Light-induced electron pairing in two-dimensional systems

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Abstract. The mechanism of electron pairing induced by a circularly polarized off-resonant electromagnetic field is suggested and examined theoretically for various two-dimensional (2D) nanostructures. Particularly, it is demonstrated that such a pairing can exist in 2D systems containing charge carriers with different effective masses. As a result of the pairing, the optically induced hybrid Bose-Fermi system appears. The elementary excitation in the system are analyzed and the possible Bose-Einstein condensation of the paired electrons and the related light-induced superconductivity are discussed.

1. Introduction
The tuning of the electronic properties of various nanostructures by an off-resonant electromagnetic field based on the Floquet theory of periodically driven quantum systems (the Floquet engineering) became the established research area of the modern condensed matter physics (see, e.g., Refs. [1, 2, 3, 4, 5, 6, 7, 8] and the references therein). Since frequency of the off-resonant field lies far from the optical absorption range, the field cannot be absorbed and only “dresses” electrons (dressing field) changing their physical properties. Among various phenomena induced by the dressing field, the dynamical localization effect [2] should be noted especially. Recently, it was demonstrated that this effect crucially modifies the Coulomb interaction in 2D systems irradiated by a circularly polarized dressing field, resulting in the attractive area in the core of the repulsive Coulomb potential [9]. As a consequence, the quasi-stationary electron states bound at repulsive scatterers appear [10, 11]. Particularly, it follows from this that the irradiation can generate the bound two-electron states (electron pairs) in 2D systems. The present study is dedicated to the theory of such a pairing and related phenomena.

2. Model
Let us consider a 2D system containing charge carriers with different effective masses, where the energy spectrum of the two ground subbands is $\varepsilon_1(k) = -\Delta_0/2 + \hbar^2 k^2/2m_1$ and $\varepsilon_2(k) = \Delta_0/2 + \hbar^2 k^2/2m_2$, $\Delta_0$ is the energy distance between the subbands, $k = (k_x, k_y)$ is the momentum of charge carrier in the 2D plane, and $m_{1,2}$ are the effective masses in the subbands. Physically, such a system can be realized in a quantum well consisting of two layers (1 and 2) filled with a two-dimensional electron gas (2DEG), which are fabricated using different semiconductor materials and isolated from each other by a buffer layer with thickness $d$ (see Fig. 1a) or in
quantum wells based on hole semiconductor materials containing heavy and light holes. In the presence of a circularly polarized electromagnetic wave incident normally to the 2D structure, the Coulomb interaction of two electrons from the subbands $\varepsilon_1(k)$ and $\varepsilon_2(k)$ is described by the Hamiltonian $H_{12}(t) = \hat{H}_1 + \hat{H}_2 + U(r_1 - r_2)$, where $\hat{H}_{1,2} = (\hat{p}_{1,2} - eA(t)/c)^2/2m_{1,2}$ are the Hamiltonians of free conduction electrons irradiated by the wave, $\hat{p}_{1,2} = -i\hbar\partial/\partial r_{1,2}$ are the plane momentum operators of the electrons, $r_{1,2}$ are the plane radius vectors of the electrons, $A(t) = (A_x, A_y) = [eE_0/\omega_0](\sin \omega_0 t, \cos \omega_0 t)$ is the vector potential of the wave, $E_0$ is the electric field amplitude of the wave, $\omega_0$ is the wave frequency, $U(r_{12})$ is the potential energy of the repulsive Coulomb interaction between the electrons, and $r_{12} = r_1 - r_2$ is the radius vector of relative position of the electrons. If the field frequency $\omega_0$ is high enough and lies far from characteristic resonant frequencies of the 2D structure (the off-resonant dressing field), the time-dependent Hamiltonian $\hat{H}_{12}(t)$ can be reduced to the effective stationary Hamiltonian,}

$$\hat{H}_0 = \hat{p}_1^2/2m_1 + \hat{p}_2^2/2m_2 + U_0(r_{12}),$$

(1)

where

$$U_0(r_{12}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} U_{12}(r_{12} - r_0(t)) d(\omega_0 t)$$

(2)

is the repulsive Coulomb potential dressed by the circularly polarized field (dressed potential), $r_0(t) = (-r_0 \cos \omega_0 t, r_0 \sin \omega_0 t)$ is the radius vector describing the classical circular trajectory of a free particle with the charge $e$ and the mass $m = m_1m_2/(m_1 - m_2)$ in the circularly polarized field, and $r_0 = |e|E_0/\omega_0^2$ is the radius of the trajectory [9, 10]. In the case of the layered nanostructure pictured in Fig. 1a, the dressed potential (2) can be written explicitly as

$$U_0(r_{12}) = \frac{2e^2}{\pi \sqrt{(r_{12} + r_0)^2 + d^2}} K \left( \frac{4r_0 r_{12}}{(r_{12} + r_0)^2 + d^2} \right),$$

(3)

Figure 1. Sketch of the system under consideration: (a) The two-layer quantum well irradiated by a circularly polarized electromagnetic wave; (b) The dressed effective potential of electron-electron interaction, $U_0$, with the local minimum which confines the electron pair with the wave function $\rho_0$ (the red curve) and the energy $\varepsilon_0$ (the yellow horizontal line). The paired electron state can decay by the tunneling through the potential barrier into the continuum of conduction electron states marked by the green wave arrow; (c) The dressed Coulomb repulsive potential in a hole semiconductor quantum well; (d) The energy spectrum of the subbands of conduction electron states marked by the green wave arrow; (c) The dressed Coulomb repulsive potential (the red curve) and the energy $\varepsilon_0$, whereas the two small green circles mark the states of decoupled electrons with the energies $\varepsilon_{1,2}$ satisfying the condition $\varepsilon_1 + \varepsilon_2 = \varepsilon_0$. The large brown circle marks the ground state of coupled electrons.
whereas the dressed potential (2) for a hole semiconductor quantum well is

\[ U_0(r_{12}) = \begin{cases} (2e^2/\pi \hbar^2)K(r_{12}/r_0), & r_{12}/r_0 \leq 1 \\ (2e^2/\pi r_{12})K(r_0/r_{12}), & r_{12}/r_0 \geq 1 \end{cases} \]

(4)

and \( K(\xi) \) is the complete elliptical integral of the first kind. It follows from Eqs. (3) and (4) that the dressed potential \( U_0(r_{12}) \) turns into the usual repulsive Coulomb potential \( U(r_{12}) \) for \( m_1 = m_2 \), what leads to disappearance of the electron coupling. Thus, the different effective masses, \( m_1 \neq m_2 \), are crucial for the effect under consideration.

3. Results and discussion

In the following, we will assume the two subbands \( \varepsilon_{1,2}(k) \) to be of electronic type. Since the dressed potentials (3) and (4) have the local minimum at \( r_{12} = 0 \) (see Figs. 1b and 1c), the solution of the Schrödinger problem with the Hamiltonian (1) results in the two-electron state with the wave function of coupled electrons \( \varphi_0 \) and their energy \( \varepsilon_0 \) pictured schematically in Fig. 1b. This state of coupled electrons is immersed into the continuum of normal electron states of the subbands \( \varepsilon_1(k) \) and \( \varepsilon_2(k) \) and separated from them by the potential barrier pictured in Fig. 1b, which confines the wave function \( \varphi_0(r_{12}) \) near the local minimum of the dressed potential \( U_0(r_{12}) \). Therefore, the coupled electron pair is quasi-stationary and can decay by tunneling electrons through this barrier into the normal electron states of the subbands, where the energies of the two decoupled electrons are denoted as \( \varepsilon_1 \) and \( \varepsilon_2 \) in Fig. 1c. Assuming the tunneling to be weak, the probability of this tunnel transition per unit area of the 2D system reads

\[ W_q = \frac{\hbar^2 \Gamma_0}{m[(\varepsilon_q - \varepsilon_0)^2 + (\Gamma_0/2)^2]}, \]

(5)

where \( \Gamma_0 \) is the tunneling-induced broadening of the energy level \( \varepsilon_0 \), \( \varepsilon_q = \hbar^2 q^2/2m \) is the kinetic energy of relative motion of the decoupled electrons, \( q \) is the momentum of the relative motion, and \( m = m_1 m_2/(m_1 + m_2) \) is the reduced effective mass. Taking the Pauli principle into account, the total probability of the decay of the electron pair in the presence of normal conduction electrons reads \( W_0^- = \sum_q W_q \left( 1 - f_{k_1} \right) \left( 1 - f_{k_2} \right) \), where \( f_{k_1,2} = 1/[\exp[(\varepsilon_{1,2}(k_{1,2}) - \varepsilon_F)/T] + 1] \) are the Fermi-Dirac distribution functions for the states of normal electrons with the momenta \( k_1 = q \) and \( k_2 = -q \) in the subbands \( \varepsilon_{1,2}(k) \). Correspondingly, the total probability of production of the coupled electrons from normal electrons is \( W_0^+ = \sum_q W_q f_{k_1} f_{k_2} \). As a result, the Fermi gas of normal electrons is instable with respect to the pairing under the condition \( W_0^+ > W_0^- \), where

\[ W_0^+ = \sum_q \left\{ \exp[\pm \varepsilon_1 \{q\} + \varepsilon_F]/T + 1 \right\} \left\{ \exp[\pm \varepsilon_2 \{q\} + \varepsilon_F]/T + 1 \right\}. \]

(6)

It follows from Eq. (6) that the Fermi system of normal electrons and the Bose system of paired electrons are in the equilibrium for the zero temperature, \( W_0^+ = W_0^- \), if \( \varepsilon_F = \varepsilon_0/2 \). Assuming the Bose gas of paired electrons to be ideal, the equilibrium density of paired electrons for the temperature \( T = 0 \) (the density of the Bose-Einstein condensate) is \( n_0 = (m_1 + m_2)\Delta \varepsilon/2\pi \hbar^2 \), where the energy difference \( \Delta \varepsilon = \varepsilon_F - \varepsilon_0/2 \) has the physical meaning of the energy gain of the total system per one produced pair, which corresponds to the transition of the system from the nonequilibrium state pictured in Fig. 1c to the equilibrium state with the Fermi energy \( \varepsilon_F = \varepsilon_0/2 \).

It follows from the aforesaid that the circularly polarized irradiation produces the hybrid Bose-Fermi system consisting of the Fermi gas of normal conduction electrons and the Bose
gas of coupled two-electron molecules. Generally, the hybrid Bose-Fermi systems substantially differs from pure Bose systems since the Fermi component changes the interaction between the Bose particles. To find the dispersion of collective modes in such a system, $\Omega_K$, let us apply the conventional condition $\zeta(\Omega, K) = 0$, where $K$ is the wave vector of the mode, $\zeta(\Omega, K) = 1 - g_K \Pi(\Omega, K)$ is the dielectric function, $\Pi(\Omega, K) = \sum_q [\epsilon_0(q) - \epsilon_0(q + K)]^2 / (\hbar \Omega + \epsilon_0(q) - \epsilon_0(q + K))$ is the polarization operator, and $g_K$ is the Fourier transform of the interaction between the Bose particles [12]. Since a small number of two-electron molecules ($p_0 r_0^2 \ll 1$) is immersed into the Fermi sea of conduction electrons, their Coulomb interaction is screened by many normal electrons and, therefore, can be described by the Fourier transform $g_K = 8 \pi e^2 / (K + 2/r_s)$, where $r_s = \zeta_0 \hbar^2 / (m_1 + m_2) e^2$ is the effective screening radius assumed to satisfy the condition $r_s / r_0 \gg 1$, and $\zeta_0$ is the static dielectric constant. Substituting the single-pair energy spectrum $\epsilon_0(K) = \epsilon_0 + h^2 K^2 / 2 (m_1 + m_2)$ and the condensate density $p_0$ into the polarization operator, we arrive at the energy spectrum of the collective mode, $\hbar \Omega_K = \sqrt{2 p_0 g_K E_K + E_K^2}$, where $E_K = h^2 K^2 / 2 (m_1 + m_2)$ is the kinetic energy of the two-electron molecule. This collective mode has the sound-like dispersion $\Omega_K = v_s K$ for small wave vectors $K \ll 1/r_s$, where $v_s = \sqrt{p_0 g_0 / (m_1 + m_2)}$ is the sound velocity. Therefore, the considered Bose-Einstein condensate is superfluid if the velocity of its flow, $v$, satisfies the Landau criterion, $v < v_s$.

For applicability of the developed theory based on the dressing field approach, the dressing field frequency, $\omega_0$, must satisfy the two conditions [9, 10, 11]: (i) $\omega_0 \tau_e \gg 1$, where $\tau_e$ is the mean free time of charge carriers; (ii) the field frequency $\omega_0$ lies far from resonant frequencies corresponding to the optical transitions between different states of the paired charge carriers. These conditions can be met simultaneously in modern quantum wells for the frequencies $\omega_0$ near the border of microwave range.

4. Conclusion
It is shown theoretically that a circularly polarized irradiation of two-dimensional systems containing charge carriers with different effective masses can produce electron pairs. Assuming that the reduced effective masses are $m, m \sim 0.1 m_e$, and the Fermi energy $\varepsilon_F$ is of tens meV, we arrive at the pair radius $r_0 \sim 10$ nm, the ground energy of coupled charge carriers $\epsilon_0$ around 100 meV and the energy broadening $\Gamma_0$ of meV scale for the relatively weak irradiation intensity $I \sim W/cm^2$ with the frequency $\omega_0 = \omega_0 / 2\pi = 100$ GHz. Correspondingly, the discussed effect can be observable in modern semiconductor quantum wells.

Acknowledgments
The reported study was funded by the Russian Science Foundation (project 20-12-00001).

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