Original Article

Dispersive Pressure, Boundary Jerk and Configurational Changes in Debris Flows

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The granular rock material within a debris flow experiences jerk (change in acceleration) as it runs over a rough basal bed or collides with sidewalls. This creates a pressure – the so-called dispersive pressure – which acts to change the configuration of the granular mass and therefore the frictional relationship of the debris flow with the basal boundary. Normal pressures are no longer hydrostatic and pressure fluctuations are created in the fluid phase. In this paper we formulate relationships between internal shear work, free mechanical energy, dispersive pressure and configurational changes within a debris flow. We associate the potential energy of the debris flow configuration with dilatant kinematic motions and show why it is necessary to integrate the shear work over time to calculate boundary jerks which cannot be represented by closed-form, analytical pressure functions. The effect of the dispersive pressure is mediated by the presence of the viscous muddy fluid which consists of two types: a) the free fluid and b) the bonded fluid attached to the solid granular phase.

\textbf{Key words:} sabo, debris flow, gravitational flow, modeling, configuration, dilatancy, non-hydrostatic pressure, dispersive pressure, jerk, free mechanical energy, two-phase, bonded fluid

1. INTRODUCTION

A debris flow consists of a muddy fluid laden with boulders, rocks and woody debris. The flow behavior of the mixture is defined by the interaction of the granular solid with the basal boundary as well as the viscous action of the muddy fluid; see Fig. 1 [Iverson, 1997]. Single phase debris flow models treat the mixture as a bulk material, using simple rheological models to describe the relationship between shear $S$ and deformation gradients in the slope parallel direction [Savage and Hutter, 1989; Hung, 1995; Rickenmann et al., 2006; Takahashi, 2007]. Two-phase models are similar except that they separate the flow material into solid and fluid components, positing separate mass and momentum balances and therefore applying different rheological laws for each phase. The components are coupled via momentum exchanges [Hutter et al., 1996].

Rheological models for debris flows do not allow for any motion of the granular solid or muddy fluid different from the mean slope-parallel velocity, see Fig. 2, [Iverson, 2003]. All debris is moving as a (sheared) bulk. Debris that encounters roughness at the basal boundary cannot move in any direction other than in the slope parallel direction defined by the terrain. Presently, models do not account for slope perpendicular movements of the debris. These movements are the cause of dilatancy and therefore the change in the internal flow configuration. Bagnold [1954] associated these slope-perpendicular movements to a “dispersive” pressure. When the dispersive pressure acts, the pressure distribution can no longer be hydrostatic. Furthermore, the dispersive pressure induces pore water pressure fluctuations that have been associated with effective stresses and therefore debris flow mobility [Iverson, 1997; McArdell et al. 2007].

Another restriction of rheological models is that the center-of-mass of a representative volume within the debris flow body is always located at the same position relative to the basal boundary (Fig. 2). For example, the center-of-mass is always located at half the debris flow height when homogeneous density distributions are assumed for the rock and muddy phases. With rheological models, two
Two-phase debris flow

Fig. 1 The debris flow is divided into streamwise segments or volumes V. Not only do the volumetric fractions of granular solid and muddy fluid vary, but also the configuration of the solid mass in the volume. The mass of the debris flow is divided into solid granular mass ($M_s$), the muddy fluid ($M_f$) and the fluid that is bonded to the solid mass ($M_b$).

Completely different flow configurations with the same rock and mud content will have the same rheological behavior. This hinders the realistic modeling of deposition induced by the sedimentation of the solid phase and dewatering [Iverson, 1997]. Bulk, rheological models are therefore “rigid” in the sense that they do not allow for non-hydrostatic pressures or a reconfiguration of the mass within the debris flow body due to the interaction with the basal boundary.

In this paper we will develop a method to include dispersive pressures (non-hydrostatic pressure) arising from interaction of the solid rock phase and the basal boundary. We show that dispersive pressures can exist only with a corresponding change in the configuration of the debris flow. That is, the result of the dispersive pressure is to change the slope-perpendicular speed and position of the center-of-mass of the granular rock mass. A product of the change in configuration is shear-induced dilatancy [Bagnold, 1954]. The mechanical description necessitates the introduction of two mechanical energies associated with the granular mass: the kinetic energy associated with velocity fluctuations [Haff, 1983] and the potential energy of the granular ensemble associated with rock locations. This energy is termed “free” in that it describes energy associated with deformations not described by the bulk movement of the flow.

We begin with a description of the mass and density in a debris flow. This is followed by a discussion of how the free mechanical energy changes the mass and density configuration of the debris flow.

2. DEBRIS FLOW MASS AND DENSITY

To model debris flow we divide the moving body into representative segments or volumes V (Fig. 1). The volume is defined by the basal area $A$ and flow height $h$. The volumes are fixed to a particular location (Eulerian formulation) and debris flow mass flows through the volume with mean velocities $u_s$ and $u_f$. The velocity $u_i$ is the speed of the muddy fluid $M_f$ that is moving at a speed different from the wetted granular solid phase.

Moreover, we assume some of the muddy material $M_b$ is bound to the solid granular phase which is moving with velocity $u_s$. Thus, the debris flow volume is divided into three distinct masses $M_f$ (free muddy phase), $M_b$ (bonded muddy material) and $M_s$ (granular solid phase, rock), see Fig. 3. The rock and the bonded material are moving with the same velocity $u_s$; the non-bonded muddy material is moving with velocity $u_f$. The total mass of the material within the representative volume is therefore $M_f + M_b + M_s = M$. We separate mass into these three categories because measured pore fluid pressure $p_f$ is associated with the free fluid phase as it is non-bonded fluid that can escape the granular matrix. The division of mass into three categories allows for fully saturated and partially saturated debris flows.

The masses are defined per unit area of the representative volume. For example, the fluid mass components are defined by the density of the muddy fluid $\rho_f$:

$$M_f = \rho_f h_f$$  \hspace{1cm} (1a)  
$$M_b = \rho_f h_b$$  \hspace{1cm} (1b)
The configuration of the solid mass within the volume can vary (Fig. 2). This results in different density distributions and heights. For example, the granular mass can be collapsed into a solid volume with height \( h_s \) measured with reference to density of the rock material \( \rho_o \):

\[
M_s = \rho_o h_s. \tag{2a}
\]

Or the granular mass can be defined with respect to a random packing density \( \rho_r \). In this case

\[
M_s = \rho_r h_s. \tag{2b}
\]

Because the height \( h_0 \) represents the height of a dry deposition pile of granules it is given a special designation, the “co-volume height”. Another option is to assume the granular mass is distributed uniformly over the flow volume. In which case

\[
M_s = \rho_s h_s \tag{2c}
\]

where \( h_s \) is the height of the solid phase in motion. In the unsaturated flow case, this height is equal to the observed height of the debris flow. The heights \( h_0 \) and \( h_s \) represent two completely different flow configurations that the debris flow may assume with the same mass.

The height \( h_0 \) is typically encountered when the debris flow has settled in the deposition zone, the random packing density being close to the deposition density of the solid phase. That is, it is encountered when the debris flow is at rest. A debris flow may have many different flow configurations \( h_s \).

3. DISPERSIVE PRESSURE AND JERK

To change the location of the solid rock material from the co-volume configuration \( h_0 \) to the flow configuration \( h_s \), requires mechanical work. The only source of work (energy) in a debris flow is the gravitational potential. Gravitational potential energy is converted into kinetic energy and the solid granular phase is set in motion. It is the interaction of the granular material with the boundary that converts the kinetic energy of the mean flow into “configurational” work energy. When the granular solid is moving downwards it encounters the terrain roughness. Rocks hit the ground and are reflected upwards back into the flow; local contact pressures are large [Iverson, 1997]. However, as we consider the total granular ensemble in the volume \( V \), the net effect of all the rocks hitting the ground is to accelerate the center-of-mass of the granular ensemble upwards in the slope perpendicular direction. We denote the position of the center-of-mass \( k_s \) and the slope-perpendicular acceleration \( k_z \) (Fig. 4). If the granular contacts with the basal boundary generate sufficient pressures to change the location of the center-of-mass there must be a corresponding reactive pressure at the basal boundary. This is the dispersive pressure \( N_g \) (see Fig. 4). This pressure is given by the mass of the rock and the bonded muddy fluid and the acceleration \( k_z \):

\[
N_g = \begin{bmatrix} M_b + M_s \end{bmatrix} k_z. \tag{3}
\]

The total reaction at the basal boundary \( N \) is the sum of the weight \( N_b \)

\[
N_b = \begin{bmatrix} M_s + M_f \end{bmatrix} g_z. \tag{4}
\]

and \( N_g \):

\[
N = N_g + N_b. \tag{5}
\]

The gravity component in the slope-perpendicular direction is denoted \( g_z \). An implicit assumption in this derivation is that the bonded muddy mass \( M_b \) is distributed uniformly over the granular solid.

Basal pressure measurements cannot distinguish between a change in mass \( M_s + M_b \) or a change in the location of the center-of-mass \( k_z \) because the total normal force \( N \) is the sum of \( N_b \) and \( N_g \). It is therefore difficult to determine the dispersive pressure experimentally. A connection to experimental measurements can nonetheless be made by noting that the time rate of change of the (measured) normal pressure \( \dot{N} \) is

\[
\dot{N} = \begin{bmatrix} M_s + M_b \end{bmatrix} k_s \tag{6}
\]

when the mass is constant in the debris flow volume \( V \). Because gravitational acceleration \( g_z \) is constant, it disappears from the time derivative. The quantity \( k_s \) is the boundary jerk, the time rate of change of the acceleration \( k_z \).

4. FREE MECHANICAL ENERGY

Shearing is the source of the energy needed to change the configuration of the debris flow. The work rate of the shear is
\[ \dot{W}_f = Su \]  

where \( S \) is the shear stress. The work rate \( \dot{W}_f \) represents the total work done per unit time in the debris flow body (W/m²). In rheological models the shear work is dissipated at the rate \( \dot{Q}_h \) to internal heat energy,

\[ \frac{D(Eh_s)}{Dt} = \dot{W}_f = \dot{Q}_h. \]  

This equation are now written using the material derivative notation \( \frac{D}{Dt} \) to indicate that we must also consider the convective transport of the any physical quantity in the debris flow.

In a model with configurational changes the shear work produces not only heat, but free mechanical energy \( R \). Free mechanical energy is divided into kinetic energy \( R_k \) and potential (configurational) energy \( R_v \).

\[ R = R_k + R_v. \]  

\( R_k \) is the kinetic energy associated with all rock particle movements different from the mean bulk velocity of the flow volume. When rocks hit the ground, they have some velocity component in the slope-perpendicular direction; that is, different from the mean downslope direction of the flow. The kinetic energy associated with the slope perpendicular movement is contained in \( R_k \). \( R_v \) is the potential energy of the center-of-mass of the granular solid. This energy is defined with respect base of the debris flow. Assuming a homogenous distribution of solid and bonded mass in the volume, results in the following definition:

\[ R_v = \frac{1}{2} (M_s + M_b) g \cdot h_s. \]  

One part of the free energy is therefore true kinetic energy \( R_k \), while the remaining part \( R_v \) describes the changed location of the particles. The production rates of \( R_k \) and \( R_v \) in the volume are denoted \( \dot{P}_k \) and \( \dot{P}_v \).

Thus, balance equations for the free mechanical energy can be written:

\[ \frac{D(R_k h_s)}{Dt} = \dot{P}_k h_s, \]  

\[ \frac{D(R_v h_s)}{Dt} = \dot{P}_v h_s \]  

and

\[ \frac{D(R_h_s)}{Dt} = \dot{P}_h_s = \dot{P}_k h_s + \dot{P}_v h_s. \]  

In configurational models, the shear work is not only dissipated to heat, but used to produce free mechanical energy:

\[ \dot{W}_f = \dot{Q}_h + \dot{P}_h_s. \]  

Different relations can be used to separate the heat dissipation from the production of free mechanical energy. In snow avalanche models the splitting is applied [Bartelt et al., 2006; Buser and Bartelt, 2009].

\[ \dot{P}_h_s = \alpha \dot{W}_f - \beta (R_k h_s) \]  

\[ \dot{Q}_h_s = (1 - \alpha) \dot{W}_f + \beta (R_k h_s) \]  

The parameter \( \alpha \) is the splitting parameter, defining the partitioning of the frictional work rate in the production of free mechanical energy \( \dot{P} \) and heat energy \( \dot{Q} \). The parameter \( \beta \) defines the dissipation of free energy of the granular solid by collisions, rubbing, abrasion, etc. It can be shown that this procedure is mass and energy conserving [Buser and Bartelt, 2009].

The sum of \( \dot{P}_k \) (change in random kinetic) and \( \dot{P}_v \) (change in configuration) therefore defines the production of total free energy \( \dot{P} \) in the volume \( V \):

\[ \dot{P} = \dot{P}_k + \dot{P}_v. \]  

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**Fig. 4** The debris flow consists of rock particles moving with mean velocity \( \mathbf{u} \). The kinetic energy associated with random particle movements is denoted \( R_k \). Rocks hit the basal boundary and are reflected back into the flow creating the dispersive pressure \( N_k \). The dispersive pressure is associated with a slope perpendicular acceleration \( \ddot{k} \) and jerk \( \dot{k} \). The jerk arises from the shear rate. To determine the dispersive pressure and therefore the dilatancy of the flow requires integrating the jerk (shear rate) over time.
We postulate that the fraction of mechanical energy $P_V$ with respect to the total production of free energy is the dimensionless coefficient $\gamma$:

$$\dot{P}_V = \gamma \dot{P} \quad \text{and} \quad \dot{P}_k = (1-\gamma) \dot{P} \quad (16)$$

The coefficient $\gamma$ resembles the coefficient of thermal expansion since it describes the tendency of a material to change its volume in response to a temperature change, in this case the granular temperature. For example when $\gamma = 1$, the entire free energy production from shearing is converted to potential energy. The coefficient $\gamma$ therefore describes the debris flow dilatancy.

5. CONFIGURATIONAL CHANGES

The problem now is to relate the configurational energy production $\dot{P}_V$ to the change in the location of the center-of-mass $k_s$. We emphasize that $\dot{P}_V$ is simply some fraction of the shear work $W_s = S u$ that is transformed into potential energy at the basal boundary. The work done by $\dot{P}_V$ is used to change the volume $V$. Therefore,

$$\frac{d(NV)}{dt} = \dot{P}_V V \quad (17)$$

or

$$NV + N\dot{V} = \dot{P}_V V \quad (18)$$

or, by substitution of Eqs. 3 - 6,

$$\left[M_b + M_s \right] \ddot{k}_s + \left[M_b + M_s \right] g_s + k_s \ddot{k}_s = \dot{P}_v \quad (19)$$

For the integration of the dispersive pressure and boundary jerk into numerical simulation programs, this equation can be conveniently written into a series of three first order differential equations:

$$\frac{Dk_s}{Dt} = w_s \quad (20a)$$

$$\frac{D(Mw_s)}{Dt} = N_k \quad (20b)$$

$$\frac{DN_k}{Dt} + N_s \frac{w_s}{k_s} = \dot{P}_v \quad (20c)$$

Moreover, the jerk associated with the convective mass transport can be considered in depth-averaged numerical models.

6. CONCLUSIONS

A longstanding problem in avalanche and debris flow mechanics is to understand how the streamwise frictional interaction with the basal boundary work.

Rheological descriptions of debris flow motion assume a rigid type bulk behavior. A single downslope velocity describes the motion of all mass. The interaction with the basal boundary is modeled entirely by linking the flow parameters to the shear. The total energy balance is given by the gravitational work rate, the mean kinetic energy of the bulk and the energy dissipation defined by the shear relationship.

![Fig. 5](image)

Rheological models assume a potential energy is converted to kinetic energy (gravitational work rate). The kinetic energy is dissipated entirely to heat (dissipation). There is no free mechanical energy. In configurational models, the shearing rate is split into dissipation and production of free energy. The free mechanical energy is partitioned into the kinetic energy associated with particle fluctuations and the potential energy. The potential energy is associated with the location of the rocks within the debris flow and...
therefore termed configurational energy. All potential energy is finally converted to heat energy.

Here we propose that rheological models should be extended to account for the free mechanical energy within the debris flow body. The free energy of the debris flow consists of two parts: (1) the kinetic energy of all the solid granular motions different from the mean downslope velocity of the flow and (2) the potential energy of the solid mass within the fluid phase. The introduction of the mechanical free energy leads to less “rigid” descriptions of debris flow motion. Rock particles now have velocities different from the mean, they can bounce, rotate and fluctuate. The product of the fluctuations is to change the mass distribution and therefore the configuration of the debris flow body.

Rheological models assume a potential energy is converted to kinetic energy, Fig. 5. The kinetic energy is dissipated entirely to heat. There is no free mechanical energy. In configurational models, the shearing rate is split into dissipation $\dot{Q}$ and production of free energy $P$, Fig. 5. The free mechanical energy is partitioned into the kinetic energy associated with particle fluctuations $R_k$ and the potential energy $R_v$. Therefore, to model the effect of free mechanical energy requires three additional model parameters. The first parameter defines the splitting of the shear work into the production of free energy (splitting parameter, dispersion parameter $\alpha$); the second parameter describes the decay of free energy by dissipative processes (the decay parameter $\beta$); the third parameter the partitioning of the free energy into kinetic and potential (dilatants) parts (partitioning parameter, dilatancy parameter $\gamma$).

The primary conclusion of our analysis is that it is impossible to derive closed form analytical functions to describe the dispersive pressure and therefore configurational changes in debris flows. The flow configuration – the distribution of solid mass in the debris flow body – is responsible for the frictional resistance and therefore the eventual stopping and deposition behavior of the flow. In the near future, it might be possible to devise numerical models that take into account streamwise variations in flow configuration, allowing a more accurate modeling of debris flow hazards.

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Received: 31 December, 2014
Accepted: 14 November, 2015