I. INTRODUCTION

Gravity, as one of the four building block forces of the universe, is the most sensible of the forces that we deal with in our every day lives. However, it also is considered as the most controversial one in modern physics. The general theory of relativity for gravitation, which was formulated by Einstein in 1915, aside from its thundering triumphs in explaining the universe, encounters important issues at the high energy levels, such as black holes information paradox and singularity problem [1, 2]. These issues arise from the fact that there is no known way to reconcile gravity with the other three forces of the nature. It seems that to rectify these issues one needs to integrate quantum physics and gravity in a unified frame, which remains as one of the most important unsettled issues in physics.

Therefore, there has been an ongoing attempt to establish a quantum theory of gravity, leading to development of different approaches such as string theory and quantum loop gravity [3, 5]. On the other hand, considering the aforementioned challenges, the search for alternative theories such as theories of emergent gravity, instead of quantum gravity has also attracted a great attention [6, 7]. The main hurdle in establishing a quantum theory for gravity is associated with the weakness of the gravitational interaction. In other words, the quantum properties of gravity becomes sensible only in the range smaller than the plank scale, requiring experiments with energies well beyond the scales that are within the near-future capabilities. This fact makes it impossible to discern a reliable route to the problem and to discriminate between various developed models of quantum gravity [8]. Therefore, laying out some empirically feasible methods for studying and testing the quantum nature of gravity is of utmost importance.

In recent years, there has been an increasing interest in the quantum-information-theoretic approaches to the study of relativistic and gravitational systems [9–11]. Recently, as a novel approach for probing the quantum nature of the gravity, it has been discovered that gravitationally-induced entanglement can indeed serve as a witness of quantumness of the gravity [12, 13]. As is demonstrated in [12, 13], the detection of entanglement, attained by gravitational interactions of massive particles, can be considered as a sufficient criteria for quanta of the gravitational field. As was demonstrated, such entanglement generations could be tailored in interferometric schemes, and considering the rapid progress in quantum information and interferometry technologies, the experimental feasibility of these approaches are within the sight.

Even though entanglement is central to the Hilbert space structure of composite quantum systems, the quantum feature of the systems is in no way restricted to the entanglement. In fact, in quantum physics, Heisenberg’s uncertainty principle [14], Bohr’s complementarity principle [15] and quantum correlations in composite quantum systems are the most fundamental aspects of systems, such that the entire fabric of the quantum weirdness can be captured by these fundamental characteristics [16]. Therefore, it is imperative to search for all these three building blocks of quantum mechanics in quantum nature of the gravity, in a feasible experimental setup.

As an other fundamental aspect of quantum physics, Bohr’s complementarity principle [15] provides one of the most fundamental aspects of the nature, which explains that the two mutually exclusive attributes, such as the waviness and the particleness, can both be imprinted in quantum systems, such that measuring one feature prohibits its dual feature to be exhibited [15, 16]. As an example, in the case of a single photon passing through an interferometer, the particleness of the photon is embed-
ded in the path predictability of the photon, while the waviness is encoded in the visibility of the interference pattern on screen [17]. The quantitative notion of such a discipline was first introduced by Wootters and Zurek in 1979 [18], which lead to a mathematical description in form of an inequality as $P^2 + V^2 \leq 1$, where $P$ represents the predictability of a quantum system, which contains the path information and indicates a quantity for particle-ness, and $V$ stands for the visibility of interference pattern, measuring the waviness of the system [19,22].

In an interesting attempt, it has been shown that the wave-particle duality has a relationship with the entanglement in the system [23–25]. This unification of the duality and entanglement in double slit (two-qubit) analyses provided an important relation as $P^2 + V^2 + C^2 = 1$ [23–25], in which $C$ is a measure of entanglement known as concurrence [26]. More recently, an interesting geometrical correspondence between this relation and stereographic projection of $S^7$ geometry was also formulated [27], which gives a full geometrical proof for the duality-entanglement relation [27].

In this paper, we put forward an experimentally feasible platform that enables testing Heisenberg’s uncertainty principle, Bohr’s complementarity principle and quantum entanglement as the nonclassical aspects of quantum mechanics in quantum gravity framework, connecting gravity and quantum physics in a broader and deeper context. As one important aspect of our study, we show that all of these three fundamental characteristics of quantum gravity can be framed and tested in an interferometric scheme.

The paper is organized as follows. In Section II we introduce the quantum gravitational interaction potential and discuss its fundamental implications. In Section III we briefly address the entanglement-complementarity relation and its main features. In Section IV we put forward the gravitationally induced complementarity and entanglement analyses in an interferometric quantum superposition scenario. In Section V we briefly analyze Bell inequality which provides a practical way for the detection of entanglement in quantum systems. In Section VI we lay out the discuss of the uncertainty principle in the context of gravitationally induced quantum phases. In Section VII we discuss the experimental feasibility of the gravitationally induced entanglement and its analyses provided in this work. We finally provide a short summary and conclude in VIII.

II. MAIN ASPECTS OF QUANTUM GRAVITY

Here, we briefly consider some important aspects of the gravitationally induced phase, and the significance of potential that results into such an entanglement. To this end, we start with the action of the gravitational field as [28]

$$S = \int d^4x \sqrt{-g} [R + L_m],$$  \hspace{1cm} (1)

where $g$ is the determinant of metric $g_{\mu\nu}$, and $R$ is Ricci scalar which depends on the derivations of the metric, and is related to the Ricci tensor by $R = g^{\mu\nu} R_{\mu\nu}$. Also, $L_m$ represent the matter part of the action.

Now, taking the variation over metric one can reach to the Einstein field equation [28]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu},$$  \hspace{1cm} (2)

where $T_{\mu\nu}$ is the stress-momentum tensor. To make the quantum nature of the gravity manifest, we study the perturbation $h_{\mu\nu}$ on the background metric of Minkowski space-time $\eta_{\mu\nu}$. Therefore, the entire metric can be written as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Considering the weak field limit, after expanding the metric and omitting the orders higher than quadratic terms and choosing the harmonic gauge we obtain [29]

$$S_{WF} = \frac{1}{2} \int d^4x ((-\frac{1}{32\pi G}) \times (\partial_\lambda h^{\mu\nu} \partial^\lambda h_{\mu\nu} - \frac{1}{2} \partial_\mu h \partial^\lambda h) + h^{\mu\nu} T_{\mu\nu}).$$  \hspace{1cm} (3)

Now, considering $h$ as a field, the first two terms refer to the free field of the gravity and the third term indicates the interaction of the gravity with the matter. The free gravitational field can be written as [29]

$$h^{\mu\nu} = \frac{1}{(2\pi)^3} \int \frac{d^3k}{\sqrt{\omega}} \sum_{\sigma = \pm} (e^{\mu\nu}_\sigma(p) a_\sigma(p) e^{\nu\sigma}_\omega + h.c.),$$  \hspace{1cm} (4)

where $\sigma$ denotes the spin of the graviton and $e^{\mu\nu}_\sigma$ is the polarization tensor with the properties $e^{\mu\nu}_\sigma e^{\mu\nu*}_\sigma = 2\delta_{\sigma\sigma'}$.

As a result, we can calculate the propagator and the scattering process of two masses. To illustrate, in one loop level, after Fourier-transformation of scattering amplitude in momentum space, and in the non-relativistic limit, the potential can be obtained as [30,32]

$$V(r) = -\frac{G m_1 m_2}{r} (1 + \frac{3}{r^2 c^2} G \frac{m_1 + m_2}{r c^2} + \frac{41Gh}{10m^2 c^2}).$$  \hspace{1cm} (5)

The second term is the General Relativistic correction, and the third term is related to the quantum gravity correction to the classical potential. These two terms are extremely small compared to the first term. More specifically the last terms is quite negligible, compared to the first two terms; therefore, the detection of quantum nature of the gravity does not seem to be feasible if one tries to detect it by using the last term of this equation. This, in fact, shows why observation of the quantum mechanical nature of gravity is such a difficult task.

Considering the extremely small contribution of quantum part in the potential in Eq. (5), the substantially important quantum is whether it is possible to observe the quantum nature of the gravity through the fist term (the Newtonian approximation) of the potential? The answer to this question is surprisingly, yes. This could be achieved by a purely information-theoretic approach.
to the gravity. In particular, as recently was discovered, an interesting route to this goal is the gravity induced entanglement, which circumvents the challenges of the weak strength of the quantum contribution to the gravitational interactions. The subtleness of the idea lies in the fact that the induced entanglement can emerge even at the low energy limits, where we can approximate the potential with its first term, i.e., the Newtonian approximation \[ E = \frac{1}{2} m v^2 - \frac{G m_1 m_2}{r} \] Interestingly, this entanglement-assisted analyses of the quantum nature of gravity offers an advantage for the accessibility of the induced quantumness in an empirical set up, compared to the effect that are directly proportional to Planck constant as appears in last term of the potential in Eq. (5). In the next sections we discuss about this remarkable approach toward the investigating of quantum gravity.

The important role of entanglement is inherited by the fact that if the gravitational interaction can generate entanglement among two masses, the gravitational field itself should be of quantum nature. In other words, we assume that two masses interact via gravity, in which the interaction generates entanglement. In this setting, gravity acts as a mediator of the entanglement between two masses. The interaction is induced by the exchange of gravitons as the mediator of the entanglement. If the entanglement is generated then gravitational field needs to be coupled quantum mechanically to each test mass in order for the generation of the entanglement. This is due to the fact that no classical mediator can generate entanglement, as was shown in [35]. A similar conclusion can be drawn from LOCC theorem which states that no local operation and classical communication (LOCC) can increase the entanglement [36]. Therefore, starting with two disentangled masses, the entire system remains disentangled unless some quantum mechanical interactions are applied. As an important conclusion of this fact, detection of entanglement in such a setting is suffix to conclude that the gravity is of quantum nature.

### III. TWO-QUBIT

**COMPLEMENTARITY-ENTANGLEMENT RELATION**

The space of a two-qubit state is the product of Hilbert space of each qubit denoted by \( H_1^C \otimes H_2^C \), offering a 4-dimensional Hilbert space for the system. Considering the fact that each Hilbert space, in this setting, can be spanned by two orthogonal basis \{0, 1\}, the basis of two-qubit Hilbert space can be written as \{00, 01, 10, 11\}, where \( |ij\rangle \) denotes the composite basis as \( |i\rangle \otimes |j\rangle \), with \( i, j = 0, 1 \). Therefore, the general form of a pure two-qubit state can be expressed as

\[
|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle.
\]

Interestingly, we can encode any two-state system, such as the paths in Young double-slit experiments or Mach

![FIG. 1: Experimental set-up of interferometer for gravitationally induced entanglement. The masses \( m_1 \) and \( m_2 \) undergo independent Mach Zehnder type interference, and interact with each other by gravity. \( BS_i \) (\( i = 1, 2 \)) indicates a beam splitter which is characterized by \( (r, t_i) \). The second beam splitter in each interferometer is assumed to be a 50:50 beam splitter.](image)

![Image](image)
of the probability. Applying this relation to the quantum state in Eq. 7, the visibility reduces to [27]

\[ V = 2 \left| \alpha_2^* \alpha_0 + \alpha_3^* \alpha_1 \right|. \]  

(9)

In a similar vein, the predictability is determined as [23–25, 27]

\[ \mathcal{P} = \left| \frac{p_u - p_d}{p_u + p_d} \right|, \]  

(10)

where the parameters \( p_u \) and \( p_d \) are the probabilities of finding first particle in each of the chosen paths (i.e., the probability of finding the first mass in the upper or the lower arm of its corresponding interferometer). Hence, from this equation the particle-ness of system can be attained as [27]

\[ \mathcal{P} = |(|\alpha_0|^2 + |\alpha_1|^2) - (|\alpha_2|^2 + |\alpha_3|^2)|. \]  

(11)

On the other hand, the amount of entanglement in the above system can be obtained using concurrence, which is defined through \( R \) matrix such that \( R = \sqrt{\hat{\rho} \hat{\sigma} \hat{\rho} \hat{\sigma}} \), with \( \hat{\rho} = (\sigma_x \otimes \sigma_x) \rho \sigma_x \sigma_x \). By organizing the eigenvalues of the \( R \) matrix in decreasing order, the concurrence is defined as [26]

\[ C = \max\{0, \lambda_0 - \lambda_1 - \lambda_2 - \lambda_3\}, \]  

(12)

In which the eigenvalues of matrix \( R \) is denoted by \( \lambda_i \) in the deceeding order. In the case of the state in Eq. 7, the concurrence is given by [27]

\[ C = 2 \left| \alpha_0 \alpha_3 - \alpha_1 \alpha_2 \right|. \]  

(13)

Recently, it was realized that concurrence plays a significant role in a quantum complementarity setting, such that [24, 25]

\[ \mathcal{P}^2 + \mathcal{V}^2 + C^2 = 1. \]  

(14)

This relation shows that entanglement can indeed control the duality in quantum system, hence, the full description of the system entails wave-particle-entanglement trility relation as above, rather than the duality description alone. This relation also can be proven geometrically as outlined in Ref. 27, where it was shown that complementarity principle can indeed be analysed from a completely geometric perspective.

**IV. GRAVITAIONALLY INDUCED COMPLEMENTARITY AND ENTANGLEMENT**

As we discussed earlier, the ultimate theory of quantum gravity should enable us to express gravity as a superposition of different states [12, 13]. This is a main feature of quantum gravity which makes it different from classical case and could be used in finding an experimental approach to test the theory in a table-top experiment.

![Fig. 2](image)

**Fig. 2:** The concurrence of the system as a function of different parameters in the setup: (a) shows the dependence of concurrence on the gravitationally induced phase and reflectivity of the first beam splitter in the first interferometer. (b) depicts the concurrence as a function of reflectivity of the first beam splitters in each interferometer, where the gravitationally induced phase is set to \( \phi = \pi \).

Following this line of thoughts, recently, it was found that quantum nature of gravity can be tested in the light of entanglement generated by the gravity [12, 13]. In these interesting works, it was shown that interferometric setup enables testing the existence of the gravitationally induced entanglement between two masses.

Here, we lay out a rather general framework to consider quantum foundation of gravity. To this end, we assume two massive particles, \( m_1 \) and \( m_2 \), each subjected to an interferometer as depicted in Fig. 1. We also assume that, in the first interferometer the first beam splitter, which particle encounters, is characterized by \( (r_1, t_1) \), where \( r_i \) and \( t_i \) represent the reflectivity and transmissivity parameters of the \( i \)th beam splitter. Similarly, in the second interferometer its first beam splitter is characterized by \( (r_2, t_2) \). For simplicity, we could assume that the arms which is perpendicular to the initial direction of motion of the particles in interferometer are negligible in comparison to the parallel arms [12, 13]. Therefore, the initial state of the two masses after passing through the first beam splitters can be expressed as the product of states of the first and second particle paths, which is given by

\[ |\psi(t = 0)\rangle = (t_1|u_1\rangle + r_1|d_1\rangle) \otimes (t_2|u_2\rangle + r_2|d_2\rangle). \]  

(15)

Note that the setting considered here is much more general compared to the previous studies [12, 13] where only 50:50 beam splitters were taken into the consideration.

The interaction of a quantum particle with gravitational field results in an induced phase, which its first experimental demonstration was attained in a famous experiment by Colella, Overhauser and Werner (which is known as the COW experiment) in 1975 [27]. Now, similar to this experiment, we consider the setting where the gravitational field can indeed induce phase shift on the quantum systems [12, 13]. In this setting gravity induces a phase shift and decouples from the system once the phase shift is attained [12, 13]. Therefore, the initial state of the systems given by \( |\psi(t = 0)\rangle \) will evolve into

\[ |\psi(t = T)\rangle = r_1r_2e^{i\phi_1}|d_1\rangle|d_2\rangle + r_1t_2e^{i\phi_2}|d_1\rangle|u_2\rangle + t_1r_2e^{i\phi_3}|u_1\rangle|d_2\rangle + t_1t_2e^{i\phi_4}|u_1\rangle|u_2\rangle, \]  

(16)
In which \( \phi \), the gravitationally induced phase is set to 0. In both cases when we choose the reflectivity of beam splitters as \( r_1 = r_2 = 1/\sqrt{2} \), the entanglement becomes maximum.

We note that the entanglement generated in this setting is indeed induced by gravitational field. Here, gravity acts as a mediator of the entanglement, and if the entanglement could be observed we can conclude that the gravity is of quantum nature. This is due to the fact that no classical mediator can generate entanglement [35].

Next, we consider the visibility of the interference in the first interferometer. Using Eq. (9), we can obtain for the visibility of the first particle

\[
\mathcal{V} = 2 \left| (r_1 r_2 e^{i \phi_1}) (t_1 t_2 e^{-i \phi_3}) + (t_1 t_2 e^{-i \phi_3}) (r_1 r_2 e^{i \phi_1}) \right|.
\]

Therefore, the square of the visibility provides

\[
\mathcal{V}^2 = 4 (r_1 t_1)^2 \left[ r_2^2 + t_2^2 e^{-i \phi} \right]^2
= 4 (r_1 t_1)^2 [r_2^2 + t_2^2 + 2 (r_2 t_2)^2 \cos(\phi)].
\]

As can readily be seen from this result, if the reflectivity in the second interferometer becomes zero \( (r_2 = 0) \), the gravitational interaction would have no effect on the visibility of the first interferometer.

In Fig. 4 the results for visibility is plotted. In Fig. 4(a), the visibility of the system as a function of reflectivity of first beam splitters in each interferometer is depicted, where the gravitationally induced phase is set to \( \phi = \pi \). Fig. 4(b) illustrates the dependence of visibility on the gravitationally induced phase and reflectivity of the first beam splitter in the first interferometer.

Finally we attain the predictability of the first mass from Eq. (11) as

\[
\mathcal{P}^2 = (r_1 - t_1)^2.
\]

Considering these relations the equality in Eq. (14) is fulfilled. As a result, the gravity induces a tradeoff between entanglement and complementarity.

In Fig. 3 the results for the predictability in terms of various involving parameters is illustrated. In Fig. 3(a) predictability as a function of reflectivity of beam splitters in each interferometer is depicted. Here the gravi-
tationally induced phase is fixed to \( \phi = \pi \). In Fig. 3(b) the dependence of predictability of the first beam splitter in first interferometer is illustrated. As we expect from Eq. 22 it only depends on the reflectivity of the first beam splitter and for \( r_1 = 1/\sqrt{2} \), the path information entirely vanishes.

Taking \( r_1 = r_2 = 1/\sqrt{2} \), the entanglement becomes maximum, while visibility reduces. Practically, choosing 50:50 beam splitter observing the quantum entanglement features of the gravity becomes more feasible. In this case, the relations for the first particle simplifies to

\[
\mathcal{C}^2 = 2 (r_1 t_1)^2 (1 - \cos(\phi)),
\]

\[
\mathcal{V}^2 = 2 (r_1 t_1)^2 (1 + \cos(\phi)),
\]

\[
\mathcal{P}^2 = (r_1^2 - t_1^2)^2.
\]
This readily provides \( P^2 + V^2 + C^2 = 1 \). Therefore, quantum nature of the gravity provides complementarity and entanglement, enabling a feasible experimental analysis of the quantum gravity, and providing important features of the quantum nature of the gravity.

V. BELL INEQUALITY AND TESTING QUANTUM GRAVITY

The non-local correlation exhibited by quantum entanglement was first addressed in 1935 by Einstein, Podolsky and Rosen in a famous paper (usually called the EPR paper) [39], which demonstrated the apparently paradoxical context of quantum mechanics. Almost three decades later, John Stewart Bell found an experimentally manageable and quantitative platform to investigate this controversial feature of composite quantum systems and how to discriminate it from classical descriptions [39]. Here, we discuss Bell inequality in the context of quantum gravity, which provides an experimentally testable approach for quantum nature of the gravity. The most commonly used Bell inequality is the so-called Bell-CHSH inequality [40, 41]. The CHSH operator is given by [40, 41]

\[
\hat{B} = \vec{a} \cdot \vec{\sigma} \otimes \left( \vec{b} + \vec{\theta} \right) \cdot \vec{\sigma} + \vec{a}' \cdot \vec{\sigma} \otimes \left( \vec{b}' - \vec{\theta} \right) \cdot \vec{\sigma},
\]

where \( \vec{a}, \vec{a}', \vec{b}, \vec{b}' \) are unit vectors. Given the above relation, the Bell-CHSH inequality reads [40, 41]

\[
|\langle \hat{B} \rangle| = |\text{tr}(\rho \hat{B})| \leq 2.
\]

Violation of this inequality, i.e., \( |\langle \hat{B} \rangle| > 2 \), indicates existence of quantum correlations in a quantum state, and hence, it provides an experimentally appealing method for the test of quanta of the gravity.

As an interesting connection, there is a relation between entanglement and the maximum violation of Bell-CHSH inequality given as [42, 43]

\[
B = 2 \left( \sqrt{1 + C^2} \right).
\]

This in turn shows that Bell inequality can be violated by all entangled states \( (C \neq 0) \). Since the quantum mechanical non-locality appears when this parameter exceeds two, we plot \( I = B - 2 \) in Fig. 5. Accordingly, Fig. 5(a) indicates the violation of Bell inequality as a function of reflectivity of first beam splitter in first interferometer and the gravitationally induced phase. In Fig. 5(b) we demonstrate Bell parameter \( I \) as a function of the reflectivity of the first beam splitters in each interferometer, where we have assumed that the gravitationally induced phase is \( \phi = \pi \). Therefore, the maximum violation of the Bell-CHSH inequality occurs when both of the beam splitters are 50:50.

To establish an interesting relation between complementarity and Bell parameter, using Eq. (24), we can attain a new relation as

\[
4(P^2 + V^2) + B^2 = 8.
\]

This relation demonstrates that in order to measure quantum non-locality, which is sufficient condition for the quantum nature of the gravity, we can simply measure the path-information and the visibility. This observation is insightful for practical detection of non-classical correlations through gravity.

VI. UNCERTAINTY RELATIONS IN INTERFEROMETERIC QUANTUM GRAVITY

Now that we have considered entanglement and complementarity as the two fundamental aspects of the quantum mechanics, we now address the uncertainty principle in the context of quantum gravity, as the other fundamental concept of the quantum mechanics [44]. The concept of quantum uncertainty was first proposed by Werner Heisenberg for the position and momentum operators [13]. Later Robertson generalized it for any pair of non-commutative operators, such that considering a pair of non-commutative operators A and B the uncertainty relation reads [45]

\[
\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|,
\]

where \( \Delta A \) and \( \Delta B \) are the standard deviations of these operators and \( |\langle [A, B] \rangle| \) is the expectation value of the commutator of the operators. We could define operators corresponding to the predictability and visibility as follows [10]

\[
\hat{P} = \sigma_z, \quad \hat{V} = \cos \theta \, \sigma_x + \sin \theta \, \sigma_y,
\]

where

\[
\begin{align*}
\sigma_z &= |u_1 \rangle \langle u_1| - |d_1 \rangle \langle d_1|, \\
\sigma_x &= |u_1 \rangle \langle d_1| + |d_1 \rangle \langle u_1|, \\
\sigma_y &= -i |u_1 \rangle \langle d_1| + i |d_1 \rangle \langle u_1|.
\end{align*}
\]

Since the expectation values of an arbitrary operator \( O \), with density matrix \( \rho \), is defined as \( \langle O \rangle = \text{Tr}(\rho O) \), one
can write the uncertainty relation for the predictability and the visibility operators as

$$\Delta P \Delta V \geq 2(r_1 t_1) \times | r_2^2 \sin(\phi_1 - \phi_3 - \theta) + t_2^2 \sin(\phi_2 - \phi_4 - \theta) | \ .$$

(29)

Even though this uncertainty relation is fundamentally significant in the context of quantum mechanics, some tighter, hence better, uncertainty relations are considered as the alternative uncertainty relations $[44, 47–49]$. One useful alternative uncertainty is defined as the sum of the uncertainties of each operator. As a result, for two Hermitian unitary operators $A = \vec{a}.\sigma$ and $B = \vec{b}.\sigma$, where $\vec{a}$ and $\vec{b}$ are the unit vectors, simultaneously acting on the density matrix $\rho = 1/2(I + \vec{r}.\sigma)$, which $\vec{r}$ is a unit vector, the uncertainty is defined as $[44]

$$\Delta A^2 + \Delta B^2 \geq 1 + | \vec{a}.\vec{b} |^2 - 2 | \vec{a}.\vec{b} | \times \sqrt{1 - \Delta A^2} \sqrt{1 - \Delta B^2} | (\vec{a} \times \vec{b}).\vec{r} |^2 .$$

(30)

Now, considering such an uncertainty in the context of the problem in hand, we attain

$$\Delta P^2 + \Delta V^2 \geq 1 + (4(r_1 t_1))^2 \times | r_2^2 \sin(\phi_1 - \phi_3 - \theta) + t_2^2 \sin(\phi_2 - \phi_4 - \theta) |^2 \ .$$

(31)

The minimum of the sum uncertainty is one, which can be obtained when one of the operators has zero uncertainty. The second term, in the right-hand-side can never exceed one, and interestingly is quadrant of the uncertainty relation in Eq. (29). Therefore, the sum uncertainty bound is more tighter than the uncertainty relation in Eq. (29).

In an experimental setting, one can control the quantum uncertainty effects of the gravity by controlling the gravitationally induced phases. This, in fact, provides a deep connection between the quantum mechanics and gravity from the uncertainty perspective.

**VII. EXPERIMENTAL CHALLENGES**

The experimental investigations of quantum gravity via induced entanglement addresses one of the most important problems of the physics. Therefore, it would be important to consider the feasibility of the induced entanglement generations via current technology. In this section, we briefly review the important challenges that such an experiments is subjected to and discuss the the feasibility of the study.

One important challenge for generating entanglement in this realm is that one needs to create a large enough induced phase shift that can be measurable in the experiments. Considering the relation for the induced phase, $Gm_1 m_2 T/\hbar d$, there are three parameters that we could adjust in reaching to a measurable phase in the order of unit. The first parameter is masses of the particles. The product of the masses must be big enough to enhance the entanglement degree. This requires preparing a large system in a quantum superposition. This problem has intensively considered in the context of quantum mechanics, and many improvements have been made thus far $[50–54]$. Therefore, we can consider a system with mass of about $10^{-14} \text{kg}$ a reasonable mass size, for which the possible candidates could be a massive molecule $[55, 56]$, micro crystals $[12]$ or Bose-Einstein condensates $[57, 58]$. The other factor in controlling the induced phase is the distance between two interferometers. In a realistic setting, if the two objects get more closer to each other than a certain limit, the other interactions such as electrostatic interactions can dominate the gravitational interaction $[12, 13]$. To overcome this challenge, it has been proposed that by using a conductor in between the two masses, hence shielding other interactions, we can adjust the distance of the interferometer as close as about $d = 1 \mu \text{m} [59]$. The last parameter that we can control is the time that each particles travels in the interferometer arms. To illustrate, if we adjust the travel time of particles in the interferometer arms in the order of $T = 0.1 \text{sec}$, the induced phase could be adjust in the order of unit and consequently the maximum entanglement of two particles through gravitational interaction would be achievable. As another practical setup, one can consider two coupled nano-mechanical oscillators with the mass $10^{-12} \text{kg}$, and the interaction time $T = 10^{-6} \text{sec}$ that would enable the phase shifts large enough to prepare maximally entangled states with the distance $d$ in the order of a few micrometers $[13]$. Therefore, in such a timescales we must protect the system from decoherence from the environment. To eliminate the environments effect on the system we need a high vacuum and a low temperature. The estimated pressure is in the order of $10^{-15} \text{pa}$ and temperature must be about $0.1 \text{K} [12]$, which is an achievable condition with the current technologies $[60]$.

**VIII. CONCLUSION**

Quantization of the gravity remains as one of the most important, yet extremely illusive, challenges at the heart of modern physics. It has been argued that a direct empirical evidence for the quantum nature of the gravity (i.e., detecting gravitons) can shed light on the characteristics of the ultimate theory of the quantum gravity. However, such a task is far beyond the current capabilities and it seems not to be achievable in terms of near future technologies. To circumvent this impasse, it was recently shown that gravitationally-induced entanglement, tailored in the interferometric frameworks, can be used to detect the quantum nature of the gravity. However, many fundamental and empirical aspects
of these schemes are yet to be discovered. Considering the fact that, beside quantum entanglement, quantum uncertainty and complementarity principles are the two other foundational aspects of quantum physics, the quantum nature of the gravity needs to manifest all of these features. In this work, we considered an interferometric setup for testing these three nonclassical aspects of quantum mechanics in quantum gravity setting, which can shed light on the connections between gravity and quantum physics in a broader and deeper discipline. As we show in this work, all of these fundamental features of quantum gravity can be framed and fully analyzed in an interferometric scheme. We showed the relation between gravitationally-induced entanglement and complementarity principle and investigated its features. We also developed a relation between the violation of Bell inequality as a sufficient criteria for the quantumness of the entanglement and complementarity principle.

[1] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Physical review letters 108, 031101 (2012).
[2] R. Brustein, Fortschritte der Physik 62, 255 (2014).
[3] S. Carlip, Reports on progress in physics 64, 885 (2001).
[4] C. Kiefer, in Approaches to fundamental physics (Springer, 2007), pp. 123–130.
[5] C. Rovelli, Living reviews in relativity 11, 1 (2008).
[6] T. Padmanabhan, Modern Physics Letters A 30, 1540007 (2015).
[7] E. P. Verlinde (2017).
[8] J. Alfaro, Physical review letters 94, 221302 (2005).
[9] A. Peres and D. R. Terno, Reviews of Modern Physics 76, 93 (2004).
[10] T. Lunghi, J. Kaniiewski, F. Bussières, R. Houlmann, M. Tomamichel, A. Kent, N. Gisin, S. Wehner, and H. Zbinden, Physical review letters 111, 180504 (2013).
[11] Y. Maleki and A. Maleki, Physics Letters B 810, 135700 (2020).
[12] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Tóróis, M. Paternostro, A. A. Geraci, P. F. Barker, M. Kim, and G. Milburn, Physical review letters 119, 240401 (2017).
[13] C. Marletto and V. Vedral, Physical review letters 119, 240402 (2017).
[14] W. Heisenberg, pp. 478–504 (1985).
[15] N. Bohr, The quantum postulate and the recent development of atomic theory 1 (1928).
[16] M. S. Zubairy, Quantum field theory in a nutshell (Prince- ton university press, 2010).
[17] B.-G. Englert, M. O. Scully, and H. Walther, Scientific American 271, 86 (1994).
[18] W. K. Wootters and W. H. Zurek, Physical Review D 19, 473 (1979).
[19] R. J. Glauber, Annals of the New York Academy of Sciences 480, 336 (1986).
[20] L. Mandel, Optics letters 16, 1882 (1991).
[21] G. Jaeger, M. A. Horne, and A. Shimony, Physical Review A 48, 1023 (1993).
[22] B.-G. Englert, Physical review letters 77, 2154 (1996).
[23] M. Jakob and J. A. Bergou, Physical Review A 76, 052107 (2007).
[24] F. de Melo, S. Walborn, J. A. Bergou, and L. Davidovich, Physical review letters 98, 250501 (2007).
[25] X.-F. Qian, A. Vamivakas, and J. Eberly, Optica 5, 942 (2018).
[26] W. K. Wootters, Physical Review Letters 80, 2245 (1998).
[27] Y. Maleki, Optics letters 44, 5513 (2019).
[28] M. P. Hobson, G. P. Efstathiou, and A. N. Lasenby, General relativity: an introduction for physicists (Cambridge University Press, 2006).
[29] A. Zee, Quantum field theory in a nutshell, vol. 7 (Princeton university press, 2010).
[30] A. F. Radkowski, Annals of Physics 56, 319 (1970).
[31] G. Kirillin and I. Khriplovich, Journal of Experimental and Theoretical Physics 95, 981 (2002).
[32] N. E. J. Bjerrum-Bohr, J. F. Donoghue, and B. R. Holstein, Physical Review D 67, 084033 (2003).
[33] R. J. Marshall, A. Mazumdar, and S. Bose, Physical Review A 101, 052110 (2020).
[34] T. Andersen, in Journal of Physics: Conference Series (IOP Publishing, 2019), vol. 1275, p. 012038.
[35] T. Krisnanda, M. Zuppardo, M. Paternostro, and T. Paterek, Physical review letters 119, 120402 (2017).
[36] M. A. Nielsen and I. Chuang, Quantum computation and quantum information (2002).
[37] R. Colella, A. W. Overhauser, and S. A. Werner, Physical Review Letters 34, 1472 (1975).
[38] A. Einstein, B. Podolsky, and N. Rosen, Physical review 47, 777 (1935).
[39] J. S. Bell, Reprinted in Quantum Theory and Measurement p. 403 (1987).
[40] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Physical review letters 23, 880 (1969).
[41] R. Horodecki, P. Horodecki, and M. Horodecki, Physics Letters A 200, 340 (1995).
[42] S. Ghosh, G. Kar, A. Sen, U. Sen, et al., Physical Review A 64, 044301 (2001).
[43] F. Verstraete and M. M. Wolf, Physical review letters 89, 170401 (2002).
[44] F. Bagchi and A. K. Pati, Physical Review A 94, 042104 (2016).
[45] H. P. Robertson, Physical Review 34, 163 (1929).
[46] G. M. Bosyk, M. Portesi, F. Holik, and A. Plastino, Physica Scripta 87, 065002 (2013).
[47] I. Bialynicki-Birula and J. Mycielski, Communications in Mathematical Physics 44, 129 (1975).
[48] L. Maccone and A. K. Pati, Physical review letters 113, 260401 (2014).
[49] Q.-C. Song, J.-L. Li, G.-X. Peng, and C.-F. Qiao, Scientific reports 7, 1 (2017).
[50] A. J. Leggett, Progress of Theoretical Physics Supplement 69, 80 (1980).
[51] B. Julesgaard, A. Kochezkin, and E. S. Polzik, Nature 413, 400 (2001).
[52] K. C. Lee, M. R. Sprague, B. J. Sussman, J. Nunn, N. K. Langford, X.-M. Jin, T. Champion, P. Michelberger, K. F. Reim, D. England, et al., Science 334, 1253 (2011).
[53] P. V. Klimov, A. L. Falk, D. J. Christle, V. V. Dobrovitski, and D. D. Awschalom, Science advances 1, e1501015 (2015).
[54] C. Ockeloen-Korppi, E. Damskägg, J.-M. Pirkkalainen, M. Asjad, A. Clerk, F. Massel, M. Woolley, and M. Sillanpää, Nature 556, 478 (2018).
[55] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. Van der Zouw, and A. Zeilinger, nature 401, 680 (1999).
[56] S. Gerlich, S. Eibenberger, M. Tomandl, S. Nimmrichter, K. Hornberger, P. J. Fagan, J. Tüxen, M. Mayor, and M. Arndt, Nature communications 2, 1 (2011).
[57] K. Helmerson and L. You, Physical review letters 87, 170402 (2001).
[58] J. Peise, I. Kruse, K. Lange, B. Lücke, L. Pezzè, J. Arlt, W. Ertmer, K. Hammerer, L. Santos, A. Smerzi, et al., Nature communications 6, 1 (2015).
[59] T. W. van de Kamp, R. J. Marshman, S. Bose, and A. Mazumdar, Physical Review A 102, 062807 (2020).
[60] H. Ishimaru, Journal of Vacuum Science & Technology A: Vacuum, Surfaces, and Films 7, 2439 (1989).