CP violation in open $t\bar{t}$ production at a linear collider

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Abstract

CP-violating asymmetries in the processes $e^+e^- \rightarrow t\bar{t}$ and $\gamma\gamma \rightarrow t\bar{t}$ can provide information about CP-violating couplings of the top quark, whose presence would signal physics beyond the standard model. Work on studies of different asymmetries has been described. Special emphasis has been laid on asymmetries in charged lepton distributions arising from top decay. The effect of longitudinal beam polarization has been discussed. Limits possible on the electric dipole moment of the top quark from different angular asymmetries have been obtained.

Linear colliders can provide a clean environment for the study of CP violation in top-quark couplings in the process $e^+e^- \rightarrow t\bar{t}$ and also in $\gamma\gamma \rightarrow t\bar{t}$. CP violation in the production process can lead to a definite pattern of deviation of $t$ and $\bar{t}$ polarizations from the predictions of the standard model (SM). A specific example is the asymmetry between the rate of production of $t_L\bar{t}_L$ and $t_R\bar{t}_R$, where $L, R$ denote helicities. Since top polarization is measured only by studying distributions of the decay products, it is advantageous to make predictions for these distributions, as far as possible, without reference to details of top reconstruction.

The usual procedure is to study either CP-violating asymmetries or correlations of CP-violating observables to get a handle on the CP-violating parameters of the underlying theory. For a given situation, correlations of optimal CP-violating variables correspond to the maximum statistical sensitivity. It is however convenient sometimes to consider variables which are simpler and can be handled more easily theoretically and experimentally.

CP violation can be studied in $e^+e^-$ collisions as well in the $\gamma\gamma$ collider option. Both these approaches are outlined below. In either case, it is seen that polarized beams help to increase the sensitivity.

SM does not predict measurable CP violation in the processes discussed here. Hence observation of a nonzero effect would signal physics beyond the standard model. Most popular extensions of SM predict electric dipole moment of the top quark to be at most $10^{-19} - 10^{-18}$ cm, which are extremely difficult to measure. As will be seen, methods described here cannot individually reach these limits, and would have to be taken in conjunction with one another to be able to reach such sensitivities.

A review of CP violation in top physics can be found in [1].

1 CP violation studies in $e^+e^- \rightarrow t\bar{t}$

CP violation in $e^+e^- \rightarrow t\bar{t}$ can mainly arise through the couplings of the top quark to a virtual photon and a virtual $Z$, which are responsible for $t\bar{t}$ production, and the $tbW$ coupling

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\[ ^1 \text{Talk presented at the 4th ACFA Workshop on Physics/Detector at the Linear Collider, Beijing, October 31 - November 2, 2001} \]
responsible for the dominant decay of the top quark into a \( b \) quark and a \( W \). The CP-violating couplings of a \( t \bar{t} \) current to \( \gamma \) and \( Z \) can be written as \( i e \Gamma^j_\mu \), where

\[
\Gamma^j_\mu = \frac{c^j_\mu}{2 m_t} i \gamma^5 (p_t - p_\tau)_\mu, \quad j = \gamma, Z, \tag{1}
\]

where \( e c^\gamma_\mu / m_t \) and \( e c^Z_\mu / m_t \) are the electric and “weak” dipole couplings.

While these CP-violating couplings may be studied using CP-violating correlations among momenta and spins which include the \( t \) and \( \bar{t} \) momenta and spins, it may be much more useful to study asymmetries and correlations constructed out of the initial \( e^+ / e^- \) momenta and the momenta of the decay products, which are more directly observable. In addition, the observables using top spin depend on the basis chosen [2, 3], and would require reconstruction of the basis which has the maximum sensitivity. In studying decay distributions, this problem is avoided.

Correlations of optimal CP-violating observables have been studied by Zhou [4]. Using purely hadronic or hadronic-leptonic variables, limits on the dipole moment of the order of \( 10^{-18} \) \( e \) \( \text{cm} \) are shown to be possible with \( \sqrt{s} = 500 \) GeV and integrated luminosity of 50 \( \text{fb}^{-1} \).

Examples of CP-violating asymmetries using single-lepton angular distributions and lepton energy correlations have been studied in [5]. In addition, other CP-violating asymmetries which are functions of lepton energy have been studied in [5]. Suitable ranges for the lepton energy, it is possible to enhance the relative contributions of CP violation in production and CP violation in decay [5]. It was shown [5, 6] that charged-lepton angular distributions are independent of any anomalous \( tbW \) couplings. Hence angular asymmetries of leptons can be used to study CP violation in the \( t \bar{t} \) production process, without pollution coming from CP violation in top decay.

One-loop QCD corrections can contribute as much as 30% to \( t \bar{t} \) production cross section at \( \sqrt{s} = 500 \) GeV [7]. It is therefore important to include these in estimates of sensitivities of CP-violating observables. The effect of QCD corrections in the soft-gluon approximation in decay lepton distributions in \( e^+ e^- \rightarrow t \bar{t} \) was discussed in [8]. These were incorporated in CP-violating leptonic angular asymmetries and corresponding limits possible at JLC with longitudinal beam polarization were presented in [9]. These are in the laboratory frame, do not need accurate detailed top energy-momentum reconstruction, and are insensitive to CP violation (or other CP-conserving anomalous effects) in the \( tbW \) vertex.

Four different asymmetries have been studied in [9]. In addition to two asymmetries \( A_{ch}(\theta_0) \) and \( A_{fb}(\theta_0) \) with the azimuthal angles integrated over, which are simple modifications of the asymmetries defined in [10], there are two others, which we call \( A_{ud}(\theta_0) \) and \( A_{lr}(\theta_0) \), and which depend on azimuthal distributions of the lepton. A cut-off \( \theta_0 \) in the forward and backward directions is assumed in the polar angle of the lepton. \( A_{ud}(\theta_0) \) and \( A_{lr}(\theta_0) \) were discussed earlier in [12], but in the absence of the cut-off \( \theta_0 \).

The charge asymmetry \( A_{ch}(\theta_0) \), which is simply the cross section asymmetry between \( l^+ \) and \( l^- \) with a cut-off \( \theta_0 \), is defined by

\[
A_{ch}(\theta_0) = \left. \int_{\theta_0}^{\pi - \theta_0} d\theta_i \left( \frac{d\sigma^+_{\theta_0}}{d\theta_i} - \frac{d\sigma^-_{\theta_0}}{d\theta_i} \right) / \left( \int_{\theta_0}^{\pi - \theta_0} d\theta_i \left( \frac{d\sigma^+_{\theta_0}}{d\theta_i} + \frac{d\sigma^-_{\theta_0}}{d\theta_i} \right) \right) \right|_{\theta_0}^{\pi - \theta_0}.
\]  

The other asymmetry \( A_{fb} \) is the leptonic forward-backward asymmetry combined with charge asymmetry, again with the angles within \( \theta_0 \) of the forward and backward directions.
Figure 1: The asymmetry $A_{ch}$ for $\text{Im } c_4^y = 1$, $\text{Im } c_2^y = 0$ (left figure), and for $\text{Im } c_4^y = 0$, $\text{Im } c_2^y = 1$ (right figure), plotted as a function of the cut-off $\theta_0$ on the lepton polar angle in the forward and backward directions for $e^-$ beam longitudinal polarizations $P_e = -0.9, 0, +0.9$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

Figure 2: The asymmetry $A_{fb}$ for $\text{Im } c_4^y = 1$, $\text{Im } c_2^y = 0$ (left figure), and for $\text{Im } c_4^y = 0$, $\text{Im } c_2^y = 1$ (right figure), plotted as a function of the cut-off $\theta_0$ on the lepton polar angle in the forward and backward directions for $e^-$ beam longitudinal polarizations $P_e = -0.9, 0, +0.9$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

excluded:

$$A_{fb}(\theta_0) = \frac{1}{2} \sigma(\theta_0) \int_{\theta_0}^{\pi-\theta_0} \int_{\theta_0}^{\pi-\theta_0} \frac{d\sigma^+}{d\theta_l} d\theta_l + \frac{d\sigma^-}{d\theta_l} d\theta_l - \frac{d\sigma^+}{d\theta_l} d\theta_l + \frac{d\sigma^-}{d\theta_l} d\theta_l + \frac{d\sigma^+}{d\theta_l} d\theta_l + \frac{d\sigma^-}{d\theta_l} d\theta_l.$$  

(3)

The up-down asymmetry is defined by

$$A_{ud}(\theta_0) = \frac{1}{2} \sigma(\theta_0) \int_{\theta_0}^{\pi-\theta_0} \left[ \frac{d\sigma^+}{d\theta_l} d\theta_l - \frac{d\sigma^-}{d\theta_l} d\theta_l + \frac{d\sigma^+}{d\theta_l} d\theta_l - \frac{d\sigma^-}{d\theta_l} d\theta_l + \frac{d\sigma^+}{d\theta_l} d\theta_l - \frac{d\sigma^-}{d\theta_l} d\theta_l \right] d\theta_l.$$  

(4)

Here up/down refers to $(p_t \pm)_y \geq 0$, $(p_t \pm)_y$ being the $y$ component of $p_t \pm$ with respect to a coordinate system chosen in the $e^+e^-$ center-of-mass (cm) frame so that the $z$-axis is along $p_e$, and the $y$-axis is along $p_e \times p_t$. The $t\bar{t}$ production plane is thus the $xz$ plane. Thus, “up” refers to the range $0 < \phi_l < \pi$, and “down” refers to the range $\pi < \phi_l < 2\pi$. 


Figure 3: The asymmetry $A_{ud}$ defined in the text, for $\text{Re } c_d^\gamma = 0.1$, $\text{Re } c_d^Z = 0$ (left figure), and for $\text{Re } c_d^\gamma = 0$, $\text{Re } c_d^Z = 0.1$ (right figure), plotted as a function of the cut-off $\theta_0$ on the lepton polar angle in the forward and backward directions for $e^-$ beam longitudinal polarizations $P_e = -0.9, 0, +0.9$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

Figure 4: The asymmetry $A_{tr}$ defined in the text, for $\text{Im } c_d^\gamma = 0.1$, $\text{Im } c_d^Z = 0$ (left figure), and for $\text{Im } c_d^\gamma = 0$, $\text{Im } c_d^Z = 0.1$ (right figure), plotted as a function of the cut-off $\theta_0$ on the lepton polar angle in the forward and backward directions for $e^-$ beam longitudinal polarizations $P_e = -0.9, 0, +0.9$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

The left-right asymmetry is defined by

$$A_{tr}(\theta_0) = \frac{1}{2 \sigma(\theta_0)} \int_{\theta_0}^{\pi-\theta_0} \left[ \frac{d\sigma^+_{\text{left}}}{d\theta_{l}} - \frac{d\sigma^+_{\text{right}}}{d\theta_{l}} + \frac{d\sigma^-_{\text{left}}}{d\theta_{l}} - \frac{d\sigma^-_{\text{right}}}{d\theta_{l}} \right] d\theta_{l}, \quad (5)$$

Here left/right refers to $(p_{l\pm})_x > 0$, $(p_{l\pm})_x$ being the $x$ component of $\vec{p}_{l\pm}$ with respect to the coordinate system system defined above. Thus, “left” refers to the range $-\pi/2 < \phi_l < \pi/2$, and “right” refers to the range $\pi/2 < \phi_l < 3\pi/2$.

Fig. 4 shows the asymmetry $A_{ch}$ arising when either of the (imaginary parts of) electric and weak dipole couplings takes the value 1, the other taking the value 0, plotted as a function of the cut-off $\theta_0$, for the polarized and unpolarized cases, for two different cm energies. Fig. 5 is the corresponding figure for $A_{fb}$.

Similarly, the asymmetries $A_{ud}$ from eq. (4) and $A_{tr}$ from eq. (5), which depend respectively on the real and imaginary parts of $c_d^{\gamma,Z}$, are shown in Figs. 3 and 4. Again, only one of the couplings takes a nonzero value, in this case 0.1, while the others are vanishing.

Tables 1-5 show the results on the limits obtainable for each of these possibilities. In all
Table 1: Individual 90% CL limits on dipole couplings obtainable from $A_{\text{ch}}$ and $A_{\text{fb}}$ for $\sqrt{s} = 500$ GeV with integrated luminosity 200 fb$^{-1}$ and for $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb$^{-1}$ for different electron beam polarizations $P_e$. Cut-off $\theta_0$ is chosen to optimize the sensitivity.

| $\sqrt{s}$ (GeV) | $P_e$ | $A_{\text{ch}}$ | $A_{\text{fb}}$ |
|------------------|-------|-----------------|-----------------|
|                  | $\theta_0$ | Im$c_{d}^{\gamma}$ | Im$c_{d}^{Z}$ | $\theta_0$ | Im$c_{d}^{\gamma}$ | Im$c_{d}^{Z}$ |
| 500              | 64$^\circ$ | 0.084 | 0.49 | 10$^\circ$ | 0.086 | 0.95 |
|                  | +0.9  | 64$^\circ$ | 0.081 | 0.17 | 10$^\circ$ | 0.075 | 0.15 |
|                  | −0.9  | 64$^\circ$ | 0.083 | 0.14 | 10$^\circ$ | 0.093 | 0.15 |
| 1000             | 64$^\circ$ | 0.029 | 0.18 | 10$^\circ$ | 0.032 | 0.36 |
|                  | +0.9  | 64$^\circ$ | 0.028 | 0.061 | 10$^\circ$ | 0.028 | 0.058 |
|                  | −0.9  | 64$^\circ$ | 0.028 | 0.047 | 10$^\circ$ | 0.034 | 0.058 |

Table 2: Simultaneous 90% CL limits on dipole couplings obtainable from $A_{\text{ch}}$ and $A_{\text{fb}}$ for $\sqrt{s} = 500$ GeV with integrated luminosity 200 fb$^{-1}$ and for $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb$^{-1}$ for unpolarized beams. Cut-off $\theta_0$ is chosen to optimize the sensitivity.

| $\sqrt{s}$ (GeV) | $\theta_0$ | Im$c_{d}^{\gamma}$ | Im$c_{d}^{Z}$ |
|------------------|------------|-------------------|---------------|
| 500              | 40$^\circ$ | 0.53              | 4.1           |
| 1000             | 40$^\circ$ | 0.20              | 1.5           |

In Table 1 are given the 90% confidence level (CL) limits that can be obtained on Im$c_{d}^{\gamma}$ and Im$c_{d}^{Z}$, assuming one of them to be nonzero, the other taken to be vanishing. The limit is defined as the value of Im$c_{d}^{\gamma}$ or Im$c_{d}^{Z}$ for which the corresponding asymmetry $A_{\text{ch}}$ or $A_{\text{fb}}$ becomes equal to $1.64/\sqrt{N}$, where $N$ is the total number of events.

Table 2 shows possible 90% CL limits for the unpolarized case, when results from $A_{\text{ch}}$ and $A_{\text{fb}}$ are combined. The idea is that each asymmetry measures a different linear combination of Im$c_{d}^{\gamma}$ and Im$c_{d}^{Z}$. So a null result for the two asymmetries will correspond to two different bands of regions allowed at 90% CL in the space of Im$c_{d}^{\gamma}$ and Im$c_{d}^{Z}$. The overlapping region of the two bands leads to the limits given in Table 2. In this case, for 90% CL, the asymmetry

| $\sqrt{s}$ (GeV) | $A_{\text{ch}}$ | $A_{\text{fb}}$ |
|------------------|-----------------|-----------------|
|                  | $\theta_0$ | Im$c_{d}^{\gamma}$ | Im$c_{d}^{Z}$ | $\theta_0$ | Im$c_{d}^{\gamma}$ | Im$c_{d}^{Z}$ |
| 500              | 64$^\circ$ | 0.11 | 0.20 | 10$^\circ$ | 0.11 | 0.20 |
| 1000             | 64$^\circ$ | 0.037 | 0.069 | 10$^\circ$ | 0.040 | 0.076 |

Table 3: Simultaneous limits on dipole couplings combining data from polarizations $P_e = 0.9$ and $P_e = −0.9$, using separately $A_{\text{ch}}$ and $A_{\text{fb}}$ for $\sqrt{s} = 500$ GeV with integrated luminosity 200 fb$^{-1}$ and for $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb$^{-1}$. Cut-off $\theta_0$ is chosen to optimize the sensitivity.
Table 4: Individual 90% CL limits on dipole couplings obtainable from $A_{ud}$ and $A_{lr}$ for $\sqrt{s}=500$ GeV with integrated luminosity $200~fb^{-1}$ and for $\sqrt{s}=1000$ GeV with integrated luminosity $1000~fb^{-1}$ for different electron beam polarizations $P_e$. Cut-off $\theta_0$ is chosen to optimize the sensitivity.

| $\sqrt{s}$ (GeV) | $P_e$ | $A_{ud}$ | $A_{lr}$ |
|------------------|-------|----------|----------|
|                  | $\theta_0$ | $\text{Re}c_{d}^\gamma$ | $\text{Re}c_{d}^Z$ | $\text{Im}c_{d}^\gamma$ | $\text{Im}c_{d}^Z$ | $\theta_0$ | $\text{Re}c_{d}^\gamma$ | $\text{Re}c_{d}^Z$ | $\text{Im}c_{d}^\gamma$ | $\text{Im}c_{d}^Z$ |
| 500              | 0     | 25°      | 0.10     | 0.034 | 30°   | 0.024 | 0.14 |
|                  | +0.9  | 30°      | 0.025    | 0.037 | 35°   | 0.024 | 0.050 |
|                  | −0.9  | 25°      | 0.022    | 0.032 | 30°   | 0.023 | 0.038 |
| 1000             | 0     | 30°      | 0.029    | 0.0096 | 60°   | 0.021 | 0.13 |
|                  | +0.9  | 35°      | 0.0068   | 0.010 | 60°   | 0.021 | 0.045 |
|                  | −0.9  | 30°      | 0.0061   | 0.0089 | 60°   | 0.021 | 0.035 |

Table 5: Simultaneous limits on dipole couplings combining data from polarizations $P_e=0.9$ and $P_e=-0.9$, using separately $A_{ud}$ and $A_{lr}$ for $\sqrt{s}=500$ GeV with integrated luminosity $200~fb^{-1}$ and for $\sqrt{s}=1000$ GeV with integrated luminosity $1000~fb^{-1}$. Cut-off $\theta_0$ is chosen to optimize the sensitivity.

| $\sqrt{s}$ (GeV) | $A_{ud}$ | $A_{lr}$ |
|------------------|----------|----------|
|                  | $\theta_0$ | $\text{Re}c_{d}^\gamma$ | $\text{Re}c_{d}^Z$ | $\theta_0$ | $\text{Im}c_{d}^\gamma$ | $\text{Im}c_{d}^Z$ |
| 500              | 25°      | 0.031    | 0.045 | 35°   | 0.031 | 0.056 |
| 1000             | 30°      | 0.0085   | 0.013 | 60°   | 0.028 | 0.052 |

CP violation studies in $\gamma\gamma \rightarrow t\bar{t}$

CP-violating dipole couplings of the top quark to photons can be studied at $\gamma\gamma$ colliders. The advantage over the study using $e^+e^-$ collisions is that the electric dipole moment is obtained independent of the weak dipole coupling to $Z$. Several proposals exist for the study of CP violation at $\gamma\gamma$ colliders.

Ma et al. [11] have discussed the CP-violating couplings of neutral Higgs in the context of
a two-Higgs doublet model. They studied the CP-violating asymmetries

\[ \xi_{\text{CP}} = \frac{\sigma_{tL\bar{t}L} - \sigma_{tR\bar{t}R}}{\sigma_{t\bar{t}}}, \]

for the case of unpolarized photon beams, and

\[ \xi_{\text{CP,1}} = \frac{\sigma_{tL\bar{t}L}^+ - \sigma_{tR\bar{t}R}^-}{\sigma_{tL\bar{t}L}^+ + \sigma_{tR\bar{t}R}^-} \]

and

\[ \xi_{\text{CP,2}} = \frac{\sigma_{tR\bar{t}R}^+ - \sigma_{tL\bar{t}L}^-}{\sigma_{tR\bar{t}R}^+ + \sigma_{tL\bar{t}L}^-} \]

for the case of circularly polarized photon beams, where the superscripts on \( \sigma \) denote the signs of the photon helicities, and the subscripts \( L \) and \( R \) denote left- and right-handed polarizations for the quarks. They found that asymmetries of the order of \( 10^{-4} - 10^{-3} \) can get enhanced to the level of a few percent in the presence of beam polarization, for reasonable values of the model parameters. Similar CP-violating observables have been identified in the MSSM by M.-L. Zhou et al. \[13\].

Choi and Hagiwara \[14\] and Baek et al. \[15\] have proposed the study of the number asymmetry of top quarks with linearly polarized photon beams, and found that a limit of about \( 10^{-17} \, \text{e cm} \) can be put on the electric dipole moment (edm) of the top quark with an integrated \( e^+ e^- \) luminosity of 20 fb\(^{-1} \) for \( \sqrt{s} = 500 \, \text{GeV} \).

| \( \lambda_e^1 \) | \( \lambda_e^2 \) | \( \lambda_t^1 \) | \( \lambda_t^2 \) | \( N \) | \( A_{ch} \) | \( A_{fb} \) | \( |A_{ch}| \) | \( |A_{fb}| \) |
|---|---|---|---|---|---|---|---|---|
| -0.5 | -0.5 | -1 | -1 | 55 | -0.019 | 0 | 3.22 | 0.25 |
| -0.5 | -0.5 | 1 | -1 | 215 | -0.025 | -0.129 | 1.28 | |
| -0.5 | -0.5 | 1 | 1 | 631 | -0.035 | 0 | 0.54 | |
| 0.5 | -0.5 | -1 | -1 | 63 | -0.024 | 0.013 | 2.49 | 4.58 |
| 0.5 | -0.5 | 1 | -1 | 23 | -0.021 | -0.080 | 4.53 | 1.20 |
| 0.5 | -0.5 | -1 | 1 | 163 | -0.021 | 0.033 | 1.72 | 1.11 |
| Unpolarized | 179 | -0.024 | 0 | 1.44 | |

Table 6: Asymmetries and corresponding 90% C.L. limits obtained on \( \text{Im } d_t \) for various combinations of initial-beam helicities. The initial electron beam energy of \( E_b = 250 \, \text{GeV} \), laser beam energy of \( \omega_0 = 1.24 \, \text{eV} \) and cut-off angle \( \theta_0 = 30^\circ \) are assumed. \( N \) is the total number of events, and the asymmetries are for a value \( \text{Im } d_t = 1/(2m_t) \).

Asymmetries of charged leptons from top decay in \( \gamma \gamma \to t\bar{t} \) with longitudinally polarized photons have been studied in \[16, 17\]. These asymmetries do not need full reconstruction of the top or anti-top. Generalizations of the asymmetries \( A_{ch} \) and \( A_{fb} \) described in the previous section (eqs. (2) and (3)) can be used to measure the electric dipole moment. The total number of events, the asymmetries and the 90% C.L. limits on the dipole moment at shown for various helicity combinations of the initial beams in Table 6. Here a cut-off angle \( \theta_0 \) of 30°...
Table 7: Number of events, asymmetries and limits on Im $d_t$, as in previous table, as a function of the beam electron beam energy $E_b$.

| $E_b$ (GeV) | N  | $A_{ch}$ | $A_{fb}$ | $|A_{ch}|$ | $|A_{fb}|$ |
|------------|----|----------|----------|-----------|-----------|
| 250        | 215| -0.025   | 0.129    | 1.282     | 0.247     |
| 500        | 1229| -0.167   | 0.420    | 0.080     | 0.032     |
| 750        | 1033| -0.223   | 0.347    | 0.065     | 0.042     |
| 1000       | 850| -0.227   | 0.244    | 0.071     | 0.066     |

is assumed, and the beam energy is taken to be 250 GeV. The geometric luminosity is assumed to be 20 fb$^{-1}$. The laser beam energy is assumed to be 1.24 eV. Table 7 shows the variation of various results with beam energy. Limits at the 90% confidence level of the order of $2 \times 10^{-17}$ e cm can be obtained with an $e^+e^-$ luminosity of 20 fb$^{-1}$ and cm energy $\sqrt{s} = 500$ GeV and suitable choice of electron beam and laser photon polarizations. The limit can be improved by a factor of 8 by going to $\sqrt{s} = 1000$ GeV.

It should be emphasized that the method relies on direct observation of lepton asymmetries rather than top polarization asymmetries, and hence does not depend heavily on the accuracy of top reconstruction.

We have also considered [18, 17] the asymmetries discussed in [16] to study the simultaneous presence of the top edm and an effective CP-violating $Z\gamma\gamma$ coupling. By using two different decay-lepton asymmetries, the top edm coupling and the $Z\gamma\gamma$ coupling can be studied in a model-independent manner.

Asakawa et al. [19] study the possibility of determining completely the effective couplings of a neutral Higgs scalar to two photons and to $t\bar{t}$ when CP is violated. They study the effects of a neutral Higgs boson without definite CP parity in the process $\gamma\gamma \rightarrow t\bar{t}$ around the pole of the Higgs boson mass. Near the resonance, interference between Higgs exchange and the continuum SM amplitude can be sizeable. This can permit a measurement in a model-independent way of 6 coupling constant combinations by studying cross sections with initial beam polarizations and/or final $t, \bar{t}$ polarizations. Using general (circular as well as linear) polarizations for the two photons, and different longitudinal polarizations for the $t$ and $\bar{t}$, in all 22 combinations could be measured, which could be used to determine the 6 parameters of the theory. Of these, half are CP-odd, and the remaining are CP-even. They also consider a specific example of MSSM, where CP-odd measurements are sensitive to the case of low $\tan\beta$.

To enable the use of observables directly measurable, we are in the process of evaluating angular and energy distributions of decay leptons in the laboratory frame [20]. Distributions are calculated for arbitrary photon polarizations and folded with the photon spectra arising from Compton scattering of polarized photons off longitudinally polarized electrons or positrons. Work is under progress to estimate CP-violating asymmetries and to examine the sensitivity of these to the CP-violating couplings.
3 Conclusions

The processes $e^+e^- \rightarrow t\bar{t}$ and $\gamma\gamma \rightarrow t\bar{t}$ can be useful probes of CP violating electric and weak dipole couplings of the top quark to $\gamma$ and $Z$, as also of CP-violating couplings to neutral Higgs.

Polarization can play a useful role in enhancing the sensitivity, as well as providing an additional parameter to tune for simultaneous model-independent determination of more than one parameter.

Results were presented for angular asymmetries of decay leptons in the laboratory frame, including $\mathcal{O}(\alpha_s)$ QCD corrections, for the $e^+e^-$ option. Only CP violation in production contributes to these asymmetries. Simultaneous model-independent 90% CL limits of order $10^{-17}$ $e$ cm can be placed on electric and weak dipole moments of the top quark for $\sqrt{s} = 500$ GeV and integrated luminosity of 200 fb$^{-1}$, with electron beam polarization of $\pm 0.9$. The limit can be improved by a large factor by going to $\sqrt{s} = 1000$ GeV and integrated luminosity 1000 fb$^{-1}$.

Using the $\gamma\gamma$ option the electric dipole coupling can be measured independent of the weak dipole coupling. Linear or circular polarization of photons can be used as an extra handle to isolate useful CP-odd asymmetries. Again asymmetries of decay-lepton angles in the laboratory frame were shown to be useful in studying CP violation. Limits on the top edm are of order of a few times $10^{-17}$ $e$ cm for the 500 GeV option, and somewhat better for higher $\sqrt{s}$.

The $\gamma\gamma$ option may also be used to study CP-violating $\gamma\gamma H$ couplings. Initial photon polarization combinations provide a number of asymmetries which could be measured. Decay lepton angular distributions in the laboratory frame could provide direct measurement of the CP-violating couplings.

It is clear that to reach sensitivities which can probe predictions of various extensions of SM which predict top electric dipole moment in the range of $10^{-19} - 10^{-18}$ $e$ cm, measurements from different asymmetries and from several decay channels, including hadronic ones, may have to be combined. Most important of all, future studies should concentrate on putting in realistic kinematic cuts against backgrounds and realistic detector efficiencies.

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