Numerical modelling of compressible viscous flow in turbine cascades

P Louda¹, K Kozel¹ and J Příhoda²
¹ Dept. of Technical Mathematics, Czech Technical University in Prague, Karlovo nám. 13, CZ-121 35 Praha 2
² Institute of Thermomechanics AS CR, Dolejškova 5, CZ-182 00 Praha 8
E-mail: petr.louda@fs.cvut.cz

Abstract. The work deals with mathematical models of turbulent flow through turbine cascade in 2D and 3D. It is based on the Favre-averaged Navier-Stokes equations with SST or EARSM turbulence models. A two-equation model of transition to turbulence is considered too. The solution is obtained by implicit AUSM finite volume method. The 2D and 3D results are shown flow through the SE1050 cascade including simulation of a range of off-design angles of attack.

1. Introduction
In this work the transonic flow through turbine is investigated numerically. The simulations are carried out in 2D and 3D. The mathematical model is based on the Favre-averaged Navier-Stokes equations with SST (eddy viscosity) turbulence model for intermittency function of Lodefier-Dick. The influence of turbulent transition is modeled by two-equation model In 3D eddy viscosity models are generally unsatisfactory and thus more sophisticated explicit algebraic Reynolds stress model (EARSM) is applied.

In 2D, the behavior of the flow-field with changing the angle of attack is investigated. The comparison with an experiment for large off-design angle indicates good agreement. Further the influence of bypass transition to the turbulence is considered, although it appeared that the results change marginally by transition model in this case. Finally the SE1050 prismatic cascade is simulated in 3D using the EARSM turbulence model. The simulation captures secondary flows in the cascade and in the wake.

2. Mathematical models
The mathematical model of turbulent flow is based on the Favre-averaged Navier-Stokes equations in Cartesian coordinates. The turbulent stress tensor and heat flux are modelled by the SST (Shear Stress Transport) model see Menter [4]. In order to model bypass transition to turbulence, the two-equation model of Lodefier and Dick [3] is used. The eddy viscosity is then multiplied by turbulence weighting factor (intermittency function). The Mayle and Abu-Ghannam, Shaw criteria of transition are employed in the present simulations.

In 3D, the EARSM model proposed by Wallin [7], is used for turbulent closure in the modification of Hellsten [1].
3. Numerical solution

The system of equations is discretized by cell-centered finite volume method with quadrilateral or hexahedral finite volumes in 2D or 3D respectively. The inviscid numerical flux is computed by the AUSMPW+ method [2] using cell center values interpolated linearly with van Leer limiter. The approximation of cell face derivatives needed in diffusive terms uses quadrilateral dual finite volumes constructed over each face of primary volume. For time discretization the backward Euler scheme (implicit) was used. The Jacobi matrices of the linearized system are obtained as derivatives of discrete expressions for fluxes with respect to nodal values from the stencil. The system is solved iteratively by a block relaxation method with direct block tri-diagonal solver on selected family of grid lines.

4. Numerical results

A transonic flow through the SE1050 cascade [6, 5] is considered here at angles of attack which differ from the design angle of attack $\alpha_1 = 19.3^\circ$ in both positive and negative direction by $\Delta \alpha_1$. The outlet Mach number is $M_{2\text{u}} = 0.905$, Reynolds number $Re_{2\text{c}} \approx 1.2 \cdot 10^6$. The model of turbulence is SST. The computational results are shown in terms of the Mach number isolines in Figs. 1 and 2. One can see that the supersonic region remains same for quite a large negative deviations from the design angle, despite large separation on the pressure side. In comparison with the measurement in Fig. 2 the structure of the flow field is predicted quite well especially above the suction side. The end of separation differs from the measurement. The diagram of loss coefficient $\xi$ shown in Fig. 3 confirms relatively low sensitivity to changes of $\alpha_1$.

The simulation of flow through SE1050 cascade taking into account bypass transition model is presented here. The $\gamma$-$\zeta$ model of Lodefier and Dick is used to predict intermittency functions. The turbulence model is SST. The inlet turbulence intensity is estimated by $Tu = 1.7\%$. The Mayle and Abu-Ghannam-Shaw criteria for onset of transition were considered. The isolines of near wall intermittency $\gamma$ are shown in Fig. 4. The free-stream factor $\zeta$ differs from unity only very near the blade surface and is not shown. The transition in both cases starts on the suction side relatively early (close to leading edge) and the flow-field as well all loss coefficient are practically same as in fully turbulent simulation.
Figure 2. SE1050 cascade at deviation from design angle of attack $\Delta \alpha_1 = -67^\circ$. left: computation, right: measurement [5] (blue: subsonic, red: supersonic)

Figure 3. Loss coefficient vs. deviation from design angle of attack, SE1050 cascade

Figure 4. Isolines of intermittency $\gamma$. Left: $M_{2ls} = 0.716$, right: $M_{2ls} = 1.007$
The 3D geometry of the SE1050 cascade is prismatic with the span equal to the chord length (100 mm). The outlet Mach number $M_{2s} = 1.012$, $Re_{2c} \approx 1.2 \cdot 10^6$, inlet turbulence intensity is 1.7\%. The turbulence is modelled by the EARSM model. The finite volume grid has 152 steps in span-wise direction and approx. 1.36 millions of finite volumes in total. The global view of the near wall Mach number is shown in Fig. 5. One can see corner effects and the re-compression zone on the suction side of the blade, typical for the SE1050 geometry.

5. Conclusions
The work presented simulations of 2D and 3D turbulent flow through the SE1050 turbine cascade. In 2D, range of off-design angles of attack has been considered and losses evaluated. The comparison with experimental data (for 1 off-design regime) is satisfactory. The influence of transition model appeared marginal in the simulated cases. The 3D simulation with EARSM turbulence model captures the secondary flow between the blades as well as in the wake.

Acknowledgments
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