Throughput Optimization Based Joint Access Point Association and Transmission Time Allocation in WLANs

ZHIWEI MAO (Senior Member, IEEE)
Gildart Haase School of Computer Sciences and Engineering, Fairleigh Dickinson University, Teaneck, NJ 07666, USA
CORRESPONDING AUTHOR: Z. MAO (e-mail: zmao@fdu.edu)

ABSTRACT In this paper, the joint access point (AP) association and transmission time allocation problem in densely deployed multirate WLANs based on time sharing medium access control (MAC) is considered. In our study, the centralized coordinated joint AP association and transmission time allocation problem with heterogeneous station (STA) throughput demands is first formulated as an NP-hard single non-zero programming (SNZP) optimization problem with proportionally fair (PF) throughput based objective function. To solve the NP-hard SNZP problem, two novel algorithms, the SNZP relaxation (SNZPR) algorithm and the iterative SNZPR (iSNZPR) algorithm, are proposed. The property and complexity of the proposed algorithms are analyzed. In addition, for dynamic network scenarios where there are new STAs arriving at and trying to join the network, a distributed joint admission, AP association and transmission time allocation (DAAA) algorithm is proposed to determine whether to admit a new STA arriving at the network or not. The performance of the proposed algorithms is then investigated and compared with that of some existing algorithms based on numerical results.

INDEX TERMS Access point association, time sharing, IEEE 802.11ax, wireless local area networks, cross-layer design and optimization.

I. INTRODUCTION

THE WIDESPREAD of mobile devices and increasing demand for low-cost wireless connectivity have promoted wide deployment of Wi-Fi wireless local area network (WLAN) based on IEEE 802.11 standard [1]. A typical WLAN consists of an access point (AP) and multiple stations (STAs) wirelessly connected and associated to the AP. An AP and its associated STAs are called a basic service set (BSS). In areas with large number of users, such as airports, stadiums, large business office buildings, multi-story apartment buildings, and other hotspots, WLANs are deployed in a dense manner, with tens to hundreds of APs closely deployed in a given area with densely and unevenly distributed STAs resulting in overlapping basic service set (OBSS). In the presence of OBSS, WLANs suffer significant performance degradation due to existing contention-based medium access control (MAC) protocols [2], increased interference [3], and uncoordinated association of STAs to APs [4], [5].

Especially, the current standard-compliant uncoordinated association of STAs to APs raises the problem of balancing high traffic of densely distributed STAs among multiple APs. In current IEEE 802.11 standard, STAs use the default strongest signal first (SSF) association scheme in which STAs associate with APs providing the strongest received signal strength (RSS) in a distributed and uncoordinated manner. The SSF association scheme is easy and efficient to implement. Nevertheless, using SSF scheme, some APs may experience congestion while others are lightly loaded, causing overall network throughput and average user experience degradation [4], [5].

In literature, in order to improve the performance of dense WLANs in the presence of OBSS, cellular technologies have been proposed to be exploited for data transmission, improving spectral efficiency and controlling overall interference level [6], and a distributed interference coordination mechanism has been developed [7]. Efficient AP association is another approach for performance improvement in WLANs.
However, lots of existing researches on AP association focus on contention-based WLANs, which provide throughput fairness among STAs associated with the same AP inside the same BSS, but cannot support heterogeneous throughput QoS levels to those STAs [8]–[17].

To alleviate the effect of channel contention and reduce power consumption, the Target Wake Time (TWT) scheduled access technique is adopted in IEEE 802.11ax standard introduced recently [18], [19]. The TWT technique adopts simple but effective time sharing mechanism such that STAs can share channel resources in deterministic and pre-designated timeslots, free of collision. With low overheads, the TWT mechanism enables us to provide services for STAs with periodic data transmissions, such as those from audio/video flows, Internet of Things (IoT) applications, etc., as well as for STAs with best-effort traffic by allowing the APs to specify a time division multiple access (TDMA)-like frame structure and a transmission time for each STA within their BSS’s [20]. Therefore, the IEEE 802.11ax TWT scheduled time sharing access provides a possible implementation method to support heterogeneous throughput QoS levels to STAs associated with the same AP inside the same BSS.

It is noted that current WLANs support various types of services with quite different throughput demands, one of the QoS requirements. For example, streaming audio/video services may have different minimal and maximal throughput demands according to different audio/video quality requirements; best-effort Web-surfing service, on the other hand, may not have an explicit minimal or maximal throughput demand, etc. However, most of existing researches on AP association based on MAC capable of supporting heterogeneous throughput levels either do not consider STAs’ throughput demands at all resulting in insufficient or wasteful throughputs for STAs, or only consider special cases of throughput demands [21]–[26].

In addition, with recent progress in network management concept of software-defined network (SDN), the SDN concept, which decouples the control plane from the data plane and centralizes network management in a single controller, is also being adopted in WLAN systems [27]. Network-wide centralized control, with global network knowledge, tends to be more efficient and is believed to be able to address the challenges of high density deployment in WLAN networks [28]. This thus brings the challenges of designing effective and efficient centralized control schemes.

Motivated by the above observations, in this paper, we consider densely deployed multirate WLANs based on time sharing MAC, which can be implemented using IEEE 802.11ax TWT mechanism. We consider centralized coordinated joint AP association and transmission time allocation with heterogeneous STA throughput demands in the considered networks. The centralized coordinated joint AP association and transmission time allocation problem is first formulated as an NP-hard single non-zero programming (SNZP) optimization problem with proportionally fair (PF) throughput based objective function. Throughput-based proportional fairness is shown to provide a good trade-off between fairness in network resource allocation among STAs and overall network throughput [17], [21]–[23], [25].

To solve the NP-hard SNZP problem, we propose the novel SNZP relaxation (SNZPR) algorithm and iterative SNZPR (iSNZPR) algorithm. The property and complexity of the proposed algorithms are then analyzed. For some STA throughput demands, it is possible that not all STAs arriving at the network can be supported. In this situation, when a new STA arrives at and tries to join the network, we need to determine whether to admit this STA or not. Thus, a distributed joint admission, AP association and transmission time allocation (DAAA) algorithm is proposed for dynamic network scenarios with new STAs arriving at and trying to join the network. The performance of the proposed algorithms is then investigated and compared with that of some existing algorithms based on numerical results.

The contributions of this paper are summarized as follows.

- We formulate the centralized coordinated joint AP association and transmission time allocation problem in densely deployed multirate WLANs based on time sharing MAC as an NP-hard SNZP optimization problem, where STAs have heterogeneous throughput demands.
- We propose two novel suboptimal algorithms, the SNZPR algorithm and the iSNZPR algorithm, to solve the SNZP optimization problem.
- We analyze the property and complexity of the proposed suboptimal SNZPR and iSNZPR algorithms.
- Based on the results of property analysis, we propose a novel algorithm, the DAAA algorithm for distributed joint admission, AP association and transmission time allocation, in the scenarios where some STAs arriving at the network cannot be supported and need to be rejected.
- Based on numerical results, we investigate and compare the fairness and average throughput per-STA performance of the proposed SNZPR and iSNZPR algorithms and those of some existing algorithms in WLANs with different STA distributions and different STA throughput demands, as well as with some of the STAs in the network using the legacy distributed SSF algorithm, instead of the proposed centralized algorithms, for AP association.
- Based on numerical results, we investigate and compare the rejection ratio for arriving STAs and fairness and average throughput per-STA for admitted STAs performance of the proposed DAAA algorithm and those of the legacy SSF algorithm.

The rest of this paper is organized as follows. Related work is reviewed in Section II. In Section III, the network model used in this paper is introduced and the problem formulation for centralized coordinated joint AP association and transmission time allocation problem is presented. The proposed SNZPR and iSNZPR algorithms and their property and complexity analyses are presented in Section IV and Section V, respectively. The proposed DAAA algorithm is discussed.
in Section VI. Numerical results on the performance of the proposed algorithms are presented, compared and discussed in Section VII. Finally, conclusions are drawn in Section VIII.

II. RELATED WORK
AP association in WLAN has been extensively studied. Efficient AP association has been investigated based on content-based WLANs using the IEEE 802.11 distributed coordinated function (DCF) MAC mechanism using either a distributed approach based on local knowledge available at STAs only [8]–[10] or a centralized approach based on global network knowledge [11]–[17].

Reference [8] proposed a practical and efficient distributed AP association scheme which balances the load by considering the traffic load of each AP estimated from the observations of frame delays. Reference [9] investigated distributed AP association based on max-min throughput fairness. In [10], the AP association problem is modeled as a distributed noncooperative dynamic game with each STA trying to maximize its achievable throughput which is a function of both the number and link states of the STAs associated with the same AP.

References [11] and [12] studied centralized throughput-based max-min fairness which distributes network resources among STAs as fairly as possible, but suffers from low network throughput in multirate WLANs. Reference [13] studied the AP re-association problem based on max-min throughput fairness. Reference [14] investigated centralized throughput-based proportionally fair AP association of BSs in the same extended service set (ESS) with mutually conflicting APs. Reference [15] proposed centralized optimal AP association to minimize the highest AP load, which is estimated through channel busy time fraction. Reference [16] proposed efficient centralized AP association to enhance throughput performance by load balancing and efficient multicasting and to reduce energy consumption by AP sleeping. Reference [17] studied throughput-based AP association and found that maximizing aggregate throughput achieves high network throughput but tends to result in unfairness in network resource allocation among STAs, and throughput-based proportional fairness provides a good trade-off between fairness and overall network throughput.

However, contention-based WLANs using DCF only provide throughput fairness among STAs associated with the same AP and cannot support heterogeneous throughput QoS levels to those STAs.

In [21]–[26], AP association was investigated based on WLANs using time sharing based MAC capable of supporting heterogeneous throughput levels to STAs associated with the same AP. In [21] and [22], the problem of centralized PF throughput based AP association and transmission time allocation is formulated as an integer nonlinear programming problem and solved using centralized and distributed approximation algorithms. Reference [23] investigated a centralized PF throughput based collaborative association where STAs can associate with and obtain transmission time from multiple APs. Reference [24] proposed an online centralized AP association and channel time allocation scheme based on reinforcement learning which minimizes the STAs dissatisfaction in terms of obtained throughput. STAs’ throughput demands are not considered in the algorithms proposed in [21]–[24] and thus these algorithms cannot guarantee the QoS requirements of certain services with specific throughput demands. Reference [25] considered WLANs with STAs each of which has an upper bound throughput demand. An algorithm named maximum aggregated bandwidth utility (MABU) was proposed for the centralized PF throughput based joint AP association and bandwidth allocation problem, in which the STAs are associated with the optimal APs and then adequate bandwidth is allocated to each STA according to its throughput demand. Reference [26] considered the joint AP association and bandwidth allocation problem by maximizing the joint overall network throughput and utilisation considering STAs’ lower bound throughput demands. However, the algorithms proposed in [25] and [26] cannot guarantee the QoS of services with general heterogeneous throughput demands which falls into a certain range with both lower and upper bounds.

AP association was also investigated based on other performance metrics such as link bandwidth/capacity, spectral efficiency, etc. Based on graph theory, [27] proposed a centralized AP association algorithm based on the performance metric of fittingness factor which indicates the suitability of the available spectrum resources to the QoS requirements of STAs. In [27], the AP is chosen as the associated AP of a STA if it provides the QoS performance requested by the STA and safeguard the overall network performance. Reference [29] studied the centralized joint AP association and bandwidth allocation problem to maximize the total bandwidth considering STAs’ constant bandwidth demands with all STAs having the same data rate. Reference [30] studied the distributed joint AP grouping and association to minimize the maximal AP load in MIMO WLANs, in which intra-group interference is mitigated using interference alignment. In [30], AP load is defined based on channel utilization calculated using traffic demand and link capacity. Reference [31] designed joint channel assignment and AP association for WLANs with both centrally controlled-APs and independently operated-APs based on the performance metric of resource efficiency which considers wireless channel quality, traffic volume, and channel interference.

III. NETWORK MODEL AND PROBLEM FORMULATION
We consider the downlink of a time sharing based multirate WLAN consisting of $M$ active STAs and $N$ APs with $M \geq N \geq 1$. Each AP transmits at the same power and thus has the same average range of coverage area. An AP can associate with and serve the STAs within its coverage area. It is assumed that each STA is covered by at least one AP, i.e., each STA is able to associate with at least one AP.
In addition, STAs have heterogeneous throughput demands, each falling into a certain throughput range. Overlapping coverage areas of adjacent APs may exist. The STAs locating inside an overlapping coverage area can thus decide to associate with any one of those adjacent APs. According to IEEE 802.11 standard, the effective bit rate between an AP and a STA is determined by the link signal to interference-plus-noise ratio (SINR) between the AP and the STA according to Table 1 [22], [32], [33]. In this paper, adequate frequency planning is assumed so that interfering APs transmit using orthogonal channels to avoid co-channel interference.

Based on the network model discussed above, the centralized coordinated joint AP association and transmission time allocation problem can be formulated as the following constrained mixed integer programming (MIP) optimization problem [21], [22], [34]

\[
\begin{align*}
\text{arg max}_{\mathbf{X}, \mathbf{T}} & \quad F(\mathbf{X}, \mathbf{T}) \\
\text{subject to:} & \quad 0 \leq \sum_{i=1}^{N} x_{ij} t_{ij} \leq 1 \quad j = 1, \ldots, N \\
& \quad 0 \leq \sum_{i=1}^{M} x_{ij} t_{ij} \leq 1 \quad i = 1, \ldots, M \\
& \quad R_{\min,i} \leq \sum_{j=1}^{N} x_{ij} r_{ij} t_{ij} \leq R_{\max,i} \\
& \quad 0 \leq t_{ij} \leq 1 \quad i = 1, \ldots, M; j = 1, \ldots, N \\
& \quad \sum_{j=1}^{N} x_{ij} = 1 \quad i = 1, \ldots, M \\
& \quad x_{ij} \in \{0, 1\} \quad i = 1, \ldots, M; j = 1, \ldots, N.
\end{align*}
\]

In (1a), \( \mathbf{X} \in \mathbb{R}^{M \times N} \) and \( \mathbf{T} \in \mathbb{R}^{M \times N} \) with \( \mathbb{R}_{\geq 0} \) representing the set of non-negative real numbers. The elements of \( \mathbf{X} \) and \( \mathbf{T} \) are denoted as \( x_{ij} \) and \( t_{ij} \) \( (i = 1, \ldots, M; j = 1, \ldots, N) \), respectively.

The variables of this MIP problem are \( x_{ij} \) and \( t_{ij} \). \( x_{ij} \) is a binary variable, as shown in (1g), with \( x_{ij} = 1 \) indicating that STA \( i \) is associated with AP \( j \), while \( x_{ij} = 0 \) indicating that STA \( i \) is not associated with AP \( j \). Equation (1f) indicates that each STA is associated with one and only one AP. \( t_{ij} \) represents the transmission time from AP \( j \) to STA \( i \). Since each AP has up to unit total transmission time as indicated in (1b), it is obvious that \( t_{ij} \) should be between 0 and 1, as shown in (1e). Equation (1c) indicates that the transmission time of each STA should be between 0 and 1. Equation (1d) indicates the throughput demands of STAs with \( R_{\min,i} \) and \( R_{\max,i} \) representing the minimal and maximal throughput demands of STA \( i \) in bits over the normalized unit time, respectively. It is assumed that \( 0 \leq R_{\min,i} \leq R_{\max,i} \).

In the cases where there is no explicit maximal throughput demand, \( R_{\max,i} \) is set as positive infinity. For practical implementations, in multirate WLANs based on IEEE 802.11ax TWT time sharing MAC, for example, a STA could send its throughput demand to the centralized controller during the negotiation phase before a TWT session is initiated [20]. The controller then runs the centralized coordinated algorithms to determine AP association and transmission time allocation and other parameters for the TWT session. Practically, a STA’s throughput demand usually keeps static for some time. If a STA has an updated throughput demand, it could just re-negotiate with the controller to initiate an updated TWT session. \( r_{ij} \) denotes the transmission bit rate from AP \( j \) to STA \( i \), which is determined by the SINR of the link according to Table 1. The link SINR can be conveniently obtained by using the received signal strength indicator (RSSI) value carried in WiFi beacon signals [1]. Therefore, it is assumed that \( r_{ij} \) is known for \( i = 1, \ldots, M \) and \( j = 1, \ldots, N \).

Using different design objectives, the cost function \( F(\mathbf{X}, \mathbf{T}) \) in (1a) can be of different formats. In this paper, the proportional fairness based throughput optimization is considered [35], that is,\n
\[
F(\mathbf{X}, \mathbf{T}) = \sum_{i=1}^{M} \log \left( \sum_{j=1}^{N} x_{ij} r_{ij} t_{ij} \right).
\]

For the convenience of discussion, we define \( \mathcal{A}_i = \{ j | r_{ij} > 0 \} \) \( i = 1, \ldots, M \) as set of potential associated APs of STA \( i \) with \( |\mathcal{A}_i| \) and \( \mathcal{A}_i^k \) representing the number of elements and the \( k \)th element in it, respectively. It is noted that \( |\mathcal{A}_i| \geq 1 \). It is also noted

| SINR (dB) | 6-7 | 7.8-9 | 9-10 | 10-11 | 11-12 | 12-13 | 13-14 | 14-15 | 15-16 |
|----------|-----|------|------|-------|-------|-------|-------|-------|-------|
| Data Rate (Mbps) | 6   | 9    | 12   | 18    | 24    | 36    | 48    | 54    |       |
that

\[ Th_i = \sum_{j \in A_i} x_{ij} r_{ij} i = 1, \ldots, M \]  

(4)

is the effective throughput of STA \( i \) in bits over the normalized unit time.

In the hope of reducing complexity of the problem in (1), we propose to define \( y_{ij} = x_{ij} r_{ij} \). That is, we use \( y_{ij} \) to represent both AP association and transmission time between STA \( i \) and AP \( j \). \( y_{ij} = 0 \) indicates that STA \( i \) is not associated with AP \( j \), while \( y_{ij} \neq 0 \) indicates that STA \( i \) is associated with AP \( j \) whose value represents the transmission time from AP \( j \) to STA \( i \). Thus, the MIP problem in (1) can be equivalently re-formulated as

\[
\arg \max_{Y} f(Y) = \sum_{i=1}^{M} \log \left( \sum_{j=1}^{N} y_{ij} r_{ij} \right) 
\]

subject to:

\[
0 \leq \sum_{j=1}^{N} y_{ij} \leq 1 \quad j = 1, \ldots, N 
\]

(5a)

\[
0 \leq \sum_{i=1}^{M} y_{ij} \leq 1 \quad i = 1, \ldots, M 
\]

(5b)

\[
R_{\min, i} \leq Th_i \leq R_{\max, i} \quad i = 1, \ldots, M 
\]

(5c)

\[
y_{ij} \leq 1 \quad i = 1, \ldots, M; j = 1, \ldots, N 
\]

(5d)

\[
y_{ij} \in \{0, 1\} \quad \text{for any} \quad i = 1, \ldots, M; \quad j = 1, \ldots, N 
\]

(5e)

\[
y_{ij} = 1 \quad \text{only one} \quad i \leq 1, \ldots, M; \quad j = 1, \ldots, N, \quad \text{is nonzero (5f)}
\]

where \( Y \in \mathbb{R}_{\geq 0}^{M \times N} \) with elements denoted as \( y_{ij} \). The cost function value corresponding to the solution provides an upper bound to the original SNZP problem in (5).

The novel formulation in (5) reduces the number of variables by half compared to the original problem formulation in (1). It is noted that the two integer constraints in (1f) and (1g) are replaced by the single non-zero constraint in (5f). Thus the problem in (5) is referred to as a single non-zero programming (SNZP) problem. In the following discussion, we will use the formulation in (5).

IV. SNZPR AND iSNZPR ALGORITHMS

Note that the SNZP problem in (5) is NP-hard in nature. In principle, its solution can only be obtained by exhaustive evaluations of the optimization problem over all possible combinations of single non-zero values of \( y_{ij} \) for any STA \( i = 1, \ldots, M \). In this section, we propose two approximation algorithms, which are termed as the SNZP algorithm and the iSNZP algorithm. The details of the algorithms are described as follows.

A. SNZP ALGORITHM

In this subsection, we discuss the proposed SNZP algorithm. There are three steps in this algorithm.

1) SNZ RELAXATION

It is noted that except for the constraint in (5f), all other constraints are linear. It is also noted that the cost function is strictly concave. Therefore, if the constraint in (5f) is removed from the problem, the MIP problem in (5) can be relaxed to

\[
\arg \max_{Y} f(Y) = \sum_{i=1}^{M} \log \left( \sum_{j=1}^{N} y_{ij} r_{ij} \right) 
\]

subject to:

\[
0 \leq \sum_{j=1}^{N} y_{ij} \leq 1 \quad j = 1, \ldots, N 
\]

(7a)

\[
0 \leq \sum_{i=1}^{M} y_{ij} \leq 1 \quad i = 1, \ldots, M 
\]

(7b)

\[
R_{\min, i} \leq Th_i \leq R_{\max, i} \quad i = 1, \ldots, M 
\]

(7c)

\[
y_{ij} \leq 1 \quad i = 1, \ldots, M; j = 1, \ldots, N 
\]

(7d)

The relaxation allows STAs to associate with multiple APs temporarily. After the relaxation, the optimization problem in (7) becomes solvable in polynomial time [36]–[38]. The solution, which is denoted as \( y^*_i \), represents the transmission time offered to STA \( i \) from AP \( j \). The cost function value corresponding to the solution provides an upper bound to the original SNZP problem in (5).

2) AP ASSOCIATION DETERMINATION

For the convenience of discussion, for any STA \( i \), we denote the AP to be associated with as AP \( a_i \). Based on the values of \( y^*_i \) obtained in Section IV-A1, \( a_i \) can be determined using the longest transmission time first (LTTT) criterion, i.e., for \( i = 1, \ldots, M \),

\[
a_i = k \quad \text{for} \quad y^*_{ij, k} \geq y^*_{ij, j}, \quad \forall j \neq k.
\]

(8)

That is, STA \( i \) is associated with the AP which offers the longest transmission time to it. The basic idea of the proposed AP association rule is that the longer the transmission time a STA obtains from an AP, the more likely the STA should associate with that AP exclusively. In other words, the transmission time offered from AP \( j \) to STA \( i \) can be understood as level of confidence for STA \( i \) to associate with AP \( j \).

It is noted that for any AP, there may be multiple STAs associate with it. Thus, we denote \( S_j \) as set of the STAs associated with AP \( j \).

3) TRANSMISSION TIME ALLOCATION

Using the AP association results from Section IV-A2, the problem in (7) can be size-reduced, resulting in a problem with \( M \) variables \( y_{i,a_i} \) (\( i = 1, \ldots, M \)) and solvable in polynomial time, which is given as

\[
\arg \max_{Y} f(Y) = \sum_{i=1}^{M} \log(y_{i,a_i r_{i,a_i}})
\]

(9a)
subject to: \[0 \leq \sum_{i \in S_j} y_{i,a_i} \leq 1 \quad a_i \in \{1, \ldots, N\}\] (9b)
\[0 \leq y_{i,a_i} \leq 1 \quad i = 1, \ldots, M\] (9c)
\[R_{\min,i} \leq y_{i,a_i}R_{i,a_i} \leq R_{\max,i} \quad i = 1, \ldots, M\] (9d)
\[y_{i,j} = 0 \quad i = 1, \ldots, M; j \neq a_i\] (9e)

Denoting the solution to the problem in (9) as \(y_{i,a_i}^*\), the effective throughput of STA \(i\) is therefore
\[T_h^* = y_{i,a_i}^*R_{i,a_i} \quad i = 1, \ldots, M.\] (10)

It is noted that the problem in (9) can be equivalently separated as up to \(N\) sub-problems of single AP transmission time allocation as shown below
\[
\arg \max_{y_i} f(y_i) = \sum_{i \in S_j} \log(y_{i,j}R_{i,j}) \quad \text{subject to: } 0 \leq \sum_{i \in S_j} y_{i,j} \leq 1 \quad j \in S_j \quad y_{i,j} = 0 \quad i = 1, \ldots, M; j \neq a_i (11a)
\]
\[0 \leq y_{i,j} \leq 1 \quad \forall i \in S_j \quad (11b)
\]
\[R_{\min,i} \leq y_{i,j}R_{i,j} \leq R_{\max,i} \quad \forall i \in S_j \quad (11c)
\]
\[y_{i,j} = 0 \quad \forall i \notin S_j, \quad y_{i,j} \neq 0 \quad (11d)
\]

which may be solved more efficiently. In (11), we have \(j = 1, \ldots, N\).

B. ISNZPR ALGORITHM

In this subsection, we discuss the proposed iSNZPR algorithm. In this proposed iterative approach, the AP associations of STAs are determined in a sequential manner. That is, those STAs whose AP associations can be determined with higher level of confidence determine their AP associations first, which helps to determine the AP associations of other STAs. This procedure is repeated until all STAs determine their AP associations. The proposed iSNZPR algorithm tries to improve the performance of the SNZPR algorithm with the help of determined AP associations obtained in previous iterations with higher level of confidence.

Denoting \(\Omega_t\) as the set of indices of the STAs whose AP associations have been determined before the \(t\)th iteration, for \(i \in \Omega_t\), we have
\[y_{i,j} = \begin{cases} 0 \text{ for } j = a_i \\ \geq 0 \text{ for } j \neq a_i \end{cases} \quad (12)\]

Denote \(Y^t \in \mathbb{R}_{\geq 0}^{M \times N}\) with elements \(y_{i,j}^t\) \((i = 1, \ldots, M; j = 1, \ldots, N)\). Due to the fact that the AP associations of STA \(i\) for \(i \in \Omega_t\) are known, the problem in (7) can be size-reduced to
\[
\arg \max_{Y^t} f(Y^t) = \sum_{i=1}^{M} \log\left(\sum_{j=1}^{N} y_{i,j}^tR_{i,j}\right) \quad \text{subject to: } 0 \leq \sum_{i=1}^{M} y_{i,j}^t \leq 1 \quad j = 1, \ldots, N \quad (13a)
\]
\[
0 \leq \sum_{j=1}^{N} y_{i,j}^t \leq 1 \quad i \notin \Omega_t \quad (13b)
\]
\[R_{\min,i} \leq y_{i,j}^tR_{i,j} \leq R_{\max,i} \quad i \notin \Omega_t \quad (13c)
\]
\[y_{i,j}^t = 0 \quad i \in \Omega_t, j \neq a_i \quad (13d)
\]

where
\[Th^* = \left\{ \begin{array}{ll}
\sum_{j=1}^{N} y_{i,j}^tR_{i,j} & \text{for } i \in \Omega_t \\
\sum_{j=1}^{N} y_{i,j}^tR_{i,j} & \text{for } i \notin \Omega_t
\end{array} \right. \] (14)

which is normalized version of \(y''_{i,j}\) and believed to be a more accurate representation of the relative level of confidence for a STA to associate with an AP. If STA \(i_0 \notin \Omega_t\) satisfies the following condition
\[\max_{j \in A_i} \sum_{j \in A_i} y''_{i,j} \geq \xi_t \quad i_0 \notin \Omega_t \quad (16)\]

where \(\xi_t\) is a predetermined threshold, then the AP association of STA \(i_0\) is determined following the rule in (8), which is given as
\[a_{i_0} = k \quad \text{for } y''_{i_0,k} \geq \xi_t, \quad \forall j \neq k. \quad (17)\]

That is, in each iteration, AP associations are determined only for the STAs that can be associated with APs with high enough level of confidence. Thus, \(\Omega_t\) is expanded to \(\Omega_{t+1}\) by adding the STAs whose AP associations are determined in the \(t\)th iteration. Then the determined AP associations are fixed in the following iterations. By following this procedure, new size-reduced problems can be formulated sequentially until all STAs determine their AP associations.

Once all STAs’ AP associations are determined, the transmission time of STAs can be obtained by solving (9) and thus the effective throughput of STAs can be found using (10).

The proposed iSNZPR algorithm is summarized in Table 2. Note that the magnitude of the threshold used in Step 3 of the algorithm directly affects the trade-off between performance and computational complexity of the iterative algorithm. In our proposed algorithm, we have
\[\xi_t = \beta \cdot \max_{i \notin \Omega_t, j \in A_i} \sum_{j \in A_i} y''_{i,j} \quad (18)\]

where \(0 < \beta < 1\). Therefore, in each iteration, at least one STA will determine its AP association. Consequently, all STAs’ AP associations can be determined in \(T\) iterations, which is less than or at most equal to \(M\), i.e., \(T \leq M\).

In the size-reduced subproblem (13), the cost function \(f(Y^t)\) is strictly concave and all constraints are linear. So
in the $t$th iteration, the solution to the subproblem $Y''$ is always the unique global maxima of the subproblem. Therefore, the iSNZPR algorithm always finds a unique suboptimal solution to the original NP-hard SNZP problem. The following proposition describes some properties of the subproblem (13).

**Proposition 1:** If the feasible region of the subproblem (13) is $G'$, then $G'' = G'$ ($t = 0, \ldots, T - 1$). Thus, the solutions to the subproblem in (13) provide monotonously non-increasing cost function values as $t$ increases, i.e., $f(Y^{(t+1)}) \leq f(Y^{(t)})$ ($t = 0, \ldots, T - 1$).

\[ Y_{i,j}^2 = \arg \max_{i,j} \; p_{i,j}^2; \quad \text{update the value of the threshold } \xi_t \text{ following (18);} \quad \text{if } ST \; i_0 \; \notin \; \Omega_t \; \text{satisfies } \max_{j \in A_{i_0}} \; p_{i_0,j}^2 \geq \xi_t; \quad \text{then determine the AP association for } ST \; i_0 \; \text{following the rule in (17).} \]

**Step 4.** Update $\Omega_{t+1}$ as the set which includes all STAs in $\Omega_t$ and those STAs whose AP associations are determined in Step 3; Update $\tau_{t+1}$; If $\tau_{t+1} = M$: go to Step 5.

**Step 5.** Set $t = t + 1$ and repeat from Step 1.

**Step 6.** Solve the problem in (9) and denote the solution as $y_{i,A,t}^*$.

Calculate the effective throughput of STA $i$, $Th_i$, following (10); Output $y_{i,A,t}^*$ and $Th_i$ ($i = 1, \ldots, M$).

\[ \text{Output } y_{i,A,t}^* \text{ and } Th_i \quad (i = 1, \ldots, M). \]

V. PROPERTY AND COMPLEXITY ANALYSES

A. PROPERTY ANALYSIS

**Proposition 2:** Denote the optimal solution to the SNZPR problem in (7) as $y^*_{i,j}$. The STAs are divided into three groups $S_A$, $S_B$ and $S_C$ according to the criteria as below. For $i = 1, \ldots, M$,

\[ i \in S_A \text{ if } R_{\min,i} < \sum_{j=1}^{N} y^*_{i,j} r_{i,j} < R_{\max,i} \quad (19) \]

\[ i \in S_B \text{ if } \sum_{j=1}^{N} y^*_{i,j} r_{i,j} = R_{\max,i} \quad (20) \]

\[ i \in S_C \text{ if } \sum_{j=1}^{N} y^*_{i,j} r_{i,j} = R_{\min,i} \quad (21) \]

If $0 < y^*_{i,j} < 1$, for STAs in group $S_A$ and $S_C$, we always have $\sum_{j=1}^{N} y^*_{i,j} = 1$ and/or $\sum_{j=1}^{N} y^*_{i,j} = 1$; for STAs in group $S_B$, we may have $\sum_{j=1}^{N} y^*_{i,j} < 1$ and $\sum_{j=1}^{N} y^*_{i,j} < 1$, simultaneously.

**Proof:** Considering the SNZPR problem in (7), we define the Lagrangian as

\[ L(Y, u, v, w, \lambda, \alpha, \beta, \Gamma, \Psi) \]

\[ = - \sum_{i=1}^{M} \log \left( \sum_{j=1}^{N} y_{i,j} r_{i,j} \right) \]

\[ + \sum_{i=1}^{N} u_i \left( \sum_{j=1}^{N} y_{i,j} - 1 \right) - \sum_{i=1}^{N} v_i \sum_{j=1}^{N} y_{i,j} \]

\[ + \sum_{i=1}^{N} w_i \left( \sum_{j=1}^{N} y_{i,j} - 1 \right) - \sum_{i=1}^{N} \beta_i \sum_{j=1}^{N} y_{i,j} \]

\[ + \sum_{i=1}^{N} \alpha_i \left( \sum_{j=1}^{N} y_{i,j} r_{i,j} - R_{\max,i} \right) \]

\[ + \sum_{i=1}^{N} \beta_i \left( R_{\min,i} - \sum_{j=1}^{N} y_{i,j} r_{i,j} \right) \]

\[ + \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{i,j} (y_{i,j} - 1) - \sum_{i=1}^{N} \sum_{j=1}^{N} \psi_{i,j} y_{i,j} \quad (22) \]

According to the KKT conditions [37], [38], the optimal solutions must satisfy

\[ \sum_{j=1}^{N} y_{i,j} r_{i,j} + u_i^* - v_i^* + w_i^* - \lambda_i^* \]

\[ + \alpha_i^* r_{i,j} - \beta_i^* r_{i,j} + y_{i,j}^* - \psi_{i,j}^* \geq 0 \quad \forall i, \forall j \quad (23) \]

\[ u_i^* \left( \sum_{j=1}^{N} y_{i,j} - 1 \right) = 0 \quad \forall j \quad (24) \]

\[ w_i^* \left( \sum_{j=1}^{N} y_{i,j} - 1 \right) = 0 \quad \forall i \quad (25) \]

\[ \lambda_i^* \sum_{j=1}^{N} y_{i,j}^* = 0 \quad \forall i \quad (27) \]

\[ \alpha_i^* \left( \sum_{j=1}^{N} y_{i,j} r_{i,j} - R_{\max,i} \right) = 0 \quad \forall i \quad (28) \]

\[ \beta_i^* \left( R_{\min,i} - \sum_{j=1}^{N} y_{i,j} r_{i,j} \right) = 0 \quad \forall i \quad (29) \]

\[ y_{i,j}^* (y_{i,j} - 1) = 0 \quad \forall i, \forall j \quad (30) \]

\[ \psi_{i,j}^* y_{i,j} = 0 \quad \forall i, \forall j \quad (31) \]

\[ u_i^*, v_i^*, w_i^*, \lambda_i^*, \alpha_i^*, \beta_i^*, y_{i,j}^*, \psi_{i,j}^* \geq 0 \quad \forall i, \forall j \quad (32) \]
For the following analyses, it is assumed that $r_{i,j} > 0$ and $0 < y^*_{i,j} < 1$. In practical situations, if some $r_{i,j}$ are zero, all the analyses are the same except that $j \in A_i$ with $A_i$ defined in (3), but not $j = 1, \ldots, N$. From (30) and (31), we have $y^*_{i,j} = 0$ and $\psi^*_{i,j} = 0$. By applying these two results and those in (24) to (29), from (23), we have

$$u^*_j \sum_{j=1}^N y^*_{j,i} + w^*_j \sum_{j=1}^N y^*_{j,i} + \sum_{j=1}^N \gamma^*_j - r_{i,j} (\alpha^*_i R_{\text{max},i} - \beta^*_i R_{\text{min},i}) = 0. \quad (33)$$

For STAs in group $S_A$, due to (19), (28) and (29), we have $\alpha^*_i = 0$ and $\beta^*_i = 0$. Thus, from (33), we can not have $u^*_j = 0$ and $w^*_j = 0$, simultaneously. Therefore, from (24) and (26), we always have $\sum_{j=1}^N y^*_{j,i} = 1$ and/or $\sum_{j=1}^N \gamma^*_j = 1$.

For STAs in group $S_C$, due to (21) and (28), we have $\alpha^*_i = 0$. Thus, from (33) and from the fact that $\beta^*_i \geq 0$, we also can not have $u^*_j = 0$ and $w^*_j = 0$, simultaneously. Therefore, similar to the case of STAs in group $S_A$, we always have $\sum_{j=1}^N y^*_{j,i} = 1$ and/or $\sum_{j=1}^N \gamma^*_j = 1$.

For STAs in group $S_B$, due to (20) and (29), we have $\beta^*_i = 0$. Thus, from (33) and from the fact that $\alpha^*_i \geq 0$, we may have $u^*_j = 0$ and $w^*_j = 0$, simultaneously. Therefore, we may have $\sum_{j=1}^N y^*_{j,i} < 1$ and $\sum_{j=1}^N \gamma^*_j < 1$, simultaneously.

It is noted that the above analyses are based on the general problem formulation in (7). Therefore, the conclusions obtained are applicable to the special cases of (7), including the size-reduced subproblem in (13) and the transmission time allocation program in (9) after all STAs’ AP associations have been determined. By applying Proposition 2, we obtain the following theorems.

**Theorem 1:** Suppose that all STAs’ AP associations have been determined and an AP is associated with only one STA. If the AP is OFF for some time during the downlink transmission time considered, the associated STA must belong to group $S_B$. On the other hand, if the associated STA belongs to group $S_B$, the AP can be either OFF for some time or always ON during the downlink transmission time considered; if the associated STA belongs to group $S_A$ or $S_C$, the AP must be always ON during the downlink transmission time considered.

**Proof:** Since all STAs’ AP associations have been determined, we need to solve (9) to find out $y^*_{i,a_i}$ ($i = 1, \ldots, M$). In the following of this proof, we consider STA $i$ and the associated AP $a_i$, which is associated with STA $i$ only. Thus, we have $\sum_{j=1}^N y^*_{j,i} = y^*_{i,a_i}$ and $\sum_{j=1}^N \gamma^*_j = y^*_{i,a_i}$. Obviously, either $0 < y^*_{i,a_i} < 1$ or $y^*_{i,a_i} = 1$.

If AP $a_i$ is OFF for some time during the downlink transmission time considered, i.e., $0 < y^*_{i,a_i} < 1$, according to Proposition 2, STA $i$ must belong to group $S_B$ because neither $\sum_{j=1}^N y^*_{j,i} = y^*_{i,a_i} \neq 1$ nor $\sum_{j=1}^N \gamma^*_j = y^*_{i,a_i} \neq 1$ and thus it cannot belong to group $S_A$ or $S_C$. In other words, if STA $i$ belongs to group $S_A$ or $S_C$, then AP $a_i$ must be always ON during the downlink transmission time considered.

If STA $i$ belongs to group $S_B$, then $0 < y^*_{i,a_i} = R_{\text{max},i}/r_{i,a_i} \leq 1$. If $r_{i,a_i} > R_{\text{max},i}$, then $0 < y^*_{i,a_i} < 1$ and AP $a_i$ is OFF for some time during the downlink transmission time considered. If $r_{i,a_i} = R_{\text{max},i}$, then $y^*_{i,a_i} = 1$ and AP $a_i$ is always ON during the downlink transmission time considered.

**Theorem 2:** Suppose that all STAs’ AP associations have been determined and an AP is associated with more than one STA. Only if all of the associated STAs belong to group $S_B$, the AP may be OFF for some time during the downlink transmission time considered; if even one of the associated STA belongs to group $S_A$ or $S_C$, then the AP is always ON during the downlink transmission time considered.

**Proof:** Based on the proof of Theorem 1, since all STAs’ AP associations have been determined, $\sum_{j=1}^N y^*_{j,i}$ and $\sum_{j=1}^N \gamma^*_j$ in Proposition 2 becomes $y^*_{i,a_i}$ and $y^*_{i,a_i}$, respectively. If AP $a_i$ is associated with more than one STA, then $0 < y_{i,a_i} < 1$. In the following of this proof, we consider STA $i$ and AP $a_i$.

According to Proposition 2, if $i \in S_A$ or $i \in S_C$, then $\sum_{j=1}^N y^*_{j,i} = 1$ must be satisfied because $\sum_{j=1}^N y^*_{j,i} \neq 1$. In other words, AP $a_i$ is always ON during the downlink transmission time considered, transmitting to the STAs associated with it.

If $i \in S_B$, on the other hand, according to Proposition 2, then we may have $\sum_{j=1}^N y^*_{j,i} < 1$, or AP $a_i$ may be OFF for some time during the downlink transmission time considered.

Using the similar analyses as in [25], it can be shown that if all STAs’ AP associations have been determined and an AP is associated with more than one STA, all the STAs associated with this AP that belong to group $S_A$ should be allocated the same transmission time.

**B. COMPLEXITY ANALYSIS**

It is known that the computational complexity in solving a continuous constrained nonlinear logarithmic concave maximization problem using the interior-point algorithm is approximately upper bounded by the product of the number of variables and the number of inequality constraints of the problem [39].

To solve the joint AP association and transmission time allocation problem discussed in this paper, for the optimal exhaustive search, we need to solve up to $N^M$ optimization problems in (9), each of which has $M$ variables. Thus the computational complexity of the exhaustive search is $O(N^M, M^2)$. For the proposed SNZPR algorithm, we need to solve one optimization problem in (7) with up to $MN$ variables and another optimization problems in (9) with $M$ variables. Thus the complexity of the proposed SNZPR algorithm is $O(M^2N)$.

For the iSNZPR algorithm, in the worst case, only one additional STA’s AP association is determined in each iteration. That is, we need $M$ iterations to determine the AP associations for all the $M$ STAs in the network. In the $r$th ($r = 0, \ldots, M - 1$) iteration, we need to solve the
optimization problem in (13) with up to \((M-t)N+t\) variables and thus the complexity of the \(t\)th iteration is
\[ C_t = \mathcal{O}((M-t)MN) \quad t = 0, \ldots, M - 1. \] (34)
After all STAs’ AP association are determined, another optimization problems in (9) with \(M\) variables is needed to be solved to calculate each STA’s allocated time. Thus the complexity of this step is
\[ C_M = \mathcal{O}(M^2). \] (35)
Therefore, the complexity of the worst case iSNZPR algorithm is
\[ C = \sum_{t=0}^{M} C_t = \sum_{t=0}^{M-1} \mathcal{O}((M-t)MN) + \mathcal{O}(M^2) \]
\[ = \mathcal{O}((M-t)MN). \] (36)
Note that based on (34) and (35), we have
\[ \exists c_t : \forall M_0 , N > M_0 : c_t \leq c_t (M-t)MN. \] (37)
Denoting \(c_{max} = \max_c \{ c_t : \forall t = 0, \ldots, M - 1\}\), then
\[ \exists c_t, c_{max} \exists M_0 : \forall M, N > M_0 : \]
\[ C = \sum_{t=0}^{M-1} C_t \leq \sum_{t=0}^{M-1} c_t (M-t)MN \]
\[ \leq c_{max} \left( \frac{M^3 N}{2} + \frac{M^3 N}{2} \right). \] (38)
From (38), we obtain
\[ C = \mathcal{O}(M^3 N). \] (39)
That is, the worst case complexity of the proposed iSNZPR algorithm is \(\mathcal{O}(M^3 N)\).

VI. DAAA ALGORITHM
Due to the existence of nonzero \(R_{\min,i} (i = 1, \ldots, M)\), it is possible that not all STAs arriving at the network can be supported. In this situation, when a new STA arrives at and tries to join the network, we need to determine whether to admit this STA or not. In this section, the DAAA algorithm is proposed for dynamic network scenarios, where there are new STAs trying to join the network.

It is assume that any STA in the network has only local information. It is also assumed that once a STA determines which AP to associate with when it joins the network, it will keep the same AP association during the whole period of its communication.

Based on the results of property analysis from Section V-A on the characteristics of AP transmission time allocation among STAs associated with the same AP, we propose a novel admission and AP transmission time allocation scheme, based on which the DAAA algorithm is proposed. The DAAA algorithm is summarized in Table 3.

TABLE 3. DAAA algorithm.

| Step | Description |
|------|-------------|
| 1.   | Find \(A_{i_0}\). |
| 2a.  | Find \(f(y_{j}) = \sum_{i \in \mathcal{S}_j} \log(y_{i,j} r_{i,j})\) for STAs currently associated with AP \(j\). |
| 2b.  | Find \(y_{\min,i,j} = \frac{R_{\min,i}}{r_{i,j}}\) and \(y_{\max,i,j} = \frac{R_{\max,i}}{r_{i,j}}\) for all \(i \in \mathcal{S}_j \cup \{i_0\}\). |
| 2c.  | If \(\sum_{i \in \mathcal{S}_j \cup \{i_0\}} y_{\min,i,j} > 1\), reject STA \(i_0\) at AP \(j\). |
| 2d.  | Otherwise, if \(\sum_{i \in \mathcal{S}_j \cup \{i_0\}} y_{\min,i,j} = 1\), let \(y_{i,j} = y_{\min,i,j}\) for all \(i \in \mathcal{S}_j \cup \{i_0\}\). |
| 2e.  | Otherwise, if \(\sum_{i \in \mathcal{S}_j \cup \{i_0\}} y_{\max,i,j} \leq 1\), let \(y_{i,j} = y_{\max,i,j}\) for all \(i \in \mathcal{S}_j \cup \{i_0\}\). |
| 2f.  | Otherwise, get \(y_{i,j}\) by solving (11) for STAs currently associated with AP \(j\) plus STA \(i_0\). |
| 3.   | If STA \(i_0\) is rejected by all APs \(j \in A_{i_0}\) : |
|      | Admit STA \(i_0\); |
|      | Calculate \(f(y_{j}) = \sum_{i \in \mathcal{S}_j \cup \{i_0\}} \log(y_{i,j} r_{i,j})\) for all AP \(j \in A_{i_0}\) not rejecting STA \(i_0\); |
|      | Find \(j_0 = \arg\ max f(y_{j})\) - \(f(y_{i_0})\); |
|      | Associate STA \(i_0\) with AP \(j_0\) by updating \(S_{\mathcal{J}_0}\) as \(S_{\mathcal{J}_0} \cup \{i_0\}\) and allocate the transmission time of AP \(j_0\) to the associated STAs according to \(y_{j_0}\). |

DAAA algorithm, when a new STA arrives at the network and tries to join, the STA first tries to find an AP that is visible to itself and can support it, without affecting the communication of the current associated STAs. If there is no such AP existing, the new STA is then rejected from admission. If there are multiple APs that can support the new STA, the new STA will join the one which provides the highest improvement in the cost function value in (11a). In addition, the transmission time of the associated AP is re-allocated among the new STA and the other STAs previously associated with this AP by solving the single AP transmission time allocation problem in (11).

Based on the above discussions, note that when there is a STA leaving the network, we only need to re-allocate the transmission time of the AP with which the leaving STA was associated by solving the single AP transmission time allocation problem in (11) for the remaining STAs still associated with that AP.

When the \(m\)th \((m = 1, \ldots, M)\) STA tries to join the network, we need to solve up to \(N\) optimization problems in (11), each of which has up to \(m\) variables. Therefore, if there are \(M\) STAs arriving at the network and trying to join, the complexity of the proposed DAAA algorithm is shown to be \(\mathcal{O}(M^3 N)\).

VII. NUMERICAL RESULTS
We consider two performance metrics in our discussion, which are average throughput per-STA and Jain’s Fairness
The wireless channel has a path loss exponent of $4$.

Shadowing with $0\text{ dB}$ mean and $10\text{ dB}$ standard deviation.

The wireless channel has a path loss exponent of $4$. Similar to the locations of STAs, the log-normal shadowing of the wireless channels between any pair of AP and STA is randomly generated and changed in different network realizations. The locations of STAs are randomly chosen within the service area considered and changed in different network realizations.

We consider WLANs with two different STA layout scenarios. In the first so-called uniform scenario or network with uniformly distributed STAs, all STAs are uniformly distributed over the service area, while in the second so-called hotspot scenario or network with hotspot distributed STAs, all STAs are clustered inside a circular subarea with center at the service area center and radius of 100 meters.

The wireless channel is assumed undergoing log-normal shadowing with $0\text{ dB}$ mean and $10\text{ dB}$ standard deviation. The wireless channel has a path loss exponent of $4$. Similar to the locations of STAs, the log-normal shadowing of the wireless channels between any pair of AP and STA is randomly generated and changed in different network realizations. The AP transmission power is $100\text{ mW}$ and the background noise power is $-80\text{ dBm}$.

In our simulations, $R_{\text{max}}$ are randomly selected between 4 and 5 Mbps, 10 and 20 Mbps, and 40 and 50 Mbps following a uniform distribution for one third of the STAs in the network, respectively, corresponding to the three STA categories with low, medium, and high throughput demands. For the proposed iSNZPR algorithm, $\beta$ in (18) is set as 0.98.

For the convenience of comparison, for the distributed algorithms including the DAAA and SSF algorithms, it is assumed that the STAs arrive at the network one after another and their admission, AP association and transmission time allocation are determined sequentially, in the same order as they arrive at the network. Once a STA determines which AP to associate with when it joins the network, it will keep the same AP association during the whole period of its communication. In addition, the transmission time of the newly associated AP is re-allocated among the new STA and the other STAs previously associated with this AP by solving the single AP transmission time allocation problem in (11). The performance of the DAAA and SSF algorithms presented in this section is that of the network after all STAs in the network have arrived and all APs’ transmission time has been allocated following the above distributed method. The performance of the centralized coordinated algorithms presented, on the other hand, is that of the network after all STAs are admitted into the network and their AP association and transmission time allocation are calculated in a centralized manner.

### B. RESULTS OF SMALL-SCALE WLAN

In this subsection, we consider a relatively small-scale WLAN with $3 \times 2$ grid and different number of STAs ranging from 18 to 60.

1) $R_{\text{MIN}} = 0$

In Figs. 1 to 4, we compare the performance of the proposed SNZPR, iSNZPR, and DAAA algorithms in the WLAN with uniformly and hotspot distributed STAs and $R_{\text{MIN}} = 0$ for all STAs. As comparison, the performance of the legacy distributed SSF algorithm, the non-linear approximation optimization (NLAO) algorithm in [22], and the MABU algorithm in [25] is also included. Note that the original NLAO algorithm does not consider throughput demands of STAs. Therefore, the algorithm is revised by adding the constraints in (1d) to fit into the problem we consider in this paper. It is also noted that due to the zero $R_{\text{MIN}}$ values for all STAs, all the STAs can be admitted and supported by the network, and thus no STA is rejected from the network.

From Figs. 1 and 3, it can be seen that in networks with uniformly distributed STAs, as the number of STAs increases, the fairness tends to increase for all the other algorithms except the DAAA algorithm. In networks with hotspot distributed STAs, as the number of STAs increases, the fairness increases for the proposed iSNZPR algorithm and the MABU algorithm, but the fairness stays rather stable for the proposed SNZPR algorithm and the NLAO algorithm and tends to decrease for the SSF and the DAAA algorithms. In networks with both uniformly and hotspot distributed STAs,
the DAAA algorithm provides the worst fairness, and the SNZPR and the NLAO algorithms provide almost the same fairness which is better than that provided by the SSF algorithm in almost all cases but worse than that provided by the iSNZPR algorithm. The iSNZPR algorithm provides the best fairness in networks with uniformly distributed STAs and the MABU algorithm provides the best fairness in networks with hotspot distributed STAs.

From Figs. 2 and 4, it can be seen that in networks with both uniformly and hotspot distributed STAs, as the number of STAs increases, the average throughput per-STA decreases for all algorithms. It can also be seen that in both networks, the DAAA algorithm has a much lower average throughput per-STA than all the other algorithms. Except the DAAA algorithm, for the network with uniformly distributed STAs, the MABU algorithm consistently has a lower average throughput per-STA than the other algorithms. The

SNZPR, the iSNZPR and the NLAO algorithm provide similar average throughput per-STA, which is higher than that provided by the SSF algorithm when the number of STAs in the network is relatively small. All these four algorithms provide similar average throughput per-STA when the number of STAs is relatively large. For the network with hotspot distributed STAs, except the DAAA algorithm, the average throughputs per-STA provided by the SSF algorithm and the iSNZPR algorithm are the lowest and the highest, respectively. The SNZPR and the NLAO algorithm provide similar average throughput per-STA, which is lower than and almost the same as that provided by the iSNZPR algorithm when the number of STAs in the network is relatively small and relatively large, respectively. The average throughput per-STA provided by the MABU algorithm is consistently between the highest and the lowest. Overall, the proposed iSNZPR algorithm provides the best performance of fairness and average throughput per-STA in both uniformly and hotspot distributed STA scenarios. The proposed DAAA algorithm does not work very well in this case.

2) $R_{\text{MIN}} = 1$ Mbps

In Figs. 5 to 8, we compare the performance of different algorithms in the WLAN with $R_{\text{MIN}} = 1$ Mbps for all STAs. Note that the MABU algorithm is not applicable in this network scenario, thus not included in these figures. It is also noted that due to the low $R_{\text{MIN}}$ values for all STAs, all the STAs can be admitted and supported by the network, and thus no STA is rejected from the network.

From Figs. 5 to 8, it is seen that in networks with both uniformly and hotspot distributed STAs, the performance of the proposed SNZPR and iSNZPR algorithms and that of the SSF and the NLAO algorithms are quite similar to those shown in Figs. 1 to 4, with slight improvements in both fairness and average throughput per-STA in most cases. It is also seen that the DAAA algorithm achieves dramatic
improvements in both the fairness and the average throughput per-STA performance in Figs. 5 to 8 compared to those shown in Figs. 1 to 4. Overall, the proposed iSNZPR algorithm again provides the best performance of fairness and average throughput per-STA in this network scenario.

3) $R_{\text{MIN}} = 1$ MBPS WITH ADVERSARY STAS

It is noted that in order to implement the proposed centralized coordinated joint AP association and transmission time allocation scheme, STAs or the software/firmware of STAs need to be upgraded accordingly. However, practically, it is almost impossible to finish the upgrades on all STAs at the same time. Thus, most likely, in practical WLANs adopting the proposed scheme, before the upgrades are completely finished, there are both STAs using the proposed centralized coordinated joint AP association and transmission time allocation scheme and STAs using the legacy SSF AP association scheme. In such a hybrid network, those STAs using the legacy SSF AP association scheme determine their AP associations in a distributed and uncoordinated manner, ignoring network-wide information, and thus tend to bring negative effect to performance of the overall WLANs. Therefore, they are referred to as adversary STAs in the following discussion of this subsection.

In Figs. 9 to 12, we consider the WLAN with adversary STAs and $R_{\text{MIN}} = 1$ Mbps for all STAs. Specifically, it is assumed that the legacy SSF algorithm is used for one third of the STAs for AP association. The AP associations and transmission time allocation of the rest of the STAs are determined by other different algorithms.

Compared to the results in Figs. 5 to 8, it is seen that with adversary STAs, the DAAA shows the most significant performance degradation in both fairness and average throughput per-STA, followed by the iSNZPR algorithm, especially when the number of STAs in the network is relatively small. The other algorithms are more robust to the
negative effect of adversary STAs. The proposed SNZPR algorithm always has almost the same performance as that of the NLAO algorithm. Overall, the proposed iSNZPR algorithm still provides the best performance in both uniformly and hotspot distributed STA cases with adversary STAs.

4) DIFFERENT $R_{\text{MIN}}$ VALUES FOR DIFFERENT STA CATEGORIES

In this subsection, $R_{\text{min}}$ values are set as 1, 2.5, and 10 Mbps for the STA categories with low, medium, and high throughput demands, respectively. It is noted that due to the relatively high $R_{\text{min}}$ values for STAs, the network may not be able to support all the STAs, and thus some STAs may be rejected from admission when trying to join the network. We investigate WLANs with different number of STAs arriving at the network ranging from 18 to 60.

In Fig. 13, the rejection ratio of arriving STAs in the WLAN with uniformly and hotspot distributed STAs by using the proposed DAAA algorithm is shown. In the figure, the number of STAs represents the total number of STAs sequentially arriving at and trying to join the network. The rejection ratio is defined as the ratio between the average number of STAs being rejected and the total number of arriving STAs. As comparison, the rejection ratio of the legacy distributed SSF algorithm is also included. It is seen from Fig. 13 that in general, the DAAA algorithm provides a lower rejection ratio than the SSF algorithm in the network with both uniformly and hotspot distributed STAs. In addition, for both algorithms, the network with hotspot distributed STAs generally has a lower rejection ratio than the network with uniformly distributed STAs.

In Figs. 14 and 15, we compare the performance of fairness and average throughput per-STA for admitted STAs. In these two figures, the number of STAs represents the total number of STAs arriving at the network. It can be seen that in the WLAN with uniformly distributed STAs,
compared with the SSF algorithm, the DAAA algorithm generally provides better fairness performance, at the expense of slightly lower average throughput per-STA. It can also be seen that in the WLAN with hotspot distributed STAs, compared with the SSF algorithm, the DAAA algorithm provides better performance in fairness and it provides higher average throughput per-STA when the number of arriving STAs is relatively small and slightly lower average throughput per-STA when the number of arriving STAs is relatively large.

Based on the results presented in this subsection, it is seen that it could be an effective approach to use the DAAA algorithm to determine admission for STAs arriving at the network, then to use the iSNZPR algorithm to determine AP association and transmission time allocation for admitted STAs to increase the overall performance of the network, as evidenced by the results in Figs. 14 and 15, where this approach is labeled as DAAA-iSNZPR. This approach may require periodic re-association for some of the admitted STAs in the network, though.

C. RESULTS OF LARGE-SCALE WLAN
We also consider a relatively large-scale WLAN with $5 \times 4$ grid and different number of STAs ranging from 40 to 160. Numerical results show that similar performance can be obtained in the large-scale WLAN as in the small-scale WLAN. As examples, Figs. 16 to 18 show the performance of different algorithms in the WLAN with $R_{\text{min}} = 1, 2.5, 10$ Mbps for the STA categories with low, medium, and high throughput demands, respectively. It can be seen that in the large-scale WLAN, the proposed DAAA algorithm provides even lower rejection ratio for arriving STAs and even better fairness for admitted STAs than the SSF algorithm, especially in the WLAN with hotspot distributed STAs. The DAAA-iSNZPR approach is shown to always provide the highest
Both small-scale and large-scale WLANs show similar performance. Numerical results have been presented to compare the fairness and average throughput per-STA performance of the proposed algorithms and those of some existing algorithms in WLANs with different STA distributions and different STA throughput demands, as well as with adversary STAs using the legacy SSF algorithm for AP association. Overall, the proposed iSNZPR algorithm provides the best performance in all the scenarios considered, although the proposed SNZPR algorithm is more robust to the negative impact from adversary STAs.

Numerical results have also been presented to investigate and compare the rejection ratio for arriving STAs and fairness and average throughput per-STA for admitted STAs performance of the proposed DAAA algorithm and those of the legacy SSF algorithm when some STAs may not be able to be admitted into the network. In general, for arriving STAs, the DAAA algorithm provides a lower rejection ratio than the SSF algorithm in the network with both uniformly and hotspot distributed STAs. For admitted STAs, compared with the SSF algorithm, the DAAA algorithm generally provides better fairness, and higher or slightly lower average throughput per-STA.

It has been shown that using the DAAA algorithm to determine admission for STAs arriving at the network followed by using the iSNZPR algorithm to determine AP association and transmission time allocation for admitted STAs could be an effective approach to the problem studied in this paper.

REFERENCES

[1] Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, IEEE Standard 802.11, 2016.
[2] F. M. Abinader, Jr. et al., “Performance evaluation of IEEE 802.11n WLAN in dense deployment scenarios,” in Proc. IEEE 80th Veh. Technol. Conf. (VTC-Fall), 2014, pp. 1–5.
[3] A. Baid and D. Raychaudhuri, “Understanding channel selection dynamics in dense Wi-Fi Networks,” IEEE Commun. Mag., vol. 53, no. 1, pp. 110–117, Jan. 2015.
[4] P. B. Oni and S. D. Blostein, “AP association optimization and CCA threshold adjustment in dense WLANs,” in Proc. IEEE Globecom Workshops, San Diego, CA, USA, Dec. 2015, pp. 1–6.
[5] P. Oni, “Access point association coordination in densely deployed 802.11 wireless networks,” M.S. Thesis, Dept. Electr. Comput. Eng., Queen’s Univ., Kingston, ON, Canada, 2015.
[6] K. Shin, I. Park, J. Hong, D. Har, and D.-H. Cho, “Per-node throughput enhancement in Wi-Fi densenets,” IEEE Commun. Mag., vol. 53, no. 1, pp. 118–125, Jan. 2015.
[7] F. M. Abinader, Jr. et al., “Distributed Wi-Fi interference coordination for dense deployments,” Int. J. Wireless Pers. Commun., vol. 97, no. 1, pp. 1033–1058, Nov. 2017.
[8] J.-C. Chen, T.-C. Chen, T. Zhang, and E. van den Berg, “Effective AP selection and load balancing in IEEE 802.11 wireless LANs,” in Proc. GLOBECOM, Nov. 2006, pp. 1–6.
[9] F. Xu, C. C. Tan, Q. Li, G. Yan, and J. Wu, “Designing a practical access point association protocol,” in Proc. IEEE INFOCOM, Mar. 2010, pp. 1–9.
[10] L.-H. Yen, J.-J. Li, and C.-M. Lin, “Stability and fairness of AP selection games in IEEE 802.11 access networks,” IEEE Trans. Veh. Technol., vol. 60, no. 3, pp. 1150–1160, Mar. 2011.
[11] Y. Bejerano, S.-J. Han, and L. Li, “Fairness and load balancing in wireless LANs using association control,” IEEE/ACM Trans. Netw., vol. 15, no. 3, pp. 560–573, Jun. 2007.
[12] S. K. Dandapat, B. Mitra, R. R. Choudhury, and N. Ganguly, “Smart association control in wireless mobile environment using max-flow,” IEEE Trans. Netw. Service Manag., vol. 9, no. 1, pp. 73–86, Mar. 2012.

[13] W. Wong, A. Thakur, and S.-H. G. Chan, “An approximation algorithm for AP association under user migration cost constraint,” in Proc. IEEE INFOCOM, Apr. 2016, pp. 1–9.

[14] M. Amer, A. Busson, and I. Guérin-Lassous, “Association optimization in Wi-Fi networks: Use of an access-based fairness,” in Proc. 19th ACM Int. Conf. Model. Anal. Simulat. Wireless Mobile Syst. (MSWIM), Nov. 2016, pp. 119–126.

[15] M. Amer, A. Busson, and I. Guérin-Lassous, “Association optimization in Wi-Fi networks based on the channel busy time estimation,” in Proc. IFIP Netw. Conf. Workshops, May 2018, pp. 298–306.

[16] M. H. Dwijaksara, W. S. Jeon, and D. G. Jeong, “User association for load balancing and energy saving in enterprise WLANs,” IEEE Syst. J., vol. 13, no. 3, pp. 2700–2711, Sep. 2019.

[17] I. Koukoutsidis and V. A. Siris, “Access point assignment algorithms in WLANs based on throughput objectives,” in Proc. 6th Int. Symp. Model. Optin. Mobile Ad Hoc Wireless Netw. (WiOpt), Apr. 2008, pp. 375–383.

[18] Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment: Enhancements for High Efficiency WLAN, IEEE Standard 802.11ax, 2017.

[19] Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment: Sub 1 GHz: License Exempt Operation, IEEE Standard 802.11ah, 2016.

[20] M. Nurchis and B. Bellalta, “Target wake time: Scheduled access in IEEE 802.11ax WLANs,” IEEE Wireless Commun., vol. 26, no. 2, pp. 142–150, Apr. 2019.

[21] L. Li, M. Pal, and Y. R. Yang, “Proportional fairness in multi-rate wireless LANs,” in Proc. IEEE INFOCOM, Apr. 2008, pp. 1678–1686.

[22] W. Li et al., “AP association for proportional fairness in multi-rate WLANs,” IEEE/ACM Trans. Netw., vol. 22, no. 1, pp. 191–202, Feb. 2014.

[23] O. B. Karimi, J. Liu, and J. Rexford, “Optimal collaborative access point association in wireless networks,” in Proc. IEEE INFOCOM, 2014, pp. 1141–1149.

[24] M. A. Kafi, A. Mouradian, and V. Vèque, “On-line client association scheme based on reinforcement learning for WLAN networks,” in Proc. IEEE Wireless Commun. Netw. Conf. (WCNC), Apr. 2019, pp. 1–7.

[25] H. Tang, L. Yang, J. Dong, Z. Ou, Y. Cui, and J. Wu, “Throughput optimization via association control in wireless LANs,” Mobile Netw. Appl., vol. 21, pp. 453–466, Jun. 2016.

[26] M. T. Islam and B. Choi, “Jointly maximizing throughput and utilization for dense enterprise WLANs,” in Proc. IEEE Int. Smart Cities Conf. (ISC2), Oct. 2019, pp. 753–759.

[27] A. Raschella, F. Bouhafs, M. Seyedebrahimi, M. Mackay, and Q. Shi, “Quality of service oriented access point selection framework for large Wi-Fi networks,” IEEE Trans. Netw. Service Manag., vol. 14, no. 2, pp. 441–455, Jun. 2017.

[28] P. Gallo, K. Kosek-Szott, S. Szott, and I. Tinnirello, “CADWAN: A control architecture for dense WiFi access networks,” IEEE Commun. Mag., vol. 56, no. 1, pp. 194–201, Jan. 2018.

[29] X. Wan, X. Guan, Y. Shen, and B.-Y. Choi, “Optimal user association in unlicensed WLANs under bandwidth constraints,” in Proc. IEEE Int. Smart Cities Conf. (ISC2), Sep. 2018, pp. 1–8.

[30] W. Wong and S.-G. Chan, “Distributed joint AP grouping and user association for MU-MIMO networks,” in Proc. Proc. IEEE INFOCOM, Apr. 2018, pp. 252–260.

[31] H. S. Oh, D. G. Jeong, and W. S. Jeon, “Joint radio resource management of channel-assignment and user-association for load balancing in dense WLAN environment,” IEEE Access, vol. 8, pp. 69615–69628, 2020.

[32] Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: High Speed Physical Layer in the 5 GHz Band, IEEE Standard 802.11a, 1999.

[33] Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Further Higher Data Rate Extension in the 2.4 GHz Band, IEEE Standard 802.11g, 2003.

[34] Y. Pochet and L. A. Wolsey, Production Planning by Mixed Integer Programming, New York, NY, USA: Springer, 2006.

[35] F. P. Kelly, “Charging and rate control for elastic traffic,” Eur. Trans. Telecommun., vol. 8, no. 1, pp. 33–37, Jan./Feb. 1997.

[36] D. S. Hochbaum, “Complexity and algorithms for nonlinear optimization problems,” Ann. Oper. Res., vol. 153, no. 1, Sep. 2007.

[37] A. Antoniou and W.-S. Lu, Practical Optimization: Algorithms and Engineering Applications. New York, NY, USA: Springer, 2007.

[38] R. Fletcher, Practical Methods of Optimization, 2nd ed. New York, NY, USA: Wiley, 2000.

[39] S. Boyd and L. Vandenberghe. Convex Optimization. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[40] R. Jain, D. Chiu, and W. Hawe, “A quantitative measure of fairness and discrimination for resource allocation in shared computer systems,” Digit. Equip. Corp., Hudson, MA, USA, Rep. DEC-TR-301, Sep. 1984.

ZHIWEI MAO (Senior Member, IEEE) received the B.Sc. and M.Sc. degrees from the Beijing University of Posts and Telecommunications, Beijing, China, in 1996 and 1999, respectively, and the Ph.D. degree in electrical engineering from the University of Victoria, Victoria, BC, Canada, in 2003. From August 2003 to June 2008, she was an Assistant Professor. In July 2008, she became an Associate Professor with the Department of Electrical Engineering, Lakehead University, Thunder Bay, ON, Canada. Since August 2008, she has been with the Gildart Haase School of Computer Sciences and Engineering, Fairleigh Dickinson University, Teaneck, NJ, USA, where she is an Associate Professor of Electrical Engineering. Her current research interests include wireless communications, digital communications, digital signal processing, and engineering design by optimization.