Estimation of resolution of dispersive correlator

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Abstract. Estimates of the maximum number of resolvable elements and the relative spectral resolution of dispersive correlators were performed. The obtained relations were used in the synthesis of Fourier holograms. Their validity was confirmed in experiments on recognition of emissive objects in the dispersive correlator setup.

1. Introduction
Optical diffractive correlators are widely used for optical data processing. However, the number of applications for these correlators is usually limited to processing of binary or grayscale images formed in monochromatic light [1]. Unlike correlators for recognizing objects in a monochromatic light, in dispersive correlators [2] recognition signals are formed by the actual radiation of recognized emitting object at its interaction with the spatial filter (optical memory). In these correlators in capacity of spatial filter computer generated Fourier-holograms are often used.

2. Spatial and spectral resolution of dispersive correlator
There are various schemes of dispersive correlators, one of them is presented in figure 1. $L_1$ is distance from the input plane of the correlator, where emissive recognizable object is located to Fourier-objective. $L_2$ is distance from objective to output plane, where signal is registered on camera. $L$ is distance from output plane to the plane of spatial frequencies, where synthesized hologram is located. In output plane correlation signal is formed. It takes into account both spatial and spectral parameters of reference and recognized objects. Its location corresponds to hologram first diffraction order.

Figure 1. The principal scheme of the dispersive correlator with single Fourier-objective.
The important informational characteristics of the dispersive correlator are: the number of resolvable elements in an image of recognized object (spatial resolution) and resolution on spectrum of radiation. In dispersive correlators part of available pixels is used for recognition of the emission spectrum, so the number of resolvable elements of recognized object image depends on characteristics of its emission spectrum. If synthesized Fourier-hologram is used in correlator, spatial and spectral resolutions are determined by the number of pixels of hologram.

At synthesis of holographic Fourier-filter for dispersive correlator, the generalized spatial pattern of the reference object is recorded on the hologram [3]. This spatial pattern contains such information as the spatial structure and the emission spectrum of the object. This spatial pattern consists of a number of multi-scale copies of reference object's image. The number of copies is defined by the number of components in the emission spectrum of reference object. Scale and location of each copy are inversely proportional to the spectrum component's wavelength

$$\frac{M_{\lambda_{\text{min}}}}{M_{\lambda_{\text{max}}}} = \frac{P_{\lambda_{\text{min}}}}{P_{\lambda_{\text{max}}}} = \frac{L_{\lambda_{\text{min}}}}{L_{\lambda_{\text{max}}}} = \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}},$$  \hspace{1cm} (1)

where $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ are the minimum and the maximum wavelengths in the spectrum of the reference object, $M_{\lambda_{\text{min}}}$ – the size of the largest copy along the axis of the carrier spatial frequency, $M_{\lambda_{\text{max}}}$ – the size of the smallest copy along the axis of the carrier spatial frequency (the sizes along other coordinate are designated as $P_{\lambda_{\text{max}}}$ and $P_{\lambda_{\text{min}}}$), $L_{\lambda_{\text{max}}}$ – the distance from coordinates origin to the center of the smallest copy, $L_{\lambda_{\text{min}}}$ – the distance from the coordinates origin to the center of the largest copy.

In figure 2 the layout of object image's copies on the calculating field of the hologram pulse response is shown. Calculation field has $N\times N$ pixels. Rectangle was used as reference object, its radiation spectrum consists of two wavelengths. $N_{\lambda_{\text{max}}}$ – the distance from the largest copy of the object to the border of the calculating field of the hologram, $N_{\lambda_{\text{min}}}$ – distance from the coordinates origin (the center of zero-order of the hologram) to the left border of the smallest copy of the object.

**Figure 2.** The layout of object image's copies on the calculating field of the hologram pulse response.

If such Fourier hologram is used as a spatial filter, resulting correlation signal is formed in output plane of dispersive correlator. It is the sum of the spectral components of the correlation signals for each wavelength of radiation of recognized object. The values of each spectral component of the correlation signal are determined by the degree of similarity of spatial forms of the reference and recognized objects. Degree of spatial convergence of correlation signal spectral components peaks is determined by degree of similarity of emission spectra of reference and recognized objects.

While estimating the maximum number of resolvable elements it was assumed that the numbers of resolvable elements in the reference and recognized object's images are the same. Correlation signal
registration requirements for determining the maximum number of resolvable elements in the image of an object are:

1) zero diffraction order containing the image of an input object should not overlap with correlation signal;
2) the highest diffraction orders should not overlap with correlation signal.

If these requirements are fulfilled, generalized spatial pattern of the reference object on the calculated field of hologram impulse response will require following number of pixels:

\[ N_{\text{PNM}} \leq \frac{2 - \frac{\lambda_{\min}}{\lambda_{\max}}}{2} \cdot \frac{N}{2}. \]  \hspace{1cm} (2)

Maximum number of resolvable elements \( M \times P \) in image of the reference object is determined by the number of pixels in the smallest copy of reference object's image contained in its generalized spatial pattern. This smallest copy corresponds to the spectral component with a maximum wavelength \( \lambda_{\max} \) and has a number of pixels \( M_{\lambda_{\max}} \times P_{\lambda_{\max}} \). Therefore, the number of resolvable elements in images of the reference and recognized objects is equal to

\[ M = \frac{N}{2} \cdot \frac{\lambda_{\min}}{\lambda_{\max}} \ , \ P = \frac{N}{2} \cdot \frac{\lambda_{\min}}{\lambda_{\max}} \]  \hspace{1cm} (3)

The obtained expression is consistent with the analogous expression for diffractive correlator with monochromatic light at \( \lambda_{\max} = \lambda_{\min} \).

Estimation of relative spectral resolution of dispersive correlator was obtained from the condition of spatial overlapping of the spectral components of the resulting correlation signal within one resolvable element:

\[ \frac{\delta \lambda}{\lambda} = \frac{2}{N} \cdot \frac{\lambda_{\max}}{\lambda_{\min}}. \]  \hspace{1cm} (4)

This expression is in agreement with the analogous expression for the spectrum analyzer based on diffraction grating, however it also accounts for the range of the emission spectrum of the reference object \( \lambda_{\max} / \lambda_{\min} \). Higher ratio \( \lambda_{\max} / \lambda_{\min} \) leads to lower spatial resolution of dispersive correlator.

The obtained expressions were used to estimate informational characteristics of dispersive correlator for recognition of self-luminous objects formed with application of radiation of RGB light-emitting diodes with \( \lambda_{\min} = 0.472 \mu m \) and \( \lambda_{\max} = 0.645 \mu m \). The number of pixels of synthesized holograms was limited by phase distortions of used transparent film and was equal 1024×1024 pixels. According to obtained estimates the maximum number of resolvable elements in image of object was equal 149×374 pixels, relative spectral resolution was about 0.003. These values are in agreement with results of experiments on dispersion correlator setup with application of synthesized holograms.

3. Conclusion

Thus, estimates of maximum number of resolvable spatial elements and relative spectral resolution of dispersive correlator were obtained. These characteristics are determined by the number of pixels of synthesized hologram and ratio of maximum to minimum wavelengths in the emission spectrum of recognized object.

References

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