Vector/tensor duality in the five dimensional
supersymmetric Green–Schwarz mechanism

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ABSTRACT: The five dimensional version of the Green–Schwarz mechanism can be invoked to cancel U(1) anomalies on the boundaries of brane world models. In five dimensions there are two dual descriptions that employ either a two–form tensor field or a vector field. We present the supersymmetric extensions of these dual theories using four dimensional $N=1$ superspace. For the supersymmetrization of the five dimensional Chern–Simons three form this requires the introduction of a new chiral Chern–Simons multiplet. We derive the supersymmetric vector/tensor duality relations and show that not only is the usual one/two–form duality modified, but that there is also an interesting duality relation between the scalar components. Furthermore, the vector formulation always contains singular boundary mass terms which are absent in the tensor formulation. This apparent inconsistency is resolved by showing that in either formulation the four dimensional anomalous U(1) mass spectrum is identical, with the lowest lying Kaluza–Klein mode generically obtaining a finite nonzero mass.

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1 Introduction

The quantum consistency of gauge theories coupled to matter, specifically the absence of anomalies, has proven to be one of the most important guiding principles for model building beyond the Standard Model. Therefore not surprisingly there have been many investigations of anomalies in brane world models, or in particular orbifold theories. The general outcome of these analyses is that the anomalies localize on even dimensional hypersurfaces and are determined by the local fermionic spectrum of the theory. (This conclusion also remains true in the context of warped compactifications). In odd dimensions anomalies do not need to cancel locally, since as long as they cancel globally, a bulk Chern–Simons term can be used to ensure that the theory is gauge invariant at the quantum level. Therefore, an interesting question is whether it is possible to have even more general localized anomalies that can still lead to consistent theories.

One of the major breakthroughs in the development of string theory was the realization of Green and Schwarz that so–called factorisable anomalies can be compensated by anomalous variations of anti–symmetric tensor fields. Studies of heterotic string compactifications to four dimensions has revealed that a four dimensional version of the Green–Schwarz mechanism is relevant for phenomenological string model building. This mechanism simultaneously cancels both pure and mixed U(1) anomalies provided that the spectrum satisfies a specific universality condition. In particular, there will be a mixed gravitational–gauge anomaly which in the $N = 1$ supersymmetric context is directly related to one–loop induced Fayet–Iliopoulos terms that has been confirmed in the string theory context. The local version of the Green–Schwarz mechanism on (strongly coupled) heterotic orbifolds has been investigated and the corresponding localized Fayet–Iliopoulos tadpoles were computed.

This provides motivation to investigate how the local Green–Schwarz mechanism can be implemented in a five dimensional setting. In odd dimensions the prime example of the Green–Schwarz–like mechanism is given by eleven dimensional Horava–Witten theory. An interesting aspect of the Green–Schwarz theory is that it allows for dual descriptions. In five dimensions the Green–Schwarz interaction can be described by two equivalent formulations using either a rank two tensor or a vector. (We do not consider the trivial possibility that boundary axion states could also cancel the anomalies.) As we will show this duality indeed occurs in our five dimensional theory even though the two descriptions are not always manifestly equivalent. For example, the vector formulation contains singular boundary mass terms for the anomalous U(1) gauge field which are absent when the two–form formulation is used. However, the proper inclusion of mixing terms in both the vector and the tensor formulations resolves this apparent inconsistency. In fact, the consistency of the duality provides confirmation of the δ(0) regularization proposed in Ref.

Supersymmetry plays a pivotal role in string theory, and therefore it is important to con-

\footnote{In the generic context of Type II and orientifold models the situation is quite different. The Fayet–Iliopoulos terms generically have a tree–level origin, whereas the Green-Schwarz field is a localized brane field arising from the twisted sector of an orbifold compactification. Since the field theory description of this phenomenon presents no subtlety, we do not consider it here in detail. There are however some instances in orientifold models (e.g., in models with branes at angles), in which the Green-Schwarz field is a bulk field and the Fayet–Iliopoulos term is one–loop induced.}
struct a manifestly supersymmetric extension of this mechanism in the context of (global) five dimensional supersymmetry. An elegant and simple way to do this is to employ four dimensional $N=1$ superspace techniques in five dimensions \cite{26, 27, 28, 29}. In particular, for the tensor formulation we use the five dimensional tensor multiplet and find that contrary to the conventional four dimensional case, the $N=1$ linear supermultiplet is insufficient to contain all the bosonic components resulting from the dimensional reduction, and it cannot reproduce $\delta_5 B_{mn}$ terms that are present in the five dimensional action. In addition the Green–Schwarz theory contains Chern–Simons interactions. The five dimensional completion of the four dimensional Chern–Simons (three–form) superfield will require an additional chiral Chern–Simons superfield.

The supersymmetric version of the vector/tensor duality is investigated in detail and leads to new duality relations. In particular the usual non–supersymmetric duality relation between one– and two–forms is modified, while a new relation appears for the scalar field components of the supermultiplets. This relation is closely related to a duality for scalars in five dimensional supersymmetric gauge theories \cite{30}. Although we are mainly concerned about a single $U(1)_A$ anomalous gauge group, it is also possible to consider the generalization to multiple $U(1)_A$ gauge groups. In fact these results can be conveniently written in terms of a prepotential, and an interesting feature of this generalization is that so–called generalized Chern–Simons interactions \cite{31, 32, 33, 34, 35, 36} will naturally appear.

The presence of anomalous symmetries in the four dimensional version of the Green–Schwarz mechanism plays an important role in phenomenological applications of various compactifications of string theories. Similarly, we expect that the five dimensional formalism developed here could be useful for the fermion mass hierarchy and supersymmetry breaking in the context of models with large extra dimensions and a low fundamental scale.

The plan of this paper is as follows: The paper is divided into two main parts. First, in section 2 we give a detailed account of the vector/tensor duality in five dimensions and explain its role in the five dimensional Green–Schwarz mechanism. This duality will then be used to resolve the seemingly ill–defined boundary $\delta(0)$ mass terms. In the second part of the paper we present the supersymmetric dual formulations of the Green–Schwarz theory using four dimensional superspace techniques. In section 3 we define the vector and tensor multiplet in this formalism and then proceed to the supersymmetric version of the vector/tensor multiplet duality in five dimensions. Next, in section 4 we systematically supersymmetrize all elements on the Green–Schwarz theory, including the Chern–Simons interactions. We derive the superfield duality relations and use them to consistently check that the dual supersymmetric descriptions are equivalent. For example, even in the supersymmetric context singular boundary terms are absent in the tensor multiplet formulation. Finally, we briefly discuss various phenomenological applications of the supersymmetric local Green–Schwarz theory in section 5 and summarize our main results in the Conclusions. Various details required for the systematic construction of the Chern–Simons superfields are given in Appendix A, while in Appendix B we present the extension of the supersymmetric Green–Schwarz theory to the case of multiple anomalous $U(1)_A$ superfields.
2 Bosonic Green–Schwarz mechanism in five dimensions

We begin with a review of the Green–Schwarz anomaly cancellation mechanism in the context of a five dimensional gauge theory on the interval $S^1/Z_2$. The spacetime coordinates are denoted by $x^M = (x^0, \ldots, x^3, x^5 = y)$, with $0 \leq y \leq \pi$ (where we have set the radius $R$ of the extra dimension and the five dimensional fundamental scale to unity). For simplicity we will consider the gauge potential $A_M$, $M = 0, \ldots, 3, 5$, that corresponds to a single $U(1)_A$ gauge group with field strength $F_{MN}$. As is well–known bulk fermions and brane chiral fermions give rise to gauge anomalies localized on the boundaries, as was shown by direct calculations \[4, 5, 6, 8\]. When the anomalies on the two boundaries are equal and opposite, they can be cancelled by a five dimensional Chern–Simons interaction. However this is a very special form for the anomaly and more general boundary anomalies can instead be cancelled by invoking the Green–Schwarz mechanism \[12\]. In five dimensions the Green–Schwarz mechanism can be described by either a rank two tensor $B_{MN}$ or a vector $A_M$ that have field strengths $H_{MNP}$ and $F_{MN}$, respectively.

Let us emphasize our notational conventions: We use calligraphic letters $\mathcal{A}, \mathcal{B}, \mathcal{F}$ to denote the non–anomalous Green–Schwarz one and two–forms, and their corresponding field strengths. The anomalous $U(1)_A$ fields will instead be denoted by the standard italic letters $A, F, \ldots$. In the following sections this notation will be extended to the super symmetric case as well. The superfields corresponding to the Green–Schwarz multiplets are denoted by calligraphic letters $(\mathcal{T}_\alpha, \mathcal{U}, \mathcal{V}, \mathcal{S})$, while standard italic letters $(\mathcal{V}, \mathcal{S})$ are reserved for the anomalous $U(1)_A$ vector multiplet.

2.1 Anomalous $U(1)_A$ gauge field in the bulk

The gauge field $A_M$ in the bulk has the standard Yang–Mills action

$$S_{YM} = \int \frac{1}{2} *F_2 F_2 = \int \frac{1}{2} *dA_1 dA_1 = - \int d^5x \frac{1}{4} F_{MN} F^{MN}, \quad (1)$$

where we have introduced form notation for brevity and compliance with the standard literature on anomalies \[37, 38\]. The field strength two–form $F_2 = \frac{1}{2} F_{MN} dx^M dx^N$ is defined as the exterior derivative $d = dx^M \partial_M$ of the one–form $A_1 = A_M dx^M$. In components $F_2 = dA_1$ is written as $F_{MN} = \partial_M A_N - \partial_N A_M$. The differentials $dx^M$ are understood to be multiplied by the anti–commuting wedge product: $dx^M dx^N = -dx^N dx^M$. On a two–form the Hodge star $*$ is defined as

$$*(dx^M dx^N) = \frac{1}{3!} \epsilon^{MNPQR} \eta_{PS} \eta_{QT} \eta_{RU} dx^S dx^T dx^U, \quad *(*(dx^M dx^M)) = -dx^M dx^N, \quad (2)$$

where $\epsilon^{MNPQR}$ is the totally anti–symmetric epsilon tensor in five dimensions with $\epsilon^{01235} = 1$. Note that the sign of a two–form changes when the Hodge star is applied twice, due to the signature of the Minkowski metric $\eta_{MN} = \text{diag}(-1, 1, 1, 1, 1)$. For more details see the textbook \[39\].

By assumption the effective action $\Gamma_{eff}(A)$, obtained by integrating out all chiral brane and bulk fermions, is not invariant under the gauge transformation $\delta A_1 = d\alpha_0$. The gauge parameter $\alpha_0$ is a real scalar function (i.e. a zero–form). However, because of the Wess–Zumino
consistency conditions \cite{10}, and the fact that gauge anomalies do not exist in five dimensions, we infer that the variation of the effective action takes the form

\[ \delta_{\alpha_0} \Gamma_{\text{eff}}(A) = \sum_I \xi_I \int \alpha_0 F_2^2 \delta(y - I) dy = \sum_I \frac{1}{2} \xi_I \int \alpha_0 F_2^2 \bigg|_I , \]

with

\[ \xi_I = \frac{1}{24\pi^2} \left( \text{tr} q_I^3 + \frac{1}{2} \text{tr} q_b^3 \right) = \frac{k}{192\pi^2} \left( \text{tr} q_I + \frac{1}{2} \text{tr} q_b \right) , \]

where \( \text{tr} q_I^3 \) denotes the cubic sum of charges of the chiral fermions at the boundary \( I = 0, \pi \), and \( \text{tr} q_b^3 \) that of the bulk fermions. Note that the bulk fermions give equal anomalies on both boundaries \[4, 5\] (assuming that the boundary conditions of bulk fields are not twisted). With the notation \( |_I \) we emphasize that all fields are to be evaluated at the boundary \( y = I \). The factor of \( 1/2 \) in the last equation arises because the delta functions \( \delta(y - I) \) are defined on the end points of the interval. Specifically, \( \int_0^\pi dy \delta(y)f(y) = f(0)/2 \), for an arbitrary function \( f(y) \), and similarly at \( y = \pi \).

In this work we will not treat the cancellation of mixed gravitational gauge anomalies via the Green–Schwarz mechanism in detail. However, if the charges satisfy the “universality” relation, expressed by the second equality in (4), then the Green–Schwarz mechanism can cancel both pure gauge and mixed gravitational anomalies simultaneously. Such “universality” relations are well-known in heterotic string models \[14, 15\]. The proportionality factor \( k \) in (4) is called the level and depends on the normalization of the anomalous \( U(1)_A \).

In the Green–Schwarz anomaly cancellation mechanism a crucial role is played by the Chern–Simons three–form

\[ \omega_3(A) = A_1 F_2 , \quad d\omega_3(A) = F_2^2 , \]

for the anomalous \( U(1)_A \) gauge potential one–form \( A_1 \). Under a gauge transformation, the gauge variation of the Chern–Simons three form is given by

\[ \delta_{\alpha_0} \omega_3(A) = d\alpha_0 F_2 . \]

### 2.2 Vector/tensor duality in five dimensions

In five dimensions a vector \( A_M \) is dual to a two–form tensor \( \mathcal{B}_{MN} \). (Aspects of the vector/tensor duality in five dimensions were discussed in the past, see for example \[41\], in the context of the Type II – heterotic duality.) A convenient starting point to describe this duality is to use the action

\[ S_{TV} = \int \left( \frac{1}{2} \star \tilde{F}_2 \tilde{F}_2 - d\mathcal{B}_2 \mathcal{F}_2 \right) , \]

which contains the non–dynamical two forms \( \mathcal{F}_2 \) and \( \mathcal{B}_2 \). This action is invariant under gauge transformations of the two–form \( \mathcal{B}_2 \) given by

\[ \delta_{\beta_1} \mathcal{B}_2 = d\beta_1 , \quad \delta_{\beta_1} \mathcal{F}_2 = 0 . \]
There are two possible equations of motion arising from the action (7). First, the equation of motion of the two–form tensor $B_2$ leads to the constraint $d\hat{F}_2 = 0$. This equation can be (locally) solved by Poincaré’s lemma for a one–form gauge potential $A_1$

$$\hat{F}_2 = F_2 = dA_1, \quad \delta_{\beta_0} A_1 = d\beta_0.$$ (9)

The gauge transformation of $A_1$ leaves its associated field strength $F_2 = dA_1$ invariant. The hatted notation $\hat{F}_2$ for the non–dynamical two–form field might appear somewhat redundant here, but we will see in the next subsection, where we discuss the dual formulations of the Green–Schwarz mechanism, that the simple relation $\hat{F}_2 = F_2$ will be modified. Using this solution in the action (7), leads to the standard kinetic Abelian Yang–Mills action

$$S_V = \int \frac{1}{2} * (dA_1) dA_1 = - \int d^5x \frac{1}{4} F_{MN} F^{MN}. \quad (10)$$

On the other hand, since the equation of motion for $\hat{F}_2$ is purely algebraic, it can be easily solved to give $-\hat{F}_2 = *dB_2$. Thus, the duality between $A_1$ and $B_2$ is established by the two equations for $\hat{F}_2$, namely

$$F_2 = dA_1 = \hat{F}_2 = - *dB_2 = - *H_3.$$ (11)

Notice that this duality equation encodes the equations of motion for $A_1$ and $B_2$: By applying $d$ to this equation leads to $d * dB_2 = 0$, while acting with $d*$ gives $d * dA_1 = 0$. In (11) we have introduced the three–form field strength $H_3 = dB_2$ of the two–form $B_2$, which has the components $H_{MNP} = \partial_M B_{NP} + \partial_N B_{PM} + \partial_P B_{MN}$. Substituting the solution for $\hat{F}_2$ back into the action (7), we find the kinetic action for a rank two tensor

$$S_T = \int \frac{1}{2} * (dB_2) dB_2 = - \int d^5x \frac{1}{12} H_{MNP} H^{MNP}. \quad (12)$$

Hence we see that in this case by eliminating $\hat{F}_2$ the two–form $B_2$ has become dynamical. Obviously, both the anti–symmetric tensor and the vector formulations describes three physical on–shell degrees of freedom.

### 2.3 Dual formulations of the Green–Schwarz mechanism

The vector/tensor action (7), described in the previous subsection, is extended to the Green–Schwarz action

$$S_{GS} = \int \frac{1}{2} F_2 F_2 + \frac{1}{2} *\widehat{F}_2 \hat{F}_2 - dB_2 \left( \widehat{F}_2 - *\omega_3(A) + X_2(A) \right) + \frac{1}{2} *\omega_3(A) \omega_3(A), \quad (13)$$

with the Chern–Simons three–form $\omega_3(A)$ given in (11). We have also included the standard kinetic term (11) of the gauge field one–form $A_1$; it will be a “spectator” as far as the duality is concerned. The two–form $X_2(A)$ only has support at the end points of the interval

$$X_2(A) = \sum_I \xi_I A_1 \delta(y - I)dy.$$ (14)

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This action has been chosen such that it cancels the anomalous variation of the effective action \( \Gamma_{\text{eff}} \) obtained by integrating out the brane and bulk fermions:

\[
\delta_{\alpha_0} \left( \Gamma_{\text{eff}} + S_{\text{GS}} \right) = 0 .
\]  

(15)

Since the anomalous variation \( \delta_{\alpha_0} \) of the effective action \( \Gamma_{\text{eff}} \) does not contain the Hodge \(*\)-dualization, we infer that the Green–Schwarz tensors \( B_{MN} \) and \( \hat{F}_{MN} \) transform (up to exact terms) as

\[
\delta_{\alpha_0} B_2 = -\alpha_0 F_2 , \quad \delta_{\alpha_0} \hat{F}_2 = - * (\delta_{\alpha_0} B_2) = * (d \alpha_0 F_2) .
\]  

(16)

By a four dimensional partial integration it follows that the term involving the gauge variation of \( X_2 \) is gauge invariant by itself, and therefore the gauge variation of the Green–Schwarz action is given by

\[
\delta_{\alpha_0} S_{\text{GS}} = \int -d(\delta_{\alpha_0} B_2) X_2(A) = \int -\alpha_0 F_2^2 \delta \xi \delta(y - I) dy .
\]  

(17)

This variation indeed cancels the anomalous contribution of \( \Gamma_{\text{eff}} \) given in \( [3] \). It should be stressed that this calculation shows that the anomalous variation is independent of which dual formulation of the theory is being used, since we did not yet choose to eliminate either \( \hat{F}_2 \) or \( B_2 \) from the theory.

As in the previous subsection we can go to the one– or two–form formulation of the Green–Schwarz mechanism by integrating out \( B_2 \) or \( \hat{F}_2 \). The equation of motion for \( B_2 \) is \( d(\hat{F}_2 - * \omega_3(A) + X_2(A)) = 0 \) which is solved by

\[
\hat{F}_2 = d A_1 + * \omega_3(A) - X_2(A) ,
\]  

(18)

using the one–form \( A_1 \). Clearly, the gauge transformation \( [9] \) associated with this one–form is still there. But in order for \( \hat{F}_2 \) to have the required gauge transformation \( [10] \) it follows that \( A_1 \) transforms under the anomalous U(1)\(_4\) as

\[
\delta_{\alpha_0} A_1 = \sum \xi_I \alpha_0 \delta(y - I) dy .
\]  

(19)

(In principle this gauge transformation could also contain the contribution \( \sum_I \xi_I d \alpha_0 \epsilon(y - I) \) with \( \epsilon(y) \) the appropriate step function. But by a suitable chosen compensating gauge transformation \( [11] \) this transformation can be turned into the one given above). Note that this anomalous gauge transformation only affects \( \delta_{\alpha_0} A_5 = \sum_I \xi_I \alpha_0 \delta(y - I) \), while the four dimensional components are invariant \( \delta_{\alpha_0} A_m = 0 \). After substituting \( [12] \) into the action \( [13] \) the one–form formulation of the Green–Schwarz mechanism becomes

\[
S_{\text{GS1}} = \int \left[ \frac{1}{2} * d A_1 d A_1 + \frac{1}{2} (d A_1 + * \omega_3(A) - X_2(A)) \left( d A_1 + * \omega_3(A) - X_2(A) \right) + \frac{1}{2} * \omega_3(A) \omega_3(A) \right] \\
= \int \left[ \frac{1}{2} * d A_1 d A_1 + \frac{1}{2} * d A_1 d A_1 - \omega_3(A) d A_1 - \left( * d A_1 - \frac{1}{2} \omega_3(A) X_2(A) \right) X_2(A) \right] .
\]  

(20)

Notice that the term \( \omega_3(A) X_2(A) \) is zero. If there are multiple non-anomalous U(1)’s then this term does not cancel and leads to the generalized Chern–Simons terms with two gauge field one–forms \( [30] \). The supersymmetrization of this possibility is described in Appendix \( [33] \).
The term $-\omega_3(A)dA_1$ is not gauge invariant, and therefore the gauge variation of (20) does not equal (17), unless the four dimensional components of the Green–Schwarz vector vanishes at the boundaries $A_\mu|_I = 0$. This action is only defined formally since it involves squares of delta functions:

$$-\int \left( *dA_1 - \frac{1}{2} *X_2(A) \right) X_2(A) = \frac{1}{2} \sum_I \xi_I \int d^5x \left( A^m \mathcal{F}_{m5} - \frac{1}{2} \xi_I \delta(0) A_m A^n \right) |_I .$$ (21)

However, since it has been obtained from the regular action (13), there is a unique and well-defined way to deal with the singular contribution $\delta(0)$ in this case.

The dual formulation using the two–form $\mathcal{B}_2$ with $\hat{\mathcal{F}}_2 = - *d\mathcal{B}_2$ is manifestly free of any singular looking terms

$$S_{GS2} = \int \left[ \frac{1}{2} *dA_1 dA_1 + \frac{1}{2} *(d\mathcal{B}_2 + \omega_3(A)) (d\mathcal{B}_2 + \omega_3(A)) - d\mathcal{B}_2 X_2(A) \right] .$$ (22)

This is the five dimensional analog of the ten dimensional action of the anti–symmetric tensor in the supergravity multiplet coupled to the super Yang–Mills theory which is, for example, relevant for the low energy description of the heterotic string on orbifolds. In this case one similarly finds that the two–form description is free of singular terms, while in the dual formulation involving a six–form the singular $\delta(0)$ term arises as well, as can be inferred from ref. [21].

Furthermore, by introducing the gauge invariant three–form field strength $\hat{\mathcal{H}} = d\mathcal{B}_2 + \omega_3(A)$ one obtains the anomalous Bianchi identity $d\hat{\mathcal{H}} = F_2^2$. Finally, the duality relation (11) between the two– and one–form formulation is modified to

$$- *d\mathcal{B}_2 = \hat{\mathcal{F}}_2 = dA_1 + *\omega_3(A) - X_2(A) .$$ (23)

2.4 The four dimensional anomalous U(1) mass spectrum

We have seen that there are two equivalent dual formulations of the Green–Schwarz mechanism in five dimensions. Let us now determine the four dimensional mass spectrum of the anomalous $U(1)_A$ photon field together with the Green–Schwarz vector or tensor fields. At first sight it seems that the two formulations lead to very different mass spectra, since in the vector formulation (21) there are divergent boundary mass terms which are absent in the dual tensor formulation (22). This would then contradict the equivalence of the two dual formulations. However we will see that both formulations give rise to the same mass spectrum in four dimensions. The key observation is that both the vector and tensor formulations contain mixing terms between the vector or tensor field and the anomalous $U(1)_A$ gauge fields that need to be correctly taken into account. This is done by showing that both formulations lead to the same recursion relations for the components of the eigenvectors, that will enable us to determine the mass spectrum explicitly. Since the method of computing with infinite dimensional mass matrices is well–known [42], only the essential steps required for demonstrating how the duality is manifesting itself will be shown.

Since theories of five dimensional vector fields with boundary masses are more familiar, we consider the vector formulation first. We only require the quadratic part of (20) and use the
where the resulting infinite divergent sums and the \( \delta A \) gauge fixing conditions
\[
\partial^m A_m + \partial_5 A_5 = 0 , \quad \partial^m A_m + \partial_5 A_5 = 0 , \tag{24}
\]
to show that the \( A_5 \) and \( A_5 \) are decoupled from the gauge fields \( A_m \) and \( A_m \). Using the standard Kaluza–Klein decompositions on the orbifold
\[
A_m = \sum_{n \geq 0} \eta_n A_m^{(n)} \cos \left( \frac{n \eta}{R} \right) , \quad A_m = \sum_{n \geq 1} \eta_n A_m^{(n)} \sin \left( \frac{n \eta}{R} \right) , \tag{25}
\]
with \( \eta_0^2 = \frac{1}{\pi R} \) and \( \eta_n^2 = \frac{2}{\pi R} \), we find the following recursion relations for the Kaluza-Klein modes
\[
n \geq 0 : \quad \left( m^2 - \left( \frac{n}{R} \right)^2 \right) A_m^{(n)} = \frac{1}{2} \sum_{n' \geq 1} \eta_{n} \eta_{n'} n' \left( \xi_0 + \xi_\pi (-1)^{n+n'} \right) A_m^{(n')} + \frac{1}{2} \delta(0) \sum_{n'' \geq 0} \eta_n \eta_{n''} \left( \xi_0^2 + \xi_\pi^2 (-1)^{n+n''} \right) A_m^{(n'')} , \tag{26}
\]
\[
n' \geq 1 : \quad \left( m^2 - \left( \frac{n'}{R} \right)^2 \right) A_m^{(n')} = \frac{1}{2} \sum_{n'' \geq 0} \eta_{n'} \eta_{n''} n' \left( \xi_0 + \xi_\pi (-1)^{n+n''} \right) A_m^{(n'')} . \tag{27}
\]

In the above expressions we have set the four dimensional momentum squared \( \partial^2 \) equal to the mass eigenvalue \( m^2 \), and we have explicitly shown the dependence on the orbifold radius \( R \). Note that \( A_m^{(n')} \) can be solved from the second equation and substituted into the first relation, where the resulting infinite divergent sums and the \( \delta(0) \) are regularised in the following way
\[
\delta(0) = \frac{1}{2\pi R} \left[ \frac{m^2}{m^2} + 2 \sum_{n' \geq 1} \frac{m^2 - (n'/R)^2}{m^2 - (n'/R)^2} \right] , \quad \frac{2}{\pi R} \sum_{n' \geq 1} (-1)^{n'} = -\frac{1}{\pi R} . \tag{28}
\]

This gives rise to an equation solely for the modes \( A_m^{(n)} \), which has the form
\[
\left( m^2 - \left( \frac{n}{R} \right)^2 \right) A_m^{(n)} = \frac{1}{4} \sum_{n',n'' \geq 0} \eta_{n} \eta_{n'} \left[ \left( \xi_0^2 + \xi_\pi^2 (-1)^{n+n''} \right) \eta_{n''}^2 \right] \left( \frac{1}{1 - \left( \frac{n'}{mR} \right)^2} \right) + \xi_0 \xi_\pi \left( (-1)^n + (-1)^{n'} \right) \frac{\eta_{n''}^2 (-1)^{n''}}{1 - \left( \frac{n'}{mR} \right)^2} A_m^{(n'')} . \tag{29}
\]

This equation encodes the mass eigenvalues of the anomalous \( U(1)_A \) photon resulting from the Green–Schwarz theory coupled to the anomalous \( U(1)_A \) gauge theory. After some algebra one finds the mass eigenvalue equation and solution
\[
\tan^2(\pi mR) = \frac{1}{4} \left( \frac{\xi_0 + \xi_\pi}{1 - \frac{1}{4} \xi_0 \xi_\pi} \right)^2 \quad \Rightarrow \quad m = \frac{n}{R} + \frac{1}{\pi R} \arctan \left( \frac{1}{2} \frac{\xi_0 + \xi_\pi}{1 - \frac{1}{4} \xi_0 \xi_\pi} \right) . \tag{30}
\]

We see that the anomalous photon mass is finite at tree level, and the only scale it depends on is the orbifold radius \( R \). In the limit that \( \xi_0 + \xi_\pi \ll 1 \) the complete Kaluza–Klein tower
shifts approximately by the amount \( \delta m \approx |\xi_0 + \xi_\pi|/(2\pi R) \). This result is compatible with the shift of the zero mode of the anomalous \( U(1)_A \) gauge field in four dimensions derived in ref. [13]. Notice also that when \( \xi_0 + \xi_\pi = 0 \) the Kaluza–Klein spectrum remains unchanged, in particular the zero mode photon stays massless.

To check that the dual formulations are equivalent let us next explain how the same recursion relation (29) is obtained in the tensor formulation. Following the same methodology as in the vector formulation, we will consider the quadratic part of (22) and use the gauge fixing conditions

\[
\partial_m A_m + \partial_5 A_5 = 0, \quad \partial_m B_{mn} + \partial_5 B_{5n} = 0,
\]

(31)
to decouple \( A_5 \) and \( B_{5n} \) from \( A_m \) and \( B_{mn} \), respectively. In terms of the Kaluza-Klein mode expansions

\[
A_m = \sum_{n \geq 0} \eta_n A_m^{(n)} \cos \left( \frac{ny}{R} \right), \quad B_{mn} = \sum_{n \geq 0} \eta_n B_{mn}^{(n)} \cos \left( \frac{ny}{R} \right),
\]

(32)
the recursion relations for \( n, n' \geq 0 \) are given by

\[
\left( \partial_4^2 - \left( \frac{n'}{R} \right)^2 \right) B_{st}^{(n')} = -\frac{1}{2} \varepsilon_{st}^{pq} \sum_{n'' \geq 0} \eta_{n'} \eta_{n''} (\xi_0 + \xi_\pi (-1)^{n' + n''}) \partial_p A_q^{(n'')},
\]

(33)
\[
\left( \partial_4^2 - \left( \frac{n}{R} \right)^2 \right) A_m^{(n)} = -\frac{1}{4} \varepsilon_{mn}^{rs} \sum_{n' \geq 0} \eta_n \eta_{n'} (\xi_0 + \xi_\pi (-1)^{n + n'}) \partial_r B_{st}^{(n')}.
\]

(34)
Note that we cannot replace \( \partial_4^2 \) by the mass eigenvalues in the above equations since there are single derivatives in these recursion relations. Instead to eliminate the single derivatives we solve the first equation for \( B_{st}^{(n')} \) and then substitute this into the second equation to obtain

\[
\left( \partial_4^2 - \left( \frac{n}{R} \right)^2 \right) A_m^{(n)} = \frac{1}{4} \sum_{n', n'' \geq 0} \eta_n \eta_{n'} \eta_{n''}^2 \left( \xi_0^2 + \xi_\pi^2 (-1)^{n + n''} + (-1)^n \xi_0 \xi_\pi (-1)^n + (-1)^{n''} \right) \frac{\delta_{m} \partial_4^2 - \partial_4 \partial_{m} \partial_4}{\partial_4^2 - \left( n'/R \right)^2} A_q^{(n'')}.
\]

(35)
Notice that the last term contains the projector onto the transverse part of the gauge field. Therefore, for the transverse polarized gauge fields, we can replace \( \partial_4^2 \) with the mass eigenvalues \( m \), and obtain an equation identical to (29). Consequently, the anomalous \( U(1)_A \) mass spectrum in the vector and tensor formulation is the same, and the two dual formulations are indeed equivalent. However, the interesting aspect of the tensor formulation is that we never had to resort to the –in principle– ill-defined regularization of \( \delta(0) \) given in (28). Thus, the agreement of the Kaluza-Klein mass spectrum in the vector and tensor formulation justifies this regularization prescription.

### 3 Supersymmetric vector/tensor multiplet duality

Before describing the full supersymmetric Green–Schwarz mechanism in five dimensions, we need to introduce the five dimensional vector and tensor multiplets. The five dimensional vector
multiplet has been discussed in various places using four dimensional $N=1$ superfields in the literature \cite{26,27,28}, while the five dimensional tensor multiplet is less known in this formalism. For completeness and to fix our notation we describe both these multiplets in the following two subsections. We also give the straightforward generalization to multiple vector multiplets which will prove convenient in later sections. We will use the conventions from Wess and Bagger \cite{14} throughout this paper. For simplicity the component forms of the supermultiplets and actions will be restricted to bosons only. Since all expressions will be given in four dimensional $N=1$ superspace, it is straightforward, though tedious, to obtain the fermionic terms. In this section we first focus on the vector multiplet that occurs in the supersymmetric Green–Schwarz mechanism, and a collection of U(1) vector multiplets in general. After that we describe the five dimensional tensor multiplet and discuss the supersymmetric extension of the duality between vector and tensor fields. (The anomalous U(1)$_A$ vector multiplet will be discussed in section 4 together with a description of the supersymmetric version of the Chern–Simons three–form.)

3.1 Five dimensional vector multiplet

A vector multiplet in five dimensions contains a vector field $A_M$, a real scalar $\varphi$ and a Dirac (or symplectic Majorana) fermion. By reducing to four dimensional supersymmetry one can infer that the five dimensional vector multiplet is described by a vector superfield $V \dagger = V$ and a chiral superfield $S$ with $\bar{D}_\dot{\alpha} S = 0$. The super covariant derivatives of the four dimensional $N=1$ superspace are denoted by $D_\alpha$ and $\bar{D}_{\dot{\alpha}}$. The gauge transformation \cite{9} of $A_M$ is lifted in $N=1$ superspace to

$$\delta_\Psi V = \Psi + \bar{\Psi}, \quad \delta_\Psi S = \sqrt{2} \partial_5 \Psi,$$

where $\Psi$ is a chiral superfield. (The gauge transformation \cite{9} is obtained by taking $\Psi = i \beta_0$.) The bosonic components of these superfields for the vector multiplet $V$ in the Wess–Zumino gauge and the chiral multiplet $S$ are identified by

$$-\frac{1}{2} [D_\alpha, \bar{D}_{\dot{\alpha}}] V ] = \sigma^{mn}_{\alpha \dot{\gamma}} A_m, \quad \frac{1}{8} D^a \bar{D}^2 D_\alpha V ] = \mathcal{D}, \quad \mathcal{S} ] = \frac{1}{\sqrt{2}} (\varphi + i A_5), \quad -\frac{1}{4} D^2 \mathcal{S} ] = \mathcal{F},$$

where we have introduced the four dimensional indices $m = 0, \ldots, 3$. As usual the notation $|$ indicates that we have set all $\bar{\theta}_\dot{\alpha} = \theta_\alpha = 0$. Out of the superfields $V$ and $S$ two independent super gauge invariant superfields can be constructed

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V, \quad V_5 = \frac{1}{\sqrt{2}} (S + \bar{S}) - \partial_5 V.$$

The bosonic components of these superfields are given by

$$D_\beta W_\alpha ] = -i (\sigma^{mn}_\alpha \epsilon)_\beta \mathcal{F}_{mn} - \epsilon_{\beta \alpha} \mathcal{D}, \quad V_5 ] = \varphi, \quad -D^2 V_5 ] = 2 \sqrt{2} \mathcal{F},$$

$$-\frac{1}{2} [D_\alpha, \bar{D}_{\dot{\alpha}}] V_5 ] = \sigma^{m5}_{\alpha \dot{\gamma}} \mathcal{F}_m, \quad \frac{1}{8} D^a \bar{D}^2 D_\alpha V_5 ] = -\partial_5 \mathcal{D},$$

(39)
where $\mathcal{F}_{mn} = \partial_m A_n - \partial_n A_m$ and $\mathcal{F}_{m5} = \partial_m A_5 - \partial_5 A_m$. The action for the five dimensional super Yang–Mills theory can be represented as \[27\]

$$S_{SV} = \int d^5x \left[ \int d^2\theta \frac{1}{4} W^a(V) W_a(V) + \text{h.c.} + \int d^4\theta V_5^2 \right] \tag{40}$$

$$\supset \int d^5x \left[ -\frac{1}{4} \mathcal{F}_{MN} \mathcal{F}^{MN} - \frac{1}{2} \partial_M \varphi \partial^M \varphi + \frac{1}{2} (\mathcal{D} + \partial_5 \varphi)^2 + \mathcal{F} \bar{\mathcal{F}} \right]. \tag{41}$$

The second line is obtained by restricting to the bosonic part of the action; we denote this by $\supset$. We have completed the square involving the auxiliary field $D$ to show that up to the auxiliary field equation of motion the component action is Lorentz invariant in five dimensions, because in later section we encounter terms that will modify the auxiliary field equations.

The present discussion can easily be extended to the case of multiple five dimensional $U(1)$ vector multiplets described by the superfields $(V^a, S^a)$ and labeled with lowercase Latin indices $a, b, \ldots$ run over all the vector multiplets, anomalous and non–anomalous. The most general gauge invariant action is encoded by a prepotential $\mathcal{P}(\varphi)$. This real function of the real scalars $\varphi = (\varphi^a)$ is at most a cubic polynomial \[30\]

$$\mathcal{P}(\varphi) = \frac{1}{2} c_{ab} \varphi^a \varphi^b + \frac{1}{6} c_{abc} \varphi^a \varphi^b \varphi^c, \tag{42}$$

where the coefficients $c_{ab}$ and $c_{abc}$ are real and totally symmetric. Denoting differentiation w.r.t. $\varphi^a$ by $\mathcal{P}_a \equiv \partial \mathcal{P}/\partial \varphi^a$, the superspace expression for the general action with an arbitrary prepotential $\mathcal{P}$ reads

$$S_V = \int d^5x \left[ \int d^2\theta \frac{1}{4} (\mathcal{P}_a (\sqrt{2} S) W^{a\alpha} W^a_\alpha - \frac{1}{6} \mathcal{P}_{abc} \bar{D}^2 (V^a D^a \partial_a V^b - D^a V^a \partial_b V^b) W^a_\alpha) + \text{h.c.} \right. \tag{43}$$

$$+ \int d^4\theta 2 \mathcal{P}(V_5)$$

$$\supset \int d^5x \left[ \mathcal{P}_a(\varphi) \left( \frac{1}{4} \mathcal{F}^{a}_{MN} \mathcal{F}^{MN} - \frac{1}{2} \partial_M \varphi^a \partial^M \varphi^b + \frac{1}{2} (\mathcal{D} + \partial_5 \varphi)^a (\mathcal{D} + \partial_5 \varphi)^b + \mathcal{F}^a \bar{\mathcal{F}}^b \right) \right.$$

$$+ \frac{1}{24} \mathcal{P}_{abc}^{\lambda MNQR} A^a_{M} \mathcal{F}^{b}_{NP} \mathcal{F}^{c}_{QR} \right]. \tag{44}$$

In order for the general action to reproduce the kinetic action for a single vector multiplet \[40\] the normalization of the prepotential is chosen such that $\mathcal{P}_{SV} = \frac{1}{2} \varphi^2$. The gauge variation of \[44\] is given by

$$\delta_\Psi S_V = \int d^5x \left( \int d^2\theta \frac{1}{3} \mathcal{P}_{abc} \Psi^{a \alpha} W^{b\alpha} W^c_\alpha + \text{h.c.} \right) \left( \delta(y - \pi) - \delta(y) \right). \tag{45}$$

### 3.2 Five dimensional tensor multiplet

A tensor multiplet contains an anti–symmetric rank two tensor $\mathcal{B}_{MN}$, a real scalar $\sigma$ and a Dirac fermion. Using the four dimensional superspace language these states can be described by a chiral spinor multiplet $T_\alpha$, with $D_\alpha T_\alpha = 0$, and a vector multiplet $U$ with $U^\dagger = U$. The
gauge transformation $\delta \mathcal{T}_\alpha = -\frac{1}{4} \partial^2 \mathcal{D}_\alpha \mathcal{C}$, $\delta \bar{\mathcal{T}}_{\dot{\alpha}} = -\frac{1}{4} \partial^2 \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{C}$, $\delta \mathcal{U} = \Psi + \bar{\Psi} - \partial_5 \mathcal{C}$, \quad (46)

where $\mathcal{C}$ and $\Psi$ denote a vector and chiral superfield, respectively. (The four dimensional analog of this transformation is well-known, see for example [45, 46].) This means that one can define a Wess–Zumino gauge for both $\mathcal{U}$ and $\mathcal{T}_\alpha$. For the chiral superfield $\mathcal{T}_\alpha$ this gauge is defined such that the $\epsilon_{\beta\alpha}$ part of $D_\beta \mathcal{T}_\alpha$ is purely imaginary. In the Wess–Zumino gauge the bosonic components of the supermultiplets $\mathcal{T}_\alpha$ and $\mathcal{U}$ are given by

$$D_\beta \mathcal{T}_\alpha = -i(\sigma^{mn}\epsilon)_{\beta\alpha} B_{mn} - \epsilon_{\beta\alpha} i\sigma, \quad -\frac{1}{2}[D_\alpha, D_{\dot{\alpha}}]\mathcal{U} = \sigma_{\alpha\dot{\alpha}} B_{m5}, \quad \frac{1}{8} D^n D^2 D_\alpha \mathcal{U} = \mathcal{T}_T. \quad (47)$$

Furthermore two gauge invariant superfields can be constructed from $\mathcal{T}_\alpha$ and $\mathcal{U}$:

$$\mathcal{T}_{5\alpha} = \partial_5 \mathcal{T}_\alpha + \mathcal{W}_\alpha(\mathcal{U}), \quad \mathcal{L} = i(\partial^a \mathcal{T}_\alpha - \bar{D}_{\dot{\alpha}} \bar{\mathcal{T}}^{\dot{\alpha}}). \quad (48)$$

The supermultiplet $\mathcal{T}_{5\alpha}$ is chiral, $\bar{D}_{\dot{\alpha}} \mathcal{T}_{5\alpha} = 0$, while $\mathcal{L}$ defines a linear multiplet $D^2 \mathcal{L} = \bar{D}^2 \mathcal{L} = 0$ [47, 48]. The bosonic components of these gauge invariant superfields read

$$D_\beta \mathcal{T}_{5\alpha} = -i(\sigma^{mn}\epsilon)_{\beta\alpha} \mathcal{H}_{mn5} - \epsilon_{\beta\alpha}(D_T + i\partial_5\sigma),$$

$$\frac{1}{2}[D_\alpha, \bar{D}_{\dot{\alpha}}]\mathcal{L} = -\frac{1}{6} \epsilon^{mnkl} \mathcal{L}_{mnk}(\sigma_l)_{\alpha\dot{\alpha}}, \quad \mathcal{L} = \sigma, \quad (49)$$

where $\mathcal{H}_{mnk} = \partial_m B_{nk} + \partial_n B_{km} + \partial_k B_{mn}$ and $\mathcal{H}_{mn5} = \partial_m B_{n5} - \partial_n B_{m5} + \partial_5 B_{mn}$. Hence the action for the five dimensional tensor multiplet can be represented as

$$S_{ST} = \int d^5x \left[ \int d^2\theta \frac{1}{4} \mathcal{T}_{5\alpha} \mathcal{T}_{5\alpha} + \text{h.c.} + \int d^4\theta (-\mathcal{L}^2) \right] \quad (50)$$

$$\mathcal{S} \int d^5x \left[ -\frac{1}{12} \mathcal{H}_{MNK} \mathcal{H}^{MNK} - \frac{1}{2} \partial_M\sigma \partial^M\sigma + \frac{1}{2} D_T^2 \right]. \quad (51)$$

The second line only contains the bosonic part of the action. Unlike the vector multiplet action the tensor superfield action is manifestly five dimensional Lorentz invariant off-shell, and the equation of motion of $D_T$ is trivial.

### 3.3 The supersymmetric vector/tensor duality

In the previous two subsections we have discussed the five dimensional vector and tensor multiplet using $N = 1$ four dimensional superspace techniques. We now wish to supersymmetrize the five dimensional vector/tensor duality described in section 2.2. Proceeding as in that section we will start with a hybrid description in which either the vector or tensor formulation has not yet been chosen. It is useful to rewrite (4) as

$$S_{TV} = \int d^5x \left[ -\frac{1}{4} \mathcal{F}_{mn} \mathcal{F}^{mn} - \frac{1}{2} \mathcal{F}_{m5} \mathcal{F}^{m5} - \frac{1}{4} \epsilon^{mnkl} \mathcal{H}_{mn5} \mathcal{F}_{kl} + \frac{1}{4} \epsilon^{mnkl} B_{mn} (\partial_k \mathcal{F}_{l5} - \partial_l \mathcal{F}_{k5}) \right]. \quad (52)$$
Therefore, to obtain the supersymmetric version we see that apart from the superfields $\mathcal{T}_\alpha$ and $\mathcal{U}$ describing the tensor multiplet degrees of freedom, we also need to introduce a chiral superfield, $\hat{\mathcal{W}}_\alpha$, and a vector superfield, $\hat{\mathcal{V}}_5$, for the non-dynamical (hatted) fields $\mathcal{T}_{mn}$ and $\mathcal{T}_{m5}$. Since only the field strength of the vector $\hat{\mathcal{T}}_{m5}$ appears in (52), the supersymmetric generalization will involve only the superfield strength $\mathcal{W}_\alpha(\mathcal{V}_5)$. Using standard superfield methods to obtain component actions we find that the supersymmetric generalization of (52) is a part of

$$S_{STV} = \int d^5x \left[ \int d^2\theta \left( \frac{1}{4} \hat{\mathcal{W}}^\alpha \hat{\mathcal{W}}_\alpha - \frac{i}{2} \hat{\mathcal{T}}_5^\alpha \hat{\mathcal{W}}_\alpha + \frac{i}{2} \hat{\mathcal{T}}_5^{2\alpha} \mathcal{W}_\alpha(\mathcal{V}_5) \right) + \text{h.c.} + \int d^4\theta \hat{\mathcal{V}}_5^2 \right] \supset S_{TV} \, .$$

(53)

Note that the last term of $S_{STV}$ is not present in (52), but is needed to ensure that the whole action is five dimensional Lorentz invariant.

The supersymmetric realization of the vector/tensor duality can now be described as follows (the related scalar/tensor duality for $N=2$ in four dimensions has been discussed in [49]): The supersymmetric version of the elimination of $\mathcal{B}_{MN}$ is implemented by varying with respect to the corresponding superfields $\mathcal{U}$ and $\mathcal{T}_\alpha$. Because of the chirality of $\hat{\mathcal{W}}_\alpha$ the equation of motion of $\mathcal{U}$ implies that $\hat{\mathcal{W}}_\alpha$ can be expressed in terms of a vector superfield $\mathcal{V}$:

$$\hat{\mathcal{D}}_\alpha \hat{\mathcal{W}}^\alpha - D^\alpha \hat{\mathcal{W}}_\alpha = 0 \quad \Rightarrow \quad \hat{\mathcal{W}}_\alpha = \mathcal{W}_\alpha(\mathcal{V}) = -\frac{1}{4} \hat{\mathcal{D}}^2 \mathcal{D}_\alpha \mathcal{V} \, .$$

(54)

To work out the consequences of the variation of $\mathcal{T}_\alpha$, it proves useful to write this superfield as $\mathcal{T}_\alpha = \hat{\mathcal{D}}^2 \mathcal{C}_\alpha$, where $\mathcal{C}_\alpha$ is an unconstrained spinor multiplet. Using the solution (54) for $\hat{\mathcal{W}}_\alpha$, we obtain constraints which are readily solved by introducing a chiral superfield $\mathcal{S}$

$$\mathcal{D}^2 \mathcal{D}_\alpha(\hat{\mathcal{V}}_5 + \partial_5 \mathcal{V}) = \mathcal{D}^2 \mathcal{D}_\alpha(\mathcal{V}_5 + \partial_5 \mathcal{V}) = 0 \quad \Rightarrow \quad \hat{\mathcal{V}}_5 = \mathcal{V}_5 = \frac{1}{\sqrt{2}} (\mathcal{S} + \bar{\mathcal{S}}) - \partial_5 \mathcal{V} \, .$$

(55)

Substituting the solutions (54) and (55) for $\hat{\mathcal{W}}_\alpha$ and $\hat{\mathcal{V}}_5$, respectively, back into the supersymmetric action (53) leads to the action $S_{SV}$ of a vector multiplet, given in (40).

Alternatively, to eliminate the supersymmetric extension of $\mathcal{T}_{MN}$, i.e. $\hat{\mathcal{W}}_\alpha$ and $\hat{\mathcal{V}}_5$, we rewrite the action (53) in the form

$$S_{STV} = \int d^5x \left[ \int d^2\theta \left( \frac{1}{4} \hat{\mathcal{W}}^\alpha \hat{\mathcal{W}}_\alpha - \frac{i}{2} \hat{\mathcal{T}}_5^\alpha \hat{\mathcal{W}}_\alpha \right) + \text{h.c.} + \int d^4\theta \left( \hat{\mathcal{V}}_5^2 - \frac{i}{2} (\mathcal{D}^\alpha \mathcal{T}_\alpha - \hat{\mathcal{D}}^\alpha \mathcal{T}_\alpha) \overline{\mathcal{V}}_5 \right) \right] \, ,$$

(56)

where we have used the chirality of $\mathcal{T}_\alpha$ and $\mathcal{W}_\alpha(\mathcal{V}_5)$, and replaced $\int d^2\theta$ by $-\frac{1}{2} \mathcal{D}^2$ under the spacetime integral. The resulting equations of motion for the non-dynamical superfields $\hat{\mathcal{W}}_\alpha$ and $\hat{\mathcal{V}}_5$ are

$$\hat{\mathcal{W}}^\alpha = i \mathcal{T}_5^\alpha \, , \quad \hat{\mathcal{V}}_5 = \mathcal{L} = \frac{i}{4} (\mathcal{D}^\alpha \mathcal{T}_\alpha - \hat{\mathcal{D}}^\alpha \mathcal{T}_\alpha) \, .$$

(57)

Substituting these expressions back into (53) gives the action $S_{ST}$ of a tensor multiplet, see (50).

By combining both expressions for $\hat{\mathcal{V}}_5$ and $\hat{\mathcal{W}}_\alpha$ the supersymmetric version of the duality relation (23) is found to be

$$\frac{1}{\sqrt{2}} (\mathcal{S} + \bar{\mathcal{S}}) - \partial_5 \mathcal{V} = \hat{\mathcal{V}}_5 = \frac{i}{4} (\mathcal{D}^\alpha \mathcal{T}_\alpha - \hat{\mathcal{D}}^\alpha \mathcal{T}_\alpha) \, , \quad \mathcal{W}_\alpha(\mathcal{V}) = \hat{\mathcal{W}}_\alpha = i (\partial_5 \mathcal{T}_\alpha + \mathcal{W}_\alpha(\mathcal{U})) \, .$$

(58)
It is not surprising that the five dimensional duality is written in terms of two equations since one also obtains two equations if the duality relation (23) is dimensionally reduced and written in four dimensional component language. Notice also that the lowest component of the first equation in (58) gives a linear version of the duality relation in [30], namely
\[ \sigma = \varphi = \frac{\partial \tilde{P}_{SV}}{\partial \varphi}. \] (59)

4 Supersymmetric Green–Schwarz mechanism

After presenting the duality between the vector and tensor multiplets in five dimensions, we now proceed to give an \( N = 1 \) four dimensional superspace discussion of the Green–Schwarz mechanism in five dimensions. We begin by introducing the Chern–Simons superfields for the anomalous vector multiplet. This will enable us to describe the supersymmetric version of the Green–Schwarz theory in the vector and tensor multiplet representation.

4.1 Chern–Simons three–form superfields

The anomalous vector multiplet in five dimensions is defined similarly to the non–anomalous vector multiplet in section 3.1, except that we use standard italic letters for the fields instead of calligraphic letters to distinguish the two. For notational completeness we begin by repeating the relevant definition of the vector multiplet in five dimensions so that the subsequent discussion on the supersymmetrization of the Chern–Simons three–form in five dimensions will be self contained. We introduce a vector multiplet \( V \) and a chiral multiplet \( S \) with bosonic components
\[ -\frac{1}{2}[D_\alpha, D_{\dot{\alpha}}]V = \sigma_{\alpha\dot{\alpha}}^m A_m, \quad \frac{1}{8} D^\alpha D^2 D_\alpha V = D, \quad |S| = \frac{1}{\sqrt{2}} (\phi + i A_5), \quad -\frac{1}{4} D^2 |S| = F, \] (60)
where \( V \) is in Wess–Zumino gauge. The bosonic components of the super gauge invariant superfields
\[ W_\alpha = -\frac{1}{4} D^2 D_\alpha V, \quad V_5 = \frac{1}{\sqrt{2}} (S + \bar{S}) - \partial_5 V, \] (61)
are given by
\[ D_\beta W_\alpha = -i (\sigma^{mn} \epsilon)_{\beta\alpha} F_{mn} - \epsilon_{\beta\alpha} D, \quad V_5 = \phi, \quad -D^2 V_5 = 2\sqrt{2} F, \]
\[ -\frac{1}{2}[D_\alpha, D_{\dot{\alpha}}]V_5 = \sigma_{\alpha\dot{\alpha}}^m F_{m5}, \quad \frac{1}{8} D^\alpha D^2 D_\alpha V_5 = -\partial_5 D. \] (62)

With these definitions we can now discuss a supersymmetrization of the Chern–Simons three–form in five dimensions. The (4+1) dimensional decomposition of the definition (5) of this three–form
\[ \frac{1}{6} \partial_m \omega_{npq} \epsilon^{mnpq} = \frac{1}{4} F_{mn} F_{pq} \epsilon^{mnpq}, \quad \frac{1}{2} \partial_m \omega_{np5} \epsilon^{mnpq} = (F_{mn} F_{p5} - \frac{1}{6} \partial_5 \omega_{mnp}) \epsilon^{mnpq}, \] (63)
implies that we need to introduce two Chern–Simons superfields in five dimensions in order to contain all the degrees of freedom. In appendix A we describe a convenient way to determine the supersymmetrizations of the definitions. For the first relation in (63) we use the four dimensional Chern–Simons superfield $\Omega$ which is a real vector superfield defined by the equations

\[
\bar{D}^2 \Omega = W^a W_a, \quad D^2 \Omega = \bar{W}_a \bar{W}^{a}. \tag{64}
\]

The solution to these equations is $\Omega = \Omega(V, V')$ where we have defined

\[
\Omega(V, V') = -\frac{1}{4} (D^a V W_a(V') + \bar{D}_a V \bar{W}^{a}(V') + V D^a W_a(V')) . \tag{65}
\]

Since $D^a W_a = \bar{D}_a \bar{W}^a$ the last term in this expression may also be written as $V \bar{D}_a \bar{W}^{a}$. In Wess–Zumino gauge the four dimensional components $\omega_{mnp} = A_m F_{np} + A_n F_{pm} + A_p F_{mn}$ of the Chern–Simons three–form (5) are obtained by the restriction

\[
\frac{1}{2} [D_\alpha, \bar{D}_\dot{\alpha}] \Omega \supset \frac{1}{12} \omega_{mnk} \epsilon^{mnl}(\sigma_l)_{\alpha \dot{\alpha}} . \tag{66}
\]

The supermultiplet containing the additional components $\omega_{mn5} = A_m F_{n5} - A_n F_{m5} + A_5 F_{mn}$ of the five dimensional Chern–Simons three–form $\omega_3$ (see the second relation in (63) and Appendix A), is obtained by solving the equation

\[
\frac{i}{2} (D^a Y_\alpha - \bar{D}_\dot{a} \bar{Y}^{\dot{a}}) = -4\partial_5 \Omega(V, V) - 8\Omega(V_5, V) , \tag{67}
\]

where $\Omega(V, V')$ is defined in (65). The solution is described by the chiral spinor multiplet $Y_\alpha$ and its conjugate, where

\[
Y_\alpha = \frac{i}{4} \bar{D}^2 (V_5 D_\alpha V - D_\alpha V_5 V + \sqrt{2} S D_\alpha V) , \quad \bar{D}_\dot{a} Y_\alpha = 0 . \tag{68}
\]

(To prove that (68) is indeed a solution of the superfield equation (67), it is useful to employ the identities (A.5) and (A.6) of Appendix A). After dropping all fermionic terms in the Wess–Zumino gauge the component $\omega_{mn5}$ appears in the projection

\[
D_3 Y_\alpha \supset -i (\sigma^{mn} \epsilon_{\beta \alpha} (\omega_{mn5} - 2i \phi F_{mn}) - \epsilon_{\beta \alpha} (A_5 D - \partial_m \phi A^m - 2i \phi D) . \tag{69}
\]

Under the five dimensional super gauge transformations $\delta_\Lambda V = \Lambda + \bar{\Lambda}$ and $\delta_\Lambda S = \sqrt{2} \partial_5 \Lambda$ the Chern–Simons superfields transform as

\[
\delta_\Lambda \Omega = -\frac{1}{4} D^a (\Lambda W_a) - \frac{1}{4} \bar{D}_\dot{a} (\bar{\Lambda} \bar{W}^{\dot{a}}) , \quad \\
\delta_\Lambda Y_\alpha = \frac{i}{4} \bar{D}^2 (V_5 D_\alpha \Lambda - D_\alpha V_5 (\Lambda + \bar{\Lambda}) + 2\partial_5 \Lambda D_\alpha V) , \tag{70}
\]

where we have exploited the chirality of various superfields.

The Chern–Simons multiplets $\Omega$ and $Y^\alpha$ will play an important role in determining the supersymmetric Green–Schwarz theory in five dimensions. Before proceeding to this discussion
in the next subsection let us mention two interesting uses of these multiplets. First, equation (64) implies that the super Yang–Mills action can be written in terms of $\Omega$ as

$$S_{\text{SYM}} = \int d^5x \int d^4\theta (-2\Omega + V_5^2) = \int d^5x \left[ \int d^2\theta \frac{1}{4} W^a W_a + \text{h.c.} + \int d^4\theta V_5^2 \right], \quad (71)$$

with the component form of this action given in (41). In the following section we will encounter the combination $2\Omega - V_5^2$ frequently.

Secondly, both Chern–Simons multiplets can be used to give a compact representation of supersymmetric Chern–Simons interactions in five dimensions. In particular, the mixed Chern–Simons interaction involving two gauge fields $A_M$ and $A_{M'}$ can be conveniently written as

$$S_{\text{CS mix}} = \int d^5x \left[ \int d^2\theta - \frac{i}{2} Y^a W_a + \text{h.c.} + \int d^4\theta 2V_5(2\Omega - V_5^2) \right]$$

$$\supset \int d^5x \left[ - \frac{1}{12} \epsilon^{MNPQR} \omega_{MNP} F_{QR} + \varphi \left( \frac{1}{2} F_{MN} F^{MN} + \partial_M \phi \partial^M \phi - (D + \partial_5 \phi)^2 - 2\bar{F} F \right) \right.$$

$$\left. + 2\phi \left( \frac{1}{2} F_{MN} F^{MN} + \partial_M \phi \partial^M \phi - (D + \partial_5 \phi)(D + \partial_5 \phi) - \bar{F} F - \bar{F} F \right) \right]. \quad (72)$$

By comparing with the prepotential action (43), we infer that this mixed Chern–Simons interaction can also be obtained from the prepotential $P_{\text{CS mix}} = -\varphi \phi^2$.

Let us close this subsection with a short discussion on the generalization to multiple vector multiplets. The Chern–Simons superfield $\Omega(V, V')$ will be used to make contact with the holomorphic prepotential language of $N=2$ supersymmetry in four dimensions. The five dimensional Chern–Simons term (the last term of the first line of equation (43)) can be rewritten using

$$\int d^4\theta \left( V^a D^a \partial_5 V^b - D^a V^a \partial_5 V^b \right) \Omega(V^a, V^b) = 8 \int d^4\theta \partial_5 V^b \Omega(V^a, V^c). \quad (74)$$

It is not difficult to see that this gives a five dimensional realization of a generalized Chern–Simons term containing two gauge fields and a field strength. In the four dimensional $N=2$ context, the existence of such terms was observed a long time ago (more recently see also [33, 34]). Their presence in $N=1$ theories was pointed out by using anomaly type arguments in phenomenological models [32], in the deconstruction of U(1) supersymmetric gauge theories [35], flux compactifications and generalized Scherk–Schwarz reductions [36]. The general superspace description of the Lagrangian and couplings was also recently worked out in [36].

Using (74) we can rewrite (43) in a form that more closely resembles the four dimensional $N=2$ action for vector multiplets

$$S_V = \int d^5x \int d^4\theta \left[ \mathcal{K}(S, \tilde{S}) + \frac{2\sqrt{2}}{3} \mathcal{F}_{abc} \partial_5 V^b \Omega(V^a, V^c) \right] + \int d^5x \left[ \int d^2\theta \frac{1}{4} \mathcal{F}_{ab}(S) W^{\alpha a} W^b_\alpha \right. \quad (75)$$

$$\left. + \int d^4\theta \frac{1}{2} \mathcal{F}_{ab}(S) \left( \partial_5 V^a \partial_5 V^b + \sqrt{2} V^a \partial_5 (S^b + \tilde{S}^b) \right) + \text{h.c.} \right],$$
where the Kähler potential $\mathcal{K}(S, \bar{S})$ is defined by the usual expression in terms of the holomorphic four dimensional prepotential $\mathcal{F}(S)$:

$$
\mathcal{K}(S, \bar{S}) = \frac{1}{2} (S^a \bar{F}_a(\bar{S}) + \bar{S}^a F^a(S)) , \quad \mathcal{F}(S) = \frac{1}{2} \mathcal{P}(\sqrt{2}S) .
$$

(76)

Since we are describing a five dimensional supersymmetric gauge theory the holomorphic prepotential $\mathcal{F}(S)$ is determined by the real prepotential $\mathcal{P}(\phi)$ in five dimensions defined in (12). Consequently, the coefficients of the holomorphic prepotential $\mathcal{F}(S)$ are real rather than complex.

### 4.2 Supersymmetrizing the dual Green–Schwarz theory

As in our discussion of the bosonic Green–Schwarz mechanism in section 2.3, we first focus on a hybrid formulation of the supersymmetric Green–Schwarz anomaly cancellation, which can serve as a basis for both the vector or tensor multiplet description. To construct the supersymmetrization of the bosonic Green–Schwarz action (13) we decompose the supersymmetric Green–Schwarz action as

$$
S_{SGS} = S_{SYM} + S_{STV} + S_{Bianchi} + S_{bdy} + S_{\omega^2} .
$$

(77)

The actions $S_{STV}$ and $S_{SYM}$, given in (3) and (71), respectively, correspond to the supersymmetrizations of the first three terms of (13). The other parts of this action, $S_{Bianchi}$, $S_{bdy}$ and $S_{\omega^2}$, contain the supersymmetric extensions of the terms $-dB_2 * \omega_3(A)$, $-dB_2 X_2(A)$ and $\frac{1}{2} * \omega_3(A) \omega_3(A)$ in (13), respectively.

The full superspace expressions and their bosonic reductions can be obtained. The supersymmetric extension of the $dB_2 * \omega_3$ term is

$$
S_{Bianchi} = \int d^5x \left[ \int d\theta \frac{1}{2} \mathcal{F}_a Y_a + h.c. + \int d^4\theta \mathcal{L}(2\Omega - V_5^2) \right]
$$

$$
\supset \int d^5x \left[ -\frac{1}{6} \mathcal{H}_{MNP} \omega^{MNP} - \frac{1}{6} \epsilon^{MNPQR} \phi \mathcal{H}_{MNP} F_{QR} + \frac{1}{2} \sigma F_{MN} F^{MN}

+ \sigma \partial_M \phi \partial^M \phi - \sigma (D + \partial \phi)^2 - 2 \sigma \bar{F} F + \mathcal{D}_T \left( A_5 D - \partial m \phi A^m \right) \right] .
$$

(79)

The term $\mathcal{L}V_5^2$ is separately gauge invariant, but it is required to obtain a five dimensional Lorentz invariant expression. The supersymmetrization of the boundary term $-dB_2 X_2(A)$ is straightforward because there we only have $N=1$ supersymmetry in four dimensions:

$$
S_{bdy} = \xi I \int d^4x \left[ \int d^2\theta \frac{i}{4} \mathcal{F}_a W_\alpha(V) \right] + h.c. \supset \frac{1}{2} \xi I \int d^4x \left[ \frac{1}{4} \epsilon^{mnkl} \mathcal{B}_{mn} F_{kl} - \sigma D \right] .
$$

(80)
Finally, the supersymmetrized version of the $\frac{1}{2} \ast \omega_3 \omega_3$ interaction is given by

$$S_{\omega^2} = \int d^5 x \left[ \int d^2 \theta \frac{1}{4} Y^\alpha Y_\alpha + \text{h.c.} - \int d^4 \theta (V_5^2 - 2\Omega)^2 \right]$$

$$\supset \int d^5 x \left[ -\frac{1}{12} \omega_{MNP} \omega^M_{NP} - \frac{1}{6} \epsilon^{MNPQR} \phi \omega_{MNP} F_{QR} + \frac{3}{2} \phi^2 F_M F^M 
+ 3\phi^2 \partial M \phi \partial^M \phi - 3\phi^2 (D + \partial_5 \phi)^2 - 6\phi^2 \bar{F} F + \frac{1}{2} (A_5 D - \partial_5 \phi A^m)^2 \right].$$

(81)

(82)

In a supersymmetric theory the gauge anomaly lifts to a full super gauge anomaly i.e. the effective action with all charged boundary and bulk matter multiplets integrated out transforms as \[53, 54\]

$$\delta \Lambda \Gamma_{\text{eff}} = \sum_I \xi_I \int d^5 x \int d^2 \theta \Lambda W^\alpha W_\alpha \delta(y - I) + \text{h.c.}.$$ 

(83)

Since this variation does not involve any Green–Schwarz superfields and it is quadratic in the vector multiplet $V$, we obtain the following super gauge transformations

$$\delta \Lambda \hat{W}_\alpha = i \delta \Lambda T_\alpha = -i \delta \Lambda Y_\alpha , \quad \delta \Lambda \hat{V}_5 = \delta \Lambda \mathcal{L} = 2 \delta \Lambda \Omega ,$$

(84)

by requiring that the action $S_{SGS}$ cancels the anomalous variation. Moreover, using the super gauge transformation $\delta \Lambda Y_\alpha$ and $\delta \Lambda \Omega$ of the Chern–Simons superfields, given in (70), we infer that the multiplets $T_\alpha$ and $U$ transform as

$$\delta \Lambda T_\alpha = 2i \Lambda W_\alpha (V) = -\frac{i}{2} D^2 (\Lambda D_\alpha V) , \quad \delta \Lambda U = i (\Lambda - \bar{\Lambda}) V_5 .$$

(85)

These equations constitute the supersymmetric version of the anomalous gauge transformations of the anti–symmetric tensor $B_{MN}$ given in (16). The first equation in (85) contains the transformation of $B_{mn}$ while the second equation contains the transformation of $B_{m5}$. With these gauge transformations it follows that the supersymmetric Green–Schwarz action is super gauge covariant

$$\delta \Lambda S_{SGS} = -\int d^4 x \int d^2 \theta \xi_I \Lambda W^\alpha W_\alpha \delta(y - I) + \text{h.c.} .$$

(86)

and this contribution precisely cancels the anomalous variation [83] of brane and bulk fermions in a supersymmetric theory. We stress that this gauge transformation is independent of the choice of using the supersymmetrized two– or one–form tensor formulation of the Green–Schwarz theory.

Having obtained all the parts which constitute the supersymmetrization of the action [13], we can now proceed as in section [83] to eliminate some of the superfields to obtain the supersymmetric analogy of the Green–Schwarz mechanism formulated using a five dimensional one– or two–form.
4.3 Vector multiplet formulation

As in section 3.3 eliminating $U$ and $T_\alpha$ gives rise to constraint equations which can be solved in terms of a vector multiplet $V$ and a chiral multiplet $S$. Because of the additional Green–Schwarz interactions the solutions (54) and (55) are modified to

$$\hat{W}_\alpha = W_\alpha(V) - iY_\alpha, \quad \hat{V}_5 = V_5 - V_5^2 + 2\Omega - \xi_I V_5 \delta(y - I),$$

(87)

where $S$ is contained in the gauge invariant superfield $V_5$ introduced in (38). The super gauge transformations (84) imply that

$$\delta_\Lambda V = 0, \quad \delta_\Lambda S = \sqrt{2} \sum_I \xi_I \Lambda \delta(y - I).$$

(88)

These variations are compatible with (19) where the four dimensional vector multiplet is invariant, and only the chiral superfield $S$ containing $A_5$ has a singular anomalous gauge transformation.

The supersymmetric one–form description of the Green–Schwarz mechanism can be encoded with these expressions as

$$S_{SGS1} = \int d^5x \left[ \int d^2\theta \frac{1}{4}(W^\alpha W_\alpha + \hat{W}_\alpha \hat{W}_\alpha + Y^\alpha Y_\alpha) + \text{h.c.} + \int d^4\theta (V_5^2 + \hat{V}_5^2 - (V_5^2 - 2\Omega)^2) \right].$$

(89)

The structure of this action is very similar to (72), since it is also written as a sum of squares. In particular it also contains a singular $\delta(0)$ term. To see this explicitly we substitute the relations (87) and rewrite the action as

$$S_{SGS1} = \int d^5x \left[ \int d^2\theta \frac{1}{4}(W^\alpha W_\alpha + \hat{W}_\alpha \hat{W}_\alpha) + \text{h.c.} + \int d^4\theta (V_5^2 + \hat{V}_5^2) \right]$$

$$+ \int d^5x \left[ \int d^2\theta - \frac{i}{2} Y^\alpha W_\alpha + \text{h.c.} + \int d^4\theta 2V_5 (2\Omega - V_5^2) \right]$$

$$+ \frac{1}{2} \xi_I \int d^4x \int d^4\theta \left[ - 2V(V_5 + 2\Omega - V_5^2) + \xi_I \delta(0)V_5^2 \right]_I.$$

(90)

The second line of this equation is identical to (72) and consequently represents a supersymmetric mixed–Chern–Simons term. The bulk actions of the vector multiplets, described by the first two lines of (90), can be obtained from the prepotential

$$P_{SGS1} = \frac{1}{2} \phi^2 + \frac{1}{2} \psi^2 - \varphi \phi^2,$$

(91)

using (13). Hence, just like in the non–supersymmetric version of the Green–Schwarz theory in the vector multiplet formulation (see below (20)), this supersymmetric action only transforms in the required way, (86), if the multiplet $V$ vanishes at the boundaries.
The last line of (90) reads
\[
\frac{1}{2} \xi_I \int d^4x \int d^4\theta \left[ -2V(\mathcal{V}_5 + 2\Omega - V_5^2) + \xi_I \delta(0)V^2 \right]_I \left. \right|_I \\
\left. \right|_I + \xi_I \int d^5x \left( -D(\varphi - \phi^2) + A^m(\mathcal{F}_{m5} - 2\phi F_{m5}) - \frac{1}{2} \xi_I \delta(0)A_m A^m \right) \delta(y - I). (92)
\]

The component form of the singular term involving the \(\delta(0)\) is exact; no fermionic terms were dropped. Comparing to the bosonic Green–Schwarz action in the vector formulation (20), we see that this singular term and the \(A^m F_{m5}\) term are correctly reproduced. Therefore the analysis of the interpretation of the \(\delta(0)\) boundary mass seems identical to the bosonic discussion in subsection 2.4. Moreover, one can show that the mass spectrum is supersymmetric. By eliminating the auxiliary \(D\) field one obtains mass terms for \(\varphi\) that lead to the same recursive mass eigenvector equation as (29). In addition, we have performed an investigation of the boundary Dirac mass term resulting from the term \(-2V\mathcal{V}_5\) in the last line of (90) in Wess–Zumino gauge, and confirmed that the mass spectrum is identical to that of the bosons derived in subsection 2.4. The calculation of the fermion mass spectrum is quantitatively similar to that obtained in a different context for boundary gravitino masses [55]. Since in the supersymmetric context we are restricting ourselves to the bosonic reduction, we do not report this calculation explicitly here.

Notice that the superspace term \(V \Omega\) does not give a contribution as it is a total derivative in superspace [50]
\[
V \Omega(V, V') = -\frac{1}{8} D^\alpha (V^2 W_\alpha(V')) + \text{h.c.}, \quad (93)
\]
for any two vector multiplets \(V\) and \(V'\). This is the supersymmetric version of the statement below (20) that \(\omega_3(A) X_2(A)\) drops out of the action.

However, if we have multiple vector multiplets, we encounter combinations like \(V' \Omega(V, V')\) as well. These are not total derivatives in superspace and give rise to generalized Chern–Simons interactions as discussed in Appendix B.

According to (43) the kinetic terms of the vector multiplets are determined by the field metric
\[
\mathcal{P}_{ab} = \begin{pmatrix} 1 - 2\varphi & -2\phi \\ -2\phi & 1 \end{pmatrix}, \quad \det \left( \mathcal{P}_{ab} \right) = 1 - 2\varphi - 4\phi^2, \quad (94)
\]
derived from the prepotential [91] (dropping the subscript). The eigenvalues of the field metric are \(\lambda_+ = 1 - \varphi \pm (\varphi^2 + 4\phi^2)^{1/2}\). Notice that the eigenvalue \(\lambda_+\) is positive; in fact \(\lambda_+ \geq 1\). The other eigenvalue \(\lambda_-\) changes sign where the Hessian \(\det \left( \mathcal{P}_{ab} \right)\) vanishes. Since some kinetic terms vanish there, the theory develops a Landau pole and near this region the anomalous \(U(1)_A\) gauge theory is strongly coupled. This signals that the effective field theory description probably breaks down.

### 4.4 Tensor multiplet formulation

The tensor formulation is obtained by eliminating the superfields \(\hat{\mathcal{W}}_\alpha\) and \(\hat{\mathcal{V}}_5\) with their algebraic equations of motion. In fact the expressions for these superfields are identical to the ones given
in (57) of section 3.3. Therefore the tensor multiplet form of the Green–Schwarz action is given by

\[
S_{SGS2} = \int d^5x \left[ \int d^2\theta \left( \frac{1}{4} W^\alpha W_\alpha + \frac{1}{4} (T_5^\alpha + Y^\alpha) (T_5_\alpha + Y_\alpha) + \frac{i}{2} \xi_i T^\alpha W_\alpha \delta(y - I) \right) + \text{h.c.} \right. \\
+ \left. \int d^4\theta \left( V_5^2 - (\mathcal{L} - 2\Omega + V_5^2)^2 \right) \right] 
\]

(95)

\[
\supset \int d^5x \left[ -\frac{1}{12} (H_{MNP} + \omega_{MNP})(H^{MNP} + \omega^{MNP}) - \frac{1}{6} \epsilon^{MNPQR} \phi (H_{MNP} + \omega_{MNP}) F_{QR} \\
- \frac{1}{2} \partial_M \sigma \partial^M \sigma - \left( \frac{1}{2} - \sigma - 3\phi^2 \right) \left( \frac{1}{2} F_{MN} F^{MN} + \partial_M \phi \partial^M \phi - (D + \partial_5 \phi)^2 - 2\bar{F}F \right) \\
+ \frac{1}{2} (D_T + A_5 D - \partial_m \phi A^m)^2 + \xi_i \left( \frac{1}{4} \epsilon^{mnlr} B_{mn} F_{kl} - \sigma D \right) \delta(y - I) \right]. 
\]

(96)

Notice that a delta function only appears in the middle term of (95). As in the non–supersymmetric case we see that no \( \delta(0) \) terms appear on the boundaries in the tensor multiplet formulation of the Green–Schwarz theory.

We have also given the complete Lorentz invariant bosonic part of the Lagrangian by combining our earlier results (51), (79), (80) and (82). Notice that the equation of motion of the auxiliary field \( D_T \) is modified, but since it still appears as a complete square it can eliminated without leaving any trace. This is not the case anymore for the auxiliary field \( D \) because of the presence of the last term in the bosonic reduction (96). We will discuss the consequences of this in section 5.

Figure 1: This plot indicates the regions where the Hessian \( \det (P_{ab}) \) and the eigenvalue \( \lambda_- \) are positive and negative. At the parabola \( \varphi(\phi) = \frac{1}{2} - 2\phi^2 \) they vanish and a Landau pole arises.
4.5 Supersymmetric duality relations

By combining (57) and (87) for \( \hat{W}_\alpha \) and \( \hat{V}_5 \) the supersymmetric generalizations of the duality relations (23) are found to be

\[
\mathcal{L} = \frac{i}{4} (D^\alpha \mathcal{T}_\alpha - D_\alpha \mathcal{T}^\alpha) = \hat{V}_5 - V_5^2 + 2\Omega - \xi_I V \delta(y - I)
\]

where we have dropped all fermionic contributions for simplicity. The latter two relations are the equation of motions of the auxiliary fields \( \mathcal{D} \) and \( \mathcal{D}_T \) in the vector and tensor formulation of the supersymmetric Green–Schwarz mechanism.

By taking the appropriate components of these superfield expressions, one retrieves the one/two–form duality. In particular, the bosonic component of the first equation in (97) gives rise to three relations

\[
\begin{align*}
\mathcal{H}_{mn5} + \omega_{mn5} &= \frac{1}{2} \epsilon_{mnkl} (\mathcal{T}^{kl} - 2\phi F^{kl}) , \\
\mathcal{D} &= -\partial_5 \sigma + 2\phi D , \\
\mathcal{D}_T &= -A_5 D + \partial_m \phi A^m ,
\end{align*}
\]

where the first equation in (98) is a part of the (4+1) dimensional decomposition of the duality between the five dimensional tensor \( B_{MN} \) and vector \( A_M \). The other part of this decomposition

\[
\frac{1}{6} \epsilon_{pnmk} (\mathcal{H}^{mnk} + \omega^{mnk}) = \mathcal{T}_{p5} - 2\phi F_{p5} - \xi_I A_\mu \delta(y - I)
\]

is obtained by taking the vector \((\frac{1}{2} [D_\beta, \bar{D}_\dot{\beta}])\) component of the second equation in (97). These two duality relations in (4+1) dimensional notation can be combined in form notation to give

\[
- \ast dB_2 = dA_1 + \ast \omega_3 - X_2(A) - 2\phi dA_1 .
\]

Notice that compared to the duality relation (24) of the non–supersymmetric Green–Schwarz mechanism discussed in section 2.3, the supersymmetrization has introduced a new term \( -2\phi dA_1 \). In addition to this bosonic term there are various fermionic modifications to this duality relation that we have dropped in our bosonic reduction. This shows that even in the bosonic reduction supersymmetry leads to a modification of the duality between the vector and tensor description of the Green–Schwarz action.

It is instructive to determine the content of the other bosonic components of the second expression in (97). The lowest and the \( D^2 \)–components give rise to

\[
\sigma = \phi - \phi^2 = \frac{\partial \mathcal{S}_{SGS1}}{\partial \phi} , \quad \mathcal{T} = 2\phi F .
\]

The expression for \( \sigma \) reproduces the duality relation between scalar components of the tensor and vector multiplet formulation in five dimensions [30]. It is an exact relation: No fermionic terms have been dropped. The second relation is the algebraic equation of the auxiliary field \( \mathcal{T} \), as can be seen by combining the actions (11) and (73). Finally the \( D \)–term component of the second relation in (97) can be cast in the form

\[
\partial_M \partial^M \sigma + \frac{1}{2} F_{MN} F^{MN} + \partial M \phi \partial^M \phi - (D + \partial_5 \phi)^2 - 2\tilde{F} F - \xi_I D \delta(y - I) = 0 ,
\]
Table 1: The even and odd superfields under the $\mathbb{Z}_2$ orbifold action.

| Even | $\Phi_+$ | $V$ | $S$ |
|------|----------|-----|-----|
| Odd  | $\Phi_-$ | $S$ | $V$ |

using the auxiliary field equation (98) for $D$. As can be seen from (95) this is the equation of motion of the scalar field $\sigma$. The fact that the supersymmetric duality relations encode various equations of motion is not surprising in the light of the observation below simplest realization of the vector/tensor duality (11), that this duality contains the equations of motion for $B_2$ and $A_1$.

At the end of subsection 4.3 we commented on the conditions for the appearance of Landau poles in the vector formulation. As a Landau pole constitutes a physical effect, it should be independent on which formulation one chooses to work in. Indeed, also in the tensor multiplet formulation we encounter the possibility of Landau poles. If in (96) the field combination $1 - 2\sigma - 6\phi^2$ vanishes, the kinetic terms of the vector multiplet disappear. Upon using the scalar duality relation (first equation in (101)) we see that this happens precisely when the Hessian $\det (P_{ab})$, given in (94), is zero. Observe that in both the vector and the tensor formulation the anomalous $U(1)_A$ vector multiplet has kinetic mixing with the Green–Schwarz fields.

5 Phenomenological aspects

In this section we investigate some simple consequences of supersymmetric five dimensional theories with an anomalous matter spectrum. As before we consider the theory on an interval or equivalently the orbifold $S^1/\mathbb{Z}_2$. The matter may be either bulk hypermultiplets or chiral multiplets on the boundaries. All matter is only charged under the anomalous $U(1)_A$, but does not couple directly to the Green–Schwarz superfield, irrespectively of whether it is described by the vector or tensor multiplet formulation. The $\mathbb{Z}_2$ parities of the various multiplets that determine the orbifold boundary conditions are given in Table 1. To fix our notation we first briefly recall how charged bulk hyper and brane chiral multiplets are described using $\mathcal{N}=1$ superfields. This will enable us to investigate vacuum solutions of anomalous theories using both the vector and tensor multiplet formulation of the Green–Schwarz theory developed in section 4. In particular we will focus on the role of the duality relations found in subsection 4.5.

Bulk hypermultiplets with charge matrix $q_b$ can be described by the chiral $\mathcal{N}=1$ multiplets $\Phi_+$ and $\Phi_-$, where the subscript refers both to the $\mathbb{Z}_2$ parity of the multiplets and the sign of
their charges. The charged hypermultiplet action reads

\[ S_{\text{hyper}} = \int d^5 x \left[ \int d^2 \theta \phi - (\partial_5 + \sqrt{2} q_b S) \Phi_+ + \text{h.c.} + \int d^4 \theta \Phi \pm e^{\pm 2i q_b V} \Phi_\pm \right] \]  

(103)

\[ \supset \int d^5 x \left[ - [\partial_5 \phi_\pm \pm i q_b A_\mu \phi_\pm] - [q_b^2 \phi_\pm \pm \phi_\pm + \phi_\pm] (D + \partial_5 \phi) \right. \\
\left. + \sqrt{2} (\phi_\pm q_b \varphi F + \text{h.c.}) + |\bar{F}_\pm| \pm (\partial_5 \pm q_b (\phi \pm i A_5)) |\phi_\pm|^2 \right], \]  

(104)

where summation over both chiral multiplets is implied. For the chiral matter \( \Phi_I \) on the boundaries one has instead

\[ S_{\text{chiral}} = \int d^5 x \int d^4 \theta \left[ \bar{\Phi}_I e^{2i q_5 V} \Phi_I \right] \delta(y - I) \]  

(105)

\[ \supset \int d^5 x \left[ - |\partial_5 \varphi_I + i q_I A_\mu \varphi_I| + \varphi_I q_I \varphi_I D \right] \delta(y - I). \]  

(106)

Finally brane localized Fayet–Iliopoulos terms are given by

\[ S_{FI} = \int d^5 x \int d^4 \theta \xi_I \kappa V \delta(y - I) = \int d^5 x \xi_I \kappa D \delta(y - I), \]  

(107)

where we have introduced a constant \( \kappa \) that is proportional to the cut–off scale if one assumes that these Fayet–Iliopoulos terms are generated at one–loop, and the coefficients \( \xi_I \) are given by \( [4] \). (We will ignore subtle issues that the \( D \)–terms induced by bulk loops extend into the bulk \([6, 56, 9]\).)

## 5.1 Vacuum structure of the Green–Schwarz theory

In the vector multiplet formulation of the Green–Schwarz theory we find the following equations for the auxiliary fields

\[ (1 - 2 \varphi)(D + \partial_5 \phi) - 2 \phi(D + \partial_5 \varphi) \pm \varphi \pm q_b \varphi \pm (\xi_I (\kappa - \varphi + \phi^2) + \varphi I q_I \varphi_I) \delta(y - I) = 0, \]

\[ D + \partial_5 \varphi - 2 \phi(D + \partial_5 \phi) = 0, \quad \mathcal{F} - 2 \phi F = 0, \quad (1 - 2 \phi) \bar{F} - 2 \phi \bar{F} + \sqrt{2} \phi_- q_b \varphi_+ = 0. \]  

(108)

In order for the background to preserve supersymmetry the auxiliary fields should have zero VEVs. In particular, from the first equation on the second line we see that this implies that \( \langle \varphi \rangle - \langle \phi \rangle^2 \) is constant.

The tensor formulation has only two auxiliary fields \( (D \) and \( F) \) relevant for the determination of the background, since the remaining auxiliary field \( \mathcal{D}_F \) is expressed in terms of gauge fields (see \([8]\)) that do not acquire VEVs. The \( D \) and \( F \) auxiliary field equations

\[ (1 - 2 \sigma - 6 \phi^2)(D + \partial_5 \phi) \pm \varphi \pm q_b \varphi \pm (\xi_I (\kappa - \sigma) + \varphi_I q_I \varphi_I) \delta(y - I) = 0, \]

\[ (1 - 2 \sigma - 6 \phi^2) \bar{F} + \sqrt{2} \varphi_+ q_b \varphi_+ = 0, \]  

(109)

are equivalent to combinations of the relations in \([10, 8]\). Indeed, by eliminating \( \mathcal{D} \) and \( F \) from those equations and using the identification \( \sigma = \varphi - \phi^2 \) one obtains \([10, 9]\). However, contrary
to the auxiliary field equations in the vector formulation, the auxiliary field equations in the
tensor formulation do not imply that $\langle \sigma \rangle$ is constant. In the tensor formulation this conclusion is
instead reached by considering the equation of motion (102) for a supersymmetric background.

Let us consider two simple of examples of matter configurations to illustrate some conse-
quences of the supersymmetrized Green–Schwarz theory in five dimensions. In the first example
we take a single chiral multiplet $\Phi_0$ with charge $+1$ at the boundary $y = 0$. The local cubic
sum of charges thus equals $\xi_0 = 1/(24\pi^2)$ and $\xi_\pi = 0$. Notice that the mixed U(1)–gravitational
anomaly is automatically cancelled since this charge assignment trivially satisfies the uni-
versality relation. Assuming that $\langle \sigma \rangle = \langle \phi \rangle - \langle \varphi \rangle^2$ is constant, we find that the first equation of
(109) (or the equivalent equations in (108)) leads to the conditions

$$\xi_0(\kappa - \langle \sigma \rangle) + \langle \varphi_0 \rangle^2 = 0, \quad (1 - 2\langle \sigma \rangle - 2\langle \phi \rangle^2)(\langle \phi \rangle = 0\). \quad (110)$$

We see that any value of the Fayet–Iliopoulos parameter $\kappa$ and the brane field VEV $\langle \varphi_0 \rangle$ can be
compensated by an appropriate VEV of $\langle \sigma \rangle$. The second equation implies that $\langle \phi \rangle = 0$ is always
the only solution. For $1 - 2\langle \sigma \rangle \geq 0$ two additional solutions arise with $\langle \phi \rangle = \sqrt{(1 - 2\langle \sigma \rangle)/2\epsilon(y)}$. This is because $\phi$ is odd and the solution will have to be proportional to the step function $\epsilon(y)$, defined by $\partial_y \epsilon(y) = 2(\delta(y) - \delta(y - \pi))$.

In the second example we consider a model with one bulk hypermultiplet $(\Phi_+, \Phi_-)$ of
charge $+3$, and single brane chiral multiplets on both boundaries with equal charge $-1$. For
the cubic sum of local charges we find $\xi \equiv \xi_0 = \xi_\pi = 25/(2 \cdot 24\pi^2)$. Again these charges satisfy
the universality relation and the mixed U(1)-gravitational anomaly is cancelled. In this case the
auxiliary field equations, (108) or (109), can be solved by

$$|\langle \varphi_+ \rangle|^2 = |\langle \varphi_- \rangle|^2 = 0, \quad |\langle \varphi_0 \rangle|^2 + |\langle \varphi_\pi \rangle|^2 = 2\xi(\kappa - \langle \sigma \rangle), \quad (1 - 2\langle \sigma \rangle - 2\langle \phi \rangle^2)(\langle \phi \rangle = \frac{1}{4}(|\langle \varphi_0 \rangle|^2 - |\langle \varphi_\pi \rangle|^2)\epsilon(y). \quad (111)$$

Notice that the VEV $\langle \sigma \rangle$ is determined by sum of the brane VEVs $|\langle \varphi_0 \rangle|^2$ and $|\langle \varphi_\pi \rangle|^2$, while
the amplitude of the jump of $\langle \phi \rangle$ is set by their difference.

In Ref. [57] bulk profiles for the field (that we call $\phi$) have been investigated when the
overall sum of Fayet–Iliopoulos terms is non–zero, i.e. $\xi_0 + \xi_\pi \neq 0$, but under the assumption
that a similar analysis can be performed as for the case in which the sum of charges vanishes
globally. In particular, as emphasized by these authors, the field(s) responsible for cancelling
the global anomaly were not specified. Our starting point was precisely to see how the local
and global anomalies can be cancelled via the five dimensional supersymmetric version of the
Green–Schwarz theory. As the above results show for the structure of supersymmetric vacua,
an additional scalar $\varphi$ (or $\sigma$) is introduced, and the auxiliary field equation for $\phi$ is cubic rather
than linear in $\phi$. However, qualitatively the analysis in Ref. [57] seems to be confirmed by our
investigation since this cubic equation also leads to non–trivial profiles for $\langle \phi \rangle$.

5.2 Possible MSSM applications

The vacuum expectation values for the scalar field $\phi$ clearly depend on the particular model
under consideration, and can play a crucial role in generating various hierarchies for super-
symmetry breaking and fermion masses in the MSSM. To briefly illustrate these possibilities
consider a model with one bulk hypermultiplet \((\Phi_+, \Phi_-)\) of charge +2, and one brane chiral multiplet \(\Phi_0\) at \(y = 0\), of charge −1 under the anomalous gauge symmetry. As shown in the previous subsection the scalar field \(\varphi_0\) will obtain a VEV, and \(\phi\) will satisfy a cubic equation leading to a solution with a nontrivial profile. Next suppose that in the MSSM all fields live on the \(y = 0\) boundary, in addition to the brane chiral field \(\Phi_0\). The Yukawa couplings must respect the (anomalous) gauge \(U(1)_A\) symmetry and are generically given by higher dimensional operators \([58]\). Besides the field \(\Phi_0\) with \(U(1)_A\) charge −1, we denote the \(U(1)_A\) charges of the superfields \((Q_i, U_j, H_2)\) by small letters \((q_i, u_j, h_2)\). These charges must be chosen such that the universality relation is satisfied and the mixed \(U(1)\) non–Abelian anomalies are cancelled. The Yukawa interactions are then described by the superpotential

\[
W = \lambda_{ij} \left( \frac{\Phi_0}{M} \right)^{q_i + u_j + h_2} Q_i U_j H_2 ,
\]

(112)

where \(M\) denotes the fundamental scale, \(Q_i\) denotes the superfield containing the left–handed \(SU(2)_L\) doublet quarks of the \(i\)th generation, \(U_j\) contains the right–handed \(SU(2)_L\) singlet up–quarks of the \(j\)th generation, and \(H_2\) contains the Higgs doublet responsible for up–type masses in the MSSM. Thus, when \(\varphi_0\) obtains a VEV, the superpotential term (112) generates Yukawa couplings for the fermions, and \(\varphi_0\) plays the role of the Froggatt–Nielsen scalar field.

Supersymmetry must also be spontaneously broken and in four dimensions this occurs by adding the gauge invariant superpotential term \(\lambda \Phi_0 \Phi_0 \Phi_+\). However, the supersymmetry breaking scale is determined by the scale of the FI term, which is typically (at least in heterotic constructions) much larger than the TeV scale. Instead, from a five dimensional perspective, supersymmetry can be broken by adding the gauge invariant superpotential term \(\lambda \Phi_0 \Phi_0 \Phi_+ (y = 0)\) on the \(y = 0\) boundary. It is indeed straightforward (but more tedious than the four dimensional case) to check that the five dimensional F and D flatness conditions \(F_0 = F_+ = F_- = D = \bar{D} = 0\) have no solution in this model. However, compared to four dimensions, the supersymmetry breaking scale will be suppressed by the volume of the compact dimension via the wave function of the bulk field \(\Phi_+\), using the mechanism proposed in a different context in \([59, 60]\). Thus, unlike in four dimensions, the FI term in this five dimensional model can induce supersymmetry breaking at the TeV scale.

If instead the charged matter fields live on different boundaries then fermion mass hierarchies can be generated in a different way. For example, if \(\Phi_0\) (\(\Phi_\pi\)) are charged matter boundary fields at \(y = 0\) (\(y = \pi\)) with \(U(1)_A\) charges −1 (+2), then the gauge invariant superpotential term that breaks supersymmetry will involve the Wilson line operator

\[
W_{\text{susy–breaking}} = \lambda \Phi_0 \Phi_0 e^{2\sqrt{2} \int_0^{\pi R} dy S(y)} \Phi_\pi .
\]

(113)

The Wilson line operator can induce a large hierarchy between the fundamental scale and the supersymmetry breaking scale \([35]\). In this case the classical field equations determining \(\text{Re} S = (1/\sqrt{2})\phi\) have a solution similar to (111). All quark and charged lepton fields should live on the \(y = 0\) boundary in order to avoid a (too large) suppression in the fermion masses arising from the Wilson line operator. On the other hand, if neutrino singlets \(N_i\) live on the \(y = \pi\) boundary, then the extra suppression from the Wilson line operator allows a nice way of getting very small Dirac neutrino masses. The relevant neutrino mass operator can be written as

\[
\lambda_{ij} \left( \frac{\Phi_0}{M} \right)^{l_i + n_j + h_2} e^{\sqrt{2} \int_0^{\pi R} dy S(y)} L_i N_j H_2 .
\]

(114)
Together with (112) the superpotential term (113) predicts a relation between the neutrino mass scale and the supersymmetry breaking scale. A similar mechanism in four dimensions was proposed in [61, 62] by using a Kahler potential operator and the Giudice–Masiero mechanism. In contrast, in our case this relation is generated by the superpotential via a large suppression occurring from the geometric separation between the various charged fields that produces the Wilson line operator.

6 Conclusions

In five dimensions the Green–Schwarz mechanism can be used to cancel $U(1)_A$ gauge anomalies. This mechanism relies on introducing a bulk field whose gauge variation is responsible for anomaly cancellation on the boundaries. In five dimensions this field can either be described by a rank two tensor field $B_2$ or a vector field $A_1$ that are dual to each other. This duality has been extended to the Green–Schwarz action where the anomalous $U(1)_A$ gauge field $A_1$ interacts with either the (non–anomalous) vector field $A_1$ or the tensor field $B_2$. In the vector formulation the Green–Schwarz action contains singular boundary mass terms for the anomalous gauge field $A_1$, which are absent in the tensor formulation. However, we showed that both formulations do give the same mass spectrum for the anomalous $U(1)_A$ gauge field. A general characteristic of this spectrum is that the lowest lying mode is massive when the global sum of charges is different from zero. In the vector formulation this involved a cancellation of the $\delta(0)$ boundary mass terms by a mixing between the anomalous and Green–Schwarz gauge fields. In the tensor formulation there are no (singular) boundary mass terms, and the Kaluza–Klein masses arise from a boundary mixing term between the Green–Schwarz tensor field and the anomalous gauge field. In this sense the tensor formulation seems better defined because it is free of any singular terms at the Lagrangian level.

A major part of our work has been to present the complete supersymmetric extension of the five dimensional Green–Schwarz mechanism using the four dimensional $N=1$ superspace. Each term in the Green–Schwarz action has been supersymmetrized and written in a manifestly $N=1$ supersymmetric form. The components of the five dimensional gauge field $A_1$ become part of a vector multiplet $V$ and a chiral multiplet $S$. From these superfields, two super gauge invariant superfields $W_a(V)$ and $V_5$ are obtained. In the dual formulation the supersymmetrization of the five dimensional tensor field $B_2$ is described by a chiral multiplet $T_\alpha$ and a vector multiplet $U$. Also in this dual formulation we have constructed two super gauge invariant combinations $T_{5a}$ and $L$ since, contrary to the four dimensional case, the supersymmetric interactions of $B_{mn}$ cannot be encoded by the linear multiplet $L$ only. In addition the supersymmetric Chern–Simons three–form interactions required the introduction of a new chiral Chern–Simons superfield $Y_\alpha$ besides the Chern–Simons superfield $\Omega$ that would only be required in a four dimensional theory.

We have also supersymmetrised the vector/tensor duality and obtained the duality relations between the supermultiplets. The usual vector/tensor duality is now modified to include scalar degrees of freedom, aside from the additional modifications due to fermionic terms. Furthermore, an interesting duality relation between the scalar components of the two formulations is obtained. If these scalar components receive VEVs then they can lead to Landau poles for the gauge field coupling. In both formulations we showed that the Landau poles occur on the same
curve in moduli space. In addition we have seen that the duality also plays a role in the equations that determine the supersymmetric vacua of the theory. In the vector multiplet formulation we encounter the same singular boundary mass terms as in the non-supersymmetric case. Hence the Kaluza–Klein mass spectrum for the anomalous $U(1)_A$ gauge field in the vector and tensor formulation is identical to the one obtained in the non-supersymmetric theory. Moreover, both formulations lead to a finite boundary Dirac mass terms for the gauginos. Thus, four dimensional supersymmetry is preserved because the resulting fermionic Kaluza–Klein mass spectrum is identical to that of the bosons.

Our analysis can be easily generalized to multiple anomalous $U(1)_A$ gauge groups. With this generalization more general Chern–Simons terms are allowed that are not present in the single $U(1)_A$ case. Anomalous $U(1)_A$ symmetries have been extensively used for phenomenological purposes in four dimensional models, such as for the fermion mass hierarchy and supersymmetry breaking. In five dimensions these models can be further enhanced by allowing supersymmetry breaking to occur at the TeV scale, and to obtain extremely suppressed Yukawa couplings for Dirac neutrino masses. These phenomenological aspects remain to be investigated further in detail. We expect the formalism developed in this paper to be useful as a starting point for further theoretical studies of the Green–Schwarz mechanism in higher dimensions. For example, generalizations to mixed $U(1)_A$ non–Abelian and mixed gravitational anomalies would be an interesting extension of our analysis. We would also expect that six or more dimensions might possibly be a more natural setting for the supersymmetric duality relations derived in this paper.

Another avenue would be to investigate the consequences of the Green–Schwarz interaction that we found in this paper for one loop computations, in particular the Fayet–Iliopoulos tadpoles. These are some of the interesting questions that remain to be studied.

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**A Deriving the defining equations for the Chern–Simons superfields**

To obtain the superfield definitions of the Chern–Simons multiplets we will use a supersymmetric version of a Lagrange multiplier method. First, using a Lagrange multiplier one–form $\tilde{A}_1$ the defining relation of the Chern–Simons three–form, $d\omega_3 = F_2^2$, can be obtained from the
action

\[ S = \int \tilde{A}_1 (d\omega_3 - F_3^2) = \int (\tilde{F}_2 \omega_3 - \tilde{A}_1 F_2^2) \]  

(A.1)

The (4+1) dimensional decomposition of this action, given by

\[ S = \int d^5 x \epsilon^{mpq} \left( \frac{1}{4} \tilde{F}_{mn} \omega_{pq} - \frac{1}{6} \tilde{F}_{m5} \omega_{npq} - \frac{1}{4} \tilde{A}_5 F_{mn} F_{pq} - \tilde{A}_m F_{np} F_{q5} \right) , \]  

(A.2)

can then be supersymmetrized in a straightforward manner. Employing various definitions given in the main text to identify the superfield terms, the supersymmetrized version of (A.2) becomes

\[ S = \int d^5 x \left[ \int d^2 \theta \left( i \sqrt{2} \tilde{W}^\alpha Y_\alpha - \sqrt{2} \tilde{S} W^\alpha W_\alpha \right) + \text{h.c.} - \int d^4 \theta \left( 4 \tilde{V}_5 \Omega + 8 \tilde{V} \Omega(V_5, V) \right) \right] . \]  

(A.3)

After a partial integration in superspace, and using \( \int d^2 \bar{\theta} = -\frac{1}{2} \), under a spacetime integral the action can be rewritten as

\[ S = \int d^5 x \left[ \int d^2 \theta \left( i \sqrt{2} \tilde{S}(\tilde{D}^2 \Omega - W^\alpha W_\alpha) + \text{h.c.} \right) \right. \]
\[ + \left. \int d^4 \theta \tilde{V} \left( \frac{i}{2} \tilde{D}_\alpha \bar{Y}^\alpha - \frac{i}{2} \tilde{D}^\alpha \tilde{Y}_\alpha - 4 \bar{\theta}_5 \Omega - 8 \Omega(V_5, V) \right) \right] . \]  

(A.4)

From this equation the defining properties, (64) and (67), of the Chern–Simons multiplets, \( \Omega \) and \( Y_\alpha \), respectively, can be read off directly.

Furthermore it is straightforward to show that \( \Omega = \Omega(V, V) \) is a solution of (64). On the other hand to prove that \( Y_\alpha \), given in (68), solves (67) it is useful to employ the following identities. For two vector superfields \( V \) and \( V' \) we have

\[ \frac{1}{2} D^\alpha \tilde{D}^2 (V D_\alpha V' - V' D_\alpha V) + \text{h.c.} = 16 (\Omega(V, V') - \Omega(V', V)) \]  

(A.5)

where we have written the identity in terms of the definition (65) of \( \Omega(V, V') \). Similarly, using the definition (61) of \( V_5 \) it is not to difficult to show that

\[ -\partial_5 \Omega(V, V) = \Omega(V, V_5) + \Omega(V_5, V) + \frac{1}{4 \sqrt{2}} \left( D^\alpha (S W_\alpha) + \tilde{D}_\alpha (\tilde{S} \tilde{W}^\alpha) \right) . \]  

(A.6)

B Generalization to multiple U(1) gauge groups

The effective Lagrangians arising from Type I and Type II string theories generically involve several fields in a generalized form of the Green–Schwarz mechanism [63, 64, 65]. In order to describe these effective theories it is useful to generalize our results to multiple U(1) vector multiplets. We will assume multiple anomalous vector multiplets \( (V^\alpha, S^\alpha) \), and non–anomalous (Green–Schwarz) vector multiplets \( (V^i, S^i) \). Their \( \mathbb{Z}_2 \) parities are the same as the once given in table I. Under the \( \mathbb{Z}_2 \) symmetry the prepotential of the theory must be even. The form of the prepotential arising from generic string theories is

\[ P = \frac{1}{2} \phi^i \varphi_i - \epsilon_{ab} \phi^a \phi^b , \]  

(B.1)
where we sum over all indices that appear as upper and lower indices. The complex scalars are defined by
\[ S^a = \frac{1}{\sqrt{2}} (\phi^a + i A_5^a) , \quad S^i = \frac{1}{\sqrt{2}} (\varphi^i + i A_5^i) . \] (B.2)

In order to work out the details of the interactions, the Chern–Simons multiplets introduced in equations (65) and (68) are generalized to
\[ \tilde{\Omega}^i = c^i_{ab} \left( \Omega(V^a, V^b) - \frac{1}{2} V_5^a V_5^b \right) , \quad Y^i = \frac{i}{4} c^i_{ab} D^2 (V_5^a D_5^b - D_5^a V_5^b + \sqrt{2} S^a D_5^b V_5^b) . \] (B.3)

For notational convenience we have absorbed the \( V_5^a V_5^b \) expression in the definition of \( \tilde{\Omega}^i \). Then the initial starting point of the generalized construction is the Lagrangian
\[ S_{SGS} = S_{STV} + S_{Bianchi} + S_{bdy} + S_{\omega^2} , \] (B.4)

where the various parts of the action are given by
\[ S_{STV} = \int d^5 x \left[ \int d^2 \theta \left( \frac{1}{4} \tilde{W}^i \tilde{W}_i - \frac{i}{2} \overline{\tau}_\alpha \tilde{W}_i + \frac{i}{2} \tau_\alpha \tilde{W}_i (\tilde{V}_5) \right) + h.c. + \int d^4 \theta \tilde{V}_i \tilde{V}_5 \right] , \] (B.5)
\[ S_{Bianchi} = \int d^5 x \left[ \int d^2 \theta \frac{1}{2} \tau_\alpha Y^i + h.c. + \int d^4 \theta 4 \tilde{\Lambda} \tilde{\Omega}^i \right] , \] (B.6)
\[ S_{bdy} = \int d^4 x \left[ \int d^2 \theta \dot{\xi}^i_{a I} \tau_\alpha W_\alpha (V) \right] + h.c. , \] (B.7)
\[ S_{\omega^2} = \int d^5 x \left[ \int d^2 \theta \frac{1}{4} Y^a Y^i + h.c. - \int d^4 \theta 4 \tilde{\Omega}^i \tilde{\Omega}^i \right] . \] (B.8)

Following the procedure used earlier to integrate out non–dynamical fields, we find that in the vector multiplet formulation the generalized Lagrangian becomes
\[ S_{SGS1} = \int d^5 x \left[ \int d^2 \theta \left( \frac{1}{4} W^a \dot{W}_a - \frac{1}{2} Y^a \dot{W}_a \right) + h.c. + \int d^4 \theta (V_5 + 4 \tilde{\Omega}_i) V_5^i \right] 
+ \frac{1}{2} \int d^4 x \left[ \int d^4 \theta \left[ - 2 \xi^i_{a I} V^a (V_5^i + 2 \tilde{\Omega}_i) + \xi^i_{a I} \dot{\xi}^b_{I I} \delta(0) V^a V_b \right] \right] . \] (B.9)

Notice in (B.9) the appearance of generalized Chern–Simons terms whose form and transformations under the gauge variations \( \delta V^a = \Lambda^a + \tilde{\Lambda}^a \) and \( \delta S^a = \sqrt{2} \partial_\theta \Lambda^a \) are given by
\[ S_{CS} = - 4 \dot{\xi}^i_{a I} c_{i a b} \int d^4 \theta \Omega(V^a, V^b) V^c \delta(y - I) , \] (B.10)
\[ \delta S_{CS} = \xi^i_{a I} c_{i a b} \int d^2 \theta (\Lambda^c W^a V^b - \Lambda^a W^b V^c) \delta(y - I) + h.c. . \] (B.11)

As we saw in Section 2.3, this term is absent in the single \( U(1)_A \) case because of the identity (B.9). The anomalous transformations of the classical Lagrangian (B.9) in the vector formulation
are given by

\[ \delta V^i = 0, \quad \delta S^i = \sqrt{2} \xi^i_\alpha \Lambda^\alpha \delta(y - I), \]

\[ \delta S_{SGS} = -2 \xi^i_\alpha \xi^a_\alpha \int d^2 \theta \Lambda^a \Lambda^b W^b_\alpha \delta(\nu + \lambda - \mu) + h.c. . \quad (B.12) \]

Interestingly, the final anomalous variation (B.12) of the action \( S_{SGS} \) is obtained by combining the gauge variation (B.11) of the generalized Chern–Simons term \( S_{CS} \) with that of the axionic coupling \( c^i \xi^{ab} S \bar{S} W^a_\alpha W^b_\alpha \) hidden in (B.9).

Alternatively, the Lagrangian in the tensor multiplet formulation is given by

\[ S_{SGS} = \int d^5 x \left[ \int d^2 \theta \left( \frac{1}{4} (T^5 \alpha + Y^\alpha) (T^5 \alpha + Y^\alpha) + \frac{i}{2} \xi^i_\alpha \Lambda^a \Lambda^b W^b_\alpha \delta(y - I) + h.c. \right) \right. \]

\[ \left. - \int d^4 \theta \left( \mathcal{L}^i - 2 \mathcal{\tilde{\Omega}}^i \right) (\mathcal{L}^i - 2 \mathcal{\tilde{\Omega}}^i) \right] . \quad (B.13) \]

Notice again the absence of ill-defined terms in the tensor formulation compared with the vector formulation, leading to a perfectly well defined description. In the tensor formulation the corresponding gauge transformations become

\[ \delta T^i_\alpha = 2 i c^i \xi^a_\alpha \Lambda^a W^b_\alpha, \quad \delta U^i = i c^i \xi^a_\alpha \Lambda^a W^b_\alpha, \]

\[ \delta S_{SGS} = -2 \xi^i_\alpha \xi^a_\alpha \int d^2 \theta \Lambda^a \Lambda^b W^b_\alpha \delta(y - I) + h.c. . \quad (B.14) \]

The dual formulations are equivalent and in particular the two anomalous gauge variations (B.12) and (B.14) match. But we emphasize that in the vector formulation the generalized Chern–Simons term (B.10) played a crucial role in obtaining this result.

Another interesting example where the generalized Chern–Simons terms play a crucial role in the anomaly cancellation is the case consisting of only one \( \mathbb{Z}_2 \) even U(1) vector multiplet and one \( \mathbb{Z}_2 \) odd Green–Schwarz multiplet in the simplest case of no localized anomalies or Fayet–Iliopoulos terms. In this case, due to parity assignments, the boundary gauge variation (B.14) is zero. Performing a dimensional reduction, the four dimensional Kaluza–Klein Lagrangian contains axionic couplings and generalized Chern–Simons terms (due to the five dimensional Chern–Simons terms), where both of these terms contain at least one massive Kaluza–Klein mode. Interestingly enough, the two terms are separately not gauge invariant, but their sum is in agreement with the original five dimensional gauge invariance of the model.

Finally, the supersymmetric duality relations (B.7) in this more general case take the form

\[ i \mathcal{J}^i_\alpha = i \left( \partial_\nu \mathcal{J}^i_\alpha + \mathcal{W}^i_\alpha (\mathcal{U}) \right) = \mathcal{W}^i_\alpha (\mathcal{V}) - i Y^i_\alpha , \]

\[ \mathcal{L}^i = \frac{i}{4} \left( D^a \mathcal{J}^i_\alpha - D^a \mathcal{J}^{\bar{i} \bar{a}} \right) = \mathcal{V}^i_\alpha + 2 \mathcal{\tilde{\Omega}}^i - c^i_\alpha V^a \delta(y - I) . \quad (B.15) \]
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