Research on $H_\infty$ Preview Control Algorithm for Industrial Wireless Sensor Networks

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Abstract. In order to solve the problem of wireless sensor network control system with network delay and packet loss, a $H_\infty$ preview control algorithm is designed. Firstly, the wireless sensor network model with delay and packet loss is established, and the system error and predictable target signal are introduced to construct the extended error system. Secondly, the state feedback controller with memory is designed. By constructing Lyapunov function and linear matrix inequality, the design method of $H_\infty$ controller and the sufficient conditions for the stability of the system are derived. The optimal $H_\infty$ preview controller algorithm is obtained by solving the LMI optimization problem. Finally, the effectiveness of $H_\infty$ preview controller algorithm is verified by simulation.

Keywords. Wireless sensor network; delay and packet loss; $H_\infty$ preview controller; extended error system.

1. Introduction

Time delay and packet loss widely exist in industrial wireless sensor network control system, which are the main causes of system instability and control performance degradation [1]. In the liquid level control of the separator in petrochemical industry, it will cause that the internal pressure of the separator exceeds the standard because of the controller delay and packet loss lead to the instability of liquid level [2-4].

Preview control is a kind of control technology which uses the future information to feedback the state of the current system [5]. At present, preview control has been widely used in automobile, robot, aircraft, electromechanical servo system and other fields [6-7]. In the field of industrial control, traditional industrial network control systems often use predictive model to describe the future process output of the system. However, due to the characteristics of nonlinearity, time-varying, strong coupling and uncertainty in modern industry, the effect of precise control is greatly reduced, which has a serious impact on the stability of the system. With the application of Internet of things technology and the maturity of 5G technology in the future, the information that will be produced in industrial production can be directly measured by intelligent sensors, and a controller with information compensation function is designed to control the system more accurately.

2. Problem Statement and Modeling

In the industrial wireless sensor network control, the industrial process is composed of the controlled object and the operation process [8]. Taking the liquid level height control of separator as an example,
in the high-pressure oil-gas separator and water gas separator equipment as shown in figure 1, for the liquid level height control of high-pressure water gas separator, FT is the flow rate sensor of inlet feeding, LT is the liquid level height sensor, Gff is the predictive feed-forward compensator, and LIC is the liquid level indicating controller [9]. In the industrial control process shown in figure 1, the control system with preview control is mainly composed of three parts: control object, state feedback controller and preview feedforward compensation. The control structure diagram is shown in figure 2. As shown in figure 2, in the wireless sensor network control system with interference input, the presence of interference in the network is considered \( \tau_d \) time delay and \( m \) packet loss, then the continuous controlled object can be expressed as:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + A_d x(t-d) + Bu(t) + D_1 w(t) \\
z(t) &= C x(t) + Fu(t) + D_1 w(t) \\
y(t) &= C x(t)
\end{align*}
\]

(1)

where \( x(t) \in \mathbb{R}^n \) is the state vector of the system; \( u(t) \in \mathbb{R}^m \) is the controller output of the system; \( z(t) \in \mathbb{R}^l \) is the output vector of the controlled object of the system; \( y(t) \in \mathbb{R}^q \) is the measurement output vector of the system; The network delay \( \tau_d \) and the packet loss \( \tau_m \) can be regarded as the system delay \( d \), and the system delay is expanded to \( d = \tau_d + \tau_m \);

**Assume 1:** Setting \( r(t) \) as the target signal, which can be previewed in \([t, t+M]\), there is \( \dot{r}(t) = r(t+1) - r(t) \), and the target value is constant after \( t+M \), that is

\[
r(t + M) = r(t + M_R)
\]

(2)

where \( M_R \geq M \). Define the error signal:

\[
\epsilon(t) = y(t) - r(t)
\]

(3)

For the system (1), the quadratic performance index function is introduced

\[
J = \int_{0}^{\infty} [e^T(t)Qe(t) + u^T(t)H_u u(t)] dt
\]

(4)

where, \( Q \in \mathbb{R}^{m \times m} > 0 \) and \( H_u \in \mathbb{R}^{n \times n} > 0 \) is a given positive definite weighted matrix. In this paper, \( \gamma > 0 \) as the given constant, it designs a state feedback preview \( H_\infty \) controller with memory, which makes the closed-loop system (1) satisfy conditions as follows:

- The system has a certain guaranteed nature, that is, there is a performance index upper bound \( J^* \), satisfying \( J \leq J^* \);
- Under zero initial condition, for all nonzero interferences \( w(t) \in L_2[0, \infty) \), where the output \( z(t) \) satisfies \( H_\infty \) norm constraint, that is\( \|z(t)\|_2 \leq \gamma \|w(t)\|_2 \);

![Figure 1. The explanatory views of high-pressure oil and water facility.](image1)

![Figure 2. The structure diagram of preview control system.](image2)
When the system is affected by disturbance, its measurement output $y(t)$ can track the target signal $r(t)$ without static error, that is, $\lim_{t \to \infty} \varepsilon(t) = \lim_{t \to \infty} (y(t) - r(t)) = 0$.

**Lemma 1 (Schur complement lemma):** If there is a matrix, 

$$A = A^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}$$

There are three equivalent conditions as follows:
- $A < 0$;
- $A_{11} < 0, A_{22} - A_{12}^T A_{11}^{-1} A_{12} < 0$;
- $A_{22} < 0, A_{11} - A_{12} A_{22}^{-1} A_{12} < 0$.

**Lemma 2:** Given the matrix $X, Y, \varepsilon > 0$, which satisfies the matrix inequality $X^TY + Y^TX \leq \frac{1}{\varepsilon}X^TX + \varepsilon Y^TY$.

### 3. Design of System Preview Feedback $H_\infty$ Controller

In this part, the original system is constructed as an extended error system, and then a preview feedback $H_\infty$ controller is designed for the system.

#### 3.1. Constructing System Augmented Matrix with Errors

According to the equation of state in system (1) and substituting into equation (3), we can get

$$\varepsilon(t) = y(t) - r(t) = Cx(t) - r(t)$$

(5)

If we derive $\varepsilon(t)$, we can get

$$\dot{\varepsilon}(t) = C\dot{x}(t) - \dot{r}(t) = CAx(t) + CA_d x(t - d) + CBu(t) + CD_1w(t) - \dot{r}(t)$$

Setting $\varphi(t) = \begin{bmatrix} \varepsilon(t) \\ x(t) \end{bmatrix}$. Combining equations (1) and (5), we can get

$$\dot{\varphi}(t) = \bar{A}_1 \varphi(t) + \bar{A}_2 \varphi(t - d) + \bar{B}_1 u(t) + \bar{D}_1 w(t) - \bar{E} \dot{r}(t)$$

(6)

where $\bar{A}_1 = \begin{bmatrix} 0 & CA \\ 0 & A \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} 0 & CA_d \\ 0 & A_d \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} CB \\ B \end{bmatrix}, \bar{D}_1 = \begin{bmatrix} CD_1 \\ D_1 \end{bmatrix}, \bar{E} = \begin{bmatrix} I \\ 0 \end{bmatrix}$.

#### 3.2. Construction of Extended Error System

The target signal information is introduced into the system (6) and the vector is defined

$$X_R(t) = \begin{bmatrix} r(t) \\ r(t + 1) \\ \vdots \\ r(t + M) \end{bmatrix}$$

According to the conditions of Assume (1), there are

$$\dot{X}_R(t) = A_R X_R(t)$$

(7)

where

$$A_R = \begin{bmatrix} -I & I & 0 & \cdots & 0 \\ 0 & -I & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -I & I \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Setting the vector $\varphi(t)$ has the following form:
\[
\begin{bmatrix}
\varphi(t) \\
X_R(t)
\end{bmatrix} = 
\begin{bmatrix}
z(t) \\
x(t) \\
\vdots \\
X_R(t)
\end{bmatrix}
\]

Combining with system (1), it observes the output equation, controlled output equation and state equation in system (6), the expanded error system is obtained as
\[
\dot{\phi}(t) = \tilde{A}\phi(t) + \tilde{A}_d\phi(t - d) + \tilde{B}u(t) + \tilde{D}w(t) 
\]
(8)
\[
y(t) = \tilde{C}\phi(t) 
\]
(9)
\[
z(t) = \tilde{C}_1\phi(t) + Fu(t) + D_1w(t) 
\]
(10)

where
\[
\tilde{A} = \begin{bmatrix}
\tilde{A}_1 & G_R \\
0 & A_R
\end{bmatrix}, \quad G_R = \begin{bmatrix} 0 & -I & \cdots & 0 \end{bmatrix}, \quad \tilde{A}_d = \begin{bmatrix} 0 & CA_d & 0 \\
0 & 0 & 0 \end{bmatrix},
\]
\[
\tilde{B} = \begin{bmatrix}
CB \\
B \\
\vdots \\
0
\end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} CD_1 \\
D_1 \\
\cdots \\
0
\end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} 0 & \vdots & 0 \end{bmatrix}, \quad \tilde{C}_1 = \begin{bmatrix} 0 & \vdots & C_1 \end{bmatrix}
\]

Accordingly, the performance index in equation (4) can also be converted into the following form:
\[
J = \int_0^\infty (\varepsilon^T Q \varepsilon + u^T Hu)dt = \int_0^\infty \begin{bmatrix} \varphi(t) \\
X_R(t) \end{bmatrix}^T Q \begin{bmatrix} \varphi(t) \\
X_R(t) \end{bmatrix} + u^T H_u u 
\]
(11)

where, \( Q = \begin{bmatrix} Q_1 & 0 \\
0 & Q_1 \end{bmatrix} \), \( Q_1 = \begin{bmatrix} Q_1 & 0 \\
0 & 0 \end{bmatrix} \).

According to the extended error system established by equation (8) and the performance index in equation (11), a state feedback controller with memory is designed as follow:
\[
u(t) = K\phi(t) + L\phi(t - d) 
\]
(12)

where \( K \) and \( L \) are undetermined matrices. Substituting equation (12) into equations (8) and (10), we can get
\[
\dot{\phi}(t) = \tilde{A}\phi(t) + \tilde{A}_d\phi(t - d) + \tilde{B}K\phi(t) + \tilde{B}L\phi(t - d) + \tilde{D}w(t) 
\]
(13)
\[
z(t) = \tilde{C}_1\phi(t) + Fu(t) + D_1w(t) = \tilde{C}_1\phi(t) + FK\phi(t) + FL\phi(t - d) + D_1w(t) 
\]
(14)

3.3. Construction of Extended Error System

**Theorem 1**: Under the assumptions 1 and 2, if there are matrices \( P > 0, R > 0 \) and \( K \) and \( L \), the following inequalities hold for all admissible disturbances
\[
\begin{bmatrix}
\tilde{A} + \tilde{B}K \quad P(\tilde{A}_d + \tilde{B}L) \\
\tilde{A}_d + \tilde{B}L \quad P \tilde{D} \\
(\tilde{C}_1 + FK)^T \\
\tilde{D}^TP \\
\tilde{C}_1 + FK \\
\end{bmatrix} < 0
\]
(15)

Then the system (13) with delay and packet loss network is asymptotically stable, and for all nonzero vectors \( w(t) \in L_2[0, \infty], \|z(t)\|_2 \leq \gamma\|w(t)\|_2 \) have establishment.

**Proof**: It selects the Lyapunov function
\[ V(\phi(t)) = \phi^T(t)P\phi(t) + \int_{t-d}^{t} \phi^T(s)R\phi(s)\,ds \tag{16} \]

where, \( P > 0, R > 0, V(\phi(t)) \) is positive definite. It derives \( V(\phi(t)) \) along time \( t \), it can get that

\[
\dot{V}(\phi(t)) = \phi^T(t)P\phi(t) + \phi^T(t)P\phi(t) + \phi^T(t)R\phi(t) - \phi^T(t - d)R\phi(t - d)
\]

\[
= \phi^T(t)[(\bar{A} + \bar{B}K)^TP + P(\bar{A} + \bar{B}K) + R] \phi(t) + \phi^T(t - d)(\bar{A}_d + \bar{B}L)^TP\phi(t)
\]

\[
+ \phi^T(t)P(\bar{A}_d + +\bar{B}L)\phi(t - d) + w^T(t)\bar{D}^TP\phi(t) + \phi^T(t)P\bar{D}w(t) - \phi^T(t - d)R\phi(t - d)
\]

When \( w(t) = 0, \dot{V}(\phi(t)) = \begin{bmatrix} \phi(t) \\ \phi(t - d) \end{bmatrix}^T \Lambda \begin{bmatrix} \phi(t) \\ \phi(t - d) \end{bmatrix}
\]

where \( \Lambda = \begin{bmatrix} (\bar{A} + \bar{B}K)^TP + P(\bar{A} + \bar{B}K) + R & P(\bar{A}_d + \bar{B}L) \\ * & -RI \end{bmatrix} \).

When \( \Lambda < 0, \dot{V}(\phi(t)) < 0 \) then the system (13) is asymptotically stable. For any \( w(t) \in L_2[0, \infty), \) it can get

\[
\dot{V}(x(t)) + z^T(t)z(t) - \gamma^2w^T(t)w(t) \leq \begin{bmatrix} \phi(t) \\ \phi(t - d) \end{bmatrix}^T \begin{bmatrix} S_1 + S_2 & S_2 \\ S_2 & S_2 \end{bmatrix} \begin{bmatrix} \phi(t) \\ \phi(t - d) \end{bmatrix}
\]

where \( S_1 = \begin{bmatrix} (\bar{A} + \bar{B}K)^TP + P(\bar{A} + \bar{B}K) + R & P(\bar{A}_d + \bar{B}L) \\ * & -R \end{bmatrix}, S_2 = [\bar{C}_1 + FK \quad FL \quad D_1]. \)

By using the matrix inequality in Theorem 1, it can get \( \dot{V}(x(t)) + z^T(t)z(t) - \gamma^2w^T(t)w(t) < 0. \)

By integrating both sides of the above formula from 0 to \( \infty, \) it can get

\[
\int_{0}^{\infty} z^T(t)z(t) \, dt - \gamma^2 \int_{0}^{\infty} w^T(t)w(t) \, dt < -V(\infty) \leq 0
\]

and obtain \( \|z(t)\|_2 \leq \gamma \|w(t)\|_2. \)

**Theorem 2:** For a given constant \( \gamma > 0, \) if there are symmetric, positive definite \( X, Y, \Pi \) Matrix and matrices \( M, N, \) for all admissible disturbances, it makes the following LMI have an optimal solution.

\[
\Pi_1 = \begin{bmatrix} \Pi_1 & \Pi_3 \\ \Pi_3^T & \Pi_4 \end{bmatrix} < 0
\]

\[
\Pi_1 = \begin{bmatrix} (\bar{A}_dY + \bar{B}N) & -Y & 0 & (\bar{C}_1X + FM)^T \\ -Y & 0 & (FN)^T & D_1^T \\ 0 & -\gamma^2I & D_1^T & -I \\ \bar{C}_1X + FM & FN & D_1 & -I \end{bmatrix}, \Pi = (\bar{A}X + \bar{B}M) + (\bar{A}X + \bar{B}M)^T.
\]

Then the controller \( u(t) = W^*X^{-1}\phi(t) + M^*Y^{-1}\phi(t - d) \) is the state feedback \( H_{\infty} \) anti-interference optimal controller with memory in the networked control systems (13) with the network delay and packet lost.

**Proof:** It multiplies the above inequality (15) by the matrix \( diag(P^{-1}, R^{-1}, I, I) \) on the left and right at the same time, and making \( X = P^{-1}, M = KP^{-1}, Y = R^{-1}, N = LR^{-1}, \) and it applies Schur’s complement lemma to obtain the inequality (18), so as to prove that the inequality has feasible solutions, then the control law \( u(t) = MX^{-1}\phi(t) + NY^{-1}\phi(t - d) \) is an optimal robust \( H_{\infty} \) controller.

5
Theorem 3: For a given constant $\gamma > 0$, if there are the following optimization problems:

$$\min J = \theta + \text{Trace}(G), (I) \left[ -\theta \quad \phi^T(0) \right] < 0, (II) \left[ -G \quad N_1^T \right] < 0$$

There is an optimal solution $(\theta, G, X, Y, M, N)$, then the controller $u(t) = W^*X^{-1}\phi(t) + M^*Y^{-1}\phi(t - d)$ is an optimal guaranteed cost controller for networked control systems (13) with delay and packet loss, and the performance index has an upper bound $J^*$, the upper bound of the corresponding performance index is $J^* = \theta + \text{Trace}(G)$, which satisfy $J \leq J^*$.

Proof: The upper bound of performance index of closed-loop system (13) is

$$J \leq \phi^T(0)P\phi(0) + \int_{-d}^{0} \phi^T(s)R\phi(s)ds$$

Because $\phi^T(0)P\phi(0) < \theta$ is equivalent to the constraint condition (I) in Theorem 3, and $Y = R^{-1}$.

$$\int_{-d}^{0} \phi^T(s)R\phi(s)ds = \int_{-d}^{0} \text{Trace}(\phi^T(s)Y^{-1}\phi(s))ds = \text{Trace}(N_1N_1^TY^{-1}) = \text{Trace}(N_1^{-1}N_1^T) < G$$

It is equivalent to the constraint condition (II) in Theorem 3, so $J \leq \theta + \text{Trace}(G)$, the proof is complete.

Theorem 4: If there is an optimal solution to the optimization problem in Theorem 3, the control input of networked control system (1) with delay and packet loss can be written as:

$$u(t) = K_C\varepsilon(t) + K_xx(t) + \sum_{i=0}^{M} K_Rr(t + i) + K_ww(t) + L_C\varepsilon(t) + L_xx(t) + \sum_{i=0}^{M} L_Rr(t + i)$$

where

$$K = (K_C, K_x, K_w, K_R(0), K_R(1), K_R(2), \ldots, K_R(M)), L = (L_C, L_x, L_w, L_R(0), L_R(1), L_R(2), \ldots, L_R(M))$$

Proof: If the optimal problem of Theorem 3 has an optimal solution $(\theta, G, X, Y, M, N)$, the controller

$$u(t) = K\phi(t) + L\phi(t - d) = W^*X^{-1}\phi(t) + M^*Y^{-1}\phi(t - d)$$

is an optimal guaranteed cost controller for networked control systems (13) with time delay and packet loss, and it decomposes the gain $K$ and $L$, it can get the results as follow

$$K = (K_C, K_x, K_w, K_R(0), K_R(1), K_R(2), \ldots, K_R(M)), L = (L_C, L_x, L_w, L_R(0), L_R(1), L_R(2), \ldots, L_R(M))$$

Then equation (19) can be written as

$$u(t) = K_C\varepsilon(t) + K_xx(t) + \sum_{i=0}^{M} K_R(i)r(t + i) + K_ww(t) + L_C\varepsilon(t) + L_xx(t) + \sum_{i=0}^{M} L_R(i)r(t + i)$$

The proof is complete.

4. Numerical Examples and Simulation Experiments

Numerical Example Consider a networked control system (1) with packet loss and delay, where

$$A = \begin{bmatrix} -0.3 & -0.1 \\ 0.1 & 0.7 \end{bmatrix}, A_d = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}, D_1 = \begin{bmatrix} 0.4 \\ -0.6 \end{bmatrix}, C_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}, F = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = [-0.08 \quad 0.08]$$

It assumes that the time delay $d=2$, and the disturbance added is random, the following three cases are compared: unforeseen signal $M=0$, two-step foreseen signal $M=2$, and four-step foreseen signal $M=4$.

Case 1: It assumes that the foreseeable target signal is the step signal below:
\[ r(t) = \begin{cases} 0, & t < 5 \\ 1, & t \geq 5 \end{cases} \]

Figure 3 shows the output response of unforeseen signal \( M=0 \), two-step predictive signal \( M=2 \), and four-step predictive signal \( M=4 \).

It can be seen from figure 3 that the output signal can track the target signal in all three cases, but there is certain overshoot in the case of no preview signal \( M = 0 \), and the tracking speed is the slowest. With the increase of preview signal, the overshoot and response speed are improved.

**Case 2:** It assumes that the foreseeable target signal is the lower slope signal

\[ r(t) = \begin{cases} 0, & t \leq 30 \\ 0.05 \times (t - 30), & 30 < t < 60 \\ t = 1.5, & t \geq 60 \end{cases} \]

Slope responses in the three cases of unforeseen signal \( M=0 \), two-step predictive signal \( M=2 \), and four-step predictive signal \( M=4 \) is shown in figure 4. It can be seen from figure 4 that the output signals of no preview signal \( M=0 \), two-step preview signal \( M=2 \) and four step preview signal \( M=4 \) can track the target signal very well, but the tracking effect of four step preview signal \( M=4 \) is the best, basically without overshoot, and can track the target signal quickly; Secondly, the tracking signal with two-step preview signal \( M = 2 \), the tracking effect of \( M=0 \) without preview signal is the worst.

![Figure 3](image3.png)  
**Figure 3.** Step output response of the system.

![Figure 4](image4.png)  
**Figure 4.** Ramp output response of the system.

5. Conclusion

In this paper, a \( H_{\infty} \) preview control method is proposed for a class of industrial wireless sensor control processes with time delay and packet loss. By introducing systematic error and target signal which can be previewed to construct an expanded error system, this method is easy to eliminate the systematic error. The theorem proves the design method of a class of state \( H_{\infty} \) preview controllers with memory and sufficient conditions for system stability by using Lyapunov function and linear matrix inequality. Finally, a simulation example shows the effectiveness of the proposed method.

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