The Local Stellar Initial Mass Function

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Abstract. This contribution describes the difficult task of inferring the IMF from local star-count data, by discussing the mass–luminosity relation, unresolved binary, triple and quadruple systems, abundance and age spreads and Galactic structure, all of which must be accounted for properly for the results to be meaningful. A consensus emerges that the local IMF may be represented by a two-part power-law, with indices $\alpha = 1 - 1.5$ for stars with mass $m \lesssim 0.5 \, M_\odot$, and the Salpeter value $\alpha = 2.3$ for more massive stars, but some uncertainties remain. Notable is also that the sensitivity of the stellar luminosity function (LF) to the derivative of the mass–luminosity relation is very evident in the (local) Hipparcos and HST, open-cluster and globular-cluster LFs, thus allowing tests of stellar structure theory. The upcoming astrometry space missions DIVA and GAIA will undoubtedly lead to significant advances in this field.

1. Introduction

The distribution of masses of stars born together, the initial mass function (IMF), determines the appearance and evolution of galaxies and star clusters, and composes the boundary condition for star-formation when it ends. Any affirmed variation of the IMF with local conditions would pose tremendously important constraints on our understanding of how stars form. The detailed shape of the IMF, let alone any variation of it between populations, has been arduous to distill from the observational data, with a large amount of serious work remaining to be done in this fundamentally important field.

This text focuses on the derivation of the IMF from solar-neighbourhood (sn) star-count data. The sn constitutes a particularly important stellar sample because of it’s proximity, and because it represents a mixture of many star-formation (sf) events. The solar-neighbourhood IMF is therefore an average IMF, being valid for the on average about 5 Gyr old Milky-Way (MW) disk stars. Furthermore, by studying the sn, methods can be developed that are of general use for the interpretation of more distant stellar populations.

Because of the restriction to the sn, the mass range covered by this contribution spans $0.1 - 3 \, M_\odot$, more massive stars being too rare. Very-low-mass stars and brown dwarfs are dealt with by Chabrier & Baraffe (2000), while Massey (1998) addresses the IMF for massive stars. Empirical evidence for systematic variations over all stellar masses ($10^{-2} - 10^2 \, M_\odot$) is studied by Kroupa (2000a), and Meyer et al. (1999) review the IMF in star-forming regions.
2. From the LF to the IMF

The mass, $m$, of an isolated main-sequence star can be determined from its absolute luminosity, $l$, and the mass–luminosity relation, $m = m(l, \tau, [\text{M/H}], s)$, where $\tau$, $[\text{M/H}]$ and $s$ are, respectively, the age, metal abundance and rotational angular momentum vector (spin). The corresponding mass–(absolute-magnitude) relation is $m(M_P)$, where $P$ represents some photometric pass band, and the other parameters have been dropped for conciseness.

The number of stars in the absolute-magnitude interval $M_P + dM_P$ to $M_P$ and in the corresponding mass interval $m$ to $m + dm$ is $dN = -\Psi dM_P = \Xi dm$, where $\Xi(m)$ is the present-day mass function (PDMF). The distribution of main sequence stars by luminosity, $\Psi(M_P)$ (the present-day stellar luminosity function, PDLF), is therefore related to the PDMF through

$$\Psi(M_P) = -\Xi(m) \frac{dm}{dM_P}. \quad (1)$$

Star-counts allow an estimate of $\Psi$ from which $\Xi$ can be derived, assuming $m(M_P, \tau, [\text{M/H}], s)$ is known.

To infer the IMF, $\xi(m)$, corrections for stellar evolution on and off the main sequence are necessary. As a star ages while burning hydrogen in the core, its effective temperature increases slightly owing to core contraction and its luminosity increases. A rotating star has a smaller internal pressure than a non-rotating star owing to the centrifugal force, leading to reduced internal temperatures, lower thermonuclear power, and subluminous stars. Models indicate that the luminosity may be reduced by about 10 per cent for rigidly rotating stars (e.g. Mendes, D’Antona & Mazzitelli 1999; Sills, Pinsonneault & Terndrup 2000). Main-sequence stars with a spectral type later than approximately F4 are slow rotators, whereas more massive stars are fast rotators. This is a result from mass loss and magnetic activity, causing the stars with convective envelopes (spectral type F4 and later) to loose spin angular momentum. Corrections for the time-evolution of $s$ should therefore also be applied. Mass-loss during the main-sequence phase of stars adds another complication, as it is mass and time dependent. Finally, stars of different chemical composition have different effective temperatures and luminosities.

Such effects lead to a widening of the main sequence. Explicit corrections for each of these effects are difficult, and in practice an average, or empirical $m(M_P)$ relation is often adopted. But in doing this, subtle but important features in the $m(M_P)$ relation may be lost (e.g. Belikov et al. 1998 for fine structure in the Pleiades LF). Additionally, $\Psi$ must be corrected for hidden companion stars, because there is not a one-to-one mapping between a system’s luminosity and it’s mass.

3. The LF

From the many attempts of constructing $\Psi$ (see Scalo 1986 for a comprehensive review), only two withstood the test of time in the sense that their respective systematic biases can be handled readily. These two fundamentally different but complementary approaches rely on constructing complete stellar samples.
from trigonometric and photometric parallax surveys. The former requires all sample-stars to have high-quality trigonometric parallax measurements, and is thus confined to a very small volume around the Sun and mostly the northern hemisphere (for historical reasons), whereas the latter extends to much larger distances through sensitive, pencil-beam surveys.

**The nearby or single star LF:** \((\Psi_{\text{near}})\) Ground-based measurements can only measure relative trigonometric parallaxes, and suffer from systematic errors, because they rely on measuring the parallactic shift relative to many background stars that must lie typically less than 1 degree from the target star to avoid refractive atmospheric effects. An astrometry satellite, on the other hand, allows a reference star to be chosen for each target star, such that the reference star is aligned along the Earth-Sun axis at the times of measurement. The reference star thus remains fixed on the celestial sphere while the target star shows the maximum parallactic shift, if the angle between the target and reference star is 90 degrees. Space astrometry thus allows the measurement of absolute trigonometric parallaxes (e.g. Perryman et al. 1995).

Distance measurements from space thus lead to a significantly improved estimate of the LF, because trigonometric-parallax-limited surveys are biased. The bias is larger for a larger uncertainty in distance measurements: The number of stars per radial shell increases as \(\propto r^2\). Thus, there are more stars lying just outside the survey distance limit but that have a distance error that places them (in the measurement) within the distance limit, than the number of stars that are within the distance limit but are measured to be outside. The result is that less-accurate measurements (i.e. ground-based parallaxes) overestimate \(\Psi_{\text{near}}\), and that the average luminosities are underestimated (Lutz & Kelker 1973; Smith & Eichhorn 1996; Oudmaijer, Groenewegen & Schrijver 1998).

| \(M_V\) | \(\Psi_{\text{near}} \times 10^3\) (stars/pc^3/mag) | \(\delta\Psi_{\text{near}} \times 10^3\) (stars/pc^3/mag) | \(N\) | \(V\) (pc^3) | \(r_{\text{compl}}\) (pc) |
|---|---|---|---|---|---|
| -1 | 0.015 | 0.015 | 1 | 65450 | 25 |
| +0 | 0.092 | 0.038 | 6 | 65450 | 25 |
| +1 | 0.24 | 0.060 | 16 | 65450 | 25 |
| +2 | 0.41 | 0.079 | 27 | 65450 | 25 |
| +3 | 1.10 | 0.13 | 72 | 65450 | 25 |
| +4 | 1.59 | 0.16 | 104 | 65450 | 25 |
| +5 | 2.92 | 0.21 | 191 | 65450 | 25 |
| +6 | 2.98 | 0.21 | 195 | 65450 | 25 |
| +7 | 3.34 | 0.26 | 164 | 49115 | 25 |
| +8 | 4.18 | 0.41 | 105 | 25147 | 20 |
| +9 | 7.00 | 1.49 | 22 | 3143 | 10 |
| +10 | 10.2 | 1.8 | 32 | 395 | 5.2 |
| +11 | 17.7 | 6.7 | 7 | 395 | 5.2 |
| +12 | 12.7 | 5.7 | 5 | 395 | 5.2 |
| +13 | 13.9 | 4.2 | 11 | 395 | 5.2 |
| +14.5 | 11.4 | 3.8 | 9 | 395 | 5.2 |

Table 1. The nearby, main-sequence PDLF estimated using Hipparcos trigonometric parallax data. For \(M_V \leq 8.5\), all declinations are used, but for \(M_V > 8.5\), \(\delta > -30°\) (Jahreiss & Wielen 1997). For \(M_V \geq 10.5\), \(\Psi\) relies on trigonometric parallax determination from the ground (Kroupa 1998, and references therein) using \(r_{\text{compl}} = 5.20\) pc and \(\delta > -20°\), giving a volume of 395 pc^3. For each magnitude bin, the actual number of stars in the survey volume is \(N\). The last four bins have been combined to two 2-magnitude wide bins.

The result of the Hipparcos mission (e.g. Perryman et al. 1997) is a re-derivation of \(\Psi\) for \(M_V \leq 11\), such that \(\Psi_{\text{Hip}} < \Psi_{\text{old}}\) (Wielen, Jahreiss & Krüger...
Kroupa (1983) by about 15 per cent. This is a direct consequence of the above mentioned Lutz-Kelker bias. Table 1 contains the PDLF estimated from Hipparcos data for $M_V < 10.5$, extended to fainter magnitudes using ground-based parallax data. Most stars have been scrutinised in much detail (e.g. Duquennoy & Mayor 1991; Fischer & Marcy 1992), so that virtually all components in multiple systems are known and counted individually.

For fainter stars ($M_V \gtrsim 11$), $\Psi_{\text{near}}$ remains defined by the sample of stars with ground-based parallaxes. Completeness of the survey volume ends near $r_{\text{compl}} = 5$ pc (Jahreiss 1994; Henry et al. 1997). The claim that the completeness limit can be extended to beyond 8 pc for M dwarfs (Reid & Gizis 1997), based on including photometric parallax estimates that allow systems with much larger distances to enter the sample (Section 5), is thus unlikely to be correct (Chabrier & Baraffe 2000). This is evident by virtue of long-term radial velocity surveys uncovering previously unknown binary systems to known primaries within $5 < r < 9$ pc and declinations $\delta > -16^\circ$ (Delfosse et al. 1999).

**The photometric or system LF:** $(\Psi_{\text{phot}})$ $\Psi_{\text{near}}$ is poorly defined for $M_V > 11$. Other techniques for estimating the LF of faint main sequence stars were consequently developed. A major driving force in this endeavour was the attempt to quantify how much mass is “hidden” in the faintest stars, given that some investigations prior-to and during 1980 arrived at significant amounts of dark matter apparently distributed like the Galactic disk (e.g. Bahcall 1984), which is now definitely known not to be the case (Crézé et al. 1998).

This approach, made possible by the advent of automatic plate measuring machines and pioneered by Reid & Gilmore (1982), involves deep photographic or CCD imaging in two or three photometric pass bands. The solid angle of such a survey is small, but the volume surveyed is large if the survey extends to $r_{\text{compl}} \gtrsim 100$ pc. Stellar distances, and thus volume number densities, are derived by using photometric parallax. Interstellar absorption is negligible within a few hundred pc if the field of view is directed out of the Galactic disk. The distance limit, $r_{\text{compl,ph}}$, to which the survey is complete, decreases with increasing $M_P$, and can be calculated given that the flux limit below which the survey becomes incomplete is known. Similar to the Lutz-Kelker bias, the Malmquist bias distorts the shape of the flux-limited $\Psi_{\text{phot}}$: an observer overestimates the number of stars, which are intrinsically brighter than the average star of a given colour. This bias arises because a colour does not uniquely specify the absolute magnitude of a star, which also depends on age, metallicity, spin and multiplicity. This bias can be corrected for (Stobie, Ishida & Peacock 1989), and only the Malmquist-corrected photometric LF, $\Psi_{\text{phot}}$, is considered from here on.

While $\Psi_{\text{near}}$ can only be defined with one sample, the many possible line-of-sights out of the Galactic disk allow many independent estimates of $\Psi_{\text{phot}}$. A consistent finding among these surveys, each yielding about 30–60 stars with $M_V \gtrsim 11$, is that $\Psi_{\text{phot}}$ has a maximum at $M_V \approx 12$ with a decline at fainter magnitudes. A weighted average, corrected to the Galactic-disk midplane density, $\overline{\Psi}_{\text{phot}}$, was calculated from the individual surveys (Kroupa 1995a).

The Hubble Space Telescope (HST) delivers diffraction-limited images down to the flux limit ($I \approx 24$). Contamination through galaxies is thus essentially removed, and the stellar distribution is probed to distances of a few kpc. For example, an M dwarf with $M_V = 16$ has $M_I = 12.1$, approximately, and is thus
Figure 1. The photometric LF corrected for Malmquist bias and at the midplane of the Galactic disk ($\Psi_{\text{phot}}$) is compared with the nearby LF ($\Psi_{\text{near}}$). The average, ground-based $\Psi_{\text{phot}}$ (dashed histogram, Kroupa 1995a) is confirmed by HST data (solid dots, Gould et al. 1997). The ground-based trigonometric-parallax sample (dotted histogram) systematically overestimates $\Psi_{\text{near}}$ due to the Lutz-Kelker bias, thus lying above the improved estimate provided by Hipparcos data (solid histogram, Table 1). The thin dotted histogram at the faint end indicates the level of refinement provided by recent stellar additions (see Kroupa 1998 and references therein).

detectable to a distance of about 2.4 kpc. Malmquist bias is negligible, because, in a field-of-view that is directed at a Galactic latitude northward of about 45 deg, the flux limit essentially corresponds to a true volume limit because the photometric distance limit lies well outside the stellar distribution. Many fields are combined to yield reasonable statistics. The Groth-strip survey (Gould, Bahcall & Flynn 1997) adds about 45 stars with $12.5 < M_V < 16.25$. The HST results confirm the finding arrived at from the ground.

**Nearby versus photometric LF:** The resulting LFs are plotted in Fig. 1. $\Psi_{\text{near}}$ increases by an order of magnitude from $M_V \approx 0$ to $M_V \approx 17$. Approximately 70 per cent of all stars have $M_V > 10.5$, demonstrating the possible importance of faint main sequence stars for the mass-budget of a galaxy and star cluster. $\Psi_{\text{near}}$ shows interesting structure. It increases monotonically with increasing $M_V$ until $M_V \approx 5$. It is flat in the interval $5.5 < M_V < 8.5$, the *Wielen dip*, but continues to rise again until $M_V \approx 12$ (Section 4.). Poisson uncertainties are too large at fainter $M_V$ to allow firm conclusions about the shape of the LF, but it is clear that $\Psi_{\text{near}}$ flattens for $M_V > 12.5$. An important point to remember is that any structure in $\Psi_{\text{near}}$ and $\Psi_{\text{phot}}$ is smeared out because of the spread in metallicities, ages, spins and distance errors (correction for Malmquist bias removes this partially in $\Psi_{\text{phot}}$).

The comparison of $\Psi_{\text{near}}$ and $\Psi_{\text{phot}}$ shows that the two are very different and appear to measure different stellar populations for $M_V > 13$. Thus, while 20 stars are counted with $13.5 < M_V < 16.5$ in $\Psi_{\text{near}}$, only 2 stars are seen
in $\Psi_{\text{phot}}$ within a volume of 395 pc. The majority of faint stars in the solar neighbourhood have ages $\tau > 1$ Gyr, and the one-dimensional velocity dispersion is about 30 km/s (Meusinger, Reimann & Stecklum 1991; KTG93; Reid, Hawley & Gizis 1995). This means that essentially all stars located within a spherical volume with a diameter of 600 pc will have been replaced within 20 Myr. The observed difference is not a local over density.

Reid & Gizis (1997) suggest that the $M_V(V-I)$ relation used for photometric parallax estimation is non-linear near $M_V = 12$, which has not been taken into account by previous work (e.g. Stobie et al. 1989; KTG93; Gould et al. 1997). The effect is such that previous work underestimates $M_V$, which leads to underestimates in the density near $M_V = 12$, since distance estimates are too large. Reid & Gizis attribute the apparent difference between $\Psi_{\text{near}}$ and $\Psi_{\text{phot}}$ to this error.

The shape and amplitude of the maximum in $\Psi_{\text{phot}}$ may thus require some revision. However, notable is that system LFs for a wide variety of star clusters also show a very pronounced maximum near $M_V = 12$ (Fig. 2) with very similar shape, which is not surprising if this structure is due to the derivative of the $m(M_P)$ relation in a ‘pure’ population, these LFs not being mired by age, metallicity and distance spreads (different spins may have an influence in young clusters). Furthermore, consulting the $M_V(V-I)$ data plotted by Baraffe et al. (1998, fig. 5), the degree of non-linearity in this relation promoted by Reid & Gizis is not evident.

4. The mass–luminosity relation

Eqn. 1 shows that any non-linear structure in this relation is mapped into observable structure in the LF, provided the MF does not have compensating structure. Such a conspiracy is very unlikely because the MF is defined through the star-formation process, but the $m(M_P)$ relation is a result of the internal constitution of stars.
A thorough understanding of $\frac{dm}{dM_P}(M_P)$ is thus necessary to avoid unphysical features entering the IMF. Much effort has gone into establishing high-quality observational constraints from binary stars (Popper 1980; Andersen 1991; Henry & McCarthy 1993; Malkov, Piskunov & Shpil'kina 1997; Henry et al. 1999). The $m(M_V)$ relation for main-sequence stars is shown in fig. 1 in KTG93. It is immediately apparent that the slope is very small at faint luminosities. This holds true in other photometric passbands as well (Henry & McCarthy 1993) and leads to large uncertainties in the MF near the hydrogen burning mass limit.

The observational data (Andersen 1991) show that the $\log_{10}[m(M_V)]$ relation is essentially linear for $m > 2 M_\odot$. However, as the mass of a star is reduced, and for effective temperatures between 6000 and 7000 K, H$^-$ opacity becomes increasingly important through the short-lived capture of electrons by H-atoms. For stars of spectral type F5 to G, H$^-$ provides more than 60 per cent of the continuous opacity. This results in reduced stellar luminosities for intermediate and low-mass stars. The $m(M_V)$ relation becomes less steep in the broad interval $3 < M_V < 8$ (fig. 1 in KTG93), leading to the Wielen dip evident in $\Psi_{\mathrm{phot}}$ (Mazzitelli 1972; Meusinger 1983; D’Antona & Mazzitelli 1986; Kroupa, Tout & Gilmore 1990, KTG90; Haywood 1994).

The modern data (Henry et al. 1993; fig. 2 in Kroupa 1998) confirm the steepening in the interval $10 < M_V < 13$ postulated by KTG90 to be the origin of the maximum in $\Psi_{\mathrm{phot}}$ near $M_V = 12$. The $m(M_V)$ relation steepens near $M_V = 10$ because the formation of H$_2$ in the outer shells of main-sequence stars causes the mean molecular weight to be larger for less massive stars, invoking core contraction. This leads to brighter luminosities and full convection for $m < 0.35 M_\odot$. The $m(M_V)$ relation flattens again for $M_V > 14$, $m < 0.2 M_\odot$, as degeneracy in the stellar core becomes increasingly important for smaller masses, thus supporting the core against further contraction (Chabrier & Baraffe 1997).

A pronounced local maximum in $-\frac{dm}{dM_V}(M_V)$ results at $M_V \approx 11.5$. Artificial suppression of H$_2$ formation eliminates this maximum (fig. 3 in KTG90). Different theoretical $m(M_P)$ relations have extrema in $\frac{dm}{dM_P}(M_P)$ at different $M_P$, suggesting the possibility of testing stellar structure theory near the critical $m \approx 0.35 M_\odot$, where stars become fully convective (Kroupa & Tout 1997; Brocato, Cassisi & Castellani 1998).

5. Additional complications

Multiple stellar systems: In addition to the non-linearities in the $m(M_P)$ relation, unresolved multiple systems affect the MF derived from an observed LF, in particular since no stellar population is known to exist that has a binary proportion smaller than 50 per cent, apart from the dynamically highly evolved globular clusters (e.g. Kroupa 2000b and references therein).

Suppose an observer sees 100 systems. Of these 40, 15 and 5 are binary, triple and quadruple, respectively. There are thus 85 companion stars which the observer is not aware of if none of the multiple systems are resolved. Since the distribution of secondary masses for a given primary mass is not uniform, but typically increases with decreasing mass (e.g. Malkov & Zinnecker 2000), the bias is such that low-mass stars are significantly underrepresented in any
survey that does not detect companions (Fig. 3 below; Piskunov & Malkov 1991; Kroupa, Tout & Gilmore 1991, KTG91; KTG93; Holtzman et al. 1997; 1998).

Counting a multiple system as a single star has the following effects in a photometric survey: (i) if the companion(s) are bright enough to affect the system luminosity noticeably, then the estimated photometric distance will be too small, and (ii) the companions are lost from the star-count analysis. The former effect enhances the apparent stellar number density at brighter magnitudes. This is evident in fig. 7 in KTG91, where the system LF lies above the single-star LF for \( M_V < 10 \). However, in reality this is countered by the larger effective photometric distance limit together with the approximately exponential stellar density fall-off perpendicular to the Galactic plane, implying that the photometric LF not corrected for Malmquist bias is about equal to or smaller than \( \Psi_{\text{near}} \) for most \( M_V \) (KTG93). The latter effect (ii) reduces the star-counts at faint magnitudes leading to a significant bias, because a G-, K- and bright M-dwarf has, on average, one or more faint M-dwarf companions. Note that a faint companion will also be missed if the system is formally resolved but the companion lies below the flux limit of the survey.

The true distance limit can be significantly larger than the nominal value when estimating a photometric parallax. If a stellar system in the star-count survey is composed of two equal-mass stars, it has an \( M_V \) brighter by 0.75 mag than a single star of the same colour. If, in addition, the system has a metallicity such that the combined absolute magnitude is \( 3\sigma_{M_V} = 1.5 \) brighter than a star of the same colour (where \( \sigma_{M_V} \) is the spread in \( M_V \) for a given colour, the 'cosmic scatter'), then the resulting absolute magnitude of the unresolved system can be brighter by as much as \( \delta M_V = 2.25 \), which implies that it can be seen \( 10^{\delta M_V/5} = 2.82 \) times as far as the nominal distance limit. For example, if the nominal distance limit is 130 pc, then systems as far away as 366 pc can in principle enter the sample. While this is an extreme case, it does demonstrate that photometric star-count surveys are contaminated by systems that are beyond the nominal distance limit, which, of course, needs to be part of any model (KTG93).

**Metallicity and Ages spreads:** The metallicity distribution, which results from the chemical evolution of the Galactic disc (see e.g. Gilmore & Wyse 1991; Samland, Hensler & Theis 1997; Tsujimoto et al. 1997; Rocha-Pinto et al. 2000), is non-Gaussian with a mean near \([\text{Fe/H}] \approx -0.2 \) dex, a width of roughly 0.3 dex, and a tail towards low iron abundance. K dwarfs have a similar distribution as G dwarfs (Flynn & Morell 1997; Rocha-Pinto & Maciel 1998). A somewhat different abundance distribution for stars of different mass is to be expected though because of the age–metallicity relation (e.g. Meusinger et al. 1991; Ng & Bertellie 1998; Carraro, Ng & Portinari 1998). Stars with short life-times usually only sample the high-metallicity range.

The distribution of stellar ages is a major source of uncertainty in the analysis of counts of stars that have a mass in the range \( 0.8 M_\odot - 3 M_\odot \) (Haywood, Robin & Crézé 1997;a; Maciel & Rocha-Pinto 1998). The SFR–\( \alpha \) degeneracy is emphasised by Binney, Dehnen & Bertelli (2000). Such stars evolve along and off the main sequence within the age of the Galactic disc (9 – 12 Gyr), so that the distribution of luminosities and colours for main-sequence stars with the same mass depend critically on the sf rate (SFR). Stars with \( m \gtrsim 3 M_\odot \) have
life-times much shorter than the age of the Galactic disc, and consequently map only the most recent.sf history. Only the ratio of the average SFR to the present SFR is important in adjusting star counts of these stars to the number of stars with \( m \lesssim 0.8 M_\odot \), which amount to all stars ever formed in the Galactic disc, assuming \( \xi(m) \) is continuous across \( m \approx 3 M_\odot \) and unchanging.

There are tentative hints at non-uniform star-formation histories. For example, Noh & Scalo (1990) discuss a marginal feature in the white-dwarf-luminosity function which may suggest a burst of star formation about \( 3 \times 10^8 \) yr ago. This burst may have had a duration of \( \lesssim 10^8 \) yr, and may have contributed about 10 per cent of the stars in the solar neighbourhood. However, only stars with \( m \lesssim 0.2 M_\odot \), that have long pre-main sequence contraction times, appear brighter by \( \delta M_V \gtrsim 0.2 \) mag if they are younger than about \( 3 \times 10^8 \) yr old (Baraffe et al. 1998). Possible bursts of the SFR about 8 and 3 Gyr ago are suggested by Rocha-Pinto & Maciel (1997), Rocha-Pinto et al. (2000), and Hernandez, Valls-Gabaud & Gilmore (2000) discuss the local sf history during the past 3 Gyr evident from Hipparcos data.

In summary, fully consistent modelling of star-count surveys must include models of the age distribution of Galactic field stars, unless only the restricted mass range \( 0.2 \lesssim m/M_\odot \lesssim 0.8 \) is studied.

**Galactic structure:** Star count surveys used to estimate the local LF of late-type main-sequence stars are restricted to distances of less than about one kpc. Galactic components such as the bulge or the stellar halo are thus not very important. A summary of Galactic structure and related topics may be found in Gilmore, Wyse & Kuijken (1989).

The structure of the Galactic disc can be described reasonably well by exponential density distributions in radial and vertical directions. The radial scale length is about 2.5 kpc (Bienaymé & Séchaud 1997; Porcel et al. 1998). The vertical structure can be fitted by two exponential distributions: the normal disc with a scale height of \( h \approx 250 \) pc (KTG93; Haywood, Robin & Crézé 1997b; Méndez & Guzmán 1998), i.e. not 300–350 pc, and the thick disc with a scale height of about 1 kpc. At the Galactic midplane, the thick disc contributes only a few per cent to the number density of disc stars (Vallenari, Bertelli & Schmidtobreick 2000), and the stellar halo contributes about 0.1 per cent (Robin & Crézé 1986). Finally, the Sun appears to be located about 10–20 pc “above” the Galactic plane (Marsakov & Shevelev 1995; Reed 1997; Minezaki et al. 1998). This off-set can usually be neglected in the analysis of star-counts, but may induce some anisotropy in \( \Psi_{\text{near}} \). Deep photometric surveys, however, have to be corrected for Galactic structure.

The contamination of \( \Psi_{\text{phot}} \) with thick-disk stars can be significant. These are, on average, metal-poorer than ‘normal’ disk stars that make-up \( \Psi_{\text{near}} \), so that photometric parallax would lead to systematically wrong distance estimates and thus space densities, since an inappropriate colour–magnitude relation is used. In a survey with distance limit \( z_o \) covering a solid angle \( \Omega \), the number of stars detected is \( N = \Omega \int_0^{z_o} \rho(z) z^2 dz \). The density fall-off perpendicular to the Galactic disk can be approximated by \( \rho(z) = \rho_o \exp(-z/h) \), \( h \) being the scale height. Thus \( N = \Omega \rho_o h^3 [2 - (2 + 2y + y^2) / \exp(y)] \), \( y \equiv z_o / h \), and about 1/3 of all stars in an HST survey with \( z_o = 1 \) kpc are thick-disk stars. The excellent agreement between the ground-based and HST \( \Psi_{\text{phot}} \), evident in Fig. [4].
is, however, in-line with the general finding that the shape of the system LF, i.e. of \( dm/dM_P \), is not very sensitive of the metallicity of the underlying population (Fig. 2 and Kroupa & Tout 1997).

6. The IMF

The above sections give an impression of the complexity required to analyse local star-counts in order to infer the underlying IMF. Such complex models have been constructed by KTG93, with the finding that \( \alpha_1 = 1.3 \pm 0.5 \) for \( m < 0.5 M_\odot \) and \( \alpha_2 \approx 2.2 \) for \( m > 0.5 M_\odot \) \((\xi(m) \propto m^{-\alpha_i})\). This is supported by the dynamical population synthesis model of the sn presented by Kroupa (1995b). This model does not rely on random association of stars into binaries, and nicely reproduces the empirical mass-ratio distribution of late-type binaries of Reid & Gizis (1997) (fig. 1 in Kroupa 2000b). Using an entirely different empirical \( m(M_V) \) relation but a simpler star-count model, leads to the same result (KTG91). These models always aim at reproducing \( \Psi_{\text{near}} \) and \( \Psi_{\text{phot}} \) simultaneously, thereby improving the constraints. A slightly shallower IMF \((\alpha = 1.05, m \lesssim 1 M_\odot)\) is arrived at by Reid & Gizis (1997), although their nearby star-count sample is incomplete (Section 3.), and stellar evolution is not modelled. Gould et al. (1997) find \( \alpha_1 \approx 0.9 \) \((m < 0.6 M_\odot)\) and \( \alpha_2 \approx 2.2 \) \((m > 0.6 M_\odot)\) by analysing their \( \Psi_{\text{phot}} \) constructed from HST data, but they apply only very crude corrections for unresolved binaries and also do not include stellar evolution. A somewhat steeper local IMF \((\alpha_1 = 2 \pm 0.5)\) is arrived at using a theoretical \( m(M_V) \) relation and \( \Psi_{\text{near}} \) (Méra, Chabrier & Baraffe 1996). This \( m(M_V) \) relation is expected to be further revised as additional opacity sources in the optical are introduced (Baraffe et al. 1998). Finally, the very extensive modelling of the sn by Haywood et al. (1994; 1997a; 1997b) arrives at \( \alpha \approx 1.7 \) for \( m < 1 M_\odot \) and \( \alpha \approx 2 \) for \( 1 - 3 M_\odot \). These models incorporate stellar evolution, unresolved binaries and Galactic-disk structure, but again only rely on \( \Psi_{\text{near}} \) to constrain \( \alpha \) for \( m \lesssim 0.5 M_\odot \).

A consensus thus appears to be emerging that the MF has \( \alpha_2 \approx 2.3 \) for \( m \gtrsim 0.5 M_\odot \) and \( \alpha_1 \approx 1 - 1.5 \) for \( 0.1 < m < 0.5 M_\odot \), the flattening of the IMF near \( 0.5 M_\odot \) becoming evident when the statistically better-defined \( \Psi_{\text{phot}} \) is used in addition to \( \Psi_{\text{near}} \).

7. The Future

The remaining discrepancies for \( m \lesssim 0.5 M_\odot \) can partially be alleviated with significantly improved trigonometric-parallax limited star-counts, as will become available with the upcoming astrometry satellites DIVA (Röser 1999) and GAIA (Lindgren & Perryman 1996; Gilmore et al. 1998b). DIVA will fly for two years in 2004, and will find all systems with \( M_V \lesssim 16 \) and with \( r_{\text{compl}} \lesssim 15 \) pc, measuring their distances and luminosities. GAIA has also been approved by the European Space Agency.

Examples of the type of single-star and system LFs expected using different \( m(M_V) \) relations are shown in Fig. 3.

The following two warnings must be stressed at this point: 1) DIVA and GAIA will provide exquisite data for the motions and positions of stellar sys-
Figure 3. Model LFs assuming $\alpha_1 = 1.6$ (dash-dotted lines) or $\alpha_1 = 1.0$ (solid lines) for $0.08 - 0.5 M_\odot$ and the Salpeter value $\alpha_2 = 2.3$ for $0.5 - 1.0 M_\odot$ ($\xi(m) \propto m^{-\alpha_i}$). Thick lines are the single-star LF, whereas thin lines show the system LF (all companions merged photometrically) for a population consisting of 8000 single stars, 8000 binaries, 3000 triples and 1000 quadruples (40:40:15:5 per cent, respectively). Companions are combined randomly from the IMF. The models assume perfect photometry, no distance errors and no metallicity or age spread. The left panel is for the semi-empirical $m(M_V)$ relation from KTG93, and the right panel uses the theoretical relation from Baraffe et al. (1998, 5 Gyr isochrone, $[M/H]=0$). The histograms are as in Fig. 1 and the models are scaled to fit the data near $M_V = 7$ (equal scaling for both the single-star and system LFs).

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