Effects of a Pre-inflationary de Sitter Bounce on the Primordial Gravitational Waves in $f(R)$ Gravity Theories

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In this work we examine the effects of a pre-inflationary de Sitter bounce on the energy spectrum of the primordial gravitational waves. Specifically we assume that the Universe is described by several evolution patches, starting with a de Sitter pre-inflationary bounce which is followed by an quasi-de Sitter slow-roll inflationary era, followed by a constant equation of state parameter abnormal reheating era, which is followed by the radiation and matter domination eras and the late-time acceleration eras. The bounce and the inflationary era can be realized by vacuum $f(R)$ gravity and the abnormal reheating and the late-time acceleration eras by the synergy of $f(R)$ gravity and the prefect matter fluids present. Using well-known reconstruction techniques we find which $f(R)$ gravity can realize each evolution patch, except from the matter and radiation domination eras which are realized by the corresponding matter fluids. Accordingly, we calculate the damping factor of the primordial de Sitter bounce, and as we show, the signal can be detected by only one gravitational wave future experiment, in contrast to the case in which the bounce is absent. We discuss in detail the consequences of our results and the future perspectives.

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I. INTRODUCTION

The two acceleration eras of our Universe, including the reheating era, are undoubtedly the most mysterious eras of all the evolutions eras we assume that our Universe experienced. The late-time acceleration era though is confirmed and the difficulty lies to pinpointing the physical theory and mechanism which controls it. However, the other two eras are speculated to have occurred in the primordial epoch of the Universe, and to date no firm evidence is provided that these eras have actually occurred. With regard to the inflationary era [1–3], the occurrence of this era could be verified by the direct detection of the B-modes in the Cosmic Microwave Background (CMB) temperature fluctuations. This for example can be verified in the stage 4 CMB experiments [5, 6] in some years from now, or alternatively, the primordial stochastic tensor modes can be directly detected in future gravitational waves experiments [7–14], see also Ref. [15] for some up to date information on cosmological studies of the LISA mission.

Now the plot may thicken with the existence or not of the inflationary era. The results of the future CMB and gravitational waves experiments will play a crucial role, in both cases of detection or not of a signal. In the unlikely event of non-observation of a signal, the scientists will be confronted with the difficult task to explain why no signal is detected. Is this non-detection because inflation did not occur, or simply because inflation is described by a theory which yields a negative tensor spectral index and a non-detectable by the current experiments signal, while it also yields a standard General Relativistic (GR) reheating era? On the antipode of this, there lies the detection of a signal. Many questions can be asked then, how strong is the signal, is it detectable by several experiments in various frequency ranges, or by some of the experiments? With regard to how strong a signal can be, this is very important. The observation of a signal in some but not all the detectors may signify some physical process which causes damping of the signal, such as supersymmetry breaking after or during reheating. This will also determine the era for which the physics change occurred in the Universe. However, with regard to how strong the signal is, many things can be said, and many questions can be asked. The answers to these questions may vary and strongly depend on the final form of the signal. Thus the strength and form of the signal may determine whether this signal is obtained by a theory with positive tensor spectral index, or by a standard inflationary theory with an abnormal reheating era. In the literature, many aspects on primordial gravitational waves are studied [16–75], and with regard to the abnormal reheating perspective and effects on the energy spectrum of the primordial gravitational waves, this aspect has recently been studied in Refs. [69, 71, 75] and it was shown that the gravitational waves energy spectrum of standard $f(R)$ gravity inflation can be enhanced significantly by the presence of an $f(R)$ gravity generated reheating era. This is in contrast to a GR compatible reheating era of course and the flatness, form and strength of the detected signal may reveal many properties regarding the underlying theory. Regarding the strength, this may vary and one mechanism.

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II. PRE-INFLATIONARY DE SITTER BOUNCE AND PRIMORDIAL EVOLUTION

Let us discuss the scenario we propose in this work, which is based on the fact that the Universe pre-inflationary was experiencing a de Sitter bounce, which is followed by a quasi-de Sitter era. Accordingly, after the quasi-de Sitter era we will assume that the Universe enters a reheating era with constant equation of state (EoS) parameter \( w \), followed by the standard patches of evolution, namely a canonical reheating era with EoS parameter \( w = 1/3 \) and finally the matter and dark energy era. Giving the Universe’s evolution in distinct patches is the best we can do as cosmologists, since it is not possible to find the exact scale factor which describes the Universe, this extends beyond the reach of the human mind. Hence, assuming several evolutionary patches for the Universe is the best we can do, and in fact, some of these patches may be directly determined, as it happens with the dark energy era and also may happen with the inflationary and post-inflationary era, via the future stage 4 CMB experiments and the future gravitational waves experiments. However, pre-inflationary evolution patches are quite hard to be probed, and our proposal in this paper is that these pre-inflationary eras may have a direct effect on the energy spectrum of the primordial gravitational waves, causing a significant damping of the spectrum. In this line of research, let us quote the scale factor for the pre-inflationary, inflationary and the first moments of the post-inflationary epoch, which is,

\[
a(t) = a_b \cosh(jt)e^{-t/t_1} + a_i e^{H_0(t-t_i) + H_1(t_0-t)} + a_w t_0 \frac{t}{(w+1)},
\]

and let us explain the different patches for the above evolution. The first term describes the de Sitter bounce [90], which is followed by the quasi-de Sitter inflationary epoch described by the second term, followed by the constant EoS parameter \( w \) reheating epoch, described by the third term. Now, \( a_b \), \( a_i \), and \( a_w \) denote the size of the Universe at the beginning of the de Sitter bounce, at the beginning of the inflationary era and at the beginning of the reheating era with constant EoS parameter \( w \). The parameters \( j \), \( H_0 \), and \( H_1 \) have mass dimensions eV, eV and eV\(^2\)\(, \) while the time instances \( t_1 \) and \( t_0 \) are characteristic and denote the time that the bounce ends and the time instance that the inflationary era ends. So for cosmic times \( t \ll t_1 \), the exponential term is practically equal to unity, while for times \( t \geq t_1 \) the exponential term causes a damping of the first term, thus the other two start to dominate. In the left and right upper plots of Fig. 1 we plot the scale factor (1) vs the cosmic time (blue curve) and the de Sitter bounce scale factor described by the first term (red curve). Also in the bottom plot we present the Hubble radius \( R_H = \frac{1}{a(t)H(t)} \).
FIG. 1. Upper plots: The scale factor (1) (blue curve) and the de Sitter bounce scale factor (red curve) vs the cosmic time. Bottom plot: the Hubble radius \( R_H = \frac{1}{a(t)H(t)} \) as a function of the cosmic time for the scale factor (1) (blue curve) and for the de Sitter bounce (red curve).

As it can be seen, the scale factor (1) is described by a bounce pre-inflationary, in which case the scale factor decreases, and the Hubble radius increases, after that the Universe experiences a short period of acceleration, in which case the Hubble radius decreases, and this short acceleration period is followed by a deceleration period, in which case the Hubble radius starts to decrease again. The blue curve has exactly the behavior described by the scale factor (1), so the Universe starts with a per-inflationary bounce, followed by a short period of inflation, followed by a power-law evolution with constant EoS parameter. For the plots we assumed that \( w = 0 \) and this is also what we will assume for the rest of the article. Hence basically, the reheating era is abnormal and has an EoS parameter \( w = 0 \), different from \( w = 1/3 \) which describes an ordinary reheating era.

III. INFLATION AND POST-INFLATION EVOLUTION WITH \( f(R) \) GRAVITY

Let us now proceed in the modified gravity description of the cosmological evolution we presented in the previous section. Our basic assumption is that \( f(R) \) gravity controls the whole evolution, from the pre-inflationary era to the late-time era. It is \( f(R) \) gravity which realizes the various evolutionary patches we described in the previous section. Schematically, the \( f(R) \) gravity which realizes the evolution patches which we described in the previous section will have the following form,

\[
f(R) = \begin{cases} 
F_B(R) & R \geq R_B, \\
R + \frac{R^2}{6M^2} & R \sim R_I, \\
F_{w}(R) & R \sim R_{PI} \ll R_I, \\
F_{DE}(R) & R \sim R_0 \ll R_{PI}, 
\end{cases}
\]

with \( R_I \) stands for the curvature scale of inflation, at the first horizon crossing, \( R_{PI} \) stands for the post-inflationary curvature scale during the abnormal reheating era, and \( R_B \) is the curvature scale near the bouncing point, when the
as a dynamical variable in our cosmological system, the Friedmann equation takes the following form,

\[ F_w(R) = \left[ \frac{c_2 \rho_1}{\rho_2} - \frac{c_1 \rho_1}{\rho_2(\rho_2 - \rho_1 + 1)} \right] R^p + \frac{1}{\rho_2(\rho_2 - \rho_1 + 1)} \sum_i \left[ \frac{c_1 S_i}{\rho_2(\delta_i + 2 + \rho_2 - \rho_1)} \right] R^{\delta_i + 2 + \rho_2} - \sum_i B_i c_2 R^{\delta_i + \rho_2} + c_1 R^{\rho_1} + c_2 R^{\rho_2}, \]

where \( c_1, c_2 \) are integration constants, and also \( \delta_i \) and \( B_i \) are,

\[ \delta_i = \frac{3(1 + w_i) - 23(1 + w)}{3(1 + w)} - \rho_2 + 2, \quad B_i = \frac{S_i}{\rho_2 \delta_i}, \]

with \( i = (r, m) \), while \( a_1 \) and \( a_2, S_i \), and \( A \) are equal to,

\[ a_1 = \frac{3(1 + w)}{4 - 3(1 + w)}, \quad a_2 = \frac{2 - 3(1 + w)}{2(4 - 3(1 + w))}, \quad S_i = \frac{\kappa^2 \rho_{i0} \delta_0}{\gamma(4 - 3(1 + w))}, \quad A = \frac{4}{3(1 + w)} \cdot \]

Now let us focus on the calculation of \( F_B(R) \), and we shall use a well known reconstruction technique \[93\] in order to find this. The \( f(R) \) gravity action in vacuum is,

\[ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m, \]

with \( \kappa^2 \) being \( \kappa^2 = 8\pi G = \frac{1}{M_p^2} \), and \( G \) is Newton’s constant, with \( M_p \) denoting the reduced Planck mass. In the metric formalism, the field equations are,

\[ f_R(R) R_{\mu\nu}(g) - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R(R) + g_{\mu\nu} \Box f_R(R) = 0, \]

with \( f_R = \frac{df}{df} \). For a flat Friedmann-Robertson-Walker (FRW) spacetime, the Friedmann equation reads,

\[ -18 \left( 4H(t)^2 \dot{H}(t) + H(t) \ddot{H}(t) \right) f_R(R) + 3 \left( H^2(t) + \dot{H}(t) \right) f_R - \frac{f(R)}{2} = 0, \]

We are interested in realizing the de Sitter bounce patch of the scale factor \( [1] \), so basically,

\[ a(t) \simeq a_b \cosh(j t), \]

near the bouncing point. Using the \( e \)-foldings number,

\[ e^{-N} = \frac{a_b}{a}, \]

as a dynamical variable in our cosmological system, the Friedmann equation takes the following form,

\[ -18 \left[ 4H^3(N)H''(N) + H^2(N)(H')^2 + H^3(N)H''(N) \right] f_R(R) + 3 \left[ H^2(N) + H(N)H'(N) \right] f_R(R) - \frac{f(R)}{2} = 0. \]

Introducing the function, \( G(N) = H^2(N) \), the Ricci scalar is written as,

\[ R = 3G'(N) + 12G(N). \]

and finally the Friedmann equation takes the final form,

\[ -9G(N(R)) \left[ 4G'(N(R)) + G''(N(R)) \right] f_R(R) + \left[ 3G(N) + \frac{3}{2} G'(N(R)) \right] f_R(R) - \frac{f(R)}{2} = 0, \]
with $G'(N) = dG(N)/dN$ and $G''(N) = d^2G(N)/dN^2$ and $f_{RR} = \frac{d^2f}{dR^2}$. Thus the $F_B(R)$ gravity which realizes the de Sitter bounce, can be found by solving Eq. (12). In our case, $G(N)$ has the following form,

$$G(N) = \left( \frac{\int \frac{\exp(N) - e^{1/2}}{5 + \exp(N)} (b + \exp(N))}{\exp(N)} \right)^2,$$

(13)

hence the Friedmann equation (12), takes the final form,

$$(-18j^2R + 72j^4 + R^2) f_{RR}(R) + 3j^2 f_R(R) - \frac{f(R)}{2},$$

(14)

which can be solved analytically, and the resulting function $F_B(R)$ which realizes the de Sitter bouncing era has the following form,

$$F_B(R) = c_1 \left( \sqrt{6j^2 - R\sqrt{12j^2 - R}}\ - 9j^2 + R \right) \sqrt{3} \sqrt{6j^2 - R\sqrt{12j^2 - R} + 15\sqrt{3}j^2 - 2\sqrt{3}R},$$

(15)

with $c_1$ being an irrelevant integration constant. Finally, the late-time era $f(R)$ gravity will be assumed to have the form,

$$F_{DE}(R) = -\gamma \Lambda \left( \frac{R}{3m_s^2} \right)^\delta,$$

(16)

where $m_s$ is $m_s^2 = \frac{\pi^2 \rho_m^{(0)}}{3}$, $\rho_m^{(0)}$ is the present day energy density of cold dark matter, $\delta$ takes values $0 < \delta < 1$, and $\gamma$ is an arbitrary dimensionless parameter, and finally $\Lambda$ is the cosmological constant at present day. The late-time phenomenology of the model (16) in the presence of an $R^2$ and subleading power-law terms was studied in Ref. [61] and it is proven to be viable, so we will not further study it here.

Having the functional forms of the total $f(R)$ gravity which realize the various evolutionary patches of the Universe, in the next section we shall investigate the effects of the bounce era on the energy spectrum of the primordial gravitational waves. For the calculation of the energy spectrum of the primordial gravitational waves, we shall use the inflationary indices of the $R^2$ model, which are,

$$r = 48 \epsilon_1^2,$$

(17)

and the corresponding tensor spectral index for the $R^2$ gravity is [61, 94],

$$n_T \simeq -2\epsilon_1^2,$$

(18)

with $\epsilon_1$ being the first slow-roll index $\epsilon_1 = -\dot{H}/H^2$. For the $R^2$ gravity, the first slow-roll index is $\epsilon_1 \simeq \frac{1}{2N}$, therefore we have,

$$n_T \simeq -\frac{1}{2N^2},$$

(19)

and

$$r = \frac{12}{N^2}.$$  

(20)

Also the post-inflationary abnormal reheating era affects the duration of the inflationary era, in the following way [93].

$$N = 56.12 - \ln \left( \frac{k}{K_s} \right) + \frac{1}{3(1 + w)} \ln \left( \frac{2}{3} \right) + \ln \left( \frac{\rho_k^{1/4}}{\rho_{ren}^{1/4}} \right) + \frac{1 - 3w}{3(1 + w)} \ln \left( \frac{\rho_{ren}^{1/4}}{\rho_{end}^{1/4}} \right) + \ln \left( \frac{\rho_k^{1/4}}{10^{16} \text{GeV}} \right),$$

(21)

with $\rho_{end}$ and $\rho_{ren}$ being the energy density of the Universe at the end of the inflationary era and at the end of the reheating era respectively, $\rho_k$ is the energy density of the Universe at the first horizon crossing, and $k_s$ is the pivot scale $k_s = 0.05 \text{Mpc}^{-1}$. Hence, for the abnormal reheating scenario with $w = 0$ post-inflationary, the inflationary era is either prolonged beyond 60 e-foldings, or it lasts for less amount of time, depending on the reheating temperature. We shall consider three reheating temperatures, $T_R = 10^{12} \text{GeV}, T_R = 10^7 \text{GeV}$ and $T_R = 10^2 \text{GeV}$, and in Table [I] we
quote the duration of the inflationary era in terms of the $e$-foldings number, and the values of the inflationary indices for the three distinct reheating temperatures. We shall use the results of Table I in the next section for the calculation of the energy spectrum of the primordial gravitational waves. We need to note that the transition from the bounce epoch to the inflationary epoch is smooth and continuous as it can be seen by looking the scale factor in Eq. (1). The realization of the scale factor \[f(T)\] could be quite difficult to do in standard single scalar field theory in the Einstein frame, this is why we chose to realize this in the context of Jordan frame description, in which both the viability of the inflationary era is guaranteed and also the resulting $f(R)$ gravity model which realizes the scale factor \[f(T)\] is quite elegant and simple.

IV. THE ENERGY SPECTRUM OF THE PRIMORDIAL GRAVITATIONAL WAVE: EFFECTS OF THE DE SITTER BOUNCE

In this section we shall quantitatively study the effect of the de Sitter bounce pre-inflationary era on the energy spectrum of the primordial gravitational waves. To this end we have to specify the duration of the pre-inflationary bounce era. Since the Planck era corresponds to a temperature of $10^{19}$ GeV, and inflation is believe to commence at $T \sim 10^{10}$ GeV, we shall assume that the pre-inflationary bounce epoch starts at $T \sim 10^{10}$ GeV and ends at $T \sim 10^{16}$ GeV. We must translate this temperature interval into a redshift interval, since this is essential for the calculation of the damping factor, so using the relation $T = T_0 (1 + z)$, where $T_0$ denotes the present day temperature $T_0 = 2.58651 \times 10^{-4}$ eV, the temperature interval $T = [10^{10} - 10^{16}]$ GeV corresponds to the redshift interval $z = [3.86621 \times 10^{28}, 3.86621 \times 10^{31}]$. After the end of the bouncing era, the inflationary era commences which is assumed to occur at a temperature $T \sim 10^{16}$ GeV and ends for example at a temperature $T_{\text{end}} \sim 10^{13}$ GeV.

The abnormal reheating era then commences which we shall assume that it lasts until the temperature drops to the order $T_{pr} = 10^{12}$ GeV. Translated in redshifts, the abnormal reheating era lasts for the redshift interval $z = [3.86621 \times 10^{15}, 3.86621 \times 10^{16}]$. These redshifts intervals are quite important for the calculation of the damping factor caused by the $f(R)$ gravity during the pre-inflationary bounce and during the abnormal reheating era.

At this point, let us review the method of extracting the overall damping or amplification effect of the modified gravity on the GR waveform. For details we refer to the review [70] and to the original paper [39] where the method appeared for the first time. The crucial parameter that quantifies the overall effect of modified gravity is the parameter $a_M$ which for $f(R)$ gravity is defined,

$$a_M = \frac{f_{RR}}{f_R H},$$

and the waveform of modified gravity in terms of the GR waveform has the following form [39, 40],

$$h = e^{-D} h_{GR},$$

where $h_{GR}$ denotes the GR waveform in which case $a_M = 0$, and $D$ is equal to,

$$D = \frac{1}{2} \int^T_0 a_M H dt_\tau = \frac{1}{2} \int_0^z \frac{a_M}{1 + z'} dz'.$$

The details on the WKB method used to derive the above solutions can be found in the review [70]. Thus, the energy spectrum for the $f(R)$ gravity is [19, 31, 32, 39, 40, 42, 61, 70].

$$\Omega_{GW}(f) = e^{-2D} \times \frac{k^2}{12H_0^2} \mathcal{P}_c(k_{re}) \left( \frac{k}{k_{re}} \right)^{n_T} \left( \frac{\Omega_{m}}{\Omega_{m}^{\text{in}}} \right)^2 \left( \frac{g_s(T_{\text{in}})}{g_s(T_{\text{end}})} \right)^{4/3} \left( \frac{3j_1(k\tau_0)}{k\tau_0} \right)^2 T_1^2(\tau_{eq}) T_2^2(\tau_{R}),$$

\[T_1^2(\tau_{eq}) T_2^2(\tau_{R}),\]
with \( k_{ref} = 0.002 \text{Mpc}^{-1} \) denoting the CMB pivot scale, \( n_T \) stands for the tensor spectral index and \( r \) is as usual the tensor-to-scalar ratio. Our main task in this section is to numerically evaluate the parameter \( D \) for all the redshifts up to the Planck era with redshift \( z_p = 3.86621 \times 10^{31} \). The main contributions which are not trivial are contributed by the redshift intervals \( z = [3.86621 \times 10^{15}, 3.86621 \times 10^{16}] \) and \( z = [3.86621 \times 10^{28}, 3.86621 \times 10^{31}] \) which correspond to the abnormal reheating and the de Sitter bounce pre-inflationary era. The rest of the redshift intervals contribute terms of the order of unity, so we shall not discuss them here, we refer the reader to Ref. [61] for details. Now recall that the de Sitter bounce and the abnormal reheating era are generated by different \( f(R) \) gravities, which plays an important role for the calculation, since the parameter \( a_M \) is essentially different for these two eras. Our numerical analysis indicates the following, the parameter \( D \) for the redshift interval \( z = [3.86621 \times 10^{15}, 3.86621 \times 10^{16}] \) is \( D = -14.5063 \) while for the redshift interval \( z = [3.86621 \times 10^{28}, 3.86621 \times 10^{31}] \) it is equal to \( D = 8.169 \). Thus the abnormal reheating leads to an amplification of the GR waveform of the order \( O(10^6) \), however the de Sitter pre-inflationary bounce causes a damping of the order \( O(10^{-4}) \). Thus the damping effect of the de Sitter pre-inflationary bounce is significant. To have a quantitative idea on how the pre-inflationary bounce affects the energy spectrum of the primordial gravitational waves, in Fig. 2 we plot the \( h^2 \)-scaled gravitational wave energy spectrum for the combined \( f(R) \) gravity model with a primordial de Sitter bounce era (upper plot) and without the pre-inflationary bounce era (bottom plot), for three reheating temperatures, namely for \( T_R = 10^{12}\text{GeV} \) (purple curves), to \( T_R = 10^7\text{GeV} \) (red curves) and (blue curves) \( T_R = 10^2\text{GeV} \). As it can be seen in the two plots of Fig. 2, in the absence of the pre-inflationary bounce, the gravitational wave signal is detectable from all the gravitational wave experiments for all the reheating temperatures, however the effect of the pre-inflationary bounce causes significant damping, and in effect it will be detectable only from one detector, the BBO, and only if the reheating temperature is larger than \( 10^7\text{GeV} \). This
V. CONCLUSIONS

In this paper we calculated the effect of a primordial de Sitter bounce on the energy spectrum of the primordial gravitational waves. Particularly we assumed that pre-inflationary the Universe experienced a de Sitter bounce phase, which is followed by a quasi-de Sitter slow-roll inflationary era, followed by a constant EoS parameter reheating era, which is followed by the standard radiation and matter domination eras, up to the late-time acceleration eras. So we assumed several cosmological evolution patches for the Universe, and we also assumed that these evolution patches are realized by vacuum $f(R)$ gravity or synergistically by $f(R)$ gravity and the perfect matter fluids. Specifically, vacuum $f(R)$ gravity generates the primordial bounce and the slow-roll inflationary eras, while the synergy of $f(R)$ gravity and the perfect fluids, realize the abnormal reheating era and the late-time acceleration eras. The quasi-de Sitter inflationary era is realized by an $R^2$ while for the rest of the eras, we investigated which $f(R)$ gravity may realize these eras, by using well-known reconstruction techniques. Then by having available the $f(R)$ gravities which realize each era, we calculated the overall effect of the primordial de Sitter bounce on the spectrum of the primordial gravitational waves, and we compared the results with the case in which the primordial bounce is absent. As we demonstrated, the primordial de Sitter bounce causes a severe damping of the energy spectrum of the primordial gravitational waves, which in the model we studied the signal would be detectable only by one future gravitational wave detector. This result is of profound importance, for the following reason: if a signal is detected in some but not all the future gravitational wave detectors, this scenario may be explained by the presence of a primordial bounce cosmology. In fact, this maybe more plausible than other scenarios which may explain the absence of a signal in some frequency range. This is because if the signal is detected only by some detectors in some frequencies, this signifies probably a global mechanism affecting all frequencies, and not a physical mechanism corresponding to a specific frequency range, such as supersymmetry breaking at some time instance during the reheating era. Thus, with this paper we offered another perspective on primordial gravitational waves physics, and one thing is certain, the detection of a signal may have multiple theories and effects that may describe it. Therefore, the plot thickens with primordial gravitational waves. Before closing, let us further comment on an interesting issue. It is known that in the bouncing cosmology scenario, there will be a huge increase of anisotropies, which eventually could give rise to large secondary gravitational waves sources, which are induced by linear scalar perturbations. This may even exceed the linear gravitational waves. This effect could also be investigated in future extensions of this work and requires a combination of linear and non-linear sources of gravitational waves. In this article however we merely focused on linear gravitational waves effects, so for modes with wavelengths well above $10^{-10}$ Hz.
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