The effect of data sampling time on the accuracy of maximum output power of a thermoelectric module

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Abstract. The instantaneous short-circuit current and open-circuit voltage of TEM (Thermoelectric Module) can be used to quickly estimate its maximum output power ($P_{\text{max}}$). Because of their instantaneous samples, the sampling time has a great influence on the accuracy of the estimated result. In this paper, the influence mechanism of sampling time is analysed systematically and a module of TEG1-127-2.8-1.2-250 is taken as a sample for verification. By means of either steady open-circuit/instantaneous short-circuit at mode A or steady short-circuit/instantaneous open-circuit at mode B, $P_{\text{max}}$ can be estimated quickly. At single mode A or B, the shorter the sampling time is, the greater the relative error of the estimation of $P_{\text{max}}$ has. This error varies with different temperature of hot plate, but the variation trend with sampling time is consistent. The valuations at model A and B are larger and smaller than the accurate values, respectively. When the sampling time is 600 to 1000ms, the error is the smallest (less than 3%). The accuracy of $P_{\text{max}}$ is significantly improved by using the average results of mode A and B. In this way, the sampling time has little influence on the estimation error, and the shortest sampling time corresponds to the highest estimation accuracy.

1. Introduction

TEM (Thermoelectric Module) converts heat energy into electrical energy by using the Seebeck effect of thermoelectric material [1-5]. An important parameter to characterize the conversion capability is $P_{\text{max}}$, the maximum output power of TEM. How to obtain it quickly has been one of the most important research works in thermoelectric application [6-12]. The traditional method to obtain $P_{\text{max}}$ is load matching method which is to achieve equal between load and the dynamic internal resistance by adjusting load resistance at stable temperature condition. It is a traditional method to obtain. Since each load adjustment requires a temperature stabilization process, this measurement method is accurate but time-consuming, which limits its application in engineering measurement of thermoelectric module and conversion system. The previous researchers proposed an engineering measurement method based on heat transfer and heat inertia of temperature field [6, 10-12]. The $P_{\text{max}}$
of TEM is obtained by utilizing short-circuit current ($I_{sc}$) and open-circuit voltage ($V_{oc}$) under two complementary maximum and minimum heat power input states corresponding to short circuit and open circuit respectively, which greatly reduces the measurement workload and realizes the engineering measurement of $P_{o\max}$ [10]. However, the instantaneous $V_{oc}$ and $I_{sc}$ are used in this method which is based on the thermal inertia i.e. the temperature field lags behind the change of electric field. Therefore, the different instantaneous sampling time will certainly affect the instantaneous $V_{oc}$ and $I_{sc}$, and then affect the accuracy of the $P_{o\max}$ of TEM. In this paper, the relevant analysis and discussion is carried out to understand and optimize the testing method.

2. Analysis of the estimation method for the maximum output power of TEM and discussion

The actual TEM was formed by connecting multiple $P$ and $N$ legs, electrically in series and thermally in parallel. The structure of TEM is shown in Figure 1. Figure 2 shows the distributions of internal temperature and thermal conductance from the hot to cold plate of TEM, where $K_h$ and $K_c$ are the integrated thermal conductance at the hot and cold side, respectively. $T_h$, $T_{ij}$, $T_c$, $T_{ij}$ are the temperatures of the hot plate, hot junction, cold plate and cold junction, respectively. It is assumed that the input heat power of hot side and the heat rejected in cold side are $Q_h$ and $Q_c$, respectively at the state of $T_h$, $T_{ij}$ and $T_c$, $T_{ij}$. According to thermoelectric technology and heat transfer characteristics in TEM, it can be obtained that [11].

$$Q_h = K_{leg}(T_h - T_{ij}) + N\alpha_{pn}I_o T_h - 0.5I_o^2R_i = K_c(T_h - T_{ij})$$

$$Q_c = K_{leg}(T_{ij} - T_{ij}) + N\alpha_{pn}I_o T_{ij} + 0.5I_o^2R_i = K_c(T_{ij} - T_{ij})$$

Figure 1. Structure of a TEM.

Figure 2. Temperature details of a TEM.

Figure 3. The electric diagram of a TEM.

Here $\alpha_{pn} = \alpha_p - \alpha_n$, and $\alpha_{pn}$, $N$, $K_{leg}$, $K_c$, $R_i$, $I_o$ are Seebeck coefficient of material, pairs, total thermal conductance of legs, thermal conductance from the leg junction to plate, internal resistance...
and output current of TEM, respectively. Figure 3 shows the electric diagram of TEM, in which $\varepsilon$ is the electromotive force of TEM.

By combining with Equations (1) and (2), the temperature difference, $\Delta T_{eg}$ of two junctions of legs can meet:

$$
\Delta T_{eg} = \frac{K_e}{2K_{leg}K_e + K_c^2} \left[ K_e(T_h - T_c) - \frac{(N\alpha_m)^2 R_I^2}{K_e} - N\alpha_m I_o(T_h - T_c) \right]
$$

(3)

Since the $I_o$ varies with the sampling time $t$, it can be seen from Equation (3) that when the structure of TEM (such as $N$, $K_e$, $K_{leg}$, $R_I$) is determined, $\Delta T_{eg}(t)$ is a function of $T_h$, $T_c$, $I_o$, $t$, which is $\Delta T_{eg}(t) = f(T_h, T_c, I_o, t)$. Then the short-circuit current $I_o$ can be expressed by the formula:

$$
I_o = \frac{\varepsilon(t)R_I}{R_i} = \frac{N\alpha_m \Delta T_{eg}(t)}{R_i} = \frac{N\alpha_m f(T_h, T_c, I_o, t)}{R_i}
$$

(4)

The $V_{oc}$ can be expressed as follow:

$$
V_{oc} = \varepsilon(t) = N\alpha_m \Delta T_{eg}(t) = N\alpha_m f(T_h, T_c, I_o, t)
$$

(5)

From the Equations (4) and (5), it can be concluded that $I_o$ and $V_{oc}$ change dynamically with $t$ even though the $T_c$ and $T_h$ are constant, which is the theoretical basis of this study.

Next we will further analyse a method of using $V_{oc} \times I_o$ to estimate the $P_{max}$ [10, 13-15] of TEM by means of the average estimation of instantaneous $V_{oc}$ and $I_o$ under the two extreme conditions which are the minimum and maximum heat inputs corresponding to switching from steady open to instantaneous short circuit and from steady short to instantaneous open circuit, respectively. For the first Mode A, i.e. from steady open circuit promptly switching to open circuit, $I_o$ will be change instantaneously from 0 to short current $I_o$ simultaneously with steady open circuit instant switching to short circuit. A closed-loop changing between electrical parameters and temperature field will be generated as $I_o$ changes, that is, as the electrical parameter $I_o$ increases from 0 to $I_o$ (maximum) instantaneously, the temperature difference $\Delta T_{eg}(t)$ decreases from $f(T_h, T_c, 0, t)$ to $f(T_h, T_c, I_o, t)$, and then cause changes of electric parameters $\varepsilon(t) = N\alpha_m \Delta T_{eg}(t)$ and $I_o(t) = \varepsilon(t)R_I^{-1}$, which would lead further to changing of the temperature field $\Delta T_{eg}(t) = f(T_h, T_c, I_o, t)$. After so many cycles like this, the dynamic equilibrium between temperature field and electrical parameters is finally achieved.

From the closed-loop change between the current and the temperature, it can be concluded that the change of $I_o$ will change the internal temperature $(T_{by}, T_{cj})$ distribution instantaneously and further affect $I_o$. However, due to thermal inertia, the change of plate temperature $T_h$ and $T_c$ will lag behind the change of internal temperature $(T_{by}, T_{cj})$, so the plate temperature $(T_h, T_c)$ is essentially the same at the switching moment from open to short circuit. Nevertheless $I_o$ always changes with the sampling time $t$ during at the switching process.

Based on the above lag characteristics of plate temperature, the short circuit current $I_{oc1}$ obtained at the switching moment can be considered as a transient output current in the condition of open circuit at constant temperature. Combining with the steady $V_{oc1}$, the $P_{max}$ can be theoretically calculated under the open circuit condition of minimum input heat power, that is $P_{max} = 0.25V_{oc1}I_{oc1}$ [10]. Similarly, for the second Mode B, i.e. from steady short circuit promptly switching to short circuit, combining with the steady output short current $I_{oc2}$, the dynamic transient open circuit $V_{oc2}$ at switching moment is used to obtain $P_{max}$ under the short circuit condition of maximum input heat power, that is $P_{max} = 0.25V_{oc2}I_{oc2}$ [10]. Because the $I_o$ in Mode A or $V_{oc}$ in Mode B is the dynamic
transient value which is closely relate to the sampling time \( t \), so the accuracy of \( P_{o_{\text{max}}} \) varies with \( t \). The correlation between sampling time and accuracy is further analysed as follow.

3. Analysis of measurement results

Based on the testing principle as shown in Figure 3, taking the typical TEM (TEG1-127-2.8-1.2-250) manufactured by the Guangdong Fuxin Technology Company, China as a testing sample, the data of different temperature conditions and sampling time are analysed, and make conclusion that the data sampling time affects the accuracy of maximum output power. The actual testing system and measurement are the same as that in Reference [16]. The only difference between them is that the sampling time of the matching internal resistance method used in Reference [16] must be less than 50ms. Under testing conditions of \( T_c=30^\circ \text{C} \) and the six different hot plate temperatures, the calculation results of sampling values at Mode A and Mode B corresponding to different sampling times (0, 200, 400, 600, 800 and 1000ms) are shown in Figure 4 and Figure 5, respectively. Figure 4 shows the relative error of \( P_{o_{\text{max}}} \) estimations at different sampling time of Mode A and B relative to the switching moment (defined as 0ms).

![Figure 4](image_url)

**Figure 4.** The curves of TEM output power with sampling time in A and B modes: (A-open to short circuit; B-short to open circuit).

Taking the working condition at \( T_c=30^\circ \text{C} \) and \( T_h=175^\circ \text{C} \) as an example. At Mode A, with the increase of sampling time, the estimation of \( P_{o_{\text{max}}} \) decreases from 14.22 W to 13.26 W and the error relative to switching moment increases gradually and finally reaches to -6.8% at 1000ms. The mechanism analysis is as follows. The output power, \( P_o (P_o = I_o V_o) \) of TEM is 0 in short circuit condition \((V_o=0V)\), and then \( Q_o = Q_o + P_o = Q \). When the steady open circuit is promptly switched to short circuit, \( I_o \) increases from 0 at open circuit to the maximum at short circuit. From Equation (2), it is obtained that: \( Q_o \) increases with the increase of \( I_o \). It can be deduced from Equation (1) and Equation (2):

\[
T_h = T_h - \frac{Q_h}{K_c} \tag{6}
\]
\[
T_c = T_c + \frac{Q}{K_c} \tag{7}
\]
Then,

$$\Delta T_{\text{leg}} = T_{bj} - T_{cj} = T_h - T_c - 2 \frac{Q_h}{K_c}$$

(8)

Figure 5. Temperature curves of TEM hot plate with sampling time in the Mode A and B (A-open to short circuit; B-short to open circuit).

Although there is thermal inertia, $T_h$ varies with the increasing of sampling time. The variation of $T_h$ with sampling time at two switching modes is shown in Figure 5. The temperature chiller is used to keep the temperature of the cold plate, $T_c$ constant during the test process. In Figure 5, we can see that the hot plate temperature varies slightly with sampling time at lower $T_h$, such as at 50°C and at 1000ms, it changes from 50 to 49.2°C (Mode A) and 50.9°C (Mode B), respectively. The variation increases with the rise of hot plate temperature. Taking the hot temperature at switching moment (0ms), 175°C and 1000ms sampling time as the reference, the maximum difference is 5.8°C. The maximum fluctuation is about 3%. In order to facilitate the analysis without affecting the conclusion, the temperature fluctuation at the hot plate $T_h$ in the sampling process is neglected that is the temperature of the hot plate is kept constant during the test process. It can be simplified to that:

$$I_o \uparrow \rightarrow Q_h \uparrow \rightarrow \Delta T_{\text{leg}} \downarrow$$

So the short-circuit current of TEM is reduced as shown in the following equation:

$$I_{sc} = \frac{\varepsilon}{R_i} = \frac{N \alpha_{\mu_i} \Delta T_{\text{leg}}}{R_i}$$

(9)

Finally, the dynamic equilibrium temperature field has been established at the dynamic current. When the open circuit is switched to short circuit, the estimation of $P_{\text{max}}$ ($P_{\text{max}} = 0.25V_{oc}I_{sc}$) decreases with the $I_{sc}$ decreasing, due to the $V_{oc}$ keeping constant. This is consistent with the actual measurement results.

Similarly, at Mode B, the initial state of short circuit correspondings to the maximum input heat power keeping $T_c$ and $T_h$ constant. As the output is switched to open circuit, the input heat power $Q_h$ decreases at once and will cause the following changes: $Q_h \downarrow \rightarrow \Delta T_{\text{leg}} \uparrow \rightarrow V_{oc} = N \alpha_{\mu_i} \Delta T_{\text{leg}} \uparrow$. Since the
$I_n$ is constant at steady short circuit, and the $V_{oc}$ increases with sampling time, the estimation of maximum output power ($0.25V_{oc}I_n$) increases with the increase of $V_{oc}$, which is also consistent with the actual measurement results.

Based on the above theoretical analysis, combined with Figure 4, it can be concluded that for the single switching Mode A or B, the trend of TEM output power decrease and increase is the consistent with the extension of sampling time at different $T_s$. Taking the output power at 0ms sampling time as the reference, the relative errors at different sampling time of 200, 400, 600, 800 and 1000ms are about ±2%, ±4%, ±6%, ±7% and ±8%, respectively. The "-" means the error is lower than the reference, and the "+" means the error is higher than it. However, in two different switching Mode A and B, the trend of TEM output power varies with sampling time. There is a downtrend of output power at Mode A. The longer the sampling time is, the more the output power decreases. On the contrary, the output power at Mode B shows an uptrend, i.e. increases with sampling time. The rising rate is greater than the decreasing rate of Mode A.

![Figure 6](image.png)

**Figure 6.** Curves of relative errors between the calculated and accurate output power of TEM with sampling time in two modes (A-open to short circuit; B-short to open circuit).

The curves in Figure 6 and Figure 4 are the same, but the relative error data on the curve have different meanings. Figure 5 shows the relative errors of the output powers of TEM at different sampling times relative to the output power in Mode A or B at 0ms sampling time. The reference in Figure 6 is the actual and accurate maximum output power of TEM in the load matching with the internal resistance. Comparing with Figure 6, at first we can see that the relative error between the output power value of TEM and the accurate value is the greatest at the 0ms switching moment regardless of any temperature or any switching mode. The errors at different temperatures are between ±5% and ±8%. The two initial steady states of open circuit and short circuit correspond to the minimum and maximum input heat power states, respectively. The input heat power at the state of actual maximum output power of TEM is close to the average of minimum and maximum one, so as to lead to the greatest difference between the initial states of minimum or maximum input heat power and average one. Therefore, the error between the output power obtained from the initial state of Mode A or B and the accurate value is the maximum.

Secondly, with the increase of sampling time, the absolute value of relative error in Mode A or B (Mode A’s is higher than the accurate value; Mode B’s is lower than the accurate value) decreases
gradually at first, and then increases gradually. In different temperature and switching modes, the sampling time points of minimum error are different, but the variation trend is consistent. With the increase of sampling time after switching, internal temperature distribution of TEM gradually reaches to the one of the maximum output power. The minimum error point will appear at the temperature distribution of TEM coinciding with it of maximum output power. At single switching Mode A, the most accurate calculation results of output power at different temperatures corresponding to the sampling time of 800 to 1000ms. Similarly, at the switching Mode B, the optimal sampling time is 600 to 800ms. There are slight differences at different temperature of hot plate. At 175°C, the optimal sampling time is 800 to 1000ms.

Table 1. The table of relative error between the average calculated and accurate values of output power sampled at different temperature and sampling time in two modes.

| T/°C | 0ms   | 200ms | 400ms | 600ms | 800ms | 1000ms |
|------|-------|-------|-------|-------|-------|--------|
| 50   | 0.4%  | 0.5%  | 0.4%  | 0.5%  | 0.6%  | 0.6%   |
| 75   | 0.3%  | 0.3%  | -0.4% | -0.3% | -0.3% | -0.4%  |
| 100  | 0.2%  | 0.5%  | 0.3%  | 0.3%  | 0.1%  | 0.3%   |
| 125  | 0.2%  | 0.5%  | 0.7%  | 0.6%  | 0.9%  | 0.8%   |
| 150  | 0.1%  | 0.1%  | 0.2%  | 0.2%  | 0.3%  | 0.3%   |
| 175  | 0.2%  | 0.2%  | 0.2%  | 0.2%  | 0.3%  | 0.4%   |

Table 1 shows the variation of the relative errors between $P_{o-AVG}$ and $P_{o-max}$ at different hot temperature and sampling time, i.e. $(P_{o-AVG} - P_{o-max}) / P_{o-max}$, where $P_{o-AVG}$ is the average value between the output power $P_{o-A}$ at Mode A and $P_{o-B}$ at Mode B, i.e. $P_{o-AVG} = 0.5(P_{o-A} + P_{o-B})$ and $P_{o-max}$ is the actual maximum output power of TEM. From the data in Table 1, it can be seen that the relative error of the average sampling output power calculation is small ($\leq 1\%$) at any temperature of hot plate and any sampling time. This is mainly due to the average input heat power condition is closer to that of the maximum output power at any temperature. Comparing with the data in Table 1, it also can be seen that the average maximum output power calculated by sampling values at the beginning of the switching time is the most accurate in different temperature states.

4. Conclusions

The maximum output power of TEM can be estimated quickly by the open-circuit voltage and short-circuit current which is steady value and the other is transient one. Using the samples to estimate the $P_{o-max}$ in single mode which mode A is from steady open circuit switching to transient short circuit or mode B is from short circuit switching to open circuit, the smaller the sampling time is, the larger the error between the calculated $P_{o-max}$ and the actual maximum output power is and its maximum error is less than $\pm 8\%$. The estimations of model A and model B are higher and lower than actual values, respectively. The minimum error occurs between 600 and 1000ms of sampling time and it is less than $\pm 3\%$. Because the complementarity input heat power is closer to the working condition of the maximum output power, the accuracy of $P_{o-max}$ obtained by using the corresponding average method of sampling calculated results of mode A and B is obviously improved and is less than 1%. The sampling time has little influence on the accuracy in this case. The smaller the sampling time is, the more accurate the estimation of $P_{o-max}$ obtained by the averaging sampling method.

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