Higher Order Perturbation Corrections of Rotating Excited States in the Standard SU(3) Skyrme Model to Baryon Mass Spectrum

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Abstract

The higher order corrections of SU(3) rotating excited states to the Gell-Mann-Okubo Relations (or GOR) are presented in the standard SU(3) Skyrme model. The Improved GOR (or IGOR) are obtained. The results show the IGOR for decuplet up to the third order and for octet up to the second order are much compatible with the experimental data. But things becomes quite inadequate for the octet to the third order. In order to overcome the inadequacy, a heuristic discussion is presented. The properties of SU(3) rotating excited states 27-let(with spin $\frac{1}{2}$ or $\frac{3}{2}$),10*-let (with spin $\frac{1}{2}$) and 35-let(with spin $\frac{3}{2}$) are also discussed.

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1 Introduction

With the extensive and intensive study of long-range QCD, Skyrme model\cite{1-10} or the chiral soliton model is believed to be a promising theory to depict the behavior of long-range strong interaction. In this theory, the baryon-multiplets emerge as topological solitons (i.e., skyrmions) in the SU(3)⊗SU(3) current algebra chiral Lagrangians. These solitons can be excited through the quantization of rotating fluctuations in terms of collective coordinates. It is believed that this chiral soliton model provides a reasonable dynamical mechanism for the mass-splitting of SU(3)-baryons. Therefore it has been applied extensively to the study of the mass relations for baryons\cite{3-10}.

Among these relations, the most famous ones are Gell-Mann-Okubo Relations (GOR in short)\cite{11}, which were originally formulated in terms of a perturbative treatment of flavor-breaking in SU(3) group theory. However, there are deviations between GOR and experimental data, which can be shown as follows,

\begin{align}
M_{\Sigma^*} - M_\Delta &= M_{\Xi^*} - M_{\Sigma^*} + \delta m_{10}^{(1)} = M_\Omega - M_{\Xi^*} + \delta m_{10}^{(2)}. \tag{1} \\
2(M_N + M_\Xi) &= 3M_\Lambda + M_\Sigma + \delta m_8, \tag{2}
\end{align}

where $\delta m_{10}^{(1)}, \delta m_{10}^{(2)}, \delta m_8$ stand for the deviations. As $\delta m_{10}^{(1)} = \delta m_{10}^{(2)} = \delta m_8 = 0$, eqs.(1) and (2) go back to the standard GOR. However, in the real world, $\delta m_{10}^{(1)}, \delta m_{10}^{(2)}, \delta m_8$ are not zero\cite{12},

\begin{align}
\delta m_{10}^{(1)} &= 3.8 MeV, \tag{3} \\
\delta m_{10}^{(2)} &= 13.6 MeV, \tag{4} \\
\delta m_8 &= -26.1 MeV. \tag{5}
\end{align}

In order to re-establish GOR and especially reveal the deviations, the deliberate consideration of the flavor breaking term is needed. Generally, three methods are often used to deal with the mass-splitting in the SU(3) Skyrme model, which are perturbative\cite{3-8}, non-perturbative\cite{9} and bound-state approach\cite{10}, respectively. In ref.\cite{5} the standard GOR has been actually re-established through one order perturbative calculations in the SU(3) Skyrme model, which could be regarded as a refurbished version of the original perturbative treatment in SU(3) group theory. Therefore it is reasonable to expect that higher order perturbative corrections in SU(3) skyrmion quantum mechanics could further improve the standard GOR. In ref.\cite{6}, the perturbative calculations have been carried out for octet-baryons up to the third order. The similar calculations for decuplet,
however, are left due to the lack of the corresponding Clebsh-Gordon Coefficients (CGC’s in short) for SU(3) group in the literatures\textsuperscript{[13, 14]}. A systematical study of GOR and its improvement through the skyrmion quantum mechanics remain also to be open. In the present paper, the Gel’fand bases are used to calculate the CGC’s for SU(3) directly, and the Improved GOR (IGOR in short) are derived for both decuplet and octet up to the second order, then the deviations of of $\delta m_{10}^{(1)}, \delta m_{10}^{(2)}$, and $\delta m_8$ are revealed which are quite compatible with experimental data. Up to the third order, $\delta m_{10}^{(1)}, \delta m_{10}^{(2)}$ are found to be more positive, but unfortunately $\delta m_8$ to be much inadequate. Therefore we expect that further study on this problem could remove the inadequacy.

This paper is organized as follows. In Sec.2, the formalism of the calculations is presented. In Sec.3, the calculations of SU(3) Clebsh-Gordon Coefficients are given. To the best of our knowledge, we find that there are not the CGC’s of $8 \otimes 35 \rightarrow 35$ in the literatures\textsuperscript{[13–14]} so far. These CGC’s are listed in the Appendix. The method employed to compute SU(3) CGC’s in this paper is practical and convenient, and is also useful for other purpose. In Sec.4, the calculations for higher order perturbation corrections to GOR of decuplet- and octet-baryons are presented, then the IGOR’s are obtained. In Sec.5, the comments and conclusions are given. A heuristic prescription is proposed which is expected to remove the inadequacy, then the role of the rotating excited states (or exotic SU(3) states) is discussed.

2 Formalism of Skyrmion Quantum Mechanics

In the present paper, the notation of ref.[7] will be adopted. The Hamiltonian of the standard SU(3) skyrmion quantum mechanics in the collective coordinate space is\textsuperscript{[7]}

$$H = H_0 + H'$$

$$H_0 = M_s + \frac{1}{2b^2} \sum_{i=1}^{8} (L_i L_i - R_i^2) + \frac{1}{2} \left( \frac{1}{a^2} - \frac{1}{b^2} \right) \sum_{A=1}^{3} R_A R_A + \frac{2\delta}{\sqrt{3}} F_\pi R_8$$

$$H' = m(1 - D_{ss}^{(ad)}(A))$$

where $M_s$ is the mass of classical soliton, $a^2$ and $b^2$ are the soliton’s “moments of inertia”, $m, F_\pi$ and $\delta$ are constants and parameters in the model, $D_{\mu\nu}^{(ad)}$ denotes the regular adjoint representation functions of SU(3), and $[L_i, L_j] = i f_{ijk} L_k, [R_i, R_j] = -i f_{ijk} R_k, [L_i, R_j] = 0$. In eq.(5), $H_0$ serves as the unperturbed Hamiltonian, and $H'$ as the perturbative part. It is easy to see that $H_0$ is diagonal.
and the eigen-wavefunctions for $H_0$ are [7]

$$|\lambda_{\mu\nu}\rangle = (-1)^{s_1+s_2} \sqrt{\lambda} D^{(\lambda)}_{\mu\nu}(A)$$

(9)

where $\lambda$ represents the Irreducible Representation (IR in short) of SU(3), and $\mu = \begin{pmatrix} I \cr Y \cr I_z \end{pmatrix}$, $\nu = \begin{pmatrix} S \cr I \cr S_z \end{pmatrix}$ with $I, Y, S$ denote isospin, hypercharge and spin respectively. The mass of the baryon for $|k\rangle \equiv |\lambda_{\mu\nu}\rangle$ can be calculated in perturbation,

$$M_k = E_k^{(0)} + E_k^{(1)} + E_k^{(2)} + E_k^{(3)} + \cdots$$

(10)

where

$$E_k^{(0)} = \langle k|H_0|k\rangle,$$

(11)

$$E_k^{(1)} = \langle k|H'|k\rangle,$$

(12)

$$E_k^{(2)} = \sum_{n\neq k} \frac{\langle n|H'|k\rangle^2}{E_k^{(0)} - E_n^{(0)}},$$

(13)

$$E_k^{(3)} = \sum_{m\neq k} \sum_{n\neq k} \frac{\langle k|H'|m\rangle \langle m|H'|n\rangle \langle n|H'|k\rangle}{(E_k^{(0)} - E_m^{(0)})(E_k^{(0)} - E_n^{(0)})} - \frac{\langle k|H'|m\rangle \langle m|H'|n\rangle \langle n|H'|k\rangle}{(E_k^{(0)} - E_m^{(0)})^2}.$$
3 The calculations of SU(3) Clebsh-Gordon Coefficients

In order to calculate the SU(3) CGC’s, the choice of basis for representations is of great importance. In the present section, we will employ Gel’fand basis\cite{15,16} and confine ourselves to discussing the calculations of only SU(3) CGC’s.

In terms of the Gel’fand symbol, any basis vector of an IR of SU(3) can be labelled as

\[ \left( \begin{array}{c} \lambda \\ \mu \end{array} \right) \equiv \left( \begin{array}{ccc} m_{13} & m_{23} & m_{33} \\ m_{12} & m_{22} & m_{11} \end{array} \right) \] \tag{17}

where the parameters \( m_{i,k} \)'s stand for the partitions of the IR’s of SU(N) \( (N \leq 3) \), e.g., \((m_{13}, m_{23}, m_{33})\) correspond to the IR’s of SU(3).

The matrix element of SU(3) generator \( E_{32} \) between the above basis is\cite{15,16}

\[ E_{32} \left( \begin{array}{ccc} m_{13} & m_{23} & m_{33} \\ m_{12} & m_{22} & m_{11} \end{array} \right) = A \left( \begin{array}{ccc} m_{13} & m_{23} & m_{33} \\ m_{12} - 1 & m_{22} & m_{11} \end{array} \right) + B \left( \begin{array}{ccc} m_{13} & m_{23} & m_{33} \\ m_{12} & m_{22} - 1 & m_{11} \end{array} \right) \] \tag{18}

where

\[ A = \left( \frac{m_{11} - m_{12}}{m_{22} - m_{12} - 1} \right)^{\frac{1}{2}} \left( \frac{m_{13} - m_{12} + 1}{m_{22} - m_{12}} \right)^{\frac{1}{2}} \tag{19} \]

\[ B = \left( \frac{m_{11} - m_{22}}{m_{12} - m_{22} + 1} \right)^{\frac{1}{2}} \left( \frac{m_{13} - m_{22} + 2}{m_{12} - m_{22} + 2} \right)^{\frac{1}{2}} \tag{20} \]

In the fundamental representation, the commutation relations of SU(3) generators \( E_{ij} \) are

\[ [E_{ij}, E_{kl}] = \delta_{jk} E_{il} - \delta_{il} E_{kj} \] \tag{21}

where \( (E_{ij})_{\alpha\beta} = \delta_{\alpha i} \delta_{\beta j} - \frac{1}{3} \delta_{ij} \delta_{\alpha\beta} \). Obviously, the matrix element of \( E_{32} \) is always non-negative, because the generalized Condon-Shortley phase convention is followed, i.e., the matrix elements of \( E_{n,n} \) of SU(N) are defined to be non-negative.

The SU(3) CGC’s can be constructed with SU(3) isoscalar factors and SU(2) CGC’s \cite{13}, i.e.,

\[ C_{CG}(SU(3)) = \left( \begin{array}{cc} \lambda_1 & \lambda_2 \\ I_1 Y_1 & I_2 Y_2 \end{array} \right| \lambda \right) \left( \begin{array}{cc} \lambda & \lambda \\ I_1 Y_1 & I_2 Y_2 \end{array} \right) \] \tag{22}

where \( I, Y, \lambda \) represent isoscalar,hypercharge and dimension of the IR of SU(3), respectively.

Then the calculations of SU(3) CGC’s can be ascribed to the calculations of SU(3) isoscalar factors

\[ \left( \begin{array}{cc} \lambda_1 & \lambda_2 \\ I_1 Y_1 & I_2 Y_2 \end{array} \right| \lambda \right). \] Making use of the above formulas, the isoscalar factors of \( 8 \otimes 35 \) can
be easily obtained, which are listed in the Appendix. All of the isoscalar factors have been checked carefully so that the unitary and orthogonal conditions are preserved. The phase conventions are also checked on which the comments are given in Appendix.

4 Higher order corrections to decuplet and octet

4.1 The baryon-decuplet sector

Considering the following decomposed formula of direct product of the SU(3) IR’s

$$8 \otimes 10 = 35 \oplus 27 \oplus 10 \oplus 8$$

we could see that the rotating excited states that acted as the intermediate states $|n\rangle$ and $|m\rangle$ in eqs.(13) and (14) can only take the following set

$$\{ |m\rangle, |n\rangle \} \in \{ 35, 27, 8 \}.$$  (24)

Making use of eq.(16) and eq.(22), we could obtain all the non-zero matrix elements for decuplet (see Table 1.)

On the other hand, noticing $C_2(8) = 3, C_2(10) = 6, C_2(27) = 8$ and $C_2(35) = 12$, we have

$$E_{10}^{(0)} - E_{8}^{(0)} = \frac{3}{2b^2}$$  (25)
$$E_{10}^{(0)} - E_{27}^{(0)} = -\frac{1}{b^2}$$  (26)
$$E_{10}^{(0)} - E_{35}^{(0)} = -\frac{3}{b^2}$$  (27)

Then the complete results of the higher order corrections to the baryons in decuplet can be obtained from eqs.(11)-(14) as follows

$$M_\Delta = M_{10} - \frac{1}{8}m - \frac{85}{672}m^2b^2 - \frac{340}{129024}m^3b^4$$  (28)
$$M_{\Sigma^*} = M_{10} - \frac{26}{336}m^2b^2 - \frac{26}{8064}m^3b^4$$  (29)
$$M_{\Xi^*} = M_{10} + \frac{1}{8}m - \frac{9}{224}m^2b^2 - \frac{35}{14336}m^3b^4$$  (30)
$$M_\Omega = M_{10} + \frac{1}{4}m - \frac{5}{336}m^2b^2 - \frac{5}{4032}m^3b^4$$  (31)

where $M_{10} = \langle H_0 \rangle_{\lambda=10} + m$. It is easy to see that there are four non-linear equations and three unknown parameters. Therefore the mass relation of the baryons in decuplet can be obtained by solving the over-determinant equations.
Table 1. Values of non-zero matrix elements of $H'_{m,n}$ for decuplet

|     | $H'_{10,10}/m$ | $H'_{27,27}/m$ | $H'_{35,35}/m$ | $H'_{10,35}/m$ | $H'_{10,27}/m$ | $H'_{27,35}/m$ |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|
| $\Delta$ | $\frac{7}{8}$ | $\frac{99}{112}$ | $\frac{13}{16}$ | $\frac{5}{18} \sqrt{7}$ | $\frac{\sqrt{30}}{16}$ | $\frac{5}{18} \sqrt{35}$ |
| $\Sigma^*$ | 1 | $\frac{57}{99}$ | $\frac{7}{8}$ | $\frac{1}{3} \sqrt{7}$ | $\frac{1}{3}$ | $\frac{5}{8} \sqrt{35}$ |
| $\Xi^*$ | 2 | $\frac{129}{112}$ | $\frac{15}{16}$ | $\frac{3}{5} \sqrt{7}$ | $\frac{\sqrt{6}}{16}$ | $\frac{5}{18} \sqrt{35}$ |
| $\Omega$ | $\frac{5}{4}$ | 0 | 1 | $\frac{1}{3} \sqrt{7}$ | 0 | 0 |

It is straightforward that the standard GOR for decuplet can be recovered at the first order, i.e.,

$$M_{\Sigma^*} - M_\Delta = M_{\Xi^*} - M_{\Sigma^*} = M_\Omega - M_{\Xi^*}$$  \hspace{1cm} (32)

At the second order, the following constrained mass relation can be obtained as the above overdeterminant equations are solved,

$$M_\Omega - M_\Delta = 3(M_{\Xi^*} - M_{\Sigma^*})$$  \hspace{1cm} (33)

eq(33) is the IGOR for decuplet, and can be also called Okubo relation which has been shown by Okubo long ago \cite{17} and by Morpurgo recently\cite{18}. Here such conclusion is again obtained in the skyrmion formalism. Apparently, eqs.(28)-(31) indicate that the equal spacing rule for decuplet (i.e.,GOR in decuplet section) no longer holds. Then a correction to the standard GOR will exist. That is, the following corrections to $\delta m_{10}^{(1)}$ and $\delta m_{10}^{(2)}$ (see eq.(1)) can be obtained from eq.(33) by comparing with eq.(1)

$$\begin{align*}
(\delta m_{10}^{(1)})_{\text{second}} &= 2M_{\Xi^*} - M_{\Xi^*} - M_\Delta. \\
(\delta m_{10}^{(2)})_{\text{second}} &= 2(2M_{\Xi^*} - M_{\Xi^*} - M_\Delta).
\end{align*}$$  \hspace{1cm} (34) (35)

Note that $(\delta m_{10}^{(2)})_{\text{second}} = 2(\delta m_{10}^{(1)})_{\text{second}}$. In practice, this is a general fact at the second order. Meanwhile, the mass-variables in eq.(34)(35) could be changed, then the other three expressions for $(\delta m_{10}^{(1)})_{\text{second}}, (\delta m_{10}^{(2)})_{\text{second}}$ read

$$\begin{align*}
(\delta m_{10}^{(1)})_{\text{second}} &= \frac{1}{3}(3M_{\Xi^*} - 2M_\Omega - M_\Delta),
\end{align*}$$  \hspace{1cm} (36)
\begin{align}
\frac{1}{3}(3M_{\Sigma^*} - 2M_{\Delta} - M_{\Omega}) &= \quad (37) \\
2M_{\Xi^*} - M_{\Sigma^*} - M_{\Omega} &= \quad (38) \\
(\delta m_{10}^{(2)})_{\text{second}} &= 2(\delta m_{10}^{(1)})_{\text{second}} \quad (39)
\end{align}

in eq. (39), we could see \((\delta m_{10}^{(2)})_{\text{second}}\) takes the double value of \((\delta m_{10}^{(1)})_{\text{second}}\) for each case.

The above four \((\delta m_{10}^{(1)})_{\text{second}}\)’s are equivalent in principle. However, since eq. (33) is just an approximate relation, the numerical values of the four \((\delta m_{10}^{(1)})_{\text{second}}\) are not exactly equal, a small error will be given. Therefore we could have

\begin{align}
(\delta m_{10}^{(1)})_{\text{second}} &= 6.8 \pm 2.6\,\text{MeV} \quad (40) \\
(\delta m_{10}^{(2)})_{\text{second}} &= 2(\delta m_{10}^{(1)})_{\text{second}} = 13.6 \pm 5.2\,\text{MeV}, \quad (41)
\end{align}

which agree well with the experimental data in eqs. (3) and (4).

As for the third order corrections, a constrained mass relation similar to eq. (33) can be obtained, which is rather complicated because eqs. (28)-(31) are non-linear. Therefore, the numerical analysis and comparison turned out to be necessary. For the sake of convenience and straightforwardness, the numerical values of the baryon masses in decuplet obtained from the constrained mass relation up to the third order are calculated, which are listed in table 2 in comparison with those of the first and second order. We only list the predictions for \(M_{\Omega}\), while \(M_{\Delta}, M_{\Sigma^*}\) and \(M_{\Xi^*}\) are input. It is easy to see that the theoretical predictions of baryon masses agree well with the experiment quantitatively.

Now we study the corrections to \((\delta m_{10}^{(1)})_{\text{third}}\) and \((\delta m_{10}^{(2)})_{\text{third}}\) in the third order. Then we could have the following equations

\begin{align}
M_{\Sigma^*} - M_{\Delta} &= M_{\Xi^*} - M_{\Sigma^*} + (\delta m_{10}^{(1)})_{\text{second}} + (\delta m_{10}^{(1)})_{\text{third}} \quad (42) \\
M_{\Sigma^*} - M_{\Delta} &= M_{\Omega} - M_{\Xi^*} + (\delta m_{10}^{(2)})_{\text{second}} + (\delta m_{10}^{(2)})_{\text{third}} \quad (43)
\end{align}

where \((\delta m_{10}^{(1)})_{\text{second}}\) and \((\delta m_{10}^{(2)})_{\text{second}}\) have been obtained in eqs. (34)-(39) and eqs. (40)-(41).

Through numerical calculations, \((\delta m_{10}^{(1)})_{\text{third}}\) and \((\delta m_{10}^{(2)})_{\text{third}}\) are found to be

\begin{align}
(\delta m_{10}^{(1)})_{\text{third}} &= -0.5 \pm 0.5\,\text{MeV}. \quad (44) \\
(\delta m_{10}^{(2)})_{\text{third}} &= -0.3 \pm 0.7\,\text{MeV}. \quad (45)
\end{align}
then we get the sums of the corrections of the second and the third order as follows,

\[(\delta m^{(1)}_{10})_{sum} = 6.3 \pm 3.1 \text{MeV}. \tag{46}\]

\[(\delta m^{(2)}_{10})_{sum} = 13.3 \pm 5.9 \text{MeV}. \tag{47}\]

It is easy to see that, the corrections of the third order are far less than those of the second order. But the corrections of the third order improve the results of \(\delta m^{(1)}_{10}\) and \(\delta m^{(2)}_{10}\). Therefore the results up to the third order are still encouraging.

**Table 2. Comparison between higher corrections and the experimental data for decuplet**

| baryons     | \(M_{\Delta (\text{input})}\) | \(M_{\Sigma^* (\text{input})}\) | \(M_{\Xi^* (\text{input})}\) | \(M_{\Omega}\) |
|-------------|-------------------------------|-------------------------------|-------------------------------|-------------|
| 1-st Order  | 1232.0                        | 1384.6                        | 1533.4                        | 1684.1      |
| 2-nd Order  | 1232.0                        | 1384.6                        | 1533.4                        | 1678.2      |
| 3-rd Order  | 1232.0                        | 1384.6                        | 1533.4                        | 1678.1      |
| Exper. data | 1232.0                        | 1384.6                        | 1533.4                        | 1672.4      |

**4.2 The baryon-octet sector**

Note that

\[8 \otimes 8 = 27 \oplus 10 \oplus 10^* \oplus 8_F \oplus 8_D \oplus 1\] \hspace{1cm} (48)

we have (see eq.(13)&(14))

\[\{|m\rangle, |n\rangle\} \in \{27, 10, 10^*, 1\}. \tag{49}\]

Employing the analysis similar to decuplet, we could obtain the non-zero matrix elements for octet (see Table 3.)
Table 3. Values of non-zero matrix elements of $H'_{m,n}$ for Octet

| baryons | $H'_{8,8}/m$ | $H'_{10^*,10^*}/m$ | $H'_{27,27}/m$ | $H'_{8,10^*}/m$ | $H'_{8,27}/m$ | $H'_{10^*,27}/m$ |
|---------|--------------|---------------------|----------------|-----------------|----------------|-----------------|
| N       | $\frac{7}{10}$ | $\frac{7}{8}$ | $\frac{423}{500}$ | $\sqrt{5}$ | $\frac{\sqrt{5}}{10}$ | $\frac{\sqrt{5}}{80}$ |
| $\Sigma$ | $\frac{11}{10}$ | 1 | $\frac{267}{280}$ | $\sqrt{5}$ | $\frac{1}{5}$ | $\frac{\sqrt{5}}{10}$ |
| $\Lambda$ | $\frac{9}{10}$ | 0 | $\frac{57}{70}$ | 0 | $\frac{3}{10}$ | 0 |
| $\Xi$ | $\frac{6}{5}$ | 0 | $\frac{33}{35}$ | 0 | $\frac{\sqrt{6}}{10}$ | 0 |

Noticing $C_2(10^*) = 6$ and making use of eq.(53), we get

$$E^{(0)}_{8} - E^{(0)}_{10^*} = -\frac{3}{2b^2}$$  \hspace{1cm} (50)
$$E^{(0)}_{8} - E^{(0)}_{27} = -\frac{5}{b^2}$$  \hspace{1cm} (51)

Similarly we could obtain the complete results for baryons in octet up to the third order

$$M_N = M_8 - \frac{3}{10}m - \frac{43}{750}m^2b^2 + \frac{3812}{1575000}m^3b^4$$  \hspace{1cm} (52)
$$M_\Lambda = M_8 - \frac{1}{10}m - \frac{9}{250}m^2b^2 - \frac{54}{43750}m^3b^4$$  \hspace{1cm} (53)
$$M_\Sigma = M_8 + \frac{1}{10}m - \frac{37}{750}m^2b^2 - \frac{1672}{196875}m^3b^4$$  \hspace{1cm} (54)
$$M_\Xi = M_8 + \frac{1}{5}m - \frac{3}{125}m^2b^2 - \frac{54}{21875}m^3b^4$$  \hspace{1cm} (55)

where $M_8 = \langle H_0 \rangle_{\lambda=8} + m$. Similar to the decuplet, the mass relation for the baryons in octet could be obtained.

At the first order, the standard GOR for octet can be easily got

$$2(M_N + M_\Xi) = 3M_\Lambda + M_\Sigma,$$  \hspace{1cm} (56)

At the second order, a modified mass relation for baryons in octet can be obtained

$$15M_\Sigma + 35M_\Lambda = 24M_N + 26M_\Xi$$  \hspace{1cm} (57)

1In ref.[6], the numerical value of the third order for $M_\Sigma$ is incorrect.
which is the IGOR for octet. Repeating the procedure for decuplet and comparing with eq.(2),
four forms of corrections to $\delta m_8$ for octet can be obtained from eq.(57)

$$(\delta m_8)_{\text{second}} = \frac{2}{13}(M_N + M_\Sigma - 2M_\Lambda), \quad (58)$$

$$= \frac{1}{12}(3M_\Sigma - 2M_\Xi - M_\Lambda), \quad (59)$$

$$= \frac{2}{35}(5M_\Sigma - M_N - 4M_\Xi), \quad (60)$$

$$= \frac{1}{15}(6M_N + 4M_\Xi - 10M_\Lambda). \quad (61)$$

The above four $(\delta m_8)_{\text{second}}$ are also equivalent in principle. However, since eq.(57) is still not an exact identity of masses, the numerical results of $(\delta m_8)_{\text{second}}$ given by eqs.(58)-(61) will give the theoretical prediction with an error as follows

$$(\delta m_8)_{\text{second}} = -15.1 \pm 1.2\, MeV \quad (62)$$

Obviously, the second order corrections to $\delta m_8$ agree well with experiment quantitatively.

As for the third order corrections, we will go on to carry out the numerical analysis and the comparison. The procedures are also similar to the decuplet and the results are listed in Table 4 in comparison with those of the first and second order. Similar to the decuplet, we only list prediction for the mass of $M_\Xi$ while others are input. It could be easily seen from Table 4 that the results at the second order agree with experiment fairly well, however it is not the case for the third order: the predictions go far away from the experiment in comparison with the second order. This can be shown more directly by studying the corrections up to the third order.

Similar to the procedure of decuplet, the sum of the corrections to $\delta m_8$ in eq.(2) up to the third order can be written as

$$2(M_N + M_\Xi) = 3M_\Lambda + M_\Sigma + (\delta m_8)_{\text{second}} + (\delta m_8)_{\text{third}} \quad (63)$$

where $(\delta m_8)_{\text{second}}$ have been obtained in eqs.(58)-(61) and eq.(62). Through numerical calculations, $(\delta m_8)_{\text{third}}$ is found to be

$$(\delta m_8)_{\text{third}} = 60.2 \pm 19.1\, MeV. \quad (64)$$

then we get the sum of the corrections of the second and the third order as follows,

$$(\delta m_8)_{\text{sum}} = 45.1 \pm 20.3\, MeV. \quad (65)$$
Obviously, the corrections are much far away from the actual deviation as is shown in eq.(5). In Fig.1, we visually display the predictions for $M_\Omega$ in decuplet and $M_\Xi$ in octet between the different order corrections.

Table 4. Comparison between higher corrections and the experimental data for octet baryons

| baryons   | $M_N$(input) | $M_\Lambda$(input) | $M_\Sigma$(input) | $M_\Xi$  |
|-----------|--------------|---------------------|-------------------|----------|
| 1-st Order| 938.9        | 1115.6              | 1193.0            | 1331.1   |
| 2-nd Order| 938.9        | 1115.6              | 1193.0            | 1323.4   |
| 3-rd Order| 938.9        | 1115.6              | 1193.0            | 1366.2   |
| Exper. data| 938.9        | 1115.6              | 1193.0            | 1318.0   |
Fig. 1 The predictions of the mass of $M_\Omega$ in decuplet and $M_\Xi$ in octet in different orders. $M_\Delta, M_\Sigma^*, M_\Xi^*$ and $M_N, M_\Lambda, M_\Sigma$ are input and the unit is $MeV$.
5 Comments and Conclusions

In the above text, the higher order corrections to GOR’s are investigated by means of perturbation in the framework of standard SU(3) Skyrme model. The results are found to be quite compatible with experiment up to the second order. However, things gets complicated at the third order. For decuplet, the results become further encouraging though the improvements are not very remarkable. Yet for octet, the results become quite disagreeable. This means that, at the perturbation level, the calculations of corrections to GOR in the standard SU(3) Skyrme model can hold to the third order for decuplet and only to the second order for octet-baryons.

As we know, the key point of the corrections to GOR’s is the calculations of the non-zero matrix elements (see eqs.(11)-(14) and eq.(16)), which are closely related to the flavor breaking term. Then it could be seen that there exist defects in present standard flavor breaking term.

The problem has been discussed by many authors \[8\], \[16\], \[19\], \[20\]. Among these discussions, in our opinion, one reasonable scheme was put forward by one of us in ref.\[16,21\]. In that model, the chiral SU(3) flavor group is embedded into a larger SU(\(N_f\)) group, and the (ud)-(s) flavor symmetry breaking term still keeps the standard form like eq.(8). Basically, in the chiral Lagrangian theory from QCD, neglecting the \(\theta\)-dependence, the standard flavor symmetry breaking term of eq.(8) comes directly from the following quark mass term \[22\]

\[
L_m = \frac{1}{8} \frac{m_u^2 F^2}{m_u + m_d} Tr[M_q(U + U^\dagger - 2)]
\]

where \(M_q=\text{diag}(m_u, m_d, m_s)\). \(L_m\) is also directly related to PCAC (partial conservation of axial vector current). In the sub-SU(3) Skyrme model, only such symmetry breaking mechanism is adopted and no non-standard terms with adjustable parameters are introduced. Therefore its success is expected and reasonable. On the other hand, it was found that the mass spectra for decuplet and octet are quite compatible with the experimental data at the first order as \(N = 5 \sim 6\). Apparently, this is interesting: in the real world, there are only six known quark flavors. Then we may speculate this idea may be possible to completely account for the deviations between GOR and the experimental data. Therefore it is still of great need to carry out the higher order perturbative calculations in the framework of ref.\[16\]. However such an investigation is out of the scope of this paper due to the lack of enough C-G coefficients for SU(\(N_f\)) with \(N_f > 3\) at present, and will be left to the further studies.

Finally we want to make some comments on the rotating excited states. As a puzzle, it was
attended long ago whether there exist some “exotic” SU(3) baryon-multiplets besides the ordinary octet and decuplet in the flavor SU(3) theory. In this paper, we could see from the procedure of the calculations that the rotating excited states have played important role in the improvement of baryon mass relations, since the higher order perturbation could be attributed to the calculations of the $H'$-matrix elements between various eigenstates of $H_0$ (see eqs. (10)-(14)). So the above results could be regarded as signals of the existence of such rotating excited SU(3)-multiplets as 10*-let (with spin $\frac{1}{2}$), 27-let (with spin $\frac{1}{2}$ or $\frac{3}{2}$) and 35-let (with spin $\frac{3}{2}$). Meanwhile, the wave-functions of these SU(3)-states satisfy the constrained condition coming from the Wess-Zumino term in the QCD effective Lagrangian, i.e., the spin-hypercharge $Y_R = 1^0$. So they should be physical states in QCD. However, up to now, no any experimental evidences have indicated the existence of these states. A possible explanation seems to be the unusually large width of the resonances which makes the identification very difficult in experiment. However, it is just because that these rotating excited states contribute to the deviations of GOR from the experiment by the method of perturbative calculations, we could “see” them indirectly through the corrections to $\delta m_{10}^{(1)}, \delta m_{10}^{(2)}, \delta m_8$. So the above positive conclusion is of course interesting.
Appendix A: Tables for Isoscalar factors of $8 \otimes 35 \rightarrow 35$

The tables in this Appendix are about the SU(3) iso-scalar factors:

$$\begin{pmatrix} \lambda_1 & \lambda_2 \\ I_1Y_1 & I_2Y_2 \end{pmatrix} \begin{pmatrix} \lambda \\ IY \end{pmatrix}$$ \hspace{1cm} (A.1)

First of all, we want to present the phase conventions adopted in this paper, because some of them are different from the corresponding ones in the ref.[13].

The phase conventions are adopted as follows:

1. The relative phase conventions within a definite iso-multiplet are determined by the Condon and Shortley phase convention. i.e.,

$$I_+ \phi(I, I_z, Y) = [(I - I_z)(I + I_z + 1)]^{1/2} \phi(I, I_z + 1, Y),$$ \hspace{1cm} (A.2)

$$I_- \phi(I, I_z, Y) = [(I + I_z)(I - I_z + 1)]^{1/2} \phi(I, I_z - 1, Y),$$ \hspace{1cm} (A.3)

2. The relative phase between the different iso-multiplets are defined in eq.(17) (see Sec.3). This is different from that of de Swart’s[13].

3. In the CG series $m_1 \otimes m_2 \rightarrow \sum n_i$, the relative phase between $n_i$ and $m_1, m_2$ are determined in this way:

First, we set all the phase factors always real. Second, we determine the sign of the phase by considering the highest eigenstates $\phi^{\lambda}_{\mu}$ of the IR’s $n_i$

$$\phi^{\lambda}_{\mu} = \sum_{I_1, I_2, I_3, Y_1, Y_2, Y_3} \begin{pmatrix} \lambda_1 & \lambda_2 \\ I_1Y_1 & I_2Y_2 \end{pmatrix} C_{CG}[SU(2)] \phi^{\lambda_1}_{\mu_1} \phi^{\lambda_2}_{\mu_2}$$ \hspace{1cm} (A.4)

Among the different SU(3) iso-scalar factors, we choose the one with the largest possible $I_1$ to be positive. If this is not sufficient, we take from the iso-scalar factors with the largest possible $I_1$ the one with the largest possible $I_2$ positive.

The conventions here are sufficient to decide all phases factors in SU(3). Conventions 2 and 3 are different from those of ref.[13]. Convention 2 is easy to understand, since we do not use $V$-spin (in ref.[13]’s notation, it is denoted with $K$.), but Gel’fand basis technique. Covention 3 is a supplement to ref.[13], since the corresponding one is not sufficient in ref.[13].

All the CGC’s (or iso-scalar factors) that are related to the calculations in this paper have been re-calculated according to the above conventions by means of Gel’fand basis. The results show that
the CGC’s by employing Gel’fand basis and the above conventions are completely consistent with the CGC’s in ref. [13]. So we could use alternatively both CGC’s in ref. [13] and those obtained by the technique in the present paper without any contradiction.

As for SU(N) with \( N \geq 4 \), new phase conventions are needed. In principle, they can be introduced according to the above method.

Table A.1: \( Y=2, I=2 \)

| \( Y_1 \) | \( I_1 \) | \( Y_2 \) | \( I_2 \) | \( 35_1 \) | \( 35_2 \) |
|----------|----------|----------|----------|----------|----------|
| 1 \( \frac{1}{2} \) | 1 \( \frac{5}{2} \) | 0 | \( -\sqrt{\frac{27}{40}} \) | \( \sqrt{\frac{2}{135}} \) |
| 1 \( \frac{1}{2} \) | 1 \( \frac{3}{2} \) | \( -\sqrt{\frac{5}{27}} \) | \( \sqrt{\frac{1}{12}} \) |
| 0 | 2 | 2 | \( \sqrt{\frac{2}{3}} \) | \( \sqrt{\frac{2}{27}} \) |
| 0 | 0 | 2 | 2 | \( \sqrt{\frac{1}{27}} \) | \( -\sqrt{\frac{49}{216}} \) |

Table A.2: \( Y=1, I=\frac{5}{2} \)

| \( Y_1 \) | \( I_1 \) | \( Y_2 \) | \( I_2 \) | \( 35_1 \) | \( 35_2 \) |
|----------|----------|----------|----------|----------|----------|
| 1 \( \frac{1}{2} \) | 0 | 2 | \( -\sqrt{\frac{2}{5}} \) | \( -\frac{1}{5} \) |
| 0 | 1 | 1 | \( \frac{5}{2} \) | \( \sqrt{\frac{140}{324}} \) | \( \sqrt{\frac{140}{324}} \) |
| 0 | 1 | 1 | \( \frac{3}{2} \) | \( \sqrt{\frac{1}{135}} \) | \( \sqrt{\frac{30}{324}} \) |
| 0 | 0 | 1 | \( \frac{5}{2} \) | \( \sqrt{\frac{4}{27}} \) | \( \sqrt{\frac{169}{864}} \) |
| -1 \( \frac{1}{2} \) | 2 | 2 | 0 | -1 |

Table A.3: \( Y=1, I=\frac{3}{2} \)

| \( Y_1 \) | \( I_1 \) | \( Y_2 \) | \( I_2 \) | \( 35_1 \) | \( 35_2 \) |
|----------|----------|----------|----------|----------|----------|
| 1 \( \frac{1}{2} \) | 0 | 2 | \( \sqrt{\frac{3}{36}} \) | \( -\sqrt{\frac{121}{216}} \) |
| 1 \( \frac{1}{2} \) | 0 | 1 | \( -\sqrt{\frac{125}{216}} \) | \( \sqrt{\frac{5}{216}} \) |
| 0 | 1 | 1 | \( \frac{5}{2} \) | \( -\sqrt{\frac{1}{90}} \) | \( -\sqrt{\frac{5}{36}} \) |
| 0 | 1 | 1 | \( \frac{3}{2} \) | \( \sqrt{\frac{961}{2160}} \) | \( -\sqrt{\frac{30}{216}} \) |
| 0 | 0 | 1 | \( \frac{3}{2} \) | \( \sqrt{\frac{15}{108}} \) | \( -\sqrt{\frac{5}{108}} \) |
| -1 \( \frac{1}{2} \) | 2 | 2 | \( \sqrt{\frac{25}{108}} \) | \( -\sqrt{\frac{1}{54}} \) |
Table A.4: $Y=0$, $I=2$

| $Y_1$ | $I_1$ | $Y_2$ | $I_2$ | $35_1$ | $35_2$ |
|-------|-------|-------|-------|--------|--------|
| 1     | $\frac{1}{2}$ | −1    | $\frac{3}{2}$ | $-\sqrt{\frac{1}{2}}$ | $-\sqrt{\frac{1}{21}}$ |
| 0     | 1      | 0     | 2      | $\sqrt{\frac{9}{21}}$ | $-\sqrt{\frac{3}{16}}$ |
| 0     | 1      | 0     | 1      | $\sqrt{\frac{1}{12}}$ | $\frac{5}{12}$ |
| 0     | 0      | 0     | 2      | $\sqrt{\frac{1}{108}}$ | $\sqrt{\frac{25}{216}}$ |
| −1    | $\frac{1}{2}$ | 1     | $\frac{5}{2}$ | $\sqrt{\frac{4}{15}}$ | $\sqrt{\frac{1}{30}}$ |
| −1    | $\frac{1}{2}$ | 1     | $\frac{3}{2}$ | $\sqrt{\frac{15}{180}}$ | $-\frac{11}{90}\sqrt{30}$ |

Table A.5: $Y=0$, $I=1$

| $Y_1$ | $I_1$ | $Y_2$ | $I_2$ | $35_1$ | $35_2$ |
|-------|-------|-------|-------|--------|--------|
| 1     | $\frac{1}{2}$ | −1    | $\frac{3}{2}$ | $\frac{1}{9}$ | $-\frac{17}{36}\sqrt{2}$ |
| 1     | $\frac{1}{2}$ | −1    | $\frac{1}{2}$ | $-\frac{5}{9}$ | $\sqrt{\frac{2}{81}}$ |
| 0     | 1      | 0     | 2      | $-\frac{1}{4}\sqrt{\frac{10}{27}}$ | $\frac{5}{36}\sqrt{15}$ |
| 0     | 1      | 0     | 1      | $\frac{13}{36}\sqrt{2}$ | $\frac{11}{36}$ |
| 0     | 0      | 0     | 1      | $-\frac{\sqrt{18}}{15}$ | $-\frac{5}{36}\sqrt{6}$ |
| −1    | $\frac{1}{2}$ | 1     | $\frac{3}{2}$ | $\frac{\sqrt{5}}{15}$ | $-\frac{\sqrt{10}}{15}$ |

Table A.6: $Y=-1$, $I=\frac{3}{2}$

| $Y_1$ | $I_1$ | $Y_2$ | $I_2$ | $35_1$ | $35_2$ |
|-------|-------|-------|-------|--------|--------|
| 1     | $\frac{1}{2}$ | −2    | 1     | $-\sqrt{\frac{3}{7}}$ | $-\sqrt{\frac{21}{7}}$ |
| 0     | 1      | −1    | $\frac{3}{2}$ | $\sqrt{\frac{5}{27}}$ | $-\sqrt{\frac{245}{864}}$ |
| 0     | 1      | −1    | $\frac{1}{2}$ | $\sqrt{\frac{1}{72}}$ | $\frac{75}{321}$ |
| 0     | 0      | −1    | $\frac{3}{2}$ | $-\sqrt{\frac{1}{27}}$ | $\frac{19}{864}$ |
| −1    | $\frac{1}{2}$ | 0     | 2     | $\sqrt{\frac{15}{36}}$ | $\sqrt{\frac{576}{30}}$ |
| −1    | $\frac{1}{2}$ | 0     | 1     | $\sqrt{\frac{1}{108}}$ | $-\frac{17}{72}\sqrt{6}$ |
Table A.7: Y=-1, I=\(\frac{1}{2}\)

| \(Y_1\) | \(I_1\) | \(Y_2\) | \(I_2\) | \(35_1\) | \(35_2\) |
|---------|---------|---------|---------|---------|---------|
| 1       | \(\frac{1}{2}\) | -2      | 1       | \(\sqrt{\frac{1}{27}}\) | \(-\sqrt{\frac{1}{3}}\) |
| 1       | \(\frac{1}{2}\) | -2      | 0       | \(-\sqrt{\frac{25}{108}}\) | \(\sqrt{\frac{1}{3}}\) |
| 0       | 1       | -1      | \(\frac{3}{2}\) | \(-\sqrt{\frac{1}{27}}\) | \(\sqrt{\frac{1}{3}}\) |
| 0       | 1       | -1      | \(\frac{1}{2}\) | \(\sqrt{\frac{49}{144}}\) | \(\sqrt{\frac{1}{3}}\) |
| 0       | 0       | -1      | \(\frac{1}{2}\) | \(-\sqrt{\frac{49}{144}}\) | \(-\sqrt{\frac{1}{3}}\) |
| -1      | \(\frac{1}{2}\) | 0       | 1       | \(\sqrt{\frac{25}{144}}\) | \(-\sqrt{\frac{1}{3}}\) |

Table A.8: Y=-2, I=1

| \(Y_1\) | \(I_1\) | \(Y_2\) | \(I_2\) | \(35_1\) | \(35_2\) |
|---------|---------|---------|---------|---------|---------|
| 1       | \(\frac{1}{2}\) | -3      | \(\frac{1}{2}\) | \(-\sqrt{\frac{2}{5}}\) | \(-\frac{1}{6}\) |
| 0       | 1       | -2      | 1       | \(\sqrt{\frac{1}{18}}\) | \(-\frac{2}{3}\) |
| 0       | 1       | -2      | 0       | \(\sqrt{\frac{1}{54}}\) | \(\sqrt{\frac{25}{108}}\) |
| 0       | 0       | -2      | 1       | \(-\sqrt{\frac{25}{108}}\) | \(\sqrt{\frac{1}{54}}\) |
| -1      | \(\frac{1}{2}\) | -1      | \(\frac{3}{2}\) | \(\frac{2}{3}\) | \(\sqrt{\frac{1}{18}}\) |
| -1      | \(\frac{1}{2}\) | -1      | \(\frac{1}{2}\) | \(\frac{1}{6}\) | \(-\sqrt{\frac{2}{5}}\) |

Table A.9: Y=-2, I=0

| \(Y_1\) | \(I_1\) | \(Y_2\) | \(I_2\) | \(35_1\) | \(35_2\) |
|---------|---------|---------|---------|---------|---------|
| 1       | \(\frac{1}{2}\) | -3      | \(\frac{1}{2}\) | \(\sqrt{\frac{1}{27}}\) | \(-\sqrt{\frac{49}{216}}\) |
| 0       | 1       | -2      | 1       | \(-\sqrt{\frac{1}{18}}\) | \(-\frac{5}{6}\) |
| 0       | 0       | -2      | 0       | \(-\sqrt{\frac{1}{3}}\) | \(-\sqrt{\frac{1}{27}}\) |
| -1      | \(\frac{1}{2}\) | -1      | \(\frac{1}{2}\) | \(\sqrt{\frac{50}{108}}\) | \(\sqrt{\frac{1}{27}}\) |

Table A.10: Y=-3, I=\(\frac{1}{2}\)

| \(Y_1\) | \(I_1\) | \(Y_2\) | \(I_2\) | \(35_1\) | \(35_2\) |
|---------|---------|---------|---------|---------|---------|
| 0       | 1       | -3      | \(\frac{1}{2}\) | 0       | \(-\sqrt{\frac{81}{108}}\) |
| 0       | 0       | -3      | \(\frac{1}{2}\) | \(-\sqrt{\frac{16}{27}}\) | \(\sqrt{\frac{5}{864}}\) |
| -1      | \(\frac{1}{2}\) | -2      | 1       | \(\sqrt{\frac{1}{27}}\) | \(\sqrt{\frac{5}{18}}\) |
| -1      | \(\frac{1}{2}\) | -2      | 0       | \(\sqrt{\frac{2}{27}}\) | \(-\sqrt{\frac{49}{432}}\) |
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