Pion couplings of the $\Delta(1232)$

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We calculate the strong couplings of pions to the $\Delta(1232)$ resonance using a QCD parameterization method that includes in addition to the usual one-quark also two-quark and previously uncalculated three-quark operators. We find that three-quark operators are necessary to obtain results consistent with the data and other QCD based baryon structure models. Our results are also in quantitative agreement with a model employing large $D$ state admixtures to the $N$ and $\Delta$ wave functions indicating that the $\pi N$ and $\pi \Delta$ couplings are sensitive to the spatial shape of these baryons.

I. INTRODUCTION

Since its discovery \cite{1} in pion ($\pi$)-nucleon ($N$) scattering the lowest excited state of the nucleon with spin $S = \frac{3}{2}$ and isospin $T = \frac{3}{2}$, called $\Delta(1232)$, has been important both for an understanding of nucleon ground state structure \cite{2} and the nucleon-nucleon interaction \cite{3}. In sufficiently energetic $NN$ collisions one or both nucleons may be excited to the $\Delta(1232)$, a process that makes a significant contribution to many nuclear phenomena \cite{4}, for example, electromagnetic properties of light nuclei \cite{5}, three-nucleon forces \cite{6}, and the binding energy of nuclear matter \cite{7}. In addition, $\Delta$ degrees of freedom are needed to explain the empirical cross sections for the $\pi N \rightarrow \pi \Delta$ \cite{8}, $\pi N \rightarrow \pi N \Delta$ \cite{9} \cite{10}, and $NN \rightarrow NN\pi\pi$ \cite{11} reactions including a novel resonance structure in the $p n \rightarrow d^{0}\pi^{0}$ channel \cite{12}.

However, a quantitative assessment of the role of $\Delta$ degrees of freedom in nuclear physics has remained difficult due to the lack of detailed knowledge of even basic $\Delta$ properties. For example, the widely used additive quark model, which is based on the assumption that observables can be calculated using a sum of one-quark operators, underpredicts the experimental $N \rightarrow \Delta$ transition magnetic moment by about 30\% \cite{13} but slightly overpredicts the $\Delta^{+}$ magnetic moment \cite{14}. Another example involves the strong coupling constants $f_{\pi N \Delta}$ and $f_{\pi \Delta \Delta}$. The former determines the decay rate $\Delta \rightarrow N + \pi$, while the latter fixes the $N \Delta$ interaction strength in nuclei (see Fig. \ref{fig:1}). Here again, the additive quark model underpredicts the empirical $N \rightarrow \Delta$ transition coupling $f_{\pi N \Delta}$ by 20\%, whereas it appears to overpredict the double $\Delta$ coupling $f_{\pi \Delta \Delta}$. A resolution of these discrepancies is necessary for a quantitative description of $\Delta$ degrees of freedom in nuclei \cite{15} \cite{16}.

Previously, the strong $\pi \Delta$ couplings were calculated with a QCD parameterization method, in which in addition to one-quark operators (additive quark model), two-quark operators were taken into account \cite{17}. It was shown that two-quark contributions amount to a 20\% increase of $f_{\pi N \Delta}$ with respect to the additive quark model results. Furthermore, the following relation between the $\pi N$, $\pi N \Delta$ and $\pi \Delta \Delta$ couplings was derived

\begin{equation}
\frac{f_{\pi p \Delta} - \frac{1}{4} f_{\pi \Delta + \Delta}}{f_{\pi \Delta + \Delta}} = \frac{\sqrt{2}}{3} f_{\pi \Delta + \Delta}.
\end{equation}

This relation connects the elusive $f_{\pi \Delta + \Delta}$ to the better known $f_{\pi p \Delta} + f_{\pi \Delta \Delta}$ couplings. Throughout this paper we use the normalization conventions of Ref. \cite{18} for the one-quark contributions, i.e., $f_{\pi \Delta \Delta} = (4/5) f_{\pi NN}$ and $f_{\pi N \Delta} = (6\sqrt{2}/5) f_{\pi NN}$ \cite{19}. These theoretical relations, which are based on the additive quark model (see sect. 3), satisfy Eq. (1). On the other hand, if the empirical relation $f_{\pi p \Delta} = 2.1 f_{\pi \Delta \Delta}$ is used in Eq. (1), one obtains $f_{\pi \Delta + \Delta} = 0.04 f_{\pi \Delta \Delta}$. However, from the viewpoint of QCD sum rules \cite{20} and 1/$N_c$ expansion \cite{21} both $f_{\pi \Delta + \Delta}$ and $f_{\pi \Delta \Delta}$ should be of the same order of magnitude.

The main purpose of this paper is to investigate if the inclusion of three-quark operators can resolve the discrepancies between theory and experiment for the $\pi \Delta$ couplings. Another motivation for this study comes from the work of Abbas \cite{22}, who found that large $D$-wave admixtures in the $N(939)$ and $\Delta(1232)$ quark wave functions reduce the additive quark model result for $f_{\pi \Delta \Delta}$ by 20\% while they increase $f_{\pi N \Delta}$ by about the same percentage. This indicates that the strong $\pi \Delta$ couplings are sensitive to the spatial shape of the quark distribution in the nucleon and its first excited state. Therefore, it is of interest to study whether the two- and three-quark terms have an analogous effect on the strong $\pi \Delta$ couplings and may thus be interpreted as describing degrees of freedom leading to nonspherical geometrical shapes of the $N$ and $\Delta$ baryons.

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are denoted as responding operator pion couplings. The most general expression for the core interaction vertices are represented as black dots.

\[
O_1 = A_1 \sum_i \tau^i_3 \sigma^i_z, \\
O_2 = A_2 \sum_{i \neq j} \tau^i_3 \sigma^j_z \\
O_3 = A_3 \sum_{i \neq j \neq k} \tau^i_3 \sigma^j_z \sigma^k_z \cdot \sigma^k,
\]

and the total operator reads

\[
O = O_1 + O_2 + O_3.
\]

III. RESULTS

Evaluating the operators in Eq. (2) between SU(6) wave functions [29] for the N and Δ we get the results compiled in Table I.
To obtain from the total quark level matrix elements $\mathcal{M}_q$ in Table I the conventional pion-baryon couplings, one proceeds as follows [19]. Conventionally, the $N \to \Delta$ transition vertices depicted in Fig. I (left) are defined as matrix elements $\mathcal{M}_B$ of baryon level $N \to \Delta$ transition spin $S$ and isospin $T$ operators. The latter are normalized so that their matrix elements are equal to the corresponding spin and isospin Clebsch-Gordan coefficients

$$\mathcal{M}_B = f_{\pi N \Delta} \langle S', S'' | T', T'' | S, T | N \rangle,$$

which gives $\mathcal{M}_B = f_{\pi N \Delta} (10 S S | S' S'' ) (10 T T' T'')$. (6)

Here, $S = T = 1/2$ refers to the spin and isospin of the $N$, and $S' = T' = 3/2$ to the spin and isospin of the $\Delta$. For the $p \to \Delta^+$ transition with $T_z = T'_z = 1/2$ and $S_z = S'_z = 1/2$ Eq. (6) gives $\mathcal{M}_B = f_{\pi N \Delta} (2/3)$. This baryon level matrix element must be equal to the corresponding total quark level matrix element, i.e., $\mathcal{M}_B = f_{\pi N \Delta} (2/3) = \mathcal{M}_q$. Consequently, the $f_{\pi N \Delta}$ coupling is obtained by multiplying the total quark level matrix element $\mathcal{M}_q$ by the second row of Table I by 3/2.

Analogously, $\Delta \Delta$ vertices depicted in Fig. I (right) are obtained from the diagonal matrix elements

$$\mathcal{M}_B = f_{\pi \Delta \Delta} \langle S', S'' | T', T'' | S, T | \Delta \rangle,$$

Evaluating Eq. (7) for the $\Delta^+$ with $T'_z = 1/2$ and maximal spin projection $S'_z = 3/2$ gives $\mathcal{M}_B = f_{\pi \Delta \Delta} (3/4)$. This baryon level matrix element must be equal to the corresponding total quark level matrix element, i.e., $\mathcal{M}_B = f_{\pi \Delta \Delta} (3/4) = \mathcal{M}_q$. Therefore, the entry for $\Delta^+$ in Table II must be multiplied by 4/3.

We then get

$$f_{\pi N \Delta} = \frac{5}{3} A_1 - \frac{2}{3} A_2 - \frac{26}{3} A_3,$$

$$f_{\pi \Delta \Delta} = \frac{4 \sqrt{2}}{3} \left( A_1 - A_2 + 2 A_3 \right) \left( \frac{3}{2} \right),$$

$$f_{\pi \Delta \Delta} = (A_1 + 2 A_2 + 2 A_3) \left( \frac{4}{3} \right).$$

Solving Eq. (8) for the constants $A_i$ leads to

$$A_1 = \frac{1}{6} f_{\pi N \Delta} + \frac{\sqrt{2}}{3} f_{\pi \Delta \Delta} + \frac{5}{24} f_{\pi \Delta \Delta},$$

$$A_2 = \frac{1}{4} f_{\pi \Delta \Delta} - \frac{\sqrt{2}}{12} f_{\pi \Delta \Delta},$$

$$A_3 = \frac{1}{12} f_{\pi N \Delta} + \frac{\sqrt{2}}{36} f_{\pi \Delta \Delta} + \frac{1}{48} f_{\pi \Delta \Delta}. \quad (9)$$

Next, we calculate numerical values for the strong $\Delta$ couplings, including successively first, second, and third order SU(6) symmetry breaking terms represented respectively by the operators $O_1$, $O_2$, and $O_3$ in Eq. (2).

First, ignoring two- and three-quark terms, we get $A_1 = (3/5) f$, where we use the abbreviation $f := f_{\pi N \Delta}$. Thus, $A_1$ is fixed by the empirical value for the strong $\pi NN$ coupling $f_{\pi N \Delta}^2/(4\pi) = 0.08$. In this first order SU(6) symmetry breaking approximation, we reproduce the well known additive quark model results for the $\pi \Delta$ couplings $\pi N \Delta = 0.08$. In this first order SU(6) symmetry breaking approximation, we reproduce the well known additive quark model results for the $\pi \Delta$ couplings $\pi N \Delta = 0.08$.

Second, if we include two-quark but still neglect three-quark terms we need the empirical relation $f_{\pi N \Delta} = 2.1 f$ to fix the additional constant $A_2$. In this case, SU(6) symmetry is broken up to second order. Eq. (3) with $A_3 = 0$ gives then $A_1 = 0.51 f$ and $A_2 = -0.24 f$ as in Ref. [18]. In this approximation we recover Eq. (1), which as shown in Table II entails an unrealistically small value for $f_{\pi \Delta \Delta} = 2 A_3$.

Finally, the inclusion of three-quark terms takes third order SU(6) symmetry breaking into account. Using the QCD sum rule value $f_{\pi \Delta \Delta} = 0.666 f$ [21], which is consistent with the data [13, 14], allows us to fix the constant $A_3$ in Eq. (5). We then get from Eq. (9) the following values for the constants: $A_1 = 0.635 f$, $A_2 = -0.081 f$, and $A_3 = 0.013 f$. By taking three-quark operators into account, we find that Eq. (1) is modified as follows

$$f_{\pi \Delta \Delta} = \frac{1}{4} f_{\pi \Delta \Delta} = \frac{\sqrt{2}}{3} f_{\pi \Delta \Delta} - 2 A_3. \quad (11)$$

Consequently, $f_{\pi \Delta \Delta}$ can be of the same magnitude as $f_{\pi \Delta \Delta} = 2 A_3$ even when the empirical value for $f_{\pi \Delta \Delta}$ is used so that the discrepancy between theory and experiment can be resolved.

| coupling  | $M_1$ | $M_1 + M_2$ | $M_1 + M_2 + M_3$ | exp. |
|-----------|------|-------------|-----------------|-----|
| $f_{\pi N \Delta}$ | 1.00 | 1.00 | 1.00 | 1.00 [17] |
| $f_{\pi \Delta \Delta}$ | 1.70 | 2.10 | 2.10 | 2.10 [18] |
| $f_{\pi \Delta \Delta}$ | 0.80 | 0.04 | 0.67 | 0.06-1.72 [9] |

TABLE II: Pion couplings of the $\Delta$(1232) in terms of the pion-nucleon coupling $f = f_{\pi N \Delta}$ with the successive inclusion of one-quark ($M_1$), two-quark ($M_2$), and three-quark ($M_3$) terms compared with experimental data. The experimental range for $f_{\pi \Delta \Delta}$ is from Table 9 in Ref. [19].
from 0.04\(f\) to 0.67\(f\), in qualitative agreement with QCD sum rule \cite{21} and \(1/N_c\) expansion \cite{22} calculations. This provides evidence for the importance of three-quark operators in axial vector quantities, such as the pion-baryon couplings.

It is of interest to compare our results to those of Abbas \cite{23}, who found the following expressions for the \(\pi N\Delta\) and \(\pi\Delta\Delta\) couplings based on one-quark axial vector operators but with \(D\) state admixtures in the \(N\) and \(\Delta\) wave functions

\[
\begin{align*}
f_{\pi N\Delta} &= \frac{6\sqrt{2}}{5} \left[ 1 - \frac{1}{6} P_D \right] \frac{1}{\sqrt{1 + P_D^2}} f, \\
f_{\pi\Delta\Delta} &= \frac{4}{5} \left[ 1 - \frac{1}{6} P_D \right] \frac{1}{1 + P_D} f,
\end{align*}
\]

(12)

where \(P_D\) is the \(D\)-state probability in the nucleon wave function \cite{29}. Note that the double \(\Delta\) coupling in Eq. (12) has been adjusted to the normalization convention used here \cite{20}. For \(P_D = 0.34\) one finds \(f_{\pi N\Delta} = 2.1 f\) and \(f_{\pi\Delta\Delta} = 0.67 f\). Incidentally, the same value \(P_D = 0.34\) also explains the empirically small quark contribution to nucleon spin \cite{31}. As a result of the large \(D\)-state admixture in the nucleon and \(\Delta\) wave functions, \(f_{\pi N\Delta}\) is increased by 20\%, and \(f_{\pi\Delta\Delta}\) is decreased by the same percentage with respect to the additive quark model values in Eq. (10). This is consistent with the data and in quantitative agreement with the present results. Apparently, nonspherical \(N\) and \(\Delta\) wave functions with large \(D\)-state probabilities have the same effect as the two- and three-quark contributions considered here.

This correspondence between large \(D\) wave admixtures and many-quark operators is not a coincidence. Via a unitary transformation it is always possible to eliminate many-body operators at the expense of more complicated wave functions without changing the observable matrix elements \cite{32}. Yet, the inclusion of many-quark operators leads overall to a more consistent and realistic description of nucleon structure for the following reasons. First, ab initio quark model calculations based on gluon and pion exchange potentials feature much smaller \(D\) state probabilities (< 0.01) \cite{33} for the \(N\) and \(\Delta\) wave functions. Second, many-quark operators are related to the quark-antiquark degrees of freedom, commonly referred to as meson cloud in physical baryons. Baryon deformation is more likely a result of these non-valence quark degrees of freedom than a consequence of massive valence quarks moving in elliptical orbits \cite{34}.

In any case, the ability of both models to describe the strong \(\pi \Delta\) couplings is closely tied to nonspherical geometric shapes of the nucleon and \(\Delta\). Other observables, such as the \(N \rightarrow \Delta\) quadrupole transition moment, baryon octupole moments, and the quark contribution to nucleon spin \cite{31} point to the same conclusion concerning the nonsphericity of both \(N\) and \(\Delta\) states.

\section{IV. SUMMARY}

In summary, we found that previously uncalculated three-quark operators make a significant contribution to the \(\pi N\Delta\) and \(\pi\Delta\Delta\) coupling constants. In particular, for the double \(\Delta\) coupling, the three-quark term reduces the influence of the negative two-quark contribution so that the final result \(f_{\pi N\Delta + \pi\Delta\Delta + \pi\Delta\Delta} = 0.67 f\) is about 20\% smaller than the additive quark model value consistent with data and other QCD based baryon structure models.

Furthermore, our theory is in quantitative agreement with results obtained in a quark model with a large \(D\)-state admixtures. Both approaches increase \(f_{\pi N\Delta}\) and simultaneously decrease \(f_{\pi\Delta\Delta}\) by about 20\% with respect to the additive quark model. This indicates that nonspherical \(N\) and \(\Delta\) states are necessary for a quantitative understanding of the strong pion couplings independent of whether the spatial deformation is described as large \(D\) state components in the valence quark wave functions or (more realistically) as two- and three-quark operators representing a nonspherical sea of quark-antiquark pairs.

Having demonstrated the importance of two- and three-quark terms for a consistent description of the \(\pi N\) and \(\pi \Delta\) couplings it will be interesting to investigate their effect on other axial vector quantities, e.g. the \(p \rightarrow \Delta^+\) transition magnetic moment and the weak axial \(N \rightarrow \Delta\) transition \cite{35} for which large discrepancies between the additive quark model and experiment persist.

\begin{thebibliography}{99}
\bibitem[1]{1} H. L. Anderson, E. Fermi, A. A. Long, D. E. Nagle, Phys. Rev. 85, 936 (1952).
\bibitem[2]{2} V. Pascalutsa, M. Vanderhaeghen, S. N. Yang, Phys. Rep. 437, 125 (2007).
\bibitem[3]{3} R. B. Wiringa, R. A. Smith, T. L. Ainsworth, Phys. Rev. C 29, 1207 (1984), R. Machleidt, Adv. Nuc. Phys. 19, 189 (1989).
\bibitem[4]{4} Steven A. Moszkowski, in Proceedings of the 11th International Workshop on Condensed Matter Theories, July 27 - August 1, 1987, Oulu, Finland, ed. J. S. Arponen et al., Plenum Press, New York, 1988.
\bibitem[5]{5} H. Arenhövel and H. G. Miller Z. Phys. 266, 13 (1974).
\bibitem[6]{6} E. Epelbaum, H. Krebs, U.-G. Meiβner, Nucl. Phys. A 806, 65 (2008).
\bibitem[7]{7} L. S. Ferreira, Nucl. Phys. A519, 269c (1990).
\bibitem[8]{8} N. Fettes and U.-G. Meiβner, Nucl.Phys. A 679, 629 (2001).
\bibitem[9]{9} R. A. Arndt et al., Phys. Rev. D 20, 651 (1979).
\bibitem[10]{10} H. Kammano and M. Arima, Phys. Rev. C 69, 025206 (2004).
\bibitem[11]{11} T. Skorodko et al., Phys. Lett. B 695, 115 (2011); M. Bashkanov et al., Phys. Rev. Lett. 102, 052301 (2009);
A. J. Buchmann, E. M. Henley, Phys. Lett. B 484 (1999).

T. Ericson and W. Weise, Pions and Nuclei, Clarendon Press, Oxford, 1988.

H. J. Weber and H. Arenhövel, Phys. Rep. 36, 277 (1978).

T. Ericson and W. Weise, Pions and Nuclei, Clarendon Press, Oxford, 1988.

Shin-Lin Zhu, Phys. Rev. C 63, 018201 (2000). The \( g_{\pi\Delta\Delta} \) and \( f_{\pi\Delta\Delta} \) couplings are related as \( f_{\pi\Delta\Delta} = f_{\pi\Delta\Delta} = \left( m_{\pi}/(2M_{\Delta}) \right) g_{\pi\Delta\Delta} \).

R. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D 49, 4713 (1994); Phys. Rev. D 51, 3697 (1995).

A. Abbas, J. Phys. G 17, L181 (1991).

G. Morpurgo, Phys. Rev. D 40, 2997 (1989); Phys. Rev. D 40, 3111 (1989).

G. Morpurgo, Phys. Rev. Lett. 68, 139 (1992); Phys. Rev. D 46, 4068 (1992); G. Dillon and G. Morpurgo, Phys. Rev. D 53, 3754 (1996).

G. Dillon and G. Morpurgo, Phys. Lett. B 448, 107 (1999); Phys. Lett. B 459, 321 (1999); Z. Phys. C 73, 547 (1997).

A.J. Buchmann and R.F. Lebed, Phys. Rev. D 62, 096005; A.J. Buchmann, J.A. Hester, and R.F. Lebed, Phys. Rev. D 66, (2002) 056002.

The operator \( \mathcal{O}_3 \) in Eq.(11) has nonvanishing matrix elements only between the spin (mixed) symmetric parts |\( \chi_S \rangle \rangle of the \( \mathcal{N}_6 \)-plet wave functions, for which one has, e.g., \( \sigma_1 \cdot \sigma_2 |\chi_S \rangle = +1 |\chi_S \rangle \) so that \( \mathcal{O}_3 \) reduces to \( \mathcal{O}_2 \) in Eq.(2).

D. B. Lichtenberg, Unitary Symmetry and Elementary Particles, Academic Press, New York, 1978; F. E. Close, An introduction to Quarks and Partons, Academic Press, London, 1979.

The \( D \) state probability in the \( \Delta \) wave function \( P_D(\Delta) \) is related to the \( D \) state probability in the nucleon wave function \( P_D \) as \( P_D(\Delta) = 2P_D/(1 + P_D) \) [22]. This has been used to eliminate \( P_D(\Delta) \) from the original expressions for the \( \pi\Delta \) couplings.

A. Abbas, J. Phys. G 15, L73 (1989).

J. L. Friar, Phys. Lett. B 59, 145 (1975).

A. Buchmann, E. Hernández, and K. Yazaki, Phys. Lett. B 269, 35 (1991), Nucl. Phys. A 569, 661 (1993).

A. J. Buchmann and E. M. Henley, Phys. Rev. C 63, 015202 (2000); Phys. Rev. D 65, 073017 (2002); Eur. Phys. J. A 35, 267 (2008); Phys. Rev. D 83, 096011 (2011).

Shi-Lin Zhu, M. J. Ramsey-Musolf, Phys. Rev. D 66, 076008 (2002); D. Barquilla-Cano, A. J. Buchmann, and E. Hernández, Phys. Rev. C 75, 065203 (2007).