Strongly Topological Interactions of Tensionless Strings

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Abstract

The tensionless limit of classical string theory may be formulated as a topological theory on the world-sheet. A vector density carries geometrical information in place of an internal metric. It is found that path-integral quantization allows for the definition of several, possibly inequivalent quantum theories. String amplitudes are constructed from vector densities with zeroes for each in- or out-going string. It is shown that independence of a metric in quantum mechanical amplitudes implies that the dependence on such vector density zeroes is purely topological. For example, there is no need for integration over their world-sheet positions.

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1 Introduction

String theory is characterized by amplitudes that can be represented as sums or integrals over two-dimensional world-sheet geometries. The known theories are critical fundamental strings and two-dimensional Yang-Mills [1, 2]. Whether Liouville or matrix-model strings are counted as a separate class of theories or not, it has proven very difficult to construct string theories which are qualitatively different from the known examples. There is an evident need for such theories if there is to be progress in the long-standing problem of relating strings and strong interactions.

Another problem which has proved surprisingly tough is the search for a huge underlying symmetry of fundamental string theory. There are many reasons to expect such symmetries: the low energy limits of string theories are field theories with large gauge symmetries, the massive string modes appear in towers governed by symmetries of two-dimensional field theories, high-energy behaviour and UV-divergences are mild even though an infinite number of fields contribute to amplitudes, etc. Field theory experience indicates that knowledge of the full symmetry of a theory is useful. String theory should not be an exception.

One approach to string symmetries has been to explore the expectation that the symmetry is spontaneously broken in the present formulation of the theory, but could become restored in some high-energy limit. Symmetries between the exponentially small high-energy amplitudes have been found by Gross [3] and recently by Moore [4]. A related but different high-energy limit has been studied at the non-interacting level by taking the string tension to zero [5-15]. In this limit all independent relativistic momentum invariants are assumed to be large compared to the tension, in contrast to the fixed mass limits of Gross and Moore. To assess the value of the limiting tensionless theory it is essential to also consider interactions.

A direct way to see the importance of an interacting tensionless string theory is to study the spectrum of the ordinary tensile string. It was found in [16] that one-loop mass level shifts are larger than the level separation for high levels, no matter how small the coupling constant is. In similar problems appearing in quantum mechanics, one turns to quasi-degenerate perturbation theory. Taken over to string physics, this recipe would mean that we start from the degenerate case with zero level separation, then introduce interactions and finally a non-zero tension, which causes the level splitting. Zero separation is precisely the case of vanishing tension.

In the present article interactions are constructed in a geometric way directly for the quantum tensionless string. It should be stressed that this procedure need not be equivalent to taking tensionless limits in string amplitudes. Such an approach would first run into the problem of properly identifying the limiting external states, since the spectra of tensile and tensionless strings cannot be the same (because there is no constant with dimensions in the tensionless case). By varying external states while increasing the energy it is possible to find high-energy limits for fundamental strings which are not exponentially decreasing, in contrast to standard fixed state limits [7]. Moreover, there could be qualitatively different high energy limits for
different string theories. For instance, in search of QCD strings one usually looks for a different behaviour than in fundamental string amplitudes \[18\] in order to accommodate partonic structure. By defining an interacting quantized tensionless string theory per se, without recourse to limits of tensile amplitudes, one has a chance to find alternative high energy theories. If one is really lucky there could be background fields in these theories which reintroduce tension and generate a new tensile string theory.

In \[13, 14\] it was demonstrated by light-cone quantization that preservation of the space-time conformal symmetry of tensionless strings by quantization is incompatible with the naively expected spectrum. Classically, the proof of conformal symmetry in the light-cone gauge makes use of the reparametrization symmetry of the full theory, so the anomaly is not to be taken lightly. Either conformal symmetry is broken or the spectrum is quite different from what one gets by standard quantization. The present geometrical approach does not by itself favour any one of these alternatives. When these techniques are developed further one will however be able to deduce the spectrum, either by calculating the partition function or by classifying the external states that couple consistently to the surface geometry. The topological classification found in this article is first step in this direction.

The article is organized as follows: in sect. 2 the action and the geometrical formulation of interactions are described, in sect. 3 the procedure for path integral quantization is discussed and in sect. 4 the topological nature of the quantum theory is examined. Section 5 contains the methods for studying the space of vector density geometries characterizing tensionless strings. In sect. 6 these methods are applied to the derivation of the main results: absence of continuous moduli associated to the insertion of external states on the surface. Finally the conclusions are summarized and set in perspective in sect. 7.

### 2 The classical action and the geometry

The geometric form of the classical action for the tensionless string \[4\] reads

\[
S_0 = \frac{1}{2} \int d^2 \xi \, V^a V^b \partial_a X \partial_b X. \tag{1}
\]

To derive it one starts from the Nambu-Goto action, goes to phase space and solves for the momenta to get a new configuration space action. In the limit of vanishing tension it may be written as above. Otherwise one obtains the ordinary tensile string action \[13, 20\]:

\[
S_T = \frac{T}{2} \int d^2 \xi \sqrt{g} g^{ab} \partial_a X \cdot \partial_b X. \tag{2}
\]

One observes that \( V^a V^b \) plays the role of \( T \sqrt{g} g^{ab} \). For the tensionless action to be diffeomorphism invariant \( V^a \) should thus transform as a vector density and \( X \) as a
The infinitesimal reparametrizations are
\[ \delta_\omega V^a = \omega^b \partial_b V^a - V^b \partial_b \omega^a + \frac{1}{2} \partial_b \omega^b V^a, \]
\[ \delta_\omega X = \omega^a \partial_a X. \] (3)

The action is also invariant under spacetime conformal transformations. While the Poincaré invariance is manifest, the conformal symmetry acts by rescaling \( V^a \). For example, the conformal boosts are
\[ X'\mu = [X^\mu + b^\mu X^2] \left[ 1 + 2 b \cdot X + b^2 X^2 \right]^{-1}, \]
\[ V'^a = V^a \sqrt{1 + 2 b \cdot X + b^2 X^2}. \] (4)

In particular, note that conformal transformations taking a point \( X_0 \) to infinity also transforms a finite \( V^a \) to zero. One may thus consistently require that asymptotic string states approaching infinity are accompanied by vanishing vector densities on the world-sheet.

From the tensionless action (eq. 1) one gets the equations of motion
\[ 0 = V^a \partial_a X \cdot \partial_b x, \]
\[ 0 = \partial_a \left( V^a V^b \partial_b x \right), \] (6)
where the first equation gives the constraints analogous to the Virasoro constraints of the tensile string, and the second equation may be interpreted as the conservation of world-sheet momentum currents parallel to \( V^a \).

A freely propagating string traces out a cylindrical world sheet. On the world sheet one can draw flow lines whose tangents are parallel to the vector density at each point. Fig. 1 shows how diffeomorphism invariance may be used to map the infinite cylinder to a twice punctured sphere. The punctures correspond to in-states or out-states, with the flow having a source or a sink at the vector density zeroes coinciding with the punctures. Vector densities on the sphere with additional sources or sinks describe tree-level interacting strings. Surfaces of higher genus represent emission and reabsorption of virtual strings, i.e. they give loop corrections.

In discussing the topologies of vector densities it is sometimes useful to relate to concepts from the theory of dynamical systems or ordinary differential equations [21]. To a vector field \( v^a \) one may associate a differential equation
\[ \frac{d\xi^a}{dt} = v^a. \] (7)

One can associate a differential equation to a vector density in the same way at the price of not having a covariant equation. A general coordinate transformation then rescales the vector density. Thus, the direction of a solution trajectory behaves covariantly, but the time parameter \( t \) is effectively rescaled in a position-dependent way. The most important topological notions in dynamical systems theory are however independent of the time parametrization, and therefore useful also for densities.
Zeroes of the vector density are then equilibrium points. They are examples of limit points, and there may also be limit cycles. The topology of a vector density is reflected in the phase portrait depicting flow-lines or solution trajectories associated with the density.

Fig. 2a illustrates simple sources, saddles and sinks of a vector density on the torus. It also exemplifies the Poincaré-Hopf theorem [23] relating the numbers of different zeroes of vector fields on a genus $g$ surface. For vector fields with only simple zeroes

$$\# \text{sources} - \# \text{saddles} + \# \text{sinks} = 2(1 - g).$$

Fig. 2b demonstrates that there are smooth vector densities on the torus without sources or sinks, but with stable and unstable cycles. We expect that all possible flows should contribute to quantum mechanical amplitudes. To determine their weights we have to define the path integral over vector densities.

## 3 The path integral

The path integral measure may be defined by considering gaussian integrals. The construction (see e.g. [24]) requires that an inner product on infinitesimal variations of fields is introduced, and that the normalization of the measure is specified [25]. Norm-preserving transformations of the fields become symmetries of the measure. For the tensile string this approach was pioneered by Polyakov [26], Alvarez [27] and Polchinski [25]. It is reviewed in [28], where further references can be found.

The inner product should be reparametrization invariant. It should also be local, which in this case means that it has to be an integral over the world-sheet of a function of the fields, but not of their derivatives. The inner product of two vector density variations then has to take the form

$$\langle \delta V_1 | \delta V_2 \rangle = \int d^2 \xi \, g_{ab}(\xi) \delta V_1^a(\xi) \delta V_2^b(\xi).$$

(9)

The function $g_{ab}$ has to transform as a tensor for the inner product to be reparametrization invariant, and it has to be non-degenerate for the inner product to be non-degenerate. We may then regard it as a metric on the world sheet, even though it does not appear in the action (1).

The metric allows for the construction of a new scalar

$$\varphi \equiv g^{-1/2} g_{ab} V^a V^b.$$  

(10)

The general reparametrization invariant and local inner product of coordinate variations is then

$$\langle \delta X_1 | \delta X_2 \rangle = \int d^2 \xi \, \sqrt{g} \delta X_1 \delta X_2 \eta(\varphi).$$

(11)

The normalization of path integrals is fixed by

$$\int DV^a e^{-\int d^2 \xi \sqrt{g} \nu(\varphi) e^{-\langle \delta V | \delta V \rangle}} = 1, \quad \int DX e^{-\int d^2 \xi \sqrt{\mu} X(\varphi) e^{-\langle \delta X | \delta X \rangle}} = 1$$

(12)
where the functions $\mu_V$ and $\mu_X$ encode our freedom in normalizing. They are the most general expressions satisfying the principle of ultralocality coined by Polchinski \cite{25}: the measure (and the ambiguity in it) should be a reparametrization invariant product of factors depending only on the fields at individual world sheet points.

Three arbitrary functions $\mu_V, \mu_X$ and $n$ enter in the definition of the measure. In the case of non-vanishing tension the corresponding ambiguities are simply constants, since no local scalar can be constructed solely from the metric. If, however, we wish to make contact with the classical theory we should ask for path integrals independent of the metric. Then at least part of the ambiguity will be removed.

4 Hyper-Weyl invariance

Weyl invariance in the tensile string means that the action (2) is independent of the conformal factor $\lambda$ when $g_{ab} \rightarrow \lambda g_{ab}$. However, the path integral measure in general depends on the conformal factor through its definition in terms of an inner product and through regularization. Only for critical strings, e.g. bosonic strings in flat $D = 26$ Minkowski space, does the Weyl invariance survive quantization. In this case one may choose to calculate in any convenient conformal factor background.

The metric in the tensionless string is precisely analogous to the conformal factor. It only enters upon quantization, with the construction of a measure. If metric independence, characteristic of topological field theories, is maintained at quantization one may choose to calculate in any background metric compatible with the surface topology. It is sometimes natural to stress the analogy to Weyl-invariance by regarding metric independence as an extension of this symmetry and call it hyper-Weyl invariance.

In the Weyl invariant string the all-important geometry is the conformal geometry which enters in the action, and loop amplitudes can be expressed as integrals over different conformal geometries. Similarly the amplitudes of the hyper-Weyl invariant tensionless string are given by integrals (or sums) over the geometries defined by the vector density $V^a$.

It will be assumed in the following that we are dealing with a hyper-Weyl invariant quantization of the tensionless string. A simple one-loop calculation in a flat background with a constant vector density suggests that hyper-Weyl symmetry is non-anomalous, and the large freedom in the definition of the path-integral parametrized by the functions $n, \mu_V$ and $\mu_X$ should help in avoiding anomalies in more complicated backgrounds. I hope to settle this issue in future work, but for the moment the most pressing question is the nature of the vector density geometry replacing the conformal geometry of the tensile string. In particular, do we have any right to call this theory topological? Even if it is independent of the metric, the density $V^a$ has appeared instead.
5 Vector density dependence

The action (11) is diffeomorphism invariant. We should fix a reparametrization gauge to select a representative from each equivalence class of vector densities. Suppose that $V^a$ has been fixed to be a definite function on the surface. A general variation $\delta V^a$ may then be separated in two parts

$$\delta V^a = (P_V \omega)^a + \delta m_j \frac{\partial V^a}{\partial m_j}, \quad (13)$$

where the first term is just an infinitesimal diffeomorphism (3), which may be written covariantly

$$(P_V \omega)^a \equiv \delta \omega V^a = \omega^b \nabla_b V^a - V^b \nabla_b \omega^a + \frac{1}{2} \nabla_b \omega^b V^a, \quad (14)$$

now that we have introduced a metric. The second term accounts for deformations of the geometry. There could be a space $\{V^a\}/Diff$ of inequivalent vector densities parametrized by moduli $m_i$. In the presence of an inner product it is easy to identify deformations caused by variations of the moduli. Since they cannot be obtained by a change of coordinates they may be chosen orthogonal to any reparametrization. Using the definition of adjoint operators this condition may be written

$$0 = \langle P_V \omega \left| \frac{\partial V}{\partial m_j} \right. \rangle \equiv \langle \omega \left| P_V^\dagger \frac{\partial V}{\partial m_j} \right. \rangle, \quad \forall \omega. \quad (15)$$

Thus, possible deformations $U^a$ of the $V^a$ geometry solve the equation

$$0 = (P_V^\dagger U)^a = \sqrt{g} \left[ V^b \nabla_b U^a - \frac{1}{2} V_b \nabla^a U^b + \nabla_b V^b U^a + \frac{1}{2} \nabla^a V_b U^b \right] \quad (16)$$

and are zero-modes of the operator $P_V^\dagger$. In this equation it is important to remember that $U^a$ is a density and that accordingly there is an extra term in the expression for the covariant derivative in terms of Christoffel symbols. The dimension of the space of geometries is the dimension of the space of normalizable zero-modes, $\text{Ker} P_V^\dagger$.

Normalizability of $U^a$ is needed to make sense of path-integral manipulations. However, the surfaces under consideration have punctures where external states can be attached. For the ordinary string this approach to amplitudes is due to D’Hoker and Giddings [29]. Around each puncture a small disc is cut out. Then it is enough to have normalizability in the remaining surface with arbitrarily small holes. This is the notion of normalizability used in the present paper. Though this definition of normalizability does not give boundary conditions for the behaviour of $U^a$ at zeroes (punctures), there are other requirements. The boundary conditions for $U^a$ are obtained by demanding that their inner products with $P_V \omega$ in the orthogonality equation (13) are finite for all infinitesimal reparametrization symmetries $\omega^a$ of the punctured surface. The limits of these inner products should be well defined as the sizes of the excluded discs approach zero. A vector field $\omega^a$ generates a symmetry only if it preserves the punctures, i.e. if it has zeroes at all punctures. Thus we shall
look for normalizable solutions to the adjoint zero-mode equation (16), belonging to the space of vector densities dual to the space of infinitesimal variations of the background vector density \( V^a \) due to such puncture-preserving diffeomorphisms.

Finding normalizable deformations \( U^a \) is not a local issue. The norm could diverge due to singularities anywhere on the world-sheet. We can, however, limit the number of normalizable \( P_V^\dagger \) zero-modes by local arguments. A local feature of the vector density \( V^a \) at a point \( P \) could force zero-modes to have non-normalizable singularities close to \( P \). Then there are no deformations in a neighbourhood of \( P \), and there can be non-trivial normalizable solutions on the surface only if their supports are separated from \( P \) by a line where Taylor expansions of the zero-modes break down.

We have seen above in equation (8) that local features like sinks, sources and saddles of \( V^a \) are unavoidable on world-sheets for interacting tensionless strings. What are the consequences of such zeroes for the existence of vector density deformations \( U^a \)?

### 6 Topological interactions

We assume independence of the metric, which could thus be taken to be flat in a neighbourhood of a zero. We also assume that world-sheets have a \( C^\infty \) differentiable structure so that it makes sense to study Taylor expansions of vector densities in any coordinate chart. Generically the topology of the flow close to the zero is determined by the first terms of the expansion, but if they vanish the topological classification of zeroes of vector fields in terms of expansion coefficients is not complete [21]. However, for non-zero real analytic vector densities there are always some leading terms and in two dimensions isolated analytic equilibrium points can be classified topologically ([22, 21]). Note that in contrast to the tensile case there is no natural complex structure, and real analyticity of a function just means that it is Taylor-expandable with a non-vanishing radius of convergence.

Before discussing the classification we study reasonably general but simple forms of the densities \( V^a \). In polar coordinates:

\[
V^r = r^{m+\frac{1}{2}} \cos(\delta + (p-1)\varphi), \quad V^\varphi = r^{m-\frac{1}{2}} \sin(\delta + (p-1)\varphi). \tag{17}
\]

These zeroes behave simply under rotations; \( p \) is a simple topological invariant, the Poincaré index, which measures how many times \( V^a \) winds around the origin when its argument rotates once around the origin in a small closed circuit, and \( m \) is the order of the zero. If \( m \) and \( p \) are integers satisfying \( m \geq 0 \) and \( m \geq |p| \) these vector densities are real analytic, the factor \( \sqrt{r} \) being due to the density factor in the transformation to polar coordinates. The phase \( \delta \) is irrelevant for \( p \neq 1 \). We then absorb it in a shift of \( \varphi \).

The zero-mode equation (16) can be solved in such backgrounds (17) by making
the ansatz
\[ U^r = r^k U_k^r(\varphi), \quad U^\varphi = r^{k-1} U_k^\varphi(\varphi). \] (18)

The solution will be a sum of such terms, each obeying
\[ 0 = \sin^2(\omega \varphi) \ddot{U}_k^r + a \sin(2\omega \varphi) \dot{U}_k^r + \left[ b + c \sin^2(\omega \varphi) \right] U_k^r, \] (19)
\[ U_k^\varphi = d \dot{U}_k^r + f \cot(\omega \varphi) U_k^r, \] (20)
for \( \delta = 0 \). Here \( \omega = p - 1 \), while \( a, b, c, d, e \) and \( f \) are somewhat complicated rational functions of \( m, p \) and \( k \). The final expressions given explicitly below are much simpler.

For each value of \( k \) there may be two, one or zero normalizable solutions, since the equation is second order. To exclude solutions by local methods we ask for which values of \( m \) and \( p \) all zero-modes are non-normalizable. In those cases there can be no deformations of the geometry. For \( k > -\frac{3}{2} - m \) the radial solution satisfies the boundary condition, but a singular angular dependence could cause a divergence of the norm. Equation (19) has regular singular points when \( \sin \omega \varphi = 0 \). The indicial equation
\[ 0 = \zeta^2 + \left( \frac{a}{2\omega} - 1 \right) \zeta + \frac{b}{\omega^2} \] (21)
then determines the exponents
\[ \zeta_\pm = \frac{1}{4\omega} \left[ (2\omega - a) \pm \sqrt{(2\omega - a)^2 - 16b} \right] = -\frac{1}{2(p-1)} [4m + 3p + 2k \mp (2m + p - 1)] \] (22)
in the power-law behaviour \( U_k^\varphi \sim \varphi^\zeta \) close to each singular point. The singular points actually represent singular rays of constant \( \varphi \) towards or away from the zero (17). The condition for divergence of the norm is \( \zeta \leq -\frac{1}{2} \). If this inequality holds for both roots of the indicial equation there can be no deformations.

Combining the conditions on angular and radial behaviour of zero-modes we find that there are no deformations of regular vector densities with \( p > 1 \). In contrast, there are always solutions with locally convergent norms for \( p < 1 \). The intermediate case \( p = 1 \) requires special attention. The zero-mode equation (19) should then be solved directly without going via equation (20), which does not make sense at \( p = 1 \).

For \( p = 1 \) the zero (17) with \( \delta = 0 \) describes the kind of simple source or sink that can be associated to asymptotic string states (Fig. 1). Nonzero \( \delta \) generically means that the flow spirals towards or away from the zero. All such flows with \( \cos \delta \neq 0 \) are topologically equivalent, and solving (19) one finds no normalizable solutions. In contrast, the exceptional case \( \cos \delta = 0 \), a ‘centre’ where the flow circles the zero without ever approaching or leaving, allows for infinitely many solutions with locally convergent norms. To summarize, there are no continuous moduli for deformations of analytic vector densities with
\[ p \geq 1, \quad \cos \delta \neq 0. \] (23)
These results tie in well with the topological classification of isolated zeroes of analytic vector fields. Each topology except the centre is represented by vector densities consisting of a finite number of sectors of elliptic, parabolic or hyperbolic type (see Fig. 3). If there are elliptic sectors they always lie between two parabolic sectors. In this language, we have found that there could be deformations around centres or in hyperbolic sectors. Zeroes with only parabolic and elliptic sectors cannot be deformed and the dependence on such vector densities is truly topological.

The zero-mode equations for the exceptional case of a centre (Fig. 3d) appears to allow an infinite number of deformations. Such infinitesimal deformations produce spiralling vector densities of the same topology as the simple source or sink. Centres should therefore be regarded as unstable configurations, and they do not give rise to any true continuous moduli.

For hyperbolic sectors the number of deformations can only be determined by global considerations. The example on the torus in Fig. 2a illustrates how moduli could be excluded: any non-zero deformation is a zero-mode also close to the parabolic source (or the sink), where the boundary conditions cannot be satisfied.

7 Conclusions and outlook

The geometry of bosonic tensionless string theory is governed by a vector density. In this article interactions have been introduced by studying vector densities with zeroes on compact surfaces. We have noted and parametrized a huge freedom in path-integral quantization. At least part of this freedom is presumably fixed when one imposes the condition that the quantum theory be topological, in the sense of retaining the independence of a two-dimensional metric which holds for the classical theory. If the theory is not fixed completely by this requirement we obtain a class of inequivalent high energy limits of string theory, some of which are not necessarily limits of critical fundamental strings. The investigation of the class of quantum topological tensionless string theories is a crucial one which I hope to return to.

On the assumption that the quantization is independent of two-dimensional metrics we have asked if the theory is what one may call ‘strongly topological’. By this we mean that the dependence on the vector density, the geometrical object in the theory, should be given by its topology, and thus not require the introduction of any continuous moduli. This was demonstrated for a large class of real analytic vector densities (sect. 6) with isolated limit points surrounded only by so-called parabolic and elliptic sectors. The case of ordinary sources and sinks (Fig. 1) corresponding to classical in-coming and out-going strings is included. There can be moduli only if there are regions on the surface to which certain zero-modes cannot be continued analytically. The possible role of regions bounded by limit cycles for this question is under study. In this article it has been established that there are no moduli associated with the external strings.
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Figure captions

1. The diffeomorphism mapping an infinite cylinder to a punctured sphere. If there had been metrics on the surfaces, a Weyl transformation would also have been needed to relate them.

2. Opposite sides of the rectangles are identified to form a torus.
   
   a) A source, a sink and two saddles on the torus represent a one-loop ‘self-energy correction’.
   
   b) One unstable and one stable limit cycle on the torus represent a one-loop vacuum diagram.

3. Zeroes of different topology with Poincaré index $p$.
   
   a) A zero with four hyperbolic sectors ($p = -1$).
   
   b) A source with a single parabolic sector ($p = 1$).
   
   c) A zero with four parabolic sectors alternating with four elliptic sectors ($p = 3$).
   
   d) A centre ($p = 1$).
This figure "fig1-1.png" is available in "png" format from:

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