Analysis of a low-energy correction to the eikonal approximation

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The relation between the dynamical eikonal approximation (DEA) and the continuum-discretized coupled-channels method with the eikonal approximation (E-CDCC) is discussed. Difference between the two models in the treatment of the Coulomb interaction is clarified. When Coulomb interaction is artificially turned off, DEA and E-CDCC are shown to give the same breakup cross section, within 3% error, of $^{15}$C on $^{208}$Pb at 20 MeV/nucleon. When the Coulomb interaction is included, the difference is appreciable and E-CDCC has a convergence problem. By including a quantum-mechanical correction to E-CDCC for lower partial waves between $^{15}$C and $^{208}$Pb, this problem is resolved and the result perfectly reproduces the result of full CDCC at a lower computational cost.

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I. INTRODUCTION

The development of radioactive-ion beams in the mid-80s has enabled the exploration of the nuclear landscape far from stability. This technical breakthrough has lead to the discovery of exotic nuclear structures, like nuclear halos and shell inversions. Halo nuclei exhibit a very large matter radius compared to their isobars. This unusual feature is explained by a strongly clusterized structure: a compact core that contains most of the nucleons to which one or two neutrons are loosely bound. Due to quantum tunneling, these valence neutrons exhibit a large probability of presence at a large distance from the core, hence increasing significantly the radius of the nucleus. Examples of one-neutron halo nuclei are $^{11}$Be and $^{15}$C, while $^{6}$He and $^{11}$Li exhibit two neutrons in their halo. Though less probable, protons halos are also possible. The exotic halo structure has thus been the subject of many theoretical and experimental studies for the last thirty years [1,2].

Due to their very short lifetime, halo nuclei must be studied through indirect techniques, such as reactions. The most widely used reaction to study halo nuclei is the breakup reaction, in which the halo dissociates from the core through the interaction with a target. The extraction of reliable structure information from measurements requires a good understanding of the reaction process. Various models have been developed to describe the breakup of two-body projectiles, i.e. one-nucleon halo nuclei (see Ref. [3] for a review).

The continuum-discretized coupled channel method (CDCC) is a fully quantum model in which the wave function describing the three-body motion—two-body projectile plus target—is expanded over the projectile eigenstates [4,5]. For breakup modeling, the core-halo continuum must be included and hence is discretized and truncated to form an approximate complete set of states. With such an expansion, the corresponding Schrödinger equation translates into a set of coupled equations [6-8]. This reaction model is very general and has been successfully used to describe several real and virtual breakup reactions at both low and intermediate energy [9-12]. However it can be very computationally challenging, especially at high beam energy. Simplifying approximations, less computationally demanding, have been developed to circumvent this difficulty.

In the time-dependent technique (TD), the projectile is assumed to follow a classical trajectory along which it feels a time-dependent potential simulating its interaction with the target. This semiclassical approximation leads to the resolution of a time-dependent Schrödinger equation (see Ref. [13] and references therein).

At sufficiently high energy, the eikonal approximation can be performed. In that approximation, the projectile-target relative motion is assumed not to deviate significantly from the asymptotic plane wave [14]. By factorizing that plane wave out of the three-body wave function, the Schrödinger equation can be significantly simplified. Both the eikonal CDCC (E-CDCC) [15,16] and the dynamical eikonal approximation (DEA) [17,18] are such eikonal models. Note that these models differ from the usual eikonal approximation in that they do not include the subsequent adiabatic approximation, in which the internal dynamics of the projectile is neglected.

The E-CDCC model solves the eikonal equation using the same discretization technique as the full CDCC model. Thanks to this, E-CDCC can be easily extended to a hybrid version, in which a quantum-mechanical (QM) correction to the scattering amplitude can be included for the low orbital angular momentum $L$ between the projectile and the target. This helps obtaining results as accurate as a full CDCC with a minimal task. In addition, E-CDCC can take the dynamical relativistic effects into account [19,20], and it has recently been extended to inclusive breakup processes [21,22].

The distinct characteristic of DEA over E-CDCC is that it describes the projectile wave function on a three-dimensional mesh, without partial-wave expansion. This prescription is expected to efficiently include components of the projectile wave function up to high orbital angular momentum between...
its constituents. Moreover it enables describing both bound and breakup states on the same footing, without resorting to continuum discretization. Since DEA treats the three-body dynamics explicitly, all coupled-channels effects are automatically included.

In a recent work [23], a comparison between CDCC, TD and DEA has been performed. The breakup of the one-neutron halo nucleus $^{15}$C on $^{208}$Pb has been chosen as test-case. At 68 MeV/nucleon, the results of the three models agree very well with each other, except for the TD calculation, which fails to reproduce the diffraction pattern of the angular distribution for breakup because of its underlying semiclassical approximation. At 20 MeV/nucleon, DEA cannot reproduce the CDCC results because the eikonal approximation is no longer valid at such low energy. It appears that the problem is due to the Coulomb deflection, which, at low energy, significantly distorts the projectile-target relative motion from a pure plane wave. Because of the computational advantage of the eikonal approximation over the CDCC framework, it is important to pin down where the difference comes from in more detail and try to find a way to correct it.

The goal of the present paper is to compare the E-CDCC and DEA models. In particular, we analyze their treatment of the Coulomb interaction and study the ability of the hybrid extension of E-CDCC to reproduce the Coulomb deflection, which is missing in the eikonal approximation. We focus on the $^{15}$C breakup on $^{208}$Pb at 20 MeV/nucleon. First, we compare the results of DEA and E-CDCC with the Coulomb interaction turned off. In this case essentially the two models solve the same Schrödinger equation. Then we include the Coulomb interaction which causes Coulomb distortion and Coulomb breakup. The comparison between DEA and E-CDCC reveals the effects of the Coulomb interaction in the two models. A comparison to the full CDCC calculation shows the importance of the QM description of the scattering waves beyond the eikonal approximation at this energy. The inclusion of the QM correction to E-CDCC enables us to take these effects into account at a much lower computational cost.

In Sec. III we briefly review DEA and E-CDCC, and clarify the relation between them. We compare in Sec. III the breakup cross sections of $^{15}$C on $^{208}$Pb at 20 MeV/nucleon, with and without the Coulomb interaction. A summary is given in Sec. IV.

II. FORMALISM

A. Three-body reaction system

We describe the $^{15}$C breakup on $^{208}$Pb using the coordinate system shown in Fig. 1. The coordinate of the center-of-mass (c.m.) of $^{15}$C relative to $^{208}$Pb is denoted by $R$, and $r$ is the neutron-$^{14}$C relative coordinate. $R_n$ and $R_{14}$ are, respectively, the coordinates of $n$ and the c.m. of $^{14}$C from $^{208}$Pb. We assume both $^{14}$C and $^{208}$Pb to be inert nuclei. In this study we neglect the spin of $n$. The Hamiltonian describing the $^{15}$C structure therefore reads

$$h = -\frac{\hbar^2}{2\mu_{n14}} \Delta_r + U_{nC}(r),$$

where $\mu_{n14}$ is the $^{14}$C-$n$ reduced mass and $U_{nC}$ is a phenomenological potential describing the $^{14}$C-$n$ interaction (see Sec. IIA). We denote by $\varphi_{\ell m}$ the eigenstates of Hamiltonian of Eq. (1) at energy $\varepsilon$ in partial wave $\ell m$, with $\ell$ the $^{14}$C-$n$ orbital angular momentum and $m$ its projection. For negative energies, these states are discrete and describe bound states of the nucleus. For the present comparison, we consider the sole ground state $\varphi_{00m_0}$ at energy $\varepsilon_0 = -1.218$ MeV. The positive-energy eigenstates correspond to continuum states that simulate the broken up projectile.

To simulate the interaction between $n$ ($^{14}$C) and $^{208}$Pb, we adopt the optical potential $U_n (U_{14})$ (see Sec. IIA). Within this framework, the study of $^{15}$C-$^{208}$Pb collision reduces to solving the three-body Schrödinger equation

$$H \Psi(R, r) = E_{tot} \Psi(R, r)$$

with the Hamiltonian

$$H = -\frac{\hbar^2}{2\mu} \Delta_R + h + U_{14}(R_{14}) + U_n(R_n),$$

where $\mu$ is the $^{15}$C-$^{208}$Pb reduced mass. Equation (2) has to be solved with the incoming boundary condition

$$\lim_{z \to -\infty} \Psi(R, r) = e^{iK_0z + \cdots} \varphi_{00m_0}(r),$$

where $K_0$ is the wave number for the initial projectile-target motion, whose direction defines the $z$ axis. That wave number is related to the total energy $E_{tot} = \hbar^2 K_0^2/(2\mu) + \varepsilon_0$. The “…” in Eq. (4) indicates that the projectile-target relative motion is distorted by the Coulomb interaction, even at large distances.

In the eikonal approximation, the three-body wave function $\Psi$ is assumed not to vary significantly from the incoming plane wave of Eq. (4). Hence the usual eikonal factorization

$$\Psi(R, r) = e^{iK_0z} \psi(b, z, r),$$

FIG. 1: Schematic illustration of the ($^{14}$C + $n$) + $^{208}$Pb three-body system.
where we have explicitly decomposed $R$ in its longitudinal $z$ and transverse $b$ components. In the following $b$ is expressed as $b = (b, \phi_R)$ with $b$ the impact parameter and $\phi_R$ the azimuthal angle of $R$.

Using factorization [5] in Eq. (3) and taking into account that $\psi$ varies smoothly with $R$, we obtain equations simpler to solve than the full three-body Schrödinger equation (2). In the following subsections, we specify the equations solved within the E-CDCC (Sec. II B) and the DEA (Sec. II C).

B. Continuum-discretized coupled-channels method with the eikonal approximation (E-CDCC)

E-CDCC expresses the three-body wave function $\Psi$ as [15, 16]

$$\Psi(R, r) = \sum_{i \ell m} \tilde{\xi}_{i \ell m}(b, z) \varphi_{i \ell m}(r) e^{i K_i z} e^{i(m_0 - m) \phi_R} \phi_C(R),$$

where the subscript $i$ is the label of the eigenenergy of $^{15}$C; $i = 0$ corresponds to the ground state and $i > 0$ to discretized continuum states of $^{15}$C. The set $\{ \varphi_{i \ell m} \}$ satisfying

$$\langle \varphi_{i' \ell' m'} | h | \varphi_{i \ell m} \rangle = \varepsilon_i \delta_{i' i} \delta_{\ell' \ell} \delta_{m' m}$$

is assumed to form an approximate complete set for the projectile internal coordinate $r$. The plane wave $e^{i K_i z}$ contains the dominant part of the projectile-target motion as explained above. The corresponding wave number varies with the energy of the eigenstate of $^{15}$C respecting the conservation of $^{15}$C during the scattering process. However, it does not change the eigenenergy of $^{15}$C; i.e., eigenenergies $\varepsilon$ up to high values in the $^{15}$C excitation energy and its position following $v_i(R)$ depends on both the projectile excitation energy and its position following

$$v_i(R) = \frac{1}{\mu} \sqrt{\hbar^2 K_i^2 - 2 \mu V_C(R)},$$

where

$$V_C(R) = \frac{Z_C Z_T e^2}{R}$$

is the projectile-target potential that slows down the projectile as it approaches the target. The coupling potential $F_{cc'}$ is defined by

$$F_{cc'}(b, z) = \langle \varphi_c | U_{14} + U_n - V_C | \varphi_{c'} \rangle e^{i(m - m') \phi_R} \delta_{ii_0},$$

and

$$R_{ii'}(b, z) = \frac{(K_{i'} R - K_{i} z)^{\eta_{ii'}}}{(K_{i} R - K_{i'} z)^{\eta_{ii'}}}.$$  (14)

Within the E-CDCC framework, the boundary condition (4) translates into

$$\lim_{z \to -\infty} \tilde{\xi}_{i \ell m}(b, z) = \delta_{i 0} \delta_{\ell 0} \delta_{m m_0}.$$  (15)

C. The dynamical eikonal approximation (DEA)

In the DEA, the three-body wave function is factorized following [17, 18]

$$\Psi(R, r) = \psi(b, z, r) e^{i K_{b} z} e^{i \chi_{C}(b, z)} e^{i \frac{\varepsilon_0 z}{(\hbar \nu_0)}},$$

where $\chi_C$ is the Coulomb phase that accounts for the Coulomb projectile-target scattering

$$\chi_C(b, z) = -\frac{1}{\hbar \nu_0} \int_{-\infty}^{z} V_C(R) \, dz'.$$  (17)

From factorization (16), we obtain the DEA equation [17, 18]

$$i \hbar \nu_0 \frac{\partial}{\partial z} \psi(b, z, r) = [h + U_{14} + U_n - \varepsilon_0 - V_C] \psi(b, z, r).$$  (18)

The initial condition of Eq. (4) translates into

$$\lim_{z \to -\infty} \psi(b, z, r) = \varphi_{000}(R).$$  (19)

The DEA equation (18) is solved for all $b$ with respect to $z$ and $r$ expanding the wave function $\psi$ on a three-dimensional mesh. This allows to include naturally all relevant states of $^{15}$C, i.e., eigenenergies $\varepsilon$ up to high values in the $^{14}$N-$^{15}$C continuum, and large angular momentum $\ell$, and its $z$-component $m$. This resolution is performed assuming a constant projectile-target relative velocity $v = \nu_0$. It should be noted that this does not mean the adiabatic approximation, because in Eq. (18) the internal Hamiltonian $h$ is explicitly included. DEA thus treats properly the change in the eigenenergy of $^{15}$C during the scattering process. However, it does not change the $^{15}$C-$^{208}$Pb velocity accordingly. This gives a violation of the conservation of the total energy of the three-body system. However, even at 20 MeV/nucleon, its effect is expected to be only a few percents as discussed below.
The calculation of physical observables requires the wave function $\Psi$ of Eq. (16) at $z \to \infty$ [17,18]. The corresponding Coulomb phase $\chi_C$ reads [24]

$$\lim_{z \to \infty} \chi_C = 2\eta_0 \ln(K_0b), \quad (20)$$

where $\eta_0$ is the Sommerfeld parameter for the entrance channel [see Eq. (9)].

D. Comparison between E-CDCC and DEA

To ease the comparison between the DEA and the E-CDCC, we rewrite the formula given in Sec. II C in a coupled-channel representation. We expand $\psi$ as

$$\psi(b, z, r) = \sum_{i,m} \xi_{i,m}(b, z) \phi_{i,m}(r) \exp \left( \frac{\xi_i - \xi_0}{i\hbar v_0} z \right) e^{i(m_0 - m) \phi_R}, \quad (21)$$

Inserting Eq. (21) into Eq. (13), multiplied by $\phi_{i}^{*} \phi_{m'}$ from the left, and integrating over $r$, one gets

$$\frac{\partial}{\partial z} \xi_{i,m}(b, z) = \frac{1}{i\hbar v_0} \sum_{c} F_{cc'}(b, z) \xi_{c'}(b, z) \exp \left( \frac{\xi_{c'} - \xi_{0}}{i\hbar v_0} z \right), \quad (22)$$

which is nothing but the DEA equation [18] in its coupled-channel representation.

The boundary condition Eq. (19) thus reads

$$\lim_{z \to \infty} \xi_{i,m}(b, z) = \delta_{i0} \delta_{t0} \delta_{mm0}, \quad (23)$$

and the total wave function is given by

$$\Psi(R, r) = \sum_{i,m} \xi_{i,m}(b, z) \phi_{i,m}(r) \exp \left( \frac{\xi_i - \xi_0}{i\hbar v_0} z \right) e^{i(m_0 - m) \phi_R} e^{iK_0z} e^{i\chi_C(b, z)}, \quad (24)$$

One may summarize the difference between Eq. (22) and Eq. (10) as follows. First, DEA uses the constant and channel-independent $^{15}$C-$^{208}$Pb relative velocity $v_0$, whereas E-CDCC uses the velocity depending on both $R$ and the channel $i$ that ensures the total-energy conservation.

Second, whereas the right-hand side of Eq. (22) involves the phase $\exp \left( i(K_{i'} - K_i)z \right)$, the E-CDCC Eq. (10) includes the phase $\exp \left( i(K_{i'} - K_i)z \right)$. The former can be rewritten as

$$\frac{\xi_{i'} - \xi_i}{i\hbar v_0} z = \frac{\hbar^2 (K_{i'}^2 - K_i^2)}{2 \mu \hbar^2 K_0} \mu z = \frac{K_{i'} + K_i}{2K_0} \mu (K_{i'} - K_i) z. \quad (25)$$

If we can assume the semi-adiabatic approximation

$$\frac{K_{i'} + K_i}{2K_0} \approx 1, \quad (26)$$

the exponent Eq. (25) becomes the same as in E-CDCC. In the model space taken in the present study, Eq. (26) holds within $1.5\%$ error at 20 MeV/nucleon of incident energy.

Third, E-CDCC equation contains $R_{i'i'}$ taking account of the channel dependence of the $^{15}$C-$^{208}$Pb Coulomb wave function, which DEA neglcts. Nevertheless, it should be noted that, as shown in Refs. [15, 16], the Coulomb wave functions in the initial and final channels involved in the transition matrix ($T$ matrix) of E-CDCC eventually give a phase $2\eta_j \ln(K_jb)$, with $j$ the energy index in the final channel. Thus, if Eq. (26) holds, the role of the Coulomb wave function in the evaluation of the $T$ matrix in E-CDCC is expected to be the same as in DEA, since DEA explicitly includes the Coulomb eikonal phase, Eq. (20).

When the Coulomb interaction is absent, we have $R_{i'i'}(b, z) = 1$ and no $R$ dependence of the velocity. Therefore, it will be interesting to compare the results of DEA and E-CDCC with and without the Coulomb interaction separately.

III. RESULTS AND DISCUSSION

A. Model setting

We calculate the energy spectrum $d\sigma/d\varepsilon$ and the angular distribution $d\sigma/d\Omega$ of the breakup cross section of $^{15}$C on $^{208}$Pb at 20 MeV/nucleon, where $\varepsilon$ is the relative energy between $n$ and $^{14}$C after breakup, and $\Omega$ is the scattering angle of the c.m. of the $n$-$^{14}$C system. We use the potential parameters shown in Table I for $U_{wc}$ (the $n$-$^{14}$C interaction), $U_{14}$, and $U_{w}$; the depth of $U_{wc}$ for the $d$-wave is changed to 69.43 MeV to avoid a non-physical $d$ resonance. The spin of the neutron is disregarded as mentioned earlier. We adopt Woods-Saxon potentials for the interactions:

$$U_x(R_x) = -V_0 f(R_x, R_0, a_0) - iW_x f(R_x, R_w, a_w) + iW_x \frac{d}{dR_x} f(R_x, R_w, a_w) \quad (27)$$

with $f(R_x, \alpha, \beta) = (1 + \exp[(R_x - \alpha)/\beta])^{-1}$; $R_x = r, R_{14}$, and $R_n$ for $x = n$, $^{14}$C, and $n$, respectively. The Coulomb interaction between $^{15}$C and $^{208}$Pb is described by assuming a uniformly charged sphere of radius $R_C$.

| TABLE I: Potential parameters for the pair interactions $U_{wc}$, $U_{14}$, and $U_n$. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $V_0$ (MeV)     | $R_0$ (fm)      | $a_0$ (fm)      | $W_w$ (MeV)     | $W_x$ (MeV)     | $U_{wc}$ (MeV)  |
| 63.02           | 2.651           | —               | —               | —               | —               |
| 37.00           | 2.840           | 0.600           | —               | —               | 9.544           |
| 64.82           | 6.932           | 0.750           | 2.840           | 21.85           | 7.466           |

In E-CDCC, we take the maximum value of $r$ to be 800 fm with the increment of 0.2 fm. When the Coulomb interaction is turned off, we take the $n$-$^{14}$C partial waves up to $l_{\text{max}} = 10$. For each $\ell$ the continuum state is truncated at $k_{\text{max}} = 1.4$ fm$^{-1}$ and discretized into 35 states with the equal spacing of $\Delta k = 0.04$ fm$^{-1}$; $k$ is the relative wave number between $n$ and $^{14}$C. The resulting number of coupled channels, $N_{\text{ch}}$, is 2311. The maximum values of $z$ and $b$, $z_{\text{max}}$ and $b_{\text{max}}$...
\( b_{\text{max}} \), respectively, are both set to 50 fm. When the Coulomb interaction is included, we use \( \ell_{\text{max}} = 6 \), \( k_{\text{max}} = 0.84 \text{ fm}^{-1} \), \( \Delta k = 0.04 \text{ fm}^{-1} \), \( z_{\text{max}} = 1000 \text{ fm} \), and \( b_{\text{max}} = 150 \text{ fm} \). We have \( N_{\text{ch}} = 589 \) in this case.

In the DEA calculations, we use the same numerical parameters as in Ref. [23]. In the purely nuclear case, the wave function \( \psi \) is expanded over an angular mesh containing up to \( N_\theta \times N_\phi = 14 \times 27 \) points, a quasi-uniform radial mesh that extends up to 200 fm with 200 points, \( b_{\text{max}} = 50 \text{ fm} \), and \( z_{\text{max}} = 200 \text{ fm} \) (see Ref. [13] for details). In the charged case, the angular mesh contains up to \( N_\theta \times N_\phi = 12 \times 23 \) points, the radial mesh extends up to 800 fm with 800 points, \( b_{\text{max}} = 300 \text{ fm} \), and \( z_{\text{max}} = 800 \text{ fm} \).

**B. Comparison without Coulomb interaction**

We show in Fig. 2 the results of \( d\sigma/d\varepsilon \) calculated by DEA (solid line) and E-CDCC (dashed line); \( d\sigma/d\varepsilon \) is obtained by integrating the double-differential breakup cross section \( d^2\sigma/(d\varepsilon d\Omega) \) over \( \Omega \) in the whole variable region. The two results agree very well with each other; the difference around the peak is below 3%.

In Fig. 3 the comparison in \( d\sigma/d\Omega \), i.e., \( d^2\sigma/(d\varepsilon d\Omega) \) integrated over \( \varepsilon \) up to 10 MeV, is shown. The agreement between the two models is excellent confirming that, when the Coulomb interaction is turned off, DEA and E-CDCC solve the same equation and give the same result, as expected from the discussion at the end of Sec. II D. In particular this comparison shows that Eq. (26) turns out to be satisfied with very high accuracy. It should be noted that the good agreement between DEA and E-CDCC is obtained only when a very large model space is taken. In fact, if we put \( \ell_{\text{max}} = 6 \) in E-CDCC, we have 30% smaller \( d\sigma/d\varepsilon \) (dotted line) than the converged value and, more seriously, even the shape cannot be reproduced. This result shows the importance of the higher partial waves of \( ^{14}\text{C} \) for the nuclear breakup at 20 MeV/nucleon.

**C. Comparison with Coulomb interaction**

The results of \( d\sigma/d\varepsilon \) calculated by DEA (solid line) and E-CDCC (dashed line) are shown in Fig. 4; the Coulomb interaction is included. In contrast to the results in Fig. 2, the difference between them is appreciable (about 10%). It suggests that this difference comes from the different treatment of the Coulomb interaction in DEA and E-CDCC. Furthermore, it is found that E-CDCC does not converge with respect to \( \Delta k \) in this case; the dashed line is obtained with the model space with which full (QM) CDCC converges. This ill-behavior of the E-CDCC result indicates that the treatment of the Coulomb interaction in E-CDCC at this energy is in-
The present study confirms the difficulty to properly describe the Coulomb interaction within the eikonal approximation at low energy. However, it is found that even at 20 MeV/nucleon, including QM corrections within the E-CDCC helps correctly reproducing the Coulomb deflection and lead to the same accuracy as that of full CDCC, with a minimal computational cost. This hybrid calculation will be useful for describing nuclear and Coulomb breakup processes in a wide range of incident energies. It could be included in other CDCC programs to increase their computational efficiency without reducing their accuracy. This could be an asset to improve the description of projectiles while keeping reasonable calculation times. Moreover, it will be interesting to find a way to include the QM corrections within the DEA.
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