BARYON ASYMMETRY
AND DARK MATTER

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Abstract
We study the implications of a large baryogenesis temperature, $T_B = \mathcal{O}(10^{10} \text{ GeV})$, on the mass spectrum of superparticles in supersymmetric extensions of the standard model. Models with a neutralino as lightest superparticle (LSP) are excluded. A consistent picture is obtained with the gravitino as LSP, followed by a higgsino-like neutralino (NSP). Gravitinos with masses from 10 to 100 GeV may be the dominant component of dark matter.
1 Introduction

In the high-temperature phase of the standard model as well as its supersymmetric extension, baryon number ($B$) and lepton number ($L$) violating processes are in thermal equilibrium [1]. As a consequence, asymmetries in baryon number and lepton number are related at high temperatures,

$$\langle B \rangle_T = C \langle B - L \rangle_T = \frac{C}{C - 1} \langle L \rangle_T,$$

(1)

where the constant $C = \mathcal{O}(1)$ depends on the particle content of the theory. Hence, the cosmological baryon asymmetry can be generated from a primordial lepton asymmetry produced by the out-of-equilibrium decay of heavy Majorana neutrinos - a mechanism referred to as leptogenesis [2].

The natural theoretical framework for extensions of the standard model with right-handed neutrinos are unified theories based on the gauge group $SO(10)$. Corresponding models of leptogenesis [3,4] can indeed explain the observed baryon asymmetry. Assuming a hierarchy of neutrino masses similar to the known mass hierarchy of up-type quarks, one obtains a rather large temperature for baryogenesis, $T_B \sim 10^{10}$ GeV [4], which can be reached in inflationary models after reheating [5]. Particularly attractive are supersymmetric models of leptogenesis [6,7] for which a consistent picture including washout processes and the generation of the initial equilibrium distribution of the heavy Majorana neutrinos has been obtained [7].

It is widely believed that the reheating temperature in the early universe cannot exceed $\mathcal{O}(10^9$ GeV) for a supersymmetric plasma [5] because of the ‘gravitino problem’ [8,9,10]. In the high-temperature plasma a large number of gravitinos is generated. The late decay of unstable gravitinos after nucleosynthesis modifies the abundances of the light elements in a way which is incompatible with observation. Stable massive gravitinos, on the other hand, may overclose the universe. In the following we shall study this problem and investigate the implications of a large baryogenesis temperature $T_B$ on the mass spectrum of superparticles.

2 Gravitino density

The production of gravitinos ($\tilde{G}$) at high temperatures is dominated by two-body processes involving gluinos ($\tilde{g}$). The corresponding cross sections have been considered for the two cases $m_{\tilde{G}} > m_{\tilde{g}}$ [11] and $m_{\tilde{G}} \ll m_{\tilde{g}}$ [11]. In the Boltzmann equation, which is used
to evaluate the gravitino density, the thermally averaged zero-temperature cross sections enter.

\[
\begin{array}{|c|c|}
\hline
\text{process} & \bar{\sigma}_i \\
\hline
A & g^a + g^b \rightarrow \bar{G} + \tilde{g}^c \\
B & g^a + \tilde{g}^b \rightarrow \bar{G} + g^c \\
C & g^a + \tilde{q}_i \rightarrow \bar{G} + q_j \\
D & q_i + \tilde{q}_j \rightarrow \bar{G} + g^a \\
E & \tilde{g}^a + \tilde{q}_i \rightarrow \bar{G} + g^a \\
F & \tilde{g}^a + \tilde{g}^b \rightarrow \bar{G} + \tilde{g}^c \\
G & \tilde{g}^a + q_i \rightarrow \bar{G} + q_j \\
H & \tilde{q}_i + \tilde{q}_j \rightarrow \bar{G} + \tilde{g}^a \\
I & \tilde{q}_i + \tilde{q}_j \rightarrow \bar{G} + \tilde{g}^a \\
J & \tilde{q}_i + \tilde{q}_j \rightarrow \bar{G} + \tilde{g}^a \\
\hline
\end{array}
\]

Table 1: Cross sections $\bar{\sigma}_i(s)$ for gravitino ($\bar{G}$) production in two-body processes involving left-handed quarks ($q_i$), scalar quarks ($\tilde{q}_i$), gluons ($g^a$) and gluinos ($\tilde{g}^a$). The cross sections are given for the specified choice of colours and averaged over spins in the initial state. $f^{abc}$ and $T_{ji}^{a}$ are the usual SU(3) colour matrices.

For processes with a gluon in the t-channel a logarithmic collinear singularity appears which has been regularized [11] by introducing either a finite gluon mass or an angular cut around the forward direction. We have calculated all partial cross sections $\sigma_i$ using both methods. The logarithmically singular terms are universal whereas the finite parts depend on the cutoff procedure. For arbitrary gravitino masses and large centre-of-mass energies, $s \gg m_{\tilde{G}}^2, m_{\tilde{g}}^2$, the different cross sections are given by

\[
\sigma_i(s) = \frac{g^2}{64\pi M^2} \left( 1 + \frac{m_{\tilde{g}}^2}{3m_{\tilde{G}}^2} \right) \bar{\sigma}_i(s),
\]

(2)

where $g$ is the QCD coupling, and $M = (8\pi G_N)^{-\frac{1}{2}} \simeq 2.4 \cdot 10^{18}$ GeV is the Planck mass. For $m_{\tilde{G}} \ll m_{\tilde{g}}$ the production cross section is enhanced since the scattering amplitude for the Goldstino component of the gravitino is inversely proportional to the supersymmetry breaking scale, $M \propto \frac{1}{\Lambda_{\text{SUSY}}} \propto \frac{1}{m_{\tilde{G}}M}$. Our results for the partial cross sections $\bar{\sigma}_i(s)$ are listed in the table. Here the collinear singularity has been regularized by a finite gluon mass $m$, and the logarithmically singular and the constant part are given for each process.
The coefficients of the $\ln(s/m^2)$ terms are in agreement with results obtained by Moroi [12].

In a consistent finite-temperature calculation, which remains to be carried out, the logarithmic singularity has to be regularized by the relevant finite-temperature mass scale which is expected to be the plasmon mass, i.e. $m \sim g(T)T$. We therefore estimate the gravitino production rate by evaluating the thermal average of the universal $\ln(s/m^2)$ term (processes B, F, G, H),

$$
\sigma_{(L)} = \frac{g^2}{2\pi M^2} \eta (1 + \eta) ((N^2 - 1)C_A + 2n_fNC_F) \left( 1 + \frac{m_g^2}{3m_G^2} \right) \ln \left( \frac{s}{m^2} \right). \tag{3}
$$

Here we have summed over all colours and all spins in the initial state, and included symmetry factors and a factor $\eta$ for each fermion in the initial state. $C_A$ and $C_F$ are the usual colour factors for the group SU(N) and $2n_f$ is the number of colour-triplet chiral multiplets, i.e. $2n_f = 12$ in the MSSM. The corresponding thermally averaged cross section reads ($\eta = 3/4$)

$$
C(T) = \left\langle \sigma_{(L)} v_{\text{rel}} \right\rangle = \frac{21g^2(T)}{32\pi \zeta(3)M^2} \left( (N^2 - 1)C_A + 2n_fNC_F \right) \left( 1 + \frac{m_g^2(T)}{3m_G^2} \right) \left( \ln \frac{1}{g^2(T)} + \frac{5}{2} + 2\ln 2 - 2\gamma_E \right), \tag{4}
$$

where we have substituted $m$ by $g(T)T$. We expect that the unknown constant part of the thermally averaged cross section contributes to $C(T)$ about the same amount as the term proportional to $\ln(1/g^2(T))$ (cf. table 1).

The production cross section $C(T)$ enters in the Boltzmann equation, which describes the generation of a gravitino density $n_{\tilde{G}}$ in the thermal bath (cf. 3),

$$
\frac{dn_{\tilde{G}}}{dt} + 3H n_{\tilde{G}} = C(T)n_{\text{rad}}^2. \tag{5}
$$

Here $H(T)$ is the Hubble parameter and $n_{\text{rad}} = \frac{\zeta(3)}{\pi^2}T^3$ is the number density of a relativistic bosonic degree of freedom. For QCD (N=3) one has

$$
C(T) \simeq 10\frac{g^2(T)}{M^2} \left( 1 + \frac{m_g^2(T)}{3m_G^2} \right) \left( \ln \frac{1}{g^2(T)} + 2.7 \right). \tag{6}
$$

From eqs. (5) and (6) one obtains for the gravitino density at temperatures $T < T_B$, assuming constant entropy,

$$
Y_{\tilde{G}}(T) \equiv \frac{n_{\tilde{G}}(T)}{n_{\text{rad}}(T)} \simeq \frac{g_s S(T)}{g_s S(T_B)} \frac{C(T_B)n_{\text{rad}}(T_B)}{H(T_B)}, \tag{7}
$$

From eqs.
where \( g_s(T) \) is the number of effectively massless degrees of freedom \[3\]. For \( T < 1 \text{ MeV} \), i.e. after nucleosynthesis, \( g_s(T) = 2 + \frac{21}{4} \left( \frac{T}{T_f} \right)^3 = \frac{43}{11} \), and \( g_s(T_B) = \frac{915}{4} \) in the MSSM. For light gravitinos \( (m_{\tilde{G}} \ll m_{\tilde{g}}(\mu), \mu \approx 100 \text{ GeV}) \) one obtains from eqs. (6)-(7) for the gravitino density and the contribution to \( \Omega h^2 \),

\[
Y_{\tilde{G}} \simeq 3.2 \cdot 10^{-10} \left( \frac{T_B}{10^{10} \text{ GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{\tilde{G}}} \right)^2 \left( \frac{m_{\tilde{g}}(\mu)}{1 \text{ TeV}} \right)^2, 
\]

\[
\Omega_{\tilde{G}} h^2 = m_{\tilde{G}} Y_{\tilde{G}}(T) n_{\text{rad}}(T) \rho_c^{-1} 
\simeq 0.60 \left( \frac{T_B}{10^{10} \text{ GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{\tilde{G}}} \right) \left( \frac{m_{\tilde{g}}(\mu)}{1 \text{ TeV}} \right)^2. 
\]

Here we have used \( g(T_B) = 0.85; \rho_c = 3H_0^2M^2 \) is the critical energy density, and \( m_{\tilde{g}}(T) = \frac{g^2(T)}{g^2(\mu)} m_{\tilde{g}}(\mu) \). If one assumes that the running masses of gluino, wino and bino (\( \tilde{b} \)) unify at the GUT scale, one has \( m_{\tilde{g}}(\mu) = \frac{3g^2(\mu)}{5g^2(\mu)} m_{\tilde{b}}(\mu) \), where \( g \) is the \( U(1)_Y \)-gauge coupling.

### 3 Constraints from nucleosynthesis

The primordial synthesis of light elements (BBN) yields stringent constraints on the amount of energy which may be released after nucleosynthesis by the decay of heavy nonrelativistic particles into electromagnetically and strongly interacting relativistic particles. These constraints have been studied in detail by several groups \[13,11\]. Depending on the lifetime of the decaying particle \( X \) its energy density cannot exceed an upper bound. From fig. 3 in \[13\] one reads off that one of the following conditions is sufficient:

\[
\text{(I)} \quad m_X Y_X(T) < 4 \cdot 10^{-10} \text{ GeV}, \quad \tau < 2 \cdot 10^6 \sec, 
\]

\[
\text{(II)} \quad m_X Y_X(T) < 4 \cdot 10^{-12} \text{ GeV}, \quad \tau \text{ arbitrary}, 
\]

where \( Y_X(T) = n_X(T)/n_{\text{rad}}(T) \).

Gravitinos interact only gravitationally. Hence, their existence leads almost unavoidably to a density of heavy particles which decay after nucleosynthesis. The partial width for the decay of an unstable gravitino into a gauge boson \( B \) and a gaugino \( \tilde{b} \) is given by \[11\] (\( m_{\tilde{b}} \ll m_{\tilde{G}} \)),

\[
\Gamma(\tilde{G} \rightarrow B\tilde{b}) \simeq \frac{1}{32\pi} \frac{m_{\tilde{G}}^3}{M^2} \simeq \left[ 4 \cdot 10^8 \left( \frac{100 \text{ GeV}}{m_{\tilde{G}}} \right)^3 \sec \right]^{-1}. 
\]
If for a fermion $\psi$ the decay into a final state with a scalar $\phi$ in the same chiral multiplet and a gravitino is kinematically allowed, the partial width reads,

$$\Gamma(\psi \rightarrow \tilde{G}\phi) = \Gamma(\psi \rightarrow \tilde{G}\phi^*) \approx \frac{1}{96\pi m_{\phi}^5} \frac{m_{\psi}^5}{m_{\tilde{G}}^2 M^2}.$$  

(13)

Given these lifetimes and the mass spectrum of superparticles in the MSSM one can examine whether one of the conditions (I) and (II) on the energy density after nucleosynthesis is satisfied.

4  Mass spectrum of superparticles

Consider first a typical example of supersymmetry breaking masses in the MSSM, $m_{\tilde{b}} < m_{\tilde{G}} \simeq 100\text{ GeV} < m_{\tilde{g}} \simeq 500\text{ GeV}$, and $T_B \simeq 10^{10}\text{ GeV}$. From eqs. (8) and (12) we conclude

$$\tau_{\tilde{G}} \simeq 4 \cdot 10^8 \text{ sec}, m_{\tilde{G}} Y_{\tilde{G}}(T) \simeq 4 \cdot 10^{-9} \text{ GeV}.$$  

According to condition (II) (11) this energy density exceeds the allowed maximal energy density by 3 orders of magnitude. This clearly illustrates the ‘gravitino problem’!

The existence of an unstable gravitino is inconsistent with a baryogenesis temperature $T_B$ as large as $10^{10}\text{ GeV}$. Consider first condition (I) (II). To satisfy the lifetime constraint $\tau < 2 \cdot 10^6 \text{ sec}$ one needs, according to (12), $m_{\tilde{G}} > 600\text{ GeV}$. Eqs. (6) and (8) then imply

$$m_{\tilde{G}} Y_{\tilde{G}} > 2.3 \cdot 10^{-8} \text{ GeV},$$

which exceeds the upper bound of condition (I) by 2 orders of magnitude. Condition (II) can also not be satisfied, since it would require an LSP mass below the experimental bounds.

Consider now the case in which the gravitino is the LSP, a possibility previously discussed in [10,14]. In this case one has to worry about the decays of the next-to-lightest superparticle (NSP) after nucleosynthesis. The lifetime constraint of condition (I), $\tau_{\text{NSP}} < 2 \cdot 10^6 \text{ sec}$, yields a lower bound on the NSP mass which depends on the gravitino mass $m_{\tilde{G}}$ (cf. eq. (13) and fig. 1). For a large range of parameters the NSP is a neutralino $\chi$, i.e. a linear combination of higgsinos and gauginos,

$$\chi = N_1 \tilde{b} + N_2 \tilde{W}_3 + N_3 \tilde{h}_1^0 + N_4 \tilde{h}_2^0.$$  

(14)

The NSP density after nucleosynthesis has been studied in great detail by a number of authors [15], since the density of stable neutralinos would contribute to dark matter. The upper bound of condition (II) on the neutralino density, $m_{\chi} Y_{\chi}(T) < 4 \cdot 10^{-12} \text{ GeV}$, corresponds to the requirement $\Omega h^2 < 0.00008$.

A systematic study of the neutralino density for a large range of the MSSM parameter space has been carried out by Edsjö and Gondolo [16]. For an interesting range of parameters, one finds a value in the ‘cosmologically interesting’ region $0.025 < \Omega_{\chi} h^2 < 1$. In
Figure 1: Upper and lower bounds on the NSP mass as function of the gravitino mass. The full lines represent the upper bound on the gluino mass $m_{\tilde{g}}(\mu) > m_{NSP}$ for different reheating temperatures. The dashed line is the lower bound on $m_{NSP}$ which follows from the NSP lifetime. A higgsino-like NSP with a mass in the shaded area satisfies all cosmological constraints including those from primordial nucleosynthesis.

In general, however, $\Omega_\chi h^2$ varies over eight orders of magnitude, from $10^{-4}$ to $10^4$. For a large part of parameter space one finds $\Omega_\chi h^2 < 0.025$. In particular this is the case for a higgsino-like neutralino, i.e. $Z_g = |N_1|^2 + |N_2|^2 < \frac{1}{2}$, in the mass range $80 \text{ GeV} < m_\chi < 450 \text{ GeV}$ [16]. For these parameters neutralino pair annihilation into $W$ boson pairs is very efficient and one therefore obtains a small neutralino density.

The bound $\Omega h^2 < 0.008$, which corresponds to the bound on the mass density $m_\chi Y_\chi(T) < 4 \cdot 10^{-10}$ GeV of condition (I), is satisfied for higgsino-like neutralinos in the mass range $80 \text{ GeV} < m_\chi < 300 \text{ GeV}$ [17]. We conclude that higgsino-like NSPs in this mass range and with a lifetime $\tau < 2 \cdot 10^6$ sec are compatible with the constraints from primordial nucleosynthesis. Note that this is a sufficient, yet not necessary condition for satisfying the bound $\Omega h^2 < 0.008$. Very small neutralino densities are also obtained for other sets of MSSM parameters.

Finally, we have to discuss the necessary condition that gravitinos do not overclose
Figure 2: Contribution of gravitinos to the density parameter $\Omega h^2$ for different gravitino masses $m_{\tilde{G}}$ as function of the reheating temperature $T_B$. The gluino mass has been set to $m_{\tilde{g}}(\mu) = 500$ GeV.

In the universe,

$$\Omega_{G\tilde{G}} h^2 < 1.$$  \hspace{1cm} (15)

Since $m_{\tilde{G}} < m_\chi < m_{\tilde{g}}$, the gravitino density is given by eq. (9). Hence, the condition (15) yields an upper bound on the NSP mass $m_\chi$ which depends on the gravitino mass and the baryogenesis temperature. The different constraints are summarized in fig. 1 which illustrates that for a wide range of MSSM parameters, where

$$m_{\tilde{G}} < m_\chi < m_{\tilde{g}},$$  \hspace{1cm} (16)

and $80 \text{GeV} < m_\chi < 300 \text{GeV}$, the baryogenesis temperature may be as large as $\mathcal{O}(10^{10})$ GeV. \footnote{The expression (9) for $\Omega_{G\tilde{G}} h^2$ is not valid for very small gravitino masses suggested by gauge-mediated supersymmetry breaking and no-scale supergravity models. This possibility has been discussed in \[38\].}

It is remarkable that for temperatures $T_B = 10^8 \ldots 10^{11}$ GeV, which are natural for leptogenesis, and for gravitino masses in the range $m_{\tilde{G}} = 10^0 \ldots 10^3$ GeV, which
is expected for gravity induced supersymmetry breaking, the relic density of gravitinos is cosmologically important (cf. fig. [3]). As an example, for $T_B \simeq 10^{10}$ GeV, $m_{\tilde{g}}(\mu) \simeq 500$ GeV, and $m_{\tilde{G}} \simeq 50$ GeV, one has $\Omega_{\tilde{G}}h^2 \simeq 0.30$.

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