Probing Higgs portals with matrix-element based kinematic discriminants in $ZZ \to 4\ell$ production

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ABSTRACT: A Higgs portal in the form of the operator $|H|^2$ provides a minimal and theoretically motivated link between the Standard Model (SM) and new physics. While Higgs portals can be constrained well by exotic Higgs decays if the beyond-the-SM states are light, testing scenarios where these particles are kinematically inaccessible is known to be challenging. We explore the sensitivity of future hadron collider measurements of $ZZ \to 4\ell$ production in constraining Higgs portal interactions. It is shown that by using a matrix-element based kinematic discriminant the reach of the high-luminosity option of the Large Hadron Collider (LHC) can be significantly enhanced compared to studies that are based on measurements of the four-lepton invariant mass spectrum alone. We also analyse the potential of the high-energy upgrade of the LHC and a Future Circular Collider in constraining new physics that couples to $|H|^2$. The obtained constraints are compared to the limits one expects to find from other single-Higgs probes. In addition, we provide an independent analysis of the relevant Higgs portal effects in double-Higgs production. We find that the constraints obtained from our $ZZ \to 4\ell$ analysis turn out to be both competitive with and complementary to the projected limits obtained using other search techniques.

KEYWORDS: Higgs Production, Higgs Properties, Models for Dark Matter, Other Weak Scale BSM Models

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1 Introduction

The discovery of a new spin-0 state by ATLAS and CMS [1, 2] with approximately the properties of the Standard Model (SM) Higgs boson has opened up new avenues in the pursuit of physics beyond the SM (BSM). In fact, there are both experimental and theoretical arguments that suggest that the Higgs boson may provide a window into BSM physics. Experimentally, the Higgs sector is far less explored and constrained compared to the gauge or fermionic sector of the SM [3, 4], while theoretically the SM Higgs doublet $H$ plays a special role because it allows to write down relevant and marginal operators of the form $|H|^2 \mathcal{O}$ with $\mathcal{O}$ itself a gauge-invariant operator with a mass dimension of two or lower.

The simplest and most studied case of such an operator is $\mathcal{O} = \phi^2$ where $\phi$ is a real scalar that is a singlet under the SM gauge group but odd under a $\mathbb{Z}_2$ symmetry [5–9]. The corresponding interaction Lagrangian reads

$$\mathcal{L}_{H\phi} = -c_\phi |H|^2 \phi^2.$$  

Notice that the $\mathbb{Z}_2$ symmetry acts on the real scalar field as $\phi \rightarrow -\phi$, which guarantees the stability of $\phi$ making it a suitable dark matter (DM) candidate. See for instance [10–12] for recent reviews of the ensuing DM phenomenology. In particular, under the assumption that $\phi$ is a relic of standard thermal freeze-out production DM direct detection experiments are known to foster stringent constraints on DM portals of the form (1.1) — see for example [12] and references therein. In theories with a non-thermal cosmological history, a real scalar $\phi$ can however be shown to be a viable DM candidate for a wide range of Higgs portal
realisations while evading existing experimental limits [13]. This opens up the possibility to probe (1.1) at high-energy colliders.

Another motivation for the existence of sizeable Higgs portal couplings to $|H|^2$ is provided by the hierarchy problem of the Higgs-boson mass. In fact, in models where the hierarchy problem is addressed by the addition of $N_r$ real scalar top partners $\phi_i$ the relevant interaction Lagrangian can be written as [14]

$$L_{H\phi_i} = -\frac{2N_c}{N_r} y_t^2 |H|^2 \sum_{i=1}^{N_r} \phi_i^2,$$

where $N_c = 3$ is the number of colours in QCD and $y_t = \sqrt{2} m_t/v \simeq 0.94$ is the top-quark Yukawa coupling with $m_t \simeq 163$ GeV the top-quark MS mass and $v \simeq 246$ GeV the Higgs vacuum expectation value. Well-known cases where (1.2) is a proxy for the resulting Higgs portal interactions are stops in the minimal supersymmetric SM (MSSM) and singlet scalar top partners in the hyperbolic Higgs [15] or tripled top model [16], if one assumes that these particles are approximately degenerate in mass. Notice that in such a case the interactions (1.1) and (1.2) are equivalent from the perspective of collider phenomenology if $|c_\phi| = 2N_c/\sqrt{N_r} y_t^2$. In the case of the MSSM, the hyperbolic Higgs and the tripled top model where $N_r = 12$, a light Higgs boson is therefore natural if one effectively has a Higgs portal of the form (1.1) with coupling strength $|c_\phi| \leq \sqrt{3} y_t^2 \simeq 1.5$.

The level of difficulty to discover or to exclude Higgs portals of the form (1.1) and (1.2) at high-energy colliders depends mainly on the mass $m_\phi$ of the new states that couple to $|H|^2$. While in the case of $m_\phi < m_h/2 \simeq 62.5$ GeV the decays of the Higgs boson into invisible [17–21] or undetected [4, 12] final states provide stringent constraints on the effective coupling strength of the Higgs portals, obtaining relevant constraints above the kinematic threshold $m_\phi > m_h/2$ turns out to be significantly more challenging. In fact, only two categories of collider measurements are known that provide sensitivity to Higgs portals above the kinematic threshold: firstly, pair-production of the new scalars in off-shell Higgs processes such as the vector-boson fusion (VBF), the $tt\bar{t}h$ and the gluon-gluon-fusion (ggF) channel [22–36], and secondly, studies of the virtual effects that these particles produce when exchanged in loop diagrams that contribute to processes such as associated $Zh$, double-Higgs and $gg \rightarrow h^* \rightarrow ZZ$ production [37–43]. The existing analyses have considered a wide range of future high-energy hadron as well as lepton colliders, including the high-luminosity (HL) and high-energy (HE) versions of the Large Hadron Collider (LHC), a Future Circular Collider (FCC), the International Linear Collider (ILC), the Compact Linear Collider (CLIC) and a muon collider.

In this article, we investigate the sensitivity of future hadron collider measurements of off-shell Higgs production in the $pp \rightarrow ZZ \rightarrow 4\ell$ channel to Higgs portal interactions such as (1.1) and (1.2). Compared to earlier studies [34, 41, 42, 44] that relied on the four-lepton invariant mass ($m_{4\ell}$) spectrum alone to separate signal from background, we instead employ a matrix-element (ME) based kinematic discriminant in our work. Being sensitive not only to $m_{4\ell}$ but also to another seven variables such as the invariant masses of the two opposite-sign lepton pairs (for details consult the articles [45–48]), ME-based discriminants
fully exploit the event kinematics. As in our recent study [49], we find that the use of a
ME method leads to a significantly improved coverage of the BSM parameter space, i.e. $c_{\phi}$ and $m_{\phi}$ in the case of (1.1), than a shape analysis of the $m_{4\ell}$ distribution. Motivated by
this finding, we analyse in detail the HL-LHC, HE-LHC and FCC potential of the proposed
method in constraining BSM physics that couples to the operator $|H|^2$.

Our work is structured as follows. In section 2 we briefly discuss the calculation of
the loop corrections to $pp \to ZZ \to 4\ell$ production arising from (1.1). The aforementioned
ME-based kinematic discriminant is introduced in section 3 where we also discuss how
higher-order QCD corrections are taken into account in our study. The numerical analysis
of the HL-LHC reach is performed in section 4 and contains a comparison between the
sensitivities obtained from a shape analysis of the $m_{4\ell}$ spectrum and the proposed ME
method. In section 5 we present our HE-LHC and FCC projections. We discuss our main
results in section 6, comparing them to the limits one expects to obtain from other single-
and double-Higgs probes, and provide a short outlook. A discussion of the impact that
different assumptions on the systematic uncertainties in our ME-based search strategy
have on the projected constraints is relegated to appendix A, while further details of the
relevant loop calculations and their implementation in the Monte Carlo (MC) code for our
double-Higgs analysis are given in appendix B.

2 Higgs portal effects in $gg \to h^* \to ZZ$

At the one-loop level the $gg \to h^* \to ZZ$ process receives contributions from Feynman
graphs such as the one displayed in figure 1 that contains a modified Higgs propagator with
insertions of the Higgs portal operator (1.1). The corresponding renormalised contribution
to the self-energy of the Higgs takes the form

$$\hat{\Sigma}(\hat{s}) = \Sigma(\hat{s}) + \left(\hat{s} - m_h^2\right) \delta Z_h - \delta m_h^2,$$  \hspace{1cm} (2.1)

where the bare Higgs self-energy, the one-loop corrections to the Higgs wave function and
the mass counterterm in the on-shell scheme are given by the following expressions

$$\Sigma(\hat{s}) = \frac{1}{(4\pi)^2} \left[ c_{\phi} A_0(m_{\phi}^2) + 2v^2|c_{\phi}|^2 B_0(\hat{s}, m_{\phi}^2, m_{\phi}^2) \right],$$

$$\delta Z_h = -\frac{2v^2|c_{\phi}|^2}{(4\pi)^2} \frac{d}{d\hat{s}} B_0(\hat{s}, m_{\phi}^2, m_{\phi}^2) \bigg|_{\hat{s}=m_h^2},$$

$$\delta m_h^2 = \frac{1}{(4\pi)^2} \left[ c_{\phi} A_0(m_{\phi}^2) + 2v^2|c_{\phi}|^2 B_0(m_h^2, m_{\phi}^2, m_{\phi}^2) \right].$$  \hspace{1cm} (2.2)

Here $\hat{s} = p^2$ with $p$ the external four-momentum entering the Higgs propagator and the $A_0$
and $B_0$ functions are one- and two-point Passarino-Veltman scalar integrals defined as in
[50, 51]. The expression in (2.2) can be easily generalised to other Higgs portals of the
form (1.1). For instance, in the case of $L_{HH\Phi} = -c_{\phi} |H|^2 |\Phi|^2$ with $\Phi$ a complex scalar field
one just has to make the substitutions $c_{\phi} \to c_{\phi}/\sqrt{2}$ and $m_{\phi} \to m_{\phi}$. 

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Notice that the contribution to the Higgs wave-function renormalisation constant $\delta Z_h$ coming from the propagator corrections exactly cancels against those of the vertices when combined to obtain the full BSM contribution to the off-shell $gg \to h^* \to ZZ$ amplitude. Similarly, the tadpole contribution proportional to $A_0(m_\phi^2)$ also cancels in the difference $\Sigma(\hat{s}) - \delta m_h^2$. In contrast, the Higgs wave-function renormalisation constant $\delta Z_h$ does not drop out in the on-shell Higgs signal strengths $\mu_{if}$ for production in channel $i$ and decay in channel $f$. In terms of the inclusive Higgs production cross sections $\sigma_i$ and the Higgs branching ratios $\text{BR}_f$, these quantities take the form

$$\mu_{if} = \frac{\sigma_i \cdot \text{BR}_f}{\sigma_i \cdot \text{BR}_{SM}^f} = 1 + \delta Z_h,$$

i.e. they receive a universal correction proportional to the Higgs wave-function renormalisation constant as given in (2.2). This feature allows to set indirect constraints on Higgs portal models by precision measurements of Higgs properties [38], which will be discussed in section 6.

3 ME-based kinematic discriminant

The Higgs propagator corrections (2.1) and the relevant vertex counterterms have been implemented into version 8.0 of the event generator MCFM [52] to obtain kinematic distributions for $pp \to ZZ \to 4\ell$ such as the $m_{4\ell}$ spectrum. In addition, our MC code is also able to calculate the following ME-based kinematic discriminant [53–55]

$$D_S = \log_{10} \left( \frac{P_h}{P_{gg} + c \cdot P_{qq}} \right).$$

Here $P_h$ denotes the squared ME for the $gg \to h^* \to ZZ \to 4\ell$ process, $P_{gg}$ is the squared ME for all $gg$-initiated channels (including the Higgs channel, the continuum background and their interference) and $P_{qq}$ is the squared ME for the $q\bar{q} \to ZZ \to 4\ell$ process. Like in [53–55] the constant $c$ is set to 0.1 to balance the $q\bar{q}$- and $gg$-initiated contributions. We stress that in the SM more than 99% of the $pp \to ZZ \to 4\ell$ cross section falls into the range of $-4.5 < D_S < 0.5$ [53]. For BSM models that predict events with $D_S < -4.5$ or $D_S > 0.5$ the variable $D_S$ therefore presents a null test.
Currently, calculations of higher-order QCD corrections to four-lepton production via $q\bar{q}$ annihilation include the full next-to-next-to-leading order (NNLO) corrections and top-quark mass effects [56–59]. Next-to-leading order (NLO) corrections to the loop-induced $gg$ channel have been computed by now as well [61–64], while for inclusive Higgs production the precision has been pushed to the next-to-next-to-next-to-leading order (N$^3$LO) in the heavy top-quark limit [65]. As the $gg \to ZZ$ process starts contributing only at $O(\alpha^2_s)$, it is part of the NNLO QCD corrections to $ZZ$ production and NLO corrections to this channel formally contribute at N$^3$LO. Lastly, NLO electroweak (EW) corrections could in principle play an important role as well. Within the SM they were combined with NNLO QCD effects for $ZZ$ production in the work [66]. However, it has been shown in the paper [67] that including NLO EW effects in the SM has only a very minor effect on the sensitivity of indirect single-Higgs analyses to modifications of the trilinear Higgs coupling. We expect a similar pattern to arise in the context of the Higgs portal models studied here. A dedicated simulation of four-lepton events including both higher-order QCD as well as EW corrections both in and beyond the SM, consistently matched to a parton shower and including detector effects is clearly beyond the scope of the present article and therefore left for future work.

In order to include higher-order QCD corrections in our $pp \to ZZ \to 4\ell$ analysis, we proceed as in our recent publication [49]. For the two relevant production channels we calculate the so-called $K$-factor defined as the ratio between the fiducial cross section at a given order in QCD and the corresponding leading order (LO) QCD prediction. In the case of the $gg$-initiated contribution we utilise the results of [60]. The ratio between the NLO and LO ggF predictions turns out to be essentially flat in $m_{4\ell}$ and by averaging we find $K_{gg}^{NLO} = 1.83$. This number agrees with the $K$-factors reported in [61, 63, 64].

In the case of the $q\bar{q}$-initiated contribution we use the NNLO results obtained in [63]. The relevant $K$-factor again turns out to be basically flat in $m_{4\ell}$ with a central value of $K_{q\bar{q}}^{NNLO} = 1.55$. This finding is in accordance with [56]. The quoted $K$-factors are then used to obtain a QCD-improved prediction for the $pp \to ZZ \to 4\ell$ cross section differential in the variable $O$ as follows:

$$
\frac{d\sigma}{dO} = K_{gg}^{NLO} \left( \frac{d\sigma}{dO} \right)_{LO} + K_{q\bar{q}}^{NNLO} \left( \frac{d\sigma}{dO} \right)_{LO}.
$$

(3.2)

Notice that (3.2) is accurate in the case of the $m_{4\ell}$ spectrum. For the $D_S$ distribution one observes [49] a close to flat $K$-factor of around 1.6 between the LO and the improved prediction (3.2). It is furthermore found that the inclusion of higher-order QCD corrections reduces the scale uncertainties by a factor of about 3 from $(7–8)\%$ to $(2–3)\%$. The fact that the central value of the improved $D_S$ spectrum lies outside the LO uncertainty bands demonstrates that the scale variations of (3.2) do not provide a reliable way to estimate the size of higher-order QCD effects. In view of this and given that the discriminant $D_S$ as defined in (3.1) is only LO accurate, we will make different assumptions on the systematic uncertainties entering our ME-based search strategy, a point we will discuss in more detail in our numerical analyses presented in sections 4 and 5 as well as in appendix A. A similar approach is also used in the projections [68, 69] that estimate the
HL-LHC reach in constraining off-shell Higgs boson production and the Higgs boson total width in \( pp \to ZZ \to 4\ell \).

4 HL-LHC analysis

In our \( pp \to ZZ \to 4\ell \) analysis we consider the window 140 GeV < \( m_{4\ell} \) < 600 GeV of four-lepton invariant masses. The charged leptons are required to be in the pseudorapidity range \( |\eta| < 2.5 \) and the lepton with the highest transverse momentum \( (p_T^{\ell}) \) must satisfy \( p_{T,\ell_1} > 20 \text{ GeV} \) while the second, third and fourth lepton in \( p_T \) order is required to obey \( p_{T,\ell_2} > 15 \text{ GeV} \), \( p_{T,\ell_3} > 10 \text{ GeV} \) and \( p_{T,\ell_4} > 6 \text{ GeV} \), respectively. The lepton pair with the mass closest to the \( Z \)-boson mass is referred to as the leading dilepton pair and its invariant mass is required to be within 50 GeV < \( m_{12} \) < 106 GeV, while the subleading lepton pair must be in the range of 50 GeV < \( m_{34} \) < 115 GeV. Notice that the ATLAS and CMS analyses [53–55, 68–71] employ similar cuts. We assume a detection efficiency of 99% (95%) for muons (electrons) that satisfy the event selections. These efficiencies correspond to those reported in the latest ATLAS analysis of off-shell Higgs production [55]. As input parameters we use \( G_F = 1/(\sqrt{2}v^2) = 1.16639 \cdot 10^{-5} \text{ GeV}^{-2} \), \( m_Z = 91.1876 \text{ GeV} \), \( m_h = 125 \text{ GeV} \) and \( m_t = 173 \text{ GeV} \). We employ NNPDF40_nlo_as_01180 parton distribution functions (PDFs) [72] with the renormalisation and factorisation scales \( \mu_R \) and \( \mu_F \) set to \( m_{4\ell} \) on an event-by-event basis. Both the different-flavour \( e^+e^-\mu^+\mu^- \) and the same-flavour \( 2e^+2e^- \) and \( 2\mu^+2\mu^- \) decay channels of the two \( Z \) bosons are included throughout our work.

In figure 2 we show our predictions for the \( m_{4\ell} \) distributions in the SM (dashed black) and three Higgs portal models (1.1). The displayed BSM benchmarks correspond to scalar masses of \( m_\phi = 70 \text{ GeV} \) (solid red), \( m_\phi = 100 \text{ GeV} \) (solid blue) and \( m_\phi = 150 \text{ GeV} \) (solid green) assuming in all cases a coupling strength of \( c_\phi = 3 \). Notice that the chosen value of \( c_\phi \) is safely below the limit \( |c_\phi| < 4\pi \) following from perturbative tree-level unitarity (see for instance [34]). In the left panel the QCD-improved predictions for \( gg \to ZZ \to 4\ell \) production including the Higgs signal, the continuum background and their interference are given. Two features of the shown BSM spectra deserve a further discussion. First, one observes peak-like structures in the distributions slightly above the threshold \( m_{4\ell} = 2m_\phi \) of two-scalar production. Second, both spectra show an enhancement at large \( m_{4\ell} \) because in the limit of partonic centre-of-mass energies \( \hat{s} \to \infty \) the correction simplifies to \( \Sigma(\hat{s}) - \delta m_h^2 \simeq -v^2|c_\phi|^2/(8\pi^2)\ln(\hat{s}/m_h^2) \). This behaviour is easily derived from (2.2). Notice furthermore that the \( gg \to h^+ \to ZZ \to 4\ell \) amplitudes interfere destructively with the \( gg \to ZZ \to 4\ell \) matrix elements so that the overall sign of the correction \( \Sigma(\hat{s}) - \delta m_h^2 \) is effectively flipped. One also sees that for the three chosen sets of Higgs portal parameters the relative corrections in the spectra amount to less than 15% over the whole range of \( m_{4\ell} \) values of interest. The same features are also observed in the right panel of figure 2 which shows the corresponding predictions for \( pp \to ZZ \to 4\ell \) production. Notice that in this case the relative modification are smaller by a factor of roughly 10 than for \( gg \to ZZ \to 4\ell \) due to the addition of the \( q\bar{q} \to ZZ \to 4\ell \) channel which receives no BSM correction.

To illustrate the discriminating power of the ME-based kinematic variable introduced in (3.1) we present in figure 3 the results for the \( D_S \) spectra in the SM and beyond. The
Figure 2. $m_{4\ell}$ spectra in the SM (dashed black) as well as for three Higgs portal model scenarios (1.1) assuming $c_\phi = 3$ and $m_\phi = 70$ GeV (solid red), $m_\phi = 100$ GeV (solid blue) and $m_\phi = 150$ GeV (solid green). The left (right) plot shows results for $gg \to ZZ \to 4\ell$ ($pp \to ZZ \to 4\ell$) production. All distributions correspond to QCD-improved predictions and LHC collisions at a centre-of-mass energy of $\sqrt{s} = 14$ TeV. The lower panels depict the ratios between the BSM distributions and the corresponding SM predictions. The shown predictions have been obtained by means of (3.2) and the choices for the Higgs portal model parameters are those from before, apart from $m_\phi = 70$ GeV which is replaced by $m_\phi = 200$ GeV. One observes that compared to the SM spectrum the BSM distributions are shifted to lower values of $D_S$. This is a simple consequence of the fact that the correction $\Sigma(\hat{s}) - \delta m_h^2$ tends to reduce the $gg \to h^* \to ZZ \to 4\ell$ amplitude and thus $P_h$ in (3.1). As a result of the sharp cut-off of the SM distribution at $D_S \simeq -3.5$, the relative BSM effects in the $D_S$ spectra for $gg \to ZZ \to 4\ell$ turn out to be large, easily exceeding 100% for the chosen benchmark values of $c_\phi$ and $m_\phi$. As illustrated in the right panel of figure 3, adding the $q\bar{q} \to ZZ \to 4\ell$ channel to the predictions for the $D_S$ distributions notably reduces the relative size of the Higgs portal corrections. Still assuming $c_\phi = 3$, the BSM effects reach the level of around 200%, 10% and 5% in the case of $m_\phi = 100$ GeV, $m_\phi = 150$ GeV and $m_\phi = 200$ GeV, respectively.

By comparing the relative modifications in the right panels of figures 2 and 3 it should be already clear that the four-lepton invariant mass $m_{4\ell}$ has a much weaker discriminating power than the variable $D_S$ in constraining interactions of the form (1.1). In order to make this statement quantitative we perform a shape analysis of both the $m_{4\ell}$ and $D_S$ spectrum following the method outlined in our earlier work [49]. Specifically, the significance $Z_i$ is calculated as a Poisson ratio of likelihoods modified to incorporate systematic uncertainties on the background using the Asimov approximation [73]:

$$Z_i = \left\{ 2 \left[ (s_i + b_i) \ln \left( \frac{(s_i + b_i)(b_i + \sigma_{b_i}^2)}{b_i^2 + (s_i + b_i)\sigma_{b_i}^2} \right) - \frac{b_i^2}{\sigma_{b_i}^2} \ln \left( 1 + \frac{s_i\sigma_{b_i}^2}{b_i(b_i + \sigma_{b_i}^2)} \right) \right] \right\}^{1/2}.$$

(4.1)
Figure 3. As figure 2 but for the QCD-improved ME-based discriminant $D_S$ as defined in (3.1) and (3.2). Furthermore, instead of $m_\phi = 70$ GeV a mass of $m_\phi = 200$ GeV is employed. For additional explanations see main text.

Here $s_i$ ($b_i$) represents the expected number of signal (background) events in bin $i$ of the $m_{4\ell}$ or $D_S$ spectrum and $\sigma_{b_i}$ denotes the standard deviation that characterises the systematic uncertainties of the associated background in that bin. To set bounds on $c_\phi$ as a function of $m_\phi$ we assume that the central values of a future measurements of the two relevant distributions will line up with the SM predictions. We hence employ

$$s_i = N_i(c_\phi) - N_i(0), \quad b_i = N_i(0), \quad \sigma_{b_i} = \Delta_i N_i(0).$$

(4.2)

The total significance $Z$ is obtained by adding the individual $Z_i$ values in quadrature. Parameter regions with a total significance of $Z > \sqrt{2}\text{erf}^{-1}(CL)$ are said to be excluded at a given confidence level CL. Here $\text{erf}^{-1}(z)$ denotes the inverse error function. In our shape analyses, we consider 23 bins of size of 20 GeV with four-lepton invariant masses in the range $140$ GeV $< m_{4\ell} < 600$ GeV and 27 bins of equal size of 0.2 that cover the range $-4.9 < D_S < 0.5$ in the case of $m_{4\ell}$ and $D_S$, respectively.

A crucial ingredient in our analysis will turn out to be the systematic uncertainties $\sigma_{b_i}$ on the background as parametrised by the parameters $\Delta_i$ in (4.2). In the case of the HL-LHC shape fits, we will employ the two different choices $\Delta_i = \Delta = 8\%$ and $\Delta_i = \Delta = 4\%$ of bin-independent systematic uncertainties. These choices can be motivated by recalling that the systematic uncertainties that ATLAS quotes in the HL-LHC study [74] for the on-shell $gg \to h \to ZZ$ signal strength amount to $5.0\%$ and $3.9\%$ in the baseline scenario S1 and S2 for the expected total systematic uncertainties. The corresponding systematic uncertainties quoted in the CMS work [69] are $7.3\%$ and $4.1\%$. Since the dominant Higgs portal corrections in $D_S$ are associated to kinematic configurations with $m_{4\ell}$ around $2m_\phi$, we believe that for not too heavy $\phi$, theoretical predictions of the $D_S$ spectra will reach
Figure 4. 95% CL limits on $|c_\phi|$ as a function of $m_\phi$ derived from the binned-likelihood analysis of the $m_4\ell$ (green lines) and the $D_S$ (blue lines) spectrum at the HL-LHC. The solid (dashed) curves are obtained assuming a systematic uncertainty of $\Delta = 8\%$ ($\Delta = 4\%$). See main text for additional details.

an accuracy that is very similar to the systematics that is expected to be achievable at the HL-LHC in the case of on-shell $gg \to h \to ZZ$ production. Notice that the BSM effects in the $m_4\ell$ spectrum also receive important corrections in the region $m_4\ell > 2m_t$ as can be seen from the plots in figure 2. Given the limitations (cf. [60, 75, 76]) of the state-of-the-art SM predictions of $pp \to ZZ$ production for kinematic configurations above the two top-quark threshold, achieving the assumed systematic uncertainties of $\Delta = 8\%$ and $\Delta = 4\%$ is certainly more challenging in the case of the $m_4\ell$ distribution. The steady progress of perturbative QCD calculations, in particular the exact evaluations of the two-loop on-shell amplitudes for $gg \to ZZ$ involving top quarks [77, 78] makes us, however, confident that systematic uncertainties in the ballpark of 10% or below are attainable till 3 ab$^{-1}$ of data are collected at the HL-LHC.

The plot in figure 4 displays the results of our binned-likelihood analysis when applied to the $m_4\ell$ (green lines) and the $D_S$ (blue lines) distribution. Given the strong constraints on $c_\phi$ from on-shell Higgs boson decays into invisible [17–21] or undetected [4, 12] final states, we only consider $m_\phi$ values above the Higgs threshold at $m_h/2$. The shown 95% CL limits correspond to our HL-LHC projections assuming the full expected integrated luminosity of 3 ab$^{-1}$ at $\sqrt{s} = 14$ TeV. The solid (dashed) exclusion lines have been obtained for a systematic uncertainty of $\Delta = 8\%$ ($\Delta = 4\%$). As anticipated, the exclusions that derive from the binned-likelihood analysis of the $m_4\ell$ spectrum are significantly weaker than those that follow from the $D_S$ distribution. It is also evident from the figure that the size of the assumed systematic uncertainties plays a non-negligible role in the extraction of the 95% CL limits in the $m_\phi$–$|c_\phi|$ plane, in particular, if the $m_4\ell$ spectrum is used to discriminate between the BSM signal and the SM background. We elaborate on this point further in appendix A. In this context, we also add that our bounds following from the
binned-likelihood analysis of the $m_{4\ell}$ distribution agree roughly with the HL-LHC limits presented in [41, 42] if one takes into account that these articles have considered the complex Higgs portal $|H|^2|\Phi|^2$. A thorough comparison with the latter results is however not possible because a discussion of systematic uncertainties is missing in the works [41, 42]. Notice finally that the bounds on $|c_\phi|$ that follow from our $D_S$ likelihood-analysis have a non-trivial behaviour for $m_\phi \lesssim 100$ GeV. This feature is related to the interference between the BSM signal and the SM background.

5 HE-LHC and FCC analyses

In the following we repeat the numerical analysis performed at the end of the last section for the HE-LHC and the FCC. In the case of the HE-LHC (FCC) we assume a centre-of-mass energy of $\sqrt{s} = 27$ TeV ($\sqrt{s} = 100$ TeV) and an integrated luminosity of $15 \text{ ab}^{-1}$ ($30 \text{ ab}^{-1}$). Apart from the $m_{4\ell}$ window which we enlarge to 1000 GeV (1500 GeV) at the HE-LHC (FCC), the selection cuts and detection efficiencies in our HE-LHC and FCC $pp \to ZZ \to 4\ell$ analyses resemble the ones spelled out at the beginning of section 4.

Possible reductions of the statistical uncertainties due to improvements in the HE-LHC and FCC detectors such as extended pseudorapidity coverages [79, 80] are not considered in our numerical analysis. We also take the values of the $K$-factors quoted in section 3 that have been obtained for LHC collisions to calculate QCD-improved predictions for the kinematic variable $D_S$ a la (3.2). In view of the fact that the assumed systematic uncertainties largely determine the HE-LHC and FCC reach in constraining Higgs portal interactions of the form (1.1), we believe that these simplifications are fully justified. Moreover, since we have seen at the end of the last section that the ME-based kinematic discriminant $D_S$ offers a significantly better sensitivity compared to $m_{4\ell}$, we will below only consider the former observable when determining the disfavoured regions in the $m_\phi - |c_\phi|$ plane.

The HE-LHC and FCC results of our shape fit to the $D_S$ distribution are displayed in figure 5. Like in the case of the HL-LHC we show results assuming different baseline scenarios for the assumed systematic uncertainties. In the case of the HE-LHC we employ $\Delta = 4\%$ and $\Delta = 2\%$, while in our FCC analysis we use $\Delta = 2\%$ and $\Delta = 1\%$. These systematic uncertainties can be motivated by noticing that the systematic uncertainties at the HE-LHC should be at least as small as those expected ultimately at the HL-LHC and that the FCC has a target precision of $1.8\%$ for the $pp \to ZZ \to 4\ell$ channel [81]. Envisaging further theoretical and experimental progress a final systematic uncertainty of $1\%$ at the FCC does therefore not seem inconceivable. From the different curves one again sees that the size of the assumed systematic uncertainties plays a notable role in determining the collider reach. Numerically, we find that halving the systematic uncertainties at the HE-LHC (FCC) leads to improvements of the 95\% CL bounds on $|c_\phi|$ of around 25\% (30\%) at $m_\phi \approx 100$ GeV and about 20\% (25\%) at $m_\phi \approx 250$ GeV. The gain in statistical power of the FCC compared to the HE-LHC is however also visible from the figure with the FCC bound at $m_\phi \approx 250$ GeV being better by roughly 25\% than that of the HE-LHC assuming the same systematic uncertainties of $\Delta = 2\%$. This trend continues at higher values of the real scalar mass reaching up to almost 35\% at $m_\phi \approx 400$ GeV.
Figure 5. 95% CL limits on $|c_\phi|$ as a function of $m_\phi$ derived from the binned-likelihood analysis of the ME-based kinematic discriminant $D_S$. The red and orange exclusions illustrate our HE-LHC and FCC projections, respectively. The systematic uncertainties that have been assumed to obtain the different bounds are shown next to the lines and vary between $\Delta = 4\%$ and $\Delta = 1\%$. Further details are given in the main text.

6 Discussion and outlook

In figure 6 we compare the HL-LHC reach of different search strategies in the $m_\phi-|c_\phi|$ plane. The solid blue exclusion line corresponds to the 95% CL limits that derives from the proposed binned-likelihood analysis of the ME-based kinematic discriminant $D_S$ assuming a systematic uncertainty of $\Delta = 4\%$. The solid green line instead indicates the bound obtained in [32] from a study of off-shell Higgs production in the VBF channel. This analysis assumes a systematic uncertainty of $\Delta = 1\%$. At the HL-LHC, measurements of the global Higgs signal strength $\mu_h$ are expected to reach an accuracy of $\Delta = 2.4\%$ in the baseline scenario S2 for the expected total systematic uncertainties [74]. Utilising the quoted precision together with (2.2) and (2.3) leads at 95% CL to the solid red line. Another process that is sensitive to Higgs portal interactions of the form (1.1) is double-Higgs production as previously demonstrated in [28, 32, 39, 40, 43]. The 95% CL bound $\kappa_\lambda \in [0.18, 3.6]$ on the modifications $\kappa_\lambda = \lambda/\lambda_{\text{SM}}$ with $\lambda_{\text{SM}} = m_h^2/(2v^2) \simeq 0.13$ of the trilinear Higgs coupling as found by the CMS projection [82] implies $\mu_{hh} \in [0.7, 1.8]$ on the signal strength in double-Higgs production at the HL-LHC. By implementing the full one-loop corrections due to (1.1) into MCFM and imposing the latter bound we obtain the solid and dashed orange lines. Consult appendix B for further details. Finally, the dashed black line corresponds to the naturalness bound $|c_\phi| = \sqrt{3}y_t^2 = 1.5$ discussed in section 1.

From figure 6 it is evident that for $m_\phi \lesssim 90$ GeV the VBF and $\mu_h$ projections provide nominally the best constraints at the HL-LHC. In the case of $m_\phi \gtrsim 90$ GeV, on the other hand, double-Higgs production at the HL-LHC typically allows to set the most stringent constraints on the parameters appearing in (1.1). Notice also that the $D_S$ constraint
Figure 6. Comparison of the HL-LHC reach of different search strategies in the $m_{\phi} - |c_\phi|$ plane. The solid blue, solid green and solid red line correspond to the 95% CL limits that derive from our binned-likelihood analysis of the ME-based kinematic discriminant $D_S$, the VBF analysis performed in [32] and a hypothetical measurement of the global Higgs signal strength $\mu_h$, respectively. If applicable the assumed systematic uncertainties or accuracies are indicated. The parameter spaces above the coloured lines are disfavoured. The region bounded by the solid (dashed) orange line follows from imposing that the signal strength in double-Higgs production obeys $\mu_{hh} \in [0.7, 1.8]$ for $c_\phi > 0$ ($c_\phi < 0$). The dotted black line corresponds to the bound $|c_\phi| = \sqrt{3} y_t^2 = 1.5$ that derives from naturalness arguments in models of neutral naturalness. For more details see main text.

provides the best sensitivity for $90 \, \text{GeV} \lesssim m_\phi \lesssim 120 \, \text{GeV}$ and stronger constraints than VBF and $\mu_h$ for $m_\phi \gtrsim 90 \, \text{GeV}$. The fact that the constraints that stem from double-Higgs production are not symmetric under $c_\phi \leftrightarrow -c_\phi$ is readily understood by noting that the Higgs portal corrections to the $gg \rightarrow hh$ amplitude involve both terms proportional to $c_\phi^3$ and $c_\phi^2$. In fact, integrating out the real scalar $\phi$ leads to the following one-loop modification of the trilinear Higgs coupling (see for instance [32, 83]):

$$\kappa \lambda \simeq 1 + \frac{v^2 c_\phi^2}{12\pi^2 m_\phi^2} \left( \frac{v^2 c_\phi}{m_h^2} - \frac{7}{12} \right), \quad (6.1)$$

where the terms in brackets interfere destructively (constructively) for $c_\phi > 0$ ($c_\phi < 0$). We add that the numerical value of the second term in brackets depends on the definition and the kinematics of the trilinear Higgs vertex and that the value in (6.1) is obtained from the full one-loop form factor (B.2) assuming two on-shell external Higgs bosons. The intricate dependence of the $gg \rightarrow hh$ amplitude on $m_\phi$ and $c_\phi$ also leads in the case of $c_\phi > 0$ to the island of disfavoured parameters starting at $m_\phi \simeq 145 \, \text{GeV}$ and $c_\phi \simeq 1.7$. This point is discussed in more detail in appendix B. Notice furthermore that all constraints shown in figure 6 depend in a non-negligible way on the assumed systematic uncertainties or accuracies. Finally, the VBF limit only applies if the new degrees of freedom produced in $h^* \rightarrow \phi\phi$ are collider stable and thus lead to a missing transverse energy ($E_T^{miss}$) signal.
Figure 7. Comparison of the HE-LHC (upper panel) and FCC (lower panel) reach of different search strategies in the $m_\phi$–$|c_\phi|$ plane. Besides the constraints shown in figure 6 also the 95% CL limit that follows from a precision measurement of the $Z\phi$ production cross section $\sigma_{Z\phi}$ is displayed in the case of the FCC as a solid magenta line. The colour coding and meaning of the other constraints resembles those in the former figure. Consult the main text for additional explanations.

at the HL-LHC. In view of these caveats one can conclude that to fully exploit the HL-LHC potential in probing Higgs portal interactions of the form (1.1) one should consider all direct and indirect probes displayed in figure 6. But even in such a case one sees that at the HL-LHC only theories compatible with the naturalness bound can be explored if the new particles that cancel the quadratic sensitivity of the Higgs mass are not heavier than $m_\phi \simeq 110$ GeV.

In the case of the HE-LHC and the FCC the sensitivity of the different search strategies to the Higgs portal parameters is shown in the two panels of figure 7. The displayed $D_S$ constraints assume systematic uncertainties of $\Delta = 2\%$ and $\Delta = 1\%$, while the VBF lim-
its taken from [32] include only statistical uncertainties. In the case of the global Higgs signal strength $\mu_h$, we employ $\Delta = 2\%$ and $\Delta = 1\%$ [81]. The 95% CL bounds on modifications of the trilinear Higgs coupling at the HE-LHC and the FCC are expected to be $\kappa_\lambda \in [0.7, 1.3]$ and $\kappa_\lambda \in [0.9, 1.1]$, respectively. See for example [84–86] for detailed discussions. The corresponding two-sided limits on the signal strength in double-Higgs production are $\mu_{hh} \in [0.80, 1.24]$ and $\mu_{hh} \in [0.93, 1.07]$. In addition, we show in the case of the FCC the exclusion that follows from an extraction of the $Z\ell$ cross section $\sigma_{Z\ell}$ with an accuracy of $\Delta = 0.2\%$ as a solid magenta line. Such a precision measurement should be possible at the $e^+e^-$ predecessor of the FCC running at a centre-of-mass energy of $\sqrt{s} = 240$ GeV with an integrated luminosity of $5 \, \text{ab}^{-1}$ [87]. The overall picture observed at the HE-LHC is very similar to that seen at the HL-LHC. Nominal the strongest constraint arises for $m_\phi \lesssim 170$ GeV ($m_\phi \gtrsim 170$ GeV) from VBF off-shell Higgs (double-Higgs) production, but the $D_S$ constraint also provides complementary sensitivity in particular for higher values of $m_\phi$. In the case of the FCC, one furthermore observes that a high precision measurement of $\sigma_{Z\ell}$ can provide additional relevant bounds in the $m_\phi - |c_\phi|$ plane. The combination of all constraints shown in the panels of figure 7 should allow to probe natural BSM theories of the form (1.2) if the new particles that cancel the quadratic sensitivity of the Higgs mass appear below approximately $m_\phi \simeq 200$ GeV ($m_\phi \simeq 300$ GeV) at the HE-LHC (FCC).

We add that the potential of CLIC and a muon collider in constraining Higgs portal interactions of the form (1.1) through VBF off-shell Higgs production has been studied in the article [32]. See also [23, 24, 26, 30] for similar analyses concerning the reach of future lepton colliders. While CLIC is not expected to improve the FCC bounds shown in the lower panel of figure 7 even when running at a centre-of-mass energy of $\sqrt{s} = 3$ TeV and collecting $3 \, \text{ab}^{-1}$ of data, a muon collider with $\sqrt{s} = 6$ TeV and $6 \, \text{ab}^{-1}$ ($\sqrt{s} = 14$ TeV and $14 \, \text{ab}^{-1}$) should allow to test natural theories of neutral naturalness up to $m_\phi \simeq 500$ GeV ($m_\phi \simeq 900$ GeV) thereby exceeding (significantly) the FCC reach.

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A Systematic uncertainties

In this appendix we discuss in more detail the prospects of the proposed binned-likelihood analyses of the $D_S$ spectra for the HL-LHC, the HE-LHC and the FCC. In particular, we examine how different assumptions on the systematic uncertainties affect the resulting constraints on the parameter space of the Higgs portal model (1.1). In figure 8, we show
the projected 95% CL limits on $|c_\phi|$ derived from our $D_S$ analysis as a function of the assumed systematic uncertainty $\Delta$ for the three aforementioned colliders. The presented limits are obtained using the benchmark numerical values for the scalar masses indicated in the figure that vary between $100 \text{ GeV} \leq m_\phi \leq 200 \text{ GeV}$.

Figure 8 further illustrates the point already made in sections 4 and 5, that the assumptions on the systematic uncertainties $\Delta$ play a crucial role in constraining the $m_\phi-|c_\phi|$ parameter space by using the $D_S$ distribution as a kinematic discriminant. In particular, one observes that the enhanced statistical power provided by the HE-LHC and the FCC, which results from the increased centre-of-mass energy and integrated luminosity of these machines compared to the HL-LHC, can only be fully exploited if systematic uncertainties are under control. For instance, in the case of $m_\phi = 100 \text{ GeV}$ the sensitivity gain between the HL-LHC and the FCC is around 17% for $\Delta = 20\%$, while for $\Delta = 1\%$ the improvement amounts to about 41%. Similar numbers of approximately 26% and 51% are found for $m_\phi = 150 \text{ GeV}$ and $m_\phi = 200 \text{ GeV}$, implying that the gain in sensitivity between different colliders is to first approximation mass-independent for the low values of $m_\phi$ considered in the figure.

B Details of the double-Higgs calculation

At the one-loop level the $gg \to hh$ process receives contributions from virtual $\phi$ exchange in propagator and vertex diagrams as well as counterterm contributions associated to wave function, mass and tadpole renormalisation (see [39, 43] for details). In the on-shell scheme
the combined corrections involving the Wilson coefficient $c_\phi$ can be written as a finite shift:

$$\lambda_{\text{SM}} \rightarrow \lambda_{\text{SM}} \left[ 1 + \delta(\hat{s}) \right].$$

(B.1)

Here $\lambda_{\text{SM}} = m_h^2/(2v^2)$ is the tree-level expression for the trilinear Higgs coupling in the SM and the $\hat{s}$-dependent form factor is given by

$$\delta(\hat{s}) = -\frac{v^2 c_{\phi}^2}{24\pi^2 m_h^2} \left( 1 + \frac{3m_\phi^2}{\hat{s} - m_h^2} \right) \left[ B_0(\hat{s}, m_\phi^2, m_\phi^2) - B_0(m_h^2, m_\phi^2, m_\phi^2) \right]$$

(B.2)

$$-\frac{v^4 c_{\phi}^4}{6\pi^2 m_h^2} C_0(m_h^2, m_h^2, \hat{s}, m_\phi^2, m_\phi^2, m_\phi^2) - \frac{v^2 c_{\phi}^2}{8\pi^2} \left. \frac{d}{d\hat{s}} B_0(\hat{s}, m_\phi^2, m_\phi^2) \right|_{\hat{s}=m_h^2},$$

with the $A_0$, $B_0$ and $C_0$ functions are one-, two-, and three-point Passarino-Veltman scalar integrals defined as in [50, 51]. Our result (B.2) agrees with [39, 43], after fixing a sign error in (12) of [43]. Notice that after integrating out the scalar field $\phi$ by expanding the on-shell form factor $\delta(2m_\phi^2)$ up to the first power in $m_h^2/m_\phi^2$, one recovers the approximate correction for $\kappa_\lambda$ as given in (6.1).

To obtain predictions for double-Higgs production we have implemented the analytic results (B.2) at the amplitude level into MCFM. We then perform sensitivity scans in the parameters $c_\phi$ and $m_\phi$, using the setup discussed at the beginning of section 4, but fixing the renormalisation and factorisation scales $\mu_R$ and $\mu_F$ to the value $2m_h$. In figure 9 we show results for the signal strength $\mu_{hh}$ in double-Higgs production for three different values of $m_\phi$ as a function of $c_\phi$. The displayed curves correspond to the results obtained at the FCC. Two feature of the shown predictions deserve some comments. First, due to the $c_\phi^3$
and $c_\phi^2$ dependence of (B.2) the signal strengths $\mu_{hh}$ are not symmetric under $c_\phi \leftrightarrow -c_\phi$. Second, the functional form of $\mu_{hh}$ depends also sensitively on the mass $m_\phi$. For low $\phi$ masses as illustrated by the choice $m_\phi = 70$ GeV in the figure, the signal strength $\mu_{hh}$ has two minima, one at around $c_\phi \simeq -1.1$ and another one at $c_\phi \simeq 1.0$. This feature leads to the orange exclusions in the lower plot in figure 7 at $|c_\phi| \simeq 1$. For larger values of $m_\phi$ the signal strengths $\mu_{hh}$ have instead only a single minimum at positive values of $c_\phi$. Notice that if the value of $\mu_{hh}$ at this minimum is incompatible with the experimental allowed range, such as happens to be the case for example for $m_\phi = 130$ GeV at the FCC, increasing/decreasing the value of $c_\phi$ will always result in $\mu_{hh}$ values that are consistent with experiment. This feature leads to the orange exclusions shown in the plots of figures 6 and 7 that are relevant for $c_\phi > 0$ and separated by a funnel of viable solutions.

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