Research article

Signal detection with spectrum windows

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A B S T R A C T

Spectrum sensing is needed in frequency agile cognitive radio systems, cognitive radar systems as well as in (cognitive) electronic warfare systems. This paper studies window based detectors where window shapes can be matched for the expected spectrum. The provided analytical results for the probabilities of false alarm and detection are valid for arbitrary windows. Based on the window shape mismatch analysis, it was concluded that the simple rectangular window is often sufficient if its bandwidth is properly set. Furthermore, based on the window bandwidth mismatch analysis, it was discussed how many different rectangular windows are needed to cover the instantaneous analysis bandwidth of the receiver without compromising too much on sensitivity. Finally, suitable constant false alarm rate threshold setting schemes were compared. It was observed that almost ideal setting schemes exist.

1. Introduction

Detecting signals buried in the radio spectrum is a common problem among many applications such as spectrum detectors in cognitive radio systems [1], cognitive radar systems [2] and cognitive electronic support measures (ESM) systems [3] in addition to traditional ESM systems. To be more specific, the problem is to detect signals from the spectrum with a receiver that has an instantaneous bandwidth (IBW). To make things more complicated, there could be several signals inside the IBW that have different spectrum shape and bandwidth, and some spectrum shapes might be unknown to the detector. Still, it would be nice to detect them all. Subsequent processing would be needed to get carrier frequencies, bandwidths and additional identification information.

One possible detector is the celebrated total power radiometer [4], also known as the energy detector, that does not even need a calculated frequency spectrum. It squeezes the whole band (actually, the samples) into one detection variable, who is compared with a detection threshold. However, it does not have capacity to detect if there were many signals since its frequency resolution is the whole IBW. At another end are detectors that compare each calculated frequency bin in a digital spectrum to a threshold. The frequency resolution of these detectors is the bandwidth of the frequency bin. Furthermore, these single bin detection variables could be grouped to see if several consecutive bins form a single signal, as proposed, e.g., in [5]. Obviously, these can estimate the number of signals.

There is also something in between these two extremes where several frequency bins are combined to form a detection variable. Indeed, since the observed spectrum variables are windowed into possibly overlapping blocks, these methods could be called spectrum window based (WIBA) detectors. The total power radiometer is a special case with a rectangular window that covers the whole observed spectrum and at the other end the window has just one non-zero element covering just one frequency bin. The frequency resolution depends on the window bandwidth and the method has capability to detect several signals within the IBW, but not within the window bandwidth.

Design challenges associated with the WIBA detectors affect resolution, complexity and sensitivity. Intuitively, a window that is matched to the signal spectrum would be optimal, except that it is not necessarily know the spectrum of all signals to be detected. In addition, if many kinds of signals are expected, many window shapes were needed that would make the detector very complex. Moreover, the sweeping of the IBW is affecting complexity. At one end, a window could sweep the whole observed spectrum advancing just one bin and at the other end, a window could divide the spectrum into non-overlapping blocks. In the middle, there could be 50% overlap between window positions. Advanced, high-end products may have nearly unlimited computation resources such that they can use several windows matched to different candidate spectrum shapes and bandwidths, but in many real life
applications the target is on very simple, yet efficient detectors such that the number of different windows is tried to keep on minimum. Indeed, there might be need to stay on a few different windows, e.g., one designed for narrowband signals (like those used for device-to-device communications), a few for mid-bandwidth signals and one for broadband signals (naturally, these bandwidths are relative to the IBW).

This paper considers these design challenges through sensitivity by addressing the following questions:

- How much sensitivity is lost if a rectangular, generic window was used instead of the exact one as a window shape?
- How much sensitivity is lost if a too wide or narrow window was used?

The problems associated with the second question are illustrated in Fig. 1 that includes an ideal window, a too narrow window, a too wide window and a window that is not totally overlapping with the signal. By answering these questions, the paper helps developers to select window bandwidths and shapes into their devices, especially from the viewpoint of a simple receiver. To our knowledge, these problems had not been addressed earlier.

Detecting signals using known spectrum is generally referred to as a spectrum matching or a spectrum (shape) feature detector. As far as the authors know, it was first applied in material science [6] and much later applied in spectrum based signal detection in [7, 8, 9, 10] and thereafter also by the authors in [11]. Theoretical results concerning the probability of false alarm were provided in [7] but the detection performance results are not available making easy equation based performance evaluations impossible. As a consequence, current understanding about the effects of spectrum shape mismatch and window bandwidth mismatch is quite limited. This paper changes the situation and provides theoretical closed form analysis results as a basis for further analysis. If rectangular windows were applied, different windows can be seen to form mini-radiometers such that rich research results concerning the radiometer can be used [4]. Alternatively, this could also be seen as a channelized radiometer [12].

The detection threshold setting is of great interest. One reason is that the energy detector is known to be sensitive to uncertainties in noise level in Gaussian and generalized Gaussian noise [13] and in fading channels [14]. Sliding window energy detection in the time domain using other than rectangular windows and related constant false alarm rate (CFAR) threshold setting are considered in [15] that implemented a few methods using software defined radios but did not provide any theoretical performance results. The results in this paper can be used to found those. Radar literature concerning CFAR detectors is rich and clutter is a main concern. New developments concerning sliding window threshold setting (with a rectangular window) were recently studied in [16]. Cell averaging CFAR techniques for the channelized radiometer are analyzed in [17]. This paper applies (and adapts) a few CFAR techniques from the last mentioned reference and compares their performance with the WIBA detector.

As a summary, the paper provides closed form results for analyzing the performance of the spectrum window detectors with arbitrary window shapes, discuss the WIBA detector design trade-offs related to shape and width mismatch and, finally, compares some CFAR threshold setting methods with the WIBA detector to see if a good one can be found among those.

The remaining of the paper is organized as follows. The signal model of this paper, as well as the detection variable and its actual and approximated distributions are introduced in Section 2. The impact of the shape mismatch is analyzed in Section 3 and the impact of bandwidth mismatch in Section 4 based on these distributions. Various threshold setting strategies are discussed in Section 5. Simulation results are shown in Section 6 and, finally, conclusions are drawn in Section 7.

2. System model and distributions

The input to the detector is the observed discrete power spectral density (PSD)

\[ Y(f) = P(f) + N(f), \]

where \( P(f) \) denotes the signal(s) and \( N(f) \) the noise part of the PSD, correspondingly, and \( f = 0, ..., N - 1 \) are the frequency bins. The signal(s) and noise are assumed to be statistically independent. Their distributions as well as assumptions related to the distributions of the time domain input signals will be discussed in a moment. The frequency resolution of the bin \( \Delta f = \text{IBW}/N \).

The exact way how the PSD (1) is calculated is not important unlike the shape of obtained signal spectrum and gathered total signal power in the estimated spectrum. Possible correlation between frequency bins is ignored in the analysis what follows, but that does not affect significantly to the results. This correlation may occur due to spectrum leakage and if overlapping temporal signal blocks are used in the spectrum estimation. It has been shown that the PSD values are asymptotically unbiased and uncorrelated if the periodogram is used for the PSD calculation [18]. Correlation could be small or zero even if the Welch method is applied with 50% overlap. Indeed, with window shapes \( 1 - i^2 \) and \( 1 - |i| \), the correlation at the adjacent bin is (about) 1/9 and 1/16, respectively, and zero in other bins [19]. Correlation within sliding window energy detection has been considered in [20] but that method is not directly applicable herein. Moreover, the leakage level is rather low, below -15 dB, if a reasonable fast Fourier transform (FFT) size is used [18].

Let \( W(f) \) be the window function that has \( M \) non-zero, positive, real valued elements that form a block (of interest) such that there are \( N/M \) non-overlapping blocks. The detection variable \( d \) concerning a block is defined as

\[ d = \sum_f W(f)Y(f). \]  

(2)

The detector decides that a signal is present if \( d > \gamma \), where \( \gamma \) is the detection threshold. The analysis of the probability of false alarm and the probability of detection are of interest, where the former refers to the case that \( d > \gamma \) if a signal is not present and the latter to the case that \( d > \gamma \) if a signal is present. Consequently, the distribution of \( d \) must be found when the signal is not present and is present.

When the window shape is rectangular and a signal (or signals) is present in the nonzero part of the window, the decision variable \( d \) in (2) follows a non-central chi-square distribution but if a signal is not present, it follows a central chi-square distribution with \( 2M \) degrees-of-freedom [4, 21]. The non-centrality parameter is the signal power inside (the nonzero part of) the block and, therefore, the total signal power only if the signal is wholly inside. This means that detection performance is independent of the actual spectrum shape, but depends just on the signal power inside the block.

The above assumption about the distribution holds quite generally due to the central limit theorem (CLT) [22], since each output of a
discrete Fourier transform used as an intermediate step in PSD calculation is basically a sum of random variables, and based on the CLT this tends to be Gaussian distributed even though the original signals are not. Since the PSD representing a frequency bin is just a power of this variable, it follows the chi-square distribution. If the input to the Fourier transform is Gaussian distributed, the assumption is exactly valid.

A non-rectangular window affects the signal and noise and, consequently, the distribution of the decision variable \( d \). An approximation of false alarm probability, \( P_{FA} \), is shown in [7] for the spectrum matching detector. However, efforts to analyze the probability of detection, \( P_{D} \), were not found. Luckily, approximated computational and analytical distributions of weighted sum of central and non-central chi-square distribution are available, and these could be utilized.

It is well known that the chi-square distribution can be presented as the gamma distribution and that the sum of chi-square variables is a chi-square distributed variable with a rectangular window, see e.g., [22]. Accordingly, the distribution of a weighted sum of chi-square distributed variables might be approximated by a gamma distribution. Indeed, by making the first two central moments of \( d \) equal to that of a gamma distribution is a well-known approximation followed herein and in [7].

When only noise is present, the approximation is sometimes called Satterthwaite–Welch approximation [23]. Let \( \Gamma(a, \theta) \) denote the gamma distribution with the shape parameter \( a \) and the scale parameter \( \theta \). Its mean and variance are \( a\theta \) and \( a\theta^2 \), respectively. The first two central moments of \( d \) are \( \kappa_1 \) and \( \kappa_2 \), respectively, and given as [23]

\[
\kappa_1 = \sum F(d) \quad \text{(3)}
\]
\[
\kappa_2 = 2 \sum F^2(d). \quad \text{(4)}
\]

Consequently, the parameters of approximating gamma distribution become

\[
a = \kappa_1^2/\kappa_2 \quad \text{(5)}
\]
\[
\theta = \kappa_2/\kappa_1 \quad \text{(6)}
\]

It is a simple exercise\(^1\) to check that with a rectangular window (of unit height) \( a = M \) and \( \theta = 2 \) in which case the gamma distribution corresponds to the central chi-square distribution with \( 2M \) degrees of freedom as it should.

If the signal is present, the approximation of the cumulative distribution function is based on results in [24, section 4.1]. Consequently, the approximating distribution is defined as

\[
F_d(y) \approx F_{\gamma}(y), \quad \text{(7)}
\]

where \( d^a = \lambda D \) and \( D \) follows the non-central chi-square distribution with \( v \) degrees of freedom and non-centrality parameter \( \omega \), where

\[
\lambda = (S_1 + 2S_2)/(S_1 + 2S_2), \quad \text{(8)}
\]
\[
v = S_1(S_1 + 2S_2)/(S_1 + 2S_2), \quad \text{(9)}
\]
\[
\omega = S_2(S_1 + 2S_2)/(S_1 + 2S_2) \quad \text{(10)}
\]

with \( S_1 = \sum W(f) \), \( S_2 = \sum W^2(f) \), \( S_3 = \sum W(f)P_f \), \( S_4 = 2 \sum W^2(f) \) and \( S_5 = \sum W^4(f)P_f \) (the fact that a complex variable has two degrees of freedom has to be used in the original equations in [24]). Straightforward calculus of parameters \( S_1, S_2, S_3, S_4 \) and \( S_5 \) and their substitution into (8) - (10) shows that with a rectangular window (7) returns to the non-central chi-square distribution with \( 2M \) degrees of freedom and the signal power \( P = \sum f P_f \) as the non-centrality parameter, as it should.

\(^1\) By noting that the complex input variables for the PSD calculation in this paper double the number of window elements and using this fact in (3) and (4) and substituting the results in (5) and (6).

Fig. 2. Comparison of sinusoidal and rectangular windows in detecting sinusoidal spectrum of different bandwidths (10, 50, 300). Window bandwidths are matched to signal bandwidths and \( P_{FA} = 10^{-5} \).

In these derivations, the noise power of both the real and imaginary part were normalized to one.

3. Spectrum shape mismatch

It is well known due to the famous Cauchy-Schwarz inequality [21] that \(|\|W(f)P_f(f)\|\| \leq |\|W(f)\|\|P(f)\|\|\) with equality if and only if the window shape is equal to the actual spectrum shape. This means that the signal power (non-centrality parameter) at the detector output is maximized only then, and otherwise there are losses. However, are these losses significant?

An example is provided to illustrate this. Let the actual signal spectrum be a half cycle of a sine signal between zero and \( \pi \). Its total power (integral) is 2. Let the window be a half sine too, but its total power normalized to one, i.e., window is \( \frac{1}{\pi} \) sin. Let another window be a rectangle with height \( 1/\pi \) such that its total power is one. Now, parameter \( S_1 \) or the signal power within the window is \( \pi/4 \) with the sine window and \( 2/\pi \) with the rectangular window. In other words, the power difference is 1 dB. Does this mean 1 dB difference in sensitivity?

Some numerical examples are provided to illustrate the difference in detection using the theoretical results given in Section 2. The results for the half sine window and the half sine and rectangular windows with bandwidths 10, 50 and 300 (frequency bins) are shown in Fig. 2, where the miss probability of detection \( (P_{miss} = 1 - P_D) \) is shown as a function of signal-to-noise-level ratio (SNLR) that denotes the total signal power compared to the noise level, say \( P/N_0 \), where \( P = \sum P(f) \) and \( N_0 \) is the noise level and is therefore expressed in the units of dB-Hz. SNLR is a convenient metric when window based detectors are compared. The average signal power to noise power ratio (SNR) per frequency bin would be \( P/MN_0 \) expressed in dB. The results in Fig. 2 show that using the ideal window shape gives somewhat better (about 0.5 dB) performance, as expected, but the difference is not that big. Indeed, the effects of window bandwidth mismatch may overcome this difference, as will be seen.

In practice, the signal spectrum could vary from rather rectangular to that of the rectangular pulsed single carrier signal though most often some band limiting filter is used to reduce adjacent channel interference. Therefore, most of the signal power is concentrated into certain bandwidth and the difference between the rectangular and ideal window depends on the selected bandwidth of the rectangular window. This aspect is further elaborated with the spectrum of the minimum shift keying (MSK) signal and the root raised cosine (RRC) shaped sig-
nal with rolloff factor 0.25 [21]. Two normalized bandwidths were used in comparison: the first one is $fT = 1$, where $f$ denotes the frequency and $T$ the symbol duration and the other is the bandwidth that includes 90% of the signal power. Note that the spectrum of the RRC shaped signal is rather rectangular in the latter case. The same comparison as for the half sin signal above gives differences 3.7 dB (MSK) and 2.4 dB (RRC) for the wider bandwidth and 0.9 dB (MSK) and 0.3 dB (RRC) for the 90% bandwidth. Also this comparison indicates that if the rectangular window bandwidth is close to the significant signal bandwidth, the difference to the ideal window is small.

Other justification to use rectangular windows is computational complexity. If rectangular windows are used, the sum of samples is needed whereas non-rectangular windows require real multiplication per sample in addition to the sum. Furthermore, rectangular windows allow the utilization of previous calculations since the outputs of narrow windows could be inputs to wider windows whereas non-rectangular windows do not allow this. This might be especially important in low end receivers that have limited computational resources. In any case, rectangular windows can be used for searching signals (almost optimally) and dedicated windows then for post-detection processing, e.g., for spectrum shape based identification.

4. Spectrum bandwidth mismatch

The spectrum bandwidth mismatch effects are explained using rectangular windows. Let $K$ be the number of frequency bins where the signal spectrum has its essential elements (all or the most of total power). The decision variable of the optimal detector would have $2K$ degrees of freedom (DoF) and the non-centrality parameter equal to the signal power $P$. If the window was too wide, say $\epsilon$ times, then the decision variable would have $2K\epsilon$ degrees of freedom, but the non-centrality parameter would be the same power $P$. If the window was $\epsilon$ times too narrow, the decision variable would have $2K\epsilon$ degrees of freedom, but the power of non-centrality parameter would be smaller. How much smaller in practice, that would depend on what part of the spectrum would be inside the window.

It is well known based on the properties of the non-central chi-square distribution that if the degrees of freedom are increasing, then more total signal power is needed for the same probability of detection. Therefore, wider spectral windows need more signal power. However, too narrow windows cannot capture all the signal power and at some point, wider windows start to be more sensitive. This is illustrated in Fig. 3 for windows whose widths are 1, 10, 50, 100, and 1024 frequency bins while the signal bandwidth varies from 1 to 300 bins and the total number of bins is 1024. The results are obtained using the theoretical results in Section 2. What is shown is how much SNLR is needed for detecting the signal with 99% probability when the false alarm rate was 1%. For simplicity, it is assumed that the signal power is uniformly distributed over frequency bins.

The results in Fig. 3 yield the following conclusions. Wideband signals are best detected using wideband windows since they are much more sensitive than narrowband windows for wideband signals. On the other hand, narrowband signals are best detected using narrowband windows, but the difference is not so large as in the opposite case. As an example of this, observe that a 100 bin window needs 6 dB more power to detect a signal within a single bin than a single bin receiver, whereas the single bin receiver needs 14 dB more power to detect a 100 bin signal than a 100 bin receiver. Therefore, a good balance with respect to the number of windows and sensitivity seems to be five to tenfold increase in the window size. Furthermore, if one should have to select just one window, it would be rather wideband. Moreover, signal bandwidth mismatch has typically much more severe impact to sensitivity than spectral shape mismatch has.

Fig. 3. Required SNLR as a function of signal spread over frequency bins for a few different rectangular window bandwidths (1, 10, 50, 100, 1024) when the desired probability of detection is 99% and the false alarm rate was set to 1% for all windows.

4.1. Total false alarm rate

So far, the false alarm rate per a test has been considered. However, in practice there might be a false alarm rate requirement per a time unit, e.g., for a time needed to collect a sample vector $Y(f)$. Let $P_{FA,\text{tot}}$ denote this total false alarm rate requirement and let there be $G = N/M$ tests within a sample vector such that the false alarm rate of the test $P_{FA} = P_{FA,\text{tot}}/G$. Since narrow windows would have a much smaller $P_{FA}$ than wider windows, it is of interest to know how this would affect sensitivity, i.e., to the results shown in Fig. 3. Additional analyses showed that the impact is not significant; the order of windows was not changed in Fig. 3, nor the positions of the results curves except that the smallest windows needed a dB or two more SNLR.

5. Setting CFAR detection threshold

The detection rule says that a signal was present in a block if

$$d > T_h \hat{\sigma}.$$  

(11)

where $T_h$ is the threshold parameter calculated based on the desired false alarm rate under the noise only hypothesis and $\hat{\sigma}$ is the estimated (or known) noise level. Consequently, the detection threshold is a function of the noise level but that is rarely known in practice. Correspondingly, the noise level has to be estimated and often preferably using the constant false alarm rate detection principles. These try to keep the desired false alarm rate since it is usually a system design criterion. Basically, CFAR methods estimate the noise power in (11) in one way or another.

CFAR methods for the radiometer are considered in [17] including order statistics (OS) and cell averaging (CA) methods. In the OS method, a percentile is often used to estimate the noise level. Reference samples, hopefully signal free and possibly around the test cell, are used to estimate the noise level in the CA method.

Robust CFAR methods based on the forward consecutive mean excision (FCME) idea introduced in [25] have been studied in [17] and [26]. These try to exclude signal containing samples from the rest used for noise level estimation. This was applied in [11] for the WIBA detector and observed performing very similar than the mean of the PSD (\(Y(f)_m\)) as a noise level estimator. However, just a small fraction, less than 20% of all samples contained a signal sample in [11] and situation may be different, in favor of FCME based methods, if more samples
Table 1. The summary of simulations and their parameters.

|                          | Known noise level | Unknown noise level |
|--------------------------|-------------------|---------------------|
| target                   | \( P_0 \)         | \( P_{PA} \)        |
| where                    | section 6.1       | section 6.2         |
| desired \( P_{PA} \)     | \( 10^{-2} \)     | \( 10^{-4} \)       |
| trials                   | 100 000           | 1000               |
| signals                  | 128, 1024, 4096   | no signal           |
| windows                  | rectangular and half sine | rectangular and half sine |
|                          |                   |                     |
|                          |                   |                     |

contain a signal sample. In [17], the FCME based threshold setting was performing similar to the OS method with a low percentile.

Since used in this paper, the operation of the FCME method is explained briefly. Let \( U(f) \) contain the PSD samples \( Y(f) \) in the increasing order. The FCME operates as follows:

1. Take the smallest \( x \% \) of \( U(f) \) to form the initial clean set. Let this be the set \( U_i \).
2. Add all samples satisfying \( U(f) < T_{FCME} S(U_j) \) in the clean set \( U_{i+1} \).
3. Variables \( U(f) < T_{FCME} S(U_L) \) belong to the last clean set \( U_L \) used to estimate the noise level. The estimated noise level is simply the mean of the power \( S(U_L) \) of the last set, but other options also occur as shown later.

The complexity of required sorting is about the same order as the complexity of the fast Fourier transform [27]. Since the noise is estimated real time from the input signal, the method reacts to noise level variations.

The OS and FCME methods are applied herein as well as the mean (of \( Y(f) \)). It is an interesting fact that the sample mean often appears to be the noise level estimate if the generalized likelihood ratio test (GLRT), a CFAR method, is applied to signal detection. A relevant example is shown in [1], where signal detection methods for cognitive radio systems are reviewed.

6. Simulations

This section validates certain interesting features with simulations. The emphasis is on the performance with CFAR threshold setting schemes. The simulation cases are listed in Table 1 together with relevant simulation parameters that are explained in the related text.

6.1. Detection with known noise level

The theory regarding the probability of detection is validated using half-sinusoidal signals and both the optimal and rectangular windows. The false alarm rate was \( 10^{-2} \) and 100 000 Monte Carlo trials were run.

The noise spectrum generated using complex Gaussian variables was added to the signal spectrum in each run, see (1). The signal bandwidth was used as the window bandwidth (the optimal case). The results are shown in Fig. 4. The simulated results have very good match for the theory to predict the performance in practice (i.e., when \( P_0 \geq 0.99 \) is achieved), except with narrow spectrum were the difference was up to 1 dB (theory is more pessimistic), i.e., rather small. Consequently, the results show that the theoretical results can be used predicting the performance of the WIBA detector without time consuming simulations. In addition, the performance difference between the optimal and rectangular window is as predicted by the theoretical analysis in section 3. At low \( P_{miss} \) values, the simulation results match better for the theoretical ones with wider windows, as expected.

6.2. False alarm rate with known noise level

It is expected that calculated detection thresholds perform properly with rectangular windows, and based on simulation results in [7] they also perform well for spectrum matching. Herein, we confirm this by simulating with different bandwidths and with different false alarm rates using half sine and rectangular windows. The threshold parameter \( T_e \) for each false alarm rate is solved using the theoretical results in Section 2, and 100/\( P_{PA} \) trials were used. Only noise spectrum was generated in each trial. The results given in Table 2 show a good match between expected and simulated \( P_{PA} \) for both half sine and rectangular windows, assuming that the noise level is known.

6.3. Threshold setting using CFAR methods

The set desired false alarm rate is an average attained if the noise level is estimated perfectly. The sensitivity of the energy detector to
noise uncertainty is well known and analyzed in [28]. Accordingly, questions using CFAR threshold setting schemes include i) how well the noise level is actually estimated (mean and variance), ii) what is the false alarm rate using the estimated noise level and iii) what is the false alarm rate in signal free windows if signals are present within the BW. We address all these questions for the OS, the mean and the FCME based threshold setting. This section covers the first two and the next one the third question.

The OS method uses tenth percentile and in the FCME method the initial clean samples set size is 10% of the sample size and the internal rejection rate $10^{-2}$. The FCME method uses two noise level estimation schemes: FCME1 uses the average of the clean set whereas FCME2 uses the average of the absolute value instead of the PSD values. Since in this case the mean is (according to the Rayleigh distribution) $\sqrt{\pi/2}$ instead of one, the results have to be scaled accordingly. Consequently, the estimated noise level in FCME1

$$\hat{\sigma} = \frac{1}{L} \sum_{f=1}^{L} U(f),$$

(12)

and in FCME2

$$\hat{\sigma} = \frac{1.1284}{L} \sum_{f=1}^{L} \sqrt{U(f)}.$$  

(13)

Scaling is also needed in the OS method. For example, since noise only variables $Y(f)$ are central chi-square distributed with two degrees of freedom (normalized such that the total power equals two), then the tenth percentile means that, on average, $U($tenth percentile$) = 0.2107$, such that the noise level estimate has to be scaled accordingly (by 9.5 due to normalized complex variables).

The results in the noise only case are shown in Table 3 when the desired false alarm rate was $10^{-2}$, the rectangular detection window was 128 bins and ten thousand iterations were used. The results show that all the methods are quite good, but the standard deviation of the OS method is the largest, 3-5 times larger than in the other methods which is obvious since it does not average. This reflects higher than expected false alarm rate while the other methods are very close to the expected false alarm rate. FCME2 is the most accurate way to set the noise level.

### Table 3. Estimated noise level, its standard deviation and obtained false alarm rate using various CFAR threshold setting methods.

| Method | Mean | Standard deviation | $P_{fa}$ |
|--------|------|--------------------|----------|
| mean   | 1.000| 0.016              | 0.009    |
| OS     | 1.003| 0.049              | 0.019    |
| FCME1  | 0.993| 0.017              | 0.012    |
| FCME2  | 0.998| 0.009              | 0.010    |

6.4. Detection performance using CFAR methods

The FFT size is 4096 frequency bins that includes a wideband (WB) signal occupying 1024 adjacent frequency bins modulated with random quadrature phase shift keying symbols. Also included is a narrowband (NB) signal occupying one frequency bin. Both signals have a random phase for each Monte Carlo trial and one thousand trials per SNRL value was used. As a practical meaning of the setup, think of the Long Term Evolution (LTE) signal that has 15 kHz subcarrier spacing (the bandwidth of a frequency bin). The used values would mean that the sample vector consists of one 15.3 MHz LTE signal and one NB signal whose bandwidth is less than 15 kHz. Furthermore, 25% of samples contain a signal and the total BW of the detector would be 61.4 MHz. The false alarm rate was set to $10^{-2}$. The FCME based method used 0.1% as its clean sample rejection rate, and 10% of the smallest samples as the initial set, and the OS method used the tenth percentile.

The noise level estimation capacity of the methods is shown first, with signals present. The estimated noise levels are shown in Fig. 5. False alarms rates, measured from signal free windows, are shown in Fig. 6 for the rectangular 1024 bins window. The former results are generally valid since noise level estimation is independent of the window whereas the latter results are representative, typical ones. The OS method tends to overestimate the noise level at high SNLR level that results in reduced sensitivity and reduced false alarm rate. The reason for this behavior is natural; when signals are present the lowest values at the ordered sample set do not necessarily present the actual lowest noise values and, consequently, the estimator tends to overestimate. The FCME methods (12) and (13) estimate the noise level quite good except at a small SNLR range (around 30 to 42 dB-Hz in this setup) where the separation of signal containing and noise only samples is not yet working so well. However, it works well at high SNLR levels. The mean method totally fails at large SNLR and the noise level is significantly overestimated, which reflects the accompanied false alarm results. Interestingly, the FCME methods keep the desired false alarm rate if signals are present and the OS method results in too many false alarms at low signal levels. It can clearly be seen from Fig. 6 where the noise estimators overestimate the noise level since therein the false alarm rate is well below the desired value.

The remaining results concentrate on the validity of the theory with respect to known noise level and sensitivity losses due to noise level estimation. With the rectangular 1024 bins window, the narrowband and wideband signals are detected equally such that results concerning just the wideband signal are shown in Fig. 7. These results show that the presented theory can be used predicting the performance of the WIBA method. Furthermore, due to uncertainties in noise level estimation, some sensitivity losses can be expected although they could be very minor, less than one dB as shown in this example using the FCME method.

In additional simulations with varying the signal bandwidth we observed the following things that might be of interest. First, if the mean method was used for noise level estimation, small windows failed to detect wideband signals even though SNRL was rather high, up to 60 dB-Hz. Consequently, this noise level estimation scheme is not universal, and it was dropped from miss probability simulations. The reason for this behavior is that small windows require higher SNRL to detect wideband signals and the estimated noise level (and the detection threshold thereafter) increases exponentially, faster than the signal power inside the detection window, such that detection fails. Using averages, the explanation is even more apparent. With the single bin window detection occurs if $P/M + 1 > T(N)/P + 1$, where ones become from normal-
Fig. 6. Measured false alarm rates for the ideal case, OS, FCME1, FCME2 and mean methods.

Fig. 7. Miss detection probability for known noise, OS, FCME1, FCME2 methods as well as according to the theory.

7. Conclusions

This paper studied the performance of window based spectrum detectors. Exact analytical tools were developed for the probabilities of false alarm and detection for arbitrary window shapes. Consequently, the results can be used to design window shapes for detecting various signals without time and resource consuming simulations and predict the performance. The idea could be used in the time domain too as temporal windows. This would be useful in detecting variable length pulses and apply various pulse shapes.

Although it is possible to use spectrum matched windows (if the shape is known to the observer) to optimize sensitivity, it was argued that the rectangular window shape is in most cases almost as sensitive (within one dB or so) and would result in a much simpler receiver that is especially important in applications where computation power is limited. Furthermore, it was discussed how to select a suitable set of windows (in a bandwidth sense) for efficiently detecting signals with different bandwidth within the instantaneous bandwidth of the receiver. As a rule of thumb, five to tenfold increase in window width was recommended as the step on size.

Another topic was threshold setting using estimated noise level based on some CFAR methods. It was observed that accurate (small bias and variance) methods keep the desired false alarm the best and there certainly are differences between the methods. The FCME method was the most suitable from the studied ones.

The future work on this topic could include verifying the results using various PSD estimation schemes that use different windows, averaging and overlapping. Due to low correlation between adjacent PSD samples and the CLT, it is expected that the results would match quite well. Other topic is developing CFAR threshold setting methods that are more accurate and, therefore, keep false alarm rate close to the desired level, are practical (in terms of source for noise level estimation), and are easy to implement. The FCME threshold setting method was observed to be a benchmark method to which new developments should be compared. Furthermore, the sensitivity of the threshold setting to non-Gaussian noise might be of interest although it has been observed in [29] that the FCME method can successfully be used with real, measured noise. Finally, the theoretical performance analysis of the detector in fading multipath channels is naturally of interest.

Declarations

Author contribution statement

Harri Saarnisaari: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Johanna Vartiainen: Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Data availability statement

No data was used for the research described in the article.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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