Aberration of light and Motion of Real Particle

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Abstract. Correct and complete (to terms of $v/c - v$ is particle’s velocity, $c$ is the speed of light) derivation of equation of motion for real dust particle under the action of electromagnetic radiation is derived. The effect of aberration of light is used. Equation of motion is expressed in terms of particle’s optical properties, standardly used in optics for stationary particles.

Key words: cosmic dust, interplanetary dust, aberration of light

1. Introduction

Equation of motion for perfectly absorbing spherical dust particle under the action of electromagnetic radiation was derived by Robertson (1937). Relativistic generalization for the case when scattered radiation is in the direction of the incident radiation – in proper reference frame of the particle – was presented by Klačka (1992, 2000a).

However, real particles scatter radiation in a more complicated manner. The consequence of this reality may be a completely different orbital evolution of dust particles. Kocifaj and Klačka (1999) show this for really shaped stationary rapidly rotating particle. Relativistically covariant equation of motion for real cosmic dust particle was presented in Klačka (2000b).

We want to derive equation of motion for real dust particle under the action of electromagnetic radiation. We want to make the explanation as simple as possible – to be
understandable to majority of scientific community working in the field of interplanetary matter. The effect of aberration of light will be used, the ideas are concentrated in Klačka (1993a, 1993b).

2. Proper reference frame of the particle – stationary particle

The term “stationary particle” will denote particle which does not move in a given inertial frame of reference, although we admit its rotational motion around an axis of rotation (with negligible rotational velocity). Primed quantities will denote quantities measured in the proper reference frame of the particle.

The flux density of the radiation energy of photons scattered into an elementary solid angle $d\Omega' = \sin \vartheta' \, d\vartheta' \, d\varphi'$ is proportional to $p'(\vartheta', \varphi') \, d\Omega'$, where $p'(\vartheta', \varphi')$ is “phase function”. Phase function depends on orientation of the particle with respect to the direction of the incident radiation and on the particle characteristics; angles $\vartheta'$, $\varphi'$ correspond to the direction (and orientation) of travel of the scattered radiation, $\vartheta'$ is polar angle and it equals zero for the case of the travel of the ray in the orientation identical with the unit vector $\hat{S}_i'$ of the incident radiation. The phase function fulfills the condition

$$\int_{4\pi} p'(\vartheta', \varphi') \, d\Omega' = 1 .$$

(1)

The momentum of the incident beam of photons which is lost in the process of its interaction with the particle is proportional to the cross-section $C'_{ext}$ (extinction). The part proportional to $C'_{abs}$ (absorption) is completely lost and the part proportional to $C'_{ext} - C'_{abs} = C'_sca$ (scattering) is again reemitted.

The momentum (per unit time) of the scattered photons into an elementary solid angle $d\Omega'$ is

$$dp'_{sca} = \frac{1}{c} \, S' \, C'_sca \, p'(\vartheta', \varphi') \, \hat{K}' \, d\Omega' ,$$

(2)

where

$$\hat{K}' = \cos \vartheta' \, \hat{S}_i' + \sin \vartheta' \, \cos \varphi' \, \hat{e}_1' + \sin \vartheta' \, \sin \varphi' \, \hat{e}_2' .$$

(3)

$S'$ is the flux density of radiation energy. The system of unit vectors used on the RHS of the last equation forms an orthogonal base. The total momentum (per unit time) of the scattered photons is

$$p'_{sca} = \frac{1}{c} \, S' \, C'_sca \, \int_{4\pi} p'(\vartheta', \varphi') \, \hat{K}' \, d\Omega' .$$

(4)

The momentum (per unit time) obtained by the particle due to the interaction with radiation is
\[
\frac{d \mathbf{p}'}{dt} = \frac{1}{c} \mathbf{S}' \left\{ C'_{\text{ext}} - C'_{\text{sca}} \int_{4\pi} p'(\vartheta', \varphi') \mathbf{K}' \, d\Omega' \right\} .
\] (5)

As for the energy, we suppose that \(dE'/dt = 0\).

For the sake of brevity, we will use “effective factors” \(Q'_{xxx}\) instead of effective cross-sections \(C'_{xxx}\): \(C'_{xxx} = Q'_{xxx} A'\), where \(A'\) is cross-section of a sphere of volume equal to the volume of the particle. Equation (5) can be rewritten to the form

\[
\frac{d \mathbf{p}'}{dt} = \frac{1}{c} \mathbf{S}' A' \left\{ \left[ Q'_{\text{ext}} - \langle \cos \vartheta' \rangle Q'_{\text{sca}} \right] \mathbf{S}'_i + \left[ - \langle \sin \vartheta' \cos \varphi' \rangle Q'_{\text{sca}} \right] \mathbf{e}'_1 + \left[ - \langle \sin \vartheta' \sin \varphi' \rangle Q'_{\text{sca}} \right] \mathbf{e}'_2 \right\} ,
\] (6)

or, in a short form

\[
\frac{d \mathbf{p}'}{dt} = \frac{1}{c} \mathbf{S}' A' \left\{ Q'_R \mathbf{S}'_i + Q'_1 \mathbf{e}'_1 + Q'_2 \mathbf{e}'_2 \right\} .
\] (7)

3. Stationary frame of reference

By the term “stationary frame of reference” we mean a frame of reference in which particle moves with a velocity vector \(\mathbf{v} = \mathbf{v}(t)\). The physical quantities measured in the stationary frame of reference will be denoted by unprimed symbols.

Our aim is to derive equation of motion for the particle in the stationary frame of reference. We will use the fact that we know this equation in the proper frame of reference – see Eq. (7) – and \(dE'/dt = 0\), which corresponds to the fact that particle’s mass \(m\) does not change – \(dm/dt = 0\).

3.1. Flux density of the radiation energy

As for the relation between \(S'\) and \(S\), we refer the reader to Eqs. (69), (113) or (116) in Klačka (1992), or, to Eqs. (9) in Klačka (1993a). We may write

\[
S' = S \left(1 - 2 \mathbf{v} \cdot \mathbf{S}'_i/c\right) .
\] (8)

3.2. Aberration of light

As for the transformation between the unit vectors \(\mathbf{S}'_i\) and \(\mathbf{S}_i\), it is given by the well-known phenomenon called aberration of light. The result is (see, e. g., Eq. (19) in Klačka (1993a)):

\[
\mathbf{S}'_i = (1 + \mathbf{v} \cdot \mathbf{S}_i/c) \mathbf{S}_i - \mathbf{v}/c ,
\]
\[
\mathbf{e}'_j = (1 + \mathbf{v} \cdot \mathbf{e}_j/c) \mathbf{e}_j - \mathbf{v}/c , \quad j = 1, 2 .
\] (9)
The inverse transformation yields

$$\mathbf{S}_i = (1 - \mathbf{v} \cdot \mathbf{S}_i'/c) \mathbf{S}_i' + \mathbf{v}/c.$$  \hspace{1cm} (10)

$$\mathbf{e}_j = (1 - \mathbf{v} \cdot \mathbf{e}_j'/c) \mathbf{e}_j' + \mathbf{v}/c, \; j = 1, 2.$$  \hspace{1cm} (10)

3.3. Transformation of force

We can immediately write

$$\frac{dp}{dt} = \frac{dp'}{dt}.$$  \hspace{1cm} (11)

3.4. Equation of motion

Putting Eqs. (7), (8) and (9) into Eq. (11), we finally obtain

$$\frac{dv}{dt} = \frac{SA'}{mc} \left\{ Q'_R \left[ \left( 1 - \mathbf{v} \cdot \mathbf{S}_i/c \right) \mathbf{S}_i - \mathbf{v}/c \right] + \sum_{j=1}^{2} Q'_j \left[ \left( 1 - 2 \mathbf{v} \cdot \mathbf{S}_i/c + \mathbf{v} \cdot \mathbf{e}_j/c \right) \mathbf{e}_j - \mathbf{v}/c \right] \right\}.$$  \hspace{1cm} (12)

4. Conclusion

We have derived equation of motion for real dust particle under the action of electromagnetic radiation. Equation of motion is represented by Eq. (8) in the proper frame of reference of the particle, and, by Eq. (12) in the stationary inertial frame of reference in which particle’s immediate velocity is \(v\). All the results may be applied to real motions if the accuracy to the first order in \(v/c\) is sufficient. As for practical applications, the terms \(v/c\) standing at \(Q'_j\) (\(j = 1, 2\)) are negligible for majority of real particles. (We want to stress that values of \(Q'\)–coefficients depend on particle’s orientation with respect to the incident radiation – their values are time dependent.)

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