Discrete breathers in dc biased Josephson-junction arrays

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We propose a method to excite and detect a rotor localized mode (rotobreather) in a Josephson-junction array biased by dc currents. In our numerical studies of the dynamics we have used experimentally realizable parameters and included self-inductances. We have uncovered two families of rotobreathers. Both types are stable under thermal fluctuations and exist for a broad range of array parameters and sizes including arrays as small as a single plaquette. We suggest a single Josephson-junction plaquette as an ideal system to experimentally investigate these solutions.

The phenomenon of intrinsic localization (intrinsic localized modes or discrete breathers (DB)) is a recent discovery in the subject of nonlinear dynamics \[\text{DB}\]. DB are solutions to the dynamics of discrete extended systems for which energy is exponentially localized in space. They appear either as oscillator localized modes, for which a localized group of oscillators librate; or rotor localized modes or rotobreathers, for which a group of oscillators rotate while the others librate \[\text{DB}\]. Recently, it has been found that DB are not restricted to periodic solutions but can also include more complex (chaotic) dynamics \[\text{DB}\].

DB have been proven to be generic solutions in hamiltonian \[\text{DB}\] and dissipative \[\text{DB}\] nonlinear lattices. It is believed that they might play an important role in the dynamics of a large number of systems, such as coupled nonlinear oscillators or rotors.

Though intrinsic localized modes have been the object of great theoretical and numerical attention in the last 10 years, they have yet to be generated and detected in an experiment. Thus, finding the best system and method for the generation, detection and study of an intrinsic localized mode in a Condensed Matter system has become an important challenge \[\text{DB}\].

Josephson-junctions arrays are excellent experimental systems for studying nonlinear dynamics \[\text{DB}\]. In this paper we propose an experiment to detect a rotating localized mode in JJ anisotropic ladder arrays biased by dc external currents \[\text{DB}\]. For this, we have done numerical simulations of the dynamics of an open ladder including induced fields \[\text{DB}\] at experimentally accessible values of the parameters of the array. We also propose a method for exciting a rotobreather in the array. We distinguish between two families of solutions which present different voltage patterns in the array. Both types are robust to random fluctuations and exist over a range of parameter values and array sizes. Unexpectedly, we have found that many of the rotobreather solutions do not satisfy the up-down symmetry usually assumed for most types of dynamical solutions in the ladder. We also show that a DB solution can be most readily studied in a single plaquette.

According to the RCSJ model, a Josephson junction is characterized by its critical current $I_c$, normal state resistance $R_n$, and capacitance $C$. The junction voltage $v$ is related to the gauge-invariant phase difference $\varphi$ as

$$v = \frac{\Phi_0}{2\pi} \frac{d\varphi}{dt},$$

where $\Phi_0$ is the flux quantum. After standard rescaling of the time by $\tau = \sqrt{\Phi_0 C / 2\pi I_c}$, the normalized current through the junction is

$$i = \dot{\varphi} + \Gamma \varphi + \sin\varphi,$$

here $\Gamma$ represents a damping and is directly related to the Stewart-McCumber parameter $\beta_c = \Gamma^{-2} = 2\pi I_c CR_n^2 / \Phi_0$.

Our anisotropic JJ ladders (see Fig. 1) contain junctions of two different critical currents: $I_{ch}$ for the horizontal junctions and $I_{cv}$ for the vertical ones. Anisotropic arrays are easily fabricated by varying the area of the junctions. In the case of unshunted junctions, the critical current and capacitance are proportional to this area. Due to the constant $I_c R_n$ product, the normal state resistance is inversely proportional to the junction area. The anisotropy parameter $h$ can then be defined as $h = I_{ch}/I_{cv} = C_h/C_v = R_c/R_h$.

To write the governing equations of an anisotropic JJ ladder array with $N$ cells, Fig. 1, we need to apply current

\[\text{FIG. 1. Anisotropic ladder array with uniform current injection. Vertical junctions have critical current $I_{cv}$ and horizontal junctions $I_{ch}$.}\]

\[\text{\,}\]
conservation at each node and flux quantization at each mesh. We are including self-induced magnetic fields so that flux quantization at mesh $j$ yields
\[ (\nabla \times \varphi)_j = -2\pi f_j. \] (3)

Here $(\nabla \times \varphi)_j = \varphi_j^v + \varphi_{j+1}^v - \varphi_j^h - \varphi_{j+1}^h$ and it represents the circulation of gauge-invariant phase differences in mesh $j = 1$ through $N$. The self-induced flux through mesh $j$ normalized by $\Phi_0$ is given by $f_j$. The resulting equation can be written compactly as,
\[ h(\varphi_j^v + \varphi_{j+1}^v - \varphi_j^h - \varphi_{j+1}^h) = -\lambda(\nabla \times \varphi)_j \]
\[ \varphi_j^v + \varphi_{j+1}^v = \lambda[(\nabla \times \varphi)_j - (\nabla \times \varphi)_{j-1}] + I \]
\[ h(\varphi_j^h + \varphi_{j+1}^h - \varphi_j^v - \varphi_{j+1}^v) = \lambda(\nabla \times \varphi)_j, \] (4)

where the open boundaries can be imposed by setting $(\nabla \times \varphi)_0 = (\nabla \times \varphi)_{N+1} = 0$ in Eqs. (3). The system has four independent parameters: $h$, $\Gamma$, the penetration depth $\lambda = \Phi_0/2\pi I_{c\text{avg}} L$ where $L$ is the mesh self-inductance, and the normalized external current $I$. On writing Eqs. (3) and (4) we assume zero external field and normalize by $I_{c\text{avg}}$. Non-zero external fields can be included in the model replacing the $(\nabla \times \varphi)_j$ terms in Eqs. (3) and (4) by $(\nabla \times \varphi)_j + 2\pi f_j^{\text{ext}}$ where $f_j^{\text{ext}}$ is the flux due to an applied external field, measured in terms of $\Phi_0$.

The parameter values we will consider are based on Nb-Al$_2$O$_3$-Nb junctions with a critical current density of 1000 A/cm$^2$. Typical values of the Stewart-McCumber parameter and the penetration depth for arrays with $h = 1/4$ are $\beta_c \sim 30$ and $\lambda \sim 0.02$. For the purposes of this work we will let $\Gamma = 0.2$, $\lambda = 0.02$ and $N = 8$.

Consider the $h = 0$ limit in Eqs. (3). In this limit the vertical junctions behave as uncoupled damped pendula driven by an external current $I$. There, we can think of a configuration in which one or a few of the phases are rotating or oscillating around their equilibrium points while the others remain at rest. Thus rotor and/or oscillator localized modes appear as solutions to the dynamics when the array is biased either by dc or ac external currents.

For a single underdamped junction driven by a constant external current, the response measured in terms of dc voltage presents a hysteresis loop between the depinning and the retrapping currents. In this range the pinned $(V = 0)$ and rotating $(V \neq 0)$ solutions coexist. Then the rotobreather solution in the $h = 0$ limit corresponds to a solution in which the phase of one of the vertical junctions is rotating while the other vertical junctions are at rest.

As $h$ is increased from zero the non-convex character of the coupling allows for the continued existence of rotobreathers in the system. Since a solution with a time increasing field cannot physically exist, the flux quantization condition (Eq. 3) implies that each cell with a rotating junction must have at least one other junction which is rotating. Thus, for the single rotobreather solution one of the vertical and some of the horizontal neighboring junctions rotate. Fig. 2 shows schematically simple rotating localized modes in a ladder and in the single plaquette. These DB are amenable to simple experimental detection when measuring the average voltage through different junctions.

Although the rotobreather solution can theoretically be continued from its $h = 0$ limit by varying $h$, we have developed a simple method of exciting it in an array. This method should be experimentally reproducible and has three steps: (i) bias all the array up to the operating point ($I = I^*$); (ii) increase the current injected into one of the junctions to a value of the current above the junction critical current ($I = I^* + \tilde{I} > 1$); (iii) go back to the operating point by decreasing to zero the value of this extra current $\tilde{I}$. Typical values of $I^*$ and $\tilde{I}$ in our simulations are 0.6.

We have checked the robustness of this method under fluctuations by simulating the equations of the ladder while adding a noise current to the junctions (this is the standard manner of including thermal effects in the system [11]). Thus we are able to excite DB in the ladder at some values of the parameters of the system. The solution showed in Fig. 3 was excited using this procedure.

Henceforth, we are going to consider ladders with an even number of cells for which one vertical junction (the central one) is rotating. We will relabel this junction as $j = 0$.

Figs. 3 shows a solution of a stable rotobreather in a JJ ladder. We plot the phase portrait $(\varphi_j^v, \varphi_j^h)$ of some of the superconducting gauge invariant phase differences of the array. The corresponding junctions are shown in Fig. 2(a). For clarity we have reduced the values of the phases to the $(-\pi, \pi]$ interval. We see that at this value of the penetration depth the solution is highly localized.

![FIG. 2. Schematic picture of rotobreather solutions: type A rotobreather in the ladder array (a), and plaquette (b); type B rotobreather in the ladder (c) and plaquette (d). Arrows are associated with rotating junctions and labels in (a) corresponds to graphs in Fig. 3](image)
while three of the junctions describe a nearly sinusoidal rotation all the others oscillate with decreasing amplitudes. The average voltage through the three rotating junctions in the array is different from zero and equals to zero for all the other junctions. Fig. 4 shows the average value of the induced field of the cells of the array.

![Graph showing the average value of the induced flux.](image)

**FIG. 4.** The average value of the induced flux \( \bar{f}_j = -f_{j-1} \) at all the cells of the ladder for the rotobreather shown in Fig. 3. This average value decreases exponentially which is characteristic of DB solutions.

Horizontal rotating junctions. Type A solutions have two possible configurations. The two rotating horizontal junctions can be both in the same side, either top or bottom, as in Fig. 3(a); or one in the top and the other in the bottom. The second family, rotobreather B [Fig. 3(c)], is characterized by one vertical and four horizontal rotating junctions. The solution shown in Fig. 3 and 4 is a type A rotobreather. Up-down symmetric solutions belong to family B but not all family B solutions satisfy this symmetry.

Figs. 3 and 4 show a solution for which the scale of localization is smaller than one cell. Thus, it is natural to study the DB solution in the simplest ladder array, the single plaquette. Obviously, the concept of exponential spatial localization is not applicable to the plaquette, but all the other characteristics of the solution remain. In particular we will also distinguish between type A and type B rotobreather solutions in the plaquette, which in this case correspond to one vertical and one horizontal rotating junctions [Fig. 3(b)], and one vertical and both horizontal rotating junctions [Fig. 3(d)] respectively. The single plaquette biased by dc external currents, is then proposed as the simplest and most convenient experimental system for detecting a rotating localized mode. The method for exciting the mode is also applicable to this system.

An important experimental issue then becomes finding the region of existence of these DB solutions with respect to the system parameters in order to investigate the feasibility of designing an array in to detect a DB. To design an array we need to calculate the junction areas, so that the anisotropy needs to be known. Since different values of the anisotropy affect the cell geometry, they also change the value of \( \lambda \). On the other hand, \( \Gamma \) is determined by the current density of the junctions and therefore it is fixed and independent of the geometry while the applied current can be easily changed while measuring. In order to make an optimal design we will fix the value of \( I \) and \( \Gamma \) to 0.6 and 0.2 respectively and study DB solutions in

![Graph showing the solution of the dynamics of the array.](image)
the $(h, \lambda)$ plane of parameters.

Type A and type B rotobreathers exist close to the $h = 0$ limit. We then calculate the maximum value of the anisotropy for which a DB exists as a stable solution at different values of $\lambda$ and $I$, respectively. Lines serve as a guide to the eyes.

![Graph of maximum anisotropy for existence of type A and B rotobreathers] (a) Ladder (b) Plaq

**FIG. 5.** Maximum values of anisotropy for the existence of type A (solid circles) and type B (open circles) rotobreather solutions at different values of $\lambda$ in an 8 cell ladder (a) and a single plaquette array (b). $\Gamma$ and $I$ are equal to 0.2 and 0.6 respectively. Lines serve as a guide to the eyes.

The rotobreather solution shown in Figs. 3(a) and 3(b) presents a mirror symmetry with respect to the rotating vertical junction: $\varphi_j^{(a)} = \varphi_{-j}^{(a)}$, and $\varphi_j^{(b)} = -\varphi_{-j-1}^{(b)}$. In the case of solutions satisfying a mirror symmetry it is possible to map the dynamics of a JJ ladder for which the rotating junction is the central one, to the dynamics of a smaller JJ ladder for which the rotating junction is on one of the ends. Then, due to the localized nature of the DB solution the dynamics can be approximated by studying a single plaquette. When doing these transformations we need to rescale two of the parameters of the equations. Thus, results for the DB solution studied above present some similarities with the dynamics of a DB in a single plaquette when $h_p = 2h_l$ and $\lambda_p = 2\lambda_l$.

By establishing a criteria for the design of simple experiments to detect these intrinsic localized modes we hope to stimulate experimental investigations.

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