Distribution Amplitudes of Heavy-Light Mesons

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Abstract

A symmetry-preserving approach to the continuum bound-state problem in quantum field theory is used to calculate the masses, leptonic decay constants and light-front distribution amplitudes of empirically accessible heavy-light mesons. The inverse moment of the B-meson distribution is particularly important in treatments of exclusive B-decay using effective field theory and the factorisation formalism; and its value is therefore computed: \( \lambda_B(\zeta = 2 \text{ GeV}) = 0.54(3) \text{ GeV} \). As an example and in anticipation of precision measurements at new-generation B-factories, the branching fraction for the rare \( B \to \gamma E_\gamma \ell \nu_\ell \) radiative decay is also calculated, retaining \( 1/m_B^2 \) and \( 1/E_\gamma^2 \) corrections to the differential decay width, with the result \( \Gamma_{B \to \gamma E_\gamma \ell \nu_\ell}/\Gamma_B = 0.47(15) \) on \( E_\gamma > 1.5 \text{ GeV} \).

Keywords: B-meson decays, heavy-light mesons, nonperturbative continuum methods in quantum field theory, parton distribution amplitudes, quantum chromodynamics

1. Introduction — In quantum chromodynamics (QCD), numerous hard exclusive processes can be analysed using the factorisation formalism. Prominent examples are the applications to elastic and transition form factors of pseudoscalar mesons \cite{1–3}. Such treatments separate the amplitude for a given scattering process into short- and long-distance components: the short-distance part is calculable using perturbative QCD; but the long-distance piece is essentially nonperturbative, deriving from the wave function of the participating hadron. It was early appreciated that factorisation can also be employed in the treatment of exclusive decays of heavy mesons \cite{4}; and the framework has subsequently been cleanly defined and widely employed — see, e.g. Refs. \cite{5–7} and citations thereof.

Constituted from a valence \( b \)-quark and either a valence \( u \) or \( d \)-quark, the \( B^{(*)0} \) are the most widely studied heavy mesons, with high-profile measurements completed, underway, and planned \cite{8–10}. Numerous exclusive processes involving this system are well suited to treatment via the factorisation approach. Each associated formula features \( \varphi_B \), the B-meson light-front distribution amplitude (DA), which is a direct analogue of the light-meson DAs that appear in the earliest factorisation formulae. However, whilst much has recently been learnt about the pointwise behaviour of leading-twist light-meson DAs \cite{11–26}, information about the B-meson DA remains sketchy \cite{27–34}.

Considered as a function of \( \xi \), the light-front longitudinal momentum fraction of the light-quark in the B-meson, it is known that at resolving scales, \( \zeta \), very much in excess of the B-meson mass, \( m_B \), \( \varphi_B(\xi) \sim 6\xi(1 - \xi) \). On the other hand, on \( \zeta \lesssim m_B \), \( \varphi_B(\xi) \) must be a very asymmetric function, whose peak lies at \( \xi \approx \hat{\omega}/m_B \), where \( \hat{\omega} > 0 \) is an intrinsic momentum-scale associated with the dressed light-quark in the B-meson.

More information is required, however, before factorised formulae for exclusive processes involving B-mesons can be useful. Herein, therefore, we will employ a continuum approach to the hadron bound-state problem in order to compute the pointwise behaviour of the B-meson DA at a typical hadronic scale, omitting radiative corrections \cite{27, 28}; the DAs of other heavy-light systems; and an array of derived quantities, including the branching fraction for the \( B \to \gamma E_\gamma \ell \nu_\ell \) radiative decay.

2. Distribution Amplitudes — Consider a heavy pseudoscalar meson with mass \( M \) and total momentum \( p = M \gamma, \gamma^2 = -1 \), constituted from a single heavy valence \( Q \)-quark and a lighter \( l \)-quark; then one may define a distribution amplitude for this system as the following light-front projection of the meson’s Poincaré-covariant Bethe-Salpeter wave function:

\[
\chi_\Lambda(k; p) = \chi_{\Lambda}(k; p)S_{\Lambda}(k - p)
\]

Here: \( f_\Lambda \) is the meson’s leptonic decay constant; the trace is over colour and spinor indices; \( \chi_\Lambda \) is a Poincaré-invariant regularization of the four-dimensional integral, with \(\Lambda \) the ultraviolet regularization mass-scale; \( Z_\Lambda(\zeta, \Lambda) \) is the mass-independent quark wave-function renormalisation constant \cite{35}, with \(\zeta \) the renormalisation scale; \(n \) is a light-like four-vector, \( n^2 = 0, n \cdot \nu = 1; w = \hat{\nu} \cdot n; S_{1, Q} \) are dressed-propagators for the meson’s valence quarks; and \( \Gamma_{\Lambda}(k; p) \) is the meson’s Bethe-Salpeter amplitude. It can be shown \cite{36, 37} that in the limit \( M_B \to \infty \), \( \Gamma_{\Lambda}(k; p) \to \Gamma_{\Lambda}(v; \sqrt{M_B}) \) and, e.g. \( f_\Lambda \sqrt{M_B} = \text{const} \).

The DA defined in Eqs. (1) has mass-dimension negative-one, support on \( w \in [0, M_B] \), and is unit normalised. It follows
that one can define
\[ \varphi_h(\xi; \zeta) = M_h \varphi_h(M_h \xi; \zeta), \quad \int_0^\zeta d\xi \varphi_h(\xi; \zeta) = 1. \quad (2) \]

QCD-evolution on \( \zeta \lesssim M_h \) actually extends the domain of support to \( w \in [0, \infty) \) by introducing a radiative tail [6]. We avoid this issue herein by computing all results at a low hadronic scale \( \zeta = \xi = 2 \text{ GeV} \), from which evolution can subsequently be employed, if desired.

3. Bound-State Problem — Our calculation of \( \varphi_h(\xi; \zeta) \) proceeds as follows. (i) Specify a symmetry-preserving truncation of the continuum bound-state problem. (ii) Using that truncation, compute the dressed-quark propagators and meson Bethe-Salpeter amplitude. (iii) Evaluate the DA by inserting the results in Eqs. (1), (2). We now elaborate on each of these steps.

The continuum bound-state problem is defined by a set of coupled integral equations [38, 39]. A tractable system is only obtained once a truncation scheme is specified. A systematic, symmetry-preserving approach is described in Refs. [40, 41]. The leading-order term is the widely-used rainbow-ladder (RL) truncation. It is accurate for ground-state light-quark- and isospin-nonzero-pseudoscalar-mesons, and related ground-state octet and decouplet baryons [38, 39, 42–44]; and, with judicious modification, heavy-heavy \( S \)-state octet and decouplet baryons [38, 39, 42–44]; and, with isospin-nonzero-pseudoscalar-mesons, and related ground-state mesons, this occurs before any moving singularity enters the domain of support. With these values one obtains a kernel in agreement with empirical values reproduced using
\[ \zeta = 0.6 \text{ GeV}. \] (5)

Herein, we employ \( \omega = 0.8 \text{ GeV} \), the midpoint of the insensitivity domain. With these values one obtains a kernel in agreement with the RGI interaction derived from analyses of QCD’s gauge sector [62, 65, 66].

With the kernel now specified, we perform a coupled solution of the dressed-quark gap- and meson Bethe-Salpeter-equations, varying the gap equations’ current-quark masses until the Bethe-Salpeter equation has a solution at \( P^2 = -M_2^2 \) following Ref. [49] and adapting the algorithm improvements from Ref. [67] when necessary. The benchmarking results in Table 1 were obtained using RGI current-masses \( \hat{m}_b = 7.4 \text{ GeV}, \hat{m}_c = 1.7 \text{ GeV} \). They correspond to the following values of the dressed-quark mass-functions:
\[ m_b := M_b(\xi_2) = 4.35 \text{ GeV}, \quad m_c := M_c(\xi_2) = 1.25 \text{ GeV}, \] (6)
defining current-quark masses which are commensurate with other determinations [68].

| Herein | Exp. [68] | IQCD [69, 70] |
|--------|----------|----------------|
| \( \eta_1 \) | 2.98 | 2.98 | 2.98 |
| \( \eta_2 \) | 9.38 | 9.39 | 9.39 |

\[ \hat{m}_q := M_q(\xi_2) = 0.0049 \text{ GeV}, \quad m_t = M_t(\xi_2) = 0.114 \text{ GeV}. \] (7)

The development of Eqs. (3), (4) is summarised in Ref. [60] and their connection with QCD is described in Ref. [62]. The kernel seemingly depends on two parameters. However, in baryons and mesons formed from heavy quarks, many observable properties are practically insensitive to variations of \( \omega \in [0.7, 0.9] \text{ GeV} \), so long as \( \zeta := D_\omega = \text{constant} \) [63, 64], with empirical values reproduced using
\[ \zeta = 0.6 \text{ GeV}. \] (5)

4. Heavy-light Mesons: Masses and Decay Constants — We focus first on the properties of mesons formed from a valence \( c \)-quark and \( \bar{q} \)-quark, \( \hat{m}_q \leq \hat{m}_c \). Namely, beginning with our \( \eta_t \) solution, we solve the gap and Bethe-Salpeter equations at a range of evenly spaced values of \( \hat{m}_q = \hat{m}_c \), directly computing the mass and decay constant of the associated bound-state until that value of \( \hat{m}_q = \hat{m}_c^2 \) is reached for which the kernel defined by Eqs. (3)–(6) is no longer reliable. For \( D^q \)-mesons, this occurs before any moving singularity enters the integration contour used in the RL Bethe-Salpeter equation because the heavy-quark parameters connected with Eq. (5) are not appropriate for light quarks. Since the \( s \)-quark defines a boundary between dominance of emergent and Higgs mass-generating mechanisms [26, 45], we terminate direct calculations at \( m_s^2 = 0.4 \text{ GeV} \approx 4m_t \). The value of any desired quantity on \( m_s < m_c \) is then estimated via extrapolation from \( m_s > m_c^2 \). The ambiguity in the value of \( m_c^2 \) is expressed in the uncertainty bands we place on our extrapolations.

In the lower panel of Fig. 1 we depict the trajectory of \( D^q \)-meson masses obtained as described above. Identifying
\[ m_q = M_q(\xi_2) = 0.0049 \text{ GeV}, \quad m_t = M_t(\xi_2) = 0.114 \text{ GeV}, \] (7)
one therefrom reads the masses in Table 2A. The lower panel of Fig. 2 depicts the associated trajectory of leptonic decay constants, from which one obtains the values listed in Table 2A. Both the masses and decay constants agree well with the empirical values.

We turn now to \( \bar{B}_q \) systems, beginning with our \( \eta_b \) solution. Here a singularity moves into the relevant integration domain for \( m_q < m_u^* = 1.3\text{ GeV} \), viz. at a current-mass just above that of the \( c \)-quark. The associated trajectory of bound-state masses is depicted in the upper panel of Fig. 1, from which one extracts the values in Table 2B: our predicted \( \bar{B}_q \)-meson masses are consistent with experiment.

Table 2: (A) Masses and decay constants of \( D_q \) mesons computed herein, using Eqs. (3)–(6); compared with averages of available experimental and lattice-QCD determinations reported in Ref. [68]. (B) As above for \( B_q \) mesons, with lQCD results for \( B_s \) taken from Ref. [71] (Quantities listed in GeV; and in our normalisation, \( f_s = 0.092\text{ GeV} \).)

| (A) | Herein | Exp. [68] | lQCD [68] |
|-----|--------|-----------|-----------|
| \( D_d \) | \( 1.88(5)\) | \( 1.87 \) | \( 1.87 \) |
| \( D_s \) | \( 1.94(4) \) | \( 1.97 \) | \( 1.97 \) |

| (B) | Herein | Exp. [68] | lQCD [68, 71] |
|-----|--------|-----------|---------------|
| \( B_u \) | \( 5.30(15)\) | \( 5.28 \) | \( 5.28 \) |
| \( B_s \) | \( 5.38(13) \) | \( 5.37 \) | \( 5.37 \) |
| \( B_c \) | \( 6.31(1) \) | \( 6.27 \) | \( 6.28(1) \) |

The upper panel of Fig. 2 displays the mass-dependence of the \( B_q \) decay constants. Since little curvature is evident, it is necessary to introduce the following physical constraints on the extrapolation. (i) Continuum [37] and lQCD [68] bound-state analyses indicate \( f_{B_s} \approx 0.85 f_{D_s} \). Hence, we require that \( f_{B_s} \), take a value in the range \((0.85 \sim 1.0) f_{D_s} \). (ii) Experiment and available calculations [25, 68] indicate that \((f_{Q} - f_{Q_0})/(f_{Q} + f_{Q_0}) \approx 0.09\), independent of the mass-average of the associated bound-states. We use this feature to constrain \( f_{B_s} \) via \( f_{B_c} \). Using the procedure just described, we obtain the curves in the upper panel of Fig. 2 and the associated results in Table 2B.

5. Heavy-light Mesons: Distribution Amplitudes — Returning to Eqs. (1), DAs for the systems discussed in the preceding section can be obtained by using the methods introduced in Refs. [11, 45]. Namely: (i) for each desired and RL-accessible value of the pair \((m_Q, m_q)\), we compute the Mellin moments

\[
\langle \xi^m \rangle := \int_0^1 d\xi \xi^m \varphi(\xi),
\]

\( m = 1, 2, 3 \); (ii) assume that the DA’s pointwise form is well represented by

\[
\varphi(\xi) = n_{\alpha \beta} 4\xi \bar{\xi} e^{-\alpha^2 4\xi \bar{\xi}} e^{-\beta^2 (\xi - \bar{\xi})^2};
\]

\( 2 \) We have validated this hypothesis by using the maximum entropy method, as described in Ref. [20], to directly determine the DA in a few, randomly selected cases.
obtained via SPM extrapolation. Our results for \((\alpha,\beta)\)-pairs relating to systems not directly accessible using RL truncation are then obtained via SPM extrapolation. Our results for \((\alpha,\beta)\)-pairs relating to systems not directly accessible using RL truncation are then obtained via SPM extrapolation. Our results for \((\alpha,\beta)\)-pairs relating to systems not directly accessible using RL truncation are then obtained via SPM extrapolation. Our results for \((\alpha,\beta)\)-pairs relating to systems not directly accessible using RL truncation are then obtained via SPM extrapolation. Our results for \((\alpha,\beta)\)-pairs relating to systems not directly accessible using RL truncation are then obtained via SPM extrapolation. Our results for \((\alpha,\beta)\)-pairs relating to systems not directly accessible using RL truncation are then obtained via SPM extrapolation. 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This measure, $\varphi_h \approx \varphi_h^\xi$ provides a fair approximation for heavy-light systems.

We have also computed $\langle w \gamma \rangle$ at $m_0 = m_\gamma$, $m_\gamma$ in the limit $m_0 \to 0$, with the results depicted in Fig. 5. They are described by a straight line, which translates into the following behaviour:

$$\langle \xi \rangle \approx \left( \frac{\langle w \rangle}{M_h} \right) \xi \approx \xi^0 + \frac{\xi^1}{M_h},$$

$$(12a) \quad \xi^0 = 0.120(11), \quad \xi^1 = 0.366(26) \text{GeV}. \quad (12b)$$

This result and related algebraic analysis using the methods of Refs. [36, 37] indicate that for each value of $\zeta$, $\langle \xi \rangle \to \xi_0^\xi$, i.e. the light-quark light-front momentum-fraction takes a finite, nonzero value in the limit $M_h \to \infty$. Naturally, at any large, fixed value of $M_h$, $\xi^\xi \to 1/2$ as $\zeta \to \infty$.

We now follow Refs. [31, 32, 34] and compute the branching fraction for the $B \to \gamma \ell \nu_\ell$ radiative decay. This process is analogous to the $\gamma^* \gamma \to \pi^\pm$ transition in the sense that it is amenable to analysis using the factorisation formalism, depends linearly upon the participating meson’s DA, and is the simplest process to probe that DA. In this calculation, we employ the formula for the $E_\gamma$-dependent differential decay width in Refs. [31, 32], which retains $1/m_b^2$ and $1/E_\gamma^2$ corrections, but our predictions for $m_b$, $M_b$, $f_b$, $A_\gamma$: Eqs. (6), (7) and Tables 2, 4. Assuming that the factorised expression is valid for $E_\gamma > E_\gamma^\text{min}$, we integrate over $E_\gamma \in [E_\gamma^\text{min}, E_\gamma^\text{max} = m_b/2]$ to obtain the branching fractions in Table 5 when $|V_{ub}| = 3.94(36) \times 10^{-3}$ [68]. Our computed $E_\gamma^\text{min}$ dependence of the branching fraction is depicted in Fig. 6. At present, for a fixed value of $\lambda_B$, the largest sources of error are $|V_{ub}|$ and $f_b$, which appear quadratically in the numerator of the differential decay-width formula. Notably, if we choose to artificially change $\lambda_B \to \xi \lambda_B$, the computed values of the branching fraction become approximately 2.6-times larger. Such marked sensitivity to the $B$-meson DA has previously been highlighted [31, 32].

6. Epilogue — Working with the leading-order, symmetry-preserving truncation of the relevant Dyson-Schwinger equations and an interaction kernel constrained by analyses of QCD’s gauge sector and tested in studies of heavy-heavy mesons and triply-heavy baryons, we delivered parameter-free predictions for the masses, decay constants and light-front distributions in Table 5 when $|V_{ub}| = 3.94(36) \times 10^{-3}$ [68]. Our computed $E_\gamma^\text{min}$ dependence of the branching fraction is depicted in Fig. 6. At present, for a fixed value of $\lambda_B$, the largest sources of error are $|V_{ub}|$ and $f_b$, which appear quadratically in the numerator of the differential decay-width formula. Notably, if we choose to artificially change $\lambda_B \to \xi \lambda_B$, the computed values of the branching fraction become approximately 2.6-times larger. Such marked sensitivity to the $B$-meson DA has previously been highlighted [31, 32].

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