Piezoelectric time delayed control for nonlinear vibration of nanobeams

Canchang Liu, Qingmei Gong, Yingchao Zhou and Changcheng Zhou

Abstract
The nonlinear vibration effect of nanobeam and problem of the pull-in between the nanobeam and driven plate are the factors which prevent nano-resonator from improving the performance. The nonlinear governing differential equation is built by considering the axial force with piezoelectric controller. The piezoelectric time-delay electrostatic pull-in control is studied and the first-order approximate solution of nonlinear equation is gotten by using the method of multi-scale. The amplitude–frequency and phase frequency response equation of nonlinear vibration system are given for the primary resonance response of the nanobeam. The relationship between the control effect and feedback control parameters is studied. It is found that the piezoelectric time-delay control can change the critical voltage but not the critical position. The axial force can affect the performance of nanobeam.

Keywords
Nanobeam, time-delay control, axial force, nonlinear vibration, pull-in control

Introduction
With the rapid development of electronic technology, the fabrication of resonators with high fundamental frequency and high quality factor has become a very active research field for more than 10 years. The applications of high frequency circuits and clock signal circuit of new generation high performance computer have put forward the new requirements for the performance of resonators. However, nano structure acted as the core components of the high-frequency resonator, its performance is affected by the pull-in effect and nonlinear vibration, which exists between nanobeam structure and drive plate. Therefore, it urgently needs a kind of control method to meet the needs of the resonator of nanobeam vibration.

In recent years, the linear vibration of structures with axial force has become one of the hot research issues. The flexural vibration frequency of the structure as a function of the axial force was investigated and the influence of axial force on vibration characteristics was given. An Euler–Bernoulli beam with constant axial force was considered in the study of the energy density and energy intensity of the beam. Lee and Lee presented a transfer matrix expression method to determine the eigenpairs of a rotating beam considering the effects of the axial force. The flexural vibration frequency in the antisymmetric mode of a thick plate was investigated considering the axial force. The determination of the internal axial force and end restraints in tie rods and cables using vibration-based methods has been studied in the area of structural health monitoring and performance assessment. Mirjavadi et al. aimed to study the buckling and free vibrational behavior of axially functionally graded nanobeam under thermal effect for the first time. However, it is difficult to control the vibration of nanobeam
with nano-size compared with macroscopic and large-scale beam. The axial force control is one of the effective methods for vibration control of nanobeam.

The nonlinear vibration of structure considering axial force has attracted the attention of researchers. Based on Euler–Bernoulli theory, Yan and Jiang\(^{10}\) and Zhang et al.\(^{11}\) studied the effect of surface stress on the vibration and buckling behavior of piezoelectric nanobeams and thin films. Based on the theory of non-local continuum, Wang and Li\(^{12}\) analyzed the influence of an axial force and small-scale effect on the nonlinear principal resonance of nanobeams. The axial force acting on the two ends of the nanobeam can produce the hardening effect of the beam and further affect the nonlinear vibration characteristics of the nanometer beam. Li et al.\(^{13}\) applied non-local elastic theory to study the stability and steady-state resonance of transverse vibration of nanobeams, and drew a conclusion that nonlocal nano-size has a significant influence on the vibration behavior by considering the axial force. When the axial force varies with time, Ghayesh and Farokhi\(^{14,15}\) found that the instability of nonlinear vibration of nanometer beam can be changed. Through theoretical analysis and numerical simulation, Younis and Nayfeh\(^{16}\) studied the nonlinear vibration response of micrometer beams under electrostatic excitation. Arani et al. concluded that increasing the axial force can improve the resonance frequency of the system, and DC electrostatic load mainly affects the bending direction of the frequency response curve.\(^{17}\)

Piezoelectric structures have been widely used in the field of vibration control because of their characteristics of both sensors and actuators. An active piezoelectric controller was used to suspend the vibration of a cantilever beam.\(^{18,19}\) The active vibration control of a composite plate using discrete piezoelectric patches has been investigated.\(^{20,21}\) One of the challenging issues in structural dynamics is to accurately and rapidly control the vibration levels.\(^{22}\) Acoustic black holes, as a new type of passive structure for vibration damping enhancement and noise attenuation, have been drawing increasing attentions of many researchers.\(^{23}\) Piezoelectric control is a common control method in linear and nonlinear vibration. However, in the field of nano-beam pull-in control, piezoelectric control strategy is rarely used.

The research on the analysis and control of electrostatic pull-in for microelectromechanical system has attracted many researchers’ attention. The exact critical pull-in voltage of the two coupled elastically restrained beams with size effect was presented for a novel model of the coupled beam-type electrodes with elastic roots.\(^{24}\) Ebrahimi and Hosseini dealt with the study of the temperature effect on the nonlinear vibration behavior of nanoplate-based nano electromechanical systems subjected to hydrostatic and electrostatic actuations.\(^{25}\) The small-scale effect on the pull-in instability and frequency of graphene sheets subjected to electrostatic and van der Waals forces was studied.\(^{26}\) Effects of surface energy on the pull-in instability and free vibration of electrostatically actuated micro/nanoscale plates were analyzed based on the modified couple stress theory.\(^{27}\) Pull-in instability of the electrostatic microstructures was a common undesirable phenomenon which implies the loss of reliability of microelectromechanical systems.\(^{28}\) Pull-in instability and free vibration of cantilever and clamped-clamped beam-type nanoactuators, which are made of functionally graded materials, were investigated using the modified strain gradient theory under the influence of electrostatic and intermolecular forces.\(^{29}\)

In this article, the dynamic response of the nanobeam vibration system is studied with a certain axial force. The main purpose of this article is to control the pull-in and nonlinear vibration of nanobeam with time delay. The influence of the axial force on the natural frequency of the system is discussed. The critical electrostatic excitation parameters changed with pull-in parameters are studied and the critical pull-in voltage value of electrostatic excitation is given. The first-order approximate solution of nonlinear equation is gotten by using the method of multi-scale. The control effect of closed piezoelectric control loop formed by piezoelectric controller on the nonlinear vibration of the system is analyzed. The amplitude–frequency and phase frequency response equation of nonlinear vibration system are given for the primary resonance of the nanobeam.

**Nonlinear dynamic for time-delay pull-in control**

The differential equation describing the electrostatically actuated nanobeam clamped at its both ends under the control of a piezoelectric closed-loop circuit with certain axial force \(F_N\) is written as\(^{30–36}\) (see Figure 1)

\[
\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = -b \frac{\partial w}{\partial t} + \frac{EA}{2l} \int_0^l \left( \frac{\partial w}{\partial x} \right)^2 \text{d}x + f_e + F_N \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 M}{\partial x^2} \tag{1}
\]
where \( f_e = \frac{aW}{2(d-w)^2} \left( 1 + 0.65 \frac{d-w}{W} \right) V^2 \), \( W \) is the area of the nanobeam, \( V \) is the nanobeam driving voltage, including the DC and AC driving voltage. \( F_N \) can be applied by the axial force controller. \( M \) is controlling torque generated at both ends of the piezoelectric system, which can be written as

\[
M = R_s p \left[ K_1 \left( \frac{w^2}{x^2} - \frac{w}{x_1^2} \right) - K_2 \left( \frac{w}{x_1^2} - \frac{w}{x_2^2} \right) \right]^3 \left( H(x^2 - x_1^2) - H(x^2 - x_2^2) \right)
\]

where \( K_1 \) is linear feedback control gain and \( K_2 \) is nonlinear feedback control gain.

Substituting \( M \), electrostatic force and dimensionless quantity, such as, \( u = \frac{w}{d_x} \), \( V' = \frac{V}{V_0} \), \( t = \frac{t'}{L^2} \sqrt{E I / \rho A} \), \( \tau = \frac{t'}{L^2} \sqrt{E I / \rho A} \), \( \mu = \mu^2 / \sqrt{\rho AE I} \), \( \Omega = \Omega^2 / \sqrt{\rho A (E I)} \), \( \mathrm{into} \) equation (1), the dimensionless control differential equation on nanobeam under axial force can be written as the followed

\[
\ddot{u}(x, t) + u(x, t) = -b \dot{u} + \delta_1 \int_0^1 u^2 \mathrm{d}x + \delta_3 u^2 + \left[ \frac{1}{(1 - u(x, t))^2} + \frac{f}{1 - u(x, t)} \right] \delta_2 V^2
\]

\[
+ \left[ k_1 \left( u'(x_2, x_2) - u'(x_1, x_1) \right) + k_2 \left( u'(x_2, x_2) - u'(x_1, x_1) \right) \right] \frac{H''(x - x_1) - H''(x - x_2)}{V^2}
\]

where \( \delta_1 = \frac{\alpha d^3}{2}, \quad \delta_2 = \frac{\alpha d^3 \mu}{2}, \quad \delta_3 = \frac{\rho d^3 \Omega^2}{E I}, \quad k_1 = \frac{R_s p d^2 K_1^2}{E I}, \quad k_2 = \frac{R_s p d^2 K_2^2}{E I} \), and \( f = \frac{0.65 d}{W} \).

The solution to the vibration (equation (2)) can be written as

\[
\begin{align*}
\sum_{k=1}^{\infty} \phi_k(x) \phi_k(t) = \sum_{k=1}^{\infty} \phi_k(x) \phi_k(t)
\end{align*}
\]

To simplify the analysis, we only study first-order pull-in model and ignore the edge effect of the electrostatic driver in the following analysis. The control differential equation of nanobeam can be written as

\[
\phi_1 \ddot{q}_1 + \phi''_1 q_1 = -\mu \phi_1 \dot{q}_1 + \delta_1 \int_0^1 u^2 \mathrm{d}x \phi''_1 + \delta_3 \phi''_1 q_1 + \left[ \frac{1}{(1 - \phi_1^2 q_1)^2} \right] \delta_2 \frac{V^2}{q_1}
\]

\[
+ k_1 \left( \phi'_1(x_2) q_1(t - \tau) - \phi'_1(x_1) q_1(t - \tau) \right) \frac{H''(x - x_1) - H''(x - x_2)}{V^2}
\]

\[ \quad + k_2 \left( \phi'_1(x_2) q_1(t - \tau) - \phi'_1(x_1) q_1(t - \tau) \right) \frac{H''(x - x_1) - H''(x - x_2)}{V^2} \]

As for a clamped-clamped nanobeam, the pull-in position of the nanobeam and the plate is always in the middle of the beam. Expressing the solution of equation (5) \( y = \phi_1 q_1(t) \), equation (5) can be written as

\[
\ddot{y} - \mu \dot{y} - \frac{\gamma}{(1 - y)^2} \frac{y}{(1 - y)^2} + \gamma \frac{y}{(1 - y)^2}
\]

where \( \alpha = \frac{\psi''(x) - \psi''(x)}{\psi''(x)}, \quad \gamma = \frac{\delta_2 V^2}{\psi''(x)}, \quad g_1 = \frac{k_1 \phi'_1(x_2) \phi'_1(x_1)}{\psi''(x_2)} \frac{H''(x - x_1) - H''(x - x_2)}{V^2}, \quad \beta = g_2 + \delta_1 \int_0^1 \frac{\phi''}{\phi''(x)} \mathrm{d}x \phi''(x), \quad g_2 = \frac{k_2 \phi'_1(x_2) \phi'_1(x_1)}{\psi''(x_2)} \frac{H''(x - x_1) - H''(x - x_2)}{V^2}. \)
Ignoring the system damping and transforming the kinetic equation into the spatial state equation, it can be written as

\[
\begin{align*}
\dot{y}_1 &= y \\
\dot{y}_2 &= -2y + \beta y^3 + \frac{\gamma}{(1-y)^2}
\end{align*}
\] (7)

The equilibrium point of the system is as follows

\[
\begin{align*}
\dot{y}_1 &= 0 \\
\dot{y}_2 &= -2y + \beta y^3 + \frac{\gamma}{(1-y)^2} = 0
\end{align*}
\] (8)

The second equation in the above formula is equivalent to

\[
f(y, g_1, g_2, \tau) = -ay + \beta y^3 + \frac{\gamma}{(1-y)^2} = 0
\] (9)

As the stability condition of the dynamic system balance is \(\frac{\partial f}{\partial y} \leq 0\), equation (9) can be written as

\[
\frac{\partial f(q_k, k_1, k_2, \tau)}{\partial y} = -a + 3\beta y^2 + \gamma \frac{2}{(1-y)^3} = 0
\] (10)

Without considering the nonlinear force, namely \(\beta\) is equal to 0 in equation (10), the pull-in critical conditions considered the electrostatic force can be written as follows

\[
\dot{y} = \frac{4x}{27}
\] (11)

Substituting equation (11) into equation (8), the critical pull-in position can be calculated as follows

\[
y = \frac{1}{3}
\] (12)

The results of equation (12) for critical pull-in position are similar with Lin and Zhao’s study.\(^{37}\) From equations (6) and (12), we can find that the pull-in position of system near the equilibrium point does not change with the change of the piezoelectric control parameter and axial forces. However, the critical electrostatic excitation parameters increase with pull-in parameters. The critical pull-in voltage value of electrostatic excitation can be increased by changing the value of the critical pull-in parameter. The hardening effect of the nanobeam can be obtained by increasing the axial force.

**Nonlinear dynamic for time delayed vibration control**

The electric-force term can be expanded in a Taylor series around \(u = 0\).\(^{38}\) The nonlinear terms of equation (3), \(1/(1-u)^2\), \(1/(1-u)\), is expanded and the higher order terms can be abandoned given that the deflection of the nanobeam is small in value. Additionally, as the nanobeam vibration system is a weak nonlinear one, the small parameter variable can be introduced to the above equation, and the dimensionless piezoelectric control equation of nanobeam can be gained as below

\[
\ddot{u} + u(4) = -bu + \delta_1 u'' + \int_0^1 (u')^2 dx + \delta_3 u'' + \delta_2 [(1+f) + (2+f)u
\]

\[
+ (3+f)u^2 + (4+f)u^3] (V_{dc}^2 + 2V_{dc}V_{ac} \cos \Omega t + V_{ac}^2 \cos^2 \Omega t)
\]

\[
+ [k_1(u'(x_2, t - \tau) - u'(x_1, t - \tau)) + k_2(u'(x_2, t) - u'(x_1, t))] [H''(x - x_1) - H''(x - x_2)]
\] (13)
The primary resonance of the nanobeam is studied under a certain axial force, which the external excitation frequency is approximately equal to the natural frequency of the system. The primary resonant piezoelectric control equation of the nanobeam, without considering the higher order term of the first harmonic part, can be written as\textsuperscript{39–41}

\[
\ddot{u} + u^{(4)} = \ddot{u} + \varepsilon \left[ 2 \delta_2 [(1 + f) + (2 + f) u] V'_{ac} \cos \Omega t \right] + \varepsilon \left[ k_1 [u'(x_2, t - \tau) - u'(x_1, t - \tau)] + k_2 [u'(x_2, t) - u'(x_1, t)] \right]^3 [H''(x - x_1) - H''(x - x_2)]
\]

Using the method of multiple scales, the approximate solution of equation (14) can be expressed as

\[
u(x, t, \varepsilon) = \sum_{i=0}^{n} v_i(x, T_0, T_1) = u_0(x, T_0, T_1) + \varepsilon u_1(x, T_0, T_1) + \cdots
\]

By substituting equation (15) into equation (14), comparing the coefficients of same power of $\varepsilon$ at both sides of the equation, and letting the coefficients of $\varepsilon$ zero, equation (14) can be written as

\[
e^0: \quad D_0^2 u_0 + u^{(4)}_0 = 0
\]

\[
e^1: \quad D_0^2 u_1 + u^{(4)}_1 = -2D_0 D_1 u_0 - bD_0 u_0 + \delta_1 u_0 \int_0^1 u_0'' dx + \delta_3 u_0''
\]

\[
+ \delta_2 [(1 + f) + (2 + f) u] V'_{ac} \cos \Omega t
\]

\[
+ \delta_2 [(1 + f) + (2 + f) u_0 + (3 + f) u'^2_0 + (4 + f) u'^3_0] V'_{ac}
\]

\[
+ \{k_1 [u'_0(x_2, t - \tau) - u'_0(x_1, t - \tau)] + k_2 [u'_0(x_2, t) - u'_0(x_1, t)] \}^3 [H''(x - x_1) - H''(x - x_2)]
\]

The approximate solution of equation (17) is expressed as

\[
u_0 = \phi(x) [A_k(T_1) e^{i \omega T_0 + A_k(T_1) e^{-i \omega T_0} = 0}
\]

\[
\cos \Omega t = \frac{1}{2} (e^{i \omega T_0 + i \omega T_1} + e^{-i \omega T_0 - i \omega T_1})
\]

Substituting equations (18) and (19) into equation (17), the equation in which the permanent term is eliminated can be obtained as below

\[
-2i \omega g_{kk} A_k' - i b \omega g_{kk} A_k + 3 \delta_1 g_{kk} A_k^2 + \delta_3 g_{kk} A_k + k_1 g_{kk} A_k e^{-i \tau}
\]

\[
+ F_2 g_{kk} A_k + 3 F_4 g_{kk} A_k^2 + I_1 g_{kk} e^{i \omega T_1} + 3 k_2 g_{kk} A_k^2 e^{-i \tau} = 0
\]

where

\[
g_{kk} = \int_0^1 \phi[\phi'(x_2) - \phi'(x_1)]^3 [H''(x - x_1) - H''(x - x_2)] dx,
\]

\[
I_1 = (1 + f) \delta_2 V'_{ac}, i = 1, 2, 3, 4\) and \(g_{kk} = \int_0^1 \phi[\phi'(x_2) - \phi'(x_1)] [H''(x - x_1) - H''(x - x_2)] dx.
\]

The polar coordinate of $A_k$ is

\[
A_k = \frac{1}{2} a_k e^{i \gamma_k}
\]

Substituting equation (21) into equation (20) and separating the real and the imaginary parts, the differential equations in which the amplitude $a_k$ and phase angle $\gamma_k$ of an approximate solution satisfying the polar coordinate
can be written as

\begin{equation}
D_1 a_k = -\mu_k a_k + F_{k1}\sin \gamma_k
\end{equation}

\begin{equation}
a_k D_1 \gamma_k = \sigma_{k41} a_k + v_{k41} a_k^3 + F_{k1}\cos \gamma_k
\end{equation}

where \( u_k = \frac{b}{2} + k_1 g_{kk}\sin \varpi / (2g_{kk} \omega_k) \), \( F_{k1} = h_{kk} / (g_{kk} \omega_k) \), \( \sigma_{k41} = \sigma + k_1 g_{kk}\cos \varpi / (2g_{kk} \omega_k) + 1/2 g_{kk} g_{kk} / (g_{kk} \omega_k) \) + 1/2 g_{kk} g_{kk} / (g_{kk} \omega_k).

In order to determine the amplitude \( a_k \) and phase angle \( \gamma_k \) of steady solutions when the nanobeam is in steady motion, assume \( D_1 a_k = D_1 \gamma_k = 0 \). The algebraic equations which satisfies with amplitude and phase angle can be written as

\begin{equation}
-\mu_k a_k + F_{k1}\sin \gamma_k = 0
\end{equation}

\begin{equation}
\sigma_{k41} a_k + v_{k41} a_k^3 + F_{k1}\cos \gamma_k = 0
\end{equation}

By eliminating \( \gamma_k \) from the sum of the square of equation (24) and (25), the amplitude–frequency response equation and the phase-frequency response equation at the primary resonance of the system are

\begin{equation}
(\mu_k a_k)^2 + (\sigma_{k41} a_k + v_{k41} a_k^3)^2 = F_{k1}^2
\end{equation}

\begin{equation}
\gamma_k = -\arctan \left( \frac{\mu_k}{\sigma_{k41} + v_{k41} a_k^2} \right)
\end{equation}

According to equation (26), the amplitude–frequency response of the nanobeam vibration system and the vibration stability of the nanobeam can be affected by the parameters of excitation voltage, damping, feedback gains.

Setting \( E_k = a_k^2 \), assuming \( v_{k41} \neq 0 \), equation (26) is expressed as the following equation

\begin{equation}
E_k^2 + \frac{2\gamma_{k41}\sigma_{k41}}{v_{k41}} E_k^2 + \frac{v_{k41}^2 + \sigma_{k41}^2}{v_{k41}} E_k - \frac{F_{k1}^2}{v_{k41}} = 0
\end{equation}

Taking the derivative of equation (28), and setting \( \partial E_k / \partial \sigma = 0 \), the equation at the resonance peak point can be written as

\begin{equation}
v_{k41} E_k + \sigma_{k41} = 0
\end{equation}

Substituting equation (29) into equation (26) and assuming \( F_{k1} \neq 0 \), then \( \cos \gamma_k = 0 \). Equation (26) is given as

\begin{equation}
-\mu_k a_k_{max} + F_{k1} = 0
\end{equation}

The maximum amplitude of the main resonance is

\begin{equation}
a_{kmax} = \frac{F_{k1}}{u_k}
\end{equation}

**Numerical analysis and discussion**

Taking the first-order vibration mode of nanobeam as an example, the discussion of the nonlinear vibration of nanobeam is carried out. This article deals with the axial force of the electrostatic excited nanobeam vibration system. Under piezoelectric control, the effects of control parameters and system parameters on the pull-in control
of nonlinear vibration, stability, and amplitude of silicon nanobeam are discussed. The length, width, and height of the silicon–sodium nanobeam are 30 μm, 500 nm, and 300 nm, respectively. The distance between nanobeams and plates is 500 nm.

**Pull-in control with time delayed control**

According to Figure 2, it is obvious that the critical pull-in point near the equilibrium point of system shifts to the right with the increase of piezoelectric control gain, while the pull-in critical position does not change. The critical electrostatic excitation parameters increase with pull-in parameters. The critical pull-in voltage value of electrostatic excitation can be increased by changing the value of the critical pull-in parameter. The pull-in position of system near the equilibrium point does not change with control parameter and axial forces. The axial force can change the hardening of the nanobeam.

**The influence of axial force on nanobeam vibration**

Figure 3 shows the piezoelectric control parameters changing with different axial force. With the increase of axial force, piezoelectric control parameters increase along with the linear increases of the axial force. The critical pull-in voltage value of electrostatic excitation can be increased with the increase of axial force $F_N$, which indicates that axial force has a hardening effect on the system. The hardening effect of the nanobeam can be got by increasing the axial force.

Figure 4 shows the variation curves of piezoelectric control parameters and piezoelectric control gain with different time delays. When the time delay is constant, the piezoelectric control parameters increase linearly with the increase of piezoelectric control gain. When the piezoelectric control gain is constant, the piezoelectric control parameter will decrease as the time delay increases in the range of $0–\pi/2$.

**Nonlinear vibration control**

Figure 5 shows the response curve of amplitude–frequency characteristics with different axial forces. The resonant frequency of the amplitude–frequency response curve moves to the right as the axial force increases. It can be seen

![Figure 2](image2.png)

**Figure 2.** Amplitude of nanobeam varying with excitation voltage parameters under the piezoelectric control.

![Figure 3](image3.png)

**Figure 3.** Piezoelectric control parameters changing with axial force.
that the axial force can change the natural frequency of the vibration system, causing the resonant region change with the axial force.

From Figure 6, when the DC excitation voltage is 2.0 V, the vibration response of the nanobeam is stable. While the DC excitation voltages are 3.0 and 4.0 V, the amplitude–frequency characteristic curves of the vibration of the nanobeam are unstable. It can be found that with the increase of the DC excitation voltage, the vibration of the nanobeam tends to be unstable from stable state and a multi-value interval appears. The vibration response is obviously nonlinear. Additionally, the amplitude–frequency characteristic response curve bends to the right, which means that the DC excitation voltage has a softening effect on the nonlinear vibration system of the nanobeam.

As the DC excitation voltage is different from damping, the maximum amplitude varies with the length of the nanobeam as shown in Figure 7. When the excitation voltage and damping are constant, the maximum amplitude increases with the length of the nanobeam. When the length of the nanobeam is constant, the maximum amplitude decreases with the increase of damping, and shows the opposite influence law for DC excitation voltage. The conclusions drawn from Figure 8 are the same as those from the amplitude–frequency response curves.

Figure 9 shows the primary resonant amplitude–frequency curves of the nanobeam with different linear control gains in the case of time delay is $\tau = \pi/6$ and nonlinear control gain $k_2 = 160$. As can be seen from the figure, the
nonlinear main resonance frequency and amplitude of the nano-resonator decrease with the increase of control gain. When the linear gain $k_1 = 0$ and $k_1 = 2$, the multi-value phenomenon will appear in the amplitude–frequency figure. While the linear gain $k_1 = 4$, the nonlinear vibration is changed into linear vibration.

Figure 10 shows the primary resonant amplitude–frequency curves of nanobeams with different time delays. When the time delayed $k_1 = 2$, the nonlinear control gain $k_2 = 160$. It can be seen from the figure that with the
increase of the time delay, the nonlinear primary resonant frequency and amplitude of the nano-resonator will decrease. When the time delayed $\tau = 0$ and $\tau = \pi/6$, the multi-value phenomenon is seen in the amplitude–frequency image. When the time delayed $\tau = \pi/3$, the nonlinear vibration is changed into linear vibration. The linear control gain and time delay both affect the nonlinear vibration of the nanobeam. The nonlinear vibration shape of the structure can be changed by changing the nonlinear gain, which can change the structural parameters, reduce or even eliminate the nonlinear effect.

Figure 11 depicts the amplitude–frequency characteristic response curve when the feedback gain is different. The feedback gain parameter $k_2$ almost has no effect on the maximum amplitude. When the frequency is far from the resonance region, the feedback gain parameter has little effect on the amplitude. When feedback control parameters are 160 and 165, the curves deviate to the right due to the geometric nonlinear term. When the excitation frequency continues to be increased, the amplitude is not strictly in accordance with the curve, and instead, the jump phenomenon appears, which means that the system vibration is unstable. When the feedback control parameter is increased to 170, the nanobeam takes on linear vibration. Therefore, the feedback gain can change the vibration characteristics of the nanobeam by changing the value of the nonlinear term.

Figure 12 depicts the amplitude–frequency characteristic response curve when the feedback gain is different. It can be seen from Figure 12 that the feedback gain $k_2$ almost has no effect on the maximum amplitude. When the frequency is far from the resonance region, the feedback gain has a little effect on the amplitude. When feedback control parameters are 160 and 180, the maximum amplitude point is not obtained at the resonant frequency point, but is deviated to the right or the left, due to the geometric nonlinear term. When the excitation frequency continues to be increased, the amplitude is not strictly in accordance with the increase of the curve, in fact, the jump phenomenon occurs because of unstable system vibration. When the feedback control parameter is increased to 170, the nanobeam shows linear vibration. The nonlinear control gain will affect the nonlinear vibration of the nanobeam. By changing the gain and the delay, the nonlinear vibration shape of the structure can be changed, and the structural parameters can also be changed to reduce and even eliminate the nonlinear influence on the structure, which means the reduction in vibration of nonlinear structures can be realized. As a result, the nonlinear vibration will be shown as linear vibration.
From the analysis on the above response equations and figures, it is found that nonlinear vibration of nanobeams can be controlled by the feedback gain parameter in the way of changing nonlinear terms' values. The feedback control gain and time delay produce a time-delay damping term by changing the phase of the control item, which can control the amplitude of nonlinear vibration. The feedback control gain and time delay can change the elastic term and the value of nonlinear principal resonance point. The feedback control gain and time-delay terms can also change the value of the nonlinear term to enhance or weaken the nonlinear term. Therefore, the steady state vibration of the nanobeam can be realized by selecting the appropriate feedback gain parameter value.

Conclusions
The Euler–Bernoulli beam with fixed ends is taken as the model. The control condition of the pull-in is obtained by analyzing the extreme value of the vibration equilibrium point of the nanometer beam. The effects of axial force on structural vibration, control gain, and time delay on nonlinear vibration are studied by response curve of amplitude–frequency. The conclusions can be drawn as:

1. The pull-in condition of the nanobeam can be changed by changing the control parameters, but the position of the pull-in cannot be changed. The amplitude–frequency response curve with the same axial force is axial symmetric. When axial force is less than zero, the inherent frequency of the system will reduce. When axial force is larger than zero, the natural frequency of the system will increase. The natural frequency increases with the increase of axial force and the axial force has a hardening effect on the system.
2. The reduction in vibration of nonlinear structures can be controlled on the basis of the effect of the linear control gain and time delayed on the nonlinear vibration of nanobeams. By analyzing the amplitude–frequency characteristic response curve, the damping, excitation voltage and the feedback gain play the main control role in the nonlinear vibration of the nanobeam. Choosing the appropriate damping value and the feedback gain value can noticeably weaken the nonlinear characteristic of the system vibration.
3. The nonlinear feedback gain can change the value of the nonlinear term to realize the control of the nonlinear vibration of the nanobeam. Therefore, by selecting the appropriate feedback gain value, the steady state vibration of the nanobeam can be realized. Increasing the damping value, reducing the excitation voltage or increasing the feedback gain can weak the nonlinear characteristic of the nanobeam. The maximum amplitude decreases with the increase of the damping value and the axial force, while it increases with the rise in the excitation voltage amplitude. The effect of the feedback gain on the maximum amplitude is smaller than that of the other parameters mentioned above. The peak amplitude varies with the length of the nanobeam, damping, and amplitude of excitation voltage.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This study is supported by the Shandong key research and development projects (Grant No. 2019GGX104066).

Figure 12. Main resonant amplitude–frequency images of nanobeams with different linear control gains.
References

1. Ghayesh MH. Functionally graded microbeams: simultaneous presence of imperfection and viscoelasticity. *Int J Mech Sci* 2018; 140: 339–350.
2. Farokhi H and Ghayesh MH. Nonlinear dynamical behaviour of geometrically imperfect microplates based on modified couple stress theory. *Int J Mech Sci* 2015; 90: 133–144.
3. Farokhi H, Ghayesh MH and Amabili M. Nonlinear dynamics of a geometrically imperfect microbeam based on the modified couple stress theory. *Int J Eng Sci* 2013; 68: 11–23.
4. Ghayesh MH. Nonlinear vibration analysis of axially functionally graded shear- deformable tapered beams. *Appl Math Model* 2018; 59: 583–596.
5. Chen Z, Yang Z, Guo N, et al. An energy finite element method for high frequency vibration analysis of beams with axial force. *Appl Math Modell* 2018; 61: 521–539.
6. Lee JW and Lee JY. An exact transfer matrix expression for bending vibration analysis of a rotating tapered beam. *Appl Math Modell* 2018; 53: 167–188.
7. Nieves FJ, Bayón A, Gascón F, et al. Nonlinear bending vibration of a prestressed thick plate. *J Mech Sci Technol* 2018; 32: 1505–1517.
8. Kernicky T, Whelan M and Al-Shaer E. Dynamic identification of axial force and boundary restraints in tie rods and cables with uncertainty quantification using set inversion via interval analysis. *J Sound Vib* 2018; 423: 401–420.
9. Mirjavadi SS, Rabby S, Shafiei N, et al. On size-dependent free vibration and thermal buckling of axially functionally graded nanobeams in thermal environment. *Appl Phys A* 2017; 123: 315.
10. Yan Z and Jiang LY. The vibrational and buckling behaviors of piezoelectric nanobeams with surface effects. *Nanotechnology* 2011; 22: 245703–245709.
11. Zhang J, Wang CY and Adhikari S. Surface effect on the buckling of piezoelectric nanofilms. *J Phys D Appl Phys* 2012; 45: 285301–285308.
12. Wang YZ and Li FM. Nonlinear primary resonance of nano beam with axial load by nonlocal continuum theory. *Int J Non Linear Mech* 2014; 61: 74–79.
13. Li C, Lim CW and Yu JL. Dynamics and stability of transverse vibrations of nonlocal nanobeams with a variable axial load. *Smart Mater Struct* 2011; 20: 1424–1428.
14. Ghayesh MH and Farokhi H. Parametric instability of microbeams in supercritical regime. *Nonlinear Dyn* 2016; 83: 1171–1183.
15. Ghayesh MH, Farokhi H and Alici G. Subcritical parametric dynamics of microbeams. *Int J Eng Sci* 2015; 95: 36–48.
16. Younis MI and Nayfeh AH. A study of the nonlinear response of a resonant microbeam to an electric actuation. *Nonlinear Dyn* 2003; 31: 91–117.
17. Arani AG, Abdollahian M and Kolahchi R. Nonlinear vibration of a nanobeam elastically bonded with a piezoelectric nanobeam via strain gradient theory. *Int J Mech Sci* 2015; 100: 32–40.
18. Bailey T and Hubbard JE. Distributed piezoelectric-polymer active vibration control of a cantilever beam. *J Guidance Control Dyn* 1985; 8: 23.
19. Zhu Q, Yue JZ, Liu WQ, et al. Active vibration control for piezoelectricity cantilever beam: an adaptive feedforward control method. *Smart Mater Struct* 2017; 26: 047003.
20. Bendine K, Boukhoulda FB, Haddag B, et al. Active vibration control of composite plate with optimal placement of piezoelectric patches. *Mech Adv Mater Struct* 2017; 26: 341–349.
21. Gomaa AR and Hai H. Analysis of piezoelectric actuator for vibration control of composite plate. *IOP Conf Ser Mater Sci Eng* 2017; 220: 012014.
22. Chang X, Ying W and Zishun L. Modeling and active vibration control of lattice grid beam with piezoelectric fiber composite using fractional order PDμ algorithm. *Compos Struct* 2018; 198: 126–134.
23. Ji H, Luo J, Qiu J, et al. Investigations on flexural wave propagation and attenuation in a modified one-dimensional acoustic black hole using a laser excitation technique. *Mech Syst Signal Process* 2018; 104: 19–35.
24. Lin SM, Lee SJ and Lin CC. The vibration and pull-in mechanism of two coupled elastically restrained beams assembly subjected to electrostatic force. *Mech Adv Mater Struct* 2018; 27: 274–285.
25. Ebrahimi F and Hosseini SHS. Effect of temperature on pull-in voltage and nonlinear vibration behavior of nanoplate-based NEMS under hydrostatic and electrostatic actuations. *Acta Mech Solida Sin* 2017; 30: 174–189.
26. Wang KF, Wang BL and Zeng S. Small scale effect on the pull-in instability and vibration of graphene sheets. *Microsyst Technol* 2017; 23: 2033–2041.
27. Wang KF, Kitamura T and Wang B. Nonlinear pull-in instability and free vibration of micro/nanoscale plates with surface energy – a modified couple stress theory model. *Int J Mech Sci* 2015; 99: 288–296.
28. Shang HL. Pull-in instability of a typical electrostatic MEMS resonator and its control by delayed feedback. *Nonlinear Dyn* 2017; 90: 171–183.
29. Ataei H, Beni YT and Shojaeian M. The effect of small scale and intermolecular forces on the pull-in instability and free vibration of functionally graded nano-switches. *J Mech Sci Technol* 2016; 30: 1799–1816.
30. Farokhi H and Ghayesh MH. Nonlinear resonant response of imperfect extensible Timoshenko microbeams. *Int J Mech Mater Des* 2017; 13: 43–55.
31. Ghayesh MH and Farajpour A. Nonlinear mechanics of nanoscale tubes via nonlocal strain gradient theory. *Int J Eng Sci* 2018; 129: 84–95.
32. Farokhi H, Ghayesh MH and Amabili M. Nonlinear resonant behavior of microbeams over the buckled state. *Appl Phys A* 2013; 113: 297–307.
33. Ghayesh MH, Amabili M and Farokhi H. Coupled global dynamics of an axially moving viscoelastic beam. *Int J Non-Linear Mech* 2013; 51: 54–74.
34. Ghayesh MH. Dynamics of functionally graded viscoelastic microbeams. *Int J Eng Sci* 2018; 124: 115–131.
35. Takács G and Rohal-Ilkiv B. *Model predictive vibration control: efficient constrained MPC vibration control for lightly damped mechanical structures*. Berlin: Springer Science & Business Media, 2012.
36. Bendine K, Boukhoulda FB, Nouari M, et al. Active vibration control of functionally graded beams with piezoelectric layers based on higher order shear deformation theory. *Earthquake Eng Eng Vib* 2016; 15: 611–620.
37. Lin WH and Zhao YP. Dynamics behavior of nanoscale electrostatic actuator. *Chin Phys Lett* 2003; 20: 2070–2073.
38. Younis MI, Abdel-Rahman EM and Nayfeh A. A reduced order model for electrically actuated microbeam-based MEMS. *J Microelectromech Syst* 2003; 12: 672–680.
39. Ghayesh MH, Kazemirad S and Darabi MA. A general solution procedure for vibrations of systems with cubic nonlinearities and nonlinear/time-dependent internal boundary conditions. *J Sound Vib* 2011; 330: 5382–5400.
40. Ghayesh MH, Kazemirad S and Reid T. Nonlinear vibrations and stability of parametrically exited systems with cubic nonlinearities and internal boundary conditions: a general solution procedure. *Appl Math Modell* 2012; 36: 3299–3311.
41. Ghayesh MH and Moradian N. Nonlinear dynamic response of axially moving, stretched viscoelastic strings. *Arch Appl Mech* 2011; 81: 781–799.