LONG RANGE COHERENT MANIPULATION OF NUCLEAR SPINS IN QUANTUM HALL FERROMAGNET

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A coherent superposition of many nuclear spin states can be prepared and manipulated via the hyperfine interaction with the electronic spins by varying the Landau level filling factor through the gate voltage in appropriately designed Quantum Hall Ferromagnet. During the manipulation periods the 2D electron system forms spatially large Skyrmionic spin textures, where many nuclear spins follow locally the electron spin polarization. It is shown that the collective spin rotation of a single spin texture is gapless in the limit of zero Zeeman splitting, and may dominate the nuclear spins relaxation and decoherence processes in the quantum well.

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I. INTRODUCTION

The emerging fields of quantum information processing and quantum computing (QC) have stimulated recently a flurry of activity in the established fields of atomic and condensed matter physics, approaching fundamental questions, such as the influence of measurement on quantum mechanical systems or the meaning of phase coherence in interacting many particle systems, from a strikingly new point of view. Experimental realization of QC has been so far successfully achieved, however, only in devices consisting of a few qubits.

The idea presented in this paper should not be considered as a proposal for building any kind of quantum computer. Instead it addresses the general problem of how to store and manipulate a large number of qubits without losing their phase coherence. This is done with respect to a concrete physical system consisting of nuclear spins in semiconducting heterojunctions under the conditions of the odd integer Quantum Hall (QH) effect\textsuperscript{5}. Our proposal has been motivated by the set of experiments, reported in\textsuperscript{1,3}, where the Knight shift, $K_S$, and the spin lattice relaxation time $T_{1S}$ of the $^{71}$Ga nuclei in GaAs multiple quantum well (MQW) structure under perpendicular magnetic field were detected by means of the optically pumped NMR technique. The electronic Landau level filling factor was varied in these experiments by tilting the magnetic field axis with respect to the 2D layers. The Knight shift was found to reduce dramatically as the filling factor was shifted slightly away from $\nu = 1$, indicating that the injection of a single charge into the 2D electron system is followed by reversal of many electronic spins. In the same interval of the filling factor the relaxation time was found to drop by several orders of magnitude with respect to its value in the quantum Hall ferromagnetic ground state.

Both effects are considered as strong evidence for the creation of skyrmionic spin textures in the electronic spin distribution as the filling factor shifts slightly away from unity, and indicate the crucial importance of the hyperfine interaction in controlling the nuclear spin dynamics. Since the hyperfine interaction is the dominant coupling of the nuclear spins to their environment they may be exploited as quantum bits (qubits), provided the environment, that is the 2D electron gas, is in a nondissipative, coherent quantum state (e.g. like the QH ferromagnetic state at LL filling factor $\nu = 1$ at low temperature\textsuperscript{6}). Furthermore, as will be shown below, near $\nu = 1$ it may be possible to manipulate coherently a large number of nuclear spins through the hyperfine interaction with the electronic spin texture by \emph{varying a single parameter}, such as the Landau level filling factor, through changes in the gate voltage.

At filling factor $\nu = 1$ the ground state of the 2D EG is ferromagnetic even in the limit of zero Zeeman energy\textsuperscript{7}. Flipping nuclear spins in this state through the hyperfine interaction is followed by the creation of spin excitons\textsuperscript{8-9}. The energy cost of this excitation can be minimized if both the electron and the hole are created at the nuclear position, where the energy gain associated with the e-h Coulomb attraction is exactly compensated by the exchange energy of the hole. Yet, the remaining small Zeeman energy (on the electronic energy scale) is a huge energy gap for the nuclear spins. The extremely long spin-lattice relaxation time observed by Barrett et al:\textsuperscript{3,4} may be due to this energy gap (see below, however). Overcoming the Coulomb attraction by increasing the e-hole distance leads to increasing the exciton transverse momentum. The corresponding excitation energy scales with the Coulomb energy, which is $\sim 100K$, that is much
larger than the Zeeman splitting. The spin exciton spectrum is strongly influenced by long range electrostatic potential fluctuations, which can trap the electron and the hole separately in local potential wells and so reduce, or even completely remove the energy gap.\(^{10}\)

Slightly away from \(\nu = 1\) the lowest energy state of the electron gas is a spin texture, in which the average spin distribution is smoothly twisted in space in order to minimize the exchange energy. The size of the twist is determined by the Zeeman energy.\(^{11,12,13}\) Microscopic calculations, based on Hartree-Fock (HF) approximation for a single, isolated Skyrmion,\(^{11,12,13}\) have found a family of low energy excitations, with an approximately quadratic relation between the energy and the number of flipped spins, \(K\), which can be associated with the kinetic rotational energy of the entire spin texture about its symmetry axis. However, except for the special case where \(K\) is a half integer, the spectrum has an excitation gap, which is some fraction of the large Coulomb energy scale. To account for the observed enhancement of the nuclear relaxation rate, these authors have suggested\(^{12}\) that at filling factor slightly away from \(\nu = 1\), where there is a finite density of Skyrmions, the ground state is a Skyrmion crystal, for which the spin waves spectrum is gapless due to the breakdown of the global spin rotation symmetry. This appealing interpretation is hard to reconcile with the latest optically pumped NMR (OPNMR) measurements.\(^{12}\) Based on this OPNMR data, the many-Skyrmion state does not appear consistent with the closed packed periodic lattice described in\(^{13}\). Instead, it was suggested\(^{12}\) that the Skyrmions’ tail is drastically reduced, e.g., due to the effect of disorder potential,\(^\text{12}\) leading to some kind of spatially inhomogeneous state of nearly independent pinned Skyrmions.

This conclusion motivates us to carefully reexamine the problem of spin excitations in a single Skyrmion.\(^{11,14,15}\) Our study has shown that the excitation gap in the collective rotational spectrum of a single Skyrmion goes to zero when the Skyrmion radius tends to infinity, and that for the characteristic Skyrmion sizes found experimentally the gap is a small fraction of the Zeeman energy scale, rather than of the large Coulomb energy scale, as claimed previously. This is done within the framework of a phenomenological approach, similar to that taken by Girvin et al,\(^{13}\) which is based on microscopic HF calculation. The influence of these low-lying electron spin excitations on the nuclear spin polarization and phase coherence via the hyperfine interactions is then discussed.

II. THE MODEL

We start our analysis by considering the Hamiltonian for nuclear spins interacting with 2D electron gas in MQW structure

\[
\hat{H} = -\hbar \gamma_n \sum_j \hat{I}_j \cdot \mathbf{B}_0 - \hbar \gamma_e \int d^2 r \mathbf{S} (r) \cdot \mathbf{B}_0 + \hat{H}_{ee} + \hat{H}_{en},
\]

where

\[
\hat{H}_{en} = A \sum_j \mathbf{S} (r_j) \cdot \hat{I}_j\]

Here \(\hat{I}_j\) is the nuclear spin operator located at \(r_j\), \(\mathbf{S} (r)\) is the electronic spin density operator, \(\mathbf{B}_0\) is the external magnetic field, which is assumed to be oriented perpendicular to the 2D electron gas (\(\mathbf{B}_0 = B_0 \mathbf{z}\)), \(\hat{H}_{ee}\) is the electron-electron interaction, \(\gamma_n = g_n \mu_n / \hbar\) and \(\gamma_e = g_e \mu_B / \hbar\) the nuclear and electronic gyromagnetic ratios respectively, and \(A = \frac{\mu_B}{2} g_n \mu_n g_0 \mu_B |u_0 (0)|^2\) is the Fermi contact hyperfine coupling constant. In this expression \(u_0 (0)\) is the periodic part of the Bloch wavefunction at the nucleus, and \(g_0\) is the g-factor of a free electron. We use the standard normalization \(\int |u_0 (r)|^2 d^2 r = v\), where \(v\) is the volume of a unit cell in the crystal.

The manipulation of the nuclear spins is carried out through spin flip-flop processes, associated with the ‘transverse’ part of the interaction Hamiltonian \(\hat{H}_{en}\) Eq. (2), i.e., \( \frac{1}{2} A \sum_j (\hat{I}_{j,+} \hat{S}_{-} (r_j) + \hat{I}_{j,-} \hat{S}_{+} (r_j))\). where \(\hat{I}_{j,+} = \hat{I}_{j,x} + i \hat{I}_{j,y}\), \(\hat{I}_{j,-} = \hat{I}_{j,x} - i \hat{I}_{j,y}\), and \(\hat{S}_{+} (r) = \psi^\dagger_\uparrow (r) \hat{\psi}_\downarrow (r)\), \(\hat{S}_{-} (r) = \psi^\dagger_\downarrow (r) \hat{\psi}_\uparrow (r)\). Here \(\hat{\psi}_\uparrow (r)\), \(\psi^\dagger_\uparrow (r)\) are the electron field operators with spin projections \(\sigma = \uparrow, \downarrow\).

The strength of the hyperfine coupling constant can be estimated by using the expression

\[
K_S \equiv \frac{1}{\hbar} A \langle \hat{S}_z (r_j) \rangle \approx \alpha (n_{2D} / \pi l)
\]

for the Knight shift at filling factor \(\nu = 1\), where \(\alpha \equiv A / \hbar\), \(n_{2D}\) is the areal density of the 2D electron gas, and \(l\) is the width of the QW. For the \(^{31}\)Ga nucleus (with \(g_n \approx 0.27\) in GaAs \(|u_0 (0)|^2 \sim 10^4\), and for the parameters characterizing the sample used by Barrett et al,\(^{12}\) i.e., \(\ell \approx 30 \text{nm}\), and \(n_{2D} = 1.5 \times 10^{11} \text{cm}^{-2}\), one finds \(K_S \sim 10^4 \text{Hz}\), in good agreement with Ref.\(^{12}\).

In the framework of the model just described, we will now show how, by varying the LL filling factor, a large number of nuclear spins can be prepared in a state appropriate to start quantum computation. A number, \(n\), stored in the memory of a hypothetical quantum computer made of nuclear spins, may be described as a direct product of \(N\) pure nuclear spin states

\[
|n\rangle = |n_1\rangle \otimes |n_2\rangle \otimes ... \otimes |n_N\rangle
\]

where \(|n_j\rangle = \sum_{\sigma = \pm 1} \delta_{n_j, \sigma} |j, \sigma\rangle\), \(\delta_{n_j, \sigma}\) is the Kronecker delta, and \(|j, \sigma\rangle\) is a nuclear state with spin projection \(\sigma\) for a nucleus located at \(r_j\). To carry out a quantum computing process, however, a coherent superposition of such
products, i.e. \(|\psi\rangle = \sum_{n=1}^{N} \alpha_n |n\rangle\), should be prepared at time \(t = 0\). This superposition may be represented more transparently for our purposes by the direct product of \(N\) mixed spin up and spin down states,

\[
|\psi\rangle \equiv \prod_{j=1}^{N} \left( |u_j \rangle \downarrow + v_j \rangle \uparrow \right)
\]

with the normalization \(|u_j|\) = \(|v_j|\) = 1.

While the hyperfine coupling with the electron spins is the dominant interaction of the nuclear spin qubits system with its environment, it is only a weak perturbation to the electron spins system. Thus, at a temperature which is much lower than any electronic energy scale in this system, the electronic spins at LL filling factor \(\nu\) should be in the corresponding ground state, \(|0; \nu\rangle\). One may, therefore, construct an effective nuclear spin Hamiltonian by projecting the combined nuclear-electronic spin Hamiltonian, Eq. (1), on the ground electronic state, \(|0; \nu\rangle\). The resulting effective nuclear spin Hamiltonian can be written as:

\[
\hat{H}_n = -\hbar \gamma_n \sum_{j=1}^{N} \hat{I}_j \cdot \mathbf{B}_0 + \sum_{j=1}^{N} \mathbf{S}(\mathbf{r}_j) \cdot \hat{I}_j
\]

where \(\mathbf{S}(\mathbf{r}) = \langle 0; \nu | \mathbf{S} | 0; \nu \rangle\) is the expectation value of the electronic spin density in the ground electronic state at filling factor \(\nu\).

The corresponding state of the nuclear spin system can be found by considering \(u_j\) and \(v_j\) as variational parameters, and then minimizing the energy functional \(E_n = \langle \psi | \hat{H}_n | \psi \rangle\), with respect to \(u_j\) and \(v_j\). As noted above, at \(\nu = \nu_0 \neq 1\) S(\mathbf{r}) has nonzero transverse components, associated with the skyrmionic spin texture, smoothly varying in space.

A simple calculation shows that

\[
E_n = \frac{1}{2} \sum_{j} \left[ \Omega_j \left( |v_j|^2 - |u_j|^2 \right) + \left( \frac{2}{\hbar^2} \left( \frac{\varepsilon_j}{\hbar \Omega_j} \right)^2 \right) \right]
\]

where \(\Omega_j = \gamma_n B_0 - \alpha S_z(\mathbf{r}_j)\) is the local nuclear Zeeman energy. For the extreme conditions (subject to the normalization \(|u_j|^2 + |v_j|^2 = 1\) ) are readily solved to yield:

\[
|v_j|^2, |u_j|^2 = \frac{1}{2} \left( 1 \pm \frac{\hbar \Omega_j}{\varepsilon_j} \right), \quad \varepsilon_j = \sqrt{\hbar^2 \Omega_j^2 + 2 \left( \frac{\hbar}{\Omega_j} \right)^2}
\]

Thus, the nuclear spins distribution follows the distribution of the electronic Skyrmion spin texture. The key parameter here is the local mixing parameter

\[
\eta_j \equiv \frac{A/\hbar \Omega_j}{|S_+(\mathbf{r}_j)|} = \left( \frac{2 \pi K_S / \Omega_j}{|S_+(\mathbf{r}_j)|} \right)
\]

which determines the local deviation of the nuclear spins state from a pure ferromagnet. Thus, for \(\eta_j \ll 1\) the many nuclear spin state is very close to a pure ferromagnet. In the opposite extreme limit, \(\eta_j \gg 1\), all individual nuclear spin states are equally probable, i.e. \(|v_j|^2 = |u_j|^2 \rightarrow 1/2\), and so one generates an ideal starting state for quantum computing. As we shall see below, this extremely strong mixing condition is not realistic. In the intermediate situation, where \(\eta_j \sim 1\) almost everywhere, the distributions \(|v_j|^2, |u_j|^2\) vary moderately around the mean value 1/2.

The condition for achieving such a desired situation is, therefore, two-fold: (1) The average Knight shift, \(K_S\), should be comparable to the average nuclear Zeeman frequency, \(\Omega\), i.e. \((2 \pi K_S / \Omega) \sim 1\); and (2) the transverse component of the normalized electronic spin density, \(|S_+(\mathbf{r}_j)|\), should be of the order one over a large spatial region (namely a region consisting of many nuclear spins). Usually the Knight shift is a small fraction of the NMR frequency, so that the first condition is not easily fulfilled. An exceptional example will be discussed toward the end of the paper. The second condition is satisfied by large skyrmionic spin texture (i.e. for sufficiently small effective \(g\)-factor).

Let us now outline very briefly a scenario for manipulating many nuclear spins in MQW by varying the LL filling factor. Very fast changes of the filling factor can be achieved without overheating the nuclear spins system, by varying the gate voltage. The process might start at an early time, \(t = -\tau_0\), when the filling factor was tuned at \(\nu = \nu_0\), slightly away from \(\nu = 1\), and then kept fixed until \(t = 0\). If the 'waiting' time \(\tau_0\) is much longer than the (relatively short) relaxation time \(T_1(\nu = \nu_0)\), then at \(t = 0\) the nuclear spins would be settled in their ground state corresponding to the electron gas at filling factor \(\nu = \nu_0\). By so doing the nuclear spin qubits are prepared in a state which is an appropriate initial state for quantum computing. However, to shield the nuclear spins from decoherence due to the low-lying electronic spin fluctuations, which are present at \(\nu = \nu_0 \neq 1\), the filling factor may be quickly switched back to \(\nu = 1\) (i.e. on a time scale shorter than \(T_1(\nu = \nu_0)\), so that the nuclear spins are trapped in their mixed, textured state, unable to relax for a long time to the pure ferromagnetic ground state dictated by the electron gas at \(\nu = 1\), since \(T_1(\nu = 1)\) is extremely long.
III. COLLECTIVE MODE

As discussed above, during the manipulation cycle, when the nuclear spins have relatively short relaxation and dephasing times, their dynamics is controlled by the low-lying spin fluctuations of the electron gas through the hyperfine interaction. For a single, isolated Skyrmion the rigid rotation of the entire spin texture about its symmetry (Z) axis is a zero mode, which can be responsible for such low energy fluctuations. The generator of this rotation, \( \tilde{L}_z \), is the Z-component of the angular momentum of the entire spin texture. To find a ‘classical’ Hamiltonian for this rotational motion in the electronic spin space, one may exploit the Hartree-Fock approximation for the Skyrmion energy near filling factor \( \nu = 1 \), consisting of Coulomb + Zeeman + nonuniformity energy, that is:\(^{13}\)

\[
E_{\text{rot}}(R) = \frac{3\pi^2 e^2}{2^6 \kappa R} + \frac{e^2}{4\kappa l_H} \sqrt{\frac{\pi}{2}} \left( \frac{R}{l_{sk}} \right)^2 \ln \left( \frac{2l_{sk}}{\sqrt{\pi} R} \right) \tag{5}
\]

where \( R \) is a variational parameter describing the Skyrmion core radius, \( l_{sk} \) is the length scale corresponding to the Skyrmion’s tail, \( l_{sk}^2 = 2\sqrt{\frac{\pi}{2}} |g| \tilde{a}_B| l_H^3 \), \( \tilde{a}_B = \kappa \hbar^2/m_0 e^2 \) is the effective Bohr radius (\( m_0 \) being the free electron mass, and \( \kappa \) the dielectric constant), and \( l_H = \sqrt{\hbar^2/eH} \) the magnetic length.

The Zeeman energy associated with the reversed spins is

\[ \Delta E_Z = g \mu_B H \tilde{L}_z = \frac{e^2}{4\kappa l_H} \sqrt{\frac{\pi}{2}} \left( \frac{R}{l_{sk}} \right)^2 \ln \left( \frac{2l_{sk}}{\sqrt{\pi} R} \right) \]

where, \( \tilde{L}_z \equiv L_z/\hbar \), and \( \tilde{e} \) stands for the natural logarithm base, so that the total number of reversed electronic spins in the Skyrmion is related to the core radius \( R \) through the expression:

\[ \tilde{L}_z = \left( \frac{R}{l_H} \right)^2 \ln \left( \frac{2l_{sk}}{\sqrt{\pi} R} \right) \tag{6} \]

Minimization with respect to \( R \) yields for the equilibrium core radius:

\[ \frac{3\pi^2 e^2}{2^6 \kappa R_{eq}^3} = \left( \frac{2}{l_H^3} \right) \ln \left( \frac{2l_{sk}}{R_{eq}} \right) |g| \mu_B H \]

while the second derivative

\[ \left[ \frac{\partial^2}{\partial R^2} E_{\text{rot}} \right]_{eq} \approx \left( \frac{|g| \mu_B H}{l_H^3} \right) \ln \left( \frac{2l_{sk}}{l_H} \right) \]

by Eq. (6), or

\[ U = \left[ \frac{\partial^2}{\partial L_z^2} E_{\text{rot}} \right]_{eq} \approx \left( |g| \mu_B H \right) \left( \frac{l_H}{R_{eq}} \right)^2 3 \ln \left( \frac{2l_{sk}}{R_{eq}} \right) \left( \frac{2l_{sk}}{\sqrt{\pi} R} \right) \frac{1}{2 \ln^2 \left( \frac{2l_{sk}}{R_{eq}} \right)} \tag{7} \]

Expanding the energy, Eq. (7), up to second order in \( \tilde{L}_z \) about its equilibrium value, \( K \), that is writing

\[ E_{\text{tot}}(\tilde{L}_z) = E_{\text{tot}}(K) + \frac{1}{2} U (\tilde{L}_z - K)^2 + \ldots \]

the second term on the RHS corresponds to the ‘classical’ rotational energy of the entire spin texture about its symmetry axis. At the classical level any deviation of \( \tilde{L}_z \) from its equilibrium value \( K \) corresponds to a continuous deformation (or more precisely a uniform contraction or expansion) of the Skyrmion with respect to its equilibrium configuration, thus conserving its topological charge, but increasing the Skyrmion energy with respect to its equilibrium value. The collective rotation of the Skyrmion in spin space is therefore reflected as a radial expansion or contraction in orbital space. Quantization of this rotational motion can be achieved by replacing \( \tilde{L}_z \to \frac{\hbar}{2} \frac{\partial}{\partial \varphi} \), where \( \varphi \) is the rotation angle, which yields

\[ \tilde{H}_{\text{rot}} = \frac{1}{2} U \left( \frac{1}{2} \frac{\partial}{\partial \varphi} - K \right)^2 \tag{8} \]

Note that since \( K = \langle 0| \nu \tilde{L}_z |0 \rangle / \hbar \), its value usually does not coincide with any (discrete) eigenvalue of the operator \( \frac{1}{2} \frac{\partial}{\partial \varphi} \), so that the spectrum of \( \tilde{H}_{\text{rot}} \) has usually a gap of the order of the rotational energy constant, \( U \).

Remarkably the above estimate, Eq. (8), shows that for a large Skyrmion, \( R_{eq} \gg l_H \), \( U \) is a small fraction of the Zeeman energy \( \varepsilon_{sp} = |g| \mu_B H \), that is \( U \ll \varepsilon_{sp} \left( \frac{l_H}{R_{eq}} \right)^2 \ll \varepsilon_{sp} \). The fraction, \( \left( \frac{l_H}{R_{eq}} \right)^2 \), tends to zero as the Skyrmion core radius becomes macroscopic, reflecting the macroscopic inertial mass associated with the collective rotation of a macroscopic spin texture. Using the equilibrium value \( R_{eq} \) as a function of the g-factor obtained above, we find that \( \left( \frac{l_H}{R_{eq}} \right)^2 \sim 2 |\tilde{g}|^2/3 \), where \( \tilde{g} \equiv g \left( \frac{l_H}{R_{eq}} \right) \). For a typical experimental value of the effective electronic g-factor, \( \tilde{g} \sim \sim 0.002 \), it is found that \( U \sim 3 \times 10^{-2} \varepsilon_{sp} \).

It is interesting to note that the magnetic field dependence of \( U \), expressed by Eq. (8), indicates similarity of the collective rotational motion to precession of a magnetic moment in a magnetic field. Indeed, by equating the classical expression for the rotational energy, \( H_{\text{rot}} = \frac{g}{2e} \left( \frac{d\varphi}{dt} \right)^2 \), to the energy scale, \( U \), of the spectrum of the rotational Hamiltonian, Eq. (8), we find for the angular velocity

\[ \left( \frac{d\varphi}{dt} \right) \sim U/\hbar = \frac{eH}{2M_{\text{scot}} c} \]

with

\[ M_{\text{scot}} = \left( 2 |\tilde{g}|^{2/3} |g| \right)^{-1} m_0 \tag{9} \]

This is an expression for an effective Larmor frequency for precession of the entire spin texture about the external magnetic field axis, with an effective mass, Eq. (9), which diverges with vanishing g-factor like \( g^{-5/3} \). For typical experimental values, \( \tilde{g} \sim 0.002 \), we find that \( M_{\text{scot}}/m_0 \sim 10^4 \).
In addition to the collective rotational motion of the entire spin texture just described, the internal degrees of freedom of the spin texture can also be excited, e.g. as spin waves associated with single electron-hole pair excitations (spin-excitons). The above consideration shows that for a sufficiently large Skyrmion the energy gap $\varepsilon_{\text{sp}}$ of the spin-waves is much larger than that of the collective rotational spectrum. This separation of energy scales may be expressed explicitly by writing the transverse electron spin density in the form:

$$ S_\rho(r, t) \equiv \frac{1}{4\pi} n(r, t) = \frac{1}{4\pi} \tilde{n}(r, t) e^{i\varphi(t)} (10) $$

where $\varphi(t)$ is the instantaneous collective rotation angle, and $\tilde{n}(r, t)$ stands for all the other degrees of freedom in the electronic spin space. It can be derived by expressing the phase of $n(r, t) = |n(r, t)| e^{i\theta(r, t)}$ as a Fourier series $\theta(r, t) = \sum_{k \neq 0} \tilde{\theta}_k(t) e^{ikr}$ + $\theta_0(t)$, and identifying the uniform term, $\theta_0(t)$, with $\varphi(t)$, so that $\tilde{n}(r, t) = |n(r, t)| \exp\left[\sum_{k \neq 0} \tilde{\theta}_k(t) e^{ikr}\right]$.

IV. NUCLEAR SPIN DYNAMICS

Let us, finally study in some detail the influence of these electron spin excitations on the dynamics of nuclear spins via the hyperfine interaction investigated in our model. The processes of nuclear spin relaxation and decoherence are reflected in the time dependence of the average $I_{++} = \langle \hat{I}_{++} \rangle$, where the brackets $\langle ... \rangle$ stand for the state of the combined system of the nuclear and electronic spins (see Ref.2). Exploiting the adiabatic approximation, which is valid when the effect of the hyperfine interaction is so weak as to be neglected beyond the leading order, which is the first order in the calculation of the nuclear spin eigen-energies, and the second order in the calculation of relaxation and decoherence. Thus we have for the transverse component of the nuclear spin located at $r$, up to second order of the corresponding perturbation theory,

$$ \left[ \frac{\partial}{\partial t} + i\Omega(r) \right] I_+(r, t) = $$

$$ -\frac{\alpha^2}{4} \int_0^t d\tau \langle 0 \left| \{ \hat{S}_+(r, t), \hat{S}_-(r, \tau) \} \right| 0 \rangle e^{i\Omega(t-\tau)} I_+(r, t) $$

where the symbol $\{,\}$ stands for anticommutator, and the averaging is performed over the ground state $|0\rangle$ of the electronic system. The local NMR frequency $\Omega(r)$ corresponds to the unperturbed precession of the nuclear spin in the external static magnetic field (with the frequency $\omega = \gamma_n B_0$) and the first order correction due to the local hyperfine interaction (the Knight shift), i.e. $\Omega(r) = \gamma_n B_0 - \alpha \langle 0 | \hat{S}_z(r) | 0 \rangle$. Note that the corresponding correction due to the transverse component of the hyperfine field is neglected in Eq. (11). Note also that

in the framework of the adiabatic approximation, used in the derivation of $\Omega(r)$, the weak time dependence of the operator $\hat{I}_+(\tau)$ due to depolarization, is neglected (so that $\hat{I}_+(\tau) \simeq I_+(t) e^{i\pi(\tau-t)}$).

The resulting equation, (11), is solved by

$$ I_+(r, t) = I_+(r, 0) e^{-\Gamma(r,t)-i\Omega(r)t} $$

(12)

where

$$ \Gamma(r, t) = \text{Re} \int_0^t dt' \xi(r, t') $$

and

$$ \xi(r, t) = \frac{\alpha^2}{4} \int_0^t d\tau e^{i\Omega(\tau-t)} \langle 0 \left| \{ \hat{S}_+(r, t), \hat{S}_-(r, \tau) \} \right| 0 \rangle $$

At filling factors slightly away from $\nu = 1$, the density of Skyrmions is small and the interaction between them can be neglected, $S_\rho(r, t)$ may be written in the form (11), describing a single Skyrmion centered at $r = 0$. On the large time scale relevant to the nuclear spin dynamics of interest here, when the internal degrees of freedom of the spin texture are essentially frozen, it is possible to neglect the time dependence of $\tilde{n}(r, t)$ in Eq. (11) (by writing $\tilde{n}(r, t) \approx \tilde{n}(r)$), so that:

$$ \xi(r, t) \approx \left( \frac{\alpha}{8\pi} |n(r)| \right)^2 \int_0^t d\tau e^{i\Omega(\tau-t)} \langle 0 \left| \{ e^{i\varphi(t)}, e^{-i\varphi(\tau)} \} \right| 0 \rangle $$

where $e^{i\varphi(t)} \equiv e^{it\hat{R}_{\text{rot}}/\hbar} e^{i\varphi} e^{-it\hat{R}_{\text{rot}}/\hbar}$. A straightforward algebra yields:

$$ e^{i\varphi(t)} = e^{i\varphi} \exp \left\{ \frac{U}{2\hbar} \left[ 1 - 2 \left( i \frac{\partial}{\partial \varphi} + K \right) \right] \right\} $$

(13)

so that the correlation function $\langle 0 \left| \{ e^{i\varphi(t)}, e^{-i\varphi(\tau)} \} \right| 0 \rangle = 2 \cos[U\delta K(t - \tau)/\hbar]$, where $\delta K \equiv |K| - K$, and $|K|$ is the integer closest to $(K - 1/2)$. Consequently, one finds that

$$ \Gamma(r, t) = 2 \left( \frac{\alpha}{8\pi} |n(r)| \right)^2 \frac{1 - \cos[(U\delta K/h - \omega)t]}{(U\delta K/h - \omega)^2} $$

(14)

This expression shows that as long as the rotational energy gap $U|\delta K|$ is much larger than the nuclear Zeeman energy $h\omega$, the off-diagonal element of the nuclear spin density matrix (i.e. the coherence) does not decay, but oscillates very quickly (i.e. with frequency $U|\delta K|/h$) between $I_+(r, 0)$ and $I_+(r, 0) e^{-\langle A[S_+(r)]/U\delta K^2 \rangle}$. It should be stressed that in deriving Eq. (14) the interaction of the electronic system to its environment was completely neglected. This coupling should lead to some energy dissipation, which results in damping of the oscillatory component of $\Gamma(r, t)$, so that for sufficiently long times, $I_+(r, t) \to I_+(r, 0) e^{-\langle A[S_+(r)]/U\delta K^2 \rangle}$.

As discussed above, the effective electron g-factor can become locally sufficiently small to make the local
Skyrmion radius large enough, so that the corresponding rotational energy gap $\Delta U$ becomes comparable to the nuclear Zeeman energy $\hbar \omega$. For such a large Skyrmionic spin texture the extremely slow collective spin rotation leads to a complete loss of coherence of nuclear spins via the hyperfine coupling. Under this condition the decay is Gaussian, $I_+(r,t) \sim e^{-(\alpha \eta(r)/2\pi)^2 t^2}$, with characteristic relaxation time $T_2 \sim \hbar / |S_+(r)| = \pi / K^S_S \left| \tilde{S}_+(r) \right|$, which is of the order of $0.1 - 1$ milliseconds for GaAs MQW. It should be stressed here that the neglect of the first order correction due to the transverse component of the hyperfine field in Eq.(11) results in the vanishing of the equilibrium solution $I_+(r,t \rightarrow \infty)$. The present dynamical approach should be therefore modified to take into account this correction in order to describe relaxation to the nonvanishing nuclear spin texture, Eq.(1).

At filling factor $\nu = 1$, where the number of Skyrmions vanishes (note that due to spatial inhomogeneity of the local filling factor some equal number of Skyrmions and anti-Skyrmions can exist even at $\nu = 1$), the nuclear spin dynamics is controlled by the coupling to the well known gapped spin waves. In the presence of the gap the virtual flip-flop excitations of electronic spin waves via the hyperfine interaction (which are the vacuum quantum fluctuations of the QH ferromagnet) lead to decoherence of the nuclear spin states, i.e.

$$\Gamma(r,t) = \Gamma(t) = (\hbar K^S_S)^2 \times \int_0^\infty k\hbar k e^{-\tilde{k}^2/2} 1 - \cos \left( \varepsilon_{ex}(\tilde{k}) \hbar / \omega t \right) / \varepsilon_{ex}(\tilde{k}) - \hbar \omega t^2 $$

(15)

where $\varepsilon_{ex}(\tilde{k}) \approx \varepsilon_{sp} + \frac{1}{2} \varepsilon_C \tilde{k}^2$, for $\tilde{k} = k_l H \ll 1$, and $\varepsilon_C = \sqrt{\pi/2} \left( e^2 / k_l H \right)$ is the Coulomb energy. Similar to the case of the collective mode with the large excitations gap, discussed below Eq.(1), in the present case the coherence does not decay to zero at any time. In contrast to the effect of the undamped collective mode, however, the presence of a continuous band of spin waves above the Zeeman gap $\varepsilon_{sp}$ results in some irreversible loss of coherence. This decoherence occurs on a very short time scale:- the precession period of the electronic spin, $2\pi / \omega_{sp}$, whereas for longer times the coherence undergoes damped oscillation (with frequency $\omega_{sp}$) about a nonzero value (see Fig.(1)), that is: $I_+(r,t) e^{i \Omega(t) t} \rightarrow I_+(r,0) \exp \left[ -2 \left( \varepsilon_C / \varepsilon_{sp} \right) (\hbar K^S_S / \varepsilon_{sp})^2 \right]$.

For $\hbar K^S_S \ll \varepsilon_{sp} (\varepsilon_C / \varepsilon_{sp})^{1/2}$ (e.g. for GaAs MQW $\hbar K^S_S / \varepsilon_{sp} \sim 10^{-7}$, and $\varepsilon_C / \varepsilon_{sp} \sim 30$ at $H = 10$ T), the corresponding decoherence is negligibly small.

In actual heterojunctions the electronic Zeeman gap is usually much smaller than the theoretical value. It can be further suppressed by applying pressure, so that the situation of gapless spin waves may not be unrealistic experimentally. In this case the integral over $\tilde{k}$ in Eq.(10) becomes in the long time limit $t \gg \hbar / \varepsilon_C :$

$$\Gamma(t) = 2 (\hbar K^S_S)^2 \int_0^\infty k \hbar k e^{-\tilde{k}^2/2} \sin^2 (\varepsilon_C \tilde{k}^2 t / 8 \hbar) / \varepsilon_{sp} \left( \varepsilon_C \tilde{k}^2 / 4 \right)^2$$

$$\sim \left( \frac{2\pi^2}{\varepsilon_C / \hbar} \right)^2 t$$

so that the decay of coherence with time is a simple exponential, $I_+(r,t) e^{i \Omega(t) t} \rightarrow I_+(r,0) \exp (-t / T_2)$, where

$$T_2 = \left[ \frac{\left( \varepsilon_C / \varepsilon_{sp} \right)}{(2\pi)^2 (\hbar K^S_S / \varepsilon_{sp})^2} \right] \left( \frac{2\pi}{\omega_{sp}} \right)$$

For GaAs MQW this expression yields $T_2 \sim 10^3$ sec. , indicating that the long relaxation times observed experimentally in the QH ferromagnetic state can be reasonably explained by a gapless spin exciton spectrum.

V. CONCLUSION

In this paper it was demonstrated how a coherent superposition of many nuclear spin states can be prepared and manipulated via the hyperfine interaction by varying the LL filling factor in appropriately designed QH Ferromagnet. During the manipulation periods the electronic spins form spatially large spin textures, where the average spin polarization in the plane perpendicular to the external magnetic field varies smoothly, and the individual spins are strongly correlated over large microscopic regions. The nuclear spins, which are coupled to their environment only via the hyperfine interaction with the electron spins, follow the changes in the electronic spin system by creating their own spin textures, which replicate the electronic ones. This effect is expected to be significant only in very special systems, where the strength of the hyperfine interaction is comparable to the nuclear Zeeman energy. The nuclear spins relaxation and decoherence processes in such states are governed by the coupling to collective spin rotational modes of the entire
electronic spin textures, which have vanishingly small excitation gap in regions where the local electronic g-factor vanishes.

It turns out that GaAs MQW, despite its remarkable features described above, is not suitable for our purpose. The reason is twofold:

1) The hyperfine coupling constant in GaAs is much too small to be effective in manipulating nuclear spins in the QW.

2) The nuclear spin dephasing time in quantum well structures based on GaAS/AlGaAs, is expected to be much smaller than the shortest value of $T_1$ found in these experiments. This drawback is due to the fact that all abundant isotopes in this compound (i.e. $^{69}$Ga, $^{71}$Ga, $^{75}$As, all with $I = 3/2$, and $^{27}$Al with $I = 5/2$) have non-zero nuclear spins, so that significant dephasing due to dipolar interactions is expected. Indeed, a rough estimate for $T_2$ for a solid in which each nuclear spin has nearby nuclear spins is in the range of milliseconds

A possible solution for both problems may be found in MQW structures composed of Si/Si$_{1-x}$Ge$_x$. The most abundant isotopes of these nuclei have zero nuclear spins, so that by purifying the host sample isotopically, and then weakly doping with, e.g. $^{31}$P donors, which has $I = 1/2$, one may reduce the dipolar dephasing to the desired low level.

Furthermore, the hyperfine coupling between the conduction electrons and the $^{31}$P nucleus in the Si host is strongly enhanced, due to the high concentration of the electron s-orbitals at the donor nucleus. Thus, a Knight shift of about 30 MHz, which is comparable to the NMR frequency at about 1T, can be obtained for Si.$^{31}$P

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