Rock mass performance monitoring with topology preserving quantised vector spaces

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Abstract. This work explains the development of a so-called rock mass system performance map, that provides a backdrop to assist in performance interpretation. The rock mass system performance maps presented in this paper are 2D graphical devices prepared using Sammon mappings, Learning Vector Quantisation, Self-Organising Topological Maps and combinations of these mathematical techniques. Using supervised or unsupervised learning methods, these algorithms project and partition high-dimensional vector spaces, of a dimension equal to the number of environmental and rock mass condition parameters considered, into 2D categories of rock mass condition. In providing 2D renderings, the techniques aim to preserve properties of the characterising vector space such as adjacency of states and topology. The condition of a rock mass can be ‘plotted’ on that map by identifying its k-nearest neighbours and interpreted relative to stability metric categories. Should the defining vector space include rock mass properties or environmental parameters that are repeatedly measured, or possibly continuously monitored, then the rock mass condition marker traces out a performance trajectory across the performance map. An example of rock mass performance map synthesis and use is presented.

1. Objectives and introductory definitions

The ultimate objective of the work reported in this paper is to show how the performance of rock engineering systems may be characterized with an instance of a state vector formed from the values of rock mass properties and geomechanical environmental factors. It will then show how these may be plotted on partitions of complete real valued vector spaces with dimension equal to that of the defining state vector, mapped or projected into a 2D charts of practical utility.

By ‘rock engineering system’ we refer to specific application domains of geomechanics for engineering purposes in the same sense conveyed by Hudson [1], exemplified by rock slopes, rock foundations, and underground excavations in rock masses, whether these be created for civil or mining engineering or other similar purposes. In using the term ‘rock properties’ we do not necessarily refer only to mechanical properties of the rock material, but take the term to generalize to mechanical properties applying to rock masses incorporating the DIANE concepts articulated by Hudson and Harrison [2] at varying engineering scales, as well as those mechanical properties resulting from introduction of stabilizing elements that render a composite rock mass engineering material. By geomechanical environmental factors we refer to the physical fields the rock engineering systems may be subjected to, depending on location, time and application. More specifically these could be...
exemplified by the stress field, water flow field, and temperature field, represented in the coupled thermo-hydro mechanical conceptual frameworks formulated by Hart & St John, [3].

Hudson’s classic diagram of analytic and synthetic approaches to rock engineering in [1] reflects two differing schools of thought to approaches to rock engineering. The first is rock engineering systems can only be understood in terms of differential equations of motion, fluid flow and thermal potential applied to discretized embodiments of the geometry of practical rock engineering projects so that change caused by disturbance of the physical system through excavation can be estimated or predicted. The alternative is that there is no point trying to rely on such numerical methods to design in real masses of rock materials because the latter are too complex in reality, the problem is data sparse, and the numerical methods must be regarded as an abstraction of reality at best; a better approach is to distill precedent practices into rock mass classification techniques and empirically established design charts. There is no correct answer to which of these is best. Sensible practitioners would rely on both approaches regarding these as complementary sources of valuable design decision support. Practical rock engineering design methodologies reside on a continuum between these extremes. What is harder to understand is why the extremes of approach are extremes: how are they related?

2. Distilling the principles of performance mapping to render maps for rock engineering

It is asserted that performance mapping schemes can be prepared to characterise rock engineering systems. In making the assertion, it is not necessary for the full dynamics of that rock engineering system to be articulated; the dynamics just have to be known to exist, and clearly, this must be true for any physical system. This is useful, as rock engineering systems are known to be physically complex, any associated mathematical description may be envisaged to be extremely complex. It could be argued that the best available prescriptions of the dynamics of a rock engineering mechanical system are those described in the software of the numerical modeling packages routinely adopted in rock engineering. Such software, ultimately, sets out a very large number of differential equations, each in a form consistent with Newton’s 2nd, and associated Laws, and solves those simultaneously, with additional equations reflecting constraint (e.g., strain / displacement compatibility for continuum approaches, slip subject to kinematic constraint for discontinuum approaches).

Anyone who has used one of those software tools will fully appreciate the futility of attempting to generate a type of computationally iterated map (or phase portrait) with basins of attraction of dynamical behaviour, characterized by Lyaponov exponents (as set out in detail in Thompson and Stewart in [4], e.g.) for a rock engineering system. However, if this was achievable, one would find that the ‘coordinates’ of that map would be the mechanical properties of the system (such as stiffness, or damping coefficient) and parameters describing the intensity of mechanical fields that system was subjected to (such as forcing magnitude or stress, and forcing frequency). If there were no computational barrier, numerical methods could be used to establish comprehensive system performance maps like those in [4] for specific cases of rock engineering systems. Lyaponov exponents could be calculated from the results of such simulations, behavioural regimes could be identified, and associated with the system properties and driving factors. Once established, the properties of the corresponding rock engineering systems could be monitored, referred to the map, so that an assessment can be made regarding whether any of the accessible regimes of behaviour are undesirable or potentially hazardous based on the properties and parameters alone, without the dynamical description. Such maps, being based on the dynamics of the rock engineering system, and objective assessment of stability measures, would present a rational conceptual framework for rock engineering performance monitoring and assessment.

Such an outcome is not just aspirational. The simplification enabled for creating the performance maps is that the dynamics of a rock engineering system could be by-passed completely, if focus is retained directly on the properties and driving factors known to affect the dynamics—only, and some method is available to associate behavioural regimes expected with the values of those properties and driving factors. Despite their starkly contrasting empiricism, the various engineering rock mass
classification schemes that have emerged from the rock engineering discipline in the decades since the 1960s could be argued to be techniques that do precisely that. They concentrate on the rock mass properties and the geomechanical driving factors and, in the simplest cases, associate these with excavation failure or stability (distinct behavioural regimes) present within their case study data sets.

3. A rock engineering performance map based on the rock mass rating system

One such example is the rock mass rating system presented in Bieniawski [5], a framework that will be adopted to illustrate the preparation of a rock engineering system performance map. Simplifying considerably for the purposes of illustration, the basic classification system contains 5 parameters. Depending on the parameter value, expressed in geomechanical units, a rating value is determined for each parameter.

Table 1. Rock Mass Rating system parameters and their ranges

| Parameter                        | Minimum rating value | Maximum rating value |
|----------------------------------|----------------------|----------------------|
| Intact rock strength (UCS)       | 0                    | 15                   |
| Rock Quality Designation (RQD)  | 3                    | 20                   |
| Discontinuity spacing (MS)       | 5                    | 20                   |
| Discontinuity condition (DC)     | 0                    | 30                   |
| Groundwater (GW)                 | 0                    | 15                   |

A city-block-type metric constituting the sum of the rating values is computed, called the rock mass rating, and this is a value that can be used to associate the rock mass, ‘measured’ by the parameter values, to a condition category, ranging from very poor (5) to very good (1), or a duration for which an opening created in that rock mass will remain stable, ranging from 30 minutes for a 1 m excavation span (5) to 20 years for a 15 m span (1). The parameters are listed and the range of their rating values are set out in table 1. The rock mass condition categories, and the corresponding ranges of the rock mass rating are set out in table 2.

Table 2. Rock Mass Class boundaries used to label RMR state vectors, and the maximum number of state vectors in each class for integer granularity

| Class Label | Lower limit of RMR value | Upper limit of RMR value | Rock Mass Class          | Number of state vectors in class |
|-------------|---------------------------|---------------------------|--------------------------|---------------------------------|
| 5           | 3                         | 20                        | V (Very poor rock)       | 42330                           |
| 4           | 21                        | 40                        | IV (Poor rock)           | 684903                          |
| 3           | 31                        | 60                        | III (Fair rock)          | 1195116                         |
| 2           | 41                        | 80                        | II (Good rock)           | 357031                          |
| 1           | 50                        | 100                       | I (Very good rock)       | 6188                            |

The five real valued parameter ratings can be taken together to form a 5D vector that measures the state of a rock mass, specific instances of which are exemplars drawn from the real, continuous vector space with bounds according to table 2. The performance map sought seeks to partition that bounded vector space respecting its 5D nature, and, for practical utility, to render it to 2D for visualization. The first step is to represent the bounded real vector space with a finite number of exemplar vectors drawn from it. Such a representation is referred to as a vector quantization. Thereafter, three distinct techniques were applied in production of the vector quantization visualizations presented: i) a Sammon mapping [6], ii) Learning Vector Quantization [7], and iii) a Self-Organizing Map [8].

3.1. RMR vector space quantization

The vector quantization can be instanced as a set, $P$, with $N$ elements, each element comprising one of the exemplar vectors, with each of those comprising 5 geomechanical parameter rating values. A set $L$ can also be formed, also with $N$ elements to contain the rock mass class labels: one of the integers from 1 to 5, as specified in table 2. $P$ and $L$ are also taken to be ordered with an order-preserving mapping, $f$, existing between them that associates elements of $L$ with elements of $P$. A computer
Program was written to systematically enumerate all possible instances of state vectors drawn from the 5D rock mass rating parameter space, produced according to a condition that the rock mass rating system parameter rating values could only assume integer values. The rock mass class labels were determined too. With this condition, the number of state vectors generated within the file for each class of the rock mass rating, were as indicated in Table 2, with \( N \) totaling 2,285,568.

3.2. Reducing the dimensionality of the vector space with a Sammon mapping

A Sammon mapping is the result of an algorithm that starts by computing the \( N(N-1)/2 \) Euclidean distances between each member of a set, \( P \), of \( N \) vectors of dimension \( K \) and all other vectors of the same set. Thereafter, a gradient descent-based iterative procedure adjusts the locations of an ordered set \( T \) of \( N \) randomly initialized 2D position vectors, where an order-preserving mapping, \( g \), exists between each element of \( T \) and each element of \( P \), so that the sum of squared errors between 2D Euclidean distances and KD Euclidean distances between corresponding pairs in \( T \) and \( P \) respectively, is minimized. At the conclusion of the procedure, the idea of closeness or spatial adjacency between a pair of vectors in 2D extends to the corresponding pair of vectors in KD. A Sammon mapping computation is intensive, both on computer memory and computer processing time; when it is used, the tendency is to map fewer vectors than many, if this is possible and practical. After the computations, because \( f \) maps \( L \) to \( P \) and \( g \) maps \( T \) to \( P \), and all are ordered, an order-preserving mapping, \( h \), exists that associates \( L \) to \( T \). The 2D vectors, \( T \), can be plotted and labeled with \( L \) for visualization.

3.3. Reducing the vector space quantization cardinality with Learning Vector Quantization

Learning Vector Quantization is a procedure through which spatial relationships on the set \( P \) of \( N \) vectors of dimension \( K \) can become faithfully represented by a set \( Q \) of \( n \) vectors of dimension \( K \), where \( n \ll N \). The idea of preserving the faithfulness of spatial relationships is based on the values of the labels, \( L \). The spatial relationships in the set \( P \) become embedded in set \( Q \) through an iterative procedure. The \( q_i \in Q \), can be initialized randomly, with random labeling defined in a label vector \( M \). Another (more rapidly converging) option is to initialize \( Q \) by sub-sampling of \( P \) and initialize \( M \) from \( L \) using \( f \). In one variant of the LVQ algorithm, the \( q_i \) are processed in turn so that for each one, a member \( p_{active} \in P \) that is ‘closest’ to \( q_i \) is identified on the basis of a distance metric on the vector components. The components of \( q_i \) are manipulated to move away from \( p_{active} \) if the labels of \( q_i \) and \( p_{active} \) are different, and are manipulated to move together if their labels agree. After this supervised learning process involving repeated presentation of \( Q \) to \( P \) is complete, the result is that the boundaries between categories or partitions of the KD space embodied in \( N \) state vectors of \( P \) and its associated labels \( L \) are well represented by just the \( n \) vectors of \( Q \) and its labels, \( M \). LVQ can thus be conceived to be a type of data compression procedure. Thereafter, to visualize those boundaries, a Sammon mapping can be applied to the \( n \) members of \( Q \) to produce a 2D position vector set \( T \) which can be labeled with \( M \) for visualization.

3.4. Reducing the dimensionality and cardinality of the vector space quantization with a Self Organising Map

A Self Organising Map (SOM) is essentially an alternative technique to visualize spatial structure present in a KD vector space in 2D. Elements, \( w_i, i = \{1...n\} \) of a set, \( W \), of KD weight vectors, are individually associated through an order preserving mapping, \( v \), to members of a set, \( S \), \( (s_i \in S, i = \{1...n\} \) of regularly spaced nodes on a lower dimensional (typically 2D) lattice, of dimensions \( n^{0.5} \times n^{0.5} \), which can be rectangular or hexagonal. Set \( W \) becomes ‘tuned’ to the set \( P \) through an unsupervised (without labeling information, \( L \)) competitive learning procedure. The (typically Euclidean) distances between a \( p_i \) and \( W \) are computed so that the ‘closest’ weight vectors in \( W \) can be identified, and those weight vectors are moved toward \( p_i \). This process for a single \( p_i \) will repeat for all
members of \( P \) and then \( P \) will be presented in its entirety again to \( W \) on each of a series of tuning iterations.

3.5. Labelling the vector space of the SOM
After the tuning process, the 2D nodes, \( S \), defined in the SOM process may be labeled. As \( N \gg n \), no one-to-one mapping can exist between labels \( L \) and a set, \( D \), of labels for nodes \( S \), and so an heuristic labelling process needs to be defined. For each \( p_i \) in \( P \), the \( k \) nearest neighbours to \( p_i \) in \( W \) are found, and the label, \( l_i \), defined by \( f \), is allocated to nodes \( s_j, j = \{1...k\} \). As multiple labels may be allocated to \( s_i \) through that procedure, at its conclusion, some means of deciding what the final label is for all the \( s_i \) must be decided upon. If the labels happen to be numeric, the final label could be defined to be the mean of labels allocated to individual nodes, or another similar arithmetic measure. If not numeric, the final label for a node could be the modal label allocated, in which case labelling would be done according to a process of majority voting.

3.6. Codebooks for rock mass performance monitoring and visualization
Rock mass performance monitoring with visualization can be achieved through the formation of set triplets, referred to as codebooks, where the constituent sets arise from the processes outlined in sections 3.1. to 3.5. The codebook: \( C_1 = \{P, T, L\} \) permits use of the complete quantized vector space of the RMR system, with \( N \) elements in each set, and visualization by means of a Sammon mapping. In comparison, codebook \( C_2 = \{Q, T, M\} \) with \( n \) elements in each set may be regarded as an approximation to \( C_1 \) that is produced using the data compression characteristic of LVQ and dimensionality reduction with a Sammon mapping. Using the computer programs from the LVQ_PAK software package [7], produced a codebook \( C_2 \) that classified 98.91\% of the state vectors in \( P \) correctly when the respective labels of the closest \( q_i \) to \( p_i \) where compared after training. Finally, \( C_3 = \{W, S, D\} \) is the codebook that arises from application of the SOM technique to \( P \) and a using majority voting labelling heuristic on \( L \) to establish \( D \). In this case, 88.93\% classification accuracy was established for \( C_3 \) using the computer programs from the SOMPAK software package [9]. Instances of \( C_2 \) and \( C_3 \) are presented in figure 1. The commands and postprocessing procedures to produce these have been archived at GitHub at: https://github.com/deanleemillar/EUROCK-21.

4. Rock engineering system performance monitoring
Either of the two quantized vector spaces for the rock mass rating system visualized in figure 1 may be used to provide context for rock engineering performance monitoring. Codebook \( C_2 \) aims to preserve the topology of the RMR system by constraining Euclidian distances between quantizing vectors; codebook \( C_3 \) aims to preserve topology by constraining adjacency between quantizing vectors. Through application of a \( k \)-nearest neighbour operator operating with \( 5D \) vectors, states of rock masses can be overlaid on the 2D visualizations.
A tunnel is considered within a rock mass with Uniaxial Compressive Strength of 150 MPa, a RQD of 83%, mean discontinuity spacing of 1.8 metres, slightly rough discontinuity surfaces, with separation < 1mm that have slightly weathered wall rock, water inflow of between 10 and 25 l/min per 10 m of tunnel length. Applying the RMR system rating tables, the state can be denoted (12,17,18,25,7), and this plots as State A on the maps. State B = (1, 20, 18, 25, 15) and C = (15, 20, 20, 9, 15) share the same RMR value (of 79) as A, and are in the same RMR class (of II), however by inspection of the individual rating values, they can be seen to have quite different character. A has moderately strong rock, is moderately fractured by discontinuities that have excellent condition but is wet. B comprised very weak rock with fewer fractures and is dry. C comprises moderately strong intact rock and is dry, and has with relatively few discontinuities, but these are in very poor condition where they occur. These three rock masses plot in different locations on C$_2$ or C$_3$; their position on the maps discriminates their character – a desirable attribute.

The process of excavating, stabilisation and construction of the tunnel is now discussed in a context of a sequence of changes to a rock mass in its undisturbed state A. The excavation method is assumed to be blasting and it is further supposed that the blasting process brings about the following changes in rock mass condition. Firstly the near field rock strength is reduced, such that the UCS rating value is reduced from 12 to 10. Secondly, blasting will introduce new fractures into the rock mass in the excavation periphery. Rating values for both the RQD and Discontinuity spacing components are thus reduced to 15 and 12 respectively. Within an uniaxial stress field, a state of tension is induced in the crown and invert of most tunnel geometries which could lead to an increase in discontinuity aperture in these areas. For the same geometry and field stress, compressive stress concentration in sidewalls will tend to reduce fracture aperture. For this example, the tensile regions are considered more important and consequently the rating value for Discontinuity condition is reduced slightly from 25 to 23. As fracture hydraulic conductivity increases with the cube of fracture aperture (see, for example, Priest [10]), it is to be expected that the increase of fracture aperture in the crown of the tunnel will lead to a greater rate of ingress of water. The rating value for Groundwater is then reduced from 7 to 4.
reflecting a change in description from “Wet” to “Dripping”. The rock mass state vector, at the end of shift, immediately after blasting is then \(\text{State } 2 = (10, 15, 12, 23, 4)\).

It is supposed that at the time the tunnel is next inspected on the following day, the increased influx of groundwater has washed out some of the highly weathered discontinuity wall rock material. Further widening of discontinuity aperture is thus evident and ground water can be described as “Flowing” into the excavation, requiring the use of an auxiliary pump. As a result, the RMR rating values for Discontinuity Condition and Groundwater are reduced further in comparison to the undisturbed state to 15 and 0 respectively. The new state vector is \((10,15,12,15,0)\); is (correctly) classified as being a member of Class III and is labelled on the rock engineering system performance map as ‘State 3’.

The tunnelling gang then set to work to inject grout into the rock mass in the tunnel periphery with two effects on the state of the rock mass. Firstly, ground water inflow is reduced so that the Groundwater component of the RMR system can be re-rated at 4, corresponding, again, to conditions described as “Dripping”. The injected grout reduces water inflow by filling the discontinuities in the rock mass acting as water conduits. Thus their aperture must be reduced and the Discontinuity Condition component of the RMR system is re-rated at 18. After the injection grouting process, the rock mass state vector is then: \((10,15,12,18,4)\). It is (correctly) classified as being a member of Class III and its location is labelled on the rock engineering system performance map as ‘State 4’.

Further tunnel stabilisation work then proceeds. It is supposed that a systematic pattern of tensioned rock bolting is installed in the crown and invert regions of the tunnel periphery. This has the effect of tightening already tight discontinuity surfaces and closing those that are open or have been opened during the excavation process. The effective RQD and mean discontinuity spacing is increased, and the reduced discontinuity apertures also improve the general discontinuity condition. As a result, the RQD, Discontinuity Spacing, and Discontinuity Condition RMR rating components are modestly re-rated upwards to 16 from 15; 13 from 12, and 20 from 18, respectively. The new rock mass state vector is thus: \((10,16,13,20,4)\). This state is (correctly) classified as being a member of Class II and is labelled on the rock engineering system performance map as ‘State 5’. It is supposed that this state represents the commissioning condition of the section of the tunnel that has been constructed.

State 5 thus represents the reference state for all future monitoring of the tunnel performance. Various scenarios can now be imagined, for example; introduction of further tunnelling works within the near field of the tunnel, environmental perturbation in the form of extreme ground water recharge due to a prolonged period of heavy rain; or dynamic loading due to a seismic event. Providing, the perturbations can be expressed in terms of changes to the state defining parameters, that is: the RMR components, the location of the current rock mass / tunnel performance can continue to be plotted on the performance map. To illustrate continued performance monitoring a final state is explained in the context of the example. It is supposed that over some years, opening of the tunnel leads to a minor degradation of the mechanical strength of the intact rock in the near field due to oxidation processes. At the same time, a few of the tensioned rock bolts may have rusted such that their reinforcing bond with the rock is broken. As a result, in places, discontinuities may have opened up and the ground water flow increased again. The state vector defining this condition may be written: \((8,16,12,17,2)\). The state vector is (correctly) classified as being a member of Class III and is plotted on the performance map as ‘State 6’. The location of State 6 happens to be the same as State 4, that is, the state prior to stabilization. Migration of the state marker on the performance map back to State 6 may thus represent a trigger that remedial work is required.

A basis for long term rock engineering system performance monitoring with automated interpretation and alerts is indicated. The procedures illustrated here may adopt alternative conceptual frameworks, such as the Q-System [11][12], where there is also an established empirical evidence base, rendering backward compatibility with precedent practice. They may also be extended to include additional variables that may directly indicate the presence of specific engineering measures (e.g. presence of stabilization structural elements, slope drainage systems), or the incorporation of a more quantitative and rational basis for partitioning the quantized vector space (e.g. with a limit equilibrium
conceptual framework for rock slopes, the factor of safety may offer a suitable label to characterize rock slope performance as in [13],[14]).

5. Conclusions
Inspired by general methodological procedures adopted in the discipline of non-linear dynamics and chaos for the simplest mechanical systems, this work has developed a similar procedure to objectively, and robustly characterize the multiplicities of behaviour of rock engineering systems which are orders of magnitude more complex. The conceptual framework of the rock mass rating system, well established in rock engineering practice, was adopted to illustrate the procedures, but other conceptual frameworks, that are equally well established within the disciplines of rock mechanics and rock engineering could be adopted instead and turned to the same purpose, for similar benefit.

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