Matter Parity Violating Dark Matter Decay in Minimal SO(10), Unification, Vacuum Stability and Verifiable Proton Decay

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Abstract
In direct breaking of non-supersymmetric SO(10) to the standard model, we investigate the possibility that dark matter (DM) decaying through its mixing with right-handed neutrino (RHν) produces high energy IceCube neutrinos having type-I seesaw masses. Instead of one universal mixing and one common heavy RHν mass proposed in a recent standard model extension, we find that underlying quark-lepton symmetry resulting in naturally hierarchical RHν masses predict a separate mixing with each of them. We determine these mixings from the seesaw prediction of the DM decay rates into the light neutrino flavors. We further show that these mixings originate from Planck-scale assisted spontaneously broken matter parity needed to resolve the associated cosmological domain wall problem. This leads to the prediction of a new LHC accessible matter-parity odd Higgs scalar which also completes vacuum stability in the Higgs potential for its mass $M_{\chi S} \simeq 177$ GeV. We have also discussed realization of relic density of decaying dark matter in relation to flux of IceCube neutrinos. Two separate minimal SO(10) models are further noted to predict such dark matter dynamics where a single scalar submultiplet from $126_H^I$ or $210_H$ of intermediate mass achieves precision gauge coupling unification. Despite the presence of two large Higgs representations and the fermionic dark matter host, $45_F$, experimentally accessible proton lifetimes are also predicted with reduced uncertainties.

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1 Introduction
Prominent drawbacks of the standard model (SM) are absence of neutrino mass [1–6], dark matter (DM) [7–30], baryon asymmetry of the universe (BAU) [31–34], gauge coupling unification [35–38] and vacuum stability of the Higgs potential [39, 40]. It is well known that

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1For recent reviews and prospects of dark matter in GUTs see Ref. [23–26, 28–30]. For reviews on neutrino masses see [9]. For lack of unification in non-supersymmetric (non-SUSY) grand desert models see Ref. [35–38].
supersymmetric (SUSY) grand unified theories (GUTs) \cite{41} can resolve most of the issues confronting SM but, in the absence of any experimental evidence of SUSY, non-SUSY SO(10) \cite{42} without any flavor symmetry is known to fulfill most of the SM limitations except for a natural resolution of the gauge hierarchy problem which may be reconciled by resorting to fine-tuning to every loop order \cite{12,14}. An understanding of different fermion families and flavor problem is expected to emerge through a recent realization of comprehensive unification in SO(18) \cite{45} which is beyond the scope of this work.

Remnants of gauged discrete symmetries \cite{16,48} such as R-parity in SUSY and matter parity in non-SUSY theories have played crucial roles in determining stability and phenomenology of dark matter \cite{23,26,28,30}. In general if a higher rank gauge theory containing $U(1)_{B-L}$ as one of its subgroups leads to SM gauge theory $SU(2) \times U(1)_{Y} \times SU(3)_{C} \equiv G_{213}$, then matter parity (MP) of the SM is conserved as a gauged discrete symmetry

$$Z_{MP} = (-1)^{3(B-L)} ,$$

if the Higgs scalar driving the symmetry breaking has $B - L = \text{even}$. Here $B(L)$ stands for baryon (lepton) number \cite{23,26,28,30}. Quite interestingly, the underlying mechanisms of type-I and type-II seesaw generation of neutrino masses at tree level in SO(10) necessarily predict matter parity conservation of the residual gauge theory. This is realized \cite{49,50} through the popular rank-5 Higgs representation $126_{H}^{\dagger} \subset SO(10)$ which has the following decompositions under respective sub-groups \cite{51}

$$126_{H}^{\dagger} \supset \Delta_{R}(1, 3, 10) + \Delta_{L}(3, 1, 10) + \xi(2, 2, 15), \quad (G_{224}),$$

$$\supset \Delta_{R}(1, 3, -2, 1) + \Delta_{L}(3, 1, -2, 1) + \Delta_{R}^{(1)}(1, 3, 2/3, 6) + \Delta_{R}^{(2)}(1, 3, -2/3, 3) + ... (G_{2213}),$$

$$\supset \Delta_{R}^{0}(1, 0, 1) + \Delta_{R}^{(1)}(1, -1, 1) + \Delta_{R}^{(2)}(1, -2, 1) + \Delta_{L}(3, -1, 1) + ... (G_{213}),$$

where $G_{2213}$ denotes left-right gauge theory $SU(2) \times SU(2)_{R} \times U(1)_{B-L} \times SU(3)_{C}$ and the second line in eq. (2) results using $Y/2 = T_{3R} + (B-L)/2$ \cite{12}. All the components carry $|3(B-L)| = \text{even number reflecting the fact that } 126_{H} \text{ is an even matter parity representation.}$

Vacuum expectation value (VEV) $V_{R} = (\Delta_{R}^{0}(1, 0, 1))$ carrying $B - L = -2$ generates heavy right-handed neutrino (RHν) Majorana mass term that drives type-I seesaw. Similarly the left-handed (LH) triplet scalar $\Delta_{L}(3, -1, 1)$ that also carries $B - L = -2$ mediates type-II seesaw. Then direct breaking of $SO(10) \rightarrow SM$ or through intermediate gauge symmetries like $G_{2213}$ or Pati-Salam symmetry $SU(2) \times SU(2)_{R} \times SU(4)_{C} (\equiv G_{224})$, SM gauge theory $G_{213}$ conserves matter parity as long as $126_{H}^{\dagger}$ is used to break $SU(2)_{R} \times U(1)_{(B-L)}$, or $U(1)_{R} \times U(1)_{(B-L)}$, or simply $U(1)_{X}$ in $SU(5) \times U(1)_{X}(X = 4T_{3R} + 3(B - L))$. We discuss $SO(10)$ breaking further in Sec. 4.

Application of matter parity conserving non-SUSY SO(10) has been extensively exploited \cite{23,26,28,30} to predict fermions or scalars as WIMP DM candidates \cite{53,54}. Gravity induced small violation of R-parity as the corresponding DM stabilising gauged discrete symmetry in SUSY theories has been extensively investigated to predict new interesting physical phenomena \cite{29}.

In this work we apply the idea of intrinsic matter parity and its gravity assisted spontaneous breaking to predict the dynamics of decaying dark matter in the minimal chain of non-SUSY SO(10) $\rightarrow SM$ where neutrino mass is given by the popular type-I seesaw mechanism \cite{55}.
Recent data from IceCube has led to the suggestion that the observed PeV energy neutrinos could be the decay product of massive dark matter which may be fermions or scalars. In one such interesting minimal standard model extension with Majorana fermion singlets proposed quite recently by Rott, Kohri, and Park (RKP), the high scale canonical seesaw mechanism explains the neutrino masses while the massive dark matter $\Sigma_F$ through its extremely small mixing with heavy degenerate right-handed neutrino (RH$\nu$) $(N)$ decays to produce the Higgs boson $(h)$ and the high energy neutrino $(\nu)$. The extreme smallness of $m^{(\text{mix})}$ needed to fit the long dark matter lifetime $(\tau_\Sigma > 10^{28} \text{ s})$ calls for a deeper theoretical explanation which is not possible within the simple SM extension. As we observe, because of underlying quark-lepton symmetry, the benchmark model proposal of type-I seesaw and single universal RH$\nu$-DM mixing with one degenerate heavy RH$\nu$ mass can not hold in the SO(10) GUT framework without imposing some hitherto unknown flavor symmetry. Also it is well known that one or more intermediate gauge symmetry breakings in SO(10) gives rise to large number of paths and models leading to standard gauge theory. As such, without using any flavor symmetry or intermediate gauge symmetry, in this work we investigate how far the underlying idea of RH$\nu$-DM mixing as origin of PeV energy IceCube neutrinos can be realised in the popular non-SUSY SO(10) GUT framework including an understanding of the dynamical origin of mixing, DM mass, vacuum stability of the scalar potential, precision coupling unification and proton lifetime prediction which are not within the purview of the benchmark model. When the SM is extended with heavy right-handed neutrino(s) to implement Type-I seesaw mechanism, the arbitrary nature of Dirac neutrino Yukawa couplings may lead to various hierarchical or degenerate form of RH$\nu$ mass spectra. Although no flavor symmetry has been explicitly mentioned, one common RH$\nu$ mass $M_{N_i} = M_N(i = 1, 2, 3) = 10^{14} \text{ GeV}$ for all three flavors and identically equal mixing, $m^{(\text{mix})} = 10^{-5} \text{ eV}$, has been used in the SM extension under the natural constraint that the DM decays with equal branching ratios to each of three light neutrino flavors. The equal branching ratio hypothesis for each neutrino flavor in the benchmark model further needs specific constraints on the equality of relevant Dirac neutrino Yukawa matrix elements for different flavors $Y^{\nu}_{\alpha i}(\alpha = e, \mu, \tau, (i = 1, 2, 3))$. Such constrained structures of RH$\nu$ and Dirac neutrino mass matrices may be possible in the SM extension under some imposed external flavor symmetry unspecified in the model. But in the non-SUSY SO(10) framework where no flavor symmetry is usually used, the type-I seesaw based degenerate RH$\nu$ and the single universal mixing hypothesis with decaying dark matter are not realisable. On the other hand type-I seesaw mechanism in SO(10) predicts the heavy RH$\nu$ masses to be predominantly hierarchical. The basic underlying reason for such hierarchical heavy RH$\nu$ masses, as against the degenerate assumption of the RKP model, is the quark-lepton symmetry in SO(10) that predicts Dirac neutrino mass matrix similar to the up-quark mass matrix. Even with such sharp contrast with the benchmark model, in this work we show how the complete dynamics of a massive decaying dark matter that explains PeV energy IceCube neutrinos is predicted by intrinsic matter parity conserving SO(10) with naturally dominant Type-I seesaw mechanism for neutrino masses originating from radically different hierarchical ansatz for Dirac neutrino masses and RH$\nu$ mass spectra. In particular, we show how the equality of branching ratios of DM $(= \Sigma_F)$ decay to three different neutrino flavors determines a separate distinct mixing of decaying dark matter (DDM) with each heavy RH$\nu$ flavor. On the other hand utilization of one universal mixing with naturally predicted Dirac neutrino mass and hierarchical RH$\nu$ masses tends to drastically reduce the DM lifetime because of more rapid decay in the other two channels: $\Sigma_F \rightarrow \nu_e h$, and $\Sigma_F \rightarrow \nu_\mu h$. 
We further show that the derived values of these three mixings can be predicted for different light neutrino mass patterns such as normally hierarchical (NH), invertedly hierarchical (IH), and quasi-degenerate (QD), consistent with oscillation data such that the $N_i - \Sigma_F$ mixings can be uniquely fixed once the light neutrino mass hierarchy is known.

Over the years implementation of gravitational effect has been found to be a very efficient mechanism in breaking all kinds of global symmetries, continuous or discrete \cite{46,48,60,64}, without causing cosmological domain wall problem \cite{47,48,65} while at the same time predicting new interesting physical phenomena such as Majoron as scalar dark matter \cite{60,61,63}. Whereas no dynamical explanation has been provided for the origin of N-$\Sigma_F$ mixing \cite{58}, in this work we further show how the three small mixings in our case originate from gravity assisted matter parity discrete symmetry breaking \cite{60,61} without causing cosmological domain wall problem \cite{47,48,65}.

We exploit Planck scale or gravity assisted spontaneous breaking of intrinsic matter parity in a manner analogous to R-Parity (RP) breaking \cite{60} in MSSM. This would require a matter parity odd singlet scalar which indeed originates from the smallest spinorial scalar representation $16_H \subset SO(10)$. Analogous to eq.(2), the contents of $16_H$ under respective gauge groups are \cite{51}

$$16_H \supset \chi_R(1,2,4) + \chi_L(2,1,4) \quad (G_{224}),$$

$$\supset \chi_R(1,2,-1,1) + \chi_R^c(1,2,-1/3,3) + \chi_L(2,1,-1,1) + \chi_L^c(2,1,-1/3,3) \quad (G_{2213}),$$

$$\supset \chi_R^0(1,0,1) = \chi_S + \chi_R^L(1,1,1) + \chi_R^{(1)}(1,1/3,3) + \chi_R^{(2)}(1,-2/3,3)$$

$$+ \chi_L(2,-1/2,1) + \chi_L(1/6,3) \quad (G_{213}).$$

Thus all the components of $16_H$ or $16_H^1$ carry $|3(B-L)| = odd$ signifying odd matter parity of these representations. Further, each of the two has only one SM singlet: $\chi_S(1,0,1) \subset 16_H$ and $\Delta_R^0 \subset 126_H$. But unlike the even matter parity of the singlet $\Delta_R(1,0,1)$, the singlet $\chi_S(1,0,1)$ has odd matter parity \footnote{The analogous fermionic singlet is the RH neutrino $N(1,0,1) \subset 16_F$ which has odd matter parity like all standard fermions.} that can acquire a VEV ($V_\chi = \langle \chi_S(1,0,1) \rangle$) without breaking SM gauge symmetry but breaking matter parity spontaneously.

As noted above matter parity of the SM gauge theory originating as intrinsic gauged discrete symmetry of non-SUSY SO(10) has played a crucial role in safeguarding stability of WIMP dark matter without requiring any adhoc discrete symmetry to be imposed by hand. In this work while explaining the dynamical origin of DDM-RH$\nu$ mixings through the matter-parity violating VEV of $\chi_S$, we preserve this ability of the SM gauge theory keeping its possibility open for WIMP DM embedding in the models (Model-I and Model-II) proposed in this work. This quality of the SM gauge theory as remnant of non-SUSY SO(10) is protected by taking care to see that matter parity as gauged discrete symmetry is left unbroken as long as the SM gauge symmetry survives till the electroweak scale. This imposes the natural upper bound on the matter parity VEV $V_\chi = \langle \chi_S \rangle \leq v_{ew} = 246$ GeV. As the VEV of this singlet Higgs is noted to be naturally bounded from above by the electroweak VEV, $v_{ew} = 246$ GeV, due to the survival of matter parity in the SM down to the electroweak scale, this SO(10) theory leads to the prediction of a new light Higgs scalar singlet with perturbative upper bound on its mass $M_{\chi_S} \leq 860$ GeV. In the process, this SO(10) theory of DDM provides its experimental testability or most desirable quality of falsifiability at LHC.

$$M_{\chi_S} \leq 860 \text{ GeV}.$$
Having only the standard Higgs doublet that defines the Higgs potential, the RKP model \cite{58} is also affected by the vacuum instability problem that needs new physics below $\sim 2 \times 10^9$ GeV \cite{39,10}. Interestingly, this new Higgs scalar singlet $\chi_S(1, 0, 1)$ that drives the Planck scale assisted matter parity breaking completes the vacuum stability in the present SO(10) models (Model-I and Model-II). The resolution of vacuum stability then predicts the $\chi_S(1, 0, 1)$ mass to be $M_{\chi_S} = 177$ GeV which is clearly detectable by accelerator searches as an evidence of this model building for decaying dark matter.

After the standard model Higgs discovery at LHC, spontaneous breaking origin of all masses in the Universe has turned out to be a very attractive universal hypothesis. While the standard fermions and gauge bosons get their masses via respective gauge and Yukawa interactions with the standard Higgs doublet, the Higgs origins of nonstandard heavy fermions including RH$\nu$, N, and dark matter, $\Sigma_F$, are not possible within the simple standard model extension alone \cite{58} unless a number of scalar singlets are added. Also like the SM, the RKP model fails to unify gauge couplings. These shortcomings are fulfilled through the present SO(10) model by minimal modification of the grand desert.

On the realization of precision gauge coupling unification, we note interesting roles of the Higgs representations $210_H$ and $126_H$, which have been used to break the GUT symmetry and generate neutrino masses. We point out that each of these two are capable of completing the desired gauge coupling unification of the SM separately by minimally populating the grand desert through only one of their respective scalar submultiplets leading to Model-I and Model-II. A specifically new finding of this work is the possibility of new minimally modified grand desert in Model-II where the popular Higgs representation $126_H \supset \eta(3, -1/3, 6)$ completes precision coupling unification with its mass $M_\eta = 10^{10.7}$ GeV. The other minimally modified grand desert model that accommodates this decaying DM phenomenology is Model-I where the intermediate mass of $\kappa(3, 0, 8) \subset 210_H$ achieves precision coupling unification for $M_\kappa = 10^{9.2}$ GeV. Although similar coupling unification in Model-I was noted earlier, its connection with decaying DM and generation of $\Sigma_F$ mass and $N_i - \Sigma_F$ mixings are new. Furthermore, the vacuum stability of the SM Higgs potential which was absent in the original suggestion is now complete in the presence of new scalar singlet both in Model-I and Model-II.

When GUT threshold effects are ignored, all single step breaking grand desert models $[E_6, SO(10), SU(5)] \rightarrow$ SM are affected by the non-unification of gauge couplings in the same fashion. This problem has been resolved by the introduction of a number of non-standard Higgs or fermion submultiplets at different mass scales in non-SUSY SU(5) \cite{66,70} with low-scale seesaw mechanisms in some cases. In these models \cite{67,70} no stable dark matter candidates can exist unless an external symmetry from outside the GUT framework is added by hand. Similarly the phenomenological suggestion of scalar or fermionic DM candidates as SM extension \cite{71} requires the imposition of an adhoc stabilising discrete symmetry externally. A very profound and attractive implementation in the popular non-SUSY SO(10) \cite{25} with minimal modification of the grand desert by a TeV scale fermionic triplet dark matter $F_T(3, 0, 1)$ and an octet fermion $O_F(1, 0, 8)$ at higher mass has led to a self sufficient SO(10) model of precision coupling unification where the traditionally used external discrete symmetries has been replaced by intrinsic matter parity underlying the GUT symmetry breaking mechanism through $126_H$ that naturally generates heavy Majorana neutrinos driving canonical seesaw. The model not only predicts verifiable proton lifetime by ongoing experiments but also it predicts the WIMP fermionic triplet DM mass $M_T \sim 2.75 \pm 0.15$ TeV which could be detected at LHC with upgraded luminosity and at future $e^+e^-$ colliders with energy $> 2M_T$. Recently, out of many WIMP DM candidates suggested, the fermion triplet $F_T(3, 0, 1)$ has
attracted much attention with well documented phenomenology for direct and indirect detection prospects [72]. In the context of matter parity conserving $SO(10)$ and dominant radiative seesaw [73] at TeV scale, precision unification with verifiable proton lifetime has been realized in the presence of wino like and Higgsino like DM and gluino like fermion near the TeV scale [26].

On the question of economic choice of particle degrees of freedom in populating the grand desert for achieving precision coupling unification in matter parity conserving $SO(10)$ with canonical seesaw, each of the two decaying dark matter (DDM) models discussed here has only one Higgs scalar submultiplet $\eta(3, -1/3, 6) \subset 126_H$ (Model-II) or $\kappa(3, 0, 8) \subset 210_H$ (Model-I). Specific roles of these two representations $210_H, 126_H$ in determining minimal SUSY $SO(10)$ model with 26 parameters in the Lagrangian was pointed out earlier [74]. Although we are dealing with non-SUSY $SO(10)$, the pivotal roles played by each member of the pair is a new property of these two as noted here. Despite the presence of these two large Higgs representations and the nonstandard fermionic representation $45_F$ hosting the dark matter, we further predict experimentally accessible proton lifetime predictions in both models with substantially reduced threshold uncertainties compared to many earlier investigations.

In one out of several mixing solutions discussed here, we have emphasized that the corresponding $SO(10)$ model is a self sufficient dynamical theory of decaying dark matter and precision gauge coupling unification as it does not need any externally imposed discrete symmetry for DM stability, or additional Planck-mass fermion singlets for the dynamical explanation of mixings, or additional representation beyond those used for GUT symmetry breaking to complete coupling unification and generate heavy DM mass. We have thus concluded that if DM decays only through its mixings with heavy RH $\nu$s mediating type-I seesaw, then matter parity violation has been observed at IceCube.

Highlights of new contributions of this work are

- First realization of decaying fermionic dark matter dynamics in non-SUSY $SO(10)$ with type-I seesaw and naturally hierarchical right-handed neutrinos for all types of light neutrino mass hierarchies.
- First determination of RH $\nu$-DM mixings from Type-I seesaw, neutrino oscillation data and IceCube neutrino data.
- Explanation of dynamical origin of mixings through Planck-scale assisted spontaneous breaking of intrinsic matter parity and derivation of couplings in the associated renormalizable and non-renormalizable Lagrangians.
- Prediction of experimentally verifiable new Higgs scalar origin of dark matter decay.
- Resolution of the vacuum instability problem of the SM scalar potential through the new light Higgs scalar underlying dark matter dynamics.
- Identification of a completely new minimal grand desert modification model (Model-II) for precision gauge coupling unification through the lone intermediate mass scalar submultiplet $\eta(3, -1/3, 6)$ rooted in the popular $SO(10)$ representation $126_H^\dagger$.
- Implementation of Dark matter dynamics and resolution of vacuum stability problem in the minimal model (Model-I) achieving precision coupling unification through the intermediate mass scalar submultiplet $\kappa(3, 0, 8) \subset 210_H$. 

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• Realization of desired relic density of decaying dark matter through a heavy scalar exchange between the DM and the SM Higgs scalar suitable for generating the expected flux of PeV energy IceCube neutrinos.

• Experimentally verifiable precision proton lifetime prediction despite the larger Higgs representations $126_H, 210_H$ and the nonstandard fermions in $45_F$.

This paper is organised in the following manner. In Sec.2 we discuss benchmark model briefly. In Sec.3 we embed decaying dark matter (DDM) in SO(10) with specific discussions on matter parity as intrinsic gauged discrete symmetry and derivation of DM mass. We derive RHν masses and the RHν-ΣF mixings numerically in Sec.4 using type-I seesaw contribution to dark matter decay rate. Dynamical generation of $N_i - Σ_F$ mixing is discussed in Sec.5 using Planck-scale assisted spontaneous breaking of matter parity. Derivation of renormalizable and non-renormalizable Yukawa couplings of the new matter parity odd light Higgs scalar $χ_S$ is derived in Sec.6. Gauge coupling unification in Model-I and Model-II is discussed in Sec.7 while proton lifetime prediction is presented in Sec.8. Prediction of new light Higgs scalar and its impact on the resolution of the vacuum stability problem is discussed in Sec.9.

In this section we also discuss advantages of low VEV of the matter parity violating Higgs scalar singlet along with a derivation from potential minimisation. We discuss how proper relic density of the DDM is realized to generate the expected flux of PeV energy IceCube neutrinos in Sec.10. We summarize and conclude in Sec.11. In the Appendix A, Sec. 12 we present diagonalisation of RHν mass matrices in the presence of mixings with decaying dark matter motivated models. In Appendix B, Sec.13 we present evolution of various gauge couplings, top-quark Yukawa coupling and GUT threshold effects on unification mass in Model-I and Model-II.

2 The Bench Mark Model

For simpler visualisation of the decay process, we present the Feynman diagram of this model in Fig. 1 where the DM $Σ_F$ is shown to decay to $ν_L h$. In ref. [58] all the three RH neutrino masses have been assumed to be identical $M_{N_i} = M_N (i = 1, 2, 3)$. Likewise, for the demonstration of the RKP model only, the mixings of all three RH neutrinos with DM $χ (≡ Σ_F$ in our notation) have been assumed to be identical $\sin β_i = \sin β = \sigma / (M_N - M_{χ}) (i = 1, 2, 3)$.

Figure 1: Feynman diagram for dark matter $Σ_F$ decaying to $ν_L$ and standard Higgs $h$ through the DM mixing with heavy RH neutrinos $N_i$ of three generations. In the bench mark model $N_i ≡ N$ under quasidegeneracy assumption and $β_i = β$. 
The DM decay predicts neutrino energy nearly equal to half the DM mass $E_\nu \sim M_{DM}/2$. Noting that the high energy neutrino flux is proportional to density $\rho$ (square of density $\rho^2$) for decaying (self annihilating) DM, RKP estimate more than 50% of events to be within $65^\circ$ ($25^\circ$) from the galactic centre for decaying (self annihilating) dark matter 58. The existing perturbative unitarity bound 54 already excludes annihilating WIMP DM masses larger than 100 TeV. Then the observed isotropy of IceCube neutrinos and their large energy lead to the suggestion that they originate from decaying DM of mass $> 100$ TeV. The other attractive part of the benchmark model is based upon right-handed neutrino (N) extended SM for canonical seesaw which is further extended by the addition of heavy Majorana singlet DM $\psi$ of assumed mass $M_\psi$. In addition a small mixing mass term $\sigma$ between N and $\psi$ has been assumed. Also all Yukawa interactions of the DM has been neglected.

Suppressing flavor indices of $N$, the suggested model Lagrangian is

$$-L^{RKP}_{Yuk} = \lambda \nu_L (h + v_{ew}) N + (N^c \bar{\psi}^c) \left( \begin{array}{c} M_N \\ \sigma \\ M_\psi \end{array} \right) \left( \begin{array}{c} N \\ \psi \end{array} \right).$$

(4)

With the assumed constraint on the masses

$$M_N > M_N - M_\psi > M_\psi \gg \sigma,$$

(5)
diagonalization of the $4 \times 4$ mass matrix in the second term in the RHS of eq.(4) results in the heavy mass eigenvalues,

$$M^\pm = \frac{1}{2} \left[ M_N + M_\psi \pm \sqrt{(M_N - M_\psi)^2 + 4\sigma^2} \right].$$

(6)

The emerging canonical seesaw formula from SM extension has been assumed in its simple form

$$m_\nu = -\lambda^2 \frac{v_{ew}^2}{M^+},$$

(7)

where $M_N$ represents bare RH neutrino mass matrix and $\lambda v_{ew}$ is the assumed Dirac neutrino mass matrix $M_D$. From eq.(6) it is clear that the largest mass eigen value $M^+ \simeq M_N$ since $\sigma$ is extremely small. For the same reason $M^- \simeq M_\psi$ which is the decaying DM mass observed at IceCube.

The mass eigen states are

$$\zeta_+ = \cos \beta N + \sin \beta \psi,$$

$$\zeta_- = -\sin \beta N + \cos \beta \psi,$$

(8)

where the mixing angle is expressed as

$$\tan 2\beta \sim 2\sigma/M_N \ll 1.$$

Now rewriting the interaction Lagrangian

$$-\mathcal{L}_{int} = \lambda h \bar{\nu}_L N$$

$$= \lambda h \bar{\nu}_L \left[ \zeta_+ + \frac{\sigma}{(M_N - M_\psi)} \zeta_- \right],$$

(9)

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leads to the suppressed effective coupling constant and the suppressed decay width

\[ \lambda_{\text{eff}} \equiv \lambda_{\text{eff}}^{\alpha} (\alpha = e, \mu, \tau), \]

\[ \lambda_{\text{eff}}^{\alpha} = \sum_i \lambda_{\alpha i}^{\sigma} \left( M_N - M_{\psi} \right), \]

\[ \Gamma_{\chi} = \Gamma^{\alpha} \simeq \frac{\lambda_{\text{eff}}^{2\alpha}}{32\pi} M^{-}. \] (10)

Accounting for the observed PeV energy neutrino excess at IceCube requires the DM lifetime

\[ \tau_{\text{DM}} = 1.9 N_\nu \times 10^{28} \text{ s}, \] (11)

where \( N_\nu = \text{number of produced neutrinos of the same type}. \) Then using \( M^{-} = 10^6 \text{ GeV}, \) eq.(10), and eq.(11) gives a very small value of the mixing parameter

\[ \lambda_{\text{eff}} \simeq 5.3 \times 10^{-29}, \]

\[ \sigma = 10^{-5} \text{ eV}. \] (12)

In the benchmark model quasidegenerate light neutrino mass \( m_\nu = 0.1 \text{ eV} \) is treated to be the result of type-I seesaw mediated by heavy RH neutrino mass \( M_N = 10^{14} \text{ GeV} \) which are also quasidegenerate. For the DM decay with assumed Dirac neutrino Yukawa coupling \( \lambda \sim 1 \) for all generations, the three different branching ratios have been also assumed to be equal,

\[ \text{Br.}(\Sigma_F \to \nu_e h) : \text{Br.}(\Sigma_F \to \nu_\mu h) : \text{Br.}(\Sigma_F \to \nu_\tau h) :: 1 : 1 : 1. \] (13)

3 Symmetry Breaking, Matter Parity and Dark Matter Mass in SO(10)

In the absence of any experimental evidence of non-standard gauge bosons below the GUT scale, here we confine to the direct minimal symmetry breaking of SO(10) to SM gauge symmetry \((G_{213})\) which is assumed to operate over the entire range of mass scale \( \mu \simeq M_W - M_{\text{GUT}} \)

\[ SO(10) \rightarrow G_{213} \rightarrow U(1)_{\text{em}} \times SU(3)_C. \] (14)

Such a direct SO(10) breaking model is prominently well established in SUSY GUTs \([74, 75]\) and also has been applied in non-SUSY SO(10) \([26, 30, 50, 76]\). In the context of protecting non-SUSY grand desert models by GUT threshold effects, particularly those due to superheavy gauge bosons, such a direct breaking model of SO(10) has been also used earlier \([77, 78]\). Decompositions of SO(10) representations 10, 16, 45, 126, and 210 under different subgroups are presented in eq.(2), eq.(3), eq.(17) and also in Appendix B. For the first step of symmetry breaking in eq.(14) we use the SO(10) Higgs representations 210\(H\) and 126\(H\) with their respective SM singlet VEVs around the same high scale. This symmetry breaking may be visualised in the following manner. The VEV of the \( G_{2213} \) singlet component contained in the \( G_{224} \) submultiplet \((1,1,15) \subset 210_H\) breaks \( SO(10) \rightarrow G_{2213} \) at \( \mu \sim M_{\text{GUT}} \). At the same scale the SM singlet component contained in the \( G_{2213} \) submultiplet \((1,3,2) \subset 126_R\) belonging to the \( G_{224} \) submultiplet \( \Delta_R(1,3,10) \subset 126_H^* \) acquires VEV \( \sim M_R \sim M_{\text{GUT}} \) to break \( SU(2)_R \times U(1)_{(B-L)} \rightarrow U(1)_Y \). The direct breaking scenario of eq.(14) is thus realised when
both these breaking processes, $SO(10) \rightarrow G_{2213}$ and $G_{2213} \rightarrow SM$, occur at the same scale $\sim M_{GUT}$ resulting in only twelve light gauge bosons of SM while making all other thirty three gauge bosons of SO(10) superheavy with masses $\sim M_{GUT}$. Even if $210_H$ is replaced by $45_H$ and $126_H$ is replaced by $16$, the two step-breaking scenario $SO(10) \rightarrow G_{2213} \rightarrow SM$ has been shown to be realizable. Although this would be inconsistent with matter parity conservation because of $(16_H)$ to break $SU(2)_R \times U(1)_{B-L}$ in stead of $(126_H)$, it also leads to direct breaking model as before when the two symmetry breaking scales are identical to $\sim M_{GUT}$. The direct breaking super-grand desert scenario of all SUSY GUTs including SU(5), SO(10) and $E_6$ including MSSM (minimal supersymmetric standard model) has been noted \cite{41} to exhibit precision gauge coupling unification when extrapolated to higher scales using the CERN-LEP data \cite{79-81}. On the other hand the higher rank non-SUSY GUTs like SO(10), and $E_6$ have been shown to require at least one intermediate gauge symmetry, such as $G_{224}$ or $G_{2213}$ \cite{50, 82, 83}. But in view of a number of interesting minimal modifications of the non-SUSY grand desert scenario \cite{25, 26, 30, 36, 76}, models without any non-standard gauge bosons below the GUT scale also appear to be simpler choices for SO(10) as discussed in this work for predictions especially on decaying dark matter (DDM).

Alternatively, this direct symmetry breaking non-SUSY SO(10) can also be visualised as a result of two different symmetry breakings occurring at the same scale $\mu = M_{GUT}$. At first SO(10) breaking to $G_{224}$ can occur by the GUT scale VEV of Pati-Salam singlet $\xi(1,1,1)$ in $210_H$ or $54_H$ accompanied by the symmetry breaking $G_{224} \rightarrow SM$ driven by the VEV of $G_{224}$ sub-multiplet $\Delta_R(1,3,\overline{10}) \subset 126_H^1$. As already noted, in all the direct breaking models a grand desert is produced without coupling unification and this problem is successfully confronted by two different ways in this work by the introduction of just one SM scalar submultiplet $\eta(3,0,8) \subset 210_H$ in Model-I and $\kappa(3,-1/3,6) \subset 126_H$ in Model-II component as discussed below in Sec.\ref{sec:7}.

Defining the corresponding fine-structure constants as $\alpha_i = \frac{g_i^2}{4\pi}$, $i = Y, 2L, 2R, B-L, 3C, 4C$, at the GUT scale $\mu = M_{GUT}$ matching conditions are $\alpha_{2-L} = \alpha_{3C} = \alpha_{4C}$ and $\alpha_{2R} = \alpha_{2L}$ leading to

$$\frac{1}{\alpha_Y(\mu)} = \frac{3}{5} \frac{1}{\alpha_{2L}(\mu)} + \frac{2}{5} \frac{1}{\alpha_{3C}(\mu)}.$$  \hspace{1cm} (15)

Renormalisation group evolution of gauge couplings and coupling unification have been discussed below in Sec.\ref{sec:7} in these two different cases (Model-I and Model-II).

In addition different SO(10) representations are identified with definite values of matter parity $Z_{MP} = \text{even}$, or odd

\begin{align*}
Z_{MP} &= \text{Even : } 10, 45, 54, 120, 126, 210, \ldots, \\
Z_{MP} &= \text{Odd : } 16, 144, \ldots
\end{align*}  \hspace{1cm} (16)

As such all SM fermions and RH neutrinos have odd $Z_{MP}$ but the SM Higgs scalar $h \subset 10_H$ has even $Z_{MP}$. Then stable DM candidates can be assigned to be in one or more of the non-standard fermion representations $10_F, 45_F, 54_F, 120_F, 126_F, 210_F, \ldots$. Similarly scalar DM candidates carrying odd matter parity can be assigned spinorial representations $16_H, 144_H, \ldots$. In the following sections we will need the SM components of $16_H$, the conjugate of the scalar representation $16_H$, $45_H$, $126_H$, $210_H$ and $45_F$ for our model descriptions out of which the components of $126_H$ and $16_H$ have been already specified in eq.\ref{eq:2} and eq.\ref{eq:3} and the remain-
ing components are

\[ 10_H = (2, 2, 1) + (1, 1, 6) \ldots \ldots \ldots (G_{224}), \]
\[ \subset (2, 2, 0, 1) + (1, 1, 2/3, 3) + (1, 1, -2/3, 3) \ldots (G_{2213}), \]
\[ \subset \phi(1, 1/2, 1) + (2, -1/2, 1) + (1, 2/3, 3) + (1, -2/3, 3) \ldots (G_{2213}), \]
\[ 210_H = (1, 1, 1) + (1, 1, 15) + (1, 3, 15) + (3, 1, 15) + \ldots \ldots (G_{224}), \]
\[ \subset \Phi_1(1, 1, 0, 1) + \Phi_2(1, 1, 0, 1) + \Phi_3(1, 1, 0, 1) + \kappa(3, 1, 0, 8) + \ldots (G_{2213}), \]
\[ 45_H = (1, 1, 15) + (3, 1, 1) + (1, 3, 1) + (2, 2, 6) \ldots (G_{224}), \]
\[ \subset S_H(1, 1, 0, 1) + (3, 1, 0, 1) + (1, 3, 0, 1) + \ldots \ldots (G_{2213}), \]
\[ \subset S_H(1, 1, 0, 1) + (3, 0, 1) + A_3(1, 0, 1) + \ldots \ldots (G_{2213}). \]

(17)

As all the components of every representation in eq. (17) and eq. (2) have been explicitly shown to possess \( 3(B - L) = +2 \) or \(-2 \), they have even matter parity satisfying \( Z_{MP} = +1 \). Thus VEVs in the respective SM singlet direction to be utilized for gauge symmetry breaking or non-standard fermion mass generation ensures conservation of matter parity in the presence of \( G_{2213} \) gauge symmetry. Another representation of lower dimension that carries odd matter parity is \( 144_H \subset SO(10) \).

Being a SM singlet which is also a SU(5) singlet \( \chi_S(1, 0, 1) \), in principle, can acquire any VEV in the range \( O(M_W) - O(M_U) \) without affecting gauge coupling evolution. But being odd under \( Z_{MP} \), conservation of matter parity in the presence of SM constrains this VEV to be at most of order \( v_{ew}, V_\chi \leq \sqrt{v_{ew}} = 246 \) GeV.

We treat \( 45_F \) as Majorana fermionic DM representation which contains the DDM as a Majorana fermion singlet. This can be decomposed under \( G_{224} \) and \( G_{2213} \) in the following manner

\[ 45_F = (1, 1, 15)_F + (3, 1, 1)_F + (1, 3, 1)_F + (2, 2, 6)_F : \text{G}_{224}, \]
\[ = \Sigma_F(1, 0, 1) + (1, 2/3, 3)_F + (1, -2/3, 3)_F + (1, 0, 8)_F + (3, 0, 1)_F + (1, 0, 1)_F + (1, \pm 1, 1)_F \]
\[ + (2, 1/6, 3)_F + (2, -1/6, 3)_F + (2, -5/6, 3)_F + (2, 5/6, 3)_F : \text{G}_{2213}. \]

(18)

Thus, the SM singlet fermion \( \Sigma_F(1, 0, 1) \) as DDM is assumed to be in the \( G_{224} \) submultiplet \( (1, 1, 15)_F \subset 45_F \). It is necessary to show how only the DDM candidate \( \Sigma_F \) can acquire PeV scale mass by Higgs mechanism while the rest 44 components are allowed to have GUT scale masses.

In order to implement such mass splitting we use the GUT scale Lagrangian where Yukawa interactions of the fermionic adjoint representation \( 45_F \) with two scalar representations \( 54_H \equiv \Phi \) have been included, although the presence of the former can be dispensed with in the minimal case.

\[ -\mathcal{L}_{Yuk}^{(MPC)} = A_F \left( \frac{m_A}{2} + \frac{h_F}{2} \Phi + \frac{h_e}{2} E \right) A_F + h.c. \]

Here \( m_A \sim M_{GUT} \) is the common bare mass of all the components of \( 45_F \). The \( SO(10) \) scalar representation \( 210_H \) has three SM singlets in Pati-Salam submultiplets \( \Phi_1(1, 1, 1), \Phi_2(1, 1, 15), \)
and \( \Phi_3(1,3,15) \). We denote these there VEVs by \( V_{210}^{(i)} = \langle \Phi^{(i)} \rangle (i = 1,2,3) \). We also denote the singlet VEV \( \langle E \rangle \subset 54_H \). Then all the component masses of 45_\( F \) are

\[
m(1, -2/3, 3) = m(1, 2/3, 3) = m_A + \frac{1}{3\sqrt{2}} h_p V_{210}^{(2)} - \frac{1}{\sqrt{15}} h_e \langle E \rangle,
\]

\[
m(1, 0, 8) = m_A - \frac{1}{3\sqrt{2}} h_p V_{210}^{(2)} - \frac{1}{\sqrt{15}} h_e \langle E \rangle,
\]

\[
m(1, 0, 1) = M_\Sigma = m_A + \frac{\sqrt{2}}{3} h_p V_{210}^{(2)} - \frac{1}{\sqrt{15}} h_e \langle E \rangle,
\]

These are the masses of all the components of \( G_{224} \) submultiplet \((1,1,15)_F \subset 45_F \). The masses of the rest 30 components are

\[
m(2, 1/6, 3) = m(2, -1/6, 3) = m_A - h_p \frac{V_{210}^{(3)}}{6} + h_e \frac{\langle E \rangle}{4\sqrt{15}},
\]

\[
m(2, -5/6, 3) = m(2, 5/6, 3) = m_A - h_p \frac{V_{210}^{(3)}}{6} + h_e \frac{\langle E \rangle}{4\sqrt{15}},
\]

\[
m(1, 1, 1) = m(1, -1, 1) = m_A + h_p \frac{V_{210}^{(1)}}{\sqrt{6}} + \frac{\sqrt{3}}{2\sqrt{5}} h_e \langle E \rangle,
\]

\[
m(3, 0, 1) = m_A - h_p \frac{V_{210}^{(1)}}{\sqrt{6}} + \frac{\sqrt{3}}{2\sqrt{5}} h_e \langle E \rangle,
\]

\[
m'(1, 0, 1) = m_A + h_p \frac{V_{210}^{(1)}}{\sqrt{6}} + \frac{\sqrt{3}}{2\sqrt{5}} h_e \langle E \rangle.
\]

These formulas have the options of finetuning two Yukawa couplings, four VEVs and \( m_A \). In case we assume all other VEVs except \( V_{210}^{(2)} \) to vanish, only one fine tuning among this VEV, \( h_p \) and \( m_A \) is sufficient to make \( M_\Sigma \simeq 1 \) PeV while keeping all other 44 component masses near the GUT scale. It is interesting to note that switching on other VEVs (e.g. \( \langle E \rangle \subset 54_H \) still offers the option of keeping only \( \Sigma_F \) mass near the PeV scale while having all other masses at the GUT scale. Similar fine tunings to keep respective component masses light have been discussed in the corresponding physical applications in \([26,27,30]\).

Thus using 210_\( H \) matter parity conserving non-standard Higgs origin of DM mass is predicted leading to the \( \Sigma_F \) mass term

\[
- \mathcal{L}_\Sigma = (1/2) M_\Sigma \bar{\Sigma}_F (1,0,1) \Sigma_F (1,0,1) + h.c.
\]

In Sec.\[10\] we will present an application of this model to predict desired relic density of DDM where the SM singlet scalar \( \xi(1,0,1) \subset \Phi_{210}(1,1,15) \) is exchanged between \( \Sigma_F \) and SM Higgs \( h \).

4 Decaying Dark Matter in SO(10) with Type-I Seesaw Dominance

4.1 Seesaw Mechanism with DM and IceCube Neutrinos from SO(10)

We have embedded the DM \( \Sigma_F \) as a Majorana fermion singlet in 45_\( F \) and also derived its mass from non-standard Yukawa interactions of SO(10). The Yukawa Lagrangian respecting
the SM gauge symmetry below the GUT scale can be written as

\[-\mathcal{L}_{Yuk} = Y^\nu \bar{l}_L N \phi + (1/2) M_N \bar{N} C N + (1/2) M_N \bar{N} C \Sigma_2 \Delta_L L + h.c.,\]  

(23)

where, in our notation, $N(S_F)$ represent the RH neutrino(DM fermion), $\phi$ = the standard Higgs scalar field, $\bar{\phi} = i \tau_2 \phi^*$, and $\Delta_L (3, -1, 1) \subset 126_H$. The mass parameter $m^{(\text{mix})}$ is the analogue of the $N - \Sigma_F$ mixing discussed in Sec.2. Using the Higgs field vacuum expectation value $\langle \phi \rangle = \frac{v_{ew}}{\sqrt{2}} \simeq 174.1$ GeV, the neutral fermion mass matrix in the $(\nu, N, \Sigma_F)$ basis turns out to be

\[\mathcal{M}_\nu = \begin{pmatrix} M_D & 0 & m^{(\text{mix})}^T \\ M_D^T & M_N & 0 \\ 0 & m^{(\text{mix})} & M_{\Sigma} \end{pmatrix}.\]  

(24)

where $M_D$ = the Dirac neutrino mass matrix = $Y^\nu \langle \phi \rangle$, $Y^\nu$ being the Yukawa coupling matrix. The type-II seesaw contribution to neutrino mass has been neglected assuming $\Delta_L$ mass at the GUT scale. Here the $M_N$ block is a $3 \times 3$ matrix and $m^{(\text{mix})}$ is a three component row matrix

\[m^{(\text{mix})} = (m_1^{\text{mix}}, m_2^{\text{mix}}, m_3^{\text{mix}}).\]  

(25)

Diagonalization of the first $2 \times 2$ block leads to the canonical seesaw formula along with heavy RH neutrino mass eigenvalues

\[m_\nu = -M_D 1 \ M_N^{-1} M_D^T, \quad m_N = M_N.\]  

(26)

The mass eigen-states are

\[\hat{\nu} = \cos \alpha \nu + \sin \alpha N, \quad \hat{N} = \sin \alpha \nu + \cos \alpha N, \quad \tan 2\alpha = 2 M_D / M_N.\]  

(27)

Similarly, diagonalisation of the second block in eq.(24) under the condition $M_N > M_N - M_{\Sigma} > M_{\Sigma} \gg m^{\text{mix}}$ gives

\[M^\pm = \frac{1}{2} \left[ M_N + M_{\Sigma} \pm \sqrt{(M_N - M_{\Sigma})^2 + 4m^{(\text{mix})}^2} \right],\]  

(28)

which gives the heavier ($\Sigma_{\text{Heavy}}$) and the lighter ($\Sigma_{\text{Light}}$) mass eigen states with respective masses

\[M_{\Sigma_{\text{Heavy}}} = M^+ \simeq M_N + m^{(\text{mix})}^2 / (M_N - M_{\Sigma}), \quad M_{\Sigma_{\text{Light}}} = M^- \simeq M_{\Sigma} - m^{(\text{mix})}^2 / (M_N - M_{\Sigma}),\]  

(29)

\[\hat{\Sigma}_{\text{Heavy}} = \cos \beta N + \sin \beta \Sigma_F, \quad \hat{\Sigma}_{\text{Light}} = -\sin \beta N + \cos \beta \Sigma_F, \quad \tan 2\beta \simeq 2 m^{(\text{mix})} / M_N.\]  

(30)
It has been shown in the next section that, in terms of individual RHν states \( N_i \) and their corresponding mixings \( m_{mix}^i \), eq. (30) can be replaced by

\[
\hat{\Sigma}_{\text{Heavy}}^i = \cos \beta_i N_i + \sin \beta_i \Sigma_F, \\
\hat{\Sigma}_{\text{Light}}^i = -\sin \beta_i N_i + \cos \beta_i \Sigma_F,
\]

\[
\tan 2\beta_i \simeq 2m_{(mix)}^i/M_{N_i}.
\]

(31)

It is clear from eq. (29) that the conditions \( M_{N_i} > M_{N_i} - M_{\Sigma} > M_{\Sigma} \) are easily satisfied in the canonical seesaw origin for neutrino masses with \( M_{N_i} \approx 10^9 - 10^{15} \) GeV.

Even if the lightest RHν \( N_1 \) is lighter than the PeV scale DM mass we note that a similar formula for mixing is also valid

\[
\tan 2\beta_1 \simeq 2m_{1{(mix)}}/M_{\Sigma_F}, \\
M_{\Sigma_F} \gg M_{N_1},
\]

(32)

### 4.2 Determination of RHν Masses

SO(10) has Pati-Salam symmetry \( \text{SU}(4)_C \) at GUT scale. As such it predicts Dirac neutrino mass matrix similar to the up-quark mass matrix. This makes Dirac neutrino Yukawa couplings and type-I seesaw prediction of RH neutrino masses predominantly hierarchical. This is the reason why the simplified picture of seesaw adopted in [58] with SM extension is not applicable in SO(10). Here we derive all the three RHν masses by fitting the type-I seesaw formula with most recent neutrino oscillation data [1–5]. At first we choose a certain hierarchy (NH/IH/QD) of light neutrino masses. Assuming numerical value of one of the mass eigenvalues, the other two are computed using best fit value of mass squared differences as shown in Table 8 of Appendix 12. The mixing matrix \( U_{PMNS} \) is then constructed according to PDG convention [79–81] with best fit values of mixing angles (\( \theta_{12}, \theta_{23}, \theta_{13} \)) and Dirac CP phase (\( \delta \)). Armed with mass eigenvalues and mixing matrix, it is now easy to obtain the effective light neutrino mass matrix \( m_\nu \)

\[
m_\nu = U_{PMNS} \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U_{PMNS}^T.
\]

(33)

Again as we are dealing with Type-I seesaw dominated scenario, the effective light neutrino mass matrix can be written as

\[
m_\nu \simeq -M_D M_N^{-1} M_D^T,
\]

(34)

from which the RHν mass matrix can be estimated as

\[
M_N = [M_D^{-1}m_\nu(M_D^T)^{-1}]^{-1}.
\]

(35)

Thus \( M_N \) can be calculated if numerical values of \( M_D \) and \( m_\nu \) are already known to us. For this purpose we determine the Dirac neutrino mass matrix using up-quark and down-quark

---

3In this context it is worthwhile to mention that the total diagonalisation matrix is given by \( U_{tot} = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) U_{PMNS} \) with \( U_{PMNS} = U(\theta_i, \delta) \text{diag}(e^{i\alpha_M/2}, e^{i\beta_M/2}, 1) \), where \( \phi, \alpha_M, \beta_M \) are unphysical phases and \( \alpha_M, \beta_M \) are Majorana phases.
diagonal basis respectively at the GUT scale $M_U \sim 10^{15}$ GeV. They are given by

$$M_D^{(u)}(\text{GeV}) = \begin{pmatrix} 0.00054 & (1.5027 + 0.0038i)10^{-9} & (7.51 + 3.19i)10^{-6} \\ (1.5027 + 0.0038i)10^{-9} & 0.26302 & 9.63 \times 10^{-5} \\ (7.51 + 3.19i)10^{-6} & 9.63 \times 10^{-5} & 81.9963 \end{pmatrix}, \quad (36)$$

$$M_D^{(d)}(\text{GeV}) = \begin{pmatrix} 0.01832 + 0.00441i & 0.08458 + 0.01114i & 0.65882 + 0.27319i \\ 0.08458 + 0.01114i & 0.38538 + 1.56 \times 10^{-5} & 3.3278 + 0.00019i \\ 0.65882 + 0.27319i & 3.3278 + 0.00019i & 81.8543 - 1.64 \times 10^{-5}i \end{pmatrix}. \quad (37)$$

This predicts almost diagonal structure of Dirac neutrino Yukawa neutrino coupling matrix $(Y_\nu = M_D^{(u)}/v_{ew})$ up to a very good approximation in the up-quark diagonal basis. But in the down-quark diagonal basis the off-diagonal elements are also quite significant. Using the Dirac neutrino mass matrix in the up-quark diagonal basis $M_D = M_D^{(u)}$, or in the down-quark diagonal basis $M_D = M_D^{(d)}$ as discussed above in eq.(35), the RH$\nu$ mass matrix $(M_N)$ is calculated. Thereafter the complex symmetric $M_N$ matrix is diagonalised by a unitarity $V_P$ matrix as

$$M_N = V_P \hat{M}_N V_P^T \quad \text{where} \quad \hat{M}_N = \text{diag}(\hat{M}_{N_1}, \hat{M}_{N_2}, \hat{M}_{N_3}). \quad (38)$$

It is to be noted that for calculation of any physical process involving RH$\nu$ (such as decay of $N$ to $\nu_\alpha, h$) we have to go to the physical basis or mass basis of the RH$\nu$s. But even after diagonalising $M_N$ with $V_P$ matrix the resulting diagonal matrix ($\hat{M}_N$) may contain complex entries. If all three diagonal elements are complex, one of them can be made real by taking out its phase which can be treated as the unphysical phase. The remaining phase parameters in the other two elements are nothing but Majorana phases which can be absorbed in the $V_P$ matrix. In this way we can get real right handed neutrino masses and the corresponding total diagonalisation matrix $V_P$ with Majorana phases included in it.

This whole exercise is repeated for each hierarchy (NH, IH, QD1, QD2) of light neutrinos taking into account both u-quark and d-quark diagonal basis. Thus, as a whole, we have analysed eight cases. The diagonalising matrix and the mass eigenvalues of the RH$\nu$s are presented systematically in Appendix[12]

### 4.3 Different $N_i - \Sigma_F$ Mixings

We now consider the mixing of the fermionic dark matter $\Sigma_F$ with the RH$\nu$ $N_i(i = 1, 2, 3)$. The Majorana type mixing term between them is given by

$$-\mathcal{L} = (N^c \Sigma_F) \mathcal{M} \begin{pmatrix} N \\ \Sigma_F \end{pmatrix}, \quad (39)$$

where $\mathcal{M}$ is a $4 \times 4$ matrix and $N$ contains three RH$\nu$ fields given by $N \equiv (N_1, N_2, N_3)^T$. The explicit form of $\mathcal{M}$ is given by

$$\mathcal{M} = \begin{pmatrix} (M_N)_{3 \times 3} & m^{(\text{mix})} \\ (m^{(\text{mix})})^T & M_\Sigma \end{pmatrix} \quad (40)$$

where $m^{(\text{mix})}$ is a column matrix with three entries: $m^{(\text{mix})} = (m_1^{\text{mix}}, m_2^{\text{mix}}, m_3^{\text{mix}})^T$. As discussed in the previous section, complex symmetric $M_N$ matrix, can be diagonalised by
$3 \times 3$ unitary $V_P$ matrix. Then we assume the $4 \times 4 M$ matrix to be block diagonalised by unitary $4 \times 4 V_P'$ matrix

$$V_P' = \left( \begin{array}{ccc} (V_P)_{3\times3} & O \\ O^T & 1 \end{array} \right)$$

where $O \equiv (0, 0, 0)^T$. 

Thus, after block diagonalisation, we are left with

$$M_{\text{block-diag}} = \left( \begin{array}{ccc} \hat{M}_N & m^{(\text{mix})}_O \\ m^{(\text{mix})}_O & M_{\Sigma} \end{array} \right).$$

For full diagonalisation the above matrix has to be rotated again by a matrix $V_S$ which can be represented as a combination of four rotation matrices as $V_S = R_{14}(\beta_1).R_{24}(\beta_2).R_{34}(\beta_3)$. For small values of $\beta_i$, $V_S$ can be presented to a good approximation as

$$V_S \simeq \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & \beta_2 \\ 0 & 0 & 1 \\ -\beta_1 & -\beta_2 & -\beta_3 \end{array} \right) \text{ where } \beta_i \simeq \frac{m^{(\text{mix})}_i}{M_N - M_{\Sigma}} \ (i = 1, 2, 3).$$

Thus we can say that the $M$ matrix is fully diagonalised through a two step rotation (first by $V_P'$ followed by $V_S$) and the total diagonalisation matrix is

$$V_N = V_P' V_S.$$ 

It should be noted that the physical or diagonal basis of these heavy fermionic fields are obtained by multiplying $(V_N)^4$ matrix with the flavor basis states $(N \Sigma_F)$.

### 4.4 Determination of Mixing Parameters

Using the three constraint equations for the three partial branching ratios of the DM decay $\Sigma_F \rightarrow \nu_\alpha + h(\alpha = e, \mu, \tau)$, we now determine the mixing parameters $m^{(\text{mix})}_i (i = 1, 2, 3)$. Equality of three branching ratios imply

$$\Gamma(\Sigma_F \rightarrow \nu_\alpha h) = \frac{M_{\Sigma}}{32\pi} \sum_{i=1,2,3} |Y_{\alpha i}(V_{N,i})|^2 = \Gamma, \ (\alpha = e, \mu, \tau).$$

This is actually a set of three equations (for $\alpha = e, \mu, \tau$) each of which contains three unknown parameters $m^{(\text{mix})}_i (i = 1, 2, 3)$. The common decay width $\Gamma$ in the RHS of the above equation is the inverse of life time ($\tau_{\Sigma}$) of the dark matter particle $\Sigma_F$ ($\tau_{\Sigma} \sim 10^{28}$ sec) which is much greater than the life time of Universe. These three equations in eq.(45) are then solved simultaneously to get the values of the unknown mixing parameters which in turn produces equal branching ratio of the decay of DM to each neutrino flavor. Following the same methodology we calculate these mixing parameters for the previously mentioned eight cases (NH, IH, QD1, QD2) with u-quark diagonal basis and d-quark diagonal basis. The results are presented in a concise manner in Table 1 and Table 2.

---

\textsuperscript{4}The complex conjugation comes because the mass matrix is written in the Majorana basis.
Solving eq. (45) we find the three mixing parameters for the normally hierarchical (NH) pattern of light neutrino masses
\[
\begin{align*}
  m_{1}^{\text{mix}} &= -2.724 \times 10^{-8} \text{eV}, \\
  m_{2}^{\text{mix}} &= -3.255 \times 10^{-8} \text{eV}, \\
  m_{3}^{\text{mix}} &= 2.395 \times 10^{-4} \text{eV}.
\end{align*}
\]

(46)

Out of these, the first two are of the same order but the third is 4 orders larger than each of them and nearly 50 times larger than the value derived in the benchmark model. Thus we have successfully derived the three different mixings the decaying dark matter is predicted to possess with the three hierarchical RH\(\nu\)s of non-SUSY SO(10) GUT that gives type-I seesaw ansatz for light neutrino masses. We have also solved for the RH\(\nu\) mass and mixing parameter spectra using neutrino oscillation data in the cases of quasi-degenerate (QD) and the inverted hierarchical (IH) light neutrino mass patterns. For the QD type solutions we have chosen one set of light neutrino masses, QD1, which satisfy the recent cosmological bound [34] and another set, QD2, expected to be reachable by Katrin experiment [84].

\[
\begin{align*}
  \text{QD1} & \quad (\hat{m}_1, \hat{m}_2, \hat{m}_3) = (0.0630079, 0.0636035, 0.0800) \text{ eV}, \\
  \text{QD2} & \quad (\hat{m}_1, \hat{m}_2, \hat{m}_3) = (0.1938, 0.1940, 0.2) \text{ eV}.
\end{align*}
\]

(47)

The QD2 choice may need priors in addition to cosmological bound.

Table 1: Predictions of mixing parameters of decaying dark matter \(\Sigma_F\) with three heavy right handed neutrinos \(N_i (i = 1, 2, 3)\) from Type-I seesaw dominance in SO(10), neutrino oscillation data with NH, QD, and IH type masses and IceCube neutrino data. Two non-vanishing Majorana phases \(\alpha_M\) and \(\beta_M\) of heavy RH\(\nu\)s needed for solutions have been indicated in each case. Dirac neutrino mass matrix \(m_D\) is taken according to up-quark diagonal basis (eq.(36)).

| Mass ordering | \(M_{N_1}\) (GeV) \((\alpha_M)\) | \(M_{N_2}\) (GeV) \((\beta_M)\) | \(M_{N_3}\) (GeV) | \(m_{1}^{\text{mix}}\) (eV) | \(m_{2}^{\text{mix}}\) (eV) | \(m_{3}^{\text{mix}}\) (eV) |
|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| NH            | \(8.9 \times 10^9\) \((147^\circ)\) | \(2.28 \times 10^9\) \((108^\circ)\) | \(1.2 \times 10^{13}\) \((174^\circ)\) | \(-2.724 \times 10^{-8}\) | \(-3.255 \times 10^{-8}\) | \(2.395 \times 10^{-4}\) |
| QD1           | \(4.75 \times 10^4\) \((179^\circ)\) | \(9.6 \times 10^8\) \((174^\circ)\) | \(9.225 \times 10^{13}\) \((174^\circ)\) | \(-2.608 \times 10^{-8}\) | \(4.282 \times 10^{-8}\) | \(-1.093 \times 10^{-5}\) |
| QD2           | \(1.54 \times 10^4\) \((178^\circ)\) | \(3.47 \times 10^8\) \((-40.18^\circ)\) | \(3.35 \times 10^{13}\) \((-40.18^\circ)\) | \(2.61 \times 10^{-8}\) | \(-1.7 \times 10^{-8}\) | \(6.97 \times 10^{-6}\) |
| IH            | \(2.976 \times 10^{15}\) \((-33^\circ)\) | \(2.5 \times 10^9\) \((-33^\circ)\) | \(6.1 \times 10^3\) \((-33^\circ)\) | \(-1.044 \times 10^{-3}\) | \(1.656 \times 10^{-7}\) | \(2.613 \times 10^{-8}\) |

Thus we have found that realistic Type-I seesaw dominance in SO(10) that fits the neutrino oscillation data and the IceCube data results in substantially different predictions on the RH\(\nu\) mass and mixing spectra compared to the simplistic assumptions of the SM extension [58]. This holds true for Dirac neutrino masses evaluated in both the up-quark or the down-quark diagonal basis. For a given light neutrino mass pattern, NH, IH, or QD, clearly there are
Table 2: Predictions of mixing parameters of decaying dark matter $\Sigma_F$ with three heavy right-handed neutrinos $N_i (i = 1, 2, 3)$ from Type-I seesaw dominance in SO(10), neutrino oscillation data with NH, QD, and IH type masses and IceCube neutrino data. Two non-vanishing Majorana phases $\alpha_M$ and $\beta_M$ of heavy RH $\nu$s needed for solutions have been indicated in each case. Dirac neutrino mass matrix $m_D$ is taken according to the down-quark diagonal basis (eq. (37)).

| Mass ordering | $M_{N_1}$ (GeV) ($\alpha_M$) | $M_{N_2}$ (GeV) ($\beta_M$) | $M_{N_3}$ (GeV) | $m_{iN}^{\text{mix}}$ (eV) | $m_{2}^{\text{mix}}$ (eV) | $m_{3}^{\text{mix}}$ (eV) |
|---------------|-------------------------------|-----------------------------|-----------------|-----------------------------|-----------------------------|-----------------------------|
| NH            | $5.85 \times 10^4$ ($-177.08^\circ$) | $3.69 \times 10^3$ ($-3.6^\circ$) | $1.14 \times 10^{13}$ | $2.948 \times 10^{-8}$ | $-1.045 \times 10^{-7}$ | $6.463 \times 10^{-4}$ |
| QD1           | $4.71 \times 10^4$ ($145.5^\circ$) | $9.9 \times 10^8$ ($-5.3^\circ$) | $9.14 \times 10^{13}$ | $3.1 \times 10^{-8}$ | $3.78 \times 10^{-8}$ | $1.75 \times 10^{-5}$ |
| QD2           | $1.55 \times 10^3$ ($9.27^\circ$) | $3.5 \times 10^8$ ($-3.35^\circ$) | $3.35 \times 10^{-13}$ | $-1.96 \times 10^{-8}$ | $2.2 \times 10^{-8}$ | $5.81 \times 10^{-6}$ |
| IH            | $3.177 \times 10^{15}$ ($-141.25^\circ$) | $2.23 \times 10^9$ ($-144.9^\circ$) | $6.59 \times 10^4$ | $-1.03 \times 10^{-3}$ | $1.045 \times 10^{-7}$ | $-3.1 \times 10^{-8}$ |

three distinct values of $N_i\Sigma_F$ mixings consistent with IceCube neutrino data and the natural hypothesis that $\Sigma_F$ decays with equal probability to each light neutrino flavor. Whereas benchmark model holds for QD type neutrino mass hierarchy in the SM extension with the stated value $\tilde{m}_{\nu_i} = 0.1$ eV with a universal heavy mass $M_N = 10^{14}$ GeV, our SO(10) ansatz matches with all types of light neutrino mass hierarchies and predominantly hierarchical $M_{N_i}$ values covering the range $10^3 - 10^{15}$ GeV.

5 Dynamical Generation of RH$\nu$-DM Mixing

In the following section we explore theoretical origin of $N_i - \Sigma_F$ mixings derived in the previous section using neutrino oscillation data and IceCube neutrino data. For convenience we choose solutions derived in the up-quark diagonal basis and for other cases similar derivations apply. As the intrinsic matter parities of $\Sigma_F \subset 45_F$ and RH$\nu$ $N_i \subset 16_{F_i}$ are even and odd, respectively, their mixing is possible if this gauged discrete symmetry is broken explicitly or spontaneously. The usual mechanisms of breaking a discrete symmetry, which might be an intrinsic gauged discrete symmetry of the theory or externally imposed upon it, are known to result in cosmological domain wall problem. A natural resolution of the domain wall problem emerges if the discrete symmetry breaking is assisted by gravity or Planck scale $46, 48, 60, 62$.

In particular, because of the redundancy of parameters of local gauge transformation, the Planck-scale assisted symmetry breaking has been noted to be more effective if the discrete symmetry is an intrinsic gauged discrete symmetry of the theory $46, 48, 60$ and the matter parity in our model being a gauged discrete symmetry ideally matches with this situation. The purpose of this section is to discuss the possibility of different renormalizable and non-renormalizable interactions for the Planck scale assisted matter parity breaking that gives rise to the extremely small values of the mixings.
5.1 Planck-Scale Assisted RHν-DM Mixing

In Sec. 3 we have shown how the non-standard Yukawa interaction in SO(10) has the capability to predict the DM ($\Sigma_F$) mass. We show how the matter parity conserving SO(10) model that predicts type-I seesaw dominance as well as its high scale, also predicts the extremely small value of $N_i - \Sigma_F$ mixing through Planck-scale assisted matter parity breaking. We assign the decaying singlet fermion DM $\Sigma_F(1,0,1)$ to the nonstandard fermionic representation 45$^F$ which has even matter parity. Similarly the RH$\nu$ being in the spinorial representation 16$^F$ possesses odd matter parity. Therefore, as the $N_i\Sigma_F$ fermion bilinear has odd matter parity, any mass-dimensioned coefficient of this term can not be generated without breaking matter parity. As the generation of this discrete symmetry breaking, either softly or spontaneously, leads to the well known domain wall problem, in this work we follow the idea that the cosmologically safe matter parity breaking can be achieved by Planck scale effects $^4$ $^6$$^{46}$ $^{48}$ $^{60}$. We assume the presence of a SO(10) singlet fermion $N'$ having Planck mass $\sim 10^{19}$ GeV. Its mixing $m^{(Br)}_i$ with the RH$\nu N_i$ which is a SU(5)-singlet and carries odd matter parity acts as a source of the matter parity breaking term. This $m^{(Br)}_i$ can emerge from the VEV $V_\chi$ of a SM scalar singlet $\chi_S(1,0,1) \subset 16^H$. 

$$-L^{(Pl)}_{mix} = m^{(Br)}_i N_i N'$$
$$\subset Y_i^\chi V_\chi N_i N'$$
$$\subset Y_i^\chi 16^F.1_F.16^H.$$ 

$$m^{(Br)}_i = Y_i^\chi \left< \chi_S(1,0,1) \right> = Y_i^\chi V_\chi.$$ (48)

In the present model, the added presence of the SO(10) singlet $N'$ of Planck mass also permits the SO(10) invariant Higgs fermion interaction

$$-L^{(Yuk)}_{Yuk} = y_{45} N' 45^F 45^H$$
$$\rightarrow y_{45} N' \Sigma_F S_H ,$$ (49)

where the SM scalar singlet $S_H \subset 45^H$ that has even matter parity can acquire VEV $\langle S_H \rangle = V_H \sim O(M_W) - O(M_{GUT})$. Thus the Planck scale assisted matter parity breaking mechanism can be visualised to originate from a seesaw type Feynman diagram shown in Fig. 2$^I$. In Fig. 2$^I$ we have also used the Yukawa interaction $f_{16^F.16^H.126^H}$. Then using VEV $\langle \Delta_R \rangle = V_R \simeq M_U$, with $M_N = f V_R$ gives the $N_i - \Sigma_F$ mixing

$$-L_{\text{seesaw}} = y_{45} m^{(Br)}_i V_H N \Sigma_F ,$$ (50)

leading to

$$m^{(mix)}_i = y_{45} \frac{m^{(Br)}_i V_H}{M_N} \simeq y_{45} \frac{m^{(Br)}_i V_H}{M_{\text{Planck}}}.$$ (51)

$^5$When $N'$ is integrated out, Fig. 2$^I$ leads to the effective 5-dim. operator scaled by Planck mass: $(\eta_{16^F.45^F.45^H})(1/M_{\text{Planck}}) \subset (\eta N.\Sigma_F.\chi_S.S_H)(1/M_{\text{Planck}})$ where $\eta$ represents product of relevant couplings. This is shown in Fig. 4$^I$ below.
Figure 2: Feynman diagram generating $N_i - \Sigma_F$ mixing in SO(10). Here $N'$ is the SO(10) singlet fermion of Planck mass and $S_H$ is the SM singlet Higgs component of 45$H$ that acquires VEV $V_H$.

It is clear that wide range of values of the explicit matter parity breaking parameter $m_i^{Br}$ in $m_i^{Br}N_iN'$ triggers the $N_i - \Sigma_F$ mixings reported in Sec. 4. We further note that keeping matter parity conservation of the SM gauge symmetry, it is possible to assign any VEV to $\chi_S$ under the constraint $V_{\chi_S} \leq O(M_W)$.

In what follows we show how the extremely small value of $N_i - \Sigma_F$ mixings are realized by Planck-scale assisted spontaneous breaking of matter parity via renormalizable and nonrenormalizable interactions.

5.1.1 Through VEV of Scalar Singlet in $16^\dagger_H$

The solutions for $m_i^{(Br)}$ stated above can be understood further on the basis of spontaneous breaking of matter parity elucidated through Feynman diagram of Fig. 3.

In eq. (48), $m_i^{(Br)}$ can be assigned its spontaneous symmetry breaking origin through the VEV $\langle \chi_S \rangle = V_{\chi_S} \sim O(M_W)$ which breaks matter parity spontaneously alone via the SO(10) invariant gauge interaction term $\chi^\dagger 16_F, 16^\dagger_H$. Here $\chi$ is the associated Yukawa coupling. The SM gauge symmetry breaks in the usual manner through the VEV of the standard Higgs doublet $\phi \subset 10_H$ that carries even matter parity with VEV $v_\phi = v_{ew} = 246$ GeV. For the purpose of this work the scalar singlet $\chi_S(1,0,1)$ is treated to be real.

In Fig. 3 we have shown how the N-N' mixing is generated through the electroweak scale VEV $\langle \chi_S \rangle$ which breaks matter parity spontaneously alone via the SO(10) invariant gauge interaction term $\chi^\dagger 16_F, 16^\dagger_H$. Here $\chi$ is the associated Yukawa coupling. The SM gauge symmetry breaks in the usual manner through the VEV of the standard Higgs doublet $\phi \subset 10_H$ that carries even matter parity with VEV $v_\phi = v_{ew} = 246$ GeV. This gives

$$m_i^{Br} = \chi^\dagger V_\chi \leq \chi^\dagger v_{ew}. \quad (52)$$

6 With the SO(10) invariant piece of the Higgs potential $V_{split} = M_U^2 16^\dagger_H 16_H + [\mu \Delta_{126}]_{16_H 16_H} + h.c.] + (other \ terms)$, where the trilinear coupling $\mu \Delta \sim M_U$, a straight forward derivation shows that any one of the real or the imaginary components of $\chi_S(1,0,1) \subset 16^\dagger_H$ can be fine tuned to remain as light as possible while the other component can acquire mass near the GUT scale. As a result the imaginary part decouples from contributing to any physical quantity below the GUT scale and self-consistency of model predictions of this work is guaranteed. This result has been discussed further in more detail in Sec. 9.2.1.
Figure 3: Feynman diagram generating N-N′ mixing that leads to Planck scale assisted matter parity breaking and extremely small value of N-Σ_F mixing. Here N′ is the SO(10) singlet fermion having Planck mass, S_H is the SM singlet Higgs component of 45_H that acquires VEV V_H, and χ_S is the singlet scalar component of 16_H with allowed VEV V_χ ≤ O(M_W).

5.1.2 Through Nonrenormalizable N-N′ Interaction

The RHν and the SO(10) singlet fermion N′ may also have a Planck scale mediated nonrenormalizable interaction through a dim.5 operator which requires the introduction of a SO(10) singlet scalar S′

\[-L_{NR-I} = K_G i 16_F .1_F .16^1_H S'/M_{Planck}.\]  (53)

Feynman diagram for this interaction and the corresponding seesaw mechanism is shown in Fig.4

Figure 4: Nonrenormalizable N-N′ interaction contributing to Planck-scale assisted matter parity breaking. Here S′ is a SO(10) singlet scalar.

Noting that V_{S′}, the VEV of S′, can be anywhere above the electroweak scale, eq.(53) gives

\[m_i^{Br} = K_G i V_\chi V_{S′}/M_{Planck}.\]  (54)

Using V_χ ~ 100 GeV, M_{Planck} = 10^{19} GeV, and V_{S′} = 10^{8} − 10^{19} GeV this predicts a wide range of values of m_i^{Br} = 1 eV − 100 GeV. Through this mechanism we have shown that small values of matter parity breaking parameter m_i^{Br} ~ 1 eV are also realizable even though V_χ ~ v_{ew}. The smallness of m_i^{Br} in this case is a result of Planck-scale suppression as well as the SM matter parity restricted smaller value of V_χ.
5.1.3 Direct Nonrenormalizable $N - \Sigma_F$ interaction

Noting that breaking matter parity as gauged discrete symmetry essentially needs assistance from gravity or Planck scale effects [46–48, 60] we introduce the following, dim.5 operator scaled by

$$-\mathcal{L}_{NR-II} = C_G, 16^i_F, 45_F, 16^i_H, 45_H/M_{Planck}$$

$$\rightarrow C_G, N_i \Sigma_F \chi_S S_H/M_{Planck}.$$  

(55)

where the constant $C_G \simeq 1$. Even though this nonrenormalizable interaction results by integrating out $N'$ that mediates the Feynman diagram of Fig.2 it is possible to write down eq.(55) independently as the Planck scale assisted dim.5 operator in the absence of $N'$. With $V_{\chi S} \sim \mathcal{O}(M_w)$, the small $N-\Sigma_F$ mixing $m_{mix} = 10^{-5}$ results for $V_H = \langle S_H \rangle \sim \mathcal{O}(10^3)$ GeV. Thus, with matter parity as its intrinsic gauge discrete symmetry, this SO(10) model without the introduction of the additional fermion singlet or external discrete symmetry becomes a self sufficient theory of unified matter and decaying dark matter. For the sake of completeness, this interaction is shown through the Feynman diagram of Fig. 5.

Figure 5: Nonrenormalizable $N-\Sigma_F$ interaction contributing to Planck-scale assisted matter parity breaking. Here $S_H$ is a SM singlet scalar in 45_H.

We emphasize that the underlying mechanism of matter parity breaking that requires Planck scale assistance [60] plays a crucial role in providing a natural explanation of $N-\Sigma_F$ mixing. Without such gravity or Planck scale effects [60], direct breaking of matter parity would give rise to cosmological domain wall problem [47,48].

6 Determination of Renormalizable and Non-Renormalizable Couplings

Our dynamical explanation of $N_i - \Sigma_F$ mixings would be complete by estimating the relevant couplings of different interaction Lagrangians discussed above which is undertaken in this section. We have shown that single and double Planck-scale suppression can occur leading to extremely small mixings between each of the three heavy RH neutrinos $N_i \subset 16^i_F (i = 1, 2, 3)$ and the decaying DM $\Sigma_F \subset 45_F$. We now estimate different Yukawa couplings of RH$\nu$ of three generations which give such mixings.
6.1 Renormalizable Solutions with Domain Wall Problem

Denoting the renormalizable Yukawa interaction of $\chi_S$ with $N_i (i = 1, 2, 3)$ through

$$-\mathcal{L}^R = h_i^{R} 45_F 16^i_F 16^\dagger_H$$

$$\rightarrow h_i^{R} V_\chi N_i \Sigma_F,$$

$$m_i^{\text{mix}} = h_i^{R} V_\chi,$$  \hspace{1cm} (56)

and using $V_\chi \leq v_{\text{ew}}$ we can estimate the limiting values of Yukawa couplings for three generations for each case of solutions given in eq.(57) and the first line of Table 1. Thus corresponding to up-quark diagonal basis and NH, we have

$$|h_i^{R}| \geq (1.10 \times 10^{-19}, 1.32 \times 10^{-19}, 9.70 \times 10^{-16})(i = 1, 2, 3).$$  \hspace{1cm} (57)

6.2 Solutions Without Domain Wall Problem

Planck scale assisted solutions expected to be free of the domain wall problem predict different values of Yukawa couplings noted below.

(a) Renormalizable Yukawa Coupling from Single Planck-Scale Suppression:

In order to deal with three different RHνs with hierarchical masses we replace our notations in all relevant figures and equations with $N \rightarrow N_i$, $Y_\chi \rightarrow Y_i^\chi$, $m^{\text{Br}} \rightarrow m_i^{\text{Br}}$, and $m^{\text{mix}} \rightarrow m_i^{\text{mix}}$ for different RHν flavors $N_i (i = 1, 2, 3)$. Integrating out all the relevant heavy fields in Fig.2 and Fig.3 we derive expression for effective mixing and Yukawa couplings due to Planck-scale suppression.

$$m_i^{\text{mix}} \simeq y_{45} m_i^{\text{Br}} V_H$$

$$= y_{45} Y_i^\chi V_H$$

$$= h_i^{\text{eff}} V_\chi,$$

$$h_i^{\text{eff}} = y_{45} Y_i^\chi V_H.$$  \hspace{1cm} (58)

In the NH case this leads to

$$h_i^{\text{eff}} \geq m_i^{\text{mix}}/v_{\text{ew}},$$

$$|Y_i^\chi| \geq (1.10 \times 10^{-8}, 1.32 \times 10^{-8}, 9.70 \times 10^{-5}),$$  \hspace{1cm} (59)

where in deriving the last step we have used $y_{45} = 1$ and $V_H = 10^8$ GeV in eq.(58). The Yukawa couplings $Y_i^\chi$ are renormalizable. With such intermediate value of scalar singlet VEV of even matter parity there will be an additional singlet scalar of mass $M_{SH} \simeq 10^8$ GeV.

(b) Effective Nonrenormalizable Coupling from Double Planck Suppression:

Integrating out all relevant fields in Fig.4 leads to corresponding expressions originating from double Planck suppression.
\[ m_i^{\text{mix}} \approx y_{45} \frac{K_G V_H V_{S'}}{M_{\text{Planck}}} \]
\[ \equiv h_i^{(\text{eff, NR})} V_{\chi}, \]
\[ h_i^{(\text{eff, NR})} = y_{45} \frac{K_G V_H V_{S'}}{M_{\text{Planck}}} \] (60)

Then for the NH case using \( y_{45} \approx 1 \), \( V_H = V_{S'} = 10^{15} \) GeV we get
\[ |K_G| \geq (1.10 \times 10^{-11}, 1.32 \times 10^{-11}, 9.7 \times 10^{-8}). \] (61)

In this case there will be no additional Higgs scalar singlets of intermediate mass except for the light \( \chi_s \) discussed in Sec.9.

(c) Effective Nonrenormalizable Coupling Without Fermion Singlet \( N' \)

We have shown in Sec.5 that extremely small mixings can be generated by non-renormalisable \( \text{dim} \cdot 5 \) interaction. As discussed above, replacing \( N \to N_i \) in Fig. 2 and correspondingly \( C_G \to C_{G_i} \) as in eq. (55) we get in the NH case
\[ |C_{G_i}| \geq (1.14 \times 10^{-4}, 1.38 \times 10^{-4}, 1.0). \] (62)

where the equality holds for \( V_{\chi} = v_{\text{ew}} \). As already noted, an interesting outcome of this estimation is that the present SO(10) theory is not only free from invoking any external discrete symmetry for DM stability, but also it does not need any Planck mass singlet fermion \( N' \) as in the case (a) and case (b) discussed above to achieve domain-wall free cosmologically acceptable matter parity breaking through Planck-scale suppressed dim.5 operator of eq.(55).

Different allowed values of couplings without the cosmological domain wall problem are summarized in Table 3 as the class of “NO” solutions.

Table 3: Renormalizable \( (h_R^i, Y_{\chi}^i) \) and nonrenormalizable \( (h_i^{(\text{eff, NR})}, C_{G_i}) \) solutions for \( \chi_S \) Yukawa couplings in the presence (labeled as “YES”) and absence (labeled as “NO”) of cosmological domain wall problem all of which predict the three \( N_i - \Sigma F \) mixings for the NH type light neutrino masses in the up-quark diagonal basis given in Table 1. The VEV of the scalar singlet has been fixed at its upper limit \( V_{\chi} = v_{\text{ew}} = 246 \) GeV. For lower allowed values of this VEV \( V_{\chi} < v_{\text{ew}} \), these couplings are enhanced by the scaling factor \( v_{\text{nev}}/V_{\chi} \).

| Domain Wall | Coupling | \( i = 1 \) | \( i = 2 \) | \( i = 3 \) |
|-------------|---------|----------|----------|----------|
| YES | \( h_R^i \) | \( 1.1 \times 10^{-18} \) | \( 1.4 \times 10^{-18} \) | \( 1.0 \times 10^{-13} \) |
| NO | \( Y_{\chi}^i \) | \( 1.1 \times 10^{-7} \) | \( 1.4 \times 10^{-7} \) | \( 1.0 \times 10^{-2} \) |
| NO | \( h_i^{(\text{eff, NR})} \) | \( 1.1 \times 10^{-7} \) | \( 1.1 \times 10^{-7} \) | \( 1.0 \times 10^{-2} \) |
| NO | \( C_{G_i} \) | \( 1.1 \times 10^{-4} \) | \( 1.4 \times 10^{-4} \) | \( 1.0 \) |

It is interesting to note that Type-I seesaw dominance in matter parity conserving SO(10) predicts decaying dark matter dynamics most generally by accommodating all the three different types of light neutrino mass hierarchies and even satisfying cosmological bound without or
with priors in the QD cases. The QD2 type solutions predict neutrinoless double beta decay rate close to the current experimental limits. In the case of mixings solutions presented for IH, QD1, and QD2 type of mass hierarchies and also for down-quark diagonal basis, the corresponding Yukawa couplings can be estimated following the same procedure. Once the light neutrino mass hierarchy is determined in future, most of these Yukawa coupling predictions would converge only to two alternative sets corresponding to up-quark or down-quark diagonal bases. On the other hand if NH or IH type hierarchy is confirmed, then the benchmark model solutions would be either ruled out or revised.

7 Gauge Coupling Unification

As pointed out in previous sections, two Higgs representations $210_H, 126_H$ have been shown to define a SUSY SO(10) GUT model [74] with minimal number of 26 parameters. In this section we show that in the direct non-SUSY SO(10) breaking to SM driven by these two representations, either $210_H$ or $126_H$ is capable of supplying just one scalar submultiplet of intermediate mass to complete minimal modification of the grand desert as discussed below in case of Model-I and Model-II.

7.1 Unification in the Minimal Model-I

It is interesting to see how the present minimal model generating DM mass gives rise to gauge coupling unification of the standard gauge theory by exercising utmost economy on the choice of lighter fields to populate the grand desert by just one nonstandard scalar submultiplet $\kappa(3,0,8) \subset 210_H$. Although such type of model was suggested briefly for coupling unification [76], the input parameters used at that time were not as accurate as available now [79–81]

\[
\begin{align*}
\alpha_s(M_Z) &= 0.1182 \pm 0.0005, \\
\sin^2\theta_W(M_Z) &= 0.23129 \pm 0.00005, \\
\alpha^{-1}(M_Z) &= 127.94 \pm 0.02.
\end{align*}
\]

Further there were no neutrino oscillation data or information on dark matter available at that time to establish natural high scale type-I seesaw dominance in this model. Also no connection with matter parity conservation, or DM candidates or their masses were discussed in the GUT framework. Neither was there any justification in favour of $210_H$ as the mediator of non-standard Yukawa origin of decaying DM mass. The choice of $\kappa(3,0,8)$ at intermediate scale was purely from curiosity to achieve unification. The experimental bound on proton lifetime has increased almost by more than one order over the years from 1993 till date that calls for estimation of uncertainties in the model predictions accurately. Apart from embedding the IceCube DM, our other motivation is to see how far the present grand unification framework can be constrained by the ongoing search experiments on proton lifetime in near future [85]. It is quite interesting to note that the DM fermion representation $45_F$ which has given prominent threshold effects elsewhere [30] has exactly vanishing contribution in the present case (Model-I).

In addition, in this work we have estimated GUT threshold effects under partially degenerate assumption which states that the superheavy masses belonging to the same SO(10) representation are degenerate in masses [86,87]. We find that under this constraint, an attractive region of parameter space requires the inclusion of threshold effects of superheavy
gauge bosons in the adjoint representation $45_V$ with their masses only few times different from $M^0_U$ \cite{77}.

We found in Sec.3 that the Higgs representation $210_H$ plays two crucial roles in the GUT symmetry breaking as well as generating the desired dark matter mass that decays to produce PeV energy IceCube neutrinos. We fine tune the parameters of the GUT scale Lagrangian in such a way that only the mass of component $\kappa(3,0,8) \subset 210_H$ is substantially lighter than the GUT scale while keeping all other superheavy component masses of $210_H$ near the GUT scale.

Using the contributions of SM particles and the scalar $\kappa(3,0,8)$ though the renormalization group (RG) equations \cite{88}, the unification of gauge couplings is shown in Fig. 6.

![Figure 6: Unification of SM gauge couplings in the presence of $\kappa(3,0,8)$ as discussed in the text. The vertical dashed lines represent the mass scales $M_\kappa = 10^{9.23}\text{GeV}$ and $M_U = 10^{15.2446}\text{GeV}$, respectively.](image)

The unification of three gauge couplings of SM at one-loop level is consistent with

\[
\begin{align*}
M^0_U &= 10^{15.2+0.0446}\text{GeV}, \\
M^0_\kappa &= 10^{9.23}\text{GeV}, \\
\alpha^{-1}_G &= 41.77.
\end{align*}
\]

The quantity $+0.0446$ in the exponent is due to the matching of the SM coupling constants at the GUT scale that occurs even if all the superheavy particle masses are exactly degenerate with $M^0_U$ \cite{77,78,89,90}. It is clear from the Fig. 6 that excellent unification of gauge couplings of the standard gauge theory below the GUT scale without the assumption of any other intermediate gauge symmetry has been achieved with only one non-standard Higgs field ($\kappa(3,0,8)$) having intermediate mass $M_\kappa = 10^{9.23}\text{GeV}$.

### 7.2 Unification in Minimal Model-II

In this case while all nonsinglet scalar components of $210_H$ and all other component masses in $126_H$ have masses at the GUT scale, only the component $\eta(3,-1/3,6) \subset 126_H$ is at
$M_\eta = 10^{10.73}$ GeV. Similar to eq.(64), the RG solutions in this case are

$$M_0^U = 10^{15.24+0.0445} \text{GeV},$$

$$M_0^\eta = 10^{10.73} \text{GeV},$$

$$\alpha^{-1}_G = 38.397$$

(65)

Excellent unification of SM gauge couplings is shown in Fig.7.

Figure 7: Unification of SM gauge couplings in the presence of $\eta(3,-1/3,6)$ as discussed in the text. The vertical dashed lines represent the mass scales $M_\eta = 10^{10.73}$GeV and $M_U = 10^{15.28}$GeV, respectively.

8 Proton Lifetime Prediction

8.1 Decay Rate

Currently the measured value on the lower limit of the proton life time for the decay modes $p \rightarrow e^+\pi^0$ and $p \rightarrow \mu^+\pi^0$ are [91,92]

$$\tau_{p}^{\text{exp}}(p \rightarrow e^+\pi^0) \geq 1.6 \times 10^{34} \text{ yrs.,}$$

$$\tau_{p}^{\text{exp}}(p \rightarrow \mu^+\pi^0) \geq 7.7 \times 10^{33} \text{ yrs.}. (66)$$

We investigate our model capabilities to account for this lower limit.

8.2 Analytic Formulas for Decay Width

Including strong and electroweak renormalization effects on the $d = 6$ operator and taking into account quark mixing, chiral symmetry breaking effects, and lattice gauge theory estimations, the decay rates are [93,96],

$$\Gamma(p \rightarrow e^+\pi^0) = \left(\frac{m_p}{64\pi f_\pi^2 M_{U}^4}\right) g^4 [A_L]^2 |\alpha_H|^2 (1 + D' + F)^2 \times R,$$

(67)
where \( R = |A_{5R}^2 + A_{SL}^2| \) for \( SU(5) \), but \( R = [(A_{5R}^2 + A_{SL}^2)(1+|V_{ud}|^2)^2] \) for \( SO(10) \). \( V_{ud} = 0.974 \) = the (1,1) element of \( V_{CKM} \) for quark mixings, and \( A_{SL}(A_{SR}) \) is the short-distance renormalization factor in the left (right) sectors. In eq.\((67)\), \( A_L = 1.25 = \) long distance renormalization factor but \( A_{SL} \approx A_{SR} = 2.542 \). These are numerically estimated by evolving the dim.6 operator for proton decay by using the anomalous dimensions of ref. \([94]\) and the beta function coefficients for gauge couplings of this model. In eq.\((67)\), \( M_U = \) degenerate mass of superheavy gauge bosons, \( \bar{\alpha}_H = \) hadronic matrix elements, \( m_p = \) proton mass = 938.3 MeV, \( f_\pi = \) pion decay constant = 139 MeV, and the chiral Lagrangian parameters are \( D = 0.81 \) and \( F = 0.47 \). With \( \alpha_H = \bar{\alpha}_H(1+D' + F) = 0.012 \) GeV\(^3\), \( \alpha_G = \bar{\alpha}_H(1+D' + F) = 0.012 \) GeV\(^3\) estimated from lattice gauge theory computations \([97,98]\), we obtain \( A_R \approx A_L A_{SL} \approx A_L A_{SR} \approx 2.726 \) and the expression for the inverse decay rate is

\[
\Gamma^{-1}(p \rightarrow e^+\pi^0) = 4 \frac{f_\pi^2 M_U^4}{\pi m_p \alpha_G^2 \alpha_H^2 A_R^2 F_q},
\]

where the GUT-fine structure constant \( \alpha_G = 0.0263 \) and the factor \( F_q = 2(1+|V_{ud}|^2)^2 \approx 7.6 \) have been used for \( SO(10) \). This formula has the same form as given in \([93]\).

### 8.3 Analytic Formula for Threshold Effects

In the single step breaking models discussed in this work, GUT threshold effects due to superheavy degrees of freedom in different \( SO(10) \) representations are expected sources of major uncertainties on unification scale and proton lifetime prediction. The underlying origin of threshold effects due to smaller quantum corrections proposed in \([78,89,90]\) has been also addressed in \( SO(10) \) \([77,86,87]\), and more recently in \([30]\). Details have been also given in the Appendix which yield the following corrections arising from different superheavy particles in the loops. Further we have estimated the threshold uncertainties following the partially degenerate assumption introduced in \([86,87,89]\) which states that the superheavy components belonging to the same GUT representation are degenerate with the same superheavy scale around \( M_U \). A new expected source of threshold uncertainty is due to fermion representation \( 45_F \). In Model-I and Model-II discussed here, we investigate possible cancellations in reducing threshold uncertainties.

Noting that the superheavy scalars, fermions, and gauge bosons contribute through small log evolutions and defining \( \eta_j = \log_{10}(M_j/M_U) \), we have the following formula for GUT threshold effects

\[
\frac{M_U}{M_U^0} = 10^{\pm C_{\text{input}} \text{ (input parameters)}}
\times 10^{\pm C_{(126)} \eta_{(126)}} \text{ (superheavy scalars in 126_H)}
\times 10^{\pm C_{(210)} \eta_{(210)}} \text{ (superheavy scalars in 210_H)}
\times 10^{\pm C_{(45)} \eta_{(45)}} \text{ (superheavy scalars in 45_H)}
\times 10^{\pm C_{(16)} \eta_{(16)}} \text{ (superheavy scalars in 16_H)}
\times 10^{\pm C_{(10)}} \eta_{(10)} \text{ (superheavy scalars in 10_H)}
\times 10^{\pm C_{\eta \eta \eta}} \text{ (superheavy gauge bosons in 45_V)}
\times 10^{\pm C_{\eta \eta \eta}} \text{ (superheavy fermions in 45_F)},
\]

(69)
where \( M_i(i = 10_H, 16_H, 45_H, \ldots) \) is the respective degenerate superheavy particle mass in the SO(10) representation \([86]\). In eq. (69) the first line represents 3σ uncertainty of input values of \( \sin^2 \theta_W(M_Z) \), \( \alpha_S(M_Z) \) and \( \alpha(M_Z) \) \([79–81]\) given in eq. (63). Other contributions represent superheavy particle contributions from respective representations associated with spontaneous symmetry breaking of gauge symmetries and also in the generation of DM mass including its mixing with RHν.

As shown in Appendix we have derived threshold corrections by providing the longitudinal modes of superheavy gauge bosons from 210\( _H \) and 126\( _H \). The relevant superheavy components of scalars can be found from representations given in Table 9. The decompositions of 54\( _H \) whose VEV contributes to DM or color octet fermion mass through eq. (21) has been skipped from Table 9 as its nonsinglet components give vanishing threshold effects in Model-I and Model-II. GUT threshold effects due to 45\( _H \) whose SM singlet component \( S_H \) enters into all the Feynman diagrams of Sec. 4 can be estimated from the decomposition given for 45\( _F \) but using the small log evolution formula for scalars given in the Appendix. Similarly superheavy gauge boson threshold effects have been estimated in Model-I and Model-II.

### 8.4 Predictions in the Minimal Model-I

In this case our estimated values of coefficients occurring in eq. (69) are

\[
\begin{align*}
C_{\text{input}} &= 0.110, \\
C_{(10)} &= -0.0196, \\
C_{(126)} &= -0.03743, \quad C_{(210)} = 0.0322, \\
C_V &= -0.9358, \quad C_{(16)} = 0.0, \\
C_{(45)} &= 0.0, \quad C_{(54)} = 0.0, \\
C_F &\simeq 0.0. 
\end{align*}
\]  

(70)

The coefficients \( C_{(16)}, C_{(45)}, \) and \( C_F \) due to superheavy components of Higgs representations 16\( _H, 45_H \) and the fermion representation 45\( _F \) in the partially degenerate case vanish \([77, 78, 86, 89, 90, 99]\).

The estimated GUT scale in the partially degenerate case turns out to be

\[
M_U = 10^{15.2446 \pm 0.1100 \pm 0.089275 |\eta_S| \pm 0.9358 |\eta_V|}.
\]  

(71)

This leads to the proton lifetime

\[
\tau_p^{SO(10)} \simeq 10^{32.940 \pm 0.440 \pm 0.3571 |\eta_S| \pm 3.743 |\eta_V|} \text{ yrs.}
\]  

(72)

It is interesting to note that, despite the two large Higgs representations 210\( _H, 126_H \), the non-standard fermion representation 45\( _F \), and other Higgs representations 45\( _H, 16_H, 10_H \), major contribution to threshold uncertainty in Model-I is only due to superheavy gauge bosons. The superheavy Higgs boson threshold effect that acts as a major source of uncertainty in intermediate scale models \([87]\) is much smaller and the fermion threshold contribution is absent in this direct symmetry breaking model.

Numerical estimations on proton lifetime for Model-I are shown in Table 11 for different splitting factors of superheavy masses.
Table 4: Upper limits on predicted proton lifetime in Model-I as a function of superheavy scalar (S) and gauge boson (V) mass splittings as defined in the text and Appendix. The factor $10^{\pm 0.44}$ represents uncertainty due to input parameters.

| $M_S/M_U$ | $M_V/M_U$ | $\tau_P \text{(yrs)}$ | $M_S/M_U$ | $M_V/M_U$ | $\tau_P \text{(yrs)}$ |
|-----------|-----------|----------------------|-----------|-----------|----------------------|
| 10        | 2         | $2.65 \times 10^{34 \pm 0.44}$ | 5         | 5         | $6.4 \times 10^{35 \pm 0.44}$ |
| 10        | 3         | $1.21 \times 10^{35 \pm 0.44}$ | 2         | 5         | $4.61 \times 10^{36 \pm 0.44}$ |
| 5         | 3         | $9.45 \times 10^{34 \pm 0.44}$ | 2         | 4         | $2 \times 10^{35 \pm 0.44}$ |
| 5         | 4         | $2.77 \times 10^{35 \pm 0.44}$ | 2         | 3         | $6.81 \times 10^{34 \pm 0.44}$ |

8.5 Predictions in the Minimal Model-II

In this case the coefficients of eq. (69) are

\[
C_{\text{input}} = 0.1334, \\
C_{(10)} = -0.02024, \\
C_{(126)} = -0.044534, \\
C_{(16)} = 0.0, \\
C_{(210)} = 0.0809717, \\
C_V = -0.9352, \\
C_{(45)} = 0.0, \\
C_{(54)} = 0.0, \\
C_F \approx 0.0. 
\] (73)

With the partially degenerate assumption that the superheavy masses of different components of a given SO(10) representation have separately degenerate masses, or with the complete degeneracy assumption of identical masses for all superheavy scalars, maximising the threshold uncertainties gives the following results.

8.6 Lifetime with Partially Degenerate Superheavy Scalar

\[
M_U = 10^{15.28 \pm 0.1334 \pm 0.1457 |\eta'_{S}| \pm 0.9352 |\eta'_{V}|} 
\] (74)

\[
\tau_P^{SO(10)} \approx 10^{33.11 \pm 0.5335 \pm 0.5828 |\eta'_{S}| \pm 3.74 |\eta'_{V}|} \text{ yrs.} 
\] (75)

In this case predictions on proton lifetime are presented in Table 5.

Table 5: Upper limits on predicted proton lifetime in Model-II as a function of superheavy scalar (S) and gauge boson (V) mass splittings defined in the text and Appendix. The factor $10^{\pm 0.53}$ represents uncertainty due to input parameters.

| $M_S/M_U$ | $M_V/M_U$ | $\tau_P \text{(yrs)}$ | $M_S/M_U$ | $M_V/M_U$ | $\tau_P \text{(yrs)}$ |
|-----------|-----------|----------------------|-----------|-----------|----------------------|
| 5         | 2         | $4.469 \times 10^{34 \pm 0.53}$ | 2         | 3         | $1.19 \times 10^{35 \pm 0.53}$ |
| 8         | 2         | $5.87 \times 10^{34 \pm 0.53}$ | 3         | 3         | $1.51 \times 10^{34 \pm 0.53}$ |
| 10        | 2         | $6.693 \times 10^{34 \pm 0.53}$ | 5         | 3         | $2.03 \times 10^{35 \pm 0.53}$ |
| 1         | 3         | $7.97 \times 10^{34 \pm 0.53}$ | 1         | 4         | $2.33 \times 10^{35 \pm 0.53}$ |

Including uncertainty due to input parameters and assuming all superheavy fermion and gauge boson masses identical to $M_U$, we find that this model predicts proton lifetime up to $3 \times 10^{34 \pm 0.58}$ yrs ($8 \times 10^{34 \pm 0.58}$ yrs) for degenerate (partially degenerate) superheavy scalar masses which is accessible to ongoing proton decay searches.
9 Vacuum Stability of the Scalar Potential

While explaining dynamical origin of $N_i - \Sigma_F$ mixings, we have found that Planck-scale assisted spontaneous symmetry breaking provides an attractive mechanism through the smaller VEV $V_\chi \leq v_{ew}$ of a matter parity odd Higgs singlet naturally present in the Higgs representation $16_I$ of SO(10). Because of its order $v_{ew}$ VEV, this Higgs scalar is predicted to be light with perturbative upper bound on its mass $\leq 860$ GeV. We discuss below how such a nonstandard Higgs singlet scalar resolves the issue of vacuum instability in Model-I and Model-II discussed in the previous sections. It is well known that the SM Higgs potential develops instability due to radiative corrections as the quartic coupling becomes negative for larger values of the Higgs field $\Lambda_I \sim |\phi| \geq 2 \times 10^9$ GeV. One popular solution to this vacuum instability has been suggested through the introduction of additional scalar field(s) of mass below the instability scale $\Lambda_I$ [39,40,100]. These may be scalar singlet candidates corresponding to WIMP [101–104] or decaying scalar DM manifesting through PeV scale IceCube neutrinos 56,57.

We have shown in Sec 5 that dynamical origin [105] of $N_i - \Sigma_F$ mixings predicts a nonstandard Higgs scalar singlet $\chi_S(1,0,1)$ with VEV $V_\chi \leq v_{ew} = 246$ GeV. To examine how the presence of this $\chi_S$ affects the evolution of the standard Higgs quartic coupling, we consider additional contributions to $V_{SM}$ due to $\chi_S$ field modified scalar potential in the presence of $\phi$ and $\chi_S$

$$V_{\phi-\chi} = -\mu_\phi |\phi|^2 + \lambda_\phi (\phi^\dagger \phi)^2 + \lambda_\chi (\chi_S^\dagger \chi_S) + 2\lambda_{\phi\chi} (\phi^\dagger \phi) (\chi_S^\dagger \chi_S)$$

(77)

After using the minimization conditions for both scalar fields $\phi$ and $\chi_S$, the entire potential $V = V_{SM} + V_{\phi-\chi}$ can be written in a convenient form

$$V = \lambda_\phi \left[ (\phi^\dagger \phi) - \frac{v_{ew}^2}{2} \right]^2 + \lambda_\chi \left[ (\chi_S^\dagger \chi_S) - \frac{V_\chi^2}{2} \right]^2 + 2\lambda_{\phi\chi} \left[ (\phi^\dagger \phi) - \frac{v_{ew}^2}{2} \right] \left( \chi_S^\dagger \chi_S - \frac{V_\chi^2}{2} \right)$$

(78)

where $V_\chi$ is the VEV of the newly added scalar singlet. For $\lambda_\phi, \lambda_\chi > 0$ and $\lambda_{\phi\chi}^2 < \lambda_\phi \lambda_\chi$, the minimum for the total potential in eq. (78) is given by $\langle \phi^\dagger \phi \rangle = \frac{v^2}{2}$ and $\langle \chi_S^\dagger \chi_S \rangle = \frac{V_\chi^2}{2}$. To know about the high scale behavior of the Higgs quartic coupling ($\lambda_\phi$) we have to solve its renormalization group (RG) equation which has been modified due to addition of the singlet scalar field ($\chi_S$). Actually the RG equation of $\lambda_\phi$ is a coupled first order differential equation which involves the quartic coupling of the singlet scalar ($\lambda_\chi$), the coupling of the interaction term ($\lambda_{\phi\chi}$), the gauge couplings ($g_{2L}, g_{1Y}, g_{3C}$) and dominant Yukawa coupling $h_t$ due to the top quark. We have considered one loop RG equations for the scalar quartic couplings shown.

---

5 It is to be noted that adding a constant term to the potential and using the minimization condition $\mu^2 = \lambda_\phi v_{ew}$, the potential can be rewritten in a convenient form as $V_{sm} = \lambda_\phi (\phi^\dagger \phi) - \frac{v_{ew}^2}{2}$ where we have omitted the constant term which does not affect the equation motions. It is evident from the above expression of potential that the minimum is at $\langle \phi^\dagger \phi \rangle = \frac{v^2}{2}$. 5
Two loop equations for gauge couplings and top quark Yukawa coupling are discussed in Appendix B of Sec.13.

\[
\frac{d\lambda_\phi}{d\ln \mu} = \frac{1}{(4\pi)^2} \left[ 12y_{\text{top}}^2 - 3g_{1Y}^2 - 9g_{2L}^2 \right] \lambda_\phi - 6g_{1op} + \frac{3}{8} 2g_{2L}^4 + (g_{1Y}^2 + g_{2L}^2)^2 + 24\lambda_\phi^2 + 4\lambda_{\phi\chi}^2, \\
\frac{d\lambda_{\phi\chi}}{d\ln \mu} = \frac{1}{(4\pi)^2} \left[ \frac{1}{2} (12y_{\text{top}}^2 - 3g_{2L}^2 - 9y_{\text{top}}^2) \lambda_{\phi\chi} + 4\lambda_{\phi\chi}(3\lambda_\phi + 2\lambda_{\phi\chi}) + 8\lambda_{\phi\chi}^2 \right], \\
\frac{d\lambda_\chi}{d\ln \mu} = \frac{1}{(4\pi)^2} \left[ 8\lambda_{\phi\chi}^2 + 20\lambda_{\chi}^2 \right].
\]

As in our Model-I and Model-II, the Dirac neutrino mass matrix has been assumed to be same as that of the up quark mass matrix and the most dominant contribution affecting RG evolution of \(\lambda_\phi(\mu)\) for mass scales \(\mu > M_{N_3}\) is the element \(Y_{33} = (M_D)_{33}/v_{\text{ew}}\). Thus the equation governing the evolution of \(\lambda_\phi\) will have a new additional term \((4Y_{33}^2\lambda_\phi - 2Y_{33}^4)\) in the RHS of of the first of eq.(79). Similarly the new term to be added in the RHS of the \(\lambda_{\phi\chi}\) equation is \(2Y_{33}^2\lambda_{\phi\chi}\).

9.1 Vacuum Stability with Spontaneously Broken Matter Parity

We now show how the presence of the Higgs scalar singlet \(\chi_S(1,0,1) \subset 16_H^\dagger\) carrying odd matter parity and having perturbative mass upper bound \(M_{\chi_S} < 860\) GeV, which has been predicted to explain the dynamical origin of extremely small value of the N-ΣF mixings, resolves the issue of vacuum instability of the SM Higgs potential. All the results derived so far and others to follow for a real scalar singlet which could be either the real or imaginary part of \(\chi_S(1,0,1)\). At the end of Sec.9.2 we show that if any one of these two component masses is made as light as the electroweak scale by allowed fine tuning of the parameters of the relevant scalar potential, the other component automatically acquires GUT scale mass. As a result only the light scalar singlet component of \(\chi_S(1,0,1)\) modifies the standard Higgs quartic coupling contributing to the resolution of vacuum stability problem while the heavy mass decouples from making any such contribution.

9.1.1 Higgs Doublet-Singlet Mixing and LHC Constraints

It is clear from the expression of the potential (eq.(78)) that the ordinary SM scalar doublet \((\phi)\) and the newly introduced scalar singlet \((\chi_S)\) mix through the \(\lambda_{\phi\chi}\) term due to which their masses also get modified little bit. Now our primary task is to diagonalize the mass matrix of the scalars which enables us to find the mass eigenvalues of the SM Higgs like state, singlet like state and the mixing angle between them. The mass of the SM Higgs like state is around 125 GeV where as that of the singlet like state is unknown. We have to analyze the phenomenological implications of this singlet scalar at LHC. Before using a certain value of the mixing angle and mass of the new scalar in our RG running analysis, we have to check the compatibility of our chosen set with present LHC data.

The mass matrix of the two scalars in \((\phi, \chi)\) basis can be derived from the expression of the potential in eq.(78)

\[
\mathcal{M}^2 = 2 \begin{pmatrix}
\lambda_{\phi v_{\text{ew}}}^2 & \lambda_{\phi\chi v_{\text{ew}}} V_{\chi} \\
\lambda_{\phi\chi v_{\text{ew}}} V_{\chi} & \lambda_{\chi} V_{\chi}^2
\end{pmatrix}.
\]

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Without any approximation this matrix is diagonalized with the 2 dimensional orthogonal rotation matrix and the mixing angle turns out to be
\[ \theta = \frac{1}{2} \tan^{-1} \left( \frac{2\lambda_{\phi \chi} v_{\text{ew}} V_{\chi}}{\lambda_{\phi} v_{\text{ew}}^2 - \lambda_{\chi} V_{\chi}^2} \right). \] (81)

Denoting the mass eigenvalue of the SM Higgs like state as \( m_1 \) and singlet scalar like state as \( m_2 \), we have
\[ m_1 = 2(\lambda_{\phi} v_{\text{ew}}^2 \cos^2 \theta + \lambda_{\chi} V_{\chi}^2 \sin^2 \theta + \lambda_{\phi \chi} v_{\text{ew}} V_{\chi} \sin 2\theta) \]
\[ m_2 = 2(\lambda_{\phi} v_{\text{ew}}^2 \sin^2 \theta + \lambda_{\chi} V_{\chi}^2 \cos^2 \theta - \lambda_{\phi \chi} v_{\text{ew}} V_{\chi} \sin 2\theta). \] (82)

Among 8 parameters in the set \( (m_1, m_2, \lambda_{\phi}, \lambda_{\chi}, \lambda_{\phi \chi}, v_{\text{ew}}, V_{\chi}, \theta) \) only two are known: the SM Higgs mass \( m_1 \sim 125 \) GeV, and the SM VEV \( v_{\text{ew}} = 246 \) GeV. Matter parity conservation down to the electroweak scale introduces a upper limit on VEV of \( \chi_S, V_{\chi} \leq v_{\text{ew}} = 246 \) GeV. The other parameters are constrained from different measurements performed at LHC [100].

We now discuss briefly about the experimental constraints on singlet scalar mixing parameters quoted above. In this model the constraints come from three different kinds of measurements: (i) Electroweak precision data, (ii) Higgs coupling measurements, and (iii) Different searches for a Higgs like scalar.

In the model under consideration, the electroweak observables are mainly modified due to the new one loop contributions to the \( W \) and \( Z \) propagators. These new contributions arise due to (i) loop diagram with the singlet scalar, (ii) modification of the coupling of the SM Higgs with the gauge bosons. Incorporating those corrections the shifts in the electroweak parameters are calculated. A global \( \chi^2 \) analysis can be carried out to get a exclusion plot for \( m_2 \) vs \( \theta \). Sizable constraint comes for \( m_2 \leq 60 \) GeV and \( m_2 \geq 170 \) GeV and strongest limit is obtained when \( m_2 \geq 450 \) GeV.

The presence of the singlet scalar or in other words the mixing between the newly added singlet scalar with the ordinary SM Higgs, modifies the coupling of the SM Higgs with other fermions and gauge bosons. Again these couplings are involved in computations of several decay widths which are observable at LHC. Taking only \( \gamma \gamma \) or \( 4l \) as final states and with the consideration that singlet scalar mass is outside \([120, 130]\) GeV a combined constraint on the singlet mass and mixing angle \( \theta \) is obtained as
\[ \sin \theta < 0.44 \text{ at } 95\% \text{ CL} \] (83)

for
\[ m_2 \geq \frac{m_1}{2} \ (62.5 \text{ (GeV)}) \text{ and } m_2 \notin [120, 130] \text{ GeV}. \] (84)

When \( m_2 \) becomes smaller than \( m_1/2 \), then the decay channel \( \phi \rightarrow \chi_S \chi_S \) opens up leading to a stringent bound on \( \theta \) which is strongly dependent on the coupling responsible for \( \phi, \chi_S \) mixing i.e, \( \lambda_{\phi \chi} \). It has been observed that for larger values of \( \lambda_{\phi \chi} \) almost whole \( m_2 - \theta \) parameter space is ruled out.

\*The \( \chi^2 \) is a function of scalar singlet mass, mixing angle between scalar singlet and SM doublet and other known parameters of SM

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Similar kinds of constraints on $m_2 - \theta$ parameter space can be obtained from search of different possible direct decay channels of the singlet state.

A combined exclusion plot for $m_2 - \theta$ parameter space can be drawn taking into account all three types of measurements discussed above. A complete analysis and the corresponding exclusion plot can be found in Ref. [100]. It is to be noted that for $m_2 = (10 - 250)$ GeV $|\sin \theta| < 0.1$ is allowed for any value of $m_2$ within these limits whereas higher values of the mixing angle ($\theta$) are permissible in few pockets of mass ranges of which the regime relevant to our analysis is $m_2 = (160 - 180)$ GeV with $|\sin \theta| < 0.4$.

In our actual numerical analysis we vary the unknown quartic couplings ($\lambda_\phi, \lambda_\chi, \lambda_{\phi\chi}$) over a wide range of values ($0.001 - 0.1$). The VEV of $\chi_S (V_\chi)$ is varied from a small value up to 246 GeV, whereas that of $\phi (v_{ew})$ is kept fixed at 246 GeV and mass of the SM Higgs like state is taken to be around 125 GeV. The highest allowed value of $\theta$ is different for different value of the mass eigenvalue $m_2$. This upper bound on $\theta$ is chosen from Fig.3 of Ref [100] and utilised appropriately for different values of the scalar singlet mass in our numerical analysis. For each allowed set of parameters ($m_1, m_2, \lambda_\phi, \lambda_\chi, \lambda_{\phi\chi}, v, V_\chi, \theta$) we analyze vacuum stability and perturbativity of the quartic couplings up to the Planck scale. After repeating this exercise for replica of many such sets, it is found that the couplings lose their perturbativity much before the Planck scale if we take the initial value of $\lambda_\chi \geq 0.3$. The problem of vacuum stability is not cured unless we take electroweak scale value of $\lambda_{\phi\chi} \geq 0.034$. To satisfy these two conditions simultaneously we have to allow $|\sin \theta| \geq 0.3$ and from Fig.3 of [100], it is clear that this value of mixing angle is only allowed in the mass range $(160 < m_2 < 180)$ GeV. The resolution for vacuum stability problem using one such set of parameters (as given in the Table 6) is shown in Fig.8 where we have also included the contribution of the Dirac neutrino Yukawa matrix which is shown by dotted lines. For SM extension with type-I seesaw extension Dirac neutrino Yukawa effect is shown at $10^{14}$ GeV as would be applicable to the benchmark model [58] which has been identified as the curve RKP in Fig.8. Our model predictions of quartic coupling shown as the upper curve in this figure naturally includes the Dirac neutrino Yukawa affecting the RG evolution for $\mu > 10^{15}$ GeV. As shown in Fig.8 the SM vacuum is indeed stable up to Planck scale when supplemented by modifications due to $h - \chi_S$ mixing even after Dirac neutrino Yukawa corrections are included. The desired quartic coupling also lies well below the perturbative limit.

It is pertinent to point out that vacuum stability in SO(10) with $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ intermediate symmetry and matter parity conservation has been carried

| $\lambda_\phi$ | $\lambda_\chi$ | $\lambda_{\phi\chi}$ | $|\sin \theta|$ | $v$ (GeV) | $V_\chi$ (GeV) | $m_1$ (GeV) | $m_2$ (GeV) |
|---|---|---|---|---|---|---|---|
| 0.141 | 0.251 | 0.037 | 0.3 | 246 | 245 | 125.25 | 177.5 |

Table 6: Values of quartic couplings, VEVs, mixing angle, SM Higgs mass ($m_1$) and scalar singlet $\chi_S$ mass ($m_2$) at electroweak scale consistent with experimental constraints which predict vacuum stability.

In our case we need not go beyond 250 GeV since the maximum value of the VEV of the scalar singlet is 246 GeV. This in turn allows us to take $m_2$ at most $\sim 250$ GeV.

\[\text{In our case we need not go beyond 250 GeV since the maximum value of the VEV of the scalar singlet is 246 GeV. This in turn allows us to take $$m_2 \sim 250$$ GeV.}\]
Figure 8: Resolution of vacuum instability problem by the predicted second Higgs scalar $\chi_S$ of mass $M_{\chi_S} \sim 177$ GeV carrying odd matter parity. The evolution of standard Higgs quartic coupling in the presence of $\chi_S$ modifications is denoted by the upper green curve and the dotted line shows the effect of Dirac neutrino Yukawa modification for $M_{N_3} = 10^{15}$ GeV. The lower red curve marked “RKP” represents the quartic coupling evolution in the benchmark model for $M_N = 10^{14}$ GeV.

out through a WIMP scalar DM near TeV scale [28]. In a different interesting approach, the imaginary part of a complex scalar field whose real part generates heavy RH$\nu$ mass for type-I seesaw has been treated as the source of IceCube neutrinos [57]. A number of other models have been also suggested for IceCube neutrino events [56]. In our present model WIMP DM as matter parity even non-standard Majorana fermionic singlet originating from $45_F$ of SO(10) is also easily accommodated in addition to the PeV scale decaying DM discussed here.
ensuring coupling unification and verifiable proton lifetime.

### 9.2 Low $V_\chi$ from Potential Minimisation

In this section we show that potential minimisation of the full Higgs potential can predict a low VEV $V_\chi = \langle Re(\chi_S) \rangle \leq v_{ew}$ where we treat only the real part of $\chi_S$ to be as light as the electroweak scale while keeping its imaginary part at the GUT scale. In the last part of this section we have proved how such a wide gap can be realized between the two component masses of $\chi_S$.

Instead of low VEV $V_\chi \leq v_{ew}$, we have checked implications of high $V_\chi \sim M_{GUT}$ assumption: (i) This violates matter parity of Lagrangian predominantly by introducing additional large contribution to heavy RH neutrino mass via Planck scale effects, thus contradicting the basic assumption of this model that matter parity violating part is a small perturbation over matter parity conserving part. (ii) Explanation of $\Sigma F$ assumption of this model that matter parity violating part is a small perturbation over matter parity conserving part. (iii) This violates matter parity of Lagrangian predominantly by introducing additional $V_H$ Higgs. We thus search for the prediction of low $V_\chi$ in such direct SO(10) breaking to SM.

We have shown that the VEV $V_\chi \leq v_{ew} = 246$ GeV is necessary for the only SM singlet scalar component $\Re(\chi_S(1,0,1))$ contained in $16^+_H$ in order to allow matter parity as gauged discrete symmetry to coexist with SM all the way down to the electroweak scale. Now we address the question whether spontaneous symmetry breaking of SO(10) can allow such small VEV through fine tuning of the model parameters in the GUT Lagrangian.

It is well known that this question is related to the stability of Higgs vacuum in the SM or the Higgs mass at the electroweak scale. While SUSY GUTs have a natural resolution to the well known gauge hierarchy problem, in non-SUSY GUTs the scalar mass is kept at the electroweak scale by fine tuning of parameters in the GUT Lagrangian to every loop order. Following the discussion of Sec. [13](#) and notations of Table [9](#) we write the full SO(10) invariant Higgs potential including $16_H, 10_H, 45_H, 126_H, 210_H$ and their conjugates noting that $10_H, 45_H$ and $210_H$ are self-conjugates. In writing such scalar potentials we have suppressed tensor indices [74](#)[75](#) for the sake of simplicity.

\[
V_{16} = M_{16}^2 16_H^\dagger 16_H + \lambda_{16}(16_H^\dagger 16_H)^2 + \lambda_{(16,10)} 16_H^\dagger 16_H 10_H 10_H \\
+ \lambda_{(16,45)} 45_H 16_H^\dagger 16_H + \lambda_{(16,126)} 126_H^\dagger 126_H 16_H^\dagger 16_H \\
+ \lambda_{(16,210)} 210_H 210_H 16_H^\dagger 16_H + m_{(16,126)} [126_H 16_H^\dagger 16_H + 126_H^\dagger 16_H 16_H] \\
+ m_{(16,10)} [16_H 16_H 10_H + 16_H^\dagger 16_H^\dagger 10_H] + m_{(16,45)} 45_H 16_H^\dagger 16_H \\
+ m_{(16,210)} 210_H 16_H^\dagger 16_H. \quad (85)
\]

\[
V_{10} = M_{10}^2 10_H^\dagger 10_H + \lambda_{10} 10_H^\dagger 10_H + \lambda_{(16,10)} 16_H^\dagger 16_H 1010_H + \lambda_{(126,10)} 126_H^\dagger 126_H 10_H 10_H \\
+ \lambda_{(45,10)} 45_H 45_H 10_H 10_H + m_{(45,10)} 45_H 10_H^\dagger 10_H + \lambda_{(210,10)} 210_H 210_H 10_H 10_H . \quad (86)
\]
As noted before we identify \( \chi_S(1, 0, 1) \) as the real part of the SM scalar singlet in \( \chi_S(1, 0, 1) \uparrow \subset 16_H^\dagger \)

\[
\chi_S^\dagger = \chi_S^R - i\chi_S^I,
\]

\[
\chi_S^R = \text{Re}(\chi_S^\dagger),
\]

\[
\chi_S^I = -\text{Im}(\chi_S^\dagger),
\] (87)

We use decompositions given in eq.(2), eq.(3), eq.(17) and Appendix B. In the generalised case at first we use GUT scale VEVs in all SM singlet directions and denote them as \((A_0, A_3) \subset 45_H^\dagger \)

\[
A_0 = V H = \sqrt{2} \langle (1, 1, 15) \rangle \subset 45_H,
\]

\[
A_3 = \sqrt{2} \langle (1, 3, 1) \rangle \subset 45_H,
\]

\[
V_{210}^{(1)} = \sqrt{2} \langle (1, 1, 1) \rangle \subset 210_H,
\]

\[
V_{210}^{(2)} = \sqrt{2} \langle (1, 1, 15) \rangle \subset 210_H,
\]

\[
V_{210}^{(3)} = \sqrt{2} \langle (1, 3, 15) \rangle \subset 210_H,
\]

\[
V_R = \sqrt{2} \langle (1, 3, 10) \rangle \subset 126_H,
\] (88)

In addition we use

\[
V_\chi = \sqrt{2} \langle \text{Re}(\chi_S) \rangle = \sqrt{2} \langle (1, 2, 4) \rangle \subset 16_H,
\]

\[
V_\phi = \sqrt{2} \langle \phi \rangle \subset \langle (2, 2, 1) \rangle \subset 10_H.
\] (89)

The VEVs of \( \text{Im}(\chi_S) \) and the left handed doublet VEV in \( 16_H \) or \( 16_H^\dagger \) are taken to be vanishing wherever necessary

\[
\langle \chi_L(2, 1, 4) \rangle = \langle \text{Im}(\chi_S) \rangle = 0.
\] (90)

Although all our derivations discussed here can apply in the presence of all GUT scale VEVs defined in eq.(88), no generality is lost by confining to the minimal case of the two models (Model-I and Model-II) for which we fix

\[
V_{210}^{(1)} = V_{210}^{(3)} = A_3 = 0.
\] (91)

Now identifying \( V_{210}^{(2)} = V_{210}, \lambda_{210}^{(2)} = \lambda_{210} \) and using eq.(2), eq.(3), eq.(17), eq.(88), eq.(89) and eq.(91), we minimise the potentials \( V_{16} \) and \( V_{10} \) to obtain

\[
\lambda_{(16)} V_\chi^2 = -[M_{16}^2 + \lambda_{(16,10)} V_\phi^2/2 + P],
\]

\[
P = \lambda_{(16,45)} A_0^2/2 + (1/2)\lambda_{(16,126)} V_R^2 + (1/2)\lambda_{(16,210)} V_{210}^2
\]

\[
+ \sqrt{2}m_{(16,126)} V_R + m_{(16,45)} A_0/\sqrt{2} + m_{(16,210)} V_{210}/\sqrt{2}.
\] (92)

\[
\lambda_{(10)} V_\phi^2 = -[M_{10}^2 + \lambda_{(16,10)} V_\chi^2/2 + Q],
\]

\[
Q = \lambda_{(10,45)} A_0^2/2 + \lambda_{(10,126)} V_R^2/2 + (1/2)\lambda_{(10,210)} V_{210}^2 + m_{(10,45)} A_0/\sqrt{2}.
\] (93)
Now solving the two equations eq.(92) and eq.(93) for simultaneously gives

\[ V_\chi^2 = -\omega_2(M_{16}^2 + P) + \omega_1(M_{10}^2 + Q). \]  

(94)

\[ V_\phi^2 = -\frac{\lambda_{16}}{\lambda_{10}}\omega_2(M_{10}^2 + Q) + \omega_1(M_{16}^2 + P). \]  

(95)

where

\[ \omega_1 = \frac{2\lambda_{(10,16)}}{(4\lambda_{10}\lambda_{16} - \lambda_{(10,16)}^2)}, \]

\[ \omega_2 = \frac{4\lambda_{10}}{(4\lambda_{10}\lambda_{16} - \lambda_{(10,16)}^2)}. \]  

(96)

These equations retain the same form in the more generalised cases when, instead of eq.(91), all GUT scale VEVs defined in eq.(88) are included. In such a generalised case the following replacements are made in the definitions of P and Q given in eq.(92) and eq.(93): \lambda_{(16,210)}V_{210}^2 \rightarrow \sum_{i=1}^3 \lambda_{(16,210)}^{(i)}V_{210}^{(i)^2}, \lambda_{(10,210)}V_{210}^2 \rightarrow \sum_{i=1}^3 \lambda_{(10,210)}^{(i)}V_{210}^{(i)^2}, m_{(16,210)}V_{210}^2 \rightarrow \sum_{i=1}^3 m_{(16,210)}^{(i)}V_{210}^{(i)} \text{ and } A_0^2 \rightarrow A_0^2 + A_0^2 \text{.}

Similarly self-consistency of model predictions discussed below in Sec.9.2.1 is guaranteed in the generalised case with the corresponding replacements in eq.(97) and eq.(100).

Besides the mass squared terms \( M_{16}^2 \) and \( M_{10}^2 \) being of order \( M_{GUT}^2 \), each of the three VEVs \( V_{210}, V_R, A_0 \) can be of order \( M_{GUT} \). In addition each of the trilinear couplings \( m_{(16,10)}, m_{(16,45)}, m_{(16,126)}, m_{(16,210)}, \text{ and } m_{(10,45)} \) is also of order \( M_{GUT} \). Then each of the terms in P and Q is proportional to \( M_{GUT}^2 \) having unknown coefficients. This suggests that our models can satisfy the low scale matter parity violating condition \( V_\chi \leq V_\phi \approx O(M_W) \) by fine tuning the mass dimensionful parameters and nine quartic couplings in eq.(94) and eq.(95). In the general non-minimal case of eq.(93), compared to eq.(92), the number of such parameters is less but they are enough to yield the well known result \( V_\phi \sim M_W^2 \) through fine tuned cancellations among different terms involving GUT scale parameters. In the sense of extended survival hypothesis \([106,107]\), the fine tunings needed to make \( V_\phi \sim v_{ew} \) which is associated with electroweak gauge symmetry breaking belong to the category of minimal fine tuning. Other fine tunings needed to keep \( \kappa(3,0,8) \) at \( M_\kappa = 10^{9.2} \text{ GeV and } Re(\chi_S) \) light are among the category of additional fine tunings.

Common to both minimal and non-minimal cases we note some interesting possibilities of cancellations which could be exploited for physical applications. Even if the input VEV \( V_H \approx 10^3 - 10^8 \text{ TeV as utilised in some cases of Sec.5 } \text{ there is provision for mutual fine-tuned cancellation of corresponding terms. For example cancellation can occur between } \lambda_{(16,15)}V_H^2 \text{ and } m_{(16,45)}V_H \text{ in the expressions for } P \text{ and } Q \text{ in eq.(94) and eq.(95) in the minimal case and similarly among corresponding terms in the nonminimal case. In our analysis we have not included radiative corrections which can be also significant in estimating the minimum of a Higgs potential more accurately }[108].

\[10\text{ In the same fashion additional effect due to GUT scale VEV of the } G_{224} \text{ singlet scalar } \langle E \rangle < 54_H \text{ discussed in Sec.3 and occuring as alternative solution to DM relic density noted in Sec.10 can be included without affecting self-consistency of model predictions discussed in Sec.9.2.1.}\]
9.2.1 GUT Scale Mass of \(\text{Im}(\chi_S)\) and Self-Consistency of Model Predictions

All our discussions presented in this section on minimisation of Higgs potential and derivation of low \(V_\chi\) apply if the real part of \(\chi_S(1,0,1)\) has mass \(\sim M_W\) while its imaginary part does not contribute to any physical quantity below the GUT scale. Further all the analyses in Sec. 9 for vacuum stability have been carried out under the assumption that \(\chi_S(1,0,1) = \chi_{SR}\) is a real singlet Higgs scalar and resolution of vacuum stability has predicted its mass to be \(M_{\chi_S} \simeq 177\) GeV. There is the possibility that the imaginary part of \(\chi_S(1,0,1)\), if light, can also contribute to relevant quantities affecting the self consistency of predictions discussed so far. On the other hand if the imaginary part has GUT scale mass, it would decouple from making any such contributions at lower scales which would guarantee self-consistency. In this section we discuss how this latter possibility is naturally realised in both of our models (Model-I and Model-II) without needing any additional fine tuning.

Using \(V_{16}\) from eq. (85) and eq. (87) we at first extract the potential contributing to the mass terms for the real and imaginary parts of \(\chi\). Although the following proof can be applied in the most general case of both the models, for the sake of simplicity we confine only to the minimal case in which all GUT scale VEVs vanish along with \(\langle \chi_L(2, -1/2, 1) \rangle = \langle \chi_{SI} \rangle = 0\) except \(V_R, V_{210}^{(2)} = V_{210}\) and \(A_0 = V_H\)

\[
V_{\text{mass}} = [M_{16}^2 + \frac{\lambda_{(16,10)}V_\phi^2}{2} + (1/2)\lambda_{(16,45)}A_0^2 + \lambda_{(16,210)}V_{210}/2] + \lambda_{(16,126)}V_R^2/2 + m_{(16,45)}A_0/\sqrt{2} + m_{(16,210)}V_{210}/\sqrt{2}[\langle \chi_{SR} \rangle + \chi_{SI}^2] + \sqrt{2}m_{(16,126)}V_R(\chi_{SR}^2 - \chi_{SI}^2) + (1/2)\lambda_16V_\chi^2\chi_{SR}^2 + \lambda_{16}V_\chi^2\chi_{SI}^2,
\]

where the last two terms follow from \(\lambda_{16}(16_H^H 16_H)^2\) under the assumption that \(\langle \chi_{SI} \rangle = 0\). Then the masses of real and imaginary parts can be separately written as

\[
M_{\chi_{SR}}^2 = M_{\theta}^2 + \lambda_{16}V_\chi^2/2 + \sqrt{2}m_{(16,126)}V_R,
\]

(97)

\[
M_{\chi_{SI}}^2 = M_{\rho}^2 + \lambda_{16}V_\chi^2 - \sqrt{2}m_{(16,126)}V_R,
\]

(98)

(99)

where, in the minimal cases,

\[
M_{\theta}^2 = M_{16}^2 + \lambda_{(16,10)}V_\phi^2/2 + m_{(16,210)}V_{210}/2 + \lambda_{(16,45)}A_0^2/2 + \lambda_{(16,126)}V_R^2/2 + m_{(16,45)}A_0/\sqrt{2} + m_{(16,210)}V_{210}/\sqrt{2}.
\]

(100)

The second line of eq. (98) follows from eq. (92). Now using eq. (98) we have

\[
M_{\theta}^2 = -\lambda_{16}V_\chi^2 - \sqrt{2}m_{(16,126)}V_R,
\]

(101)

which through eq. (99) gives GUT scale mass to the imaginary part of the scalar singlet

\[
M_{\chi_{SI}}^2 = -2\sqrt{2}m_{(16,126)}V_R \sim M_{GUT}^2.
\]

(102)

Thus, once we realize \(V_\chi \sim M_W\) by fine tuning or, equivalently, \(\text{Re}(\chi_S)\) is made to acquire mass of \(O(M_W)\) to implement low scale matter parity breaking and resolve vacuum stability
issue of the scalar potential, the GUT scale mass of $I m(\chi_S)$ automatically follows. Because of such high value of its mass, $I m(\chi_S)$ decouples from making any contribution to the standard Higgs quartic couplings below the GUT scale reported in Sec. 9 and also all other relevant quantities predicted in Sec. 5 and Sec. 6. Thus self-consistency of model predictions is guaranteed. As already noted in Sec. 9.2, the self consistency of model predictions is maintained in the generalised case even when the effects of all additional GUT scale VEVs defined in eq. (88) are included.

10 Relic Density of Decaying Dark Matter and Flux of Ice-Cube Neutrinos

In this section we discuss how the proper relic density of the decaying dark matter candidate $\Sigma_F$ is realized within the present SO(10) framework to generate the expected flux of PeV energy IceCube neutrinos. It was shown by Griest and Kaminkowski [54] that any elementary particle having mass greater than 340 TeV can not have a thermal origin while the upper limit on thermal dark matter mass has been derived to be $\sim 200$ TeV [109]. As we are dealing with a very massive ($\sim$ PeV) Majorana type decaying dark matter (DDM), its relic density must have a non-thermal origin. In the following sections we shall discuss one such suitable mechanism compatible with our theoretical framework which can successfully address the observed relic density of the DDM.

10.1 Relic Density through the Exchange of a Heavy Scalar Singlet

As discussed earlier, the SO(10) representation $45_F$ has two singlets under SM. We have used the singlet fermion contained in the Pati-Salam sub-multiplet $(1, 1, 15)_F \subset 45_F$ as the Majorana fermion DM component. The present mechanism of generating relic density is similar to the non-equilibrium thermal dark matter (NETDM) mechanism discussed in a number of recent works in matter parity conserving non-SUSY SO(10) with high scale intermediate gauge symmetries [28, 110]. In the present work in the absence of any intermediate gauge symmetry, the desired heavy scalar singlet mediating the Higgs exchange process is naturally present at the GUT scale. At some early epoch during the evolution of the Universe, when the temperature of the thermal bath containing the SM particles is very high, the Majorana type fermionic DM ($\Sigma_F \subset 45_F$) can be produced due to the scattering of SM particles by the exchange of a very heavy scalar ($\xi \subset 210_H$) of mass $M_\xi \gg M_\Sigma$. This exchanged scalar can be the scalar singlet in $(1, 1, 15)_H \subset 210_H$. Alternatively, this exchanged scalar could be the $G_{224}$ singlet $(1, 1, 1)_H \subset 54_H$. Therefore the production rate of DM particles is kept at extremely small level with negligible self interaction probability among them. One notable aspect of this mechanism is that the DDM fermion $\Sigma_F$ having no renormalisable interaction with the SM particles is prevented from being in the thermal bath. Thus, while the DM particles are produced by SM particles in the thermal bath via heavy scalar exchange, they do not annihilate due to mutual interactions. Further they do not attain thermal equilibrium justifying their nomenclature as NETDM. In [28, 110], one main reason for the need of $G_{224}$ as intermediate gauge symmetry has been gauge coupling unification [50, 82] and the heavy scalar mass exchanged to achieve NETDM has been fine tuned to be at the $G_{224}$ breaking intermediate scale. Now that we achieve precision gauge coupling unification in Model-I and Model-II with minimally modified grand desert in each case, we need not adopt additional
fine tuning to assign the mass of $\xi$ at the intermediate scale. Its natural presence around the GUT scale successfully drives the DDM generation process. As discussed in Sec.3 $\Sigma_F$ possesses nonstandard Yukawa couplings $h_p$ and $h_e$ with $210_H$ and $54_H$, respectively. But only $h_p$ with $210_H$ of our minimal models has been shown to be enough to keep $\Sigma_F$ mass at PeV scale.

Thus, although the decaying dark matter $\Sigma_F \subset 45_F$ can not have renormalisable interaction directly with SM particles, it does possess such Yukawa interaction of the type $f_{210} \Sigma_F \Sigma_F \xi \subset f_{210} 45_F 45_F X_H$ where $X_H = 210_H$ or $54_H$. Thus $f_{210} \simeq O(h_p)$, or $\simeq O(h_e)$ of eq.(20) and eq.(21) of Sec.3. At the other end, the exchanged DM does interact via renormalisable quartic interaction $\lambda_{\xi} \phi \phi^\dagger \xi \subset \lambda_{\xi} 10_H 10_H X_H^\dagger X_H$ where $\phi$ = SM Higgs doublet and $\lambda_{\xi} \sim \lambda_{(10,210)}$ which has been also defined in Sec.3. Alternatively, when the $G_{224}$ singlet of $54_H$ is used instead of the singlet in $210_H$, this quartic coupling can be replaced correspondingly. No generality is lost even if this quartic coupling is treated to be small in the cases of the two SO(10) models. From these two interactions, we can easily construct the Feynman diagram shown in Fig.9 which elucidates the underlying NETDM type mechanism without any intermediate gauge symmetry for the production of the DDM. As explained above the DM interacts with the SM particle (the SM Higgs $h$) at high temperature bath whereas the DM itself can not be in the thermal bath.

To find out the relic density of the DM produced in this mechanism we have to solve the Boltzmann equation

$$\frac{dY_{\Sigma}}{dz} = \sqrt{\frac{\pi}{45}} \sqrt{g_\ast} M_{\Sigma} M_{pl} \frac{\langle \sigma v \rangle}{z^2} Y_{eq}^2,$$

where $Y_{\Sigma} = n_{\Sigma}/s$, $Y_{eq} = n_{eq}/s$, $n_{\Sigma}$ is the number density of the dark matter particle, $n_{eq}$ is the equilibrium number density of the initial state standard model particle and $s$ is the entropy density. In eq. \(103\) $z = M_{\Sigma}/T$ where $T$ is the temperature of the Universe, $M_{pl}$ is the Planck mass and $g_\ast = 107$ is the effective number of massless degrees of freedom. The thermally averaged cross section of the above process multiplied by the equilibrium density squared is given by

$$\langle \sigma v \rangle n_{eq}^2 \simeq \frac{T}{512 \pi^5} \int_{4M_{\Sigma}^2}^{\infty} d\hat{s} \sqrt{\hat{s} - 4M_{\Sigma}^2} K_1 \left( \frac{\sqrt{\hat{s}}}{T} \right) \sum |M|^2,$$

Figure 9: Feynman diagram for the decaying dark matter ($\Sigma_F$) production through the exchange of heavy scalar singlet $\xi$ contained in $210_H \subset$ SO(10). The mechanism also operates if the Higgs representation $210_H$ is replaced by $54_H \subset$ SO(10) as indicated in the parenthesis.

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where $\hat{s}$ represents the square of centre of mass energy, $K_1(x)$ is the Bessel function of second kind and $\mathcal{M}$ denotes the Feynman amplitude of the process under consideration. The summation is over all possible initial states as well as the spin of the final states. The sum of the modulus squared amplitude is calculated as

$$\sum |\mathcal{M}|^2 \simeq F_\xi \left( \frac{\hat{s} - 4M_{\xi}^2}{M_{\xi}^2} \right)$$  \hspace{1cm} (105)$$

where $F_\xi$ is a numerical factor which takes into account the couplings of the two vertices of the Feynman diagram of Fig. [3] i.e approximately

$$F_\xi \sim f_\Sigma \lambda_\xi.$$  \hspace{1cm} (106)$$

Substituting the expressions of thermally averaged annihilation cross section from eq.(104) and modulus squared amplitude from eq.(105) in the Boltzmann equation (eq.(103)), we integrate it to very high value of $z$ (or equivalently from $T_{RH}$ to temperature of present epoch), to get the present day dark matter density as

$$Y^{(\text{inf})}_{\Sigma} = \frac{F_\xi}{1024\pi^7} \left( \frac{45}{\pi g_*} \right)^{3/2} \frac{M_{pl}M_{\Sigma}}{M_{\xi}^2} \int_{T_{RH}}^{T_0} \frac{x^2}{x^2} \left( \frac{\hat{s}}{4x^2} \right)^{3/2} \frac{t(t^2 - 4x^2)^{3/2}K_1(t)}{t} dt dx$$ \hspace{1cm} (107)$$

where $T_0$ is the temperature at the present epoch. So the quantity $M_{\Sigma}/T_0$ tends to infinity and the dark matter density (scaled by entropy density) at present epoch is termed as $Y^{(\text{inf})}_{\Sigma}$. Under the plausible assumption that the dark matter mass is much smaller than the reheating temperature, $M_{\Sigma} \ll T_{RH}$. Then the abundance of dark matter particle at present epoch comes out to be

$$Y^{(\text{inf})}_{\Sigma} \simeq \left( \frac{45}{\pi g_*} \right)^{3/2} \frac{F_\xi M_{pl}T_{RH}}{64\pi^7 M_{\xi}^2}.$$ \hspace{1cm} (108)$$

Denoting values at the present epoch by subscript "(inf)", $Y^{(\text{inf})}_{\Sigma}$ is also related to known quantities

$$Y^{(\text{inf})}_{\Sigma} s^{(\text{inf})} M_{\Sigma} = (\Omega_{DM} h^2) \left[ \frac{\rho_{c}^{(\text{inf})}}{h^2} \right],$$ \hspace{1cm} (109)$$

where $h$ denotes the Hubble parameter and

$$\begin{align*}
(\Omega_{DM} h^2) &= 0.12, \\
\rho_{c}^{(\text{inf})}/h^2 &= 1.05 \times 10^{-5} \text{GeVcm}^{-3}, \\
s^{(\text{inf})} &= 2.89 \times 10^{-3} \text{cm}^{-3}. 
\end{align*}$$ \hspace{1cm} (110)$$

Then using eq.(110) and eq.(109) in eq.(108) we have

$$T_{RH} = 1.3 \times 10^7 \times \left[ \frac{\Omega_{DM} h^2}{0.12} \right]^{1/3} \left[ \frac{F_\xi^{-1} g_*^{3/2}}{10^6} \right] \left[ \frac{M_{\xi}}{10^{16}} \right]^2 \left[ \frac{M_{\Sigma}}{10^6} \right]^{-1}. \hspace{1cm} (111)$$
where $M_\xi, M_\Sigma$ are in GeV. As an illustration of resulting analytic solutions for the heavy scalar mass $M_\xi (\xi \subset 210_H)$ and reheating temperature $T_{RH}$ under this approximation we get for $\Omega_{DM} h^2 = 0.12$ and $M_\Sigma = 10^6$ GeV

$$T_{RH} = 1.3 \times 10^7 \text{ GeV}, M_\xi = 3.1 \times 10^{15} \text{ GeV}, F_{\xi}^{-1} g_{*}^{3/2} = 10^7,$$

$$T_{RH} = 1.3 \times 10^7 \text{ GeV}, M_\xi = 10^{16} \text{ GeV}, F_{\xi}^{-1} g_{*}^{3/2} = 10^6,$$

$$T_{RH} = 1.3 \times 10^9 \text{ GeV}, M_\xi = 10^{17} \text{ GeV}, F_{\xi}^{-1} g_{*}^{3/2} = 10^6,$$

$$T_{RH} = 1.3 \times 10^8 \text{ GeV}, M_\xi = 10^{17} \text{ GeV}, F_{\xi}^{-1} g_{*}^{3/2} = 10^5. \quad (112)$$

As discussed in Sec.7 the threshold corrected GUT scale in our model can easily acquire the range of values $M_{GUT} \approx 10^{15.5} - 10^{16.5}$ GeV. Therefore the desired superheavy Higgs scalar component of $\Phi^{(2)}(1,1,15)$ left over after spontaneous symmetry breaking naturally acts as the $\xi$ particle which is exchanged between the DM and the SM Higgs as shown in the Feynman diagram of Fig.9 to generate the desired relic density. Then this mass can be easily of the same order as $M_{GUT}$ Since $g_* \sim 100$, and $f_\Sigma \sim 0.1$, the desired quartic coupling of the exchanged scalar $\xi$ with SM Higgs scalar are $\lambda_{\xi} \sim 0.01$ when $F_{\xi}^{-1} g_{*}^{3/2} = 10^6, \lambda_{\xi} \sim 0.01 - 0.1$ when $F_{\xi}^{-1} g_{*}^{3/2} = 10^5$ and $\lambda_{\xi} \sim 0.0001 - 0.001$ when $F_{\xi}^{-1} g_{*}^{3/2} = 10^7$. These results are presented in Table 7.

Table 7: Allowed analytic solutions for exchanged heavy scalar mass $M_\xi$ and reheating temperature $T_{RH}$ as a function of quartic coupling $\lambda_{\xi}$ consistent with relic density $\Omega h^2 = 0.12$.

| $M_\xi$ (GeV) | $F_{\xi}^{-1} g_{*}^{3/2}$ | $\lambda_{\xi}$ | $T_{RH}$ (GeV) |
|---------------|-----------------|----------------|--------------|
| $3.1 \times 10^{15}$ | $10^6$ | $\sim 0.001$ | $1.3 \times 10^7$ |
| $10^{16}$ | $10^9$ | $\sim 0.01$ | $1.3 \times 10^7$ |
| $10^{17}$ | $10^9$ | $\sim 0.01$ | $1.3 \times 10^7$ |
| $10^{18}$ | $10^5$ | $\sim 0.1$ | $1.3 \times 10^8$ |

Alternatively, without using any approximation we have also carried out numerical integration of eq. (107) to express $T_{RH}$ as a function of DDM mass as shown in Fig.10 for $M_\xi = 10^{15.5}$ GeV.

In Fig. 11 numerical solutions for two different sets of $M_\xi, F_\xi$ combinations are presented showing variation of reheating temperature with DM mass.

In the region of interest the solutions presented in Fig. 11 are found to be similar to the analytic solutions given in eq. (112). Thus, it is clear that $T_{RH} \approx 10^7 - 10^9$ GeV could reproduce the desired relic density for DDM mass $M_{DM} = M_\Sigma = 10^6$ GeV with the exchanged heavy particle masses $M_\xi \sim 10^{15} - 10^{17}$ GeV predicted within this non-SUSY SO(10) GUT framework.

10.2 Flux of IceCube Neutrinos

In this subsection we try to explain the two PeV energy neutrino events [18] detected by IceCube through the decay of heavy Majorana type fermionic DM whose mass is also in the
The differential flux generated due to the decay of the DM in the Milky Way halo is given by

\[
\frac{d\Phi}{dE_\nu} = \frac{1}{4\pi M_\Sigma \tau_\Sigma} \frac{dN_\nu}{dE_\nu} \int_0^\infty ds \rho[r(s,l,b)]
\]

where \(M_\Sigma\) and \(\tau_\Sigma\) are mass and decay life time of the DDM particle \(\Sigma_F\) and \(\frac{dN_\nu}{dE_\nu}\) is the energy spectrum of the neutrinos produced due to the DM decay. Since in the present work we are concentrating only on one decay channel of the DM, i.e. \(\Sigma_F \to \nu h\), the spectrum would be a delta function \(\frac{dN_\nu}{dE_\nu} = \delta(E_\nu - \frac{M_\Sigma}{2})\). In eq. (113), \(\rho(r)\) is the density profile of the dark matter particle in our Galaxy where \(r\) is the distance from the galactic centre. For all numerical estimations we have used NFW profile for the density \(\rho(r)\), which is expressed as

\[
\rho(r) = \frac{\rho_h}{r_c(1 + \frac{r}{r_c})^2}.
\]

Here the critical radius \(r_c \simeq 20\) kpc and \(\rho_h \simeq 0.33\) GeVcm\(^{-3}\). The integral of eq. (113) over \(s\) variable is known as line of sight integral which has to be evaluated to obtain the flux at
Figure 11: Variation of reheating temperature as a function of decaying dark matter mass for two values of exchanged scalar masses $M_\xi = 10^{17}$ GeV with $\lambda_\xi = 0.1$ (upper curve) and $M_\xi = 10^{16}$ GeV with $\lambda_\xi = 0.01$ (lower curve) consistent with $\Omega_{DM} h^2 \sim 0.12$

earth and $s$ is related to $r$ through the relation

$$ r(s, l, b) = \sqrt{s^2 + R^2 + 2sR \cos b \cos l} $$

(115)

where $R \approx 8.5$ kpc is the distance of sun from the galactic centre and $l, b$ are galactic coordinates known as longitude and latitude, respectively. Now upon evaluating the integral eq.(113) can be written in a simpler form as

$$ \frac{d\Phi}{dE_\nu} = D_h \delta(E_\nu - M_\Sigma/2) $$

(116)

where

$$ D_h = 1.7 \times 10^{-13} \left( \frac{1 \text{PeV}}{M_\Sigma} \right) \left( \frac{10^{28} \text{s}}{\tau_\Sigma} \right) \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}. $$

(117)

The number of expected neutrino events at Icecube in small energy bin $\Delta E_\nu$ can be obtained by evaluating the following integral over the differential flux:

$$ N = \int_{\Delta E_\nu} \frac{d\Phi}{dE_\nu} A(E_\nu) dE_\nu $$

(118)

where $A(E_\nu)$ is the exposure at energy $E_\nu$. From the plot provided by Icecube collaboration [18], it is clear that the number of events near PeV energy for data accumulated during 662 days is nearly two, or more specifically at $E_\nu = 1.3$ PeV, $N = 2$ where both contain
certain experimental uncertainties. To explain this data point we take the mass of the DM to be $M_{\Sigma} = 2.6$ PeV and from [114] we find the 662 days exposure at $E_{\nu} = 1.3$ PeV as $A(E_{\nu} = 1.3 \text{PeV}) \simeq 10^{14} \text{cm}^{2} \text{s sr}$. Using these values in the above integral (eq.(118)), the number of events can be easily estimated. Now in the LHS of eq.(118) if we set the number of events to be $\sim 2$, then we get $\tau_{\Sigma} \simeq (2 - 3) \times 10^{28} \text{ s}$ which is in reasonable agreement with the value used in Sec.2 and [56].

11 Discussion, Summary, and Conclusion

In this work we have suggested non-SUSY SO(10) theory with extremely small matter parity violation as the underlying origin of decaying Majorana fermion dark matter that manifests as PeV scale high energy neutrino flux at IceCube. Since the Dirac neutrino Yukawa couplings in SO(10) are as hierarchical as the up-quark Yukawa couplings leading to predominantly and naturally hierarchical RH$\nu$ masses through type-I seesaw we have noted that equal branching ratio constraint on DM decay to every neutrino flavor can never be achieved with one common mixing as proposed in the benchmark model of extended SM with one common heavy RH$\nu$ mass [58]. Despite such diversities we have shown here that dark matter decay operates with separate distinct mixing with each of the RH$\nu$s. We have determined these three mixings for the first time by solving the underlying constraint relations on DM decay to three different neutrino flavors. The patterns of these three mixings are also found to depend upon the light neutrino mass hierarchy such as NH, IH or QD. In the SM extension [58], the RH$\nu$s and the DM fermion $\Sigma_{F}$ are externally added singlets with their assumed bare masses having no Higgs origins. Whereas in SO(10) models the RH$\nu$ is a member of matter unified spinorial representation $16_{F}$ of odd matter parity, we have identified the decaying DM $\Sigma_{F}$ to be a Majorana fermion singlet of even matter parity contained in $45_{F}$ of SO(10). Whereas the Higgs origin of RH$\nu$ is well known in SO(10), we have shown how the $\Sigma_{F}$ mass is predicted through matter parity conserving Higgs Yukawa interaction involving $210_{H}$ that drives GUT symmetry breaking in both the minimal models suggested here: Model-I and Model-II.

The direct breaking of SO(10) to SM predicts type-I seesaw dominance over type-II seesaw through a mild fine tuning of $126^{\dagger}_{H}$ Yukawa coupling which is also corroborated by the neutrino oscillation data and the underlying quark-lepton symmetry of SO(10).

Deeper theoretical origin of extremely small $N_{i} - \Sigma_{F}$ mixings is suggested in this work to be due to Planck-scale assisted intrinsic matter parity discrete symmetry breaking which is most desired for the resolution of the associated cosmological domain wall problem. This spontaneous symmetry breaking origin is also modeled to occur due to the bounded vacuum expectation value $V_{\chi} \leq v_{ew} = 246$ GeV of a matter-parity odd matter real scalar singlet $\chi_{S}(1, 0, 1)$. As a result, we predict a light Higgs scalar singlet of perturbative mass bound $M_{\chi_{S}} \leq 860$ GeV accessible to experimental searches at LHC and planned accelerators. The renormalizable and non-renormalizable values of different Yukawa couplings of $\chi_{S}$ underlying the small mixings have been explicitly derived by us for NH type light neutrino masses and the method can be used to predict the corresponding sets of values for other hierarchies like QD and IH. Interestingly, we have also shown how the mixings can be dynamically generated through a dim.5 operator but without using a Planck-mass fermion singlet $N'$. Whereas the SM extended model [58] rests upon the QD type light neutrino masses and is likely to be severely constrained if other types of hierarchies such as NH or IH are established in future, our SO(10)

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ansatz fits all types of mass hierarchies. Once the light neutrino mass hierarchy is established, this analysis will fix the associated Yukawa coupling matrix needed to explain the origin of $N_i - \Sigma_F$ mixings.

Quite interestingly, we have further shown that this light Higgs scalar $\chi_S$, besides generating the spontaneous breaking origin of the dark matter mixings with $N_i$, also resolves the issue of vacuum stability that persists in the SM and its extension. This scalar singlet has been identified as the real part of the complex scalar singlet in $16^\dagger_H$ and its $O(M_W)$ mass needed for model predictions has been realized by fine tuning of parameters in the SO(10) invariant potential. Self-consistency of the model predictions is guaranteed from the fact that the same fine tuning that gives $O(M_W)$ mass to $\text{Re}(\chi_S)$ also renders GUT scale mass to the imaginary part of the scalar singlet which, thus, decouples from contributing to all relevant physical quantities at all lower scales discussed in this work.

We have also shown that this theory of decaying dark matter in SO(10) predicts two different minimal modifications of the grand desert through Model-I and Model-II which achieve precision gauge coupling unification with vacuum stability of the SM in each case which are absent in the bench mark model. In Model-I the presence of the scalar component $\kappa(3,0,8) \subset 210_H$ of mass $M_\kappa = 10^{9.23}$ GeV in the grand desert achieves precision unification at $M_U^0 = 10^{15.224}$ GeV. This representation $210_H$ also provides the Higgs origin of the $\Sigma_F$ mass in addition to driving the direct breaking of SO(10) to SM. Although only an incomplete and limited unification aspect of this grand desert modification as in Model-I was noted earlier in 1993, the precision fitting with neutrino oscillation data, determination of $M_\kappa$, $\tau_p$, and the model association with decaying dark matter dynamics are new including the three different mixings and the prediction of the new Higgs scalar singlet $\chi_S$. Furthermore the completion of vacuum stability which was absent in the earlier work has been achieved in this work because of its decaying dark matter dynamics that predicts light Higgs scalar $\chi_S$ with mass $m_2 = M_{\chi_S} = 177$ GeV for which the perturbative upper bound is $M_{\chi_S} < 860$ GeV.

The pattern of precision coupling unification in the minimal Model-II is the first observation as noted in this work. In this case the new minimal modification of the grand desert is achieved by supplying the single scalar component $\eta(3,-1/3,6) \subset 126_H$ of mass $M_\eta = 10^{10.7}$ GeV again with precision proton lifetime predictions and solution to vacuum stability emerging from the model explanation of the dynamics of decaying dark matter. Like Model-I this model also predicts LHC detectable new light scalar with mass upper bound $M_{\chi_S} \leq 860$ GeV.

Our vacuum stability resolution predicts a lower mass for this Higgs scalar $m_2 = M_{\chi_S} = 177$ GeV.

Despite two large sized representations $126_H$ and $210_H$, the fermionic representation $45_F$, and also $45_H,16^\dagger_H$, Model-I and Model-II are noted to predict proton lifetime prediction up to $\tau_p \simeq 10^{35}$ yrs with reduced threshold uncertainties. This value is clearly within the accessible limit of ongoing Superkamiokande and Hyperkamiokande experiments.

Heavy Higgs scalar ( mass $M_\ell \sim M_{\text{GUT}}$) exchange mechanism that operates between the DDM and the SM Higgs has been shown to successfully generate the desired relic density of $\Sigma_F$. Its decay at the galactic center is also found to yield suitable IceCube neutrino flux.

In this work we have followed direct spontaneous symmetry breaking of SO(10)→ SM as explained in Sec.3. Realization of scalar or fermionic WIMP DM through $SU(5) \times U(1)_X$ intermediate symmetry at high scale has been discussed while advancing the application of matter parity in SO(10) [23–25]. All our results can apply also through such high scale $SU(5) \times U(1)_X$ intermediate breaking. An alternative interesting possibility of spontaneous
symmetry breaking of the Abelian subgroup $U(1)_X$ near the electroweak scale resulting in an extra low-mass neutral gauge boson is beyond the scope of the present work.

In conclusion, we have found that this non-SUSY SO(10) with its intrinsic matter parity is a self sufficient theory of decaying dark matter and neutrino physics that predicts all the particle content, the Higgs or seesaw origin of their masses and mixings, and the minimal modifications of the grand desert by a single scalar of intermediate mass for precision gauge coupling unification without the assistance of any externally imposed stabilising discrete symmetry or fermion singlets of SO(10) such as $N$. The $N_i - \Sigma_F$ mixings are extremely small because of the underlying intrinsic matter parity of broken SO(10) as gauged discrete symmetry whose breaking must be assisted by gravity or the Planck-scale for cosmologically safe acceptable solutions. Thus, a Planck scale suppression of mixings naturally emerges in both Model-I and Model-II. Another factor contributing to the suppression of this mixings is the matter parity conservation constraint of SM that restricts the singlet VEV $V_\chi \leq v_{ew}$ leading to the experimentally testable new Higgs scalar mass with perturbative upper bound $m_\chi S \leq 860$ GeV. The resolution of vacuum instability issue predicts its actual mass $m_\chi S = 177$ GeV. Starting from parity and matter parity invariant SO(10) gauge theory, in this work we have suggested experimental evidence of matter parity nonconservation at IceCube.

Prospects of this model for scalar singlet WIMP dark matter including the detection possibility of the new Higgs scalar $\chi_S(1, 0, 1)$ would be discussed elsewhere.

12 APPENDIX A: Diagonalisation of RH\$ Mass Matrices

It is to be noted that the light neutrino mixing matrix in PMNS parametrization is given by

$$U_{\text{PMNS}} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \text{diag}(e^{-i\frac{m_\nu}{2}}, e^{i\frac{m_\nu}{2}}, 1).$$

For the sake of simplicity, primarily during the calculation of the light neutrino mass matrix, the Majorana phases are assumed to be zero and unphysical phases are also not considered. The best fit values of the solar and atmospheric mass squared differences [81] are used to calculate the light neutrino mass eigenvalues for different hierarchies as shown in table 8. Using these mass eigenvalues and best fit values [81] of the mixing angles and Dirac CP phase, light neutrino mass matrix $m_\nu$ is calculated. It is then straightforward to calculate the RH$\nu$ mass matrix ($M_N$) using $m_\nu$ and $m_D$ in the u-quark or d-quark diagonal basis. Further $M_N$ is diagonalised by unitarity $V_P$ matrix. This $V_P$ matrix is written in two parts, one $3 \times 3$ matrix which includes mixing angles, Dirac CP phase and unphysical phases) and the other is the multiplicative diagonal Majorana phase matrix ($P_m$).

The $V_P$ matrices for different cases are given below

**Case 1: NH with u-quark diagonal basis**

$$V_P = \begin{pmatrix}
    -0.513 - 0.858i & -0.0023 - 0.0011i & (0.8921 + 8.991i) \times 10^{-6} \\
    -0.0016 + 0.0019i & 0.130 - 0.99i & 0.00035 - 0.0027i \\
    (0.17 + 1.4i) \times 10^{-5} & -0.0003 - 0.0027i & 0.1217 + 0.992i
\end{pmatrix} \times P_m$$

(120)
Table 8: Light neutrino mass eigenvalues for different mass hierarchies (taking into account best fit values of the mass squared differences)

| Mass ordering | $m_{\nu 1}$ (eV) | $m_{\nu 2}$ (eV) | $m_{\nu 3}$ (eV) |
|---------------|------------------|------------------|------------------|
| NH            | 0.00127          | 0.008809         | 0.049815         |
| QD1           | 0.0630           | 0.0636           | 0.08             |
| QD2           | 0.1938           | 0.1940           | 0.2              |
| IH            | 0.0476           | 0.0484           | 0.00127          |

where $P_m = \text{diag}(e^{i\alpha M/2}, e^{i\beta M/2}, 1)$ with $\alpha_M = 79.83^\circ, \beta_M = -19.1^\circ$ and RH$\nu$ mass eigenvalues are $M_{N_1} = 8.9 \times 10^9$ GeV, $M_{N_2} = 2.28 \times 10^9$ GeV, $M_{N_3} = 1.19 \times 10^{15}$ GeV.

Case 2: QD1 with u-quark diagonal basis

$$V_P = \begin{pmatrix}
-0.992 - 0.118i & -0.0003 - 0.00007i & (0.1193 + 1.314i) \times 10^{-6} \\
-9.38 \times 10^{-6} + 0.0003i & 0.0944, -0.995i & 0.0004 - 0.004727i \\
9.363 \times 10^{-9} + 1.35 \times 10^{-6}i & -2.921 \times 10^{-6} - 0.00042361i & 0.0068 + 0.999i
\end{pmatrix} \times P_m$$

(121)

where $P_m = \text{diag}(e^{i\alpha M/2}, e^{i\beta M/2}, 1)$ with $\alpha_M = 167.08^\circ, \beta_M = -11.57^\circ$ and RH$\nu$ mass eigenvalues are $M_{N_1} = 4.7 \times 10^4$ GeV, $M_{N_2} = 9.64 \times 10^8$ GeV, $M_{N_3} = 9.22 \times 10^{15}$ GeV.

Case 3: QD2 with u-quark diagonal basis

$$V_P = \begin{pmatrix}
-0.999 - 0.0285i & -0.0003 - 0.0001i & (0.011 + 2.23i) \times 10^{-8} \\
0.00033i & 0.3389 - 0.940i & 0.000034 - 0.00009i \\
7.196 \times 10^{-9} + 1.20i \times 10^{-6} & -0.000100356i & 0.0059 + 0.999i
\end{pmatrix} \times P_m$$

(122)

here $P_m = \text{diag}(e^{i\alpha M/2}, e^{i\beta M/2}, 1)$ with $\alpha_M = -171.31^\circ, \beta_M = -40.18^\circ$ and RH$\nu$ mass eigenvalues are $M_{N_1} = 1.54 \times 10^3$ GeV, $M_{N_2} = 3.47 \times 10^8$ GeV, $M_{N_3} = 3.35 \times 10^{13}$ GeV.

Case 4: IH with u-quark diagonal basis

$$V_P = \begin{pmatrix}
(-3.101 - 0.01281i) \times 10^{-6} & 0.00001 + 0.00047i & 0.0133 - 0.999i \\
0.0022 + 0.00147i & -0.8378 - 0.5459i & -0.00038 - 0.00027i \\
0.837 + 0.5459i & 0.00225 + 0.00147i & 0
\end{pmatrix} \times P_m$$

(123)

here $P_m = \text{diag}(e^{i\alpha M/2}, e^{i\beta M/2}, 1)$ with $\alpha_M = 113.83^\circ, \beta_M = 113.83^\circ$ and RH$\nu$ mass eigenvalues are $M_{N_1} = 2.97 \times 10^{15}$ GeV, $M_{N_2} = 2.49 \times 10^9$ GeV, $M_{N_3} = 6.15 \times 10^3$ GeV.
**Case 5: NH with d-quark diagonal basis**

\[
V_P = \left( \begin{array}{ccc}
-0.9348 - 0.2723i & -0.0282 - 0.225i & -0.00219 + 0.00803i \\
0.2187 + 0.0637i & -0.119 - 0.9655i & 0.00554 + 0.0386i \\
-0.000445 - 0.00351i & 0.00499 + 0.0394i & 0.1254 + 0.9912i
\end{array} \right) \times P_m
\]

where \( P_m = \text{diag}(e^{i\alpha M/2}, e^{i\beta M/2}, 1) \) with \( \alpha_M = -177.08^\circ, \beta_M = -3.6^\circ \) and RH\( \nu \) mass eigenvalues are \( \hat{M}_{N_1} = 5.85 \times 10^4 \text{ GeV}, \hat{M}_{N_2} = 3.69 \times 10^8 \text{ GeV}, \hat{M}_{N_3} = 1.14 \times 10^{15} \text{ GeV}. \)

**Case 6: QD1 with d-quark diagonal basis**

\[
V_P = \left( \begin{array}{ccc}
-0.9046 - 0.3615i & 0.00073 - 0.225i & -0.00322 + 0.00801i \\
0.2095 + 0.0840i & 0.00504 - 0.973i & 0.00053 + 0.0403i \\
-0.000031 - 0.0035i & 0.00036 + 0.0411i & 0.0088 + 0.9991i
\end{array} \right) \times P_m
\]

where \( P_m = \text{diag}(e^{i\alpha M/2}, e^{i\beta M/2}, 1) \) with \( \alpha_M = 145.5^\circ, \beta_M = -5.3^\circ \) and RH\( \nu \) mass eigenvalues are \( \hat{M}_{N_1} = 4.71 \times 10^3 \text{ GeV}, \hat{M}_{N_2} = 9.9 \times 10^8 \text{ GeV}, \hat{M}_{N_3} = 9.14 \times 10^{13} \text{ GeV}. \)

**Case 7: QD2 with d-quark diagonal basis**

\[
V_P = \left( \begin{array}{ccc}
0.9044 + 0.361i & 0.00102 - 0.225i & -0.00323 + 0.0080i \\
-0.2094 - 0.084i & 0.00621 - 0.9733i & 0.00048 + 0.0406i \\
0.00002 + 0.0035i & 0.0003 + 0.0414i & 0.0078 + 0.999i
\end{array} \right) \times P_m
\]

where \( P_m = \text{diag}(e^{i\alpha M/2}, e^{i\beta M/2}, 1) \) with \( \alpha_M = 9.27^\circ, \beta_M = -3.35^\circ \) and RH\( \nu \) mass eigenvalues are \( \hat{M}_{N_1} = 1.55 \times 10^3 \text{ GeV}, \hat{M}_{N_2} = 3.5 \times 10^8 \text{ GeV}, \hat{M}_{N_3} = 3.35 \times 10^{13} \text{ GeV}. \)

**Case 8: IH with d-quark diagonal basis**

\[
V_P = \left( \begin{array}{ccc}
0.00920815 + 0.000266017i & -0.213401 + 0.0719151i & 0.0459181 - 0.973188i \\
0.040555 - 0.0143812i & -0.922805 + 0.309508i & -0.0107965 + 0.225095i \\
0.942383 - 0.33163i & 0.0413766 - 0.0145607i & 0.00331518 - 0.00116663i
\end{array} \right) \times P_m
\]

where \( P_m = \text{diag}(e^{i\alpha M/2}, e^{i\beta M/2}, 1) \) with \( \alpha_M = -141.25^\circ, \beta_M = -144.9^\circ \) and RH\( \nu \) mass eigenvalues are \( \hat{M}_{N_1} = 3.177 \times 10^{15} \text{ GeV}, \hat{M}_{N_2} = 2.23 \times 10^9 \text{ GeV}, \hat{M}_{N_3} = 6.59 \times 10^9 \text{ GeV}. \)
13 APPENDIX B: Renormalization Group Solutions for Mass Scales and Threshold Effects

The RG equations for SM gauge couplings and top quark yukawa coupling at two loop level are given by

\[
\frac{d\alpha_i}{d\ln\mu} = \frac{1}{16\pi^2} \left( \frac{9}{2} y_{i\alpha}^2 - \frac{17}{12} g_1^2 Y - \frac{9}{4} g_2^2 L - 8 g_3^2 C \right) y_{i\alpha} + \frac{1}{(16\pi^2)^2} \left[ -\frac{23}{4} g_2^4 L + \frac{3}{4} g_2^2 g_1^2 Y + \frac{1187}{216} g_1^4 Y + 9 g_2^2 g_3^2 C + \frac{19}{9} g_3^2 g_1^2 Y - 108 g_3^4 C \right] \]
\[
+ \left( \frac{225}{16} g_2^2 L + \frac{131}{16} g_1^2 Y + 36 g_3^2 C \right) y_{i\alpha}^2 + 6(-2 g_1^4 Y - 2 g_{i\alpha}^2 \lambda \phi^2 + \lambda_s^2) \tag{128}
\]

\[
\frac{d\lambda_i^S}{d\ln\mu} = \frac{1}{16\pi^2} \left( \frac{41}{6} g_1^3 Y + \frac{1}{(16\pi^2)^2} \left( \frac{199}{18} g_1^2 Y + \frac{9}{2} g_2^2 L + \frac{44}{3} g_3^2 C - \frac{17}{6} y_{i\alpha} \right) g_1^3 Y \right) \tag{129}
\]

\[
\frac{d\lambda_i^L}{d\ln\mu} = \frac{1}{16\pi^2} \left( -\frac{19}{6} g_3^3 C \right) + \frac{1}{(16\pi^2)^2} \left( \frac{3}{2} g_1^2 Y + \frac{35}{6} g_2^2 L + 12 g_3^2 C - \frac{3}{5} y_{i\alpha} \right) g_2^3 L \tag{130}
\]

\[
\frac{d\lambda_i^C}{d\ln\mu} = \frac{1}{16\pi^2} \left( -7 g_3^3 C \right) + \frac{1}{(16\pi^2)^2} \left( \frac{11}{6} g_1^2 Y + \frac{9}{2} g_2^2 L - 26 g_3^2 C - 2 y_{i\alpha} \right) g_3^3 C \tag{131}
\]

The matching formula for different gauge couplings\(\alpha_i^{-1}, i = 2L, Y, 3C\) at the unification scale is given by

\[
\alpha_i^{-1}(M_U) = \alpha_i^{-1} - \frac{\lambda_i(M_U)}{12\pi}, \tag{132}
\]

where \(\lambda_i, i = 2L, Y, 3C\) are matching functions due to superheavy scalars (S), Majorana fermions (F) and gauge bosons (V),

\[
\lambda_i^S(M_U) = \sum_j Tr \left( t_{iSj} \hat{p}_{Sj} \ln \frac{M_j^S}{M_U} \right),
\]

\[
\lambda_i^L(M_U) = \sum_k 4Tr \left( t_{iFk} \ln \frac{M_k^F}{M_U} \right),
\]

\[
\lambda_i^C(M_U) = \sum_l Tr \left( t_{iVL}^L \right) - 21 \sum_l Tr \left( t_{iVL}^C \right), \tag{133}
\]

where \(t_{iS}, t_{iF}\) and \(t_{iV}\) represent the matrix representations of broken generators for scalars, Majorana fermions, and gauge bosons, respectively. The term \(\hat{p}_{Sj}\) denotes the projection operator that removes the Goldstone components from the scalar that contributes to spontaneous symmetry breaking.

Decomposition of different SO(10) representations under \(G_{213}\) with respect to their superheavy components are given in Table 9. We use the following notations for the respective components \(10_H \supset H_i, 126_H \supset H_i', 16_H \supset H_i''\), \(45_H \supset S_i, 210_H \supset S_i', \) and \(45_F \supset F_i\).

Using Table 9, we calculate values of matching functions \(\lambda_i(M_U), i = 2L, Y, 3C\).
Analytic formulas for the unification scale and \( \kappa \) mass constrained by gauge coupling unification are

\[
\begin{align*}
\ln \frac{M_U}{M_Z} &= \frac{16\pi}{187\alpha} \left( \frac{7}{8} - \frac{10\alpha}{3\alpha_3 C} + s_W^2 \right) + \Delta_{ij}^U \\
\ln \frac{M_\kappa}{M_Z} &= \frac{4\pi}{187\alpha} \left( \frac{15 + 23\alpha}{3\alpha_3 C} - 63s_W^2 \right) + \Delta_{ij}^\kappa \\
\frac{1}{\alpha_G} &= \frac{3}{8\alpha} + \frac{1}{187\alpha} \left( \frac{347}{8} + \frac{466\alpha}{3\alpha_3 C} - 271s_W^2 \right) + \Delta_{ij}^{\alpha_G}
\end{align*}
\]

where \( s_W^2 = \sin^2 \theta_W (M_Z) \) and the first term in the above equation represent one-loop contributions. The terms \( \Delta_{ij}^i, i = U, \kappa, \alpha_G \) denoting the threshold corrections due to unification scale \( (M_U) \), intermediate scale \( (M_\kappa) \) and GUT fine structure constant \( (\frac{1}{\alpha_G}) \) are given by

\[
\begin{align*}
\Delta_{ij}^U &= \Delta \ln \frac{M_U}{M_Z} = \frac{1}{561} (-48\lambda_{2L} + 25\lambda_Y + 23\lambda_{3C}) \\
\Delta_{ij}^\kappa &= \Delta \ln \frac{M_\kappa}{M_Z} = \frac{5}{3366} (9\lambda_{2L} + 7\lambda_Y - 16\lambda_{3C}) \\
\Delta_{ij}^{\alpha_G} &= \Delta \left( \frac{1}{\alpha_G} \right) = \frac{1}{13464\pi} (-945\lambda_{2L} + 1135\lambda_Y + 932\lambda_{3C}),
\end{align*}
\]

Here \( a_i \), \( b_{ij} \) and \( a_{ij}' \), \( b_{ij}' \) are one loop and two loop beta function coefficients for the range of mass scales \( M_Z - M_U \) and \( M_\kappa - M_U \), respectively, they are given in the Table 10.
Table 10: One loop and two loop beta function coefficients for RG evolution of gauge couplings

| Mass scales(µ) | a_i | b_ij |
|---------------|-----|------|
| M_Z < µ < M_κ | \begin{pmatrix} 11 \\ -19 \\ -7 \end{pmatrix} | \begin{pmatrix} 41 \\ 19 \\ 10 \end{pmatrix} |
| M_κ < µ < M_U | \begin{pmatrix} 11 \\ -1 \\ -1 \end{pmatrix} | \begin{pmatrix} 77 \\ 9 \\ 10 \end{pmatrix} |

Using one loop beta function coefficients from Table 10 in eq.(134), we get

\begin{align*}
M_U &= 10^{15.2446} \text{ GeV}, \\
M_κ &= 10^{9.23} \text{ GeV}, \\
α^{-1}_G &= 41.77. 
\end{align*}

(136)

Contribution due to gauge coupling matching at the GUT scale that occurs even when all superheavy masses are identical to M_U [82] has been included. Using eq.(135) and values of matching functions, we estimate the corrections to mass scales due to superheavy masses in partially degenerate case when all the superheavy component masses belonging to a definite SO(10) representation are degenerate [77,86,87],

\begin{align*}
∆ \ln \frac{M_U}{M_Z} &= -0.0196078η_{10} - 0.037433η_{126} + 0.0322341η_{210} \\
∆ \ln \frac{M_κ}{M_Z} &= 0.05882η_{10} - 0.160428η_{126} + 1.13815η_{210} \\
∆ \left( \frac{1}{α_G} \right) &= 0.059293η_{10} + 1.62758η_{126} + 1.72778η_{210}.
\end{align*}

(137)

Maximising the threshold uncertainty in M_U leads to

\begin{align*}
∆ \ln \left( \frac{M_U}{M_Z} \right) &= ±0.089275|η_{SH}|, \\
∆ \ln \left( \frac{M_κ}{M_Z} \right) &= ±1.23975|η_{SH}|, \\
∆ \left( \frac{1}{α_G} \right) &= ±0.040907|η_{SH}|.
\end{align*}

(138)

where η_{SH} = ln(M_{SH}/M_U) and (M_{SH}/M_U) = n(1/n) with plausible allowed values of real number n = 1 − 10.

Similarly the threshold effects due to superheavy gauge boson components in 45_V have been estimated as shown in eq.(69) and eq.(73) in Sec.7. Threshold effects due to 45_F, 45_H, 54_H and 16_H are noted to vanish in the minimal model due to the theorem [99]. In the case of complete degeneracy in superheavy scalar masses from all representations, including the threshold
effects are

\[
\Delta \ln \left( \frac{M_U}{M_Z} \right) = \pm 0.0248 |\eta_{SH}|, \\
\Delta \ln \left( \frac{M_\eta}{M_Z} \right) = \pm 1.0365 |\eta_{SH}|.
\]

(139)

Most dominant threshold uncertainty on the unification mass and proton lifetime occurs due
to superheavy gauge bosons

\[
[\Delta \ln \left( \frac{M_U}{M_Z} \right)]_V = -0.9358 \eta_V,
\]

(140)

where \( \eta_V = \ln (M_V/M_U) \). Thus degenerate superheavy gauge boson masses few times lighter
than \( M_U \) can cause substantial enhancement in proton lifetime prediction.

13.2 Minimal Model-II

Analytic formulas for the two mass scales \( M_U \) and \( M_\eta \) are

\[
\ln \frac{M_U}{M_Z} = \frac{18\pi}{247\alpha} \left( 1 + \frac{4}{3} s_W^2 - \frac{4}{3} \frac{\alpha}{\alpha_3C} \right) + \Delta_{UII}^V, \\
\ln \frac{M_\eta}{M_Z} = \frac{4\pi}{247\alpha} \left( 16 + \frac{55}{3} \frac{\alpha}{\alpha_3C} - 61 s_W^2 \right) + \Delta_{\eta II}^V, \\
\frac{1}{\alpha_G} = \frac{1}{494\alpha} \left( 241 - 502 s_W^2 + \frac{1060}{3} \frac{\alpha}{\alpha_3C} \right) + \Delta_{GII}^V.
\]

(141)

where \( \Delta_{Ii}^V, i = U, \eta, \alpha_G \) denote threshold corrections to unification scale\( (M_U) \), intermediate
scale\( (M_\eta) \), and inverse GUT fine structure constant\( (1/\alpha_G) \)

\[
\Delta_{UII}^V = \Delta \ln \left( \frac{M_U}{M_Z} \right)_V = \frac{1}{494}(5\lambda_Y + 7\lambda_{2L} - 12\lambda_{3C}) \\
\Delta_{\eta II}^V = \Delta \ln \left( \frac{M_\eta}{M_Z} \right)_V = \frac{5}{2223}(27\lambda_{2L} - 16\lambda_Y - 11\lambda_{3C}) \\
\Delta_{GII}^\alpha = \Delta \left( \frac{1}{\alpha_G} \right)_V = \frac{1}{17784\pi}(-783\lambda_{2L} + 1205\lambda_Y + 1060\lambda_{3C}).
\]

(142)

One loop and two loop beta function coefficients for different ranges of mass scales are
given in the Table.11

Using one loop beta function coefficients from Table.11 in eq.(141), we get respective
solutions for mass scales

\[
M_U = 10^{15.2835} \text{GeV}, \\
M_\eta = 10^{10.73} \text{GeV}, \\
\alpha_G^{-1} = 38.397.
\]

(143)

We estimate threshold corrections to different mass scales and GUT gauge coupling fol-
lowing procedures similar to model-I
Table 11: One loop and two loop beta function coefficients for RG evolution of gauge couplings

| Mass scales($\mu$) | $a_i$ | $b_{ij}$ |
|---------------------|-------|---------|
| $M_Z < \mu < M_\eta$ | $\left(\begin{array}{c} \frac{11}{6} \\ \frac{9}{10} \\ -\frac{1}{2} \\
\end{array}\right)$ | $\left(\begin{array}{ccc} \frac{19}{10} & \frac{19}{10} & \frac{22}{5} \\
\frac{9}{10} & \frac{9}{10} & 12 \\
-\frac{7}{10} & -\frac{7}{10} & -26 \\
\end{array}\right)$ |
| $M_\eta < \mu < M_U$ | $\left(\begin{array}{c} \frac{5}{2} \\ \frac{1}{2} \\
\frac{1}{2} \\
\end{array}\right)$ | $\left(\begin{array}{ccc} \frac{5}{4} & \frac{19}{10} & \frac{47}{5} \\
\frac{5}{4} & \frac{19}{10} & \frac{47}{5} \\
\frac{1}{2} & \frac{1}{2} & 172 \\
\end{array}\right)$ |

\[\Delta \ln \frac{M_U}{M_Z} = -0.020243\eta_{10} + 0.044534\eta_{126} - 0.0809717\eta_{210}\]

\[\Delta \ln \frac{M_\eta}{M_Z} = -0.039136\eta_{10} - 0.7139\eta_{126} - 0.323212\eta_{210}\]

\[\Delta \left(\frac{1}{\alpha_G}\right) = 0.0541256\eta_{10} + 1.50961\eta_{126} + 1.17143\eta_{210}\]

(144)

Maximising the uncertainty in $M_U$ leads to

\[\Delta \ln \left(\frac{M_U}{M_Z}\right) = \pm 0.145749|\eta_{SH}|,\]

\[\Delta \ln \left(\frac{M_\eta}{M_Z}\right) = \pm 0.3515517|\eta_{SH}|,\]

\[\Delta \left(\frac{1}{\alpha_G}\right) = \pm 0.284|\eta_{SH}|,\]

(145)

where $\eta_{SH} = \ln\left(\frac{M_{SH}}{M_U}\right)$ and $M_{SH}/M_U = n(1/n)$ with plausible allowed values of real number $n = 1 - 10$. Neglecting threshold effects due to superheavy gauge bosons, our estimation gives for completely degenerate superheavy scalar masses of all SO(10) representations

\[\Delta \ln \left(\frac{M_U}{M_Z}\right) = \pm 0.05668|\eta_{SH}|,\]

\[\Delta \ln \left(\frac{M_\eta}{M_Z}\right) = \pm 1.07625|\eta_{SH}|.\]

(146)

As in Model-I, the most dominant contribution to the GUT scale and proton lifetime uncertainties is due to superheavy gauge boson masses

\[\left[\Delta \ln \left(\frac{M_U}{M_Z}\right)\right]_V = -0.9352\eta_V,\]

(147)

where $\eta_V = \ln\left(\frac{M_V}{M_Z}\right)$. Thus degenerate superheavy gauge boson masses only few times lighter than $M_U$ can cause substantial enhancement in proton lifetime prediction. An well known potential of SO(10) for fitting all charged fermion masses [116][118] is beyond the scope of the present work.
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