THEORETICAL ASPECTS OF HADRON PHOTOPRODUCTION

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ABSTRACT

We review recent developments in the theoretical description of inclusive single-hadron production at next-to-leading order in the parton model of quantum chromodynamics. Fragmentation functions are extracted from fits to data of inclusive pion and kaon production in $e^+e^-$ annihilation at different centre-of-mass energies. Exploiting sensitivity to the scaling violation, one can simultaneously fit the asymptotic scale parameter $\Lambda$ so as to obtain an independent determination of the strong coupling constant $\alpha_s$. Owing to the factorization theorem, the fragmentation functions only depend on the species of the produced particles, but not on the process by which they are produced. This allows one to make absolute theoretical predictions for inclusive pion and kaon production in other types of experiments such as photon-photon, photon-hadron, or hadron-hadron scattering. Recent data of photoproduction taken by the H1 and ZEUS Collaborations at DESY HERA nicely agree with such next-to-leading-order predictions.

1. Introduction

The Lagrangian of quantum chromodynamics (QCD) contains quarks and gluons as elementary fields. Allowing for these particles to appear as asymptotic states, we can evaluate scattering amplitudes perturbatively, in principle with arbitrary precision. Of course, this picture needs to be complemented by the principle of confinement of colour; experiments detect hadrons rather than quarks and gluons. Nevertheless, this simplified computational procedure is very successful in describing the production of jets of hadrons at high centre-of-mass (c.m.) energies ($\sqrt{s}$) in $e^+e^-$ annihilation and at high transverse momenta ($p_T$) in scattering processes. Due to parton-hadron duality, clustering partons in the final state according to certain jet definitions yields a useful approximation, although this does not account for any details of hadronization.

On the other hand, experiments are providing us with copious information on the inclusive production of single hadrons, which cannot be interpreted along these lines. In this case, we need a detailed concept for describing how partons turn into
hadrons. In the framework of the QCD-improved parton model, this is achieved by introducing fragmentation functions (FF’s), $D_a^h(x, \mu^2)$. The value of $D_a^h(x, \mu^2)$ corresponds to the probability for the parton $a$ produced at short distance $1/\mu$ to form a jet that includes the hadron $h$ carrying the fraction $x$ of the longitudinal momentum of $a$. Unfortunately, it is not yet understood how the FF’s can be derived from first principles, in particular for hadrons with masses smaller than or comparable to the asymptotic scale parameter, $\Lambda$. However, given their $x$ dependence at some scale $\mu$, the evolution with $\mu$ may be computed perturbatively in QCD using the Altarelli-Parisi (AP) equations. This allows us to test QCD quantitatively within one experiment observing single hadrons at different values of $\sqrt{s}$ (in the case of $e^+e^-$ annihilation) of $p_T$ (in the case of scattering). Moreover, the factorization theorem guarantees that the $D_a^h$ functions are independent of the process in which they have been determined, and represent a universal property of $h$. This enables us to make quantitative predictions for other types of experiments as well. To summarize, having extracted FF’s from fits to experimental data, we may test their $\mu$ dependence as predicted by the AP equations and their universality as postulated by the factorization theorem.

After the pioneering leading-order (LO) analyses of pion, kaon, and charmed-meson FF’s in the late 70’s, there had long been no progress on the theoretical side of this field. This may partly be attributed to the advent of general-purpose Monte Carlo (MC) event generators based on LO parton-level matrix elements in the early 80’s, which were soon to become very popular in the experimental community. In these computer programs, the fragmentation into hadrons is simulated according to certain phenomenological model assumptions, e.g., the cluster algorithm in the case of HERWIG or the LUND string model in the case of PYTHIA. Although such MC packages often lead to satisfactory descriptions of the data, from the theoretical point of view, their drawback is that this happens at the expense of introducing a number of ad-hoc fine-tuning parameters, which do not originate in the QCD Lagrangian. Furthermore, in the MC approach, it seems impossible to implement the factorization of final-state collinear singularities, which impedes a consistent extension to next-to-leading order (NLO). On the contrary, in the QCD-improved parton model, such singularities are absorbed into the bare (infinite) FF’s so as to render them renormalized (finite) in a way quite similar to the procedure for the parton density functions (PDF’s) at the incoming legs. Therefore, a meaningful quantitative test of QCD can only be performed in the parton model endowed with FF’s at NLO.

Some time ago, NLO FF sets for $\pi^0$, $\pi^\pm$, $K^\pm$, and $\eta$ mesons have been constructed through fits to data of $e^+e^-$ annihilation generated with HERWIG. By contrast, our procedure, and partly that of Ref. 4, has been to fit to genuine experimental $e^+e^-$ data. We have introduced LO and NLO FF sets for charged pions and kaons and for neutral kaons. The results presented here are obtained with the up-to-date sets of Refs. 7 and 8, which are based on data from SLAC PEP with $\sqrt{s} = 29$ GeV.
and CERN LEP1 with $\sqrt{s} = M_Z$. In contrast to Refs. 4 and 5, Refs. 6–8 also provide ready-to-use parameterizations of the $\mu$ dependence of the FF’s.

This presentation is organized as follows. In Section 2, we shall report the essential features of our fitting procedure for charged pions and kaons and neutral kaons. In Sections 3 and 4, we shall confront recent data of inclusive pion and kaon production collected by the H1 and ZEUS Collaborations in $\gamma p$ scattering at DESY HERA and by the UA1 Collaboration in $p\bar{p}$ scattering at the CERN SppS collider, respectively, with NLO predictions based on our FF’s. Our conclusions will be summarized in Section 5.

2. Fit to data of $e^+e^-$ annihilation

The most direct way to obtain information on the FF’s of hadrons is to analyze their energy spectrum measured in $e^+e^-$ annihilation, where the theoretical predictions are not obscured by additional nonperturbative input, e.g., in the form of PDF’s for the incoming particles. At NLO in the parton model with $n_f$ massless quark flavours, the differential cross section of $e^+e^- \to \gamma/Z \to h + X$, normalized to the total hadronic cross section $\sigma_{\text{tot}}$, is given by

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dx} = \sum_a \int_x^1 \frac{dz}{z} D_h^a \left( \frac{x}{z}, M_h^2 \right) \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_a}{dz} \left( z, \mu^2, M_h^2 \right),$$

where $x = 2E_h/\sqrt{s}$ is the fraction of the beam energy carried by $h$, $a = g, q_1, \ldots, \bar{q}_{n_f}$, $\mu$ is the renormalization scale, and $M_h$ is the factorization scale. The parton-level cross sections read

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{q_i}}{dx} (x, \mu^2, M_h^2) = \frac{e_{q_i}^2}{\sum_{i=1}^{n_f} e_{q_i}^2} \left\{ \delta(1 - x) + \frac{\alpha_s(\mu^2)}{2\pi} \left[ P_{q\to q}^{(0,T)}(x) \ln \frac{s}{M_h^2} + C_q(x) \right] \right\},$$

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{g}}{dx} (x, \mu^2, M_h^2) = 2\frac{\alpha_s(\mu^2)}{2\pi} \left[ P_{q\to g}^{(0,T)}(x) \ln \frac{s}{M_h^2} + C_g(x) \right],$$

where $e_{q_i}$ is the effective coupling of $q_i$ to the photon and the Z boson including propagator adjustments and $C_a$ are the NLO corrections. Here, $P_{a\to b}^{(0,T)}$ are the LO terms of the timelike $a \to b$ splitting functions,

$$P_{a\to b}^{(T)} (x, \alpha_s(\mu^2)) = \frac{\alpha_s(\mu^2)}{2\pi} P_{a\to b}^{(0,T)} (x) + \left( \frac{\alpha_s(\mu^2)}{2\pi} \right)^2 P_{a\to b}^{(1,T)} (x) + O(\alpha_s^3),$$

which control the $\mu$ evolution of the FF’s via the AP equations,

$$\frac{\mu^2}{d\mu^2} D_h^a (x, \mu^2) = \sum_b \int_x^1 \frac{dz}{z} P_{a\to b}^{(T)} \left( \frac{x}{z}, \alpha_s(\mu^2) \right) D_b^h (x, \mu^2).$$
Analytic expressions for $P^{(T)}_{a \rightarrow b}$ may be found, e.g., in Ref. 16. The integro-differential equation (4) may be solved either via the Mellin transform technique or with brute force as it stands. Good agreement is found between these two methods.

It is natural to choose $\mu = M_h = \sqrt{s}$. This eliminates the terms involving $P^0_{a \rightarrow b}$ in Eq. (4). In $e^+e^-$ annihilation, gluon fragmentation occurs only at NLO and beyond. To increase the sensitivity to $D_h^g$, one may select longitudinal polarization, so that the delta function in Eq. (2) does not contribute, or concentrate on gluon-tagged three-jet events. Then, the respective NLO terms in Eq. (2) constitute the Born approximation, and it is desirable to include the next-to-next-to-leading-order corrections in order to reduce the scale and scheme dependences. Unfortunately, these were not yet available when the analyses of Refs. 6–8 were carried out.

The fitting procedure in Refs. 7 and 8 was as follows. For each of the hadron channels $h = \pi^+ + \pi^-, K^+ + K^-, K^0_S + K^0_L$ and each parton type $a = g, u, d, s, c, b$, we made the ansatz

$$D^h_a(x, \mu_0^2) = N x^\alpha (1 - x)^\beta,$$

at the respective starting scale

$$\mu_0 = \begin{cases} \sqrt{2} \text{ GeV} & \text{if } a = g, u, d, s, \\ M(\eta_c) = 2.98 \text{ GeV} & \text{if } a = c, \\ M(\Upsilon) = 9.46 \text{ GeV} & \text{if } a = b. \end{cases}$$

According to the flavour composition of $h$, we respectively imposed

$$D^\pm_u(x, \mu_0^2) = D^\pm_d(x, \mu_0^2),$$

$$D^K_u(x, \mu_0^2) = D^K_s(x, \mu_0^2),$$

$$D^{K^0}_u(x, \mu_0^2) = D^{K^0}_s(x, \mu_0^2).$$

The analysis of Ref. 7 was based on charged-pion, charged-kaon, and unidentified charged-hadron data. Apart from charged pions and kaons, mainly protons and antiprotons contribute to the charged-hadron yield. Inspired by Ref. 21, we approximated

$$\frac{d\sigma^{h^\pm}}{dx} = [1 + f(x)] \frac{d\sigma^{\pi^\pm}}{dx} + \frac{d\sigma^{K^\pm}}{dx},$$

where $f(x) = 0.195 - 1.35 (x - 0.35)^2$. In their charged-hadron analysis, the ALEPH Collaboration distinguished between $uds$, $c$, and $b$-enriched samples. Also exploiting information on identified gluon jets, we were thus able to treat $g, u, s, c, b \rightarrow \pi^\pm$ and $g, u, d, c, b \rightarrow K^\pm$ fragmentation separately. Due to the large gap in $\sqrt{s}$ between PEP and LEP, we could simultaneously determine $\Lambda^{(5)}_{\overline{MS}}$. Thus, we had a total of $2(\pi, K) \times 5(\text{partons}) \times 3(N, \alpha, \beta) + 1(\Lambda^{(5)}_{\overline{MS}}) = 31$ independent fit parameters. These turned out to be tightly constrained by our LO and NLO fits. In fact, we obtained $\chi^2/\text{d.o.f.}$ values of 134.4/136 = 0.99 and 125.3/136 = 0.92, respectively. We found
\[ \Lambda_{\text{MS}}^{(5)} = 108 \text{ MeV (227 MeV)} \text{ at LO (NLO), which corresponds to } \alpha_s (M_Z^2) = 0.122 \text{ (0.118). This nicely agrees with the latest LEP value, (0.121 \pm 0.003).} \]

In our neutral-kaon analysis, we adopted the \( \Lambda_{\text{MS}}^{(5)} \) values from Ref. 7. Furthermore, appealing to the flavour blindness of the gluon, we assumed that

\[ D^{K^0}_g (x, \mu^2_0) = D^{K^\pm}_g (x, \mu^2_0), \]

at \( \mu_0 = \sqrt{2} \text{ GeV} \). Thus, the number of independent fit parameters was \( 4 \times 3(N, \alpha, \beta) = 12 \). Our combined fit to the MARK II and ALEPH neutral-kaon samples yielded \( \chi^2 / \text{d.o.f.} = 9.9 / 20 \text{ at LO and 8.6/20 at NLO.} \)

3. Comparison with data of photoproduction in ep scattering

According to present HERA conditions, \( E_e = 27.5 \text{ GeV} \) positrons collide with \( E_p = 820 \text{ GeV} \) protons in the laboratory frame, so that \( \sqrt{s} = 300 \text{ GeV} \) is available in the c.m. frame. It has become customary to take the rapidity of hadrons travelling in the proton direction to be positive. The rapidities measured in the ep laboratory and c.m. frames are related through

\[ y_{\text{c.m.}} = y_{\text{lab}} - \frac{1}{2} \ln \frac{E_p}{E_e}. \]

In photoproduction, the electron or positron beam acts like a source of quasi-real photons, with low virtualities \( -Q^2 \), so that HERA is effectively operated as a \( \gamma p \) collider. The appropriate events may be discriminated from deep-inelastic-scattering events by electron tagging or anti-tagging. The photon flux is well approximated by the Weizsäcker-Williams formula,

\[ F^e_\gamma (x, Q^2_{\text{max}}) = \frac{\alpha}{2 \pi} \left[ 1 + \frac{(1-x)^2}{x} \ln \frac{Q^2_{\text{max}}}{Q^2_{\text{min}}} + 2m_e^2 x \left( \frac{1}{Q^2_{\text{max}}} - \frac{1}{Q^2_{\text{min}}} \right) \right], \]

where \( x = E_\gamma / E_e, Q^2_{\text{min}} = m_e^2 x^2 / (1-x), \) and \( Q^2_{\text{max}} = 0.01 \text{ GeV}^2 (0.02 \text{ GeV}^2) \) for tagged events in the case of H1 (ZEUS). The cross section of \( ep \rightarrow h + X \) emerges from the one of \( \gamma p \rightarrow h + X \) by convolution with \( F^e_\gamma \). By kinematics, \( x_{\text{min}} \leq x \leq 1, \) where \( x_{\text{min}} = p_T \exp(-y_{\text{c.m.}})/[\sqrt{s} - p_T \exp(y_{\text{c.m.}})] \). In conformity with the H1 and ZEUS event-selection criteria, we impose \( 0.3 < x < 0.7 \) and \( 0.318 < x < 0.431, \) respectively.

It is well known that \( \gamma p \rightarrow h + X \) proceeds via two distinct mechanisms. The photon can interact either directly with the partons originating from the proton (direct photoproduction) or via its quark and gluon content (resolved photoproduction). Both contributions are formally of the same order in the perturbative expansion. Leaving aside the proton PDF’s, \( F^e_p \), and the FF’s, \( D^h \), which represent common factors, the LO cross sections are of \( \mathcal{O}(\alpha_\alpha s) \) in both cases. In the case of the resolved
mechanism, this may be understood by observing that the $ab \to cd$ cross sections, which are of $\mathcal{O}(\alpha_s^2)$, get dressed by photon PDF's, $F^\gamma_a$, whose leading terms are of the form $\alpha \ln(M^2_/\Lambda^2) \propto \alpha/\alpha_s$, with $M_\gamma$ being the corresponding factorization scale. Here, $a, b, c, d$ denote quarks and gluons. In fact, the two mechanisms also compete with each other numerically. Resolved photoproduction dominates at small $p_T$ and positive $y_{ab}$, while direct photoproduction wins out at large $p_T$ and negative $y_{ab}$.

LO calculations suffer from significant theoretical uncertainties connected with the freedom in the choice of the renormalization scale, $\mu$, in $\alpha_s$ and the factorization scales, $M_\gamma$, $M_p$, and $M_h$, in $F^\gamma_a$, $F^p_b$, and $D^h_c$, respectively. In order to obtain reliable predictions, it is indispensable to proceed to NLO. Let us first consider resolved photoproduction, which is more involved. Starting out from the well-known LO cross section of $\gamma p \to h + X$, one needs to include the NLO corrections, $K_{ab\to c}$, to the parton-level cross sections, $d\sigma_{ab\to c}/dt$, to substitute the two-loop formula for $\alpha_s$, and to endow $F^\gamma_a$, $F^p_b$, and $D^h_c$ with NLO evolution. This leads to

$$
\frac{d^3\sigma}{dy dp_T} = \frac{1}{\pi} \sum_{a,b,c} \int dx_1 dx_2 \frac{dx_h}{x_h^2} F^\gamma_a \left(x_\gamma, M^2_\gamma\right) F^p_b \left(x_p, M^2_p\right) D^h_c \left(x_h, M^2_h\right) \left[\frac{d\sigma_{ab\to c}}{dt} \left(s, t, \mu^2\right) \right.
\times \left. \delta \left(1 + \frac{t + u}{s}\right) + \frac{\alpha_s(\mu^2)}{2\pi} K_{ab\to c} \left(s, t, u, \mu^2, M^2_\gamma, M^2_p, M^2_h\right) \theta \left(1 + \frac{t + u}{s}\right) \right],
$$

(12)

where $a,b,c = q_1, \ldots, q_{n_f}$, $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$, and $u = (p_b - p_c)^2$. The parton momenta are related to the photon, proton, and hadron momenta by $p_a = x_\gamma p_\gamma$, $p_b = x_p p_p$, and $p_c = p_H/x_h$. The $K_{ab\to c}$ functions may be found in Ref. 26 for $M_\gamma = M_p$. This restriction was relaxed in Ref. 27.

The NLO cross section of direct photoproduction emerges from Eq. (12) by substituting $F^\gamma_a(x_\gamma, M^2_\gamma) = \delta(1 - x_\gamma)$, replacing $d\sigma_{ab\to c}/dt$ and $K_{ab\to c}$ with $d\sigma_{ab\to c}/dt$ and $K_{ab\to c}$, respectively, and omitting the sum over $a$. The $K_{ab\to c}$ functions were first derived in Ref. 28 setting $M_\gamma = M_p = M_h$ and taking the spin-average for incoming photons and gluons to be $1/2$. In Ref. 27, the scales were disentangled and the spin-average convention was converted to the $\overline{\text{MS}}$ scheme, i.e., to be $1/(n - 2)$, with $n$ being the dimensionality of space-time. Analytic expressions for the $K_{ab\to c}$ functions are listed in Ref. 29.

The theoretical predictions presented here are calculated at NLO in the $\overline{\text{MS}}$ scheme with $n_f = 5$ quark flavours and $\Lambda^{(5)}_{\overline{\text{MS}}} = 202$ MeV. Unless otherwise stated, we use the CTEQ4M proton PDF's, the GRV photon PDF's, and the BKK FF's for charged hadrons and neutral kaons. We set $\mu = M_\gamma = M_p = M_h = \xi p_T$, with $\xi = 1/2, 1, 2$, in order to estimate the theoretical uncertainty.

In Fig. 1, we compare the ZEUS and preliminary H1 data on $ep \to h^\pm + X$ via photoproduction with the corresponding NLO predictions. We find good agreement as for both normalization and shape. In the upper $p_T$ range, our central prediction slightly overshoots the ZEUS data (see Fig. 1a), while it tends to be a tiny bit below the centres of the H1 data points, but well within their errors bars (see Fig. 1b).
The study of the $y_{\text{lab}}$ spectrum in Fig. 1c nicely illustrates the interplay of direct and resolved photoproduction. The direct-photon (resolved-photon) contribution peaks at negative (positive) $y_{\text{lab}}$. At NLO, both contributions strongly depend on the factorization scheme and scale associated with the incoming photon leg, while their sum represents a meaningful physical observable. In the $\overline{\text{MS}}$ scheme with $\xi = 1$, the resolved-photon contribution clearly dominates for a minimum-$p_T$ cut as low as 2 GeV. This offers the opportunity to probe the photon PDF’s. In particular, $F^\gamma_g$ is only feebly constrained experimentally. In the forward direction, at $y_{\text{lab}} > 1$, it makes up more than 50% of the cross section (see Fig. 1d). Thus, a dedicated experimental study in the forward direction could help to pin down $F^\gamma_g$ and to distinguish between the various available photon PDF sets. Figure 1d also shows the predictions for the ACFGFP set with massless charm quark and for the updated GS set. Unfortunately, the minimum-$p_T$ cut in the data is still too low to allow for a sufficiently
Fig. 2. Comparison of H1 data on $ep \to K^0/\bar{K}^0 + X$ via photoproduction with NLO predictions.

precise theoretical description within the QCD-improved parton model.

The H1 data on $ep \to K^0/\bar{K}^0 + X$ via photoproduction are in good agreement with the corresponding NLO predictions (see Fig. 2). In Fig. 2b, the vertical error bars include the statistical errors quoted in Table 4.11 of Ref. 11 and an overall systematic uncertainty of $10\%$ added in quadrature. Strictly speaking, the minimum-$p_T$ cut of 2 GeV is too low for the parton-model prediction to be usefully precise. Nevertheless, Fig. 2b nicely exposes the potential of the $y_{lab}$ distribution to probe $F_{g}^\gamma$ and $D_{K^0}^g$. In view of the general circumstance that the gluon FF’s are less tightly constrained by the $e^+e^-$ data than the quark FF’s and the particular assumption underlying Eq. (9), an independent experimental determination of $D_{K^0}^g$ would be especially desirable.

4. Comparison with data of $p\bar{p}$ scattering

The high-statistics data on $p\bar{p} \to h^\pm + X$ and $p\bar{p} \to K_S^0 + X$ recently published by the UA1 Collaboration offer yet another opportunity to test the universality of the FF’s predicted by the factorization theorem in a nontrivial way (see Fig. 3). At low $p_T$, the bulks of the cross sections are due to the gluon FF’s, which makes $p\bar{p}$ scattering complementary to $e^+e^-$ annihilation. The charged-hadron data are spread over a wide range in $p_T$, way up to $p_T = 25$ GeV, and thus carry intrinsic information on the scaling violation of fragmentation. This is nicely illustrated by the dotted line in Fig. 3a, which emerges from the upper solid line by suspending the AP evolution of the FF’s and evaluating them at the fixed scale $\mu_0 = \sqrt{2}$ GeV instead. This leads to a significant increase at high $p_T$, by more than a factor of 5 at $p_T = 25$ GeV. The experimental data clearly favour the scaling violation encoded in the solid line.
Fig. 3. Comparison of UA1 data on $p\bar{p}\to h^\pm + X$ and $p\bar{p}\to K_0^0 + X$ with NLO predictions.

5. Conclusions

The comparative study of inclusive single-hadron production in $e^+e^−$, $ep$, and $p\bar{p}$ collisions allows for a meaningful quantitative test of the QCD-improved parton model and, in particular, of the scaling violation and universality of fragmentation. Furthermore, photoproduction experiments at HERA provide useful information on the interplay of the direct- and resolved-photon mechanisms. Extensions of these measurements to higher values of $p_T$ and $y_{lab}$ would render an independent determination of $\alpha_s$ possible and considerably improve our knowledge of $F^\gamma_g$.

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1. V.N. Gribov and L.N. Lipatov, Yad. Fiz. 15 (1972) 781 [Sov. J. Nucl. Phys. 15 (1972) 438]; G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298; Yu.L. Dokshitser, Zh. Eksp. Teor. Fiz. 73 (1977) 1216 [Sov. Phys. JETP 46 (1977) 641].
2. V. Barger, T. Gottschalk, and R.J.N. Phillips, Phys. Lett. 70B (1977) 51; R. Baier, J. Engels, and B. Petersson, Z. Phys. C2 (1979) 265; M. Anselmino, P. Kroll, and E. Leader, Z. Phys. C18 (1983) 307.
3. L.M. Sehgal and P.M. Zerwas, Nucl. Phys. B108 (1976) 483; C. Peterson, D. Schlatter, I. Schmitt, and P.M. Zerwas, Phys. Rev. D27 (1983) 105.
4. P. Chiappetta, M. Greco, J.-Ph. Guillet, S. Rolli, and M. Werlen, Nucl. Phys. B412 (1994) 3.
5. M. Greco and S. Rolli, Z. Phys. C60 (1993) 169; Phys. Rev. D52 (1995) 3853; M. Greco, S. Rolli, and A. Vicini, Z. Phys. C65 (1995) 277.
6. J. Binnewies, B.A. Kniehl, and G. Kramer, Z. Phys. C65 (1995) 471.
7. J. Binnewies, B.A. Kniehl, and G. Kramer, Phys. Rev. D52 (1995) 4947.
8. J. Binnewies, B.A. Kniehl, and G. Kramer, Phys. Rev. D53 (1996) 3573.
9. I. Abt et al. (H1 Collaboration), Phys. Lett. B328 (1994) 176; M. Erdmann, private communication.
10. C. Adloff et al. (H1 Collaboration), Report Nos. DESY 97–095 and hep-ex/9705018 (May 1997), Z. Phys. C (in press).
11. K. Johannsen, Ph.D. thesis, University of Hamburg, Internal Report No. DESY FH1–96–01 (June 1996).
12. M. Derrick et al. (ZEUS Collaboration), Z. Phys. C67 (1995) 227.
13. G. Arnison et al. (UA1 Collaboration), Phys. Lett. 118B (1982) 167; G. Bocquet et al. (UA1 Collaboration), Phys. Lett. B366 (1996) 434.
14. G. Bocquet et al. (UA1 Collaboration), Phys. Lett. B366 (1996) 441.
15. G. Altarelli, R.K. Ellis, G. Martinelli, and S.-Y. Pi, Nucl. Phys. B160 (1979) 301; R. Baier and K. Fey, Z. Phys. C2 (1979) 339.
16. J. Binnewies, B.A. Kniehl, and G. Kramer, Report Nos. DESY 97–012, MPI/PhT/97–009, and hep-ph/9702408 (February 1997), Z. Phys. C (in press).
17. G.D. Cowan, in Proceedings of the XXVII International Conference on High Energy Physics, 20–27 July 1994, Glasgow, Scotland, UK, edited by P.J. Bussey and I.G. Knowles (IOP, Bristol, 1995), p. 883; D. Buskulic et al. (ALEPH Collaboration), Phys. Lett. B357 (1995) 487; B364 (1995) 247 (E); C. Padilla Aranda, Ph.D. thesis, University of Barcelona (September 1995).
18. P.D. Acton et al. (OPAL Collaboration), Z. Phys. C58 (1993) 387.
19. P.J. Rijken and W.L. van Neerven, Phys. Lett. B386 (1996) 422; Nucl. Phys. B487 (1997) 233; J. Binnewies, Report Nos. DESY 97–128 and hep-ph/9707269 (July 1997).
20. H. Aihara et al. (TPC/Two Gamma Collaboration), Report No. LBL–23737 (March 1988); Phys. Rev. Lett. 61 (1988) 1263.
21. R. Akers et al. (OPAL Collaboration), Z. Phys. C63 (1994) 181; D. Buskulic et al. (ALEPH Collaboration), Z. Phys. C66 (1995) 355.
22. R. Clare et al. (LEP Electroweak Working Group), Report No. LEPEWWG/97–02 (August 1997).
23. H. Schellman et al. (MARK II Collaboration), Phys. Rev. D31 (1985) R3013.
24. D. Buskulic et al. (ALEPH Collaboration), Z. Phys. C64 (1994) 361.
25. E.J. Williams, Proc. Roy. Soc. London A139 (1933) 163; C.F. v. Weizsäcker, Z. Phys. 88 (1934) 612.
26. F. Aversa, P. Chiappetta, M. Greco, and J.Ph. Guillet, Phys. Lett. B210 (1988) 225; B211 (1988) 465; Nucl. Phys. B327 (1989) 105.
27. B.A. Kniehl and G. Kramer, *Z. Phys.* C62 (1994) 53.
28. P. Aurenche, R. Baier, A. Douiri, M. Fontannaz, and D. Schiff, *Nucl. Phys.* B286 (1987) 553.
29. L.E. Gordon, *Phys. Rev.* D50 (1994) 6753.
30. H.L. Lai, J. Huston, S. Kuhlmann, F. Olness, J. Owens, D. Soper, W.K. Tung, and H. Weerts, *Phys. Rev.* D55 (1997) 1280.
31. M. Glück, E. Reya, and A. Vogt, *Phys. Rev.* D46 (1992) 1973.
32. P. Aurenche, P. Chiappetta, M. Fontannaz, J.P. Guillet, and E. Pilon, *Z. Phys.* C56 (1992) 589.
33. L.E. Gordon and J.K. Storrow, *Nucl. Phys.* B489 (1997) 405.