Influence of the cutting precision of the cams on the dynamic performances of the cam mechanisms

D Popa\textsuperscript{1,2} and C–M Popa\textsuperscript{2}

\textsuperscript{1} Department of Automotive and Transport, University of Pitești, 1, Târgu din Vale, 110040, Pitești, Argeș, Romania
\textsuperscript{2} Department of Manufacturing and Industrial Management, University of Pitești, 1, Târgu din Vale, 110040, Pitești, Argeș, Romania

Email: dinel_popa@yahoo.com

Abstract. In this paper, we study the influence of the determination of the external shape of the cam, and of the precision cutting on the imposed values of the reduced acceleration. The study is performed considering a circular cam at which one may analytically determine the displacement and the reduced velocity and acceleration. In the first part of the paper, one performs simulations to determine the external shape with a precision of 0.01\text{mm} and 0.001\text{mm}, that is, the same precision obtained by using a comparison measurer or an indicating snap gauge. For the cam we use an angular step of 1\text{°}, 2\text{°}, 5\text{°}, and 10\text{°}, and one studies the influence of the rotational step on the values of the reduced acceleration. In the second part of the paper, for the study of the dimensional deviation on the reduced acceleration, one performs two numerical simulations for a periodical deviation of maximum magnitude of 0.05\text{mm} and for a periodical deviation of maximum magnitude of 0.02\text{mm}, respectively. Further on, we also realize a study of the influence of roughness of the external profile of the cam on the reduced acceleration. We consider the case in which the roughness is constant on all profile of the cam and the case in which the roughness has random values.

1. Introduction

Several angular steps are used for the cam to study the influence of the cam's rotation step on the reduced acceleration values.

The cam mechanisms are widely used in all fields of technique. At these mechanisms, the motion is transmitted from the cam to the follower by direct contact or by using a roll. The use of cam mechanisms is justified by the advantage of simplicity, small dimensions and the possibility to obtain very complicated laws of motion at the driven element (follower). Albeit, from the structural point of view, the cam mechanisms are simple ones, from the technological point of view they require a high precision in the manufacturing process of the cam. In the automotive industry, the cams of the thermal engines are responsible for their performances. For this reason the cams are usually manufactured with a precision of micrometer or tenth of micrometer. The cam manufacturing processes are different [1, 2, 3, 4, 5, 6, 7]. A modern method consists in the obtaining of the cams by sintering from metallic powders or hardened steels [1, 4, 7]. They are thermal treated, grinded in pack and then assembled on an axis, which thus becomes a camshaft. In this way, one obtains chipper camshafts with lighter weight. Some manufacturers fixed the cams on tubular axes by using hot air with high pressure,
increasing thus the diameter of the tube in the assembling zone of the come. Other producers assemble the heated cams by positioning on chilled axes [1, 4]. The traditional method implied the cast or the beat of the camshaft followed by mechanical procedures to obtain the cams and the spindles of the bearings, thermal treatment and, finally, the grinding of the external profile of the cam [1, 2, 3, 4, 5, 6, 7]. In both situations, the grinding of the external shape of the came is performed by copying. One thus obtains a small roughness of (0.4K 0.08)μm on the cam’s surface. After the manufacturing of the cam, one has to verify the values of the displacements and of the reduced accelerations, which have to be within some limits. A great value of the reduced acceleration implies also great values for the forces in the mechanism.

2. Influence of the precision of the determination of the external profile on the reduced acceleration

The question at which we want to answer is with which minimum precision one has to determine the external profile of the cam in order to obtain a correct value for the reduced acceleration. In the case when the external shape of the cam is measured with a comparison measurer or an indicating snap gauge, as in figure 1, then the obtained mechanism is one with rotational cam and axial follower having translational motion for its top. In figure 1 the rotation of the cam 1 starts in the position in which the point $A_o$ coincides to the point $A^*$, where $OA^* = O\min = r_0$. We denoted by $r_0$ the radius of the base circle, by $\psi$ the angle of cam, by $\phi$ the rotational angle of the cam, and by $s$ the displacement of the comparison measurer where the displacement vanishes on the base circle. In this case, the rotational angle $\phi$ of the cam is equal to the cam’s angle ($\phi = \psi$).

In order to study the influence of the precision of the determination of the external profile of the cam 1, we will use a cam mechanism at which one easily determines the displacement of the follower. For these mechanisms one will determine the analytical calculation relations for the displacement of the follower, the reduced velocity and accelerations of the follower. With the aid of a calculation program one gives values to the angle $\phi$ in the interval $[0, 360^\circ]$, with constant step $\Delta\phi$. One thus obtains numerical values for the displacement of follower. These values are saved in a script file based one which, in AutoCAD one draws the variation graphic $s = s(\phi)$. In the files, the values have two or three decimal digits for the simulation of the values given by a comparison measurer or an indicating snap gauge. Based on these data, the function $s = s(\phi)$ is numerically determined and, with the aid of finite differences, the values for the reduced velocity and acceleration. These values are also saved in corresponding script files in order to draw, in AutoCAD, the graphics $\frac{ds}{d\phi}(\phi)$ and $\frac{d^2s}{d\phi^2}(\phi)$. On the

![Figure 1. Determination of the external cam with the comparison measurer.](image1)

![Figure 2. Mechanism with circular cam and translational follower](image2)
same diagram one draws the graphic of the angular acceleration obtained using the analytical relation and the graphic obtained by derivation using finite differences. One thus can perform comparisons between the two graphics.

The study is performed by considering a circular cam of radius \( R \) and eccentricity \( e \). The mechanism has the kinematic schema in figure 2.

The cam will rotate with constant angular step in the interval \([0, 360^\circ]\). For the angular step, we will consider the values 10°, 20°, 50°, and 100°. We will read the indications of a comparison measurer and of an indicating snap gauge with a precision of 0.001 mm, and 0.001 mm, respectively, in the four cases. To be out to do this, we will perform the kinematic analysis of the mechanism with circular cam and translational follower with top in translational motion (figure 2). For a circular cam of radius \( R \) and eccentricity \( e \), taking into account the notations in figure 2, we will obtain the parametric expressions of the polar radius \([8]\)

\[
\rho(\varphi) = -e \cos \varphi + \sqrt{R^2 - e^2 \sin^2 \varphi}
\]  

and of the displacement of the axial follower

\[
s = \rho(\varphi) - r_0 = -e \cos \varphi + \sqrt{R^2 - e^2 \sin^2 \varphi} - r_0.
\]

The radius of the base circle is \( r_0 = R - e \).

By derivation of expression (2) with respect to time, we obtain the expression of the reduced velocity,

\[
\frac{ds}{d\varphi} = e \sin \varphi - \frac{e^2 \sin 2\varphi}{2\sqrt{R^2 - e^2 \sin^2 \varphi}}.
\]

The reduced acceleration is obtained by derivation of the relation (4) with respect to \( \varphi \),

\[
\frac{d^2s}{d\varphi^2} = e \cos \varphi - \frac{8e^2 \cos 2\varphi \left(R^2 - e^2 \sin^2 \varphi\right) + e^4 \sin^2 2\varphi}{8 \left(R^2 - e^2 \sin^2 \varphi\right)^{3/2}}.
\]

The values of the reduced velocity and accelerations may be also obtained by using finite differences. We consider the notations

\[
\varphi_i = \varphi_i + (i - 1) \Delta \varphi, \quad s_i = s(\varphi_i)
\]

and, after the development into Taylor series (limited to the first three terms), we obtained, at the step \( i \), the expressions of the reduced velocity and acceleration

\[
\left| \frac{ds}{d\varphi} \right| = \frac{s_{i+1} - s_{i-1}}{2 \Delta \varphi}, \quad \left| \frac{d^2s}{d\varphi^2} \right| = \frac{s_{i+1} - 2s_i + s_{i-1}}{\Delta \varphi^2}.
\]

In a numerical application in which \( R = 35 \text{ mm}, \ e = 12 \text{ mm}, \ r_0 = 23 \text{ mm} \), based on a calculation program, one gives numerical values for the angle \( \varphi \) in the interval \([0, 360^\circ]\); from the relations (2) – (4) we obtain numerical values used to draw the graphics in figure 3. The se graphics are drawn, in AutoCAD, based on the script files generated by the calculation program.

For the study of the influence of the precision of the reading of cam’s profile, one uses the same calculation program. The values for the displacement \( s = s(\varphi) \) are saved in two files, the first one in which \( s(\varphi) \) has two decimal digits, and the second one in which \( s(\varphi) \) has three decimal digits. With these data, one numerically calculates the reduced acceleration by using the relation (9). The values
are transferred in AutoCAD with the aid of two script files for a graphical representation. As reference, one uses the representation of the reduced acceleration given by relation (5). For the angular step of $1^\circ$, one obtains the graphics in figure 4, for the angular step of $2^\circ$, the graphics in figure 5, for the angular step of $5^\circ$ the graphics in figure 6, and for the angular step of $10^\circ$ the graphics in figure 7.
The curve denoted by 1 was obtained using the values of displacement approximated with two decimals, the curve denoted by 2 was obtained using the same values approximated with three decimals, while the curve 3 is the theoretical curve.

From the analysis of the obtained curves one observes that the highest deviation from the nominal values appears at the curve drawn with angular step of $10^0$ and a precision of hundredth of millimetre in reading of the profile of the cam.

Practically, one may not use the values of the reduced acceleration obtained by the method of finite differences. A better situation is observed when the precision in reading of the cam’s shape is of thousandth of millimetre.

The situation becomes better when the angular step $\Delta \phi$ increases. For an angular step of $10^0$, the values obtained by using the method of finite differences are closed to the theoretical values.

For the explanation of these values, we will refer to relation (9). The numerator of the fraction $(s_{i+1} - 2s_i + s_{i-1})$ is multiplied by the ratio $\frac{1}{\Delta \phi^2}$, the values of which are 3282.8 for $1^0$, 820.7 for $2^0$, 525.3 for $5^0$, and 113.3 for $10^0$.

The values of the expression $s_{i+1} - 2s_i + s_{i-1}$ are obtained with a deviation of $\pm 0.020 \text{ mm}$ for a precision of the cam’s profile of hundredth of millimetre, and a deviation of $\pm 0.0020 \text{ mm}$ for a precision of thousandth of millimetre obtained at the measurement of the cam. One thus obtains the deviations of the reduced accelerations from the theoretical values:

- for the profile of the cam given with a precision of hundredth of millimetre: $\pm 65.66 \frac{\text{mm}}{\text{rad}^2}$ for an angular step of $1^0$, $\pm 16.416 \frac{\text{mm}}{\text{rad}^2}$ for an angular step of $2^0$, $\pm 10.506 \frac{\text{mm}}{\text{rad}^2}$ for an angular step of $5^0$, and $\pm 2.266 \frac{\text{mm}}{\text{rad}^2}$ for an angular step of $10^0$;

- for the profile of the cam given with a precision of thousandth of millimetre: $\pm 6.566 \frac{\text{mm}}{\text{rad}^2}$ for an angular step of $1^0$, $\pm 1.641 \frac{\text{mm}}{\text{rad}^2}$ for an angular step of $2^0$, $\pm 1.050 \frac{\text{mm}}{\text{rad}^2}$ for an angular step of $5^0$, and $\pm 0.226 \frac{\text{mm}}{\text{rad}^2}$ for an angular step of $10^0$.

From those presented above, it results that it is not recommended to use a small integration step for the determination of the reduced acceleration.

The numerical values obtained by derivation are strongly affected by the decreasing of the step. On the other hand, if one increases the precision of the determination of the external profile, then one may decrease the step.
For instance, at a precision of the determination of the profile equal to $10^{-4}$ mm and an angular step of $2^\circ$, the deviations of the reduced acceleration are situated in the interval $\pm 0.1624$ mm/rad$^2$, which is an acceptable error for our numerical example.

3. Influence of the precision of the manufacturing of external shape of cam on the values of the reduced acceleration

To be out to study this influence, we will consider that the cams are manufactured by grinding. Usually, this is the last manufacturing mechanical operation. Sometimes, one also performs a super finishing by polish with paper band or cloth on special machine tools [4, 6].

The control of camshaft consists in a verification of the precision of the manufacturing of the cams. Usually, one verifies [2, 3, 5, 6] their heights, angular position, taper, dimensional precision of the shape and eccentricity of the spindle, deviation from the keyway etc. For cams, one admits deviations of 0.002K 0.05 mm on the profile [1, 2, 6] and a deviation of 1K $2^\circ$ from the angular position [1, 2, 3, 5, 6]. In order to simulate the dimensional deviation from the profile, we considered that it is a sinusoidal function of period $4\phi$.

We studied the influence of the dimensional deviation on the reduced acceleration. We performed numerical simulations using the cam mechanism in figure 2.

In the first simulation one considered a deviation of maximum magnitude equal to 0.05 mm (curve 1 in figure 8), while in the second simulation the maximum magnitude of the deviation is equal to 0.002 mm (curve 2 in figure 8). In figure 8, the curve denoted by 3 is the theoretical curve.

The diagrams were obtained by using the calculation program previously used, and completing the relation (2) for the calculation of the displacement $s$ with the term $a \sin(4\phi)$, while at the expression of the reduced acceleration (5) we added the term $-16a \sin(4\phi)$, where $a$ is the deviation.

From the analysis of the three graphics, one observes that the dimensional deviation of profile equal to 0.05 mm slightly modifies the values of the reduced acceleration.

In the case of the dimensional deviation of the profile equal to 0.002 mm, the graphic of the reduced acceleration practically coincides with the theoretical graphic.

For the study of the influence of the roughness of the external shape of the cam, we used the same calculation program.

One studies two cases. In the first one, we consider that the roughness is constant and equal to $\mu_{max}$ on the entire profile of the cam, while in the second one, we consider that it has a random value in the interval $[0, \mu_{max}]$.

One analyzes the influence of the roughness on the reduced acceleration. For the roughness we consider the values $\mu_{max} = 0.08 \mu m$ and $\mu_{max} = 0.4 \mu m$.

From the calculation program, result script files based on which one draws the graphics in figure 9 and figure 10.

In the two graphics, the curve denoted by 1 was obtained with the aid of the value of displacement at which we added the constant value $0.08 \mu m$ (figure 9), and $0.2 \mu m$ (figure 10), respectively; the
curve denoted by 2 was obtained using the value of the displacement at which we added a random value in the intervals $[0...0.08] \mu m$ (figure 9), and $[0...0.2]\ mm$ (figure 10), respectively. The curve 3 is the theoretical one.

![Figure 9](image1.png)  
**Figure 9.** Cam manufactured with a deviation of 0.08 $\mu m$.

![Figure 10](image2.png)  
**Fig. 10.** Cam manufactured with a deviation of 0.4 $\mu m$.

From the analysis of the graphics in the two figures it results that a roughness of 0.08 $\mu m$ does not significantly influences the values of the reduced accelerations. The curve for which one considered a constant roughness on the entire external shape practically superposes on the curve for which one considered a random roughness.

In the case in which the roughness has the value 0.4 $\mu m$, the curves of the reduced acceleration have significant deviations from the theoretical curve. Greater values for the deviation are obtained in the case of random variation of roughness.

4. Conclusions

In this paper one studied the conditions that have to be fulfilled in the manufacturing process of the cam and during the process of the verification of the accuracy of the cam’s profile in order to avoid errors of manufacturing or errors which affect the functioning of the cam mechanism.

The cam mechanism used for the simulation is a mechanism with circular cam and axial follower with top. We used this mechanism because it is used for the determination of the external profile of the cam when the follower is replaced a comparison measurer or an indicating snap gauge, and because it presents simple relation for the determination of the reduced acceleration.

The reduced acceleration was used as a calibrator in the performed study. From the analysis of the influence of the precision of the determination of the external shape of the cam on the reduced acceleration calculated with the aid of the finite differences, one may conclude:

- a great angular step ($5^0, 10^0$) for the rotation of the cam slightly affects the reduced acceleration when the precision of the external profile is hundredth or thousandth of millimeter;
- a small angular step ($1^0, 2^0$) for the rotation of the cam highly affects the reduced acceleration when the precision of the external profile is hundredth or thousandth of millimeter;
- in the case in which the angular step is $\Delta \phi=1^0$, and the precision of the external shape of the cam is hundredth of millimetre (comparison measurer) one practically cannot use the values of the reduced acceleration; the situation becomes better when the values of the displacements are given with a precision of thousandth of millimeter.
For an acceptable error of the values of the reduced acceleration, numerically determined by finite
differences, with an angular step $\Delta \phi = 2^0$, the precision of the determination of the profile must be at
least $10^{-4} \text{mm}$.

In the manufacturing process of the cams, deviations from the nominal values are accepted. Further
on, we studied the influence of the dimensional deviations on the functioning of the cam mechanism.
As calibrator, one used the reduced acceleration too.

One considered two values for the deviations from the profile: $0.05 \text{mm}$ and $0.002 \text{mm}$.
Simulations revealed that these values have not a significant influence on the reduced acceleration.
The deviation equal to $0.05 \text{mm}$ slightly modifies the values of the reduced acceleration, while the
deviation equal to $0.002 \text{mm}$ practically does not influence the values of the reduced acceleration.

In the study concerning the influence of the roughness on the values of the reduced acceleration,
we considered two cases. In the first one we considered two constant values for the roughness:
$\mu_{\text{max}} = 0.08 \mu\text{m}$ and $\mu_{\text{max}} = 0.4 \mu\text{m}$, while in the second case, one gave random values in the interval
$[0..\mu_{\text{max}}]$. It resulted that a roughness of $0.08 \mu\text{m}$ does not significantly influence the values of the reduced acceleration, while a roughness of $0.2 \mu\text{m}$ has a significant influence on the reduced acceleration.

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