Gauge-independent resummed gluon self-energy in hot QCD

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In memory of Russell H. Swanson

Abstract

The pinch technique (PT) is applied to obtain the gauge-independent resummed gluon self-energy in a hot Yang-Mills gas. Calculation is performed at one-loop level in linear gauges which preserve rotational invariance and the resummed propagators and vertices are used. The effective gluon self-energy, which is obtained as the sum of the resummed gluon self-energy and the resummed pinch contributions, is not only gauge-independent but also satisfies the transversality relation. Using this gauge-independent effective gluon self-energy, we calculate the damping rate for transverse gluons in the leading order and show that the result coincides with the one obtained by Braaten and Pisarski.

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1 Introduction

The knowledge of the behaviour of the QCD effective coupling constant $\alpha_s(=g^2/4\pi)$ at high temperature is very important for the study of the quark-gluon plasma and/or the evolution of the early Universe. The running of $\alpha_s$ with the temperature $T$ and the external momentum $k = |k|$ is governed by the thermal $\beta$ function $\beta_T$ [1]. However, the previous calculations of $\beta_T$ have exposed various problems [2]-[4], a serious one of which is that the results are gauge-fixing dependent [5]. To circumvent the difficulty concerning the gauge dependence, it was then proposed to use [6] the Vilkovisky-DeWitt effective action [7][8] or to use [9]-[11] the background field method (BFM) for the calculation of $\beta_T$ at one-loop. (In Yang-Mills theories the Vilkovisky-DeWitt effective action formalism coincides with BFM in the background Landau gauge [8].)

The thermal $\beta$ function $\beta_T$ was calculated in BFM at one-loop level for the cases of the gauge parameter $\xi_Q = 0$ [6][10], $\xi_Q = 1$ [9] and $\xi_Q =$ an arbitrary number [11]. The results are expressed in a form,

$$\beta_T^{BFM} = \frac{g^3N}{2} \left\{ \frac{7}{16} - \frac{1}{8}(1 - \xi_Q) + \frac{1}{64}(1 - \xi_Q)^2 \right\} \frac{T}{k}, \quad (1.1)$$

where $N$ is the number of colors. Contrary to the case of the QCD $\beta$ function at zero temperature, $\beta_T^{BFM}$ is dependent on the gauge-parameter $\xi_Q$. This dependence comes from the $\xi_Q$-dependence of the finite part of $\Pi_{\mu\nu}$ calculated in BFM. The notion that BFM gives $\xi_Q$-dependent finite part for $\Pi_{\mu\nu}$ had already been known [7][8].

Quite recently, it was shown [12] by the present author that the pinch technique (PT) gives the gauge-independent result for $\beta_T$. Calculations are performed at one-loop level in four different gauges, (i) the background field method with an arbitrary gauge, (ii) the Feynman gauge, (iii) the Coulomb gauge, and (iv) the temporal axial gauge, and they yield the same result $\beta_T = g^3N\frac{T}{32k}$ in all four cases. This gauge-independent result for $\beta_T$ from PT corresponds to the result from BFM in Eq.(1.1) with the special value $\xi_Q = 1$ of the gauge parameter.

However, the above gauge-independent result for $\beta_T$ is incomplete. As Elmfors and Kobes pointed out [11], the leading contribution to $\beta_T$, which gives a term $T/k$, does not come from the hard part of the loop integral, responsible for a $T^2/k^2$ term,
but from soft loop integral. Hence they emphasized that it is not consistent to stop the calculation at one-loop order for soft internal momenta and that the resummed propagator and vertices \cite{13} must be used to get the complete leading contribution. The need for resummation is urged also by the following observation: The fact that in Ref.\cite{12} the same $\beta_T$ was obtained at one-loop level in four different gauges implies that the effective gluon self-energy $\hat{\Pi}^{\mu\nu}$, constructed in the framework of PT without resummation, is gauge-fixing independent and universal. Provided that we use $\hat{\Pi}^{\mu\nu}$ for calculation of the damping rate $\gamma_t$ for transverse gluons at zero momentum, we would obtain $\gamma_t = -\frac{11}{24\pi} Ng^2 T \ [14]$, a negative damping rate, which is not acceptable today \cite{15} \cite{16}.

The PT is an algorithm to construct, order by order in the coupling constant $g$ (in other words, loop by loop), modified *gauge-independent* off-shell amplitudes through the rearrangement of the Feynman graphs. When a theory contains more than one scales like thermal field theories, then the contributions from the higher-loop diagrams cannot be neglected anymore and the PT fails to give a right answer. On the other hand, the resummation in thermal field theories collects systematically all leading higher-loop contributions \cite{17} but the effective amplitudes obtained by resummation, in general, contain terms which are *gauge-dependent*. For the calculation of $\beta_T$, we need not only the resummation but also some prescription such as the PT to pick up the gauge-independent pieces from the effective *off-shell* gluon self-energy.

In this paper we will employ the PT and construct the gauge-independent resummed gluon self-energy at one-loop order in hot QCD. This is the first step toward obtaining the gauge-independent thermal $\beta$ function $\beta_T$ in the complete leading order. Calculation is performed in linear gauges which preserve rotational invariance and the resummed gluon propagator and vertices are used. We find that the resummed effective gluon self-energy, which is the sum of the gluon self-energy and the pinch contributions, is not only gauge-independent but also satisfies the transversality relation. Then using this gauge-independent effective gluon self-energy, we calculate the damping rate for transverse gluons in the leading order and show that the result coincides with one obtained by Braaten and Pisarski \cite{15}. It should be
emphasized that our approach to calculate the gluon damping rate is quite different from the one taken by Braaten and Pisarski. In Ref. [15] they calculated, in the Coulomb gauge, the effective gluon self-energy $\Pi^{\mu\nu}$ using the resummed gluon propagators and vertices. The gluon self-energy $\Pi^{\mu\nu}$ thus obtained contains both gauge-independent and dependent pieces. To isolate the gauge-independent pieces in $\Pi^{\mu\nu}$ and to calculate the gluon damping rate, Braaten and Pisarski constructed the two-gluon $T$-matrix element by putting $\Pi^{\mu\nu}$ on the mass-shell and sandwiching it between physical wave functions. And the gluon damping rate came from the imaginary part of the $T$. Our approach is to first construct the gauge-independent effective resummed gluon self-energy $\hat{\Pi}^{\mu\nu}$ with recourse to the PT and then to calculate the gluon damping rate by putting $\hat{\Pi}^{\mu\nu}$ on the mass-shell and taking its imaginary part. Since $\hat{\Pi}^{\mu\nu}$ is already gauge independent, we do not have to construct the two-gluon $T$-matrix element.

The PT was proposed some time ago by Cornwall [18] for an algorithm to form new gauge-independent proper vertices and new propagators with gauge-independent self-energies. First it was used to obtain the one-loop gauge-independent effective gluon self-energy and vertices in QCD [19][20] and then it has been applied to the standard model [21]. Recently the independence of the PT results on the gauge-fixing procedure and the uniqueness of the PT algorithm were discussed by Papavassiliou and Pilaftsis [22]. Also the PT has been employed to show the dual gauge fixing property of the $S$-matrix [23]. The application of PT to QCD at high temperature was first made by Alexanian and Nair [24] to calculate the gap equation for the magnetic mass to one-loop order. The issue of gauge independence of several quantities in hot QCD was recently discussed from the PT point of view [25].

The paper is organized as follows. Throughout this paper we work with linear gauges which secure rotational symmetry (in the rest frame of the heat bath) [26]. So in the next section we present the Feynman rules in this class of gauges. Then we review some properties of the resummed two-point function and resummed three- and four-point vertices of gluons and the Ward-Takahashi identities satisfied by these functions which will be in full use in the following sections. We divide the resummed gluon propagator into four pieces; two pieces are gauge independent and
the other two are gauge dependent. In Sect. 3, we consider the resummed gluon
self-energy at one loop. We decompose it into the sum of several terms according to
its gauge dependence. In Sect. 4, we first develop the general prescription necessary
for extracting the resummed pinch contributions to the resummed gluon self-energy
from the one-loop quark-quark scattering amplitude. Resorting to this prescription,
we then calculate the one-loop resummed pinch contributions. In Sect. 5, we show
that the effective gluon self-energy, which is the sum of the resummed gluon self-
energy obtained in Sect. 3 and pinch contributions in Sect. 4, is gauge-independent
and also satisfies the transversality relation. Using the effective gluon self-energy, in
Sect. 6, we calculate the damping rate for the transverse gluons at rest in the leading
order and show that the result coincides with one obtained by Braaten and Pisarski.
Sect. 7 is devoted to the conclusions and discussions. In addition, we present three
Appendices. In Appendix A, we give one-loop resummed pinch contributions to the
gluon self-energy from the vertex diagrams of the first kind, of the second kind and
box diagrams, separately, in the linear gauges which preserve rotational invariance.
In Appendix B, we present a decomposition of the hard thermal loop for three-gluon
vertex into the sum of terms with different tensor bases. In Appendix C, we give
the expressions of effective three- and four-gluon vertices evaluated at \( k = 0 \) and
\( k_0 = m_g \), where \( m_g \) is a thermal gluon mass. These expressions are necessary for
the calculation of the gluon damping rate in Sect. 6.

2 Preliminaries

In this section we introduce all the “ingredients” which we will use in the following
sections. Throughout this paper we use the Minkowski metric \((+−−−)\) and the
notation that upper-case letters represent four-momenta: \( K^\mu = (K^0, \mathbf{k}) \) and \( k = |\mathbf{k}| \).

For simplicity we consider only gluons. In fact, in Sect. 4 we introduce quark
fields and study the quark-quark scattering at one-loop level. However, this is only
for picking up the pinch contributions of gluons to the resummed gluon self-energy.
We follow Baier, Kunstatter and Schiff [26] for the choice of gauges. We restrict our
consideration to linear gauges in the following form:

\[ I_{\text{gauge fixing}} = -\frac{1}{2\xi} \int d^4x (\mathcal{F}^a)^2 , \]  

(2.1)

with

\[ \mathcal{F}^a = \mathcal{J}^\mu(x) A^a_\mu(x) . \]  

(2.2)

It may be natural to work in the rest frame of the heat bath where rotational symmetry is unbroken. The most general gauge fixing condition \( \mathcal{J}^\mu \) which preserves rotational invariance is (in the momentum space representation)

\[ \mathcal{J}^\mu = c(K) K^\mu + b(K) n^\mu \]  

(2.3)

with \( n^\mu = (1, 0, 0, 0) \). Now we introduce a four-vector \( \tilde{n}_\mu(K) \) which is the component of \( n_\mu \) orthogonal to the four-momentum \( K_\mu \),

\[ \tilde{n}_\mu(K) = n_\mu - \frac{k_0 K_\mu}{K^2} . \]  

(2.4)

so that \( \tilde{n}_\mu(K) K^\mu = 0 \). Note

\[ \tilde{n}^2(K) = \tilde{n}_0(K) = -\frac{k^2}{K^2} \]  

(2.5)

In terms of the resulting orthogonal bases, the gauge fixing condition \( \mathcal{J}^\mu \) is rewritten as

\[ \mathcal{J}^\mu = a(K) K^\mu + b(K) \tilde{n}^\mu(K) , \]  

(2.6)

where

\[ a(K) = c(K) + b(K) \frac{k_0}{K^2} . \]  

(2.7)

Here we give a few examples of linear gauges:

(i) The covariant gauge: The gauge fixing condition is given by \( \partial^\mu A^a_\mu = 0 \). Thus \( \mathcal{J}^\mu_{COV} = K^\mu \) and we have

\[ a_{COV} = 1, \quad b_{COV} = 0 . \]  

(2.8)

(ii) The Coulomb gauge : Since \( \partial_i A^a_i = 0 \) is the gauge fixing condition, we have \( \mathcal{J}^\mu_{CG} = K^\mu - k_0 n^\mu \) and

\[ a_{CG} = -\frac{k^2}{K^2} = \tilde{n}^2(K), \quad b_{CG} = -k_0 . \]  

(2.9)
The temporal axial gauge: The gauge fixing condition is given by \( n^\mu A^a_\mu = 0 \). Thus \( J^\mu_{TAG} = -i n^\mu \) and

\[
a_{TAG} = -i \frac{k_0}{K^2}, \quad b_{TAG} = -i.
\]

In the temporal axial gauge the gauge parameter \( \xi \) has a dimension of mass\(^{-2} \).

For later convenience we introduce two projection operators:

\[
P_{\mu\nu}(K) &= g_{\mu\nu} - \frac{K_\mu K_\nu}{K^2} - \frac{1}{\tilde{n}^2(K)} \tilde{n}_\mu(K)\tilde{n}_\nu(K) \\
&= - \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \frac{k_i k_j}{k^2} \end{pmatrix} \\
Q_{\mu\nu}(K) &= \frac{1}{\tilde{n}^2(K)} \tilde{n}_\mu(K)\tilde{n}_\nu(K).
\]

The operator \( P_{\mu\nu}(K) \) projects out spatially transverse modes, and is orthogonal to both \( K^\mu \) and \( \tilde{n}^\mu(K) \). On the other hand, the operator \( Q_{\mu\nu}(K) \) picks up longitudinal modes. These operators satisfy

\[
P^2 = P, \quad Q^2 = Q, \quad PQ = QP = 0
\]

\[
K^\mu P_{\mu\nu}(K) = n^\mu P_{\mu\nu}(K) = 0
\]

\[
K^\mu Q_{\mu\nu}(K) = 0, \quad n^\mu Q_{\mu\nu}(K) = \tilde{n}_\nu(K)
\]

\[
P_{\mu\nu}(K) + Q_{\mu\nu}(K) = g_{\mu\nu} - \frac{K_\mu K_\nu}{K^2}.
\]

In this class of linear gauges, the inverse of the bare gluon propagator \( iD^{(0)ab}_{\mu\nu} = i\delta^{ab}D^{(0)}_{\mu\nu} \) is given by

\[
D^{(0)-1}_{\mu\nu}(K) = \Gamma_{\mu\nu}(K) - \frac{1}{\xi} (aK_\mu + b\tilde{n}_\mu)(aK_\nu + b\tilde{n}_\nu)
\]

where

\[
\Gamma_{\mu\nu}(K) = -K^2 g_{\mu\nu} + K_\mu K_\nu \\
= -K^2 [P_{\mu\nu}(K) + Q_{\mu\nu}(K)]
\]
is the bare two-point function. The ghost propagator is

\[ iG^{ab}(K) = i\delta^{ab} \frac{-1}{a(K)K^2} \]  

(2.19)

and the ghost-gluon vertex (see Fig.1) is expressed as

\[ gf^{abc}\Gamma_\mu = gf^{abc}\left[a(P)P_\mu + b(P)\tilde{n}_\mu(P)\right]. \]  

(2.20)

The bare gluon three- and four-point vertices depend only on the classical action owning to the linearity of the gauge fixing condition. Thus the bare three-point vertex \( \Gamma^{abc}_{\mu\nu\lambda}(P,Q,R) \equiv gf^{abc}\Gamma^{\mu\nu\lambda}_{\mu\nu\lambda}(P,Q,R) \) and the bare four-point vertex \( \delta^{cd}\Gamma^{\mu\nu\alpha\beta}_{\mu\nu\alpha\beta}(P,Q,R,S) \equiv -i\delta^{ab}g^2N\Gamma^{\mu\nu\alpha\beta}_{\mu\nu\alpha\beta}(P,Q,R,S) \) are, respectively, expressed as

\[ \Gamma^{abc}_{\mu\nu\lambda}(P,Q,R) = (P - Q)_{\lambda}g_{\mu\nu} + (Q - R)_{\mu}g_{\nu\lambda} + (R - P)_{\nu}g_{\lambda\mu} \]  

(2.21)

\[ \Gamma^{\mu\nu\alpha\beta}_{\mu\nu\alpha\beta}(P,Q,R,S) = 2g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha} \]  

(2.22)

where it is understood that each momentum flows inward and thus the relations \( P + Q + R = 0 \) and \( P + Q + R + S = 0 \) hold, respectively, in the three- and four-point vertices. Also we have traced over the last two color indices in the four-point vertex. These bare two-point function and three- and four-point vertices satisfy the following Ward-Takahashi identities:

\[ K^\mu\Gamma_{\mu\nu}(K) = 0 \]  

(2.23)

\[ R^\lambda\Gamma_{\mu\nu\lambda}(P,Q,R) = \Gamma_{\mu\nu}(P) - \Gamma_{\mu\nu}(Q) \]  

(2.24)

\[ S^\beta\Gamma_{\mu\nu\alpha\beta}(P,Q,R,S) = \Gamma_{\mu\nu\alpha}(P + S,Q,R) - \Gamma_{\mu\nu\alpha}(P,Q + S,R) \]  

(2.25)

In gauge theories at finite temperature we lose the usual connection between the loop expansion and the powers of \( g \). In hot QCD there appear two independent mass scales, \( T \) and \( gT \ll T \). Momenta which are of order \( T \) are called “hard”, while those which are of order \( gT \) are called “soft”. If all of the external legs in a bare amplitude are soft, then one-loop corrections to that amplitude, in which all internal momenta are hard, are of the same order in \( g \). In order to calculate consistently, we must take into account these one-loop corrections called the hard thermal loops. The higher-order effects coming from the hard thermal loops are systematically resummed into effective propagators and vertices [13].
Let $\delta \Pi_{\mu\nu}$, $\delta \Gamma_{\mu\nu\lambda}$ and $\delta \Gamma_{\mu\nu\alpha\beta}$ denote the hard thermal loop corrections to the two-point function and three- and four-point vertices, respectively. It was found [13][17][30] that these hard thermal loops are gauge independent and obey the same Ward-Takahashi identities as the tree-level functions. Then if the corrected two-point function and three- and four-point vertices are defined as

\begin{align*}
\Gamma^*_{\mu\nu} &= \Gamma_{\mu\nu} + \delta \Pi_{\mu\nu} \\
\Gamma^*_{\mu\nu\lambda} &= \Gamma_{\mu\nu\lambda} + \delta \Gamma_{\mu\nu\lambda} \\
\Gamma^*_{\mu\nu\alpha\beta} &= \Gamma_{\mu\nu\alpha\beta} + \delta \Gamma_{\mu\nu\alpha\beta} ,
\end{align*}

(2.26) (2.27) (2.28)

it is obvious that these corrected functions also satisfy the same Ward-Takahashi identities. More precisely, we have

\begin{align*}
K^\mu \Gamma^*_{\mu\nu}(K) &= 0 \quad (2.29) \\
R^\lambda \Gamma^*_{\mu\nu\lambda}(P, Q, R) &= \Gamma^*_{\mu\nu}(P) - \Gamma^*_{\mu\nu}(Q) \quad (2.30) \\
S^\beta \Gamma^*_{\mu\nu\alpha\beta}(P, Q, R, S) &= \Gamma^*_{\mu\nu\alpha}(P + S, Q, R) - \Gamma^*_{\mu\nu\alpha}(P, Q + S, R) . \quad (2.31)
\end{align*}

Since the hard thermal loop $\delta \Pi_{\mu\nu}$ satisfies the identity $K^\mu \delta \Pi_{\mu\nu}(K) = 0$, it can be decomposed as

$$
\delta \Pi_{\mu\nu}(K) = \delta \Pi_T(K) P_{\mu\nu}(K) + \delta \Pi_L(K) Q_{\mu\nu}(K) . \quad (2.32)
$$

Thus $\Gamma^*_{\mu\nu}$ can be written as

$$
\Gamma^*_{\mu\nu}(K) = -K_T^2 P_{\mu\nu}(K) - K_L^2 Q_{\mu\nu}(K) , \quad (2.33)
$$

where

\begin{align*}
K_T^2 &= K^2 - \delta \Pi_T(K) \quad (2.34) \\
K_L^2 &= K^2 - \delta \Pi_L(K) \quad (2.35)
\end{align*}

and it satisfies

\begin{align*}
\eta^\mu \Gamma^*_{\mu\nu}(K) &= \Gamma^*_{0\nu}(K) = -K_L^2 \tilde{n}_\nu(K) \quad (2.36) \\
\Gamma^*_{\mu\lambda}(K) \Gamma^*_{\nu\nu}(K) &= K_T^4 P_{\mu\nu}(K) + K_L^4 Q_{\mu\nu}(K) . \quad (2.37)
\end{align*}
The explicit forms of \( \delta \Pi_T(K) \) and \( \delta \Pi_L(K) \) are given in Appendix B.

The resummed three-point vertex \( \Gamma_{\mu\nu\lambda} \) has the same properties as the bare one \( \Gamma_{\mu\nu\lambda} \): 

\[
\begin{align*}
\Gamma_{\mu\nu\lambda}(P, Q, R) &= \Gamma_{\nu\lambda\mu}(Q, R, P) = \Gamma_{\lambda\mu\nu}(R, P, Q) \\
\Gamma_{\mu\nu\lambda}(-P, -Q, -R) &= -\Gamma_{\mu\nu\lambda}(P, Q, R) \\
\Gamma_{\mu\nu\lambda}(P, Q, R) &= -\Gamma_{\mu\nu\lambda}(P, R, Q)
\end{align*}
\] 

(2.38)

Also the resummed four-point vertex \( \Gamma_{\mu\nu\alpha\beta}(P, Q, R, S) \) is, just like the bare one, symmetric under interchange of momenta and Lorentz indices of the first two lines, the last two lines, and interchange of the first pair with the second pair. The Ward-Takahashi identities Eqs. (2.29)-(2.31) satisfied by the corrected two-point function and three- and four-point vertices and the properties of the corrected three-point vertex given in Eq. (2.38) will be frequently used in the following sections to show the gauge-independence of the resummed gluon self-energy obtained by the PT.

The inverse of the resummed propagator \( i \Gamma_{\mu\nu} = i\delta_{\mu\nu} \Gamma_{\mu\nu} \) is given by 

\[
\begin{align*}
\Gamma_{\mu\nu}^{-1}(K) &= \Gamma_{\mu\nu}(0) - \delta_{\mu\nu} \Pi_{\mu\nu}(K) \\
&= -K_T^2 P_{\mu\nu}(K) - K_L^2 Q_{\mu\nu}(K) - \frac{1}{\xi} \left( aK_{\mu} + b\bar{n}_{\mu} \right) \left( aK_{\nu} + b\bar{n}_{\nu} \right).
\end{align*}
\] 

(2.39)

Inverting \( \Gamma_{\mu\nu}^{-1} \), we have for the resummed propagator 

\[
\begin{align*}
\Gamma_{\mu\nu}(K) &= -\frac{1}{K_T^2} P_{\mu\nu}(K) - \frac{1}{K_L^2} \left[ Q_{\mu\nu}(K) - \frac{b}{a} \frac{1}{K_T^2} \left( \bar{n}_{\mu} K_{\nu} + K_{\mu} \bar{n}_{\nu} \right) - \frac{K_{\mu} K_{\nu}}{K^4} \right] \\
&= -\frac{\xi K_{\mu} K_{\nu}}{a^2 K^4}.
\end{align*}
\] 

(2.40)

For later convenience, we rewrite \( \Gamma_{\mu\nu} \) as follows:

\[
\begin{align*}
\Gamma_{\mu\nu}(K) &= A(K) g_{\mu\nu} + B(K) \frac{K_{\mu} K_{\nu}}{K^2} + S(K) \left[ n_{\mu} n_{\nu} - \frac{k_0}{K^2} (n_{\mu} K_{\nu} + K_{\mu} n_{\nu}) \right] \\
&+ T(K) (n_{\mu} K_{\nu} + K_{\mu} n_{\nu})
\end{align*}
\] 

(2.41)

where

\[
A(K) = -\frac{1}{K_T^2}
\] 

(2.42)
\[ B(K) = -\frac{1}{K_T^2} K^2 + \frac{1}{K_L^2} \left( \frac{k_0^2}{k^2} - \frac{2k_0 b}{K^2 a} + \frac{k^2 b^2}{K^4 a^2} \right) - \frac{\xi}{K^2 a^2} \] (2.43)

\[ S(K) = \left( \frac{1}{K_T^2} - \frac{1}{K_L^2} \right) \frac{1}{\bar{n}^2(K)} \] (2.44)

\[ T(K) = \frac{1}{K^2 L} b \] (2.45)

Since the ratio \( b(K)/a(K) \) is a odd function of \( K \), we easily see that \( A(K), B(K), \) and \( S(K) \) are even functions of \( K \), while \( T(K) \) is an odd function. Also \( A(K) \) and \( S(K) \) do not depend on \( a, b \), nor the gauge parameter \( \xi \) and thus they are gauge-independent. It is noted that the gauge parameter \( \xi \) only appears in \( B(K) \). If we take the limit \( \delta \Pi_T = \delta \Pi_L = 0 \), the function \( S(K) \) vanishes. So the \( S(K) \) term is a unique one for the resummed gluon propagator. In the next two sections we use the decomposed form of the gluon propagator given in Eq.(2.41) and divide the one-loop resummed gluon self-energy and the corresponding resummed pinch contributions into terms according to its dependence on the functions \( A, B, S, \) and \( T \). In doing so, we can easily see the cancellation of the gauge-dependent parts when the pinch contributions are added to the resummed gluon self-energy.

### 3 Resummed Gluon Self-Energy

In this section we consider the resummed gluon self-energy \( *\Pi_{\mu\nu} \). We assume that the external and the loop momenta are soft. Then there are three diagrams which contribute at one-loop:

\[ *\Pi_{\mu\nu}(K) = *\Pi_{\mu\nu}^3(K) + *\Pi_{\mu\nu}^4(K) + \Pi_{\mu\nu}^6(K) . \] (3.1)

The graph in Fig.2(a) with two resummed three-gluon vertices gives

\[ *\Pi_{\mu\nu}^3(K) = \frac{N g^2}{2} \int dP \ *\Gamma_{\lambda\mu\nu}(P, K, Q) \ *D^{\alpha\beta}(Q) \ *\Gamma_{\beta\nu\tau}(-Q, -K, -P) \ *D^{\tau\lambda}(P) , \] (3.2)

with

\[ \int dP = \int \frac{d^3p}{8\pi^3} T \sum_n , \] (3.3)
where the summation goes over the integer \( n \) in \( p_0 = i2\pi n T \) and the spatial integration is implicitly assumed to be over soft momenta only. We have chosen the variables as \( K + P + Q = 0 \) so that there holds a relation
\[
\int dP f(P, Q) = \int dP f(Q, P) .
\] (3.4)

In the following we make extensive use of this symmetry property of the integrands under interchange of \( P \) and \( Q \). The graph in Fig.2(b) with a resummed four-gluon vertex gives
\[
^*\Pi^{3g}_{\mu\nu}(K) = \frac{N g^2}{2} \int dP\; ^*\Gamma_{\mu\nu\alpha\beta}(K, -K, P, -P)\; ^*D^{\alpha\beta}(P) .
\] (3.5)

Finally, the contribution of the ghost loop in Fig.2(c) gives
\[
\Pi^{gh}_{\mu\nu}(K) = N g^2 \int dP \sum_{i \geq j} I^{ij}_{\mu\nu} \quad \text{with} \quad i, j = A, B, S, T ,
\] (3.7)

where
\[
I^{AA}_{\mu\nu} = -\frac{1}{2} A(P) A(Q) \; ^*\Gamma_{\lambda\mu}^\alpha(P, K, Q) \; ^*\Gamma_{\alpha\nu}^\lambda(Q, K, P) ,
\] (3.8)
\[
I^{BB}_{\mu\nu} = \frac{1}{2} B(P) B(Q) \frac{1}{P^2 Q^2} P^\lambda P^\tau \; ^*\Gamma_{\mu\lambda}(K) \; ^*\Gamma_{\nu\tau}(K) ,
\] (3.9)
\[
I^{SS}_{\mu\nu} = -S(P) S(Q) \left[ \frac{1}{2} \; ^*\Gamma_{0\mu\sigma}(P, K, Q) \; ^*\Gamma_{0\nu\sigma}(Q, K, P) \right]
\]
\[ + \frac{p_0}{p^2} \left\{ [K_L^2 \bar{n}_\mu(K) - Q_L^2 \bar{n}_\mu(Q)] \ast \Gamma_{0:0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]

\[ + \frac{p_0 q_0}{p^2 Q^2} \left\{ [K_L^2 \bar{n}_\mu(K) - Q_L^2 \bar{n}_\mu(Q)][K_L^2 \bar{n}_\nu(K) - P_L^2 \bar{n}_\nu(P)] \right. \\
\left. - \frac{1}{2} [P^\lambda \ast \Gamma_{\lambda \mu}(K) \ast \Gamma_{0:0}(Q, K, P) + (\mu \leftrightarrow \nu)] \right\}, \quad (3.10) \]

\[ I_{\mu \nu}^{TT} = T(P)T(Q) \left[ -[K_L^2 \bar{n}_\mu(K) - P_L^2 \bar{n}_\mu(P)][K_L^2 \bar{n}_\nu(K) - Q_L^2 \bar{n}_\nu(Q)] \right. \\
\left. + \frac{1}{2} \left\{ P^\lambda \ast \Gamma_{\lambda \mu}(K) \ast \Gamma_{0:0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \right], \quad (3.11) \]

\[ I_{\mu \nu}^{AB} = A(Q)B(P) \frac{1}{p^2} \left\{ [\Gamma_{\mu \alpha}(K) - \ast \Gamma_{\mu \alpha}(Q)] \ast \Gamma_{\nu \alpha}(K) - \ast \Gamma_{\nu \beta}(Q) \right\}, \quad (3.12) \]

\[ I_{\mu \nu}^{AS} = -A(Q)S(P) \left\{ [\Gamma_{0:0}(P, K, Q) \ast \Gamma_{0:0}(Q, K, P) \right. \\
\left. - \frac{p_0}{p^2} \left\{ [\Gamma_{\mu \alpha}(K) - \ast \Gamma_{\mu \alpha}(Q)] \ast \Gamma_{\nu \alpha}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \right\}, \quad (3.13) \]

\[ I_{\mu \nu}^{AT} = -A(Q)T(P) \left\{ [\Gamma_{\mu \alpha}(K) - \ast \Gamma_{\mu \alpha}(Q)] \ast \Gamma_{0:0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\}, \quad (3.14) \]

\[ I_{\mu \nu}^{BS} = B(P)S(Q) \frac{1}{p^2} \left\{ [K_L^2 \bar{n}_\mu(K) - Q_L^2 \bar{n}_\mu(Q)][K_L^2 \bar{n}_\nu(K) - Q_L^2 \bar{n}_\nu(Q)] \right. \\
\left. + \frac{q_0}{Q^2} \left\{ [K_L^2 \bar{n}_\mu(K) - Q_L^2 \bar{n}_\mu(Q)] \ast \Gamma_{\lambda \nu}(K) + (\mu \leftrightarrow \nu) \right\} \right\}, \quad (3.15) \]

\[ I_{\mu \nu}^{BT} = -B(P)T(Q) \frac{1}{p^2} \left\{ Q^\lambda \ast \Gamma_{\lambda \mu}(K)[K_L^2 \bar{n}_\nu(K) - Q_L^2 \bar{n}_\nu(Q)] + (\mu \leftrightarrow \nu) \right\}, \quad (3.16) \]

\[ I_{\mu \nu}^{ST} = S(Q)T(P) \left\{ [K_L^2 \bar{n}_\mu(K) - Q_L^2 \bar{n}_\mu(Q)] \ast \Gamma_{0:0}(Q, K, P) + (\mu \leftrightarrow \nu) \right. \\
\left. + \frac{q_0}{Q^2} \left\{ [K_L^2 \bar{n}_\mu(K) - P_L^2 \bar{n}_\mu(P)][K_L^2 \bar{n}_\nu(K) - Q_L^2 \bar{n}_\nu(Q)] + (\mu \leftrightarrow \nu) \right\} \right. \\
\left. + \frac{q_0}{Q^2} \left\{ Q^\lambda \ast \Gamma_{\lambda \mu}(K) \ast \Gamma_{0:0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \right]. \quad (3.17) \]

For an illustration, let us show the derivation of \( I_{\mu \nu}^{BB} \) term. The product of two propagators \( \ast D^{\alpha \beta}(Q) \ast D^{\tau \lambda}(P) \) in Eq.(3.2) has a term

\[ B(P)B(Q) \frac{P^\lambda P^\tau Q^\alpha Q^\beta}{p^2 Q^2}, \quad (3.18) \]

which gives

\[ I_{\mu \nu}^{BB} = \frac{1}{2} B(P)B(Q) \frac{P^\lambda P^\tau Q^\alpha Q^\beta}{p^2 Q^2} \ast \Gamma_{\lambda \mu}(P, K, Q) \ast \Gamma_{\beta \nu}(Q, K, P) \ast \Gamma_{\beta \nu}(-Q, -K, -P). \quad (3.19) \]
Using the properties of the corrected three-point vertex $\Gamma_{\mu\nu\lambda}$ in Eq.(2.38) and the Ward-Takahashi identities (2.29) and (2.30) satisfied by $\Gamma_{\mu\nu}$ and $\Gamma_{\mu\nu\lambda}$, we find that
\[ P^\lambda Q^\alpha Q^\beta \Gamma_{\lambda\mu\alpha}(P, K, Q) \Gamma_{\beta\nu\tau}(-Q, -K, -P) = P^\lambda P^\tau \Gamma_{\lambda\mu}(K) \Gamma_{\nu\tau}(K) \] (3.20)
and thus we reach the expression for $I^{BB}_{\mu\nu}$ given in Eq.(3.9). The other terms in Eq.(3.8) through Eq.(3.17) are derived in a similar way.

Likewise $\Pi^{4g}_{\mu\nu}$ is expressed in terms of $A$, $B$, $S$, and $T$ as
\[ \Pi^{4g}_{\mu\nu}(K) = N g^2 \int dP \sum_{i} J^i_{\mu\nu} \] with $i = A, B, S, T$, (3.21)
where
\[ J^A_{\mu\nu} = \frac{1}{2} A(P) \Gamma_{\mu\nu\alpha}(K, -K, P, -P), \] (3.22)
\[ J^B_{\mu\nu} = B(P) \frac{1}{P^2} [\Gamma_{\mu\nu}(K) - \Gamma_{\mu\nu}(Q)], \] (3.23)
\[ J^S_{\mu\nu} = S(P) \left\{ \frac{1}{2} \Gamma_{\mu\nu00}(K, -K, P, -P) \right. \]
\[ + \frac{P_0}{P^2} \left\{ \Gamma_{\mu\nu0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \], \] (3.24)
\[ J^T_{\mu\nu} = -T(P) \left\{ \Gamma_{\mu\nu0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\}. \] (3.25)

4 Resummed Pinch Contributions

4.1 Pinch Technique

In this section we obtain the one-loop resummed pinch contributions to the resummed gluon self-energy. We proceed in the same way as we did before in the second paper of Ref.[12]. The only difference is that here we use the resummed gluon propagators and resummed vertices instead of bare ones. Let us consider the $S$-matrix element $\hat{T}$ for the elastic quark-quark scattering at one-loop, assuming that quarks have the same mass $m$. We introduce quarks just as technical devices to extract the pinch contributions. We assume that both the momentum transfered
in the $t$-channel and the loop momenta are soft. Besides the self-energy diagram in Fig.3, the vertex diagrams of the first and second kind and the box diagrams contribute to $\hat{T}$. They are shown in Fig.4(a), Fig.5(a), and Fig.6(a), respectively. Note that gluon propagators and vertices are resummed ones. For quark sectors, however, we still use bare quark propagators and bare quark-gluon vertices, because we are only interested in gluonic parts. Although each contribution from diagrams in Fig.3, Fig.4(a), Fig.5(a), and Fig.6(a) is, in general, gauge-dependent, the sum is gauge-independent. This can be seen from the following observation: When we set $T = 0$, all the hard thermal loop contributions vanish. Then the sum reduces to the ordinary zero-temperature $S$-matrix element for the elastic quark-quark scattering at one-loop, which is obviously gauge-independent. At finite temperature the hard thermal loop contributions are switched on. Since these contributions do not depend on the gauge choices and thus the sum remains gauge-independent.

Now we single out the “pinch parts” of the vertex and box diagrams, which are depicted in Fig.4(b), Fig.5(b), and Fig.6(b). They emerge when a $\gamma^\mu$ matrix on the quark line is contracted with a four-momentum $K^\mu$ offered by a resummed gluon propagator or a resummed three-gluon vertex. Such a term triggers an elementary Ward identity of the form

$$K = (P + K - m) - (P - m).$$

(4.1)

The first term removes (pinches out) the internal quark propagator, whereas the second term vanishes on shell, or vice versa. This procedure leads to contributions to $\hat{T}$ with one or two less quark propagators and, hence, we will call these contributions $\hat{T}_P$, “pinch parts” of $\hat{T}$.

Next we extract from $\hat{T}_P$ the pinch contributions to the resummed gluon self-energy $\ast \Pi_{\mu\nu}$. First note that the contribution of the resummed gluon self-energy diagram to $\hat{T}$ is written in the form (see Fig.3)

$$\hat{T}^{(S.E)} = [T^a \gamma_\alpha]^\ast D^{\alpha\mu}(K) \ast \Pi_{\mu\nu} \ast D^{\nu\beta}(K)[T^a \gamma_\beta],$$

(4.2)

where $T^a$ is a representation matrix of $SU(N)$, and $\gamma_\alpha$ and $\gamma_\beta$ are $\gamma$ matrices on the external quark lines. The pinch contribution $\ast \Pi_{\mu\nu}^P$ to $\hat{T}_P$ should have the same form.
Thus we must take away $[T^a \gamma_\alpha] * D^\alpha_\mu (K)$ and $* D^\mu_\beta (K)[T^a \gamma_\beta]$ from $\hat{T}_P$. For that purpose we use the following identity satisfied by the resummed gluon propagator and its inverse:

$$g_{\alpha \beta} = * D^\alpha_\mu (K) * D^{-1}_\mu\nu (K) * D^\nu_\beta (K) + K_\alpha \text{ term} = * D_{\alpha\mu}^{-1}(K) * D^\mu_\beta (K) = * \Gamma_{\alpha\mu}(K) * D^\mu_\beta (K) + K_\beta \text{ term}, \quad (4.3)$$

where $* \Gamma_{\mu\nu}(K)$ is given in Eq. (2.33). The $K_\alpha$ and $K_\beta$ terms give null results when they are contracted with $\gamma_\alpha$ and $\gamma_\beta$, respectively, of the external quark lines.

The pinch part of the one-loop vertex diagrams of the first kind depicted in Fig.4(b) plus their mirror graphs has a form

$$\hat{T}^{(V_1)}_P = A [T^a \gamma_\alpha] * D^\alpha_\beta (K)[T^a \gamma_\beta], \quad (4.4)$$

where $A$ (also $B^0$, $B^1_{\lambda\nu}$, $B^2_{\nu\lambda}$, $C^0$, and $C_{ij}$ in the equations below) contains a loop integral. Using Eq. (4.3) we find

$$\gamma_\alpha * D^\alpha_\beta (K) \gamma_\beta = \gamma_\alpha * D^\alpha_\mu (K) * \Gamma_{\mu\nu}(K) * D^\nu_\beta (K). \quad (4.5)$$

Thus the contributions to $* \Pi_{\mu\nu}$ from the vertex diagrams of the first kind are written as

$$* \Pi_{\mu\nu}^{(V_1)} = * \Gamma_{\mu\nu}(K) A. \quad (4.6)$$

The pinch part of the one-loop vertex diagrams of the second kind depicted in Fig.5(b) has a form

$$\hat{T}^{(V_2)}_P = [T^a] \left\{ [\gamma_\nu] B^0 + [\gamma^\lambda] B^1_{\lambda\nu} + \sum_i [P_i] B^2_{\nu\lambda} \right\} * D^\mu_\beta (K)[T^a \gamma_\beta], \quad (4.7)$$

where $P_i$ is a four-momenta appearing in the diagrams. By redefinition of the loop-integral momentum we can choose $P_i = P$ or $n$ where $P$ is the loop-integral momentum and $n$ is a unit vector $n^\mu = (1, 0, 0, 0)$. Using Eq. (4.3) and

$$[\gamma^\lambda] = [\gamma_\alpha] * D^\alpha_\mu (K) * \Gamma^\lambda_\mu(K) \quad (4.8)$$

$$[P_i] = [\gamma_\alpha] * D^\alpha_\mu (K) * \Gamma^\lambda_{\mu\lambda}(K) P^\lambda_i \quad (4.9)$$
we obtain for the contributions to $^*\Pi_{\mu\nu}$ from the vertex diagrams of the second kind

\[
^*\Pi_{\mu\nu}^{(V_2)} = \Gamma_{\mu\nu}(K)B^0 + \Gamma_{\mu}^{\lambda}(K)B_{\lambda\nu}^{1} + \Gamma_{\mu\nu}(K)\sum_{i}B_{i\nu}P_{i}^{\lambda}
\]

\[+ (\mu \leftrightarrow \nu) , \tag{4.10}\]

where $(\mu \leftrightarrow \nu)$ terms are the contributions from mirror diagrams.

The pinch part of the one-loop box diagrams depicted in Fig.6(b) has a form

\[
\hat{T}_{P}^{(Box)} = [T^a]\{[\gamma_\alpha][\gamma^\alpha]C^0 + \sum_{i,j}C_{ij}[P_i][P_j]\}[T^a]. \tag{4.11}\]

Again from Eq. (4.13) we see that $[\gamma_\alpha][\gamma^\alpha]$ and $[P_i][P_j]$ are rewritten as

\[
[\gamma_\alpha][\gamma^\alpha] = [\gamma_\alpha] * D^{\mu\alpha}(K)[* \Gamma_{\mu\lambda}(K) * \Gamma_{\lambda\nu}(K)] * D^{\nu\beta}(K)[\gamma^\beta] \tag{4.12}\]

\[
[P_i][P_j] = [\gamma_\alpha] * D^{\alpha\mu}(K)[* \Gamma_{\mu\lambda}(K) * \Gamma_{\nu\tau}(K)P_{i}^{\lambda}P_{j}^{\tau}] * D^{\nu\beta}(K)[\gamma^\beta] \tag{4.13}\]

and thus we obtain for the contributions to $^*\Pi_{\mu\nu}$ from the box diagrams

\[
^*\Pi_{\mu\nu}^{(Box)} = \Gamma_{\mu\lambda}(K) * \Gamma_{\lambda\nu}(K)C^0 + \Gamma_{\mu\lambda}(K) * \Gamma_{\nu\tau}(K)\sum_{i,j}C_{ij}P_{i}^{\lambda}P_{j}^{\tau}. \tag{4.14}\]

### 4.2 Resummed Pinch Contributions

Following the prescription developed in Sec.4.1, we now obtain the resummed pinch contributions to the resummed gluon self-energy. First we present the results. The individual contributions from the vertex diagrams of the first kind, of the second kind and the box diagrams are presented in Appendix A. In total they are expressed as

\[
^*\Pi_{\mu\nu}^{(P)}(K) = Ng^2 \int dP\{J_{\mu\nu}^{(P)B} + \sum_{i\geq j}I_{\mu\nu}^{(P)ij}\} \quad \text{with} \quad i, j = A, B, S, T \tag{4.15}\]

where

\[
J_{\mu\nu}^{(P)B} = -B(P)\frac{1}{P^2} * \Gamma_{\mu\nu}(K) \tag{4.16}\]

and
\[ I_{\mu \nu}^{(P)AA} = -A(P)A(Q) \left[ 2 \Gamma_{\mu \nu}(K) + \left\{ \Gamma_{\mu \alpha}(K) V^\alpha_\nu(Q, K) + (\mu \leftrightarrow \nu) \right\} \right], \quad (4.17) \]

\[ I_{\mu \nu}^{(P)BB} = -\frac{1}{2} B(P)B(Q) \frac{1}{P^2 Q^2} P^\lambda P^\tau \Gamma_{\mu \lambda}(K) \Gamma_{\nu \tau}(K), \quad (4.18) \]

\[ I_{\mu \nu}^{(P)SS} = S(P)S(Q) \left[ -\frac{p_0 q_0}{P^2 Q^2} K_L^4 \tilde{\eta}_{\mu}(K) \tilde{\eta}_{\nu}(K) \right. \]
\[ + \frac{p_0}{P^2} \left\{ K_L^2 \tilde{\eta}_{\mu}(K) \tilde{\eta}^\alpha(Q) \Gamma_{\alpha \nu 0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]
\[ - \frac{1}{2} \frac{p_0 q_0}{P^2 Q^2} \left\{ P^\lambda \Gamma_{\lambda \mu}(K) \Gamma_{0 \nu 0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\}, \quad (4.19) \]

\[ I_{\mu \nu}^{(P)TT} = T(P)T(Q) \left\{ \Gamma_{\mu}^4 \tilde{\eta}_{\mu}(K) \tilde{\eta}_{\nu}(K) + \left[ K_L^2 P_L^2 \tilde{\eta}_{\mu}(K) \tilde{\eta}_{\nu}(Q) - \frac{1}{2} \left\{ P^\lambda \Gamma_{\lambda \mu}(K) \Gamma_{0 \nu 0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \right] \right\}, \quad (4.20) \]

\[ I_{\mu \nu}^{(P)AB} = A(Q)B(P) \frac{1}{P^2} \left\{ -\Gamma_{\mu \alpha}(K) \Gamma^\alpha_{\nu}(K) \Gamma_{\mu \alpha}(K) \Gamma^\alpha_{\nu}(K) \right\}, \quad (4.21) \]

\[ I_{\mu \nu}^{(P)AS} = A(Q)S(P) \left\{ K_L^2 \tilde{\eta}_{\mu}(K) \tilde{\eta}^\alpha(P) [g_{\alpha \nu} + V_{\alpha \nu}(Q, K)] + (\mu \leftrightarrow \nu) \right\} \]
\[ - \frac{p_0}{P^2} \left\{ \Gamma_{\mu}^\alpha(K) \Gamma_{\alpha \nu 0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\}, \quad (4.22) \]

\[ I_{\mu \nu}^{(P)AT} = A(Q)T(P) \left\{ \Gamma_{\mu \alpha}(K) \Gamma^\alpha_{\nu 0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]
\[ + \left\{ \left[ -Q_T^2 + \frac{q_0}{Q^2 \tilde{n}^2(Q)} \tilde{\eta}_{\mu}(Q) + \frac{Q_\mu}{Q^2} \tilde{\eta}_{\nu}(K) + (\mu \leftrightarrow \nu) \right] \right\}, \quad (4.23) \]

\[ I_{\mu \nu}^{(P)BS} = B(P)S(Q) \frac{1}{P^2} \left\{ -K_L^4 \tilde{\eta}_{\mu}(K) \tilde{\eta}_{\nu}(K) + \left\{ K_L^2 \tilde{\eta}_{\mu}(K) Q_L^2 \tilde{\eta}_{\nu}(Q) + (\mu \leftrightarrow \nu) \right\} \right\} \]
\[ - \frac{q_0}{Q^2} \left\{ Q^\lambda \Gamma_{\lambda \mu}(K) \left[ K_L^2 \tilde{\eta}_{\nu}(K) - Q_L^2 \tilde{\eta}_{\nu}(Q) \right] + (\mu \leftrightarrow \nu) \right\}, \quad (4.24) \]

\[ I_{\mu \nu}^{(P)BT} = B(P)T(Q) \frac{1}{P^2} \left\{ [K_L^2 \tilde{\eta}_{\mu}(K) - Q_L^2 \tilde{\eta}_{\mu}(Q)] Q^\lambda \Gamma_{\lambda \nu}(K) + (\mu \leftrightarrow \nu) \right\}, \quad (4.25) \]

\[ I_{\mu \nu}^{(P)ST} = -S(Q)T(P) \left\{ \left[ K_L^2 \tilde{\eta}_{\mu}(K) + \frac{q_0}{Q^2} Q^\lambda \Gamma_{\lambda \mu}(K) \right] \Gamma_{0 \nu 0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]
\[ + \frac{q_0}{Q^2} \left\{ 2K_L^4 \tilde{\eta}_{\mu}(K) \tilde{\eta}_{\nu}(K) - \left\{ P_L^2 \tilde{\eta}_{\nu}(P) + Q_L^2 \tilde{\eta}_{\nu}(Q) \right\} + (\mu \leftrightarrow \nu) \right\}, \quad (4.26) \]
The function $V_\alpha(P,K)$, which appeared in Eqs. (4.17) and (4.22), is defined in Appendix B.

Now we explain how the above terms are obtained. Let us consider the pinch contribution from the vertex diagram of the second kind. The diagram of Fig.5(a) gives

$$\hat{T}^{(V_2)} = N^2 g^2 \int dP \left[ T^a \right] \left[ \gamma_\kappa \frac{1}{L-P-m} \gamma_\lambda \right] * D^{\lambda\tau}(P) * D^{\kappa\alpha}(Q) \times * \Gamma_{\tau\nu\alpha}(P,K,Q) * D^{\nu\beta}(K) [T^a\gamma_\beta] \tag{4.27}$$

The contribution of $\hat{T}^{(V_2)}$ to $I^{(P)BB}_{\mu\nu}$ is extracted as follows. The product of two propagator $* D^{\lambda\tau}(P) * D^{\kappa\alpha}(Q)$ contains a term

$$B(P)B(Q) \frac{P^\lambda P^\tau Q^\kappa Q^\alpha}{P^2 Q^2} \tag{4.28}$$

According to the pinch prescription, the product

$$\left[ \gamma_\kappa \frac{1}{L-P-m} \gamma_\lambda \right] P^\lambda Q^\kappa = \left[ (L-P-m) - (L+K-m) \right] \frac{1}{L-P-m} P$$

(4.29)

gives $P$, since a term $(L+K-m)$ vanishes on mass shell. Also due to the Ward-Takahashi identities (2.29) and (2.30) satisfied by the effective two-point function and three-point vertex, we find

$$P^\tau Q^\kappa * \Gamma_{\tau\nu\alpha}(P,K,Q) = -P^\tau * \Gamma_{\tau\nu}(K) \tag{4.30}$$

Thus a component of $\hat{T}^{(V_2)}_P$ which is relevant to $I^{(P)BB}_{\mu\nu}$ is written as

$$\left. \hat{T}^{(V_2)}_P \right|_{BB} = -N^2 g^2 \int dP \left[ T^a \right] B(P)B(Q) \frac{1}{P^2 Q^2} [P^\tau] P^\tau * \Gamma_{\tau\nu}(K) \times * D^{\nu\beta}(K) [T^a\gamma_\beta] \tag{4.31}$$

The final step is to use Eqs. (4.7) and (4.10) and we obtain for the contribution of the vertex diagram of the second kind to $I^{(P)BB}_{\mu\nu}$,

$$I^{(P)BB}_{\mu\nu} \left|_{V_2} = -B(P)B(Q) \frac{1}{P^2 Q^2} * \Gamma_{\mu\lambda}(K) P^\lambda P^\tau * \Gamma_{\tau\nu}(K) \right. \tag{4.32}$$
where we have added the mirror diagram contribution. The analysis of the pinch contributions from the box diagram can be done in a similar way and we find
\[ I_{\mu \nu}^{(P)BB} \big|_{\text{Box}} = -\frac{1}{2} I_{\mu \nu}^{(P)BB} \big|_{V_2}. \]
Then the sum of \( I_{\mu \nu}^{(P)BB} \big|_{V_2} \) and \( I_{\mu \nu}^{(P)BB} \big|_{\text{Box}} \) gives the expression in Eq. (4.18) for \( I_{\mu \nu}^{(P)BB} \). The other terms in Eq. (4.16) through Eq. (4.26) are obtained similarly, except for \( I_{\mu \nu}^{(P)AA}, I_{\mu \nu}^{(P)SS} \) and \( I_{\mu \nu}^{(P)AS} \).

The derivation of \( I_{\mu \nu}^{(P)AA} \), \( I_{\mu \nu}^{(P)SS} \) and \( I_{\mu \nu}^{(P)AS} \) are more involved. Let us start with \( I_{\mu \nu}^{(P)AA} \) term. This term emerges from the vertex diagram of the second kind. The product \( * D_\lambda \tau (P) * D_\kappa \alpha (Q) * \Gamma_{\tau \nu \alpha} (P, K, Q) \) contains a term
\[ A(P) A(Q) * \Gamma_{\lambda \nu \kappa} (P, K, Q). \] (4.33)

We expand \( * \Gamma_{\lambda \nu \kappa} (P, K, Q) \) into the sum of terms with different tensor structures, such as terms proportional to \( P_\lambda \), terms proportional to \( Q_\kappa \) and others. Then \( * \Gamma_{\lambda \nu \kappa} (P, K, Q) \) is rewritten as
\[ * \Gamma_{\lambda \nu \kappa} (P, K, Q) = P_\lambda \{ g_{\kappa \nu} + V_{\kappa \nu} (P, K) \} - Q_\kappa \{ g_{\lambda \nu} + V_{\lambda \nu} (Q, K) \} + \cdots \] (4.34)

where the dots \( \cdots \) represent terms which are neither proportional to \( P_\lambda \) nor to \( Q_\kappa \).

The functions \( V_{\kappa \nu} (P, K) \) and \( V_{\lambda \nu} (Q, K) \) are given by Eqs. (B.9)-(B.11) in Appendix B. It is symmetric in indices \( \lambda \) and \( \nu \) and satisfies the following identity (see also Eq. (B.12)):
\[ K^\nu V_{\lambda \nu} (Q, K) = \left( 1 - \frac{Q_T^2}{Q^2} \right) Q_\lambda + \left( Q_T^2 - Q_L^2 \right) \frac{q_0 n_{\lambda} (Q)}{Q^2 n^2 (Q)}. \] (4.35)

Now the pinch prescription gives
\[ \gamma^\kappa \frac{1}{\ell - P - m} P \implies - \gamma^\kappa, \] (4.36)
\[ - Q_\kappa \frac{1}{\ell - P - m} \gamma^\lambda \implies - \gamma^\lambda. \] (4.37)

Thus a pinch part \( \tilde{T}_P^{(V_2)} \) which is relevant to \( I_{\mu \nu}^{(P)AA} \) is expressed as
\[ \tilde{T}_P^{(V_2)} \big|_{AA} = - N g^2 \int dP \left[ T^a \right] A(P) A(Q) \left[ \gamma^\lambda \right] \left[ g_{\lambda \nu} + V_{\lambda \nu} (Q, K) \right] \times * D^\nu_\beta (K) \left[ T^a \gamma_\beta \right], \] (4.38)
where we have used the symmetry property of the integrand under the interchange of \( P \) and \( Q \). The mirror diagram contribution is obtained by interchanging indices \( \mu \) and \( \nu \) in the above expression of \( \hat{T}_P^{(V_2)} \). Then the formulas (4.7) and (4.10) give \( I_{\mu\nu}^{(P)AA} \) in Eq.(4.17).

Now we proceed to the derivation of \( I_{\mu\nu}^{(P)AS} \) term, which emerges from the vertex diagram of the second kind. It is reminded that the \( S \) term in the resummed gluon propagator is typical at finite temperature. Indeed the \( S \) term will vanish at \( T = 0 \).

The product \( ^*D_\lambda^e(P) \cdot ^*D_\kappa^e(Q) \cdot ^*\Gamma_{\tau\nu\alpha}(P, K, Q) \) in Eq.(4.27) contains terms

\[
A(Q)S(P) \left\{ n_\lambda \tilde{n}^\tau(P) - \frac{p_0}{P_2} [n_\lambda P^\tau + P_\lambda n^\tau] \right\} \cdot ^*\Gamma_{\tau\nu\alpha}(P, K, Q)
\]

\[
+ A(P)S(Q) \left\{ n_\kappa \tilde{n}^\alpha(P) - \frac{q_0}{Q_2} [n_\kappa Q^\alpha + Q_\kappa n^\alpha] \right\} \cdot ^*\Gamma_{\lambda\nu\alpha}(P, K, Q).
\]

The second line gives the same contribution as the first one. The first line is rewritten as

\[
A(Q)S(P) \left\{ n_\lambda \tilde{n}^\tau(P) \cdot ^*\Gamma_{\tau\nu\kappa}(P, K, Q) - \frac{p_0}{P_2} P_\lambda \cdot ^*\Gamma_{0\nu\kappa}(P, K, Q) \right\}
\]

\[
= A(Q)S(P) \left\{ n_\lambda \tilde{n}^\tau(P) \{-Q_\kappa [g_{\tau\nu} + V_{\tau\nu}(Q, K)] \} \right.
\]

\[
- \frac{p_0}{P_2} P_\lambda \cdot ^*\Gamma_{0\nu\kappa}(P, K, Q) + \cdots \right\},
\]

where a decomposed form for \( ^*\Gamma_{\tau\nu\kappa}(P, K, Q) \) such as given in Eq.(4.34) and an identity \( \tilde{n}^\tau(P)P_\tau = 0 \) were used. The dots \( \cdots \) represent irrelevant terms which are not proportional to \( P_\lambda \) nor to \( Q_\kappa \) and, therefore, do not yield the pinch parts. Then the pinch technique prescription gives a pinch part \( \hat{T}_P^{(V_2)} \) which is relevant to \( I_{\mu\nu}^{(P)AS} \) as follows:

\[
\hat{T}_P^{(V_2)} \big|_{AS} = Ng^2 \int dP \left[ T^a \right] A(Q)S(P) \left\{ -[\gamma^\nu] \tilde{n}^\tau(P) [g_{\tau\nu} + V_{\tau\nu}(Q, K)] \right.
\]

\[
+ [\gamma^\kappa] \frac{p_0}{P_2} \cdot ^*\Gamma_{0\nu\kappa}(P, K, Q) \right\},
\]

where the contribution of the second line in Eq.(4.33) has been added. Now it is straightforward to obtain the expression of Eq.(4.22) for \( I_{\mu\nu}^{(P)AS} \). The derivation of \( I_{\mu\nu}^{(P)SS} \) can be done in a similar way.

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5 Gauge-independent Resummed Gluon Self-Energy

In this section we show that when we combine the resummed gluon self-energy calculated in Sect.3 with the resummed pinch contributions in Sect.4, we will obtain the effective gluon self-energy which is \textit{gauge independent} and also \textit{satisfies the transversality relation}.

5.1 Gauge-independence

First we show that once the pinch contributions are added, the $B$-terms are cancelled out. From the expressions for the $B$-terms obtained in Sect.3 and Sect.4, we see that

\[ I^{BB}_{\mu\nu} + I^{(P)BB}_{\mu\nu} = 0 \] (5.1)
\[ I^{BT}_{\mu\nu} + I^{(P)BT}_{\mu\nu} = 0 \] (5.2)

and

\[ I^{AB}_{\mu\nu} + I^{(P)AB}_{\mu\nu} + I^{BS}_{\mu\nu} + I^{(P)BS}_{\mu\nu} = B(P) \frac{1}{P^2} \Gamma_{\mu\nu}(Q) \] (5.3)

On the other hand, we find

\[ J^B_{\mu\nu} + J^{(P)B}_{\mu\nu} = -B(P) \frac{1}{P^2} \Gamma_{\mu\nu}(Q) \] (5.4)

Thus, all the $B$-terms, when added together, cancel out, which means that the gauge parameter $\xi$-dependence disappears.

Next we show that the $T$-terms, when the pinch contributions are added, also cancel out and thus all the gauge-dependent terms disappear. First we have

\[ I^{TT}_{\mu\nu} + I^{(P)TT}_{\mu\nu} = -T(P)T(Q)P_L^2Q_L^2 \bar{n}_{\mu}(P)\bar{n}_{\nu}(Q) \]
\[ = - \frac{1}{P^2Q^2 a(P) a(Q)} b(P) b(Q) L_{\mu\nu}(Q) \] (5.5)

where, in the second line, Eq.(2.43) has been used for $T(P)T(Q)$. It is noted that the sum of $I^{TT}_{\mu\nu} + I^{(P)TT}_{\mu\nu}$ cancels against a term proportional to $\frac{b(P) b(Q)}{a(P) a(Q)}$ in $\Pi^{gh}_{\mu\nu}$ given
in Eq.(3.6). Using the decomposed expression

\[ *\Gamma_{\mu\alpha}(Q) *\Gamma^\alpha_{\nu\beta}(Q, K, P) = -Q_T^2 \left\{ *\Gamma_{\mu\nu}(Q, K, P) + \frac{Q_\mu}{Q^2} \left[ K^2 L_n(\nu) - P^2 L_{\tilde{n}}(\nu) \right] \right\} \]

\[ + (Q_T^2 - Q_L^2) \frac{\tilde{n}_\mu(Q)}{\tilde{n}(Q)} \left\{ *\Gamma_{0\nu}(Q, K, P) + \frac{q_0}{Q^2} \left[ K^2 L_n(\mu) - P^2 L_{\tilde{n}}(\nu) \right] \right\} \] (5.6)

we find that the sum of \( I_{\mu\nu}^{AT} + I_{\mu\nu}^{(P)AT} \) is written as

\[ I_{\mu\nu}^{AT} + I_{\mu\nu}^{(P)AT} = T(P) \left\{ *\Gamma_{\mu\nu}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]

\[ - \left( 1 - \frac{Q_L^2}{Q_T^2} \right) \frac{1}{\tilde{n}(Q)} \left\{ \tilde{n}_\mu(Q) *\Gamma_{0\nu}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]

\[ + \left( 1 - \frac{Q_L^2}{Q_T^2} \right) \frac{q_0}{Q^2} \frac{P^2 L_n(\mu) \tilde{n}_\nu(P) + (\mu \leftrightarrow \nu)}{\tilde{n}(P)} \]

\[ - \frac{P_L^2}{Q^2} \left\{ Q_\mu \tilde{n}_\nu(P) + (\mu \leftrightarrow \nu) \right\} \] (5.7)

where we have used Eq.(2.44) for \( S(Q) \). The first line cancels against \( J_T^{\mu\nu} \), and the second and third lines cancel against the sum of \( I_{\mu\nu}^{ST} \) and \( I_{\mu\nu}^{(P)ST} \). Thus we have

\[ [I_{\mu\nu}^{AT} + I_{\mu\nu}^{(P)AT}] + [I_{\mu\nu}^{ST} + I_{\mu\nu}^{(P)ST}] + J_T^{\mu\nu} \]

\[ = -T(P) \frac{P_L^2}{Q^2} \left\{ Q_\mu \tilde{n}_\nu(P) + (\mu \leftrightarrow \nu) \right\} \]

\[ = - \frac{1}{P^2 Q^2} \frac{b(P)}{a(P)} \frac{b(P)}{a(P)} \left\{ Q_\mu \tilde{n}_\nu(P) + \tilde{n}_\mu(P) Q_\nu \right\} , \] (5.8)

which cancels against a term proportional to \( \frac{b(P)}{a(P)} \) in \( \Pi_{\mu\nu}^{gh}(K) \) given by Eq.(3.6). Therefore, the \( a- \) and \( b- \)dependence of the \( T \)-terms and \( \Pi_{\mu\nu}^{gh}(K) \) completely cancel out.

Summing up the remaining terms, we find for the gauge-independent resummed gluon self-energy,

\[ *\Pi_{\mu\nu}(K) = *\Pi_{\mu\nu}(K) + *\Pi_{\mu\nu}^P(K) \]

\[ = Ng^2 \int dP \left\{ [I_{\mu\nu}^{AA} + I_{\mu\nu}^{SS} + I_{\mu\nu}^{AS}] + [J_{\mu\nu}^A + J_{\mu\nu}^S] \right\} \]

\[ + [I_{\mu\nu}^{(P)AA} + I_{\mu\nu}^{(P)SS} + I_{\mu\nu}^{(P)AS}] + I_{\mu\nu}^{rest} \} \] (5.9)
with

\[ I_{\mu\nu}^{\text{rest}} = \frac{1}{P^2Q^2}P_\mu Q_\nu. \quad (5.10) \]

where \( I_{\mu\nu}^{AA} \), \( I_{\mu\nu}^{SS} \), \( I_{\mu\nu}^{AS} \), \( J_{\mu\nu}^A \), \( I_{\mu\nu}^{(P)AA} \), \( I_{\mu\nu}^{(P)SS} \), and \( I_{\mu\nu}^{(P)AS} \) are given in Eqs. (3.8), (3.10), (3.13), (3.22), (4.17), (4.19), and (4.22), respectively. Explicitly, \( \hat{\Pi}_{\mu\nu}(K) \) is written as

\[
\hat{\Pi}_{\mu\nu}(K) = Ng^2 \int dP \left\{ -A(P)A(Q) \left[ \frac{1}{2} \Gamma_\lambda^\mu\alpha(P,K,Q) \Gamma_\alpha^\nu(Q,K,P) + 2 \Gamma_{\mu\nu}(K) + \{ \Gamma_{\mu\alpha}(K) V^\alpha_\nu(Q,K) + (\mu \leftrightarrow \nu) \} \right] \\
- S(P)S(Q) \left[ \frac{1}{2} \Gamma_{0\mu\alpha}(P,K,Q) \Gamma_{0\alpha\nu}(Q,K,P) \\
+ \frac{p_0q_0}{P^2Q^2} \tilde{\eta}_\mu(Q) K^2_{\mu} \tilde{\eta}_\nu(K) + (\mu \leftrightarrow \nu) \right] \\
- A(Q)S(P) \left[ \frac{1}{2} \Gamma_{\mu\alpha\nu}(P,K,Q) \Gamma^{\mu\alpha\nu}(Q,K,P) \\
- \{ K^2_{\mu} \tilde{\eta}_\mu(K) \tilde{\eta}_\alpha(P) [g_{\alpha\nu} + V_{\alpha\nu}(Q,K)] + (\mu \leftrightarrow \nu) \} \right] \\
+ \frac{1}{2} A(P) \Gamma_{\mu\nu\alpha}(K,-K,P,-P) + \frac{1}{2} S(P) \Gamma_{\mu\nu\alpha}(K,-K,P,-P) \\
- S(P) \frac{p_0}{P^2Q^2} \left\{ Q_\mu \left[ K^2_{\mu} \tilde{\eta}_\nu(K) - P^2_{\mu} \tilde{\eta}_\nu(P) \right] + (\mu \leftrightarrow \nu) \right\} \\
+ \frac{1}{P^2Q^2}P_\mu Q_\nu \right\}. \quad (5.11) 
\]

Since \( \hat{\Pi}_{\mu\nu}(K) \) does not depend on \( a, b \) and \( \xi \), it is clear that \( \hat{\Pi}_{\mu\nu}(K) \) is gauge independent. Also it is emphasized that inclusion of pinch contributions, \( I_{\mu\nu}^{(P)AA} \), \( I_{\mu\nu}^{(P)SS} \), and \( I_{\mu\nu}^{(P)AS} \), to \( \hat{\Pi}_{\mu\nu}(K) \) is indispensable. Otherwise, \( \hat{\Pi}_{\mu\nu}(K) \) does not satisfy the transversality relation, which will be shown in the next subsection.
5.2 Transversality of $\hat{\Pi}_{\mu\nu}(K)$

The effective resummed gluon self-energy $\hat{\Pi}_{\mu\nu}(K)$ obtained in the above subsection is not only gauge-independent, but also it satisfies the transversality relation

$$K^\mu \hat{\Pi}_{\mu\nu}(K) = 0 \quad (5.12)$$

This can be shown by explicit calculation. By applying $K^\mu$ to the sum $(I^{AA}_{\mu\nu} + I^{SS}_{\mu\nu} + I^{AS}_{\mu\nu} + J^A_{\mu\nu} + J^S_{\mu\nu} + I^{rest}_{\mu\nu})$, we find

$$K^\mu [(I^{AA}_{\mu\nu} + I^{SS}_{\mu\nu} + I^{AS}_{\mu\nu}) + (J^A_{\mu\nu} + J^S_{\mu\nu}) + I^{rest}_{\mu\nu}]$$

$$= A(P) \frac{Q^\lambda}{Q^2} *\Gamma_{\lambda\nu}(K)$$

$$- S(P) \left\{ \frac{Q \cdot \tilde{n}(P)}{Q^2} K^2 L \tilde{n}_\nu(K) + \frac{p_0 q_0}{P^2 Q^2} P^\lambda *\Gamma_{\lambda\nu}(K) \right\} \quad (5.13)$$

where we have discarded terms which are odd functions in $P$.

On the other hand, using the identity Eq.(4.35) satisfied by the function $V^\alpha_{\mu}(Q, K)$, we find

$$K^\mu I^{(P)AA}_{\mu\nu} = A(P) \frac{Q^\lambda}{Q^2} *\Gamma_{\lambda\nu}(K)$$

$$- A(P) \frac{Q^\lambda}{Q^2} *\Gamma_{\lambda\nu}(K)$$

$$+ A(P) (1 - \frac{Q^2}{Q^2_T}) \frac{1}{\tilde{n}^2(Q) Q^2} \tilde{n}^\lambda(Q) *\Gamma_{\lambda\nu}(K). \quad (5.14)$$

The first term turns out to vanish due to the symmetry property of the integrand under the interchange of $P$ and $Q$, i.e.,

$$A(P) \frac{Q^\lambda}{Q^2_T} *\Gamma_{\lambda\mu}(K) \implies - \frac{1}{2} \frac{P^\lambda + Q^\lambda}{P^2 Q^2_T} *\Gamma_{\lambda\mu}(K)$$

$$= \frac{1}{2} \frac{1}{P^2 T^2} K^\lambda *\Gamma_{\mu\lambda}(K) = 0. \quad (5.15)$$

In a similar way we obtain

$$K^\mu I^{(P)SS}_{\mu\nu} = S(P) S(Q) \left\{ \frac{p_0}{P^2} \left[ Q^2 L \tilde{n}^2(Q) - P^2 L \tilde{n}(P) \cdot \tilde{n}(Q) \right] K^2 L \tilde{n}_\nu(K) \right.$$
\[ K^\mu I^{(P)}_{\mu\nu} = A(Q)S(P) \left\{ \left[ -\frac{Q^2}{Q^2} Q \cdot \bar{n}(P) + (Q_L^2 - Q^2) \frac{q_0}{Q^2} \frac{\bar{n}(P) \cdot \bar{n}(Q)}{\bar{n}(Q)^2} \right] K_L^2 \bar{n}_\nu(K) + \frac{p_0 q_0}{P^2 Q^2} \left[ P_\mu^2 \bar{n}^\lambda(P) - Q_L^2 \bar{n}^\lambda(Q) \right] *\Gamma_\lambda(K) \right\}, \]

(5.16)

Then the sum of Eqs. (5.14), (5.16) and (5.17) becomes

\[- A(P) \frac{Q^2}{Q^2} *\Gamma_\lambda(K) + S(P) \left\{ \frac{Q \cdot \bar{n}(P)}{Q^2} K_L^2 \bar{n}_\nu(K) + \frac{p_0 q_0}{P^2 Q^2} P^\lambda *\Gamma_\lambda(K) \right\}, \]

(5.18)

which is just an opposite of Eq. (5.13). Thus we reach the result of Eq. (5.12).

Since the effective resummed gluon self-energy \( *\hat{\Pi}_{\mu\nu}(K) \) satisfies the transversality relation, it can be decomposed as

\[ *\hat{\Pi}_{\mu\nu}(K) = *\hat{\Pi}_\perp(K) P_{\mu\nu}(K) + *\hat{\Pi}_\parallel(K) Q_{\mu\nu}(K) \]

(5.19)

where \( *\hat{\Pi}_\perp(K) \) and \( *\hat{\Pi}_\parallel(K) \) are refered to the transverse and longitudinal functions, respectively. Both functions can be extracted by applying the projection operators to \( *\hat{\Pi}_{\mu\nu}(K) \) as follows:

\[ *\hat{\Pi}_\perp(K) = \frac{1}{2} P_{\mu\nu}(K) *\hat{\Pi}_{\mu\nu}(K) \]

(5.20)

\[ *\hat{\Pi}_\parallel(K) = Q_{\mu\nu}(K) *\hat{\Pi}_{\mu\nu}(K). \]

(5.21)

To show the usefulness of the effective resummed gluon self-energy \( *\hat{\Pi}_{\mu\nu}(K) \) obtained here, we will calculate in the next section the damping rate for the transverse gluons in the leading order.

### 6 Gluon Damping Rate in the Leading Order

There had been much controversy over the gluon damping rates in hot QCD. “Naive” one-loop calculations for the damping rate of gluons showed that it is gauge dependent in both magnitude and sign. It was then realized \([17]\) that there are higher-loop diagrams which contribute to the same order in the coupling constant \( g \) as the
one-loop diagram. These higher-order effects were resummed into effective propagators and vertices in a systematic way. The complete calculation in the leading order was first made by Braaten and Pisarski \[13\]. As we already know, the resummed gluon self-energy $*\Pi^{\mu\nu}$ contains gauge-dependent and gauge-independent pieces. To isolate the gauge-independent pieces in $*\Pi^{\mu\nu}$ and to calculate the gluon damping rate, Braaten and Pisarski constructed the two-gluon $T$-matrix element by putting $*\Pi^{\mu\nu}$ on the mass-shell and sandwiching it between physical wave functions. The gluon damping rate came from the imaginary part of the $T$ and the actual calculation was made in the Coulomb gauge.

We now calculate the damping rate for transverse gluons at zero momentum in a different way from the one taken by Braaten and Pisarski \[13\]. In the previous section, we have constructed the gauge-independent resummed gluon self-energy $*\hat{\Pi}_{\mu\nu}(K)$ in the leading order. Since $*\hat{\Pi}_{\mu\nu}(K)$ satisfies the transversality relation, it is written in terms of the transverse function $*\hat{\Pi}_\perp(K)$ and the longitudinal function $*\hat{\Pi}_\parallel(K)$. Then, in the leading order in $g$, the damping rate $\gamma_t(0)$ for the transverse gluon at zero momentum is given by

$$\gamma_t(0) = \frac{1}{4m_g} \text{Disc}^* \hat{\Pi}_\perp(k_0 = m_g, k = 0).$$  \hspace{1cm} (6.1)

where $m_g = \frac{1}{3} \sqrt{NgT}$ is the gluon mass induced by the thermal medium.

In the limit $k \to 0$, it follows from $O(3)$ rotational invariance that $*\hat{\Pi}_{ij} = *\hat{\Pi}_\perp \delta_{ij}$ so that

$$*\hat{\Pi}_\perp(k_0, k = 0) = \frac{1}{3} *\hat{\Pi}_{ii}(k_0, k = 0)$$  \hspace{1cm} (6.2)

To evaluate $*\hat{\Pi}_{ii}(k_0, k = 0)$, we require the information on the effective vertices which appear in the sum

$$-\frac{1}{2} A(P)A(Q) *\Gamma_{\lambda i}^\alpha(P, K, Q) *\Gamma_{\alpha i}^\lambda(Q, K, P)$$

$$-\frac{1}{2} S(P)S(Q) *\Gamma_{0i0}(P, K, Q) *\Gamma_{0i0}(Q, K, P)$$

$$-A(Q)S(P) *\Gamma_{0i\alpha}(P, K, Q) *\Gamma_{\alpha i0}(Q, K, P)$$

$$= \frac{1}{2} \frac{1}{\mathbf{p}^2 \mathbf{q}^2} \left[ \frac{p_0^2}{P_T^2} - \frac{P^2}{P_L^2} \right] \left[ \frac{q_0^2}{Q_T^2} - \frac{Q^2}{Q_L^2} \right] *\Gamma_{00i}(P, Q, K) *\Gamma_{00i}(P, Q, K)$$

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Finally the pinch contributions $I_{ij}^{(P)AA}$, $I_{ij}^{(P)SS}$, and $I_{ij}^{(P)AS}$ do not contribute, because they vanish at $k_0 = m_g$ and $k = 0$.

Summing up each contribution, the discontinuity in the transverse function is written as

$$\text{Disc}^* \hat{\Pi}_\perp(k_0 = m_g, k = 0) = \frac{3}{2} g^2 N \text{Disc} \int dP \left[ \frac{1}{P_T^2 Q_T^2} \frac{2}{P^2} (p_0 q_0)^2 + \frac{1}{P_L^2 Q_L^2} \frac{Q^4}{P^2 m_g^2} \right]$$
\[
+ \frac{1}{P_L^2 Q_L^2} \frac{P^2 Q^2}{p^2 q^2 p^2} - \frac{P_L^2 P^2}{Q_T^2 p^2 9m_g^2} \]

(6.9)

Since \( P_L^2 = P^2 - \delta \Pi_L(P) \) and

\[
\delta \Pi_L(P) = \frac{3m_g^2 P^2}{p^2} \left[ \frac{1}{2} \frac{p_0 + p}{p_0 - p} - 1 \right],
\]

(6.10)

the fourth term in Eq.(6.9) is rewritten as

\[
\text{Disc} \int dP \left[ -\frac{P_L^2 P^2}{Q_T^2} \frac{1}{p^2 9m_g^2} \right] = \text{Disc} \int dP \left[ \frac{1}{Q_T^2} \frac{1}{3} \frac{1}{p^5} \left\{ \frac{1}{2} \frac{p_0 + p}{p_0 - p} \right\} \right]
\]

(6.11)

Thus we find that the expression of \( \text{Disc} \hat{\Pi}_\perp \) in Eq.(6.9) is equivalent to the one in Eq.(22) of Ref.[15].

It is once again emphasized that our approach for the calculation of the damping rate for the transverse gluons is quite different from the one taken by Braaten and Pisarski. We first constructed the gauge-independent resummed gluon self-energy, and then extracted the transverse function and evaluated the discontinuity in the transverse function at \( k_0 = m_g \) and \( k = 0 \). By construction, the gauge-independence of our result is manifest.

7 Summary and Discussion

In this paper we have applied the \( S \)-matrix PT to hot QCD and calculated the effective gluon self-energy at one-loop order. We have found that the effective gluon self-energy, which is the sum of the resummed gluon self-energy and the resummed pinch contributions, is not only gauge-independent but also satisfies the transversality relation. Using this gauge-independent resummed gluon self-energy, we have

\footnote{A factor \( \frac{1}{3} \) is missing in the last term, which is proportional to \( Q_0 \left( \frac{u_0}{k} \right) \), in Eq.(22) of Ref.[15]. It is only a typographical error and does not propagate to Eq.(25), and thus their conclusions are intact. I would like to thank Eric Braaten for providing me with this information.}
calculated the damping rate for transverse gluon in the leading order and have shown that the result coincides with the one obtained by Braaten and Pisarski.

It was once pointed out by Baier, Kunstatter and Schiff [26] that in covariant gauges (i.e., $a_{COV} = 1$, $b_{COV} = 0$) there appear mass-shell singularities in the terms proportional to the gauge-fixing parameter $\xi$. In fact, they showed explicitly that the imaginary part of the following integral

$$\text{Im} \int dP \frac{1}{2} D^{\mu \nu}\{I_{\mu \nu}^{BB} + I_{\mu \nu}^{AB} + I_{\mu \nu}^{BS} + J_{\mu \nu}^{B}\}_{a=1,b=0}$$

(7.1)

develops poles on the physical mass shell and gives gauge-dependent gluon damping rate, unless an infrared regulator is maintained throughout the calculation. Note that the gauge parameter $\xi$ appears only in the $B$-related terms. (This issue also appeared in the calculation of the quark damping rate in the leading order [27]). We now know that the $B$-related term in the resummed gluon propagator in covariant gauges also gives rise to pinch contributions, and that this subtlety on mass-shell singularities in the covariant gauges dissapears once the pinch contributions are added to the resummed gluon self-energy.

We considered the one-loop quark-quark scattering amplitude to extract the resummed pinch contributions. Then there may arise an argument on the process- dependence of our result for the gauge-independent resummed gluon self-energy. Now let us inspect closely the expressions of the resummed pinch contributions given in Eqs.(4.16)-(4.26). They are all made up of the terms which have at least one of the following factors:

$$^*\Gamma_{\mu \nu}(K), ~ ^*\Gamma_{\mu \alpha}(K), ~ ^*\Gamma_{\nu \alpha}(K), ~ K_{L}^{2}\tilde{n}_{\mu}(K), ~ K_{L}^{2}\tilde{n}_{\nu}(K).$$

(7.2)

Those factors have come out when we used the identity given in Eq.(4.3) and took away the resummed gluon propagator part, $^*D^{\mu \nu}(K)$ or $^*D^{\nu \beta}(K)$. This fact reminds us of an idea of the intrinsic PT [19]. Since the gauge-independent resummed gluon self-energy has been obtained by adding the pinch contributions to the one-loop resummed gluon self energy $^*\Pi_{\mu \nu}(K)$ given in Eq.(3.1), there should be such terms in $^*\Pi_{\mu \nu}(K)$ that cancel against the pinch contributions. Those terms necessarily possess at least one of those factors given in Eq.(7.2). Now the intrinsic PT algorithm tells us to start with three one-loop diagrams for the $^*\Pi_{\mu \nu}(K)$ depicted in
Fig. 2.(a)-(c), then to apply the Ward-Takahashi identities Eqs. (2.30)-(2.31) to the corrected three- or four-point vertices and to throw out the factors given in Eq. (7.2) which appeared as a result of the Ward-Takahashi identities. This intrinsic PT algorithm works fine for the case of the resummed gluon self-energy [28] and we can reach the same expression given in Eq. (5.11). Although Eq. (5.11) still contain terms with such factors as given in Eq. (7.2), these terms exactly cancel against the corresponding ones which come out of the products $\Gamma^\lambda_{\mu\alpha}(P, K, Q) \Gamma^{\alpha\lambda}_{\nu}(Q, K, P)$ and $\Gamma^0_{\mu\alpha}(P, K, Q) \Gamma^{\alpha\nu}_0(Q, K, P)$. The intrinsic PT do not resort to the consideration of any specific scattering process. And it succeeds to give the same expression that was obtained by the S-matrix PT in which we examined the one-loop quark-quark scattering. This supports implicitly the notion that our PT result on the resummed gluon self-energy is process independent.

Furthermore the intrinsic PT can be applied to obtain the resummed gluon three-point function $\Pi^\lambda_{\mu\nu}(P, Q, R)$. Then it can be shown that this effective three-point function $\Pi^\lambda_{\mu\nu}(P, Q, R)$ is gauge independent and also satisfies the tree-level Ward-Takahashi identity [28]

$$R^\lambda \Pi^\lambda_{\mu\nu}(P, Q, R) = \Pi^\lambda_{\mu\nu}(P) - \Pi^\lambda_{\mu\nu}(Q).$$

(7.3)

This means that the wave-function renormalization for the resummed gluon self-energy given in Eq. (5.11) contains the running of the QCD coupling and that, using its expression, we can obtain the correct thermal $\beta$ function in the leading order [23].

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A Pinch Contributions

(i) The contribution of the vertices of the first kind:

\[ *\Pi_{\mu\nu}^{P(V_1)}(K) = -Ng^2 *\Gamma_{\mu\nu}(K) \int dPB(P) \frac{1}{P^2} \quad \text{(A.1)} \]

(ii) The contribution of the vertices of the second kind:

\[ *\Pi_{\mu\nu}^{P(V_2)}(K) = Ng^2 \int dP \times \left\{ -A(P)A(Q) \left[ 2 *\Gamma_{\mu\nu}(K) + \left\{ *\Gamma_{\mu\alpha}(K) V^\alpha_{\nu}(P, K) + (\mu \leftrightarrow \nu) \right\} \right] \right. \]

\[ -B(P)B(Q) \left\{ \frac{1}{P^2 Q^2} P^\lambda P^\tau *\Gamma_{\lambda\mu}(K) *\Gamma_{\nu\tau}(K) \right\} \]

\[ + S(P)S(Q) \left\{ \frac{P_0}{P^2} \left\{ K^2_L \bar{n}_{\mu}(K) \bar{n}^\alpha(Q) *\Gamma_{\alpha\nu}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \right\} \]

\[ - \frac{1}{2} P^\lambda \frac{P_0 Q_0}{P^2 Q^2} \left\{ *\Gamma_{\lambda\mu}(K) *\Gamma_{0,\nu}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]

\[ + T(P)T(Q) \left\{ \left[ K^2_L \bar{n}_{\mu}(K) [K^2_L \bar{n}_{\nu}(K) - P^2_L \bar{n}_{\nu}(P)] + (\mu \leftrightarrow \nu) \right] \right. \]

\[ + \left\{ -\frac{1}{2} P^\lambda *\Gamma_{\lambda\mu}(K) *\Gamma_{0,\nu}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]

\[ - A(Q)B(P) \left\{ *\Gamma_{\mu\alpha}(K) \left[ *\Gamma_{\alpha\nu}(K) - *\Gamma_{\alpha\nu}(Q) \right] + (\mu \leftrightarrow \nu) \right\} \]

\[ + A(Q)S(P) \left\{ K^2_L \bar{n}_{\mu}(K) \bar{n}^\alpha(P) \left[ g_{\alpha\nu} + V_{\alpha\nu}(Q, K) \right] + (\mu \leftrightarrow \nu) \right\} \]

\[ - \left\{ \frac{P_0}{P^2} \left\{ *\Gamma_{\mu\alpha}(K) *\Gamma_{\alpha\nu}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \right\} \]

\[ + A(Q)T(P) \left\{ *\Gamma_{\mu\alpha}(K) *\Gamma_{\alpha\nu}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} \]

\[ + \left\{ \left[ (-Q^2 + Q^2_L) \frac{q_0}{Q^2} \bar{n}_{\mu}(Q) + Q^2 \frac{Q_0}{Q^2} K^2_L \bar{n}_{\nu}(K) \right] (\mu \leftrightarrow \nu) \right\} \]

\[ - B(P)S(Q) \left\{ \left[ K^2_L \bar{n}_{\mu}(K) + \frac{q_0}{Q^2} Q^\lambda \Gamma_{\lambda\mu}(K) \right] \left[ K^2_L \bar{n}_{\nu}(K) - Q^2_L \bar{n}_{\nu}(Q) \right] + (\mu \leftrightarrow \nu) \right\} \]

\[ + \left\{ \frac{q_0}{Q^2} \left[ K^2_L \bar{n}_{\mu}(K) Q^\lambda \Gamma_{\lambda\nu}(K) + (\mu \leftrightarrow \nu) \right] \right\} \]
\[ + B(P)T(Q) \frac{1}{P^2} \left[ \left\{ K_L^2 \tilde{n}_\mu(K) Q^\lambda * \Gamma_{\lambda \nu}(K) + (\mu \leftrightarrow \nu) \right\} + \left\{ Q^\lambda * \Gamma_{\lambda \mu}(K) \left[ K_L^2 \tilde{n}_\nu(K) - Q^2 L_{\tilde{n}_\nu}(Q) \right] + (\mu \leftrightarrow \nu) \right\} \right] - S(Q)T(P) \left[ \left\{ \left[ K_L^2 \tilde{n}_\mu(K) + \frac{q_0}{Q^2} Q^\lambda \Gamma_{\lambda \mu}(K) \right] * \Gamma_{0 \alpha 0}(Q, K, P) + (\mu \leftrightarrow \nu) \right\} + \frac{q_0}{Q^2} \left\{ K_L^2 \tilde{n}_\mu(K) \left[ 2 K_L^2 \tilde{n}_\nu(K) - P^2 L_{\tilde{n}_\nu}(P) - Q^2 L_{\tilde{n}_\nu}(Q) \right] + (\mu \leftrightarrow \nu) \right\} \right] \]  

\[(A.2)\]

(iii) The box contribution:

\[
* \Pi_{\mu \nu}^{(Box)}(K) = N g^2 \int dP \times \left[ \frac{1}{2} B(P)B(Q) \frac{1}{P^2 Q^2} P^\lambda P^\tau * \Gamma_{\mu \lambda}(K) * \Gamma_{\nu \tau}(K) - S(P)S(Q) \frac{q_0 q_0}{P^2 Q^2} K^4 L_{\tilde{n}_\mu}(K) \tilde{n}_\nu(K) - T(P)T(Q)K^4 L_{\tilde{n}_\mu}(K) \tilde{n}_\nu(K) + A(Q)B(P) \frac{1}{P^2} * \Gamma_{\mu \alpha}(K) * \Gamma^\alpha_{\nu}(K) + B(P)S(Q) \frac{1}{P^2} \left[ K^4 L_{\tilde{n}_\mu}(K) \tilde{n}_\nu(K) - \frac{q_0}{Q^2} \left\{ P^\lambda * \Gamma_{\lambda \mu}(K) K^2 L_{\tilde{n}_\nu}(K) + (\mu \leftrightarrow \nu) \right\} \right] + B(P)T(Q) \frac{1}{P^2} \left\{ P^\lambda * \Gamma_{\lambda \mu}(K) K^2 L_{\tilde{n}_\nu}(K) + (\mu \leftrightarrow \nu) \right\} + S(Q)T(P) \frac{2q_0}{Q^2} K^4 L_{\tilde{n}_\mu}(K) \tilde{n}_\nu(K) \right] \]  

\[(A.3)\]

**B Hard thermal loop for three-gluon vertex**

The hard thermal loop for the three-gluon amplitude is expressed as [30]

\[
\delta \Gamma_{\mu \nu \lambda}(P, Q, K) = - \left[ W_{\mu \nu \lambda}(P, K) - W_{\mu \nu \lambda}(Q, K) \right] . \]  

\[(B.1)\]
The function \( W_{\mu\nu\lambda}(P, K) \) is given by
\[
W_{\mu\nu\lambda}(P, K) = \frac{3m_g^2}{4\pi} \int d\Omega \frac{U_\mu U_\nu U_\lambda}{[P \cdot U][K \cdot U]} [p_0] \tag{B.2}
\]
where \( m_g^2 = \frac{1}{9}Ng^2T^2 \) is the gluon mass induced by the thermal medium and \( \hat{U} = (1, \hat{U}) \) and \( \hat{U} \) is a three-dimensional unit vector. It is noted that \( W_{\mu\nu\lambda}(P, K) \) is totally symmetric in indices \( \mu, \nu, \) and \( \lambda \) and satisfies the following identities:
\[
g^{\mu\nu}W_{\mu\nu\lambda}(P, K) = 0 \tag{B.3}
\]
\[
K^\lambda W_{\mu\nu\lambda}(P, K) = -\delta \Pi_{\mu\nu}(P) + 3m_g^2n_\mu n_\nu \tag{B.4}
\]
\[
P^\lambda W_{\mu\nu\lambda}(P, K) = \frac{p_0}{k_0}[-\delta \Pi_{\mu\nu}(K) + 3m_g^2n_\mu n_\nu] \tag{B.5}
\]
where
\[
\delta \Pi_{\mu\nu}(K) = \frac{3m_g^2}{4\pi} \int d\Omega \frac{k_0}{[K \cdot U]} + 3m_g^2n_\mu n_\nu
= \delta \Pi_T(K)P_{\mu\nu}(K) + \delta \Pi_L(K)Q_{\mu\nu}(K) \tag{B.6}
\]
with [31]
\[
\delta \Pi_T(K) = 3m_g^2 \left\{ -\frac{1}{4k} \left[ \left( \frac{k_0}{k} \right)^2 - 1 \right] \ln \frac{k_0 + k}{k_0 - k} + \frac{1}{2} \left( \frac{k_0}{k} \right)^2 \right\} \tag{B.7}
\]
\[
\delta \Pi_L(K) = 3m_g^2K^2 \left\{ \frac{1}{2k} \ln \frac{k_0 + k}{k_0 - k} - 1 \right\}
= -2\delta \Pi_T(K) + 3m_g^2. \tag{B.8}
\]
and \( k = |k| \).

The function \( W_{\mu\nu\lambda}(P, K) \) may be expanded, in general, as
\[
W_{\mu\nu\lambda}(P, K) = \alpha[n_\mu n_\nu n_\lambda] \\
+ \beta[n_\mu n_\nu P_\lambda + n_\mu n_\lambda P_\nu + n_\lambda n_\mu P_\nu] + \beta'[n_\mu n_\nu K_\lambda + n_\nu n_\lambda K_\mu + n_\lambda n_\mu K_\nu] \\
+ \delta[n_\mu P_\nu P_\lambda + n_\nu P_\lambda P_\mu + n_\lambda P_\mu P_\nu] + \delta'[n_\mu K_\nu K_\lambda + n_\nu K_\lambda K_\mu + n_\lambda K_\mu K_\nu] \\
+ \zeta[n_\mu P_\nu K_\lambda + n_\mu K_\nu P_\lambda + n_\nu P_\lambda K_\mu + n_\nu K_\lambda P_\mu + n_\lambda P_\mu K_\nu + n_\lambda K_\mu P_\nu] \\
+ \eta[P_\mu P_\nu P_\lambda] + \theta[K_\mu K_\nu K_\lambda] \\
+ \kappa[P_\mu P_\nu K_\lambda + P_\nu P_\lambda K_\mu + P_\lambda P_\mu K_\nu] + \xi[P_\mu K_\nu K_\lambda + P_\nu K_\lambda K_\mu + P_\lambda K_\mu K_\nu] \\
+ \psi[g_{\mu\nu}n_\lambda + g_{\nu\lambda}n_\mu + g_{\lambda\mu}n_\nu] \\
+ \chi[g_{\mu\nu}P_\lambda + g_{\nu\lambda}P_\mu + g_{\lambda\mu}P_\nu] + \omega[g_{\mu\nu}K_\lambda + g_{\nu\lambda}K_\mu + g_{\lambda\mu}K_\nu]. \tag{B.9}
\]
Picking up the terms which are proportional to $P_\mu$ from $W_{\mu\nu\lambda}(P, K)$, we find

$$W^{\mu\nu\lambda}(P, K) = P_\mu V_{\nu\lambda}(P, K) + \cdots$$  \hspace{1cm} (B.10)

with

\begin{align*}
V_{\nu\lambda}(P, K) &= \beta(P, K)[n_\nu n_\lambda] + \delta(P, K)[n_\nu P_\lambda + P_\nu n_\lambda] \\
&+ \zeta(P, K)[n_\nu K_\lambda + K_\nu n_\lambda] + \eta(P, K)[P_\mu P_\lambda] + \kappa(P, K)[P_\nu K_\lambda + K_\nu P_\lambda] \\
&+ \xi(P, K)[K_\nu K_\lambda] + \chi(P, K) g_{\nu\lambda}. \quad (B.11)
\end{align*}

Obviously the function $V_{\nu\lambda}(P, K)$ is symmetric in indices $\nu$ and $\lambda$.

The identities (B.3)-(B.5) provide many relations satisfied by the functions $\beta$, $\delta$, $\cdots$ in $V_{\nu\lambda}(P, K)$. In particular, (B.4) gives

\begin{equation}
K^\nu V_{\nu\lambda}(P, K) = \delta \Pi_T(P) \left[ \frac{P_\nu}{P^2} + \frac{P_0}{p^2} \tilde{n}_\nu(P) \right] - \delta \Pi_L(P) \frac{P_0}{p^2} \tilde{n}_\nu(P), \quad (B.12)
\end{equation}

which leads to the following relations:

\begin{align*}
k_0 \beta + (K \cdot P) \delta + K^2 \zeta &= [\delta \Pi_T(P) - \delta \Pi_L(P)] \frac{P_0}{p^2} \quad (B.13) \\
k_0 \delta + (K \cdot P) \eta + K^2 \kappa &= -\delta \Pi_T(P) \frac{1}{p^2} + \delta \Pi_L(P) \frac{P_0}{p^2} \frac{1}{P^2} \quad (B.14) \\
k_0 \zeta + (K \cdot P) \kappa + K^2 \xi + \chi &= 0. \quad (B.15)
\end{align*}

It is a formidable task to find out, in general, the expressions of the functions $\beta$, $\delta$, $\zeta$, $\eta$, $\kappa$, $\xi$ and $\chi$ in $V_{\nu\lambda}(P, K)$. However, when we restrict ourselves to the special cases such as $k = 0$ or $k_0 = 0$, it is not so hard to obtain the relevant expressions.

\section*{C Effective three- and four-gluon vertices at $k = 0$ and $k_0 = m_g$}

Evaluating Eq. (B.2) at $k = 0$, we find

\begin{equation}
W_{000}(P, K) \bigg|_{k=0} = \frac{P_0 P_0}{k_0 P^2} \left[ \frac{2}{3} \frac{p^2}{P^2} X(P) + m_g^2 \right]. \quad (C.1)
\end{equation}
\[ W_{0ji}(P,K) \bigg|_{k=0} = \frac{p_j p_i}{k_0 p^2} \left[ \left( 1 + \frac{2}{3} \frac{p^2}{P^2} \right) X(P) + m_g^2 \right] + \frac{\delta_{ji}}{k_0} \left[ -\frac{1}{3} \frac{p^2}{P^2} X(P) + m_g^2 \right] \] (C.2)

\[ W_{iji}(P,K) \bigg|_{k=0} = \frac{p_j p_i p_0}{k_0 p^2 P^2} \left[ \left( \frac{5}{3} + \frac{2}{3} \frac{p^2}{P^2} \right) X(P) + m_g^2 \right] - \left[ p_i \delta_{ji} + p_j \delta_{il} + p_l \delta_{ij} \right] \frac{p_0}{3k_0 P^2} X(P) , \] (C.3)

where

\[ X(P) = \frac{p^2}{p^2} \left\{ \delta \Pi_L(P) - \delta \Pi_T(P) \right\} . \] (C.4)

With these informations we obtain

\[ \Gamma_{00i}(P,Q,K) \Gamma_{00i}(P,Q,K) \bigg|_{k=0,k_0=m_g} = \frac{q^2}{m_g^2} \left\{ 3m_g^2 \left[ \frac{p_0}{P^2} + \frac{q_0}{Q^2} \right] + 2 \left[ \frac{p_0}{P^2} \frac{P^2}{P_T^2} + \frac{q_0}{Q^2} \frac{Q^2}{Q_T^2} \right] \right\}^2 \] (C.5)

\[ = \frac{q^2}{m_g^2} \left\{ 3 \left[ p_0 + \frac{q_0}{Q^2} m_g^2 \right] - \frac{p_0}{P^2} \frac{P^2}{P_L^2} + 2 \frac{q_0}{Q^2} \frac{Q^2}{Q_L^2} \right\}^2 \] (C.6)

\[ = q^2 \left\{ 3 + \frac{1}{k_0} \left[ \frac{p_0}{P^2} P^2_L + \frac{q_0}{Q^2} Q^2_L \right] \right\}^2 , \] (C.7)

\[ \Gamma_{0ji}(P,Q,K) \Gamma_{0ji}(P,Q,K) \bigg|_{k=0,k_0=m_g} = 2 \left\{ -3q_0 + \frac{1}{k_0} \left( P_T^2 - Q_T^2 \right) \right\}^2 \]

\[ + \left\{ 3p_0 + 3(p_0 - q_0) m_g^2 \frac{p^2}{P^2 Q^2} + 2 \frac{p_0}{k_0} \left[ \frac{p_0}{P^2} P_T^2 - \frac{q_0}{Q^2} Q_T^2 \right] \right\}^2 \] (C.8)

\[ = \frac{1}{2m_g^2} \left\{ 3Q^2 - (P^2_T + 2Q^2_T) \right\}^2 \]

\[ + \frac{1}{m_g^2} \left\{ 3 \left[ \frac{q_0}{Q^2} m_g^2 + p_0 q_0 \right] + \left[ \frac{p_0}{P^2} P^2_L + 2 \frac{q_0}{Q^2} Q^2_L \right] \right\}^2 , \] (C.9)

\[ \Gamma_{iji}(P,Q,K) \Gamma_{iji}(P,Q,K) \bigg|_{k=0,k_0=m_g} \]
\[
\frac{1}{p^2} \left\{ \frac{2}{k_0} \left[ \frac{p_0^3}{P^2} + \frac{q_0^3}{Q^2} \right] - 6p_0q_0 + 3\frac{m_g^2p^2}{P^2Q^2}(p^2 - p_0q_0) \right\}^2 + \frac{6}{p^2} \left\{ \frac{1}{k_0} \left( p_0P_T^2 + q_0Q_T^2 \right) - 3p_0q_0 - p^2 \right\} + 12p^2.
\] (C.10)

Since the required hard thermal loop for the four-gluon vertex is given by
\[
\delta \Gamma_{ii00}(K, -K, P, -P) \bigg|_{k=0, k_0=m_g} = -3 \left[ \frac{p_0}{p} \ln \frac{p_0 + p}{p_0 - p} - \frac{q_0}{q} \ln \frac{q_0 + q}{q_0 - q} \right],
\] (C.11)

we obtain
\[
* \Gamma_{ii00}(K, -K, P, -P) \bigg|_{k=0, k_0=m_g} = -6 - 6p^2 \left[ \frac{1}{P^2} - \frac{1}{Q^2} \right] - \frac{4p^2}{m_g^2} \left[ \frac{P_T^2}{P^2} - \frac{Q_T^2}{Q^2} \right].
\] (C.12)
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**Figure Caption**

Fig.1
The ghost-gluon vertex in linear gauges which preserve rotational invariance. The wavy and dashed lines denote gluon and ghost fields, respectively.

Fig.2
(a) The resummed gluon self-energy diagram with three-gluon interactions.
(b) The tadpole diagram for the resummed gluon self-energy.
(c) The ghost diagram for the resummed gluon self-energy.
The blobs in the gluon propagators and vertices represent that they are effective quantities with the hard thermal loop corrections included.

Fig.3
The gluon self-energy diagram for the quark-quark scattering. The solid lines represent quark fields.

Fig.4
(a) The vertex diagrams of the first kind for the quark-quark scattering.
(b) Their pinch contribution.

Fig.5
(a) The vertex diagram of the second kind for the quark-quark scattering. (b) Its pinch contribution.

Fig.6
(a) The box diagrams for the quark-quark scattering. (b) Their pinch contribution.
\[ g f^{abc} \Gamma_\mu(P) = g f^{abc} \left[ a(P) P_\mu + b(P) \bar{n}_\mu(P) \right] \]

Figure 1
Figure 2
Figure 4

Figure 5

Figure 6