Analysis of Neutron Fission Reaction Rate in the Nuclear Fuel Cell Using Collision Probability Method with Non Flat Flux Approach

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Abstract. Neutron fission reaction rate in the nuclear reactor depends on macroscopic cross section and neutron flux distribution. The macroscopic cross section depends on the type of nuclide, the type of reaction, and the group energy of the neutrons relative to the nuclides. Flux distribution is very important in a nuclear reactor, because it is closely related to power distribution. In general, the integral neutron transport equation is solved using a collision probability (CP) method with a flat flux (FF) approach. Consequently, the CP matrix is also assumed constantly, therefore, the distribution of the neutron flux throughout the cell becomes flat. In the non-flat flux (NFF) approach, the neutron flux is modelled by linear interpolation as a function of mesh in the cylindrical nuclear fuel cell of a fast reactor type. This study uses the CP method with a NFF approach and it is applied to analyze the neutron fission reaction rate of a cylindrical nuclear fuel cell of a fast reactor type. Nuclear data library that is used in this study is JFS-3-J33 which belongs to the SLAROM computer code. Calculation results of the fission reaction rate shows that it is decrease in the high energy region due to the events of elastic collision that caused the neutron easier to lose of energy. The same fission reaction rate pattern occurs in the FF and NFF approaches.

1. Introduction
The most difficult part of the nuclear reactor analysis is to solve the neutron distribution system as it describes the integral transport equation with variable of energy, space and time [1]. In general, the integral transport is solved using collision probability (CP) method with flat flux (FF) approach [2]. CP method has advantage for simple and symmetrical geometry such as cylindrical cell. Flat flux means that the neutron flux is every points in the region of nuclear fuel cell is considered to be constant and the neutron cross section in all region of the cell is homogeneous. Consequently, the CP matrix is also assumed constantly, therefore, the distribution of the neutron flux throughout the cell becomes flat. Transport of neutron in this study covers two perspectives. First, calculation of homogenization of the nuclear fuel cell is performed using CP method with the FF approach [3]. Second, the points of mesh in cell region is assumed not flat, rather they follow neutron interpolation model based on non-flat flux (NFF) approach [4]. The selected flux interpolation is linear, quadratic or spline cubic function and it is applied to analyze flux distribution of a cylindrical nuclear fuel cell of fast reactor type. The method itself requires fine spatial subdivision of the geometrical domain into so called flat-flux zones where the material properties are assumed to be constant [5].

Neutron interaction with the nuclear fuel cell is presented through the CP process from one nucleus to another until it is absorbed or escapes the boundary of a system. One of the important and characteristic features of neutron interactions with matter that proceed through a compound nucleus formation is that cross sections exhibit maximum values at certain incident neutron energies [5].
Fission process represents a type of nuclear interactions that produces neutrons and energy and as such is a basic principle of nuclear power generation. In a nuclear reactor, neutrons not only have mono energetic, it covers a wide range of energy, and this is referred to as multi group energy. The reaction rates as well as neutron flux and cross sections are energy dependent. At a given neutron energy, the neutron flux at a given time varies with the spatial position in the reactor core. The spatial distribution of fissile material is not entirely uniform initially and is not uniform after the reactor has been operating for a certain time. In order to determine a fission rate at a given time, the neutron transport equation must be integrated over all neutron energies and spatial positions in the reactor. In thermal reactors, the majority of fissions occur in the thermal energy region where flux and macroscopic cross sections are both very large. The fission rate can be roughly estimated assuming the average values of space and energy for flux and cross section. A representative geometry of a complex reactor assembly is selected to show the distribution of neutron flux and reaction rates as a function of neutron energy group and spatial coordinates. Neutron flux at a certain point in the reactor core will depend on the distribution of nuclear properties such as cross section and the position where the flux is calculated. The reaction rate for various types of interactions is found from the appropriate cross section type: fission, absorption and total. In order to get a good fission reaction rate, cross section of fission must be dominant.

This study uses the CP method with a NFF approach and it is applied to analyze the neutron fission reaction rate of a cylindrical nuclear fuel cell of a fast reactor type for 70 energy group. Library data that is used in the research is the JFS-3-J33 JAEA (Japan Atomic Energy Agency) of group constants for the SLAROM code.

2. Theoretical Background
Probability of neutron that uniformly spreading out and isotropic in the region $i$ and suffer first collision in region $j$ of cylindrical nuclear fuel cell is follow [6]

$$P_g = \frac{2}{\Sigma_i} \int d\rho \{ K'_i(\lambda^2) - K_i(\lambda_i + \lambda_j) - K_i(\lambda_i + \lambda_j) + K_i(\lambda_i + \lambda_j + \lambda_i + \lambda_j) \}$$

where $\lambda_k = \sum_{k=1}^{l} (x_k - x_{k-1})$, $\lambda^2 = \sum_{k=1}^{l} \lambda_k$ and $\lambda^1 = \sum_{k=1}^{l} \lambda_k \cdot K_v(\lambda)$ is Bickley-Nayler function of third order. Therefore, if energy range of neutron is divided into multi group energy, then the average flux in the energy interval $\Delta E_g$ is described by $\varphi_{ig}$, so the neutron transport equation is performed as follow [7]

$$\Delta E_g \Sigma_{g'g} V_{g'} \varphi_{g'} = \sum_j P_{g'g} V_{g'} \left[ \sum_{g''} \Delta E_{g''} \Sigma_{g''g} \varphi_{g''} + \Delta E_{g'} \Sigma_{g'g} \right]$$

(2)

where $\Delta E_g$ and $\Delta E_{g'}$ are energy width in the group $g$ and $g'$, $\Sigma_{g'g}$ is scattering cross section in the region $i$ from the group $g'$ to $g$. For the computational process, equation (2) can be rewritten as multi group energy [2]

$$\Sigma_{g'g} V_{g'} \varphi_{g'} = \frac{1}{k_{ef}} \sum_i V_i P_{g'g} S_{g'}$$

(3)

Neutron reaction rate proportional to neutron flux and target area. The reaction rate for various types of interactions is found from the appropriate cross section type: Fission, absorption and total. In order to get a good fission rate, cross section fission must be the dominant in the interaction. Reaction rate of neutron fission of through shielding material of fuel in the region $i$ with energy group $g$ is defined by

$$R_{ig} = \varphi_{ig} \Sigma_{ig}$$

(4)

where $\varphi_{ig}$ is neutron flux and $\Sigma_{ig}$ is macroscopic cross section. In the case of NFF, neutron flux in the equation (4) is taken from [3]
\[ \phi_{eg} = \frac{1}{\alpha_{eg}} (Q_{eg} - \beta_{eg} \phi_{e-1g}) \] (5)

where

\[ \alpha_{eg} = \left\{ \sum_{jg} \left( \frac{1}{2} x_j^2 x_i - \frac{1}{2} x_j^2 x_{i-1} - \frac{1}{3} x_j^3 \right) \right\} + \sum_{i,g} P_{ig} \sum_{jg} \left( \frac{1}{6} x_i^3 - \frac{1}{2} x_i^2 x_{i-1} + \frac{1}{3} x_i^3 \right) \]

\[ \beta_{eg} = \left\{ \sum_{jg} \left( \frac{1}{2} x_j^2 x_{i-1} - \frac{1}{2} x_j^2 x_{i-1} - \frac{1}{3} x_j^3 \right) \right\} + \sum_{i,g} P_{ig} \sum_{jg} \left( \frac{1}{6} x_i^3 - \frac{1}{2} x_i^2 x_{i-1} + \frac{1}{3} x_i^3 \right) \]

\[ Q_{eg} = \frac{1}{2} \sum_{i,g} P_{ig} S_{ig} \left( x_i^3 - x_i^2 x_{i-1} - x_i^2 x_{i-1} + x_i^3 \right). \]

3. Design and Computational Model

The geometry of cylindrical nuclear fuel cell is divided into three regions: region 1 is fuel, region 2 is cladding and region 3 is coolant [2]. The composition of mesh follows [3]. Specification design of nuclear fuel cell is shown in Table 1. Library data that is used in the research is the JFS-3-J33 of group constants for the SLAROM code in 70 energy group.

| Parameters                              | Specification                               |
|-----------------------------------------|---------------------------------------------|
| Type of nuclear fuel cell              | Uranium-Plutonium Nitride                  |
| Geometry of cell                       | Cylindrical 1D                              |
| Material Structure                     | Stainless steel                             |
| Coolant                                | Lead-Bismuth (Pb-Bi)                       |
| Diameter of pin cell                   | 1.134 cm                                    |
| Cladding thickness                     | 0.11 cm                                     |
| Temperature Average                    | 1183 K                                      |
| Volume fraction: fuel structure        | 61.73%                                      |
| Volume fraction: coolant               | 19.40%                                      |
|                                         | 18.87%                                      |

The computational program to calculate fission reaction rate in the nuclear fuel cell using CP method with FF and NFF approach has been written in Delphi. The procedure to get fission reaction rate are follow the mechanism, after reading input data, microscopic cross section is calculated for each nuclide by considering temperature and homogeneous background cross section. Macroscopic cross section is calculated using corrected background cross section for width of cell. The collision probability matrix \( p_{ij} \) is calculated for all regions that divided into several mesh uses equation (1). For case NFF approach, equation (5) is inserted to equation (3) to get the \( k_{eff} \) and then neutron flux in the equation (5) is inserted to equation (4) to get the fission reaction rate. The iteration is executed for 6 meshes and 70 group of energy. The detail of mesh composition, the radius of each region cell and number of nuclide in each region are shown in [3].

4. Results and Discussion

Figure 1 shows that the fission reaction rate has high value in the region of high energy and then decreased in the intermediate energy region and increase in the low energy region for FF and NFF approch. Fast reactor is working at high energy, between 0.1 eV to 10 MeV (energy group 1 to 30). Fission reaction rate has decreased in the region of high energy due to elastic collision events, therefore neutrons easier to lose energy so the fission reaction rate also decreased. Energy group increase to right, but the energy decreases. The number 1 to 19 are fast neutron energy groups covers...
the cross section data for neutron energy range from 0.11109 to 10 MeV, while the 20 to 70 are thermal neutron energy group cover the neutron energy range from 0.32242 eV to 86.517 keV [8]. This is in accordance with the fact that the formation of neutron flux spectrum only occur at a high energies, because the fast region occurred in a high energy, while the thermal region occurred at low energy. Since the fast reactor is used, then the influence of thermal region can be neglected. It is noticed that the behavior of thermal region, in order to calculate constant group for fast reactor, occasionally avoids the differentiation.

Figure 1. Neutron fission reaction rate in each energy group for FF and NFF approach.

In the intermediate energy range or in the resonant energy region, for all reactors, neutrons fission is absorbed in the resonant energy region. Resonant energy region occurs in the energy group 30 to group 65, or are in the energy range of 1 eV to 1 keV. Fission resonant region is where the energy characteristics of fission nuclides easily excited that indicated the presence of the cross. As may be seen, the NFF approach result is in good agreement with the FF approach, both have same pattern. In the region of low energy or often referred to as the resolved resonance region will depend on the nature of the radionuclide used. For fertile nuclides resolved resonance region until a few keV energy, while for the fissile nuclides in the region of about 50 eV. In this incident is demonstrated how the scattering dominates in the MeV range for most of the fission neutrons have an energy that is just above 1 MeV. For commercial fast reactors, most of the neutrons will have a population in the region around 100 keV. For heavy nuclides, energy is lost in the event of elastic collisions is much smaller than in the inelastic collision. Neutron in the event of elastic collisions with the heavy nuclei, energy is almost equal to the initial energy. In the process of inelastic collision, neutron energy mostly used for excitation.

Figure 2 shows that in the fast energy region, the fission reactions rate occurs only in the fuel region, whereas the cladding and the coolant region does not occur because the both region does not occur fission. For thermal energy region, same pattern of fission reactions rate are shown in the Figure 3. This pattern follows the neutron flux distribution in each mesh using NFF and FF approach. Neutron flux spectrum behavior in the fuel cell region for 6 meshes distribute neutron flux in three regions, i.e. fuel, cladding and coolant. In the NFF and FF approach, neutron flux distribution tends to form linearly in the fuel region, then slowdown in the cladding and in the coolant. Neutron flux distribution tends to decrease in the cladding and coolant for fast energy region, in accordance situation come from the thermal energy region, although in the thermal energy region, flux distribution tends to increase in the cladding and coolant region [3].
Figure 2. Fission reactions rate of the nuclear fuel cell for 6 mesh in the fast region for FF and NFF approach.

Figure 3. Fission reactions rate of the nuclear fuel cell for 6 mesh in the thermal region for FF and NFF approach.

5. Conclusion
Solution of neutron transport equation using the collision probabilities method with NFF approach that implemented in cylindrical cell calculation yields fairly good results comparing with FF approach. The neutron fission reaction rate has high value in the region of high energy and then decreased in the intermediate energy region and increase again in the low energy region for FF and NFF approach. The decreasing of neutron fission reaction rate in the high energy region due to the events of elastic collision that caused the neutron easier to lose of energy. The neutron fission reactions rate in the fast energy region, occurs only in the fuel region, whereas the cladding and the coolant region does not occur because the both region does not occur fission. Same situation also occurs for thermal energy region using FF and NFF approaches, because the pattern follows the neutron flux distribution in each mesh of region.
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