses are anyway about three orders of magnitude above the full-year doses for a person or bird when they are not surrounded by dense material.

Because of the dilute beam halo from showers induced in the atmosphere, the radiation dose at the site boundary would not be strictly zero even if the neutrino radiation disk was well above ground level. Speculatively, the halo might extend down below the neutrino disk by perhaps of order the interaction length of air, \( \lambda_{\text{air}} = 700 \text{ m} \). On naively comparing with equations 14 and 12, the disk average dose might be expected to be down by of order \((0.5 \text{ m})/\lambda_{\text{air}}\), which is about three orders of magnitude, from the in-disk prediction of equation 12. Similarly, comparison with equation 13 suggests that the additional doses from straight sections might speculatively be predicted to be diluted by the square of this ratio, i.e. by about six orders of magnitude. (These predictions are essentially just repeating the arguments leading up to equations 15 and 16, but now with a limiting radial extent of 700 meters rather than 0.5 meters.) If these tentative predictions are valid then both the average dose and the straight section dose would be safely below radiation limits. However, the predictions are very speculative and detailed Monte Carlo showering calculations are called for to predict the true level of the beam halo for this geometry.

On the positive side, a site diameter of order 100 km is also an appropriate size scale for jointly housing the largest potentially plausible proton or electron colliders along with the many-TeV muon colliders. This is the case in the “proof-of-plausibility” scenario for future energy frontier colliders that is presented in reference (10) and that might lead to a site layout that is illustrated conceptually in figure 6. As a detail on the site layout, the various muon acceleration and collider rings could plausibly be placed in the same plane to minimize the radiation zone.
FIGURE 8. A PeV-scale linear muon collider (12) would shoot two neutrino beams upwards and towards the sky at angles to the horizontal of perhaps a few tens of milliradians, depending on the length of the linacs.

to perhaps of order 100 square kilometers.

Just to illustrate the size scale, figure 7 shows a circle with a 400 km circumference (i.e. a 64 km radius) drawn in an unpopulated desert region of Australia. The content of both figures 6 and 7 is further discussed in reference (10). Note that the figures also include a linear collider, whose radiation hazards we now discuss.

V NEUTRINO RADIATION AT MANY-TEV LINEAR MUON COLLIDERS

It was speculated during this workshop that the ultimate potential energy reach for muon colliders could be extended by using linear $\mu^+\mu^-$ colliders, and straw-man parameter sets were presented for such single pass colliders (12).

From the standpoint of neutrino radiation, a linear muon collider has two advantages over circular muon colliders. Firstly, the radiation is confined to two pencil beams which would naturally be oriented at an upward tilt of perhaps tens of milliradians, as illustrated in figure 8. Secondly, the spent muons after each single-pass collision can be immediately ranged out in a beam dump rather than surviving until they decay into high energy neutrinos. This gives a large reduction in the radiation dose within the pencil beams.

We now derive the dose for the unpleasant and very artificial situation of a person living full-time in dense material immediately downstream from a linac, in the center of one of the neutrino beams. This will then be used to assess doses for more realistic situations.
The calculation proceeds by considering the total dose as an integral over the dose contributions, $\delta D(E)$, received in each energy interval, $[E, E + dE]$, as the muon bunch accelerates to the beam energy, $E_\mu$:

$$D^{\text{linac}} = \int_0^{E_\mu} dE \delta D(E).$$  \hfill (20)

We already have the means to estimate $\delta D(E)$, since it can be compared to the dose from a straight section in a circular collider. After scaling by the relative fraction of muons decaying in the two cases, one obtains:

$$\delta D(E) = D_{s}^{ss}(E_{\text{many TeV}}) \times \frac{1}{f^{ss}} \times \frac{df}{dE} (E) dE,$$  \hfill (21)

where $\frac{df}{dE} (E) dE$ is the fraction of muons that decay in the energy interval $[E, E + dE]$. This is given by:

$$\frac{df}{dE} (E) = \frac{1}{\gamma_\mu \beta c \tau} \times g,$$  \hfill (22)

where the product $\gamma_\mu \beta c \tau = 660$ is the characteristic decay length of ultra-relativistic muons, with $\beta c \tau = 660$ meters, and $g = dE/dz$ is the acceleration gradient.

Substituting in the expressions and constants from equations 16, 2, 5 and 7 gives, after some algebra and on taking care with units:

$$D^{\text{linac}}[\text{mSv}] = 0.67 \times N_\mu^{[10^{20}]} \times \int_0^{E_\mu[\text{TeV}]} dE E X(E).$$  \hfill (23)

To solve this equation analytically, it is an adequate approximation to replace the energy-weighted integral of $X(E)$ by its value at, say, $E = E_\mu/2$, giving finally:

$$D^{\text{linac}}[\text{mSv}] \sim 0.33 \frac{N_\mu^{[10^{20}]}}{g[\text{GeV/m}]} \times X(E_\mu/2) \times (E_\mu[\text{TeV}])^2.$$  \hfill (24)

As a numerical example, the straw-man parameters for the 1 PeV muon linear collider of reference (12) assume $E_\mu = 500$ TeV and $N_\mu = 0.064 \times 10^{20}$ in a $10^7$ second year. Substituting in these values, with the additional assumption that $g = 1$ GV/m, gives:

$$D^{\text{linac}}[2 \times 500 \text{ TeV example}] = 1400 \text{ mSv/year}.$$  \hfill (25)

To put this figure for a whole-year dose in perspective, it is approximately 30 times the recommended maximum dose for a U.S. radiation worker, i.e. 50 mSv/year (3), so such a radiation worker would be able to work directly in the beam for of order 100 hours of accelerator running time. This shows that, while the dose would be well above legal limits for long-term occupancy, any doses from transient exposure would still be relatively small. In particular, the calculation gives reassurance on the negligible dose that would be received by a bird or plane flying through the beam.
VI SUMMARY

Neutrino radiation is a very serious problem and design constraint for muon colliders, particularly at very high center-of-mass energies. A characterization of the neutrino hazard has been presented that quotes the numerical formulae from reference (2) applying for TeV energy scales and below, and that extends the numerical predictions up to many-TeV energies.

It has been shown that the radiation dose in the plane of the muon collider ring rises quickly as the cube of the beam energy up to around the TeV collider energy scale before leveling off for many-TeV colliders to a slightly less than quadratic rise with energy on average and slightly less than linear for the radiation hot spots downstream from a fixed length of straight section.

To solve the neutrino radiation problem, many-TeV muon colliders and their associated neutrino radiation disks may be located within a new world laboratory with site diameter of order 100 km and located at a carefully chosen site in either the U.S., Canada, Australia, Northern Europe or elsewhere where large isolated tracts of land can be found with resources for a large-scale high technology laboratory.

REFERENCES

1. B.J. King, *Assessment of the prospects for muon colliders*, paper submitted in partial fulfillment of requirements for Ph.D., Columbia University, New York (1994), available from LANL preprint archive as physics/9907026.
2. B.J. King, *Potential Hazards from Neutrino Radiation at Muon Colliders*, available from LANL preprint archive as physics/9908017. An abbreviated version of this paper is available as Proc. PAC’99, New York, 1999, pp. 318-320.
3. C. Caso et al., *The Review of Particle Physics*, The European Physical Journal C3 (1998) 1.
4. The Muon Collider Collaboration, *Status of Muon Collider Research and Development and Future Plans*, Phys. Rev. ST Accel. Beams, 3 August, 1999.
5. B.J. King, *Discussion on Muon Collider Parameters at Center of Mass Energies from 0.1 TeV to 100 TeV*, Proc. EPAC’98, BNL–65716. available from LANL preprint archive as physics/9908016.
6. B.J. King, *Parameter Sets for 10 TeV and 100 TeV Muon Colliders, and their Study at the HEMC’99 Workshop*, these proceedings. Also available from http://pubweb.bnl.gov/people/bking/heshop/hemc_papers.html.
7. J.D. Cossairt, N.L. Grossman, E.T. Marshall, *Fermilab-Conf-96/324* (1996) and *Fermilab-Pub-97/101* (1997); Private communications with N.V. Mokhov.
8. C.J. Johnstone and N.V. Mokhov, Proc. PAC’97 (1997) 414; Nikolai Mokhov and Andreas Van Ginneken, *Neutrino Radiation at Muon Colliders and Storage Rings*, to appear in Proc. ICRS-9 Int. Conf. on Radiation Shielding, Tsukuba, Ibaraki, Japan, 1999.
9. See, for example, Chris Quigg, *Neutrino Interaction Cross Sections*, FERMILAB-Conf-97/158-T.
10. B.J. King, *Prospects for Colliders and Collider Physics to the 1 PeV Energy Scale*, these proceedings. Also available from http://pubweb.bnl.gov/people/bking/heshop/hemc_papers.html.

11. Colin Johnson, Siting Options for a High Energy Muon Collider, oral presentation at this workshop. Transparency copies can be viewed at http://pubweb.bnl.gov/people/bking/heshop/hemc_papers.html.

12. F. Zimmermann, *Final Focus Challenges for Muon Colliders at Highest Energies*, these proceedings. Also available from http://pubweb.bnl.gov/people/bking/heshop/hemc_papers.html.