Accepted to the Astrophysical Journal Letters

The Discovery of the Radio Afterglow
From the Optically Dim $\gamma$-Ray Burst of March 29, 1998

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ABSTRACT

We report on the discovery of a variable radio source, VLA J070238.0+385044, associated with the proposed x-ray counterpart, 1SAX J0702.6+3850 of GRB 980329. The source was monitored from one day after the burst to one month later at centimeter wavelengths (1.4, 4.9, 8.3 and 15 GHz; Very Large Array – VLA), and in the 3-mm band (90 GHz; Owen Valley Radio Observatory – OVRO). Based on its variability, compactness and spectrum, we identify VLA J070238.0+385044 as the afterglow from GRB 980329. We interpret the rapid flux density variations as interstellar scintillation and the sharp turnover in the radio spectrum below 13 GHz as arising from synchrotron self-absorption. This suggests that the angular diameter of the fireball from GRB 980329 was of order a few microarcseconds in the first two weeks. The absence of a readily detectable afterglow in the optical, but clear detections in both the radio and infrared, can be understood as the result of extinction by dust. In this context it is interesting to note that half of all well-localized gamma-ray bursts have no optical counterpart. Based on our study of GRB 980329 we suggest that the progenitors of a sizable fraction of gamma-ray bursts are associated with regions of moderate to high gas density.

Subject headings: gamma rays: bursts - radio continuum: general

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1. Introduction

The gamma-ray burst GRB 980329 was detected on 1998 March 29.1559 UT by the gamma-ray burst monitor on the BeppoSAX satellite and localized to an error circle of 3′ radius by the Wide Field Camera (Frontera et al. 1998). The burst lasted a total of 55 s and was bright – a fluence of $5 \times 10^{-5}$ erg cm$^{-2}$ in the range 50–300 keV was measured by the Burst and Transient Experiment on board the Compton Gamma Ray Observatory satellite (Briggs et al. 1998).

Followup observations with the Narrow Field X-ray Instruments (NFI) on board BeppoSAX began 7 hrs after the burst. in’t Zand et al. (1998) detected a previously uncataloged X-ray source 1SAX J0702.6+3850 (with an error circle of 1′ radius) which was seen to fade by a factor of 3 over the 14 hr observation period. The burst position was further constrained by a timing annulus derived between the Ulysses and BeppoSAX satellites (Hurley et al. 1998).

A power law decay in the X-ray flux, $S_x \propto t^\delta$ appears to be a universal signature of the X-ray afterglows; $\delta$ ranges from $-1.1$ to $-1.5$ (e.g. Costa et al. 1997, Piro et al. 1998). This phenomenon of long-lived post-burst emission – afterglow – is nicely explained by GRB models known as fireballs, in which relativistically expanding debris shells interact with a surrounding medium and accelerate a power law distribution of particles (Mészáros & Rees 1997). As the fireball decelerates the peak of the emitted radiation shifts in time to lower energies producing power law decays in the X-ray, optical and radio bands (e.g. Waxman 1997a, Sari, Piran & Narayan 1998).

The successful detections at X-ray wavelengths have not been matched by equal numbers of afterglows at optical wavelengths. Only 4 of the $\sim 10$ GRBs with X-ray afterglows have detected optical afterglows (van Paradijs et al. 1997, Bond et al. 1997, Halpern et al. 1998, Groot et al. 1998). GRB 970828 is one notable non-detection; it exhibited a typical X-ray decay (Murakami et al. 1998), yet no optical transient was associated with this burst to a limit 100 times lower than that of the optical afterglow from GRB 970508 (Odewahn et al. 1997). Several authors have attempted to explain the diversity of afterglow behavior by invoking more complex fireball models (Mészáros, Rees, & Wijers 1998, Panaitescu, Mészáros, & Rees 1998), including such effects as non-isotropic expansion, non-uniform distributions of external media, and a time dependence in the radiative efficiency of the shock. Paczyński (1998) suggested that foreground extinction could account for the absence of optical afterglows from GRBs. Indeed, modest extinction has been inferred for the optical afterglow from GRB 971214 (Reichart 1998, Halpern et al. 1998, Ramaprakash et al. 1998). A logical extension of this extinction hypothesis is that, in some cases, the afterglow may be seen in the radio or infrared bands but not in the optical.

In this paper we report the detection of a variable radio source in the reduced error circle of GRB 980329 and argue that it is the radio afterglow from an optically obscured burst.
2. Observations

The first radio observations at 8 GHz with the Very Large Array (VLA) began on 1998 March 30.212 UT. Absolute flux calibration was derived from short observations of 3C 286 in the standard fashion, or by using J0713+438 which also has a very stable flux density (Taylor, Readhead & Pearson 1996). Phase calibration was derived from observations of the nearby calibrators J0653+370 or J0713+438.

Continuum observations in the 3 mm band were made with the Owens Valley Radio Observatory (OVRO) six element array between 1998 April 6 and 1998 April 11. A total of four runs, each with approximately 6 hours of on-source time, were obtained under good conditions. Projected baselines ranged from 30 to 120 m. Two 1 GHz bandwidth continuum channels with central frequencies of 88.6 and 91.6 GHz were observed simultaneously in the upper and lower side bands. Flux calibration was derived using 3C 273 with an absolute uncertainty of $\sim 20\%$. We used J0552+398 and J0646+448 for phase calibration.

3. Results

On 1998 April 1 (see Table 1), a modestly bright radio source, VLA J070238.0+385044 lying within the NFI error circle was detected. The precise position of this radio source is $\alpha$(J2000) = 07h02m38.0217, $\delta$(J2000) = 38$^\circ$50′44.017 with an uncertainty of 0.05 arcsec in each coordinate. Re-inspection of the March 30 observation revealed a 3$\sigma$ detection at the location of VLA J070238.0+385044. The flux density of the source dramatically decreased on April 2 (Table 1). This clear variability led us to announce that VLA J070238.0+385044 was potentially a radio afterglow of GRB 980329 (Taylor et al. 1998).

The source continued to exhibit variability as can be seen from Figure 1. The modulation index (defined as the standard deviation divided by the mean) for the period 1998 March 30 – April 20 is 0.43 at 8.3 GHz and 0.63 at 4.9 GHz. Beginning approximately three weeks after the initial gamma-ray burst the modulation in the flux density decreased (modulation index from April 21.91–30.10 is 0.30 and 0.39 at 8.3 and 4.9 GHz respectively). Perhaps the best evidence that the amplitude of the rapid flux variations has subsided is seen in the distribution of spectral indices in Table 1. Between April 6 and April 21 $\alpha$ varies from 0.37 to 2.96, whereas after this time $\bar{\alpha} \simeq 2.6$, suggesting that the underlying intrinsic spectrum has been revealed.

VLA J070238.0+385044 is unresolved in all our observations. The dataset of 1998 April 1 with its high signal-to-noise ratio allowed us to place an upper limit on the angular size of VLA J070238.0+385044 of $<50$ milliarcseconds. Based on observations on April 19.00 the linear polarization of VLA J070238.0+385044 is less than 21% (2$\sigma$) at 8.3 GHz.

A summary of the OVRO results at 90 GHz is presented in Table 2. In the first two runs, there were marginal (2$\sigma$) detections of a source within 2″ of VLA J070238.0+385044. The rms
in the sum of the first three days is 0.45 mJy beam$^{-1}$. A source with a peak flux of 1.45 mJy is found within 0.4$''$ of VLA J070238.0+385044.

Smith & Tilanus (1998) reported detections of a radio source coincident with VLA J070238.0+385044 using the 15-m James Clerk Maxwell Telescope (JCMT) at 350 GHz. Preliminary flux densities, on April 5.2, 6.2 and 7.2 UT were reported as $5 \pm 1.5$ mJy, $4 \pm 1.2$ mJy, and $2 \pm 0.8$ mJy, respectively. Combining the first two nights Smith & Tilanus claim a mean flux density of 4.5$\pm$1 mJy. Within the stated uncertainties, the JCMT source decayed very rapidly or the measurements errors are larger than reported. In view of this somewhat confusing picture, we will restrict the discussion below to measurements obtained from our VLA and OVRO efforts.

In Figure 2 we plot the time-averaged flux densities for the VLA J070238.0+385044. A linear least squares fit gives a spectral index $\alpha \simeq 0.9$ (where $\alpha$ is defined by $S_\nu \propto \nu^\alpha$). Closer inspection shows that while the radio spectrum rises with increasing frequency, $\alpha$ flattens, with $\alpha \simeq 1.7$ between 4.9 and 8.3 GHz, and $\alpha \simeq 0.8$ from 15 GHz to 90 GHz. We then fit the predicted $\nu^{1/3}$ power-law spectrum (Waxman 1997b), attenuated by a synchrotron self-absorption component of the form $(1 - e^{-\tau})/\tau$ where $\tau = \tau_o (\nu/\nu_o)^{-5/3}$. As we discuss in §4.2, Fig. 2 is sufficient to illustrate that a $\nu^{1/3}$ spectrum is not inconsistent with the data at high frequencies and that the synchrotron self-absorption turnover frequency (where $\tau = 1$) is close to 13 GHz. The extrapolated flux density at 350 GHz for this fit is 1.7 mJy.

What, then, is the nature of this compact, time-variable centimeter radio source? The probability of finding an unrelated radio source at 8.3 GHz in the NFI error circle is small, 8%, but it is not negligible (Windhorst et al. 1993). However, the properties of VLA J070238.0+385044 are at odds with the majority of sub-milliJansky radio sources at 5–10 GHz, which have flat spectral indices ($\bar{\alpha} = -0.4$), and are extended, ($\bar{\theta} = 2.6''$) (Windhorst et al. 1995, Richards et al. 1998). Optically they are usually identified with normal nearby spirals or disturbed, faint blue galaxies ($R \simeq 22$) whose radio emission is thought to originate from recent bursts of star formation.

It is conceivable that VLA J070238.0+385044 could be a member of that small fraction of the sub-mJy population that are identified with low-luminosity active galactic nuclei (AGN). The statistics of AGN flux density variability are poor at these low flux density levels (see summary in Frail 1998). However, the variability for these sources is broad-band and not the chromatic variability seen in Figure 1.

The properties of VLA J070238.0+385044 most closely resemble that of the early-time behavior of the radio afterglow from the GRB of 1997 May 8 (Frail et al. 1997, Taylor et al. 1997). Like VLA J070238.0+385044, the radio source VLA J065349.4+791619 was initially undetected at 1.43 GHz, exhibited large flux variations at 4.86 and 8.46 GHz (Frail et al. 1997), and had a rising spectrum toward higher frequencies (Shepherd et al. 1998). This marks only the second time a radio afterglow has been detected from a GRB. However, unlike the case for GRB 970508, no clear optical signature was detected from the afterglow of GRB 980329. In the days prior to the report of the radio detection several groups initially reported the absence of any optical variability in
the R-band in the range of $R > 20–23$ (Guarnieri et al. 1998, Klose, Meusinger, & Lehmann 1998, Djorgovski et al. 1998a). Following our announcement of the variable radio source (Taylor et al. 1998), we and others re-examined the optical and IR data at the precise sub-arcsecond position of the radio source. We identified a faint optical counterpart of VLA J070238.0+385044 (Djorgovski et al. 1998b). Evidence for a decaying counterpart was then found and reported as follows: I-band (Klose 1998), J-band (Cole et al. 1998), R-band (Palazzi et al. 1998a), and K-band (Larkin et al. 1998, Metzger 1998). These observations provide ample confirmation that VLA J070238.0+385044 is the radio afterglow from GRB 980329.

4. Discussion and Conclusion

4.1. The Mystery of Optically Dim GRBs

In §1 we alluded to the fact that about half of the well-localized GRBs appear not to have any associated optical transient. There are two solutions to this mystery (1) the afterglow decays rapidly in such GRBs or (2) Many GRBs suffer from extinction. Photoelectric attenuation at keV energies decreases as $\nu^{-3}$ whereas optical extinction $\propto \nu^q$ where $q$ is one or even larger. Thus, a modest column density, say $10^{21}$ cm$^{-2}$, in the host has little affect on the SAX passband of 1–60 keV (Wide Field Camera and the Narrow Field Instruments) but can suppress the observed R-band flux by almost 5 mag. Indeed the absence of detectable optical flux accompanying strong X-ray emission may simply be due to the strong redshift dependence of the suppression at observed optical wavelengths in comparison to the X–ray wavelengths, $\tau_{\text{opt}}/\tau_{\text{X–ray}} \propto (1+z)^{3+q}$.

The standard afterglow model of Mészáros & Rees (1997) predicts that the peak flux density will be independent of frequency. Our average 8.3 GHz flux density of 325 $\mu$Jy from the period April 21–30 implies a peak R-band magnitude $\sim 17$, yet only 15 hours after the burst R was greater than 20 (Klose, Meusinger, & Lehmann 1998). Observations of very red colors for GRB 980329 show that, at least for this GRB, dust extinction is the cause of the dim optical emission (Djorgovski et al. 1998c, Palazzi et al. 1998b). Evidence for dust has been inferred in GRB 971214 (Reichart 1998, Halpern et al. 1998, Ramaprakash et al. 1998); however, the inferred column density, while uncertain, is rather modest, $A_R \sim 1$ mag. In the case of GRB 980329, the R-band extinction is higher, making the detection of the afterglow easier at IR wavelengths.

If dust is indeed the main cause of optically dim GRBs then we conclude that extinction is significant in at least half of the GRBs. At face value this is not compatible with the currently popular model of coalescing neutron stars (Narayan, Paczyński, & Piran 1992). Such ns-ns binaries are expected to live long enough ($10^9$ yr) and acquire sufficiently high velocities to remove them from their high gas density birthplaces. An alternative model for GRBs is the “hypernova” model (Woosley 1993, Paczyński 1998), in which very massive stars produce a “dirty” fireball with $\sim 300$ times the luminosity of a supernova. Since such massive stars die young (age $\sim 10^6$ yr), a natural consequence of this model is that GRBs should be associated with high density, dusty
regions actively undergoing star formation. The large energies inferred for the γ-ray burst and optical afterglow of GRB 971214 (Kulkarni et al. 1998; Ramaprakash et al. 1998) present a further challenge to the coalescing ns-ns models. The hypernova model is capable of producing a more energetic and long-lived afterglow via its “dirty fireball” than current ns-ns models which produce a “clean fireball” (Paczyński 1998).

4.2. Future Evolution of VLA J070238.0+385044

Taking cues from the radio afterglow of GRB 970508 we expect that the current chromatic intensity variations will die and be replaced by broad-band lower modulation-index scintillation. Analyzing existing and future data in terms of diffractive and refractive scintillation will enable us to trace the angular evolution of the fireball (Frail et al. 1997, Waxman, Kulkarni, & Frail 1997). The character of the fluctuations from GRB 980329 are strikingly similar to GRB 970508 for which a size of 3 μarcsec was inferred (Goodman 1997, Frail et al. 1997). The angular size of GRB 980329 may be even smaller than this since it is at a lower Galactic latitude (b = 18.7° vs 26.7°) and the quenching of scattering scales inversely with sin(b) (Goodman 1997).

When these flux modulations die down (due to the fireball expansion) the underlying spectrum is revealed. Between 8.3, 15 and 90 GHz the spectral slope α is roughly consistent with the canonical 1/3 value expected for the low energy tail of the electron distribution (Fig. 2). However, between 4.9 and 8.3 GHz a value of α ≃ 2 is suggested at late times (see Table 1). A similar slope was seen for GRB 970508 between 1.43 GHz and 4.86 GHz (Shepherd et al. 1998), and was attributed to synchrotron self-absorption (Katz & Piran 1997). Again, as with GRB 970508, we determine from this an angular size for GRB 980329 of a few μarcsec (Katz & Piran 1997). The higher self-absorption frequency inferred for GRB 980329 suggests that it is more compact than GRB 970508, or in a denser environment, or that there exists an additional component of low energy electrons (Waxman 1997b) that increase the opacity at radio wavelengths. In keeping with what has been observed from GRB 970508 and model predictions, the radio spectrum should evolve in time as the synchrotron peak shifts to lower energies. Within a few months we will have sufficient data to carry out a comprehensive quantitative analysis of the radio observations with respect to fireball models.

We thank E. Waxman and the referee, D. Helfand, for helpful suggestions. The NRAO is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc. Research at the Owens Valley Radio Observatory is supported by the National Science Foundation through NSF grant number AST 96-13717. SRK’s research is supported by the National Science Foundation and NASA.
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# TABLE 1
VLA Observations of GRB 980329

| Epoch (UT 1998) | ∆t (d) | S(4.9) (µJy) | S(8.3) (µJy) | α (4.9 – 8.3) |
|-----------------|--------|--------------|--------------|--------------|
| Mar. 30.23      | 1.07   | 166 ± 50     | -            | -            |
| Apr. 01.13      | 2.97   | 256 ± 16     | -            | -            |
| Apr. 02.15      | 3.99   | 84 ± 23      | -            | -            |
| Apr. 03.14      | 4.98   | 109 ± 47     | -            | -            |
| Apr. 04.11      | 5.95   | 135 ± 21     | -            | -            |
| Apr. 05.12      | 6.96   | 194 ± 26     | -            | -            |
| Apr. 06.89      | 8.73   | 58 ± 37      | 22 ± 37      | 1.81 ± 1.80  |
| Apr. 08.06      | 9.90   | 179 ± 41     | 146 ± 49     | 0.37 ± 0.41  |
| Apr. 11.09      | 12.93  | 274 ± 34     | 143 ± 45     | 1.19 ± 0.34  |
| Apr. 18.04      | 19.88  | 227 ± 22     | 107 ± 25     | 1.41 ± 0.25  |
| Apr. 19.00      | 20.84  | 205 ± 17     | 42 ± 21      | 2.96 ± 0.51  |
| Apr. 20.27      | 22.11  | 88 ± 21      | 52 ± 31      | 0.98 ± 0.64  |
| Apr. 21.91      | 23.75  | 291 ± 17     | 54 ± 20      | 3.15 ± 0.37  |
| Apr. 23.10      | 24.94  | 245 ± 37     | 65 ± 43      | 2.48 ± 0.68  |
| Apr. 27.02c     | 28.86  | 465 ± 90     | 128 ± 42     | 2.41 ± 0.38  |
| Apr. 30.10      | 31.94  | 299 ± 51     | 85 ± 44      | 2.35 ± 0.55  |

\(^a\)A 1.4 GHz flux density of 38 ± 45 µJy was also measured.  
\(^b\)A 1.4 GHz flux density of 48 ± 33 µJy was also measured.  
\(^c\)A 15 GHz flux density of 350 ± 130 µJy was also measured.

**Notes to Table 1:** All observations were taken with the VLA in the A configuration, providing a resolution of 1.5″, 0.5″, 0.25″, and 0.18″ at 1.4, 4.9, 8.3, and 15 GHz respectively. The total duration of each observing run ranged between 0.5 and 5 hours. The 1σ errors for the flux density measurements are given, which in general are proportional to the square root of the time-on-source.
### TABLE 2
OVRO Observational Summary

| Epoch    | S(90) (mJy) | α(J2000) (h m s) | δ(J2000) (° ′ ″) | weather |
|----------|-------------|------------------|------------------|---------|
| Apr. 06  | 1.65 ± 0.70 | 07 02 38.03      | 38 50 44.77      | clear   |
| Apr. 08  | 1.67 ± 0.85 | 07 02 37.85      | 38 50 44.28      | clear   |
| Apr. 10  | 1.24 ± 0.79 | 07 02 37.96      | 38 50 44.34      | cloudy  |
| Apr. 11  | 0.72 ± 0.80 | ...              | ...              | cloudy  |
| First 3 runs |            |                  |                  |         |
| combined | 1.45 ± 0.45 | 07 02 38.02      | 38 50 44.39      | ...     |

**Notes to Table 2:** The images had a synthesized beam of $4.86'' \times 3.90''$ (FWHM) at position angle $-89.3^\circ$ with RMS noise level $\sim 0.8 \text{ mJy beam}^{-1}$ in a single run.
Fig. 1.— Radio light curve for GRB 980329. The time of the 4.9 GHz observations have been offset by 0.2 days to avoid overlap with the 8.3 GHz measurements. Upper limits (at 2σ) are shown for those measurements with a significance of less than 2σ.
Fig. 2.— The radio spectrum of GRB 980329 during the first month. The average flux densities are plotted at 4.9 and 8.3 GHz. The curve shows a $\nu^{1/3}$ spectrum with a low frequency cutoff imposed by synchrotron self-absorption. The turnover frequency (where $\tau_\circ = 1$) is $\sim 13$ GHz.