Proposal for a Satellite-Borne Experiment to Test Relativity of Simultaneity in Special Relativity

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Abstract

An orbiting ‘photon clock’ is proposed to test directly the relativity of simultaneity of special relativity. This is done by comparison of the arrival times at a ground station of three microwave signals transmitted by two satellites following the same low Earth orbit.

PACS 03.30.+p

Einstein’s original paper on special relativity (SR) [1] discussed three physical effects which changed completely our conceptual understanding of space and time. These are: relativity of simultaneity (RS), length contraction (LC) and time dilatation (TD). RS and LC are closely related. In the space-time Lorentz transformation (LT) \( a \):

\[
x' = \gamma(x - vt) \\
t' = \gamma(t - \frac{\beta x}{c})
\]

both effects result from the spatial dependence (the term \(-\gamma\beta x/c\)) of the time transformation equation (2). LC is given by a \( \Delta t = 0 \) projection of the LT [2]. Because of RS, events which are simultaneous in S (i.e. have \( \Delta t = 0 \)) are not so in \( S' \), resulting in LC [2, 3]. One hundred years after the publication of Einstein’s paper only the TD effect has been experimentally confirmed. For a concise review of experimental tests of SR see Reference [4]. Unlike LC, the TD effect (a \( \Delta x' = 0 \) projection of the LT [2]) does not involve RS since the space-time events concerned occur always at a fixed position in \( S' \) – the spatial coordinate of the clock under consideration.

The purpose of this letter is to propose a direct experimental test of RS. The enormous improvement, in recent decades, of the precision of time measurements due to the widespread application of atomic clocks much facilitates the test. A test of the related LC effect seems, in contrast, to be much more difficult [4]. The proposed experiment is an actual realisation, in space, of the light-signal clock synchronisation procedure proposed by Einstein in [1]. However, no actual experimental clock synchronisation is needed. At the practical level the experiment can be considered as a sequel to the Spacelab experiment.

\(^a\)The frame \( S' \) (space-time coordinates \( x',t' \)) moves along the positive \( x \)-axis of S (space-time coordinates \( x,t \)) with velocity \( v \). \( Ox' \) is parallel to \( Ox \). Clocks in S and \( S' \) are synchronised so that \( t = t' = 0 \) when the origins of S and \( S' \) coincide. \( \beta \equiv v/c, \; \gamma \equiv 1/\sqrt{1-\beta^2} \).
NAVEX [5] in which special and general relativity (TD and the gravitational red-shift) were tested by comparing a caesium clock in a space shuttle in a low, almost-circular, orbit around the Earth with a similar, synchronised, clock at a ground station. The experiment requires two satellites, one of which could conveniently be the International Space Station (ISS) which has orbit parameters similar to those of the NAVEX shuttle, the other a shuttle, or other satellite, following the same orbit as the ISS but separated from it by a few hundred kilometers.

A scheme of the proposed experiment is shown in Fig.1. Two satellites, A and B, in low Earth orbit, separated by the distance $L$, pass near to a ground station C. Cartesian coordinate systems are defined in the co-moving inertial frame of the satellites (S’) and the ground station (S). The origin of S’ is chosen midway between A and B with $x'$-axis parallel to the direction of motion of the satellites and $y'$ axis outwardly directed in the plane of the orbit. Ox and Oy are parallel to O’x and O’y at the position of closest approach (culmination) of O’ to C. Clocks in S and S’ are synchronised at $t = t’ = 0$ at culmination, where the coordinates of O’ in S are: $(x,y,z) = (0,0,0)$. The signal is detected and reflected promptly back towards B. After a delay $\delta t = 2\frac{d}{c} + \frac{L^2}{2cR}(d_B + d_A)$, as calculated using the LT (1) and (2) are presented in Table 1. Taking into account the propagation times of the signals from A and B to C the following differences of arrival times of the signals at C are found:

$$\delta t_{BA} \equiv t(S_B) - t(S^{(1)}_A) = \frac{L}{c} + \frac{L}{c\beta}(d_B - d_A) + \frac{L^2}{2cR}(d_B + d_A) + \frac{\beta L}{c} - \frac{\beta L^2}{2cR} \quad (3)$$

$$\delta t_{AB} \equiv t(S^{(2)}_A) - t(S_B) = \frac{L}{c} - \frac{L}{c\beta}(d_B - d_A) - \frac{L^2}{2cR}(d_B + d_A) - \frac{\beta L}{c} - \frac{\beta L^2}{2cR} \quad (4)$$

where $R \equiv \sqrt{H^2 + D^2 + L^2/4}$, $d_{A,B} \equiv v t_D(A,B)/L$ and only terms of $O(\beta)$ have been retained. Hence:

$$\Delta t \equiv \delta t_{BA} - \delta t_{AB} = \frac{2\beta L}{c} + \frac{2L}{c\beta}(d_B - d_A) + \frac{L^2}{cR}(d_B + d_A) \quad (5)$$

It is interesting to note that RS, as manifested in the non-vanishing value of $\Delta t$ in (5) when $t_D(A) = t_D(B) = 0$, is an $O(\beta)$ effect, not an $O(\beta^2)$ one as for LC and TD. The term $2\beta L/c$ in (5) originates from the second (spatially-dependent) term in the LT of time,(2), responsible for RS. The orbital velocity of the ISS is 7.674 km/s ($\beta = 2.56 \times 10^{-5}$) [6]. Since the ground station velocity in much less than this $^b$, this is essentially the same as the relative velocity in (5). Choosing $L = 400$km (for the ISS $H \simeq 350$km [6]) and setting

$^b$For the NAVEX experiment it was 0.311km/s.
Figure 1: Scheme of an experimental realisation of Einstein’s clock synchronisation procedure using two satellites in low Earth orbit. The x-y projection is shown. ‘Relativity of Simultaneity’ is directly tested in the experiment by observation at the ground station C of the times of arrival of the ‘photon clock’ signals $S_A^{(1)}$ and $S_A^{(2)}$ from the satellite A {a) and c)} and $S_B$ from the satellite B {b)}. C is viewed from the co-moving frame of A and B. Coordinate systems and geometrical and temporal parameters used in the analysis are defined.
Table 1: Coordinates of space time events in $S'$ and $S$. The origin of $S'$ is midway between the satellites $A$ and $B$. The origin of $S$ is at $C$.

| Event       | $x'$ | $t'$  | $x$                  | $t$                        |
|-------------|------|-------|----------------------|----------------------------|
| $S_A^{(1)}$ emitted | $-\frac{L}{2}$ | $-\frac{L}{c} + t_D(A)$ | $-\gamma L(\frac{1}{2} + \beta - \frac{vt_D(A)}{L})$ | $-\frac{\gamma L}{c}(1 + \frac{\beta}{2} - \frac{ct_D(A)}{L})$ |
| $S_B$ emitted | $\frac{L}{2}$ | $t_D(B)$ | $\gamma L(\frac{1}{2} + \frac{vt_D(B)}{L})$ | $\frac{\gamma L}{c}(\frac{\beta}{2} + \frac{ct_D(B)}{L})$ |
| $S_A^{(2)}$ emitted | $-\frac{L}{2}$ | $\frac{L}{c} + t_D(A)$ | $-\gamma L(\frac{1}{2} - \beta - \frac{vt_D(A)}{L})$ | $\frac{\gamma L}{c}(1 - \frac{\beta}{2} + \frac{ct_D(A)}{L})$ |

t_D(A) = t_D(B) = 0 in (5) gives $\Delta t = 2\beta L/c = 68.3$ns. Such a time difference is easily measurable with modern techniques. Signal arrival times in the NAVEX experiment were quoted with 1ns precision. The uncertainties in the clock rates for the relativity tests in NAVEX corresponded to an experimental time resolution of $\approx 0.1$ns over one rotation period (1.6h) of the shuttle. The contribution of the last term on the right side of (5) is negligible. For $L/R = 1$ and delays as long as $1\mu$s it contributes only 0.05ns to $\Delta t$ for $\beta = 2.56 \times 10^{-5}$. Thus $\Delta t$ is essentially independent of the distance between the satellites and the ground station at culmination. During the total transit time of the microwave signals in the ‘photon clock’ constituted by the satellites, they move in $S$ only a distance $\approx 2L\beta = 10.2$m. Different times of emission of the signal sequence are easily taken into account by a suitable choice of the delay times $t_D(A, B)$.

Although a particular coordinate system and clock synchronisation are used to calculate the entries of Table 1 and the time differences $\delta t_{BA}$ and $\delta t_{AB}$ in (5), the quantity $\Delta t$ is independent of this choice, so no clock synchronisation is required to measure $\Delta t$. If, however, pre-synchronised clocks are available in the satellites they may be used to generate the signal sequence: $S_A^{(1)}$, $S_B$, $S_A^{(2)}$, without the necessity of ‘photon clock’ signals between the satellites. The latter in fact may be considered to effect a real-time synchronisation of hypothetical clocks in $A$ and $B$. In this case, a simpler direct measurement of RS is possible. Sending signals $S_A$ and $S_B$ at the same time in $S'$, when $O'$ is at culmination, the LT (1) and (2) predict that the signals will be observed at $C$ with a time difference of $\gamma \beta L/c$, half the value of $\Delta t$ in the photon clock experiment.

The ease of measurement of the $O(\beta)$ RS effect may be contrasted with the difficulty of measuring, in a similar experiment, the $O(\beta^2)$ LC effect. Using the value $\beta = 2.56 \times 10^{-5}$ appropriate for the ISS, the apparent contraction of the distance between the satellites $A$ and $B$, as viewed at some instant in $S$, of $(1 - 1/\gamma)L$, amounts to only $131 \mu m$ for $L = 400$km. It is hard to conceive any experiment using currently known techniques with sufficiently good spatial resolution to measure such a tiny effect.

In a recent paper by the present author [4] it has been suggested, in order to avoid certain casual paradoxes of SR, and to ensure translational invariance, that the origin of the frame $S'$ in the LT (1) and (2) should be chosen to coincide with the position of the transformed event (a ‘local’ LT). In this case it is predicted that $\Delta t = 0$ for $t_D(A) = t_D(B) = 0$ in (5) and that signals emitted simultaneously in $S'$ from $A$ and $B$ at culmination will be received at the same time in $S$ at $C$. 

References

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