HAUSDORFF-(2n – 2) DIMENSIONAL MEASURE ZERO SET AND COMPACTNESS OF THE ∂-NEUMANN OPERATOR ON (0, n – 1) FORMS

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Abstract. By using a variant Property ($P_q$) of Catlin, we discuss the relation of small set of weakly pseudoconvex points on the boundary of pseudoconvex domain and compactness of the ∂-Neumann operator. In particular, we show that if the Hausdorff (2n – 2)-dimensional measure of the weakly pseudoconvex points on the boundary of a smooth bounded pseudoconvex domain is zero, then the ∂-Neumann operator $N_{n-1}$ is compact on (0, n–1)-level $L^2$-integrable forms.

1. Introduction

On a bounded pseudoconvex domain $\Omega$ in $\mathbb{C}^n$, an important question in the ∂-Neumann problem is to study whether there exists a bounded inverse of the complex Laplacian $\Box_q = \Box_{q-1} + \Box_q$ on the $L^2$-integrable $(0, q)$-type forms of the domain $\Omega$ ($1 \leq q \leq n$) and discuss the regularity property of the inverse if it exists. To be precise, given a $L^2$-integrable $(0, q)$ form $v$ on $\Omega$, the ∂-Neumann problem is to find $u \in \text{dom}(\Box_q)$ such that $\Box_q u = v$ and further study regularity property of the solution operator on $L^2$-integrable forms. We call the (bounded) inverse of $\Box_q$ as the ∂-Neumann operator and denote it as $N_q$. For classical results about the regularity properties of $N_q$, one may check [5], [6], [11], [16] and [17].

In this paper, we focus on the study of compactness of the ∂-Neumann operator on specific level forms. In this regard, Kohn and Nirenberg ([12]) proved that compactness of $N_q$ implies the global regularity of $N_q$ on smooth bounded pseudoconvex domains, here the global regularity means that $N_q$ maps the space of forms with components smooth up to the boundary of $\Omega$ to itself. It is well known that compactness of $N_q$ is equivalent to a quantified estimate on $L^2$-integrable forms (see section 2), hence analysis on compactness of $N_q$ is more robust and has its own interest. For useful applications of such analysis results, one can check [4], [7], [8], [9], [14], [18] and references there.

Within the viewpoint of potential analysis theory, there are numerous sufficient conditions for compactness of $N_q$ on a smooth bounded pseudoconvex domain. For instance, Property $(P_q)$ in Catlin’s work ([3]) and Property $(\tilde{P}_q)$ in McNeal’s work ([13]) are well known so far. In [20], the author introduced several variant conditions of Property $(P_q)$ and Property $(\tilde{P}_q)$, which also imply compactness of $N_q$ on high level $L^2$-integrable forms on a smooth bounded pseudoconvex domain. These variant conditions are obtained by proving a unified estimate of the twisted

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Kohn-Morrey-Hörmander estimate (see in [13] or section 2.6 in [16]) and the $q$-pseudoconvex Ahn-Zampieri estimate (see section 1.9 in [17] or [1]) on a smooth bounded domain.

In this article, we focus on applying the conditions in [20] on $(0, n - 1)$ forms and we discuss the relation of small set of infinite-type points on the boundary of pseudoconvex domain and compactness of the $\partial$-Neumann operator $N_{n-1}$.

This subject is motivated by the results of Sibony ([15]) and Boas ([2]) on general pseudoconvex domains: let $q = 1$ and assume that the set $K$ of the weak pseudoconvex points on the boundary $\partial \Omega$ has Hausdorff 2-dimensional measure zero in $\mathbb{C}^n$, then the $\partial$-Neumann operator $N_1$ is compact on $L^2_{(0,1)}(\Omega)$. Boas ([2]) has an explicit construction of the function $\lambda$ involved in the proof. Due to the lack of biholomorphic invariance on Property ($P_q$) when $q > 1$, the approach cannot be generalized to the case $q > 1$ and hence $N_q$ is not known to be compact in the $q > 1$ case.

By applying the variant Property ($P_q$) when $q = n - 1$ in [20], we prove the following theorem which generalizes above result of Sibony and Boas to the case of $q = n - 1$:

**Theorem 1.1.** Let $\Omega$ be a smooth bounded pseudoconvex domain in $\mathbb{C}^n$. If the Hausdorff $(2n - 2)$-dimensional measure of weakly pseudoconvex points of $\partial \Omega$ is zero, then the $\partial$-Neumann operator $N_{n-1}$ is compact on $L^2_{(0,n-1)}(\Omega)$ forms.

Our result on the $(0, n - 1)$-forms is interesting, since under this case, the variant of Property ($P_q$) we used in proof only involves with the diagonal entries in the complex Hessian, rather than the sum of eigenvalues in the complex Hessian. This fact, in turn, explains why Property ($P_q$) of Catlin or Property ($P_q$) of McNeal is not convenient to apply in the proof of above result.

The paper is organized as follows: in section 2, we list some facts and background materials about the $\partial$-Neumann problem and related potential analysis results; in section 3, we prove the main result and mention one example.

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## 2. Preliminaries

Let $L^2_{(0,q)}(\Omega)$ be the space of $(0,q)$-forms $(1 \leq q \leq n)$ with $L^2$-integrable coefficients on a bounded domain $\Omega$ in $\mathbb{C}^n \ (n \geq 2)$. The $L^2$-norm of a $(0,q)$-form $u$ is defined as $\| u \|_{L^2} = \left( \sum_j u_j d\bar{z}_j \right)^2$. Similarly, the weighted $L^2$-norm of $u$ is defined by $\| u \|_{L^2} = \left( \sum_j u_j e^{-\varphi} d\bar{z}_j \right)^2$. Let $\mathcal{J} : L^2_{(0,q)}(\Omega) \rightarrow L^2_{(0,q+1)}(\Omega)$ by: $\mathcal{J}(\sum_j u_j d\bar{z}_j) = \sum_j \sum_j \frac{\partial u_j}{\partial \bar{z}_j} d\bar{z}_j \wedge d\bar{z}_j$. Let $\text{dom}(\mathcal{J}) = \{ u \in L^2_{(0,q)}(\Omega) | \mathcal{J} u \in L^2_{(0,q+1)}(\Omega) \}$ and $\text{dom}(\mathcal{J}^*) = \{ v \in L^2_{(0,q+1)}(\Omega) | \exists C > 0, \| v \|_{L^2} \leq C \| u \|_{L^2}, \forall u \in \text{dom}(\mathcal{J}) \}$ be the domain of $\mathcal{J}$ and $\mathcal{J}^*$ respectively. The weighted $\mathcal{J}$-complex is defined similarly in the weighted $L^2$-integrable forms. We denote the resulting adjoint by $\mathcal{J}^*_\varphi$ and its domain is $\text{dom}(\mathcal{J}^*_\varphi)$. It is well known that $\text{dom}(\mathcal{J}^*_\varphi) = \text{dom}(\mathcal{J}^*)$ if $\varphi \in C^1(\Omega)$. The formal adjoint of $\mathcal{J}$ is $\mathcal{J}_\varphi$ such that $(u, \mathcal{J}^*_\varphi v)_{\varphi} = (\mathcal{J}_\varphi u, v)_{\varphi}$ for every $C^\infty$ smooth compactly supported form $v$ on $\Omega$. And $\mathcal{J}_\varphi u = \mathcal{J}_\varphi u$ if $u \in \text{dom}(\mathcal{J}_\varphi)$.  

Given a boundary point $P$ of $\Omega$, we choose vector fields $L_1, \cdots, L_n$ of type $(1, 0)$ which are orthonormal and span $T_z^c(\partial \Omega)$ for $z$ near $P$, where $\Omega = \{ z \in \Omega : \rho(z) < -\epsilon \}$, $\Omega$ is defined to be the complex normal which can be normalized to be 1 on the boundary. We use above vector fields to induce a special boundary chart such that $\{\omega_j\}^n_{j=1}$ is the dual basis of $\{L_j\}^n_{j=1}$ near $P$. It is then clear that $\overline{\partial}f = \sum_{j=1}^n (\overline{L}_j f) \omega_j$ for a $C^1$ smooth function $f$. Let $c_{jk}'$ be defined by $\overline{\partial} \omega_i = \sum_{j,k}^n c_{jk}' \omega_j \wedge \omega_k$.

**Definition 2.1.** let $f$ be a $C^2$ smooth function, define $f_{jk} = L_j \overline{L}_k f + \sum_i c_{jk}' \overline{L}_i f$.

It is then clear that $\overline{\partial} \overline{\partial} f = \sum_{j,k} f_{jk} \omega_j \wedge \omega_k$. For a general $L^2$-integrable form $u = \sum'_{|J|=q} u_J \overline{\omega}_J$ in the special boundary chart, we have:

$$\overline{\partial} u = \sum'_{|K|=q-1} (\overline{L}_i u_{JK} - \overline{L}_j u_{IK}) \overline{\omega}_i \wedge \overline{\omega}_j \wedge \overline{\omega}_K + \cdots,$$

$$\partial_e u = - \sum'_{|K|=q-1} \sum_{j \leq n} \delta_{\omega_j} (u_{JK}) \overline{\omega}_K + \cdots,$$

where $\delta_{\omega_j} f = e^{\overline{\partial} f} L_j (e^{-\overline{\partial} f})$ for $L^2$-integrable functions $f$. The dots in above two equations are the terms that only involve with the coefficients of $u$ and the differentiation of the coefficients of $L_j$ or $\overline{\omega}_K$.

We define the complex Laplacian as $\square u := \overline{\partial} \overline{\partial} u + \overline{\partial} \partial_e u$ on $L^2_{(0,q)}$ forms. Here we suppress the subscript of the level of the form in $\overline{\partial}$ and $\partial_e$ for simplicity. We call the inverse operator of $\square$ as the $\overline{\partial}$-Neumann operator, and denote it as $N_q$.

Hörmander ([10, 11]) showed that $\square$ has a bounded inverse $N_q$ on $L^2_{(0,q)}(\Omega)$ when $\Omega$ is a bounded pseudoconvex domain. $N_q$ is said to be compact on $L^2_{(0,q)}(\Omega)$ if the image of the unit ball in $L^2_{(0,q)}(\Omega)$ under $N_q$ is relatively compact in $L^2_{(0,q)}(\Omega)$. We can characterize the compactness of $N_q$ by the following well known fact (see [13] or [10], Proposition 4.2):

**Proposition 2.1.** Let $\Omega$ be a bounded pseudoconvex domain in $\mathbb{C}^n$, $1 \leq q \leq n$. Then the following are equivalent:

(i) $N_q$ is compact as an operator on $L^2_{(0,q)}(\Omega)$.

(ii) For every $\epsilon > 0$, there exists a constant $C_\epsilon$ such that we have the compactness estimate:

$$||u||^2 \leq \epsilon(||\overline{\partial} u||^2 + ||\partial_e u||^2) + C_\epsilon ||u||^2_{-1} \text{ for } u \in \text{dom}(\overline{\partial}) \cap \text{dom}(\partial_e).$$

(iii) The canonical solution operators $\overline{\partial} N_q : L^2_{(0,q)}(\Omega) \cap \ker(\overline{\partial}) \to L^2_{(0,q-1)}(\Omega)$ and $\square N_{q+1} : L^2_{(0,q+1)}(\Omega) \cap \ker(\overline{\partial}) \to L^2_{(0,q)}(\Omega)$ are compact.

Catlin ([3]) showed that if $\Omega$ be a smooth bounded pseudoconvex domain and $b\Omega$ satisfies Property $(P_q)$, then $N_q$ is compact on $L^2_{(0,q)}(\Omega)$. McNeal ([13]) showed that Property $(P_q)$ can be weakened to Property $(\tilde{P}_q)$ on individual function level, and still implies compactness of $N_q$. We list the definition of Property $(P_q)$ here for use in section 3:

**Definition 2.2.** A compact set $K \subset \mathbb{C}^n$ has Property $(P_q) \ (1 \leq q \leq n)$ if for any $M > 0$, there exists an open neighborhood $U$ of $K$ and a $C^2$ smooth function $\lambda$
on $U$ such that $0 \leq \lambda \leq 1$ on $U$ and $\forall z \in U$, the sum of any $q$ eigenvalues of the complex Hessian \( \left( \frac{\partial^2 \lambda}{\partial z_j \partial \bar{z}_k} \right)_{j,k} \) is at least $M$.

In [20], the author introduced several variant conditions of Property ($P_q$) and Property ($\tilde{P}_q$) which still imply compactness of $N_q$ on smooth bounded pseudoconvex domains. We list the definition of a variant of Property ($P_{n-1}$) in [20], which will be used in this article.

**Definition 2.3.** For a smooth bounded pseudoconvex domain $\Omega \subset \mathbb{C}^n (n > 2)$, $b\Omega$ has Property ($P_{n-1}^\#$) if there exists a finite cover \( \{V_j\}_{j=1}^N \) of $b\Omega$ with special boundary charts and the following holds on each $V_j$: for any $M > 0$, there exists a neighborhood $U$ of $b\Omega$ and a $C^2$ smooth function $\lambda$ on $U \cap V_j$ such that $0 \leq \lambda(z) \leq 1$ and there exists $t (1 \leq t \leq n-1)$ such that $\lambda_{tt} \geq M$ on $U \cap V_j$.

Here, as in Definition [2.1], $\lambda_{tt} = L_t \bar{L}_t \lambda + \sum_i c_{it} \bar{L}_i \lambda$ is the diagonal entry in the Hessian matrix $(\lambda_{jk})$. We have the following result in [20]:

**Theorem 2.2 ([20]).** Let $\Omega \subset \mathbb{C}^n (n > 2)$ be a smooth bounded pseudoconvex domain. If $b\Omega$ has Property ($P_{n-1}^\#$), then the $\bar{\partial}$-Neumann operator $N_{n-1}$ is compact on $L^2_{(0,n-1)}(\Omega)$.

We also need the following result due to Sibony ([15]):

**Proposition 2.3.** Let $K$ be a compact subset in $\mathbb{C}^n (n \geq 1)$ and $K$ has Lebesgue measure zero in $\mathbb{C}^n$. Then $K$ has Property ($P_n$) in $\mathbb{C}^n$.

The original result is formulated for $n = 1$ case. But the sum of any $n$ eigenvalues of the complex Hessian of $\lambda$ in $\mathbb{C}^n$ is equal to the real Laplacian of $\lambda$ in $\mathbb{R}^{2n}$, and most of the classical potential results which were used in the proof of this result can also be formulated in $\mathbb{R}^{2n}$, hence the result can be generalized to $n > 1$ case trivially.

3. PROOF OF MAIN THEOREM

**Proof of Theorem 1.1.** Let $\{\xi_j\}_{j=1}^{n-1}$ be the orthonormal coordinates which span the complex tangent space $Z$ in the special boundary chart at a boundary point $P$. Let $V$ be a neighborhood of the boundary point $P$, and $K$ be the weakly pseudoconvex points on the boundary $b\Omega$. Let $\pi^Z : \mathbb{C}^n \to \mathbb{C}^{n-1}$ be the projection map from $\mathbb{C}^n$ onto the complex tangent space $Z$ at $P$.

The set $\pi^Z(K \cap V)$ has Hausdorff-$2n-2$ dimensional measure zero in a copy of $\mathbb{C}^{n-1}$, since any continuous map preserves Hausdorff measure zero set. Since Hausdorff-$2n-2$ dimensional measure is equivalent to Lebesgue measure in $\mathbb{C}^{n}$ (modulo a constant), by Proposition 2.3 the set $\pi^Z(K \cap V)$ has Property ($P_{n-1}$) of Catlin. That is, for any $M > 0$, there exists a neighborhood in $\mathbb{C}^{n-1}$ of $\pi^Z(K \cap V)$ and a $C^2$ smooth function $\lambda^M(\xi_1, \cdots, \xi_{n-1})$ such that $0 \leq \lambda^M \leq 1$ and the real Laplacian $\Delta \lambda^M(\xi_1, \cdots, \xi_{n-1}) \geq M$ on the above neighborhood of $\pi^Z(K \cap V)$. Here the Laplacian is taken with respect to the coordinates $\{\xi_1, \cdots, \xi_{n-1}\}$ in $\mathbb{C}^{n-1}$. Define $\lambda^M_{jk}$ which is same in Definition 2.4 therefore $\Delta \lambda^M(\xi_1, \cdots, \xi_{n-1}) = \sum_{j=1}^{n-1} \lambda^M_{jj}$ by using the invariance of real Laplacian under orthonormal coordinates change.

On the neighborhood $V$, define the trivial extension function $\eta^M(\xi_1, \xi_2, \cdots, \xi_n) = \lambda^M(\xi_1, \cdots, \xi_{n-1})$. Then the real Laplacian $\Delta \eta^M$ on the boundary is equal to the
real Laplacian $\Delta \lambda^M$. Consider the entries in the complex Hessian of $(\eta^M_{j,k})$, the size of this matrix is $n \times n$. For $1 \leq j \leq n - 1$, $\eta^M_{j,j} = \lambda^M_{j,j}$ by using Definition 2.1.

Now let the set $E^M_j = \pi^Z(K \cap V) \cap \{\eta^M_{j,j} \geq \frac{\lambda^M_{j,j}}{1}\}, 1 \leq j \leq n$. By definition of $\lambda^M$, we have $\pi^Z(K \cap V) \subseteq \bigcup_{j=1}^{n-1} E^M_j$. Then $\bigcup_{j=1}^{n-1} (\pi^Z^{-1}(E^M_j) \cap V) \supseteq K \cap V$, here $\pi^Z^{-1}$ is the inverse map of $\pi^Z$.

The diagonal entry $\eta^M_{j,j}$ in the complex Hessian of $(\eta^M_{j,k})$ satisfies the conditions in the definition of Property $(P^M_n \#)$ on each $\pi^{-1}_Z(E^M_j) \cap V$ when $1 \leq j \leq n - 1$. Now since $\bigcup_{j=1}^{n-1} (\pi^{-1}_Z(E^M_j) \cap V) \supseteq K \cap V$ by the previous paragraph, we can apply Property $(P^M_n \#)$ together with partition of unity to prove the compactness estimate locally on $V$. The cut-off functions in the partition should produce extra partial derivatives by hitting $\overline{\partial}$ and $\partial$, but those derivatives can be handled in the same way as the proof of Theorem 2.2 hence the desired compactness estimate (see (ii) in Proposition 2.1) will not be affected. Also for the strongly pseudoconvex points on $V$, they are naturally of D'Angelo's finite type and hence compactness estimate holds there (see [3, 5] or [16]). Since compactness of the $\overline{\partial}$-Neumann operator is a local property, the conclusion follows.

**Remark 3.1.** For the case of Hausdorff 2-dimensional measure and compactness of $N_1$, as we pointed out in the introduction section, the essential argument in Sibony and Boas’s work ([2] and [15]) is to show that the infinite-type points on the boundary satisfy Property $(P_1)$. In such argument, the idea is to project the set $K$ of infinite-type points to each $z_j$-plane and the resulting set satisfies Property $(P_1)$ on each complex 1-dimensional plane, hence summing all involved functions in the definition of Property $(P_1)$ will give the desired conclusion. Now in our case of Theorem 3.1, such summation of functions does not work since eigenvalues from each respective complex Hessian interfere the summation of eigenvalues in the whole complex Hessian. Therefore, verifying Property $(P_q)$ or Property $(\overline{P}_q)$ under such case appears not to work. A detailed explanation of such phenomenon under potential analysis background can also be found in the author’s recent work (see remarks after Corollary 3.2 in [19]).

Our result in Theorem 3.1 shows that small set of weakly pseudoconvex points (or infinite-type points) on the boundary in the sense of Hausdorff-$2n-2$ dimensional measure is benign in the compactness of $N_{n-1}$. When $1 < q < n - 1$, whether similar conclusion holds in the sense of Hausdorff-$2q$ dimensional measure is not known yet. In such case, a certain arrangement on projections onto each $q$-dimensional subspace needs to be found.

For an example when Theorem 3.1 holds, we give one example from [19] and refer the reader to there for details of calculation.

**Proposition 3.1.** Define a smooth complete Hartogs domain $\Omega \subset \mathbb{C}^3$ by:

$$\Omega = \{(z_1, z_2, z_3) \mid |z_3|^2 < e^{-\varphi(z_1) - \psi(z_2)}, z_1 \in \mathbb{D}(0,1), z_2 \in \mathbb{D}(0,1)\}.$$

Assume that $\varphi, \psi \in C^\infty(\mathbb{D}(0,1))$ and subharmonic on $\mathbb{D}(0,1)$ in the respective complex plane. Assume further that the boundary points $(z_1, z_2, z_3)$ are strictly pseudoconvex when $(z_1, z_2)$ is close to $b(\mathbb{D}(0,1) \times \mathbb{D}(0,1))$. If the Hausdorff 4-dimensional measure of the weakly pseudoconvex points of $b\Omega$ is zero, then the $\overline{\partial}$-Neumann operator $N_2$ is compact.
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