Abstract—This paper presents a new approach to phased array null-steering using randomly distributed antenna arrays. Classically, null-steering is done by applying adaptive amplitude tapers along individual phased array elements; however, this can be difficult and cumbersome to apply in a distributed system. To achieve the same effect, we propose a novel method which employs parasitic subarrays of circular topologies to precisely place nulls in specified directions of interest. Unlike typical adaptive beamforming algorithms, this process does not require the estimation of second order statistical metrics or knowledge of precise element placement; rather, it utilizes a shared aperture approach to generate null beams simultaneously. The analysis that proceeds examines the mean radiation pattern and mean distributions from finite element models to demonstrate the steering capability. The results show that shell annular and circular annular subarrays of the spherical random array composed of monopole elements steer deep nulls in precise increments.

Keywords—null steering, randomly distributed antenna arrays, subarray beamforming, and topological beamforming.

I. INTRODUCTION

Modern wireless communication systems such as mesh ad-hoc networking require adaptive and reconfigurable antennas to handle high data-rate throughputs and ensure quality of service. To achieve the demands of these systems, individual elements can be combined to form a “sampled” large aperture or phased array in order to increase their performance and attain enhanced capabilities such as pattern control and beam steering [1]. Classically, phased arrays were built by placing the antenna elements in a periodic or lattice structure to resemble a uniform sampling of the spatial aperture. This approach allowed engineers to apply principles of digital filter design to the development of the phased array architecture. However, it was well known that using a spatial deterministic sampling function could lead to undesirable effects such as grating lobes, scan-blindness, and surface wave propagation along the array manifold. Designers would then need to account for additional criterion in their optimization models to prevent these phenomena, adding a level of complexity to the already difficult engineering problem [1,5].

To alleviate these conditions, we developed the method of randomizing the sampling function or randomly distributing the elements according to some known probability density function. Since the sampling is no longer periodic, the Nyquist alias-free scan criterion disappears, and random arrays could achieve total pattern control and steerability, limited only by its element radiation pattern [2-5]. The extensive work in [4,5] showed that pattern steerability is realizable and that the random array’s radiation pattern converges in a probabilistic sense to its expected (mean) radiation pattern. In the proceeding sections, we extend the mathematical framework developed in [2-5] to include the additional capability of null-steering using a subarray approach.

II. ANALYSIS OF RADIATION FROM SUBARRAYS

Consider a uniform distribution of isotropic elements bound to a spherical topology as shown in Fig. 1; this is the generalized spherical random array (SRA). In [5], it was shown that the mean array factor for this structure is given by

$$\overline{AF}(\theta, \phi) = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \tint \left(\xi_z(\theta, \phi)\right)\tint \left(\xi_z(\theta, \phi)\right)$$

where the notation $\xi_{\{x,y,z\}}(\theta, \phi) = \{x, y, z\} \cdot \cos(\psi)$ denotes the directional cosine of the steering angle [5]. The tinc functions in Eq. (1) correspond to the characteristic function (Fourier transform) of a spherical random variable; mathematically,
it is given by $j_1(x)/x$. Similarly, consider a planar circular random array (CRA); the mean array factor in this case is [5]

$$\overline{\text{AF}}(\theta, \phi) = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \left[\text{jinc}(\zeta_1(\theta, \phi)) \cdot \text{jinc}(\zeta_1(\theta, \phi))\right]^2$$  \hspace{1cm} (2)

where directional cosines are again used, and the jinc function is the cylindrical extension given by $J_1(x)/x$. In both cases, the notation for tinc and jinc were introduced to demonstrate how the array factors in Eq. (1)-(2) relate back to the linear array factors for periodic arrays [1].

Now, we extend the analysis of the SRA and CRA to the derivative topologies of a shell annular random array (SARA) and circular annular random array (CARA). We assume that $A_{i,s}$ in Fig. 1 converges in the limit to $A_{i,o}$. These are also selected such that the SARA $(0, A_{i,s}, A_{o,i})$ has an area large enough to include at least $N$ elements, where $K$ elements are inside of the shell and $L$ elements are outside so that $K + L = N$. Likewise, the same selection criterion applies for the CARA $(0, A_{i,s}, A_{o,i})$ with $K$ denoting the number of elements inside of the ring and $L$ denoting the number of elements outside. These topologies provide enhanced stealth and point to point architectures and are critical to achieving very narrow (directive) beamwidths. They also reduce probabilities of interception from unintended receivers and provide greater energy efficiency [5].

Characteristic functions were thoroughly derived in [5] and are shown in Eq. - for completeness with $M=1$. Additionally, if multiple shells or rings are compounded as a function of $M$ (the number of rings or shells), the kernel of these characteristic functions can be found in such a way that each subarray factor compouds on each other, broadening the effective null depth for a designed spatial angle. In fact, results of these expressions indicate contributions of all rings (circular or spherical) apply independently to the average beampattern.

$$\overline{\text{AF}}(\theta, \phi) = \frac{1}{K} + \left(1 - \frac{1}{K}\right) \left[\frac{1}{M} \sum_{m=1}^{M} \left(\overline{\text{AF}}_m(\theta, \phi)\right)\right]^2$$  \hspace{1cm} (3)

$$\overline{\text{AF}}_m(\theta, \phi) = \frac{1}{K} + \left(1 - \frac{1}{K}\right) \left[\frac{1}{M} \sum_{m=1}^{M} \left(\text{sinc}(\overline{\text{AF}}_m(\theta, \phi)) - \text{tinc}(\overline{\text{AF}}_m(\theta, \phi))\right)\right]^2$$  \hspace{1cm} (4)

$$\overline{\text{AF}}_m(\theta, \phi) = \frac{\lambda (m-1/4)}{2 \sin(\phi')} \cdot m=1,\ldots,M$$  \hspace{1cm} (5)

$$\overline{\text{AF}}_m(\theta, \phi) = \frac{\lambda (m-1/8)}{2 \sin(\phi')} \cdot m=1,\ldots,M$$  \hspace{1cm} (6)

A comparison of the distributed ring is shown in Figure 2. The ring null-steers at designed spatial angles $\phi^o = \{3^\circ, 4^\circ, 5^\circ, 6^\circ\}$ by modulating its unit less or normalized aperture size $2A/\lambda = \{14.33, 10.75, 8.6, 7.17\}$. The scan is endfire, but the same result could be produced in the azimuth. In Figure 3, simulated results demonstrate the null deepening for both the SARA and CARA topologies as the order of the subarray increases. The net effect when combined with the null-placement capability shown in Figure 2 is total pattern controllability with deep, broad nulls at precise locations.

### III. CONCLUSIONS

This paper analyzes a new null-steering method for aperiodic random arrays, utilizing a shared aperture approach. The inherent aperiodicity and symmetry of the sphere and circle allow for pattern control and steerability; applying a parasitic subarray using a subset of the topology allows nulls to be steered at precise angular offsets of the main beam. Moreover, when successive subarrays are combined, a compounding effect of deep and broad nulling is observed in both the SARA and CARA topologies. This additional capability of arbitrary null-steering adds to the already versatile aspects of the random array.

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