### Table 1:

| Sample | $R_e$ (kΩ) | $R_T$ (kΩ) | $C$ (fF) |
|--------|------------|------------|----------|
| 1      | 0.10       | 39.4       | 1.85     |
| 2A     | 0.25       | 24.7       | 3.75     |
| 3      | 0.26       | 26.8       | 3.45     |
| 4C     | 0.35       | 15.8       | 5.25     |
| 5B     | 0.37       | 23.0       | 3.85     |
| 6F     | 0.66       | 35.2       | 2.40     |
| 7C     | 0.83       | 13.4       | 5.20     |
| 8A     | 0.94       | 44.4       | 2.90     |
| 9A     | 1.12       | 44.9       | 2.25     |
| 10A    | 1.17       | 27.4       | 3.00     |
| 11B    | 4.76       | 52.0       | 2.50     |

| Sample | $R_e$ (kΩ) | $R_T$ (kΩ) | $C$ (fF) |
|--------|------------|------------|----------|
| 1B     | 0.32       | 110.1      | 3.30     |
| 2B     | 3.11       | 268.6      | 2.25     |
| 3F     | 3.48       | 170.8      | 1.95     |

| Sample | $R_e$ (kΩ) | $R_T$ (kΩ) | $C$ (fF) |
|--------|------------|------------|----------|
| 1C     | 0.76       | 145.6      | 5.20     |
| 2C     | 1.30       | 103.1      | 6.00     |
| 3C     | 2.14       | 117.5      | 7.10     |
| 4D     | 2.31       | 123.6      | 6.10     |
| 5D     | 2.49       | 122.9      | 5.00     |
| 6E     | 2.56       | 214.2      | 3.40     |
| 7E     | 2.91       | 193.9      | 3.90     |

**Figure 1:**
Figure 2:
Figure 3:
Effect of the Electromagnetic Environment on Arrays of Small Normal Metal Tunnel Junctions: Numerical and Experimental Investigation

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Abstract

We present results of a set of experiments to investigate the effect of dissipative external electromagnetic environment on tunneling in linear arrays of junctions in the weak tunneling regime. The influence of this resistance decreases as the number of junctions in the chain increases and ultimately becomes negligible. Further, there is a value of external impedance, typically \( \sim 0.5 \, \text{k}\Omega \), at which the half-width of the zero-voltage dip in the conductance curve shows a maximum. Some new analytical formulae, based on the phase-correlation theory, along with numerical results will be presented.

73.23.Hk, 73.40.Gk, 73.40 Rw
In recent years large attention has been paid to the role of electromagnetic environment on charging effects in small tunnel junctions, both theoretically and experimentally \[1\]- \[7\]. Yet, arrays of such tunnel junctions with well-defined external impedances have not been extensively discussed. This is partly because the theoretical formulation of such arrays is more elaborate: there are, e.g., cotunneling effects, inhomogeneities, and background charges, which along with the effects of environment make such an investigation rather difficult in general terms. This lack of theoretical predictions, in turn, has decelerated experimental search for observation of new features in arrays. In this letter we will attempt to fill part of this gap by demonstrating a set of experimental observations and a comparison of them to the results obtained from the already existing phase-correlation (PC) theory for single tunnel junctions, which we have now extended to analyze junction arrays numerically. This analysis is important in setting limits to the systematic error of the reading of the Coulomb blockade primary thermometer \[8\].

According to the PC theory, the tunneling rate through the \(k\)th junction of a completely symmetric array with \(C_k \equiv C\), \(R_{T,k} \equiv R_T\) and \(C_{0,k} = 0\) \[9\] (see Fig. 1), in the weak-tunneling regime, \(R_{T,k} \gg R_K \equiv h/e^2\), can be written as a convolution integral of the form

\[
\Gamma_k^\pm(\{n\}) \equiv \Gamma(\delta F_k^\pm, \{n\}) = \frac{1}{e^2 R_T} \int_{-\infty}^{+\infty} dE E \frac{E}{1 - e^{-\beta E}} P_k(-\delta F_k^\pm - E). \tag{1}
\]

Here \(\delta F_k^\pm\) is the change in free energy of the array when an electron tunnels to right (left), \(\{n\} \equiv \{n_1, n_2, \ldots, n_{N-1}\}\) designates the charge configuration on the islands, and \(P_k(E) \equiv (2\pi \hbar)^{-1} \int_{-\infty}^{+\infty} dt e^{i J_k(t) + i E t}\) is the probability density for the electron to exchange energy \(E\) with the environment. The correlation function \(J_k(t)\), which accounts for the environment of the \(k\)th junction, is given by

\[
J_k(t) = 2 \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \frac{\text{Re}[Z_t^k(\omega)]}{R_K} \frac{e^{-i\omega t} - 1}{1 - e^{-\beta h \omega}}, \tag{2}
\]

where \(Z_t^k(\omega)\) is the total impedance of the circuit as seen by this junction, and \(\beta \equiv (k_B T)^{-1}\).

By applying Fourier transform techniques to Eq. (1) one obtains
\[ \Gamma_k^\pm (\{n\}) = \frac{1}{e^2 R_T} \left\{ \frac{1}{\beta} + \frac{-\delta F_k^\pm - i\hbar J'_k(0)}{2} - \frac{\pi}{2\beta^2 \hbar} \int_{-\infty}^{+\infty} dt e^{i[J_k(t) - i\delta F_k^\pm \hbar]} \right\}. \tag{3} \]

This equation is central in the following numerical calculations. With \( \Gamma_k^\pm \)'s and the algorithm in [10] with \( R_{\Sigma} \equiv \sum_{k=1}^{N_k} R_{T,k} \), we find the current \( I \) through the array in equilibrium as

\[ I = \sum_{\{n\}_{\text{visited}}} \left\{ e^{\sum_{k=1}^{N_k}[\Gamma_k^+(\{n\}) - \Gamma_k^-(\{n\})]} \frac{R_{T,k}}{R_{\Sigma}} \right\} \frac{1}{\sum_{\{n\}_{\text{visited}}} (\sum_{k=1}^{N_k}[\Gamma_k^+(\{n\}) + \Gamma_k^-(\{n\})])^{-1}}. \tag{4} \]

In the expression above, starting initially from an arbitrary configuration, the states visited, \( \{n\}_{\text{visited}} \), over which the outer sums run through, are obtained by dividing the interval \([0, 1]\) into segments proportional to \( \Gamma_k^\pm \)'s in each current state, and drawing a random number \( r \) in the interval. For sequential tunneling, the segment to which \( r \) corresponds to will specify the junction through which the tunneling event happens and the tunneling direction. This way the distribution of \( \{n\}_{\text{visited}} \) will be statistically collected, and it will allow one to calculate the sums (now weighted by the distribution) according to Eq. (4) similarly to what has been done in [10].

In the case of a symmetric two-junction array we will also use a simpler algorithm, described in [10], to obtain the probability of finding \( n \) excess electrons on the island, \( \sigma(\{n\}) \), and finally the equilibrium current \( I \) through the array

\[ I = I_k = e \sum_{n=-\infty}^{\infty} \sigma(\{n\}) [\Gamma_k^+(\{n\}) - \Gamma_k^-(\{n\})]. \tag{5} \]

Assuming a completely symmetric array and a purely resistive environment \( R_e \), the common real part of the total impedance, \( Z_t(\omega) \equiv Z_t^k(\omega) \), in Eq. (2) reduces to the simple form

\[ \text{Re}[Z_t(\omega)] = \frac{R_e}{(\omega \omega_c)^2 + N^2}, \tag{6} \]

with \( \omega_c \equiv 1/R_c C \). It is already suggested by Eq. (6) that the effect of the environment decreases with increasing \( N \) and becomes vanishingly small for long arrays. Furthermore, using the partial sum expansion of \( \coth(x) \) (or by applying Cauchy’s integral theorem), one can evaluate \( J(t) \equiv J_k(t) \) in Eq. (2) as
\[ J(t) = \frac{\pi}{N^2 R_K} \left\{ (1 - e^{-|N\omega_c t|}) \left[ \cot\left( \frac{\beta h N\omega_c}{2} \right) - i \right] - \frac{2|t|}{\beta h} + 4 \sum_{n=1}^{\infty} \frac{(N\omega_c)^2(1 - e^{-|N\omega_n t|})}{2\pi n[\omega_n^2 - (N\omega_c)^2]} \right\} , \] (7)

where \( \omega_n \equiv 2\pi n/\beta h \) are Matsubara frequencies. The above equation is a straightforward extension of the result obtained for a single tunnel junction in resistive environment \[4,11\].

In our numerical calculations we have used the generally valid formula in Eq. (2), but equivalence of results is checked both by direct comparison of the numerical values derived from Eqs. (2) and (7), and by comparing the final conductance curves; the results from the two methods are indistinguishable.

Figure 1 shows a schematic view of an array with its bias circuitry. To check for the consistency of results for arrays with different number of junctions, each sample included one pair of arrays with about 3 \( \mu \)m space in between. Each pair had a different number of junctions, typically \( N = 1, 2, 2, 8 \), and \( N = 2, 20 \) (only samples with \( N = 2, 8 \) and \( N = 2, 20 \) are shown in Table I). The array consisted of Al/AlO\(_x\)/Al tunnel junctions (0.01 – 0.05 \( \mu \)m\(^2\)) with four chromium resistors, \( Z_{e,j}(\omega) = R_j \), at a distance of 2 \( \mu \)m, two at each end of the array. Cr resistances of 1 – 20 k\( \Omega \) can be easily obtained by adjusting the width (\( \sim 100 \) nm), length (\( \sim 2 \) \( \mu \)m), and thickness (\( \sim 3 – 8 \) nm) of chromium films. The equivalent environment resistance (additional to the natural free space like impedance of \( \sim 100 \) \( \Omega \)) will, then, be \( R_e = R_1 R_2/(R_1 + R_2) + R_3 R_4/(R_3 + R_4) \). Samples were made by e-beam lithography and three-angle evaporation techniques. All measurements were carried out at 4.2 K. More details of measurement techniques are presented in [3].

Figure 2 shows measured data for a typical sample with \( N = 2, R_T = 44.9 \) k\( \Omega \), \( R_e = 1.12 \) k\( \Omega \) and \( C = 2.25 \) fF (sample 9A in Table I (a)). \( C \) was determined by searching for the best fit of the measured depth of the zero-bias anomaly to the value given by Eqs. (1 – 3); those equations are in agreement with the zero-bias results of [4]. Each point in the simulated \( IV \)-curve (not shown in the figure) was obtained as a result of 1000 (pseudo)random tunneling events. Conductance curve (CC) was obtained by numerical derivation of the \( IV \)-curve. It is worth mentioning that even with much smaller number of draws, e.g. 10, the result agrees within better than 1.5% accuracy (with respect to the height and width of the CC dip) with
those obtained from "long simulations". The time needed for each step, with a reasonable partition of voltage interval and using a regular desktop computer, was about 1 minute. The result with \( N = 2 \) obtained from Eq. (5), based on the algorithm presented in [10], is identical with (within, at least, three digits with respect to the height and the half-width of the conductance curve at zero bias voltage) that of a more comprehensive method described above.

In Fig. 3 we have drawn the normalized half-width of the zero-bias minimum of the conductance curve, \( V_{1/2} \), as a function of the environment resistance for different pairs of samples. By normalization we mean scaling the half-widths by those derived for arrays without an external impedance, i.e., scaling by \( V_{1/2,0} \equiv 5.439 N k_B T / e \) [8]. Usually, we have done measurements for each sample in six different combinations of current and voltage probes (four-probe measurement) and there are variations between the different combinations shown by the error bars. The unequal height of the error bars for different samples is due to this. For \( N = 2 \), \( V_{1/2} \) shows a sharp maximum at around \( R_e \approx 1 \) kΩ. For longer arrays, \( N = 8 \) and \( N = 20 \), the dependence is much weaker, and \( V_{1/2} \) stays close to \( V_{1/2,0} \), which is directly demonstrating the advantage of using long arrays in Coulomb blockade thermometry. In each case (unambiguously only for \( N = 2 \), though), the normalized \( V_{1/2} \) approaches unity with \( R_e \to 0 \) and for \( R_e \to \infty \). This is in agreement with predictions of the PC theory. We will discuss this in further detail below.

In the same figure the results obtained by numerical simulation, and for \( N = 2 \) by the direct calculation described above, are depicted. In spite of the overall agreement between experiment and theory, there is a noticeable discrepancy between the measured and the predicted value of \( V_{1/2,\text{max}} \), the widest half-width, of the two-junction array. In this case (\( N = 2 \)) we could calculate the half-width with the two methods described above, without a noticeable difference between them. For \( N = 20 \) the absolute difference between experiment and theory is smaller; the predicted peak itself is very small, less than 1% (see inset of Fig. 3). Comparison of experimental data to those derived from the theory for \( N = 2 \) shows that the former produce even 15% wider conductance curves (Fig. 3). Cotunneling and
other higher order tunneling effects may play some role in our samples, because the junction resistances are not large as compared to $R_K$. While in the simpler case of a two-junction array without electromagnetic environment higher order tunneling tends to broaden the half-width of the CC dip \[12\], making such comparison between our data and the theoretical study of cotunneling effects in an array with dissipative environment \[13\] is beyond the scope of this work and is not done here. There is also a difference in the shape of the “shoulders” of the measured conductance curves for $N = 2$ in particular (see Fig. 2) as compared to the theory. Such a distortion of shape can be caused by the nonuniform size distribution of junctions in the array, which we have studied in the case of no external impedance \[8,14\]. Experimentally the intercomparison of the depths in different samples is difficult because the size of the junctions varies from sample to sample, and this gives the main contribution to the depth variation.

Next, let us consider the conductance curve in more detail. Using the time-domain formulation of single electron tunneling presented in \[4,15\] we repeated calculation of the high-temperature conductance $G(V)$ of the symmetric $N$-junction array without stray capacitance in the limit of large $R_e$ ($Nu_N \ll R_e/R_K$; $u_N \equiv [(N - 1)/N][e^2/Ck_B T]$). The result reads:

$$\frac{G(V)}{G_T} = 1 - u_N g(v) \quad (8)$$

with $G_T \equiv 1/N R_T$, $v \equiv eV \beta/N$ and $g(x) \equiv [x \sinh x - 4\sinh^2(x/2)]/8\sinh^4(x/2)$. Comparison of the above expression with the corresponding one derived for the perfectly conducting environment ($R_e = 0$) in \[8\], $G(V)/G_T = 1 - [(N - 1)/N]u_N g(v)$, indicates that

$$\frac{(\Delta G)}{(G_T)_{R_e\to\infty}} = \frac{N}{N - 1} \frac{(\Delta G)}{(G_T)_{R_e\to0}}, \quad (9)$$

where $\Delta G/G_T \equiv 1 - G(0)/G_T$ stands for the depth of the conductance dip. Here, again, the half-width has a universal value given by $V_{1/2} = V_{1/2,0}$. Equation (9) together with the last expression for $V_{1/2}$ confirms that the effect of a dissipative environment on conductance becomes increasingly suppressed by large $N$ and is most noticeable for a two-junction array.
Finally, let us have a closer look at the results above in view of Coulomb blockade thermometry (CBT) which is a primary (and secondary) thermometer based on single electron charging effects in arrays of tunnel junctions [8]. The main parameter is the half-width of the CC dip. We conclude that the inaccuracy in temperature measurements arising from environmental effects can be made small by increasing the number of junctions, \( N \). Our numerical simulations along with the experimental results depicted above show that for \( N = 20 \) the temperature determined by CBT is very close to the thermodynamic temperature. In practice, in a sample with no intentional \( R_e \) the effective value of the external impedance is of the order of free space impedance \( R_{e,eff} < Z_0 \simeq 377 \, \Omega \), and therefore the agreement is better than \( \pm 0.5\% \) (see the experimental data point marked by an open circle in Fig. 2).

In summary, we have studied the effect of the resistive electromagnetic environment on transport in arrays of normal metal tunnel junctions within the high temperature and the weak tunneling regime. Special attention has been paid to the half-width of the conductance curve. Overall agreement between numerical results based on the extension of the PC theory and experimental data has been observed. We cannot explain the quantitative discrepancy between theory and experiment for the strong enhancement of the width at intermediate values of \( R_e \) in the \( N = 2 \) case: higher order tunnelling effects have not, however, been included in the present theory. As a practical conclusion, we verify that the effect of environment is mostly emphasized in a two-junction array and can be made sufficiently small for thermometric applications by increasing the number of junctions in the array.

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TABLE CAPTION Table I. (a) Parameters of the measured samples with $N = 2$. Samples with the same capital letter, e.g., 2A, 8A, 9A and 10A, belong to the same chip. (b) and (c) Parameters of the measured samples with $N = 8$ and $N = 20$, respectively. Samples 4C and 1C, 7C and 2C, and 5B and 1B constitute pairs of samples, with a space of only about 3 $\mu$m between the respective arrays. Capacitances are obtained by fitting the theoretical conductance curves to the experimental ones. The only fitting parameter is the capacitance of the sample.
FIGURE CAPTIONS

Fig. 1. An array of $N$ tunnel junctions in an electromagnetic environment together with its bias circuitry. For a symmetric array in a purely dissipative environment, $Z_{e,j}(\omega) = R_j$, and with negligible stray capacitances, $C_{0,k} = 0$, the real part of the total impedance as seen by the $k$th junction, $\text{Re}[Z_k^e(\omega)]$, reduces to the simple form presented in the text (Eq. (6)).

Fig. 2. The measured conductance curve for a two-junction array with $R_e = 1.12$ kΩ (sample 9A). The curve under the data points is the theoretical conductance curve with $C = 2.25$ fF (see text).

Fig. 3. Measured half-width of the conductance curve, normalized by $5.439 N k_B T/e$ (see text), for samples with $N = 2$ (solid diamonds), $N = 8$ (open triangles) and $N = 20$ (open squares) as a function of extra (i.e., that in addition to the unintentional space impedance of $\sim 100$ Ω) external resistance. For comparison, a sample ($N = 20$) without any intentional on-chip impedance has been shown in the plot by an open circle. The error bars show differences in half-widths measured in different four probe configurations, which exceed the scatter within each individual configuration. The lowermost solid curve is obtained by simulation for completely symmetric 20-junction array with $C \equiv C_k = 5.0$ fF. The uppermost solid curve is the result of simulation for a two-junction array with $C = 2.2$ fF, whereas dotted and dashed lines correspond to $C = 5.0$ fF and $C = 1.4$ fF (the largest and the smallest capacitance of our samples), respectively. The middle solid curve indicates the result of simulation for an eight-junction array with $C = 5.0$ fF. The capacitances were obtained from the depth of the dip of the conductance curves. The inset shows the normalized half-width together with the depth of the conductance curve ($\Delta G/G_T$), obtained from the theory, for a 20-junction array. Note that in order to compare this figure with the theoretical results, additional resistance of $\sim 100$ Ω should be added to the experimental values on the horizontal scale, because the unintentional impedance cannot be evaluated reliably.
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