Star formation efficiency in turbulent clouds

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ABSTRACT

Here we present a simple, but nevertheless, instructive model for the star formation efficiency (SFE) in turbulent molecular clouds. The model is based on the assumption of log-normal density distribution which reflects the turbulent nature of the interstellar medium (ISM). Together with the number count of cloud cores, which follows a Salpeter-like core mass function (CMF), and the minimum mass for the collapse of individual cloud cores, given by the local Jeans mass \( M_J \), we are able to derive the SFE for clouds as a function of their Jeans masses. We find a very generic power-law, \( \text{SFE} \propto N_1^{-2.6} \), where \( N_1 = N_{\text{cloud}}/N_1 \) and a maximum SFE \( \text{SFE}_{\text{max}} \approx 1/3 \) for the Salpeter case. This result is independent of the turbulent Mach number but fairly sensitive to variations of the CMF.

Key words. ISM: clouds – ISM: structure – ISM: kinematics and dynamics – Turbulence

1. Introduction

Molecular clouds, the birthplaces of stars in galaxies, are pervaded by turbulent motions, which to a large extend determine the cloud’s density distribution (see e.g. reviews by Mac Low & Klessen 2004; Ballesteros-Paredes et al. 2007; Dobbs et al. 2013; Padoan et al. 2013, and references herein). The distribution function of those density fluctuations is commonly described by a log-normal distribution (see e.g. Vázquez-Semadeni 1994; Padoan & Nordlund 2002; Federrath et al. 2008). Furthermore, the mass distribution of cores and clumps i.e. the core mass function (CMF), within the molecular cloud seems to follow a power-law distribution, similar to the stellar initial mass function (IMF) (see e.g., Klessen 2004; Ballesteros-Paredes et al. 2007; Dobbs et al. 2013; André et al. 2010; Padoan et al. 2013, and references herein). The distribution function (PDF) of density fluctuations in a turbulent cloud, we start with the canonical form of the probability distribution function (PDF) of density fluctuations in a turbulent cloud, $p(s) = \frac{1}{\sqrt{2\pi} \sigma^2} \exp\left(-\frac{(s-s_0)^2}{2\sigma^2}\right)$ (1) with $s = \ln(\rho/\rho_0)$ and $s_0 = -1/2 \sigma^2$. The variance of this PDF depends on the turbulence $b = \sigma^2 = \ln(1 + b^2 M_J^2)$, where for simplicity we neglect the impact of magnetic fields (see e.g., Vázquez-Semadeni 1994; Padoan & Nordlund 2002; Federrath et al. 2008). The parameter $b$ is related to the type of turbulence (see again Federrath et al. 2008), but does not play a crucial role in our consideration as we will see later on.

Obviously only those cloud cores will collapse and form stars which exceed the Jeans mass $M_J = \left(\frac{c_s^3}{\sqrt{G}}\right) \frac{1}{\sqrt{\rho}} \propto \rho^{-1/2}$ (2) at their mean density $\rho$ and temperature $T \propto c_s^2$ ($c_s$ is the speed of sound and $G$ the gravitational constant).

Using the mass of the cloud that exceeds the density $\rho$ $M(s) = M_{\text{cloud}} \int_s^\infty ds \ p(s)$ (3) \begin{equation}
\frac{M_{\text{cloud}}}{2} \left[ 1 - \text{erf}\left(\frac{s-s_0}{\sqrt{2} \sigma}\right)\right]
\end{equation}
we can calculate the minimum mass $M_{\text{min}}$ of a turbulent cloud which is Jeans unstable, i.e.

$M_{\text{min}} = M(s) = M_J(s)$. (4)

Given the fact that the cloud is fragmented by the same nature of turbulence, the mass at a given density is not located in the entire cloud which is able to collapse by gravitational instability.

2. Model description

We start with the canonical form of the probability distribution function (PDF) of density fluctuations in a turbulent cloud, $p(s) = \frac{1}{\sqrt{2\pi} \sigma^2} \exp\left(-\frac{(s-s_0)^2}{2\sigma^2}\right)$ (1) with $s = \ln(\rho/\rho_0)$ and $s_0 = -1/2 \sigma^2$. The variance of this PDF depends on the turbulence $b = \sigma^2 = \ln(1 + b^2 M_J^2)$, where for simplicity we neglect the impact of magnetic fields (see e.g., Vázquez-Semadeni 1994; Padoan & Nordlund 2002; Federrath et al. 2008). The parameter $b$ is related to the type of turbulence (see again Federrath et al. 2008), but does not play a crucial role in our consideration as we will see later on.

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single cloud cores but rather distributed according to a core mass function (CMF). Interestingly, the CMF has a very similar shape the stellar initial mass function (IMF) and is often assumed to follow a Salpeter distribution, i.e.

$$\text{CMF} \equiv \frac{dN}{d\log M} \propto M^{-\alpha}$$

(5)

with \(\alpha = 1.35\) (Alves et al. 2007). Now the key point is the normalisation of this number distribution which differs from cloud to cloud. The normalisation of the CMF is given by the total mass of the cloud:

$$M_{\text{cloud}} = C \int_{M_{\text{low}}}^{M_{\text{high}}} dM \ M^{-\alpha}.$$  

(6)

It follows that (\(\alpha \neq 1\))

$$M_{\text{cloud}} = C \frac{M^{-\alpha+1}}{\alpha - 1}$$

(7)

assuming that \(M_{\text{cloud}} \gg M_{\text{low}}\). Unfortunately on first sight, this result depends strongly on the mass of the cores, \(M_{\text{low}}\) which still contribute to the (Salpeter) distribution. But fortunately, this mass can easily be determined as we expect that the CMF is largely governed by the impact of self-gravitating cloud cores (e.g., Kainulainen et al. 2011; Kainulainen & Tan 2013; Girichidis et al. 2014). In this case the lowest-mass core which gives rise to the CMF is given by \(M_{\text{min}}\), i.e. the core which is just Jeans unstable. Now the number distribution of the cores within the cloud which masses are larger than \(M\) is given by

$$N(M) = C \int_{M_{\text{thres}}}^{M_{\text{cloud}}} dM \ M^{-\alpha-1}$$

(8)

with \(C = (\alpha - 1)/(M_{\text{cloud}}/M_{\text{min}}) \ M_{\text{min}}^{-\alpha}\) from Eq. (7). Hence,

$$N(M) = \frac{\alpha - 1}{\alpha} \left( \frac{M_{\text{cloud}}}{M_{\text{min}}} \right) \left( \frac{M}{M_{\text{min}}} \right)^{-\alpha} - \left( \frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-\alpha}.$$ 

(9)

Now we can search for the largest (locally connected) cloud core which is found by the condition

$$M_{\text{thres}} : N(M_{\text{thres}}) = 1.$$ 

(10)

This condition tells us that there is only one core with mass \(M_{\text{thres}}\), whereas cores that exceed this mass do not exist in the cloud, i.e. \(N(M) < 1\). Hence, cores with \(M_{\text{core}} > M_{\text{thres}}\) can not contribute to star formation, solely because they are not present. Otherwise, cores that are smaller than \(M_{\text{thres}}\) become increasingly more abundant for decreasing core masses (as long as \(\alpha > -1\)). That means one could, in principle, determine the smallest cores that might contribute to star formation by \(M(s)/M(s) > M_{\text{thres}}(s)\). But this conditions does ignore that high density cores (the ones with the smallest masses) are embedded in larger, more massive cores which are able to form stars (Vázquez-Semadeni 1994). Therefore, this condition does not apply for our consideration of the SFE.

At this point we briefly have to comment on a similar approach discussed by Padoan (1995), (see also Padoan & Nordlund 2002). Here the efficiency to from stars is assumed to be essentially \(M(m) \times N(M)\) (see Eqs. (21) and (24) of Padoan 1995), i.e. by the total mass \(M\) of all clumps with mass \(m\) in the entire cloud times the frequency of those clumps in the cloud. Hence,

$$\text{SFE} \propto N_{V}^{-1/\alpha}$$

(14)

which results in \(\text{SFE} \propto N_{V}^{-0.26}\) for the Salpeter case.

3. Results

Together with Eq. (9) and the condition Eq. (11) the star formation efficiency can be expressed as

$$\text{SFE} \approx \left( \frac{\alpha - 1}{\alpha} \right) \left( \frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-1} \left( \frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-\alpha} \left( \frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-1} \left( \frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-1}$$

$$\approx \left( \frac{\alpha - 1}{\alpha} \right)^{1/\alpha} \left( \frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-1},$$

(13)

where we assumed \(\alpha > 1\) for the approximation. Measuring the cloud mass in terms of its Jeans mass at the mean density we see from Eq. (13) that

$$\text{SFE} \propto N_{J}^{-1-\alpha/\alpha}$$

(14)

This is particular plausible if we assume that the IMF and CMF have a similar origin.
Fig. 2. SFE for our fiducial model, i.e. $M = 1$, $\alpha = 1.35$ (solid line), and for $\alpha = 2$ and $\alpha = 1.1$ (upper and lower dashed line, respectively).

Fig. 3. SFE as a function of the CMF-slope $\alpha$ for different instability parameters. The lines from top to bottom are for $N_J = 1, 10, 100$ and 1000, respectively. Up to $N_J \lesssim 10$ there is a clear decreasing trend of the SFE with decreasing concentration of the cloud. For massive clouds, this trend is reversed up to a minimal value of $\alpha = \alpha_{\text{min}}(N)$ (see text).

Obviously, the definition Eq. (11) is the largest value a cloud could achieve if all the mass of unstable cores are converted instantaneously into stars ignoring all kinds of additional effects like feedback from the stars themself. But this picture incorporates the effect of reduced accretion onto stars by fragmentation of the cloud, i.e. this model quantifies the consequence of Fragmentation Induced Starvation (FIS) (Peters et al. 2010 Girschidis et al. 2012), which can be seen from Fig. 2. The more unstable the cloud, quantified by $N_J$, the less efficient it can form stars because it is more prone to fragmentation with a number of fragments which are not Jeans unstable anymore. Interestingly, in the Salpeter case ($\alpha = 1.35$) the maximal SFE is $\sim 1/3$ for clouds with $M_{\text{cloud}} \approx M_J$. This applies, for instance, to isolated Bok globules like Barnard 68 which might be barely unstable (Alves et al. 2001). Even such low-mass clouds could only convert at most $\sim 1/3$ of their mass into stars, even without any feedback, because they will fragment while they are collapsing.

Also interesting is the fact that the SFE decreases with decreasing concentration of the cloud (decreasing $\alpha$) for less unstable systems ($N_J \lesssim 10$). Again the reason is the fragmentation property of the cloud. Less concentrated clouds are more susceptible to fragmentation than clouds with a high density concentration. This behaviour is intensively studied in Girschidis et al. (2011) and Girschidis et al. (2012) where the collapse of clouds with various density profiles were investigated. Nevertheless, the situations gets a bit more complicated for more unstable clouds ($N_J > 10$) as seen in Fig. 3. Here, less concentration of the CMF helps to increase the SFE up to a certain maximal value depending on $N_J$ and $\alpha$. Hence, for more unstable clouds, the enhanced fragmentation helps to a certain degree to increase the SFE as such fragments are still Jeans unstable and therefore contribute to star formation. This competition between constructive fragmentation and rapid collapse is only efficient up to a minimal concentration, $\alpha_{\text{min}}$ of the cloud (e.g., for $N_J = 1000 \rightarrow \alpha_{\text{min}} \approx 1.2$). For less concentrated clouds, $\alpha \lesssim 1.1$, the SFE becomes essentially independent of $N_J$ and approaches zero in the limiting case $\alpha \rightarrow 1$.

Another interesting aspect which one obtains from the above is the threshold mass, $M_{\text{thres}}$, of the cloud (in the $\alpha = 1.1$ case it becomes almost independent of $N_J$) and $\rho_{\text{thres}}/\rho_0$ is of order unity. This is not too surprising as we only consider globally unstable clouds in the first place. Again, only for $N_J \lesssim 10$ we find a clear trend of $\rho_{\text{thres}}$ with the cloud concentration $\alpha$. Less concentrated clouds need a larger threshold density to produce stars compared to those with a steeper CMF. For more unstable clouds, $N_J > 10$, the competition between fragmentation and collapse does not lead to such a clear trend with the cloud concentration.

Mach number dependence. So far we presented our results for transonic molecular clouds with $M = 1$ (assuming $b = 1$, see also Eq. (11) and below). But it turns out that neither the type of turbulence nor its strength has a large impact on our results. This can already be seen from Eq. (11) which has only a very weak dependence on $M_{\text{thres}}$, i.e. on the quantity which depends on $M$. We tested the Mach number dependence of the SFE number 3 of the more precise number is 36.6%
Fig. 5. The threshold density for star formation for $\alpha = 1.35$ (solid line), $\alpha = 1.1$ (dashed line) and $\alpha = 2$ (dotted line). Similar to the SFE, only for $N_J < \sim 10$ there is a trend of a decreasing threshold with increasing CMF concentration (see also text).

numerically using the basic equations, but could not see any visible difference. Hence, we omit a plot showing SFE as a function of $M$. The weak dependency of SFE on $M$ can be understood as follows: Larger Mach numbers result in wider distributions of the density-PDF and therefore would give rise to a larger density threshold (or smaller $M_{\text{min}}$) for the same Jeans mass. But a wider PDF also reduces the overall instability of the cloud, i.e. reduces $N_J$. Both effects almost compensate each other. But please note, that already the dependency of $M_{\text{min}}$ on $M$ is very weak as can be seen from Fig. 1.

4. Conclusions

Here we presented a simple model for the star formation efficiency in turbulent molecular clouds. The model is based on the assumption of log-normal density distribution which reflects the turbulent nature of the ISM. Similar to previous analytic studies, we use this distribution to estimate the minimum mass which can actually collapse by gravitational instability. Any cores that mass exceeds $M_{\text{min}}$ are also Jeans-unstable, but, according to the density distribution, have a lower mean density and are less frequent than lower-mass cores. The latter statement reflects the observed distribution of core masses. But following the CMF, not all low-density regions exist as connected clumps, and hence are not Jeans-unstable. Combining the density-PDF and the CMF we calculate largest core within the cloud which is still able to collapse. This in turn can be used to infer an upper limit for the SFE. For a given slope of the CMF, we find a very generic power-law, $\text{SFE} \propto N_J^{-(\alpha-1)/\alpha}$ and a maximum $\text{SFE}_{\text{max}} \approx 0.37$ for the Salpeter case. Again, this result is independent of the turbulent Mach number.

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