Experimental Investigation and Bifurcation Set Equation Modeling of Chip-Splitting Catastrophe

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Experimental investigation and bifurcation set equation modeling of chip-splitting catastrophe

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Abstract

This paper presents the methods of designing experiments for observation of the critical conditions of chip-splitting catastrophe (CSC) based on the dichotomy principle and the methodology of modelling CSC with a bifurcation set equation on the basis of catastrophe theory. 355 groups of experiments are carried out for observing critical conditions of CSC in the symmetrical straight double-edged cutting. It is found that the occurrence of CSC is of obvious regularity. Besides, the specific cutting force is reduced by a maximum of 64.68% after CSC. The bifurcation set equation is established, which can predict the critical conditions of CSC in the symmetrical cutting with a straight double-edged tool with any combinations of cutting edge angles and rake angles. With verification,

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the predicted values of the critical cutting thickness of CSC based on the established equation are in good agreement with experimental values and yield an average absolute prediction error of 5.34%, which is satisfactorily low. This study lays the groundwork for energy-saving optimization of tool structure and process parameters through reasonable utilization of CSC.

**Keywords:** Chip-splitting catastrophe; Critical conditions; Bifurcation set; Double-edged cutting; Swallowtail catastrophe

1 Introduction

The chip-splitting catastrophe (CSC) is a kind of abrupt change in chip morphology, which can often be observed when cutting with a double-edged tool and the technological (control) parameters changing continuously. For example, when using a straight double-edged tool with a 45° cutting edge angle to symmetrically transverse feed an AISI 1045 steel disc workpiece at a specific spindle speed and feed rate, the following CSC can be observed: during the cutting process, when the cutting speed continuously reduces to a certain critical value, a single strip chip that remains intact will suddenly split from the middle, from one strip chip to two strip chips, and eventually flow out in two different directions [1].

It is different from artificial chip splitting in cutting with multi-tooth cutting tools with chip-splitting structure, such as multifacet drills [2], milling cutters [3], a dedicated cutting tool in dry orbital drilling process [4] and broaching [5], etc., where artificial chip splitting is caused by the discontinuity of the cutting layout [6]. CSC, however, is generated by the inherent law of the cutting process, which belongs to the natural chip splitting [1]. Furthermore, the cutting force caused by the CSC is substantially reduced (reduced much larger than the artificial chip splitting) [7], which makes people aware of the considerable utilization potential of the CSC in energy-saving machining and in the design of energy efficient cutting tools. Therefore, an in-depth study of systematic experimental investigation of the critical conditions and the modeling technique of CSC in double-edged cutting is of great significance for promoting the development of metal cutting theory and energy conservation and
discharge reduction technology, and to solve the practical engineering problems. For example, excessive deformation of the workpiece [8], tool wear and other issues produced by superfluous cutting force in the processing of numerous thin-walled parts [9] and crucial parts, such as the turning of oil well thin-walled pipe joint/casing thread of high-strength steel [10] and in the machining of aircraft landing gear in the petroleum and aviation industries.

For a long time, people have been committed to the research on rules of change in chip morphology and on modeling techniques under different processing conditions and have made fruitful achievements. For instance, through his experimental observation of chip morphology in the machining of EN16MnCr5 Steel with the straight double-edged tool, Monkova et al. [11] found that the larger the cutting thickness and the edge inclination angle, the longer the chip. Polvorosa et al. [12] compared the tool wear and its corresponding chip morphology when turning Alloy 718 with cemented carbide inserts. They concluded that tool wear can be predicted by observing the chip morphology under specific processing conditions. During the turning experiment of AISI 1045 steel, Dilip Jerold [13] reported that using cryogenic carbon dioxide as the cutting fluid can reduce the chip thickness leads to the decrease of chip compression ratio and cutting force, and thus a better surface quality of the workpiece. Based on the experimental data, Patwari et al. [14] developed the chip serration frequency prediction model in high-speed end milling of S45C steel using TiN inserts, and its prediction accuracy was 95%. Under other operating conditions, such as the step shoulder down-milling of Ti-6Al-4V [15], the combination of friction drilling and form tapping process for making ‘nutless’ bolted joints [16] and the orthogonal cutting of hardened low alloy steel 51CrV4 + Q [17], plentiful valuable research findings regarding chip morphology have also been yielded. Unfortunately, these findings were obtained under the condition of fixed process parameters and do not involve the sudden change of chip morphology, so they can not be applied to the modeling of CSC.

The research on CSC began 50 years ago. In his experiment with symmetrical V-shaped double edge cutting tool in a V-trough for right angle fixed width cutting workpiece, Luk [18] observed that
when the ratio between incomplete deep cut thickness (2nd cut) and complete deep cut thickness (1st cut) is gradually reduced to less than 0.1, CSC will occur during the 2nd cutting. On this basis, Yamamoto and Nakamura [19] have further experimentally studied CSC by using the straight double-edged V-shaped tools with two different cutting edge angles and three different rake angles combinations to plane brass blocks with five different widths and achieved six sets of CSC critical conditions with the same rake angle. It was also identified that the specific cutting force in CSC was up to 30% lower than that in no CSC. A simple mechanical model was established in their investigations to qualitatively explain the mechanism of CSC. To examine the influence of CSC on process parameter design, Shi [20] conducted cutting experiments by traversing low-carbon steel disc workpieces with a high speed steel straight double-edged tool with different cutting edge angles, maintaining both edge inclination angle and rake angle at 0. It was discovered that the cutting force before and after the CSC precipitously decreased, with the maximum reduction up to 40.55%. His research also found that the CSC is more likely to occur as the cutting edge angle becomes larger and the rake angle and cutting thickness becomes smaller within a certain range of process parameters. In utilizing the same style tools with a non-zero degree in the cutting edge inclination angle and rake angle to perform similar experiments, Bi et al. [21] discovered that the occurrence of CSC led to a lowering of the maximum specific cutting force by 27.78%. Rezayi Khoshdarregi and Altintas [22] also noticed CSC in thread turning experiments with V-profile insert, but did not conduct further investigation on that.

Obviously, the above mentioned studies on CSC have all been in the stage of preliminary experimental observations and qualitative explanations of the mechanism on CSC. There were no systematic experiments to obtain complete experimental data for the critical conditions of CSC, nor can a mathematical model of CSC be established based on current experimental data, to realize accurate prediction of the critical conditions for CSC. One of the main factors for this is that people, for the longest time, have been employing calculus approach to research the continuous and gradual
changing phenomena of metal cutting [23], but calculus approach is not suitable for the research on cutting catastrophe phenomenon which is non-continuous and abrupt changing like CSC.

The mathematical tool applicable to the modeling of cutting catastrophe phenomenon is the catastrophe theory [24], which was established in the 1970s. It adopts seven standard catastrophe mathematical models, which are composed of the potential function, the manifold surface equation and the bifurcation set equation (to predict the critical conditions of catastrophe) to describe plenty of catastrophe phenomena when control variables change continuously [25]. Catastrophe theory and its two recommended modeling methods, theoretical modeling and experimental modeling methods, have been successfully applied to modeling, control and utilization of catastrophe phenomena in many different fields [26].

In the field of mechanical engineering, the successful application of theoretical modeling methods based on catastrophe theory can be seen in many catastrophe phenomena, such as shear angle catastrophe [27], friction coefficient catastrophe [28], chip flow angle catastrophe phenomenon when cutting with a double-edged tool of 0 edge inclination angle [29] and of arbitrary edge inclination angle [30], etc. In the aspect of experimental modeling research based on experimental data, Luo [31] assumed that the relationship between the actual control variables of the abrupt change of tool wear state and the theoretical control variables of cusp catastrophe [24] was a linear function, the coefficients of the functions were fitted according to the experimental data, and the cusp catastrophe model of tool wear state catastrophe was established. The prediction error of this model for tool wear value was less than 10%. Unfortunately, there have not been any reports on the modeling and prediction of the CSC exercising catastrophe theory.

In this paper, the complete data for the critical conditions of CSC are obtained by systematic experimental study on CSC. According to this result, based on the standard bifurcation set equation of the swallowtail catastrophe [24] proposed by the catastrophe theory, this paper assumes that the mapping from the actual control variables of CSC to the theoretical control variables of swallowtail
catastrophe is a linear function. The 12 coefficients of the functions are fitted by means of modern optimization technique. The bifurcation set equation of CSC suitable for straight double-edged tools with any combinations of cutting edge angles and rake angles is established. Through actual cutting experiments, the effectiveness of the bifurcation set equation is verified, and the accurate prediction of the critical cutting thickness of CSC is realized.

2 Experimental investigation and acquisition of experimental data for critical conditions of CSC

The investigated material was a normalized medium carbon steel AISI 1045 (C45E+N). Table 1 shows the chemical composition of the batch as received in weight-% [32].

| Chemical composition of steel AISI 1045 in weight-% |
|--------------------------------------------------|
| C    | Si   | Mn  | P    | S    | Mg  | Cr  | Ni  | Mo  |
| 0.450| 0.176| 0.602| 0.018| 0.027| <0.0005| 0.065| 0.026| 0.003|
| Cu   | Al   | Ti  | V    | Co   | As  | Sn  | N   | Fe  |
| 0.096| 0.016| 0.0005| 0.0011| 0.0039| 0.0029| 0.006| 0.002| >98.5|

It was proposed to use the approach in shooting and analyzing the three-round chip shape video for the HSS straight double-edged tool transverse feeding the AISI 1045 steel disc workpieces (Fig. 1) to examine whether the CSC occurs under given cutting conditions, and to obtain the experimental data on the critical conditions of CSC, namely, the cutting edge angle $\theta$, rake angle $\gamma_0$, and cutting thickness $a_c$ during CSC, and at the same time measure the three-dimensional dynamic cutting force.

![Fig. 1 Schematic diagram of symmetrical transverse cutting](image)

AISI 1045 steel disc workpiece with the HSS straight double-edged tool
2.1 Experimental design

2.1.1 Experimental devices

The experiment was carried out on a CW6163E precision lathe equipped with a stepless frequency regulator. The experimental setup consisted of a disc workpiece, a tool, a video recording device and a Kistler turning force measuring system as shown in Fig. 2. Wherein, the diameter of the disc workpiece was 180 mm, the thickness was \( b \), and it was clamped between the scroll chuck and the tailstock; the tool was a self-made straight double-edged tool with a cutting edge angle \( \theta \) and a rake angle \( \gamma_0 \). The tool holder was horizontally mounted, perpendicular to the workpiece axis to ensure that the corner passed through the spindle rotation center; the video recording device was an iPhone 6 mobile phone with a slow-motion recording function, which was fixed by a bracket placed on the machine's cross slide carriage and at a guaranteed level; Kistler turning force measurement system consisted of a Kistler 9257A three-dimensional force gauge, a Kistler 5070 charge amplifier, a Kistler 5697 data acquisition system and a DynoWare data analysis software.

![Fig. 2 Schematic diagram of experimental devices and cutting force measurement system](image)

2.1.2 Experimental conditions and related instructions

1) Dry cutting was adopted to facilitate the observation and video shooting of chip morphology. The cutting speed was set to \( V = 5 \) m/min to avoid the formation of the built-up edge [33]. Similar low speeds are also often used in processing operations such as thread turning, planing and broaching [34].

2) To ease the comparison of experimental results, the cutting edge inclination angle \( \lambda_s \) of all tools
in the experiment was set to 5°, and with the cutting width $a_w$ uniformly set at 3 mm for both cutting edges of all the tools in the experiment. In this way, the workpiece thickness $b$ and the edge angle $\theta$ of the disc workpiece must satisfy:

$$b = 2a_w\cos\theta = 6\cos\theta$$

That is to say, the thickness $b$ of the workpiece must be calculated and determined according to the edge angle $\theta$ set in the experiment.  

3) During the experiment, the adjustment of the cutting thickness $a_c$ was realized by changing the transverse feed $f$ of the machine tool, and

$$f = \frac{a_c}{\cos\theta}$$

For the CW6163E precision lathe, the adjustment of $f$ is graded, so the adjustment of $a_c$ is also graded. Due to this limitation, only an approximate value of the critical cutting thickness can be obtained by the cutting experiment.  

4) The cutting angle parameters of the self-made cutting tool for the experiment, i.e., the cutting edge angle $\theta$ and the rake angle $\gamma_0$, were picked up by manually adjusting the rotation angle of the universal clamp when grinding the high speed steel straight double-edged tools.  

2.2 Methods and processes for obtaining experimental data of critical conditions  
As mentioned above, three-round cutting experiments were used to gain the critical conditions of CSC, namely, the cutting edge angle $\theta$, rake angle $\gamma_0$, and cutting thickness $a_c$. The specific methods and processes are as follows:

1) Taking full advantage of the discoveries made by previous researchers on the regularity of the occurrence of the CSC [1] and the related experimental experience, and with sufficient consideration given to the minimum interval between two consecutive feedings in the machine tool, the dichotomy method was applied to gradually reduce the interval of control variable value and experimental value interval used in three-round experiments. The experimental data of critical conditions for CSC were procured.  

2) After appropriate amplification, the interval ranges of the cutting edge angle $\theta$, the rake angle...
$\gamma_0$ and the cutting thickness $a_c$ of the three control variables were set according to the range of approximate critical conditions of CSC obtained in the prior experimental research. For $\theta \in [40^\circ, 60^\circ]$, $\gamma_0 \in [-20^\circ, 0^\circ]$, $a_c \in [0.05, 0.20]$ mm, gave three control variables to 5, 5, and 4 level values with equal spacing, i.e., cutting edge angle $\theta$: 40°, 45°, 50°, 55°, and 60°, rake angle $\gamma_0$: -20°, -15°, -10°, -5°, and 0°, cutting thickness $a_c$: 0.05 mm, 0.10 mm, 0.15 mm, and 0.20 mm. The corresponding tool number used in the first round of experiments, the accessible machine feed value $f$, the actual cutting thickness $a_{ce0}$ calculated in reverse according to the $f$ value, and the workpiece width $b$ calculated according to the cutting edge angle $\theta$ are all shown in Table 2.

| Cutting edge angle $\theta$ (deg) | Actual cutting thickness $a_{ce0}$ (mm) | Feed $f$ (mm/r) | Workpiece thickness $b$ (mm) | Cutting edge angle $\theta$ (deg) | Actual cutting thickness $a_{ce0}$ (mm) | Feed $f$ (mm/r) | Workpiece thickness $b$ (mm) |
|----------------------------------|----------------------------------------|----------------|-----------------------------|----------------------------------|----------------------------------------|----------------|-----------------------------|
| 40                               | 0.0498                                 | 0.065          | 4.60                        | 45                               | 0.0495                                 | 0.070          | 4.25                        |
|                                  | 0.0996                                 | 0.130          |                             |                                  | 0.0990                                 | 0.140          |                             |
|                                  | 0.1532                                 | 0.200          |                             |                                  | 0.1414                                 | 0.200          |                             |
|                                  | 0.1922                                 | 0.260          |                             |                                  | 0.1980                                 | 0.280          |                             |
| 50                               | 0.0546                                 | 0.085          | 3.86                        | 55                               | 0.0487                                 | 0.085          | 3.00                        |
|                                  | 0.0964                                 | 0.150          |                             |                                  | 0.0975                                 | 0.170          |                             |
|                                  | 0.1446                                 | 0.225          |                             |                                  | 0.1491                                 | 0.260          |                             |
|                                  | 0.1928                                 | 0.300          |                             |                                  | 0.1950                                 | 0.340          |                             |
| 60                               | 0.0500                                 | 0.100          | 3.00                        | 60                               | 0.1500                                 | 0.300          | 3.00                        |
|                                  | 0.1000                                 | 0.200          |                             |                                  | 0.2000                                 | 0.400          |                             |

3) Twenty-five straight double-edged tools were ground (Fig. 3) according to the full factor experimental design method, and four cutting experiments were completed with each tool in the first round of experiment, which is 25 × 4 experiments in total for all tools to observe and judge whether CSC occurred in the cutting process. If all four experiments of a tool occur CSC, or all do not occur CSC, it indicates that the critical cutting thickness is not within the value range, and the cutting experiment of the tool ends here; if both CSC and no CSC are observed in the four experiments of a tool, the two adjacent cutting thicknesses that have and do not have CSC are set as new cutting thicknesses interval. The second round cutting experiment is carried out with this tool.
4) For the tools that need the second round of cutting experiments, use 3 level values with equal spacing within the value range of cutting thickness to accomplish another three experiments with the same method as step 3 (experiments for two of the three boundary level values have already been completed, only the experiment for the middle-level value needs to be done). The results observed from these three experiments will not be the same, that is, they will not be all chip-splitting, nor will they be all non-chip-splitting at the same time. Using the same method as in step 3, the boundary value of the renewed cutting thickness is obtained; and from the boundary values choose the one with CSC as the critical cutting thickness of the tool and then is added to the list of obtained critical conditions together with the corresponding edge angle $\theta$ and rake angle $\gamma_0$.

5) After the completion of the second round of experiments, if any set of critical condition data in the critical condition list is the same as two of the three cutting parameters of a non-chip-splitting experiment in the previous two rounds of experiments, i.e., the cutting thickness $a_c$ and the rake angle $\gamma_0$, or the cutting thickness $a_c$ and the edge angle $\theta$ are the same, and the other tool angle parameters (edge angle $\theta$ or rake angle $\gamma_0$) is different but adjacent, then the critical value of this angle parameter should be included within the two adjacent values. This group of critical condition data would be removed from the list of critical conditions, and use the two values from this angle parameter as the boundary values of the new value range. From this angle parameter, such as the rake angle $\gamma_0$, 3
horizontal values (two of which are interval boundary values) are obtained using the equal interval principle, and to grind out a new tool with intermediate horizontal values. The cutting thickness $a_c$ of the new tool is the critical cutting thickness obtained from the previous two tools. A third round of the experiments is carried out with this new tool.

6) Based on the results from observations in the third round of cutting experiments, in line with the same method mentioned above, we can obtain a new boundary value for the rake angle $\gamma_0$, and taking the greater absolute value of the two boundary values to be the critical value of the rake angle $\gamma_0$ (or cutting edge angle $\theta$), together with the corresponding cutting edge angle $\theta$ (or rake angle $\gamma_0$) and cutting thickness $a_c$, combine them into the list of obtained critical conditions.

2.3 Experimental results
2.3.1 Observed chip-splitting catastrophe phenomenon

A total of 122 cutting experiments were performed successfully in 3 rounds, of which 41 observed with CSC and 81 without. It was also discovered that when the CSC occurred, each of the two cutting edges produced a chip, and the two chips were curled and discharged along the rake face in a V-shaped form in the direction of basic symmetry relative to the tool tip, as shown in Fig. 4. Moreover, when its length reached a certain value, the chip would break naturally from near the tool tip and some of them remained V-shaped, which was shown in Fig. 5. This scenario is entirely consistent with the experimental description from previous researchers regarding the phenomenon of CSC [18].

To verifying whether the occurrence of CSC is affected by accidental factors, portions of the experiments were repeated 1 - 3 times. The results show that within experimental range, chip-splitting or non-chip-splitting are repeatable and stable.
Fig. 4 Chip-splitting Catastrophe observed in the experiments: Cutting edge angle $\theta = 45^\circ$, Rake angle $\gamma_0 = -10^\circ$, Critical cutting thickness $a_{ce} = 0.0495\text{mm}$

Fig. 5 The chip morphology after chip-splitting catastrophe: Cutting edge angle $\theta = 45^\circ$, Rake angle $\gamma_0 = -10^\circ$, Critical cutting thickness $a_{ce} = 0.0495\text{mm}$
2.3.2 The impact of tool geometry parameters on CSC

It is found that CSC is affected by both tool geometry parameters and cutting conditions. Among them, of several tool angles of tool geometries which have an impact on CSC, the two deserving of attention are the cutting edge angle and the rake angle. Firstly, in a given process parameters frame, CSC is more likely to occur as the cutting edge angle becomes more immense. This is aligned with the research results of Shi [1] as mentioned above. The main reason for this phenomenon probably lies in that the larger the cutting edge angle is, the higher the degree of deviation from the natural chip ejection direction of the chips generating from each of the two straight cutting edges to the chip ejection direction of all the chips produced by the entire tool to be discharged as a whole, that is to the direction of the symmetrical line of two cutting edges. Thus the severer the chip-ejection interference is, the more tremendous the force to maintain all the chips generated by the entire tool to be discharged as a whole. When this force exceeds the tear strength of the workpiece material, CSC becomes inevitable.

Secondly, within the given range of process parameters, CSC is less likely to be observed when cutting with a tool with a positive rake angle. However, when cutting with a tool with a negative rake angle, CSC is more often to be observed, and the larger the absolute value of the negative rake angle is, the more likely the CSC is to be observed. This accords substantially with the conclusion drawn by Luk [18], but it is not in perfect accord with the results observed on the cutting experiments of brass by Yamamoto and Nakamura [19]. Yamamoto and Nakamura [19] reported that CSC is also easy to occur under the condition of cutting with a tool with big positive rake angles. This shows the mechanism of CSC is worth further research.

The further cutting experiments found that the CSC was unexceptionally observed when we used the cutting tools which were made of different materials, such as high-speed steel tools, cemented
carbide inserts and PCD tools to cut workpieces of various materials like AISI 1045 steel, copper and aluminium alloy. The implication is that the CSC investigated herein shows a certain degree of universality. The experiments conducted by Luk [18] and Yamamoto and Nakamura [19] also support this view.

2.3.3 Experimental data of critical conditions of CSC

Using the aforementioned method, 22 groups of experimental data on the critical conditions of CSC for symmetrical transverse cutting of AISI 1045 steel disc workpieces with straight double-edged tools were obtained. These data were specifically listed in Table 3 as the middle row of each group of data and marked in bold characters. Table 3 also lists the boundary values of the control variables from the second and third rounds of experiments corresponding to these 22 groups of experimental data (placed above or below the bold letter row of the experimental critical condition data), along with the cutting force and chip-splitting states. It should be noted that the critical cutting thickness $a_{ce}$ in Table 3 was obtained by reverse calculation according to the actual feed amount $f$ and the cutting edge angle $\theta$, keeping to four places after the decimal.
Table 3
 Cutting force and chip-splitting states corresponding to the critical conditions on the CSC and its boundary values

| Data number | Rake angle | Cutting edge angle | Critical cutting thickness | Specific main cutting force | Specific cutting force | Y  | Data number | Rake angle | Cutting edge angle | Critical cutting thickness | Specific main cutting force | Specific cutting force | Y  |
|-------------|------------|--------------------|----------------------------|-----------------------------|------------------------|----|-------------|------------|--------------------|----------------------------|-----------------------------|------------------------|----|
| No.         | γ0 (°)     | θ (°)              | \(a_{ce}\) (mm)            | \(F_t\) (N/mm²)             | \(F_c\) (N/mm²)        |    | No.         | γ0 (°)     | θ (°)              | \(a_{ce}\) (mm)            | \(F_t\) (N/mm²)             | \(F_c\) (N/mm²)        |    |
| 1           | -15 40     | 0.0498             | 3635                       | 2544                        | Y 12                    | -5 | 55          | 0.0516     | 3298               | 1712                        | Y                           | 1550                   |    |
| -13 40      | 0.0498     | 3537               | 2155                       | Y 4                         | 55                       | 0.0516     | 3349                    | 1473                    | Y                             |
| -10 40      | 0.0498     | 3477               | 3126                       | N                           | 0                        | 55                       | 0.0516     | 5039               | 2190                        | N                           | 1516                   |    |
| 2           | -15 40     | 0.0498             | 3635                       | 2544                        | Y 13                     | -10| 55         | 0.0516     | 3388               | 1744                        | Y                           | 1516                   |    |
| -15 40      | 0.0766     | 3218               | 2252                       | Y -10                       | 55                       | 0.0803     | 2592                    | 1727                    | Y                             |
| -15 40      | 0.0996     | 4404               | 2619                       | N -10                       | 55                       | 0.0975     | 5176                    | 3844                    | N                             |
| 3           | -10 40     | 0.0498             | 3477                       | 3025                        | N 14                     | -15| 55         | 0.0975     | 3133               | 1692                        | Y                           | 1516                   |    |
| -10 43      | 0.0512     | 3395               | 2021                       | Y -12                       | 55                       | 0.0975     | 3056                    | 1574                    | Y                             |
| -10 45      | 0.0512     | 3382               | 1810                       | Y -10                       | 55                       | 0.0975     | 5176                    | 3844                    | N                             |
| 4           | -20 45     | 0.0990             | 1928                       | 1367                        | Y 15                     | -20| 55         | 0.1459     | 5351               | 4216                        | Y                           | 1516                   |    |
| -18 45      | 0.0990     | 3145               | 1524                       | Y -18                       | 55                       | 0.1459     | 5516                    | 4772                    | Y                             |
| -15 45      | 0.0990     | 4798               | 3259                       | N -15                       | 55                       | 0.1459     | 5206                    | 3860                    | N                             |
| -15 45      | 0.0495     | 3673               | 3441                       | Y 16                        | -10                       | 55                       | 0.0975     | 5176               | 3844                        | N                           | 1516                   |    |
| -15 45      | 0.0707     | 3614               | 2181                       | Y -10                       | 58                       | 0.1007     | 2898                    | 2349                    | Y                             |
| -15 45      | 0.0990     | 4798               | 3259                       | N -10                       | 60                       | 0.1007     | 2915                    | 1703                    | Y                             |
| 6           | -10 45     | 0.0495             | 3498                       | 1872                        | Y 17                     | -15| 55         | 0.1491     | 5094               | 3777                        | N                           | 1516                   |    |
| -8 45       | 0.0495     | 3441               | 2057                       | Y -15                       | 58                       | 0.1484     | 2782                    | 2325                    | Y                             |
| -5 45       | 0.0495     | 4923               | 2532                       | N -15                       | 60                       | 0.1484     | 2782                    | 2260                    | Y                             |
| 7           | -10 50     | 0.0482             | 3658                       | 2099                        | Y 18                     | -5 | 60         | 0.0500     | 3470               | 2923                        | Y                           | 1516                   |    |
| -6 50       | 0.0482     | 3510               | 1317                       | Y -5                        | 60                       | 0.0750     | 3078                    | 1816                    | Y                             |
| -5 50       | 0.0482     | 5425               | 1829                       | N -5                        | 60                       | 0.1000     | 5083                    | 5140                    | N                             |
| 8           | -10 50     | 0.0482             | 3658                       | 2099                        | Y 19                     | -10| 60         | 0.1000     | 2935               | 1715                        | Y                           | 1516                   |    |
| -10 50      | 0.0707     | 3484               | 2494                       | Y -8                        | 60                       | 0.1000     | 2915                    | 2555                    | Y                             |
| -10 50      | 0.1010     | 4625               | 3592                       | N -5                        | 60                       | 0.1000     | 5083                    | 5140                    | N                             |
| 9           | -15 50     | 0.1010             | 2893                       | 1792                        | Y 20                     | -10| 60         | 0.1000     | 2935               | 1715                        | Y                           | 1516                   |    |
| -13 50      | 0.1010     | 2965               | 1789                       | Y -10                       | 60                       | 0.1200     | 2785                    | 1532                    | Y                             |
| -10 50      | 0.1010     | 3437               | 3592                       | N -10                       | 60                       | 0.1500     | 4932                    | 4109                    | N                             |
| 10          | -15 50     | 0.1010             | 2893                       | 1792                        | Y 21                     | -15| 60         | 0.1500     | 2725               | 2236                        | Y                           | 1516                   |    |
| -15 50      | 0.1221     | 2864               | 1906                       | Y -13                       | 60                       | 0.1500     | 2696                    | 1861                    | Y                             |
| -15 50      | 0.1446     | 4832               | 4157                       | N -10                       | 60                       | 0.1500     | 4932                    | 4109                    | N                             |
| 11          | -5 50      | 0.0546             | 4789                       | 1615                        | N 22                     | -15| 60         | 0.1500     | 2725               | 2236                        | Y                           | 1516                   |    |
| -5 53       | 0.0512     | 3324               | 2116                       | Y -15                       | 60                       | 0.1700     | 2656                    | 1830                    | Y                             |
| -5 55       | 0.0512     | 3324               | 1725                       | Y -15                       | 60                       | 0.2000     | 5124                    | 3078                    | N                             |

Note: 1) “Y” means there is CSC; “N” means there is not CSC. 2) The 22 groups of experimental critical condition data are the middle row of each group of data, marked with bold letter. 3) The boundary values of the control variables are placed above or below the bold letter row of the experimental critical condition data. 4) The reduction of cutting force mentioned in the text is underlined and bolded.
2.3.4 Cutting force before and after CSC

Analyzing the cutting force data in Table 3, the following conclusions can be obtained:

1) For the same tool (data numbers in Table 3 are 2, 5, 8, 10, 13, 18, 20, and 22), its specific main cutting force and specific feed force during CSC were much reduced than those without CSC. Between them, the reduction range of the specific main cutting force was 24.67% (8) - 49.92% (13), while the range of reduction of the specific feed force was 14.01% (2) - 64.68% (18). It has also been remarked that the tool with the data number of 13 had the largest reduction of 49.92% in the specific main cutting force, and tool number 18 had the largest reduction of 64.68% in the specific feed force.

2) Given the same cutting edge angle and cutting thickness (data numbers in Table 3 are 1, 4, 6, 7, 9, 12, 14, 15, 19, and 21), tools with a larger negative rake angle accompanied by CSC had smaller specific main cutting force and specific feeding force when compared to tools with a smaller negative rake angle and without CSC. Among them, as data number 14 shows, for the tool with a larger negative rake angle (-12°) and CSC, when compared to tool with a smaller rake angle (-10°) and no CSC, the specific main cutting force and the specific feed force were reduced by 40.95% and 59.05% respectively.

3) Given the same cutting edge angle and cutting thickness (data numbers in Table 3 are 3, 11, 16, and 17), tools with a larger cutting edge angle and CSC had a smaller specific main cutting force and specific feed force when compared to tool with a smaller cutting edge angle and no CSCs among them, as data number 17 shows, for the tool with a larger cutting edge angle (58°) and CSC, when compared to the tool with a smaller cutting edge angle (55°) and without CSC, the specific main cutting force and the specific feeding force were reduced by 45.39% and 38.45% respectively after the CSC.

The above conclusions evidently deviate from people's common understanding of the cutting process, but are consistent with the previous researchers' experimental results on CSC [1]. These outcomes again testified that CSC has great application potential in the energy-saving design of tool structure and cutting parameter optimizations.
3 Experimental data based model establishment of the bifurcation set equation of CSC

3.1 Modeling principle and method of bifurcation set equation in CSC

3.1.1 Introduction to experimental modeling method of catastrophe phenomena based on catastrophe theory

It is pointed out in the catastrophe theory that any catastrophic phenomena can be described by a mathematical model consisting of a potential function ($E$) determined by control variables and state variables, manifold surface ($M$) equation determined by the potential function and a bifurcation set ($B$) equation containing only the control variables and capable of predicting the critical conditions of the catastrophe. Catastrophe phenomena can be divided into seven types under the conditions of no more than four control variables and no more than two state variables, and the mathematical model of each catastrophe type can be converted into a standard form by a specific mapping function [35].

There are two methods to establish the mathematical model of catastrophe, namely, the theoretical modeling method and the experimental modeling method. The experimental modeling method is to select a kind of standard form of catastrophe mathematical model according to the number of independent actual change control variables and state variables observed in the experiments. It is assumed that there is a functional mapping relationship between the theoretical control variables and the actual control variables used in the standard mathematical model. Based on the experimental data with finite points on the bifurcation set curve/surface ($B$) or manifold surface ($M$), coefficients of these mapping functions are computed, and then the complete mathematical model of the catastrophe is established. In this paper, the experimental modeling method is used to establish the bifurcation set equation of the CSC when the straight double-edged tool symmetrically transverse feeds the AISI 1045 steel disc workpieces.

3.1.2 Modeling of bifurcation set equation of CSC based on the swallowtail catastrophe

It is shown in previous experiments that when a straight double-edged tool symmetrically transverse feeds an AISI 1045 steel disc workpieces, the cutting conditions that can be controlled artificially will affect the generation of CSC [1], such as the cutting edge angle $\theta$, rake angle $\gamma_0$, cutting edge inclination angle $\lambda_s$, cutting speed $V$, cutting width $a_w$, and cutting thickness $a_c$, etc. In order to
simplify the problem, with the help of the prior research experience and observation results of multi-
round related experiments completed by the author, only the cutting edge angle $\theta$, rake angle $\gamma_0$, and
cutting thickness $a_c$ are taken as control variables, whereas the CSC occurrence is taken as the state
variable, and the remaining cutting conditions are fixed. The aforementioned experimental modeling
method of the catastrophe phenomenon is used to establish the model of the bifurcation set equation
of CSC.

The catastrophe theory asserts that when the number of control variables is 3 and the number of
state variables is 1, any actual catastrophic phenomena all belong to the swallowtail catastrophe within
the 7 standard catastrophe types, and its mathematical model can be expressed in the following
standard form through topological equivalence transformation, namely the differential homomorphism
[36], where $u$, $v$, and $w$ are the theoretical control variables, and $x$ is a theoretical state variable:

$$\begin{align*}
E : E &= x^5 + ux^3 + vx^2 + wx \\
M : 5x^4 + 3ux^2 + 2vx + w &= 0 \\
B : 4096u^6 + 46629v^4 + 4096w^3 &= 0 
\end{align*}$$

Based on this, the approaches to establish the bifurcation set equation model of CSC are as follows:
the mapping function between actual control variables and theoretical control variables is presumed to
be the linear function of the actual control variables with coefficients to be determined [37]. Then a set
of experimental data on the critical conditions of the CSC are collected through systematic cutting
experiments, that is, the coordinates of a series of points on the surface of the bifurcation set of CSC
(referred to as the experimental points); and the coordinates of the experimental points are substituted
into the mapping function to obtain the coordinates of the corresponding theoretical points. By
substituting the coordinates of these theoretical points into the bifurcation set equation $B$ in equation
(3), a set of bifurcation set equations $B'$ expressed in experimental coordinates with undetermined
coefficients is deduced. Take the sum of the squares of the left-hand polynomial of these equations $B'$,
and get a function indicated by the aforementioned undetermined coefficients and the coordinates of
the experimental points. Taking this as the objective function, the undetermined coefficients which
make the objective function take the minimum value are gained by the optimization method.

3.2 Mapping function and objective optimization function for solving its coefficients

Referring to the relevant exposition of catastrophe theory, the mapping function between the cutting edge angle $\theta$, the rake angle $\gamma_0$, and the cutting thickness $a_c$ to the three theoretical control variables, $u, v$, and $w$ was considered as a set of linear functions (Saunders, 1980), that is

\[
\begin{align*}
    u &= f_1(\theta, \gamma_0, a_c) = p_1 + p_2 \theta + p_3 \gamma_0 + p_4 a_c = h_1(X, p_1, p_2, p_3, p_4) \\
    v &= f_2(\theta, \gamma_0, a_c) = p_5 + p_6 \theta + p_7 \gamma_0 + p_8 a_c = h_2(X, p_5, p_6, p_7, p_8) \\
    w &= f_3(\theta, \gamma_0, a_c) = p_9 + p_{10} \theta + p_{11} \gamma_0 + p_{12} a_c = h_3(X, p_9, p_{10}, p_{11}, p_{12})
\end{align*}
\]

(4)

Where $X = (\theta, \gamma_0, a_c), p_i \in R, i=1, 2, 3, ..., 12$. By substituting $u, v$, and $w$ of equation (4) into the bifurcation set equation of the standard swallowtail catastrophe mathematical model [38], i.e., equation $B$ of equation (1), we get:

\[
4096 h_1^5(X, p_1, p_2, p_3, p_4) + 46629 h_1^4(X, p_5, p_6, p_7, p_8) + 4096 h_1^3(X, p_9, p_{10}, p_{11}, p_{12}) = 0
\]

(5)

The coordinates $X_i$ of $n$ practical points (here, $n = 22$) obtained by the experiments are $X_i = (\theta_i, \gamma_0, a_c_i) (i=1, 2, 3, ..., n)$, instead of $X$ in formula (5), $n$ equations about coefficients $p_1, p_2, p_3, ..., p_{12}$ can be get:

\[
4096 h_1^5(X_i, p_1, p_2, p_3, p_4) + 46629 h_1^4(X_i, p_5, p_6, p_7, p_8) + 4096 h_1^3(X_i, p_9, p_{10}, p_{11}, p_{12}) = 0
\]

(6)

Make $P = (p_1, p_2, p_3, ..., p_{12})$, write the left side of equation (6) as a new function of $P$ and $X_i$, and get:

\[
g_i(P, X_i) = 0 \quad (i=1, 2, 3, ..., n)
\]

(7)

Constructing an unconstrained optimization objective function:

\[
\min F = \sum_{i=1}^{n} g_i^2(P, X_i) \quad (i=1, 2, 3, ..., n)
\]

(8)

The coefficients of the mapping function can be obtained by computing $P$, which takes the minimum value of the objective function $F$ of formula (8).

3.3 Establishment of the bifurcation set equation of CSC

The objective function of equation (8) is figured out by a modern optimization technology, and the coefficient $P$ (in 5 decimal places) of the mapping function procured is exhibited in Table 4.
Table 4
Optimized parameters $P$

| Parameter | Final value | Parameter | Final value |
|-----------|-------------|-----------|-------------|
| $p1$      | -0.31745    | $p7$      | -0.00034    |
| $p2$      | 0.00647     | $p8$      | -0.05225    |
| $p3$      | -0.01364    | $p9$      | 0.00955     |
| $p4$      | -1.84204    | $p10$     | -0.00020    |
| $p5$      | -0.01249    | $p11$     | 0.00038     |
| $p6$      | 0.00027     | $p12$     | 0.04931     |

Bringing it into equation (5), the CSC bifurcation set equation is achieved:

\[
\begin{align*}
4096(-0.31745 + 0.00647\theta - 0.01364\gamma_0 - 1.84204a_c)^6 \\
+46629(-0.01249 + 0.00027\theta - 0.00034\gamma_0 - 0.05225a_c)^4 \\
+4096(0.00955 - 0.00020\theta + 0.00038\gamma_0 + 0.04931a_c)^2 = 0
\end{align*}
\]

where the unit of both tool rake angle $\gamma_0$ and edge angle $\theta$ is deg, and the unit of cutting thickness $a_c$ is mm.

3.4 Analysis of Bifurcation Set of CSC

According to this equation and the experimental data of critical conditions and its boundary values of CSC given in Table 3, the theoretical control parameters $u$, $v$ and $w$ can be calculated by using the mapping function from the actual control parameters to the theoretical control parameters, and the bifurcation set surface of swallowtail catastrophe of CSC [39] can be drawn according to the range of $u$, $v$ and $w$ values, as shown in Fig.6. It is a complex surface in the three-dimensional control space, and a set of all critical points in the control space that cause the sudden change of the system. Consistent with the conclusion of the catastrophe theory (Castrigiano and Hayes, 2018), the bifurcation set surface divides the control space into 5 regions, namely regions I, II, III, IV and V, and can be combined with the phenomena observed in the experiments to determine the equilibrium state that each region corresponds to. The theoretical point is determined by the three theoretical control parameters, which can be calculated with the mapping functions from the three actual control parameters. When the changing pathway of the theoretical point passes through the bifurcation set surface and changes from
the region where the chips do not split to the region where the chips split, the CSC happens.

As an example, Fig. 7 shows the distribution of a theoretical critical point (Point 2) and its two boundary value points (Points 1 and 3) in the theoretical control space. Among them, Point 1 is located in region III, where the chips split; Point 3 is in region I, where the chips do not split; Point 2 is in the bifurcation set surface, and it is a critical point between the chip-splitting and the non-chip-splitting regions. According to the above analysis, when the theoretical point changes along the pathway $3 \rightarrow 2 \rightarrow 1$ and enters Region III from Region I, the system changes from one equilibrium state (in which the minimum value of the potential function is relatively high and the chips do not split) to another equilibrium state (in which the minimum value of the potential function is relatively low and the chips split). This is the reason for the CSC and the fundamental law governing the occurrence of the CSC. With the established bifurcation set equation and its corresponding bifurcation set surface, the cause of CSC can be convincingly explained, and thus the occurrence of CSC can be predicted and controlled.
4 Prediction results of critical conditions and experimental verification of bifurcation set equation

4.1 Prediction of critical cutting thickness

According to the established CSC bifurcation set equation, the predicted critical cutting thickness $a_{c01}$ and $a_{c02}$ of CSC are expressed in Table 5. The straight double-edged tool with a combination of different cutting edge angles $\theta$ and rake angles $\gamma_0$ (Table 5) was used for symmetrical transverse feeding AISI 1045 steel disc workpieces in the prediction process. The following needs to be clarified:

1) In the prediction, the cutting edge angle $\theta$ and the rake angle $\gamma_0$ were taken at three levels following regular change variations which vary according to the law. To investigate the extrapolation prediction ability of the established bifurcation set equation, the three levels of the cutting angle $\theta$ were taken at 35º, 50º, and 65º; the three levels of the rake angle $\gamma_0$ were taken at - 20º, - 10º, and 0º.

2) The established bifurcation set equation is a sixth-order algebraic equation of the control variables $\theta$, $\gamma_0$, and $a_c$. When $\theta$ and $\gamma_0$ are known, they are converted to a sixth-order algebraic equation of $a_c$. Theoretically speaking, there are at the most six roots, that is, up to six critical cutting thicknesses $a_{c0}$ can be predicted. However, the Matlab program compiled to deal with the sixth-order algebraic equation roots of $a_c$ mentioned above can work out at the most two non-repeated positive real roots ($a_{c01}$, $a_{c02}$).

| Tool number | Cutting edge angle | Rake angle | Predicted critical cutting thickness $a_{c01}$ (mm) | Predicted critical cutting thickness $a_{c02}$ (mm) | Feed (mm/r) | Actual cutting thickness $a_{0}$ (mm) | Workpiece thickness $b$ (mm) | Experimental critical cutting thickness $a_{\gamma}^c$ (mm) | Relative error $\delta$ (%) |
|-------------|--------------------|------------|---------------------------------|---------------------------------|-------------|---------------------------------|-----------------|---------------------------------|--------------------------|
| 1           | 35                 | -20        | 0.0848                          | 0.0926                          | 0.050       | 0.0410                          | 4.91            | 0.0819                          | 3.54                     |
|             |                    |            |                                 |                                 | 0.130       | 0.1065                          |                 |                                 |                          |
| 2           | 50                 | -20        | 0.1322                          | 0.1559                          | 0.140       | 0.0900                          | 3.86            | 0.1285                          | 2.88                     |
|             |                    |            |                                 |                                 | 0.280       | 0.1800                          |                 |                                 |                          |
| 3           | -10                |           | 0.0600                          | 0.0804                          | 0.050       | 0.0321                          | 0.0578          |                                 | 3.81                     |
|             |                    |            |                                 |                                 | 0.150       | 0.0964                          |                 |                                 |                          |
| 4           | 65                 | -20        | 0.1817                          | 0.2122                          | 0.380       | 0.1606                          | 2.54            |                                 | /                        |
|             |                    |            |                                 |                                 | 0.450       | 0.1902                          |                 |                                 | /                        |
| 5           | -10                |           | 0.1095                          | 0.1330                          | 0.130       | 0.0549                          | 0.1099          |                                 | -0.36                    |
|             |                    |            |                                 |                                 | 0.280       | 0.1183                          |                 |                                 |                          |
| 6           | 0                  |           | 0.0390                          | 0.0486                          | 0.050       | 0.0211                          | 0.0465          |                                 | -16.13                   |
|             |                    |            |                                 |                                 | 0.150       | 0.0634                          |                 |                                 |                          |

Table 5
Prediction results and verification experimental parameters and results of critical cutting thickness

Note: 1) The maximum and minimum relative errors between the predicted value and the experimental value are underlined and bolded. 2) The slash”/” indicates that the critical cutting thickness is not obtained in the actual experiment, so the corresponding prediction error for the critical cutting thickness cannot be calculated.
4.2 Design for experimental verification of the bifurcation set equation

A group of verification experiments were designed to verify the effectiveness of the established bifurcation set equation model and the accuracy of the critical conditions (such as the critical cutting thickness) predicted by the CSC model. For this group of experiments, the operating process and the measurement method for the critical cutting thickness were the same as those of critical conditions experimental data acquisition experiments. The difference was in the setting of tool parameters in the verification experiments, which were the edge angle $\theta$, rake angle $\gamma_0$, workpiece thickness $b$, feed amount $f$ and the actual cutting thickness $a_{c0}$ corresponding to the feed amount $f$, as shown in Table 5.

4.3 Verification experiments results and discussions

According to the above design, a group of 6 experiments were completed to measure the actual critical cutting thickness $a_{ce}$ by using symmetrical transverse feeding AISI 1045 steel disc workpieces by self-made high speed steel straight double-edged tools. The column of $a'_{ce}$ in Table 5 is the experimental critical cutting thickness value obtained. It should be clarified:

1) During the cutting by tool number 4 in Table 5, the workpiece was severely deformed, and the experiment cannot be completed as planned; therefore, the critical cutting thickness cannot be measured, and thus cannot calculate prediction error for the critical cutting thickness for this tool, and it was indicated by slash “/”. The reason for this occurrence was that, for this tool, the two critical cutting thicknesses predicted by the bifurcation set equation were both large, $a_{c01} = 0.1817$ mm and $a_{c02} = 0.2122$ mm to be specific, so the actual cutting thickness used for this experiment was larger than 0.2122 mm, leading to a very large cutting force. The diameter of the workpiece was 180 mm, while the workpiece thickness calculated by the tool's cutting edge angle was only 2.5 mm, and the rigidity was obviously insufficient.

2) As can be seen from Table 5, almost all the predicted values $a_{c02}$ of the critical cutting thickness were greater than the actual experimental values. A possible reason for the cause of this situation is: Catastrophe theory asserts that the occurrence of catastrophe is determined by both the final value of the control variables and the change paths to reach this final value. In the verification experiments of
the critical conditions of the CSC, the path change for the control variables of the cutting thickness is a single direction from small to large (that change naturally occurs when the tool cuts into the workpiece), therefore it can only trigger the sudden change of the smaller critical cutting thickness [29]. The machine tool used in the experiment cannot realize the variable feed cutting, which means the change of the cutting thickness cannot be from large to small. So it is theoretically impossible to trigger the catastrophe corresponding to the more immense predicted value of the critical cutting thickness $a_{c,02}$.

Table 5 also shows the relative error $\delta$ between the predicted value and the experimental value of the CSC bifurcation set equation. The error range is $-16.13\% \sim 3.81\%$, the average error is $-1.25\%$, and the average absolute error is $5.34\%$. As shown in Fig. 6, the predicted values are in good agreement with those by experiments.

![Comparison of predicted critical cutting thickness with experimental data](image)

Fig. 8 Comparison of predicted critical cutting thickness with experimental data

5 Conclusions

The CSC during the symmetrical cutting of the double-edged tool is accompanied by a significant reduction in the cutting force, which has excellent potential for improving the process effect. The premise of the utilization is to model the CSC. Based on the catastrophe theory, a modeling method of the bifurcation set equation of CSC in symmetrical cutting with a double-edged tool was proposed. By
utilizing this method, the bifurcation set equation, which could predict the critical conditions of CSC in symmetrical cutting with double-edged tools was established. The validity of this equation and the accuracy in predicting the critical cutting thickness of the CSC was experimentally verified. The conclusions for this paper can be summarized as follow:

(1) The method of measuring the critical conditions of CSC has been proposed. 3-round cutting experiments were designed based on gradually reducing the value range of control variables, and then used the captured chip flow videos to determine the critical conditions of CSC during the symmetrical cutting of the AISI 1045 steel disc workpieces by a straight double-edged tool.

(2) A total of 122 sets of three rounds of cutting experiments were carried out to survey the critical conditions of CSC when using a straight double-edged tool symmetrical transverse fed AISI 1045 steel disc workpieces. Among them, 22 groups of critical conditions experimental data were acquired.

(3) It was found in analyzing the experimental data that with the same tool, both the specific main cutting force and the specific feed force with CSC were much lower than the specific main cutting force and the specific feed force without CSC, and the maximum reduction under the experimental conditions reached 64.68%. It was also found that the larger the cutting edge angle and the absolute value of the negative rake angle is, the more likely the CSC is to be observed within a certain range of process parameters.

(4) An experimental modeling method of the CSC bifurcation set equation has been developed based on the standard mathematical model of the swallowtail catastrophe and the experimental data of critical conditions. This method transformed the modeling process into a set of optimization solution processes of mapping function coefficients from actual control variables (cutting edge angle $\theta$, rake angle $\gamma_0$, and cutting thickness $a_c$) to theoretical control variables ($u$, $v$, $w$).

(5) Applying the above method, the function coefficients were solved when the above mapping function was linear by using the experimental data of the critical conditions of CSC when the straight double-edged tool was symmetrically traversing the AISI 1045 steel disc workpieces. On this basis,
the bifurcation set equation of CSC was established when the straight double-edged tool symmetrically transverse fed the AISI 1045 steel disc workpieces.

(6) Using the model, the critical cutting thickness value of CSC was forecasted under the conditions of the AISI 1045 steel disc workpieces were symmetrically transverse cut by straight double-edged tools with a combination of different cutting edge angles and rake angles.

(7) A set of verification experiments for the bifurcation set equation were completed to measure the critical cutting thickness prediction error of CSC. The results show that the critical conditions prediction error range of CSC is -16.13%~3.81%, with the average error of -1.25%, and the average absolute error of 5.34%.

This study laid a foundation for controlling and utilizing CSC, and also found a new way for energy-saving optimization design of tool structure and cutting process parameters.

Declarations

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**Availability of data and material:** The datasets used or analysed during the current study are available from the corresponding author on reasonable request. All experimental data in this paper are obtained in the laboratory.

**Code availability:** The codes used during the current study are available from the corresponding author on reasonable request.

**Authors' contributions:** Conceptualization, Methodology, Software, Validation, Investigation, Data Curation, Writing - Original Draft, Project administration.
References

1. Shi H (2018) Metal Cutting Theory: New Perspectives and New Approaches. Springer International Publishing, Cham, pp 110–146
2. Wang G, Fuh K, Yan B (2001) A new mathematical model for multifacet drills derived by using angle-solid model. Int J Mach Tools Manuf 41:103–132
3. Liu G, Wang Y, Zhang H et al. (2014) Research on helical milling specialized tool based on chip-splitting principle. J Mech Eng 50:176–184
4. Zhou L, Dong H, Ke Y et al. (2016) Analysis of the chip-splitting performance of a dedicated cutting tool in dry orbital drilling process. Int J Adv Manuf Technol 90:1–15
5. Stephenson DA, Agapiou JS (2016) Metal Cutting Theory and Practice. third ed. Taylor & Francis, CRC Press, Boca Raton, pp 44–46
6. Zhou W, Li S, Liu R et al. (2019) Experimental and simulation investigation of multi-tooth cutting process of long fiber using copper wire continuous feeding. J Mater Process Technol 273:116–252
7. Shi H, Wang J (1995) A model for non-free-cutting. Int J Mach Tools Manuf 35:1507–1522
8. Gao H, Zhang Y, Wu Q et al. (2018) Investigation on influences of initial residual stress on thin-walled part machining deformation based on a semi-analytical model. J Mater Process Technol 262:437–448
9. Zhang Y, Guo S, Zhang Z et al. (2019) Simulation and experimental investigations of complex thermal deformation behavior of wire electrical discharge machining of the thin-walled component of Inconel 718. J Mater Process Technol 270:306–322
10. Khoshdarregi RM, Altintas Y (2018) Dynamics of multipoint thread turning—Part I: General formulation. J Manuf Sci Eng 140:061003-1—11
11. Monkova K, Peter MP, Sekerakova A et al. (2019) Comparative Study of Chip Formation in Orthogonal and Oblique Slow-Rate Machining of EN 16MnCr5 Steel. metals 9:698–720
12. Polvorosa R, Suárez A, Lacalle LL de et al. (2017) Tool wear on nickel alloys with different coolant pressures: Comparison of Alloy 718 and Waspaloy. J Manuf Process 26:44–56. https://doi.org/10.1016/j.jmapro.2017.01.012
13. Dilip Jerold B, Pradeep Kumar M (2011) Experimental investigation of turning AISI 1045 steel using cryogenic carbon dioxide as the cutting fluid. J Manuf Process 13:113–119. https://doi.org/10.1016/j.jmapro.2011.02.001
14. Patwari MAU, Amin AKMN, Faris W (2010) Identification of Instabilities of the Chip Formation and It’s Prediction Model During End Milling of Medium Carbon Steel (S45C). Am J Eng Appl Sci 3:193–200
15. Shyha I, Gariani S, Ahmed M et al. (2018) Analysis of Microstructure and Chip Formation When Machining Ti-6Al-4V. metals 8:185–205
16. Urbikain G, Perez JM, López de Lacalle LN et al. (2018) Combination of friction drilling and form tapping processes on dissimilar materials for making nutless joints. P I Mech Eng B-J Eng 232:1007–1020. https://doi.org/10.1177/0954405416661002
17. M. Tiffe, J. Saelzer, A. Zabel (2019) Analysis of mechanisms for chip formation simulation of hardened steel. Procedia CIRP 82:71–76
18. Luk WK (1969) The mechanics of symmetrical vee form tool cutting. Int J Mach Tool Des Res 9:17–38
19. Yamamoto A, Nakamura S (1978) On the Chip Parting at V-shaped Groove Cutting. J Jpn Soc
20. Shi H (1999) Chip-ejection interference in cutting processes of modern cutting tools. Sci China 42:275–281
21. Bi H, Xiong L, Huang R (2014) Experimental investigation on the effect of chip flow interference on main cutting force. Int. J. Mach. Mach. Mater 16:38–64
22. Rezayi Khoshdarregi M, Altintas Y (2015) Generalized modeling of chip geometry and cutting forces in multi-point thread turning. Int J Mach Tools Manuf 98:21–32. https://doi.org/10.1016/j.ijmachtools.2015.08.005
23. Weng J, Zhuang K, Hu C, Zhu D, Guo S, Ding H (2019) A novel approach to thermal modeling based on three-dimensional analysis in turning Inconel 718 with round insert. J. Mater. Process. Technol 266:588–598
24. Castrigiano DPL, Hayes SA (2018) Catastrophe Theory, Second ed. CRC Press, New York, pp 1–9
25. Poston T, Stewart I, Plaut RH (1978) Catastrophe Theory and Its Applications. Pitman, London, pp 1–7
26. Woodcock AER, Davis M (1978) Catastrophe theory. Penguin, London, pp 76–92
27. Klamecki B E (1982) Catastrophe Theory Models of Chip Formation. J Eng Ind 104:369–374
28. Bao J, Liu J, Yin Y et al. (2015) Characterization and experiments on the friction catastrophe behaviors of brake material during emergency braking. Eng Fail Anal 55:55–62
29. Cui H, Wan X, Xiong L (2019) Modeling of the catastrophe of chip flow angle in the turning with double-edged tool with arbitrary rake angle based on catastrophe theory. Int J Adv Manuf Technol 104:2705–2714. https://doi.org/10.1007/s00170-019-04114-1
30. Zhu B, Xiao YMH, Wan X et al. (2020) Theoretical modeling and experimental verification of chip flow angle catastrophe in double-edged cutting considering non-linear effects. Int J Mech Sci 172:105394–105406. https://doi.org/10.1016/j.ijmecsci.2019.105394
31. Luo Z (1994) Study on catastrophe theory-based modeling and prediction of tool life. China J Mech Eng 30:103–112
32. Buchkremer S, Klocke F, Veselovac D (2016) 3D FEM simulation of chip breakage in metal cutting. Int J Adv Manuf Technol 82:645–661. https://doi.org/10.1007/s00170-015-7383-9
33. Arsecularatne JA, Fowle RF, Mathew P (1996) Nose radius oblique tool: Cutting force and built-up edge prediction. Int J Mach Tools Manuf 36:585–595
34. Klocke F, Kuchle A (2011) Manufacturing Processes 1: Cutting (RWTHedition), 63-149. Springer-Verlag, Heidelberg, Germany, pp 63–149
35. Arnold VI (1992) Catastrophe Theory, third ed. Springer-Verlag Berlin Heidelberg, New York, pp 77–88
36. Saunders PT (1980) An introduction to catastrophe theory. Cambridge University Press, Cambridge, pp 32–39
37. Gilmore R (1981) Catastrophe theory for scientists and engineers. John Wiley & Sons, New York, pp 34–47
38. Sun J, Tan Q (eds) (2011) Research on catastrophe model of logistics capacity for logistics system of national economy mobilization
39. Piyaratne MKDK, Zhao H, Meng Q (2013) APHIDSim: A population dynamics model for wheat aphids based on swallowtail catastrophe theory. Ecol Model 253:9–16
Figure captions

Fig. 1 Schematic diagram of symmetrical transverse cutting AISI 1045 steel disc workpiece with the straight double-edged tool

Fig. 2 Diagram of experimental devices and cutting force measurement system

Fig. 3 25 cutting tools in the first round of experiment

Fig. 4 Chip-splitting catastrophe observed in the experiments: Cutting edge angle $\theta = 45^\circ$, Rake angle $\gamma_0 = -10^\circ$, Critical cutting thickness $a_{ce} = 0.0495\text{mm}$

Fig. 5 The chip morphology after chip-splitting catastrophe: Cutting edge angle $\theta = 45^\circ$, Rake angle $\gamma_0 = -10^\circ$, Critical cutting thickness $a_{ce} = 0.0495\text{mm}$

Fig. 6 Three-dimensional diagram of swallowtail catastrophe bifurcation set of CSC

Fig. 7 The position of the critical point (Point 2) on the bifurcation set surface

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Process parameters corresponding to different cutting edge angles

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Figure 1

Schematic diagram of symmetrical transverse cutting AISI 1045 steel disc workpiece with the HSS straight double-edged tool
Figure 2

Schematic diagram of experimental devices and cutting force measurement system
Figure 3

25 cutting tools in the first round of experiment

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