Full Counting Statistics of Charge Transfer in Coulomb Blockade Systems

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Full counting statistics (FCS) of charge transfer in mesoscopic systems has recently become a subject of significant interest, since it proves to reveal an important information about the system which can be hardly assessed by other means. While the previous research mostly addressed the FCS of non-interacting systems, the present paper deals with the FCS in the limit of strong interaction. In this Coulomb blockade limit the electron dynamics is known to be governed by a master equation. We develop a general scheme to evaluate the FCS in such case, this being the main result of the work presented. We illustrate the scheme, by applying it to concrete systems. For generic case of a single resonant level we establish the equivalence of scattering and master equation approach to FCS. Further we study a single Coulomb blockade island with two and three leads attached and compare the FCS in this case with our recent results concerning an open dot either with two and three terminals. We demonstrate that Coulomb interaction suppresses the relative probabilities of large current fluctuations.

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I. INTRODUCTION

The current fluctuations in the various mesoscopic systems have been the subject of both theoretical and experimental research in the last two decades. Traditionally, the attention was focused on the shot noise phenomenon. The shot noise is the main fundamental source of current noise at low temperatures. In classical systems shot noise unambiguously related to the discreteness of the electron charge. In quantum system the shot noise can be used as unique tool to reveal the information about the electron correlations and entanglement of different kind. The investigation of the quantum shot noise cross-correlations in the multi-terminal mesoscopic devices is the new trend in this field which has attracted much attention as well. The most achievements in the study of the shot noise phenomena have been summarized in the recent review article.

Alternative way to investigate the current correlations in the mesoscopic system has been proposed in the pioneering work by Levitov et al. This new fascinating theoretical approach, known as the full counting statistics, yields not only shot noise power but also all possible correlations and momenta of charge transfer. The essence of this method is an evaluation of the probability distribution function of the numbers of electrons transferred to the given terminals during the given period of time. The first and the second moments of this distribution correspond to the average currents and the shot-noise correlations, respectively. The probability distribution also contains the fundamental information about large current fluctuations in the system.

Initially, FCS method made use of the scattering approach to mesoscopic transport. It was assumed that the mesoscopic system was completely characterized by its scattering matrix. This method enabled the study the statistics of the transport through the disordered metallic conductor and the two-terminal chaotic cavity. Muzykantskii and Khmelnitskii generalized the original approach to the case of the normal metal/superconducting contacts. The very recent development in this field is the counting statistics of the charge pumping in the open quantum dots.

The use of multichannel scattering matrix of the system was crucial to obtain the results of the above mentioned works. However, such approach leads to the difficulties in case of practical layouts, where the scattering matrix is random and cumbersome. They become apparent especially in case of multi-terminal geometry. To circumvent these difficulties one evaluates the FCS with the semiclassical Keldysh Green function method or with its simplification called the circuit theory of mesoscopic transport. The Keldysh method to FCS was first proposed by one of the authors in order to treat the effects of the weak localization corrections onto the FCS in the disordered metallic wires. The method proves to be very flexible and has been recently applied to the FCS in superconducting heterostructures, multi-terminal normal metal systems as well in the three-terminal superconducting beam splitter.

The above research addressed the FCS of non-interacting electrons. Since the interaction may bring correlations and entanglement of electron states the theory of FCS in mesoscopic systems places in a strong Coulomb blockade limit.

Note, that the shot-noise in the Coulomb blockade devices has attracted the significant attention. Korotkov et al. and Hershfield et al. presented the first theory in the framework of “orthodox” approach to single electron transport. Later on Korotkov also studied the frequency dependence of the shot noise by means of Langevin approach both in low (classical) and very high (quantum)
frequency limits. The frequency dependence of the shot
noise in the single electron transistor was also investigated in Ref. 13–14. The ferromagnetic single electron
transistor was considered by Bulka et. al. 15. The shot noise experiments were performed by Birk et. al.16,17. In this
work, the nano-particle in between the STM (scanning
tunneling microscope) tip and the metallic electrode
was used to form the Coulomb blockade island and the
quantitative agreement with the theory of Hershfield et. al.
was found.

The electrons dynamics in Coulomb blockade limit is
fortunately relatively simple. When the cotunneling phe-
omena is disregarded, the evolution of the system is gov-
erned by a master equation. The charge transfer is thus
typical stochastic process rather than the quantum
mechanical one. Nevertheless the FCS is by no means
trivial and has not been studied yet. In the present pa-
per we have developed the general approach to FCS in
the Coulomb blockade regime. This is the central result
of the paper. Our method turns out to be an elegant
extension of the usual master equation approach. We
also predict the FCS in different Coulomb blockade sys-
tems. The previous results for the zero frequency shot
noise power can be evaluated in our approach as the sec-
ond moment of charge transfer probability distribution
function.

The paper is organized as follows. In the section II
we start by presenting the two physical systems to be treated
within the master equation. Basing on these prototypes
we formulate the general model. We derive our approach
to the FCS in the section III. In the section IV we applied
to the single resonant level and consider its relation to
the scattering approach. Two- and three-terminal single
electron transistors are considered in section V. We also
compare their FCS in the Coulomb blockade regime with
that one of non-interacting electrons. We summarize
the results in section VI.

II. SYSTEMS UNDER CONSIDERATION

The dynamics of many different physical systems can
be described by master equation. For our purposes it is
convenient to write it down as follows

\[ \frac{\partial}{\partial t} |p(t)\rangle = -\hat{L}|p(t)\rangle \quad (1) \]

where each element \( p_n(t) \) of the vector \( |p(t)\rangle \) corresponds
to the probability to find the system in the state \( n \). The
matrix elements of operator \( \hat{L} \) are given by

\[ L_{mn} = \delta_{nm} \gamma_n - \Gamma_{m\rightarrow n}, \quad \Gamma_m = \sum_{m \neq n} \Gamma_{m\rightarrow n} \quad (2) \]

Here \( \Gamma_{m\rightarrow n} \) stands for the transition rate from the state
\( m \) to the state \( n \). \( \gamma_n \) stands for the total transition rate
from the state \( n \). Thus defined operator \( \hat{L} \) always has a
zero eigenvalue, the corresponding eigenvector being the
stationary solution of the master equation.

![Diagram](https://via.placeholder.com/150)

FIG. 1: The single resonant level system, formed by the two
tunnel barriers. The resonant level in the quantum well is
shown by the dashed line.

A variety of Coulomb blockade mesoscopic systems
obey Eq. (1) under appropriate conditions. Here the
main advantage of the master equation approach is a
possibility of non-pertubative treatment of the interac-
tion effects. In what follows, we first remind the master
equation description of two simple systems: single reso-
nant level and many-terminal Coulomb blockade island.
On the basis of these examples we will sketch the master
equation for the general Coulomb blockade system. This
will prepare us to the next section where we derive the
FCS method.

A. Resonant level model.

An elaborated model of the resonant center was pre-
 Aerated in Ref. 19. It was subsequently improved in the work20 to include the Coulomb interaction. One the
physical realization is disordered tunneling barrier which
is placed between two leads.21 At sufficiently low tem-
peratures the main mechanism of transport in this sys-
tem is the resonant tunneling via localized states formed
by impurity centers. Another physical realization is the
Coulomb blockade quantum dot in the low temperature
regime \( k_BT \ll \Delta E \), where \( \Delta E \) is a mean separation be-
tween the energy levels in the dot.22 By applying the
gate voltage, one can tune the given level to be between
the chemical potentials of the leads. (See Fig.1). We
consider below two limiting cases where one disregards
either double occupancy of the level or on-cite Coulomb
interaction.

In the strongly interacting case the double occupancy
of the resonant level is entirely excluded due to the
Coulomb repulsion \( U \). Then the system can be found
only in two different microscopic states: one with no elec-
trons, and another with a single electron. The transport
through the level can be described by master equation
approach, provided the applied voltage or the tempera-
ture are not too low, i.e. \( \max\{eV, k_BT\} \gg h\Gamma_L(R) \). Here \( \Gamma_L(R) \) are the quantum-mechanical tunneling rates from
the left (right) electrode onto the resonant level. We will also assume that at the relevant energy scale, given by \( \max\{eV, k_BT\} \), the rates \( \Gamma_{L(R)} \) are energy independent.

Under above assumptions the transition rates in Eq. (1) are given by

\[
\begin{align*}
\Gamma_{1\to 0} &= 2\Gamma_L f_L(\epsilon_i) + 2\Gamma_R f_R(\epsilon_i) \\
\Gamma_{0\to 1} &= \Gamma_L[1 - f_L(\epsilon_i)] + \Gamma_R[1 - f_R(\epsilon_i)]
\end{align*}
\]

The microscopic states \( \{0\} \) and \( \{1\} \) denote the situation with no and one electron, respectively. Fermi function \( f_L(R) = (1 + \exp[(\epsilon - \mu_L(R))/kT])^{-1} \) accounts for the filling in the left (right) lead and \( \epsilon_i \) is the position of the resonant level. The factor 2 in the rate \( \Gamma_{1\to 0} \) stems from the fact that two quantum states, with spin up and down, are available for tunneling. The description in terms of rates is correct when the Coulomb repulsion is strong enough: \( U \gg \max\{eV, k_BT\} \).

The opposite limit is the case of vanishing Coulomb interaction. In this case the spin up and down channels can be treated independently. Each of them can be described by the master equation, provided the same condition as before is fulfilled: \( \max\{eV, k_BT\} \gg \hbar \Gamma_{L(R)} \). For both spin directions the rates are written as

\[
\begin{align*}
\Gamma_{1\to 0} &= \Gamma_L f_L(\epsilon_i) + \Gamma_R f_R(\epsilon_i) \\
\Gamma_{0\to 1} &= \Gamma_L[1 - f_L(\epsilon_i)] + \Gamma_R[1 - f_R(\epsilon_i)]
\end{align*}
\]

Here the indices \( \{0\} \) and \( \{1\} \) denote the filling factor of the level by electron with a chosen spin.

Disregarding of Coulomb interaction is not adequate for a realistic system. However the latter model is worth to consider as well. The point is that the statistics of the charge transfer in this case can be also evaluated in the framework of the non-interacting scattering approach, thus providing the way to establish the consistency of two approaches to FCS.

**B. Many-terminal Coulomb blockade island.**

The electrical circuit incorporating the quantum dot with several terminals is shown in Fig. 2. This circuit is an extension of the usual set-up in case of conventional two-terminal quantum dot. At the present stage of nanotechnology the mesoscopic system, associated with this circuit, can be realized with the use of two-dimensional electron gas (2DEG) in the GaAs/AlGaAs heterostructures.

The essential elements of the circuit shown in Fig.2 are the resistances \( R_k \) of the contacts, the mutual capacitances \( C_k \) between the leads and the island and the external dc voltage sources \( V_k \). Correspondingly, \( C_g \) and \( V_g \) denote the gate capacitance and gate voltage, which is used to vary the offset change on the island. We assume that the dot is placed in the Coulomb blockade regime, \( R_k \gg R_Q = 2\pi\hbar/e^2 \). In order the Coulomb blockade effect will be observable the condition \( k_BT \ll E_c = e^2/2C_\Sigma \) is also required. Here \( E_c \) is a charging energy of the island, \( C_\Sigma = \sum_{k=1}^{N}C_k + C_g \) is a sum capacitance of the system and \( N \geq 2 \) is a number of leads attached to the dot. We also assume the temperature to be rather high, \( k_BT \gg \Delta E \), with \( \Delta E \) being the mean level spacing in the dot, so that the discreteness of the energy spectrum in the island is not important. The possible effects of co-tunneling will not be discussed in the paper. Therefore the characteristic scale of applied voltage \( eV \) is assumed to be greater than the Coulomb blockade threshold, \( eV \geq E_c \).

Under the above conditions the multi-terminal dot is fairly well described by the "orthodox" Coulomb blockade theory. One can consider the excess number of electrons on the island \((-n)\) as a good quantum number, corresponding to the macroscopic state of the system. The tunneling of electrons will occur one by one, increasing or decreasing the charge \( Q_0 = ne \) on the island by \( \pm e \). The corresponding tunneling rate \( \Gamma_{n\pm 1 \to -n}^{(k)} \) across the junction \( k \) is expressed via the electrostatic energy difference \( \Delta E_{n\pm 1 \to -n}^{(k)} \) between the initial \((n)\) and final \((n \pm 1)\) configurations

\[
\Gamma_{n\pm 1 \to -n}^{(k)} = \frac{1}{e^2R_k} \Delta E_{n\pm 1 \to -n}^{(k)} \left[1 - \exp[-\Delta E_{n\pm 1 \to -n}^{(k)}/k_BT]\right]
\]

The evaluation of \( \Delta E_{n\pm 1 \to -n}^{(k)} \) can be done along the same lines as in the case of two-terminal dot. The result reads

\[
\Delta E_{n\pm 1 \to -n}^{(k)} = \pm e(V_k - V_0(n)) - \frac{e^2}{2C_\Sigma}
\]

where \( V_0(n) \) is the electrostatic potential on the island. It is written as

\[
V_0(n) = \frac{1}{C_\Sigma}(en + C_g V_g) + \frac{1}{C_\Sigma} \sum_{i=1}^{N} \tilde{C}_i V_i
\]
Here \( C_t \equiv C_t + C_g / N \) and we also assumed that the dot is biased such a way, that external voltages are subjected to the condition \( \sum_{i=1}^{N} V_i = 0 \). In this case the gate voltage \( V_g \) can be used to influence the offset charge \( q = C_g V_g \) on the island in a controlled way.

Neglecting the quantum correlations between different tunneling processes, we may write down the master equation (9). It connects the states with different island charge, the total transition rate from the state \( n \) to \( n \pm 1 \) being the sum of tunneling rates over all junctions:

\[
\Gamma_{n \pm 1 \rightarrow n} = \sum_{k=1}^{N} \Gamma^{(k)}_{n \pm 1 \rightarrow n}.
\]

C. Universal model.

We now outline the universal model, which is an extension of the preceding two. The physical realization of this model can be either an array of Coulomb blockade quantum dots or a mesoscopic system with a number of resonant levels. We assume that all the relevant conditions, mentioned previously, are satisfied and therefore the description in terms of master equation is valid. It is convenient schematically to represent the system as a graph (see Fig. 3), so that each node \( \alpha \) would correspond either to the single resonant level or the external terminals and the line \((\alpha, \beta)\) would be associated with the possible transition. Let \( M \) be the total number of nodes in this graph. We denote nodes by the Greek indices and numerate each line by the Latin indices, i.e. we will write \( k = (\alpha, \beta) \), where \( k = 1 \ldots L \) and \( L \) is a total number of lines. In case of many-dot system each line \( k \) unambiguously corresponds to the tunnel junction. In case of mesoscopic system with many resonant levels one has no a direct one-to-one correspondence between the line of the graph and the junction. Still, for the sake of convenience, we will sometimes refer below to each line of the graph as to the "junction" when it may not lead to contradiction. The lines are assumed to be directed, thus specifying the sign convention for a current \( I_k \) through the "junction" \( k \). There are \( N \) external junctions \( k = 1 \ldots N \), \((N \leq L)\), connecting the terminals with the system. The currents through these junctions are directly measurable and hence are of our main interest in this paper.

The macro- or microscopic state of the universal model is given by a set of occupation numbers \(|n\rangle = |n_1, \ldots, n_M\rangle \); \( n_\alpha \) is equal to any integer for the array of quantum dots and refers to the excess charge on the island \( \alpha \); in case of many resonant levels system \( n_\alpha = 0, 1 \), since it is assumed that the double occupancy is totally suppressed due to the strong Coulomb interaction.

We now consider the general properties of the master equation (9) describing the above model. Owing to the fact \( \sum_{n} L_{nm} = 0 \), the \( \hat{L} \) operator has the right, \(|p_0\rangle\), and the left, \(|q_0\rangle\), eigenvectors corresponding to zero eigenvalue

\[
\hat{L}|p_0\rangle = 0,
\]

\[
|q_0\rangle = \gamma |L\rangle, \sum_{k} |p_k\rangle\langle q_k| = \hat{I}
\]

where \( \hat{I} \) is a unitary operator in the absorbing subspace. For any physically reasonable \( \hat{L} \) operator \( \lambda_0 = 0 \) and \( \text{Re} \lambda_k > 0 \) for \( k \neq 0 \).

For the following it is also useful to represent \( \hat{L} \) operator in the form

\[
\hat{L} = \hat{\gamma} - \hat{\Gamma}, \quad \hat{\Gamma} = \sum_{k=1}^{L} (\hat{\Gamma}_k^{(+)\dagger} + \hat{\Gamma}_k^{(-)})
\]

where \( \hat{\gamma} \) is the diagonal operator in the basis \(|n\rangle\) of the system configuration and \( \hat{\Gamma}_k^{(\pm)} \) are associated with the tunneling transitions through the "junction" \( k = (\alpha, \beta) \): \n
\[
\hat{\gamma} = \sum_{\{n\}} |n\rangle\langle n|, \quad \hat{\Gamma}_k^{(\pm)} = \sum_{\{n\}} |n'\rangle\Gamma_k^{(\pm)}(n)\langle n|
\]

The state \(|n'\rangle\) = \(|n_1, \ldots, n'_\alpha, \ldots, n'_\beta, \ldots, n_M\rangle\) results from the state \(|n\rangle\) by appropriate changing the corresponding occupation numbers: \( n'_\alpha = n_\alpha - \sigma_k, \quad n'_\beta = n_\beta + \sigma_k \), where \( \sigma_k = \pm 1 \) denotes the direction of the transition.

FIG. 3: The graph of universal model (See the main text). The terminals are connected with the system via external junctions 1, 2 and 3. The nodes \( \alpha, \beta \) and \( \gamma \) are either resonant levels or dots, linked with each other by internal junctions \( k \)'s. The arrows denote the conventional direction of a current through each junction.
III. THE FCS IN THE MASTER EQUATION

In this section we derive the general scheme to evaluate the FCS in the arbitrary mesoscopic system placed in the strong Coulomb blockade regime. In what follows, we assume that the mesoscopic system under study can be described by the universal model outlined in the preceding section. We also assume that the transition rates of the corresponding master equation are known. The physical origin of these transition rates is a pure quantum mechanical phenomenon of tunneling. However, the reduced description of the system dynamics by means of master equation corresponds to the Markov classical stochastic process. The mutual quantum correlations between subsequent tunneling events are not taking into account in such approach. It is a price one has to pay to go from the quantum-mechanical to classical description. On the other hand, it enables us to use classical probability methods to derive the FCS.

In the following we will partially use the notation of the book. We assume that for each outcome of the experiment one can put into correspondence a random set of points at the time axis, obeying the condition

\[ +T/2 > \tau_1 > \tau_2 > \cdots > \tau_{s-1} > \tau_s > -T/2 \quad (12) \]

Here \( \tau_i \) is the instantaneous moment of \( i \)'s transition in the system, and \( s = 0, 1, \ldots \) is a non-negative integer, corresponding to the total number of transitions during the time of measurement \( T \). In the end of calculation this time should be understood as infinitely large, \( T \rightarrow +\infty \), so that in average \( s \gg 1 \). Given this set of points we introduce the elementary random event \( \zeta_s = (\tau_1, k_1, \sigma_1; \ldots ; \tau_s, k_s, \sigma_s) \). It corresponds to the experimental outcome, when at time \( \tau_i \) the tunneling happens through the junction \( k_i, \sigma_i = \pm 1 \) being the direction of the transition. The events \( \zeta_s \) constitute the set \( \Omega \) of all possible experimental outcomes.

At the next step one should define the measure (or the probability) \( d\mu(\zeta) \) at the set \( \Omega \). For this purpose we may very generally introduce the sequence of non-negative probabilities \( Q_s((\tau_1, k_1, \sigma_1) = Q(\tau_1, k_1, \sigma_1; \ldots ; \tau_s, k_s, \sigma_s) \geq 0 \) defined at the domain (12) so that

\[ d\mu(\zeta) = Q_0 + \sum_{s=1}^{+\infty} \sum_{\{k_1, \sigma_1\}} Q_s((\tau_1, k_1, \sigma_1)) d\tau_1 \cdots d\tau_s \quad (13) \]

The functions \( Q \) are normalized according to the condition

\[ \int_\Omega d\mu(\zeta) \equiv Q_0 + \sum_{s=1}^{+\infty} \sum_{\{k_1, \sigma_1\}} \cdots \int_{+T/2 > \tau_1 > \cdots > \tau_s > -T/2} Q_s((\tau_1, k_1, \sigma_1)) \prod_{i=1}^{s} d\tau_i = 1 \quad (14) \]

Each term in Exp. (13) corresponds to the probability of an elementary event \( \zeta_s \).

To complete the preliminaries it is also necessary to define the concept of a stochastic process. Mathematically speaking, it can be any integrable function \( \bar{A}(t) \equiv A(t, \zeta) \) defined at the space of all experimental outcomes \( \Omega \) and parametrically depending on time. It is sometimes convenient to omit the explicit \( \zeta \) dependence. We will use a "check" in this case to stress that the quantity in question is a random variable. Each stochastic process \( A(t, \zeta) \) generates the sequence of time dependent functions \( \{A_0(t), A_1(t, \tau_1, k_1, \sigma_1), \ldots, A_s(t, \tau_s, k_s, \sigma_s)\} \). Its average \( \langle \bar{A}(t) \rangle_\Omega \) over the space \( \Omega \) is defined as

\[ \langle \bar{A}(t) \rangle_\Omega = \int_\Omega A(t, \zeta) d\mu(\zeta) \equiv A_0(t) Q_0 + \sum_{s=1}^{+\infty} \sum_{\{k_1, \sigma_1\}} A_s(t, \tau_1, k_1, \sigma_1) \cdot Q_s((\tau_1, k_1, \sigma_1)) \prod_{i=1}^{s} d\tau_i \quad (15) \]

The analogous prescription should be used, for instance, to define the correlations \( \langle \bar{A}(t_1) \bar{B}(t_2) \rangle_\Omega \) between any two stochastic processes.

For the subsequent analysis we define the random process \( I^{(k)}(t) \), corresponding to the classical current through the external junction \( k \leq N \). It is given by the sequence of the shot \( \delta \)-pulses in time

\[ I_k^{(k)}(t, \zeta_s) = \sum_{i=1}^{s} e\sigma_i \delta(t - \tau_i) \delta(k - k_i) \quad (16) \]

where \( m \) is included to take into account the direction of the jump and \( \delta(k - k_i) \equiv \delta_{k_i} \). It is the Kronecker \( \delta \) symbol. Given this definiton at hand, we introduce the generating functional \( S[\{\chi_i(t)\}] \) depending on \( N \) counting fields \( \chi_i(\tau) \) of each of them associating with a given terminal \( i \):

\[ \exp(-S[\{\chi_i(t)\}]) = \left\langle \exp \left\{ \sum_{n=1}^{+\infty} \sum_{\tau} d\tau \chi_{\alpha}(\tau) \bar{I}^{(n)}(\tau)/e \right\} \right\rangle \quad (17) \]

The above average is assumed in the sense of definitions (13) and (15). We will refer to \( S[\{\chi_i(t)\}] \) as the action. Its evaluation is the main goal of this section.

The \( m \)-order functional derivatives of \( S[\{\chi_i(t)\}] \) with respect to \( \chi_i(t) \) give the irreducible \( m \)-order current correlations. First derivatives correspond to the average currents through terminals; the second derivatives give the shot noise and noise correlations. In the low-frequency limit of current correlations one may use the time-independent counting fields \( \chi_i \). In this case the action \( S[\{\chi_i(t)\}] \) allows to express the probability of \( N \) electrons to be transferred through the terminal \( i \) during the
The above definitions were rather general, but non-constructive, since the probabilities $Q$ have not been specified so far. One has to relate them with transition rates of the master equation. We will show below how this can be achieved with use of Markov property of the system. Our approach will have much in common with the path integral method in statistical physics. The advantage of such scheme is that it will readily give an explicit algorithm for calculation the generating function (13).

To proceed with the construction of measure (13) we assume that at initial time $t = T/2$ the system was in the state $\{n^{(i)}\}$. Starting from this state it is readily to reconstruct the subsequent time evolution of the charge configuration $\{n^{(i)}\} \rightarrow \{n^{(i+1)}\}$ for any given outcome $\zeta_s$. (See Fig.3) The choice of $\zeta_s$ specifies that the transition between neighbouring charge states $\{n^{(i-1)}\}$ and $\{n^{(i)}\}$ happens through the junction $k_i = (\alpha_i, \beta_i)$. Therefore the sequence $\{n^{(i)}\}$ is given by the relation $n^{(i-1)} = n^{(i)} - \sigma_{k_i}$, $n^{(i)} = n^{(i)} + \sigma_{k_i}$, and $\sigma_{k_i} = \sigma_i$ for all $\gamma \neq \alpha_i$ and $\beta_i$. To write down the probability $Q_s(\{\tau_i, k_i, \sigma_i\})$ we assume that our event $\zeta_s$ constitutes the Markov chain and rely on two facts: (i) the conditional probability of the system to survive at state $n^{(i)}$ between the times $\tau_{i+1}$ and $\tau_i$ is proportional to $\exp[-\gamma(n^{(i)})(\tau_i - \tau_{i+1})]$; (ii) the probability that the transition occurs through the junction $k_i$ during the time interval $d\tau_i$ at the moment $\tau_i$ is given by $\Gamma^{(\gamma)}_{k_i}(n^{(i)})d\tau_i$. These arguments suggest that $Q_s$ have the form

$$Q_s = Z_0^{-1} \exp[-\gamma(n^{(s)})T] \exp[-\gamma(n^{(0)})(T/2 - \tau_i)] \Gamma^{(\gamma)}_{k_i}(n^{(i)}) \exp[-\gamma(n^{(i)})(\tau_i - \tau_{i+1})] \Gamma^{(\gamma)}_{k_{i+1}}(n^{(i+1)}) \exp[-\gamma(n^{(i+1)})(\tau_{i+1} - \tau_{i+2})] \Gamma^{(\gamma)}_{k_{i+2}}(n^{(i+2)}) \ldots \exp[-\gamma(n^{(s-1)})(\tau_{s-1} - \tau_s)] \Gamma^{(\gamma)}_{k_s}(n^{(s)}) \exp[-\gamma(n^{(s)})(\tau_s + T/2)]$$

where the constant $Z_0$ should be found from the normalization condition (14). As we will see below, $Z_0 = 1$. In Appendix A we show how the usual description of system dynamics in terms of master equation can be established on the basis of probabilities (13).

The above correspondence between the random Markov chain $\zeta_s$ and the probabilities $Q_s$ gives the key to evaluate the generating function (17). By definition (16) for any given $\zeta_s$ we have

$$\exp\left\{\sum_{n=1}^{N} \int_{-\infty}^{\infty} d\tau \chi_n(\tau) \left(P^{(n)}(\tau, \zeta_s)/\rho\right)\right\} = \prod_{i=1}^{N} \exp\{i\sigma_i \chi_{k_i}(\tau_i)\}$$

It is assumed here that $\chi_{k_i} = 0$ if the transition happens through internal junction $k_i > N$ whereas no physically measurable current is generated in this case. On averaging the latter expression over all possible configurations $\Omega$ with the weight $d\mu(\zeta)$ we see that the generating function takes the structure of normalization condition (14)

$$Z(\{\chi(\tau)\}) = \exp(-S(\{\chi(\tau)\})) = Q_0 + \sum_{n=1}^{\infty} \sum_{\{k_i, \sigma_i\}} \int \cdots \int Q_s^{\sigma_i}(\{\tau_i, k_i, \sigma_i\}) \prod_{i=1}^{N} d\tau_i$$

Here the $\chi$-dependent functions $Q_{s}(\{\tau_i, k_i, \sigma_i\})$ are defined similar to probabilities (19) with the only difference that the rates $\Gamma^{(\gamma)}_{k_i}$(n) should be substituted by $\Gamma^{(\sigma)}_{k_i}(\tau_i)$ if $k \leq N$.

The expression (23) can be written in the more compact and elegant way. For that we introduce the $\chi$-dependent linear operator $\hat{L}_{\chi}$ defined as

$$\hat{L}_{\chi}(\tau) = \hat{\chi}(\tau)$$

$$\hat{\chi}(\tau) = \sum_{k=1}^{N} \left(\hat{\Gamma}^{(+)}_{k_i} e^{i\chi_k(\tau)} + \hat{\Gamma}^{(-)}_{k_i} e^{-i\chi_k(\tau)}\right)$$

Following the above consideration we have attributed the $\chi$-dependent factor to each operator $\hat{\Gamma}^{(\sigma)}_{k_i}$ $(k = 1 \ldots N)$, corresponding to the transition through the external junction. The diagonal part and internal transition operators $\hat{\Gamma}^{(\sigma)}_{k_i}$ with $k > N$ remained unchanged. Then we consider the evolution operator $\hat{U}_1(t_1, t_2)$ associated with $\hat{L}_{\chi}(\tau)$. Since $\hat{L}_{\chi}(\tau)$ is in general time-dependent, $\hat{U}_1(t_1, t_2)$ is given by the time-ordered exponent

$$\hat{U}_1(t_1, t_2) = T_{\tau} \exp\left\{-\int_{t_1}^{t_2} (\hat{\gamma}(\tau) - \hat{\Gamma}_{\chi}(\tau)) d\tau\right\}$$

The similar construction is widely used in quantum statistics. The difference in the present case is that at $\chi = 0$ the operator $\hat{U}_1(t_1, t_2)$ gives the evolution of probability rather than the amplitude of probability.

With the use of evolution operator (24) the generating function (20) can be cast into the form

$$Z(\{\chi(\tau)\}) = \langle q_0 | \hat{U}_1(T/2, -T/2) | n_s \rangle$$

To prove it we will argue in the following way. We explore the fact, that $\hat{\gamma}(\tau)$ and $\hat{\Gamma}_{\chi}(\tau)$ commute under the time-ordered operator in Eq. (24) and regard $\hat{\Gamma}_{\chi}(\tau)$ as a perturbation. This gives the matrix element $\langle q_0 | \hat{U}_1(T/2, -T/2) | n_s \rangle$ in the form of series

$$\sum_{s=1}^{\infty} \langle q_0 | T_{\tau} \exp\left\{-\int_{t_1}^{t_2} (\hat{\gamma}(\tau) d\tau\right\} \sum_{k_i, \sigma_i} \hat{\Gamma}^{(\sigma_i)}_{k_i}(\tau_1) e^{i\sigma_i \chi_{k_1}(\tau_1)} \cdots \hat{\Gamma}^{(\sigma_s)}_{k_s}(\tau_s) e^{i\sigma_s \chi_{k_s}(\tau_s)} | p_0 \rangle \prod_{i=1}^{N} d\tau_i$$

where $\chi_{k_i} = 0$ if the transition happens through internal junction $k_i > N$ whereas no physically measurable current is generated in this case.
It follows from the definition (19) that each term in this series corresponds to the function \(Q_\tau^0(\{\tau_i, k_i, \sigma_i\})\), namely

\[
Q_0 = \langle q_0 | e^{-\gamma T} | n_s \rangle \\
Q_\tau^\nu(\{\tau_i, k_i, \sigma_i\}) = \langle q_0 | T_\tau \exp\left\{-\int_{-T/2}^{T/2} \hat{\gamma}(\tau) d\tau\right\} | n_s \rangle
\]

Therefore \(\text{Exp} \[24\] \) and \(\text{Exp} \[25\] \) are reduced to the previous result \(\text{Exp} \[20\] \). This finishes the proof. Note, that owing to the property \(\text{Exp} \[8\] \) one has the identity \(Z_0 = \langle q_0 | \exp(-T\hat{L}) | n_s \rangle = 1 \) at \(\chi = 0\). Therefore the probabilities \(\text{Exp} \[19\] \) are correctly normalized.

The way \(\text{Exp} \[23\] \) the generating function \(Z(\{\chi_i(t)\})\) is written depends on the initial state \(|n_s\rangle\) of the system. It looks artificial and we may show that the choice of \(|n_s\rangle\) does not affect the final results. It is naturally to assume that \(\chi_k(t) \to 0\) when \(t \to -\infty\). i.e. physically speaking, the measurement is limited in time. To be specific one may suppose that \(\chi_k(t) = 0\) when \(-T/2 < t < -T/2 + \Delta t\). If the time interval \(\Delta t\) is sufficiently large as compared with the typical transition time \(\Gamma^{-1}\) then the system will reach the steady state during this period of time. It follows directly from the fact that \(\exp(-\hat{L}\Delta t)|n_s\rangle \to |p_0\rangle\) when \(\Delta t \gg \Gamma^{-1}\). Thus one can substitute \(|n_s\rangle\) to \(|p_0\rangle\) in \(\text{Exp} \[23\] \). Assuming also the limit \(T \to \infty\), we arrive to the main result of this section

\[
\exp(-S(\{\chi_i(t)\})) = \langle q_0 \| T_\tau \exp\left\{-\int_{-\infty}^{\infty} \hat{L}_\chi(\tau) d\tau\right\} |p_0\rangle
\]

We see that the generating function can be written in the form of the averaged evolution operator which corresponds to the modified operator \(\hat{L}_\chi\), depending on the counting fields \(\chi_i(\tau)\).

In the rest of the paper we will deal only with the low frequency limit of the current correlations, \(\omega \ll \Gamma\), and concentrate on the particle statistics \(\text{Exp} \[18\] \). We are interested in the probability \(P(\{N_i\})\) of \(N_i\) electrons to be transferred through the corresponding terminal during a large time interval \(t_0 \gg \Gamma^{-1}\). In this case one can put \(\chi_k(t) = \chi_k\) when \(0 \leq t \leq t_0\) and \(\chi_k(t) = 0\) otherwise. The action \(\text{Exp} \[26\] \) then reduces to the

\[
S(\{\chi_i\}) = t_0 \Lambda_{\text{min}}(\{\chi_i\})
\]

where \(\Lambda_{\text{min}}(\{\chi_i\})\) is a minimal eigenvalue of the operator \(\hat{L}_\chi\). Thus the problem of statistics in question, provided the transition rates in the system are known, is merely a problem of the linear algebra.

In the rest of this section we consider the question of the current conservation in the nodes. For this purpose we associate the counting fields \(\chi_i\) with each line \(k = \{\alpha, \beta\}\) of the graph and in the appropriate way modify the \(\hat{L}_\chi\) operator. Then we define the classical current operator \(\hat{j}_\chi^{(k)} = \hat{j}_\chi^{(\alpha, \beta)}\) through each line by means of the relation

\[
\hat{j}_\chi^{(k)} = ie \frac{\partial \hat{L}_\chi}{\partial \chi_k} \equiv e \left(\hat{\Gamma}^{(k)+} e^{i\chi_k} - \hat{\Gamma}^{(k)-} e^{-i\chi_k}\right)
\]

Its average value can be found via relation

\[
I_k(\{\chi_i\}) = \frac{ie}{t_0} \frac{\partial S}{\partial \chi_k} = \langle q_0^{(0)} | \hat{j}_\chi^{(k)} | p_0^{(0)}\rangle
\]

It follows from the Eq. (27) and the fact that \(A_{\text{min}}(\{\chi_i\}) = \langle q_0^{(0)} | \hat{L}_\chi | p_0^{(0)}\rangle\) with \(|q_0^{(0)}\rangle\) and \(|p_0^{(0)}\rangle\) being the eigenvectors of the \(\hat{L}_\chi\) operator.

The average physical currents for \(k \leq N\) are given by \(I_k = I_k(\{\chi_i\})|_{\chi=0}\). Expanding the vector notation in \(\text{Exp} \[23\] \) one gets the usual relation for the current in the master equation method. The current is expressed via transition rates and the steady probability distribution \(p_0(\{n_i\})\).

We also introduce the particle number operator \(\hat{n}_\chi^{(\alpha)}\) in each node, given by a usual formula

\[
\hat{n}_\chi^{(\alpha)} = \sum_{\{n\}} |n\rangle n_\alpha (n)
\]

Then after few algebra one see that the relation

\[
\sum_\beta \pm \hat{j}_\chi^{(\alpha, \beta)} = -e \left[ \hat{n}_\chi^{(\alpha)}, \hat{L}_\chi \right]
\]

always holds at any node \(\alpha\). Here the summation is going over all nodes \(\beta\), connected to \(\alpha\). The choice of the sign in front of each term under the sum depends on the situation whether the given directed line \(k = \{\alpha, \beta\}\) is going out or coming into the chosen node \(\alpha\). The Eq. (31) gives the charge conservation law in the operator language. On averaging the latter expression over the steady distribution, \(|q_0\rangle \ldots |p_0\rangle\), and using \(\text{Exp} \[18\] \) and \(\text{Exp} \[23\] \) we arrive at the conservation law for the \(\chi\)-dependent currents at each node \(\alpha\)

\[
\sum_\beta \pm I_\chi^{(\alpha, \beta)}(\{\chi_i\}) = 0
\]

This also ensures the conservation of the physical current in the model \(\sum_k I_k = 0\), where the sum is extended only to the external junctions \(k\). It follows from summing up the relations \(\text{Exp} \[32\] \) over all internal nodes \(\alpha\) and setting \(\chi = 0\) afterwards.

**IV. RESONANT LEVEL MODEL**

In this section we consider the current statistics of the resonant level model. First we focus on the non-interacting case. Then we apply the general result of the section III to the strongly interacting case and compare the statistics in these two different regimes. In the end of the section we re-derive the results of the previous works concerning the shot noise in these systems.
Following the definition (21) and the expression for the rates (3), the $\hat{L}_\chi$-matrix of the single resonant level model in the non-interacting regime reads as

$$\hat{L}_\chi = \begin{pmatrix} \Gamma_{1\rightarrow 0} & -\Gamma_{0\rightarrow 1}(\chi) \\ -\Gamma_{1\rightarrow 0}(\chi) & \Gamma_{0\rightarrow 1} \end{pmatrix}$$  (33)

where

$$\Gamma_{1\rightarrow 0}(\chi) = \Gamma_L f_L e^{-i\chi_1} + \Gamma_R f_R e^{-i\chi_2}$$
$$\Gamma_{0\rightarrow 1}(\chi) = \Gamma_L (1 - f_L) e^{i\chi_1} + \Gamma_R (1 - f_R) e^{i\chi_2}$$  (34)

Calculating the minimal eigenvalue of this matrix one obtains the current statistics in the form

$$S(\chi) = \frac{\Gamma_L + \Gamma_R - \sqrt{\mathcal{D}(\chi)}}{\pi}$$  (35)

$$\mathcal{D}(\chi) = (\Gamma_L + \Gamma_R)^2 + 4\Gamma_L \Gamma_R \times \left[ f_{(\rightarrow)}(\epsilon_i)(e^{-i\chi} - 1) + f_{(\rightarrow)}(\epsilon_i)(e^{i\chi} - 1) \right]$$

Here $f_{(\rightarrow)}(\epsilon_i) = f_L(\epsilon_i)[1 - f_R(\epsilon_i)]$, $f_{(\rightarrow)}(\epsilon_i) = f_R(\epsilon_i)[1 - f_L(\epsilon_i)]$ and $\chi = \chi_1 - \chi_2$. We have also accounted for the double occupancy of the level by multiplying the result by two.

Since the Coulomb blockade phenomenon is completely disregarded in this model, one might have come to the same result in the framework of the pioneering approach by Levitov and co-workers. We will show now that it is indeed the case.

Following Ref. the general expression for the current statistics through a single contact is written as

$$S(\chi) = -\frac{t_0}{\pi} \sum_n \int d\epsilon \ln \left\{ 1 + T_n(\epsilon) \times \left[ f_L(\epsilon)(1 - f_R(\epsilon))(e^{-i\chi} - 1) + f_R(\epsilon)(1 - f_L(\epsilon))(e^{i\chi} - 1) \right] \right\}$$  (36)

It is valid for any two-terminal geometry provided the region in between two electrodes can be described by the one-particle scattering approach and the effects of interaction are of no importance. $T_n(\epsilon)$ is a set of transmission eigenvalues which are in general energy-dependent. Fermi functions include the effects of applied voltage and the temperature. In the particular case of a single resonant level the only resonant transmission eigen-channel $T_r(\epsilon)$ plays the essential role, its energy dependence is being given by the Breit-Wigner formula

$$T_r(\epsilon) = \frac{\Gamma_L \Gamma_R}{(\epsilon - \epsilon_i)^2 + (\Gamma_L + \Gamma_R)^2/4}$$  (37)

Here $\epsilon_i$ denotes a position of a resonant level. The result (36) is more general, than Exp. (35). When electrons are supposed to be non-interacting the former is valid for any temperature. We will show below that one can reproduce the statistics (35) on substituting $T_r(\epsilon)$ into the Exp. (34) and assuming the regime $k_B T \gg \hbar \Gamma$. As it was discussed previously, this is the condition, when the master equation approach, and hence its consequence (35), are valid.

It is easier to perform the calculation if one first evaluates the $\chi$-dependent current $I(\chi) = (ie/t_0)\partial S/\partial \chi$. It reads

$$I(\chi) = \frac{1}{\pi} \int d\epsilon \left[ f_{(\rightarrow)}(\epsilon)e^{i\chi} - f_{(\rightarrow)}(\epsilon)e^{-i\chi} \right] \times$$
$$\left\{ T_r^{-1}(\epsilon) + \left[ f_{(\rightarrow)}(\epsilon)(e^{-i\chi} - 1) + f_{(\rightarrow)}(\epsilon)(e^{i\chi} - 1) \right] \right\}$$  (38)

In what follows we assume that the resonant level is placed between the chemical potentials $\mu_L(\Omega)$ in the leads. Since $k_B T \gg \Gamma_L(\Omega)$, the main contribution comes from the Lorentz peak and one can put $\epsilon = \epsilon_i$ in the Fermi functions. Therefore we left only with the two poles $\epsilon_{1(2)} = \epsilon_i \pm i \sqrt{\mathcal{D}(\chi)/2}$ under the integrand (38).

Closing the integration contour in the upper or lower half-plane we arrive at

$$I(\chi) = 2e \Gamma_L \Gamma_R \left[ f_{(\rightarrow)}(\epsilon_i)e^{i\chi} - f_{(\rightarrow)}(\epsilon_i)e^{-i\chi} \right] / \sqrt{\mathcal{D}(\chi)}$$

On integrating it over $\chi$ one finds for the $S(\chi) = (t_0/ie)\int_0^\chi I(\chi')d\chi'$ the result (35) obtained by means of master equation. Thus, we have verified the correspondence between two approaches to statistics in the non-interacting regime.

To proceed we address the strongly interacting regime. In this case the $(2 \times 2)$ $\hat{L}_\chi$-matrix is formed with the use of rates (3). It has a structure similar to Eq. (33), provided the $\chi$-dependent rates are written as

$$\Gamma_{1\rightarrow 0}(\chi) = 2\Gamma_L f_L e^{-i\chi_1} + 2\Gamma_R f_R e^{-i\chi_2}$$
$$\Gamma_{0\rightarrow 1}(\chi) = \Gamma_L (1 - f_L) e^{i\chi_1} + \Gamma_R (1 - f_R) e^{i\chi_2}$$  (39)

On evaluating the corresponding eigenvalue $\hat{L}_\chi$ one can write down the expression for statistics in the strongly interacting limit

$$S(\chi) = (t_0/2) \left\{ \Gamma_L[1 + f_L(\epsilon_i)] + \Gamma_R[1 + f_R(\epsilon_i)] - \sqrt{\mathcal{D}(\chi)} \right\}$$
$$\mathcal{D}(\chi) = \left\{ \Gamma_L[1 + f_L(\epsilon_i)] + \Gamma_R[1 + f_R(\epsilon_i)] \right\}^2 + 8\Gamma_L \Gamma_R \times \left[ f_{(\rightarrow)}(\epsilon_i)(e^{i\chi} - 1) + f_{(\rightarrow)}(\epsilon_i)(e^{-i\chi} - 1) \right]$$  (40)

To proceed we consider the shot noise regime $eV \gg k_B T$ and assume that the voltage is applied to the right electrode as shown in Fig.2. Then the temperature fluctuations become non-essential and both statistics (35) and (40) take a rather simple form

$$S(\chi)|_{U=0} = t_0 \left\{ \Gamma_L + \Gamma_R \right. - \sqrt{(\Gamma_R - \Gamma_L)^2 + 4\Gamma_L \Gamma_R e^{-i\chi}} \right\}$$  (41)
$$S(\chi)|_{U\rightarrow \infty} = \frac{1}{2t_0} \left\{ 2\Gamma_L + \Gamma_R - \sqrt{(2\Gamma_L - \Gamma_R)^2 + 8\Gamma_L \Gamma_R e^{-i\chi}} \right\}$$  (42)

Given the latter expressions at hand one can easily re-derive the known result for the average current
Poisson statistics

Then one has to find the saddle point by means of steepest descent method. It is applicable we consider, since both the action $P$ one in the non-interacting case is an effective suppression $\Omega(\chi)$.

FIG. 4: The current statistics through the single resonant level. 1 and 4 - non-interacting model; 2 - interacting model; 4 - Poisson statistics

$I = (ie/t_0)\partial S/\partial \chi|_{\chi=0}$ and the shot noise power $S_{\text{shot}} = (e^2/t_0)\partial^2 S/\partial \chi^2|_{\chi=0}$ in these models. It is conventional to represent $S_{\text{shot}}$ in the form $S_{\text{shot}} = eIF$, where $F$ is the so-called Fano factor. Then one obtains $F = (\Gamma_F^L + \Gamma_F^R)/\Gamma_L + \Gamma_R)^2$ in the non-interacting regime and $F = (\Gamma_F^L + 4\Gamma_F^R)^2/\Gamma_L + 2\Gamma_R)^2$ in the Coulomb blockade limit.

As one can see from the Eq. (41), at low temperatures, the difference of statistics in the large $U$ limit from the one in the non-interacting case is an effective suppression of $\Gamma_R$ rate by a factor of two. To find the probability distribution $P(N, t_0)$ one can estimate the integral by means of steepest descent method. It is applicable in the given case of low frequency regime $\omega \ll \Gamma$, which we consider, since both the action $S(\chi) \gg 1$ and the average number of transmitted electrons $\bar{N} = \bar{t}_0/e \gg 1$.

Then one has to find the saddle point $\chi_\ast$ of the function $\Omega(\chi) = S(\chi) + i\chi \bar{t}_0/e$, which is defined by the equation $I = (ie/t_0)\partial S/\partial \chi$. It is turned out that $\chi_\ast$ always lies on the imaginary axis. This equation can be regarded as a parametric relation between $I$ and $\chi_\ast$, and with the exponential accuracy we obtain the estimation for the probability $P(I) \sim \exp[-\Omega(\chi_\ast)]$.

The results for the statistics are shown in Fig. 4. The statistics are compared with the Poisson type statistics $S(\chi) = 2\Gamma_0\exp(i\chi) - 1$ with the effective rate given by $\Gamma^{-1} = \Gamma_L^{-1} + \Gamma_R^{-1}$. Both statistics approach the Poisson one, provided the system is strongly asymmetric, $\Gamma_L \gg \Gamma_R$.

V. THE FCS IN THE COULOMB BLOCKADE QUANTUM DOTS.

In this section we discuss the application of the method to the many-terminal Coulomb blockade quantum dot. The consideration will be limited to the two- and three-terminal layouts. Our treatment will be mainly numerical, though some analytical results in the two-terminal setup are also plausible. In the beginning of the section a few technical details, which are common for both cases, are given. In particular, we establish the relation of the FCS approach with the preceding papers, concerning the shot noise in the conventional two-terminal Coulomb blockade dots. In the following we consider the FCS, first for two-terminal, and then for three-terminal configurations. We will also compare the FCS in the strongly Coulomb blockade limit with our recent results, concerning the FCS in many-terminal chaotic quantum dot with contacts being the tunnel junctions.

A. General remarks

In case of Coulomb blockade island the macroscopic state of the system is characterized by the excess charge $Q = ne$, which is quantized in terms of electron charge ($-e$). As we have pointed out in the section II, within the “orthodox theory”, the charge $Q$ can be changed only by $\pm e$ in course of one tunneling event. Therefore the master equation connects the given macroscopic state $n$ only with the neighboring states $n\pm1$. The corresponding rates $\Gamma_{n\pm1\rightarrow n}$ of these transitions are equal to the sum of $N = 2$ or $3$ independent probabilities $\Gamma_{n\pm1\rightarrow n}^{(k)}$ through the junctions, those are given by $\Gamma_{n\pm1\rightarrow n}$ and $\Gamma_{n\pm1\rightarrow n}^{(k)}$. Along with the lines of section III, in order to find the FCS of the charge transfer through the island, we have to modify the rates $\Gamma_{n\pm1\rightarrow n}$ into $\chi$-dependent quantities $\Gamma_{n\pm1\rightarrow n}^{(k)}(\chi)$, in accordance with the rule (21) and to evaluate the minimal eigenvalue $\Lambda_{\text{min}}$ of the three-diagonal matrix $\hat{L}\chi$ afterwards. In the given case it is convenient to write down the latter problem as the eigenvalue problem for the following set of linear equations

$$ (\Lambda - \gamma_n)p_n + \Gamma_{n\pm1\rightarrow n+1}^\chi p_{n+1} + \Gamma_{n\pm1\rightarrow n-1}^\chi p_{n-1} = 0 \quad (43) $$

where $\gamma_n = \Gamma_{n\rightarrow n-1} + \Gamma_{n\rightarrow n+1}$, and $\Gamma_{n\rightarrow n\pm1}^{(k)} = \sum_{k=1}^{N}\Gamma_{n\pm1\rightarrow n}^{(k)}e^{\pm ik\chi}$. Sign index denotes the outcome (income) of an electron from (to) the island.

In general, we have treated the problem numerically. At sufficiently low temperatures $k_B T \ll E_c$, which is mainly the case of the following discussion, the temperature dependence in rates is non-essential. Then one can set $\Gamma_{n\pm1\rightarrow n}^{(k)}(\chi) = \Delta E_{n\pm1\rightarrow n}(e^2R_k)$ when $\Delta E_{n\pm1\rightarrow n}(e^2R_k) > 0$ and $\Gamma_{n\pm1\rightarrow n}^{(k)}(\chi) = 0$ otherwise. Thus defined rates are linear functions in $n$. The possible set of $\{n\}$, corresponding to nonvanishing rates, is limited to some interval $n_{\text{min}} \leq n \leq n_{\text{max}}$. Hence Eq. (43) becomes a finite linear problem. At higher temperatures
\[ k_B T \leq E_c \] we have found that the increase (decrease) of both \( n_{\text{min}} \) and \( n_{\text{max}} \) to extra 7 \( \div \) 8 states gives the results up to \( 10^{-15} \) degree of accuracy in course of the numerical procedure.

The matrix \( \hat{L}_\chi \) of Eq. (13) is non-Hermitian. This fact may cause an instability in the numerical algorithm when the range \([n_{\text{min}}, n_{\text{max}}]\) is large. However, in most practical cases this problem can be circumvented by transforming \( \hat{L}_\chi \) to Hermitian form. First we note, that one only needs to work with pure imaginary counting fields \( \chi_k \), as long as the probability \( P(\{N_i\}, t_0) \) is estimated in the saddle point approximation. (See the discussion in the end of the section III.) Hence the rates \( \Gamma_n^{\pm} \) become the positive real numbers. Then we can apply the linear transformation \( p'_n = A_n p_n \). It leads to the rates in the new gauge \( \Gamma_n^{\pm} = A \Gamma_n^{\pm} A^{-1} \). The unknown \( A_n \)'s may be chosen in a way that the symmetry relation \( \Gamma_n^{\pm} = \Gamma_n^{\mp} \) would hold. This gives the recurrent relation \( A_{n+1}/A_n = (\Gamma_{n+1}^{\pm}/\Gamma_{n-1}^{\pm})^{1/2} \). With the use of latter the Eq. (13) takes the Hermitian form when it is written in terms of \( p'_n \), and transformed rates \( \Gamma_n^{\pm} = (\Gamma_{n+1}^{\pm} \Gamma_{n-1}^{\pm})^{1/2} \). The diagonal term \( \gamma_n \) is not affected under this transformation.

Let us also discuss a useful relation for the shot noise correlations \( S_{km} = (e^2/t_0) \partial^2 S/\partial \chi_k \partial \chi_m \big|_{\chi=0} \). One can use the identity \( \Lambda_{\text{min}}(\{\chi_n\}) = \langle \hat{J}_0 | \hat{J}_0 \rangle \) in order to express them in terms of eigenvectors \( |p_n\rangle, \langle q_n| \) and eigenvalues \( \Lambda_n \) of the matrix \( \hat{L} \). With the use of standard algebra and assuming the normalization \( \langle q_n|p_n'\rangle = \delta_{n,n'} \) we can write \( S_{km} \) in the form

\[ S_{km} = e^2 \langle q_0 | \hat{S}^{(k,m)} | p_0 \rangle + e^2 \sum_{l>0} 1/\lambda_l \left( \langle q_0 | \hat{J}_l | p_0 \rangle \langle q_l | \hat{J}_l | p_0 \rangle + \langle q_0 | \hat{J}_m | p_0 \rangle \langle q_l | \hat{J}_l | p_0 \rangle \right) \]

where the \( \hat{J}_k \) operator was defined by (18) and \( \hat{S}^{(k,m)} = \partial^2 \hat{L}_\chi/\partial \chi_k \partial \chi_m \big|_{\chi=0} \). Note, that the relation (14) holds in any basis. One may, in particular, use it for the basis \( p'_n \) discussed above. In this case one must define the matrix elements of \( \hat{J}_k \) as \( [\hat{J}_k]_{n+1,n} = (i \partial/\partial \chi_k) \Gamma_{n+1}^{\pm} \) and to evaluate \( \hat{S}^{(k,m)} \) in the same manner.

The relation (14) represents the fact, that the shot noise correlations are defined by the whole spectrum of the relaxation times \( \tau_{k-1} = \lambda_k \) in the system. In the case of two-terminal geometry it coincides with preceding results of Ref. 13.

### B. Two-terminal Coulomb blockade island.

The electrical circuit with a two-terminal Coulomb blockade island is shown in Fig. 5. The dot is biased such a way, that \( V_2 = -V_1 = V/2 \). At low temperatures

\[ k_B T \ll e^2/C_\Sigma \] the \( \chi \)-dependent rates \( \Gamma^{\pm} \) reads as

\[ \Gamma_{n+1}^{\pm} = \left( \frac{\tilde{C}_1 V}{e} - (n + \frac{C_2 V_g}{e} + \frac{1}{2}) \right) e^{i\chi_2} \frac{1}{R_2 C_\Sigma} \]

\[ \Gamma_{n-1}^{\pm} = \left( \frac{\tilde{C}_1 V}{e} + (n + \frac{C_2 V_0}{e} - \frac{1}{2}) \right) e^{-i\chi_1} \frac{1}{R_1 C_\Sigma} \]

where \( \tilde{C}_1(2) = C_1(2) + C_2/2 \) are effective capacitances. The gate voltage \( V_g \) can be used to control the offset charge \( q_0 = C_g V_g \) on the dot. It can be varied continuously in the range \(-e/2 \leq q_0 \leq e/2\). The resulting dimension of the matrix \( \hat{L}_\chi \) is given by the number of absorbing states \( n_{\text{max}} - n_{\text{min}} \), where \( n_{\text{min}} \) (\( n_{\text{max}} \)) is the maximal (minimal) integer closest to the points \( n_{1,2} \) where the rates \( \Gamma_{n+1}^{\pm} \) vanish.

First we briefly consider the voltage dependence of the shot noise in the system. It was calculated with the

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**FIG. 5:** The equivalent circuit of the two-terminal Coulomb blockade island.

**FIG. 6:** The Fano factor versus the offset charge in the 2 - terminal Coulomb blockade dot. The parameters are: \( C_1 = C_2, R_1 = 10 R_2, T = 0.01 e^2/2C_\Sigma; \) (1) - \( V = 1.5 e/C_\Sigma \), (2) - \( V = 2.0 e/C_\Sigma \), (3) - \( V = 4.0 e/C_\Sigma \), (4) - \( V = 6.0 e/C_\Sigma \).
The results for the noise-to-current ratio (Fano factor) are presented in Figs. 6-8. The Coulomb blockade features are strongly pronounced in case of asymmetric junctions only. In the experimental situation, when the dot is made up with the use of 2D-electron gas in the semiconducting heterostructure, the resistances of the contacts are much easier to vary than the mutual capacitances.

Therefore we have chosen $C_1 = C_2$ and plotted the Fano factor for different values of ratio $R_2/R_1$ and offset charge $q_0$. The curves are truncated below the Coulomb blockade threshold, where the considered "orthodox" theory is not applicable. At high values of the ratio $R_2/R_1$ they exhibit the strong characteristic Coulomb blockade oscillations. The special points at the voltage dependences occur when either $n_{\text{min}}$ or $n_{\text{max}}$ are changed by 1. At high bias voltages the noise-to-current ratio saturates to the value $F = (R_1^2 + R_2^2)/(R_1 + R_2)^2$ independently of the capacitances $C_k$ and the offset charge $q_0$. (See the discussion below as well.) An increase of a temperature leads to the smearing of oscillations due to the additional thermal noise. The above results coincide with those, obtained previously by Hershfield et al.

To proceed we turn to the question of the FCS. For the sake of clarity we first present our recent analytical results for the FCS of the chaotic quantum dot with two tunnel junctions, when their resistances $R_k \ll \pi \hbar/e^2$. In this case the effects of Coulomb interaction are negligible. Then we trace the differences in the FCS, when the dot is placed in the strongly interacting regime $R_k \gg \pi \hbar/e^2$.

In the non-interacting limit the action $S(\chi)$ is expressed via the voltage $V$ and the resistances $R_k$ only. At low temperatures $k_B T \ll eV$ it has a form similar to \cite{11}

$$S(\chi) = \frac{Vt_0}{2e} \left\{ \frac{1}{R_1} + \frac{1}{R_2} - \frac{\sqrt{(1/R_1 - 1/R_2)^2 + 4(1/R_1 R_2)^{-1} e^{(\chi_2 - \chi_1)}}}{2} \right\}$$

It would be completely equivalent to the charge transfer by non-interacting particles through the resonant level if one regards the ratios $\Gamma_{L,R} = V/(eR_{1,2})$ as the effective tunneling rates. The generating function \cite{16} gives the above mentioned value $F = (R_1^2 + R_2^2)/(R_1 + R_2)^2$ for the Fano factor.

In the strongly Coulomb blockade limit the action $S(\chi)$, in general, remarkably deviates from Exp. \cite{16}. Still, there are two exceptions, when $S(\chi)$ resembles the statistics \cite{11} and \cite{11}. The first case occurs at low voltages, slightly above the Coulomb blockade threshold value, when only one charging state is available for tunneling. This situation can be easily realized in the asymmetric dot with $R_2 \neq R_1$. Then mere two states with $n = 0$ and $n = 1$ are involved and $\hat{L}_\chi$ is reduced to the $2 \times 2$ matrix \cite{15}. The only difference is that the rates $\Gamma_{L,R}$ contain the voltage dependence as given by Exp. \cite{15}. Thus the action $S(\chi)$ reproduces the result \cite{11}, where the rates $\Gamma_{L,R}$ are assumed to be voltage dependent.

To proceed we describe the second exceptional situation when the action $S(\chi)$ can be found analytically. Let $n_{1,2}$ be zeros of rates \cite{15}, i.e. $\Gamma_{n\pm 1 \pm n}(n_{1,2}) = 0$. We now interested in the situation when both zeroes $n_{1,2}$ simultaneously become integers. This situation may occur at the limited number of special points $V_t$ at the Coulomb blockade staircase when the ratio $C_1/C_2$ is close to a rational along with the special choice of the offset charge $q_0$. (E.g., for the configuration $C_1 = C_2$, shown in Figs. 6,8,

FIG. 8: The Fano factor versus the applied voltage at different temperatures. Parameters are shown on the plot. Temperature is given in terms of charging energy. Figure 7: The Fano factor versus the applied voltage in the 2-terminal Coulomb blockade dot. The parameters are: $C_1 = C_2$, $k_BT \ll e^2/2C_\Sigma$, $q_0 = 0$; (1) - $R_1/R_2 = 1$, (2) - $R_1/R_2 = 2$, (3) - $R_1/R_2 = 4$, (4) - $R_1/R_2 = 10$. Figure 8: The Fano factor versus the applied voltage at different temperatures. Parameters are shown on the plot. Temperature is given in terms of charging energy.
this is the case when (i) $q_0 = 0$, $V_k = (2k + 1)e/C_S$ and (ii) $q_0 = \pm e/2$, $V_k = 2k(e/C_S)$ with $k$ being integer.) In this situation we may show analytically (see Appendix B for the proof), that the action $S(\chi)$ at points $V_k$ takes the form similar to the statistics in the non-interacting regime \([18]\). The only difference that the voltage $V$ has to be substituted to $\pm (V_k - e/C_S)$. The reduction of $V$ by the amount of the threshold voltage value $e/C_S$ is thus the manifestation of the Coulomb interaction. The given statistics is also valid as a limit at high voltages $V \gg e/C_S$. One may conclude it from the physically reasonable arguments that the result for the action in this limit should be linear function in voltage and must not depend on the capacitance ratio $C_1/C_2$. Hence, the statistics is insensitive to the fact, whether $n_{1,2}$ are integers or not. This also explains the saturation of the Fano factor at Figs. 6-8 to the non-interacting current-to-noise ratio.

To access the general situation, the whole problem has been treated numerically. We have evaluated the probability \([18]\) in the saddle point approximation along with the same lines as it was done for the resonant level model. The Eq. \((29)\) was used as the parametric relation between the current $I$ and the counting field $\chi = \chi_2 - \chi_1$ in the saddle point $\chi_s$. In Fig. 9 we give the example of the logarithm of probability distribution $P(I)$ for the number of different voltages $V$ and offset charge $q_0 = 0$. All curves are normalized by the reduced value of the voltage $(V - e/C_S)$. For the value $V = 3e/C_S$ the renormalized logarithm of probability coincides with the one, obtained from the non-interacting limit \([18]\). We have also plotted the same statistics in Fig. 4 in the resonant level model for the ratio of rates $\Gamma_L/\Gamma_R = 10$, when the interaction effects are disregarded. We see, that in general the probability distribution is strongly affected by the Coulomb blockade phenomenon, as compared to the non-interacting regime. It approaches to the non-interacting limit only at rather high voltages $V \gg e/C_S$.

C. Three-terminal Coulomb blockade island

The electrical circuit with a three-terminal Coulomb blockade island is presented in Fig. 10. It is biased by three external voltage sources so that the current, flowing through the third terminal, would split into the first and the second ones. The voltages $U_{1(2)}$ are used to control the bias between the 3d and the 1st (the 2nd) terminals: $U_{1(2)} = V_3 - V_{1(2)}$. The third terminal is biased at voltage $V_3 = (U_1 + U_2)/3$ with respect to the ground. Such type of setup assures the condition $V_1 + V_2 + V_3 = 0$. Hence the gate voltage, $V_g$, can be used as before to control the offset charge $q_0 = C_p V_g$ on the island. As in the previous subsection we discuss the only low temperature regime $k_B T \ll e^2/C_S$. In what follows it is assumed that $U_2 > U_1$. Then, according to Eq. \((44)\) and \((21)\) the $\chi$-dependent rates of the system are written as follows

$$
\Gamma_{n+1-n}^x = \Gamma_3^{(x)}(n)e^{ix_3} + \Gamma_1^{(+)}(n)\theta(m - 1/2 - n)e^{ix_1},
$$
$$
\Gamma_{n-1-n}^x = \Gamma_2^{(-)}(n)e^{-ix_2} + \Gamma_1^{(-)}(n)\theta(n - m - 1/2)e^{-ix_1},
$$

where

$$
\Gamma_k^{(+)(-)} = a_k^{(+)(-)} \pm \left(n + \frac{C_p V_g}{e} \pm \frac{1}{2} \right) \frac{1}{R_k C_S}, \quad (47)
$$

and

$$
a_3^{(+)} = \frac{C_1 U_1 + C_2 U_2}{e R_3 C_S}, \quad a_3^{(-)} = \frac{(C_1 + C_3) U_2 - C_1 U_1}{e R_2 C_S},
$$
$$
a_1^{(+)} = \pm \frac{C_2 U_2 - (C_3 + C_2) U_1}{e R_1 C_S}
$$

FIG. 9: The statistics of current in the two-terminal quantum dot. Parameters are shown on the plot. Curves 3 and 5 coincide but correspond to different axes. 1) $V = 1.5 e/C_S$, 2) $V = 2.0 e/C_S$, 3) $V = 3.0 e/C_S$, 4) $V = 4.0 e/C_S$, 5) non-interacting regime.

FIG. 10: The equivalent circuit of the three-terminal Coulomb blockade island.
The point \( m \) is determined by the relation \( \Gamma_1(-m + 1/2) = \Gamma_1^{(+)}(m - 1/2) = 0 \) (\( m \) is non-integer in general). The dimension of the \( \tilde{L}_\chi \)-matrix is equal to \( n_{\text{max}} - n_{\text{min}} \), where \( n_{\text{max}}(n_{\text{min}}) \) can be found from the conditions \( \Gamma_1(-n) \geq 0 \) \( (\Gamma_1^{(+)}(n) \geq 0) \). The effective capacitances \( C_k \) are defined as \( C_k = C_k + C_0/3 \).

We can see from the Exp. \( 47 \) that there are four elementary processes of charge transfer in the system at low temperatures, each of them is being associated with the pre-factor \( e^{\pm \chi_k} \). The presence of the exponents \( e^{\pm \chi_k} \) corresponds to the charge transfer from the third terminal into the island and from the island into the second terminal, respectively. Hence, the random current through the 3d (2nd) junctions always have the positive (negative) sign. Two factors \( e^{\pm \chi_k} \) stems from the charge transfer through the first junction in the direction either from the island into the first contact or vice versa. Therefore the random current \( I_1 \) is able to fluctuate in both directions.

Let us first consider the shot noise correlations in the system. For that it is useful to introduce the \((3 \times 3)\) matrix \( F \) with elements \( F_{km} = S_{km}/e^2 \), where the current correlations \( S_{km} \) are given by \( 14 \) and \( S_2 = \sum_{i=1}^{3} |I_i| \). The matrix \( F \) is a generalization of the Fano factor for the multiterminal system. It is symmetric and obeys the relation \( \sum_{i=1}^{3} F_{ik} = 0 \). It follows from the general law of the current conservation in the system. For the considered 3-terminal dot we have found that the cross-correlations \( F_{km} \) \( (k \neq m) \) are always negative for any set of parameters.

In Fig. 11 we give the illustrative example of the voltage dependence of the shot noise correlations \( F_{km} \) for the certain choice of parameters. (For the cross-correlations the modulus \( |F_{km}| \) are given.) As in the 2-terminal case the Coulomb blockade features are strongly pronounced only for the asymmetric setup. The results in Fig. 10 corresponds to \( R_1 = R_3 = R_2/10, \bar{C}_1 = \bar{C}_2 = \bar{C}_3 \) and \( U_2/U_1 = 4 \). The latter ratio of voltages has been chosen on the basis of arguments that for a given value of resistances \( R_k \) it would split the average current \( \bar{I}_3 \) into two equal currents \( |I_1| = |I_2| = \bar{I}_3/2 \) provided one could apply the usual linear Kirchhoff rules to this circuit. In Fig. 12 the dependence of the shot noise correlations on the offset charge is shown for the same set of parameters and the value of \( U_1 = 1.25 \epsilon/C_2 \). The special points of both these dependences occur when either \( n_{\text{min}}, n_{\text{max}} \) or the integer part of \( m \) are changed by \( \pm 1 \). As the result we observe multi-periodic Coulomb blockade oscillations in the offset charge dependences in contrast to the single periodic oscillations in the two-terminal case.

We now proceed with the consideration of the FCS. As before, the action \( S(\{|\chi_i|\}) \) has been calculated with the use of \( 27 \) and afterwards the probability \( 18 \) has been estimated by means of the steepest descent method. The difference with the two-terminal geometry is that three currents \( I_k \) and three counting fields \( \chi_k \) are now involved. Due to the current conservation \( \sum_k I_k = 0 \) for any plausible fluctuation, only two currents are independent and thus the action \( S(\{|\chi_i|\}) \) depends on the differences \( \chi_{ik} = \chi_i - \chi_k \) only. In what follows we have chosen \( I_1 \) and \( I_2 \) as the independent variables to plot the logarithm of probability \( \ln P(I_1, I_2) \). With the exponential accuracy it is given by \( \ln P(I_1, I_2) \sim e^{-\Omega(\chi')} \), where \( \chi' \) is a saddle point of the function \( \Omega(\chi) = S(\chi) + i\chi_1 I_1 t_0 / e + i\chi_2 I_2 t_0 / e \). The results for \( \ln P(I_1, I_2) \) are shown in Figs. 13, 14, and...
and $G_k = R_k^{-1}$ are the conductances of the junctions.

The logarithm of probability $\ln P_0(I_1, I_2)$, evaluated with the use of statistics (49), is shown by the dashed line in addition to the previous curves in Figs. 13, 14. Its contour map for the same values of parameters is also separately presented in Fig. 16. The maximum of $\ln P_0(I_1, I_2)$, as expected, occurs at $I_1 = I_2 = U_1/3R_1$.

The two main conclusion which one can derive on comparing the statistics in the two limiting regimes (strongly interacting in case of the Coulomb blockade quantum dot and the non-interacting one in the open chaotic quantum dot) are as follows. First, we see, that in spite of the different regimes, the qualitative dependence of probabilities versus the currents is similar for both statistics. Second, we may conclude, that the relative probability of the big current fluctuations in the Coulomb blockade limit is suppressed with respect to the situation in the non-interacting regime. This is a characteristic feature of both two- and three-terminal quantum dots. This behavior is rather natural. It stems from the fact, that any big current fluctuation in Coulomb blockade dot is related with the large accumulation (or depletion) of the charge on the island. The latter costs the extra electrostatic energy and hence the probability of such fluctuation is drastically decreased. The suppression of the average current due to the presence of the Coulomb gap in the interacting system is the particular example of this behavior.

![Plot](image-url)  
**FIG. 14:** The logarithm of current distribution $\ln P(I_1, I_2)$ in the 3-terminal quantum dot as a function of current $I_2$, under condition $I_1 = (I_2)$. Parameters are the same as in Fig. 13. (1) - $U_1 = 1.25 e/C_S$, (2) - $U_1 = 2.0 e/C_S$, (3) - $U_1 = 4.0 e/C_S$; curve (4) corresponds to the non-interacting regime.
level model we have established the equivalence of the different systems. For a generic case of a single resonant proportional to the minimal eigenvalue of the operator the charge transfer through the whole system is then processed. To take into account the electron jump through the junction $k$ during the tunneling event. The generating function of the charge transfer through the whole system is then proportional to the minimal eigenvalue of the operator $L_\chi$.

We have applied this scheme to study the FCS in two different systems. For a generic case of a single resonant level model we have established the equivalence of the new method with the scattering approach to the FCS, when the particles in the system are non-interacting.

Afterwards we have considered the FCS and the shot noise correlations in the two- and three-terminal Coulomb blockade quantum dots. The consideration was limited to the temperature regime when the orthodox theory of Coulomb blockade phenomenon is applicable. For the case of two-terminal dot we have re-established all the known results for the shot noise in this system. In the three-terminal case we have shown that the auto- and cross- shot noise correlations exhibit the characteristic Coulomb blockade oscillations as the functions of the applied voltages and the offset charge. We have considered the question of the FCS as well. In general situation we evaluated the probability distribution numerically. However, at some special values of parameters in two-terminal dot we have managed to find FCS analytically. In these exceptional cases the FCS resembles the statistics of the charge transfer through the single resonant level. Then we compared the statistics in the Coulomb blockade dots with our previous results concerning an open dot with two and three terminals. We found that the Coulomb interaction suppresses the relative probability of the big current fluctuations.

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### APPENDIX A

In this Appendix we show that the construction of the probability measure on the basis of Markov chains $\zeta$, which was used to derive the main result of section III, leads to the usual description of the system dynamics in terms of master equation. This correspondence is achieved in the standard way of probability theory by introducing the stochastic process $\hat{n}^{(k)}(t)$ corresponding to the island charge at a given time $t$

$$n(t, \zeta_s) = n_s + \sum_{i=1}^s \sigma_i \theta(t - \tau_i) \quad (A1)$$

Similarly one can consider the random number of electrons $\hat{n}^{(k)}(t)$ transferred through the junction $k$ after $t \geq -T/2$

$$n^{(k)}(t, \zeta_s) = \sum_{i=1}^s \sigma_i \theta(t - \tau_i) \delta(k - k_i) \quad (A2)$$
The random variables \( \tilde{n}(t) \) and \( \tilde{n}^{(k)}(t) \) are subjected to the relations

\[
\begin{align*}
n(t, \zeta) &= n_s + \sum_{k=1}^{N} n^{(k)}(t, \zeta) \\
I^{(k)}(t, \zeta) &= \frac{\partial}{\partial t} n^{(k)}(t, \zeta)
\end{align*}
\]

After that we can introduce the probability distribution \( P(n, t) \) and the joint probability distribution \( P(n_1, t_1; n_2, t_2) \) \((t_1 \geq t_2)\) of the process \( \tilde{n}(t) \)

\[
P(n, t) = \int_\Omega \delta(n - n(t, \zeta))d\mu(\zeta) \quad (A4)
\]

Their ratio \( P(n_1, t_1 | n_2, t_2) = P(n_1, t_1; n_2, t_2)/P(n_2, t_2) \) gives the conditional probability to find the system at state \( n_1 \) at time \( t_1 \), given that at time \( t_2 \) it was at state \( n_2 \). The integrals \( (A4) \) can be efficiently evaluated along with the same reasoning as we have used to prove the normalization condition. As the result one ends up with

\[
P(n_1, t_1 | n_2, t_2) = \langle n_1 | \hat{U}(t_1, t_2) | n_2 \rangle \quad (A5)
\]

The latter expression is the usual way to describe the system in terms of master equation. The conditional probability \( P(n_1, t_1 | n_2, t_2) \) regarded as a function of \( n_1 \) and \( t_1 \) obeys this equation with the initial condition \( P(n_1, t_1) = \delta_{n_1, n_2} \) at \( t_1 = t_2 \).

APPENDIX B

This appendix contains the derivation of the action \( S(\chi) \) at low temperatures at some special points \( V_k \) in the Coulomb blockade staircase in the two-terminal dot. We introduce the notation \( \Gamma^{(\pm)}(n) = \Gamma^{(\pm)}_n \), that enables to write down Eq. \( (B3) \) in the form

\[
\begin{align*}
\left(\Lambda - \gamma_n\right)p_n + \Gamma^{(-)}_n(n + 1)p_{n+1} + \Gamma^{(+)}_n(n - 1)p_{n-1} &= 0
\end{align*}
\]

If all \( \chi_k = 0 \) then the stationary solution of this equation, corresponding to \( \Lambda = 0 \), satisfies the detailed balance condition \( p_{n+1} \Gamma^{(-)}_0(n + 1) = p_n \Gamma^{(+)}_0(n) \). In general situation, when \( \chi_k \neq 0 \), one may try to resolve \( (B3) \) making use of the substitution

\[
\begin{align*}
\frac{p_{n+1}}{p_n} = \frac{\Gamma^{(+)}_n(n)}{y \Gamma^{(-)}_n(n + 1)}, \quad \frac{p_{n-1}}{p_n} = \frac{y \Gamma^{(-)}_n(n)}{\Gamma^{(+)}_n(n - 1)}
\end{align*}
\]

with unknown constant \( y \) to be found. This reduces the difference equation \( (B1) \) to the relation

\[
\left(\Lambda - \gamma_n\right) + \Gamma^{(-)}_n(n) + \Gamma^{(+)}_n(n) y^{-1} = 0. \quad (B3)
\]

Here \( \gamma_n = \Gamma^{(0)}_0(n) + \Gamma^{(0)}_0(n) \), and \( \Gamma^{(\pm)}_n(n) \) are linear functions in \( n \), given by \( (B5) \). Then one might find the two unknown \( y \) and \( \Lambda \) on comparing the constant and linear in \( n \) terms in relation \( (B3) \). It yields

\[
y = \left( R_1^{-1} - R_2^{-1} + \sqrt{D(\chi)} \right) / 2R_1^{-1}e^{-i\chi_1}
\]

\[
\Lambda(\chi) = \frac{1}{2e}(V - e/C\Omega) \left( R_1^{-1} + R_2^{-1} - \sqrt{D(\chi)} \right) \quad (B4)
\]

It looks like we have found in such a way the required solution. However, it is not valid at all possible values of parameters. The matter is that in the most general situation the expressions \( (B5) \) are not correct at points \( n = n_{\text{min}} \) and \( n = n_{\text{max}} \). One have to set \( \Gamma^{(-)}_n(n_{\text{min}}) = \Gamma^{(+)}_n(n_{\text{max}}) = 0 \) by hand that breaks the analytical \( n \)-dependence of equations \( (B1) \) and \( (B4) \) at the boundaries. The only exceptional situation, when the above way to solve the Eq. \( (B3) \) is indeed true, corresponds to the case \( n_1 = n_{\text{min}}, n_2 = n_{\text{max}} \), with \( n_{1(2)} \) being the zeros of functions \( \Gamma^{(\pm)}_n(n) \). In this case the substitution \( (B2) \) gives \( p_{n_{\text{max}} - 1} = p(n_{\text{min}} - 1) = 0 \) and hence the actual values of \( \Gamma^{(-)}_n(n_{\text{max}} + 1) \) and \( \Gamma^{(+)}_n(n_{\text{min}} - 1) \) in Eq. \( (B4) \) play no role. Then we arrive to the action \( S(\chi) = t_0 \Lambda(\chi) \) in the form which was claimed in section V (a).

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