Sphaleron in the dilatonic electroweak theory

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ABSTRACT

A numerical study of static, spherically symmetric sphaleron solutions in the standard model coupled to the dilaton field is presented. We show that sphaleron is surrounded by strong dilaton cloud which vanishes inside the sphaleron.

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1 INTRODUCTION

In this paper the electroweak theory is extended by the inclusion of dilatonic fields. Such fields appear in a natural way in Kaluza-Klein theories [1], superstring inspired theories [2, 3] and in theories based on the noncommutative geometry approach [4].

As previous studies have already shown the inclusion of a dilaton in a pure Yang-Mills theory has consequences already at the classical level. In particular the dilaton Yang-Mills theories possess ‘particle - like’ solutions with finite energy which are absent in pure Yang - Mills case. On the other hand, the sphaleron was introduced by Klinkhamer and Manton [5] to describe a static electroweak gauge field configuration that constitutes a saddle point between two vacua differing by non trivial topology (the hedgehog topology). Analogous equations have recently been obtained for the t’Hooft-Polyakov monopole model coupled to dilatonic field [6]. There is also growing interest [7] in baryon number violation within the standard model induced by sphalerons. The rate of baryon number violating processes depends on the energy of sphaleron [8].

The aim of this paper is to examine the properties of the sphaleron solution in the presence of dilatonic field in the electroweak theory. We demonstrate the existence of spherically symmetric dilatonic cloud surrounding the sphaleron. We also discuss very interesting properties of the Higgs field in the dilaton electroweak theory.
2 THE DILATONIC ELECTROWEAK THEORY

One of the interesting features of the standard model is the scale invariance of highly symmetric phase. Quantum effects anomalously cause its break up and produce nonvanishing cosmological constant. The classical scale invariance offers a link between the standard model and gravity, which is successfully implemented in the Jordan-Brans-Dicke theory [9] of scalar-tensor theory of gravity.

In this paper we consider the Glashow-Weinberg-Salam dilatonic model with $SU_L(2) \times U_Y(1)$ symmetry

\[ \mathcal{L} = \frac{-1}{4} e^{2\varphi(x)/f_0} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4} e^{2\varphi(x)/f_0} B_{\mu\nu} B^{\mu\nu} + \frac{1}{4} \partial_{\mu} \varphi \partial^{\mu} \varphi + (D_{\mu} H)^{+} D_{\mu} H - U(H) e^{-2\varphi(x)/f_0} \quad (1) \]

with the $SU_L(2)$ field strength tensor $F_{\mu\nu}^a = \partial_{\mu} W_{\nu}^a - \partial_{\nu} W_{\mu}^a + g \epsilon_{abc} W_{\mu}^b W_{\nu}^c$ and the $U_Y(1)$ field tensor $B_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}$. The covariant derivative is given by $D_{\mu} = \partial_{\mu} - \frac{1}{2} ig W_{\mu}^a \sigma^a - \frac{1}{2} g' Y B_{\mu}$, where $B_{\mu}$ and $W_{\mu} = \frac{1}{2} W_{\mu}^a \sigma^a$ are local gauge fields associated with $U_Y(1)$ and $SU_L(2)$ symmetry groups respectively. $Y$ denotes the hypercharge. The gauge group is a simple product of $U_Y(1)$ and $SU_L(2)$ hence we have two gauge couplings $g$ and $g'$. The generators of gauge groups are: a unit matrix for $U_Y(1)$ and Pauli matrices for $SU_L(2)$. In the simplest version of the standard model a doublet of Higgs fields is introduced...
\[ H = \left( \begin{array}{c} H^+ \\ H^0 \end{array} \right), \text{ with the Higgs potential} \]

\[ U(H) = \lambda \left( H^+ H - \frac{1}{2} v_0^2 \right)^2 \]  

(2)

The \( f_0 \) and \( v_0 \) parameters in the Lagrangian function (1) determine the dilaton scale \( f_0 = 10^7 \text{GeV} \) [10], and the electroweak symmetry breaking scale \( v_0 = 250 \text{GeV} \). The form of the potential (2) leads to vacuum degeneracy, nonvanishing vacuum expectation value of the Higgs field and consequently to fermion and boson masses. In this process of spontaneous symmetry breaking the Higgs field acquires nonzero mass.

The Euler-Lagrange equations for the lagrangian (1) are scale-invariant:

\[ x^\mu \rightarrow x'^\mu = e^{\frac{x}{f_0}} x^\mu, \]

\[ \varphi \rightarrow \varphi' = \varphi + u, \]

\[ H \rightarrow H' = H, \]

\[ W_{a\mu} \rightarrow W_{a\mu}' = W_{a\mu}, \]

\[ B_{\mu} \rightarrow B_{\mu}' = B_{\mu}. \]

These transformations change the Lagrange function in a following way

\[ L \rightarrow L' = e^{-\frac{2x}{f_0}} L \]  

(3)

3 THE DILATONIC SPHALERON

Let us now consider the sphaleron type solution in the electroweak theory with dilatons. The sphaleron may be interpreted as inhomogeneous bosons
condensate $\varphi(x), W^a_\mu(x)$. Let us assume for simplicity that $g' = 0$. (In the sphaleron theory was also considered also for $g' \neq 0$.) Firstly, we make the following anzatz for the sphaleron Higgs field

$$H = \frac{1}{\sqrt{2}} v_0 U(x) h(r) \left( \begin{array}{c} 0 \\ 1 \end{array} \right)$$

(4)

where $U(x) = i \sum \sigma^a n^a$ and $n^a = \frac{r^a}{r}$ describe the hedgehog structure. This produces a nontrivial topological charge of the sphaleron. The topological charge is equal to the Chern-Simons number. Such a hedgehog structure determines the asymptotical shape of the sphaleron with gauge fields different from zero

$$W^a_i = \epsilon_{aij} n^j \frac{1 - f(r)}{gr}, \varphi(x) = f_0 s(r).$$

(5)

It is convenient to introduce the dimensionless variable $x$ defined as $x = M_W r = r/r_W$, where $M_W^2 = \frac{1}{g^2} v_0^2 \sim 80 \text{GeV}$, $r_W = \frac{1}{M_W} \sim 10^{-18} \text{cm}$. Spherical symmetry is assumed for the dilaton field $s(r)$, as well as for the Higgs field $h(r)$ and the gauge field $f(r)$, leading to the following expression for the total energy

$$E = \frac{4\pi M_W}{g^2} \int \rho_0(x) x^2 dx,$$

(6)

where the energy density is:

$$\rho_0(x) = 2h'(x)^2 + \frac{1}{\alpha} s'(x)^2 + \frac{1}{x^2} e^{2s(x)} \left\{ f'(x)^2 + \frac{1}{2x^2} (f(x) - 1)^2 (f(x) - 3)^2 \right\} + \varepsilon (h(x)^2 - 1)^2 e^{-2s(x)} + \frac{1}{x^2} (f(x) - 3)^2 h(x)^2.$$ 

(7)

$M_H^2 = 2\lambda v_0^2$ determines the Higgs mass; $\alpha = \frac{M_H^2}{f_0 g^2}$, and $\varepsilon = \frac{M_H^2}{2M_W}$ are dimensionless parameters which completely determine the sphaleron system.
The resulting Euler-Lagrange equations are following, where the $s(x)$ function describes the gauge field inside the sphaleron,

$$f''(x) + 2f'(x)s'(x) + (3 - f(x))h(x)^2 e^{-2s(x)} - \frac{1}{x^2}(f(x) - 1)(f(x) - 2)(f(x) - 3) = 0. \quad (8)$$

$h(x)$ function describes the Higgs field in our theory, it satisfies the following equation:

$$h''(x) + \frac{2}{x}h'(x) + \epsilon e^{-2s(x)}(1 - h^2(x))h(x) - \frac{1}{2x^2}(3 - f(x))^2 h(x) = 0. \quad (9)$$

The $s(x)$ function describing the dependence of a dilaton field on $x$ in the extended electroweak theory obeys the equation:

$$s''(x) + \frac{2}{x}s'(x) + \alpha e^{2s(x)} \left\{ -\frac{2}{x^2}f'(x)^2 - \frac{1}{x^4}(f(x) - 1)^2(f(x) - 3)^2 \right\} + 2\epsilon\alpha e^{-2s(x)}(1 - h^2(x))^2 = 0. \quad (10)$$

with a dimensionless constant $\alpha \sim \left(\frac{M_W}{g_f^0}\right)^2 \sim 10^{-9}$. This means that the dilaton field is essentially a free field. The simplest solutions are the global ones corresponding to the vacuum with broken symmetry in the standard model. It is obvious that far from the center of the sphaleron our solutions should describe the normal broken phase which is very well known from the standard model. From the known asymptotic solutions for $x \to 0$:

$$f(x) = 1 + 2t^2 x^2 + O(x^3), \quad (11)$$

$$h(x) = ux + O(x^3), \quad (12)$$

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\[ s(x) = s_0 - \frac{1}{2} s_0 v^2 x^2 + O(x^3), \]  

(13)

and for \( x \to \infty \):

\[ f(x) = 3 - f_\infty e^{-x}, \]  

(14)

\[ h(x) = 1 - \frac{h_\infty}{x} e^{-\sqrt{2} x}, \]  

(15)

\[ s(x) = s_\infty - \frac{d_\infty}{x}, \]  

(16)

we are able to construct a four-parameter family of trial solutions:

\[ f(x) = 1 + 2 \tanh^2(t x), \]  

(17)

\[ h(x) = \tanh(u x), \]  

(18)

\[ s(x) = s_0 e^{-\frac{1}{2} v^2 x^2}, \]  

(19)

where \( t, u, v, s_0 \) are parameters to be determined by the variational procedure. Parameters \( u, t, v \) may be determined in this way to satisfy boundary conditions (12), (13), (14). For parameter \( s_0 \), by minimizing the sphaleron mass (6) with respect to \( s_0 \) we may choose the trial sphaleron configuration with the minimal energy (mass). In so doing, the trial solutions have a proper behaviour for \( x \to 0 \) and describes the sphaleron configuration with minimal energy. Then this trial function can serve as an initial solutions for solving the coupled systems of differential equation (9), (10), (11) by the use of *Mathematica* and *Scientific Workplace* with Maple library.

Specifically, we substituted trial functions (18), (19), (20) into the differential equations (9), (10), (11), then performed Taylor series expansion for
$x \to 0$ and solved system of algebraic equation for $u, t$ and $v$,

$$u = \sqrt{\frac{3}{10}} \sqrt{\varepsilon + \frac{6}{5} (3 + \sqrt{9 + 5\varepsilon}) e^{-s_0}}$$  \hspace{1cm} (20)$$

$$t = \sqrt{\frac{3}{10}} e^{-s_0} \sqrt{3 + \sqrt{9 + 5\varepsilon}}$$  \hspace{1cm} (21)$$

$$v = \sqrt{\frac{2}{3}} e^{-s_0} \sqrt{\frac{\alpha}{s_0} (\varepsilon - \frac{54}{25} (3 + \sqrt{9 + 5\varepsilon})}$$ \hspace{1cm} (22)$$

The numerical solutions are close to the trial minimal functions (18), (19), (20), which are presented on Figs. 2-4. The relevant values of the parameters are those which minimize the energy, Fig.1. For example, with the standard values of $M_W = 80.6 GeV$, $M_Z = 91.16 GeV$, $M_H = 350 GeV$ we found the numeric solutions $t, u, k$, as functions depending on the initial conditions of the dilaton field $s(0) = a$ in the center of the sphaleron. Our solutions describe both the behavior of Higgs field and gauge field inside the sphaleron and the shape of the dilaton cloud surrounding the sphaleron. Such a cloud is large and extends far outside the sphaleron \[.]\]

The sphalerons might be created during the first order phase transition in the expanding universe as inhomogeneous solutions of the equations of motion. The bubbles left after the phase transition probably took place in the early universe, break the CP and C symmetry on their walls and can cause the breaking of baryonic symmetry. Detailed consideration of this problem will be the subject of a separate paper.
4 CONCLUSIONS

The aim of this paper was to present a numerical study of the classical sphaleron solutions of the SU(2) Yang-Mills theory coupled to the dilaton fields.

Dilaton field appears naturally in low energy sector of the effective field theories derived from superstring theories or noncommutative field theory approach. Numerical solutions suggest that sphaleron possess an ‘onionlike’ structure. In the small inner core the scalar field is decreasing with global gauge symmetry restoration $SU(2) \times U(1)$. In the middle layer the gauge field undergoes sudden change. It is very interesting that sphaleron coupled to dilaton field has also an outer shell, where dilaton field changes drastically. The spherically symmetric dilaton solutions coupled to the gauge field or gravity are interesting in their own and may moreover influence the monopole catalysis of baryogenesis induced by sphaleron.

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FIGURE CAPTIONS

Figure 1. The sphaleron energy $E_o$ as a function of the variarional parameter $s_o$.

Figure 2. The dependence of the Higgs field $h(x)$ on $x$.

Figure 3. The dependence of the gauge field $f(x)$ on $x$.

Figure 4. The dependence of the dilaton field $s(x)$ on $x$. 

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