Localized modes in orientation-disordered uniaxial medium

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Abstract. Using a 4×4 transfer matrix method, localized modes in orientation-disordered uniaxial medium have been investigated. We confirm that localized modes origin from the randomness of spatial orientation of optical axes. The misalignment of the optical axe provides the opportunity for a rearrangement of the localized modes. The number of localize mode also closely relative to the spatial orientation of optical axis. Numerical results indicate that it is possible to adjust the localized modes through altering the relative orientation of the optical axes of scatterers. This study is an importance for well understanding of localization of light wave and lasing action in anisotropic random media.

1. Introduction

Since Letokhov predicted that combination of multiple scattering and light amplification would lead to a lasing phenomenon (random lasers) [1], the diffusion and transport of light waves in complex dielectric structures have spurred a vast range of experimental and theoretical work during the past decade. LAWANDY’s group observed a narrowing of the spontaneous emission spectrum from a methanol solution of rhodamine 640 perchlorate dye and TiO₂ microparticles when excitation pump light intensity exceeded a threshold [2]; Wiersma’s group reported the coherent backscattering measurement from amplifying random media using optically pumped Ti:sapphire powders [3]; CAO’S group extensively studied the lasing process in disordered semiconductor polycrystalline films, powders and clusters [4,5]. In all the previous study, localization and interference effects which survive to multiple scattering events have been invoked to explain the random lasing observed in many exotic and complex systems. It is confirmed that the random laser origin from the amplification of such localized modes.

All the above study is relative to the isotropic scatterers; as a result, the spatial disorder of scatterers is the unique factor to yield the localization of lightwave. However, recently, coherent backscattering experiments performed with high accuracy apparatus, manifested weak localization of light even in tensorial systems characterized by high optical anisotropy, like nematic liquid crystals.
In such a system, the randomness of spatial orientation of optical axes of uniaxial scatterers should be another factor to lead to localization of lightwave.

In this paper, by using the \(4 \times 4\) transfer matrix method, we investigate wave localization behaviour in one-dimensional random systems made of the orientation-disordered uniaxial scatterers, in which uniaxial scatterers are periodic in spatial location but disordered in spatial orientation of the optical axes.

Our results show that the localization behaviour in a propagating band is in general similar to that found in the random systems made of the isotropic scatterers and can be described by the standard localization theory developed for isotropic materials. However, the wave transport mechanism of such system is very different from that of isotropic materials. In the case of an anisotropic model, the localized modes origin from disorder in spatial orientation of the optical axes which are sharp contrast to the case of isotropic random system. The number of localized modes closely relative to the misalignment of the optical axial of scatterers. This study is an importance for well understanding of localization of lightwave and lasing action in anisotropic random media.

2. Theoretical models

We consider the propagation of electromagnetic (EM) waves through a one-dimensional random sample consisting of the uniaxial layers. The anisotropic layers have the same dielectric tensor \(\varepsilon\) and thickness \(L\), but not the same orientation of optical axis. Randomness is introduced into system by angle of optical axes \(\alpha = \theta (1 + \omega r)\), where \(r\) is random number distributed uniformly between -0.5 and 0.5, \(0 \leq \omega \leq 1\) gives the amplitude of randomness, and \(\theta\) is angle between optical axes.

Suppose that optical axis is oriented along the X direction, the dielectric tensor of the uniaxial layer is defined as

\[
\begin{bmatrix}
\varepsilon_{\|} & 0 & 0 \\
0 & \varepsilon_{\perp} & 0 \\
0 & 0 & \varepsilon_{\perp}
\end{bmatrix}.
\]  

(1)

As optical axis now is rotated in the XY plane, the dielectric tensor, is immediately written as

\[
\begin{bmatrix}
\varepsilon_{\|} \cos^2 \alpha + \varepsilon_{\perp} \sin^2 \alpha & (\varepsilon_{\|} - \varepsilon_{\perp}) \cos \alpha \sin \alpha & 0 \\
(\varepsilon_{\|} - \varepsilon_{\perp}) \cos \alpha \sin \alpha & \varepsilon_{\perp} \cos^2 \alpha + \varepsilon_{\|} \sin^2 \alpha & 0 \\
0 & 0 & \varepsilon_{\perp}
\end{bmatrix}.
\]  

(2)

Here \(\alpha\) is the angle between the optical axis and the X direction. The random structure is constructed by stacking these uniaxial layers with random \(\alpha\).

Assume that the EM wave is incident on one surface of the sample. The transmission coefficient \(T\) of a layered system can be calculated by using the standard \(4 \times 4\) transfer matrix method \[8,9\].

The field in the \(i\)th layer can be described by a vector \(\varphi(i) = [E_{i}(i), H_{i}(i)]\). The transfer matrix relates the field in the \((i+1)\)th layer to \(i\)th layer through \(\varphi(i+1) = M \varphi(i)\), where \(M\) can be calculated by following a similar approach in \[9\] and \(T\) can be obtained by \(M^{-1} = \prod_{i=1}^{N} M\) and \(N\) is the total number of layers \[10\].

3. Results and discuss

For compare with random structure, we first study a periodic structure in which two kinds of anisotropic layers with \(\alpha_{1}\) and \(\alpha_{2}\) are stacked in order. The band structures of such a periodical structure can be obtained by using the standard \(4 \times 4\) transfer matrix method. We find band structure. The band structure is shown in Fig.1 (a), where we have chosen \(n_{1} = 3\), \(n_{2} = 9\), \(\alpha_{1} = 0\), \(\alpha_{2} = \pi/2\) and \(\Lambda = 120\)mm. We investigate the effect of the angle of optical axis on the band structure while the optical axis in the first layer is kept constant \(\alpha_{1} = 0\) and rotated optical axis in the second layer. Fig.1 show the progressive change in the band structure as the angle \(\alpha_{2}\) is made less. We find that band structure closely relative to
the misalignment of optical axes that can be described by \( \Delta \alpha = \alpha - \alpha \). We obtain a wider stopband as \( \alpha = \pi / 2 \).

**Figure 1.** Transmission spectrum for different angle \( \alpha \), (a) \( \alpha = 90 \), (b) \( \alpha = 40 \) and (c) \( \alpha = 20 \).

When the above systems are random, we are interested in studying wave localization behaviours. For this purpose, we consider a finite system with any given number of periods. To introduce the randomness, we rearrange angle \( \alpha \) of the uniaxial layers in a random manner as mentioned in Sec.2. By using a \( 4 \times 4 \) transfer matrix method, we calculate the transmission coefficient \( T \) for each random configuration. For any chosen frequency and sample thickness \( L \), \( T \) is calculated for 500 configurations to study the localization behaviour.

Figure 2 shows the transmission spectra with various degrees of randomness, where we have chosen \( n_i = 3 \), \( n_e = 9 \), \( \theta = \pi \) and \( \Lambda = 120 \text{nm} \). The introduction of randomness to a perfectly ordered structure decreased transmissivity in the passbands slightly and changed drastically the transmissivity in the stopbands. Transmission peaks appear in the stopbands as angle increase. This peak indicates the frequency of localized modes and its linewidth reflects the decay rate of the localized mode. It is clearly seen that number of localized mode increased as we increase the angle. As \( \omega = 1.0 \), the localization behavior in a propagating band is in general similar to that found in the random systems made of the isotropic scatterers.

Although the localization of waves for 1D system is well known, there are some interesting differences between the random isotropic and anisotropic system. We find all passbands still exist even \( \omega = 0.9 \) as shown in Figure 2. This is different from the situation found in isotropic disordered
medium. It is interesting to see the behaviour of our anisotropic disordered medium like a partially ordered system. This property may be useful for effectively pumping of random media, which is one of the practical issues of random laser.

![Figure 2. Transmission spectrum for different randomness strength](image)

**Figure 2.** Transmission spectrum for different randomness strength $w$ for (a) $w = 0.5$, (b) $w = 0.7$, (c) $w = 0.8$, and (d) $w = 1.0$

4. **Summary and conclusions**
We have investigated the wave localization behaviour in one-dimensional random systems made of the orientation-disordered uniaxial scatterers. We find the localization behaviour in a propagating band is in general similar to that found in the random systems made of the isotropic scatterers. However, the wave transport mechanism of such system is very different from that of isotropic materials. We confirm that localized modes origin from randomness of optical axe. The misalignment of the optical axes provides the opportunity for a rearrangement of the localized modes. This study is an importance for well understanding of localization of lightwave and lasing action in anisotropic random media.

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