FIVE-BRANE CONFIGURATIONS, CONFORMAL FIELD THEORIES
AND THE STRONG-COUPLING PROBLEM *

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Abstract

Decoupling limits of physical interest occur in regions of space–time where the string coupling diverges. This is illustrated in the celebrated example of five-branes. There are several ways to overcome this strong-coupling problem. We review those which are somehow related to two-dimensional conformal field theories. One method consists of distributing the branes over transverse space, either on a circle or over a sphere. Those distributions are connected to conformal field theories by T-dualities or lead to a new kind of sigma model where the target space is a patchwork of pieces of exact conformal-field-theory target spaces. An alternative method we discuss is the introduction of diluted F-strings, which trigger a marginal deformation of an AdS3 × S3 × T4 background with a finite string coupling. Our discussion raises the question of finding brane configurations, their spectrum, their geometry, and their interpretation in terms of two-dimensional conformal models.

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1 Some motivations and ideas

String theory unifies gauge interactions and gravity. Until recently, it was developed along two main streams of quite distant goals. On the one hand, string phenomenology aimed at recovering the standard model and other supersymmetric low-energy theories. On the other hand, string gravity was focusing on the search for a variety of backgrounds that could help in understanding cosmology and probe gravity at short distances (study quantum effects, singularities, black-hole thermodynamics ...).

The discovery of branes has deeply modified the previous landscape of interests in a variety of ways. Fundamental constituents are now BPS objects like F1-, NS5-, or Dp-branes, and the understanding of string theory goes through the investigation of these objects, in all possible ways [1, 2].

A prime conceptual achievement of the above discovery was to capture the string spectrum beyond perturbation theory. In turn this allowed for the unification of various string vacua through dualities, and led to M-theory that has brought its own fundamental objects: the M2- and M5-branes [3].

All these extended objects have introduced new insights and techniques. In this framework, phenomenology and gravity motivations and methods have become very close. The safest tool for string phenomenology is exact conformal field theory (CFT), since it guarantees an absolute handling over the (perturbative) string spectrum. This includes orbifold models, fermionic constructions ... For gravity purposes these methods are not convenient because they often lack of clear geometrical interpretation which is, however, straightforward in $O(\alpha')$ two-dimensional sigma-models. Those turn out to be more useful for cosmology searches.

The presence of branes, blurs the latter Cartesian picture. Indeed, branes act like impurities, which alter the string spectrum and modify the background. A plethora of possibilities for phenomenology with an up-to-date point of view on low supersymmetry-breaking scale, hierarchies ..., as well as new geometrical set-ups with non-trivial gravitational (and other background) fields therefore appear. Hence, a fundamental and unifying aim emerges: find brane configurations, their spectrum, their geometry, and CFT interpretation [4, 5, 6, 7, 8]. The Randall–Sundrum model is a good illustration of this line of thought: it provides a brane configuration with both geometrical and phenomenological intrinsic value.

Another important drawback of the study of branes was the discovery of decoupling limits different from the usual low-energy limits [9, 10, 11]. The existence of such limits is quite unexpected for simple reasons. String spectra are highly constrained (GSO projections, modular invariance, supersymmetry issues ...). Gravity and gauge sectors appear therefore in an intricate way. It looks very unlikely that one could find a limit – other than going to low energies – which would enable us to disentangle this imbroglio of states and separate those sectors; tracing back their respective origins to some geometric feature looks even more out of reach. Much like orbifold fixed points, branes contribute part of the spectrum, though they truly appear to carry it in the semi-classical limit only. Put differently, not only many sectors must be considered to create a string, but these sectors cannot be chosen at wish or designated to originate from a particular geometrical object. Any ad hoc construction of...
this type is not expected to survive – as a string – $O(\alpha')$ corrections. The Randall–Sundrum model is again a good example of this situation since no reliable string realization has been provided so far, which can reproduce its features. The role of brane configurations which possess a clear CFT interpretation appears again to be of major importance.

Despite this highly constrained structure, there are limits such that the spectrum (i) is dominated by excitations leaving on the branes, (ii) cannot be described by means of ordinary low-energy quantum field theory, and (iii) does not contain the gravitational sector. This decoupling of gravity is paradoxically an asset. The reason for that is that, under these circumstances, old ideas about holography become operational.

Unfortunately, these decoupling limits occur in regions of space–time where the string coupling diverges. Although these divergences are physical (they are even intimately related to the very existence of the decoupling) they set the limits to the confidence of perturbative string theory. Alternative brane configurations or regulated descriptions must be found, which can be analyzed as much as possible, by means of exact CFT.

The motivation of this short note is to illustrate these issues in the celebrated example of five-branes, and review various ways to overcome the strong-coupling problem within frameworks that are related, one way or the other, to two-dimensional conformal models.

2 The five-brane solutions in type II superstring

The ten-dimensional effective action reads, in the Einstein frame:

$$S^{(10)} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g^{(10)}} \left( R^{(10)} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{12} e^{-\gamma \phi} H^2 \right).$$

(2.1)

Here $\phi_\gamma$ is the dilaton field and $\gamma = \pm 1$ corresponds to the two distinct NS-NS or R-R three-form field strengths $H$ in type IIB theory (type IIA allows only for $\gamma = +1$). We do not introduce any gauge field, which means in particular that the branes under consideration carry no other charge than NS-NS or R-R.

The canonical five-brane solutions are of the form

$$ds^2 = h(r)^{-1/4} (-dt^2 + d\vec{x}^2) + h(r)^{3/4} (dr^2 + r^2 d\Omega_3^2),$$

(2.2)

where $\vec{x} \equiv \{x^5, x^6, \ldots, x^9\}$ are Cartesian coordinates in a five-dimensional Euclidean flat space and $d\Omega_3$ is the metric on a unit-radius three-sphere. Together with the radial (dimensionless) coordinate $r$, the latter is transverse to the five-brane. Poincaré invariance within the five-brane world-volume is here automatically implemented.

The ansatz (2.2) indeed minimizes (2.1) provided the dilaton and antisymmetric tensor fields are also expressed in terms of the function $h(r)$:

$$\phi_\gamma(r) = \frac{\gamma}{2} \log h(r),$$

(2.3)

$$H = -r^3 h' \Omega_3,$$

(2.4)

where $\Omega_3 \equiv \sin^2 \theta \sin \varphi \, d\theta \wedge d\varphi \wedge d\omega$ is the volume form on the three-sphere, and $dH = 0$ except at the location of the branes which act like sources. Finally, $h(r)$ is a harmonic
function satisfying
\[ h = 0. \]

The general solution is therefore
\[ h(z) = h_0 + \frac{N}{r^2}, \quad (2.5) \]

with \( N \) and \( h_0 \) two integration constants, which are both positive for \( h(r) \) be positive. The first one, \( N \geq 0 \), is interpreted as the total number of five-branes, sitting at \( r = 0 \), (integral of \( dH \), vanishing everywhere except for \( r = 0 \)).

If no five-branes are present, we recover flat-space with constant dilaton and no antisymmetric tensor. This background is the target space of an exact, albeit trivial, two-dimensional conformal sigma model.

For \( h_0 = 0 \), the transverse geometry is an \( S^3 \) of radius \( L = \sqrt{N} \) with a covariantly constant antisymmetric tensor (proportional to the three-sphere volume form) plus a linear dilaton:
\[
\begin{align*}
  ds^2 &= -dt^2 + dx^2 + dy^2 + N \Omega_3^2, \\
  \phi_\gamma &= \frac{\gamma}{2} \log N - \frac{\gamma y}{\sqrt{N}}
\end{align*}
\]
(we have introduced \( y = \sqrt{N} \log r \), and (2.6) holds in the sigma-model frame).

Type-II strings in the geometry (2.6), (2.7) is an exact \( N = 4 \) superconformal theory \[ \{1, 12, 13, 14\} \], which implies the existence of \( N = 2 \) space-time supersymmetry in six dimensions (1/2 of the initial supersymmetry). From the world-sheet point of view, this is an exactly conformal two-dimensional sigma model if \( \gamma = +1 \) (NS). The target space of the latter is the ten-dimensional manifold \( U(1)_Q \times SU(2)_k \times M^6 \). The last factor is the flat six-dimensional longitudinal space-time, \( k = N - 2 \) is the level of the \( SU(2) \) current algebra, and \( Q = -\gamma/\sqrt{k+2} \) is the background charge of the radial transverse coordinate. This conformal field theory has been extensively investigated in the past. Its spectrum is obtained by appropriate combinations of \( SU(2)_k \) and Liouville characters. Discrete Liouville representations generate short \( N = 4 \) multiplets, while continuous representations lead to long (massive) multiplets. In the semiclassical limit, i.e. at large \( k \), one can trace back the origin of the various states: discrete states are mostly confined in the vicinity of the brane, while states from the continuous Liouville spectrum are delocalized in the transverse directions. The latter are called \textit{bulk states}, while the former are the \textit{brane states}, which are those that survive in the decoupling limit, as excitations of the little string theory. It should however be stressed once again that separating the spectrum into “bulk” and “brane” states is usually arbitrary, and at best valid in certain regimes only. In fact, for an exact conformal field theory, even the geometry of the target space itself is not clearly defined in all regimes.

The physical solution for the neutral five-brane \( \Pi \) has non-vanishing \( N \) and \( h_0 \). The corresponding background is therefore asymptotically flat. This background does not correspond to any (known) exact conformal field theory. However, it smoothly connects two regions of space–time where the string propagation leads to two different, and both exact
conformal field theories (at least for the NS branes). Other backgrounds, interpolating between various exact-CFT target spaces, will be discussed in the Secs. 3 and 5. Somehow, this seems to be a natural feature of physical geometries.

We now come to the strong-coupling problem. In the background described by Eqs. (2.6) and (2.7), the string coupling constant, $g_s \equiv \exp \phi = h^{\gamma/2}$, becomes infinitely large at the location of the NS5-branes ($r = 0$), while for the D5-brane background ($\gamma = -1$) the same phenomenon occurs at $r \to +\infty$, i.e. in the asymptotic region, far away from the sources (see Fig. 1). In these regions of space–time the string perturbation breaks down and the very concept of worldsheet becomes questionable.

Figure 1: The string coupling of solution (2.5) diverges at $r \to 0$ for the NS5-branes ($\gamma = +1$). When the sources are D5-branes ($\gamma = -1$) the divergence occurs at $r \to \infty$, provided $h_0 = 0$ (we assume $h_0 < 1$).

Although technically worrisome, the divergence of the string coupling is not a pathology. Consider type-IIB string theory in the background of NS5-branes. The $r \to 0$ limit corresponds to the infra-red regime of the NS5’s little string theory. In this regime, the little strings are described by a $(1, 1)$ six-dimensional super-Yang–Mills, which is free. Since the latter is holographically dual to the original string theory in the NS5-brane background, the string coupling must diverge where the super-Yang–Mills coupling vanishes, i.e. at $r \to 0$. This consistency check shows that the behavior of the string coupling and in particular its divergence, is deeply related to the structure of the theory itself. It can even be avoided within the theory by performing an S-duality that switches to the D5-brane background, where the $g_s \to 0$ in the vicinity of the horizon. The caveats in this method are (i) the absence of exact conformal field theory techniques for R–R backgrounds, and (ii) the treatment of the regions where $g_s \sim 1$.

Similar considerations also apply to the type-IIA NS5-brane backgrounds. There, the little string theory in the infra-red is the fixed point of a $(2, 0)$ six-dimensional superconformal theory. By holography, this is dual to a configuration of M5-branes spread over a circle of vanishing radius. This is the setup that should be worked out in order to capture the strong-coupling regime of NS5-branes in type IIA. It necessitates a complete understanding of M theory in the AdS$_7 \times S^4$ background, which is presently out of reach.
In the absence of a satisfactory non-perturbative treatment of string theory, the latter comments exhaust what can be done for understanding the strong-coupling regime, within the above five-brane backgrounds. One can however slightly deviate from these backgrounds, or embed them in more general webs of five-brane configurations. This can be helpful and will be the subject of the following analysis, where our discussion will mostly focus on Neveu–Schwartz backgrounds since those can possibly lead to exact conformal field theories.

3 Scanning the web of NS5 (or D5) branes

A rich variety of configurations can be reached by distributing the five-branes in transverse subspaces. This generates new backgrounds, which create remarkable webs of exact conformal field theories, allow for new decoupling limits, and are ultimately useful for understanding the strong-coupling problem.

The paradigm we will be following here is the distribution of branes on a circle [15]. There are several degrees of approximation at which this system can be studied. We will assume, as previously, a large number \( N \) of branes, which makes sensible the semi-classical treatment of the system. One can consider the full geometry, or magnify the resolution in the vicinity of the circle. In this region the geometry will be sensitive to the nature of the distribution. If the \( N \) five-branes are distributed on \( \ell \) points carrying \( n \) branes each (\( N = \ell \times n \)), several regimes (decoupling limits) are possible [15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. In particular, if the discrete structure remains visible, the corresponding little string theory undergoes a Higgs mechanism, where a \( U(1) \) (translation along the circle) is broken to \( \mathbb{Z}_n \).

For the purposes of the present note, we will consider the simplest situation, where we are close enough to the horizon to avoid the asymptotic structure (such as e.g. \( h_0 \) terms in Eq. 2.5)), but at a distance where the discrete distribution is not visible. In this regime, the transverse near-horizon metric and Kalb–Ramond field read:

\[
\begin{align*}
\text{ds}^2 &= N \left( d\rho^2 + d\theta^2 + \frac{\tan^2 \theta \, d\psi^2 + \tanh^2 \rho \, d\tau^2}{1 + \tanh^2 \rho \tan^2 \theta} \right), \\
B &= \frac{N}{1 + \tanh^2 \rho \tan^2 \theta} \, d\tau \wedge d\psi.
\end{align*}
\]

We have used the transverse coordinates \( 0 \leq \rho < \infty \), \( 0 \leq \theta \leq \pi/2 \) and \( 0 \leq \psi, \tau \leq 2\pi \), in which the sources are located at \( \rho = 0 \) and \( \theta = \pi/2 \).

For NS5-branes, the dilaton field is given in

\[
e^{-2\phi} = e^{-2\phi_0} \left( \cosh^2 \rho \cos^2 \theta + \sinh^2 \rho \sin^2 \theta \right),
\]

and \( g_s \) diverges on the circle carrying the sources\(^1\).

Much like the simpler configuration of branes analyzed in Sec. 2, the background (3.1)–(3.3) turns out to interpolate between various corners, where it coincides with the target

\(^1\)The constant \( \phi_0 \) is related to the coupling \( g_{\text{YM}} \) of the little string theory in the usual manner: \( \exp -2\phi_0 = \left( g_{\text{YM}}^2 U_0^2 / N \right)^1 \), where \( U_0 \) is a constant.
space of certain exact two-dimensional sigma models. The above space can be scanned as follows:

1. The $\rho = \text{constant}$ subspace

The three-dimensional background obtained by setting $\rho = \rho_0$ is a squashed three-sphere. This is the target space of an $SU(2)_k$ WZW model, marginally deformed with an exact $(1, 1)$ operator, bilinear in some left and right $SU(2)_k$ currents: $J \bar{J}$. In the four-dimensional transverse space under consideration, this marginal deformation is dynamical, in the sense that the corresponding parameter is promoted to genuine coordinate, i.e. a two-dimensional dynamical field [25, 26, 27, 28].

There are two limits of interest in the continuous line of $SU(2)_k$ marginal deformations: (a) $\rho \to 0$. The limiting sigma model is in this case $U(1) \times SU(2)_k/U(1)$. The first factor is a line\(^2\), while the target space of the coset factor is the bell geometry (see Fig. 2):

$$ds^2 = N \left( d\theta^2 + \tan^2 \theta d\psi^2 \right).$$

(3.4)

The string coupling

$$g_s = e^{\phi} = \frac{e^{\phi_0}}{\cos \theta}$$

(3.5)

diverges on the boundary of the bell, which is precisely the circle where the branes are distributed.

(b) $\rho \to \infty$. The corresponding three-dimensional target space is now the undeformed $S^3$ described by the $SU(2)_k$ WZW model.

2. The $\theta = \text{constant}$ subspace

The interpretation of the three-dimensional space $\theta = \theta_0$ as the target space of a conformal sigma model is not clear except for two limiting values of $\theta_0$ [29, 30, 31].

(a) $\theta = 0$. The sigma model is now $U(1) \times SL(2, \mathbb{R})_k/U(1)_{\text{axial}}$ and the coset factor describes the cigar geometry:

$$ds^2 = N \left( d\rho^2 + \tanh^2 \rho d\tau^2 \right),$$

(3.6)

It corresponds to the coordinate $\tau$ in the metric [81], appropriately rescaled, though.
with coupling
\[ g_s = e^\phi = \frac{e^{\phi_0}}{\cosh \rho} \]
everywhere finite (see Fig. 3).

(b) \( \theta = \pi/2 \). The difference with the previous case resides in the type of gauging which is used in the coset. In the case at hand, this is a vector gauging with \textit{trumpet} geometry
\[ ds^2 = N \left( d\rho^2 + \cotanh^2 \rho d\psi^2 \right), \]
and coupling
\[ g_s = e^\phi = \frac{e^{\phi_0}}{\sinh \rho} \]
diverging at \( \rho \to 0 \).

Figure 3: The trumpet and cigar geometries, with angular and radial coordinates \( \psi \) or \( \tau \) and \( \rho \) respectively. The dilaton is finite everywhere on the cigar. It diverges, however, at the boundary of the trumpet \( \rho \to 0 \).

The underlying two-dimensional conformal structure of the background (3.1)–(3.3) becomes somehow more transparent after a chain of orbifold operations and T-dualities: \( \mathbb{Z}_N^\tau \to T^\tau \to \mathbb{Z}_N^\psi \to T^\psi \). The resulting geometry,
\[ ds^2 = N \left( d\rho^2 + d\theta^2 + \tan^2 \theta \, d\tau^2 + \tanh^2 \rho \, d\psi^2 \right) \]
is the target space of the following conformal model:
\[ \frac{SU(2)_k}{U(1)} \times \frac{SL(2, \mathbb{R})_{k+4}}{U(1)_{\text{axial}}}. \]

The latter has \( N = 4 \) superconformal invariance, and \textit{no strong-coupling problem} since the dilaton is constant; it has been analyzed in [13, 14, 32]. Moreover, the model (3.10) is believed
to be T-dual of the conformal model $SU(2)_k \times U(1)_Q$ met in previous Sec. 2 as near-horizon geometry of $N$ coinciding NS5-branes. With this starting point, several holographic issues of such a distribution have been investigated in [16, 17, 33].

Although T-duality in curved space is a subtle issue, and despite the perturbative nature (in $\alpha'$) of many of the previous results\(^3\), the whole picture appeals for further investigation of the connections that might relate the various brane distributions, and the role played by the underlying two-dimensional sigma models, even if they appear as exactly conformal only in some corners of the target space. This will clearly help in understanding the issue of the strong-coupling regime, which does not systematically occur.

4 Spreading the NS5 (or D5) branes over an $S^3$

So far we have discussed the effect of distributing five-branes on a circle in transverse space. The near-horizon geometries of the backgrounds generated in that way exhibit remarkable properties, and although they do not seem to be directly related to any known conformal model, they possess smoothly connected corners (subspaces) where we indeed recover CFT features. Exact conformal description, reproducing the entire target space is possible only after T-dualities.

Concerning the issue of the dilaton behavior, strong-coupling regions still persist at the location of the sources. They disappear only after some T-duality operations, which lead, remarkably, to the target space of a CFT, $SU(2)_k/U(1) \times SL(2, \mathbb{R})_{k+4}/U(1)_{\text{axial}} \times M^6$, that turns out to be T-dual to the original canonical five-brane near-horizon, $U(1)_Q \times SU(2)_k \times M^6$, studied in Sec. 2. In some sense, this provides a way out to the original strong-coupling problem, although not a satisfactory one: in a curved non-compact background, T-duality may [27, 34] or may not be an exact symmetry\(^4\).

The picture in terms of five-branes on a circle may be therefore an oversimplification. In this section we shall focus on another option: distribute the branes over a three-dimensional transverse sphere. This configuration respects the transverse $SO(4) \simeq SU(2) \times SU(2)$ symmetry, regulates truly the coupling, and introduces a new kind of two-dimensional sigma-model whose target space is a “patchwork” of CFT target-space pieces.

The divergence of the ordinary Coulomb field can be avoided by assuming a spherically symmetric distribution of charge over a two-sphere centered at the original point-like charge. We can similarly introduce a distribution of five-branes over the transverse three-sphere \(^4\), at some finite radius, say $r = R$. This amounts in adding to the bulk action (2.1) a source term of the form:

\[
S_{\text{five-brane}} = -\frac{NT_5}{2\pi^2} \int d^{10}x \sin^2 \theta \sin \varphi \delta(r - R) \left( e^{-\gamma \phi_7/2} \sqrt{-\hat{g}(6)} + \hat{C}_6 \right),
\]

\(^3\)Coset metrics such as (3.4), (3.6) or (3.8) are expected to receive higher-order $\alpha'$ corrections.

\(^4\)An NS5-brane with one longitudinal direction wrapped on a circle is T-dual to flat space [35], although we have serious reasons to believe that the dynamics in this case is non-trivial. The Nappi–Witten pp-wave background [36], which is also T-dual to flat space [37], is not equivalent to flat space or a standard orbifold of it, and this can be asserted since its exact solution is known [38, 39].
where $\tilde{C}_6$ is the dual of the two-index antisymmetric tensor. Several remarks are in order here. In writing (4.1), we have chosen a gauge in which $(t, \vec{x})$ are the world-volume coordinates of the five-branes. Thus, the induced metric $\hat{g}^{(6)}_{ij}$ is just the reduction of the background metric $g^{(10)}_{\mu\nu}$ ($\mu, \nu, \ldots \in 0, 1, \ldots, 9$ and $i, j, \ldots \in 0, 5, \ldots, 9$). All five-branes are sitting at $r = R$, and are homogeneously distributed over the $S^3$. Their density is normalized so that the net number of five-branes be $N$.

The energy–momentum tensor of the source term (4.1) is

$$T^\mu_{\text{five-brane}}(x) = \frac{2}{\sqrt{-g^{(10)}}} \frac{\delta S_{\text{five-brane}}}{\delta g^{(10)}_{\mu\nu}(x)} = -\frac{NT_5^5}{2\pi^2} \sin^2 \theta \sin \varphi \delta(r - R) \epsilon^{\alpha\gamma\phi_\gamma} \delta_i^\mu \delta_j^\nu \hat{g}^{(6)}_{ij} \sqrt{\hat{g}^{(6)}_{\mu\nu}} g^{(10)}_{\mu\nu}.$$  

This enables us to write the full equations of motion resulting from action (2.1) plus (4.1). Expressed in the sigma-model frame, Eq. (4.1) exhibits the following dilaton coupling:

$$\exp^{-\frac{3+\gamma}{2} \phi_\gamma}.$$ For $\gamma = +1$ this is indeed the coupling of an NS5-brane, while for $\gamma = -1$ we recover the D5-brane. Introducing the same ansatz as before for the metric (Eq. (2.2)), the dilaton and the three-index tensor are given respectively by Eqs. (2.3) and (2.4), in terms of $h(z)$, which now solves

$$h(z)^\frac{3}{4} e^{4z} \Box h = -2N \delta(z - Z). \quad (4.2)$$

In writing the latter, we have introduced a new radial variable, $z = \log r \in \mathbb{R}$, with $Z = \log R$ being the location of the branes. We have also expressed\(^5\) $T_5^5$ in terms of $\kappa_{10}$. The result is independent of the nature of the brane.

Replacing a point-like charge with a spherical distribution leads to the same configuration outside the two-sphere, while the electric field vanishes inside (Gauss’s law), avoiding thereby the Coulomb divergence. This is depicted in Fig. 4. The simplest solution to Eq. (4.2), where we set for simplicity $Z = 0$ ($R = 1$), is precisely an analogue of that electrostatic example, as we have advertised previously:

$$h(z) = h_0 + N e^{-\left(z + |z|\right)}.$$  

For $r > 1$ ($z > 0$) we recover (2.3), while for $0 < r < 1$ ($z < 0$) the space is flat since $h = h_0 + N$. Moving the brane sources from $r = 0$ to a uniform $S^3$ distribution at $r = 1$ amounts therefore in excising a ball which contains the would-be near-horizon geometry, and replacing it with a piece of flat space. The price to pay for this matching is the introduction of sources uniformly distributed over $S^3$ and localized at $z = 0$.

The NS–NS or R–R flux (Eq. (2.4)) now reads:

$$H = 2N \Theta(z) \Omega_3. \quad (4.4)$$

\(^5\)When $\alpha'$ is restored (it has been set equal to one), the five-brane tensions read $T_5^{\text{NS}} = \frac{2\pi^2 \alpha'}{\kappa_{10}}$ and $T_5^D = \frac{1}{4\pi \kappa_{10} \alpha'}$. They turn out to be equal, once $\kappa_{10}$ is expressed in terms of $\alpha'$: $2\kappa_{10}^2 \equiv 16\pi G_{10} = (2\pi)^7 \alpha'^4$, where $G_{10}$ is the ten-dimensional Newton’s constant.
It vanishes inside the ball. Consequently \( dH \sim \delta(z) \), and its integral counts the total number of five-branes.

Concerning the dilaton field, Neveu–Schwarz and Dirichlet sources lead to different pictures, according to Eqs. (2.3) and (4.3). For NS5-branes, the excised ball removes altogether the divergent-coupling region of the canonical neutral five-brane, and replaces it with a constant one, \( g_s^2 = h_0 + N \). In the case of D5-branes, the coupling inside the ball \( (r < 1) \) becomes also constant, \( g_s^2 = (h_0 + N)^{-1} \). These results are summarized in Fig. 5.

As we have already stressed, a remarkable situation is provided by \( h_0 = 0 \). For negative \( z \), the transverse space is flat, as for generic \( h_0 \). For positive \( z \), the geometry is that of a three-sphere of radius \( L = \sqrt{N} \) plus linear dilaton (see Eqs. (2.6) and (2.7)).

When the sources are of the Neveu–Schwartz type, both patches are type-II string backgrounds described in terms of exact \( N = 4 \) superconformal theories. In this case, the background is free of strong-coupling singularities. This is to be compared to the canonical situation where all branes are located at the origin. As pointed out in Sec. 2, exact CFT interpretation is only possible (i) in the vicinity of the origin, where the near-horizon transverse geometry is precisely that of a three-sphere of radius \( L = \sqrt{N} \) plus linear dilaton, and (ii) in the asymptotic region, which is flat. By moving the five-branes at finite \( z \), we create a patchwork where we sew finite (i.e. not only asymptotic) pieces of space–time, each being a portion of some exact CFT target space. Although this method can be easily generalized to more involved configurations (see [10]), it is not clear how this “patchwork CFT’s” could be treated beyond the \( \alpha' \) expansion. Preliminary (perturbative) results about the spectrum and holography are available though.
Figure 5: The string coupling of solution (4.3) is finite everywhere for the NS5-branes ($\gamma = +1$) and its electrostatic analogue is given in Fig. [4]. However, it diverges when the sources are D5-branes and $h_0 = 0$.

The case of Ramond–Ramond fields is more involved in all respects. Firstly, there is no exact conformal model reproducing such background fluxes. Secondly, spreading them over an $S^3$ at finite distance might be interesting *per se*, but does not help in removing the strong-coupling region which, for $h_0 = 0$, is at $z \to \infty$. Another electrostatic analogue is useful here in order to resolve the divergence.

When a point-like charge is surrounded by a homogeneous spherical shell of opposite charge, outside of the shell the potential is constant whereas it is Coulomb inside. This is the screening phenomenon. It can be transposed to the five-brane sources by introducing negative-tension objects. These are orbifold planes in the case of NS–NS backgrounds, and orientifold planes for R–R. They cannot have fluctuations in a unitary theory because the corresponding modes would be negative-norm.

In order to regulate the strong coupling in the configurations where $N$ D5-branes are located at $z \to -\infty$ (i.e. at the origin, $r = 0$), we should introduce an equal number of orientifold planes at some finite $z = Z$, so that the coupling remains constant for $z \geq Z$. Whether many O5 planes could be included in a theory is, to some extent, an open question. Obstructions do exist for O9 planes, whereas there is in principle a small window for accommodating a few O5’s. The number of such planes might not be allowed to exceed some finite value, though. Hence, the whole scheme should be revisited, since the above geometrical description assumes a large number of branes, especially when those are distributed over an $S^3$ in a continuous fashion. If such D5/O5 configurations are possible, they indeed regulate the strong-coupling problem, without promoting the whole set-up to some, even unconventional, conformal two-dimensional model.

On the contrary, there is no any constraint on the number of orbifold planes that can be introduced for screening the charges of NS5-branes. Although these planes are not necessary for solving the strong-coupling problem, they allow to generate a large variety of patchwork CFT backgrounds.
5 Adding fundamental (or D) strings: null deformations of $SL(2, \mathbb{R})$

There is another setup which offers a variety of possibilities for supersymmetric string backgrounds, exact CFT realizations, new decoupling limits and holographic pictures, and where the string coupling is naturally regulated. This is the NS5/F1 or its S-dual version (in type IIB), the D1/D5 [12, 43, 44]. Those are $N=2$ backgrounds, with $N=4$ enhancement in the near-horizon limit, where the geometry is $AdS_3 \times S^3 \times T^4$.

The appearance of some near-horizon geometry is closely related to a specific choice for the decoupling limit. The key observation is here that one can introduce a tuning parameter that measures the dilution of the strings and allows for finite and controllable deformations of the AdS$_3$ factor [45]. The remarkable fact is that those deformations turn out to be exactly marginal deformations of the underlying conformal field theory, namely the $SL(2, \mathbb{R})$ WZW model.

In order to be more specific, let us consider the D1/D5 picture. The D5-branes extend over the coordinates $x \equiv x^5, x^6, \ldots, x^9$, whereas the D1-branes are smeared along the four-torus spanned by $x^6, \ldots, x^9$. The volume of this torus is asymptotically $V = (2\pi)^4 \alpha'^2 v$ (we restore $\alpha'$ in this chapter). With these conventions, in the sigma-model frame, the supergravity solution at hand reads:

\begin{align}
\tilde{s}^2 &= \frac{1}{\sqrt{H_1 H_5}} (-dt^2 + dx^2) + \sqrt{\frac{H_1}{H_5}} \sum_{\ell=6}^9 (dx^\ell)^2 + \sqrt{H_1 H_5} \left( dr^2 + r^2 d\Omega_3^2 \right), \\
\tilde{e}^{2\tilde{\phi}} &= g_s^2 H_1 \frac{H_5}{H_5}, \\
H &= -\frac{1}{g_s} dH_1^{-1} \wedge dt \wedge dx + 2\alpha' N_5 \Omega_3
\end{align}

($H$ is now the Ramond–Ramond three-form field strength) with

\begin{align}
H_1 &= 1 + \frac{g_s \alpha' N_1}{vr^2}, & H_5 &= 1 + \frac{g_s \alpha' N_5}{r^2}.
\end{align}

The near-horizon ($r \to 0$) string coupling constant and the ten-dimensional gravitational coupling constant are

\begin{align}
g_{10}^2 &= g_s^2 \frac{N_1}{v N_5}, & 2\kappa_{10}^2 &= (2\pi)^7 e^{2\tilde{\phi}} \alpha'^4.
\end{align}

The standard decoupling limit, which leads to the AdS$_3$/CFT$_2$ correspondence, is

\begin{align}
\alpha' \to 0, \\
U \equiv r/\alpha' \text{ fixed}, \\
v \text{ fixed}.
\end{align}

In this limit, the holographic description is a two-dimensional superconformal field theory living on the boundary of AdS$_3$ that corresponds to the world-volume theory of the D1/D5 system compactified on a $T^4$ whose volume is held fixed in Planck units [44, 46].
In order to reach a decoupling limit that corresponds to the near-horizon geometry for the D5-branes only, one has to consider the limit:

\[
\begin{align*}
\alpha' &\to 0, \\
U &\to r/\alpha' \text{ fixed,} \\
g_s \alpha' &\text{ fixed,} \\
\alpha'^2 v &\text{ fixed.}
\end{align*}
\]

The last condition is equivalent to keeping fixed the six-dimensional string coupling constant:

\[g_s^2 = \frac{g_s^2}{v}.
\]

Since the gravitational coupling constant vanishes in this limit, the world-volume theory decouples from the bulk. The geometrical picture of the setup is the following: as \(v \to \infty\), the torus decompactifies and the density of D-strings diluted in the world-volume of the D5-branes goes to zero.

The string coupling remains finite in this near-horizon limit, while the asymptotic region is strongly coupled. A perturbative description, valid everywhere is obtained by S-duality. The supergravity solution (5.1), (5.2) in the S-dual frame reads:

\[
ds^2 = e^{-\phi} ds^2 = \frac{1}{g_s} \left\{ \frac{1}{H_1} (-dt^2 + dx^2) + \sum_{i=6}^9 (dx^i)^2 + \alpha'^2 H_5 (dU^2 + U^2 d\Omega_3^2) \right\},
\]

\[
e^{2\phi} = \frac{1}{g_s^2} \frac{H_5}{H_1},
\]

with (in the limit (5.4) under consideration)

\[
H_1 = 1 + \frac{g_s N_1}{\alpha' v U^2}, \quad H_5 = \frac{g_s N_5}{\alpha' U^2}.
\]

The expression (5.3) for the antisymmetric tensor remains unchanged but it stands now for a NS flux.

Finally, we define the new variables:

\[
u = \frac{1}{U}, \quad X^\pm = X \pm T = \frac{x \pm t}{g_s \sqrt{N_1 N_5}},
\]

in which the horizon is located at \(u \to \infty\), while \(u \sim 0\) corresponds to the asymptotic region. We also introduce the following mass scale (in \(\alpha'\) units):

\[
M^2 = \frac{g_s N_1}{\alpha' v},
\]

which measures the “renormalized” dilution of D-strings. In these coordinates, the solution
\[ ds^2 = N_5 \left\{ \frac{du^2 + dX^2 - dT^2}{u^2 + 1/M^2} + d\Omega_3^2 \right\} + \frac{1}{\alpha' g_s} \sum_{i=6}^{9} (dx^i)^2, \]

\[ e^{2\phi} = \frac{u^2}{g_0^2 u^2 + 1/M^2}, \]

\[ H = N_5 \left\{ \frac{2u}{(u^2 + 1/M^2)^2} du \wedge dT \wedge dX + 2\Omega_3 \right\}. \]

This is the geometry of a deformed AdS$_3$ times an S$^3 \times T^4$. At highest possible D-string concentration ($M \to \infty$), we recover the ordinary NS5/F1 near-horizon geometry, AdS$_3 \times S^3 \times T^4$. In the opposite limit, namely for infinitely diluted D-strings, the AdS$_3$ factor factorizes into two light-cone coordinates plus a space direction with linear dilaton. The remarkable fact is that not only these limiting geometries are target spaces of conformal sigma models (respectively $SL(2, \mathbb{R})_{k+4} \times SU(2)_k \times U(1)^4$ and $\mathbb{R}^{1,1} \times U(1)_Q \times SU(2)_k \times U(1)^4$, $k = N_5 - 2$), but any geometry at finite $M$ originates from a conformal two-dimensional theory. The latter turns out to be a marginal deformation of $SL(2, \mathbb{R})$, driven by an exact $(1,1)$ operator bilinear in null left and right currents. Supersymmetry is $N = 2$ along the line, with $N = 4$ enhancement in the two limits as well as at a subset of discrete values of $M$.

One can analyze the geometry (5.9) for arbitrary, finite values of $M$. The asymptotic region ($u \to 0$) is not sensitive to the presence of the fundamental strings. Hence, it is not surprising that this region describes the near-horizon geometry of the pure NS5-brane background, $\mathbb{R}^{1,1} \times U(1)_Q \times SU(2)_k \times T^4$ in its weakly coupled region – i.e. away from the horizon. In the opposite radial limit ($u \to \infty$), one is getting close to the horizon and always feeling the F1’s: the background becomes effectively that of the NS5/F1 near-horizon: $SL(2, \mathbb{R})_{k+4} \times SU(2)_k \times T^4$, with a finite constant dilaton. For any finite $u$ the geometry is the deformed one, with a $u$-dependent dilaton, bounded everywhere.

In some sense, we are here regulating the strong-coupling region of the NS5-brane background by adding an appropriate condensate of fundamental strings. This regularization is an alternative to the one proposed in [40] and described in Sec. 4; it avoids the spherical target-space wall of the latter, and replaces it by a smooth transition, driven by a marginal worldsheet deformation. Again, one can see how interrelated are the issues of brane configurations, exact CFT backgrounds and string-coupling behavior.

6 Summary

The near-horizon geometry of the canonical five-brane configuration has been extensively investigated. It allows for a decoupling limit, where the surviving degrees of freedom are the brane excitations which exhibit a new dynamics, the little string theory. Moreover, at least in the case of NS5-branes, the perturbative string theory in this region is an exact conformal field theory. Unfortunately, the perturbation theory is expected to break down in this limit due to strong-coupling ambiguities.
In this note, we have tried to illustrate some pieces of relationship among brane configurations, conformal models and the strong-coupling problem, and reviewed the various methods that are presently known for facing the latter, namely:

(i) Embed the original configuration in a web of other configurations, related by T-dualities;
(ii) Distribute the five-branes over a three sphere in transverse space;
(iii) Adding fundamental strings (or D-strings in the S-dual picture), with a dilution parameter which acts like a cut-off.

Further investigation is needed in all cases: control the T-dualities in curved spaces, incorporate the asymptotic regions in a CFT framework, understand the decoupling limits in these more exotic configurations, analyze the “patchwork” conformal models . . .

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