High spin proton emitters in odd-odd nuclei and shape changes

D. S. Delion

Institute of Physics and Nuclear Engineering,
Bucharest Măgurele, POB MG-6, Romania

R. J. Liotta, and R. Wyss

KTH (Royal Institute of Technology)
AlbaNova University Center,
S-10691, Stockholm, Sweden

Abstract

We present a formalism to describe proton emission from odd-odd nuclei based on a scattering like approach. Special emphasis is given to the case of transitions between states with different deformations. As an example we estimate the proton half-life of the odd-odd nucleus $^{58}$Cu. Our calculations show that the change of deformation in the decay process has a significant influence on the half-life. In addition, the angular momentum coupling of proton and neutron orbitals can result in an important $K$-hindrance of the decay. To fully account for the observed half-life in $^{58}$Cu, one has to consider a shape mixing in the final state.

PACS numbers: 23.50.+z, 24.10.Eq, 27.40.+z
The detection and study of nuclei on the proton drip line are being pursued intensely at present [1]. Many spherical as well as deformed nuclei in the region \( 50 < Z < 82 \) have been shown to be proton emitters [2,3,4,5,6,7,8,9]. Recently, an interesting example of prompt proton decay was reported: the proton decay of the medium mass odd-odd nucleus \(^{58}\text{Cu}\) which includes a transition from a highly deformed state into a near spherical state, thus showing a pronounced shape change during the transition from the mother to the daughter nucleus [10].

The study of proton emission are mainly based on a scattering like approach [11,12,13,14,15] or on the R-matrix formalism [16,17,18]. A comparison of the two approaches was recently performed in Refs. [14,19]. These studies were restricted to the case of decay from odd-even to even-even nuclei. Only recently odd-odd nuclei have been considered [20]. However, in all cases it has been assumed that the shapes of the mother and the daughter nuclei are the same.

In this letter we present a formalism based on the scattering like approach to study proton emission from deformed odd-odd nuclei including transitions between excited states as well as different deformations between the mother and daughter nuclei. In addition, we investigate the role played by the angular momentum orientation of the odd particle in the intrinsic system. As an example we will consider the case of \(^{58}\text{Cu}\), which life time has been established recently [21].

We will present in some detail the case corresponding to proton emission processes connecting two axially symmetric deformed nuclei.

The odd-odd mother wave function in the laboratory frame is given by the ansatz [22]

\[
\Psi_{M,K_i}^{L} = \frac{1}{\sqrt{2(1 + \delta_{K_i,0})}} \left[ D_{M_i,K_i}^{J_i}(\Omega) \psi_{K_i}(r'_\pi, r'_\nu) + (-)^{J_i+K_i} D_{M_i,-K_i}^{J_i}(\Omega) \bar{\psi}_{K_i}(r'_\pi, r'_\nu) \right],
\]

where \( \Omega \) are the Euler angles and the normalised Wigner function

\[
D_{M_i,K_i}^{J_i}(\Omega) \equiv \sqrt{\frac{2J_i + 1}{8\pi^2}} D_{M_i,K_i}^{J_i}(\Omega)
\]

describes the rotation of the core. The prime coordinates (e. g. \( r'_\pi \)) refers to the intrinsic frame.

The intrinsic wave function of the odd-odd (proton-neutron) system is described by

\[
\psi_{K_i}(r'_\pi, r'_\nu) = \Phi_{K_\pi}(r'_\pi) \Phi_{K_\nu}(r'_\nu),
\]

where the initial intrinsic spin projection is \( K_i = K_\pi + K_\nu \). The functions \( \Phi \) are standard Nilsson wave functions which can be written in terms of spherical components, i. e.

\[
\Phi_{K_\tau}(r_\tau) = \sum_{ij} g_{ijK_\tau}(r_\tau) Y_{ijK_\tau}(\hat{r}_\tau'),
\]

\[
Y_{ijK_\tau}(\hat{r}_\tau') \equiv \left[ i^j \Gamma_{i}(\hat{r}_\tau') \chi_{k_{\tau}}^{2} \right]_{jK_\tau}, \quad \tau = \pi, \nu .
\]

In order to evaluate the proton decay width we express the mother wave function in terms of the coordinates of the outgoing proton in the laboratory frame. By performing standard manipulations the wave function (1) becomes a superposition of terms corresponding to the proton wave function coupled to the rotational band built on the odd neutron state with spin projection \( K_\nu \), i. e.
\[ \Psi_{M_iK_i}^{J_i} = \sum_{J'_f} \sum_{lj} \frac{g_{ljK_{\pi}}(r_{\pi})}{r_{\pi}} \phi_{J'_f lj}(\Omega, \hat{r}_{\pi}, r'_{\nu}), \]

where

\[ \phi_{J'_f lj}(\Omega, \hat{r}_{\pi}, r'_{\nu}) = (-)^{j+K_i} \langle J_fK_\nu j - K_\pi | J_fK_\nu \rangle \left[ Y_{lj}(\hat{r}_{\pi}) D_{lj}^{J_f}(\Omega) \right]_{J_iM_i} \times \frac{1}{\sqrt{2(1 + \delta_{K_{\pi}, 0})}} \left[ \Phi_{K_{\nu}}(r'_{\nu}) + (-)^{J_f + K_\nu} \Phi_{K_{\nu}}(r'_{\nu}) \right]. \]

By using the orthogonality relation of these functions

\[ \langle \phi_{J'_f lj} | \phi_{J'_f l'j'} \rangle = \langle J_i, K_i; j, -K_\pi | J_f, K_\nu \rangle^2 \delta_{J_f, l'j'} \delta_{lj} \delta_{lj'}, \]

the proton partial decay width to a fixed final state \( J_f \) becomes \[23\]

\[ \Gamma_{J_f} = \hbar \nu \sum_{lj} \langle J_i, K_i; j, -K_\pi | J_f, K_\nu \rangle^2 \lim_{r_{\pi} \to \infty} |g_{ljK_{\pi}}(r_{\pi})|^2. \]

To obtain the radial parts of the wave functions, i.e. the functions \( g_{ljK_{\pi}}(r_{\pi}) \) in Eq. \[5\], one has to solve the corresponding Schrödinger equation, i.e.

\[ \hat{H}_{\pi} \Psi_{M_iK_i}^{J_i} = E_{\pi} \Psi_{M_iK_i}^{J_i}, \]

where \( \hat{H}_{\pi} \) is the Hamiltonian describing the motion of the emitted proton with positive energy \( E_{\pi} \). We assume that the neutron is a spectator. That is, the Hamiltonian \( \hat{H}_{\pi} \) will not affect the neutron coordinates.

One can solve Eq. \[9\] by transforming to the intrinsic frame, as in Eq. \[1\]. One then integrates over the neutron coordinates and all angular coordinates to obtain the system of equations corresponding to the radial proton wave functions, i.e.

\[ \frac{d^2 g_{ljK_{\pi}}}{dr_{\pi}^2} = \left\{ \frac{l(l+1)}{r_{\pi}^2} - \frac{2\mu}{\hbar^2} E_{\pi} \right\} g_{ljK_{\pi}} \]

\[ + \frac{2\mu}{\hbar^2} \sum_{lj'} \left[ \langle Y_{ljK_{\pi}} | V_1 | Y_{lj'K_{\pi}} \rangle g_{lj'K_{\pi}} + \langle Y_{ljK_{\pi}} | V_2 | Y_{lj'K_{\pi}} \rangle \frac{dg_{lj'K_{\pi}}}{dr_{\pi}} \right]. \]

Here \( V_2 \) is given by the non-spherical part of the spin-orbit interaction \[14\].

Our integration procedure using all possible channels confirms that the initial state in the odd-odd nucleus has a "stretched" configuration \( \pi g_{9/2} \otimes \nu g_{9/2} \) \[15\] with an amplitude 0.98. For the initial state one therefore has \( J_i = 2j = 9 \). As a result the system of equations \[14\] practically contains only that component. That is, the outgoing proton escapes as a wave containing all angular momenta but the contribution of the configuration \( (l, j) = (4, 9/2) \) is clearly dominant. Therefore, in our radial equations we match the external solution with only that internal wave function. We impose on the radial component \( g_{ljK_{\pi}}(r_{\pi}) \) outgoing boundary conditions at large distances, i.e.

\[ g_{ljK_{\pi}}(r_{\pi}) = C_{ljK_{\pi}} \mathcal{H}_{ljK_{\pi}}^{(+)}(kr_{\pi}) \rightarrow C_{ljK_{\pi}} \left[ G_{l}(kr_{\pi}) + IF_{l}(kr_{\pi}) \right], \]

where \( F_l(kr_{\pi}) \) and \( G_l(kr_{\pi}) \) are the regular and irregular Coulomb functions, respectively. \( \mathcal{H}_{ljK_{\pi}}^{(+)}(kr_{\pi}) \) denotes the numerical solution found using backward integration. The wave number is given by
The constant $C$ in Eq. (11) is obtained by matching the external and internal solutions at some point $R$

$$C_{ljK_\pi} = \mathcal{P}_{ljK_\pi}(R)/|\mathcal{H}^{(+)}_{ljK_\pi}(kR)|.$$  

(13)

where $\mathcal{P}_{ljK_\pi}(R)$ is the proton formation amplitude.

Since $\lim_{r_\pi \to \infty} |F_i(kr_\pi)|^2 + |G_i(kr_\pi)|^2 = 1$ one can write the final expression for the proton decay width (8) as

$$\Gamma_{jf} = \hbar \nu \left[ \langle J_i, K_i; j, -K_\pi | J_f, K_\nu \rangle \mathcal{P}_{ljK_\pi}(R)/|\mathcal{H}^{(+)}_{ljK_\pi}(kR)| \right]^2.$$  

(14)

A similar formula was already presented in Ref. [20].

We will assume that the mother and daughter even-even cores can be described within the BCS approach. As already mentioned the odd neutron acts as a spectator. The proton-neutron interaction is included in a phenomenological manner by adjusting the proton and neutron pairing gaps to their experimental values. With these approximations the formation factor $\mathcal{P}$ is computed as the internal component multiplied by the amplitude to find the final proton Nilsson state ($c_{ljK_\pi}^\dagger$) in the initial quasiparticle state ($a_{ljK_\pi}^\dagger$), i.e.

$$\mathcal{P}_{ljK_\pi}(R) = g^{(int)}_{ljK_\pi}(R) \langle BCS_f | c_{ljK_\pi}^\dagger a_{ljK_\pi}^\dagger | BCS_i \rangle,$$

(15)

where $g^{(int)}_{ljK_\pi}$ is the internal solution of the system (10) and $\langle BCS_i(f) \rangle$ refers to the initial (final) $^{57}$Ni core state. One finally obtains

$$\mathcal{P}_{ljK_\pi}(R) = g^{(int)}_{ljK_\pi}(R) u_{K_\pi}^{(f)} U_{K_\pi K_\nu} \langle BCS_f | BCS_i \rangle.$$  

(16)

Here $u_{K_\pi}^{(f)}(\tau)$ is the proton BCS amplitude of the final nucleus. The elements of the matrix $U$ connect the quasiparticle operators corresponding to the initial and final deformations (for details see Appendix (E.3) of Ref. [24]). The overlap integral of the two cores is given by

$$I \equiv \langle BCS_f | BCS_i \rangle = |det[U(\pi)]det[U(\nu)]|^{1/2},$$

(17) $$U_{K_\pi \nu} = [u_{K_\pi}^{(f)} + v_{K_\pi}^{(f)}] \langle \Phi_{K_\pi}^{(f)} | \Phi_{K_\nu}^{(i)} \rangle.$$

where $\Phi_{K_\pi}^{(f/i)}$ denote the single particle Nilsson wave functions. In Eq. (14) we assume that the $K$-value for both the odd-proton and neutron states are conserved during the decay process.

The proton wave function necessary to evaluate the formation amplitude in Eq. (14), will be calculated by using a Woods-Saxon mean field with universal parametrisation [25]. We adjust the depth of the potential in order to reproduce the $Q$-value.

The decay width (14), or the half-life $T = \hbar \ln 2/\Gamma$, should be independent of the radius $R$ in a region beyond the nuclear surface. This condition is automatically satisfied in our case because the internal and external solutions satisfy the same Schrödinger equation.

To analyse the $K$-dependence and to determine the shape of the mother and daughter nuclei, we performed total Routhian surface (TRS) calculations, where quadrupole and hexadecapole deformations are minimized with respect to the energy in the rotating frame of reference. Pairing correlations are treated selfconsistently by means of the Lipkin-Nogami
method [24,27,28]. The shape of the mother (initial) nucleus $^{58}$Cu is calculated to be prolate and axially symmetric. The hexadecapole deformation is calculated to be $\beta_4 = 0.05$ while the quadrupole deformation parameter yields a value of $\beta_2 = 0.35$. The bandhead spin of the mother nucleus, assigned to be $J = 9$, corresponds to a predominantly $K = 1/2$ configuration for both protons and neutrons, that is fully rotationally aligned.

The initial $K$-value of $^{58}$Cu is given by $K_i = |K_\pi \pm K_\nu|$ yielding a value of $K_i = 1, 0$. If one assumes that the daughter (final) nucleus $^{57}$Ni has the same prolate deformation as $^{58}$Cu the overlap is unity and the half-live becomes $T \approx 0.8 \times 10^{-16}$ s, which underestimates the corresponding experimental value, i. e. $T_{\text{exp}} \approx 0.5 \times 10^{-12}$ s [21], by four orders of magnitude.

Actually the nucleus $^{57}$Ni is expected to be spherical. However, our calculations yield the shape of the $J_f = 9/2^+$ state to be oblate deformed, with a deformation value of $\beta_2 \approx -0.2$. This oblate deformed state is approximately 600 keV lower in energy as compared to the spherical minimum. We thus expect the ground state of the $J_f = 9/2^+$ state to be oblate deformed and not spherical. Since the moment of inertia at oblate shapes is rather small, non collective excitations can compete or mix into the corresponding collective rotations. According to the TRS calculations, the oblate collective minimum disappears with increasing angular momenta, whereas the non-collective oblate minimum stays stable up to high frequencies. The quadrupole deformation lifts the degeneracy of the $J_f = 9/2^+$ multiplet, forming a distinct state with a $K$-value of 9/2 as ground state. For collective rotational states one expects a strongly coupled band involving M1 transitions. Indeed, the data on $^{57}$Ni shows a M1 structure above the proposed $J_f = 9/2^+$ state, distinctly different from the rest of the level scheme [24]. The first excited state with spin 11/2$^+$ is also $\approx 1$ MeV lower in energy than the corresponding excitations of the negative parity spherical states. Although the structure resembles that of a strongly coupled band, it does not follow a $J(J+1)$ pattern, pointing towards non collective excitations, in agreement with the calculations.

Assuming that the mother nucleus is prolate and the daughter spherical the overlap (17) becomes $I^2 \approx 3.2 \times 10^{-2}$, giving a half-life of $T \approx 0.24 \times 10^{-14}$ s, which is still two orders of magnitude below the experimental value. If one assumes the oblate shape discussed above for the daughter nucleus the half life increases further and one obtains $I^2 \approx 10^{-2}$ and $T \approx 0.7 \times 10^{-14}$ s. The change is not large as compared with the prolate-spherical transition assumed above.

In order to understand the discrepancy with the experimental halflife $T$, we analysed in detail the different elements entering the expression of the decay width. We thus noticed that the $K$-values corresponding to the odd proton and neutron which may couple to the initial projection $K_i$, ranging from $K=1/2$ to $K=9/2$, can change drastically the values of the Clebsch-Gordan coefficient in Eq. (14). In Fig. 1 we show the half-life computed according to the decay width given by Eq. (8) as a function of the final (neutron) spin projection in the intrinsic system $K_f = K_\nu$. The solid lines correspond to decays where the mother and daughter nuclei have the same deformation, i. e. where $\beta_i = \beta_f = 0.35$ and the overlap factor entering Eq. (14) is virtually unity. In addition to $K_i = 1, 0$ (upper and middle line) we also considered the value $K_i = 2$ (lower line). In the same figure we show the effect of the change in deformation due to the overlap, Eq. (17), for prolate and oblate shapes. The dashed lines correspond to the case where the daughter nucleus has an oblate deformation, i. e. where $\beta_i = 0.35$ and $\beta_f = -0.2$.

The oblate deformation has a profound influence on the wave function of the daughter nucleus: Whereas for spherical nuclei all $K$-values are degenerate and equally probable, at
oblate shapes the $K_\pi = 9/2$ orbit of the $g_{9/2}$ subshell is at the Fermi surface. This implies, according to Eq. (4), that the transition is forbidden since $K_\pi = 1/2$ and $K_\nu = 9/2$ would result in a vanishing Clebsch Gordan coefficient. Thus, in addition to the shape hindrance we encounter a $K$-hindrance.

Therefore the only possibility to account for the proton decay of $^{58}$Cu is to assume that the initial and final $K$-values are not pure. This is in fact expected, since the Coriolis interaction will induce $K$-mixing. To further investigate the role of this interaction, we performed additional particle plus rotor calculation for the initial and final nuclei by using the Nilsson basis functions given by Eq. (4), i.e.

\[ \bar{\Phi}_{\nu}^{(i/f)} = \sum_{K_\nu} d_{K_\nu}^{(i/f)} \Phi_{K_\nu}^{(i/f)}. \]  

(18)

The odd neutron is not anymore a spectator and its preformation factor is given by

\[ P_\nu = \langle \bar{\Phi}_{\nu}^{(f)} | \bar{\Phi}_{\nu}^{(i)} \rangle = \sum_{K_\nu} d_{K_\nu}^{(f)} d_{K_\nu}^{(i)} \langle \Phi_{K_\nu}^{(f)} | \Phi_{K_\nu}^{(i)} \rangle, \]  

(19)

Our calculation shows that different eigenstates with the same projection give a small contribution to $P_\nu$.

The effect of the Coriolis mixing is two-fold:

i) The initial and final $K_\nu$ will now acquire components of higher and lower values, as can be seen in Table 1. From the same Table it becomes clear that indeed the predominant component in the initial prolate nucleus has $K_\nu = 1/2$.

ii) The value of $K_i$ corresponding to the band-head will change accordingly allowing additional coupling possibilities. The new term arising from the Coriolis mixed states implies that the $K_\pi = \pm 1/2$ can couple to $K_\nu = \mp 5/2$ yielding a $K_i = 2, 3$ state. According to Table 1, this is the only common component in both initial and final nuclei. Higher $K$-values can be possible although with even smaller amplitudes.

Then, assuming that the daughter nucleus is deformed with $\beta_f = -0.2$ the calculated half-life for the case of $K_i = 2$ and $K_f = K_\nu = 5/2$ (see Fig. 1) becomes $T \approx 1.8 \times 10^{-14}$ s. The neutron preformation factor is $P_\nu^2 \approx 10^{-5}$. As a result the lifetime acquires the value $T \approx 1.8 \times 10^{-9}$ s, which overestimates the experimental value by three orders of magnitude.

Therefore, the experimental value would correspond to a decay in the daughter nucleus where the $9/2^+$ state has components in the wave function corresponding to oblate and spherical shapes, respectively. In other words, shape fluctuations due to the relative $\beta$-softness of the final nucleus provides a suitable scenario to explain the actual lifetime. Assuming that the spherical and oblate states in $^{57}$Ni are mixed, one obtains a lifetime between the spherical $T \approx 10^{-14}$ s and oblate prediction $T \approx 10^{-9}$ s. A simple estimate shows that the amplitudes of spherical and oblate components should correspond to $X_{sph} \approx 0.1$ and $X_{obl} \approx 0.9$, respectively, in order to reproduce the experimental half-life.

In conclusion we have presented in this letter a formalism to analyse proton decay in situations where the initial and final nuclei are in excited states with different deformations. We applied the formalism to study the decay of the odd-odd nucleus $^{58}$Cu and found that due to the difference of deformations between the mother and daughter nuclei the calculated half-life increases by two orders of magnitude, but still remains two orders of magnitude below the experimental value.

Due to the prolate shape of the mother and oblate shape of the daughter nucleus, the decay is K-hindered and can proceed only via a very weak component originating from the
Coriolis mixing of initial and final states. The half-life in this case becomes three orders of magnitude larger than the experimental value. A possible explanation of the measured half life is to assume mixing of spherical and oblate shapes in the daughter nucleus, i.e. in $^{57}$Ni. The spherical component has a small amplitude $\approx 0.1$ but it is essential to describe the experimental half-life.

Given the large changes in life times, associated with the different physical processes involved in the decay, implies of course a substantial uncertainty in our analysis. Still, these unexpected features indicate that proton decay is a powerful tool to investigate fine details of the intrinsic structure of nuclei at the drip line.

Acknowledgements

One of us (D S D ) thanks for the hospitality extended to him by the Department of Physics of KTH, AlbaNova University Centre, Stockholm, where part of this work was performed. This work is supported by the Swedish Science Council.

Figure captions

Fig. 1.

Half-life of $^{58}$Cu as a function of the final spin projection in the intrinsic system $K_f = K_\nu$. The solid lines correspond to transitions without deformation change, i.e. with $\beta_f = \beta_i = 0.35$. The dashed lines correspond to $\beta_i = 0.35$ and an oblate final shape with $\beta_f = -0.2$. The labels of the lines to the right of the figure are the corresponding values of $K_i$.

Table. 1

Amplitudes defined by Eq. (18) corresponding to the different $K_\nu$-values for the $9/2^+$ state at prolate and oblate shapes, respectively.

| $K_\nu$ | 1/2 | 3/2 | 5/2 | 7/2 | 9/2 |
|--------|-----|-----|-----|-----|-----|
| $d_{K_\nu}^{(i)}$ | 0.901 | 0.405 | 0.089 | - | - |
| $d_{K_\nu}^{(f)}$ | - | - | 0.053 | 0.212 | 0.986 |
REFERENCES

[1] P.J. Woods, and C.N. Davids, Ann. Rev. Nucl. Part. Sci. 47, 541 (1997).
[2] P.J. Sellin et al., Phys. Rev. C47, 1933 (1993).
[3] R.D. Page et al., Phys. Rev. Lett. 72, 1798 (1994).
[4] C.N. Davis et al., Phys. Rev. C55, 2255 (1997).
[5] J.C. Batchelder et al., Phys. Rev. C57, R1042 (1998).
[6] C.N. Davis et al., Phys. Rev. Lett. 80, 1849 (1998).
[7] C.R. Bingham et al., Phys. Rev. C59, R2984 (1999).
[8] K. Rykaczewski et al., Phys. Rev. C60, 011301 (1999).
[9] A.A. Sonzogni et al., Phys. Rev. Lett. 83, 1116 (1999).
[10] D. Rudolph et al., Phys. Rev. Lett. 80, 3018 (1998).
[11] V.P. Bugrov, and S.G. Kadmenski, Sov. J. Nucl. Phys. 49, 967 (1989).
[12] S.G. Kadmenski, and V.P. Bugrov, Phys. At. Nucl. 59, 399 (1996).
[13] S. Aberg, P.B. Semmes, and W. Nazarewicz, Phys. Rev. C56, 1762 (1997).
[14] C.N. Davis, and H. Esbensen, Phys. Rev. C61, 054302 (2000).
[15] H. Esbensen and C.N. Davis, Phys. Rev. C63, 014315 (2001).
[16] E. Maglione, L.S. Ferreira, and R.J. Liotta, Phys. Rev. C59, R589 (1999), and references therein.
[17] A.T. Kruppa, B. Barmore, W. Nazarewicz, and T. Vertse, Phys. Rev. Lett. 84, 4549 (2000).
[18] B. Barmore, A.T. Kruppa, W. Nazarewicz, and T. Vertse, Phys. Rev. C62, 054315 (2000).
[19] A. Bianchinni, R. J. Liotta and N. Sandulescu, Phys. Rev. C63, 24610 (2001).
[20] L.S. Ferreira and E. Maglione, Phys. Rev. Lett. 86, 1721 (2001).
[21] D. Rudolph et al., Nucl. Phys. A694, 132 (2001).
[22] A. Bohr, and B.R. Mottelson, Nuclear Structure vol. II (Benjamin, London, 1975).
[23] P. O. Fröman, Mat. Fys. Skr. Dan. Vid. Selsk 1 no. 3 (1957).
[24] P. Ring, and P. Schuck, The Nuclear Many Body Problem (Springer-Verlag, Berlin, 1980).
[25] J. Dudek, Z. Szymanski, and T. Werner, Phys. Rev. C23, 920 (1981).
[26] W. Satu/suppress la, R. Wyss and P. Magierski, Nucl Phys. A578, 45 (1994).
[27] W. Satu/suppress la and R. Wyss, Phys. Script. T56, 159 (1995).
[28] R. Wyss and W. Satu/suppress la, Phys. Lett. B351, 393 (1995).
[29] D. Rudolph, C. Baktash, M.J. Brinkman, M. Devlin, H.-Q. Jin, D.R. LaFosse, L. Riedinger, D.G. Sarantites, C.-H. Yu, Eur. Phys. J. A4, 115 (1999).
\[ \beta_i = -0.2 \]
\[ \beta_i = \beta_i = 0.35 \]