The Casimir effect for parallel plates involving massless Majorana fermions at finite temperature

Hongbo Cheng*
Department of Physics, East China University of Science and Technology,
Shanghai 200237, China
The Shanghai Key Laboratory of Astrophysics,
Shanghai 200234, China

Abstract

We study the Casimir effect for parallel plates with massless Majorana fermions obeying the bag boundary conditions at finite temperature. The thermal influence will modify the effect. It is found that the sign of the Casimir energy keeps negative if the product of plate distance and the temperature is larger than a special value or the energy will change to be positive. The Casimir energy rises with the stronger thermal influence. We show that the attractive Casimir force between two parallel plates becomes greater with the increasing temperature. In the case of piston system involving the same Majorana fermions with the same boundary conditions, the attractive force on the piston will weaker in the hotter surrounding.

PACS number(s): 03.70.+k, 03.65.Ge

*E-mail address: hbcheng@sh163.net
I. Introduction

The Casimir effect is essentially a direct consequence of quantum field theory subject to a change in the spectrum of vacuum oscillations when the quantization volume is bounded, or some background field is inserted. Historically H. B. G. Casimir originated the effect of boundaries [1]. Twenty years later T. H. Boyer found that the Casimir force for a conducting spherical shell is repulsive, showing that the circumstances will determine the force nature [2]. Afterwards more efforts have been paid for the problem and related topics and more and more results and methods have been put forward [3-10]. The precision of the measurements has been greatly improved experimentally. Recently the sign of Casimir energy and the nature of Casimir force have been applied in many subjects because the Casimir effect depends on various factors. For example the Casimir effect can be utilized to study the high-dimensional spacetimes. We can research on the Casimir effect for parallel plates in the spacetimes with extra compactified dimensions to show that the extra-dimension influence is manifest and distinct while the model was introduced [11-21]. The research on the Casimir effect for parallel plates or piston in the braneworld such as Randall-Sundrum models and etc. have also made great progress [22-31]. It is interesting that we can estimate the properties of the world with more than four dimensions. The Casimir effect has been also explored in the context of string theory [32].

It is significant to study the Casimir effect for Majorana fermions under some kinds of boundary conditions. Majorana fermions have been discussed widely in many areas of physics. As a Majorana fermion the Kaluza-Klein neutrino could be thought as a candidate of weakly interacting massive particle [33]. In the physics of superconductors and superfluids a Majorana bound state is theoretically predicted in rotating superfluid $^3\text{He} - A$ between parallel plates with the gap that is about 10$\mu$m and in triplet superconductors [34]. The Majorana vortex bound state is subject to a singular vortex in chiral p-wave superfluid [35]. Further the Casimir effect for Majorana fermions in parallel plates with bag or chiral boundary conditions is studied [36]. It was pointed that the Casimir energy is singularity free. In addition the Casimir effect for a massive fermionic field in the geometry of two parallel plates in the Minkowski spacetime with an arbitrary number of toroidally compactified spatial dimensions was also evaluated [37].

The quantum field theory at finite temperature shares a lot of effects. The thermal influence on the Casimir effect can not be neglected and the influence certainly modifies the effect. The Casimir energy for a rectangular cavity under a nonzero temperature environment was considered and the temperature controls the energy sign [38]. The Casimir effect for parallel plates including thermal corrections in the spacetime with additional compactified dimensions was explored and the magnitude of Casimir force as well as the sign of Casimir energy change with the temperature [39-42]. In addition the Casimir effect for a scalar field within two parallel plates under thermal influence in the bulk region of Randall-Sundrum models was also evaluated [43].

In this paper we are going to consider the thermal corrections to the Casimir effect for parallel plates as well as pistons containing massless Majorana fermions in detail to generalize some results...
of Ref. [36]. The description of the Casimir effect for massless Majorana fermions in the confine region such as parallel plates will change at different temperature. At first we derive the frequency of massless Majorana fermions subject to bag boundary conditions with thermal corrections by means of finite-temperature field theories. We regularize the frequency to obtain the Casimir energy density with the help of zeta function technique. Further the Casimir force between the parallel plates involving the Majorana fermions can also be gained from the Casimir energy density at finite temperature. Certainly the discussions can be generalized to the case of piston. It is necessary to explore the nature of Casimir energy and Casimir force for massless Majorana fermions within the parallel-plate system. In the end the conclusions are listed.

II. The Casimir effect of massless Majorana fermions in the parallel-plate system with bag boundary conditions at finite temperature

In finite-temperature field theories the imaginary time formalism can be utilized to describe the fermion field in the thermal equilibrium. We introduce a partition function for a system containing Dirac fields,

$$Z = N \int_{\text{antiperiodic}} D\bar{\psi} D\psi \exp\left[\int_0^\beta \int d^3x \mathcal{L}(\psi, \partial_\mu \psi)\right]$$

(1)

where $\mathcal{L}$ is the Lagrangian density for the system under consideration. $N$ a constant and "antiperiodic" means,

$$\psi(0, \mathbf{x}) = -\psi(\tau = \beta, \mathbf{x})$$

(2)

where $\beta = \frac{1}{T}$ is the inverse of the temperature and $\tau = it$. Here the massless Majorana fermions can be described through the Dirac equation as follow,

$$i\gamma_\mu \partial_\mu \psi = 0$$

(3)

where $\mu = 0, 1, 2, 3$. These massless fields satisfying the MIT bag boundary conditions which are expressed as,

$$in^\mu \gamma_\mu \psi = \psi$$

(4)

are confined to the interior of parallel-plate system. In Eq. (4) $n^\mu = (0, \mathbf{n})$ and $\mathbf{n}$ the vectors normal to the surface of plates and directed to the interior of the slab configuration. According to the solutions to the equations of motion (3) and the boundary conditions (4), the generalized zeta function can be written as,
\[ \zeta(s, -\partial_E) = Tr(-\partial_E)^{-s} \]
\[ = -\int \frac{d^2k}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{l=-\infty}^{\infty} \left[k^2 + \frac{\pi^2(n + \frac{1}{2})^2}{R^2} + \frac{4\pi^2(l + \frac{1}{2})^2}{\beta^2}\right]^{-s} \] (5)

where
\[ \partial_E = \frac{\partial^2}{\partial t^2} + \nabla^2 \] (6)

and \( k \) denote the transverse components of the momentum. \( R \) is the distance of the plates. We obtain the total energy density of the system with thermal corrections in virtue of \( \varepsilon = -\frac{\partial}{\partial \beta}\left(\frac{\partial \zeta(s, -\partial E)}{\partial s}\right)\big|_{s=0} \) and regularize the expression by means of zeta function technique to obtain the Casimir energy per unit area for parallel plates with massless Majorana fermions at finite temperature as follow,

\[ \varepsilon_C = -\frac{7}{8} \frac{\pi^2}{720R^3} + \frac{1}{\sqrt{2}} \frac{1}{\beta^2 R^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{3}{2}}(n_2 + \frac{1}{2})^{\frac{3}{2}} K_{\frac{3}{2}}[\frac{\pi \beta}{R} n_1(n_2 + \frac{1}{2})] \]
\[ + \frac{\pi}{\sqrt{2}} \frac{1}{\beta^2 R^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{3}{2}}(n_2 + \frac{1}{2})^{\frac{3}{2}} \]
\[ \times \left[K_{\frac{5}{2}}(\frac{\pi \beta}{R} n_1(n_2 + \frac{1}{2})) + K_{\frac{1}{2}}(\frac{\pi \beta}{R} n_1(n_2 + \frac{1}{2}))\right] \] (7)

where \( K_{\nu}(z) \) is the modified Bessel function of the second kind. Here we hire the zeta function regularization. In Eq. (7) the terms with series converge very quickly and only the first several summands need to be taken into account for numerical calculation to further discussion. If the temperature approaches zero, the Casimir energy density will turn to be the results of Ref. [36], so will the Casimir force. When the temperature becomes extremely high the asymptotic behaviour of the Casimir energy per unit area for massless Majorana fermions within the parallel plates is,

\[ \lim_{T \rightarrow \infty} \varepsilon_C = \frac{7}{240} \frac{\pi}{\Gamma\left(\frac{3}{2}\right)} \frac{R}{\beta^4} \] (8)

meaning that the Casimir energy rises as the temperature grows. Having performed the burden calculation, we find that the sign of the Casimir energy will be positive if the plates separation and temperature satisfy as \( RT > 0.37 \) in natural unit. The special value has something to do with the systems and boundary conditions.

The Casimir force on the plates is given by the derivative of the Casimir energy with respect to the plate distance. Here the Casimir force per unit area on the plates enclosing the massless Majorana fermions with bag boundary conditions can be written as,

\[ f_C = -\frac{\partial \varepsilon_C}{\partial R} \]
\[ f_C = -\frac{7\pi^2}{1920} \frac{1}{R^4} + \frac{3}{2\pi^2} \frac{1}{\beta^2 R^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{3}{2}} (n_2 + \frac{1}{2})^\frac{3}{2} K_\frac{3}{2} \bigg( \frac{\pi R}{n_1} (n_1 + \frac{1}{2}) \bigg) \]
\[ -\frac{\pi}{2\pi^2} \frac{1}{\beta^2 R^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{1}{2}} (n_2 + \frac{1}{2})^\frac{1}{2} \]
\[ \times \left[ K_\frac{1}{2} \left( \frac{\pi \beta}{R} n_1 (n_2 + \frac{1}{2}) \right) + K_\frac{3}{2} \left( \frac{\pi \beta}{R} n_1 (n_2 + \frac{1}{2}) \right) \right] \]
\[ + \frac{5}{2\pi^2} \frac{1}{\beta^2 R^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{1}{2}} (n_2 + \frac{1}{2})^\frac{1}{2} \]
\[ \times \left[ K_\frac{1}{2} \left( \frac{\pi \beta}{R} n_1 (n_2 + \frac{1}{2}) \right) + K_\frac{3}{2} \left( \frac{\pi \beta}{R} n_1 (n_2 + \frac{1}{2}) \right) \right] \]
\[ -\frac{\pi^2}{2\pi^2} \frac{1}{\beta^2 R^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{1}{2}} (n_2 + \frac{1}{2})^\frac{1}{2} \]
\[ \times \left[ K_\frac{1}{2} \left( \frac{\pi \beta}{R} n_1 (n_2 + \frac{1}{2}) \right) + 2K_\frac{3}{2} \left( \frac{\pi \beta}{R} n_1 (n_2 + \frac{1}{2}) \right) \right] \]
\[ + K_\frac{7}{2} \left( \frac{\pi \beta}{R} n_1 (n_2 + \frac{1}{2}) \right) \] (9)

Similarly if the temperature vanishes, the Casimir pressure shown in Eq. (9) keeps the first term that is consistent with that from Ref. [36]. When the two plates leave apart from each other, the Casimir force per unit area does not vanish because of the temperature and is expressed as,

\[ \lim_{R \to \infty} f_C = \frac{-7\pi^2}{120} T^4 \] (10)

It is evident that the Casimir force between the two plates where the Majorana fermions satisfy the bag boundary conditions becomes greater during the process that the temperature grows while the force nature remains attractive. As a function of plate distance for a definite temperature the Casimir force per unit area with thermal influence is plotted in Fig. 1. The curves of Casimir pressure associated with the plates separation for different magnitude of temperature are similar. As the temperature becomes higher the whole Casimir pressure curve will lower, which means that the stronger thermal influence leads the attractive Casimir force for two parallel plates containing massless Majorana fermions to be greater. It is interesting that here the asymptotic value of the Casimir force with thermal corrections does not vanish unless the temperature is equal to zero. It should also be pointed out that the asymptotic behaviour of the Casimir force shown in Eq. (10) is independent of plates position.

III. The Casimir piston of massless Majorana fermions with bag boundary conditions at finite temperature

It is significant to explore the Casimir effect for massless Majorana within the device of piston
at finite temperature. More efforts have been contributed to the topics related to the piston [15-22, 36]. Under the environment with nonzero temperature we discuss the massless Majorana fermions in the system consisting of three parallel plates and the bag boundary conditions are imposed on the plates. Having found and regularized the total vacuum energy per unit area of the three-parallel-plate system, we obtain the Casimir energy per unit area

$$\varepsilon_C = \varepsilon_1^C(R,T) + \varepsilon_2^C(L-R,T) + \varepsilon_{\text{out}}^C(T),$$

where $$\varepsilon_1^C(R,T)$$, $$\varepsilon_2^C(L-R,T)$$ and $$\varepsilon_{\text{out}}^C(T)$$ represent the Casimir energy per unit area of two inner parts and the Casimir energy per unit area outside the system respectively. Here $$\varepsilon_1^C(R,T) = \varepsilon_C$$ presented in Eq. (7) and $$\varepsilon_2^C(L-R,T) = \varepsilon_C|_{R\rightarrow L-R}$$. The Casimir energy per unit area outside the three-parallel-plate system has nothing to do with the plates distance. Further the Casimir force per unit area on the piston is given with the help of derivative of the Casimir energy per unit area with respect to the plates separation like

$$f_{PC}' = -\frac{\partial \varepsilon_C}{\partial R} = -\frac{\partial \varepsilon_1^C}{\partial R} - \frac{\partial \varepsilon_2^C}{\partial R} - \frac{\partial \varepsilon_{\text{out}}^C}{\partial R}.$$

As one outer plate is moved to the extremely distant place or equivalently $$L \rightarrow \infty$$, the Casimir pressure between the remaining two plates with Majorana fermions is,

$$f_{PC} = \lim_{L \rightarrow \infty} f_{PC}'$$

$$= \frac{3}{\sqrt{2} R^2 \beta^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{3}{2}}(n_2 + \frac{1}{2})^\frac{3}{2} K_n^\frac{1}{2}(\frac{4\pi R}{\beta} n_1(n_2 + \frac{1}{2}))$$

$$+ \frac{(6 + 2\pi)\sqrt{2}}{R^4 \beta^2} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{1}{2}}(n_2 + \frac{1}{2})^\frac{1}{2}[K_n^\frac{1}{2}(\frac{4\pi R}{\beta} n_1(n_2 + \frac{1}{2}))$$

$$+ K_n^\frac{3}{2}(\frac{4\pi R}{\beta} n_1(n_2 + \frac{1}{2}))]

$$- 2\pi^2 R^2 \beta^2 \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-\frac{3}{2}}(n_2 + \frac{1}{2})^\frac{3}{2}[K_n^\frac{1}{2}(\frac{4\pi R}{\beta} n_1(n_2 + \frac{1}{2}))$$

$$+ 2K_n^\frac{3}{2}(\frac{4\pi R}{\beta} n_1(n_2 + \frac{1}{2})) + K_n^\frac{5}{2}(\frac{4\pi R}{\beta} n_1(n_2 + \frac{1}{2}))]$$

(11)

In fact the piston device can help us to cancel the terms which are independent of the plate gap. According to Eq. (11), we discover that,

$$\lim_{R \rightarrow \infty} f_{PC} = 0$$

(12)

which is favoured by the observational results and,

$$\lim_{T \rightarrow \infty} f_{PC} = 0$$

(13)

which means that the Casimir force is weaker with stronger thermal influence. The dependence of the Casimir force per unit area for the piston on the distance between the piston and the other plate is shown in Fig. 2. It is manifest that the attractive Casimir force for Majorana fermion piston will be weaker as the temperature grows because the parts in the Casimir pressure like Eq. (10) are plate-position-independent and are certainly cancelled in the piston device although
their magnitude is an increasing function of temperature. The curves of the force with different temperature are similar.

IV. Conclusion

In this work we investigate the Casimir effect with thermal modification for parallel plates and piston filled with massless Majorana fermions while there are bag boundary conditions imposed at the plates. At first we discover that the Casimir energy for two parallel plates rises with the increase in the temperature. When the product of the plates separation and the temperature is larger than a special value like $RT > 0.37$, the sign of the Casimir energy will change to be positive. The special value will be different in different system with different boundary conditions. It is also found that the magnitude of Casimir force between two parallel plates with massless Majorana fermions satisfying the bag boundary conditions at the plates become greater as the thermal influence is stronger while the Casimir force keeps attractive. In the case of piston system which contains the massless Majorana fermions with the same bag boundary conditions at the plates, the attractive Casimir force on the piston will be weaker when the surrounding is hotter. As the temperature is extremely high, the Casimir force on the piston will vanish. When the plates distance is extremely large, the asymptotic value of the Casimir force between two parallel plates depends on the temperature and does not disappear. In the context of piston model the Casimir force between the piston and the other plate will approach to be zero when the two plates leave each other much farther. Our results can return to be those of Ref. [36] when the temperature is equal to zero. Here we declare how the Casimir effect for massless Majorana fermions within the parallel system subject to the bag boundary conditions depends on the temperature.

Acknowledgement

This work is supported by NSFC No. 10875043 and is partly supported by the Shanghai Research Foundation No. 07dz22020.
References

[1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. 51(1948)793

[2] T. H. Boyer, Phys. Rev. 174(1968)1764

[3] G. Plunien, B. Muller, W. Greiner, Phys. Rep. 134(1986)87

[4] J. Ambjorn, S. Wolfram, Ann. Phys. (N. Y.)147(1983)1

[5] E. Elizalde, S. D. Odintsov, A. Romeo, A. A. Bysenko, S. Zerbini, Zeta Regularization Techniques with Applications, World Scientific, Singapore, 1994

[6] E. Elizalde, Ten Physical Applications of Spectral Zeta Functions, Springer-Verlag, Berlin, 1995

[7] K. A. Milton, The Casimir Effect, Physical Manifestation of Zero-Point Energy, World Scientific, Singapore, 2001

[8] V. M. Mostepanenko, N. N. Trunov, The Casimir Effect and its Applications, Oxford University Press, Oxford, 1997

[9] M. Bordag, U. Mohideen, V. M. Mostepanenko, Phys. Rep. 353(2001)1

[10] K. Kirsten, Spectral Function in Mathematics and Physics, Chapman and Hall, London, 2002

[11] K. Poppenhaeger, S. Hossenfelder, S. Hofmann, M. Bleicher, Phys. Lett. B582(2004)1

[12] H. Cheng, Chin. Phys. Lett. 22(2005)2190
    H. Cheng, Mod. Phys. Lett. A21(2006)1957

[13] H. Cheng, Phys. Lett. B643(2006)311

[14] R. M. Cavalcanti, Phys. Rev. D69(2004)065015

[15] M. P. Hertzberg, R. L. Jaffe, M. Kardar, A. Scardicchio, Phys. Rev. Lett. 95(2005)250402

[16] A. Edery, Phys. Rev. D75(2007)105012

[17] A. Edery, N. Graham, I. MacDonald, Phys. Rev. D79(2009)125018

[18] H. Cheng, Phys. Lett. B668(2008)72

[19] K. Kirsten, S. A. Fulling, Phys. Lett. B671(2009)179
    K. Kirsten, S. A. Fulling, Phys. Rev. D79(2009)065019

[20] K. Milton, J. Wagner, Phys. Rev. D80(2009)125028

[21] E. Elizalde, S. D. Odintsov, A. A. Saharian, Phys. Rev. D79(2009)065023
[22] A. Flachi, D. Toms, Nucl. Phys. B610(2001)144
[23] A. A. Saharian, M. R. Setare, Phys. Lett. B552(2003)119
[24] E. Elizalde, S. Nojiri, S. D. Odintsov, S. Ogushi, Phys. Rev. D67(2003)063515
[25] J. Garriga, A. Pomarol, Phys. Lett. B560(2003)91
[26] M. Frank, I. Turan, L. Ziegler, Phys. Rev. D76(2007)015008
[27] R. Linares, H. A. Morales-Tecotl, O. Pedraza, Phys. Rev. D77(2008)066012
[28] M. Frank, N. Saad, I. Turan, hep-th/0807.0443(2008)
[29] H. Cheng, Chin. Phys. Lett. 27(2010)031101
   H. Cheng, Commun. Theor. Phys. 53(2010)1125
[30] M. P. Hertzberg, R. L. Jaffe, M. Kardar, A. Scardicchio, Phys. Rev. D76(2007)045016
[31] A. Flachi, T. Tanaka, Phys. Rev. D80(2009)124022
[32] A. A. Saharian, M. R. Setare, Phys. Lett. B552(2003)119
   H. Cheng, X. Li, Chin. Phys. Lett. 18(2001)1163
   L. Hadasz, et.al., Phys. Rev. D62(2000)025011
[33] V. K. Oikonomou, J. D. Vergados, C. Moustakidis, Nucl. Phys. B773(2007)19
[34] H. Y. Keep, A. Raghavan, K. Maki, arXiv: 0711.0929
[35] Y. Tsutsumi, T. Kawakami, T. Mizushima, M. Ichioka,K. Machida, Phys. Rev. Lett. 101(2008)135302
[36] V. K. Oikonomou, N. D. Tracas, arXiv:0912.4825
[37] S. Bellucci, A. A. Saharian, Phys. Rev. D80(2009)105003
[38] H. Cheng, J. Phys. A: Math. Gen. 35(2002)2205
[39] H. Cheng, Chin. Phys. Lett. 22(2005)3032
[40] L. P. Teo, Phys. Lett. B672(2009)190
[41] L. P. Teo, Nucl. Phys. B819(2009)431
[42] L. P. Teo, JHEP 0906(2009)076
[43] M. Rypestol, I. Brevik, New J. Phys. 12(2010)013022
Figure 1: The solid, dot and dashed curves of the Casimir force per unit area between two parallel plates as functions of plate separation $R$ for $\beta = 0.02, 0.03, 0.04$ respectively.
Figure 2: The solid, dot and dashed curves of the Casimir force per unit area for piston as functions of plate separation $R$ for $\beta = 0.02, 0.025, 0.03$ respectively.