Aerodynamic Thermodynamic Modeling and Simulation of Turbofan Engine

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Abstract. The mathematical model of turbofan engine is established and simulation calculation is carried out in this paper. An engine component model is established by means of the gas flow path of the engine. Then, according to the common working conditions, common working equations are established and the N+1 residual method is used to solve the nonlinear equations. The general model of an engine is established based on the design point parameters, and the specific performance parameters of the intermediate state are calculated under standard atmospheric conditions. The simulation results are compared with the measured performance data of the engine, which shows that: there are errors between the calculated value of the model and the measured value of the engine. The maximum error is 2.85%, which meets the requirement of model error less than 3% in general engineering.

1. Introduction

There are many kinds of aeroengine models, and the classification criteria are also different. From a mathematical point of view, engine models can be divided into non-linear aerothermodynamic model and linear small deviation model [1]. The former is an accurate description of the working process of the engine. It is based on the main components along the flow path of the engine, the characteristic data of engine components and aerothermodynamic laws [2]. It is applicable to both steady and dynamic processes. The latter assumes that the input parameters vary in a very small range. Based on the overall engine, the model obtained by linearizing the engine non-linear model with given input parameters, output parameters and control parameters is usually applicable to the dynamic characteristics of the engine near a given working state [3]. In this paper, the steady-state mathematical model of the engine is studied, and the non-linear aerothermodynamic model is adopted.

2. Establishment of Engine Component Level Model

The basic idea of engine component level model is to model the relatively independent components separately according to the sequence of gas flow based on the known component characteristic data [4]. Then, according to the conditions of continuous flow rate and power balance, the common working equations are established. Finally, the non-linear equations are solved simultaneously to determine the common working point of the engine. The section diagram and definition of the engine studied in this paper are shown in Figure 1 and Table 1.
Figure 1. The sketch map of engine section

Table 1. Definition of Engine Sections

| Section Number | Actual definition                                      | Section Number | Actual definition                                      |
|----------------|--------------------------------------------------------|----------------|--------------------------------------------------------|
| 0              | Far ahead of engine inlet                              | 1              | Entrance inlet                                         |
| 2              | Fan inlet                                              | 22             | Fan outlet                                             |
| 26             | Connotation inlet of high pressure compressor          | 13             | Outer culvert entrance                                 |
| 3              | Combustor inlet                                        | 4              | High pressure turbine inlet                            |
| 44             | Low pressure turbine inlet                             | 5              | Low pressure turbine outlet                            |
| 15             | Outlet of outer culvert                                | 6              | Heat flow inlet of mixing chamber                      |
| 16             | Cold flow inlet of mixing chamber                      | 65             | Mixing chamber outlet                                  |
| 67             | Afterburner inlet                                      | 68             | Afterburner outlet                                     |
| 7              | Nozzle inlet                                           | 8              | Nozzle outlet                                          |

When the engine components work together, the working conditions of aerothermodynamics and rotor dynamics must be followed. That is the basic equations of flow balance, power balance and pressure balance [5]. The common working equations of engine being made up of these basic equations are specifically as follows:

1) Flow balance of low pressure turbine/fan
The inlet flow rate of low pressure turbine can be obtained by linear interpolation of low pressure turbine characteristics. The equilibrium equation is:

\[(M_{44} - M'_{44})/M_{44} = 0\]  \hspace{1cm} (1)

2) Flow balance equation of high pressure turbine/compressor
The flow rate \(M_{40}\) can be obtained by linear interpolation of high pressure turbine characteristics. The equilibrium equation is:

\[(M_{40} - M'_{40})/M_{40} = 0\]  \hspace{1cm} (2)

3) Power balance equation of low pressure turbine/fan is:

\[(N_{TL} - N_{cL})/N_{TL} = 0\]  \hspace{1cm} (3)
(4) Power balance equation of high pressure turbine/compressor

\[
\frac{N_{TH} - N_{cH}}{N_{TH}} = 0
\]

(5) Static pressure balance at mixing chamber inlet:

\[
\frac{P_5 - P_{15}}{P_5} = 0
\]

(6) Air flow balance of afterburner outlet/tail nozzle:

\[
\frac{M_8 - M_7}{M_8} = 0
\]

Two parameters are needed to determine the working state of fan, compressor, high and low pressure turbines. \( n_L \), \( \pi_{cl}^* \), \( n_H \) and \( \pi_{cH}^* \) are selected respectively as the parameters to characterize the working state of fan and compressor. Because of the mechanical connection between compressor and turbine, \( n_L \) and \( n_H \) have been determined [6]. So enthalpy drop \( DH_H \) and \( DH_L \) are selected as parameters to characterize the working state of high and low pressure turbines. Since the four components work together and are subject to the constraints of common working conditions, the values of the six parameters \((n_L, n_H, \pi_{cl}^*, \pi_{cH}^*, DH_H, DH_L)\) determining the four components’ states must meet the common working equations, which is:

\[
\begin{align*}
G_1(n_H, n_L, \pi_{cH}^*, \pi_{cl}^*, DH_H, DH_L) &= (N_{TH} - N_{cH}) / N_{TH} = 0 \\
G_2(n_H, n_L, \pi_{cH}^*, \pi_{cl}^*, DH_H, DH_L) &= (N_{TH} - N_{cl}) / N_{TH} = 0 \\
G_3(n_H, n_L, \pi_{cH}^*, \pi_{cl}^*, DH_H, DH_L) &= (M_{44} - M_{44}^f) / M_{44} = 0 \\
G_4(n_H, n_L, \pi_{cH}^*, \pi_{cl}^*, DH_H, DH_L) &= (M_{40} - M_{40}^f) / M_{40} = 0 \\
G_5(n_H, n_L, \pi_{cH}^*, \pi_{cl}^*, DH_H, DH_L) &= (P_5 - P_{15}) / P_5 = 0 \\
G_6(n_H, n_L, \pi_{cH}^*, \pi_{cl}^*, DH_H, DH_L) &= (M_8 - M_7) / M_8 = 0
\end{align*}
\]

3. Algorithms for Solving Engine Model

When the engine is in steady state, the model solving problem is transformed into solving a set of nonlinear equations composed of six common working equations. N-R method and N+1 residual method are commonly used to solve the common working equations of engines. N-R method has high accuracy, but its operation speed is slow [7][8]. N+1 residual method has the advantages of high accuracy and fast convergence in solving nonlinear equations. The principle and solving process of N+1 residual method are briefly introduced below.

The systems of non-linear equations are:

\[
\begin{align*}
G_1(x_1, x_2, ..., x_N) &= 0 \\
G_2(x_1, x_2, ..., x_N) &= 0 \\
G_3(x_1, x_2, ..., x_N) &= 0 \\
G_4(x_1, x_2, ..., x_N) &= 0 \\
G_5(x_1, x_2, ..., x_N) &= 0 \\
G_6(x_1, x_2, ..., x_N) &= 0
\end{align*}
\]

There is an approximate solution \( X = (x_1, x_2, ..., x_N) \). Substituting it into equations (8) will obtain:
\[
\begin{align*}
G_1(x_1, x_2, \ldots, x_N) = e_1 \\
G_2(x_1, x_2, \ldots, x_N) = e_2 \\
G_3(x_1, x_2, \ldots, x_N) = e_3 \\
G_4(x_1, x_2, \ldots, x_N) = e_4 \\
G_5(x_1, x_2, \ldots, x_N) = e_5 \\
G_6(x_1, x_2, \ldots, x_N) = e_6
\end{align*}
\]  

(9)

Noting \( E = (e_1, e_2, \ldots, e_6)^T \), which is a residue of the approximate solution \( X \) of the system of equation (9).

Assume that equations (8) have \( N+1 \) approximate solutions: \( X_0, X_1, \ldots, X_N \). Their corresponding residues are \( E_0, E_1, \ldots, E_N \). Let \( Y = (y_0, y_1, \ldots, y_N) \), then a system of linear equations of order \( N+1 \) can be constructed:

\[
\begin{bmatrix}
1 & 1 & \ldots & 1 \\
E_0 & E_1 & \ldots & E_N
\end{bmatrix}
\begin{bmatrix}
y_0 \\
y_1 \\
\vdots \\
y_N
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]  

(10)

From this linear equations we can obtain \( y_0, y_1, \ldots, y_N \). A new approximate solution \( X_{N+1} \) of the equations (8) can be formed by using them.

\[
X_{N+1} = \sum_{i=1}^{N} y_i \cdot X_i
\]  

(11)

Check whether the corresponding residual satisfies the following inequalities:

\[
\| E_{N+1} \| = \sqrt{\sum_{i=1}^{N} e_i^2} \leq \varepsilon
\]  

(12)

If the inequality is satisfied, the solution vector corresponding to the norm is the solution. Otherwise, compare the norm of \( N+1 \) residue and select the maximum value:

\[
\| E_{\text{max}} \| = \max(\| E_0 \|, \| E_1 \|, \ldots, \| E_{N+1} \|)
\]  

(13)

In that way, the corresponding solution vector \( X_{\text{max}} \) is the worst solution. Then latest solution \( X_{N+1} \) and its norm are used to replace the worst solution, and the calculation is restarted in the order of formula (10) ~ (13) until the inequality (12) is satisfied. The new one is the solution that satisfies the accuracy requirement [9].

How to determine the \( N+1 \) approximate solutions are the first problem in the application of \( N+1 \) residual method. Generally, an approximate solution is given and its corresponding residual is calculated.

\[
X_0 = (x_0^0, x_2^0, \ldots, x_N^0)
\]  

(14)

\[
E_0 = (e_1^0, e_2^0, \ldots, e_N^0)
\]  

(15)

Then each component of \( X_0 \) is given a random increment \( \Delta_i \) in turn, which forms \( N \) approximate solutions and their corresponding residual vectors.
\[ X_i = (X_1^0, ..., X_i^0 + \Delta_1, ..., X_N^0) \]  
\[ E_i = (e_1^i, e_2^i, ..., e_N^i) \]

Since N approximate solutions are generated randomly on the basis of \( X_0 \), the initial solution vectors generated by each running simulation are different. Therefore, the key to the application of N+1 residual method is to give a set of better initial solutions. In this paper, the experience value in the process of engine design is taken as the initial guess value, and the calculation results are fine.

4. Simulation Results and Analysis

Taking an engine for an example, a general model of the engine is established based on its design point parameters, and the specific performance parameters of the engine are calculated in the intermediate state under standard atmospheric conditions. The simulation results are compared with the measured performance data of the engine. The results are shown in Table 2. It can be seen that there are some deviations between the calculated value of the model and the measured value of the engine. The maximum error is 2.85%, which meets the requirement of the model error within 3% in general engineering. The error is mainly due to the simplification of many conditions such as heat exchange between air flow and engine components in the thermodynamic modeling process.

| performance parameter | relative error |
|-----------------------|---------------|
| P22                   | -0.50%        |
| P3                    | -1.08%        |
| T3                    | 1.66%         |
| T55                   | -1.91%        |
| P55                   | 1.85%         |
| F                     | 2.85%         |
| WFT                   | -2.05%        |

5. Conclusions

A component model is established according to the gas flow path of the engine. A set of common working equations is established according to the common working conditions of aerothermodynamics and rotor dynamics. And the N+1 residual method is used to solve the nonlinear equations. Taking an engine for an example, the performance of intermediate state in standard atmosphere is calculated. Comparing with the measured data, the maximum error between the calculated and measured values is 2.85%, which meets the requirement of model error less than 3% in general engineering.

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