Adaptive Sliding Mode Control Design for the Attitude of the Quadrotor Unmanned Aerial Vehicle (UAV)

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Abstract. In this article, the adaptive sliding mode controller (ASMC) is developed for the quadrotor attitude subsystem. The proposed ASMC controller aims to reduce/decrease the unwanted chattering phenomena associated with the conventional sliding mode controller, and meanwhile achieving a robust trajectory tracking for the attitude. The stability of the proposed (ASMC) controller has been verified based on Lyapunov stability theorem. The quadrotor UAV model and the performance of the proposed controller have been simulated and tested by simulation using MATLAB/SIMULINK environment. The simulation result is proof that the chattering has been reduced significantly.

1. Introduction
The quadrotor unmanned aerial vehicles (UAV) systems have been getting observable attention from researchers because of its wide applications in civilian and military sectors. The quadrotor classifies as a complex system, due to the high nonlinear dynamics, under-actuated and coupled dynamics, and these challenges must be considered in the controller design stage. There are numerous control techniques applied to the quadrotor, such as PID [1], [2], feedback linearization [3], [4], adaptive control [5], [6], and sliding mode controllers [7].

The SMC control is a nonlinear control technique which drives the system’s state trajectories to reach the sliding surface in a limited time and stay on it thereafter. The advantage of the SMC, the robustness against parameter variations and the finite-time to reach the sliding surface [8]–[10]. The main disadvantage of the SMC is the chattering phenomena. There are many proposed methods in the literature to overcome or reducing the chattering effects to some undisturbed limit or rang [11]–[14]. Chattering usually causes many problems such as vibration in the mechanical parts and heating in electronics kits which leads to power consumption. In addition, the discontinuous control signal in SMC may excite high-frequency dynamic of the system neglected in the course of modelling such as unmodeled structural modes, and time delays [15].

In this paper, the proposed ASMC controller has been developed based on Lyapunov theorem. There are two main objectives of the proposed ASMC controller. Firstly, achieving a robust trajectory tracking taking, and secondly reduce the impact of the chattering problem by designing an adaptive
gain of switching function, by which the proposed ASMC controller will be able to achieve a robust trajectory tracking and reducing the chattering impact.

2. Quadrotor Dynamics Model
The dynamic equations of quadrotor UAV in 6-degree of freedom (DOFs) are given as follows:

\[
\begin{align*}
\ddot{x} & = \cos \phi \sin \theta \cos \psi + \sin \phi sin \psi u_4 \\
\ddot{y} & = \cos \phi \sin \theta \cos \psi - \sin \phi \sin \psi u_4 \\
\ddot{z} & = -g + \cos \phi \cos \theta \frac{m}{I_x} u_4 \\
\ddot{\phi} & = \dot{\theta} \psi \frac{I_y - I_z}{I_x} + \dot{\theta} \Omega_d \frac{J_r}{I_x} + \frac{1}{I_x} u_1 \\
\ddot{\theta} & = \phi \psi \frac{I_z - I_x}{I_y} + \phi \Omega_d \frac{J_r}{I_y} + \frac{1}{I_y} u_2 \\
\ddot{\psi} & = \dot{\theta} \psi \frac{I_x - I_y}{I_z} + \frac{1}{I_z} u_3
\end{align*}
\]

where \(u_1, u_2, u_3, u_4\) are the control inputs, \(x, y, z\) denote the position of the quadrotor UAV, while \(\phi, \theta, \psi\) represent the quadrotor attitude, roll, pitch and yaw angles, respectively. \(J_r\) represents the rotor inertia, and \(I_x, I_y, I_z\) denote the inertia on \(x, y, z\) axis respectively. \(l\) is the arm length. \(\Omega_d\) represents the disturbances.

3. The Adaptive Sliding Mode Control (ASMC) Design
The scope of the ASMC controller design will cover the quadrotor’s attitude subsystem dynamics equations only, as follows:

\[
\begin{align*}
\ddot{\phi} & = a_1 \dot{\theta} \psi + a_2 \dot{\theta} \Omega_d + \frac{1}{I_x} u_1 + \mu_\phi \\
\ddot{\theta} & = a_3 \dot{\phi} \psi + a_4 \dot{\phi} \Omega_d + \frac{1}{I_y} u_2 + \mu_\theta \\
\ddot{\psi} & = a_5 \dot{\psi} \theta + \frac{1}{I_z} u_3 + \mu_\psi
\end{align*}
\]

where,

\[
\begin{align*}
a_1 & = \frac{I_y - I_z}{I_x}, a_2 = \frac{J_r}{I_x}, a_3 = \frac{I_z - I_x}{I_y}, a_4 = \frac{J_r}{I_y}, \text{ and } a_5 = \frac{I_x - I_y}{I_z}
\end{align*}
\]

And, \(\mu_\phi, \mu_\theta, \mu_\psi\) are the lumped uncertainties for \(\phi, \theta, \psi\) dynamics, respectively. Therefore, the control aim is to design ASMC controller to stabilize the attitude error dynamics. The desired attitude given by \((\phi_d, \theta_d, \psi_d)\), while the actual attitude is \((\phi, \theta, \psi)\). The error dynamics is given as follows:

\[
\begin{align*}
e_\phi & = \phi - \phi_d \\
e_\theta & = \theta - \theta_d \\
e_\psi & = \psi - \psi_d
\end{align*}
\]

The proposed ASMC controller has been designed based on the following steps:

**Step 1** is to define the tracking errors as in (2).

**Step 2** is to select the sliding surface as follows, [10]:

Thus, the sliding surfaces for attitude angles are

\[ s_\phi = \dot{\phi} + k_\phi e_\phi \\
= \dot{\theta} + k_\theta e_\theta \\
= \dot{\psi} + k_\psi e_\psi \]

where, \( s_\phi, s_\theta \) and \( s_\psi \) represent the sliding surfaces for roll, pitch and yaw, respectively. While \( k_\phi, k_\theta \) and \( k_\psi \) are positive constants.

**Step 3** is to apply the sliding mode condition as follows,

\[ \dot{s} = -k_1 \text{sgn}(s) - k_2 s \]  

Substituting (5) into (6), yields to,

\[ \dot{\phi} + k_\phi \dot{\phi} = -k_1 \phi \text{sgn}(s_\phi) - k_2 \phi s_\phi \]

\[ \dot{\theta} + k_\theta \dot{\theta} = -k_1 \theta \text{sgn}(s_\theta) - k_2 \theta s_\theta \]

\[ \dot{\psi} + k_\psi \dot{\psi} = -k_1 \psi \text{sgn}(s_\psi) - k_2 \psi s_\psi \]

Substitute (2) and (2) into (7), yields to,

\[ a_1 \dot{\psi} + a_2 \dot{\theta} \Omega_d + \frac{1}{l_x} u_1 + \mu_\phi - \dot{\phi}_d + k_\phi \dot{\phi} = -k_1 \phi \text{sgn}(s_\phi) - k_2 \phi s_\phi \]

\[ a_3 \phi \dot{\psi} + a_4 \phi \Omega_d + \frac{1}{l_y} u_2 + \mu_\theta - \dot{\theta}_d + k_\theta \dot{\theta} = -k_1 \theta \text{sgn}(s_\theta) - k_2 \theta s_\theta \]

\[ a_5 \phi \dot{\theta} + \frac{1}{I_z} u_3 + \mu_\psi - \dot{\psi}_d + k_\psi \dot{\psi} = -k_1 \psi \text{sgn}(s_\psi) - k_2 \psi s_\psi \]

where \( k_{1_\phi}, k_{1_\theta}, k_{1_\psi} > 0 \) and \( k_{2_\phi}, k_{2_\theta}, k_{2_\psi} > 0 \) are the SMC control gains.

**Step 4** is to cancel the nonlinear terms and uncertainty in the parameters in (8), the control input \( u_1, u_2, u_3 \) are selected as follows:

\[ u_1 = l_x (\dot{\phi} - a_1 \phi \dot{\psi} - a_2 \dot{\theta} \Omega_d - k_\phi \dot{\phi} + \mu_\phi + U_1) \\
u_2 = l_y (\dot{\theta} - a_3 \phi \dot{\psi} - a_4 \phi \Omega_d - k_\theta \dot{\theta} + \mu_\theta + U_2) \\
u_3 = l_z (\dot{\psi} - a_5 \phi \dot{\theta} - k_\psi \dot{\psi} + \mu_\psi + U_3) \]

Substitute (9) into (8) leads to

\[ \dot{s}_\phi = U_1 + \zeta_\phi \]

\[ \dot{s}_\theta = U_2 + \zeta_\theta \]

\[ \dot{s}_\psi = U_3 + \zeta_\psi \]

where,

\[ \zeta_\phi = l_x \mu_\phi \\
\zeta_\theta = l_y \mu_\theta \\
\zeta_\psi = l_z \mu_\psi \]
Step 5 is to obtain the estimated uncertainty $\zeta_\phi, \zeta_\theta, \zeta_\psi$ based on the following selected Lyapunov functions:

\[
\begin{align*}
V_\phi &= \frac{1}{2} s_\phi^2 + \frac{1}{2} \zeta_\phi y_\phi \zeta_\phi \\
V_\theta &= \frac{1}{2} s_\theta^2 + \frac{1}{2} \zeta_\theta y_\theta \zeta_\theta \\
V_\psi &= \frac{1}{2} s_\psi^2 + \frac{1}{2} \zeta_\psi y_\psi \zeta_\psi
\end{align*}
\]  

(12)

where, $\zeta_\phi, \zeta_\theta, \zeta_\psi$ are the error between the actual uncertainty and the estimated uncertainty $\tilde{\zeta}_\phi = \zeta_\phi - \hat{\zeta}_\phi, \tilde{\zeta}_\theta = \zeta_\theta - \hat{\zeta}_\theta, \text{ and } \tilde{\zeta}_\psi = \zeta_\psi - \hat{\zeta}_\psi$, while, $y_\phi, y_\theta, and y_\psi$ are positive constant. Thus, by differentiating the both sides of equation (12) yields to:

\[
\begin{align*}
\dot{V}_\phi &= s_\phi \dot{s}_\phi + \dot{\zeta}_\phi y_\phi \zeta_\phi \\
\dot{V}_\theta &= s_\theta \dot{s}_\theta + \dot{\zeta}_\theta y_\theta \zeta_\theta \\
\dot{V}_\psi &= s_\psi \dot{s}_\psi + \dot{\zeta}_\psi y_\psi \zeta_\psi
\end{align*}
\]  

(13)

According to (13) the adaption laws are set as follows

\[
\begin{align*}
\dot{\hat{\zeta}}_\phi &= \frac{1}{y_\phi} s_\phi \\
\dot{\hat{\zeta}}_\theta &= \frac{1}{y_\theta} s_\theta \\
\dot{\hat{\zeta}}_\psi &= \frac{1}{y_\psi} s_\psi
\end{align*}
\]  

(14)

Thus, from equation (10), the control inputs will be as follows

\[
\begin{align*}
U_1 &= -\hat{\zeta}_\phi - k_{1_\phi} sgn(s_\phi) - k_{2_\phi} s_\phi \\
U_2 &= -\hat{\zeta}_\theta - k_{1_\theta} sgn(s_\theta) - k_{2_\theta} s_\theta \\
U_3 &= -\hat{\zeta}_\psi - k_{1_\psi} sgn(s_\psi) - k_{2_\psi} s_\psi
\end{align*}
\]  

(15)

Step 6 is to reduce/eliminate the chattering effects associate with the switching function, the controller parameters $k_{1_\phi}, k_{1_\theta}, \text{ and } k_{1_\psi}$ which represent the uncertainty bounds can be estimated as follows [16]:

\[
\begin{align*}
\dot{k}_{1_\phi} &= \beta_\phi |s_\phi| \\
\dot{k}_{1_\theta} &= \beta_\theta |s_\theta| \\
\dot{k}_{1_\psi} &= \beta_\psi |s_\psi|
\end{align*}
\]  

(16)

where, $\beta_\phi, \beta_\theta, \text{ and } \beta_\psi$ are positive constants.

4. Simulation Results and Discussion

The Quadrotor mathematical model was simulated using MATLAB/SIMULINK environment, based on (1) and the model parameter are taken from [17], as listed in table 1. While the parameters of the conventional SMC and the proposed ASMC are presented in table 2 and table 3, respectively.
Table 1. Quadrotor’s model parameters.

| Name                  | Parameter | Value  | Unit   |
|-----------------------|-----------|--------|--------|
| The mass              | $m$       | 0.650  | kg     |
| Inertia on x axis     | $I_x$     | 7.5e-3 | kgm²   |
| Inertia on y axis     | $I_y$     | 7.5e-3 | kgm²   |
| Inertia on z axis     | $I_z$     | 1.3e-2 | kgm²   |
| The thrust coefficient| $b$       | 3.13e-5| Ns²    |
| The drag coefficient  | $d$       | 7.5e-7 | Nms²   |
| The rotor inertia     | $J_r$     | 6e-5   | kgm²   |
| The arm length        | $l$       | 0.23   | m      |

Table 2. SMC controller parameters.

| Parameter | $\phi$ | $\theta$ | $\psi$ |
|-----------|--------|----------|--------|
| $k$       | 3      | 3        | 3      |
| $k_1$     | 30     | 30       | 30     |
| $k_2$     | 15     | 15       | 15     |

Table 3. Adaptive SMC controller parameters.

| Parameter | $\phi$ | $\theta$ | $\psi$ |
|-----------|--------|----------|--------|
| $k$       | 3      | 3        | 3      |
| $k_2$     | 15     | 15       | 15     |
| $\gamma$  | 0.1    | 0.1      | 0.1    |
| $\alpha$  | 1      | 1        | 1      |

The simulation results proof that the proposed ASMC controller shows a robust and fast response in term of the trajectory tracking for the quadrotor attitude as shown in figure 1 and figure 2 compare to the conventional SMC and Quasi sliding mode control (QSMC) [10]. Figures 3, figure 4 and figure 5 show the chattering effects in the control signals for conventional SMC, QSMC and the proposed ASMC, respectively as can be seen the proposed ASMC showed a satisfactory performance in term of chattering reduction. The adaptive tuned gains for the proposed controller are presented in figure 6.

Figure 1. The quadrotor’s attitude.

Figure 2. The quadrotor attitude error.
5. Conclusion and Future work
The proposed adaptive SMC controller has been developed to control and stabilize the quadrotor’s attitude by taking into consideration both a robust trajectory tracking and reduce the chattering impact. The performance of the proposed ASMC controller has been compared with the conventional SMC and QSMC controllers, and the results showed a remarkable improvement, particularly in term of the chattering reduction. The future work has been planned to extend the work to include two major tasks, control the quadrotor UAV in 6 degree-of-freedom (6-DOF), and test the proposed ASMC against the parameter’s uncertainty.

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References
[1] Reizenstein A. Position and Trajectory Control of a Quadcopter Using PID and LQ Controllers. 2017; Available from: http://urn.kb.se/resolve?urn=urn%3Anbn%3Ase%3Aliu%3Adiva-139498
[2] Salih AL, Moghavvemi M, Mohamed HAF, Gaeid KS. Flight PID controller design for a UAV quadrotor. Sci Res Essays [Internet]. 2010;5(23):3660–7. Available from: http://www.academicjournals.org/SRE
[3] Lee D, Kim HJ, Sastry S. Feedback linearization vs. adaptive sliding mode control for a quadrotor helicopter. Int J Control Autom Syst. 2009;7(3):419–28.
[4] Mukherjee P, Waslander S. Direct Adaptive Feedback Linearization for Quadrotor Control. AIAA GuidNavig Control Conf 2012;2012;(August):1–10.

[5] Zuo Z. Adaptive trajectory tracking control design with command filtered compensation for a quadrotor. JVC/Journal Vib Control. 2013;19(1):94–108.

[6] Li S, Li B, Geng Q. Adaptive sliding mode control for quadrotor helicopters. In: Proceedings of the 33rd Chinese Control Conference, CCC 2014. IEEE; 2014. p. 71–6.

[7] Runcharoon K, Srichatrapimuk V. Sliding Mode Control of quadrotor. 2013 IntConf TechnolAdv Electr Electron ComputEng TAECE 2013.2013;(1):552–7.

[8] Mazinana h, Kazemimf, Shirzadh. An efficient robust adaptive sliding mode control approach with its application to secure communications in the presence of uncertainties, external disturbance and unknown parameters. Trans InstMeas Control. 2014;36(2):164–74.

[9] Zhu J, Khayati K. A new approach for adaptive sliding mode control: Integral/exponential gain law. Trans InstMeas Control. 2016;38(4):385–94.

[10] Vaidyanathan S, Lien CH. Applications of sliding mode control in science and engineering [Internet]. Vol. 709, Studies in Computational Intelligence. 2017. Available from: http://link.springer.com/10.1007/978-3-319-55598-0

[11] Suleiman HU, Murazu MB, Zarra TA, Salawudeen AT, Thomas S, Galadima AA. Methods of chattering reduction in sliding mode control: A case study of ball and plate system. IEEE IntConf Adapt SciTechnol ICAST. 2018;2018–Augus:1–8.

[12] Chang J-L. On Chattering-Free Dynamic Sliding Mode Controller Design. J Control SciEng [Internet]. 2012;2012:1–7. Available from: http://www.hindawi.com/journals/jcse/2012/564906/

[13] Morioka H, Wada K, Sabanovic A, Jezernik K. Neural network based chattering free sliding mode control. SICE ’95 Proc 34th SICE AnnuConfIntSess Pap. 1995;1303–8.

[14] Baek J, Jin M, Han S. A New Adaptive Sliding-Mode Control Scheme for Application to Robot Manipulators. IEEE Trans Ind Electron. 2016;63(6):3628–37.

[15] Rafimanzeli M, Yazdanpanah MJ. A Novel Approach for Chattering Minimization in Sliding Mode Control.

[16] Soltani J, Rezaei MM. Robust control of an islanded multi-bus microgrid based on input–output feedback linearisation and sliding mode control. IET GenerTransmDistrib. 2015;9(15):2447–54.

[17] Bouabdallah S. Design and control of quadrotors with application to autonomous flying. Thesis [Internet]. 2007;3727(3727):61. Available from: http://biblion.epfl.ch/theses/2007/3727/EPFL_TH3727.pdf

[18] Munir M, Singh S. A new approach for adaptive sliding mode control of a quadrotor using backstepping technique. IET Gener Transm Distrib. 2017;11(8):1609–17.