Gravitational wave astronomy and cosmology

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Abstract

The first direct observation of gravitational waves’ action upon matter has recently been reported by the BICEP2 experiment. Advanced ground-based gravitational-wave detectors are being installed. They will soon be commissioned, and then begin searches for high-frequency gravitational waves at a sensitivity level that is widely expected to reach events involving compact objects like stellar mass black holes and neutron stars. Pulsar timing arrays continue to improve the bounds on gravitational waves at nanohertz frequencies, and may detect a signal on roughly the same timescale as ground-based detectors. The science case for space-based interferometers targeting millihertz sources is very strong. The decade of gravitational-wave discovery is poised to begin. In this writeup of a talk given at the 2013 TAUP conference, we will briefly review the physics of gravitational waves and gravitational-wave detectors, and then discuss the promise of these measurements for making cosmological measurements in the near future.

Keywords: gravitational waves : cosmology

1. Introduction and overview

Although often introduced as a consequence of Einstein’s theory of general relativity (GR), gravitational radiation is in fact necessary in any relativistic theory of gravity. These waves are simply the mechanism by which changes in gravity are causally communicated from a dynamical source to distant observers. In GR, the curvature of spacetime (which produces tidal gravitational forces) is the fundamental field characterizing gravity. Gravitational waves (GWs) are propagating waves of spacetime curvature, tidally stretching and squeezing as they radiate from their source into the universe.

Tidal fields are quadrupolar, so GWs typically arise from some source’s bulk, quadrupolar dynamics. Consider a source whose mass and energy density are described by $\rho$. Choosing the origin of our coordinates at the source’s center of mass, its quadrupole moment is given by

$$Q_{ij} = \int \rho \left( x_i x_j - \frac{1}{3} \delta_{ij} r^2 \right) dV ,$$

where the integral is taken over the source. The gravitational-wave potential, $h_{ij}$, comes from the second time derivative of $Q_{ij}$:

$$h_{ij} = \frac{2G}{c^2} \frac{1}{r} \frac{d^2 Q_{ij}}{dt^2} ,$$

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$$h_{ij} = \frac{2G}{c^2} \frac{1}{r} \frac{d^2 Q_{ij}}{dt^2} ,$$
where $r$ is distance from the source to the observer. The magnitude of a typical component of $h_{ij}$ is

$$h \approx \frac{G m v^2}{c^4 r},$$  \hspace{1cm} (3)$$

where $v$ is the typical speed associated with the source’s quadrupolar dynamics, and $m$ is the mass that participates in those dynamics. Notice the combination of constants appearing here,

$$\frac{G}{c^4} = 8.27 \times 10^{-50} \text{gm}^{-1} \text{cm} \left(\frac{\text{cm}}{\text{sec}}\right)^{-2}. \hspace{1cm} (4)$$

This is rather small, reflecting the fact that gravity is the weakest of the fundamental forces. To overcome it, one must typically have large masses moving very quickly. A short-period binary in which each member is a compact object (white dwarf, neutron star, or black hole) is a perfect example of a strong quadrupolar radiator. For many of the sources we discuss, $m$ is of order solar masses (or even millions of solar masses), and $v$ is a substantial fraction of the speed of light.

(In addition to quadrupole dynamics, there is one other well-known mechanism for producing GWs: the amplification of primordial ground-state fluctuations by rapid cosmic expansion. We will briefly discuss this way of producing GWs in Sec. 2.4.)

The GWs a source emits backreact upon it, which appears as a loss of energy and angular momentum. The “quadrupole formula” predicts that a system with a time changing quadrupole moment will lose energy to GWs according to

$$\frac{dE}{dt} = -\frac{1}{5} \frac{G}{c^5} \sum_{ij} \frac{d^3Q_{ij}}{dt^3} \frac{d^3Q_{ij}}{dt^3}. \hspace{1cm} (5)$$

This loss of energy from, for example, a binary star system will appear as a secular decrease in the binary’s orbital period — orbital energy is lost to GWs and the stars fall closer together. This effect was first seen in the first known binary neutron star system, PSR 1913+16 (the famed “Hulse-Taylor” pulsar) [1]. In this system, each neutron star has a mass slightly over 1.4 $M_\odot$, and they orbit each other in less than 8 hours. The period has been observed to decrease by about 40 seconds over a baseline of nearly 40 years of observation. Similar period evolutions have now been measured in about 10 galactic binaries containing pulsars [2], and has even been seen in optical measurements of a close white dwarf binary system [3].

From these “indirect” detections, a major goal now is to directly detect GWs. Except at the longest wavelengths (where direct detection has recently been reported), almost all measurement schemes use the fact that a GW causes oscillations in the time of flight of a light signal; the basic idea was sketched by Bondi in 1957 [4]. Imagine an emitter located at $x = x_e$ that generates a series of very regular pulses, and a sensor at $x = x_s$. Ignoring the nearly static contribution of local gravitational fields (e.g., from the Earth and our solar system), the spacetime metric through which the light pulses travel can be written

$$ds^2 = -c^2 dt^2 + [1 + h(t)] dx^2. \hspace{1cm} (6)$$

Light moves along a null trajectory for which $ds^2 = 0$, which means that that the speed of light with respect to these coordinates is (bearing in mind that $h \ll 1$)

$$\frac{dx}{dt} = c \left[1 - \frac{1}{2}h(t) \right]. \hspace{1cm} (7)$$
The time it takes light to travel from the emitter to the receiver is
\[
\Delta T = \int_{x_e}^{x_s} \frac{dx}{dx/dt} = \frac{x_s - x_e}{c} + \frac{1}{2c} \int_{x_e}^{x_s} h(t) \, dx.
\] (8)

The gravitational wave thus enters as an oscillation in the arrival time of pulses. If the emitter is regular enough to be a precise clock, one may measure the GW by measuring this oscillation.

Before rushing out to build our detector, we should estimate how strong the gravitational waves we seek are. We use the formula for \( h \) given above, substituting fiducial values for the physical parameters that are likely to characterize the sources we aim to measure:
\[
h \approx \frac{G}{c^4} \frac{mv^2}{r} \approx 10^{-22} \times \left( \frac{200 \text{ Mpc}}{r} \right) \times \left( \frac{M}{3M_\odot} \right) \times \left( \frac{v}{0.3c} \right)^2
\]
\[
= 10^{-20} \times \left( \frac{6 \text{ Gpc}}{r} \right) \times \left( \frac{M}{10^6 M_\odot} \right) \times \left( \frac{v}{0.1c} \right)^2.
\] (9)

The first set of numbers characterizes stellar mass sources that are targets for ground-based high-frequency detectors, discussed in Sec. 2.1; the second characterizes massive black holes that are targets of space-based low-frequency detectors discussed in Sec. 2.2 and (at somewhat higher \( M \), lower \( v \), and smaller \( r \)) of pulsar timing arrays discussed in Sec. 2.3.

The numbers for \( h \) are tiny. Measuring timing oscillations at this level of precision might seem crazy. However, there is no issue of principle that prevents us from measuring effects at this level; the real challenge is to ensure that noise does not obscure the signal we hope to measure. Recall that a gravitational wave acts as a tidal force. The tide per unit mass for a GW of amplitude \( h \) and frequency \( \omega \) is \( R \approx \omega^2 h \). Considering a light source and sensor separated by distance \( L \), this means that we must control against stray forces on our test mass \( m \) of magnitude
\[
F \approx mL\omega^2 h = 6 \text{ piconewtons} \left( \frac{m}{40 \text{ kg}} \right) \left( \frac{f}{100 \text{ Hz}} \right)^2 \left( \frac{L}{4000 \text{ m}} \right) \left( \frac{h}{10^{-22}} \right).
\] (10)

(These fiducial parameters correspond to the LIGO observatories.) Six piconewtons is small, but it is well within our reach to isolate against forces of this magnitude — this is roughly the weight of a single animal cell. Though challenging, measuring a GW of \( h \sim 10^{-22} \) is within our grasp.

In the remainder of this article, we discuss some of the science of GWs. We break up our discussion by frequency band. We begin with the high frequency band, with wave frequencies ranging from Hz to kHz, which are targeted by ground-based interferometers; then move to low frequency, waves with periods of minutes to hours, which are targets of space-based interferometers; then very low frequency, waves with periods of order months to years, which are targets of pulsar timing arrays; and finally conclude with ultra low frequency, with wavelengths comparable to the size of the universe.

2. The spectrum of gravitational waves

2.1. High frequency

The high-frequency band of roughly 1 – 1000 Hz is targeted by ground-based laser interferometers. The lower end of this band is set by gravitational coupling to local seismic disturbances,
which can never be isolated against \[5\]; the upper end is set by the fact that 1 kHz is roughly the highest frequency that one expects from astrophysical strong GW sources. In laser interferometry, the laser’s very stable frequency serves as the clock for the measurement procedure sketched in Sec. 7. GWs are detected by their action on light propagating between widely separated (hundreds to thousands of meters) test masses.

Several facilities around the world are involved in the search for GWs. Some of these facilities are presently offline as they undergo upgrades to “advanced” sensitivity, but will begin active GW searches again in about two years. There is very close collaboration among the facilities’ research groups; combining data from multiple observatories greatly increases the ability to discriminate against noise and to insure detection. The most sensitive instruments in the worldwide network are associated with the Laser Interferometer Gravitational-wave Observatory, or LIGO. LIGO has a pair of four kilometer, L-shaped interferometers located in Hanford, Washington and Livingston, Louisiana. Closely associated with LIGO is GEO600, a 600 meter interferometer near Hannover, Germany. Because of its shorter arms, GEO cannot achieve the same sensitivity as the LIGO detectors. However, it has been used as a testbed for advanced interferometry techniques, which has allowed it to maintain its role as an important part of the worldwide detector network. Completing the present network is Virgo, a three kilometer interferometer located in Pisa, Italy, and operated by a French-Italian collaboration. Its sensitivity is fairly close to that of the LIGO instruments. Discussion of recent performance and upgrade plans for these three instruments can be found here [6].

A source of mass \(M\) and size \(R\) has a natural GW frequency of \(f \sim (1/2\pi) \sqrt{GM/R^3}\). A compact source has size \(R \sim\) several \(\times GM/c^2\). For such sources, the natural GW frequency is in the high-frequency band if \(M \sim 1 \sim 100 M_\odot\). For this reason, the high-frequency band largely targets objects like neutron stars and black holes. One of the most important sources in this band is the coalescence of binary neutron star systems — essentially, the last several minutes of systems like the Hulse-Taylor binary pulsar. Binaries containing black holes may also be important sources, though our poorer understanding of the formation of compact binaries with black holes make their rates substantially less certain.

As mentioned above, the LIGO and Virgo instruments are presently undergoing an upgrade to “advanced” sensitivity, which will give them a reach to binary neutron star inspiral of about 200 Mpc. This is far enough that astrophysical models suggest they should measure multiple coalescence events per year [7]. The rate for events involving black holes could plausibly be even higher: the signal from black hole binaries is stronger, greatly increasing the observable distance (and hence sensitive volume) [8]. The LIGO instruments are expected to begin observations at the first stage of advanced sensitivity in 2016 (see discussion of detector commissioning timetables and associated references in Ref. [9]), and should reach their final advanced design by 2018. Virgo is expected to follow LIGO by about two years.

2.2. Low frequency

The low frequency band extends from as low as \(10^{-5}\) Hz up to about 1 Hz, and is targeted using laser interferometry between spacecraft. This band is particularly source rich. Low frequency GW detectors are expected to measure signals from dozens of coalescing massive binary black holes [10] (similar to the binaries targeted by pulsar timing arrays, though at lower masses and at the tail end of the GW-driven inspiral); from dozens to possibly hundreds of stellar mass compact objects captured onto strong-field orbits of \(\sim 10^6 M_\odot\) black holes [11]; and from millions of close binary star and binary white-dwarf systems in our galaxy [12]. There may even be strong signals from processes related to phase transitions in the early universe: if the electroweak transition
occurring at temperatures of a few TeV is first order (as some scenarios for baryogenesis suggest it could be \[13\]), then we expect a stochastic background signal peaked at \(f \sim 1\) mHz(\(T/\text{TeV}\)) from collisions of domain walls associated with the transition.

The promise of this band has been known for quite some time, and has motivated several proposed missions to measure GWs at these frequencies. From the late 1990s until early 2011, the focus was LISA, the Laser Interferometer Space Antenna. LISA was proposed as a joint ESA-NASA mission, consisting of a three-spacecraft constellation orbiting the sun in an equilateral triangle with sides of \(5 \times 10^6\) km. Each spacecraft was to be placed into an orbit such that the constellation orbited the sun once per year, lagging the Earth by \(20^\circ\), and inclined \(60^\circ\) with respect to the ecliptic. By measuring the separation between drag-free proof masses in the spacecraft using phase-locked laser transponders with picometer accuracy, LISA would have achieved sufficient sensitivity to measure a rich spectrum of sources in this band over a multiyear mission lifetime. See Ref. \[14\] and references therein for detailed discussion.

Sadly for those of us in the United States, funding constraints have forced NASA to withdraw from this mission. The European LISA partners have forged ahead with plans for eLISA (“evolved LISA”, \[15\]). The European Space Agency has selected “The Gravitational Universe” as the science theme for their L3 launch opportunity, which is currently scheduled for 2034; eLISA is the leading mission concept under development to implement this theme. The design of eLISA can be expected to evolve in the next decade or so, but the present design envisions a somewhat smaller LISA-like constellation (\(10^6\) km arms), with most likely a shorter mission lifetime. This design should achieve an impressive fraction of the original LISA source science \[15\]. Within the US, NASA’s Physics of the Cosmos Program Advisory Group (PhysPAG \[16\]) formed a study group \[17\] to evaluate what options might be possible should budgets allow NASA to rejoin a space-based GW mission (perhaps after the launch of the James Webb Space Telescope). Options being considered range from junior partner with ESA in eLISA to the development of NASA-only mission similar to eLISA (for example, “SGO-Mid” \[18\], the middle range of a suite of Space-based Gravitational-wave Observatories that were examined in a study of possible GW missions).

2.3. Very low frequency

Very low frequency GWs are targeted by timing of pulsars. This technique uses the fact that millisecond pulsars are very precise clocks; indeed, the stability of some pulsars rivals laboratory atomic clocks. Using a well-characterized millisecond pulsar as the light source and a radio telescope on the Earth as the sensor, this technique implements Bondi’s idea for measuring gravitational waves in the band from roughly \(10^{-9} \sim 10^{-7}\) Hz. These boundaries of this band are set by practical considerations: One must integrate a pulsar’s signal for a few months (i.e., a time \(\sim 10^7\) seconds) in order for a GW signal to stand above the expected noise level; and data on pulsars that are best suited to this analysis only goes back a few decades (\(\sim 10^9\) seconds).

In this frequency band, the two most plausible sources are the coalescence of massive binary black holes, and a high-frequency tail of the primordial GWs described in Sec. 2.4. We will defer discussion of this tail of primordial GWs to Sec. 2.3 and briefly describe here GWs from binaries containing massive black holes. Such binaries are formed by the merger of galaxies which themselves have massive black holes at their cores. Population synthesis estimates based on models of structure formation and galaxy growth suggest there should be a substantial population of such binaries whose members are black holes of \(10^6 \sim 10^8 M_\odot\). The GWs produced by these binaries combine to form a stochastic background in the very low frequency band \[21\]. This background is targeted by pulsar timing observations.
In the past several years, the promise of measuring this background has motivated the formation of three collaborations to precisely time a large number of pulsars to measure this background: NANOGrav, the North American Nanohertz Observatory for Gravitational Waves \[22\]; EPTA, the European Pulsar Timing Array \[23\]; and PPTA, the Parkes Pulsar Timing Array \[24\]. These three collaborations together form IPTA, the International Pulsar Timing Array. They are presently timing about 40 pulsars, and have set upper limits on a background of GWs in the nanohertz frequency band \[25\]. As they find additional pulsars that are “good timers” and build a longer baseline of timing data, these limits will grow stronger, and either begin cutting into predictions from galaxy formation and growth models (which will begin to limit the space of possible binary formation models \[25\]), or produce a detection in this band.

2.4. Ultra low frequency

The ultra low frequency GW band, $10^{-13} \text{ Hz} \lesssim f \lesssim 10^{-18} \text{ Hz}$, is best described using wavelength: it consists of GWs with $c/H_0 \gtrsim \lambda \gtrsim 10^{-5} c/H_0$. In other words, these are waves that vary on lengthscales comparable to the size of our universe. To make strong GWs with quadrupole dynamics on these scales would require relativistic masses that stretch across much of the sky. Such masses would upset the observed homogeneity of the universe on these scales. Some mechanism other than quadrupole dynamics must be invoked to describe these GWs.

Such a mechanism is provided by cosmic inflation \[26\], the hypothesized epoch of false-vacuum-driven expansion when the universe repeatedly doubled in size, with a doubling time of $\sim 10^{-37}$ seconds. For an intuitive picture of how inflation does this, consider the following argument due to Allen \[19\]. Consider a ground state quantum simple harmonic oscillator in 1-D, with potential $V = \frac{m}{2} x^2 \omega_i^2$. This system’s wavefunction is

$$\psi \equiv \phi_0(x) = \left(\frac{m\omega_i}{\pi \hbar}\right)^{1/4} \exp\left(-\frac{m\omega_i x^2}{2\hbar}\right). \quad (11)$$

Now imagine that the potential very suddenly changes to $V = \frac{m}{2} x^2 \omega_f^2$. The change is so rapid that the wavefunction cannot adiabatically evolve with the potential; indeed, to first approximation, the wavefunction is left unchanged. However, it is no longer a ground state, but is instead a highly excited state of the final potential. To see this, we write

$$\psi = \sum_{n=0}^{\infty} c_n \psi_n^f(x), \quad (12)$$

where $\psi_n^f = N_n^f H_n(x \sqrt{m \omega_f/\hbar}) \exp(-m \omega_f x^2/2\hbar)$ are basis functions corresponding to states of the final potential; $H_n$ is a Hermite polynomial. A straightforward exercise yields the expansion coefficients $c_n$, from which we deduce the energy of the final state to be

$$E = \hbar \omega_f \left(\frac{1}{2} + \frac{(\omega_f - \omega_i)^2}{4\omega_i \omega_f}\right) \approx \hbar \omega_f \left(\frac{1}{4} \frac{\omega_i}{\omega_f}\right). \quad (13)$$

(We use $\omega_i \gg \omega_f$ in the last step.) The change in the potential created $N = \omega_i/4\omega_f$ quanta.

With this cartoon-level sketch in mind, consider now cosmic inflation. Prior to inflation, the universe is in a vacuum state, filled with the ground state of various fields, including gravity. Inflation acts like the suddenly changing potential, sharply reducing the frequencies associated with modes of the field, doing so rapidly enough that the evolution is non-adiabatic. GWs in
particular are created by this process; this is the only known mechanism for producing GWs with wavelengths near the Hubble scale while maintaining the homogeneity and isotropy of the universe. For more detailed discussion that goes beyond this heuristic picture, see Refs. [19, 20].

Following inflation, the GWs that are produced by this process propagate through the universe. Because of gravity’s weakness, they barely interact with matter as they propagate, just stretching and squeezing the primordial plasma in the young expanding universe. In particular, the GWs stretch and squeeze the plasma at the moment of recombination, when the plasma has cooled enough that atoms can form, and photons begin to free stream, forming the cosmic microwave background (CMB). This stretching and squeezing creates a quadrupolar temperature anisotropy in the plasma at recombination, which causes the CMB to be linearly polarized [27]. The GWs thus leave an imprint on the cosmic microwave background.

Other processes polarize the CMB as well. In particular, the density inhomogeneities primarily responsible for the famous temperature fluctuations in the CMB also cause quadrupolar anisotropies that lead to linear polarization. One can however detangle these two sources of polarization in a model-independent fashion. Polarization is a vector, and can be written as the gradient of a scalar potential plus the curl of a vector potential. The contributions from the gradient of the scalar potential are known as “E modes,” and those from the curl of the vector potential as “B modes.” Because density perturbations have no handedness associated with them, they can only create E modes. GWs can have a handedness, and so they can source both E and B modes. The B modes are thus a unique and powerful signature of primordial GWs. Since inflation is the only mechanism we know of to create GWs with wavelengths close to the Hubble length, their detection is considered to be a “smoking gun” for cosmic inflation.

Prior to 17 March 2014, the standard lore was that these GWs were in all likelihood so weak that we were quite some time away from measurement of these waves. Bounds inferred from the temperature spectrum by the WMAP and Planck satellites [28, 29] were pointing to relatively small levels of primordial GWs; also, foreground effects, which can transform an E-mode signal into a B-mode [30], were thought to be potentially quite daunting. It was thus quite stunning when the BICEP2 collaboration announced a 7-σ detection of B-modes from their telescope at the South Pole [31]. This result needs to be confirmed by other experiments, and it must be understood why they are (apparently) in discord with previous upper limits. Their results suggest a GW strength high enough that confirmation should be likely fairly quickly.

As we write this article, the BICEP2 announcement is only a month old. We can expect a lot of work in this field, with (hopefully) confirmation very soon, and future work allowing us to begin probing the nature of inflation directly. We conclude this section by noting that, if confirmed, the BICEP2 result represents the first time that the influence of GWs on matter (other than the waves’ own source) has been measured. This will be the first of many examples of GWs being exploited for astronomy and cosmology.

3. Cosmology with gravitational-wave measurements

When the author was asked to speak at the 2013 TAUP meeting, the invitation requested a review of GWs and cosmology. Any review of this subject, prior to the direct detection of gravitational waves, was bound to contain a lot of speculation. But that has all changed, most dramatically, with the BICEP2 announcement of 17 March 2014.

When I originally presented this material at the September 2013 TAUP Conference, I gave the standard line that these GWs would likely require years of study to understand foregrounds and other systematic effects before any discovery. Fortunately, my tendency to procrastinate meant that I didn’t write up this article until well after the BICEP2 announcement. This has given me a chance to wipe a little bit of egg off my face.
GWs, necessarily must be speculative to some degree. The reader should take the discussion here to indicate what cosmological applications of GW physics have been seriously thought about to date. It will be interesting (and possibly amusing) to compare this discussion with the applications that actually develop once GW measurement becomes routine.

3.1. Standard sirens

Chief among the sources across several GW bands are the inspiral and merger of compact binary sources. A binary is a nearly perfect quadrupole radiator and, unless general relativity fails in the deep strong field, its waves have a form that depends only on physical parameters of the system. The waveform depends most strongly on the source binary’s masses and spins, the angles which determine its position on the sky and orientation with respect to the line of sight, and the distance to the source. Schematically, a measured binary waveform takes the form

\[ h_{\text{meas}} = \frac{G(1 + z)M/c^2}{D_L(z)} [\pi(1 + z)M f(t)]^{2/3} F(\text{angles}) \cos \Phi(m_1, m_2; S_1, S_2; t). \]  

(14)

The binary’s masses and spins strongly affect the waveform’s phase evolution \( \Phi \) (note that \( f = (1/2\pi) d\Phi/dt \)). Because data analysis is based on phase coherently matching data to a model, \( \Phi \) typically will be measured to within a fraction of a radian. The masses and spins can thus be determined to good accuracy (where details of “good” depend on the measurement’s signal-to-noise ratio, and how well certain near degeneracies between parameters are broken; see [32] for more detailed discussion).

The mass parameter \( M \equiv (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5} \), known as the “chirp mass,” is determined extremely well. The wave’s amplitude effectively depends only on the angles which determine the binary’s orientation and sky position, and on the binary’s luminosity distance \( D_L(z) \). If there are enough GW detectors to measure both GW polarizations, then inclination angles can be determined. If it is likewise possible to determine sky position, then all of the important source angles are determined. In this case, measuring the binary’s waveform directly determines its luminosity distance \( D_L \). Binary inspiral thus has the potential to act as a standard siren\(^2\) — a precisely calibrated source whose measured characteristics encode distance to the source [33, 34].

In the high-frequency band, standard sirens are likely to be coalescing binaries, measured out to a distance of a few hundred Mpc. These events may be accompanied by an “electromagnetic” counterpart [35], which opens the possibility that they could be measured simultaneously in GWs and with telescopes in various wavebands [9]. In the low-frequency band, standard sirens are likely to be merging black holes out to a redshift \( z \sim 1 \) — perhaps even further. All of these measurements promise a new way of pinning down cosmic expansion [33, 36] in a manner which would have total different systematics from other techniques.

3.2. Tracing massive black hole growth

It appears that the galaxies which populate our universe grew in a hierarchical manner, through the repeated merger of smaller galaxies or protogalaxies. At some point in this process, black holes formed in the cores of at least some of these structures. When these galaxies or protogalaxies merge, the black holes will eventually come close enough to one another to bind into a tight

\(^2\)In most astronomical applications, this would be called a standard candle. However, in many respects, GWs can be regarded as sound-like, and the use of “siren” rather than “candle” has been adopted to reflect this.
binary that evolves through GW emission (although multiple evolutionary steps are needed to reach the point that GW emission is important \[37\]). The GW signal from such black holes (with masses from \(10^5 - 10^7 \, M_\odot\)) can be measured by space interferometers like LISA and eLISA to high redshift. In many cases, enough signal will be measured (many months or even a few years of inspiral, through to final merger into a single black hole) that the system’s parameters can determine the rest frame masses and spins, as well as the source redshift \[38\].

Black holes are completely determined by their masses and spins \[39\]. If we determine these two parameters, we have determined everything that can be known about them. How a black hole’s mass and spin evolves is quite sensitive to the details of how the black hole gains mass: accretion tends to spin up black holes, and mergers tend to spin them down (with significant variation depending on the detailed mode in which accretion is presumed to operate). Precision data on merging black holes’ masses and spins over a range of redshifts will provide a tremendous amount of information clarifying how black holes formed and grew from very early cosmic epochs \[10\].

As discussed in Sec. 2.2, observations of these merging black holes in the low frequency band are quite some time in the future. Fortunately, very low frequency observations with pulsar timing arrays are likely to begin telling us about a related population of merging black holes relatively soon: the prime source for these arrays are merging massive black holes which likewise form from the merger of galaxies \[21\]. These black hole binaries differ in several important ways from those targeted by space interferometry: they are at rather higher masses than the targets of interferometers (\(\sim 10^8 \, M_\odot\) rather than \(\sim 10^6 \, M_\odot\); they typically come from much lower redshift; and they involve binaries that are millions of years away from their final merger. However, they are similar in that the measurement of these waves directly probes a dynamical consequence of galaxy assembly and evolution. Recent papers make it clear that there is much that can be learned by a discovery of GWs in this band (e.g., \[40, 41\]).

3.3. Echoes from the early universe

Finally, there is much that can be learned from GWs produced in the early universe. We have already described the process by which inflation produces GWs, and are eagerly waiting for confirmation of the BICEP2 results announced in Ref. \[31\]. If confirmed, it will soon be possible to measure this spectrum at different scales, making it possible to begin probing the detailed physics of the inflationary potential. It will then be possible to begin phenomenology of processes at the roughly \(10^{15} \, \text{GeV}\) scale associated with inflation.

It is worth noting here that inflation does not just produce GWs near the Hubble scale, but yields a very broad-band spectrum of fluctuations. A very simple estimate predicts a flat spectrum from about \(10^{-15} \, \text{Hz}\) to well-above the high frequency band. A more careful analysis shows that the spectrum actually rolls off at high frequencies, with a value that depends on \(n_T\), the spectral index of tensor modes \[42\]. For the purposes of this article, this high-frequency tail produces waves that are well below the projected sensitivity of any measurement that is foreseeable in the next decade or two. If the amplitude of GWs found by BICEP2 is confirmed, then there will be a very strong case to begin developing experiments or missions to measure this background (for example, BBO \[43\] or DECIGO \[44\]).

Finally, we reiterate that although inflation is guaranteed to produce GWs, there are other early universe processes that could produce such a signal. Perhaps the most interesting possibility is

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3This frequency is related to the transition from the post-inflationary radiation-dominated universe to a matter-dominated universe \[19, 20\].
that of GWs from a first-order electroweak phase transition \cite{13}, discussed in Sec. 2.3. Such a signal would require space interferometry, but success in other parts of the GW spectrum will strengthen the case for such a mission.

4. Outlook

For the past two or so decades, GW has been described as a field of great promise. It is now on the threshold of delivering on that promise. If the BICEP2 results are soon confirmed, the first delivery has in fact already arrived. With advanced ground-based detectors soon to begin operations, and with pulsar timing arrays continuing to advance in capability, we can expect to begin using information from three of the four major GW frequency bands in the next several years. The fourth band will probably take somewhat longer (anything involving space missions involves a long lead time), but solid detections in the other bands will build enthusiasm for probing the rich low-frequency band’s data stream.

The decade of gravitational-wave discovery has begun.

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