Tensor Excitations in Nambu – Jona-Lasinio Model

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Abstract

It is shown that in the one-flavour NJL model the vector and axial-vector quasi-particles described by the antisymmetric tensor field are generated. These excitations have tensor interactions with quarks in contrast to usual vector ones. Phenomenological applications are discussed.

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1 Introduction

The Nambu – Jona-Lasinio (NJL) model \cite{1, 2} was proposed 35 years ago. But up to now there is still a great interest in this model \cite{3}. The main feature of the NJL model is that it provides an explanation of chiral symmetry breaking in particle physics in analogy with superconductivity \cite{4}. As far as elementary excitations in a superconductor can be described by means of a coherent mixture of electrons and holes, one can try to explain the specter of meson states in the framework of quark’s degrees of freedom. The relativistic theory offers many possibilities to construct rich specter of hadron physics.

In this letter I will show that new quasi-particles can be introduced in the one-flavour NJL model from the very beginning. These particles correspond to vector and axial-vector meson modes with quantum numbers $J^P C = 1^{-+}$ and $1^{+-}$, respectively. The latter is not mentioned at all in applications of the NJL model. They are described by the second rank antisymmetric tensor fields and allow vector description with nonlocal interactions. These fields appear in conformal theories \cite{5} and have not been yet used in the low-energy phenomenology. These excitations were missed and they are not considered as real particles at the present time. The reasons for that will be clarified later.

I must say that tensor currents appeared in the NJL model when a multiflavour case was discussed. These tensor terms generate the $\rho - \omega$ mass splitting through an intermediate bound state. In the original paper \cite{2} the authors have found even the vector excitations in the tensor channels. But the lack of known meson states did not allow linking these modes with real particles and they were completely forgotten. There is an opinion \cite{6} that such “anomalous” terms can appear only through the $U_A(1)$ breaking from the ’t Hooft instanton induced vertex \cite{7} and the presence of tensor mesons at nuclear length scale is elusive \cite{8}. This is a wrong concept. A tensor description of the vector mesons in the chiral field theory was also considered \cite{9}. But the final conclusion was that vector and tensor descriptions are equivalent \cite{10}. This is the right conclusion when we consider the interactions independently for vector and tensor fields and this is not the case when they interact with the other matter fields.

To crystallize the idea I deal with only the one-flavour NJL model. The generalization for many flavours is straightforward and will be presented elsewhere.

2 The effective Lagrangian.

One of the most important symmetries of the real world and the QCD, which is kept in the NJL model, is the chiral symmetry. Following the classical paper \cite{1} I require that the primary fermion interaction must be invariant under $\gamma^5$- and ordinary phase transformations

$$\psi \rightarrow \exp[i\alpha\gamma^5] \psi, \quad \bar{\psi} \rightarrow \bar{\psi} \exp[i\alpha\gamma^5],$$

$$\psi \rightarrow \exp[i\alpha] \psi, \quad \bar{\psi} \rightarrow \bar{\psi} \exp[-i\alpha],$$

where $\alpha$ is a constant and $\psi$ is the Dirac spinor corresponding to a quark field. I restrict myself to consideration of quark-antiquark bound state formations as real particles. These states are explicitly invariant under transformations \cite{2}.
As far as the Dirac spinor has four components one can construct 16 independent bilinear forms in quark-antiquark channel: \( \bar{\psi} \psi, \bar{\psi} \gamma^5 \psi, \bar{\psi} \gamma_\mu \psi, \bar{\psi} \gamma_\mu \gamma^5 \psi \) and \( \bar{\psi} \sigma_{\mu\nu} \psi \). Under the Lorentz group they transform as scalar, pseudoscalar, vector, axial-vector and antisymmetric tensor, correspondingly. To deal with the chiral properties of these bilinear forms it is useful to define chiral currents:

\[
V_\mu^{\pm} A_\mu = \bar{\psi} \gamma^\mu (1 \pm \gamma^5) \psi, \quad S_\mu^{\pm} = \bar{\psi} (1 \pm \gamma^5) \psi, \quad T_\mu^{\pm}_{\alpha\beta} = \bar{\psi} \sigma_{\mu\nu} (1 \pm \gamma^5) \psi.
\]

The vector \( V_\mu^{\pm} \) and axial-vector \( A_\mu \) currents obviously satisfy the chiral invariance. The last two terms transform under (1) as follows:

\[
S_\mu^{\pm} \to \exp[\pm 2i\alpha] S_\mu^{\pm}, \quad T_\mu^{\pm}_{\alpha\beta} \to \exp[\pm 2i\alpha] T_\mu^{\pm}_{\alpha\beta}.
\]

Now it is easy to construct the chiral invariant Lagrangian choosing scalar \( S_\mu^{\pm} \) and tensor \( T_\mu^{\pm}_{\alpha\beta} \) current-current interactions with opposite chiralities and various \( V_\mu \) and \( A_\mu \) interactions. The former is the primary interaction in the original work [1] of Nambu and Jona-Lasinio. The latter is used in the extensions of the NJL model to achieve a sufficient attractive force in the axial-vector channel [11]. What about the tensor one? It is easy to see that its Lorentz invariant form

\[
T_\mu^{+}_{\alpha\beta} T_\mu^{-}_{\alpha\beta} \equiv 0
\]

identically equals zero, because \( T_\mu^{+}_{\alpha\beta} \) and \( T_\mu^{-}_{\alpha\beta} \) belong to different irreducible representations of the Lorentz group, namely \((1,0)\) and \((0,1)\). To my opinion this is the main reason that these degrees of freedom were missed. To incorporate these currents into the NJL model I will carry out dynamical analysis of the modes associated with them. In general all collective modes become dynamical ones through the self-energy quantum corrections from the fermionic loops.

Let us consider the self-energy quantum correction to the tensor fields \( T_\mu^{\pm}_{\alpha\beta} \) from the Lagrangian

\[
\mathcal{L} = i \bar{\psi} \phi \psi + \frac{t}{4} \bar{\psi} \sigma_{\mu\nu} (1 + \gamma^5) \psi T_\mu^{+}_{\alpha\beta} T_\mu^{-}_{\alpha\beta} + t \bar{\psi} \sigma_{\mu\nu} (1 - \gamma^5) \psi T_\mu^{+}_{\alpha\beta} T_\mu^{-}_{\alpha\beta}.
\]

The divergent part of this contribution

\[
\mathcal{P}^{\pm}_{\mu\nu\alpha\beta}(q) = i \left( \frac{t}{4} \right)^2 \int \frac{d^4 p}{(2\pi)^4} \text{Tr}[\sigma_{\mu\nu} (1 + \gamma^5) (\gamma_5 \gamma_\alpha \gamma_\beta - \gamma_\alpha \gamma_\beta) (1 - \gamma^5) \gamma_\nu \gamma_\mu] \equiv \frac{1}{12\pi^2} \frac{t^2}{4\pi} \Pi^{\pm}_{\mu\nu\alpha\beta}(q) q^2
\]

determines the kinetic term of the free Lagrangian for the tensor fields

\[
\mathcal{L}^T = \frac{q^2}{2} T_\mu^{+}_{\alpha\beta}(q) \Pi^{\pm}_{\mu\nu\alpha\beta}(q) T_\mu^{-}_{\alpha\beta}(q),
\]

where the operator \( \Pi^{\pm}_{\mu\nu\alpha\beta}(q) \)

\[
\Pi^{\pm}_{\mu\nu\alpha\beta}(q) = 1^\pm_{\mu\nu\lambda\sigma} \Pi_{\lambda\sigma\alpha\beta}(q) = \Pi_{\mu\nu\lambda\sigma}(q) I^\pm_{\lambda\sigma\alpha\beta},
\]

is expressed through projectors

\[
1^\pm_{\mu\nu\alpha\beta} = \frac{1}{2} (1_{\mu\nu\alpha\beta} \pm \frac{i}{2} \epsilon_{\mu\nu\alpha\beta}),
\]

and

\[
1_{\mu\nu\alpha\beta} = \frac{1}{2} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}),
\]

\[
\Pi_{\mu\nu\alpha\beta}(q) = 1_{\mu\nu\alpha\beta} - (q_\mu q_\nu g_{\alpha\beta} - q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\alpha g_{\mu\beta} + q_\nu q_\beta g_{\mu\alpha})/q^2.
\]
There is a well known relation between nonlinear current-current interaction and bosonized interaction form expressed through the path integral

$$\exp[-\frac{i}{2}JK^{-1}J] = \int[d\varphi]\exp[iJ\varphi + \frac{i}{2}\varphi K\varphi].$$  \hspace{1cm} (11)

Therefore, one can apply directly this result to the NJL model. Removing the dynamic pole $q^2$ in (7) and using the property

$$\Pi_{\mu\nu\lambda\sigma}(q)\Pi_{\lambda\sigma\alpha\beta}(q) = \frac{1}{\mu\nu\alpha\beta},$$  \hspace{1cm} (12)

one obtains nontrivial chiral invariant tensor interaction in the NJL model

$$L_{\text{eff}}^T = -G\bar{\psi}_{\mu\lambda}(1 + \gamma^5)\psi \frac{q_{\mu\nu}}{q^2} \bar{\psi}_{\nu\lambda}(1 - \gamma^5)\psi$$

$$= -G \left[ \bar{\psi}_{\mu\lambda}\gamma^5\psi \cdot \bar{\psi}_{\nu\lambda}\psi - \bar{\psi}_{\mu\lambda}\gamma^5\psi \cdot \bar{\psi}_{\nu\lambda}\gamma^5\psi \right] \frac{q_{\mu\nu}}{q^2}. \hspace{1cm} (13)$$

The coupling constant $G$ is assumed to be positive, so that the forces between quarks and antiquarks are attractive.

I want to note here, that such kind of terms were missed also in the effective four-fermion interaction of the weak lepton decay [12] and are not taken into account when the phenomenological parameters are discussed [13]. The most general effective interaction must include such tensor terms which lead to new Michel parameters [14].

Let us define vector and axial-vector currents as

$$R_{\mu} = \hat{\partial}_{\nu} \left( \bar{\psi}_{\mu\nu}\psi \right), \hspace{1cm} B_{\mu} = i\hat{\partial}_{\nu} \left( \bar{\psi}_{\mu\nu}\gamma^5\psi \right),$$  \hspace{1cm} (14)

where $\hat{\partial}_{\mu} = \partial_{\mu}/\sqrt{-\partial^2}$. These are conserved nonlocal currents with quantum numbers $J^{PC}$ are $1^{--}$ and $1^{++}$. This conservation law is not a product of the gauge symmetry as in the Noether case, rather it is a topological conservation law. Usual vector $V_{\mu}$ and axial-vector $A_{\mu}$ currents are associated with quantum numbers $1^{--}$ and $1^{++}$, respectively. Now this is a complete set of basic vector-meson excitations [3]

$$I = 0 \hspace{1cm} I = 1 \hspace{1cm} I = 1/2$$

$$1^{--} : \omega, \phi \hspace{1cm} \rho \hspace{1cm} \rho^{*}$$

$$1^{++} : \ h_{1} \hspace{1cm} b_{1} \hspace{1cm} K_{1}$$

$$1^{++} : \ f_{1} \hspace{1cm} a_{1} \hspace{1cm} K_{1}$$

(15)

if the NJL model with three flavours is used.

### 3 The tensor fields.

In the previous section I have introduced unusual chiral tensor fields $T_{\mu\nu}^\pm$ and their Yukawa interaction with quarks [3]. Let me describe here in more detail the properties of these fields. Now if I apply once again the transformation [11] for bosonization of the effective Lagrangian [13], I must introduce vector $R_{\mu}$ and axial-vector $B_{\mu}$ fields, associated with currents [14]

$$L_{\text{Yukawa}} = t \hat{\partial}_{\nu} \left( \bar{\psi}_{\mu\nu}\psi \right) \cdot R_{\mu} + i t \hat{\partial}_{\nu} \left( \bar{\psi}_{\mu\nu}\gamma^5\psi \right) \cdot B_{\mu}. \hspace{1cm} (16)$$

2The $C$ assignment in $J^{PC}$ only refers to the neutral members of the integer isospin multiplets.
As far as these fields interact with conserved currents \( R_\mu \rightarrow R_\mu - \partial_\mu \eta \) and \( B_\mu \rightarrow B_\mu - \partial_\mu \xi \) with arbitrary \( \eta \) and \( \xi \) are allowed. Therefore, we have three physical degrees of freedom for each of the fields. Although the new fields have nonlocal tensor interactions with the quarks, such interactions can be rewritten in local manner \( (5) \) and do not spoil renormalizability \( (13) \). In order to obtain a relation between eqs. \( (5) \) and \( (16) \) I rewrite it through one real antisymmetric tensor field \( T_{\mu\nu} \)

\[
T_{\mu\nu} = \hat{R}_{\mu\nu} - \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \hat{B}_{\alpha\beta},
\]

using identity \( \frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma_{\alpha\beta} = \gamma^5 \sigma_{\mu\nu} \):

\[
L_{\text{Yukawa}} = \frac{t}{2} \bar{\psi} \sigma_{\mu\nu} \psi \cdot T_{\mu\nu},
\]

where \( \hat{R}_{\mu\nu} = \hat{\partial}_\nu R_\mu - \hat{\partial}_\mu R_\nu, \hat{B}_{\mu\nu} = \hat{\partial}_\nu B_\mu - \hat{\partial}_\mu B_\nu, \) and

\[
T_{\mu\nu}^\pm = 1_{\mu\nu\alpha\beta} T_{\alpha\beta}.
\]

The antisymmetric tensor field \( T_{\mu\nu} \) have six independent components: three-vector and three-axial-vector, that match with physical degrees of freedom for the newly introduced vector \( R_\mu = \hat{\partial}_\mu T_{\nu\mu} \) and the axial-vector \( B_\mu = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \hat{\partial}_\nu T_{\alpha\beta} \) fields. I will stress here that all interactions of the vector \( R_\mu \) and the axial-vector \( B_\mu \) fields can be described by only one antisymmetric tensor field \( T_{\mu\nu} \). This is the main difference between the new vector and axial-vector fields and the usual ones.

Using eq. \( (15) \) the free Lagrangian for tensor field \( (6) \) now can be written in more conventional form \( (16) \)

\[
L_{\text{free}}^T = \frac{1}{4} (\partial_{\lambda} T_{\mu\nu})^2 - (\partial_\mu T_{\mu\nu})^2.
\]

This Lagrangian differs from those used in \( (3) \) \( (10) \) and the gauge invariant ones \( (17) \). I do not put any constraint on the tensor field and consider all its degrees of freedom as physical. If we substitute here \( T_{\mu\nu} \) from \( (17) \) we return back to our fields \( R_\mu \) and \( B_\mu \):

\[
L_{\text{free}}^T = -\frac{1}{4} R_{\mu\nu}^2 - \frac{1}{4} B_{\mu\nu}^2,
\]

where \( R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu \) and \( B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \) This is nothing but the free Lagrangian for usual vector fields. In other words for the free fields an equivalence between tensor and vector fields exists. But it is not the case when interaction turns on and chiral symmetry breaking takes place.

### 4 The interactions.

First of all I will write down the Lagrangian for Yukawa interactions, initial fermionic and induced bosonic kinetic terms

\[
\mathcal{L} = g_S \bar{\psi} \gamma_5 \psi S + ig_P \bar{\psi} \gamma_5 \gamma \psi P + g_V \bar{\psi} \gamma_\mu \psi V_\mu + g_A \bar{\psi} \gamma_\mu \gamma_5 \psi A_\mu + \frac{t}{2} \bar{\psi} \sigma_{\mu\nu} \psi \cdot T_{\mu\nu}
\]

\[
+ i \bar{\psi} \partial_\mu \psi + \frac{1}{2} (\partial_\mu S)^2 + \frac{1}{2} (\partial_\mu P)^2 - \frac{1}{4} V_{\mu\nu}^2 - \frac{1}{4} A_{\mu\nu}^2 + \frac{1}{4} (\partial_\lambda T_{\mu\nu})^2 - (\partial_\mu T_{\mu\nu})^2,
\]

(22)
where I define the fields strength tensors $V_{\mu \nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $A_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ for the vector and the axial-vector fields as usually. Due to the dynamic appearance of the kinetic terms, the coupling constants in one-loop approximation turn out to be related

$$g^2_S = g^2_P = \frac{2}{3} g^2_V = \frac{2}{3} g^2_A = \frac{1}{3} = 8\pi^2 / N_c \varepsilon,$$  \hspace{1cm} (23)

where $N_c$ is the number of colours and $\varepsilon$ is the parameter of the dimensional regularization: $d = 4 - 2\varepsilon$. All fields are massless and could acquire masses through symmetry breaking with non-zero expectation value of the scalar field: $<S> \neq 0$.

It is known that global transformations become localized when the dynamic degrees of freedom are generated \[20\]. For the local gauge transformations \[24\], the vector field $V_\mu$ must transform as $V_\mu \rightarrow V_\mu + \partial_\mu \alpha / g_V$ to preserve the Lagrangian \[22\] invariant. Let us consider the more interesting case of the local chiral transformations. It is expected that the following transformations

$$\psi \rightarrow \exp[i \alpha \gamma^5] \psi, \quad S \rightarrow S \cos 2\alpha + P \sin 2\alpha, \quad T_{\mu \nu} \rightarrow T_{\mu \nu} \cos 2\alpha + \tilde{T}_{\mu \nu} \sin 2\alpha, \quad A_\mu \rightarrow A_\mu + \partial_\mu \alpha / g_A, \quad P \rightarrow P \cos 2\alpha - S \sin 2\alpha, \quad \tilde{T}_{\mu \nu} \rightarrow \tilde{T}_{\mu \nu} \cos 2\alpha - T_{\mu \nu} \sin 2\alpha,$$  \hspace{1cm} (24)

where $\tilde{T}_{\mu \nu} = \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} T_{\alpha \beta}$, do not preserve the invariance of the Lagrangian \[22\], because induced noninvariant kinetic terms exist and additional interactions among these fields must be introduced. These interactions can be written down using symmetry properties respect to the transformations \[24\] or can be derived directly from one-loop fermionic contributions. There exist two groups of terms. The first group includes interactions arising from substitution of covariant derivatives, which restore the local chiral invariance and obligatory contain interactions with the axial-vector field $A_\mu$:

$$L^A_{\text{int}} = 2g_A A_\mu [S \partial_\mu P - P \partial_\mu S] + 2g_A^2 A_\mu^2 [S^2 + P^2] - 2g_A A_\mu [T_{\mu \nu} \partial_\lambda \tilde{T}_{\lambda \nu} - \tilde{T}_{\mu \nu} \partial_\lambda T_{\lambda \nu}] + g_A^2 [(A_\lambda T_{\mu \nu})^2 - 4(A_\mu T_{\mu \nu})^2].$$  \hspace{1cm} (25)

The second one consists of explicitly invariant terms:

$$L_{\text{int}} = -g_A (ST_{\mu \nu} + P \tilde{T}_{\mu \nu}) V_\mu - g_A^2 (ST_{\mu \nu} + P \tilde{T}_{\mu \nu})^2 - \frac{1}{3} g_A^2 (S^2 + P^2)^2 - \frac{1}{4} g_A^2 [(T_{\mu \nu} T_{\mu \nu})^2 - 4T_{\mu \nu} T_{\nu \alpha} T_{\alpha \beta} T_{\beta \mu}].$$  \hspace{1cm} (26)

These are all terms invariant under the $C$, $P$- and gauge transformations. In general the coupling constants in \[26\] could be different, but our choice is imposed by the quantum contributions from the one-loop diagrams. It is common rule for the composite models like the NJL model, that the whole dynamics is managed by only one coupling constant. The renormalization group fixed point approach \[13\] and the reduction method in the number of coupling parameters \[21\] are the other side of the coin called supersymmetry.

5 Summary and conclusions.

In conclusion I want to discuss particular features of the antisymmetric tensor mesons. The detail analysis and complete phenomenological applications will be the aim of another work. Here I restrict myself to present the main differences between the tensor and the vector mesons. The milestone of my approach consist in the different forms of the quark
interactions with vector and tensor mesons. They have unlike chiral properties. On the mass-shell the Gordon decomposition reads

\[ \partial_\nu (\bar{\psi}_1 \sigma_{\mu \nu} \psi_2) = (m_1 + m_2)\bar{\psi}_1 \gamma_\mu \psi_2 + i[(\partial_\mu \bar{\psi}_1) \psi_2 - \bar{\psi}_1 (\partial_\mu \psi_2)] \]

\[ i\partial_\nu (\bar{\psi}_1 \sigma_{\mu \nu} \gamma^5 \psi_2) = i(m_1 - m_2)\bar{\psi}_1 \gamma_\mu \gamma^5 \psi_2 - [(\partial_\mu \bar{\psi}_1) \gamma^5 \psi_2 - \bar{\psi}_1 \gamma^5 (\partial_\mu \psi_2)] \].

(27)

Only due to nonvanishing and different quark masses we can get usual pieces of the vector and axial-vector interactions for tensor mesons. The pseudoscalar current with derivatives like in the right side of \( (27) \) is used usually in the quark interactions with axial-vector mesons \( 1^{+-} \), which on the mass-shell is related to tensor current. But it is not case when intermediate quarks states are involved. From my point of view the quark interactions with the axial-vector mesons \( 1^{+-} \) must be tensorial.

Now if we have two different vector particles with the same quantum numbers \( 1^{--} \) they can be mixed. Let us investigate this problem in the framework of the NJL model. For this purpose I will write down the mass terms arising under bosonization of four-fermion interactions and the bilinear terms come from the interactions \( (25) \) and \( (26) \) after the chiral symmetry breaking with substitution \( S \rightarrow S - m/g_S \):

\[ L_M = \frac{M_A^2 + 6m^2}{2} - \sqrt{6}m A_\mu (\partial_\mu P) - \frac{(2m)^2}{2}S^2 + \frac{M_T^2 - 6m^2}{2}B_\mu^2 + \frac{M_A^2}{2}V_\mu^2 + \sqrt{\frac{3}{2}}m V_{\mu \nu} \hat{R}_{\mu \nu} + \frac{M_T^2 - 6m^2}{2}R_\mu^2. \]

(28)

Here \( M_V, M_A, M_T \) and \( m \) masses can be independent. But if we believe that the effective four-fermion interactions of the quarks could originate in QCD by gluon exchange in \( 1/N_c \) limit one obtain \( M_V = M_A \). In this case for usual NJL model we get very heavy \( \rho \)-meson for the reasonable constituent quark mass. Account of the tensor mesons improves this situation.

Indeed, as far as the isospin triplets consist of up and down quarks with approximately the same masses I can apply my one-flavour model to the real world. The last three terms in \( (28) \) describe mixing of tensor and vector mesons. Let us suppose that \( \rho \)-meson mass \( m_\rho = 768.5 \pm 0.6 \text{ MeV} \) and the mass of the near \( \rho \)-meson state \( m_{\rho'} = 1465 \pm 25 \text{ MeV} \) are solutions of matrix equation

\[ q^2 - U M^2 U^{-1} = 0, \]

(29)

where

\[ U(q^2) = \begin{pmatrix} \cos \theta(q^2) & -\sin \theta(q^2) \\ \sin \theta(q^2) & \cos \theta(q^2) \end{pmatrix}, \quad M^2(q^2) = \begin{pmatrix} \frac{M_V^2}{\sqrt{6m^2q^2}} & \sqrt{\frac{6m^2q^2}{m_{b_1}^2}} \\ \sqrt{\frac{6m^2q^2}{m_{b_1}^2}} & \frac{m_{b_1}^2}{m_{b_1}^2} \end{pmatrix} \]

(30)

are mixing and square mass matrices. Here I have introduced the mass for pure state of \( b_1 \)-meson: \( m_{b_1} = 1231 \pm 10 \text{ MeV} \), because the relation \( m_{b_1}^2 = M_T^2 - 6m^2 \) holds. From \( (28) \) I can determine the two unknown masses: the vector meson mass \( M_V = 914.6 \pm 23.6 \text{ MeV} \) and the constituent quark mass \( m = 253.3 \pm 46.7 \text{ MeV} \). Therefore \( \rho(770) \) and \( \rho(1450) \) mesons occur to be dynamical mixed states of vector and tensor mesons with

\[ \tan^2 2\theta(q^2) = \frac{24m^2q^2}{\lambda(m_\rho, m_{\rho'}, \sqrt{6m})}, \]

(31)

where \( \lambda(m_\rho, m_{\rho'}, \sqrt{6m}) = (m_\rho^2 + m_{\rho'}^2 - 6m^2)^2 - 4m_{\rho'}^2m_{\rho}^2 \) is the triangle function. Now I can predict the \( a_1 \)-meson mass. Using the relation \( M_V = M_A \) one obtains \( m_{a_1} = 1105.2 \pm 83.7 \).
MeV, that is in good agreement with all the data of $a_1$-mass measurements from hadronic production and $\tau$-lepton decay experiments, but disagrees with its reanalysis [21]. The first two terms in (28) describe the well known mixing between axial-vector and pseudoscalar mesons. This leads to an extra renormalization of the pseudoscalar field: $P' = Z^{-1/2} P$ with $Z = (1 - 6m^2/m_{a_1}^2)^{-1}$.

Due to the presence of three-point interaction $-g_A P V_{\mu}\tilde{T}_{\mu}$ in (26) the meson states with quantum number $1^{++}$ can decay into vector and pseudoscalar mesons: $b_1 \rightarrow \omega \pi$; $h_1 \rightarrow \rho \pi$; $K_1 \rightarrow \rho K$, $K^*(892)\pi$. This interaction contains also “anomalous” term $-g_A P\varepsilon_{\mu\nu\alpha\beta} V_{\mu\nu} R_{\alpha\beta}$, which appears for the usual vector fields from the chiral anomaly. For the tensor fields it is naturally presented in the Lagrangian. Moreover such an interaction gives additional contribution [16] to the Adler-Bell-Jackiw anomaly. The structure of the tensor and vector meson interactions looks like very similar but this analogy is over when quarks or baryons are included. May be the acceptance of that fact that in Nature antisymmetric tensor particles may exist will help us to understand more deeply hadron and electroweak physics.

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