A model for the common origin of Jupiter family and Halley type comets

V.V. Emel’yanenko · D.J. Asher · M.E. Bailey

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Abstract A numerical simulation of the Oort cloud is used to explain the observed orbital distributions and numbers of Jupiter-family and Halley-type short-period comets. Comets are given initial orbits with perihelion distances between 5 and 36 au, and evolve under planetary, stellar and Galactic perturbations for 4.5 Gyr. This process leads to the formation of an Oort cloud (which we define as the region of semimajor axes $a > 1000$ au), and to a flux of cometary bodies from the Oort cloud returning to the planetary region at the present epoch. The results are consistent with the dynamical characteristics of short-period comets and other observed cometary populations: the near-parabolic flux, Centaurs, and high-eccentricity trans-Neptunian objects. To achieve this consistency with observations, the model requires that the number of comets versus initial perihelion distance is concentrated towards the outer planetary region. Moreover, the mean physical lifetime of observable comets in the inner planetary region ($q < 2.5$ au) at the present epoch should be an increasing function of the comets’ initial perihelion distances. Virtually all observed Halley-type comets and nearly half of observed Jupiter-family comets come from the Oort cloud, and initially (4.5 Gyr ago) from orbits concentrated near the outer planetary region. Comets that have been in the Oort cloud also return to the Centaur ($5 < q < 28$ au, $a < 1000$ au) and near-Neptune high-eccentricity regions. Such objects with perihelia near Neptune are hard to discover, but Centaurs with characteristics predicted by the model (e.g. large semi-major axes, above 60 au, or high inclinations, above $40^\circ$) are increasingly being found by observers. The model provides a unified picture for the origin of Jupiter-family and Halley-type comets. It predicts that the mean physical lifetime of all comets in the region $q < 1.5$ au is less than ~200 revolutions.

Keywords Comets · Oort cloud · Centaurs · Solar system formation · celestial mechanics

1 Introduction

1.1 Origin of short-period comets

Explaining the origin of short-period (SP) comets (periods $P < 200$ yr) is a long-standing problem. The main difficulty lies in the differences and apparent inconsistency between the respective numbers and orbital distributions of Jupiter-family (JF) and Halley-type (HT) comets. These we classify using the Tisserand parameter $T$ with respect to Jupiter (Carusi et al. 1987), JF comets having $T > 2$ (and $P$ usually below 20 years), HT comets having $T < 2$ (and $P$ usually between 20 and 200 years). When SP comets are classified this way the number of observed HT comets is found to be less than, or at most comparable to, the number of observed JF comets (see Section 2.1 below). However, most dynamical theories of their origin from the near-parabolic flux predict a far greater proportion of HT comets (Emel’yanenko and Bailey 1998), with the overall number of observed JF comets conversely being much too large relative to the calculated number (Joss 1973; Delsemme 1973). This discrepancy is associated with the well-known fading problem for long-period comets originating in the Oort cloud and has,
at least in part, led to the idea that the two classes of SP comet may have different physical structures and different proximate sources in the present Solar system.

Thus, although there have been many advances in understanding the diverse populations of small bodies in the Solar system, neither a single source dominated by trans-Neptunian objects nor one dominated by the traditional Oort cloud near-parabolic flux at small perihelion distances seems capable of explaining the entire distribution of orbital elements of SP comets. In particular the observed JF comet inclination distribution was recognized to have too many comets at low $i$ relative to the calculated distribution (Duncan et al. 1988; Quinn et al. 1990).

For these reasons, the majority of authors nowadays consider JF and HT comets to be physically as well as dynamically distinct classes, presumably formed in separate regions of the early Solar system and having different dynamical and physical evolutionary histories. Under this viewpoint, JF comets are often regarded as originating largely in the proto-planetary disc beyond Neptune, for example in or close to the Edgeworth-Kuiper belt (EKB). The idea that JF comets might originate in a primordial disc or ‘belt’ of comets located near or beyond the orbit of Neptune was investigated by a number of authors (e.g. Fernández 1980, 1982; Duncan et al. 1988; Torbett 1989; Torbett and Smoluchowski 1990; Quinn et al. 1990). The discovery of 1992 QB$_1$ (Jewitt and Luu 1993) and of further Edgeworth-Kuiper objects played a pivotal role in theories of the origin of SP comets, and important advances building on this evidence were made in particular by Duncan et al. (1995), Duncan and Levison (1997), Levison and Duncan (1997), and Levison et al. (2001). A key point (Duncan and Levison 1997) was recognition of the potentially important role played by the ‘scattered’ disc, introduced by Torbett (1989) and detected a few years later (Luu et al. 1997), in which it appears that the scattered disc of primordial objects originally formed in the region of the major planets is the principal source of observed JF comets, rather than the EKB. Under the viewpoint of distinct JF and HT classes, HT comets are regarded as objects captured from the Oort cloud (Levison et al. 2001), a structure that would have been produced inevitably as a by-product of planetary, stellar and other perturbations acting on planetesimals or cometary nuclei originally formed by accretion within the planetary region of the proto-planetary disc.

However, a rather unsatisfactory feature of this general picture is the assumption that HT comets coming from the Oort cloud must disintegrate very quickly in order to explain the small number of objects observed (Emel’yanenko and Bailey 1998; Levison et al. 2001). The number of observed inert HT ‘asteroids’ is also very small, and it seems as if the disintegration of a kilometre-size comet nucleus, into presumably an initial trail of much smaller boulder-size objects and then finally dust, must proceed fairly rapidly and lead to eventual extinction of the original comet. On the other hand, dynamical theories appear to require that a high proportion of the JF comet source flux should survive dynamical transfer into the inner Solar system to become active JF comets and that these JF comets should survive for $\sim 10^3$ revolutions in the inner Solar system. This difference in the physico-dynamical evolution of the two types of objects is the fading problem for SP comets.

It is probably not unreasonable to assume that comets that formed in different parts of the proto-planetary disc have different physical properties and therefore different lifetimes in the observable region, and it appears that this idea has become very deeply rooted. What is missing, however, is direct observational evidence to support the idea of two qualitatively distinct types of SP comet, correlating with dynamical class. Thus, present theories of the origin of SP comets rely on a poorly understood fading hypothesis to accommodate the observations, and there is no satisfactory physico-dynamical explanation as to why two very different types of SP comet should exist and yet appear observationally almost indistinguishable. Indeed, although comets show a very diverse range of properties, covering a very broad range of sizes, densities, dust-to-gas ratios and so on, there is as yet no compelling observational evidence for the expected bimodality of physical characteristics corresponding to HT versus JF dynamical class (Lamy et al. 2004).

In this work, whilst recognizing that comets may have different physical properties depending for example on their sizes or where they might have formed in the proto-planetary disc, we present a model for the common origin and evolution – from the Oort cloud – of the majority of comets in the Solar system.

1.2 Role of the Oort cloud

We define the Oort cloud as the region containing objects with semimajor axes $a > 10^3$ au (i.e., objects from the Oort cloud have at some point during their evolution reached $a > 10^3$ au). This definition is consistent with those used by other authors; e.g. Wiegert and Tremaine (1999) and Rickman et al. (2008) used similar values of semimajor axis, i.e. $a \approx 1-3 \times 10^3$ au, to define the inner boundary of the Oort cloud. Dones et al. (2004) introduced a further restriction, namely
that the maximum value of perihelion distance $q$ during an object’s orbital evolution should exceed 45 au for it to be counted as an ‘Oort cloud’ object. However, objects with $a \gtrsim 10^3$ au spend nearly all their time at large heliocentric distances, whatever their value of $q$, and therefore in the Oort cloud.

In this paper we have chosen to define Oort-cloud objects solely according to $a$ because the influence of stellar and Galactic perturbations is determined mainly by $a$ for near-parabolic orbits. It has been shown (e.g. Emel’yanenko 2005) that the dynamical pathways by which objects with $a > 10^3$ au reach the planetary region are different from those of typical trans-Neptunian objects (TNOs). While the evolution of TNOs is largely determined only by planetary perturbations, stellar and Galactic perturbations play a more substantial role in the process that drives the perihelia of objects with $a > 10^3$ au towards and through the planetary region, regardless of their previous $q$.

In the present paper (Section 2 onwards) we numerically integrate a much larger number of objects than in the Oort cloud model of Emel’yanenko et al. (2007), in particular to obtain statistically significant numbers of SP comets captured from the Oort cloud and allow a comparison of the model SP numbers and orbital distributions with the corresponding distributions of observed HT and JF comets. First (Section 2), in order that our model parameters can be constrained by observations, we assess the known characteristics of the various populations of cometary bodies.

2 Principal features of observed cometary populations

2.1 Short-period comets

We took data from the MPC (Minor Planet Center) and JPL (Jet Propulsion Laboratory) lists of discovered comets with $P < 200$ yr and $q < 1.5$ au near the present epoch. The completeness level in the discovery of SP comets is slightly uncertain, especially for HT comets and when results are extrapolated to fainter magnitudes and larger perihelion distances. However, many discussions (e.g. Fernández et al. 1999; Levison et al. 2001) have indicated a relatively high degree of completeness in the observed sample of active comets at small perihelion distances ($q < 1.5$ au). This level of completeness is supported too by studies of long-period comets, essentially none of which have been missed at $q < 1.3$ au since 1985 (Fernández and Sosa 2012).

We excluded SOHO comets because these have rather uncertain physical and dynamical characteristics; this only affects the distribution near very small $q$, a region that we do not study here. We also excluded multiple-apparition comets that have not been observed for a number of revolutions and are now treated as dead or inert. For split comets we took only the orbit of the main nucleus. In the end we obtained a list of 103 observed objects that we regard as representing the present-day set of active SP comets with $q < 1.5$ au. Of these, 75 have $T > 2$ (JF comets) and 28 have $T < 2$ (HT comets).

Figures 1 and 2 present orbital element distributions of these observed objects. The inclinations (Figure 2) show JF comets ($T > 2$) are concentrated close to the ecliptic and prograde HT comets outnumber retrograde ones (Fernández and Gallardo 1994; Levison et al. 2001).

Additionally the intrinsic numbers of JF and HT comets are a key constraint for our model. Fernández et al. (1999) found that about a hundred active JF comets should currently exist in the region $q < 1.5$ au, down to nuclear radius $R \sim 0.7$ km. The number appears to drop very rapidly for smaller bodies (Fernández et al. 1999; Snodgrass et al. 2011). This estimate could be modified...
to take account of more recent comet discoveries (cf. Section 2.1 of Di Sisto et al. 2009) but the result would not be significantly changed.

For HT comets, their longer average orbital periods constitute the principal bias against their discovery relative to JF comets. That is, although we expect that most active comets passing perihelion with sufficiently small $q$ will be found at the current level of observational searches, many HT comets have not yet returned to perihelion during the last few decades when searches have been at such levels. In this way, taking account of the HT period distribution, Levison et al. (2001), from 22 observed HT comets with $q < 1.3$ au, estimated a population of 57 active HT comets ($q < 1.3$). This result can be extrapolated to about a hundred objects with $q < 1.5$ au. In a later paper (Levison et al. 2006) the observed number has only increased to 24, suggesting the estimate is reliable.

We conclude that there are roughly a hundred active JF comets and a comparable number, i.e. approximately a hundred, of active HT comets to be explained in the region $q < 1.5$ au at times near the present epoch. Certainly the number of already known active JF comets shows that their intrinsic number cannot be much below a hundred, while the intrinsic HT number cannot be much above a hundred without implausibly many bright comets being missed by observational searches.

2.2 Near-parabolic flux

The flux of dynamically new comets from the Oort cloud is a fundamental parameter underpinning all dynamical models of the small-body populations in the Solar system, including the estimates in this paper. There are uncertainties in the frequency, $\nu_{\text{new}}$, of comets with $a > 10^4$ au passing perihelion per au in $q$ per year, but $\nu_{\text{new}}$ is usually estimated to lie in the range 2 to 4 for present-day comets in near-Earth space (Bailey and Stagg 1988; Fernández and Gallardo 1999; Wiegert and Tremaine 1999; Francis 2005). For quantitative estimates in this paper we adopt $\nu_{\text{new}} = 2.5$, within the observable region $q < 1.5$ au.

Francis (2005) undertook a detailed discussion of the objects that the LINEAR survey should discover for a given intrinsic cometary population. Considering also the question of the cometary absolute magnitude distribution, he found that very faint (on average presumably smaller) comets are only slightly more abundant than somewhat brighter (presumably larger) ones. Thus statements about cometary numbers, while evidently depending in detail on the adopted absolute-magnitude cutoff, are not strongly dependent on the precise value of that cutoff. In order to fix ideas, our adopted value $\nu_{\text{new}} = 2.5$ comets with $a > 10^4$ au passing perihelion per au in $q$ per year may be assumed to apply to comets with total visual absolute magnitudes $H_{10} \lesssim 11$. The quantity $H_{10}$ is the magnitude normalized to 1 au from Earth and Sun (e.g. Everhart 1967). The inclusion of fainter comets (e.g. extrapolating results from $H_{10} = 11$ to $H_{10} = 16$) makes very little practical difference to our results (Francis 2005; Sosa and Fernández 2011), although the calibration factor, $\nu_{\text{new}}$, would of course increase. The relative lack of very small (diameters $d \lesssim 0.5$ km) comets (Fernández and Sosa 2012) suggests that the physical response of the smallest dynamically ‘new’ comets from the Oort cloud to the thermal shock of their first passage at small perihelion provides a clue to the underlying rapid fading of new comets from the Oort cloud, necessary to explain the detailed shape of the observed $1/a$-distribution (cf. Bailey 1984).
Table 1  Centaurs (objects with $5 < q < 28$ au and $a < 1000$ au, excluding a few resonant trans-Neptunian objects and Trojans) that have a probable source in the Oort cloud. The majority of such objects have $a > 60$ au and after observational debiasing would be extremely numerous. Centaurs with $a < 60$ au are listed if $i > 40^\circ$. Only Centaurs with an observational arc larger than 100 days (asteroid orbits from MPC) and comets of orbital Classes 1 and 2 (Marsden and Williams 2008) are included. A unified classification scheme for Centaurs was proposed by Horner et al. (2003).

| $a$ (au) | $q$ (au) | $i$ (deg) |
|---------|---------|----------|
| (29981) 1999 TD$_{10}$ | 99.4 | 12.3 | 6 |
| (87269) 2000 OO$_{67}$ | 653 | 20.8 | 20 |
| (127546) 2002 XU$_{33}$ | 66.8 | 21.0 | 78 |
| 2003 FH$_{129}$ | 71.3 | 27.6 | 19 |
| (65489) 2003 FX$_{128}$ | 100 | 17.8 | 22 |
| 2004 VH$_{131}$ | 60.8 | 22.3 | 12 |
| 2005 VD | 6.7 | 5.0 | 173 |
| (308933) 2006 SQ$_{272}$ | 906 | 24.2 | 19 |
| 2007 JK$_{13}$ | 46.1 | 23.6 | 45 |
| 2007 UM$_{12}$ | 12.9 | 8.5 | 42 |
| 2008 KV$_{12}$ | 41.7 | 21.2 | 104 |
| (315898) 2008 QD$_{4}$ | 8.4 | 5.4 | 42 |
| 2008 YB$_{3}$ | 11.7 | 6.5 | 105 |
| 2009 MS$_{9}$ | 386 | 11.0 | 68 |
| 2009 YD$_{7}$ | 129 | 13.4 | 31 |
| 2010 BK$_{118}$ | 447 | 6.1 | 144 |
| 2010 JJ$_{124}$ | 82.9 | 23.6 | 38 |
| 2010 NV$_{12}$ | 294 | 9.4 | 141 |
| 2010 WG$_{9}$ | 53.8 | 18.8 | 70 |
| C/1984 U1 | 646 | 5.5 | 179 |
| C/1998 M6 | 972 | 6.0 | 92 |
| C/1999 K2 | 145 | 5.3 | 82 |
| C/2001 Q1 | 176 | 5.8 | 67 |
| C/2002 K2 | 561 | 5.2 | 131 |
| C/2002 P1 | 497 | 6.5 | 35 |
| C/2002 VQ$_{84}$ | 189 | 6.8 | 71 |
| C/2003 J1 | 514 | 5.1 | 98 |
| C/2005 R4 | 914 | 5.2 | 164 |
| C/2007 D3 | 765 | 5.2 | 46 |
| C/2007 K1 | 425 | 9.2 | 108 |

2.3 Centaurs

Centaurs are an intermediate cometary population (including active comets and inactive apparent asteroids), some of them being en-route from the outer Solar system to near-Earth space and the SP comet region. As a transition population the Centaurs must be replenished from a more distant source, presumably located either in the trans-Neptunian region of the Oort cloud, and they play a pivotal role in constraining theories of the origin of SP comets.

There is however no abiding consensus on the exact definition of a Centaur. Many authors (e.g. Stern and Campins 1996; Gladman 2002; Gladman et al. 2008; Jewitt 2009) adopt the criterion that a Centaur should orbit largely in the region of the outer planets. This has often been taken to mean $a \lesssim 30$ au, i.e. less than the semimajor axis of Neptune. In contrast, following Emel’yanenko et al. (2007), we define Centaurs as small bodies moving in heliocentric orbits with $5 < q < 28$ au and $a < 1000$ au (with any value of $i$), excluding a few resonant trans-Neptunian objects and Trojans. Thus, many objects that we call Centaurs (cf. Horner et al. 2003, 2004a,b) would be classified by some other authors as scattered-disc objects.

The condition $q < 28$ au separates Centaurs from the NNHE region described in Section 2.3. Our Centaur definition reflects the fact that this entire region of orbital element phase space ($a < 1000$ au and any $i$) constitutes a transition region of dynamically short-lived orbits in which population numbers and orbit distributions provide vital evidence about the outer Solar system source regions. So whereas a significant number of Centaurs are produced by dynamical evolution from the Kuiper belt or the trans-Neptunian region (e.g. Tiscareno and Malhotra 2003; Volk and Malhotra 2008), we emphasize that using a similar definition of a Centaur to that used in this paper, Emel’yanenko et al. (2005) showed the debiased distribution of observed Centaurs contradicts the idea that Centaurs primarily originate from a flattened disc-like population. They inferred instead that the Oort cloud produces $\sim 90$% of Centaurs, specifically well over 90% of Centaurs that have $a > 60$ au (which themselves constitute 90% of the Centaur population after observational debiasing) and $\sim 50$% of Centaurs with $a < 60$ au. Of these $a < 60$ Centaurs, the Oort cloud contributes especially to those with $i > 40^\circ$.

Observational evidence for Centaurs with these orbital characteristics is growing (Table I), consistent with predictions (Emel’yanenko 2005; Emel’yanenko et al. 2005) that a significant number of Centaurs have a proximate source in the Oort cloud. Emel’yanenko et al. (2005) concluded that there were two separate but overlapping dynamical classes of Centaurs, one originating in the Oort cloud and the other from the observed near-Neptune high-eccentricity region, each source region producing $\sim 50$% of Centaurs with $a \lesssim 60$ au and $\sim 50$% of JF comets. A bimodal colour distribution is observed in Centaurs (Peixinho et al. 2003). The only presently apparent difference in the two groups’ orbital properties is that red Centaurs tend to have lower $i$ (Tegler et al. 2008), while Peixinho et al. (2012) instead find that the bimodality is only pronounced in smaller objects. A dynamical evolution study suggests red Centaurs have spent less time at small $q$ (Melita and Licandro 2012).
2.4 Trans-Neptunian objects

As with Centaurs the nomenclature is not universal. For example (Gladman et al. 2008) in some classification schemes the term ‘Kuiper belt’ can mean the union of the ‘classical’ Kuiper belt, the scattered disc, the ‘extended’ (or detached) scattered disc and resonant objects exterior to the Neptune Trojans, the whole region sometimes being described simply as the trans-Neptunian region.

We define the trans-Neptunian region as the part of the Solar system in the vicinity of and beyond Neptune but interior to the Oort cloud, containing trans-Neptunian objects (TNOs) with \( a < 10^3 \) au. This region contains a complex, overlapping population of dynamically distinct classes of small bodies.

First there is the classical Edgeworth-Kuiper belt (EKB), a region estimated to contain a current total mass of the order of 0.01–0.02 \( M_\oplus \) (Bernstein et al. 2004; Fuentes and Holman 2008). The observed EKB objects are widely believed to represent the remains (perhaps less than 1%) of a massive primordial population of objects originally formed in low to moderate-eccentricity orbits in the extended proto-planetary disc beyond Neptune (Stern 1995, 1996; Morbidelli and Brown 2004). Non-resonant EKB objects cannot be the dominant source of observed JF comets as there are too few observed low-eccentricity orbits in this region with perihelia close enough to the orbit of Neptune to be captured in sufficient numbers (see Emel’yanenko et al. 2005). Resonant EKB objects can diffuse to other dynamical populations over Gyr time-scales (Morbidelli 1997; Tiscareno and Malhotra 2009), but their escape rate is rather less than that of ‘scattered disc’ objects (Volk and Malhotra 2008), so that this scattered disc, a declining and dynamically unstable population introduced by Duncan and Levison (1997), is a more important source of JF comets. For these reasons the classical EKB is not part of our present model.

A second class of 'primordial' TNO (i.e. TNOs that have never reached the Oort cloud) is a subset of the 'scattered disc' population. In this picture (Torbett 1989), objects originally formed in the region of the giant planets are gravitationally scattered outwards in \( a \) to produce an extended, flattened disc-like structure. Whereas a primordial disc of objects beyond Neptune would be characterized by low eccentricities and inclinations, according to many theories of cometary origin, the scattered disc is expected to contain objects on orbits having much higher eccentricities and substantial inclinations, perhaps merging smoothly into the unobserved but massive inner Oort cloud described by Hills (1981).

This second class of TNO therefore comprises objects that may have encountered Uranus and Neptune during an early phase of evolution of the Solar system and somehow survived to the present day without ever having evolved as far as the Oort cloud (\( a > 10^3 \) au). In our model (Section 3.1), for example, 6% of particles that had initial perihelion distances in the range \( 25 < q < 36 \) au survived to the present day without entering the Oort cloud or reaching any other end-state of the model (Emel’yanenko et al. 2007). This means that there is likely to be a significant number of surviving objects in this region whose orbits would appear to be very long-lived and which previous work has shown might possibly be a significant source of SP comets (Duncan and Levison 1997, Emel’yanenko et al. 2004).

A third class of TNO comprises bodies that were formed with original orbits in or close to the proto-planetary disc, but which at some time in their orbital history became part of the Oort cloud (\( a > 10^3 \) au) and are thus not 'primordial' in the sense of the second class above. Although most objects reaching the Oort cloud still have \( a > 10^3 \) au at the present epoch, a few evolve back to \( a < 10^3 \) au and so into the trans-Neptunian region. Our model produces many such objects, which we define as 'Oort Scattered Disc' (OSD) in Emel’yanenko et al. (2007).

We define also the near-Neptune high-eccentricity (NNHE) region, by \( 28 < q < 35.5 \) au and \( 60 < a < 1000 \) au. This region has an important dynamical characterization, covering objects that come close enough to Neptune’s orbit to be captured. The \( q \) cutoff at \( 28 \) au, just within Neptune’s orbit and below which an object becomes a Centaur, acknowledges the importance for coming under a planet’s control of a particle’s perihelion distance (Horner et al. 2003).

Observed NNHE objects are an important source of SP comets coming from the trans-Neptunian region (Emel’yanenko et al. 2004, 2005). Whether these observed NNHE objects are the same as NNHE objects produced as a result of dynamical evolution of objects into and subsequently from the Oort cloud remains to be determined. Section 6.1 concludes they are not, and therefore that the observed NNHE objects come from another source than that considered here.

3 Integrations

3.1 Model and methods

To construct our Oort cloud model, following Emel’yanenko et al. (2007), particles’ initial conditions after the formation and migration of the planets had the original semimajor axes uniformly distributed in the range \( 50 <
The original inclinations were distributed following a 'sine law' scaled to the interval 0 < \(i_0\) < 40°; the original perihelion distances were distributed uniformly in the range 5 < \(q_0\) < 36 au; and the original arguments of perihelion and original ascending nodes were distributed uniformly between 0 and 360°. The inclination distribution (peaked at \(i_0 = 20°\) falling to zero at 0 and 40°) is similar to the model scattered disc \(i\) distribution adopted by Volk and Malhotra (2008) following Brown (2001). Our choice of \(q_0 < 36\) au is connected with the assumption that the Oort cloud was created by objects coming from the planetary region or its nearest vicinity. Although some objects with \(q_0 > 36\) au may reach the near-Neptune region (Duncan et al. 1995; Emel’yanenko et al. 2003), it is evident that their contribution to the Oort cloud is small because the rate of diffusion in perihelion distance is slow.

While our choice of \(q_0\) assumes that comets were scattered to the Oort cloud region mainly by planetary perturbations, we do not use as initial conditions near-circular orbits in the planetary region (in contrast, for example, with Dones et al. 2004). Thus although it may be natural to assume that planetesimals formed in near-circular orbits are a source of Oort cloud comets, the accretional model of planetary formation still has so many difficulties and unclear questions that we deliberately avoid considering any particular hypothesis of comet formation a priori. Indeed the real situation with the initial orbital distribution of comets could be much more complicated than that described in Dones et al. (2004) even if comets were formed in near-circular, co-planar orbits. For example, planetary migration in the early Solar system appears to have been important in shaping the outer Solar system (Tsiganis et al. 2005). Moreover, the Sun may have formed in a denser stellar environment than it occupies now (Fernández and Brunini 2000; Levison et al. 2010). This makes assumptions about the distribution of comets in the early Solar system very uncertain.

Instead our approach is to constrain some features of the cometary distribution in the early Solar system by analysing observed distributions of cometary objects in the present Solar system. The main aim is to show that there are models of the Oort cloud that can explain the observed distributions of JF and HT comets. Our Oort cloud model can be interpreted as providing some general constraints on aspects of the cometary orbital distribution during early stages of the Solar system’s evolution. While details of the earliest stages of planetary and Oort cloud formation are beyond the scope of the present paper, we regard our Oort cloud as representing a general class of model in which cometary planetesimals, formed in the proto-planetary disc, have been scattered outwards by the planets to become subject to stellar and Galactic perturbing forces (cf. Duncan et al. 1987; Fernández 1997; Dones et al. 2004; Dybczyski et al. 2008; Leto et al. 2008). Hahn and Malhotra’s (1999) finding (their Section 4) that the total mass reaching the Oort cloud is quite insensitive to the orbital histories of the migrating planets tentatively supports our assertion that the precise details of planetary migration and comet formation are not relevant to our present purpose. It is for these reasons that we regard the ‘initial conditions’ of our integrations as applying to the time after the Solar system’s planetary migration phase.

There are several further motivations for the choice of initial high-eccentricity (50 < \(a_0\) < 300 au; \(q_0\) in or near planetary region) rather than near-circular orbits. This range of \(a_0\) is sufficiently large that objects can reach it at an early stage of evolution on the way to the Oort cloud under a wide range of different assumptions of cometary formation. The choice of initial conditions also allows particles to experience planetary perturbations for a long time before reaching the Oort cloud.
region, the model’s maximum value of $a_0$ being much smaller than that used in a similar approach by Duncan et al. (1987). The choice of initial $i$, and initial $e$ ranging above 200 au, is moreover expected in the scattered disc model with migrating Neptune (Gomes 2003). The main reason for our choice of initial orbits, however, is that the majority of high-eccentricity trans-Neptunian objects have orbits with $50 < a < 300$ au and $i < 40^\circ$. Figure 3 shows the distribution of $a$ and $i$ for discovered multiple-opposition objects with $q > 36$ au. This population of trans-Neptunian objects may preserve at least some memory of its original early Solar system distribution. Results for different initial models can be obtained by applying appropriate weights (Section 5.3).

The initial orbits were integrated in a model Solar system taking full account of planetary perturbations. All objects that reached the Oort cloud ($a > 10^3$ au) were then evolved for the remaining age of the Solar system under the combined action of planetary, stellar and Galactic perturbations.

In the present work, the 8925 objects that survived after 4.5 Gyr were cloned 200 times and integrated for a further 300 Myr including planetary, stellar and Galactic perturbations. The initial orbital distribution of these objects is shown in Figures 1 and 2 of Emel’yanenko et al. (2007). In order to suppress any possible artefacts associated with the initial conditions of the 300 Myr integrations we analysed our results on the interval 50–300 Myr. We took account of perturbations from the four large planets Jupiter to Neptune, using the secular perturbation theory of Brouwer and van Woerkom (1950) and Sharaf and Budnikova (1967), adding the terrestrial planets’ masses to the Sun. Objects were removed when $q < 0.005$ au or $1/a < 10^{-5}$ au$^{-1}$, or if they collided with planets.

The orbital calculations used the symplectic integrator described in the papers Emel’yanenko (2002) and Emel’yanenko et al. (2003) unless and until the orbit reached $q < 2.5$ au and the symplectic integrator of Emel’yanenko (2007) beyond that. The former method solves the Hamiltonian equations of barycentric motion for test particles moving in the field of the Sun and planets. It uses an adaptive time-step that is a function of the distance $r$ from the centre of and of the magnitude of perturbations, and so can deal with both highly eccentric orbits and close planetary encounters. The time-step is almost proportional to $r$ at small distances and in the absence of close encounters: in general it was 15 days at $r = 5$ au, and it did not exceed 900 days at any distance.

For objects reaching $q < 2.5$ au, the time-step of the integrator was approximately equal to $4.99989 \varphi / \varphi$, where $\varphi = 1 + Br + \gamma \sum_{j=1}^{4} b_j / \Delta_j + \gamma_1 / r^{3/2}$, $B=0.005549$, $\gamma=3$, $\Delta_j$ is the distance between the object and the perturbing planet, and $a_j$ and $m_j$ are the mass and the semimajor axis of the perturbing planet ($j=1,2,3,4$ for Jupiter, Saturn, Uranus and Neptune, respectively).

The Galactic model is taken from Byl (1986), but with the Sun’s angular speed $\Omega_0 = 26$ km s$^{-1}$ kpc$^{-1}$ and the mid-plane density of the Galactic disc in the Solar neighbourhood $\rho_0 = 0.1 M_\odot$ pc$^{-3}$ following Levison et al. (2001). To model stellar perturbations the procedure of Heisler et al. (1987) was used.

3.2 Initial results

We have previously shown that objects that have visited the Oort cloud ($a > 10^3$ au) at some time in their orbital history make a significant contribution to the observed classes of cometary objects in the Solar system (Emel’yanenko et al. 2007). Table 2 updates the results of that work using the present, more extensive simulations, adopting a present-day near-parabolic flux $\nu_{\text{new}} = 2.5$. The difference between the first three lines of this Table and the corresponding results in Table 2 of Emel’yanenko et al. (2007) are partly due to the assumed $\nu_{\text{new}} = 2$ in that paper and partly also due to statistical fluctuations in the relatively small number of objects considered in the earlier work.

In the present Table 2, $N_{\text{OC}}$ is the total number of objects in the Oort cloud ($a > 10^3$ au) at the present epoch; and $N_I$ and $N_O$ are the corresponding numbers in the relatively flattened inner Oort cloud ($10^3 < a < 10^4$ au) and the more isotropic outer Oort cloud ($a > 10^4$ au) respectively.

$N_S$ is the number of OSD objects (objects from the Oort cloud in the region $q > 30$ au, $60 < a < 1000$ au, the ‘$S$’ suffix indicating that they are located in the analogous region to the scattered-disc objects discussed by authors such as Duncan and Levison 1997), $N_N$ is the number in the NNHE region and $N_C$ is the number of Centaurs, also at the present epoch. In our model, $N_S$, $N_N$ and $N_C$ represent the numbers of objects in these respective regions which have previously visited the Oort cloud. In order of magnitude, $N_S \approx 3$–4$N_N$, the majority in orbits that do not strongly interact with Neptune, and $N_N \approx 7$–8$N_C$.

Finally, $\nu_JF$ and $\nu_HT$ are the corresponding present-day annual injection rates of cometary objects coming from the Oort cloud into JF and HT orbits with $q < 1.5$ au. The values $\nu_JF$ and $\nu_HT$ are ‘dynamical’ injection rates, i.e. obtained by ignoring any effects of physical fading or disintegration. The total number of active JF and HT comets will depend (see below) on their respective dynamical and physical lifetimes as SP comets. It
is noteworthy that \( \nu_{HT} \) is relatively insensitive to the initial frequency distribution of objects versus perihelion distance. Many Halley-types come from long-period Oort cloud orbits with perihelion distances in the inner planetary region (i.e. roughly within the orbit of Jupiter), but others (roughly 20% of the total) originate from the high-eccentricity Oort cloud cometary flux through the outer planetary region (Emelyanenko and Bailey 1998; Emelyanenko et al. 2007) and have a correspondingly more complex dynamical history. Some of these comets reaching JF or HT orbits pass through the \( N_S \) or \( N_N \) regions en route from the Oort cloud.

### Table 2

The number of cometary objects in different dynamical classes at the present epoch. All figures are calibrated with an assumed near-parabolic flux \( \nu_{new} = 2.5 \). The first three lines show the total number in the Oort cloud and the contributions to this number from objects in the inner and outer Oort cloud respectively. The second three lines show the numbers of OSD objects (\( N_S \)), NNHE objects (\( N_N \)) and Centaurs (\( N_C \)) coming from the Oort cloud. The final two lines indicate the present-day rate of production of new JF and HT comets from this Oort-cloud source into orbits with \( q < 1.5 \) au, neglecting any effects due to fading. The columns provide results for four different frequency distributions of initial perihelion distance, with relative numbers in the outer Solar system increasing from left to right.

For clarity the dynamical definitions used for these classes are then summarized. The \( N_S \) and \( N_N \) classes overlap; we primarily use \( N_S \) to analyse data (see especially Section 6.1) but calculate \( N_S \) for extra comparisons with other work.

| \( q^{-2} \) | \( q^{-1} \) | \( q^0 \) | \( 25 < q < 36 \) |
|---|---|---|---|
| \( N_{OC} \) | \( 4.8 \times 10^{11} \) | \( 5.3 \times 10^{11} \) | \( 5.8 \times 10^{11} \) | \( 7.1 \times 10^{11} \) |
| \( N_1 \) | \( 1.7 \times 10^{11} \) | \( 2.2 \times 10^{11} \) | \( 2.6 \times 10^{11} \) | \( 4.1 \times 10^{11} \) |
| \( N_O \) | \( 3.1 \times 10^{11} \) | \( 3.1 \times 10^{11} \) | \( 3.1 \times 10^{11} \) | \( 3.0 \times 10^{11} \) |
| \( N_{OC} \) | \( 9.0 \times 10^9 \) | \( 18.0 \times 10^9 \) | \( 21.0 \times 10^9 \) | \( 43.0 \times 10^9 \) |
| \( N_1 \) | \( 3.0 \times 10^9 \) | \( 5.6 \times 10^9 \) | \( 6.5 \times 10^9 \) | \( 12.9 \times 10^9 \) |
| \( N_O \) | \( 4.6 \times 10^8 \) | \( 7.7 \times 10^8 \) | \( 8.4 \times 10^8 \) | \( 15.2 \times 10^8 \) |

\( \nu_{JF} \) | 0.043 | 0.069 | 0.100 | 0.203 |
| \( \nu_{HT} \) | 0.073 | 0.079 | 0.082 | 0.083 |

\( N_{OC} \quad a > 1000 \)
\( N_1 \quad 1000 < a < 10000 \)
\( N_O \quad a > 10000 \)
\( N_{OC} \quad q > 30, \ 60 < a < 10000 \)
\( N_1 \quad 28 < q < 35.5, \ 60 < a < 10000 \)
\( N_O \quad 5 < q < 28, \ a < 1000 \) (not resonant TNOs, Trojans)
\( \nu_{JF} \quad P < 200, \ T > 2 \)
\( \nu_{HT} \quad P < 200, \ T < 2 \)

### 4 Short-period comet problems

#### 4.1 Numbers

It is well known that, with a population of only \( \sim 100 \) HT comets with \( q < 1.5 \) au (as constrained by observations), if we try to explain their origin by capture from the present-day Oort-cloud near-parabolic flux with initial perihelion distances \( q_{\text{init}} < 5 \) au, then it is necessary to place a very tight limit on the physical lifetime of such comets. This limit is further strengthened by the inclusion of HT comets originating from Oort-cloud source orbits with initial perihelion distances \( q_{\text{init}} > 5 \) au. Since comets are typically active at larger distances than 1.5 au, we must also consider restrictions on their physical lifetime in the region \( q < 2.5 \) au. Thus, for particles reaching \( q < 1.5 \) au, our integrations record also the preceding length of time spent with \( q < 2.5 \) au.

Although highly volatile ices, such as carbon monoxide CO, can sublimate at large distances \( \sim 10 \) au, the main driver of cometary activity, as recognized long ago by Whipple (1950), is the sublimation of water \( H_2O \) ice. The mass loss rate for sublimating water ice has a fast decrease for heliocentric distances larger than \( 20 \) au (Jewitt 2004). Therefore, in our model we apply restrictions on the cometary lifetime only in the region \( q < 2.5 \) au, assuming that outside this region the fading of comets is negligible in comparison to that when \( q < 2.5 \) au.

In order that the steady-state number of active HT comets should be \( \lesssim 100 \), our results imply that objects from the Oort cloud (\( a > 10^3 \) au at some time during their history) should survive as active comets for an average of \( \lesssim 150 \) revolutions in the region \( q < 2.5 \) au, in the model where the number of objects per unit perihelion distance is proportional to \( q_0^{-2} \). The result is much the same for other models, as indicated by the relatively weak dependence of \( \nu_{HT} \) versus dynamical model given in Table 2.

However, when we apply the same physical-lifetime constraint to the Oort-cloud objects that eventually become JF comets, we predict too few JF comets by a factor of around 30. That is, we predict only about three JF comets in the region \( q < 1.5 \) au compared to the \( \sim 100 \) to be explained. This illustrates the well-known problem of explaining the number of JF comets captured from the Oort cloud if the two classes of SP comet are assumed to have broadly the same physical properties and lifetimes, a result (as we have indicated) at the heart of what we have called the SP comet fading problem.

There is an extensive literature on possible ways to overcome this ‘number’ problem, including the as-
served NNHE region is approximately 0.93 × 10^{-9} \text{ yr}^{-1} \text{ km}^{-2} \text{ sr}^{-1} \text{ au}^{-2}. Here \( N_q^N \) is the intrinsic (i.e., observationally debiased) number of objects in the NNHE region represented by the then observed sample.

If it is assumed that most JF comets come from this region and have broadly the same fading behaviour as the observed Halley-types (i.e., mean lifetimes of the order of 150 revolutions in the region \( q < 2.5 \text{ au} \)), then our calculations would require \( N_q^N > 3 \times 10^{10} \). This value, which as in Section 2 may be assumed to apply to \( H_{10} \lesssim 11 \text{ cometary bodies (nuclear diameter } \gtrsim 1 \text{ km)} \), is greater than all previous estimates of the number of objects in this NNHE region, for example the \( 4 \times 10^6 \) scattered-disc objects with \( q \) in the range 34–36 au estimated by Trujillo et al. (2000). Furthermore, our result is a lower limit, for example because some of the comets that might have reached \( q < 1.5 \text{ au} \) in the absence of fading will be removed from the distribution of active comets by the lifetime limit of 150 revolutions within \( q < 2.5 \text{ au} \). Thus, if we take account of physical fading, the rate of injection of active JF comets to the region \( q < 1.5 \text{ au} \) is less than \( 0.18 \times 10^{-10} N_q^N \text{ yr}^{-1} \), requiring an even larger number of objects in the observed NNHE region to explain the observed number of JF comets.

A second argument comes from the predicted inclination distribution of the resulting JF comets. Emel’yanenko et al. (2004) found that the observed JF comets could in principle be explained by the evolution of objects captured from the observed NNHE region provided that the maximum lifetime of the resulting JF comets in the region \( q < 2.5 \text{ au} \) was not too long, i.e. approximately 2500 years (~360 revolutions). However, such a lifetime (i.e. 360 revolutions) is already 2–3 times longer than that required to explain the active HT comets from our Oort-cloud source, again highlighting the SP comet fading problem.

A third general argument leading to the same conclusion arises because the estimates in Emel’yanenko et al. (2004) were based on the assumptions that (a) the number of objects in the observed NNHE region is of the order of \( 10^{10} \) and (b) the physical behaviour of all JF comets is broadly the same. If the number of objects in the observed NNHE region is smaller than this, as seems likely (e.g., Levison et al. 2006), we would have to invoke longer average JF comet lifetimes in the region \( q < 2.5 \text{ au} \) to explain the observed number. Alternatively, if there are two types of JF comet, for example one with a mean lifetime in the region \( q < 2.5 \text{ au} \) comparable to that (~150 revolutions) required to explain the number of HT comets, then the other must have a much longer average lifetime to compensate. This would exacerbate the SP comet fading problem, not just by highlighting a difference in the physical properties of some JF comets and Halley-types, but by introducing a new (and arbitrary) difference between two different assumed types of JF comet.

Various arguments could of course be invoked to justify possible physical differences between different types of comet, for example that Oort-cloud objects might have visited the Jupiter-Saturn region many times before being finally ejected into the outer Solar system, whereas NNHE objects might never have come close to the inner Solar system before finally evolving into the observed Jupiter family. In this case, and whatever one’s view of the merit of such speculations, it is evident that we should not dismiss lightly the possibility that there may be two or more distinct types of SP comet.

However, in order to accommodate the twin constraints of the number of JF comets (tending to require a long lifetime) and their inclination distribution (tending to require a shorter lifetime), such models are
also subject to fine tuning and a very strict observational test. That is, the dynamically distinct HT and JF classes of SP comet should, on average, have very different fading properties and rates of decay in the observable region. In particular, the JF comets originating from a flattened source distribution other than the Oort cloud must, if they are to dominate the observed distribution of JF comets, have much longer lifetimes in the observable region than their HT counterparts originating from the Oort cloud. In principle, such a major physical difference between the two dynamical classes of SP comet (or even within the dynamically defined Jupiter family if the latter come from originally distinct sources) should be amenable to an observational test.

In summary, the SP comet fading problem remains an obstacle to understanding the origin of SP comets. Although it may be reasonable to suggest that the comets which formed in different regions of the primordial Solar system might have different fading properties after they eventually evolve into the observable region, it is important to emphasize that there is as yet no clearcut observational evidence to support such a view, nor even for any clear physical differences between the two main classes of SP comet. Rather than two physically different types of SP comet, behaving in statistically different ways in the inner planetary region so far as fading is concerned, we therefore instead develop in the remainder of this paper a unified model for the origin of SP comets. In this unified model all comets, whether coming from the Oort cloud or trans-Neptunian region, display broadly similar physical behaviour in the inner planetary region.

5 Unified model

We return to the idea that the key factor linking the two classes of SP comet, and perhaps all classes of comet, is their singular lack of strength and associated rapid fading. We thus seek a unified model for the origin and evolution of cometary bodies in the Solar system (particularly the observed SP comets) in which the majority of observed SP comets (though perhaps not all) originate from an Oort-cloud source which itself has an origin primarily in the dynamical evolution of objects left behind after the period of planet formation and planetary migration. In this case it is reasonable to assume that, to first order, the majority of comets will have broadly similar characteristics, though not necessarily identical physical properties, including those relating to fading.

In developing this unified physical picture for the origin of comets, we obtain new constraints on their required fading properties within the observable region. In particular, we use dynamical information provided by the results of our integrations and the link between Centaurs and SP comets to constrain the cometary numbers and lifetimes. In broad terms, our unified model predicts that essentially all the HT comets and nearly half the JF comets come from the Oort cloud. A flattened trans-Neptunian disc source is, however, required for the remaining ∼50% of JF comets. However, these objects too are predicted to have relatively short physical lifetimes within the observable region in order not to produce too many active JF comets. Thus, all comets have essentially the same fading properties within the observable region.

5.1 Centaurs and the NNHE region

In principle, understanding the relative contributions of different outer Solar system source regions to the SP comet population requires a full description of the number and orbital distribution of all objects in the trans-Neptunian region. Unfortunately our present knowledge of this complex region is limited by the precision with which the observed orbits are known and by severe observational selection effects. We therefore use the observed distribution of Centaurs (objects with 5 < q < 28 au and a < 1000 au) to constrain our results. Centaurs are an important transition population providing valuable information. Emel’yanenko (2005) and Emel’yanenko et al. (2007) presented various characteristics of the orbital distribution of Centaurs from the Oort cloud, results which are supported by our present work. The more extensive integrations of our current paper are necessary to provide a sufficient number of integrated particles transferred from the outer Solar system to SP orbits.

We recall that Emel’yanenko et al. (2005) predicted the orbital distribution of Centaurs originating from the observed NNHE region (28 < q < 35.5 au and 60 < a < 1000 au). These early results were based on the orbits of seven well-determined observed TNOs in the NNHE region suitably weighted by an observational debiasing procedure (Emel’yanenko et al. 2004). Let us denote as \( N'_N \) the intrinsic (i.e. debiased) number of objects in the NNHE region represented by this observed sample. Note that \( N'_N \) introduced above in Section 2.3 is defined in terms of exactly the same region of orbital element phase space. However, whereas \( N_N \) refers to objects that have been in the Oort cloud (a > 10^3 au), \( N'_N \) is the intrinsic (observationally debiased) number of NNHE objects represented by the discovered population. By this definition, \( N'_N \) and \( N_N \) could comprise the same population, or be disjoint, or partially overlap.
If disjoint, then $N'_C$ could represent the number of objects
in the NNHE region associated with a primordial
source distribution in the trans-Neptunian disc and so
not be included in our Oort-cloud model. We can dis-
cover how $N'_N$ really relates to $N_N$ by using Centaurs
as a constraint, as follows.

In a steady state, the number of Centaurs $N'_C$ ori-
ginating from the observed NNHE source region is a fixed
proportion of the total number $N'_N$ of such objects.
Emel’yanenko et al. (2005), using the integrations of
Emel’yanenko et al. (2004), calculated the constant of
proportionality $f_{N'N'\rightarrow C'} \simeq 0.008$, i.e. $N'_C \simeq 0.008N'_N$.
They also showed that these Centaurs were split in the
ratio 0.003 to 0.005 between orbits having respectively
$a > 60$ au and $a < 60$ au, nearly all the latter having
$20 < a < 60$ au (Emel’yanenko et al. 2005, fig. 2).

In order to compare these dynamical results with
observations it is necessary to apply an appropriate de-
biasing correction to the observed distribution of Cen-
taurs. The results of Emel’yanenko et al. (2005), based
on a sample of 42 well-determined Centaur orbits ex-
cluding objects in the 2/3 mean-motion resonance with
Neptune, showed that the intrinsic number of Centaurs
$N_{\text{Cobs}}$ is overwhelmingly dominated by objects with $a > 60$ au (roughly 90% of Centaurs having such orbits),
and that $N_{\text{Cobs}} \approx 0.13N_N$. This ratio, namely 0.13, is
much larger than the dynamical prediction $f_{N'N'\rightarrow C'} \simeq
0.008$, and this fact alone implies that the majority of
Centaurs, particularly the majority of those with $a > 60$
au, must have another source, i.e. a source other than
the $N'_N$ objects representing the observed NNHE re-

region. In this case, because it is an inescapable part of
any successful model, such a source is most likely the
Oort cloud.

Emel’yanenko et al. (2005) also showed (their fig. 5)
that, after debiasing, only 10% of Centaurs with $a < 60$
au have $40 < a < 60$ au. On the other hand, if the
principal source of Centaurs had been the observed
NNHE region, the dynamically predicted fraction would
have been around 50% (loc. cit. fig. 2). This is further
evidence that the $N'_N$ objects representing the observed
NNHE region cannot explain all the observed Centaurs.
Indeed, it raises the possibility that the Oort cloud may
contribute significantly to Centaurs with $a < 60$ au as
well.

In summary, the dynamically predicted number of
Centaurs with $a > 60$ au coming from the observed
NNHE region is roughly $0.003N'_N$, whereas observations
require this number to be of the order of $90\% \times 0.13 =
0.117N'_N$. The difference between these two results (i.e.
$0.114N'_N$) can be attributed to an Oort-cloud flux, i.e.
the flux of Oort-cloud objects through the planetary
system irrespective of whether they have gone through
the NNHE region. At this stage we make no assumption
as to whether any or all of the $N'_N$ objects represented
by the observed NNHE population come from the Oort
cloud. In any case, their contribution to Centaurs with
$a > 60$ au, i.e. $\simeq 0.003N'_N$, is insignificant.

Our new integrations provide a value for the steady-
state ratio of the number of Centaurs produced from
the Oort cloud with $a < 60$ au to the number with $a > 60$
au (cf. Table 2) later). Specifically, for every Cen-
taur with $a > 60$ au, approximately 0.07 Centaurs are
produced with $a < 60$ au. Therefore, for every $0.114N'_N$
Centaurs with $a > 60$ au that the Oort cloud produces, it
also produces $\sim 0.008N'_N$ Centaurs with $a < 60$ au.

As we have noted, the dynamically predicted num-
ber of Centaurs with $a < 60$ au coming from the ob-
served NNHE region is $N'_C(a < 60) \simeq 0.005N'_N$ and
the debiased number of Centaurs with $a < 60$ au is
$N_{\text{Cobs}}(a < 60) \simeq 0.10 \times 0.13N'_N \simeq 0.013N'_N$. Thus, the
additional population of Centaurs with $a < 60$ au pro-
duced by the Oort-cloud flux through the planetary sys-
tem is sufficient to account for this difference of $0.008N'_N$.
However, to a good approximation, the same Oort cloud
flux does not explain the entire number of $N_{\text{Cobs}}(a < 60)
\simeq 0.013N'_N$ Centaurs with $a < 60$ au, the $0.005N'_C$
objects from the observed NNHE region being unac-
counted for.

We conclude that the observed $N'_N$ objects are not
produced from the Oort cloud. In other words, the ob-
served NNHE objects studied by Emel’yanenko et al.
(2004) illustrate the dynamical features of near-Neptune
high-eccentricity objects that have never visited the
Oort cloud. In contrast, the predicted $N_N$ NNHE objects
originating from the Oort cloud in our model rep-

resent a sample of objects which owing to discovery bi-
ases are under-represented in the observed population.

Thus, although we defined $N'_N$ in terms of the obser-
vationally debiased known population, we may now in-
terpret it as referring to a ‘primordial’ trans-Neptunian
population that has never become part of the Oort
cloud ($a > 10^3$ au). So while the numbers $N_N$ and $N'_N$
describe objects in the same region of orbital element
phase space, they are essentially disjoint sets of objects.
The $N_N$ objects coming from a proximate source in the
Oort cloud are largely unobserved, i.e. are not yet rep-

resented in the $N'_N$ population of observed objects in
the NNHE region.

These results allow us to estimate the number $N'_N$ of
NNHE objects that have never visited the Oort cloud.
Thus, because the two sources are disjoint, $N_{\text{Cobs}} =
N_C + N'_C$, and hence $N_C = 0.122N'_N$ where $N_C$ is listed
in Table 2. This in turn allows us to determine the
additional contribution of these ‘primordial’ NNHE ob-
jects to the number of Centaurs ($N'_C = 0.008N'_N$) and
to the flux \( \nu_{JF} \) of JF comets with \( q < 1.5 \) au, taking \( \nu_{JF}/N_{\mathcal{N}} = 0.18 \times 10^{-10} \) from Emel’yanenko et al. (2004).

These values are given in Table 3 for the same distributions of initial \( q_0 \) as in Table 2. As with Table 2, \( \nu_{JF} \) is a ‘dynamical’ annual injection rate, i.e. assuming no physical lifetime limit. For comparison, the scattered disc proposed as a source of JF comets by Duncan and Levison (1997) corresponds to objects whose evolution was dominated by initial close encounters with Neptune during the early dynamical history of the Solar system, with no restriction on their subsequent evolution in semimajor axis. What we term the ‘primordial’ NNHE region overlaps this scattered disc to a large extent but does not include objects that ever reached \( a > 10^3 \) au.

### 5.2 Initial perihelion distribution

A further important factor that allows us to discover features of the dynamical and physical evolution of comets is the orbital distribution of JF comets. In particular, the predicted distribution of inclinations is very sensitive to the physical lifetime of JF comets (Levison and Duncan 1997). On this basis, we obtained limits of 2500 yr for the physical lifetime of JF comets in the region \( q < 2.5 \) au and 1200 yr in the region \( q < 1.5 \) au (Emel’yanenko et al. 2004), assuming all JF comets come from the trans-Neptunian region. In our present calculations, we have found that JF comets coming from the Oort cloud have similar dynamical characteristics and that the modelled \( i \) distribution of JF comets is close to the observed \( i \) distribution if the above physical lifetime limits are imposed.

| \( q_0^{-2} \) | \( q_0^{-1} \) | \( q_0^0 \) | \( 25 < q_0 < 36 \) au |
|-------------|-------------|-------------|------------------|
| \( N_{\mathcal{N}} \) | \( 3.7 \times 10^6 \) | \( 6.3 \times 10^9 \) | \( 6.9 \times 10^9 \) | \( 12.5 \times 10^9 \) |
| \( N_{\mathcal{C}} \) | \( 3.0 \times 10^7 \) | \( 5.0 \times 10^7 \) | \( 6.0 \times 10^7 \) | \( 10.0 \times 10^7 \) |
| \( \nu_{JF} \) | \( 0.067 \) | \( 0.113 \) | \( 0.124 \) | \( 0.225 \) |

But if we impose these limits on all SP comets, we have the problem of numbers described above (Section 4.1): the resulting ratio of the number of HT to JF comets is too large. From observational constraints, this ratio is around 1 – maybe below 1 but unlikely to be more than 1.5 (Section 2.3). We find the ratio ranges from 3.2 for the Oort-cloud model with initial perihelia within \( 25 < q_0 < 36 \) au to 12.3 for the model with the initial distribution proportional to \( q_0^{-2} \). In addition, the absolute number of JF comets is too small in models where objects are initially concentrated towards lower \( q_0 \), e.g. the number is only 12 in the case of the \( q_0^{-2} \) distribution. An additional SP comet contribution from the ‘primordial’ trans-Neptunian region does not solve these difficulties: adding these SP comets (based on the data of Table 3 but with the physical lifetime limits imposed) the HT/JF ratio ranges from 1.5 to 4.6, the number of JF comets being 32 for the \( q_0^{-2} \) distribution. Overall these constraints favour models where the initial number of objects increases with \( q_0 \) and are against models where the number decreases with \( q_0 \).

### 5.3 Best-fitting models

In order, therefore, to explore a suitable family of models, we assume firstly that the initial number of objects versus perihelion distance follows a power-law distribution, i.e. the number of objects in the range \( (q_0, q_0 + dq_0) \) is proportional to \( q_0^{-\beta} dq_0 \). To obtain consistency with both the numbers and orbital distributions of observed SP comets we also introduce a model for the physical lifetime in the observable region \( q < 2.5 \) au. Protoplanetary disc models suggest the snow line (boundary beyond which ice can condense) gradually moves inwards from distant regions (Davis 2005; Ciesla and Cuzzi 2006; Garaud and Lin 2007; Oka et al. 2011; Martin and Livio 2012) implying that the water distribution in the early Solar system would have been a function of heliocentric distance. It follows that comets’ composition could depend on their initial perihelion distance \( q_0 \) in the early Solar system. We assume the physical lifetime – within \( q < 2.5 \) au for comets that reach this region at the present epoch – is a constant number \( n_2 \) of revolutions for all objects formed in the outer \( q_0 \) range \( (25,36) \) au and varies as \( q_0^{-\beta} \) for \( q_0 < 25 \) au (with no discontinuity at \( q_0 = 25 \)). We impose an equivalent restriction, with the same \( \beta \), for the lifetime in the region \( q < 1.5 \) au at the present epoch, i.e. \( n_1 \) revolutions when the initial \( q_0 \) is within \( (25,36) \) au and \( n_1(q_0/25)^{\beta} \) for \( q_0 < 25 \) au.

We have explored which values of these four parameters \( \alpha, \beta, n_1 \) and \( n_2 \) are consistent with the observational constraints. The total steady-state number of JF
comets (to be compared to the number derived from observations) is a sum of the $N_{JF}$ which we calculate here, originating from the Oort cloud, and the additional contribution $N'_{JF}$ from the ‘primordial’ trans-Neptunian population. $N'_{JF}$ ranges from $\sim 50$ for $\alpha = 1$ to $\sim 70$ for the model where objects are initially concentrated in the outer region $25 < q_0 < 36$ au.

As we saw (Section 5.2), models with $\alpha < 0$ produce unsatisfactory results, namely too few JF comets as well as an incorrect value for the HT/JF ratio. Thus $\alpha > 0$ is implied, i.e. a greater initial concentration of comets towards larger initial $q_0$. Moreover for values of $\alpha$ larger than 2 (i.e. a strong initial concentration of comets towards the outer region), we need to introduce very strict restrictions on the cometary lifetime, and the resulting number of HT comets in retrograde orbits becomes too small in comparison with the observed number.

Our calculations show that models with $\beta \geq 1$ give results close to observations. But provided $\beta \gtrsim 1$, it is less tightly constrained than $\alpha$ and can even increase to infinity (formally $\beta = \infty$ means that all comets that do not originate within the outer region $25 < q_0 < 36$ au die after the first perihelion passage with $q < 2.5$ au).

Overall it is impossible to derive unique constraints on the cometary lifetimes and the values of $\alpha$ and $\beta$ simultaneously because of uncertainties in the number and the orbital distribution of SP comets. A range of possible solutions for $N_{JF}$ and $N_{HT}$ is presented in Table 4 representative of the allowed combinations of parameters $\alpha$, $\beta$, $n_1$ and $n_2$. The best solutions correspond to a lifetime limit $n_1 \approx 150$ revolutions, and $n_2 \approx 400$ revolutions, with $\alpha$ being in the approximate range 1 to 2, although there are other possibilities (e.g. the first solution in Table 4) with $n_1$ or $n_2$ differing by up to a few tens of per cent.

Table 4 summarizes our results for one of the best-fitting models. The parameters are $\alpha = 1$, $\beta = 2$, $n_1 = 150$ and $n_2 = 420$. Our model is consistent with the observed features of SP comets, Centaurs and TNOs, and Table 5 estimates the numbers of present-day cometary objects coming from the various original source regions.

### Table 4

| $\alpha$ | $\beta$ | $n_1$ | $n_2$ | $N_{JF}$ | $N_{HT}$ |
|---|---|---|---|---|---|
| 0.5 | $\infty$ | 170 | 600 | 45 | 112 |
| 1 | 1 | 150 | 420 | 42 | 118 |
| 1 | 2 | 150 | 420 | 42 | 108 |
| 1 | $\infty$ | 150 | 420 | 41 | 101 |
| 2 | 2 | 140 | 400 | 46 | 112 |
| 2 | $\infty$ | 140 | 400 | 45 | 107 |

### Table 5

| Initial region: | 5–10 au | 10–25 au | 25–36 au | TN |
|---|---|---|---|---|
| $N_{OC}$ | $1.0 \times 10^9$ | $1.8 \times 10^{11}$ | $4.3 \times 10^{11}$ | – |
| $N_1$ | $8.0 \times 10^7$ | $5.0 \times 10^{10}$ | $2.5 \times 10^{11}$ | – |
| $N_0$ | $1.0 \times 10^9$ | $1.3 \times 10^{11}$ | $1.8 \times 10^{11}$ | – |
| $N_S$ | 0 | $1.0 \times 10^9$ | $2.6 \times 10^{10}$ | – |
| $N_C$ | $3.0 \times 10^4$ | $9.0 \times 10^7$ | $9.3 \times 10^8$ | – |
| $N_C(\alpha < 60)$ | 0 | $1.0 \times 10^6$ | $6.6 \times 10^7$ | – |

The best solutions correspond $\alpha = 1$, $\beta = 2$, $n_1 = 150$ and $n_2 = 420$. Our model is consistent with the observed features of SP comets, Centaurs and TNOs, and Table 5 estimates the numbers of present-day cometary objects coming from the various original source regions.

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**Fig. 4** The model distribution of $T$ and $a$ in perihelia for SP comets with $q < 1.5$ au coming from the Oort cloud.
The regions correspond to three ranges of initial $q_0$ for objects that have visited the Oort cloud, with objects originating from the ‘primordial’ trans-Neptunian population that have never been in the Oort cloud listed in the final column (TN). The notation in Table 5 is as introduced earlier, with data also listed for the subset of Centaurs having $a < 60$ au.

Whereas the model strongly constrains the initial $q_0$ distribution, results are not highly sensitive to the initial $a_0$ distribution, adopted as uniform in the range 50–300 au. For example, for the best-fitting model of Table 5, changing the distribution from uniform to $a_0^{-1}$ per unit interval of $a_0$, by applying appropriate weights to the integrated particles, changes $N_{JF}$ from 42 to 41 and $N_{HT}$ from 108 to 77 (cf. Table 4).

Although our paper is mainly concerned with the origin of SP comets, we can compare our results for various cometary populations with other work. Fernández et al. (2004), following Trujillo et al. (2000) estimate $7.5 \times 10^9$ objects with radius $R > 1$ km and $q > 30$ au $a > 50$ au but note an order of magnitude uncertainty in this number. Moreover, our estimates are based on the flux of new comets with $H_{10} < 11$ corresponding to $R > 0.3$ km according to Fernández and Sosa (2012). The number of such objects should be larger than the number of objects with $R > 1$ km. Thus the estimate of Fernández et al. (2004) does not contradict our possible values of $N_S$. Estimates for the number of comets in the outer Oort cloud range up to $10^{12}$ (cf. Heisler 1990; Weissman 1996; Section 2.4 of Dones et al. 2004), while the distribution of comets in different parts of the Oort cloud is consistent with other models (cf. Emel’yanenko et al. 2007; Dybczyński et al. 2008; Leto et al. 2008).

Our data correspond to an initial population of approximately $1.6 \times 10^{12}$ objects with $R > 0.3$ km in the region $25 < q_0 < 36$ au, $50 < a_0 < 300$ au. This is quite consistent with the value of $\sim 3 \times 10^{12}$ objects with $R > 0.5$ km and cometary albedos in the original trans-Neptunian planetesimal disc, presented in Figure 1 of Morbidelli et al. (2009).

The data of Table 5 show that almost all JF comets originate from orbits with initial perihelia in the outer planetary system, and that over 90% of the steady-state number of HT comets come from the same $25 < q_0 < 36$ au region. This indicates that the majority of observed HT comets would have had initial orbits with perihelion distances largely overlapping the range of perihelia of the objects that eventually became JF comets. This is in contrast to the general picture described in Section 1, where JF comets largely originate from initial orbits in the trans-Neptunian region and HT comets from initial orbits in the region of the giant planets, with subsequent very different dynamical histories.

For all the models in Table 4 the orbital distributions of SP objects with $q < 1.5$ au coming from the Oort cloud have similar characteristics. Figures 4 and 5 show the orbital element distributions in perihelia (i.e. equal weight to each perihelion passage) for SP objects with $q < 1.5$ au coming from the Oort cloud, applying the restrictions $n_2 = 420$, $n_1 = 150$, $\beta = 2$ (all objects are equally presented, thus formally $\alpha = 0$ in these plots). The Figures show that in our model, JF comets ($T > 2$) are concentrated near the ecliptic plane, approximately 70% of them having $i < 15^\circ$. Regarding HT comets ($T < 2$), although the model reveals both prograde and retrograde orbits, prograde HT comets outnumber retrograde ones. In these ways the basic features of these distributions are consistent with those of the observed distributions in Figures 1 and 2.

In our model, all the modelled objects with periods under 20 yr have inclinations $i < 60^\circ$. There are several reasons for this. First, the majority of objects captured to the JF population originate from the inner Oort cloud (Emel’yanenko 2005). In our model, the inner Oort cloud is a rather flattened source of comets

![Fig. 5](image-url) The model distribution of $T$ and $i$ in perihelia for SP comets with $q < 1.5$ au coming from the Oort cloud.
(Emel’yanenko et al. 2007). Secondly, the majority of such objects are injected from the inner Oort cloud on to orbits with perihelia in the region of the outer planets by external perturbations. Their subsequent evolution is similar to the scheme described for trans-Neptunian objects by Kazimîrchoskî Polonskaya (1972) and Levison and Duncan (1997). The latter showed that preferentially objects with Tisserand parameters near 3 with respect to a planet cross the orbit of this planet. This suggests that mainly objects on prograde orbits are transferred to the inner planetary region.

Our results – from analysing observed SP comets – about the initial distribution of objects that form the Oort cloud are consistent with the standard picture of the origin of the Solar system. The conclusion was that \( \beta \geq 1 \): this corresponds to comets originally from the outer planetary region having a greater probability of survival and thus a longer lifetime as active comets, with objects originating from regions with small heliocentric distances conversely becoming extinct more quickly. This accords with the amount of water (as the main driver of cometary activity) being larger for more distant objects in the early Solar system.

6 Summary and conclusions

We have developed a model of the origin and evolution of the Oort cloud which is consistent with the basic observed orbital distributions of comets, Centaurs and high-eccentricity trans-Neptunian objects. Rather than requiring intrinsically different fading properties for Jupiter-family and Halley-type short-period comets, the model instead adopts the hypothesis that the physical lifetime of objects as active comets in the inner planetary region at the present epoch is a function of their initial perihelion distance in the early Solar system, and is the same for both JF and HT comets. The observed JF and HT populations also constrain the initial distribution of objects versus perihelion distance. Our results show that:

1. The mean physical lifetime of comets is \( \lesssim 200 \) revolutions in the region \( q < 1.5 \) au. This implies a significant cometary contribution to the distribution of small bodies (‘boulders’ and dust) making up the near-Earth interplanetary complex.
2. No model in which the initial number of comets is a decreasing function of their initial perihelion distance \( q_0 \) in the early Solar system can explain the present observed distribution of short-period comets.
3. Models in which the initial distribution of objects versus perihelion distance is concentrated more towards the outer planetary region, and in which their present active physical lifetime is an increasing function of \( q_0 \), are consistent with the present orbital distributions and numbers of both HT and JF comets.
4. Essentially all the observed HT comets and nearly half the observed JF comets come from a proximate Oort-cloud source (i.e. have experienced orbits with \( a > 10^3 \) au). The remaining \( \sim 50\% \) of observed JF comets come from the observed near-Neptune high-eccentricity (NNHE) population, a dynamically unstable region in which the cometary numbers decline by 95\% over 4 Gyr. In addition, more than 90\% of all Centaurs \((5 < q < 28 \text{ au}, a < 1000 \text{ au})\) come from the Oort cloud.
5. The model predicts that there is a significant Oort-cloud contribution to the NNHE population. The number of such objects is comparable to the debiased number of objects already discovered in the NNHE region, but they are still undetected owing to observational biases (e.g. considering large semimajor axes or high inclinations).

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