Δ resonance contribution to two-photon exchange in electron-proton scattering

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Abstract

We calculate the effects on the elastic electron-proton scattering cross section of the two-photon exchange contribution with an intermediate Δ resonance. The Δ two-photon exchange contribution is found to be smaller in magnitude than the previously evaluated nucleon contribution, with an opposite sign at backward scattering angles. The sum of the nucleon and Δ two-photon exchange corrections has an angular dependence compatible with both the polarisation transfer and the Rosenbluth methods of measuring the nucleon electromagnetic form factors.

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The electromagnetic form factors reflect the essentially non-local nature of the nucleon in its interactions with photons. As the basic observables parametrising nucleon compositeness, the form factors have long been studied both experimentally and theoretically. This interest has been renewed recently due to the increased precision of electron-proton scattering experiments and the availability of two alternative methods of extracting the form factors from the data: the Rosenbluth method – also known as the longitudinal-transverse (LT) separation technique [1, 2] – and the polarisation-transfer (PT) technique [3]. If one uses the traditional one-photon exchange calculation to extract the form factors, the two methods lead to apparently incompatible results: while the PT method yields a ratio of the electric to magnetic form factors which falls off linearly with the square of the momentum transfer $Q^2$, the LT separation experiments give an approximately constant ratio [3, 4, 5]. Finding an explanation of this discrepancy is important for the use of electron-proton scattering as a precise and reliable tool in hadronic physics.

Several theoretical studies [6, 7] have suggested that the problem could be at least partially resolved by including higher-order two-photon exchange corrections in the analysis of electron-proton scattering data, in addition to the lowest order one-photon exchange (Born) approximation. The recent explicit calculation [6] has shown that with the two-photon exchange taken into account in the analysis of electron-proton scattering, the ratio of the form factors extracted from the LT separation measurements becomes more compatible with the ratio from the PT experiments. However, the two-photon exchange diagrams calculated in Ref. [6] contained only nucleons in the intermediate state; the contribution of other hadrons has not been included until now. In view of the prominent role of the $\Delta$ resonance (unlike other excited states) in many hadronic reactions, it is essential to evaluate its contribution to the two-photon exchange in electron-proton scattering. Without an explicit calculation the results with only the nucleon intermediate state can only be viewed as suggestive in resolving the discrepancy. Some aspects of the $\Delta$ contribution were addressed before [8, 9], using various approximate approaches. These earlier studies demonstrated the importance of treating the $\Delta$ on a par with the nucleon in considering higher-order corrections to electron-proton scattering.

This letter presents a quantum field theoretical calculation of the two-photon exchange “box” and “crossed-box” diagrams with a $\Delta$ resonance in the intermediate state. We will show that the $\Delta$ two-photon exchange correction is somewhat smaller in magnitude than that of the nucleon. At backward scattering angles the $\Delta$ and nucleon contributions tend to partially cancel each other, their sum nevertheless yielding a predominantly negative two-photon exchange correction. We will show that the modified cross section has an angular dependence consistent with both the LT separation and PT measurements of the form factors.

We consider scattering of electrons (mass $m_e \approx 0.511 \times 10^{-3}$ GeV) off protons (mass $M_N \approx 0.938$ GeV) with the four-momenta assigned as $e(p_1) + p(p_2) \rightarrow e(p_3) + p(p_4)$. The differential cross section for this process is written in the form $d\sigma = d\sigma_B(1 + \delta_N + \delta_\Delta)$ where $d\sigma_B$ is the lowest-order Born contribution (i.e. the cross section obtained from the one-photon exchange tree diagram) and $\delta_N$ ($\delta_\Delta$) is the higher-order correction obtained from two-photon exchange diagrams containing nucleons ($\Delta$’s) in the intermediate state. (Other higher-order effects which should be included in the formula for $d\sigma$ – such as the vacuum polarisation and the electron-photon vertex corrections – have been extensively studied in the past and are known [10] to be irrelevant to the differences between the PT and LT analyses; we therefore focus here on the two-photon exchange effects only.) It is convenient
to divide $d\sigma$ by the well-known factor describing the scattering from a structureless "proton" (see, e.g., [11]) and thus use the reduced cross section

$$d\sigma_R = \left[ G_M^2(Q^2) + \frac{\epsilon}{\tau} G_E^2(Q^2) \right] (1 + \delta_N + \delta_\Delta). \quad (1)$$

Here the Born contribution is written in terms of the electric and magnetic form factors of the proton, $G_E(Q^2)$ and $G_M(Q^2)$, which are functions of the momentum transfer squared $Q^2 \equiv -q^2 \equiv 4\tau M_N^2 = -(p_1 - p_3)^2$. The kinematic variable $\epsilon$ is related to the scattering angle $\theta$ through $\epsilon = [1 + 2(1 + \tau)\tan^2(\theta/2)]^{-1}$, which is equal to the photon polarisation in the Born approximation.

We denote the Born scattering amplitude as $M_B$ and the two-photon exchange amplitudes with the nucleon and $\Delta$ intermediate states as $M_N^{\gamma\gamma}$ and $M_\Delta^{\gamma\gamma}$, respectively. From the equation $d\sigma = d\sigma_B(1 + \delta_N + \delta_\Delta) = |M_B + M_N^{\gamma\gamma} + M_\Delta^{\gamma\gamma}|^2$, we derive to first order in the electromagnetic coupling $e^2/(4\pi) \approx 1/137$:

$$\delta_{N,\Delta} = 2 \frac{\text{Re} \left( M_B^\dagger M_N^{\gamma\gamma} \right)}{|M_B|^2}. \quad (2)$$

The nucleon part $\delta_N$ of the two-photon exchange was analysed in Ref. [6]. Below we will evaluate the $\Delta$ two-photon exchange contribution $\delta_\Delta$. The scattering amplitude $M_\Delta^{\gamma\gamma}$ is given by the sum of the box and crossed-box loop diagrams depicted in Fig. 1.

![FIG. 1: Two-photon exchange box and crossed-box graphs for electron-proton scattering with a $\Delta$ intermediate state, calculated in the present letter.](image)

We use the $\gamma N\Delta$ vertex of the following form [12]:

$$\Gamma_{\gamma\Delta \rightarrow N}(p, q) \equiv iV_{\Delta m}^{\alpha\nu}(p, q) = i\frac{eF_\Delta(q^2)}{2M_\Delta^2} \left\{ g_1 \left[ g^{\nu\alpha}p^\nu - p^\nu\gamma^\alpha - \gamma^\nu\gamma^\alpha p \cdot q + \gamma^\nu p q^\alpha \right] 
+ g_2 \left[ p^\nu q^\alpha - g^{\nu\alpha}p \cdot q \right] + (g_3/M_\Delta) \left[ q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}p) + q^\nu(q^\nu p - \gamma^\alpha p \cdot q) \right] \right\} \gamma_5 T_3, \quad (3)$$

where $M_\Delta \approx 1.232$ GeV is the $\Delta$ mass, $p_\alpha$ and $q_\nu$ are the four-momenta of the incoming $\Delta$ and photon, respectively, and $g_1$, $g_2$ and $g_3$ are the coupling constants. An analysis of Eq. (3) in the $\Delta$ rest frame suggests that $g_1$, $g_2$ and $g_3$ may be interpreted as magnetic, electric and Coulomb components, respectively, of the $\gamma N\Delta$ vertex. The form factor in Eq. (3) is necessary for ultraviolet regularisation of the loop integrals evaluated below; we use the simple dipole form

$$F_\Delta(q^2) = \frac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2}, \quad (4)$$

1 We use the notation and conventions of Ref. [11] throughout.
where $\Lambda_\Delta$ is the cutoff. The form factor entails some model-dependence of our results, which is unavoidable in any dynamical hadronic calculation. The isospin transition operator $T_3$ is defined by the relations $\sum_{\alpha=1}^{3} T_3^a T_\alpha = 1$ and $T_\alpha T_\beta^\dagger = \delta_{\alpha\beta} - \tau_\alpha \tau_\beta/3$, where $\tau_{1,2,3}$ are the usual Pauli matrices. The vertex with an outgoing $\Delta$ is given by the Dirac conjugate of Eq. (3),
\[
\Gamma_{\gamma N\to\Delta}^\alpha(p, q) \equiv iV_{\Delta out}^{\alpha\mu}(p, q) = \gamma_0 \left[ \Gamma_{\gamma N\to N}^\alpha(p, q) \right]^\dagger \gamma_0,
\]
with $p_\alpha$ and $q_\nu$ the four-momenta of the outgoing $\Delta$ and incoming photon, respectively. The $\gamma N\Delta$ vertex is orthogonal to the four-momenta of both the photon and the $\Delta$:
\[
q_\nu \Gamma_{\gamma \Delta \to N}^{\alpha\mu}(p, q) = 0, \quad p_\alpha \Gamma_{\gamma \Delta \to N}^{\alpha\mu}(p, q) = 0.
\]
The first of these equations ensures the usual electromagnetic gauge invariance of the calculation while the second allows us to use only the physical spin $3/2$ component of the $\gamma N\Delta$ propagator as its influence on the unpolarised cross section should be small.

The first and second loop integrals in Eq. (7) must be mutually related by crossing symmetry, which can be formulated in terms of the numerator of Eq. (4) using the Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2 = 2M_N^2 + 2m_e^2 - t - s$. Denoting
\( f^{\gamma\gamma}(s, t) \equiv M_B^{\dagger} M_{\Delta}^{\gamma\gamma} \) and writing it as the sum \( f^{\gamma\gamma}(s, t) = f^{\gamma\gamma}_{\text{box}}(s, t) + f^{\gamma\gamma}_{x-\text{box}}(s, t) \), where the first (second) term is calculated using only the first (second) integral in Eq. (7), the crossing symmetry requires that

\[
 f^{\gamma\gamma}_{x-\text{box}}(s, t) = -f^{\gamma\gamma}_{\text{box}}(u, t)|_{u=2M_N^2+2m_e^2-t-s} \Leftrightarrow f^{\gamma\gamma}(s, t) = -f^{\gamma\gamma}(2M_N^2+2m_e^2-t-s). \tag{12}
\]

We calculated the integrals in Eq. (7) explicitly and checked that our results obey the crossing symmetry constraint Eq. (12).

The \( \Delta \) two-photon exchange correction to the differential cross section can be expressed as a quadratic form in the \( \gamma N \Delta \) coupling constants \( g_M = g_1; g_E = g_2 - g_1 \) and \( g_C = g_3 \):

\[
 \delta_{\Delta} = C_M g_M^2 + C_{ME} g_M g_E + C_E g_E^2 + C_C g_C^2 + C_{EC} g_E g_C + C_{MC} g_M g_C, \tag{13}
\]

with the coefficients depending on the kinematical variables. The relative contributions of the coupling constants \( g_M \), \( g_E \) and \( g_C \) to \( \delta_{\Delta} \) can be assessed from Table I where the \( C_M \), \( C_{ME} \), etc. are given as functions of \( \epsilon \) at two fixed \( Q^2 \) values. In this calculation we used the dipole \( \gamma N \Delta \) form factor Eq. (11) with the cutoff \( \Lambda_\Delta = 0.84 \) GeV.

| \( \epsilon \) | \( Q^2 = 3 \) GeV\(^2 \) | \( Q^2 = 6 \) GeV\(^2 \) |
|---------|----------------|----------------|
| \( C_M \times 10^4 \) | \( C_{ME} \times 10^4 \) | \( C_E \times 10^4 \) | \( C_C \times 10^4 \) | \( C_M \times 10^4 \) | \( C_{ME} \times 10^4 \) | \( C_E \times 10^4 \) | \( C_C \times 10^4 \) |
| 0.1 | 2.92 | 1.49 | -1.64 | -1.09 | 3.95 | 3.54 | -5.98 | -5.58 |
| 0.2 | 2.53 | 0.94 | -1.61 | -1.00 | 2.07 | 1.72 | -5.74 | -4.98 |
| 0.3 | 2.17 | 0.50 | -1.57 | -0.88 | 0.69 | 0.49 | -5.45 | -4.29 |
| 0.4 | 1.83 | 0.14 | -1.52 | -0.72 | -0.21 | -0.22 | -5.11 | -3.48 |
| 0.5 | 1.54 | -0.11 | -1.45 | -0.50 | -0.81 | -0.63 | -4.72 | -2.52 |
| 0.6 | 1.23 | -0.32 | -1.37 | -0.21 | -1.18 | -0.85 | -4.25 | -1.35 |
| 0.7 | 0.95 | -0.46 | -1.27 | 0.18 | -1.35 | -0.89 | -3.69 | 0.16 |
| 0.8 | 0.65 | -0.55 | -1.15 | 0.79 | -1.31 | -0.77 | -2.99 | 2.33 |
| 0.9 | 0.31 | -0.57 | -0.98 | 1.98 | -0.94 | -0.42 | -1.99 | 6.38 |

In the following we will discuss the results obtained with the fixed coupling constants \( g_M = 7 \) and \( g_E = 2 \). These couplings were used in the Dressed K-matrix Model [12] (adjusted for a different normalisation of the vertex used in the present calculation), yielding a good coupled-channel description of pion-nucleon scattering, pion photoproduction and Compton scattering at low and intermediate energies. For example, the \( E2/M1 \) ratio obtained in Ref. [12] from the pion photoproduction multipoles at the position of the \( \Delta \) resonance, is \( R_{EM} = \text{Im} M_{1+}^{2}/\text{Im} M_{1+}^{3} \times 100\% \approx -3\% \), in agreement with the PDG [13] value: \(-2.5 \pm 0.5\)%). Recent analyses [20] of pion electroproduction suggest that the Coulomb coupling constant \( g_C \) is small and negative. In our calculation we will vary \( g_C \) in the range \([-2, 0]\). With these values of \( g_M \), \( g_E \) and \( g_C \) one can see from Eq. (13) and Table I that the magnetic
coupling dominates the $\Delta$ two-photon exchange correction whereas the electric coupling has a much smaller effect. Since the contribution of the Coulomb component is strongly suppressed (not exceeding 0.2%) we will omit it from further discussion, setting $g_C = 0$ in the rest of the paper.

The $\epsilon$ dependence of the sum of the $\Delta$ and nucleon two-photon exchange corrections is shown in Fig. 2 for two fixed values of $Q^2$. The dependence on the $\gamma N\Delta$ form factor can be seen by comparing the results obtained with the cutoffs $\Lambda_\Delta = 0.84$ GeV and $\Lambda_\Delta = 0.68$ GeV (the latter choice corresponds to a $\Delta$ which is spatially “bigger” than the nucleon). The purely nucleon contribution, shown for comparison, was calculated as in Ref. [6] using the $\gamma NN$ form factors extracted from the PT experiments [3, 4]. The $\Delta$ correction is more prominent at higher momentum transfers. The $\Delta$ tends to reduce the effect of the nucleon two-photon exchange, making the modulus of the negative nucleon correction somewhat smaller at backward angles (i.e. at low $\epsilon$). The combined effect of the nucleon and $\Delta$ two-photon exchanges produces a negative correction to the cross section at small $\epsilon$, decreasing in magnitude as $\epsilon$ increases.\(^2\) The main features of the $\Delta$ contribution – its smallness and its tendency to attenuate the nucleon contribution at backward angles – are insensitive to the $\gamma N\Delta$ form factor, being to that extent model-independent. The detailed interplay between

\(^2\) The diminishing of the two-photon exchange correction at forward angles is consistent with the analysis of electron-proton and positron-proton scattering data [19].
the $\Delta$ and the nucleon contributions can be more complicated, especially at forward angles, as can be seen from Fig. 2.

The calculated differential cross section is shown by the solid lines in Fig. 3 including the Born term and the sum of the two-photon exchange corrections $\delta_N + \delta_\Delta$ with the nucleon and the $\Delta$ intermediate states. The reduced cross section Eq. (1), scaled for convenience by the square of the standard dipole form factor $G_D(Q^2) = 1/(1 + Q^2/0.84)^2$, is compared in Fig. 3 with the LT separation measurements from SLAC [1] (at $Q^2 = 4$ and 6 GeV$^2$) and JLab [2] (at $Q^2 = 2.64$ GeV$^2$). The dotted lines show the Born contribution alone, using the nucleon form factors $G_{E,M}(Q^2)$ taken from the analysis of the JLab PT experiment [3, 4]. One can see that including only the Born term is inadequate in the analysis of the data. The addition of the two-photon exchange correction increases the slope of the cross section, also exhibiting some nonlinearity in $\epsilon$. Thus the results of the PT and LT separation experiments become essentially compatible by including the nucleon and $\Delta$ two-photon exchange corrections.

To summarise, we calculated the correction to the electron-proton scattering cross section due to the two-photon exchange with a $\Delta$ intermediate state, treated on the same footing as the intermediate nucleon contribution. For realistic choices of the $\gamma N \Delta$ vertex we found that the $\Delta$ contribution alters the cross section by an amount from $-1\%$ to $+2\%$, and is largest at backward scattering angles. For the cross section obtained using the LT separation technique, the $\Delta$ two-photon exchange contribution slightly reduces the magnitude of the (negative) nucleon correction. Generally, the cross section including the nucleon and $\Delta$ two-photon exchange corrections has the angular dependence which can accommodate the

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3}
\caption{Effect of adding the two-photon exchange (with the indicated choice of the $\gamma N \Delta$ form factor) to the Born cross section, the latter evaluated with the nucleon form factors from the PT experiment [3, 4]. The reduced cross section is scaled as described in the text. The curves for $Q^2 = 2.64$, 4 and 6 GeV$^2$ have been shifted vertically by $-0.04$, $+0.04$ and $+0.09$, respectively. The data points at three fixed momentum transfers are taken from Refs. [1, 2].}
\end{figure}
results of both the LT separation and PT methods of measuring the nucleon form factors. This calculation therefore provides explicit and compelling evidence that the two-photon exchange contribution (with the lowest mass, \(N\) and \(\Delta\) intermediate states) can resolve the form factor discrepancy. To reconcile these two methods completely, theoretical analyses of the data might need additional ingredients. For example, one may take into account the dependence of the \(\gamma NN\) and \(\gamma N\Delta\) vertices on the nucleon and \(\Delta\) off-shell momenta (as was suggested in \([21]\)). Heavier hadron resonances or quark degrees of freedom should also become important at higher momentum transfers (see e. g. \([22]\)).

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