Proton Spin Content From Lattice QCD*

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We calculate the form factor of the quark energy momentum tensor and thereby extract the quark orbital angular momentum of the nucleon. The calculation is done on a quenched $16^3 \times 24$ lattice at $\beta = 6.0$ and with Wilson fermions at $\kappa = 0.148$, 0.152, 0.154 and 0.155. We calculate the disconnected insertion stochastically with an unbiased subtraction. This proves to be an efficient method of reduce the error from the noise. We find that the total quark contribution to the proton spin is $0.29\pm 0.07$. From this we deduce that the quark orbital angular momentum is $0.17\pm 0.08$ and predict the gluon spin to be $0.21\pm 0.07$, i.e. about 40% of the proton spin is due to the glue.

To understand the spin content of the proton remains a challenging problem in QCD.\textsuperscript{[1]} Experimental\textsuperscript{[2],[3]} and lattice results\textsuperscript{[4],[5]} suggest that the quark contribution ($\frac{1}{2} \Sigma$) to the proton spin is about $25\pm 10\%$. But to date, we have very little knowledge about the remaining part of the proton spin. We do not have reliable estimate about the spin contribution from the gluons or the orbital angular momentum of the quark. In this talk, I will show our lattice results on the total angular momentum of the quarks and thereby deduce the quark orbital angular momentum and predict the gluon contribution to the proton spin.

Recently it was shown\textsuperscript{[6]} that one can decompose the total angular momentum of QCD in a gauge invariant way, i.e.

$$J = \int d^3x \frac{1}{2} \bar{\psi} \gamma_5 \psi + \int d^3x \bar{\psi} \left\{ \vec{x} \times (-i \vec{D}) \right\} \psi + \int d^3x \left[ \vec{x} \times (\vec{E} \times \vec{B}) \right]. \quad (1)$$

The forward matrix element of this operator in the proton defines the decomposition of the proton spin $\frac{1}{2} = \frac{1}{2} \Sigma + L_q + J_g$, where $\frac{1}{2} \Sigma$ is the quark spin contribution, $L_q$ is the quark orbital angular momentum and $J_g$ is the total angular momentum of the glue. To calculate the angular momentum of the quark and the gluon in the proton, one first notices that the gauge invariant quark-gluon energy momentum tensor\textsuperscript{[6]} is

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu} = \frac{1}{2} \left[ \bar{\psi} \gamma^{(\mu} i \partial^{\nu)} \psi + \bar{\psi} \gamma^{(\nu} i \partial^{\mu)} \psi \right] + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} - F^{\mu\alpha} F^{\nu}_\beta, \quad (2)$$

where the first part is the quark energy momentum tensor and the second one is that of the gluon. Form factors of this energy momentum tensor current can be defined as

$$\langle p, s | T^{\mu\nu}_q (0) | p', s' \rangle = \bar{u}(p, s) T_1 q, g (q^2) \gamma^{(\mu} (p'^\nu - \bar{\nu}^{\nu} q^2) + T_2 q, g (q^2) g^{\mu\nu} m \bar{u}(p', s') \rangle, \quad (3)$$

where $p' = (p^\mu + p'^\mu)/2$, $q^\mu = p^\mu - p'^\mu$ and $u(p)$ is the nucleon spinor. It is proved\textsuperscript{[6]} that the total angular momentum of the quark or gluon is related to the sum of the $T_1$ and $T_2$ form factors at zero momentum transfer, i.e.

$$J_{q,g} = \frac{\langle p, s | T^{\mu\nu}_q (0) | p, s \rangle}{\langle p, s | p, s \rangle} = \frac{1}{2} \left[ T_1 q, g (0) + T_2 q, g (0) \right] = T q, g (0). \quad (4)$$

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Therefore, by calculating the form factor $T^{q,g}(q^2)$ at different $q^2$ and then extrapolating to $q^2 \rightarrow 0$ limit one can obtain the total angular momentum of the quark and the gluon separately. In this talk I shall present our calculation of the total angular momentum of the quark, $J_q = \frac{1}{2} \Sigma + L_q$.

To compute the matrix element, one needs to calculate the two and three point functions. Three point function has two parts: connected insertion (CI) due to valence and cloud quarks, and disconnected insertion (DI) arising out of sea quarks. For CI, we evaluate the form factor $T(q^2)$ in terms of the following ratio (for notation see ref. [1]):

$$\frac{\text{Tr} \left[ \Gamma_m G_{NT_{01},N}(t_2,t_1,\bar{0},-\bar{q}) \right]}{\text{Tr} \left[ \Gamma_e G_{NN}(t_2,\bar{0}) \right]} \frac{\text{Tr} \left[ \Gamma_e G_{NN}(t_1,\bar{0}) \right]}{\text{Tr} \left[ \Gamma_e G_{NN}(t_1,\bar{q}) \right]} = \frac{1}{2} \epsilon_{jkm} q_k T^{q,g}(q^2).$$

(5)

For CI, we calculate $T^q(q^2)$ at different $q^2$ and then extrapolate that to the $q^2 \rightarrow 0$ limit. Results are obtained for relatively light quarks with $\kappa = 0.148, 0.152, 0.154$ and 0.155. We follow the same technique used in [2] and [3]. Fig.1 shows the dipole fitting of $T(q^2)$ at different $q^2$ for $\kappa = 0.152$ and 0.155. Plots for the other $\kappa$’s are similar.

Figure 1. Dipole fitting for CI. Extrapolated ($q^2 \rightarrow 0$) values are shown by $T_L(0)$.

The chiral limit for $J^{CI}_q$ is taken with a linear dependence on the quark mass $m_q$ for these four $\kappa$ (fig.3). To account for the correlations, both fittings in $q^2$ and $m_q$ are done by using the covariant matrix and the final error at the chiral limit is obtained from the Jackknife procedure. Finally, to get the continuum result we multiply the lattice $T_L(0)$ by the tadpole improved renormalization constant $[3]$ for the operator $T^{q,g}$ and obtain the CI part of the total angular momentum of quarks $J^{CI}_q = 0.43 \pm 0.07$, which almost saturates the proton spin. The calculation is done on a quenched $16^3 \times 24$ lattice at $\beta = 6.0$ with Wilson fermions and with 100 configurations. For DI contribution, we use the relation [4].

$$\frac{1}{3} \sum_{i=1}^{3} \sum_{\tau} \frac{\Gamma_i^{\beta\alpha} G^{i}_{PP}(t_f \tau)}{G^{i}_{PP}(t_f)} (t_{i,f} > q) \rightarrow C + t_f \cdot T_{DI}^L (6)$$

where $C$ is a constant and DI is obtained from the slope of the sum on the left hand side as a function of $t_f$. To calculate the trace of the three point function in DI, we follow the same methodology as in ref. [4] and use the same stochastic algorithm, which employs the $Z_2$ noise estimator and provides the estimate with minimum variance. In addition, we use two more techniques to reduce the errors. First one is the removal of the unwanted real or imaginary part of the three point function through charge conjugation and hermiticity (CH) symmetry. Secondly, we use an unbiased subtraction method [10] to reduce the variance due to the stochastic estimator, i.e. we use the following estimator

$$\text{Tr} \left( A^{-1} \right) = E[<\eta^\dagger (A^{-1} - \sum_{i=1}^{P} \lambda_i O^{(i)} ) \eta >],$$

(7)

where $\eta$’s are $Z_2$ noise vectors, $O^{(i)}$’s are a set of $P$ traceless matrices and $\lambda_i$’s are variational constants which need to be tuned to reduce the variance of $\text{Tr} (A^{-1})$. In principle, one can take any set of traceless matrices as a choice of $O^{(i)}$.

But we choose a set of traceless matrices obtained from the hoping parameter expansion of the propagator in order to match the off-diagonal behavior of the matrix $A^{-1}$ so that they can cancel the off-diagonal contributions to the variance [10]. A variational program which minimizes the variance from the 100 gauge configurations is used to determine the optimum set of $\lambda$’s. With these two techniques, we obtain a reduction of error by as much as 3-4 times. The subtraction method is also proved to be very efficient for the calculations of fermion determinants [10] and for the traces of
many other operators \[3\]. Fig.2 shows a plot of sum vs \( t_f \) (Eq.\[3\]) for \( \kappa = 0.154 \) and \( q^2 = 1 \) with and without subtraction. Finally, we fit \( T'_{dis} \) as a function of \( q^2 \) by a monopole form which is shown in Fig.2 for \( \kappa = 0.154 \). Similar to CI, we also use covariant matrix fitting and the final errorbar is obtained by the jackknife method. A finite mass correction factor \[12\] is introduced while extrapolating to the chiral limit. For strange quark contribution we follow the same procedure as in ref. \[4\].

![Figure 2](image)

Figure 2. (a) The ratio of Eq.\[3\] for \( \kappa = 0.154 \) at \( q^2 = 1 \). Ratios with subtraction are shifted a little towards right. \( m \) is the slope. (b) The monopole fitting of \( T(q^2)_{dis} \) for several \( q^2 \).

![Figure 3](image)

Figure 3. The lattice \( T_{CI}(0) \) and \( T_{DI}(0) \) for CI and DI as a function of quark mass \( m_q \). The chiral result is indicated by .

From DI calculation, we find that \( J_{q(DI)} \), like the DI part of the quark spin \( \frac{1}{2} \Sigma^{DI} \), is also flavor symmetric, i.e., \( J_{q(DI)} = J_{d(DI)} \simeq J_{u(DI)} = -0.047 \pm 0.013 \). Therefore, the total DI contribution \( J_{q(DI)} = -0.14 \pm 0.04 \). Here one should note that in ref. \[4\], results for \( \frac{1}{2} \Sigma^{DI} = -0.18 \pm 0.03 \), which suggests that \emph{the sea quarks give very little orbital angular momentum contribution}. Adding CI and DI contributions, we obtain \( J_q = 0.29 \pm 0.07 \) and therefore we can predict the gluon angular momentum content from the spin sum rule, i.e., \( J_g = 1/2 - 0.29 \pm 0.07 = 0.21 \pm 0.07 \). From our results one can draw the following conclusions:

1. The total angular momentum content of the quark in the proton is \( J_q = 0.29 \pm 0.07 \), i.e., about 60% of the proton spin is contributed by the quarks. Since the lattice result in ref. \[4\] gives \( \Sigma = 0.25 \pm 0.12 \), one can deduce that the quark orbital angular momentum is \( 0.17 \pm 0.08 \). Thus, one concludes that about 25% of the proton spin comes from the quark spin and about 35% comes from the quark orbital angular momentum.

2. Gluon angular momentum contribution is predicted to be \( J_g = 0.21 \pm 0.07 \), i.e., about \( \sim 40\% \) of the proton spin is attributable to the glue.

3. For the sea quarks, almost all the contribution comes from the quark spin and the total sea quark contribution nearly cancels the contribution due to glue. As a result, the proton spin is almost saturated by the CI (valence & cloud quark contribution) of the quark angular momentum alone.

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