Formation keeping method of weakly driven multi robot in time-varying flow field

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Abstract. In order to solve the problem that group formation control and optimal path cannot be considered simultaneously for weakly driven multi robot under the influence of environmental flow field, a method combining time-optimal path planning and leader-follower formation control is proposed in this paper. The time-optimal path of the leader is obtained by level set method, and the formation keeping is based on this path. According to the position of the leader and the reachable set of the follower at a certain time, the position of the follower is determined by solving the formation optimization function, and the complete path is completed by solving the formation optimization problem iteratively. While the formation optimization function maintains the group formation, the reachable set generated by the level set method ensures the feasibility of the path obtained in the time-varying strong flow.

1. Introduction
Formation keeping is one of the basic problems in multi robot applications. For long-term and large-scale reconnaissance and detection tasks, how to make multiple small robots with simple functions form a cluster to perform the tasks is a hot topic in current research.

The classical formation control methods include leader follower method, virtual structure method and behavior-based method, etc., which are widely used in the field of robots on the ground [1], in the air [2] and underwater [3]. The leader-follower method is the most commonly used framework for multi robot formation control. Jing LUO et al. [4] introduced the virtual follower on the basis of the leader-follower structure in combination with the virtual structure method and the leader-follower method. It transformed the formation control problem of multi robots into the real follower's tracking and control of the trajectory of the preset virtual follower. The algorithm needs to capture the location information of each robot in real time and transmit it to each robot through wireless network. Based on the leader-follower structure, Sunxin WANG et al. [5] combined path planning and trajectory control with formation control. The leader robot uses the improved artificial potential field method for online local path planning, and uses the sliding mode motion control method to solve the real-time trajectory tracking of the follower. The artificial potential field method generates a repulsive force field between the robots and the target point or obstacle, effectively avoid the obstacle and move towards the target with directivity, but it does not optimize the path time or energy consumption. Yunhui LAI et al. [6] proposed a formation control method of quadrotor UAV with triangle structure based on the graph theory method. The robots occupy the points of the directed digraph, and the distance and
communication state between robots are the edges of the directed digraph. Combined with the leader-follower structure, the distance feedback rate is designed for the formation control of multiple UAVs.

In the atmosphere or ocean, the navigation of the weakly driven robot is highly affected by flow. In the researches above, the influence of flow field is not considered in formation control, and the researches on the combination of optimal path and formation keeping are not deep enough.

Baoxia CUI et al. [7] expanded the search neighbor nodes of A* to 24, but under the influence of strong flow, the neighbor nodes cannot be crossed, resulting in search failure. After the external force constraint is added in RRT algorithm, the path shows greater randomness [8]. Path planning of weakly driven robot is a new research field [9].

Based on the continuous path planned by the level set method, this paper proposes a leader-follower framework based weakly driven multi robot formation keeping method. The core problem to be solved by this method is how to use the level set method to plan an effective path in the time-varying strong flow field while keeping the observation formation of multiple robots.

2. Path planning method based on level set

The path planning adopts the level set method. In the process of computing the path, the level set method can also give the region set that the robot can reach at a certain time, which is referred to as the reachable set. The reachable set provides theoretical basis for the formation keeping algorithm of multiple robots. Given the configuration space $\Omega \subseteq \mathbb{R}^2$, the space point in $\Omega$ is marked as $x$ and the time point is marked as $t$. Now it is assumed that there is a time-varying flow field $V(x, t)$ in $\Omega$, and the robot $R$ starts to move in $\Omega$ under the influence of the flow field $V$ from the time $t_0 (\geq 0)$. $v$ is defined as the velocity of the robot relative to the flow field, the motion model followed by the trajectory of the robot can be written as

$$\frac{dx_R}{dt} = vh(t) + V(x_R(t), t), \quad t > t_0$$

(1)

Where, $h(t)$ is the control rate of the robot at the time $t$.

2.1. Level set method

In classical robot path planning, the particle tracking optimization method is often used, that is, the position of the particle at each time is determined by solving the above differential equation to generate the path. The level set method is an implicit interface evolution method. "Implicit" means that the interface is implicit in the function $\phi(x, t)$ defined in $\Omega$, and the evolution of interface is the numerical computation of $\phi$. Compared with the particle tracking method, the level set method does not analyze the shape of the specific curve (plane), thus avoiding a variety of adverse situations such as particles are too dispersed or clustered.

In the level set method, $\phi(x)$ uses the symbolic distance function, which is numerically equal to the Euclidean distance from each point in the space to the interface to be tracked. The closed interface to be tracked is defined as $\mathcal{F}(t)$, and the point on $\mathcal{F}(t)$ at the time $t$ is recorded as $x_i$. The sign of the point inside the closed interface is negative, and that of the external point is positive. Obviously, for all points $x_i$ on the interface, $\phi(x_i) = 0$, that is, the interface to be tracked is implied in the zero level set in $\phi(x)$.

Over time, $\phi(x, t)$ follows the Hamilton-Jacobi equation

$$\frac{\partial \phi(x, t)}{\partial t} + v | \nabla \phi(x, t) | + V(x, t) \phi(x, t) = 0$$

(2)

2.2. Time-optimal path planning

For robot path planning based on level set, the starting point $x_s$ and the target point $x_d$ need to be defined in advance. In $\phi(x_s, t_0) = 0$, that is, at the initial time $t_0$, the zero level set of $\phi$ is the starting point $x_s$. Similarly, in $\phi(x_d, t_d) = 0$, $t_d$ is the time required for the interface $\mathcal{F}$ to pass through $x_d$ for the first time, and the path obtained is the time-optimal path.
Path planning is divided into two stages: forward evolution and backward tracking. The forward evolution stage starts from the starting point, and the evolution of $\Phi$ follows equation (2). It can be understood that under the influence of flow field $\mathbf{V}(x, t)$, the robot moves along the normal direction of the interface at the velocity $v$ until the interface reaches $r_d$ for the first time. The backward tracking stage starts from the target point, and the whole path is reversely deduced according to the normal direction of the flow field and the front. The reverse deduction process meets the following

$$\frac{dx}{dt} = -\nabla \Phi(x, t) - v \left| \nabla \Phi(x, t) \right|$$  (3)

The numerical implementation of the algorithm is to convert continuous equations (2) and (3) into expressions that can be computed by the computer. The finite difference method is used in the numerical computation, so a key point is to select the time step $\Delta t$ for computation, which must meet the CFL conditions. When computing the gradient of $\Phi$, the upwind mechanism is used to ensure the validity of the data computed.

3. Multi robot formations keeping based on level set method

Based on the concept of reachable closed interface $\mathcal{F}(t)$ given by the level set method, this section derives the multi robot formation keeping method under the influence of dynamic strong flow field. This method adopts the leader-follower collaborative framework, and integrates the time-optimal path given by the level set method and $\mathcal{F}(t)$. The goal of the method is to generate a multi robot path satisfying the formation constraints in the dynamic flow field, and the level set method ensures the feasibility of the path. The accuracy of environmental dynamic flow field information is the key to whether the level set method and the formation keeping method discussed in this paper can be applied in practice. With the development of numerical models of atmospheric and marine environment, the accuracy of flow field information is gradually improved. This paper assumes that the environmental information flow field is accurate, and focuses on the keeping formation algorithm.

Formation refers to regular polygons with robots as the vertices, such as equilateral polygons or equal interior angle polygons. Keeping these formations for multi robot detection or patrol has many advantages, such as environment characteristic field gradient estimation, characteristic field reconstruction, hazard detection and other production scenes. In this paper, the leader-follower collaborative framework is selected to discuss the formation keeping of multi robots. This problem can be easily transformed into other collaborative frameworks or multi robot clusters. The baseline of the formation is the time-optimal path of the leader, which is obtained by the level set method. For any given time, the leader's position is fixed, and the level set method also gives the reachable set of the follower. The leader's position is fixed as a vertex of the polygon, and the intersection point between other points and the corresponding robot's reachable set is determined by rotating the polygon. The key of formation keeping method is to design the algorithm for rotation and the determination of intersection point.

3.1. Problem description

In the leader-follower framework, for the convenience of narration, it is assumed that there are $1 + N(N > 0)$ robots in the robot team, and the motion of each robot follows equation (1). One robot in the team is designated as the leader and the remaining $N$ robots are followers. Specify the starting point $r^0_l$ and target point $r^0_d$ of the leader and the starting point $r^0_i$ of the $i^{th}$ follower, $i = 1, \ldots, N$. According to $r^0_l$ and $r^0_d$, the time-optimal path $r^*_{i}$ of the leader can be computed by equations (2) and (3). The target point of the follower is not specified in advance, and the key of the problem is changed into how to determine $r^*_i$ by $r^*_i$, $i = 1, \ldots, N$.

Starting from the starting point $r^0_i$ of the follower, compute its position $r^m_i$ after each time step $\Delta t$ in sequence, $m = 1, \ldots, M$. $M$ is obtained when computing the leader's optimal path, and the follower's path $r^*_i$ is connected by $r^m_i$. For the convenience of expression, the symbols of variables related to $m \times \Delta t$ are marked with the subscript $m$, and the time after $t_0 + m \times \Delta t$ is called $m$ time.
The reachable set of the $i$th follower at time $m$ is defined as $\mathcal{R}_m^i$, $\mathbf{r}_m^i \in \mathcal{R}_m^i$, that is, the path point of the follower should be located in the reachable set of the robot at the same time. The whole computation is completed by iteration. Each step of the iteration computes a path point, and the next computation is based on the results of that of the previous one.

### 3.2. Formation keeping algorithm

At the initial time $t_0$, each follower is in its initial position $\mathbf{r}_i^0$, $i = 1, \ldots, N$. As an iterative algorithm, we first discuss how to get the first position $\mathbf{r}_1^0$ after the starting point. For any robot $i$, the corresponding value function $\phi_1^i$ after $\Delta t$ time can be obtained by solving equation (2), then the zero level set of reachable set $\mathcal{R}_1^i$, i.e. $\phi_1^i$, can be obtained by the equipotential line extraction method. $\mathbf{r}_1^i$ is located on $\mathcal{R}_1^i$, and satisfies the following formation optimization function

$$\mathbf{r}_1^i = \arg \min_{\mathbf{x}_1^i \in \mathcal{R}_1^i} \varphi(\mathbf{x}_1^0, \mathbf{x}_1^2, \ldots, \mathbf{x}_1^N)$$

Where, $\varphi$ is a standardized function of polygons, which can be written as

$$\varphi(\mathbf{x}_1^0, \mathbf{x}_1^1, \mathbf{x}_1^2, \ldots, \mathbf{x}_1^N) = \max(\lambda_1 \varphi_1(\mathbf{x}_1^0, \mathbf{x}_1^1, \mathbf{x}_1^2, \ldots, \mathbf{x}_1^N), \lambda_2 \varphi_2(\mathbf{x}_1^0, \mathbf{x}_1^1, \mathbf{x}_1^2, \ldots, \mathbf{x}_1^N))$$

Where, $\lambda_1$ and $\lambda_2$ are adjusting parameters, and the functions $\varphi_1$ and $\varphi_2$ can be written as

$$\varphi_1(\mathbf{x}_1^0, \mathbf{x}_1^1, \mathbf{x}_1^2, \ldots, \mathbf{x}_1^N) = \max((\rho_{P_0-P_1} - \rho_{P_1-P_2})^2, (\rho_{P_1-P_2} - \rho_{P_2-P_3})^2, \ldots, (\rho_{P_{N-1}-P_N} - \rho_{P_N-P_0})^2)$$

(6)

$$\varphi_2(\mathbf{x}_1^0, \mathbf{x}_1^1, \mathbf{x}_1^2, \ldots, \mathbf{x}_1^N) = \max((\rho_{P_0-P_1} - \rho_{P_1-P_2})^2, (\rho_{P_1-P_2} - \rho_{P_2-P_3})^2, \ldots, (\rho_{P_{N-1}-P_N} - \rho_{P_N-P_0})^2)$$

(7)

Where, $\rho_{a,b}$ is the distance from $x_a$ to $x_b$. Euclidean distance in space is used in this paper. $c_p$ is the geometric center of the polygon $P$, which is mathematically expressed as $c_p = \frac{1}{N+1} \sum_{k=0}^{N} p_k$, and $p_k$ is the vertex of a sequentially connected polygon.

![Figure 1](image-url)  

Figure 1. The illustration of formation optimization at $m^{th}$ time and the meaning of signs when the number of followers is 2. The black curve is the time-optimal path of the leader, the gray dashed line depicts the reachable set at $m^{th}$ time.

In equation (6), $\varphi_1$ describes the difference between the sides of the polygon, $0 \leq \varphi_1 \leq 1$. When the polygon is a regular polygon, $\varphi_1 = 0$. When the length of one side of the polygon approaches 0, that is, when two adjacent vertices approach to each other, $\varphi_1$ approaches 1. In equation (7), $\varphi_2$ describes the difference from adjacent vertices to the center of a polygon, $0 \leq \varphi_2 \leq 1$. When the polygon is a regular polygon, $\varphi_2 = 0$. When the polygon is squeezed and two opposite vertices approach, $\varphi_2$ approaches 1. $\lambda_1$ and $\lambda_2$ ($0 \leq \lambda_1, \lambda_2 \leq 1$) are used to adjust the role of $\varphi_1$ and $\varphi_2$ in...
optimization. To sum up, the optimization equation (4) makes the polygon formed close to or equal to a regular polygon.

Solve the optimization equation (4) to obtain the position $\mathbf{r}_i^t$ of each follower robot after $\Delta t$ time, $i = 1, \cdots, N$. Repeat the above steps with $\mathbf{r}_i^t$ as the starting point to get the position of the follower $\mathbf{r}_i^t$ after $2 \times \Delta t$ time. Repeat this process until the leader's target point is reached, and the path of the $i^{th}$ follower can be obtained by connecting $\mathbf{r}_m^t$, $m = 1, \cdots, M$.

4. Simulation results

The simulation experiment is carried out in a two-dimensional plane, and the time-varying flow field in the plane is generated by the model. The Cartesian coordinate system is used, and the grid density is $100 \times 100$. For the convenience of computation, the ranges of two coordinate axes in space are both $[-1,1]$. In the experiment, the velocity of the weakly driven robot is set as a constant value $v = 0.28 \text{ m/s}$.

Fig. 2 shows the dynamic flow field at different times. The maximum velocity of the flow field is $0.6 \text{ m/s}$, which is twice the velocity of the robot. The black solid circle in the figure represents the starting point of the leader, and the pentagram represents the target point of the leader. The gray curve in Fig. (b, c, d) shows the evolution of the leader's reachable set $\mathcal{R}_m^t$ over time. The black solid line in Fig. 2 (d) is the time-optimal path of the leader from the starting point to the target point obtained by solving equations (2) and (3).

Figure 2. The quivers depict the time varying flow field where the maximum velocity is 0.6 m/s. Black solid circle and pentagram represent the starting and ending points of the leader respectively. Gray lines in a, b and c show the evolution of the reachable sets over time. The black solid line is the time-optimal path of the leader.

With the leader's optimal path $\gamma_s^*$, according to the algorithm description, taking $\gamma_s^0$ as the benchmark, solve the optimization problem (4) and gradually determine the follower's path. Fig. 3 shows the screenshots of the three time windows in the follower path determination process. If the formation is of a regular triangle, $\lambda_1 = 1$ and $\lambda_2 = 0$ is selected in the simulation, that is, the optimal solution with equal side length is more likely to be found. There are two grey curves in Fig. 3 (a, b, c), representing the reachable sets of two followers. The reachable sets are given by the level set method. The computation of reachable set is based on the result of the last computation. The reachable set in Fig. (a) is computed from the initial points given by experiments, and the reachable set in Fig. (b) is computed from the path points computed in Fig. (a). The solution of equation (4), that is, the point on
the path, falls on the reachable set of the corresponding robot. Because equation (4) only optimizes the equality of the side lengths of the polygon, and does not maintain the formation of robots unchanged, the side lengths of the polygon formation are variable during the process from the starting point to the target point. The side length of the starting formation is 1 km, and the side lengths of the formation computed in Fig. 3 (a, b, c) are $1.581km$, $1.326km$ and $1.297km$ respectively.

**Figure 3.** In the case where there exist three members in a swarm, i.e. one leader and two followers, the computation of path points that form a certain coordination. The path points fall into the reachable sets which are computed by level set method. The mth computation is based on the results of that of (m-1)th.

Fig. 4 shows the formation of quadrangle and hexagon, where streamline is the flow field at the time of reaching the target point. The increase of the number of followers will undoubtedly lead to the increase of algorithm computation. The computational complexity is mainly composed of two parts: level set algorithm and formation keeping algorithm. For the level set algorithm, $\bar{F}$ is assumed to be the velocity of the robot moving a unit grid point, and $\bar{F}$ can represent the sum of the effective velocities of the robot in motion. $n_x$ is the number of grids in the direction of a coordinate axis ($n_x = 20$ in simulation), and $T^0_k$ is the time required to plan the path of the leader, $\bar{F}n_xT^0_k$ is the time required to solve equation (3) to obtain the path of each robot, and the algorithm complexity is $O(\bar{F}^2n_x^2T^0_k^2)$. Use the formation keeping algorithm to search the points on the reachable sets of multi robots which can keep the formation of a regular polygon. Suppose $l_m$ the length of path points from $m$th to $m + 1$th time, and the complexity of formation keeping algorithm is $O(\bar{F}^3n_x^4M)$. The meaning of $N, M$ is the same as above, where $N$ is the number of followers and $M$ is the number of computation steps.

**Figure 4.** Paths of the leader and the followers in the formation of triangle and hexagon.

5. **Conclusions**

For weakly driven robots, the time-varying flow field will have a strong impact on the motion of the robot, and even restrict the behavior of the robot. The time-optimal path given by the level set method ensures the effectiveness of the planned path in theory, and the reachable set of the robot at a certain time is a guarantee of multi robot formation keeping algorithm. By solving the optimization problem,
the formation keeping algorithm proposed by the simulation results ensures that the path obtained is feasible under the premise of keeping the regular polygon formation.

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