Semi Supervised Image Segmentation Based on Markov Random Field and Kernel K-means

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Abstract: Semi supervised image segmentation uses a small amount of supervised information to improve the performance of image segmentation. In the first part of this paper, we construct a submodular regularization term based on Markov random field (MRF). The regularization term is combined with kernel k-means (KKM), and a semi supervised kernel k-means algorithm (SKKM) is designed based on graph cut technique. In the second part of this paper, we combine SKKM with smooth regularizer to get MRF & SKKM. The experimental results show that, compared with the original SKKM, the segmentation results of MRF & SKKM achieve better results in common evaluation indicators; moreover, compared with the binary image of segmentation results, MRF & SKKM has higher and smoother edge fitting.

1. Introduction

Semi supervised learning (SSL) is a learning method between supervised learning and unsupervised learning. In actual problems, the number of labeled samples is much smaller than the number of unlabeled samples. Unsupervised learning and supervised learning do not use unlabeled samples, so the generalization ability is difficult to guarantee. To avoid the problem of traditional supervised learning, SSL introduces unlabeled samples in training, which greatly improves learning efficiency [1].

In machine learning and computer vision, the clustering algorithm KKM [2] solves the problem that KM algorithm [3] (K-means, referred to as km) requires data sets to be linearly separable in the input space. In order to use graph cut [4] technique to iteratively optimize KKM, Tang et al. [5] constructed linear upper bound function for KKM, and MRF regularization and KKM algorithm fusion.

Inspired by the above work, in order to express the bound pair constraints as semi supervised information, we construct a regularization term with submodular property based on MRF framework. Combining KKM with the regularization term, a SKKM algorithm based on graph cuts is designed. Next, this paper combines smooth regularizer with SKKM, and proposes MRF & SKKM algorithm, which achieves good segmentation effect.

2. Related theories

The main idea of KKM is to map the linearly inseparable data of the original input space to a high-dimensional space, so that the original problem can be solved by a simple linear separator. The work of Tang et al. [6] gives the expression form of KKM when clustering two or more clusters. This paper considers that image segmentation is limited to two classifications, that is, it is divided into disjoint pixel sets S and S. Given the data points \( \{ I_p \mid p \in \Omega \} \) that cannot be correctly divided by the KM in the input...
space, these data points are mapped into the Hilbert space through the kernel function \(k\). Then we can get the expression of KKM's energy function with respect to two clusters:

\[
E_k(S) = \frac{\sum_{pq \in S} k_{pq}}{2|S|} + \frac{\sum_{pq \notin S} k_{pq}}{2|\bar{S}|}
\]

(1)

The kernel function is \(k_{pq} = k(I_p, I_q)\), and the number of data points contained in cluster \(S\) and cluster \(\bar{S}\) is \(|S|\) and \(|\bar{S}|\), respectively.

In the work of Tang et al., KKM is combined with MRF regularization \([5,6]\), and a new objective energy function is obtained:

\[
E(S) = E_k(S) + \gamma \sum_{c \in F} E_c(S_c)
\]

(2)

Among them, the first term \(E_k(S)\) is the energy function of KKM, see formula (1). \(\gamma \sum_{c \in F} E_c(S_c)\) is a MRF regularized potential energy similar to the second-order Potts model. Its role is to ensure that the boundaries are aligned and are sub-modular. The correlation weight \(\gamma\) value of the second term is a constant. For large images, due to the high computational complexity of the KKM method, Tang et al. \([5,6]\) proposed a bound optimization algorithm that approximates the global optimal value of the original objective function by optimizing the auxiliary (upper bound) function of the objective function. As shown below, formula (3) is an auxiliary (upper bound) function \(A(S)\) of KKM, which corresponds to the energy function formula (1). Among them, the target energy function \(E_k(S)\) is less than or equal to the auxiliary function at any time \(t\), see formula (4).

\[
A(S) = -2\sum_{p \in S} \frac{k_{pq}}{|S|} - 2\sum_{q \in \bar{S}} \frac{k_{pq}}{|\bar{S}|} + |S| \sum_{p \in S} \frac{k_{pq}}{|S|} + |\bar{S}| \sum_{q \in \bar{S}} \frac{k_{pq}}{|\bar{S}|}
\]

(3)

\[
E_k(S) \leq A(S)
\]

(4)

3. Semi supervised kernel k-means

Inspired by the work done by Tang et al. \([5]\), this paper presents a semi supervised kernel k-means (SKKM) model based on the MRF framework to express bound pairwise constraints for semi supervised information.

3.1. SKKM energy function based on paired constraints

When two samples must be in the same class cluster, we call it bound pairwise constraint. In the field of image segmentation, the bound constraint can be obtained by user interaction or automatically detected by saliency detection.

If \(M\) is used to represent the paired must-have set, when \((p, q) \in M\), then there is \(S_p = S_q\). In layman's terms, they are both in the foreground or in the background. We use the mathematical formula \(\beta_{pq}[S_p - S_q]\) to express: (1) impose a penalty for not satisfying the constraint; (2) do nothing when the constraint is satisfied. details as follows:

\[
E_{L1}(S) = \sum_{(p, q) \in M} \beta_{pq}[S_p - S_q]
\]

(5)

In the above formula, \(\beta_{pq}\) is used as a normal number indicating the strength of punishment. In this paper, the energy function is constructed so that the clustering result satisfies the pairwise bound constraints as much as possible, and the pairwise bound constraints are shown in formula (6):

\[
E_{SKKM}(S) = E_k(S) + E_{L1}(S)
\]

(6)
3.2. Proof of L1 term Model of SKKM energy function

According to formula (6), it is found that the L1 term and the second-order Potts model are both MRF regularization potentials, and they are very similar. We prove according to the definition of regularity. \( E_{L1} \) represents a graph with binary variables. \( p \) and \( q \) are arbitrary pixels, then both \( p \) and \( q \) satisfy:

1. If \( S_p = S_q = 0 \), then \( E^{p,q}(0,0) = 0 \)
2. If \( S_p = S_q = 1 \), then \( E^{p,q}(1,1) = 0 \)
3. If \( S_p = 0, S_q = 1 \), then \( E^{p,q}(0,1) = \beta_{pq} \)
4. If \( S_p = 1, S_q = 0 \), then \( E^{p,q}(1,0) = \beta_{pq} \)

Because \( \beta_{pq} \) is a normal number, all \( E^{p,q} \) satisfy inequality (7), thus proving that \( E_{L1} \) is a regularization potential with submodularity.

\[
E^{p,q}(0,0) + E^{p,q}(1,1) \leq E^{p,q}(0,1) + E^{p,q}(1,0)
\]  (7)

3.3. Auxiliary functions of SKKM

In Section 2, the auxiliary (upper bound) function \( A_t(S) \) of KKM is introduced (see formula (3)); in Section 3.1, the SKKM energy function is proposed (see formula (6)), in Section 3.2, according to the definition of regularity, L1 term is submodular. In summary, we give an auxiliary (upper bound) function \( \alpha_{SKKM}^{t}(S) \) of SKKM:

\[
\alpha_{SKKM}^{t}(S) = A_t(S) + E_{L1}(S)
\]  (8)

4. MRF & SKKM that merges MRF and SKKM

In order to make the edge part appear smoother in the segmentation result, this paper combines SKKM and smooth regularizer to obtain MRF&SKKM, and conduct a comparative experiment.

4.1. MRF regularizer

Formula (9) is a MRF regularizer using the second-order Potts model, and its role is to make the boundary smoother.

\[
\sum_{c \in F} E_c(S_c) = \sum_{p,q \in N} w_{pq}[S_p \neq S_q]
\]  (9)

Among them, the edge \( c\{p,q\} \) composed of neighboring nodes in the t graph is contained by the binary factor \( F = N \). The discontinuous penalty value between the node \( p \) and the node \( q \) is expressed by the weight \( w_{pq} \), and can be set as a decreasing function of the brightness difference \( I_p - I_q \) or using a constant, so that the boundary where the image contrast difference is large tends to be more divided.

4.2. Energy function and auxiliary function of MRF&SKKM

The MRF regularizer is combined with SKKM to construct the energy function of MRF&SKKM. Formula (10) is the target energy function of MRF&SKKM.

\[
E_{MRF&SKKM}(S) = E_{SKKM}(S) + \gamma \sum_{c \in F} E_c(S_c)
\]  (10)

\( \alpha_{t}^{SKKM&MRF}(S) \) in formula (11) is an auxiliary function corresponding to the MRF&SKKM objective function, which is also derived from the second-order Potts model’s submode.
4.3. Use maximum flow algorithm to solve auxiliary functions

In this paper, the maximum flow algorithm is used to solve the auxiliary function. Through many iterations, the auxiliary function can approach the optimal value of the original objective function step by step.

Algorithm flow:

Initialization: $T = 0, S_t$

Before the shutdown conditions are met, cycle through the following steps:

1. The auxiliary function is solved by maximum flow algorithm:

   \[ S^* = \arg \min_s \alpha (S) \]

2. To update $t = t + 1, S_t = S^*$

end

output $S_t$

5. Experiment

The experiment in this section is based on image data segmentation. In the image, the part that people care about is often called the foreground, and the other part is the background. In the semi-supervised binary classification segmentation, the goal is to divide all the pixels in the image into the front scene set or background point set. Some known front attractions and background points are used to construct the bound constraint and the unconnected constraint. The bound constraint means that two points are both the front sight or the background point set. The non-joint constraint means that the two points belong to the front sight and the background point respectively. Pairwise constraints can be obtained through user interaction, such as scribing, or can be automatically detected by other tools, such as saliency detection. In this paper, we use user interaction to draw lines.

The first step of this work is to combine the submodular L1 term with the energy function of KKM to obtain the energy function SKKM $E$ of SKKM. And, based on the auxiliary upper bound function of the original KKM, this paper constructs the auxiliary upper bound function of the SKKM energy function. In the second step, the MRF regularizer with sub-mode is combined with SKKM to construct the energy function of MRF&SKKM, and based on the sub-mode of the second-order Potts model, the auxiliary function of MRF&SKKM energy function is obtained. The solution method uses the maximum flow the minimum cut algorithm.

5.1. Experimental setup

(1) Data set and code

Data set: MSRA1K image data set.

Code address: http://www.csd.uwo.ca/~yuri/Abstracts/iccv15-kernel-abs.shtml

(2) Measures

ER (error rate), F1 (F1 measure \(^8\)), JC (Jaccard coefficient \(^9\)) and MHD (Modified Hausdorff Distance \(^10\))

(3) Initialization

Modeling: Gaussian mixture model with two Gaussian branches; initial segmentation: image segmentation results obtained by EM algorithm.

(4) Optimal value of weight parameter $\gamma$ of second order Potts model

In order to find the optimal value of the weight parameter $\gamma$ of the second-order Potts model as the parameter value of SKKM and MRF & SKKM. The value of $\gamma$ is set to 0.001, 0.01, 0.1, 1, 10 in turn, and the effect of image segmentation is recorded when different $\gamma$ values are recorded. The indicators
were Er, F1, JC and MHD. At the same time, the mean and variance of measurement indexes were recorded. The experimental data were 100 randomly selected images in MSRA1K.

Table 1 evaluation table of MRF & SKKM image segmentation results with different gamma values

| γ   | ER    | variance | F1    | variance | JC    | variance | MHD   | variance |
|-----|-------|----------|-------|----------|-------|----------|-------|----------|
| 0.001 | 0.0442 | 0.0037  | 0.8803 | 0.0229  | 0.8151 | 0.0363  | 2.1875 | 4.9035  |
| 0.01  | 0.0445 | 0.0035  | 0.8796 | 0.0232  | 0.8146 | 0.0376  | 2.1923 | 4.9243  |
| 0.1   | 0.0358 | 0.0031  | 0.9011 | 0.0197  | 0.8452 | 0.0320  | 1.8003 | 4.0294  |
| 1     | 0.0306 | 0.0027  | 0.9109 | 0.0228  | 0.8533 | 0.0322  | 1.6329 | 5.5391  |
| 10    | 0.1056 | 0.0239  | 0.7262 | 0.0882  | 0.6443 | 0.1101  | 6.2137 | 45.6702 |

In Table 1, we observe that when γ = 1, the measurement index of image segmentation results is the best. Therefore, the experimental parameter γ is set to 1.

5.2 Analysis of experimental results
This section is the image segmentation experiment of MRF & SKKM algorithm and SKKM algorithm. The measurement indexes are ER, F1, JC and MHD. At the same time, the mean and variance of measurement indexes were recorded. The experimental data were 100 randomly selected images in MSRA1K.

Table 2 evaluation table of SKKM and MRF & SKKM

|        | ER    | variance | F1    | variance | JC    | variance | MHD   | variance |
|--------|-------|----------|-------|----------|-------|----------|-------|----------|
| SKKM   | 0.0463 | 0.0042  | 0.8691 | 0.0257  | 0.8092 | 0.0387  | 2.3875 | 4.9275  |
| MRF&SKKM | 0.0317 | 0.0031  | 0.9125 | 0.0221  | 0.8615 | 0.0316  | 1.6932 | 5.5206  |

According to table 2, MRF & SKKM is obviously better than SKKM algorithm in the mean and variance of Er, F1, JC and MHD.

![Image segmentation binary graph between SKKM and MRF & SKKM](image)

Fig. 1 Image segmentation binary graph between SKKM and MRF & SKKM

Figure 1 shows the segmentation results. The first line to the fifth line are the original image, marked image, standard binary image, MRF & SKKM image segmentation binary image, SKKM image.
segmentation binary image. Compared with the image segmentation results of SKKM, the segmentation results of MRF & SKKM are: (1) the segmentation effect is closer to the standard binary image; (2) the edge part is smoother. MRF & SKKM achieves better results than SKKM in terms of the results of Er, F1, JC and MHD, as well as the accuracy and edge smoothness of binary image.

6. Conclusion
The work of this paper is mainly divided into two parts. The first part, based on MRF framework, constructs a submodular regularizer which is used to express semi-supervised information and must be connected into pairs of constraints. The energy function of SKKM is constructed by combining the regularization term with KKM. The auxiliary functions of SKKM are obtained based on the submodality of second-order Potts model. In the second part, the smooth regularizer is combined with SKKM to construct the energy function of MRF&SKKM, and the auxiliary function of MRF&SKKM is also obtained based on the submodality of second-order Potts model. And through experimental comparison, it is found that MRF&SKKM improves the segmentation effect compared with SKKM algorithm. The next step is to try to apply this method to other semi-supervised clustering problems to verify its wide application value.

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Reference
[1] Tu En-mei, Yang Jie. A Review of Semi-Supervised Learning Theories and Recent Advances[J]. Journal of Shanghai Jiaotong University, 2018, 52(10): 1280-1291.
[2] Marimuthu, S., Ponnambalam, S. G., & Jawahar, N. Evolutionary Algorithms for Scheduling M-machine Flow Shop with Lot Streaming[J]. Robotics and Computer-integrated Manufacturing, 2008, 24(1): 125-139.
[3] Duda, R. O., Hart, P. E., & Stork, D. G. Pattern Classification(2nd Edition)[M]. Wiley, 2001: 55-88.
[4] Jain, A. K. Data Clustering: 50 Years Beyond K-means[C]. European Conference on Machine Learning and Knowledge Discovery in Databases, 2008: 3-4.
[5] Tang, M., Ayed, I. B., Marin, D., & Boykov, Y. Secrets of GrabCut and Kernel K-means[C]. IEEE International Conference on Computer Vision, 2016: 1555-1563.
[6] Tang, M., Marin, D., Ayed, I. B., & Boykov, Y. Normalized Cut Meets MRF[J]. European Conference on Computer Vision. 2016:748-765.
[7] Muller, K., Mika, S., Ratsch, G., Tsuda, K., Scholkopf, B., & M"{u}ller, K. R., et al. An introduction to kernel-based learning algorithms[J]. IEEE Transactions on Neural Networks, 2001, 12(2):181.
[8] Mitchell. Machine Learning[M]. McGraw-Hill, 2003.
[9] Jain, A. K., & Dubes, R. C. Algorithms for Clustering Data[J]. Prentice Hall, 1988, 32(2): 227-229.
[10] Dubuisson, M. P., & Jain, A. K. A Modified Hausdorff Distance for Object Matching[M]. Int.conf.pattern Recognition Jerusalem Israel, 1994