Critical wetting of a class of nonequilibrium interfaces: A computer simulation study

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I. INTRODUCTION

Much scientific effort has gone into the study of equilibrium wetting since, in the late seventies, Cahn introduced the idea that it can be described as a phase transition [1]. Among the various theoretical approaches developed, interface displacement models have proved particularly useful [2]. Within this perspective, the focus is on the interface that separates two coexisting (bulk) phases confined by a wall or substrate, and the wetting transition corresponds to the unbinding of the interface from the wall. This happens upon a rise of the temperature when the wall adsorbs preferentially one of the phases leading to a divergence of the thickness of the adsorbed layer. The dynamics of such an interface can be described at a coarse-grained level by the following continuum stochastic growth equation [3]

\[ \frac{\partial h(x, t)}{\partial t} = D \nabla^2 h - \frac{\delta V(h)}{\delta h} + \eta(x, t). \] (1)

Here, \( h(x, t) \) is the local height of the interface from the wall, the regions \( y > h(x) \) and \( y < h(x) \) corresponding to the two bulk phases. \( D \) is the interfacial tension coefficient and \( \eta \) is a Gaussian white noise with zero mean and variance \( \langle \eta(x, t)\eta(x', t') \rangle = 2\sigma \delta(x - x')\delta(t - t') \) that mimics thermal fluctuations. \( V(h) \) accounts for the net interaction between the wall and the interface and its form depends on the nature of the forces between the particles in the bulk phases and with the wall, its rigorous derivation from microscopic Hamiltonians being far from trivial. If all the interactions are short-ranged, one may take in the limit of large \( h \) at phase coexistence [2]

\[ V(h) = \int dx \left[ b(T)e^{-h(x)} + \frac{c}{2}e^{-2h(x)} \right], \] (2)

where \( T \) is the temperature [4]. The amplitude \( c > 0 \) is a repulsion whereas \( b(T) \) vanishes linearly with the mean-field wetting temperature, \( T_w \), as \( T - T_w \), and can represent either an effective repulsion or attraction between the interface and the wall (see Fig. 1). At sufficiently low temperatures, \( b < 0 \), the equilibrium thickness of the wetting layer as given by the stationary configurational average \( \langle h \rangle \) is finite (pinned interface). This corresponds to an attractive potential (see Fig. 1). As the temperature is raised the potential becomes less attractive and eventually, above a certain value \( b = b_w \), it no longer binds the interface and \( \langle h \rangle \) diverges. Within mean-field approximation, ignoring spatial correlations, \( \langle h \rangle \) follows from \( \partial V(h)/\partial h = 0 \), whereby one finds for an attractive wall \( b < 0 \) \( \langle h \rangle = \ln(-c/b) \) and, consequently, a critical wetting transition takes place as \( b \rightarrow b_w = 0 \). Recently, effective short-ranged, equilibrium critical wetting showing mean-field like exponents seems to have been experimentally observed [5].

Extensions of equilibrium, interface displacement models to nonequilibrium conditions have only been recently addressed and constitute a topic of ongoing research activity [6, 7]. Supplementing equation (1) with the most relevant nonequilibrium nonlinear term \( \lambda(\nabla h)^2 \) [8], leads to a natural generalization of equation (1) that assumes that the velocity of the interface depends on its local-slope,

\[ \frac{\partial h(x, t)}{\partial t} = D \nabla^2 h + \lambda(\nabla h)^2 + be^{-h} + ce^{-2h} + \eta(x, t), \] (3)

which is a Kardar-Parisi-Zhang (KPZ) interface [8] interacting via a short-ranged potential with a wall. KPZ interfaces have a nonzero average velocity, \( v = \lambda(\nabla h)^2 \), and hence steady-state interfaces move on average thereby favoring one of the phases over the other. Wetting, on the other hand, by definition occurs at coexistence, i.e. zero average velocity of the free (no wall) interface. This agrees with the thermodynamic picture that at bulk coexistence any arbitrary fraction of the system may be in one phase, with the remainder in the other.
Therefore, a constant $a_c = -\nu$ needs to be included in equation (3) to study wetting transitions driven by the wall. The nonequilibrium analog of equilibrium critical wetting corresponds to the depinning transition at $a_c$ as $b \to b_w$. Clearly, for $\lambda = 0$ and $a_c = 0$, the model reduces to the equilibrium one. In equilibrium, a constant force term, $a$, in the interfacial equation measures the deviation from bulk coexistence (i.e. it represents the chemical potential difference between the two phases) while here it plays a similar role by balancing the force exerted by the KPZ nonlinearity on the tilted parts of the interface, thereby guaranteeing that the average velocity of the free interface is zero.

Owing to the lack of $h \leftrightarrow -h$ symmetry of the KPZ dynamics, it is necessary to specify either the relative position of the wall with respect to the interface, upper or lower, for a fixed sign of $\lambda$, or reversely, the sign of $\lambda$ after a wall position has been arbitrarily chosen. In earlier studies of nonequilibrium (complete) wetting (see below), these two different physical situations lead to the existence of by-now-well-documented two different universality classes, called multiplicative noise 1 (MN1) and multiplicative noise 2 (MN2) respectively [9]. Here, we take $\lambda < 0$ in Eq. (3) (i.e. the critical wetting counterpart of MN1) which has shown to have a much richer phenomenology than that of $\lambda > 0$ in other studies of nonequilibrium wetting [1,10]. The analysis of the case $\lambda > 0$ will be tackled elsewhere.

Thus defined, this model system, arguably the simplest nonequilibrium one, has served for the study of universality issues in nonequilibrium wetting. In particular, by fixing $b > b_w$, i.e. in the presence of a repulsive wall, and letting $a \to a_c^-$ (path 1 in figure 2) nonequilibrium complete wetting transitions (MN1) were investigated. For $b < b_w$ (attractive wall) and varying $a$ there is a rich phenomenology: the pinned and the depinned phases lose their stability at different values of $a$, giving rise to a continuous depinning transition at $a^*(b) > a$ in the directed-percolation universality class (see Fig. 2 path 2) [11], and a first-order phase-transition along path 3, at $a = a_c$. In the broad interval $a_c < a < a^*(b)$ both phases coexist (see [9] and references therein). Tricritical behavior along path 4 was analyzed by Ginelli et al. [12], and a preliminary study of nonequilibrium critical wetting (path 5) was presented in [13].

In this paper we investigate numerically and analytically nonequilibrium critical wetting (path 5 in figure 2) as defined by equation (3). The focus will be on one-dimensional systems only. Higher system dimensionalities were studied in [10] by a mean-field analytic approximation to Eq. (3) which revealed the existence of three different regimes of scaling behavior. Their connection with our findings (or the lack of it) is discussed in the last section.

II. MODELS AND OBSERVABLES

Our continuous model is defined by the stochastic growth equation

$$\frac{\partial h(x,t)}{\partial t} = D \nabla^2 h + \lambda (\nabla h)^2 + a_c + b e^{-h} + c e^{-2h} + \eta(x,t),$$

(4)

with, as explained above, $a_c = -\lambda (\nabla h)^2$. At the critical wetting transition, i.e. as $b$ approaches $b_w$ from below, the average stationary thickness of the wetting layer diverges continuously as $\langle h \rangle \sim |b - b_w|^{\beta_h}$, where $\beta_h$ is a
critical exponent. At \( b = b_w \), \( \langle h(t) \rangle \sim t^{\beta_h} \) for asymptotically long times. Two other exponents we study are the dynamic and the correlation length exponents, \( z \) and \( \nu \) respectively, defined through their usual expressions \( \xi(t) \sim t^{1/z} \), \( \delta h \sim |b - b_w|^{-\nu} \), where \( \xi \) is the correlation length. They are related to the previous ones by the scaling form \( \theta_h = \beta_h / z \nu \).

Some of these exponents may be written in terms of known KPZ exponents. In particular, since at \( b = b_w \) the interface is asymptotically free, the dynamic exponent retains its one-dimensional free KPZ value \( z = 3/2 \) \( [3] \).

Also, as we illustrate now, \( \theta_h \) is given by the exponent characterizing the growth of the interfacial width in the wall, as a result of which there is an effective fluctuation-induced repulsion between the wall and the interface. The latter can be estimated by noting that interface fluctuations are cutoff owing to the presence of local contacts between the interface and the wall. In this case, as said before, \( \theta_h \) is common to paths 4 and 5 in figure 2, therefore \( \theta_h = \beta_w = 1/3 \) in one dimension. This can be understood as follows: interfacial fluctuations are cutoff owing to the presence of the wall, as a result of which there is an effective fluctuation-induced repulsion between the wall and the interface. The latter can be estimated by noting that the wall makes itself felt when the mean interfacial separation \( \langle h \rangle \) is of the same order as the average extent of the interface fluctuations, \( \delta h \). From \( \delta h \sim \xi \), where \( \xi \) is the usual KPZ roughness exponent, and the definition of \( \nu \) we find that the effective repulsion force has the form \( h^{-1/\nu} \) and is therefore long-ranged in \( d = 1 \) where \( \xi = 1/2 > 0 \). Comparing this force with the deterministic one in the Langevin equation \( [1] \), which is short-ranged, it is straightforward to conclude that in \( d = 1 \) fluctuations dominate the unbinding of the interface. As fluctuations are governed, in the regime where the interface is asymptotically free, by the growth exponent of the KPZ, then \( \theta_h = 1/3 \). This result, has been verified in our computer simulations (see below). Note that this, as well as any other exponent computed exactly at the critical point, is path independent and, therefore, holds also for complete wetting.

Of more interest is the behavior of the surface order-parameter defined as \( \langle e^{-h} \rangle \) or, equivalently as the density of local contacts between the interface and the wall. Indeed, considering equation \( [4] \) with \( D = -\lambda \), the change of variables \( h = -\ln n \) leads to

\[
\frac{\partial n(x,t)}{\partial t} = D \nabla^2 n - a_n n - b n^2 - c n^3 + m \eta(x,t), \tag{5}
\]

which is a multiplicative noise Langevin equation for the surface order-parameter \( [4] \) to be interpreted in the Stratonovich sense \( [14] \). For sufficiently low values of \( b < b_w \) the interface remains pinned and the stationary density of locally pinned sites at the wall is high \( \langle n \rangle \leq 1 \). As the transition is approached (increasing \( b \) following path 5 in Fig. 3), the stationary density of pinned segments goes to zero in a continuous manner as \( \langle n(b, t = \infty) \rangle \sim |b - b_w|^{\beta_h} \). At \( b = b_w \), the interface depins and therefore \( \langle h(t) \rangle \) diverges and \( \langle n(t) \rangle \) vanishes with the characteristic exponent \( \langle n(b = b_w, t) \rangle \sim t^{-\theta_n} \). Obviously, as said before, \( \theta_n \) is common to paths 4 and 5 in figure 3 and the value \( \theta_n = 0.5(1) \) has been reported previously \( [12] \). Here we focus here on the determination of the path dependent exponents \( \beta_h, \beta_n, \) and \( \nu \). Before proceeding further, we refer the reader to \( [13] \) for a detailed analysis of the critical behavior of the above observables in equilibrium critical wetting.

To study numerically the model defined above, owing to well documented numerical instability problems \( [16] \), it is more convenient to integrate numerically Eq. \( [5] \) than the equivalent form Eq. \( [4] \). Equation \( [5] \) can be efficiently integrated by means of a recently introduced split-step scheme specifically designed to deal with Langevin equations with non-additive noise \( [17] \). Setting \( D = -\lambda = 0.1 \), \( \sigma = 1 \) we can use the result \( a_c = 0.143668(3) \) obtained from previous investigations of the analogous nonequilibrium complete-wetting transition \( [18] \). As will be illustrated below, estimates of the critical exponents are severely hindered by uncertainties in the value of \( a_c \).

To circumvent this problem and to confirm universality we have carried out simulations of a discrete interfacial growth model which (i) in the absence of walls is known to belong to the KPZ universality class, (ii) has been successfully used in nonequilibrium complete wetting analyses \( [19] \) and, most importantly, (iii) allows for an exact determination of the velocity of the free interface, and therefore permits to extract the critical exponents with good accuracy. To be more specific, we consider a single step plus wall model (SSW) defined as follows. At time \( t \) interface positions above sites \( i \) of a one-dimensional line of length \( L \) are given by integer height variables \( h_i(t) \), satisfying the solid-on-solid constrain \( |h_i(t) - h_i(t+1)| = 1 \). Initially, we take \( h_0(2t) = 0 \) and \( h_0(2t+1) = 1 \). New height configurations are generated by choosing at random a site \( i \) and growing it to \( h_i(t) \rightarrow h_i(t) + 2 \) if and
only if a local minimum existed at $i$. It can be shown that this rule generates a KPZ-like interface with $\lambda = -1/2$ moving with an asymptotic long-time, average velocity $v = (1 + L)/(2L)$ [21]. Additionally, taking advantage of the exact knowledge of $v$, the interface is globally pulled down by one unit every $L/v$ growth trials, in such a way that the interface has zero average velocity. A wall at $h = 0$ is then introduced by precluding the interface from overtaking the wall that is behind it. This is achieved by implementing the previous global, downwards movement as $h_i(t) = |h_i(i) - 1|$ for each $i$. Finally, in order to implement an attractive wall the growing rates at the bottom layer are reduced from 1 to $1 - q$ with $0 \leq q \leq 1$. The parameter $q$ represents the short-ranged attraction exerted by the wall on the interface. If $q = 1$ no growth is possible at the local minima located at the wall and hence an interface at the wall does not move, whereas if $q = 0$ the short-range attraction is switched off.

In all, the SSW algorithm is as follows: (i) A site $i$ is randomly chosen and grown from $h(i)$ to $h(i) + 2$ if a local minimum exists at $i$ ($h(i + 1) + h(i - 1) - 2h(i) = 2$). This is done with probability $1$ if $h(i) > 0$, or with probability $1 - q$ if $h(i) = 0$, with $0 \leq q \leq 1$. No action is taken if $i$ does not correspond to a minimum. Time is increased by $1/v$ after $L$ of such attempts. (ii) Every $L/v$ growth trials $h(i) = |h(i) - 1|$ for each $i$. Given that $L/v$ is generally not an integer, this is done by using $\lfloor L/v \rfloor$ with probability $L/v - \lfloor L/v \rfloor$ and $\lfloor L/v \rfloor + 1$ with probability $\lfloor L/v \rfloor + 1 - L/v$, where $\lfloor \cdot \rfloor$ denotes the integer part. Periodic boundary conditions are imposed. By tuning the parameter $q$ a critical wetting transition is observed.

III. NUMERICAL RESULTS

We next summarize our main findings for both the discrete model (SSW) and the stochastic differential equation (SDE).

To determine the critical points, $q_w$ (for the SSW) and $b_w$ (for the SDE), we take a system-size as large as possible ($L = 2^{17}$ here), plot the order-parameter $\langle n(t) \rangle$ versus $t$ in a double logarithmic scale, and look for the separatrix between curves converging to a constant value and those bending downward (not shown). In this way, the critical values $q_w = 0.4445(5)$ (SSW) and $b_w = -1.0815(9)$ (SDE) are determined. In figure 3 results are shown in log-log for the time decay of the order-parameter $\langle n(t) \rangle$ at the critical point of the SSW model for different system-sizes ranging from $L = 2^7$ to $L = 2^{17}$. From the slope of a straight-line fit to the lowest curve one finds $\theta_{\text{SSW}} = 0.49(2)$ ($\theta_{\text{SDE}} = 0.50(5)$ for the SDE; not shown), where the error is computed by comparing the slopes corresponding to the upper and lower bounds of $q_w$ ($b_w$). The inset of Fig. 3 shows estimates of the crossover times, $\tau_{xx}(L)$, from time decay to saturation as a function of the system-size. Using $\tau_{xx}(L) \sim L^\beta$ we find the temporal exponent values $\beta_{\text{SSW}} = 1.4(1), \beta_{\text{SDE}} = 1.3(2)$ (not shown), which are in agreement with those previously reported in [12] following path 4 of Fig. 2 and compatible with the theoretical considerations described above.

From the scaling of the saturation value at the critical point, $\langle n(t) \rangle$ versus $t$ in a double logarithmic scale, and look for the separatrix between curves converging to a constant value and those bending downward (not shown). In this way, the critical values $q_w = 0.4445(5)$ (SSW) and $b_w = -1.0815(9)$ (SDE) are determined. In figure 3 results are shown in log-log for the time decay of the order-parameter $\langle n(t) \rangle$ at the critical point of the SSW model for different system-sizes ranging from $L = 2^7$ to $L = 2^{17}$. From the slope of a straight-line fit to the lowest curve one finds $\theta_{\text{SSW}} = 0.49(2)$ ($\theta_{\text{SDE}} = 0.50(5)$ for the SDE; not shown), where the error is computed by comparing the slopes corresponding to the upper and lower bounds of $q_w$ ($b_w$). The inset of Fig. 3 shows estimates of the crossover times, $\tau_{xx}(L)$, from time decay to saturation as a function of the system-size. Using $\tau_{xx}(L) \sim L^\beta$ we find the temporal exponent values $\beta_{\text{SSW}} = 1.4(1), \beta_{\text{SDE}} = 1.3(2)$ (not shown), which are in agreement with those previously reported in [12] following path 4 of Fig. 2 and compatible with the theoretical considerations described above.

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Finally, we have confirmed that for both models at the tricritical point the width, $W$, of the interface grows with time with an exponent compatible with that of the
dependent ones, evidence in the numerical values of two of them (the path 2 β and components at equilibrium critical wetting are θ. L numbers KPZ, bars as in Fig. 4. h β point gives a direct estimation of β and 2 7 distance to the wall vs time, yielding FIG. 6: (Color online) Main: log-log plot of the average SDE β and 2 n/h = 1 − In summary, the exponent values determined above aperture sets that observed at high system dimensionalities from a self-consistent, mean-field approximation to equation 10. In that approximation, a sequence of three scaling regimes was reported to exist depending on the relative importance of the noise strength as compared to the spatial coupling. The first two have a Gaussian character, while in the third regime, i.e. the strong noise one (in which the wetting temperature is shifted away from zero) all moments, mk = ⟨n⟩b, for k > 2D/σ2 + 1 scale with the same exponent, while simple scaling mk ∼ mk is obtained for k ≤ 2D/σ2 + 1. This is a rather curious type of anomalous scaling not very different from that observed in analogous mean-field approximations for non-equilibrium complete wetting 22. However, we have verified numerically that in the one-dimensional system moments of arbitrary order scale as ⟨n⟩ itself, as happens in other Langevin equations with multiplicative noise 9. This is a consequence of the large fluctuations occurring at d = 1, notwithstanding which it is possible to make some analytic predictions, as we now discuss.

First, we explicitly show how the critical value of b is depressed from its mean-field value bw = 0 to bw < 0 once fluctuations are included. Taking spatial and noise averages in the stationary state of equation 4 for a generic value of a, and denoting the average squared slope of the interface by s2 ≡ ⟨(∇h)2⟩, we obtain

\[ a - s^2 + b⟨n⟩ + ⟨n^2⟩ = 0 \]  

(6)

where, without loss of generality we have taken D = λ = −1 and c = 1. At coexistence ac = s2, and hence s2 = s2 + b⟨n⟩ + ⟨n^2⟩ = 0. Additionally, in the pinned phase (i.e. ⟨n⟩ ≠ 0) s2 < s2 and therefore equation 6 has a solution only if b < 0. Still, bw could be 0, but by noting that on approaching the tricritical point along path 5 the moments of ⟨n⟩ scale in the same way, ⟨n⟩ ~

IV. THEORETICAL CONSIDERATIONS

In summary, the exponent values determined above appear to be compatible with the set of simple rational numbers θn = 1/2, z = 3/2, βn = 3/2, ν = 2, θh = 1/3, and βh = 1. For the sake of comparison, the critical exponents at equilibrium critical wetting are θn = 1/4, z = 2, βn = 1, ν = 2, θh = 1/4 and βh = 1 [21]. The coincidence in the numerical values of two of them (the path dependent ones, βh and ν) is somehow intriguing.

KPZ, W(t) ∼ tθw, with θW = 1/3, and saturates in finite system-sizes to a non-vanishing value given by W ∼ L−βh/ν ∼ L−1/2 as in the KPZ.

FIG. 5: (Color online) Main: log-log plot of the saturation values of ⟨n⟩ for L = 217 vs the distance to the critical point gives a direct estimation of βSSW = 1.50(9) and (inset) βSDE = 1.46(6). Error bars as in Fig. 4.

FIG. 7: (Color online) Main: the scaling of the saturation value of ⟨h_sat⟩ yields βSSW = 0.0(1) for data collected from the SSW model. Inset: A value βSDE = 1.0(1) results for data collected from the SDE. Error bars as in Fig. 4.

FIG. 6: (Color online) Main: log-log plot of the average distance to the wall vs time, yielding θSSW = 0.35(2). System-sizes are (from top to bottom) L = 217, 212, 211, 210, 29, 28, and 27. Inset: finite-size scaling of ⟨h_sat⟩(q = qw) indicating βw/v = 0.52(4). All values are for the SSW model. Error bars as in Fig. 4.
\( \langle n^2 \rangle \sim |b - b_w|^{\beta_n} \), it is easy to see that \( b_w \neq 0 \) because otherwise the negative term \( b(n) \) would be subdominant in comparison with the positive \( \langle n^2 \rangle \), and no solution could exist for small \( b \). Clearly, by continuity \( b_w \) cannot be positive and hence \( b_w < 0 \) ensues.

It is illuminating to show how a similar reasoning leads to a more predictive analysis when applied to the analogous complete wetting transition (path 1 in Fig. 9) known to be in the MN1 class. In that case, the condition \( a - a_c + s_c^2 - s^2 + b(n) = 0 \) must be satisfied, with the negative term \( \delta a = a - a_c \) balancing the two positive terms \( s_c^2 - s^2 \) and \( b(n) \sim |\delta a|^{\beta_n} \) in the pinned phase. This implies that \( \beta_n > 1 \) if a solution for \( \langle n \rangle \) to exist for small \( \delta a \). Furthermore, by noticing that if KPZ scaling is applicable, then \( s_c^2 - s^2 \sim \xi^{2(1-\zeta)} \), where \( \zeta \) is the roughness exponent of a free KPZ. Recalling that \( \zeta = 1/2 \) in 1d, immediately entails \( \nu = 1 \) for complete wetting.

On the contrary, we have not been able to derive the value of \( \nu \) for nonequilibrium critical wetting, nor does it seem immediate that \( \nu = 1 \) along path 4 as found numerically in [12]. The main difference with the complete wetting case appears to rest on the behavior of the slopes \( s^2 \). According to our measurements for equation \( s_c^2 - s^2 \sim |b - b_w|^{1.51(2)} \) for critical wetting (path 5) and \( s_c^2 - s^2 \sim |a - a_c|^{0.74(2)} \) for tricritical complete wetting (path 4), pointing to an anomalous scaling of the slopes, i.e. \( s^2 = \langle (\nabla h)^2 \rangle \) does not have the same scaling dimension as \( |L^{-1}|^2 |h|^2 \), as happens in the complete wetting case.

That \( s_c^2 - s^2 \) scales along path 4 with an exponent less than unity could have been anticipated from the condition \( |\delta a| - b_w|\delta a|^{\beta_n} = s_c^2 - s^2 \) at the tricritical point, implying \( s_c^2 - s^2 \sim |\delta a|^{\beta_s} \) with \( \beta_s \leq 1 \).

The exponent value \( \beta_n = 0.74(5) \) reported in [12] for the multicritical complete wetting transition along path 4 (confirmed in our own measurements), along with \( \beta_n \approx 1.5 \) for critical wetting as referred above, indicates that the anomalous scaling of the slopes is ultimately controlled by \( \beta_n \) in both cases, rather than by the roughness exponent of the KPZ as in the complete wetting case. Notice the constancy along path 4 and 5 of the ratio \( \beta_n / \nu \approx 0.75 \), which is a property of the tricritical point and therefore must be independent of the path.

The fact that the slopes acquire a scaling not directly derivable from the free KPZ can be argued to be a consequence of the effect of the potential well on the interface. Figure 8 shows snapshots of configurations \( h(x) \) that result from solving the SDE in the stationary state for different parameters that correspond to approaching the critical points from different paths:

- **Panel A** for \( (b = -1.2 < b_w, a = -0.25 < a_c) \), corresponds a situation slightly below the complete wetting (MN1) transition (path 1 of Fig. 2).

- **Panel B** for \( (b = b_w, a = -0.153 < a_c) \), slightly below the multicritical complete wetting transition (path 4).

- **Panel C** for \( (b = -1.3 < b_w, a = a_c) \), slightly below the critical wetting transition (path 5).

The distances to the transition points are chosen so that \( \langle h \rangle \approx 5 \) in all cases. Note the clear qualitative difference between panel A, for which standard scaling holds, and the rest. Observe that in panel C, the interface consists of patches of essentially free KPZ interfaces separated by regions of sites pinned by the potential well. It then seems plausible to conclude that, following path 5, regions locally trapped within the potential well develop slopes different from that of a free KPZ (indeed, roughness is severely restricted within the potential well) even at points arbitrarily close to the tricritical point, thereby inducing an anomalous scaling controlled by \( \beta_n \). In panel B, i.e. upon approaching the tricritical point along path 4, the effect of the bounding wall is less apparent as the potential well is marginally disappearing at \( b = b_w \), but a similar effect, induced by the potential shape, should be at work in this borderline case.

V. SUMMARY AND DISCUSSION

We have investigated the universal properties of nonequilibrium critical wetting transitions in one spatial dimension. For that, we study effective interfacial models in the KPZ universality class with \( \lambda < 0 \) bounded by a lower wall, and determine the average height \( \langle h \rangle \) and the surface order-parameter \( \langle n \rangle = \langle e^{-b h} \rangle \).

A scaling analysis leads to the prediction \( z = 3/2 \) and a time behavior of the average height governed by the growth exponent of the free KPZ, i.e. \( \langle h(t) \rangle \sim t^{1/3} \). These results have been verified numerically. Other exponents have been computed from extensive numerical simulations of the Langevin equation and a discrete model in the same universality class that enables a more precise numerical analysis, as a result of which we find \( \nu \approx 2 \), \( \beta_h \approx 1 \), and \( \beta_n \approx 3/2 \), suggesting that actually the exponents take rational values (see table 1). Interestingly enough, the first two values agree with those of equilibrium critical wetting.

Simple analytical arguments allow us to show that the critical value of the control parameter \( b \) is depressed by fluctuations from its mean-field value \( b_w = 0 \) to \( b_w < 0 \). We have also shown that in critical wetting, as well as in multicritical complete wetting, the average interface slopes do show anomalous scaling, not controlled by the free KPZ equation: local regions pinned by the binding potential generate anomalous scaling.

We have not been able to predict the value of \( \nu \) using the same analytical considerations that yield \( \nu = 1 \) in the complete wetting (MN1) case, nor is it trivial to obtain \( \nu = 1 \) for multicritical complete wetting. It is nevertheless possible to conclude that in this latter case the exponent governing the scaling of the interface slopes is less than unity, \( s_c^2 - s^2 \sim |a - a_c|^{\beta_s} \), in agreement with the value \( \beta_s \approx 0.75 \) obtained. There is numeric evidence that
TABLE I: Summary of the critical exponents for nonequilibrium, critical wetting transitions with short-range forces. Results are shown for the discrete model SSW, the stochastic differential equation SDE, and from other sources when available.

| Exponent | SSW | SDE | Others |
|----------|-----|-----|--------|
| $z$      | 1.4(1) | 1.3(2) | 1.5(1) [11] |
| $\theta_n$ | 0.49(2) | 0.50(5) | 0.50(1) [11] |
| $\theta_h$ | 0.35(2) | 0.33(1) | - |
| $\nu$    | 2.0(2) | 2.4(5) | - |
| $\beta_n$ | 1.50(9) | 1.46(6) | - |
| $\beta_h$ | 0.9(1) | 1.0(1) | - |

The slopes actually scale with $\beta_s = \beta_n$ for both critical wetting and multicritical complete wetting (paths 5 and 4 of Fig. 2 respectively), a result that violates naive scaling. In effect, dimensional analysis demands that $s^2 - s^2$ scales as $|a - a_c|^{2(1 - \zeta)}$, $\zeta$ being the roughness exponent of the free KPZ [23]. This is obeyed at the complete wetting transition, and indeed was used to derive $\nu = 1$ in MN1 [7], but does not hold for critical wetting nor for multicritical complete wetting.

The cause of the deviation from standard scaling can be sought in the interfacial profiles shown in figure 8. For critical and multicritical wetting, patches of free KPZ interfaces ($n(x) \approx 0$) are separated by regions of sites that lie in the potential well, plausibly hindering the standard KPZ scaling from setting in gradually as the tricritical point is approached. This is in contrast to typical interfacial profiles for nonequilibrium complete wetting (MN1) where no potential well is at play.

According to a recent self-consistent, mean-field approximation to equation 3 three different scaling regimes of critical behavior for the surface order-parameter can be distinguished [10]. The first two are of Gaussian type, while the third one is a highly nontrivial strong-fluctuating regime. This rich structure is completely washed out by fluctuations in one-dimensional systems, where a unique and universal scaling regime emerges. Moreover, we have verified numerically that in the one-dimensional system moments of arbitrary order scale as $\langle n \rangle$ itself, as happens in other Langevin equations with multiplicative noise [9]. The fact that the numerical values of the exponents are changed is not surprising at all, given the presence of severe fluctuations in one-dimension but what is more striking is that out of the three regimes appearing in the mean-field approach, only one survives. A possible explanation for such an abrupt change might come from a recent claim that the strong-coupling renormalization group fixed point of the KPZ dynamics is essentially different above and below $d = 2$ [24]. This point remains to be further studied, as well as some other aspects of nonequilibrium critical wetting including its subtle relation to its equilibrium counterpart (signaled by the coincidence of some exponents), and the possible existence of various nonuniversal scaling regimes in higher dimensions. It also stands as a main experimental challenge to observe in the laboratory the phenomenology reported on here and in previous theoretical works of nonequilibrium wetting.

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