Research Article

A Novel Mathematical Model for Radio Mean Square Labeling Problem

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A radio mean square labeling of a connected graph is motivated by the channel assignment problem for radio transmitters to avoid interference of signals sent by transmitters. It is an injective map \( h \) from the set of vertices of the graph \( G \) to the set of positive integers \( \mathbb{N} \), such that for any two distinct vertices \( x, y \), the inequality \( d(x, y) + \left( h(x)\right)^2 + \left( h(y)\right)^2/2 \geq \text{dim}(G) + 1 \) holds. For a particular radio mean square labeling \( h \), the maximum number of \( h(v) \) taken over all vertices of \( G \) is called its spam, denoted by \( \text{rmsn}(h) \), and the minimum value of \( \text{rmsn}(h) \) taking over all radio mean square labeling \( h \) of \( G \) is called the radio mean square number of \( G \), denoted by \( \text{rmsn}(G) \). In this study, we investigate the radio mean square numbers \( \text{rmsn}(P_n) \) and \( \text{rmsn}(C_n) \) for path and cycle, respectively. Then, we present an approximate algorithm to determine \( \text{rmsn}(G) \) for graph \( G \). Finally, a new mathematical model to find the upper bound of \( \text{rmsn}(G) \) for graph \( G \) is introduced. A comparison between the proposed approximate algorithm and the proposed mathematical model is given. We also show that the computational results and their analysis prove that the proposed approximate algorithm overcomes the integer linear programming model (ILPM) according to the radio mean square number. On the other hand, the proposed ILPM outperforms the proposed approximate algorithm according to the running time.

1. Introduction

In wireless networks, each radio station assigns a number called frequency. When different transmitters of district stations send signals, the receiver might get unnecessarily interference of the signals sent by transmitters in particular with close frequencies. This is the channel assignment problem introduced by Hale [1] in 1980 to minimize such interference. In 2001, Chartrand et al. [2] proposed converting this problem to graph theoretical problem using vertex labeling. Many researchers involved with this problem [3–16] and produced different methods to minimize the interference of signals [7]. Recently, Ramesh et al. [8] proposed a new method called radio mean square labeling, which is defined as follows. A radio mean square labeling of a connected graph \( G \) is an injective function \( h \) from its vertex set \( V \) to the set of natural numbers \( \mathbb{N} \), such that for any two distinct vertices \( x \) and \( y \), \( (h(x))^2 + (h(y))^2/2 \geq \text{diam}(G) + 1 - d(x, y) \) holds, where \( d(x, y) \) denotes the distance between the two vertices, and \( \text{diam}(G) \) represents the diameter of the graph [8]. For a radio mean square labeling \( h \), the maximum number of \( h(v) \) taken over all vertices of \( G \) is called its spam, denoted by \( \text{rmsn}(h) \), and the minimum value of \( \text{rmsn}(h) \) taking over all radio mean square labeling \( h \) of \( G \) is called the radio mean square number of \( G \), denoted as \( \text{rmsn}(G) \). The radio mean square number of \( h \), denoted by \( \text{rmsn}(h) \) is the maximum number assigned to any vertex of \( G \). Ramesh et al. [8] determined the radio mean square number for some graphs such as in the central subdivision of spoke wheel graph and biwheel graph.
Due to most of nontrivial coloring models, graph coloring is an NP-hard problem. Therefore, we take into consideration a graph coloring algorithm [9–11]. The judgment of the performance of the used algorithm does include its effectiveness and accuracy for a large number of vertices and the level of complexity regarding suboptimal solutions [12, 13]. Here, we introduce an approximate algorithm that leads to an upper bound of the radio mean square for a large number of vertices. Finally, we turn our attention to reformulate the radio square labeling as a linear programming model and then minimize the suggested linear function. We use of transforming the non-linear constraint to become a linear constraint by using of some large integers dependably on the coefficient range of the given problem under path calculating conditions. Some illustrated examples and comparison between the techniques will be given.

It should be noted that all the considered graphs in this study are finite, simple, connected, and undirected.

The organization of the study goes as follows: in Section 2, the radio mean square number of cycles and paths are given. Section 3 is devoted to present an approximate algorithm that finds the upper bound of the radio mean square number of a given graph and an illustrative example is included. Section 4 deals with a new mathematical model for finding the upper bound of the radio mean square number of the given graph. Section 5 provides the experimental results. Analysis and statistical tests between the mathematical model and the proposed approximate algorithm are provided. The last section is considered for conclusion.

2. Results

The following theorem is about the rmsn for the path $P_n$, and next, we will get the rmsn for cycle $C_n$.

**Theorem 1.** For the path $P_n$, $n \geq 1$, and the radio mean square number is

$$\text{rmsn}(P_n) = n + k;$$

$$\begin{cases}
1 \leq n \leq 15 \text{ and } k = 0, \\
16 \leq n \leq 22 \text{ and } k = 1, \\
23 \leq n \leq 32 \text{ and } k = 2, \\
33 \leq n \leq 44 \text{ and } k = 3, \\
45 \leq n \leq 58 \text{ and } k = 4, \\
59 \leq n \leq 73 \text{ and } k = 5, \\
74 \leq n \leq 92 \text{ and } k = 6, \\
k^2 + 5k + 9 \leq n \leq k^2 + 7k + 14, k \geq 7.
\end{cases}$$

**Proof.** Clearly, $\text{diam}(P_n) = n - 1$. Then, one can define $h: V(P_n) \longrightarrow \mathbb{N}$ as follows:

$$1 \leq n \leq 15, \quad k = 0,$$

$$16 \leq n \leq 22, \quad k = 1,$$

$$23 \leq n \leq 32, \quad k = 2,$$

$$33 \leq n \leq 44, \quad k = 3,$$

$$45 \leq n \leq 58, \quad k = 4,$$

$$59 \leq n \leq 73, \quad k = 5,$$

$$74 \leq n \leq 92, \quad k = 6.$$ (2)

One may label the vertices of $P_n$ as follows:

Subcase a.1: $n$ is even:

$$h(x_1) = 1, h(x_n) = k + 2,$$

$$h(x_{2i}) = n + k - 2i, 0 \leq i < \frac{n}{2} - 1,$$ (3)

$$h(x_{n-1}) = n + k - 2j, 0 \leq j < \frac{n}{2} - 1.$$ (4)

Subcase a.2: $n$ is odd:

$$h(x_1) = 1, h(x_n) = k + 2,$$

$$h(x_{2i}) = n + k - 2i, 0 \leq i < \frac{n - 1}{2},$$ (5)

$$h(x_{n-1}) = n + k - 2j, 0 \leq j < \frac{n - 1}{2} - 1.$$ (6)

Therefore, for any pair $(x_i, x_j), i \neq j, 0 \leq i, j \leq n$, we have

$$d(x_i, x_j) + \frac{[h(x_i)^2 + h(x_j)^2]/2} {\geq 1 + n - 1 = 1 + \dim (P_n),}$$
Hence, \( h \) is a valid radio mean square labeling for \( P_n \), and therefore, \( \text{rmsn}(P_n) \leq \text{rmsn}(h) = n + k \). Since \( h \) is injective, \( \text{rmsn}(P_n) = n + k, n \geq 1 \) for all radio mean square labeling \( h \), and hence, \( \text{rmsn}(P_n) = n + k, n \geq 1 \). Therefore, the labeling \( h \) defined above satisfies the radio mean square condition.

**Example 1.** The radio mean square numbers of \( P_3, P_{10}, P_{96}, \) and \( P_{115} \) are shown in Figure 1. It is clear that \( k = 0 \) for \( P_3 \) and \( k = 7 \) for \( P_{96} \) and \( P_{115} \).

**Theorem 2.** The radio mean square numbers of the cycles \( C_n, n \geq 3 \), are given by

\[
\text{rmsn}(C_n) = \begin{cases} n, & \text{if } 3 \leq n \leq 7, \\ n + k, & \text{if } n \geq 8, \end{cases}
\]

\( \therefore \text{rmsn}(C_n) \leq \text{rmsn}(h) = \begin{cases} n, & \text{if } 3 \leq n \leq 7, \\ n + k, & \text{if } n \geq 8, \end{cases} \)

Since \( h \) is injective,

\[
\text{rmsn}(C_n) \geq \begin{cases} n, & \text{if } 3 \leq n \leq 7, \\ n + k, & \text{if } n \geq 8, \end{cases}
\]

\( \therefore \text{rmsn}(C_n) = \begin{cases} n, & \text{if } 3 \leq n \leq 7, \\ n + k, & \text{if } n \geq 8, \end{cases} \)

For all radio mean square labeling \( h \),

\[
\text{Case 4. bFor } n \geq 8, 2k^2 + 2k + 4 \leq n \leq 2k^2 + 6k + 7, k \geq 0. \text{ We may label the vertices of } C_n \text{ by one of the following subcases:}
\]

Subcase b.1: \( n \) is even:

\[
h(x_{n/2}) = k + 1, \quad h(x_{n+1}) = k + 3 + 2i, 0 \leq i < \frac{n}{2} - 1, \quad (9)
\]

\[
h(x_{n-j}) = k + 2 + 2j, 0 \leq j < \frac{n}{2}.
\]

Therefore, for any pair \( (x_i, x_j), i \neq j, 1 \leq i, j \leq n \), we have \( d(x_i, x_j) + \frac{h(x_i)^2 + h(x_j)^2}{2} \geq 1 + \lceil n/2 \rceil \geq 1 + \dim(C_n). \)

Subcase b.2: \( n \) is odd:

\[
h(x_{n/2}) = k + 1, \quad h(x_{n+1}) = k + 2 + 2i, 0 \leq i < \frac{n+1}{2} - 1, \quad (10)
\]

\[
h(x_{n-j}) = k + 3 + 2j, 0 \leq j < \frac{n+1}{2} - 1.
\]

So, for any pair \( (x_i, x_j), i \neq j, 1 \leq i, j \leq n \), the following inequality holds \( d(x_i, x_j) + \frac{h(x_i)^2 + h(x_j)^2}{2} \geq 1 + \lceil n/2 \rceil \geq 1 + \dim(C_n). \)

Hence, \( h \) is a valid radio mean square labeling for \( C_n \), and therefore,
Therefore, the labeling \( h \) defined above satisfies the radio mean square condition.

**Example 2.** The radio mean square numbers of cycles \( C_4, C_9, C_{16}, \) and \( C_{17} \) are shown in Figure 2. It is clear that \( k = 1 \) for \( C_{16} \) and \( C_{17} \), but \( k = 7 \) for \( C_{9} \) and \( C_{112} \).

### 3. A Novel Graph Radio Mean Square Algorithm

Here, we introduce an approximated algorithm. This algorithm finds an upper bound of the radio mean square for arbitrary graph \( G \). The main idea is to labeling some vertices (initial vertices) by \( \lfloor \sqrt{\text{diam}} \rfloor \). On the other hand, the algorithm chooses a different vertex as an initial vertex in each iteration.

The time complexity of an algorithm is defined as the number of instructions of this algorithm multiplied by the running time of each instruction. The time complexity is considered as a good metric to evaluate the given algorithm. Thus, Algorithm 1 has nine steps, and both Step 1 and Step 2 have the same instruction. Step 3 has a nested loop that has \( O(n^2) \) time complexity. On the other hand, Step 4, Step 5, and Step 6 have \( O(n) \) time complexity. Step 7 has one instruction, while Step 8 and Step 9 have \( O(n^3) \) and \( O(n^4) \), respectively. Therefore, Algorithm 1 has the time complexity \( O(n^5) \).

In the coming example, we show and explain how to compute the radio mean square labeling problem for \( P_5 \).

**Example 3.** Suppose that \( x_i \) is the label of the vertex \( v_i \), \( 1 \leq i \leq 5 \). Therefore, \( 1 \) explores an upper bound of the radio mean square labeling problem as follows:

It is known that \( \text{diam}(P_5) = 4 \). We select a vertex \( x_1 \) and \( \text{col}(x_1) = 2 \). Let \( S = \{ x_1 \} \), and for all \( v \in V(G) - S \), compute

\[
\begin{align*}
temp(x_2) &= \max_{x_i} \left\{ 2 + \text{ceil}\left( \frac{\text{max}\left(\sqrt{4} + 1 - 4, 1\right)}{4} \right) \right\} = 3, \\
temp(x_3) &= \max_{x_i} \left\{ 2 + \text{ceil}\left( \frac{\text{max}\left(\sqrt{4} + 1 - 2, 1\right)}{4} \right) \right\} = 3, \\
temp(x_4) &= \max_{x_i} \left\{ 2 + \text{ceil}\left( \frac{\text{max}\left(\sqrt{4} + 1 - 3, 1\right)}{4} \right) \right\} = 3, \\
temp(x_5) &= \max_{x_i} \left\{ 2 + \text{ceil}\left( \frac{\text{max}\left(\sqrt{4} + 1 - 4, 1\right)}{4} \right) \right\} = 3.
\end{align*}
\]

(14)

Let \( \min = \min_{v \in V(G) - S}\{ \text{temp}(v) \} = 3 \); we choose a vertex \( x_2 \in V(G) - S \), such that \( \text{temp}(x_2) = 3 \). Give \( \text{col}(x_2) = 3 \) and \( S = \{ x_1, x_2 \} \).

\[
\begin{align*}
\text{temp}(x_4) &= \max_{x_i} \left\{ 2 + \text{ceil}\left( \frac{\text{max}\left(\sqrt{4} + 1 - 3, 1\right)}{4} \right) \right\} = 5, \\
\text{temp}(x_5) &= \max_{x_i} \left\{ 2 + \text{ceil}\left( \frac{\text{max}\left(\sqrt{4} + 1 - 2, 1\right)}{4} \right) \right\} = 5.
\end{align*}
\]

(15)

Let \( \min = \min_{v \in V(G) - S}\{ \text{temp}(v) \} = 4 \); we choose a vertex \( x_3 \in V(G) - S \), where \( \text{temp}(x_3) = 4 \). Give \( \text{col}(x_3) = 4 \) and \( S = \{ x_1, x_2, x_3 \} \).

\[
\begin{align*}
\text{temp}(x_4) &= \max_{x_i} \left\{ 2 + \text{ceil}\left( \frac{\text{max}\left(\sqrt{4} + 1 - 3, 1\right)}{4} \right) \right\} = 5, \\
\text{temp}(x_5) &= \max_{x_i} \left\{ 2 + \text{ceil}\left( \frac{\text{max}\left(\sqrt{4} + 1 - 2, 1\right)}{4} \right) \right\} = 5.
\end{align*}
\]

(16)
Let \( \min = \min_{v \in V(G) - S} \{ \text{temp}(v) \} = 5 \); we choose a vertex \( x_4 \in V(G) - S \), where \( \text{temp}(x_4) = 5 \). Give \( \text{col}(x_4) = 5 \) and \( S = \{x_1, x_2, x_3, x_4\} \).

\[
\begin{align*}
\text{temp}(x_5) &= \max_{x_i} \left\{ 2 + \text{cell} \left( \frac{\text{max} \{ \sqrt{4 + 1 - 4}, 1 \} \} }{4} \right) \right. \\
& \quad \left. + 3 + \text{cell} \left( \frac{\text{max} \{ \sqrt{4 + 1 - 3}, 1 \} \} }{4} \right) \right. \\
& \quad \left. + 4 + \text{cell} \left( \frac{\text{max} \{ \sqrt{4 + 1 - 2}, 1 \} \} }{4} \right) \right. \\
& \quad \left. + 5 + \text{cell} \left( \frac{\text{max} \{ \sqrt{4 + 1 - 1}, 1 \} \} }{4} \right) \right. \\
&= 6
\end{align*}
\]

Let \( \min = \min_{v \in V(G) - S} \{ \text{temp}(v) \} = 6 \); we select a vertex \( x_5 \in V(G) - S \), where \( \text{temp}(x_5) = 6 \). Give \( \text{col}(x_5) = 6 \) and \( S = \{x_1, x_2, x_3, x_4, x_5\} \). Plainly, all vertices are labelled and \( \text{rmsn}(P_5) = 6 \).

### 4. Formulation of the Radio Mean Square Labeling as a Mathematical Model

In this section, we present the integer linear programming model (ILPM) for the radio mean square labeling problem.

Let \( G \) be a connected graph of order \( n \) with \( V(G) = \{v_1, v_2, \ldots, v_n\} \) and let \( D = [d_{ij}] \) be the distance matrix of \( G \), that is, \( d_{ij} = d(v_i; v_j) \) for \( 1 \leq i, j \leq n \). Suppose that \( x_i \) is the label of the vertex \( v_i \), \( 1 \leq i \leq n \). Now, we can propose the mathematical model for the radio mean square labeling problem as the ILPM. Let us suppose the function \( F \) is

\[
F = x_1 + x_2 + \cdots + x_n.
\]

#### 4.1. Formulation 1. Minimize \( F \) subject to the \( \left( \frac{n}{2} \right) \) constraints

\[
\left( x_i \right)^2 + \left( x_j \right)^2 / 2 \geq \text{diam} + 1 - d(v_i; v_j) \quad \text{for } 1 \leq i \leq n - 1, 2 \leq j \leq n, \text{ and } i < j,
\]

and as mentioned before, the following steps will transform nonlinear constraints to become linear which is easy to deal with.

#### 4.2. Formulation 2. Since

\[
\left( x_i \right)^2 + \left( x_j \right)^2 / 2 \leq \left( x_i \right)^2 + \left( x_j \right)^2 / 2,
\]

we have the following inequalities:

\[
\begin{align*}
\left( x_i \right)^2 + \left( x_j \right)^2 / 2 &\geq \text{diam} + 1 - d(v_i; v_j), \\
\left( x_i \right)^2 + \left( x_j \right)^2 / 2 &\geq \sqrt{\text{diam} + 1 - d(v_i; v_j)}, \\
\left( x_i \right)^2 + \left( x_j \right)^2 / 2 &\geq \sqrt{\text{diam} + 1 - d(v_i; v_j)}, \\
\left( x_i \right)^2 + \left( x_j \right)^2 / 2 &\geq \sqrt{\text{diam} + 1 - d(v_i; v_j)},
\end{align*}
\]

\[
M\left( x_i - x_j \right) \geq \frac{\left( x_i \right)^2 + \left( x_j \right)^2 / 2}{2} + 1 \geq \text{diam} + 1 - d(v_i; v_j).
\]

\( M \) is a large integer number which depends on the coefficients range of the problem.

The absolute value notation is used to get distinct values for \( x_i \), where \( i = 1, 2, \ldots, n \).

Now, we can reformulate the radio mean square problem as follows:

\[
\min F = x_1 + x_2 + \cdots + x_n.
\]

Subject to

\[
\begin{align*}
M\left( x_i - x_j \right) &\geq \sqrt{\text{diam} + 1 - d(v_i; v_j)}, \\
1 \leq i \leq n - 1, \\
2 \leq j \leq n \text{ and } i < j.
\end{align*}
\]

Here, the floor function is used because the values of \( x_i \), \( 1 \leq i \leq n \), are integers.

#### Example 4. The details of the ILPM to compute the radio mean square labeling for \( P_3 \). Assume that \( x_i \) is the label of the vertex \( v_i \), such that \( 1 \leq i \leq 3 \). Thus, the mathematical model for the radio mean square labeling problem as the ILPM is prepared as follows:

\[
\min f = x_1 + x_2 + x_3,
\]

Subject to

\[
\begin{align*}
M\left( x_i - x_j \right) &\geq \sqrt{\text{diam} + 1 - d(v_i; v_j)}, \\
1 \leq i \leq n - 1, \\
2 \leq j \leq n \text{ and } i < j.
\end{align*}
\]
**Input:** \( G \) be an \( n \)-vertex graph, simple connected graph, and the diameter of \( (\text{diam}) \)

**Output:** an upper bound of radio mean square number of \( G \)

**Begin**

**Step 1:** choose a vertex \( u \) and \( \text{col}(u) = \text{floor}(\sqrt{\text{diam}}) \)

**Step 2:** \( S = \{u\} \)

**Step 3:** for all \( v \in V(G) - S \), compute

\[
\text{temp}(v) = \text{max}(\text{col}(t) + \text{ceil}(\frac{\sqrt{(D + 1 - d(u,v)\text{,}1)}/\text{diam}})),\text{col}(v))
\]

**Step 4:** let \( \text{min} = \text{min}_{v \in V(G) - S} \text{temp}(v) \)

**Step 5:** choose a vertex \( v \in V(G) - S \), such that \( \text{temp}(v) = \text{min} \)

**Step 6:** give \( \text{col}(v) = \text{min} \)

**Step 7:** \( S = S \cup \{v\} \)

**Step 8:** repeat **Step 3–Step 6** until all vertices are labelled

**Step 9:** repeat **Step 1–Step 7** for every vertex \( x \in V(G) \)

**End**

**Algorithm 1:** Finding an upper bound of the radio mean square number of a graph \( G \).

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**Table 1:** Description of the computing environment.

| CPU                  | Intel (R) Core (TM) i5-2430 CPU at 2.40 GHz |
|----------------------|--------------------------------------------|
| RAM size             | 4 GB RAM                                   |
| MATLAB version       | R2019a (9.6.0.1072779)                     |

**Table 2:** Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the path graph.

| \( N \) | Standard RMS | Proposed algorithm | Integer linear programming |
|--------|--------------|--------------------|----------------------------|
|        | rmsn \((P_n)\) | CPU time           | rmsn \((P_n)\) CPU time    |
| 1      | 1            | —                  | —                          |
| 2      | 2            | 0.0001565          | 2                          | 0.0135961 |
| 3      | 3            | 0.0007495          | 3                          | 0.0137607 |
| 4      | 4            | 0.0009242          | 4                          | 0.0142838 |
| 5      | 5            | 0.0009259          | 9                          | 0.0148360 |
| 6      | 6            | 0.0011930          | 11                         | 0.0149701 |
| 7      | 7            | 0.0028154          | 13                         | 0.0151607 |
| 8      | 8            | 0.0031659          | 15                         | 0.0153437 |
| 9      | 9            | 0.0086583          | 17                         | 0.0153530 |
| 10     | 10           | 0.0092995          | 28                         | 0.0153602 |
| 11     | 11           | 0.0178153          | 31                         | 0.0154630 |
| 12     | 12           | 0.0179739          | 34                         | 0.0158122 |
| 13     | 13           | 0.0192524          | 37                         | 0.0159264 |
| 14     | 14           | 0.0214671          | 40                         | 0.0160664 |
| 15     | 15           | 0.0323685          | 43                         | 0.0165312 |
| 16     | 16           | 0.0397062          | 46                         | 0.0165756 |
| 17     | 17           | 0.0488390          | 65                         | 0.0166756 |
| 18     | 18           | 0.0521773          | 69                         | 0.0168593 |
| 19     | 19           | 0.0635412          | 73                         | 0.0168673 |
| 20     | 20           | 0.0773853          | 77                         | 0.0173017 |
| 21     | 21           | 0.0926470          | 81                         | 0.0173982 |
| 22     | 22           | 0.1082337          | 85                         | 0.0174649 |
| 23     | 23           | 0.1321028          | 89                         | 0.0176519 |
| 24     | 24           | 0.1680530          | 93                         | 0.0178209 |
| 25     | 25           | 0.1819503          | 97                         | 0.0178319 |
| 26     | 26           | 0.2174177          | 126                        | 0.0178408 |
| 27     | 27           | 0.2432315          | 131                        | 0.0178502 |
| 28     | 28           | 0.2838989          | 135                        | 0.0179323 |
| 29     | 29           | 0.3306090          | 142                        | 0.0181244 |
| 30     | 30           | 0.3907487          | 146                        | 0.0182150 |
| 31     | 31           | 0.3907487          | 146                        | 0.0182150 |
| 32     | 32           | 0.3907487          | 146                        | 0.0182150 |
| 33     | 33           | 0.3907487          | 146                        | 0.0182150 |
| 34     | 34           | 0.3907487          | 146                        | 0.0182150 |
| 35     | 35           | 0.3907487          | 146                        | 0.0182150 |
| 36     | 36           | 0.3907487          | 146                        | 0.0182150 |
| 37     | 37           | 0.3907487          | 146                        | 0.0182150 |
| 38     | 38           | 0.3907487          | 146                        | 0.0182150 |
| 39     | 39           | 0.3907487          | 146                        | 0.0182150 |
| 40     | 40           | 0.3907487          | 146                        | 0.0182150 |
| 41     | 41           | 0.3907487          | 146                        | 0.0182150 |
| 42     | 42           | 0.3907487          | 146                        | 0.0182150 |
| 43     | 43           | 0.3907487          | 146                        | 0.0182150 |
| 44     | 44           | 0.3907487          | 146                        | 0.0182150 |
| 45     | 45           | 0.3907487          | 146                        | 0.0182150 |
| 46     | 46           | 0.3907487          | 146                        | 0.0182150 |
| 47     | 47           | 0.3907487          | 146                        | 0.0182150 |
| 48     | 48           | 0.3907487          | 146                        | 0.0182150 |
| 49     | 49           | 0.3907487          | 146                        | 0.0182150 |
| 50     | 50           | 0.3907487          | 146                        | 0.0182150 |
\[
\begin{align*}
|x_1 - x_2| & \geq \left\lceil \sqrt{\text{diam} + 1 - d(v_1, v_2)} \right\rceil, \\
|x_1 - x_3| & \geq \left\lceil \sqrt{\text{diam} + 1 - d(v_1, v_3)} \right\rceil, \\
|x_2 - x_3| & \geq \left\lceil \sqrt{\text{diam} + 1 - d(v_2, v_3)} \right\rceil, \\
x_1, x_2, x_3 & \geq 0.
\end{align*}
\] (22)

Since \(\text{diam} = n - 1\) and \(\text{diam} = 2\) for \(P_3, M = 1\), the distance matrix of \(P_3\) is

\[
D = \begin{bmatrix}
0 & 1 & 2 \\
1 & 0 & 1 \\
2 & 1 & 0
\end{bmatrix}.
\] (23)

Therefore, the above mathematical model can be written as follows:

\[
\min f = x_1 + x_2 + x_3.
\] (24)

Subject to

\[
|x_1 - x_2| \geq 1; |x_1 - x_3| \geq 1; |x_2 - x_3| \geq 1; x_1, x_2, x_3 \geq 0.
\] (25)

The solution of the above model equals to 3.

5. Computational Study

In this article, we propose the analysis of the computational results that show the superiority of Algorithm 1 on the ILPM according to the radio mean square number. On the other hand, the proposed ILPM outperforms Algorithm 1 according to CPU time.

Paths and cycles are used to evaluate the proposed models. The computation environment is given in Table 1. MATLAB solver is used to solve the ILPM. In Tables 2 and 3, the following symbols standard RMS, rmsn, and CPU times are used to indicate the exact radio mean square number, the calculated mean square number, and the running time for path and cycles, respectively. The convergence between the calculated and exact upper bounds of the radio mean square number of paths is given in Table 2. Figures 3 and 4 show that the superiority of the proposed Algorithm 1 on the ILPM according to the radio mean square number. For example, the standard radio mean square number for \(P_{50}\) is 54, but it is 56 and 344 by Algorithm 1 and the ILPM, respectively. Figures 5 and show the superiority of the
**Figure 3**: Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the path graph.

**Figure 4**: Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the cycle graph.

**Figure 5**: Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the path graph according to CPU time.
proposed Algorithm 1 on the ILPM according to CPU time. Table 3 provides that the gap between the ILPM and the proposed Algorithm 1 is large according to the radio mean square number. It is clear that 1 is better than the ILPM according to the radio mean square number. According to the CPU time, Tables 2 and 3 explain the superiority of the proposed ILPM on Algorithm 1.

6. Conclusions

In this work, we determined the radio mean square numbers \( \text{rmsn}(P_n) \) and \( \text{rmsn}(C_n) \) for paths and cycles. Then, the proposed approximate algorithm is introduced to obtain \( \text{rmsn}(G) \) for graph \( G \). In addition, a new mathematical model is proposed in order to find the upper bound of \( \text{rmsn}(G) \) for graph \( G \), and a comparison between the proposed approximate algorithm and the proposed mathematical model is introduced. Finally, the computational results and their analysis have proved that the proposed approximate algorithm overcomes the ILPM according to the radio mean square number.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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