Dissociation of heavy quarkonium states in rapidly varying strong magnetic field

Partha Bagchi,1 Nirupam Dutta,2 Bhaswar Chatterjee,3‡ and Souvik Priyam Adhya1§

1Theoretical Physics Division, Variable Energy Cyclotron Centre, 1/AF, Bidhan Nagar, Kolkata 700064, India
2School of Physical Sciences, National Institute of Science Education and Research Bhubaneswar, P.O. Jatni, Khurda 752050, Odisha, India
3Department of Physics, Indian Institute of Technology Roorkee, Roorkee 247667, India
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In recent time, it has been argued that a very high intensity magnetic field is expected [1–4] to be formed in non central high energy nucleus-nucleus collisions. This realisation already has motivated several investigations searching interesting perturbative and non-perturbative phenomena [5–7] of QCD matter in the laboratory. On the other hand, the magnetic field can modify several issues dramatically which previously have been understood without it. For example, the issue of heavy quarkonia suppression in the deconfined Quark Gluon Plasma [8–10] can be modified greatly if one considers the magnetic field into account. A very obvious modification in this area is the Zeeman splitting of quarkonium states in constant magnetic field which essentially creates various quarkonium states [11, 12] differing by their spin degrees of freedom which is very similar to the case of positronium in quantum electrodynamics [13]. Then, there are possibilities for spin mixing in homogeneous [10, 12] and inhomogeneous [14] magnetic field environment. Besides that, ionisation [15] of bound states due to the tunnelling caused by the magnetic field can lead to suppression of quarkonium states. Furthermore, the static quark anti-quark potential in medium can be modified up to a big extent if the magnetic field can persist for a longer time. Depending on non-centrality, the magnetic field can be as strong as $B \simeq 50 \, m_\pi^2$ where $m_\pi^2 = 10^{18}$ Gauss. This field strength decays very quickly as the spectator quarks move away from the fireball and it has been estimated that at time $t \lesssim 0.4 \, fm$, the magnetic field is practically negligible. However, if QGP is formed, then it can trap the magnetic field because of its high electrical conductivity. So the formation of QGP can increase the persistence time [10] of magnetic field in Relativistic Heavy Ion Collision (RHIC). Nevertheless, the field will decay to few orders of magnitude within few $fm/c$ time. Hence, the produced magnetic field is time dependent and in turn would significantly affect the production of particles and their subsequent dynamics. So it is worth studying the properties of quarkonia in presence of such transient (or time varying) magnetic field.

This is true that there are several view points regarding the nature of the magnetic field generated through Heavy Ion Collisions (HIC) and hence, whatever we predict at the moment by considering the speculative ideas of the magnetic field may not lead us to a proper quantitative predictions of observables. Nevertheless, the qualitative aspects of various phenomena can be understood well enough. In this article, we have considered a magnetic field which is decaying with time and have calculated the transition of quarkonia to the continuum states from the bound one. This leads to further suppression of quarkonia which is completely different from the ionisation process discussed earlier [15]. In a time varying magnetic field, quarkonia evolves non-adiabatically because the quark anti-quark potential becomes time dependent from the bound one. This leads to further suppression of quarkonia in quantum electrodynamics [13]. Then, there are possibilities for spin mixing in homogeneous [10, 12] and inhomogeneous [14] magnetic field environment. Besides that, ionisation [15] of bound states due to the tunnelling caused by the magnetic field can lead to suppression of quarkonium states. Furthermore, the static quark anti-quark potential in medium can be modified up to a big extent if the magnetic field can persist for a longer time. Depending on non-centrality, the magnetic field can be as strong as $B \simeq 50 \, m_\pi^2$ where $m_\pi^2 = 10^{18}$ Gauss. This field strength decays very quickly as the spectator quarks move away from the fireball and it has been estimated that at time $t \lesssim 0.4 \, fm$, the magnetic field is practically negligible. However, if QGP is formed, then it can trap the magnetic field because of its high electrical conductivity. So the formation of QGP can increase the persistence time [10] of magnetic field in Relativistic Heavy Ion Collision (RHIC). Nevertheless, the field will decay to few orders of magnitude within few $fm/c$ time. Hence, the produced magnetic field is time dependent and in turn would significantly affect the production of particles and their subsequent dynamics. So it is worth studying the properties of quarkonia in presence of such transient (or time varying) magnetic field.

In this work, we will restrict ourselves within the strong magnetic field approximation which essentially means that the magnetic field will act as the dominant scale and will prevail over other scales present in the system such as mass and temperature as because $eB/m^2 >> 1$ and $eB/T >> 1$, where $m$ is the mass of the particle affected by magnetic field and $T$ is the temperature of the system. This is obviously above the Schwinger’s critical limit [19] that makes it possible to have a classical description of the magnetic field. The effects of magnetic field is incorporated through the propagator of the charged particles present in the medium which in our case are the light quarks. Though there is no effect of magnetic field on the gluon propagator at the zeroth order, it gets affected in the next order through vacuum fluctuation. The fermion
propagator in the strong field limit is given by

\[ S_0(k) = \frac{i m + \gamma \cdot k}{k_\parallel^2 - m^2} \left( 1 - i \gamma_1 \gamma_2 e^{\frac{-i k_\parallel B}{m}} \right) \]  

(1)

for zero temperature. Here we have assumed the magnetic field, B to be along a fixed direction (lets say z). \( q_f \) is the electric charge of the fermion of flavor \( f \) and \( K \) is the fermion 4-momentum expressed as \( k^2 = -(k_x^2 + k_y^2) \), \( k_\parallel^2 = k_0^2 + k_z^2 \) and \( \gamma \cdot k_\parallel = \gamma_0 k_0 - \gamma_3 k_z \). The split in the 4-momentum occurs due to the Landau quantization in the plane transverse to the magnetic field as the fermion energy is given by

\[ E = \sqrt{m^2 + k^2_\parallel + 2n|q_f|B} \]  

(2)

with \( n \) being the number of Landau levels which is equal to zero in the strong field limit. At finite temperature, the propagator in real time \([20]\) becomes

\[ iS_{11}(p) = \left[ \frac{1}{p^2 - m^2 + i\epsilon} + 2\pi i n_p \delta(p^2 - m^2) \right] (1 + \gamma^0 \gamma^3 \gamma^5) \times (\gamma^0 p_0 - \gamma^3 p_z + m) e^{\frac{-p_0 B}{m}}, \]  

(3)

where the distribution is

\[ n_p(p_0) = \frac{1}{e^{\beta |p_0|} + 1}, \]

with the Boltzmann factor \( \beta \). The Debye screening mass \( (m_D) \) heavy quark potential in strong magnetic field can be obtained by taking the static limit \((|p| = 0, p_0 \rightarrow 0)\) of the longitudinal part of the gluon self energy \( \pi_{m_D} \). If there is no magnetic field in medium then \( m_D \) can be written for three flavor case as \( m_D = gT \sqrt{1 + N_f/6} \). In presence of magnetic field, The Debye mass \([21]\) becomes,

\[ m_D^2 = g^2 T^2 + \frac{g^2}{4\pi^2 T} \sum_f \int_0^\infty |q_f B | dp_z \left( \frac{e^{\beta \sqrt{p_z^2 + m_f^2}}}{(1 + e^{\beta \sqrt{p_z^2 + m_f^2}})^2} \right)^2 \]  

(4)

Where the first term is the contribution from the gluon loops and this is solely dependent on temperature and magnetic field doesn’t affect it. The second term is the contribution from the fermion loop and this term strongly depends on magnetic field and is not much sensitive to the temperature of the medium. In the first term, \( g^2 = 4\pi \alpha_s'(T) \) where \( \alpha_s'(T) \) is the usual temperature dependent running coupling where the renormalization scale is taken as \( 2\pi T \). It is given by

\[ \alpha_s'(T) = \frac{2\pi}{(11 - \frac{2}{3} N_f) \ln \left( \frac{\Lambda}{\Lambda_{QCD}} \right)} \]  

(5)

Where \( \Lambda = 2\pi T \) and \( \Lambda_{QCD} \sim 200 \text{ MeV} \)

In the second term, \( g^2 = 4\pi \alpha_s''(k_z, q_f B) \), where \( \alpha_s''(k_z, q_f B) \) is the magnetic field dependent coupling and doesn’t depend on temperature. This is given by \([22, 23]\)

\[ \alpha_s''(k_z, q_f B) = \frac{1}{\alpha_s''(\mu_0)^{-1} + \frac{11\pi}{12\pi} \ln \left( \frac{k_z^2 + M^2}{\mu_0^2} \right) + \frac{1}{3\pi} \sum_f \frac{q_f B}{\sigma}} \]  

(6)

where

\[ \alpha_s''(\mu_0) = \frac{12\pi}{11N_c \ln \left( \frac{\mu_0^2 + M^2}{\Lambda^2} \right)} \]  

(7)

All the parameters are taken as \( M_B = 1 \text{ GeV} \), the string tension \( \sigma = 0.18 \text{ GeV}^2 \), \( \mu_0 = 1.1 \text{ GeV} \) and \( \Lambda = 0.385 \text{ GeV} \).

In the strong field limit, the temperature dependence of the Debye mass is almost negligible. Now one has to see the nature of the magnetic field which decreases with time and that essentially makes the Debye screening mass a time dependent quantity. The intensity of the initial magnetic field \( B_0 \) is of the order of few \( m_\pi^2 \) and decays with time in the following way,

\[ B = B_0 \frac{1}{1 + at}, \]  

(8)

using the fitting of the result provided in the article by K. Tuchin \([24]\) with the value of the parameter \( a = 0.5 \).

The heavy quark potential in medium can be written as,

\[ V(r) = -\frac{\alpha}{r} \exp(-m_Dr) + \frac{\sigma}{m_D} \left( 1 - \exp(-m_Dr) \right) \]  

(9)

The effect of the temperature and magnetic field is incorporated in the Debye mass given in eq[4]. This is obvious that potential becomes time dependent due to the time dependence of the magnetic field and temperature. We consider that initially at \( t = t_i \), there are only ground states of charmonia \((J/\Psi)\) and bottomonia \((\Upsilon(1S))\). These two states evolve in a time dependent potential which causes transition to other excited states and as well as to the dissociated continuum. We would like to calculate the transition probabilities of the ground states to the continuum which gives us the dissociation probabilities of \((J/\Psi)\) and \((\Upsilon(1S))\). This is a very difficult task because solving Schröedinger equation for a time dependent potential is cumbersome. We have adopted time dependent perturbation theory in this context in order to calculate the dissociation probability up to the first order. The perturbation at any instant \( t \) considered to be as \( H^1(t) = V(r, t) - V(r, t_i) \). We want to calculate the transition probability to the unbound states which are obviously plane wave states given by,

\[ \Psi_k = \frac{1}{\sqrt{\Omega}} e^{ik \cdot r} \]  

(10)
The binding energy of $J/\psi$ and $\Upsilon(1S)$ as a function of the magnetic field intensity.

which is box normalised over a volume $\Omega$ and can have all possible values of the momentum $\vec{k}$. The first order contribution to the transition amplitude can be expressed as,

$$a_{ik} = \int \frac{d}{dt} \langle \Psi_k | H^1(t) | \Psi_i \rangle \frac{e^{i(E_i - E_k)} dt}{(E_i - E_k)}.$$  \hspace{1cm} (11)

$|\Psi_i\rangle$, $E_i$ are initial quarkonium state and the corresponding energy eigenstates respectively and $E_k$ is the energy of the dissociated state $|\Psi_k\rangle$. The total transition probability to all continuum states is given by,

$$= \int_{k=0}^{\infty} |a_{ik}|^2 \frac{\Omega}{(2\pi)^3} k^2 dk,$$ \hspace{1cm} (12)

where the number of unbound states between the momentum continuum $k$ and $k + dk$ over $4\pi$ solid angle is

$$dn = \left( \frac{L}{2\pi} \right)^3 k^2 dk = \frac{\Omega}{(2\pi)^3} k^2 dk \hspace{1cm} (13)$$

We know that $J/\psi$ and $\Upsilon(1S)$ can survive in the thermal medium (QGP) almost up to $2.2T_c$ and $4T_c$ respectively but in the presence of magnetic field, the binding energies of these states get modified. The binding energy is given by,

$$E_{\text{disso}} = E_B - 2m_q - \frac{\sigma}{m_D},$$ \hspace{1cm} (14)

where $m_q$ is mass of quark and $E_B$ is the energy eigenvalue calculated from time independent Schrödinger equation by using Neumerovs method. We have plotted the binding energy of $J/\psi$ at a temperature $1.7T_c$ and $\Upsilon(1S)$ at a temperature $3T_c$ as a function of the magnetic field intensity in fig[1]. The binding energies do not change much over a span of magnetic field intensity from $1 - 15m_{\pi}^2$. In other words, these quarkonium states can survive at a higher temperature if there is magnetic field present in the medium. Within the specified range of the magnetic field intensity the dissociation temperature of $J/\psi$ and $\Upsilon(1S)$ becomes $2.73 - 2.94T_c$ and $8.12 - 8.89T_c$ respectively. In the current experimental scenario the medium temperature does not go up to $8T_c$ and therefore we have not considered the medium temperature above $500MeV$ for the calculation of dissociation probability.

We have employed first order perturbation theory to evaluate the dissociation probabilities of both the ground states first by considering a purely thermal QGP which cools off to the temperature $T_c$ of the medium and then the same has been calculated by considering the time dependent magnetic field in the evolving QGP. For $J/\psi$, we have started at a temperature of the medium which is $1.7T_c$ and then we allow the medium temperature to reduce according to the power law given by,

$$T(t) = T_0 \left( \frac{\tau_0}{\tau_0 + t} \right)^{\frac{1}{\tau_0}},$$ \hspace{1cm} (15)

with $T_0$, the initial temperature and $\tau_0$ be the equilibration time, taken to be approximately $5fm/c$ for QGP. We have calculated the dissociation probability when the medium temperature falls off to $T_c$ from an initial value in presence of the time dependent magnetic field. The initial value of the magnetic field is not known exactly and therefore we have used various initial values of the magnetic field intensity and have shown the dissociation probabilities as a function of initial magnetic field. The same has been done for the $\Upsilon(1S)$ state by considering the initial temperature around $3T_c$. In fig[2] the solid black line denotes the dissociation probability of $J/\psi$ which increases with the initial field intensity. The state $J/\psi$ can be dissociated 12 to 50 percent within the range of the field intensity $1 - 15m_{\pi}^2$. The dotted blue line shows that the dissociation probability for $\Upsilon(1S)$ is almost zero over the specified span of the field strength.

Summarising the article, we conclude that due to the modification of the heavy quark potential in presence of magnetic field, the bound states $J/\psi$ and $\Upsilon(1S)$ become more strongly bound compared to those in a pure thermal QGP. As a result, the bound states can survive much higher temperature than we have expected previously. All though $J/\psi$ can be dissociated by making non-adiabatic transitions to the unbound states but $\Upsilon(1S)$ is still remains bound. We have estimated the dissociation probability within the limits of first order perturbation theory. For a better prediction one must solve the Schröedinger equation for a time dependent potential which by any means seems extremely challenging.
Figure 2: Dissociation probability of $J/\psi$ and $\Upsilon(1S)$ as a function of the intensity of the initial magnetic field.

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* E-mail: p.bagchi@vecc.gov.in
† E-mail: nirupamdu@gmail.com
‡ E-mail: bhaswar.mph2016@iitr.ac.in
§ E-mail: sp.adhya@vecc.gov.in

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