Peculiarities of the acousto-optical interactions of Bessel light beams in crystals

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Abstract. The noncollinear acousto-optical filtration of diffracted Bessel light beams in uniaxial paratellurite crystals has been investigated. With the use of the method of overlap integrals it is shown that independently on the order of Bessel light beams in the conditions of transverse phase-matching of diffracted waves in the range of optical spectrum of 0.4-1.1 nm in paratellurite crystals the bandwidth of transmission of ~0.52 nm is reached due to detuning from Bragg’s synchronism only. It is shown that in uniaxial gyrotropic crystals polarization-independent diffraction of Bessel light beams is possible, i.e. Bragg diffraction efficiency is independent of the polarization state of the incident beam. It has been stated that the physical reason of such diffraction is simultaneous realization of two processes of anisotropic scattering under which the Bragg synchronism conditions are realized for the orthogonally polarized elliptical Bessel beams.

1. Introduction

Acousto-optic (AO) diffraction of light waves in plane-wave approach or Gaussian light beams is quite well studied [1]. Features of the collinear acousto-optic transformation by ultrasound of Bessel light beams (BLB) are investigated [2, 3]. Owing to narrow diffraction structure of BLB of the highest orders and narrow dark central spot, Bessel beams of high order can be used for steering of atoms at long distances and focusing of cold atoms. AO tunable filters (AOTF) have emerged in present time as convenient, all-electronic, light weight and higher speed means of spectral filtering [4]. AOTF are based on the interaction of acoustic and optical waves in certain crystals, the most popular of them is paratellurite (TeO₂) [5].

In this paper with the use of the method of overlap integrals the noncollinear AO filtration of polychromatic Bessel light beams of higher orders at propagating in the uniaxial crystals is considered. At the same time as an example, the AO interaction of azimuthally uniform BLB of zero order (m = 0) and the azimuthal non-uniform different orders (m ≠ 0) in a paratellurite crystal is considered. In this work the features of the AO Bragg polarization diffraction of BLB propagating nearly to the optical axis of a gyrotropic uniaxial crystal on ultrasonic traveling wave is investigated theoretically and experimentally.

2. Theoretical investigations and discussion

We will consider the geometry of the AO interaction for which slow shear ultrasonic (US) wave propagates in TeO₂ a crystal in the direction which has angle α to axis X∥[110] and also occupies space between the plane z = 0 and z = l. The axis of incident BLB is located in the AZ plane at an angle θe to X. At the same time in the direction of a diffraction order the longitudinal phase-matching conditions of diffracted waves as \(k_o + K = k_e + \Delta k\) (anisotropic diffraction of o-type wave into e-type wave)
takes place, where \( \tilde{k}_o \), \( \tilde{k}_e \) are the wave vectors of the incident and diffracted waves; \( \tilde{K} \) are the wave vectors of US wave; \( \Delta \tilde{k} \) represents the phase velocity mismatch between the incident and diffracted optical wave (longitudinal phase mismatch). An AOTF in imaging systems usually requires a large angular aperture that is achieved under the noncritical phase matching conditions. These conditions in the wave vector diagram are realized when the tangents of the wave vector surfaces, corresponding to incident and diffracted light are parallel. The gyrotropy property of crystal is not taken into account for such interaction geometry. The axis of incident BLB is located in the \( AX \) plane at an angle \( \theta \) to the front of the ultrasonic wave. The cross section of the surface of wave vectors of a gyrotropic uniaxial crystal by the \( XZ \) diffraction plane and the location of the incident \( \tilde{k}_o \) and \( \tilde{k}_e \) and diffracted (\( \tilde{k}_o' \) and \( \tilde{k}_e' \)) wave vectors Bessel beams are used. At the same time two types of vector interactions, namely, \( \tilde{k}_o + \tilde{K}_1 = \tilde{k}_e, \tilde{k}_e + \tilde{K}_2 = \tilde{k}_o' \), where \( \tilde{K}_1, \tilde{K}_2 \) components of the US wave, are seen to contribute to the diffracted field. In further calculations a simplified model of the acoustic beam was adopted, which is a superposition of two plane waves with wave vectors \( \tilde{K}_1 \) and \( \tilde{K}_2 \), propagating at a small angle to the \( X \) axis \( \alpha \).

The diffraction efficiency is calculated using the expression

\[
\eta = \frac{\chi^2 \sin^2 \left[ l_d \sqrt{\frac{\chi^2 + (\Delta k_z / 2)^2}{\chi^2 + (\Delta k_z / 2)^2}} \right]}{\chi^2 + (\Delta k_z / 2)^2},
\]

where

\[
\chi = \frac{\pi n_o^4 \delta_m p_{ef}}{2 n_o \sin(\theta_o + \alpha) \sin(\theta_e + \alpha)} \left[ \frac{2 P_a}{l_1 l_2 \sigma u^3} \right],
\]

\[
\Delta k_z = b \left\{ \left[ \frac{\xi(\theta_o) - \eta}{2} \right]^2 + 2 \eta \xi(\theta_o) (1 - \cos(\theta_o + \alpha)) \right\} - 1,
\]

\[
\xi(\theta_o) = \frac{n_e}{\sqrt{n_o^2 \cos^2 \theta_o + n_e^2 \sin^2 \theta_o}}, \quad \eta = \frac{\lambda f}{n_o \sigma}, \quad b = \left\{ - \frac{2 \pi n_o \lambda}{\lambda_0^2} \right\},
\]

and \( p_{ef} = \left[ (p_{12} - p_{11}) - p_{44} \sin(2\alpha) / 8 \right] \) \( (p_{11}, p_{12}, p_{44} \) is the photoelastic constants, \( \sigma \) is the density of a crystal, \( \nu \) is the phase speed of US wave, \( \lambda_0 \) is the central length of a light wave in vacuum; \( \Delta \lambda \) is the detuning of the wavelength from central Bragg \( \lambda_0 \); \( f \) is the frequency of ultrasound; \( g_m \) is the overlap integral; \( l_d \) is the length of the AO interaction taking into account the direction of US beam group velocity; \( n = (n_o + n_e) / 2 \), where \( n_o \) \((n_e) \) is the ordinary (extraordinary) index of refraction of a crystal; \( p_s \) is the power of a US wave, \( l_1 \) is the length of a piezoelectric transducer along a direction \( X \) \([110], l_2 \) is the width of a piezoelectric transducer). We must consider overlap integral corresponding to \( o-e \) type Bessel light beam transformation from the relation

\[
g_m = \frac{2 \pi R_x}{\sqrt{\int_0^{2 \pi} \int_0^{R_x} \rho \, d\phi \, d\rho}} \frac{\int_0^{2 \pi} \int_0^{R_x} \rho \, d\phi \, d\rho}{\int_0^{2 \pi} \int_0^{R_x} \rho \, d\phi \, d\rho},
\]

where vector-function polarizations have the form
\[ \bar{e}_o = e_{o1} \bar{e}_1 + e_{o2} \bar{e}_2 + e_{o3} \bar{e}_3, \]
\[ \bar{e}_e = e_{e1} \bar{e}_1 + e_{e2} \bar{e}_2 + e_{e3} \bar{e}_3, \]

and
\[ e_{o1} = i q_o \cos \theta_o (J_{m-1}(q_o \rho) e^{-i \varphi} + J_{m+1}(q_o \rho) e^{i \varphi} )/2 + i q_o |\sin \theta_o| J_m(q_o \rho) , \]
\[ e_{o2} = q_o \cos \theta_o (J_{m-1}(q_o \rho) e^{-i \varphi} - J_{m+1}(q_o \rho) e^{i \varphi} )/2 \sqrt{1 + a^4 \sin^2 \theta_o} + q_o |\sin \theta_o| J_m(q_o \rho), \]
\[ e_{o3} = q_o \cos \theta_o (J_{m-1}(q_o \rho) e^{-i \varphi} - J_{m+1}(q_o \rho) e^{i \varphi} )/2 \sqrt{1 + a^4 \sin^2 \theta_o} + q_o |\sin \theta_o| J_m(q_o \rho), \]
\[ e_{e1} = k^{-1} n_o^{-2} n_e^{-2} [a q_o^2 \sin \theta_e^2 (J_{m-2}(q_e \rho) e^{-i \varphi} - J_{m+2}(q_e \rho) e^{i \varphi} )/4 ] + \{ a q_e q_o |\cos \Theta_e|((J_{m+1}(q_e \rho) e^{i \varphi} - J_{m-1}(q_e \rho) e^{-i \varphi} )/2) , \]
\[ e_{e2} = i k^{-1} n_o^{-2} n_e^{-2} [a q_e^2 \sin \theta_e^2 (J_{m+2}(q_e \rho) e^{i \varphi} + J_{m-2}(q_e \rho) e^{-i \varphi} )/4 ] + \{ a q_e q_o |\cos \Theta_e|((J_{m-1}(q_e \rho) e^{-i \varphi} + J_{m+1}(q_e \rho) e^{i \varphi} )/2) - a k^{-2} - 3 q_e^2 / 2 \sin \theta_e J_m(q_e \rho) , \]
\[ e_{e3} = i k^{-1} n_o^{-4} [a q_e^2 \sin \theta_e^2 (J_{m+2}(q_e \rho) e^{i \varphi} + J_{m-2}(q_e \rho) e^{-i \varphi} )/4 ] + \{ q_e q_o |\sin \Theta_e|((J_{m-1}(q_e \rho) e^{-i \varphi} + J_{m+1}(q_e \rho) e^{i \varphi} )/2) + q_e^2 / 1 + \cos \theta_e + q_e^2 / \sin \theta_e J_m(q_e \rho) . \]

Here \( \bar{e}_1, \bar{e}_2, \bar{e}_3 \) is the single vector of crystallographic coordinate \( X_1, X_2, X_3 \); \( \rho, \varphi \) are the cylindrical coordinates; \( J_m(\xi) (\rho = m, m \pm l, m \pm 2) \) is the Bessel functions.

Figure 1 presents the dependences of the diffraction efficiency \( \eta \) on the transverse phase mismatch parameter \( q_o = \Delta q / \alpha_0 \) (\( \Delta q = |q_e - q_o| \)) for different BLB mode value.

![Figure 1](image.png)

**Figure 1.** Dependence of \( \eta \) on \( q_o \) for mode \( m = 0(1), 1(2), 2(3), 3(4) \) (TeO\(_2\) crystal is considered; \( \alpha = 10^6, \gamma_e = 0.53, R_m = 6 \text{ mm}, P_e = 24 \text{ mW}, f = 159 \text{ MHz}, \lambda_0 = 0.75 \text{ \mu m}, l_1 = 1 \text{ cm}, l_2 = 0.5 \text{ cm} \).
Under longitudinal and transversal conditions the diffraction efficiency reaches maximal value of \( \eta = 1 \). For US intensities reaching the same values for longitudinal phase synchronism, the diffraction efficiency became lower till zero value and then we see enhancement of it till the maximal value. With the use of the relations (1)-(3) the physical characteristic of the acousto-optical filtration in the optical spectral region 0.4-1.1 \( \mu \text{m} \) has been studied. The central optical wavelength would be \( \lambda_0 = 0.75 \mu \text{m} \). The index of refraction is \( n_o = 2.24, n_e = 2.237 \). For the length of a piezoelectric transducer \( l_1 = 1 \text{ cm} \) along a direction \( X || [110] \) and the width of a piezoelectric transducer \( l_2 = 0.5 \text{ cm} \) the maximal diffraction efficiency is \( \eta = 1 \) for \( P_a = 24 \text{ mW} \).

Figure 2 presents the dependence of the diffraction efficiency \( \eta \) on the transmission bandwidth \( \Delta \lambda \) of the AO device, in the region of 0.4-1.1 nm.

![Figure 2](image)

**Figure 2.** Dependence of \( \eta \) on \( \Delta \) under condition of cross phase synchronism \( (q_n = 0) \) for different BLB modes \( m = 0 \div 30 \) (TeO\(_2\) crystal is considered; \( \alpha = 10^6 \), \( \gamma_o = \gamma_e = 0.5^0 \), \( R_m = 6 \text{ mm} \), \( P_a = 24 \text{ mW} \), \( f = 159 \text{ MHz} \), \( \lambda_0 = 0.75 \mu \text{m} \), \( l_1 = 1 \text{ cm} \), \( l_2 = 0.5 \text{ cm} \)).

The full width of the filter passband has been calculated at the level of 50% from maximal value of diffraction efficiency. In the conditions of transverse phase synchronism filter passband is \( \Delta \lambda_{1/2} = 0.52 \text{ nm} \). Under maximal diffraction efficiency and absence of transversal phase-matching the acoustooptical filtration was not realized during the higher level of sideband maxima in diffracted orders.

Let BLB with \( r \) ellipticity and azimuth polarization \( \psi \) (orientation of a big polarization axis of the ellipse relative to the diffraction plane) incident on the boundary of the crystal. The crystal active BLB are represented as own elliptically polarization waves with ellipticity parameter \( r_o \) and \( r_e \). Then, taking into account both \( e-o \) and \( o-e \) anisotropic diffraction, we obtain the following expression for diffraction efficiency:

\[
\eta = \frac{1}{1+r^2_1}[\cos^2\psi + r^2\sin^2\psi]\sin^2(g_m\chi_{oe}) + \sin^2\psi + r^2\cos^2\psi\sin^2(g_m\chi_{oe})],
\]

where

\[
\chi_{oe} = \Delta[\tau_\psi \tau_\psi \cos\phi_2 + \cos\phi_1] + (\sin\phi_1 + \tau_o \tau_e \sin\phi_2)\Delta_2, \quad \Delta_1 = \left[n_e^4(p_{1111} - p_{1112})/\tau_{oe}\right], \quad \Delta_2 = \left(n_e^2 n_o^2 p_{444} / 2\tau_{oe}\right), \quad \Delta = 2\pi U_{12}\left[2\lambda_0 (n_r n_e \cos\phi_1 \cos\phi_2) / 2 \right]^{1/2}.
\]
Here strain tensor component $U_{12} = |\nabla_{2} U| = \left| \frac{I_{a}}{\sigma_{0}} \right|^{3/2}$, \(\tau_{oo} = \left[ 1 + \tau_{o} \right] \left[ 1 + \tau_{e} \right]^{-1/2} ; p_{11}, p_{12}, p_{44}\) photoelastic constants of the crystal.

3. Experimental results and discussions

For an AO cell on TeO$_2$ crystal, BLB diffraction with a wavelength of 632.8 nm was considered during the propagation of beams near the optical axis of the crystal. Here linearly polarization of Gaussian beam from He-Ne laser with the power of 25 mW means of the plate turned into circularly polarized first, and then using a conical lens (an axicon with an angle at the base of 2 degrees) a zero-order Bessel light beam was formed. Film polarizer at the entrance to the crystal sets up necessary states of linear polarization of incident ($\psi$) Bessel beam. The intensity distribution of the diffracted BLB in the far-field zone is shown in figure 3.

![Figure 3](image1.png)

**Figure 3.** Ring intensity distribution of two diffracted BLBs in the far field when a light beam is incident at a small angle $\phi_1$ to the optical axis $Z$ of a TeO$_2$ gyrotropic uniaxial crystal: (a) the total field of the diffracted light beam at the crystal output; (b), (c) the diffracted field after the polarizer is installed in front of the camera in two mutually orthogonal positions, respectively.

It can be seen that in the far-field zone the diffracted beam is spatially divided into two annular fields, corresponding to $o$-$e$ and $e$-$o$ AO interactions. The ring character of the fields indicates that the diffracted field is a superposition of two Bessel beams with the angles of the cones coinciding with the angle of the cone of the incident BLB. The location of the film polarizer directly behind the exit face of the AO cell showed that the ring fields observed in the far-field zone have elliptical polarization orthogonal to each other (figure 3) in this case, the ring intensity distribution corresponds to $e$-$o$ diffraction in figure 3 (b), and diffraction in figure 3 (c). This fact confirms the simultaneous implementation of two processes ($e$-$o$ and $o$-$e$) of the interaction of Bessel beams with approximately equal diffraction efficiency. The diffraction efficiency $\eta$ from the polarization azimuth $\psi$ of a linearly polarized incident BLB was investigated. Figure 4 shows the experimental dependence of the diffraction efficiency $\eta$ of the polarization azimuth $\psi$ of incident linearly polarized Bessel beam on ultrasound at different intensities $I_u$.

Acoustic power was estimated considering less power in coordination with the piezoelectric transducer and generator losses on gluing with sound-line. The maximum width $h$ of the piezoelectric transducer used to excite an ultrasonic beam with divergence $2\alpha$ satisfies the equation $h \approx \lambda_{0} / 2\alpha$, where $\lambda_{0}$ is the length of the ultrasonic wave. For the considered geometry of the AO interaction and the central frequency $f_0 = 30$ MHz of the ultrasonic wave in a TeO$_2$ crystal, the value $h \approx 11$ mm. In the experiment a piezoelectric transducer with a width of $h_{0} = 8.5$ mm was used. Therefore, an acoustic beam with the divergence of $\Delta \theta \approx 0.26^0$ that exceeds the angle value of $2\alpha \approx 0.22^0$. Consequently, adopted in the calculation model of the acoustic beam with low divergence is adequate to experiment
and allows theoretical description of the complex phenomenon of diffraction in gyrotropic uniaxial crystal as a superposition of two processes of anisotropic scattering of elliptically polarizing BLB in terms of Bragg synchronism.

![Figure 4](image.png)

**Figure 4.** The dependence of the diffraction efficiency $\eta$ on the azimuth of the incident light polarization $\psi$ at different intensities of ultrasound $I_0$: 1-10, 2-30, 3-60 mW/cm$^2$ ($l = 8.5$ mm, $n_a = 2.26; n_e = 2.1; \phi_l = 0.7^0, \phi_e = 0.4^0, \gamma = 0.5^0, \tau = 0; \alpha = 0.11^0, f_0 = 30$ MHz, $p_{11} = 0.0074, p_{12} = 0.87, p_{44} = -0.17; \tau_0 = 0.92, \tau_e = 0.9; \sigma = 5.72$ kg/cm$^3, R_g = 1$ mm, $g_0 = 1$, TeO$_2$; 1-3 is the theory, 1’- 3’ is the experiment).

From figure 4 it follows that with these parameters of the AO interaction there is a dependence of the diffraction efficiency of the BLB on the polarization state of the incident light. The change in diffraction efficiency due to the polarization of the incident light is ~25%. By reducing the triple level supplied from the signal generator the polarization independence of diffraction (curve 1) has been obtained. In this case, the dependence of diffraction efficiency on the polarization state of the incident light is in the level of error and is less than 4%. From the above figure 4 dependence implies that optimum conditions for polarization-independent diffraction can be realized due to the choice of a suitable length of AO interaction and intensity of ultrasonic waves. It should be noted that bandwidth investigated for anisotropic AO diffraction was $\Delta f \sim f_0$, where $f_0$ is the center frequency of the ultrasound source. On the basis of formula (4) dependence of diffraction efficiencies $\eta$ on the azimuth of the polarization of the incident BLB was theoretically investigated. Figure 4 shows the calculated dependence of the diffraction efficiency $\eta$ on polarization azimuth $\psi$ of linearly polarized incident Bessel beam of ultrasound at different intensities $I_0$. The results obtained at low ultrasound intensities are in good agreement with the experimental data presented in figure 4 (curves 1, 2 - theory and curves 1’, 2’ - experiment).

It should be noted that polarization-independent AO BLB diffraction can be implemented in other gyrotropic uniaxial crystals (for example, in quartz and tellurium [1], [4], [5]) based on perspective geometries of AO interaction. Polarization-independent acousto-optical modulation of the nearly-non-diffracted BLB of infrared spectral range, traveling at the vicinity of tellurium optical axis on the slowly shear ultrasonic waves is investigated. It is shown that under any ultrasonic intensity for XZ-cut of tellurium crystals less dependence of diffraction efficiency of BLB from the light polarization is taken place. Polarization-independent modulation of BLB under the Bragg diffraction by ultrasound in YZ crystal plane on the shear ultrasonic wave polarized under $-63^0$ to Z axis in a YX plane is stated.

### 4. Conclusion

With the use of the method of overlap integrals it is shown that independently on the order of vortex Bessel light beams in the conditions of transverse phase-matching of diffracted waves in the range of optical spectrum of 0.4-1.1 nm in TeO$_2$ crystals the bandwidth of transmission of ~0.52 nm is reached due to detuning from Bragg synchronism. The features of AO diffraction of Bessel light beams propagating at small angles to the optical axis of a gyrotropic uniaxial crystal were theoretically and experimentally investigated. The distinctive feature of the polarization-independent modulation is the
independence of the diffraction efficiency on the polarization state of the incident BLB. Such polarization-independent AO devices for Bessel beams are promising for applications in laser technology (in particular for laser cutting and welding), for probing, absorbing, and scattering media to great depth, laser location, high-resolution microscopy and other areas, as well as for solving problems in laser control and diagnostics, micro- and nanotechnology.

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