Commutators and propagators of Moyal star-products and microcausality for free scalar field on noncommutative spacetime

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Abstract

We study the Moyal commutators and their expectation values between vacuum states and non-vacuum states for free scalar field on noncommutative spacetime. Then from the Moyal commutators, we find that the microcausality is satisfied for the linear operators of the free scalar field on noncommutative spacetime. We construct the Feynman propagator of Moyal star-product for noncommutative scalar field theory.

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1 Introduction

Spacetime may have discrete and noncommutative structures under a very small microscopic scale. The concept of spacetime noncommutativity was first proposed by Snyder [1]. About ten years ago, Doplicher et al. proposed the uncertainty relations for the measurement of spacetime coordinates and the simplified noncommutative algebra for the spacetime coordinates [2]. In recent years, spacetime noncommutativity was discovered again in superstring theories [3]. The end points of open strings trapped on a D-brane with a nonzero NSNS two form background B-field turn out to be noncommutative, noncommutative field theories then appear as the low energy effective theory of these D-branes. It has resulted a lot of researches on noncommutative field theories [4,5].

In noncommutative spacetime, we can regard the Moyal star-product as the basic product operation. Thus it is necessary to study the commutators and propagators of Moyal star-products for quantum fields on noncommutative spacetime. In this paper we will study the commutators and propagators of Moyal star-products for scalar field on noncommutative spacetime. In Sec. II, we construct the Moyal commutators and evaluate their expectation values between vacuum states and non-vacuum states for noncommutative scalar field theory.

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We find that the microcausality is satisfied for the linear operators of the free scalar field on noncommutative spacetime. In Sec. III, we calculate the Feynman propagator of Moyal star-product for scalar field on noncommutative spacetime. In Sec. IV, we discuss some of the problems for the microcausality of noncommutative scalar field theory.

2 Commutators of Moyal star-products and their expectation values

In noncommutative spacetime, spacetime coordinates satisfy the commutation relation

\[ [x^\mu, x^\nu] = i\theta^{\mu\nu}, \quad (2.1) \]

where \( \theta^{\mu\nu} \) is a constant real antisymmetric matrix that parameterizes the noncommutativity of the spacetime. For field theories on noncommutative spacetime, they can be obtained through introducing the Moyal star-product, i.e., all of the products between field functions are replaced by the Moyal star-products in the Lagrangian. The Moyal star-product of two functions \( f(x) \) and \( g(x) \) is defined to be

\[
\begin{align*}
  f(x) \star g(x) &= e^{\frac{i}{2}\theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu}} f(x + \alpha)g(x + \beta)|_{\alpha=\beta=0} \\
  &= f(x)g(x) + \sum_{n=1}^{\infty} \left( \frac{i}{2} \right)^n \frac{1}{n!} \theta^{\mu_1\nu_1} \cdots \theta^{\mu_n\nu_n} \partial_{\mu_1} \cdots \partial_{\mu_n} f(x) \partial_{\nu_1} \cdots \partial_{\nu_n} g(x). 
\end{align*}
\]

(2.2)

The Moyal star-product of two functions of Eq. (2.2) is defined at the same spacetime point. We can generalize Eq. (2.2) to two functions on different spacetime points [5]:

\[
\begin{align*}
  f(x_1) \star g(x_2) &= e^{\frac{i}{2}\theta^{\mu\nu} \frac{\partial}{\partial x_1^\mu} \frac{\partial}{\partial x_2^\nu}} f(x_1 + \alpha)g(x_2 + \beta)|_{\alpha=\beta=0} \\
  &= f(x_1)g(x_2) + \sum_{n=1}^{\infty} \left( \frac{i}{2} \right)^n \frac{1}{n!} \theta^{\mu_1\nu_1} \cdots \theta^{\mu_n\nu_n} \partial_{\mu_1} \cdots \partial_{\mu_n} f(x_1) \partial_{\nu_1} \cdots \partial_{\nu_n} g(x_2). 
\end{align*}
\]

(2.3)

Equation (2.3) can be established through generalizing the commutation relation of spacetime coordinates at the same point to two different points:

\[ [x_1^\mu, x_2^\nu] = i\theta^{\mu\nu}. \quad (2.4) \]

We can also expect to search for a grounds of argument for Eq. (2.4) from superstring theories.

In noncommutative spacetime, we can regard the Moyal star-product as the basic product operation. Thus it is necessary to analyze the the commutators of Moyal star-products for quantum fields on noncommutative spacetime. To consider the noncommutative \( \varphi^4 \) scalar field theory, its Lagrangian is given by

\[
\mathcal{L} = \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - \frac{1}{2} m^2 \varphi \varphi - \frac{1}{4!} \lambda \varphi \varphi \varphi \varphi. \quad (2.5)
\]
The Fourier expansion of the free scalar field is given by
\[
\varphi(x, t) = \int \frac{d^3k}{\sqrt{(2\pi)^32\omega_k}} [a(k)e^{ik\cdot x - i\omega t} + a^\dagger(k)e^{-ik\cdot x + i\omega t}].
\] (2.6)

Here we adopt the usual Lorentz invariant spectral measure for the Fourier expansion of the scalar field [6,7]. In Eq. (2.6), we take the spacetime coordinates to be noncommutative. They satisfy the commutation relations (2.1) and (2.4). The commutation relations for the creation and annihilation operators are still the same as that in the ordinary commutative spacetime:
\[
[a(k), a(k')] = \delta^3(\mathbf{k} - \mathbf{k}') ,
\]
\[
[a(k), a(k')] = \delta^3(\mathbf{k} - \mathbf{k}') ,
\]
\[
[a(k), a(k')] = \delta^3(\mathbf{k} - \mathbf{k}') .
\] (2.7)

We define the commutator of the Moyal star-product to be
\[
[\varphi(x), \varphi(y)]_* = \varphi(x) \ast \varphi(y) - \varphi(y) \ast \varphi(x) .
\] (2.8)

We call Eq. (2.8) the Moyal commutator for convenience. From the Fourier expansion of Eq. (2.6) for the free field, we can calculate the Moyal commutator of two scalar fields. It is given by
\[
[\varphi(x), \varphi(y)]_* = \int \frac{d^3kd^3k'}{(2\pi)^3\sqrt{2\omega_k2\omega'_k}} \left\{ a(k)e^{-ikx} + a^\dagger(k)e^{ikx}, a(k')e^{-ik'y} + a^\dagger(k')e^{ik'y} \right\}_*
\]
\[
= \int \frac{d^3kd^3k'}{(2\pi)^3\sqrt{2\omega_k2\omega'_k}} \left\{ a(k)e^{-ikx}, a(k')e^{-ik'y} \right\}_* + \left\{ a(k)e^{-ikx}, a^\dagger(k')e^{ik'y} \right\}_* 
\]
\[
+ \left\{ a^\dagger(k)e^{ikx}, a(k')e^{-ik'y} \right\}_* + \left\{ a^\dagger(k)e^{ikx}, a^\dagger(k')e^{ik'y} \right\}_*. 
\] (2.9)

For the reason that there are two kinds of noncommutative structures, i.e., field operators and spacetime coordinates, the spacetime coordinates now is noncommutative, we cannot apply the commutation relations for the creation and annihilation operators of Eq. (2.7) directly to obtain a c-number result for the Moyal commutator.

In order to obtain the c-number result for the Moyal commutator, we can calculate its vacuum state expectation value. We have
\[
\langle 0 | [\varphi(x), \varphi(y)]_* | 0 \rangle 
\]
\[
= \langle 0 | \int \frac{d^3kd^3k'}{(2\pi)^3\sqrt{2\omega_k2\omega'_k}} \left\{ a(k)a^\dagger(k')e^{-ikx} \ast e^{ik'y} - a(k')a^\dagger(k)e^{-ik'y} \ast e^{ikx} \right\} | 0 \rangle 
\]
\[
= \int \frac{d^3k}{(2\pi)^32\omega_k} \left[ e^{-ikx} \ast e^{iky} - e^{-iky} \ast e^{ikx} \right]
\]
\[
= \int \frac{d^3k}{(2\pi)^32\omega_k} \left[ \exp \left( \frac{i}{2} k \cdot k \right)e^{-ik(x-y)} - \exp \left( \frac{i}{2} k \cdot k \right)e^{ik(x-y)} \right] ,
\]
where we have applied Eq. (2.3) and we note
\[ k \times p = k_\mu \theta^{\mu\nu} p_\nu . \]  
Because \( \theta^{\mu\nu} \) is antisymmetric, \( k \times k = 0 \), we obtain
\[ \langle 0 | [\varphi(x), \varphi(y)] \rangle = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \left[ e^{-ik(x-y)} - e^{ik(x-y)} \right] \]
\[ = -\frac{i}{(2\pi)^3} \int \frac{d^3k}{\omega_k} e^{ik(x-y)} \sin \omega_k(x_0 - y_0) = i\Delta(x - y) . \]

So the result of Eq. (2.12) is equal to the commutator of scalar field in ordinary commutative spacetime:
\[ [\varphi(x), \varphi(y)] = -\frac{i}{(2\pi)^3} \int \frac{d^3k}{\omega_k} e^{ik(x-y)} \sin \omega_k(x_0 - y_0) = i\Delta(x - y) . \]  

It is obvious to see that this equality relies on the antisymmetry of \( \theta^{\mu\nu} \).

We can also calculate the expectation values between non-vacuum states for the Moyal commutators. Let \( |\Psi\rangle \) represent a normalized non-vacuum physical state which is in the occupation eigenstate:
\[ |\Psi\rangle = |N_{k_1}N_{k_2} \cdots N_{k_i} \cdots, 0\rangle , \]  
where \( N_{k_i} \) represents the occupation number of the momentum \( k_i \). We can suppose that the occupation numbers are nonzero only on some separate momentums \( k_i \). For all other momentums, the occupation numbers are zero. We use 0 to represent that the occupation numbers are zero on all the other momentums in Eq. (2.14). The state vector \( |\Psi\rangle \) has the following properties:
\[ \langle N_{k_1}N_{k_2} \cdots N_{k_i} \cdots |N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle = 1 , \]  
\[ \sum_{N_{k_1}N_{k_2} \cdots} |N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle \langle N_{k_1}N_{k_2} \cdots N_{k_i} \cdots | = 1 , \]  
\[ a(k_i) |N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle = \sqrt{N_{k_i}} |N_{k_1}N_{k_2} \cdots (N_{k_i} - 1) \cdots \rangle , \]  
\[ a^\dagger(k_i) |N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle = \sqrt{N_{k_i} + 1} |N_{k_1}N_{k_2} \cdots (N_{k_i} + 1) \cdots \rangle . \]

Equation (2.16) is the completeness expression for the state vector \( |\Psi\rangle \). Therefore Eq. (2.14) can represent an arbitrary scalar field quantum system. Then from Eq. (2.9), the expectation value between any non-vacuum physical states for the Moyal commutator is given by
\[ \langle \Psi | [\varphi(x), \varphi(y)] |\Psi\rangle \]
\[ = \langle \Psi | \int \frac{d^3k d^3k'}{(2\pi)^3 \sqrt{2\omega_k 2\omega_{k'}}} \left\{ [a(k)e^{-ikx}, a^\dagger(k')e^{ik'y}]_\ast + [a^\dagger(k)e^{ikx}, a(k')e^{-ik'y}]_\ast \right\} |\Psi\rangle \]
\[ = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \left[ e^{-ikx} \ast e^{iky} - e^{-iky} \ast e^{ikx} \right] = i\Delta(x - y) , \]
which is just equal to the vacuum state expectation value of Eq. (2.12). The properties of the commutation relations for the creation and annihilation operators of Eq. (2.7) are still reflected in the above evaluations for the non-vacuum state expectation values for the Moyal commutators.

The vacuum state expectation value and non-vacuum state expectation value of the equal-time Moyal commutator can be obtained from Eqs. (2.12) and (2.19). They are

\[
\langle 0 | [\varphi(x, t), \varphi(y, t)]_\star | 0 \rangle = \Delta(x - y, 0) = 0 , \tag{2.20}
\]

\[
\langle \Psi | [\varphi(x, t), \varphi(y, t)]_\star | \Psi \rangle = \Delta(x - y, 0) = 0 . \tag{2.21}
\]

Because \(\Delta(x - y)\) is a Lorentz invariant singular function, it satisfies

\[
\Delta(x - y) = 0 \quad \text{for all} \quad (x - y)^2 < 0 , \tag{2.22}
\]

we have

\[
\langle 0 | [\varphi(x), \varphi(y)]_\star | 0 \rangle = 0 \quad \text{for all} \quad (x - y)^2 < 0 , \tag{2.23}
\]

and similarly

\[
\langle \Psi | [\varphi(x), \varphi(y)]_\star | \Psi \rangle = 0 \quad \text{for all} \quad (x - y)^2 < 0 . \tag{2.24}
\]

For quantum fields in ordinary commutative spacetime, they satisfy the microcausality principle [6,7]. For scalar field theory we have

\[
[\varphi(x), \varphi(y)] = 0 \quad \text{for all} \quad (x - y)^2 < 0 , \tag{2.25}
\]

which means that any two fields as physical observables commute with each other when they are separated by a spacelike interval. Or we can say any two physical measurements separated by a spacelike interval do not interfere each other. In noncommutative spacetime, we can regard the Moyal star-product as the basic product operation. Although we cannot obtain that \([\varphi(x), \varphi(y)]_\star = 0\) for \((x - y)^2 < 0\) from Eqs. (2.23) and (2.24), because any physical measurement is taken under certain physical state, what the observer measures are certain expectation values in fact, Eqs. (2.23) and (2.24) can still represent the satisfying of microcausality for free scalar field on noncommutative spacetime. However in the above we have only analyzed the linear operator \(\varphi(x)\). For the other linear operators of the scalar field such as \(\partial^\mu \varphi(x)\), we can obtain that their microcausality properties on noncommutative spacetime are similar to that of the linear operator \(\varphi(x)\). They also satisfy the microcausality. The calculation and result are similar as above. Thus we omit to write down them here explicitly. For the quadratic operators of free scalar field on noncommutative spacetime such as \(\varphi(x) \star \varphi(x)\), their microcausality properties need to be studied further.

### 3 Feynman propagator of Moyal star-product for noncommutative scalar field theory

The same as quantum fields in ordinary commutative spacetime [6,7], we can decompose the Fourier expansion of the free scalar field into positive frequency part and negative
frequency part:

\[ \varphi(x) = \varphi^+(x) + \varphi^-(x), \quad (3.1) \]

where

\[ \varphi^+(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} a(k)e^{ik\cdot x - i\omega t} = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} a(k)e^{-ikx}, \quad (3.2) \]
\[ \varphi^-(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} a^\dagger(k)e^{-ik\cdot x + i\omega t} = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} a^\dagger(k)e^{ikx}. \quad (3.3) \]

According to Eq. (2.9), we can decompose the vacuum expectation value of the Moyal commutator into two parts:

\[ \langle 0 |[\varphi(x), \varphi(y)]_s|0 \rangle = \langle 0 |[\varphi^+(x), \varphi^-(y)]_s|0 \rangle + \langle 0 |[\varphi^-(x), \varphi^+(y)]_s|0 \rangle. \quad (3.4) \]

For the two parts of Eq. (3.4), we obtain

\[ \langle 0 |[\varphi^+(x), \varphi^-(y)]_s|0 \rangle = \langle 0 | \int \frac{d^3k}{(2\pi)^3 2\omega_k} [a(k)e^{-ikx}, a^\dagger(k)e^{iky}]_s|0 \rangle = \int \frac{d^3k}{(2\pi)^3 2\omega_k} e^{-ikx} \ast e^{iky} = \int \frac{d^3k}{(2\pi)^3 2\omega_k} e^{-i(kx-y)} = i\Delta^+(x-y), \quad (3.5) \]

and

\[ \langle 0 |[\varphi^-(x), \varphi^+(y)]_s|0 \rangle = \langle 0 | \int \frac{d^3k}{(2\pi)^3 2\omega_k} [a^\dagger(k)e^{ikx}, a(k)e^{-iky}]_s|0 \rangle = - \int \frac{d^3k}{(2\pi)^3 2\omega_k} e^{-iky} \ast e^{ikx} = - \int \frac{d^3k}{(2\pi)^3 2\omega_k} e^{-ik(y-x)} = -i\Delta^+(y-x) = i\Delta^-(x-y). \quad (3.6) \]

Thus we can write

\[ \langle 0 |[\varphi(x), \varphi(y)]_s|0 \rangle = i\Delta(x-y) = i(\Delta^+(x-y) + \Delta^-(x-y)). \quad (3.7) \]

We can also see that if we replace the vacuum state by any non-vacuum state in the above formulas the results will not change. The above results are also equal to the corresponding commutators in the ordinary commutative spacetime.

For the commutator (3.5), we can rewrite it furthermore:

\[ \langle 0 |[\varphi^+(x), \varphi^-(y)]_s|0 \rangle = \langle 0 |[\varphi^+(x) \ast \varphi^-(y)]|0 \rangle = \langle 0 |[\varphi(x) \ast \varphi(y)]|0 \rangle = i\Delta^+(x-y). \quad (3.8) \]
We define the time-ordered Moyal star-product of two scalar field operators to be

\[ T \phi(x) \star \phi(x') = \theta(t - t') \phi(x) \star \phi(x') + \theta(t' - t) \phi(x') \star \phi(x), \]

(3.9)

where \( \theta(t - t') \) is the unit step function. We can calculate the vacuum expectation value of Eq. (3.9):

\[ \langle 0 | T \phi(x) \star \phi(x') | 0 \rangle = \theta \langle 0 | \phi(x) \star \phi(x') | 0 \rangle + \theta \langle 0 | \phi(x') \star \phi(x) | 0 \rangle. \]

(3.10)

Equation (3.10) is just the Feynman propagator of Moyal star-product for scalar field. We can call it the Feynman Moyal propagator for convenience. To introduce the singular function \( \Delta_F(x) \), we can write the Feynman Moyal propagator (3.10) as

\[ \langle 0 | T \phi(x) \star \phi(x') | 0 \rangle = i \Delta_F(x - x'). \]

(3.11)

From Eqs. (3.6), (3.8), and (3.10), we have

\[ \Delta_F(x - x') = \theta(t - t') \Delta^+(x - x') - \theta(t' - t) \Delta^-(x - x'), \]

(3.12)

where the momentum integral representation of the singular function \( \Delta_F(x - x') \) is given by

\[ \Delta_F(x - x') = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\epsilon} e^{-ik \cdot (x - x')} . \]

(3.13)

From the above results, we can see the Feynman Moyal propagator of noncommutative scalar field is just equal to the Feynman propagator of scalar field on ordinary commutative spacetime. However it is necessary to point out that in Eq. (3.9) for the definition of the time-ordered Moyal star-product of two scalar fields, we have applied a simplified manipulation. This is because the Moyal star-products are not invariant generally under the exchange of the orders of two functions, for the second term of the right hand side of Eq. (3.9) we need to consider this fact. In the Fourier integral representation, it can be seen clearly that the second term of the right hand side of Eq. (3.9) will have an additional phase factor \( e^{ik \cdot k'} \) in contrast to the first term of the right hand side of Eq. (3.9) due to the exchange of the order of \( \phi(x) \) and \( \phi(x') \) for the Moyal star-product. However in Eq. (3.10) when we calculate the vacuum expectation value for Eq. (3.9), we can see that the non-zero contribution comes from the \( k = k' \) part inside the integral (cf. Eqs. (3.5) and (3.6)). This will make the phase factor to be \( e^{ik \cdot k} \), which is 1 due to the antisymmetry of \( \theta^{\mu \nu} \). Thus in the right hand side of Eq. (3.9), we can omit this effect in the exchange of the order of two scalar fields for their Moyal star-product equivalently.

Just like that in ordinary commutative spacetime, the physical meaning of the Feynman Moyal propagator (3.10) can also be explained as the vacuum to vacuum transition amplitude for quantum fields on noncommutative spacetime. We can also construct the Feynman Moyal propagators for other fields such as the fermion field and electromagnetic field. The reason why we would like to construct the Feynman propagators of Moyal star-products for quantum fields on noncommutative spacetime is that: for noncommutative field theories we can establish their \( S \)-matrix where the products between field operators in \( \mathcal{H}_{\text{int}} \) are Moyal
star-products. From the Wick’s theorem expansion for the time ordered products of field operators, there will occur the Feynman Moyal propagators. Therefore it is necessary to study Feynman propagators of Moyal star-products for quantum fields on noncommutative spacetime.

4 Discussion

In this paper, we studied the Moyal commutators and their expectation values between vacuum states and non-vacuum states for free scalar field on noncommutative spacetime. We also studied the Feynman propagator of Moyal star-product for scalar field on noncommutative spacetime. From the Moyal commutators, we find that the microcausality is satisfied on noncommutative spacetime for the linear operators of the free scalar field. For the quadratic operators of the free scalar field on noncommutative spacetime, their microcausality properties need to be studied further. For the microcausality of the quadratic operators of the scalar field on noncommutative spacetime, their microcausality properties have been studied by some authors in the literature [8,9]. In Ref. [8], Greenberg obtained that for the commutators $[\varphi(x)\star\varphi(x) : \varphi(y)\star\varphi(y)]_\star$ and $[\varphi(x)\star\varphi(x) : \partial_\mu : \varphi(y)\star\varphi(y)]$, they do not satisfy the microcausality on noncommutative spacetime. However the result of Ref. [8] is based on an incomplete expansion and quantization of the scalar field. In Ref. [9], through analyzing the expectation value $\langle 0 | [\varphi(x)\star\varphi(x) : \varphi(y)\star\varphi(y)] | p, p' \rangle$, Chaichian et al. obtained that microcausality is violated for quantum fields on noncommutative spacetime when $\theta_{0i} \neq 0$. However, for the microcausality problem, it is more reasonable that we should analyze the expectation values between the same state vectors for the field operators. In Ref. [10], through supposing the spectral measure to be the form of $SO(1,1) \times SO(2)$ invariance, the authors obtained the result that microcausality is violated for quantum fields on noncommutative spacetime generally. However, such a conclusion is a necessary result of the breakdown of the Lorentz invariance in Ref. [10]. In Ref. [11], the authors obtained that even the $SO(1,1)$ microcausality may be violated for quantum fields on noncommutative spacetime through calculating the propagators with quantum corrections.

It is necessary to point out that the microcausality problem discussed in this paper is only restricted to free fields. For quantum field theories on noncommutative spacetime, there exist some other possibilities to violate the microcausality. For example because there exist the UV/IR mixing phenomena [12], the infrared singularities that come from non-planar diagrams may result the existence of nonlocal and instantaneous interactions [13]. These nonlocal and instantaneous interactions do not satisfy the usual microcausality of quantum field theories on ordinary commutative spacetime. For the classical theory of fields on noncommutative spacetime, there also exist the possibilities that violate the causality principle. For example in Ref. [14], Durhuus and Jonsson proved that for nonlinear noncommutative waves, their propagation speed is infinite. They found that if the initial conditions have a compact support then for any positive time the support of the solution can be arbitrarily large. In Ref. [15], the authors have found that for the solitons in noncommutative gauge theories, they can travel faster than the speed of light with an arbitrary speed. These phenomena mean that for noncommutative field theories, there exist
the instantaneous interactions even though for their classical field theories.

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