Neutrino Oscillations Induced by Two-loop Radiative Mechanism

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(TOKAI-HEP/TH-0001, June, 2000)

Two-loop radiative mechanism, when combined with an $U(1)_{L'}$ symmetry generated by $L_e - L_\mu - L_\tau (=L')$, is shown to provide an estimate of $\Delta m^2_{23}/\Delta m^2_{atm} \sim \epsilon m_e/m_\tau$, where $\epsilon$ measures the $U(1)_{L'}$-breaking. Since $\Delta m^2_{23} \sim 3.5 \times 10^{-3} \text{ eV}^2$, we find that $\Delta m^2_{23} \sim 10^{-6} \text{ eV}^2$, which will fall into the allowed region of the LOW solution to the solar neutrino problem for $\epsilon \sim 0.1$.

PACS: 12.60.-i, 13.15.+g, 14.60.Pq, 14.60.St
Keywords: neutrino oscillations, neutrino mass, radiative mechanism

Recent evidence for atmospheric neutrino oscillations [1] has promoted new theoretical activities of understanding properties of neutrinos, especially concerning the long-standing theoretical issue of their masses and mixings [2]. It is known that there have been so far two main ideas to account for the smallness of neutrino masses, which are, respectively, called as seesaw mechanism [3] and as radiative mechanism [4]. Various possibilities of realizing the radiative mechanism in neutrino physics have been discussed [5, 6]. Especially, in recent analyses on neutrino mass matrix of the Zee-type [7], the usefulness of the conserved quantum number of $L_e - L_\mu - L_\tau (=L')$ has been recognized [8]. This $U(1)_{L'}$ symmetry works to yield maximal mixing in both atmospheric and solar neutrino oscillations [9]. The maximal mixing in atmospheric neutrino oscillations has been supported by the data indicating that $\sin^2 2\theta_{23} \sim 1$ with $\Delta m^2_{23} \sim 3.5 \times 10^{-3} \text{ eV}^2$ [10], where $\theta_{ij}$ stands for the mixing angle and $\Delta m^2_{ij}$ stands for the squared mass difference for $\nu^i \leftrightarrow \nu^j$. Solar neutrino oscillations have also been considered to exhibit the maximal mixing if the oscillations are described by $\Delta m^2_{21} \sim 3 \times 10^{-5} \text{ eV}^2$ as a large mixing angle solution (LMA), $\sim 10^{-7} \text{ eV}^2$ as a less probable solution with low probability and low mass (LOW) and $\sim 10^{-10} \text{ eV}^2$ as a vacuum oscillation solution (VO) [11].

In the present article, we further apply the ansatz of the $L'$-conservation to models of neutrino masses based on two-loop radiative mechanism [6]. It is anticipated to provide more natural explanation of the tiny neutrino mass without enhanced suppression in couplings, which is experimentally of order 0.01 eV [12]. Furthermore, some flavor-changing interactions receive extra suppression owing to the presence of the approximate $U(1)_{L'}$ symmetry. It should be also noticed that, in the radiative mechanism of the Zee type, which is based on one-loop diagrams, fine-tuning of lepton-number violating couplings is necessary to yield bimaximal mixing even if one invokes the $L'$-conservation. The fine-tuning can be characterized by “inverse hierarchy in the couplings”, namely, $f_{13}m^2_2 \sim f_{12}m^2_1$ [10], where $f$’s are to be defined in Eq.(1). In the present model, it will be shown that nearly bimaximal structure is dynamically guaranteed by the heaviness of the $\tau$ lepton and by the lightness of the electron. Therefore, no fine-tuning is necessary.

The two-loop radiative mechanism can be embedded in the standard model by employing two $SU(2)_L$-singlet charged Higgs scalar, $h^+$ and $k^{++}$, in addition to the standard Higgs, $\phi$. The extra Higgses, $h^+$ and $k^{++}$, respectively, couple to charged lepton-neutrino pairs and charged lepton-charged lepton pairs. Their interactions are described by

$$-\mathcal{L}_h = \sum_{i,j=1,2,3} \frac{1}{2} f_{ij} \overline{\psi^i_L} \psi^i_L h^+ + f_{(ij)} (\ell^i_R) \ell^j_R k^{++} + \text{(h.c.)},$$

where $\psi^i_L$ and $\ell^i_R$ (i=1,2,3) stand for three families of leptons and the Yukawa couplings, $f$’s, satisfy $f_{i|j} = -f_{|j}$ and $f_{(ij)} = f_{(ji)}$. Now, let us introduce the $U(1)_{L'}$ symmetry into $\mathcal{L}_h$. By envisioning the import of its breaking effect,
we employ an additional $k^{\pm\pm}$ to be denoted by $k^{\pm\pm}$.

The quantum number, $L'$, is assigned to be 1 for $(\psi^+_L, \ell^+_R)$, 0 for $(\phi, h^+, k^{\pm\pm})$, -1 for $(\psi^L, \ell^L)$ and -2 for $k^{\pm\pm}$. The ordinary lepton number, to be denoted by $L$, can also be assigned to be 1 for leptons, 0 for $\phi$ and -2 for $(h^+, k^{\pm\pm}, k^{\pm\pm})$.

Yukawa interactions take the form of

$$-\mathcal{L}_Y = \sum_{i=1,2,3} f^i_\phi \overline{\psi}_L^i \phi \ell^i_R + \sum_{i=2,3} \left( \frac{1}{2} f^{i11}_{1i} \overline{\psi}_L^i \psi^+_L h^+ + f^{i11}_{1i} \overline{\ell}_R^i \ell^+_R k^{\pm\pm} \right) + f^{i11}_{1i} \overline{\ell}_R^i \ell^+_R k^{\pm\pm} + (\text{h.c.}),$$

and Higgs interactions are described by self-Hermitian terms composed of $\phi \phi^\dagger$ ($\phi = \phi, h^+, k^{\pm\pm}, k^{\pm\pm}$) and by the non-self-Hermitian terms in

$$V_0 = \mu_0 h^+ h^+ k^{\pm\pm} + (\text{h.c.}),$$

where $\mu_0$ represents a mass scale. This coupling softly breaks the $L$-conservation but preserves the $L'$-conservation. To account for solar neutrino oscillations, the breaking of the $L'$-conservation should be included and is assumed to be furnished by

$$V_0 = \mu_0 h^+ h^+ k^{\pm\pm} + (\text{h.c.}),$$

where $\mu_0$ represents a breaking scale of the $L'$-conservation. One can instead introduce a neutral Higgs scalar, which spontaneously breaks $U(1)_L$ (or $U(1)_{L'}$) by acquiring vacuum expectation value related to $\mu_0$ (or $\mu_0$) \cite{3}. However, there necessarily appears a Nambu-Goldstone boson called Majoron, whose coupling to matter should be kept sufficiently small. Thus, the dangerous massless Majoron is to include soft $U(1)$-breaking interactions such as Eqs.(3) and (4), which generate its mass of order of the breaking mass scale.

Neutrino masses are generated by interactions corresponding to Fig.1 for the $U(1)_L$-conserving processes and Fig.2 for the $U(1)_{L'}$-breaking processes. The resulting neutrino mass matrix is found to be

$$M_\nu = \begin{pmatrix} 0 & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix},$$

where $m_{ii}$ and $m'_{ij}$ ($i,j = 2,3$) are calculated to be

$$m_{1i} = 2 \sum_{j=2,3} f_{1j}^i f_{j1}^i \frac{m_{\ell} m_e \mu_0}{m^2_k} F(m^2_{\ell}, m^2_k, m^2_{h}),$$

$$m'_{ij} = -f_{1i}^j f_{j1}^i \frac{m_{\ell} m_e \mu_0}{m^2_k} F(m^2_{\ell}, m^2_k, m^2_{h}),$$

under the approximation that $m^2_{k,h} \gg m^2_{\ell,e,h}$. Mass parameters, $m_{k,h}$, respectively, stand for the masses of Higgs scalars, $k^{\pm\pm}$, $k^{\pm\pm}$ and $h$, and the function of $F$ is defined by

$$F(x,y,z) = \frac{1}{16\pi^2} \frac{x \ln (x/z) - y \ln (y/z)}{x - y}.$$

The outline of its derivation can be seen from the Appendix.

The entries of $m_{12}$ and $m_{13}$ receive contributions from both $\mu$- and $\tau$-exchange as in Eq.(1). Since $m_\tau \gg m_\mu$, the $\tau$-exchange gives dominant contributions to $m_{12}$ and $m_{13}$, which result in the same mass-dependence. One can, then, observe that $m_{12} \sim m_{13}$ is a natural consequence without fine-tuning of the couplings. In fact, nearly bimaximal mixing is reproduced by $f_{12}^i \sim f_{13}^i$. Other entries, $m'_{ij}$'s, are further suppressed by the factor of $m_{\ell}/m_\tau$. Thus, the form of our neutrino mass matrix is consistent with the one described by nearly bimaximal mixing.

We find that $\Delta m^2_{\text{atm}}$ for atmospheric neutrino oscillations and $\Delta m^2_{\odot}$ for solar neutrino oscillations are calculated to be:

$$\Delta m^2_{\text{atm}} = m^2_{12} + m^2_{13} (\equiv m^2_\odot), \quad \Delta m^2_{\odot} = 4m_\nu \delta m$$

with

$$\delta m = \frac{1}{2} |m'_{22} \cos^2 \theta_\nu + 2m'_{23} \cos \theta_\nu \sin \theta_\nu + m'_{13} \sin^2 \theta_\nu |.$$
where the mixing angle $\theta_{\nu}$ is defined by $\cos \theta_{\nu} = m_{12}/m_{\nu}$ (sin $\theta_{\nu} = m_{13}/m_{\nu}$) and the anticipated relation of $m_{13}' \ll m_{ik}$ for $i,j,k = 2,3$ have been used. As far as mass scales are concerned, we reach

$$\Delta m_{\odot}^2 \sim \frac{\mu_b m_e}{\mu_m m_f} \Delta m_{atm}^2.$$  \hfill (11)$$

It turns out to be $\Delta m_{\odot}^2/\Delta m_{atm}^2 \sim 3.5 \times 10^{-3}$ eV$^2$ [17], which is the announced result. The experimental value of $\Delta m_{\odot}^2 \sim 10^{-7}$ eV$^2$ for $\epsilon \sim 0.1$, which lies in the region corresponding to the LOW solution to the solar neutrino problem [12].

To see order of magnitude estimates of our parameters, we have to first recognize possible constraints on masses and couplings since the interactions mediated by $h^+, k^{++}$ and $k^{++}$ disturb the well established low-energy phenomenology. The most stringent constraints on $f's$, which are relevant for our discussions, are listed as

$$\xi \frac{|f_{[11]} f_{[12]}|}{m_k^2} < 2.9 \times 10^{-11} (6 \times 10^{-9}) \text{ GeV}^{-2},$$  \hfill (12)$$

from $\mu^- \rightarrow e^- e^- e^+$ with $\text{BR}(\mu^- \rightarrow e^- e^- e^+) < 10^{-12}$ [14] ($\mu^- \rightarrow e^- + \gamma$ with $\text{BR}(\mu^- \rightarrow e^- + \gamma) < 4.9 \times 10^{-11}$ [14]) [13], where $\xi \sim (16 \pi^2)^{-1}(\mu_0/m_k)(\mu_b/m_{k'})$ arising from the loop for the $k^{++}-k^{'+}$ mixing, which represents the extra suppression factor due to $U(1)_{L'}$ and $\bar{m}_k^2$ stands for the averaged mass of $k^{++}$ and $k^{'+}$, and

$$\frac{|f_{[11]}|}{m_k^2} < 1.2 \times 10^{-5} \text{ GeV}^{-2},$$  \hfill (13)$$

from $e^- e^- \rightarrow e^- e^- [16]$. The contributions to this process via the $k^{'+}$-exchanges turn out to be higher loop-effects since $k^{'+}$ does not directly couple to $e^- e^-$ and are expected to be well suppressed. The $\mu$ decay of $\mu^- \rightarrow \nu_\mu \bar{\nu}_\mu e^-$ is used to determine the value of the Fermi constant, which includes the extra $h^+$-contributions, thus, providing slight deviation of the electroweak gauge coupling of $g$ from the standard value; therefore, the constraint should be deduced from that on $g$ [17], which can be translated into

$$\frac{|f_{[12]}|}{m_k^2} < 1.7 \times 10^{-6} \text{ GeV}^{-2},$$  \hfill (14)$$

for $\nu_\mu(\bar{\nu}_\mu)e^- \rightarrow \nu_\mu(\bar{\nu}_\mu)e^-$. For the present analysis, the couplings of $f's$ are kept as small as $\mathcal{O}(\epsilon)$. We adopt the following parameter values that satisfy these constraints, where $f_{[12]} \sim f_{[13]}$ is assumed to yield nearly bimaximal mixing: $f_{[12]} \sim f_{[13]} \sim 2e$ yielding $m_h > 350$ GeV by Eq.(14), from which $m_h \sim 350$ GeV is taken, $f_{[11]} \sim f_{[13]} \sim e$, $m_h \sim 2$ TeV with $m_k - m_{k'} \sim m_h/10$ and $\mu_0 \sim 1.5$ TeV with $\mu_b \sim \mu_0/10$ giving $\epsilon \sim 0.1$. The constraint of Eq.(12) is satisfied by $f_{[12]} < 1$. These parameters, in fact, reproduce $\Delta m_{atm} \sim 2.4 \times 10^{-3}$ eV$^2$ and $\Delta m_2^2 \sim 10^{-7}$ eV$^2$, which is relevant for the LOW solution to the solar neutrino problem. The mass scale of the heaviest neutrino mass is characterized by $(16 \pi^2)^{-2}(m_e m_\tau \mu_0/m_{k'}^2) \sim 0.01$ eV.

To conclude, we have demonstrated that two-loop radiative mechanism well works to account for neutrino oscillation phenomena when it is combined with the $U(1)_{L'}$ symmetry. Thanks to the well known loop-factor of $(16 \pi^2)^{-2}$, neutrino masses are well suppressed to yield $\mathcal{O}(0.01)$ eV. The couplings of $f's$ can be chosen to be $\mathcal{O}(\epsilon)$ as $f_{[12]} \sim f_{[13]} \sim 2e$ and $f_{[11]} \sim f_{[13]} \sim e$. Solar neutrino oscillations are controlled by the factor of $(m_e/m_{k'})^2$, leading to the relation of $\Delta m_{2}^2 \sim (m_e \mu_b/m_{k'} \mu_0)\Delta m_{atm}^2$, which provides the LOW solution for $\mu_b \sim 0.1 \mu_0$.

The work of M.Y. is supported by the Grant-in-Aid for Scientific Research No 12047223 from the Ministry of Education, Science, Sports and Culture, Japan.

Note added: While preparing this manuscript, we are aware of the article [18] that has treated the same subject and has reached slightly different conclusion.

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1. The possible contributions to the $\nu_e - \nu_\mu$ entry of $M_e$ in Eq.(6), arising from the three loop diagrams found in Ref. [13], come from four-loop diagrams involving the loop for the $k^{'+}-k^{++}$ mixing characterized by the factor of $\xi \sim (16 \pi^2)^{-1}(\mu_0/m_k)(\mu_b/m_{k'})$. This diagram at most yields $\delta \sim (16 \pi^2)^{-1}(m_{k'}^2/m_{k'}^2)$, which should be compared with $m_{k'}^2/m_{k'}^2$. Our estimate of $\Delta m_2^2 \sim m_{k'}^2$ is not drastically altered by including this effect since the present parameter set gives $\delta \sim 2m_{k'}^2/m_{k'}^2$, which turns out to be $\mathcal{O}(m_{k'}^2/m_{k'}^2)$.

2. Of course, $\epsilon \sim 10^{-4}$ gives $\Delta m_{2}^2 \sim 10^{-10}$ eV$^2$, corresponding to the VO solution [13]. However, it is not suitable for our discussions to obtain a tiny mass-splitting without such enhanced suppression in couplings.
Appendix

In this Appendix, we describe the outline of obtaining the integral of Eq.(8) used in Eqs.(16) and (17). From the diagram in Fig.1, we write the relevant integration to be:

\[
I = \int \frac{d^4k \, d^4q}{(2\pi)^4} \frac{1}{(k^2 - m^2_f) (k^2 - m^2_h) (q^2 - m^2_f) (q^2 - m^2_h) ((k-q)^2 - m^2_k)}.
\]

(15)

By performing the integration over \(k\) supplemented by

\[
\frac{1}{abc} = \frac{\Gamma (3)}{\Gamma (1) \Gamma (1) \Gamma (1)} \int_0^1 dx \int_0^1 y dy \frac{1}{[c + (b - c) y + (a - b) y]}.
\]

(16)

and by noticing the formula for the one-loop integral

\[
\int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - a) (q^2 - b) (q^2 - c)} = -\frac{i}{16\pi^2} \left[ \frac{a \ln a}{(a-b)(a-c)} + \frac{b \ln b}{(b-a)(b-c)} + \frac{c \ln c}{(c-a)(c-b)} \right],
\]

(17)

we reach

\[
I = \int dx dy \frac{i}{16\pi^2[y(1-y)]} I(x, y),
\]

(18)

where

\[
I(x, y) = -\frac{i}{16\pi^2} \left[ \frac{m^2_e \ln m^2_e}{(m^2_e - m^2_h) (m^2_e - M^2)} + \frac{m^2_h \ln m^2_h}{(m^2_h - m^2_f) (m^2_h - M^2)} + \frac{M^2 \ln M^2}{(M^2 - m^2_h) (M^2 - m^2_f)} \right].
\]

(19)

with

\[
M^2 = \frac{m^2_e - (m^2_f - m^2_h) y - (m^2_h - m^2_f) xy}{y(1-y)}.
\]

(20)

Under the approximation of \(m^2_k \gg m^2_{i,e,h}\), we find that

\[
y(1-y)(a-M^2) \approx -a(y-\alpha)(y-\beta)
\]

(21)

with

\[
\alpha = \frac{m^2_e}{a} \left[ 1 - \frac{m^2_h - (m^2_f - m^2_i) x}{m^2_k} \right], \quad \beta = 1 + \frac{m^2_h - (m^2_f - m^2_i) x}{m^2_k},
\]

(22)

which yield

\[
J(a) = \int dx dy \frac{1}{y(1-y)(a-M^2)} \approx \frac{1}{m^2_k} \frac{m^2_e \ln (m^2_e/m^2_h) - m^2_i \ln (m^2_i/m^2_f)}{m^2_i - m^2_h}
\]

(23)

and

\[
\int dx dy \frac{M^2 \ln M^2}{y(1-y)(a-M^2) (b-M^2)} \approx -\frac{aJ(a) - bJ(b)}{a-b} \ln m^2_k,
\]

(24)

where we have used \(M^2 \approx \ln m^2_k\). The parameters of \(a\) and \(b\) should satisfy the condition of \(a, b \ll m^2_k\). The function of \(J(a)\) turns out to be independent of \(a\) in the present approximation. Collecting these results, we finally obtain

\[
I = \frac{F(m^2_e, m^2_f, m^2_h) F(m^2_i, m^2_f, m^2_h)}{m^2_k},
\]

(25)

where

\[
F(x, y, z) = \frac{1}{16\pi^2} \frac{x \ln (x/z) - y \ln (y/z)}{x - y},
\]

(26)

which is the expression of Eq.(8).
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**Figure Captions**

Fig.1: $U(1)_L$-conserving two loop radiative diagrams for $\nu^i$-$\nu^j$ ($i=2,3$) via $\mu^-$ ($i=2$) and $\tau^-$ ($i=3$).

Fig.2: $U(1)_L$-breaking two loop radiative diagrams for $\nu^i$-$\nu^j$ ($i,j=2,3$) via $e^-$. 
Fig. 1: $U(1)_{L'}$-conserving two loop radiative diagrams for $\nu^i - \nu^j$ ($j=2,3$) via $\mu^-(i=2)$ and $\tau^-(i=3)$.

Fig. 2: $U(1)_{L'}$-breaking two loop radiative diagrams for $\nu^i - \nu^j$ ($i,j=2,3$) via $e^-$. 