3D modelling of macroscopic force-free effects in superconducting thin films and rectangular prisms

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Abstract
In superconductors, when the magnetic field has a parallel component to the current density \( \mathbf{J} \) force-free effects appear due to flux cutting and crossing. This results in an anisotropic \( \mathbf{E}(\mathbf{J}) \) relation, with \( \mathbf{E} \) being the electric field. Understanding force-free effects is interesting not only for the design of superconducting power and magnet applications but also for material characterization. This work develops and applies a fast and accurate computer modeling method based on a variational approach that can handle force-free anisotropic \( \mathbf{E}(\mathbf{J}) \) relations and perform fully three-dimensional (3D) calculations. We present a systematic study of force-free effects in rectangular thin films and prisms with several finite thicknesses under applied magnetic fields with arbitrary angle \( \theta \) with the surface. The results are compared with the same situation with the isotropic \( \mathbf{E}(\mathbf{J}) \) relation. The thin film situation shows gradual critical current density penetration and a general increase of the magnitude of the magnetization with the angle \( \theta \) but a minimum at the remnant state of the magnetization loop. The prism model presents current paths with 3D bending for all angles \( \theta \). The average current density over the thickness agrees very well with the thin film model except for the highest angles. The prism hysteresis loops reveal a peak after the remnant state, which is due to the parallel component of the self-magnetic-field and is implicitly neglected for thin films. The presented numerical method shows the capability to take force-free situations into account for general 3D situations with a high number of degrees of freedom. The results reveal new features of force-free effects in thin films and prisms.

Keywords: force free effects, 3D modelling, anisotropy, thin films, superconducting bulks

(Some figures may appear in colour only in the online journal)

1. Introduction

Type II superconductors are essential for large bore or high-field magnets [1–4] and are promising for power applications, such as motors for aeroplanes [5, 6] or ship propulsion [7, 8], generators [9–11], grid power-transmission cables [12, 13], transformers [14–17], and fault-current limiters [18–22]. The critical current density, \( J_c \), of type II superconductors depends on the magnitude and angle of the local magnetic field. There are three types of anisotropy which we call ‘intrinsic’, ‘de-pinning’, and ‘force free’ anisotropy.

The ‘intrinsic’ anisotropy is the following. Certain superconductors present an axis with suppressed superconductivity, where the critical current density is lower. In cuprates, for instance, the critical current density in the \( c \) crystallographic axis is much smaller than in the \( ab \) plane. There is also important anisotropy in REBCO vicinal films due to flux channeling [23, 24].

The ‘de-pinning’ anisotropy of \( J_c \) is due to anisotropic maximum pinning forces caused by either anisotropic pinning centres or anisotropic vortex cores [25]. When the current density \( \mathbf{J} \) is perpendicular to \( \mathbf{B} \) and the electric field \( \mathbf{E} \) is parallel to \( \mathbf{J} \), the anisotropy of \( J_c \) is always due to de-pinning anisotropy. This kind of anisotropy is important for high-temperature superconductors, such as (Bi,Pb)$_2$Sr$_2$Ca$_2$Cu$_2$O$_{10}$ and REBa$_2$Cu$_3$O$_{7−x}$, and iron-based superconductors. The magnetic field dependence and anisotropy has an impact on the performance of magnets and power applications.
Another type of anisotropy is ‘force-free’ anisotropy, which appears when the current density presents a substantial parallel component with the local magnetic field. The parallel \( J \) component does not contribute to the macroscopic driving force (or Lorentz force) on the vortices, \( \mathbf{F} = \mathbf{J} \times \mathbf{B} \), being the microscopic vortex dynamics for \( \mathbf{B}=\mathbf{J} \) a complex process that includes flux cutting and crossing [26–29]. Many power applications with rotating applied fields are influenced with force-free effects. Force-free anisotropy also appears for intrinsically isotropic materials.

There are many macroscopical physical models on force-free anisotropy that regard both flux cutting and de-pinning, such as the double critical state model (CSM) [28], the general double CSM [30], Brant and Mikitet extended double CSM [31] and the elliptic CSM [32]. A valuable comparison of these models can be found in [29]. There are many experimental works on de-pinning anisotropy, such as state of the art REBCO commercial tapes [33–39], Bi2223 tapes [40, 41] and iron based [42–45] conductors, as well as a database of anisotropic \( J \), measurements [46]. Correction of self-magnetic field in critical current, \( I_c \), measurements is also important [47, 48].

In this article, we focus on force-free effects, which cause anisotropy when \( J \) has a parallel component to \( B \) (or \( E \) is not parallel to \( J \)). We also base our study in modeling only. The object of study are thin films and rectangular prisms of several thicknesses with various angles of the applied fields, with a special focus on the current path and hysteresis loops. We compared results with the isotropic situation, in order to understand the observed behavior. The modeling is performed by minimum electromagnetic entropy production in three dimensions [49], which is suitable for 3D calculations, and avoids spending variables in the air. Moreover, the method enables force-free anisotropic power laws [50], which is the core of this study.

Magnetic field mapping has been proven to be a useful tool to obtain the current density and electric field in thin films or thin bulks [23, 24, 51, 52]. However, most works about anisotropy regard intrinsic anisotropy. Only [27] observes the magnetic flux penetration under force-free situations. The calculated current density in this article could motivate experimentalists to measure these force-free situations in detail. In addition, magnetization measurements including force-free effects have not been published. Magnetization measurements under tilted applied field either avoid the contribution from force-free effects occurring at the ends of long samples [53] or suppress that contribution by analysis [54]. Again, the results of this article may motivate these experiments.

2. Mathematical model

2.1. Minimum electromagnetic entropy production in 3D (MEMEP 3D) method

This study is based on MEMEP 3D [49], which is a variational method. The method solves the effective magnetization \( \mathbf{T} \), defined as

\[
\mathbf{\nabla} \times \mathbf{T} = \mathbf{J},
\]

where \( \mathbf{J} \) is the current density. In addition to the magnetization case, MEMEP 3D can also take transport currents into account, after adding an extra term in (1) (see [49]). We take the interpretation that \( \mathbf{T} \) is an effective magnetization due to the screening currents. The \( \mathbf{T} \) vector is non-zero only inside the sample, and hence the method avoids discretization of the air around the sample. The advantages of MEMEP 3D are reduction of computing time, enabling an increase of total number of degrees of freedom in the sample volume, and efficient parallelization. The general equation of electric field \( \mathbf{E} \) is derived from Maxwell equations

\[
\mathbf{E}(\mathbf{J}) = -\mathbf{\nabla} \phi - \mathbf{\nabla} \phi, \quad (2)\\
\mathbf{\nabla} \cdot \mathbf{J} = 0, \quad (3)
\]

where \( \phi \) is the scalar potential.

In the Coulomb’s gauge, we can split the vector potential \( \mathbf{A} \) into \( \mathbf{A}_f \) and \( \mathbf{A}_a \), where \( \mathbf{A}_f \) is the vector potential contributed by the applied field and \( \mathbf{A}_a \) is the vector potential contributed by the current density inside the sample. Then, \( \mathbf{A}_a \) is

\[
\mathbf{A}_a(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V d^3\mathbf{r}' \cdot \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}' - \mathbf{r}|} = \frac{\mu_0}{4\pi} \int_V d^3\mathbf{r}' \cdot \mathbf{\nabla}' \times \mathbf{T}(\mathbf{r}'),
\]

where \( \mathbf{r} \) and \( \mathbf{r}' \) are position vectors.

According to the definition of \( \mathbf{T} \), we can rewrite equations (2) and (3) into

\[
\mathbf{E}(\mathbf{\nabla} \times \mathbf{T}) = -\mathbf{\nabla} \phi, \quad (5)\\
\mathbf{\nabla} \cdot (\mathbf{\nabla} \times \mathbf{T}) = 0. \quad (6)
\]

The second equation is always satisfied, and hence we must solve only the first equation. As it was shown in [49], solving equation (5) is equivalent to minimizing the functional

\[
L_T = \int_V d^3\mathbf{r} \left\{ \frac{1}{2} \frac{\Delta \mathbf{A}_f}{\Delta t} \right\} \cdot (\mathbf{\nabla} \times \Delta \mathbf{T}) + \frac{\Delta \mathbf{A}_a}{\Delta t} \cdot (\mathbf{\nabla} \times \Delta \mathbf{T}) + U(\mathbf{\nabla} \times \mathbf{T}),
\]

where \( U \) is the dissipation factor, defined as

\[
U(\mathbf{J}) = \int_0^J \mathbf{E}(\mathbf{J}^\prime) \cdot d\mathbf{J}^\prime.
\]

The functional can include any \( \mathbf{E}(\mathbf{J}) \) relation with its corresponding dissipation factor.

The functional is solved in the time domain in time steps like \( t = t_0 + \Delta t \), where \( t \) is the present time, \( t_0 \) is the previous time step and \( \Delta t \) is the time between two time steps. Then, we define \( \mathbf{T}, \mathbf{A}_f, \mathbf{A}_a \), as the value of the corresponding variables at the present time step; \( \Delta \mathbf{T}, \Delta \mathbf{A}_f, \Delta \mathbf{A}_a \) are the change of the variables between two time steps; and \( t_0, \mathbf{A}_{f0}, \mathbf{A}_{a0} \) are the variables from the previous time step. In this work, the applied magnetic field \( \mathbf{B}_0 \) is uniform and \( \Delta t \) is constant, although the method enables non-uniform \( \mathbf{B}_0 \) and variable \( \Delta t \).
Figure 1. Using a magnetic field dependence for \( J_{c,\parallel} \) and \( J_{c,\perp} \) avoids indeterminations at \( B = 0 \). (a) Elliptic double CSM. (b) Anisotropic Kim model.

In this work, we mesh the sample uniformly into rectangular prism-shaped cells, as detailed in [49]. However, the formalism enables to use non-uniform mesh and/or tetrahedral meshing.

The magnetic moment \( \mathbf{m} \) is calculated by equation

\[
\mathbf{m} = \frac{1}{2} \int d^3 \mathbf{r} \mathbf{J} \times \mathbf{r},
\]

where \( \mathbf{r} \) is a position vector of interpolated \( \mathbf{J} \) at the centre of the cell. The magnetization is \( \mathbf{M} = \mathbf{m}/V \) and \( V \) is volume of the sample.

### 2.2. \( E(J) \) relation

In a previous study [55], we used the isotropic power law as \( E(\mathbf{J}) \) relation in the functional (7)

\[
E(\mathbf{J}) = E_i \left( \frac{\mathbf{J}}{J_i} \right)^n \mathbf{J},
\]

where \( \mathbf{B} \cdot \mathbf{J}, E_i, \mathbf{J} \), and \( J_i \) is the critical electric field \( 10^{-4} \text{ V m}^{-1} \), \( J_i \) is the critical current density, and \( n \) is the power law exponent or \( n \) factor. The \( n \) factor depends on the quality of the superconducting materials, temperature and local magnetic field \( \mathbf{B} \).

Usually, these \( E(\mathbf{J}) \) relations are extracted from global measurements of a certain sample, such as the current–voltage of wires or tapes or relaxation in magnetization measurements, and from where local \( E(\mathbf{J}) \) relations are derived. The Bean CSM [56, 57] corresponds to \( n \to \infty \), but real superconductors present smaller \( n \) factors, ranging from around 10 to the order of 100. The case of \( n = 100 \) is practically equivalent to the CSM. The dissipation factor for isotropic \( E(\mathbf{J}) \) relation of (10) is

\[
U(\mathbf{J}) = \frac{E_i J_c}{1 + n} \left( \frac{\mathbf{J}}{J_i} \right)^{n+1}.
\]

In this article, we focus on the anisotropic case, in order to model the force-free effects with anisotropic power law [50]

\[
E(\mathbf{J}) = 2m_0 U_0 \left( \frac{J_{\parallel}}{J_{\parallel,0}} \right)^2 \left( \frac{J_{\perp}}{J_{\perp,0}} \right)^{n-1} \left( \frac{J_{\parallel}}{J_{\parallel,0}} \mathbf{e}_\parallel + \frac{J_{\perp}}{J_{\perp,0}} \mathbf{e}_\perp \right),
\]

where

\[
m_0 = (n + 1)/2, \quad U_0 = E_i J_c/(n + 1), \quad J_0 = \mathbf{J} \cdot \mathbf{B}/|\mathbf{B}|, \quad J_{\parallel} = (\mathbf{J} \times \mathbf{B})/|\mathbf{B}|, \quad J_{\perp} = (\mathbf{J} \cdot \mathbf{B})/|\mathbf{B}|
\]

\( J_{\parallel} \) and \( J_{\perp} \) are critical current densities parallel and perpendicular to \( \mathbf{B} \), respectively. Vector \( \mathbf{B} \) is the local magnetic field and \( \mathbf{e}_\parallel, \mathbf{e}_\perp \) are unit vectors of the current density, where \( \mathbf{e}_\parallel = \mathbf{B}/|\mathbf{B}|, \mathbf{e}_\perp = \mathbf{J}_\parallel/|\mathbf{J}_\parallel| \) and \( \mathbf{J}_\parallel = \mathbf{J} - J_0 \mathbf{e}_\parallel \). Notice that \( \mathbf{J} = J_0 \mathbf{e}_\parallel + J_{\parallel} \mathbf{e}_\parallel + J_{\perp} \mathbf{e}_\perp \) is always positive. The applied magnetic field \( \mathbf{B}_s \) is not always perpendicular to the sample surface (figure 3(a)). The corresponding anisotropic dissipation factor is

\[
U(\mathbf{J}, \mathbf{B}) = U_0 \left[ \left( \frac{J_{\parallel}}{J_{\parallel,0}} \right)^2 + \left( \frac{J_{\perp}}{J_{\perp,0}} \right)^{n-1} \right].
\]

The anisotropic power law becomes the elliptic CSM for large enough \( m_0 \) or \( n \) with two critical current densities \( J_{\parallel,0} \), \( J_{\perp,0} \), which apply according to direction of the local magnetic field \( \mathbf{B} \). The problem of the anisotropic \( U(\mathbf{J}, \mathbf{B}) \) relation is the uncertainty of the unit vector of local magnetic field \( \mathbf{B} \) with very low or zero \( |\mathbf{B}| \). In the samples there exist places where the local magnetic field vanishes. We suggest the following two options in order to remove this uncertainty.

The first option is to use a sharp \( J_{\parallel,0} \) (\( \mathbf{B} \)) and \( J_{\perp,0} \) (\( \mathbf{B} \)) dependence, where at \( |\mathbf{B}| \to \infty \) they follow \( J_{\parallel,0} \to J_{\parallel,0} \) and at \( |\mathbf{B}| = 0 \), \( J_{\perp,0} \to J_{\perp,0} \) with a linear transition between \( |\mathbf{B}| = 0 \) and a certain magnetic field \( |\mathbf{B}| = B_0 \), being \( B_0 \) a small magnetic field (figure 1(a)). The limit of \( B_0 \to 0 \) corresponds to the elliptic critical state model [58]. For simplicity, we consider only this linear dependence of \( \mathbf{B} \) for \( J_{\perp,0} \), keeping \( J_{\perp,0} \) as constant. The reason is to reproduce the Bean CSM for perpendicular applied fields.

The magnetic field is calculated from the current density after the functional is minimized. The functional is solved iteratively [49]; at the first iteration, \( \mathbf{T} \) is calculated with \( \mathbf{B}_0 = \mathbf{B}_0 \) and \( \mathbf{B}_0 = 0 \), being \( \mathbf{B}_0 \) the magnetic field generated by \( \mathbf{J} \) at the previous time step, the second iteration starts with \( \mathbf{B}_0 = 0 \) calculated from \( \mathbf{J} \) at the previous iteration, where \( \mathbf{J} = \nabla \times \mathbf{T} \); iterations are repeated until we find a solution with given tolerance in each component of \( \mathbf{J} \). The sharp \( J_{\parallel,0}(\mathbf{B}) \) dependence causes numerical problems in this iterative method, since a small error in \( \mathbf{B} \) causes a large error in \( \mathbf{J} \) in the next iteration step.

In order to avoid this numerical problem, the functional is minimized in a certain time \( t \) with the total magnetic field \( \mathbf{B} \)
from the previous time step \( \mathbf{B}(t - \Delta t) \). The vector potential \( \mathbf{A} \) is still calculated according the present time \( t \). This is the reason why the remanent state is shifted by \( \Delta t \) in the results. The negative effect of that assumption can be decreased by increasing the number of time steps in one period of applied magnetic field.

Another option to avoid the problems at \( |\mathbf{B}| \to 0 \) is to assume Kim’s model for both \( J_{\|}(B) \) and \( J_{\perp}(B) \) dependences, where \( J_{\|}(B = 0) = J_{\perp}(B = 0) \equiv J_{0} \) and \( J_{\|} = J_{\perp} \) for \( B \to \infty \) (figure 1(b)). This Kim model is

\[
J_{\|}(B) = \frac{J_{0}}{\left(1 + \frac{|B|}{B_{0\|}}\right)^{m}},
\]

\[
J_{\perp}(B) = \frac{J_{0}}{\left(1 + \frac{|B|}{B_{0\perp}}\right)^{m}},
\]

where in this article we choose \( m = 0.5 \), \( B_{0\perp} = 20 \text{ mT} \), \( J_{0} = 3 \times 10^{10} \text{ A} \text{ m}^{-2} \) and \( B_{0\|} = 9B_{0\perp} \). so that \( J_{\|}(B \to \infty) = 3J_{\perp}(B \to \infty) \). For this case, the \( J_{\|}(B) \) and \( J_{\perp}(B) \) dependences are not sharp, and hence we use the original iterative method for magnetic field dependent \( J_{c} \). Then, \( \mathbf{J} \) at time \( t \) uses \( \mathbf{B} \) of the same time \( t \). Moreover, this smooth \( J_{\perp}(B) \), \( J_{\|}(B) \) dependence is more realistic than the elliptic CSM of figure 1(a), since the typical measured critical current densities decrease with the applied field.

2.3. Sector minimization

Reduction of computing time is of essential importance for 3D calculations. We already studied the case of parallel minimization by sectors, where sectors are overlapping by one cell [49]. In this article, we increase the overlapping of sectors in the following way. Now, the sectors are not overlapping to each other, and hence they share only the edge on the border, which are not solved (figure 2(a)). Then, we added another two sets of sectors, but the boundary in each set of sectors is shifted along the diagonal by 1/3 of the sector-diagonal size (figures 2(b), (c)). The edge in the boundary in the first set is solved at least once in some of the other two sets. The additional sets increase the memory usage, which is still low, but they decrease the computing time. Sets of the sectors are minimized in series one after the other, but sectors within each set are solved in parallel to achieve high efficiency of parallel computing. Although computing all three sets of sectors in parallel could further enhance parallelization, we have found that solving each set sequentially reduces computing time. The process of solving all three sets subsequently is repeated until the maximum difference in any component of \( \mathbf{T} \) between two iterations of the same set is below a certain tolerance. We use elongated cells, in order to improve the accuracy for a given number of cells, as detailed in the appendix.

3. Results and discussion

As a result of the variational model, we calculated two geometries like an infinitesimally thin film (or simply, ‘thin film’) and a thin prism with finite thickness (or ‘thin prism’) (figures 3(a), (b)). The force-free effects are modeled with the anisotropic power law in combination of either constant \( J_{\perp}, J_{\|} \) or Kim model \( J_{\perp}(B), J_{\|}(B) \). We calculated as well the pure isotropic case of a thin film and thin prism for comparison. The calculations are performed with two values of the \( n \) factor, 30 and 100, in order to have results close to the realistic values and analytical critical-state formulas, respectively.
For constant \( J_{\perp} \) and \( J_{\parallel} \), we actually use the sharp magnetic field dependence of figure 1(a), which requires to use the magnetic field calculated at the previous time step to evaluate \( J_{\perp}(B) \) and \( J_{\parallel}(B) \). We define the ‘quasi-remanent state’ as the time when the applied field in the \( J_{\perp}(B) \) evaluation vanishes. For constant \( J_{\perp}, J_{\parallel} \) this happens one step after the actual applied field vanishes. For Kim-like \( J_{\perp}(B) \) and \( J_{\parallel}(B) \), we use the same \( B \) as for the solved time step, and hence the quasi-remanent state corresponds to the actual remanent state.

3.1. Anisotropic force-free effects in films

In this section, we study square thin films of dimensions \( 12 \times 12 \text{ mm}^2 \) and thickness \( 1 \mu \text{m} \). We also take the common assumption of the thin film limit, which consists on averaging the electromagnetic properties over the sample thickness. For our method, this is achieved by taking only one cell along the sample thickness. We used a total number of degrees of freedom of 4200.

3.1.1. Power device situation. In this section, the magnetic applied field \( B_{a} \) has a sinusoidal waveform of 50 Hz and the same perpendicular, \( B_{a,z} \), component for all angles \( \theta \) with amplitude \( B_{a,z,0} = 50 \text{ mT} \) (figure 3(a)). The angle \( \theta = 0^\circ \) is completely perpendicular to the surface of the thin film. We calculated the cases with \( \theta = 0^\circ, 45^\circ, 60^\circ, 80^\circ \). For this study, the perpendicular critical current density \( J_{\perp} \) is equal to \( 3 \times 10^{10} \text{ A m}^{-2} \) and \( J_{\parallel} \) is three times higher. The dependence of \( J_{c} \) on the magnetic field is on figure 1(a), where we choose \( B_{s} = 1 \text{ mT} \). The \( n \) factor of the anisotropic power law is equal to 30, which is a realistic value for REBCO tapes in self-field.

The first case, with \( \theta = 0^\circ \), is shown on figure 4. The penetration of the current density to the film strip is explained by colour maps of \( |\mathbf{J}| \) normalized to \( J_{\perp} \), while the lines are current flux lines. The current density gradually penetrates to the sample after increasing the applied field (figure 4(a)), until it reaches almost saturated state at the peak of applied field (figure 4(b)). During the decrease of the applied field, current starts penetrating again from the edges of the sample with opposite sign till the centre. The quasi-remanent state, at \( B_{a} \approx 0 \text{ mT} \), presents symmetric penetration of \( J \) along both \( x \) and \( y \) axis (figure 4(c)). We show the first time step after remanence, \( B = 0 \), for comparison with the cases with \( \theta = 0^\circ \), where we use \( B \) of the previous time step in order to obtain \( J_{\parallel} \) (figure 4(a)).

The second case is for \( \theta = 45^\circ \) and applied field amplitude \( B_{a,m} = 70.7 \text{ mT} \) (figure 5). The force-free effects appear during the increase of the applied field (figure 5(a)). The current lines parallel to the \( x \) axis are more aligned with the direction of the applied field. Therefore, \( J_{||} \) becomes relevant, and hence current density at that direction is higher compared to the current density along the \( y \) axis. The current penetration depth is smaller from top and bottom at the peak (figure 5(b)) compared to that from the sides. The penetration depth of \( J_{x} \) from right and left is the same as for \( \theta = 0^\circ \), because \( J_{x} \) is still perpendicular to \( B_{a} \). The quasi-remanent state (figure 5(c)) with the applied field close to zero experiences the self-field as dominant component of the local magnetic field. Then, the self-field in the thin film approximation has only \( B_{s} \) component, which is completely perpendicular to the surface and the current density. Therefore, only \( J_{\perp} \) is relevant and the maximum \( J \) in the sample is decreased back to that value.

The last two cases, \( \theta = 60^\circ \) and \( 80^\circ \), present similar behavior. The penetration of \( J_{x} \) to the sample is even smaller during the increase of the applied field (figures 6(a), 7(a)), because of the higher angles \( \theta = 60^\circ, 80^\circ \). The maximum \( J_{x} \) component at the peak of the applied field (figures 6(b), 7(b)) is reaching 2.5 and 3 times \( J_{\perp} \), which is the value of \( J_{\parallel} \). Again, at remanent state (figures 6(c), 7(c)) the maximum \( J_{x} \) component is decreased back to values around \( J_{\perp} \) because of the self-field without any parallel component of the local magnetic field.
Figure 5. The same as figure 4 but for $\theta = 45^\circ$.

Figure 6. The same as figure 4 but for $\theta = 60^\circ$.

Figure 7. The same as figure 4 but for $\theta = 80^\circ$. For sufficiently large applied fields (a), (b) zones appear with $|\mathbf{J}| \approx J_{\perp}$ and $|\mathbf{J}| \approx J_{\parallel}$, while at the quasi-remanent state (c), $|\mathbf{J}|$ is limited to $J_{\perp}$. 
The hysteresis loops for all angles $\theta$ of the applied field are shown in figure 8(a). The larger the applied field angle, the higher the impact of $J_c$, and hence there exist places with the current density around $J_c$. The current density around $J_c$ creates higher magnetic moment in comparison to $\theta = 0^\circ$ where $J_c$ is limited by $J_{c\perp}$. The self-field is dominant at the range of the applied field $\pm 5 \text{ mT}$, causing a mostly perpendicular local magnetic field, and hence $J_c$ is again limited to $J_{c\perp}$. This is the reason why the magnetization is decreasing back to the same value as in the case of $\theta = 0^\circ$. We calculated the same situation with isotropic power law. The results of the isotropic case are the same for each angle $\theta$ because the perpendicular applied field is the same as for $\theta = 0^\circ$ (see magnetization loops in figure 8(b)). Consistently, these magnetization loops also agree with the anisotropic case with $\theta = 0^\circ$, since $J_c = J_{c\perp}$ for the whole loop (figure 8(a)).

3.1.2. Magnet situation. The next calculation assumes the same parameters and geometry as the previous cases. The difference is in the $n$ factor, with value 100, triangular waveform of the applied field of $1 \text{ mHz}$ frequency and amplitude $B_{a,\text{m}} = 150 \text{ mT}$. This magnetization is qualitatively similar to magnet charge and discharge. Moreover, the situation is qualitatively similar to measurements on SQUID or vibrating-sample magnetometers. Many magnetometers enable to measure both components of the magnetic moment, enabling to extract the component perpendicular to the sample surface.

The angles of applied field are the same as previous calculations, being $\theta = 0^\circ$, $45^\circ$, $60^\circ$, $80^\circ$. The high $n$ factor reduces the current density to values equal or below $J_{c\perp}$ or $J_{c||}$. Another reason for reduction of current density is the very low frequency of the applied field, which allows higher flux relaxation. The constant ramp rate causes that the magnetization loops are flat after the sample is fully saturated (figure 9(b)). The case of $\theta = 0^\circ$ induces only current density perpendicular to the applied field, and hence magnetization loop is horizontal at the remanent state. Again, we see a minimum at remanence for higher $\theta$.

The last thin film example assumes anisotropic power law with two critical current densities, which depends on the magnetic field according Kim model $J_{c\perp}(B)$, $J_{c||}(B)$. The dependence is on figure 1(b). The magnetic field $\mathbf{B}$ is calculated in the same time step $\mathbf{B}(t)$ as the functional is minimized, and hence now the remanent state is straightforwardly for $\mathbf{B} = 0$ as it is shown on figure figure 10(b).
$B_x$ component of the maximum applied field is 300 mT and it is the same for all angles $\theta$. The magnetization of the sample (figure 10(a)) is higher close to the remanent state, since the applied field is close to zero and the self-field only slightly decreases the critical current density. With increasing the applied field from the zero-field-cool situation, the sample becomes fully saturated already at 40 mT. With further increase of the applied field, the Kim dependence causes a decrease in $Jc_{||}$, decreasing the magnitude of the magnetization. The highest magnetization is at the applied field with $\theta = 80^\circ$, in spite of $|B_x|$ being the largest and hence reducing the most $Jc_{\perp}$ and $Jc_{||}$. The cause is that there still exist areas with current density around $Jc_{||}$, at the remanent state, we can see again reduction of magnetization to the level of $\theta = 0^\circ$ (figure 10(b)).

3.2. Anisotropic force-free effects in prisms

3.2.1. Current density in prisms. In the following, we analyze the force-free effects in prisms. We model the prisms with the same dimensions as thin film $12 \times 12$ mm but thickness 1 mm. The mesh of the sample is created by elongated cells, which we explain in the appendix. The total number of cells is $31 \times 31 \times 15$, which corresponds to around 43 000 degrees of freedom. The frequency of the applied field is 50 Hz and the amplitude of the $z$ component of $B_x$ is 50 mT for all angles $\theta = 0^\circ, 45^\circ, 60^\circ, 80^\circ$; and hence the total amplitude is $B_{x,m} = 50$, 70.7, 100 and 287.9 mT, respectively. The critical current densities are chosen so that the sheet critical current density $K_i \equiv Jc d$ is the same as for the thin film, being $d$ the sample thickness. Further values are $Jc_{\perp} = 3 \times 10^3$ A mm$^{-2}$, $Jc_{||} = 3Jc_{\perp}$ and $n = 30$.

The force-free effects are modeled with the anisotropic power law and the sharp dependence of $Jc_{||}$ with the magnetic field of figure 1(a). Then, the functional is minimized with the magnetic field from the previous time step like in the case of thin film.

The first case is with applied field $\theta = 0^\circ$. We calculated the average current density over thickness. The penetration of the average current density into the prism at the peak of applied field is on figure 11, where we add the case of thin film for comparison. There is a small difference in penetration depth of the current density, which might be caused by a different number of elements in the $x$ and $y$ directions. The result for the prism is coarser in the $xy$ plane, but we solved 10 times higher number of degrees of freedom compared to the thin film, since the prism is a 3D object. For isotropic materials with constant $Jc$, the thickness average of the current density is exactly the same as the thin film approximation, as long as the same thickness is used [55]. The reason is that in our model the thin film approximation only consists on averaging all electromagnetic quantities along the thickness. For anisotropic materials, where the local $B$ is relevant, there could be a small difference. The reason is that the parallel component of the magnetic field is anti-symmetric along the thickness, and hence its thickness average vanishes, while it is locally non-zero. This parallel component could increase $|Jc|$ due to force-free effects. This increase of $|Jc|$ is not present at the saturated regions because $Jc$ is roughly uniform. This uniform $Jc$ creates $B$ perpendicular to $Jc$, except close to the regions where $Jc$ changes direction. Then, the increase of $|Jc|$ should only be present at the non-saturated regions, as observed, and the turning places of $Jc$.

The second prism case is $\theta = 45^\circ$. The penetration depth of the average current density in the prism (figure 12(b)) agrees with the thin film case (figure 12(a)). The agreement is as well in the lines of $x = 0$ and $y = 0$ (figures 12(c), (d)). The $Jc$ component of the current density is around $Jc_{\perp}$ (figure 12(d)), but $Jc$ is 2 times higher. The reason of the higher magnitude of $Jc$ is that the applied field has a
component in the \( x \) direction, causing force-free effects. This also causes that \( J_x \) at the penetration front reaches \( J_{c}\) in the thin film, since \( B_z \) there vanishes (figure 12). The penetration depth in the prism is smaller, because of the thicker cells.

The last two cases with \( \theta = 60^\circ, 80^\circ \) are similar to the appropriate cases of thin film, although with certain differences. For the angle \( \theta = 60^\circ \) there is lower penetration depth from the right and left sides (figure 13(b)) than the thin film (figure 13(a)). The angle \( \theta = 80^\circ \) has an even lower penetration depth from these sides (figure 14(b)) compared to thin film (figure 14(a)). The current profiles along the \( x \) and \( y \) directions show the same behavior of lower current penetration (figures 13(c), (d), 14(c), (d)). The cause of lower penetration depth along both \( x \) and \( y \) directions is due to the prism finite thickness. Since \( \theta \approx 0^\circ \), there is a significant \( J_z \) component of the current density, which is around \( J_{c,\perp} \) (figure 16(c), \( \theta = 80^\circ \)).

Finally, we compare the 3D current paths in the prism at the peak of the applied field for the anisotropic case and two applied field angles \( \theta = 0^\circ \) and \( 80^\circ \). For the first angle, the sample is fully saturated as seeing the mid planes perpendicular to the \( x \) and \( y \) axis (figures 15(a), (b)) and hence \( J_z \) almost vanishes (figure 15(c)). For the second angle (\( \theta = 80^\circ \)), the \( J_y \) component of the current density is also saturated in most of the volume (figure 16(b)). Now, the border between positive and negative \( J_y \) component follows roughly the direction of the applied magnetic field. The \( J_x \) component is not saturated in the sample (figure 16(a)) and the highest penetration depth is at the centre of the prism. Since the current loops are almost perpendicular to the angle of the applied field, a substantial \( J_z \) component exists (figure 16(c)).

3.2.2. Magnetization loops in prisms. We calculated the hysteresis loops for all previous cases (figure 17). In order to explain all effects, we also analyzed the same situation with isotropic power law (figure 18). The \( M_z \) component of the magnetization is lower for higher applied magnetic field angle \( \theta \) (figure 18(b)). This is because the path of the screening current loops tilts away from the \( xy \) plane. The \( M_z \) component
is zero for $\theta = 0^\circ$ (figure 18(a)), since the current path is only in the $xy$ plane. This also causes and increase of the $M_z$ component with increasing $\theta$. This geometry effect can be reduced by decreasing the prism thickness. Consistently, $M_x$ vanishes at $\theta = 0$ because the current loops are mainly in the $xy$ plane and the remaining bending in the $z$ direction is symmetric (see figure 5 of [55]).

The hysteresis loops with anisotropic $E(J)$ relation have more effects. On one hand, increasing the angle $\theta$ enlarges the region with $J \approx J_c$, increasing also $M_z$. On the other hand, by increasing $\theta$, the tilt increases, reducing $M_z$. The result is an increase in $M_z$ with $\theta$ but for $\theta = 80^\circ$ this increase is smaller than for the thin film (figure 17(b)). The magnetization in the $x$ direction (figure 17(a)) shows mostly the same behavior as isotropic case. The difference is only in a peak of the magnetization around the zero applied field. This peak of the magnetization appears for both components, $M_x$ and $M_z$. The reason of this peak is the following. For very small magnetic fields, the self-field dominates. Close to the top (highest $z$) and bottom (lowest $z$) of the sample, the self-field is parallel to the surface. Then, part of the sample experiences a local magnetic field parallel to the current density, increasing $J_c$ towards $J_{cP}$. For applied fields much larger than the self-field, the magnetic field follows the direction of the applied field. This applied field is not perfectly parallel to the surface, causing a lower $J_c$.

Another calculation with isotropic $E(J)$ relation shows the geometry effects due to different thickness of the prism ($d = 1, 0.6, 0.5, 0.1$) while keeping a constant sheet current density. First we check that for only perpendicular applied field, the prism results approach to the thin film by reducing the thickness. Figure 19 shows that the normalized $z$ component $M_z/J_{cw}$ is roughly the same for all thicknesses $d$. This figure also tells us that the magnetic moment $m_z$ almost does not depend on $d$, since $M_z/J_{cw} = m_z/J_c w^3$ and we keep both $J_c d$ and $w$ constant. For a magnetic field angle $\theta$ of $80^\circ$, $M_z$ increases with decreasing the sample thickness, since the screening current is forced to flow closer to the $xy$ plane (figure 20(b)). However, the other normalized component, $M_x/J_{cd}$, increases with the thickness $d$, due to the increase of the area of the projection of the current loops in the $yz$ plane.
4. Conclusions

This article systematically studied the anisotropic force-free effects in superconducting thin films and prisms under uniform applied magnetic field making an angle $\theta$ with the surface. In order to better understand all effects, we performed modeling with isotropic and anisotropic $E(J)$ relation due to force-free effects.

For this purpose, we use the MEMEP 3D numerical method \[49\]. We further developed the model in order to enable elongated cells, to reduce the total number of elements or enable to model relatively long or thin structures without further increasing the total number of elements. In particular, we studied the elliptical double critical-state model with a continuous $E(J)$ relation due to force-free effects.

In the thin film force-free model, we calculated the gradual penetration of the current density. We found at the remanent state that $J$ decreases to $J_{c,\perp}$ and the magnetization increases with the angle $\theta$. The magnetization of the isotropic film is the same for all applied field angles, when comparing for the same perpendicular component of the applied magnetic field and its amplitude. The anisotropic model, both with and without $J_c(B)$ dependence, shows a minimum of the magnetization at the remanent state for $\theta \neq 0$. The cause is the absence of any parallel component of the local magnetic field to the current density, avoiding $J_c$ enhancement due to force-free effects. In superconducting prisms, we observed 3D current paths. The average current density over thickness shows good agreement with thin film sample. However, for high applied magnetic field angles there appear small differences. The $M_x$ component is increasing with the angle $\theta$, because of the significant increase of $J_z$. The $M_y$ slightly decreases due to the tilt in the screening currents. The magnetization loops show a peak after the remanent state due to the influence of the parallel component of the self-field, increasing $J_c$ up to $J_{P}$ at part of the sample. This effect is not present for the thin film approximation because the parallel component of the self-field is neglected. Calculations for several prism thicknesses down to 100 $\mu$m support the validity of the results.

Figure 13. The same as figure 11 but for $\theta = 60^\circ$ ($B_{ic,m} = 50$ mT and $B_{im} = 100$ mT). A slightly lower critical current penetration can be observed from the prism (b), (d) compared to the film (a), (c).
We expect that the thin film geometry may not be a good approximation for study force-free effects in magnetization measurements. This study confirmed that the MEMEP 3D method is suitable for any $E(J)$ relation and it can be solved with a relatively high number of degrees of freedom and relatively thin samples in 3D space. Further work could be to investigate different shapes of the sample and speed up the calculation, maybe by multi-pole expansion. This modeling work could motivate measurements in the force-free situation. Since intrinsic anisotropy should be avoided, measurements

Figure 14. The same as figure 11 but for $\vartheta = 80^\circ$ ($B_{\vartheta,m} = 50$ mT and $B_{\varphi,m} = 287.9$ mT). The prism (b), (d) presents a substantially lower penetration of the critical current density than the film (a), (c).

Figure 15. Current density in a prism with force-free anisotropy at the peak of the applied field and $w = 12$ mm, $d = 1$ mm, $\vartheta = 0^\circ$, and $B_{\varphi,m} = 50$ mT. The current density components are: (a) $J_x$, (b) $J_y$, (c) $J_z$. Note that the plotted planes in (b) are not the same as in (a) or (c).
Figure 16. The same as figure 15 but for $\theta = 80^\circ$ ($B_{\text{ext}} = 287.9$ mT).

Figure 17. The influence of the applied field angle to the magnetization loops in a prism is somewhat different than that of a thin film with the same sheet critical current density $J_c$. Shows the case of an anisotropic $E(J)$ relation and a constant $J_{||}, J_{\perp}, n = 30$, and a sinusoidal applied magnetic field with $B_{\text{ext}} = 50.0$ mT and $f = 50$ Hz. (a) The $x$ component of the magnetization $M_x$ component. (b) $M_z$.

Figure 18. The same as figure 17 but for isotropic $E(J)$. Several differences appear from the anisotropic case, such as the peaks close to the remanence in figure 17.
need to be on macroscopically isotropic samples, such as multi-granular Ba112 superconductors or MgB2 bulks. Low-temperature superconductors like NbTi or Nb3Sn may also be the object of study.

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Appendix. Elongated cells

The elongated cells are cells with different geometry ratio than square (for 2D) or cubic (for 3D). These cells allow to model geometries such as long thin film, or thin/thick bulk. The elongated cells enable to reduce the total number of elements, and hence reduce the computing time. A key issue is the calculation of the interaction matrix between elemental surfaces (or ‘surfaces’). The interaction matrix between surfaces \(i\) and \(j\) of type \(s\) \(\in\{x, y, z\}\) is generally [49]

\[
a_{ij} = \frac{\mu_0}{4\pi V_i V_j} \int_{V_i} d^3r \int_{V_j} d^3r' \frac{h_{ij}(r) h_{ij}(r')}{|r - r'|}.
\]

with

\[
V_i \equiv \int_V d^3r h_{ii}(r) .
\]

The first-order interpolation functions \(h_{ij}(r)\) are defined as in figure A1(b) for coordinate \(r_i = r_r\), vanishing outside the two neighboring cells in the \(r_i\) direction.

In the case of square or cubic cells or square sub-elements, the self-interaction term \(a_{ii}\) can be calculated by the approximated analytical formula

\[
a_{ii} \approx \frac{\mu_0}{4\pi L} \left\{ \frac{1 + \sqrt{2} - 2\sqrt{3}}{5} \right. \left. - \frac{\pi}{3} + \ln[(1 + \sqrt{2})(3 + \sqrt{3})] \right\} \]  
\]  

(A.3)

for a cube of side \(L_{ii}\) [60] and

\[
a_{ii} \approx \frac{\mu_0}{\pi L_{ii}} \left\{ \frac{1 - \sqrt{2}}{3} + \ln(1 + \sqrt{2}) \right\} \]  
\]  

(A.5)

for a thin prism [61] with thickness \(d\) much smaller than its side \(L_{ii}\). For equation (A.3) we assumed that the current density is uniform in the volume of influence, defined as the volume between surface \(i\) of type \(s\) and the centre of the neighbouring cells in the \(s\) direction (see figure A1(a) for \(s = x\)). The average vector potential \(a_{ij}\) is calculated by approximation everywhere else, \(i \neq j\)

\[
a_{ij} \approx \frac{\mu_0}{4\pi |r_i - r_j|} .
\]

(A.6)

where \(r_i\) is the centre of surface \(i\) of type \(s\).

In the case of elongated cells, the interaction matrix of the vector potential, \(a_{ij}\), needs to be calculated numerically. The numerical calculation splits the surrounded area of two surfaces into small square sub-elements (figure A1(a)). The average vector potential of the two surfaces is integrated over all sub-elements, which contain surfaces again. The sub-elements are calculated in the same way as square cells, but sub-elements are multiplied by the linear interpolations functions \(h_{ij}(r)\) at the centre of the sub-element surfaces with indexes \(l, m\), being \(r_l\) and \(r_m\). Elongated cells contain as many sub-elements in order to reach as square as possible shape. In general, the average vector potential generated by sub-element \(l\) on sub-element \(m\) is

\[
a_{ijlm} = \frac{\mu_0}{4\pi V_i V_m} \int_{V_i} d^3r \int_{V_m} d^3r' \frac{h_{ij}(r) h_{ij}(r')}{|r - r'|} .
\]

(A.7)

\[
\text{where } V_i \text{ and } V_m \text{ are the volume of influence of the sub-elements, as defined in figure A1(a). For } l \neq m, \text{ we}
\]
approximate the integral above by

$$a_{ijlm} \approx \frac{\mu_0 h_d(r_i) h_d(r_m)}{4\pi|\mathbf{r}_i - \mathbf{r}_m|}. \quad (A.8)$$

When \( l \) corresponds to the sub-element \( m \) both in position and size, we use the approximated formula for uniform current density in the sub-element

$$a_{ijll} \approx \frac{\mu_0 h_d(r_i) h_d(r_i)}{4\pi V_{sl}} \int_{V_i} d^3r \int_{V_i} d^3r' \frac{1}{|\mathbf{r} - \mathbf{r}'|}. \quad (A.9)$$

Following the same steps as for equation \( (A.3) \), \( a_{ijll} \) becomes

$$a_{ijll} \approx \frac{\mu_0 h_d(r_i) h_d(r_i)}{4\pi L_{sl}} \left\{ 1 + \sqrt{2} - 2\sqrt{3} \right\} = \frac{\pi}{3} + \ln\left(1 + \sqrt{2}\right)(2 + \sqrt{3}) \right\} \quad (A.10)$$

for a cube of side \( L_{sl} \) and

$$a_{ijll} \approx \frac{\mu_0 h_d(r_i) h_d(r_i)}{\pi L_{sl}} \left\{ 1 - \frac{\sqrt{2}}{3} + \ln\left(1 + \sqrt{2}\right) \right\} \quad (A.11)$$

for a thin prism of side \( L_{sl} \).

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