Deterministic Bell State Discrimination

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We make use of local operations with two ancilla bits to deterministically distinguish all the four Bell states, without affecting the quantum channel containing these Bell states.

Entangled states play a key role in the transmission and processing of quantum information [1, 2]. Using entangled channel, an unknown state can be teleported [3] with local unitary operations, appropriate measurement and classical communication; one can achieve entanglement swapping through joint measurement on two entangled pairs [4]. Entanglement leads to increase in the capacity of the quantum information channel, known as quantum dense coding [5]. The bipartite, maximally entangled Bell states provide the most transparent illustration of these aspects, although three particle entangled states like GHZ and W states are beginning to be employed for various purposes [6, 7].

Making use of single qubit operations and the Controlled-NOT gates, one can produce various entangled states in a quantum network [1]. It may be of interest to know the type of entangled state that is present in a quantum network, at various stages of quantum computation and cryptographic operations, without disturbing these states. Nonorthogonal states are impossible to discriminate with certainty [8]. A number of results have recently been established regarding distinguishing various orthogonal Bell states [9, 10, 11, 12]. It is counter intuitive to know that, multipartite orthogonal states may not be discriminated with only local operations and classical communications (LOCC) [8]. However, any two multipartite orthogonal states can be unequivocally distinguished through LOCC [11]. If two copies belonging to the four orthogonal Bell states are provided, LOCC can be used to distinguish them with certainty. It is not possible to discriminate, either deterministically or probabilistically the four Bell states, if only a single copy is provided. It is also known that, any three Bell states cannot be discriminated deterministically, if only LOCC is allowed.

Making use of only linear elements, it has been shown that a never failing Bell measurement is impossible [13]. A number of theoretical and experimental results already exist in this area of unambiguous state discrimination [14, 15, 16]. Appropriate unitary transforms and measurements, which transfer the Bell states into disentangled basis states, can unambiguously identify all the four Bell states [15, 16, 17]. However, in the process of measurement the entangled state is vandalized. The above is satisfactory, where the Bell state is not required further in the quantum network.

We present in this letter, a scheme which discriminates all the four Bell states deterministically and is able to preserve these states for further use. As LOCC alone is insufficient for this purpose, we will make use of two ancilla bits, along with the entangled channels. Throughout the protocol, we will only employ local unitary operations. At the end measurements are carried out on the two ancilla bits, therefore we are able to preserve the Bell states for further operations.

It should be noted that, the Bell states:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),$$
$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),$$
$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),$$
$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle),$$

(1)

when operated on by single qubit operators such as Hadamard and Pauli matrices, in solitude or in combination, get transformed into each other. This is an interesting property which can easily transform one Bell state to the other on demand. This property of Bell states proves very handy in distinguishing them. Exploiting the above nature of the Bell states, we have designed a circuit for the Bell state discrimination, as shown in Fig.1. It consists of two quantum channels, depicted as $|x_{12}\rangle$, carrying the entangled state; two ancilla qubits are used for carrying out local operations. In the end, measurement is taken on these ancilla bits to know with certainty, the type of Bell state that exists in the channel. Measurement on the first ancilla will differentiate the four Bell
states into two pairs i.e., either $|\psi^+\rangle/|\phi^+\rangle$ or $|\psi^-\rangle/|\phi^-\rangle$ as given in Eq.2. While the measurement on the second ancilla differentiates the Bell states within these two groups as stated in Eq.3. The remarkable property of this circuit is that, the Bell states in first two quantum channels retain their initial states, even after being discriminated. Here, we have used Hadamard operation on the entangled channel while differentiating the Bell states in Eqs.2 and 3, though one can also use other suitable single qubit operations. In Table-I we have shown the results of the measurements on both the ancillas when different Bell states are present in the given circuit (Fig.1). Before measurement the states can be explicitly written as,

$$|R_{A1}\rangle = [I_2 \otimes I_2 \otimes H] \ast [(x_1 \oplus A_1) \otimes (x_2 \oplus A_1) \otimes I_2] \ast$$

$$|I_2 \otimes I_2 \otimes H] \ast [(x_{12} \otimes |A_1\rangle]$$

and

$$|R_{A2}\rangle = [H^\otimes^3] \ast [(x_1 \oplus A_2) \otimes (x_1 \oplus A_2) \otimes I_2] \ast$$

$$[H^\otimes^3] \ast [(x_{12} \otimes |A_2\rangle]$$

Table. I:

| Bell State | Measurement $A_1$ | Measurement $A_2$ |
|------------|------------------|------------------|
| $|\psi^+\rangle$ | 0 | 0 |
| $|\psi^-\rangle$ | 1 | 0 |
| $|\phi^+\rangle$ | 0 | 1 |
| $|\phi^-\rangle$ | 1 | 1 |

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