Application of the theory of strength of materials to determine the shear stresses in beams with round holes

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Abstract. A study was carried out of the possibility of applicability of the theory of strength of materials to determine the shear stresses arising in sections weakened by perforation in the form of round holes, depending on the perforation parameters with a constant shearing force along the length of the beam. A method was proposed for the implementation of the Zhuravsky’s formula for a section weakened by round holes, in the form of dividing the cross section into two independent parts and determining the maximum shear stresses by methods of strength of materials. The reliability of this approach is confirmed by finite element analysis in the ANSYS Mechanical APDL software package. The results obtained can be used to determine the maximum shear stresses in perforated beams in cross-sections along the center of the cut.

1. Introduction
The shear stresses in beams depend on the geometric characteristics of the section and the magnitude of the shearing force. In engineering calculations, the well-known formula of the theory of strength of materials is used, which in the future we will call the Zhuravsky’s formula [1-2]. This formula was obtained for a beam with a narrow constant cross-section. For complex cross-sections, the shear stresses are not recommended to be calculated using this formula, since the results obtained differ greatly from the real ones. In some cases, it is possible to use methods of elasticity theory, and in case of mathematical difficulties – numerically, in particular, using the finite element method [3-16]. Modern computer technology makes it possible to obtain appropriate numerical solutions, but they are not quite convenient for ordinary engineering or estimation calculations. It is desirable for an engineer to have a simple formula or a simple way to find stresses. Therefore, the Zhuravsky’s formula proposed in the XIX century is still used today, and not only for calculating rectangular beams.

The use of perforated beams allows you to reduce the cost of structures and reduce their weight parameters while meeting the conditions of bearing strength. However, the presence of perforation significantly changes the nature of stress distribution. The use of a simple method for determining the maximum stresses in perforated beams would simplify the design calculations and increase the efficiency of the design decisions. In this work, the problem was posed on the applicability of the Zhuravsky’s formula for determining the shear stresses arising in sections weakened by perforation in the form of round holes, depending on the perforation parameters at a constant shearing force along the length of the beam.
To reliably determine the highest shear stresses, a parametric computer simulation of the operation of a perforated beam was carried out using the ANSYS Mechanical APDL software package.

2. Research
A cantilever beam of rectangular section is considered, rigidly clamped on one side. In order to obtain a shearing force $Q_y$, constant along the length, the beam was loaded at the free end with a concentrated force $P = 1000$ kN, perpendicular to the beam axis. The cross-section, as well as a fragment of the perforation in the beam web for parametric stress state studies is shown in Figure 1. For the study, the parameters of the hole radius $R$ and the distance between them (pitch) $L$ were varied. The value of the beam height was taken $H = 0.9$ m, the beam thickness $T = 0.1$ m. The beam material is steel with an elastic modulus $E = 2.1 \times 10^5$ MPa and a Poisson's ratio $\nu = 0.3$. The beam's own weight was not taken into account. The material model is selected as linear-elastic.

![Figure 1. The cross-section (left) and fragment of a beam perforation (right).](image)

In the classical representation, the Zhuravsky’s formula is written as [1]:

$$\tau_{yz} = \frac{Q_y S_z}{J_z b_y},$$

where: $Q_y$ – shearing force acting in the considered cross-section, $S_z$ – static moment of the cut-off part of a cross-section about the axis $z$, $J_z$ – axial moment of inertia of the section about the axis $z$, $b_y$ – function of section thickness in height.

Considering the Zhuravsky’s formula in relation to solid beams, it can be seen that the maximum shear stresses are at the same level at which the static moment of the cut-off part takes the greatest value, which corresponds to the position of the center of gravity of the section. However, the hole, the center of which usually passes directly along the center of gravity of the section, strongly changes the nature of the shear-stress diagram along this section. Figure 2 shows a diagram of shear stresses in a section along the center of the hole in accordance with the Zhuravsky’s formula.
Figure 2. Diagram of shear stresses in the cross-section along the center of the hole in accordance with the Zhuravsky’s formula.

Studying the distribution of shear stresses obtained by the numerical method, which will be presented below, it was noticed that in the upper and lower parts of the beam along the section of the center of the hole, the shear-stress diagrams are close to parabolic. Based on this, it can be assumed that the upper and lower parts of the beam work as if independently of each other (Figure 3). Then, taking into account that the average shear stress for each part is determined by the formula \( \tau_{xy}^{av} = \frac{Q_y}{2A} \), the maximum shear stress at the center of gravity of the section of each part was calculated (Figure 2). Considering that for a rectangular section of the beam, a transition from the average value to the maximum one according to the formula \( \tau_{xy}^{max} = 1.5 \tau_{xy}^{av} \) is possible, and then the greatest shear stress for each of the beam branches will be determined as:

\[
\tau_{xy}^{max} = \frac{3Q_y}{4A},
\]

where: \( A \) – cross-sectional area of one beam branch, \( Q_y \) – shearing force, acting in the considered section.

To reliably determine the maximum shear stresses in a cross-section with perforation, we will carry out parametric computer modeling of the operation of a beam with round holes using the ANSYS Mechanical APDL software package. To carry out a numerical study, a rectangular beam rigidly clamped at one end with perforations in the form of round holes was considered, the radius of which varies parametrically to the beam height. The finite element calculation was carried out in a two-dimensional formulation for a generalized plane stress state. The size of the finite element changes depending on the perforation parameters and is determined based on the number of finite elements with local thickening in the zones of holes. Due to the constancy of the shearing force, the shear stresses along the perforated beam (far from the kinematic and static boundary conditions) [10-16] will change only due to the influence of the parameters of the holes, which makes it possible to study the effect of perforation on the distribution of shear stresses in local zones near the holes.

Figure 3 shows the shear stresses isofields \( \tau_{xy} \) over the section at \( R = H/6 \) and \( L = 5R \), determined by the finite element method in the ANSYS Mechanical APDL software package for the beam shown in Figure 1.
Figure 3. Isofields of shear stresses $\tau_{xy}$ and shear stress diagram over a cross-section with a hole in the ANSYS Mechanical APDL software package at $R = H/6$ and $L = 5R$, MPa.

As can be seen from the shear-stress diagram in Figure 3, the shear stresses on the upper and lower parts of the beam have the form of curves with an extremum slightly shifted to the hole. The proposed solution of the resistance of materials leads to a symmetric parabolic function with an extremum on each of the branches of the perforated beam (Figure 2), which is consistent with the proposed application of Zhuravsky’s formula.

Let us analyze, based on the carried out parametric studies, the change in the shear stresses at different radii of the holes $R$. Figure 4 shows the graphs of the change in the maximum shear stresses in the cross-section along the center of the circular hole depending on the radius of the holes $R$ with the distance between them $L = 5R$. These graphs show the values of the maximum shear stresses $\tau_{xy}$, determined numerically by the finite element method in the ANSYS Mechanical APDL software package, as well as using the Zhuravsky’s formula.

Figure 4. The graph of the change in the maximum shear stresses $\tau_{xy}$ over the section passing through the center of the round hole depending on the radius $R$ at $L = 5R$ for the beam shown in Figure 1.
As can be seen from the graph in Figure 4, the proposed method of using Zhuravsky’s formula to determine the maximum shear stresses in a cross-section along the center of a round hole gives good convergence with a numerical solution for significant dimensions of the radius of the holes $R$, which indicates the legitimacy of using the method for calculating the strength of materials. However, as is known, the stress-strain state of perforated beams depends not only on the ratio of the perforation radius $R$ to the cross-section height $H$, but also on the hole pitch $L$ [16]. We present some results of parametric studies with varying the hole pitch $L$ to determine the sensitivity of the results of computational studies to the parameter $L$. Figure 5 shows the diagrams of shear stresses $\tau_{xy}$ in a cross-section along the center of a circular hole at $R = H/6$ and different distances $L$ between the holes.

![Figure 5](image)

**Figure 5.** Diagrams of shear stresses $\tau_{xy}$ in the section along the center of the holes at $R = H/6$ and different pitches between the holes $L$: a) $L = 5R$, b) $L = 3R$, c) $L = 2R$, d) $L = R$.

As can be seen from the above diagrams, the shear stresses over the cross-section with a hole are practically insensitive to changes in the parameter $L$. This fact allows us to conclude that the application of the Zhuravsky’s formula is possible with sufficiently wide parameters of the pitch between the holes $L$.

3. **Conclusions**

1. The study was carried out of the possibility of applicability of the theory of strength of materials to determine the shear stresses arising in sections weakened by perforation in the form of circular cuts, depending on the perforation parameters with a constant shearing force along the length of the beam.
2. It was found that, using the assumption that in the cross-section along the center of the hole, the two branches of the beam work independently of each other, it is possible to obtain a simple and reliable method for determining the maximum shear stresses using the Zhuravsky’s formula.

3. The maximum shear stresses practically do not depend on the pitch between the holes $L$, which makes it possible to apply the Zhuravsky’s formula to determine the maximum shear stresses on the assumption that the shear stresses are independent of the parameter of the pitch between the perforations $L$.

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