Mathematical simulation of dynamics for the temperature regime of the fire with the account of air flows motion according to free convection principle

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Abstract. The article presents an original solution to the problem of predicting the dynamics of the temperature regime of a fire by means of a mathematical model of the main processes occurring in case of fire in a room. The simulation of the dynamics of heat processes occurring in fire conditions in a room equipped with aeration lights was carried out on the basis of the theory of convective heat exchange and gas dynamics. The obtained model allows determining the time when the critical value of the fire hazard factor (temperature) occurs taking into account the operation of the aeration lights.

1. Introduction

Temperature, rate of the fire development, the height of the inflammation source of the floor level, natural ventilation of a room – cross-section and the height of location of the ventilation hole are known to have a significant effect on dynamics of the primary step of the fire in premises [1–6].

Prediction of dynamics of the temperature conditions of the fire enables to obtain science-based values for the time of occurrence of the critical values for such a dangerous fire factor as the temperature. Mathematical simulation lets the description of the physical processes proceeding under conditions of non-controllable burning thus allowing to obtain preliminary data on the dynamics of the parameters of interest. As a result, it is possible to get a prediction on the fire development, dynamics of the dangerous fire factors and possible fire damage at the certain objects [2–10].

Considering physical processes taking place during the fire it is necessary to account for a number of parameters which have a significant effect of the burning process. Since the fires rather often occur in the premises of different designation it is required to take into account its characteristics, for example, to consider it a ventilated one due to its incomplete sealing. Ventilation flows can appear as a result of slots and leakages in assembly of the building structures and materials. It is also necessary to consider the presence and fan equipment delivery; its parameters depend on the configuration and power consumption of the ventilation installations [11–14].

In the conditions of the fire combustion quite often only smoke protection ventilation performance is provided, as well as the operation of the ventilation equipment with the natural impetus. So, when
predicting the fire temperature behavior the effect of the smoke protection ventilation on the required parameter was considered as well as the influence of the ventilating skylights.

2. Mathematical simulation of temperature dynamics
Simulation of the dynamics of parameters characterizing thermal processes proceeding in the conditions of the fire inside a premise equipped with ventilating skylights was based on the theory of convective heat exchange and gas dynamics.

This is connected with the requirement of obtaining of the data on the dynamics of the gas environment temperature with the account of the thermal flows motion and thermal losses through the enclosure.

Taking into account the presence of the open flame and the emission of radiant heat it seems possible to accept assumptions that permit simplification of the formulated problem and at the same time that do not exclude the effect of the parameters on the thermal process. Thus, the main parameters of the process that have a considerable influence on the required value were taken into account and justified while less significant ones were excluded [15, 16].

The boundary conditions in this approximation include the following information – the premise is equipped with ventilation system with a natural impetus; the source of heat and radiant emission is in the center of a confined space; combustion source emits radiant heat; temperature of the gas environment temperature is evaluated as a mean-bulk one and saturation of the gas environment by combustion products is not accounted.

Assumptions taken in the process of mathematical simulations look as follows:

- gaseous exchange in the premise is realized only through the ventilation skylights, meaning that the volume of gas removed from the ventilation system is equal to the volume of the air intake to the premise;
- heating of the gas environment occurs due to convection, heat transfer by enclosure which are heated as a result of radiant flux impact from combustion source;
- the heat radiated by the source of burning is uniformly distributed over the premise volume;
- motion of the gas environment in the premise is due to the change of the air density as a result of heating from the source of burning.

Air fluxed motion with the account of a heat transfer within the fluid medium from the heat source in the conditions of up-flow and down-flow free air motion within the confined space can be described by the following system of equations [17, 18]:

\[
\begin{align*}
\frac{d\mathbf{V}}{d\tau} & = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \mathbf{V} \\
\frac{\partial \rho}{\partial \tau} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) & = 0
\end{align*}
\]

(1)

This system of equations includes three equations: Fourier- Kirchhoff equation, describing energy transfer in the fluid medium; Navier-Stokes equation describing the motion of viscous incompressible liquid; continuity equation.

Since a free convection is considered along an axis $x$, it is possible to accept that $v = 0$, $w = 0$, $\frac{\partial u}{\partial x} = 0$, $\nabla p = 0$. Bulk force of $F_y = F_z = 0$, and the action of force along $x$ axis is determined by the lifting force or by up thrust
\[ F_x = g\beta(t - t_o) \]  
(2)

where \( g \) – is a free fall acceleration, \( \text{m/sec}^2 \);
\( \beta \) – thermal expansion coefficient, \( ^\circ\text{C}^{-1} \);
\( t_o \) – initial temperature, \( ^\circ\text{C} \).

The current value of the density is determined by the expression

\[ \rho = \rho_o(1 - \beta(t - t_o)) \]  
(3)

where \( \rho_o \) – is the initial value of density, \( \text{kg/m}^3 \). Then the system of equations (1) takes the form

\[
\begin{bmatrix}
\frac{\partial t}{\partial \tau} + u \frac{\partial t}{\partial x} = \frac{\lambda}{\rho C_p} \frac{\partial^2 t}{\partial x^2} + \frac{Q}{\rho C_p} \\
\frac{\partial u}{\partial \tau} = g\beta(t - t_o) \\
\frac{\partial t}{\partial \tau} + u \frac{\partial t}{\partial x} = \left[1 - \beta(t - t_o)\right]\frac{\partial u}{\partial x}
\end{bmatrix}
\]  
(4)

Taking into account additional expressions (2) and (3), system of equations (4) can be transformed to the following form

\[
\frac{\lambda}{\rho C_p} \frac{\partial^2 t}{\partial x^2} - g\tau[(1 - \beta(t - t_o))]\frac{\partial t}{\partial x} + \frac{Q}{\rho C_p} = 0
\]  
(5)

If one substitutes the value of dimensionless temperature into equation (5)

\[ z = 1 - \beta(t - t_o) \]  
(6)

We obtain

\[
\frac{\partial t}{\partial x} = \frac{1}{\beta} \frac{\partial z}{\partial x}, \frac{\partial^2 t}{\partial x^2} = \frac{1}{\beta} \frac{\partial^2 z}{\partial x^2}
\]  
(7)

Taking into account (6), (7) equation (5) becomes as follows

\[
\frac{\partial^2 z}{\partial x^2} = \frac{\rho C_p g}{\lambda} \tau \cdot z \frac{\partial z}{\partial x} - \frac{Q\beta}{\lambda} = 0
\]  
(8)

If one accepts that

\[ \xi = \frac{x}{h} \]  
(9)

Then, substituting dimensionless height into equation (8) we arrive to

\[
\frac{\partial^2 z}{\partial \xi^2} - \frac{\rho C_p g h}{\lambda} \tau \cdot z \frac{\partial z}{\partial \xi} - \frac{Q\beta h^2}{\lambda} = 0
\]  
(10)
Basing from the accepted assumptions let us derive the value of the heat energy source that depends only on the characteristics of the considered fire load

$$Q = \frac{\psi_{sp}Q_H^pF}{V_{Pr}}$$  \hspace{1cm} (11)$$

where $\psi_{sp}$ is a specific rate of a substance burning-out, kg/(m²·sec);

$Q_H^p$ is a calorific value of a substance, J/kg;

$F$ – is a surface area of burning, m²;

$V_{Pr}$ – volume of the premise, m³.

As a result, equation (10) takes the form

$$\frac{\partial^2 z}{\partial \xi^2} - \frac{\rho C_p gh}{\lambda} \cdot \tau \cdot z \cdot \frac{\partial z}{\partial \xi} - \frac{\psi_{sp}Q_H^pFh^2}{V_{Pr}\lambda} = 0$$  \hspace{1cm} (12)$$

Let us apply the substitutions representing dimensionless complexes:

$$\alpha_1 = \frac{\rho C_p gh}{\lambda} \cdot \tau \text{ and } \alpha_2 = \frac{\psi_{sp}Q_H^pFh^2}{V_{Pr}\lambda}$$

hence

$$\frac{\partial^2 z}{\partial \xi^2} - \alpha_1 \cdot z \cdot \frac{\partial z}{\partial \xi} - \alpha_2 = 0$$  \hspace{1cm} (14)$$

Equation (14) includes derivatives only from one independent variable value so it is possible to proceed from the partial derivatives to the total ones

$$z'' - \alpha_1 \cdot z \cdot z' - \alpha_2 = 0$$  \hspace{1cm} (15)$$

Let us decrease the order of equation (15) in our problem at the expense of the following:

$$z' - \frac{1}{2} \alpha_1 \cdot z^2 - \alpha_2 \xi = 0$$  \hspace{1cm} (16)$$

Function of $z(\xi)$ is the solution of equation (16)

$$z(\xi) = \frac{\frac{3}{\sqrt[4]{4\alpha_1\alpha_2}}}{\alpha_1} \left[ C_1 \text{AiryAi} \left( \frac{1}{2} \frac{3}{\sqrt[4]{4\alpha_1\alpha_2}} \cdot \xi \right) + \text{AiryBi} \left( \frac{1}{2} \frac{3}{\sqrt[4]{4\alpha_1\alpha_2}} \cdot \xi \right) \right] + C_2 \left[ \alpha_1 \left[ C_1 \text{AiryAi} \left( \frac{1}{2} \frac{3}{\sqrt[4]{4\alpha_1\alpha_2}} \cdot \xi \right) + \text{AiryBi} \left( \frac{1}{2} \frac{3}{\sqrt[4]{4\alpha_1\alpha_2}} \cdot \xi \right) \right] + C_2 \right]$$  \hspace{1cm} (17)$$

where AiryAi, AiryBi are the wave functions related to the special functions:

$$\text{AiryAi} \left( \frac{1}{2} \frac{3}{\sqrt[4]{4\alpha_1\alpha_2}} \cdot \xi \right) = -\frac{\sqrt[3]{3} \cdot \gamma \left( \frac{2}{3} \right)}{2\pi} + \frac{\sqrt[3]{6} \sqrt[4]{\alpha_1\alpha_2} \xi^2}{12 \cdot \gamma \left( \frac{2}{3} \right)} \cdot \xi^2 + \frac{\sqrt[3]{3} \cdot \gamma \left( \frac{2}{3} \right)}{12\pi} \alpha_1\alpha_2\xi^3$$  \hspace{1cm} (18)$$
Airy B\(\left(1,-\frac{1}{2}\sqrt{4\alpha_1\alpha_2}\cdot\xi\right)\) = \(\frac{3\sqrt{9}\cdot\gamma\left(\frac{2}{3}\right)}{2\pi} + \frac{6\sqrt{72}\sqrt{(\alpha_1\alpha_2)}^2}{12\cdot\gamma\left(\frac{2}{3}\right)}\cdot\xi^2 - \frac{3\sqrt{9}\cdot\gamma\left(\frac{2}{3}\right)}{12\pi}\alpha_1\alpha_2\xi^3\)  

\(19\)

Airy A\(\left(-\frac{1}{2}\sqrt{4\alpha_1\alpha_2}\cdot\xi\right)\) = \(\frac{\sqrt{3}}{3\cdot\gamma\left(\frac{2}{3}\right)} + \frac{\sqrt{9}\cdot\gamma\left(\frac{2}{3}\right)}{4\pi}\sqrt{4\alpha_1\alpha_2}\cdot\xi - \frac{\sqrt{3}\cdot\alpha_1\alpha_2\xi^3}{36\cdot\gamma\left(\frac{2}{3}\right)}\)  

\(20\)

Airy B\(\left(-\frac{1}{2}\sqrt{4\alpha_1\alpha_2}\cdot\xi\right)\) = \(\frac{\sqrt{243}}{3\cdot\gamma\left(\frac{2}{3}\right)} - \frac{3\sqrt{9}\cdot\gamma\left(\frac{2}{3}\right)}{4\pi}\sqrt{4\alpha_1\alpha_2}\cdot\xi - \frac{\sqrt{243}\cdot\alpha_1\alpha_2\xi^3}{36\cdot\gamma\left(\frac{2}{3}\right)}\)  

\(21\)

where \(\gamma\left(\frac{2}{3}\right) = 1.3541\) is gamma function from the field of the special functions.

Dimensionless coefficient \(\alpha_1\) involves an independent parameter \(\tau\), therefore, the out parameter in (18) represents a two-parameter function \(z(\xi, \tau)\).

Let us introduce the boundary conditions for the problem:

For \(\xi = 0\) and \(\tau = 0\), \(z(\xi, \tau) = 1\)  

\(22\)

With the account of the accepted assumptions it seems possible to determine constant values in equations (17)

\[C_1 = \frac{2\alpha_1 \cdot \pi \sqrt{243} - 3\sqrt{9} \cdot \gamma\left(\frac{2}{3}\right)^2 \sqrt{4\alpha_1\alpha_2}}{3\sqrt{9} \cdot \gamma\left(\frac{2}{3}\right)^2 \sqrt{4\alpha_1\alpha_2} + 2\alpha_1 \cdot \pi \sqrt{243}}\]  

\(23\)

Taking into account the wave functions of (18)-(21) let us transform equation (17)

\[z(\xi, \tau) = \frac{\sqrt{\alpha_1(\tau)\alpha_2}}{k} \left(\varphi_1(\tau)\xi^3 + \varphi_2(\tau)\xi^2 + \varphi_3(\tau)\right) + 1\]  

\(24\)

where

\[\varphi_2(\tau) = -0.112\left(-18.16 \cdot \sqrt{\alpha_1(\tau)\cdot\alpha_2} + 15.7\alpha_1(\tau)\right)^2 + 0.194\sqrt{\left(\alpha_1(\tau)\cdot\alpha_2\right)^3} \]

\[\varphi_3(\tau) = 0.26\left(-18.16 \cdot \sqrt{\alpha_1(\tau)\cdot\alpha_2} + 15.7\alpha_1(\tau)\right)^2 + 0.45\]
3. Conclusions
As a result, the equation was derived including the dimensionless values and dimensionless complexes, so it can be considered as a criterion one.

This equation represents a one-dimensional mathematical model describing temperature dynamics of the gas medium during the fire in the premise, taking into account the motion of the air flows according to the principle of the free convection.

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