ABSTRACT

Students learn best when the teacher uses a variety of teaching methods. The aim of this conceptual paper is to highlight the need to move from traditional methods to methods which can bring life to the classroom. One way to improve your own teaching is to try to apply schematic learning theory building on learners’ schema to teach mathematics better and then think about how well the activity improved students’ learning.

Key words: Schema ; Teaching mathematics.

Academic Discipline: Mathematics Education

Mathematics Subject Classification

Introduction

One of the most widely accepted ideas within the mathematics education community is the idea that students should understand mathematics. The goal in many research and implementation efforts in mathematics education has been to promote learning with understanding. But achieving this goal has been like searching for the Holy Grail (The Holy Grail is a dish, plate, stone, or cup that is part of an important theme of Arthurian literature. According to legend, it has special powers, and is designed to provide happiness, eternal youth and food in infinite abundance) Hiebert & Carpenter (1992).

Inadequacy of traditional methods

Most mathematics teaching from elementary school through college courses teaches what might be called rituals: “do this, then do this, then do this …..”. Teachers will typically accept the correctly –performed ritual as enough success for the time being. In other words what most students learn in their mathematics courses is to carry out a large number of standardized procedures, cast in precisely defined formalisms, for obtaining answers to clearly delimited classes of exercise questions (Dreyfus, 1991). They end up with a considerable amount of mathematical knowledge but without the working methodology of the mathematician, that is they lack the know-how that allows them to use their knowledge in a flexible manner to solve problems of a type unknown to them.

Mathematical thinking

Mathematicians clearly know that mathematics is not being created in final and polished form, but through trial and error, through partially correct (and partially wrong) statements, through intuitive formulations in which loose terms and imprecisions have intentionally been introduced, through drawings that try to visually present parts the mathematical structures being thought about, through dynamic changes being made to these drawings etc.
Why instructors stick to traditional methods?

The polished formalism manner as a manner of teaching has several advantages: for example, it guarantees that most of the material in the syllabus can be covered and the instructor is not deviated from his plan of action. Unfortunately, it also has at least one serious disadvantage: it is inflexible in terms of adaptability to the learners. At advanced mathematical thinking level, the method may work for students who major in mathematics or those students with a high mathematical aptitude.

Pass rates

Zimbabwe is viewed as a continental leader in many aspects including education, sports and culture. However, the recent results of The Rapid Assessment of Primary and Secondary Schools (Chakanyuka, S. et al. 2009) conducted by the National Education Advisory Board indicate that Zimbabwean students’ achievements in mathematics are far below world class standards. One of biggest challenges in mathematics education is closing the achievement gap. Although students may have learnt to find answers when solving problems, there is evidence from provincial, national and international assessments that many students do not understand the procedures they use and cannot monitor their problem solving. These factors can lead to difficulties in solving new problems. The question to be addressed in this paper reads: How can instructors of mathematics build on learners’ schemata to teach mathematics better?

Conceptual framework

The current research shall be guided by the following conceptual frameworks:

- the use of schematic theory to enhance learners’ conceptual understanding in mathematics (Skemp, 1962; Piaget, 1975)
- Reflective abstraction (Dubinsky, 1986)

Schematic learning theory

A schema (schemata; in plural) is a cognitive framework or concept that helps organize and interpret information (Cherry, 2015). It can be a diagrammatic representation or in a broad sense, a structural framework or plan or a mental codification of experience that includes a particular organized way of perceiving cognitively and responding to a complex situation or set of stimuli. One of the eminent principle of scientific learning is starting from the known to the unknown, from simple to complex and from concrete to abstract knowledge. A majority of our contemporary learning theories fail to take account of the way in which existing knowledge influences, and also makes possible subsequent learning. Mathematical knowledge is sequentially arranged in such a way it cannot be taught haphazardly. The knowledge of the operations of addition, subtraction, multiplication, and division with numbers makes possible, or influences the learning of solving equations. The knowledge of Calculus concepts influences the learning of Analysis concepts. This hierarchical arrangement appears in other subjects to varying degrees, but to a less extent.

Ignorance of the history of the Bantu migration matters little to a student learning the history of Cuban revolution.

Mathematics is probably the most interdependent and hierarchical of any structure of knowledge currently taught. This effect of previous learning on a particular task has been called by psychologists “transfer,” and has been the subject of a limited amount of rather scattered research. Piaget calls these structures of existing knowledge schemata. Closely related to this are two other important ideas, those of assimilation and accommodation. Assimilation means the fitting of new experiences into the existing schema; and by accommodation means the changes in a schema which enable it to take in new experiences. It is these two processes separately, alternately, or in combination, which preserve the continuity of the schema from its early simple beginning to the final complexity which it may reach. Skemp (1962) carried out an experiment to investigate the effectiveness of schematic learning. Two artificial schemata were devised and used for a controlled experiment to compare the results of schematic learning with learning in which an appropriate schema had not previously been learnt. Schematic learning was twice as effective for immediate recall, which a month later seven times as much was recalled of the material schematically learnt as of the other. These results make it strikingly clear that the presence or absence of a suitable schema has a very great effect on the success of learning and recall, and indicate that further research is urgently needed into the nature, varieties, and mode of formation of these schemata. For mathematics above all subjects a schematic learning theory is necessary.

Wiseman (2008) posits that schema theory lays out a picture of how people organize the truly enormous amount of background knowledge which they accumulate about the world. Such knowledge is organized into mental units, called schemas. When people learn, when they build knowledge, they are either creating new schemas, or linking together preexisting schemas in new ways. Schema theory states that, when students reconstruct information, they fit it into information that already exists in their minds. As they attempt to retrieve information from their long-term memory, the searches sometimes are not very exact and they do not always locate precisely what they want. When asked to retrieve information, students often fill in the gaps between fragmented memories, sometimes not well organized and/or with inaccurate information, with a variety of accuracies and inaccuracies. Finally, sound learning can go forward well or be misdirected based on what a student not organized or learned well previously. Teachers need to teach their students how to store information for more efficient and accurate retrieval; they must guide their students in the development of accurate schemas and teach them so that they logically build upon knowledge already gained.

The key elements of a Schema are:
1. An individual can memorize and use a schema without even realizing of doing so.
2. Once a schema is developed, it tends to be stable over a long period of time.
3. Human mind uses schemata to organize, retrieve, and encode chunks of important information.
4. Schemata are accumulated over time and through different experiences.

**Reflective Abstraction (APOS)**

- A theory of the development of concepts in Advanced Mathematical Thinking.
- Reflective Abstraction is the construction of mental objects and of mental actions on these objects.
- A schema is more or less coherent collection of objects and processes.
- New mathematical knowledge is constructed by the acts of recognizing and solving problems, asking new questions and creating new problems.

**Structure of Schema**

(Dubnisky, 1986)

![Structure of Schema](image)

**Schemas and their construction**

- **Objects:** mathematical objects e.g. numbers, variables, functions, topological spaces, groups etc.
- **Actions:** calculations, proving, representations etc.
- **Process:** interiorized action. Interiorization permits one to be conscious of an action, to reflect on it and to combine it with other actions.
- **Encapsulation:** In general, encapsulation is the inclusion of one thing within another thing so that the included thing is not apparent. Decapsulation is the removal or the making apparent a thing previously encapsulated (e.g. relationship between derivative and integration).

**Building on learner’s schema to teach mathematics better**

Schema theory emphasizes the importance of generic knowledge that will help the formation of mental representations. In the teaching process, the task of teachers would be to help students develop new schemata and establish connections between them. The teaching process should start with relevant background information and prior knowledge which is a vital component. For students to effectively process information, their existing schemata related to the new information need to be activated. This can be done by using visualizations, visual manipulative, outlines, resource material, find out what have done before they start the course/lesson, decide on the order in which you will teach the topics etc.

![Observation of learner’s schema](image)
Fostering conceptual thinking in mathematics (Dubnisky, 1986)

- Observe students in the process of learning a particular topic or set of topics to see their developing conceptual structures, that is, their concept images.
- Analyze the data and, using these observations, along with the theory of reflective abstraction and the instructor’s understanding of the mathematics involved, develop a genetic decomposition for each topic of concern that represents one possible way in which students might construct the concept.
- Design instruction that attempts to move the student along the cognitive steps in the genetic decomposition; develop activities and create situations that will induce students to make the specific reflective abstractions that are called for.
- Repeat the process, revising the genetic decompositions and the instructional treatment, and continue as long as possible or until stabilization occurs. [the process is like priming a diesel pump]

Figure 4: Enhancing teaching using learners’ schema

An important aspect on schema based teaching is that teachers should use familiar examples in their teaching instead of abstract information, provide students feedback in the form of numerous, fully worked out classical examples. Always the teacher should take into consideration the philosophical, psychological and sociological foundations aspect of the learner in order to build effective schemas in mathematics. Students should be taught about the different parts of mathematics and how they fit together. Mathematics can be taught using step-by-step approach to a topic but it is important to show that many topics are linked. It is also important to acknowledge both the language and culture of the learner.

Critique of schema learning theory

While the use of schemas to learn mathematics in most situations is spontaneous, sometimes an existing schema can hinder the learning of new information. The most serious problem occurs when the new ideas are not satisfactorily accommodated. Papert (1980) argue that “new knowledge often contradicts the old, and effective learning requires strategies to deal with such conflicts. Sometimes the conflicting pieces of knowledge can be reconciled, sometimes one or the other must be abandoned, and sometimes the two can both be kept around if safely maintained in separate compartments”. Cornu (1983) termed these pieces of knowledge as “obstacles”. An obstacle is a piece of knowledge; it is part of the knowledge of student. This knowledge was at one time generally satisfactory in solving certain problems. It is precisely this satisfactory aspect which has anchored the concept in the mind and made it an obstacle. This knowledge later proves to be inadequate when faced with new problems and this inadequacy may not be obvious.

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