**D-Meson Mixing in 2+1-Flavor Lattice QCD**

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We present results for neutral $D$-meson mixing in 2+1-flavor lattice QCD. We compute the matrix elements for all five operators that contribute to $D$ mixing at short distances, including those that only arise beyond the Standard Model. Our results have an uncertainty similar to those of the ETM collaboration (with 2 and with 2+1+1 flavors). This work shares many features with a recent publication on $B$ mixing and with ongoing work on heavy-light decay constants from the Fermilab Lattice and MILC Collaborations.

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1. Introduction

These proceedings contain a status update of an ongoing calculation of $D^0$-$\bar{D}^0$ mixing matrix elements [1], similar to our published work on $B^0$-$\bar{B}^0$ mixing [2]. We present nearly final results for all five matrix elements, sufficient to describe $D^0$-$\bar{D}^0$ mixing not only in the Standard Model, but also in any high-energy extension that modifies only the local $\Delta C = 2$ interaction.

In the Standard Model, neutral-meson mixing is mediated by one-loop, GIM-suppressed processes, shown in Fig. 1. In extensions of the Standard Model, other particles could appear in the boxes; there could even be tree-level flavor-changing neutral currents. Mixing has been observed in all four neutral-meson systems—$K^0$, $D^0$, $B^0$, and $B_s^0$—but the pattern of internal quark masses and CKM factors explains why the phenomenology differs so greatly from one system to another.

Because the $W$ bosons and $b$ quarks have masses well above the QCD scale, mixing can be re-expressed as stemming both from a local $\Delta C = 2$ interaction and two $\Delta C = 1$ interactions separated by a distance of order $1/\Lambda_{QCD}$. From degenerate perturbation theory, the off-diagonal term in the mass-width matrix is [3]

$$M_{12} - \frac{i}{2} \Gamma_{12} \propto \langle D^0 | \mathcal{L}^{\Delta C = 2} | \bar{D}^0 \rangle + \sum_n \frac{\langle D^0 | \mathcal{L}^{\Delta C = 1} | n \rangle \langle n | \mathcal{L}^{\Delta C = 1} | \bar{D}^0 \rangle}{M_D - E_n + i0^+}. \quad (1.1)$$

The second term is very difficult to estimate. For $D^0$ mesons it is also not negligible, unlike for $B^0$ and $B_s^0$, where $t$, $c$, and $u$ quarks appear in the box. (For kaons, the second term is important but not dominant.) One can relate the measured mass and width differences, $\Delta M$ and $\Delta \Gamma$, to $|M_{12}|$, $|\Gamma_{12}|$, and the relative phase $\arg(\Gamma_{12}/M_{12})$ [4]. In some extensions of the Standard Model, only the first term and, thus, $M_{12}$ is altered [5].

The effective Lagrangian $\mathcal{L}^{\Delta C = 2}$ (at energies below the $b$-quark mass) is built out of the following operators (and their Wilson coefficients) [6, 7, 8]:

$$\mathcal{O}_1 = \bar{c} \gamma^\mu L u \bar{c} \gamma_\mu L u, \quad \hat{\mathcal{O}}_1 = \bar{c} \gamma^\mu R u \bar{c} \gamma_\mu R u, \quad (1.2)$$
$$\mathcal{O}_2 = \bar{c} L u \bar{c} L u, \quad \hat{\mathcal{O}}_2 = \bar{c} R u \bar{c} R u, \quad (1.3)$$
$$\mathcal{O}_3 = \bar{c} \sigma^{\mu \nu} L \gamma^\nu u \bar{c}, \quad \hat{\mathcal{O}}_3 = \bar{c} \sigma^{\mu \nu} R \gamma^\nu u \bar{c}, \quad (1.4)$$
$$\mathcal{O}_4 = \bar{c} L u \bar{c} R u, \quad (1.5)$$
$$\mathcal{O}_5 = \bar{c} \sigma^{\mu \nu} L \gamma^\nu R u, \quad (1.6)$$

where $L$ ($R$) denotes a left- (right-)handed projector on the Dirac indices, and $\alpha$ and $\beta$ are color indices. By parity conservation in QCD, $\langle D^0 | \hat{\mathcal{O}}_i | \bar{D}^0 \rangle = \langle D^0 | \mathcal{O}_i | \bar{D}^0 \rangle$, $i = 1, 2, 3$. Thus, the five matrix elements $\langle D^0 | \mathcal{O}_i | \bar{D}^0 \rangle$, $i = 1, \ldots, 5$, suffice to describe the short-distance part of all $\Delta C = 2$ processes, whether their origin is $W$-$b$ box or something else. In these proceedings, we report on a calculation of all five matrix elements using lattice QCD with 2+1 flavors of sea quarks.

![Figure 1: Box diagrams mediating $D^0$-$\bar{D}^0$ mixing in the Standard Model.](image-url)
2. Lattice-QCD calculation

Our $D$-meson calculations have much in common with our published $B$-meson work [2]. We use the same ensembles (generated by the MILC collaboration) with 2+1 flavors of sea quark [9]. The light quarks (valence and sea) are based on the staggered asqtad action; the heavy $c$ (or $b$) quark on the Fermilab interpretation of the clover action. The lattice spacings for the ensembles satisfy $a \approx 0.045$ fm, $\approx 0.06$ fm, $\approx 0.09$ fm, and $\approx 0.12$ fm. The sea-quark masses yield pions with

\begin{align}
177 \text{ MeV} & \lesssim M_\pi \lesssim 555 \text{ MeV}, \\
257 \text{ MeV} & \lesssim M_\pi^{\text{rms}} \lesssim 670 \text{ MeV},
\end{align}

(2.1) (2.2)

The ensembles contain 600–2200 gauge-field configurations, and we use 4 or 8 sources/config.

To carry out the chiral-continuum extrapolation, we take into account the subtle way in which spin emerges for staggered fermions with staggered-Wilson four-fermion lattice operators. The three-point correlation function, it turns out, contains contributions not only from the continuum-limit operator of desired spin, but also some of the wrong spin [10]. Because only the five operators in Eqs. (1.2)–(1.6) can arise, we automatically have the information needed to disentangle this effect. We use the one-loop chiral-perturbation-theory formulas of Ref. [10] to remove the wrong-spin contribution in the course of our chiral-continuum fit.

The operators in Eqs. (1.2)–(1.6) require renormalization for any ultraviolet regulator. We carry out the renormalization of the lattice operators corresponding to Eqs. (1.2)–(1.6) together with matching to $\overline{\text{MS}}$ schemes in continuum QCD. We use a mostly nonperturbative method to handle the largest lattice-to-continuum matching corrections [11, 12], supplemented with a one-loop calculation of the remaining, small renormalization parts [13, 2]. We choose the renormalization scale for $D$-meson matrix elements to be 3 GeV, while we chose $m_b$ for $B_{(s)}$ mesons.

The main difference between our work on $D$ vs. $B_{(s)}$ mesons is the analysis of the correlation functions. The signal-to-noise ratio is much better for $D$-meson correlators. For the two-point correlators, the optimal time range $t_{\text{min}} \lesssim t \lesssim t_{\text{max}}$ differs: $t_{\text{min}} \approx 0.7 \ (0.2)$ fm, $t_{\text{max}} \approx 3.0 \ (2.4)$ fm for $D \ (B_{(s)})$ mesons. The difference for the three-point correlators is more striking. We fix the four-quark operators at $t = 0$ and the meson creation (annihilation) operator at time $t_x < 0 \ (t_y > 0)$. As

\begin{figure}
\centering
\includegraphics[width=\textwidth]{D-meson-mixing}
\caption{Fitting ranges for three-point correlators: triangular (green) and-or fan-shaped (magenta) regions for $B$ mixing (left); two-strip diagonal region (green) for $D$ mixing (right). Background color shows the signal-to-noise ratio from good (blue) to bad (red).}
\end{figure}
shown in Fig. 2, we use a triangular and-or fan-shaped region in the \(|t_x| - t_y\) plane for \(B_{(s)}\) mesons [2], while we use a long diagonal of width 2 for \(D\) mixing, \(\{|t_x| = t_y\} \cup \{|t_x| = t_y + 1\}\). The long diagonal makes it easier to disentangle the lowest-lying state, if the signal persists that far. A simultaneous fit to two- and three-point functions is used to extract the matrix elements \(\langle O_i \rangle \equiv \langle D^0|O_i|\bar{D}^0\rangle\).

3. Chiral-continuum extrapolation

To carry out the chiral-continuum extrapolation, we develop a fit function based on chiral perturbation theory (\(\chi PT\)), Symanzik effective field theory, and heavy-quark effective theory (HQET). It takes the form

\[
F_i = F_i^{\text{logs}} + F_i^{\text{analytic}} + F_i^{\text{HQ disc}} + F_i^{\alpha_s a^2 \text{ gen}} + F_i^{\text{renorm}} + F_i^\kappa,
\]

(3.1)

where \(F_i^{\text{logs}}\) denotes the next-to-leading order description from heavy-meson rooted staggered \(\chi PT\), with nonanalytic terms including those that disentangle the wrong-spin contributions [10]; \(F_i^{\text{analytic}}\) is a polynomial of various terms that arise in \(\chi PT\) at next-to-leading or higher order; \(F_i^{\text{HQ disc}}\) describes heavy-quark discretization effects using HQET as a theory of cutoff effects [12]; \(F_i^{\alpha_s a^2 \text{ gen}}\) parametrizes generic cutoff effects of light quarks and gluons, à la Symanzik; and \(F_i^{\text{renorm}}\) allows the fit to be sensitive to higher orders in \(\alpha_s\) for matching and renormalization. Finally, \(F_i^\kappa\) incorporates a correction for tuning the charm-quark hopping parameter \(\kappa\), based on extra runs at \(a \approx 0.12\) fm.

Figure 3: Stability of the chiral-continuum extrapolation for several variants of the fit function \(F_i\): \(\langle O_1 \rangle\) (left), minimized \(\chi^2_{\text{aug}}/\text{dof}\) (right). Stability plots for the other \(\langle O_i \rangle\) look similar.
MS scheme with cross checks. The same applies to the other (above) depend on the renormalization scheme; the tabulated results are in the fits that omit important information. Our nearly final results for $\Delta$ see from Fig. 3. We express the discretization errors; we substitute infinite-volume one-loop integrals for the finite-volume sums $\chi$ instead of $\chi$ by discretization effects and higher-order matching effects). We then augment this $\chi^2$ with Gaussian priors for the fit parameters implied in Eq. (3.1), choosing a central value of 0 and width of $\pm 1$ in natural units for $\chi$PT and HQET [15] and minimize the resulting $\chi^2$ aug. We reconstitute the fit function at zero lattice spacing and physical quark masses to obtain our estimate of the $\langle \hat{O}_i \rangle$ and their uncertainty.

We have 510 data points for $\langle \hat{O}_i \rangle$, ranging over the ensembles, valence-quark masses, and five operators. In our base version of $F_i$, there are 127 parameters. To check whether the final results are robust, we repeat the procedure with several variants of $F_i$, as illustrated in Fig. 3. We express the $\chi$PT with $f_\pi$ instead of $f_\pi$: we choose different orders of $\alpha_s$ in $F_i^{\text{renorm}}$ and even replace the mostly nonperturbative (mNPR) matching with a fully perturbative (PT) one; we check various alternatives for the polynomial $F_i^{\text{analytic}}$ (NLO, NNLO, N3LO); we check what happens when the $\chi$PT prior widths in $F_i^{\text{analytic}}$ are doubled; we check alternatives for the heavy-quark discretization errors; we substitute infinite-volume one-loop integrals for the finite-volume sums in one-loop $\chi$PT; we omit the data from the coarsest or finest lattice spacing; we fit each matrix element separately, thereby ignoring data constraints on wrong-spin contributions. As one can see from Fig. 3, the results for the $\langle \hat{O}_i \rangle$ are very stable, so we take these variations in the fit as cross checks. The same applies to the other $\langle \hat{O}_i \rangle$. The largest deviations are $\sim 1\sigma$ and come from fits that omit important information. Our nearly final results for $D$ mixing are given in Table 1, together with published results for $B_{(s)}$ mixing from Ref. [2]. These matrix elements (as noted above) depend on the renormalization scheme; the tabulated results are in the MS scheme with naive (fully commuting) $\gamma^5$ and the evanescent-operator basis used by Beneke, Buchalla, Greub, Lenz, and Nierste (BBGLN) [16].

The MILC asqtad ensembles omit the charmed-quark sea. As in Ref. [2], we assign an additional 2% uncertainty to account for this omission. This uncertainty is given separately, in the second set of parentheses, in Table 1.

| $\langle \hat{O}_i \rangle$/(GeV$^3$) | $f_{B_{(s)}}B_{B_{(s)}}$/(GeV$^2$) |
|------------------------|-------------------------------|
| $q = d$ | $q = s$ |
| $\hat{O}_1$ | 0.0432(29)(9) | 0.0342(29)(7) | 0.0498(30)(10) |
| $\hat{O}_2$ | -0.0833(38)(17) | 0.0303(27)(6) | 0.0449(29)(9) |
| $\hat{O}_3$ | 0.0248(16)(5) | 0.0399(77)(8) | 0.0571(77)(11) |
| $\hat{O}_4$ | 0.1469(69)(30) | 0.0390(28)(8) | 0.0534(30)(11) |
| $\hat{O}_5$ | 0.0554(38)(11) | 0.0361(35)(7) | 0.0493(36)(10) |
| $\mu$ | 3 GeV | $m_b$ | $m_b$ |

Table 1: Results for $D$ [this work] and $B$ [2] mixing in the renormalization scheme of Ref. [16].

Both the renormalization and wrong-spin effects mix operators 1, 2, and 3 with each other, and also 4 and 5 with each other. It is thus natural to fit the matrix elements in each sector simultaneously. Some ingredients in $F_i^{\text{logs}}$ are common for all $i$, such as masses, $f_\pi$, light-meson $\chi$PT constants [14], and the $D^*-D-\pi$ coupling. We introduce these external inputs with Gaussian priors, for example $g_{D^*-D\pi} = 0.53 \pm 0.8$. Because of these common ingredients, we choose to fit all five matrix elements simultaneously. We form a $\chi^2$ function from $F_i - \langle \hat{O}_i \rangle$ and the sample covariance matrix of the $\langle \hat{O}_i \rangle$, where $\hat{O}_i$ denotes the renormalized lattice operators (which differ from the continuum $\hat{O}_i$ by discretization effects and higher-order matching effects). We then augment this $\chi^2$ with Gaussian priors for the fit parameters implied in Eq. (3.1), choosing a central value of 0 and width of $\pm 1$ in natural units for $\chi$PT and HQET [15] and minimize the resulting $\chi^2_{\text{aug}}$. We reconstitute the fit function at zero lattice spacing and physical quark masses to obtain our estimate of the $\langle \hat{O}_i \rangle$ and their uncertainty.
4. Outlook

Our results agree well with and have similar uncertainty as previous lattice-QCD results from the ETM collaboration, with 2 [17] or 2+1+1 [18] flavors in the sea. The comparison of these results tests not only the flavor-dependence of the matrix elements but also the sensitivity to lattice fermion formulation: ETM employs twisted-mass Wilson fermions, while we employ staggered fermions. All these calculations use several lattice spacings and take the continuum limit. References [17, 18] report the so-called “bag factors” often used in phenomenology [7]; a detailed comparison would require choices of quark masses and decay constants (and their uncertainties) that would obscure the error budget of one or the other set of results. We have a set of calculations underway [19] to compute the \( D^- \) and \( B_s^- \) meson decay constants on the same ensembles and will report the bag factors then.

Estimates of the contribution to \( M_{12} \) of the second term in Eq. (1.1) range over \((10^{-3} – 10^{-2})\Gamma \) [20], where \( \Gamma \) is the total width of the neutral \( D \) meson. It turns out, however, that all Standard-Model phases appearing in Eq. (1.1) are small. Thus, in a TeV-scale model that might produce a large phase in \( M_{12} \), the results for the \( \langle \mathcal{O}_i \rangle \) can be used to constrain the model’s parameters. Furthermore, until a method is developed to tame the second term in Eq. (1.1), the accuracy achieved in this work and Refs. [17, 18] should suffice for this purpose.

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