Spin waves in disordered magnetic systems

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Abstract. Long-wavelength spin waves in disordered magnetic systems have been investigated. In the framework of the Heisenberg model with magnetic dipole and exchange interactions between spins it is found that an additional longitudinal spin wave mode appears. This mode is characterized by variations of the value of the magnetic moment density. In order to analyse influence of the magnetic disorder on spin wave dispersion relations, the special case of volume and surface spin waves in the Damon-Eshbach (DE) geometry in films with magnetic disorder is considered. It is revealed strong influence of the magnetic disorder on surface spin waves, which consist of two branches – the DE mode and the longitudinal spin wave mode. Decrease of the ordering parameters leads to a decrease of the initial frequency and the curve slope of the DE mode dispersion curve and to an increase of the dispersion curve slope of the longitudinal mode. If the spin noncollinearity is high, then the DE mode dispersion assumes a curve with the backward character. It is found that the dispersion relation of the longitudinal mode is weak temperature dependent. The developed model can explain the observed double-peak structure of FMR spectra in magnetic nanocomposites.

1. Introduction

Amorphous magnetic materials and nanocomposites consisted of magnetic nanoparticles in insulating matrix are important types of disordered magnetic systems. Due to the amorphous structure, amorphous magnetic materials possess unusual properties such as the influence of fluctuations of the exchange interaction on the magnetic state, the appearance of the roton spin wave branch and the giant $\Delta E$-effect [1, 2, 3, 4]. Nanocomposites consisting of ferromagnetic metal nanoparticles embedded in an insulating matrix exhibit interesting properties such as giant magnetoresistance due to spin-dependent tunneling [5, 6, 7, 8, 9, 10, 11], the occurrence of additional modes in the ferromagnetic resonance (FMR) spectrum in a narrow range near the percolation threshold [12, 13, 14], sharp increase in the FMR linewidth with decreasing metal content [12, 13, 15], excellent soft magnetic properties governed by the interaction between ferromagnetic nanoparticles [16, 17] and the influence of inter-cluster dipolar interactions on the magnetic relaxation of nanoclusters [18, 19]. The effect of the injection magnetoresistance has been observed on heterostructures of silicon dioxide with cobalt nanoparticles grown on GaAs substrates [20, 21]. Owing to this, these heterostructures are of great interest for spintronic devices.

In order to analyze the above-mentioned properties and effects, it is utterly important to get information about magnetic structures of disordered systems. One of the effective tools, which can be used for the characterization of magnetic structures is spin waves propagating in these disordered magnetics. Spin waves are very sensitive to the magnetic disorder. This exhibits in changes of spin wave dispersion curves. For investigation of changes of dispersion curves, one can use spin waves generated by antennae [22, 23, 24] or thermally excited spin waves observed by the
Brillouin light scattering technique \cite{25, 26, 27, 28, 29}. In order to interpret experimental results, it is need to determine theoretical dependencies between spin wave dispersion curves and disordered parameters. This leads us to develop a model of spin waves in disordered systems.

In this paper, in the framework of the Heisenberg model with magnetic dipole and exchange interactions between spins we consider long-wavelength spin waves in disordered magnetic systems. Averaging pseudodifferential Landau-Lifshitz equations \cite{30}, in section 2 we derive the equation describing spin waves. In order to analyse influence of the magnetic disorder on spin wave dispersion relations, in section 3 we consider particular solutions for the case of spin waves in the Damon-Eshbach (DE) geometry. We have found that in disordered magnetic systems a longitudinal spin wave mode appears. This mode is analogous to plasmon modes in solid states \cite{31, 32}. In contrast with disordered systems, in ferromagnetically ordered ones the longitudinal mode is degenerated and is not observed. Taking into account the existence of the longitudinal spin wave mode in disordered systems, in section 4 it is supposed that the double-peak structure of FMR spectra observed in magnetic nanocomposites \cite{33, 34, 35, 36} can be explained by excitations of usual spin waves and the longitudinal spin wave mode.

2. Derivation of long-wavelength spin wave equation for disordered magnetic systems

In order to derive equation describing long-wavelength spin waves in disordered magnetics, we consider the Heisenberg model with magnetic dipole and exchange interactions between spins \cite{30}. We will use the approximation of effective propagators and interaction lines of the spin operator diagram expansion. In the framework of this approximation the local dynamics of the spin system is described by Landau-Lifshitz equations. The ac magnetic field originated from spins is described by the equation for the magnetostatic potential. The pseudodifferential Landau-Lifshitz equations have the generalized form and take into account longitudinal temperature dependent variations of the variable magnetization. Averaging over a small volume $\delta V$ and using the transformation from the local basis to the global one, we will obtain the average value of variations of the magnetic moment density. Substitution for this average value in the equation for the magnetostatic potential gives us the spin wave equation for disordered magnetic systems for spin waves with wavelength $\lambda \gg (\delta V)^{1/3}$.

In order to analyze disordered spin systems and to derive spin wave equation, we choose the local basis $(x', y', z')$ such that the axis $Oz'$ is parallel to the self-consistent field $\vec{H}^{(c)} = \vec{H} + \vec{H}^{(ex)} + \vec{H}^{(dip)}$ (figure 1), where $\vec{H}$ is the applied magnetic field, $\vec{H}^{(ex)}$ is the exchange field and $\vec{H}^{(dip)}$ is the dipole magnetic field originated from neighbouring spins. The exchange field can be written as $\vec{H}^{(ex)}(\vec{r}) = (g\mu_B)\sum_{\vec{r}'} \nabla' \cdot \vec{I}_{\mu \nu} \langle \langle S_{\nu}(\vec{r}') \rangle \rangle$, where $\mu, \nu = \{+, -, z'\}$, $\vec{I}_{\mu \nu}$ is the exchange interaction between spins, $g$ is the Landé factor, $\mu_B$ is the Bohr magneton and $\langle \langle S_{\nu}(\vec{r}') \rangle \rangle = \{\langle \langle S_x(\vec{r}') \rangle \rangle, \langle \langle S_y(\vec{r}') \rangle \rangle, \langle \langle S_z(\vec{r}') \rangle \rangle \}$ is the statistical average spin. It is supposed that the summation in this relation and in all following relations is performed over all repeating indices. The dipole magnetic field can be written as \cite{30}

$$H^{(dip)}_{\mu}(\vec{r}) = g\mu_B \nabla_{\mu} \sum_{\vec{r}'} \frac{1}{|\vec{r} - \vec{r}'|} \nabla' \cdot \langle \langle S_{\nu}(\vec{r}') \rangle \rangle$$

$$= \frac{g\mu_B}{V_a} \nabla_{\mu} \int_{V} \frac{1}{|\vec{r} - \vec{r}'|} \nabla' \cdot \langle \langle S_{\nu}(\vec{r}') \rangle \rangle \, d^3r' + H^{(a)}_{\mu}(\vec{r}),$$

(1)
where the first term is the depolarizing magnetic field of the continuum magnetic sample, $H^{(a)}_\mu(\vec{r})$ is the anisotropy magnetic field, $V_a$ is the atomic volume. Without action of ac magnetic fields at the site $\vec{r}$ the average value of spin $\langle\langle \vec{S}(\vec{r}) \rangle\rangle$ is parallel to the field $\vec{H}^{(c)}$. In this spin orientation the local energy minimum is occurred. We assume that in the vicinity of this local energy point the exchange interaction is isotropic, $2I_{zz}^{(0)} = 2I_{x-x}^{(0)} = I_{y-y}^{(0)}$. Then, if the ac magnetic field $h_\nu(\vec{r},\omega)$ acts on spins, in the local basis $(x',y',z')$ the Landau-Lifshitz equations give relations between small variations of the magnetic moment density, $m_\nu(\vec{r},\omega) = g\mu_B \delta(\langle\langle S_\nu(\vec{r},\omega)\rangle\rangle) / V_a$, and the field $h_\nu(\vec{r},\omega)$

\[ m_\pm(\vec{r},\omega) = 2\gamma \hat{E}_\pm^{-1} M(\vec{r}) h_\pm(\vec{r},\omega) \]  
\[ m_z(\vec{r},\omega) = \gamma \hat{E}_z^{-1} \frac{B^{[1]}(p)}{B(p)} M(\vec{r}) h_z(\vec{r},\omega), \]  

where $h_\pm = 1/2(h_x \mp ih_y)$, $\omega$ is the frequency, $\delta(\langle\langle S_\nu(\vec{r},\omega)\rangle\rangle)$ is the variation of the statistical average spin $\langle\langle S_\nu(\vec{r})\rangle\rangle$, $\gamma = g\mu_B / \hbar$ is the gyromagnetic ratio and $M(\vec{r}) = g\mu_B B(p) / V_a$ is the magnetic moment density. Functions

\[ B(p) = SB_S(Sp) \]  
\[ B^{[1]}(p) = S \frac{\partial B_S(Sp)}{\partial p} \]

are expressed by the Brillouin function $B_S$ for the spin $S$: $B_S(x) = (1 + 1/2S) \coth[(1 + 1/2S)x] - (1/2S) \coth(x/2S)$. $p = \beta g\mu_B H^{(c)}(\vec{r})$, $\beta = 1/kT$, $k$ is the Boltzmann constant, $T$ is the temperature. At low temperature the derivative of the Brillouin function $B^{[1]}(p)$ tends to 0 and the longitudinal variable magnetization $m_z$ is negligible. The Landau-Lifshitz operators have the form

\[ \hat{E}_\pm m_\pm(\vec{r},\omega) = [\gamma H^{(mag)}(\vec{r}) \pm \omega] m_\pm(\vec{r},\omega) \]
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\[ + \frac{4\pi\gamma\alpha M(\vec{r})}{V_b} \sum_{\vec{r}'} \int_{V_b} k^2 \exp[i\vec{k}(\vec{r} - \vec{r}')] m_{z}(\vec{r}', \omega) \, d^3k \]

\[ \hat{E}_z m_z(\vec{r}, \omega) = \omega m_z(\vec{r}, \omega), \]  

where \( H^{\text{mag}} = |\vec{H} + \vec{H}^{\text{dip}}| \) is the value of the sum of the external magnetic field and the dipole magnetic field acting on the spin \( \vec{S}(\vec{r}) \), \( V_b = (2\pi)^3/V_a \), \( \alpha = w V_a/4\pi(g\mu_B)^2 \) is the exchange interaction constant and \( w \) is the coefficient in the Fourier transform of the exchange interaction in the local energy minimum with respect to the space variables: \( \tilde{I}^{(0)}(\vec{k}) = \sum_{\vec{r}} I^{(0)}(\vec{r}) \exp(-i\vec{k}\cdot\vec{r}) = \tilde{I}^{(0)}(0) - w k^2 \). The Landau-Lifshitz equation \([3]\) is considered in the linear approximation with respect to \( B^{[1]}(p)/B(p) \), therefore, we have left the main factor \( \omega \) in the operator \( \hat{E}_z \) \((5)\) and have dropped out factors proportional to \( B^{[1]}(p) \) in this operator. In order to simplify notations, we write the Landau-Lifshitz equations \((2), (3)\) in the local basis in the short form

\[ m_{\mu}(\vec{r}, \omega) = \hat{\chi}^{(\text{loc})}_{\nu\mu}(\vec{r}, \omega), \]

where \( \hat{\chi}^{(\text{loc})} \) is the pseudodifferential tensor operator.

In the considered Heisenberg model the interaction between spins contains the magnetic dipole interaction. From the form of the magnetic dipole interaction \([30]\) it follows that the ac magnetic field \( h_{\nu} \) originated from neighbouring spins is magnetostatic, i.e. it is expressed in terms of the magnetostatic potential \( \varphi \): \( h_{\nu} = -\nabla_{\nu}\varphi \). In the approximation of effective propagators and interaction lines in the global basis \((x, y, z)\) the ac magnetic field of spin waves is described by the equation for the magnetostatic potential

\[ -\Delta \varphi(\vec{r}, \omega) + 4\pi \nabla_i m_i(\vec{r}, \omega) = 0, \]

where the index \( i \) is \((x, y, z)\).

Spin wave equation for disordered magnetic systems is derived by averaging the Landau-Lifshitz equations \((2), (3)\) over random variables and by the substitution of the average value of magnetic moment density variations for \( m_i \) in equation \((6)\). In order to substitute \( m_i \) in equation \((6)\), we must perform the transformation from the local basis \((x', y', z')\) to the global basis \((x, y, z)\) (figure \[1\]) and average the pseudodifferential tensor operator \( \hat{\chi}^{(\text{loc})}_{\nu\mu} \) over angles \( \theta, \psi, \phi \), the exchange interaction \( I^{(0)} \) and the value of the dipole magnetic field \( H^{\text{dip}} = |\vec{H}^{\text{dip}}| \). Averaging is performed in a small volume \( \delta V \) of the disordered system. The volume \( \delta V \gg V_a \) and must contain a great number of spins, but its size is considerably smaller then the size of the sample. The transformation \( U \) between the bases \((x', y', z')\) and \((x, y, z)\) are expressed through the Euler angles \( \theta, \psi, \phi \) \([37]\). Without loss of generality, the Euler angle \( \phi \) can be assumed equal to 0. Then, in the global basis \((x, y, z)\) the average tensor operator \( \hat{\chi}^{(\text{av})}_{ij} \), which gives relations between \( m_i \) and \( h_j \), \( m_i(\vec{r}, \omega) = \hat{\chi}^{(\text{av})}_{ij} h_j(\vec{r}, \omega) \), has the form

\[ \hat{\chi}^{(\text{av})}_{ij} = \int_{-\infty}^{\infty} \int_{0}^{\pi} \int_{0}^{2\pi} U^{-1}_{i\nu} \hat{\chi}^{(\text{loc})}_{\nu\mu} U_{\mu j} u(I^{(0)}, H^{\text{dip}}) f(\theta) \rho(\psi) \sin \theta \, dI^{(0)} \, dH^{\text{dip}} \, d\theta \, d\psi, \] 

where indices \( i, j \) are \((x, y, z)\), indices \( \nu, \mu \) are \((x', y', z')\) and \( u(I^{(0)}, H^{\text{dip}}), f(\theta), \rho(\psi) \) are distributions in the volume \( \delta V \). Distributions must be normalized by relations

\[ \int_{-\infty}^{\infty} \int_{0}^{\infty} u(I^{(0)}, H^{\text{dip}}) \, dI^{(0)} \, dH^{\text{dip}} = 1 \]
\[ \int_0^\pi f(\theta) \sin \theta \, d\theta = 1 \]

\[ \int_0^{2\pi} \rho(\psi) \, d\psi = 1. \]

We suppose that the angle \( \psi \) is the random variable with the distribution \( \rho(\psi) = (2\pi)^{-1} \). In this case, taking into account equations (2), (3), (4), (5) and well-known expressions of the transformation \( U \) versus the Euler angles, we have

\[
U = \begin{pmatrix}
\cos \psi & \sin \psi & 0 \\
-\cos \theta \sin \psi & \cos \theta \cos \psi & \sin \theta \\
\sin \theta \sin \psi & -\sin \theta \cos \psi & \cos \theta
\end{pmatrix}
\]

and integrating over \( \psi \) in relation (7), we find the average tensor operator

\[
\bar{\chi}^{(av)} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \begin{pmatrix}
F\xi + D\eta & G\zeta & 0 \\
-G\zeta & F\xi + D\eta & 0 \\
0 & 0 & 2F\eta + D(1 - 2\eta)
\end{pmatrix} u(I^{(0)}, H^{(dip)}) \, dI^{(0)} \, dH^{(dip)},
\]

where

\[
The \xi, \eta, \zeta \text{ are defined as in equation (7). Substituting } m_i = -\bar{\chi}_{ij}^{(av)} \nabla_j \varphi \text{ in equation (8), we obtain the equation describing spin waves in disordered magnetic systems }
\]

\[
\Delta \varphi(\vec{r}, \omega) + 4\pi \nabla_i \bar{\chi}_{ij}^{(av)} \nabla_j \varphi(\vec{r}, \omega) = 0.
\]

In connection with equation (9) it is need to make the following remarks.

1. Equation (9) describes long-wavelength spin waves with wavelength \( \lambda \gg (\delta V)^{1/3} \). Effects related to short-wavelength spin waves such as the appearance of the roton spin wave branch and the spin wave localization are not described by equation (9) and are out of our consideration.

2. Relaxation of spin waves can be taken into account in the one-loop approximation of the considered Heisenberg model, which is the next approximation after the approximation of effective
propagators and interaction lines \(^{[30]}\). Self-energy diagrams containing \(n\) loops give correction terms to Green functions and interaction lines proportional to \((V_a/R_{int}^3)^n\), where \(R_{int}\) is the radius of the interaction between spins. For \(V_a/R_{int}^3 \ll 1\) one-loop diagrams give greatest correction terms in comparison with self-energy diagrams with \(n > 1\) loops.

It is need to note that in the approximation of effective propagators and interaction lines the average value of the magnetic moment density is given as

\[
M^{(av)}(\vec{r}) = \int_{-\infty}^{\infty} \int_{0}^{\infty} M(\vec{r})\zeta u(I^{(0)}, H^{(dip)}) dI^{(0)} dH^{(dip)},
\]

where in the magnetic moment density \(M^{(av)}(\vec{r})\) the vector \(\vec{r}\) is the center of the small volume \(\delta V\). At the full randomness of the spin disorder the ordering parameter \(\zeta = 0\) and, therefore, \(M^{(av)}(\vec{r}) = 0\). But, at the same time, the tensor operator \(\hat{\chi}^{(av)}\) \(^{[8]}\) takes on the diagonal form and does not tend to zero.

3. Spin waves in films with magnetic disorder

In order to clarify peculiarities of long-wavelength spin waves in disordered magnetic systems, we consider spin waves in the DE geometry in films with magnetic disorder. In the DE geometry the orientation of the applied magnetic field \(\vec{H}\) is parallel to the film surface, spin waves propagate along the axis \(Ox\) and the wavevector \(\vec{q}\) is orthogonal to the field \(\vec{H}\) (figure \(2\)). We consider the case, when the spin wavelength \(\lambda \gg (\delta V)^{1/3}\) and the magnetic disorder is due to a noncollinearity of spins. Therefore, we suppose that in relations \(^{[7]}\) and \(^{[8]}\) the distribution \(u(I^{(0)}, H^{(dip)})\) is equal to \(\delta(I^{(0)} - I_0^{(0)})\delta(H^{(dip)} - H_0^{(dip)})\), where \(I_0^{(0)}\) is the average value of the exchange interaction between spins and \(H_0^{(dip)}\) is the average value of the dipole magnetic field acted from neighbouring spins. Since the depolarizing magnetic field of the continuum in-plane magnetized film is equal to zero \(^{[23]}\), in accordance with relation \(^{[11]}\) the dipole magnetic field \(\vec{H}_0^{(dip)}(\vec{r})\) can be reduced to the anisotropy magnetic field \(\vec{H}_0^{(a)}(\vec{r})\). Taking into account that the film is homogeneous through the thickness \(d\), we will find the solution of equation \(^{[9]}\) in the form

\[
\varphi(x, y, z, \omega) = \exp(iqx) \begin{cases} 
A_1 \exp(|q|y), & y < 0 \\
A_2 \exp(Qy) + A_3 \exp(-Qy), & 0 < y < d \\
A_4 \exp(-|q|y), & y > d
\end{cases}
\]

where \(Q\) is the transverse wavevector, \(q = 2\pi/\lambda\). The magnetostatic potential \(\varphi(\vec{r}, \omega)\) and the normal component of the ac magnetic induction must be continuous at boundaries. This gives the boundary conditions

\[
\varphi(\vec{r}, \omega)|_{+\partial} = \varphi(\vec{r}, \omega)|_{-\partial}
\]

\[
4\pi \hat{\chi}^{(av)}_{yx} \nabla_x \varphi(\vec{r}, \omega) + (1 + 4\pi \hat{\chi}^{(av)}_{yy}) \nabla_y \varphi(\vec{r}, \omega)|_{+\partial} = 4\pi \hat{\chi}^{(av)}_{yx} \nabla_x \varphi(\vec{r}, \omega) + (1 + 4\pi \hat{\chi}^{(av)}_{yy}) \nabla_y \varphi(\vec{r}, \omega)|_{-\partial},
\]

where \(\partial\) is the notation of the boundaries at \(y = 0\) and \(y = d\). The boundary conditions determine spin wave dispersion relations. In the DE geometry two types of solutions of equation \(^{[12]}\) exist – volume and surface spin waves.
3.1. Volume spin waves

Volume spin waves (figure 2) are characterized by imaginary values of the transverse wavevector $Q$. Sewing $\varphi(\vec{r}, \omega)$ in accordance with the boundary conditions (12), we determine the coefficients $A_1, \ldots, A_4$ in the form (11)

$$A_1 = A_4 = 0 \quad A_2 = -A_3$$

and get the spin wave dispersion relation

$$\omega_n = \{[\Omega_H + \alpha \Omega_M(q^2 + |Q_n|^2)][\Omega_H + \alpha \Omega_M(q^2 + |Q_n|^2) + \xi \Omega_M]\}^{1/2} + \delta \omega_n,$$

where the transverse wavevector $Q_n = i\pi n/d, n = 1, 2, 3, \ldots$ is the mode number, $\Omega_H = \gamma H^{(mag)} = \gamma |\vec{H} + \vec{H}_0^{(a)}|, \Omega_M = 4\pi\gamma M$. The term $\delta \omega_n$ is due to the longitudinal variations of the magnetic moment density $m_z$ in equation (3) caused by nonzero temperature. In the linear approximation with respect to $B^{(1)}(p)/B(p)$, this term is written as

$$\delta \omega_n = -\frac{\xi \eta B^{(1)}(p)\Omega_M^2}{2B(p)[\Omega_H + \alpha \Omega_M(q^2 + |Q_n|^2) + \xi \Omega_M]}.$$

For the case of ferromagnetic films at zero temperature, the term $\delta \omega_n = 0$, the parameter of ordering $\xi = 1$ and relation (13) coincides with the well-known relation of spin wave spectra for in-plane magnetized ferromagnetic films [23].

3.2. Surface spin waves

Surface spin waves (figure 2) are characterized by real values of the wavevector $Q$. Taking into account relation (8) and the form of the solution (11) in the interior region of the film, from equation (9) we find that $Q = q$. Sewing $\varphi(\vec{r}, \omega)$ in accordance with the boundary conditions (12), we can express the coefficients $A_1, A_3, A_4$ in the form (11) versus the coefficient $A_2$

$$A_3 = \frac{(\Omega_H^2 - \omega^2)(1 + a - \kappa) + \Omega_M(\xi \Omega_H + \zeta \omega)}{(\Omega_H^2 - \omega^2)(1 + a + \kappa) + \Omega_M(\xi \Omega_H - \zeta \omega)} A_2$$

$$A_1 = A_2 + A_3$$

(14)
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\[ A_4 = A_2 \exp(|q| + q) d + A_3 \exp(|q| - q) d, \]

where the parameter \( \kappa = q/|q| = \pm 1 \) defines the direction of the spin wave propagation and

\[ a = \eta B^{[1]}(p) \Omega_M / B(p) \omega. \]

The frequency \( \omega \) is determined by the spin wave dispersion relation

\[
\omega = \left\{ \Omega_H^2 + \frac{\Omega_M}{8} (4 \xi \Omega_H + \zeta^2 \Omega_M u) \right. \\
\left. \pm \frac{\Omega_M}{8} \left[ (4 \xi \Omega_H + \zeta^2 \Omega_M u)^2 + 16 \Omega_H^2 \left( \zeta^2 - \xi^2 \right) u \right]^{1/2} \right\}^{1/2} + \delta \omega,
\]

where \( u = 1 - \exp(-2|q| d) \). The term \( \delta \omega \) is due to the longitudinal variations of the magnetic moment density \( m_z \) in equation (3) and is given by

\[
\delta \omega = -\eta B^{[1]}(p) \Omega_M (\omega^2 - \Omega_H^2)(2 \omega^2 - 2 \Omega_H^2 - \xi \Omega_H \Omega_M u) / B(p) \omega^2 (8 \omega^2 - 8 \Omega_H^2 - 4 \xi \Omega_H \Omega_M - \zeta^2 \Omega_M^2 u).
\]

From relation (14) one can see that the magnetostatic potential \( \varphi(\vec{r}, \omega) \) of surface waves depends on the direction of the wave propagation – for the wavevector \( q > 0 \) the wave propagates along one surface and for \( q < 0 \) the wave propagates along the other surface (figure 2). The spin wave dispersion relation (15) determines two branches of surface spin waves. In order to elucidate influence of the spin noncollinearity on these branches, we have carried out calculations of dispersion curves (15) without the term \( \delta \omega \). Dispersion curves are presented in figure 3 for the magnetic film with the saturation magnetization \( 4\pi M = 5 \) kOe for different parameters of ordering, \( \xi \) and \( \zeta \). The magnetic field \( H^{(mag)} = |\vec{H} + \vec{H}_0^{(a)}| \), which is the sum of the applied magnetic field and the anisotropy magnetic field, is equal to 2 kOe for all curves and the gyromagnetic ratio \( \gamma \) is equal to \( 2 \pi \cdot 2.83 \) MHz/Oe. Spin waves propagating in opposite directions have identical dispersion curves. For \( \xi = \zeta = 1 \) spins have ferromagnetic ordering. In this case, the upper branch is the DE mode [23], the lower branch at the frequency \( F = \omega/2\pi = \Omega_H/2\pi = 5.66 \) GHz is degenerated. Usually, the lower branch is not taken into account and is neglected. Decrease of the ordering parameters \( \xi \) and \( \zeta \) leads to a decrease of the initial frequency of the DE mode dispersion curve and to a decrease of its slope. At the same time, the degeneration of the lower branch is removed. For films with the spin noncollinearity of high values, the DE mode dispersion assumes a curve with the backward character. The limiting case of the spin disorder – full randomness – corresponds to the ordering parameters \( \xi = 2/3 \) and \( \zeta = 0 \). At these ordering parameters the presented curves in figure 3 can be regarded as the limit of surface spin wave dispersion relations.

4. Discussion

Consideration of dispersion curves of long-wavelength spin waves presented above is carried out for films in the DE geometry and exhibits two branches of surface spin waves. In consequence of this consideration, it is arisen the following question: what is the nature of these two spin wave branches, which are observed in disordered magnetic systems? It is need to note that only one of these branches is observed in ferromagnets. In order to explain this problem, we consider rotation
Figure 3. Influence of the spin noncollinearity on two branches of long-wavelength surface spin wave dispersion curves in the film with a magnetic disorder with the saturation magnetization $4\pi M = 5 \text{kOe}$ at the magnetic field $H^{(mag)} = 2 \text{kOe}$. The upper branch is the DE mode and the lower branch is the longitudinal spin wave mode. The wavevector $q$ is normalized by the thickness $d$. The parameters of ordering are: (1) $-\xi = 1, \zeta = 1$; (2) $-\xi = 0.9, \zeta = 0.7$; (3) $-\xi = 0.75, \zeta = 0.3$; (4) $-\xi = 2/3, \zeta = 0$.

of two noncollinear spins in the field $\vec{H}^{(c)}(\vec{r})$ in the volume $\delta V$ (figure 4b). In the common case, due to different orientations and values of the field $\vec{H}^{(c)}(\vec{r})$ at different points 1 and 2 of the space variable $\vec{r}$, variations $\delta \vec{S}_1, \delta \vec{S}_2$ and their phases are different

$$
\vec{S}_1(t) = \vec{S}_1^{(0)} + \delta \vec{S}_1 \exp(i\omega_1 t)
$$

$$
\vec{S}_2(t) = \vec{S}_2^{(0)} + \delta \vec{S}_2 \exp(i\omega_2 t),
$$

(16)

where $\vec{S}_1^{(0)}, \vec{S}_2^{(0)}$ are nonperturbed spins in the points 1 and 2, respectively. Averaging in relations (11) and (12) in the volume $\delta V$ corresponds to the summation of rotating spins. According to relations (16), this leads to changes in the value of the sum of spins during their rotation (figure 4b)

$$
|\vec{S}_1(t) + \vec{S}_2(t)| = |\vec{S}_1^{(0)} + \vec{S}_2^{(0)}|^2 + (\vec{S}_1^{(0)} \cdot \delta \vec{S}_2) \exp(i\omega_2 t) + (\vec{S}_2^{(0)} \cdot \delta \vec{S}_1) \exp(i\omega_1 t)|^{1/2}
$$

(17)

and, consequently, the value of the total spin of the volume $\delta V$ can be changed. For simplicity, in relation (17) we restrict ourself by the linear approximation with respect to $|\delta \vec{S}_{1(2)}|/|\vec{S}_{1(2)}|$. From relation (17) one can conclude that in disordered systems the average magnetic moment density $M^{(av)}(\vec{r})$ given by relation (10) can be variable. This leads to the existence of spin excitations of two types: (1) rotations of the total spin of the volume $\delta V$ and (2) variations of the value of this total spin or variations of the value of the magnetic moment density $M^{(av)}(\vec{r})$. The first excitations are
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(a) Rotation of spins $\vec{S}_1$ and $\vec{S}_2$ at different points 1 and 2 of the space variable in the volume $\delta V$. (b) Changes in the value of the sum of spins.

Figure 4. (a) Rotation of spins $\vec{S}_1$ and $\vec{S}_2$ at different points 1 and 2 of the space variable in the volume $\delta V$. (b) Changes in the value of the sum of spins.

analogous to excitations in ferromagnets. The second ones can be called as longitudinal excitations. They are not observed in ferromagnets. Longitudinal excitations are more expressed in the case of full spin randomness. In this case, ordering parameters reach the values: $\xi = 2/3$, $\zeta = 0$, $\eta = 1/3$, and the average tensor operator $\hat{\chi}^{(av)}$ has the diagonal form. Due to the pole singularity at $\omega = \Omega_H$ in the operator $\hat{\chi}^{(av)}$, for the longitudinal spin wave (the lower branch in figure 3) the magnetic moment density variations $m_i = -\hat{\chi}^{(av)}_{ij}\nabla_j \phi$ have high values.

Longitudinal spin excitations are analogous to plasmon excitations in solid states. Plasmons are collective oscillations of the electron density. In plasma the Coulomb interaction between electrons is dominant and is determined by the electrical potential. The longitudinal spin excitations in disordered magnetic systems are characterized by variations of the value of the magnetic moment density and are described by the magnetostatic potential given by equation (9), which are analogous to the electron density and to the electrical potential in plasma, respectively. In this sense, the longitudinal spin excitations can be called as the plasmon-like spin wave modes.

It is need to note differences between DE and longitudinal modes. (1) According to spin wave dispersion relation (15) for in-plane magnetized films, the frequency $\omega$ of the DE mode is higher than the frequency of the longitudinal spin wave mode. This increase of the DE mode frequency is due to the nonzero value of the saturation magnetization $4\pi M$. This leads to the nonzero value of $\Omega_M$ in relation (15). In contrast with the DE mode, the frequency of the longitudinal mode is close to $\Omega_H$ determined by the magnetic field $H^{(mag)}$, which is the sum of the applied magnetic field and the anisotropy magnetic field. (2) Owing to the inequality $\omega > \Omega_H$ for the DE mode, the term $\delta \omega$ in the dispersion relation (15) depends on temperature through the Brillouin function factor.
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$B^{(1)}(p)/B(p)$ and the frequency of the DE mode decreases with temperature increasing. In contrast with this, since $\omega \to \Omega_H$ for the longitudinal mode, the dispersion relation of the longitudinal mode slightly depends on temperature.

FMR spectrum is the limit case of spin waves – the spin wave wavevector $q$ is equal to zero. In this limit case, due to the pole singularity at $\omega = \Omega_H$ in the operator $\chi^{(av)}$, for the longitudinal spin wave mode in disordered magnetic systems the magnetic moment density variations $m_i = -\chi^{(av)}_{ij} \nabla_j \varphi$ are not zero at $\nabla_j \varphi \to 0$. This leads to an additional peak in the FMR spectrum. In magnetic nanocomposites the double-peak structure of FMR spectra is observed [33, 34, 35, 36]. A weak secondary peak appears in FMR spectra in nanogranular films composed of ferromagnetic Fe nanoparticles embedded in SiO$_2$ glass matrices [33]. FMR profiles for nanocomposites Ni/ZnO consisted of nanosized Ni and ZnO particles shows a secondary peak which appears at the high field tail part of the absorption curve [33]. Two resonance peaks are observed in the $\mu''$ spectra for Co/ZnO nanocomposites [34]. Microwave spectra of Ni/ZnO and Ni/γ-Fe$_2$O$_3$ nanocomposites possess a double-peak behavior in the losses over 2 - 16 GHz frequency range [36]. Taking into account that the observed effect can be caused by oxide structures formed in the nanoparticle interfacial regions, another explanation of the double-peak structure of FMR spectra can be proposed on the base of the developed model of spin waves in disordered magnetic systems. According to this model, one peak is the usual rotation of the total spin. In the FMR spectra this peak corresponds to usual spin waves with the zero wavevector, $q = 0$. The second peak is due to variations of the value of the magnetic moment density $M^{(av)}(\vec{r})$. This peak corresponds to the longitudinal spin wave mode at $q = 0$.

5. Conclusion

We have investigated long-wavelength spin waves in disordered magnetic systems. The results of the investigations performed can be summarized as follows.

(1) In disordered magnetic systems a longitudinal spin wave mode is appeared. In ferromagnets this mode is degenerated and is not observed. The longitudinal spin excitation in disordered systems is characterized by variations of the value of the magnetic moment density. In this sense, this spin excitation can be called as the plasmon-like spin wave mode.

(2) The special case of long-wavelength spin waves in the DE geometry in films with magnetic disorder are considered. It is found that the magnetic disorder influences volume and surface spin waves. Surface spin waves have two branches – the DE mode and the longitudinal spin wave mode. Decrease of the ordering parameters leads to a decrease of the initial frequency of the DE mode, to a decrease of the slope of the DE mode dispersion curve and to an increase of the dispersion curve slope of the longitudinal mode. For films with the spin noncollinearity of high values, the DE mode dispersion assumes a curve with the backward character. It is found that the dispersion relation of the longitudinal mode is weak temperature dependent.

(3) The observed double-peak structure of FMR spectra in magnetic nanocomposites can be explained in the framework of the developed model.

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References

[1] Petrakovskii G A 1981 Sov. Phys. Uspekhi 24 511
[2] Kaneyoshi T 1992 Introduction to Amorphous Magnets (Singapore: World Scientific)
[3] The Magnetism of Amorphous Metals and Alloys 1995 ed. J A Fernandez-Baca and Wai-Yim Ching (Singapore: World Scientific)
[4] O’Handley R C 2000 Modern Magnetic Materials: Principles and Applications (New York: John Wiley & Sons)
[5] Dieny B, Sankar S, McCartney M R, Smith D J, Bayle-Guillemaud P and Berkowitz A E 1998 J. Magn. Magn. Mater. 185 283
[6] Mitani S, Takanashi K, Yakushiji K and Fujimori H 1998 J. Appl. Phys. 83 6524
[7] Yakushiji K, Mitani S, Takanashi K, Ha J-G and Fujimori H 2000 J. Magn. Magn. Mater. 212 75
[8] Lutsev L V, Kalinin Yu E, Sitnikov A V and Stognii O V 2002 Phys. Solid State 44 1889
[9] Aronzon B A, Kovalev D Yu, Varfolomeev A E, Likal’ter A A, Ryl’kov V V and Sedova M A 1999 Phys. Solid State 41 857
[10] Granovsky A B, Kozlov A A, Bagmut T V, Nedukh S V, Tarapov S I and Clerc J P 2005 Phys. Solid State 47 738
[11] Sankar S, Dender D, Borchers J A, Smith D J, Erwin R W, Kline S R and Berkowitz A E 2000 J. Magn. Magn. Mater. 221 1
[12] Wang Wen-Nai, Jiang Zheng-Sheng and Du You-Wei 1995 J. Appl. Phys. 78 6679
[13] Butera A, Zhou J N and Barnard J A 1999 Phys. Rev. B 60 12270
[14] Butera A, Zhou J N and Barnard J A 2000 J. Appl. Phys. 87 5627
[15] Lutsev L V, Kazantzseva N E, Tchmutin I A, Ryvkina N G, Kalinin Yu E and Sitnikoff A V 2003 J. Phys.: Condensed Matter 15 3665
[16] Morikawa T, Suzuki M and Taka Y 1998 J. Appl. Phys. 83 6664
[17] Ohnuma S, Kobayashi N, Masumoto T, Mitani S and Fujimori H 1999 J. Appl. Phys. 85 4574
[18] Luis F, Bartolome J, Petroff F, Garcia L M, Vaures A and Carrey J 2003 J. Appl. Phys. 93 7032
[19] Luis F, Petroff F and Bartolome J 2004 J. Phys.: Condensed Matter 16 5109
[20] Lutsev L V, Stognij A I and Novitskii N N 2005 JETP Letters 81 514
[21] Lutsev L V, Stognij A I, Novitskii N N and Stashkevich A A 2006 J. Magn. Magn. Mater. 300 e12
[22] Stancil D D 1993 Theory of Magnetostatic Waves (New York: Springer)
[23] Gurevich A G and Melkov G A 1996 Magnetization Oscillations and Waves (New York: CRC Press)
[24] Kabos P and Stalmachov V S 1994 Magnetostatic Waves and Their Applications (New York: Chapman and Hall)
[25] Jorzik J, Demokritov S O, Mathieu C, Hillebrands B, Bartenlian B, Chappert C, Rousseaux F and Slavin A N 1999 Phys. Rev. B 60 15194
[26] Roussigné Y, Cheri M, Dugautier C and Moch P 2001 Phys. Rev. B 63 134429
[27] Demokritov S O, Hillebrands B and Slavin A N Phys. Rep. 2001 348 441
[28] Grimsditch M, Guedes I, Vavassori P, Methushko V, Ilic B, Neuzil P and Kumar R 2001 J.Appl. Phys. 89 7096
[29] Gubbio G, Candeloro P, Businaro L, Di Fabrizio E, Geradino A, Zivieri R, Conti M and Carlotti G 2003 J.Appl. Phys. 93 7595
[30] Lutsev L V 2005 J. Phys.: Condens. Matter 17 6057
[31] Platzman P M and Wolff P A 1973 Waves and Interactions in Solid State Plasmas (London: Academic Press)
[32] Maier S A 2007 Plasmonics. Fundamentals and Applications (New York: Springer)
[33] Tomita S, Hagiwara M, Kashiwagi T, Tsuruta C, Matsui Y, Fujii M and Hayashi S 2004 J.Appl. Phys. 95 8194
[34] Brosseau C and Talbot P 2005 J.Appl. Phys. 97 104325
[35] Castel V, Youseff J B and Brosseau C 2007 Journal of Nanomaterials ID 27437
[36] Lutsev L, Yakovlev S and Brosseau C 2007 J.Appl. Phys. 101 034320
[37] http://mathworld.wolfram.com/EulerAngles.html