Generation of high-frequency radiation in semiconductor superlattices with suppressed space-charge instabilities

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We theoretically investigated the scheme allowing to avoid destructive space-charge instabilities and to obtain a strong gain at microwave and THz frequencies in semiconductor superlattice devices. Superlattice is subjected to a microwave field and a generation is achieved at some odd harmonics of the pump frequency. Gain arises because of parametric amplification seeded by harmonic generation. Negative differential conductance (NDC) is not a necessary condition for the generation. For the mode of operation with NDC, a limited space-charge accumulation does not sufficiently reduce the gain.

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There exists a strong demand for miniature, solid-state, room temperature operating sources and detectors of THz radiation (0.2-10 THz). The need is caused by a rapid progress of THz sciences and technologies ranging from the astronomy to the biosecurity. Semiconductor superlattices (SLs) operating in the miniband transport regime, are interesting electronic devises demonstrating properties of both nonlinear and active media. Nonlinearity of voltage-current (UI) characteristic of SL gives rise to a generation of harmonics of microwave and THz radiation: for a high electron mobility and a high doping, even such strongly nonlinear phenomena as chaos and symmetry breaking have been predicted. On the other hand, Bloch oscillations of electrons within a miniband of SL cause an appearance of negative differential conductance (NDC) for dc fields (voltages) larger than the critical Esaki-Tsu $E_c$. Remarkably, the static NDC is accompanied by a small-signal gain for ac fields of very broad frequency range from zero up to several THz. The feasible device, employing such active media properties of SL, is known as the THz Bloch oscillator. However, an existence of gain at low frequencies causes space-charge instabilities and a formation of high-field domains inside SL. These domains are believed to be destructive for the Bloch gain. Recent experiments demonstrate that even a use of sophisticated design of SL device or a development of delicate spectroscopy techniques enable to observe only a very weak Bloch gain in dc-biased SLs.

In these respects important questions arise: Is it possible to obtain a gain for a high-frequency field employing only a nonlinearity of UI-characteristic of SL? Can we avoid formation of domains in the schemes based on nonlinear mechanisms of gain?

In the present Letter, we examine a feasibility of high-frequency superlattice oscillator supporting the regimes either with strongly suppressed or even without space-charge instabilities. We analyze the situation when a pure ac pump field of microwave frequency range is applied to SL. Generation arises at third, seventh etc. harmonics of the pump. In the weak-probe field limit, the mechanism for a gain consists in a combination of parametric amplification of the probe seeded by a frequency multiplication of the pump, as well as of nonparametric absorption. Remarkably, the oscillator can operate even if the amplitude of pump field does not reach $E_c$ preventing an instability of charge distributions inside SL. Thus, we demonstrate the possibility to obtain a gain for a high-frequency field employing only nonlinear characteristics of SL without using its active medium properties. Efficiency of the oscillator increases if SL switches partly to NDC. For the regime of operation with NDC, we clearly show that a space-charge accumulation can only limit but not destroy the gain. This mechanism of space-charge instability stabilization is rather general and applicable to other SL devices, like the Bloch oscillator.

Part of the presented here results has been reported at the conferences.

The paper is organized as follows. (1) Analytic consideration of a small-signal gain; (2) 1D and 3D numerical study of a large-signal gain; (3) Effect of a finite $Q$ of resonator on the oscillator efficiency; (4) Analytic analysis of space-charge wave stability in the linear approximation and within the framework of drift-diffusion model; (5) Numerical analysis of nonlinear stage of the instability within the drift-diffusion model; (6) Direct comparison of the gain found in spatially-homogeneous approximation with the gain computed with an account of space-charge effects: Calculations using both the 1D drift-diffusion model and the 3D ensemble Monte Carlo method; (7) Effects arising beyond the quasistatic approximation; (8) Brief historical review. Some details of the calculations are presented in the separate appendices.

We consider the response of miniband electrons to the action of ac field $E(t) = E_p + E_{pr}$, where $E_p = E_{p1}$ $\cos \omega t$ is the strong pump and the probe, $E_{pr} = E_n \cos \omega_n t$, is the $n$th odd harmonic of the pump, $\omega_n = n \omega$ ($n = 3, 5, 7, \ldots$). SL is placed in a cavity providing a feedback only for the field with frequency $\omega_n$. In the quasistatic limit $\omega \tau < 1$, $\omega_n \tau < 1$, the dependence of...
the drift electron velocity \( V \) on the electric field can be well described by the Esaki-Tsu formula \( V(E) = 2V_p(E/E_c)[1 + (E/E_c)^2] \), where \( E_c = \hbar/e\tau \) is the critical field (\( \tau \) is the SL period), and \( V_p \) is the peak electron velocity. We suppose an operation at room temperature. The absorption of the probe field in SL is defined in scaled units as \( A = \langle v(t) \cos(\omega_t t) \rangle \), where \( v = V(E)/V_p \) and averaging \( \langle . . . \rangle \) is performed over the period \( T = 2\pi/\omega \).

First, we are interested in a small-signal gain in SL \( (E_p \ll E_c) \). Expanding \( v(E) \approx v(E_p) + v'(E_p) \times E \), we present the total absorption \( A \) as the sum of \( A_h = \langle v(E_p) \cos(\omega_t t) \rangle \), \( A_{coh} = \langle v'(E_p) \cos(2\omega t) \rangle E_n/2 \), and \( A_{inc} = \langle v'(E_p) \rangle E_n/2 \) terms, which can be called the harmonic, the coherent and the incoherent absorption components, correspondingly (for details see Appendix A).

We see that while \( A_{coh} \) and \( A_{inc} \) are dependent on both \( E_p \) and \( E_n \), the term \( A_h \) is a function of only pump strength \( E_p \), and therefore \( A_h \) gives the main contribution to the absorption of a weak probe. Substituting the Esaki-Tsu dependence in the definition of \( A_h \), we find \( A_h = (-1)^k[2((b - 1)^{2k+1})/(b E_p^{2k+1})] \), where \( b = (1 + E_p^2)^{1/2} \), \( E_p \equiv E_p/E_c \) and \( 2k + 1 = n \) \( (k = 1, 2, 3, \ldots) \). This equation describes odd harmonics of the current \( (\text{cf Eq. 17 in Appendix A}) \); in the limit of weak pump \( E_p \ll 1 \) it takes familiar form \( E_n^n \). Importantly, \( A_h \) is negative for the odd values of \( k \). Therefore, generated harmonics with \( n = 3, 7, \ldots \) can provide seeding gain for an amplification of a probe field. It has no threshold in the amplitude of pump \( E_p \). Therefore, if the pump amplitude is less than the critical field, \( E_p/E_c < 1 \), the gain at harmonics will not be accompanied by the space-charge instabilities in SL.

We turn to the analysis of the term \( A_{coh} \) that describes a parametric amplification of the probe field due to a coherent interaction of the pump and the probe fields in SL. Numerical calculation of the integral demonstrates that \( A_{coh} \) is always negative for odd \( n = 3, 5, 7 \). In the limit of weak pump \( E_p \ll 1 \), we find \( A_{coh} \propto E_p^{2n} E_n \). Now we can describe the amplification of a weak signal at \( 3\omega \) under the action of a weak pump. Third harmonic of a weak pump is generated at cubic nonlinearity of SL characteristic, then it can get a seeding gain at the same nonlinearity \( A_h \propto E_p^3 \), but the next step of parametric gain, \( A_{coh} \), uses already the seventh order \( (\propto E^7) \) nonlinearity in the \( V(E) \)-dependence. The 5th harmonic also can be generated in SL. However, in contrast to the case of the 3d harmonic, it cannot be further amplified \( (A_h > 0) \) and field in the cavity, tuned to \( 5\omega \), eventually evolves to zero.

Finally, nonparametric effects are described by the term \( A_{inc} = E_p/(1 + E_p^2)^{3/2} \). We should notice that this is the only term which can exist in the expression for total absorption of SL in the case \( \omega_p/\omega \) is not an integer. The absorption \( A_{inc} \) is always positive.

We will see that the absorption due to the incoherent interactions of fields plays an important role in the stabilization of space-charge instability in SL.

![Fig. 1](image-url)  
**Fig. 1:** The dependence of absorption at \( 3\omega \) on the pump amplitude \( E_p \) for the different relative probes, \( \eta \). Inset: Maximal relative amplitude of the probe possessing gain, \( \eta_0 \), as a function of the pump \( E_p \). Calculation with Esaki-Tsu characteristic (solid) and Monte Carlo technique (symbols).

To find the efficiency of oscillator we need to consider the effects of both a large-signal gain and a finite Q-factor of resonator. The stationary value of the cavity field, \( \bar{E}_{st} \), is determined by the balance of gain and loss \( (\text{cf. Eq. 17 in Appendix A}) \) as \( \bar{E}_{st} = -A(E_{st}, \bar{E}_{st}) \), where \( Q = (\omega_{pl}\tau/\omega_0)Q_p \), \( E_{st} = E_{st}'/E_c \), and \( \eta_0 = (4\pi e^2 N/\epsilon m_0)^{1/2} \) is the miniband plasma frequency \( N \) is the doping density, \( m_0 \) is the electron mass at the bottom of miniband and \( \epsilon \) is the averaged dielectric constant of SL), (for details see Appendix B). We found a large-signal absorption \( A(E_p, E_n) \) using two methods: (i) simple 1D calculations with Esaki-Tsu characteristic and (ii) 3D single-particle Monte Carlo computations with an account of electron scattering at optical and acoustic phonons. For the Monte Carlo computations we consider GaAs/AlAs SL of the period \( a = 6.22 \text{ nm} \) and the miniband width \( \Delta = 24.4 \text{ meV} \). Static U1-characteristic of this SL can be well described by the Esaki-Tsu formula with \( E_c = 4.89 \text{ kV/cm} \), \( V_p = 1.44 \times 10^6 \text{ cm/s} \) and \( \tau = 220 \text{ fs} \). We took the frequency of pump field \( \omega = 100 \text{ GHz} \). These are our default parameters for all computations.

We start with the case of an ideal cavity \( (Q \to \infty) \). The dependence \( A(E_p) \) for \( 3\omega \)-generation and for the different values of relative probe amplitude, \( \eta = E_p/E_p \), are shown in Fig. 1. Results of simple 1D theory and 3D Monte Carlo simulations are in a good agreement. For \( \eta = 0 \) and for small \( \eta \ll 1 \), the dependence of \( A(\bar{E}_p) \) follows to the corresponding dependencies for the seeding gain, \( A_h(\bar{E}_p) \), and the small-signal gain. With a further
increase of \( \eta \), the gain decreases and finally the absorption becomes zero for some \( \eta_0 \): \( A(E_\omega, \eta_0) = 0 \). The value \( \eta_0^2 = [E_{\omega n}^2/E_\omega]^2 \) determines the maximal SL oscillator efficiency for the given pump. Inset in Fig. 1 shows the dependence of \( \eta_0 \) on the pump amplitude. If the amplitude of pump does not reach NDC, \( E_\omega < E_c \), the maximal oscillator efficiency is less than 5%. For the oscillations in NDC regime, \( \eta_0^2 \) reaches 23% for \( E_\omega/E_c \approx 4 \). The maximal efficiency for \( \tau_\omega \)-oscillations is 5% (\( E_\omega/E_c \approx 5 \)). We also evaluated the oscillator efficiency in the case of finite \( Q \), Fig. 2 represents the dependencies of \( \eta_0 = E_{\omega n}^2/E_\omega \) on \( Q \) for the doping densities \( N = 10^{15} \) and \( 10^{16} \) cm\(^{-3} \). We see that for a heavy doped SL the efficiency of generation can reach the maximal efficiency (\( \approx 23\% \)) even in a cavity with \( Q < 100 \) for both operational regimes: with NDC and without NDC (cf Figs. 2 and 1). We should also note that oscillator’s start up also does not require a high-\( Q \) resonator because of large small-signal gain in SL. Although the oscillator can operate even without NDC, the maximum of efficiency within spatially-homogeneous approximation is reached for \( E_\omega/E_c > 1 \). Therefore we turn to the consideration of an evolution of space-charge instabilities in SL. For \( \omega \tau < 1 \) a space-time evolution of the electron density \( \rho(x,t) \), the current density \( j(x,t) \) and the field \( E(x,t) \) inside a SL of the total length \( L \), driven by the given voltage \( U_p(t) = U_c \cos \omega t \) (\( U_c = E_c L \)), can be well described by the drift-diffusion (DD) model 13. Set of equations consists of the current equation \( j = e\rho V(E) - D(E)\partial \rho/\partial x \), the Poisson equation \( \partial E/\partial x = 4\pi e^{-1}(\rho - N) \), the relation to the applied voltage \( U(t) = \int_0^L E(x,t)dx \), and the continuity equation \( \partial \rho/\partial t + \partial j/\partial x = 0 \) 14. Following the Einstein relation the diffusion coefficient is \( D = k_B T V/E \). Linearizing these equations, we find that small fluctuations of space-charge with long wavelength will grow, if the dielectric relaxation increment, \( \omega_d \int_0^t [\partial V(E_p(t))/\partial E]dt \) (with \( \omega_d = \omega_{pl}^2 \tau_d \), is negative. Taking the integral over the period of pump, \( t = T \), we see that the increment is proportional to \( A_{inc} \), which is always positive for all \( n \) and \( E_\omega \). Therefore the system is stable against small fluctuations of charge or field. That is the limited-space-charge accumulation (LSA) mode of SL oscillator operation. However, in contrast to the traditional LSA mode in Gunn diodes driven by dc and ac voltages 15, LSA in SL does not require a large amplitude of ac field. We also considered the influence of voltage harmonics calculating numerically the increment for \( U(t) = U_p(t) + U_n \cos \omega_n t \). For the practically important range \( U_n < U_n^\ast \) we found that LSA still works. Next, solving numerically DD-model, we consider dynamics of large fluctuations. We find that even large fluctuations of charge-density are resolved during the time of dielectric relaxation \( \propto \omega_d^{-1} \). Therefore this type of LSA in SL works even in the nonlinear regime. Finally, we directly calculated the influence of inhomogeneous distributions of space-charge on a gain in SL. We determined the absorption at the nth harmonic of voltage as \( A_n = \langle \langle (j(x,t)/j_0) \cos \omega_n t \rangle \rangle \), where \( j_0 = eV_p N \) and averaging \( \langle \langle \ldots \rangle \rangle \) is performed both over the period \( T \) and the length \( L \). In the computations a formation of domains was caused by a Gaussian spread in the value of \( E_c \) (or \( \tau \)). Domains were periodically created during the part of period \( T \) when the SL was switched to the state with NDC, and then they were annihilated during another part of \( T \). We computed the relative decrease in gain, \( \delta = (A - A_0)/A \), for different \( \omega \) and \( U_n \). Dependence of \( \delta \) on \( U_n/U_c \), for a long SL of 130 periods and for \( \omega/\omega_d = 0.1 \), is shown in Fig. 3. Reduction of gain due to LSA is 2% for optimal \( U_n/U_c = 4 \) and it is less than 8% overall. For our default parameters with \( \omega = 100 \) GHz, the value \( \omega/\omega_d = 0.1 \) corresponds to the doping \( N = 2 \times 10^{16} \) cm\(^{-3} \). For the same SL parameters but for the pump frequency of tens of GHz, the condition \( \omega/\omega_d = 0.1 \) is satisfied for \( N \approx 10^{15} \) cm\(^{-3} \). As is evident from Fig. 2 (lower subplot), resonators with a moderate \( Q \) still can provide a reasonable efficiency of generation in NDC-regime for SLs with the electron densities \( N = 10^{15} - 10^{16} \) cm\(^{-3} \). We also found that with an increase of \( \omega/\omega_d \) the value of \( \delta \) quickly decreases (see Inset in Fig. 3). Therefore, for higher frequencies or lower doping (\( \omega_d \propto N \)) providing \( \omega/\omega_d > 0.1 \), the influence of domains on the gain is practically negligible. We have made additionally the ensemble Monte Carlo simulations of space-charge dynamics 16. These 3D simulations confirmed, in general, our main conclusions
made in the framework of 1D DD-model. Moreover, within the Monte Carlo approach we went beyond the LSA conditions considering a short (18 periods) but heavy doped \((N = 10^{17} \text{ cm}^{-3})\) SL. In this case we found that \(\delta\) is only \(\approx 14\%\) for \(U_\omega = 4U_c\).

In practical terms, the SL oscillator with the parametric cascading generation, i.e. the frequency conversion followed by the parametric amplification, can operate in the whole microwave and THz frequency bands. In the present work we restrict our treatment to the microwave and low-THz frequency range since there exists plenty of serial radiation emitters which can provide a necessary power to pump the SL oscillator. Probably the most interesting and easily realizable case is to use a pump with \(\omega \approx 100\) GHz in order to obtain a generation in the important frequency band of hundreds of GHz \[17\]. In order to suppress electrical domains and to use resonator with a reasonable \(Q\), the doping of SL should be \(\approx 10^{16} \text{ cm}^{-3}\). The interaction of miniband electrons with the electric field for such frequencies is still quasi-static.

However, it is instructive to discuss briefly the effects arising beyond the quasistatic approximation. Using the exact solution of Boltzmann transport equation in the constant relaxation time approximation \[18\], we have found the following two effects. First, if \(\omega\) is higher than some critical \(\omega_{cr}\), even in an ideal cavity a gain arises only if the pump \(E_p\) is larger than some threshold value \(E_{th}\). For the gain at third harmonic \(\omega_{cr} \approx 0.34\tau^{-1}(\approx 250\) GHz); importantly, we have \(E_{th} < E_c\) for the frequencies satisfying \(\omega < 0.38\tau^{-1}(\approx 270\) GHz). Therefore, the generation of radiation at 3\(\omega\) without NDC and, as consequence without an influence of domains, is possible for the pump field with frequencies \(\omega < 270\) GHz. The second effect, which can arise beyond the quasistatic approximation, is the possibility to have a gain at the 5th and the 9th harmonics. For example, a small gain at 5th harmonic arises for \(\omega > 0.14\tau^{-1}(\approx 100\) GHz).

Theoretical research devoted to a gain arising at frequency multiplication in SLs has some history. To the best of our knowledge, Pavlovich presented the first calculations showing a principle possibility of a small-signal gain in the presence of a strong pump with \(\omega_\tau \geq 1\), if the frequency of the probe is some half-integer of \(\omega\) \[13\, 20\]. Romanov pointed out that such a gain should originate from a multiphoton parametric amplification \[22\, 23\]. Possibility to get a gain at harmonics of microwave pump also has been discussed in \[22\, 23\]. Because references to each other are absent in the papers \[12\, 21\, 22\], we suppose that all these works were done independently. This interesting activity did not receive much attention so far. We should also underline that a possible role of space-charge instabilities in SL has not been analyzed in these previous contributions.

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**APPENDIX A. WEAK-SIGNAL ABSORPTION IN THE QUASISTATIC LIMIT**

Here we derive the expression for absorption of the weak probe field \(E_{pr} = E_\omega \cos(n\omega t)\) in the presence of strong pump field \(E_p = E_\omega \cos \omega t, (E_n \ll E_0)\).

Introduce the Fourier expansions for the electron velocity \(V(E(t))\) and its derivative \(V'(E(t))\) (prime means derivation with respect to \(E\)) as

\[
Y(E_{pump}(t)) = \frac{1}{2} \tilde{Y}_0 + \sum_{l=1}^{\infty} \tilde{Y}_l \cos(l\omega t),
\]

where \(\tilde{Y}\) stands either for \(V\) or for \(V'\). Fourier transform \(\tilde{Y}_l\) is defined as

\[
\tilde{Y}_l = \frac{2}{T} \int_0^T Y(E_{pump}(t)) \cos(l\omega t) dt \tag{2}
\]

Substituting the Fourier expansion \[11\] and the Taylor expansion

\[
V(E) \approx V(E_p) + V'(E_p) \times E_{pr}
\]
in the definition of absorption, we obtain

\[
A = V_p^{-1} \langle V(E_p) \cos(n \omega t) \rangle + V_p^{-1} \langle V'(E_p) \cos^2(n \omega t) \rangle E_n = \\
V_p^{-1} \left[ \sum_{l=1}^{\infty} \hat{V}_l \cos(l \omega t) \cos(n \omega t) \right] + E_n \left( \sum_{l=1}^{\infty} \hat{V}_l' \cos^2(n \omega t) \cos(l \omega t) \right) + \frac{1}{2} \hat{V}_0' \cos^2(n \omega t) \right] \\
= \frac{\hat{V}_n}{2V_p} \left[ \hat{V}_n' / V_p + \hat{V}'_{2n} / V_p \right] E_n / 4 = A_{\text{harm}}(E_\omega) + A_{\text{inc}}(E_\omega, E_n) + A_{\text{coh}}(E_\omega, E_n). \quad (3)
\]

Using Esaki-Tsu characteristic, its derivative

\[
V'(E) = \frac{2V_p}{E_c} \frac{1 - E^2}{(1 + E^2)^2}, \quad \bar{E} = E / E_c \quad (4)
\]

and the definition of Fourier coefficients \( \hat{A}_k \), we calculate

\[
A_h = \frac{\hat{V}_n}{2V_p} = (-1)^k \frac{2}{k} \left( \frac{2k + 1}{b E_c^{2k+1}} \right), \quad b = (1 + E_c^2)^{1/2}
\]

\( (2k + 1 = n, \ k = 0, 1, 2, \ldots, \ n = 3, 5, 7, \ldots; \ \bar{E}_{\omega,n} \equiv E_{\omega,n} / E_c \) for the harmonic component of total absorption (see Fig. \( \text{Fig. 4} \) upper),

\[
A_{\text{inc}} = \frac{E_n}{4V_p} \hat{V}_0' = \frac{\bar{E}_n}{(1 + \bar{E}_c^2)^{3/2}} \quad (6)
\]

for the incoherent component of total absorption, and

\[
A_{\text{coh}} = \frac{E_n}{4V_p} \hat{V}_0' = \frac{\bar{E}_n}{2 \pi} \int_0^{2\pi} \left( \frac{1 - \bar{E}_c^2 \cos^2 x}{(1 + \bar{E}_c^2 \cos^2 x)} \right) \cos(2nx) dx \quad (7)
\]

for the coherent component of total absorption.

Note that \( A_{\text{inc}} / E_n \), Eq. \( (6) \), is independent on \( n \); it is always positive. The integral \( (7) \) can be calculated analytically in two limiting cases \( \bar{E}_\omega \ll 1 \) and \( \bar{E}_\omega \gg 1 \). For instance, for \( n = 3 \) and \( \bar{E}_\omega \ll 1 \), we expand \( V'(E) \) up to the term of \( \bar{E}_c^6 \) and get \( A_{\text{coh}} \approx -(7/64) \bar{E}_c^6 \bar{E}_3 \). In the limit \( \bar{E}_\omega \gg 1 \) it easy to obtain \( A_{\text{coh}} \approx -(2n \bar{E}_c) / \bar{E}_c^2 \) for any \( n \). In general case the integral \( (7) \) can be easily calculated numerically, see Fig. \( \text{Fig. 4} \) lower. Note that \( A_{\text{coh}} \) is always negative.

It is also instructive to compare \( A_{\text{inc}} \) and \( A_{\text{coh}} \) (see Fig. \( \text{Fig. 5} \)). For \( \bar{E}_\omega \ll 1 \) we have \( |A_{\text{coh}} / A_{\text{inc}}| \approx \bar{E}_c^2 / 2 \ll 1 \). In the opposite limit \( \bar{E}_\omega \gg 1 \), we have \( |A_{\text{coh}} / A_{\text{inc}}| \approx \bar{E}_c \gg 1 \). Thus, a contribution of the coherent component to the total absorption is larger than a contribution of the incoherent component for a large pump amplitude. Of course, a contribution of both the coherent and the incoherent components is still less than a contribution of the harmonic component.
APPENDIX B. DYNAMICS OF THE FIELD IN THE CAVITY

Here we present the equations describing dynamics of electric field in the cavity and derive formula for the SL oscillator efficiency with an account of finite $Q$-factor of the cavity.

For simplicity we consider a single-mode cavity with the eigenfrequency $\omega_c$ tuned to the $n$th harmonic: $\omega_c \approx \omega_n$. We search the field in the cavity $E_{field}$ in the form

$$E_{field} = E_n(t) \cos(\omega_n t + \phi_n(t)), \quad (8)$$

where the amplitude $E_n(t)$ and the phase $\phi(t)$ are slowly-varying in comparison to the carrier frequency $\omega_n$, i.e.

$$|\dot{E}_n| \ll \omega_n |E_n|, \quad |\dot{\phi}_n| \ll \omega_n |\phi_n|. \quad (9)$$

Substituting (8) in the Maxwell equations and using the so-called slow varying envelope approximation (SVEA), which is valid in conditions (9), one can obtain [11]

$$\frac{\partial E_n}{\partial t} = -\frac{\gamma_c}{2} E_n - 2\pi e J_c(k_n, t), \quad (10)$$

$$\frac{\partial \phi_n}{\partial t} E_n + (\omega_n - \omega_c)E_n = -\frac{2\pi}{e} J_s(k_n, t), \quad (11)$$

$$J_{c,s}(k_n, t) = \frac{1}{L} \int_0^L dx R(x, k_n)J_{c,s}, \quad (12)$$

where $\gamma_c$ is the phenomenological loss constant, $R(x)$ is the cavity mode with wavenumber $k_n$ ($R(x) \approx \exp(ik_n x)$ and $\omega_c = ck_n$ for travelling wave mode; $R(x) \approx \sin(ik_n x)$ for standing wave mode). In what follows we will neglect spatial dependencies of $J_{c,s}$ because wavelength of microwave or THz radiation is larger than nanostructure length, $k_nL \approx \lambda/\lambda \ll 1$. Involved in Eq. (12) cosine and sine Fourier transformations of the current are defined as

$$j_c = 2j_0 \langle v(t) \cos \omega_n t \rangle, \quad j_s = 2j_0 \langle v(t) \sin \omega_n t \rangle,$$

where $j_0 = eV_p N$ and $v(t)$ is the scaled velocity of an electron in SL arising under the action of ac field $E(t) = E_p(t) + E_{field}(t)$.

Next, we suppose that $\phi_n(0) = 0$ and the exact resonance, $\omega_n = \omega_c$. Interaction of miniband electrons with the field $E(t)$ is quasistatic and it is determined by the Esaki-Tsu dependence $V(t) = V[E(t)]$. In these conditions we have $j_s(t) = j_s(0) = 0$, $\phi_n(t) = \phi_n(0) = 0$ and $j_c = 2j_0 A$. This approach means that for the quasistatic interactions of electrons with fields we can neglect dispersion effects in the cavity. Now we have single equation for the field in the cavity

$$\frac{\partial \bar{E}_n}{\partial t} = -\frac{\gamma_c}{2} \bar{E}_n - \frac{\omega_n^2}{2} \bar{E}_n A(t), \quad (13)$$

with $\omega_n^2 = 4\pi e^2 N/(em_0)$, $m_0 = (2\hbar^2)/(\Delta a^2)$ and $\bar{E}_n(t) = E_n(t)/E_c$. Stationary solution of Eq. (13), $\bar{E}_n^{st}$, has the form

$$\frac{\bar{E}_n^{st}}{Q} = -\frac{\omega_n^2}{\omega_n} \frac{\bar{E}_n A}{(E_\omega, \bar{E}_n^{st})}, \quad (14)$$

where $Q = \omega_n/\gamma_c$. This formula has been used to plot Fig. 3.

As follows from (13), the stationary value of the field inside the cavity is reached during the characteristic time $\sim \gamma_c$, i.e. $|\bar{E}_n/E_n| \approx \gamma_c$. Therefore the condition of applicability of SVEA [8] is satisfied in a cavity with enough high $Q$: $\gamma_c/\omega_n = 1/Q \ll 1$.

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