Lightfront QED, Stueckelberg field and Infrared divergence

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Abstract

Stueckelberg mechanism introduces a scalar field, known as Stueckelberg field, so that gauge symmetry is preserved in the massive abelian gauge theory. In this work, we show that the role of the Stueckelberg field is similar to the Kulish and Faddeev coherent state approach to handle infrared (IR) divergences. We expect that the light-front quantum electrodynamics (LFQED) with Stueckelberg field must be IR finite in the massless limit of the gauge boson. We have explicitly shown the cancellation of IR divergences in the relevant diagrams contributing to self-energy and vertex correction at leading order.

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I. INTRODUCTION

It has been known for years, computation of the transition matrix element in gauge theories like QED or QCD inherits infrared divergences (IR) due to massless gauge boson [1-3]. There are different remedies to cure IR divergences. For example, one can treat these IR divergences by mass regularization where a small mass is introduced for the gauge particle. The other standard way is to perform dimensional regularization. Or traditionally one introduces a small energy cutoff for the in and out states, since the instruments come with natural limitations. We find the divergence gets canceled as we remove the cutoff [4].

The standard Lehmann-Symanzik-Zimmermann (LSZ) formalism is based on the assumption that at large time ‘$t > T$’ the coupling can be “switched off” or in other words the particles can be treated as free particles in the scattering process in the limit $|t| \to \infty$. Thus, in that case, the initial and the final state can be considered as Fock state to calculate the matrix element. However, in gauge theories with massless particles, in particular, $U(1)$ gauge theory, the asymptotic states are not the free states. The charged particles are dressed by soft or long wavelength photons as pointed out by Kulish and Faddeev (KF) [5]. Experimentally, these low energy photons cannot be detected by the detector and lead to soft divergences or IR divergences if one tries to do a theoretical computation. The cancellation of IR divergences at amplitude level was first summarized by Chung [6], which says that IR divergences can be eliminated if one chooses the initial and final states to be charged particles with a suitable superposition of an infinite number of photons. KF demonstrated that the asymptotic interaction Hamiltonian in QED is non-vanishing. They obtained the appropriate initial and final state thereby modifying the Hilbert space, which is nothing but the new asymptotic states. They constructed the asymptotic state for QED by defining modified gauge invariant $S$-matrix and showed that the IR divergences cancel at amplitude level using this new basis.

Bagan et al. [7] analogous to KF described that the coupling in QED does not asymptotically vanish hence the matter field is unphysical. Thus if one does not take into account carefully the asymptotic behavior then the IR divergences appear. They constructed a gauge invariant fermion wavefunction which handles the soft photon to obtain the IR finite result.

The KF method was later applied to obtain a set of asymptotic states in the asymptotic region of perturbative quantum chromodynamics (pQCD) by Nelson and Butler [8]. It was
shown that the asymptotic states constructed leads to cancellation of IR divergences in certain matrix elements in the lowest order in pQCD. KF approach was utilized by various authors [9, 10] to develop the coherent states in QED and QCD, in order to study the IR behavior of abelian as well as non-abelian gauge theories. The matrix elements using these coherent states were shown to be IR finite.

The canonical field theory methods reviewed so far can also be analyzed in light front (LF) formalism devised by Dirac [11]. One of the advantages of LF quantization is that it provides the understanding of Feynman’s infinite momentum frame, in which all finite mass particles behave like massless particles. The smooth massless limit can be perceived in this context.

Harindranath and Vary [12] applied coherent state formalism to light front field theory (LFFT) for the first time and showed that a coherent state may be a valid vacuum in LFFT. A coherent state formalism was developed by one of us to deal with true IR divergences in light-front QED (LFQED) to calculate fermion mass renormalization [13–15]. In the coherent state approach, the initial and final Fock state is replaced by superposition of an infinitely large number of soft photons in terms of new Hilbert space. It was shown that IR divergences cancel up to $O(e^4)$ in LFQED using coherent state basis in light front gauge [13] and Feynman gauge [14]. Later, this method was generalized for cancellation of IR divergence in fermion mass renormalization to all orders in LFQED [15].

It is known that IR divergences in gauge invariant QED is due to massless vector bosons. It will be interesting to look for alternate theories, which are free from IR divergences, giving physics of QED. For example, consider the case of Stueckelberg formalism wherein an additional scalar field led to massive gauge boson with a salient feature of gauge invariance [16, 17]. This was in contrast to massive vector Proca field which was just the extension of $U(1)$ gauge theory with the additional mass term with a drawback of violation of abelian gauge symmetry. The additional triumph of Stucekelberg theory was that it is a renormalizable theory [17]. Not only the Stueckelberg mechanism, but there are other ways of generating vector boson mass like the well-known Higgs mechanism in field theories and $B \wedge F$ terms as in BF topological theories.

Recently the infrared question and soft photon theorems have been linked to new asymptotic symmetries that emerge for massless particles in gauge theories [18, 19]. The Stueckelberg QED has an additional degree of freedom which exists for the massive gauge bosons.
Preserving the degrees of freedom at null infinity, while taking the limit of gauge boson mass to zero, we can get additional global or asymptotic symmetry. In fact, one of us discussed modified soft photon theorems due to massive photons and analyzed the subtle procedure of taking massless limit.

The paper is organized as follows: In Sec. II we give the QED Lagrangian after the addition of Stueckelberg field and then obtain a generalized Stueckelberg Hamiltonian. In Sec. III it is shown that IR divergences cancel when one takes the limit \( m \to 0 \) for the scalar field using light-front formalism up to leading order for self-energy correction. We also checked that IR divergences up to \( O(g^3) \) cancel in the massless limit of the scalar field. B is discussed in Sec. IV. We conclude with our remarks and future plans in Sec. V.

II. STUECKELBERG LAGRANGIAN

We start by writing QED Lagrangian with Stueckelberg field \[16, 17\]

\[ \mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_{Stueck} + \mathcal{L}_{gf}, \]  

where

\[ \mathcal{L}_\psi = \bar{\psi} \left[ (i \partial_\mu + g A_\mu) \gamma^\mu - M \right] \psi, \]  

\[ \mathcal{L}_{Stueck} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 \left( A_\mu - \frac{1}{m} \partial_\mu B \right) \left( A^\mu - \frac{1}{m} \partial^\mu B \right) \]  

\[ \mathcal{L}_{gf} = -\frac{1}{2\alpha} \left( \partial_\mu A^\mu + \alpha m B \right) \left( \partial_\nu A^\nu + \alpha m B \right) \]

\( \psi, A \) and \( B \) describe the fermion field, gauge vector field and Stueckelberg scalar fields respectively. The field-strength tensor is given by: \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) The term in Eq. 4 is the gauge fixing term to remove the redundancy. For simplification, we choose Feynman gauge which corresponds to \( \alpha = 1 \). It should be noted that the Stueckelberg field decouples to conventional QED in the mass going to zero limit. In fact, to start with, electromagnetic potential \( A_\mu \) has four components. The field equations led to the massless particle, the photon with two transverse physical degrees of freedom due to gauge invariance. However, the addition of mass term spoils the gauge invariance. But, by introducing an extra scalar field \( B \) we have five fields now. This is Stueckelberg trick which gives Lorentz covariant and gauge invariant massive spin-1 theory.
The Lagrangian without the gauge fixing term is invariant under the gauge transformation:

\[ A_\mu \to A_\mu + \partial_\mu \lambda, \quad B \to B + m\lambda, \quad \psi \to e^{ig\lambda} \psi. \]  

(5)

The complex gauge function satisfies the field equation congruent with \( A_\mu \) and \( B \):

\[ (\partial^2 + m^2)\lambda = 0. \]  

(6)

The Stueckelberg field is not coupled to the fermion, which is actually not a gauge invariant remark. We make the following gauge transformation using the Stueckelberg field itself as gauge parameter:

\[ A_\mu = \tilde{A}_\mu + \frac{1}{m} \partial_\mu B. \]  

(7)

After this transformation we have the fermion field coupled to the Stueckelberg field through derivative interaction. The Lagrangian in the new variables is:

\[
\mathcal{L} = \bar{\psi} \left[ \gamma^\mu \left( i \partial_\mu + g \tilde{A}_\mu + \frac{g}{m} \partial_\mu B \right) - M \right] \psi - \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} m^2 \left( \tilde{A}_\mu \tilde{A}^\mu \right)^2 - \frac{1}{2} \left( \partial_\mu \tilde{A}_\mu \right)^2 \\
+ \frac{1}{2} (\partial_\mu B) (\partial^\mu B) - \frac{1}{2} m^2 B^2
\]  

(8)

The fermion wavefunction \( \psi \) can be decomposed into independent and dependent component of \( \xi \) and \( \eta \) respectively (for details cf. Ref [21]).

\[ \psi = \xi + \eta \]  

(9)

The components of four vector \( A_\mu \) are chosen as

\[ \tilde{A}_+ = a_+ + \alpha_+, \quad \tilde{A}_- = a_- = 0, \quad \tilde{A}_k = a_k \]

This Hamiltonian can be obtained generalizing [12] for massive photon QED in light front as

\[ P^- = H_0 + V_1 + V_2 + V_3 + V_B \]  

(10)

where

\[
H_0 = \int d^2x_\perp dx \left[ \frac{i}{2} \bar{\xi} \gamma^- \partial^- \xi + \frac{1}{2} (F_{12})^2 - \frac{1}{2} a_+ \partial^- \partial_k a_k \right]
\]  

(11)
is the free Hamiltonian,
\[ V_1 = g \int d^2x \, dx^\perp \tilde{\xi} \gamma^\mu \xi a_\mu \]  \hspace{1cm} (12)
is the \( O(g) \), standard 3-point interaction vertex,
\[ V_2 = g \int d^2x \, dx^\perp \bar{\eta} \gamma^\perp \tilde{\partial}_\perp \eta \]
\[ = -\frac{i}{4} g^2 \int d^2x \, dx^\perp \, dy^\perp \epsilon(x^\perp - y^\perp) (\tilde{\xi} a_k \gamma^k)(x) \gamma^+(a_j \gamma^j \xi)(y) \]  \hspace{1cm} (13)
is an \( O(g^2) \) non-local effective 4-point vertex corresponding to instantaneous fermion exchange and
\[ V_3 = \frac{g}{2} \int d^2x \, dx^\perp \tilde{\xi} \gamma^+ \xi \varphi_+ \]
\[ = -\frac{g^2}{4} \int d^2x \, dx^\perp \, dy^\perp \gamma^+(\tilde{\xi} \gamma^+ \xi)(x) |x^\perp - y^\perp| |(\tilde{\xi} \gamma^+ \xi)(y) \]  \hspace{1cm} (14)
is an \( O(g^2) \) non-local effective 4-point vertex corresponding to an instantaneous photon exchange.
\[ V_B = \frac{g}{m} \int d^2x \, dx^\perp \tilde{\xi} \gamma^\mu \xi \partial_\mu B \]  \hspace{1cm} (15)
is the \( O(g) \) 3-point interaction term due to the Stueckelberg field after gauge transformation.
\( \xi \) and \( a_\mu \) have standard expansions in terms of creation and annihilation operators:
\[  \xi(x) = \int \frac{d^2p^\perp}{(2\pi)^{3/2}} \int \frac{dp^+}{\sqrt{2p^+}} \sum_{s=\pm \frac{1}{2}} u(p, s) e^{-i(p^+ x^\perp - p^\perp x^\perp)} b(p, s, x^+) \]
\[ + v(p, s) e^{i(p^+ x^\perp - p^\perp x^\perp)} d^\dagger(p, s, x^+), \]  \hspace{1cm} (16)
\[ a_\mu(x) = \int \frac{d^2k^\perp}{(2\pi)^{3/2}} \int \frac{dk^+}{\sqrt{2k^+}} \sum_{\lambda=1,2} e^\lambda(kq) e^{-i(k^+ x^\perp - k^\perp x^\perp)} a(k, \lambda, x^+) \]
\[ + e^{i(k^+ x^\perp - k^\perp x^\perp)} a^\dagger(k, \lambda, x^+), \]  \hspace{1cm} (17)
\[ \partial_\mu B(x) = \int \frac{d^2q^\perp}{(2\pi)^{3/2}} \int \frac{dq^+}{\sqrt{2q^+}} \sum_{\lambda'=1,2} q_\mu e^{-i(q^+ x^\perp - q^\perp x^\perp)} a(q, \lambda', x^+) \]
\[ + e^{i(q^+ x^\perp - q^\perp x^\perp)} a^\dagger(q, \lambda', x^+), \]  \hspace{1cm} (18)
and satisfy
\[ \{ b(p, s), b^\dagger(p', s') \} = \delta(p^+ - p'^+) \delta^2(p^\perp - p'^\perp) \delta_{ss'}, \]
\[ = \{ d(p, s), d^\dagger(p', s') \}, \]
\[ [a(k, \lambda), a^\dagger(k', \lambda')] = \delta(k^+ - k'^+) \delta^2(k^\perp - k'^\perp) \delta_{\lambda\lambda'}. \]  \hspace{1cm} (19)
These commutation/anticommutation relations hold at equal light-front time \( x^+ \).
III. SELF ENERGY CORRECTION UP TO $O(g^2)$

We obtain the transition matrix element in light-front time ordered perturbation theory using perturbative expansion as follows

$$T = V + V \frac{1}{p^- - H_0} V + \cdots$$

(20)

FIG. 1. Self energy diagram. In the figure on the left, wavy lines indicate massive gauge boson while the double line (in the figure on right) represents the Stueckelberg field. The fermion field is denoted by a straight line.

The contribution to self-energy $O(g^2)$ correction is obtained from

$$T^{(1)}(p, p) = T_{1a}(p, p) + T_{1b}(p, p) + T_{1c}(p, p)$$

$$= \langle p, s | V_1 \frac{1}{p^- - H_0} V_1 | p, s \rangle + \langle p, s | V_B \frac{1}{p^- - H_0} V_B | p, s \rangle + \langle p, s | V_2 | p, s \rangle$$

(21)

In Eq. [21] on the right hand side, we obtain the $O(g^2)$ contributions to fermion self-energy correction. The first term corresponds to the standard three-point vertices, the second term corresponds to three-point vertices due to the Stueckelberg field while the third term corresponds to four-point instantaneous vertex which arises in light front quantization. We will focus on the true IR divergences in the massless limit which shows up due to the vanishing energy denominator. The third term can be dropped as it is not contaminated by IR divergence. It is very important to understand here that in the second term the energy denominator gets IR divergence, when one takes $m \rightarrow 0$ limit. Then the second term contribute when longitudinal polarization is used, equal and opposite to the first term. In order to calculate the transition matrix element $T_{1a}$ and $T_{1b}$ contributing to fermion self-energy correction to $O(g^2)$, we insert a complete sets of states to account for the intermediate
state. Details of the calculation follow:

\[ T_{1a}(p, p) = \sum_{\text{spins}} \int d^3p_1' d^3k_1' \langle p, s | V_1 | p_1', s_1', k_1', \lambda_1' \rangle \langle p_1', s_1', k_1', \lambda_1' | \frac{1}{p^--H_0} | p_2', s_2', k_2', \lambda_2' \rangle \times \langle p_2', s_2', k_2', \lambda_2' | V_1 | p, s \rangle \]  

On substituting for \( V_1 \) and further simplification gives

\[ T_{1a}(p, p) = g^2 \frac{(2\pi)^3}{p^+} \int \frac{dk^+}{k^+} \frac{(p \cdot \epsilon(k))^2}{(p^--p_1^- - k^-)} \]

\[ = -\frac{g^2}{2(2\pi)^3 p^+} \int d^2k_\perp \int\frac{dk^+}{k^+} \frac{Tr[\epsilon^\lambda(k, \lambda)(\not{p}_1 + m)\epsilon^\lambda(k, \lambda)(\not{p} + m)]}{4(p^--p_1^- - k^-)} \]  

\[ = -\frac{g^2}{2(2\pi)^3 p^+} \int d^2k_\perp \int\frac{dk^+}{k^+} \frac{Tr[\epsilon^\lambda(\not{p}_1 + m)\epsilon^\lambda(\not{p} + m)]}{4 M^2 (p^--p_1^- - k^-)} \]  

In going from Eq. 23 to Eq. 24 we have written the longitudinal polarization vector as

\[ \epsilon_\mu(k, \lambda) = \frac{k_\mu}{m} + O\left(\frac{\mu}{k}\right) + \cdots \]  

Since we focus on IR divergence in the massless limit which come from the disappearance of the longitudinal mode we use corresponding polarization.

In the similar manner, we can calculate the diagram in Fig. 1(b)

\[ T_{1b}(p, p) = \langle p| V_B \frac{1}{p^- - H_0} V_B | p \rangle \]

\[ = \frac{g^2}{2(2\pi)^3 p^+} \int d^2q_\perp \int \frac{dq^+}{q^+} \frac{Tr[\epsilon^\lambda(\not{p}_1 + m)\epsilon^\lambda(\not{p} + m)]}{4 M^2 (p^--p_1^- - q^-)} \]  

In the limit \( m \to 0, k = q \) and we observe that IR divergences in Eq. 24 and Eq. 26 cancel exactly each other.

It was shown that IR divergences cancel when we use coherent state basis instead of Fock state to calculate the same matrix element in Ref [13]. Now, we have calculated self-energy correction up to \( O(g^2) \) and shown that the IR divergences cancel when we use Stueckelberg field and take \( m \to 0 \) limit. This explains up to \( O(g^2) \) the contribution for the terms responsible for the cancellation for IR divergences in the coherent state basis are provided by the Stueckelberg field.

IV. VERTEX CORRECTION UP TO \( O(g^3) \)

In this section, we discuss the lowest order radiative correction for 3-point interaction in light front formalism. It was shown in Ref [23] the IR divergences cancel when one uses
coherent state basis instead of Fock state to calculate the matrix element. Now we compute using the Stueckelberg field.

The $O(g^3)$ correction terms due to the three-point vertex contributing to IR divergences are given by

$$T_{12}^{(1)}(p', p, q) = T_{12(a)} + T_{12(b)} + T_{12(c)} + T_{12(d)}$$

$$= \langle p', s', q, \lambda | V_1 \frac{1}{p^- - H_0} V_1 \frac{1}{p^- - H_0} V_1 | p, s \rangle$$

$$+ \langle p', s', q, \lambda | V_B \frac{1}{p^- - H_0} V_B \frac{1}{p^- - H_0} V_1 | p, s \rangle$$

$$+ \langle p', s', q, \lambda | V_2 \frac{1}{p^- - H_0} V_1 | p, s \rangle + \langle p', s', q, \lambda | V_3 \frac{1}{p^- - H_0} V_1 | p, s \rangle$$

FIG. 2. The relevant vertex correction diagrams contributing to IR divergence up to $O(g^3)$.

where $T_{12(a)}$ corresponds to vertex correction contribution coming from three 3-point vertices $V_1$. As we have an additional three-point vertex $V_B$ due to Stueckelberg field interaction with the fermion field which is represented by $T_{12(b)}$. The subscript 12 corresponds to one particle state going to two particle states. There are also contributions for vertex correction coming from the vertex due to instantaneous fermion vertex $V_2$ and instantaneous boson vertex $V_3$ corresponding to $T_{12(c)}$ and $T_{12(d)}$ respectively. We consider only the diagrams shown in Fig. 2 and also we limit our calculation for the vertex correction $\Lambda^+(p', p)$ hence the last term do not contribute due to the tensor structure.
The contribution to vertex correction up to $O(g^3)$ is given by

$$T_{12(a)} = \epsilon_\mu \Lambda_{21(a)}^\mu$$

$$= g^3 \int \frac{[dk]}{k^+ k_1^+ k_2^+} \bar{u}(p', s') \gamma^\alpha (k_1 + m) \gamma^\mu (k_2 + m) \gamma^\beta u(p', s) \epsilon_\alpha(k, \lambda) \epsilon_\beta(k, \lambda) \epsilon_\mu(q, \lambda')$$

$$= \frac{g^3}{M^2} \int \frac{[dk]}{k^+ k_1^+ k_2^+} \frac{Tr \left[ \gamma^\alpha (k_1 + m) \gamma^\mu (k_2 + m) \gamma^\beta \right]}{(p^- - k^- - k_1^-)(p^- - k^- - k_2^- - k'^-)}$$

$$T_{12(b)} = \epsilon_\mu \Lambda_{21(b)}^\mu$$

$$= -\frac{g^3}{M^2} \int \frac{[dq']}{q'^+ k_1'^+ k_2'^+} \frac{\bar{u}(p', s') q'^\alpha (k_1' + m) \gamma^\mu (k_2' + m) q'^\beta u(p, s) \epsilon_\mu(q, \lambda')}{(p'^- - q'^- - k_1'^-)(p'^- - q'^- - k_2'^- - k'^-)}$$

$$= -\frac{g^3}{M^2} \int \frac{[dq']}{q'^+ k_1'^+ k_2'^+} \frac{Tr \left[ q'^\alpha (k_1' + m) \gamma^\mu (k_2' + m) q'^\beta \right]}{(p'^- - q'^- - k_1'^-)(p'^- - q'^- - k_2'^- - k'^-)}$$

We use again longitudinal polarization from Eq. 25. We observe the addition of the Stueckelberg field leads to the cancellation of IR divergences in the limit $m \to 0$ at the amplitude level itself as in the self-energy computation. Coherent states led to the similar cancellation of IR divergence was shown earlier in Ref [23]. Again it is clear the role of soft photon in coherent states is played by Stueckelberg field. Using coherent states IR divergence cancellation was extended to all orders in Ref [15]. We do not anticipate difficulty in establishing similar cancellation using Stueckelberg field.

V. CONCLUSION

In this work, we have shown that IR divergences get cancelled if one adds Stueckelberg field to QED lagrangian using light-front formalism. Massive Stueckelberg QED has been studied earlier and shown to be renormalizable. For very low mass it has also been shown to reproduce results of conventional QED as long as the interaction is through a conserved current. Our main goal was to study $m \to 0$ limit in Stueckelberg QED where IR divergences could make its appearance. Interestingly, we could reproduce the leading order results expected by KF approach. It appears that the arguments can be extended to establish the cancellation of IR divergences to all orders. We hope to pursue in future such a generalization where the tools discussed in Ref. [15] will be useful.

Applying the Stueckelberg mechanism to QCD has limitations due to self coupling amongst gauge bosons. In fact, the non-abelian Stueckelberg theory is non-renormalisable. Hence, the problem of achieving IR divergence cancellation in QCD needs a novel approach.
The recent paper [25] attempts IR issues through Higgs mechanism in abelian theory. Comparing their methods with our approach may give us a tangible idea to handle IR problems in non-abelian theories.

Another interesting aspect to explore will be the interplay between IR divergences and supersymmetry in the context of supersymmetric formulation of Stueckelberg QED [26]. Also it is known that SUSY Stueckelberg QED is renormalizable.

We have confined to gauge theories in this work. It will be challenging to investigate Stueckelberg mechanism in gravity theories. For example, the linearised massless gravity theory studied by van Dam, Veltman, and Zakharov (vDVZ) has a discontinuity known as vDVZ discontinuity[24]. The addition of new fields into the massive gravity theory could remove such discontinuities. These new fields, similar to the Stueckelberg field, are required so that the number of degrees of freedom are unchanged even after taking the massless limit. These additional Stueckelberg degrees of freedom can play non-trivial role due to its gravitational interaction. This will be presented elsewhere [27].

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[1] F. Bloch and A. Nordsieck, Phys. Rev. 52 (1937) 54.
[2] D. R. Yennie, S. C. Frautschi and H. Suura, Annals Phys. 13 (1961) 379.
[3] N. Nakanishi, Prog. Theor. Phys. 19 (1958) 159.
[4] G. Leibbrandt, Rev. Mod. Phys. 47 (1975) 849.
[5] P. P. Kulish and L. D. Faddeev, Theor. Math. Phys. 4, 745 (1970).
[6] V. Chung, Phys. Rev. 140 (1965) B1110.
[7] E. Bagan, M. Lavelle, D. McMullan and S. Tanimura, Phys. Rev. D 65 (2002) 105004.
[8] D. R. Butler and C. A. Nelson, Phys. Rev. D 18, 1196 (1978).
[9] M. Greco, F. Palumbo, G. Pancheri-Srivastava and Y. Srivastava, Phys. Lett. 77B (1978) 282.
[10] H. D. Dahmen and F. Steiner, Z. Phys. C 11, 247 (1981).
[11] P. A. M. Dirac, Rev. Mod. Phys. 21 (1949) 392.
[12] A. Harindranath and J. P. Vary, Phys. Rev. D 37 (1988) 3010.
[13] J. D. More and A. Misra, Phys. Rev. D 86, 065037 (2012), Nucl. Phys. Proc. Suppl. 251-252 (2014) 68.
[14] J. D. More and A. Misra, Phys. Rev. D 87, 085035 (2013), Few Body Syst. 55 (2014) 519.
[15] J. D. More and A. Misra, Phys. Rev. D 89, no. 10, 105021 (2014), Few Body Syst. 56, no. 6-9, 551 (2015).
[16] J. L. Jacquot, Phys. Lett. B 631, 83 (2005).
[17] H. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. A 19, 3265 (2004)
[18] D. Kapec, M. Perry, A. M. Raclariu and A. Strominger, Phys. Rev. D 96, no. 8, 085002 (2017).
[19] A. Laddha and P. Mitra, JHEP 1805 (2018) 132.
[20] T. R. Govindarajan and N. Kalyanapuram, Mod. Phys. Lett. A 34 (2019) 1950009.
[21] D. Mustaki, S. Pinsky, J. Shigemitsu and K. Wilson, Phys. Rev. D 43, 3411 (1991).
[22] Tom Banks (2008), "Modern Quantum Field Theory", Cambridge University Press.
[23] A. Misra, Phys. Rev. D 50,4088 (1994).
[24] K. Hinterbichler, Rev. Mod. Phys. 84 (2012) 671.
[25] S. D. Glazek, arXiv:1811.09728 [hep-th].
[26] B. Kors and P. Nath, JHEP 0507 (2005) 069.
[27] T R Govindarajan and Nikhil Kalyanapuram (under preparation).