Finite size effects on the galaxy number counts: evidence for fractal behavior up to the deepest scale

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Abstract

We introduce and study two new concepts which are essential for the quantitative analysis of the statistical quality of the available galaxy samples. These are the dilution effect and the small scale fluctuations. We show that the various data that are considered as pointing to a homogenous distribution are all affected by these spurious effects and their interpretation should be completely changed. In particular, we show that finite size effects strongly affect the determination of the galaxy number counts, namely the number versus magnitude relation ($N(<m)$) as computed from the origin. When one computes $N(<m)$ averaged over all the points of a redshift survey one observes an exponent $\alpha = D/5 \approx 0.4$ compatible with the fractal dimension $D \approx 2$ derived from the full correlation analysis. Instead the observation of an exponent $\alpha \approx 0.6$ at relatively small scales, where the distribution is certainly not homogeneous, is shown to be related to finite size effects. We conclude therefore that the observed counts correspond to a fractal distribution with dimension $D \approx 2$ in the entire range $12 \lesssim m \lesssim 28$, that is to say the largest scales ever probed for luminous matter. In addition our results permit to clarify various problems of the angular catalogs, and to show their compatibility with the fractal behavior. We consider also the distribution of Radio-galaxies, Quasars and $\gamma$ ray burst, and we show their compatibility with a fractal structure with $D \approx 1.6 \div 1.8$. Finally we have established a quantitative criterion that allows us to define and predict the statistical validity of a galaxy catalog (angular or three dimensional).
1 Introduction

The crucial question we are going to discuss in detail is the definition of the minimal statistics that is necessary to characterize correctly the large scale distribution of matter. The clarification and the definition of this fundamental point will allow us to reinterpret and clarify the conflictual situation arising from observations of different nature.

The best information on galaxy distribution would be to know the position in space of all the galaxies. The data that are closest to this ideal situation are the so-called "volume-limited" (VL) samples [1], [2]. The VL samples can be extracted from a redshift survey that contains information on the three spatial coordinates (angular position plus redshift). Such samples avoid the luminosity selection effect related to the observational point: one defines a maximum depth and includes in the sample only those galaxies that would be visible from any point of this volume. In such a way this VL sample is observer independent and its correlation properties can be studied directly.

The galaxy distribution in all the redshift catalogues currently available [3]-[17] show a highly inhomogeneous distribution characterized by the presence of clusters, superclusters, filaments walls and large voids up to the boundaries of such surveys.

The modern statistical analysis of these three dimensional samples [18] show that this irregular distribution is characterized by having long range fractal correlations. In particular Coleman & Pietronero [2], [19] found that the galaxy distribution in the CfA1 survey [4] is fractal with \( D \approx 1.6 \) up to \( R_s \approx 20h^{-1}Mpc \). This depth is of the order of the radius of the maximum sphere that is fully contained in the sample volume. This limit is imposed by the request that the correlation function should be computed only in spherical shell ([2]). There have been attempts to push \( R_s \) to larger values by using various weighting schemes for the treatment of boundary conditions [20]. These methods however, unavoidably introduce artificial homogenization effects and therefore should be avoided [2]. Sylos Labini et al. [21] found that the Perseus-Pisces galaxy redshift surveys [3] has fractal long-range correlations up to \( 30h^{-1}Mpc \) with dimension \( D \approx 2 \). The fractal dimension in this case is somewhat larger than in CfA1. This tendency for a higher value of the fractal dimension is related to the poor statistics and small depth of the CfA1 catalogue and it is confirmed by various other redshift surveys. For example a similar behavior has been found by Di Nella et al. [22] and [23] in the LEDA database [7] up to \( \sim 150h^{-1}Mpc \). A first apparent contradictory result has been found by Strauss et al. [24] (see also [25]) analyzing the IRAS catalogues, where they found an evidence towards homogenization at \( R_s \approx 15 \div 20h^{-1}Mpc \).
Fig. 1. The galaxy number counts in the B-band, from several surveys. In the range $12 \lesssim m \lesssim 19$ the counts show an exponent $\alpha \approx 0.6$, while in the range $19 \lesssim m \lesssim 28$ the exponent is $\alpha \approx 0.4$.

A different way to get information for larger scales is to compute the conditional average (in VL samples) from the observation point only. This allows us to extend $R_s$ to the full depth of the catalogue, at the expenses of having a reduced statistics. We are going to see that in this way it is possible to detect fractal correlations up to $130 h^{-1} Mpc$ in Perseus-Pisces (PP) [21] and up to $800 \div 900 h^{-1} Mpc$ in the new ESP catalogue [26].

Historically [27] [28] the oldest type of data about galaxy distribution is given by the relation between the number of observed galaxies $N(> f)$ and their apparent brightness $f$. It is easy to show that [28]

$$N(> f) \sim f^{-D/2} \quad (1)$$

where $D$ is the fractal dimension of the galaxy distribution. Usually this relation is written in terms of the apparent magnitude $f \sim 10^{-0.4m}$ (note that bright galaxies correspond to small $m$). In terms of $m$, Eq.(1) becomes

$$\log N(< m) \sim \alpha m \quad (2)$$

with $\alpha = D/5$ [26] [28]. Note that $\alpha$ is the coefficient of the exponential behavior of Eq.(2) and we will call it “exponent” even though it should not be confused with the exponents of power law behaviors. In Fig.1 we have collected all the recent observations of $N(< m)$ versus $m$ [29]-[35] in the B-spectral-band ($m_B$). At bright and intermediate magnitude ($12 \lesssim m_B \lesssim 18$), corresponding to small redshift ($z < 0.2$), one obtains $\alpha \approx 0.6$, while from $m_B \sim 19$ up to $m_B \sim 28$ the counts are well fitted by a smaller exponent with $\alpha \approx 0.4$. The usual interpretation [28], [36]-[42] is that $\alpha \approx 0.6$ corresponds to $D \approx 3$ consistent with homogeneity, while at large scales galaxy evolution and space time expansion effects are invoked to explain the lower value $\alpha \approx 0.4$.

On the basis of the previous discussion of the VL samples we can see that this interpretation is untenable. In fact, we know for sure that, at least up to $R_s \sim 150 h^{-1} Mpc$ there are fractal correlations [4], [21], [22] so one would
eventually expect the opposite behaviour. Namely small value of $\alpha \approx 0.4$ (consistent with $D \approx 2$) at small scales followed by a crossover to an eventual homogeneous distribution at large scales ($\alpha \approx 0.6$ and $D \approx 3$).

The GNC in the (red) $R$-band shows an exponent $\alpha \approx 0.37 - 0.41$ in the range $20 < R < 23$ [30],[33] and [35]. Moreover Gardner et al. [13] have studied the GNC in the (infrared) $K$-band in the range $12 \lesssim K \lesssim 23$, and they show that the slope of the counts changes at $K \approx 17$ from 0.67 to 0.26 (see also [14]-[18]). Djorgovski et al. [17] found that the slope of the GNC is little bit higher than [13], i.e. $\alpha = 0.315 \pm 0.02$ between $K = 20$ and 24 magn. The situation is therefore quite similar in the different spectral bands. The puzzling behaviour of the GNC represents the second contradiction we find in the data analysis.

An additional argument in favour of homogeneity at rather small scales has been proposed as arising from an appropriate rescaling of the angular correlations [28]. This is the third evidence that seems to be conflictual with the properties observed in the VL correlations analysis.

We argue here that this apparently contradictory experimental situation can be fully understood on the light of the small scale effects in the space distribution of galaxies. For example a fractal distribution is non analytic in every occupied point: it is not possible to define a meaningful average density because we are dealing with intrinsic fluctuations that grow with as the scale of the system itself. This situation is qualitatively different from an homogeneous picture, in which a well defined density exists, and the fluctuations represent only small amplitude perturbations. The nature of the fluctuations in these two cases is completely different, and for fractals the fluctuations themselves define all the statistical properties of the distribution. This concept has dramatic consequences in the following discussion as well as in the determination of various observable quantities, such as the amplitude of the two point angular correlation function.

It is worth to notice that small scale effects are usually neglected in the study of fractal structures because one can generate large enough structures to avoid these problems. In Astrophysics the data are instead intrinsically limited and we are going to see that an analysis of finite size effects is very important. We discuss in detail the problems of finite size effects in the determination of the asymptotic properties of fractal distributions, considering explicitly the problems induced by the lower cut-off (Section 3).

We discuss in detail the case of real galaxy redshift survey (Sec.4 and Sec.5), while in Section 6 we consider the case of the magnitude limited samples. In particular, we find a criterion which allows us to define the statistical validity of the various samples with respect to the finite size effects. In particular it results that the apparent exponent $\alpha \approx 0.6$ at small scales (bright magnitudes
m < 19) arises purely from finite size effects and cannot be related to the real correlation properties of the sample.

In Section 7 we discuss the concept of a statistically fair sample, i.e. a sample which contains enough statistical information to be representative of the whole distribution from which it has been extracted. In particular we define the lower number density of points needed to recover the genuine statistical properties of a certain distribution (homogeneous or fractal). Moreover we find that the all-sky IRAS surveys [10], [24] do not contain enough points to be statistically fair samples.

The implications of our analysis is that it provides a quantitative criterion to define the statistical validity of a redshift and an angular survey, and for the optimization of its geometry to obtain the maximum reliable information (Sec.8). These results can be useful also to design future programs for redshift or angular surveys. On this basis we can predict the expected statistical properties of several surveys such as CfA2 [8], Las Campanas [12], ESP [13], etc.

In Section 9 we discuss the angular correlation functions, on the light of the previous analysis. It results that the scaling of the amplitude of the angular correlation function with the depth of the sample, that is considered to be a possible evidence for of homogeneity [28], is actually due to finite size effects just as the exponent of the number counts. In addition in Section 10 we consider the distribution of Radio-galaxies, Quasars and γ ray bursts. We show that these objects are fractally distributed with $D \approx 1.6 \pm 1.8$.

Finally in Section 11 we present our main conclusion, stressing in particular the dramatic and puzzling consequences on the general theoretical framework and, in particular on the Big-Bang model.

2 Galaxy number counts: Basic Relations

The basic assumption that we use to compute all the following relations is that:

$$\nu(L, \vec{r}) = \phi(L)D(\vec{r}) \tag{3}$$

i.e. that the number of galaxies for unit luminosity and volume $\nu(L, \vec{r})$ can be separated as the product of the space density $D(\vec{r})$ and the luminosity function $\phi(L)$. This is a crude approximation in view of the multifractal properties of the distribution (correlation between position and luminosity), and a
detailed discussion is found in [49] and [26]. However, for the propose of the present discussion the approximation of Eq.3 is rather good and the explicit consideration of the multifractal properties would have a minor effect on the properties we are going to discuss ([49]).

The integrated space number density, i.e. the total number of points inside a sphere of radius \( R \), in the general case, has the following property

\[
N(< R) = \int_0^R D(\vec{r}) d^3r = \int_0^R \sum_{i=1}^N \delta(\vec{r} - \vec{r}_i) d^3r = BR^D
\]

where \( D = 3 \) for the homogeneous case, while if \( D < 3 \) the distribution is fractal and \( B \) is the prefactor (see Sec. 3). This is an asymptotic relation and the problem is to recover this behavior from finite size portions of real structures (we discuss in detail this point in Section 3).

We briefly introduce some basic definitions. If \( L \) is the absolute or intrinsic luminosity of a galaxy at distance \( r \), this will appears with an apparent flux

\[
f = \frac{L}{4\pi r^2}.
\]

Suppose all the galaxies have the same luminosity \( L \). If the number of points inside a sphere of radius \( r \) grows as \( N(< r) \sim r^D \), then it is simple to show that the number of galaxies with apparent flux greater than \( f \) goes as \( N(> f) \sim f^{-\frac{D}{2}} \). This the basic relation for the GNC. Now we consider the case in which there is a certain distribution of galaxy luminosities.

For historical reasons the apparent magnitude \( m \) of an object with incoming flux \( f \) is defined to be ([28])

\[
m = -2.5 \log_{10} f + \text{constant},
\]

while the absolute magnitude \( M \) is instead related to its intrinsic luminosity \( L \) by

\[
M = -2.5 \log_{10} L + \text{constant}.
\]

From Eq. [3] that the difference between the apparent and the absolute magnitudes of an object at distance \( r \) is (at relatively small distances, neglecting relativistic effects)

\[
m - M = 5 \log_{10} r + 25
\]

where \( r \) is expressed in Megaparsec (1Mpc = 3.26 \cdot 10^6 \text{ light years}.)

We now compute the expected GNC in the simplest case of a magnitude limited (ML) sample. A ML sample is obtained measuring all the galaxies
with apparent magnitude brighter than a certain limit $m_{\text{lim}}$. In this case we have (for $m < m_{\text{lim}}$)

$$N(< m) = B \Phi(\infty) 10^{Dm}$$

(9)

where

$$\Phi(\infty) = \int_{-\infty}^{\infty} \phi(M) 10^{-\frac{D}{5}(M+25)} dM$$

(10)

We consider now the case of a volume limited (VL) sample. A VL sample contains every galaxy in the volume which is more luminous than a certain limit, so that in such a sample there is no incompleteness for an observational luminosity selection effect ([1], [2]). Such a sample is defined by a certain maximum distance $R$ and an absolute magnitude limit given by:

$$M_{\text{lim}} = m_{\text{lim}} - 5 \log_{10} R - 25$$

(11)

($m_{\text{lim}}$ is the survey apparent magnitude limit). By performing the calculations for the number-magnitude relation, we obtain

$$N(< m) = A(m) \cdot 10^{\frac{D_{5}}{m}} + C(m)$$

(12)

where $A(m)$ is

$$A(m) = B \int_{M(m)}^{M_{\text{lim}}} \phi(M) 10^{-\frac{D}{5}(M+25)} dM$$

(13)

and $M(m)$ is given by Eq.8 (with $R_{\text{VL}}$ in the place of $r$), and it is a function of $m$. The second term is

$$C(m) = B R^{D} \int_{-\infty}^{M(m)} \phi(M) dM$$

(14)

This term, as $A(m)$, depends from the VL sample considered. We assume a luminosity function with a Schechter shape [50]

$$\phi(M) dM \sim 10^{-0.4(\delta+1)M} e^{-10^{0.4(M^* - M)}} dM$$

(15)

where $\delta \sim -1.1$ and the cut-off $M^* \sim -19.5$ [14]. For $M(m) \gtrsim M^*$ we have that $C(m) \approx 0$, and $A(m)$ is nearly constant with $m$. This happens in particular for the deeper VL samples for which $M_{\text{lim}} \sim M^*$. For the less deeper VL ($M_{\text{lim}} > M^*$) samples these terms can be considered as a deviation from a power law behavior only for $m \to m_{\text{lim}}$. In Fig.2 we show the behavior of Eq.12 for the case $m_{\text{lim}} = 15.5$ and we consider VL samples at VL40:
Fig. 2. Behaviour of \( N(< m) \) for various VL samples of an ideal survey with \( m_{lim} = 15.5 \). For the deeper VL sample \((120h^{-1}Mpc)\) (large squares) a straight power law is found, while for the \(40h^{-1}Mpc\) (small squares) sample there is a flattening for \( m \to m_{lim} \), even if the true exponent \( \alpha = D/5 \) is found at lower magnitudes. The sample at \(80h^{-1}Mpc\) (crosses) show an intermediate situation.

\((R = 40h^{-1}Mpc, M^* = -17.53)\); \(VL80 \ (R = 80h^{-1}Mpc, M^* = -19.07)\); \(VL120 \ (R = 120h^{-1}Mpc, M^* = -19.93)\). We can see that the first subsample for \( m \to m_{lim} \) show a curved behavior while in the other cases the \( \log(N(< m)) - m \) relation is well fitted by a power law with the right exponent \( \alpha = D/5 \). This is the ideal case where no finite size effects are present.

If one has \( \phi(M) = \delta(M - M_0) \) then it is simple to show that \( \log(N(< m)) \sim (D/5)m \) also each VL sample. This relation will be useful in the following (Section 5).

3 The problem of finite size effects

In this section we discuss the general problem of the minimal sample size that is able to provide us with a statistically meaningful information. For example the mass-length relation for a fractal, that defines the fractal dimension is ([51] and [52])

\[
D = \lim_{r \to \infty} \frac{\log(N(< r))}{\log(r)}
\]  

(16)

However this relation is properly defined only in the asymptotic limit, because only in this limit the fluctuations of the fractal structure are self-averaging. A fractal distribution is characterized by large fluctuations at all scales and these fluctuations determine all the statistical properties of the structure. If the structure has a lower cut-off, as it is the case for any real fractal, one needs a "very large sample" in order to recover the statistical properties of the distribution itself. Indeed, in any real physical problem we would like to recover the asymptotic properties from the knowledge of a finite portion of a
Fig. 3. Behavior of the density computed from one point. At small distances, inside the Voronoi’s length $V$, one finds almost no galaxies because the total number is rather small. Then the number of galaxies starts to grow, but this regime is strongly affected by finite size fluctuations. Finally the correct scaling region $r \approx \lambda$ is reached. In the intermediate region the density can be approximated roughly by a constant value. This leads to an apparent exponent $D \approx 3$ (in terms of GNC $\alpha \approx 0.6$). This exponent is not real but just due to the size effects.

Fig. 4. Definition of the Voronoi volume (see text)

fractal and the problem is that a single finite realization of a random fractal is affected by finite size fluctuations due to the lower cut-off.

Considering homogeneous distribution we can define, an average, a characteristics volume associated to each particle. This is the Voronoi volume $v_v$ whose radius $\ell_v$ is of the order of the mean particle separation. It is clear that the statistical properties of the system can be defined only in volumes much larger than $v_v$. Up to this volume in fact we observe essentially nothing. Then one begins to include a few (strongly fluctuating) points, and finally, the correct scaling behavior is recovered (Fig.3). For a Poisson sample consisting of $N$ particles inside a volume $V$ then the Voronoi volume (Fig.4) is of the order

$$v_v \sim \frac{V}{N}$$  \hspace{1cm} (17)
and $\ell_v \approx v_v^{1/3}$. In the case of homogeneous distribution, where the fluctuations have small amplitude with respect to the average density, one readily recovers the statistical properties of the system at small distances, say, $r \gtrsim 5\ell_v$.

The case of fractal distribution is more subtle. For a self-similar distribution one has, within a certain radius $r_0$, $N_0$ objects. Following [3] we can write the mass-length relation between $N(< R)$, the number of points inside a sphere of radius $R$, and the distance $R$ of the type

$$N(< R) = BR^D$$

where the prefactor $B$ is related to the lower cut-offs $N_0$ and $r_0$

$$B = \frac{N_0}{r_0^D}$$

In this case the prefactor $B$ is defined for spherical samples. If we have a portion of a sphere characterized by a solid angle $\Omega$, we write Eq.(18) as

$$N(< R) = BR^D\frac{\Omega}{4\pi}$$

Suppose we have only a finite portion of a fractal structure characterized by a volume

$$V(R) = \frac{\Omega R^3}{3}$$

In this case the Voronoi’s volume can be written as

$$v_v = \frac{4\pi}{3} R^{3-D} \frac{\Omega}{B}$$

We define

$$V_\Omega = \frac{\lambda^3}{3} \Omega$$

as the minimal volume which allows us to recover the statistical properties of the system. From a series of numerical tests on artificial distributions we will conclude that $V_\Omega \sim 10^3 v_v$. From this we can define the minimal statistical length as

$$\lambda \sim \left(\frac{4\pi}{\Omega} 10^3 \frac{R^{3-D} \Omega}{B} \right)^{1/3}$$

The minimal statistical length $\lambda$ is an explicit function of the prefactor $B$ and of the depth of the survey $R$. Essentially this discussion implies that $\lambda$ should be about one order of magnitude larger than $\ell_v \sim v_v^{1/3}$, apart from solid angle.
effects. We note that in order to optimize the region of the sample beyond the minimal statistical length, we have to maximize the ratio

\[
\frac{R}{\lambda} \sim R^{\frac{D}{4}} \Omega^{\frac{1}{2}}
\]

hence if \( D > 1 \) it is clearly convenient to increase the depth of the sample rather than the solid angle in order to improve the statistics as we discuss in detail in the following (see in particular Sec.7).

In the case of real galaxy catalogs we have to consider the luminosity selection effects. In a volume limited (VL) sample, characterized by an absolute magnitude limit \( M_{\text{lim}} \), (minimal absolute flux) the mass-length relation Eq.20, can be generalized as (see Sec. 2)

\[
N(R, M_{\text{lim}}) = BR^D \Omega \frac{\psi(M_{\text{lim}})}{4\pi}
\]

where \( \psi(M_{\text{lim}}) \) is the probability that a galaxy has an absolute magnitude brighter than \( M_{\text{lim}} \)

\[
0 < \psi(M_{\text{lim}}) = \frac{\int_{-\infty}^{M_{\text{lim}}} \phi(M) dM}{\Psi(\infty)} < 1
\]

where \( \phi(M) \) is the Schechter luminosity function (Sec.2) and \( \Psi(\infty) \) is the normalizing factor

\[
\Psi(\infty) = \int_{-\infty}^{M_{\text{min}}} \phi(M) dM
\]

where \( M_{\text{min}} \) is the fainter absolute magnitude surveyed in the catalog (usually \( M_{\text{min}} \approx -10 \div -11 \)).

It is possible to compute the intrinsic prefactor \( B \) from the knowledge of the average conditional density \( \Gamma(r) \) \cite{2,21} as

\[
\Gamma(r) = \frac{D}{4\pi} Br^{D-3}
\]

computed in the VL samples and normalized for the luminosity factor (Eq.27). In the various VL subsamples of Perseus-Pisces and CfA1 redshift surveys we find that

\[
B \approx 10 \div 15(h^{-1}\text{Mpc})^{-D}
\]
Fig. 5. The spatial density \( n(r) \) and \( N(<r) \) computed in two VL sample cut at 60\(h^{-1}\)Mpc \((a, b)\), and 70\(h^{-1}\)Mpc \((c, d)\). The density is dominated by large fluctuations and it has not reached the scaling regime.

depending on the parameters of the Schechter function \( M^* \) and \( \delta \). From Eq.24 and Eq.30 we obtain

\[
\lambda \approx \left( \frac{10^3 R^{3-D}}{\Omega} \right)^{\frac{1}{3}} \tag{31}
\]

and for \( R \) in the range 100 ÷ 1000\(h^{-1}\)Mpc we find that a good approximation to Eq.31 is

\[
\lambda \approx \frac{(40 ÷ 60)h^{-1}\text{Mpc}}{\Omega^{\frac{1}{3}}} \tag{32}
\]

This is the value of the minimal statistical length that we will use in the following (see Table 1). This length depends also, but weakly, from the particular morphological features of the sample. In the case of Perseus-Pisces \((\Omega = 0.9)\) we find \( \lambda \approx 50h^{-1}\text{Mpc} \) as shown in Fig.5 and Fig.6, and for CfA1 \((\Omega = 1.8)\) \( \lambda \approx 30h^{-1}\text{Mpc} \) (Fig.7): in both cases the agreement with Eq.32 is quite good. Moreover we have done the following test: we have cut the Perseus-Pisces survey at various solid angles and we have checked that Eq.31 holds with good accuracy for various values of \( R \) and \( \Omega \).

For relatively small volumes it is possible to recover the correct scaling behavior for scales of order of \( \ell_v \) (instead of \( \sim 10\ell_v \)) by averaging over several samples or, as it happens in real cases, over several points of the same sample when this is possible. Indeed when we compute the correlation function we perform an average over all the points of the system even if the VL sample is not deep enough to satisfy the condition expressed by Eq.32. Even in this case the lower cut-off introduces a limit in the sample statistics as we point out in Sec.7 and Sec.8.

In Section 4 we show how these considerations can be applied to the case of a real redshift survey (Perseus-Pisces) as well as in the case of artificial catalogs with a priori assigned properties.
| Survey   | $\Omega(sr)$ | $\lambda(h^{-1}Mpc)$ | References                                      |
|----------|--------------|----------------------|------------------------------------------------|
| CfA1     | 1.8          | 30                   | Huchra et al., (1983); Davis et al., (1982)     |
| CfA2     | 1.8          | 30                   | Geller & Huchra, (1988); Park et al., (1994)    |
| SSRS1    | 1.13         | 40                   | Da Costa et al., (1988; 1991)                    |
| SSRS2    | 1.13         | 40                   | Da Costa et al., (1994)                          |
| PP       | 1            | 50                   | Haynes et al., (1988); Sylos Labini et al., (1995) |
| LEDA     | $4 \pi$      | 20                   | Paturel et al., (1988)                          |
| ORS      | 8            | 25                   | Santiago et al., (1994)                         |
| APM      | 1.36         | 50                   | Loveday et al., (1992a, b)                       |
| LCRS     | 0.12         | 100                  | Schectman, (1992)                               |
| KOSS     | 0.01         | 250                  | Kirshner et al., (1981)                         |
| ESP      | 0.006        | 300                  | Vettolani et al., (1994)                        |
| KOO      | $1.5 \cdot 10^{-4}$ | 950              | Koo, (1988)                                     |
Fig. 6. The spatial density $n(r)$ and $N(< r)$ computed in two VL sample VL110 ($a, b$) and VL130 ($c, d$). In this case the density is dominated by large fluctuations only at small distances, while at larger distances, after the Perseus Pisces chain at $50h^{-1}Mpc$, a very well defined power law behavior is shown, with the same exponent of the correlation function of Fig.8 ($D = 2$).

Fig. 7. The spatial density ($a$) $n(r)$ and the integral number of points ($b$) $N(< r)$ computed in a VL sample (VL60) of CfA1. A very well defined power law behavior with $D \approx 2$ is shown for $r \gtrsim 30h^{-1}Mpc \approx \lambda$.

4 Integral from the vertex in Perseus-Pisces

To clarify the effects of the spatial inhomogeneities and finite size effects we have studied the GNC in the Perseus-Pisces redshift survey [5]. In a previous paper ([21]) we have analyzed the spatial properties of galaxy distribution in this sample and we briefly summarize our main results. We find that the correlation function $\Gamma(r)$ (conditional average density - [2]), computed in several VL samples, is

$$\Gamma(r) = \frac{< n(\vec{r}_0) n(\vec{r} + \vec{r}_0) >}{< n >} \sim r^{-\gamma}$$

(33)

where $\gamma \approx 1$ up to $\sim 30h^{-1}Mpc$. This result means that the galaxy distribution in this sample is fractal up to this depth with dimension $D = 3 - \gamma \approx 2$. This trend of larger value of $D$ with respect to CfA1 ($D \approx 1.5$) is confirmed by all the deeper samples (CfA2 [55, 56], LEDA [22, 23], ESP [57]) and it is
probably due to a more stable statistics (see Sec.7) of this larger sample with respect to CfA1. We have also studied the behavior of the galaxy number density in the VL samples, i.e. the behavior of:

\[ n(r) = \frac{N(< r)}{V(r)} \sim r^{D-3} \tag{34} \]

One expects that, if the distribution is homogeneous the density is constant, while if it is fractal it decays with a power low behavior.

When one computes the correlation function of Eq.33, one indeed performs an average over all the points of the survey. In particular we limit our analysis to the size defined by the radius of the maximum sphere fully contained in the sample volume, so that we do not introduce any weighting scheme in the treatment of the boundaries of the sample [21]. Several authors (e.g. [20]) have extended the effective depth for the computation of \( \Gamma(r) \) making use of the weighting schemes for the treatment of the boundary conditions: Guzzo et al. [20] found a tendency towards homogenization at \( \sim 40h^{-1}Mpc \) in PP, while we have limited our analysis at \( R_s \approx 30h^{-1}Mpc \) As we show in the following, such a method introduces a spurious tendency towards homogenization, and must be avoided for system characterized by long-range correlation (see also [2] and [21]).

On the contrary Eq.34 is computed only from a single point, the origin. This allows us to extend the study of the spatial distribution up to very deep scales and in fact we find that Eq.34 holds up to \( \sim 130h^{-1}Mpc \), that is the maximum depth surveyed by this catalog, with the same exponent \( \gamma \approx 1 \) as before. The price to pay is that this method is strongly affected by statistical fluctuations and finite size effects. Analogously, when one computes \( N(< m) \), one does not perform an average but just counts the points from the origin. As in the case of \( n(r) \) also \( N(< m) \) will be strongly affected by statistical fluctuations due to finite size effects. We now clarify how the behaviour of \( N(< m) \), and in particular its exponent, are influenced by these effects.

We have computed the \( n(r) \) in the various VL sample, and we show the results in Fig.5 and Fig.6. In the less deeper VL samples (VL60, VL70) (Fig.5) we see that the density does not show any smooth behavior because in this case the finite size effects dominate the behaviour as we are at distances \( r < \lambda \) (Eq.32), while at about the same scales we can find a very well defined power law behavior with the correlation function analysis (Fig.8).

In the deeper VL samples (VL110, VL130) we can see (Fig.6) that a smooth behavior is reached for distances larger than the scaling distance \( r \approx \lambda \sim 50h^{-1}Mpc \). The fractal dimension turns out to be \( D \approx 2 \) as in the case of the correlation function (Fig.8). Also the amplitude of the density matches quite well that of \( \Gamma(r) \) (if properly normalized).
Fig. 8. The correlation function for VL60 (small squares) and VL70 (crosses) and VL110 (large squares). In this case the correlation function $\Gamma(r)$, that is the conditional density averaged from each point of the sample, shows a very well defined power law behaviour. The finite size effects are eliminated exactly for the averaging procedure. The reference line has a slope of $-\gamma = D - 3 = -1$.

Fig. 9. The Number counts $N(<m)$ for the VL samples VL60 and VL70. The slope is $\alpha \approx 0.6$. In this case occurs a flattening for $m \to m_{lim}$.

We show in Fig.8, Fig.10 and Fig.11 the behaviour of $N(<m)$ respectively for the VL samples of Fig.5 (VL60 and VL70), of Fig.8 (VL110, VL130) and finally for the whole magnitude limit sample. We can see that for VL60 and VL70 there are very strong inhomogeneities in the behaviour of $n(r)$ and these are associated with a slope $\alpha \approx 0.6$ for the GNC. (The flattening for $m \to m_{lim}$ is just to a luminosity selection effect that is explained in Section 2). For VL110 and VL130 the behaviour of the density is much more regular and smooth, so that it shows indeed a clear power law behavior. Correspondingly the behaviour of $N(<m)$ is well fitted by $\alpha \approx 0.4$. Finally the whole magnitude limit sample is again described by an exponent $\alpha \approx 0.6$.

We have now enough elements to describe the behaviour of the GNC. The first point is that the exponent of the GNC is strongly related to the space distribution. Indeed what has never been taken into account before is the role of finite size effects. The behaviour of the GNC is due to a convolution of the space density and the luminosity function (Sec.2), and the space density enters in the GNC as an integrated quantity. The problem is to consider the
The Number counts $N(< m)$ for the VL samples VL110 and VL130. The slope is $\alpha \approx 0.4$, apart from the initial fast growth. This behavior corresponds to a well-defined power law behavior of the density with exponent $D \approx 5\alpha \approx 2$ (Fig.6).

The Number counts $N(< m)$ for the whole magnitude limit sample. The slope is $\alpha \approx 0.6$ and it is clearly associated only to fluctuations in the spatial distribution rather than to a real homogeneity in space.

right space density in the data analysis. In fact, if we have a very fluctuating behavior for the density in a certain region, as in the case shown in Fig.5, its integral over this range of length scales is almost equivalent to a flat one. This can be seen also in Fig.6: at small distances one finds almost no galaxies because the total number is rather small. Then the number of galaxies starts to grow, but this regime is strongly affected by finite size fluctuations. Finally the correct scaling region $r \approx \lambda$ is reached. This means for example that if one has a fractal distribution, there will be first a raise of the density, due to finite size effects and characterized by strong fluctuations, because no galaxies are present before a certain characteristic scale. Once one enters in the correct scaling regime for a fractal the density becomes to decay as a power law. So in this regime of raise and fall with strong fluctuations there will be a region in which the density can be approximated roughly by a constant value. This leads to an apparent exponent $D \approx 3$, so that the integrated number grows as $N(< r) \sim r^D$ and it is associated in terms of GNC, to $\alpha \approx 0.6$. This exponent is therefore not real but just due to the finite size fluctuations. Only when a well defined statistical scaling regime has been reached, i.e. for $r > \lambda$, one can
The question of the difference between the integration from the origin and correlation properties averaged over all points lead us to a subtle problem of asymmetric fluctuations in a fractal structure. From our discussion, exemplified by Fig.3, the region between $\ell_v$ and $\lambda$ corresponds to an underdensity with respect to the real one. However we have also showed that for the full correlation averaged over all the points ($\Gamma(r)$) the correct scaling is recovered at distances appreciably smaller than $\lambda$. This means that in some points one should observe an overdensity between $\ell_v$ and $\lambda$. However, given the intrinsic asymmetry between filled and empty regions in a fractal, only very few points will show the overdensity (a fractal structure is asymptotically dominated by voids). These few points nevertheless will have an important effect on the average values of the correlations. This means that, in practice, a typical points shows an underdensity up to $\lambda$ as shown in Fig.3. The full averages instead converge at much shorter distances. This discussion shows the peculiar and asymmetric nature of finite size fluctuations in fractals as compared to the symmetric Poissonian case.

For homogeneous distribution (Fig.12) the situation is in fact quite different. Below the Voronoi length $\ell_v$ there are finite size fluctuations, but for distances $r \gtrsim (2 \div 4)\ell_v$ the correct scaling regime is readily found for the density, the integrated density and the number counts. In this case the finite size effects do not affect too much the properties of the system because a Poisson distribution is characterized by small amplitude fluctuations.

In the VL samples where $n(r)$ scales with the asymptotic properties (Fig.3) the GNC grows also with the right exponent ($\alpha = D/5$). If we now consider
instead the behaviour of the GNC in the whole magnitude limit survey, we fund
that the exponent is $\alpha \approx 0.6$ (Fig.11). This behavior can be understood by
considering that at small distances, well inside the distance $\lambda$ defined by Eq.32,
the number of galaxies present in the sample is large because there are galaxies
of all magnitudes. Hence the majority of galaxies correspond to small distances
($r < \lambda$) and the distribution has not reached the scaling regime in which the
statistical self-averaging properties of the system are present. For this reason in
the ML sample the finite size fluctuations dominate completely the behavior
of the GNC. Therefore this behaviour in the ML sample is associated with
spurious finite size effects rather than to real homogeneity. We discuss in a
more quantitative way the behavior in ML surveys in Sec.6, while in Sec.5
we perform a test to proof that the exponent $\alpha \approx 0.4$ is the real statistical
property of the galaxy distribution.

5 A test for the finite size effects: average $N(< m)$

To prove that the behaviour found in Fig.1, and then the exponent $\alpha \approx
0.6$, is connected to large fluctuations due to finite size effects in the space
distribution and not for real homogeneity, we have done the following test. We
have adopted the same procedure used for the computation of the correlation
function ([3], [21]), i.e. we make an average for $N(< m)$ from all the points of
the sample rather than counting it from the origin only.

To this aim we have considered a VL sample with $N$ galaxies and we have built
$N$ independent flux-limited surveys in the following way. We consider each
galaxy in the sample as the observer, and for each observer we have computed
the apparent magnitude of all the other galaxies. To avoid any selection effect
we consider only the galaxies that lie inside a well defined volume around the
observer. This volume is defined by the maximum sphere fully contained in
the sample volume with the observer as a center.

Moreover we have another selection effect due to the fact that our VL sample
has been built from a ML survey done with respect to the origin. To avoid
this incompleteness we have assigned to each galaxy a constant magnitude $M$.
In fact, our aim is to show that the inhomogeneity in the space distribution
plays the fundamental role that determines the shape of the $N(< m)$ relation,
and the functional form of the luminosity function enters in Eq.13 only as an
overall normalizing factor.

Once we have computed $N_i(< m)$ from all the points $i = 1, ..., N$ we then
compute the average. We show in Fig.12 and Fig.13 the results for VL60 and
VL110: a very well defined exponent $\alpha = D/5 \approx 0.4$ is found in both cases.
This is in fully agreement with the average space density (the conditional
average density $\Gamma(r)$ that shows $D \approx 2$ in these VL samples.

We have also performed various tests on artificial distributions with a priori assigned properties. Using the random $\beta$-model algorithm \cite{4} and we have performed the analysis by assigning to each point of the system the same absolute magnitude. The results are in complete agreement with the previous findings: if we do not perform any average the exponent of the GNC is strongly affected by the presence of fluctuations due to finite size effects and we obtain $\alpha \approx 0.6$, while if we compute the GNC by making an average over all the points of the structure we find again that the relation $\alpha = D/5$ holds in a very well approximation (Fig.15 and Fig.16). On the contrary in the homogeneous case (Fig.12) one does need to perform any average to recover the correct scaling properties of $N(<m)$, because the system reaches very soon ($r \gtrsim \ell_v$) the correct scaling properties.
Fig. 15. (a) The space density computed from the vertex in an artificial fractal sample with $D = 2.5$. (b) The $N(< m)$ relation for the same sample. The reference line has a slope $\alpha = 0.6$.

Fig. 16. (a) The average space density computed in the artificial fractal sample with $D = 2.5$ of Fig. 15. The reference line has a slope $D - 3 = -1$. (b) The average $N(< m)$ for the same sample. The reference line has a slope $\alpha = D/5 = 0.5$.

6 GNC in Magnitude limited catalogs

We are now able to clarify the problem of ML catalogs. Suppose to have a certain survey characterized by a solid angle $\Omega$ and we ask the following question: up to which apparent magnitude limit $m_{\text{lim}}$ we have to push our observations to obtain that the majority of the galaxies lie in the statistically significant region ($r \gtrsim \lambda$) defined by Eq. 31. Beyond this value of $m_{\text{lim}}$ we should recover the genuine properties of the sample because, as we have enough statistics, the finite size effects self-average. From the previous condition for each $\Omega$ we can find a solid angle $m_{\text{lim}}$ so that finally we are able to obtain $m_{\text{lim}} = m_{\text{lim}}(\Omega)$ in the following way.

We assume a Schecther luminosity function (Eq. 15) and that the fractal dimension is $D = 2$ (we have tested that the final result depends very weakly on some reasonable values for the three parameters used: $D$ for the space distribution and $\delta$ and $M^*$ for the luminosity function). As shown in Fig. 17, the requested condition happens when the area $I_2 > I_1$. We have evaluated
Fig. 17. An ideal survey in the $d-M$ space. If the majority of galaxies lie in the statistically significant region then we can obtain a meaningful statistical information from the survey. This condition implies that the integral $I_2$ is greater than $I_1$.

Fig. 18. If a survey defined by the apparent magnitude limit $m_{\text{lim}}$ and the solid angle $\Omega$ lie in the statistically significant region it is possible to obtain the self-averaging properties of the distribution also with the integral from the vertex. Otherwise one needs a redshift survey that contains the three dimensional information, and then one can perform average. Only in this way it is possible to smooth out the finite size effects.

numerically these integrals and in Fig.18 we show the result that satisfies the requested conditions. From the previous figure it follows that for $m > 19$ the statistically significant region is reached for almost any reasonable value of the survey solid angle. This implies that in the deep surveys, if we have enough statistics, we readily find the right behavior ($\alpha = D/5$) while it does not happens in a self-averaging way for the nearby samples. Hence the exponent $\alpha \approx 0.4$ found in the deep surveys ($m > 19$) is a genuine feature of galaxy distribution, and corresponds to real correlation properties. In the nearby surveys $m < 17$ we do not find the scaling region in the ML sample for almost any value of the solid angle. Correspondingly the value of the exponent is subject to the finite size effects, and to recover the real statistical properties of the distribution one has to perform an average.

From the previous discussion it appears now clear why a change of slope is found at $m \sim 19$: this is just a reflection of the lower cut-off of the fractal
structure and in the surveys with $m_{\text{lim}} > 19$ the self-averaging properties of the distribution cancel out the finite size effects. This result depend very weakly on the fractal dimension $D$ and on the parameters of the luminosity function $\delta$ and $M^*$ used. Our conclusion is therefore that the exponent $\alpha \approx 0.4$ for $m > 19$ is a genuine feature of the galaxy distribution and it is related to a fractal dimension $D \approx 2$, that is found for $m < 19$ in redshift surveys only performing averages.

We note that this result is based on the assumption that the Schechter luminosity function (eq.(15)) holds also at high redshift, or, at least to $m \sim 20$. This result is confirmed by the analysis of Vettolani et al. [15] who found that the luminosity function up to $z \sim 0.2$ is in excellent agreement with that found in local surveys [14].

Finally we report in Table 2 the exponents of the galaxy counts in different frequency bands, at faint magnitudes. We can see that the exponents is lower than 0.6 in all the case, and it is in the range $0.3\div0.5$, so that $D$ is in the range $1.5\div2.5$. These differences can be probably related to the multifractal behavior of the luminous matter distribution [13] or to a poor statistics. Only a three dimensional analysis allows one to decide between these two possibilities.


7 Definition of a statistically fair sample

In the previous sections we have defined the condition to select a sample large enough to manifest the self-averaging properties of a fractal distribution. We now study in detail a different but related question. Given a sample with a well defined volume, which is the minimum number of points that it should contain in order to have a statistically fair sample even if one computes averages over all the points.

Consider a sample which contains a portion of a fractal structure with a lower cut-off \( B \). Given the geometry of the sample (depth \( R \) and solid angle \( \Omega \)) we obtain the value \( \lambda \) of the minimal statistical length according to Eq.24 and Eq.31. Now we investigate what happens if we eliminate randomly more and more points from the sample. The fractal dimension and the lower cut-off (minimal statistical length) will not be affected by this depletion process, because they are related only to the intrinsic properties of the fractal structure, i.e to \( B \). Instead, the correlation properties are more and more affected by a statistical noise as we cut the points that contribute to the statistics. This noise is superimposed to the genuine signal so that \( D \) and \( \lambda \) are not changed at all, but the estimation of their values becomes noisy. Obviously, given a finite portion of the original system characterized by a lower cut-off \( B \), it will exist a maximum value of the number of points that we can eliminate from the structure in order to conserve the genuine statistical properties of the original distribution.

At this point we can characterize the statistical information in each VL sample more quantitatively. Suppose that the sample volume is a portion of a sphere with solid angle \( \Omega \) and radius \( R_{VL} \), and that the number of points inside this volume, \( N_{VL} \), is

\[
N_{VL} = B_{VL} \frac{\Omega}{4\pi} R_{VL}^D.
\]  

(35)

where \( B_{VL} \) takes into account the luminosity selection effect. The original system inside the same volume contains

\[
N = B \frac{\Omega}{4\pi} R_{VL}^D.
\]  

(36)

Hence the percentage of galaxies present in the sample can be written as

\[
p = \frac{N_{VL}}{N} = \frac{B_{VL}}{B}.
\]  

(37)

In this way we can associate to each VL sample a well defined value of \( p \) (see Table 3).
Table 3
The percentage of galaxies $\frac{B_{VL}}{B} \%$ in some volume limited sample of several surveys. $R_{VL}$ is the depth of the volume limited sample and $N_{VL}$ is the number of points contained. The all-sky IRAS catalogs have a very poor statistics (see text).

| Survey         | $\Omega (sr)$ | $R_{VL} (h^{-1} Mpc)$ | $N_{VL}$ | $\frac{B_{VL}}{B} \%$ |
|----------------|---------------|-----------------------|----------|------------------------|
| CfA1           | 1.8           | 40                    | 442      | 13 %                   |
| CfA1           | 1.8           | 80                    | 226      | 1.7 %                  |
| PP             | 1             | 60                    | 990      | 23 %                   |
| PP             | 1             | 100                   | 688      | 5.7 %                  |
| IRAS (2Jy)     | $4\pi$        | 40                    | 300      | 1.1 %                  |
| IRAS (2Jy)     | $4\pi$        | 80                    | 250      | 0.2 %                  |
| IRAS (1.2Jy)   | $4\pi$        | 60                    | 876      | 1.6 %                  |
| IRAS (1.2Jy)   | $4\pi$        | 80                    | 766      | 0.8 %                  |
| IRAS (1.2Jy)   | $4\pi$        | 100                   | 704      | 0.5 %                  |

The crucial point is that the random cut of point must stop at a certain limit: beyond this limit one does not have in the sample enough points to recover the real statistical properties of the distribution. We call statistical fair sample a sample that contains a number of points for unit volume larger than this limit. The problem is how to define this limit, or, in other words, to determine the minimal value of the percentage of Eq.37 that allows one to recover the genuine information, for example, by the two points correlation analysis. We can proceed in two independent way. The first is by analyzing the correlation properties of the VL samples of Perseus-Pisces and CfA1, the second is by
Fig. 19. (a) The average conditional density $\Gamma(r)$ for VL100 of Perseus-Pisces and the whole ML survey. The reference line has a slope of $-\gamma = -1.1$. The percentage of galaxies present in the sample is $\sim 6\%$. (b) The same of (a) but the percentage of galaxies present in the sample is $\sim 1.2\%$. This is the limiting case to recover the statistical properties of the sample, according to the condition of Eq. 37. (c) The same of (a), (b) but the percentage of galaxies present in the sample is $\sim 0.5\%$. In this case we are well below of the condition of Eq. 37. We can see that at small scale there is a residual power law behavior, while at large scale the power law behavior is broken.

studying artificial distributions.

In the VL limited samples of the Perseus-Pisces survey we can eliminate randomly points up to reach the fraction contained in the various IRAS samples. We note that the correlation function has a clear power law behaviour up to $30h^{-1}Mpc$ if the percentage remain larger than

$$p \geq 1 \div 2\%$$

(38)

then it shows a cut off towards homogenization (Fig. 19). This is clearly spurious and due to the fact that we have reached the limit of statistical validity or fairness of the sample. The apparent homogeneous behaviour is related to the fact that in a given sample, the mean separation among galaxies $\lambda_v$ grows when the number of points decreases ($\lambda_v \sim (V/N)^{1/3}$), and when it becomes of the same order of the largest void present in the sample volume, the correlation properties are destroyed. This means that the artificial noise introduced by the depletion of points, have erased the intrinsical fluctuations of the original system. In this situation see the system as an homogeneous one.

We have considered also an artificial catalog with a priori assigned properties generated with the random $\beta$—model algorithm [54]. We show the results in Fig. 20: also in this case we can recover the right statistical properties only in the limit of Eq. 38. Clearly Eq. 38 depends from the morphological features of the realization of the fractal structure and the percentage can weakly fluctuate from a realization to another.

It is possible to compute the $B_S$ in the VL samples of several surveys and
Fig. 20. (a) The average conditional density $\Gamma(r)$ for an artificial fractal sample with dimension $D = 2$. The reference line has a slope of $-\gamma = -1$. The percentage of galaxies present in the sample is $\sim 100\%$ (diamonds), $\sim 5\%$ (crosses) (this is the limiting case to recover the statistical properties of the sample, according to the condition of Eq. 37) $\sim 1\%$ (squares) In this case we are well below of the condition of Eq. 37. We can see that at small scale there is a residual power law behavior, while at large scale the power law behavior is broken.

Fig. 21. The average conditional density for some VL samples of the IRAS 1.2 Jy redshift survey. The reference line has a slope of $D \sim 2$ ($\gamma = 1$). The crossover towards homogenization is spurious and due to a poor sampling (see Table 3). We find that in CfA1, PP, ESP and LEDA Eq. 38 is well satisfied in almost all the VL samples (see Table II). On the contrary the VL samples extracted from the IRAS 2Jy survey \cite{9, 24} and the IRAS 1.2Jy \cite{25} survey are well beyond the limit of Eq. 38 (Table II): this is the reason why \cite{9} and \cite{25} find that in these catalogs $r_0$ does not scale with sample depth as it should be for the fractal case. In fact, in these cases one observes a constant density, as the power law correlation have been destroyed by a very poor sampling beyond a certain scale ($\sim 15h^{-1}Mpc$) (Fig. 21). On the other hand Strauss et al., \cite{9} stress that IRAS galaxies belong to the same structures as the optical ones, and in particular, they point out that the infrared galaxies do not fill the voids that define the same highly irregular patterns seen in the optical samples. However \cite{9} and \cite{25} conclude that the IRAS galaxies seem to be less correlated than the optical ones, as the value of the so-called correlation length $r_0$ is smaller than that of CfA1. Moreover they claim that the sample
is homogeneous and that their analysis disprove that galaxy distribution is fractal, at least for the infrared galaxies.

On the contrary our results imply that the correlation properties as the IRAS galaxies are the same of the optical ones, even if in the infrared catalogs it is not possible to recover the correct statistical features because of a very poor statistics (Table II).

8 Experimental implications

In Sect. 5 we have defined the *minimal statistical length* and in Sec.7 we have defined what is a *statistically fair sample*. We consider now the implications of these considerations for the optimization of real redshift surveys. In fact we are able to establish a quantitative criterion that defines the statistical validity of a survey, so that it is possible to optimize its geometry in order to obtain the maximum reliable information. Eq.32 defines the scaling region of a certain survey with solid angle $\Omega$ and Eq.37 Eq.38 give the conditions for its statistical validity.

Considering a ML sample, the survey becomes statistically meaningful, if its depth, defined by the apparent magnitude limit, and its solid angle satisfy the condition that the majority of galaxies lie in the scaling region. In Fig.18 we have identified the sample fairness condition for angular surveys. Only the deep surveys satisfy this condition, while the nearby samples are all affected by finite size effects. This means that one cannot detect statistically meaningful properties from such surveys. Only in the case in which one also measure the redshifts, as for CfA1 for example, one can barely recover the genuine properties of the (3d) distribution as the *minimal statistical length* $\lambda$ is of the order of the depth of the catalog. In the case of Perseus-Pisces and CfA2 one can find some VL samples, in which the effective depth $R_s$ is appreciably larger than $\lambda$. In these cases it is possible to study the integral from the vertex (i.e. $N(<r)$ or $n(r)$) and this allows us to extend the analysis up to the deeper depth observed ($R_s \sim 130Mpc$). But for these same catalogs the ML properties are strongly affected by finite size effects because most galaxies are close.

We report in Table 1 the *minimal statistical lengths* for several redshift surveys [3, 7, 8 - 15, 58 - 61], some of which are published, while others are in progress, and this provides a precise prediction that can be tested in these surveys. We have tested such a condition in Perseus-Pisces, CfA1, LEDA and ESP and the agreement is very good.

Related to the definition of a redshift catalog we consider the following problem: given the solid angle of the survey (and hence the *minimal statistical
length $\lambda$) the crucial point is that the sample should extend enough beyond $\lambda$, having a significant statistics in agreement with Eq. 38. In order to optimize a redshift survey we can define a quantitative criterion to select the optimum geometry so that the finite size effects are minimized. A redshift survey is characterized by two parameters: the solid angle $\Omega$ and the apparent magnitude limit $m_{\text{lim}}$.

In order to study the statistical properties of the galaxy space distribution we have to select from the whole ML survey a certain Volume Limited (VL) subsample. The number density of galaxies ($n_{\text{VL}}$) in such a sample is given by Eq. 26 and Eq. 27. In particular this depends on the absolute magnitude limit of such a sample which is defined by Eq. 11, and is given by (Sec. 2)

$$M_{\text{lim}} = m_{\text{lim}} - 5 \log_{10} R_{\text{VL}} - 25.$$  \hspace{1cm} (39)

Hence $n_{\text{VL}}$ directly depends on $m_{\text{lim}}$, $\Omega$ and $R_{\text{VL}}$, where $R_{\text{VL}}$ is the maximum depth of the VL sample. In particular, the first condition is that $n_{\text{VL}}$, which refers to the deepest VL sample obtainable from the ML survey, is such that

$$\frac{B_{\text{VL}}}{B} \gtrsim (2 \div 3)\%$$ \hspace{1cm} (40)

according to Eq. 38, so that the sample is statistically fair. The second condition is that

$$R_{\text{VL}} \gtrsim (2 \div 4)\lambda$$ \hspace{1cm} (41)

In this way the VL extends more than the minimal statistical length so that the statistical properties can be recovered. These conditions can be enough to determine the best values of ($\Omega, m_{\text{lim}}$) but, in order to limit the number of redshifts measured we can impose also that the total number of galaxies in the whole Magnitude Limited (ML) catalog is at least of the order of $N = (2 \div 3)10^3$ (this number is a reasonable choose considering the real experimental surveys). Hence from Eq. 9 and Eq. 10 we have (third condition)

$$N(<m_{\text{lim}}) = B_\Omega \frac{\Omega}{4\pi} \Phi(\infty) 10^B m_{\text{lim}} \gtrsim N$$ \hspace{1cm} (42)

It is possible to find numerically the best solution that satisfies the three previous conditions numerically. In Fig. 22 we show our results in the ($m_{\text{lim}}, \Omega$) space: for a given $m_{\text{lim}}$ the straight line represents the best solid angle $\Omega$ that maximizes the three conditions. The solid angle $\Omega$ decreases with the increasing magnitude as

$$\Omega \sim 10^{-0.4m_{\text{lim}}}$$ \hspace{1cm} (43)

In Fig. 23 we show the correspondent depth of the deepest VL sample; $R_{\text{VL}}$
Fig. 22. The optimum match between the solid angle $\Omega$ and the apparent magnitude limit $m_{\text{lim}}$, of a redshift survey that satisfies the three conditions explained in the text. For any of these surveys the number of galaxies in the ML catalog is constant ($N \sim (3 \div 5)10^3$) as well as it is constant the number of galaxies the deepest VL sample. The depth of such VL sample is the order $\sim (2 \div 4)\lambda$.

Fig. 23. The depth of the deepest VL sample as a function of the apparent magnitude (see Fig.17)

grows as

$$R_{VL} \sim 10^{0.2m_{\text{lim}}}$$

(44)

We can use these results in the following way: suppose that our aim is to study the space distribution of galaxies up to, say $\sim 800h^{-1}Mpc$. From Fig.23 we can find the $m_{\text{lim}}$ of the redshift survey corresponding to such a distance and from Fig.22 the relative solid angle $\Omega$. If we follow these conditions we will obtain the best statistical information from the minimum number of redshifts ($\sim 3000$). From Fig.22 we can note that the ESP survey ($m_{\text{lim}} = 19.4$ and $\Omega = 0.006$ [13]) is almost optimal in this respect, and it can be used to study the large scale space distribution from $300h^{-1}Mpc$ up to $\approx (800 \div 900)h^{-1}Mpc$ ([26] and [57]).
9 The angular two-point correlation function

Dogterom & Pietronero [62] (see also [2]) studied in detail the surprising and subtle properties of the angular projection of a fractal distribution. They find that the angular projection produces an artificial crossover towards homogenization with respect to the angular density. This crossover is artificial (just due to the projection) as it does not correspond to any physical features of the three dimensional distribution. Moreover they showed that there is an explicit dependence of the angular two point correlation function $\omega(\theta)$ on $\theta_M$ the sample angle: this effect has never been taken into account in the discussion of real angular catalogs. These arguments show that it is very dangerous to make any definite conclusion just from the knowledge of the angular distribution. However, there is a point of the discussion that remains still open. In fact, some authors ([28], [31]) claim that one of the most important facts that disproves the existence of fractal correlations at large scales, is the scaling of the amplitude of the two point angular correlation function (ACF) with sample depth, in the small angles approximation. We can now clarify this puzzling situation. The ACF is defined as [28]

$$\omega(\theta) = \frac{< n(\theta_0)n(\theta_0 + \theta)>}{< n >^2} - 1 \quad (45)$$

Assuming that the fractal correlation are only present at small scales, i.e. that $\xi(r) = (r_0/r)^\gamma$, it is possible to show that in the small angle approximation ($\theta << 1$) one has for the homogeneous case that [28]

$$\omega(\theta) \sim \theta^{1-\gamma}(r_0/D_*)^\gamma \quad (46)$$

where the depth $D_*$ is

$$D_* = \left( \frac{L_*}{4\pi f} \right)^{1/2} \quad (47)$$

$L_*$ is the cut-off of the Schechter luminosity function and $f$ is the limiting flux density of the survey. In the case of a fractal distribution with $D < 2$ it is easy to show that instead of Eq.46 we have [28]

$$\omega(\theta) \sim \theta^{1-\gamma} \quad (48)$$

so that the difference between the homogeneous and the fractal case is that in the first case the amplitude of the ACF depends on the sample depth $D_*$ while in the second case it is constant. Peebles [28] claims that, since in the real angular catalogs one observes the scaling of the amplitude [31] and [63], this provides an evidence against the fractal behavior. We show now that this conclusion is not correct because it does not take into account the finite size effects in real galaxy surveys, as it is the case of the GNC. In fact, the
The angular correlation function for Perseus-Pisces limited at \( m_{\text{lim}} = 14.5 \) (\( m_{145i} \)) and 15.5 (\( m_{155i} \)). The points refer to different samples of the APM catalog scaled to the ACF of the Lick survey (Maddox et al., 1990). The scaling with depth of the amplitude of the angular correlation function in this case is due to a finite size effects and it is not a proof of homogeneity in space, as we know from the space analysis that this sample has fractal behavior up to its deeper depth.

The amplitude of the ACF is strongly related to the behaviour of the angular density, i.e. to \( N(< m) \).

Eq. (48) is obtained under the assumption that the density for a fractal scales as \( r^{-\gamma} \): this is true for the average conditional density in the case of an ideal fractal distribution, if the correct scaling regime is reached. The conditional density computed from a single point is instead strongly affected by finite size effects up to the characteristic minimal statistical length \( \lambda \).

In order to illustrate this point we present the analysis of the ACF for the Perseus-Pisces redshift survey [5]. In Fig. 24 we show the behavior of the ACF \( \omega(\theta) \) for the whole ML survey. We can see that there is a clear scaling of the amplitude with the apparent magnitude limit of the survey (\( m_{\text{lim}} = 14.5, 15.5 \) respectively). We know from the space analysis that the galaxy distribution is fractal in this sample and therefore this trend is not a consequence of homogeneity, but only of the finite size effects that are especially large for the counting from the vertex. In summary the apparent homogeneity inferred from the angular catalogs has the same origin as the exponent \( \alpha \approx 0.6 \) of the galaxy counts at bright magnitudes (small scales). Both arise from finite size effects (for a more detailed study on the ACF we refer the reader to [64]) and do not correspond to the real statistical properties.

We note that the exponent of \( \omega(\theta) \) is also affected by the projection and somewhat higher than \( 1 - \gamma \approx 0 \), corresponding to \( D \approx 2 \) obtained in the 3d catalog. We find in fact that for \( \theta << 1 \)

\[
\omega(\theta) \sim \theta^{-0.7}
\]  

\text{(49)}
Fig. 25. Normalized differential sources counts at $\nu = 1.4\,GHz$. *Abscissa* log flux density (Jy). *Ordinate* log differential number of sources $n(S)$. The solid line represents the behaviour of a fractal structure with $D = 1.8$. The agreement is excellent, except in the bright fluxes region.

as it is confirmed in various angular surveys [31] and [63]. The approximation of the Limber equation for small angles, that gives the exponent of the ACF is just $1 - \gamma$, does not hold exactly, may be because we are close to the limiting case $D \approx 2$.

Finally we stress that Park et al., [55], analyzing the CfA2 redshift surveys, found the space two point correlation function has an index $\gamma \approx 1$ corresponding to $D \approx 2$. The CfA2 surveys is just the three dimensional counterpart of the Zwicky angular catalog for which Groth & Peebles [63] found that the ACF, at small angles, scales as $\omega(\theta) \sim \theta^{-0.7}$. This again shows the difficulties of recovering the genuine physical properties from the angular analysis alone.

10 Other astrophysical data

In Observational Astrophysics there are a lot of data that are only angular ones, as the measurements of distances is general a very complex task. We briefly discuss here the distribution of radiogalaxies, quasars and the $\gamma$-ray burst distribution. Our conclusion will be that all these data are compatible with a fractal structure with $D \approx 1.6 \div 1.8$ similar to that of galaxies.

The majority of catalogued radio sources are extragalactic and that some of the strongest are at cosmological distances. One of the most important information on radio galaxies distribution has been obtained from the sources counts as a function of the apparent flux [65]. Extensive surveys of sources have been made at various frequencies in the range $0.4 \div 5\,GHz$. In fig.25 we show a compendium of sources counts at $\nu = 1.4\,GHz$ [63]. The differential counts is plotted against the apparent flux. We can see that in the bright flux region there is a deviation from a power law function, while for four decades
the agreement with a fractal distribution with $D \approx 1.8$ is excellent. Such a behaviour has been usually explained in literature as an effect of sources evolution. Here we propose that the radio galaxies are fractally distributed, as galaxies, with almost the same fractal dimension. The deviation at bright fluxes in this picture is explained as a spurious effect due to the small scale fluctuations, as in the case of galaxy counts. In the other frequency bands the situation is nearly the same (5). The simple picture of a fractal distribution of radio sources is therefore fully compatible with the experimental situation.

In the case of Quasars the situation is almost the same. In fact we find that at bright magnitudes ($14.75 < B < 18.75$) the exponent of the counts is $\alpha \approx 0.88$, while at faint magnitude it is $\alpha \approx 0.3$ (6). Even in this case we can interpret such a behavior as due to the fractal distribution in space with $D \approx 1.5$.

Finally we comment the distribution of $\gamma$-ray burst (GRB). This is a long-standing problem in Astrophysics and after 20 years of intense studies and observations it is still mysterious: the origin of GRBs, in our galaxy or from the Cosmos, is a matter of debate (67) (68) (69). We argue here that from the present angular and intensity data it is possible to show that the space distribution of $\gamma$-ray bursts sources is fully compatible with a fractal structure with $D \approx 1.7$. This result clarifies the statistical analysis of the available data and points out a fundamental aspect of the $\gamma$-ray bursts sources distribution.

From the angular catalogs recent available (70) we have substantially it is possible to study three statistical quantities. The first one is the number versus apparent intensity distribution that shows a deviation from the Euclidean behaviour at low fluxes (71) (70). The second is the $V/V_{max}$ test (72) that again provides an evidence that the spatial distribution of sources is not homogenous (71) (70). Finally the angular distribution is isotropic within the statistical limits (70) (73) and there is not any evidence for an angular correlation or a clustering of bursts towards the galactic center or along the galactic plane of bursts (70) (74) (72) (76). These results together indicate that the bursts sources are distributed isotropic but not homogeneously (70). We argue here that these three evidences are fully compatible with a fractal distribution of sources with $D \approx 1.7$.

The first observational fact is the number of burst as a function of the apparent flux $N(> f)$. At bright apparent flux, that are associated to small distance of the sources, one sees an exponent $-3/2$, that seems to be in agreement with the homogeneous case. This is just a spurious effect that arises form the fact this quantity is computed without performing any average. At faint apparent fluxed, one is integrating the density in the correct scaling regime, and in this region the genuine statistical properties of the system can be detected. From the $N(> f)$ relation in the limit of low $f$ we can estimate a fractal dimension
$D \approx 1.7 \pm 0.1$.

An equivalent test on the homogeneity versus fractal properties is given by the $V/V_{\text{max}}$ distribution [72], where

$$\frac{V}{V_{\text{max}}} = \left( \frac{C}{C_{\text{lim}}} \right)^{-\frac{2}{3}} \quad (50)$$

In this ratio $V$ is the volume contained in a sphere extending to the location of the bursts and $V_{\text{max}}$ is the volume of the sphere extending to the maximum distance at which the same burst would be detectable by the instrument, whose limiting flux is $C_{\text{lim}}$. It is simple to show that if the spatial distribution of sources is described a fractal structure, then we have for the average

$$< \frac{V}{V_{\text{max}}} > = \frac{D}{D + 3} \quad (51)$$

Even in this case the data show [70] that $< \frac{V}{V_{\text{max}}} > = 0.33 \pm 0.01$ that in terms of fractal dimension means $D \approx 1.5 \pm 0.1$ This value is somewhat higher than the fractal dimension estimated with the $N(> f)$. This difference is probably due to the fact that this test is integrated while the $N(> f)$ is a differential one.

Let us came to the third evidence, i.e. a substantially isotropic angular distribution and a lack of any correlation at small angles. As we have seen in the previous section, the projection of a fractal distribution on the unit sphere conserves the correlations properties only in the small angles approximation, while at large angular scale the long range correlation are destroyed by projection effects. The angular correlation function $\omega(\theta)$ has a power law behaviour in the small angle approximation ($\theta < 10^\circ$). In the available sample [70] the number of points is $N = 1122$ distributed over the whole sky. This means that at angular distance smaller than $\sim 20^\circ$ one does not see any other object in average, and therefore it is not possible to study the angular correlation function at such low angular separation.

These results indicate together that the $\gamma$-ray bursts Sources are fractally distributed in space with $D \approx 1.7$. This is value is very similar to that the fractal dimension of the galaxy distribution is space that is $D \approx 2$ up to some hundreds megaparsec. This "coincidence" can be seen as an indication that the $\gamma$-ray bursts Sources are associate to a population of objects distributed as the visible galaxies. We stress that a larger sample of bursts will allow one a better determination of the fractal dimension and, if the number of objects for steradians will became larger, it will be possible to study also the angular correlations in the small angle approximation.
11 Discussion and conclusions

Since the Hubble’s pioneering work \cite{27} the galaxy number counts (GNC) has been considered as a classical cosmological test and one of the most powerful methods to study the galaxy space distribution as well as the effect of luminosity and space time evolution in different spectral regions \cite{28}-\cite{42}.

Moreover the apparent exponent $\alpha \approx 0.6$ at small scale appears to be in contradiction with the fractal distribution derived from the full correlation analysis in the redshift catalogs. However the discussion of the behaviour of the GNC neglects the effect of the non-analyticity of fractal distributions. A fractal distribution is characterized by having fluctuations at all scales and, in dealing with real fractals characterized by a lower cut-off, a crucial point is the effect of finite size fluctuations. The lower cut-off can be defined by the Voronoi volume for a Poisson distribution. In case of galaxy surveys the Voronoi distance represents the minimal average distance between galaxies, and the condition that the survey volume should contain a large number of Voronoi volumes, defines the minimal statistical length $\lambda$ beyond which the statistical properties are detectable.

A Poisson distribution is characterized by small amplitude fluctuations that represent small perturbations of a well defined average. A fractal distribution is characterized by fluctuations that can be as large as the system itself, so that the concept of average density looses its physical meaning. These fluctuations define all the statistical properties of the distribution. The qualitative difference between the fluctuations in these two cases is crucial to understand the importance of finite size effects. While for a homogeneous distribution one readily recovers the genuine statistical properties of the sample at distance, say, three times the Voronoi length, in the fractal case it is necessary to have a linear size of at least ten times that of the Voronoi volume. For averages $(\Gamma(r), \Gamma^*(r))$ one recovers the correct properties at appreciably smaller length.

The main problem of the standard analysis of the GNC is related to the fact that one computes $N(< m)$ only from one point, the origin. In this situation the fluctuations due to the finite size of the system dramatically affect the behaviour of the GNC to rather large scales.

With the aim of clarifying this point, we have studied in detail the behaviour of the GNC in a redshift survey, the Perseus-Pisces, where it is possible to control the behaviour of the real space density. We find that the space distribution is well described by a fractal with dimension $D \approx 2$ up to $130h^{-1}Mpc$ \cite{21}. From the GNC analysis, we find that the exponent $\alpha \approx 0.6$ is instead related to the presence of strong finite size fluctuations in the density $n(r)$. These strong fluctuations affect the behaviour not only of $n(r)$ but also of $N(< m)$ and
make the exponent of the GNC similar to the case of a homogeneous spatial distribution, i.e. $\alpha \approx 0.6$. This is because a very fluctuating behavior in the density can be roughly approximated by a constant density. In fact at small distances ($r << \lambda$) one finds almost no galaxies. Then the number of galaxies start to grow but this regime is strongly affected by fluctuations. Finally the correct scaling region is reached for $r \approx \lambda$, and the regime of raise and fall with strong fluctuations can be roughly approximated to a constant one. To show that this is indeed the case, we consider in detail the GNC is some VL samples of the Perseus-Pisces survey. In the deeper VL samples, where the spatial extension of the survey allows the self-averaging of the finite size fluctuations ($r > \lambda$) and it allows one to recover the smooth power law behavior for the density, the exponent of the GNC is simply $\alpha = D/5 \approx 0.4$. If instead one computes the density only from one, the exponent is $\alpha \approx 0.6$ because of the finite size fluctuations. This happens also in the whole magnitude limit survey. The crucial point is that the exponent is $\alpha \approx 0.6$ in this case is not associated to homogeneity in space, but it is due only to the presence of finite size fluctuations in the space distribution that do not self-average because the $N(< m)$ relation is computed only from one point, the origin. On the contrary if we perform an average from all the points of the survey for $N(< m)$ we can readily recover the same exponent found with the spatial analysis ($\alpha = D/5 \approx 2$). We have tested these results in some artificial catalogs with a priori assigned properties (fractal and homogeneous), finding clear confirmations for such a behavior.

Having this result in mind we can clarify now the problem of the GNC behaviour. In deeper optical samples it is found that $\alpha \approx 0.4$. Now we can interpret this result as the genuine one of a large enough sample where, as in the case of the deeper VL samples, the finite size effects are averaged out for the presence of structures and voids in such a manner that the scaling region is reached and the self-averaging properties of the fractal structure are present. This situation can be described in a more quantitative way. We have introduced in Section 4 the minimal statistical length $\lambda$ that defines the length beyond which this behavior occurs. In particular this length is directly related to the lower cut-off of the galaxy distribution and to the solid angle of the survey. In Section 5 we have considered in detail the change of slope in ML surveys in the $B$-band, that are the most significant from a statistical point of view.

The crucial point that arises from our analysis is that for $m > 19$ the ML survey extends enough in the statistically significant region, i.e. $r > \lambda$, for almost all the reasonable values of the solid angle of the surveys, so that the genuine behavior can be recovered also in this case, because the the finite size fluctuations due to the lower cut-off are averaged out. So we can relate the exponent found in the deep galaxy catalogs ($m > 19$) with what we find at small magnitude and redshift in the appropriate way, concluding that $\alpha \approx 0.4$. 

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is connected to the fractal distribution in space with dimension \( D \approx 2 \) \cite{58} that we observe at small redshifts and magnitudes (Fig.1).

Moreover all the different values of the exponent of the GNC \( \alpha \) found in different spectral regions are associated with the presence of these spatial fluctuations and are not intrinsic features of the galaxy luminosity distribution. For example, in \( R \)-band the results are quite in agreement with those in the \( B \)-band as the exponent is \( \alpha \approx 0.4 \) in both cases. In the \( K \)-band we find \( \alpha \approx 0.67 \) in the range \( K < 17 \) and this behavior is clearly due to finite size fluctuations only. For \( K > 17 \) the exponent \( \alpha \approx 0.3 \), and this can be probably related to the multifractal behavior of the luminous matter distribution \cite{49} or to a poor statistics. Only a three dimensional analysis allows one to decide between these two possibilities.

Therefore our main conclusion is that the analysis of the GNC shows a clear and strong evidence that the fractal distribution seen in the correlation analysis in local redshift surveys, with dimension \( D \approx 2 \), extends to the deepest depths so that we have a unique fractal structure in the magnitude range \( 12 \leq m \leq 28 \) with \( D \approx 2 \), that is to say the largest scales ever probed for luminous matter.

In addition we have shown that the counts of radio galaxies, Quasars and \( \gamma \) ray bursts are fully compatible with a fractal distribution in space with dimension \( D \approx 1.7 \). The behavior of the real data for these objects looks like the case of galaxy counts. For radio galaxies and Quasars we can conclude that the genuine scaling behaviour can be found only at faint fluxes, and it is related to the fractal properties of the space distribution rather than to the effects of evolution of space time. In the case of \( \gamma \) ray bursts our conclusion is that there is a strong evidence that the sources are associate to a population of objects distributed as the visible galaxies, as they have almost the same fractal dimension.

The minimal statistical length, that is related to the fractal lower cut-offs and the solid angle of a survey has important consequences also from an experimental point of view. In fact, only beyond such a length one can find the correct scaling properties. This gives a quantitative criterion to define the statistical validity of a survey and for the optimization of its geometry in order to derive the maximum reliable information. On this basis we can predict the expected statistical properties of several angular and redshift surveys (Table 1, Fig.18 and Fig.22). Moreover from the knowledge of the lower cut-off, we can establish a quantitative condition to define a statistically fair sample, i.e. a sample that is large enough to allow one to recover the genuine statistical properties of the distribution. Such a condition is related to the number of points present in the sample to its space extension, and it depends explicitly on the lower cut-off of the fractal structure.
Finally have we considered the consequences of the finite size effects on the
determination of the amplitude of the angular two point correlation function.
Our analysis implies that this quantity is strongly affected by the presence of
finite size effects, as it is determined from a single point, so that it suffers of
the same problems of the GNC. Indeed the amplitude of the GNC is directly
related to the angular density, i.e. to $N(< m)$. As a consequence of these
spurious effects one has the impression of an apparent homogenization shown
by the scaling with depth of the amplitude of the ACF, in the small angles
approximation. This scaling has been considered [28] as an evidence for an
homogeneous distribution of galaxies, while our results show that it arises
only from finite size fluctuations due to the presence of a lower cut-off in
the galaxy distribution. This analysis shows again that the analysis of the
angular properties for distribution characterized by long-range correlations is
very subtle and requires a careful treatment.

The result that the galaxy distribution can be described as fractal structure
with $D \approx 2$ from small scales, up to the deepest scale ever probed for visible
matter has dramatic consequences for the standard scenario [26]. At relatively
small scales the evolutionary effects are certainly negligible. However the ob-
servation of the same behavior in the entire range $12 \lesssim m \lesssim 28$ raises a
puzzling problem for the standard framework because one would expect at
large scales some modifications due to the space-time evolution effects and
the galaxy evolution mechanism.

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