Rough Gauge Fields, Smearing and Domain Wall Fermions

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At a fixed lattice spacing, as determined by say $m_\rho$, adding additional fermion flavors to a dynamical simulation produces rougher gauge field configurations at the lattice scale. For domain wall fermions, these rough configurations lead to larger residual chiral symmetry breaking and larger values for the residual masses, $m_{\text{res}}$. We discuss ongoing attempts to reduce chiral symmetry breaking for $N_f = 3$ dynamical domain wall fermion simulations by different smoothing choices for the gauge fields.

1. INTRODUCTION

As is widely discussed (see, for example, Ref. [1]), topological lattice dislocations give rise to a non-vanishing density of near-zero modes of the hermitian Wilson-Dirac operator (HWDO), which is considered to be an origin of large chiral symmetry breaking for domain wall fermions (DWF) and a sign of the theory being close to or inside the Aoki phase. In quenched simulations, we are able to suppress the creation of these lattice dislocations by using improved gauge actions [2] (DBW2 for instance) and have achieved better chiral symmetry in numerical simulations. However, if the lattice spacing, as determined by, say, the $\rho$ mass, is kept fixed as more flavors of fermions are added, asymptotic freedom predicts a rougher gauge field at the single-lattice spacing scale. As a consequence of the rougher fields, the residual mass $m_{\text{res}}$, becomes larger at a comparable lattice spacing than in the quenched theory.

Recently there have been various attempts [4] to improve the chiral properties of DWF in dynamical simulations by modifying the gauge action. Here we study a smearing technique on $N_f = 3$ lattices, i.e., replacing each gauge link by a weighted sum of the original link and products of gauge matrices along 3, 5 and 7 link paths, to see if chiral symmetry breaking is reduced.

2. SMEARING ON GAUGE LINKS

A smeared link is constructed as:

$$c_1 + c_3, \Sigma + c_5, \Sigma + c_7, \Sigma$$

As we can see, the new links are not necessarily elements of $SU(3)$. Although this may make the interpretation of the unrenormalized observables obscure, it does have the advantages of cheap computer time cost and simple numerical implementation in full QCD simulations. Yet the $c$’s should be appropriately chosen so that the conventional field normalization in weak coupling is preserved.

The $c$’s can be simply normalized as:

$$c_1 + 6c_3 + 24c_5 + 48c_7 = 1$$

(1)

While this preserves the field normalization in weak coupling, at the lattice spacings of current simulations a better choice has been seen to be the so-called tadpole improvement condition, in which the smearing coefficients are chosen so that

$$c_1 + 6c_3u_0^2 + 24c_5u_0^4 + 48c_7u_0^6 = 1$$

(2)

where $u_0$ is the quartic root of the average plaquette. We explore these and other normalization possibilities by measuring $m_{\text{res}}$ with a fixed $M_5 = 1.8$ for different combinations of $c_1$ and $c_3$ without enforcing a pre-defined normalization. Coefficients which give smaller $m_{\text{res}}$ are then further studied.

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Table 1

$m_{\text{res}}$ with different smearings. All the measurements have valence $L_s = 8$ and $M_5 = 1.8$ except the one with $c_7 \neq 0$, which has $M_5 = 2.2$. Data for unsmeared lattices are from 84 configurations; others are all from 30 configurations.

| $c_1$ | $c_3$ | $c_5$ | $c_7$ | $m_{\text{val}}$ | $m_{\text{res}}$ |
|-------|-------|-------|-------|-----------------|-----------------|
| 1.0   |       |       |       | 0.02            | 0.01135(7)      |
|       | 0.03  |       |       | 0.02            | 0.01115(6)      |
|       | 0.04  |       |       | 0.02            | 0.01097(6)      |
|       | 0.05  |       |       | 0.02            | 0.01083(5)      |
| 1/8   | 1/16  | 1/64  | 1/384 | 0.02            | 0.04811(28)      |
|       | 0.04  |       |       | 0.02            | 0.04633(27)      |
| 0.25  | 0.051 |       |       | 0.02            | 0.00651(13)      |
|       | 0.04  |       |       | 0.02            | 0.00609(11)      |
| 0.4   | 0.12  |       |       | 0.02            | 0.00904(14)      |
|       | 0.04  |       |       | 0.02            | 0.00855(12)      |
| 0.8   | 0.06  |       |       | 0.02            | 0.00580(11)      |
|       | 0.04  |       |       | 0.02            | 0.00551(9)       |

3. SIMULATION RESULTS

3.1. Residual masses

Smearings are performed on our $16^3 \times 32$, $N_f = 3$ lattices, which were generated with the DWF and DBW2 gauge action. The parameters used were $\beta = 0.72$, $M_5 = 1.8$, $L_s = 8$ and $m_{\text{sec}} = 0.04$, yielding a lattice spacing (from $m_\rho$) of $a^{-1} \approx 1.6$ to 1.7 GeV [5]. Choices for the smearing coefficients we have studied include (i) one set normalized according to eq.(1): $c_1 = 1/8, c_3 = 1/16, c_5 = 1/64, c_7 = 1/384$, a la MILC’s Fat7; (ii) one set following eq.(2): $c_1 = 0.25, c_5 = 0.051$; (iii)&(iv) two sets with arbitrary normalizations for comparison: $c_1 = 0.4, c_3 = 0.12$ and $c_1 = 0.8, c_3 = 0.06$. In Table 1, we see that $m_{\text{res}}$ has generally decreased for different choices of smearing coefficients except set (i).

For DWF, the value of $m_{\text{res}}$ is affected by the value for $M_5$. However, smearing can change the normalization of terms in the action which involve the gauge field, relative to the normalization of the mass term $M_5$. We can see this by considering eigenspectra of the HWDO (spectral flows) on the smeared gauge configurations. Shown in Figure 1 are representative spectral flows from smeared and unsmeared lattices\(^1\). The pinch point ($\kappa_c$ for Wilson fermions) on the top graph (Fat7 smearing) has shifted from $-M_5 \sim -0.9$ to $-1.8$. The larger $m_{\text{res}}$ for this smearing choice is a result of non-optimal $M_5$ and possibly rougher gauge configurations. On the bottom graph (tadpole smearing), the density of small eigenvalues is visibly decreased and the gap is widely opened near $-M_5 \sim -1.8$, leading to the smaller residual masses with coefficient choice (ii).

However, it is also worth noting that the major crossing at $-M_5 \sim -1.6$ still persists after smearing. The failure to eliminate these crossings indicates that the topological lattice dislocations are not changed. While $m_{\text{res}}$ has decreased for this $L_s$, the persistence of the topological dislocations makes studying the large-$L_s$ behavior of $m_{\text{res}}$ important. We present results on this in Sec. 3.3.

\(^1\)The author thanks S.D.Cohen for providing the data necessary for this figure.
Table 2

$m_{\rho a}$ in valence quark chiral limit and mass renormalization. $m_{\text{val}} = 0.02, 0.03, 0.04, 0.05$ are used in the extrapolation for unsmeared lattices; $m_{\text{val}} = 0.02, 0.04$ are used otherwise.

| $c_1$ | $c_3$ | $c_5$ | $c_7$ | $m_{\rho a}$ | $Z_m$ |
|------|------|------|------|-------------|------|
| 1.0  |      |      |      | 0.539(4)    | 1.0  |
| $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{32}$ | $\frac{1}{64}$ | 0.522(28) | 0.76(3) |
| 0.25 | 0.051 |      |      | 0.564(34)   | 0.79(2) |
| 0.4  | 0.12  |      |      | 0.563(31)   | 0.88(2) |
| 0.8  | 0.06  |      |      | 0.563(26)   | 0.83(3) |

3.2. Lattice scales and mass renormalization

To solidify our conclusions about the effects of smearing on $m_{\text{res}}$, we have studied the lattice spacing on smeared lattices using $m_{\rho a}$, since it is known that $m_{\text{res}}$ is strongly dependent on lattice spacing. We determine the lattice scales by taking $m_{\rho a}$ in the valence quark chiral limit. As shown in Table 2, $m_{\rho a}$ is essentially unaffected by smearings.

Another important effect when comparing $m_{\text{res}}$ with different smearing choices is the change in mass renormalization introduced by smearing. Noting the fact that $m_{\pi}^2 \propto m_{q}$, where $m_q = m_{\text{val}} + m_{\text{res}}$ for DWF, we can extract the smeared lattice mass renormalization factors relative to those on the unsmeared lattices by

$$Z_m = \frac{(m_{\pi}^s)^2}{m_{\pi}^u} \frac{m_{\pi}^u}{m_{\pi}^s}$$

where the quantities on unsmeared lattices are superscripted by $u$ and those on smeared lattices by $s$. The renormalized residual masses are thus $m_{\text{res}}^r = Z_m m_{\text{res}}$. The $Z_m$’s are also shown in Table 2.

3.3. $L_s$ dependence of $m_{\text{res}}$

Given that smearing has not removed the crossings at $-M_5 = -1.6$ (lower panel of Fig. 1), we have done further investigations on the decay of $m_{\text{res}}$ with $L_s$. As shown in Figure 2, $m_{\text{res}}$ on the smeared lattices decays essentially the same as on the unsmeared lattices. Contrary to the quenched DBW2 case [2], where the residual masses show simple exponential decay with $L_s$, here both smeared and unsmeared lattices show a more complicated behavior. The same phenomenon has also been seen in our quenched simulations with Wilson gauge action [3], where many topological dislocations are also observed.

4. CONCLUSIONS

We have found that for DWF, the smearings we have chosen do not change the lattice scale, but introduce additional mass renormalization factors. The smeared gauge fields are smoother, but we conclude that the smearings we have chosen do not change the topological dislocations present on the dynamical lattices, thus the large $L_s$ behavior of $m_{\text{res}}$ stays the same. Smearing does not appear to help DWF simulations at strong coupling. However, for $a^{-1} \sim 2$ GeV, where dislocations are suppressed due to weaker coupling, and a practical choice of $L_s$ (say, 16), dynamical 3 flavor simulations with DWF are ready to do with current techniques.

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