Di-neutron correlation in light neutron-rich nuclei

K. Hagino\textsuperscript{1}, H. Sagawa\textsuperscript{2}, and P. Schuck\textsuperscript{3,4}

\textsuperscript{1} Department of Physics, Tohoku University, Sendai, 980-8578, Japan
\textsuperscript{2} Center for Mathematical Sciences, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8560, Japan
\textsuperscript{3} Institut de Physique Nucléaire, CNRS, UMR8608, Orsay, F-91406, France
\textsuperscript{4} Université Paris-Sud, Orsay, F-91505, France

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Using a three-body model with density-dependent contact interaction, we discuss the root mean square distance between the two valence neutrons in \textsuperscript{11}Li nucleus as a function of the center of mass of the neutrons relative to the core nucleus \textsuperscript{9}Li. We show that the mean distance takes a pronounced minimum around the surface of the nucleus, indicating a strong surface di-neutron correlation. We demonstrate that the pairing correlation plays an essential role in this behavior. We also discuss the di-neutron structure in the \textsuperscript{8}He nucleus.

1. Introduction

The idea of di-neutron correlation can date back to the Migdal’s paper in 1972\textsuperscript{11}, in which he argued that two neutrons may be bound in finite nuclei even if they are not bound in the vacuum. This idea was explicitly exploited by Hansen and Jonson \textsuperscript{2}, where they proposed the di-neutron cluster model and successfully analysed the matter radius of \textsuperscript{11}Li. They also predicted a large Coulomb dissociation cross section of the \textsuperscript{11}Li nucleus. Although the di-neutron correlation in the \textsuperscript{11}Li nucleus has been discussed for two decades since the publication of Hansen and Jonson, it is only recently that a strong indication of its existence has been obtained experimentally in the Coulomb dissociation of \textsuperscript{11}Li\textsuperscript{8}. The new measurement has stimulated lots of theoretical discussions on the di-neutron correlation, not only in the 2n halo nuclei, \textsuperscript{11}Li and \textsuperscript{6}He\textsuperscript{4}, but also in medium-heavy neutron-rich nuclei \textsuperscript{8,9,10} as well as in infinite neutron matter \textsuperscript{11,12}.

In this contribution, we present our recent activities on the di-neutron correlation in light neutron-rich nuclei, focusing especially on the size of Cooper pair in the \textsuperscript{11}Li nucleus. We also discuss the di-neutron correlation in a 4n nucleus, \textsuperscript{8}He.

2. Surface di-neutron correlation in \textsuperscript{11}Li

In Ref.\textsuperscript{6}, we have used a three-body model with density-dependent contact interaction\textsuperscript{13,14} to study the two-neutron Cooper pair in \textsuperscript{11}Li at various positions $R$ from...
the center to the surface of the nucleus (this calculation is essentially equivalent to the particle-particle Tamm-Dancoff approximation\textsuperscript{[15,16]}. We have found that i) the two-neutron wave function oscillates near the center whereas it becomes similar to that for a bound state around the nuclear surface, and ii) the local pair coherence length has a well pronounced minimum around the nuclear surface. This is qualitatively the same behavior as found in neutron matter\textsuperscript{[11]}, and has subsequently been confirmed in heavier superfluid nuclei as well\textsuperscript{[10]}.\

![Figure 1](image)

Fig. 1. The root mean square distance $r_{\text{rms}}$ for the neutron Cooper pair in $^{11}$Li as a function of the nuclear radius $R$. The solid line shows the result of three-body model calculation with density-dependent contact pairing force, while the dashed and the dotted lines are obtained by switching off the neutron-neutron interaction and assuming the $[(1p_{1/2})^2]$ and $[(2s_{1/2})^2]$ configurations, respectively. The local expectation value of the neutron-neutron interaction is also shown in the lower panel.

It is important to notice here that the pairing interaction changes the structure of the Cooper pair strongly and qualitatively. To demonstrate this, we show in Fig. 1 the local coherence length of the Cooper pair in $^{11}$Li as a function of the nuclear radius $R$ obtained with and without the neutron-neutron $(nn)$ interaction. For the uncorrelated calculations, we consider both the $[(1p_{1/2})^2]$ and $[(2s_{1/2})^2]$ configurations. One can see that, in the non-interacting case, the Cooper pair continuously expands, as it gets farther away from the center of the nucleus. In marked contrast, in the interacting case it becomes smaller going from inside to the surface before expanding again into the free space configuration. This is nothing more than the pronounced strong coupling, \textit{i.e.}, the BEC-like feature of a Cooper pair on the nuclear surface, as described in Ref.\textsuperscript{[6]}

We also show in the lower panel the local expectation value of the $nn$-interaction, $E_{\text{pair}}(R)$. This quantity is defined as

$$
\langle \Psi_{\text{gs}} | v_{\text{pair}} | \Psi_{\text{gs}} \rangle = \int dR dr |\Psi_{\text{gs}}(\mathbf{R}, r)|^2 v_{\text{pair}}(r_1, r_2) \equiv \int R^2 dR E_{\text{pair}}(R),
$$

(1)
where $r_1$ and $r_2$ are the coordinates of the valence neutrons relative to the core nucleus, and $r = r_1 - r_2$ and $R = (r_1 + r_2)/2$. The ground state wave function is denoted as $\Psi_{gs}$. We see that there is maximal attraction where the the Cooper pair is smallest ($R \sim 3$ fm). Note that this surface enhancement of the $nn$ force is not primarily an effect of the density effect because it also happens with the density independent Gogny force. It namely is just the other way round: the density dependence of a zero range force is needed to mock up the finite range of a density independent force, as is explained in Ref. [17]. In fact, the similar compact Cooper pair on the nuclear surface has recently been shown in Ref. [10] using the finite range Gogny interaction as well as in an old study with a simple Yukawa interaction.[18].

![Fig. 2. (Color online) A two-dimensional (2D) plot for the two-particle density for the correlated pair (the upper panel) and for the uncorrelated \([(1p_{1/2})^2] \) configuration (the lower panel). It represents the probability distribution for the spin-up neutron when the spin-down neutron is at $(z, x) = (3.4, 0) \text{ fm.}]

The dramatic influence of the pairing interaction can be clearly seen also in the two-particle density. Figure 2 shows the two-particle density for the correlated and the uncorrelated \([(1p_{1/2})^2] \) configurations in the total spin $S = 0$ channel when the spin-down neutron is located at $(z, x) = (3.4, 0) \text{ fm.}$ As can be seen, in the non-interacting case, the distribution has a symmetric two bump structure with respect to the origin of the nucleus. This originates from the absence of mixing of wave functions of opposite parity. On the contrary, in the interacting case, the bump on the far side almost disappears and only the bump on the side of the test
particle survives. This is the essential effect of major shell mixing, which is a clear indication of strong pairing correlations\cite{10}.

3. Di-neutron structure in $^8$He

Let us now discuss the di-neutron structure in the $^8$He nucleus. An important question is how the spatial structure of valence neutrons evolves from that in the $2n$-halo nucleus, $^{11}$Li, when there are more numbers of neutrons. To address this, we study the $^8$He nucleus using a core+$4n$ five-body model\cite{8}. We again use the density-dependent contact pairing interaction among the valence neutrons, and diagonalize the five-body Hamiltonian with the Hartree-Fock-Bogoliubov (HFB) + particle number projection method. See Ref.\cite{8} for further details.

![Fig. 3. The two-particle density for the $^8$He nucleus as a function of $r_1 = r_2 = r$ and the relative angle $\theta$ between a spin-up and a spin-down neutrons (the upper panel). The lower panel is for the four-particle density for the dineutron-dineutron configuration.](image)

The top panel in Fig. 3 shows the two-particle density, $\rho_2(r, \hat{r} = 0, \uparrow; r, \hat{r}, \downarrow)$. One clearly finds a strong concentration of two-particle density around $\theta \sim 0$ at around the nuclear surface. This is similar to what has been found in the Borromean nucleus $^{11}$Li, and clearly indicates the strong di-neutron correlation in this nucleus. Since the strong di-neutron structure is apparent for a spin-up and spin-down neutrons, we next plot in the lower panel the four-particle density for the two-dineutron configuration. The four-particle density for the dineutron-dineutron configuration
has a peak around $\theta \sim \pi/2$. This peak arises from the main component of the wave function, that is, the $[(1p_{3/2})^4]$ configuration, for which the four-particle density is proportional to $\sin^4\theta \propto |Y_{11}|^4$. Therefore, two di-neutrons seem to move rather freely in the core+$4n$ nuclei respecting solely the Pauli principle.

4. Conclusion

We studied the two-neutron wave function in the Borromean nucleus $^{11}$Li by using a three-body model with a density-dependent pairing force, and explored the spatial distribution of the two neutron wave function as a function of the center of mass distance $R$ from the core nucleus. We showed that the relative distance between the two neutrons has a pronounced minimum on the nuclear surface, that is, a compact BEC-like di-neutron structure. We also studied the di-neutron structure in the $^8$He nucleus. We showed that two neutrons with the coupled spin of $S=0$ exhibit a strong di-neutron correlation around the surface of this nucleus, whereas the correlation between the two di-neutrons is much weaker.

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