Azimuthal asymmetries of hadrons produced in muon SIDIS off longitudinally polarized deuterons

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Abstract. The azimuthal asymmetries in the cross sections of charged-hadron production in the muon SIDIS reactions off the longitudinally polarised deuterons are determined using the COMPASS data of 2006 and combined data of 2002-2006. The asymmetries are presented as functions of the hadron azimuthal angle $\phi$ in two ways: first, for hadrons integrated over the kinematic variables and, second, as a function of one of the kinematic variables $x$, $z$ or $p_T^h$ while integrating over two other. In each case asymmetries were fitted by functions included the $\phi$-independent terms and terms amplitude of which are modulated with $\phi$ as predicted by the theory: $\sin \phi$, $\sin 2\phi$, $\sin 3\phi$ and $\cos \phi$. Results for amplitudes are presented and discussed.

1. Introduction.

Semi-Inclusive Deep Inelastic Scattering (SIDIS)

$$\mu + N \rightarrow \mu' + nh + X, \quad n = 1, 2 \ldots$$

of high energy polarised muons $\mu$ off polarised nucleons $N$ in the initial state with scattered muons $\mu'$, $n = 1, 2 \ldots$ hadrons $h$ and unobserved particles $X$ in the final state give an access to the spin dependent Parton Distribution Functions PDFs of nucleons. The SIDIS cross section depends in particular on the azimuthal angle $\phi$ of the produced hadron (see e.g. [1]). This dependence leads to the azimuthal asymmetries in hadron production which are related to the Transverse-Momentum-Dependent Parton Distribution Functions (TMD PDFs) and Parton Fragmentation Functions (PFFs) in nucleons.

The TMD PDFs were measured in a number of experiments. The most recent data are obtained by the COMPASS collaboration at CERN using the unpolarised [2] and transversely polarised [3] LiD (“deuteron”) and NH$_3$ (“proton”) targets. The data on hadron production off the longitudinally polarised deuterons were collected in 2002-2004 and 2006 and off protons in 2007 and 2011. Results on azimuthal asymmetries in hadron production off longitudinally polarised deuterons, based on the data of 2002-2004, were published in [4]. The analysis of 2006 and combined 2002-2006 data are presented here.

2. Theoretical framework.

The kinematics of SIDIS reactions are illustrated in figure 1. The 4-momentum of the incident and scattered muons are denoted by $l$ and $l'$, respectively. The 4-momentum of the virtual photon is given by $q = l - l'$ with $Q^2 = -q^2$. The angle of the momentum $q^h$ of the virtual photon with respect to the incident muon in the laboratory frame is denoted by $\theta_q$. The vectors $p_T^h$ and $P_T$ are the hadron momentum and the longitudinal target polarisation, respectively. Their transverse components $p_T^h$ and $P_T$ are defined with respect to the virtual photon momentum. The longitudinal component of target polarisation $|P_T| = |P| \cos \theta_q$ is approximately equal to $|P|$ due to the smallness of the angle $\theta_q$. Its small transverse component is equal to $|P_T| = |P| \sin \theta_q$ with $\sin \theta_q = 2(Mx/Q)(1-y)^{1/2}$, where $M$ is the nucleon mass, $y = (q\nu)/(l\nu)$ is a fractional energy of the virtual photon and $p$ is the 4-momentum of the target nucleon. The angle $\phi$, the azimuthal angle between the lepton scattering plane and hadron production plane, $\phi = 0, \pi$ is the angle of the target polarisation vector with respect to the lepton scattering plane. The Bjorken variable $x \equiv x_{ Bj} = Q^2/(2p\cdot q)$, the fraction of the virtual photon energy taken by hadron $z = (p\cdot p^h)/(p\cdot q)$ and...
The contributions of the cross section $d\sigma$ denote the target polarisation, which is parallel (positive) or antiparallel (negative) to the beam.

For the 2006 data taking with the $^6\text{LiD}$ longitudinally polarised target, a modified COMPASS set up was used that is described elsewhere [5]. Major improvements, influencing the present analysis, were, first, the replacement of two 60-cm long target cells denoted as U and D by three cells (U, M and D) of lengths 30, 60 and 30 cm, respectively, second, replacement of the target cells (U, M and D) of lengths 30, 60 and 30 cm, respectively.

### 3. The 2006 data analysis.

For the 2006 data taking with the $^6\text{LiD}$ longitudinally polarised target, a modified COMPASS set up was used that is described elsewhere [5]. Major improvements, influencing the present analysis, were, first, the replacement of two 60-cm long target cells denoted as U and D by three cells (U, M and D) of lengths 30, 60 and 30 cm, respectively, second, replacement of the target...
solenoid magnet by a new one with a wider aperture, third, installation of the electromagnetic calorimeter ECAL1 in front of the hadron calorimeter HCAL1. These modifications aimed to further minimizing systematic uncertainties, to increasing an acceptance of the spectrometer and improving its e/γ identification capabilities.

In 2006 the data taking have been performed in two groups of periods. Each group is characterized by an initial setting of the target cell polarizations using different frequencies of the microwave field to polarize the target material. We call one group of periods as Freezing 1 (FR1) and another as Freezing 2 (FR2). Each period includes certain number of time intervals (runs) of the continuous data taking. The FR1 data taking periods started with initial setting of positive deuteron polarization in the target cells U and D and negative one in the cell M corresponding to the positive direction of the holding solenoid field \( f = + \) (coincided with the beam direction). After taking some number of runs, the solenoid field was reversed to \( f = - \) causing the reversal of the target cell polarizations and taking of data with opposite deuteron polarizations in the cells. The periodic rotation of polarizations continued up to the end of FR1 periods. Within the period the polarisation values, measured for each run, remained constant at the level of \( \sim 55 \% \). If not, polarisations are restored by the microwave field before beginning of the next period. For FR2 periods the procedure was analogous but initial polarization setting in cells at the same field was opposite to one in FR1.

Events detected by the COMPASS spectrometer [10] were conserved for the analysis under condition \( Q^2 > 1 \) (GeV/c)^2. The statistic of 2006, excluding runs that not passed the data stability tests (see below), comprise about \( 64.2 \times 10^6 \) events.

Selection of SIDIS events was similar to that of [4]. For each event a reconstructed vertex with incoming and outgoing muons and one or more additional hadron tracks are required. Applying cuts on the quality of reconstructed tracks and vertices, the target cell effective length (28, 56, 28 cm), effective beam diameter (2.8 cm), the momentum of incoming muons (140 - 180 GeV/c), the fractional energy carried by all hadrons from the event \( (z < 1) \) and the fractional virtual-photon energy \( (0.1 < y < 0.9) \), about \( 36.6 \times 10^6 \) SIDIS events selected for further analysis.

The tracks from SIDIS events have been identified as hadron tracks if each of them has \( p_T^h > 0.05 \) GeV/c, comes from the current fragmentation region of \( x_F > 0 \) \( (x_F \) is the c.m. Feynman variable), hits one of the calorimeters HCAL1 or HCAL2 and calorimeter has an associated cluster with energy greater than 5 GeV in HCAL1 or greater than 7 GeV in HCAL2. All hadrons from each SIDIS event are included in the analysis. The total number of the hadrons used for 2006 asymmetry calculations is about \( 15.6 \times 10^6 \) compared to \( 53 \times 10^6 \) in 2002-2004.

Using advantages of the 3-cell polarised target, the 2006 data stability tests are performed for a number of observables using their ratios, \( R_i \) per run from cells U and D (U+D) and M. Requiring that the beam crosses all cells of the target and the acceptance of the spectrometer is stable, one can expect that \( R_i = (U_i + D_i)/M_i \), which is luminosity independent, will be stable and close to 1. A number of plots representing \( R_i \) was considered. Example of the \( R_i \) values as function of the run number were fitted by a constant and the \( R_i \)-distributions are fitted by the Gaussian. All plots have shown that \( R_i \) are mostly stable within the \( \pm 3\sigma \) limits.

![Figure 2.](image.png)

**Figure 2. Left:** the \( R_{ev} = R_{i} = (U_i + D_i)/M_i \) ratios of SIDIS events per run vs. the run number. Its fit by a constant and the \( \pm 3\sigma \)-corridor are shown lines. **Right:** the distribution of \( R_{ev} \) and its fit by the Gaussian with Mean \( <R_{ev}> \) and \( \sigma \) values equal to \( 1.05 \pm 0.001 \) and \( 0.015 \pm 0.01 \), respectively.
around the average value for all runs, except for separate runs and runs of the period W35.

Stability of measurements of the hadron azimuthal angle $\phi$ is very important for determination of asymmetries. Distributions of $\phi$-values are obtained for each run of data taking and average values per run $<\phi>$ are determined. Tests of $<\phi>$-values per run as a function of run number have shown that its distribution has the Gaussian shape and for most of runs its values are within the 3$\sigma$ limits around the mean value.

The method of asymmetry calculations described below requires stability of the spectrometer acceptance. Special tests have confirmed that acceptances are indeed mostly stable.

The acceptance free method of asymmetry calculations have been used in [4]. To cancel acceptances, the method utilizes double ratios of events. For the 3-cell target the method was modified as following. The target cell M was conditionally divided in sub-cells M1 and M2, each of 28 cm long. So, conditionally we have got a pair of targets with the cells (U and M1) and (M2 and D). In these conditional targets the cells have equal lengths and opposite polarisations at the fixed holding solenoid field direction $f$. For each conditional target one can obtain double ratios of SIDIS events (or selected from them hadrons), $N^p_{i\text{pf}}$, from the cell $i$ with polarisation $p$ (+ or $-$) at the field $f$ (+ or $-$. At $f=+$ these double ratios, $DR^+_i$ and $DR^-_i$, are:

$$DR^+_i = [(N^U_i - C^U_i)/(N^{M1}_i - C^{M1}_i)]_{FR1} \cdot [(N^{M1}_i + C^{M1}_i)/(N^U_i + C^U_i)]_{FR2} = s^2_{1+}, \quad (4.1)$$

$$DR^-_i = [(N^D_i + C^D_i)/(N^{M2}_i + C^{M2}_i)]_{FR1} \cdot [(N^{M2}_i - C^{M2}_i)/(N^D_i - C^D_i)]_{FR2} = s^2_{1-}, \quad (4.2)$$

where $C^U_i$ is the “acceptance factor” (inverse product of a luminosity and acceptance) of the target cell $i$ at the field $f = +$. For the first ratios events are taken from FR1 runs and for the second ones from FR2 runs. If the acceptance factors $C^U_i$ do not depend on the freezing they cancel in (4.1, 4.2). By definition, each of these double ratios is equal to the squared value of $s = \sigma_+ / \sigma_-$ (or $dr_+ / dr_-$) which is the ratio of the total (or differential) SIDIS cross sections for positive and negative target polarisations. So, within the statistical errors the values $s^2_{1+}$ and $s^2_{1-}$ calculated with events obtained from runs with $f = +$ must be equal. Similar expressions for the negative field $f = -$ (with cancel acceptances) are the following:

$$DR^+_1 = [(N^{M1}_+ / N^U_+)]_{FR1} \cdot [(N^U_+ / N^{M1}_-)]_{FR2} = s^2_{1+}, \quad (4.1a)$$

$$DR^-_1 = [(N^{M2}_+ / N^D_+)]_{FR1} \cdot [(N^D_+ / N^{M2}_-)]_{FR2} = s^2_{1-}. \quad (4.2a)$$

As one expects, all $s_i$ have the same values within the errors.

Equations DR$^+_1$ and DR$^-_1$, being combined, include full statistic of measurements. To be combined, all these double ratios should be stable and equal within the statistical errors.

Because determination of the acceptances for SIDIS events and for hadrons involves different criteria, stability tests of each double ratio were performed both for SIDIS events (DR$^i_{ev}$) and for hadrons (DR$^i_h$) using the corresponding data from consecutive pairs of FR1 and FR2 selected runs. Examples of these tests for DR$^i_{ev}$ and DR$^i_h$ calculated with SIDIS events are shown in figure 3. The difference in the DR values calculated for full statistics is visible. It is due to different contributions to them of pairs from the period W35. When these pairs are partially excluded by fitting the distributions in the narrow ranges of DR (see the bottom plots), the average values of these double ratios became practically equal to each other within errors. Removal of all pairs of this period brings distributions and their average values in a perfect agreement which is necessary for calculation of asymmetries.

So, tests of the DR stability have shown that most of the DR$^i_{ev}$ and DR$^i_h$ are stable over the whole data taking periods and statistically well inside the 3$\sigma$-limits except the period W35.

The final selection of the 2006 data was performed rejecting runs for which the acceptance ratios, values of the angle $<\phi>$ and values of the double ratio DR$^i_3$ and DR$^i_4$ are outside the 3$\sigma$-limits of average, as well as all runs of the W35 period. In total about 10% of 2006 events were rejected.
Figure 3. **Top plots**: the double ratios of SIDIS events, $\text{DR}_{\text{ev}}^{+} \equiv \text{DR}_{1}^{+}$ vs. the number of the run pair for the positive (left) and $\text{DR}_{\text{ev}}^{-} \equiv \text{DR}_{1}^{-}$ for the negative (right) solenoid field directions. The vertical lines show borders of the selected runs. The DR values calculated for full statistic are equal to 1.017 ± 0.001 and 0.991 ± 0.001, respectively. **Bottom plots**: distributions of the $\text{DR}_{\text{ev}}^{+}$ and $\text{DR}_{\text{ev}}^{-}$ values fitted by Gaussians in the DR ranges 0.8 – 1.2. Mean values are equal to 1.013 ± 0.003 and 0.995 ± 0.003, respectively. The σ values are 0.032 ± 0.003 and 0.038 ± 0.05, respectively. The ±3σ-corridors are shown in the upper plots.

For calculations of azimuthal asymmetries $a_{h\pm}(\phi)$, hadrons, separately $h^+$ and $h^-$, have been distributed over 10 $\phi$-bins in the range ±180°. These distributions are used for calculations of double ratios $\text{DR}_{h\pm}(\phi)$ combining statistically the double ratios $\text{DR}_{1}^{\pm}$ and $\text{DR}_{2}^{\pm}$. As it was shown in [4], values $\text{DR}_{h\pm}(\phi)$ in the linear approximation are related to $a_{h\pm}(\phi)$ multiplied by polarisation terms:

$$\text{DR}_{h\pm}(\phi) = [(\text{DR}_{1}^{+} \oplus \text{DR}_{2}^{+})_{h\pm} \oplus (\text{DR}_{1}^{-} \oplus \text{DR}_{2}^{-})_{h\pm}](\phi) \cong 1 + a_{h\pm}(\phi) \cdot \left[\sum_{i,f} (f^{i} \cdot P_{pf}^{i})\right]_{h\pm}, \quad (5)$$

where symbol $\oplus$ means statistical sum. The polarisation terms are given by the statistical sum of the $f^{i} \cdot P_{pf}^{i}$ values which are expressed via the weighted product of the dilution factor $f^{x, y}$, calculated as in [4], and absolute value of the polarisations $P_{pf}^{i}$ (p = + and −) at $f = +$ and $f = −$ in the target cells $i = U, M1, M2, D$. The values $P_{pf}^{i}$ are averaged over a number of hadrons ($h^+$ or $h^-$) from the cell $i$. So, the asymmetries in the linear approximation are given by expression:

$$a_{h\pm}(\phi) \cong \frac{[\text{DR}_{h\pm}(\phi) - 1]}{[\sum_{i,f} (f^{i} \cdot P_{pf}^{i})]_{h\pm}}. \quad (6)$$

The asymmetries $a_{h\pm}(\phi)$ are calculated either for hadrons integrated over all kinematic variables (integrated asymmetries) or as functions of one of the variables $x, p^{T}$ or $z$ while integrating over the other two. To be compatible with the data of 2002-2004, the same binning of variables as in [4] has been used for the 2006 data. In each case the data were fitted by (3).

Figure 4. The amplitudes of the integrated azimuthal asymmetries $a \equiv a_{h\pm}(\phi)$ for 2002, 2003, 2004 and 2006 and combined 2002-2006 (SUM).
The modulation amplitudes of the 2006 integrated asymmetries, together with those of the 2002-2004 [4], are shown in figure 4. They are in agreement between themselves. The values of amplitudes combined from the fits of 2002-2004 and 2006 samples (SUM) are also shown. Within the errors, all amplitudes except \( a_{h^0} \) are consistent with zero. Values \( a_{h^0} \) for \( h^+ \) and \( h^- \), as expected, are equal to each other within the statistical errors.

Behaviour of the \( a_{h^\pm} (\phi) \) amplitudes vs. the variables \( x, z \) and \( p_T^h \) for the 2006 data was compared to that of the 2002-4 data and found them to be in agreement within statistical errors.

4. Azimuthal asymmetry for combined deuteron data.
The \( a_{h^\pm} (\phi) \) modulation amplitudes of the combined 2002-2006 data vs. the variable \( x, z \) and \( p_T^h \) are presented in figure 5. Fits of all distributions by constants, except the \( a_{h^0} (x) \), have produced results consistent with zero. The \( x \)-dependence of terms \( a_{h^0}^T (x) \) is not shown. Instead the \( x \)-dependence of terms \( a_{h^0}^T (x) / D_0 (x, y) \), where \( D_0 \) is the virtual-photon depolarisation factor calculated as in [11] for each \( x \)-bin, is presented. If the main contributions to the terms \( a_{h^0}^T (x) \) came from the helicity PDF \( g_1 (x) \) then the values of \( a_{h^0}^T (x) / D_0 (x, y) \), by definition, should be equal to the virtual photon asymmetries \( A_{h^1d} (x) \): \( a_{h^0}^T (x) / D_0 (x, y) = A_{h^1d} (x) \). Figure 5 shows that there is a good agreement between our data on \( a_{h^0}^T (x) / D_0 (x, y) \) and the data on \( A_{h^1d} (x) \) [11] confirming correctness of the results on asymmetries calculated by the acceptance-free method. The values of \( A_{h^1d} (x) \) were obtained with the data of 2002-2004. Similar \( x \)-behaviour has also asymmetry \( A_{d^1d} (x) \) and \( A_{K^+d} (x) \) for identified pions and Kaons [5] (data of 2002-2006).

![Figure 5. Azimuthal asymmetries as functions of \( x, z \) and \( p_T^h \) for the \( h^\pm \) combined 2002-2006 data.](image)

5. Estimation of systematic uncertainties.
Possible sources of additive systematic errors in the asymmetry evaluation can be connected with variations of the spectrometer acceptance. The compatibility of data collected during four years and their stability tests indicated absence of large systematic effects. It is therefore assumed that the systematic uncertainties of the asymmetry measurements due to not completely cancel acceptance are smaller than the statistical ones. As a quantitative measure of possible
additive systematic errors in the asymmetry measurements the value $\Delta (a_{h1}(\phi)) = \pm 0.003$ is obtained. The uncertainties due to the beam and target polarizations, estimated to be less than 5% each, and uncertainties of the dilution factor, estimated to be $\sim 2\%$, introduce multiplicative systematic uncertainties in the asymmetry measurements. When combined in quadrature, they give a global systematic multiplicative uncertainty of less than 6%.

6. Conclusions and discussions.
As one can see from figure 4, the $\phi$-independent terms $a_{h10}$ of integrated azimuthal asymmetries are about equal for $h^+$ and $h^-$. The amplitudes of all $\phi$-modulating terms are very small, i.e. of the per-mil-orders, and consisted with zero within statistical errors.

There are some comments concerning the asymmetries $a_{h1}(\phi)$ as functions of the kinematic variables $x$, $z$ and $p_T^h$. The first comment concerns the $\phi$-independent terms $a_{h10}(x)$. Figure 5 shows that there is a good agreement between the COMPASS data on $a_{h10}(x)/D_0(x, y)$ and $A_{h1d}(x)$ [5, 11] confirming expectations that the main contributions to $a_{h10}(x)$ are coming from the helicity PDF $g_1(x)$.

Fits by the constants of the $x$-, $z$- and $p_T^h$-dependences of the terms modulating in $\phi$ have not shown any statistically significant peculiarities and produced results consistent with zero. Tendencies in possible $x$-dependence of the $\sin 2\phi$ and $\cos \phi$ amplitudes are visible. The $\sin 2\phi$ amplitude for $h^+$ is mostly positive and rising with increasing $x$, while for $h^-$ it is mostly negative and decreasing with $x$. This behaviour agrees with those discussed in [6, 12]. The rising with $x$ of absolute values of the $\cos \phi$ amplitudes, connected with the Cahn effect [13] and already visible with the 2002-2004 data [4], persists with all COMPASS data.

Finally one can conclude that contributions of the TMD PDFs to the asymmetries of the hadron production in the SIDIS reactions from the longitudinally polarised deuterons are small either due to possible cancellations of the deuteron constituent quark and anti-quark contributions to the asymmetries, or due to smallness of transverse component of the target polarisation and suppressing factors $\sim M/Q$. Some of these conclusions will be checked studying asymmetries from longitudinally polarised protons.

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