Neutrino mass models
with an abelian family symmetry

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Abstract

Abelian family symmetries provide a predictive framework for neutrino mass models. In seesaw models based on an abelian family symmetry, the structures of the Dirac and the Majorana matrices are derived from the symmetry, and the neutrino masses and mixing angles are determined by the lepton charges under the family symmetry. Such models can lead to mass degeneracies and large mixing angles as well as mass hierarchies, the squared mass difference between quasi-degenerate neutrinos being determined by the symmetry. We present two models illustrating this approach.

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\section{Introduction}

Fermion masses are one of the most fundamental problems of particle physics. While the origin of the observed hierarchy between quark and charged lepton masses remains unexplained in the Standard Model and most of its extensions, the question of whether the neutrinos are massive or not is still open. On the theoretical side, since neutrino masses are not protected by any fundamental symmetry\footnote{Neutrino masses are only protected by lepton number symmetry, which is an accidental global symmetry of the Standard Model, and turns out to be violated in most of its extensions.}, there is no reason to expect them to be zero. Now, if the neutrinos are massive, the rather unnatural suppression of their masses relative to the quarks and charged leptons of the same family has to be explained. On the phenomenological side, massive neutrinos could solve in a natural way several astrophysical and cosmological problems.

Family symmetries, which have been first introduced in order to explain the quark mass and mixing hierarchies, provide a predictive framework for neutrino mass models. We present two seesaw models based on an abelian family symmetry, the one leading to a hierarchical mass spectrum, the other yielding two quasi-degenerate neutrinos with a large mixing angle.

\section{Neutrino oscillations}

There is no direct laboratory evidence for non-zero neutrino masses (the present upper bounds are $m_{\nu_e} < 5.1 \text{ eV}$, $m_{\nu_\mu} < 160 \text{ keV}$, $m_{\nu_\tau} < 24 \text{ MeV}$), but some experimental data suggest neutrino oscillations. Before discussing them, let us briefly review neutrino oscillations in the simple case of two flavours. Neutrinos oscillate when the weak eigenstates $\nu_{\alpha,\beta}$ are mixtures of the mass eigenstates $\nu_{1,2}$:

$$
\begin{pmatrix}
\nu_{\alpha} \\
\nu_{\beta}
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
$$

(1)

Suppose a weak eigenstate $\nu_{\alpha}$ is produced at $t = 0$: $\nu(0) = \nu_{\alpha} = \cos \theta \nu_1 + \sin \theta \nu_2$. At $t$ it will have evolved into $\nu(t) = \cos \theta e^{-iE_1 t} \nu_1 + \sin \theta e^{-iE_2 t} \nu_2$. Assuming relativistic neutrinos, the probability of detecting the other weak eigenstate is:

$$
P(\nu_{\alpha} \rightarrow \nu_{\beta}) = |<\nu_{\beta} |\nu(t)>|^2 = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)
$$

(2)
The strongest indication in favour of neutrino oscillations comes from the solar neutrino deficit. All solar neutrino experiments have observed a suppression of the $\nu_e$ flux relative to the predictions of the standard solar models. The most convincing explanation of this deficit is provided by the Mikheyev-Smirnov-Wolfenstein (MSW) conversion of the electron neutrino into another species inside the sun. There are two allowed regions in the $(\sin^2 2\theta, \Delta m^2)$ plane, one with $\Delta m^2 \sim 6 \times 10^{-5} - 10^{-4} eV^2$ and $\sin^2 2\theta \sim 10^{-3} - 10^{-2}$, the other with a large mixing angle $(\Delta m^2 \sim 10^{-6} - 10^{-5} eV^2$ and $0.2 \leq \sin^2 2\theta \leq 0.9$). Another hint for neutrino oscillations is the atmospheric neutrino anomaly. Some experiments have measured a significantly lower ratio of the $\nu_\mu$ to the $\nu_e$ flux than predicted, which could be a signature of $\nu_\mu$ oscillations into another flavour with a large mixing angle $(\Delta m^2 \sim 10^{-2} eV^2$ and $\sin^2 2\theta \geq 0.5$). However, this anomaly has not been observed by all experiments, and some uncertainties remain. Finally, there may be indications in favour of $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations with a small mixing angle ($\sin^2 2\theta \sim$ a few $10^{-3}$ for $\Delta m^2$ in the few $eV^2$ region) from the LSND experiment. This interpretation needs to be confirmed.

Massive neutrinos are also interesting for cosmology. Structure formation requires, in addition to cold dark matter, a small amount of hot dark matter, which could be composed of a neutrino with mass between 1 and 10 $eV$, or several degenerate neutrinos in the few $eV$ range.

3 Models of neutrino masses

3.1 Generalities

There are numerous models of neutrino mass. All of them need an extension of the particle content of the Standard Model. A Dirac mass term ($\mathcal{L}_m = - m_D \bar{\nu}_L N_R + h.c.$) requires the introduction of a right-handed (RH) neutrino $N_R$ in addition to the standard left-handed (LH) neutrino $\nu_L$. Since $N_R$ is a $SU(2)_L$ singlet, such a mass term violates the weak isospin by $\Delta I_W = 1/2$, and must therefore be generated by a Yukawa coupling to a Higgs doublet: $\mathcal{L}_{\text{Yuk}} = - h_{\nu} (\bar{\nu}_L \bar{e}_L) H_2 N_R + h.c.$, with $m_D = h_{\nu} < H_2^0 >$. A Dirac neutrino is then like other fermions, but its Yukawa coupling $h_{\nu}$ has to be unnaturally small. A Majorana mass term ($\mathcal{L}_m = - 1/2 m_M \bar{\nu}_L \nu_R + h.c.$), which violates lepton number by two units ($\Delta L = 2$), involves a transition from the standard $\nu_L$ ($I_W = 1/2$) to its CP conjugate $\nu_R^c$ ($I_W = -1/2$). Such a mass term has $\Delta I_W = 1$ and must therefore originate from a Yukawa coupling to a weak Higgs triplet (Gelmini-Roncadelli model) or from an effective interaction.

In the presence of a RH neutrino, both Dirac and Majorana mass terms can be present, as well as a $\Delta I_W = 0$ Majorana mass term for the RH neutrino, $\mathcal{L}_m = -1/2 M_R \bar{N}_R^c N_R + h.c.$. The full mass term takes then the following form
(in the absence of a Higgs triplet):

\[- \frac{1}{2} (\bar{\nu}_L \bar{N}_L) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \bar{N}_R \end{pmatrix} + h.c. \]  

(3)

The physical neutrino states, which are two Majorana neutrinos, are obtained from the diagonalization of this 2x2 matrix. Particularly interesting is the seesaw limit \( m_D \ll M_R \), in which one eigenstate has a mass far below the weak scale.

\[
\begin{aligned}
m_1 &\simeq m^2_D/M_R \\
m_2 &\simeq M_R \\
\tan \theta &\simeq m_D/M_R
\end{aligned}
\]  

(4)

Since the mixing angle is small, the light eigenstate is mainly the Standard Model neutrino.

3.2 Seesaw models

The seesaw mechanism is very popular, because it naturally generates neutrino masses much lighter than the weak scale. Moreover, it can be easily implemented in numerous extensions of the Standard Model, like SO(10) GUT’s or string models, where such Standard Model singlets as \( N_R \) with masses in the phenomenologically interesting range (typically, \( M_R \sim 10^{12} - 10^{16} \text{ GeV} \), which corresponds to a light neutrino mass \( m_\nu \sim 10^{-3} - 10 \text{ eV} \)) can be present. Assuming one RH neutrino per family, the Dirac (Majorana) mass in (3) is replaced by a 3x3 matrix in family space \( M_D (M_M) \), and the light neutrino spectrum is obtained by diagonalizing the symmetric matrix:

\[
M_\nu = -M_D M_M^{-1} M_D^T = R_\nu \text{Diag} (m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) R^T_\nu
\]  

(5)

Note that the mixing angles relevant for neutrino oscillations are given by the analog of the CKM matrix, which also involves the charged lepton sector:

\[
V_L = R^L_\ell R_\nu
\]  

(6)

In general, the entries of both the Dirac and the Majorana matrices are free parameters, and one has to choose a specific ansatz in order to make any definite prediction in the neutrino sector. It is often assumed that the Dirac mass matrix has the same structure than the up quark mass matrix \( M_D \sim M_U \). For the

The Dirac mass \( m_D \), which is protected by the electroweak symmetry, is expected to be of the order of the breaking scale \( M_{\text{weak}} = 246 \text{ GeV} \), whereas the Majorana mass \( M_R \), being not constrained by any symmetry, can be much larger than \( M_{\text{weak}} \). Thus \( m^2_D/M_R \ll M_{\text{weak}} \).

The charged lepton mass matrix is in general not hermitian, so it is diagonalized by two unitary matrices: \( \text{Diag}(m_1, m_2, m_3) = R^L_\nu M_\nu R^R_\nu \).

This arises naturally in Standard Model extensions with a quark/lepton symmetry, like SO(10).
Majorana matrix, however, no such simplifying assumption can be done, and it is necessary to assume a specific form. It follows that the neutrino spectrum of a given model depends on the ansatz that has been chosen, which is not very satisfactory.

Alternatively, one can try to derive the structures of the Dirac and the Majorana matrices from a symmetry. This symmetry has to act in a different way on the three neutrino families, otherwise the matrices would be unconstrained. Such a symmetry is called a family symmetry. This approach has proved to be successful in the quark sector, where, following the original idea by Froggatt and Nielsen, several groups [4, 5, 6, 7, 8, 13] have shown that an abelian family symmetry can reproduce the observed mass and mixing hierarchies.

4 Neutrino mass models with a $U(1)$ family symmetry

The class of models we consider are extensions of the Minimal Supersymmetric Standard Model (MSSM) with: (i) a gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$, where $U(1)_X$ is an abelian family symmetry; (ii) a SM singlet field $\theta$ with $X$-charge $X_\theta = -1$, which is used to break $U(1)_X$ and to generate fermion masses; and (iii) three RH neutrinos $\bar{N}_i$ ($i$ is a family index), in addition to the MSSM spectrum, which are needed to generate neutrino masses by the seesaw mechanism. We require that the family symmetry reproduce the experimental data on quarks and charged leptons, which forces it to be anomalous [6, 13], and that its anomalies be compensated for by an appropriate mechanism (the Green-Schwarz mechanism).

In the following, we concentrate on the neutrino sector. We denote the lepton doublets by $L_i$ (with their $I_W = +1/2$ components $\nu_i$), and the right-handed neutrinos by $\bar{N}_i$, and their charges under $U(1)_X$ respectively by $l_i$ and $n_i$. We note $h_1$ and $h_2$ the $X$-charges of the two MSSM Higgs doublets $H_1$ and $H_2$.

4.1 Dirac and Majorana matrices

Let us show how the Dirac ($M_D$) and Majorana ($M_M$) matrices are constrained by the family symmetry. Each Dirac mass term $L_i \bar{N}_j H_2$ carries an $X$-charge $p_{ij} = l_i + n_j + h_2$. If $p_{ij} \neq 0$, the coupling is forbidden by $U(1)_X$, and the corresponding entry of $M_D$ is zero. However, if the excess charge $p_{ij}$ is positive, one can write non-renormalisable interactions involving the chiral...
singlet $\theta$:

$$L_i \bar{N}_j H_2 \left( \frac{\theta}{M} \right)^{p_{ij}}$$

(7)

where $M$ is a large scale characteristic of the underlying theory (typically $M \sim M_{\text{Planck}}$ or $M_{\text{GUT}}$). When $\theta$ acquires a vev, $U(1)_X$ is spontaneously broken and effective Dirac masses are generated:

$$(M_D)_{ij} \sim v_2 \left( \frac{< \theta >}{M} \right)^{p_{ij}}$$

(8)

where $v_2 = < H_2 >$. Since $U(1)_X$ is broken below the scale $M$, $\epsilon \equiv < \theta > / M$ is a small parameter. Thus the Dirac matrix obtained has a hierarchical structure with the order of magnitude of its entries fixed by their excess charges under $U(1)_X$.

The entries of the Majorana matrix $M_M$ are generated in the same way, with non-renormalizable interactions of the form:

$$M_R \bar{N}_i \bar{N}_j \left( \frac{\theta}{M} \right)^{q_{ij}}$$

(9)

giving rise to effective Majorana masses

$$(M_M)_{ij} \sim M_R \left( \frac{< \theta >}{M} \right)^{q_{ij}}$$

(10)

provided that $q_{ij} = n_i + n_j$ is a positive integer (otherwise $(M_M)_{ij} = 0$). Thus, the light neutrino mass matrix $M_\nu = -M_D M_M^{-1} M_D^T$, and consequently the neutrino masses and mixing angles, is determined by the charges of the leptons under $U(1)_X$. No particular ansatz for $M_D$ nor $M_M$ is required. Note, however, that each of the entries is determined only up to an arbitrary factor of order one by the family symmetry.

Of course, there is a large variety of models, depending on the charges one assigns to the lepton fields. Contrary to the quark charges, whose possible values are strongly restricted by the experimental data on quark masses and CKM angles, the lepton charges are poorly constrained. In the following, we present two classes of models. The first one leads to a hierarchical mass spectrum, the second one has two quasi-degenerate neutrinos with a large mixing angle.

4.2 Model 1: hierarchical mass spectrum

By analogy with the quark and charged lepton mass matrices, we start from

\[10\] Remember that the first motivation for introducing a family symmetry was to understand the quark and charged lepton mass hierarchies.
a matrix with only one coupling allowed by the family symmetry:

$$\mathcal{M}_\nu = m_{\nu_3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(11)

The breaking of $U(1)_X$ fills in the zero entries with powers of the small parameter $\epsilon$, leading to a hierarchical spectrum. Assuming that (a) the $X$-charges of all mass terms are positive ($p_{ij} \geq 0$, $q_{ij} \geq 0$) and (b) the dominant entry of each mass matrix is the (3,3) entry, one automatically obtains pattern (11). The light neutrino masses are then:

$$m_{\nu_1} \sim \frac{m_3^2}{M_3} \epsilon^2 |l_1 - l_3| \quad m_{\nu_2} \sim \frac{m_3^2}{M_3} \epsilon^2 |l_2 - l_3| \quad m_{\nu_3} \sim \frac{m_3^2}{M_3}$$

(12)

The $\nu_\tau$ mass is given by the usual seesaw formula ($M_3$ is the mass of the heaviest RH neutrino, $m_3$ the largest Dirac mass), whereas the other neutrino masses are suppressed relative to $m_{\nu_\tau}$ by powers of the small breaking parameter $\epsilon$. Note that the hierarchy depends only on the $X$-charges of the lepton doublets $L_i$. The lepton mixing matrix is:

$$V_L \sim \begin{pmatrix} 1 & \epsilon |l_1 - l_2| & \epsilon |l_1 - l_3| \\ \epsilon |l_1 - l_2| & 1 & \epsilon |l_2 - l_3| \\ \epsilon |l_1 - l_3| & \epsilon |l_2 - l_3| & 1 \end{pmatrix}$$

(13)

This model has several remarkable features. First, the neutrino mass and mixing hierarchies do not depend on the particular form of the Majorana matrix. This is a great difference with most seesaw models. The reason for this is that the dependences of $\mathcal{M}_D$ and $\mathcal{M}_M$ on the heavy neutrino charges compensate for each other in $\mathcal{M}_\nu$. Secondly, the mass spectrum obtained is naturally hierarchical without hierarchy inversion: $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$. Finally, the mixing angles and the mass ratios are related by:

$$\sin^2 \theta_{ij} \sim \frac{m_{\nu_i}}{m_{\nu_j}}$$

(14)

These relations, which are common to numerous seesaw models, show that small mixing angles are associated with mass hierarchies. They also imply $V_{\nu_\mu} V_{\nu_\tau} \sim V_{\nu_e}$ in lepton charged current, in analogy with $V_{u_\mu} V_{u_\tau} \sim V_{u_e}$ in quark charged current.

The experimental data on solar neutrinos and atmospheric neutrinos put constraints on the parameters of the model. For example, if one wants to explain simultaneously the solar neutrino deficit by MSW $\nu_e \rightarrow \nu_\mu$ transitions, and the atmospheric neutrino anomaly by $\nu_\mu \rightarrow \nu_\tau$ oscillations, one must choose:

$$l_1 - l_3 = 3 \quad l_2 - l_3 = 1$$

(15)

The possibility of mass degeneracies will be discussed in the next section.
which leads to the following spectrum:

\[
\begin{align*}
    m_{\nu_e} &\sim 10^{-5} \text{ eV} \\
    m_{\nu_\mu} &\sim 5.10^{-3} \text{ eV} \\
    m_{\nu_\tau} &\sim 0.1 \text{ eV}
\end{align*}
\]

The uncertainties in the mixing angles are due to the fact that the mass matrix entries are determined only up to a factor of order one by the family symmetry. It is quite difficult to obtain a large mixing angle with a hierarchical spectrum [see (14)], as required by the atmospheric neutrino data. Furthermore, the tau neutrino is too light to be a good candidate for hot dark matter. However, if one ignores the atmospheric neutrino problem, it is possible to obtain a cosmologically relevant tau neutrino and to account for the solar neutrino deficit at once.

### 4.3 Model 2: quasi-degenerate neutrinos

As suggested above, in the context of abelian family symmetries, large mixing angles are naturally related to mass degeneracies. It thus seems rather difficult for the previous model to account for the atmospheric neutrino anomaly, or to accommodate the large angle branch of the MSW effect. Yet mass degeneracies and large mixing angles are not excluded: according to (12) and (13), \( l_2 = l_3 \) yields \( m_{\nu_\mu} \sim m_{\nu_\tau} \) and \( V_{\mu\nu} \sim V_{\tau\nu} \sim 1 \). But the presence of unconstrained factors of order one in each entry of \( \mathcal{M}_\nu \) can upset these formulae. Also, an accurate mass degeneracy requires fine-tuning of these factors. This leads us to consider another class of family symmetries, allowing for two equal\(^{12}\) couplings, e.g.:

\[
\mathcal{M}_\nu = m_{\nu_3} \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}
\]

Such a matrix has two degenerate eigenvalues\(^{13}\) \( m_{\nu_2} = -m_{\nu_3} \), with a maximal mixing angle, \( \sin^2 2\theta = 1 \). This degeneracy is slightly lifted by the breaking of \( U(1)_X \). In order to reproduce pattern (17), we choose the following assignment of lepton charges:

\[
\begin{align*}
    l_1 &= l' > l \\
    l_2 &= -l_3 = l
\end{align*}
\]

and, for simplicity, we assume \( h_1 = h_2 = 0 \). The light neutrino mass matrix is then:

\[
\begin{pmatrix}
\epsilon^{2l'} & \epsilon^{l'+l} & \epsilon^{l'-l} \\
\epsilon^{l'+l} & \epsilon^{2l} & 1 \\
\epsilon^{l'-l} & 1 & \epsilon^{2l}
\end{pmatrix}
\]

\(^{12}\)The equality of the couplings in (17) follows from the symmetry of Majorana mass terms.

\(^{13}\)The relative sign simply means that the mass eigenstates have opposite \( CP \) parities.
As expected, the spectrum contains two heavy, strongly degenerate neutrinos

\[ m_{\nu_1} \sim \epsilon^{2l'} \ll m_{\nu_2} \simeq -m_{\nu_3} \]  

(20)

with a squared mass difference determined by the family symmetry:

\[ \Delta m^2_{23} \sim m^2_{\nu_3} \epsilon^{2l} \]  

(21)

Taking into account the charged lepton sector, one obtains the lepton mixing matrix:

\[
V_L \sim \begin{pmatrix}
1 & \epsilon^{l'-l} & \epsilon^{l'+l} \\
\epsilon^{l'-l} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\epsilon^{l'-l} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}
\]  

(22)

The quasi-degenerate neutrinos are almost maximally mixed, while the third neutrino, which is lighter, has small mixings with the other ones.

Such a mass and mixing pattern can account for the hot dark matter of the Universe, and simultaneously explain the atmospheric neutrino deficit in terms of \( \nu_\mu \leftrightarrow \nu_\tau \) oscillations. This requirement, together with the experimental limits on \( \bar{\nu}_\mu \leftrightarrow \bar{\nu}_e \) oscillations and the charged lepton masses, fixes the parameters of the model to be:

\[ l = 2 \quad l' = 5 \quad m_{\nu_3} = 2 - 3 \text{ eV} \]  

(23)

Thus, the mu and tau neutrinos are in the relevant mass range for hot dark matter:

\[ m_{\nu_\mu} \sim (5 - 8)\times 10^{-7} \text{ eV} \quad m_{\nu_\tau} \simeq m_{\nu_3} = 2 - 3 \text{ eV} \]  

(24)

and, being strongly degenerate, they can oscillate with the parameters needed in order to solve the atmospheric neutrino problem:

\[ \Delta m^2_{\mu\tau} \sim (0.9 - 2)\times 10^{-2} \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta_{\mu\tau} \simeq 1 \]  

(25)

Furthermore, \( \bar{\nu}_\mu \leftrightarrow \bar{\nu}_e \) oscillations are found to be in the domain of sensitivity of the LSND and KARMEN experiments:

\[ \Delta m^2_{e\mu} = 4 - 9 \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta_{e\mu} \sim 10^{-3} \]  

(26)

Finally, let us note that the solar neutrino problem cannot be solved by this model, unless a sterile neutrino \( \nu_s \) is added.

5 Conclusion

We have presented two seesaw models based on an abelian family symmetry, the one leading to a hierarchical mass spectrum, the other yielding an accurate mass degeneracy and a large mixing angle between the two heaviest
neutrinos. In such models, the neutrino masses and mixing angles are determined in terms of the lepton charges under the family symmetry. Also, the squared mass difference between quasi-degenerate neutrinos is predicted. No ansatz for the Dirac nor the Majorana matrix is needed. Furthermore, the fact that the same symmetry is able to explain the observed fermion mass hierarchy and simultaneously constrains the neutrino spectrum sets an interesting connection between two fundamental problems in particle physics. Unfortunately, the lepton charges, though constrained by experimental data, are not fully determined by the model, which leads us to consider different classes of models, corresponding to different mass patterns. However, some generic properties of abelian family symmetries suggest that the model may originate from a more fundamental theory, which would fix all of its parameters.

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