Improved Reduced Models for Single-Pass and Reflective Semiconductor Optical Amplifiers

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Abstract—We present highly accurate and easy to implement, improved lumped semiconductor optical amplifier (SOA) models for both single-pass and reflective semiconductor optical amplifiers (RSOA). The key feature of the model is the inclusion of the internal losses and we show that a few subdivisions are required to achieve an accuracy of 0.12 dB. For the case of RSOAs, we generalize a recently published model to account for the internal losses that are vital to replicate observed RSOA behavior. The results of the improved reduced RSOA model show large overlap when compared to a full bidirectional travelling wave model over a 40 dB dynamic range of input powers and a 20 dB dynamic range of reflectivity values. The models would be useful for the rapid system simulation of signals in communication systems, i.e. passive optical networks that employ RSOAs, signal processing using SOAs and for implementing digital back propagation to undo amplifier induced signal distortions.

Index Terms—Semiconductor optical amplifier, Simulations, Four-wave mixing, Reflective semiconductor optical amplifier, Passive optical networks, Nonlinear optics, Signal processing.

I. INTRODUCTION

Modelling semiconductor optical amplifiers (SOA) has been a topic for over two decades [1]-[4]. Very recently, models of reflective SOAs (RSOA) have emerged, [3][4], mainly driven by RSOA exploitation within passive optical networks (PON) [5],[6]. Reduced (or lumped) SOA models, [1]-[3], allow for solving the gain and refractive index dynamics without having to solve computationally intensive propagation equations as was done in [4]. In all the reduced SOA models presented, the inclusion of internal scattering losses in the lumped SOA models have proven to be elusive due to the fact that no analytical solution arises when the internal scattering losses are non-zero [1],[7].

In this letter we propose an improved reduced model for both SOAs and RSOAs that approximates the inclusion of the internal scattering losses. The assumption is based on regarding the SOA’s gain coefficient to be constant over a certain length of SOA. This assumption is certainly valid for short SOA sections; when the optical power is much less than the SOA saturation power and under strong saturation conditions when the gain is depleted to the extent such that there are no large longitudinal variations in the gain coefficient. For single pass SOAs over a 50 dB dynamic range of input powers: we show that by even considering a single section; that the maximum discrepancy of 1 dB was found when calculating output power by considering a single calculation step over the entire SOA as opposed to splitting the SOA up into 40 subsections. The method obviates the need for a fine-grained SOA model, allowing for rapid and accurate system calculations of signal propagation through SOAs.

The improved reduced model for SOAs is extended to RSOAs and builds upon the simpler of two recently published reduced RSOA models [3], allowing for the inclusion of the internal scattering losses. The results from the improved reduced model are compared with the full travelling wave model (TWM) [4], showing excellent agreement with discrepancies < 1 dB over a 40 dB dynamic range of input powers combined with a 20 dB dynamic range of reflectivity values. In addition, we also show how the losses are incorporated in accounting for the intraband contributions to the nonlinear gain. The inclusion of these effects enables the simulation of all-optical signal processing using four-wave mixing (FWM).

II. Improved Reduced SOA Model

We begin the analysis by transcribing the SOA propagation and gain dynamics equations from [1].

$$\frac{d}{dz}E(z,t) = (g(z,t) - a_{int})E(z,t)$$  \hspace{1cm} (1)

$$\frac{d\phi(z,t)}{dz} = -\frac{1}{t_s}a_t g(z,t)$$  \hspace{1cm} (2)

$$\frac{dg(z,t)}{dt} = g_0 - g(z,t) - \frac{g(z,t)E(z,t)}{\tau_s P_{sat}}$$  \hspace{1cm} (3)

Eqs.-(1) and (2) describe the amplification and phase shift accumulation of the optical field $E(z,t)$ along the SOA with $z$ and $t$ being the spatial and temporal variables; the optical power is given by $|E(z,t)|^2$. $a_{int}$ describes the internal scattering losses. $g$ is the gain coefficient whose dynamics are described by Eq-(3); the second term on the right hand side of which describes gain depletion due to stimulated emission while the first term describes gain recovery back to the unsaturated value $g_0$. The gain recovery time, $\tau_s$, is the carrier lifetime. $P_{sat}$ is the saturation power. The gain-phase coupling is determined by $a_t$. The reduced models rely on integrating the gain over a length, $L$, of the SOA. Equating $h$ as the spatially-integrated SOA gain coefficient over $L$.
we define $g_w(t)$ as the spatially-averaged gain coefficient. The assumption is valid as long as the spatial profile of the gain coefficient is constant. In principle, unidirectional signal amplification along the SOA causes the gain coefficient to monotonically decrease along the length of the SOA, thus requiring for the SOA to be broken up into many sections in order to capture the correct gain profile. Assuming a constant gain coefficient allows us to write an approximate analytical expression for the integral of the second term on the right hand side of Eq.-(3). The input optical field to the SOA is given as $E_u(t) = E(0,t)$; using (4), Eqs.- (1) and (3) can be re-written as:

$$h(t) = \int_0^L g(z,t) \, dz = g_w(t)L ,$$  

(4)

between the output power calculation because the SOA gain profile is constant because the power in the SOA is much less than $P_{sat}$. Though when the input power increases, the gain profile no longer remains flat thus requiring more subdivisions to get an accurate value for the output power. Though as is clear from Fig. 2, a maximum discrepancy of just 1 dB is found over the entire input power range up to 10 dBm by considering a single subdivision, this is an acceptable error in most circumstances. For the cases when greater accuracy is required, the discrepancy could be reduced below 0.12 dB by only considering 3 subdivisions, as is evident from Fig. 2.

The reduced model in [2] also accounted for the intraband contributions to the nonlinear SOA gain [7]. FWM is the only 3rd order nonlinear process that is transparent to modulation format and has been used to process a variety of signals with amplitude and/or phase encoding [2],[8],[9]. We will now show how the intraband effects can be included in the improved model. Starting with the rate equation describing the dynamics of carrier heating (CH) [2],[7]:

$$\frac{d \Delta g_{ch}(t)}{dt} = - \frac{\Delta g_{ch}(t) - g(t)|E(z,t)|^2}{\tau_{ch} P_{sat,ch}^2}$$

(10)

where $\Delta g_{ch}$ is the gain change due to CH, $\tau_{ch}$ is the associated time constant with carrier-phonon collisions and is ~500 fs. $g$ is the optical gain defined in Eq.- (1) and $P_{sat,ch}$ is the saturation powers associated with CH. Using the technique outlined in (4)-(7) and invoking the adiabatic condition that changes in $|E|^2$ occur over timescales longer than $\tau_{ch}$ i.e. $d\Delta g_{ch}/dt = 0$, then the spatially integrated version of (10) yields the contribution to the gain of:

$$\Delta h_{ch}(t) \approx \frac{h(t)}{h(t) - \alpha_{loss,L}} \left[ \exp[h(t) - \alpha_{loss,L} - 1]E_u(t) \right]^2$$

(11)
Two pumps, the pumps and idlers are extracted from the carefully obtained experimental results [8]. For detunings < 30 GHz, both idlers become commensurate with the pumps indicating that the device exhibits fast carrier dynamics.

### III. REFLECTIVE SOAS

RSOAs are finding applications as low-cost upstream transmitters within PONs [5]-[6],[10]; therefore targeting accurate, yet simple, models that describe their behavior is a laudable goal for system analysis simulations. In general, the output power characteristics of reflective amplifiers differ from single pass amplifiers in that a maximum output power is reached at a certain input power [11], and this was shown experimentally and numerically for RSOAs [10],[12]. Simplified travelling wave models for RSOAs have appeared [3],[4], with the model in [3] introducing a lumped RSOA model that shows nice agreement with the full travelling wave model in [4]. We show how the losses could be implemented in a reduced RSOA model using the techniques outlined in section II and we assume sub-unity reflectivity values for the reflective facet, as they are preferred to avoid lasing [13]. A depiction of an RSOA is shown in Fig. 3. The fields at the input and reflective facets are defined, and the superscript [+,-] defines the shown directions of propagation. A typical plot of the distributed nature of the localized RSOA gain is shown and the gain (carrier density) is greatest in the center of the device because the gain saturation is strongest at the input and reflective facets because the counter-propagating fields are strongest at the facets [12]. To model the RSOA gain dynamics we invoke the criterion for the reduced RSOA model in [3] that the intensity of the input signal does not vary over timescales of signal time-of-flight in RSOAs (~10 ps); such criterion is easily met by signals in WDM-PON scenarios with baudrates ≤ 10 Gb/s. Noting that the gain is depleted by two counter propagating waves originating from the two facets and including the internal losses, the integrated gain emerging considerably stronger. The two idlers also behave as was measured [8]. For detunings < 30 GHz, both idlers become commensurate with the pumps indicating that the device exhibits fast carrier dynamics.

### TABLE II

| Symbol | Definition | Value |
|--------|------------|-------|
| $h_0$ | Unsatuated gain parameter | 11 |
| $P_{sat}$ | Saturation power | 40 mW |
| $\tau_s$ | Carrier lifetime | 60 ps |
| $\alpha_{ch}$ | Gain-phase coupling parameter | 6 |
| $\alpha_{ch}L$ | Internal losses | 2 |
| $P_{sat,ch}$ | Saturation power for carrier heating | 300 mW |
| $\alpha_{ch}E_t$ | Carrier heating gain-phase coupling | 3 |

### TABLE III

| Symbol | Definition | Value |
|--------|------------|-------|
| $h_0$ | Unsatuated gain parameter | 10.5 |
| $P_{sat}$ | Saturation power | 10 mW |
| $\tau_s$ | Carrier lifetime | 100 ps |
| $\alpha_{ch}$ | Gain-phase coupling parameter | 5 |
| $\alpha_{ch}L$ | Internal losses | 4.5 |
| $R$ | Reflectivity | 0.01 – 1 |

Fig. 3 Schematic of amplification within an RSOA. The input field $E_{in}$ travels along the RSOA. At $z = L$ the field is partially reflected back in the opposite direction and is amplified until the field re-emerges. $E_{out}$ at the $z=0$ facet. The maximum gain occurs near the middle of the RSOA [12].

A similar expression could be written for spectral hole burning (SHB) [2][7]. For FWM, solving (10) in the adiabatic limit restricts signal-pump detunings to be less than 300 GHz, though this allows us to include the intraband effects without having to excessively oversample the input field to calculate $\Delta h_{ch}(t)$ using the spatially integrated form of (10). The output optical field is given by:

$$E_{out}(t) = E_{in}(t)\exp\left\{\frac{1}{2}(\Delta h_{ch}(t))^{2} \right\}$$

With $\alpha_{ch}$ describing the refractive index dynamics associated with CH. The contribution arising from SHB is given by $\Delta h_{ch}$. We replicate the carefully obtained experimental results presented in [8] using the SOA parameters given in Table II. The situation is shown in Fig. 1 with two equal power pumps $P_1$ and $P_2 = 100 \mu W$ at the SOA input. The two pumps interact in the SOA creating two SOA idlers, $I_0$, and $I_1$, via FWM. The SOA input field is given as:

$$E_{in}(t) = \sqrt{P_1} + \sqrt{P_2} \exp(j2\pi f_{ch}t)$$

The detuning, $f_{ch}$, is varied from 8 to 300 GHz. The power of the pumps and idlers are extracted from the calculated output spectrum. The results are shown in Fig. 2 and agree quite well with the experimental and travelling-wave simulation results in [8]. The output power for both pumps show quite strong cross-gain modulation for detunings <100 GHz, with $P_1$ emerging considerably stronger. The two idlers also behave as was measured [8]. For detunings < 30 GHz, both idlers become commensurate with the pumps indicating that the device exhibits fast carrier dynamics.

![Input Spectrum](image1.png) ![Output Spectrum](image2.png)

**Fig. 1** Scenario for FWM. Two pumps $P_1$ and $P_2$ are input into the SOA. Idlers $I_0$, $I_1$, and $I_2$ are created due to the nonlinear interaction between the pumps.

![Output Power vs Detuning](image3.png)

**Fig. 2** Calculated output power of the pumps and idlers using the improved SOA model.

![RSOA Parameters Table](image4.png)

**Table II**

| Symbol | Definition | Value |
|--------|------------|-------|
| $h_0$ | Unsatuated gain parameter | 11 |
| $P_{sat}$ | Saturation power | 40 mW |
| $\tau_s$ | Carrier lifetime | 60 ps |
| $\alpha_{ch}$ | Gain-phase coupling parameter | 6 |
| $\alpha_{ch}L$ | Internal losses | 2 |
| $P_{sat,ch}$ | Saturation power for carrier heating | 300 mW |
| $\alpha_{ch}E_t$ | Carrier heating gain-phase coupling | 3 |

![RSOA Parameters Table](image5.png)

**Table III**

| Symbol | Definition | Value |
|--------|------------|-------|
| $h_0$ | Unsatuated gain parameter | 10.5 |
| $P_{sat}$ | Saturation power | 10 mW |
| $\tau_s$ | Carrier lifetime | 100 ps |
| $\alpha_{ch}$ | Gain-phase coupling parameter | 5 |
| $\alpha_{ch}L$ | Internal losses | 4.5 |
| $R$ | Reflectivity | 0.01 – 1 |

![Fiber-RSOA Coupling Loss](image6.png)

**Table III**

| Symbol | Definition | Value |
|--------|------------|-------|
| $h_0$ | Unsatuated gain parameter | 10.5 |
| $P_{sat}$ | Saturation power | 10 mW |
| $\tau_s$ | Carrier lifetime | 100 ps |
| $\alpha_{ch}$ | Gain-phase coupling parameter | 5 |
| $\alpha_{ch}L$ | Internal losses | 4.5 |
| $R$ | Reflectivity | 0.01 – 1 |

![Fiber-RSOA Coupling Loss](image7.png)

**Table III**

| Symbol | Definition | Value |
|--------|------------|-------|
| $h_0$ | Unsatuated gain parameter | 10.5 |
| $P_{sat}$ | Saturation power | 10 mW |
| $\tau_s$ | Carrier lifetime | 100 ps |
| $\alpha_{ch}$ | Gain-phase coupling parameter | 5 |
| $\alpha_{ch}L$ | Internal losses | 4.5 |
| $R$ | Reflectivity | 0.01 – 1 |
The boundary condition at the leftmost facet is \( E_0 = 1 \) and \( \alpha_{st} = 0 \), in terms of the input \( h(t) \) of \( E_0(t) \) in terms of the input \( E_0(t) \):

\[
E_0(t) = \frac{1}{1 + R \exp \left( -\alpha_{st} L \right)} E_0(t)
\]

Inserting (14) into (13) and rearranging gives the RSOA gain dynamics in response to an arbitrary input optical field:

\[
\frac{dh(t)}{dt} = \frac{h_0 - h(t)}{\tau_h} - \frac{h(t)}{\tau_h} \exp \left( -\alpha_{st} L \right) \times \left[ \exp \left( h(t) - \alpha_{st} L \right) - 1 \right] \left[ E_0(t)^2 + \left| E_0(t) \right|^2 \right]
\]

(15)

Eq. (15) reduces to the model in [3] when \( R = 1 \) and \( \alpha_{st} = 0 \). The output optical field is simply as the amplification of the input field on the rightwards journey, reflected by the reflective facet (14) and amplified on the leftwards journey:

\[
E_{out}(t) = E_{in}(t) \sqrt{R} \exp \left( -\alpha_{st} L \left( 1 - j \alpha_{st} \right) h(t) \right)
\]

(16)

Using Eqs. (15) and (16) we now present results of RSOA gain saturation for CW input signals varying from -30 to +10 dBm for with reflectivity \( R \) (-21 \leq R \leq 0 \, \text{dB}) set to mimic the experimentally obtained curves in [10]. The calculated results in Fig. 6 were obtained using the RSOA parameters given in Table III for both the improved model and for the full TWM with the RSOA split into 30 sections and internal losses included with the TWM described in [3][4]. The RSOA length was taken to be 800 \( \mu \text{m} \) when implementing the TWM. There is excellent agreement between the obtained input-output power transfer characteristics for both models and both models replicate the findings of RSOA with differing values of reflectivity [10]. The remarkable thing is that there is considerable overlap between the curves with a maximum discrepancy of 1 dB despite the fact that the reduced RSOA model neither considers spatially-resolved counter-propagating fields nor spatially resolves the gain profile. All discrepancies are within \( \pm 1 \, \text{dB} \) over the entire input power range from -30 to 10 dBm and the reflectivity range from -21 to 0 dB, and thus the improved model would allow for accurate and rapid simulations of signal amplification in WDM PON scenarios. The importance of including the internal losses is shown by the red lines in Fig. 4 with the parameters in Table III adjusted to allow for the same net small signal gain that generated the blue curves i.e. \( h_0 = 6 \) and \( \alpha_{st} = 0 \). The results from both the reduced and TWM models show excellent agreement and show a departure from the measured RSOA behavior in [10][12], thus highlighting the necessity to include internal RSOA losses.

IV. Conclusion

We presented an improved reduced model for both SOAs and RSOAs showing that rapid and accurate simulations can be achieved in a single calculation step.

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