CHARMLESS $B \rightarrow VP$ DECAYS USING FLAVOR SU(3) SYMMETRY

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The decays of $B$ mesons to a charmless vector ($V$) and pseudoscalar ($P$) meson are analyzed within a framework of flavor SU(3) in which symmetry breaking is taken into account through ratios of decay constants in tree ($T$) amplitudes. The magnitudes and relative phases of tree and penguin amplitudes are extracted from data; the symmetry assumption is tested; and predictions are made for rates and $CP$ asymmetries in as-yet-unseen decay modes. A key assumption for which we perform some tests and suggest others is a relation between penguin amplitudes in which the spectator quark is incorporated into either a pseudoscalar meson or a vector meson. Values of $\gamma$ slightly restricting the range currently allowed by fits to other data are favored, but outside this range there remain acceptable solutions which cannot be excluded solely on the basis of present $B \rightarrow VP$ experiments.

I. INTRODUCTION

$B$ meson decays are a rich source of information on fundamental phases of weak charge-changing couplings, as encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Decays to charmless final states, many of which occur at branching ratios exceeding $10^{-5}$, are particularly useful, since many of them involve more than one significant quark subprocess and thus have the potential for displaying direct $CP$ asymmetries. To interpret such data one must disentangle information on CKM (weak) phases from strong-interaction final-state phases.

In $B \rightarrow PP$ decays, where $P$ is a charmless pseudoscalar meson, flavor SU(3) symmetries have been employed 1,2,3,4,5 to extract weak phases in such decays as $B^0 \rightarrow \pi^+\pi^-$ and various charge states of $B \rightarrow K\pi$ (see, e.g., the recent reviews of 6 and a recent analysis 7 of $B \rightarrow K\pi$). The decays $B \rightarrow VP$, where $V$ is a charmless vector meson, involve more invariant amplitudes, since one cannot use Bose statistics to simplify the decays, in contrast to the case of two spinless final pseudoscalars in the same meson multiplet 1,2,3,4. Nonetheless, after the first report of a charmless $B \rightarrow VP$ decay 8, it became possible to perform such analyses by using rates and $CP$ asymmetries in some decays to predict others 5,9,10,11,12.

In the present paper, following upon our recent analysis of $B \rightarrow PP$ decays 13, we analyze $B \rightarrow VP$ decays within flavor SU(3), incorporating symmetry breaking through ratios of meson decay constants in tree ($T$) amplitudes. The magnitudes and relative phases of invariant amplitudes are extracted from data; the symmetry assumption is tested; and predictions are made for rates and $CP$ asymmetries in as-yet-unseen decay modes.

Our approach differs from ones involving a priori calculations of $B \rightarrow VP$ decay rates and $CP$ asymmetries involving QCD and factorization. Factorization was applied to these decays in Refs. 14,15,16,17. The QCD factorization methods of Refs. 18,19 were considered for $B \rightarrow VP$ decays in Refs. 20,21,22,23,24. Many of these authors were able to fit some data but could not reproduce those processes dominated by strangeness-changing penguin amplitudes, which others have argued should be enhanced 25,26,27,28. Our method, by contrast, relies on assumptions of isospin and SU(3) flavor symmetry, provides tests of these assumptions, and is capable of extracting strong final-state phases from data rather than needing to predict them. It is similar to the analysis of $B \rightarrow \rho^\mp\pi^\mp$ and $B \rightarrow \rho^\mp K^\pm$ in Ref. 22, and of $B \rightarrow K^{\pm\pi^\mp}$ in Ref. 30 (which uses extensive data on $B \rightarrow \rho^\mp\pi^\pm$), but we are concerned with a wider set of $B \rightarrow VP$ decays.

The present analysis has considerable sensitivity to the CKM phase $\gamma$. This is driven in part by the pattern of tree-penguin interference in a wide variety of $B \rightarrow VP$ decays, and in part by the incorporation of time-dependent information on $B \rightarrow \rho\pi$, as has also been noted in Ref. 24.

We review notation and conventions for amplitudes in Section II. We average currently known experimental rates and $CP$ asymmetries from the CLEO, BaBar, and Belle Collaborations and use these averages to obtain magnitudes of amplitudes in Section III. We then show how to extract invariant amplitudes (identified with specific flavor topologies) in Section IV by fitting the experimental amplitudes and $CP$ asymmetries. The simplest fit assumes a relation between penguin amplitudes in which the spectator quark is incorporated into either a pseudoscalar meson or a vector meson. We sug-
Notation

We use the following quark content and phase conventions:

- **Bottom mesons**: \(B^0 = \bar{d}b, \overline{B}^0 = bd, B^+ = ub, B^- = -\bar{b}u, B_s = sb, \overline{B}_s = bs;\)
- **Charmed mesons**: \(D^0 = -\bar{c}u, \overline{D}^0 = uc, D^+ = cd, D^- = -\bar{c}d, D_s = cs, \overline{D}_s = sc;\)
- **Pseudoscalar mesons**: \(\pi^+ = ud, \pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}, \pi^- = -\bar{d}u, K^+ = us, K^0 = d\bar{s}, \overline{K}^0 = \bar{d}\bar{s}, K^- = -\bar{s}u, \eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}, \eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6};\)
- **Vector mesons**: \(\rho^+ = ud, \rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}, \rho^- = -\bar{d}u, \omega = (u\bar{u} + d\bar{d})/\sqrt{2}, K^{*+} = us, K^{*0} = d\bar{s}, \overline{K}^{*0} = \bar{d}\bar{s}, K^{*-} = -\bar{s}u, \phi = s\bar{s}.\)

In the present approximation there are four types of independent amplitudes: a “tree” contribution \(t\); a “color-suppressed” contribution \(c\); a “penguin” contribution \(p\); and a “singlet penguin” contribution \(s\), in which a color-singlet \(q\bar{q}\) pair produced by two or more gluons or by a \(Z\) or \(\gamma\) forms an SU(3) singlet state. We neglect smaller contributions from an “exchange” amplitude \(e\), an “annihilation” amplitude \(a\), and a “penguin annihilation” amplitude \(pa\). The amplitudes we retain contain both the leading-order and electroweak penguin contributions:

\[
\begin{align*}
\Gamma(b \to M_1 M_2) &= \frac{p_v}{8\pi m_B^2} |\langle B \to M_1 M_2 \rangle|^2, \quad (2)
\end{align*}
\]

where \(p_v\) is the momentum of the final state meson in the rest frame of \(B\), \(m_B\) is the \(B\) meson mass, and \(M_1\) and \(M_2\) can be either pseudoscalar or vector mesons. Using Eq. (2), one can extract the magnitude of the invariant amplitude of each decay mode from its experimentally measured branching ratio. To relate partial widths to branching ratios, we use the world-average lifetimes \(\tau^+ = (1.653 \pm 0.014)\ \text{ps}\) and \(\tau^- = (1.534 \pm 0.013)\ \text{ps}\) computed by the LEPS group [34]. Unless otherwise indicated, for each branching ratio quoted we imply the average of a process and its CP-conjugate.

Two phase conventions are in current use for the penguin amplitudes, depending on whether one considers them to be dominated by the CKM factors \(V_{tb}V_{tq}\) \((q = s, d)\), or integrates out the \(t\) quark, uses the unitarity relation \(V_{ub}V_{tq} = -V_{cb}V_{tb} - V_{ub}V_{tq}\), and absorbs the \(V_{ub}V_{tq}\) term into a redefined tree amplitude. Here we adopt the latter convention. For a discussion of the relation between the two see, e.g., Ref. [32]. Thus both the strangeness-changing and strangeness-preserving penguin amplitudes will be taken to have real weak phases in this discussion.

Experimental Data and Amplitude Decompositions

The experimental branching ratios and \(CP\) asymmetries from the CLEO, BaBar, and Belle Collaborations are summarized and averaged in Tables II (for \(\Delta S = 0\) transitions) and III (for \(\Delta S = 1\) transitions). Data are current up to and including the 2003 Lepton-Photon Symposium at Fermilab. We use the Particle Data Group method [30] for performing averages, including a scale factor \(S = [x^2/(N - 1)]^{1/2}\) when the \(x^2\) for an average of \(N\) data points exceeds \(N - 1\). (The Heavy Flavor Averaging Group [37] does not use this scale factor. In other respects our averages agree with theirs when inputs are the same.) The corresponding experimental amplitudes, extracted from partial decay rates using Eq. (2), are shown in Tables III and IV. In these tables we also give the theoretical expressions for these amplitudes (see also Refs. [8], [12]) and, anticipating the results of the next section, the magnitudes of contributions to the observed amplitude of the invariant amplitudes \(|T|\) and \(|T'|\) and \(|P|\) in one fit to the data. These contributions include Clebsch-Gordan coefficients. \(CP\)
asymmetries are defined as

\[ A_{CP}(B \rightarrow f) \equiv \frac{\Gamma(B \rightarrow f) - \Gamma(B \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(B \rightarrow \bar{f})}. \]

By comparing the magnitudes of individual contributions with experimental amplitudes, one can tell whether one contribution dominates or whether constructive or destructive interference between two contributions is favored.

\[ \text{IV. EXTRACTING AMPLITUDES} \]

In the present section we show how a global fit to decay rates and CP asymmetries can determine many (though not all) of the invariant amplitudes governing \( B \rightarrow VP \) decays. We shall be able to determine from experimental data their magnitudes and relative strong phases and the weak phase \( \phi \). We shall assume a universal ratio \( p'_V/p_P = -e^{i0.01} \), initially assuming \( c = 1 \) and \( \phi = 0 \) in accord with Ref. [31], presenting also results with arbitrary \( c \) and \( \phi \). We interpret \( \phi \) as a relative strong phase between \( p'_V \) and \( -p_P \), so that it does not change sign under CP-conjugation. We now explain in some detail the inputs and fit parameters.

We base the present fit on the following processes (see Tables [III] and [IV]):

- The \( B^+ \rightarrow K^{*0}\pi^+ \) amplitude involves \( |p'_P| \) alone. The decay rate provides one data point. No CP asymmetry is expected or seen.

- The decays \( B^0 \rightarrow \rho^-\pi^+ \) and \( \overline{B}^0 \rightarrow \rho^+\pi^- \) (equivalently, their CP-averaged branching ratio and CP asymmetry quoted in Table [I]) involve \( t_V \) and \( p_V \). These processes thus provide two data points.

- The decays \( B^0 \rightarrow \rho^+\pi^- \) and \( \overline{B}^0 \rightarrow \rho^-\pi^+ \) involve \( t_P \) and \( p_P \) and provide two data points.

- In the time-dependent study of \((B^0, \overline{B}^0) \rightarrow \rho^\pm\pi^\mp\), the asymmetry parameters \( S_{+/-} \) and \( S_{-/+} \), to be defined at the end of this section, provide two more data points. (Other time-dependent parameters are related to those already included.)

- The decays \( B^0 \rightarrow \phi K^+ \) and \( \overline{B}^0 \rightarrow \phi K^- \) involve \( t'_V \) and \( p'_V \) and provide two data points, since the CP-averaged decay rate and CP asymmetry have been presented.

- The decays \( B^0 \rightarrow K^{*+}\pi^- \) and \( \overline{B}^0 \rightarrow K^{*-}\pi^+ \) involve \( t'_P \) and \( p'_P \) and similarly provide two data points.

- The decays \( B \rightarrow K^*\eta \) (for both charge states) play an important role in constraining the phase \( \phi' \) of \( -p'_V/p'_P \), since this phase must be small in order that \( p'_P \) and \( p'_V \) contribute constructively to the large decay rate, as anticipated in Ref. [31]. We include two decay rates and two CP asymmetries, adding a total of four data points. Since our scheme predicts a very small CP asymmetry for \( B^0 \rightarrow K^{*0}\eta \), the parameters of the fit will not be affected by this observable.

- The rate and CP asymmetry for \( B^+ \rightarrow \rho^0\pi^+ \) and \( B^+ \rightarrow \omega\pi^+ \) have been measured. The two decay rates are dominated by \( t_V \) but provide some information about the magnitude of the amplitude \( c_P \), about which we shall have more to say below. These processes thus add four more data points to our fits.

- The rate and CP asymmetry for \( B^+ \rightarrow \rho^+\pi^0 \) have been measured, adding two data points.

- The decay rates for \( B \rightarrow \phi K \) (both charge states) have been measured. The corresponding decay widths are expected to be equal. They are measured to be within 7% when one takes into account the difference in lifetimes of the \( B^+ \) and \( B^0 \) mesons. [Note added: The branching ratio \( B(P^0 \rightarrow \phi K^0) \) quoted in Table [IV] has been updated [47]. The central values of the world-averaged decay widths now are exactly equal.] We include the \( B \rightarrow \phi K \) decay rates as two more data points. Since both the amplitudes \( p'_P \) and \( s'_P \) contributing to these processes are expected to have the same weak phase, we predict zero CP asymmetry in any \( B \rightarrow \phi K \) decay. This is certainly true for the charged mode, whose CP asymmetry we include as another data point.

- Taking the average of BaBar and Belle values [64], we find the time-dependent parameters in \( B^0 \rightarrow \phi K_S \) to be \( S_{\phi K_S} = -0.147 \pm 0.697 \) (\( S = 2.11 \)) and \( A_{\phi K_S} = 0.046 \pm 0.256 \) (\( S = 1.08 \)), whereas we predict the standard model values \((\sin 2\beta, 0)\). The average of BaBar and Belle determinations via the subprocess \( \bar{b} \rightarrow ec\bar{s} \) is \( \sin \beta = 0.736 \pm 0.049 \) [64]. The parameter \( A_{\phi K_S} \) is equivalent to the direct CP asymmetry \( A_{CP}(B^0 \rightarrow \phi K_S) \). (The corresponding asymmetry for \( B^+ \rightarrow \phi K^+ \) is seen in Table II to be very small.) These observables thus contribute \( \Delta \chi^2 = (1.61, 0.03) \) to our fit, without affecting the fit parameters. We include them in our \( \chi^2 \) total, adding two more data points. In view of the large \( S \)-factor, contributing to a considerable amplification of the experimental error in \( S_{\phi K_S} \), it is premature to regard the deviation of this observable from its standard model expectation as a signal of new physics. In Ref. [55] we discussed some scenarios which could give rise to such a deviation. One should add a penguin amplitude (e.g., for \( \bar{b} \rightarrow s\bar{s}s \)) with arbitrary magnitude and weak and strong phases to the present global fit to see if one can describe all the \( B \rightarrow VP \) data with any greater success.
Based on asymmetries quoted in Ref. [50] and BaBar value of $\rho \to \pi \eta$ being strangeness-preserving amplitude and positive. This choice will be seen in our fits of $B^+ \to \omega K^+$, while we are aware only of a decay rate for $B^0 \to \omega K^0$. We thus add three more data points for these processes.

The BaBar Collaboration has recently reported observation of the decay modes $B^+ \to \rho^+ \eta$ and $B^+ \to \rho^+ \rho^0$ at levels indicating a significant role for the $C_V$ amplitude. We include the branching ratios for these processes as averages between the BaBar and older CLEO [38] values. In addition we include the new BaBar value of $A_{CP}(B^+ \to \rho^+ \eta)$.

The decay rate for $B^+ \to \rho^0 K^+$ was recently measured by the Belle collaboration with high significance [52]. We include the average between the Belle result and the previous measurements by BaBar and CLEO as another data point.

Although only an upper limit exists so far for $B(B^0 \to K^{*0} \pi^0)$, we use the Belle central value and error [42] in order to enforce this upper limit in the fits.

The grand total of fitted data points is thus 34, including some quantities such as $A_{CP}(B^+ \to \phi K^+)$, $S_{\phi K_S}$ and $A_{S\phi K_S}$ which do not affect our fit. We now count the parameters of the fit.

The amplitude $p_\rho$ is taken to have a strong phase of $\pi$ by definition. Its weak phase, since it is dominated by $-V_{ts}V_{cs}$ (see the discussion at the end of Sec. III), also is $\pi$, so we will have $p_\rho$ real and positive. This choice will be seen in our favored solution to entail tree amplitudes with positive real and small imaginary parts (when their weak phases are neglected), in accord with expectations from factorization [24]. The corresponding strangeness-preserving amplitude $p_\rho$ is determined by $p_\rho = (V_{cd}/V_{cs})p_{\rho'} = -\lambda p_\rho$, where $\lambda =$...
TABLE II: Same as Table I for $|\Delta S| = 1$ decays of $B$ mesons.

| Mode | CLEO | BaBar | Belle | Avg. |
|------|------|-------|-------|------|
| $B^{+}\to K^{*0}\pi^{+}$ | $7.6^{+1.3}_{-0.9} \pm 1.6$ ( < 16) | $15.5 \pm 1.8^{+0.5}_{-1.0}$ | $8.5 \pm 0.9^{+0.8}_{-0.7} \pm 0.5$ | $9.0 \pm 1.4$ ($S = 1.11$) |
| $K^{+}\pi^{0}$ | $1.3^{+0.8}_{-0.7} \pm 1.0$ ( < 38) | - | - | $< 31$ |
| $K^{+}\eta$ | $26.4^{+9.6}_{-8.2} \pm 3.3$ | $2.5^{+1.8}_{-1.7} \pm 1.8$ | $2.6^{+1.8}_{-1.7} \pm 3.0$ | $25.9 \pm 3.4$ |
| $K^{+}\eta^{'}$ | $11.1^{+12.7}_{-8.0}$ ( < 35) | $6.1^{+3.9}_{-3.2} \pm 1.2$ | $< 90$ | $< 12$ |
| $\rho^{0}K^{+}$ | $8.4^{+10.0}_{-3.6} \pm 1.8$ ( < 17) | $3.9 \pm 1.2^{+1.3}_{-1.3}$ ( < 6.2) | $3.9 \pm 0.6^{+0.4}_{-0.3}$ | $4.1 \pm 0.8$ |
| $\rho^{0}K^0$ | $< 45$ | - | - | $< 48$ |
| $\omega K^+$ | $3.2^{+2.4}_{-1.9} \pm 0.8$ ( < 7.9) | $5.0 \pm 1.0 \pm 0.4$ | $6.7^{+1.3}_{-1.2} \pm 0.6$ | $5.4 \pm 0.8$ |
| $\rho^{0}K^+$ | $< 45$ | - | - | $< 48$ |
| $\phi K^+$ | $5.5^{+2.1}_{-1.8} \pm 0.6$ | $10.0^{+0.9}_{-0.8} \pm 0.5$ | $8.6 \pm 0.8^{+0.6}_{-0.6} \pm 0.3$ | $9.0 \pm 0.9$ ($S = 1.39$) |
| $B^{0}\to K^{*0}\pi^{0}$ | $16^{+6}_{-5} \pm 2$ | - | $14.8^{+4.6}_{-4.4} \pm 1.0 \pm 0.9$ | $15.3 \pm 3.8$ |
| $K^{*0}\pi^{0}$ | $0.26^{+0.19}_{-0.04} \pm 0.08$ | - | - | $0.26 \pm 0.35$ |
| $K^{*0}\eta$ | $13.8^{+8.5}_{-5.6} \pm 1.6$ | $19.0^{+2.2}_{-2.1} \pm 1.3$ | $16.5^{+4.6}_{-4.2}$ | $17.8 \pm 2.0$ |
| $K^{*0}\eta^{'}$ | $< 4.2$ | $< 20$ | $< 6.4$ | $< 6.4$ |
| $\rho^{-}K^{0}$ | $16.0^{+7.6}_{-6.2} \pm 2.8$ ( < 32) | $7.3^{+1.3}_{-1.2} \pm 1.3$ | $15.1^{+4.1}_{-4.1} \pm 2.0$ | $9.0 \pm 2.3$ ($S = 1.41$) |
| $\rho^{0}K^0$ | $< 45$ | - | - | $< 48$ |
| $\omega K^0$ | $10.0^{+4.2}_{-4.2} \pm 1.4$ ( < 21) | $5.3^{+1.4}_{-1.2} \pm 0.5$ | $4.0^{+1.3}_{-1.0} \pm 0.5$ | $5.2 \pm 1.1$ |
| $\phi K^0$ | $5.4^{+3.2}_{-2.7} \pm 0.7$ ( < 12.3) | - | - | $7.8 \pm 1.1$ |

\(^{\circ}\)Value utilized in order to stabilize fits. See text.

$\lambda/(1 - \lambda^2) = 0.230$ and $\lambda = 0.224$ [60]. Thus the weak phase of $p'_{p}$ in the present convention is zero, while its strong phase is $\pi$. When we assume that $p'_{p}/p'_{p} = p_{V}/p_{P} = -1$, as suggested in Ref. [31] and as done in Refs. [10] and [12], we will have one free parameter $|p'_{P}|$. More generally, we shall consider fits with this ratio real or complex, adding one or two new parameters $c$ and $\phi$ defined by $p'_{V}/p'_{P} = p_{V}/p_{P} = -c$ or $p'_{V}/p'_{P} = p_{V}/p_{P} = -\cos^{10}$. We do not introduce SU(3) breaking in penguin amplitudes.

- The magnitudes $|t_{P,V}|$ of the tree amplitudes and strong relative phases $\delta_{P,V}$ between them and the corresponding penguin amplitudes $p_{P,V}$ are free parameters: four in all. We make strangeness-changing tree amplitudes to those for $\Delta S=0$ using ratios of decay constants: $t_{P} = \lambda(f_{K}/f_{P})t_{V} \simeq 0.281t_{V}$ and $t_{P} = \lambda(f_{K}/f_{P})t_{P} \simeq 0.240t_{P}$. We thus assume the same relative strong phases between tree and penguin amplitudes in $\Delta S = 0$ and $|\Delta S| = 1$ processes. We use the following values of the decay constants [12] [32]: $f_{K} = 130.7$ MeV, $f_{K} = 199.8$ MeV, $f_{P} = 208$ MeV, $f_{K} = 217$ MeV.

- The weak phase $\gamma$ of $t_{V}$ and $t_{P}$ is a free parameter. We assume it to be the same for both tree amplitudes.

- We take the electroweak penguin amplitude $P_{EW,P}'$ to have the same weak and strong phases as $p_{P}'$. Then the electroweak penguin contribution to $s_{P}'$, $-P_{EW,P}'$, interferes destructively with $p_{P}'$, as was anticipated by explicit calculations [67]. Thus $-p_{P}'/p_{P}'$ is one real positive parameter. We ignore any contribution from the singlet penguin $S_{P}'$, which we expect to involve gluonic coupling to SU(3) flavor-singlet components of vector mesons and thus to be suppressed by the Okubo-Zweig-Iizuka (OZI) rule. We did not find a stable fit if we allowed the strong phase of the EWP contribution to $s_{P}'$ to vary. The contribution $-P_{EW,P}'$ appearing in $s_{P}'$ then implies a corresponding contribution $+P_{EW,P}'$ in $c_{P}'$.

- The term $c_{P}$ appears to play a key role in accounting for the deviation of the $B^{+} \to \rho^{0}\pi^{+}$ and $B^{+} \to \omega\pi^{+}$ decay rates from the predictions based on $tV$ alone. It contains two terms: a term $C_{P}$ which we choose to have the same weak and strong phases as $t_{V}$, and a small EWP contribution $P_{EW,P}' = -\lambda f'_{P} f_{P}$ associated with the term taken in $s_{P}'$ above. Thus $C_{P}/t_{V}$ will be one additional real parameter, which will turn out to be positive in all our fits.

- We include a contribution from the amplitude $C_{V}$,
motivated by the large $B^+ \to \rho^+ \eta$ and $B^+ \to \rho^+ \eta'$ branching ratios. We choose $C_V$ to have the same strong and weak phases as $p_P$. We do not introduce SU(3) breaking in $C_P$ and $C_V$ amplitudes and assume $C_P = \lambda C_P$, $C_V = \lambda C_V$.

- We take the electroweak penguin contribution to $s_V$, $-\frac{1}{2}P_{EW,V}$, to have the same strong and weak phases as $p_P'$. This contribution then implies corresponding contributions $+ \lambda P_{EW,V}$ in $c'_V$ and $+ P_{EW,V} = -\lambda P_{EW,V}$ in $c_V$. The apparent suppression of the decay $B^0 \to K^{*0}\rho^0$ (see Sec. VI) suggests that $P_{EW,V}'$ and $p_P'$ are interfering destructively in this process, implying constructive interference in both charge states of $B \to K^*\eta$. We ignore any contribution from the singlet penguin $S'_V$ in the absence of information about its magnitude and phase.

There are thus ten, eleven, or twelve parameters to fit 34 data points, depending on the assumption for the $p'_V/p'_{P'}$ ratio: −1, real, or complex. In fact, not all partial decay rates are independent, as we must have the following equalities between rate differences:

\[
\Gamma(B^0 \to \rho^-\pi^+) = \Gamma(B^0 \to \rho^+\pi^-) = (f_s/f_K)[\Gamma(B_0 \to \rho^+K^-) - \Gamma(B_0 \to \rho^-K^+)] , \tag{3}
\]

\[
\Gamma(B^0 \to \rho^+\pi^-) = \Gamma(B^0 \to \rho^-\pi^+) = (f_P/f_K)[\Gamma(B^0 \to K^{*-}\rho^+) - \Gamma(B^0 \to K^+\rho^-)] . \tag{4}
\]

When transcribed into relations among branching ratios, these read, respectively,

\[
(11.1 \pm 3.8) \times 10^{-6} \mp (2.8 \pm 1.9) \times 10^{-6} , \tag{5}
\]

\[
(4.4 \pm 4.1) \times 10^{-6} \mp (7.6 \pm 10.4) \times 10^{-6} . \tag{6}
\]

The first of these is violated at the 2σ level. One reaps little gain in relaxing the assumption $p'_V = -p'_{P'}$ since these relations are expected to hold regardless of $p'_V/p'_{P'}$.  

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**TABLE III: Summary of predicted contributions to $\Delta S = 0$ decays of $B$ mesons to one vector and one pseudoscalar mesons.** Amplitude magnitudes $|A_{\text{exp}}|$ extracted from experiments are quoted in units of eV. The results are based on a fit with $p'_V = -p'_{P'}$ and $\gamma \approx 65^\circ$ (see Table V).

| Mode | Amplitudes | $|T|^a$ | $|P|^a$ | $p_B$ (GeV) | $|A_{\text{exp}}|^b$ | $A_{CP}$ |
|------|------------|--------|--------|-------------|----------------|--------|
| $B^+ \to K^{0*}K^+$ | $p_P$ | 0 | 7.5 | 2.539 | $< 24.1$ | - |
| $K^-\bar{K}^0$ | $p_V$ | 0 | 7.5 | 2.539 | - | - |
| $\rho^0\pi^+$ | $-\frac{1}{\sqrt{2}}(t_V + c_P + p_V - p_P)$ | 25.1 | 10.6 | 2.582 | 31.4 ± 1.9 | -0.17 ± 0.11 |
| $\rho^+\pi^0$ | $-\frac{1}{\sqrt{2}}(t_P + c_V + p_P - p_V)$ | 39.7 | 14.4 | 2.582 | 34.5 ± 4.2 | 0.23 ± 0.17 |
| $\rho^0\eta$ | $-\frac{1}{\sqrt{2}}(t_p + c_V + p_V + s_V)$ | 32.4 | 2.1c | 2.554 | 31.2 ± 4.7 | 0.06 ± 0.29 |
| $\rho^+\eta'$ | $\frac{1}{\sqrt{2}}(t_p + c_V + p_V + s_V)$ | 22.9 | 0.7c | 2.493 | 38.7 ± 6.6 | - |
| $\omega\pi^+$ | $\frac{1}{\sqrt{2}}(t_V + c_P + p_V + 2s_P)$ | 25.1 | 0.066d | 2.580 | 25.3 ± 2.3 | 0.11 ± 0.21 |
| $\phi\pi^+$ | $\frac{1}{\sqrt{2}}s_P$ | 0.009d | 2.539 | - | $< 6.7$ | - |

- $T$ is the sum of all tree and color-suppressed amplitudes that contribute to a process.
- $P$ is the sum of all penguin amplitudes, including electroweak ones.
- $|A_{\text{exp}}|$ is defined by Eq. 2 as an amplitude related to a $CP$-averaged branching ratio quoted in Table I.
- No $S_V$ contribution included. $^d$ No $S_P$ contribution included.
- Based on $CP$-averaged branching ratios quoted in Table I.
- $^d$ Takes account of the relative phase between $C_P$ and $C_V$ amplitudes.
When we assume $p'_V = -p'_P$, the specific expressions entering our fits include

$$A(B^+ \rightarrow K^{*0} \pi^+) = |p'_P| |p'_V| t_{PV}$$

The phase convention is such that $\delta_{\nu,\rho} = 0$ corresponds to $t_{PV}$ having a phase of $\gamma$ with respect to $p_{PV}$. Amplitudes associated with the charge-conjugate modes can be obtained by flipping the sign of $\gamma$ in the above expressions. The expressions lead to the rate relations and if one squares them and takes appropriate differences.

We now discuss two additional parameters in $(B^0, \bar{B}^0) \rightarrow \rho^+ \pi^+$ which provide further constraints. These are measured in a time-dependent study by the BaBar Collaboration:

$$S_{\rho\pi} = -0.13 \pm 0.18 \pm 0.04$$

$$\Delta S_{\rho\pi} = 0.33 \pm 0.18 \pm 0.03$$

where

$$S_{+-} = \frac{2|\lambda^{+-}|}{1 + |\lambda^{+-}|^2}$$

$$S_{-+} = \frac{2|\lambda^{+-}|}{1 + |\lambda^{+-}|^2}$$

and $q/p = e^{-2i\beta}$ with $\beta = 23.7^\circ$. Since our fits predict the phases and magnitudes of all the relevant decay amplitudes, it is a simple matter to calculate the $S$'s. They provide crucial information on the relative strong phases, among other things. As mentioned, other observables $C_{\rho\pi}$, $\Delta C_{\rho\pi}$, and $A_{\rho\pi}$ as defined in Refs. 26 and 28 are related to information we already use in our fits and need not be considered separately. (See the Appendix.)

To see explicitly the constraints provided by $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$ it is helpful to calculate them in the limit in which the small penguin contributions to the $B \rightarrow \rho \pi$ amplitudes can be neglected. Defining $r \equiv |t_{PV}/t_{P}|$ and $\delta \equiv \text{Arg}(t_{PV}/t_{P})$, one finds $S_{+-} = 2r \sin(2\alpha + \delta)/(1 + r^2)$ and

$$S_{\rho\pi} = \frac{2r}{1 + r^2} \sin 2\alpha \cos \delta$$

$$\Delta S_{\rho\pi} = \frac{2r}{1 + r^2} \cos 2\alpha \sin \delta$$

\(\text{TABLE IV: Same as Table III for} |\Delta S| = 1 \text{ decays of} B \text{ mesons.}\)

| Mode | Amplitudes | $|T'|$ | $|P'|$ | $p_c$ (GeV) | $|A_{exp}|$ | $A_{CP}$ |
|------|------------|--------|--------|-------------|-------------|-----------|
| $B^+ \rightarrow K^{*0} \pi^+$ | $p'_P$ | 0 | 32.6 | 2.561 | 31.2 $\pm$ 2.4 | - |
| $K^{*0} \pi^0$ | $-\frac{1}{\sqrt{2}}(t'_P + c'_V + p'_P)$ | 9.4 | 39.7 | 2.562 | < 58.1 | - |
| $K^{*0} \pi^0$ | $-\frac{1}{\sqrt{2}}(t'_P + c'_V - p'_P + s'_V)$ | 7.7 | 46.7a | 2.534 | 53.4 $\pm$ 3.5 | 0.10 $\pm$ 0.12 |
| $K^{*0} \pi^0$ | $\frac{1}{2}(t'_P + c'_V + p'_P + 2s'_V)$ | 5.4 | 16.5a | 2.472 | < 62.8 | - |
| $\rho^* K^+$ | $-\frac{1}{\sqrt{2}}(t'_V + c'_P + p'_V)$ | 6.9 | 22.9 | 2.559 | 21.2 $\pm$ 2.1 | - |
| $\rho^* K^+$ | $\rho'_V$ | 0 | 32.6 | 2.559 | < 72.3 | - |
| $\omega K^+$ | $\frac{1}{\sqrt{2}}(t'_V + c'_P + p'_V + 2s'_P)$ | 6.9 | 23.0b | 2.557 | 24.3 $\pm$ 1.8 | -0.003 $\pm$ 0.122 |
| $\phi K^+$ | $p'_P + s'_P$ | 0 | 32.5b | 2.516 | 31.6 $\pm$ 1.6 | 0.030 $\pm$ 0.072 |

\(\text{a No} S'_V \text{ contribution included. b No} S'_P \text{ contribution included.}\)
Both these quantities are small experimentally, consistent with solutions in which \( \alpha = \pi - \beta - \gamma \) is near \( \pi/2 \) while \( \delta \) is near zero or \( \pi \). The non-zero central value of \( \Delta S_{\rho\pi} \), in our favored solution to be discussed below, combines with a value of \( \cos 2\alpha \) near \(-1\) and other constraints to favor a small negative value of \( \delta \).

V. TESTS OF THE \( p'_{\nu} = -p''_{p} \) ASSUMPTION

In this section, we note that the relation \( p'_{\nu} = -p''_{p} \), proposed in Ref. [31] and used in previous discussions [10, 12], can be tested experimentally, and discuss the status of such tests.

As described before, information on \( |p'_{p}| \) is directly obtained from the \( B^+ \rightarrow K^{0}\pi^+ \) decay rate, which is predicted to be equal to that of its \( CP \) conjugate. (The absence of a \( CP \) asymmetry in this mode is one test of the present picture.) In principle, one can also extract \( |p'_{\nu}| \) directly from the \( B^+ \rightarrow \rho^+K^0 \) rate (for which one also expects a vanishing \( CP \) asymmetry). Currently, only the CLEO group has reported an upper bound on the averaged branching ratio, \( \overline{B}(\rho^+K^0) < 48 \times 10^{-6} \) based on a sample of 2.6 million \( B\bar{B} \) pairs. The present BaBar and Belle sample of approximately 100 times as much data would enable this mode to be observed at the predicted branching ratio of \( \overline{B}(\rho^+K^0) \approx 12.6 \times 10^{-6} \) with good significance.

An indirect way to test \( p'_{p} = -p''_{\nu} \) is to compare the \( \omega K^0 \) and \( \phi K^0 \) modes. Using \( c'_{p} = C'_{p} + p'_{EW,P} \) and \( s'_{p} = S'_{p} - p'_{EW,P}/3 \) [Eq. (21)] and the facts that \( C'_{p} \) is smaller than \( p'_{EW,P} \) by a factor of about 0.22 (see Table I in Ref. [12]) and that \( S'_{p} \) is \( \text{OZI} \)-suppressed, we can safely neglect them and have

\[
A(\omega K^0) = \frac{1}{\sqrt{2}} \left( p'_{\nu} + \frac{1}{3} p'_{EW,P} \right), \tag{23}
\]
\[
A(\phi K^0) = p'_{p} - \frac{1}{3} p'_{EW,P}. \tag{24}
\]

Therefore, the amplitude magnitudes are related by a factor of \( \sqrt{2} \) if \( p'_{p} = -p''_{\nu} \). Current data, using the average for \( B^+ \rightarrow \phi K^+ \) and \( B^0 \rightarrow \phi K^0 \) amplitudes (expected to be equal within our approximations) show that \( \sqrt{2} A(\omega K^0)/A(B \rightarrow \phi K) = 1.13 \pm 0.13 \), consistent with 1. [Note added: The updated branching ratio \( B(\rho^+K^0) \) results in a slightly smaller ratio of amplitudes: \( \sqrt{2} A(B^0 \rightarrow \omega K^0)/A(B \rightarrow \phi K) = 1.11 \pm 0.13 \).

Observation of the \( \rho^+K^0 \) mode and more precise determination of \( \omega K^0 \) and \( \phi K^0 \) modes thus will be very helpful in justifying the assumption of \( p'_{p} = -p''_{\nu} \). However, we find in the next section that global fits to data in which this assumption is relaxed are not very different from those in which \( p'_{p} = -p''_{\nu} \) is assumed.

VI. DISCUSSION OF PREDICTIONS

Plots of \( \chi^2 \) as a function of \( \gamma \) for three fits are shown in Fig. 1. Three local minima are found, around \( \gamma = 26^\circ, 63^\circ \), and \( 162^\circ \). The fit with \( p'_{\nu}/p''_{p} \) real gives a \( \chi^2 \) very similar to that with \( p'_{\nu}/p''_{p} = -1 \) for \( \gamma \approx 26^\circ \) and to that with \( p'_{\nu}/p''_{p} \) complex for the other two minima, so we shall not consider it further. The magnitudes of individual amplitudes and the strong phases determined in the fits with \( p'_{\nu}/p''_{p} = -1 \) and \( p'_{\nu}/p''_{p} \) complex are compared with one another for the three local minima in Table VI. The corresponding predictions of these fits are compared with one another and with experiment in Tables VII and VIII.

![Graph](image-url)
errors for all solutions with complex \( p'_V/p_P \) have been estimated using a Monte Carlo method in which parameter sets were generated leading to \( \chi^2 \) values no more than 1 unit above the minimum, and the spread in predictions was studied. For any prediction depending on a single parameter, the error in that parameter was used to obtain the error in the prediction. 

Within the range 38° ≤ γ ≤ 80°, the present fits specify γ to within a few degrees at the 1σ level. Values corresponding to \( \Delta \chi^2 \leq 1 \) above the minimum for \( p'_V/p_P = -1 \) and complex are γ = (65±6)° and (63±6)°, respectively. Ranges for \( \Delta \chi^2 \leq 3.84 \) above the minimum (95% c.l. limits) are 54°–75° and 51°–73° in the two fits. 

The local minima found at γ ≈ (26, 162)° are associated with larger \( C_V \) and \( P_{EW,V} \), leading to larger predicted branching ratios \( B(B^+ \to K^{++}π^0) = (22.1^{+5.1}_{-5.1} \times 10^{-6}) \) versus the 15.0^{+3.3}_{-2.8} prediction for γ ≈ 63°. These predictions do not conflict with the present CLEO bound of 31 × 10^{-6} and are not too far below it. The branching ratio of this decay should be measurable very soon. The solution for γ ≈ 162° also predicts a larger \( CP \) asymmetry for \( B^+ \to ρ^++π^0 \), closer to the present central value which is slightly disfavored in the fits with γ ≈ 63°. The fits with γ ≈ (26, 162)° predict larger branching ratios and \( CP \) asymmetries for the color-suppressed all-neutral decay modes \( B^0 \to ρ^0(η, η') \) and \( B^0 \to ωπ^0 \) than do the fits with γ ≈ 63°. 

Several observables provide the main contributions to \( \chi^2 \). These are summarized in Table IX. The large \( CP \) asymmetry in \( B^0 \to ρ^-π^+ \) provides the largest \( \Delta \chi^2 \) in all three cases. The only other contributions with \( \Delta \chi^2 \geq 1 \) occur for γ = 162°, with \( \Delta \chi^2[A_{CP}(B^+ \to K^{++}π^-)] = 1.3 \), and for γ = 26°, with \( \Delta \chi^2[B(B^0 \to ωK^0)] = 1.3 \). 

We comment on several predictions of the fits with γ ≈ 63° which appear to be of general nature, not depending on specific assumptions about \( p'_V/p_P \) or symmetry breaking. 

All fits predict \( |t_V| < |t_P| \). In a factorization picture \( t_V \) involves the production of a \( π^\pm \) \((f_π = 130.7 \text{ MeV}) \) by the weak current, whereas \( t_P \) involves production of a \( ρ^\pm \) \((f_ρ = 208 \text{ MeV}) \). This inequality is therefore not so surprising. One has \( |t_V/t_P| \approx 0.68 \) while \( f_π/f_ρ \approx 0.63 \), suggesting within factorization that the \( B \to ρ \) and \( B \to π \) form factors are similar. The values of \( |t_V| \) we find are comparable to that of \( |t| \), the tree amplitude in \( B \to PP \), which was found in [13] to be \( |t| = 27.1 \pm 3.9 \text{ eV} \). One also finds \( |t_P/t| \approx f_π/f_ρ \), as expected from factorization when the \( ρ−π \) mass difference is neglected. 

All fits predict \( p'_V \approx −p'_P \), even if this equality is not enforced. This is largely due to the need for constructive interference between \( p'_V \) and \( −p'_P \) in the decays \( B \to K^+η \). The corresponding ratio \( C_V/|t| \approx 0.2 \), and a small amount of constructive interference between \( C \) and \( T \) terms appears to account for an enhancement of the \( B^+ \to π^+τ^0 \) decay (see, e.g., [13]). 

Small strong phases (mod π) are favored, implying that the relative strong phase between \( t_V \) and \( t_P \) is small. This is consistent with the prediction of QCD factorization methods [22]. 

The pattern of strong and weak phases obtained here is such that there is a small amount of constructive tree-penguin interference in the \( CP \)-averaged branching ratios for \( B^0 \to ρ^-π^+ \) and \( B^0 \to ωπ^0 \), and a small amount of destructive interference in \( B^0 \to ρ^+π^- \) and \( B^0 \to ρ^-K^+ \). The preference of the fits for large values of γ (small values of cos(γ), when final-state phases are small mod π, is due in part to the fact that these interference effects are relatively small. 

We shall comment below on details which depend on specific assumptions. First we discuss some specific decay modes for which predictions are fairly stable over the
TABLE V: Comparison of parameters extracted in fits to branching ratios and $CP$ asymmetries under various assumptions. Values of the topological amplitudes are quoted in units of eV. Probabilities are those for $\chi^2$ to exceed the value shown for the indicated number of degrees of freedom.

| Quantity | $\phi$ | $p'_v/p'_p = -1$ | $p'_v/p'_p = -e^{i\delta}$ |
|----------|--------|------------------|------------------|
| Value in fit | $\gamma$ | $(24 \pm 5)^\circ$ | $(65 \pm 6)^\circ$ | $(164 \pm 5)^\circ$ | $(26 \pm 5)^\circ$ | $(63 \pm 6)^\circ$ | $(162 \pm 5)^\circ$ |
| $\phi$ | 0 (input) | 0 (input) | 0 (input) | $(351 \pm 12)^\circ$ | $(2 \pm 18)^\circ$ | $(-20 \pm 32)^\circ$ |
| $|p'_v/p'_p|$ | $32.4^{+11}_{-15}$ | $32.6^{+1.5}_{-1.5}$ | $32.4^{+14}_{-14}$ | $32.5^{+1.7}_{-1.6}$ | $32.3^{+1.4}_{-1.5}$ | $32.0 \pm 1.6$ |
| $|p'_v|$ | $32.4^{+11}_{-15}$ | $32.6^{+1.5}_{-1.5}$ | $32.4^{+14}_{-14}$ | $31.3^{+3.6}_{-3.0}$ | $37.1^{+2.3}_{-2.4}$ | $35.3^{+3.4}_{-3.6}$ |
| $|s'_p|$ | $0.9 \pm 1.3$ | $0.0 \pm 1.3$ | $0.5 \pm 1.3$ | $1.7^{+1.8}_{-1.6}$ | $1.1 \pm 1.3$ | $1.1^{+1.7}_{-1.5}$ |
| $|s'_v|$ | $6.2^{+2.2}_{-2.1}$ | $7.9^{+2.4}_{-2.3}$ | $6.9^{+2.3}_{-2.2}$ | $8.5^{+3.5}_{-3.4}$ | $5.8^{+2.4}_{-2.1}$ | $6.1^{+3.8}_{-2.9}$ |
| $|t'_v|$ | $46.7^{+11}_{-3.4}$ | $43.1^{+3.4}_{-3.4}$ | $46.1^{+3.2}_{-3.4}$ | $46.2^{+3.2}_{-3.4}$ | $43.3^{+3.2}_{-3.4}$ | $47.0^{+3.1}_{-3.4}$ |
| $|t'_p|$ | $32.5^{+3.6}_{-3.9}$ | $30.3^{+3.4}_{-3.7}$ | $31.9^{+3.6}_{-3.7}$ | $33.8^{+3.7}_{-3.9}$ | $29.6^{+3.4}_{-3.7}$ | $30.9^{+4.2}_{-4.7}$ |
| $\delta_p$ | $(184 \pm 12)^\circ$ | $(181 \pm 8)^\circ$ | $(36 \pm 13)^\circ$ | $(199 \pm 14)^\circ$ | $(182 \pm 14)^\circ$ | $(162 \pm 20)^\circ$ |
| $\delta_v$ | $(-87 \pm 11)^\circ$ | $(-18 \pm 7)^\circ$ | $(-100 \pm 10)^\circ$ | $(-102 \pm 19)^\circ$ | $(-20 \pm 9)^\circ$ | $(-109 \pm 26)^\circ$ |
| $|C_p|$ | $6.0^{+4.3}_{-4.1}$ | $5.3^{+4.3}_{-4.0}$ | $5.7^{+4.4}_{-4.1}$ | $6.8^{+4.5}_{-4.2}$ | $5.6^{+4.2}_{-4.0}$ | $6.0^{+4.4}_{-4.1}$ |
| $|C_v|$ | $16.8^{+3.5}_{-5.7}$ | $13.1^{+5.7}_{-6.0}$ | $15.0^{+5.6}_{-5.8}$ | $17.3^{+5.6}_{-5.8}$ | $13.1^{+5.7}_{-6.0}$ | $14.9^{+5.6}_{-5.8}$ |

Fit properties:

| $\chi^2$/d.f. | 23.9/24 | 25.5/24 | 22.1/24 | 20.6/22 | 20.5/22 | 18.9/22 |
|----------------|---------|---------|---------|---------|---------|---------|
| % c.l. | 47% | 38% | 57% | 55% | 55% | 65% |

Derived quantities:

| Derived quantities | $|p'_v/p'_p|$ | $-s'_v/p'_p$ | $s'_v/p'_v$ | $t'_v/t'_p$ | Arg($t'_v/t'_p$) | $C_p/t_V$ | $C_v/t_P$ | $|p'_p|$ | $|p'_v|$ | $|s'_p|$ | $|s'_v|$ | $|t'_p|$ | $|t'_v|$ | $|t'_p|$ | $|C'_p|$ | $|C'_v|$ |
|-------------------|---------|---------|---------|----------|----------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Value in fit | 1 (input) | 0.03 ± 0.04 | 0.21 ± 0.08 | 0.70 ± 0.10 | $(-91 \pm 10)^\circ$ | $0.19^{+0.17}_{-0.13}$ | $0.36^{+0.15}_{-0.13}$ | $7.4 \pm 0.3$ | $7.4 \pm 0.3$ | $0.2 \pm 0.3$ | $1.4 \pm 0.5$ | $9.1^{+1.0}_{-1.1}$ | $11.2^{+0.7}_{-0.8}$ | $1.4^{+0.10}_{-0.09}$ | $3.8 \pm 1.3$ | $3.0^{+1.4}_{-1.3}$ |
| $|p'_p|$ | $0.6^{+0.3}_{-0.3}$ | $0.0 \pm 0.3$ | $1.8^{+0.5}_{-0.5}$ | $8.5^{+1.0}_{-1.1}$ | $10.3^{+0.7}_{-0.8}$ | $1.2^{+0.0}_{-0.0}$ | $3.4 \pm 1.3$ | $3.0^{+1.4}_{-1.3}$ | $3.4 \pm 1.3$ |

*EWP contribution only.

range of assumptions.

A. $\Delta S = 0$ decays

The decay $B^+ \rightarrow \bar{K}^0 K^+$ is predicted to be dominated by the $p_p$ amplitude and thus to have zero $CP$ asymmetry. Its branching ratio is expected to be about $0.5 \times 10^{-6}$, considerably below present upper limits. Any deviations from these predictions could indicate the importance of an annihilation amplitude or, equivalently, important rescattering effects. One expects a similar or slightly larger prediction for $B^+ \rightarrow K^{*+} \bar{K}^0$.

A non-zero negative $CP$ asymmetry is predicted for $B^+ \rightarrow \rho^0 \pi^+$, as a result of the interference of the amplitudes $t_V + c_P$ and $p_V - p_p$. In our favored solution (with $\gamma \approx 63^\circ$) this arises as the result of a small but non-negligible relative final-state phase $\delta_V$. The same phase contributes a $CP$ asymmetry of opposite sign to $B^+ \rightarrow \rho^0 K^+$ and $B^+ \rightarrow \omega K^+$, as we shall see below. It appears to be generated in our fit by the appreciable $CP$ asymmetry in $B^0 \rightarrow \rho^- \pi^+$, and also leads to a non-zero prediction for $\Delta S_{\rho^0}$.

The decay $B^+ \rightarrow \rho^+ \pi^0$ is expected to have a branching ratio of about $12 \times 10^{-6}$, consistent with the recently reported level $33$. The decay $B^+ \rightarrow \phi \pi^+$, dominated by an amplitude coming from the electroweak penguin in $s_p$, is included
for completeness. We do not expect it to be observed any time soon. The corresponding predicted rates for $B^+ \to \rho^+ \pi^-$ is well reproduced, in part because of the enhancement associated with the constructive interference between $t_P$ and $C_V$. The large value of $C_V$ is driven by the attempt to fit an even larger branching ratio for $B^+ \to \rho^+ \eta'$ reported in the same experiment. The penguin contributions are expected to cancel if $p_T = -p_T$, so both of these decays are expected to have zero or very small $CP$ asymmetries. This stands in contrast to the large asymmetries expected for $B^+ \to \pi^+ \eta$ and $B^+ \to \pi^+ \eta'$. In $B^+ \to \rho^+ \eta^{(i)}$, the QCD penguin contributions associated with the $u\bar{u}$ and $d\bar{d}$ components of the $\eta^{(i)}$ nearly cancel one another if $p_T \simeq -p_T$, while in $B^+ \to \pi^+ \eta^{(i)}$, these contributions reinforce one another.

We predict $B(B^+ \to \rho^+ \eta')/B(B^+ \to \rho^+ \eta) \simeq 1/2$, whereas the observed ratio exceeds 1. This may reflect a shortcoming of our description of the $\eta'$ wave function, which has been argued in Ref. [13] to contain important symmetry-breaking effects.

We are unable to accommodate the large central value of the $CP$ asymmetry in $B^0 \to \rho^- \pi^+$. This will be true of any formalism which respects the rate difference relation [4], which is seen in Eq. (6) to be poorly obeyed by central values.

**B. $|\Delta S| = 1$ decays**

The decay $B^+ \to K^{*0} \pi^+$, dominated by $p_T$, is the main source of information on that amplitude. It is expected to have zero $CP$ asymmetry. Better measurement of its branching ratio would reduce the errors on $|p_T|_p$.

Electroweak penguin contributions play an important role in the large predicted value $B(B^+ \to K^{*+} \pi^0) \simeq 15 \times 10^{-6}$. As mentioned, one may expect detection of this mode in the near future, and it may help to choose among various local $\chi^2$ minima in Fig. 1.

The predictions for $B^+ \to K^{*+} \eta$ and $B^+ \to K^{*+} \eta'$ include a small tree contribution whereas no such contribution is expected for the corresponding decays $B^0 \to K^{*0} \eta$ and $B^0 \to K^{*0} \eta'$. This leads us to expect a slight enhancement of $B(B^+ \to K^{*+} \eta)$ with respect to $B(B^0 \to K^{*0} \eta)$, as suggested by the data. The successful predic-
TABLE VIII: Observables providing $\chi^2 \geq 1.5$ in at least one of the three fits with complex $p'_p/p_V$. 

| Observable | $\gamma = 26^o$ | $\gamma = 63^o$ | $\gamma = 162^o$ |
|------------|----------------|----------------|----------------|
| $A_{CP}(B^+ \rightarrow \rho^+ \pi^0)$ | 1.4 | 2.0 | 0.2 |
| $B(B^+ \rightarrow \rho^0 \eta')$ | 2.4 | 3.5 | 2.8 |
| $A_{CP}(B^0 \rightarrow \rho^- \pi^+)$ | 3.7 | 4.9 | 5.0 |
| $A_{CP}(B^0 \rightarrow \rho^- \pi^0)$ | 0.4 | 1.0 | 1.7 |
| $A_{CP}(B^+ \rightarrow \omega K^+)$ | 4.0 | 2.6 | 2.4 |
| $S^m_{KS}$ | 1.6 | 1.6 | 1.6 |
| Sum: | 13.5 | 15.6 | 13.7 |

(see Table VI) do not considerably affect the branching ratio of this decay. A small but non-negligible positive $CP$ asymmetry of about 0.2 is expected in both these $S^m_C$, due to the interference of the $p'_p$ and $t'_V$ terms.

The process $B^+ \rightarrow \rho^+ K^0$, as mentioned, is expected to be governed solely by the $p'_p$, term. Measurement of its branching ratio would provide valuable information on $|p'_V|$. As noted, the only upper limit on the branching ratio, $B(B^+ \rightarrow \rho^+ K^0) < 48 \times 10^{-6}$, comes from the CLEO Collaboration, so it should be improved (or the decay discovered) very soon.

The rate difference sum rule [5] involving $B^0 \rightarrow K^{*+} \pi^-$ is seen in Eq. [7] to be satisfied, though with large errors. We predict small values for $A_{CP}(B^0 \rightarrow K^{*+} \pi^-)$ and $A_{CP}(B^0 \rightarrow \rho^+ K^-)$.

We predict a branching ratio for $B^0 \rightarrow K^{*0} \pi^0$ of only about $10^{-6}$, below the present experimental upper bound of $3.5 \times 10^{-6}$. The electroweak penguin component of the $\epsilon'_V$ amplitude is responsible for interfering destructively with the $p'_p$ component in this process. We expect a corresponding enhancement in $B(B^+ \rightarrow K^{*0} \pi^0)$, since the relative signs of $\epsilon'_V$ and the dominant penguin amplitude $p'_p$ are opposite in the two processes. One expects the sum rule (analogous to one discussed recently in [8])

$$B(B^+ \rightarrow K^{*+} \pi^0) = \frac{\tau^+}{\tau_0} B(B^0 \rightarrow K^{*0} \pi^0) = \frac{1}{2} \left[ B(B^+ \rightarrow K^{*0} \pi^+) + \frac{\tau^+}{\tau_0} B(B^0 \rightarrow K^{*+} \pi^-) \right]$$

(25)

to hold to first order in $|p'_p|$ and $|t'_V|$. The right-
hand side is $(12.7 \pm 2.2) \times 10^{-6}$. The sum rule is only approximately obeyed by the predicted branching ratios since quadratic terms in $|r_p'/p_p'|$ and $|c_p'/p_p'|$ are non-negligible.

The branching ratio and $CP$ asymmetry for $B^0 \to \rho^- K^+$ are reproduced satisfactorily. Since these quantities enter into the rate difference relation \[4\], which is poorly obeyed [see \[5\]], one suspects that it is the experimental $CP$ asymmetry $A_{CP}(B^0 \to \rho^- \pi^0) = -0.54 \pm 0.19$ which is slightly off line, as mentioned earlier.

The decay $B^0 \to \rho^0 K^0$ is predicted to have a branching ratio of about $7 \times 10^{-6}$, not far below its experimental upper limit of $12 \times 10^{-6}$. The $CP$ asymmetry is expected to be very small.

We already noted the comparison of $B^0 \to \omega K^0$ and $B \to \phi K$ amplitudes in Section \[V\] as one test for $p_{\omega} = -p_{\phi}$. Zero $CP$ asymmetry is expected. We predict $A_{sPK^0} = 0$, consistent with observation, but, as mentioned \[6\], are unable to account for $S_{\phi K^0} = -0.15 \pm 0.70$ ($S = 2.11$), predicting instead the value $\sin(2\beta) = 0.736 \pm 0.049$.

C. Processes sensitive to assumptions

Most parameters of the fits appear to be relatively stable. This stability is due in part to the inclusion of $S_{\rho\tau}$ and $\Delta S_{\rho\tau}$, which are the only quantities in which the interference between $t_{\bar{V}}$ and $t_p$ is probed directly. Small changes in relative strong phases occur when we relax the assumption that $p_{\omega} = -p_{\phi}$. The changes in predicted branching ratios and $CP$ asymmetries appear to be so small that they will not be detected in the near future.

The least stable aspect of the fits is associated with the amplitudes $C_{\rho\tau}$ (color-suppressed tree) and $P_{EW,V}'$, contributing to $s_{\rho\tau}$ and $c_{\rho\tau}$. The need for a $C_{\rho\tau}$ amplitude is associated with the large branching ratios for $B^+ \to \rho^+(\pi^0, \eta, \eta')$, but we still cannot fit the large branching ratio for the last process. The $\chi^2$ minima at $\gamma = (26, 162)^\circ$ are associated with larger values of $C_{\rho\tau}$ and $P_{EW,V}'$, which lead to the predictions $B(B^+ \to K^{*+} \pi^0) = (22.1^{+2.4}_{-5.1}, 18.2^{+5.0}_{-4.1}) \times 10^{-6}$.

We have assumed no $SU(3)$ symmetry breaking in relating the $\Delta S = 0$ penguin amplitudes $|p_{P,V}|$ to the $|\Delta S = 1$ amplitudes $|p_{P,V}'|$. This assumption will be checked in the future when the appropriate $B \to K^- \pi^+$ or $B \to K^+ K^-$ decay rates are compared with the predictions in Table VI. The corresponding assumption $|P/P'| = |V_{cd}/V_{cs}| = 0.230$ is close to being checked in $B^+ \to K^- \pi^0$ decays, where it entails the prediction $B(B^+ \to K^- \pi^0) = (0.75 \pm 0.11) \times 10^{-6}$ \[13\], to be compared with the experimental upper limit of $2.2 \times 10^{-6}$ \[14\].

We have assumed nonet symmetry and a particular form of octet-singlet mixing in describing the decays $B \to K^+ \eta$. When an independent measurement of the $p_{\rho\tau}$ amplitude becomes available through the decay $B^+ \to \rho^+ K^0$, this assumption will receive an independent check.

As mentioned earlier, we assumed that the strong phase of the electroweak penguin contribution $P_{EW,V}'$ to $s_{\rho\tau}$ and $c_{\rho\tau}$ is the same as that of $p_{\rho\tau}$, and that the $P_{EW,V}'$ contribution differs in phase by $180^\circ$ with respect to $p_{\rho\tau}$. It may be necessary to relax these assumptions in future fits once more data become available involving these contributions.

VII. U-SPIN RELATIONS

In the previous sections we have employed the complete flavor $SU(3)$ symmetry group, neglecting small annihilation-type amplitudes. A best fit was performed in order to calculate magnitudes and phases of $SU(3)$ amplitudes. In the present section we will rely only on U-spin \[15, 16\], an important subgroup of $SU(3)$, introducing U-spin breaking in terms of ratios of decay constants. U-spin will be shown to imply two quadrangle relations among $|\Delta S| = 1$ amplitudes and two quadrangle relations among $\Delta S = 0$ amplitudes. This exhausts all sixteen $B^+ \to VP$ decays given in Tables III and IV. Relations will also be presented among penguin amplitudes in strangeness changing and strangeness conserving decays, and among tree amplitudes in these decays. Such relations may be used with branching ratio measurements to constrain tree amplitudes in $|\Delta S| = 1$ and penguin amplitudes in $\Delta S = 0$ decays. This could give an indication about potential $CP$ asymmetries in certain modes. Expressions and values for decay amplitudes calculated in previous sections, where assumptions stronger than U-spin and U-spin breaking were made, must obey the quadrangle relations as well as these constraints.

The U-spin subgroup of $SU(3)$ is the same as the I-spin (isospin) except that the doublets with $U = 1/2, U_3 = \pm 1/2$ are

\[
\text{Quarks : } \begin{bmatrix} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} |d\rangle \\ |s\rangle \end{bmatrix},
\]

\[
\text{Antiquarks : } \begin{bmatrix} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} |\bar{s}\rangle \\ -|\bar{d}\rangle \end{bmatrix}.
\]

$B^+$ is a U-spin singlet, while $\pi^+(\rho^+)$ and $K^+(K^{*+})$ belong to a U-spin doublet,

\[
|0 \ 0\rangle = |B^+\rangle = |u\bar{b}\rangle,
\]

\[
\begin{bmatrix} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} |u\bar{s}\rangle = |K^+ (K^{*+})\rangle \\ -|u\bar{d}\rangle = -|\pi^+ (\rho^+)\rangle \end{bmatrix}.
\]

Nonstrange neutral mesons belong either to a U-spin triplet or a U-spin singlet. The U-spin triplet residing in
the pseudoscalar meson octet is
\[ \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} |\eta_8\rangle - \frac{1}{2} |\pi^0\rangle = \frac{\sqrt{2}}{2} |s\bar{d} - d\bar{s}\rangle \\ \frac{1}{\sqrt{3}} |\eta_3\rangle = \frac{1}{\sqrt{2}} |s\bar{d} + d\bar{s}\rangle \end{bmatrix}, \tag{30} \]
and the corresponding singlet is
\[ |0 \ 0\rangle = \frac{1}{2} |\eta_8\rangle + \frac{\sqrt{3}}{2} |\pi^0\rangle = \frac{1}{\sqrt{6}} |s\bar{s} + d\bar{d} - 2u\bar{u}\rangle. \tag{31} \]
In addition the \( \eta_1 \) is, of course, a U-spin singlet. We take \( \eta_8 = (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6} \) and \( \eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \). The physical \( \eta \) and \( \eta' \) are mixtures of the octet and singlet,
\[ \eta = \frac{2\sqrt{2}}{3} |\eta_8\rangle - \frac{1}{3} |\eta_1\rangle, \quad \eta' = \frac{2\sqrt{2}}{3} |\eta_1\rangle + \frac{1}{3} |\eta_8\rangle. \tag{32} \]
The U-spin triplet in the vector meson octet is
\[ \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} |\phi\rangle - \frac{1}{\sqrt{2}} |\rho^0\rangle - \frac{1}{\sqrt{2}} |\omega\rangle = \frac{\sqrt{2}}{2} |s\bar{d} - d\bar{s}\rangle \\ \frac{1}{\sqrt{3}} |\eta_3\rangle = \frac{1}{\sqrt{6}} |s\bar{s} + d\bar{d} - 2u\bar{u}\rangle \end{bmatrix}, \tag{33} \]
and the corresponding singlet is
\[ |0 \ 0\rangle_s = \frac{1}{\sqrt{6}} |\phi\rangle + \frac{\sqrt{2}}{2} |\rho^0\rangle - \frac{1}{2\sqrt{3}} |\omega\rangle = \frac{1}{\sqrt{6}} |s\bar{s} + d\bar{d} - 2u\bar{u}\rangle. \tag{34} \]
The SU(3) singlet vector meson is \( |0 \ 0\rangle_1 = (|\phi\rangle + \sqrt{2}|\omega\rangle)/\sqrt{3} \).

The \( \Delta C = 0, \Delta S = 1 \) effective Hamiltonian transforms like a \( \bar{s} \sim |1 \frac{1}{2}\rangle \) component \((\Delta U_3 = \frac{1}{2})\) of a U-spin doublet, while the \( \Delta C = 0, \Delta S = 0 \) Hamiltonian transforms like a \( \bar{d} \sim |\frac{1}{2} - \frac{1}{2}\rangle \) component \((\Delta U_3 = -\frac{1}{2})\) of another U-spin doublet. Since the initial \( B^+ \) meson is a U-spin singlet, the final states are U-spin doublets. The VP states can be formed from a \( K^+ \) or a \( \rho^+ \) belonging to a U-spin doublet \( |0 \ 0\rangle \), while the pseudoscalar meson belongs to the two U-spin singlets, \( |1 \ 1\rangle \) and \( |0 \ 0\rangle \), and to the U-spin triplet \( |1 \ 0\rangle \). These VP states resemble the corresponding PP states studied within U-spin in Ref. 18, the only difference being that the \( K^+ \) and \( \pi^+ \) are now replaced by \( K^{*+} \) and \( \rho^+ \). Alternatively, one may choose the pseudoscalar \( (K^+ \) or \( \pi^+) \) in a U-spin doublet, while the vector meson resides in the two U-spin singlets \( |1 \ 1\rangle \) and \( |0 \ 0\rangle \) and in the triplet \( |1 \ 0\rangle \).

One finds four cases in which four physical amplitudes are expressed in terms of three U-spin amplitudes, corresponding to final states in which one of the final mesons is a member of a U-spin doublet while the other belongs to the two U-spin singlets and the U-spin triplet. This implies two quadrangle relations among \( \Delta S = 1 \) amplitudes and two quadrangle relations among \( \Delta S = 0 \) amplitudes:
\[ 2\sqrt{2}A(K^{*+}\eta) + A(K^{*+}\eta^{'}) = \sqrt{3}A(\rho^+K^0) + \sqrt{3}A(K^{*+}\pi^0), \tag{35} \]
\[ 2\sqrt{2}A(\rho^+\eta) + A(\rho^+\eta^{'}) = \sqrt{3}A(\rho^+\pi^0) + \sqrt{6}A(K^{*+}\bar{K}^0) \tag{36} \]
\[ A(\rho^0K^+) + A(\omega K^+) = \sqrt{2}A(\phi K^+) - \sqrt{2}A(K^{*0}\pi^+) \tag{37} \]
\[ A(\rho^0\pi^+) + A(\omega\pi^+) = \sqrt{2}A(\phi\pi^+) + \sqrt{2}A(K^{*0}K^+) \tag{38} \]
The first two relations are straightforward generalizations of corresponding relations obtained for \( B \to PP \). The last quadrangle is expected to be squashed, since the two terms on the right-hand-side contain no tree amplitude and are expected to be smaller than each of the two terms on the left-hand-side. (See expressions and values in Table III.) All four relations among complex amplitudes hold separately for \( B^+ \) and \( B^- \) decays.

One may decompose the \( \Delta S = 1 \) and \( \Delta S = 0 \) effective Hamiltonians into members of the same two U-spin doublets multiplying given CKM factors 73, 74.

\[ \mathcal{H}_{\text{eff}}(\Delta S = 1) = V_{ub}^*V_{us}O_{\text{eff}} + V_{cb}^*V_{cs}O_{\text{eff}}, \tag{39} \]
\[ \mathcal{H}_{\text{eff}}(\Delta S = 0) = V_{ub}^*V_{us}O_{\text{eff}} + V_{cb}^*V_{cd}O_{\text{eff}}. \tag{40} \]

Hadronic matrix elements of the two U-spin doublet operators, \( O_{\text{eff}}^{\mu \nu} \), will be denoted \( A^\mu \) and \( A^\nu \) and will be referred to as tree and penguin amplitudes, where the latter include electroweak penguin contributions. Note that these amplitudes multiply different CKM factors in \( |\Delta S| = 1 \) and \( |\Delta S| = 0 \) processes. The expressions \( \text{(39)} \) and \( \text{(40)} \) imply relations among penguin amplitudes \( A^\mu \) in strangeness changing and strangeness preserving processes and identical relations among corresponding tree amplitudes \( A^\nu \).

Starting with processes involving \( \eta \) and \( \eta^{'}, \) one may simply transcribe results obtained for \( B \to PP \), replacing \( \pi^+ \) and \( K^+ \) by \( \rho^+ \) and \( K^{*+} \). Thus, one finds expressions for \( \Delta S = 0 \) penguin amplitudes in terms of sums of two \( |\Delta S| = 1 \) penguin amplitudes which are expected to dominate these processes 18.

\[ A^\nu(K^{*+}\eta) = A^\nu(K^{*+}\eta^{'}) - \frac{2}{\sqrt{3}}A^\nu(K^0\pi^0) \tag{41} \]
\[ A^\nu(\rho^+\eta) = A^\nu(K^{*+}\eta^{'}) - \frac{1}{\sqrt{6}}A^\nu(K^0\pi^+) \tag{42} \]

Since all amplitudes involve unknown strong phases, these are in general triangle relations. Assuming that the two penguin amplitudes on the right-hand-sides of each of Eqs. \( \text{(41)} \) and \( \text{(42)} \) dominate the respective processes, the rates of these processes may be used to obtain constraints on the penguin amplitudes on the left hand sides. For this purpose one would need to measure \( \text{B}(B^+ \to \rho^+ K^0) \) and improve the upper bound on \( \text{B}(B^+ \to K^{*+}\eta^{'}) \).

Similarly, one obtains expressions for \( \Delta S = 1 \) tree amplitudes in terms of sums of two \( \Delta S = 0 \) tree amplitudes,
\[ A^\mu(K^{*+}\eta) = A^\mu(\rho^+\eta) + \frac{2}{\sqrt{3}}A^\mu(K^{*+}\bar{K}^0) \tag{43} \]
The second terms on the right-hand-sides vanish in the approximation of neglecting annihilation amplitudes. This provides two equalities between tree amplitudes in $B^+ \to K^{*+} \eta'$ and $B^+ \to \rho^+ \eta'$. (In Tables III and IV these amplitudes are given by $t_\rho'$ and $t_\rho$, respectively.) The observed amplitude $A(B^+ \to \rho^+ \eta') = 31.2 \pm 4.7$ eV (see Table III) then implies, via Eq. (54), that the tree contribution in $B^+ \to K^{*+} \eta$ is $(31.2 \pm 4.7) \times \lambda(f_K/f_\rho) \simeq (8 \pm 1)$ eV. This is approximately the value calculated in Table IV.

Another set of U-spin relations, applying separately to penguin and tree amplitudes, can be derived for decay amplitudes involving $\rho^0$, $\omega$, and $\phi$. Physical amplitudes, consisting of penguin and tree contributions, may be decomposed into U-spin amplitudes,

$$A^u(K^{*+} \eta') = A^u(\rho^+ \eta') + \frac{1}{\sqrt{6}} A^u(K^{*+} \bar{K}^0) \ .$$

$$A^s(K^{*0} \pi^+) = A^s(\rho^0 K^+) - A^s(\bar{K}^0 K^+)/\sqrt{2} \ ,$$

$$A^c(\omega \pi^+) = A^c(\rho^0 K^+) + A^c(K^{*0} \pi^+)/\sqrt{2} \ ,$$

$$A^c(\phi \pi^+) = A^c(\rho^0 K^+) + A^c(K^{*0} \pi^+)/\sqrt{2} \ ,$$

$$A^c(\bar{K}^0 K^+) = A^c(\bar{K}^0 K^+) \ .$$

In the approximation of neglecting annihilation amplitudes the second terms in Eqs. (57)–(59) vanish. Thus, tree amplitudes within each of the three pairs of $\Delta S = 0$ and $|\Delta S| = 1$ processes involving a $K^+$ and a $\pi^+$ are equal. Assuming that the tree amplitude dominates $B^+ \to \omega \pi^+$, where the penguin amplitudes $p_\rho$ and $p_\rho$ interfere destructively, Eq. (58) implies a sizable tree amplitude in $B^+ \to \omega K^+$, $\approx 23$ eV $\times \lambda(f_K/f_\rho) \simeq 7$ eV. This value, calculated earlier in Table IV, permits a sizable tree-penguin interference in this decay. Table IV shows equal tree amplitudes in $B^+ \to \omega K^+$ and $B^+ \to \rho^0 K^+$. This result is beyond U-spin. All the tree amplitudes in Eqs. (59) and (60) vanish in our approximation.

VIII. CONCLUSIONS

We have analyzed the decays of $B$ mesons to a charmless vector ($V$) and pseudoscalar ($P$) meson in the framework of flavor SU(3). The relative magnitudes of tree and color-suppressed amplitudes extracted from data appear consistent with the factorization hypothesis. For example, the ratio of the tree amplitude in which the current produces a vector meson to that in which it produces a pseudoscalar is approximately $f_\rho/f_\pi$, and the ratio of color-suppressed to tree amplitudes is approximately that in $B \to PP$ data.

Penguin amplitudes are also extracted from data. Here we are not aware of successful a priori predictions of their magnitudes. For solutions compatible with other determinations of $\gamma$ [69], we find a fairly stable pattern of small final-state phases (mod $\pi$), implying small $CP$ asymmetries in all processes. In particular, we do not expect a large $CP$ asymmetry in $B^0 \to \rho^+ \pi^-$. We find a small relative strong phase between $t_\rho$ and $t_\rho$. There exist also solutions for $\gamma$ outside the expected range; these have larger final-state phases but cannot be excluded by present experiments. Our preferred $\gamma \approx 63^\circ$ fits favor a weak phase $\gamma$ within the range $57^\circ$–$69^\circ$ at the $1\sigma$ level, and $51^\circ$–$73^\circ$ at 95% c.l. if one restricts attention to the range $35^\circ$–$80^\circ$ allowed in fits to other data [54].

Predictions have been made for rates and $CP$ asymmetries in as-yet-unseen decay modes. Some of these modes, such as $B^+ \to \rho^+ K^0$, $B^0 \to \rho^0 K^0$, and $B^+ \to K^{*+} \pi^0$ should be seen soon.

A key assumption for which we have performed some tests and suggested others is the relation $p_\rho'/p_\rho \approx -1$ between penguin amplitudes in which the spectator quark is incorporated into either a pseudoscalar meson or a vector meson. This relation is quite well satisfied, with the question of a small relative strong phase between $p_\rho'$ and $-p_\rho'$ still open.
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APPENDIX: $B \to \rho^\mp \pi^\mp$ RATES AND ASYMMETRIES

In Table I the $CP$-averaged branching ratio $B_{\rho\pi}^\pm$ quoted for the decay $B^0 \to \rho^\mp \pi^\pm$ is the sum of the $CP$-averaged branching ratios for $B^0 \to \rho^- \pi^+$ and $B^0 \to \rho^+ \pi^-$. The $CP$ asymmetry $A_{CP}(\rho^\mp \pi^\pm) = -0.14 \pm 0.08 \equiv A_{\rho\pi}$ is

$$A_{\rho\pi} = \frac{B(\rho^+ \pi^-) - B(\rho^- \pi^+)}{B(\rho^+ \pi^-) + B(\rho^- \pi^+)} , \quad (61)$$

where

$$B(\rho^\pm \pi^\mp) \equiv B(B^0 \to \rho^\pm \pi^\mp) + B(B^0 \to \rho^\mp \pi^\pm) . \quad (62)$$

These quantities are related to the individual $CP$-averaged branching ratios and $CP$ asymmetries by \ref{CP-averaged branching ratios and CP asymmetries}

$$\frac{1}{2}[B(B^0 \to \rho^+ \pi^-) + B(B^0 \to \rho^- \pi^+)] = \frac{1}{2}(1 \pm \Delta C \pm A_{\rho\pi} C)B_{\rho\pi}^\pm , \quad (63)$$

where $C = 0.35 \pm 0.13 \pm 0.05$ and $\Delta C = 0.20 \pm 0.13 \pm 0.05$ are measured in time-dependent decays \ref{CP-averaged branching ratios and CP asymmetries}. The individual $CP$ asymmetries are

$$A_{CP}(B^0 \to \rho^- \pi^+) = \frac{A_{\rho\pi} - C - A_{\rho\pi} \Delta C}{1 - \Delta C - A_{\rho\pi} C} , \quad (64)$$

$$A_{CP}(B^0 \to \rho^+ \pi^-) = -\frac{A_{\rho\pi} + C + A_{\rho\pi} \Delta C}{1 + \Delta C + A_{\rho\pi} C} . \quad (65)$$

In calculating the entries in Table I for the individual branching ratios and asymmetries we have used the correlations among the input variables \ref{correlations among the input variables} to evaluate the experimental errors.

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