Search for astro-gravity correlations

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Abstract

A new approach in the gravitational wave experiment is considered. In addition to the old method of searching for coincident reactions of two separated gravitational antennae it was proposed to seek perturbations of the gravitational detector noise background correlated with astrophysical events such as neutrino and gamma ray bursts which can be reliably registered by correspondent sensors. A general algorithm for this approach is developed. Its efficiency is demonstrated in reanalysis of the old data concerning the phenomenon of neutrino-gravity correlation registered during of SN1987A explosion.

1 JOINT SCENARIOS FOR ASTRO-GRAVITY EVENTS

A conventional scheme of the gravitational wave experiment on searching for stochastic bursts of gravitational radiation from astrophysical sources supposes a registration of coincident reactions of two or more spatially separated gravitational detectors. It was considered as only way to establish a global nature of the detected signal which probably could be a metric perturbation associated with gravitational wave if the detector’s isolation was good enough [1].

A realization of this scheme requires at least two identical gravitational antennae located in different points of the globe with good synchronized clocks, good communication etc. Although this ideology is known already thirty years the coincident experiment in automatic regime was performed only by J.Weber during his first observation with room temperature bar detectors located in Chicago and Maryland [2]. Later the ”coincidence searching” episodically have been done by several groups as a rule in the form of joint data analysis of the electronic records of both setups \textit{a posteriori} but not on line. The recent example of such procedure with cryogenic antennae EXPLORER and ALLEGRO is presented in the paper [3]. A reason why the detection of coincidences ”on line” was
replaced with analysis *a posteriori* is obvious. The "on line" regime (although it’s very convenient and effective) requires an additional electro-communication equipment. Besides it could be easy realized if the same research group would have two equivalent detectors in disposal (like it was in "time of room temperature bar detectors") but a complication and large cost of modern cryogenic and interferometrical set up makes it difficult in general. In nearest future the automatic selection of coincidences probably will be realized with two large scale interferometric antennae which are under construction now in the LIGO project [4]. At present however the coincidence analysis *a posteriori* is considered as the only way of investigation stipulated by a presence of two gravitational antennae in simultaneous operation with equivalent sensitivity.

In last years another type of gravitational wave experiment was discussed. The idea is to search weak perturbations of the gravitational detector’s noise background correlated with some astrophysical events such as neutrino and gamma ray bursts [5,6,7,8]. The reason of this approach lies in the understanding that last stages of star evolution (such as supernova explosion, binary coalescence, collapse etc.) traditionally considered as the gravitational burst sources have to be accompanied also by neutrino and very likely gamma radiation. It means in general that a detection of neutrino or gamma ray bursts by appropriate sensors defines time marks around which one might hope to find also exitations of the gravitational detectors. An advatage of this method consists first of all in a remarkable reduction of the observational time interval and second in a potential opportunity to accumulate weak signals. The last point is especially interesting taking into account a deficit of required sensitivity of the gravitational detectors available at present in the world laboratories.

The theoretical presentation of the neutrino bursts produced by collapsing stars at the end of stellar evolution is well known, see for example [9,10,11]. According to the theory a total energy released in the form of neutrino radiation of all flavors has the order of value $0.1M_\odot c^2$ and a time scale of several seconds (2–20 s). This radiation can be detected (mainly due to the inverse $\beta$- decay reaction) if a source is located not too far from the Earth (10–100) kpc. Correspondent experimental programms ("Supernova Watcher") are accepted and carried out by the all neutrino groups having appropriate liquid scintillation detectors [12,13] or water cherenkov detectors [14,15]. Moreover the first registration of neutrino flux from supernova as it believes was fixed during of SN1987A explosion [16-19]. All this programms are orientated on the search of collapsing stars in the Galaxy and close local groups i.e. expected average rate of events is 3 per 100 years [16]. It is unlikely to wait a large increasing of penetrating power from the neutrino telescopes in nearest future. So Super Kamiokande detector with effective mass in ten times larger allows a detection of 150 neutrino events per year from LMC but only one event from Andromeda [20]. It is unrealistic to relay on a detection neutrino from supernova in the Virgo Cluster (15–20 Mpc) which considered as one of the principal sources of a signal for gravitational detectors. Thus a search of correlations between noise
backgrounds of neutrino and gravitational wave detectors is limited by the condition of very low event rate \((3-10)10^{-2} y^{-1}\) and an opportunity of "signal-noise enhancing" through some integrating procedure practically is absent. Although an expected amplitude of a solitary gravitational pulse signal might be relatively large up to \(10^{-18}\) in term of metric perturbation from a source in the center of Galaxy.

The other astrophysical phenomenon of our interest, gamma-ray bursts, looks more propitious although it still remains to be confused [21]. The main attractive feature of this phenomenon is a relatively high event rate, on average one per day. The large energy emission evaluated for some registered gamma bursts up to the \(0,1 M_c c^2\) together with amplitude short time variations on order of \(0,1\ s\) implies to relativistic stars as burst sources.

In process of study of this phenomenon two principal scenarios have been considered in respect of the gamma-ray bursts nature. The first one suggests its galactic origin associated with high velocity pulsars distributed not only in the galactic disc but also in the Halo [22]. The second scenario appeals to a cosmological picture in which gamma bursts are produced during catastrophic processes with relativistic stars such as collapses, binary coalescences, supernova explosions in distant galaxies [23]. Thus the both scenarios deal with objects that have been considered also as sources of gravitational radiation. Galactic pulsars could produce only very weak GW-bursts as a result of "starquakes" with equivalent metric perturbation on the Earth of order of \(10^{-23} ÷ 10^{-24}\) [24] for a source in center of Galaxy. However authors of the papers [25,26] believe that even a more close pulsar population in vicinity 100 pc might provide an observable rate of gamma events \(\sim 5\) per month through mechanism of "starquake". Then a correspondent GW burst amplitude would be awaited on the level of \(10^{-21} ÷ 10^{-22}\). In the cosmological picture, if one includes into consideration binaries with back hole components the astrophysical forecast gives the GW-burst event rate up to 30 per year at a metric amplitude level of \(10^{-21}\) in the solar vicinity of 50–100 Mpc [27,28]. This estimation was found supposing that only \(10^{-4}\) part of stellar rest mass energy could be converted into gravitational radiation. A more optimistic value of the conversion coefficient \(10^{-2}\) used in the other papers [29,30] would increase the expected metric amplitude up to \(10^{-20}\).

The recent results obtained with BeppoSAX satellite and Keck II telescope permitted to confront the gamma-ray burst GRB971214 with a galaxy having the redshift of \(z = 3.4\). The other case is the burst GRB970508 with an optical counterpart at \(z \geq 0.835\) [31]. That is the strong evidence of the cosmological nature at least for a part of the registered bursts. Along with these very far sources (1-10) Gpc. more close events were registered. For example the burst GRB980425 probably was associated with an optical object type of supernova explosion at the distance 40 Mpc \((z = 0.08)\) [32]. It is not completely clear how the gamma radiation could penetrate through envelope of supernova, how the black hole coalescence could release the gamma burst, but the energetic
of observable events definitely requires scenarios with a crash of relativistic stars and therefore an expectation of the gravitational radiation accompaniment seems reasonable. Moreover the energetic estimation of the GRB971214 burst $\sim 2 \cdot 10^{53}$ erg even exceeds a conventional theoretical electromagnetic energy release $10^{51}$ erg for supernova or neutron star binary merging [33]. It makes the models of black hole binary mergers or rapidly rotating massive black hole with accretion, so called "hypernova" [34], more attractive and at the same time they are more promising in respect of the gravitational wave output.

Thus there are serious theoretical prerequisites to search for gravitational bursts around time marks defined by correspondent events of neutrino and gamma-ray detectors. Now lists of desirable events can be provided by the four world neutrino telescopes and cosmic CGRO (BATSE) and BepoSAX satellites. In this situation the key question is a sensitivity of the gravitational detectors which are in operation at present. In fact this is only supercryogenic resonance detector "NAUTILUS" (INFN,Frascati) and similary set up "AURIGA" (INFN,Legnaro) [58] could achieve the sensitivity level $10^{-21}$ for short bursts $\sim 10^{-3}$ s [35]. The two cryogenic detectors mentioned above "ALLEGRO" [59] and "EXPLORER" have the short burst sensitivity $6 \cdot 10^{-19}$ i.e. of 2,5 orders less the desirable value. However it worth to note here that for more long signals the estimation of its sensitivity must be increased up to $10^{-21}$ for burst duration close to 1 sec due to accumulation of signal cycles (see details in [36]).

Generally an improvement of detection sensitivity depends on our knowledge of the signal structure, arrival time etc. In this sense a theory does not provide us a large assortment of models for gravitational signal. Mostly its energetic part might be presented by a short pulse with several cycles of carrier frequency ($10^2 - 10^3$) Hz [24]. There is a deficit of models with joint description of the gravitational, neutrino, and gamma radiation output. Some examples one can find in the papers [24,29,30,37] where multi-stage scenarios of gravitational collapse were considered in the processes of neutron star formation and star remnants coalescence. In such approach a packet of the neutrino pulses separated by time intervals from few seconds up to several days accompanied by gravitational bursts was predicted with a total energy release up to one percent of the rest mass. The multi-stage scenario is also typical for collapse of massive star with large initial angular momentum [24]. A radial matter compression there might be interrupted by repulsing bounces, fragmentation, fragments mergers or ejection of one of them etc. In principle each of these stage could produce gravitational, electromagnetic and neutrino bursts but a detailed description of such models has not yet been developed. Entirely inspite of obvious uncertainty of joint scenarios and unknown event rate of complex collapses in the Universe an expectation of the multi-pulse structure for a gravitational signal associated with a packet of neutrino and gamma ray bursts is enough grounded at present.

The argumentation above stimulates one to define an optimal data processing of the gravitational detector output in parallel with a record of astrophysical events registered by neutrino or gamma ray observatories. A simple compar-
ison with an attempt to find coincidences is insufficient due to an inevitable unknown time delay between events of different nature [57] but mainly due to a deficit of gravitational and neutrino detector sensitivity. Partly for this reason the attempts of searching for correlation between neutrino-gamma data [38] and gamma-gravity data [39] were not successful. It has to be done according to the optimal filtration theory taking into account all available information concerning of noise background and conceivable model of signal [40].

The goal of this paper is to formulate some optimal algorithm of searching for a correlation of neutrino as well as gamma-ray events with perturbations of gravitational bar detector. We consider this case because a noise statistics of the resonance bar detector at present can be defined more accurately than the statistics of a free mass gravitational antenna.

The associated goal is to apply the optimal algorithm to old data concerning the neutrino-gravity correlation phenomenon registered during of S N1987A supernova explosion. The results reported in the series papers [41-44] have not found any clear astrophysical explanation, have met some objections, and up to now continue to be subject for discussion.

2 MLP-detection algorithm for incoherent packets of GW pulses

In a general frame of the filtration theory it is necessary to define principal properties of expected signal and noise background in order to find an optimal filtering procedure. Following the argumentation above we take as a signal model so called "incoherent packet of pulses" in which gravitational events are given by an irregularly group of short impulses with two principal parameters: arrival times \( \tau_i \) and amplitudes \( a_i \). Intervals between individual impulses might be varied in wide limits according to the rate of astrophysical events. The form of individual pulse is ignored besides its duration \( \hat{\tau} \) which is supposed to be enough short i.e. it contains only a few periods of the carrier frequency \( \omega \) so that \( \omega \hat{\tau} \sim 1 \) and \( \omega \sim (\omega_0 \mp 1/\hat{\tau}) \) where \( \omega_0 \) is a central frequency of the receiver bandwidth.

A stochastic background is defined by the noises of gravitational bar antenna. The structure of modern cryogenic antenna contains of a cooled bar-detector, electromechanical transducer as a read out, amplifier and a preliminary filtration link with limited bandwidth \( \Delta \omega \leq \hat{\tau}^{-1} \): a differential cell, Winer-Kolmogorov filter etc. The principle point is that one can take the Gaussian model of output antenna noise as a good approximation having in the mind a perfect acoustical, seismic and electrical isolation of modern cryogenic set ups.

After these remarks one can give a mathematical formulation of the optimal detection procedure. The antenna output \( x(t) \) is an additive mixture of the noise \( \xi(t) \) and signal \( S(t) \) where the last one might be described by incoherent
sequence of short "gravitational" bursts $s_k$ so that

$$x(t) = \lambda S(t) + \xi(t) \quad S(t) = \sum_k s_k(t).$$  \hspace{1cm} (1)

Here $\xi(t)$ is supposed to be a stationary gaussian noise with the spectral density $W(\omega)$ defined by the antenna structure; $\lambda = (1, 0)$ is a formal parameter marking a presence or absence of the signal. The individual pulse signal in the $S(t)$ sequence can be presented in the complex space as

$$s_k(t) = \text{Re}[\tilde{s}_k(t)e^{j\omega_0 t}],$$
$$\tilde{s}_k(t) = a_k \tilde{H}(t - t_k)e^{j(\Theta_k - \omega_0 t_k)}.$$  \hspace{1cm} (2)

The new notations in these expressions $\tilde{s}_k(t)$ and $\tilde{H}(t)$ are complex overlapes of the "gravitational" bursts and impulse characteristics (Green function) of the linear antenna track

$$H(t) = \text{Re}[\tilde{H}(t)e^{j\omega_0 t}] = H_0(t) \cos[\omega_0 t + \psi(t)]$$

with $\omega_0$ as a resonance frequency of the bar antenna.

In addition to the mentioned signal parameters, — pulse amplitudes and arrival times, the expression (2) contains also the third parameter, — initial phases $\Theta_k$. Of course an optimal data processing algorithm depends on apriori suppositions concerning these values.

The amplitude parameter $a_k$, if it is small, does not produce any remarkable influence on the structure of data processing algorithm. In contrast the two other parameters, initial phase and pulse arrival time essentially affect on the optimal detection procedure. In particular a principal specifics of the "astrogavity correlation hypothesis " should be expressed in the apriori supposition that arrival times of gravitational bursts are located in vicinity of astrophysical event times registered by some independent way, i.e.

$$t_k = t_{ak} + \tau, \quad k = [1, 2...n].$$  \hspace{1cm} (3)

Here $\tau_{ak}$ are the time-marks of "astrophysical events" a total number of which was $n$ on the observational interval $[0, T]$; $\tau$ is an unknown shift between "astrophysical" and "gravitational" events. Admissible values of this shift have to be limited apriori by some interval $(\tau_{\min}, \tau_{\max})$ which must be defined specially.

The problem of optimal data processing algorithm for a detecting of packet of GW-pulses on the output of gravitational bar antenna can be solved in the frame of Maximum Likelihood Principal. According to MLP one has to construct a special variable, some function of antenna output process $x(t)$, maximization of which can provide a maximum probability to register a signal aposteriori, i.e. referring only to the factual realization $x(t)$ on the observational time interval $[0, T]$ and having in the mind an available apriori information.
In the case of a signal on the gaussian noise the answer is well known: MLP-variable $z$ is proportional to the logarithm of likelihood ratio functional $\Lambda[x]$ [40,45]

$$\Lambda[x] = \left\langle \exp \left[ \int_0^T x(t)u(t)\,dt - \frac{1}{2} \int_0^T S(t)u(t)\,dt \right] \right\rangle$$ (4)

where the reference function $u(t)$ is a solution of the integral equation

$$\int_0^T K_\xi(t-\tau)u(\tau)\,d\tau = S(t),\quad 0 < t < T$$ (5)

with $K_\xi(t)$ — the correlation function of the $\xi(t)$ process; the symbol $\langle \ldots \rangle$ means a statistical averaging.

In this point we must introduce one more hypothesis concerning on apriori signal information: namely we suppose that pulses in our packet are rare enough and can not recover each other in time. It completely corresponds to astrophysical expectation of small events rate for catastrophic phenomena with relativistic stars. This hypothesis of ”unrecovering pulses” immediately leads to the factorization of likelihood ratio functional

$$\Lambda[x] = \prod_{k=1}^n \Lambda_k[x]$$ (6)

where $\Lambda_k$ is the likelihood ratio functional for individual k-pulse. It obeys to the formulae (4,5) with substitution $u(t) \rightarrow u_k(t)$ and $S(t) \rightarrow s_k(t)$. Thus finally the MLP-variable can be presented in the form

$$Z = \sum_{k=1}^n \ln \Lambda_k[x] = \sum_{k=1}^n z_k, \quad z_k = \ln \Lambda_k[x].$$ (7)

The equations (4), (5), written for the individual pulse $s_k(t)$ and formula (6) represent a general solution of the problem MLP-variable for the incoherent packet of signal pulses on the gaussian noise background. To reduce it on a practical level one has to find a manifest form of the reference function $u_k(t)$ and then to calculate the value $z_k$. Below we give some approach to such procedure.

Under the natural conditions that a correlation time of the bar antenna noise as well as a signal pulse duration are much less the observational interval $T$ one can expand the upper limit of integrand in the equation (5) to infinity; then a spectral transformation of this expansion leads to

$$u_k(\omega) \simeq s_k(\omega)/N_\xi(\omega)$$ (8)

where the correspondent Fourier transformants are introduced: $u_k(\omega) \leftrightarrow u_k(t)$, $s_k(\omega) \leftrightarrow s_k(t)$ and $N_\xi(\omega) \leftrightarrow K_\xi(t)$. 

7
Then having in mind the Parseval identity
\[
\int_{-\infty}^{\infty} a(t)b(t) \, dt = (1/2\pi) \int_{-\infty}^{\infty} a(\omega)b^*(\omega) \, d\omega
\]
one can rewrite (4) for individual pulse in the following form
\[
\Lambda_k[x] \simeq \left\langle \exp \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{x(\omega + \omega_0)\hat{s}_k^*(\omega)}{N_\xi(\omega + \omega_0)} d\omega - \frac{1}{8\pi} \int_{-\infty}^{\infty} \frac{|\hat{s}_k^*(\omega)|^2}{N_\xi(\omega + \omega_0)} d\omega \right\} \right\rangle
\]
(9)

Here \( \hat{s}_k(\omega) \) is the transformant of the complex overlap of the pulse \( \tilde{s}_k(t) \).

After substitution this overlap from (2) in (9) the last one is reduced to
\[
\Lambda_k[x] \simeq \left\langle \exp \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{x(\omega + \omega_0)\hat{H}^*(\omega)}{N_\xi(\omega + \omega_0)} e^{j\omega t_k} d\omega \right. \\
\left. - \frac{a_k^2}{8\pi} \int_{-\infty}^{\infty} \frac{\left| \hat{H}(\omega) \right|^2}{N_\xi(\omega + \omega_0)} d\omega \right\} \right\rangle
\]
(10)

with \( \hat{H}(\omega) \leftrightarrow \tilde{H}(t) \) and \( \chi_k = \omega_0 t_k - \Theta_k \).

It is convenient to present the expression (10) in terms of the output antenna variable \( \tilde{y}(t) \) which is a result of passing the input variable \( x(t) \) through some optimal "data processing filter" with transfer function \( K_{opt} = [\hat{H}^*(\omega)/N_\xi(\omega)] \exp(-j\omega t_0) \), where \( t_0 \) is a filter time delay. Then (10) can be converted into
\[
\Lambda_k[x] \simeq \left\langle \exp \left\{ a_k \text{Re} \left[ e^{j\psi_k} \tilde{y}(t_k) \right] - a_k^2 \sigma^2 / 2 \right\} \right\rangle
\]
(11)

with notations: \( \psi_k = \omega_0(t_0 + t_k) - \Theta_k \) and \( \sigma^2 = (1/\pi) \int_{-\infty}^{\infty} |K_{opt}(\omega)|^2/N_\xi(\omega) \, d\omega \) — the output noise dispersion.

Finally, introducing an amplitude of the output signal reaction \( A_k = | \langle \tilde{y}(t_k) \rangle | = a_k \sigma^2 \) one comes to the following expression for the k-likelihood ratio
\[
\Lambda_k[x] \simeq \left\langle \exp \left\{ (A_k/\sigma^2) \text{Re} \left[ e^{j\psi_k} \tilde{y}(t_k) \right] - A_k^2 / 2\sigma^2 \right\} \right\rangle
\]
(12)

The formula (12) gives in principle an answer to the question about the structure of MLP-variable, but it contains signal pulse parameters \( A_k, \Theta_k, t_k \) which are unknown apriori. To avoid this problem one can use a so called "generalized form of MLP" [45] when unknown parameters are replaced by their "maximum likelihood evaluations" \( \hat{A}_k, \hat{\Theta}_k, \hat{t}_k \) which can be taken as solutions of the following extremum equations
\[
\partial z_k/\partial A_k = 0, \quad \partial z_k/\partial \Theta_k = 0, \quad \partial z_k/\partial t_k = 0
\]
(13)
A direct calculation with $z_k = \ln \Lambda_k[x]$ from (12) leads to conclusions that
a) the MLP-evaluation of the amplitude coincides with the overlope of a narrow bandwidth process on the antenna output $R(t)$

$$
\hat{A}_k^2 = \{Re[e^{j\hat{\psi}_k} \hat{y}(t_k)]\}^2 = |y(t_k)|^2 = R^2(t_k)
$$

(14)

and then a recipe for construction of MLP-variable is

$$
z_k = (\hat{A}_k^2/2\sigma^2) = (R^2(t_k)/2\sigma^2)
$$

(15)

b) the MLP-evaluation of the unknown time shift $\tau$ between the "astrophysical" time mark $\tau_{ak}$ and arrival time of "gravitational" signal $\tau_k$ is defined by a position of maximum of the function $z_k(\tau_{ak} + \tau)$ in the $\tau$ space.

The conclusions above correspond to the supposition that parameters of signal pulse are definite but unknown values. There is also other conceivable case when one considers initial signal phases $\Theta_k$ as stochastic variables with uniform distribution in the interval $[0,2\pi]$. Then after a statistical averaging one can find a different form of the MLP-variable

$$
< \Lambda_k[x] > = \exp\left[ -\frac{\hat{A}_k^2}{2\sigma^2} \right] I_0\left( \frac{\hat{A}_k R(t_k)}{\sigma^2} \right)
$$

(16)

with the following equation for amplitudes $\hat{A}_k$

$$
\hat{A}_k = R(t_k) \frac{I_0[\hat{A}_k R(t_k)]}{I_1[A_k R(t_k)]}
$$

(17)

$I_0, I_1$ in the (16),(17) are the modified Bessel functions.

A correspondent MLP-variable for the case of stochastic initial phase $\Theta_k$ looks like

$$
z_k = \ln I_0\left( \frac{\hat{A}_k R(t_k)}{\sigma^2} \right) - \frac{\hat{A}_k^2}{2\sigma^2}
$$

(18)

A solution of the equation (17) is given on the Fig.1. It demonstrates that a difference between estimations (14) and (17) is essential only for small signals with amplitudes $A_k > \sigma$. For the amplitudes $A_k \geq 2\sigma$ the both estimations practically coincides and recommends to take a value of the output overlope $R(t_k)$ as a MLP-evaluation of the $A_k$. Then the difference between MPL variables (15) and (18) also vanishes.

Now coming back to the expression (7) we can summarize the results. In the frame of the model of incoherent packet of signal pulses on gaussian narrowband noise background the MLP-algorithm recommends to compose a following variable

$$
Z = \sum_{k=1}^n (R^2(t_k)/2\sigma^2)
$$

(19)
which is the sum of quadratic counts of the antenna output overlap taken in
times of astrophysical events with some small shift \( \tau \) \(^3\); the sum is accumu-
lated on the interval of observation which \textit{aposteriori} contained \( n \) events.

Then it is recommended to find an absolute maximum of \( Z \) through vari-
atations of the shift \( \tau \) (see \(^{13}\) and point \( b \)), i.e. to get over a new so called
absolute maximum — variable

\[
Z_{\text{max}} = \max_{\tau} Z(\tau), \quad \tau \in [\tau_{\text{min}}, \tau_{\text{max}}]
\]  

(20)

A value of \( \tau_{\text{opt}} \) which provides a maximum of \( Z(\tau) \) should be taken as MLP-
evaluation of the real time shift between astrophysical event and gravitational
signal (in our simple approach the shift is supposed to be the same for all events,
— a hypothesis of “homogeneity of events”). As we remarked already there is
no a definition of the \( \tau \)-interval limits inside of the statistical model; it has to
be chosen on a base of additional physical arguments.

A strategy of the operator performing a data processing of gravitational
antenna output and having a list of ”astrophysical events” can be thought in
the frame of Neuman-Pirson approach (under condition of \textit{appriori information
deficit}). After composing the \( Z_{\text{am}} \) variable one has to compare it with a \textit{threshold}
defined by statistical properties of \( Z_{\text{max}} \). A crossing of the threshold would mean
”a presence of signal” with an accuracy of the ”false alarm” error. Thus such
strategy supposes a preliminary knowledge of \( Z_{\text{max}} \) statistics. It might be taken
from theoretical suppositions or to be a result of some empirical study of the
output antenna realization.

3 STATISTICAL PROPERTIES OF THE MEA-
SURABLE VARIABLES

There are three observant variables involved in the MLP data processing. These
are the squared overlap of the antenna output \( R^2(t) \) \(^{14}\), the sum of overlap
counts \( Z \) \(^{15}\), taken in the moments of astrophysical events on the observational
interval \([0,T]\) and the maximum value of this sum \( Z_{\text{max}} \) \(^{20}\) corresponded to
the optimal time shift. Statistics all of them can be calculated analytically if
we accept the gaussian approximation of the bar antenna noise. As experiment
has shown this supposition is very close to reality with an exception of large
energetic thresholds where the thermal statistics can be distored by stochastic
nongravitational hindrances (a correction which could be introduced in this case
we discuss in section 5).

The formulae \(^{15},^{20}\) were derived in dimentionless form. For a comparison
with experiments it is useful to have also dimentional expressions for these
variables in kelvin degrees.
Figure 1: MLP-estimation of the signal amplitude
3.1 Statistics of the squeared overlap

It is well known that thermal oscillations of the resonance bar detector are described as a narrow band gaussian stochastic process \(x(t) = A(t) \cos \omega_0 t - B(t) \sin \omega_0 t\) with slow changing quadratures \(A(t), B(t)\) having the correlation function \(k(\tau) = \sigma_0^2 \exp(-\gamma|\tau|)\), where \(\sigma_0^2 = kT_0/m\omega_0^2\) is the brownian dispersion and \(\gamma = \omega_0/2Q\) is the relaxation index (\(m, T_0, Q\) are the bar equivalent mass, absolute temperature and quality factor).

After preliminary filtration (a differential link, Winer-Kolmogorov filter etc.) \(x(t)\) reduces to some narrowband process inside a limited bandwidth \(\Delta \omega\) with squeared overlap \(R^2 = (\Delta A)^2 + (\Delta B)^2\). This value is proportional to an "energy innovation" (or variation of energy) of the bar \(E(t)\) during the time \(\Delta t = \Delta \omega^{-1} \ll \gamma^{-1}\).

\[
E(t) = m\omega_0^2 R^2(t)/2, \quad <E(t)> = kT_0 2\gamma \Delta t.
\]

Correspondent variation of the quadrature \(\Delta A, \Delta B\) has a correlation function type of \(k_{\Delta}(\tau) = \sigma^2 \rho(\tau)\) where

\[
\rho = \begin{cases} 
1 - (|\tau|/\Delta t), & |\tau| \leq \Delta t \\
0, & |\tau| > \Delta t.
\end{cases}
\]

and the value of dispersion \(\sigma^2\) is coupled with the brownian dispersion \(\sigma_0^2\) through an effective noise temperature \(T_e\) of the bar

\[
\sigma^2 = (kT_e/m\omega_0^2) = \sigma_0^2 (T_e/T_0), \quad T_e = T_0(2\gamma \Delta t).
\]

The correlation function of the squeared output overlap can be easy calculated in the usual form \(K(\tau) = <R^2(t)R^2(t+\tau)> - <R^2(t)>^2\) which results in

\[
K(\tau) = 4\sigma^4 \rho^2(\tau) \quad (21)
\]

The formulae above show that the correlation coefficient of the squeared antenna overlap \(\rho^2(\tau)\), falls down to zero at the "innovation time scale" \(\Delta t\). Thus independent counts under a discrete presentation of the output overlap \(R(t) \rightarrow R(t_k)\) must be separated by time distances \((t_{k+1} - t_k) \geq \Delta t\).

3.2 Statistics of the sum of overlap counts

The next variable of our interest is the sum of counts of output overlap taken in the times of astrophysical events \([19]\). It is convenient to normalize this variable dividing it on a total number of events on the observational interval \([0, T]\). Then the new variable \(C = Z/n\) will be proportional to a "selected mean value" of energy innovation

\[
\bar{E} = (1/n) \sum_{k=1}^{n} E(t_k)
\]
collected on the observational interval in the special time marks — astrophysical events so as

\[ C = Z/n = (1/n) \sum_{k=1}^{n} R^2(t_k)/2\sigma^2 = \]

\[ = (1/nkT_e) \sum_{k=1}^{n} E(t_k) = (1/kT_e)\bar{E} \] (22)

If the number of events in the sum (22) is larger than thirty a distribution of the C-variable asymptotically should be the gaussian one according to the "central limiting theorem" with the mean value \(< C(t) > = 1\) or \(< \bar{E} > = kT_e\).

The correlation function \( \rho_c = \langle C(t_1, t_2, \ldots, t_n) \rangle = \langle t_1 + \tau, t_2 + \tau, \ldots, t_n + \tau \rangle \rangle - \langle C(t_1, t_2, \ldots, t_n) \rangle^2 > \) has a structure

\[ \rho_c(\tau) = \frac{1}{n^2} \left[ n\rho^2(\tau) + \sum_{i=1}^{n} \sum_{k=1, k \neq i}^{n} \rho^2(t_i - t_k + \tau) \right] \]

which demonstrates the presence of a principal peak in the region \( 0 \leq \tau \leq \Delta t \) with a parabolic degeneration in time and a series peaks in the points where \( \tau = (t_i - t_k) \). Such nontrivial structure produces some peculiarity in a definition and calculation of the "correlation time" for C-variable. Here we would like only remark that under a supposition that the sequence of astrophysical events is a poissonian flux of pulses, the expression of \( \rho_c(\tau) \) can be reduced to the following form (where \( \Delta t \leq \tau \leq T \))

\[ \rho_c(\tau) = \frac{1}{n} \left[ \rho^2(\tau) + \frac{1}{\pi} (n - 1) \left( \frac{\Delta t}{T} \right) \left( 1 - \frac{\left| \tau \right|}{T} \right) \right] \] (23)

Under reasonable suppositions that the total number of events \( n \) on the observational interval \( T \) is not too large \( n(\Delta t/T) \ll 1 \) and the correlation time of \( C(\tau) \) is limited \( |\tau| \ll T \) the formula (23) might be simplified

\[ \rho_c(\tau) \simeq \rho^2(\tau)/n, \quad \rightarrow \quad K_\bar{E}(\tau) = \rho^2(\tau) (kT)^2/n \] (24)

A dispersion of the C-variable (and \( \bar{E} \)) is depressed in the factor of \((1/n)\) in compare with the \( R^2 \)-variable in agreement with the statistical property of the sum of identical independent counts.

3.3 Statistics of the absolute maximum of C-variable

As above we will consider the normalized sum of the envelope counts i.e. instead of \( Z_{max} \) (24) one deals with \( C_{max} = \max_{\tau} Z(\tau)/n \). The maximum has to be found through time shift variations on the apriori given time interval (20). Let’s accept that the time shifts are produced by discrete steps \( \delta t \). Then we have
the output combination of values \( \{ C(tak + m\delta t) \} \), \( m = 1, 2, \ldots L \) with a total number \( L = (\tau_{\text{max}} - \tau_{\text{min}})/\delta t \).

In the case of gaussian statistics of the \( C \)-variable a solution for its absolute maximum distribution might be taken from literature. In particular one can use the Cramer formula [46] which presents the absolute maximum statistics \( C_{\text{max}} \) through another auxiliary stochastic parameter \( \xi \).

\[
C_{\text{max}} \quad < C \gtrsim \quad \sqrt{\frac{1}{n} \left[ \sqrt{2\ln \mu(\Delta \tau)} + \xi/\sqrt{2\ln \mu(\Delta \tau)} \right]}
\]

(25)

where \( \xi \) has a probability density

\[
w(\xi) = e^{-\xi} \exp(-e^{-\xi})
\]

(26)

with a mean value \( < \xi > = 0.577 \) and dispersion \( \sigma_{\xi}^2 = \pi^2/6 \).

The formulae (25), (26) are true in the asymptotical sense, i.e. under \((\Delta \tau/\delta t) \to \infty\).

The parameter \( \mu \) in the formula (25) depends on the region of time shift variations \( \Delta \tau = (\tau_{\text{max}} - \tau_{\text{min}}) \) and a second derivative of the correlation coefficient of \( C \)-variable \( \rho^2(\tau) \) in the point \( \tau = 0 \) so that

\[
\mu(\Delta \tau) = (1/2\pi)\Delta \tau \sqrt{-2\rho''(0)}
\]

(27)

A calculation the value \( \hat{R}_c(0) \) for the processes of Markov type is always a nontrivial procedure. In our case an estimation can be done through the approximation of Owen functions [47] and results in

\[
\mu(\Delta \tau) = \frac{1}{\pi} \frac{\Delta \tau}{\delta t} \sqrt{\frac{1 - \rho^2}{1 + \rho^2}}, \quad \rho^2 = (1 - \delta t/\Delta \tau)^2.
\]

(28)

The formulae (25), (26), (28) in principle solve the problem of calculation a "probability of chance" to exceed some threshold level \( C_{\text{th}} \) for the absolute maximum variable \( C_{\text{max}} \).

4 NEUTRINO-GRAVITY CORRELATION EFFECT OF SN1987A

As a test of the proposed algorithm we consider its application to the phenomenon of "neutrino-gravity correlation effect" reported in the series papers by the RTM-collaboration (INFN, Univ. "La Sapienza", "Tor Vergata" (Roma), Inst. Cosmogeofisica CNR (Torino), Univ. Maryland (Washington) and Inst. Nuclear Res. Rus. Ac (Moscow)) [41-44].
The effect consists in fixation of remarkable correlation during of SN1987A phenomenon between the unified noise background of room temperature gravitational bar detectors in Roma and Maryland at the one side and a neutrino background registered by the Mont Blanc neutrino scintillator at the other side. A direct interpretation of this correlation as an affect of gravitational and neutrino radiations from a collapsing star have met objections from the point of view a required energy of gravitational wave. There was a deficit of two order of value in a conventional estimation of the gravitational radiation output from supernova at the distance of BMC in compare with the room temperature bar detector sensitivity [41]. Later several other investigations were carried out in attempts to clarify a nature of this effect which probability of chance was evaluated as extremely small, order of $10^{-6}$ [42]. In that number a searching of any correlation with other elementary particle backgrounds [48], with seismic noise background [49] etc. Besides a dynamics of joint antenna pattern of gravitational detectors in Roma and Maryland was calculated [50] and some hypothesis of a new physics also were considered (see examples in [51]. Nevertheless it did not lead to any definit model of the phenomenon. Then a computer simulation of the neutrino and gravity data was carried out to prove that the "$\nu g$-correlation effect" could be a usual statistical fluctuation if one would be correct in probability of chance estimation [52]. However RTM collaboration did not accept this critics arguing that the authors of [52] did not use the real experimental data and presented some contrary argumentation in favour of the objective character of the effect [53].

In this section we present the results of our analysis of the real data kindly provided for us by the RTM group. With it we follow the algorithm developed in the previous sections making a coparison with the RTM methodics.

4.1 Method and results of RTM group

The bank of data containing joint records of "energy innovations" of the gravitational detectors in Roma and in Maryland was limited by the time interval from UT, 12h00m, Feb. 22 — UT, 06h00m, Feb. 23. At the same time interval there was a list of neutrino events corresponded to the stochastic background counts of the LSD neutrino scintillator in the program "Supernova Watcher". All data were presented in the digital form. A sampling time of the gravitational records $\Delta t = 1$ s. was also equal to the "innovation time" interval (i.e. a bandwidth of the filtering tract was $\Delta \omega = 1/\Delta t$). A sampling time of the neutrino counts (an accuracy of the event time marks) was 0,01 s. There were no joint data after UT, 07h00m because the Maryland detector had stoped an operation for technical reasons.

The neutrino list had a singularity in the region 2h52m, Feb. 23: there was a group of five $\nu$-pulses with very small poissonian probability of chance. These neutrino was detected independetly and beforehand an information about optical observation of the supernova was received.
Side by side with traditional "coincidence methodics" RTM group have applied an original method of analysis composing from gravitational data an auxiliary statistics which was the sum of energy innovations taken in the neutrino time marks normalized to the number of events. In fact the RTM group have anticipated the optimal strategy of MLP approach resulting in the $C$-variable (22) as "a sufficient statistics". The reason of such choice RTM group have seen in the physical sense of the C-variable as a value proportional to the correlation function between two stochastic serieses: counts of the gravitational detector energy variations and neutrino events (in the last series only time marks of events was essential because the amplitudes were fixed by the threshold selection). As we mentioned in the previous section there is also another important physical sense of the C-variable as a mean value of "gravitational energy innovation" calculated on the base of "neutrino times".

A generalization introduced by RTM group consisted also of the decision to use a combined energy innovation composed by a linear combination of counts of two appriori independent set ups in Roma and Maryland. This so called "a net excitation method" in therminology of the paper [52] has a clear argumentation considering the two gravitational detectors as links of the one united wideband gravitational antenna under a global gravitational wave influence (frequencies of Roma and Maryland detectors were different so the unified antenna received an energy from different spectral component of GW-pulse). Thus RTM group have dealt with C-variable in the dimensional form

$$C(\tau) = \frac{1}{n} \sum_{k=1}^{n} [E_R(t_k + \tau) + E_M(t_k + \tau)]$$

(29)

which corresponds in fact $\bar{E}$ in our notation (22), but for a convenient comparison we will keep the RTM definitions in this section. The summation (29) was made with some normalization factor, reflected in noise temperatures of the both detectors.

The value of $C(\tau)$ have been collected into two hours time window according to neutrino marks inside of it. The procedure was repeated after displacement the window on half hour along the observation time interval and so on. The time shift $\tau$ was taken equal $-1.4$ s according to the estimation made in the first paper [41] through a digital filter applied to the Roma detector data and repeating the structure of the "five-neutrino group" registered by LSD at 2h52m. Later in the paper [42] this shift was reduced to $-1.2$ s as a more optimal value resulting in the largest meaning of the $C$-variable.

The main result of the RTM group analysis consists in the statement that the $C$ statistics reaches the maximum value $C_{exp} = 72.5K^0$ on the two hours interval around the point 2h52m where the number of registered neutrino events was $n = 96$.

To prove it RTM group have made a simulation of the neutrino events using a generator of stochastic numbers which provided neutrino time marks according
to the poissonian low (the total number of events have been fixed by the real neutrino list). Having this artificial ”neutrino flux” RTM group could calculate corresponded values of $C$-variable for each two hours interval with variation of the time shift if necessary.

A presence of the ”$\nu g$-correlation” effect was demonstrated on two type of graphs. The first was a relative number of cases when a simulated ”artificial” $C(\tau = -1.2s)$ exceeded of the observable in experiment value $C_{exp}$ versus of consequent two hours intervals, see Fig. 2, a. The second was an analogical relative number but calculated on the two hours interval around of 2h52m (the interval of ”neutrino singularity”) versus of time shift which was variated around the stationary meaning (–1.2 s), see Fig.2 c. The both graphs have shown an exclusivity of the experimentaly observed data: there were deep downfalls at the place around of 2h52m, and in the point of zero time shift at the Fig.2 a. A presence of these downfalls means a registration of a very rare event. The RTM group used two ways for a probability of chance estimation. First, a simple utilization of the ”binominal formula” $p = m/n$, where $m$-the number of cases when $C \geq C_{exp}$ and $n$-the total number of tests. Second, an empirical construction of the $C$ statistics distribution through the simulation of the neutrino events flux (each simulated neutrino series gave a definit value of $C$ — a one point in the empirical differential distribution graph). Having this distribution in disposal one could easy evaluate the chance probability to get a definit $C$ value. The $C$ distribution from the paper [42] is presented on the Fig.2 e together with position of the observable in experiment value $C_{exp}$.

The both ways gave a chance probability $p \simeq 10^{-3}$ for the effect on Fig.2 a and extremely small $p \simeq 10^{-6}$ for Fig.2 c. Espessially the last fact was interpreted as the detection of abnormal correlation between gravitational and neutrino data in the time around 2h52m UT — so called ”neutrino-gravity correlation effect” associated with SN1987A.

4.2 Method and results of SAI group

The theory of the method we used in our reanalysis was given above in the sections II. It corresponds to the RTM method besides the fact of replacement the $C$-variable by the $C_{max}$ statistics but also applied to the combined R&M data in the manner of ”net exitation” approach.

4.2.1 Gaussian predictions

First of all we had possibility to forecast an expected result of our reanalysis on the base of gaussian approximation in the section III, C. A good agreement of the experimental data (output realization of the gravitational detectors) with hypothesis of gaussian distribution was demonstrated more then once and in particular in the paper [42]. The calculated and measured noise temperatures $T_e$ were for Roma detector $T_e = T_R = 28.6K^0$ and for Maryland detec-
Figure 2: a, b — A time evolution of number of cases $C \geq C_{exp}$; c, d — Number of cases $C \geq C_{exp}$ versus of time shift on the interval 1h52m–3h52m; e, f — $C$-distribution under $\nu$-set simulation

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tor $T_e = T_M = 22.1K^0$ (the normalization factor in our reanalysis was equal $\varepsilon = T_M/T_R \simeq 0.77$ which is very close to the value of RTM group 0.75). Then estimations of the mean value and dispersion of C-variable for the "net excitation" structure and dimensional form $(C \to \bar{E})$ (29) according to formulae (21), (24) has to be

$$k^{-1} < C > = (T_R + T_M) \simeq 51K^0$$
$$k^{-1}\sqrt{K_E(0)} = \sqrt{(T_R^2 + T_M^2)/n} \simeq 3.7K^0,$$

(n = 96). (30)

Thus in the region of the effect (two hours around 2h52m) the theory forecasts the C-variable distribution in the gaussian form with central point 51$K^0$ and effective width $3.7K^0$.

As we have seen a principal statistics of MLP algorithm is the absolute maximum of $C$ under time shift variations i.e. $C_{\text{max}}$. An expected mean value of the $C_{\text{max}}$ is given by the formula (25) after its statistical averaging

$$<C_{\text{max}}> = <C> + \sqrt{K_E(0)} \left[\sqrt{2\ln \mu} + <\xi> / \sqrt{2\ln \mu}\right]$$

(31)

It was mentioned that $<\xi> = 0.577$. In the $\mu$ estimation a principal role belongs to the range $\Delta \tau$ and step $\delta t$ of time shifts. There is no recommendation for a choice of them inside the MLP algorithm. It has to be done on physical arguments. In our reanalysis it was taken $\Delta \tau = \mp 100s$ and $\delta t = 0.01$ s corresponding to the experimental data specifics. Then the formula (28) gives $\rho^2 \simeq 0.1$ and $\mu \simeq 6.46$. A substitution of these numbers into (31) results in the evaluation of mean value $<C_{\text{max}}>$: in the region of the effect $<C_{\text{max}}> \simeq 65K^0$ (if $k = 1$).

The formulae (25–28) permit to estimate also a width of $C_{\text{max}}$-distribution as well as its form and then to find a "false alarm error" or "chance probability" for any value of $C$ realized in experiment. However due to dependence of these estimations on a choice of characteristic times we do not do it here but instead we present below results of our empirical data analysis in the manner similar to RTM group.

4.2.2 Empirical analysis

In principle an empirical analysis has a conventional advantage of refusing from any hypothesis apriori on respect with a distribution low of the data under consideration. At the same time a task of reconstruction statistical properties of observable variables on the base of only one unique realization of the stochastic process belongs to the family of "ill posed" problems and uncertainties of reconstruction might be large enough to make this method ineffective. So each step in empirical data analysis must be estimated in respect of possible errors.
The procedure of MLP algorithm factually is a very delicate filtration process in attempt of detecting a weak signal strongly covered by the noise. As illustration of this idea one can look at the output realization of Roma gravitational detector during the time Feb. 22–23 (Fig. 3) (a computer reconstruction of the digital data). It is unlikely to extract a signal from this background without special very sophisticated recipes.

In our "real data" reanalysis we started with repetition of two tests of RTM group. i) Using "neutrino two hour serieses" simulated by poissonian generator it was checked how often an immitational "two hour $C$-variable" taken with fixed shift $\tau = -1.2$ s exceeded the experimental value of $C$. Our results completely confirmed the results of RTM group, see Fig. 2 b: we have got a singularity at the two hour interval around of 2h52m. ii) At the region of the effect it was checked how often a "simulated $C$ with different shifts" exceeded the experimental $C_{exp}$ with selected shift $\tau = -1.2$ s. Again our results confirmed a presence of singularity with slightly different estimation of the chance probability ($\simeq 10^{-5}$), see Fig. 2 c.

We used for estimation of the chance probability the same "binominal formula" $p = m/n$ as RTM group although a rightfullness of it was criticized in the paper [52] by the referring to the "absence of independency" between different counts of $C$ variable. However one can show that for enough high values of $C_k$ they might be considered as independent [54]. More in detail: the number of
independent counts $n^*$ in the total sample number $n$ is defined as

$$n^* = \frac{n}{1 + (n - 1)r}, \quad r = f(C/\sqrt{K_c(0)})R(C_k),$$

$$f(x) \approx x(\Phi)'(x).$$

where $\Phi(x)$ is the probability integral and $R(C_k) \approx n(\Delta t/T) \ll 1$ is the correlation coefficient of $C_k$.

For $C \geq \sqrt{K_c(0)}$, $r(x) \to 0$, and the $n^* \to n$ i.e. for relatively large values of $C$ practically all samples are independent.

We also reconstructed the empirical $C$ distribution at the region of the effect and found the same graph as RTM result on the Fig.2 f which was centered in the point $52K^0$ in a good agreement with theoretical forecast of gaussian approach ($51K^0$).

Having got these confirmations we must not forget however general properties of solution of ill posed problems: a reliability falls down on the wings of reconstructed distribution. So we could accept the empirical estimations of chance probability above not literally but only on the order of value.

After all of this we can consider the key point of our reanalysis which is the following: the "binominal formula" and "$C$" distribution are not adequate statistics for the "probability of chance" evaluation in the experiment under consideration. The matter is the estimations above did not take into account a selection of data through time shift variations. As the general MLP algorithm recommends it has to be done with help of the absolute maximum distribution $C_{max}$. In the empirical method a reconstruction of this distribution on the interval of the effect goes through the following procedure: one simulates a "neutrino series" with $n = 96$ and then variates the time shift to find an optimal one provided a maximum value of $C = C_m$. This value will be the one point of the $C_{max}$ distribution. Independent repetitions of the procedure lead to reconstruction of the complete distribution. It is naturally to wait that the selection will move the $C_{max}$ distribution to the region of larger values of $C$ and increase the chance probability.

At the Fig. 4 we present the results of reconstruction of $C_{max}$ distribution together with $C$ distribution and mark the value of experimentally registered effect $C_{exp} = 72, 5K^0$. A reconstruction of $C_{max}$ requires a much more computer time then for $C$ variable, so the graph on Fig.4 contains $10^3$ points. Two characteristic times the range and step of time shift variations were choosed on the base of following arguments.

The range of time shift can not be too larger of an average time distance between poissonian neutrino events so as in opposit case a "time shift operation" could capture additional "neutrino" from neighbour two hours intervals. The average interpulse distance in the experimetal LSD neutrino record was 70–80 s so we took the range of time shift $\Delta \tau = \mp 100$ s.

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Figure 4: Empirical distributions $C$ and $C_{max}$
The step of time shift was taken equal to the shortest sample time of the data available i.e. to the "neutrino sample time" $\delta t = 0.01$ s. It is clear that a more detailed time examining would exceed the time accuracy of the data.

As one can see from the Fig.4 the empirical $C_{max}$ distribution was shifted to the right side to meet the experimentally registered value $C_{exp} = 72.5K^0$. A center of the distribution is located close to the points $65 - 66K^0$ that is again in a good agreement with the forecast of gaussian approximation of data statistics $(65K^0)(!)$. A new estimation of the chance probability on respect with this distribution fits to the numbers $10^{-4} - 10^{-3}$. This estimation increases essentially the RTM value $10^{-6}$ but remains to be enough small to recognize an objectivity of the "some correlations" between the "gravity and neutrino" data.

However having in the mind of "ill posed" character of the problem of reconstruction of statistical distribution we have to consider once more all sources of possible uncertainties. The main one is of course a dependence of the result on the choice of characteristic times which was made refering to the experiment specifics but not to objective mathematical restriction. It is clear that expanding a range of time shifts infinitely and shortening a step one increases enormously a "number of occasions" and drives the probability of chance to the unit. Thus our conclusions above have a conventional character suspended on our argumentation for a choice of characteristic times.

Together with this there is another, particular in the given case but very serious, source of uncertainty which was recognized in the process of empirical analysis. The matter is a different scale of sampling for neutrino and gravity data: 1 s for the gravitational detector counts but 0.01 s for accuracy of neutrino time marks. It means that an operator has to make an interpolation procedure trying to find a correspondent gravitational count (value of energy innovation) to the definite neutrino event. The simplest way to do it is a "step kind" interpolation but generally it can be done by some optimal manner [55]. The most important point for us here is a recognition that any interpolation introduces an uncertainty in our calculation of the statistical parameters and in particular in the position of the center of $C_{max}$ distribution.

An estimation of the interpolation error [55] leads us to the value $\Delta C_{int} = 4.6K^0$. This error box is shown at the Fig. 4 around $C_{max}$ center position. A calculation gives that a displacement of this center to the right side of the error box provides the chance probability on the order of 0.01 or larger. This is a relatively typical level for stochastic measurements and does not mean any extraordinary event.

Thus our reanalysis shows that the available experimental data are unsufficient to make a reliable conclusion in favour of the detection a "remarkable $\nu g$-correlation" during of SN1987A explosion.
5 Discussion

The example with SN1987A gives enough presentation how the MPL-algorithm works exhibiting clearly at the same time its weak point: a dependence on the unknown range of time shift between astrophysical and gravitational events. An apriori estimation of it on physical arguments is desirable to provide an efficiency of the algorithm. Any attempts to limit this range appealing to some peculiarity of experimental data or particular manner of operator behaviour under searching for the "signal exitation $C_{exp}$" do not lead to "objective boundaries" for time shift and thus a correspondent evaluation of the chance probability remains to be suspended. Only an apriori knowledge of the time shift range could introduce some certainty (deterministic elements) in our ill posed problem. In the extremely favourable case when the value of shift is known exactly the estimation of chance probability can be taken just from $C$-distribution which is much more robust then $C_{max}$-distribution. The last one however has to be used obligatory if the shift was not given beforehand. We can remark here that the authors of the paper [52] have came very close to this idea introducing of so called "$q$-parameter" to define how often a realization with rare statistical properties occurs in the process of computer simulation of experimental data. It can be shown that such approach leads directly to the absolute maximum distribution.

Having in the mind a phenomenon of joint neutrino and gravitational radiation from SN1987A one could be limited in the range of time shift by the theoretical restriction of the neutrino rest mass which is less then 10 ev; then a delay of neutrino signal would not exceed 2.7 s. That is just the hypothesis which was adopted as a starting point for the data processing in the papers [41–44], where the maximum time shift range was taken on order of $\pm 2$ s. However due to a large uncertainty of joint scenario for supernova radiation dynamics [24,29,30,37] as well as due to a general tend to avoid any hypothetical propositions concerning a "nature of the source" we used in our reanalysis the maximum time shift compatible with the structure of experimental data $\pm 100$ s. The interesting fact was that even for this large time shift interval the chance probability of the "correlation effect" was kept on small level $10^{-3} - 10^{-4}$ and only the "interpolation uncertainty" did not permit to confirm a presence of the RTM-correlation.

Some alternative hypothesis for explanation of the observed experimental data was proposed in the paper [51] where a time evolution of the $C$-variable on the interval of observation was presented separately for Roma, Maryland and combined (R+M) antennue. So the evolution diagrams with big peak at the region of 2–4 h 23 Feb. were similiary for the combined (R+M) and Roma antennue, but the diagram for the Maryland antenna was different (more smooth and no big peak). Early a correlation between seismic data and (R+M) antennae background during of SN1987A was reported [49]. Thus the hypothesis [51] is that it was registered a correlation between Mont Blanc neutrino scintillator and
Roma detector backgrounds produced by a small scale earthquake in the south Europe region occurred roughly in the SN1987A time. The data of Maryland detector has no evident coupling with this phenomenon.

Coming back to the general algorithm of searching for "astro-gravity correlations" we would like to make several remarks.

1) The $C$-variable in the form (19) used in our reanalysis is the exact MLP-variable for a signal with unknown but deterministic parameters: $A_k, \Theta_k, \tau_k$. It also approximately corresponds to the case of stochastic uniformly distributed phase; a correct phase averaged expression of MLP-variable in this case is given by the formula (18) with summation over all astrophysical events. Using (18) one could wait a decrease of chance probability for two reasons: a) the expression (18) gives a more optimal estimation for small signals $A < \sigma$, b) this is one step from pure MLP-method to the Bayesian approach which has in general a lower false alarm error.

2) A next step to the Bayesian approach could be associated with an averaging (18) also over unknown time shifts supposed to be uniformly distributed in the $a priori$ given time interval. This would produce a following essential decrease of chance probability but the payment will be a refuse from evaluation of time shift between astrophysical and gravitational data.

3) The MLP algorithm (19),(20) contains in principle a possibility of signal accumulation. However for the "post demodulation" read out an accumulation of small incoherent pulses $A_k \leq \sigma$ increases a signal-noise ratio proportionally to $n^{1/4} \ i.e. \ i t \ c a n \ n o t \ b e \ e f f e c t i v e$. In the opposite case of large pulses the accumulation tends to usual law of independent stochastic counts $n^{1/2}$ but here it is unlikely to expect a big value for $n$ on a reasonable observational time according to astrophysical scenario.

4) A search of "astro-gravity correlations" as a new form of gravitational wave experiment has a clear advantage of sharp reduction of the observational time interval involved in the data processing. This leads to an equivalent diminution of the chance probability proportionally to the factor $n\Delta \tau/T$ but on a threshold signal-noise ratio it produces a small influence increasing this ratio only in the $(1/2) \ln (T/n\Delta \tau)$ times which is insignificant.

5) We have seen in our reanalysis that the gaussian approximation gave a good agreement with statistics found empirically. However on the wings of empirical distributions an uncertainty of estimations grows. A possible way to improve a quality of empirical estimation consists in using a family of Pirson statistics to approximate more correctly a distribution of the experimental data how it was proposed in [8].

6) The Pirson statistics could be also used to prognosticate of expected probability of chance and other statistical values under a generalization of MLP-algorithm for the case of non gaussian noises. At practice there is always an excess of large nonthermal pulses at the tail of integral energy distribution for gravitational bar detectors. Using the Pirson approximation for a density probability of such non gaussian noise it is possible to make some preliminary filtra-
tion to suppress nongaussian hindrances. Then a generalized MLP-algorithm will have the same form (19),(20) with substitution of some known function of the output overlope $f(R_k^2)$ instead of overlope itself $R_k^2$ [56].

In conclusion we would like to note that a simple translation of the developed algorithm to the laser interferometrical antenna on free masses is impossible without serious modification. The matter is a response of this wide frequency band set up to gravitational signals can not be presented in some universal form like it was done for "quasi $\delta$-excitations" of the bar detector. One has to take into account a complex structure of individual GW-pulses. Thus a construction of optimal algorithm for detection of a packet of such pulses correlated with astrophysical events becomes a multi-parametrical problem and has to be studied specially.

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