Causal unitary qubit model of black hole evaporation

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Abstract

A new simple qubit model of black hole evaporation is proposed. The model operates on four qubits and is defined in terms of quantum gates and a quantum circuit. The chief features of the model include explicit unitarity and (most notably) causality which is understood as the impossibility of the transfer of information from the interior of the black hole through its horizon. The corresponding von Neumann entanglement entropy yields a crude (four-qubit) approximation of the Page curve.

Keywords: black hole information loss paradox, qubit model of black hole evaporation, causal and unitary black hole evaporation, von Neumann entanglement entropy, the Page curve

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1. Introduction

The black hole (BH) information (loss) paradox is a problem concerning difficulties with the unitarity of the process of BH evaporation and evolution (see e.g. Hawking [9], or reviews Chakraborty and Lochan [4], Harlow [8], Polchinski [16], Marolf [10]). The assumption (ours, in particular) that the unitarity is conserved implies several general scenarios. For example, one can adopt the scenario (as also we do) that information is being (somehow) gradually released during BH evaporation. However this point of view (as obviously any other) requires an indication of some convincing physical mechanism, or (in the case of the lack of thereof) at least some working abstract algorithm for the transfer of information. One of the obvious approaches to study the paradox, abstracting from a particular physical mechanism, consists in an analysis of the problem in terms of qubits. In literature, we can find numerous qubit models more or less successfully reproducing various steps involved in BH evolution (see e.g. Broda [2, 3], Giddings [6, 5], Giddings and Shi [7], Mathur [11, 12], Mathur and...
Plumberg [13], Osuga and Page [14], or review Avery [1]). Unfortunately, it seems that in all these models the important issue of causality has not attracted due attention, and consequently the possibility of faster-than-light communication has not been explicitly excluded. In contrast to this is our present treatment, where causality is a priority. More precisely, in our approach, we impose strict control on the direction of information transfer through the BH horizon. As a (sufficient) minimum, we assume that any transfer of information from a BH through its horizon is forbidden. On the other hand, any transfer of information through the horizon in the opposite direction (from outside to inside), as well as between particles (qubits) residing exclusively outside or inside the BH is not restricted. Fortunately, working in terms of qubits (and consequently, in terms of quantum gates and a quantum circuit) gives us an excellent possibility to control the direction of the transfer of information. In particular, the CNOT (controlled NOT) gate acting between qubits on both sides of the BH horizon is only allowed provided the control qubit is exactly the outer qubit, and the target qubit is the inner one. Clearly, the SWAP gate in this position (acting across the horizon) is forbidden. However, all kinds of gates acting exclusively outside or inside the BH are allowed (see Fig. 1).

![Figure 1: Four examples of quantum gates and their specific locations in the context of the presence of the BH horizon. The two left panels present the CNOT gates, whereas the two right ones, the SWAP gates. In our approach, the two upper gates are causally forbidden in their proposed locations, whereas the two lower gates are causally allowed. The double gray line denotes the BH horizon, with its thicker component corresponding to the inner part of the BH.](image)

The scenario with the gradual release of information during BH evaporation suggests the following subscenarios:

**Information passes through the BH horizon.** In principle, this is the most obvious possibility, but in consequence information seems to be permanently locked inside the BH (and possibly also lost), or faster-than-light communication should be invoked for its release (information cannot escape the BH horizon without traveling faster than light). So, we could ask after Polchinski [16]: “How does information travel from inside the black hole to the outside?”

**Information does not pass through the BH horizon.** This subscenario evidently solves the problem with information retrieval, but it creates problems with the BH itself. Apparently, the BH does not appear in this subscenario, or
it is not being populated by matter.

Information is both reflected at the BH horizon and passes through the horizon. On first sight, the above difficulties with information retrieval and the BH itself seem to be solved. Unfortunately, the existence of two copies of information (the first copy inside the BH and the second one outside the BH) requires convincing arguments that we are not conflicted with the no-cloning theorem (and at later stages, possibly also, with the no-deleting theorem). For the corresponding discussion in the framework of the, so-called, BH complementarity see e.g. Susskind and Lindesay [17].

Information resides in the entangled states of (pairs of) qubits on both sides of the BH horizon. This is the point of view assumed in this paper.

2. The four-qubit model

The model is defined in a finite dimensional Hilbert space $\mathcal{H}$, in the language of qubits. To automatically ensure unitarity of the model, the quantum evolution is described in terms of quantum gates operating in the framework of a quantum circuit. We have fixed four qubits as a minimal number of qubits necessary to demonstrate the proposed mechanism (a huge tensor power of the four-qubit block could be considered as an approximation to a possible fuller model). Each of the four qubits has assigned a specific task:

1st qubit, denoted by $m$ ($m$ like “matter”), is the “original” information carrying qubit (one of those) initially forming the BH or passing through the BH horizon of the already formed BH.

2nd qubit, denoted by $g$ ($g$ like “graviton”), is an auxiliary qubit for some time being entangled with the qubit $m$. Its principal task consists in collective transport (together with the qubit $m$) of the information initially contained in $m$ without violating the no-cloning theorem (and at later stages, possibly also, the no-deleting theorem).

3rd qubit, denoted by “$-$” (the minus sign), corresponds to a Hawking particle with the negative energy $-\omega$, which contributes to the negative energy flux required for BH evaporation.

4th qubit, denoted by “$+$” (the plus sign), corresponds to a Hawking particle with the positive energy $\omega$, which compensates (according to energy conservation) negative energy of the “$-$” particle.

The quantum circuit in Fig. 2 performs the algorithm proposed in the paper, and it works as follows. Initially, the qubit $m$ (“matter”) is in the arbitrary but fixed state

$$|\psi_m\rangle = \lambda |0_m\rangle + \mu |1_m\rangle,$$

(1)

whereas the rest of the qubits, $g$, “$-$”, “$+$”, is in the “vacuum” state

$$|0_g\rangle, \quad |0_-\rangle, \quad |0_+\rangle,$$

(2)

respectively. Nowhere do we refer to the formalism of the (antisymmetric) Fock space, but our physics picture of qubits is close to a fermionic description of
matter. Hence, in our terminology the state $|0\rangle$ is called the vacuum state, and so interpreted. The total initial state is then
\[
|\Psi_{mg-+}\rangle = |\psi_m\rangle |0_g0_{-0+}\rangle \equiv (\lambda |0_m\rangle + \mu |1_m\rangle) |0_g0_{-0+}\rangle.
\] (3)

Next a short series of quantum operations (gates) acts independently in the first pair of the qubits ($m$ and $g$), and in the second one (“−” and “+”), respectively, as described in the framework of Stage 1 in Fig. 2. As a result, the quantum information initially contained only in the qubit $m$ is now contained in the following state of the first pair of the entangled qubits ($m$ and $g$),
\[
|\phi_{mg}\rangle = \lambda |0_m0_g\rangle + \mu |1_m1_g\rangle.
\] (4)

In turn, the evolution of the second pair of the qubits (“−” and “+”) yields a Hawking pair given by the state
\[
|\chi_{-+}\rangle = \alpha |0_{-0+}\rangle + \beta |1_{-1+}\rangle.
\] (5)

The symbol “$U_H$” in Fig. 2 denotes a one-qubit unitary transformation ($U_H^{-1}$ is its inverse) defined by (or defining) the Hawking process specified by the complex coefficients $\alpha$ and $\beta$. Finally, the total state of the system produced in the framework of Stage 1 is
\[
|\Psi_{mg-+}\rangle = |\phi_{mg}\rangle |\chi_{-+}\rangle \equiv (\lambda |0_m0_g\rangle + \mu |1_m1_g\rangle) (\alpha |0_{-0+}\rangle + \beta |1_{-1+}\rangle).
\] (6)

Once the qubit $m$, and (independently) “−”, passes through the BH horizon, a corresponding contribution to the von Neumann entanglement entropy increases by
\[
S' = -|\lambda|^2 \ln |\lambda|^2 - |\mu|^2 \ln |\mu|^2,
\] (7)
and

$$S'' = -|\alpha|^2 \ln |\alpha|^2 - |\beta|^2 \ln |\beta|^2,$$

respectively. We could interpret $S'$, coming from the first pair of qubits ($m$ and $g$), as the Bekenstein–Hawking entropy of the BH due to its matter content, and $S''$, coming from the second pair ("−" and "+") as entanglement entropy due to Hawking radiation, respectively. The total four-qubit contribution to entropy is then $S_1 = S' + S''$.

![Figure 3](https://example.com/figure3.png)

Figure 3: A sample four-qubit “Page curve”. The upper (a) and the middle (b) panel presents a sample contribution to the von Neumann entanglement entropy from the first ($m$ and $g$) and the second ("−" and "+") qubit pair, respectively. The lower panel (c) presents their sum, i.e. the total value of the von Neumann entanglement entropy. The instants $\tau_1$, $\tau_2$, $\tau_3$ and $\tau_4$ are defined in Fig. 2.

For graphical definiteness, we have assumed a particular order of elements in Fig. 3. However, some flexibility in their order is allowed. Actually, positions of the elements can be (continuously) deformed, at least within each of the three Stages (i.e. between vertical dashed lines), without any consequences for our discussion.

Stage 2 in Fig. 3 containing two SWAP operations, one entirely inside the BH and one entirely outside the BH, yields the total state

$$|\Psi_{mg-+}^2\rangle = (\alpha |0_m 0_g\rangle + \beta |1_m 1_g\rangle) (\lambda |0_- 0_+\rangle + \mu |1_- 1_+\rangle).$$

(9)

Obviously the entropy is left unchanged, i.e. $S_2 = S_1$. The (final) Stage 3 completes the information transfer, and the total state assumes the form

$$|\Psi_{mg-+}^3\rangle = |0_m 0_g 0_-\rangle (\lambda |0_+\rangle + \mu |1_+\rangle) \equiv |0_m 0_g 0_-\rangle |\psi_+\rangle.$$

(10)
The total entropy is now \( S_3 = 0 \).

It is interesting to analyze our model from the perspective of the Page curve (see Page [15]). Since the model consists of only four qubits we should not expect too much and require the emergence of a genuine Page curve. For our four-qubit model a sample “Page curve” is presented in Fig. 3c. The entropy changes (abruptly) only at the instants \( \tau_1, \tau_2, \tau_3 \) and \( \tau_4 \) corresponding to crossings of the BH horizon (see Fig. 2). The staircase-shape curve in Fig. 3c can be interpreted as a crude approximation of the genuine smooth tent-shape Page curve.

3. Summary

We have proposed a new simple four-qubit model of BH evaporation, formulated in terms of quantum gates operating in the framework of a quantum circuit. The unitarity of the model is automatically satisfied by virtue of the gate construction. Moreover, most notably, the model is causal, which is understood as the impossibility of the transfer of information from the interior of the BH through its horizon. The causality is built in the model by appropriate selection of gates and their positions with respect to the BH horizon (see Fig. 1 and the accompanying discussion). Thus, the model unitarily and causally transfers quantum information from the “original” ingoing “matter” qubit \( m \) to the outgoing Hawking qubit “+”.

The corresponding von Neumann entanglement entropy yields a crude (four-qubit) staircase-shape approximation (see Fig. 3c) of the Page curve. Alternatively, one can consider only the part \( S'' \) of entropy (coming from the Hawking radiation) as a contribution to the Page curve. This even seems to be more in the original spirit of Page, and moreover qualitatively changes nothing in our reasoning. The step-shape curve in Fig. 3b is a crude approximation of the Page curve as well. One could further argue that a possible huge tensor power of the four-qubit block (interpreted as an approximation to a hypothetical fuller model with a huge number of qubits) could yield a smooth curve as a result of the summation of the huge number of the four-qubit staircase-shape curves, or step-shape ones, corresponding to Fig. 3c, or Fig. 3b, respectively, with the sets of instants \( \tau_1, \tau_2, \tau_3, \tau_4 \) a bit different for each independent four-qubit block.

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