Abstract
Photometric and spectroscopic observations and analysis of the eccentric eclipsing binary V459 Cassiopeiae ($e = 0.0244$) were performed by Lacy et al. (2004). Observations of minimum light show the presence of apsidal motion. In order to find the observed rate of apsidal motion, I followed the procedure described by Guinan & Maloney (1985). A new observed rate of apsidal motion of $15^\circ/100$ yr with a period of 2400 yr is computed which is not in agreement with the one reported earlier. Also the advance of the periastron is calculated theoretically by taking into account the Newtonian (classical) and general-relativistic effects according to the physical and orbital parameters of the system. The theoretical value of $2^\circ .64/100$ yr is obtained which is 5.75 times smaller than the observed rate of the apsidal motion.

Key words: stars: binaries: eclipsing: V459 Cas – stars: apsidal motion

1 Introduction
V459 Cassiopeiae (BV5, GSC 04030-01001; $V_{max} = +10.33$; $P=8.45$ days) is a double-lined eclipsing binary where its variability was discovered by Strohmeier (1955). A measurement on the eccentricity of the system was done by Busch (1976) after Meununger & Wenzel (1967) recognized the eccentricity of its orbit. Through a series of spectroscopic and photometric observations (in UBV and $uvby$) carried out from 1984 to 2004, the absolute dimensions of the physical and orbital parameters of V459 Cas was obtained after precise, simultaneous analysis of the light and velocity curves of the system (Lacy et al., 2004; hereafter L04). According to L04, the corresponding radii and masses of the components are $R_1 = 2.009 \pm 0.013 R_\odot$, $R_2 = 1.965 \pm 0.013 R_\odot$ and $M_1 = 2.02 \pm 0.03 M_\odot$, $M_2 = 1.96 \pm 0.03 M_\odot$ respectively with an eccentricity of $e=0.0244$. But the rate of apsidal motion, $\dot{\omega}$, or the period of apsidal revolution, $U$, is not well determined which is an important parameter for probing the stellar structure of the eclipsing binary components. I investigate the observed rate of apsidal motion in Sec. 2 by following the method of Guinan & Maloney (1985)(hereafter GM85) to measure the observed rate of apsidal motion, $\dot{\omega}_{obs}$. The comparison between the theoretical and the observational values of $\dot{\omega}$ is presented in Sec. 3. The final results and conclusions are given in Sec. 4.

2 Observed rate of apsidal motion
Due to the deep, narrow eclipses of V459 Cas the rate of apsidal motion can be determined by analysis of primary and secondary eclipse timings. In their paper, GM85 described the procedure that must be followed to determine the apsidal motion rate from the change in the displacement of the secondary minimum from the half point (0.5 phase) according to

$$D = (t_2 - t_1) - 0.5 \times Period,$$

(1)

where $t_2$ and $t_1$ are times of secondary and primary minima, respectively and $D$ is related to the longitude of periastron $\omega$ by the formula given by Sterne (1939,a,b)

$$D = \frac{P}{\pi} \left[ \tan^{-1} \left( \frac{e \cos \omega}{1 - e^2} \right) + \frac{e \cos \omega}{1 - e^2 \sin^2 \omega} (1 - e^2)^{1/2} \right].$$

(2)

In Eq. (2), $e$ is the orbital eccentricity and $P$ is the orbital period in days. Fig. 1, presents the (O-C) diagram from all the observed times of primary and secondary minima. To plot the (O-C) diagram for the primary minima (upper panel of Fig. 1), I used the times of minima listed by L04 as well as two other minima reported by Lacy et al. (1998) and Hübischer (2005). The residuals are calculated according to the
Min.I = HJD \[2452565.67170 + 8.45825381d \times E.\] 

For all the secondary minima in Table 1, \(D\) and from that the \(\omega\) are calculated using Eq. (2). As you can see from Table 1, the slow increase (or decrease) in \(D\) (or \(\omega\)) is due to the advance of the line of apsides of the orbit and by computing the slope of a line fitted to the secondary minima on \(\omega\) versus Julian Day number diagram, I determined a new rate of apsidal motion \(\dot{\omega}_{\text{obs}} = 15.18 \pm 6.75\) deg/100 yr. It must be noted that among the observed secondary minima listed in Table 1, the error values of the last five minima are at least 100 times smaller than the other minima. Therefore, I use only these five minima to measure the \(\dot{\omega}_{\text{obs}}\). Also in some cases the error in secondary minima are so high and Eq. (2) doesn’t give any \(\omega\) correspond to the \(D\), calculated from Eq. (1) since as it is obvious from Fig. 3, for any set of values of \(e\) and \(P\) in Eq. (2), \(D\) is always limited between its maximum and minimum.
values (e.g. $D_{\text{min}} < D < D_{\text{max}}$). Thus, I can not find any $\omega$ correspond to $D > D_{\text{max}}$ or $D < D_{\text{min}}$. Finally Fig. 4 shows the relative orbit of V459 Cas, drawn to scale. Actually what can be measured from Eq. (2) is $\dot{\omega}$ and according to Fig. 4, $\dot{\omega}_{\text{obs}} < 0$ but L04 have considered $\dot{\omega}'$ as the angle of line of apside. One can reveal from Fig. 4 that $\omega = 2\pi - \omega'$ and $|\dot{\omega}_{\text{obs}}| = |\dot{\omega}'|$.}

### 3 Comparison with theory

Basically the theoretical rate of apsidal motion is due to the contribution of two terms; a classical term as well as the general-relativistic term. Then the total rate of apsidal motion is equal to

$$
\dot{\omega}_{\text{tot}} = \dot{\omega}_{\text{CL}} + \dot{\omega}_{\text{GR}}.
$$

where $\dot{\omega}_{\text{CL}}$ denotes the classical or Newtonian term and $\dot{\omega}_{\text{GR}}$ is the relativistic contribution. I am going now to calculate each term in equation (4) with the assumption that the rotation equators of both stars are coincident with their orbital plane. In classical mechanics, the advance of the line of apsides arises due to the lack of spherical symmetry in the shape of the stars. Tidal and rotational distortion are two mechanisms lead to this asymmetry which depend on the fractional radii, the internal mass distribution, and the axial rotation of the stars. $\dot{\omega}_{\text{CL}}$ can be determined using the expressions given by Cowling (1938), Sterne (1939), and Kopal (1959)

$$
\dot{\omega}_{\text{CL}}(\text{deg yr}^{-1}) = 365.25 \left(\frac{360}{P}\right) \left(k_{2,1} r_1^5 [15f_2(e) \left(\frac{M_2}{M_1}\right)]ight)
+ \left(\frac{\omega_{r,1}}{\omega_k}\right)^2 \left(1 + \frac{M_2}{M_1}\right)
+ k_{2,2} f_2^5[15f_2(e) \left(\frac{M_1}{M_2}\right)]
+ \left(\frac{\omega_{r,2}}{\omega_k}\right)^2 \left(1 + \frac{M_1}{M_2}\right),
$$

where $f_2(e) = (1 + \frac{3}{10} e^2)(1 - e^2)^{-5}$. $P$ is the orbital period in days, $M_1$ and $M_2$ are the masses of stars in terms of $M_\odot$, $k_{2,1}$ and $k_{2,2}$ are known as the apsidal motion constants of the component 1 and 2 respectively, $r_1$ and $r_2$ are the fractional radii of the stars ($r = \frac{R}{A}$; $A$ = semimajor axis of the orbit), $\omega_{r,1}$ and $\omega_{r,2}$ are the star’s angular rotation speeds, and $\omega_k$ is the mean angular Keplerian velocity equal to $\frac{2 \pi}{P}$. In series expansion $f_2(e)$, I did not keep the terms involving $e^4$ and higher orders since they have not any significant contribution to the result of summation in $f_2(e)$. L04 indicated that the components of V459 Cas are main-sequence stars with an age of about 525 Myr. Therefore in order to calculate $k_2$ which is correspond to the second harmonic of the mutual tidal distortion, I used Table 1 of Jeffery (1984) where he has computed $k_2$ from modern stellar-interior models of evolving main sequence stars. From the estimated values for chemical composition ($X = 0.712, Z = 0.012$), I adopt a nearly equal value of $k_{2,1} \approx k_{2,2}=0.0042$ for both components. Using the profiles of absorption lines in the star’s spectrum, it is possible to estimate the angular rotation velocities $\omega_r$ of the stars from their projected rotational velocity, $V_r \sin i$. L04 have estimated $V_r \sin i$ from the absorption lines of $\text{Mg}^\dagger\lambda4481$ in the spectrogram of V459 Cas. From these measurements, $V_r \sin i = 54 \text{ kms}^{-1}$ and $43 \text{ kms}^{-1}$ for the first and the second components respectively. Therefore $\dot{\omega}_{r,1}/\dot{\omega}_k = 4.49$ and
\[ \frac{\dot{\omega}_{r,2}}{\dot{\omega}_k} = 3.66 \] are obtained from \( \dot{\omega}_{r,i} = \frac{V_{\text{rot}}}{R_i} \). Substituting all of the parameters and the values from L04 into equation (5), I find

\[ \dot{\omega}_{\text{CL}}^{\text{theo}} = 1^\circ.22 \pm 0.12/100 \text{ yr}. \]

The expression for the advance of the line of apsides due to general relativity has a simple form and according to Levi-Civita (1937) and Kopal (1959) is equal to

\[ \dot{\omega}_{\text{GR}}^{\text{theo}} = 9.2872 \times 10^{-3} \frac{(M_1 + M_2)^{2/3}}{(P/2\pi)^{5/3}(1-e^2)}, \]

where \( M_1 \) and \( M_2 \) are in \( M_\odot \) and \( P \) is in days. Putting again the value of the parameters into equation (6) yields

\[ \dot{\omega}_{\text{GR}}^{\text{theo}} = 1^\circ.42 \pm 0.01/100 \text{ yr}. \]

Thus the combined classical and general relativistic rate of apsidal motion is equal to

\[ \dot{\omega}_{\text{CL+GR}}^{\text{theo}} = 2^\circ.64 \pm 0.12/100 \text{ yr}, \]

which is calculated according to equation (6). To calculate the standard deviations, I used the errors assigned to each parameter given in the references. The above value indicates that in comparison to \( \dot{\omega}_{\text{obs}}^{\text{obs}} \), \( \dot{\omega}_{\text{CL+GR}}^{\text{theo}} \) is about 5.75 times smaller with a discrepancy of 12\(^\circ.54\)deg/100 yr.

4 Results and discussion

V459 Cas has been observed and analyzed carefully by L04. In this paper I followed the method of GM85 to measure a value for the observed rate of apsidal motion. Table 2 presents the results of independent determinations of the rate of apsidal motion together with the corresponding period of apsidal revolution \( U \). The relation between the apsidal motion period \( U \) and \( \dot{\omega}_{\text{obs}} \) has a simple form

\[ U = \frac{360P}{\dot{\omega}_{\text{obs}}}, \]

where \( \dot{\omega}_{\text{obs}} \) is expressed in degrees per cycle and \( P \) is the anomalistic period expressed in days. Also the observed rate of apsidal motion is 5.75 times greater from the expected theoretical value. It is not the first time that such a discrepancy is observed. For example in the case of V1143 Cygni the observed rate of apsidal motion is about one-third of the value estimated from the theory but according to the new studies it seems that this deviation is decreasing with the expansion of observational time (Dariush et al., 2004). On the other hand in the case of highly eccentric eclipsing binary, DI Herculis, the observed rate of apsidal motion is one-seventh of the estimated theoretical value. Tough a series of investigations have been made during the past decades to find a clear explanation for this high discrepancy (GM85; Claret, 1998; and Claret & Willems, 2002) but the case of DI Herculis is still unsolved. The method of GM85 is expected to give the value of \( \dot{\omega}_{\text{obs}}^{\text{obs}} \) exactly but to determine a more accurate value for \( \dot{\omega}_{\text{obs}}^{\text{obs}} \) we need more accurate timings of secondary minima since up to now, our observational baseline covers only a very small fraction of the period of apsidal revolution \( U \). Therefore it seems that the reason for such a large discrepancy between the theory and the observation is mainly due to the small number of observational data points.

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Table 1: The photoelectric times of secondary minima for V459 Cas, together with the values of $D$ and $\omega$, computed using Eqs. (1) and (2) according to the light element given in Eq. (3).

| HJD. (2400000.+) | error | Epoch | $D$ | $\omega$(deg) |
|------------------|-------|-------|-----|---------------|
| 34714.447        | 0.05  | -2110.5 | -0.08 | 127          |
| 35856.253        | 0.05  | -1975.5 | -0.13 | –            |
| 36245.275        | 0.05  | -1929.5 | -0.19 | –            |
| 36541.37         | 0.05  | -1894.5 | -0.13 | –            |
| 36820.533        | 0.05  | -1861.5 | -0.09 | 139          |
| 37945.472        | 0.05  | -1728.5 | -0.10 | 145          |
| 37945.504        | 0.05  | -1728.5 | -0.07 | 125          |
| 39070.443        | 0.05  | -1595.5 | -0.08 | 130          |
| 40068.542        | 0.05  | -1477.5 | -0.05 | 117          |
| 40914.331        | 0.05  | -1377.5 | -0.09 | 136          |
| 41210.459        | 0.05  | -1342.5 | -0.01 | 93           |
| 41599.428        | 0.05  | -1296.5 | -0.11 | 153          |
| 41599.453        | 0.05  | -1296.5 | -0.09 | 134          |
| 41988.428        | 0.05  | -1250.5 | -0.19 | –            |
| 42741.339        | 0.05  | -1161.5 | -0.07 | 122          |
| 51867.7992       | 5E-4  | -82.5  | -0.0665 | 120.4        |
| 51918.5502       | 4E-4  | -76.5  | -0.0650 | 119.6        |
| 52586.75193      | 1.3E-4| 2.5    | -0.06540 | 119.84      |
| 52992.74846      | 1.9E-4| 50.5   | -0.06506 | 119.67      |
| 53026.58164      | 1.4E-4| 54.5   | -0.06489 | 119.58      |

Table 2: Determined rate of apsidal motion for V459 Cas.

| $\omega_{obs}$ (deg/100 yr) | $U_{yr}$ | Source |
|----------------------------|----------|--------|
| 6.04 ± 4.74                | 6100     | L04    |
| 15.18 ± 6.75               | 2400     | Present study |

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