Application of PEEC Model in Calculation of Transformer High Frequency Voltage

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Abstract. Overvoltage will occur on transformer windings when high frequency waves such as lightning strike and Very fast transient overvoltage(VFTO) intrude into transformer windings. The model and parameters is the basis of calculating transient overvoltage in transformer windings. In this paper, partial element equivalent circuit (PEEC) is used to calculate High Frequency Voltage on transformer windings. the inner grid of conductor through which conductive current passes and the surface grid of conductor with charge distribution is used to transform into the inductance unit and capacitance unit of PEEC respectively. The mutual capacitance between different windings is solved by PEEC. The change regulation of resistance coefficient of transformer windings under different frequency waveforms is analyzed, and the internal voltage of transformer under different frequency applied voltage is solved. The results show that the self-inductance of winding decreases slightly with the increase of applied voltage frequency, and basically remains unchanged in high frequency band. With the increase of frequency, the voltage waveform on winding turns decreases rapidly.

1. Introduction
When high frequency waves such as lightning strike and VFTO intrude into transformer windings, there will be non-uniform distribution on transformer windings, and a large proportion of voltage may fall on the windings near the transformer ports, which will damage the insulation at the transformer ports. In addition, the high frequency waves may also cause oscillation in the inner turns of transformer windings which leads to internal overvoltage. Therefore, It is necessary to establish a transformer winding equivalent transient model in order to provide a basis for transformer insulation design which can be used for quantitative analysis and determine the voltage distribution in transformer under high frequency waves[1-3].

The determination of model and parameters is the basis of calculating transient overvoltage in transformer windings. Analytical calculation method and numerical analysis method is used to model large-scale electromagnetic equipment at home and abroad. The analytical calculation method is more intuitive than numerical analysis method. According to the frequency range of the transient voltage acting on the electromagnetic equipment, the winding of the equipment can be equivalent to two kinds of lumped parameter circuit and distributed parameter model by analytical calculation method. The analytical calculation method is efficient, but its application range is narrow [4]. At present, finite difference time domain method, finite element method, moment method are used as the commonly numerical analysis methods. The finite-difference time-domain method (FDTD) is a method proposed by K.S. Yee for numerical calculation of electromagnetic field. The intensity of electric field and magnetic field is discretized alternately in space and time. The Maxwell curl equation with time
variable is transformed into difference equation, and the spatial electromagnetic field in time domain is solved analytically. Recently, dielectric surface impedance has been used to deal with the boundary problem of electromagnetic field, which simplifies the processing of finite difference time domain method[5]. However, the skin effect of conductor is obvious under the action of high frequency wave, and the requirement of finite element analysis for partition is high, which leads to the slow calculation of electromagnetic distribution in large equipment by finite element method[6].

In view of the above situation, the experts put forward Partial Element Equivalent Circuit (PEEC) to establish the equivalent model of equipment windings, and a numerical calculation method to solve the field-circuit coupling method of coupling parameters between windings[7,8]. PEEC is one of the field-circuit coupling methods. It converts the solution of Maxwell's electromagnetic field integral equation into the solution of corresponding equivalent circuit equation. It simplifies the solution and improves the calculation accuracy while improving the efficiency, such as dynamic large-capacity converter. A general method for extracting inductance matrix of large-scale complex multi-conductor systems, such as series compensator, synchronous motor, etc[9,10]. The above literature mainly considers the influence of inductance matrix and neglects the function of capacitance unit. In this paper, the internal grid of conducting current passing through the conductor body and the surface grid of conducting charge distribution are transformed into the inductance cell and capacitance cell of PEEC respectively, the electromagnetic field problem can be transformed into the corresponding path problem. The internal voltage is solved under different frequencies of high frequency wave.

2. The theoretical basis of PEEC model
On the basis of Coulomb's law, Ampere's law and Faraday's law, Maxwell put forward the hypothesis of displacement current and obtained the basic equations of time-varying electromagnetic field according to the law of charge conservation. There are three independent equations in Maxwell's electromagnetic field equation system.

\[ \nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t} \]  
\( (1) \)

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  
\( (2) \)

\[ \nabla \times \vec{J}_c = -\frac{\partial \rho_f}{\partial t} \]  
\( (3) \)

In the formula: magnetic field intensity \( \vec{H} \), conductive current density \( \vec{J}_c \), electric flux density \( \vec{D} \), electric field intensity, magnetic flux density \( \vec{B} \), the above five parameters are vectors. \( \rho_f \) is charge density. In order to establish the model by PEEC method, formula (1)-(3) can be reconstructed as follows:

\[ \vec{E}_0(t) = \vec{E}(t) + \frac{\partial \vec{A}}{\partial t} + \nabla \varphi \]  
\( (4) \)

In the formula: applies electric field of external circuit \( \vec{E}_0(t) \). Formula 4 is the basic equation for constructing PEEC model. The inductance and capacitance of the module unit correspond to the current unit and the potential node in the circuit. Figure 1 shows the meshing of module units.
The resistance of cores is larger than the resistance value of windings and increases obviously at low frequency of wave. With the increase of frequency, the resistance increases and cores with the unit length (1m) have almost DC (1Hz) winding while the self-inductance of the core is lower than that of the winding when the current is 10MHz with the amplitude of 1A.

Table 1 shows the variation of PEEC equivalent parameters of square windings and cores with the unit length (1m), surface area (1cm²) under the current of 1Hz, 50Hz, 1kHz, 1MHz and 10MHz with the amplitude of 1A.

From Table 1, it can be seen that the resistance of the core in unit field is higher than that of the winding. The change law of the per unit length resistance is basically the same to windings and cores. With the increase of frequency, the resistance increases, but the increase range decreases. The resistance of cores is larger than the resistance value of windings and increases obviously at low frequency of wave.

3. Internal overvoltage of Transformer

Lightning stroke and VFTO are composed of several voltage with 10 kHz to 10 MHz. In order to simplify the effect of external high frequency wave on internal overvoltage of transformer, they are regarded as voltage with single frequency. Assuming that the permeability of transformer core does not change with frequency, the resistance and inductance of transformer winding and core will change with the frequency of high frequency wave.

Table 1. Variation of parameters of winding and core under different frequencies

| frequencies | winding resistance(10⁻⁴) | winding inductance(10⁻⁵) | core resistance(e10⁻⁴) | core inductance(10⁻⁶) |
|-------------|--------------------------|------------------------|-----------------------|-----------------------|
| 1Hz         | 1.7                      | 1.11                   | 4.29                  | 1.09                  |
| 50Hz        | 1.72                     | 1.11                   | 27                    | 1.06                  |
| 1kHz        | 3.39                     | 1.10                   | 121                   | 1.06                  |
| 1MHz        | 92                       | 1.06                   | 3819                  | 1.06                  |
| 10MHz       | 290                      | 1.06                   | 12076                 | 1.06                  |

It can be seen from formula (5) that the resistance and inductance of module units will change with the frequency of wave. Table 1 shows the variation of PEEC equivalent parameters of square windings and cores with the unit length (1m), surface area (1cm²) under the current of 1Hz, 50Hz, 1kHz, 1MHz, 10MHz and 10MHz with the amplitude of 1A.

The PEEC model corresponding to module units can be obtained by formula (5) as shown in Figure 2.

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frequency (less than 50 Hz). The self-inductance of windings and cores decreases with the increase of frequency, but the reduction is small. Similarly, the change rate of core self-inductance at low frequency is higher than that of winding self-inductance.

According to the given transformer winding model, the high-voltage winding of the main transformer is 1-1000 numbered from the inner layer to the outer layer, and the low-voltage winding is 1001-1100 according to the position of the line turns. The mutual inductance between the line turns of No. 1 and No. 1001 and other lines is calculated by using the formula given in Formula (5) as shown in Figure 3.

![Figure 3. The mutual inductance between the line turns](image)

It can be seen that the mutual inductance between the rings of single-turn conductors between high-voltage and low-voltage windings is similar to that between single-turn conductors from figure 3. With the increase of the distance between the coils, the mutual inductance between the turns of high-voltage windings decreases rapidly, and the reduction rate decreases with the increase of the distance. Therefore, when calculating the voltage distribution using inductance parameters, only the mutual inductance between turns in the adjacent line segments needs to be considered, and the mutual inductance between turns outside a certain distance can be neglected. The variation of mutual inductance between the axial equivalent of turns, between other windings and between rings is similar, which is not discussed here. The ground capacitance of single-turn conductors with high and low voltage windings is shown in Fig. 4 and Fig 5.

![Figure 4. Ground capacitance of single-turn wire with high voltage winding](image)

![Figure 6. Mutual capacitance of winding](image)

It can be seen that the ground capacitance of single-turn wire with the high voltage winding can be changed from 0 to 48 pF from figure 4. The single-turn capacitance of the high-voltage winding increases nonlinearly with the decrease of the distance between the winding turns and the ground, and the growth rate increases continuously. The high-voltage winding is designed by layers, with the high-voltage winding far away from the core, the area of the line will increase slightly and the capacitance to the ground will increase. Because the current carrying capacity of the low-voltage winding single-turn conductor is higher than that of the high-voltage winding, the current carrying capacity is proportional to the cross-sectional area, which makes its ground capacitance higher than that of the high-voltage winding. The variation law of the ground capacitance of the low-voltage winding single-turn conductor is similar to that of the single-layer high-voltage winding. It increases nonlinearly with
the decrease of the distance between the winding turns and the ground. And the growth rate is increasing. Similarly, the mutual capacitance between single-turn conductors with high and low voltage windings is shown in Figure 6.

From figure 6, it can be seen that the maximum mutual capacitance between winding is similar to ground capacitance of winding in figure 5. With the increase of distance between winding conductors, the reduction rate decreases, and the maximum mutual capacitance between high-voltage outer winding conductors is higher than that between inner conductors. The oscillation voltage will be induced on the winding when the high frequency wave enters the transformer, the oscillation voltage is composed of sinusoidal voltages with different natural frequencies. Transient voltage of winding No.1 when the high frequency wave of $10^6$ V enters is shown in Figure 7.

**Figure 7.** Transient Voltage of coil No.1

It can be seen that in the process of high frequency wave enters the transformer, an oscillating voltage higher than the source wave will be generated in the winding NO.1 from figure 7. The maximum value of the oscillating voltage is about 1.45MV. Due to the damping effect of resistance, the voltage wave will be attenuated.

**Figure 8.** Voltage waveforms of windings under high frequency waves of different frequencies

(a) high frequency wave of 0.1MHz

(b) high frequency wave of 1MHz

(c) high frequency wave of 1.7MHz

(d) high frequency wave of 5MHz
Fig. 8 shows the voltage variation of winding No.1, No.249 and No.450 under high frequency wave with fixed voltage and frequency of 0.1MHz, 1MHz, 1.7MHz and 5MHz. From figure 8, it can be seen that under the action of 0.1MHz of high frequency wave, the peak oscillation voltage of the windings is nearly proportional to the distance from the winding to the neutral point, and the voltage of the winding turns is not attenuated, which shows that the voltage distribution inside the winding is dominated by inductance and is basically not affected by resistance. When the frequency of high frequency wave rises to 1MHz, the turn voltage of line 249 increases obviously, because the oscillation frequency of VFTO is close to the natural frequency of transformer, and resonance occurs inside transformer, the peak value of winding voltage is even close to three times of the voltage of high frequency wave. When the high frequency wave frequency is 1.7 MHz, although it deviates from the natural frequency, the deviation is not large. The peak value of winding of No.249 is still close to twice that of VFTO voltage, and the voltage on the winding does not change significantly. When the frequency of high frequency wave is 5 MHz, it will have a greater degree of attenuation.

4. Conclusion
The distribution of winding voltage in transformer is affected by the winding parameters and their structure under the action of high frequency wave. The following conclusions can be get through calculation:

1) Under the action of high frequency wave of different frequencies, the resistance of the transformer winding and core increases with the increase of high frequency wave frequency, while the self-inductance of the winding decreases slightly with the frequency of high frequency wave increases and basically remains unchanged in high frequency band.

2) The mutual inductance between windings decreases with the increase of the distance between windings, and the reduction rate decreases with the increase of the distance. The maximum mutual capacitance between winding is similar to ground capacitance of winding. With the increase of distance between winding conductors, the reduction rate decreases, and the maximum mutual capacitance between high-voltage outer winding conductors is higher than that between inner conductors.

3) Self-vibration will occur in transformer after high frequency wave intrudes. When the frequency of high frequency wave is similar to the natural frequency of transformer, the peak voltage acting on the turn of high voltage wire can reach several times of high frequency wave. With the increase of high frequency wave frequency, the voltage waveform on the turn of winding wire decreases rapidly.

Acknowledgement
Support projects: Scientific and Technological Projects of Shandong Electric Power Company (2018A-010).

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