Detection of an appropriate pharmaceutical company to get a suitable vaccine against COVID-19 with minimum cost under the quality control process

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Abstract
Some international pharmaceutical companies have succeeded in producing vaccines against COVID-19. Countries all over the world have aimed to obtain these vaccines with minimum cost. We consider a set of $K$-independent Markovian waiting lists. Each list contains a set of countries, where each one of them has an exponential service time and a Poisson arrival process. These companies differ in some characteristics such as the vaccine production cost and the speed of the required quantity delivery. We present a new detection model that helps in providing an appropriate decision to choose a suitable company. Moreover, the concept of balking and the retention of reneged countries is taken into consideration under the quality control process of each waiting list. Under steady state, we face an interesting and difficult discrete stochastic optimization problem. Its solution gives an optimal distribution of the searching effort, which is bounded by a known probability distribution. A simulation study has been derived to get the minimum value of the paid cost random values. The highest service rate, the total expected profit of each queuing system, and the optimum performance measures, which depend on this cost, have been obtained to show the effectiveness of this model.

KEYWORDS
management decision making, optimal performance measures, optimal planning, queues and service in operations research, queuing systems, search theory

1 | INTRODUCTION

Since the emergence of the deadly COVID-19, competition has intensified between global research and pharmaceutical companies to find an early vaccine against this virus. Currently, a group of giant companies have completed their efforts in clinical trials for potential vaccines against the virus. Indeed, the efforts of these companies have been crowned with success to obtain this vaccine. The experiments have shown that the effectiveness of this vaccine exceeds 90\%, which is an extremely high rate, as declared by the World Health Organization. This vaccine was produced by the U.S. company Pfizer, which worked with the German company Pew & Tech. They said that the data showed stimulation of a high level of T-cell responses against the COVID-19. After that, the announcement by the Moderna company claimed that the produced vaccine was safe and raised immune responses in the body in all healthy volunteers. Also, it claimed that the effectiveness of this vaccine exceeds 90\%. This company announced that no volunteer during the study suffered serious side effects,
but more than half of them complained of mild or moderate effects such as fatigue, headache, chills, muscle aches, or pain at the injection site. Also, AstraZeneca plc is considered one of the most prominent companies in this field. The vaccine, which was developed by the Oxford University with AstraZeneca plc, helped more than a thousand people to develop antibodies and white blood cells that can fight against COVID-19. Many other companies have also entered in this competition, such as Johnson & Johnson, the American Gilead company, the British Glasgow Smith company, and other international companies and institutions.

This virus's danger on the human life has led to a great competition among the countries of the world to quickly obtain a vaccine and with minimum cost. This prompted us to present an efficient model that helps the country in taking an appropriate decision to choose a suitable company among these international companies and institutions. Therefore, the optimal search theory for the lost targets is more effective here to take this decision. This theory has a remarkable importance in our real life where it is used to detect the lost targets (randomly located or moved) with the maximum probability and minimum cost. In an earlier work, many types of search plans have been derived on these line by Beck et al.,1–3 El-Hadidy et al.,4–12 Balkhi,13 and Reyniers.14,15 Many studies have presented different search techniques on the plan and space, such as El-Hadidy et al..16–23,40–41 For more recent and interesting search plans, see El-Hadidy et al..24–26 All these studies have aimed to detect the lost targets with minimum cost and maximum probability. Another aim is to study the finiteness of the first meeting time between the randomly moving target and the searchers as in El-Hadidy et al..27–29

In this work, we present an effective probabilistic model to overcome the waiting problem to get a vaccine where time is an important factor in containing the spread of the virus, especially in the winter season. This situation is similar to the queuing problem, where clients (countries) seek to obtain a good service at a suitable time with minimum cost. Due to the large number of countries which wish to obtain this vaccine, the country may be balking and retention of reneging the waiting list (queue). Waiting for a long time in a list (queue) may lead to a situation when a country may become impatient. Consequently, it leaves the waiting list before receiving the required service (this state is called a reneged case; see Shortle et al.39). In this model, we can assist countries in making the appropriate decision to choose the appropriate pharmaceutical company in terms of the required cost and time to obtain the vaccine. This choice also depends on the financial capabilities of each country. There exists another benefit of this model helps countries to take an appropriate economic decision in any economic field. Here, we use the same model which was recently used in El-Hadidy,30 but with quality control for each queue through the presence of an inspector. If any defect happened, then this inspector will return the country to the waiting list again. This puts us in the face of a difficult discrete stochastic optimization problem that was previously studied in El-Hadidy.35,26 The main goal here is to obtain the optimal distribution of the searching effort to get the suitable pharmaceutical company with the maximum probability. In the case of multiple M/M/1 cooperative queues where each one of them contains a single server and one access flow, Anily and Haviv31 provided a collaboration model between them. Also, Timmer and Scheinhardt32,33 studied another example of improving the service levels with a good number of clients. In addition, the study, which is based on the controlling process of the client access, is derived from Anily and Haviv34 where the total access flow was divided among all the servers in the system.

The ability to manage the available resources in an ideal way is one of the most important reasons for the institution’s success. Therefore, the right decision making of managing the waiting list problem to obtain the vaccine is one of the factors to overcome the spread of COVID-19. It also helps the country to succeed in managing its institution where this decision takes into account the minimum waiting time and cost to obtain this vaccine. Thus, the main objective here is to determine the appropriate company that fulfills the country’s desires.

Our probabilistic model is more different and flexible, where it helps the countries to choose the suitable company. It allows us to select a suitable queue from a set of K-independent M/M/1/N queues (waiting lists). The quality control is applied for each queue by using an inspector. This inspector returns the country (client) back to the service when something is wrong. In addition, this model takes into account the probability of the reneged country and the balking country to enter the waiting list without obtaining the service. This model provided us the ability to find a suitable company with the maximum probability and minimum cost. We consider this cost as a random variable effort that has a known probability distribution. This model contributes to establish a strong link between search theory, queuing theory, operations research, management, and decision-making science. Also, it presents the solution of one of the most difficult stochastic and discrete optimization problems to make an appropriate decision and then increasing the control over the management of the system.

This paper is organized as follows: Section 2 provides the assumptions and the notations which are used in our model. Section 3 describes our model and presents a system of basic probabilistic differential-difference equations corresponding to each company. More than solving this system of the equation under a steady-state situation, we provide the detection probability function which depends on this solution. In Section 4, we present a difficult discrete stochastic optimization...
problem and its solution gives the minimum searching effort and the maximum service rate for the suitable company under the quality control process. Besides that, an application provides the effectiveness of our model in Section 5, the optimization of the economic model which gives the total expected profit of each company. Finally, the results, discussion, and the future work are presented.

2 BASIC NOTATIONS AND ASSUMPTIONS

This system is formulated from a set of $K$-independent queues $Q_i, \ i = 1, 2, ..., K$, where the quality is monitored on their performance. We consider the following notations:

| Notation | Description |
|----------|-------------|
| $Q_i$    | The queue number $i, i = 1, 2, ..., K$ |
| $\mu_i$  | The queue number $i$ service rate |
| $\lambda_i$ | The queue number $i$ arrival rate |
| $p_{ni}$ | Steady-state probability when the queue number $i$ has $n_i$ units |
| $p_0$    | Steady-state total probability when there are no units in all companies (the system) |
| $p_0_i$  | Steady-state probability when there are no units in the queue number $i$ |
| $N$      | Total capacity of the system (the summation of all company capacities) |
| $N_i$    | The capacity of the queue number $i, i = 1, 2, ..., K$ |
| $n$      | Units number in the system, $0 \leq n \leq N$ |
| $n_i$    | Units number in $Q_i, 0 \leq n_i \leq N_i$ |
| $L_i$    | The expected value of the number of units in the queue number $i, i = 1, 2, ..., K$ |
| $L_{qi}$ | The expected value of the waiting units to be served in the queue number $i$ (i.e., the ability of introducing the service for at least one country in the queue number $i$) |
| $L_{ni}$ | $L_i - L_{qi}$, The countries average number that actually serves in the queue number $i$ |
| $H^*_i$  | The minimum excepted value of the waiting time for a selected queue number $i$ from the system |
| $H^*_q$  | The minimum expected value of the waiting time in the queue number $i$ |
| $H^*_s$  | The minimum expected value of the service time in the queue number $i$ |
| $W(Z)$  | The probability of detection for the suitable company under the quality control process |
| $\bar{W}(Z)$ | The complement of $W(Z)$ |
| $R_{ni}$ | The queue number $i$ average rate of reneging |
| $R_{ni}$ | The queue number $i$ average rate of retention |
| $C_{si}$ | The queue number $i$ service cost per unit time |
| $C_{hi}$ | The queue number $i$ holding cost per unit time |
| $C_{li}$ | The lost country cost in the queue number $i$ |
| $C_{ri}$ | The reneged country cost in the queue number $i$ |
| $C_{si}$ | Retaining a reneged country cost in the queue number $i$ |
| $C_{ri}$ | S serving a feedback country cost in the queue number $i$ |
| $R_i$   | Revenue from the service provided to the queue number $i$ |
| $TEC_i$ | The total expected cost per unit time of the company number $i$ |
| $TER_i$ | The total expected revenue per unit time of the company number $i$ |
| $TEP_i$ | The total expected profit per unit time of the company number $i$ |

The following assumptions are used to describe the behavior of each $Q_i, i = 1, 2, ..., K$ as follows:

1. Countries (units or clients) enter $Q_i, i = 1, 2, ..., K$ one by one according to a Poisson process with the rate $\lambda_i (> 0)$ such that $1/\lambda_i$ is the mean interarrival time.
2. The service times of the countries are exponentially distributed random variables (independent and identical distributed random variables) with the rate $\mu_i (> 0)$ and mean service time $1/\mu_i$, where $0 < \lambda_i < \mu_i$.
3. Obtaining the vaccine from each company (queue) is subject to the principle of first come first served.
4. In $Q_i$, $i = 1, 2, ..., K$, after the country gets the vaccine, it either joins at the end of the original $Q_i$, $i = 1, 2, ..., K$ as a feedback country with probability $(1 - q_i)$ (the probability that a defect done) or depart the queue with probability $q_i$, $0 \leq q_i < 1$.

5. The inspection event (responsible for controlling the quality of the service) in the queue number $i$ when there are $n_i$ jobs:

$$\omega_{i}^{(n_i)} = \begin{cases} 
1 & \text{if the unit is inspected,} \\
0 & \text{if the unit is not inspected,}
\end{cases}$$

where $0 \leq \sum_{i=1}^{K} n_i = n \leq N = \sum_{i=1}^{K} N_i$. This event is defined from a given policy where the inspector examines all serviced units to ensure the quality of service.

1. During this race to obtain the vaccine, the country waits a period until the service will start with a probability $1 - p_i$.

   In this situation, if the country waits for a long period of time and does not start this service, then the country will lose and leave $Q_i$, $i = 1, 2, ..., K$ without obtaining the service with a probability $(n_i - 1) \alpha_i p_i$, $2 \leq n_i \leq N_i$ where the time is a very important factor.

2. The country may join $Q_i$, $i = 1, 2, ..., K$ with a probability $\beta_i$ or balk with a probability $1 - \beta_i$ when $n_i$ countries already exist in $Q_i$, $0 \leq \beta_i < 1$ for $1 \leq n_i \leq N_i - 1$ and $\beta_i = 1$ otherwise.

### 3 MODEL FORMULATION

Naturally, there is an increase in the demand for a safe and inexpensive vaccine against COVID-19. Thus, its price will be varied from one company to another. Each country determines the specifications and the vaccine type based on its capabilities. Due to some problems such as delays in performing the service or the high costs, some countries refrain from purchasing the vaccine that they contracted for after a waiting period.

The country is striving to choose an appropriate vaccine in terms of the costs and the effectiveness against COVID-19. All companies and institutions are considered independent from each other in this regard. Therefore, the country chooses an appropriate company that guarantees the quality of the vaccine. Each company (i.e., queue or the waiting list) contains a server and an inspector to ensure the quality of the provided services as shown in Figure 1. Among these independent companies, an appropriate company is selected. According to the previous assumptions, we have the basic probabilistic differential-difference equations corresponding to each queue $Q_i$, $i = 1, 2, ..., K$ (see Kotb and El-Ashkar\(^{35}\)):

$$p_{0i}'(t) = -\lambda_i p_{0i}(t) + \mu_i q_i \omega_{1}^{(n_i)} p_{1i}(t), \ n_i = 0 \quad (1)$$

$$p_{1i}'(t) = -(\beta_i \lambda_i + \mu_i q_i \omega_{1}^{(n_i)}) p_{1i}(t) + \lambda_i p_{0i}(t) + (\mu_i q_i \omega_{2}^{(n_i)} + \alpha_i p_i) p_{2i}(t), \ n_i = 1 \quad (2)$$

$$p_{(n_i)}'(t) = -(\beta_i \lambda_i + \mu_i q_i \omega_{n_i}^{(n_i)} + (n_i - 1) \alpha_i p_i) p_{(n_i)}(t) + \beta_i \lambda_i p_{(n_i-1)}(t) + (\mu_i q_i \omega_{n_i+1}^{(n_i)} + n_i \alpha_i p_i) p_{(n_i+1)}(t), \ 1 < n_i < N_i \quad (3)$$

$$p_{(N_i)}'(t) = -(\mu_i q_i \omega_{N_i}^{(n_i)} + (N_i - 1) \alpha_i p_i) p_{(N_i)}(t) + \beta_i \lambda_i p_{(N_i-1)}(t), \ n_i = N_i \quad (4)$$

Under the steady-state situation, the above system will take the following form:

$$-\lambda_i p_{0i} + \mu_i q_i \omega_{1}^{(n_i)} p_{1i} = 0, \ n_i = 0 \quad (5)$$

$$-(\beta_i \lambda_i + \mu_i q_i \omega_{1}^{(n_i)}) p_{1i} + \lambda_i p_{0i} + (\mu_i q_i \omega_{2}^{(n_i)} + \alpha_i p_i) p_{2i} = 0, \ n_i = 1 \quad (6)$$

$$-(\beta_i \lambda_i + \mu_i q_i \omega_{n_i}^{(n_i)} + (n_i - 1) \alpha_i p_i) p_{(n_i)} + \beta_i \lambda_i p_{(n_i-1)} + (\mu_i q_i \omega_{n_i+1}^{(n_i)} + n_i \alpha_i p_i) p_{(n_i+1)} = 0, \ 1 < n_i < N_i \quad (7)$$
Choosing an appropriate $M/M/1/N_i$ queue from $K$-independent queues under the quality control process

\[-(\mu_i q_i \omega(N_i) + (N_i - 1) \alpha_i p_i) p(N_i) + \beta_i \lambda_i p(N_i - 1) = 0, \quad n_i = N_i\]  

(8)

Kotb and El-Ashkar\(^{35}\) solved this system by using the iterative method and obtained the probability of $n_i$ units or countries $Q_i$, $i = 1, 2, \ldots, K$ as follows:

\[
P(n_i) = \begin{cases} 
  p_{0i}, & n_i = 0 \\
  \frac{\left(\beta_i \lambda_i \alpha_i p_i\right)^n}{\beta_i \prod_{j=0}^{n-1} \left(\frac{\mu_i q_i \omega(N_i)}{\alpha_i p_i} + j\right)} p_{0i}, & 1 \leq n_i \leq N_i
\end{cases}
\]  

(9)

where

\[
P_{0i} = \left[1 + \frac{1}{\beta_i} \sum_{n_i=1}^{N_i} \frac{\left(\beta_i \lambda_i \alpha_i p_i\right)^n}{\prod_{j=0}^{n-1} \left(\frac{\mu_i q_i \omega(N_i)}{\alpha_i p_i} + j\right)} \right]^{-1}.
\]  

(10)

Since $\lambda_i$ and $\mu_i$ give the average rates of entering and serving the country in $Q_i$, $i = 1, 2, \ldots, K$, respectively, then one can consider a measure of traffic congestion (traffic intensity) for $Q_i$-server as $\rho_i$ where $0 < \rho_i < 1$. Consequently, (9) and (10)
will become functions of $\mu_i$ and $\rho_i$ as follows:

$$
P(n_i) = \begin{cases} 
\frac{p_0_i}{\beta_i \prod_{j=0}^{n_i-1} \left( \frac{\mu_i q_i(n_j)}{\alpha_i p_i} + j \right)} & n_i = 0 \\
\frac{\left( \frac{\beta_i \mu_i \rho_i}{\alpha_i p_i} \right)^{n_i}}{p_0_i \prod_{j=0}^{n_i-1} \left( \frac{\mu_i q_i(n_j)}{\alpha_i p_i} + j \right)} & 1 \leq n_i \leq N_i \end{cases},
$$

where

$$
p_0_i = \left[ 1 + \frac{1}{\beta_i} \sum_{n_i=1}^{N_i} \frac{\left( \frac{\beta_i \mu_i \rho_i}{\alpha_i p_i} \right)^{n_i}}{\prod_{j=0}^{n_i-1} \left( \frac{\mu_i q_i(n_j)}{\alpha_i p_i} + j \right)} \right]^{-1},
$$

It is clear that the events of choosing one from $Q_i$, $i = 1, 2, \ldots, K$ are mutually exclusive events. Thus,

$$
p_0 = \prod_{i=0}^{K} p_{0_i} = \prod_{i=0}^{K} \left[ 1 + \frac{1}{\beta_i} \sum_{n_i=1}^{N_i} \frac{\left( \frac{\beta_i \mu_i \rho_i}{\alpha_i p_i} \right)^{n_i}}{\prod_{j=0}^{n_i-1} \left( \frac{\mu_i q_i(n_j)}{\alpha_i p_i} + j \right)} \right]^{-1},
$$

when there are no countries in the system (i.e., there is no country in $Q_1, Q_2, \ldots, Q_K$).

If $Q_i$ $i = 1, 2, \ldots, K$ is full then it is normal for all courtiers to leave it upon their arrival. This is one of the important reasons that the country aims to search for a suitable company to get the vaccine. Therefore, we assume that the country moves according to the Markov process under favorable conditions to choose the suitable company. If the service is done, then the conditional probability of detecting the suitable company $Q_i$, $i = 1, 2, \ldots, K$ will be $1 - b(i, n_i, Z(n_i))$ where $Z(n_i)$ shows the amount of the searching effort (see El-Hadidy). Thus, one can consider the function $L(Z) = \sum_{i=1}^{K} \sum_{n_i=1}^{N_i} L_i(Z)$ as a cost function of detection where $0 \leq L(Z) \leq X$, $L_i(Z) \leq X_i$ such that $\sum_{i=1}^{K} L_i(Z) \leq X$. As in El-Hadidy, we can get the probability of detection for the suitable company under the quality control process by

$$
W(Z) = \sum_{i=1}^{K} \sum_{n_i=1}^{N_i} p_{n_i} \left( 1 - b(i, n_i, Z(n_i)) \right).
$$

From the independence principle, each country aims to minimize the cost of detection for a suitable company. This will participate in maximizing the quality control of the system. As in Hong et al., we consider the exponential detection function, $b(i, n_i, Z(n_i)) = e^{-Z(n_i)}$. Thus, (14) will become

$$
W(Z) = \sum_{i=1}^{K} \sum_{n_i=1}^{N_i} p_{(n_i)} \left( 1 - e^{-Z(n_i)} \right).
$$
From (11) and (12) in (15), we can obtain the probability of no detection for the suitable company under the quality control process by

\[
\hat{W}(Z) = \prod_{i=1}^{K} \sum_{n_i=1}^{N_i} p(n_i) e^{-Z(n_i)k} = \prod_{i=1}^{K} \sum_{n_i=1}^{N_i} \left[ \frac{\beta_i \mu_i \rho_i}{\alpha_i \pi_i} \right]^{n_i} \prod_{j=0}^{n_i-1} \left( \frac{\mu_j q_j \omega_j(n_i)}{\alpha_i \pi_i} + j \right) \left[ 1 + \frac{1}{\hat{\beta}_i} \sum_{n_i=1}^{N_i} \prod_{j=0}^{n_i-1} \left( \frac{\mu_j q_j \omega_j(n_i)}{\alpha_i \pi_i} + j \right) \right]^{-1} e^{-Z(n_i)k} \]  

(16)

4 | THE MAXIMUM SERVICE RATE AND THE MINIMUM DETECTION COST UNDER THE QUALITY CONTROL PROCESS

The decision-makers need a set of factors that the country tries to strike a balance between them in order to be able to find a suitable company. One of the important factors is the detection cost of the suitable company under the quality control process. This factor participates in maximizing the service quality. Therefore, we need to maximize \( W(Z) \) which is equivalent to the minimization of \( \hat{W}(Z) \). This will minimize the values of the searching effort \( Z(n_i)k, i = 1, 2, \ldots, K \). On the other hand, the minimization of the traffic intensity \( \rho_i \) is equivalent to the maximization of the service rate \( \mu_i \), where \( \rho_i = \frac{\lambda_i}{\mu_i} \). This is another objective that we strive to find the appropriate (not crowded) queue. This will provide the service as quickly as possible.

In our problem, we have the condition \( L_i(Z) \leq X_i \), then one can let \( P(L_i(Z) \leq X_i) \leq \hat{\beta}_i, \hat{\beta}_i \in [0, 1] \). From the central limit theorem, we can obtain \( \frac{X_i - E(X_i)}{\sqrt{\text{var}(X_i)}} \leq \hat{\beta}_i \). If \( X_i - E(X_i) \leq \Omega_ip \sqrt{\text{var}(X_i)} \) then \( X_i - E(X_i) \leq \Omega_ip \). Finally, we present a difficult convex multiobjective nonlinear discrete stochastic optimization problem (where \( W(Z) \) is an exponential function):

\[
P(1) : \min_{Z(n_i)k, \rho_i} W(Z),
\]

subject to:

\[
L(Z) = \left( Z \in R^K \left| L(Z) = \sum_{i=1}^{K} \sum_{n_i=1}^{N_i} Z(n_i)k - E(X_i) - \Omega_ip \sqrt{\text{var}(X_i)} \right. \right) \leq 0,
\]

\( p(n_i) - 1 \leq 0, -Z(n_i) \leq 0, \rho_i - 1 < 0, \) and \( i = 1, 2, \ldots, K, 1 \leq n_i \leq N_i \) where

\[
W(Z) = \prod_{i=1}^{K} \sum_{n_i=1}^{N_i} p(n_i) e^{-Z(n_i)k}
\]

\[
= \prod_{i=1}^{K} \sum_{n_i=1}^{N_i} \left[ \frac{\beta_i \mu_i \rho_i}{\alpha_i \pi_i} \right]^{n_i} \prod_{j=0}^{n_i-1} \left( \frac{\mu_j q_j \omega_j(n_i)}{\alpha_i \pi_i} + j \right) \left[ 1 + \frac{1}{\hat{\beta}_i} \sum_{n_i=1}^{N_i} \prod_{j=0}^{n_i-1} \left( \frac{\mu_j q_j \omega_j(n_i)}{\alpha_i \pi_i} + j \right) \right]^{-1} e^{-Z(n_i)k} \]

\[
p(n_i) = \left[ \frac{\beta_i \mu_i \rho_i}{\alpha_i \pi_i} \right]^{n_i} \prod_{j=0}^{n_i-1} \left( \frac{\mu_j q_j \omega_j(n_i)}{\alpha_i \pi_i} + j \right) \left[ 1 + \frac{1}{\hat{\beta}_i} \sum_{n_i=1}^{N_i} \prod_{j=0}^{n_i-1} \left( \frac{\mu_j q_j \omega_j(n_i)}{\alpha_i \pi_i} + j \right) \right]^{-1}.
\]
To get the optimal (minimum) values of $Z(n_i)_i$, $i = 1, 2, ..., K$, we apply the necessary Kuhn–Tucker conditions to solve the above problem. Therefore, we have

$$
- \sum_{n_i=1}^{N_i} \left( \frac{\beta_i \mu_i p_i}{\alpha_i p_i} \right)^{n_i} \left( \frac{\beta_i \prod_{j=0}^{n_i-1} ((\mu_i q_i \omega_{j+1}^{(n_i)}/\alpha_i p_i) + j)}{\prod_{j=0}^{n_i}} (\mu_i q_i \omega_{j+1}^{(n_i)}/\alpha_i p_i) + j \right) - Z(n_i)_i
$$

$$
\left[ 1 + \frac{1}{\beta_i} \sum_{n_i=1}^{N_i} \left( \frac{\beta_i \mu_i p_i}{\alpha_i p_i} \right)^{n_i} \left( \prod_{j=0}^{n_i-1} ((\mu_i q_i \omega_{j+1}^{(n_i)}/\alpha_i p_i) + j) \right) \right]^{-1}
$$

$$
\left[ 1 + \frac{1}{\beta_i} \sum_{n_i=1}^{N_i} \left( \frac{\beta_i \mu_i p_i}{\alpha_i p_i} \right)^{n_i} \left( \prod_{j=0}^{n_i-1} ((\mu_i q_i \omega_{j+1}^{(n_i)}/\alpha_i p_i) + j) \right) \right]^{-1} \left[ \prod_{i=i+1}^{K} \frac{N_i}{n_i} \right] \left[ 1 + \frac{1}{\beta_i} \sum_{n_i=1}^{N_i} \left( \frac{\beta_i \mu_i p_i}{\alpha_i p_i} \right)^{n_i} \left( \prod_{j=0}^{n_i-1} ((\mu_i q_i \omega_{j+1}^{(n_i)}/\alpha_i p_i) + j) \right) \right]^{-1}
$$

$$
\left[ 1 + \frac{1}{\beta_i} \sum_{n_i=1}^{N_i} \left( \frac{\beta_i \mu_i p_i}{\alpha_i p_i} \right)^{n_i} \left( \prod_{j=0}^{n_i-1} ((\mu_i q_i \omega_{j+1}^{(n_i)}/\alpha_i p_i) + j) \right) \right]^{-1} \left[ \prod_{i=i+1}^{K} \frac{N_i}{n_i} \right] \left[ 1 + \frac{1}{\beta_i} \sum_{n_i=1}^{N_i} \left( \frac{\beta_i \mu_i p_i}{\alpha_i p_i} \right)^{n_i} \left( \prod_{j=0}^{n_i-1} ((\mu_i q_i \omega_{j+1}^{(n_i)}/\alpha_i p_i) + j) \right) \right]^{-1}
$$

$$
\left\{ \sum_{i=1}^{K} \left\{ \sum_{n_i=1}^{N_i} Z(n_i)_i - E(X_i) - \Omega_i p \sqrt{\text{Var}(X_i)} \right\} = 0, \quad (19) \right\}
$$
where $U_{\xi}, \xi = 1, 2, \ldots, n_{\sigma}$ is the Lagrange multiplies. Since $Z_{(n_i)\sigma} > 0$ for any $Q_i, i = 1, 2, \ldots, K$, then from (21) one can get $U_{3\sigma} = 0$. It is known that $\lambda_i < \mu_i$ (i.e., $\rho_i < 1$) for each $Q_i, i = 1, 2, \ldots, K$ at any time $t > 0$, then from (22), one can have $U_{4\sigma} = 0$. For optimality, the probability $1 - p_{(n_i)\sigma}$ should be greater than zero at any time $t > 0$ then from (20), one can get $U_{2\sigma} = 0$. Since $\sum_{i=1}^{K} \sum_{n_{i1}=1}^{N_{i1}} Z_{(n_i)\sigma} \leq E(X_i) + \Omega_i p_{\sqrt{var(X_i)}} < 0$, then from (19), one can get $U_{1\sigma} = 0$. Thus, the case $U_{1\sigma} > 0, U_{2\sigma} = U_{3\sigma} = U_{4\sigma} = 0$ is the only case that considers to get the minimum cost $Z_{(n_i)\sigma}^* > 0$ and the minimum traffic intensity $\rho_i^*$. Consequently, (17) and (18) will become

$$
1 - \frac{N_{n_{\sigma}1}}{\beta_i \prod_{j=0}^{\mu_i \phi_{n_{\sigma}1} / \alpha_i p_i + j}} \left[ \begin{array}{l}
1 + \frac{1}{\beta_i} \sum_{n_{\sigma}=1}^{n_{\sigma}1} \left( \frac{\beta_i \mu_i \phi_{n_{\sigma}1} / \alpha_i p_i + j}{\alpha_i p_i} \right) \left( \rho_i \sum_{n_{i1}=1}^{N_{i1}} \left( \frac{\beta_i \mu_i \phi_{n_{i1}} / \alpha_i p_i + j}{\alpha_i p_i} \right) \right) \right]^{-1} e^{-Z_{(n_i)\sigma}}
\end{array}
$$

$$
\leq 0,
$$

(20)

$$
U_{3\sigma} \{ -Z_{(n_i)\sigma} \} = 0,
$$

(21)

$$
U_{4\sigma} \{ \rho_i - 1 \} = 0,
$$

(22)

$$
U_{\xi} \geq 0,
$$

(23)

and

$$
\sum_{n_{\sigma}=1}^{N_{n_{\sigma}}} \left( \frac{\beta_i \mu_i \phi_{n_{\sigma}} / \alpha_i p_i + j}{\alpha_i p_i} \right) \left( \rho_i \sum_{n_{i1}=1}^{N_{i1}} \left( \frac{\beta_i \mu_i \phi_{n_{i1}} / \alpha_i p_i + j}{\alpha_i p_i} \right) \right) \left( n_{\sigma}e^{-Z_{(n_i)\sigma}} \right) e^{-Z_{(n_i)\sigma}}
\end{array}
$$

$$
\leq 0,
$$

(24)

$$
\sum_{n_{\sigma}=1}^{N_{n_{\sigma}}} \left( \frac{\beta_i \mu_i \phi_{n_{\sigma}} / \alpha_i p_i + j}{\alpha_i p_i} \right) \left( \rho_i \sum_{n_{i1}=1}^{N_{i1}} \left( \frac{\beta_i \mu_i \phi_{n_{i1}} / \alpha_i p_i + j}{\alpha_i p_i} \right) \right) \left( n_{\sigma}e^{-Z_{(n_i)\sigma}} \right) e^{-Z_{(n_i)\sigma}}
\end{array}
$$

$$
\leq 0,
$$

(25)
respectively. From (24), we have

\[
\begin{align*}
&\left[ \sum_{n_{c}=1}^{N_{c}} \left( \frac{\beta_{c} \mu_{c} p_{c}}{\alpha_{c} p_{c}} \right)^{n_{c}} \left( \beta_{c} \prod_{j=0}^{n_{c}-1} \left( \frac{(\mu_{c} q_{c} \omega_{j+1}/\alpha_{c} p_{c}) + j}{\alpha_{c} p_{c}} \right) \right) \right]^{-1} \\
&\left[ 1 + \frac{1}{\beta_{c}} \sum_{n_{c}=1}^{N_{c}} \left( \frac{\beta_{c} \mu_{c} p_{c}}{\alpha_{c} p_{c}} \right)^{n_{c}} \left( \prod_{j=0}^{n_{c}-1} \left( \frac{(\mu_{c} q_{c} \omega_{j+1}/\alpha_{c} p_{c}) + j}{\alpha_{c} p_{c}} \right) \right) \right]^{-1} e^{-Z(n_{c})_{c}} \\
&\times \prod_{i=1, i \neq c}^{K} \sum_{n_{i}=1}^{N_{i}} \left[ \frac{\beta_{i} \mu_{i} p_{i}}{\alpha_{i} p_{i}} \right]^{n_{i}} \left[ 1 + \frac{1}{\beta_{i}} \sum_{n_{i}=1}^{N_{i}} \left( \frac{\mu_{i} q_{i} \omega_{j+1}/\alpha_{i} p_{i}}{\alpha_{i} p_{i}} \right) \right]^{-1} e^{-Z(n_{i})_{i}} \right] = U_{c}.
\end{align*}
\]

From (26) in (19), we get

\[
\begin{align*}
&\left\{ \sum_{i=1}^{K} \sum_{n_{i}=1}^{N_{i}} Z(n_{i})_{i} - E(X_{i}) - \Omega_{i} p \sqrt{Var(X_{i})} \right\} \left[ \sum_{n_{c}=1}^{N_{c}} \left( \frac{\beta_{c} \mu_{c} p_{c}}{\alpha_{c} p_{c}} \right)^{n_{c}} \left( \beta_{c} \prod_{j=0}^{n_{c}-1} \left( \frac{(\mu_{c} q_{c} \omega_{j+1}/\alpha_{c} p_{c}) + j}{\alpha_{c} p_{c}} \right) \right) \right]^{-1} \\
&\left[ 1 + \frac{1}{\beta_{c}} \sum_{n_{c}=1}^{N_{c}} \left( \frac{\beta_{c} \mu_{c} p_{c}}{\alpha_{c} p_{c}} \right)^{n_{c}} \left( \prod_{j=0}^{n_{c}-1} \left( \frac{(\mu_{c} q_{c} \omega_{j+1}/\alpha_{c} p_{c}) + j}{\alpha_{c} p_{c}} \right) \right) \right]^{-1} e^{-Z(n_{c})_{c}} \\
&\times \prod_{i=1, i \neq c}^{K} \sum_{n_{i}=1}^{N_{i}} \left[ \frac{\beta_{i} \mu_{i} p_{i}}{\alpha_{i} p_{i}} \right]^{n_{i}} \left[ 1 + \frac{1}{\beta_{i}} \sum_{n_{i}=1}^{N_{i}} \left( \frac{\mu_{i} q_{i} \omega_{j+1}/\alpha_{i} p_{i}}{\alpha_{i} p_{i}} \right) \right]^{-1} e^{-Z(n_{i})_{i}} \right] = 0.
\end{align*}
\]

Since \( \sum_{i=1}^{K} \sum_{n_{i}=1}^{N_{i}} Z(n_{i})_{i} - E(X_{i}) - \Omega_{i} p \sqrt{Var(X_{i})} \neq 0 \), then

\[
\begin{align*}
&\left[ \sum_{n_{c}=1}^{N_{c}} \left( \frac{\beta_{c} \mu_{c} p_{c}}{\alpha_{c} p_{c}} \right)^{n_{c}} \left( \beta_{c} \prod_{j=0}^{n_{c}-1} \left( \frac{(\mu_{c} q_{c} \omega_{j+1}/\alpha_{c} p_{c}) + j}{\alpha_{c} p_{c}} \right) \right) \right]^{-1} \\
&\left[ 1 + \frac{1}{\beta_{c}} \sum_{n_{c}=1}^{N_{c}} \left( \frac{\beta_{c} \mu_{c} p_{c}}{\alpha_{c} p_{c}} \right)^{n_{c}} \left( \prod_{j=0}^{n_{c}-1} \left( \frac{(\mu_{c} q_{c} \omega_{j+1}/\alpha_{c} p_{c}) + j}{\alpha_{c} p_{c}} \right) \right) \right]^{-1} e^{-Z(n_{c})_{c}} \\
&\times \prod_{i=1, i \neq c}^{K} \sum_{n_{i}=1}^{N_{i}} \left[ \frac{\beta_{i} \mu_{i} p_{i}}{\alpha_{i} p_{i}} \right]^{n_{i}} \left[ 1 + \frac{1}{\beta_{i}} \sum_{n_{i}=1}^{N_{i}} \left( \frac{\mu_{i} q_{i} \omega_{j+1}/\alpha_{i} p_{i}}{\alpha_{i} p_{i}} \right) \right]^{-1} e^{-Z(n_{i})_{i}} \right] = 0.
\end{align*}
\]

In the beginning, some companies will announce that they have developed a vaccine, and most countries will turn to them. But during testing the vaccine, a defect was found and it needs some more careful studies. At this time, countries leave these companies and will head towards other companies that have passed all the tests on the vaccine. Therefore, we
can assume here that some lists are completely empty (i.e., \( n_i = 0 \)). Then, from the formula:

\[
\prod_{i=1}^{K} \sum_{n_{i}=1}^{N_{i}} \left( \frac{\beta_{i} \mu_{i} \rho_{i}}{\alpha_{i} p_{i}} \right)^{n_{i}} \left( \frac{n_{i} + 1}{\beta_{i} \prod_{j=0}^{n_{i}-1} \left( \frac{\mu_{i} q_{\omega_{i}^{(n_{i})}}}{\alpha_{i} p_{i}} + j \right)} \right) \left( 1 + \frac{1}{\beta_{i} \prod_{j=0}^{n_{i}-1} \left( \frac{\mu_{i} q_{\omega_{i}^{(n_{i})}}}{\alpha_{i} p_{i}} + j \right)} \right)^{-1} e^{-Z(n_{i})},
\]

there exist at least one of \( n_i, i = 1, 2, \ldots, K \), equals to zero. Hence, (25) and (27) will become

\[
\sum_{n_{z}=1}^{N_{z}} \left( \frac{\beta_{z} \mu_{z} \rho_{z}}{\alpha_{z} p_{z}} \right)^{n_{z}} \left( \frac{n_{z} + 1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right) \left( n_{z} \rho_{z}^{n_{z}-1} \right) e^{-Z(n_{z})}
\]

\[
\times \left\{ \left[ 1 + \frac{1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right]^{-1} \right\}
\]

\[
- \left\{ 1 + \frac{1}{\beta_{z}} \sum_{n_{z}=1}^{N_{z}} \left( \frac{\beta_{z} \mu_{z} \rho_{z}}{\alpha_{z} p_{z}} \right)^{n_{z}} \left( \frac{n_{z} + 1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right) \left( n_{z} \rho_{z}^{n_{z}-1} \right) e^{-Z(n_{z})} \right\} = 0.
\]

and

\[
\left\{ Z(n_{z}) - E(X_{z}) - \Omega_{z} \sqrt{Var(X_{z})} \right\} \left[ \sum_{n_{z}=1}^{N_{z}} \left( \frac{\beta_{z} \mu_{z} \rho_{z}}{\alpha_{z} p_{z}} \right)^{n_{z}} \left( \frac{n_{z} + 1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right) \right]^{-1} e^{-Z(n_{z})}
\]

\[
\times \left\{ \left[ 1 + \frac{1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right]^{-1} \right\}
\]

\[
- \left\{ 1 + \frac{1}{\beta_{z}} \sum_{n_{z}=1}^{N_{z}} \left( \frac{\beta_{z} \mu_{z} \rho_{z}}{\alpha_{z} p_{z}} \right)^{n_{z}} \left( \frac{n_{z} + 1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right) \left( n_{z} \rho_{z}^{n_{z}-1} \right) e^{-Z(n_{z})} \right\} = 0.
\]

Also, since \( e^{-Z(n_{z})} \neq 0 \) and \( \rho_{z} = \frac{\lambda_{z}}{\mu_{z}} \) then in (29) and (30), we have

\[
\sum_{n_{z}=1}^{N_{z}} \left( \frac{\beta_{z} \mu_{z} \rho_{z}}{\alpha_{z} p_{z}} \right)^{n_{z}} \left( \frac{n_{z} + 1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right) \left( n_{z} \left( \frac{\lambda_{z}}{\mu_{z}} \right)^{n_{z}-1} \right)
\]

\[
\times \left\{ \left[ 1 + \frac{1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right]^{-1} \right\}
\]

\[
- \left\{ 1 + \frac{1}{\beta_{z}} \sum_{n_{z}=1}^{N_{z}} \left( \frac{\beta_{z} \mu_{z} \rho_{z}}{\alpha_{z} p_{z}} \right)^{n_{z}} \left( \frac{n_{z} + 1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right) \left( n_{z} \rho_{z}^{n_{z}-1} \right) e^{-Z(n_{z})} \right\} = 0,
\]

and

\[
\left\{ Z(n_{z}) - \lambda_{z} \right\} > \left\{ \left[ 1 + \frac{1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right]^{-1} \right\}
\]

\[
\sum_{n_{z}=1}^{N_{z}} \left( \frac{\beta_{z} \mu_{z} \rho_{z}}{\alpha_{z} p_{z}} \right)^{n_{z}} \left( \frac{n_{z} + 1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right) \left( n_{z} \left( \frac{\lambda_{z}}{\mu_{z}} \right)^{n_{z}-1} \right)
\]

\[
\times \left\{ \left[ 1 + \frac{1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right]^{-1} \right\}
\]

\[
- \left\{ 1 + \frac{1}{\beta_{z}} \sum_{n_{z}=1}^{N_{z}} \left( \frac{\beta_{z} \mu_{z} \rho_{z}}{\alpha_{z} p_{z}} \right)^{n_{z}} \left( \frac{n_{z} + 1}{\beta_{z} \prod_{j=0}^{n_{z}-1} \left( \frac{\mu_{z} q_{\omega_{z}^{(n_{z})}}}{\alpha_{z} p_{z}} + j \right)} \right) \left( n_{z} \rho_{z}^{n_{z}-1} \right) e^{-Z(n_{z})} \right\} = 0.
\]
where \( r_\sigma = E(X_\sigma) + \Omega_\sigma \sqrt{\text{Var}(X_\sigma)} \). Consequently, from (31) and (32) we get

\[
\{Z_{(n,\sigma)} - r_\sigma\} \sum_{n_\sigma = 1}^{N_\sigma} \left( \frac{\beta_\sigma \lambda_\sigma}{\alpha_\sigma p_\sigma} \right)^{n_\sigma} \left( \frac{\prod_{j=0}^{n_\sigma-1} }{((\mu_\sigma q_\sigma \omega j+1)/\alpha_\sigma p_\sigma) + j} \right)^{-1} \\
\left(1 + \frac{1}{\beta_\sigma} \sum_{n_\sigma = 1}^{N_\sigma} \left( \frac{\beta_\sigma \lambda_\sigma}{\alpha_\sigma p_\sigma} \right)^{n_\sigma} \left( \frac{\prod_{j=0}^{n_\sigma-1} }{((\mu_\sigma q_\sigma \omega j+1)/\alpha_\sigma p_\sigma) + j} \right)^{-1} \right) \\
- \sum_{n_\sigma = 1}^{N_\sigma} \left( \frac{\beta_\sigma \mu_\sigma}{\alpha_\sigma p_\sigma} \right)^{n_\sigma} \left( \frac{\prod_{j=0}^{n_\sigma-1} }{((\mu_\sigma q_\sigma \omega j+1)/\alpha_\sigma p_\sigma) + j} \right)^{-1} \left( \frac{\lambda_\sigma}{\mu_\sigma} \right)^{n_\sigma-1} \\
\times \left(1 + \frac{1}{\beta_\sigma} \sum_{n_\sigma = 1}^{N_\sigma} \left( \frac{\beta_\sigma \lambda_\sigma}{\alpha_\sigma p_\sigma} \right)^{n_\sigma} \left( \frac{\prod_{j=0}^{n_\sigma-1} }{((\mu_\sigma q_\sigma \omega j+1)/\alpha_\sigma p_\sigma) + j} \right)^{-1} \right) \\
- \left(1 + \frac{1}{\beta_\sigma} \sum_{n_\sigma = 1}^{N_\sigma} \left( \frac{\beta_\sigma \lambda_\sigma}{\alpha_\sigma p_\sigma} \right)^{n_\sigma} \left( \frac{\prod_{j=0}^{n_\sigma-1} }{((\mu_\sigma q_\sigma \omega j+1)/\alpha_\sigma p_\sigma) + j} \right)^{-1} \right) \left( \frac{\lambda_\sigma}{\mu_\sigma} \right)^{n_\sigma-1} \right)^2 = 0.
\]

If the values of \( N_\sigma \) and \( \alpha_\sigma, \beta_\sigma, q_\sigma, p_\sigma, \lambda_\sigma, n_\sigma \), and \( \omega^{(n_\sigma)} \), \( j = 1, 2, \ldots, n_\sigma \) are generated randomly, then one can use (33) to get the minimum values of \( \rho_\sigma^* \) and \( Z_{n_\sigma}^* \) numerically. The minimum value of the traffic intensity \( \rho_\sigma^* \) means the value of the service rate attains its maximum \( \mu_\sigma^* \) which eases the crowding in the suitable company under the quality control process.

### 5 APPLICATION

Since COVID-19 swept all the world, competition has raged between international research and pharmaceutical companies to produce the vaccine earlier. These giant companies have intensified their efforts to finish the clinical trials of the potential vaccines. Therefore, we assume that six international companies with different absorptive capacities \( N_\sigma \), \( \sigma = 1, 2, \ldots, 6 \) reach the production stage of the vaccine. Naturally, the countries choose vaccines that are safer and less expensive. Some vaccines are safer, but they are more expensive because there are some difficulties such as the way of storage (e.g., Pfizer-Pew&Tech COVID-19 vaccine). Each country chooses the appropriate vaccine, according to its financial capabilities. During the clinical trial periods of the vaccine, some countries retreated their requests from some companies, which had already booked and moved to reserve the same quantity from another company. Our model contributes the country to choose the appropriate company to provide it with the appropriate vaccine at minimum time and cost. For all these hypotheses, let the values of \( \sigma, \beta_\sigma, q_\sigma, p_\sigma, \lambda_\sigma, n_\sigma \), and \( \omega^{(n_\sigma)} \), \( j = 1, 2, \ldots, n_\sigma \) be generated randomly by using Maple 13 (see Table 1). We use (33) to obtain the maximum values of \( \mu_\sigma^* \) and the minimum values of \( Z_{n_\sigma}^* \) numerically as given in Table 1. Figure 2 presents the plotting of three dimensions of (33). It presents the relationship between \( \mu_\sigma \) and \( Z_{n_\sigma} \) for \( N_\sigma, \sigma = 1, 2, \ldots, 6 \) as two intersected planes, where all points in the interested curve are satisfied (33). Thus, we have an infinite number of solutions that give the optimal values \( \mu_\sigma^* \) and \( Z_{n_\sigma}^* \). By using Maple 13, we deduce an equation of \( \mu_\sigma \) and \( Z_{n_\sigma} \) (equation of the intersected curve) which presents the intersection between the two planes. Also, the relationship between \( \mu_\sigma \) and \( Z_{n_\sigma} \) appears in Figure 3. Since \( 0 < \lambda_\sigma < \mu_\sigma \) then one can determine the minimum value \( Z_{n_\sigma}^* \) which corresponds to the maximum value \( \mu_\sigma^* \) for all \( N_\sigma, \sigma = 1, 2, \ldots, 6 \). These optimal values depend on the maximum probabilities \( p_{(n_\sigma)\sigma}^* \) and \( p_{(n_\sigma)\sigma}^* \) for each queue \( Q_\sigma \), \( \sigma = 1, 2, \ldots, 6 \). We use (9) and (10) to get these probabilities (see Table 1). In addition, to obtain the maximum probability of detection \( W^* (Z) \), we substitute in (16) to get \( \hat{W} (Z) = 0.00330037 \) and then obtain the maximum value of \( W^* (Z) = 1 - \hat{W} (Z) = 0.9966996308 \).

From Table 1, some countries consider \( Q_6 \) as a suitable company where its cost is minimum \( (Z_{n_6}^* \approx 4.967348633) \) and it has a service rate \( \mu_3^* \approx 0.825 \) (see Figure 3F). There are many factors that made this company the best despite its large
| $\sigma$ | $N_\sigma$ | $\lambda_\sigma$ | $\alpha_\sigma$ | $\beta_\sigma$ | $q_\sigma$ | $p_\sigma$ | $r_\sigma$ | $n_\sigma$ | $\omega^{(n_\sigma)}_j$ | $\mu_\sigma^*$ | $Z_{n_\sigma}^*$ | $p_{0\sigma}^*$ | $p_{(n_\sigma)\sigma}^*$ |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|-----------------|-----------|-------------|--------------|------------------|
| 1     | 15     | 7.859854202 | 0.4349854305 | 0.5119686803 | 0.5379125549 | 0.4620874451 | 3.882435139 | 6      | 1, 1, 1, 0, 1 | 7.863     | 18.83584083  | 2 x 10^{-13} | 0.511969       |
| 2     | 20     | 6.659373677 | 0.8754630337 | 0.007913860755 | 0.176707474  | 0.8223292526 | 6.398451650 | 6      | 1, 1, 0, 0, 1 | 6.659     | 7.006323135  | 0.004459     | 0.436566       |
| 3     | 25     | 5.538871347 | 0.8437148787 | 0.1838989170 | 0.1378970236 | 0.8621029764 | 5.841354561 | 15     | 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1 | 5.593     | 5.841396166  | 2.2 x 10^{-7} | 0.999998       |
| 4     | 30     | 5.762906241 | 0.5103203005 | 0.1783351410 | 0.6232007062 | 0.3767992938 | 5.813652052 | 4      | 1, 0, 1, 1    | 5.769     | 35.61496709  | 3.1 x 10^{-20} | 0.178335       |
| 5     | 35     | 1.311287770 | 0.9581034987 | 0.8028754624 | 0.6788365468 | 0.3211634352 | 2.868143393 | 3      | 1, 0, 1       | 1.317     | 37.60583203  | 1.3 x 10^{-18} | 0.802875       |
| 6     | 40     | 0.7593374481 | 0.3081840326 | 0.4645548800 | 0.4822218558 | 0.517781442  | 4.967348635 | 27     | 1, 0, 1, 1, 1 | 0.825     | 4.967348633  | 8 x 10^{-15}  | 1               |
capacity $N_6 = 40$. Although $n_6 = 27$ (it is a large number of countries corresponds to $Q_6$ capacity) the probabilities $p^*_{06} \cong 1$ and $p^*_{(n_6)6} \cong 8 \times 10^{-15}$ (i.e., there is no unit in $Q_6$ or all units serve until its arrival). Also, this queue has a high rate of departure where $q_3 = 0.482218558$. On the other hand, the inspection process was carried out many times with this service rate. This means that this company accomplishes the required quantities with minimum time, cost, and with the highest possible accuracy. But its balking rate $\beta_6 = 0.4645488$ may be high compared with the other companies such as $Q_2$, $Q_3$, and $Q_4$. This may be due to the spread of bad reports about vaccine safety after some clinical trials. Anyway, choosing the suitable company is the decision for the country.
Figure 3: The optimal values of $\mu^*$ and $Z^*_\sigma$ for different values of $N_\sigma$. 

(A) $N_1 = 15$
(B) $N_2 = 20$
(C) $N_3 = 25$
(D) $N_4 = 30$
(E) $N_5 = 35$
(F) $N_6 = 40$
TABLE 2  The optimal values of the performance measures for each queue $Q_\sigma, \sigma = 1, 2, \ldots, 6$

| $\sigma$ | $L_0^\sigma \cong$ | $L_{q0}^\sigma \cong$ | $L_{q1}^\sigma \cong$ | $H_0^\sigma$ | $H_{q0}^\sigma$ | $H_{q1}^\sigma$ | $R_{r\sigma}$ | $R_{R\sigma}$ |
|---------|----------------|----------------|----------------|-------------|-------------|-------------|--------------|--------------|
| 1       | 3              | 2              | 1              | 0.3908232801| 0.2635494569| 0.1272288231| 10.80516922  | 12.57821706  |
| 2       | 3              | 2              | 1              | 0.3933396933| 0.2438450039| 0.1494946895| 0.6099139088 | 0.1317767301 |
| 3       | 15             | 14             | 1              | 2.708127534 | 2.527585377 | 0.1805421569| 0.0000492871 | 0.0000078837 |
| 4       | 1              | 0              | 1              | 0.1237812954| 0.1237812954| 14.91691831 | 24.67158028  |
| 5       | 2              | 1              | 1              | 1.836839369 | 1.074230258 | 0.1805421569| 0.0000492871 | 0.0000078837 |
| 6       | 0              | 0              | 0              | 0            | 2.8 × 10^{-3} | 2.8 × 10^{-3} | 0             | 1.007 × 10^{-12} | 9.3 × 10^{-13} |

Figure 4 presents a nonlinear relationship between $L_0^\sigma$ and $\alpha_\sigma, \sigma = 1, 2, \ldots, 6$. This shows the effect on the optimal expected number of units in the system number $\sigma = 1, 2, \ldots, 6$ with the random values $\alpha_\sigma$. This relationship is insufficient to judge on the suitable queue.

In the above calculations, we take into consideration the profit of the clients only, and we neglect it for the company. It is assumed that there will be a profit of the company producing the vaccine in order to continue providing its services. Thus, we need to strike a balance between all these hypotheses. To do this balance, we deal with an optimization model which was used by Kotb and El-Ashkar\textsuperscript{35} to select a suitable company.

5.1 An optimal decision with maximum profit and minimum searching effort

Adequate decision making is a measurement method that presents the governments’ ability to accomplish difficult tasks. The previous part concerns only with finding the suitable company with the minimum cost $Z_{n\sigma}$ and the maximum service rate $\mu_{c\sigma}$ and do not take into its consideration the profit which returns to the company. This profit is an important factor...
TABLE 3 The values of $T_{EC_{\sigma}}$, $T_{ER_{\sigma}}$, and $T_{EP_{\sigma}}$ for each queue $Q_{\sigma}, \sigma = 1, 2, \ldots, 6$

| $\sigma$ | $N_{\sigma}$ | $T_{EC_{\sigma}}$ | $T_{ER_{\sigma}}$ | $T_{EP_{\sigma}}$ |
|--------|--------------|----------------|----------------|----------------|
| 1      | 15           | 46.56678681    | 84.96104558    | 38.39425877    |
| 2      | 20           | 209.6911516    | 2.288339893    | $-$207.4028117 |
| 3      | 25           | 493.1860906    | $3.385137 \times 10^{-10}$ | $-$493.1860906 |
| 4      | 30           | 34.3331459     | 86.05570173    | 51.72238714    |
| 5      | 35           | 155.1527190    | $3.385137 \times 10^{-10}$ | $-$155.1527190 |
| 6      | 40           | 20.54596928    | 0              | $-$20.54596928 |

FIGURE 5 The relationship between $T_{EP_{\sigma}}$ and $q_{\sigma}, \sigma = 1, 2, \ldots, 6$

that contributes to the best decision. With this profit, the company will have the ability to continue with the vaccine production. Thus, we take this condition in our consideration when we choose the suitable company.

To identify the suitable company, we calculate (i) the total expected cost per unit time for each queue $Q_{\sigma}, \sigma = 1, 2, \ldots, 6$ from the following:

$$T_{EC_{\sigma}} = \mu_{\sigma}(C_{S\sigma} + p_{1\sigma}^{*}C_{1S\sigma}) + C_{h\sigma}L_{\sigma}^{0} + C_{l\sigma}\lambda_{\sigma}p_{(N_{\sigma})\sigma} + C_{r\sigma}R_{r\sigma} + C_{R\sigma}R_{R\sigma}$$

$$= \mu_{\sigma}\left(C_{S\sigma} + \frac{\lambda_{\sigma}C_{1S\sigma}}{\mu_{\sigma}q_{\sigma}\omega(1)_{\sigma}}p_{0\sigma}\right) + \frac{C_{h\sigma}}{\beta_{\sigma}} \sum_{n_{\sigma}=1}^{N_{\sigma}} \left(\frac{\beta_{\sigma}\lambda_{\sigma}}{\alpha_{\sigma}p_{\sigma}}\right)^{n_{\sigma}} \left(\beta_{\sigma} \prod_{j=0}^{n_{\sigma}-1} \left(\frac{(\mu_{\sigma}q_{\sigma}\omega(N_{\sigma})_{j+1}/\alpha_{\sigma}p_{\sigma})}{\alpha_{\sigma}p_{\sigma}} + 1\right)\right)^{-1} p_{0\sigma}^{*}$$

$$+ \left(C_{l\sigma}p_{\sigma} + C_{R\sigma}q_{\sigma}\right) \sum_{n_{\sigma}=1}^{N_{\sigma}-1} \left(\frac{\beta_{\sigma}\lambda_{\sigma}}{\alpha_{\sigma}p_{\sigma}}\right)^{n_{\sigma}} \left(\prod_{j=0}^{N_{\sigma}-1} \left(\frac{(\mu_{\sigma}q_{\sigma}\omega(N_{\sigma})_{j+1}/\alpha_{\sigma}p_{\sigma})}{\alpha_{\sigma}p_{\sigma}} + 1\right)\right)^{-1} p_{0\sigma}^{*}$$

(see Koth and El-Ashkar$^{35}$); (ii) the total expected revenue per unit time $T_{ER_{\sigma}} = R_{\sigma}\mu_{\sigma}(1 - p_{0\sigma}^{*})$; and (iii) the total expected profit per unit time $T_{EP_{\sigma}} = T_{ER_{\sigma}} - T_{EC_{\sigma}}$. Besides the above randomly generated values of $\alpha_{\sigma}, \beta_{\sigma}, q_{\sigma}, p_{\sigma}, \lambda_{\sigma}, n_{\sigma}, \omega_{(n_{\sigma})_{j}, j = 1, 2, \ldots, n_{\sigma}, \sigma = 1, 2, \ldots, 6}$ from Table 1 and the values $R_{r\sigma}, R_{R\sigma}$ from Table 2, we consider $C_{3S\sigma} = 2, C_{3S\sigma} = 12, C_{l\sigma} = 3, C_{l\sigma} = 1, C_{r\sigma} = 2, and C_{R\sigma} = 3$. Thus, the values of $T_{EC_{\sigma}}, T_{ER_{\sigma}}, and T_{EP_{\sigma}}$ are given in Table 3.

As in Figure 5, the companies with the waiting list $Q_{4}$ and $Q_{5}$ have the maximum total expected profit per unit time. With these companies, the country ensures that the production continues with high safety ratios of the produced vaccine. Now, the decision is to choose the best among them. From Table 1, we found the company with the waiting list $Q_{4}$ is more suitable, where $Z_{n_{4}^{*}}$ is smaller than $Z_{n_{5}^{*}}$. The departure rates $q_{4}$ and $q_{5}$ are some convergent, respectively. The values of $R_{r4}, R_{R4}$ are smaller than $R_{r5}, R_{R5}$.
This model presents more quality control in finding the suitable company with minimum effort. At the same time, it is keeping the company from losing.

6 | CONCLUDING REMARKS AND THE FUTURE WORK

A model that presented here increases the speed of finding the suitable company with minimum effort to get a vaccine of COVID-19. It also provides more quality control on the production process of this vaccine. The country’s financial and economic conditions are taken into consideration. The waiting list of each company contains a set of countries. Each country has an exponential service time and a Poisson arrival process. Under a steady-state situation, we solved an interesting and difficult discrete stochastic optimization problem to get the minimum detection effort. This effort has a known bounded probability distribution. An application has been presented to show how this model presented more quality control in finding the suitable company with minimum effort. At the same time, it is keeping this company from losing. Also, the optimal performance measures, the highest service rate, and the total expected profit of each queuing system are obtained.

In the future work, we can consider the solution of this model under transient behavior to support the decision making about choosing the suitable company with maximum quality control. If we consider the stochastic volatility parameter in this model then the companies will be avoiding the sudden loss. Moreover, this model can be extendable to cover a multiserver setting in the case of multiple production lines for each company.

DATA AVAILABILITY STATEMENT

The data used in the application example are available from the MAPLE 13 programs used here can be requested from the authors.

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