More about Dynamical Reduction and the Enumeration Principle

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ABSTRACT

In view of the arguments put forward by Clifton and Monton [1999] in a recent preprint, we reconsider the alleged conflict of dynamical reduction models with the enumeration principle. We prove that our original analysis of such a problem is correct, that the GRW model does not meet any difficulty and that the reasoning of the above authors is inappropriate since it does not take into account the correct interpretation of the dynamical reduction theories.

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1 Introduction

In a recent paper (Lewis [1997]) the compatibility of dynamical models of spontaneous reduction (in particular of the so called GRW model (Ghirardi, Rimini, Weber [1986])) with the enumeration principle has been questioned. We have replied to the arguments of Lewis [1997] on this journal (Ghirardi, Bassi [1999]). Subsequently, we have received a preprint by R. Clifton and B. Monton [1999] in which the whole matter is reconsidered. Their conclusions can be summarized in the following terms: the authors agree with us that Lewis’ criticisms to GRW are not cogent but they consider our argument as wrong and feel the necessity of presenting the correct way to derive the desired conclusion. Such a way requires a quite elaborated (even though in our opinion not appropriate and superfluous) argument proving that the ‘counting anomaly’ cannot, in principle, be put into evidence. In this paper we reconsider the whole matter pointing out that both Lewis [1997] and Clifton and Monton [1999] have not taken the correct attitude about the interpretation of the GRW theory and that the central proof of Clifton and Monton is quite inappropriate. We strengthen our argument by resorting to explicit examples to make clear to the readers some specific dynamical aspects of reduction theories which seem to have been misinterpreted, and which must be taken into account when dealing with systems containing an enormous number of macroscopic subsystems.

2 A Brief Account of the Enumeration Anomaly

The essential point which has given rise to various critical remarks (Shimony [1991]; Albert and Loewer [1996]) about dynamical reduction theories and is the very starting point of the arguments of Lewis [1997] and Clifton and Monton [1999] is the so called ‘tails problem’. It consists in the fact that the collapse processes characterizing the GRW theory (as well as its more refined versions such as CSL (Pearle [1989]; Ghirardi, Pearle, Rimini...
[1990]) unavoidably lead to wavefunctions which have a non compact support in configuration space, thus giving rise to problems with the attribution of determined locations to physical systems.

The alleged puzzling implications of this fact for the problem we are interested in can be exemplified in a very simple way. One considers a macroscopic marble (which, for simplicity, we take to have a radius of about one centimeter and normal density) in an initial state at time \( t = 0 \) denoted as

\[
|\Psi(0)\rangle = \frac{1}{\sqrt{2}}[|in\rangle + |out\rangle],
\]

(1)

the states \(|in\rangle\) \((|out\rangle)\) being eigenstates of the marble being well inside (outside) an extremely large box, and one recalls, as just mentioned, that dynamical reduction theories lead to wavefunctions which, in the case of macroscopic objects, are certainly very well localized but always possess ‘tails’ going off to infinity. To give a synthetic but significative description of the state of affairs one must recall that GRW’s dynamics implies that a macroscopic object cannot remain (Bell [1987]) for more than a split second in a state like (1), but it will always be in one of the following two states:

\[
|\Psi_{in}(t)\rangle = \alpha |in\rangle + \beta |out\rangle,
\]

(2)

or

\[
|\Psi_{out}(t)\rangle = \beta |in\rangle + \alpha |out\rangle,
\]

(3)

with \(|\alpha|^2 + |\beta|^2 = 1\), \(|\alpha|^2 >> |\beta|^2\). Lewis [1997] and Clifton and Monton [1999] agree that one can interpret state (2) as one in which the marble is inside the box, and state (3) as one in which it is outside the box, even though in the two papers slightly different reasons are presented to justify the above claims. Lewis [1997] resorts to the consideration of the scalar product to decide when two states are ‘near to each other’ and consequently can be considered as describing similar properties:

Those who defend the GRW theory point out that (2) is very close to the state \(|in\rangle\) in which the marble is determinately in
the box, and (3) is very close to the eigenstate $|\text{out}\rangle$ in which
the marble is determinately outside the box. The scalar product
provides a convenient measure of the proximity of the two states...

Intuitively, it seems that if two states are sufficiently close,
they will be macroscopically equivalent, at least for all practical
purposes (Lewis [1997] p.316).

On the other hand Clifton and Monton [1999] adopt the criterion PosR pro-
posed by Albert and Lower [1996] to decide whether a particle is inside or
outside a given box:

‘Particle $x$ is in region $R$’ if and only if the proportion of the
total squared amplitude of $x$’s wave function which is associated
with points in $R$ is greater than or equal to $1 - p$ (Clifton and
Monton [1999] p.4), $p$ being an appropriately chosen positive number smaller than 0.5.

One then takes into account a system of a large number $n$ of non-interacting
marbles, each of which is in a state like (2):

$$|\Psi\rangle_{\text{all}} = \left(\alpha |\text{in}\rangle_1 + \beta |\text{out}\rangle_1\right) \otimes \left(\alpha |\text{in}\rangle_2 + \beta |\text{out}\rangle_2\right) \otimes \cdots \otimes \left(\alpha |\text{in}\rangle_n + \beta |\text{out}\rangle_n\right). \quad (4)$$

Intuitively, due to the fact that for such a state, according to the above
discussion, each marble is inside the box, one would like to be entitled to
conclude that all marbles are inside the box.

Lewis [1997] as well as Clifton and Monton [1999] call attention to the
fact that according to any one of the previous criteria this cannot be the
case and, consequently, the GRW theory violates the enumeration principle.
Actually Lewis states:

... the proximity of $|\Psi\rangle_{\text{all}}$ to the eigenstate of all $n$ marbles
being in the box is given by

$$|_{\text{all}}\langle \Psi | \text{all in}\rangle|^2 = |\alpha|^2n. \quad (5)$$
Recall that $|\alpha|^2$ is slightly less than 1. This means that as $n$ becomes very large, $|\alpha|^{2n}$ becomes small; ... Consequently, when $n$ is sufficiently large $|\Psi\rangle_{alt}$ is not close to the eigenstate of all $n$ marbles in the box ... Given this, we certainly cannot claim that $|\Psi\rangle_{alt}$ is a state in which all $n$ marbles are in the box\footnote{We stress that the above expression (5) is precisely the probability that Standard Quantum Mechanics (SQM) with the Wave Packet Reduction Postulate (WPR) attributes to the outcome $n$ of a measurement aimed to ascertain how many particles are in the box.} (Lewis [1997], p.318).

Clifton and Monton [1999] argue in a similar way making reference to a generalization of PosR which they denote as the fuzzy link:

‘Particle $x$ lies in region $R_x$ and $y$ lies in $R_y$ and $z$ lies in $R_z$ and ...’ if and only if the proportion of the total squared amplitude of $\psi(t, r_1, r_2, ..., r_n)$ that is associated with points in $R_x \times R_y \times R_z \times ...$ is greater than or equal to $1 - p$ (p.4)

and they conclude:

...by applying the fuzzy link for $|\alpha|^{2n} \leq p$ to $|\Psi\rangle_{alt}$, one obtains the result that not all the marbles are in the box. And this seems to contradict the results one obtains when one applies the fuzzy link on a marble-by-marble basis, where one gets the results that marble 1 is in the box, marble 2 is in the box, and so on through to marble $n$ (p.9).

We stress that Lewis [1997] in his paper insists in adopting the scalar product criterion to characterize the nearness of a statevector to another one and makes reference to this formal ‘nearness’ to attribute precise properties
concerning the location of a macroscopic physical system\footnote{To be precise, we have to mention that Lewis [1997] in his paper has taken into account the ‘objective mass density’ interpretation of the GRW theory which will represent the basis of the analysis of the next section and the associated criterion of ‘macroscopic nearness’ which has been introduced by Ghirardi, Grassi and Benatti [1995]. However, to stress that these interpretations do not allow to overcome the difficulties, he has to resort to analyzing ‘the chance of finding all $n$ marbles on observation’. This position corresponds to mixing up the ontology which is the appropriate one for the GRW theory with the one which characterizes SQM with the WPR postulate, a misleading and inappropriate procedure.}. Similarly, Clifton and Monton in their paper stick always to the fuzzy link criterion. Instancies of this precise attitude can be found, e.g., after their eq. (15) and in various other places of their paper.

Before coming to analyze our answer to Lewis [1997], the criticisms to it by Clifton and Monton [1999], and their central argument aiming to prove that the counting anomaly cannot, in principle, ever become manifest, we consider it important to call attention on the peculiar attitude that the positions of Lewis [1997] and of Clifton and Monton [1999] imply with respect to the ontology of dynamical reduction models.

\section{The Interpretation of the GRW Theory}

In our opinion the most serious drawback of (Lewis [1997]) and (Clifton and Monton [1999]) derives from their misleading and inconsistent use of the dynamical reduction formalism. Such a formalism finds its very conceptual motivations in the desire to overcome the measurement problem of quantum mechanics. The most characteristic trait of the GRW theory consists, in J.S. Bell’s words, in the fact that:

There is nothing in this theory but the wavefunction. It is in the wavefunction that we must find an image of the physical world, and in particular of the arrangement of things in ordinary
three-dimensional space. [Schrödinger] would have liked the complete absence of particles from the theory, and yet the emergence of ‘particle tracks’, and more generally of the ‘particularity’ of the world, on the macroscopic level (Bell [1987], p. 44).

Accordingly, there is no ‘measurement process’ as distinct from any other physical process. This fact is so universally recognized, even by those who do not share the dynamical reduction point of view, that the GRW and CSL theories have been classified among the ‘quantum theories without observers’. Actually, as a consequence of a lively debate about the meaning of the theory, the appropriate interpretation (now universally accepted) of the theory has been precisely formulated by Ghirardi, Grassi and Benatti [1995]. It is based in an essential way on the consideration of the average mass density c-number function $M(r, t)$ in ordinary three-dimensional space defined as:

$$M(r, t) = \langle \Psi(t) | M(r) | \Psi(t) \rangle,$$

where $|\Psi(t)\rangle$ is the statevector describing the individual physical system at time $t$ and $M(r)$ is the average mass density operator at point $r$, the average being taken on the typical localization volume $[1/\sqrt{\alpha}]^3$ which characterizes the theory:

$$M(r) = \sum_k m_k N^{(k)}(r)$$

$$\equiv \sum_k m_k \left\{ \left[ \frac{\alpha}{2\pi} \right]^\frac{3}{2} \sum_s \int d\mathbf{q} e^{-\frac{1}{\alpha} (\mathbf{q} - r)^2} a_k^\dagger(\mathbf{q}, s) a_k(\mathbf{q}, s) \right\}. \quad (7)$$

In the above equation $a_k^\dagger(\mathbf{q}, s)$ and $a_k(\mathbf{q}, s)$ are the creation and annihilation operators of a particle of type $k$ ($k = \text{electron, proton,...}$) at point $\mathbf{q}$, with spin component $s$.

The most important feature of the GRW dynamics (contrary to what happens within SQM) is that of making, in the case of macroscopic objects, the value of the quantity $M(r, t)$ objective or, resorting to a physically more expressive term which has been introduced by Ghirardi and Grassi [1996], accessible. The very idea of accessibility is appropriately expressed as follows:
A property corresponding to a value (a range of values) of a certain variable in a given theory is objectively possessed or accessible when, according to the predictions of that theory, experiments (or physical processes) yielding reliable information about the variable would, if performed (or taking place), give an outcome corresponding to the claimed value. Thus, the crucial feature characterizing accessibility (as far as statements about individual physical systems are concerned) is the matching of the claims and the outcomes of physical processes testing the claims\(^3\) (Ghirardi [1997], p. 227).

The fundamental feature of the GRW’s dynamics of making accessible, in the macroscopic case, precisely the mass density function \(M(r, t)\) has been discussed in all details by Ghirardi, Grassi and Benatti [1995]. In that paper it has been proven that within such a theory any macroscopic body ends up, in extremely short times, in such a state that all physically testable effects related to the mass density distribution actually agree with the statement that such a distribution is the one which is actually present, independently of any test being actually performed or not. This is why it is generally accepted that the ontology of the dynamical reduction program implies (at the macroscopic level) to answer to John Bell’s [1990] question: \textit{Probability of what exactly are the probabilities of your theory?} by stating: probabilities that the mass density at the various points of ordinary space be the one given by \(M(r, t)\). Ghirardi, Grassi and Benatti [1995] have presented a precise mathematical criterion to evaluate whether the mass density is accessible at

\(^3\)We stress the crucial role played in the above sentence by the specification ‘according to the prediction of that theory’. Obviously, we perfectly agree that any theory has to be subjected to experimental tests and, consequently, that those physical processes designed to test it have an extreme relevance. But we also stress that when a theory allows to make precise statements about physical properties (as it happens e.g., for classical mechanics) one does not need to actually perform a test every time one makes claims about the values possessed by the physical quantities one is interested in.
point \( r \): one takes into account the ratio \( R^2(r, t) \) of the variance \( \mathcal{V}(r, t) \) to the square of \( \mathcal{M}(r, t) \):

\[
R^2(r, t) = \frac{\mathcal{V}(r, t)}{\mathcal{M}(r, t)} = \frac{\langle \Psi(t) | [M(r) - \langle \Psi(t) | M(r) | \Psi(t) \rangle]^2 | \Psi(t) \rangle}{\langle \Psi(t) | M(r) | \Psi(t) \rangle^2}.
\]

(8)

Then, when \( R^2(r, t) \ll 1 \) the mass density at \( r \) is accessible. If the mass density is accessible at all points of a region \( R \), all physical effects agree with the assumption that the considered mass density objectively characterizes the situation within the region.

The reader will easily understand (see Ghirardi, Grassi and Benatti [1995] for the rigorous detailed proofs) that, for a state like (2), the mass density is accessible at all points of the volume of 1 cubic centimeter around the centre of mass position (which, as we will show in section 4 turns out to be extremely well definite) of the marble. Exactly the same argument shows that at all space points lying outside this region the mass is non accessible. Actually, by the argument of Ghirardi, Grassi and Benatti [1995] it can be shown that for all points ‘occupied’ by the marble, \( R(r, t) \approx e^{-[10^{15}]} \) and that the total mass outside the just mentioned region turns out to be of the order of \( e^{-[10^{15}]} \) nucleon masses.

If one takes into account the correct perspective about the interpretation of the GRW theory and the explicit proofs of all above mentioned facts, one should have clear why the enumeration anomaly cannot arise within such a theory. The very universal dynamics characterizing the theory guarantees that in the state (4) the mass is objective precisely in the regions where the various marbles are, and that all physical tests one can imagine will confirm precisely that the actual mass density distribution is the one corresponding to the above statement. The marbles can therefore be claimed to be all within the box, and the total mass within the box is actually the one corresponding to all of them being in the box. There is absolutely no need of actually performing any test to be sure that these statements are correct: the very formal structure of the theory guarantees that this is the case.
Obviously, due to the dynamics of the theory, even a state like \( \ket{4} \) can change and, at a subsequent time (see however section 4 for a detailed analysis) could evolve in a state of the type:

\[
\ket{\Psi}_{\text{all}} = (\alpha \ket{\text{out}}_1 + \beta \ket{\text{in}}_2) \otimes (\alpha \ket{\text{in}}_2 + \beta \ket{\text{out}}_2) \otimes \ldots \otimes (\alpha \ket{\text{in}}_n + \beta \ket{\text{out}}_n). \tag{9}
\]

However, this fact (i.e., the change of the space region where the mass density distribution referring to marble 1 is accessible) does not give rise to any difficulty: for such a state the mass is objective where the marbles are, i.e., around the centre of mass of the \( n - 1 \) marbles in the box, and around the centre of mass of marble 1 which is outside the box. And any conceivable test, be it actually performed or not, will agree with these statements.

Here a short digression to reply to one of the criticisms of Clifton and Monton [1999] to our paper (Ghirardi and Bassi [1999]) is at order. When, to describe the above possible evolution of the statevector \( \ket{4} \), in section 3 of our paper we have made explicit the result of taking the products of the terms of the state \( \ket{\Psi}_{\text{all}} \)

\[
\ket{\Psi}_{\text{all}} = \alpha^n \ket{\text{in}}_1 \otimes \ket{\text{in}}_2 \ldots \ket{\text{in}}_n + \alpha^{n-1} \beta \ket{\text{out}}_1 \otimes \ket{\text{in}}_2 \ldots \ket{\text{in}}_n + \ldots + \beta^n \ket{\text{out}}_1 \otimes \ket{\text{out}}_2 \ldots \ket{\text{out}}_n, \tag{10}
\]

and we have stated that:

the precise GRW dynamics will lead in about one millionth of a second to the suppression of the superposition and to the ‘spontaneous reduction’ of the state \( \ket{\Psi}_{\text{all}} \) to one of its terms (Ghirardi and Bassi, p. 708),

we wanted simply to call attention to the fact that what could happen was just the transition from a state like \( \ket{4} \) to one like \( \ket{9} \) — or even a similar one in which \( k \) particles are outside the box. Actually, Clifton and Monton [1999] have pointed out (more appropriately) that under the GRW dynamics only states like \( \ket{4} \) and \( \ket{9} \) - or similar ones in which more marbles are ‘out of the
box’ - can occur. Thus, one could say that we have been a little bit sloppy in using the above expression ‘one of the terms (of Eq. (10))’ to make reference to states like (4) or (9). However, the point we wanted to make is that a rigorous interpretation of states of this type requires the consideration of the mass density functional (as we have made clear in this Section) and cannot be based on the fuzzy link criterion or other inappropriate criteria. Therefore, the criticism of Clifton and Monton [1999] to our previous paper misses the crucial point of our argument just because it does not make reference to valid criteria for interpreting the wavefunction within dynamical reduction models.

Concluding, within theories like the GRW model no violation of the enumeration principle can occur if one correctly looks at it in terms of the accessibility of the mass density distribution. But something more must be said with reference to the paper by Clifton and Monton [1999]. Even though, as we have just discussed, they have misinterpreted our argument, they agree with us in rejecting Lewis’ [1997] criticisms because, in their opinion, if one operationalizes the process of counting marbles by explicitly modelling the process itself in terms of collapsing the GRW wavefunction (Clifton and Monton [1999], p.19),

the violation of the enumeration principle cannot be, even in principle, put into evidence. For those who have followed our discussion about the ontology of the theory it should be clear that the GRW model does not in any way require to operationalise the process of counting the marbles, since the theory makes precise (and testable if one wants to do so) statements about the actual location of the marbles at any time. This shows that the arguments of Clifton and Monton [1999] are superfluous. But this is not the whole story: in section 5 we will show that the treatment of these authors is imprecise and misleading.

Before coming to such an analysis we would like to present a very simple study of the actual dynamics of a system like the one under consideration
to have the opportunity of analyzing better the various situations which can occur. This will allow us to stress how careful one must be if one wants to consider (unphysical) limits like those based on the consideration of a number of marbles as large as one wants. In order to avoid being misunderstood we warn the reader that we have no objections about taking the considered limit, but we want to show how one has to deal with it if one wants to argue in a consistent way. Our discussion will show that while the GRW theory does not, in any case, meet difficulties with the enumeration principle, the attempt to operationalize the counting of the particles can become a nonsensical task. This is another reason which makes the analysis of Clifton and Monton [1999] quite inappropriate.

4 A Simplified Realistic Example of the GRW Dynamics for a Macroscopic Object

We consider it extremely useful, to give a precise sense to the problem under investigation, to present an explicit analysis of the dynamical evolution of a single macroscopic system such as one of the marbles of the previous sections. Suppose that at the initial time \( t = 0 \) its normalized centre-of-mass wavefunction is:

\[
\Phi(x, 0) = \left[ \frac{a}{\pi} \right]^{\frac{1}{4}} e^{-\frac{a}{2} x^2} \tag{11}
\]

We disregard the internal motion and we confine our attention to the spontaneous localization process. We recall that for a macro-object the localization frequency \( \lambda \) is amplified according to the number \( N \) of its constituent nucleons, which, for the case under consideration \( (N \approx \text{Avogadro's number}) \) means that:

\[
\lambda \approx N \lambda_{\text{micro}} \approx 10^7 \text{ sec}^{-1}. \tag{12}
\]

\(^4\)For simplicity, we treat the system as if the centre-of-mass motion takes place in one dimension. This assumption does not change in any way the conclusions we will draw.
We consider now the effect of a localization, taking place at \( x_0 \) at time \( t_1 \) on the state \( \Phi(x,0) \). We have:

\[
\Phi(x,0) \Rightarrow \Phi_{x_0}(x,t_1) = \left[ \frac{a + \alpha}{\pi} \right]^{\frac{1}{4}} e^{-\frac{a + \alpha}{2}(x-x_0)^2}
\]

(13)

Taking into account the various localizations processes which take place in the interval \((0,t)\), the wavefunction \( \Phi_{x_0(t)}(x,t) \) at time \( t \) will be:

\[
\Phi_{x_0(t)}(x,t) = \left[ \frac{a + n(t)\alpha}{\pi} \right]^{\frac{1}{4}} e^{-\frac{a + n(t)\alpha}{2}(x-x_0(t))^2}
\]

(14)

In the above equation \( n(t) \) is the number of localization processes in the considered time interval. Such a number is a Poisson process with average value \( \langle n(t) \rangle = \lambda t \). Similarly \( x_0(t) \) is a stochastic process with zero mean value: \( \langle x_0(t) \rangle = 0 \). The variance of the final Gaussian wave function and the rate at which it decreases with time are given by:

\[
\sigma_{Loc}^2(t) = \frac{1}{a + n(t)\alpha} \approx \frac{1}{a + \alpha \lambda t}; \quad \frac{d\sigma_{Loc}^2(t)}{dt} \approx -\frac{\alpha \lambda}{(a + \alpha \lambda t)^2}.
\]

(15)

The above equations show that the wavefunction tends, for \( t \to +\infty \), to a Dirac’s delta function. However, one has to take into account that the free dynamics of the centre of mass implies, according to Schrödinger’s equation, an increase of the spread of the wavefunction (which has always a Gaussian shape) which is more rapid the narrower is the wavefunction itself. The rate of increase in the case under consideration is given by:

\[
\frac{d\sigma_{Sch}^2(t)}{dt} = \frac{4\pi^2\hbar^2(a + \lambda \alpha t)}{m^2}.
\]

(16)

A regime condition is reached when the rate of decrease due to the localizations equals the rate of increase due to the free spread, which happens for a time \( \tilde{t} \):

\[
\tilde{t} = \sqrt{\frac{m}{2\pi\hbar\lambda \alpha}} \approx 10^5 \text{ sec},
\]

(17)
the indicated value referring to the case of a macroscopic object of the kind we are considering. If this value is replaced into the expression for the width one gets for the standard deviation in position:

$$\Delta x = \left[ \frac{2\pi \hbar}{m\lambda \alpha} \right]^{\frac{1}{4}} \simeq 10^{-11} \text{cm}. \quad (18)$$

It is useful to note that this value matches almost exactly the one which has been identified by Ghirardi, Rimini and Weber ([1986], Section 8) by a much more complex and realistic analysis. It has also to be remembered that in the same paper it has been proved that the GRW dynamics leads (obviously in the case of a macroscopic system) to a momentum spread such that the Heisemberg relations are almost satisfied with the equality sign.

The conclusion of this first part has to be kept in mind to grasp the real situation one is dealing with: according to the GRW theory a macro-object will find itself, just as a consequence of the dynamics governing all natural processes, in a state such that its centre of mass wavefunction has a spread of the order of $10^{-11} \text{cm}$. The typical wavefunction of the centre of mass of each of our marbles will therefore be:

$$\Phi(x) = \left[ \frac{10^{22}}{\pi} \right]^{\frac{1}{4}} e^{-\frac{10^{22}}{\pi} (x-\overline{x})^2} \quad (19)$$

for an appropriate $\overline{x}$.

Starting with such a state, let us suppose that it suffers repeated localizations all at the same place $x_0$ at a macroscopic distance from $\overline{x}$. As we have seen, the wavefunction will keep its Gaussian structure whose equilibrium width does not change, while its position (taking into account that $10^{22} \gg \alpha = 10^{10}$) is shifted according to:

$$\overline{x} \Rightarrow \tilde{x} \simeq \overline{x} + \frac{\alpha \lambda t}{10^{22} x_0}.$$

Therefore, to have a displacement of the order of $x_0$ of the centre of mass of one of our marbles it takes a time of the order of 1 day and a number of localizations (all of them occurring around $x_0$) of the order of $10^{12}$. 
Since the probability of a localization at a distance larger than $x_0 - \bar{x}$ equals $\text{erfc}[10^{11}(x_0 - \bar{x})]$, one sees that for a distance $x_0 - \bar{x}$ of the order of 10 cm, such a probability is of the order of $e^{-[10^{22}]}$. Concluding the probability that our marble be displaced of 10 cm in one day is much smaller than $e^{-[10^{34}]}$!

We stress once more that we have not made this calculation to prove how unphysical is to think that a marble is drawn out of the box by the spontaneous localizations, but to point out that such a process, which can certainly occur in principle for an unphysically large number of particles, requires a remarkable time. This means that, in the conditions we are envisaging, the state will keep the form (7) for quite long times and that, provided the number of marbles is incredibly large, it will change, e.g., into the state

$$|\Psi\rangle_{\text{alt}} = (\beta|\text{in}\rangle_1 + \alpha|\text{out}\rangle_1) \otimes (\alpha|\text{in}\rangle_2 + \beta|\text{out}\rangle_2) \otimes ... \otimes (\alpha|\text{in}\rangle_n + \beta|\text{out}\rangle_n).$$

(21)

smoothly in time. Thus, the marbles go out of the box quite slowly.

In the just considered case the situation turns out to be perfectly reasonable. At the beginning, starting from a state like (4), the mass will be objective at the points (within the box) where the marbles are, and correspondingly the total mass contained within the box will objectively match the one of the $n$ marbles which are in it. The situation will change with time and, after some time, one (or more) marble will go out of the box. Once more the theory guarantees that the mass is accessible precisely in the regions where the marbles are located and guarantees the accessibility of the total mass within the box and the matching of these accessible values. There is no need to test these claims: they have been proved to be logically implied by the formalism. Obviously, if one wants to do so, one can check, by resorting to appropriate tests which are governed by the precise laws of the theory, that things are just as we have stated even though this procedure is conceptually irrelevant. In spite of the fact that tests for the positions of all marbles and for the total mass within the box could require a remarkable time, let us assume that one can cope with this program within the indicated
time interval.

However we have now to take into account other possibilities. Since we have accepted the challenge of assuming that the number of marbles is as large as one wants, we have to point out that a single localization of a marble can occur at a place which is very far from its centre of mass position. Equation (20) shows that a single localization taking place at a point separated from the centre of mass of the marble by a distance of $10^{14} \text{cm}$ (i.e. of about 1 light-day) will displace the marble about 1 m. Obviously, a localization has a probability of about $e^{-10^{50}}$ of occurring at such a distance but, since we are playing the peculiar game of putting no limit to the number of marbles, one can very well claim that some marble will certainly jump in and out at every localization. And this remark (even though we do not like to play with science fiction arguments) gives more strength to our position than to the one of Clifton and Monton [1999].

According to the GRW theory with its ontology, the situation is perfectly clear: at any time a certain number of marbles are in the box and a certain number are out of the box and the corresponding statements about the mass within and outside the box match with those concerning the locations of the individual marbles. The situation, however, changes every millionth of a second. Thus, if one thinks that to tackle the conceptual problem of the counting anomaly requires a line (which we will analyze in the next section) of the type of the one of Clifton and Monton [1999] of operationalizing the counting process through measuring instruments, then he must first of all assume that it is possible to perform the incredible task of testing the positions of all particles and the total mass within the box in much less than a millionth of a second.

The general picture should now be clear: for any state like (21) the assertions concerning each particle being in or out of the box and those about the number of particles which are in the box agree perfectly. The agreement is clearly implied and guaranteed by the very formal structure of the theory.
and by the only physically meaningful interpretation which allows to speak of accessible mass distribution within the theory (i.e. by the very ontology of the model). To reach this conclusion there is no need to actually consider practical ways to perform tests identifying whether each given marble is in or out and the total number of particles which are in the box. At any rate, if one has the time and the opportunity to perform the tests, one will never meet a contradiction with the enumeration principle.

We pass now to analyze the arguments of Section 4 of Clifton and Monton’s paper.

5 A Critical Analysis of the Request to Operationalize the Process of Counting Marbles

The line chosen by Clifton and Monton [1999] to tackle the problem under discussion consists in proving, by operationalizing the counting process, that the violation of the enumeration principle within theories of the GRW type cannot be put into evidence. In fact, as already mentioned, the authors claim that

The trouble is that Lewis fails to operationalize the process of counting marbles by explicitly modelling the process itself in terms of collapsing GRW wavefunctions (Clifton and Monton [1999], p. 19).

The unappropriateness of such a point of view has already been stressed in section 3. Within GRW’s theory no measurement process ever occurs, the only physical processes being interactions among physical systems governed by universal laws. In a case like the one under consideration, since both the marbles to be counted and the measurement instruments which are devised to count them are macroscopic systems which have to be put exactly
on the same grounds from the point of view of the spontaneous localization processes affecting them, to operationalize the counting process amounts to make uselessly more complicated the problem. The theory is absolutely explicit about the position properties of the marbles and there is no need to resort to measuring apparata.

Leaving aside this obvious remarks, we feel the necessity of stressing further serious drawbacks which affect the central section 4 of the Clifton and Monton [1999] paper. In order to do this we have to follow the authors in their attempt to operationalize the counting process. They consider first of all \( n \) apparataxes, one for each marble, devised to detect whether each marble is or is not in the box. The process involves correlating the \(|in\rangle\) and \(|out\rangle\) states of the marble to orthogonal states of a macroscopic measuring apparatus. This process should lead from the state

\[
[(\alpha |in\rangle_1 + \beta |out\rangle_1) \otimes (\alpha |in\rangle_2 + \beta |out\rangle_2) \otimes ... \otimes (\alpha |in\rangle_n + \beta |out\rangle_n)] \otimes |ready\rangle_{M1} \otimes ... \otimes |ready\rangle_{Mn}
\]

(22)

to the state

\[
(\alpha |in\rangle_1 |IN\rangle_{M1} + \beta |out\rangle_1 |OUT\rangle_{M1}) \otimes ... \\
\otimes (\alpha |in\rangle_n |IN\rangle_{Mn} + \beta |out\rangle_n |OUT\rangle_{Mn})
\]

(23)

with obvious meaning of the symbols.

Here we cannot avoid stressing a fundamental fact: the apparataxes, to allow one to know whether they indicate IN or OUT must exhibit some macroscopic difference\(^5\). Such macroscopic differences, in perfect agreement with the GRW ontology, will be related to different locations of some macroscopic part of the apparataxes themselves in order to allow the observer’s reading of the ‘pointer’ to get the desired information. However, since no

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\(^5\)In fact, if the orthogonality of the final apparatus states would be correlated, e.g., to one of their atoms being in the fundamental or in the first excited state, respectively, such systems would not be of any use for our purposes.
state of a macroscopic object can have compact support in configuration space, also the states of the apparatuses will be superpositions of states corresponding to different locations of ‘their pointer’. Thus, one must accept that the correct final apparatus states will be of the type:

\[
|IN\rangle_{Mi} \rightarrow |\tilde{IN}\rangle_{Mi} = \gamma |IN\rangle_{Mi} + \delta |OUT\rangle_{Mi}, \tag{24}
\]

\[
|OUT\rangle_{Mi} \rightarrow |\tilde{OUT}\rangle_{Mi} = \delta |IN\rangle_{Mi} + \gamma |OUT\rangle_{Mi}, \tag{25}
\]

with \(|\gamma|^2 \gg \gg \delta^2\). Thus, what is the usefulness of resorting to measuring instruments to identify the positions of the marbles if one has then to identify the position of the pointer of the instrument to become aware of the outcome?

According to Clifton and Monton [1999], once one has a state like (23), two further steps are still necessary to acquire knowledge about the number of marbles which are in the box. First of all one needs a further apparatus \(M\) whose associated observable \(O\) has \(n+1\) eigenvalues \(o_i\) where

the \(o_i\)-eigenspace of the operator associated with \(O\) is the subspace spanned by all the terms in the superposition (16) [our eq. (23)] which have as coefficient \(\alpha^i \beta^{n-i}\) (Clifton and Monton [1999], p. 21).

When this is done one gets a state which is a superposition of products of the (marbles+counting apparatus) states and states of the kind \(|O = k\rangle_M\) specifying that precisely \(k\) marbles are in the state \(|in\rangle\) and the \(k\) associated operators \(M_i\) are in the state \(|IN\rangle_{Mi}\) (obviously, if \(k < n\) the state associated to \(|O = k\rangle_M\) will be a superposition of states in which the individual marbles are in different positions but precisely \(k\) of them are in the state \(|in\rangle\)). Since the state is a superposition, a violation of the enumeration principle is still there. But, the authors state:

this does not mean that a failure of the rules of counting has now become manifest. (Clifton and Monton [1999], p. 21)
Why it is so? According to Clifton and Monton:

[because] the [resulting] state is highly unstable given the GRW dynamics, since we see from [its form] that it is an entangled superposition of states of macroscopic systems, where [its] various terms markedly differ as to the location of the pointer on M’s dial that register the value of $O$ (Clifton and Monton [1999], p. 21)

Accordingly, the GRW dynamics dictates a collapse on one of the terms of the superposition. If the collapse is on the term containing the state $|O = n'\rangle_M$, then there is no problem: in it exactly $n$ marbles are in the state $|\text{in}\rangle$, exactly $n$ apparata are in the state $|\text{IN}\rangle$, and $M$ has registered $O = n$. However, if a different value of $O$ occurs, e.g., $O = k$, reduction will produce an entangled state. Once more this does not give rise to any problem:

since its terms (pairwise) differ as to location of at least one of the marbles, and since the $M_i$ apparatuses and marbles are macroscopic, ..., there will a further quick, effective collapse to one of the terms [with precisely $k$ marbles in state $|\text{in}\rangle$ and precisely $k$ $M_i$ apparatuses in state $|\text{IN}\rangle$] (Clifton and Monton [1999], p. 22).

Thus, once more, the number of particles which are in the box matches the reading of the apparatus $M$. Concluding, the violation of the counting rule is still there, but it cannot be revealed.

We stress that the above argument is quite puzzling: the reasons for which the time hierarchy suggested by the authors should be respected are totally obscure. The marbles are macroscopic systems and their masses may very well be comparable or even larger that those of the pointers both of the apparatuses $M_i$ and of $M$. Why the GRW dynamics has to be suspended up to the time in which the pointer of $M$ is localized, and subsequently (in
the case that \( O \neq n \) up to the moment in which all pointers of the \( M_i \)'s are localized, is really a big mistery.

A final remark is at order. Since \( M \)'s reading is given by its pointer location (as explicitly stated by Clifton and Monton [1999]), a consistent use of the GRW dynamics implies that it cannot be perfectly localized: it will be affected by the ‘tails problem’ just as all macroscopic systems. Thus, if one follows the authors in their strange argument, one should conclude that (with a small probability) the state can be the one in which \( n \) marbles are in the state \( |\text{in}\rangle \), \( n \) apparatuses \( M_i \) register \( |\text{IN}\rangle \), but the pointer of the apparatus \( M \) can be reduced on a state corresponding to its pointing at \( k \neq n \)!

The reader should have understood that the reasons for the peculiar account of the whole process described by Clifton and Monton [1999] derive uniquely from the fact that the authors have betrayed, from the very beginning, the real spirit of the dynamical reduction models, i.e. the basic fact that what these models claim to be true of the world out there is the mass distribution in the whole universe in those regions in which it is accessible. All arguments must be developed keeping in mind this fundamental point which Lewis [1997] and Clifton and Monton [1999] have disregarded. If one does so, a very clear and simple picture of the problem emerges, and no conceptual difficulties whatsoever arise.

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