Family Symmetry, Gravity, and the Strong CP Problem*

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Abstract

We show how in a class of models Peccei–Quinn symmetry can be realized as an automatic consequence of a gauged $U(1)$ family symmetry. These models provide a solution to the strong CP problem either via a massless $u$–quark or via the DFSZ invisible axion. The local family symmetry protects against potentially large corrections to $\theta$ induced by quantum gravitational effects. In a supersymmetric extension, the ‘$\mu$–problem’ is shown to have a natural solution in the context of gravitationally induced operators. We also present a plausible mechanism which can explain the inter–generational mass hierarchy in such a context.

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I. Global symmetries have lately come under suspicion. There are arguments that quantum gravitational effects violate global symmetries and will induce in the effective low–energy theory all possible operators that respect the local symmetries of the theory.\textsuperscript{1} The magnitudes of the coefficients of these operators are expected to be set by the appropriate powers of the Planck scale, but beyond that no quantitative statements are possible at present. In superstring theory there are firmer arguments against the possibility of exact global continuous symmetries.\textsuperscript{2}

This suspicion has led various authors to reevaluate theoretical ideas that involve global symmetries, such as Peccei–Quinn symmetries,\textsuperscript{3} CP,\textsuperscript{4} baryon\textsuperscript{5} and lepton numbers\textsuperscript{6} and cosmological texture.\textsuperscript{7} The idea has been to see if such global symmetries can arise as an automatic consequence of local symmetries.

In this paper we will reexamine Peccei–Quinn symmetry.\textsuperscript{8} Peccei–Quinn symmetry can be realized in two ways: either in the Wigner–Weyl way which leads to a massless quark (or quarks) usually taken to be the $u$–quark; or in the Nambu–Goldstone way giving rise to an axion.\textsuperscript{9} We will first show how local family symmetries can lead to a $u$–quark light enough to solve the strong CP problem. Then we will show how, in a closely related fashion, very simple DFSZ axion models\textsuperscript{10} can arise from local family symmetry realizing an idea of Wilczek.\textsuperscript{11} Generalization to grand unification and supersymmetry will be presented. A natural solution to the ‘$\mu$’–problem’ in SUSY models is found in the context of quantum gravity induced operators. We also construct a scheme in this context which can explain the inter–generational mass–hierarchy.

We adopt the philosophy in this paper that the operators presumably induced by quantum gravitational effects are not further highly suppressed (beyond the powers of $M_{Pl}$ expected on dimensional grounds). There are no obvious small parameters which would lead us to expect such a suppression.
We therefore take the dimensionless coefficients in front of these operators to be of order one.

II. Massless $u$–quark: Let us assume that the low energy theory looks like the standard model, but that there is secretly a $U(1)$ local symmetry broken at some high scale. Call this $U(1)'$. Let us assign the $U(1)'$ charges of the standard model fields as shown in Table I. The $U(1)'$ charges of the right–handed quarks and leptons ($D^c_L(i), U^c_L(2, 3), l^c_L(i)$) are determined by the usual standard model Yukawa couplings in terms of the charges of the other fields ($i =$ family index). Note that an exception is the right–handed $u$–quark since we do not want its usual Yukawa coupling to be present. (Choosing the $U(1)'$ charge of the left-handed quark doublet $Q^L(1)$ could also lead to a massless $u$–quark, but in this case there will be no Cabibbo mixing involving the first family.) Its $U(1)'$ charge we require to differ from those of the right–handed $c$ and $t$ by an amount $\Delta$. Now, if we suppose that the unknown and presumably heavy fermions that are required to exist for anomaly freedom are all color singlets, we find from the $SU(3)_C \times U(1)'$ anomaly condition

$$0 = 6a + 3(-a + q) + 2(-a - q) + (-a - q + \Delta),$$

or $\Delta = 0$. In other words, the $U(1)'$ fails to distinguish the $u$–quark and protect it from having a tree–level mass. There are two simple ways out.

(1) One can allow exotic colored fermions to contribute to the anomaly. The simplest model of this type is the following: Have the $U(1)'$ charge, $Q'$, vanish for all standard model fermions except $u^c_L$ which has $Q' = 1$, and add a pair of quarks with $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ quantum numbers $(3, 1, -2/3, 0)_L + (\overline{3}, 1, +2/3, -1)_L$. Note that these have exotic electric charges—the quark has electric charge minus $2/3$. All anomalies cancel. A Higgs field, $S(1, 1, 0, 1)$ can give Dirac mass to the exotic pair of quarks. A dimension–4 mass term for $u$ is forbidden; however, a term

$$\frac{f}{M_{Pl}} (Q_L u^c_L \varphi S^*),$$

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would be expected to be induced by quantum gravity. In order to solve the strong CP problem, $m_u$ should be less than about $10^{-9}$ of its usually assumed value of 5 to 10 MeV, so that $f\langle S\rangle/\mpl \simeq 10^{-14}$. One therefore expects the exotic quarks to have mass of order $\langle S\rangle \sim (100\ TeV)/f$. Unfortunately such exotically charged quarks (unless they could be inflated away) would run a foul of terrestrial searches for anomalously charged matter.

(2) To us a more interesting possibility is that we modify the standard model by having two Higgs doublets. Suppose $\varphi$ couples as usual to the $c$ and $t$ quarks (but not $u$), and $\varphi'$ couples as usual to charge $-1/3$ quarks and leptons. (In our notation $\varphi$ and $\varphi'$ both have hypercharge $= +1/2$.) The $U(1)'$ quantum numbers of the various fields are now given as in Table II. The same exercise as before shows that the $SU(3)C \times U(1)'$ anomaly cancels if

$$\Delta = 3(q - q') \ .$$

So the $u$ quark mass is forbidden by $U(1)'$ only if $\varphi$ and $\varphi'$ have different $U(1)'$ charges. This means that the $\varphi^\dagger \varphi'$ and the $(\varphi^\dagger \varphi')^2$ terms are also forbidden. There is then the danger of a weak scale axion. If that happens, there would be overkill: two accidental PQ symmetries would result, the usual Weinberg–Wilczek one realized in the Nambu–Goldstone way, and another PQ symmetry that takes only $u^c \rightarrow e^{i\alpha} u^c$ realized in the Wigner–Weyl way. However, the axion is easily avoided if the singlet field $S$ needed to break $U(1)'$ above the weak scale has a coupling

$$\varphi^\dagger \varphi' S^n/\mpl^{n-2} \ .$$

Here $n$ is some appropriate integer determined by the $U(1)'$ charges. If $n = 1$ or 2, eq. (2) is a part of the renormalizable Lagrangian, for $n > 2$, eq. (2) will only be induced by quantum gravitational effects. From eq. (1) and (2), one sees that there will also be a possible term induced by gravity

$$Q_L u^c \varphi(S^*)^{3n}/\mpl^{3n} \ .$$
We require $\langle S \rangle^n / M_{Pl}^{n-2} \sim M_W^2$ to avoid either an axion or the disruption of the gauge hierarchy. Then

$$m_u \sim M_W \left( M_W^2 / M_{Pl}^2 \right)^3$$

which is certainly small enough to solve the strong CP problem. We should note also that the usual color–instanton contribution to $m_u$ will still arise as in the standard model as these arise from operators like $Q_L u \phi' \left( \phi^\dagger \phi' \right)^2$ which are allowed by $U(1)'$, not surprisingly since $U(1)'$ is constructed to have no color anomaly.

It is worth showing that a set of extra fermions that is not too wild can be found that ensures cancellation of all gauge anomalies (see Table III). These extra fermions are singlets under color and $SU(2)_L$. The $SU(3)^2 \times U(1)'$ and $SU(2)^2 \times U(1)'$ and gravity $\times U(1)'$ anomaly cancellation imply $\Delta = 3(q - q')$, $b = -3a$, and $q' = -b$, respectively (see Tables II and III). The $U(1)_Y^2 \times U(1)'$ anomaly then automatically cancels, and the $U(1)_Y \times U(1)^2$ and $U(1)^3$ anomaly conditions are respectively

$$2yz(x_1 - x_2 - z) = 12(q - q')^2$$
$$3(x_1 + x_2)z(x_1 - x_2 - z) = 18(q - q')^2(5q' - q) .$$

These give $x_1 + x_2 = y(5q' - q)$ and

$$z^2 + (x_2 - x_1)z + \frac{6}{y}(q - q')^2 = 0 . \quad (4)$$

The heavy fermions $\psi_1$ and $\psi_2$ have electric charge $y$ which has to be different from zero. The simplest choice is $y = 1$, so that they are heavy leptons of the ordinary type. $\psi_1$ and $\psi_2$ can both get mass from a scalar $S$ of charge $Q' = z/m$ if the mass terms are of the form $\psi_1 \psi_1 (S^*)^m + \psi_2 \psi_2 S^m$. Note that for $m = 1$, these mass terms are part of the renormalizable Lagrangian. It would be nice if the same scalar, $S$, coupled to $\phi^\dagger \phi'$ as in eq. (2), and prevented an axion. The scalar $S$ from eq. (2) we see has $U(1)'$ charge
So if \( z/m = (q - q')/n \), eq. (4) gives \((x_2 - x_1) = (q' - q) \left( \frac{6n}{ym} + \frac{m}{n} \right) \), which together with \((x_2 + x_1) = y(5q' - q) \) gives nice rational values for the \( U(1)' \) charges of the extra fermions. Note that the masses of these extra fermions are expected to be of order \( M_{\text{Pl}}(M_W/M_{\text{Pl}})^{2m/n} \). The special choice \( m = 1, n = 2 \), which guarantees that the axion mass as well as \( \psi_{1,2} \) masses arise at tree level would imply that the masses of \( \psi_{1,2} \) are of order \( M_W \).

It should be pointed out that “discrete gauge symmetries”\(^{13} \) that survive at low energy are useless in making \( m_u = 0 \) as the same would also lead to weak axions. If a \( \phi^\dagger \phi' \) term is present, it means any residual \( Z_N \) must be such that \( q - q' = 0 \mod N \). But then \( \Delta = 3(q - q') = 0 \mod N \) and the \( Z_N \) fails to distinguish the \( u \) from the \( c \) and \( t \).

**III. Family symmetry and automatic DFSZ axions:** From the foregoing we see that there is a quite natural link between gauged family symmetries and axions. We tried to impose a \( U(1)' \) symmetry that distinguished \( u \) from \( c \) and \( t \) (i.e., a simple family symmetry) and found that we were led to consider two–Higgs–doublet models where the same family symmetry distinguished the two Higgs doublets and prevented a \( \phi^\dagger \phi' \) term. We will now show how to exploit this to construct very simple DFSZ axion models.

Consider the very model we discussed in the last section where the fermion content is displayed in Tables II and III. Let there be two singlet scalars \( S \) and \( T \). \( T \) we give \( Q' = \Delta = 3(q - q') \) (the latter equality following, again, from the \( SU(3)^2 \times U(1)' \) anomaly freedom). \( S \) we give \( Q' = \frac{1}{n}(q - q') \). Then there are the terms

\[
\varphi^\dagger \varphi' \left( S^n/M_{\text{Pl}}^{n-2} \right) \quad (5a)
\]
\[
Q_L u^\dagger \varphi \left( T^*/M_{\text{Pl}} \right) \quad (5b)
\]
\[
\left( T^* S^{3n}/M_{\text{Pl}}^{3n-3} \right) \quad (5c)
\]

(There is also the term \( Q_L u^\dagger \varphi' (\varphi^\dagger \varphi')^2 \) discussed earlier, but this is negligible.) Let us ignore the term (5c) for the moment since for the cases of interest it
will be of high order in \((1/M_{Pl})\). We are interested, now, in solving the strong CP problem via an axion rather than a massless or very light \(u\) quark. So we choose \(\langle T \rangle\) to be large enough so that \(m_u\) arising from (5b) is about 5 to 10 MeV. Thus \(\langle T \rangle \approx 10^{15}\) GeV. Now there are (if we neglect (5c)) two \(U(1)\) symmetries (besides \(U(1)_Y\)) to consider: the local symmetry \(U(1)'\) and the anomalous (and accidental) global \(U(1)\) symmetry that takes \(T \rightarrow e^{i\alpha}T\) and \(u^c \rightarrow e^{i\alpha}u^c\). We will call the latter symmetry \(U(1)_T\). These are both broken by \(\langle S \rangle\) and \(\langle T \rangle\). So the \(U(1)'\) gauge boson will become massive, and there will also be an axion. What is \(f_a\)? Assume \(\langle T \rangle \gg \langle S \rangle\). Then \(\langle T \rangle\) will break \(U(1)' \times U(1)_T\) down to a global \(U(1)\) and the \(U(1)'\) gauge boson will eat the phase of \(T\). (This is the so–called ’tHooft mechanism.) The residual global \(U(1)\), which we will call \(U(1)_{PQ}\), will be that linear combination of \(U(1)'\) and \(U(1)_T\) under which \(T\) is neutral. The \(U(1)_{PQ}\) charges of all the fermions will be the same as their \(U(1)'\) charges, except for \(u^c\). From (5b) and the fact that \(Q_{PQ}(T) = 0\) we see that \(u^c\) has the same PQ charge as \(c^c\) and \(t^c\). But this PQ symmetry is then just the familiar DFSZ kind of \(U(1)\). It gets broken by \(\langle S \rangle\), so that for cosmological and astrophysical reasons \(10^{10}\) GeV \(\ll \langle S \rangle \ll 10^{12}\) GeV.

The coefficient of \(\phi^\dagger\phi'\) will be of order \(M_W^2\) if \(n\) is chosen to be 4 (see eq. (5a)). Recall that in the usual DFSZ models \(n = 2\) (or 1), which would require fine–tuning the coefficient of \(\phi^\dagger\phi'\) to be of order \(M_W^2\).

Up to this point we have neglected the term (5c). This term explicitly violates the Peccei–Quinn symmetry and therefore, as emphasized in ref. (3), contributes to \(\bar{\theta}\). One expects that this contribution will be

\[
\bar{\theta} \sim (m^2_\pi f^2_\pi)^{-1} \frac{(T^*) \langle S \rangle^{3n}}{M_{Pl}^{3n-3}} .
\]

Using \((S/M_{Pl})^n \sim (M_W/M_{Pl})^2\) (from (5a)), \(\langle T \rangle / M_{Pl} \sim m_u/M_W\) (from (5b)), one finds

\[
\bar{\theta} \approx 10^{-26} .
\]
This relation depends on the power $3n$ that appears in (5c). The 3 comes from the $SU(3)^2 \times U(1)'$ anomaly condition $\Delta = 3(q - q')$, and depends on the charge assignment of the quarks under the $U(1)'$ family group.

IV. GUT embedding: The family group we have used is somewhat peculiar: it distinguishes the $u'$ quark only. One might ask whether more general family groups are possible, including ones that would commute with grand unified gauge groups. To see that this is indeed the case we will describe a simple $SU(5) \times U(1)'$ example. Consider three families of quarks and leptons, each contained in a $10 + \bar{5}$ of $SU(5)$. Let $i = 1, 2, 3$ be the family index. Assign to the fermion representations $10^{(i)}_L U(1)'$ charges $a + p_i \Delta$, to the $\bar{5}^{(i)}_L U(1)'$ charges $b + q_i \Delta$, and to the Higgs representations $5_H$ and $\bar{5}_H$ $U(1)'$ charges $q = -2a$ and $-q' = -(a + b)$ respectively. If the integers $p_i$, $q_i$ all vanished, then the usual Yukawa couplings $10^{(i)}_L 5^{(j)}_H$ and $10^{(i)} \bar{5}^{(j)}_H$ would be allowed, and freedom from the $SU(5)^2 \times U(1)'$ anomaly would imply $9a + 3b = 0$. One recognizes the resulting $U(1)'$ as just being contained in $SO(10)$: $SU(5) \times U(1)' \subset SO(10)$. (Of course, other fermions need to be added, here $\nu_R$'s would do, to cancel the $U(1)'^a$ anomaly.) But if the $p_i$, $q_i$ are not all zero, $U(1)'$ is a family symmetry and forbids certain $d = 4$ Yukawa terms. New $SU(5)$--singlet fermions are in some cases required for $U(1)'$ anomaly cancellation, but their presence will not affect our results. As before, we introduce $SU(5)$ singlet fields $S$ and $T$ with $U(1)'$ charges $(q' - q)/n = (3a + b)/n$ and $\Delta$ respectively. Then the couplings

\begin{align}
(5_H \bar{5}_H) S^n/M_{Pl}^{n-2} \tag{6a} \\
(10^{(i)} \bar{5}^{(j)} \bar{5}_H) (T^*/M_{Pl})^{p_i + q_j} \tag{6b} \\
(10^{(i)} 10^{(j)} 5_H) (T^*/M_{Pl})^{p_i + p_j} \tag{6c}
\end{align}

should be induced by gravity. That is, the missing $d = 4$ Yukawa interactions appear in the effective low energy theory suppressed by appropriate power of $\langle T \rangle / M_{Pl}$. 

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Now, the $SU(5)^2 \times U(1)'$ anomaly tells us that
\[ \sum_i 3(a + p_i \Delta) + \sum_i (b + q_i \Delta) = 0 , \]
or
\[ \Delta = -\frac{3a + b}{3[\Sigma p_i + \Sigma q_i]} \]
where $(3a + b) = (q' - q)$. Therefore one has a term allowed by $U(1)'$ and, hence presumably induced by quantum gravity
\[ T[3\Sigma p_i + \Sigma q_i] S^{3n}/M_{Pl}^{5n-4} . \] (7)

As in the previous example, this generates a $\bar{\theta}$ which is very small since it is proportional to $(\langle S \rangle / M_{Pl})^{3n} \sim (M_{Pl}^2 / M_{Pl}^2)^3$. However, it should be noted that a problem would arise if the integer $[3\Sigma p_i + \Sigma q_i]$ were a multiple of 3. Then an operator that is the 3rd root of eq. (7) would be induced by gravity that would in general give too large a $\bar{\theta}$. Moreover, in that case an operator $5_H \tilde{5}_H T[3\Sigma p_i + \Sigma q_i]/^3$ would endanger the gauge hierarchy. The same danger exists if $[3\Sigma p_i + \Sigma q_i]$ and $3n$ have any common divisor. Note the significant fact that if all the $p_i$ are equal and all $q_i$ are equal, then $[3\Sigma p_i + \Sigma q_i]$ is a multiple of 3, so that to avoid the aforementioned problems $U(1)'$ must truly be a family symmetry, i.e., it should distinguish among families.

V. Supersymmetric extension: All of the above considerations apply to supersymmetric models as well, with certain significant changes. Consider, for example, the supersymmetric version of the model we just discussed. (6a)-(6c) are then to be interpreted as terms in the superpotential (with one more power of $M_{Pl}$ in the denominator of (6a) to make the dimensions come out right). Note that $R$–parity violating couplings such as $10^{(i)} \tilde{5}^{(j)} \tilde{5}^{(k)}$ and $10^{(i)} \tilde{5}_H \tilde{5}_H$ are forbidden by the $U(1)$ family symmetry. Eq. (6a) implies that the $\mu$ parameter is
\[ \mu \sim \langle S \rangle^n / M_{Pl}^{n-1} . \]
or

$$\left( \frac{\langle S \rangle}{M_{Pl}} \right)^n \sim \frac{M_W}{M_{Pl}}$$

(instead of $M_W^2 / M_{Pl}^2$ as in the non–supersymmetric case). So for $10^{10} \text{ GeV} \lesssim \langle S \rangle \lesssim 10^{12} \text{ GeV}$, choosing $n = 2$ would ‘explain’ why the $\mu$ parameter is of the weak scale (and not the Planck scale). The anomaly condition, including now the effect of $\bar{5}_H$, $5_H$ is

$$\Delta = -(a + b) \frac{2}{[3\Sigma p_i + \Sigma q_i]}$$

and the gravity induced term contributing to $m_a$ is

$$T[3\Sigma p_i + \Sigma q_i] S^{2n} / M_{Pl}^{2n + [3\Sigma p_i + \Sigma q_i] - 3}.$$  

We see that to make $\bar{\theta} \lesssim 10^{-9}$ requires $\left( \frac{\langle T \rangle}{M_{Pl}} \right)[3\Sigma p_i + \Sigma q_i] \lesssim 10^{-55}$, if $[3\Sigma p_i + \Sigma q_i]$ and $2n = 4$ have no common divisor. This is obviously a rather stringent condition, but in other models the condition would be different.

A further remark is in order as regards the ‘doublet–triplet mass–splitting’ in SUSY $SU(5)$. Since the Higgs doublet has a mass of order $M_W$ in our scheme, question may be raised as to the origin of the superheavy mass of its color–triplet partner. Other known mechanisms, such as the ‘missing partner mechanism’ are compatible with our scheme and could give superlarge mass to the color triplets.

VI. Fermion mass hierarchy: It is possible in the present context to have a natural explanation of the inter–generational mass hierarchy. Take for example the SUSY model of the previous section. The quark and lepton masses arise from eq. (6b) and (6c). Suppose we choose the $U(1)'$ charges such that $p_i = (2, 1, 0)$ and $q_i = (1, 1, 1)$ in the notation introduced earlier. Then if $x \equiv \langle T \rangle / M_{Pl} \sim \sqrt{m_e/m_t} \simeq 1/15$, and $\tan\beta$ (the ratio of the Higgs vacuum expectation values) $\simeq 3$, the fermion mass matrices will have the
form

\[
M_u \sim v \sin \beta \begin{pmatrix}
  x^4 & x^3 & x^2 \\
  x^3 & x^2 & x \\
  x^2 & x & 1
\end{pmatrix} ;
M_{d,l} \sim v \cos \beta \begin{pmatrix}
  x^3 & x^3 & x^3 \\
  x^2 & x^2 & x^2 \\
  x & x & x
\end{pmatrix},
\]

where \( v \approx 175 \text{ GeV} \). (Numbers of order one multiplying various entries in the matrices have been dropped.) Such matrices give a nice hierarchy of masses as well as mixing angles. Note that the mass ratios in the up-sector are smaller by a factor \( x \) relative to those in the down sector, in agreement with observations. We have not attempted to reconcile the strong CP problem simultaneously with the mass hierarchy, but models which accomplish both are not inconceivable.

**VII. Conclusion:** We found that a local family symmetry can make \( m_u \) light enough to solve the strong CP problem. However, if this symmetry has a residue at low energy that is a “discrete gauge symmetry” there is the tendency to get a weak axion as well. We also found that a DFSZ kind of Peccei–Quinn symmetry can arise very naturally as a consequence of local \( U(1) \) family symmetries. This approach has several appealing features: (i) there is a direct connection between the scale of Peccei–Quinn breaking and the value of the \( \mu \) parameter; (ii) the \( \mu \) parameter arises as a result of gravitationally induced terms and its smallness is in some sense explained; (iii) the choice of family group, the value of \( f_a \), the value of \( \theta \) and the size of certain light quark and lepton masses are linked together. Of course, these models suffer the great defect of all the DFSZ models that they are hard (impossible?) to test. But perhaps the ideas suggested here will allow further progress on the idea of family symmetry.
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**Table I.** $U(1)'$ charges of the standard model fermions and the Higgs doublet.

| $Q_L^{(i)}$ | $D_L^{(i)}$ | $U_L^{c,(2,3)}$ | $u_L^c$ | $L^{(i)}$ | $l^{c(i)}$ | $\varphi$ |
|------------|--------------|-----------------|----------|----------|------------|---------|
| $a$        | $(-a + q)$   | $(-a - q)$      | $(-a - q + \Delta)$ | $b$      | $(-b + q)$ | $q$     |

**Table II.** $U(1)'$ charge assignment in the two Higgs doublet model.

| $Q_L^{(i)}$ | $D_L^{(i)}$ | $U_L^{c,(2,3)}$ | $u_L^c$ | $L^{(i)}$ | $l^{c(i)}$ | $\varphi$ | $\varphi'$ |
|------------|--------------|-----------------|----------|----------|------------|---------|-----------|
| $a$        | $(-a + q')$  | $(-a - q)$      | $(-a - q + \Delta)$ | $b$      | $(-b + q')$ | $q$     | $q'$      |

**Table III.** Hypercharge and $U(1)'$ quantum numbers of heavy leptons needed for anomaly cancellation.

| $Y$ | $Q'$ | $\psi_{1L}^c$ | $\psi_{1L}^c$ | $\psi_{2L}$ | $\psi_{2L}^c$ |
|-----|------|---------------|---------------|-------------|--------------|
| $y$ | $x_1$ | $-y$          | $y$           | $y$          | $-y$         |
|     |      | $-x_1 + z$   | $x_2$         | $-x_2 - z$  |              |