Calculation of the performance of the electromagnetic magnetic fluid separator non-magnetic materials

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Abstract. Magnetic fluid separation is a promising area in the field of disposal and recycling of non-magnetic materials. Performance and separation accuracy are important features of any separator. The methods for calculating the productivity offered at this moment have a number of assumptions that can significantly affect the result (constant magnetic permeability of the magnetic fluid, linear change in tension along the height of the gap, etc.), and are applicable only to specific models of separators. The aim of the study is to develop a methodology for calculating the performance of an electromagnetic magnetic fluid separator of non-magnetic materials, taking into account the distribution of the magnetic field in the gap and the influence of the hydrodynamic properties of the magnetic fluid on the movement of particles in the separation zone. The proposed approach allows us to calculate the mass and volumetric capacities of the magnetic fluid separator, the dependences of the capacities for each fraction separately, and the total on various parameters of the separator (for example, on the angle of inclination of the pole pieces). The obtained results can be used to assess the optimality of the adopted parameters when designing a magnetic fluid separator. Also, the results obtained will help optimize the installations already in operation.

1. Introduction.
Every year, the volumes of electronic waste in the world increase and in 2016 more than 40 million tons were produced. In the future, the situation will only worsen it is assumed that at the beginning of the next decade, at least 50 million tons of electronic waste will be released in the world posing a threat to human health and the environment [1].

Unsuitable for further use devices and materials may contain valuable natural resources, such as gold, silver, copper, platinum, etc. Their competent processing can provide reduction of environmental pollution, conservation of natural resources and return to the production of non-ferrous, precious metals and other materials. So, for example, in alluvial deposits, the gold content is up to 0.1 - 0.5 g / t of ore, in primary ores 1 - 5 g / t. And in the secondary raw materials, the gold content reaches 100 g / t and even 100 kg / t. The proportion of precious metals that can be obtained from recycled materials reaches 20 - 50%. So, in 2016, the cost of such materials not extracted from discarded electronics amounted to $ 55 billion [2].

During secondary processing, the performance of the technological scheme is determined by the productivity of each stage. The performance of magnetic fluid separation determines the power of the previous plants for grinding, magnetic separation and the required capacity for the following operations [3,4].

Despite this, the issue of performance is often resolved by empirical studies of an already created device. So, in [5] the dependences of the separation efficiency on the productivity of the experimental setup are given (figure 1).
Separation efficiency is evaluated by Zemlyansky’s criterion:

\[ E_Z = \varepsilon_1 + \varepsilon_2 - 100, \]

where \( \varepsilon_1 \) is the extraction of light fractions into tails, \%, \( \varepsilon_2 \) is the extraction of heavy fractions into concentrate, \%.

In [6], the issue of the performance of a magnetic fluid separator with poles diverging in the horizontal plane is considered (Figure 2). This design provides additional strength, which helps to move the light fraction in the horizontal direction.

The following expressions are used to calculate the performance of the separator.

The final velocity of the non-magnetic particle in the horizontal direction:

\[ v_x = k_3 \sqrt{\frac{d}{\rho_{kh}} \left( \frac{\mu_0 \chi H}{\partial x} + \rho_p g \sin \alpha \right)}, \]

where \( k_3 \) is the empirical coefficient, \( d \) is the particle diameter, \( \rho_{kh} \) is the horizontal component of the quasi-density of the magnetic fluid, and \( \chi \) is the magnetic susceptibility of the magnetic fluid.

The final velocity of a non-magnetic particle in the vertical direction:

\[ v_y = k_1 k_2 \sqrt{\frac{\rho_p - \rho_{ky}}{\rho_{ky}}}, \]

where \( k_1, k_2 \) are empirical coefficients, \( \rho_{ky} \) is the vertical component of the quasi-density of the magnetic fluid, and \( \rho_p \) is the particle density.

Required separation zone length:

\[ L = k_4 h \sqrt{\frac{\rho_{kh} \mu_0 \chi H}{\partial x} + \rho_p g \sin \alpha}{(\rho_p - \rho_{ky}) \rho_{kh}}, \]

where \( k_4 \) is the empirical coefficient.
Separator capacity:
\[ Q = \rho \Delta h - \theta \beta \cdot v. \]

where \( \Delta h \) is the thickness of the layer of material being transported, \( \theta \) is the degree of loosening of the fraction, the working width of the separator channel, \( \beta \) is the fraction of the light fraction in the feedstock.

The above methods for determining the performance are suitable exclusively for specific types of magnetic fluid separators and include a few empirical coefficients, which makes it difficult to use them in the development of new devices. Also, several assumptions are applied that may affect the accuracy of determining this parameter.

The use of numerical modeling of physical fields to calculate the performance of magnetic-liquid separators can give higher accuracy and the possibility of applying a new technique in the design of new ones.

2. Research Methods.
To calculate the particle trajectory, when moving in the working gap of the magnetic fluid separator (MFS), the following model was adopted. Four forces act on a non-magnetic particle with density and volume \( V \) in a magnetic fluid with density and magnetization \( M \) (Fig. 3): gravity force \( F_g = \rho \cdot g \cdot V \), Archimedes force \( F_A = -\rho_k \cdot g \cdot V \), buoyancy from the side of the magnetic fluid
\[ F_M = -\mu_0 \cdot M \cdot \nabla H \cdot V, \]

where \( \nabla H \) is the gradient of the magnetic field, and the resistance force \( F_V \), due to the viscosity of the magnetic fluid (MF), determined by the Stokes formula
\[ F_V = -3 \pi \eta \cdot d \cdot \bar{v}, \]

where: \( \eta \)-dynamic viscosity of the MF; \( \bar{v} \)-particle velocity.

**Figure 3.** The movement of non-magnetic particles in a magneto-liquid separator: 1 - pole pieces forming a working gap; 2 - magnetic fluid; 3 - device for supplying non-magnetic particles; 4 - container for heavy particles; 5 - container for light particles.

The equation of motion of a particle can be obtained based on Newton’s second law.
\[ F_T + F_A + F_M + F_V = m \frac{dv}{dt}, \]  
where: \( m = \rho V \) is the mass of the particle. We assume that the particles are spherical and have a diameter \( d \). After substituting all the values into Newton’s formula, the equation of motion of a nonmagnetic particle in the working gap of the MFS will have the form

\[ \rho V \frac{dv}{dt} + 3\pi \cdot \eta \cdot d \cdot \vec{v} + \left[ -\mu_0 M \cdot \nabla H \right] \cdot \vec{V} + \left( \rho - \rho_{mF} \right) \cdot g \vec{V} = 0. \]  

(4)

We accept the following assumptions:
- MF has a linear rheological characteristic and constant dynamic viscosity (\( \eta = \text{const} \));
- the working gap of the MFS is designed correctly, as a result of which the third term on the left side of equation (4) is constant (it represents the resulting force that acts on the particle in statics).

We write equation (4) in the form:

\[ \frac{a}{d} \frac{dv}{dt} + b \cdot \vec{v} + c = 0, \]  

(5)

where \( a, b, c \) are constant values independent of the magnitude of the velocity.

In projections on the axis, equation (4) has the form:

\[ \rho g V(-e_y) - \rho_{mF} g V(-e_y) - \mu_0 M \cdot \left[ \nabla H \right] \sin \alpha (-e_x) + \left[ \nabla H \right] \cos \alpha (-e_y) V - 
-3\pi \eta d (v_x e_x + v_y e_y) = \rho V \frac{d}{dt} (v_x e_x + v_y e_y) \]

\[ \rho V \frac{dv_x}{dt} + 3\pi \cdot \eta \cdot d \cdot v_x - \mu_0 M \cdot \left[ \nabla H \right] \sin \alpha V = 0 \]  

(6)

\[ \rho V \frac{dv_y}{dt} + 3\pi \cdot \eta \cdot d \cdot v_y + \left[ -\mu_0 M \cdot \nabla H \right] \cos \alpha + \left( \rho - \rho_{mF} \right) g \cdot V = 0 \]  

(7)

As a result of integration of equations (6) and (7), we obtain expressions for the particle velocity along the axes

\[ v_x = \left( v_{ox} + \frac{c_x}{b} \right) e^{-\frac{a}{b}} - \frac{c_x}{b}, \]  

(8)

\[ v_y = \left( v_{oy} + \frac{c_y}{b} \right) e^{-\frac{a}{b}} - \frac{c_y}{b}, \]  

(9)

where: \( a = \rho V; \) \( b = 3\pi \cdot \eta \cdot d \cdot k_b; \)

\[ c_x = k_c \left( -\mu_0 M \cdot \nabla H \cdot V \cos \alpha - \rho_{mF} \cdot g V \right) + \rho \cdot g V; \]

\[ c_y = -k_c \cdot \mu_0 M \cdot \nabla H \cdot V \sin \alpha, \]

where \( k_b, k_c \) – coefficients taking into account the degree of immersion of the particle in the MF;

\[ k_b = \frac{S_k}{S} = \frac{h_b}{d} \cdot k_c = \frac{V_k}{V} = \frac{6(d - \frac{h_b}{3})}{2 \cdot 3 d^3}, \]  

(10)

where \( h_b \) is the height of the submerged part of the particle, \( d \) is its diameter.

The following parameters are set in a computer program for calculating the particle trajectory:
- particle density \( \rho \);
- fluid density $\rho_{MF}$;
- the angle of the poles (surface of the magnetic fluid) $\alpha$;
- the length of the active part of the poles $L$;
- the height of the column of magnetic fluid $h$;
- the height of the minimum working gap relative to the lower surface of the magnetic fluid $h_1$ (the boundary of the change in the direction of the magnetic force $FM$);
- magnetization curve of magnetic fluid;
- the value of the dynamic viscosity of the breast $\eta$;
- particle diameter $d$;
- the initial position of the particle $X_0, Y_0$;
- projections of the initial velocity vector $v_0X, v_0Y$.

When calculating the trajectory by known coordinates at the previous iteration, it is determined whether the particle is in magnetic fluid. If not, then the change in speed is determined under the action of only gravity. If the particle is in the MF, then the degree of its immersion in the MF is determined, the coefficients $k_b$ and $k_c$ are determined by (10), and the velocity is calculated using the above formulas (7-8). The known speeds determine the change in the coordinates of the particles at the considered step of the change in time. The coordinates are written to the array and are ultimately displayed on the graphs (Figures 4 - 5) [7].

![Figure 4](image1.png)  
**Figure 4.** The calculated value of the trajectory of an aluminum particle with a diameter of 5.6 mm at an angle of inclination of the poles of 10°. Dotted line - particle trajectory, solid line - MF borders, dash-dot line - boundary of the change in the direction of action of the magnetic force.

![Figure 5](image2.png)  
**Figure 5.** The calculated value of the trajectory of motion of a copper particle with a diameter of 4.9 mm at an angle of inclination of the poles of 10°. Dotted line - particle trajectory, solid line - MF borders, dash-dot line - boundary of the change in the direction of action of the magnetic force.

The performance of the separator when moving the shared material in a monolayer can be calculated by the following formula:

$$Q = \rho d s v / k,$$

where $d$ is the average particle diameter of the material to be separated, $b$ is the width of the layer of the material to be separated, $v$ is the average speed of the mixture to be shared towards the unloading device, and $k$ is the fill factor of the material [8]. Since the movement of the separated mixture in the magnetic fluid separator occurs in two directions: in the horizontal plane for the light fraction, and in the vertical for the heavy fraction, two capacities should be considered - in the light and heavy fractions.

Thus,

$$Q_L = \rho_L d_s b v_{sx} / k_L$$

- productivity for a light fraction;
\[ Q_{H} = \rho_{H} d_{H} \delta v_{yav}/k_{H} \] - performance for the heavy fraction, where \( d_{L} \) and \( d_{T} \) are the diameters of light and heavy particles, respectively, \( b \) is the arc length of the upper surface of the magnetic fluid, \( \delta \) is the minimum clearance of the magnetic fluid separator (figure 6), \( v_{xav} \) and \( v_{yav} \) are the average speeds along the corresponding axes, \( k_L \) and \( k_H \) are the filling factors of the light and heavy fractions, respectively [5,6]. To obtain volumetric performance, you must use the following formula:

\[ Q_{V} = Q/\rho, \] where \( \rho \) is the density of the corresponding fraction. Using the models described above, particle velocities in the separation zone of the separator can be obtained.

Since the particle velocity during its movement in the separation zone is not constant, it is necessary to calculate the average velocity:

\[ v_{av} = \frac{\int_{0}^{T} v \, dt}{T}, \]

where \( T \) is the time of movement in the separation zone.

Numerically average speed:

\[ v_{av} = \frac{\sum_{i}^{N} v_{i} \Delta t}{T}, \]

where \( v_{i} \) is the velocity at the i-th iteration, \( N \) is the iteration number at which the particle exits the separation zone. If \( T = N \cdot \Delta t \), then

\[ v_{av} = \frac{\sum_{i}^{N} v_{i}}{N}. \]

3. The results of the study.

As an example, the performance of a magnetic fluid separator was calculated for different angles of inclination of the pole pieces. The following magnetic fluid parameters were taken for calculation: viscosity 7.5 Pa \( \cdot \) s, density 1.15 g/cm\(^3\), saturation magnetization 25 kA/m [9]. The separated mixture is two-component with particles with a diameter of 1.5 mm, and densities of 7.26 g/cm\(^3\) (tin) and 2.7 g/cm\(^3\) (aluminum) for heavy and light fractions, respectively. The parameters of the separator correspond to the laboratory setup: the length of the separation zone is 15 cm, the height of the separation zone is 7 cm, the arc length of the upper surface of the magnetic fluid is 2.5 cm, the minimum gap is 1 cm.

As a result of the calculation, the speeds and residence time in each section were obtained, based on which the dependences of the mass (figure 7) and volumetric (figure 8) capacities on the angle of inclination of the pole pieces were calculated (the calculation results are summarized in table 1).
Table 1. Dependences of the mass and volumetric performance of the magnetic fluid separator on the angle of the poles.

| α, deg | 0   | 10  | 20  | 30  | 40  | 45  |
|--------|-----|-----|-----|-----|-----|-----|
| Q_H, kg/h | 75,48 | 78,74 | 83,15 | 88,94 | -   | -   |
| Q_L, kg/h | -   | 17,85 | 42,09 | 53,3 | 55,83 | 55,95 |
| Q_Σ, kg/h | 75,48 | 96,59 | 125,24 | 142,24 | 55,83 | 55,95 |
| Q_V_H, l/h | 10.40 | 10.85 | 11.45 | 12.25 | 0.00 | 0.00 |
| Q_V_L, l/h | 0.00 | 6.61  | 15.59 | 19.74 | 20.68 | 20.72 |
| Q_Σ, l/h | 10.40 | 17.46 | 27.04 | 31.99 | 20.68 | 20.72 |

4. Discussion and conclusions.
In contrast to the methods considered earlier, the developed technique uses the full picture of the field distribution in the gap; in calculating the productivity, instantaneous values of particle velocities are used, which undoubtedly increases the accuracy of the calculation. Also, the developed technique can be applied to various types of magnetic fluid separators.

The dependences obtained make it possible to select the optimal parameters of the magneto-liquid separator to ensure maximum performance and energy efficiency of the separation process. They also help to evaluate the adequacy of some parameters of the magneto-liquid separator. So, for example, from the graphs (figure 7, figure 8), it can be seen that at an angle of inclination of the pole tips of more than 40 degrees, the heavy fraction begins to fall into the light and this position of the poles is unacceptable. Thus, using the program developed based on this algorithm [10], it is possible to design a magnetic fluid separator that maximally meets the requirements of the separation process in the processing of secondary materials.

Figure 7. The dependence of the performance of the MFS on the angle of the poles: 1 - total, 2 - for the heavy fraction, 3 - for the light fraction.

Figure 8. The dependence of the volumetric capacity of the MFS on the angle of the poles: 1 - total, 2 - for the light fraction, 3 - for the heavy fraction.

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