Complexity reduction by sign prediction in tree traversal of MIMO decoder

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Abstract: The Breadth First Signal Decoder (BSIDE) algorithm can be expressed as tree searching class of algorithm that does not require an estimate of SNR and provides an optimal BER performance. However, the average number of nodes to be searched by BSIDE in a tree is more that leads to higher implementation complexity. In this letter, a sign prediction technique exploited over the similarity property of QAM that reduces the computational complexity of the algorithm without any degradation in the performance is proposed. Simulation results show that the approach with sign prediction reduces 50\% reduction in computational complexity compared to BSIDE for 2 \times 2 and 66\% for 4 \times 4 MIMO systems.

Keywords: MIMO, QAM, BSIDE, PED, ML, SD

Classification: Electron devices, circuits, and systems

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1 Introduction

The requirements of reliable high data rate services for the future communication systems are readily served by MIMO systems [1, 2]. The BER performance and the capacity of the MIMO system can be improved by deploying spatial diversity and spatial multiplexing techniques respectively. The improvement in performance of wireless links is achieved by MIMO at the cost of high computational complexity at both transmitter and receiver sides. In particular at the receiver end designing an efficient detector for MIMO systems is a challenging task. The Maximum Likelihood (ML) is theoretically considered to be the optimal receiver for Uncoded MIMO systems [3, 4]. However, the implementation of ML methods results in exponential computational complexity which makes its realization unfeasible. The Sphere Decoder, (SD) [5, 6, 7] is the one, with reduced complexity which was recently applied to ML detection problem. SD reduces the computational complexity, by only considering symbols that lie inside a sphere. The drawback of SD is that the complexity is highly dependent on the SNR of the received signal resulting in variable throughput. Though K-Best decoder [8, 9] results in constant throughput with implementation possible, its complexity is higher than the SD. In this letter, a sign prediction technique is proposed to reduce the computational complexity of MIMO decoder without sacrificing the performance of the system.

2 System model

In a MIMO system with $N_T$ transmit antennas and $M_R$ receive antennas, the received signal vector $Y$ is given by

$$Y = HX + v$$

(1)

Where $H$ denotes $M_R \times N_T$ channel matrix, $X$ is the transmitted signal and $v$ is the i.i.d. complex additive white Gaussian noise (AWGN) vector. At the receiver side, a real value representation is used for $X$ and $Y$ vector with $N = 2N_T$ and $M = 2M_R$ elements respectively transforming $H$ to an $M \times N$ matrix. The ML estimate of the signal vector $X_{ML} \in \Omega$ minimizes, $\|Y - HX\|^2$, where $\Omega$ denotes the set of real entries in the constellation. Assuming $N_T = M_R$ and introducing QR decomposition of $H$, equation (1) can be written in a recursive manner as

$$\|Y - HX\|^2 = \sum_{i=1}^{N} Y_i - \sum_{j=1}^{N} R_{ij}X_j^2$$

(2)
where \( \hat{Y}_i \) represent \( i \)th element of \( \hat{Y} = Q^H Y \), \( R_{ij} \) denotes the \((i,j)\)th element of upper triangular matrix \( R \), and \( X_j \) is the \( j \)th element of \( X \).

### 3 Existing algorithm

The existing algorithm [10] can be well understood with a m-ary tree of \( N \) layers as shown in Fig. 1, where \( m \) is the number of elements in the lattice. Let \( \mathbf{X}^{(s)} = [X_1^{(s)}, X_2^{(s)}, \ldots, X_{N-l+1}^{(s)}]^T \) be a vector corresponding to the \( s \)th node of the \( l \)th layer, where \( 1 \leq s \leq m^{N-l+1} \) and \( 1 \leq l \leq N \). BSIDE algorithm searches for the minimum valued node, layer by layer so that decision for received signals is made at their corresponding levels. Initially all the node distance at the \( N \)th layer is computed, among which a node with minimum valued Partial Euclidean Distance (PED) is traced further. The procedure is executed for all layers until first to get the ML solution. The parameter \( d_l \) which is the distance at each layer should satisfy \( d_{N-1} \leq d_N \). To reduce the complexity, Decision Feedback Equalization (DFE) detection algorithm and ML detection are combinely used where \( \mathbf{\hat{X}} = [\hat{X}_1, \hat{X}_2, \ldots, \hat{X}_N]^T \) and \( \mathbf{\hat{d}} = \| \mathbf{\hat{Y}} - R \mathbf{\hat{X}} \|^2 \) are the DFE solution and distance respectively. The parameter \( d_l \) is obtained as

\[
d_l = \min\{d_{l+1}, \| \mathbf{\hat{Y}} - R p_l(\| \|^2) \} \quad l = N, N-1, \ldots, 2
\]

with \( d_{N+1} = \hat{d} \) and \( p_l = [\hat{X}_1, \hat{X}_2, \ldots, \hat{X}_{l-1}, X_l^{(i)}, X_l^{(j)}, \ldots, X_{N-l+1}^{(q)}]^T \). Let \( b_l \) be the undiscarded node at the \( l \)th layer with \( b_{N+1} = 1 \). The node distance at the \( p \)th layer is given as

\[
\Psi([X_1^{(j)}, X_{1,j+1}^{(q)} \ldots X_{N-l+1}^{(q)}]^T) = \left( \mathbf{\hat{Y}}_l - R_{lj}X_1^{(j)} - \sum_{i=l+1}^{N} R_{lj}X_i^{(q)} + \ldots \right.
\]

\[
\sum_{j=l+1}^{N} \left( \mathbf{\hat{Y}}_j - \sum_{i=l+1}^{N} R_{lj}X_i^{(q)} \right) + \ldots
\]

\[
= (\mathbf{\hat{Y}}_l - R_{lj}X_1^{(j)} - \Theta(X_l^{(q)}))^2 + \Psi(X_l^{(q)})
\]

with \( j = 1, 2 \ldots m, q = 1, 2 \ldots b_{l+1} \). Where \( \Psi \) and \( \Theta \) are the node distance and signal residue of an \((N - l + 1)\) dimensional vector.

### 4 Proposed algorithm

The complexity of the MIMO decoder with tree representation can be reduced by either reducing the number of nodes visited per level or the number of operations required per visited node. Since, the complexity reduction by reducing the number of operations required is already analyzed [11], in this paper, an attempt has been made to reduce the complexity of the BSIDE by reducing the number of nodes to be searched per level.

Using the fact that, \( R_{ii} \) is a positive real value [12] and hence \( \hat{Y} \) will take the sign of \( X \). Using the similarity property of QAM the set \( \Omega \) is divided into \( \Omega_1 = \{-1, -3\} \) and \( \Omega_2 = \{1, 3\} \). The sign of \( \hat{Y} \) is predicted before the symbol is decoded. If \( \hat{Y} \) is positive then the search for the transmitted symbol is carried out with the set \( \Omega_2 \) or else with \( \Omega_1 \). This can be well explained with an example.
EXAMPLE: Consider a MIMO systems having \( M_R = N_T = 2 \) and employing 16 QAM modulation scheme. A tree with \( 2N_T = 4 \) levels with each node having 4 branches is constructed as shown in Fig. 1. The Input output relation after real value and QR decomposition is given by

\[
\hat{Y} = RX + \bar{v}
\]

while computing \( \hat{X}_4 \),

\[
\hat{X}_4 = \arg\min_{\hat{X} \in \Omega} || \hat{Y}_4 - R_{44} \hat{X} ||^2
\]

\( \hat{X}_4 \) takes the value from the set \( \Omega = \{-3, -1, 1, 3\} \), the PED is calculated with respect to each value in set \( \Omega \). While \( \hat{Y}_4 \) remains same at that level of tree. Let

\[
\begin{align*}
\hat{X}_{4,1} &= \hat{Y}_4 - R_{44}(-3) + \bar{V}_4 \text{ wrt } \hat{X} = -3 \\
\hat{X}_{4,2} &= \hat{Y}_4 - R_{44}(-1) + \bar{V}_4 \text{ wrt } \hat{X} = -1 \\
\hat{X}_{4,3} &= \hat{Y}_4 - R_{44}(3) + \bar{V}_4 \text{ wrt } \hat{X} = 3 \\
\hat{X}_{4,4} &= \hat{Y}_4 - R_{44}(1) + \bar{V}_4 \text{ wrt } \hat{X} = 1
\end{align*}
\]

Considering \( R_{ii} \) is a positive real number [12], \( \hat{Y}_4 \) will take the sign of \( \hat{X}_4 \). The proposed algorithm starts dividing the set \( \Omega \) in to two smaller sets \( \Omega_1 = \{-1, -3\} \) and \( \Omega_2 = \{1, 3\} \) by using similarity property of QAM. If the predicted \( \hat{Y}_4 > 0 \) then it is enough to make a search over set \( \Omega_2 \) in the tree or else \( \Omega_1 \). This proposed technique allows the algorithm to compute the PED that corresponds only to one set, resulting in significant reduction of number of nodes to be searched in the tree, to decode the transmitted complex symbols. Therefore the equation (8) can be rewritten with respect to \( \hat{Y}_4 \) as...
If $\hat{Y}_4 > 0$,
\[
\begin{align*}
\hat{X}_{4,1} &= \hat{Y}_4 - R_{44}(-3) + \bar{V}_4 \text{ wrt } \bar{X} = -3 \\
\hat{X}_{4,2} &= \hat{Y}_4 - R_{44}(-1) + \bar{V}_4 \text{ wrt } \bar{X} = -1
\end{align*}
\] (9)

If $\hat{Y}_4 < 0$,
\[
\begin{align*}
\hat{X}_{4,1} &= \hat{Y}_4 - R_{44}(-3) + \bar{V}_4 \text{ wrt } \bar{X} = -3 \\
\hat{X}_{4,2} &= \hat{Y}_4 - R_{44}(-1) + \bar{V}_4 \text{ wrt } \bar{X} = -1
\end{align*}
\] (10)

The noise becomes a dominating factor at low SNR and increased modulation order. To overcome this and maintain the performance the proposed method is implied on BSIDE (without estimating the SNR) whose intrinsic nature is capable of transforming between DFE and ML detection. When detection is undergone with ML, the sign prediction technique become inactive and complexity increases to its maximum level as in BSIDE. To overcome this, the similarity property of QAM with sign change is exploited [11], where for example in a 16 QAM constellation, the real valued entries is given as $[-3, -1, 1, 3]$ in which detection is carried out with transmitted symbol of $[1, 3]$ and for remaining signal, sign change technique is adopted. This results in reduced complexity than BSIDE algorithm. It is worth noting that the reduction in complexity increases as the constellation size becomes larger. This is because the number of tree nodes increases exponentially with the constellation size.

5 Analysis of computational complexity

The computational complexity [13] of the MIMO decoder is indexed by Number of Multiplication (NOM) required. The analysis for the proposed algorithm with respect to NOM is carried out. As in the case of BSIDE algorithm, to perform QR decomposition and DFE, the NOM is approximately given as [10]
\[
MN^2 - \frac{N^3}{3} + M^2 + \frac{N(N + 5)}{2}
\] (11)

Let $U_l$ and $D_l$ be the NOMs required to find $\{\Theta(X_{(i)})\}_{q=1}^{b_l}$ and $d_l$ respectively. Then the NOM for the proposed work at the $l^{th}$ layer can be given as
\[
\frac{m}{4} + \frac{m}{2} b_{l+1} + U_l + D_l
\] (12)

With $U_N = 0$ and $D_l = 0$, thus the NOMs required for the proposed algorithm is given as
\[
\sum_{l=1}^{N} \frac{m}{4} + \frac{m}{2} b_{l+1} + U_l + D_l
\] (13)

The parameters $\{b_l\}_{l=1}^{N}$, $\{U_l\}_{l=1}^{N-1}$, $\{D_l\}_{l=2}^{N}$ depend on the SNR, the size of the signal constellation, and the numbers of the transmit/receive antennas and satisfy
\[
1 < b_l \leq \frac{m^{N-l+1}}{2}
\] (14)
\[
N - l \leq U_l \leq (N - l) \frac{m^{N-l}}{2}
\] (15)
\[
0 \leq D_l \leq N + \sum_{l=1}^{N} i
\] (16)
The maximum and minimum NOMs required for the proposed algorithm can be given as

\[
NOM_{\text{Max}} = \frac{m^N}{N} \left\{ \left(\frac{m}{2}\right)^2 + \frac{m}{2}(N - 2) - N \right\} - \left(\frac{m}{2}\right)^2 + 2 \frac{m}{2} + \frac{N(2N^2 + 9N + 3\frac{m}{2} + 1)}{6} \right. \\
NOM_{\text{Min}} = \frac{N\left(N + 3\frac{m}{2} - 1\right)}{2}
\]  

(17)  

(18)

6 Simulation results

The performance and the computational complexity of the proposed algorithm are evaluated through the metrics BER and number of real multiplication required in the tree search. The Fig. 2 shows the performance curves for the BSIDE and proposed algorithm for 2 × 2, 4 and 16 QAM. From the simulation result, it is observed that the proposed algorithm closely approaches the BSIDE algorithm without any degradation in the performance.

The computational complexity for the proposed algorithm is compared with BSIDE algorithm for QAM of order 4, 16 respectively. Since ML-MIMO detection technique is a NP hard, here the comparison is made with BSIDE which is considered to be an alternative method with reduced complexity.

By exploiting the similarity property of QAM, the complexity reduction for the proposed algorithm is achieved through reducing the number of nodes to be searched in the tree using sign prediction technique.

Fig. 3 and 4 show the complexity trend for both BSIDE and proposed algorithms for 2 × 2 and 4 × 4 MIMO systems. From the simulation results it is observed that the complexity decreases as the order of the modulation scheme and number of antenna’s get increased. This reduction in complexity is equal to 50% for 2 × 2 and 66% for 4 × 4 MIMO systems.

Fig. 2. BER performance of proposed and BSIDE algorithm
7 Conclusion

An efficient tree search technique for MIMO decoding is proposed in this paper. The performance of the proposed algorithm is similar as that of BSIDE algorithm. A significant complexity reduction is witnessed by exploiting the sign prediction technique, thereby ensuring better complexity reduction for larger constellation size and antenna numbers.