Invariant Regularization of Supersymmetric Chiral Gauge Theory

Hiroshi SUZUKI

Department of Physics, Ibaraki University, Mito 310-8512, Japan

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We present a regularization scheme which respects the supersymmetry and the maximal background gauge covariance in supersymmetric chiral gauge theories. When the anomaly cancellation condition is satisfied, the effective action in the superfield background field method automatically restores the gauge invariance without counterterms. The scheme also provides a background gauge covariant definition of composite operators that is especially useful in analyzing anomalies. We present several applications: The minimal consistent gauge anomaly; the super-chiral anomaly and the superconformal anomaly; as the corresponding anomalous commutators, the Konishi anomaly and an anomalous supersymmetric transformation law of the supercurrent (the “central extension” of \(N = 1\) supersymmetry algebra) and of the \(R\)-current.

§1. Introduction

Behind the recent non-perturbative analyses of the supersymmetric gauge theories, the existence of an invariant regularization that respects the supersymmetry and the gauge symmetry always seems to be assumed. Such a regularization should be vital also in practical calculations because it avoids the introduction of counterterms necessary to recover Ward-Takahashi identities. Therefore, it seems yet to be important to seek for such an invariant regularization in supersymmetric gauge theories.\(^1\) If one knows how to restore the preferred symmetries by counterterms, of course, any regularization may be employed. However, it is also true that with a non-invariant regularization a naive analysis easily leads to wrong physical predictions in view of the preferred symmetries. For example, one might miss the fermion number non-conservation in chiral gauge theories,\(^2\) with use of the gauge non-invariant Pauli-Villars regularization. An invariant regularization allows one to obtain correct predictions in the naive way.

In this talk, I like to introduce the recent attempt of our group\(^3\) for obtaining an invariant regularization scheme in supersymmetric chiral gauge theories. Our scheme is perturbative in nature but it possesses the following properties. The supersymmetry is manifest; the scheme is based on the superfield formalism in an exactly four-dimensional spacetime. The effective action in the superfield background field method above one-loop order is always background gauge invariant. Being consistent with the existence of the gauge anomaly, the one-loop effective action is not background gauge invariant in general. However the breaking is kept to be minimal;
the consistent gauge anomaly is proportional to the anomaly \( d^{abc} = \text{tr} T^a \{ T^b, T^c \} \).
As the result, when the anomaly cancellation condition \( d^{abc} = 0 \) is satisfied, the effective action restores the background gauge invariance without any counterterms. The scheme also provides a background gauge covariant supersymmetric definition of composite operators, that is especially useful in analyzing anomalies. Unfortunately the quantum gauge (or BRS) symmetry is not manifest in our scheme and one has to verify the Slavnov-Taylor identity order by order (this is possible at least in the one-loop level\(^5\)). We note that however even with respect to the background gauge symmetry it is not trivial to obtain such an invariant scheme, due to the gauge anomaly in chiral gauge theories.

Our convention is basically that of Ref.\(^3\); our particular convention will be noted when necessary. For simplicity of presentation, we assume the gauge representation \( R \) of the chiral multiplet, which will be denoted as \( T^a \), is irreducible. The normalization of the gauge generator is \([ T^a, T^b ] = i \epsilon^{abc} T^c \), \( \text{tr} T^a T^b = T(R) \delta^{ab} \), \((T^a)_{ij}(T^a)_{jk} = C(R) \delta_{ij} \) and \( i^{acd}b_{dcd} = C_2(G) \delta^{ab} \).

§2. Superfield background field method

We consider a general renormalizable supersymmetric model\(^[5]\)

\[
S = \frac{1}{2T(R)} \int d^6z \text{ tr } W^a W_a + \int d^8z \Phi \bar{\Phi} + \int d^8z \left( \frac{1}{2} \Phi T m \Phi + \frac{1}{3} \Phi \Phi \Phi \right) + \text{h.c.} \tag{2.1}
\]

To apply the notion of the superfield background field method,\(^\delta\) we split the gauge superfield and the chiral superfield as\(^\gamma\)

\[
e^V = e^{V_B} e^{V_Q}, \quad \Phi = \Phi_B + \Phi_Q. \tag{2.2}
\]

Here, the subscripts \( B \) and \( Q \) represent the background field and the quantum field, respectively. We shall regard \( V_B \) as a vector superfield, and thus \( V_Q \) is not a vector superfield: \( V_Q = e^{V_B} V_Q e^{-V_B} \). With the splitting (2.2), the original gauge transformation\(^\beta\)

\[
e^{V'} = e^{-iA} e^{V_B} e^{iA}, \quad \Phi' = e^{-iA} \Phi, \tag{2.3}
\]

where \( A = T^a A^a \) is a chiral superfield \( D_a A = 0 \), is realized in the following two different ways. (i) By the quantum gauge transformation:

\[
V_B' = V_B, \quad e^{V_Q'} = (e^{-V_B} e^{-iA} e^{V_B} e^{iA}, \quad \Phi' = e^{-iA} \Phi. \tag{2.4}
\]

(ii) By the background gauge transformation:

\[
e^{V_B'} = e^{-iA} e^{V_B} e^{iA}, \quad V_Q' = e^{-iA} V_Q e^{iA}, \quad \Phi' = e^{-iA} \Phi. \tag{2.5}
\]

Next, we introduce the background covariant derivative symbol:

\[
\nabla_a = e^{-V_B} D_a e^{V_B}, \quad \overline{D}_{\dot{a}}, \quad \{ \nabla_a, \overline{D}_{\dot{a}} \} = -2i \sigma_m^{\alpha \dot{a}} \nabla_m. \tag{2.6}
\]

\(^\dagger\) \( d^8z = d^4x d^2 \theta d^2 \bar{\theta}, d^8z = d^4x d^2 \theta, d^8\bar{\sigma} = d^4x d^2 \bar{\sigma}. \)
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Since the gauge parameter $\Lambda$ in Eq. (2.5) is chiral, these derivatives transform as $\nabla' = e^{-i\Lambda}\nabla e^{i\Lambda}$ under the background field transformation (2.5). On the gauge representation $R$, the covariant derivative is defined by

$$D_\alpha \Phi = \nabla_\alpha \Phi, \quad D_m \Phi = \nabla_m \Phi. \quad (2.7)$$

On the other hand, the quantum field $V_Q$ transforms as the adjoint representation under the background transformation and the covariant derivative for the adjoint representation is defined as

$$D_\alpha V = [\nabla_\alpha, V], \quad D_m V = [\nabla_m, V], \quad (2.8)$$

where $a(n)$ (anti-)commutator is used when $V$ is Grassmann-even(-odd). Expressions become even simpler with use of the adjoint gauge representation matrix, which is

$$(T^a)^{bc} = -i\epsilon^{abc}, \quad \text{tr} T^a T^b = C_2(G)\delta^{ab}. \quad (2.9)$$

With this convention, the covariant derivative in the adjoint representation (2.8) can be written as

$$D_\alpha V = T^a (\nabla_\alpha V)^a, \quad D_m V = T^a (\nabla_m V)^a, \quad (2.10)$$

where a component of the covariant derivative is defined by

$$(\nabla_\alpha V)^a = (e^{-V_B})^{ab} D_\alpha (e^{V_B})^{bc} V^c, \quad \{\nabla_\alpha, T_\delta\} = -2i\epsilon_{\alpha\delta}^{ac} \nabla_c, \quad (2.11)$$

and $V_B$ is the background gauge superfield in the adjoint representation

$$V_B = T^a V^a_B. \quad (2.12)$$

The essence of the background field method is to use the gauge fixing condition that is covariant under the background gauge transformation (2.5). We impose the Lorentz-type gauge fixing condition and its conjugate

$$\nabla^2 V_Q = f, \quad \nabla^2 V_Q = e^{-V_B} f^1 e^{V_B}. \quad (2.13)$$

Then the standard procedure gives rise to the gauge fixing term and the ghost-anti-ghost term:

$$S' = -\frac{\xi}{8T(R)} \int d^8 z \, \text{tr}(\nabla^2 V_Q)(\nabla^2 V_Q)$$

$$+ \frac{1}{T(R)} \int d^8 z \, \text{tr}(e^{-V_B} c^1 e^{V_B} + c')$$

$$\times \mathcal{L}_{V_Q/2} \cdot [(c + e^{-V_B} c^1 e^{V_B}) + \coth(\mathcal{L}_{V_Q/2}) \cdot (c - e^{-V_B} c^1 e^{V_B})]$$

$$- \frac{2\xi}{T(R)} \int d^8 z \, \text{tr} e^{-V_B} b^1 e^{V_B} b, \quad (2.14)$$

where $\xi$ is the gauge parameter. By construction, this action is invariant under the background field transformation, Eq. (2.5) and $b' = e^{-i\Lambda b} e^{i\Lambda}$ etc. Note that, since the parameter $\Lambda$ of the quantum field transformation (2.4) is chiral and the gauge fixing function $f$ in Eq. (2.13) is also chiral, all the ghost $c$, anti-ghost $c'$, and Nielsen-Kallosh ghost $b$ are simply chiral superfields $\nabla_\alpha c = \nabla_\alpha c' = \nabla_\alpha b = 0$. 


§3. Supersymmetric gauge covariant regularization

In calculating radiative corrections to the effective action in the background field method, i.e., the generating functional of 1PI Green’s functions with all the external lines being the background field $V_B$ or $\Phi_B$, we expand the total action $S_T = S + S'$ in powers of the *quantum* field, $S_T = S_{T0} + S_{T1} + S_{T2} + S_{T3} + \cdots$. (Hereafter, a number appearing in the subscript indicates the power of the quantum fields.) The quadratic action is further decomposed as $S_{T2} = S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}} + S_{T2}^{\text{mix}}$.

The first part of the quadratic action, which is composed purely of the gauge superfields, is given by

$$S_{T2}^{\text{gauge}} = \int d^8 z V_Q^a \left[ -\bar{\nabla}^m \nabla_m + \frac{1}{2} W_B^a \nabla_a - \frac{1}{2} W_B^a \bar{D}^a + \frac{1}{16} (1 - \xi) (\bar{\nabla}^2 \bar{D}^2 + \bar{D}^2 \bar{\nabla}^2) \right] \delta^{ab} \delta \theta^2,$$

where the field strength in the adjoint representation is defined by

$$W_{Ba} = T^a W_{Ba} = -\frac{1}{4} \bar{D}^2 (e^{-V_B} D_{\alpha} e^{V_B}) = -\frac{1}{4} \{ D_{\alpha}, \bar{D}_{\alpha} \},$$

$$\bar{W}_{B\dot{a}} = T^a \bar{W}_{B\dot{a}} = \frac{1}{4} e^{-V_B} D^2 (e^{V_B} D_{\dot{a}} e^{-V_B})e^{V_B} = \frac{1}{4} \{ \bar{\nabla}_{\alpha}, \{ \bar{\nabla}_{\alpha}, \bar{D}_{\dot{\alpha}} \} \}.$$ (3.2)

The ghost-anti-ghost action, to second order in the quantum fields, is given by

$$S_{T2}^{\text{ghost}} = \int d^8 z [\bar{\phi}^a (e^{V_B})^{ab} c^b + c^a (e^{V_B})^{ab} \bar{c}^b - 2 \xi b^a (e^{V_B})^{ab} b^b].$$ (3.3)

There are two kinds of action which contain the chiral multiplet. One is the part that survives even for $\Phi_B = 0$,

$$S_{T2}^{\text{chiral}} = \int d^8 z \bar{\phi}^a e^{V_B} \phi_Q + \int d^6 z \frac{1}{2} \bar{\phi}^a \phi_Q m \phi_Q + \text{h.c.},$$ (3.4)

and the other is the part that disappears for $\Phi_B = 0$,

$$S_{T2}^{\text{mix}} = \int d^8 z \left( \bar{\phi}^a e^{V_B} V_Q \phi_Q + \bar{\phi}^a e^{V_B} V_Q \phi_B + \frac{1}{2} \bar{\phi}^a e^{V_B} V_Q^2 \phi_B \right) + \int d^6 z g \bar{\phi} B \phi^2 + \text{h.c.}$$ (3.5)

Our regularization is then implemented as follows: We take propagators of the quantum fields that are given by formally diagonalizing $S_2 = S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}}$. Then, for a finite ultraviolet cutoff $M$, we modify the propagators so as to improve the ultraviolet behavior and simultaneously preserve the background gauge covariance. For example, for the quantum gauge superfield, we use

$$\left\langle T^a V_Q^a(z)V_Q^a(z') \right\rangle = \frac{i}{2} \left\langle \left\{ \bar{\nabla}^m \nabla_m + W_B^a \nabla_a / 2 - \bar{W}_{B\dot{a}} \bar{D}^{\dot{a}} / 2 + (1 - \xi) (\bar{\nabla}^2 \bar{D}^2 + \bar{D}^2 \bar{\nabla}^2) / 16 \right\} / (\xi M^2) \right\rangle \overset{\text{\footnotesize\textsuperscript{a)}}}{\delta(z) = \delta(x) \delta(\theta) \delta(\bar{\theta})}.$$
and, for the ghost superfields,
\[ \left\langle T^* c^a(z) c^{ib}(z') \right\rangle = -2\xi \left\langle T^* b^a(z) b^{ib}(z') \right\rangle = \frac{i}{16} f(-D^2/16M^2) \frac{1}{\nabla^2 D^2 / 16 - m^\dagger m} \nabla^2 e^{-V_B} \delta(z - z'), \tag{3.7} \]
and, for the quantum chiral superfield,
\[ \left\langle T^* \Phi_Q(z) \Phi_Q^T(z') \right\rangle = \frac{i}{4} f(-D^2/16M^2) \frac{1}{\nabla^2 D^2 / 16 - m^\dagger m} m^\dagger \delta(z - z'). \tag{3.8} \]

In these expressions, \( f(t) \) is the regulator, which decreases sufficiently rapidly \( f(\infty) = f'(\infty) = f''(\infty) = \cdots = 0 \) in the ultraviolet, and \( f(0) = 1 \) to reproduce the original propagators in the infinite cutoff limit \( M \to \infty \). In this way, the propagators obey the same transformation law as the original ones under the background gauge transformation on the background gauge superfield \( V_B \). In fact, it can easily be seen that the propagator of the vector superfield, even with the modification (3.6), transforms covariantly under the background gauge transformation (2.5):
\[ \left\langle T^* V_Q^a(z) V_Q^b(z') \right\rangle' = \left[ e^{-i\tilde{A}(z)} \right]^{ac} \left\langle T^* V_Q^c(z) V_Q^d(z') \right\rangle \left[ e^{i\tilde{A}(z')} \right]^{db}, \tag{3.10} \]
where \( \tilde{A} = T^a A^a \) is the gauge parameter in the adjoint representation. Similarly, the propagator (3.8) transforms as
\[ \left\langle T^* \Phi_Q(z) \Phi_Q^T(z') \right\rangle' = e^{-i\tilde{A}(z)} \left\langle T^* \Phi_Q(z) \Phi_Q^T(z') \right\rangle e^{i\tilde{A}(z')}, \tag{3.11} \]
under the background gauge transformation. These properties are crucial for the gauge covariance of the scheme.

Using the above propagators of the quantum fields, 1PI Green’s functions are evaluated as follows. There are two kinds of contributions, because we have diagonalized \( S_2 = S_{T2}^{\text{gauge}} + S_{T2}^{\text{ghost}} + S_{T2}^{\text{chiral}} \) in constructing the propagators. (I) Most of radiative corrections are evaluated (as usual) by simply connecting quantum fields in \( S_{T2}^{\text{mix}} \), \( S_{T3} \), \( S_{T4} \), etc., by the modified propagators. This defines the first part of the effective action, \( \Gamma_1[V_B, \Phi_B] \), which is given by the 1PI part of
\[ \left\langle T^* \exp[i(S_T - S_2)] \right\rangle. \tag{3.12} \]
(II) However, since the quadratic action \( S_2 \) depends on the background gauge superfield \( V_B \) (but not on \( \Phi_B \)) non-trivially, the one-loop determinant arising from
the Gaussian integration of \( \exp(iS_2) \) has to be taken into account. We define this one-loop effective action as:

\[
\Gamma_{II}[gV_B] = \int_0^1 dg \int d^8z V_B^a(z) \left( \frac{\delta \Gamma_{II}[gV]}{\delta gV^a(z)} \right)_{V_B \to gV_B}.
\]

(3.13)

Here the indication \( V_B \to gV_B \) implies that all \( V_B \)-dependences involved are replaced by \( gV_B \). The quantum fields in the vacuum expectation value are connected by the modified propagators. This prescription for the one-loop effective action is characteristic of our scheme and the prescription maximally respects the gauge covariance.

We will discuss this point in more detail in the next section.

The total effective action is then given by the sum

\[
\Gamma[V_B, \Phi_B] = \Gamma_{I}[V_B, \Phi_B] + \Gamma_{II}[V_B].
\]

(III) When a certain composite operator \( O(z) \) is inserted in a Green’s function, it is computed as usual (by using the modified propagators):

\[
\langle T^*O(z) \exp[i(S_T - S_2)] \rangle.
\]

(3.14)

It is easy to see that the first part of the effective action \( \Gamma_{I}[V_B, \Phi_B] \) (which contains all the higher loop diagrams) is always supersymmetric and background gauge invariant. Also a composite operator which behaves classically as a gauge covariant superfield is regularized as a background gauge covariant superfield. On the other hand, the one-loop effective action \( \Gamma_{II}[V_B] \) defined by Eq. (3.13) is not necessarily gauge invariant. This is assuring because if the whole effective action were always gauge invariant then there would be no possibility of the gauge anomaly that may arise from chiral multiplet’s loop. However, \( \Gamma_{II}[V_B] \) automatically restores the gauge invariance without supplementing counterterms, when the gauge anomaly \( d^{abc} = \text{tr}T^a\{T^b, T^c\} \) vanishes. We will explain this mechanism in the next section. The prescription (3.13), on the other hand, is quite useful in evaluating the quantum anomaly while preserving the supersymmetry and the gauge covariance (or invariance). We will present several applications in the later section.

§4. Peculiarity of the one-loop effective action

Our prescription (3.13) is a natural supersymmetric generalization of the gauge covariant regularization in Ref. 9). The idea behind Eq. (3.13) is the following. We first introduce an auxiliary gauge coupling parameter \( g \) by \( V_B \to gV_B \). Then we may differentiate the effective action \( \Gamma_{II}[gV_B] \) with respect to the parameter \( g \) and integrate it over this parameter. Noting that the \( g \)-dependences arise only through the combination \( gV_B \), we have the first line of Eq. (3.13) as the formal representation. In the second line, the regularization is specified with use of the modified propagators in the preceding section. The crucial point for the property of the prescription is the
where the prime stands for the background gauge transformation (2.3).

To see how the prescription (3.13) works, we consider the gauge variation of \(I_{\Pi}[V_B]\) under the infinitesimal gauge transformation

\[
\delta_{\lambda}I_{\Pi}[V_B] = \int_0^1 dg \int d^8z \delta_{\lambda}V_B^a(z) \left( \frac{\delta S_2}{\delta V_B^a(z)} \right)_{V_B \to gV_B}
\]

\[
\quad + \int_0^1 dg \int d^8z \int d^8z' V_B^a(z) \delta_{\lambda}V_B^b(z') \left( \frac{\delta S_2}{\delta V_B^b(z')} \right)_{V_B \to gV_B} \frac{\delta S_2}{\delta V_B^a(z)}
\]

(4.3)

We then insert \(dg/dg = 1\) into the first term and perform the integration by parts with respect to \(g\). By noting again that the \(g\)-dependences arise only through the combination \(gV_B\), we have the following representation

\[
\delta_{\lambda}I_{\Pi}[V_B] = \int d^8z \delta_{\lambda}V_B^a(z) \left( \frac{\delta S_2}{\delta V_B^a(z)} \right)
\]

\[
\quad + \int_0^1 dg \int d^8z' \delta_{\lambda}V_B^b(z')
\]

\[
\times \int d^8z \left\{ V_B^a(z) \left[ \frac{\delta}{\delta V_B^b(z')} \left( \frac{\delta S_2}{\delta V_B^a(z)} \right) - \frac{\delta}{\delta V_B^a(z')} \left( \frac{\delta S_2}{\delta V_B^b(z)} \right) \right] \right\}
\]

(4.4)

This representation shows that our consistent anomaly consists of two pieces: The first piece is the covariant gauge anomaly [11], [12]

\[
\int d^8z \delta_{\lambda}V_B^a(z) \left( \frac{\delta S_2}{\delta V_B^a(z)} \right)
\]

\[
\quad \overset{M \to \infty}{=} -\frac{1}{64\pi^2} \int d^6z \ tr i\Lambda W_B^a W_Ba + \frac{1}{64\pi^2} \int d^6z \ tr e^{-V_B} i\Lambda^\dagger e^V_B \mathbf{W}_{B^a} \mathbf{W}_{B^a}^\dagger
\]

(4.5)

where the conjugate of the background field strength has been defined by

\[
\mathbf{W}_{B^a} = e^{-V_B} \mathbf{W}_{B^a} e^{V_B} = \frac{1}{4} [\nabla^a, \{\nabla_a, D_\alpha\}] \quad \text{and} \quad \mathbf{W}_{B_\alpha} = \frac{1}{4} D^2(e^{V_B} D_\alpha e^{-V_B})
\]

(4.6)

Note that the covariant anomaly (4.5) is proportional to the anomaly \(d_{abc}\), because \(\Lambda, W_{B^a}, \Lambda^\dagger, \text{ and } \mathbf{W}_{B^a}\) are Lie algebra valued. The second piece in Eq. (4.4), on the other hand, provides a difference between the consistent anomaly and the covariant
anomaly. The difference is expressed by the functional rotation of the covariant gauge current

$$\frac{\delta}{\delta V^a_B(z')} \left\langle \frac{\delta S_2}{\delta V^a_B(z)} \right\rangle - \frac{\delta}{\delta V^b_B(z)} \left\langle \frac{\delta S_2}{\delta V^b_B(z')} \right\rangle,$$

(4.7)

which represents the non-integrability of the covariant gauge current. The important point to note here is that the gauge covariance (4.1) implies the following property of the functional rotation:

$$\int d^8 z \delta \Lambda V^a_B(z) \left[ \frac{\delta}{\delta V^a_B(z')} \left\langle \frac{\delta S_2}{\delta V^a_B(z)} \right\rangle - \frac{\delta}{\delta V^b_B(z)} \left\langle \frac{\delta S_2}{\delta V^b_B(z')} \right\rangle \right] = \frac{\delta}{\delta V^b_B(z')} \int d^8 z \delta \Lambda V^a_B(z) \left\langle \frac{\delta S_2}{\delta V^a_B(z)} \right\rangle.$$

(4.8)

The right hand side is nothing but the covariant anomaly (4.5). Quite interestingly, the functional rotation (4.7) is a local functional of the gauge superfield, being proportional to (a derivative of) the delta function $\delta(z - z')$, as we will see shortly. This fact and Eq. (4.8) imply that the functional rotation vanishes and consequently our consistent anomaly (4.4) entirely vanishes, when the covariant anomaly vanishes. Namely, when the gauge representation of the chiral multiplet is anomaly-free, i.e., when $d^{abc} = 0$, the one-loop effective action (3.13) automatically restores the gauge invariance without supplementing any counterterms. In this sense, a breaking of the gauge symmetry is kept to be minimal with the present prescription.

The direct calculation shows that for arbitrary variations $\delta_1$ and $\delta_2$,

$$\delta_1 \langle \delta_2 S_2 \rangle - \delta_2 \langle \delta_1 S_2 \rangle \rightarrow_{M \rightarrow \infty} -\frac{1}{64\pi^2} \int d^8 z \text{tr} \Delta_1 \left( [D^\alpha \Delta_2, W^\alpha_{B\alpha}] + [\bar{D}_\dot{\alpha} \Delta_2, \bar{W}^\dot{\alpha}_{B\dot{\alpha}}] + \{ \Delta_2, D^\alpha W^\alpha_{B\alpha} \} \right),$$

(4.9)

with the notation $\Delta = e^{-V_B} \delta e^{V_B}$. From this, we see that the functional rotation (4.7) is a local object. The consistent gauge anomaly (4.4) is consequently given by

$$\delta_\Lambda \Gamma_{II}[V_B] \rightarrow_{M \rightarrow \infty} -\frac{1}{64\pi^2} \int d^8 z \int_0^1 dg \int_0^1 d\beta \text{tr} e^{-\beta g V_B} \delta_\Lambda V_B e^{\beta g V_B} \times \left( [D^\alpha V_B, W^\alpha_{B\alpha}] + [\bar{D}_\dot{\alpha} V_B, \bar{W}^\dot{\alpha}_{B\dot{\alpha}}] + \{ V_B, D^\alpha W^\alpha_{B\alpha} \} \right)_{V_B \rightarrow gV_B}.$$

(4.10)

It is obvious that our consistent anomaly is proportional to the anomaly $d^{abc}$, as expected. This anomaly must satisfy the Wess-Zumino consistency condition because it is the gauge variation of the functional (3.13). In fact, Eq. (4.10) coincides the consistent anomaly due to Marinković, which was obtained as a solution of the consistency condition.

We are interested in anomaly-free models for which the gauge anomaly (4.10) vanishes. Nevertheless it is interesting to examine the form of the anomaly (4.10) in...
the Wess-Zumino (WZ) gauge. We first set $\Lambda(z) = a(y)$ to reproduce the usual gauge transformation ($a$ is real). Then we have

$$
\delta_\Lambda \Gamma_{\Pi}[V_B] \big|_{M \to \infty} = -\frac{1}{96\pi^2} \int d^4x \ tr a \left[ \varepsilon^{m n k l} \partial_m \left( v_{Bn} \partial_k v_{Bl} + \frac{i}{4} v_{Bn} v_{Bk} v_{Bl} \right) \right. \\
\left. - \frac{1}{2} \partial_m (\bar{\lambda}_B \sigma^m \lambda_B - \lambda_B \sigma^m \bar{\lambda}_B) \right]. 
$$

(4.11)

This expression of the usual gauge anomaly in the WZ gauge is surprisingly simple compared to the result of existing field theoretical calculations. The first term is Bardeen’s anomaly; the second term may be eliminated by adding a non-supersymmetric local counterterm

$$
C = \frac{1}{384\pi^2} \int d^4x \ tr v_{Bm} (\bar{\lambda}_B \sigma_m \lambda_B - \lambda_B \sigma_m \bar{\lambda}_B). 
$$

(4.12)

as $\delta_\Lambda \Gamma_{\Pi}[V_B] + \delta_\epsilon C$, where $\delta_\epsilon$ is the usual gauge transformation.

As another interesting case, we may consider the anomalous breaking of the supersymmetry in the WZ gauge, the so-called $\varepsilon$-SUSY anomaly. The supersymmetry transformation in the WZ gauge is a combination of the supersymmetric transformation generated by $\varepsilon Q + \bar{\varepsilon} \bar{Q}$ (which is not anomalous in the present formulation) and the gauge transformation $\delta_\Lambda$ with the gauge parameter $\Lambda(z) = -i\theta\sigma^m \bar{v}_{Bm}(y) - \theta^2 \varepsilon \bar{\lambda}_B(y)$. Therefore we have the (apparent) breaking of supersymmetry as the consequence of the gauge anomaly. By setting the gauge parameter $\Lambda$ to this form in Eq. (4.10), we have

$$
\delta_\Lambda \Gamma_{\Pi}[V] \big|_{M \to \infty} = \frac{i}{384\pi^2} \int d^4x \ tr (\bar{\varepsilon} \sigma^m \lambda_B - \bar{\lambda}_B \sigma^m \varepsilon) \\
\times \left[ 3\bar{\lambda}_B \sigma_m \lambda_B - \varepsilon^m_{\ n k l} \left( 2\bar{v}_{Bn} (\partial_k v_{Bl}) + 2(\partial_n v_{Bk}) v_{Bl} + \frac{3i}{2} v_{Bn} v_{Bk} v_{Bl} \right) \right] \\
- \delta_\epsilon C, 
$$

(4.13)

where $\delta_\epsilon$ is the super-transformation in the WZ gauge $\delta_\epsilon v_{Bm}^m = i\bar{\varepsilon} \sigma^m \lambda_B + h.c., \delta_\epsilon \lambda_B = \sigma^m \varepsilon v_{Bmn} + i\varepsilon D_B$. Eq. (4.13) shows that Eq. (4.10) reproduces the $\varepsilon$-SUSY anomaly given in Ref. [13] again with the non-supersymmetric local counterterm $C$ [12]. The structure of Eq. (4.12) is quite simple, compared to that of the counterterm required in Ref. [13] for obtaining the above “minimal” form. Our anomaly is proportional to $d_{abc}$ from the beginning and thus the possible (non-supersymmetric) counterterm also must be proportional to $d_{abc}$. This fact severely restricts the possible form of (non-supersymmetric) counterterms.

For anomaly-free cases, the above prescription for the one-loop effective action (3.13) can be shown to be equivalent to (a supersymmetric generalization of) the generalized Pauli-Villars regularization in Ref. [14]. Since this is a Lagrangian level regularization whose Hamiltonian is Hermitian, the S-matrix is manifestly unitary. (Note that, in the $M \to \infty$ limit, negative norm Pauli-Villars regulators cannot contribute to the out-state of the physical S-matrix.)
§5. One-loop Green’s functions

It is easy to evaluate one-loop two point Green’s functions with our scheme. To carry out actual calculations, we have to choose a form of the regulator. A simple choice is

\[ f(t) = \frac{1}{(t+1)^2}. \]  

(5.1)

The self-energy part of the chiral multiplet, for example, is evaluated according to the rule (3.12). This yields in terms of the effective action,

\[ \Gamma_I[V_B, \Phi_B] = - \frac{1}{32\pi^2} C(R) \int d\theta \int d^4x d^4x' \Phi_B^\dagger(x, \theta, \theta') \Phi_B(x', \theta, \theta') \times \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \left( \ln \frac{M^2}{p^2} - \frac{5}{6} \right) + O(\Phi_B). \]  

(5.2)

The constant \(-5/6\) depends on the choice of the regulator \(f(t)\). In fact, since we “know” that the effective action \(\Gamma_I[V_B, \Phi_B]\) is always gauge invariant, we may covariantize the local part of the effective action (which is proportional to \(\ln M^2\)) as \(\int d^6z \Phi_B^\dagger e^{V_B} \Phi_B\).

For the vacuum polarization tensor of \(V_B\), we have from the prescription (3.13),

\[ \Gamma_{II}[V_B] = \frac{M^2}{16\pi^2} \int d^8 z \tr V_B + \frac{1}{64\pi^2} \frac{T(R) - 3C_2(G)}{2T(R)} \int d^4 \theta \int d^4 x d^4 x' \times \tr V_B(x, \theta, \theta') \left[ \frac{1}{4} D^a D^a V_B(x', \theta, \theta') \right] \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \left( \ln \frac{M^2}{p^2} + 1 \right) + O(\Phi_B^3). \]  

(5.3)

Equations (5.3) and (5.3) reproduce the well-known one-loop result. If one is interested in the divergent part of the effective action \(\Gamma_{II}[V_B]\), it is easy to obtain the general result for an arbitrary \(f(t)\): \(\int d^6z \Phi_B^\dagger e^{V_B} \Phi_B\).

\[ M \frac{d}{dM} \Gamma_{II}[V_B] \]

\[ = -\infty \int_0^\infty dt f(t) \int d^8 z \tr V_B + \frac{T(R) - 3C_2(G)}{64\pi^2} \int d^6 z \tr W_B^a W_B^a. \]  

(5.4)

This shows that the one-loop \(\beta\)-function of the gauge coupling is independent of the choice of \(f(t)\).

§6. Super-chiral and superconformal anomalies

Since our scheme gives a supersymmetric gauge covariant definition of composite operators, it also provides a simple and reliable method to evaluate quantum anomalies. In this section, we present several examples in the one-loop approximation. For simplicity, we neglect the effect of the superpotential by setting \(m = g = 0\). Throughout this section, we assume that the background chiral superfield \(\Phi_B\) and the Yukawa coupling \(g\) vanish, for simplicity of analysis.
The first example is the super-chiral anomaly, which is defined as a breaking of the Ward-Takahashi identity:

$$-\frac{1}{4}D^2 \langle \Phi^\dagger e^V \Phi(z) \rangle + \langle \Phi^T m \Phi(z) \rangle = 0.$$  (6.1)

This identity is associated with the chiral symmetry of the massless action, $\Phi(z) \to e^{i\alpha} \Phi(z)$. We first take in Eq. (6.1) the quadratic terms in the quantum fields (i.e., the one-loop approximation). Then, according to Eq. (3.14), we define the regularized super-chiral current as

$$\langle \Phi^\dagger Q e^V B \Phi Q(z) \rangle = i \frac{1}{16} \lim_{z' \to z} \text{tr} f(-D^2 \nabla^2 / 16M^2) D^2 \frac{1}{\nabla^2 D^2 / 16 - m^\dagger m} \nabla^2 \delta(z - z'),$$  (6.2)

and similarly, from Eq. (3.9),

$$\langle \Phi^T Q m \Phi Q(z) \rangle = i \frac{4}{4} \lim_{z' \to z} \text{tr} f(-D^2 \nabla^2 / 16M^2) D^2 \frac{1}{\nabla^2 D^2 / 16 - m^\dagger m} m^\dagger m \delta(z - z').$$  (6.3)

We then directly apply $-D^2/4$ on the composite operator in Eq. (6.2). Then, by noting the chirality of Eq. (6.2) with respect to the $z$ variable, we have

$$-\frac{1}{4}D^2 \langle \Phi^\dagger Q e^V B \Phi Q(z) \rangle + \langle \Phi^T Q m \Phi Q(z) \rangle$$

$$= -i \frac{1}{4} \lim_{z' \to z} \text{tr} f(-D^2 \nabla^2 / 16M^2) D^2 \delta(z - z')$$

$$M \to \infty = \frac{1}{64\pi^2} \text{tr} W^A_B W_{Ba}(z),$$  (6.4)

which reproduces the well-known form of the super-chiral anomaly. (Note that the anomaly is independent of the choice of $f(t)$.) For the actual calculation of the second line, see Ref. 3). The expression (6.4) holds even in chiral gauge theories, and in this sense Eq. (6.4) may be viewed as a supersymmetric version of the fermion number anomaly.

Since we have defined the regularized composite operator in Eqs. (6.2) and (6.3), an anomalous supersymmetric commutation relation associated with the super-chiral anomaly, the Konishi anomaly, can be derived straightforwardly. First we note in the Wess-Zumino gauge,

$$\Phi^\dagger e^V \Phi = A^\dagger A + \sqrt{2} \bar{\psi} A + \cdots,$$  (6.5)

and thus classically,

$$\sqrt{2} \bar{\psi} A = D^\dagger (\Phi^\dagger e^V \Phi) \bigg|_{\theta = \bar{\theta} = 0}.$$  (6.6)

Therefore the supersymmetric transformation of the composite operator $\bar{\psi} A$ may be defined as

$$\frac{1}{2\sqrt{2}} \langle \{ Q_a, \bar{\psi} Q A_Q(x) \} \rangle = \frac{1}{4} Q_a D^\dagger \langle \Phi^\dagger e^V B \Phi Q(z) \rangle \bigg|_{\theta = \bar{\theta} = 0}$$
\[
\left. \frac{1}{4} D^2 \left\langle \Phi^4 e^{\Phi} \Phi \right\rangle \right|_{\theta = \overline{\theta} = 0} ^{M \to \infty} = \frac{1}{64\pi^2} \mathrm{tr} \lambda_B^a \lambda_B^a (x). \quad (6.7)
\]

This is the Konishi anomaly.\[3\] The point in the above derivation is that we have defined the regularized composite operator first; moreover, we did so in terms of the superfield. Therefore the supersymmetric transformation of the regularized composite operator can be performed by one stroke of the differential operator, \[\overline{Q}_\alpha = -\partial / \partial \overline{\theta}^\alpha + i \theta^\alpha \sigma^m \partial_m.\[4\]

Another interesting example is the superconformal anomaly.\[5\] It is a breaking of the Ward-Takahashi identity:

\[
\mathcal{D}_k \left\langle R_{\alpha \dot{\alpha}} (z) \right\rangle - 2 \left\langle \Phi^T m e^{-V} D_\alpha e^V \Phi \right\rangle + \frac{2}{3} D_\alpha \left\langle \Phi^T m \Phi \right\rangle = 0. \quad (6.8)
\]

The superconformal current \( R_{\alpha \dot{\alpha}} \) is defined by \( R_{\alpha \dot{\alpha}} = R_{\alpha \dot{\alpha}}^{\text{chiral}} + R_{\alpha \dot{\alpha}}^{\text{gauge}} \), where

\[
R_{\alpha \dot{\alpha}}^{\text{chiral}} = - \mathcal{D}_{\dot{\alpha}} (\Phi^T e^{\Phi}) = - \frac{1}{3} \left[ D_\alpha, \mathcal{D}_{\dot{\alpha}} \right] (\Phi^T e^{\Phi}) = - \overline{\mathcal{D}_{\dot{\alpha}}} (\Phi^T e^{\Phi}) \nabla_\alpha \Phi_Q - \frac{1}{3} \left[ D_\alpha, \overline{\mathcal{D}_{\dot{\alpha}}} \right] (\Phi^T e^{\Phi}) Q + \cdots, \quad (6.9)
\]

and

\[
R_{\alpha \dot{\alpha}}^{\text{gauge}} = - \frac{2}{T(R)} \mathrm{tr} W_\alpha e^{-V} W_{\dot{\alpha}} e^V. \quad (6.10)
\]

From (6.9), the regularized superconformal current of the chiral multiplet is defined to the one-loop level by

\[
\left\langle R_{\alpha \dot{\alpha}}^{\text{chiral}} (z) \right\rangle = - \frac{i}{16} \lim _{z \to z'} \mathrm{tr} \nabla_\alpha f (- D^2 \nabla^2/16M^2) \mathcal{D}_\alpha \frac{1}{\nabla^2 \mathcal{D}^2 / 16 - m^2 m} \nabla^2 \mathcal{D}_\alpha \delta (z - z')
\]

\[
- \frac{1}{3} \frac{i}{16} \left[ D_\alpha, \overline{\mathcal{D}_{\dot{\alpha}}} \right] \lim _{z \to z'} \mathrm{tr} f (- D^2 \nabla^2/16M^2) D^2 \frac{1}{\nabla^2 D^2 / 16 - m^2 m} \nabla^2 \delta (z - (z')) \quad (6.11)
\]

To deal with this expression, we note the identity

\[
D_\alpha \lim _{z \to z'} \mathrm{tr} A (z) \delta (z - z') = \lim _{z \to z'} \mathrm{tr} \left[ \nabla_\alpha, A(z) \right] \delta (z - z'), \quad (6.12)
\]

where \( A(z) \) is an arbitrary operator. Then by using this identity and noting the chirality, we find

\[
\left\langle R_{\alpha \dot{\alpha}}^{\text{chiral}} (z) \right\rangle = \frac{1}{3} \frac{i}{16} \lim _{z \to z'} \left[ - \nabla_\alpha f (- D^2 \nabla^2/16M^2) \mathcal{D}_\alpha \frac{1}{\nabla^2 \mathcal{D}^2 / 16 - m^2 m} \nabla^2 \mathcal{D}_\alpha
\]

\[
+ f (- D^2 \nabla^2/16M^2) D^2 \frac{1}{\nabla^2 D^2 / 16 - m^2 m} \nabla^2 \mathcal{D}_\alpha \nabla_\alpha
\]

\[
+ \overline{\mathcal{D}_{\dot{\alpha}}} \nabla_\alpha f (- D^2 \nabla^2/16M^2) D^2 \frac{1}{\nabla^2 D^2 / 16 - m^2 m} \nabla^2 \right] \delta (z - (z')). \quad (6.13)
\]
By applying $\mathcal{D}^i$ further, we have
\[
\mathcal{D}^i \langle R^{\text{chiral}}_{\alpha\dot{\alpha}}(z) \rangle = \frac{1}{3!^{i}} \lim_{z' \to z} \left[ \nabla_{\alpha} f (-\mathcal{D}^2 \nabla^2/16M^2) \mathcal{D}^2 - f(-\mathcal{D}^2 \nabla^2/16M^2) \mathcal{D}^2 \right] \delta(z - z') - \frac{1}{2} \nabla_{\alpha} \mathcal{D}^2 \nabla_{\alpha} - 2 \nabla^2 W_{B\alpha},
\]

and
\[
\mathcal{D}^i \nabla_{\alpha} \mathcal{D}^2 = -4W_{B\alpha} \mathcal{D}^2,
\]

to yield
\[
\mathcal{D}^i \langle R^{\text{chiral}}_{\alpha\dot{\alpha}}(z) \rangle = -2 \langle \Phi^T \nabla_{\alpha} \Phi(z) \rangle + \frac{2}{3} D_{\alpha} \langle \Phi^T m \Phi(z) \rangle
\]

\[
M \to \infty \quad \lim_{z' \to z} \left[ M^2 \int_0^\infty dt f(t) + \frac{1}{6} \square \right] \text{tr} W_{B\alpha}(z) + \frac{1}{192\pi^2} D_{\alpha} \text{tr} W_B^\beta W_{B\beta}(z)
\]

\[
+ 2 \text{tr} W_{B\alpha}(z) \langle \Phi(z) \Phi_{\dot{\alpha}}(z) \rangle e^{V_B(z)}.
\]

For the gauge sector, a similar calculation shows
\[
\mathcal{D}^i \langle R^{\text{gauge}}_{\alpha\dot{\alpha}}(z) \rangle \to -2 \text{tr} W_{B\alpha}(z) \langle \Phi(z) \Phi_{\dot{\alpha}}(z) \rangle e^{V_B(z)} - \frac{1}{64\pi^2} \frac{C_2(G)}{T(R)} D_{\alpha} \text{tr} W_B^\beta W_{B\beta}(z),
\]

up to the BRS exact piece which can be neglected due to the Slavnov-Taylor identity.

Finally, we have
\[
\mathcal{D}^i \langle R_{\alpha\dot{\alpha}}(z) \rangle = -2 \langle \Phi^T \nabla_{\alpha} \Phi(z) \rangle + \frac{2}{3} D_{\alpha} \langle \Phi^T m \Phi(z) \rangle
\]

\[
M \to \infty \quad \lim_{z' \to z} \left[ M^2 \int_0^\infty dt f(t) + \frac{1}{6} \square \right] \text{tr} W_{B\alpha}(z) - \frac{1}{192\pi^2} \frac{3C_2(G) - T(R)}{T(R)} D_{\alpha} \text{tr} W_B^\beta W_{B\beta}(z),
\]
where the first divergent piece can be removed by adding the Fayet-Iliopoulos $D$-term.

As we have done for the Konishi anomaly, we may derive from the superconformal anomaly (6.19) the anomalous “central extension” of the $N = 1$ supersymmetry algebra which has recently been advocated by Shifman et al.\textsuperscript{20} An analysis of this problem from the viewpoint of path integrals and the Bjorken-Johnson-Low prescription can be found in Ref.\textsuperscript{21}. We first note the definition of the supercharge:

$$Q_\dot{\alpha} = \int d^3 x \mathcal{J}_\dot{\alpha}, \quad \mathcal{J}_\dot{\alpha}^m = -\frac{1}{2} \bar{\sigma}^{m\dot{\alpha}\dot{\beta}} \mathcal{J}_{\dot{\alpha}\dot{\beta}}. \tag{6.20}$$

The improved supercurrent $\mathcal{J}_{\dot{\beta}\dot{a}\alpha}$ is related to the superconformal current $R_{\alpha\dot{a}}$ as\textsuperscript{18}

$$R_{\alpha\dot{a}} = R_{\alpha\dot{a}}^{(0)} - i \bar{\theta}_\beta \left( \mathcal{D}_{\dot{\beta}} R_{\dot{\beta}\dot{\gamma}} - 2 \varepsilon_{\dot{\alpha}\dot{\beta}} \mathcal{D}_{\dot{\gamma}} R_{\dot{\gamma}\dot{\alpha}} \right) + \cdots, \tag{6.21}$$

where the first component $R_{\alpha\dot{a}}^{(0)}$ is the $R$-current. From these relations, we have classically

$$\mathcal{J}_{\dot{\alpha}} = -i \frac{1}{2} \bar{\sigma}^{\dot{\alpha}\dot{\beta}} \left( \mathcal{D}_{\dot{\beta}} R_{\dot{\beta}\dot{\gamma}} - 2 \varepsilon_{\dot{\alpha}\dot{\beta}} \mathcal{D}_{\dot{\gamma}} R_{\dot{\gamma}\dot{\alpha}} \right) |_{\theta = \bar{\theta} = 0}. \tag{6.22}$$

Therefore we may define the supersymmetric transformation of the supercurrent operator as

$$\left< \{ Q_\dot{\alpha}, \mathcal{J}_{\dot{\beta}}(x) \} \right> = \left< \frac{i}{2} \bar{\sigma}^{\dot{0}\dot{\gamma}} \mathcal{Q}_\dot{\alpha} \left[ \mathcal{D}_{\dot{\beta}} \left< R_{\dot{\gamma}\dot{\delta}}(z) \right> - 2 \varepsilon_{\dot{\beta}\dot{\gamma}} \mathcal{D}_{\dot{\delta}} \left< R_{\dot{\gamma}\dot{\delta}}(z) \right> \right] \right> |_{\theta = \bar{\theta} = 0}$$

$$= \left< \frac{i}{2} \bar{\sigma}^{\dot{0}\dot{\gamma}} \left( \varepsilon_{\dot{\alpha}\dot{\beta}} \mathcal{D}_{\dot{\gamma}} + 2 \varepsilon_{\dot{\alpha}\dot{\beta}} \mathcal{D}_{\dot{\gamma}} \right) \left< R_{\dot{\gamma}\dot{\delta}}(z) \right> \right> |_{\theta = \bar{\theta} = 0}$$

$$= \frac{M \rightarrow \infty}{48 \pi^2} \left< \frac{3C_2(G) - T(R)}{T(R)} \bar{\sigma}^{\dot{0}\dot{\gamma}} \theta \alpha \beta \gamma \delta \right> |_{\theta = \bar{\theta} = 0}, \tag{6.23}$$

where we have used the superconformal anomaly (6.19).\textsuperscript{3} The “central extension” of the $N = 1$ supersymmetry algebra\textsuperscript{20} is obtained by integrating Eq. (6.23) over the spatial coordinate $x$. In deriving the second line from the first line in Eq. (6.23), we have used the identity

$$\mathcal{D}_{\dot{\alpha}} \mathcal{D}_{\dot{\beta}} X_{\dot{\gamma}\dot{\delta}} |_{\theta = \bar{\theta} = 0} = -\varepsilon_{\dot{\alpha}\dot{\beta}} \mathcal{D}_{\dot{\gamma}} \mathcal{D}_{\dot{\delta}} X_{\dot{\gamma}\dot{\delta}} |_{\theta = \bar{\theta} = 0}, \tag{6.24}$$

which may be confirmed component by component.

In a similar way, we may study the anomalous commutator between the supercharge and the $R$-current.\textsuperscript{23} The $R$-current is defined from the superconformal current by

$$R_{\alpha\dot{a}}^{(0)m} = -\frac{1}{2} \bar{\sigma}^{m\dot{\alpha}\dot{a}} R_{\alpha\dot{a}} |_{\theta = \bar{\theta} = 0}. \tag{6.25}$$

Then the supersymmetric transformation of the $R$-current is related to the superconformal anomaly as

$$\left< \{ Q_\dot{\alpha}, R_{\beta\dot{a}}^{(0)}(x) \} \right> = \left< -\frac{1}{2} \bar{\sigma}^{\dot{0}\dot{\beta}} \mathcal{Q}_\dot{\alpha} \left< R_{\beta\dot{\delta}}(z) \right> \right> |_{\theta = \bar{\theta} = 0}

\textsuperscript{3} We have set $m = 0$ for simplicity.
\[
= -i \langle \mathcal{J}^0_\alpha (x) \rangle - \sigma^{0 \dot{\alpha} \dot{\beta}} \varepsilon_{\dot{\alpha} \dot{\beta}} \mathcal{D}^\dagger \langle R_{\beta 
abla} (z) \rangle \bigg|_{\theta = \bar{\theta} = 0} \\
= -i \langle \mathcal{J}^0_\alpha (x) \rangle + \frac{i}{8\pi^2} \left[ M^2 \int_0^\infty dt f(t) + \frac{1}{6} \Box \right] \text{tr} \lambda^0_B (x) \sigma^0_{\alpha \dot{\alpha}} \\
+ \frac{i}{48\pi^2} \frac{3C_2 (G) - T (R)}{T (R)} \text{tr} \lambda^0_B (x) \left[ \frac{1}{2} \sigma^0_{\alpha \dot{\alpha}} D_B (x) + \sigma^m_{\alpha \dot{\alpha}} v^+_{B m n} (x) \right], 
\]

where we have used the relation (6.22) and the superconformal anomaly (6.19). In the last line, \( v^+_{B m n} \) is the self-dual part of the field strength:

\[
v^+_{B m n} = \frac{1}{2} \left( v_{B m n} + \frac{i}{2} \varepsilon_{m k l} v^k_B \right).
\]

§7. Conclusion

In this talk, I introduced the recent attempt of our group for obtaining an invariant regularization scheme in supersymmetric chiral gauge theories. I also presented several applications in the one-loop level. Although our scheme provides a unified treatment of various problems in the one-loop level, the application to higher-loop problems seems practically difficult. One reason is that use of the regulator \( f(t) \) introduces complication in the Feynman rule. Another reason is the lack of the manifest BRS symmetry. Nevertheless, we believe the idea to use the covariant gauge current, such as in Eq. (3.13), is basically the correct one. We are thus supposing that a certain variation of the dimensional reduction, along the above idea might make the scheme more practical.

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