Alignment procedures for the CMS silicon tracker

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Abstract. The CMS all-silicon tracker consists of 16588 modules. Therefore its alignment procedures require sophisticated algorithms. Advanced tools of computing, tracking and data analysis have been deployed for reaching the targeted performance. Ultimate local precision is now achieved by the determination of sensor curvatures, challenging the algorithms to determine about 200k parameters simultaneously. Systematic biases in the geometry are controlled by adding further information into the alignment workflow, e.g. the mass of decaying resonances. The orientation of the tracker with respect to the magnetic field of CMS is determined with a stand-alone chi-square minimization procedure. The geometries are finally carefully validated. The monitored quantities include the basic track quantities for tracks from both collisions and cosmic muons and physics observables.

1. Introduction
One of the main components of the multi-purpose detector CMS [1] at the pp collider LHC is the central all-silicon tracker. It is used to reconstruct the trajectories of charged particles traversing the detector volume as well as to determine the position of interaction or decay vertices.

The CMS tracker is located in the centre of CMS\(^1\) and it is operated in a magnetic field of 3.8 T which is provided by a superconducting solenoid. The tracker design is based on silicon sensors segmented into pixels in the innermost region and strips in the outermost one; it is mechanically divided into several subdetectors which are briefly discussed in the following.

The pixel detector surrounds the nominal interaction region. In this detector, silicon pixel modules are utilised to provide a very high resolution for vertex reconstruction. This is achieved measuring two-dimensional positions with a resolution of up to 9 \(\mu\)m. In the barrel region the modules of the pixel barrel detector (BPIX) are arranged in three cylindrical layers. In the endcap region the BPIX is supplemented by the forward pixel detector (FPIX) which consists of two discs on either side of the interaction region.

The pixel detector is surrounded by the silicon strip modules providing one-dimensional measurements mainly in the transverse plane\(^2\) \(r\phi\). The tracker inner barrel (TIB), which forms the inner part of the strip detector, consists of four layers of modules, while the tracker outer barrel (TOB) is subdivided into six layers of modules. In the inner part, the tracker inner

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\(^1\) The origin of the CMS coordinate system is located at the nominal interaction point. The z axis points along the beam pipe, while the y axis is pointing upwards. The x axis is oriented horizontally and points towards the centre of LHC.

\(^2\) Measurements in global z direction are provided by stereo modules that consist of two sensors which are mounted back-to-back with a stereo angle of 100 mrad.
disc (TID) is located in the gap between TIB and the tracker endcaps (TEC). For strip modules the resolution ranges from 23 to 60 µm.

Since many physics analyses at CMS depend on an accurate reconstruction of charged particle trajectories (also referred to as tracks) and interaction vertices, precise knowledge of the sensor positions of the order of typically better than 10 µm is mandatory. Due to the limited mounting precision, the offline alignment is crucial at CMS. Sophisticated alignment algorithms, which make use of the measured tracks, are exploited extensively. Due to the large available dataset in 2011, the alignment is limited by systematic effects and not by the statistical precision. To cope with these systematic limitations, several improvements have been developed and integrated into the CMS alignment procedure, such as the special treatment of tracks from Z⁰ boson decays into µ⁺µ⁻, the determination of sensor surface deformations, and the determination of the orientation of the tracker with respect to the magnetic field.

In this article some of the most recent achievements of the CMS alignment performed in summer 2011 are presented after a short introduction into track-based alignment. In addition, some of the latest alignment validations and studies of systematic distortions are also discussed.

2. Track-Based Alignment

In track-based alignment algorithms, particle trajectories are reconstructed taking into account parameters which are used to describe the track, as well as corrections to the positions and orientations of modules in which the hits are reconstructed. This is achieved by constructing a \( \chi^2 \) function according to

\[
\chi^2(\tau, p) = \sum_{j} \sum_{i} \left( \frac{m_{ij} - f_{ij}(\tau_j, p)}{\sigma_{ij}} \right)^2,
\]

where \( m_{ij} \pm \sigma_{ij} \) corresponds to the measurement \( i \) associated to trajectory \( j \). The measurements comprise of e.g. positions and also scattering angles. The prediction from the track model \( f_{ij}(\tau_j, p) \) depends on the track parameters, \( \tau_j \), and the alignment parameters, \( p \). Since the parameters in equation 1 are overdetermined, a minimisation can be performed to extract the optimal track and alignment parameters.

In this article all presented alignment results have been obtained using the Millepede-II program [2] which minimises equation 1 taking into account all correlations between track and alignment parameters. The minimisation is performed after approximating the track model \( f_{ij} \) with a Taylor expansion while keeping only the linear terms. This leads to the normal equations of least squares \( Ca = b \) with \( a = (p, \tau)^T \). The quantity \( C \) is a large \( n \times n \) matrix where \( n \) can be in the order of \( O(10^8) \), making it impossible to directly solve the equation. In the Millepede approach the distinction between the track and alignment parameters, which are referred to as local and global parameters, is used to reduce the problem to the sparse equation system \( C'p = b' \) by using block matrix algebra [2]. In this equation the dimension of \( C' \) is reduced to the number of global parameters, making it feasible to solve the equation system. However, it was a breakthrough to determine more than 200 000 alignment parameters simultaneously. Millepede-II offers several algorithms to solve the matrix equation such as inversion, diagonalisation, and the MINRES algorithm [3]. The latter iteratively solves the sparse equation system, while the former two algorithms can typically only be used for a relatively small number of global parameters, one reason being that for inversion the computing time needed grows as \( n^3 \).

The trajectory of a charged particle in a magnetic field can be fully described using five helix parameters. But due to the active and non-active material in the tracker, the effect of multiple scattering of particles in the detector material has to be taken into account in the track reconstruction. Since the progressive Kalman filter technique is less suited for the integration...
into global alignment methods, a track model referred to as “General Broken Lines” \cite{4, 5} is implemented.

2.1. Alignment parameters
In the CMS software the sensor translations $u, v, w$ as well as the sensor rotations $\alpha, \beta, \gamma$ around these axes are defined in the local reference system of the sensors. For single sensors the $u$ axis is always pointing into the direction of the most sensitive coordinate (perpendicular to the strip direction). The $v$ axis is pointing along the direction of the insensitive coordinate (parallel to the strip direction). The alignment parameters along $v$ are not determined for strip modules. Thus, for each strip (pixel) module up to 5 (6) parameters have to be estimated in total in the alignment procedure.

![Figure 1. a,b) Illustration of two of the three modified Legendre polynomials used to describe bowed sensor surfaces as a function of the relative position in $u$ and $v$ direction \cite{7}. c) Schematic visualisation of a kink between two sensors \cite{7}.](image)

The parameters defined above correspond to sensors described by a parametrisation assuming flat sensor surfaces. In reality, however, the surfaces of the sensors are not flat. To take this effect into account, for each sensor the sum of second order modified Legendre polynomials \cite{6} has been introduced in order to parametrise the sensor surface. The curvatures of the sensor surfaces are referred to as “bows”. Two of the three polynomials are illustrated in figure 1.

All TOB modules and the TEC modules at radii $r > 60$ cm consist of two individual daisy-chained sensors (figure 1c). With respect to alignments of the CMS tracker performed in earlier years, the treatment of these double sensor modules has been improved by allowing the separate determination of the alignment parameters for both sensors. This improvement is referred to as the determination of “kinks”.

Furthermore, it is possible to incorporate the hierarchical structure of the CMS tracker by employing linear equality constraints.

2.2. The weak mode problem
Generally, by minimising the $\chi^2$ function defined in equation 1 not all possible distortions of the tracker can be resolved \cite{8, 9}. The residuals can be insensitive to certain global deformations referred to as “weak modes”. Weak modes correspond to global transformations that (at least approximately) preserve the validity of the track model for the track sample under consideration, i.e. the transformed hits are to a high degree still consistent with a valid trajectory. Such systematic distortions have very little impact on the goodness-of-fit of the track. However, these distortions might significantly bias the track parameters and other quantities such as invariant masses. Consequently, physics analyses might suffer from these systematic limitations of the alignment.

Weak modes cannot be resolved by increasing the number of tracks, since the $\chi^2$ function does not exhibit a distinct minimum with respect to these modes, and are intrinsic to the topology
of the tracks in the used datasets. Therefore, by using e.g. tracks from collisions together with
tracks from cosmic rays, which typically connect the upper and the lower part of the tracker, it
is possible to suppress some of these weak modes. Another possibility is the inclusion of other
quantities in the alignment, such as beam spot constraints or physics observables, whose values
are sensitive to the tracker geometry.

At CMS the information from the decay of $Z^0$ bosons into two muons is included into the
alignment procedure. This is achieved by re-parametrisising the two tracks associated to the
decay into one common object [10]. The $2 \times 5$ parameters$^3$ of these two tracks are transformed
into nine other parameters based on the kinematics of the decay. These parameters include the
position of the decay vertex, the momentum of the $Z^0$ boson in the laboratory frame, the polar
and azimuthal angle defining the direction of the muons in the rest frame of the $Z^0$, and the mass
of the $Z^0$ boson. The known $Z^0$ mass is imposed as a virtual measurement with an uncertainty
equivalent to the root mean square around the position of the peak of the $Z^0$ line-shape taking
into account radiative corrections.

3. The 2011 alignment campaign and the CMS tracker alignment workflow

The summer 2011 alignment$^4$ has been performed using data recorded until end of June. The
data samples comprise about 15 million tracks of loosely selected isolated muons, about 3 million
low momentum tracks, around 3.6 million cosmic ray tracks that were taken between LHC fills
and during as well as before regular collisions data taking, and about 375 thousand muon pairs
forming $Z^0$ candidates.

The CMS tracker alignment workflow using Millepede-II is divided into two independent
steps. First, dedicated binary files for all selected tracks are created. The collection of these is
largely parallelised by using the “CMS CERN Analysis Facility” (CAF) [11]. For the summer
2011 alignment described here, 246 parallel jobs have collected the required technical ingredients
such as the residuals, the derivatives of the residuals with respect to the global and local
parameters, and the measurement uncertainties. All these jobs have typically been done within
two hours. In a second step, the binary data files are read by the so-called “Pede” program,
which sets up and solves the matrix equation $C'p = b'$.

In the summer 2011 alignment campaign, more than 200 000 alignment parameters$^5$ have
been determined which would have resulted in a usage of about 160 GB of RAM if the full
matrix $C'$ had been stored in double precision. Due to the application of MINRES in the
minimisation procedure the required amount of memory can be reduced by storing only non-
zero matrix elements in conjunction with a bit-packed addressing of continuous non-zero blocks
in a row. In addition, for some elements of the matrix the corresponding sums run only over a
few tracks. These elements are stored in single instead of double precision. With these measures,
the memory usage has been approximately 32 GB.

However, not only the memory usage induces strong requirements on the computing
infrastructure, but also the possible input/output throughput and available CPU resources
are crucial components in the alignment workflow. To reduce the time devoted to reading
the data files or performing the minimisation with MINRES, the Millepede-II program is largely
parallelised using the OpenMP® library [12]. The input data have been compressed to reduce the
total size of the files to 46.5 GB, which have been read 13 times in total during the minimisation
procedure. Overall, the MINRES algorithm has been executed four times using continuously
tighter cuts for the rejection of outliers.

$^3$ In addition, also the track parameters used to describe multiple scattering are considered.
$^4$ At CMS the outcome of the tracker alignment is often also called “geometry”, which describes the absolute
positions, orientations and surface deformations of all sensors of the tracker.
$^5$ For each strip sensor eight parameters have been determined, whereas for the pixel sensors typically nine
alignment parameters have been estimated.
The CPU usage on an Intel® Xeon® L5520 with 2.27 GHz running eight threads in total was 44.5 h corresponding to 9:50 h wall clock time.

3.1. Validation of curved sensors and double sensors

The parametrisation of the curved sensor surfaces and the treatment of the double sensors in the alignment is validated [7] by determining average residual distributions in \( w \) direction defined as \( \langle (u_{\text{pred}} - u_{\text{hit}})/\tan \psi \rangle \) where \( \psi \) corresponds to the angle between the track and the surface normal in the local \( u-w \) plane. Since tracks with large \( \psi \) have a higher sensitivity to deviations from the assumption of flat sensor surfaces, the tracks are weighted by \( \tan^2 \psi \) in the validation procedure presented here to give more weights to tracks with large incident angle.

**Figure 2.** The quantity \( \langle w \rangle = \langle (u_{\text{pred}} - u_{\text{hit}})/\tan \psi \rangle \) as a function of the relative position on the module, \( v_r \), in TOB and BPIX for different module shape parametrisation [7].

In figure 2 the quantity \( \langle (u_{\text{pred}} - u_{\text{hit}})/\tan \psi \rangle \) as a function of the relative position on the module, \( v_r \), along the \( v \) axis is shown in TOB and BPIX. The summer 2011 geometry is denoted as “Curved Sensors” whereas the three other presented geometries have been obtained by neglecting either the bows (“Flat sensors”), the kinks (“Curved Modules”), or both (“Flat Modules”) in the alignment procedure. For the “Flat Modules” geometry a strong track angle and position dependence on the hit residuals is evident. This dependence is largely eliminated by considering of bows and kinks simultaneously. The determination of parameters describing bowed sensor surfaces also removes the structure visible in the BPIX which is particularly important for forward tracks due to their large incident angle.

Figure 3 shows the average track fit probability for cosmic ray tracks as a function of the absolute distance of closest approach of the track to the beam line, \( |d_0| \). The quantity \( |d_0| \) is strongly correlated with \( \psi \), because tracks originating from the nominal interaction region have typically small \( |d_0| \) and therefore tend to be normal to the sensor surfaces due to the geometry of the tracker. However, cosmic rays are coming mainly from above and are not constrained to the nominal interaction region. If those tracks have large \( |d_0| \), then \( \psi \) is typically large for some modules depending on their location. As demonstrated in the figure, only the summer 2011 alignment mostly removes the dependence of the average track fit probability on \( |d_0| \). This improvement of the parametrisation increases the consistency between cosmic ray and collision tracks in the alignment procedure and thus allows to fully exploit their complementarity.

On average, the size of the sagitta of the sensor surface is of the order of 30 \( \mu \text{m} \) [13] in BPIX whereas the angle between the two sensors surfaces forming one compound module is typically of the order of 0.8 mrad [13] in the TOB.
3.2. Sensitivity to weak modes

In the summer 2011 alignment campaign the information from the decay of $Z^0$ bosons into muons has been exploited for an improved control of weak modes. In order to demonstrate the impact of this approach an alignment has been performed where the decay information from $Z^0$ bosons is neglected.

In figure 4a the dependence of the dimuon mass peak, $M_{\mu\mu}$, on the pseudorapidity, $\eta$, of the positively charged muon is presented. For the geometry where the information from the $Z^0$ decay has not been used in the alignment (red points) a distinct dependence of $M_{\mu\mu}$ on $\eta$ is observed. This dependence can be attributed to a twist of the whole tracker defined as $\Delta \phi_i = c \cdot z_i$, where $\Delta \phi_i$ is the change of the azimuthal angle of a module $i$. The twist transformation changes the track curvature of positively and negatively charged particles oppositely. For decays where the two muons have similar $\eta$ the impact on the dimuon mass therefore largely cancels. On the other hand, for larger pseudorapidity differences of the two muons, the bias caused by the twist can be sizable. Due to the decay kinematics, this bias is more pronounced for higher mass resonances. However, as clearly shown in figure 4 the inclusion of the $Z^0$ boson decay information into the alignment effectively suppresses the $\eta$ dependence of $M_{\mu\mu}$. The small offset between data and
Monte Carlo design is currently under investigation as is the dependence of $M_{\mu\mu}$ on $\phi$ seen in figure 4b.

In order to study the sensitivity of the alignment to weak modes, the possible deformations of the geometry are parametrised in the cylindrical coordinates $r$, $z$, and $\phi$. A subset of nine basic distortions [8, 9] is investigated by deforming the geometry according to these modes [7]. In a second step the full alignment procedure for each of these misaligned geometries is repeated. Afterwards, to test the capabilities of the alignment to correct for the introduced misalignment, the module-by-module position differences with respect to the summer 2011 geometry are determined subtracting global movements and rotations of the whole tracker.

![Figure 5](image)

**Figure 5.** The module-by-module position difference of the re-aligned geometries with respect to the summer 2011 geometry [7] after subtracting global movements and rotations of the tracker (left column). Track $\chi^2$ distributions for the two different investigated systematic distortions of the tracker [7] (right column). For further details see text.

The outcome of the study is shown in figure 5 for the twist deformation\(^6\) and the sagitta deformation along global $x$ direction\(^7\) defined as $\Delta x = c \cdot r$. In the geometry comparisons the induced misalignments are indicated as red straight lines. In addition, track $\chi^2$ distributions for collision tracks are also presented for the summer 2011 geometry, the misaligned and the re-aligned geometry. It can be noticed that the twist and the sagitta deformations are indeed weak modes for collision tracks, because after the introduced misalignment the $\chi^2$ distributions for collision tracks remain basically unchanged. However, the introduced twist deformation is fully corrected by the alignment which can be attributed to the usage of the $Z^0$ boson decay information. This result is a large improvement compared to previous alignments [8, 15].

\(^6\) For the twist deformation the pixel detector is not shown due to a large statistical scattering, since at small radii small shifts cause large $\Delta \phi$.

\(^7\) A similar expression for a sagitta deformation in global $y$ direction can be defined accordingly.
However, the alignment procedure is not able to fully correct the introduced sagitta which is currently subject of further detailed investigations.

4. Tracker tilt angles

In order to reduce systematic biases in the reconstruction of tracks and vertices arising from a tilt of the whole tracker with respect to the magnetic field, a standalone approach developed to determine these angles is used [14]. In this analysis the rotation around the $x$ ($y$) axis is defined as $\theta_x$ ($\theta_y$) which corresponds to a vertical (horizontal) tilt.

The tilt angles are determined by scanning the parameter space in $\theta_x$ and $\theta_y$ utilising three different track quality estimators such as the average normalised track-$\chi^2$ distribution denoted as $\langle \chi^2/N_{df} \rangle$, where the quantity $N_{df}$ is equal to the number of degrees of freedom, the average $\chi^2$ probability method, $\langle \text{Prob}(\chi^2, N_{df}) \rangle$, and the total $\chi^2$ defined as $\sum_i \chi^2_i$. The optimal values of $\theta_x$ and $\theta_y$ are determined by minimising the track quality estimators described above taking into account all selected tracks.

![Figure 6.](image)

The outcome of this study is shown in figure 6 where $\theta_x$ and $\theta_y$ are shown as a function of $\eta$. The shaded areas indicate the systematic uncertainties, which have been estimated by determining the spread between the three employed methods. Additionally, the transverse momentum cut has been varied and the change of the result has been taken into account in the determination of the systematic uncertainties.

It can be noted that the tracker tilt angles in the Monte Carlo simulation using the design tracker geometry are, within the systematic uncertainties, consistent with zero. In contrast to this, the data show a distinct deviation from zero for $\theta_x$ of the order of $0.3 \pm 0.1$ (sys.) mrad. The latter value was included into the CMS tracker geometry description used in the reconstruction procedure to reduce systematic biases.

5. Summary

In this article some of the most recent achievements of the CMS tracker alignment using the global fit approach as implemented in Millepede-II were presented. In the discussed alignment, which has been performed for summer 2011, more than 200 000 parameters have been determined taking into account surface deformations of sensors. Weak modes in the alignment are suppressed by exploiting cosmic ray tracks and by utilising the information from the decay of $Z^0$ bosons into muons. This effectively results in controlling the twist deformation. Additionally, information from the $Z^0$ decay are also suited for validation purposes. The sensitivity of the alignment to other global distortions such as the sagitta deformation has also been studied. The orientation of
the tracker with respect to the magnetic field has been determined with a stand-alone method. A small vertical tilt of the tracker was observed and taken into account in the full CMS reconstruction. The achievements presented here have significantly improved the systematic precision of CMS tracker alignment compared to previous alignment campaigns.

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