The decay $h^0 \rightarrow A^0 A^0$:

a complete 1-loop calculation in the MSSM

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Abstract
In the minimal supersymmetric standard model the decay $h^0 \rightarrow A^0 A^0$ of the light neutral scalar $h^0$ is kinematically allowed for low values of $\tan \beta$ when radiative corrections to the neutral Higgs masses are taken into account. The width of this decay mode is revisited on the basis of a complete 1-loop diagrammatic calculation. We give the analytical expressions and numerical results and compare them with the corresponding ones from the simpler and compact approximations of the effective potential method and the renormalization group approach.
1 Introduction

The search for Higgs bosons is a basic challenge for particle physics with the aim to shed light on the mechanism of electroweak symmetry breaking. The Minimal Supersymmetric Standard Model (MSSM) predicts at least one light neutral scalar Higgs particle. In order to experimentally detect possible signals of Higgs bosons and to trace back as far as possible the physical origin of a produced scalar particle, detailed studies of production and decay processes of Higgs bosons are required.

The Higgs sector of the MSSM consists of two neutral CP-even states $h^0, H^0$, one neutral CP-odd state $A^0$, and a pair of charged bosons $H^\pm$. At tree level, there are only two independent input parameters and thus the Higgs masses are strongly correlated. As a special consequence, the bound $M_{h^0} < M_{A^0}$ kinematically forbids the decay mode $h^0 \rightarrow A^0 A^0$ of the lightest neutral scalar boson $h^0$. As has been found few years ago and worked out in various stages of approximations \[1,2,3,4,5,6,7,8,9,10\], radiative corrections in the MSSM Higgs sector are large, proportional to $m_{\text{top}}^4$ at 1-loop in their leading terms. They significantly modify the tree level relations between masses and couplings and hence have to be taken into account for phenomenological studies and for interpretation of the results from experimental searches.

A specific consequence of the large radiative corrections to the $h^0$ mass is the possibility of having $M_{h^0} > 2M_{A^0}$, which makes the decay $h^0 \rightarrow A^0 A^0$ kinematically allowed. In refs. \[3,7\] it was shown that it can be the dominant decay mode of the $h^0$ and thus is of crucial importance for the experimental search for supersymmetric Higgs bosons. The calculation of the decay width was performed in various approximations: in the effective potential approach \[3\] and by the use of renormalization group equations \[7\]. The expressions obtained in this way are of impressive simplicity and thus most suitable for fast handling in Monte Carlo studies.

Besides these approximations, a diagrammatic calculation complete at the 1-loop level is desirable. It allows for the virtual contributions from all particles from the MSSM spectrum and the momentum dependence of the 2- and 3-point functions. This method is the technically most complicated one, but also most accurate at the 1-loop level. It provides a reference frame for checking the quality of the compact approximate formulae and allows to study the full parameter dependence of the decay rate. In the analysis of experimental data for the MSSM Higgs search done so far \[12\] only the available approximations were used.

In this article the width for the decay mode $h^0 \rightarrow A^0 A^0$ of the light MSSM neutral scalar boson is revisited on the basis of a complete 1-loop diagrammatic calculation. In addition, we compare the results with the corresponding ones from the simpler approximations and discuss the typical size of the differences. The calculation is performed in the on-shell renormalization scheme in the version worked out in ref. \[9\]. Section 2 contains a brief summary of the structure of the propagator corrections and corrections to the neutral Higgs masses. The set of vertex corrections to the $h^0 A^0 A^0$ vertex is given in section 3 with the explicit analytical expressions in the appendix. Results for the decay widths and branching ratios are presented and discussed in sections 4 and 5. The appendices provide all conventions and necessary formulae.
2 The MSSM Higgs sector

2.1 Tree-level structure

The Higgs potential of the MSSM reads \[1\]

\[
V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 + m_{12}^2 (\epsilon_{ab} H_a^1 H_b^2 + \text{h.c.}) + \frac{1}{8} (g'^2 + g^2) (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 - \frac{g^2}{2} |H_1 \bar{H}_2|^2,
\]

where \(m_i, m_{12}\) are mass parameters, and \(g, g'\) the \(SU(2)\) and \(U(1)\) gauge couplings. \(H_1\) and \(H_2\) are decomposed in the following way:

\[
H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}, \quad (2.2)
\]

\[
H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}.
\]

The vacuum expectation values \(v_1\) and \(v_2\) define the angle \(\beta\) in terms of the ratio

\[
\tan \beta = \frac{v_2}{v_1} \quad (2.3)
\]

In order to obtain the CP-even neutral mass eigenstates, the rotation

\[
\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \quad (2.4)
\]

is performed. The spectrum consists of:

- 2 neutral bosons, \(CP=1\): \(h^0, H^0\)
- 1 neutral boson, \(CP=-1\): \(A^0\)
- 2 charged bosons: \(H^+, H^-\)
- 3 unphysical Goldstone bosons: \(G^0, G^+, G^-\)

The potential (2.1) can be fixed with the help of two independent parameters: \(\tan \beta\) and

\[
M_{A^0}^2 = -m_{12}^2 (\tan \beta + \cot \beta), \quad (2.5)
\]

where \(M_{A^0}\) is the mass of the \(A^0\) boson. The mixing angle of the \((h^0, H^0)\) system at tree level can be obtained from

\[
\tan 2\alpha = \tan 2\beta \frac{M_{A^0}^2 + M_Z^2}{M_{A^0}^2 - M_Z^2}, \quad -\frac{\pi}{2} < \alpha \leq 0, \quad (2.6)
\]

and the masses of all other physical Higgs bosons are given by

\[
m_{h^0}^2 = \frac{1}{2} \left[ M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta} \right], \quad (2.7)
\]

\[
m_{H^\pm}^2 = M_A^2 + M_W^2.
\]
2.2 Radiative corrections to the Higgs masses

The simple tree-level results are changed significantly when radiative corrections are taken into account \([1, 2, 3, 4, 5, 6, 7, 8, 9]\). In this paper we make use of the on-shell renormalisation scheme worked out in \([9]\). The inverse of the \((h^0, H^0)\) propagator matrix

\[
\Delta^{-1} = -i \begin{pmatrix} k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}^0(k^2) & \hat{\Sigma}_{h^0H^0}^0(k^2) \\ \hat{\Sigma}_{h^0H^0}^0(k^2) & k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}^0(k^2) \end{pmatrix},
\]

contains the renormalized self-energies \(\hat{\Sigma}_{h^0}^0, \hat{\Sigma}_{H^0}^0, \hat{\Sigma}_{h^0H^0}^0\), documented in \([9]\). The individual propagators in the matrix

\[
\Delta = i \begin{pmatrix} \Delta_{h^0} & \Delta_{h^0H^0} \\ \Delta_{h^0H^0} & \Delta_{H^0} \end{pmatrix}
\]

are given by

\[
\Delta_{h^0} = \frac{1}{k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}^0(k^2) - \frac{\Sigma_{h^0H^0}^0(k^2)}{k^2 - m_{h^0}^2 + \hat{\Sigma}_{H^0}^0(k^2)}},
\]

\[
\Delta_{H^0} = \frac{1}{k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}^0(k^2) - \frac{\Sigma_{h^0H^0}^0(k^2)}{k^2 - m_{h^0}^2 + \hat{\Sigma}_{H^0}^0(k^2)}},
\]

\[
\Delta_{h^0H^0} = \frac{-\hat{\Sigma}_{h^0H^0}^0(k^2)}{(k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}^0(k^2))(k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}^0(k^2)) - (\hat{\Sigma}_{h^0H^0}^0(k^2))^2}.
\]

In these expressions, \(m_{h^0}\) and \(m_{H^0}\) are the formal parameters of \((2.7)\). The physical one-loop masses are determined by the pole positions of the matrix \((2.9)\). They can be obtained as the solutions of the equation

\[
(k^2 - m_{h^0}^2 + \hat{\Sigma}_{h^0}^0(k^2))(k^2 - m_{H^0}^2 + \hat{\Sigma}_{H^0}^0(k^2)) - (\hat{\Sigma}_{h^0H^0}^0(k^2))^2 = 0.
\]

The physical Higgs masses are denoted by \(M_{h^0}\) and \(M_{H^0}\) in order to distinguish them from the formal tree-level masses \((2.7)\).

As a consequence of the sizeable loop contributions there is a part of the parameter space with \(M_{h^0} > 2M_{A^0}\), where thus the decay \(h^0 \to A^0A^0\) is kinematically allowed. The correlation of \(M_{h^0}, M_{A^0}\) and \(\tan \beta\) for allowed \(h^0 \to A^0A^0\) decays is shown in Figure 2.1 (all masses in GeV, the same applies for all figures) for a typical set of parameters. \(M_{SUSY}\) is the scalar mass parameter, as explained in \([A.4]\).

2.3 \(m_{\text{top}}^4\)-approximation

For the Higgs masses, the dominant 1-loop contributions are the leading terms \(\sim m_{\text{top}}^4\). In the simplest approximation, assuming unmixed left and right stop squarks, the renormalised self energies can be expressed as

\[
\hat{\Sigma}_{h^0}^0(k^2) = -\omega_l \cos^2 \alpha, \\
\hat{\Sigma}_{H^0}^0(k^2) = -\omega_l \sin^2 \alpha, \\
\hat{\Sigma}_{h^0H^0}^0(k^2) = -\omega_l \sin \alpha \cos \alpha,
\]

\[(2.12)\]
Figure 2.1: $M_{h^0} - M_{A^0}$-dependence where $h^0 \rightarrow A^0 A^0$ is kinematically allowed

with ($N_C$ is the number of the colors, $s_w^2 = 1 - M_W^2/M_Z^2$)

$$
\omega_t = \frac{2N_C e^2 m_{top}^4}{(4\pi)^2 s_w^2 M_W^2 \sin^2 \beta} \log \left( \frac{m_h m_A}{m_{top}} \right). \tag{2.13}
$$

Accordingly, equation (2.7) is modified in the following way:

$$
M_{H^0,A^0}^2 = \frac{M_{A^0}^2 + M_Z^2 + \omega_t}{2} \pm \sqrt{\left( \frac{M_{A^0}^2 + M_Z^2}{4} + \omega_t^2 \right)^2 - M_{A^0}^2 M_Z^2 \cos^2 2\beta + \frac{\omega_t \cos 2\beta}{2} (M_{A^0}^2 - M_Z^2)}. \tag{2.14}
$$

3 Vertex corrections for $h^0 \rightarrow A^0 A^0$

3.1 Complete 1-loop terms

The elementary $h^0 A^0 A^0$ -vertex arises from the following term of the Higgs potential (2.1):

$$
\frac{1}{8} (g^2 + g^2) (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 \tag{3.1}
$$

which leads to

$$
-ig M_Z \frac{\cos 2\beta \sin(\alpha + \beta)}{2 c_w} = : i T_0, \quad c_w = \cos \theta_W. \tag{3.2}
$$

The $H^0 A^0 A^0$ -vertex is obtained by the substitution $\sin(\alpha + \beta) \rightarrow -\cos(\alpha + \beta)$. 

4
At the one-loop level the $h^0A^0A^0$-vertex is given by $T_0 + \Delta T_{h^0}$ with $T_0$ from (3.2) and the renormalized vertex correction $\Delta T_{h^0}$. This expression consists of the sum of all vertex correction diagrams and the counter term:

$$i\Delta T_{h^0} = \sum_i \Delta T_{h^0,i} + CT_{h^0} \tag{3.3}$$

The $H^0A^0A^0$-vertex at the one loop level is obtained in an analogous way:

$$i\Delta T_{H^0} = \sum_i \Delta T_{H^0,i} + CT_{H^0} \tag{3.4}$$

All diagrams contributing to the vertex corrections are shown in Figures 3.1 and 3.2. Their analytical expressions are explicitly given in appendix B (for $h^0A^0A^0$).

The counter term derived from (3.1) reads in the scheme of $\left[9\right]$:

$$CT_{h^0} = -\frac{ig}{2c_w} \left[ -\Sigma'_{A}(M_{A^0}^2)M_Z \left\{ \frac{3}{2} \sin(\alpha + \beta) \cos 2\beta \right\} \right. \right.$$  
$$+ \left. \left. \Re \Sigma_{AZ}(M_{A^0}^2) \left( \cos(\beta - \alpha) - \frac{\cos^2 2\beta \sin(\alpha + \beta)}{\sin 2\beta} \right) \right] \right. \tag{3.5}$$

Figure 3.1: vertex corrections, part 1
Figure 3.2: vertex corrections, part 2
\[ + \Sigma'_\gamma(0) \frac{M_Z}{2} \cos 2\beta \sin(\alpha + \beta) \]
\[ + \text{Re}\Sigma_Z(M_Z^2) \frac{1}{M_Z} \left( \cos 2\beta \sin(\alpha + \beta) - \frac{c_w^2}{2s_w^2} \cos 2\beta \sin(\alpha + \beta) \right) \]
\[ + \text{Re}\Sigma_W(M_W^2) \frac{1}{M_Z} \cos 2\beta \sin(\alpha + \beta) \left( \frac{1}{2s_w^2} - \frac{1}{2c_w^2} \right) \]
\[ - \Sigma_{ij}(0) \frac{1}{M_Z} \cos 2\beta \sin(\alpha + \beta) \frac{s_w}{c_w} \].

Analogously, the \( H^0 A^0 A^0 \) counterterm is given by
\[ CT_{H^0} = + \frac{ig}{2c_w} \left[ \frac{\Sigma'_A}{M_Z} \left( \frac{3}{2} \cos(\alpha + \beta) \right) \right. \]
\[ + \text{Re}\Sigma_{AZ}(M_A^2) \left( \sin(\beta - \alpha) - \frac{\cos^2 2\beta \cos(\alpha + \beta)}{\sin 2\beta} \right) \]
\[ + \Sigma'_\gamma(0) \frac{M_Z}{2} \cos 2\beta \cos(\alpha + \beta) \]
\[ + \text{Re}\Sigma_Z(M_Z^2) \frac{1}{M_Z} \cos 2\beta \cos(\alpha + \beta) \left( \frac{1}{2s_w^2} - \frac{1}{2c_w^2} \right) \]
\[ - \Sigma_{ij}(0) \frac{1}{M_Z} \cos 2\beta \cos(\alpha + \beta) \frac{s_w}{c_w} \].

\( \Sigma_a, \Sigma_{ab} \) denote the diagonal and non-diagonal self energies, and \( \Sigma' \) the derivative with respect to the momentum squared. As has been checked analytically, the sum of the vertex diagrams and the counterterm is UV-finite.

### 3.2 Approximations

For practical purposes it is convenient to consider in particular the subset of vertex correction diagrams containing terms \( \sim m^4_{\text{top}} \). There are three diagrams of this type, shown in Figure 3.3.

![Figure 3.3: The leading \( h^0 A^0 A^0 \) vertex diagrams](image-url)
The coefficient of the $\sim m_{\text{top}}^4$ term in the sum of these diagrams is UV-finite. In the approximation $m_{\text{top}}, m_{tR}, m_{tL} \gg M_{h^0}, M_{A^0}$ one obtains:

\[
\Delta T_{h^0,\text{top}} = \frac{3 g^3 \cos \alpha}{4 M_W^3 \sin \beta (4\pi)^2} \left[ \cot^2 \beta \left( 2m_{\text{top}}^4 (M_{h^0}^2 - 2M_{A^0}^2) \right) \frac{1}{m_{\text{top}}^2} \right. \\
+ 4m_{\text{top}}^4 \ln \left( \frac{m_{\text{top}}^2}{m_{tL}^2 m_{tR}^2} \right) \\
+ 2(\mu - A_t \cot \beta)^2 m_{\text{top}}^4 \ln \frac{m_{tR}^2 - m_{tL}^2}{m_{tL}^2 - m_{tR}^2} \left. \right].
\] (3.7)

The further simplification $m_{tL} \approx m_{tR}$ yields

\[
\Delta T_{h^0,\text{top}} = \frac{3 g^3 \cos \alpha}{4 M_W^3 \sin \beta (4\pi)^2} \left[ \cot^2 \beta \left( 2m_{\text{top}}^4 (M_{h^0}^2 - 2M_{A^0}^2) \right) \frac{1}{m_{\text{top}}^2} \right. (I) \\
+ 4m_{\text{top}}^4 \ln \frac{m_{\text{top}}^2}{m_{t}^2} \right. (II) \\
- 2(\mu - A_t \cot \beta)^2 m_{\text{top}}^4 \left. \right]. (III) .
\] (3.8)

This very simple formula approximates the full 1-loop result surprisingly well. The size of the vertex corrections and the quality of the approximation (3.8) are displayed in Figure 3.4.

Term (II) reproduces exactly the result given in [3] for the case $A_t = \mu = 0$, as well as the $m_{\text{top}}^4$-term of the expression in [7]. In the general case for $A_t, \mu \sim O(m_t)$, the contribution (III) can be of similar size.

In the case of mixed sfermions, the left and right-handed $\tilde{t}_{L,R}$ have to be replaced by the mass eigenstates of the sfermion mass matrix (A.2):

\[
\tilde{t}_{L,R} \rightarrow \tilde{t}_1, \tilde{t}_2.
\] (3.9)

Each vertex bilinear in the left- and/or right-handed sfermions is replaced by four vertices involving $\tilde{t}_1, \tilde{t}_2$ (see app. A1):

\[
V(\tilde{t}_\alpha, \tilde{t}_\beta) = \sum_{i,j=1,2} T_{\alpha i} T_{\beta j} V(\tilde{t}_i \tilde{t}_j) \quad (\alpha, \beta = L, R).
\] (3.10)

The diagrams of Figure 3.3 have been computed also for mixed sfermions. The result can be cast into the same form as (3.7) by substituting $m_{\tilde{t}_{L,R}} \rightarrow m_{\tilde{t}_{1,2}}$, the eigenvalues of the nondiagonal sfermion mass matrix.

4 The decay $h^0 \rightarrow A^0 A^0$

The one-loop decay amplitude for $h^0 \rightarrow A^0 A^0$, graphically depicted in Figure 4.1, can be written as follows:
Figure 3.4: The $h^0 A^0 A^0$ vertex corrections $\Delta T_{h^0}$ and the approximation (3.7)

\[
T_1 = \sqrt{Z_{h^0}} \left[ - \frac{gM_Z}{2c_w} \cos 2\beta \sin(\alpha + \beta) + \Delta T_{h^0} \\
+ Z_{h^0 H^0} \left( \frac{gM_Z}{2c_w} \cos 2\beta \cos(\alpha + \beta) + \Delta T_{H^0} \right) \right].
\]

(4.1)

$\Delta T_{h,H}$ are the vertex corrections (3.3) and (3.4). $Z_{h^0}$ is the residue of the $h^0$-propagator in (2.9), which determines the wave-function renormalisation of the external $h^0$:

\[
Z_{h^0} = \text{Res}_{M_{h^0}} \Delta_{h^0} = \frac{1}{1 + \tilde{\Sigma}_{h^0}(k^2) - \left( \frac{\Sigma_{h^0 H^0}(k^2)}{k^2 - m_{h^0}^2 + \Sigma_{H^0}(k^2)} \right)} |_{k^2 = M_{h^0}^2}
\]

(4.2)

The mixing between $h^0$ and $H^0$ enters via

\[
Z_{h^0 H^0} = -\frac{\tilde{\Sigma}_{h^0 H^0}(M_{h^0}^2)}{M_{h^0}^2 - m_{H^0}^2 + \Sigma_{H^0}(M_{h^0}^2)}.
\]

(4.3)
A simple approximation for the full amplitude includes only the leading term $\omega_t$ in eq. (2.13). The mass matrix

$$M_{Higgs}^2 = \sin 2\beta \left( \begin{array}{cc} \cot \beta M_Z^2 + \tan \beta M_{A^0}^2 & -M_Z^2 - M_{A^0}^2 \\ -M_Z^2 - M_{A^0}^2 & \tan \beta M_Z^2 + \cot \beta M_{A^0}^2 + \omega_t \end{array} \right)$$

(4.4)

can be diagonalised by $\alpha_{eff}$ [3], which is determined by

$$\tan \alpha_{eff} = \frac{-M_{A^0}^2 - M_{h^0}^2 \tan \beta}{M_Z^2 + M_{A^0}^2 \tan^2 \beta - (1 + \tan^2 \beta) M_{h^0,\omega_t}}.$$  

(4.5)

$\alpha_{eff}$ replaces the lowest order quantity in the tree level vertex (3.2). The use of $\alpha_{eff}$ is equivalent to the diagrammatic inclusion of the $h^0H^0$ mixing term with respect to the $m_t^4$ top content. The corresponding approximation for the one-loop amplitude is obtained by

$$T_{1,\alpha_{eff}} = \left[ -\frac{g M_Z}{2 c_w} \cos 2\beta \sin(\alpha + \beta) + \Delta T_{h^0} \right]_{\alpha \rightarrow \alpha_{eff}}$$

(4.6)

In the further simplification of this expression, $\Delta T_{h^0}$ is replaced by $\Delta T_{h^0,top}$ of Eq. (3.7). The comparison of the full one-loop amplitude and the two approximations is shown in Figure 4.2.

The decay width for $h^0 \rightarrow A^0A^0$

$$\Gamma_1(h^0 \rightarrow A^0A^0) = \frac{1}{32\pi M_{h^0}} \sqrt{1 - \frac{M_{A^0}^2}{M_{h^0}^2}} |T_1|^2$$

(4.7)

is determined at the 1-loop level by the mass $M_{h^0}$ and the decay amplitude $T_1$ in [3,4].

Of practical importance are the branching ratios

$$\frac{\Gamma_1(h^0 \rightarrow A^0A^0)}{\Gamma_{tot}}, \quad \frac{\Gamma_1(h^0 \rightarrow b\bar{b})}{\Gamma_{tot}}$$

(4.8)

where the total width is calculated as

$$\Gamma_{tot} = \Gamma_1(h^0 \rightarrow A^0A^0) + \Gamma_1(h^0 \rightarrow b\bar{b}) + \Gamma_1(h^0 \rightarrow c\bar{c}) + \Gamma_1(h^0 \rightarrow \tau^+\tau^-) + \Gamma_1(h^0 \rightarrow gg).$$

(4.9)
Figure 4.2: Full 1-loop amplitude $T_1$ in (4.1) and the approximations with $\alpha_{\text{eff}}$ in (4.6).

The decay $h^0 \to \tilde{\chi}^+_i \tilde{\chi}^-_j$ does not occur because such light charginos are excluded from the direct search. This also suppresses the decay $h^0 \to \tilde{\chi}^0_i \tilde{\chi}^0_j$.

The decay widths for $h^0 \to f \bar{f}$ have been calculated taking into account the full set of propagator corrections given in ref. [9] and the QCD corrections given in ref. [13]. The full set of electroweak vertex corrections is also available [10], but they are numerically of subleading size and can be neglected at a sufficient level of accuracy. The decay widths for $h^0 \to gg$ has been computed taking into account the top- and the bottom-loop, as pointed out in ref. [13].

5 Discussion

Loop effects are of basic importance for the decay $h^0 \to A^0 A^0$ in a twofold respect: The loop contributions to the mass spectrum make the decay kinematically possible, and the vertex
corrections give a sizeable enhancement of the effective $h^0 A^0 A^0$ coupling over the tree level coupling, which would even vanish for $\tan \beta = 1$. The partial decay width $\Gamma_1(h^0 \rightarrow A^0 A^0)$ from the full 1-loop calculation, Eq. (4.7), is shown in Figure 5.1. It is worth to note that the simplified approximation based on Eq. (4.6) with $\alpha_{\text{eff}}$ from (4.5) and the vertex corrections (3.7) yield results which deviate not more than 10% from the full calculation. The quality of the approximations is made more explicit in Table 5.1, where for various sets of SUSY parameters the approximate and full 1-loop results are compared. As a general feature it appears that the approximations are suitable for the most cases. Only for unrealistically small values of $\tan \beta$ the approximation becomes unreliable (see Figure 4.2).

The dependence on the top mass is much weaker as naively expected from the expressions in (3.7). The powers of $m_t$ are actually compensated by the $m_{\text{top}}$-dependence of $\cos \alpha_{\text{eff}}$, thus leaving only a logarithmic behaviour with $m_{\text{top}}$, as pointed out in ref. [7].

The results for the branching ratios

$$BR(A^0 A^0) = \frac{\Gamma_1(h^0 \rightarrow A^0 A^0)}{\Gamma_{\text{tot}}}, \quad BR(b \bar{b}) = \frac{\Gamma_1(h^0 \rightarrow b \bar{b})}{\Gamma_{\text{tot}}}$$

are displayed in Figures 5.2 and 5.3. For practically the entire range where the decay $h^0 \rightarrow A^0 A^0$ is allowed, it is the dominant decay mode. The branching ratio $BR(\tau^+ \tau^-)$ for decays into $\tau$ pairs does not exceed 6%. The decay modes into $c\bar{c}$ and $gg$ are suppressed via the $m_{\text{top}}$-dependence of $\cos \alpha_{\text{eff}}$. They never exceed 1% and can thus be neglected.

| $\mu$ [GeV] | $M_{\text{SUSY}}$ [MeV] | $\Gamma_1$ [MeV] | $\Gamma_{1,\alpha_{\text{eff}}}$ [MeV] |
|-----------|-----------------|----------------|------------------|
| 50        | 1000            | 15.9           | 14.6             |
| 300       | 1000            | 17.3           | 17.0             |
| 50        | 500             | 14.4           | 15.1             |
| 300       | 500             | 29.9           | 31.2             |
| 50        | 300             | 12.2           | 14.0             |
| 300       | 300             | 88.0           | 90.2             |

Table 5.1: Decay width $\Gamma_1$, complete at 1-loop, and the approximation $\Gamma_{1,\alpha_{\text{eff}}}$ from the amplitude (4.6) with $\Delta T_{h^0, \text{top}}$ in (3.7). $m_{\text{top}} = 200\text{GeV}, M_{A^0} = 25\text{GeV}, \tan \beta = 0.8, M = 500\text{GeV}$, diagonal sfermion mass matrix.
Figure 5.1: One-loop decay width $\Gamma_1(h^0 \to A^0 A^0)$

$m_t = 140, 170, 200$

$M_A = 20$
$M = 500$
$\mu = 200$

$M_{SUSY} = 1000$
$M_{SUSY} = 750$
$M_{SUSY} = 500$
Figure 5.2: Branching ratio $\Gamma_1(h^0 \to A^0 A^0)/\Gamma_{tot}$

Figure 5.3: Branching ratio $\Gamma_1(h^0 \to b\bar{b})/\Gamma_{tot}$
| $\mu$ [GeV] | $M_{SUSY}$ [GeV] | $r_{h^0 A^0 A^0}$ |
|---------|----------------|----------------|
| 0       | 1000           | 0.989          |
| 50      | 1000           | 0.989          |
| 300     | 1000           | 1.042          |
| 0       | 500            | 1.044          |
| 50      | 500            | 1.055          |
| 300     | 500            | 1.403          |
| 0       | 300            | 1.389          |
| 50      | 300            | 1.443          |
| 300     | 300            | 3.244          |

Table 5.2: The ratio $r_{h^0 A^0 A^0}$ of the effective $h^0 A^0 A^0$ couplings of this paper and ref [7] (see text). The parameters are as in table 5.1.

Our results agree fairly well with those of refs. [3, 7]. The reason is the already mentioned good quality of an approximate calculation with terms involving only top and stops. For comparison, we put together the results of the complete calculation and of the RG method of ref. [7] in table 5.2 in terms of the ratio $r_{h^0 A^0 A^0}$ of the 1-loop $h^0 A^0 A^0$ vertex in Eq. (3.2), (3.3) and the corresponding expression in ref. [7]. Both results are very close for large values of $M_{SUSY}$. They deviate more for lower values of the stop masses and with increasing $\mu, A_t$, as expected from the last term of Eq. (3.8).

We have not implemented the recently calculated leading 2-loop results to the neutral MSSM Higgs boson mass [14], based on effective potential and renormalization group methods. They would modify the complete 1-loop results in the same way as the approximations and thus do not influence the differences discussed here which are only obtained by an explicit diagrammatic calculation.

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Appendix

A Masses and constants

A.1 Sfermions

The sfermion mass term of the Lagrangian

\[ \mathcal{L}_{sferm, mass} = -\frac{1}{2} (\tilde{f}_L, \tilde{f}_R) Z (\tilde{f}_L, \tilde{f}_R) \]  

(A.1)

contains the mass matrix

\[ Z = \begin{pmatrix} \tilde{M}_Q^2 + M_Z^2 \cos 2\beta (I_3 - Q_f s_w^2) + m_f^2 & m_f (A_f + \mu \{\cot, \tan\} \beta) \\ m_f (A_f + \mu \{\cot, \tan\} \beta) & \tilde{M}_{U,D}^2 + M_Z^2 \cos 2\beta Q_f s_w^2 + m_f^2 \end{pmatrix}. \]  

(A.2)

\{\cot, \tan\} refer to the corresponding fermions with isospin \{+\frac{1}{2}, -\frac{1}{2}\}. Unmixed sfermions are obtained by putting

\[ A_d = -\mu \tan \beta, \quad A_u = -\mu \cot \beta \]  

(A.3)

which, however, is only important for stops since the non-diagonal terms are suppressed for the light quarks and low tan \( \beta \). For the simplest case we assume equal soft breaking parameters for all sfermions:

\[ \tilde{M}_Q = \tilde{M}_U = \tilde{M}_D =: M_{SUSY} \]  

(A.4)

For mixed sfermions the coefficients of the transformation to the mass eigenstates \( \tilde{t}_{1,2} \)

\[ \tilde{t}_\alpha = \sum_{j=1,2} T_{\alpha j} \tilde{t}_j \quad (\alpha = L, R) \]  

(A.5)

enter the bilinear stop vertices of the diagrams in section 3, according to Eq. (3.10).

A.2 Charginos

The chargino mass term

\[ \mathcal{L}_{Char, mass} = -\frac{1}{2} (\psi^+, \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c.} \]  

(A.6)

with

\[ X = \begin{pmatrix} M & M_W \sqrt{2} \sin \beta \\ M_W \sqrt{2} \cos \beta & -\mu \end{pmatrix}, \]  

(A.7)

contains the SU(2) gaugino mass parameter \( M \) and the Higgsino doublet mixing parameter \( \mu \). The mass matrix is diagonalized with the help of the unitary \( (2 \times 2) \)-matrices \( U \) and \( V \):

\[ \tilde{\chi}_i^+ = V_{ij} \psi_j^+, \quad \tilde{\chi}_i^- = U_{ij} \psi_j^- . \]  

(A.8)
$U$ and $V$ are given by
\begin{align}
U &= O_-
onumber \tag{A.9} \\
V &= \begin{cases} 
O_+, & \text{det } X > 0 \\
\sigma_3O_+, & \text{det } X < 0
\end{cases}
\end{align}

with
\begin{align}
O_\pm &= \begin{pmatrix}
\cos \phi_\pm & \sin \phi_\pm \\
-\sin \phi_\pm & \cos \phi_\pm
\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\
0 & -1 \end{pmatrix}. \tag{A.10}
\end{align}

$\cos \phi_\pm$ and $\sin \phi_\pm$ are given by ($\epsilon = \text{sgn}(\text{det } X)$)
\begin{align}
\tan \phi_+ &= \frac{\sqrt{2} M_W (\sin \beta M_+ + \epsilon \cos \beta M_-)}{M_+ + \epsilon \mu M_-} \\
\tan \phi_- &= \frac{-\mu M_+ - \epsilon \mu M_-}{\sqrt{2} M_W (\sin \beta M_+ + \epsilon \cos \beta M_-)}. \tag{A.11}
\end{align}

If $\phi_+ < 0$ then $\phi_+ \rightarrow \phi_+ + \pi$.

$M_+$ and $M_-$ are the entries in the diagonalized chargino mass-matrix
\begin{align}
M_{D,\text{Char}} &= U^* X V^{-1} = \begin{pmatrix} M_+ & 0 \\
0 & M_-
\end{pmatrix}. \tag{A.12}
\end{align}

The physical neutralino mass-eigenstates can be computed with the help of a unitary diagonalization matrix $N$:
\begin{align}
\tilde{\chi}_i^0 &= N_{ij} \psi_j^0 \tag{A.16}
\end{align}

The diagonal mass matrix is given by:
\begin{align}
M_{D,\text{Neutr}} &= N^* Y N^{-1}. \tag{A.17}
\end{align}

The matrix $Y$ contains the U(1) gaugino mass $M'$ parameter as a further input quantity. As conventionally done, we assume the GUT constraint
\begin{align}
M' &= \frac{5}{3} \frac{s_w^2}{c_w^2} M. \tag{A.18}
\end{align}
B  Vertex corrections for $h^0 \rightarrow A^0 A^0$

This section contains the analytical expressions for all one-loop corrections which contribute to the $h^0 A^0 A^0$-vertex. For the Feynman rules we adopt the convention of [11]. The enumeration refers to section 3, fig. 3.1 and 3.2. The indices denote the particles inside the loop with assignment of masses as follows:

An overall factor $g^3/(4\pi)^2$ is suppressed. The symmetry factors are included. The color factor $N_f$ which occurs for fermion and sfermion loops is 3 for quarks and squarks, 1 otherwise. $f$ denotes the fermions with weak isospin $+\frac{1}{2}$, $f'$ their isospin partners with isospin $-\frac{1}{2}$. Summation over the various generations is understood.

For given external momenta the first two arguments of the $C_0$-functions are the same in all diagrams and thus have also been dropped: $C_0(-q_1, q_2, m_1, m_0, m_2) \equiv C_0(m_1, m_0, m_2)$. Here $-q_1$ and $q_2$ denote the momenta of the two outgoing $A^0$ and $(-q_1 + q_2) = k$ denotes the momentum of the incoming $h^0$ with $k^2 = M_{h^0}^2$. In order to get the corresponding results for $\Delta T_{H^0,i}$ one has to replace the $h^0$-coupling by the corresponding $H^0$-coupling according to the Feynman rules of ref. [11].

\[
\Delta T_{1, h^0 h^0} = \frac{3iM_Z}{2(2\pi)^3} \cos 2\beta \cos^2 2\alpha \sin(\alpha + \beta) B_0(M_{h^0}^2, m_{h^0}, m_{h^0})
\]

\[
\Delta T_{1, h^0 H^0} = \frac{iM_Z}{(2\pi)^3} \cos 2\beta \sin 2\alpha (2 \sin 2\alpha \sin(\alpha + \beta) - \cos(\alpha + \beta) \cos 2\alpha ) B_0(M_{h^0}^2, m_{h^0}, m_{H^0})
\]

\[
\Delta T_{1, H^0 H^0} = \frac{iM_Z}{2(2\pi)^3} \cos 2\beta \cos 2\alpha (2 \sin 2\alpha \cos(\alpha + \beta) + \sin(\alpha + \beta) \cos 2\alpha )
\]
\[\Delta T_{2, A^0 A^0} = \frac{3iM_Z}{2(2c_w)^3} \cos^2 2\beta \sin(\alpha + \beta) B_0(M_{h^0}^2, M_{A^0}, M_{A^0})\]  

(B.3)

\[\Delta T_{2, A^0 G^0} = \frac{3iM_Z}{(2c_w)^3} \cos 2\beta \sin^2 2\beta \sin(\alpha + \beta) B_0(M_{h^0}^2, M_{A^0}, M_Z)\]  

(B.4)

\[\Delta T_{2, G^0 G^0} = \frac{-iM_Z}{2(2c_w)^3} \cos 2\beta \sin(\alpha + \beta) (3 \sin^2 2\beta - 1) B_0(M_{h^0}^2, M_Z, M_Z)\]  

(B.5)

\[\Delta T_{3, H^+ H^-} = \frac{i}{(2c_w)^3} \cos^3 2\beta \sin(\alpha + \beta) \left(M_W \sin(\beta - \alpha) + \frac{M_Z}{2c_w}\right) B_0(M_{h^0}^2, M_{H^+}, M_{H^+})\]  

(B.6)

\[\Delta T_{3, H^+ G^-} = \frac{2i}{(2c_w)^3} \cos 2\beta \sin 2\beta \left(c_w M_W \cos(\beta - \alpha) - M_Z \sin 2\beta \sin(\alpha + \beta)\right) B_0(M_{h^0}^2, M_{H^+}, M_W)\]  

(B.7)

\[\Delta T_{3, G^+ G^-} = \frac{-iM_Z}{(8c_w)} \cos 2\beta \sin(\alpha + \beta) \left(1 + \sin^2 2\beta - \frac{s_w^2}{c_w} \cos^2 2\beta\right) B_0(M_{h^0}^2, M_{H^+}, M_W)\]  

(B.8)

\[\Delta T_{4, \tilde{f}_R \tilde{f}_R} = \frac{i}{2} \left[ M_Z c_w Q_f s_w^2 \sin(\alpha + \beta) - \frac{m_f^2}{M_W \sin \beta} \cos \alpha \right] \left[ Q_f c_w \cos 2\beta - \frac{m_f^2}{M_W} \tan^2 \beta \right] N_f B_0(M_{h^0}^2, m_{\tilde{f}_R}, m_{\tilde{f}_R})\]  

(B.9)

\[\Delta T_{4, \tilde{f}_R' \tilde{f}_R'} = \frac{i}{2} \left[ M_Z c_w Q_f' s_w^2 \sin(\alpha + \beta) + \frac{m_f'^2}{M_W \cos \beta} \sin \alpha \right] \left[ Q_f' c_w \cos 2\beta - \frac{m_f'^2}{M_W} \tan^2 \beta \right] N_f' B_0(M_{h^0}^2, m_{\tilde{f}_R'}, m_{\tilde{f}_R'})\]  

(B.10)

\[\Delta T_{5, \tilde{f}_L \tilde{f}_L} = \frac{i}{2} \left[ M_Z c_w \left(\frac{1}{2} - Q_f s_w^2\right) \sin(\alpha + \beta) - \frac{m_f^2}{M_W \sin \beta} \cos \alpha \right] \left[ \frac{1}{2} - Q_f c_w \cos 2\beta - \frac{m_f^2}{M_W} \tan^2 \beta \right] N_f B_0(M_{h^0}^2, m_{\tilde{f}_L}, m_{\tilde{f}_L})\]  

(B.11)

\[\Delta T_{5, \tilde{f}_L' \tilde{f}_L'} = \frac{i}{2} \left[ M_Z c_w \left(\frac{1}{2} - Q_f' s_w^2\right) \sin(\alpha + \beta) + \frac{m_f'^2}{M_W \cos \beta} \sin \alpha \right] \left[ \frac{1}{2} - Q_f' c_w \cos 2\beta - \frac{m_f'^2}{M_W} \tan^2 \beta \right] N_f' B_0(M_{h^0}^2, m_{\tilde{f}_L'}, m_{\tilde{f}_L'})\]  

(B.12)

\[\Delta T_{6, Z^0 Z^0} = \frac{iM_Z}{2c_w^3} \sin(\beta - \alpha) \left[B_0(M_{h^0}^2, M_Z, M_Z) - \frac{1}{2}\right]\]  

(B.13)

\[\Delta T_{7, W^+ W^-} = iM_W \sin(\beta - \alpha) \left[2B_0(M_{h^0}^2, M_W, M_W) - 1\right]\]  

(B.14)

\[\Delta T_{8, A^0 A^0} = \frac{iM_Z}{(2c_w)^3} \cos^2 2\beta \sin(\alpha + \beta) \cos 2\alpha B_0(M_{A^0}^2, m_{A^0}, M_{A^0})\]  

(B.15)

\[\Delta T_{8, h^0 G^0} = \frac{iM_Z}{(2c_w)^3} \sin^2 2\beta \cos(\alpha + \beta) \cos 2\alpha B_0(M_{h^0}^2, m_{h^0}, M_Z)\]  

(B.16)

\[\Delta T_{8, H^0 A^0} = \frac{-iM_Z}{(2c_w)^3} \cos^2 2\beta \sin(\alpha + \beta) \sin 2\alpha B_0(M_{A^0}^2, m_{H^0}, M_{A^0})\]  

(B.17)
$$\Delta T_{8,H^0G^0} = \frac{-iM_Z}{(2c_w)^3} \sin^2 2\beta \cos(\alpha + \beta) \sin 2\alpha B_0(M_{A^0}^2, m_{H^0}, M_Z)$$  \hspace{1cm} (B.19)

$$\Delta T_{9,H^0G^0} = \frac{iM_W}{8} \sin(\beta - \alpha) B_0(M_{A^0}^2, M_{H^+}, M_W)$$  \hspace{1cm} (B.20)

$$\Delta T_{10,A^0h^0h^0} = \frac{3iM_Z^2}{(2c_w)^3} \cos^2 2\beta \sin^3(\alpha + \beta) \cos 2\alpha C_0(m_{h^0}, M_{A^0}, m_{h^0})$$  \hspace{1cm} (B.21)

$$\Delta T_{10,A^0h^0H^0} = \frac{-iM_Z^2}{(2c_w)^3} \cos^2 2\beta \sin(\alpha + \beta) \cos(\alpha + \beta) \cos 2\alpha C_0(m_{h^0}, M_{A^0}, m_{H^0})$$  \hspace{1cm} (B.22)

$$\Delta T_{10,A^0H^0H^0} = \frac{-iM_Z^2}{(2c_w)^3} \cos^2 2\beta \cos^2(\alpha + \beta) \cos 2\alpha C_0(m_{h^0}, M_{A^0}, m_{H^0})$$  \hspace{1cm} (B.23)

$$\Delta T_{10,G^0h^0h^0} = \frac{3iM_Z^2}{(2c_w)^3} \cos^2 2\beta \sin^3(\alpha + \beta) \cos 2\alpha C_0(m_{h^0}, M_{Z}, m_{h^0})$$  \hspace{1cm} (B.24)

$$\Delta T_{10,G^0h^0H^0} = \frac{-iM_Z^2}{(2c_w)^3} \sin^2 2\beta \sin(\alpha + \beta) \cos(\alpha + \beta) \cos 2\alpha C_0(m_{h^0}, M_{Z}, m_{H^0})$$  \hspace{1cm} (B.25)

$$\Delta T_{10,G^0H^0H^0} = \frac{3iM_Z^2}{(2c_w)^3} \sin^2 2\beta \cos^2(\alpha + \beta) \cos 2\alpha C_0(m_{h^0}, M_{Z}, m_{H^0})$$  \hspace{1cm} (B.26)

$$\Delta T_{11,G^+H^+H^+} = \frac{iM_W^2}{4} \left( M_W \sin(\beta - \alpha) + \frac{M_Z}{2c_w} \right) \cos 2\beta \sin(\alpha + \beta) \cos 2\alpha C_0(M_{H^+}, M_W, M_{H^+})$$  \hspace{1cm} (B.27)

$$\Delta T_{12,H^+G^+G^+} = \frac{-iM_Z^3}{(2c_w)^3} \cos 2\beta \sin(\alpha + \beta) C_0(M_{H^+}, M_W, M_{H^+})$$  \hspace{1cm} (B.28)

$$\Delta T_{13,f_Lf_Rf_R} = -i \left[ \frac{M_Z}{c_w} Q_f s_{w}^2 \sin(\alpha + \beta) - \frac{m_f^2 \cos \alpha}{M_W \sin \beta} \right] \frac{m_f^2}{4M_W} \left( \mu - A_f \cot \beta \right)^2$$  \hspace{1cm} (B.29)

$$\Delta T_{13,f_Lf_Rf_L} = -i \left[ \frac{M_Z}{c_w} Q_f s_{w}^2 \sin(\alpha + \beta) + \frac{m_f^2 \sin \alpha}{M_W \cos \beta} \right] \frac{m_f^2}{4M_W} \left( \mu - A_f \tan \beta \right)^2$$  \hspace{1cm} (B.30)

$$\Delta T_{14,f_Rf_Lf_L} = -i \left[ \frac{M_Z}{c_w} \left( \frac{1}{2} - Q_f s_{w}^2 \right) \sin(\alpha + \beta) - \frac{m_f^2 \cos \alpha}{M_W \sin \beta} \right] \frac{m_f^2}{4M_W} \left( \mu - A_f \cot \beta \right)^2$$  \hspace{1cm} (B.31)

$$\Delta T_{14,f_Rf_Lf_L} = -i \left[ \frac{M_Z}{c_w} \left( \frac{1}{2} - Q_f s_{w}^2 \right) \sin(\alpha + \beta) - \frac{m_f^2 \sin \alpha}{M_W \cos \beta} \right] \frac{m_f^2}{4M_W} \left( \mu - A_f \tan \beta \right)^2$$  \hspace{1cm} (B.32)

$$\Delta T_{15,fff} = \frac{i m_f^4 \cos \alpha \cot^2 \beta}{(2M_W)^3 \sin \beta} \left\{ C_0(m_f, m_f, m_f) [2M_{A^0}^2 - M_{h^0}^2] + 2B_0(M_{h^0}^2, m_f, m_f) \right\} N_f$$  \hspace{1cm} (B.33)
\[
\Delta T_{15, f' f'} = \frac{m_1^4 \sin \alpha \tan^2 \beta}{(2M_W)^3 \cos \beta} \left\{ C_0(m_{f'}, m_{f'}, m_f) \left[ 2M_{A_0}^2 - M_{R^0}^2 \right] \\
-2B_0(M_{R^0}^2, m_{f'}, m_f) \right\} N_f' 
\] (B.34)

In the following, vertex corrections \(U, V\) and \(N\) denote the diagonalising matrices given in appendix A. The convention for \(A, A_+, A_-\) etc. are shown below \((\omega_+ = \frac{1 + \gamma_5}{2})\):

\[
\begin{align*}
\Delta T_{16, \tilde{\chi}_i^+ \tilde{\chi}_k^+ \tilde{\chi}_j^+} &= -ABC \left\{ C_0(m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_j^+}) \right. \\
&\left. \left[ [B_+ A_+ C_+ + B_- A_- C_-] m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} m_{\tilde{\chi}_k^+} \\
&- [B_+ A_- C_+ + B_- A_+ C_-] \frac{M_{R^0}^2 - m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2}{2} \right] \\
&\left. - [B_+ A_+ C_- + B_- A_- C_+] \frac{M_{A_0}^2 - m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2}{2} \right] \\
&\left. - [B_+ A_- C_- + B_- A_+ C_+] \frac{M_{A_0}^2 - m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2}{2} \right] \\
&\left. + B_0(M_{R^0}^2, m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_j^+}) \left[ [B_+ A_+ C_+ + B_- A_- C_-] \frac{m_{\tilde{\chi}_i^+}^2}{2} + [B_+ A_- C_- + B_- A_+ C_+] \frac{m_{\tilde{\chi}_j^+}^2}{2} \right] \\
&\left. + B_0(M_{A_0}^2, m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_j^+}) \left[ [B_+ A_+ C_- + B_- A_- C_+] \frac{m_{\tilde{\chi}_i^+}^2}{2} + [B_+ A_- C_- + B_- A_+ C_+] \frac{m_{\tilde{\chi}_j^+}^2}{2} \right] \\
&\left. + B_0(M_{A_0}^2, m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_j^+}) \left[ [B_+ A_+ C_- + B_- A_- C_+] \frac{m_{\tilde{\chi}_i^+}^2}{2} + [B_+ A_- C_- + B_- A_+ C_+] \frac{m_{\tilde{\chi}_j^+}^2}{2} \right] \right. \\
&\left. \right\} \right.
\] (B.35)

The following couplings have to be inserted:

- \(A = i\), \(A_+ = Q_{ij} \sin \alpha - S_{ij} \cos \alpha\), \(A_- = Q_{ji}^* \sin \alpha - S_{ji}^* \cos \alpha\)
- \(B = -g\), \(B_+ = Q_{ki} \sin \beta - S_{ki} \cos \beta\), \(B_- = Q_{ik}^* \sin \beta - S_{ik}^* \cos \beta\)
- \(C = -g\), \(C_+ = Q_{jk} \sin \beta - S_{jk} \cos \beta\), \(C_- = Q_{kj}^* \sin \beta - S_{kj}^* \cos \beta\)
- \(Q_{ij} = \frac{1}{\sqrt{2}} V_{1i} U_{j2}\)
- \(S_{ij} = \frac{1}{\sqrt{2}} V_{12} U_{j1}\)
\[ \Delta T_{17,\tilde{\chi}_k^0,\tilde{\chi}_j^0} = -ABC \left\{ C_0(m_{\chi_i^0}, m_{\chi_j^0}, m_{\tilde{\chi}_k^0}) \right\} \]

\[ \left[ [B_+ A_+ C_+ + B_- A_- C_-] m_{\chi_i^0} m_{\chi_j^0} m_{\tilde{\chi}_k^0} \right. \]

\[ - [B_+ A_- C_+ + B_- A_+ C_-] m_{\chi_i^0} \frac{M_{H_0}^2 - m_{\chi_i^0}^2 - m_{\chi_j^0}^2}{2} \]

\[ - [B_+ A_+ C_- + B_- A_- C_+] m_{\chi_i^0} \frac{M_{A_0}^2 - m_{\chi_i^0}^2 - m_{\chi_k^0}^2}{2} \]

\[ - [B_+ A_- C_- + B_- A_+ C_+] m_{\tilde{\chi}_j^0} \frac{M_{A^0}^2 - m_{\chi_i^0}^2 - m_{\tilde{\chi}_j^0}^2}{2} \]

\[ + B_0(M_{H_0}^2, m_{\chi_i^0}, m_{\tilde{\chi}_j^0}) \left( [B_+ A_+ C_+ + B_- A_- C_-] \frac{m_{\chi_i^0}^2}{2} + [B_+ A_- C_+ + B_- A_+ C_-] \frac{m_{\chi_j^0}^2}{2} \right) \]

\[ + B_0(M_{A_0}^2, m_{\chi_i^0}, m_{\tilde{\chi}_j^0}) \left( [B_+ A_+ C_+ + B_- A_- C_-] \frac{m_{\chi_i^0}^2}{2} + [B_+ A_- C_+ + B_- A_+ C_-] \frac{m_{\chi_j^0}^2}{2} \right) \]

\[ + B_0(M_{A_0}^2, m_{\chi_i^0}, m_{\tilde{\chi}_j^0}) \left( [B_+ A_+ C_+ + B_- A_- C_-] \frac{m_{\chi_i^0}^2}{2} + [B_+ A_- C_+ + B_- A_+ C_-] \frac{m_{\chi_j^0}^2}{2} \right) \}

The following couplings have to be inserted:

\[ A = i, \quad A_+ = Q''_{ij} \sin \alpha - S''_{ij} \cos \alpha, \quad A_- = Q''_{ji} \sin \alpha - S''_{ji} \cos \alpha \]

\[ B = -g, \quad B_+ = Q''_{ki} \sin \beta - S''_{ki} \cos \beta, \quad B_- = Q''_{ik} \sin \beta - S''_{ik} \cos \beta \]

\[ C = -g, \quad C_+ = Q''_{jk} \sin \beta - S''_{jk} \cos \beta, \quad C_- = Q''_{kj} \sin \beta - S''_{kj} \cos \beta \]

\[ Q''_{ij} = \frac{1}{2} [N_{i3}(N_{j2} - N_{j1} \tan \theta_W) + N_{j3}(N_{i2} - N_{i1} \tan \theta_W)] \]

\[ S''_{ij} = \frac{1}{2} [N_{i4}(N_{j2} - N_{j1} \tan \theta_W) + N_{j4}(N_{i2} - N_{i1} \tan \theta_W)] \]

\[ \Delta T_{18,H^0Z^0A^0} = \frac{2i M_Z}{(2\pi)^3} \cos^2(\beta - \alpha) \cos 2\beta \sin(\alpha + \beta) \]

\[ \left\{ C_0(M_Z, m_{h_0^0}, m_{A^0}) [m_{h_0^0}^2 - M_Z^2 + M_{h_0^0}^2] \right\} \]

\[ + B_0(M_{A_0}^2, m_{h_0^0}, M_Z) - B_0(M_{A_0}^2, m_{h_0^0}, M_{A^0}) + B_0(M_{H_0}^2, M_Z, M_{A^0}) \}

\[ \Delta T_{18,H^0Z^0A^0} = \frac{2i M_Z}{(2\pi)^3} \cos(\beta - \alpha) \cos 2\beta \sin(\beta - \alpha) \cos(\alpha + \beta) \]

\[ \left\{ C_0(M_Z, m_{H_0^0}, M_{A^0}) [-M_Z^2 + M_{H_0^0}^2 + m_{H_0^0}^2] \right\} \]

\[ + B_0(M_{A_0}^2, m_{H_0^0}, M_Z) - B_0(M_{A_0}^2, m_{H_0^0}, M_{A^0}) + B_0(M_{H_0^0}^2, M_Z, M_{A^0}) \}

\[ \Delta T_{18,H^0Z^0G^0} = \frac{2i M_Z}{(2\pi)^3} \cos(\beta - \alpha) \sin 2\beta \sin(\beta - \alpha) \sin(\alpha + \beta) \]

\[ \left\{ C_0(M_Z, m_{h_0^0}, M_Z) [-M_{A^0}^2 + 2M_{h_0^0}^2] + B_0(M_{h_0^0}^2, M_Z, M_Z) \right\} \]

\[ \Delta T_{18,H^0Z^0G^0} = \frac{2i M_Z}{(2\pi)^3} \sin 2\beta \sin^2(\beta - \alpha) \cos(\alpha + \beta) \]
$$\Delta T_{19,H+W+G+} = \frac{-2iM_W}{8} \sin(\beta - \alpha)$$

(B.41)

$$\Delta T_{20,Z^0H^0} = \frac{-iM_Z}{2c_w^3} \cos(\beta - \alpha) \sin(\beta - \alpha) \left( 2\sin 2\alpha \sin(\alpha + \beta) - \cos(\alpha + \beta) \cos 2\alpha \right)$$

$$\Delta T_{20,Z^0H^0} = \frac{3iM_Z}{2c_w^3} \cos^2(\beta - \alpha) \sin(\alpha + \beta) \cos 2\alpha$$

(B.43)

$$\Delta T_{20,Z^0H^0} = \frac{-iM_Z}{2c_w^3} \cos(\beta - \alpha) \sin(\beta - \alpha) \left( 2\sin 2\alpha \sin(\alpha + \beta) - \cos(\alpha + \beta) \cos 2\alpha \right)$$

$$\Delta T_{20,Z^0H^0} = \frac{-iM_Z}{2c_w^3} \cos^2(\beta - \alpha) \sin(\alpha + \beta) \cos 2\alpha$$

(B.44)

$$\Delta T_{20,Z^0H^0} = \frac{-iM_Z}{2c_w^3} \cos(\beta - \alpha) \sin(\beta - \alpha) \left( 2\sin 2\alpha \sin(\alpha + \beta) - \cos(\alpha + \beta) \cos 2\alpha \right)$$

$$\Delta T_{20,Z^0H^0} = \frac{-iM_Z}{2c_w^3} \cos^2(\beta - \alpha) \sin(\alpha + \beta) \cos 2\alpha$$

(B.45)

$$\Delta T_{21,W^+H^+} = \frac{i}{2} \left( M_W \sin(\beta - \alpha) + \frac{M_Z}{2c_w} \cos 2\beta \sin(\alpha + \beta) \right)$$

(B.46)

$$\Delta T_{22,H^0Z^0} = \frac{2iM_Z}{(2c_w^3)} \sin^3(\beta - \alpha)$$

(B.47)

$$\Delta T_{22,H^0Z^0} = \frac{2iM_Z}{(2c_w^3)} \sin^3(\beta - \alpha)$$

(B.48)

$$\Delta T_{23,H^+W+W^+} = \frac{iM_W}{2} \sin(\beta - \alpha)$$

(B.49)
\[ \Delta T_{24,A^0, A^0} = -2B_0(M_{H^0}^2, M_W, M_W) + B_0(M_{A^0}^2, M_{H^+, M_W}) \]

(B.50)

\[ \Delta T_{24, h^0, A^0} = iM_Z^3 \left( \frac{c\omega}{2} \right)^3 \frac{2}{(2\cosh)^3} \sin^2(\alpha + \beta) \cos 2\beta C_0(M_{A^0}, m_{h^0}, M_{A^0}) \]  

(B.51)

\[ \Delta T_{24, h^0, G^0, A^0} = iM_Z^3 \left( \frac{c\omega}{2} \right)^3 \frac{2}{(2\cosh)^3} \sin^2(\alpha + \beta) \cos 2\beta C_0(M_{G^0}, m_{h^0}, M_{A^0}) \]  

(B.52)

\[ \Delta T_{24, h^0, G^0, A^0} = \frac{iM_Z^3}{(2\cosh)^3} \sin^2(\alpha + \beta) \cos 2\beta C_0(M_{G^0}, m_{h^0}, M_{A^0}) \]  

(B.53)

\[ \Delta T_{24, h^0, G^0, A^0} = iM_Z^3 \left( \frac{c\omega}{2} \right)^3 \frac{2}{(2\cosh)^3} \sin^2(\alpha + \beta) \cos 2\beta C_0(M_{A^0}, m_{h^0}, M_{A^0}) \]  

(B.54)

\[ \Delta T_{24, h^0, G^0, A^0} = iM_Z^3 \left( \frac{c\omega}{2} \right)^3 \frac{2}{(2\cosh)^3} \sin^2(\alpha + \beta) \cos 2\beta C_0(M_{A^0}, m_{h^0}, M_{A^0}) \]  

(B.55)

\[ \Delta T_{24, h^0, G^0, A^0} = -iM_Z^3 \left( \frac{c\omega}{2} \right)^3 \frac{2}{(2\cosh)^3} \sin^2(\alpha + \beta) \cos 2\beta C_0(M_{Z}, m_{h^0}, M_{A^0}) \]  

(B.56)

\[ \Delta T_{24, h^0, G^0, A^0} = iM_Z^3 \left( \frac{c\omega}{2} \right)^3 \frac{2}{(2\cosh)^3} \sin^2(\alpha + \beta) \cos 2\beta C_0(M_{Z}, m_{h^0}, M_{A^0}) \]  

(B.57)

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