Cosserat continuum model and its application to the studies of progressive failure

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ABSTRACT

The finite element analysis, in which an elastoplastic Cosserat continuum model for the soil is incorporated, is implemented to simulate the strain localization phenomena due to strain softening or non-associated plasticity of the material. Based on the finite element procedure developed for proposed pressure-dependent elastoplastic Cosserat continuum model, progressive failure phenomena which occurring in the shear layer and the full scale test of filling embankment on soft foundation characterized by strain localization due to material softening and the material dilatancy, i.e. non-associated plasticity, are numerically simulated respectively. Numerical results indicate that the classical continuum finite element may suffer from pathological mesh dependence and be incapable of completing the analysis of the whole failure process, while Cosserat continuum finite elements possess better performance in preserving the well-posedness of the localization problems and in completing the simulation of the entire progressive failure process occurring in geotechnical engineering structures.

Keywords: Cosserat continuum model, strain localization, strain softening, non-associated plasticity

1 INTRODUCTION

According to the researches of Potts et al. (1990,1997) and Troncone (2005), for progressive failure and strain localization phenomena to occur in some geotechnical engineering where soil exhibits brittle or strain softening properties, simple analysis methods such as limit equilibrium, are incapable of quantifying the possible effects of progressive failure, while the finite element method (FEM) does. So, there is a tendency to use the FEM to analyse such geotechnical engineering.

Unfortunately, when strain softening constitutive behavior is incorporated into a computational model in the frame of classical plastic continuum theories, the initial and boundary value problem of the model will become ill-posed, resulting in pathologically mesh-dependent solutions. Furthermore, the energy dissipated at strain softening is incorrectly predicted to be zero, and the finite element solutions converge to incorrect, physically meaningless ones as the element mesh is refined. This is one case encountered for FEM to simulate such practical engineering. For another case that the non-associated yield criterions are adopted to describe the plastic behaviour for most of soils, Pande and Pietruszczak (1986) and Li et al (2002) indicated that Druck’s hypothesis for stable materials were violated and the initial and boundary value problem of the model would become ill-posed even for the strain hardening behaviour under certain conditions.

To accurately simulate strain localisation phenomena characterized by occurrence and severe development of the deformation localised into narrow bands of intense irreversible strain caused by strain softening or the non-associated plasticity, it is required to introduce some type of regularisation mechanism into the classical continuum model to preserve the well-posedness of the localisation problem. One of the radical approaches to introduce the regularisation mechanism into the model is to utilise the Cosserat micro-polar continuum theory, in which high-order continuum structures are introduced. Among the work, which utilise the Cosserat continuum model as the regularisation approach to analyse strain localisation problems in geotechnical engineering, are contributions of Tejchman and Bauer (1996), Iordache and Willam (1998), Cramer et al. (1999), Manzari (2004), Li and Tang (2005), Khoei et al. (2007). Tang and Li (2007) developed a consistent algorithm of the pressure-dependent elastoplastic model in the framework of Cosserat continuum theory, which was used to analyse the progressive failure problem of the soil foundation characterized by strain localisation due to strain softening.

In the present study, finite element analysis of the landslide of the filling embankment on soft foundation characterised by strain localisation due to the material dilatancy, i.e. non-associated plasticity, are performed.
The parameters used in the analysis are studied detailedly. The results indicate that the soil mass involved in the sliding and the location of the failure surface deduced from the numerical simulation are found to be very similar to those observed in the field. Comparisons are made between classical elasto-plastic finite element analysis and Cosserat elasto-plastic finite element analysis. It illustrates that the main advantage of the latter approach, which has been successfully adopted to solve varied strain localization problems, is that the achieved solution is not affected by convergence problems.

2 THE FEM NUMERICAL MODEL FOR COSSE FERAT CONTINUUM MODEL

![Stress and couple-stress in a two-dimensional Cosserat continuum.](image)

Each material point in the two dimensional Cosserat continuum has three degrees of freedom, i.e. two translational degrees of freedom \( u_x, u_y \) and one rotational degree of freedom \( \omega_z \) with the rotation axis orthogonal to the two dimensional plane,

\[
u = [u_x, u_y, \omega_z]^T
\] (1)

Correspondingly, the strain and stress vectors are defined as

\[
\varepsilon = [\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{zx}, \varepsilon_{zy}, \kappa_{xx}, \kappa_{yy}, \kappa_{xy}, \kappa_{yx}, \kappa_{yz}, \kappa_{zy}, \kappa_{zx}, \kappa_{zy}, \kappa_{zz}, \kappa_{yz}, \kappa_{zy}, \kappa_{zz}]^T
\] (2)

\[
\sigma = [\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \sigma_{yx}, \sigma_{zx}, \sigma_{zy}, m_{xx}/l_c, m_{yy}/l_c, m_{xy}/l_c, m_{yx}/l_c, m_{zx}/l_c, m_{zy}/l_c]^T
\] (3)

where \( \kappa_{xx}, \kappa_{yy} \) are introduced as micro-curvatures in Cosserat theory, \( m_{xx}, m_{yy} \) are the couple stresses conjugate to the curvatures \( \kappa_{xx}, \kappa_{yy} \), \( l_c \) is defined as the internal length scale, as can be seen in Fig. 1.

The relation between strain components and displacement components and the equilibrium equations can be written in matrix–vector forms as

\[
\varepsilon = Lu
\] (4)

\[
L^T \sigma + f = 0
\] (5)

in which the operator matrix

\[
L^T = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial y} & 0 & 0 \\
0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & I_c & \frac{\partial}{\partial y}
\end{bmatrix}
\] (6)

It is assumed that the strain vector \( \varepsilon \) is decomposed into both the elastic and the plastic parts, i.e. \( \varepsilon_e \) and \( \varepsilon_p \), and the elastic strain vector \( \varepsilon_e \) is linearly related to the stress vector,

\[
\sigma = D_e \varepsilon_e
\] (7)

in which the elastic modulus matrix \( D_e \) for isotropic media can be given in the form

\[
D_e = \begin{bmatrix}
\lambda + 2G & \lambda & 0 & 0 & 0 & 0 \\
\lambda & \lambda + 2G & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2G & 0 & 0 & 0 \\
0 & 0 & 0 & G + G_t & G - G_t & 0 \\
0 & 0 & 0 & G - G_t & G + G_t & 0 \\
0 & 0 & 0 & 0 & 0 & 2G \\
0 & 0 & 0 & 0 & 0 & 2G
\end{bmatrix}
\] (8)

with the Lame constant \( \lambda = 2G\nu/(1-2\nu) \), \( G \) and \( \nu \) are the shear modulus and Poisson’s ratio in the classical sense, while \( G_t \) is introduced as the Cosserat shea modulus.

In the framework of Cosserat continuum theory, a consistent algorithm of the pressure-dependent elastoplastic model, i.e. the return mapping algorithm for the integration of the rate constitutive equation and the closed form of the consistent elastoplastic tangent modulus matrix, has been derived (Li and Tang, 2005).

3 EFFECTIVENESS OF THE COSSE FERAT CONTINUUM FINITE ELEMENTS

To illustrate the effectiveness of the Cosserat continuum finite elements stated above, a shear layer with infinite length in the \( z' \) axis, modeled as a one dimensional plane strain problem, is considered to illustrate the performance of the Cosserat continuum model. Two regular uniform meshes with different mesh densities, i.e. \( 20 \times 1, 40 \times 1 \) eight nodded finite element discretizations, are utilized as illustrated in Fig. 2(a) and Fig. 2(b). All nodal displacements in the \( y' \) axis are prevented. The bottom of the shear layer is fixed and a monotonously increasing displacement in the \( x' \) axis enforced at the nodes on the upper boundary is prescribed. Each two nodes with a same \( y \) coordinate value are enforced to have identical displacement in the \( x' \) direction.

With the piecewise linear hardening/softening assumption for the cohesion, we have
where $c_0$ is the initial cohesion, $h_p^c$ the softening parameter for cohesion, $\varepsilon_p$ the equivalent plastic strain. The material parameters used in this example are chosen as: $E=1.0\times10^{10}\text{Pa}$, $\nu=0.25$, $G_c=2.0\times10^9\text{Pa}$, $c_0=1.0\times10^8\text{Pa}$, $h_p^c=-5.0\times10^8\text{Pa}$.

Different values of internal length scale $l_c$ are used in the analysis. The rotational degrees of freedom at the nodes on the top and the bottom boundaries are fixed to trigger the shear band in the shear layer.

\begin{equation}
  c = c(\varepsilon_p) = c_0 + h_p^c\varepsilon_p
\end{equation}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{Deformed configuration of a shear layer subjected to a prescribed transverse displacement $u=7.5\text{mm}$ in the $x$-axis at the top of the layer: (a) classical continuum; (b) Cosserat continuum.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Curves of the transverse load applied to the top of the layer with increasing prescribed transverse displacement $U$ of the top surface with different mesh densities.}
\end{figure}

Fig. 2 and Fig.3 illustrate deformed configuration and gradual reduction of the load-carrying capability for different mesh densities of shear layer due to strain softening with the development of plastic strains, where the shear layer is treated as the classical continuum or Cosserat continuum. They illustrate that the load-displacement curves for the same value of the internal length scale $l_c$ but with different mesh densities converge to a physically realistic solution independent from the mesh density for Cosserat continuum, while pathologically mesh-dependent solutions for classical continuum.

\section{Numerical Analysis for the Failure of the Full Scale Filling Test}

\subsection{The full scale filling test}

In the 1980’s, a large number of embankments against the seawater had been filled in the seaport of Shengli Oil Field. But later, the project had to be cancelled because some embankments collapsed and slid before they achieved the necessary height to obstruct the seawater. The reason is that bearing capacity of the foundation is insufficient to undertake the filling embankment above it. The upper layer of the embankment foundation is constituted of 1.0–2.0 m thick sandy loam, and the lower layer is constituted of 2.6–9.3 m thick soft mucky loam. The instability and failure of embankments are mainly related to the insufficient bearing capacity of soft mucky loam layer (Tang and Wang, 2006).

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig5.png}
\caption{The natural foundation and finite element mesh}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig6.png}
\caption{Embankment and the distribution of surface settlement plates and border piles: (a) schematic geological section of filling embankment and distribution of surface settlement plates (A1-A7); (b) floor plan of embankment and distribution of border piles.}
\end{figure}

To obtain essential data to guide the design and construction of the embankment on the soft foundation, the full scale filling test had been carried out. The foundation of the filling embankment is constituted of 2.0 m thick sandy loam and 8.0 m thick soft mucky
loam (Fig. 5). The schematic geological section of filling embankment, and observation instruments such as border piles and surface settlement plates are buried in the embankment and the foundation (Fig. 6(a), (b)). In addition, the geological prospecting and vane shear test are carried out in field to get the material parameters of soils.

The filling material for the testing embankment is sandy loam. The whole filling process is completed in 16 layers, each layer with the thickness of 0.3 m. After compaction, the dry density of the filling material is 1.55 g/cm³. The displacements of border piles and surface settlement plates during the filling progress can be seen from Fig. 7(a) and (b). It shows that as the height of filling embankment achieves 3.9 m, the horizontal displacement indicated by border piles and the vertical settlement indicated by surface settlement plates increase a little. As the filling embankment achieves the height of 4.2 m, the horizontal displacement and vertical settlement increase rapidly. When the filling embankment achieves 4.78 m high, a majority of embankment together with part of foundation slides towards south and the global instability occurs, as can be seen from the horizontal displacement and the vertical settlement on the surface of embankment and foundation.

4.2 Soil parameters and the numerical model

An elasto-plastic model with the Drucker-Prager non-associated plastic law is used to describe all the soils. Referring to the testing report (Tang and Wang, 2006), the different soil parameters used in the analysis are illustrated in Table 2. The parameters which are the same for each soil are: $E = 5.0 \times 10^6$ Pa, $G_c = 1.0 \times 10^6$ Pa, $l_c = 0.12$ m, $c = 5.0 \times 10^3$ Pa. According to Vermeer and de Borst (1984), the dilatancy angles are approximately between 0-20° for soils, rocks and concrete. Especially, the dilatancy angles are less than 15° for typical soils and nil for normal consolidated clay. In the present study, the soils are basically normal consolidated. For conservative consideration of decreasing the numerical difficulty of classical continuum finite element model, the dilatancy angles have been set 12° (not nil) for mucky loam and 10° (not nil) for soft mucky loam, which implies the non-associated flow rule incorporated during plasticity yielding. The parameters $G_c$ and $l_c$, which are related to Cosserat continuum model, are chosen in accordance with Li and Tang (2005), Tang and Li (2007, 2008), i.e. Cosserat shear modular $G_c \geq 0.5G$ and the internal scale $l_c \geq \sqrt[3]{H}$, where H is the scale of the structure considered. Fig. 8 shows the mesh adopted in the calculation after completing the filling process. In order to simulate the slippage towards south, the north fixed boundary is chosen more closely to the center of the embankment.

Table 2. Soil parameters used in the analysis.

| Soil parameters       | $\gamma$ (kN/m³) | $\nu$ | $\phi$ (°) | $\psi$ (°) |
|-----------------------|------------------|------|-----------|-----------|
| filling soil          | 19.8             | 0.3  | 32        | 12        |
| mucky loam            | 19.6             | 0.4  | 16        | 10        |
| soft mucky loam       | 18.8             | 0.3  | 16        | 10        |

4.3 Numerical analysis for the filling process

Considering the filling process, the progressive failure phenomena of the full scale test, characterized by strain localization and its development due to the non-associated yield criterion adopted, are numerically simulated. Before the filling, assuming that
all the soils in the foundation exhibit elastic perfectly plastic behavior with the Drucker-Prager failure criterion, the initial stress state within the foundation under the embankment is reproduced by progressively increasing the gravity acceleration up to the value of 9.81 m/s². At the end of the gravity loading, the corresponding displacements and strains are reset to zero. The filling process is simulated by adding 9 layers of elements to the filling zone above the initial finite element mesh of the foundation. For 1~7 layers, 0.6 m thick soil for each layer is filled, then 0.3 m thick soil for the 8th layer and 0.28 m thick soil for 9th layer.

Fig. 9. Effective plastic strain distributions in the embankment and foundation after the filling of the 8th layer with classical continuum

As non-associated plastic behavior is involved in the problem, it may exhibit an effectively strain softening behavior in case of $\nu > 1/3$ (Pande and Pietruszczak, 1986), which accords with soft mucky loam and the initial and boundary value problem of the classical continuum model will become ill-posed. At first, analysis has been performed assuming an elasto-plastic model with the Drucker-Prager non-associated plastic law for all the soils involved. Fig.9 shows the evolution of the effective plastic strain occurring after the filling of the 8th layer. It can be seen that the plastic strain field develops with a clear localisation within the foundation and the embankment. At this stage, the whole development of the failure surface is not completed. As the filling process continues (add the 9th layer), the classical finite element numerical solution faces significant difficulties of the increasing number of negative eigenvalues in the system stiffness matrix and the numerical calculation can not be carried out any more. Therefore, the numerical analysis based on this elasto-plastic model with non-associated plastic law is not fully effective in capturing the entire failure process that occurs in the present study even if the difference between the dilatancy angle and the friction angle is not too much for conservative consideration. If the dilatancy angles were chosen as nil, the numerical difficulty would be met earlier.

Fig. 10. Effective plastic strain distributions in the embankment and foundation during the filling process with Cosserat continuum: (a) after the filling of the 4th layer; (b) after the filling of the 5th layer; (c) after the filling of the 6th layer; (d) after the filling of the 7th layer; (e) after the filling of the 8th layer; (f) after the filling of the 9th layer.
The calculations have been repeated using the elasto-plastic Cosserat continuum model. Fig.10(a), (b), (c), (d), (e) and (f) show the evolution of the more significant effective plastic strain during the filling process. It can be seen from Fig.10(a), (b) and (c) that the effective plastic strain is concentrated in the middle of foundation and embankment after completing 6 filling layers. As the filling process continues, the effective plastic strain propagates up in the embankment and in the foundation, and finally causes the collapse of the embankment and the foundation when the filling is completed (Fig.10(d), (e) and (f)). The ultimate filling height and the position of the failure surface at both the slope of the embankment and the surface of the foundation agree well with the phenomena observed in the field. Therefore the numerical analysis based on Cosserat elasto-plastic model with non-associated plastic law is effective in capturing the entire failure process that occurs in the present study.

5 CONCLUSIONS

In the present work, the Cosserat continuum theory is introduced into the FEM numerical model, which is used to simulate the strain localisation problems due to strain softening or non-associated plasticity of soils. The effectiveness of the Cosserat continuum model is illustrated in simulation of strain localization and post peak response by a shear layer.

The full scale filling test, which was carried out to obtain the essential data to guide the design and construction of the embankment on the soft foundation, is introduced. Considering the filling process, the progressive failure of the full scale test, characterised by strain localisation due to the material dilatancy, i.e., non-associated plasticity, is numerically simulated with classical continuum model and Cosserat continuum model. Numerical results indicate the inability of classical continuum model in simulating the whole progressive failure process even for conservative consideration, while the capability and performance of Cosserat continuum model in keeping the well-posedness of the boundary value problems with non-associated plastic law incorporated and in continuing the numerical simulation until a clear collapse mechanism achieved. According to the numerical results of Cosserat continuum model, the ultimate filling height and the position of the failure surface at both the slope of the embankment and the surface of the foundation are in agreement with the results of the full scale test.

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