String breaking mechanisms induced by magnetic and electric condensates

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The normal confining phase of gauge theories is characterised by the condensation of magnetic monopoles and center vortices. Sometimes in coupled gauge system one finds another phase with simultaneous condensation of electric and magnetic charges. In both phases the confining string breaks down at a given scale because of pair creation, however the mechanism is different. In the former case the string breaking is a mixing phenomenon which is invisible in the Wilson loop. On the contrary, in presence of both electric and magnetic condensates the string breaking can be observed even in the Wilson loops. Numerical experiments on a 3D $Z_2$ gauge-Higgs system neatly show this new phenomenon.

1. INTRODUCTION

Center vortices and magnetic monopoles are widely believed to be the most important degrees of freedom for confinement in Yang Mills theories. Plausibility arguments suggest that large center vortices or magnetic condensates imply area law for the Wilson loop.

It is worth noting that while these arguments are generally applied to pure Yang Mills models, they hold true also in gauge theories coupled to matter. This can explain a surprising phenomenon observed in all these kind of coupled systems: although the potential between static sources flattens at large distances because of the screening produced by pair creation, this flattening (called string breaking) is invisible in the Wilson loop: it continues to obey an area law in full QCD even at distances where the static charges are completely screened. The point is that in gauge theories coupled to matter the basis of the operators has to be enlarged in order to get a reliable estimate of the potential. In this way it has been observed the breaking of the confining string in Higgs models and in QCD. So the fact that large Wilson loops obey an area law even in coupled systems may be considered as a further support to the plausibility arguments for the confinement mechanisms.

In this work we use such a special property, which is also suggested by the underlying string picture of the confining phase, as a tool to test the validity of various confinement criteria. Indeed in coupled gauge systems there are different vacua, distinguished by different entities which condense. In the simple model we use as a guide, namely the 3D $Z_2$ gauge-Higgs model, there are five kinds of vacua (see Fig.1). A simple, rigorous, argument shows that the only confining vacua (i.e. those with Wilson loop obeying an area law) are those in which both magnetic monopoles and center vortices condense. Luckily there is a non-confining vacuum with monopoles but without vortices (region III of Fig.1), where this result has been checked.

This model has two different confining vacua. Both have center vortex and magnetic monopole condensates, but one of them has also an electric condensate, i.e. the Higgs field has a vacuum expectation value different from zero.

The former fulfils all the requirements of the confinement criteria, so an area law is expected; here we found that the Wilson loop obeys a perfect area law even at distances larger than five times the string breaking scale.

The latter can be identified with the torn phase predicted in Ref.: the Wilson loop obeys an area law below a given scale. Above this threshold we observed for the first time a clear signal of string breaking (see Fig.3). A similar phase in 4D SU(2)-Higgs model has been reported at this conference.
2. THE MODEL

The action of a 3D $\mathbb{Z}_2$ gauge theory coupled to matter can be written as

$$S(\beta_G, \beta_I) = -\beta_I \sum_{(ij)} \sigma_i U_{ij} \sigma_j - \beta_G \sum_{\text{plaq.}} U_\Box,$$

where both the link variable $U_{ij} \equiv U_\ell$ and the matter field $\sigma_i$ take values $\pm 1$ and $U_\Box = \prod_{\ell \in \Box} U_\ell$. In this model the construction of center vortex configurations is straightforward: to each frustrated plaquette (i.e. $U_\Box = -1$) assign a vortex in the dual link. Since the product of the plaquettes belonging to any elementary cube is 1, center vortices form closed subgraphs of even coordination number. Thus a connected vortex subgraph contributes to a given Wilson loop $W(C)$ only if an odd number of lines are linked to it. The Wilson loop is a vortex counter modulo 2. Finite vortex subgraphs contribute only to the perimeter term of $\langle W(C) \rangle$. This expectation value obeys an area law only if there is an infinite, percolating, FK cluster in the dual phase. An infinite FK cluster in the dual description corresponds exactly to the dual Higgs mechanism of 't Hooft and Mandelstam: a disorder field $\tilde{\sigma}$ carrying a $\mathbb{Z}_2$ magnetic charge acquires a non-vanishing vacuum expectation value. Thus confinement implies both infinite center vortex subgraph and magnetic monopole condensation. In pure gauge theory these two requirements coincide, while in the coupled system the situation is more tricky.

![Phase diagram](image.png)

Figure 1. Phase diagram of the $\mathbb{Z}_2$ gauge Higgs model.

We can now apply the same line of reasoning used for the center vortices. Finite clusters can link with the 't Hooft loop only along its perimeter. Therefore they contribute only to the perimeter term. Owing to the duality relation (3), the Wilson loop obeys an area law only if there is an infinite, percolating, FK cluster in the dual phase. An infinite FK cluster in the dual description corresponds exactly to the dual Higgs mechanism of 't Hooft and Mandelstam: a disorder field $\tilde{\sigma}$ carrying a $\mathbb{Z}_2$ magnetic charge acquires a non-vanishing vacuum expectation value. Thus confinement implies both infinite center vortex subgraph and magnetic monopole condensation. In pure gauge theory these two requirements coincide, while in the coupled system the situation is more tricky.

In the coupled theory we have two kind of interesting subgraphs: center vortices or FK clusters in the dual lattice describe the gauge field degrees of freedom; FK clusters in the direct lattice describe the charged Higgs matter. There are three kind of infinite clusters: i) FK cluster in the dual lattice (magnetic condensate), ii) FK cluster in the direct lattice (electric condensate), iii) large center vortices in the dual lattice. The size of these clusters in finite lattices is proportional to the volume. Straightforward numerical experiments show that they are distributed in the phase diagram according to Fig.1 and Tab.1

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1 This is a $\mathbb{Z}_2$ analogue of the Meissner effect.
Tab. 1

| phase | magnetic condensate | electric condensate | large vortices |
|-------|---------------------|---------------------|---------------|
| I     | yes                 | no                  | yes           |
| II    | yes                 | yes                 | yes           |
| III   | yes                 | yes                 | no            |
| IV    | no                  | yes                 | no            |
| V     | no                  | no                  | no            |

Region III has a magnetic condensate but no large center vortices then there is no confinement, as confirmed by numerical tests. The region I and II are confining. The former is a normal confining phase [7]: the potential extracted from an enlarged basis shows the expected string breaking (see Fig.2), while the Wilson loop obeys a perfect area law even at large scale. This has been checked up to distances of the order of five times the string breaking scale [13].

Figure 2. The static potential in the region I

The latter is a torn phase produced by the Meissner effect we alluded before: the simultaneous presence of infinite clusters in the direct and the dual lattices with the constraint of no frustration induces strong correlations on vortex lines which produce a visible string breaking effect also in the Wilson loop, as Fig.3 clearly shows.

Figure 3. $-\ln W(R, R)$ as a function of $R$ in the region II (torn phase).

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