Non-Abelian Antisymmetric-Vector Coupling from Self-Interaction

Adel Khoudeir

Centro de Astrofísica Teórica, Departamento de Física, Facultad de Ciencias, Universidad de los Andes, Mérida, 5101, Venezuela.

Abstract

A non-abelian coupling between antisymmetric fields and Yang-Mills fields proposed by Freedman and Townsend several years ago is derived using the self-interaction mechanism.

1 INTRODUCTION

Abelian second-rank antisymmetric fields \[1\] play an essential role in strings and supergravity theories and have been extensively studied in the last decades \[2, 3, 4, 5\]. In free theories they describe massless and spinless particles and appear in many contexts, for instance, arising as mediators of the interaction between open strings with charged particles \[2\] and in ten dimensions, coupling with the Chern-Simons 3-form to achieve an elegant unification of Yang-Mills and supergravity \[6\]. In particular the Cremmer-Scherk theory \[3\] has received considerable attention \[7, 8\] due to the fact that the coupling between the abelian antisymmetric field and a Maxwellian field through a topological \(BF\) term leads to massive propagations which are compatible with gauge invariances. Moreover, Allen, et. al. \[7\] have shown unitarity and renormalizability of the Cremmer-Scherk theory. This fact motivates the non-abelian generalization of the model and several attempts have been proposed \[9\]. Simultaneously, other alternatives for non-abelian massive vector bosons without the presence of Higgs field have been proposed in the last year \[10\].

The non-abelian extension of antisymmetric theories was achieved by Freedman and Townsend \[4\] starting from a first-order formulation where

\[1\] e-mail: adel@ciens.ula.ve
the antisymmetric field $B^m_{mn}$ and an auxiliary vector potential are independent variables. It is worth recalling that the non-abelian generalization of the abelian S-duality theory [11] is a Freedman-Townsend theory [12]. In their work, Freedman and Townsend proposed the non-abelian generalization of the Cremmer-Scherk theory. In this letter, starting from an appropriate first-order formulation for the Cremmer-Scherk theory, we will derive the non-abelian generalization using the self-interaction mechanism [13], which has been successfully applied to formulate Yang-Mills, gravity [13], supergravity [14], topologically massive Yang-Mills [15] and Chapline-Manton [16] theories.

2 THE ABELIAN MODEL

Our starting point will be a first-order formulation for the Cremmer-Scherk theory. This is realized introducing an auxiliary vector field $(v_m) a la$ Freedman-Townsend. The action is written down as [17]

$$I = \langle -\frac{1}{4}\mu\epsilon^{mnpq}B_{mn}[\partial_p v_q - \partial_q v_p] - \frac{1}{2}\mu^2 v^m v_m - \frac{1}{2}\mu\epsilon^{mnpq}B_{mn}\partial_p A_q \rangle$$

$$+ \frac{1}{4}F_{mn}F^{mn} - \frac{1}{2}F_{mn}[\partial_mA_n - \partial_n A_m] >$$

where $<>$ denotes integration in four dimensions. All the fields involved have mass dimensions and $\mu$ is a mass parameter. There are two sets of abelian gauge invariances:

$$\delta_\lambda A_m = \partial_m \lambda, \quad \delta_\lambda F_{mn} = 0$$

$$\delta_\zeta B_{mn} = \partial_m \zeta_n - \partial_n \zeta_m, \quad \delta_\zeta v_m = 0.$$  

Independent variations in $v_m, B_{mn}, F_{mn}$ and $A_m$ lead to the following equations of motion

$$v^m = -\frac{1}{6\mu}\epsilon^{mnpq}H_{npq},$$

$$\epsilon^{mnpq}\partial_p [v_q + A_q] = 0,$$

$$F_{mn} = \partial_m A_n - \partial_n A_m,$$

$$\partial_p F^{pm} = \frac{1}{6}\mu\epsilon^{mnpq}H_{npq}.$$
where $H_{mn} \equiv \partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn}$ is the field strength associated with the antisymmetric field. The Cremmer-Sherk action is obtained after substituting equations (4) and (6) in (1):

$$I_{CrSc} = \frac{1}{4} F_{mn[A]} F_{mn[A]} - \frac{1}{12} H_{mn[B]} H_{mn[B]} - \frac{1}{4} \mu \epsilon^{mnpq} B_{mn} F_{pq[A]}.$$  

(8)

On the other hand, equation (5) can be solved (locally) for the $v$ field,

$$v_m = -[A_m + \frac{1}{\mu} \partial_m \phi],$$  

(9)

where $\phi$ is a scalar field. Substituting this solution in the action $I$, the Stuckelberg formulation for massive abelian vector bosons is obtained

$$I_{St} = \frac{1}{4} F_{mn[A]} F_{mn[A]} - \frac{1}{2} \mu^2 [A_m + \frac{1}{\mu} \partial_m \phi] [A^m + \frac{1}{\mu} \partial^m \phi].$$  

(10)

As it is well known, both formulations (Stuckelberg and Cremmer-Sherk) are equivalent descriptions of massive abelian gauge invariant vectorial theories and propagate three degrees of freedom. This equivalence is reflected by the fact that they are connected by duality [18]. Indeed, since the scalar field appears in equation (10) only through its derivative, we can apply the dualization method due to Nicolai and Townsend [19], which consist in replacing $\partial_m \phi$ by $\frac{1}{2} l_m$ and adding a new term to equation (10): $\epsilon B \partial l$, i.e.

$$I_{Stmod} = \frac{1}{4} F_{mn[A]} F_{mn[A]} - \frac{1}{2} \mu^2 [A_m + \frac{1}{2 \mu} l_m] [A^m + \frac{1}{2 \mu} l_m] + \frac{1}{4} \epsilon^{mnpq} B_{mn} \partial_p l_q.$$  

(11)

At this stage, $B_{mn}$ is a Lagrange multiplier forcing the constraint $\partial_m l_n - \partial_n l_m = 0$ whose local solution is $l_m = 2 \partial_m \phi$. Now, if we eliminate $l_m$ via its equation of motion

$$l^m = \frac{1}{3} \epsilon^{mnpq} H_{npq} - 2 \mu A^m$$  

(12)

and go back to equation (11), the Cremmer-Sherk action is recovered.

Finally, let us recall that the second-order field equations can be written as

$$\partial_p F^{pm} = J^m, \quad \partial_p H^{mnn} = J^{mn},$$  

(13)

where

$$J^m = \frac{1}{6} \mu \epsilon^{mnpq} H_{npq} \quad \text{and} \quad J^{mn} = \frac{1}{2} \mu \epsilon^{mnpq} F_{pq}$$  

(14)

are "topological" currents in the sense that they are conserved without using the equations of motion.
3 THE SELF-INTERACTION PROCESS

Now, we extend the first-order action, equation (1), by introducing a triplet of free abelian antisymmetric fields $B_{mn}^a$, coupled with a triplet of free abelian vector fields $A_m^a$; ($a = 1, 2, 3$)

$$I_o = -\frac{1}{4}\epsilon^{mpq}B_{mn}^a[\partial_p v_q^a - \partial_q v_p^a] - \frac{1}{2}\mu^2 v_m^am - \frac{1}{2}\mu\epsilon^{mpq}B_{mn}^a\partial_p A_q^a$$

$$+ \frac{1}{4}F_{mn}^aF^{amn} - \frac{1}{2}F^{amn}[\partial_mA_n - \partial_n A_m]$$

Besides the local gauge transformations

$$\delta_\lambda A_m^a = \partial_mA_1^a, \quad \delta_\lambda F_{mn}^a = 0$$

$$\delta_\xi B_{mn}^a = \partial_mB_{sn}^a - \partial_n B_{sm}^a, \quad \delta_\xi v_m^a = 0,$$

our action has two global invariances: one is a global $SU(2)$ rotation and the other is a a global symmetry associated with the Freedman-Townsend theory:

$$\delta_\omega X^a = g_1\epsilon^{abc}X^b\omega^c$$

where $X^a = (A_m^a, F_{mn}^a, v_m^a, B_{mn}^a)$ and

$$\delta_\rho B_{mn}^a = g_2\epsilon^{abc}[v_m^b + A_m^b]\rho_n^c - m \leftrightarrow n,$$

$$\delta_\rho v_m^a = \delta_\rho A_m^a = \delta_\rho F_{mn}^a = 0,$$

$\omega$ and $\rho$ being global parameters. In principle the coupling constants $g_1$ and $g_2$ are different. We note that under type II transformations the action changes by a total derivative. The Noether currents associated to these invariances are given by

$$g_1^{-1}j^a_{mn} = \epsilon^{abc}F^{bmn}A_n^c + \frac{1}{2}\mu\epsilon^{mpq}\epsilon^{abc}B_{pqr}[A_n^c + v_n^c]$$

and

$$g_2^{-1}K_{mn}^a = \frac{1}{2}\mu\epsilon^{mpq}\epsilon^{abc}[A_p^b + v_p^b][A_q^c + v_q^c].$$

These are conserved on-shell. In order to couple these currents to the action $I_o$ we must add the corresponding self-interaction terms: $I_1$ and $I_2$ defined by:

$$j^a_{mn} \equiv \frac{\delta I_1}{\delta A_m^a}; \quad K_{mn}^a \equiv -2\frac{\delta I_2}{\delta B_{mn}^a}.$$
These functional differential equations can easily be integrated. In fact, we find that

$$I_1 = -g_1 < \frac{1}{2} \epsilon^{abc} F_{mn}^a A^b_m A^c_n + \frac{1}{4} \mu \epsilon^{mpq} \epsilon^{abc} B^a_{mn} A^b_p A^c_q >$$

and

$$I_2 = -g_2 < \frac{1}{4} \epsilon^{mpq} \epsilon^{abc} B^a_{mn} v^b_p v^c_q + \frac{1}{4} \mu \epsilon^{mpq} \epsilon^{abc} B^a_{mn} A^b_p A^c_q >$$

However, these two terms have overlapping parts. This situation is akin to what happens in the derivation of supergravity from self-interaction [14]. In order to overcome this obstacle we must require equality of the coupling constants: $g \equiv g_1 = g_2$ and write down the self-interaction action as

$$I_{SI} \equiv -g < \frac{1}{2} \epsilon^{abc} F_{mn}^a A^b_m A^c_n + \frac{1}{4} \epsilon^{mpq} \epsilon^{abc} B^a_{mn} v^b_p v^c_q + \frac{1}{4} \mu \epsilon^{mpq} \epsilon^{abc} B^a_{mn} A^b_p A^c_q >$$

Actually, we have that

$$j^{am} \equiv \frac{\delta I_{SI}}{\delta A^m_a} \quad \text{and} \quad K^{amn} \equiv -2 \frac{\delta I_{SI}}{\delta B^a_{mn}}.$$  

The self-interaction mechanism stops here since no other derivative terms appear in $I_{SI}$. Finally, the full non-abelian theory is

$$I = I_o + I_{SI} = -\frac{1}{4} \mu \epsilon^{mpq} B^a_{mn} [F^a_{pq} + f^a_{pq} + 2 \epsilon^{abc} A^b_p v^c_q] - \frac{1}{2} \mu^2 v^a_m v^a_m - \frac{1}{4} F^a_{mn} F^{amn},$$

where

$$F^a_{mn} \equiv \partial_m A^a_n - \partial_n A^a_m + g \epsilon^{abc} A^b_m A^c_n$$
and
\[ f^a_{mn} \equiv \partial_m v^a_n - \partial_n v^a_m + g \epsilon^{abc} v^b_m v^c_n \]  (29)

which is just that proposed by Freedman and Townsend (equation (2.15) in their paper). As usual, the self-interaction process combines the abelian gauge transformations with the global ones giving rise to non-abelian local gauge transformations. In our case, we have

\[ \delta_\alpha A^a_m = \partial_m \alpha^a + g \epsilon^{abc} A^b_m \alpha^c \]  (30)
\[ \delta_\alpha B^a_{mn} = g \epsilon^{abc} B^b_{mn} \alpha^c \]  (31)
\[ \delta_\alpha v^a_m = g \epsilon^{abc} v^b_m \alpha^c \]  (32)

and

The action of Freedman-Townsend, equation (27), is equivalent to massive Yang-Mills (locally) as can be shown after elimination of \( B^a_{mn} \) through its equation of motion, which said us that \( A_m + v_m \) is a pure gauge.

4 CONCLUSION

In this letter, by starting with a nice abelian first-order formulation, and through the application of the self-interaction mechanism we have obtained the Freedman-Townsend theory and its corresponding gauge transformation rules through self-interaction. The first order abelian formulation allowed us to find Cremmer-Sherk and Stuckelberg formulations for massive spin-1 theories, these later formulations are connected by duality. The BRST quantization of the massive Freedman-Townsend has been performed by Thierry-Meig [20]. Since massive Freedman-Townsend theory is equivalent (in topologically trivial manifols) to massive Yang Mills it should be interesting to attempt to connect Friedman-Townsend with others approaches dealing with massive gauge bosons without the presence of Higgs field [14].
5 ACKNOWLEDGEMENT

I thank P.J. Arias for useful discussions and U. Percoco, M. Caicedo, N. Pantoja and L. Labrador also for carefully reading the manuscript.

6 REFERENCES

References

[1] V.I. Ogievetsky and I.V. Polubarinov, Sov. J. Nucl. Phys. 4 (1967) 156; S. Deser, Phys. Rev. 187 (1969) 1931.

[2] M. Kalb and P. Ramond, Phys. Rev. D9 (1974) 2273.

[3] E. Cremmer and J. Scherk, Nucl. Phys. B72 (1974) 117.

[4] D.Z. Freedman and P.K. Townsend, Nucl. Phys. B177 (1981) 282.

[5] S. Deser and E. Witten, Nucl. Phys. B178 (1981) 491; S. Deser, P.K. Townsend and W. Siegel, Nucl. Phys. B184 (1981) 333; J. Thiery-Mieg and L. Baulieu, Nucl. Phys. B228 (1983) 259.

[6] E. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuizen, Nucl. Phys. B195 (1982) 97; G. F. Chapline and N.S. Manton, Phys. Lett. B120 (1983) 105.

[7] T. J. Allen, M. J. Bowick and A. Lahiri, Mod. Phys. Lett. A6 (1991) 559.

[8] A. Lahiri, Mod. Phys. Lett. A8 (1993) 2403; R. Amorin and J. Barcelos-Neto, Mod. Phys. Lett. A10 (1995) 917.

[9] A. Lahiri, hep-th/9301060; D. S. Hwang, C.Y. Lee, hep-th/ 9512216; A. Lahiri, hep-th/9602009; J. Barcelos-Neto, A. Cabo and M.B.D. Silva, preprint IC/95/245.

[10] V. Periwal, hep-th/9509084 and hep-th/950985; J. de Boer, K. Skenderis and P. van Nieuwenhuizen, Phys. Lett. B367 (1996) 175; A. J. Niemi, Phys. Lett. B374 (1996) 119.
[11] E. Witten, hep-th/9506011

[12] Y. Lozano, Phys. Lett. B364 (1995) 19; N. Mohammedi hep-th/9507040; C. Hong-Mo, J. Faridani and T.S. Tsun, hep-th/9503106; O. Ganor and J. Sonnenschein, hep-th/9507036

[13] S. Deser, Gen. Rel. Grav. 1 (1970) 9; Class. Quant. Grav. 4 (1987) L99.

[14] D. G. Boulware, S. Deser and J.H. Kay Physics 96A (1979) 141.

[15] C. Aragone and E. Araujo, Acta Científica Venezolana, 36 (1985) 207.

[16] C. Aragone and J. Stephany, Rev. Bras. Fis. 16 (1987) 287; Quantum Mechanics of Fundamental Systems 1, edited by C. Teitelboim(Plenum Press, N.Y, 1988) p. 27.

[17] We use the metric $\eta_{mn} = \text{diag}(-1, +1, +1, +1)$.

[18] E. Harikumar and M. Sivakumar, hep-th/9604181

[19] H. Nicolai and P.K. Townsend, Phys. Lett. B98 (1981) 257.

[20] J. Thierry-Meig, Nucl. Phys. B335 (1990) 334.