Critical current density in anisotropic superconductors containing columnar defects

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Abstract. In order to reveal the influence of the tilt angle of columnar defects on angular dependence of critical current density the Monte Carlo simulation of vortex system in three-dimensional layered high-temperature superconductor (HTSC) was performed. For simulations, the three-dimensional model of layered HTSC generalized for the case of tilted magnetic field was used. The angular dependencies of the critical current were calculated at different splay angle of columnar defects and different anisotropy of HTSC. It was shown that simulation results correspond qualitatively to the experimental results for REBCO coated conductors.

1. Introduction

Nowadays, the improvement of critical current density in high-temperature superconductors is still required. It is also important to reduce the anisotropy of critical current since in most practical applications the superconductor is imposed in the magnetic fields of various orientations. Columnar pinning centers (columnar defects) have been shown to be considerably effective to prevent the motion of vortex lines and to increase the critical current density. Columnar defects can be either perpendicular to superconducting planes or tilted for an arbitrary angle. The angular dependencies of critical current on external magnetic field have been widely studied experimentally [1-5]. The maxima of experimental dependencies are normally associated with pinning of vortex lines on tilted columnar defects or superconducting planes [1,4], but despite the large number of experimental works the maxima do not have an explicit explanation.

The vortex lattice in layered HTSC in tilted magnetic field is widely studied theoretically [6]. In case of high anisotropy of HTSC (γ) it was shown that vortex lattice is composed of two interacting lattices of Abrikosov and Josephson vortices [6]. Abrikosov vortex lattice is produced by component of magnetic field perpendicular to superconducting layers and Josephson vortex lattice — by parallel component. The crossing lattices have been also observed experimentally [7].

Real HTSC are anisotropic materials with arbitrary configurations of pinning centers. To predict magnetic and transport properties of real HTSC-tapes it is necessary to take into account interaction of vortex lattice in tilted magnetic field with pinning centers at different values of anisotropy parameter. The aim of our work is numerical simulation of vortex lattice in tilted magnetic field and analysis of angular dependencies of critical current at different tilt angles of columnar defects.

2. Model of layered superconductor in tilted magnetic field
The simulations were performed in the framework of 3D-model of layered HTSC, by using Monte Carlo method. Gibbs energy of three-dimensional vortex system in this model can be given in the following form [8,9]:

$$G = \sum_z \left[ N_z \epsilon + \sum_{i,j} U_{in-plane}(r_{ij}) + \sum_{i,j} U_{surf}(r_{ij}) + \sum_{i} U_{inter-plane}(r_{ij}^{z+1}) + U_m + \sum_i U_{inter-plane}(r_i^{z+1}) \right]$$ (1)

where \( \epsilon = s \epsilon_0 (\ln[\lambda(T)/\xi(T)] + 0.52) \) is a self-energy of vortex line, \( \lambda(0), \xi(0) \) - London penetration depth and coherence length at \( T = 0 \), respectively, \( N_z \) - is a number of vortices in \( z \)-plane, the second term describes a pairwise interaction between pancake vortices, third term is an interaction between vortex and pinning center, next two terms correspond to interaction of vortex with sample surface and meissner and transport current respectively, the last term in (1) – the inter-plane interaction between pancakes. \( \epsilon_0 = \Phi_0^2/(4\pi\lambda)^2 \), \( \Phi_0 = \pi\hbar c/e \) is a magnetic flux quant. The summation is carried out over superconducting layers. For a detailed description of the inter-plane interaction of vortices, see [9,10].

The last term describes the inter-plane interaction. For electromagnetic and Josephson interaction between pancakes in adjacent layers we used equations obtained in [11,12]:

$$U_{inter-plane}(r_{ij}^{z+1}) = U_{em}(r_{ij}^{z+1}) + U_{jos}(r_{ij}^{z+1})$$ (2)

$$U_{em}(r_{ij}^{z+1}) = 2s\epsilon_0 \left[ C + \ln(r_{ij}^{z+1}/2\lambda) + K_0(r_{ij}^{z+1}/\lambda) \right]$$ (3)

$$U_{jos}^{z+1}(r_{ij}^{z+1}) = \begin{cases} \epsilon_0 d[1 + \ln(\lambda/\lambda_s)]0.25(r_{ij}^{z+1}/r_g)^2 \ln(\theta r_g/r_{ij}^{z+1})r_{ij}^{z+1} \leq 2r_g, \\ \epsilon_0 d[1 + \ln(\lambda/\lambda_s)](r_{ij}^{z+1}/r_g) - 0.5r_{ij}^{z+1} > 2r_g, \end{cases}$$ (4)

\( C=0.5772 \) is the Euler constant, \( r_g=\gamma s \) is the Josephson length.

According to theoretical works [6,13], the magnetic field parallel to superconducting layers penetrate the superconductor in the form of Josephson vortices. Josephson vortices are located in the inter-layer spacing. In the absence of Abrikosov vortices Josephson vortices form a triangular lattice. We use the results [6,13] to obtain the energy of Josephson vortex system for Monte Carlo simulations.

The self-energy of Josephson vortex:

$$\epsilon_1 = \frac{\Phi_0^2}{4\pi} H_{cl} = \frac{\Phi_0^2}{16\pi^2 \lambda_b \lambda_c} \left[ \ln \frac{\lambda_b(T)}{\lambda_s} + 1.12 \right], \lambda_b^2 = \frac{s}{d_s} \lambda_c^2, \lambda_c^2 = (\gamma s)^2 + \frac{d_s}{s} \lambda_c^2,$$

The interaction energy of Josephson vortices (per unit length of Josephson vortex):$U_{1,2}^{jos} = \frac{\Phi_0^2}{8\pi^2 \lambda_b \lambda_c} K_0 \left( \sqrt{\frac{(y_2 - y_1)^2}{\lambda_c^2} + \frac{s^2(n_2 - n_1)^2}{\lambda_b^2}} \right)$.

For the interaction between Abrikosov and Josephson vortex

$$U_{1,2}^{r} = -\frac{\Phi_0^2}{8\pi^2 \lambda_b \lambda_c} K_0 \left( \frac{x_0 \pm d/2}{\lambda_b} \right) \left( \sqrt{\frac{y^2}{\lambda_c}} + \frac{z}{\lambda_b} \right) \left( \sqrt{\frac{y^2}{\lambda_c}} + \frac{z}{\lambda_b} \right).$$

\( s = 2.7 \) nm – the sum of inter-layer spacing and superconducting layer thickness \( d_s = 0.27 \) nm, \( \gamma \) is anisotropy parameter, \( d \) is the size of the superconductor in the direction perpendicular to the transport current.
The simulations were performed for the typical parameters of bismuth HTSC, for the sample size 5 µm x 3 µm. The algorithm takes into account the processes of production, annihilation and movement of Josephson vortices. The absolute value of magnetic field is \( H = 320 \) Gs.

### 3. Results: angular dependencies of critical current

Using the model described above we obtained angular dependencies of critical current for two different values of anisotropy parameter. We made the calculations for several different values of tilt angle of the columnar defects. The results are represented in fig. 1.

**Figure 1.** The angular dependencies: a) \( \gamma=40 \), b) \( \gamma=10 \). \( \beta \) is the tilt angle of columnar defects.

The angular dependencies for \( \gamma=10 \) have pronounced minima at \( \theta=90^\circ \) and maxima at \( \theta \) close to 0 and 180°. This result is in qualitative agreement with experimental results for REBCO superconductors in high magnetic field [1-5]. The \( j_c(\theta) \) curves for \( \gamma=40 \) have a maximum at certain angle (\( \theta_{\text{max}} \)) which value depends on tilt angle of columnar defects. In fig. 2 we plot the \( \theta_{\text{max}} \) and \( j_c(\theta_{\text{max}}) \) as a function of \( \beta \). These results show that \( \theta_{\text{max}} \) is independent of tilt angle of the defects and maximal value of critical current decreases at high \( \beta \) when \( \gamma=40 \) and for \( \gamma=10 \) the critical current is about two times higher than for \( \gamma=40 \).

**Figure 2.** \( \theta_{\text{max}} \) and \( j_c(\theta_{\text{max}}) \) as a function of \( \beta \).

The maximum of critical current at \( \theta=30^\circ \) has been observed experimentally for YBCO layer in [3]. For qualitative explanation of this result, we calculate a critical current in our model for \( \gamma=40 \) in magnetic field perpendicular to superconducting layers. The magnitude of the field equals to perpendicular component of tilted field. In fig. 3 we plot the critical current in perpendicular field on the angular dependence of the critical current at two different values of \( \beta \).
The field dependencies for perpendicular magnetic field (fig. 3) have minima at $H = 320 \text{ Gs}$ (which corresponds to $\theta = 0$) and maxima at $H = 0$ ($\theta = 90^\circ$). In the absence of parallel component of magnetic field Josephson vortices do not form. Hence, the maxima of $j_c(\theta)$ curves can be explained by interaction between Abrikosov and Josephson vortices and are not directly associated with the tilt angle of columnar defects.

![Figure 3](image_url)

**Figure 3.** Values of critical current in perpendicular field are marked by crosses at the corresponding values of $\theta$.

4. **Conclusion**

The 3D-model of layered HTSC was generalized for the case of layered superconductor in tilted magnetic field. The angular dependencies of critical current have been calculated at different tilt angle of columnar defects and at two values of $\gamma$. It was shown that $j_c(\theta)$ curves for $\gamma = 10$ correspond qualitatively to experimental results for REBCO in high magnetic fields. The dependencies for $\gamma = 40$ have additional maxima at the direction of magnetic field $\theta \sim 30^\circ$. It was shown that the maxima are due to the interaction between Abrikosov and Josephson vortices and are not directly associated with tilt angle of columnar defects.

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