Standard Model predictions for weak decays of $\eta$ mesons

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Abstract

The branching ratios of weak decays of $\eta$ mesons are estimated in the framework of the Standard Model. To observe such decays, $\eta$ meson sources with $N_\eta > 10^{13}$ per year are needed.
The comparatively big number of $\eta$ mesons at the acting $\eta$ factory (SATURN, $N_\eta \sim 10^{12}$ per year) and at future accelerators (CELSIUS, $N_\eta \sim 3 \cdot 10^9$ and DAPhNE, $N_\eta \sim 3 \cdot 10^7$ per year) makes pertinent the question on the possibility to observe weak decays of $\eta$ mesons.

These decays possess some specific properties which would be good to verify. Namely, because of $G$-parity conservation \[2\], \[3\] the decays

\begin{align*}
\eta &\rightarrow \pi^\pm l^\mp \nu \\
\eta &\rightarrow \pi^0 \pi^\pm l^\mp \nu
\end{align*} \hspace{1cm} (1)

are suppressed in comparison with the analogous decays of $K_L^0$ mesons possessing the same CP-properties as $\eta$.

The decays

\begin{equation}
\eta \rightarrow K^\pm e^\mp \nu
\end{equation} \hspace{1cm} (3)

are $G$-parity allowed, but they are suppressed compared to $K_L^0 \rightarrow \pi^\pm e^\mp \nu$ by the momentum-space factor $\sim 10^{-3}$.

Of the nonleptonic weak decays, the decay

\begin{equation}
\eta \rightarrow \pi \pi
\end{equation} \hspace{1cm} (4)

is the most interesting one as it needs CP violation and it can not be masked by strong interaction contributions as in the case of $\eta \rightarrow 3\pi$ decays.

Let’s consider the above mentioned processes in detail.

1 \hspace{1cm} $\eta \rightarrow \pi^\pm l^\mp \nu$

The hadronic part of the matrix element is

\begin{equation}
< \pi^\pm | J_1^\mu + iJ_2^\mu | \eta > = f_+^{(\eta)}(q^2)(p_\eta + p_\pi)_\mu + f_-^{(\eta)}(q^2)(p_\eta - p_\pi)_\mu
\end{equation}

In the Standard Model, where the second-class currents \[2\] are absent, the form factors $f_\pm$ can be different from zero only due to isospin breakdown occurring through the virtual electromagnetic interactions or due to the $m_d - m_u$ mass difference. As the last mechanism gives the largest effect, one can estimate the probability of this decay using a Chiral Theory of low-energy mesonic processes. The effective Lagrangian approach, based on the idea that momentum dependence of the form factors is determined mainly by spin 0 and spin 1 intermediate resonances \[4\], gives the result \[3\]

\begin{itemize}
\item For references see Table 5 in paper \[1\]
\item The form factor $f_+^{(\eta)}$ is originated by successive transitions $\eta \rightarrow \pi^0 \rightarrow \pi^\pm l^\mp \nu$ with

\begin{equation}
L(\eta \rightarrow \pi^0) = \frac{\sqrt{3}}{4}(m_\eta^2 - m_\pi^2) \frac{m_d - m_u}{m_s - \frac{1}{2}(m_d + m_u)} \cdot (\cos \theta_P - \sqrt{2} \sin \theta_P)
\end{equation}

For $f_-^{(\eta)}$, the dependence on SU(3) breaking parameter is really absent because the quantity $(m_\eta^2 - m_\pi^2)$ itself is proportional to $m_s - \frac{1}{2}(m_d + m_u)$.
\end{itemize}
\begin{align}
&f_+^{(\eta)}(q^2) = -\sqrt{3}
&\cdot \left[ \frac{m_d - m_u}{8 m_s - \frac{1}{2} (m_d + m_u)} \right]
&\cdot \left[ 1 + q^2 / (M_\rho^2 - q^2) \right] \cdot (cos \theta_P - \sqrt{2} \sin \theta_P) \\
&f_-^{(\eta)}(q^2) = \sqrt{3}
&\cdot \left[ \frac{(m_d - m_u)(m_u^2 - m_\pi^2)}{8 m_s - \frac{1}{2} (m_d + m_u)} \right]
&\cdot \left[ (M_\rho^2 - q^2)^{-1} - (M_{a_0(980)}^2 - q^2)^{-1} \right] \cdot (cos \theta_P - \sqrt{2} \sin \theta_P) 
\end{align}

where \( \theta_P \) is the mixing angle in the pseudoscalar nonet. Its preferred value [5] seems to be \( \theta_P \simeq -19.5^\circ \) from which one gets \( \cos \theta_P - \sqrt{2} \sin \theta_P \simeq \sqrt{2} \); other values (such as \( \theta_P \simeq -10^\circ \)) have also been proposed, but our results are essentially independent of these details.

Therefore, the form factors of the decay (1) are suppressed by the factor

\[
\beta = \sqrt{3}
\cdot \left[ \frac{m_d - m_u}{8 m_s - \frac{1}{2} (m_d + m_u)} \right]
\cdot \left[ \frac{1}{ctg \theta_C} \cdot (cos \theta_P - \sqrt{2} \sin \theta_P) \right]
\]

in comparison with the form factors of \( K_L^0 \rightarrow \pi^\pm l^\mp \nu \) decay.

Using the most conservative estimate [6] for the quantity (8), we come to the result

\[
\beta < \sim 0.1
\]

Then

\[
B.r.(\eta \rightarrow \pi^\pm l^\mp \nu) \simeq 2 \beta^2 \frac{(m_u)}{m_K} \frac{\Gamma(K_L^0 \rightarrow \pi^\pm l^\mp \nu)}{\Gamma_{tot}(\eta)} (\cos \theta_P - \sqrt{2} \sin \theta_P)^2 \\
\simeq 2 \cdot 10^{-13} \cdot (\cos \theta_P - \sqrt{2} \sin \theta_P)^2
\]

Therefore, an observation of the decay (1) with a rate considerably larger than \( 10^{-13} \) would be an evidence on the existence of some new physics beyond the SM. The estimates of possible contributions of the second-class currents to this process are contained in refs. [7] and [8], but the results are expressed in terms of unknown coupling constants of the second-class current interaction.

2 \( \eta \rightarrow \pi^0 \pi^\pm l^\mp \nu \)

The hadronic part of the matrix element is

\[
< \pi^0 \pi^\pm |A_\mu|\eta> = f_1(q^2)(p_\pi + p_\pi')\mu + f_2(q^2)(p_\pi - p_\pi')\mu + f_3(q^2)(p_\pi - p_\pi - p_\pi')\mu
\]
\begin{equation}
< \pi^0 \pi^+ | V_\mu | \eta > = f_4 \frac{i \varepsilon_{\mu \nu \alpha \beta}}{M_K^2} (p_\pi + p_\pi')_\alpha (p_\pi - p_\pi')_\beta (p_\eta)\nu
\end{equation}

Again, as in the case of the decay (1), the form factors \( f_{1,2,3} \) are suppressed by \( G \)-parity conservation and they are smaller than the corresponding form factors for \( K_L^0 \rightarrow \pi^0 \pi^\pm l^\mp \nu \) by a factor \( \beta \). This is not the case for the form factor \( f_4 \) which is \( G \)-parity allowed. But the contribution of this form factor to the \( K_L^0 \rightarrow \pi^0 \pi^\pm e^\mp \nu \) decay rate is approximately 0.5%.

Then the estimate for B.r. of \( \eta \rightarrow K^\pm e^\mp \nu \) is

\begin{equation}
B.r.(\eta \rightarrow \pi^0 \pi^\pm l^\mp \nu) \cong 2(\beta^2 \text{ or } \frac{2}{3} \cot^2 \theta_c \cdot 0.005)(\cos \theta_P - \sqrt{2} \sin \theta_P)^2 \frac{\Gamma(K_L^0 \rightarrow \pi^0 \pi^\pm l^\mp \nu)}{\Gamma_{tot}(\eta)} \lesssim 1.7 \cdot 10^{-16} \cdot (\cos \theta_P - \sqrt{2} \sin \theta_P)^2
\end{equation}

\section{3 \( \eta \rightarrow K^\pm e^\mp \nu \)}

The hadronic part of the matrix element is of the form of (5). But

\begin{equation}
f_+(\eta \rightarrow K^- e^+ \nu) = \sqrt{\frac{3}{2}} f_+(K^0 \rightarrow \pi^- e^+ \nu)
\end{equation}

and

\begin{equation}
f_-(\eta \rightarrow K^- e^+ \nu) = \sqrt{\frac{3}{2}} \left( \frac{m_\eta^2 - m_K^2}{m_K^2 - m_\pi^2} \right) f_-(K^0 \rightarrow \pi^- e^+ \nu)
\end{equation}

With these values of the form factors

\begin{equation}
\frac{\Gamma(\eta \rightarrow K^\pm e^\mp \nu)}{\Gamma(K_L^0 \rightarrow \pi^\pm e^\mp \nu)} \approx 0.867 \cdot 10^{-3}
\end{equation}

or

\begin{equation}
B.r.(\eta \rightarrow K^\pm e^\mp \nu) = 4 \cdot 10^{-15}
\end{equation}

with negligible \( \eta - \eta' \) mixing effects.

\section{4 \( \eta \rightarrow \pi \pi \)}

Like \( K_L^0 \rightarrow 2\pi \) decay, this decay violates CP invariance \[3\], but the strangeness does not change in \( \eta \rightarrow 2\pi \) transitions.

For this reason, in the Standard Model, the amplitude of the \( \eta \rightarrow 2\pi \) decay must be suppressed at least by the factor \( G_F A^2 \sin \theta_C \) (with \( A \lesssim 1 \text{ GeV} \)) in comparison with the \( K_L^0 \rightarrow 2\pi \) amplitude. This estimate follows from the fact that CP violation in SM
occurs due to imaginary parts in the Yukawa couplings and to have the observable effect of CP violation one needs to include flavour-changing transitions like $\eta \rightarrow K^0 \bar{K}^0$ containing non-self-conjugated product of the Yukawa couplings. Considering then the decay of the $\{K^0, K^0\}$ system into $2\pi$ states we come to the above estimate.

## 5 Conclusions

The estimates obtained in the framework of the Standard Model

$$B.r.(\eta \rightarrow \pi^\pm l^\mp \nu) \lesssim 2 \cdot 10^{-13} \cdot (\cos \theta_P - \sqrt{2} \sin \theta_P)^2$$

$$B.r.(\eta \rightarrow \pi^0 \pi^\pm l^\mp \nu) \approx 1.7 \cdot 10^{-16} \cdot (\cos \theta_P - \sqrt{2} \sin \theta_P)^2$$

$$B.r.(\eta \rightarrow K^\pm l^\mp \nu) \approx 4 \cdot 10^{-15}$$

$$B.r.(\eta \rightarrow 2\pi) \lesssim 4[G_F^2 \Lambda^2 \sin \theta_C]^2 \cdot 10^{-14}$$

show that the observation of these decays at $\eta$ factories with $N_\eta < 10^{13}$ per year would be the evidence of some new physics beyond the Standard Model.

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References

[1] V.S.Demidov, E.P.Shabalin. The DAΦNE Physics Handbook. Ed. by L.Maiani, G.Pancheri, and N.Paver. INFN-LNF 1992, p. 45.

[2] S.Weinberg. Phys. Rev. 112 (1958) 1375.

[3] L.B.Okun, I.S.Tsukerman. JETP 47 (1964) 349.

[4] E.P.Shabalin. Yad. Fiz. 51 (1990) 464 [Sov. J. Nucl. Phys. 51 (1990) 296].

[5] F.Gilman and R.Kauffmann, Phys. Rev. D36 (1987) 2761; A.Bramon, Phys. Lett. B51 (1974) 87.

[6] N.H.Fuchs, H.Sazdjian, and J.Stern. Phys. Lett. B238 (1990) 381.

[7] P.Singer. Phys. Rev. 139B (1965) 483.

[8] W.Lucha et al. Phys. Rev. D46 (1992) 2255.

[9] I.Yu.Kobzarev, L.B.Okun. JETP 46 (1964) 1418.

[10] H.Neufeld, H.Rupertsberger. Preprint UWThPb - 1994 - 15, July 1, 1994.