It’s all About Consistency: A Study on Memory Composition for Replay-Based Methods in Continual Learning

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Abstract

Continual Learning methods strive to mitigate Catastrophic Forgetting (CF), where knowledge from previously learned tasks is lost when learning a new one. Among those algorithms, some maintain a subset of samples from previous tasks when training. These samples are referred to as a memory. These methods have shown outstanding performance while being conceptually simple and easy to implement. Yet, despite their popularity, little has been done to understand which elements to be included into the memory. Currently, this memory is often filled via random sampling with no guiding principles that may aid in retaining previous knowledge. In this work, we propose a criterion based on the learning consistency of a sample called Consistency AWare Sampling (CAWS). This criterion prioritizes samples that are easier to learn by deep networks. We perform studies on three different memory-based methods: AGEM, GDumb, and Experience Replay, on MNIST, CIFAR-10 and CIFAR-100 datasets. We show that using the most consistent elements yields performance gains when constrained by a compute budget; when under no such constrain, random sampling is a strong baseline. However, using CAWS on Experience Replay yields improved performance over the random baseline. Finally, we show that CAWS achieves similar results to a popular memory selection method while requiring significantly less computational resources.

1 Introduction

Deep Learning models have repeatedly shown themselves to be the state of the art in numerous tasks, including image recognition[1][2], Natural Language Processing (NLP) [3][4] or games previously thought to be intractable to solve, such as Go [5] and Starcraft II [6]. But for all of their strengths, these models lack in versatility: when trained to perform on new tasks they rapidly forget how to solve previous ones. This condition is known as catastrophic forgetting [7][8] and is the main problem tackled by Continual Learning methods.

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A variety of methods have been proposed to approach this problem. Some have focused on allocating parameter usage for each new task [9, 10], others define restrictions on gradients learned [11, 12], while others use metalearning to learn reusable weights for all tasks [13, 14]. Out of all of these, memory-based methods like Experience Replay [15, 16] have consistently exhibited greater performance, with relative simplicity. In these methods, a memory of samples from previous tasks is kept during training of the current task. Each particular method uses this memory as it sees fit to avoid forgetting how to solve previous tasks. But for all the relevance and popularity of memory-based methods, no studies have been conducted on how populating the memory affects the CL methods leveraging it.

Intuitively, one would think that populating the memory with some reasonable criteria should be desirable. However, recent studies [17, 18, 19] show that when populating the memory by focusing solely on sample diversity or class balance, random selection of elements ends up performing nearly or just as well without adding an extra computation. How can we go about finding such criteria?

Recent work has introduced the idea of learning consistency [20] of a sample. This concept seeks to measure how hard is to learn a given sample for a family of models for a certain task. Critically, samples with high learning consistency are learned faster and forgotten more rarely than inconsistent samples. Based on these findings, we propose a criterion for memory population based on prioritizing samples that show greater learning consistency. To quantify it, we use the C-Score [20]. C-Score is a metric that measures the ratio of training iterations a sample is correctly classified or not. This is then calculated for and averaged over a range of model initializations and architectures.

To test our hypothesis, we conduct experiments on three memory-based methods that use the memory in different ways: AGEM [21], GDumb [22] and Experience Replay [16] on a class incremental setting. Our findings include:

- In concordance with recent work [23], we observe that for limited compute scenarios populating the memory with the most consistent samples improves performance.
- When not constrained by a compute budget, selecting items randomly is a strong baseline.
- We introduce Consistency-Aware Sampling (CAWS)- which samples randomly from the most consistent scores. For Experience Replay, we find that CAWS significantly improves performance over the random baseline.
- When using CAWS to add elements to the memory, we achieve equal or better results than MIR without requiring extra computation during training.

The rest of the document is divided as follows: In Section 2 we discussed related works in the area of Continual Learning and Learning Consistency. Section 3 explain the experiments setups, methods, dataset and strategies. Then, in Section 4 we discuss the results when using C-Score on Continual Learning, what gives us cause to propose CAWS. Finally, we discuss limitations, future directions and conclusions.

2 Related Work
2.1 Memory-Based Continual Learning

Memory-based methods mitigate CF by inserting data from previous tasks into the training process of the current one [24, 25]. These approaches can either use raw samples [15, 26], minimize gradient interference [12, 21] or train generative models such as GANs or autoencoders [27, 28, 29] to generate elements from previously-seen distributions.

Studies around memory-based methods have focused on understanding different aspects of memory usage, such as: example selection for the mini-batch [30, 31], size of the external memory [15], different methods of coding information into the memory [19], among others [32, 33]. Other works have measured the impact of hyperparameters on certain methods [34], or studied the effect that rehearsal methods have on the loss functions [35]. A different line of work has focused on how to select elements from the memory, either by how much the loss of an element is affected [30] or by ranking based on the importance of preserving prior knowledge [32].

Yet, in spite of the popularity of memory-based methods, little has been studied about the impact of memory composition on model behaviour. Some proposals along this line are based on applying reservoir strategies [36], while others have proposed to use entropy-based functions to increase
memory diversity \cite{37,38}. Others have increased diversity by minimizing the angles of the gradients between different elements \cite{31}. Despite improving performance in certain scenarios, few studies have been done targeting how to improve memory representativeness, since simply increasing diversity assumes that there is a uniform distribution in the task space, which is not always true.

2.2 Learning Consistency

Learning Consistency and the C-Score \cite{20} come from a line of work analyzing deep neural network training dynamics. One landmark study \cite{39} showed that deep neural networks had the capacity to learn even random noise. Later studies \cite{40}, showed that natural images were learned faster than noise. Others analyzed how examples are forgotten during training \cite{41}. Other metrics have been proposed for measuring learning dynamics such as model confidence, learning speed, holdout retraining and ensemble agreement \cite{42} which correlate well with each other. Learning speed in particular has been shown to correlate well with C-Score. Finally, a recent alternative for understanding per sample difficulty from the model’s perspective is to measure the prediction depth in which a sample is correctly predicted at \cite{43}.

3 Experimental Setup

3.1 Continual Learning Scenario

In this work, we focus on a Class-Incremental setting, as has been done in much recent work in Continual Learning \cite{44}. It is a much harder and realistic setting than the traditional Task Incremental setting \cite{45}. In Class Incremental scenarios, each task $t$ consists of a new data distribution $D^t = (X^t, Y^t)$, where $X^t$ denotes the input instances and $Y^t$ denotes the instance labels. The goal is to train a classification model $f : X \rightarrow Y$ using data from a sequence of $T$ tasks: $D = \{D^1, \ldots, D^T\}$. Each task is presented sequentially to the model and trained for $E$ epochs. Crucially, unlike the Task Incremental setting, a task descriptor is only available during training. In our setup, however, we don’t use it in training as well.

3.2 Methods

To better understand the effect of populating memory, we analyze 3 commonly used methods that utilize the memory buffer in different ways: 1) AGEM \cite{21} which uses the memory as a regularization by comparing the gradients of elements in the buffer with the current task, 2) GDumb \cite{22} that only uses the memory to train the model during each task, and 3) Experience Replay (ER) which trains the model jointly using current task data with the memory. Following previous works \cite{15,30}, we use the memory in ER by concatenating it with the current batch.

3.3 Learning Consistency

The main focus of this study is to understand the effects of memory composition in model performance through the lens of learning consistency. To test the impact of learning consistency, we use the
recently proposed metric of C-Score [20] for each sample. In a nutshell, high C-Score samples are related to samples that are more consistently learned by models while low C-Scores are related to samples which models tend to learn with difficulty. Therefore, we will modulate which samples enter the memory by choosing the top/bottom N scoring samples against a random baseline. We devise 3 different memory population methods:

- **High Consistency (high-c)**: memory is populated with the top C-Score samples.
- **Low Consistency (low-c)**: memory is populated with the bottom C-Score samples.
- **Random**: our baseline in which elements are selected at random, using a Reservoir sampling [46] to balance the memory.

### 3.4 Datasets

We train our models on different splits of MNIST [47], CIFAR-10 and CIFAR-100 [48]. These datasets have available precomputed C-Scores and are traditionally used in Continual Learning. They also provide instances of different distributions of C-Scores, with MNIST and CIFAR-10 having highly skewed distributions, while CIFAR-100 shows a much more uniform distribution of C-Scores, as shown in Figure 1.

### 3.5 Implementation Details

All experiments are run with 3 different seeds, with each seed inducing a different ordering of sequences. For MNIST we use a simple Neural Network with 2 linear layers of 512 and 256 units respectively. In the case of CIFAR-10 and CIFAR-100 we use a simple convolutional architecture proposed in [49], and multiplying the amount of channels of each layer by $N = 1$. We use SGD as our optimizer with learning rate 0.001, momentum 0.9 and batch size 32, unless otherwise mentioned. All methods are trained using Avalanche [50].

We ran our experiments with different memory sizes and amounts of tasks. This in order to verify that the behavior found is robust to the selected hyper-parameters. Details can be found in the Appendix.

### 4 Results

#### 4.1 Populating Memory by Consistency

We begin by analyzing the impact of using our learning consistency-based criterion on the population of the memory. As can be seen in Figure 2, in the early stages of training the high-c strategy beats other strategies in most CL methods. However, the random strategy, given enough compute, eventually beats most of other memory population strategies, for every continual learning method. It should be noted that the more uniform distribution of the C-Score in CIFAR-100 helps to distinguish between different phases of difficulty in the selected data. This, along with the good results of random sampling in CIFAR-10, encourage us to consider whether it is relevant to expand the sampling space.

Previous work on curriculum learning has studied the effect of using C-Scores on images and has found that its impact is relevant when training with a compute budget restriction or when training with noisy samples [23]. While Continual Learning literature has suggested that the random baseline is hard to beat [19].

We believe these results are a consequence of always using the same top scoring samples in the high-c strategy, which does not provide the model with enough data diversity to model previous task distributions adequately. On the other hand, the random baseline allows the model to consume data samples drawn from the whole distribution.

We also analyze how forgetting behaves in GDumb and ER. Results are shown in Figure 3, since is here where the biggest differences are seen. In GDumb, we observe a lower forgetting in high-c (green columns), mainly because the forgetting is low for later tasks. When there is a balanced set of elements by class, this strategy tends to forget previous tasks less rapidly. For ER, we see that forgetting in high-c starts low, consistent with the hypothesis that these elements are easy to learn and are rarely forgotten. However, as we increase the number of epochs, forgetting increases considerably.
In COBS, we try to ensure there is a balanced set of samples from different C-Score ranges. In 4.2 Outperforming random by Consistency-Aware Sampling (CAWS)

Considering the results obtained in the previous sections, and because Random sampling has shown to be a rather strong baseline. We propose two alternative enhancements to our method: 1) COnsistency Balanced Sampling (COBS) and 2) Consistency AWARE Sampling (CAWS). These enhancements were tested using ER, however, it can be used in any method that uses memory.

In COBS, we try to ensure there is a balanced set of samples from different C-Score ranges. In particular, we partition our memory into equally sized bins. We populate each bin with elements from a specific C-Score range. COBS tries to ensure that elements from all of the training distribution are
Figure 4: Difference in how the evaluated methods select samples from different sections of the C-Score distribution for a memory of size = 3. low-c only selects elements with the lowest C-Score. high-c selects only those with the highest C-Score. On the other hand, COBS divides the distribution in equal-length bins and samples an equal number of elements from each bin. CAWS creates a sub-group of elements with C-Score greater than a given threshold, and samples randomly from this group.

always available to the memory and tries to address C-Score imbalance from the training set. COBS attains the random baseline’s diversity but without perpetuating C-Score distribution imbalances. See Algorithm 1 for details. CAWS, on the other hand, tries to enhance random sampling by restricting its domain to a set of the most consistent samples. We set a threshold $\delta$ to create a subset of the most consistent elements. Please see Algorithm 2 for more details. A visual explanation of both methods can be found in Figure 4.

Algorithm 1: COBS

Components:
- $D^t$: Dataset for task $t$.
- $M$: Memory.
- $N^t$: # of elements to add per class.
- $N^b$: Number of bins.
- $N^c$: # of elements in memory of class $c$.
- $C$: C-Score for sample $x$.  

$\alpha \leftarrow \frac{1}{N^b} ; N^b_i \leftarrow \lfloor \frac{N^t}{N^b} \rfloor$

for $i \leftarrow 1$ to $N^b$ do
  for classes in $M$ do
    $M \leftarrow$ remove $N^c - N^b_i$ from bin $i$
  end
  for classes in $D^t$ do
    $x \leftarrow$ Sample $N^b_i$ from $D^t_i$ where:
    \[ \alpha \cdot i \leq C(x) \leq \alpha \cdot (i+1) \]
    \[ M.add(x) \]
  end
end
Output: Populated memory $M$.

Algorithm 2: CAWS

Components:
- $D^t$: Dataset for task $t$.
- $M$: Memory.
- $N^c$: # of elements in memory of class $c$.
- $N^t$: # of elements to add per class.
- $\delta$: C-Score threshold.
- $C$: C-Score

for classes in $M$ do
  $M \leftarrow$ remove $N^c - N^t$ elements
end
for classes in $D^t$ do
  $x \leftarrow$ Sample $N^t$ from $D^t_c$ where $C(x) \leq \delta$
  \[ M.add(x) \]
end
Output: Populated memory $M$.

Results for these two methods compared against the random baseline can be seen in Table 1. CAWS outperforms random for every dataset and is the top performing method except in MNIST. We see this difference to be especially marked in CIFAR-100, which has a more uniform C-Score distribution. This is expected as with a more uniform C-Score distribution more data is required to model the dataset correctly. random is forced to select from the whole of the C-Score spectrum which may include samples that are not good for generalizing. CAWS on the other hand as it chooses a C-Score threshold can select from samples that generalize better.

If we analyze the forgetting shown in Figure 6, we see that CAWS is the method with the least forgetting for the different memory sizes. Using CAWS, forgetting tends to be more evenly divided, being consistently lower across all tasks, which suggests that this method manages to select representative elements of the different tasks, hence causing less forgetting.
Table 1: Accuracy results for random, COBS and CAWS for three different memory sizes. Different strategies to populate the memory. For CAWS, we show results from the best threshold in parentheses. CAWS significantly outperforms random on CIFAR-10 and especially on CIFAR-100. We attribute these differences to the more uniform distribution of CIFAR-100 scores versus CIFAR-10. COBS performs better on MNIST because its distribution is so skewed towards high C-Scores that the memory contains all of the long tail elements and a significant part of consistent elements as well.

|          | 2000     | 4000     | 6000     |
|----------|----------|----------|----------|
| MNIST    |          |          |          |
| Random   | 93.91 ± 0.45 | 94.85 ± 0.02 | 96.27 ± 0.19 |
| COBS     | 95.19 ± 0.50 | 96.79 ± 0.30 | 97.01 ± 0.21 |
| CAWS     | 94.37 ± 0.21 (0.4) | 95.51 ± 0.11 (0.3) | 96.27 ± 0.19 (0.0) |
| CIFAR-10 |          |          |          |
| Random   | 60.61 ± 0.08 | 64.94 ± 0.59 | 65.81 ± 1.02 |
| COBS     | 53.77 ± 0.92 | 59.82 ± 0.15 | 62.57 ± 0.87 |
| CAWS     | 62.41 ± 0.36 (0.7) | 65.8 ± 1.14 (0.6) | 66.72 ± 0.28 (0.7) |
| CIFAR-100|          |          |          |
| Random   | 24.41 ± 0.26 | 30.47 ± 0.44 | 32.88 ± 0.23 |
| COBS     | 22.81 ± 0.22 | 28.37 ± 0.44 | 32.15 ± 0.28 |
| CAWS     | 28.68 ± 0.21 (0.7) | 33.74 ± 0.80 (0.5) | 36.07 ± 0.64 (0.5) |

Figure 5: Accuracy for models trained using CAWS for different C-Score thresholds. The dotted lines represent the random baseline performance. We observe that for all models and datasets there is a certain threshold that outperforms the random baseline, especially when the dataset has a more uniform C-Score distribution as is the case of CIFAR-100.

COBS is the top performing method on MNIST because this dataset’s C-Score distribution is so severely skewed that it can oversample from the long tail distribution, while still having access to enough consistent data. On the other datasets, we see it significantly underperforms compared to the other methods. This tells us that while diversity is important, preserving the statistical properties of previous tasks is important to some degree.

4.3 Proper population makes proper selection redundant

As part of this work, we have studied how populating the memory affects different methods. However, previous work has shown that the strategy used to retrieve elements from memory is also relevant.

Figure 6: Forgetting per task for different memory population methods for different memory sizes. Numbers at the bottom indicate results for the first task, while results above that indicate results for later tasks.
When using MIR, results improve for all memory population methods except CAWS. When using
We can see the results in Table 2. Results without using MIR are equal to ER since randomly selected
To test this, we study the behavior of MIR [30], a method for selection of memory samples. This
Table 2: Accuracy results for different memory population methods with and without using MIR. Results are calculated for 20, 50, 100 elements per class in the memory. It can be seen that by using MIR to retrieve data from the memory we improve the results in most scenarios, regardless of how this memory is populated. The best results are obtained with CAWS (except in MNIST), where applying MIR only brings small benefits at a high cost.

| Method     | CIFAR-10 | CIFAR-100 | MNIST   |
|------------|----------|-----------|---------|
|            | 20       | 50        | 100     |
| Random     | 35.8 ± 2.0 | 33.1 ± 6.5 | 39.1 ± 0.9 |
| high-c     | 29.5 ± 0.8 | 29.9 ± 1.7 | 34.7 ± 3.1 |
| low-c      | 19.2 ± 0.5 | 22.9 ± 1.3 | 24.0 ± 2.0 |
| COBS       | 28.8 ± 1.6 | 36.5 ± 2.2 | 38.1 ± 0.9 |
| CAWS - 0.2 | 36.9 ± 2.2 | 42.8 ± 1.3 | 47.8 ± 1.6 |
| CAWS - 0.5 | 37.4 ± 3.3 | 41.8 ± 2.5 | 47.1 ± 1.8 |
| CAWS - 0.7 | 38.4 ± 1.5 | 44.3 ± 1.4 | 48.3 ± 1.4 |
|            | 20       | 50        | 100     |
| Random     | 20.6 ± 0.3 | 16.2 ± 0.8 | 15.4 ± 0.8 |
| high-c     | 21.5 ± 0.7 | 18.1 ± 0.3 | 17.0 ± 2.2 |
| low-c      | 11.5 ± 0.9 | 9.5 ± 0.7  | 11.2 ± 0.8 |
| COBS       | 18.7 ± 0.5 | 18.7 ± 0.2 | 18.9 ± 0.4 |
| CAWS - 0.2 | 21.0 ± 0.4 | 22.7 ± 0.5 | 22.7 ± 0.4 |
| CAWS - 0.5 | 21.7 ± 0.9 | 22.8 ± 0.5 | 23.2 ± 1.7 |
| CAWS - 0.7 | 23.2 ± 0.4 | 24.5 ± 0.8 | 25.1 ± 1.2 |
|            | 20       | 50        | 100     |
| Random     | 81.7 ± 1.0 | 88.8 ± 0.6 | 92.4 ± 0.8 |
| high-c     | 81.5 ± 0.6 | 86.9 ± 0.9 | 90.6 ± 1.1 |
| low-c      | 36.3 ± 3.3 | 56.9 ± 3.3 | 75.1 ± 1.3 |
| COBS       | 71.0 ± 2.2 | 83.5 ± 2.2 | 91.4 ± 1.9 |
| CAWS - 0.2 | 82.5 ± 1.4 | 89.5 ± 1.9 | 92.8 ± 0.9 |
| CAWS - 0.5 | 81.8 ± 1.6 | 87.7 ± 2.6 | 91.3 ± 1.3 |
| CAWS - 0.7 | 84.4 ± 1.0 | 89.0 ± 1.1 | 91.0 ± 1.8 |

Tables 30, 31. Thus, we expect that coupling our method for memory population with a method for sampling from such a method, would yield increased benefits.

To test this, we study the behavior of MIR [30], a method for selection of memory samples. This method emphasizes selecting samples that would achieve the greatest loss differential if one were to train with or without them. Our implementation of MIR is based on the released code and adapted to work on Avalanche. We also replicate the hyperparameter settings from the original paper and apply them to our method for a fair comparison.

We can see the results in Table 2. Results without using MIR are equal to ER since randomly selected elements are added to the batch. It is important to note that these results are different from those in the previous section because the hyperparameters used in MIR are very different from ours. However, the outcomes are consistent with those presented in the previous section, verifying that these results are transversal to the hyper-parameters.

When using MIR, results improve for all memory population methods except CAWS. When using CAWS to populate the memory, we see there’s almost no improvement or even a decrease in performance as seen in CIFAR-10. This shows that selecting proper elements in the memory can be equivalent to selecting which elements from the memory to train with. This might seem trivial, however, using MIR requires recalculating which elements to select at every step, while CAWS just requires selecting elements once. This is quite an important difference in required computation.

5 Discussion

By showing how learning consistency affects performance when populating the memory with different strategies, this work has explained why random is such a reliable baseline. The intuition behind these results is the following: each class is represented by numerous concepts to form the class, and elements with a high C-Score help represent those popular concepts. However, some of those instances represent the same group of concepts, leaving out some less popular but equally helpful
concepts. Because it comes to popularity, a random selection can select based on this distribution, leaving out part of these elements with a high C-Score, but selecting more diverse groups.

By selecting the most popular (or consistent) instances, the update reinforces a clear idea about the previous tasks. With few iterations on the model, it achieves relatively high accuracy and benefits from a low forgetfulness of these elements learned in the previous tasks. However, when populating with a random strategy, many concepts constitute the memory, the update direction is perturbed by outliers. This causes lower accuracy and higher forgetting.

A different picture emerges as we increase the number of epochs per task: the limited representativeness of the high-c strategy decreases accuracy achieved. For ER, it also causes greater forgetfulness since what is popular in a task ceases to be so when sampled and trained together with another. On the other hand, when adding elements to memory that represent different sectors of the data distribution, we can achieve more updates to the model that can correctly accommodate the various concepts of previous datasets.

CAWS mitigates the shortcomings of high-c because it retains access to a pool of representative samples while still retaining their statistical properties. Thus, it samples more and from a more varied group of representative samples. While on the other hand, the random strategy also includes elements that may be too unique or outliers in its memory, hurting its generalization capabilities.

**Limitations:** This method requires having precomputed C-Scores for each example for the datasets on which one wishes to train. Precomputing C-Scores is a compute intensive task which requires training several models to obtain a decent estimate of the C-Score. However, some approximations of the C-Score can be computed, as proposed by the authors of [20], but they still require training. Also C-Scores make more sense when training on the original task: when dividing a dataset into substasks, the scores may change for a given sample as the task is different. CAWS also requires choosing a C-Score threshold which is task dependent and requires additional hyperparameter search.

**Future Directions:** Given the potential benefits of using C-Score for Continual Learning, future work should focus on finding computationally lighter approximations for it. This would not only be a substantial contribution to Continual Learning, but also for Curriculum Learning where obtaining generic difficulty measures from the model’s perspective is usually costly.

**Societal Impact:** We foresee no direct potential applications from this work that have negative impact on society.

### 6 Conclusions

In this work, we have analyzed how different ways of populating the memory through a criterion based on learning consistency affects different Continual Learning methods. We find that using only the most consistent samples in the memory is useful solely when having a limited compute budget. Otherwise, a random selection of elements is quite a strong baseline. However, selecting elements randomly from a set of the most consistent elements - a procedure we named Consistency AWare Sampling (CAWS)- does help and outperforms this random baseline. We also analyze if CAWS can be combined with an algorithm that prioritizes certain elements from the memory and find that no substantial increase in performance is attained. CAWS attains the same performance than using MIR but without the extra computational cost. We believe this study is a first step on finding principles on which to populate memories in Continual Learning methods.

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A Appendix

A.1 More results on populating Memory

Accuracy results for 5-Split MNIST can be seen in Figure 7. Similar to CIFAR-10 and CIFAR-100, random obtained good results compared to the other memory populating strategies. However, unlike previous datasets, low-c beats the high-c strategy. We believe these results reflect the vertical distribution of MNIST. When we sample from the lower part of the distribution, we sample from the entire data distribution since a large part of these have a C-Score greater than 0.9, similar to CAWS. This idea is reinforced when looking at the forgetting results in Figure 8 where regardless of the number of epochs, low forgetting is seen when we apply ER.

![Figure 7: Accuracy obtained in 5-Split MNIST dataset when training each task for different amount of epochs.](a) AGEM (b) GDumb (c) ER

Resuming with the results shown in Section 4.1, in Figure 9 we show the evolution of the forgetting when training tasks over a different number of epochs in CIFAR-10 and CIFAR-100. These figures show similar results to Figure 3 but reflect what happens in AGEM, in addition to showing the standard deviation of each experiment.

Similar to Figure 3, we can divide the accuracy obtained in each task at the end of the training process. These results are shown in Figure 10. We can see that low-c has lower accuracy than other strategies, regardless of the number of epochs, showing the limited learning capacity of this subset. On the other hand, high-c obtains a higher accuracy in the last tasks, but with higher forgetting in the previous ones.

A.2 Ablation Studies

**Memory Size**: To check the consistency of our experiments, we tested the accuracy obtained with different memory sizes in the methods. As shown in Figure 11, we observed that there is no significant difference in the behavior on the way to populate the memory between the different sizes of memories. We study forgetting in Figure 12. We show that there is no change in the behavior, more than expected, when training with a larger number of elements.

**Amount of tasks**: We run experiments only in CIFAR-100 since the number of tasks that can be generated in Split-CIFAR-10 and Split-MNIST is limited by the number of classes. We use a memory buffer of 4000 elements for the whole sequence, and 15 epochs train each task. Accuracy and Forgetting can be seen in 13 and 14 respectively, showing similar patterns than previous results. As

![Figure 8: Forgetting of the experiment shown in Figure 7](a) AGEM (b) GDumb (c) ER

Figure 13
Figure 9: Forgetting that occurs when training with 5-Split CIFAR-10 and 5-split CIFAR-100. We can see a clear relationship between accuracy and forgetfulness achieved by each method. When there is a rise in forgetfulness, the accuracy stays the same, showing that forgetfulness diminishes performance despite learning more in each task.

Figure 10: Accuracy divided by task when training for different amount of epochs.

expected, as we increase the number of tasks, the scenario becomes more challenging, so the average accuracy drops naturally.

A.3 CO2 Emission Related to Experiments

Experiments were conducted using a private infrastructure, which has a carbon efficiency of 0.432 kgCO₂eq/kWh. A cumulative of 1727 hours of computation was performed on hardware of type GTX 1080 Ti (TDP of 250W).

Total emissions are estimated to be 186.52 kgCO₂eq of which 0 percents were directly offset. Estimations were conducted using the Machine Learning Impact calculator presented in [31].
Figure 11: Accuracy obtained when using different sizes of the memory buffer. Each column represent a strategy to populate the memory. These experiments were run in CIFAR-10, CIFAR-100 and MNIST with 5 task, and with 15 epochs per task.

Figure 12: Forgetting obtained in experiments shown in Figure 11.
Figure 13: Accuracy obtained by various methods when testing with different amounts of tasks in Split CIFAR-100.

Figure 14: Forgetting from the experiment shown in Figure 13