Constraining $f(Q, T)$ gravity from energy conditions

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We are living a golden age for experimental cosmology. New experiments with high accuracy precision are been used to constrain proposals of several theories of gravity, as it has been never done before. However, important roles to constrain new theories of gravity in a theoretical perspective are the energy conditions. Throughout this work, we carefully constrained some free parameters of two different families of $f(Q, T)$ gravity using different energy conditions. This theory of gravity combines the gravitation effects due to the torsion through the nonmetricity function $Q$, and manifestations from the quantum era of the Universe in the classical theory (due to the presence of the trace of the energy-momentum tensor $T$). Our investigation unveils the viability of $f(Q, T)$ gravity to describe the accelerated expansion our Universe passes through. Besides, one of our models naturally provides a phantom regime for dark energy and satisfies the dominant energy condition. The results here derived strength the viability of $f(Q, T)$ as a promising complete theory of gravity, lighting a new path towards the description of the dark sector of the Universe.

I. INTRODUCTION

Since the remarkable measurements from Supernova Cosmology Project [1] and High Redshift Supernova Team [2], we have consciously known that our Universe passes through an accelerated phase of expansion, whose agent is named dark energy. Dark energy corresponds to approximately 70% of the content of the so-called dark sector of the Universe, and its understanding is one of the actual biggest problems in science. A simple path to describe the nature of the dark energy consists in to add a cosmological constant to Einstein’s General Relativity (GR), yielding to the ΛCDM model. However, the cosmological constant brings several other issues related to its nature. Among them, we highlight the cosmic coincidence problem, and its huge discrepancy between cosmological observations and quantum field theory predictions, which is about 120 orders of magnitude [3].

Apart from these listed problems, GR stills the most well succeed theory to describe the Universe. It was confirmed by several surveys such as PLANCK Collaboration [4], Dark Energy Survey [5], besides the recent beautiful measurements of gravitational waves from LIGO/VIRGO Collaboration [6], and the first image of a black hole obtained by the Event Horizon Telescope [7].

Although, GR does not yield to a renormalizable quantum theory for gravity, opening space to several alternative theories desiring to describe gravity at a quantum level.

A promising theory of gravity was introduced by Jimenez et al. [8], and called symmetric teleparallel gravity or $f(Q)$, where the gravitation interaction is mediated the nonmetricity term $Q$. Such a theory rapidly inspired several works and it has been constantly tested. Among such tests, we highlight the work done by Lazkoz et al [9], where several $f(Q)$ models were constrained through redshift comparison with data from the expansion rate, Type Ia Supernovae, Quasars, Gamma-Ray Bursts, Baryon Acoustic Oscillations data, and Cosmic Microwave Background distance. Besides, the $f(Q)$ model also unveiled a compatible description of an accelerated phase when submitted to energy conditions constraints as shown in [10].

In the search for a complete theory of gravity emerged the $f(Q, T)$, recently presented by Yixin Xu et al. [11]. Such a theory couples the gravitation effects due to the torsion generated by the tetrad fields (through the nonmetricity function $Q$), and manifestations from the quantum domain in the classical theory (due to the presence of the trace of the energy-momentum tensor $T$). The $f(Q, T)$ gravity has been shown compatible with the accelerating expansion phase [11], besides it is also in agreement with important different phases our Universe passes through as the baryogenesis [13]. Moreover, such a theory is compatible with measurements of the Hubble parameter for different redshifts as one can see in [12]. Beyond these successful tests, an important role any alternative theory of gravity should obey is the energy condition constraints [14]. These constraints are crucial to determine the proper regimes allowed for

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a new theory of gravity, to describe its attractive nature, and to assign the causal and the geodesic structure of space-time. Furthermore, the energy conditions also allow us to confront a new theory of gravity against ΛCDM model.

Therefore, in this work, we intend to study carefully all the energy conditions constraints on different forms of \( f(Q, T) \) gravity. Our analyses were carried using the actual values of the Hubble, and the deceleration parameters. The energy conditions enable us to impose constraints over our free parameters, unveiling the viability of the \( f(Q, T) \) models. We also verify the compatibility of our results with ΛCDM model. The discussions along this study are organized in the following nutshell: in section II we introduce generalities about the \( f(Q, T) \) gravity. In section III we use the Raychaudhuri equations to find our energy conditions embedding the nonmetricity and the trace of the energy-momentum tensor contributions. The constraints on \( f(Q, T) \) models are discussed in details in section IV. A comparison between the \( f(Q, T) \) models and the ΛCDM model is presented in section V, where we also depicted the equation of state parameters for the models here studied. Section VI is dedicated to our final remarks and perspectives.

II. OVERVIEW OF \( f(Q, T) \) GRAVITY

The \( f(Q, T) \) gravity is described through the following action [11],

\[
S = \int \left( \frac{1}{16\pi} f(Q, T) + L_m \right) d^4x \sqrt{-g}. \quad (1)
\]

where \( f \) is an arbitrary function of the nonmetricity \( Q \), and of the trace of the energy-momentum tensor \( T \), besides \( L_m \) represents the Lagrangian of a given matter, and \( g = \text{det}(g_{\alpha\beta}) \). As it was discussed by Jimenez et al. [8], the nonmetricity function is such that

\[
Q \equiv -g^{ab}(L^\mu_{va}L^\nu_{b\mu} - L^\mu_{v\mu}L^\nu_{a\beta}), \quad (2)
\]

where \( L^\mu_{v\gamma} \) is the disformation tensor whose explicit form is

\[
L^\mu_{v\gamma} = -\frac{1}{2}g^{\mu\lambda}(\nabla_\gamma g_{v\lambda} + \nabla_{\lambda} g_{v\gamma} - \nabla_{\gamma} g_{v\lambda}). \quad (3)
\]

Another key ingredient to describe the symmetric teleparallel is the nonmetricity tensor, which is defined as

\[
Q_{\gamma\mu\nu} = \nabla_\gamma g_{\mu\nu}, \quad (4)
\]

and whose traces are

\[
Q_\mu = Q_{\mu a}^a, \quad \tilde{Q}_\mu = Q^a_{\mu a}. \quad (5)
\]

We can also define a superpotential related with the nonmetricity tensor as

\[
4P^\mu_{a\beta} = -Q^\mu_{a\beta} + 2Q_{(a\beta}^\mu \delta^\alpha_{(\beta} g_{\alpha\beta} - \tilde{Q}^\mu_{a\beta} \delta^\mu_{(a\beta} Q_{\beta)}, \quad (6)
\]

yielding to the quadratic form for the nonmetricity function [8]

\[
Q = -Q_{\mu a\beta} P^\mu_{a\beta}. \quad (7)
\]

Moreover, as it is known the energy-momentum tensor can be written as

\[
T_{a\beta} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^a_{\alpha\beta}} \quad (8)
\]

and its variation in respect to the metric tensor is such that

\[
\frac{\delta g^{\mu\nu} T_{\mu\nu}}{\delta g^a_{\alpha\beta}} = T_{a\beta} + \Theta_{a\beta}, \quad (9)
\]

where

\[
\Theta_{a\beta} = g^{\mu\nu} \delta T_{\mu\nu} / \delta g^a_{\alpha\beta}. \quad (10)
\]

Therefore, taking the variation of action (1) with respect to the metric, we find the field equations

\[
8\pi T_{a\beta} = -\frac{2}{\sqrt{-g}} \nabla_\mu (f_Q \sqrt{-g} P^\mu_{a\beta} - \frac{1}{2} f Q_{a\beta} + f_T (T_{a\beta} + \Theta_{a\beta}) - f_Q (P_{a\mu\beta} Q_{\mu\nu} - 2Q_{a\mu}^\nu P_{\nu\beta})). \quad (11)
\]

where \( f_Q \) is a standard Friedmann-Lemaitre-Robertson-Walker metric (FLRW) given by,

\[
ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad (12)
\]

where \( a(t) \) is the scale factor of the Universe. Besides, for such a metric, the nonmetricity function \( Q \) is reduced to \( Q = 6H^2 \). Furthermore, the matter content of the Universe is assumed as been a perfect fluid, whose energy-momentum tensor is \( T_{a\beta} = \text{diag}(-\rho, p, p, p) \).

Therefore, substituting Eqs. (7), (8), and (10) into (9), we yield to the modified Friedmann equations for such a theory, which are explicitly represented as

\[
3H^2 = \frac{1}{2F} \left( -8\pi\rho + \frac{f}{2} - \frac{2G}{1+G} (\dot{F} + H F) \right), \quad (13)
\]
The previous equations can be rewritten in analogy to GR in the following way

\[ H + 3H^2 = \frac{8\pi}{2F} + \frac{f}{4F} \ddot{F} H. \]  

(14)

The Raychaudhuri equation is explicitly given by

\[ 3H^2 = \frac{8\pi}{2} \rho_{\text{eff}}, \]  

(15)

and

\[ H + 3H^2 = \frac{8\pi}{2} p_{\text{eff}}, \]  

(16)

resulting in

\[ \rho_{\text{eff}} = \frac{\rho}{F} - \frac{f}{16\pi F} + \frac{G}{1 + \frac{C}{4\pi F}} \dot{F} H + \frac{F}{4\pi F} H, \]  

(17)

\[ p_{\text{eff}} = \frac{p}{F} + \frac{f}{16\pi F} - \frac{\dot{F}}{4\pi F} H, \]  

(18)

as the effective density, and pressure. Here \( \cdot \)dot represents a derivative with respect to time, besides \( F = f_0 \), and \( 8\pi G = f_0 \) denote differentiation with respect to \( Q \), and \( T \), respectively. Moreover, we are able to observe that the contributions coming from the \( f(Q,T) \) model are embedded into \( \rho_{\text{eff}} \), and \( p_{\text{eff}} \).

### III. ENERGY CONDITIONS

Energy conditions in modified gravity are the tools which empower the casual and geodesic structure of space-time. These conditions are formulated with the help of Raychaudhuri equations which describe the action of congruence and attractiveness of the gravity for timelike, spacelike, or lightlike curves. The first Raychaudhuri equation [15] is explicitly given by

\[ \frac{d\theta}{d\tau} = -\frac{1}{3} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu} - R_{\mu\nu} u^\mu u^\nu, \]  

(19)

where \( \theta \), \( \sigma_{\mu\nu} \), and \( \omega_{\mu\nu} \) are the expansion, shear, and rotation, associated to the vector field \( u^\mu \), respectively. Besides, \( R_{\mu\nu} \) denotes the Ricci tensor. Furthermore, in the case of null vector \( k^\mu \), the Raychaudhuri equation has the form

\[ \frac{d\theta}{d\tau} = -\frac{1}{2} \theta^2 - \sigma_{\mu\nu} \omega^{\mu\nu} + \rho_{\mu\nu} \omega^{\mu\nu} - R_{\mu\nu} k^\mu k^\nu. \]  

(20)

So, from Eqs. (13), and (14), the attractive gravity condition demands the constraints

\[ R_{\mu\nu} u^\mu u^\nu \geq 0, \]  

(21)

and

\[ R_{\mu\nu} k^\mu k^\nu \geq 0. \]  

(22)

Following the methodology presented in [10], the \( f(Q,T) \) models are going to be restricted to the set of energy conditions bellow

- null energy condition (NEC) \( \Leftrightarrow \rho + p \geq 0; \)
- weak energy condition (WEC) \( \Leftrightarrow \rho + p \geq 0; \)
- dominant energy condition (DEC) \( \Leftrightarrow \rho \geq |p|; \)
- strong energy condition (SEC) \( \Leftrightarrow \rho + 3p \geq 0. \)

Then, by substituting Eqs. (11), and (12) in the previous relations, we established the following set of energy conditions

- null energy condition (NEC) \( \Leftrightarrow \rho + p \geq 0; \)
- weak energy condition (WEC) \( \Leftrightarrow \rho + p \geq 0; \)
- dominant energy condition (DEC) \( \Leftrightarrow \rho \geq |p|. \)

Moreover, WEC, DEC and SEC energy conditions demand the extra constraints

\[ \text{DEC} \Leftrightarrow F \leq 0; \]  

(23)

\[ \text{WEC and DEC} \Leftrightarrow \rho - \frac{f}{16\pi} + \frac{G}{1 + \frac{C}{4\pi F}} \frac{\dot{F} H + F H}{4\pi F} \geq 0, \]  

(24)

and

\[ \text{SEC} \Leftrightarrow \rho + 3p - \frac{3 - C}{8\pi} \frac{\dot{F} H}{4\pi} + \frac{G}{1 + \frac{C}{4\pi F}} \frac{F H}{4\pi} \geq 0. \]  

(25)

Through these constraints, we can realize how different \( f(Q,T) \) models modify the standard energy conditions derived from the Raychaudhuri equations.

### IV. CONSTRAINING \( f(Q,T) \) GRAVITY MODELS

In the framework of FRW metric, one can also use the constraints of energy conditions to restrain certain models in \( f(Q,T) \) gravity. A cosmological quantity which is essential to properly describe the energy conditions in a phenomenological perspective is the deceleration parameter, whose definition is [16]

\[ q = -\frac{1}{H^2} \frac{\ddot{a}}{\dot{a}}. \]  

(26)
Alternatively, the time derivative of the Hubble parameter can be rewritten as

\[ H = -H^2(1 + q). \quad (27) \]

Beyond these ingredients, in order to constraint the energy conditions with phenomenological observations, we are going to consider that \( H = H_0 = 67.9 \), and \( q = q_0 = -0.503 \) as the present values for the Hubble, and the deceleration parameters, respectively [4, 17].

We can observe the behavior of \( \rho \), WEC, DEC, and SEC energy conditions in the graphics depicted in Fig. 1. There we realize how the density \( \rho \) decreases for specific values of \( m \) and \( b \), corroborating with the exponential expansion behavior derived by Xu et al. [11]. Moreover, the energy conditions allow us to constrain the free parameters \( m \) and \( b \). Through DEC we found that \( m \) should be negative. Also (28), (29) and (31) assure the range of model parameters as \( b > -4 \pi \) and \( m \leq 0 \), satisfying NEC, WEC, and DEC. Moreover, the constrained parameters result in the violation of SEC, which is compatible with the accelerated phase our Universe passes through [18]. Another remarkable feature coming from the energy conditions, is that the constrained parameters \( m \) and \( b \) corroborate, and fine tune the observational bounds for \( f(Q, T) \) gravity investigated in [12].

As a second model, let us deal with \( f(Q, T) = Q^{n+1} + bT \) where \( m \) and \( b \) are free parameters. Such a model was proposed by Xu et al. [11], and considers nonlinear contributions due to the torsion in the gravity sector, moreover, it was constrained through measurements of the Hubble parameter for different redshifts [12]. The present model yields to \( f = f_Q = (n + 1)Q^n \) and \( 8\pi\mathcal{G} = f_T = b \), and by working with Eqs. (13), (14), (26), and (27) we are able to derive the energy conditions below

\[ NEC \Leftrightarrow -\frac{2^{n-1}3^n(2n + 1) (H_0^2)^n+1 (b(nq_0 + n + q_0 + 4) + 24\pi)}{b^2 + 12\pi b + 32\pi^2} - \frac{2^{n-1}3^n(2n + 1) (H_0^2)^n+1 (3b(nq_0 + n + q_0) + 8\pi(2nq_0 + 1) + 2q_0 - 1))}{b^2 + 12\pi b + 32\pi^2} \geq 0, \quad (32) \]
FIG. 1. Density parameter, and Energy conditions for $f(Q) = mQ + bT$. The graphics were depicted with the present values of $H_0$ and $q_0$ parameters.

$$WEC \iff -\frac{2^{n-1}3^n(2n+1)(H_0^n)^{n+1}}{b^2 + 12\pi b + 32\pi^2} \geq 0,$$

and

$$DEC \iff -\frac{2^{n-1}3^n(2n+1)(H_0^n)^{n+1}}{b^2 + 12\pi b + 32\pi^2} \geq 0.$$ (33)

$$SEC \iff -\frac{2^{n-2}3^n(n+1)(H_0^n)^{n+1}}{\pi} \geq 0.$$ (35)

Analogously to our first case, we depicted the density parameter, as well as the WEC, DEC, and SEC en-
Density parameter, and Energy conditions for $f(Q) = Q^{n+1} + bT$. The graphics were depicted with the present values of $H_0$ and $q_0$ parameters.

energy conditions, whose features can be appreciated in Fig. 2. There we observe a slow decreasing of $\rho$ in respect of parameters $n$, and $b$ corresponding to an expansion regime smoother than our first case, such a behavior corroborates with features analyzed by Xu et al. [11]. Moreover, Eqs.(32), (33), (34), and (35), unveil that NEC and DEC are satisfied, while WEC is partially obeyed ($\rho > 0$) if $n \leq -1$, and $b > -4\pi$. Yet in the energy conditions, we can also see that SEC is again violated, confirming that our Universe experiences an accelerated phase. Furthermore, the WEC violation along with positive density, makes this $f(Q,T)$ gravity naturally behaves like scalar-tensor gravity models [10, 19]. Despite the viability of such a theory in respect to the energy conditions, the constrained values of parameter $n$ are out of the phenomenological bounds established in [12], creating a tension in use such a $f(Q,T)$ model as a proper description of gravity. We are going to present some extra comments concerning this tension in the next section.

V. COMPARISON WITH $\Lambda$CDM MODEL

As a matter of completeness, let us compare our constraints with the $\Lambda$CDM model. This model is so far the most well succeed to describe the evolution of the Universe at different phases. A direct way to link an $f(Q,T)$ gravity with the $\Lambda$CDM model consists to take the special case $f(Q,T) = f_\Lambda(Q) = -Q$ [9]. Such a regime yields to the following energy conditions

- NEC: $2(1 + q) H^2 \geq 0$,
- WEC: $3H^2 \geq 0$, and $2(1 + q) H^2 \geq 0$,
- SEC: $6q H^2 \geq 0$,
- DEC: $2(2 - q) H^2 \geq 0$.

One can observe that all energy conditions are satisfied with the present values of $H$ and $q$ except SEC, corroborating with the description of an accelerated expansion. This behavior is compatible with the first model here analyzed for the $f(Q,T)$ gravity.
Another interesting cosmological parameter which is bounded by experiments is the equation of state parameter $\omega$. Recent observations from Planck Collaboration inform that $\omega \lesssim -1$ [4]. Therefore, the EoS parameter is considered a suitable candidate for comparing our models with $\Lambda$CDM. The EoS parameter ($\omega$) is defined as $\omega = \frac{p}{\rho}$.

By taking our previous relations for density, and pressure, we are able to find that the EoS parameter for the model A i.e. $f(Q, T) = mQ + bT$ and model B i.e. $f(Q, T) = Q^{\omega+1} + bT$ are respectively written as

$$\omega = \frac{4H_0^2m(3bq_0 + 8\pi(2q_0 - 1))}{b - 6}\left(\frac{6(3b+16\pi)H_0^2m}{b} + 4H_0^2m(q_0 + 1) - 6H_0^2m\right) - 1.03^{+0.03}_{-0.03}, \text{ SNe data [4]}, \ (38)$$

we still have room for a phantom description of dark energy [22]. Despite these problems concerning the phantom era, $f(Q, T) = Q^{\omega+1} + bT$ gravity satisfies DEC if $n \leq -1$, and $b > -4\pi$. Therefore, such a theory of gravity naturally enables us to describe a phantom era for the dark energy, without the need of extra dimensions or phantom scalar fields. Therefore, we believe that the tension between this $f(Q, T)$ model and observational data for the Hubble parameter at different redshifts, lies in the compatibility of this $f(Q, T)$ with a phantom era description for the dark energy.

**VI. CONCLUSION**

An essential role to establish a consistent theory of gravity is the energy condition. As new theories of gravity are bubbling in the literature, it is relevant to put them up to test through constraints over different energy conditions. In this work, we computed the strong, the weak, the null, and the dominant energy conditions for two $f(Q, T)$ gravity models. The $f(Q, T)$ is a promising new theory for gravity based on the combination of the nonmetricity function $Q$ with the trace of the energy-momentum $T$.

The models here considered were proposed by Xu et al. [11], and constrained by observational data of the Hubble parameter in [12]. Firstly we worked with $f(Q, T) = mQ + bT$, where $m$, and $b$ are free parameters. The energy conditions yield us to constraint these free parameters as $b > -4\pi$, and $m \leq 0$. The previous values result in the violation of SEC, corroborating with

\[ \omega = \frac{3b(nq_0 + n + q_0) + 8\pi(nq_0 + 1) + 2q_0 - 1)}{b(nq_0 + n + q_0 + 4) + 24\pi}. \]
an accelerated phase of expansion for the Universe. Besides, such a model is suitable to describe the Universe in respect to energy conditions as $\Lambda$CDM.

As a second case, we worked with $Q^{n+1} + bT$, whose free parameters should be constrained to $n \leq -1$, and $b > -4\pi$, to satisfy DEC, and NEC energy conditions. In this case, WEC energy condition is partially obeyed while SEC is again violated. The violation of WEC makes this model naturally behaves like scalar-tensor gravity theories. Moreover, the model is compatible with the dark energy era once SEC is not satisfied. A surprisingly feature comes from the equation of state parameter for this model, which describes a phantom regime for the dark energy, allowing extra acceleration for the expansion of the Universe without violates DEC.

The results here presented allowed us to verify the viability of different families of $f(Q, T)$ gravity models, lighting new paths for a complete description of gravity compatible with the dark energy era, which embeds effects from the quantum era of the Universe. The constraints for our free parameters yield to several testable families for $f(Q, T)$ gravity, opening space even for models compatible with a phantom regime for the dark energy. Moreover, it would be interesting to investigate carefully the coupling of $f(Q, T)$ with inflaton fields, looking for possible analytic models or for cosmological parameters constraints. It would be also interesting to impose constraints on such theories of gravity with observational data from low redshifts, such as BAO measurements at $z = 0.1 - 2.5$ which are expected to be performed in the near feature by BINGO [23], and CHIME [24] telescopes. We hope to report on some of these investigations in the near future.

ACKNOWLEDGMENTS

S. A. acknowledges CSIR, Govt. of India, New Delhi, for awarding Junior Research Fellowship. JRLS would like to thank CNPq (Grant no. 420479/2018-0), CAPES, and PRONEX/CNPq/FAPEQ-PB (Grant nos. 165/2018, and 0015/2019) for financial support. PKS acknowledges CSIR, New Delhi, India for financial support to carry out the Research project[No. 03(1454)/19/EMR-II Dt.02/08/2019].

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