Diffractive production at high energies in the Miettinen–Pumplin model

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Abstract

The model of soft diffractive dissociation proposed some time ago by Miettinen and Pumplin is shown to describe correctly the data at FERMILAB energies. The comparison with Goulianos model of Renormalized Pomeron Flux is also presented.

The Regge description of diffractive dissociation at high energies encounters a serious problem related to unitarity. When the parameters determined below 100 GeV center of mass energy are applied to FERMILAB energies, the estimated cross section for diffraction dissociation exceeds total cross section in blatant violation unitarity and the data. To save the Regge picture K.Goulianos proposed to renormalize the pomeron flux in a way which restores unitarity and the agreement with the data.

The Goulianos prescription, although very elegant and effective, is not easily justifiable from theory. Therefore it seems interesting to consider other possibilities. In the present paper the high-energy data were analyzed using the parton model of Miettinen and Pumplin which was shown to work very well at ISR energies. The model, based on the Good-Walker picture of diffractive processes satisfies, by construction, all unitarity constraints. We have found that a straightforward extrapolation of the Miettinen and Pumplin model to the FERMILAB energies describes very well the data. No further adjustments are necessary. One may thus conclude that the Miettinen and Pumplin description of diffractive processes provides a correct framework for analysis of the present experimental results.

In the Miettinen and Pumplin model, following the state of the incident hadron is expanded into a superposition of “diffractive” states

\[ |B\rangle = \sum_k C_k |\psi_k\rangle, \]

which are eigenstates of the scattering operator

\[ \text{Im}T |\psi_k\rangle = t_k |\psi_k\rangle. \]

If the different eigenstates are absorbed by the target with different intensity, the outgoing state is no longer \(|B\rangle\) and inelastic production of particles takes place. The relevant formulae for the cross sections take the form.
\[ \frac{d\sigma_{el}}{d^2b} = | \langle B | ImT | B \rangle |^2 = \sum_k |C_k|^2 t_k |^2 = \langle t \rangle^2, \quad (3) \]

\[ \frac{d\sigma_{tot}}{d^2b} = 2 \langle t \rangle. \quad (4) \]

\[ \frac{d\sigma_{diff}}{d^2b} = \sum_k | \langle \psi_k | ImT | B \rangle - \frac{d\sigma_{el}}{d^2b} |^2 = \langle t^2 \rangle - \langle t \rangle^2. \quad (5) \]

The basic assumption of Miettinen and Pumplin is that the eigenstates of diffraction are parton states [6]

\[ | \psi_k \rangle \equiv | \vec{b}_1, ..., \vec{b}_N, y_1, ..., y_N \rangle, \quad (6) \]

where \( N \) is the number of partons; \( y_i \) is the rapidity of parton \( i \) and \( \vec{b}_i \) is the impact parameter of parton \( i \) relative to the impact parameter of the incident particle.

Therefore, equation (1) takes the form

\[ | B \rangle = \sum_{N=0}^{\infty} \int \prod_{i=1}^{N} d^2\vec{b}_i dy_i C_N(\vec{b}_1, ..., \vec{b}_N, y_1, ..., y_N) | \vec{b}_1, ..., \vec{b}_N, y_1, ..., y_N \rangle. \quad (7) \]

The probability \( | C_N |^2 \) associated with \( N \) partons, which are assumed to be independent, is given by Poisson distribution with mean number \( G^2 \)

\[ | C_N(\vec{b}_1, ..., \vec{b}_N, y_1, ..., y_N) |^2 = e^{-G^2} \frac{G^{2N}}{N!} \prod_{i=1}^{N} | C(\vec{b}_i, y_i) |^2. \quad (8) \]

To specify the eigenvalues \( t_k \), Miettinen and Pumplin assumed that partons interact independently with the target. This implies that if the probability for a parton \( i \) to interact is denoted \( \tau_i \), the probability that none of \( N \) partons interacts is \( \prod_{i=1}^{N} (1 - \tau_i) \), hence the probability for anyone of them to interact is \( 1 - \prod_{i=1}^{N} (1 - \tau_i) \).

Miettinen and Pumplin took \( | C(\vec{b}_i, y_i) |^2 \) and \( \tau_i(\vec{b}_i, y_i) \) in a form

\[ | C(\vec{b}_i, y_i) |^2 = \frac{1}{2\pi\beta\lambda} \exp \left( -\frac{|y_i|}{\lambda} - \frac{b_i^2}{\beta} \right), \quad (9) \]

\[ \tau_i(\vec{b}_i, y_i) = A \exp \left( -\frac{|y_i|}{\alpha} - \frac{b_i^2}{\gamma} \right), \quad (10) \]

with

\[ A = 1 \quad \frac{\alpha}{\lambda} = 2.0 \quad \frac{\gamma}{\beta} = 2.0 \quad (11) \]

Taking all this into consideration, the total, elastic and diffractive cross sections are given by

\[ \frac{d\sigma_{tot}}{d^2b} = 2 \left( 1 - \exp \left( -G^2 \frac{4}{9} e^{-\frac{4}{3} \frac{b^2}{\beta}} \right) \right), \quad (12) \]
As we see, the model depends on two parameters $G^2$ and $\beta[fm^2]$. Those parameters can be determined for a given energy $\sqrt{s}$ from experimental data for $\sigma_{tot}$ and $\sigma_{el}$ using (12) and (13).

In 1978 Miettinen and Pumplin performed calculations for two colliding protons with the center of mass energy $\sqrt{s} = 53 GeV$. They obtained the result for $\sigma_{SD}$ which were in good agreement with the data.

We have applied the Miettinen and Pumplin model to energies $\sqrt{s} = 546 GeV$ and $\sqrt{s} = 1800 GeV$ using the CDF [7] and E811 [8] data for $\sigma_{tot}$ and $\sigma_{el}$.

The obtained values of $G^2$ and $\beta[fm^2]$, presented in table 1 allow to calculate the cross section for single diffractive production. Figure 1 shows the results compared with experiments and with the Goulianos model. To take into account the beam and the target dissociation, $\sigma_{SD}$ is multiplied by the factor of two.

![Figure 1: Total pp single diffraction cross section data compared with predictions based on Miettinen and Pumplin model and Goulianos model.](image)

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| references | $\sqrt{s}[GeV]$ | $\sigma_{tot}[mb]$ | $\sigma_{el}[mb]$ | $G^2$ | $\beta[fm^2]$ | $2\sigma_{diff}[mb]$ |
|-----------|-----------------|-------------------|-------------------|-------|--------------|-------------------|
| [4]       | 53              | 43                | 8.7               | 2.91  | 0.235        | 6.51              |
| CDF [7]   | 546             | 61.26±0.93        | 12.87±0.30        | 3.12  | 0.319        | 8.82              |
| E811 [8]  | 1800            | 71.71±2.02        | 15.79±0.87        | 3.38  | 0.351        | 9.63              |
| CDF [7]   | 1800            | 80.03±2.24        | 19.70±0.85        | 4.20  | 0.337        | 8.87              |

Table 1: Total and elastic cross sections together with the obtained values of $G^2$ and $\beta[fm^2]$. The last column includes calculated values of diffractive cross sections.
The two values for $\sqrt{s} = 1800 GeV$ are consequence of two different results for $\sigma_{tot}$ and $\sigma_{el}$ measured by CDF and E811. We see from the table that Miettinen and Pumplin model is valid in a considerable range of energies. Similarly to the Goulianos model it predicts a slow rise of $\sigma_{SD}$ with $\sqrt{s}$. It gives, however the values which are a little higher.

![Figure 2: The momentum-transfer dependence of a beam dissociation obtained within Miettinen and Pumplin model.](image)

The dependence of diffractive dissociation cross section, shown in figure on the momentum transfer $t$ is obtained by applying Fourier transform to (14). One sees that the slope increases with energy. The calculated values of the slope for energy $\sqrt{s} = 1800 GeV$ are 9.9 (CDF) and 10.2 (E811), which is consistent with measurement of E710 i.e. 10.5 ± 1.8. We have also checked that the elastic slopes, calculated from (13), are consistent with existing experimental data.

In conclusion, we have shown that the Miettinen and Pumplin model correctly describes diffusion dissociation in hadron-hadron collisions with the energies of the order of TeV. Calculated values of $\sigma_{SD}$ are in reasonable agreement with experimental data. Moreover, the dependence on energy is similar to that calculated by Goulianos within his model of renormalized Pomeron flux.

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