Effect of $\beta^-$–charged eradiation and its calculation in the nuclear electrodynamics theory

V Yu Tertychny-Dauri
Saint-Petersburg National Research University of Information Technologies, Mechanics & Optics
Department of Physics and Engineering
49 Kronverkskiy Prospect, Saint-Petersburg 197101, Russia
E-mail: tertychny-dauri@mail.ru

Abstract. The study of own fields and charged particles motion and also charged fission splinters of a heavy nucleuses into nonrelativistic approximation is the subject of this paper research. The main efforts are concentrated in quest of charged share components by the radioactive $\beta^-$–disintegration. The corresponding field equations and equations of motion in the nuclear electrodynamics processes are obtained and their solutions are found. Analysis of the microscopic equations is generalized to the level of the macroscopic description of continuous medium electrodynamics and is accompanied by quantumomechanical additions.

1. Introduction
The important peculiarity of presented work is a definition and solution of fields equations and motion equations for fission splinters in consideration of charged share of radioactive inside nuclon $\beta^-$–disintegration effect, i. e. in consideration of emitted cascaded electrons but without consideration of $\gamma$–eradiation [1], [2], [3]. The obtained theoretical results can be a basis for the further mathematical modelling of generation phenomenon of a virulent directional electromagnetical fields on the toroid (the nuclear electrogenerator) in the process of nuclear chain fission [4], [5], [6]. We propose the toroid is the solenoid with the external winding. Moreover inside of this toroid all fragments of the nuclear disintegration remain in force. That is why light charges don’t may escape the system and therefore enter the formalism.

2. The fields and motion of charged fission particles
We are interested by the calculation recipe of electrical $E_*(R, t) \in \mathbb{R}^3$ and magnetical $B_*(R, t) \in \mathbb{R}^3$ fields in the point with coordinate $R \in \mathbb{R}^3$ at time moment $t$ which are produced by the particles with charges $Z_i$, coordinates $R_i(t)$ and velocities $\dot{R}_i(t) = v_i(t)$, $i = 1, 2, \ldots$. It is known that given fields into Gaussian units system (taking into account of values in order $1/c$, where $c$ is velocity of light, and of multiplicator $4\pi$ in the expression for potentials) can be written with the aid of microscopic Maxwell-Lorenz equations system in the form

$$\nabla E_* = \sum_i Z_i \delta (R_i - R), \quad -E_*' + \nabla \times B_* = \frac{1}{c} \sum_i Z_i \dot{R}_i \delta (R_i - R),$$

$$\nabla B_* = 0, \quad B_*' + \nabla \times E_* = 0. \quad (1)$$
We denote here: for the vector ∇ (the differential Hamiltonian operator) is the differentiation
on coordinate R, the feature on top is the differentiation on ct; δ (Rt − R) is the delta-function
from Rt − R included in the field sources.

Taking into account of the vector potential a(R,t) : B* = ∇ × a, the scalar potential
φ(R,t) : E* = −∇φ − a' and the Lorentzian calibrated condition: ∇a + φ' = 0 also, the solution
for system (1) in the form of nonrelativistic electromagnetical field can be found

E* = ∑ Ei, Ei = −∇ Zi/4πσi, B* = ∑ Bi, Bk = 1/c ∇ × Zk ∂H/∂Pk,

where through σi = |Ri − R| is denoted the distance between points with coordinates
Ri and R in the space R3.

The equation of i–th particle motion with the mass mi under the influence Lorentzian force
has the form

mi Ri = Zi [E(Ri,t) + 1/c Ri × B(Ri,t)],

where E(Ri,t) = ∑ j Ej + E0, B(Ri,t) = ∑ j Bj + B0; (E, B) is full electromagnetical field in
the point Ri at time moment t; (E0,B0) is external field, (Ei, Bi) is internal field of the charged
particles in the given point, moreover i ≠ j.

For equation (2) can been attached the canonical form: ∂H/∂P1 = Ri, ∂H/∂Ri = −Ri with
the aid of the Hamiltonian

H = ∑ i P2i/2mi + ∑ i ∑ j ZiZj 8πσij

+ ∑ i Zi [φ0(Ri,t) − Pj/cmij A0(Ri,t)], σij = |Ri − Rj |,

where i ≠ j, into the the terms of impulse variables Pi, coordinate variables Ri and potentials
(scalar φ0 and vector A0) of external field.

3. The fields and motion of charged fission splinters

Let us set the problem about conclusion of field equations and motion equations of the charged
splinters (the united particles with internal nuclear structure) which are made in consequence of
the chain nuclear fission reaction. These compound united particles can be considered as many
time ionized positive ions for the reason electrons upsetting of the atom outer skin of divided
substance.

We proceed from the field equation of individual particles (1). Add to the index i the index
k. Then instead of vector Ri we take the vector RKi, where RKi = Rk + rk; where k is the
splinter index, i is the index of the particle of given splinter, rk is the internal coordinate of
ki-th particle concerning the fixed point of k-th splinter.

The charged fission splinters are not ”the stable complexes” but are the powerfully nonsteady
particles groups are exposed the pronounced instantaneous radioactive fission (the β−-fission
accompanied by the γ-eradiation). We have in the k-th fission splinter the individual charged
particles in the point RKi with the charges Zk and the charged nucleus in the point Rk is exposed
β−-fission with the charge Zk : Zk = Xk + Yk, where Xk is the proton charge of nucleus (i. e.
the original proton charge of splinter + the proton charge of β−-fission products), Yk is the
electron charge of β−-fission products.

The solutions E* and B* of field equations can be approximated as converged rows on
parameter |rk|/|Rk − R|, i.e. the dimensions of a splinter RKi smaller a distance σk from the
observation point R to fixed point (nucleus) Rk of k-th splinter. Transform equations (1) taking
into account all expansions, included the \( \delta \)-function expansion into the Taylor’s row on \( r_{ki} \) in the locality of the point \( (R_k - R) \)

\[
\nabla E_s = \sum_k \sum_i Z_{ki} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (r_{ki} \nabla)^n \delta(R_k - R) + \sum_k X_k \delta(R_k - R) + \sum_k Y_k \delta(R_k - R),
\]

\[-E'_s + \nabla \times B_s = \frac{1}{c} \sum_k \sum_i Z_{ki} (\dot{R}_k + r_{ki}) \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (r_{ki} \nabla)^n \delta(R_k - R)
\]

\[+ \frac{1}{c} \sum_k X_k \dot{R}_k \delta(R_k - R) + \frac{1}{c} \sum_k Y_k \dot{R}_k \delta(R_k - R), \quad \nabla B_s = 0, \quad B'_s + \nabla \times E_s = 0.\]

The motion equation of \( ki \)-th particle into \( k \)-th splinter with the charge \( Z_{ki} \), the mass \( m_{ki} \) and the coordinate \( R_{ki} \) at the time moment \( t \) into full electromagnetical field \( (E(R_{ki}, B(R_{ki}, t))) \)

has a form of equation (2), where \( R_i \to R_{ki} \), i. e. \n
\[
m_{ki} \ddot{R}_{ki} = Z_{ki} \left[ E(R_{ki}, t) + \frac{1}{c} \dot{R}_{ki} \times B(R_{ki}, t) \right]
\]

and

\[
E(R_{ki}, t) = - \sum_j \nabla_k \left\{ \frac{Z_{kj}}{4\pi \sigma_{k,ij}} - \nabla_k \left( \frac{X_k + Y_k}{4\pi \sigma_{k,ik}} \right) \right\} - \sum_l \sum_j \nabla_k \left\{ \frac{Z_{lj}}{4\pi \sigma_{k,l,jj}} \right\}
\]

\[+ \sum_l \nabla_k \left( \frac{X_l + Y_l}{4\pi \sigma_{k,l,il}} \right) + E_0(R_{ki}, t), \quad B(R_{ki}, t) = B_0(R_{ki}, t), \]

where \( j \neq i, l \neq k \); \( \nabla_k \) is the gradient vector on the elements of vector \( R_{ki} \). In the system (4) are denoted

\[
\sigma_{k,ij} = |R_{ki} - R_{kj}|, \quad \sigma_{k,ik} = |R_{ki} - R_{kk}| = |R_{ki} - R_k|,
\]

\[
\sigma_{kl,ij} = |R_{ki} - R_{ij}|, \quad \sigma_{kl,il} = |R_{ki} - R_{il}| = |R_{ki} - R_l|,
\]

and moreover \( |R_{ki} - R_k| = |r_{ki}| \).

On the analogy of (2) \((i \to k)\) can be written the motion equation of the nucleus of \( k \)-th splinter with the charge \( Z_k = X_k + Y_k \), the mass \( m_k \), the coordinate \( R_k \) into full electromagnetical field \( (E(R_k, t), B(R_k, t)) \)

\[
m_k \ddot{R}_k = Z_k \left[ E(R_k, t) + \frac{1}{c} \dot{R}_k \times B(R_k, t) \right],
\]

where

\[
E(R_k, t) = - \sum_j \nabla_k \left\{ \frac{Z_{kj}}{4\pi \sigma_{k,kj}} - \sum_l \sum_j \nabla_k \left( \frac{Z_{lj}}{4\pi \sigma_{k,l,kj}} \right) \right\}
\]

\[- \sum_l \nabla_k \left( \frac{X_l + Y_l}{4\pi \sigma_{k,l,kl}} \right) + E_0(R_k, t), \quad B(R_k, t) = B_0(R_k, t), \quad l \neq k. \]

After the summarizing of equations (3) on \( i \) and the adding with equation (5) for description of the motion of \( k \)-th splinter with the mass \( \bar{m}_k \) and the coordinate of masses center \( \bar{R}_k \)

\[
\bar{m}_k = \sum_i m_{ki} + m_k, \quad \bar{R}_k = \frac{\sum_i m_{ki} R_{ki} + m_k R_k}{\bar{m}_k}
\]
we obtain the equation
\[ \bar{m}_k \ddot{R}_k = \sum_i Z_{ki} \left[ E(R_{ki}, t) + \frac{1}{c} \dot{R}_{ki} \times B(R_{ki}, t) \right] + Z_k \left[ E(R_k, t) + \frac{1}{c} \dot{R}_k \times B(R_k, t) \right] . \] (7)

Substitute the expressions (4) and (6) into the equation (7). Then the resultant force corresponding the intrasplinter field for the set of central forces is equal zero. Consequently the equation (7) acquires the form
\[
\bar{m}_k \ddot{R}_k = - \sum_l \sum_i \sum_j \nabla_{ki} \frac{Z_{ki} Z_{lj}}{4\pi \sigma_{kl,ij}} - \sum_i \sum_j \nabla_{ki} \frac{Z_{ki} (X_i + Y_i)}{4\pi \sigma_{kl,il}} - \sum_l \sum_j \nabla_{k} \frac{Z_{lj} (X_k + Y_k)}{4\pi \sigma_{kl,kj}} \\
- \sum_l \nabla_k \left( \frac{(X_i + Y_i)(X_k + Y_k)}{4\pi \sigma_{kl,kl}} + \sum_i Z_{ki} \left[ E_0(R_{ki}, t) + \frac{1}{c} \dot{R}_{ki} \times B_0(R_{ki}, t) \right] \right) \\
+ (X_k + Y_k) \left[ E_0(R_k, t) + \frac{1}{c} \dot{R}_k \times B_0(R_k, t) \right] , \quad l \neq k. \] (8)

For all that external field \((E_0, B_0)\) satisfies the homogeneous equations
\[ \nabla B_0 = 0, \quad B'_0 + \nabla \times E_0 = 0. \]

Thus, the equation (8) is the motion equation of \(k\)-th splinter into electromagnetical field of other splinters and external sources.

It is important to note the received equations on a level with the traditional items in the right part of the Lorenzian force contain also the items are made a force connected with radioactive charged eradiation of nucleuses into fission splinters (the accounting of \(\beta^-\)–charged eradiation by the radioactive electrodynamical effect).

4. Conclusions
Chain fission of heavy nuclei is one of many physical phenomena which proceed in an avalanche scheme. In fission there is an impetuous increase in the numbers of neutrons, charged particles and fission splinters with enormous kinetic energy. The present theory aims to describe the above process quantitatively. In the future it could find an application in powerful reactors operating exclusively on the basis of electromuclear conversions.

References
[1] de Groot S R, Suttorp L G 1972 Foundations of Electrodynamics (Amsterdam: North-Holland Publishing Company) 560 p
[2] Artsimovich L A, Lukyanov S Yu 1979 Motion of Charged Particles into Electric and Magnetic Fields (Moscow: Nauka) 224 p
[3] Tertychny-Dauri V Yu 2014 Foundations of Nuclear Electrodynamics (Saarbrucken: Lambert Academic Publishing) 304 p
[4] Keepin G R 1965 Physics of Nuclear Kinetics (Massachusetts, London: Addison-Wesley Publishing Company) 428 p
[5] Tertychny-Dauri V Yu 2006 Optimal stabilization in the problems of adaptive nuclear kinetics Different. Equat. 42 374-384
[6] Tertychny-Dauri V Yu 2008 Galamech Vol 3 Hyperreactive Mechanics (Moscow: Fizmatlit) 576 p