Effect of dynamic stiffness of fasteners on vibration and acoustic radiation of a ballastless track

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Abstract
A new model for a single wheel rolling over a metro ballastless track is developed. It is used to carry out the analysis on the effect of dynamic characteristics of fastener on the vibration and noise radiation of the wheel and the track in the vertical direction, under the excitation of the wheel/rail uneven surfaces in detail. In this analysis, a rail is modeled as a Timoshenko beam resting on discrete rubber booted short sleepers, and the sleepers are connected with the slab through linear springs and damping units, the slab is modeled by using the FE method, the fastener is characterized by using the Poynting-Thomson model which takes into account that the stiffness and dumping of the fasteners vary with vibration frequency in their service. The dynamic characteristics of the fastener include the variation of its stiffness and damping with frequency. The analysis considers that the fastener dynamic stiffness increases with the excitation frequency while its damping decreases. The vibration and acoustic radiation of the wheel/track is, to varying degree, affected by the dynamic property of the fastener. The vibration and acoustic radiation of the sleeper and the slab is greatly affected by the dynamic property of the fastener. But the effect of the total noise level of the wheel and the track by the dynamic property of the fastener is not so large because the wheel and rail noise is dominant in the whole analyzed system. These conclusions have certain reference values for the study of the vibration and noise reduction measures of the wheel and track coupled system using the fastening characteristic.

Keywords
Fastener, dynamic stiffness, wheel/rail rolling noise, p-t model, rubber booted short sleeper track

Introduction
The rapid development of China’s railways has changed the travel efficiency of its citizens. With train speed increase, wheel/rail interaction becomes fiercer. This leads to the increase of vibration and sound radiation in wheel-rail rolling contact. To address this drawback, a variety of vibration damping tracks were bred, and rubber booted short sleeper track is one of them. Compared to the slab track, the vibration damping tracks can improve track vibration isolation performance. Many researches were published on the vibration reduction of the rubber booted sleeper track, and its vibration reduction effects were found through the field tests and theoretical analysis. But the good vibration isolation capability results in intense wheel-rail interaction, which leads to fatigue failure of key components of train and track, high-level wheel/rail noise and rail corrugation. Through the field test and simulation of an analytical model, researchers found that the rubber booted sleeper track shows a higher wheel/rail acoustic power than the slab track. And the generation of rail corrugation on the rubber booted sleeper track section causes a severer wheel/rail interaction and higher level of wheel/rail noise. So a deep study regarding this problem is necessary.

Although, models were built by many researchers for analyzing the vibration and noise characteristics of rubber booted sleeper track, very few works have considered the effect of fastener dynamic stiffness on the dynamical behavior of the track. In fact, the rail pad inserting in the fastener is made of rubber. Laboratory measurements of the fastener dynamic stiffness has showed that the stiffness and damping of it could be affected by the external excitation frequency, environmental temperature, preload generated by the clips and the uncertainty of fastening system, and in turn influence the vibration and noise radiation of the track, and the specific impact of each variable is strongly material-dependant.
In order to have a better understanding of the effect of the fastener dynamic properties on vehicle/track dynamic performance, various fastener models were proposed. In the time domain, a nonlinear fractional derivative viscoelastic model was used to analyze the influence of frequency-dependent and amplitude-dependent properties of the fastener on the dynamic characteristics of vehicle-track coupling system.\(^\text{16}\) Then a nonlinear spring-dashpot element was built by Sadeghi for analyzing the frequency of load and magnitude of preload on fastener vertical dynamic behavior, results showed that the above parameters have great influence on the fastener vibration characteristics.\(^\text{17}\) Wei used the logarithmic fitting function to characterize the stiffness and damping frequency-dependent characteristics of the fastener, and used this model to analyze the vibration characteristics of the track and the vehicle/track coupling system, and their impact on the subway tunnel environment vibration.\(^\text{18–20}\) At the same time, the temperature characteristic of the fastener stiffness was studied and its influence on the random frequency response characteristics of wheel/rail interaction was analyzed. Through laboratory tests and simulation, Liu found that at a high temperature the temperature-dependent stiffness of the WJ-7B small resistance fastener rubber pad has little influence on the wheel/rail contact force and the wheels accelerations.\(^\text{21}\)

The amplitude-, temperature-, and frequency-dependent characteristics and nonlinearity of fastener have been analyzed by many researchers, but wheel/rail rolling noise is mainly analyzed on the dynamic behavior of structure with small vibration amplitude at high frequency. At the same time, in most cases, the temperature in China’s metro operation areas shows a small temperature difference which means that temperature has limited influence on the fastener stiffness, so the excitation frequency becomes the main factor of wheel/rail rolling noise. This is also the research hotspot at present.

The vertical and lateral dynamic frequency-dependent stiffness and loss factor of high-speed railway fastener at 5–1250 Hz were obtained by Gao, and the wheel/rail vibration and noise radiation are analyzed based on the test data. The results indicate that when considering the fastener dynamic stiffness wheel/rail interaction forces, total sound pressure levels of rail and wheel/rail exhibit a significant differences compared to ordinary fastener model.\(^\text{22}\) And the wheel/rail vibration and noise in frequency domain were derived by Yin, which accounts for the characteristics of frequency-dependent stiffness of rail fastener by adopting the logarithmic fitting function.\(^\text{23}\)

A review of the literature indicates that there are still some limitations in the investigation of the effect of rail pad dynamic stiffness on the wheel/rail vibration. The analysis of the influence of the dynamic stiffness characteristics of the fastener on wheel/rail vibration characteristics has mainly been conducted in low frequency domain. Although researches have been conducted on the wheel/rail noise radiation when considering the dynamic stiffness of fastener, the track models used are non-vibration damping tracks. It is rarely mentioned in published papers regarding the analysis of the stiffness and damping frequency-dependent characteristics of fasteners influencing on the vibration and noise of the wheel rolling on the rubber

![Figure 1. Wheel/track coupling system.](image)

boorted short sleeper track. Besides rail corrugation often occurred in vibration reduction track, like rubber booted sleeper track. Although plenty of researches have been conducted on the mechanism of rail corrugation generation and its influence on wheel-rail interaction, failure of train and track components, few studies have analyzed the influence of rail corrugation on wheel/rail rolling noise.

In this paper, a new model for a single wheel rolling over a metro ballastless track is developed in order to find out the high frequency vibration and noise characteristics of the wheel and the track parts with considering the influence of the dynamic stiffness and damping of the fasteners. The P-T mechanical model is used to represent the frequency-dependent characteristics of damping and stiffness of rail fastener, and the track model is validated through the experimental test data and FE model, and the linear contact stiffness is also justified to be suitable for the study of wheel/rail vibration and noise in this paper.

**Wheel/rail interaction model**

The wheel/track interaction model used in predicting the vibration and sound radiation of the wheel/track is shown in Figure 1. And the model is built in the frequency domain, based on the assumption that, the wheel/rail noise is analyzed under the condition of a steady state random process, and the vibration amplitude of the wheel/rail system is low.\(^\text{24,25}\)

This model considers that a single wheel rolls on a rubber booted short sleeper track, and their coupling is replaced with a contact spring. When the wheel moves along the rail, the irregularity excitation between them is replaced with the “moving” roughness, which means the roughness “strip” is pulled through the gap between the wheel and the rail.

**Track model with frequency-dependent stiffness and damping of fasteners**

**Frequency-dependent fastener model.** The ordinary fastener models, the Kelvin model, considers frequency-independent stiffness and damping only. In this paper, the P-T model is used to capture the frequency-dependent characteristics of the fastener. The P-T model is shown in Figure 1, in which a spring element and the Maxwell model are considered in the calculation model of fastener. And the force of the P-T model, can be calculated by equation (1),
then the total stiffness \((K_z)\) of the P-T model can be expressed by equation (2), in the equation (2), the real terms represent stiffness, and the imaginary term denotes the damping coefficient. 

\[
F = K_1 x_1 + \left(1 - \frac{K_2}{K_2 + i\omega C_2}\right) K_2 x_2
\]  

\[
K_2 = K_1 + K_2 - \frac{\omega^2 C_2}{(K_2)^2 + (\omega C_2)^2} + i \frac{\omega C_2 (K_2)^2}{(K_2)^2 + (\omega C_2)^2}
\]  

According to the equation (2), the parameters of the P-T model are expressed by equations (3) and (4). 

\[
K_1 = K_1 + K_2 - \frac{\omega C_2}{(K_2)^2 + (\omega C_2)^2} \Rightarrow K_1 + K_2 \cdot 2\alpha
\]  

\[
C_1 = \frac{\omega C_2 (K_2)^2}{(K_2)^2 + (\omega C_2)^2} \Rightarrow C_2 \frac{\omega z^2}{\omega^2}
\]  

\(K_1\) is the frequency-independent stiffness; \(K_2\) is the frequency-dependent stiffness; \(C_2\) is the frequency-dependent viscous damping, the coefficient \(\alpha = \omega^2 / (\omega^2 + z^2)\) and \(z = K_2 / C_2\), \(\omega\) is the radian frequency.

Fastener dynamic stiffness can be obtained by rail pad tester TNO. During simple harmonic vibration, force \((F)\) on the blocked output side of a vibration isolator, displacement \((u)\), and acceleration \((a)\) on the input side can be measured. Then the dynamic stiffness \(k(f)\) of rail pad can be derived by 

\[
k(f) = \frac{F}{u} = - (2\pi f)^2 \frac{F}{a}
\]  

The loss factor can be calculated from 

\[
\eta(f) = \frac{|k(f)|}{\text{Re}[k(f)]}
\]  

The equivalent viscous damping \(C\) can be calculated from the loss factor 

\[
C = \frac{\eta \cdot k}{\omega}
\]  

where \(f\) is the excitation frequency, \(\omega\) is the radian frequency.

The engineering optimization software isight software is used to obtain the parameters that match the experimental results. The objective function (ERRO) adopted in the optimization was

\[
\text{ERRO} = \sqrt{\sum_{i=0}^{N} (P_i - E_i)^2}
\]  

where \(P_i\) and \(E_i\) are the simulation results and the corresponding measured results at a given frequency \(f_i\), \(N\) is the number of data points.

Figure 2(a) shows the comparison of the fastener dynamic stiffness results of the P-T model and the laboratory measurement data. In the figure, three different fastener stiffness data, considering the preloads under the working condition, are used to verify the P-T model, and their static stiffness are, respectively, 6 kN/mm, 75 kN/mm, and 280 kN/mm.

It is difficult to accurately obtain the dynamic damping value of the fastener in a laboratory. The stiffness \(K_i\) and damping \(C_i\) of the P-T modal are proportionable, which means if the dynamic stiffness is approximately consistent with the experimental results, the damping matches the field measurement data in the range of the error permitted. As it can be seen in Figure 2(b), the frequency-dependent characteristic of the damping and stiffness of the fastener can be captured by using the P-T model in the frequency range of 0–1500 Hz. It can be concluded that the P-T model has good applicability for characterizing the fastener of dynamic stiffness and damping, and can reflect the vertical frequency-dependent variation characteristics of the stiffness and damping of the fastener. Compared to the rail pad, the dynamic stiffness of the under sleeper pad, with low static stiffness, is less sensitive to the external excitation frequency, so the dynamic stiffness characteristics of the under sleeper pad are not considered in this paper.

**Track model of rubber booted short sleeper.** For the model of rubber booted short sleeper track, the rail is simulated by using the Timoshenko beam, the rubber booted short sleeper is simplified as a mass block, the rubber support under the sleeper is simulated by using a linear damping-spring, and the slab is modeled with the FE method. In the model, the rail density \(\rho_r = 7850 \text{ kg/m}^3\), its cross-section area \(A_r = 7.69 \times 10^{-3} \text{ m}^2\), the shear factor \(\kappa = 0.4\), the shear modulus \(G_r = 0.77 \times 10^{11} \text{ Pa}\), Young's modulus
E_r = 2.1 \times 10^{11} \text{ Pa}; \text{ the sleeper mass } M_{sn} = 90 \text{ kg}, \text{ the sleeper space is } 0.57 \text{ m}, \text{ the rubber support stiffness } k_s \text{ and damping } c_s \text{ are respectively } 30 \text{ kN/mm} \text{ and } 1 \times 10^{4} \text{ Ns/m}.

The track motion equation in frequency domain is denoted by equation (9)−(12), equation (9) and equation (10) are the rail equations, equation (5) is the equation of the sleeper, and equation (12) is the slab equation. In these equations, \( N \) is the total number of the fasteners involved in the numerical simulation, \( N_p \) is the number of the fasteners on one slab, \( \omega \) is the radian frequency, \( u_r \) is the rail receptance, \( u_{slab} \) is the slab receptance, and \( \phi \) indicates the rotation of the cross-section relative to the un-deformed axis.

\[
l - \rho_r A_r \omega^2 u_r + G_r A_r \kappa(\phi' - u_r'') + \sum_{n=1}^{N} K_p (u_r - u_{sn}) \delta(z - z_{sn}) = F \delta(z)
\]

In equation (9)−(12), \( \rho, E, \text{ and } G \) with foot marks are, respectively, the density, the Yong’s modulus, and shear modulus, the subscripts \( r \) and \( s \) means the rail and the slab, respectively. \( \kappa \) is the shear coefficient of the rail. \( K \) is the rubber structure between the sleeper and the slab, \( K_p = K_r - i\omega C_r \) is the total stiffness of the fastener (\( K_r \) and \( C_r \) are the frequency-dependent stiffness and damping of the fastener, respectively), \( K_s = k_s + i\omega c_s \) is the total stiffness of the elastic support between the sleeper and the slab (\( k_s \) and \( c_s \) are, respectively, the stiffness and damping of the elastic support between the sleepers and slab), and \( K_b = k_b + (1 + i\eta_b) \) is the total stiffness of the elastic support under the slab, \( k_b \) is, respectively, the stiffness and loss factor of the elastic support structure under the slab, the slab parameter in this paper can be found in Ref.16.

The vertical frequency response function of the rail is indicated by equation (13)

\[
u(z) = Fa(z,0) - \sum_{n=1}^{N} F_{p_n} a(z_{\text{sn}}) \delta f = 1,2,\ldots,N \tag{13}
\]

\[
F_{p_n} = K_p [u_r(z_{\text{pn}}) - u_{\text{mn}}] 
\tag{14}
\]

In the equation (13), \( F_{p_n} \) is the vertical force between rail and sleeper, which can be obtained by using equation (14), and \( u_r (z_{\text{pn}}) \) and \( u_{\text{mn}} \) are the receptances of the rail and the sleeper, respectively, at the fastener.

The vertical frequency response function of the slab is denoted by equation (15), and the force between the sleeper and slab is indicated by equation (16). \( z_{\text{slm}} \) is the position of \( F_{\text{slm}} \) imposed on the slab, \( l \) is the serial number of the slabs.

\[
u_{\text{slab}}(z) = \sum_{n=1}^{N} F_{\text{slm}} a(z_{\text{slm}}) \delta f = 1,2,\ldots,N \tag{15}
\]

\[
F_{\text{slm}} = K_s [u_s(z_{\text{slm}}) - u_{\text{slm}}] 
\tag{16}
\]
used in the calculation, $m$ is the serial number of the fasteners imposed on the slab, and the slab displacement can be obtained by using equation (15)  

$$u_{slab}(z_i) = \sum_{n=1}^{N_u} F_{slab}(z_i) \beta(z_i,z_{ln})$$  

(15)

$$F_{slab} = K_s[u_{slab}(z_{ln}) - u_{wa}], l = 1, 2, ..., N/10 \quad m = 1, 2, ..., N_p$$  

(16)

By means of the modal superposition method, $\beta(z_i,z_{ln})$ in equation (15) can be obtained, as indicated by equation (17)  

$$\beta(z_i,z_2) = \sum_{n=1}^{N_u} W_n(z_i) W_n(z_2) \left(1 + \eta_{slab})\alpha_n^2 - \omega^2 + (1 + \eta_{f})\alpha_f^2 \right)$$  

(17)

In equation (17), $W_n(z)$ is the modal shape function of the plate with free edge, $\omega_n$ is the natural frequency of rectangular plates with free edges, $\omega_f$ is the natural frequency of the rectangular plate resting on the Winkler foundation with free edges, $\eta_{slab}$ is the loss factor of the slab, and $N_u$ is the number of the free rectangular plate modes.

**Model calibration.** The finite element method is used to model the slab and its size is 6250 mm $\times$ 2800 mm $\times$ 260 mm, the thickness of the track plate is 0.26 m. According to current research, the influence of slab vibration characteristics on wheel/rail vibration noise is mainly around 100 Hz. Thus, in this study, the highest mode superposition frequency of the slab is 253 Hz which is proper for the analysis of the wheel vibration characteristics.

The rubber booted sleeper track FE model (RFE) is used to verify the accuracy of the track model (TM) developed in this paper. The FE track model is shown in Figure 3, rail pad, rubber boot in the FE model are modeled by linear 1D spring and damping element, which is frequency-independent. The harmonic force is applied in the middle of the rail span, and the mobility of the rail, sleeper, and slab near the excitation point is compared. In order to ensure comparability, the rail pad stiffness is constant in both models. The comparison results are shown in the Figure 4.

As can be seen in Figure 4, TM simulation results are matched well with the RFE, except for the sleeper and slab at higher frequency range. Modal analysis of the RFE shows that the flexural vibration of sleeper is the main cause of sleeper and slab mobility discrepancy between the two models at 1430 Hz. But the under rail structure mainly affects the wheel/rail vibration and noise radiation in the low frequency band, while in the higher frequency, the rail and wheel become the dominator, which means that the model established in this paper is suitable for wheel/rail vibration and noise radiation analyzing.

**Model of the wheel**

The wheel is modeled by using the finite element method, and the model accuracy is justified through laboratory tests, the wheel eigenmodes and natural frequencies are obtained. Based on the modal superposition method, the frequency response of the wheel can be predicted. The wheel hub is fully constrained, and the unit force is input at the wheel/rail contact point. Then the receptance of the wheel can be calculated by using equation (18)  

$$a_{\beta}(\omega) = \sum_{r=1}^{N} \frac{\phi_r \theta_r}{M_r \omega_r^2 (1 - \omega_r^2 + 2(i \omega_r) \zeta_r \cos(nz))}$$  

(18)

In equation (18), $r$ indicates the $r$th modal, $\phi_r$ and $\theta_r$ are the $r$th modal shape functions at node $x_i$ and $\phi_r$, respectively. $M_r$ is defined as the modal mass, $\omega_r$ is the angular frequency of the wheel, and $\omega$ is the circular frequency. $\omega_m = \omega/\omega_r$ is the frequency ratio, and $\zeta_r$ is the modal damping ratio which can be derived from measurement. The wheel radius $R$ is 0.43 m, its density is 7850 kg/m$^3$, Poisson’ ratio $\mu$ is 0.3, and young’s modulus $E$ is $2.1 \times 10^{11}$ Pa.

**Model of wheel/rail interaction**

**Rail/wheel combined roughness.** In this paper, the vertical vibration and acoustic radiation of the wheel and the track are analyzed only. According to the TWINS model, it can be known that the irregularities on the surfaces of wheel and rail can induce wheel/rail dynamic force which can be calculated by the surface roughness of the wheel and the rail (the roughness wavelength related to the vibration and noise of the wheel and the rail is mainly concentrated in the range of 5 mm–500 mm). When analyzing the wheel/rail interaction, it is necessary to select appropriate roughness values of the wheel and the rail. The roughness spectra used in the calculation are the “irregularity” spectra of rail (class C), and wheel (class C) surfaces, which were obtained through a large number of field test in the HARMONOISE project.

**Contact filter.** The contact area between wheel and rail is an ellipse with half axis lengths of $a$ and $b$. Then if the roughness wavelength of the rail and wheel surfaces in the longitudinal direction is shorter than or equal to $2a$, the effect of the roughness could be attenuated through the filter function. The estimated filter function of the contact region is expressed with equation (19)  

$$H(k)^2 = \frac{4}{\alpha(kb)^2} \int_0^{\pi/2} \tan \alpha [J_1(kb \sec \alpha)]^2 d\alpha$$  

(19)

$J_1(x)$ is the first-order Bessel function, $b$ is the contact circle radius, $k$ is the roughness wave number, and $\alpha$ is the correlation coefficient of the wheel and rail surface roughness.

**Contact stiffness.** The vertical contact stiffness is indicated by equation (20)  

$$k_h = \frac{3}{2\xi} \left( \frac{E}{(1 - \mu^2)} \right)^2 P_0 \frac{4R_w R_k}{R_w + R_k}$$  

(20)

$R_w$ is the wheel radius, $R_k$ is the transverse railhead curvature radius, $E$ is the young’s modulus of the rail and the wheel, $\mu$ is the Poisson’s ratio, $P_0 = 60$ kN is the static load of a single wheel, $\xi$ is a dimensionless quantity dependent on the curvature radii of the two surfaces, the relation between $\xi$ and $\theta$ can be seen in Ref. $\theta$ is written as
Then the contact stiffness can be calculated and it reads as $k_H = 1197$ kN/mm. The vertical dynamic force between the wheel and the rail can be calculated by using

$$ F = -\frac{R}{\alpha^{W} + \alpha^{R} + \alpha^{C}} $$  (22)

In Equation (22), $R$ is the combined roughness spectrum, $\alpha^{W}$ is the flexibility of the wheel, $\alpha^{R}$ is the rail flexibility, $\alpha^{C}$ is the contact stiffness flexibility, and $\alpha^{C} = 1/k_H = 8.36 \times 10^{-10}$ m/N.

**Wheel/rail sound radiation model**

The sound power of the rail and the sleeper can be calculated by using equation (23)\textsuperscript{30}

$$ W_r = \rho_r c_r L h \langle V^2 \rangle \sigma $$  (23)

where $\rho_r$ is the air density, $c_r$ is the velocity of sound travel in the air, $L = 6.25$ m is the length of the track used for calculation, $h$ is the total projected length of the rail section contour in the horizontal direction, $h = 0.22$ m, $\langle V^2 \rangle$ is the mean square value of time and space of the normal velocity of the rail surface vibration, it can be calculated by using

$$ \langle V^2 \rangle = \frac{1}{L} \int |V|^2 dz $$  (24)

For the sleeper, the sound radiation surface area is $L \times h = 0.32 \times 0.7$ m$^2$. The radiation ratio $\sigma$, is determined by the size and shape of the vibrating structure.

Analytical methods can be used to obtain the radiation ratio $\sigma$, when the structure is not complex. But for the rail and the sleeper, the radiation ratio can be calculated by the boundary element method (BEM).\textsuperscript{30} Also by using the boundary element method, the wheel acoustic radiation model is established. The wheel sound power can be calculated by using the boundary element method and the wheel-track coupled model.

**Analysis of the nonlinear effects**

A linear approximation of wheel/track coupling system is suitable for low wheel/rail surface roughness level and the wheel and rail is always in contact. However, short rail corrugation wavelength may cause a loss of contact between wheel and rail, under this situation the linearized contact stiffness can no longer be used. Therefore, a single-wheel/track coupling model in the time domain is developed\textsuperscript{31,32} the excitation of two rail corrugation wavelengths (50 mm and 80 mm) with amplitude of 0.03 mm, which are the common wavelengths occurred in the rubber booted sleeper track section, are evaluated to find out the dynamic behavior differences between the wheel/rail coupling model with nonlinear (NLM) and linear contact model (LM). The expressions of the two contact models can be found in Reference.\textsuperscript{32} All the parameters used here are the same as the mentioned above.

The total time calculated is 3 s, and the integral step is 0.000,001,440 s\textsuperscript{29}. The wheel/rail force of wheel/track coupling system under rail corrugation excitation of wavelength 80 and 50 mm are compared in Figures 5(a) and (b), as can be seen in the figure, when the wheel runs into the uneven rail surface, there is a phenomenon of loss contact between the wheel and rail, wheel/rail force equal to 0, which cause an impact. And significant differences are found between the NLM and LM at this period. As the impact energy of the wheel and rail diminish, the differences of wheel/track coupling system dynamic behavior caused by the two model almost disappeared.

The Evaluation Index $EI(t)$ is used to evaluate the wheel/rail force differences between the LM and NLM under different corrugation wavelength excitation, the function is written as\textsuperscript{32}

$$ EI(t) = \frac{NLH(t) - LH(t)}{\text{MAX} |NLH(t)|} $$  (25)

where $NLH(t)$ and $LH(t)$ are the time history of the wheel/rail force with the nonlinear and linear contact stiffness model, respectively. And the time period of calculation is selected under the condition that the impact effect almost disappeared.
Based on the results in Figure 5, the Evaluation Index of the wheel/track force under rail corrugation excitation of wavelength 80 and 50 mm are calculated and compared in the Figure 6. According to Figure 6(a), for the corrugation excitation of wavelength 50 mm, $|E(t)|$ decreased over time. And the maximum value is about 0.092 when $t = 2.2$ s. And for the corrugation wavelength of 80 mm, the maximum value of $|E(t)|$ is about 0.047 when $t = 2.05$ s, which is lower than that of rail corrugation excitation wavelength of 50 mm. But when $t$ is close to 3 s, the maximum value of $|E(t)|$ is decreased to 0.047 for the rail corrugation wavelength of 50 mm and 0.046 for the rail corrugation wavelength of 80 mm. The results indicate that the differences between LM and NLM are negligible under the rail corrugation excitation of wavelength 80 and 50 mm (amplitude is 0.03 mm).

Analysis on influence of frequency-dependent characteristic of fastener

In this chapter, the vibration and noise radiation of the wheel and the track parts is analyzed by using the P-T model and the K-V model. The common subway operation speed in China is at 30–80 km/h; thus, in this paper, the train speed is considered as 60 km/h. Compared with the slab track, the rubber booted short sleeper track can effectively reduce the vibration of the slab, but vibration is transmitted from the rail to the sleeper, which brings new structural vibration and noise radiation. So, the further research is needed.

Mobility characteristic of track parts

Using the two models, the vertical track mobility is calculated again and the frequency increases, under unit impulse force excitation. They are shown in Figure 7.

Figure 7(a) is the mobility of the rail. The frequency-dependent characteristic of stiffness and damping of the fastener has a great influence on the rail second-order bending vibration. The frequency of the rail second-order bending vibration modal increases from 250 Hz to 280 Hz, and the amplitude of the modal shape also increases. It is caused by the fastener stiffness increase and the damping decrease, rail corrugations are often found at this frequency band, therefore, attentions should be paid to the vibration characteristics of the fastener. The rail modal shape of 3020 Hz is shown in Figure 7(a). It can be seen that at 3020 Hz, the rail vibration wave peak is at the fastener, so the decrease of the fastener damping can increase the rail vibration amplitude greatly.

Figure 7(b) shows the mobility of the sleeper. It can be observed that the ordinary fastener model underestimates the amplitude of the sleeper vibration at 190–630 Hz and overestimate above 630 Hz. In the lower frequency range, the rail vibration transmitted to the sleeper is mainly affected by the fastener stiffness, and the stiffer fastener can transmit more energy to the sleeper, while at higher frequencies, the transmission is influenced by the damping, and the decrease of the fastener damping results in less energy transmission.

Figure 7(c) is the mobility of the slab. It can be seen that the dynamic effect of the fastener spreads to the slab. Their difference of the results obtained by using the two models can be observed from 100 Hz, and with the increase of the excitation frequency the slab mobility calculated with the P-T model is higher than that by using the K-V model in 240–1180 Hz range. Especially at 3020 Hz, its dynamic influence is similar to that on the sleeper.

Vibration and acoustic radiation of wheel, track parts, correlation coupling parts

This section analyzes the effect of the dynamic stiffness and damping of the fastener on the vibration and noise of the wheel, the track parts, the correlation coupling parts, with considering the effect of wheel and rail surface roughness excitation. The roughness of Class C is used as the roughness excitation of the wheel and the rail in the analysis. In the calculation, the train operation speed is 60 km/h.

Firstly, by using the two models, the vertical vibration velocities of the rail, the sleeper, and the slab are analyzed with considering the dynamic stiffness and damping of the fastener. Secondly, their acoustic radiation power is calculated in turn.

The results of the rail are shown in Figure 8(a). The frequency-dependent characteristics of the fastener mainly
affect the vertical rail velocity in the area around 3 020 Hz. This is because the fastener damping is reduced in the frequency area, which means the lower constrain to the rail. Figure 8(a) indicates that the variation of stiffness and damping of the fastener with frequency effects on the rail vibration only in the higher frequency range under the wheel/rail roughness excitation.

The sleeper vibration velocity under the wheel/rail (roughness excitation) force is shown in Figure 8(b). The difference between the results calculated with the two fastener models is obvious above 150 Hz. The vertical sleeper velocity calculated with the P-T model is higher than that with the K-V model in 150–630 Hz range, which is caused by the fastener stiffness increase. At higher frequencies, the fastener damping decrease results in the low vibration energy transmitted from the rail to the sleeper, which also means that the K-V model could overestimate the vibration level of the sleeper at higher frequencies, as shown in Figure 8(b). But it could underestimate the sleeper vibration in the middle frequency range.

Compared to the Figure 8(b), the velocity of the slab in the Figure 8(c) has the similar phenomenon occurred in the middle frequency range. In the frequency range of 250–1150 Hz, the slab velocity is higher than the result with the K-V model when considering the fastener dynamic effects. With the decrease of the damping, less energy transmitted from the rail to the sleeper results in lower slab vibration, in
other words, the vibration isolation effect between the rail and the sleeper is overrated by using the K-V model.

From Figure 8, the conclusion can be drawn: the fluctuation features of the rail, sleeper, and slab under wheel/rail roughness excitation are quite similar to the mobility of the rail, sleeper, and slab (without roughness excitation), respectively. The P-T fastener model could be used to carry out more reasonable and accurate calculations in track dynamical behavior.

The wheel vibration under the wheel/rail roughness excitation is studied. It is found that the vibration of the wheel is not sensitive to the change of the dynamic stiffness and damping of the fastener. This means that the change of the dynamic stiffness and damping of the fastener has little effect on the sound radiation of wheel. So, the relevant calculation results will be omitted.

Figure 9(a) indicates that the rail sound power calculated with the P-T model is higher than that obtained by using the K-V model in 315–4500 Hz (except for 800 Hz). In this frequency band, the radiated sound power is about 1.2 dBA higher than that of the K-V model on average. Especially in the frequency band around 3150 Hz, the rail sound power is about 1.7 dBA higher than that calculated with the K-V model. This is because the decrease of the fastener damping above 400 Hz results in a lower rail decay rate, which causes higher rail noise radiation power.

Figure. 9(b) shows the sleeper sound radiation obtained by using the two models. It can be seen that the sleeper sound power obtained by using the P-T model is higher in 160–630 Hz. Their difference is about 2.2 dBA on average. But, using the K-V model could overestimate the sleeper sound radiation level in the frequency range of 1000–4500 Hz. The sleeper radiation sound power of the K-V model at 4500 Hz frequency band is 24.7 dBA, which is 14.4 dBA higher than that of the P-T model. The vibration transmitting from the rail to the sleeper is increased by the fastener stiffness increase in the middle frequency range (160–630 Hz). At high frequencies, the vibration transmission is influenced by the fastener damping, and the fastener damping decrease can result in a weak connection between the rail and the sleeper.

The slab vibration has a limited influence on the noise of the wheel/rail, and so the slab noise radiation is not considered in this analysis.

Figure 10 indicates the sound power levels of the related parts and the total wheel/track coupling system, and the difference of the results obtained by using the two models. In low frequency range, the wheel/rail combined noise is mainly dominated by the sleeper, in the middle frequency range (250–1600 Hz), the rail becomes the main source of the total wheel/track noise, while in high frequency range, the wheel is the major noise source. And the difference between the results of the two models is mainly in the frequency ranges of 160–1000 Hz, 1600–2000 Hz, and above 4500 Hz. In the frequency of 160–2500 Hz, the wheel/rail rolling noise shows a higher level of the P-T model, compared to that of the K-V model. Their difference is about 1.5 dBA on average.

Table 1 shows the total radiation noise levels of the wheel/rail under the combined roughness excitations of wheel/rail, which are calculated by using the two models. In
the table, A-A indicates the combination of the wheel roughness A and the rail roughness A, B-B indicates the combination of the wheel’s B and the rail’s B, and so on. The surface roughness classification of the wheel and the rail are obtained from the HARMONOISE project.²⁹,³⁰ When the combined wheel/rail surface roughness excitation drops from the A-A to the D-D, the total wheel/track sound power level increases from 98.4 dBA to 115.0 dBA. The average level of difference of the numerical results obtained by the two fastener models is from 1.3 dB to 1.4 dBA. In this case, the impact of the dynamic stiffness and damping of the fastener is not so large on the total noise level of the wheel/track coupling system. This is because that although the dynamic stiffness and damping of the fastener has a big influence on the vibration and radiated noise of the sleeper and the slab the total noise of them is not dominant in the total noise of the wheel/track coupling system.

**Effect of rail corrugation on wheel/track noise radiation**

Surface irregularities of rail and wheel give rise to the noise, ground-borne vibration, more additional dynamic loading, and fierce wheel/rail interaction which leads to the larger noise radiation and causes the damage on the parts of the vehicle and the track.³³–³⁹ In this chapter, the radiation noise analysis uses the three classical roughness of rail and wheel used in the HARMONOISE project and the rail corrugation roughness spectrum measured from a rubber booted short sleeper track in China subway line, as shown in Figure 11. The train speed is 60 km/h.

With the increase of the track operation time, the interaction between the wheel/rail will cause rail corrugation, especially in the sharp curved track. Rail corrugation will lead to severe wheel-rail interaction. Rail corrugation wavelength depends on train operation speed and structural characteristics of track. So in this paper, the field measured corrugation data of the rubber booted sleeper track (operating speed of 60 km/h) is used to analyze the wheel/rail rolling noise, combined with considering the effect of the wheel surface roughness of different classifications. The corrugation with wavelengths of 50 mm and 80 mm, shown in Figure 11, is used in the analysis. For the operation speed of 60 km/h and the corrugation of wavelengths of 50 mm and 80 mm, the corresponding corrugation passing frequencies are, respectively, 333.3 Hz and 208.3 Hz. According to previous study, the longitudinal diameter of wheel-rail contact patch is less than 10 mm for new wheel and rail. If the worn wheel and track are considered, the longitudinal diameter of contact patch will be smaller. Therefore, for rail corrugation with a wavelength of 50 mm, the point contact model can ensure the calculation accuracy.⁴⁰

Figures 12 and Figure 13 indicate the sound power levels under the excitations of rail corrugation of wavelengths of 80 mm and 50 mm, respectively. (a), (b), (c), and (d) of the Figures denote the numerical results, with considering the excitations of the wheel roughness levels of A, B, C, and D, respectively. As can be seen in Figures, when the corrugation occurs on the rail, it will definitely and significantly increase the noise levels of the wheel/track and the related parts. For the corrugation excitation of 80 mm and 50 mm wavelengths, the sound power levels reached their maxima at about the corrugation passing frequencies, 208.3 Hz and 333.3 Hz, respectively. But, for the corrugation excitation of the two different wavelengths, the noise level of the wheel is less than those of the rail and the sleeper at the frequencies less than about 1300 Hz. This is an open problem.

Due to the existence of the rail corrugation, the contribution of the rail noise to the total noise in the discussed full frequency domain is dominant for any wheel surface roughness. Above 2500 Hz, the contribution of the wheel noise will dominate like the rail. Compared to the results of the rail corrugation excitation of 80 mm wavelength, as shown in Figure 12, under the corrugation excitation of 50 mm wavelength, the noise levels increase more sharply in approaching the corrugation passing frequency of 333.3 Hz, as shown in Figure 13. The maximum difference of the noise levels calculated with the two fastener models is close to two dBA, and the average level of the difference is 0.7–0.9 dBA, with considering the combined excitation of the rail corrugation with different wavelength and the different wheel surface roughness.

In the Figure 14, wheel/rail and related parts noise with multiple and single wavelength rail corrugation excitation is compared, and the wheel roughness level is C. As can be seen in the Figure, the noise levels of the wheel/rail and the related parts show maximum at the corrugation passing frequency. And the total acoustic power of wheel/rail noise is 113.3 dBA under the corrugation excitation wavelength of 50 mm, which is 1.5 dBA higher than that of rail corrugation wavelength of 80 mm. Although the rail corrugation excitation wavelength of 80 mm will cause the surge of wheel/rail noise around 208.3 Hz, it is not in the important frequency range of wheel/rail noise (315–2000 Hz), so its influence on the total acoustic power of wheel/rail is limited. And the wheel/rail noise increases sharply at 333.3 and 208.3 Hz when the rail corrugation excitation of two wavelengths (50 and 80 mm) is applied at the same time, and the total acoustic power of wheel/rail noise is 113.5 dBA, which is only 0.3 dBA higher than that of rail corrugation.
Figure 12. Sound power levels under rail corrugation excitation of 80 mm wavelength, with the combined examinations of (a) wheel roughness A, (b) wheel roughness B, (c) wheel roughness C, and (d) wheel roughness D.

Figure 13. Sound power levels under rail corrugation excitation of 50 mm wavelength, with the combined examinations of (a) wheel roughness A, (b) wheel roughness B, (c) wheel roughness C, and (d) wheel roughness D.
Conclusion

This paper presents the study on the effect of the dynamic stiffness of the fastener system on the vibration and acoustic radiation of a single wheel running on a rubber booted short sleeper track. In the study, the P-T model is used to improve the prediction accuracy of vibration and noise of the wheel coupled with the track. And the nonlinear effects of the wheel/rail contact are analyzed under the different rail corrugation wavelength. The influence of the fastener frequency-dependent characteristics is considered in the model, which is used to in detail analyze the vibration and acoustic of the wheel rolling on the rubber booted short sleeper track, with considering the combined excitation of the rail corrugation with different wavelength and different surface roughness of the wheel/rail. The conclusion can be drawn as:

1. In the analyzed frequency range of 5000 Hz, the change of the dynamic stiffness and damping of the fastener has the greatest influence on the resonance frequencies and amplitudes of the modal shapes of the rail, the influence on the sleeper is second, and the influence on the short sleeper is least. Special attention is required for the two bending resonant frequencies of the rail affected, 250 Hz and 1150 Hz (Rail Pinned-pinned resonant frequency), which are often the sources of rail corrugation.

2. The influence of wheel/rail contact nonlinear effects can be ignored when the rail corrugation amplitude is low. Significant differences are found between the NLM and LM when wheel and rail are loss of contact between each other. When \( t \) is close to 3 s, the maximum value of \( |E(t)| \) is decreased to 0.047 for the rail corrugation wavelength of 50 mm and 0.046 for the rail corrugation wavelength of 80 mm. The results indicate that as the impact energy of the wheel and rail diminish, the differences of wheel/track coupling system dynamic behavior caused by the two models almost disappeared.

3. The vibration and sound radiation level of the wheel coupled with the track is influenced to varying degrees by the dynamic stiffness and damping of the fastener system under the excitation of the wheel/rail irregularity (including rail corrugation). The influence range of the rail vibration velocity is above 2000 Hz, the influence range of the sleeper and the slab is above 200 Hz. The influence on the noise level of the sleeper is greatest (due to the change of energy transfer characteristics of the fastener), the influence on the rail noise is second, and the influence on the track is least. The maximum influence on the total noise level of the wheel/track is close to 2 dBA, and the average level is 0.7–0.9 dBA.

4. Due to the existence of the rail corrugation excitation, the contribution of the rail noise to the total noise of the wheel coupled the track, in the discussed full frequency domain, is dominant, compared to the excitation of any standard wheel surface roughness. Above 2500 Hz, the contribution of the wheel noise will dominate like the rail. For the corrugation wavelength of 50 mm. The results show that the wheel/rail noise is larger than that of rail corrugation with single wavelength excitation, and the rail corrugation excitation of shorter wavelength leads to higher wheel/rail noise.
excitation, the sound power levels of all the discussed parts reaches their maxima at about the corrugation passing frequencies, and the higher the corrugation passing frequency (the shorter the wavelength of the corrugation), the faster the sound power level of the relevant analyzed parts increases in approaching the passing frequency.

5. When the rail corrugation passing frequency is in the range of 300–2000 Hz, it can lead to a higher wheel/rail noise, and the existence of rail corrugation excitation with multiple wavelengths caused a higher noise radiation than that of single wavelength excitation, and radiation. The wheel/rail total acoustic power is 113.3 dBA for the corrugation excitation wavelength of 50 mm, which is 1.5 dBA higher than that of rail corrugation wavelength of 80 mm. And the total acoustic power of wheel/rail noise is 113.5 dBA, under rail corrugation excitation with multiple wavelengths (50 and 80 mm), which is only 0.3 dBA higher than that of rail corrugation wavelength of 50 mm.

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Authors’ Contributions
XH, JH, XL, and ML analyzed the data; H Xu wrote the paper; All authors read and approved the final manuscript.

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Availability of data and materials
The datasets supporting the conclusions of this article are included within the article.

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