Flow shear suppression of pedestal ion temperature gradient turbulence-A first principles theoretical framework

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Abstract

A combined analytic and computational gyrokinetic approach is developed to address the question of the scaling of an important class of pedestal turbulent transport with arbitrary levels of $E \times B$ shear. Due to strong gradients and shaping in the pedestal, the ion temperature gradient driven instabilities of interest are not curvature-driven like typical core instabilities. By extensive numerical (gyrokinetic) simulations, it is demonstrated that pedestal modes respond to shear suppression very much like the predictions of a basic analytic decorrelation theory. The quantitative agreement between the two provides us with a new, first principles (physics-based) theoretical framework for pedestal shear suppression in regimes ranging from present-day experiments to future burning plasmas that lie in low shear regimes.

Keywords: gyrokinetics, shear suppression, pedestal, pedestal turbulence

(Some figures may appear in colour only in the online journal)

1. Introduction

The interplay between shear flow and turbulence is a central component in the self-organization of wide-ranging fluid and plasma systems. In neutral fluids, for example, shear flow is a common driver of turbulence (i.e., Kelvin–Helmholtz instabilities). In contrast, the primary source of turbulence in a typical fusion plasma is the immense free energy contained in extreme temperature and density gradients. As a result, the shear flow, in fact, suppresses drift turbulence [1], and is thought to be the main mechanism underlying the formation and sustenance of the edge transport barrier characteristic of the tokamak H-mode [2]. Since the residual turbulence mediates the structure of the pedestal and, consequently, largely determines plasma confinement, it (and its interplay with shear flow) is of central importance to fusion energy.

The earliest theoretical investigations of shear suppression [3–6], which we will call decorrelation theories, predicted reduced fluctuation amplitudes due to the combined advection by background shear flow and self-consistent turbulent flow. Amongst these, the analytic theory of Zhang and Mahajan [5, 7] has compared rather favorably with experimental observations of shear suppression [8, 9], albeit in experimental setups, perhaps, less challenging than a fusion-relevant H-mode pedestal.

It turns out, however, that the predictions of these basic theories are in striking agreement with gyrokinetic simulations (using the GENE code [10, 11]) of the pedestal, providing a sound basis for a deep fundamental understanding of the reduction of turbulence by shear flow. We can, thus, with greater confidence, apply a combination of analytical and numerical approaches to study the scaling of turbulence with flow shear, even in the extreme environment of the H-mode pedestal. In effect, we will seek a first principles (physics-based) answer to the crucial question: how does turbulence in the pedestal react to a systematic reduction of shear rate? Since most proposed burning plasma machines are expected to approach the low shear regime, a reliable answer to this question is crucial to the future of fusion energy via Tokamaks operating in H-modes.

After the decorrelation theories of shear suppression (worked out in simple geometry) were proposed, subsequent work emphasized the importance of toroidal effects (e.g., the strong dependence of growth rates with ballooning angle) [12–17]. Toroidal effects are indeed prominent for the conventional instabilities in the plasma core. Driven by toroidal
curvature (i.e., via resonances with the magnetic drift frequencies), such fluctuations peak in the low magnetic field region of the torus. Interestingly, however, the dynamics of shear suppression manifests differently under conditions characteristic of the pedestal where, due to steep gradients and geometric shaping, an important class of relevant modes are typically not curvature-driven, and consequently are insensitive to toroidal effects [18, 19] (having, e.g., little dependence of growth rate on ballooning angle). It is expected, then, that the influence of shear flow on pedestal turbulence may be very different from what could be extrapolated from the notions pertinent to the core plasma. In fact, we reach the surprising conclusion that, despite the substantial complexity and computational challenges involved in pedestal turbulence simulations, the early decorrelation theories of shear suppression become highly relevant.

The importance of this work should be framed as follows. First, it verifies via kinetic simulation the applicability of a simple analytic theory for shear suppression in a system that is much more encompassing than assumed by the theory. This suggests that such theories will likely find application far beyond the fusion plasmas that are the topic of this paper. Notably, the theory does not assume asymptotic limits in shear rate but is uniquely applicable from the weak to strong shear limit. Second, building on recent numerical studies [19, 20], this work establishes the theoretical underpinnings necessary to understand pedestal transport over the transition to lower shear burning plasma regimes.

2. Decorrelation theories

The decorrelation theories of shear suppression begin with a generic fluid equation of the form

$$\partial_t \xi + \bar{v}(x) \partial_x \xi + \tilde{v}(x, y, t) \partial_y \xi = q(x, y, t),$$  \hspace{1cm} (1)

where $x$ is a radial coordinate, $y$ is the corresponding binormal coordinate, $\bar{v}$ is the fluctuating $E \times B$ velocity, $\tilde{v}(x)$ is the macroscopic steady state shear flow, $q$ is a gradient driven source term, and $\xi$ is a fluid quantity like density or temperature. Here we analyze the perpendicular temperature fluctuations (i.e. $\xi = T_p$—hereafter denoted by $T$) so that equation (1) may be viewed as a simple analog to equation (11) from [21], which is derived from a moment expansion of the gyrokinetic equations. The decorrelation theories are based on the so-called clump theory described in [22], and apply basic turbulence closures to solve for properties of the two point correlation function (note that [7] derives similar results via an alternative approach to clump theory).

For the purpose of this study, we have reproduced a calculation very similar to the original Zhang–Mahajan (ZM) theory [5] (due to the close connection, we will refer to our model simply as the ZM theory—details can be found in the appendix). The calculation arrives at the following relation describing the reduction of turbulence by shear flow

$$P^{\left(P - \frac{1}{3}\right)}(P - 1) = \frac{2}{3}W^2p^{2\alpha},$$  \hspace{1cm} (2)

where $P^{-1}$ represents the reduction in fluctuation amplitudes

$$P^{-1} \equiv \frac{\Delta^2}{\Delta^2_0} \frac{(\bar{T})^2}{(\bar{T}_0)^2},$$  \hspace{1cm} (3)

and $W$ is the normalized shear rate

$$W = \gamma_{E \times B}^2 \Theta.$$  \hspace{1cm} (4)

In these expressions, 0 subscripts denote shear-free quantities (i.e. quantities calculated from zero shear simulations), $\gamma_{E \times B} = dv/dx$ is the shear rate, the brackets represent ensemble averages (in practice, averages over space and time), $\tau_v$ is the shear-free correlation time, $\Delta_v$ is the correlation length in the $x, y$ direction (respectively), $\Theta = \Delta_y/\Delta_x$ accounts for anisotropy, and $\alpha$ is a near-unity scaling parameter, which will be described below.

The ZM theory has two features that distinguish it from other decorrelation theories. Both are indispensable for the quantitative comparisons that will be described below. First, the theory is non-asymptotic in shear rate, describing shear suppression seamlessly across the weak ($W \lesssim 1$) and strong ($W \gg 1$) shear limits. Second, and most importantly, it accounts for the fact that fluctuation levels and nonlinear diffusivity are intimately connected and are both sensitively dependent on shear rate. This is accomplished via an ad hoc expression relating the two:

$$D = D_0 (\bar{T}^2)\alpha,$$  \hspace{1cm} (5)

where $D$ is the nonlinear diffusivity, $D_0$ is a constant proportionality factor, and $\alpha$ is the relevant scaling parameter related to the strength of the turbulence ($\alpha \sim 0.5/1.0$ for strong/weak turbulence, respectively). In summary, given a normalized shear rate, correlation lengths, and the scaling parameter $\alpha$, equation (2) predicts the relative suppression of turbulent fluctuation amplitudes. In essence, the theory captures the nonlinear decorrelation of turbulence when subject simultaneously to a background shear flow and a self-consistent turbulent flow. By balancing this decorrelation with a generic gradient drive, an expression is derived for the reduction of turbulence by shear flow. Notably, the theory neglects parallel dynamics (e.g. Landau damping), zonal flows [23], toroidal effects, non-local (i.e., global) effects [24–27], coupling with damped eigenmodes [28–31], and non-monotonic flow profile variation, all of which are included in our simulations. Thus, to the extent that simulation and theory agree, it can perhaps be concluded that the underlying mechanism of shear suppression in the pedestal is described by a few relatively simple ingredients. Presently, we make such comparisons.

3. Background on pedestal transport

The gyrokinetic GENE simulations described here are designed to include a wide range of relevant pedestal effects while also facilitating clear comparisons with theory. To this end, we employ an adiabatic electron approximation in order to reduce computational demands and limit the dynamics to the ion temperature gradient (ITG) driven turbulence of
interest (as discussed below in the context of more comprehensive simulations, the adiabatic electron assumption does not appear to substantively change the shear scaling). We note that this ITG turbulence is not the dominant pedestal transport mechanism in many present-day experiments precisely due to its suppression by shear flow. The most important fluctuations are likely electron temperature gradient (ETG) turbulence [32–36, 38], microtearing modes [20, 37, 38], and low-n (toroidal mode number) magnetic fluctuations [37, 39–43]—all of which are expected to be much less sensitive to shear flow than ITG. Effectively, we are focusing directly on the mechanism that is perhaps least understood and is expected to become increasingly important as \( \rho_s \) decreases, as discussed below.

Recent related work [19, 20] explores the implications of the expected \( \rho_s \) scaling of pedestal flow shear (\( \rho_s \) is the ratio of the sound gyroradius to the minor radius \( \rho \)). The pedestal shear rate is effectively determined by the self-organization of the pedestal by means of force balance between the radial electric field and the pressure gradient. This force balance, which is well-described by neoclassical theory and well-founded experimentally [44], results in shear rates that scale linearly with \( \rho_s : \gamma_{E \times B} \Delta \alpha \propto \rho_s \) (\( \gamma_{E \times B} \) is the ion thermal velocity). [19, 20] identify two classes of pedestal transport from gyrorhekinetic simulations. The first class scales close to the expected gyroBohm \( \rho_s \) scaling and, accordingly, encompasses mechanisms that are largely insensitive to shear flow: ETG turbulence due to its small scales in space and time, microtearing turbulence likely due to its reliance on magnetic as opposed to electrostatic fluctuations, and neoclassical transport. The second class of transport, from ITG turbulence, is small throughout most of the experimentally accessible parameter space but has an unfavorable \( \rho_s \) scaling due to its high sensitivity to shear flow. References [19, 20] predict the latter mechanism—shear-sensitive ITG turbulence—to be relevant on JET (which has access to the lowest values of \( \rho_s \) among active experiments) and to become increasingly important in the transition to low-\( \rho_s \) regimes. To summarize, ITG turbulence in the pedestal is suppressed by the strong shear rates characteristic of present-day experiments. We are addressing the question of the manner in which it emerges as the shear rate (i.e., \( \rho_s \)) decreases.

### 4. Comparisons between theory and simulation

In order to make comparisons with the ZM theory (equation (2)), the time- and box-averaged squared temperature fluctuation amplitude, \( \langle \tilde{T}^2 \rangle \), and the radial and binormal correlation lengths (\( \Delta_r \) and \( \Delta_b \)) are calculated from simulation data. The correlation lengths are calculated for temperature fluctuations at the top of the torus where the fluctuation levels peak. The heat flux, which is embedded in the corresponding temperature moment of the gyrokinetic nonlinearity, provides an appropriate proxy for the nonlinear diffusivity \( D \). The scaling factor \( \alpha \) (recall equation (5)) is extracted from a comparison of \( Q_i \) and \( \langle \tilde{T}^2 \rangle \) and lies in the range \( \alpha = [0.81, 0.97] \) for the cases studied here. In order to connect the shear rate \( W \) (equation (4)) with its corresponding quantity from the simulations, we use the inverse linear growth rate at \( k_r \rho_s = 0.1 \) as a proxy for the shear-free correlation time \( \tau_{\text{corr}} \), the standard definition of the \( E \times B \) shear rate \( \gamma_{E \times B} = \tau_{\text{corr}}^{-1} \frac{d}{dr} \frac{E_r}{\mu_0 \rho_b} \) used in the GENE code [36] (\( \rho_b \) is the square root of the normalized toroidal magnetic flux, \( q \) is the safety factor, \( E_r \) is the radial electric field, \( R \) is the major radius, and \( B_b \) is the poloidal magnetic field), and an anisotropy factor \( \Theta = \Delta_r/\Delta_b \) defined by the correlation lengths. One free parameter is used to scale the shear rate and is selected to minimize the discrepancy between simulation and theory. Encouragingly, this free parameter remains of order unity in all cases studied (varying from 0.58 to 2.1). The simulations are based on the low \( \rho_s \) pedestal setup described in [19], which uses JET profile shapes [45] in conjunction with ITER geometry and projected ITER pedestal parameters. Representative profiles, shear rates, gradient scale lengths and the \( q \) profile are shown in figure 1.

The extensive simulation campaign described below entails scans of \( E \times B \) shear rate for four different scenarios, which are designed to isolate various effects and gauge variation in shear suppression dynamics. Typical resolution was (384, 64, 64, 24) gridpoints in the \((r, z, v_{||}, \mu)\) coordinates. In \( \rho_s \) scans, the radial resolution was adjusted to keep the number of gridpoints per gyroradius roughly constant. GENE employs a Fourier representation for the binormal \( y \) coordinate, for which our simulations resolve \( k_{y,\text{min}} \rho_s = 0.04 \) and \( k_{y,\text{max}} \rho_s = 3.16 \). The first case, the local constant shear (LCS) case, is designed to match the assumptions of the ZM theory as closely as possible by employing a local approximation (i.e., taking plasma parameters, gradients, and shear rate at a single radial location and neglecting effects from radial profile variation). A comparison between theory and
shear rate. Note that the two low shear cases were devoted to this low shear regime. Multiple low shear simulations were run for extended periods of time in order to reduce statistical uncertainty. These simulations exhibit only non-monotonic shear profiles, suggesting that the non-monotonic shear profile acts as a singular perturbation to the length scales. In effect, a reduction in the scale length effectively increases \( P^{-1} \). Consequently, we normalize to the low shear case and implement a small corresponding offset in the equation for the radial electric field. The GRS scenario involves an additional level of complexity since it conflates the effects of shear suppression with intrinsic \( \rho_\text{s} \) effects, which are well-known to independently affect turbulence levels (i.e., produce deviations from gyro-Bohm scaling) as \( \rho_\text{s} \) is raised above a certain threshold [24–27]. We address this additional complexity with a straightforward modification to the ZM theory. We assume that finite \( \rho_\text{s} \) effects are limited to two mechanisms—(1) \( E \times B \) flow shear, and (2) \( \rho_\text{s} \) effects manifest in the linear instability drive. The direct effect of the shear rate is accounted for automatically by using the self-consistent shear profile as described above. The effect on the linear instability drive is incorporated by accounting for the \( \rho_\text{s} \) dependence of the linear growth rate (calculated without \( E \times B \) shear), which we add to the theory when balancing the decorrelation and gradient drive

\[
\frac{\langle T^2/T_0^2 \rangle}{\tau_c} = \gamma_\text{lim}(\rho_\text{s}) (\alpha/\nu_T) \frac{D}{L^2},
\]

(6)

where \( \gamma_\text{lim} \) is the \( \rho_\text{s} \)-dependent linear growth rate, \( \tau_c \) is the decorrelation time, and \( L \) is a macrosopic gradient scale length. Note that the decorrelation theories (e.g., [4, 5]) use equation (6) without the inclusion of the linear growth rate simulations for the GFS case also find very good agreement with the ZM theory, as seen in figure 2(c).

5. Scan of \( \rho_\text{s} \)

While valuable for the purpose of theoretical verification, the three cases examined thus far may be characterized as idealized and somewhat artificial setups that exploit the flexibility of our simulations to independently scan \( E \times B \) shear rates. In an experimental context there is little external control over the shear rate. As described above, the shear rate is set by the self-organization of the pedestal by means of force balance between the radial electric field and the pressure gradient resulting in direct proportionality between \( \rho_\text{s} \) and pedestal shear rates. Consequently, the most experimentally relevant simulation scenario (called the GRS case) involves a fully self-consistent scan of \( \rho_\text{s} \). To conduct such a scan, we construct a series of equilibria with varying values of \( \rho_\text{s} \) while holding pedestal profile shapes and all other dimensionless parameters fixed (i.e., safety factor \( q \), \( \nu_\text{e}, \beta \)). This is achieved by varying magnetic field \( B \) and major radius \( R \), representing typical variations in machine parameters, while adjusting other parameters (density, temperature) to keep \( \beta \) and collisionality fixed. This procedure is described in detail in appendix A of [19]. The pedestal width is also held fixed (in radial magnetic flux coordinates) consistent with empirical observations that the pedestal width is insensitive to \( \rho_\text{s} \) [46–48] (this is consistent also with theory-based predictions like EPED [39]). For each equilibrium, the \( E \times B \) shear profile is determined self-consistently from the density and temperature profiles using the standard neoclassical expression [49, 50].

The GRS scenario involves an additional level of complexity since it conflates the effects of shear suppression with intrinsic \( \rho_\text{s} \) effects, which are well-known to independently affect turbulence levels (i.e., produce deviations from gyro-Bohm scaling) as \( \rho_\text{s} \) is raised above a certain threshold [24–27]. We address this additional complexity with a straightforward modification to the ZM theory. We assume that finite \( \rho_\text{s} \) effects are limited to two mechanisms—(1) \( E \times B \) flow shear, and (2) \( \rho_\text{s} \) effects manifest in the linear instability drive. The direct effect of the shear rate is accounted for automatically by using the self-consistent shear profile as described above. The effect on the linear instability drive is incorporated by accounting for the \( \rho_\text{s} \) dependence of the linear growth rate (calculated without \( E \times B \) shear), which we add to the theory when balancing the decorrelation and gradient drive

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either the weak turbulence asymptotic scaling is strongly dependent on the and \( \rho \) (fi).

Comparisons between equation (7) and simulations.

\[ \frac{\xi}{P} \approx 2^{2/3} W^{-2/3} \]

Figure 3 shows a comparison between this high shear scaling and fits to the asymptotic simulation data points. Note that the asymptotic scaling is strongly dependent on the \( \alpha \) factor. The self-consistent values of \( \alpha \) produce a much better match than either the weak turbulence (\( \alpha = 1 \) translating to \( W^{-2} \)) or strong turbulence (\( \alpha = 0.5 \) corresponding to \( W^{-1} \)) limits.

6. Experimental application

The details (including scale lengths, anisotropy, etc) of the quantities \( P^{-1} \) and \( W \) are necessary to achieve the quantitative agreement between theory and simulation described above but are not easily accessible via experimental diagnostics. Fortunately, the more-intuitive and experimentally accessible quantities gyroBohm-normalized heat flux \( Q/Q_{GB} \) and \( \rho_* \) are sufficient to extract a rough, yet robust, scaling upon which experimental predictions can be based. As shown in figure 4, the heat flux scales approximately as the square of the inverse shear rate. Figure 4 also includes simulations described in [19] that include kinetic electrons and electromagnetic effects suggesting that this scaling is not limited to the adiabatic electron approximation explored in this paper. Noting the direct proportionality between shear rate and \( \rho_* \), we can predict the following strong shear scaling for pedestal heat flux from ITG turbulence: \( Q/Q_{GB} \propto \rho_*^{2} \). Since ion thermal transport is often thought to be roughly neoclassical in the pedestal, this theoretical scaling can be tested experimentally by determining the anomalous (i.e., beyond neoclassical) component of the ion heat flux over a dedicated \( \rho_* \) scan. Though challenging, such a measurement may be feasible via a combination of precision edge diagnostics and neoclassical modeling. Such a campaign would optimally be centered on a high \( \eta = L_\alpha/L_T \) scenario with a weak particle source in the pedestal and extend to the lowest accessible values of \( \rho_* \) (i.e., values accessible on JET). In light of the observed weak scaling of pedestal structure with \( \rho_* \) [46–48], this campaign should match (by design) quite closely the setup of our numerical \( \rho_* \) scans described here and in [19, 20]. After a transition to a weak shear regime, a rough scaling closer to \( Q/Q_{GB} \propto \rho_*^{4} \) is predicted.
7. Discussion

The present study demonstrates that the underlying mechanism of pedestal shear suppression involves a relatively small set of transparent physical ingredients, which are insensitive to parameter variations and difficult to modify. We briefly note that various effects may modify the scaling discussed here, including non-trivial coupling between ion and electron scales, feedback from ITG turbulence on pedestal width, or systematic changes in the SOL boundary condition with \( \rho_e \). Nonetheless, we have the makings of a first principles, physics-based theory of turbulence suppression (via shear flow) that can serve as a foundation for understanding turbulent transport in regimes that have not been experimentally probed yet. In fact, ITER and most other future burning plasma experiments are expected to approach the low shear regime not accessed in most current experiments.

We end the paper by pointing out that the simulations/theory suggest several possible routes for controlling turbulent transport in pedestals. Since the relevant suppression parameter \( W \) is the shear rate normalized to the linear growth rate, the most promising route to optimized pedestal performance in low \( \rho_e \) regimes is through the minimization of ITG growth rates. As demonstrated in [19, 20], this can be accomplished by at least two mechanisms—(1) ion dilution via impurity seeding, and (2) reduction of \( \eta_i \) (the ratio of the density and temperature gradient scale lengths) through the separatrix boundary condition. The latter route will rely heavily on optimization of divertor performance. We note also that these and similar mechanisms unrelated to shear flow are capable of reducing transport from other pedestal instabilities (see, e.g., [20, 35, 37, 51–53]). Such non-shear suppression mechanisms should be vigorously studied and understood in hopes of exploiting them in low shear regimes.

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Appendix

Here we succinctly outline a derivation of the ZM shear suppression theory [5, 7] used in this study. This derivation is intended to be more accessible than the rigorous but very involved calculation described in [7]. Our simplified derivation first reproduces the orbit equations from Biglari–Diamond–Terry [4] using the standard clump theory [22]. Thereafter, the distinctive elements of the ZM theory are applied, namely a non-asymptotic (in shear rate) treatment of the problem and the use of an ansatz relating the nonlinear diffusivity and fluctuation amplitude (described below). The original ZM theory builds on an alternative set of orbit equations, which stem from an independent approach that does not rely on the standard clump theory. The model described below exhibits only minor quantitative differences from the original ZM theory.

We use coordinates \((x, y)\), where \(x\) represents the radial direction and \(y\) varies in some direction (other than that of \(B\)) on the magnetic surface. Parallel gradients, along with such geometrical details as magnetic curvature, are neglected.

A perturbed fluid quantity, such as temperature or density, is denoted by \(\xi(x, y, t)\) and assumed to satisfy the equation

\[
\partial_t \xi + \bar{v}(x) \partial_x \xi + \nu(x, y, t) \partial_y \xi = q(x, y, t). \tag{8}
\]

Here \(\bar{v}(x)\) is the shear flow, a slowly varying equilibrium flow in the \(y\)-direction, while \(\nu(x, y, t)\) is a turbulent flow. We assume that the turbulent flow is incompressible, and that the variation of \(\xi\) on flux surfaces is smaller than its radial variation, so the \(x\)-component of the turbulent velocity dominates. Rather than trying to solve (8), we derive from it an approximate equation for the correlation function

\[
C_{12} \equiv \langle \xi(x_1, y_1, t) \xi(x_2, y_2, t) \rangle \equiv \langle \xi \xi \rangle,
\]

where the angle brackets denote a statistical average. Standard renormalization methods, based primarily on the random phase approximation, yield the evolution equation

\[
(\partial_t + \omega_{i} x \partial_x - \partial_y (k_0^2 x^2) D \partial_x) C_{12} = Q, \tag{9}
\]

where \(x_1 = x_2 = \bar{x}_1\) is the relative coordinate, \(Q\) is the source, \(\omega_{i} \equiv \nu_{i} / c_{i}\) is the shearing rate, \(D\) is a turbulent diffusion coefficient and \(k_0\) is a spectral-averaged wave number, related to the width \(\Delta\) of an eddy: \(k_{0,x} = 1/\Delta_{x,y} \). The corresponding Green’s function \(G(x, t; \bar{x}, t')\) satisfies the homogeneous version of (9) with initial data \(G(x, 0; \bar{x}, t) = \delta(x - \bar{x})\). We are content to study the moments

\[
M^n(t) \equiv \int dx G(x, t; x_0, 0)x^n, \tag{i}
\]

where \((x^1, x^2) = (x, y)\) and \(x_0\) is some initial value. Integration by parts yields the dynamical moment equations

\[
\partial_t M^{n1} = 2Dk_0^2 (3M^{n1} + \sin^2 \theta M^{22}), \tag{10}
\]

\[
\partial_t M^{n2} = \omega_{i} M^{n1} + 2Dk_0^2 M^{22}, \tag{11}
\]

\[
\partial_t M^{22} = 2\omega_{i} M^{22}. \tag{12}
\]

Here \(k_0^2 \equiv k_{0,x}^2\) and \(\sin \theta \equiv k_{0,y} / k_{0,x}\). Denoting the characteristic time for change in \(M^n\) by \(\tau_c\) and introducing the nominal diffusion time \(\tau_D \equiv (k_0^2 D)^{-1} = \Delta^2 / D\) we obtain the characteristic equation

\[
\tau(z - 2) (z - 6) = 4(\omega_{i} \tau_D)^2 \sin^2 \theta, \tag{13}
\]

where \(\tau \equiv \tau_D / \tau_c\). Since the gradients relax through turbulent diffusion, the source for turbulence is measured by \(D / L^2\). This observation leads to the estimate
\[
\frac{1}{\tau_c} \left( \frac{T^2}{T_0^2} \right) = D/L^2.
\]

We concentrate on the fastest relaxation rate, \( z_0 = 6 \), where the subscript refers to zero shear. The effects of shear are displayed through the ratio

\[
z = \frac{\Delta z}{\Delta z_0} \left( \frac{T}{T_0} \right)^\gamma 
\]

Following [5], we adopt the ansatz \( D = D_\ast (T^2)_0^\gamma \), where \( D_\ast \) is independent of the turbulence level and scale. Then (13) becomes

\[
P \left( P - \frac{1}{3} \right) \left( P - 1 \right) = \frac{2}{3} W^2 p^{2\gamma},
\]

where \( W = \tau_\ast (\Delta z/\Delta z_0)^{1-\gamma} \). Aside from the numerical details, (16) agrees with [5].

In the case of a self-consistent \( \rho_s \) scan, equation (14) must be modified to account for the intrinsic \( \rho_s \) effects manifest in the linear growth rate

\[
\frac{1}{\tau_c} \left( \frac{T^2}{T_0^2} \right) = \gamma \ln(\rho_s) (v_{Ti}/a) D/L^2.
\]

This generalization translates in the high shear limit into an alternative normalization for the shear rate: \( W \) is now normalized to the \( \rho_s \)-dependent linear growth rate.

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