Field-induced superconductivity with enhanced and tunable paramagnetic limit

A. Buzdin*, S. Tollis, and J. Cayssol
Condensed Matter Theory Group, CPMOH,
UMR 5798, Université Bordeaux I,
33405 Talence, France
(*)also Institut Universitaire de France, Paris

We demonstrate that in a superconducting multilayered system with alternating interlayer coupling a new type of nonuniform superconducting state can be realized under in-plane magnetic field. The Zeeman effect in this state is compensated by the energy splitting between bonding and antibonding levels. Such compensation mechanism at low temperature leads to the field-induced superconductivity. We discuss the conditions for the experimental observation of the predicted phenomena.

There are two mechanisms of the superconductivity destruction by a magnetic field: orbital and paramagnetic effects [1, 2]. Usually it is the orbital effect that is more restrictive. However in the systems with large effective mass of electrons [3, 4] or in low-dimensional compounds, like quasi-one-dimensional or layered superconductors under in-plane magnetic field [5], the orbital magnetism is weakened and it is the paramagnetic effect which is responsible for the superconductivity destruction. The Chandrasekhar-Clogston paramagnetic limit [6, 7] is achieved when the energy of the polarization of the normal electron gas, \(-\chi_n H^2/2\), equals the superconducting condensation energy \(-N(0)\Delta_0^2/2\), where \(N(0)\) is the density of states of the normal electron gas, \(\chi_n\) its spin susceptibility and \(\Delta_0 = 1.76T_c\) is the zero temperature superconducting gap. This gives the critical field \(H_p = \Delta_0/(\sqrt{2}\mu_B)\) of the first order transition at \(T = 0\), \(\mu_B\) being the Bohr magneton. Later Larkin and Ovchinnikov [8] and Fulde and Ferrell [9] (FFLO) predicted the appearance at low temperature of a nonuniform superconducting state with the zero temperature critical field \(H^2_{FFLO} = 0.755\Delta_0/\mu_B\), i.e. somewhat higher than the paramagnetic limit \(H_p\). This prediction was made for three-dimensional superconductors. In quasi-two-dimensional superconductors the critical field of the FFLO state is even higher, namely \(H^2_{FFLO} = \Delta_0/\mu_B\) [10], while in quasi-one-dimensional systems there is no paramagnetic limit at all [11]. The appearance of the modulated FFLO state is related to the pairing of electrons with opposite spins which do not have the opposite momenta anymore due to the Zeeman splitting.

In this Letter, we demonstrate that in a ballistic superconducting bilayer at low temperature and strong enough coupling \(t_1 \gg \Delta_0\) between the conducting planes, the paramagnetic limit is enhanced up to \(H_c \sim t_1/\mu_B\) far above the usual limit \(H^2_{FFLO} = \Delta_0/\mu_B\). More precisely, a very unusual superconducting phase is settled between a lower and an upper critical fields given by \(\mu_B H_c = t_1 \pm \mu_B^2 H^2_0/t_1\) and below a maximal temperature of the order of \(T_c^2/\Delta_1\). Thus one obtains field-induced superconductivity above the lower field while the upper one provides the enhanced paramagnetic limit which may be tuned by varying the electronic coupling \(t_1\). This is due to the compensation of the Zeeman splitting by the energy splitting \(t_1\) between bonding and antibonding electronic states of the bilayer. As another important feature of this new phase, adjacent layers support opposite signs of the order parameter. Note that such a so-called \(\pi\) phase was predicted before [12, 13] for the superconductor-ferromagnet multilayered systems where the atomic superconducting and ferromagnetic layers alternated and were weakly coupled in contrast to the present system.

\[ \Delta \phi^r \]
\[ \Delta, \psi^r \]
\[ \Delta_\pi, \varphi \]
\[ \Delta, \varphi \]
\[ \Delta_\pi, \psi \]
\[ \Delta, \psi \]

FIG. 1: Multilayered system. The \(n^{th}\) unit cell contains two superconducting planes \(\varphi_n\) and \(\psi_n\). The transfer integrals are very different, \(t_2 \ll t_1\). The superconducting phase difference \(\chi\) between adjacent planes can be either 0 or \(\pi\).

We consider a model multilayered system with a crystallographic structure similar to those of the high-\(T_c\) superconductors [14], as shown in Fig. 1. Namely, we assume that the electrons are confined in the atomic planes with the same zero-field dispersion relation \(\xi(p) = p^2/2m - E_F\), \(E_F\) being the Fermi energy. The transfer integrals between the planes are different \(t_1 \gg t_2\), both being smaller than the Debye energy \(\hbar \omega_D \ll E_F\). In this structure the \(n^{th}\) unit cell contains two conducting planes, labelled by \(\psi_n\) and \(\varphi_n\). The coordinate in the plane is \(r\) and the cells are separated by a distance \(a\) along the \(z\)-axis which is chosen perpendicular to the
planes. We suppose that the Cooper pairing occurs in the planes and perform our analysis in the framework of the standard mean field BCS Hamiltonian,

$$H = H_{\psi 0} + H_{\varphi 0} + H_t + \frac{1}{|\lambda|} \int d^2r \Delta_n^2(r), \tag{1}$$

with

$$H_{\psi 0} = \sum_{\mathbf{p},n} \xi(\mathbf{p}) \psi_{n,\alpha}^\dagger(\mathbf{p}) \psi_{n,\alpha}(\mathbf{p})$$

$$+ \frac{1}{2} \Delta_{n,\alpha\beta}(\mathbf{q}) \psi_{n,\alpha}(\mathbf{p} + \mathbf{q}) \psi_{n,\beta}(-\mathbf{p}) + h.c.,$$

$$H_t = \sum_{\mathbf{q}} \frac{\psi_{n,\alpha}(\mathbf{p})}{(t_1 \varphi_{n,\alpha}(\mathbf{p}) + t_2 \psi_{n+1,\alpha}(\mathbf{p}))} + h.c.,$$

where $\lambda$ is the BCS coupling constant, summation over repeated spin indexes $\alpha, \beta$ is implied, and $\Delta_{n,\alpha\beta}(\mathbf{q})$ is the Fourier transform of the superconducting order parameter $\Delta_{n,\alpha\beta}(\mathbf{r}) = \Delta_{n}(\mathbf{r}) i\sigma^y_{\alpha\beta}$, $\sigma^y$ being the second Pauli matrix. The operators $\psi_{n,\alpha}(\mathbf{p})$ and $\varphi_{n,\alpha}(\mathbf{p})$ destroy one electron with spin $\alpha$ and momentum $\mathbf{p}$, respectively in planes $\psi_n$ and $\varphi_n$. Our model includes both translational and gauge symmetry breaking. Indeed, the superconducting order parameter in the planes $\psi_n$ and $\varphi_n$ are respectively given by $\Delta_n(\mathbf{r}) = \Delta e^{i\mathbf{q}\cdot\mathbf{r} + i\kappa n}a$ and $\Delta_n(\mathbf{r}) e^{i\kappa}$, where $\mathbf{q}$ and $\kappa$ are respectively the in-plane and the perpendicular modulation wave vectors. These vectors and the superconducting phase difference $\chi$ must be determined from the minimum energy condition.

The Gor'kov equations corresponding to the Hamiltonian \[1\] are solved exactly. In the small $\Delta$ limit, the linearized anomalous Gor'kov Green function

$$F_{\omega}^1 = \langle \psi_{\omega}^\dagger(\mathbf{p}) \psi_{\omega}^{}(-\mathbf{p}) \rangle$$

reduces to the form \[15\]

$$F_{\omega}^1 = \frac{\Delta(t_k^* \tilde{t}_k e^{-i\kappa} - \omega_- \omega_+)}{(\omega_- - |t_k|^2)(\omega_+ - |t_k|^2)}, \tag{2}$$

where $t_k = t_1 + t_2 e^{i\kappa}$, $\omega_\pm(\mathbf{p}) = i\omega \pm \xi(\mathbf{p}) - \mu_B H$, $\tilde{t} = t_{k+\kappa}$ and $\omega_\pm(\mathbf{p}) = \omega_\pm(\mathbf{p} + \mathbf{q})$, $\omega$ being the Matsubara frequencies.

We first solve the isolated bilayer problem $t_2 = 0$. After integration of Eq. \[2\] over $\xi(\mathbf{p})$, one obtains the anomalous Eilenberger propagator $f_{\omega}^1(\mathbf{v}_F) = \int (d\xi/\pi) F_{\omega}^1 = \sum_{a=\pm1} f_{\omega}^{1,a}$ with

$$f_{\omega}^{1,a} = \frac{\omega + i\mu_B H + i\Sigma_{\omega} e^{-i\kappa} + i\sigma_1 t_1 e^{-i\kappa/2} \cos(\chi/2)}{2(\omega + i\mu_B H + i\Sigma_{\omega})(\omega + i\mu_B H + i\Sigma_{\omega} + i\sigma_1 t_1)} \tag{3}$$

for positive $\omega$, where $a = \pm1$ labels bonding and antibonding states while $\mathbf{v}_F$ is the Fermi velocity vector in the plane. Then the self-consistency relation \[11\] below implies that the superconducting phase difference $\chi$ between neighboring layers is either zero or $\pi$. In the absence of Zeeman splitting $H = 0$, the superconducting order parameter is naturally the same in both layers, namely $\chi = 0$. In this case, $f_{\omega}^1 = \Delta/\omega$ coincides with the well-known anomalous Green function of a superconductor in the limit $\Delta \to 0$, and the self-consistency relation gives the bare critical temperature $T_c$ of the isolated layer. More generally for $\chi = 0$ and finite $H$, the interlayer coupling $t_1$ drops from Eq. \[3\] and one retrieves $f_{\omega}^1 = \Delta/(\omega + i\mu_B H)$ for a two dimensional superconductor under parallel magnetic field. The other possible choice is $\chi = \pi$, when the superconducting order parameter is opposite on adjacent layers. For small values of the magnetic field, this later $\pi$ phase exhibits naturally a lower critical temperature than the $\chi = 0$ phase.

![FIG. 2: Excitation spectrum. Usual singlet pairing (thin line circles) between opposite-spin electrons occupying the same orbital is affected by Zeeman effect. In contrast, $\pi$ coupling (thick line) between two electrons occupying a bonding and an antibonding orbitals may lead to the cancellation of the Zeeman splitting.](image)

However for relatively large interlayer coupling $t_1 > T_c$ and high field, the situation becomes drastically different. Indeed, the excitation spectrum consists of four different branches $\epsilon = \pm \xi + \mu_B H \pm t_1$ in the limit $\Delta \to 0$, see Fig. 2. The singlet pairing may occur here between one electron in the bonding orbital and the other electron in the antibonding orbital. This results in a very special coupling where, if $\mu_B H = t_1$, the Zeeman splitting is exactly compensated. Therefore enhanced superconductivity is expected in the vicinity of $\mu_B H = t_1$, at least at zero temperature.

In order to derive rigourously this prediction, we analyse the self-consistency relation,

$$\Delta = 2\pi |\lambda| N_{2D}(0) T \sum_{\omega > 0} \text{Re} \langle f_{\omega,a}^1(\mathbf{v}_F) \rangle, \tag{4}$$

in the $\pi$ phase where $f_{a,a}^1(\mathbf{v}_F)$ depends on the coupling $t_1$ in the following way

$$f_{\omega,a}^1(\mathbf{v}_F) = \frac{\Delta}{2(\omega + i\mu_B H + i\sigma_1 t_1 + i\mathbf{v}_F \cdot \mathbf{q})/2}, \tag{5}$$

Here $N_{2D}(0) = m/(2\pi)$ is the two-dimensional density of states per unit surface and per one spin orientation, and the brackets $\langle \ldots \rangle$ denotes averaging over the polar angle $\theta = (\mathbf{v}_F, \mathbf{q})$. 

We first discuss the zero temperature second-order phase transition between the normal metal and the \( \pi \) phase, as a function of the magnetic field. From Eqs. (4,5), the critical field \( H \) is shown to satisfy

\[
|H + t_1/\mu_B + \sqrt{(H + t_1/\mu_B)^2 - X^2}| \cdot |H - t_1/\mu_B + \sqrt{(H - t_1/\mu_B)^2 - X^2}| = 4H_0^2, \tag{6}
\]

where \( X = |q|v_F/(2\mu_B) \), and \( H_0 = \Delta_0/2\mu_B \) is the critical field of the second-order superconducting phase transition in a two-dimensional monolayer. One must then find the value of \( X \) which maximizes the critical field \( H \). If the \( \pi \) phase is assumed to be uniform inside each plane, namely if \( q = 0 \), Eq. (6) merely reduces to \( |H^2 - t_1^2/\mu_B^2| = H_0^2 \) and we obtain a lower and an upper critical fields respectively given by \( \mu_B H_c = t_1 \pm \mu_B^2 H_0^2/2t_1 \), in the limit \( t_1 \gg \mu_B H_0 \). Thus at zero temperature and strong enough coupling \( t_1 \gg \Delta_0 \), the superconductivity destruction follows a very special scenario. At low fields, superconductivity is first suppressed in the usual manner at the paramagnetic limit \( H_{FLO}^{1D} = \Delta_0/\mu_B \) leading to the normal metal phase. Then further increase of the field leads to a normal to the superconducting \( \pi \) phase. The upper critical field is maximal for the choice \( X = |q|v_F/2\mu_B = |H - t_1/\mu_B| \), and then Eq. (6) reduces to

\[
|H - t_1/\mu_B| \cdot |H + t_1/\mu_B + 2\sqrt{H_1/\mu_B}| = 4H_0^2, \tag{7}
\]

which gives the upper and lower fields \( \mu_B H_c = t_1 \pm \mu_B^2 H_0^2/2t_1 \) in the \( t_1 \gg \Delta_0 \) limit. Note that the period of the modulated order parameter \( |q|^{-1} = \xi_0(t_1/\Delta_0) \) is larger than the corresponding period in the two-dimensional FFLO phase which coincides with the ballistic coherence length \( \xi_0 = v_F/\Delta_0 \). [10].

Furthermore one may derive the full temperature-field phase diagram using Eqs. (6,7) and the result is shown in Fig. 3. When the temperature is increased, the lower critical field increases whereas the upper one decreases. Along the upper (resp. lower) critical line the FFLO modulation is lost at some temperature \( T^{*\uparrow} \) (resp. \( T^{*\downarrow} \)). For higher temperatures a uniform \( \pi \) phase (U-\( \pi \)) is recovered and the temperature dependence of the critical field is given by

\[
\ln \frac{T}{T_c} = \frac{1}{2} \sum_{\alpha = \pm 1} \Re \left[ \Psi \left( \frac{1}{2} \right) - \Psi \left( \frac{1}{2} + i \frac{\mu_B H_c(T) + \alpha t_1}{2\pi T} \right) \right], \tag{8}
\]

where \( \Psi(x) \) is the Digamma function and \( \Psi(1/2) = -C - 2 \ln 2 \approx -1.963 \), \( C \) being the Euler constant. Finally the lower and upper critical lines merge at field \( H_c = t_1/\mu_B \) and temperature \( T_M = \pi e^{-C} T_c^2/(4t_1) \) in the limit \( t_1 \gg T \). Therefore the field induced \( \pi \) superconductivity is confined to temperatures lower than \( T_M \). The structure of these U-\( \pi \) and the FFLO-\( \pi \) phases is reminiscent of the corresponding U-0 and the FFLO-0 phases although the former are shifted to higher fields and lower temperatures than the later.
phases as shown in Fig. 4, and finally disappear for $t_1$ slightly smaller than $\Delta_0$. From an experimental point of view, one might choose a system with intermediate coupling $t_1$ small enough to settle the $\pi$ phase island in an available range of temperatures but also large enough to separate the $\pi$ phase island from the usual superconducting phases with $\chi = 0$.

Hereafter we discuss various physical mechanisms limiting the above predicted $\pi$ superconductivity: role of finite $t_2$, impurity and orbital effects.

For finite coupling $t_2$ between the bilayers, the self-consistency relation \[4\] together with Eq. (2) leads to the following equation

$$\left\langle \ln \left( \frac{2\mu_B H_0}{\sum_{a=\pm 1} \sqrt{(\mu_B H + X \cos \theta)^2 - (t_1 + at_2)^2}} \right) \right\rangle = 0.$$  \[9\]

For $X = |H - (t_1 + t_2)/\mu_B|$, the singularity around $\theta = \pi$ produces a corrective term to the upper and lower critical fields found previously. However this correction is negligible if $t_1 t_2 \ll \mu_B^2 H_0^2$. In the opposite regime, that is for larger values of the inter-bilayers coupling $t_2$, the bonding and the antibonding electronic levels form bands whose dispersion avoids exact compensation between the intra-bilayer coupling $t_1$ and the Zeeman splitting. When $t_1 = t_2$, the quasi-two-dimensional case \[10\] is retrieved: the presently studied $\pi$ phases are lost in favor of FFLO phases as shown in Fig. 4, and finally disappear for $t_1$ slightly smaller than $\Delta_0$. From an experimental point of view, one might choose a system with intermediate coupling $t_1$ small enough to settle the $\pi$ phase island in an available range of temperatures but also large enough to separate the $\pi$ phase island from the usual superconducting phases with $\chi = 0$.

We are grateful to M. Houzet and A. Koshelev for useful comments.

\[1\] A.A. Abrikosov, L. P. Gor’kov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York) (1963).

\[2\] D. Saint-James, G. Sarma, and E.J. Thomas, *Type II Superconductivity*, Pergamon, New York (1969).

\[3\] A. Bianchi *et al.*, Phys. Rev. Lett. 91, 187004 (2003).

\[4\] C. Capan *et al.*, Phys. Rev. B 70, 134513 (2004).

\[5\] S. Uji *et al.*, Nature (London) 410, 908 (2001).

\[6\] B. Chandrasekhar, Appl. Phys. Lett. 1, 7 (1962).

\[7\] A.M. Clogston, Phys. Rev. Lett. 9, 266 (1962).

\[8\] A. I. Larkin and Y. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47, 1136 (1964) [Sov. Phys. JETP 20, 762 (1965)].

\[9\] Fulde, P., and R. A. Ferrell, Phys. Rev. 135, A550 (1964).

\[10\] L.N. Bulaevskii, Zh. Eksp. Teor. Fiz. 64, 2241 (1973) [Sov. Phys. JETP 37, 1133 (1973)].

\[11\] A.I. Budzink and V.V. Tugushev, Zh. Eksp. Teor. Fiz. 85, 735 (1983)[Sov. Phys. JETP 58, 428 (1983)].

\[12\] A.V. Andreev, A. I. Budzin, and R. M. Osgood III, Phys. Rev. B 43, 10124 (1991).

\[13\] M. Houzet and A.I. Budzin, Europhys. Lett. 58, 596 (2002).

\[14\] L.N. Bulaevski and M.V. Zyskin, Phys. Rev. B 42, 10230 (1990).

\[15\] Exact expressions for the Green functions will be published elsewhere and then used to study thoroughly the low coupling $t_1 \ll \Delta_0$ limit where various nonuniform phases also appear, but not the presently studied $\pi$-FFLO phase.
[16] T.M. Rice, Phys. Rev. 140, A1889 (1965).
[17] A.I. Larkin and A.A. Varlamov, p 95 in The Physics of Superconductors, Vol. I, edited by K.H. Bennemann and J.B. Ketterson, Springer (2003).
[18] L.I. Glazman and A.E. Koshelev, Zh. Eksp. Teor. Fiz. 97, 1371 (1990) [Sov. Phys. JETP 70 (4), 774 (1990)].
[19] L.G. Aslamazov, Zh. Eksp. Teor. Fiz. 55, 1477 (1968) [Sov. Phys. JETP 28 (4), 773 (1969)].
[20] V. Jaccarino and M. Peter, Phys. Rev. Lett. 9, 290 (1962).