Similarity measures of intuitionistic fuzzy soft sets and their decision making

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Abstract

In this article, we define some types of distances between two intuitionistic fuzzy soft (IFS) sets and proposed similarity measures of two IFS-sets. We then construct a decision method which is applied to a medical diagnosis problem that is based on similarity measures of IFS-sets. Finally we give two simple example to show the possibility of using this method for diagnosis of diseases which could be improved by incorporating clinical results and other competing diagnosis.

Keyword: Soft sets; intuitionistic fuzzy soft sets; Hamming distances; Euclidean distances; similarity measure.

1 Introduction

In 1999, Molodtsov [30] has introduced the concept of soft sets. The soft set theory successfully models the problems which contains uncertainties. In literature, there are theories, such as probability, fuzzy sets [35], intuitionistic fuzzy sets [7], rough sets [33] that are dealing with the uncertain data.

In this work we use soft set theory. The operations (e.g. [3, 15, 27, 31]) and applications (e.g. [3, 11, 13]) on soft set theory have been studied by some researcher. In recent years, many decision making on soft set theory have been expanded by embedding the ideas of fuzzy sets (e.g. [5, 9, 13, 16, 17, 19, 20, 23, 29, 32]), intuitionistic fuzzy sets (e.g. [7, 8, 10, 21, 25, 31]) and rough sets [5, 21].

Majumdar and Samanta [28] give two types of similarity measure between soft sets and have shown an application of this similarity measure of soft sets. Kharal [23] give counterexamples to show that Definition 2.7 and Lemma 3.5...
contain errors in \[28\]. In \[23\], a new measures have been presented and this measures have been applied to the problem of financial diagnosis of firms.

In this paper, we first present the basic definitions and theorem of soft sets, fuzzy sets, intuitionistic fuzzy sets and intuitionistic fuzzy soft sets that are useful for subsequent discussions. We then define distances and similarity measures between two intuitionistic fuzzy soft (IFS) sets. By using the similarity we construct a decision making method. We finally give an application, which shows that the similarity measures can be successfully applied to a medical diagnosis problem that contains uncertainties.

2 Preliminary

In this section, we present the basic definitions of soft set theory \[15, 30\], fuzzy set theory \[35\], intuitionistic fuzzy set theory \[7\] and intuitionistic fuzzy soft set theory \[10\] that are useful for subsequent discussions.

**Definition 2.1** \[15\] Let \( U \) be a universe, \( E \) be a set of parameters that are describe the elements of \( U \), and \( A \subseteq E \). Then, a soft set \( F_A \) over \( U \) is a set defined by a set valued function \( f_A \) representing a mapping

\[
f_A : E \rightarrow P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \in E - A
\]

where \( f_A \) is called approximate function of the soft set \( F_A \). In other words, the soft set is a parametrized family of subsets of the set \( U \), and therefore it can be written a set of ordered pairs

\[
F_A = \{(x, f_A(x)) : x \in E, f_A(x) = \emptyset \text{ if } x \in E - A\}
\]

**Definition 2.2** \[35\] Let \( U \) be a universe. Then a fuzzy set \( X \) over \( U \) is a function defined as follows:

\[
X = \{\mu_X(u)/u : u \in U\}
\]

where \( \mu_X : U \rightarrow [0, 1] \)

Here, \( \mu_X \) called membership function of \( X \), and the value \( \mu_X(u) \) is called the grade of membership of \( u \in U \). The value represents the degree of \( u \) belonging to the fuzzy set \( X \).

**Definition 2.3** \[7\] Let \( E \) be a universe. An intuitionistic fuzzy set \( A \) on \( E \) can be defined as follows:

\[
A = \{< x, \mu_A(x), \gamma_A(x) > : x \in E\}
\]

where, \( \mu_A : E \rightarrow [0, 1] \) and \( \gamma_A : E \rightarrow [0, 1] \) such that \( 0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \) for any \( x \in E \).

Here, \( \mu_A(x) \) and \( \gamma_A(x) \) is the degree of membership and degree of non-membership of the element \( x \), respectively.
If $A$ and $B$ are two intuitionistic fuzzy sets on $E$, then

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for $\forall x \in E$.
2. $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$ $\forall x \in E$.
3. $A^c = \{< x, \gamma_A(x), \mu_A(x) > : x \in E \}$
4. $A \cup B = \{< x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x)) > : x \in E \}$
5. $A \cap B = \{< x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) > : x \in E \}$
6. $A + B = \{< x, \mu_X(x) + \mu_Y(x) - \mu_X(x)\mu_Y(x), \gamma_X(x)\gamma_Y(x) > : x \in E \}$
7. $A \cdot B = \{< x, \mu_A(x)\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x)\gamma_B(x) > : x \in E \}$.

**Definition 2.4** An intuitionistic fuzzy soft set (or namely IFS-set) is defined by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in \hat{\mathcal{F}}(U)\}$$

where $\gamma_A : E \to \hat{\mathcal{F}}(U)$ such that $\gamma_A(x) = \emptyset$ if $x \notin A$ and $\emptyset$ is intuitionistic fuzzy empty set. Moreover $\gamma_A(x)$ is an intuitionistic fuzzy set. So it is denoted by

$$\gamma_A(x) = \{(u, \mu_A(u), \nu_A(u)) : u \in U\}$$

for all $x \in E$. Moreover, $\mu_A : U \to [0, 1]$ and $\nu_A : U \to [0, 1]$ with the condition $0 \leq \mu_A(u) + \nu_A(u) \leq 1$, for all $u \in U$. The numbers $\mu_A(u)$ and $\nu_A(u)$ denote the membership degree end non-membership degree of $u \in U$ to the intuitionistic fuzzy set $\gamma_A(x)$, respectively.

**Example 2.5** Suppose that there are five car in the universe $U = \{u_1, u_2, u_3, u_4, u_5\}$ under consideration “$x_1 =$large”, “$x_2 =$costly”, “$x_3 =$secure”, “$x_4 =$strong”, “$x_5 =$economic” and”. “$x_6 =$repair”. Therefore parameter set is $E = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. Let $A = \{x_1, x_2, x_3, x_4\}$. Then IFS-set $\Gamma_A$ is represented the following tabular form:

$$\Gamma_A = \{(x_1, \{(u_1, 0.5, 0.2), (u_2, 0.5, 0.2), (u_3, 0.5, 0.2), (u_4, 0.5, 0.2)\}),
\ (x_2, \{(u_1, 0.6, 0.4), (u_2, 0.9, 0.1), (u_3, 0.5, 0.3), (u_4, 0.1, 0.9)\}),
\ (x_3, \{(u_1, 0.7, 0.2), (u_2, 0.8, 0.1), (u_3, 0.2, 0.16), (u_4, 0.4, 0.5)\}),
\ (x_4, \{(u_1, 0.4, 0.3), (u_2, 0.2, 0.7), (u_3, 0.8, 0.2), (u_4, 0.2, 0.1)\})\}$$

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### 3 Similarity Measures of IFS-Sets

In this section, we first present the basic definitions of distances between two intuitionistic fuzzy sets [8] and two soft sets [28] that are useful for subsequent discussions. We then define some distances and similarity measures of IFS-sets.

**Definition 3.1** [8] Let $U = \{x_1, x_2, x_3, \ldots, x_n\}$ be a universe and $A, B$ be two intuitionistic fuzzy sets over $U$ with their membership function $\mu_A, \mu_B$ and non-membership function $\nu_A, \nu_B$, respectively. Then the distances of $A$ and $B$ are defined as,

1. **Hamming distance**;

$$d(A, B) = \frac{1}{2} \sum_{i=1}^{n} [|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|]$$

2. **Normalized Hamming distance**;

$$l(A, B) = \frac{1}{2n} \sum_{i=1}^{n} [|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|]$$

3. **Euclidean distance**;

$$e(A, B) = \sqrt{ \frac{1}{2} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2] }$$

4. **Normalized Euclidean distance**;

$$q(A, B) = \sqrt{ \frac{1}{2n} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2] }$$

**Definition 3.2** [28] Let $U = \{u_1, u_2, u_3, \ldots\}$ be a universe, $E = \{x_1, x_2, x_3, \ldots\}$ be a set of parameters, $A, B \subseteq E$, and $F_A$ and $G_B$ be two soft sets on $U$ with their approximate functions $f_A$ and $g_B$, respectively.

If $A = B$, then similarity between $F_A$ and $G_B$ is defined by

$$S(F_A, G_B) = \frac{\sum_{i=1}^{n} f_A(x_i) \cdot g_B(x_i)}{\sum_{i=1}^{n} \max[f_A(x_i), g_B(x_i)]^2}$$

where

$$f_A(x_i) = (\chi_{f_A(x_i)}(u_1), \chi_{f_A(x_i)}(u_2), \chi_{f_A(x_i)}(u_3), \ldots)$$

$$g_B(x_i) = (\chi_{g_B(x_i)}(u_1), \chi_{g_B(x_i)}(u_2), \chi_{g_B(x_i)}(u_3), \ldots)$$

and

$$\chi_{f_A(x_i)}(u_j) = \begin{cases} 1, & u_j \in f_A(x_i) \\ 0, & u_j \notin f_A(x_i) \end{cases} \quad \chi_{g_B(x_i)}(u_j) = \begin{cases} 1, & u_j \in g_B(x_i) \\ 0, & u_j \notin g_B(x_i) \end{cases}$$
Note 3.3 If \( A \neq B \) and \( C = A \cap B \neq \emptyset \), then \( f_A(x_i) = 0 \) for \( x_i \in B \setminus C \) and \( g_B(x_i) = 0 \) for \( x_i \in A \setminus C \).

If \( A \cap B = \emptyset \), then \( S(F_A, G_B) = 0 \) and \( S(F_A, F^c_A) = 0 \) as \( f_A(x_i) \cdot f^c_A(x_i) = 0 \) for all \( i \).

**Definition 3.4** [28] Let \( F_A \) and \( G_B \) be two soft sets over \( U \). Then, \( F_A \) and \( G_B \) are said to be \( \alpha \)-similar, denoted as \( F_A \approx^\alpha G_B \), if and only if \( S(F_A, G_B) \geq \alpha \) for \( \alpha \in (0, 1) \).

**Definition 3.5** [28] Let \( U = \{u_1, u_2, u_3, \ldots\} \) be a universe, \( E = \{x_1, x_2, x_3, \ldots\} \) be a set of parameters, \( A, B \subseteq E \) and \( F_A, G_B \) be two soft sets on \( U \) with their approximate functions \( f_A \) and \( g_B \), respectively. Then, the distances of \( F_A \) and \( G_B \) are defined as,

1. Hamming distance;
   \[
d^h(F_A, G_B) = \frac{1}{m} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} |f_A(x_i)(u_j) - g_B(x_i)(u_j)| \right\}
   \]

2. Normalized Hamming distance;
   \[
l^h(F_A, G_B) = \frac{1}{mn} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} |f_A(x_i)(u_j) - g_B(x_i)(u_j)| \right\}
   \]

3. Euclidean distance;
   \[
e^h(F_A, G_B) = \sqrt{\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} (f_A(x_i)(u_j) - g_B(x_i)(u_j))^2}
   \]

4. Normalized Euclidean distance;
   \[
q^h(F_A, G_B) = \sqrt{\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} (f_A(x_i)(u_j) - g_B(x_i)(u_j))^2}
   \]

**Definition 3.6** [28] Let \( F_A \) and \( G_B \) be two soft sets over \( U \). Then, by using the Euclidean distance, similarity measure of \( F_A \) and \( G_B \) is defined as,

\[
s'(F_A, G_B) = \frac{1}{1 + e^h(F_A, G_B)}
\]

Another similarity measure of \( F_A \) and \( G_B \) can be defined as,

\[
s''(F_A, G_B) = e^{-\alpha e^h(F_A, G_B)}
\]

where \( \alpha \) is a positive real number called the steepness measure.
Definition 3.7: Let $U = \{u_1, u_2, ..., u_n\}$ be a universe, $E = \{x_1, x_2, ..., x_m\}$ be a set of parameters, $A, B \subseteq E$ and $\Gamma_A, \Lambda_B$ be two IFS-sets on $U$ with their intuitionistic fuzzy approximate functions $\gamma_A(x_i) = \{(u, \mu_A(u), \nu_A(u)) : u \in U\}$ and $\lambda_B(x_i) = \{(u, \mu_B(u), \nu_B(u)) : u \in U\}$, respectively.

If $A = B$ and $\mu_A(x_i)(u_j) - \nu_A(x_i)(u_j) \neq 0$ or $\mu_B(x_i)(u_j) - \nu_B(x_i)(u_j) \neq 0$ for at least one $i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, ..., m\}$, then similarity between $\Gamma_A$ and $\Lambda_B$ is defined by

$$S_{IFS}(\Gamma_A, \Lambda_B) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} |(\mu_A(x_i)(u_j) - \nu_A(x_i)(u_j)) \cdot (\mu_B(x_i)(u_j) - \nu_B(x_i)(u_j))|}{\sum_{i=1}^{m} \sum_{j=1}^{n} \max\{|\mu_A(x_i)(u_j) - \nu_A(x_i)(u_j)|^2, |\mu_B(x_i)(u_j) - \nu_B(x_i)(u_j)|^2\}}$$

where

- $\mu_A(x_i)(u_j) = (\mu_A(x_i)(u_1), \mu_A(x_i)(u_2), ..., \mu_A(x_i)(u_n))$
- $\nu_A(x_i)(u_j) = (\nu_A(x_i)(u_1), \nu_A(x_i)(u_2), ..., \nu_A(x_i)(u_n))$
- $\mu_B(x_i)(u_j) = (\mu_B(x_i)(u_1), \mu_B(x_i)(u_2), ..., \mu_B(x_i)(u_n))$
- $\nu_B(x_i)(u_j) = (\nu_B(x_i)(u_1), \nu_B(x_i)(u_2), ..., \nu_B(x_i)(u_n))$

If $A = B$ and $\mu_A(x_i)(u_j) - \nu_A(x_i)(u_j) = 0$ and $\mu_B(x_i)(u_j) - \nu_B(x_i)(u_j) = 0$ for all $i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, ..., m\}$, then $S_{IFS}(\Gamma_A, \Lambda_B) = 1$.

Example 3.8: Assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set, $E = \{x_1, x_2, x_3, x_4\}$ is a set of parameters, $A = \{x_1, x_2, x_4\}, B = \{x_1, x_2, x_4\}$ are subsets of $E$. If two IFS-sets $\Gamma_A$ and $\Lambda_B$ over $U$ are contracted as follows:

$$\Gamma_A = \left\{ (x_1, \{(u_1, 0.5, 0.5), (u_2, 0.4, 0.5), (u_3, 0.7, 0.2), (u_4, 0.8, 0.1)\}), (x_2, \{(u_1, 0.4, 0.6), (u_2, 0.2, 0.7), (u_3, 0.2, 0.8), (u_4, 0.2, 0.2)\}), (x_3, \{(u_1, 0.2, 0.7), (u_2, 0.1, 0.9), (u_3, 0.5, 0.4), (u_4, 0.7, 0.2)\}) \right\}$$

$$\Lambda_B = \left\{ (u_1, 0.2, 0.7), (u_2, 0.1, 0.9), (u_3, 0.5, 0.4), (u_4, 0.4, 0.4)\}), (x_2, \{(u_1, 0.5, 0.5), (u_2, 0.4, 0.5), (u_3, 0.3, 0.6), (u_4, 0.4, 0.5)\}), (x_3, \{(u_1, 0.4, 0.6), (u_2, 0.2, 0.7), (u_3, 0.2, 0.8), (u_4, 0.2, 0.5)\}) \right\}$$

Then we can obtain

$$\mu_A(x_1)(u_j) = (0.5, 0.4, 0.7, 0.8), \quad \nu_A(x_1)(u_j) = (0.5, 0.5, 0.2, 0.1),$$
$$\mu_A(x_2)(u_j) = (0.4, 0.2, 0.2, 0.2), \quad \nu_A(x_2)(u_j) = (0.6, 0.7, 0.8, 0.2),$$
$$\mu_A(x_3)(u_j) = (0.2, 0.1, 0.5, 0.7), \quad \nu_A(x_3)(u_j) = (0.7, 0.9, 0.4, 0.2),$$
$$\mu_B(x_1)(u_j) = (0.2, 0.1, 0.5, 0.4), \quad \nu_B(x_1)(u_j) = (0.7, 0.9, 0.4, 0.4),$$
$$\mu_B(x_2)(u_j) = (0.5, 0.4, 0.3, 0.4), \quad \nu_B(x_2)(u_j) = (0.5, 0.5, 0.6, 0.5),$$

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\overline{\mu_B(x_3)(u_j)} = (0.4, 0.2, 0.2, 0.2), \quad \overline{\nu_B(x_3)(u_j)} = (0.6, 0.7, 0.8, 0.5).

and

\begin{align*}
\langle \mu_A(x_1)(u_j) - \nu_A(x_1)(u_j) \rangle &= (0.0, -0.1, 0.5, 0.7), \\
\langle \mu_A(x_2)(u_j) - \nu_A(x_2)(u_j) \rangle &= (-0.2, -0.5, -0.6, 0.0), \\
\langle \mu_A(x_3)(u_j) - \nu_A(x_3)(u_j) \rangle &= (-0.5, -0.8, 0.1, 0.5), \\
\langle \mu_B(x_1)(u_j) - \nu_B(x_1)(u_j) \rangle &= (-0.5, -0.8, 0.1, 0.0), \\
\langle \mu_B(x_2)(u_j) - \nu_B(x_2)(u_j) \rangle &= (0.0, -0.1, -0.3, -0.1), \\
\langle \mu_B(x_3)(u_j) - \nu_B(x_3)(u_j) \rangle &= (-0.2, -0.5, -0.6, -0.3)
\end{align*}

Now the similarity between \( \Gamma_A \) and \( \Lambda_B \) is calculated as

\[ S_{IFS}(\Gamma_A, \Lambda_B) = 0.31 \]

**Theorem 3.9** Let \( E \) be a parameter set, \( A, B \subseteq E \) and \( \Gamma_A \) and \( \Lambda_B \) be two IFS-sets over \( U \). Then the followings hold;

i. \( S_{IFS}(\Gamma_A, \Lambda_B) = S_{IFS}(\Lambda_B, \Gamma_A) \)

ii. \( 0 \leq S_{IFS}(\Gamma_A, \Lambda_B) \leq 1 \)

iii. \( S_{IFS}(\Gamma_A, \Gamma_A) = 1 \)

**Proof:** Proof easily can be made by using Definition 3.7.

**Theorem 3.10** Let \( E \) be a parameter set, \( A, B, C \subseteq E \) and \( \Gamma_A, \Lambda_B \) and \( \Upsilon_C \) be three IFS-sets over \( U \) such that \( \Gamma_A \) is a intuitionistic fuzzy soft subset of \( \Lambda_B \) and \( \Lambda_B \)is a Intuitionistic fuzzy soft subset of \( \Upsilon_C \) then,

\[ S_{IFS}(\Gamma_A, \Upsilon_C) \leq S_{IFS}(\Lambda_B, \Upsilon_C) \]

**Proof:** The proof is straightforward.

**Definition 3.11** Let \( U = \{u_1, u_2, ..., u_n\} \) be a universe, \( E = \{x_1, x_2, ..., x_m\} \) be a set of parameters, \( A, B \subseteq E \) and \( \Gamma_A, \Lambda_B \) be two IFS-sets on \( U \) with their intuitionistic fuzzy approximate functions \( \gamma_A(x_i) = \{(u, \mu_A(u), \nu_A(u)) : u \in U\} \) and \( \lambda_B(x_i) = \{(u, \mu_B(u), \nu_B(u)) : u \in U\} \), respectively. Then the distances of \( \Gamma_A \) and \( \Lambda_B \) are defined as,

1. **Hamming distance**, 

\[ d^*_{IFS}(\Gamma_A, \Lambda_B) = \]

\[ \frac{1}{2m} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| \right\} \]
2. Normalized Hamming distance,
\[
l_{IFS}(\Gamma_A, \Lambda_B) = \frac{1}{2mn} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| \right\}
\]

3. Euclidean distance,
\[
e_{IFS}(\Gamma_A, \Lambda_B) = \left( \frac{1}{2mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ (\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j))^2 + (\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j))^2 \right] \right)^{\frac{1}{2}}
\]

4. Normalized Euclidean distance,
\[
q_{IFS}(\Gamma_A, \Lambda_B) = \left( \frac{1}{2mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ (\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j))^2 + (\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j))^2 \right] \right)^{\frac{1}{2}}
\]

**Example 3.12** Let us consider the Example 3.8. Then, the distances of \(\Gamma_A\) and \(\Lambda_B\) are calculated as follows;
\[
\begin{align*}
d_{IFS}(\Gamma_A, \Lambda_B) &= 0.07 \\
l_{IFS}(\Gamma_A, \Lambda_B) &= 0.37 \\
e_{IFS}(\Gamma_A, \Lambda_B) &= 0.28 \\
q_{IFS}(\Gamma_A, \Lambda_B) &= 0.19
\end{align*}
\]

**Theorem 3.13** Let \(E\) be a parameter set, \(A, B \subseteq E\) and \(\Gamma_A\) and \(\Lambda_B\) be two IFS-sets over \(U\). Then the followings hold;
\[
\begin{align*}
i. \ d_{IFS}(\Gamma_A, \Lambda_B) &\leq n \\
ii. \ l_{IFS}(\Gamma_A, \Lambda_B) &\leq 1 \\
iii. \ e_{IFS}(\Gamma_A, \Lambda_B) &\leq \sqrt{n} \\
iv. \ q_{IFS}(\Gamma_A, \Lambda_B) &\leq 1
\end{align*}
\]

**Proof:** Proof easily can be made by using Definition 3.11.

**Theorem 3.14** Let \(IFS(U)\) be a set of all IFS-sets over \(U\). Then the distances functions \(d_{IFS}, l_{IFS}, e_{IFS}\) and \(q_{IFS}\), defined from \(IFS(U)\) to the non-negative real number \(R^+\), are metric.

**Proof:** We give only proof for \(l_{IFS}\). If \(\Gamma_A, \Lambda_B\) and \(\Upsilon_C\) \(\in IFS(U)\), then
for \( \alpha \) Hamming distance, similarity measure of \( \Gamma \) where

\[
\begin{align*}
\text{Definition 3.15} & \\
\text{Triangle inequality follows easily from the observation that for any three IFS-sets } \Gamma_A, \Lambda_B, \text{ and } \Upsilon_C, \\
\forall i = \{1, 2, \ldots, m\}, j = \{1, 2, \ldots, n\} & \\
& \left| \mu_A(x_i)(u_j) - \mu_B(x_i)(u_j) \right| + \left| \nu_A(x_i)(u_j) - \nu_B(x_i)(u_j) \right| = 0 \\
& \Rightarrow |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| = 0 \\
& \Rightarrow \Gamma_A = \Lambda_B \\
& \text{Conversely, let} \\
& \Gamma_A = \Lambda_B \\
& \Rightarrow |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| = 0 \\
& \Rightarrow l_{IFS}^1(\Gamma_A, \Lambda_B) = 0 \\
& \text{Clearly, } l_{IFS}^1(\Gamma_A, \Lambda_B) = l_{IFS}^1(\Lambda_B, \Gamma_A) \\
& \text{Triangle inequality follows easily from the observation that for any three IFS-sets } \Gamma_A, \Lambda_B, \text{ and } \Upsilon_C, \\
\forall i = \{1, 2, \ldots, m\}, j = \{1, 2, \ldots, n\} & \\
& |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| = |\mu_A(x_i)(u_j) - \mu_C(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_C(x_i)(u_j)| \\
& \Rightarrow |\mu_A(x_i)(u_j) - \mu_B(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_B(x_i)(u_j)| = |\mu_A(x_i)(u_j) - \mu_C(x_i)(u_j)| + |\nu_A(x_i)(u_j) - \nu_C(x_i)(u_j)| \\
& \Rightarrow l_{IFS}^1(\Gamma_A, \Lambda_B) \leq l_{IFS}^1(\Gamma_A, \Upsilon_C) + l_{IFS}^1(\Upsilon_C, \Lambda_B) \\
& \text{The others proofs can made similarly.} \\
\text{Definition 3.15} & \\
& \text{Let } \Gamma_A \text{ and } \Lambda_B \text{ be two IFS-sets over } U. \text{ Then, by using the Hamming distance, similarity measure of } \Gamma_A \text{ and } \Lambda_B \text{ is defined as,} \\
& S_{IFS}^1(\Gamma_A, \Lambda_B) = \frac{1}{1 + d_{IFS}^1(\Gamma_A, \Lambda_B)} \\
& \text{Another similarity measure of } F_A \text{ and } G_B \text{ can be defined as,} \\
& S_{IFS}^\alpha(\Gamma_A, \Lambda_B) = e^{-\alpha d_{IFS}^1(\Gamma_A, \Lambda_B)} \\
& \text{where } \alpha \text{ is a positive real number called the steepness measure.} \\
\text{Definition 3.16} & \\
& \text{Let } \Gamma_A \text{ and } \Lambda_B \text{ be two IFS-sets over } U. \text{ Then, } \Gamma_A \text{ and } \Lambda_B \text{ are said to be } \alpha \text{-similar, denoted as } \Gamma_A \approx^\alpha \Lambda_B, \text{ if and only if } S'(\Gamma_A, \Lambda_B) \geq \alpha \text{ for } \alpha \in (0, 1). \\
& \text{We call the two IFS-sets significantly similar if } S'(\Gamma_A, \Lambda_B) > \frac{1}{2}. \\
\end{align*}
\]
Example 3.17 Let us consider the Example 3.12. Similarity measure of $\Gamma_A$ and $\Lambda_B$ is obtained as,

$$S'_{IFS}(\Gamma_A, \Lambda_B) = \frac{1}{1 + d'_{IFS}(\Gamma_A, \Lambda_B)} = 0.73$$

$\Gamma_A$ and $\Lambda_B$ is significantly similar because $S'_{IFS}(\Gamma_A, \Lambda_B) = 0.73 > \frac{1}{2}$

Theorem 3.18 Let $E$ be a parameter set, $A, B \subseteq E$ and $\Gamma_A$ and $\Lambda_B$ be two IFS-sets over $U$. Then the followings hold;

i. $0 \leq S'_{IFS}(\Gamma_A, \Lambda_B) \leq 1$

ii. $S'_{IFS}(\Gamma_A, \Lambda_B) = S'_{IFS}(\Lambda_B, \Gamma_A)$

iii. $S'_{IFS}(\Gamma_A, \Lambda_B) = 1 \iff \Gamma_A = \Lambda_B$

Proof: Proof easily can be made by using Definition 3.15.

4 Decision Making Method

In this section, we construct a decision making method that is based on the similarity measure of two IFS-sets. The algorithm of decision making method can be given as;

Step 1. Constructs a IFS-set $\Gamma_A$ over $U$ based on an expert,

Step 2. Constructs a IFS-set $\Lambda_B$ over $U$ based on a responsible person for the problem,

Step 3. Calculate the distances of $\Gamma_A$ and $\Lambda_B$,

Step 4. Calculate the similarity measure of $\Gamma_A$ and $\Lambda_B$,

Step 5. Estimate result by using the similarity.

Now, we can give an application for the decision making method. By using the Hamming distance, similarity measure of two IFS-sets can be applied to detect whether an ill person is suffering from a certain disease or not.

5 Application

In this applications, we will try to estimate the possibility that an ill person having certain visible symptoms is suffering from cancer. For this, we first construct a IFS-set for the illness and a IFS-set for the ill person. We then find the similarity measure of these two IFS-sets. If they are significantly similar, then we conclude that the person is possibly suffering from cancer.
Example 5.1 Assume that our universal set contain only two elements cancer and not cancer, i.e. $U = \{u_1, u_2\}$. Here the set of parameters $A = B = E$ is the set of certain visible symptoms, let us say, $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ where $x_1 = \text{jaundice}$, $x_2 = \text{bone pain}$, $x_3 = \text{headache}$, $x_4 = \text{loss of appetite}$, $x_5 = \text{weight loss}$, $x_6 = \text{heat wounds}$, $x_7 = \text{handle and shoulder pain}$, $x_8 = \text{lump}$ anywhere on the body for no reason and $x_9 = \text{chest pain}$.

**Step 1.** Constructs a IFS-set $\Gamma_A$ over $U$ for cancer is given below and this can be prepared with the help of a medical person:

\[
\Gamma_A = \big\{ (x_1, \{(u_1, 0.5, 0.5), (u_2, 0.4, 0.5)\}), (x_2, \{(u_1, 0.7, 0.2), (u_2, 0.8, 0.1)\}), (x_3, \{(u_1, 0.4, 0.6), (u_2, 0.2, 0.7)\}), (x_4, \{(u_1, 0.2, 0.8), (u_2, 0.2, 0.2)\}), (x_5, \{(u_1, 0.2, 0.7), (u_2, 0.1, 0.9)\}), (x_6, \{(u_3, 0.5, 0.4), (u_4, 0.7, 0.2)\}), (x_7, \{(u_1, 0.3, 0.7), (u_2, 0.4, 0.4)\}), (x_8, \{(u_1, 0.5, 0.2), (u_2, 0.7, 0.1)\}), (x_9, \{(u_1, 0.3, 0.4), (u_2, 0.7, 0.1)\}) \big\}
\]

**Step 2.** Constructs a IFS-set $\Lambda_B$ over $U$ based on data of ill person:

\[
\Lambda_B = \big\{ (x_1, \{(u_1, 0.9, 0.1), (u_2, 0.9, 0.0)\}), (x_2, \{(u_1, 0.1, 0.9), (u_2, 0.1, 0.8)\}), (x_3, \{(u_1, 0.7, 0.1), (u_2, 0.8, 0.9)\}), (x_4, \{(u_1, 0.9, 0.1), (u_2, 0.9, 0.8)\}), (x_5, \{(u_1, 0.9, 0.1), (u_2, 0.9, 0.2)\}), (x_6, \{(u_3, 0.1, 0.9), (u_4, 0.1, 0.8)\}), (x_7, \{(u_1, 0.9, 0.1), (u_2, 0.7, 0.9)\}), (x_8, \{(u_1, 0.9, 0.9), (u_2, 0.1, 0.9)\}), (x_9, \{(u_1, 0.8, 0.1), (u_2, 0.1)\}) \big\}
\]

**Step 3.** Calculate Hamming distances of $\Gamma_A$ and $\Lambda_B$,

\[
d_{IFS}(\Gamma_A, \Lambda_B) \cong 1.1
\]

**Step 4.** Calculate the similarity measure of $\Gamma_A$ and $\Lambda_B$,

\[
S_{IFS}(\Gamma_A, \Lambda_B) = \frac{1}{1 + d_{IFS}(\Gamma_A, \Lambda_B)} \cong 0.48 < \frac{1}{2}
\]

**Step 5.** Hence the two IFS-sets, i.e. two symptoms $\Gamma_A$ and $\Lambda_B$ are not significantly similar. Therefore, we conclude that the person is not possibly suffering from cancer.

Example 5.2 Let us consider Example 5.1 with different ill person.

**Step 1.** Constructs a IFS-set for cancer $\Gamma_A$ is in the Example 5.1.
Step 2. A person suffering from the following symptoms whose corresponding IFS-set $\Upsilon_C$ is given below:

$$\Upsilon_C = \left\{ (x_1, \{(u_1, 0.5, 0.4), (u_2, 0.4, 0.4)\}), (x_2, \{(u_1, 0.7, 0.1), (u_2, 0.8, 0.1)\}) \right\},$$

$$\left\{ (x_3, \{(u_1, 0.4, 0.5), (u_2, 0.2, 0.6)\}), (x_4, \{(u_1, 0.2, 0.7), (u_2, 0.2, 0.1)\}) \right\},$$

$$\left\{ (x_5, \{(u_1, 0.2, 0.6), (u_2, 0.1, 0.8)\}), (x_6, \{(u_3, 0.5, 0.3), (u_C, 0.7, 0.1)\}) \right\},$$

$$\left\{ (x_7, \{(u_1, 0.2, 0.6), (u_2, 0.1, 0.8)\}), (x_8, \{(u_1, 0.5, 0.3), (u_2, 0.7, 0.1)\}) \right\},$$

$$\left\{ (x_9, \{(u_1, 0.5, 0.3), (u_2, 0.7, 0.1)\}) \right\}$$

Step 3. Calculate Hamming distances of $\Gamma_A$ and $\Lambda_B$,

$$d^{\text{IFS}}_I(\Gamma_A, \Lambda_B) \approx 0.41$$

Step 4. Find the similarity measure of these two IFS-sets as:

$$S^{\text{IFS}}_I(\Gamma_A, \Upsilon_C) = \frac{1}{1 + d^{\text{IFS}}_I(\Gamma_A, \Upsilon_C)} \approx 0.71 > \frac{1}{2}$$

Step 5. Here the two IFS-sets, i.e. two symptoms $\Gamma_A$ and $\Upsilon_C$ are significantly similar. Therefore, we conclude that the person is possibly suffering from cancer.

6 Conclusion

Majumdar and Samanta give two types of similarity measure between soft sets and have shown an application of this similarity measure of soft sets. In [23], Kharal give counterexamples to show that Definition 2.7 and Lemma 3.5 contain errors in [28]. In [23], a new measures have been presented and this measures have been applied to the problem of financial diagnosis of firms. In this paper, we have defined four types of distances between two IFS-sets and proposed similarity measures of two IFS-sets. Then, we construct a decision making method based on the similarity measures. Finally, we give two simple examples to show the possibility of using this method by using Hamming distance for diagnosis of diseases. In these example, if we use the other distances, we can obtain similar result.

The method can be applied to problems that contain uncertainty such as problems in social, economic systems, pattern recognition, medical diagnosis, game theory, coding theory and so on.

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