Impact-Angle and Terminal-Maneuvering-Acceleration Constrained Guidance against Maneuvering Target

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Abstract: A new, highly constrained guidance law is proposed against a maneuvering target while satisfying both impact angle and terminal acceleration constraints. Here, the impact angle constraint is addressed by solving an optimal guidance problem in which the target’s maneuvering acceleration is time-varying. To deal with the terminal acceleration constraint, the closed-form solutions of the new guidance are needed. Thus, a novel engagement system based on the guidance considering the target maneuvers is put forward by choosing two angles associated with the relative velocity vector and line of sight (LOS) as the state variables, and then the system is linearized using small angle assumptions, which yields a special linear time-varying (LTV) system that can be solved analytically by the spectral-decomposition-based method. For the general case where the closing speed, which is the speed of approach of the missile and target, is allowed to change with time arbitrarily, the solutions obtained are semi-analytical. In particular, when the closing speed changes linearly with time, the completely closed-form solutions are derived successfully. By analyzing the generalized solutions, the stability domain of the guidance coefficients is obtained, in which the maneuvering acceleration of the missile can converge to zero finally. Here, the key to investigating the stability domain is to find the limits of some complicated integral terms of the generalized solutions by skillfully using the squeeze theorem. The advantages of the proposed guidance are demonstrated by conducting trajectory simulations.

Keywords: generalized closed-form solutions; maneuvering target; stability domain; trajectory shaping guidance

1. Introduction

In some engagement scenarios, in order to penetrate the weak part of an armored maneuvering target, or to keep the target in the field of view (FOV) of the seeker, the missile is required to attack the target from a specified orientation. Additionally, according to the requirement of penetrating the armored target [1] and minimizing the influence of the parasitic effect [2], the angle of attack (AOA) of the missile at the moment of impact should be near zero. As a small AOA generates a small maneuvering acceleration, the AOA constraint can be converted into that on terminal maneuvering acceleration. Due to the aerodynamic constraints, the allowable maneuverability of intercepting vehicle is limited, a sufficiently small terminal maneuvering acceleration at the moment of impact helps to remain control margin to cope with sudden disturbances [3]. This is of great benefit in some typical scenarios, such as attacking a target with decoys [4,5]. Therefore, this paper aims at designing a highly constrained guidance against a maneuvering target while considering both the terminal impact angle and acceleration limitations.

At present, for intercepting stationary targets, there exist many guidance laws that can address the terminal impact angle constraint, which are collectively referred to as trajectory shaping guidance (TSG). In 1964, Cherry [6] suggested the earliest TSG by designing the acceleration command as a polynomial function of time and called it explicit guidance (E...
guidance). Bryson [7] first used the optimal control theory to demonstrate the optimality of E guidance. By assuming that the velocity is constant, Idan et al. [8] and Glizer [9] derived the analytical solutions for the planar interception problems. Zarchan [2] developed the TSG by using the Schwartz inequality under the hypothesis of small angles. Ben-Asher and Yaesh [10] improved the TSG by solving the minimum effort multiple-target interception problem. Using the optimal control theory, Yu et al. [11] developed a TSG for exo-atmospheric interception considering the final velocity vector constraint. A generalized vector explicit guidance (GENEX) was proposed by Ohlmeyer and Phillips [12] for attacking stationary targets by taking the reciprocal of the n-th power of the time-to-go of the missile as the cost function. The extensions of the well-known proportional navigation guidance (PNG) have also been applied well in trajectory shaping [13–16]. By varying the value of the navigation constant, Ratnoo and Ghose [15] improved the PNG to ensure the terminal angle constraint. Erer and Merttopcuoglu [16] suggested a biased PNG with a constant additional term to guarantee terminal angle constraint. The non-linearity of dynamics is also considered by some researchers. The sliding mode control (SMC)-based guidance was proposed by Kumar et al. [17] for the interception of stationary targets satisfying impact angle constraint. By utilizing the nonlinear mapping-based backstepping technique, Liu et al. [18] developed a non-switching guidance strategy for attacking a stationary ground target from the desired direction. Considering non-linear coupled dynamics, Hu et al. [19] developed an analytical impact angle guidance by using two cubic polynomials to create a reference LOS profile. However, the scenario of intercepting maneuverable target is not considered in the above approaches [6–19]. It is challenging because large maneuverability requirements may degrade the performance of the guidance accuracy and lead to saturated accelerations in the terminal phase.

There are some guidance laws capable of addressing other practical constraints in addition to the impact angle constraint, He et al. [20] proposed an adaptive backstepping-based integrated guidance to intercept the maneuverable target without knowing the target’s information. Biswas et al. [21] used a finite-time SMC for impact angle guidance design to achieve finite-time interception of targets. Based on an integral Lyapunov control algorithm, Lin et al. [22] presented a robust adaptive impact angle guidance method for maneuvering targets. Hu et al. [23] proposed a twisting control-based guidance to satisfy the impact angle constraint against a maneuvering target. Although these guidance laws [20–23] can reduce the sensitivity against maneuvering target by assuming the target’s maneuver as a disturbance term, they are based on constant-velocity models and pay less attention to the fuel consumption and terminal acceleration constraints, which are crucial for practice engineering. Furthermore, some guidance laws [24–26] were developed by considering the impact angle, fuel consumption, and terminal acceleration requirements, simultaneously. By designing a typical linear quadratic optimal controller, Ryoo et al. [24] developed an optimal guidance considering both the impact angle and terminal acceleration constraints. Motivated by this work, Li et al. [25] developed a new adaptive optimal impact angle guidance model, which can address the seeker’s FOV angle constraint. Wang et al. [26] designed an impact-angle-constrained suboptimal guidance law using the state-dependent Riccati equation (SDRE) technique and addressed terminal acceleration constraint by adjusting the guidance coefficients. However, these guidance laws [24–26] were proposed by assuming that the missile’s velocity is constant and the target is stationary, and thus do not perform well when the target is maneuvering.

In this paper, a new guidance model is developed for intercepting a maneuvering target with time-varying maneuvering acceleration while satisfying both the impact angle and terminal maneuvering acceleration constraints. To develop the guidance, a vector-form optimal guidance problem considering the time-varying maneuvering acceleration of target is posed and solved analytically. In order to deal with the terminal maneuver limit, we further try to obtain the analytical solutions of the new guidance. Thus, an engagement system based on the guidance is created and linearized using small angle assumptions, which yields a special LTV system, the system matrix of which can be ex-
pressed as the product of a time-varying scalar function and a constant matrix. Due to this characteristic, the spectral-decomposition-based method [27] can be applied to solve the LTV system analytically, and thus semi-analytical solutions of flight-path angle, heading angle and acceleration command are obtained successfully. Compared with the existing solutions [2–25], the new solutions have two major advantages: (1) the solutions are applicable to the case that the target is maneuvering and has a time-varying maneuvering acceleration; (2) the closing speed is allowed to vary with time arbitrarily. In particular, if the closing speed changes linearly with time, the completely closed-form solutions for the new guidance are derived. By comparison, the solutions for TSG [2] are derived by assuming that the target is stationary and the closing speed is constant. Therefore, we name the new solutions the generalized solutions. Using these solutions, we further attempt to investigate the stability domain of the coefficients for the new guidance. However, the investigation is difficult because it is not easy to find the limits of some complicated integral terms of the generalized solutions. Here, we overcome the difficulty by creating some novel inequalities and then employing the Squeeze theorem [28]. Thus, the stability domain is obtained successfully, in which the terminal maneuvering acceleration can converge to zero no matter whether the closing speed keeps constant or changes with time. As verified by conducting the trajectory simulations, in the stability domain, the proposed guidance is stable and can effectively satisfy the impact angle and terminal maneuvering acceleration constraints even if the target performs sinusoidal evasive maneuvers.

The structure of this paper is as follows. The flight dynamics are stated in the Section 2. Next, the new guidance is designed in Section 3 and the stability domain of the guidance coefficients is derived in Section 4. Then, Section 5 shows the scheme of the guidance. The simulation results are presented in Section 6. Finally, the main contributions of this paper are summarized in Section 7. The Appendix A deduces the complete closed-form solutions for the case where the closing speed varies linearly with time for a solid-propellant homing missile with fixed thrust.

2. Guidance Problem

As shown in Figure 1, a general engagement geometry is presented where a missile $M$ is attempting to attack a maneuvering target $T$. In the $xoy$ frame, $\mathbf{X}_M$ denotes the position vector of the missile, $\mathbf{V}_M$ denotes the velocity vector of the missile, $\gamma_{LOS}$ denotes the LOS angle, $\mathbf{a}_M$ denotes the missile’s acceleration vector, $\mathbf{X}_T$ denotes the position vector of the target, $\mathbf{V}_T$ denotes the velocity vector of the target, $\mathbf{a}_T$ is the time-varying maneuvering acceleration of the target.

![Figure 1. The engagement geometry.](image)

The equations of motion for the missile and target are given as:

\begin{align}
\mathbf{X}_M &= \mathbf{V}_M \\
\mathbf{V}_M &= \mathbf{a}_M \\
\mathbf{X}_T &= \mathbf{V}_T
\end{align}
The velocity magnitude of missiles are assumed to be higher than that of the target.

3. Development of Highly Constrained Guidance

In this section, the impact angle constraint is addressed first by solving an optimal guidance problem against a maneuvering target. Subsequently, to deal with the terminal maneuvering acceleration constraint, the generalized solutions of the new guidance are derived, and then the stability domain of the guidance coefficients is investigated by analyzing the generalized solutions.

3.1. Formulaic of Guidance Law

Using Equations (1)–(4), the equations for the motion of the target relative to the missile can be obtained by:

\[
X_{TM} = V_{TM} \tag{5}
\]

\[
V_{TM} = a_T - a_M \tag{6}
\]

where \( X_{TM} = X_T - X_M \) and \( V_{TM} = V_T - V_M \) are the position and velocity vectors of the target relative to the missile, respectively. By contrast with the existing studies \([24–26]\), the incoming target considered here is a hypersonic vehicle (HV) \([29]\), which is very difficult to intercept because HV not only has very fast speed, but also has highly maneuvering ability. Since the maneuvering acceleration of such a target is variable, \( a_T \) is modeled as an \( m \)-degree polynomial of time without losing generality, which is a common function used by the target’s trajectory prediction technique \([30]\), as below:

\[
a_T = \sum_{k=0}^{m} a_k T^k \hat{u}_k \tag{7}
\]

As we hope the missile intercepts the target from a desired direction, the desired terminal conditions are:

\[
X_{TM}(t_f) = 0, V_{TM}(t_f) = V_{TMf} \tag{8}
\]

where \( V_{TMf} \) stands for the desired terminal relative velocity.

To derive the optimal solution that achieves the regulation of Equation (8), we proposed the proposition as follows.

**Proposition 1.** By referring to the work in \([12]\), a quadratic cost function is considered as:

\[
J = \int_0^T \frac{a_T^T a_M}{2 T^n} dT \tag{9}
\]

where \( n \) is a parameter. Then, the optimal solution to minimize Equation (9) subject to the desired boundary condition of Equation (8) and considering the target’s maneuver is:

\[
a_M = \frac{K_1}{T^2} [X_{TM} + V_{TM}T] + \frac{K_2}{T^2} [V_{TM}T - V_{TMf}T] + \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} \right) a_k T^k \hat{u}_k \tag{10}
\]

where,

\[
K_1 = (n+2)(n+3) \tag{11}
\]

\[
K_2 = -(n+1)(n+2) \tag{12}
\]
**Proof.** The Hamiltonian of the above optimal guidance problem is:

\[ H = \frac{a_M^T a_M}{2T} + \lambda_1^T V_{TM} + \lambda_2^T (a_T - a_M) \]  

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multiplier vectors. Then the co-state equations are:

\[ \frac{d\lambda_1}{dT} = -\dot{\lambda}_1 = \frac{\partial H}{\partial X_{TM}} = 0 \]  

(14)

\[ \frac{d\lambda_2}{dT} = -\dot{\lambda}_2 = \frac{\partial H}{\partial V_{TM}} = \lambda_1 \]  

(15)

Integrating Equations (14) and (15) gives:

\[ \lambda_1 = C_1 \]  

(16)

\[ \lambda_2 = C_1 T + C_2 \]  

(17)

Furthermore, we can obtain the control equation from the Hamiltonian as below:

\[ \frac{\partial H}{\partial a_M} = a_M^T n - \lambda_2 = 0 \]  

(18)

Solving Equation (18) yields:

\[ a_M = \lambda_2 T^u \]  

(19)

Substituting Equation (17) into Equation (19) gives:

\[ a_M = C_1 T^{u+1} + C_2 T^u \]  

(20)

By substituting Equation (20) into Equations (5) and (6) and then integrating them from 0 to \( T \), we can obtain:

\[ V_{TMf} = V_{TM} + \int_0^T a_T dT - \frac{C_1}{n+2} T^{n+2} - \frac{C_2}{n+1} T^{n+1} \]  

(21)

\[ X_{TMf} = X_{TM} + \int_0^T V_{TMf} dT - \int_0^T \int_0^T a_T dT + \frac{C_1}{(n+2)(n+3)} T^{n+3} + \frac{C_2}{(n+1)(n+2)} T^{n+2} \]  

(22)

Incorporating the boundary conditions, Equation (8) and the target's maneuver function Equation (7) in Equations (21) and (22) yields:

\[ \frac{C_1}{n+2} T^{n+2} + \frac{C_2}{n+1} T^{n+1} = V_{TM} - V_{TMf} + \sum_{k=0}^{m} \frac{1}{k+1} a_{k} T^{k+1} \hat{u}_{k} \]  

(23)

\[ \frac{C_1}{(n+2)(n+3)} T^{n+3} + \frac{C_2}{(n+1)(n+2)} T^{n+2} = -X_{TM} - V_{TMf} T + \sum_{k=0}^{m} \frac{1}{(k+2)(k+1)} a_{k} T^{k+2} \hat{u}_{k} \]  

(24)

Solving Equations (23) and (24) for \( C_1 \) and \( C_2 \), and then substituting them into Equation (20), the optimal solution can be obtained as Equation (10). □

**Remark 1.** The result of Proposition 1 is also the optimal guidance command that achieves at the end of homing. Now we further simplify the guidance command in Equation (10) by predicting the time-to-go by \( T = \frac{R}{V} \), where \( V = |V_{TM}| \) and \( R = |X_{TM}| \), as:

\[ a_M = \frac{V^2}{R} \left[ K_1 \left( -\hat{r} + \hat{v} \right) + K_2 \left( -\hat{v} + \hat{v_f} \right) \right] + \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} \right) a_{k} T^{k} \hat{u}_{k} \]  

(25)
where $\hat{r}, \hat{v}$ and $\hat{v}_f$ are the unit vectors satisfying $X_{TM} = -X_{MT} = -\hat{r}$, $V_{TM} = -V_{MT} = -\hat{v}$, and $V_{TMf} = -V_{MTf} = -\hat{v}_f$ respectively.

**Remark 2.** For a vehicle with no longitudinal control capability, the guidance command can be rewritten as the scalar-form expression perpendicular to the velocity vector. We define $n_c$ as the component of the vehicle’s commanded acceleration vector perpendicular to $V_{MT}$. $\hat{u}$ as the unit vectors co-directional with $n_c$. Let $b_k = a_k(\hat{u}_k \cdot \hat{u})$, the scalar-form expression of $n_c$ can be obtained as:

$$n_c = a_m \cdot \hat{u} = -\frac{V^2}{R} \left[ K_1 \sin(\gamma - \gamma_{LOS}) + K_2 \sin(\gamma - \gamma_f) \right] + \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} \right) b_k T^k$$  \hspace{1cm} (26)

where, $\gamma$ is the angle of $V_{MT}$ with respect to the x-axis, $\gamma_f$ is the desired final value of $\gamma$, and $\gamma_{LOS}$ is the angle of the LOS measured counterclockwise from the horizontal reference line as shown in Figure 2.

![Figure 2. Definitions of variables.](image)

### 3.2. Generalized Solutions of the New Guidance

In order to determine the stability domain of the guidance coefficients (i.e., $K_1$ and $K_2$) such that the maneuvering acceleration of the vehicle can converge to zero finally, we now need to derive the generalized solutions of the new guidance.

Obviously, there is,

$$V \frac{d\gamma}{dt} = n_c - n_T$$  \hspace{1cm} (27)

where $n_T$ is the magnitude of $n_T$. Substituting Equation (26) into Equation (27) yields:

$$V \frac{d\gamma}{dt} = -\frac{V^2}{R} \left[ K_1 \sin(\gamma - \gamma_{LOS}) + K_2 \sin(\gamma - \gamma_f) \right] + \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k T^k$$  \hspace{1cm} (28)

In addition, according to the geometry, there is,

$$\frac{d\gamma_{LOS}}{dt} = -\frac{V \sin(\gamma - \gamma_{LOS})}{R}$$  \hspace{1cm} (29)

Then we need to obtain the generalized solutions under the following assumption.

**Assumption 1.** To improve the probability of success, the flight vehicle generally needs to perform a head-on interception at the end of the engagement, which means that the terms $\gamma - \gamma_{LOS}$ and $\gamma - \gamma_f$ are approximately zero.
Using Assumption 1, Equations (28) and (29) can be linearized as an LTV system, as:

\[ \dot{x} = f_1 A_1 x + B_1 + C_1 \]  

(30)

where

\[
x = \begin{bmatrix} \gamma \\ \gamma_{LOS} \end{bmatrix}, f_1 = \frac{V}{R} A_1 = \begin{bmatrix} -(K_1 + K_2) & K_1 \\ -1 & 1 \end{bmatrix}, B_1 = \frac{V}{R} \begin{bmatrix} K_2 \\ 0 \end{bmatrix}, C_1 = \frac{1}{V} \left[ \sum_{k=0}^{m} \left( \frac{K_1}{K_{f+2}} + \frac{K_2}{K_{f+1}} - 1 \right) b_k T^k \right]
\]

(31)

Let,

\[ \Delta = (K_1 + K_2 - 1)^2 + 4K_2 \]  

(32)

Then, we present the solution of the LTV system in the proposition below.

**Proposition 2.** The solution of the system in Equation (30), which is the generalized solutions of \( \gamma \) and \( \gamma_{LOS} \), can be discussed in the following three aspects:

1. If \( \Delta > 0 \), let \( \lambda_1 \) and \( \lambda_2 \) (\( \lambda_1 > \lambda_2 \)) be the two different real roots Equation (46), which are:

\[
\lambda_1 = \frac{-(K_1 + K_2 - 1) + \sqrt{\Delta}}{2}
\]

(33)

\[
\lambda_2 = \frac{-(K_1 + K_2 - 1) - \sqrt{\Delta}}{2}
\]

(34)

Then the system in Equation (30) has the following solutions:

\[
\gamma(t) = \frac{1}{\lambda_1 - \lambda_2} \left[ R(t_0)^{\lambda_1} (\lambda_1 - 1) - R(t_0)^{\lambda_2} (\lambda_2 - 1) \right] + \frac{K_2 \gamma_f}{\lambda_1 - \lambda_2} \left( \frac{R(t_0)^{\lambda_1}}{R(t_0)^{\lambda_2}} - 1 \right) + \frac{1}{\lambda_1 - \lambda_2} \sum_{k=0}^{m} \left( K_1 k + K_2 k + 1 - 1 \right) b_k \int_{t_0}^{t} \left( \frac{t_f - \tau}{V(\tau)} \right)^k \left[ R(\tau)^{\lambda_1} (\lambda_1 - 1) - R(\tau)^{\lambda_2} (\lambda_2 - 1) \right] d\tau
\]

(35)

\[
\gamma_{LOS}(t) = \frac{-\gamma_0}{\lambda_1 - \lambda_2} \left[ R(t_0)^{\lambda_1} (\lambda_1 - 1) - R(t_0)^{\lambda_2} (\lambda_2 - 1) \right] + \frac{K_2 \gamma_f}{\lambda_1 - \lambda_2} \left( \frac{R(t_0)^{\lambda_1}}{R(t_0)^{\lambda_2}} - 1 \right) - \frac{1}{\lambda_1 - \lambda_2} \sum_{k=0}^{m} \left( K_1 k + K_2 k + 1 - 1 \right) b_k \int_{t_0}^{t} \left( \frac{t_f - \tau}{V(\tau)} \right)^k \left[ R(\tau)^{\lambda_1} (\lambda_1 - 1) - R(\tau)^{\lambda_2} (\lambda_2 - 1) \right] d\tau
\]

(36)

2. If \( \Delta < 0 \), define the complex conjugates roots of Equation (46) as:

\[
\begin{cases} 
\lambda_1 = p + iq \\
\lambda_2 = p - iq
\end{cases}
\]

(37)

Then the generalized solutions are:

\[
\gamma(t) = R(t_0)^p x \left[ \cos \left( q \ln \left( \frac{R(t_0)}{R(t)} \right) \right) \right] \left( \gamma_0 + \frac{K_2 \gamma_f}{p^2 + q^2} \right) + \frac{1}{2} \sin \left[ q \ln \left( \frac{R(t_0)}{R(t)} \right) \right] \times \left[ (p - 1) \gamma_0 + K_1 \gamma_{LOS} + K_2 \gamma_f \frac{p^2 + q^2}{p^2 + q^2} \right] + \gamma_f
\]

\[
+ \frac{1}{\lambda_1 - \lambda_2} \sum_{k=0}^{m} \left( K_1 k + K_2 k + 1 - 1 \right) b_k \int_{t_0}^{t} \left( \frac{t_f - \tau}{V(\tau)} \right)^k \left[ R(\tau)^{\lambda_1} (\lambda_1 - 1) - R(\tau)^{\lambda_2} (\lambda_2 - 1) \right] d\tau
\]

(38)
\[
\begin{align*}
\gamma_{LOS}(t) &= \frac{R(t_0)}{R(t)} \left[ \cos \left( \frac{\beta}{p^2 + q^2} \right) \right] \times \left( \gamma_{LOS0} + \frac{K_2 \gamma_f}{R(t)} \right) + \frac{1}{q} \frac{R(t_0)}{R(t)} \sin \left( \frac{\beta}{p^2 + q^2} \right) \\
&\quad - \left[ - \gamma_0 + (p - 1) \gamma_{LOS0} - \frac{p}{p^2 + q^2} K_2 \gamma_f \right] \right) + \gamma_f \\
&= - \sum_{k=0}^m \left( \frac{K_1 + K_2}{k+1} - 1 \right) b_k \int_{t_0}^t \left( \frac{R(t_0) \gamma_f}{R(t)} \right)^k \left[ \gamma_{LOS0} + \left( 1 - \gamma_0 \right) \right] d\tau \\
\end{align*}
\]

(3) If \( \Delta = 0 \), there is \( \lambda_1 = \lambda_2 \). The solutions can be expressed as:

\[
\gamma(t) = \frac{R(t_0)}{R(t)} \left[ 1 + (\lambda_1 - 1) \ln \left( \frac{R(t_0)}{R(t)} \right) \right] \gamma_0 + K_1 \frac{R(t_0)}{R(t)} \ln \left( \frac{R(t_0)}{R(t)} \right) \gamma_{LOS0} \\
- K_2 \gamma_f \left[ \frac{1}{\lambda_1} \left( 1 - \frac{R(t_0) \gamma_0}{R(t)} \right) \right] + (\lambda_1 - 1) \left[ - \ln \left( \frac{R(t_0)}{R(t)} \right) \right] \left[ 1 - \frac{R(t_0) \gamma_0}{R(t)} \right] \right) \right] \\
+ \sum_{k=0}^m \left( \frac{K_1 + K_2}{k+1} - 1 \right) b_k \int_{t_0}^t \left( \frac{R(t_0) \gamma_f}{R(t)} \right)^k \left[ 1 + (\lambda_1 - 1) \ln \left( \frac{R(t)}{R(t)} \right) \right] d\tau \\
\gamma_{LOS}(t) &= - \left[ \lambda_1 \left( 1 - \frac{R(t_0) \gamma_0}{R(t)} \right) \right] + \frac{1}{\lambda_1} \left( \frac{R(t_0) \gamma_0}{R(t)} \right) \right] \\
&= - \sum_{k=0}^m \left( \frac{K_1 + K_2}{k+1} - 1 \right) b_k \int_{t_0}^t \left( \frac{R(t_0) \gamma_f}{R(t)} \right)^k \left[ \gamma_{LOS0} + \left( 1 - \gamma_0 \right) \ln \left( \frac{R(t)}{R(t)} \right) \right] d\tau \\
\gamma_0 \quad \text{and} \quad \gamma_{LOS0} \text{ represent the initial values of } \gamma \text{ and } \gamma_{LOS} \text{ respectively; } t_0 \text{ and } t \text{ denote the initial time and current time, respectively.}
\]

**Proof.** Equation (30) is a special LTV system, in which the system matrix can be expressed as the product of a time-varying scalar function and a constant matrix. It has been proved in the literature [27] that such an LTV system has the following solution:

\[
x(t) = M_1(t, t_0)x(t_0) + \int_{t_0}^t M_1(t, \tau) \left[ f(t, \tau) \right] d\tau
\]

where,

\[
M_1(t, t_0) = \exp(A_1 f_2(t, t_0))
\]

where,

\[
f_2(t, t_0) = \int_{t_0}^t f_1 d\tau = \int_{t_0}^t \frac{V(t)}{R} d\tau = \int_{t_0}^t - \frac{dR}{R} = \ln \frac{R(t_0)}{R(t)}
\]

Substituting Equation (31) into Equation (42) yields:

\[
\begin{bmatrix}
\gamma \\
\gamma_{LOS}
\end{bmatrix} = \exp(A_1 f_2(t, t_0)) \begin{bmatrix}
\gamma_0 \\
\gamma_{LOS0}
\end{bmatrix} + \int_{t_0}^t \exp(A_1 f_2(t, \tau)) \\
\times \left[ \frac{V}{p} \begin{bmatrix} K_2 \\ 0 \end{bmatrix} \right] \gamma_f + \frac{1}{V} \left[ \sum_{k=0}^m \left( \frac{K_1 + K_2}{k+1} - 1 \right) b_k \left( \frac{t_f - \tau}{V(t)} \right)^k \right] d\tau
\]

where \( \exp(A_1 f_2(t, t_0)) \) can be solved by the minimum polynomial method [31]. The minimum polynomial of \( A_1 \) is:

\[
|\lambda I - A_1| = \begin{vmatrix} \lambda + (K_1 + K_2) & -K_1 \\ 1 & \lambda - 1 \end{vmatrix} = \lambda^2 + (K_1 + K_2 - 1)\lambda - K_2
\]

According to the knowledge of quadratic polynomial, the roots of Equation (46) depend on \( \Delta \) defined in Equation (32). Then we discuss the solution as follows:

(1) If \( \Delta > 0 \), we have,
\[ K_2 = -\lambda_1 \lambda_2 \] (47)
\[ K_1 = -\lambda_1 - \lambda_2 + 1 - K_2 = (\lambda_1 - 1)(\lambda_2 - 1) \] (48)

For Equation (45), according to the minimum polynomial method [31], there are:

\[
\exp(A_1 f_2(t, t_0)) = a_0(t, t_0)I + a_1(t, t_0)A_1
\]
\[
= \begin{bmatrix}
    a_0(t, t_0) - a_1(t, t_0)(K_1 + K_2) & a_1(t, t_0)K_1 \\
    -a_1(t, t_0) & a_0(t, t_0) + a_1(t, t_0) \\
\end{bmatrix}
\] (49)

where,

\[ a_1(t, t_0) = \frac{e^{\lambda_1 f_2} - e^{\lambda_2 f_2}}{\lambda_1 - \lambda_2} = \frac{1}{\lambda_1 - \lambda_2} \left[ \frac{R(t_0)^{\lambda_1}}{R(t)}^{\lambda_1} - \frac{R(t_0)^{\lambda_2}}{R(t)}^{\lambda_2} \right] \] (50)
\[ a_0(t, t_0) = \frac{\lambda_1 e^{\lambda_1 f_2} - \lambda_2 e^{\lambda_2 f_2}}{\lambda_1 - \lambda_2} = \frac{1}{\lambda_1 - \lambda_2} \left[ \frac{\lambda_1 R(t_0)^{\lambda_2}}{R(t)}^{\lambda_2} - \frac{\lambda_2 R(t_0)^{\lambda_1}}{R(t)}^{\lambda_1} \right] \] (51)

Substituting Equation (49) into Equation (45) gives the generalized solutions of \( \gamma \) and \( \gamma_{LOS} \) under the condition \( \Delta > 0 \), shown in Equation (35) and Equation (36) respectively.

(2) If \( \Delta < 0 \), according to Equations (47) and (48), there are:

\[ K_2 = -\lambda_1 \lambda_2 = -p^2 - q^2 \] (52)
\[ K_1 = -\lambda_1 - \lambda_2 + 1 - K_2 = p^2 - 2p + q^2 + 1 \] (53)

By substituting Equation (37) into Equations (35) and (36), the closed-form solution of \( \gamma \) and \( \gamma_{LOS} \) under the condition \( \Delta < 0 \) can be obtained as Equations (38) and (39).

(3) If \( \Delta = 0 \), the minimum polynomial of \( A_1 \) can be obtained as:

\[
|\lambda I - A_1| = \begin{vmatrix}
    \lambda + (K_1 + K_2) & -K_1 \\
    1 & \lambda - 1 \\
\end{vmatrix} = (\lambda - \lambda_1)^2 = \lambda^2 + (K_1 + K_2 - 1)\lambda - K_2
\] (54)

and there are,

\[ \lambda_1 = \lambda_2 = \frac{-(K_1 + K_2 - 1)}{2} \] (55)
\[ K_2 = -\lambda_1^2 \] (56)
\[ K_1 = -2\lambda_1 + 1 - K_2 = (\lambda_1 - 1)^2 \] (57)

Thus, by adopting the minimal polynomial method [31] again, we obtain:

\[
\exp(A_1 f_2(t, t_0)) = a_0(t, t_0)I + a_1(t, t_0)A_1
\]
\[
= \begin{bmatrix}
    a_0(t, t_0) - a_1(t, t_0)(K_1 + K_2) & a_1(t, t_0)K_1 \\
    -a_1(t, t_0) & a_0(t, t_0) + a_1(t, t_0) \\
\end{bmatrix}
\] (58)

where,

\[ a_1(t, t_0) = f_2(t, t_0) \exp(\lambda_1 f_2(t, t_0)) = \ln \left( \frac{R(t_0)}{R(t)} \right) \frac{R(t_0)^{\lambda_1}}{R(t)^{\lambda_1}} \] (59)
\[ a_0(t, t_0) = \left[ 1 - \lambda_1 \ln \left( \frac{R(t_0)}{R(t)} \right) \right] \frac{R(t_0)^{\lambda_1}}{R(t)^{\lambda_1}} \] (60)

By substituting Equation (58) into Equation (45), the generalized solutions of \( \gamma \) and \( \gamma_{LOS} \) can be obtained as Equation (40) and Equation (41) respectively. \( \square \)
Remark 3. Using the result of Proposition 2, the generalized solution for the acceleration command can be obtained by $n_c = V\dot{\gamma} + n_T$ from Equation (27), which is also shown in three aspects as

1. If $\Delta > 0$, substituting Equation (35) into $n_c = V\dot{\gamma} + n_T$ yields:

$$n_c(t) = \frac{V(t)^2}{R(t)^{p+1}} \left\{ \ln \left( \frac{R(t)}{R(t_0)} \right) \right\} \left[ \lambda_1 (\lambda_1 - 1) \gamma_0 + K_1 \lambda_1 \gamma_{LOS0} + K_2 \gamma_f (\lambda_1 - 1) \right]$$

$$+ \frac{V(t)^2}{R(t)^{p+1}} \left\{ \lambda_2 (\lambda_2 - 1) \gamma_0 + K_1 \lambda_2 \gamma_{LOS0} + K_2 \gamma_f (\lambda_2 - 1) \right\}$$

$$+ \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} \right) b_k T^k + \frac{V(t)^2}{R(t)^{p+1}} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \times$$

$$\int_{t_0}^{t} \frac{t - \tau}{V(\tau)} \left[ \lambda_1 (\lambda_1 - 1) \frac{R(\tau)^{\lambda_1}}{R(t)^{\lambda_1}} - \lambda_2 (\lambda_2 - 1) \frac{R(\tau)^{\lambda_2}}{R(t)^{\lambda_2}} \right] d\tau$$

2. If $\Delta < 0$, we have,

$$n_c(t) = \frac{V(t)^2}{R(t)^{p+1}} \left\{ \sin \left[ q \ln \left( \frac{R(t)}{R(t_0)} \right) \right] \left[ \frac{1}{q} \left( p^2 - p - q^2 \right) \gamma_0 + K_1 p \gamma_{LOS0} + K_2 (p - 1) \gamma_f \right] + K_2 (p - 1) \gamma_f \right\}$$

$$+ \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} \right) b_k T^k + \frac{V(t)^2}{R(t)^{p+1}} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \times$$

$$\int_{t_0}^{t} \frac{t - \tau}{V(\tau)} \left[ \lambda_1 (\lambda_1 - 1) \frac{R(\tau)^{\lambda_1}}{R(t)^{\lambda_1}} - \lambda_2 (\lambda_2 - 1) \frac{R(\tau)^{\lambda_2}}{R(t)^{\lambda_2}} \right] d\tau$$

3. If $\Delta = 0$, the below solution can be obtained by substituting Equation (40) into $n_c = V\dot{\gamma} + n_T$, as:

$$n_c(t) = \frac{V(t)^2}{R(t)^{p+1}} \left\{ \ln \left( \frac{R(t)}{R(t_0)} \right) \right\} \left[ \lambda_1 (\lambda_1 - 1) \gamma_0 + K_1 \lambda_1 \gamma_{LOS0} + K_2 (\lambda_1 - 1) \gamma_f \right]$$

$$+ (2\lambda_1 - 1) \gamma_0 + K_1 \gamma_{LOS0} + K_2 \gamma_f \right\} + \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} \right) b_k T^k$$

$$+ V(t)^2 \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \int_{t_0}^{t} \frac{t - \tau}{V(\tau)} \left[ \lambda_1 (\lambda_1 - 1) \frac{R(\tau)^{\lambda_1}}{R(t)^{\lambda_1}} + (2\lambda_1 - 1) + (\lambda_1 - 1) \lambda_1 \right] \left( \frac{R(\tau)}{R(t)} \right) d\tau$$

Remark 4. Compared with the existing solutions [2–25] the generalized solutions in Proposition 2 and Remark 3 are very complex and even have some integral terms. This is because the generalized solutions further consider the case of target maneuver and allow the relative speed to be an arbitrary function of time. The closed-form expressions of the integral terms in the above equations depend heavily on the relative speed profile and are very complicated. In the Appendix A, we derive the exact closed-form expressions for the case that the relative speed is assumed to change linearly with time for a solid-propellant missile that cannot adjust thrust actively.

4. Stability Domain of Guidance Coefficients

In order to make the maneuvering acceleration converge to zero finally, the stability domain of the guidance coefficients (i.e., $K_1$ and $K_2$) is investigated here. In consideration of the different types of the generalized solutions obtained in the Proposition 2, the stability domain is investigated from three aspects.

Proposition 3. If the guidance coefficients satisfy the condition $1 - K_2 + 2\sqrt{-K_2} < K_1 < -2K_2 + 2$ and $\Delta > 0$, the terminal maneuvering acceleration $n_c(t_f)$ will converge to zero.
Proof. According to Equations (32)–(34), if \( \Delta > 0 \), \( \lambda_1 \) and \( \lambda_2 \) are different real roots and \( \lambda_1 > \lambda_2 \). Substituting Equations (47) and (48) into the above condition \( 1 - K_2 + 2\sqrt{-K_2} < K_1 < -2K_2 + 2 \) gives:

\[
\begin{align*}
\left\{ \begin{array}{l}
1 + \lambda_1\lambda_2 + 2\sqrt{\lambda_1\lambda_2} < -\lambda_1 - \lambda_2 + 1 + \lambda_1\lambda_2 \\
-\lambda_1 - \lambda_2 + 1 + \lambda_1\lambda_2 < 2\lambda_1\lambda_2 + 2 
\end{array} \right. 
\end{align*}
\]  

(64)

Simplifying Equation (64) yields:

\[
\begin{align*}
\left\{ \begin{array}{l}
\lambda_1 + \lambda_2 + 2\sqrt{\lambda_1\lambda_2} < 0 \\
\lambda_1 + \lambda_2 + 1 + \lambda_1\lambda_2 > 0 
\end{array} \right. 
\end{align*}
\]  

(65)

Solving Equation (65), we can obtain the following two groups of inequalities:

\[
\begin{align*}
\left\{ \begin{array}{l}
(\sqrt{\lambda_1} + \sqrt{\lambda_2})^2 < 0 \\
(\lambda_1 + 1)(\lambda_2 + 1) > 0 
\end{array} \right. 
\end{align*}
\]  

(66)

or

\[
\begin{align*}
\left\{ \begin{array}{l}
(\sqrt{\lambda_1} - \sqrt{\lambda_2})^2 > 0 \\
(\lambda_1 + 1)(\lambda_2 + 1) > 0 
\end{array} \right. 
\end{align*}
\]  

(67)

Obviously, \( (\sqrt{\lambda_1} + \sqrt{\lambda_2})^2 < 0 \) cannot hold. Thus, the domain of \( \lambda_1 \) and \( \lambda_2 \) is only determined by Equation (67). By solving Equation (67), we obtain:

\[ \lambda_1 < -1, \lambda_2 < -1 \]  

(68)

To obtain the terminal maneuvering acceleration, we decompose a limit of Equation (61) into three parts, as below.

\[ n_c(t_f) = \lim_{\tau \to t_f} n_c(t) = n_{c1}(t_f) + n_{c2}(t_f) + n_{c3}(t_f) \]  

(69)

where,

\[
n_{c1}(t_f) = \lim_{\tau \to t_f} \frac{V(t)}{\lambda_1 - \lambda_2} \sum_{k=0}^{m} \frac{K_1}{k+2} + \frac{K_2}{k+1} \left[ \lambda_1(\lambda_1 - 1)\gamma_0 + K_1\lambda_1\gamma_{\text{LOS}} + K_2\gamma_f(\lambda_1 - 1) \right] \\
- \frac{R(t)\lambda_2}{R(t)^2} \left[ \lambda_2(\lambda_2 - 1)\gamma_0 + K_1\lambda_2\gamma_{\text{LOS}} + K_2\gamma_f(\lambda_2 - 1) \right] \right] 
\]  

(70)

\[
n_{c2}(t_f) = \lim_{\tau \to t_f} \frac{V(t)^2}{\lambda_1 - \lambda_2} \sum_{k=0}^{m} \left[ \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right] b_k (t_f - \tau)^k 
\]  

(71)

\[
n_{c3}(t_f) = \lim_{\tau \to t_f} \frac{V(t)^2}{\lambda_1 - \lambda_2} \sum_{k=0}^{m} \left[ \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right] b_k \\
\times \int_{0}^{t_f} \frac{(t_f - \tau)^k}{\lambda_1(\lambda_1 - 1)\gamma_0 + K_1\lambda_1\gamma_{\text{LOS}} + K_2\gamma_f(\lambda_1 - 1)} - \frac{R(\tau)\lambda_2}{R(\tau)^2} \left[ \lambda_2(\lambda_2 - 1)\gamma_0 + K_1\lambda_2\gamma_{\text{LOS}} + K_2\gamma_f(\lambda_2 - 1) \right] d\tau 
\]  

(72)

With the vehicle approaching the target, there is \( \lim_{\tau \to t_f} R(t) = 0 \). Thus, according to Equation (70), when \( \lambda_1 < -1 \) and \( \lambda_2 < -1 \), there is \( n_{c1}(t_f) = 0 \). In addition, it is easy to obtain \( n_{c2}(t_f) = (K_1/2 + K_2)b_0 \). Therefore, to determine \( n_c(t_f) \), the key is to obtain \( n_{c3}(t_f) \). However, this is difficult because of the complicated integral terms resulting from the target’s maneuver.

According to Equation (72), \( n_{c3}(t_f) \) can be divided into:

\[ n_{c3}(t_f) = n_{c30}(\lambda_1, t_f) - n_{c30}(\lambda_2, t_f) \]  

(73)
When, 

\[ n_{c30}(\lambda_1, t_f) = \lim_{t \to t_f} V(t)^{2} \frac{K_1}{k+2} \frac{K_2}{k+1} - 1 \frac{d_1(\lambda_1, t)}{d_0(\lambda_1, t)} \]  

(74) 

\[ n_{c30}(\lambda_2, t_f) = \lim_{t \to t_f} V(t)^{2} \frac{K_1}{k+2} \frac{K_2}{k+1} - 1 \frac{d_1(\lambda_2, t)}{d_0(\lambda_2, t)} \]  

(75) 

\[ d_0(\lambda_i, t) = R(t)^{\lambda_i + 1}; \quad i = 1, 2 \]  

(76) 

\[ d_1(\lambda_i, t) = \int_{t_0}^{t} \left( \frac{t_f - \tau}{V(\tau)} \right)^k R(\tau)^{\lambda_i} d\tau; \quad i = 1, 2 \]  

(77) 

We analyze \( n_{c30}(\lambda_1, t_f) \) first, where the key is to determine \( \lim_{t \to t_f} d_0(\lambda_1, t) \) and \( \lim_{t \to t_f} d_1(\lambda_1, t) \). When \( \lambda_1 < -1 \), it is obvious that \( \lim_{t \to t_f} d_0(\lambda_1, t) = \infty \). However, \( \lim_{t \to t_f} d_1(\lambda_1, t) \) is hard to determine due to the complex non-linear integrand. To resolve this difficulty, we create some inequalities subtly to simplify the integral. Denote the maximum and minimum closing speeds during the engagement as \( V_{\text{max}} \) and \( V_{\text{min}} \), respectively. Obviously, there is \( V_{\text{max}} \geq V(\tau) \geq V_{\text{min}} > 0 \), and then the following inequalities can be obtained:

\[ \left( \frac{t_f - \tau}{V_{\text{max}}} \right)^k > \frac{R(\tau)}{V_{\text{max}}}^k \]  

(78) 

\[ \left( \frac{t_f - \tau}{V_{\text{min}}} \right)^k < \frac{R(\tau)}{V_{\text{min}}}^k \]  

(79) 

Then, from Equation (77), we can establish the following inequalities:

\[ \lim_{t \to t_f} D_{1\text{max}}(t) = \lim_{t \to t_f} d_1(\lambda_1, t) \geq \lim_{t \to t_f} D_{1\text{min}}(t) \]  

(80) 

where,

\[ D_{1\text{min}}(t) = \int_{t_0}^{t} \left( \frac{V(t)}{V_{\text{max}}}^2 \right)^k \frac{R(t)}{V(t)}^{\lambda_1} d\tau = - \frac{1}{\lambda_1 + k + 1} \frac{R(t)^{\lambda_1 + k + 1} - R(t_0)^{\lambda_1 + k + 1}}{V_{\text{max}}^{k+2}} \]  

(81) 

\[ D_{1\text{max}}(t) = \int_{t_0}^{t} \left( \frac{V(t)}{V_{\text{min}}}^2 \right)^k \frac{R(t)}{V(t)}^{\lambda_1} d\tau = - \frac{1}{\lambda_1 + k + 1} \frac{R(t)^{\lambda_1 + k + 1} - R(t_0)^{\lambda_1 + k + 1}}{V_{\text{min}}^{k+2}} \]  

(82) 

By observing Equations (80)–(82), we can find that the magnitude of \( \lim_{t \to t_f} d_1(\lambda_1, t) \) depends on the parameter \( k \) (See Equation (7)), which is associated with the target’s maneuvering acceleration. We define \( k_1 \) as an integer meeting that \( 0 \leq \lambda_1 + k_1 + 1 < 1 \).

1. If \( 0 \leq k < k_1 \), there is \( \lambda_1 + k + 1 < 0 \), which means \( \lim_{t \to t_f} R(t)^{\lambda_1 + k + 1} = \infty \). Thus we have:

\[ \infty = \lim_{t \to t_f} D_{1\text{max}}(t) \geq \lim_{t \to t_f} d_1(\lambda_1, t) \geq \lim_{t \to t_f} D_{1\text{min}}(t) = \infty \]  

(83)
In addition, \( \lim_{t \to t_f} d_0(\lambda_1, t) = \infty \) for \( \lambda_1 < -1 \). Therefore, the following limit can be determined by applying the L’Hospital’s rule:

\[
\lim_{t \to t_f} \frac{d_1(\lambda_1, t)}{d_0(\lambda_1, t)} = \lim_{t \to t_f} \frac{d(d_1(\lambda_1, t))/dt}{d(d_0(\lambda_1, t))/dt} = -\frac{1}{(\lambda_1 + 1)^2(t_f)}; \quad \text{if } k = 0
\]
\[
\lim_{t \to t_f} \frac{(t_f-t)^k}{(\lambda_1+1)^2(t_f)} = \begin{cases} 
0 & \text{if } 0 < k < k_r \n\end{cases}
\]

(2) If \( k \geq k_r \), there is \( \lambda_1 + k + 1 \geq 0 \), and so \( \lim_{t \to t_f} R(t)^{\lambda_1+k+1} = 0 \). Therefore, we can obtain \( \lim_{t \to t_f} D_{1\text{min}}(t) \) and \( \lim_{t \to t_f} D_{1\text{max}}(t) \) easily, as below:

\[
\lim_{t \to t_f} D_{1\text{max}}(t) = \frac{1}{\lambda_1^{1+k+1}} \frac{R(t_0)^{1+k+1}}{V_{\text{max}}^{1+k+1}}
\]
\[
\lim_{t \to t_f} D_{1\text{min}}(t) = \frac{1}{\lambda_1^{1+k+1}} \frac{R(t_0)^{1+k+1}}{V_{\text{min}}^{1+k+1}}
\]

Since \( \lim_{t \to t_f} d_0(\lambda_1, t) = \infty \), there is:

\[
0 = \lim_{t \to t_f} \frac{D_{1\text{max}}(t)}{d_0(\lambda_1, t)} \geq \lim_{t \to t_f} \frac{d_1(\lambda_1, t)}{d_0(\lambda_1, t)} \geq \lim_{t \to t_f} \frac{D_{1\text{min}}(t)}{d_0(\lambda_1, t)} = 0
\]

Substituting Equations (84) and (86) into Equation (74), we can obtain \( n_{c30}(\lambda_1, t_f) \):

\[
n_{c30}(\lambda_1, t_f) = -\frac{1}{\lambda_1 - \lambda_2} \frac{\lambda_1(\lambda_1 - 1)}{(\lambda_1 + 1)^2} \left( \frac{K_1}{2} + K_2 - 1 \right) b_0
\]

Similarly, \( n_{c30}(\lambda_2, t_f) \) can be obtained by:

\[
n_{c30}(\lambda_2, t_f) = -\frac{1}{\lambda_1 - \lambda_2} \frac{\lambda_2(\lambda_2 - 1)}{(\lambda_2 + 1)^2} \left( \frac{K_1}{2} + K_2 - 1 \right) b_0
\]

Substituting Equations (87) and (88), Equations (47) and (48) into Equation (73) gives:

\[
n_c(t_f) = -\left( \frac{K_1}{2} + K_2 \right) b_0 \]

Substituting Equation (89) into Equation (69), there is:

\[
n_c(t_f) = \lim_{t \to t_f} n_c(t) = 0 + \left( \frac{K_1}{2} + K_2 \right) b_0 - \left( \frac{K_1}{2} + K_2 \right) b_0 = 0
\]

Therefore, it can be concluded that if \( 1 - K_2 + 2\sqrt{-K_2} < K_1 < -2K_2 + 2 \), \( \lim_{t \to t_f} n_c(t) \) converges to zero finally. □

**Proposition 4.** If the guidance coefficients satisfy the condition \( 3 - K_2 < K_1 < 1 - K_2 + 2\sqrt{-K_2} \) and \( \Delta < 0 \), the terminal maneuvering acceleration \( n_c(t_f) \) converges to zero.

**Proof.** For the condition that \( \Delta < 0 \), \( \lambda_1 \) and \( \lambda_2 \) are complex conjugates and described by Equations (37)–(53). Substituting Equations (37)–(53) into the above condition \( 3 - K_2 < K_1 < 1 - K_2 + 2\sqrt{-K_2} \) yields:

\[
\begin{cases}
3 + p^2 + q^2 < p^2 - 2p + q^2 + 1 \\
p^2 - 2p + q^2 + 1 < 1 + p^2 + q^2 + 2\sqrt{p^2 + q^2}
\end{cases}
\]

(91)
Simplifying Equation (91) yields:

\[
\begin{align*}
 p < -1 & \quad \text{or} \\
 p + \sqrt{p^2 + q^2} > 0
\end{align*}
\]

By solving Equation (92), we obtain:

\[ p < -1 \]  \hspace{1cm} (93)

Then the terminal maneuvering acceleration can be obtained by taking the limit of Equation (62), as:

\[
\begin{align*}
n_c(t_f) &= \lim_{t \to t_f} n_c(t) = n_{c1}(t_f) + n_{c2}(t_f) + n_{c3}(t_f)
\end{align*}
\]

where,

\[
\begin{align*}
n_{c1}(t_f) &= \lim_{t \to t_f} V(t)^2 \frac{R(t_0)}{R(t)} \
&= \cos \left[ q \ln \left( \frac{R(l_0)}{R(t_0)} \right) \right] \left( 2p - 1 \right) \gamma_0 + K_1 \gamma_{LOS} + K_2 \gamma_f \hspace{1cm} (95)\sin \left[ q \ln \left( \frac{R(t_0)}{R(t_0)} \right) \right] \left( 2p - q^2 \right) \gamma_0 + K_1 p \gamma_{LOS} + K_2 (p - 1) \gamma_f \right] \\
&+ \sum_{k=0}^{m} \left( \frac{K_1}{k + 2} + \frac{K_2}{k + 1} \right) b_k (t_f - t)^k
\end{align*}
\]

\[
\begin{align*}
n_{c2}(t_f) &= \lim_{t \to t_f} \frac{V(t)^2 \int_{t_0}^{t} \frac{R(t)}{R(t_0)} \left( \frac{K_1}{r + 2} + \frac{K_2}{r + 1} \right) b_k (t_r - t)^k}{V(t)} \\
&= \left[ \lambda_1 R(t)^{\lambda_1} R(t)^{\lambda_1} (\lambda_1 - 1) - \lambda_2 R(t)^{\lambda_2} R(t)^{\lambda_2} (\lambda_2 - 1) \right] d\tau
\end{align*}
\]

\[
\begin{align*}
n_{c3}(t_f) &= \lim_{t \to t_f} \left[ \lambda_1 R(t)^{\lambda_1} R(t)^{\lambda_1} (\lambda_1 - 1) - \lambda_2 R(t)^{\lambda_2} R(t)^{\lambda_2} (\lambda_2 - 1) \right] d\tau
\end{align*}
\]

Obviously, as \( R(t_f) = 0 \), if the real part of the roots meets that \( p < -1 \), there are:

\[
\begin{align*}
\lim_{t \to t_f} \left[ \frac{R(t)}{R(t_0)} \right]^{p - 1} \cos \left[ q \ln \left( \frac{R(t_0)}{R(t_0)} \right) \right] - \left| \lim_{t \to t_f} \frac{R(t)}{R(t_0)} \right|^{p - 1} = 0 \\
\lim_{t \to t_f} \left[ \frac{R(t)}{R(t_0)} \right]^{p - 1} \sin \left[ q \ln \left( \frac{R(t_0)}{R(t_0)} \right) \right] - \left| \lim_{t \to t_f} \frac{R(t)}{R(t_0)} \right|^{p - 1} = 0
\end{align*}
\]

According to Equations (95) and (98), we can conclude that when the real part of \( \lambda_1 \) is less than \(-1\) (i.e., \( \text{Re}(\lambda_1) < -1 \)), \( n_{c1}(t_f) = 0 \) holds. Since Equation (72) is the same as Equation (72), the limit of Equation (97) can also be determined by Equation (89). Meanwhile, we have \( n_{c2}(t_f) = (K_1/2 + K_2)b_0 \) from Equation (96). As a result, it can be concluded that \( \lim_{t \to t_f} n_c(t) = 0 \) can be guaranteed when the stability domain of the guidance coefficients is specified by \( 3 - K_2 < K_1 < 1 - K_2 + 2\sqrt{-K_2} \). □

**Proposition 5.** If the guidance coefficients satisfy the condition \( K_2 < -1, K_1 = 1 - K_2 + 2\sqrt{-K_2} \) and \( \Delta = 0 \), the terminal maneuvering acceleration \( n_c(t_f) \) converges to zero.

**Proof.** If \( \Delta = 0 \), we have \( \lambda_1 = \lambda_2 \). Substituting Equations (56) and (57) into the above two conditions \( K_2 < -1 \) and \( K_1 = 1 - K_2 + 2\sqrt{-K_2} \) gives:

\[
\begin{align*}
-\lambda_1^2 < -1 \\
(\lambda_1 - 1)^2 = 1 + \lambda_1^2 + 2\sqrt{\lambda_1^2}
\end{align*}
\]
Simplifying Equation (99), we obtain:

\[\begin{align*}
\left\{ \begin{array}{l}
\lambda_1^2 > 1 \\
-\lambda_1 = \sqrt{\lambda_1^2}
\end{array} \right. \quad (100)
\end{align*}\]

Solving Equation (100) yields:

\[\lambda_1 < -1 \quad (101)\]

Then the terminal maneuvering acceleration can be obtained from Equation (63), as:

\[n_c(t_f) = \lim_{t \to t_f} n_c(t) = n_{c1}(t_f) + n_{c2}(t_f) + n_{c3}(t_f) \quad (102)\]

where,

\[n_{c1}(t_f) = \lim_{t \to t_f} V(t)^2 \frac{R(t_0)^{\lambda_1}}{R(t)^{\lambda_1+1}} \left\{ (2\lambda_1 - 1)\gamma_0 + K_1\gamma_{\text{LOS}} + K_2\gamma_f \\
+ \ln\left(\frac{R(t_0)}{R(t)}\right) \left[ \lambda_1(\lambda_1 - 1)\gamma_0 + K_1\lambda_1\gamma_{\text{LOS}} + K_2(\lambda_1 - 1)\gamma_f \right] \right\} \quad (103)\]

\[n_{c3}(t_f) = \lim_{t \to t_f} V(t)^2 \int_{t_0}^{t_f} \sum_{k=0}^{m} \frac{K_1(k+1)}{k+1} b_k (t_f - \tau)^k \\
\times \frac{R(t_0)^{\lambda_1}}{R(t)^{\lambda_1+1}} \left[ (2\lambda_1 - 1) + \lambda_1(\lambda_1 - 1) \ln\left(\frac{R(t_0)}{R(t)}\right) \right] d\tau \quad (104)\]

\[n_{c2}(t_f) = \lim_{t \to t_f} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} \right) b_k (t_f - t)^k \quad (105)\]

In Equation (103), as \(R(t_f) = 0\), when \(\lambda_1 < -1\), we can obtain the following limit by the L’Hospital’s rule.

\[\lim_{t \to t_f} \left| \frac{\ln(R(t_0)/R(t))}{R(t)^{\lambda_1+1}} \right| = \lim_{t \to t_f} \left| \frac{d[\ln(R(t_0)/R(t))]/dt}{d[R(t)^{\lambda_1+1}]/dt} \right| = \lim_{t \to t_f} \left| -\frac{1}{(\lambda_1 + 1)R(t)^{\lambda_1+1}} \right| = 0 \quad (106)\]

Substituting Equation (106) into Equation (103), we can conclude that if and only if \(\lambda_1 < -1\), then \(n_{c1}(t_f) = 0\). In addition, apparently, there is \(n_{c2}(t_f) = (K_1/2 + K_2)b_0\) from Equation (104). Now we need to obtain \(n_{c3}(t_f)\) under the condition that \(\lambda_1 < -1\).

Let,

\[n_{c3}(t_f) = n_{c31}(t_f) - n_{c32}(t_f) \quad (107)\]

where,

\[n_{c31}(t_f) = \lim_{t \to t_f} V(t)^2 (2\lambda_1 - 1) \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \frac{d_1(\lambda_1,t)}{d_0(\lambda_1,t)} \quad (108)\]

\[n_{c32}(t_f) = \lim_{t \to t_f} V(t)^2 \lambda_1(\lambda_1 - 1) \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \frac{d_2(\lambda_1,t)}{d_0(\lambda_1,t)} \quad (109)\]

\[d_2(\lambda_1,t) = \int_{t_0}^{t} \left( \frac{t_f - \tau}{V(\tau)} \right)^k R(\tau)^{\lambda_1} \ln\left(\frac{R(\tau)}{R(t)}\right) d\tau \quad (110)\]
\( d_0(\lambda_1, t) \) and \( d_1(\lambda_1, t) \) have been shown by Equations (76) and (77), respectively. Similar to the derivation process from Equations (74) to (87), the following equation can be obtained using the L’Hospital’s rule:

\[
\begin{align*}
\lim_{t \to t_f} n_{C31}(t_f) &= \lim_{t \to t_f} V(t)^2(2\lambda_1 - 1) \sum_{k=0}^{k_1} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \frac{d(d_1(\lambda_1, t))}{dt} \\
&= - \left( \frac{2\lambda_1 - 1}{\lambda_1 + 1} \right) \left( \frac{K_1}{2} + K_2 - 1 \right) b_0
\end{align*}
\]

(111)

Next, we analyze \( n_{C32}(t_f) \). If \( \lambda_1 < -1 \), obviously \( \lim_{t \to t_f} d_0(\lambda_1, t) = \infty \) holds. By adopting the inequalities shown in Equations (78) and (79), we can create:

\[
\lim_{t \to t_f} D_{2\max}(t) \geq \lim_{t \to t_f} d_2(\lambda_1, t) \geq \lim_{t \to t_f} D_{2\min}(t)
\]

(112)

where \( D_{2\min}(t) \) and \( D_{2\max}(t) \) are given in Equation (113) and Equation (114) respectively.

\[
\begin{align*}
D_{2\min}(t) &= \int_{t_0}^{t_f} V(t)^2 \frac{R(t)}{V(t)^{\lambda_1} \ln \left( \frac{R(t)}{R(t_0)} \right)} \frac{d\tau}{\lambda_1 + k = -1} \\
&= \left\{ \begin{array}{ll}
\frac{1}{2V(t)^{k+2}} \ln \left( \frac{R(t_0)}{R(t)} \right) & \lambda_1 + k = -1 \\
- \frac{1}{V(t_0)^{k+2}} \left[ \frac{R(t)^{\lambda_1} R(t_0)^{\lambda_1+1} - R(t_0)^{\lambda_1+1}}{(\lambda_1 + k + 1)^2} + \frac{R(t)^{\lambda_1} R(t_0)^{\lambda_1+1} \ln \left( \frac{R(t)}{R(t_0)} \right)}{R(t_0)^{\lambda_1+1}} \right] & \lambda_1 + k \neq -1
\end{array} \right.
\end{align*}
\]

(113)

\[
\begin{align*}
D_{2\max}(t) &= \int_{t_0}^{t_f} V(t)^2 \frac{R(t)}{V(t)^{\lambda_1} \ln \left( \frac{R(t)}{R(t_0)} \right)} \frac{d\tau}{\lambda_1 + k = -1} \\
&= \left\{ \begin{array}{ll}
\frac{1}{2V(t)^{k+2}} \ln \left( \frac{R(t_0)}{R(t)} \right) & \lambda_1 + k = -1 \\
- \frac{1}{V(t_0)^{k+2}} \left[ \frac{R(t)^{\lambda_1} R(t_0)^{\lambda_1+1} - R(t_0)^{\lambda_1+1}}{(\lambda_1 + k + 1)^2} + \frac{R(t)^{\lambda_1} R(t_0)^{\lambda_1+1} \ln \left( \frac{R(t)}{R(t_0)} \right)}{R(t_0)^{\lambda_1+1}} \right] & \lambda_1 + k \neq -1
\end{array} \right.
\end{align*}
\]

(114)

From Equations (112)–(114), it can be seen that \( \lim_{t \to t_f} \frac{d_2(\lambda_1, t)}{d_0(\lambda_1, t)} \) appearing in Equation (109) also depends on the target-maneuver parameter \( k \). Define \( k_{r2} \) as an integer meeting that \( 0 < \lambda_1 + k_{r2} + 1 < 1 \).

(a) If \( 0 \leq k < k_{r2} \) or \( k = -\lambda_1 - 1 \), we have \( \lambda_1 + k + 1 \leq 0 \), which leads to \( \lim_{t \to t_f} R(t)^{\lambda_1+k+1} = \infty \).

Thus there is:

\[
\infty = \lim_{t \to t_f} D_{2\max}(t) \geq \lim_{t \to t_f} d_2(\lambda_1, t) \geq \lim_{t \to t_f} D_{2\min}(t) = \infty
\]

(115)

Note that \( \lim_{t \to t_f} d_0(\lambda_1, t) = \infty \) for \( \lambda_1 < -1 \). Therefore, in this condition, the magnitude of \( \lim_{t \to t_f} \frac{d_2(\lambda_1, t)}{d_0(\lambda_1, t)} \) can be solved by the L’Hospital’s rules.

\[
\begin{align*}
\lim_{t \to t_f} \frac{d_2(\lambda_1, t)}{d_0(\lambda_1, t)} &= \lim_{t \to t_f} \frac{d(d_2(\lambda_1, t))/dt}{d(d_0(\lambda_1, t))/dt} \\
&= - \frac{1}{(\lambda_1 + 1)} \lim_{t \to t_f} \int_{t_0}^{t_f} \frac{(V(t)^{\lambda_1})^2}{R(t)^{\lambda_1+1} V(t)^{\lambda_1+k+1}} \frac{d\tau}{R(t)^{\lambda_1+1}} \\
&= - \frac{1}{(\lambda_1 + 1)} \lim_{t \to t_f} \int_{t_0}^{t_f} \frac{(V(t)^{\lambda_1})^2}{R(t)^{\lambda_1+1}} \frac{d\tau}{R(t)^{\lambda_1+1}} \\
&= - \frac{1}{(\lambda_1 + 1)} \lim_{t \to t_f} \int_{t_0}^{t_f} \frac{d_2(\lambda_1, t)}{d_0(\lambda_1, t)}
\end{align*}
\]

(116)
In the previous calculation, \( \lim_{t \to t_f} \frac{d_1(\lambda_1, t)}{d_0(\lambda_1, t)} \) has been obtained by Equation (84). Substituting Equation (84) into Equation (116) gives:

\[
\lim_{t \to t_f} \frac{d_2(\lambda_1, t)}{d_0(\lambda_1, t)} = \begin{cases} 
- \frac{1}{(\lambda_1 + 1)^2 V(t_f)^2} & \text{if } k = 0 \\
0 & \text{if } 0 < k < k_2 \text{ or } k = -\lambda_1 - 1
\end{cases}
\]  

(117)

(b) If \( k \geq k_2 \), there is \( \lambda_1 + k + 1 > 0 \), which means \( \lim_{t \to t_f} R(t)_{\lambda_1+k+1} = 0 \). Using these, Equations (113) and (114) can be rewritten as:

\[
\begin{align*}
\lim_{t \to t_f} D_{2\text{min}}(t) &= \frac{R(t)_{\lambda_1+k+1}}{(\lambda_1+k+1)^2 V(t_f)^2} - \lim_{t \to t_f} \frac{1}{(\lambda_1+k+1)^2 V(t_f)^2} R(t)_{\lambda_1+k+1} \ln \left( \frac{R(t)}{R(t_f)} \right) \\
\lim_{t \to t_f} D_{2\text{max}}(t) &= \frac{R(t)_{\lambda_1+k+1}}{(\lambda_1+k+1)^2 V(t_f)^2} - \lim_{t \to t_f} \frac{1}{(\lambda_1+k+1)^2 V(t_f)^2} R(t)_{\lambda_1+k+1} \ln \left( \frac{R(t)}{R(t_f)} \right)
\end{align*}
\]  

(118)

Therefore, by substituting Equation (118) into Equation (112), dividing Equation (112) by \( \lim_{t \to t_f} d_0(\lambda_1, t) = \lim_{t \to t_f} R(t)_{\lambda_1+1} = \infty \) and applying Equation (106), we obtain:

\[
0 = \lim_{t \to t_f} \frac{D_{2\text{max}}(t)}{d_0(t)} \geq \lim_{t \to t_f} \frac{d_2(t)}{d_0(t)} \geq \lim_{t \to t_f} \frac{D_{2\text{min}}(t)}{d_0(t)} = 0
\]  

(119)

Substituting Equations (117) and (119) into Equation (109), \( n_{C2} \left( t_f \right) \) is obtained, as below:

\[
n_{C2} \left( t_f \right) = \frac{\lambda_1}{(\lambda_1+1)^2} \left( \frac{K_1}{2} + K_2 - 1 \right) b_0
\]  

(120)

Substituting Equations (47), (48), (111) and (120), into Equation (107) yields:

\[
n_{C3} \left( t_f \right) = - \left( \frac{K_1}{2} + K_2 \right) b_0
\]  

(121)

Then combining Equation (121) with Equation (102), we obtain:

\[
n_c \left( t_f \right) = \lim_{t \to t_f} n_c(t) = 0 + \left( \frac{K_1}{2} + K_2 \right) b_0 - \left( \frac{K_1}{2} + K_2 \right) b_0 = 0
\]  

(122)

Therefore, the stability domain of the guidance coefficients under the condition of \( \Delta = 0 \) can be obtained and described by \( K_2 < -1 \) and \( K_1 = 1 - K_2 + 2\sqrt{-K_2} \).

**Remark 5.** According to the results of Proposition 3, Proposition 4 and Proposition 5, the stability domain of the guidance coefficients making the terminal acceleration converge to zero finally is:

\[
\begin{cases} 
K_2 < -1 \\
3 - K_2 < K_1 < -2K_2 + 2
\end{cases}
\]  

(123)

*If the guidance coefficients are located in the stability domain, the maneuvering acceleration will converge to zero finally, no matter whether the closing speed keeps constant or changes with time.*

5. **The Scheme of the New Guidance**

The procedure for implementing the new guidance law is summarized in Figure 3 and Table 1. Figure 3 shows the flowchart of the new guidance. The input parameters are the states of the missile (i.e., \( \mathbf{X}_M, \mathbf{V}_M \)) measured by the inertial navigation system (INS) [32], relative states measured by the seeker (i.e., \( R, \gamma_{LOS} \)), and the terminal constraints.
$X_{TMf}$, $V_{TMf}$, which are shown in the yellow block in Figure 3. By employing tracking filters [33,34], the target motion can be estimated in a reliable way. Then, substituting the relative states into Equation (26), the acceleration command can be obtained.

**Figure 3.** The flowchart of the new guidance.

**Table 1.** The scheme of the new guidance.

| Input parameters: The states of the missile (i.e., $X_M, V_M$) measured by the INS and relative states measured by the seeker (i.e., $R, \gamma_{LOS}$); the terminal constraints $X_{TMf}$, $V_{TMf}$ |
| Control input parameters: The relative states and target maneuvers obtained by the filter. |
| Output: The miss distance, impact angle, and terminal maneuvering acceleration. |

**Step 1:** Select guidance coefficients beforehand:

a. Linearize the relative dynamics and derive the closed-form solutions of acceleration command;

b. By analyzing the solutions, obtain the stability domain of the guidance coefficients, shown in Equation (123).

c. Choose guidance coefficients in the domain.

**Step 2:** Calculate the guidance command from Equation (26);

**Step 3:** Determine whether to continue the guidance loop;

a. Employ the guidance command to the nonlinear dynamics;

b. Update the current states of missile and target;

If missile attacks the target, go to the next step, if not, go to step 2.

**Step 4:** Output the terminal states.

### 6. Simulation Results

#### 6.1. Accuracy Verification of Generalized Solutions

Now we verify the accuracy of the generalized solutions. As a comparison, the results of the solutions proposed by [2] are also provided here. In this example, the initial conditions of the vehicle are $x_0 = 0$ km, $y_0 = 0$ km, $V_{m0} = 2500$ m/s, $\gamma_{m0} = 0^\circ$. The initial conditions of the target are $x_{t0} = 80$ km, $y_{t0} = 0$ km, $V_{t0} = 1800$ m/s, $\gamma_{t0} = 180^\circ$. The target’s maneuvering acceleration is set to $n_T = \sum_{k=0}^{6} b_k \left( t_f - t \right)^k$, where $\{b_0, \ldots, b_6\} = \{-5.51, 3.55, -0.82, 0.07, 7.19 \times 10^{-4}, -3.23 \times 10^{-4}, 1.06 \times 10^{-5}\}$. The change rate of the closing speed is assumed to be $-50$ m/s². In Appendix A, the completely closed-form solutions for the new guidance are derived under the condition that the closing speed changes linearly
with time. The desired impact angle is set to $\gamma_f = \gamma_{1f} - 180^\circ$. Two cases about the guidance coefficients are considered as below.

Case 1: The guidance coefficients are set to $K_1 = 20$ and $K_2 = -12$, where the corresponding roots are $\lambda_1 = -3$ and $\lambda_2 = -4$, and the closed-form solution of the acceleration command is described by Equation (A16).

Case 2: The guidance coefficients are $K_1 = 16$ and $K_2 = -9$, where the corresponding roots are $\lambda_1 = \lambda_2 = -3$, and the analytical solution of the acceleration command is described by Equation (A24).

The simulation results are presented below. It can be seen from Figures 4 and 5 and that the new closed-form solutions are more accurate than TSG solutions. This is because TSG solutions are only applicable to the case that the target is stationary and the vehicle’s speed is constant. It can also be seen from these figures that, by properly selecting the guidance coefficients, the acceleration command can converge to zero finally. In addition, the average computing time for the new analytical solutions is $3.21 \times 10^{-5}$ s, which is 3 orders of magnitude less than that of trajectory simulation.

![Figure 4](image1.png)

**Figure 4.** Commanded acceleration profiles for the trajectory simulation and closed-form solutions when $\lambda_1 = -3$ and $\lambda_2 = -4$.

![Figure 5](image2.png)

**Figure 5.** Commanded acceleration profiles for the trajectory simulation and closed-form solutions when $\lambda_1 = \lambda_2 = -3$.

6.2. Performance of the New Guidance

In order to demonstrate the performance of the new guidance, we compare the results of new guidance, TSG [2], adaptive weighting optimal guidance law (OGL/AIAW) [25], and state-dependent Riccati equation-based impact angle constrained guidance (SDRE-
based IACG) [26]. In this example, let the guidance coefficients of the new guidance be $K_1 = 7.5$ and $K_2 = -3$, where the corresponding roots are $\lambda_1 = -1.5$ and $\lambda_2 = -2$.

The initial conditions of intercepting vehicle are set to $x_0 = 0$ km, $y_0 = 0$ km, $V_{m0} = 3000$ m/s and $\gamma_{m0} = 20^\circ$, the change rate of speed of the vehicle is assumed to be $-100$ m/s$^2$. Considering the limitation of the vehicle’s maneuverability, we set $|n_c| \leq 200$ m/s$^2$, which has been verified in [35,36]. The initial conditions of target are $x_{t0} = 50$ km, $y_{t0} = 2$ km, $V_{t0} = 2000$ m/s and $\gamma_{t0} = 180^\circ$. Let $n_T = \sum_{k=0}^{5} b_k t^k$, where \( \{b_0, \ldots, b_5\} = \{-20.34, -0.27, -0.30, 0.11, -0.01, 2.35 \times 10^{-4}\} \). As a head-on collision is expected, the desired final flight-path angle of the vehicle is set to $\gamma_f = \gamma_{tf} = 180^\circ$.

The simulation results of the four guidance laws mentioned above are shown in Figures 6–9 and Table 2. Figure 6 shows that the trajectories for OGL/AIAW and SDRE-based IACG bend more than the new guidance, which indicates that OGL/AIAW and SDRE-based IACG requires more control effort (shown in Figure 7). Although the forpart trajectory of TSG is smoother than the other three guidance laws, Figure 7 shows that TSG has a large magnitude of terminal commanded acceleration, which may lead to a large miss distance if the control system constraint is stricter or the maneuver of the target is higher. A similar situation also degrades the performance of the OGL/AIAW. Because the guidance command of OGL/AIAW is saturated finally (see Figure 7), the accuracy of OGL/AIAW is much lower than that of the other three guidance laws, as shown in Table 2. Figure 7 also shows that the SDRE-based guidance law has the fastest reduction of guidance command when approaching the target, resulting in the straightest trajectory at the end of engagement shown in Figure 6. But the command profile of terminal guidance has a big fluctuation at the end due to the high-speed target’s maneuvers, which is unfavorable to attack accuracy and control tracking, especially for a high-speed vehicle. In contrast, in the scenario of intercepting a maneuvering high-speed target, only the terminal acceleration command of the new guidance law can converge to zero. Figure 8 shows the profile of $\gamma - \gamma_t$. Due to the command saturation, the OGL/AIAW fails to meet the terminal impact angle constraint. Figure 9 shows the cumulative control effort profiles for the four guidance laws, from which we can see that the control effort of the new guidance is slightly larger than that of TSG but far less than that of the other two guidance laws. In summary, among the four guidance laws, the new guidance is comprehensively optimal for the case of intercepting a maneuvering target.
**Figure 7.** Comparison of the commanded acceleration profiles for new guidance, TSG, OGL/AIAW, and SDRE-based IACG.

**Figure 8.** Comparison of the $\gamma - \gamma_t$ profiles for new guidance, TSG, OGL/AIAW, and SDRE-based IACG.

**Figure 9.** Comparison of the control effort profiles for new guidance, TSG, OGL/AIAW, and SDRE-based IACG.
### Table 2. Terminal results of new guidance, TSG, OGL/AIAW, and SDRE-based IACG.

| Terminal States       | Miss Distance (m) | Impact Angle Error (deg) | Terminal Acceleration Command (m/s²) | Terminal Control Efforts (m²/s³) |
|-----------------------|-------------------|--------------------------|-------------------------------------|----------------------------------|
| New Guidance          | 2.02 × 10⁻⁹       | 4.21 × 10⁻⁶              | −0.22                               | 1.81 × 10⁵                      |
| TSG                   | 1.00 × 10⁻³       | 4.00 × 10⁻³              | −200.00                             | 1.79 × 10⁵                      |
| OGL/AIAW              | 304.94            | 4.53                     | 24.66                               | 3.54 × 10⁵                      |
| SDRE-based IACG       | 1.21 × 10⁻⁶       | 7.38 × 10⁻⁶              | 12.88                               | 2.60 × 10⁵                      |

6.3. Waving Maneuvering Target

In order to verify the performance of the new guidance more adequately, four more challenging examples are given where the vehicles are intended to intercept waving maneuvering targets. In the four cases, the vehicles have the same initial conditions, which are \(x_0 = 0\) km, \(y_0 = 0\) km, \(V_{m0} = 3000\) m/s and \(\gamma_{m0} = 0°\). The change rates of the vehicles’ velocities are no longer constant, but change nonlinearly with time and are determined by \(\dot{V}_m = -C_D(0.5\rho V^2)S_{ref}/m\), where the CAV-H vehicle model [37] is used to specify the drag coefficient \(C_D\), the reference area \(S_{ref}\), and the vehicle’s mass \(m\). As shown in Table 3, \(\lambda_1\) and \(\lambda_2\) are set to different values for the 4 cases. The targets’ maneuvering acceleration are designed as \(n_T = A_T \sin(\omega_T(t_f - t))\), where the parameters \(A_T\) and \(\omega_T\) are shown in Table 3. The common initial conditions of the targets are \(x_{t0} = 60\) km, \(y_{t0} = 0\) km, \(V_{t0} = 1800\) m/s and \(\gamma_{t0} = 170°\). The desired impact angle is set to \(\gamma_f = \gamma_{tf} - 180°\).

### Table 3. Setting of the simulation parameters for different cases.

| Parameters | Case 1 | Case 2 | Case 3 | Case 4 |
|------------|--------|--------|--------|--------|
| \(\lambda_1\) | −2.5   | −2 + 2i | −2.6   | −1     |
| \(\lambda_2\) | −4.1   | −2 − 2i | −2.6   | −2     |
| \(K_1\)     | 17.85  | 13     | 12.96  | 6      |
| \(K_2\)     | −10.25 | −8     | −6.76  | −2     |
| \(A_T\)     | −8 g   | −7 g   | −9 g   | −10 g  |
| \(\omega_T\) | 3      | 3      | 3      | 3      |

The simulation results are shown in Table 4 and Figures 10–13. In Case 1–3, as the selected guidance coefficients fall within the stability domain, the vehicles can successfully intercept the targets while satisfying all the constraints. However, in Case 4, the guidance coefficients are located at the boundary of the stability domain but do not belong to the stability domain. Due to this, the commanded acceleration for Case 4 fails to converge to zero finally, as shown in Figure 11. This indicates that the analysis taken on the stability domain is rigorous.

### Table 4. Terminal results for the four cases.

| Case | 1                  | 2                  | 3                  | 4                  |
|------|--------------------|--------------------|--------------------|--------------------|
| Miss distance (m) | 8.69 × 10⁻¹⁸     | 9.72 × 10⁻¹⁸     | 9.35 × 10⁻¹⁸     | 1.90 × 10⁻⁸     |
| Impact angle errors (deg) | 6.74 × 10⁻¹¹ | 2.48 × 10⁻¹⁰ | 7.72 × 10⁻¹⁰ | 2.62 × 10⁻⁵ |
| Terminal acceleration command (m/s²) | 2.00 × 10⁻³    | −6.40 × 10⁻³   | 2.70 × 10⁻⁴    | −45.65           |
Figure 10. Trajectories for the four cases.

Figure 11. Acceleration command profiles for the four cases.

Figure 12. The profiles of $\gamma - \gamma_t$ for the four cases.
6.4. Robustness of the New Guidance  

6.4.1. Robustness to Time Delays

Since the acceleration command generated by the guidance law is realized by an autopilot which tracks the acceleration command by modulating fins or thrusters, there always exists a lag between the acceleration command and an achieved acceleration. To verify the robustness of the new guidance law to autopilot lag, a first-order control system is considered here, that is:

\[
\frac{n_{I}}{n_{C}} = \frac{1}{1 + \tau s}
\]

where \( n_{C} \) is the acceleration command, \( n_{I} \) is the actual acceleration, \( \tau \) is the time constant. Accordingly, the commanded acceleration \( n_{C} \) in Equation (26) should be replaced by \( n_{I} \).

Now we present four cases with different time delays, which are \( \tau = 0 \), \( \tau = 0.1 \) s, \( \tau = 0.3 \) s and \( \tau = 0.5 \) s respectively. The initial conditions of intercepting vehicle are set to \( x_{0} = 0 \) km, \( y_{0} = 0 \) km, \( V_{m0} = 3000 \) m/s and \( \gamma_{m0} = 50^\circ \). The change rate of the vehicles’ velocity is \(-20 \) m/s\(^{2}\). The limitation of the vehicle’s maneuverability is \(|n_{C}| \leq 200 \) m/s\(^{2}\). The initial conditions of target are \( x_{f0} = 80 \) km, \( z_{f0} = 20 \) km, \( V_{t0} = 1000 \) m/s and \( \gamma_{t0} = 165^\circ \). Let \( n_{T} = \sum_{k=0}^{4} b_{k} t^{k} \), where \( \{b_{0}, \cdots, b_{4}\} = \{17.51, -3.77, 0.64, -0.04, 7.57 \times 10^{-4}\} \). The desired final flight-path angle of the vehicle is set to \( \gamma_{f} = \gamma_{tf} = 180^\circ \).

The simulation results under different values of \( \tau \) are shown in Figures 14–16 and Table 5. Figure 14 shows the trajectories under different values of \( \tau \). Figure 15 indicates the time delays, and it can be seen that all load factors finally converge to near zero. Figure 16 shows that the impact angle constraints can be achieved. Table 5 shows that without any compensation incorporated for the delays, the new guidance still has strong robustness and high precision against time delays.
To further show the robust performance of the new guidance, an example considering the time delay of the control system (i.e., $\tau = 0.5$) and stricter acceleration constraint (i.e., $|n_c| \leq 150 \text{ m/s}^2$) is given, where the new guidance is compared with the SDRE-based IACG [26], TSG [2], OGL/AIAW [25]. Both the last two guidance laws satisfy the optimal condition (i.e., Equation (9)) with different weights but have no limitation on terminal acceleration. The initial conditions of intercepting vehicle are set to $x_0 = 0 \text{ km}$, $y_0 = 0 \text{ km}$, $V_{x0} = 3000 \text{ m/s}$ and $\gamma_{m0} = 15^\circ$, the change rate of speed of the vehicle is assumed to be $-100 \text{ m/s}^2$. The initial conditions of target are $x_{t0} = 60 \text{ km}$, $y_{t0} = 0 \text{ km}$, $V_{t0} = 2000 \text{ m/s}$ and $\gamma_{t0} = 173^\circ$. Let $n_T = \sum_{k=0}^{5} b_k t^k$, where $\{b_0, \cdots, b_5\} = \{19, -0.08, -0.01, 0.01, -1.30 \times 10^{-3}, 1.00 \times 10^{-4}\}$. Also, a head-on collision is expected here.

The simulation results of the four guidance laws mentioned above are shown in Figures 17–21 and Table 6. Figure 17 shows that the OGL/AIAW and SDRE-based IACG fails to intercept the target, thus the impact angle constraints also cannot be satisfied (shown in Figure 18). Figure 20 indicates the main reason for the failures of OGL/AIAW and SDRE-based IACG is that when the acceleration limit is stricter, their acceleration command is saturated. Also, we can see from Figure 20 that even if the guidance command of TSG is not saturated, it still causes larger miss distance than the new guidance (shown...
in Table 6). Figure 19 shows the commanded acceleration of the new guidance changes slowly compared with other guidance laws, so the actual acceleration tracks well and the miss distance is small (shown in Table 6). This also verifies that a smaller commanded acceleration leads to smaller miss distance. Figure 21 shows the cumulative control effort profiles for the four guidance laws, from which we can see that the control effort of the new guidance is smaller than that of other three guidance laws for this case. Therefore, imposing constraints on terminal maneuvering acceleration leads to good performance in the face of time delay. Otherwise, the guidance command may diverge when facing a maneuvering target and leads to low guidance accuracy.

Figure 17. Comparison of the trajectories for new guidance, TSG, OGL/AIAW, and SDRE-based IACG when \( \tau = 0.5 \).

Figure 18. Comparison of the \( \gamma - \gamma_t \) profiles for new guidance, TSG, OGL/AIAW, and SDRE-based IACG when \( \tau = 0.5 \).

Figure 19. Comparison of the commanded acceleration profiles for new guidance, TSG, OGL/AIAW, and SDRE-based IACG when \( \tau = 0.5 \).
Figure 20. Comparison of the actual acceleration profiles for new guidance, TSG, OGL/AIAW, and SDRE-based IACG when $\tau = 0.5$.

Figure 21. Comparison of the control effort profiles for new guidance, TSG, OGL/AIAW, and SDRE-based IACG when $\tau = 0.5$.

Table 6. Terminal results of new guidance, TSG, OGL/AIAW, and SDRE-based IACG when $\tau = 0.5$

| Terminal States        | Miss Distance (m) | Impact Angle Error (deg) | Terminal Acceleration Command (m/s²) | Terminal Control Efforts (m²/s²) |
|------------------------|-------------------|--------------------------|-------------------------------------|---------------------------------|
| New Guidance           | 0.15              | 0.06                     | 0.10                                | $1.13 \times 10^5$              |
| TSG                    | 1.76              | 0.20                     | 33.25                               | $1.30 \times 10^5$              |
| OGL/AIAW               | 210.01            | 6.59                     | 58.50                               | $2.21 \times 10^5$              |
| SDRE-based IACG        | 25.41             | 1.19                     | 149.47                              | $2.18 \times 10^5$              |

6.4.2. Monte Carlo Simulations

In this case, 100-run Monte Carlo simulations are carried out to demonstrate the robustness of the new guidance. The initial conditions of intercepting vehicle are set to $x_0 = 0 \text{ km}$, $y_0 = 10 \text{ km}$, $V_{m0} = 3000 \text{ m/s}$ and $\gamma_{m0} = 50^\circ$. The change rates of the vehicles’ velocities are determined by aerodynamic drag. The limitation of the vehicle’s maneuverability is set as $|n_c| \leq 200 \text{ m/s}^2$. The initial conditions of target are $x_{t0} = 80 \text{ km}$, $y_{t0} = 50 \text{ km}$, $V_{t0} = 1000 \text{ m/s}$ and $\gamma_{t0} = 165^\circ$. The maneuvering accelerations of the targets are specified by $n_T = \sum_{k=0}^{4} b_k t^k$, where its parameters $\{b_0, \cdots, b_4\} = \{18.76, -4.04, 0.69, -0.04, 8.11 \times 10^{-4}\}$. The desired final flight-path angle of the vehicle is set to $\gamma_f = \gamma_{tf} - 180^\circ$. The uncertainties of the initial states of vehicle and target, the aerodynamic coefficients, the atmospheric density, and target maneuver parameters are supposed to follow Gaussian distributions. The amplitude of each uncertainty is given in Table 7.
Table 7. The perturbation settings in guidance simulation.

| Disturbance Parameters | $\Delta x_0$ | $\Delta y_0$ | $\Delta V_{m0}$ | $\Delta \gamma_{m0}$ | $\Delta x_f$ | $\Delta y_f$ | $\Delta V_{f0}$ | $\Delta \gamma_{f0}$ | $\Delta C_D$ | $\Delta \rho$ | $\Delta b_k$ |
|------------------------|--------------|--------------|------------------|----------------------|--------------|--------------|------------------|----------------------|--------------|--------------|--------------|
| Value                  | 5 km         | 5 km         | 10%              | $5^\circ$            | 5 km         | 5 km         | 10%              | $5^\circ$            | 10%          | 10%          | 10%          |

Figures 22–27 show the simulation results in dispersed cases. Figure 22 displays the trajectories in dispersed cases, where the blue lines denote the trajectories of vehicles and the black dash lines denote that of the targets. Figures 23 and 27 show the commanded acceleration profiles can converge to zero finally in dispersed cases. Figures 24 and 26 show sufficiently small miss distance and impact angle errors, which indicates that the guidance with the impact position and impact angle constraints can be achieved. Figure 25 shows the velocity profiles in dispersed cases.

**Figure 22.** Trajectories in dispersed cases.

**Figure 23.** Acceleration command profiles in dispersed cases.

**Figure 24.** The profiles of $\gamma - \gamma_f$ in dispersed cases.
6.5. Three Dimensional Case

A three-dimensional (3D) engagement example is given to verify the effectiveness of the guidance. Since the guidance law in Equation (25) is in the vector form, we can easily apply the guidance to a 3D engagement scenario.

As shown in Figure 28, for the horizontal plane, $\psi_m$ denotes the heading angle, which is the angle between the velocity vector and the vertical plane; $\psi_{LOS}$ denotes the azimuth angle of the line of sight. For the vertical plane, $\gamma_m$ denotes the flight-path angle, which is the angle between the velocity vector and the horizontal plane; $\gamma_{LOS}$ denotes the pitch angle of the line of sight.
Table 8. Setting of the simulation parameters of different cases for 3D scenarios.

| Parameters | Case 1       | Case 2       | Case 3       |
|------------|--------------|--------------|--------------|
| $\lambda_1$ | $-3.5$       | $-2.3$       | $-2.5 + 1.3i$ |
| $\lambda_2$ | $-4.2$       | $-2.3$       | $-2.5 - 1.3i$ |
| $K_1$       | $23.40$      | $10.89$      | $13.94$      |
| $K_2$       | $-14.70$     | $-5.29$      | $-7.94$      |
| $[x_0, y_0, z_0]/\text{km}$ | $(0,0,0)$ | $(10,-10,0)$ | $(-10,10,0)$ |
| $\gamma_0/\circ$ | $45$       | $10$         | $15$         |
| $\psi_0/\circ$ | $45$       | $50$         | $40$         |
| $\gamma_1/\circ$ | $190$      | $180$        | $180$        |
| $\psi_1/\circ$ | $200$      | $220$        | $190$        |

The simulation results are shown in Figures 29–33 and Table 9. Figure 29 displays the trajectories. Figure 30 shows that all the acceleration commands can finally converge to zero because the guidance coefficients are within the stability domain. Figures 31 and 32 show that in all the 3D cases the impact angle constraints are satisfied. The velocity profiles for different cases are shown in Figure 33.

Figure 28. Three-dimensional (3D) engagement.

The desired velocity vector is:

$$\hat{v}_f = \begin{bmatrix} \cos \gamma_f \cos \psi_f \\ \cos \gamma_f \sin \psi_f \\ \sin \gamma_f \end{bmatrix}$$

(125)

where $\gamma_f$ and $\psi_f$ denote the desired flight-path angle and heading angle respectively. Substituting this vector into Equation (25) obtains a 3D guidance law.

Here 3D examples are given. The initial conditions are shown in Table 8. The change rates of the vehicles’ velocities are governed by aerodynamic drag. The limitation of the vehicle’s maneuverability is $|n_c| \leq 200$ m/s$^2$. The maneuvering accelerations of the targets are specified by $n_T = \sum_{k=0}^{5} b_k t^k$, where $\{b_0, \ldots, b_5\} = \{2.44, 1.28 \times 10^{-3}, 2.24 \times 10^{-2}, -8.00 \times 10^{-4}, 2.40 \times 10^{-4}, -1.78 \times 10^{-5}\}$. The desired velocity vectors of the vehicles are set to $\gamma_f = \gamma_T - 180^\circ$ and $\psi_f = \psi_T - 180^\circ$, which means head-on collision. The guidance coefficients are selected within the stability domain shown in Equation (123), as shown in Table 8.
The simulation results are shown in Figure 29–33 and Table 9. Figure 29 displays the trajectories. Figure 30 shows that all the acceleration commands can finally converge to zero because the guidance coefficients are within the stability domain. Figures 31 and 32 show that in all the 3D cases the impact angle constraints are satisfied. The velocity profiles for different cases are shown in Figure 33.

Figure 29. Trajectories for 3D scenarios.

Figure 30. Acceleration command profiles for 3D scenarios.

Figure 31. The profiles of $\gamma - \gamma_t$ for 3D scenarios.
Table 9. Terminal results for 3D scenarios.

| Case | 1          | 2          | 3          |
|------|------------|------------|------------|
| Miss distance (m)      | $1.20 \times 10^{-17}$ | $1.22 \times 10^{-17}$ | $1.19 \times 10^{-17}$ |
| Terminal $\gamma - \gamma_t$ errors (deg) | $1.77 \times 10^{-10}$ | $2.89 \times 10^{-9}$ | $2.59 \times 10^{-8}$ |
| Terminal $\psi - \psi_t$ errors (deg) | $1.43 \times 10^{-8}$ | $8.47 \times 10^{-9}$ | $2.67 \times 10^{-8}$ |
| Terminal acceleration command (m/s²) | 0.76 | 0.32 | 0.43 |

7. Conclusions

This paper presents the design of a new guidance for intercepting a maneuverable target while considering both impact angle and terminal maneuvering acceleration constraints. The guidance is developed by employing optimal control theory. After that, an LTV system is developed by linearizing the new nonlinear engagement system and solved to obtain the generalized solutions of flight-path angle, heading angle and acceleration command accounting for target maneuvers. By skillfully adopting the squeeze theorem to analyze the limits of some complex integral terms in these solutions, the stability domain of the guidance coefficients is derived. The simulation results show that, in the stability domain, the new guidance can accurately satisfy the impact angle and terminal...
maneuvering acceleration constraints and has a comprehensive better performance than TSG, OGL/AIAW and SDRE-based IACG if the target maneuvers strongly.

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Appendix A

In Section 3, the semi-analytical solutions are developed for the generalized case where the closing speed is allowed to be an arbitrary positive function of time. Here, a special case for a solid-propellant missile is considered where the closing speed is assumed to change linearly with time since the thrust cannot be adjusted actively, and then the completely closed-form solutions are derived. Let \( V(t) = -a_D t + V_0 \) and \( a_D \neq 0 \). The derivation process is presented below

(1) If \( \Delta > 0 \)

In this case, the key to deriving the completely closed-form solutions is to solve the integral term of Equation (61). We divide Equation (61) into two parts:

\[
n_c(t) = n_{ca}(t) + n_{ci}(t) \tag{A1}
\]

where,

\[
n_{ca}(t) = \frac{V(t)^2}{\lambda_1 - \lambda_2} \left\{ \frac{R(t_0)}{R(t)} \right\}^{\lambda_1} \left[ \lambda_1 (\lambda_1 - 1) \gamma_0 + K_1 \lambda_1 \gamma_{LOS} + K_2 \gamma_f (\lambda_1 - 1) \right]
- \frac{R(t_0)^{\lambda_2}}{R(t)^{\lambda_2}} \left[ \lambda_2 (\lambda_2 - 1) \gamma_0 + K_1 \lambda_2 \gamma_{LOS} + K_2 \gamma_f (\lambda_2 - 1) \right] + \sum_{k=0}^{m} \left( \frac{K_1}{\tau^2} + \frac{K_2}{\tau} \right) b_k T^k \tag{A2}
\]

\[
n_{ci}(t) = \frac{V(t)^2}{\lambda_1 - \lambda_2} \left\{ \frac{R(t_0)}{R(t)} \right\}^{\lambda_1} \left[ \lambda_1 (\lambda_1 - 1) \frac{R(t)}{R(t)^{\lambda_1}} - \lambda_2 (\lambda_2 - 1) \frac{R(t_0)^{\lambda_2}}{R(t)^{\lambda_2}} \right] d\tau \tag{A3}
\]

In order to obtain the analytical expression of Equation (A1), the substitution method is used here. Let \( \tau = -\frac{V(t)}{a_D} + \frac{V_0}{a_D} \) and \( R(\tau) = \frac{V(t)^2}{2a_D^2} + R(t_0) - \frac{V_0^2}{2a_D^2} \), which can be deduced from \( V(\tau) = -a_D \tau + V_0 \). By substituting the two formulae into \( n_{ci}(t) \), and then after some algebra, we obtain:

\[
n_{ci}(t) = -\frac{V(t)^2}{\lambda_1 - \lambda_2} \frac{1}{a_D} \sum_{k=0}^{m} A_k \int_0^v \frac{(Dv + B)^k}{v} \left[ \lambda_1 (\lambda_1 - 1) \frac{v^2 + C}{R(t)^{\lambda_1}} - \lambda_2 (\lambda_2 - 1) \frac{v^2 + C}{R(t)^{\lambda_2}} \right] d\nu \tag{A4}
\]

where,
\[ v = \frac{V}{\sqrt{2aD}}, \quad A_k = \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k, \quad D = \sqrt{\frac{2}{aD}}, \quad B = t_f - \frac{V_0}{aD}, \quad C = R(t_0) - \frac{V_0^2}{2aD} \quad (A5) \]

Using the technique of integration by parts and after a large number of algebraic operations, Equation (A4) can be converted into Equation (A6).

\[ n_c(t) = \frac{V(t)^2}{2aD} \sum_{k=0}^{m} A_k \int_0^t k^k R(t_0)^{k+1} R(t)^{k+1} \left[ \frac{\lambda_1(\lambda_1 - 1) \gamma + K_1 \gamma \text{LOS} + K_2 \gamma f(\lambda_1 - 1)}{2C(\lambda_1 + 1)R(t)^{\lambda_1 + 1}} \right] \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( t_f - t_0 \right)^k \quad (A8) \]

\[ a_1(t) = \frac{V(t)^2}{\lambda_1 - \lambda_2} \left[ \frac{R(t_0)^{\lambda_1} R(t_0)^{k+1}}{R(t)^{\lambda_1 + 1}} \right] \left[ \frac{\lambda_1(\lambda_1 - 1) \gamma + K_1 \gamma \text{LOS} + K_2 \gamma f(\lambda_1 - 1)}{2C(\lambda_1 + 1)R(t)^{\lambda_1 + 1}} \right] \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( t_f - t_0 \right)^k \]

\[ a_2(t) = \frac{R(t)}{C} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k T^k \frac{K_1 + 2K_2}{K_1 + 2K_2 - 2} \quad (A9) \]

\[ a_3(t) = -\frac{V(t)^2}{\lambda_1 - \lambda_2} \frac{1}{2D} \sum_{k=0}^{m} A_k \int_0^t V^k R(t_0)^{k+1} R(t)^{k+1} \left[ \frac{\lambda_1(\lambda_1 - 1) \gamma + K_1 \gamma \text{LOS} + K_2 \gamma f(\lambda_1 - 1)}{2C(\lambda_1 + 1)R(t)^{\lambda_1 + 1}} \right] \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k T^k \quad (A10) \]

The closed-form expressions of the integral terms in Equation (A10) can be derived once the values of \( \lambda_1 \) and \( \lambda_2 \) are given. For example, we consider a case that \( \lambda_1 = -3 \) and \( \lambda_2 = -4 \). For this case, the analytical expression of Equation (A10) can be obtained as:

\[ a_3(t) = \frac{3}{20C} \sum_{k=0}^{m} A_k \int_0^t V^k R(t_0)^{k+1} R(t)^{k+1} f_\nu(\lambda = -3) \]

\[ -V(t)^2 \frac{10}{2D} \sum_{k=0}^{m} A_k \int_0^t V^k R(t_0)^{k+1} R(t)^{k+1} f_\nu(\lambda = -4) \quad (A12) \]
where,

\[
f_v(\lambda = -3) = \int_{v_0}^{v} v^{k-r-1} (v^2 + C)^{\lambda+1} dv = \int_{v_0}^{v} v^{k-r-1} (v^2 + C)^{-2} dv
\]

\[
= \frac{v^{k-r}}{2C R(t)} - \frac{v^{2}}{2C R(t)} \times \left[ \sum_{p=1}^{n} (-C)^{p-1} \left( \frac{2p-2}{k-r-2p} \right) \right] + (-C)^{p} f_{p}(v)
\]

\[
f_v(\lambda = -4) = \int_{v_0}^{v} v^{k-r-1} (v^2 + C)^{\lambda+1} dv = \int_{v_0}^{v} v^{k-r-1} (v^2 + C)^{-3} dv
\]

\[
= \frac{v^{k-r}}{4C R(t)} - \frac{v^{2}}{4C R(t)} \times \left[ \sum_{p=1}^{n} (-C)^{p} \left( \frac{2p-4}{k-r-4p} \right) \right] f_v(\lambda = -3)
\]

where, \( p_k = f_C((k - r - 1)/2) \) is a ceiling function that gives the smallest integer greater than \((k - r - 1)/2\). \( f_{p}(v) \) is a function of \( v \) and relies on the value of \( k - r - 2p_k - 1 \), as below:

\[
f_{p}(v) = \begin{cases} 
\frac{1}{2} \ln \left[ \frac{v^2}{R(t)} \right] - \frac{1}{2} \ln \left[ \frac{v^2}{R(t)} \right] & \text{if } k - r - 2p_k - 1 = 0 \\
\frac{1}{2} \ln \left[ \frac{v^{2} - v^{1}}{v^{2} + v^{1}} \right] - \ln \left[ \frac{v^{0} - v^{1}}{v^{0} + v^{1}} \right] & \text{if } k - r - 2p_k - 1 = 1 
\end{cases}
\]

Substituting Equation (A12) into Equation (A7), the completely analytical expression of \( n_c \) can be obtained as Equation (A16).

\[
n_c(t) = V(t) R(t) \left[ 12 \gamma - 60 \gamma_{LOS} + 48 \gamma_{f} + \frac{3R(t)}{d_0} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( t_f - t_0 \right) ^k \right]
\]

\[
- V(t)^2 R(t)^3 \sum_{k=0}^{m} A_k \sum_{r=0}^{k} C_{k}^{r} D^{k-r} B_r (k - r - 4) R(t)^2 f_v(\lambda = -3)
\]

\[
+ V(t)^2 \frac{3}{8} \sum_{k=0}^{m} A_k \sum_{r=0}^{k} C_{k}^{r} D^{k-r} B_r (k - r - 4) R(t)^2 f_v(\lambda = -3)
\]

\[
+ V(t)^2 \frac{2R(t)}{d_0} \sum_{k=0}^{m} \sum_{r=0}^{k} A_k \sum_{r=0}^{k} C_{k}^{r} D^{k-r} B_r (k - r - 6) R(t)^3 f_v(\lambda = -4)
\]

(2) If \( \Delta < 0 \)

In this case, \( \lambda_1 \) and \( \lambda_2 \) are complex conjugates, and the closed-form solution of the acceleration command can be obtained as:

\[
n_c(t) = V \gamma + n_T(t) = a_1^*(t) + a_2(t) + a_3(t) + a_4(t)
\]

where \( a_2(t) - a_4(t) \) are the same as Equations (A9)–(A11), but \( a_1^*(t) \) is defined as Equation (A18).

\[
a_1^*(t) = V^2 R(t)^{p} \cos \left[ \frac{q \ln \left( \frac{R(t)}{R(t)} \right)}{R(t)^{p+1}} \right] \left[ 2p - 1 \gamma_{LOS} + K_1 \gamma_{f} + K_2 \gamma_{f} \right]
\]

\[
- \frac{1}{2} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( t_f - t_0 \right) ^k
\]

\[
+ V^2 R(t)^{p} \sin \left[ \frac{q \ln \left( \frac{R(t)}{R(t)} \right)}{R(t)^{p+1}} \right] \left[ \frac{1}{2} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( t_f - t_0 \right) ^k \right]
\]

(3) If \( \Delta = 0 \)
Similarly, we divide Equation (63) into two parts:

\[ n_c(t) = n_{ca}(t) + n_{ci}(t) \]  \hspace{1cm} (A19)

where,

\[ n_{ca}(t) = V(t)^2 \frac{R(t_0)^{y_f}}{R(t)^{y_f}} \left\{ \ln \left( \frac{R(t)}{R(t_0)} \right) \right\} \frac{1}{(1 + \gamma_0)} \]  \hspace{1cm} (A20)

\[ + \left( 2\lambda_1 - 1 \right) \gamma_0 + K_1 \gamma_{LOS} + K_2 \gamma_f \]  \hspace{1cm} (A20)

\[ + \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} \right) b_k \left( t_f - t \right)^k \]

\[ n_{ci}(t) = V(t)^2 \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \int_{t_0}^{t_f} \left( \frac{R(t)}{R(t_0)} \right)^{y_f} \]  \hspace{1cm} (A21)

\[ \times \left( (2\lambda_1 - 1) + \lambda_1 (\lambda_1 - 1) \ln \left( \frac{R(t)}{R(t_0)} \right) \right) dt \]

Substituting \( \tau = \frac{V(t)}{\sigma_D} + \frac{V_0}{\sigma_D} \) and \( R(t) = V(t)^2 \frac{\sigma_D}{\sigma_D} + R(t_0) - \frac{V_0^2}{\sigma_D} \) into \( n_{ci}(t) \) yields Equation (A22).

\[ n_{ci}(t) = -V(t)^2 \frac{1}{\sigma_D} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \int_{t_0}^{t_f} \left( \frac{\nu^2 + C}{R(t)} \right)^{y_f} \]  \hspace{1cm} (A22)

\[ d\nu + \lambda_1 \left( \lambda_1 - 1 \right) \int_{t_0}^{t_f} \left( \frac{\nu^2 + C}{R(t)} \right)^{y_f} \ln \left( \frac{(\nu^2 + C)}{R(t)} \right) d\nu \]

Using the technique of integration by parts again and after a large number of algebraic operations, \( n_{ci}(t) \) can be re-expressed as Equation (A23).

\[ n_{ci}(t) = V(t)^2 \frac{\lambda_1^2 + 2\lambda_1 - 1}{(\lambda_1 + 1)^2} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( \frac{R(t)}{R(t_0)} \right)^{y_f} \]  \hspace{1cm} (A23)

\[ - \frac{V(t)^2}{\sigma_D} \frac{3\lambda_1^2 + 3\lambda_1 - 2}{2(\lambda_1 + 1)} \sum_{k=0}^{m} \frac{A_k}{\nu} \int_{t_0}^{t_f} \frac{B_r}{2(\nu^2 + C)^{y_f}} \]  \hspace{1cm} (A23)

\[ + V(t)^2 \frac{\lambda_1 (\lambda_1 - 1)}{2(\lambda_1 + 1)} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \int_{t_0}^{t_f} \left( \frac{\nu^2 + C}{R(t)} \right)^{y_f} \ln \left( \frac{R(t)}{R(t_0)} \right) d\nu \]

\[ + V(t)^2 \frac{\lambda_1 (\lambda_1 - 1)}{2(\lambda_1 + 1)} \sum_{k=0}^{m} \frac{A_k}{\nu} \int_{t_0}^{t_f} \frac{B_r}{2(\nu^2 + C)^{y_f}} \]  \hspace{1cm} (A23)

\[ \times \left( C \lambda_1^2 + 2\lambda_1 - 1 \right) b_k \left( \frac{R(t)}{R(t_0)} \right)^{y_f} \]  \hspace{1cm} (A23)

\[ \int_{t_0}^{t_f} \left( \frac{\nu^2 + C}{R(t)} \right)^{y_f} \ln \left( \frac{(\nu^2 + C)}{R(t)} \right) d\nu \]

\[ \]  \hspace{1cm} (A23)

Substituting Equations (A23) and (A20) into Equation (A19), the closed-form solution of \( n_c \) can be obtained as:

\[ n_c(t) = V(\gamma) + n(\gamma) = a_5(t) + a_6(t) + a_7(t) + a_8(t) \]  \hspace{1cm} (A24)

where,

\[ a_5(t) = V^2 \frac{R(t_0)^{y_f}}{R(t)^{y_f}} \left\{ \left( 2\lambda_1 - 1 \right) \gamma_0 + K_1 \gamma_{LOS} + K_2 \gamma_f \right\} \]  \hspace{1cm} (A25)

\[ - \frac{R(t_0)}{\sigma_D} \left( \lambda_1^2 + 2\lambda_1 - 1 \right) \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( t_f - t_0 \right)^k \]

\[ + \ln \left( \frac{R(t)}{R(t_0)} \right) \left\{ \lambda_1 (\lambda_1 - 1) \gamma_0 + K_1 \lambda_1 \gamma_{LOS} + K_2 (\lambda_1 - 1) \gamma_f \right\} \]  \hspace{1cm} (A25)

\[ + \frac{R(t_0)}{\nu(t_0)^2} \left( \lambda_1 (\lambda_1 - 1) \right) \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( t_f - t_0 \right)^k \}

\[ a_6(t) = \frac{R(t)}{C} \frac{\lambda_1^2 + 2\lambda_1 - 1}{(\lambda_1 + 1)^2} \sum_{k=0}^{m} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) b_k \left( t_f - t_0 \right)^k \]  \hspace{1cm} (A26)
\[ a_7(t) = -V(t) \frac{\frac{1}{2} \lambda_1^{\frac{k}{2}} \lambda_2^l}{2(\lambda_1+\lambda_2)} \sum_{k=0}^{m} A_k \sum_{r=0}^{k} C_i^r D^{k-r} B^r \times \frac{k-r+2(\lambda_1+1)}{2c(\lambda_1+1)R(t)^{\lambda_1+1}} \int_{v_0}^{v} v^{k-r-1} (v^2 + C)^{\lambda_1+1} dv \]

\[ + V(t) \frac{1}{2\lambda_1} (1 - A_1) \sum_{k=0}^{m} A_k \sum_{r=0}^{k} C_i^r D^{k-r} B^r \times \frac{(k-r-2)}{2(\lambda_1+1)R^{\lambda_1+1}} \int_{v_0}^{v} (v^2 + C)^{\lambda_1+1} v^{k-r-3} \ln \left( \frac{(v^2+C)}{R(t)} \right) dv \]

\[ a_8(t) = \sum_{k=1}^{m} \left[ \frac{2}{(\lambda_1+1)^2} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) + 1 \right] b_k (t_f - t)^k \]

The analytical expression of the integral terms in Equation (A27) depends on the values of \( \lambda_1 \) and \( \lambda_2 \). For example, if we let \( \lambda_1 = \lambda_2 = -3 \) and substitute it into Equation (A27), there is:

\[ a_7(t) = -V(t) \frac{1}{6\lambda_1^2} \sum_{k=0}^{m} A_k \sum_{r=0}^{k} C_i^r D^{k-r} B^r \times \frac{k-r+2(\lambda_1+1)}{2c(\lambda_1+1)R(t)^{\lambda_1+1}} \int_{v_0}^{v} v^{k-r-1} (v^2 + C)^{\lambda_1+1} dv \]

\[ + V(t) \frac{1}{2\lambda_1} (1 - A_1) \sum_{k=0}^{m} A_k \sum_{r=0}^{k} C_i^r D^{k-r} B^r \times \frac{(k-r-2)}{2(\lambda_1+1)R^{\lambda_1+1}} \int_{v_0}^{v} (v^2 + C)^{\lambda_1+1} v^{k-r-3} \ln \left( \frac{(v^2+C)}{R(t)} \right) dv \]

\[ a_8(t) = \sum_{k=1}^{m} \left[ \frac{2}{(\lambda_1+1)^2} \left( \frac{K_1}{k+2} + \frac{K_2}{k+1} - 1 \right) + 1 \right] b_k (t_f - t)^k \]

where,

\[ F_p(a, \beta) = \int_{v_0}^{v} \frac{v^{k-r-\beta}}{(v^2 + C)^{\beta}} dv = \begin{cases} F_{p1}(a, \beta) & \text{if } k-r-\beta < 0 \\ F_{p2}(a, \beta) & \text{if } 0 \leq k-r-\beta < 4 \\ F_{p3}(a, \beta) & \text{if } k-r-\beta \geq 4 \end{cases} \]

From \( C = R(t_0) - \frac{\sqrt{v_0^2}}{2\lambda_1^2} \) and \( R(t_f) = \frac{V(t_f)^2}{2\lambda_1^2} + R(t_0) - \frac{\sqrt{v_0^2}}{2\lambda_1^2} = 0 \), we can obtain \( C \leq 0 \), which means the denominator \((v^2 + C)\) has two real roots \( \pm \sqrt{-C} \). Therefore, the above equation can be solved by using the method of partial fraction decomposition [38], which allows us to decompose the integrand into the sum of simpler, more easily integrated rational functions, as below.

\[ F_{p1}(\lambda, \phi) = \int_{v_0}^{v} \left[ \sum_{i=1}^{\lambda} \frac{Q_{iu}}{(v + \sqrt{-C})} + \sum_{j=1}^{\lambda} \frac{Q_{ju}}{(v - \sqrt{-C})} + \sum_{j=1}^{\lambda} \frac{Q_{ju}}{v^2} \right] dv \]

\[ = Q_{00} \ln \left( \frac{v + \sqrt{-C}}{v_0 + \sqrt{-C}} \right) - Q_{11} \left( \frac{1}{v + \sqrt{-C}} - \frac{1}{v_0 + \sqrt{-C}} \right) + Q_{20} \ln \left( \frac{v - \sqrt{-C}}{v_0 - \sqrt{-C}} \right) - Q_{21} \left( \frac{1}{v - \sqrt{-C}} - \frac{1}{v_0 - \sqrt{-C}} \right) + Q_{30} \ln \left( \frac{v}{v_0} \right) - \sum_{j=2}^{\lambda} \frac{Q_{ju}}{v^{j-1}} - C^{v_0^{-1}} \]

References

1. Rusnak, I.; Weiss, H.; Eliav, R.; Shima, T. Missile guidance with constrained intercept body angle. *IEEE Trans. Aerosp. Electron. Syst.* 2014, 50, 1445–1453. [CrossRef]
2. Zarchan, P. *Tactical and Strategic Missile Guidance*, 6th ed.; AIAA: Reston, VA, USA, 2012.
3. Mehta, S.S.; Ton, C.; MacKunis, W. Acceleration-free nonlinear guidance and tracking control of hypersonic missiles for maximum tactical and strategic missile guidance. *AIAA Guidance, Navigation, and Control Conference*, Los Angeles, CA, USA, 24–26 August 2014.
4. Dionne, D.; Michalska, H.; Rabbath, C.A. Predictive guidance for pursuit-evasion engagements involving multiple decoys. *J. Guid. Control Dyn.* 2007, 30, 1277–1286. [CrossRef]
5. Wang, S.; Guo, Y.; Wang, S.; Liu, Z.; Zhang, S. Cooperative guidance considering detection configuration against target with a decoy. *IEEE Access.* 2020, 8, 66291–66303. [CrossRef]
6. Cherry, G. A general, explicit optimal guidance law for rocket-propelled spacecraft. In *Proceedings of the AIAA/Ion Astrodynamics Guidance and Control Conference*, Los Angeles, CA, USA, 24–26 August 1964.
7. Bryson, A.E. *Applied Optimal Control*; Blaisdell: Waltham, MA, USA, 1969.

8. Idan, M.; Golan, O.M.; Guelman, M. Optimal planar interception with terminal constraints. *J. Guid. Control Dyn.* 1995, 18, 1273–1279. [CrossRef]

9. Glizer, V.Y. Optimal planar interception with fixed end conditions: Closed-form solution. *J. Optim. Theory* 1996, 88, 503–539. [CrossRef]

10. Ben-Asher, J.Z.; Yaesh, I. *Advances in Missile Guidance Theory*; AIAA Progress in Aeronautics and Astronautics: Reston, VA, USA, 1998.

11. Yu, W.; Chen, W.; Yang, L.; Liu, X.; Zhou, H. Optimal terminal guidance for exoatmospheric interception. *Chin. J. Aeronaut.* 2016, 29, 1052–1064. [CrossRef]

12. Ohlmeyer, E.J.; Phillips, C.A. Generalized vector explicit guidance. *J. Guid. Control Dyn.* 2006, 29, 261–268. [CrossRef]

13. Murtaugh, S.A.; Criel, H.E. Fundamentals of proportional navigation. *IEEE Spectrum* 1966, 3, 75–85. [CrossRef]

14. Budiyono, A.; Rachman, H. Proportional guidance and CDM control synthesis for a short-range homing surface-to-air missile. *J. Aerosp. Eng.* 2012, 25, 168–177. [CrossRef]

15. Ratnoo, A.; Ghose, D. Impact angle constrained interception of stationary targets. *J. Guid. Control Dyn.* 2008, 31, 1817–1822. [CrossRef]

16. Erer, K.S.; Merttopcuoglu, O. Indirect impact-angle-control against stationary targets using biased pure proportional navigation. *J. Guid. Control Dyn.* 2012, 35, 700–704. [CrossRef]

17. Kumar, S.R.; Rao, S.; Ghose, D. Sliding-mode guidance and control for all-aspect interceptors with terminal angle constraints. *J. Guid. Control Dyn.* 2012, 35, 1230–1246. [CrossRef]

18. Liu, B.; Hou, M.; Yu, Y.; Wu, Z. Three-dimensional impact angle control guidance with field-of-view constraint. *Aerosp. Sci. Technol.* 2020, 105, 106014. [CrossRef]

19. Yu, W.; Chen, W.; Yang, L.; Liu, X.; Zhou, H. Optimal terminal guidance for exoatmospheric interception. *Chin. J. Aeronaut.* 2016, 29, 1052–1064. [CrossRef]

20. He, S.; Song, T.; Lin, D. Impact angle constrained integrated guidance and control for maneuvering target interception. *J. Guid. Control Dyn.* 2017, 40, 2652–2660. [CrossRef]

21. Biswas, B.; Maitya, A.; Kumar, S.R. Finite-time convergent three-dimensional nonlinear intercept angle guidance. *J. Guid. Control Dyn.* 2019, 43, 146–153. [CrossRef]

22. Lin, D.; Ji, Y.; Wang, W.; Wang, Y.; Wang, H.; Zhang, F. Three-dimensional impact angle-constrained adaptive guidance law considering autopilot lag and input saturation. *Int. J. Robust Nonlinear Control* 2020, 30, 3653–3671. [CrossRef]

23. Hu, Q.; Han, T.; Xin, M. Three-dimensional guidance for various target motions with terminal angle constraints using Twisting control. *IEEE Trans. Ind. Electron.* 2020, 67, 1242–1253. [CrossRef]

24. Ryoo, C.K.; Cho, H.; Takih, M.J. Time-to-go weighted optimal guidance with impact angle constraints. *IEEE Trans. Control Syst. Technol.* 2006, 14, 483–492. [CrossRef]

25. Li, R.; Wen, Q.; Tan, W.; Yijie, Z.H. Adaptive weighting impact angle optimal guidance law considering seeker’s FOV angle constraints. *J. Syst. Eng. Electron.* 2018, 29, 146–155. [CrossRef]

26. Wang, C.; Dong, W.; Wang, J.; Shan, J. Nonlinear suboptimal guidance law with impact angle constraint: An SDRE-based approach. *IEEE Trans. Aerosp. Electron. Syst.* 2020, 56, 4831–4840. [CrossRef]

27. Yu, W.; Chen, W. Entry guidance with real-time planning of reference based on analytical solution. *Adv. Space Res.* 2015, 55, 2325–2345. [CrossRef]

28. Surname, M.L.; Tennoe, M.T.; Henssonow, S.F. *Squeeze Theorem*; Betascript Publishing: Beau Bassin, Mauritius, 2010.

29. Speie, E.; Nacouzi, G.; Lee, C.; Moore, R.M. *Hypersonic Missile Nonproliferation-Hindering the Spread of a New Class of Weapons*; RAND Corporation: Santa Monica, CA, USA, 2017.

30. Han, C.; Xiong, J. Method of trajectory prediction for unpowered gliding hypersonic vehicle in gliding phase. In *Proceedings of the 2016 IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC), Xi’an, China, 3–5 October 2016*; pp. 262–266.

31. Meyer, C.D. *Matrix Analysis and Applied Linear Algebra*; The Society for Industrial and Applied Mathematics: Philadelphia, PA, USA, 2000.

32. Bezick, S.M.; Pue, A.J.; Patzelt, C.M. Inertial Navigation for Guided Missile Systems. *Johns Hopkins APL Tech. Digest.* 2010, 4, 331–342.

33. Singer, R.A. Estimating Optimal Tracking Filter Performance for Manned Maneuvering Targets. *IEEE Trans. Aerosp. Electron. Syst.* 1970, 6, 473–483. [CrossRef]

34. Ekstrand, B. Tracking filters and models for seeker applications. *IEEE Trans. Aerosp. Electron. Syst.* 2001, 37, 965–977. [CrossRef]

35. Hoxner, G.; Shima, T.; Weiss, H. LQC guidance law with bounded acceleration command. *IEEE Trans. Aerosp. Electron. Syst.* 2008, 44, 77–86. [CrossRef]

36. Zhou, D.; Xu, B. Adaptive dynamic surface guidance law with input saturation constraint and autopilot dynamics. *J. Guid. Control Dyn.* 2016, 39, 1152–1159. [CrossRef]

37. Phillips, T.H. *A Common Aero Vehicle Model, Description, and Employment Guide Arlington*; Schafer Corporation for AFRL and AFSPC: Arlington, VA, USA, 2003.

38. Pipes, L.A.; Harvill, L.R. *Applied Mathematics for Engineers and Physicists*, 3rd ed.; Dover Publications: New York, NY, USA, 2014.