Medium-induced gluon emission via transverse and longitudinal scattering in dense nuclear matter

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We study the medium-induced gluon emission from a hard quark jet traversing the dense nuclear matter within the framework of deep inelastic scattering off a large nucleus. We extend the previous work and compute the single gluon emission spectrum including both transverse and longitudinal momentum exchanges between the hard jet parton and the medium constituents. On the other hand, with only transverse scattering and using static scattering centers for the traversed medium, our induced gluon emission spectrum in the soft gluon limit reduces to the Gyulassy-Levai-Vitev one-rescattering-one-emission formula.

I. INTRODUCTION

The study of parton energy loss and jet quenching has been regarded as a very useful tool to probe the properties of the quark-gluon plasma (QGP) produced in ultra-relativistic heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC). After they are produced from early stage hard collisions, high transverse momentum partonic jets propagate through the dense nuclear medium and interact with the medium constituents via binary elastic and radiative inelastic collisions before fragmenting into hadrons. Jet-medium interaction not only changes the energy of the leading parton, but also modifies the internal structure of the jets, such as the distribution of jet momentum among different jet constituents. The picture of jet-medium interaction and parton energy loss has been confirmed by many experimental observations at RHIC and the LHC, such as the significant suppression of high transverse momentum hadron and jet production in nucleus-nucleus collisions as compared to the expectations from independent nucleon-nucleon collisions [5–11].

Several theoretical schemes have been founded to study the radiative component of energy loss experienced by the hard jet partons propagating through the dense nuclear medium, such as Baier-Dokshitzer-Mueller-Peigne-Schiff-Zakharov (BDMPS-Z) [12–19], Gyulassy, Levai and Vitev (GLV) [20–22], Armesto-Salgado-Wiedemann (ASW) [23–25], Arnold-Moore-Yaffe (AMY) [26–27], and higher twist (HT) [28–30] formalisms. For a more detailed comparison of different jet energy loss schemes, the reader is referred to Ref. [31] and references therein. There have also been many literatures studying the effect of binary elastic collisions between the hard partons and the medium constituents [32–35]. Also, extensive studies have been performed for various jet quenching observables, such as the suppression of large transverse momentum hadron production [36–39], the nuclear modification of back-to-back dihadron and photon-hadron pair correlations [40–42], and the full jet modification [43–45]. By comparing phenomenological jet modification calculations to experimental measurements, one may obtain the quantitative values of jet transport coefficients, such as the transverse momentum broadening rate \( \dot{q} = -d\langle (\Delta p_T)^2 \rangle / dt \) and the longitudinal momentum loss rate \( \dot{e} = -dE/dt \) [38], from which a lot of information about the dense nuclear medium traversed by the hard jets can be inferred.

While the studies of jet quenching in heavy-ion collisions have already entered the quantitative era, there still exist many theoretical uncertainties in detailed calculations of the effects caused by jet-medium interaction. For example, most current studies of inelastic radiative contribution mainly focus on the gluon emission induced by the transverse momentum exchange between the propagating hard jet partons and the dense nuclear matter. However, when a hard parton interacts with the traversed dense medium, both transverse and longitudinal momenta are exchanged between them [44–50]. While there have been many studies on the longitudinal momentum exchange (loss) experienced by the propagating jet partons, the main focus was on the evaluation of purely collisional energy loss either from the leading parton [51–53] or by the shower partons of the full jet [54–57]. In Refs. [58–60], the effect of longitudinal momentum transfer between the hard parton and the medium constituents on the medium-induced emission vertex has been studied.

In this work, we study the medium-induced gluon emission from a hard quark jet which scatters off the medium constituents during the propagation through the dense nuclear medium, within the framework of deep-inelastic scattering (DIS). We extend the HT radiative energy loss approach [28–29] and include the contributions from both transverse and longitudinal momentum exchanges to the gluon emission vertex. It is also an extension of Refs. [62–63] which study the medium-induced photon emission from longitudinal and transverse scatterings. Here we derive a closed formula for the medium-induced single gluon emission spectrum with the inclusion of the contributions from both transverse and longitudinal momentum transfers between the hard parton and the medium constituents. We further show that if one neglects the
longitudinal momentum transfer and only considers the transverse scattering, our medium-induced gluon emission spectrum in the soft gluon limit can reduce to the GLV one-scattering-one-vertex formula.

The paper is organized as follows. In Sec. II, we provide some details for the derivation of the medium-induced gluon emission spectrum for a quark jet parton traversing the dense nuclear medium. Other details may be found in the Appendix. Sec. III contains our summary.

II. MEDIUM-INDUCED GLUON EMISSION

Here we study the gluon emission from a hard jet parton in dense nuclear matter in the framework of deep inelastic scattering (DIS) off a large nucleus. We consider the following DIS process:

\[ e(L_1) + A(P_A) \rightarrow e(L_2) + q(l_q) + g(l) + X, \quad (1) \]

where \( L_1 \) and \( L_2 \) are the momenta of the incoming and outgoing leptons, and \( P_A = Ap \) is the momentum of the incoming nucleus, with \( p = [p^+, p^-, p_L] = [p^+, 0, 0_L] \) being the momentum carried by each nucleon in the nucleus, \( l_q \) and \( l \) are the momenta of the produced hard quark and the radiated gluon. Here the light-cone notation is used for four-vectors, e.g., \( q = (q^+, q^-) \) being the momentum carried by each nucleon in the nucleus. The light-cone gauge is given by:

\[ \tilde{g}^\alpha\beta(l) = -g^{\alpha\beta} + \frac{n^\alpha l^\beta + n^\beta l^\alpha}{n \cdot l}. \quad (5) \]

In the limit of very high energy and collinear emission, one may neglect the \( (\perp) \) component of the incoming quark field operators and factor out the one-nucleon state from nucleus state. After some straightforward calculation, the contribution to the hadronic tensor at leading twist can be obtained as follows:

\[ \frac{dW}{d^2 l} = \sum_q Q_q^2 (-g_\perp^{\mu\nu}) A_B(g_\perp(2\pi)f_q(x_B + x_L)) \times \frac{\alpha_s}{2\pi} C_F P(y) \frac{1}{l_\perp^2}. \quad (6) \]

Here \( g_\perp^{\mu\nu} = g^{\mu\nu} - g^{\mu\perp}g^{\perp\nu} - g^{\mu\perp}g^{\perp\nu} \), \( C_F \) denotes the probability of finding a nucleon state with a momentum \( p \) inside the nucleus \( A \), \( \alpha_s \) is the strong coupling, and \( y = l^\perp / q^- \) and \( l_\perp \) are the fractions of the forward momentum and transverse momentum carried by the radiated gluon with respect to the parent quark. For convenience, the momentum fraction \( x_L = l_\perp^2 / \frac{2p^+q^-y}{(1-y)} \) is also defined. The quark parton distribution function \( f_q(x) \) is defined as:

\[ f_q(x) = \int \frac{dy_0}{2\pi} e^{-ixp^+y_0} \langle p | \bar{\psi}(y_0) \gamma^\perp(x) \psi(0) | p \rangle. \quad (7) \]

where \( x \) is the fraction of the forward momentum carried by the quark from the nucleon. The leading order quark-to-gluon (photon) splitting function \( P(y) \) is given by:

\[ P(y) = \frac{1 + (1 - y)^2}{y}. \quad (8) \]

Note the color factor \( C_F \) for quark to gluon splitting vertex is factored out. Therefore, the differential single gluon emission spectrum at leading order (without
rescattering with the medium constituents) is given as:

$$\frac{dN_g^{\text{res}}}{dy d\ell^2_{\perp}} = \frac{\alpha_s}{2\pi} C_F \frac{P(y)}{\ell^4_{\perp}} \frac{1}{1 - y} \frac{1 - y}{\ell^2_{\perp}}. \quad (9)$$

We now study the medium modification on the above gluon emission spectrum due to the re-scattering with the medium constituents. We will compute the medium-induced gluon emission spectrum from a hard quark jet traversing the dense nuclear medium in the DIS framework. In this work, we use the following power counting scheme and notations: $Q$ for the hardest momentum scale, and $\lambda$ for a small dimensionless parameter. Considering the scattering of a nearly on-shell projectile parton carrying a momentum $q = (q^+, q^{-}, q_{\perp}) \sim (\lambda^2 Q, Q, 0)$ off a nearly on-shell target parton carrying a momentum $\sim (Q, \lambda^2 Q, 0)$, the exchanged gluons is then the standard Glauber gluon carrying a momentum $\sim (\lambda^2 Q, \lambda Q, \lambda Q)$ \(^7\). If the target parton is allowed to be off shell, the longitudinal momentum component of the exchanged gluon may be the same order as the transverse components; such type of gluons is often referred to as the longitudinal-Glauber gluon which carries a momentum $\sim (\lambda^2 Q, \lambda Q, \lambda Q)$ \(^6\). In this work, we will investigate the influence of both transverse and longitudinal momentum transfers on the single gluon emission from a hard quark jet which interacts with the constituents of the dense nuclear medium.

Figure 2 shows one of central-cut diagrams that contributes to the hadronic tensor at the twist-four level. It describes the process with a single rescattering on the radiated gluon in both the amplitude and the complex conjugate. The other 20 diagrams at the twist-four level are presented in Appendix. In this section, we provide the detailed calculation of the hadronic tensor and the medium-induced gluon emission spectrum for Figure 2. The calculations for the other 20 diagrams are completely analogous and their main results are listed in Appendix.

![Figure 2](image-url)
To simplify the hadronic tensor $W_{(2)}^{A\mu
u}$, one first isolates the phase factors associated with the gluon insertions: $e^{-i(l_q + l_1 - q_1):z_1}e^{i(l_q' + l_1') - z_1'}$. After carrying out the integration over $z$ and $z'$, we obtain two $\delta$ functions which may be used to integrate over the momenta $q_1$ and $q_1'$, rendering the relations: $q_1 = l_q - l_1$ and $q_1' = l_q' - l_1'$. From the momentum conservation at each vertex, we can obtain the following relations among various momenta in Figure 2

\[ p_0 = q_1 - q, \quad k_1 = l - l_1, \quad p_0' = q_1' - q, \quad k_1' = l - l_1'. \]  

(14)

Re-introducing the momentum $p_0 = l + l_q - k_1 - q$ and changing the integration variables $l_1 \rightarrow k_1$ and $l_1' \rightarrow k_1'$, the phase factor can be expressed as: $e^{-i p_0 \cdot y e^{-ik_1 \cdot z_1} e^{ik_1' \cdot z_1'}}$.

In this study, we work in the very high energy and collinear emission limit, and the dominant component of the rescattered gluon field is the forward (+) component. In such limit, one may factor out one-nucleon state from the nucleus state and ignore the ($\perp$) component of the quark field operators,

\[ \langle A|\bar{\psi}(y_0)\gamma^\mu \hat{O}\gamma^\nu\psi(0)|A \rangle \approx AC_p^A(p|\bar{\psi}(y_0)\gamma^+ \psi(0)|p) \text{Tr}[\gamma^- \gamma^\mu \gamma^\nu \gamma^\nu + \frac{1}{2}] \text{Tr}[\gamma^- (\hat{O}|A)]. \]  

(15)

The gluon propagators together with the three-gluon vertices in the hard trace part may be simplified as:

\[ \tilde{G}^{\alpha\beta\gamma} (l_1) \tilde{\Gamma}^{\alpha\gamma\beta} (l_1) \tilde{A}_c^{\alpha'} (z_1) \tilde{G}^{\beta\alpha'\gamma} (l_1) \tilde{A}_c^{\beta} (z_1) \tilde{G}^{\gamma\beta\alpha'} (l_1) \tilde{A}_c^{\gamma} (z_1) = \tilde{G}^{\alpha\beta\gamma} (l_1) [g_{\alpha\beta} (l + l_1 - A_{c}^{\alpha} (z_1)) ] \tilde{G}^{\beta\alpha'\gamma} (l_1) [g_{\alpha'\beta} (l + l_1' - A_{c}^{\beta} (z_1)) ] \tilde{G}^{\gamma\beta\alpha'} (l_1). \]  

(16)

With the above simplification, the hadronic tensor may be written as follows:

\[ W_{(2)}^{A\mu
u} = \sum Q_q^2 g^4 \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_1'}{(2\pi)^4} 2\pi \delta (l_1^2) \int \frac{d^4 l_q}{(2\pi)^4} 2\pi \delta (l_q^2) \int d^4 y_0 \int d^4 z_1 \int d^4 z_1' \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_1'}{(2\pi)^4} \int \frac{d^4 p_0}{(2\pi)^4} \int d^4 q_0, \]

\[ \times \left( \frac{1}{q_1^2 - i\epsilon} \frac{1}{l_1^2 + i\epsilon} \frac{1}{l_1'^2 + i\epsilon} \right) \frac{1}{q_1'^2 - i\epsilon} \frac{1}{l_1'^2 + i\epsilon} \frac{1}{l_1'^2 + i\epsilon} \times (-g_{\mu\nu}) A_C^A (p|\bar{\psi}(y_0)\gamma^+ \psi(0)|p) \langle A|A_c^{\gamma} (z_1) A_c^{\beta} (z_1)|A \rangle \times \frac{1}{N_c} \text{Tr}[T^{\alpha_0} f^{a_0\alpha_1} f^{a_1\alpha_2} T^{\alpha_2}]. \]  

(17)

We now look at the internal propagators and external lines. For the quark propagator before the gluon emission,

\[ q_1^2 = (q + p_0)^2 = 2p^+ q^- (1 + x_0)^{-1} \lambda_B + x_0 - x_B, \]  

(18)

where we have defined the momentum factors,

\[ x_0 = \frac{p_0^+}{p^+}, \quad x_0' = \frac{p_0^-}{q^-}, \quad x_{D0} = \frac{p_0^2}{2p^+ q^- (1 + x_0)}. \]  

(19)

For the internal gluon propagator,

\[ l_1^2 = (l - k_1)^2 = 2p^+ q^- (y - \lambda^-_1) [x_L (1 - y) - \lambda_1 - \lambda_{D1}], \]  

(20)

where we have defined the momentum factors,

\[ \lambda_1 = \frac{k_1^+}{p^+}, \quad \lambda_1^- = \frac{k_1^-}{q^-}, \quad \lambda_{D1} = \frac{(1 - k_{1\perp})^2}{2p^+ q^- (y - \lambda_1^+)}. \]  

(21)

For the final outgoing quark, the on-shell condition gives:

\[ (2\pi)\delta (l_1^2) = \frac{1}{2p^+ q^- (1 + x_0 + \lambda_1 - y)} (2\pi)\delta (-x_B + x_0 + \lambda_1 - x_L (1 - y) - \eta_{D1}). \]  

(22)

where we have defined the momentum factors,

\[ \eta_{D1} = \frac{(1 - k_{1\perp} - p_{0\perp})^2}{2p^+ q^- (1 + x_0 + \lambda_1 - y)}. \]  

(23)
Combining the internal quark and gluon lines with the final outgoing quark,

\[
D_q = \frac{C_q}{(2q^-)^3(p^+)^3} \frac{1}{-x_B + x_0 - x_{D0} - i\epsilon (y - \lambda^-_1)(x_L (1 - y) - \lambda_1 - \lambda_{D1} - i\epsilon) - x_B + x'_0 - x'_{D0} + i\epsilon} \frac{1}{y - \lambda^-_1} (y - \lambda^-_1)(x_L (1 - y) - \lambda'_1 - \lambda'_{D1} + i\epsilon) (2\pi) \delta(-x_B + x_0 + \lambda_1 - x_L (1 - y) - \eta_{D1}),
\]

where

\[
C_q = \frac{1}{1 + x_0^-} \frac{1}{1 + x_0^-} \frac{1}{1 + x_0^- + \lambda_1 - y}.
\]

The trace part of the hadronic tensor [the last line of Eq. (17)] can be simplified as:

\[
N_q = \frac{4q^-}{C_q} \frac{1 + (1 - y - \lambda^-_1/1 + x_0^-)(1 - y - \lambda^-_1/1 + x_0^-) (1 - \lambda_1 - \lambda_{D1} + k_1 - q_0, p_0) \cdot \left( \begin{array}{c}
1 - k_1 \perp \frac{y - \lambda^-_1}{1 + x_0^-} p_0 \perp \end{array} \right)}{(y - \lambda^-_1)(y - \lambda^-_1) (1 - y - \lambda'_1 - \lambda'_{D1} + i\epsilon)},
\]

With the above simplifications, the hadronic tensor now reads:

\[
W_{(2)}^{\mu
u} = \sum_q Q_q^2 g_4 \int \frac{d^4l}{(2\pi)^4} 2\pi \delta(l^2) \int \frac{d^4l'}{(2\pi)^4} 2\pi \delta(l + q_0 - p_0 - k_1 - q_0) \int d^4y_0 \int d^4z_1 \int d^4z_1' \times
\]

\[
\times \frac{1}{-x_B + x_0 - x_{D0} - i\epsilon} \frac{1}{x_L (1 - y) - \lambda_1 - \lambda_{D1} - i\epsilon} \frac{1}{x_B + x'_0 - x'_{D0} + i\epsilon} \frac{1}{x_L (1 - y) - \lambda'_1 - \lambda'_{D1} + i\epsilon} \frac{1}{(2\pi)^4} \frac{(e^{-ix_0 p^+ y_0} e^{-i\lambda_1 p^+ z_1^+} e^{i\lambda'_1 p' z_1^+})}{(y - \lambda^-_1)(y - \lambda^-_1) (1 - y - \lambda'_1 - \lambda'_{D1} + i\epsilon)} \frac{1}{(1 - y - \lambda^-_1/1 + x_0^-)(1 - y - \lambda'_1/1 + x_0^-)}
\]

\[
\times \left( \begin{array}{c}
1 - k_1 \perp \frac{y - \lambda^-_1}{1 + x_0^-} p_0 \perp \end{array} \right) \left( \begin{array}{c}
1 - k_1' \perp \frac{y - \lambda^-_1}{1 + x_0^-} p'_0 \perp \end{array} \right),
\]

where for convenience, we have used the three-vector notations for momentum and coordinate: \( k = (k^-, k^\perp) \) and \( z = (z^+, z_\perp) \), and \( k \cdot z = k^- z^+ - k_1^\perp \cdot z_\perp \).

Now we perform the integration over the momentum fractions \( x_0, \lambda_1, \) and \( \lambda'_1 \). Using the on-shell condition for the outgoing quark and the overall momentum conservation \( p_0 = p_0 + k_1 - k'_1 \), we can carry out the integration over \( x_0 \),

\[
\int \frac{dx_0}{2\pi} \frac{e^{-ix_0 p^+ y_0}}{-x_B + x_0 - x_{D0} - i\epsilon} \frac{1}{x_B + x'_0 - x'_{D0} + i\epsilon} \frac{(2\pi) \delta(-x_B + x_0 + \lambda_1 - x_L (1 - y) - \eta_{D1})}{(2p^+ q^-)^2} \frac{1}{y - \lambda^-_1} \frac{1}{(y - \lambda^-_1)}
\]

\[
= \frac{e^{-i(x_B + x_L (1 - y) + \eta_{D1}) p^+ y_0}}{x_L (1 - y) - \lambda_1 + \eta_{D1} - x_{D0} - i\epsilon} \frac{e^{+i\lambda'_1 p' z_1^-}}{x_L (1 - y) - \lambda'_1 + \eta'_{D1} - x'_{D0} + i\epsilon}. \]

The remaining integration over \( \lambda_1 \) may be performed with a counterclockwise semicircle in the upper half of the complex plane:

\[
\int \frac{d\lambda_1}{2\pi} \frac{e^{-i\lambda_1 p^+(z_1^- - y_0^-)}}{x_L (1 - y) - \lambda_1 + \eta_{D1} - x_{D0} - i\epsilon}(x_L (1 - y) - \lambda_1 - \lambda_{D1} - i\epsilon) = \frac{i\theta(z_1^- - y_0^-) e^{-ix_0 (1 - y) p^+(z_1^- - y_0^-)} e^{i(\eta_{D1} - x_{D0}) p^+ y_0} e^{i\lambda_{D1} p^+ z_1^-} e^{-i\chi_D p^+ y_0} - e^{-i\chi_D p^+ z_1^-}}{\chi_{D10}},
\]

where we have defined the variable \( \chi_{D10} = \eta_{D1} + \lambda_{D1} - x_{D0} \) for convenience. The integration over the momentum fraction \( \lambda'_1 \) is completely analogous. After performing the integration over the quark lines and the gluon lines, we
obtain the hadronic tensor as:

\[
W_{(2)}^{\mu\nu} = \sum_q Q_q^2 (-g_\mu^\nu) A_{\mu}^A C_F g^A \int \frac{d^4 l}{(2\pi)^4} 2\pi\delta(l^2) \int \frac{d^4 l_q}{(2\pi)^4} (2\pi)^4 \delta^4(l_+ - l_0 - k_1 - q)
\]

\[
\times \int d\theta_0 \int d^3 y_0 \int \frac{d^3 p_0}{(2\pi)^3} e^{-ip_0 \cdot y_0} e^{-ix_P p^+ y_0} \langle p | \bar{\psi}(y_0) \gamma^+ \frac{1}{2} \psi(0) | p \rangle
\]

\[
\times \int d\theta_1 \int d\delta z_1 (-i) \theta(z_1 - y_0') \int d^3 z_1 \int \frac{d^3 k_1}{(2\pi)^3} \int d^3 k'_1 \int \frac{d^3 k''_1}{(2\pi)^3} (e^{-ik_1 \cdot z_1} e^{ik'_1 \cdot z_1})
\]

\[
\times \langle A | A_{\mu}^A(z_1) A_{\nu}^A(z_1') | A \rangle \times \frac{1}{N_c} \text{Tr}[T^{a_0} f^{a_0 c_1 a_1} f^{a_1 c_1' a_0'} T^{a_0'}]
\]

To proceed, one may write the expectation of two gluon operators in the nucleus state as follows:

\[
\langle A | A_{\mu}^A(z_1) A_{\nu}^A(z_1') | A \rangle = \frac{1}{d(R)} \text{Tr}[T_{c_1}(R) T_{c_1'}(R)] \langle A | A^+(z_1) A^+(z_1') | A \rangle = \frac{\delta c_1 c_1'}{N_c} \frac{C_2(R)}{N_c^2 - 1} \langle A | A^+(z_1) A^+(z_1') | A \rangle,
\]

where \(d(R)\) and \(C_2(R)\) are the dimension and the Casimir factor of the representation \(R\), with \(C_2(R) = C_F\) and \(C_2(R) = C_A\) for exchanging the gluon field initiated by the on-shell quark and gluon, respectively. Note that two quark operators have been factored out, and two gluon insertions have the same color. Then the color factor for the hadronic tensor may be evaluated as:

\[
\frac{\delta c_1 c_1'}{N_c} \text{Tr}[T^{a_0} f^{a_0 c_1 a_1} f^{a_1 c_1' a_0'} T^{a_0'}] = C_F C_A.
\]

Now we make transformation for the coordinate variables \((z_1, z_1') \rightarrow (Z_1, \delta z_1): Z_1 = (z_1 + z_1')/2, \delta z_1 = z_1 - z_1'\). The translational invariance of the correlation functions gives:

\[
\langle A | A^+(z_1) A^+(z_1') | A \rangle = \langle A | A^+(\delta z_1) A^+(0) | A \rangle.
\]

With the above transformation, the integration for the phase factor may now be carried out, 

\[
\int d^3 z_1 \int d^3 z'_1 e^{-ik_1 \cdot z_1} e^{ik_1' \cdot z'_1} = (2\pi)^3 \delta^3(k_1 - k_1').
\]

The above \(\delta\) function implies that the pair of gluon field insertions in each nucleon state carry the same momentum, \(k_1' = k_1\). In addition, one may carry out the integration over the space coordinate \(y_0\) and obtain \(p_0 = p_0' = 0\). Then the hadronic tensor may be written as:

\[
W_{(2)}^{\mu\nu} = \sum_q Q_q^2 (-g_\mu^\nu) A_{\mu}^A \frac{C_F}{N_c^2 - 1} \frac{\alpha_s}{\pi} \int \frac{dy_0}{y_0} \int d^3 y_0 e^{-i(x_B + x_L) p^+ y_0} \langle p | \bar{\psi}(y_0) \gamma^+ \frac{1}{2} \psi(0) | p \rangle
\]

\[
\times \int dZ_1 \int d\delta z_1 \int d^3 k_1 \frac{(2\pi)^3}{(2\pi)^3} e^{-ik_1 \cdot \delta z_1} \langle A | A^+(\delta z_1) A^+(0) | A \rangle
\]

\[
\times e^{iL_P y_0} e^{-i x_D p^+ y_0} e^{-i(x_L - y - \lambda_D) p^+ \delta z_1} \left( e^{-i\chi_{10} p^+ y_0} - e^{-i\chi_{10} p^+ \delta z_1} \right) (1 - e^{i\chi_{10} p^+ \delta z_1})
\]

\[
\times C_A C_F \frac{1}{(x_D^{10})^2} \frac{2}{(2\pi)^3} \left( 1 + \left( 1 + \frac{y}{\lambda_l} - \frac{y}{\lambda_l'} \right)^2 \left( \frac{y - \frac{y}{\lambda_l}}{\left( \frac{y}{\lambda_l'} - y \right)^2} \right)^2 \right) (\mathbf{l}_1 - \mathbf{k}_{1\perp})^2.
\]

Now we simplify the phase factor (the second last line in the above hadronic tensor, denoted as \(S_{(2)}\)). We first note that \(Z_1\) is the location of the gluon insertion point which can span over the nucleus size, while \(y_0'\) and \(\delta z_1\) are
confined within the nucleon size, thus $y_0^\perp, \delta z_1^\perp \ll Z_1^-$. This may be used to simplify the phase factor:

$$S_{(2)} = e^{ix_L p^+ y_0^\perp} e^{-ix_D p^+ y_0^\perp} e^{-i(x_L (1-y) - \lambda_D 1)p^+ \delta z_1^-} (e^{-i x_D 10 p^+ y_0^\perp} - e^{-i x_D 10 p^+ Z_1^- + \frac{1}{2} \delta z_1^-}) (1 - e^{i x_D 10 p^+ Z_1^- - \frac{1}{2} \delta z_1^-})$$

$$\approx [2 - 2 \cos(\chi_{D10} p^+ Z_1^-)].$$

Now we recall the expression of $\chi_{D10}$:

$$\chi_{D10} = \eta_{D1} + \lambda_{D1} - x_{D0} = \frac{(1_\perp - k_\perp)^2}{2 p^+ \delta \lambda_1 (1 + \lambda_1 - y)} - x_L \frac{y(1 - y)}{(y - \lambda_1) (1 + \lambda_1 - y)} \frac{(1_\perp - k_\perp)^2}{l_\perp^2}. \tag{37}$$

The hard matrix element (the last line of the above hadronic tensor, denoted as $\delta T_{(2)}$) may be simplified as:

$$\delta T_{(2)} = \frac{2 y P(y)}{l_\perp^2} \mathcal{A} C_F \left[ \frac{1 + (1 - \lambda_1 - y)^2}{1 + (1 - y)^2} \mathcal{A} \left( \frac{y - \lambda_1}{y - \lambda_1} \right)^2 \frac{l_\perp^2}{(1_\perp - k_\perp)^2} \right] = \frac{2 y P(y)}{l_\perp^2} \delta T_{(2)}', \tag{38}$$

where we have defined $\delta T_{(2)}'$ for convenience. With the above simplification, the hadronic tensor reads:

$$W_{A_{(2)}}^{\mu\nu} = \sum_q Q_q^2 (-g_{\mu\nu}) A_C p (2\pi) f_q(x_B + x_L) C_2(R) \frac{2\alpha_s}{N_c^2 - 1} \int dy P(y) \int \frac{d^2 l_\perp}{\pi l_\perp^2} \times \int dZ_1^- \int d\delta z_1^- \int d^3 \delta z_1 \int \frac{d^3 k_1}{(2\pi)^3} e^{-ik_1 \cdot \delta z_1} \langle A| A^p(\delta z_1^-) A^+ (0)| A \rangle S_{(2)}(y, l_\perp, k_\perp, Z_1^-) \delta T_{(2)}'(y, l_\perp, k_\perp).$$

The above formula is quite general in the sense that the property of the dense nuclear medium that the hard jet parton probes is contained in the gluon field correlator $\langle A| A^p(\delta z_1^-) A^+ (0)| A \rangle$. As long as the gluon field correlator (in momentum space) is known, one can use it to study the medium modification effect on the gluon emission process. To perform the Fourier transformation for the gluon field, we come back to the Cartesian coordinate $z = (z^0, z^1, z^2, z^3)$:

$$\langle A| A^\mu(z_1) A^\nu(z_1')| A \rangle = \langle A| A^\mu(z_1, z_1) A^\nu(z_1', z_1')| A \rangle = \langle A| A^\mu(z_1) A^\nu(z_1')| A \rangle,$$

where for simplicity we have taken the gluon fields to be time-independent. Then one may transform the gluon field correlation function into the momentum space as follows:

$$\langle A| A^\mu(z_1) A^\nu(z_1')| A \rangle = \int \frac{d^3 p}{(2\pi)^3} e^{ip \cdot z_1} A^\mu(p) \int \frac{d^3 p'}{(2\pi)^3} e^{ip' \cdot z_1'} A^\nu(p')$$

$$= \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 \delta p}{(2\pi)^3} e^{ip \cdot \delta z_1} A^\mu(p) A^\nu(p + \delta p/2). \tag{41}$$

Here, we have changed the integral variables from $d^3 p d^3 p'$ to $d^3 p d^3 \delta p$. Using the translational invariance, the above gluon field correlator is not dependent on $Z_1$, which implies $\delta p = 0$. Therefore, the gluon field correlation function may be written as:

$$\langle A| A^\mu(\delta z_1) A^\nu(0)| A \rangle = \rho \int \frac{d^3 p}{(2\pi)^3} e^{ip \cdot \delta z_1} A^\mu(p) A^\nu(p) = \rho \int \frac{d^3 \delta p}{(2\pi)^3} e^{i(p \cdot \delta z_1) + p \cdot \delta z_1} A^\mu(p^3, p^\perp) A^\nu(p^3, p^\perp), \tag{42}$$

where $\rho$ is the density of the medium constituents (scattering centers) that the hard jet parton interacts. Now we substitute the above gluon field correlator and obtain the hadronic tensor as follows:

$$W_{A_{(2)}}^{\mu\nu} = \sum_q Q_q^2 (-g_{\mu\nu}) A_C p (2\pi) f_q(x_B + x_L) C_2(R) \frac{2\alpha_s}{N_c^2 - 1} \int dy P(y) \int \frac{d^2 l_\perp}{\pi l_\perp^2} \times \int dZ_1^- \int d\delta z_1^- \int \frac{\sqrt{2} d k_1}{(2\pi)^3} \rho A^+(\sqrt{2} k_1^\perp, k_\perp) A^+(\sqrt{2} k_1^\perp, k_\perp) S_{(2)}(y, l_\perp, \lambda_1, k_\perp, Z_1^-) \delta T_{(2)}'(y, l_\perp, \lambda_1, k_\perp).$$

Now we have to specify the form for the gluon field. For simplicity we take the static Yukawa potential:

$$A_0^\mu(p) = \frac{g}{p^2 + \mu^2}, \quad A(p) = 0. \tag{44}$$
where $\mu$ is the mass of the exchanged gluon field. Note that the color factor for the gluon field has been factored out. The gluon field correlation function now becomes:

$$\langle A|A^\mu(\delta z^0, \delta z)A^\nu(0)|A \rangle = \delta^\mu_0\delta^\nu_0\rho \int \frac{d^3p}{(2\pi)^3} e^{i p \cdot \delta z} \frac{g^2}{(p^2 + \mu^2)^2}. \tag{45}$$

Then the hadronic tensor for Figure 2 takes the following expression:

$$W_{(2)}^{\mu\nu} = \sum_q Q^2_q (-g^\mu\nu) A^\mu_C A^\nu_p (2\pi) f_Q(x_B + x_L) \frac{C_2(R)}{N_c^2 - 1} C_A C_F (2\alpha_s^2) \int dy P(y) \int \frac{d^2l_{\perp}}{\pi l_{\perp}^2} \times \int dZ_1^\perp \frac{P(y)}{2\pi l_{\perp}^2} \int \frac{dk_{1,\perp}^\perp}{2\pi} \int \frac{d^2k_{1,\perp}^\perp}{(2\pi)^2} \frac{1}{[(\sqrt{2}k_{1,\perp}^\perp)^2 + k_{1,\perp}^2 + \mu^2]^2} \times \left[ 2 - 2 \cos \left( \frac{y(1 - y)}{(y - \lambda_1^0)(1 + \lambda_1^0 - y)} \frac{(l_{\perp} - k_{1,\perp})^2}{l_{\perp}^2} \tau_{\text{form}} \right) \right] \times \left[ 1 + (1 + \lambda_1^0 - y)^2 \frac{y - \lambda_1^0}{y - \lambda_1^0} \frac{l_{\perp}^2}{(l_{\perp} - k_{1,\perp})^2} - \frac{1 + (1 + \lambda_1^0 - y)^2}{1 + (1 - y)^2} \frac{y - \lambda_1^0}{y - \lambda_1^0} \frac{l_{\perp}^2}{(l_{\perp} - k_{1,\perp})^2} \right], \tag{46}$$

where $r_N = \int d\delta z$ is the interaction time, which should depend on the energy scale of the jet parton: $r_N \sim (2\pi)/q^0$, and $\tau_{\text{form}} = 1/(x_L p^+)$ is the formation time of the radiated gluon.

The calculations for the other 20 diagrams are completely analogous; their main results are provided in Appendix. After combining the contributions of all 21 diagrams, the medium-induced single gluon emission spectrum reads:

$$\frac{dN_N^\text{med}}{dy d^2l_{\perp}} = \frac{C_F}{2\pi} \int dZ_1^\perp \int \frac{d^2k_{1,\perp}^\perp}{(2\pi)^2} \frac{1}{[(\sqrt{2}k_{1,\perp}^\perp)^2 + k_{1,\perp}^2 + \mu^2]^2} \times \left\{ C_A \left[ 2 - 2 \cos \left( \frac{y(1 - y)}{(y - \lambda_1^0)(1 + \lambda_1^0 - y)} \frac{(l_{\perp} - k_{1,\perp})^2}{l_{\perp}^2} \tau_{\text{form}} \right) \right] \times \left[ 1 + (1 + \lambda_1^0 - y)^2 \frac{y - \lambda_1^0}{y - \lambda_1^0} \frac{l_{\perp}^2}{(l_{\perp} - k_{1,\perp})^2} - \frac{1 + (1 + \lambda_1^0 - y)^2}{1 + (1 - y)^2} \frac{y - \lambda_1^0}{y - \lambda_1^0} \frac{l_{\perp}^2}{(l_{\perp} - k_{1,\perp})^2} \right] \right.$$
Using the static Yukawa potential, the medium-induced single gluon emission spectrum (after summing over all 21 diagrams) reads:

\[
\frac{dN^\text{med}}{dyd^2l_\perp} = \frac{\alpha_s}{2\pi} \frac{C_2(R)}{\lambda_{\text{mfp}}^2} \frac{P(y)}{\lambda_{\text{mfp}}} \int dZ_1^- \left( 8\sqrt{2\pi^2\alpha_s^2} \rho \right) \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{1}{(k_{1\perp}^2 + \mu^2)^2} \times \left\{ C_A \left[ 2 - 2\cos \left( \frac{(l_\perp - k_{1\perp})^2}{l_{\perp}^2} Z_1^- \frac{Z_1^-}{\tau_{\text{form}}} \right) \right] \right. \\
+ \left( \frac{C_A}{2} - C_F \right) \left[ 2 - 2\cos \left( \frac{Z_1^-}{\tau_{\text{form}}} \right) \right] \left[ \frac{1}{(l_\perp - k_{1\perp})^2} - 1 \right] + \left. C_F \left[ \frac{1}{(l_\perp - yk_{1\perp})^2} - 1 \right] \right\}. 
\]

(49)

We note that the differential elastic cross section for a quark jet scattering with the Yukawa potential is:

\[
\frac{d\sigma_{\text{el}}}{d^2p_{\perp}} = \frac{C_2(R)}{N_c^2 - 1} \frac{|gA^0(p_{\perp})|^2}{4\pi^2} = \frac{C_2(R)}{N_c^2 - 1} \frac{4\alpha_s^2}{(p_{\perp}^2 + \mu^2)^2}. 
\]

(50)

In very high energy limit, the elastic cross section in very high energy limit is:

\[
\sigma_{\text{el}} = \frac{C_2(R)}{N_c^2 - 1} \frac{4\alpha_s^2}{\mu^2}. 
\]

(51)

Therefore, one may write the elastic scattering probability as follows:

\[
\frac{dP_{\text{el}}}{d^2p_{\perp}} = \frac{1}{\sigma_{\text{el}}} \frac{d\sigma_{\text{el}}}{d^2p_{\perp}} = \frac{\mu^2}{\pi(p_{\perp}^2 + \mu^2)^2}. 
\]

(52)

Now note that \( \rho\sigma_{\text{el}} = 1/\lambda_{\text{mfp}} = \sqrt{2}/\lambda_{\text{mfp}}^- \). Therefore, we obtain the single gluon emission spectrum induced by transverse scattering as follows:

\[
\frac{dN^\text{med}}{dyd^2l_\perp} = \frac{\alpha_s}{2\pi} \frac{P(y)}{\lambda_{\text{mfp}}} \int dZ_1^- \int d^2k_{1\perp} \frac{1}{\sigma_{\text{el}}} \frac{d\sigma_{\text{el}}}{d^2k_{1\perp}} \\
\times \left\{ C_A \left[ 2 - 2\cos \left( \frac{(l_\perp - k_{1\perp})^2}{l_{\perp}^2} Z_1^- \frac{Z_1^-}{\tau_{\text{form}}} \right) \right] \right. \\
+ \left( \frac{C_A}{2} - C_F \right) \left[ 2 - 2\cos \left( \frac{Z_1^-}{\tau_{\text{form}}} \right) \right] \left[ \frac{1}{(l_\perp - k_{1\perp})^2} - 1 \right] + \left. C_F \left[ \frac{1}{(l_\perp - yk_{1\perp})^2} - 1 \right] \right\}. 
\]

(53)

The above formula is another main result of our work which represents the medium-induced single gluon emission spectrum when considering only the transverse momentum exchange between the hard jet parton and the dense nuclear medium. The transport property of the nuclear medium that the jet parton interacts is contained in the differential elastic scattering cross section \( d\sigma/d^2k_{1\perp} \) and the elastic scattering rate \( \Gamma_{\text{el}} = \rho\sigma_{\text{el}} = 1/\lambda_{\text{mfp}}^- \). We note that although the above result only considers the contribution from the transverse scattering and neglects the longitudinal momentum transfer, there is no assumption for the radiated gluon. If we further takes the soft gluon emission limit \( (y = l^-/q^- \ll 1) \), the medium-induced single gluon emission spectrum becomes:

\[
\frac{dN^\text{med}}{dyd^2l_\perp} = \frac{\alpha_s}{2\pi} \frac{P(y)}{\lambda_{\text{mfp}}} \int dZ_1^- \int d^2k_{1\perp} \frac{1}{\sigma_{\text{el}}} \frac{d\sigma_{\text{el}}}{d^2k_{1\perp}} C_A \left[ 2 - 2\cos \left( \frac{(l_\perp - k_{1\perp})^2}{l_{\perp}^2} Z_1^- \frac{Z_1^-}{\tau_{\text{form}}} \right) \right] \left[ \frac{1}{(l_\perp - k_{1\perp})^2} \right]. 
\]

(54)

Now our result agrees with the GLV one-rescattering-one-emission formula \[20\,22\,70\]. Therefore, we recover the GLV result when considering only the transverse momentum exchange and taking the limit of soft gluon emission.

### III. SUMMARY

In this work, we have studied the medium-induced gluon emission from a hard quark jet traversing the dense nuclear medium within the framework of deep-inelastic scattering off a large nucleus. In particular, we have extended the higher twist radiative energy loss approach and computed the medium-induced single gluon emission spectrum including both transverse and longitudinal momentum transfers between the hard jet parton and the constituents of the nuclear medium. We have also shown that our medium-induced gluon emission spectrum in
the soft gluon limit can reduce to the Gyulassy-Levai-Vitev one-rescattering-one-emission formula if one considers only the contribution from transverse momentum exchange and assumes static scattering centers for the traversed medium. Our study constitutes a significant progress in understanding the medium-induced radiative process during the interaction of the hard jet with the dense nuclear matter. Phenomenological studies for parton energy loss and jet quenching in relativistic heavy-ion collisions will be presented in future publication.

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**APPENDIX**

In this Appendix, we present the main results (kernels) for the other 20 cut diagrams, as shown in Figures 3-13.

FIG. 3.

The phase factor for Figure 3 reads:

\[
S_{3, L} = e^{i x_L p^+ y_0 e^{-i(x_L(1-y) - \frac{\Delta p_1}{2} - \frac{\Delta p_2}{2}) p^+ \delta z_1^+}} (e^{i \chi_{D10} p^+ Z_{1}^- - 1})
\]

\[
S_{3, R} = e^{i x_L p^+ y_0 e^{i(x_L(1-y) - \frac{\Delta p_1}{2} - \frac{\Delta p_2}{2}) p^+ \delta z_1^+}} (e^{-i \chi_{D10} p^+ Z_{1}^- - 1})
\]

\[
S_3 = S_{3, L} + S_{3, R} \approx 2 \cos(\chi_{D10} p^+ Z_{1}^-) - 2.
\]

The matrix element for Figure 3 reads:

\[
\delta T_{3, L} = \delta T_{3, R} = C_F \frac{2}{2} \left[ \frac{1 + (1 + \lambda_1^- - y)(1 - \frac{y}{1 + \lambda_1})}{1 + (1 - y)^2} \left( \frac{y - \lambda_1^-}{y - \lambda_1} \right) \frac{i \vec{p}_1 \cdot \left( \vec{l} - \vec{k}_1 \right)^2 \left( 1 - \frac{y}{1 + \lambda_1} \cdot \vec{k}_1 \right)^2}{(1 - \vec{k}_1)^2} \right].
\]

FIG. 4.
The matrix element for Figure 4 reads:

\[ S_{4,L} = e^{ixLP_y0} e^{-ixDLP^{-1}y_0} e^{-i(x_L(1-y) + \eta_{D1})p^+\delta z_i^1} (1 - e^{-i\delta z_i^1} - e^{-i\chi_{D10}p^+Z_i^1} + e^{-i(x_{D01} - x_L)p^+Z_i^1}), \]

\[ S_{4,R} = e^{ixLP_y0} e^{-i(x_L(1-y) + \eta_{D1})p^+\delta z_i^1} (1 - e^{-i\delta z_i^1} - e^{i\chi_{D10}p^+Z_i^1} + e^{i(x_{D01} - x_L)p^+Z_i^1}), \]

\[ S_4 = S_{4,L} + S_{4,R} \approx 2 - 2\cos(x_{L}p^+Z_i^1) - 2\cos(\chi_{D10}Z_i^1) + 2\cos[(\chi_{D01} - x_{L})p^+Z_i^1]. \]  

(57)

The phase factor for Figure 4 reads:

\[ \delta T_{(4),L} = \delta T_{(4),R} = -C_F \frac{C_A}{2} \left[ 1 + \frac{(1 + \lambda^1 - y)(1 - y)}{1 + (1 - y)^2} \left( \frac{y - \frac{\lambda^1}{2}}{y - \frac{\lambda^1}{1}} \right) \frac{l_{\parallel} \cdot (l_{\perp} - k_{1\perp})}{(l_{\perp} - k_{1\perp})^2} \right]. \]  

(58)

The phase factor for Figure 5 reads:

\[ S_5 = e^{ixLP_y0} e^{-ixDLP^{-1}y_0} e^{-i(\eta_{D1} - \eta_{D0})p^+\delta z_i^1} (e^{-i\delta z_i^1} - e^{-ixLP^+Z_i^1})(1 - e^{ixLP^+Z_i^1}) \]

\[ \approx 2 - 2\cos(x_{L}p^+Z_i^1), \]  

(59)

where

\[ \eta_{D0} = \frac{l_i^2}{2p^+q^-(1 - y)}. \]  

(60)

The matrix element for Figure 5 reads:

\[ \delta T_5 = C_F^2. \]  

(61)

The phase factor for Figure 6 reads:

\[ S_6 = e^{ixLP_y0} e^{-ixDLP^{-1}y_0} e^{-i\delta z_i^1} \approx 1, \]  

(62)

where

\[ x_{D1} = \frac{k_{1\perp}^2}{2p^+q^-(1 + \lambda_{1}^1)}. \]  

(63)
The matrix element for Figure 7 reads:

$$\delta T_{[q]} = C_F^2 \left[ 1 + \left( 1 - \frac{\eta}{1+\lambda_1^*} \right) \prod \frac{p^2}{1 + (1 - y)^2 \left( \eta_1 - \frac{y}{1+\lambda_1^*}k_1 \right)^2} \right].$$

(64)

![Diagram for Figure 7](image)

**FIG. 7.**

The phase factor for Figure 8 reads:

$$S_{[q]} L = e^{i x L P^+ y_0} e^{-i (x L (1 - y) + \eta_{D2}) P^+ \delta z_1} (e^{i x L P^+ Z_1^*} - 1),$$

$$S_{[q]} R = e^{i x L P^+ y_0} e^{i (x L (1 - y) + \eta_{D2}) P^+ \delta z_1} (e^{-i x L P^+ Z_1^*} - 1),$$

$$S_{[q]} = S_{[q]} L + S_{[q]} R \approx 2 \cos(x L P^+ Z_1^*) - 2.$$  

(65)

The matrix element for Figure 8 reads:

$$\delta T_{[q]} L = \delta T_{[q]} R = C_F \left( C_F - \frac{C_A}{2} \right) \left[ 1 + \left( 1 - \frac{\eta}{1+\lambda_1^*} \right) \prod \frac{p^2}{1 + (1 - y)^2 \left( \eta_1 - \frac{y}{1+\lambda_1^*}k_1 \right)^2} \right].$$

(66)

![Diagram for Figure 8](image)

**FIG. 8.**

The phase factor for Figure 8 reads:

$$S_{[q]} L = \frac{1}{2} e^{-i (x L (1 - y) - \lambda_{D2}) P^+ \delta z_1} (e^{i x L P^+ Z_1^*} - 1),$$

$$S_{[q]} R = \frac{1}{2} e^{i x D0 P^+ y_0} e^{i (x L (1 - y) - \lambda_{D2}) P^+ \delta z_1} (e^{-i x L P^+ Z_1^*} - 1),$$

$$S_{[q]} = S_{[q]} L + S_{[q]} R \approx \cos(x L P^+ Z_1^*) - 1.$$  

(67)

Here,

$$\lambda_{D2} = \frac{(l_1 - k_2)}{2p^+q^-(y - \lambda_2^*)}.$$  

(68)
The matrix element for Figure 8 reads:

$$
\delta T_{8,L} = \delta T_{8,R} = C_F C_A \left( \frac{y - \frac{\lambda^2}{2}}{y(y - \lambda^2)} \right)^2
$$

(69)

FIG. 9.

The phase factor for Figure 9 reads:

$$
S_{9,L} = e^{-i \frac{1}{2} \left[ (yL - 1 - \lambda D_2 + \eta D_2 - \bar{\eta} D_2) p^+ \delta z^+ \right]} (e^{-i x_L p^+ Z^+} - e^{-i (x_L - \chi D_20) p^+ Z^+}),
$$

$$
S_{9,R} = e^{-i x_{d0} p^+ y^+} e^{i \frac{1}{2} \left[ (yL - 1 - \lambda D_2 + \eta D_2 - \bar{\eta} D_2) p^+ \delta z^+ \right]} (e^{i x_L p^+ Z^+} - e^{i (x_L - \chi D_20) p^+ Z^+}),
$$

$$
S_{9} = S_{9,L} + S_{9,R} \approx 2 \cos(x_L p^+ Z^+) - 2 \cos[(x_L - \chi D_20) p^+ Z^+],
$$

(70)

where

$$
\eta_{D2} = \frac{(1_L - k_{1L})^2}{2 p^+ q^+ (1 + \lambda_1 - y)}, \quad \bar{\eta}_{D2} = \frac{1_L^2}{2 p^+ q^+ (1 - y)} = y x_L,
$$

$$
\chi_{D20} = \eta_{D2} + \lambda_{D2} - x_{D0} = \frac{(1_L - k_{1L})^2}{2 p^+ q^+ (y - \lambda_1) (1 + \lambda_1 - y)} = \chi_{D^10}.
$$

(71)

(72)

The matrix element for Figure 10 reads:

$$
\delta T_{10,L} = \delta T_{10,R} = -C_F C_A \left[ \frac{1}{2} \left( \frac{1 + \lambda_1^2}{1 - y} \right) \left( \frac{y - \lambda^2}{1 - y} \right) \frac{1_L \cdot (1_L - k_{1L})}{(1_L - k_{1L})^2} \right].
$$

FIG. 10.

The phase factor for Figure 10 reads:

$$
S_{10,L} = \frac{1}{2} e^{i x_{dL} p^+ y^+} e^{-i x_L p^+ Z^+} \left( e^{i (x_L - 2\eta D_2 - 2\lambda D_2) p^+ \frac{1}{2} \delta z^+} - e^{i (x_L - 2z+ D_1) p^+ \frac{1}{2} \delta z^+} \right),
$$

$$
S_{10,R} = \frac{1}{2} e^{i x_{d0} p^+ y^+} e^{-i x_{d0} p^+ y^+} e^{i x_L p^+ Z^+} \left( e^{-i (x_L - 2\eta D_2 - 2\lambda D_2) p^+ \frac{1}{2} \delta z^+} - e^{-i (x_L - 2z+ D_1) p^+ \frac{1}{2} \delta z^+} \right),
$$

$$
S_{10} = S_{10,L} + S_{10,R} \approx 0.
$$

(73)
The matrix element for Figure 10 reads:

$$\delta T_{10}^L = \delta T_{10}^R = C_F \frac{C_A}{2} \left[ 1 + \left( 1 - \frac{y - \frac{\lambda}{1 + \lambda_1}}{1 - \lambda_1} \right) (1 - y) \left( \frac{y - \frac{\lambda}{1 + \lambda_1}}{y - \frac{\lambda}{1 + \lambda_1}} \right) \frac{1}{1 + (1 - y)^2} \left( \left( 1 - \frac{y - \frac{\lambda}{1 + \lambda_1}}{1 + \lambda_1} \right) k_{1\perp} \right)^2 \right]. \quad (74)$$

The matrix element for Figure 11 reads:

$$\delta T_{11}^L = \delta T_{11}^R = C_F^2. \quad (76)$$

The phase factor for Figure 11 reads:

$$S_{11}^L = -\frac{1}{2} e^{-i(yx_L - \eta D_1)p^+ \delta z_i^-} (1 - e^{ixLP^+ Z_i^-}),$$
$$S_{11}^R = -\frac{1}{2} e^{ixLP^+ y_0^- e^{-ixDP^+ y_0} e^{i(yx_L - \eta D_1)p^+ \delta z_i^-} (1 - e^{-ixLP^+ Z_i^-}),}$$
$$S_{11} = S_{11}^L + S_{11}^R \approx \cos(xLP^+ Z_i^-) - 1. \quad (75)$$

The phase factor for Figure 12 reads:

$$S_{12}^L = \frac{1}{2} e^{-ixD_1 p^+ \delta z_i^-} (e^{ixLP^+ Z_i^-}),$$
$$S_{12}^R = \frac{1}{2} e^{ixLP^+ y_0^- e^{ixD_1 p^+ \delta z_i^-} (e^{-ixLP^+ Z_i^-}),}$$
$$S_{12} = S_{12}^L + S_{12}^R \approx -\cos(xLP^+ Z_i^-). \quad (77)$$

The phase factor for Figure 13 reads:

$$S_{13}^L = -\frac{1}{2} e^{ixLP^+ Z_i^-} \left( e^{i((1-2y)x_L + 2\eta D_1)p^+ \frac{1}{2} \delta z_i^- - e^{-i((1-2y)x_L - 2\eta D_1)p^+ \frac{1}{2} \delta z_i^-}}, \right)$$
$$S_{13}^R = -\frac{1}{2} e^{ixLP^+ y_0^- e^{-ixDP^+ y_0} e^{-ixLP^+ Z_i^-} \left( e^{-i((1-2y)x_L - 2\eta D_1)p^+ \frac{1}{2} \delta z_i^- - e^{i((1-2y)x_L - 2\eta D_1)p^+ \frac{1}{2} \delta z_i^-}).} \right.$$
The matrix element for Figure 13 reads:

$$\delta T^{[13]}_{L} = \delta T^{[13]}_{R} = C_F \left( C_F - \frac{C_A}{2} \right) \left[ \frac{1 \cdot (1 - y k_{1\perp})}{(1 - y k_{1\perp})^2} \right].$$

(80)
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