Charged Higgs bosons in the Next-to MSSM (NMSSM)

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Abstract

The charged Higgs boson decays $H^\pm \to W^\pm A_1$ and $H^\pm \to W^\pm h_1$ are studied in the framework of the next-to Minimal Supersymmetric Standard Model (NMSSM). It is found that the decay rate for $H^\pm \to W^\pm A_1$ can exceed the rates for the $\tau^\pm\nu$ and $tb$ channels both below and above the top-bottom threshold. The dominance of $H^\pm \to W^\pm A_1$ is most readily achieved when $A_1$ has a large doublet component and small mass. We also study the production process $pp \to H^\pm A_1$ at the LHC followed by the decay $H^\pm \to W^\pm A_1$ which leads to the signature $W^\pm A_1 A_1$. We suggest that $pp \to H^\pm A_1$ is a promising discovery channel for a light charged Higgs boson in the NMSSM with small or moderate $\tan\beta$ and dominant decay mode $H^\pm \to W^\pm A_1$. This $W^\pm A_1 A_1$ signature can also arise from the Higgsstrahlung process $pp \to W^\pm h_1$ followed by the decay $h_1 \to A_1 A_1$. It is shown that there exist regions of parameter space where these processes can have comparable cross sections and we suggest that their respective signals can be distinguished at the LHC by using appropriate reconstruction methods.

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I. INTRODUCTION

An attractive extension of the Minimal Supersymmetric Standard Model (MSSM) is the Next-to MSSM (NMSSM) in which an additional singlet neutral complex scalar field $S$ is added. The presence of this singlet field provides an elegant solution to the $\mu$ problem of the MSSM. The $\mu$ parameter in the MSSM superpotential, which does not break supersymmetry (SUSY) and is present when SUSY is unbroken, is completely unrelated to the electroweak or SUSY breaking scales. In some models like Supergravity, $\mu$ is naturally expected to be of the order $M_{\text{Planck}}$. However, the radiative electroweak symmetry breaking conditions require the $\mu$ parameter to be of the same order as $M_Z$. Such a conflict is called the $\mu$ problem [1].

The superpotential of the NMSSM contains the term $\lambda \hat{S} \hat{H}_u \hat{H}_d$, and the $\mu$ term of the MSSM which mixes the two doublet fields $\hat{H}_u$ and $\hat{H}_d$ is not present explicitly. When the singlet field acquires a vacuum expectation value $< s >$ of the order of the SUSY breaking scale, an effective $\mu$ parameter $\mu_{\text{eff}} = \lambda s$ of the order of the electroweak scale is then dynamically generated. Moreover, it has been shown that with the additional singlet Higgs field the MSSM fine-tuning (or “little hierarchy problem”) problem can be ameliorated in regions of the NMSSM parameter space [2, 3].

A charged Higgs boson ($H^\pm$) appears in any extension of the Standard Model with two hypercharge $Y=1$ doublets. Its phenomenology has been extensively studied in both the Two Higgs Doublet Model (2HDM) and MSSM. The phenomenology of $H^\pm$ in the NMSSM is similar in many ways to that in the MSSM since no charged singlet fields have been added. The increased parameter content of the NMSSM scalar potential compared to that of the MSSM permits large mass splittings among the Higgs spectrum, which allows other decay modes of $H^\pm$ to be important which were substantially suppressed in the context of the MSSM. In the MSSM the coupling $H^\pm A W$ (where $A$ is the CP-odd neutral Higgs boson) contains no mixing angle suppression but the relation $M_A \sim M_{H^\pm}$ ensures that the decay $H^\pm \rightarrow AW$ is greatly suppressed in most of the parameter space [4], [5]. In the NMSSM there are two pseudoscalars $A_1$ and $A_2$ which are mixtures of the doublet and singlet fields. There exists regions in the theoretical parameter space where $A_1$ is predominantly doublet and light, and hence the decay $H^\pm \rightarrow A_1 W$ is unsuppressed.

The importance of the decay $H^\pm \rightarrow A_1 W$ in the NMSSM was emphasized in [6] where it was shown that dominance over $H^\pm \rightarrow cs, \tau \nu$ is possible and branching ratios close to 100% can be attained for intermediate values of $\tan \beta$. A LHC simulation was performed in [7] and concluded that such a decay offers very good detection prospects for $H^\pm$ if the branching ratios of $t \rightarrow H^\pm b$ and $H^\pm \rightarrow A_1 W$ are sufficiently large. In this work we perform a comprehensive scan of the NMSSM parameter space using the publicly available code NMHDECAY [8] in order to identify the regions where $H^\pm \rightarrow A_1 W$ can be sizeable.

The strength of the coupling $H^\pm A_1 W$ can also have an application to the production of $H^\pm$ via $pp \rightarrow H^\pm A_1$ which has been studied in the CP conserving MSSM [9], [10] and CP violating MSSM [11]. If the branching ratio for the decay $H^\pm \rightarrow A_1 W$ were also sizeable such a production mechanism would lead a final state of $Wb b b b$ (for $M_{A_1} > 2m_b$) [11] which has been simulated in [12] in the context of the LHC with promising conclusions. This $Wb b b b$ signature can also arise from the process $pp \rightarrow W h_1 \rightarrow W A_1 A_1$ which was simulated in [13] and shown to provide a clear signal at the LHC. We compare the magnitude of both mechanisms and discuss how they may be distinguished.

Our work is organized as follows: in section II we present a short review of the Higgs sector of the NMSSM; in section III the limits that lead to a light $A_1$ in the NMSSM
In section IV the phenomenology of $H^\pm$ is introduced; section V contains our numerical results for the branching ratios of $H^\pm \to A_1 W, h_1 W$ and cross-sections $pp \to H^\pm A_1 \to W bbbb(W\tau\tau\tau\tau)$ and $pp \to W h_1 \to W bbbb(W\tau\tau\tau\tau)$. Conclusions are given in section VI.

II. A BRIEF REVIEW ON THE HIGGS SECTOR OF THE NMSSM

For detailed discussions of the Higgs sector of the NMSSM the reader is referred to [14, 15, 16, 17, 18]. In this section we follow the notation of Ref. [19]. The NMSSM Higgs sector differs from that of the MSSM by the addition of an extra complex scalar field, $S$. The Higgs fields of the model then consist of the usual two Higgs doublets $\hat{H}_u$ and $\hat{H}_d$ together with this extra Higgs singlet.

In the NMSSM Lagrangian, the extra singlet field is allowed to couple only to the Higgs doublets of the model and consequently the couplings of the new field $S$ to gauge bosons and fermions will only be manifest via their mixing with the doublet Higgs fields. The superpotential of the NMSSM is given by

$$W = W_{\text{MSSM}} + \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{1}{3} \kappa S^3,$$

where $W_{\text{MSSM}}$ is the usual MSSM superpotential and only terms that depend on the singlet field are explicitly written. The soft breaking terms for both the doublet and singlet are included in $V_{\text{soft}}$:

$$V_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + [\lambda A_\lambda S H_u H_d + \frac{1}{3} \kappa A_\kappa S^3] + \text{h.c.},$$

where $\lambda, \kappa, A_\lambda, A_\kappa, m_S$ and the vacuum expectation value of the singlet field, $s$, which will generate the effective $\mu$ term given by $\mu_{\text{eff}} = \lambda s$. As in the MSSM, $m_S$ can be fixed by the minimization condition of the scalar potential.

After electroweak symmetry breaking the Higgs spectrum of the NMSSM consists of three neutral scalars ($h_1, h_2, h_3$), two pseudoscalars ($A_1, A_2$) and a pair of charged Higgs bosons $H^\pm$. In both the CP-odd and CP-even sector the physical eigenstates are ordered as $M_{h_1} \lesssim M_{h_2} \lesssim M_{h_3}$ and $M_{A_1} \lesssim M_{A_2}$. The mass of $H^\pm$ at tree-level is given by [14, 20]:

$$M_{H^\pm}^2 = \frac{2\mu_{\text{eff}}}{\sin 2\beta}(A_\lambda + \kappa s) + M_W^2 - \lambda^2 v^2$$

where $\tan \beta = v_u/v_d$ and $v^2 = v_u^2 + v_d^2$. This differs from the corresponding MSSM expression in which $M_A$ and $M_{H^\pm}$ are strongly correlated and become roughly equal for $M_A \geq 140$ GeV.

The CP-odd mass matrix can be obtained as follows: Firstly, as in MSSM one rotates the bare fields ($3m_{H_u}, 3m_{H_d}, 3m_S$) into a basis $(A, G, 3m_S)$ where $G$ is a massless Goldstone boson. Then one eliminates the Goldstone mode and the remaining $2 \times 2$ CP-odd mass matrix in the basis $(A, 3m_S)$ is given by:

$$M_{P,11}^2 = \frac{\lambda s}{\sin \beta \cos \beta} (A_\lambda + \kappa s),$$

$$M_{P,22}^2 = (2\lambda\kappa + \frac{\lambda A_\lambda}{2s}) \sin 2\beta v^2 - 3\kappa A_\kappa s,$$

$$M_{P,12}^2 = \lambda v (A_\lambda - 2\kappa s).$$

3
Here $A = \cos \beta \Re \mH_u + \sin \beta \Im \mH_d$ is the CP-odd MSSM Higgs boson while $\Im \mS$ comes from the singlet $S$ field. The pseudoscalars fields are further rotated to the diagonal basis $(A_1, A_2)$ by an orthogonal $2 \times 2$ matrix such that:

\[ A_1 = \cos \theta_A A + \sin \theta_A \Re \mS \]
\[ A_2 = -\sin \theta_A A + \cos \theta_A \Im \mS \]

where

\[ \cos \theta_A = \frac{M^2_{P,12}}{\sqrt{M^4_{P,12} + (M^2_{A_1} - M^2_{P,11})^2}} \quad \sin \theta_A = \frac{M^2_{A_1} - M^2_{P,11}}{\sqrt{M^4_{P,12} + (M^2_{A_1} - M^2_{P,11})^2}} \]

The Higgs boson-gauge boson couplings originate from the covariant derivative of the kinetic energy term. Those relevant for our study are described by the following Lagrangian:

\[ \mathcal{L}_{VVH,VHH} = g m_W g_{VV h_i} W^{+\mu} W^{-h_i} - g W^+ \left( \frac{ig W^+ \bar{h}_i}{2} h_i + \frac{P_{11}}{2} A_1 \right) \partial^\mu H^- + h.c \]

where $g_{VV h_i} = \sin \beta S_{i1} + \cos \beta S_{i2}$, $g_{W^+ H^- h_i} = \cos \beta S_{i1} - \sin \beta S_{i2}$, $P_{11} = \cos \theta_A$ and $P_{21} = -\sin \theta_A$, $S$ and $P$ are orthogonal matrices which diagonalize respectively the CP-even and CP-odd scalar mass matrix. From the last term in eq. (7) one can see that the vertex $W^\pm H^\mp A_1$ is directly proportional to $P_{11}$ i.e. the doublet component of the mass eigenstate $A_1$. Consequently, if $A_1$ is entirely composed of doublet fields this coupling is maximized and if $A_1$ is purely singlet the coupling vanishes.

As in the MSSM one can easily derive the following sum rules:

\[ \sum_{i=1}^{3} g^2_{WW h_i} = 1 \]
\[ g^2_{WW h_i} + g^2_{W^+ H^- h_i} + S^2_{i3} = 1 \quad i = 1,2,3 \]

Here $S_{i3}$ is the singlet component of $h_i$. From the second sum rule it follows that if $h_i$ is purely doublet ($S_{i3} \approx 0$) then the MSSM sum rule, $g^2_{WW h_i} + g^2_{W^+ H^- h_i} = 1$, is recovered where $h_i$ is entirely composed of doublet fields. Conversely, if $h_i$ is purely singlet ($S^2_{i3} \approx 1$) then one has $g^2_{WW h_i} + g^2_{W^+ H^- h_i} \approx 0$ and both $h_i VV$ and $h_i H^+ W^-$ must be suppressed, and this will present a real challenge for the detection of Higgs bosons. This sum rule will be explored in our numerical analysis.

### III. A LIGHT $A_1$ IN THE NMSSM PARAMETER SPACE

The parameter space of the NMSSM can naturally accommodate a light $A_1$ which is of great phenomenological interest. To identify such regions it is instructive to examine the vanishing limits of the determinant of the mass matrix of the pseudoscalar, which can be expressed as:

\[ \det M^2_P = -\frac{3\kappa \lambda s}{\sin 2\beta} \left( 2\kappa s^2 A_\kappa + 2s A_\kappa A_\lambda - 3\lambda A_\lambda v^2 \sin 2\beta \right) \]

It is then straightforward to identify four distinct cases where $\det M^2_P$ approaches 0:
• Case 1: $A_\lambda \to 0$ and $A_\kappa \to 0$ \cite{21},
• Case 2: $\kappa \to 0$ \cite{22},
• Case 3: $\lambda \to 0$,
• Case 4: $s \to 0$.

Moreover, it is evident that combinations of these basic cases can also lead to a light $A_1$. The requirement of perturbativity up to the grand unification scale restricts $\lambda < 0.8$ \cite{23}. Therefore Case 4 ($s \to 0$) is ruled out since it would lead to a very small $\mu_{eff}$ which is excluded by the mass bound for charginos from direct searches. However, if one gives up this perturbative requirement up to grand unification scale and considers $\lambda \gg 1$, as in the so-called $\lambda$SUSY model \cite{24} (which can be realized in the supersymmetric fat Higgs models \cite{25}), then Case 4 might be viable.

The first two limits are related with the discrete symmetries of Higgs potential: one is called the R-axion limit with $A_\lambda \to 0$ and $A_\kappa \to 0$ \cite{21}; the other is called the PQ-axion limit with $\kappa \to 0$ the superpotential eq.\(1\) and its associated Lagrangian contains an extra global $U(1)$ symmetry \cite{22}. In both cases, these symmetries are spontaneously broken by the Higgs vev leading to Pseudo-Goldstone boson in the spectrum.

At tree-level, in the R-axion limit \cite{21}, the mass spectra and mixing of the CP-odd Higgs sector can be expressed as:

\[
m^2_{A_1} = 3s(-\kappa A_\kappa \sin^2 \theta_A + \frac{3}{2 \sin 2\beta} \lambda A_\lambda \cos^2 \theta_A) + O(\kappa^2 A_\kappa^2, \lambda^2 A_\lambda^2),
\]

\[
m^2_{A_2} = \frac{2\lambda\kappa v^2}{\cos^2 \theta_A} \sin 2\beta + O(\kappa^2 A_\kappa^2, \lambda^2 A_\lambda^2),
\]

\[
\tan \theta_A = \frac{s}{v \sin 2\beta} + O(\kappa A_\kappa, \lambda A_\lambda).  \tag{10}
\]

In the R-axion limit scenario, as can be seen from eq.(10), a light pseudoscalar is obtained for small $\kappa A_\kappa$ and $\lambda A_\lambda$ or a combination of small $\kappa A_\kappa$ and $\lambda A_\lambda$.

At tree-level, in the PQ-axion limit \cite{22}, one has:

\[
m^2_{A_1} = 3s\kappa(-A_\kappa \sin^2 \theta_A + \frac{6}{\sin 2\beta} \lambda s \cos^2 \theta_A) + O(\kappa^2),
\]

\[
m^2_{A_2} = -\frac{2\lambda A_\lambda v}{\sin 2\theta_A} + O(\kappa^2),
\]

\[
\tan \theta_A = -\frac{2s}{v \sin 2\beta} + O(\kappa).  \tag{11}
\]

It is interesting to see that in eq. (11) the limit $\kappa \to 0$ gives $m_{A_1} \to 0$. This is actually the case where the $U(1)$ PQ symmetry is left unbroken in the superpotential. The spontaneous breaking of such PQ symmetry by a Higgs vev leads to a massless Goldstone boson, the axion. To obtain a light pseudoscalar $A_1$ one needs to introduce a small $\kappa$ which only slightly breaks the PQ symmetry.
The third case is also related with a discrete symmetry of two Higgs doublet models. In this limit one has:

\[
m^2_{A_1} = \frac{2\lambda s}{\sin 2\beta} (A_\chi + \kappa s) + O(\lambda^2),
\]

\[
m^2_{A_2} = -3\kappa s A_\kappa + O(\lambda),
\]

\[
\tan \theta_A = \frac{\lambda(A_\chi - 2\kappa s)v}{3\kappa A_\kappa s} + O(\lambda^2).
\]  

(12)

When \( \lambda \to 0 \), a large value for \( s \) is needed to keep \( \mu_{\text{eff}} \) of the order of the electroweak scale. In this case \( \lambda \to 0 \), and for \( \mu_{\text{eff}} \) fixed, \( A_1 \) is mainly doublet and this is the exact MSSM limit.

IV. \( H^\pm \) IN THE NMSSM

In this section we describe the phenomenology of the \( H^\pm \) in the NMSSM and highlight its differences with the phenomenology of \( H^\pm \) in the MSSM. The phenomenology of \( H^\pm \) in the NMSSM has many similarities with that of \( H^\pm \) in the MSSM (the latter recently reviewed in [26]). This is to be expected since the fermionic couplings are identical in the two models. The main differences in their phenomenology originate from the possibility of large mass splittings among the Higgs bosons in the NMSSM which permits decay channels like \( H^\pm \to A_1 W \) to proceed on-shell [6]. In the MSSM such a decay can only be open for extreme choices of certain SUSY parameters (e.g. for \( \mu > 4M_{\text{SUSY}}[27] \)) which induce large quantum corrections in the effective scalar potential. Moreover, in the NMSSM a light CP-even \( h_1 \) is also allowed and one can have the opening of the decay \( H^\pm \to h_1 W \) both below and above the top-bottom threshold. This latter channel may change the NMSSM phenomenological predictions for the charged Higgs with respect to the MSSM. In the MSSM the decay \( H^\pm \to h_1 W \) is also open but the coupling \( g_{W^+H^-h_1} \sim \cos^2(\beta - \alpha) \) is strongly suppressed when \( M_{H^\pm} \gg m_{h_1} + m_W \) and thus its branching ratio is very small for such \( M_{H^\pm} \). For \( M_{H^\pm} < m_{h_1} + m_W \) and just above the threshold the branching ratio for this channel can reach 10% at most for small values of \( \tan \beta \) [4], [5], [7].

The phenomenology of \( H^\pm \) in the NMSSM has received considerably less attention than its neutral Higgs sector. In recent years much effort has been focused on establishing a "no-lose theorem" at the LHC in which detection of at least one Higgs boson in the NMSSM is guaranteed. However, the potential importance of the decay \( h_1 \to A_1 A_1 \) has prevented such a theorem being established [19], [29], [30], [31], [32]. Moreover, it has been shown that a large branching ratio for \( h_1 \to A_1 A_1 \) would weaken the LEP bounds for a SM like \( h_1 \) in the NMSSM [2].

For \( M_{A_1} < 2m_\chi \) [3], [33] dominance of \( h_1 \to A_1 A_1 \) has the virtue of allowing \( h_1 \) as light as \( 90 \to 100 \) GeV and can realize the "LEP excess scenario" easily. Such values of \( M_{h_1} \) can be accommodated in the NMSSM with little fine-tuning, in contrast to the MSSM case where considerable fine-tuning is necessary in order to comply with the LEP limit \( M_{h_1} > 114 \) GeV from the Higgsstrahlung channel. However, a large branching ratio for \( h_1 \to A_1 A_1 \) followed by \( A_1 A_1 \to 4\tau \) is challenging for detection at the Tevatron (see [34]). For the final states \( V2b2\tau \) and \( V4b \) at the Tevatron, the observation is difficult due to the limited statistics, as shown in [35]. At the LHC, by utilizing the central exclusive production process and high-resolution low-angle sub-detectors, it is shown in [36] that it is possible to reconstruct
the masses of $h_1$ and $A_1$. In [37] it was suggested that production of $A_1$ in association with charginos followed by the possibly dominant decay $A_1 \to \gamma \gamma$ could offer good detection prospects for an almost purely singlet $A_1$. An alternative probe is the decay $\Upsilon \to A_1 \gamma$ at B factories [38]. A high-energy $e^+e^-$ linear collider would easily probe the scenario of dominant decay $h_1 \to A_1 A_1$ for $m_{A_1} < 2 m_b$ via the recoil mass technique which is insensitive to the decay of $h_1$.

For $M_{A_1} > 2 m_b$ one would have the dominant decay $A_1 A_1 \to b b b b$ for which a LEP limit of $M_{A_1} > 110$ GeV was derived. In such a scenario the fine-tuning problem is not greatly ameliorated but detection prospects at the LHC are much better. In partonic level analyses it has been shown that a signal with high significance and full Higgs mass reconstruction can be obtained from the process $p p \to W h_1 \to WA_1 A_1 \to W b b b b$ [13, 35]. The main challenge in reconstructing the full decay chain is to retain an adequate tagging efficiency of $b$’s in the low $p_T$ region where signal events are located, as shown in [35].

In many of the studies which are concerned with establishing a no-lose theorem the charged Higgs mass is taken to be very heavy $M_{H^\pm} > 400$ GeV (e.g. the benchmark points in [31]). It has been known for some time that a moderately light $M_{H^\pm} < m_t$ is possible in the NMSSM. A first detailed study appeared in [6], and this possibility has recently been emphasized in [39]. However, such a $H^\pm$ would contribute sizably to the rare decay $b \to s \gamma$ whose branching ratio has been measured and is consistent with the SM expectation. In the context of the NMSSM a contribution to $b \to s \gamma$ from another New Physics particle (usually the lightest chargino, $\chi^\pm_1$) is needed to partially cancel the large $H^\pm$ contribution for $M_{H^\pm} < m_t$ [40]. If flavour violation induced by gluinos ($\tilde{g}$) is considered [41], the NMSSM parameter space for a light $H^\pm$ can be enlarged while keeping the branching ratio for $b \to s \gamma$ consistent with the measured value. This merely requires a suitable cancellation among the contributions from $H^\pm$, $\chi^\pm_1$ and $\tilde{g}$, the latter being of essentially arbitrary magnitude. In light of this possibility we do not impose the $b \to s \gamma$ constraint in our numerical analysis.

Another potentially important constraint on the scenario of $M_{H^\pm} \lesssim m_t$ comes from the measurement of the decay $B^\pm \to \tau^\pm \nu$ [42]. This decay is mediated at tree-level [43] by $H^\pm$ and its contribution cannot be canceled by any other new particle in the model. Current data excludes two regions in the parameter space of $[M_{H^\pm}, \tan \beta]$. However, the non-holomorphic contribution [44] would shift the location of these two regions and thus we do not impose such a constraint in our analysis. Importantly, for $\tan \beta \lesssim 20$ of most interest to us $M_{H^\pm} \lesssim m_t$ is almost always allowed. Moreover, a recent analysis [45] shows that a light charged Higgs boson in the NMSSM is compatible with the constraints from $b \to s \gamma$, $\Delta M_q$, $B^\pm \to \tau^\pm \nu$ and $B_s \to \mu^+ \mu^-$ even without invoking extra sources of flavour violation from gluinos.

V. CHARGED HIGGS DECAY $H^\pm \to SW$ AND THE PRODUCTION MECHANISM $p p \to H^\pm S$, $S = A_1, h_1$

A. Charged Higgs decay modes $H^\pm \to SW$

The decay $H^\pm \to AW$, where $A$ is a CP-odd Higgs boson, may be sizeable in a variety of models with a non-minimal Higgs sector such as Two Higgs doublet models (Type I and II) [46, 47, 48] and in SUSY models with Higgs triplets [49]. Two LEP collaborations (OPAL and DELPHI) performed a search for a charged Higgs decaying to $AW^*$ (assuming $m_A > 2 m_b$) and derived limits on the charged Higgs mass [50] comparable to those obtained
from the search for $H^\pm \to cs, \tau\nu$. In the MSSM the decay width for $H^\pm \to AW$ is very suppressed in most of the parameter space [4, 5] because the charged Higgs and the CP-odd Higgs are close to mass degeneracy.

The importance of the decays $H^\pm \to A_1W$ and $H^\pm \to h_1W$ in the NMSSM was first pointed out in [4]. Their branching ratios may be close to 100% which can provide a clear signal at the LHC. Simulations of the process $pp \to t\bar{t}$ followed by $t \to H^\pm b$ and $H^\pm \to A_1W$ have been performed for the NMSSM [4], CP conserving MSSM [51] and CP violating MSSM [52]. The partial width is given by:

$$\Gamma(H^\pm \to A_1W) = \frac{\alpha \cos^2 \theta_A}{16s^2_W M^2_W M^2_{H^\pm}} \lambda^2(M^2_{H^\pm}, M^2_{A_1}, M^2_W)$$  \hspace{1cm} (13)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$ is the two–body phase space function. The decay width of $H^\pm \to h_1W$ can be obtained from eq. (13) by replacing $\cos \theta_A$ by $g_{H^\pm W^\pm h_1}$ and $M_{A_1}$ by $M_{h_1}$.

As can be seen from (13), the decay width of $H^\pm \to A_1W$ is directly proportional to $\cos \theta_A$ which is the doublet component of $A_1$. This decay width can be substantially enhanced if $A_1$ is predominantly composed of doublet fields. However, even with small doublet (large singlet) component of $A_1$ it is possible that $H^\pm \to A_1W$ is the dominant decay mode. We perform a scan of the parameter space using the code [8] (NMSSM-Tools incorporates the LEP2 bounds for 4b and 6b final states) in order to quantify the importance of $H^\pm \to A_1W$ and $H^\pm \to h_1W$.

Hereafter we assume that all scalar superparticles share the same soft mass term $M_{SUSY}$, and the ratios of gaugino masses satisfy $M_1 : M_2 : M_3 = 1 : 2 : 6$; the trilinear couplings are related to $M_{SUSY}$ but the sign is not fixed, i.e. $A_{t,b} = \pm 2M_{SUSY}$. We scan the parameter space of the model by varying the free parameters within the following region:

$$\lambda = [0, 1], \ k = [-1, 1], \ \tan \beta = [0.2, 60], \ \mu = [-1, 1] \text{TeV}, \ A_\lambda = [-1.0, 1.0] \text{TeV}, \ A_k = [-1.0, 1.0] \text{TeV}, \ M_{SUSY} = [0.2, 3] \text{TeV}, \ M_1 = [0.07, 3] \text{TeV}.$$  \hspace{1cm} (14)$$

While varying these parameters, we take into account the experimental constraints on the MSSM spectrum e.g., charged Higgs mass $\geq 80 \text{ GeV}$, chargino and scalar fermions $\geq 100 \text{ GeV}$. We also apply the full set of LEP constraints obtained from searches for neutral Higgs bosons decaying to final states like $Z2b, Z4b, 6b, 6\tau, Z2b2\tau, Z4\tau, 2b2\tau$.

In Fig. (1) we display the branching ratios of $W^\pm A_1 \to \tau\nu$ and top-bottom modes. Before the opening of the $H^\pm \to tb$ channel, the full dominance of $W^\pm A_1$ over $\tau\nu$ requires light $M_{A_1} \lesssim 100 \text{GeV}$, large doublet component of $A_1$ and $\tan \beta$ not too large. Note that at large $\tan \beta \approx 15 - 25$, the $W^\pm A_1$ and $\tau\nu$ channels become comparable in size. Once the decay $H^\pm \to tb$ is open, it competes strongly with $W^\pm A_1$ for $\tan \beta \lesssim 15$. As can be seen from Fig. (1) upper left, the branching ratio of $H^\pm \to W^\pm A_1$ is less than 90%. It is interesting to see also that for $\cos^2 \theta_A \lesssim 0.05$ there is not a single point with $Br(H^\pm \to W^\pm A_1) \gtrsim 50\%$. Note also that at large $\tan \beta \gtrsim 25$, it is hard for $H^\pm \to W^\pm A_1$ to compete with $\tau\nu$ and top-bottom modes.

The case of the analogous decays $H^\pm \to W^\pm h_{1,2}$ are displayed in Fig. (2) as a function of $M_{H^\pm}$ and $\tan \beta$. One can see from the upper right panel of Fig. (2) that $W^\pm h_1$ dominates over $\tau\nu$ only for moderate $\tan \beta \lesssim 5$ and before the opening of $H^\pm \to tb$ decay, which strongly competes with $H^\pm \to W^\pm h_1$ mode.
FIG. 1: Comparison of the branching ratios of $H^\pm \rightarrow \{W^\pm A_1, \tau \nu, tb\}$ as a function of $M_{H^\pm}$ (upper left), $\cos^2 \theta_A$ (upper right), $M_{A_1}$ (lower left) and $\tan \beta$ (lower right). In all panels only points with $\text{Br}(H^\pm \rightarrow W^\pm A_1) \geq 50\%$ are selected.

From the lower panel of Fig. (2) one can see that the branching ratio for $H^\pm \rightarrow W^\pm h_2$ can only be larger than 20\% for charged Higgs mass larger than about 220 GeV. This is mainly due to the fact that $m_{h_2}$ is most of the time larger than 140 GeV. It is clear that both $H^\pm \rightarrow W^\pm h_2$ and $H^\pm \rightarrow tb$ are of comparable size except in the case of large $\tan \beta$ where $H^\pm \rightarrow tb$ mode dominates.

Importantly, we note that if $S_{13}^2 \approx 1$ the second sum rule in Eq. (8) requires $g_{VVh_1}^2 \approx 0$ and $g_{W^\pm H^\pm h_1}^2 \approx 0$. In this case, $S_{13}^2 \approx 1$, both modes $H^\pm \rightarrow W^\pm h_{1,2}$ are suppressed and hence the full dominance of $W^\pm h_{1,2}$ requires small $S_{13}$.

In our numerical analysis we have explicitly checked that if $h_1$ is predominantly singlet, i.e., $S_{13}^2 \gtrsim 0.9$, both couplings $g_{VVh_1}^2, g_{W^\pm H^\pm h_1}^2 \lesssim 0.1$, in accordance with this sum rule. The larger $S_{13}^2$ is, the smaller are the couplings $g_{VVh_1}^2$ and $g_{W^\pm H^\pm h_1}^2$. When $S_{13}^2 \gtrsim 0.9$, $h_1$ is almost purely singlet and even a very light $h_1$ can be allowed by LEP experimental constraints. In this case, both the vertices of $ZZh_1$ and $f \bar{f} h_1$ are suppressed, as shown in Fig. (3a). In this case, the $h_2$ will be the Standard Model like Higgs boson and the coupling $g_{VVh_2}^2$ can be large, as indicated by the first sum rule in Eq. (8) and demonstrated in Fig. (3a).

In the converse case when $S_{13}^2 \approx 0 \rightarrow 0.1$, $g_{VVh_1}^2$ and $g_{W^\pm H^\pm h_1}^2$ have to share the quantity
1 - S^2_{13}. Since h_1 is dominantly doublet the coupling g_{W^+H^-h_1} can be maximal, and hence the branching ratio of H^± → W^±h_1 can be large.

B. The cross-sections for pp → H^±h_1, pp → W^±h_1 and pp → H^±A_1 in the NMSSM

Searches for Higgs bosons at the LHC suffer from large QCD backgrounds. However, detailed studies have shown that multiple signals for the MSSM Higgs bosons are possible in a sizeable region of the plane [tan β, M_{H^±}] [53]. Much of these studies for the MSSM can be applied to the NMSSM with some caveats which were discussed in Section IV. The most problematic region for H^± discovery in the MSSM is for moderate values of tan β, since the production mechanisms which rely on a large bottom quark or top quark Yukawa coupling (e.g. gb → H^±t) are least effective. Hence alternative mechanisms which could offer good detection prospects for H^± at moderate values of tan β are desirable.

The cross sections for the pair production mechanisms pp → H^±A_1 and pp → H^±h_1 fall quickly with increasing scalar masses but for relatively light masses (∼ 200 GeV) they can provide promising signal rates which might enable their detection at the LHC. One common
feature is that the produced scalars enjoy large transverse momenta, which are crucial for the trigger and event selection. The cross section for $pp \rightarrow W^\pm h_1$ was first studied [9] at both the LHC and Tevatron in the CP conserving MSSM for $M_{A_1} > 100$ GeV. The analogous process $pp \rightarrow H^\pm h_1$ for a very light $h_1$ with unsuppressed coupling $h_1 H^\pm W^\mp$ was studied in the 2HDM and the CP violating MSSM in [11] at the Tevatron. In [10] it was shown that $pp \rightarrow H^\pm h_0, H^\pm A^0$ can be important in specific regions of parameter space (i.e., very light $h_0, A^0$) in the CP conserving MSSM.

In the NMSSM, if the coupling $H^\pm W^\mp A_1$ is sizeable, so will be the cross section for $pp \rightarrow W^\pm \rightarrow H^\pm A_1$ provided that $H^\pm$ and $A_1$ are not too heavy. The production mechanism $pp \rightarrow H^\pm A_1$ followed by the decay $H^\pm \rightarrow W^\pm A_1$ would give rise to a signal $W^\pm A_1 A_1 \rightarrow W bbbb$ [11] or $W^\pm A_1 A_1 \rightarrow W\tau\tau\tau\tau$. The signature $W^\pm A_1 A_1 \rightarrow W bbbb$ was simulated at the LHC in [12] in the context of the CP violating MSSM with the conclusion that a sizeable signal essentially free of background could be obtained. We use NMSSM-TOOLS1.1.1 to calculate the mass spectrum and couplings of the NMSSM Higgs bosons, and we link CTQ6.1M PDF distribution to this code in order to calculate the cross sections of $pp \rightarrow H^\pm A_1$, $pp \rightarrow H^\pm h_1$ and $pp \rightarrow W^\pm h_1$. All cross sections are evaluated at a scale which is the sum of the masses in final states and do not include next-to-leading order QCD enhancement factors (K factors) of around 1.2 → 1.3 [9,54].

For our numerical analysis, we have done a systematic scan with NMSSM-TOOLS1.1.1 [8]. Firstly, we explore the phenomenological implication of the sum rule Eq. (8) with Fig. (3b). There are several comments in order:
1) All points in Fig. (3b) respect the following constraint $M_{H^\pm} \gtrsim M_W$, this leads to a
smaller cross section for $\sigma(pp \rightarrow H^\pm h_1)$.

2) As $S_{13}^2$ increases both processes are suppressed due to the decrease of the couplings $W^\pm W^\pm h_1$ and $W^\pm H^\pm h_1$.

3) When $S_{13}^2 > 0.8$, $h_1$ is dominated by singlet component, therefore it can be very light see Fig. (3h). In this cases, according to sum rule Eq. $S_8$, the vertex $VVh_1$ suffers a severe suppression. However, some points with large $\sigma(pp \rightarrow W^\pm h_1)$ arise due to the fact that a very light $h_1$ is allowed.

In Fig. (3h) we study the cross section of $pp \rightarrow H^\pm h_1$ at the LHC and select points which simultaneously satisfy the following conditions:

$$\sigma(pp \rightarrow H^\pm h_1) > 0.1 \text{ pb} \quad \text{and} \quad \text{Br}(H^\pm \rightarrow W^\pm A_1) > 0.5 \ . \quad (15)$$

We require points in parameter space with cross sections larger than 0.1 pb as a conservative threshold of observability for this channel at the LHC. From the figure it is clear that $\sigma(pp \rightarrow H^\pm h_1) < 0.5 \text{ pb}$ when the charged Higgs boson decays dominantly to $W^\pm A_1$.

In Fig. (4b), we study the cross section of $pp \rightarrow W^\pm h_1$ at LHC and select points which satisfy the following conditions:

$$\sigma(pp \rightarrow W^\pm h_1) > 0.1 \text{ pb} \quad \text{and} \quad \text{Br}(h_1 \rightarrow A_1 A_1) > 0.5 \ . \quad (16)$$

The typical cross section for $\sigma(pp \rightarrow W^\pm h_1)$ is around a few pb, which is considerably larger than $\sigma(pp \rightarrow H^\pm h_1)$. The larger cross sections correspond to the larger branching ratios for $h_1 \rightarrow A_1 A_1 (\sim 90\%)$. The numerical results in Fig. (4b) are in good agreement with analogous results presented in $^{32}$.

In Fig. (5) we analyze the components of $H_1$ and $A_1$. Points which satisfy the following condition are selected:

$$\text{Br}(h_1 \rightarrow A_1 A_1) \geq 0.5 \ . \quad (17)$$

As expected, when both $h_1$ and $A_1$ are dominantly composed of doublet fields the region of light Higgs bosons is ruled out from searches for $e^+e^- \rightarrow Zh_1 \rightarrow Z2A_1 \rightarrow Z4b$, and $m_{h_1}$ should be heavier than around 100 $\sim 110$ GeV. When $M_{A_1} \lesssim 2m_b$, $m_{h_1}$ can be lighter than 100 GeV due to the fact that the LEP2 sensitivity to the channel $e^+e^- \rightarrow Z4\tau$ was less robust than that for $e^+e^- \rightarrow Z4b$. Interestingly, when both $h_1$ and $A_1$ are mainly singlet and hence the vertex of $VVh_1$ is greatly suppressed, much lighter values for $m_{h_1} (\lesssim 80$ GeV) are still allowed, as shown by points with red stars in Fig. (5a) and blue crosses in Fig. (5b).

This process $pp \rightarrow H^\pm A_1 \rightarrow W^\pm A_1 A_1$ leads to the same signature as the process $pp \rightarrow Wh_1 \rightarrow WA_1 A_1 \rightarrow Wbbbb$. The latter has been simulated in $^{13}$ and also offers very good detection prospects. We will compare the magnitude of these two distinct mechanisms which lead to the same Wbb signature. In addition, the mechanism $pp \rightarrow H^\pm h_1$ followed by the decay $H^\pm \rightarrow W^\pm A_1$ would also lead to the same final state $W^\pm A_1 h_1 \rightarrow Wbbbb$. We will concentrate on the scenario where $h_1 \rightarrow A_1 A_1$ is large and thus $h_1 \rightarrow bb$ will be kinematically suppressed. We will discuss the magnitude of $pp \rightarrow H^\pm h_1 \rightarrow W^\pm A_1 h_1 \rightarrow Wbbbb$ later.

In Fig. (6a) we study the process $pp \rightarrow H^\pm A_1$ by choosing points which satisfy the following conditions:

$$\sigma(pp \rightarrow H^\pm A_1) > 0.1 \text{ pb} \quad \text{and} \quad \text{Br}(H^\pm \rightarrow W^\pm A_1) > 0.5 \ . \quad (18)$$

It is apparent that the magnitude of $\sigma(pp \rightarrow H^\pm A_1)$ can reach a few pb and thus is within the detection capability of the LHC. The analysis of $^{12}$ (for the CP violating MSSM)
For $\sigma(pp \rightarrow H^\pm h_1)$ we sum over $\sigma(pp \rightarrow H^+ h_1)$ and $\sigma(pp \rightarrow H^- h_1)$; for $\sigma(pp \rightarrow W^\pm h_1)$ we sum over $\sigma(pp \rightarrow W^+ h_1)$ and $\sigma(pp \rightarrow W^- h_1)$. We show the two decay modes of $A_1$: $A_1 \rightarrow b\bar{b}$, and $A_1 \rightarrow \tau\bar{\tau}$, which corresponds to two mass regions: $2m_b < M_{A_1} < m_{h_1}/2$, and $2m_{\tau} < M_{A_1} < 2m_b$, respectively.

FIG. 5: Parameter space satisfying $Br(H_1 \rightarrow A_1 A_1) \geq 0.5$ in the plane $[m_{h_1}, m_{A_1}]$. The components of both $h_1$ and $A_1$ are displayed.
suggested that $\sigma(pp \rightarrow H^\pm A_1) \gtrsim 0.1$ pb with a large $Br(H^\pm \rightarrow W^\pm A_1)$ would be sufficient for an observable $Wbbb$ signal at the LHC. Most strikingly, the cross section of the process $pp \rightarrow H^\pm A_1$ can be comparable to that of $pp \rightarrow W^\pm h_1$.

The majority of the points in Fig. (6a) correspond to the parameter space where $tan\beta$ is located in the range $0.2 \lesssim tan\beta \lesssim 20$. As seen in the previous section, when $tan\beta \gtrsim 20$ the decay channel $H^\pm \rightarrow \tau^\pm \nu_\tau$ (or $H^\pm \rightarrow tb$) will dominate over $H^\pm \rightarrow W^\pm A_1$. It is evident from Fig. (6a) that there are plenty of points with $\sigma(pp \rightarrow H^\pm A_1) \gtrsim 0.1$ pb and $Br(H^\pm \rightarrow W^\pm A_1) \gtrsim 90\%$.

In Fig. (6b) we show the dependence of $\sigma(pp \rightarrow H^\pm A_1)$ on $tan\beta$ and $cos\theta_A$. The figure clearly shows that when $A_1$ is mainly doublet the cross section $\sigma(pp \rightarrow H^\pm A_1)$ can reach a few pb. Importantly, the cross section can be sizeable in the whole region $1 < tan\beta < 30$, and thus this mechanism can be applied to the region $5 \lesssim tan\beta \lesssim 20$ for which $H^\pm$ discovery in the conventional production mechanisms (which utilize the $t$ and $b$ quark Yukawa couplings) are least effective. Thus $H^\pm$ production via $pp \rightarrow H^\pm A_1$ might offer the best prospects for the detection of a light NMSSM charged Higgs boson in the region of intermediate $tan\beta$.

It is clear from Fig. (6b) that there are no points at all with $0 \lesssim cos^2\theta_A \lesssim 0.4$, the reason being that such points do not satisfy the requirement $\sigma(pp \rightarrow H^\pm A_1) \gtrsim 0.1$ pb.

Fig. (7) shows the dependence of $\sigma(pp \rightarrow H^\pm h_1(A_1))$ on $m_{h_1}(m_{A_1}) + m_{H^\pm}$. Points in Fig. (7a) satisfy the conditions given in Eq. (15), while points in Fig. (7b) satisfy the conditions given in Eq. (18). Clearly the points with large cross section correspond to the region in the parameter space where both $H^\pm$ and $h_1(A_1)$ are light and the couplings $W^\pm H^\pm h_1$ and $W^\pm H^\pm A_1$ are near maximal. In Fig. (7b), it is evident that the cross section for $pp \rightarrow H^\pm A_1$
FIG. 7: Left panel: the cross section of \( pp \rightarrow H^+ h_1 \) against \( M_{h_1} + M_{H^\pm} \); all points satisfy the condition in Eq. (15). Right panel: the cross section of \( pp \rightarrow H^+ A_1 \) against \( M_{A_1} + M_{H^\pm} \); all points satisfy the condition in Eq. (18). We show the two decay modes of \( A_1 \): \( A_1 \rightarrow b\bar{b} \) and \( A_1 \rightarrow \tau\bar{\tau} \), which corresponds to two mass regions: \( 2m_b < M_{A_1} < m_{h_1}/2 \), and \( 2m_\tau < M_{A_1} < 2m_b \), respectively.

can reach a few pb when \( A_1 \) is as light as 10 GeV. In contrast, in Fig. (7a) one can see that that there are only points for \( m_{h_1} + m_{H^\pm} \gtrsim 170 \text{ GeV} \) which corresponds to \( m_{H^\pm} \gtrsim 80 \text{ GeV} \) and \( m_{h_1} \gtrsim 90 \text{ GeV} \). The lack of sample points with large cross section for \( pp \rightarrow H^+ h_1 \) is due to difficulties in finding points with relatively light \( h_1 \) and \( H^\pm \) (i.e., \( m_{h_1} + m_{H^\pm} \lesssim 170 \text{ GeV} \)) which can satisfy the experimental constraints.

As emphasized earlier, the processes \( pp \rightarrow H^\pm A_1 \) and \( pp \rightarrow V h_1 \) could lead to the same final state, \( Wb\bar{b}b\bar{b} \) or \( W\tau\bar{\tau}\tau\bar{\tau} \). Hence a numerical comparison of their cross sections is of particular interest and is shown in Fig. (8), where all points satisfy the following conditions:

\[
\sigma(pp \rightarrow H^\pm A_1) > 0.1 \text{ pb} \quad \text{and} \quad \sigma(pp \rightarrow W^\pm h_1) > 0.1 \text{ pb} .
\]  

(19)

Superimposed on Fig. (8a) and Fig. (8b) are the main decay modes of the charged Higgs boson and the decay neutral Higgs boson \( H_1 \) respectively. We further impose the following conditions:

\[
Br(H^\pm \rightarrow W^\pm A_1) > 0.5 \quad \text{and} \quad Br(h_1 \rightarrow A_1 A_1) > 0.5 ,
\]  

(20)

and the surviving points are displayed in Fig. (9a). Importantly, there are many points where the two cross sections are of comparable size. We note that for these points in Fig. (9a) the pseudoscalar \( A_1 \) can be both R-axion like or a mixture of the three allowed basic axions. If the magnitude of the cross sections of both \( pp \rightarrow H^\pm A_1 \) and \( pp \rightarrow V h_1 \) are similar then the interference of the two channels (i.e., the same \( Wb\bar{b}b\bar{b} \) signature arising from distinct production mechanisms) should be taken into account. We have neglected such effects in the present study.
FIG. 8: Left panel: comparison of $\sigma(pp \rightarrow H^\pm A_1)$ and $\sigma(pp \rightarrow W^\pm h_1)$ with two $H^\pm$ decay modes. Right panel: comparison of $\sigma(pp \rightarrow H^\pm A_1)$ and $\sigma(pp \rightarrow W^\pm h_1)$ with two $h_1$ decay modes. The dotted line corresponds to $\sigma(pp \rightarrow W^\pm h_1) = \sigma(pp \rightarrow H^\pm A_1)$.

We now discuss whether the $Wb\bar{b}bb$ signatures can be distinguished experimentally by comparing the strategies adopted in [12] (for $pp \rightarrow H^\pm A_0$) and [13] (for $pp \rightarrow W^\pm h_1$). In order to reconstruct the peak of the CP-even Higgs $h_1$, one can select events with a charged lepton and four tagged $b$ quark jets as shown in [13]. This enables both a clean Higgs signal with high significance and a measurement of $M_{h_1}$ given by the invariant mass of the four $b$ quark jets, $m_{4b}$. The process $pp \rightarrow H^\pm A_1$ might be an irreducible background but presumably could be significantly suppressed with the aforementioned cut on $m_{4b}$ e.g., $m_{h_1} - 15\text{GeV} < m_{4b} < m_{h_1} + 15\text{GeV}$.

Regarding detection of $pp \rightarrow H^\pm A^0$, it was demonstrated in [12] (for the analogous process $pp \rightarrow H^\pm H_1 \rightarrow WH_1H_1$ in the CP violating MSSM) that the mass of $H^\pm$ can be reconstructed. This is achieved by defining a tranverse mass ($M_T$) which is a function of the momenta of the two secondary $b$ jets (i.e., those originating from the decay $H^\pm \rightarrow A_1W \rightarrow Wbb$) and the momenta of the lepton and missing energy coming from the $W$ boson. It was shown that $M_T$ is sensitive to the underlying charged Higgs mass and thus can be used for the determination of $M_{H^\pm}$. The pair of $b$ jets from $pp \rightarrow W^\pm h_1$ might be an irreducible background but presumably could be suppressed with a cut on $M_T$.

To reconstruct the peak of the light CP-odd neutral Higgs $A_1$ one can require events with four tagged $b$ jets, construct the three possible double pairings of $b\bar{b}$ invariant masses, and then select the pairing giving the least difference between the two $b\bar{b}$ invariant masses values [12]. $W4b$ signatures from the process $pp \rightarrow W^\pm h_1$ also contribute constructively to the reconstruction of $A_1$. Thus we conclude that it is promising to reconstruct the peaks of the CP-even neutral Higgs ($h_1$), charged Higgs ($H^\pm$) and CP-odd neutral Higgs ($A_1$) and thus experimentally distinguish the $Wb\bar{b}bb$ signatures arising from the two distinct production mechanisms. We defer a detailed simulation to a future work.
$\sigma(p p \to H^\pm A_1)$ (PB) 

**FIG. 9:** Left panel: comparison of $\sigma(pp \to H^\pm A_1)$ and $\sigma(pp \to W^\pm h_1)$ with different $A_1$ decay modes. Points are selected with the condition given in Eqs. (19-20). Right panel: comparison of $\sigma(pp \to H^\pm A_1)$ and $\sigma(pp \to H^\pm h_1)$ with the same set of points. The dotted line corresponds to $\sigma(pp \to H^\pm A_1) = \sigma(pp \to H^\pm h_1)$; the dashed line corresponds to $\sigma(pp \to H^\pm A_1) = 3\sigma(pp \to H^\pm h_1)$; the solid line corresponds to $\sigma(pp \to H^\pm A_1) = 10\sigma(pp \to H^\pm h_1)$.

Finally, we also compare the cross sections of $pp \to H^\pm A_1$ and $pp \to H^\pm h_1$ in Fig. (9b). The points are from the same data sample used in Fig. (9a). It is clear that $\sigma(pp \to H^\pm A_1)$ is around one order of magnitude larger than $\sigma(pp \to H^\pm h_1)$, and the underlying reason is that $M_{h_1} > 2M_{A_1}$. Consequently, $pp \to H^\pm h_1 \to W^\pm A_1 h_1 \to Wbbb$ will also be suppressed and can be safely neglected. Another interesting feature from Fig. (9b) is that points satisfying the conditions listed in Eq. (20) lead to $A_1$ composed mainly of the doublet fields. The conditions in Eq. (20) together with the dominance of $h_1$ by doublet component (small $S_{13}$) can give large cross sections for both channels.

**VI. CONCLUSION**

In summary, we have studied the phenomenology of light charged Higgs bosons in the framework of NMSSM. We performed a comprehensive study of the magnitude of the branching ratios for the decays $H^\pm \to W^\pm A_1$ and $H^\pm \to W^\pm h_1$ (first considered in [6]). It was shown that such decays can dominate over the standard decays $H^\pm \to \tau^\pm \nu$ and $H^\pm \to tb$ both below and above the top-bottom threshold. This is due to the fact that $A_1$ can have a large doublet component and small mass. Large branching ratios for $H^\pm \to W^\pm A_1$ and $H^\pm \to W^\pm h_1$ would affect the anticipated search potential for $H^\pm$ at the LHC.

We also studied the production process $pp \to H^\pm A_1$ and showed that sizeable cross sections (> 1 pb) are possible. We compared the magnitude of the cross sections for both $pp \to H^\pm A_1$ and the Higgsstrahlung process $pp \to W^\pm h_1$ and showed that they can be
of similar size. If $H^\pm$ and $h_1$ decay via $H^\pm \rightarrow W^\pm A_1$ and $h_1 \rightarrow A_1 A_1$ respectively, the above two processes would lead to the same final state, $Wb\bar{b}b$ or $W\tau\tau\tau\tau$. We stressed that the interference term for $Wb\bar{b}b$ and $W\tau\tau\tau\tau$ might not be negligible and should be taken into account in any simulation study. In particular, the signature $Wb\bar{b}b$ affords promising detection prospects at the LHC and we discussed how to distinguish the distinct contributions from $pp \rightarrow H^\pm A_1$ and $pp \rightarrow W^\pm h_1$ by using appropriate cuts.

It is known that intermediate values of $\tan \beta$ (e.g., $5 < \tan \beta < 20$) are most problematic for discovery of $H^\pm$ at the LHC since the $H^\pm tb$ Yukawa coupling (which is employed in the conventional production processes) takes its lowest values. In such a region the process $pp \rightarrow H^\pm A_1$ can have a sizeable cross section if $m_{H^\pm} + m_{A_1} < 200$ GeV. Therefore we propose $pp \rightarrow H^\pm A_1$ as a unique mechanism to probe the parameter space of intermediate $\tan \beta$ and light charged Higgs boson in the NMSSM.

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