Experimental evidence of the Strato-Rotationnal Instability

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Abstract. This experimental study is devoted to the analysis of the Strato-rotational Instability (SRI) that affects the classical cylindrical Couette flow when the fluid is stably stratified in the axial direction. In agreement with recent theoretical and numerical predictions, we describe for the first time in detail the destabilization of the stratified flow below the Rayleigh line (i.e. the stability threshold without stratification). We confirm that the unstable modes of the SRI have an helicoidal shape and that the SRI takes place as soon as the azimuthal linear velocity decreases along the radial direction as it was claimed by Shalybkov and Rüdiger in 2005. A direct consequence of this departure from the Rayleigh criterion is that centrifugally stable stratified Keplerian flows, as those met in accretion disks, are unstable to the SRI.

1. Introduction
In 2001, Molemaker et al. [1] predicted that cylindrical Couette flow in a stratified fluid may become unstable under the Rayleigh line, i.e. in the stable Couette regime of pure fluid co-rotating flow. Moreover, and contrary to the classical Taylor vortices, the most unstable modes should be non-axisymmetric. The theoretical analysis of Molemaker et al. was then continued in an astrophysical context by Shalybkov and Rüdiger [2] and extended to the stability of accretion disk Keplerian flows by Dubrulle et al. [3]. To the best of our knowledge, no experimental validation of these theoretical predictions have been published yet, apart from a short comment that can be found in Withjack and Chen (1974) [4] and a stability curve published by Boubnov et al. in 1997 [5]. We present here the first experimental evidence of this so-called Strato-Rotationnal Instability and its expected helicoidal modes. Kalliroscope visualizations of the flow are performed in a classical Couette device. A salt stratification is obtained via the double bucket technique and the Froude number is chosen equal to 0.5 which is the classical value met in accretion disk studies. The particularity of our study comes from the possibility to rotate separately both cylinders and therefore to reach the new regime. Non axisymmetric helicoidal modes are indeed observed under the Rayleigh line as predicted by the theories. Moreover, our experimental determination of the Strato-Rotationnal Instability threshold is in excellent agreement with the theoretical value proposed by Shalybkov and Rüdiger [2].

2. Experimental set-up and protocol
Our Couette device consists of two co-axial cylinders whose length is 168mm. The inner radius of the outer cylinder is \( Ro = 69\, \text{mm} \) and the radius of the inner cylinder is \( Ri = 55\, \text{mm} \), conferring a 14mm gap between the two concentric cylinders. The ratio of the inner to outer radius of the fluid cavity is equal to \( \eta = 0.8 \) explicitly chosen to allow some comparisons with the results of Shalybkov and Rüdiger[2]. The cylinders are positioned vertically. The inner cylinder is made of black polished acetal plastic and the outer one is of transparent glass. Both cylinders are driven by a d.c. servo-motor. Angular
velocities are measured by an optical encoder with an accuracy better than 1%. The top and bottom lids of the device rotate with the external cylinder. The classical double-bucket technique [6] is used to obtain a vertical stratification of salt water. The vertical density gradient is measured before each run with the help of a density meter or a conductivity meter during the filling phase. A linear gradient is quite easily achieved and its value leads to the determination of the Brunt-Väisälä pulsation $N$ with an accuracy better than 5%. It determines the Froude number $Fr = \Omega_i/N$. Figure 1-a presents the experimental set-up and figure 1-b is an example of salt stratification measured along the vertical axis of the Couette chamber.

![Experimental set-up for stratified Taylor-Couette flow.](Image)

**Figure 1:** a) Experimental set-up for stratified Taylor-Couette flow. b) Example of density stratification measure.

### 3. Visualization of the helicoidal modes of the SRI

Visualizations are performed with the help of Oiliroscope flakes. In order to complement these bulk visualizations that use a classical light bulb for illumination, a laser sheet is also installed to get a description of the hydrodynamical structure in a vertical meridional plane.

Having determined the Brunt-Väisälä pulsation by measuring the salt concentration on the vertical axis, the value $\Omega_i$ of the inner cylinder rotation rate is given by the choice of the desired Froude number (i.e. $Fr = 0.5$). Both cylinders are then run at this same angular speed to set the flow in a solid body rotation. The Reynolds number $Re$ is calculated using this fixed inner cylinder angular speed $\Omega_i$, the gap between the cylinders and water viscosity ($Re = \Omega_i R_i (R_o - R_i)/\nu$). The outer cylinder rotation rate is then slowly decreased in order to shear the flow. When the SRI threshold is reached, a simple helix grows in the flow. Figure 2-a shows this helicoidal mode that grows for some minutes. Both axial wave number $k > 0$ or $k < 0$ can be excited at threshold depending on initial conditions. After this transient regime, a saturated state is reached in the form of a braid pattern created by the superimposition of both $k > 0$ and $k < 0$ waves. This saturated state where the +/- z symmetry is restored, is presented in figure 2-b.

Figure 3-a) shows the space-time diagram of the braid pattern observed at SRI threshold and shown in figure 2-b). These diagrams are built by assembling lines along vertical axis $Z$ from successive
a) 

Figure 2: a) At threshold a simple helix grows in the flow. b) After the transient, a saturated braid pattern takes place. \( \text{Re} = 1155 \ (\Omega_0 = 1.07 \text{rad/s}, \Omega_i = 1.5 \text{rad/s}), \ Fr = 0.5. \)

...video images, and allow to extract in a systematic way the temporal frequency \( \omega \) and the axial wavenumber \( k \) of the hydrodynamic structures using a 2D-Fourier transform of the image. If we suppose that the helices travel at a constant phase velocity given by the spatial average \( <\Omega> \) of the angular velocity in the flow, it is quite easy to extract from the space-time diagrams the azimuthal wavenumber \( m = \omega /<\Omega> \). Figure 3-b) shows this discretization. Comparisons of the wave characteristics \((\omega, m, k)\) with theoretical results of [2] are excellent.

Further away from the threshold, when \( \Omega_0 \) is still decreased, the braid pattern transforms in a nearly axisymmetric mode \((m=0)\) but with localized defects. These defects are aligned on a vertical direction and are visible on figure 4-a). They induce an periodic dynamics represented by the amazing chess pattern in the space time recordings (see figure 4-b). This regime should be reminiscent of the vortex mode state observed by Boubnov et al. [5] in the case where the outer cylinder is at rest (the flow was centrifugally unstable in this case).

4. Conclusion

For a given stratification, we can also determine the SRI threshold for different inner cylinder rotation rates. Eventhough the Froude number is different in each data point, it is quite clear on Figure 5-a that the stratified Taylor-Couette flow is unstable under the Rayleigh line. If we focus on the case of a fixed Froude number \((Fr=0.5)\), each Reynolds number requires a particular density stratification. Ten series of experiments corresponding to ten different Reynolds numbers ranging from 339 to 1210 have been performed. Figure 5-b summarize the experimental stability diagram in the parameter plane.
\(\mu = \Omega_i / \Omega_0 \text{Re}\). It shows our data points (\(x\) for instability, \(o\) for stable flow) superimposed on the stability diagram (solid curves) calculated in [2].

![Figure 4](image)

Figure 4. a) for small \(\Omega_0\) the braid pattern is transformed in a nearly \(m=0\) mode but some defects persist and generate a periodic dynamics as seen by the chess pattern of the space-time diagram in b).

As can be observed an excellent agreement is obtained. In particular, we confirm that the inviscid threshold (dotted line) proposed in [2], is above the Rayleigh line (solid line). A direct consequence of this new instability threshold is the possibility for Keplerian flow, where the angular velocity decreases as \(r^{-3/2}\), to be centrifugally stable but unstable versus the SRI. The SRI might then be the source of turbulence in these astrophysical flows that might explain the outward diffusion of angular momentum required to get the observed accretion rate of matter in their centers.

![Figure 5](image)

Figure 5. a) Threshold of the Taylor Couette flow with and without stratification in the \((\Omega_0, \Omega_i)\) plane for varying Froude numbers with \(N = 2.84\) rad/s. b) Stability diagram for \(Fr=0.5\). Stratified Keplerian like flows are unstable to the SRI.

References

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