This paper presents a brief review of the evidence for dark matter in the Universe on the scales of galaxies. In the interests of critically and objectively testing the dark matter paradigm on these scales, this evidence is weighed against that from the only other game in town, modified Newtonian dynamics. The verdict is not as clear cut as one might have hoped.

1. Introduction

In the study of dark matter, its properties on the galactic scale are of abiding interest. It is, after all, the signature of a galaxy’s dark halo that almost all of the current dark matter experiments are seeking to detect — they are trying to interact with the rather small part of the Milky Way’s massive halo that happens to be passing through the experimenters’ laboratories at the moment. Any predictions as to detection rates, etc, are therefore dependent on our understanding the properties of galactic-scale dark matter halos.

Galaxies have also played a pivotal role in the entire development of the dark matter paradigm. In one of the founding papers on the subject,\(^1\) Zwicky used the galaxies in the Coma Cluster as test particles to which he applied the virial theorem. This calculation led to an unexpectedly high mass for the cluster, and Zwicky pointed out that if this total were divided amongst the constituent galaxies, then they would each have an average mass of $5 \times 10^{10} M_{\odot}$ (where $M_{\odot}$ is the mass of the Sun), whereas their average luminosity was estimated to be only $9 \times 10^7 L_{\odot}$ (where $L_{\odot}$ is the Sun’s luminosity). Clearly, such galaxies could not be made up of stars like the Sun, and some extra contributor to the mass is required. In this prescient paper, Zwicky also pointed out that gravitational lensing should
provide another approach to determining the masses of clusters, and that the rotation curves of individual galaxies—their speed of rotation as a function of radius, \( v_c(r) \)—should allow one to probe the distribution of mass within these systems by equating the centripetal acceleration to the gravitational acceleration due to the mass.

Early studies of rotation curves were hampered by the fact that the optical emission lines used to measure Doppler shifts, and hence rotation speeds, could only be detected in the inner parts of galaxies. In general, at these small radii the inferred mass is quite consistent with the rather uncertain census of luminous components. However, the discovery that spiral galaxies are surrounded by huge disks of atomic hydrogen completely changed the picture. Using the Doppler shifts in this gas’ 21cm radio emission, it became clear that the rotation speeds of such galaxies remain roughly constant out to the largest radii measured, \( v_c \sim \) constant. This is certainly not what would have been predicted on the basis of the known luminous mass, as the measurements were made at radii well beyond most of these components, so one would have expected the velocities to have entered a Keplerian decline, with \( v_c \propto r^{-1/2} \). These incontrovertible observations were fundamental ingredients in a huge shift of paradigm, taking astronomers from a position in which evidence for dark matter could be quietly swept under the cosmic rug to one in which it was recognised that dark matter seems to dominate the Universe on almost all scales.

Over the last twenty-five years, this recognition has spawned an entire industry of simulators trying to model the formation of structure in the Universe through the gravitational collapse of dark-matter-dominated material, concentrating on the idea that this mass comprises gravitationally-interacting non-relativistic “cold dark matter” (CDM). In the early days, these simulations lacked the resolution to study the properties of individual galaxies, but more recently very large computer simulations have achieved the dynamic range necessary to study the formation and properties of individual galaxies in CDM cosmology. These simulations now make some fairly definite and robust predictions as to the distribution of dark matter on galactic scales. In particular, they predict that the dark matter should be distributed such that its density distribution has a central power-law cusp, \( \rho \propto r^{-\gamma} \). Authors differ somewhat as to the steepness of this cusp, but it seems to have a power-law index of between \( \gamma = 1 \) and \( \gamma = 1.5 \). However, all seem to agree that at large radii the density profile steepens to a power-law index of \( \gamma = 3 \). This all fits reasonably well with the observed flat rotation curves, which would be produced by a density profile
with $\gamma = 2$, as the measured rotation curves typically probe intermediate radii between these two extreme regimes.

The simulations also predict the overall shapes of the dark halos, which are typically oblate with a range of shortest-to-longest axis ratios of $\sim 0.5 \pm 0.2$. This quantity is hard to determine observationally — the rotation curve of a spiral galaxy, for example, provides a useful measure of the radial distribution of mass in a galaxy, but not its distribution perpendicular to the plane of the galaxy’s disk. However, probes of this third dimension do exist in the form of the orbits of material in merging satellites, the thickness of the gas layer, and the shapes of hot X-ray emitting gas halos around elliptical galaxies. As far as can be ascertained from these limited data, the observed shapes of halos are consistent with the CDM predictions.

Not everything in the CDM simulations fits so well with the observations, though. In particular, the repeated merging of sub-halos that produces galaxies in these simulations does not produce a simple smooth density distribution. Instead, the simulations predict that one should see large amounts of substructure in the form of small satellite systems around every large galaxy. Although big objects like the Milky Way are accompanied by a retinue of smaller satellites, there seems to be a problem in that CDM simulations predict far more companions than are observed. This galaxy formation picture is also beginning to run into trouble with observations at high redshift: in this hierarchical scenario, the largest most massive galaxies should form last, yet observations indicate that some fraction of very luminous galaxies are already in place quite early in the Universe’s history.

2. The Alternative to Dark Matter

Although the dark matter paradigm is almost universally accepted in the astronomical community, it does seem to have its limitations, so it still pays to step back from it occasionally and try to weigh it up against competing theories. In this regard, the only real alternative is the Modified Newtonian Dynamics or MOND hypothesis. In this theory, the Newtonian acceleration due to gravity, $a_n$, is replaced by an acceleration $a$ that obeys the equation $a_n = a \times \mu(|a|/a_0)$. The function $\mu$ is chosen so that $a$ varies smoothly from $a_n$ when $a \gg a_0$ to $\sqrt{a_n \times a_0}$ when $a \ll a_0$. The only free parameter in the theory is the characteristic acceleration constant $a_0$ at which the transition occurs.

This modification to Newton’s laws neatly explains the flatness of the outer parts of galaxy rotation curves. At these large radii, Newtonian
acceleration due to gravity is simply \( a_n \approx \frac{GM}{r^2} \), where \( M \) is the galaxy’s total mass. Centripetal acceleration here is sufficiently low that we are in the MOND regime, so we can write

\[
\frac{v_c^2}{r} = a \approx \sqrt{\frac{GM}{r^2}} \times a_0.
\] (1)

The \( r \) dependence cancels from the equation, so we find that

\[
v_c \approx (GMa_0)^{1/4},
\] (2)

independent of radius.

This amendment to Newton’s laws is somewhat ad hoc, and was clearly reverse-engineered to deal with the issue of flat rotation curves. However, it is quite legitimate to ask whether this solution is any more arbitrary than postulating the existence of invisible mass of an unspecified nature to explain away the perceived problem of galaxies’ rotation curves. It is also worth noting that MOND provides explanations for various other astronomical phenomena at no extra charge. For example, there is observed to be a tight correlation between asymptotic rotation speeds and luminosities in galaxies, such that \( L \propto v_c^4 \). This proportionality, known as the Tully–Fisher relation, has no particularly fundamental explanation in standard Newtonian gravity. However, if all that a galaxy contains is its luminous stars, so that \( L \propto M \), then the Tully–Fisher relation would follow trivially from Eq. (2) if MOND were correct.

A further criticism arising from the ad hoc nature of MOND is that it has no satisfactory relativistic generalisation. However, this situation changed dramatically earlier this year when a fully relativistic field theory was developed that reduces to Newtonian dynamics in the low-velocity regime and to MOND at low accelerations. This theory for the first time makes a robust prediction as to what gravitational lensing signature one might expect from MOND — another shortcoming often pointed out in the theory — and shows that it will be of the same amplitude as that predicted on the basis of dark matter models.

With this new relativistic formulation, it is also now possible to carry out fully self-consistent tests of MOND such as performing simulations of structure formation in a cosmological context, to see if the theory fits with other astronomical observations as closely as the CDM simulations. Although no-one has yet carried out such simulations, there are already some interesting insights that one can obtain into the subject. The longer range nature of gravity in MOND means that the two-body relaxation time and
dynamical friction timescale are dramatically shorter than in Newtonian gravity.\textsuperscript{13} One might therefore expect the over-abundance of substructure found in the CDM simulations (see Sec. 1) to be effectively wiped out in a MOND universe, as such structure should relax into a smooth distribution on a relatively short timescale. However, whether this simple heuristic argument works in detail will only become clear once some full cosmological MOND simulations are performed.

3. Case Studies

To further test the dark matter paradigm on galactic scales, let us now turn to a few specific types of galaxy to see how well the data fit the theory. There is not much point in using normal spiral galaxies for such tests, as these systems played a key role in the development of the dark matter paradigm (and, indeed, of MOND as well); it would be very surprising if the theory failed to reproduce the properties that so strongly motivated it in the first place. Instead, we look to other types of galaxy for some independent confirmation or refutation of the theory.

3.1. Low Surface Brightness Galaxies

Actually, spiral galaxies are rather poor places to test the dark matter paradigm for other reasons, too. One of the clear predictions in CDM is that the dark matter should have a central cusp. However, the large amount of luminous matter at small radii in these systems means that such cusps will have very little impact on the rotation curves of spiral galaxies. Further, the gravitational interaction between luminous and dark matter could well have redistributed the dark matter, so any primordial central cusp may have been completely wiped out.

Fortunately, a class of galaxies exists in which these issues should not arise. These “low surface brightness galaxies” contain a very low density of luminous material even in the inner parts, so that the observed dynamics should be dominated by the gravitational forces of the dark halo at small radii as well as large radii. Further, the small amount of luminous matter should not have had much ability to redistribute the dark matter, so the central cusp should still be there.

Rather disturbingly, as illustrated in Fig. 1, the rotation curves of these systems do not seem to match up to the predictions of the CDM models. Although they flatten off to the constant rotation velocity characteristic of a dark matter halo, they rise significantly more slowly in their central parts
Figure 1. Rotation curve (circular rotation velocity as a function of radius) for the low surface brightness galaxy NGC 6822. The points show data obtained from the 21cm emission of atomic hydrogen, while the line gives the best-fit model assuming that the galaxy’s mass is dominated by a centrally-cusped dark matter halo. (Figure kindly provided by W.J.G. de Blok.)

than one would expect for a system with a centrally-concentrated cusped mass distribution.\textsuperscript{14}

Once again, MOND seems to do rather better.\textsuperscript{15} Not only does it reproduce the observed slowly-rising rotation curves, but in many cases it can also explain the match up between localized features in the rotation curves and small-scale structure in the photometry — if these low surface brightness galaxies were dominated by dark matter halos, then the faint features in the luminous light distribution should have almost no direct impact on the rotation curve. However, one has to keep a critical eye on how illustrative examples are selected: in the case of Ref. [10], the highlighted
low surface brightness galaxy in their Fig. 3 shows a beautiful match in
the small-scale structure of the rotation curve and the MOND predictions,
whereas the broader range of cases in their Figs. 4 and 5 show a number of
examples where there is no such correspondence.

3.2. Elliptical Galaxies

The other main class of luminous galaxy are the rather dull-looking el-
liptical systems. Studies of the dark matter in these objects have been
hampered for several reasons. First, they tend to be found in clusters, so it
becomes difficult (or even an issue of semantics) to separate any dark mat-
ter associated with galaxies from that which should be attributed to the
cluster over all. Second, they lack the extensive gas disks that sur
round spiral galaxies. They therefore do not have a simple kinematic tracer that
allows their masses to be measured out to large radii.

For the largest elliptical galaxies, alternative probes such as gravita-
tional lensing\textsuperscript{16} and X-ray emitting gas\textsuperscript{17} have been used to infer that these
systems have extensive massive halos. However, such bright galaxies tend
to lie at the centres of clusters, raising the issue of whether the halo belongs
to the galaxy or the cluster.

For more “normal” ellipticals, comparable in luminosity to the Milky
Way, not a lot of data have been available. However, we have recently devel-
oped a new instrument, the Planetary Nebula Spectrograph, specifically to
study such objects.\textsuperscript{18} This instrument detects and measures the kinematics
of planetary nebulae (PNe) in the outer parts of elliptical galaxies. Since
PNe are simply stars that have reached the ends of their lives, their kine-
matics will be representative of the over-all stellar population (but much
easier to measure due to the bright emission lines in PNe spectra). The
initial results that we have obtained using this instrument were not at all
what we expected.\textsuperscript{19} As Fig. 2 shows, the random velocities of the PNe do
not stay constant out to large radii, as one might expect by analogy with
the rotation curve of a comparable spiral galaxy; instead they seem to go
into Keplerian decline, as would be expected in the complete absence of
dark matter.

There are a number of possible explanations for this result that would
not involve making as radical a change as to suggest that these systems are
devoid of dark matter. First, there is an extra degree of freedom that we did
not have to worry about for spiral galaxies: while the gas in disks around
spirals all follows orbits that are close to circular, we have no \textit{a priori} way
Figure 2. A compilation of the stellar line-of-sight velocity dispersion as a function of projected radius for four intermediate-luminosity elliptical galaxies. The data for different galaxies have been normalized by the central velocity dispersion and the effective radius (the radius within which half of the galaxy's light appears projected) to make them directly comparable. The dashed line shows the predicted trend in this relation if these galaxies were embedded in massive halos comparable to those found around similar luminosity spirals. The dotted line shows the Keplerian decline that would be found in the absence of a dark halo.

of knowing whether the stars in ellipticals (and hence their PNe) follow circular orbits, radial orbits, or something in between. Since objects on radial orbits will move mostly transverse to the line of sight at large radii, one would expect the measured line-of-sight velocity dispersion to drop for a system consisting of such objects. However, it would be a surprising coincidence if such an extreme collection of orbits managed to mimic the behaviour of a Keplerian decline; indeed, detailed orbit modelling indicates that such a solution is not consistent with the data.19 Alternatively, these
galaxies could be surrounded by halos that are so extended that, even at the unprecedentedly large radii to which we are now looking, the dark matter still does not dominate the mass. However, this scenario also seems to conflict with CDM models, which predict that these moderate-luminosity galaxies should have dark halos that are more centrally concentrated than their brighter kin, not less so.\textsuperscript{20}

Although this result was not what we were expecting, it did not come as a surprise to the advocates of MOND. Indeed, Ref. [10] had already made the definite prediction that lower-luminosity elliptical galaxies should have velocity dispersion profiles that fall with radius, and subsequent fitting to our data confirmed that the observed approximately Keplerian decline is what would be expected in MOND.\textsuperscript{21} Essentially, the absence of any dark matter signature in these systems arises because elliptical galaxies are more centrally concentrated than their spiral cousins, which means that the characteristic accelerations within them stay safely above the MOND $a_0$ threshold. Thus, their observed kinematics should be consistent with ordinary Newtonian physics, with no non-standard dynamics to be misinterpreted as dark matter.

4. Conclusions

The idea that we live in a Universe dominated by dark matter is so deeply embedded in most astronomers’ World views that it is not something that many of us ever question. Nonetheless, all of the evidence that we currently have for dark matter is highly circumstantial, so we should at least compare the hypothesis critically with any alternatives.

In this regard, a comparison between the dark matter hypothesis and modified Newtonian dynamics, at least on the galactic scale, produces a rather equivocal answer. In a number of cases, MOND seems to fit the observations rather better than dark matter. There are also examples where MOND has passed the ultimate test of a scientific theory by making predictions that differ from the dark matter theory, which subsequently turn out to be true. On scales other than the galactic, perhaps MOND does a little less well: even its proponents recognise that there seems to be a problem at the size of clusters, with MOND predicting more mass than can be found within the known luminous components.\textsuperscript{22} However, until some serious effort is invested in proper cosmological MOND simulations, we will not be able to establish the context that will determine quite how serious any problems might be with this theory.
As things stand, it is principally a matter of aesthetics as to which idea Occam’s Razor favours. Personally, I would still go for the dark matter theory: it is astoundingly arrogant to assume that everything in the Universe should glow in the dark for the benefit of astronomers, so invoking dark matter appeals to me as a way to remind us of our own insignificance. Nonetheless, I recognise the appeal of a simple modification to the law of gravity over the invocation of an entirely unknown form of matter, so am unable to draw any definite conclusions even on this aesthetic issue.

In the context of this particular volume, however, the message is much more clear cut: the direct laboratory detection of massive particles from the Milky Way’s halo would provide by far the most convincing confirmation of the whole dark matter paradigm, and would lay this issue to rest once and for all.

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