Spectra and radial flow at RHIC with Tsallis statistics in a Blast-Wave description

Zebo Tang,1 Yichun Xu,1 Lijuan Ruan,2 Gene van Buren,2 Fuqiang Wang,3 and Zhangbu Xu2

1University of Science & Technology of China, Hefei 230026, China
2Brookhaven National Laboratory, Upton, New York 11973, USA
3Purdue University, West Lafayette, Indiana 47907, USA
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We have implemented the Tsallis statistics in a Blast-Wave model and applied it to mid-rapidity transverse-momentum spectra of identified particles measured at RHIC. This new Tsallis Blast-Wave function fits the RHIC data very well for \( p_T < 3 \text{ GeV}/c \). We observed that the collective flow velocity starts from zero in \( p+p \) and peripheral \( Au+Au \) collisions growing to 0.470 ± 0.009(c) in central \( Au+Au \) collisions. The \( (q−1) \) parameter, which characterizes the degree of non-equilibrium in a system, changes from 0.100 ± 0.003 in \( p+p \) to 0.015 ± 0.005 in central \( Au+Au \) collisions, indicating an evolution from a highly non-equilibrated system in \( p+p \) collisions toward an almost thermalized system in central \( Au+Au \) collisions. The temperature and collective velocity are well described by a quadratic dependence on \( (q−1) \). Two sets of parameters in our Tsallis Blast-Wave model are required to describe the meson and baryon groups separately in \( p+p \) collisions while one set of parameters appears to fit all spectra in central \( Au+Au \) collisions.

I. INTRODUCTION

Identified particle spectra in transverse momenta provide pillars in the discoveries in relativistic heavy ion collisions \([1,2,3,4]\). In \( Au+Au \) collisions at RHIC, identified particle yields per unit of rapidity integrated over the transverse momentum range have provided information on chemical potential and temperature at the chemical freeze-out in a statistical analysis \([1,2]\). The transverse momentum \( (p_T) \) distributions of particles with different masses can be described in a Boltzmann-Gibbs Blast-Wave (BGBW) model with a compact set of parameters of temperature \((T)\), flow velocity \((\beta)\) and flow profile \((\rho)\) \([1,4]\). With the BGBW model applied to the data in a limited \( p_T \) range, a large radial flow \( \beta \approx 0.6 \) is obtained in central \( Au+Au \) collisions, while much smaller flow is obtained in peripheral \( Au+Au \) collisions \([3,7]\). The description of heavy ion collisions using these freeze-out conditions have been used as evidence of collective motion in relativistic heavy ion collisions \([1,2,3,10]\) (which has further allowed extraction of drag and diffusion coefficient of heavy quarks in the expanding bulk \([11,12,13,14]\)), provided necessary interplay with elliptic flow effect for the observed mass ordering in \( v_2 \) \([8]\), and as a necessary condition for explaining the ridge phenomenon in many models \([9,16,17,18,19]\).

However, such BGBW descriptions have limitations. Modeling nuclear collisions in ideal hydrodynamics is limited to low \( p_T \) (\( \lesssim 1 \text{ GeV}/c \)) because it is generally believed that the equilibrium description fails at high \( p_T \), where particle production may be dominated by non-equilibrium or hard processes and exhibits a characteristic power-law tail \([20]\). The blast-wave model has a strong assumption on local thermal equilibrium so that a Boltzmann distribution can be applied \([3]\). This results in an arbitrary choice of \( p_T \) range of the spectra where the function is able to fit the data and requires low and high \( p_T \) cuts \([3,7]\). The BGBW model also lacks non-extensive quantities to describe the evolution from \( p+p \) to central \( A+A \) collisions. For example, one would expect that the energy deposited at mid-rapidity for particle production fluctuates significantly from event to event in \( p+p \) collisions while the current BGBW treats all \( p+p \) events the same with a heat bath at a fixed temperature. The resulting finite flow velocity \( \beta \approx 0.2 \) in \( p+p \) collisions from the same fitting procedure obscures the interpretation that collective flow is large and unique in \( A+A \) collisions \([3]\). In \( Au+Au \) collisions, the fluctuations at initial impact due to Color-Glass Condensate (CGC) formation or individual nucleon-nucleon collision may not be completely washed out by subsequent interactions at either the QGP phase or hadronic phase \([21,22,23]\). All these effects leave footprints in the spectra at low and intermediate \( p_T \) (\( p_T \lesssim a \text{ few GeV}/c \)). For example, the variation of \( \phi \) and \( K^∗ \) spectra from an \( m_T \) power-law (Levy) function in \( p+p \) and peripheral \( Au+Au \) collisions to an \( m_T \) exponential (Boltzmann) function in central \( Au+Au \) collisions can be clearly observed \([24,25,26,27]\), where \( m_T = \sqrt{m_0^2 + p_T^2} \) is the transverse mass of particle with mass \( m_0 \) at a given \( p_T \).

With its development and success of Tsallis statistics in dealing with complex systems in condensed matter, many authors have utilized Tsallis statistics to understand the particle production in high-energy and nuclear physics \([28,29,30,31,32]\). Although the implications and understanding of the consequences of such an application are still under investigation, the function is relatively easy to understand. The usual Boltzmann distribution in an \( m_T \) exponential form is re-written as an \( m_T \) power-law function:

\[
\frac{d^2N}{2\pi m_T dm_T dy} \propto (1 + \frac{q - 1}{T} m_T)^{-1/(q-1)} \tag{1}
\]

where the left-hand side is the invariant differential par-
particle yield and \( q \) is a parameter characterizing the degree of non-equilibrium. The distribution can be derived from the usual procedure in statistical mechanics, starting from a non-equilibrium \( q \)-entropy. If particle production is not distributed at a fixed temperature but rather as a system where \( T \) varies with total energy \( (E) \) as \( T \propto (q - 1)E \), this will result in a negative binomial distribution (NBD) for particle number and temperature fluctuations. Their relations to \( q \) are the skewness in NBD \( \kappa = 1/(q - 1) \) and the fluctuation of temperature \( \sigma^2(1/T) = (q - 1)/(1/T)^2 \). Regardless of the physical interpretation, Eq. 1 does provide a necessary power-law behavior at high \( p_T \) and an exponential behavior at low \( p_T \). This is exactly what has been observed in p+p and peripheral Au+Au collisions at RHIC for \( K^* \) and \( \phi \), whose measurable \( p_T \) range can reach low and high \( p_T \). When \( q \to 1 \), Eq. 1 becomes the familiar Boltzmann distribution again.

In addition to the features of the spectral evolution necessary to describe the \( p_T \) spectra observed at RHIC, the physical interpretation of the results can provide quantitative insight into the non-equilibrium processes in relativistic heavy ion collisions and whether the system has thermalized in central Au+Au collisions. We expect that individual nucleon-nucleon collisions inside a nucleus-nucleus collision are in non-equilibrium at the initial impact and/or have a large energy fluctuation or a large \( (q - 1) \) value, which produce a large power-law tail in the \( m_T \) spectra. Alternatively, in a CGC scenario, the system as a whole produces a strong color field with large fluctuations. This can be treated as many hot spots in a nucleus-nucleus collision at the initial impact. In a viscous hydrodynamic evolution, the hot spots are smoothed (dissipated) into producing collective flow, creating more particles and increasing temperature. It has been argued in the Tsallis statistics that the increase of temperature and flow velocity during the evolution is connected to the decrease of \( (q - 1) \) by (shear and bulk viscosity) in linear or quadratic proportion, such as \( T = T_0 + a \xi (q - 1)^2 \). This can provide quantitative insight into the bulk viscosity, which is predicted to peak at the phase transition and is much larger than the shear viscosity.

In this paper, we present the procedure of implementing Tsallis statistics in the Blast-Wave model (TBW) and use it to fit the identified particle spectra at mid-rapidity at RHIC. The physics implications are discussed and preparation for future work is also presented. Good TBW fits can also provide a practical experimental tool to extract particle yields \( (dN/dy) \) by extrapolating to unmeasured kinematic ranges since most of the experimental measurements only cover a limited \( p_T \) range for any given particle.

II. IMPLEMENT TSALLIS STATISTICS INTO BLAST-WAVE MODEL

To take into account collective flow in both longitudinal and transverse directions in relativistic heavy ion collisions, a simple Tsallis distribution needs to be embedded in the framework of hydrodynamic expansion. We follow the recipe of the Blast-Wave model provided by Schnedermann et al. The formula has been adopted by many authors to implement a fit to the data in relativistic heavy ion collisions. It is relatively trivial to change sources of particle emission from a Boltzmann distribution to a Tsallis distribution in the Blast-Wave model.

\[
\frac{dN}{m_T dm_T} \propto m_T \int_{-Y}^{+Y} \cosh(y)dy \int_{-\pi}^{+\pi} d \phi \int_{0}^{R} dr (1 + \frac{q - 1}{T} (m_T \cosh(y) \cosh(\rho) - p_T \sinh(\rho) \cos(\phi)))^{-1/(q - 1)},
\]

where \( \rho = \tanh^{-1}(\beta_r (\frac{r}{R})^n) \) is the flow profile growing as \( n \)-th power from zero at the center of the collisions to \( \beta_r \) at the hard-spherical edge \( (R) \) along the transverse radial direction \( (r) \), and \( \beta = \beta_r (1 + 1/(n + 1)) \) is the average flow velocity. We have used \( n = 1 \) in this study. However, the integrations after the replacement are hypergeometric functions and the integrals over rapidity \( (y) \) and azimuthal angle \( (\phi) \) cannot be decoupled into two Bessel functions as in BGBW. The computing program typically used to provide conventional Blast-Wave fits to RHIC data has been modified to numerically calculate the integration in the above spectral function and fit the resulting distributions to the data. There are a few assumptions which do not change when a Tsallis distribution replaces a Boltzmann distribution:

1. Bjorken longitudinal expansion is assumed so that the measured particle yield does not depend on rapidity due to the integration over rapidity of the source. This is approximately true at mid-rapidity at RHIC or LHC. On the other hand, this assumption can be lifted in future analyses with a more complicated integration provided an emitting source along rapidity is known.

2. Isotropic emission in azimuth is assumed for each local source. However, the distribution of the source can have an azimuthal dependence in reality. In the future, this can be implemented as has been done in the conventional BGBW model to fit the azimuthal dependence of HBT radius and
3. The emission source everywhere has the same density and degree of non-equilibrium \((q)\) at the time of kinetic freeze-out. This may not be true since the high-\(p_T\) particles (jet) tend to have surface emission\([42, 43]\). This kind of corona effect has been implemented in many other models and can be adopted in this framework as well.

4. Resonance decay contributions to the stable particle yields have been treated as part of the source emission. The detailed decay kinematics and its effect on the spectra have been studied in Ref.\([4, 30]\). Incorporation of resonance effects on spectra can be made in future improvements.

The goal of this paper is to provide a first implementation of the TBW model to exercise a trial case with the RHIC data. Future work can change the above assumptions to resemble more realistic conditions by comparing the assumptions to data and hydrodynamic calculations.

FIG. 1: (Color Online) Identified particle transverse momentum spectra in Au+Au collisions at \(\sqrt{s_{NN}} = 200\) GeV in0-10% central (a) and in peripheral 60-80% collisions (b). The symbols represent experiment data points. The solid curves represent the TBW fit.

III. RESULTS OF FIT TO RHIC DATA

The STAR Collaboration has published a series of particle spectra at mid-rapidity. The most complete set is for \(p+p\) and Au+Au collisions at \(\sqrt{s_{NN}} = 200\) GeV. The identified particle spectra include \(\pi^\pm, K^\pm, K^0, \Lambda, \bar{\Lambda}, \Xi, \bar{\Xi}, p, \bar{p}, \phi, K^*, K^+, p, p, K^+, \) and \(\Xi^+, \Xi^0, \Xibar, \bar{\Xi}^+, \bar{\Xi}^0\). The invariant differential yields were measured as \(dN/d\eta/dp_T\). In the new TBW model, three parameters are common for all particles: temperature \(T\), non-equilibrium parameter \(q\), and maximum flow velocity \(\beta_s = \beta(1 + n/2)\) where \(n = 1\) and average flow velocity \(\beta\) is bounded to the range \([0, 0.7]\)\([39]\) to aid in fit convergence and avoid non-physical results. An additional parameter provides the overall normalization of \(dN/dy\) for each species. We choose the Minuit in Root\([50]\) to perform a least-\(\chi^2\) fit used in ref.\([1]\). Figure\(1\) shows the \(p_T\) spectrum data together with our fit results in two selected centrality bins (0-10% and 60-80%) in Au+Au collisions. The fit parameters and \(\chi^2/\text{DoF}\) are tabulated in Tab.\([1]\). As stated earlier in our model’s third assumption, surface emission could become important at high \(p_T\); we limit our fits to \(p_T < 3\) GeV/c to avoid this region, which still extends the fit range well beyond previous BGBW fits. The curves from our model generally describe the data very well, especially in central Au+Au collisions. For peripheral Au+Au collisions, the meson spectra are well described by the model while the baryons are in general over-predicted at higher \(p_T\). On the other hand, the \(\chi^2/\text{DoF}\) show good fits in all cases. The main results are:

1. \((q - 1)\), a measure of the degree of non-equilibrium, decreases by a factor of 5 from 0.086 to 0.018. This means the power in the \(m_T\) power-law increases from about 12 to 56, attaining an almost Boltzmann distribution.

2. \(T\), the average temperature of the local source, shows a small increase from 114 MeV to 122 MeV. This trend is in contrast to the conventional BGBW result, where a decrease of temperature was observed\([39]\).

3. \(\beta\), the average flow velocity, increases from 0 in peripheral to 0.47c in central Au+Au collisions. That the minimum \(\chi^2/\text{DoF}\) is found at the lower bound of 0 for peripheral collisions indicates that either the model is incomplete (some approximations may not be sufficiently true), or that no flow has developed in peripheral collisions within the context of the model. This also coincides with a large \(q-1\), indicating a very non-equilibrated system if the description as a unified system applies.

Figure\(2\) shows the temperature and flow velocity versus \((q-1)\) for \(p+p\) and Au+Au collisions. Each shaded region represents a one-\(\sigma\) contour from the error matrix obtained from the TBW fit for a given centrality. The dependence is clearly non-linear and has a negative correlation. There is a jump of flow velocity from zero in \(p+p\) and 60-80% Au+Au centrality to 0.28 at 40-60% Au+Au centrality, coinciding with the transition behavior in several other observables\([35]\). We fit the distributions with a quadratics and obtain \(T = (0.123 \pm 0.0014) - (1.2 \pm 0.4)(q - 1)^2\) and \(\beta = (0.49 \pm 0.01) - (61 \pm 5)(q - 1)^2\), as shown in the figure.

Since the TBW model can be used to describe systems at non-equilibrium, it is natural to extend the fit to \(p+p\) collisions. However, a very poor \(\chi^2/\text{DoF}\) was obtained if we include all of the mesons and baryons in a common
TABLE I: Values of parameters from TBW fit to identified particle transverse spectra in Au+Au collisions of different centralities and in p+p collisions at RHIC. Quoted errors are quadratical sum of statistical and uncorrelated systematic errors. The limits of $\beta$ is set to [0, 0.7].

| Centrality | $\beta$     | $T$     | $q - 1$ | $\chi^2/\text{DoF}$ |
|------------|-------------|---------|---------|---------------------|
| 0-10%      | 0.470±0.009 | 0.122±0.002 | 0.018±0.005 | 130/125          |
| 10-20%     | 0.475±0.008 | 0.122±0.002 | 0.015±0.005 | 119/127          |
| 20-40%     | 0.441±0.009 | 0.124±0.002 | 0.024±0.004 | 159/127          |
| 40-60%     | 0.282±0.017 | 0.119±0.002 | 0.066±0.003 | 165/135          |
| 60-80%     | 0.125±0.005 | 0.114±0.003 | 0.086±0.002 | 138/123          |

For central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, the TBW fits find a value of $q$ approaching unity, which implies results very similar to those from BGBW with a large radial flow velocity $\beta$. The progression of the TBW fit parameters with centrality is generally smooth but is strikingly different from BGBW fits, culminating in a notably non-equilibrium description of $p+p$ and peripheral Au+Au collisions where a preference is found for zero collective flow. Qualitatively, increasing centrality produces a strong increase in flow velocity, a mild increase in temperature, and a dramatic decrease in ($q-1$). This is consistent with a picture of increased thermalization with centrality but disfavors complete thermal equilibrium in all systems, a requirement for applicability of the BGBW model. The BGBW model appears to translate the non-equilibrium features, indicated by our model, into non-zero collective flow velocity and higher temperatures in $p+p$ and peripheral Au+Au collisions.

IV. DISCUSSIONS AND OUTLOOKS

Modifying the Blast-Wave model to utilize Tsallis statistics instead of the conventional Boltzmann-Gibbs statistics has allowed high quality fits (see $\chi^2/\text{DoF}$ in Table I) over a broader transverse momentum range and has altered the conclusions which can be drawn from the fits. The extended $p_T$ range is enabled by the Tsallis statistics’ capacity to evolve from an $m_T$ exponential source into a power law through increasing $q$ values, though the physical interpretation of this statistical model in the context of high energy nuclear collisions remains to be fully understood.

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Hydrodynamics with space-time evolution from an initial condition is so far the most realistic simulation of what happens in relativistic heavy ion collisions. However, even with implementation of initial conditions and an interface to hadronic cascade models at late stage of the evolution, it is hard to obtain an intuitive picture, in contrast to that offered by an analytical parametrization in the Blast-Wave model. It seems unlikely that hydrodynamics is applicable at all for $p+p$ and very peripheral A+A collisions at RHIC. Being able to provide a systematic comparison between $p+p$ and central A+A collisions in one model framework is still valuable and in some cases may be necessary.

We have additionally observed that $p+p$ spectra continue to be well described by our TBW curves from 3 to
due to increased relative contributions of hard and semi-hard processes; the conventional BGBW is not expected to be very meaningful for \( p+p \) and peripheral \( \text{Pb+Pb} \) collisions. If \((q-1)\) is not larger for LHC \( p+p \) collisions and a non-zero flow velocities is observed, the question of thermalization or even QGP in such collisions might be raised. The results of TBW fits at LHC will certainly be informative.

V. CONCLUSIONS

In summary, we have implemented the Tsallis statistics in a Blast-Wave model and applied it to sets of identified particle spectra versus transverse momenta at mid-rapidity at RHIC. This new TBW function fits the RHIC data quite well for \( p_T < 3 \text{ GeV}/c \). We observe that the collective flow velocity starts from zero in \( p+p \) and peripheral \( \text{Au+Au} \) collisions and rises to \( 0.470 \pm 0.009 \text{ c} \) in central \( \text{Au+Au} \) collisions. The parameter \((q-1)\), which characterizes the degree of non-equilibrium in a system, changes from 0.100 \( \pm 0.003 \) to 0.015 \( \pm 0.005 \) systemati-
cally from \( p+p \) to central \( \text{Au+Au} \) collisions, indicating an evolution from a highly non-equilibrated system in \( p+p \) collisions toward an almost thermalized system in central \( \text{Au+Au} \). TBW fits using all species from multiple centralities demonstrate a quadratic dependence of the temperature and collective flow velocity on \((q-1)\), diverging into two different fits for mesons and baryons necessary to best describe \( p+p \) spectra.

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