Curvature perturbation from symmetry breaking the end of inflation

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Abstract. We consider a two-field hybrid inflation model, in which the curvature perturbation is predominantly generated at the end of inflation. By finely tuning the coupling of the fields to the waterfall we find that we can get a measurable amount of non-Gaussianity.

Keywords: CMBR theory, inflation

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1. Introduction

We consider the scenario [1]–[3] in which a possibly dominant contribution to the curvature perturbation is generated at the end of inflation. As illustrated in figure 1, this contribution is generated if the end of inflation (taken to be abrupt) occurs on a spacetime slice which is not one of uniform energy density.

This mechanism can operate most cleanly in hybrid inflation, where slow roll is valid right up to the end of inflation so that slices of uniform density corresponds to slices of equal potential, or in other words of uniform inflaton field. Inflation in that case ends when the mass squared of the waterfall field falls through zero. The mechanism operates if the mass squared depends not only on the inflaton, but also on some other light field. The general formalism for this scenario was worked out in [2].

As it stands, this mechanism may be deemed unattractive because the introduction of the extra light field seems ad hoc, designed only to produce the desired effect. In this paper we propose that inflation actual takes place in the space of a two-component object $\phi, \sigma$. During inflation the potential depends only on $\phi^2 + \sigma^2$, corresponding to an SO(2) symmetry. But as illustrated in figure 2 the end of inflation occurs on an ellipse, which breaks the symmetry and generates a contribution to the curvature perturbation at the end of inflation. We choose inverted hybrid inflation [4] instead of the original model [5] so that we can reproduce the observed negative spectral tilt [6].

2. The model

The potential is

$$ V(\phi, \sigma, \psi) = V_0 - \frac{1}{2} m^2 \left( \phi^2 + \sigma^2 \right) - \left( f \phi^2 + g \phi \sigma + h \sigma^2 \right) \frac{\psi^2}{2} + \frac{m^2}{2} \psi^2. $$

The waterfall field $\psi$ is held at 0 during inflation and rolls rapidly to its true minimum when

$$ m^2_\psi = f \phi^2 + g \phi \sigma + h \sigma^2, $$

where $\phi_e(\sigma)$ is the value of $\phi$ at the end of inflation.
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Figure 1. Single-field inflation ends on a surface of constant energy density; we however are considering the case of multiple fields driving inflation, and thus inflation ends on a surface of non-constant energy density, as represented by the dotted line. It is clear that $t_1$ and $t_2$ produce different amounts of inflation; thus the family of trajectories is still curved at the end, and the curvature perturbation is increased by an amount $N_e = \delta N$.

During inflation, we take the trajectory to be along the $\sigma = 0$ path, and the potential behaves as a quadratic one $V = V_0 - m^2 \phi^2/2$. We allow the elliptic surface of the end of inflation to have any orientation (see figure 2), which is equivalent to fixing the ellipse and varying the trajectory. This is accounted for in the cross-coupling term in the potential ($-g \phi \sigma \psi^2/2$). For such a set-up, either all slow roll conditions are satisfied until $\psi$ is destabilized, or the violation of one ‘pauses’ inflation until the amplitude of the perturbations decays and inflation resumes. In this paper we consider the former. To analyse this case, we will use the equations derived in [2].

3. The equations

The curvature perturbation is $\zeta = \zeta_{\text{inf}} + \zeta_{\sigma}$. The first term is the constant contribution of $\delta \phi$, and the second term is the constant contribution of $\delta \sigma$ which is generated only at the end of inflation. From [2, 7],

$$
\zeta_{\sigma} = N'_c \phi_c' \delta \sigma + \frac{1}{2} \left[ 2N''_c (\phi_c')^2 + N'_c \phi_c'' \right] (\delta \sigma)^2, \quad (3.1)
$$
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**Figure 2.** Representation of the set-up, where the surface at the end of inflation is an ellipse, therefore varying the trajectory (or the orientation of the ellipse) varies the amount of inflation. The dotted circles correspond to scenarios in which all trajectories are equal, as in single-field inflation.

where

\[
N_e' = \frac{dN_e}{d\phi_e} = \frac{1}{\sqrt{2\epsilon_e}},
\]

(3.2)

\[
N_e'' = \frac{d^2N_e}{d\phi_e^2} = \frac{2\epsilon_e - \eta}{2\epsilon_e}.
\]

(3.3)

The slow roll parameters are

\[
\eta = -\frac{m^2}{V_0},
\]

(3.4)

\[
\epsilon_e = \frac{m^4}{2V_0^2} \phi_e^2 = \eta^2 \frac{m^2}{2f}
\]

(3.5)

and the spectral index for this model is

\[
n - 1 = 2\eta.
\]

(3.6)

From (2.2), for \(\sigma = 0\) we have \(\phi_e^2 = m_\psi^2/f\). Thus:

\[
\phi_e' = \frac{g\phi_e + 2h\sigma}{2f\phi_e + g\sigma} = -\frac{g}{2f}
\]

(3.7)
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\[
\phi''_e = -\frac{2h}{2f\phi_e + g\sigma} - \frac{2f}{2f\phi_e + g\sigma} \left( \frac{g\phi_e + h\phi}{2f\phi_e + g\sigma} \right)
= \frac{1}{m_\psi \sqrt{f}} \left( h - \frac{g}{2} \right). 
\tag{3.8}
\]

Substituting equations (3.2)–(3.8) in equation (3.1) we get

\[
\zeta_\sigma = -\frac{g}{2f\sqrt{2\epsilon_e}} \delta\sigma + \frac{1}{2} \left[ \left( 2 - \frac{\eta}{\epsilon_e} \right) \frac{g^2}{4f^2} + \left( h - \frac{g}{2} \right) \frac{(h - (g/2))}{m_\psi \sqrt{2f\epsilon_e}} \right] (\delta\sigma)^2.
\tag{3.9}
\]

4. Parameter constraints I

We analyse first the case \( \mathcal{P}_\zeta \gg \mathcal{P}_{\zeta_{\text{inf}}} \), where \( \zeta_{\text{inf}} \) is the inflaton contribution to \( \zeta \) and \( \mathcal{P} \) are the spectra. Then \( \mathcal{P}_\zeta \simeq \mathcal{P}_\zeta \). To agree with the observation \( \zeta_\sigma \) must be almost Gaussian, so the first term of equation (3.9) must dominate giving

\[
\mathcal{P}_\zeta = \frac{g^2}{4f^2} \left( \frac{1}{2\epsilon_e} \right) \left( \frac{H_k}{2\pi} \right)^2.
\tag{4.1}
\]

As \( \mathcal{P}_{\zeta_{\text{inf}}} = (1/2\epsilon_e)(H_k/2\pi)^2 \), thus \( \mathcal{P}_\zeta = (g^2/4f^2)(\epsilon/\epsilon_e)\mathcal{P}_{\zeta_{\text{inf}}} \). From slow roll we have \( 3H\dot{\phi} = -(dV/d\phi) \), and \( N = H\Delta t \), thus \( \phi_e/\phi = \exp(\eta N) \sim 1 \). Since \( \epsilon/\epsilon_e = (\phi_k/\phi_e)^2 \), \( \mathcal{P}_\zeta \) simplifies to

\[
\mathcal{P}_\zeta = \frac{g^2}{4f^2} \mathcal{P}_{\zeta_{\text{inf}}}.
\tag{4.2}
\]

Since the slow roll parameters in this model satisfy \( \epsilon \ll \eta \), the spectral index is \( n = 1 + 2\eta \) whether or not \( \zeta_\sigma \) dominates. We can choose \( \eta \) so that \( n = 0.95 \) in agreement with observation.

Non-Gaussianity is specified by parameters \( f_{NL} \) and \( \tau_{NL} \), defining respectively the bispectrum and trispectrum of \( \zeta \). We write

\[
\zeta_\sigma = \zeta_k + x\zeta_k^2,
\tag{4.3}
\]

where \( \zeta_k = N_e\phi_e\delta\sigma \) is practically Gaussian. Then

\[
(3/5)f_{NL} = x = \frac{2\eta f}{g^2} \left( h - \frac{g}{2} \right)
\tag{4.4}
\]

and \( \tau_{NL} = 4x^2 \). For \( f_{NL} \) to be observable we need \( h/g \gg 1 \), which means that \( f_{NL} \) is positive while \( \tau_{NL} \) is bounded to be \( \lesssim 10^4 \) [8]. Its observational bound is \( -54 < f_{NL} < 114 \) [6].

In figure 3 we plot \( f_{NL} \) versus \( h/g \), taking \( g/f = 10 \) and \( 2\eta = 0.05 \). The interesting result is that the multi-field hybrid model can generate a significant amount of non-Gaussianity through the termination of inflation. Note that for \( h > 500g \) we get \( f_{NL} \gtrsim 1 \), which is the condition for eventual detectability.
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Figure 3. Plot of $\frac{3}{5}f_{NL}$ versus $h/g$ for three values of $g/f$. It is clear from the graph that we have a measurable (significant) non-Gaussianity for $h \gg g$. The shaded region corresponds to the region which can be measured, and which has not yet been ruled out.

Now we come to the case $P_{\zeta_\sigma} \ll P_{\zeta_{\text{inf}}}$. Then
\[ \zeta \approx \zeta_{\text{inf}} + \delta \sigma^2, \] (4.6)
with
\[ \delta \sigma^2 \approx \frac{h - g/2}{m_\psi \sqrt{2f}} \delta \sigma^2. \] (4.7)

The non-Gaussian fraction is
\[ r_{ng} \equiv \left( \frac{P_{\delta \sigma}}{P_{\zeta_{\text{inf}}}} \right)^{1/2} \approx \left( \frac{h - g/2}{m_\psi \sqrt{f}} \right)^{1/2}, \] (4.8)
which is related to the non-Gaussianity parameters by
\[ r_{ng} \sim \left( |f_{NL}| P_{\zeta}^{1/2} \right)^{1/3} \approx (\tau_{NL} P_{\zeta})^{1/4}. \] (4.9)

The observational bound on $f_{NL}$ requires $r_{ng} < 0.2$ while the bound on $\tau_{NL}$ requires $r_{ng} < 0.07$ [9].

5. Parameter constraints II

Now we fix the orientation of the ellipse, and allow a variable unperturbed trajectory making an angle $\theta$ with the $x$-axis. If $\theta$ is regarded as a parameter of the theory this is equivalent to the previous formulation. But if inflation has lasted for a long time before the observable Universe leaves the horizon, the observable Universe will be surrounded by a huge inflated patch, and for a random location of the observable Universe $\theta$ will have a flat probability distribution.
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![Graphs showing $\frac{3}{5}f_{NL}$ versus $\theta$ for various degrees of symmetry breaking the end of inflation.](image)

**Figure 4.** $\frac{3}{5}f_{NL}$ versus the angle $\theta$ for various degrees of symmetry breaking the end of inflation.

We convert to a coordinate system in which the ellipse is aligned with the axes, i.e. where the ellipse equation is given by

$$f\tilde{\phi}^2 + h\tilde{\sigma}^2 = 1 \quad (5.1)$$

and $f, g, h$ are related to $\tilde{f}, \tilde{h}$ by

$$f = \tilde{f}\cos^2\theta + \tilde{h}\sin^2\theta,$$
$$g = 2\sin\theta\cos\theta(\tilde{h} - \tilde{f}),$$
$$h = \tilde{f}\sin^2\theta + \tilde{h}\cos^2\theta. \quad (5.2)$$

In this new coordinate system the spectrum equation (4.2) is given by

$$P_{\zeta_{\sigma}} = \left(\frac{X}{\cot\theta +(X+1)\tan\theta}\right)^2 P_{\zeta_{\text{inf}}}, \quad (5.3)$$

where $\theta$ is the angle between the new and old coordinate systems, and $X = (h/\tilde{f}) - 1$ is the symmetry breaking term. Since we require $P_{\zeta_{\sigma}} > P_{\zeta_{\text{inf}}}$ then $X > (\tan\theta + \cot\theta)/(1 - \tan\theta)$ which translates to a constraint on the range of $\theta$:

$$\frac{X - \sqrt{X^2 - 4X - 4}}{2(X+1)} < \tan\theta < \frac{X + \sqrt{X^2 - 4X - 4}}{2(X+1)}. \quad (5.4)$$
The non-Gaussianity from end of inflation (4.5) becomes
\[
\frac{3}{5} f_{NL} = \frac{2\eta}{X^2 \sin^2(2\theta)} \left\{ -\frac{X}{4} \sin(2\theta)(X + 2) + X + 1 + \frac{X^2}{4} \sin(2\theta)(\cos(2\theta) - \sin(2\theta)) \right\},
\]
which we plot in figure 4 for various degrees of symmetry breaking.

6. Discussion

Judging from figure 4, we note that for a measurable \( f_{NL} \) in the case where \( P_{\zeta_{0}} > P_{\zeta_{inf}} \), we require a high degree of symmetry breaking the end of inflation \( X \gg 1 \) and a small angle of orientation \( \theta \ll \pi/4 \). If inflation lasted for a long time before the observable Universe left the horizon, such a small angle requires the observable Universe to be in a special location. Anthropic selection cannot be invoked to favour that location, because it applies only to the normalization \( P_{\zeta} \) of the spectrum [9]. We conclude that within this model, an observable value of \( f_{NL} \) is allowed but not favoured.

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