W-shaped profile and breather-like soliton of the fractional nonlinear Schrödinger equation describing the polarization mode in optical fibers

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Abstract
We use the fractional nonlinear Schrödinger equation (FNLSE) to describe the polarization mode in optical fiber with Self-Steepening, Self-Frequency Shift, and Cubic-quintic terms to analyze the effects of the fractional time parameter (FTP) on bright and dark solitons as well as breather-like solitons. We use the transformation hypothesis and auxiliary equations method to obtain three families of solutions such as combined bright soliton, dark solitons, and rational solitons. We have shown the effects of the fractional parameter (FP) on the W-shaped profile, bright and dark optical soliton solutions as well as the corresponding chirp component. It is observed that for small values of the FP, optical soliton shape is affected and the soliton is unstable. Moreover, one observes the effects of fraction time on Modulation Instability (MI) gain spectra and MI bands. For certain values of the FP, it is formed sides lobes and for specific small values of the FP, both stability zone increases and amplitude of the MI gain increase while the stability zones increase. To confirm the robustness of the analytical results, we have used a numerical investigation. One exhibits the formation of breathers-like soliton with stable amplitude for small values of the FTP. It results from this study that the FP is efficient and can be used as an energy source where soliton or breathers-like soliton are involved for communication in optical fibers.

Keywords W-shaped profile · Breather-like soliton · Fractional nonlinear Schrödinger equation

1 Introduction

During the last past 10 years, diverse research has successfully emerged in different fields of science such as biology, plasma, fluid mechanic, metamaterials, optical fiber (OF) and so on (Yusuf et al. 2019; Gómez et al. 2021; Jhangeer et al. 2020, 2021a, b; Houwe et al. 2020; Li et al. 2011; Tanemura et al. 2004). One of the exact traveling waves solutions most investigated during the last past years is a solitary wave, called “soliton” which arises from the interaction between nonlinearity and dispersion. Soliton can maintain its shape during
propagation with a constant speed and can also conserve amplitude after collision with another soliton. Nowadays, soliton investigation has achieved the highest level in OF owing to the fact that many parameters are involved during its propagation namely Group velocity Dispersion (GVD), nonlinearity Kerr, Self-phase modulation (SPM), Cross-phase modulation (XPM) just to name a few (Tanemura et al. 2004; Murdoch et al. 1995; Rothenberg 1990; Drummond et al. 1990). It should be noted that solitons facilitated the transport of information, as well as the design of components serving to enhance the signal amplifier, stabilizer, and sound level detectors during communication. Certainly, optical parameter’s effects on solitons were deeply investigated and a large variety of the nonlinear evolution equations (NLEE) which describe the propagation of pulse and ultrashort pulse in OF have been enrolled. We underline that NLEE with conformable derivative order (CFDO) are the most involved today owing to the fact that the conformable fractional derivative (CFD) parameter can play an important role during the propagation of soliton and in the well-known aspect of the memory. Most often the NLSE with CFD order is used to stress analytical results in optical fibers. During the last decades, fractional differential equations have become important in diverse areas of science. Several definitions of fractional derivatives have been pointed out in many works, such as a new definition of a local fractional derivative of order \(a\) (Yépez-Martínez et al. 2022), the fractional-order spatial derivatives of the Riesz type, fractional diffraction operator with the Lévy index \(\alpha\) based on Levy \(\alpha\)-stable processes in quantum mechanics (Boris 2021), Grunwald–Letnikov definitions and Riemann–Liouville, Caputo, Atangana–Balchenu derivative in Caputo sense, Atangana–Balchenu fractional derivative in Riemann–Liouville sense and the well known CFD (Podlubny 1999; Abdon and Dumitru 2016; Khalil et al. 2014). For example, in Houwe et al. (2021a, 2021b), it has been established the effects of the fraction time (FT) order on optical solitons and MI gain. The authors show for the weak value of CFP, bright and dark optical soliton shapes are modified. More recently, in Bienvenu et al. (2021), the authors use the Chen Lee–Liu model with CF derivative time to figure out the importance of the fractional derivative on several analytical results. Many other works are concerned in literature to show the interesting features of the CFP, but just a few have focussed on the numerical investigation to fit with the predictions made on analytical investigation.

In this work, we aim to show the effects of the conformable time-fractional (CTF) parameter (Khalil et al. 2014) on exact traveling waves solution and optical soliton. We assume the dimensionless NLSE with CTF describing the polarization mode in OF. The model parameters are Self-Steepening (SS), Self-Frequency Shift (SFS), and the nonlinearities Kerr, Cubic-quintic (CQ) in the presence of lower GVD. It has been shown by Eslami et al. (2021) and coworkers obtained analytical solutions such as chirped optical soliton. The model was also recently studied by Tebue et al. (2019) to investigate chirped soliton in OF. Bright, dark, and cnoidal solitons are obtained. Out of this, several others works have been done in literature to find analytic exact traveling waves solutions by employing a huge class of mathematical methods such as Ansatz method, Auxiliary equation method and its modification, Rational \((1/G(\xi))\)-Expansion Method, \((G'/G)\) Expansion Method and its modification, \((G'/G)\) Expansion Method and its modification, Generalized exponential rational function method, Generalized Auxiliary equation method, Kudryashov method and its modification, Sardar sub-equation method, sine-Gordon expansion approach, double Laplace transform method, Rational method, \((G'/G)\)-Expansion Method, Fan-Extended Sub Equation (FESE) technic, Extended Modified Method, Modified Simple Equation Method, \(\exp(-\varphi(q))\) Method, Sub-ODE Method, Generalized Extended Algebraic Method (GEAM) and so on (Yusuf et al. 2019; Gómez
et al. 2021; Jhangeer et al. 2020, 2021a, b; Li et al. 2011; Tanemura et al. 2004; Murdoch et al. 1995; Rothenberg 1990; Drummond et al. 1990; Houwe et al. 2020a, b, 2021a, b, c, d, e; Bienvenu et al. 2021; Eslami et al. 2021; Tebue et al. 2019; Khalid and Gómez-Aguilar 2021; Yépez-Martínez et al. 2022; Tarikul et al. 2021a, b, c; Khater et al. 2021; Mohamed et al. 2022; Akinyemi et al. 2022; Kumar et al. 2022; Melike et al. 2016; Hosseini et al. 2017; Mohammed et al. 2021; Desaix et al. 2002; Susanto and Malomed 2021; Rao et al. 2020; Souleymanou et al. 2019a, b, 2021a, b, c, d, e; Ablowitz and Horikis 2017; Mukame et al. 2018; Yepez-Martinez et al. 2019).

In our context, to examine the behavior of the analytic solutions and propagation of the CW in the normal and anomalous regimes, we use New Generalized Auxiliary Equation Method (NGAEM) and the linearizing technic. By varying the FP and fixing the values of the model, we establish how the shape of the W-shape profile and dark soliton are affected. Using the same procedure we show the variation of the MI gain. Throughout the numerical simulation, we establish the propagation of the breather-like soliton and chaotic breather-like soliton for certain values of the FTP.

The followin is how we organize our work: In Sect. 2 we summarize the NGAEM. In Sect. 3, we apply the method and show the effects of the FTP on W-shaped, bright, dark optical soliton. The linearizing scheme is used in Sect. 4 to establish the MI growth rate and set out the stable/unstable domains. To manage the robustness of the analytical results, we use numerical simulation in Sect. 5. We conclude the work in the last section.

2 Methodology

We stress the main steps of the NGAEM for finding traveling wave solutions for NLPDEs.

Step 1. Given a nonlinear physical model governed by partial differential equation

\[ G(\Phi, \Phi_t, \Phi_{tt}, \Phi_{zz}, \ldots) = 0. \]  

Using the wave transformation \( \Phi(z, t) = \Gamma(\zeta), \zeta = z - vt \), Eq. (1) is transformed into an ordinary differential equation (ODE) given by

\[ F(\Gamma, \Gamma', \Gamma'', \ldots) = 0. \]  

Step 2. Suppose that the solution of ODE Eq. (2) can be expressed as follows

\[ \Gamma(\zeta) = \sum_{j=0}^{n} Y_j(\ell(\zeta))^j, \]  

where \( Y_j(0 \leq j \leq n) \) are constant coefficients to be determined later and \( \ell(\zeta) \) satisfies the following auxiliary equations

\[ \ell_{\zeta}^2 = 2(Ln(B))^2 \times \left( H_0 + H_1 \ell(\zeta) + H_2 \ell(\zeta)^2 + H_3 \ell(\zeta)^3 + H_4 \ell(\zeta)^4 \right), \]  

\[ \ell_{\zeta\zeta} = (Ln(B))^2 \times \left( H_1 + 2H_2 \ell(\zeta) + 3H_3 \ell(\zeta)^2 + 4H_4 \ell(\zeta)^3 \right). \]

Step 3: It is supposed that solution of Eq. (3) can be written in the following form

\[ \Gamma(\zeta) = Y_0 + Y_1 \ell(\zeta) + Y_2 \ell(\zeta)^2 + Y_3 \ell(\zeta)^3 + \cdots + Y_n \ell(\zeta)^n, \]
where $Y_0, Y_1, Y_3, Y_4$ and $Y_n$ are reals constant to be determined later. Using the principle of the balance between the high-order nonlinear and derivative terms (2), we get the value of the integer $n$.

Step 4: With $n$ determined, we then collect all coefficients of powers of $\ell(\xi)$ in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters $Y_0, Y_j, j = (1, 2, \ldots, n)$.

Step 5: To obtain the exact solutions of Eq. (2), the following solutions of Eqs. (4) or (5) are used.

Case 1: for $H_0 = H_1 = H_3 = 0$, $H_2 > 0$, $H_4 < 0$,

$$\ell(\xi) = \sqrt{-\frac{ghH_2}{H_4}} \text{sech}_B(\sqrt{2H_2} \xi),$$  \hfill (7)

Case 2: for $H_0 = \frac{H_2^2}{4H_4}$, $H_1 = H_3 = 0$, $H_2 < 0$, $H_4 > 0$,

$$\ell(\xi) = \sqrt{-\frac{H_2}{2H_4}} \tanh_B(\sqrt{-H_2} \xi).$$  \hfill (8)

Case 3: for $H_0 = H_1 = 0$, $H_2 > 0$, $H_4 > 0$, $\tau = \pm 1$

$$\ell(\xi) = \frac{ggh_2 \text{sech}^2_B(\sqrt{2H_2} \xi)}{2\tau \sqrt{H_2} H_4 \tanh_B(\sqrt{2H_2} \xi) - H_3}.$$  \hfill (9)

Case 4: for $H_0 = H_1 = 0$, $H_2 > 0$, $H_3^2 - 4H_2H_4 > 0$,

$$\ell(\xi) = \frac{2\sqrt{ggh_2 \text{sech}(\sqrt{2H_2} \xi)}}{\sqrt{H_3^2 - 4H_2H_4} - \sqrt{ghH_3 \text{sech}(\sqrt{2H_2} \xi)}}.$$  \hfill (10)

Case 5: for $H_0 = H_1 = 0$, $H_2 > 0$, $\tau = \pm 1$

$$\ell(\xi) = \frac{ggh_2H_3 \text{sech}^2_B(\tau \sqrt{2H_2} \xi)}{H_2H_4(1 - \tanh_B(\tau \sqrt{2H_2} \xi))^2 - H_3^2},$$  \hfill (11)

where $H_0$, $H_1$, $H_2$, $H_3$ and $H_4$ are arbitrary constants. Therefore, using Eqs. (7–11) and Eqs. (3) or (6), the exact solutions to Eq. (2) can be obtained.

The generalized hyperbolic and triangular functions are defined as

$$\sinh_B(\xi) = \frac{gB^\xi - hB^{-\xi}}{2}, \quad \cosh_B(\xi) = \frac{gB^\xi + hB^{-\xi}}{2}, \quad \tanh_B(\xi) = \frac{gB^\xi - hB^{-\xi}}{gB^\xi + hB^{-\xi}},$$

$$\coth_B(\xi) = \frac{gB^\xi + hB^{-\xi}}{gB^\xi - hB^{-\xi}}, \quad \text{sech}_B(\xi) = \frac{2}{gB^\xi + hB^{-\xi}}, \quad \csc_B(\xi) = \frac{2}{gB^\xi + hB^{-\xi}},$$

$$\sin_B(\xi) = \frac{gB^{i\xi} - hB^{-i\xi}}{2i}, \quad \cos_B(\xi) = \frac{gB^{i\xi} + hB^{-i\xi}}{2}, \quad \tan_B(\xi) = -\frac{gB^{i\xi} - hB^{-i\xi}}{gB^{i\xi} + hB^{-i\xi}},$$

$$\cot_B(\xi) = \frac{gB^{i\xi} + hB^{-i\xi}}{2i}, \quad \sec_B(\xi) = \frac{2}{gB^{i\xi} + hB^{-i\xi}}, \quad \csc_B(\xi) = \frac{2i}{gB^{i\xi} + hB^{-i\xi}},$$

where $\xi$ is an independent variable, the constants $g > 0$ and $h > 0$ are called deformation parameters.
2.1 Effects of the fractional parameter on soliton

We aim in this section, to examine the effects of the CFTP and the physical parameters (PP) of NLSE with the conformable time-fractional which describes the polarization mode on analytical and numerical chirp and chirped soliton. Sundry prospective definitions of the fractional derivative have been obtained and more details of the conformable fractional derivative properties are given in Khalil et al. (2014), Raza et al. (2020), Kaplan et al. (2017), Kaplan (2017). For analytical investigation, we use the NGAEM integration algorithm. To consolidate the analytical investigation, we employ the Split-step Fourier and Runge Kutta algorithm to show step by step the effects of the FP associated with the PP of the model. It is worth indicating that the NLSE describing the polarization mode has been recently handled analytically by Eslami et al. (2021). Chirped optical soliton has been studied by using the rational method. The authors show the effects of several parameters of the model on the chirp and the corresponding chirped optical soliton. More in ref. Desaix et al. (2002), significant results on chirped femtosecond solitons and kink-like solitons have been pointed out. The figure of merit of SS and SFS parameters was highlighted. The governing equation with FTP which describes the polarization mode is set as follows (Eslami et al. 2021; Tebue et al. 2019):

\[
\frac{i}{c} \frac{\partial \psi(z,t)}{\partial z} + xD_t^\beta \psi(z,t) + s \left( |\psi(z,t)|^2 \psi(z,t) \right) + i\gamma D_t^\beta \left( |\psi(z,t)|^2 \psi(z,t) \right) \\
+ ie \psi(z,t)D_t^\beta \left( |\psi(z,t)|^2 \right) \\
+ \theta |\psi(z,t)|^4 \psi(z,t) = 0, \quad 0 < \beta \leq 1.
\]  

The quantity \( \psi(z,t) \) is the complex polarization mode amplitude, while the GVD coefficient is \( x \), SS term and SFS are respectively \( \gamma \) and \( \epsilon \). However, \( z \) is the space parameter, and \( t \) is the delayed time. The parameter \( s \) is real and denotes the Kerr nonlinear coefficient. In addition, the nonlinear coefficient which counts for CQ nonlinearity is denoted by \( \theta \). Some analytic investigations have been recently done by using the model (Eslami et al. 2021; Tebue et al. 2019). The authors have established the existence of chirp components and Chirped solitons as solutions. These results have certainly opened the way to another analytic investigation, but to our knowledge, no more work has been done to confirm the stability of these results. As the mathematical model contains nonlinear terms and a weak dispersion coefficient, we employ tiny perturbations of the CW as the solution of Eq. (12) to analyze the MI spectra with the effects of the FP. Assuming the Riemann–Liouville form (Raza et al. 2020; Kaplan et al. 2017; Kaplan 2017), we assume the following transformation with the chirp components as follows:

\[
\psi(z,t) = L(\xi)e^{i(C(\xi) - \kappa z)},
\]  

and \( \xi = \frac{\vartheta}{\eta} - uz \), where \( u = \frac{1}{\vartheta} \), \( \vartheta \) denotes the soliton speed. The chirp component can be expressed as

\[
\delta \omega(z,t) = -\frac{\partial}{\partial z} (C(\xi) - \kappa z) = \frac{1}{\vartheta} C_\xi = 0.
\]  

We transform Eq. (12) to the ODE by using Eq. (13), we obtain respectively, the real and imaginary components, having the form,
\[ aL_{\xi \xi} + \kappa L + s L^3 - \gamma C_\xi L^3 - aL(C_\xi)^2 + uLC_\xi + 0L^5 = 0, \quad (15) \]
\[ uL_{\xi} - (3\gamma + 2\epsilon)L\xi L^2 - 2aL\xi C_\xi - 2C_\xi L = 0. \quad (16) \]

Now, we extract the chirp component by multiplying Eq. (16) by \( L \) and then integrating once, we show the following equation in terms of \( L^2 \)
\[ C_\xi = -\frac{(3\gamma + 2\epsilon)}{4\alpha} L^2 + \frac{1}{2\alpha \theta^3}, \quad (17) \]
and the corresponding chirp parameters gives
\[ \delta \omega(z, t) = -\frac{(3\gamma + 2\epsilon)}{4\alpha \theta} L^2 + \frac{1}{2\alpha \theta^2}, \quad (18) \]
with \( \alpha \neq 0 \) and \( \theta \neq 0 \). It results that the chirp components depends on the GVD coefficient, SS and the SFS parameters. We can insert Eq. (17) into Eq. (15) and the ODE in the elliptic form is obtained as
\[ L_{\xi \xi} + \left( \frac{4\alpha \kappa + u^2}{4\alpha^2} \right) L + \left( \frac{2\alpha s - \gamma u}{2\alpha^2} \right) L^3 \left( \frac{16\alpha \theta - 4\epsilon^2 - 4\epsilon \gamma + 3\gamma^2}{16\alpha^2} \right) L^5 = 0. \quad (19) \]

We adopt the transformation \( L(\xi) = \sqrt{E(\xi)} \) to reduce Eq. (19) to the integrable form as
\[ (E_\xi)^2 - \left( \frac{4\alpha \kappa + u^2}{4\alpha^2} \right) E^2 - 2\left( \frac{2\alpha s - \gamma u}{2\alpha^2} \right) E^3 \]
\[ - 4\left( \frac{16\alpha \theta - 4\epsilon^2 - 4\epsilon \gamma + 3\gamma^2}{16\alpha^2} \right) E^4 - 2EE_{\xi \xi} = 0, \quad (20) \]
the constraint relation of Eqs. (19) and (20) is \( \alpha \neq 0 \). Now, we use the homogeneous balanced principle between \( EE_{\xi \xi} \) and \( E^4 \) in Eq. (20), which leads to \( n = 1 \). Subsequently, it is introduced into Eqs. (3) or (5) the value of \( n \), and taking into account Eqs. (4–5), a system of equation in terms of \( \ell(\xi) \) is obtained. Thus, we set all the coefficients of each individual term \( \ell(\xi) \) to zero, which gives the results below:

- **Set 1**

\[ A_0 = 0, \ A_1 = \sqrt{\frac{-24H_4(z\ln(B))^2}{16\alpha \theta - 4\epsilon^2 - 4\epsilon \gamma + 3\gamma^2}} \]
\[ H_2 = \frac{-2(zs^2 + \gamma^2 \kappa)}{(\ln(B))^2 \gamma^2 \alpha}, \quad u = \frac{2zs}{\gamma}, \quad (21) \]
\[ H_4 = H_4, \ \theta = \theta, \]
with 

\[-24H_{4}(16\alpha \theta - 4\epsilon^{2} - 4\epsilon\gamma + 3\gamma^{2}) > 0 \text{ and } \gamma \neq 0, \alpha \neq 0.\]

\[A_{0} = \frac{4}{5} \frac{4\kappa \alpha + u^{2}}{-2\kappa s + \gamma u}, \quad H_{2} = \frac{1}{10} \frac{4\kappa \alpha + u^{2}}{2(Ln(B))}, \quad H_{4} = -\frac{5A_{1}^{2}(-2\kappa s + (\gamma)u)^{2}}{(128\kappa \alpha + 32u^{2})\alpha^{2}(Ln(B))^{2}},\]

\[\theta = \frac{-60\alpha^{2}s^{2} - 64\alpha^{2}\kappa - 64\alpha\epsilon\gamma\kappa + 48\gamma^{2}\kappa + 60\gamma u \epsilon - 16\epsilon u^{2} - 16\epsilon u^{2} - 3\gamma^{2}u^{2}}{(256\kappa \alpha + 64u^{2})\alpha},\]

\[A_{1} = A_{1}, \quad u = u.\]

(22)

with the constraint condition as \((-2\kappa s + \gamma u) \neq 0, \quad (128\kappa \alpha + 32u^{2})\alpha^{2} \neq 0, \quad (-24H_{4}(\alpha Ln(B))^{2}) \times (16\alpha \theta - 4\epsilon^{2} - 4\epsilon\gamma + 3\gamma^{2}) > 0, \quad (Ln(B))^{2}\gamma^{2}\alpha \neq 0, \quad (128\kappa \alpha + 32u^{2})\alpha^{2}(Ln(B))^{2} \neq 0.\]

- Case 1: For \(H_{0} = H_{1} = H_{3} = 0, \text{ and } H_{2} > 0, H_{4} < 0\) using Eqs. (21–22) respectively, the bright soliton in the governing model (12) is acquired as

\[\psi_{1,1,1}(z, t) = \sqrt{A_{1}} \left[ -\frac{ghH_{2}}{H_{4}} \text{sech}_{B}(\sqrt{2H_{2}\left(\frac{t}{\beta} - \frac{2\kappa s}{\gamma}\right)}) e^{j(C_{(\beta - 2\alpha s) - \kappa z})},\right]\]

(23)

\[\psi_{1,1,2}(z, t) = \sqrt{A_{0} + A_{1}} \left[ -\frac{ghH_{2}}{H_{4}} \text{sech}_{B}(\sqrt{2H_{2}\left(\frac{t}{\beta} - uz\right)}) e^{j(C_{(\beta - uz) - \kappa z})},\right]\]

(24)

and the corresponding chirp gives

\[\delta \omega_{1,1,1}(z, t) = -\frac{(3\gamma + 2\epsilon)A_{1}}{4\alpha \beta} \left[ -\frac{ghH_{2}}{H_{4}} \text{sech}_{B}(\sqrt{2H_{2}\left(\frac{t}{\beta} - \frac{2\kappa s}{\gamma}\right)}) + \frac{1}{2\alpha \beta} \right],\]

(25)

\[\delta \omega_{1,1,2}(z, t) = -\frac{(3\gamma + 2\epsilon)}{4\alpha \beta} \left[ A_{0} + A_{1} \left[ -\frac{ghH_{2}}{H_{4}} \text{sech}_{B}(\sqrt{2H_{2}\left(\frac{t}{\beta} - \frac{2\kappa s}{\gamma}\right)}) \right]\right] + \frac{1}{2\alpha \beta}.\]

(26)

In the plotted Figs. 1, 2 and 3 we show the effects of the conformable derivative time fractional (CDTF) order and fixed values of OF parameters \((\alpha = 0.1, \epsilon = -0.02, s = 0.25, \theta = 0.1)\) on the contour plot evolution (Fig. 1a–f), W-shaped profile (Fig. 2a–d) and spatial evolution of the bright optical soliton (Fig. 3). We observed the propagation of soliton with the variation of the FTP. For \(\beta = 0.45\) the width of W-shaped is important between \(10 \leq t \leq 20\) (see Fig. 1a,b). When we increase the value of the CFP to \(\beta = 0.65, \beta = 0.75, \beta = 0.85\) and \(\beta = 0.95\) illustrated through Fig. 1b–f, it is observed that the width of the W-shaped shrinks and takes the valued of \(t = 10\). Moreover, we have shown the impact of the CFP on the evolution of W-shaped profile in Fig. 2. For \(\beta = 0.6\) the width of the W-shaped increases and the width shrinks when the value of the CFP increases (see Fig. 2b–d). In Fig. 3, we set out once again the clout of the CFP on W-shaped profile. For small value of the CFP, there is modification on the width of the W-shaped which seem to propagate in the direction of the red arrow (see Fig. 3a–c). When we increase the value of the CFP, it appears that the width of the W-shaped profile shrinks.
Beside, in plot Fig. 4 we show the evolution of the bright optical soliton with fixed value of the model parameters and fixed valued of the CFP ($\beta = 0.25$) with the variation of times. We also, noticed the effects of the CFP on the corresponding chirp for $\beta = 0.2$ in Fig. 5a,b and $\beta = 0.4$ in Fig. 5c,d. We observe a stretching of the amplitude of the chirp bright which seems to dissect according to the direction of the red arrow during propagation (see Fig. 5a,b). In plot Fig. 5c,d the red circle shows the fragments of the chirp caused by the effects of the FTP. These fragments indicate the dispersion of wave energy during propagation of the corresponding chirp signal.

Considering now the constraint condition $H_2 < 0$ and $H_4 > 0$, we plot in Fig. 6 the behavior of the bright optical soliton with the effects of the FTP and fixed values of the model parameters (MP). It is observed that when the value of the CFP increases the bright optical soliton shape is well formed and shift from left to right by maintaining without any deformation. In Fig. 7 with set the constraint condition $A_0 \neq 0$, $\alpha \neq 0$, $H_4 > 0$ and $H_2 > 0$ we want to show the breather like soliton evolution under the effects of the CFP. We increase the value of the CFP and we see that the obtained breather increases in amplitude (see Fig. 7b,c), which denotes a gain of energy during propagation. This scenario explain energy transfer caused by the effects of CFP during propagation of W-shaped and breather like soliton in OF. These results could give new opportunities to data processing and communication improvement. Compared these results with refs. Eslami et al. (2021), Tebue et al. (2019), Desaix et al. (2002), news behavior of the W-shaped profile are obtained when the valued of the FT parameter decreases.
Case 2: For $H_0 = H_2 = H_4 = 0$, and $H_2 < 0$, $H_4 > 0$, using Eqs. (21–22) respectively, the dark soliton is revealed as

$$\psi_{1,2}(z, t) = \sqrt{A_1 \sqrt{-H_2/2H_4}} \tanh B \left( \sqrt{-H_2 \left( \frac{t^\beta}{\beta} - \frac{2gs}{\gamma} \right)} \right) e^{i \left( c \left( \frac{z}{\beta} - \frac{2gs}{\gamma} \right) - \kappa z \right)},$$

(27)
Fig. 4 Evolution of bright soliton $|\psi_{1,2}(z,t)|^2$ with the effects of the CFP ($\beta = 0.25$) at $B = e$, $\varkappa = 0.1$, $g = 0.042$, $h = 0.00078$, $e = -0.02$, $s = 0.25$, $H_4 = -0.095$, $\theta = 0.1$, $H_2 = 1.84$, $A_0 = -0.06$, $A_1 = 0.5$

$$\psi_{1,2}(z,t) = \sqrt{A_0 + A_1}\sqrt{-\frac{H_2}{2H_4}}\tanh_B\left(\sqrt{-\frac{H_2}{2H_4}}\left(\frac{\beta}{(\gamma - uz)}\right)\right)e^{i\left(C\left(\frac{\beta}{(\gamma - uz)}\right)\right)} , \quad (28)$$

the corresponding chirp expression

$$\delta\omega_{1,2,1}(z,t) = -\frac{(3\gamma + 2e)A_1}{4\varkappa^2}\sqrt{-\frac{H_2}{2H_4}}\tanh_B\left(\sqrt{-\frac{H_2}{2H_4}}\left(\frac{\beta}{\gamma} - 2\gamma s\right)\right) + \frac{1}{2\varkappa^2} , \quad (29)$$
\[ \delta \omega_{1,2}(z,t) = -\frac{1}{2\pi i^2} \left[ A_0 + A_1 \sqrt{\frac{-H_2}{2H_4}} \tanh_B \left( \sqrt{-H_2} \left( \frac{\nu}{\beta} - uz \right) \right) \right] (30) \]
• **Set 2**

\[
A_0 = 0, \quad A_1 = A_1, \quad H_2 = -\frac{1}{2} \frac{4\pi k + u^2}{(\ln(B))^2},
\]

\[
H_3 = \frac{1}{2} \frac{A_1(-2\pi s + \gamma u)}{\pi^2 (\ln(B))^2}, \quad \theta = 0, \quad u = u,
\]

\[
H_4 = -\frac{1}{24} \frac{A_1^2(16\pi \theta - 4e^2 - 4\gamma \pi + 3\gamma^2)}{\pi^2 (\ln(B))^2},
\]

\[
A_0 = A_0, \quad A_1 = A_1, \quad H_2 = -\frac{1}{2} \frac{2\pi sA_0 + \gamma uA_0 - 4\pi k - u^2}{\pi^2 (\ln(B))^2}, \quad u = u,
\]

\[
H_3 = A_1(10\pi sA_0 - 5\gamma uA_0 + 16\pi k + 4u^2),
\]

\[
H_4 = -\frac{1}{6} \frac{A_1^2(-4\pi sA_0 + 2\gamma uA_0 - 4\pi k - u^2)}{A_0^2 \pi^2 (\ln(B))^2},
\]

\[
\theta = -\frac{1}{16} \frac{4\pi^2 A_0^2 - 4\gamma \pi A_0^2 + 3\gamma^2 A_0^2 + 16\pi sA_0 - 8\gamma uA_0 + 16\pi k + 4u^2}{A_0^2}
\]

• **Case 3:** For \(H_0 = H_1 = 0, H_2 > 0 H_4 > 0, \tau = \pm 1\) using Eqs. (31–32) respectively, is revealed as

\[
\psi_{2,3,1}(z,t) = \sqrt{\frac{ghA_1H_2 \text{sech}^2_B(\sqrt{2H_2} \frac{\phi - u\tau}{2})}{2\tau \sqrt{H_2H_4 \text{tanh}_B(\sqrt{2H_2} \frac{\phi - u\tau}{2})} - H_3}} e^{i(C(\frac{\phi - u\tau}{2}) - \kappa z)},
\]

\[
\psi_{2,3,2}(z,t) = \sqrt{\frac{ghA_1H_2 \text{sech}^2_B(\sqrt{2H_2} \frac{\phi - u\tau}{2})}{2\tau \sqrt{H_2H_4 \text{tanh}_B(\sqrt{2H_2} \frac{\phi - u\tau}{2})} - H_3}} e^{i(C(\frac{\phi - u\tau}{2}) - \kappa z)},
\]

thereafter the corresponding chirp components gives

• **Case 3:** For \(H_0 = H_1 = 0, H_2 > 0 H_4 > 0, \tau = \pm 1\) using Eqs. (31–32) respectively, is revealed as

\[
\delta \omega_{2,3,1}(z,t) = -\frac{(3\gamma + 2e)}{4\pi \theta} \frac{ghA_1H_2 \text{sech}^2_B(\sqrt{2H_2} \frac{\phi - u\tau}{2})}{2\tau \sqrt{H_2H_4 \text{tanh}_B(\sqrt{2H_2} \frac{\phi - u\tau}{2})} - H_3} + \frac{1}{2\pi \theta^2},
\]
\[ \delta \omega_{2,3,2}(z, t) = -\frac{(3\gamma + 2\epsilon)}{4\tau \theta} \left[ A_0 + \frac{ghA_1H_2 \text{sech}^2 \left( \sqrt{2H_2} \frac{(\theta - uz)}{2} \right)}{2\tau \sqrt{H_2H_4} \tanh_B \left( \sqrt{2H_2} \frac{(\theta - uz)}{2} \right) - H_3} \right] + \frac{1}{2\tau \theta^2}. \]  

**Case 4:** For \( H_0 = H_1 = 0 \), and \( H_2 > 0 \), \( H_3^2 - 4H_2H_4 > 0 \), using Eqs. (31–32) respectively, is revealed as

\[ \psi_{2,4,1}(z, t) = \frac{2\sqrt{ghA_1H_2 \text{sech}_B \left( \sqrt{2H_2} \left( \frac{\theta}{\tau} - uz \right) \right)}}{\sqrt{H_3^2 - 4H_2H_4 - \sqrt{ghH_3 \text{sech}_B \left( \sqrt{2H_2} \left( \frac{\theta}{\tau} - uz \right) \right)}}} e^{i \left( c \left( \frac{\theta}{\tau} - uz \right) - \kappa z \right)}, \]

\[ \psi_{2,4,2}(z, t) = \frac{2\sqrt{ghA_1H_2 \text{sech}_B \left( \sqrt{2H_2} \left( \frac{\theta}{\tau} - uz \right) \right)}}{\sqrt{H_3^2 - 4H_2H_4 - \sqrt{ghH_3 \text{sech}_B \left( \sqrt{2H_2} \left( \frac{\theta}{\tau} - uz \right) \right)}}} e^{i \left( c \left( \frac{\theta}{\tau} - uz \right) - \kappa z \right)}, \]

and the corresponding chirp gives

\[ \delta \omega_{2,4,1}(z, t) = -\frac{(3\gamma + 2\epsilon)}{4\tau \theta} \left[ A_0 + \frac{2\sqrt{ghA_1H_2 \text{sech}_B \left( \sqrt{2H_2} \left( \frac{\theta}{\tau} - uz \right) \right)}}{\sqrt{H_3^2 - 4H_2H_4 - \sqrt{ghH_3 \text{sech}_B \left( \sqrt{2H_2} \left( \frac{\theta}{\tau} - uz \right) \right)}}} + \frac{1}{2\tau \theta^2}, \]

\[ \delta \omega_{2,4,2}(z, t) = -\frac{(3\gamma + 2\epsilon)}{4\tau \theta} \left[ A_0 + \frac{2\sqrt{ghA_1H_2 \text{sech}_B \left( \sqrt{2H_2} \left( \frac{\theta}{\tau} - uz \right) \right)}}{\sqrt{H_3^2 - 4H_2H_4 - \sqrt{ghH_3 \text{sech}_B \left( \sqrt{2H_2} \left( \frac{\theta}{\tau} - uz \right) \right)}}} \right] + \frac{1}{2\tau \theta^2}. \]

**Case 5:** For \( H_0 = H_1 = 0 \), and \( H_2 > 0 \), \( \tau = \pm 1 \), using Eqs. (31–32) respectively, is revealed as

\[ \psi_{2,5,1}(z, t) = \frac{\sqrt{ghA_1H_2 \text{sech}^2 (\tau \sqrt{2H_2} (\theta - uz) / 2)}}{\sqrt{H_2R_4 \left( 1 - \tanh_B \left( \tau \sqrt{2H_2} \frac{(\theta - uz)}{2} \right) \right)} - H_3^2} e^{i \left( c \left( \frac{\theta}{\tau} - uz \right) - \kappa z \right)}, \]
The chirp components is set out as

$$\psi_{2,5,2}(z,t) = A_0 + \frac{ghA_1H_2H_3 sech^2_B \left( \frac{\tau \sqrt{2H_2 (\frac{\phi}{\tau} - \omega_c)}}{2} \right)}{H_2H_4 \left( 1 - \tanh_B \left( \frac{\tau \sqrt{2R_2 (\frac{\phi}{\tau} - \omega_c)}}{2} \right) \right)^2} e^{c \left( \frac{\phi}{\tau} - \omega_c \right) - \omega_c}. \quad (42)$$

The constraint condition of Eq. (31) is $\alpha Ln(B) \neq 0$ and Eq. (32) is valid for $\alpha A_0 \neq 0$.

### 3 Modulation instability

#### 3.1 Linear stability analysis

We investigate the MI growth rate with the effects of the FTP. MI is the processus that shows the behavior of the CW with tiny perturbations during the confrontation between nonlinearity and dispersion terms. Many studies have been done to show the effects of optical parameters on MI such as the normal and anomalous dispersion regime combined with SS, Self-phase modulation (SPM), Cross Phase modulation (XPM) in birefringent OF, Higher-Order dispersions (HOD) and so on (Li et al. 2011; Tanemura et al. 2004; Murdoch et al. 1995; Rothenberg 1990; Drummond et al. 1990; Houwe et al. 2021a, b, c; Souleymanou et al. 2021a, c; Houwe et al. 2021d, e; Souleymanou et al. 2021f). It is worth mentioning that the effects of CFP have been recently set out on the MI growth rate associated with Third-Order dispersions (TOD). These results have shown the formation of instability zones (instable CW) when the value of the CFP increases and for small values the stability zones. In this work, we fixed the value of the NLSE parameters and varied the CFP to stress the behavior of MI gains spectra. We use the linearizing technic, to establish the dispersion relation and the MI gain. We assume the CW with small disturbances as the solution of the set Eq. (12):

$$\psi(z,t) = (P_0 + F(z,t)) e^{ikz}. \quad (45)$$

Here $D(z, t)$ is the tiny perturbation introduced to examine the MI gain. Inserting Eq. (45) into the set of coupled Eq. (12), the linearizing technic gives
\[ iF_z + \frac{\alpha}{(2\beta)} \frac{\partial^2 F}{\partial t^2} + ieP^2 \left( \frac{\partial^2 F}{\partial t}\frac{\partial F}{\partial t} + \frac{i}{\beta} \right) + ivF^2 \left( 2 \frac{\partial^2 F}{\partial t} + \frac{\partial F}{\partial t} \right) \]
\[ + sP^2 (2F + F^*) + \theta P^4 (3F + 2F^*) = 0, \]

\( F(z, t)^c \) is the complex conjugated of \( F(z, t) \). We consider the solution of Eq. (46) in the form of

\[ F(z, t) = f_1 e^{i(kz - \Omega t)} + f_2 e^{-i(kz - \Omega t)}, \]

where \( f_1 \) and \( f_2 \) are the disturbance amplitudes, while \( K \) and \( \Omega \) are wave number and angular frequency of the MI respectively. Making some algebraic manipulations, the dispersion relation with second-order polynomial gives

\[ m_2K^2 + m_1K + m_0 = 0, \]

where

\[ m_2 = -((2\beta))^2(\beta)^2, \quad e_1 = e^{-i\text{sign}(\Omega)/2}, \quad e_2 = e^{i\text{sign}(\Omega)/2}, \]
\[ m_1 = (\Omega)^2((2\beta)(\beta)^2\xi(e_1 - e_2)(e_1 + e_2)) + (\Omega)^2((2\beta)^2\beta^2P^2\xi(e_1 - e_2)(2\gamma + e)), \]
\[ m_0 = (\Omega)^2((\beta)^2(2\beta)^2\xi e_1^2 e_2^2) + (\Omega)^2((2\beta)^2\beta^2P^2\xi e_1 e_2(\xi e_1 + e_2)(2\gamma + e)) \]
\[ + (\Omega)^2((2\beta)^2(2\beta)^2\xi e_2^2 e_1^2) + (\Omega)^2((2\beta)^2\xi e_1^2 e_2^2(2\beta)^2P^2(3\beta)^2 - 2(\beta)^2(2\beta)^2(2\beta)^2P^2(3\beta)^2) \]
\[ + (\Omega)^4((2\beta)^2\beta^2P^2(\xi e_1 + e_2)(\xi e_1^2 e_2^2 + 4\gamma^3P^2 + 3\gamma^3P^2 + 3\gamma^3P^2 + 3\gamma^3P^2) \]
\[ + \left( (2\beta)^2(2\beta)^2P^2(5\beta^2 + 3\beta^2 + 3\beta^2) \right) \]

In plots Fig. 8a–d, we show the effects of the CFP on the MI gain in anomalous dispersion regime. We fixed the values of the model parameters. For \( \beta = 0.1 \), it is observed the appearance of one side lobe. After increasing the value of the CFP to \( \beta = 0.3 \), the side lobe decreases and the MI growth rate amplitude decreases. Thereafter, we increase the value of the CFP to \( \beta = 0.8 \), the side lobe tends to vanish and the MI band shrinks, while the amplitude increases. Now, in normal dispersion regime, we observe the formation of tiny instability zones for \( \beta = 0.2 \) (see Fig. 9a). By increasing the value of the CFP, in Fig. 9b–d we observed that the stability zones increase and the amplitude of the MI gain increases. To confirm these results, we will use numerical study in the next section by using pseudospectral technic associated with the fourth-order Runge–Kutta.

### 4 Numerical simulation

We numerically investigate the robustness of the above analytical results of the NLSE describing the polarization mode in OF. We firstly used Eqs. (24) and (28) with their corresponding chirp components to confirm the effects of the CFP by fixing the MP. Thereafter, to verify the prediction made on the MI growth rate.
4.1 Effects of the fractional parameter on numerical soliton solutions

We use the Split-step Fourier which is based on the pseudospectral technic associated with the fourth-order Runge–Kutta integration alongside time direction to verify the effects of the CFP and stability of the obtained analytical results. We use for this purpose the discrete Fourier mechanism to assess the spatial derivative. We set the Fourier transform of NLSE Eq. (12) with FT order as

\[ \text{Fig. 8} \text{ MI growth rate with the variation of the CFP in normal dispersion regime a } [\beta = 0.1], \text{ b } [\beta = 0.3], \text{ c } [\beta = 0.5] \text{ and d } [\beta = 0.8] \text{ at } P_0 = 0.25, \text{ } z = 0.01, \text{ } \epsilon = 0.8, \text{ } s = 0.25, \text{ } \theta = 0.45, \text{ } \gamma = 0.06 \]

\[ \text{Fig. 9} \text{ MI growth rate with the variation of the CFP in anomalous dispersion regime a } [\beta = 0.2], \text{ b } [\beta = 0.6], \text{ c } [\beta = 0.7] \text{ and d } [\beta = 0.85] \text{ at } P_0 = 0.25, \text{ } z = -0.1, \text{ } \epsilon = 0.8, \text{ } s = 0.25, \text{ } \theta = 0.45, \text{ } \gamma = 0.06 \]

4.1 Effects of the fractional parameter on numerical soliton solutions

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\[
\hat{\psi}_z - i\tilde{\varepsilon} (i\omega)^{2\beta} \hat{\psi} = \mathcal{F} \left[ s \left( |\psi(z,t)|^2 \psi(z,t) \right) + i\gamma D_t^\beta \left( |\psi(z,t)|^2 \psi(z,t) \right) \right] \\
+ i\varepsilon \psi(z,t) D_t^\beta \left( |\psi(z,t)|^2 \right) + \theta |\psi(z,t)|^4 \psi(z,t) ,
\]
with
\[
\hat{q} = \hat{\psi} e^{-i\xi (i\omega)^{2\beta} z} .
\]

We consider the fourth-order Runge–Kutta with time step \(\Delta t = 0.001\). The following set of analytical parameters of the model are used: \(A_0 = -0.06, A_1 = 0.5, H_2 = 1.84, H_4 = -0.09, h = 0.0078, g = 0.042, s = 0.25, \theta = 0.1, \alpha = 0.01, \gamma = 0.045, \epsilon = -0.02\). The assumption consists in keeping the values of the model and NGEAM parameters fixed and then varying the CFP to observe the behavior of the optical soliton during propagation.

In Fig. 10, we show the behavior of the W-shaped profile with the variation of CFP in anomalous dispersion regime. The constraint conditions are the same on the analytical results (i.e. \(H_2 < 0\) and \(H_4 > 0\)). We notice that when the CFP increases, the W-shaped profile propagated from right to left without any deformation and maintains shaped. However, in Fig. 11a,b it is shown the propagation of the corresponding chirp of bright optical soliton in anomalous dispersion regime (\(\alpha = -0.01\)) with fixed value of the model parameter and variation of the CFP. In Fig. 11a,b we display the chirp bright for \(\beta = 0.25\) and then when we increase the value of the CFP to \(\beta = 0.6\), its position changes. In the meantime, in Fig. 13a–d we show the corresponding chirp anti-kink-like soliton with the variation of the FP and fixed value of the MP. The constraint condition remain the same in Fig. 11. At the end we notice that: (i) the FD parameter induces the movement of the optical soliton; (ii) optical soliton propagates while retaining its amplitude; (iii) the presence of the
FD parameter stimulates energy at the soliton to allow it to propagate in the optical medium (Figs. 12 and 13).

4.2 Numerical analysis of MI growth rate

We examine the MI growth rate by using the numerical simulation in this section. Let’s assume the initial condition in the form of the perturbed plane wave as follows:

$$\psi(z, 0) = (P_0 + \xi \cos(K_0 z))e^{iKz}.$$  \hspace{1cm} (51)

To moreover examine the stability or instability zones, we set $P_0 = 1.5$, $\xi = 0.001$, $\alpha = 0.1$, $\theta = 0.5$, $\gamma = 0.45$, $\epsilon = 0.02$, $k = 0.5$. The time step is $\Delta t = 0.001$ and the grid with $N = 2^{10}$ nodes. In plots Fig. 14, the evolution of the $|\psi|^2$ anomalous dispersion regime ($\alpha = -0.8$) with the effects of the CFP and the excitation parameter $K_0 = 1.05$. We realize

Fig. 11 Propagation of the bright chirp soliton profile of $\delta \omega_{1,2}(z, t)$ with the effects of CFP in normal dispersion regime a [$\beta = 0.25$], b [$\beta = 0.4$]

Fig. 12 Propagation of dark soliton solutions $\psi_{1,2,2}(z, t)$ with the effects of CFP in normal dispersion regime a [$\beta = 0.2$] and b [$\beta = 0.6$]

FD parameter stimulates energy at the soliton to allow it to propagate in the optical medium (Figs. 12 and 13).

4.2 Numerical analysis of MI growth rate

We examine the MI growth rate by using the numerical simulation in this section. Let’s assume the initial condition in the form of the perturbed plane wave as follows:

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To moreover examine the stability or instability zones, we set $P_0 = 1.5$, $\xi = 0.001$, $\alpha = 0.1$, $\theta = 0.5$, $\gamma = 0.45$, $\epsilon = 0.02$, $k = 0.5$. The time step is $\Delta t = 0.001$ and the grid with $N = 2^{10}$ nodes. In plots Fig. 14, the evolution of the $|\psi|^2$ anomalous dispersion regime ($\alpha = -0.8$) with the effects of the CFP and the excitation parameter $K_0 = 1.05$. We realize

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that the amplitude does not increase and the maximum value is 1.12 despite the increasing of the CFP. However, it is observed the formation of breather like-soliton when we increase the value of the CFP (see Fig. 14c–f). Otherwise, for $\beta = 0.85$ in Fig. 14f, we observe chaotic breathing, which means that the whole system vibrates and therefore the system becomes unstable and difficult to control. In Fig. 15, we use the same time step in normal dispersion regime $a [\beta = 0.2]$, $b [\beta = 0.6]$, $c [\beta = 0.85]$ and $d [\beta = 1]$. 

Fig. 13 Propagation of anti-kink-like soliton of $\delta \omega_{1,2}(z,t)$ with the effects of CFP in normal dispersion regime $a [\beta = 0.1]$, $b [\beta = 0.3]$, $c [\beta = 0.5]$, $d [\beta = 0.75]$, $e [\beta = 0.85]$ and $f [\beta = 0.95]$ the breather-like soliton

Fig. 14 Numerically evolution of breather-like soliton $|\psi|^2$ Eq. (51) in anomalous dispersion regime with the effects of CFP top panel $a $ $\beta = 0.1$, $b $ $\beta = 0.3$, $c $ $\beta = 0.5$, $d $ $\beta = 0.75$, $e $ $\beta = 0.85$ and $f $ $\beta = 0.95$ the breather-like soliton.
dispersion regime ($\alpha = 0.8$), we observe that the breather like-soliton propagates very fast when the CFP increases and the maximum amplitude does not grow up to 1.12 (see Fig. 15a–d). It performs harsh oscillations in the range of $[0.97–1.12]$, which confirms the stability of the system.

5 Conclusion and remarks

This present work shows the effects of the FP on optical soliton solutions and MI growth rate. We employ the NGEAM to deal with three families of optical soliton solutions and their corresponding chirp such as bright, dark, and the combined bright-dark optical solitons. Throughout Figs. 1, 2, 3, 4 and 5, we have pointed out the FP effects on analytical results. For the small value of the FP, we observe the deformation of the bright and dark soliton’s shape. We have equally examined the MI growth rate with the effects of the FP. One observes that the FP can provoke instability zones and increases MI bands. For small values of the FP, we have displayed MI gain where the MI bands increase. One notices also that when the FP increases, the W-shaped profile can propagate very fast and its shape is kept stable. In addition, the effects of the FP on the corresponding chirp of the dark optical soliton have been obtained. We use numerical simulation to corroborate and confirm the stability and the effective effects of the FP on the obtained analytical results. Throughout the numerical simulation, we show the effects of the fractional order on MI gain. We observe the formation of breather-like soliton in normal and anomalous dispersion regimes. In the case of small values of the FP, we obtain stable breather-like solitons which are formed during strong oscillations where both amplitude and shape are stable during the propagation. Compared these results with refs. Houwe et al. (2021a), Bienvenu et al. (2021), Eslami et al. (2021), Houwe et al. (2021c), Souleymanou et al. (2021a), Houwe et al. (2020), new behavior of optical soliton solutions and W-shaped profile is fulfilled. It results that the features of the obtained results where the FP is used can open the way for further applications in optical fibers.
Declarations

Conflict of interest  The authors declare that they have no conflict of interest.

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