Measurements and Simulations of the Spill Quality of Slowly Extracted Beams from the SIS-18 Synchrotron

S Sorge, P Forck and R Singh
GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstr 1, 64291 Darmstadt, Germany
E-mail: S.Sorge@gsi.de

Abstract. In this contribution, results of recent measurements of the spill structure of slowly extracted beams out of the GSI heavy ion synchrotron SIS-18 are presented and compared to results of simulations. Aim of the study is the determination of spill structures at several kHz which arise from ripples in the fields of the accelerator magnets due to imperfections of the magnets’ power supplies. The goal of the study is to understand how the ripple is transferred from the magnets to the spill and to find possible ways to mitigate it.

1. Introduction
The present GSI heavy ion synchrotron SIS-18 has a circumference of $C = 216.72$ m and is used for serving beams of ion species from protons to uranium ions with a rigidity up to $B\rho = 18$ Tm to various experiments. Slow extraction is a very important operation mode necessary to perform many kinds of fixed target experiments. One of the key characteristics of slowly extracted beams is the spill quality, where spill structures should be reduced as well as possible in order to satisfy the requirements of experiment detectors. The spill quality was experimentally studied within a measurement campaign in the year 2016 [1]. Resonant extraction, where the horizontal tune is moved towards the tune of a third integer resonance excited by resonance sextupoles, as well as rf knock-out (rf ko) extraction, where the beam width is slowly increased by a horizontal rf field, were investigated. More details on experimental conditions and detector systems can be found in the contribution to this proceedings of R. Singh et al. [2]. Several parameters were varied in order to study their influence on spill structures. Experiments with resonant extraction were performed with varied extraction duration, chromaticity, momentum spread, and strength of the resonance sextupoles. The horizontal rf field strength was varied in the measurements using rf ko extraction. The strength of the resonance sextupoles is found to have the strongest influence on the spill quality. In addition, spills of coasting and bunched beams were measured because applying bunched beams had been shown to yield a significant improvement of the spill [3].

Recently, theoretical studies using particle-tracking simulations have been started in order to reproduce experimental results. In this contribution we focus on the influence of sextupole strength and bunching on the spill quality of beams extracted by changing the tune with quadrupoles because of the strong impact. These studies are still ongoing.
Figure 1. Duty factors as function of time (graph above) and spectral power density of the ripple (graph below) of the spills of coasting beams measured with the maximum strengths of the resonant sextupoles $(k_2L)_{\text{max}} = (0.03, 0.1, 0.18) \text{ m}^{-2}$.

2. Results

2.1. Coasting beam

The spill quality can be characterised by evaluating the duty factor defined by

$$F = \frac{\langle N \rangle^2}{\langle N^2 \rangle}$$

as well as the spectral power density of the spill ripple determined by Fourier analysis of the spills. $\langle \ldots \rangle$ in Equation (1) denotes the average of the number of particles in time intervals of a duration which is given by the resolution of the measurements within longer time intervals. The spills were measured with the time resolution of 10 $\mu$s and the duty factors were calculated in time bins of 10 ms. Figure 1 shows duty factors and the corresponding spill spectra for various sextupole strengths. The most significant observations are that the duty factors are higher for weaker sextupoles and increase in time towards the end of spill independently of sextupole strengths. Furthermore the spill spectra were found to drop above a certain limiting frequency, where the limiting frequency is lower for weaker sextupoles. Both observations indicate a better spill quality for weaker sextupoles.

Particle-tracking simulations have been performed in order to model the slow extraction process for conditions similar to those of the experiments. The simulation interval is reduced to 500000 revolutions which is $t_{\text{sim}} = 0.55$ s at the experimental beam energy $E_{\text{kin}} = 0.3 \text{ GeV/u}$. For the purpose to introduce a field ripple, several ripple signals are applied to the strengths of focusing and defocusing quadrupoles. In this contribution we show only results obtained with a white noise signal with a limited bandwidth $f_{\text{bw}} = 10 \text{ kHz}$ which is applied to visualise the damping of the spill spectra at high frequencies. The amplitude of the field ripple signal is normalised to $|\Delta(k_1L)_{r,a}/(k_2L)_{\text{quad}}| = 10^{-5}$ which corresponds to the relative ripple of the current $\Delta I_{r,a}/I_{\text{quad}}$ in the quadrupole magnets which was determined experimentally [2].
Figure 2. Graph above: Duty factors of the spills simulated for coasting beam for the same sextupole strengths as in Figure 1. Graph: below: Corresponding spill spectra. The black graphs are simulated for the same conditions as the green but the beam was bunched with an rf voltage of amplitude $V_a = 2$ kV and harmonic number $h = 4$.

2 shows the duty factors and the spill spectra obtained for the sextupole strengths applied. One can see that the dependence on the sextupole strength found in the measurements is qualitatively reproduced.

A possible explanation for the observations is served by a spread of the times $T_{tr}$ the particles need for the transit from the stable phase space area to the extraction septum (ES). The spread $\Delta T_{tr}$ arises from the chromatic tune spread and translates into a finite time interval for the arrival of particles which leave the stable phase space area together. That results in the limiting frequency $f_{lim} \approx 1/\Delta T_{tr}$. Spill structures of frequencies $f > f_{lim}$ are washed out. The transit time for a dynamically changing stable phase space area reads according to Equation (4.17) in [4]

$$T_{tr} = \frac{1}{\sqrt{3} \varepsilon} \ln \left| \frac{n}{n + \frac{3}{\varepsilon} - \frac{1}{\sqrt{3} \varepsilon^2}} \right|,$$

where $\varepsilon(t) = 6\pi |Q(t) - Q_{x,\text{res}}|$ and $n = |\vec{X}_{ES}|/|\vec{X}_{sep}|$.

$$|\vec{X}_{sep}| = \frac{2}{3} \frac{\varepsilon}{S}$$

is the distance between the edge of the stable phase space area enclosed by the separatrices (sep) and its centre, $S$ is the strength of a virtual sextupole according to the equation on p. 21 above in [4], and $|\vec{X}_{ES}|$ is the distance of the extraction septum from the edge of the stable
phase space area, everything written in normalised coordinates. $|\vec{X}_{\text{sep}}|$ defines the size of the stable phase space area and is not a free parameter because the stable phase space area has to be larger than the phase space area of the beam. Hence, it is set approximately equal for all sextupole strengths applied apart from deviations due to the momentum spread. Consequently, the relation $\varepsilon \propto |S|$ is approximately valid according to Equation (3) so that the average transit time at each instant is increased for weaker sextupoles. In addition, the spread of the transit times, approximately given by

$$\Delta T_{\text{tr}} \approx \frac{dT_{\text{tr}}}{d\varepsilon} \Delta \varepsilon$$

(4)

with the spread $\Delta \varepsilon$ arising from the momentum spread of the particles extracted at each instant, is increased for weaker sextupoles. For that reason, the limiting frequency of a spectrum is lower for weaker sextupoles and the spectrum starts to drop at a lower frequency. The washed out spill structures do not contribute to the reduction of the duty factor. Furthermore, the duty factor increases during the extraction process towards the end of spill because $\varepsilon \to 0$.

There are limitations to the reduction of the sextupole strengths. Firstly, there is a minimum distance between machine and resonance tunes given by the chromatic tune spread, $\Delta Q = |\xi \delta_{\text{max}}|$. Secondly, applying weak sextupoles reduces the spiral step resulting in increased particle loss at the extraction septum.

### 2.2. Bunched beam

Spills of slowly extracted bunched beams were measured only for the maximum sextupole strength $(k_2 L)_{\text{max}} = 0.1$ m$^{-2}$. A strong improvement of the spill structure was found which one can see in Fig. 3 with duty factors and spectra of measured spills. The improvement could be reproduced in tracking simulations which is shown by the green and black curves in Figure 2. The duty factors in Figures 2 and 3 obtained with bunches are clearly larger than those obtained with coasting beam and the spectra for the bunched beams have much less spectral power density at low frequencies. Possible mechanisms of spill improvement due to bunch application are still under discussion.

A reasonable explanation can be provided by assuming that the extraction rate is determined by the tune change rate and the tune ripple causes a spill modulation, where particles are “collected” while the ripple tune moves away from the resonance tune and extracted in a bunch when the ripple tune approaches the resonance tune. Then the intensity of the spill ripple depends on the machine tune change and the tune change caused by the ripple during a ripple half period.

Realistic ripple frequencies are in the range $f_r = 150$ Hz ... 7 kHz. The lower frequency is the lowest field ripple frequency which arises from the circuitry of the power supplies. The higher frequency is approximately the maximum frequency of the spectrum shown in the graph below of Figure 2 represented by the green curve. The corresponding ripple half periods are in the range $T_{\text{r, half}} = (3.3 \ldots 0.07)$ ms. The tune change due to the ripple during the ripple half period for the maximum relative field ripple amplitude $|\Delta (k_1 L)_{\text{max}}/(k_1 L)| = 10^{-5}$ is $\Delta Q_{\text{r, max}} = 2 \cdot 10^{-4}$.

The machine tune change during $T_{\text{r, half}}$ is determined by initial and final tunes which were $Q_i = 4.322$, $Q_f = 4.337$ in the simulations, as well as the extraction time $t_{\text{ex}} = 0.55$ s. One finds $\Delta Q = 9.0 \cdot 10^{-5}$ for $T_{\text{r, half}} = 3.3$ ms and $\Delta Q = 1.9 \cdot 10^{-6}$ for $T_{\text{r, half}} = 0.07$ ms. That is less than the tune change due to the ripple. Therefore, the tunes of particles in a coasting beam will stay at least a ripple half period near the resonance resulting in the formation of inhomogeneities on the spill.

On the other hand, the change of the chromatic tune of a particle due to synchrotron motion is given by

$$|\Delta Q_{s,p}| = \left| \xi \delta_{p,\text{max}} \left[ \sin \left( \frac{2\pi T_{\text{r, half}}}{T_s} + \psi \right) - \sin \psi \right] \right|,$$

(5)
Figure 3. Duty factors as function of time (graph above) and spectral power density of the ripple (graph below) of the spills of coasting and bunched beams measured with \( (k_2L)_{\text{max}} = 0.1 \, \text{m}^{-2} \).

where \( \xi = -5.7 \) and \( \delta_{p,\text{max}} \) are the horizontal chromaticity and the maximum momentum deviation of a particle during a synchrotron period. The synchrotron period according to an rf voltage with the amplitude \( V_a = 2 \, \text{kV} \) and the harmonic number \( h = 4 \) is \( T_s = 1.37 \, \text{ms} \). \( \psi \) is the phase of the particle in the longitudinal phase space. Particles will preferentially be extracted when they are near the cusp of synchrotron at \( \psi = \pi/2 \) where the resulting chromatic tune deviation is near the resonance. Considering a particle at \( \psi = \pi/2 \) with the maximum momentum deviation \( \delta_{p,\text{max}} = 7.5 \cdot 10^{-4} \) which equal to that of the bunch, one finds the tune change \( |\Delta Q_{s,p}| = 2.2 \cdot 10^{-4} \) during \( T_{r,\text{half}} = 0.07 \, \text{ms} \). The longest ripple half period \( T_{r,\text{half}} = 3.3 \, \text{ms} \) is even longer than a synchrotron period so that the total tune change will be \( \Delta Q_{s,p} = |2\xi\delta_{p,\text{max}}| = 8.6 \cdot 10^{-3} \). Tune change in both the corner cases are much larger than the corresponding values arising from the machine tune change, so that the particle will cross the resonance much faster when performing synchrotron motion. Therefore bunched beam extraction is less affected by the tune ripple.

3. Outlook
Open points in the simulation work which will be subject of future work are studies on the influence of the parameters chromaticity and momentum spread on the spill structure which were investigated in the measurements, whereas there are limitations to the variation of the extraction duration due to computing time. The influence of the chromaticity seems to be particularly interesting for the extraction of bunched beams because it affects the tune spread and, hence, the rate of changing the chromatic tune due to synchrotron motion. Furthermore, rf ko extraction will be simulated.
References
[1] Singh R et al. 2016 Observation of SIS-18 spill structure by systematic parameter variations GSI scientific report FAIRPROJECT-SIS100-SIS18-21 10.15120/GR-2017-1
[2] Singh R, Boutachkov P, Forck P, Sorge S and Welker H 2018 Slow extraction spill characterization from micro to milli-second scale Proc. Int. Part. Acc. Conf. (Vancouver) Contr. WEPAK007
[3] Forck P, Eickhoff H, Peters A and Dolinskii A 2000 Measurements and improvements of the time structure of slowly extracted beam from a synchrotron Proc. of Europ. Part. Acc. Conf. (Vienna) Contr. MOP4B03
[4] Badano L, Benedikt M, Bryant P J, Crescenti M, Holy P, Maier A, Pullia M, Rossi S and Knaus P 1999 Proton-Ion Medical Machine Study (PIMMS), Part I, (Geneva: CERN/PS 99-010 (DI))