Experimental signatures of cosmological neutrino condensation

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Superfluid condensation of neutrinos of cosmological origin at a low enough temperature can provide simple and elegant solution to the problems of neutrino oscillations and the accelerated expansion of the universe. It would give rise to a late time cosmological constant of small magnitude and also generate tiny Majorana masses for the neutrinos as observed from their flavor oscillations. We show that carefully prepared beta decay experiments in the laboratory would carry signatures of such a condensation, and thus, it would be possible to either establish or rule out neutrino condensation of cosmological scale in laboratory experiments.

The flavour oscillations of neutrinos and the accelerated expansion of the universe are two fundamental problems in particle physics and cosmology. The neutrino oscillations indicate that neutrinos are massive and their masses are slightly different for the different flavors. On the other hand the accelerated expansion of the universe requires an existence of some form of dark energy or a cosmological constant. Due to the proximity of the dark-energy scale with the neutrino masses, there have been a lot of efforts in finding an unified solution to both the problems.

Recently, there has been a lot of interests in neutrino condensation on the cosmological scales. It has been argued that the background neutrinos under certain conditions can become superfluid. The neutrino condensates can then contribute towards the accelerated expansion of the universe and they can also generate the Majorana mass term. In this work we investigate this scenario further with the aim of investigating experimental signatures of the background neutrino-condensates.

Our arguments are based on the phenomenon called the "Andreev reflection" which is extensively studied in the literatures of superconductivity. Before explaining the Andreev reflection, we first mention the general ideas for the neutrino condensation. Our analysis is based on a very simple and elegant model given by Caldi and Chodos, although the results are more general and can be applied to the other mechanisms also. This picture requires that the background neutrinos to be in a degenerate gas/liquid state. The idea of degenerate neutrinos in the universe is not inconsistent with the standard model of cosmology. It is well known that at a low enough temperature a gas of degenerate fermions undergo a phase transition to the superfluid state, if there is an attractive-interaction between the fermions in any angular-momentum channel. However such transition for the Dirac neutrinos within the standard electroweak interaction may not be possible (see also Ref. [1]). Interestingly, they also suggest that the degenerate Dirac-neutrinos in the presence of some new (unknown) attractive interaction, howsoever weak, can form condensates.

The condensation can be studied by using the Lagrangian density for the four-fermion interaction within the the mean-field theory frame work. From this one can find the BCS-type superfluid condensation with the gap parameter given by $\Delta \sim \mu e^{-\frac{\mu}{2\Delta}}$, where $\mu$ is the neutrino chemical potential and $G$ signifies the strength of the four-fermions coupling. Such condensates can naturally give rise to Majorana mass of the neutrinos. In fact for the values of $\mu$ allowed by the big-bang nucleosynthesis, it is always possible to choose the values of $G$ such that the cosmological constant, $\Lambda \sim G^2 |<\nu\nu>|^2$, is of the order $(10^{-3} eV)^4 = (meV)^4$ and the value of neutrino mass, $m_\nu \sim \Delta = G^2 |<\nu\nu>|$, can be anywhere between a $meV$ and few hundred $MeV$. Neutrino mass generated through such a mechanism will not have any significant contribution to the dark matter in the universe. However, if we allow the chemical potentials for different neutrino species to vary, the condensates could depend non-trivially on flavor, leading to an interesting spectrum of neutrino masses and mixing. This suggests a possible mechanism for neutrino oscillations without violation of the lepton number conservation at the microscopic level.

ANDREEV REFLECTION

It is not difficult to understand the Andreev reflection phenomenon qualitatively. Generally the quasi-particle states propagating in a superfluid with energy $\epsilon$ below the gap energy $\Delta$ are forbidden. But the sub-$\Delta$ transfer is possible if an incoming electron with $\epsilon$ is transferred along with another electron by forming a Cooper pair into the superconductor. In terms of single particle state this can be described by the reflection of a hole in the normal metal. Thus the Andreev reflection of a hole (or an electron) is equivalent to the transfer of a Cooper pair in (or out of) the superconductor. But the single electron transfer with the sub-gap energy is not allowed in the superconductor. Quasi-particles of energy $\epsilon > \Delta$, propagate in the condensate as a massive particles, of mass equal to $\Delta$. In the mean field description, the starting point is an...
effective Hamiltonian with four-fermion interaction given by,
\[
H = \int d^3x \left[ \sum_{\alpha=\uparrow,\downarrow} \psi_{\alpha}^\dagger(r,t) \left( -\frac{\nabla^2}{2m} - E_F \right) \psi_{\alpha}(r,t) + \Delta(r,t) \psi_{\uparrow}^\dagger(r,t) \psi_{\downarrow}(r,t) + \Delta^*(r,t) \psi_{\downarrow}^\dagger(r,t) \psi_{\uparrow}(r,t) \right]
\]
(1)
where, \(E_F\) is the Fermi energy (\(\sim\) chemical potential), and the "order parameter" field \(\Delta\) is
\[
\Delta_{\alpha\beta}(r,t) = \langle \psi_{\alpha}(r,t) \psi_{\beta}(r,t) \rangle = \Delta(r,t) e^{i\epsilon_{\alpha\beta}}
\]
(2)
The equation of motion obtained from the Hamiltonian above is known as Bogoliubov-de Gennes equation in the theory of superconductivity/superfluidity and is given by,
\[
u \partial_t \begin{pmatrix} \psi_{\uparrow}(r,t) \\ \psi_{\downarrow}(r,t) \end{pmatrix} = \begin{pmatrix} -\frac{\nabla^2}{2m} - E_F & \Delta(r,t) \\ \Delta^*(r,t) & \frac{\nabla^2}{2m} + E_F \end{pmatrix} \begin{pmatrix} \psi_{\uparrow}(r,t) \\ \psi_{\downarrow}(r,t) \end{pmatrix}
\]
(3)
These linear equations can be treated as equations for single particle wave functions instead of operators. For \(\Delta = 0\), the equations decouple and we have the plane wave solutions, with the dispersion relation,
\[
E = \epsilon_q (\text{for particles}) \quad \text{and} \quad E = -\epsilon_q (\text{for holes (antiparticles)}), \quad \epsilon_q = q^2/2m - E_F.
\]
(4)
The quasiparticle wave functions are,
\[
\begin{pmatrix} \phi_p \\ \phi_h \end{pmatrix} = \begin{pmatrix} A_+ & \epsilon \exp(i\delta) \\ A_- & \exp(-i\delta) \end{pmatrix} \exp(i\epsilon \cdot r - i\nu F t) + \begin{pmatrix} A_- & \epsilon \exp(i\delta) \\ A_+ & \exp(-i\delta) \end{pmatrix} \exp(i\epsilon \cdot r - i\nu F t)
\]
(5)
\(\epsilon = \sqrt{E^2 - |\Delta|^2}\), \(q^2/2m = E_F \pm \xi\), \(\delta\) is the phase of the complex order parameter, and \(D\) and \(F\) are some constants. The wave functions, \(\phi_p\) and \(\phi_h\) correspond to particle and hole like excitations of mass \(\Delta\). However, note that they are not in the eigenstate of particle number or electric charge. For \(E >> |\Delta|\), they reduce to wave functions for particles and holes respectively. When the quasi-particle excitation energy, \(E < |\Delta|\), the momenta are complex, and therefore, these modes do not propagate in the condensate medium. When such excitations reach the normal conductor-condensate junction from the conductor side of the arrangement, we have, what is known as Andreev reflection in the theory of superconductivity.

Let us consider a normal conductor-superconductor slab aligned along the \(z - \text{axis}\). Assume \(r = (0,0,z)\). The junction of the materials is located at \(z = 0\). At the conductor side \((z < 0)\), \(\Delta = 0\) while at the superconductor side \((z > 0)\), \(\Delta = \text{constant}\). Let us now suppose that particles of energy, \(E\), from conductor side of the slab, hits the junction. To see the details of what happens at the boundary, we need to solve the stationary Bogoliubov - de Gennes equation, Eq.(3), with the boundary conditions : (1) for \(z \to -\infty\) excitations are particles or holes (conductors), (2) for \(z \to \infty\) excitations are quasiparticles (superconductors), (3) for \(z = 0\) wave functions and their first spatial derivatives are continuous at the boundary at \(z = 0\). It is not hard to solve this problem and detailed solutions can be found in reference \[7\]. The net result is that the probability current through the junction is,
\[
j_z = \begin{cases} \frac{2\nu F}{E_F^2 + \xi^2} + O(1/F^2) & \text{for } E < \Delta \\ \frac{2\nu F}{E_F^2 + \xi^2} & \text{for } E > \Delta \end{cases}
\]
(6)
where \(\nu F = \sqrt{2E_F/m}\) is the Fermi velocity. The reflection and transmission coefficients are,
\[
\begin{align*}
R_{\text{hole}} &= 1, \quad T_{\text{quasi}} = 0 & \text{for } E < \Delta \\
R_{\text{hole}} &= \frac{2\nu F}{E_F^2 + \xi^2}, \quad T_{\text{quasi}} = \frac{2\nu F}{E_F^2 + \xi^2} & \text{for } E > \Delta
\end{align*}
\]
(7)
These equations clearly show that an incoming particle hitting the junction from the conductor side with energy below the gap can not cross the junction, instead a a hole is reflected back in the opposite direction. This is not difficult to understand physically- near the boundary, the incoming particle creates a particle and a hole of equal and opposite momenta from the Fermi sea, pairs up with the particle and disappears in the condensate and the hole is reflected back. This is the Andreev reflection phenomenon. For \(E >> \Delta\), \(R_{\text{hole}} \to 0, T_{\text{quasi}} \to 1\). For energy, \(E\), not too high above the gap, \(\Delta\), there is partial transmission of quasiparticles and partial reflection of holes.

**EXPERIMENTAL SIGNATURES**

To understand the role of Andreev reflection in the context of neutrinos, we have to find the fate of the condensate within materials. Cosmological neutrinos are of low energy and large wavelength, and therefore, the size of the cooper pairs in the condensate would be large. Weak interaction of neutrinos with matter is mediated through charge currents as well as neutral currents. However, the cross section of such interactions is extremely small. For \(E_\nu = 1\ MeV\), this cross section is \(\sim 10^{-44} \text{cm}^2\), and for \(E_\nu = 1\ eV\), it is \(\sim 10^{-56} \text{cm}^2\). Thus it would seem that matter is almost transparent for neutrinos. However, it turns out that for low energy neutrinos there is coherent neutrino processes in matter\[12\]. For momentum \(\sim\) a few hundred \(MeV/c\), neutrinos would elastically scatter
from nuclei with cross section proportional to $A^2$. For lower momenta it will coherently scatter off the entire atom, meaning all the nucleons and electrons in the atom. Subsequent lowering of momenta involves, coherent scattering of neutrinos from all the atoms contained in the wavelength. Thus the scattering cross section would grow as the square of number elementary particles (quarks & leptons) contained within the wavelength. Thus, it would seem that the cooper pairs could be broken and condensate could be disordered by coherent scattering processes. We know from work of Caldi and Chodos \textsuperscript{2} that the value of the order parameter is decided by the chemical potential and the coupling constant of the assumed "new interaction". From the big bang nucleosynthesis arguments, we get only the upper bound on the combined value of the chemical potentials of all the three flavors of neutrinos. We may get an estimate of the range of value of the chemical potentials of all the three flavors.

The following two possiblities:

1. The cooper pairs of the cosmological condensate are formed by antineutrinos of low energy and large wavelength. The pair binding is not too strong even by the standard of weak interactions, and therefore, within the radioactive material (tritium) the condensate is disordered by the process of coherent scattering. The condensate outside the material remains unchanged. We have antineutrinos both from the disordered cosmological condensate and the radioactive beta decay within the tritium sample. We encounter here a situation similar to one in condensed matter system with metal-condensate boundary. We, therefore, refer to Andreev reflection phenomenon to understand the physical processes in the material, condensate and their boundary. Antineutrinos ( and neutrinos if any) of energy lower than the value of the condensate order parameter $|\Delta|$, are trapped inside the material. If any of these sub-barrier antineutrinos, of momentum $\vec{p}$, reaches the condensate boundary, a neutrino-antineutrino pair of equal and opposite momentum, $-\vec{p}$ and $\vec{p}$, is created at the junction. The newly created antineutrino combines with the old incoming antineutrino, forms a cooper pair and disappears in the condensate. The neutrino (hole) is reflected back. Similar phenomenon takes place when the reflected neutrino reaches the boundary. However, those with energy greater than $|\Delta|$, move into the condensate and propagate as quasiparticles of mass $|\Delta|$, their dispersion relation being, $E = \sqrt{p^2 + |\Delta|^2}$. Thus the energy of the antineutrinos from the beta decay in the sample covers the entire spectrum of the released energy and thus the Kurie plot in this case is a straight line (as in the case of massless neutrinos) with end point at the maximum energy, $E_0 = 18.6\, KeV$ (with a few extra events near the end point which will be explained latter). Such a mechanism of mass generation
does not lead to violation of lepton number at the microscopic level, and therefore, also implies that there is no neutrinoless double beta decay [13].

Andreev reflection process creates a mixture of sub-barrier energy neutrinos (holes ) and antineutrinos within the sample. The process $^3H_1 + \nu \rightarrow ^3H_e2 + e^- \nu$ is energetically feasible and has the same cross-section as

$^3H_1 \rightarrow ^3H_e2 + e^- + \bar{\nu}$. Reaction cross section for such a processes involving low energy neutrinos is very small. However, in an experiment run over very long time some low energy antineutrino excess event, which would show up as excess electrons near the end point of Kurie plot, should be seen. The estimated number of such electrons is approximately 50% more than what is expected from the standard model. From the end point of the spectrum where the excess events start appearing, one can estimate the value of the order parameter. It should be noted that the background cosmological neutrinos, in the absence of condensation would also create excess events near the end of of the beta decay spectrum, however unlike in our case, the energy spectrum will have a monoenergetic peak at $E = E_0 + m_\nu$, where $E_0$ is end point of spectrum and $m_\nu$ is the neutrino mass [10]. Experiments such as KATRIN is designed to search for beta decay to low energy antineutrinos and it is in these experiments that one would expect such events to be established or ruled out.

(2). The cosmological condensate penetrates the tritium sample and there is no change either in the superfluid ordering or the value of the order parameter. As explained earlier, single particle states of energy less than the value of the order parameter can not be sustained in the condensate. Because of this constraint on the available phase space, radioactive tritium can not decay into sub-barrier energy antineutrinos. However, it can decay into antineutrinos of energy greater than the value of the order parameter, $|\Delta|$. These supra-barrier neutrinos produced in the beta decay in the nucleus have wavelength shorter than the neutrinos in the cooper pair and they are in eigenstate state of particle number operator within the cooper pair length scale. Therefore, in the calculation of beta decay amplitude they are considered massless, with dispersion relation, $E_\nu = cp_\nu$ (elsewhere, $c = 1$). In the derivation of Kurie function in this case, the important point that we have to keep in mind is that $E_\nu > \Delta$, because there is no single particle state below this energy. Thus the Kurie plot is still a straight-line but the end point has a cut off at energy equal to $E_0 - |\Delta|$. However, within the condensate these antineutrinos will propagate as quasi-particles (in Bogolubov state) of mass $|\Delta|$, with dispersion relation given by, $E = \sqrt{p^2 + |\Delta|^2}$. As in the previous case, the the antineutrinos (and neutrinos) propagate as massive quasi-particles having Majorana mass but there is no violation of the lepton number conservation at the microscopic level, and therefore, there is no possibility of neutrinoless double beta decay. The massive quasi-particles are not in the eigenstate states of particle number or the electronic lepton number operators. This is also the necessary requirement for neutrino oscillations. Thus, slightly deferent values of the condensate order parameters for three flavors of neutrinos would nicely account for neutrino oscillations.

In both the cases discussed above, we find that the value of the order parameter (and thus the mass of the neutrinos) can be estimated from the end point behaviour of the beta decay spectrum. The rate of neutrinoless double beta decay is known to be proportional to the square of neutrino mass. Thus, we can estimate this decay rate and expect that in a suitably designed experiment, it can either be established or ruled out.

CONCLUSIONS

We have discussed in some details how the superfluid condensation of background neutrinos of cosmological origin can generate neutrino mass for flavor oscillations and also account for the accelerated expansion of the universe. Such a mechanism does not seem to be in conflict with any aspect of the standard model of particle physics. However, it indicates the existence of new physics in the form of new attractive interaction among the neutrinos. We have shown that, in carefully prepared beta decay experiments, this model would lead to experimental signatures as listed below:

(i). Absence of neutrinoless double beta decay is a necessary requirement. If neutrinoless double beta decay is observed, the mechanism of mass generation and oscillation discussed in this paper should be considered redundant.

(ii). The Kurie plot is a straight line with either of the two possibilities, (a) the end point of the plot extends up to the total beta decay energy released, $E_0 = 18.6KeV$, with a few extra low energy events in an experiment run over sufficiently long time (such events are known to exist, however, they are still within the experimental error bars and can be settled only by future experiments [14]), (b) the straight line terminates at $E_0 - |\Delta|$, where $|\Delta|$ is the superfluid order parameter.

Thus, it should be possible to either establish or rule out cosmological neutrino condensation from carefully prepared beta decay experiments in laboratory.

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