Luneburg-lens-like structural Pauli attractive core of the nuclear force at short distances

Gerald A. Miller
Department of Physics, University of Washington, Seattle, WA 98195-1560

Abstract
A recent paper [S. Ohkubo, Phys. Rev. C 95, 044002 (2017)] found that the measured $^1S_0$ phase shifts can be reproduced using a deeply attractive nucleon-nucleon potential. We find that the deuteron would decay strongly via pion emission to the deeply bound state arising in this potential. Therefore the success of a deeply attractive potential in describing phase shifts must be regarded only as an interesting curiosity.

Keywords: deeply attractive nucleon-nucleon potential, pion emission, deuteron stability

1. Introduction

A recent paper [1] finds a nuclear force with an attractive potential at short distances that reproduces the experimental $^1S_0$ phase shifts well. Such a potential can be motivated by early quark-model ideas [2], but later work [3] showed that quark model ideas lead to short distance repulsion between nucleons. Here we show that the deep attraction causes a deeply bound state to exist, with the drastic consequence that the deuteron would not be stable.

The $^1S_0$ potential $V(r)$ of [1] is given by

$$V(r) = -5e^{-(r/2.5)^2} - 270e^{-(r/0.942)^2} - 1850e^{-(r/0.447)^2}$$

$$\equiv \sum_{n=1}^{3} V_n e^{-r^2/r_n^2}. \quad (1)$$

*corresponding author: Gerald A. Miller, miller@uw.edu
The strength parameters of $V$ are given in units of MeV, and range parameters are in units of fm. This purely attractive potential has a depth of 2125 MeV at $r = 0$ and a half-width $r_0$ of about 0.4 fm. The corresponding uncertainty principle estimate of the kinetic energy, $\hbar^2/(Mr_0^2)$, with $M$ as the nucleon mass, is 259 MeV, so that the quickest look at this potential leads to the conclusion that the existence of a deeply bound state is an immediate consequence of using Eq. (1).

The easiest analytic way to show that a bound state must exist is to use the variational principle. The single-parameter trial wave function $u(r)$ used here takes the form:

$$u(r) = \frac{2r e^{-r^2/2R^2}}{\sqrt{\pi} \sqrt{R^3}}.$$  \hspace{1cm} (2)

with the normalization $\int_0^\infty dr \, u^2(r) = 1$. If the expectation value of the Hamiltonian, $H$, within this (or any) wave function is less than zero, the potential must yield a bound state. The expectation value of the $H$, defined as $B(R)$ is given by

$$B(R) = \frac{3\hbar^2}{2MR^2} + \sum_{n=1}^3 V_n \frac{1}{(1 + R^2/r_n^2)^{3/2}},$$  \hspace{1cm} (3)

with the first, positive term arising from the kinetic energy much smaller than the negative potential energy terms. This can be seen immediately using only the $V_3 = -1850$ MeV term of Eq. (3). With $R = r_3$ the $V_3$ term is $V_3/(2\sqrt{2}) = -650$ MeV, while the kinetic energy term is about 390 MeV. Fig. 1 shows that $\langle H \rangle \equiv B(R)$ bottoms out at about -620 MeV. Thus there must be a bound state, and its binding energy must be greater than or equal to 620 MeV. Numerical solution of the Schroedinger equation yields a binding energy of about 640 MeV.

In Ref. [1], this state is denoted as “unphysical” and “Pauli forbidden”. However, the Pauli principle does not forbid a $^1S_0$ bound state. For example 6 quarks each in the lowest orbital of the MIT bag model form a bound state in that channel if gluon exchange effects are neglected [4] and such states could play an important role in nucleon-nucleon scattering [3, 5].

A deeply bound $^1S_0$ state has never been found and our very existence shows that this bound state cannot be real. This is because the deuteron would decay strongly to this bound state by the emission of a pion.
2. Deuteron instability

We proceed to compute the width $\Gamma$ and lifetime using first order perturbation theory. The pion ($\pi^+$) emission interaction Hamiltonian, $H_I$, is given in first-quantized notation by,

$$H_I = \frac{g}{2M} \sum_{i=1,2} \sigma_i \cdot k \tau_i^-, \quad (4)$$

where the pion nucleon coupling constant, $g$ is taken to be, $g^2/(4\pi) = 13.5$, $k$ is the pion momentum in the center-of-mass frame, and the operators $\sigma_i, \tau_i^-$ are usual Pauli spin and isospin operators that act on nucleon $i$. The operator $H_I$ connects the initial deuteron state (of spin $m$) with the final two-body state, with a matrix element

$$\mathcal{M}_m \equiv \langle B | H_I | D, m \rangle, \quad (5)$$

and the total decay width $\Gamma$ is given (after evaluating the phase space integral) by

$$\Gamma = \frac{1}{2\pi} \frac{1}{3} \sum_m |\mathcal{M}_m|^2 k. \quad (6)$$

Evaluation yields the expression

$$\Gamma = \frac{2}{3} \frac{g^2}{4\pi} \frac{k^3}{M^2} (I_0 + \frac{\sqrt{2}}{2} I_2)^2 \quad (7)$$

where

$$I_l \equiv \int dr u_B(r) u_l(r) j_l(kr/2), \quad (8)$$
with $u_B(r)$ is the radial $^1S_0$ bound state wave function produced by the potential of Eq. (1), $u_{0.2}$ are the $S$ and $D$ state deuteron radial functions, and $j_{0.2}$ are spherical Bessel functions. With a bound state of 640 MeV, $k = 728$ MeV/c. Numerical evaluation using the deuteron wave function of the Argonne V18 potential [6] gives $I_0 = 0.0925$, $I_2 = 0.0161$, so that the net result is $\Gamma = 42$ MeV, which corresponds to a lifetime $T = \frac{\hbar}{\Gamma} = 1.6 \times 10^{-23}$ s, so that deuterons could not exist.

3. Discussion

There are many other possible reactions for which the use of this potential would have drastic erroneous consequences. Immediate examples are the transition amplitudes for $np \to d\gamma$ and more importantly the $pp \to De\bar{\nu}_e$ reaction that is essential for understanding the energy radiated by our sun. The low-energy nucleon-nucleon wave functions of the potential of Eq. (1) have a node [1], which arises from the necessary orthogonality of the bound state with all scattering states. The nodes in the wave function would vastly reduce the mentioned transition amplitudes.

The possibility of a purely attractive nucleon-nucleon potential, with its connection to a Luneburg lens [7] is very interesting. But such an interaction can only be regarded as an oddity unrelated to the real Universe.

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