1. Introduction

Various detection methods of physical quantities employ optical fibers where typical dependence of intensity, wavelength or phase of a propagating electromagnetic radiation on a measured physical quantity is monitored. Different devices have been proposed for the measurement of a large number of physical parameters including temperature, acoustic, electric and magnetic fields, pressure, strain rotation, displacement, etc.

A subset of group of optical fiber sensors is represented by sensors which use intermodal interference of propagating modes of electromagnetic radiation. The most attractive from this group are sensors using intermodal interference of fundamental mode LP(01) and first higher order mode LP(11) which propagate in optical fibers with an elliptical core. Various sensors of pressure, temperature and distributed strain based on interference of presented modes were published [1, 2].

Another group of optical fiber sensors based on intermodal interference typically uses interference of modes LP(01) a LP(02) propagating in a cylindrical optical fiber. From this group of optical fiber sensors were published sensors of acoustics waves, fiber elongation, longitudinal strain, temperature and hydrostatic pressure [3, 4, 5].

Hollow-core optical fibers belong to a special class of optical fibers with an air core surrounded by a higher refractive index material. A hollow-core optical fiber filled with certain kinds of liquids forms the liquid-core optical fiber.

In this paper an exploitation of intermodal interference between LP(01) and LP(02) modes in a liquid-core optical fiber for temperature measurement is theoretically described. We suppose the hollow-core optical fiber with a fused silica cladding and a core filled with toluene. In such a liquid-core optical fiber the waveguide properties are sensitive to temperature due to a different thermal coefficient of a refractive index core and cladding. It can be used for sensing applications where the intermodal interference of defined modes is measured in a wide temperature range.

2. Theoretical approach

Light in an optical fiber propagates by means of modes of electromagnetic fields. An individual mode propagates with a phase constant \( \beta \) which depends on parameters of an optical fiber and can be different for individual modes. The difference between phase constants allows for observation of intermodal interference under special conditions.

The interference signal \( s \) for monochromatic light detected at the end of an optical fiber is in the case of two-mode interference proportional to term [6]

\[
s = \cos(\beta_m - \beta_n) l,
\]

where \( l \) is the length of optical fiber and \( \beta_m, \beta_n \) are the phase constants of the individual interfering modes. For the cylindrical optical fiber the phase constant of the mth mode can be expressed as [6]

\[
\beta_m = \frac{2\pi}{\lambda} \sqrt{n_m^2 - \frac{V_m'(V)}{V^2} (n_m^2 - n_l^2)}.
\]
where $\lambda$ is the wavelength of light in vacuum, $n_{co}$ and $n_{cl}$ are maximal and minimal refractive indices of the core and cladding of optical fiber. $V_{m}$ is the normalized propagation constant in the transverse direction of the $m$th mode and $V$ is the normalized frequency defined by [7]

$$V = \frac{2\pi r}{\lambda} \sqrt{n_{co}^{2} - n_{cl}^{2}}. \quad (3)$$

where $r$ is the core diameter of optical fiber.

Taken into account the optical fiber filled in with toluene, the core refractive index $n_{co}$ can be described by the dispersion equation for toluene at the temperature $T = 20 \degree C$ [8]

$$n_{co}(\lambda, 20 \degree C) = 1.474775 + \frac{6990.31}{\lambda^2} + \frac{2.117810}{\lambda^4} \quad (4)$$

where $\lambda$ is the light wavelength in nanometers.

Dispersion of the cladding refractive index $n_{cl}$ can be described by the Sellmeier equation for fused silica at the temperature $T = 20 \degree C$ [9]

$$n_{cl}(\lambda, 20 \degree C) = 1 + \frac{0.6961663}{\lambda^2 - 0.0684043} + \frac{0.4079426}{\lambda^2 - 0.1162414} + \frac{0.8974794}{\lambda^2 - 9.896161} \quad (5)$$

where $\lambda$ is the light wavelength in micrometers.

Toluene was chosen as an appropriate core medium in this approach because of a higher thermal coefficient of refractive index compared to fused silica. While the thermal coefficient of refractive index for fused silica at the temperature $T = 20 \degree C$ and at wavelength 644 nm is $1 \times 10^{-4} \degree C^{-1}$ [10], the refractive index for toluene at the temperature $T = 20 \degree C$ and at wavelength 633 nm is $3.55 \times 10^{-4} \degree C^{-1}$ [11]. This considerable difference of the thermal coefficient of the refractive index favors this material combination for designing the optical fiber sensor sensitive to temperature.

Generally, the material and geometrical parameters of optical fibers are changing with temperature. For an optical fiber where the core is represented by toluene and cladding by fused silica, the change of geometrical parameters with temperature can be neglected because the coefficient of thermal expansion for fused silica is $5.5 \times 10^{-7} \degree C^{-1}$ [10]. Then only changes of the material parameters with temperature are taken into account. The phase constants of the propagating modes are a function of material parameters and then they depend on temperature. The change of the phase constants of the propagating modes with temperature will modify the interference dependence of interfering modes. Investigation of an interference signal as a dependence on temperature can reveal a relation between parameters of intermodal interference and temperature.

The temperature dependence of the refractive index for toluene and fused silica near $T = 20 \degree C$ using eq. (4) and (5) can be expressed

$$n_{co}(\lambda, T) = n_{co}(\lambda, 20 \degree C) + \frac{dn_{co}}{dT}(T - 20) \quad (6)$$

and

$$n_{cl}(\lambda, T) = n_{cl}(\lambda, 20 \degree C) + \frac{dn_{cl}}{dT}(T - 20) \quad (7)$$

where $dn/dT$ are the thermal coefficients of refractive index for toluene and fused silica and $T$ is the temperature in °C.

Fig. 1 shows the dependence of the refractive index contrast $\Delta n = n_{co} - n_{cl}$ between toluene and fused silica given by (6) and (7) in the wavelength range of 600–1500 nm for the temperatures of 0, 20 and 40 °C. We suppose a constant thermal coefficient of the refractive index of both materials in the calculated wavelength range and for all the temperatures.

The refractive index contrast between toluene and fused silica is positive and small ($\Delta n < 0.05$) in the all considered wavelength range what allows to use the weakly guiding approximation for a description of waveguide properties of the mentioned optical fiber [7].

3. Numerical results

The calculated difference $\Delta \beta$ for modes LP01 and LP02 is shown in Fig. 2 at the temperatures of 0, 20 and 40 °C. The weakly guiding approximation was used in this calculation for a step-index optical fiber with cladding from fused silica and core with radius $r = 2.5 \mu m$ filled in with toluene.

As shown in Fig. 2 the evident decrease of $\Delta \beta$ is caused by a temperature increase. Maxima in these $\Delta \beta$ spectral dependences correspond to the equalization wavelength $\lambda_0$. The equalization wavelength of the fiber is the wavelength at which the group velocities of two modes are the same [12]. Its value for the optical
A fiber with a constant profile of refractive index and radius is a function of only refractive indices of core and cladding [13]. Then the refractive index changes caused by the temperature change the equalization wavelength \( \lambda_0 \). Therefore, the measurement of equalization wavelength in spectral domain can be used for the temperature sensing applications. In the spectral dependencies of \( \Delta \beta \) (Fig. 2) the equalization wavelength \( \lambda_0 \) shows the 440 nm blue shift as the temperature increases in the temperature range of 40 °C.

Fig. 3 summarizes calculated results of the equalization wavelength \( \lambda_0 \) in the temperature dependence from 0 °C to 40 °C. In this graph the nearly linear decrease of \( \lambda_0 \) is demonstrated as the temperature increases.

In order to characterize the sensitivity, the \( d\lambda_0/dT \) dependence on temperature in the temperature range of 0–40 °C was calculated from data shown in Fig. 3 (Fig. 4). For the temperature of 0 °C \( d\lambda_0/dT = -9.7 \text{ nmK}^{-1} \) and for the temperature of 40 °C \( d\lambda_0/dT = -12.2 \text{ nmK}^{-1} \).

Methods of the intermodal interference of modes LP
01
and LP
02
are well described enabling exact measurement of an interference signal in spatial and spectral domains [14]. In Fig. 5 numerically calculated interference curves of modes LP
01–LP
02 given by (1) for the considered optical fiber 5 cm long at the temperatures of 0, 20 and 40 °C are shown.

It is evident that the equalization wavelength \( \lambda_0 \) can be well identified with resolution better than 1 nm. If \( d\lambda_0/dT \) is in the interval from \(-9.7 \text{ nmK}^{-1}\) to \(-12.2 \text{ nmK}^{-1}\) in the investigated temperature range, then the temperature can be determined with resolution better than 0.1 °C.

The exact detection of the equalization wavelength \( \lambda_0 \) is conditioned by the intermodal interference of only modes LP
01
and LP
02
. Therefore, this method requires the confinement of propagation of higher cylindric-symmetrical mode LP
03
near \( \lambda_0 \). For the designed optical fiber sensor for \( \lambda_0 \) at the temperature of 0 °C the cut-off wavelength for LP
03
is 795 nm and at the temperature of 40 °C it is 562 nm. The position of \( \lambda_0 \) from the presented figures is in the range from 844 nm to 1284 nm in all investigated temperature range. Thus a good identification of the equalization wavelength \( \lambda_0 \) is guaranteed for such an optical fiber sensor in the proposed temperature range.

4. Conclusion

The temperature sensor based on an optical fiber consisting of fused silica and core of 2.5 μm radius filled in with toluene is proposed in this paper. For temperature sensing the dependence
of the equalization wavelength $\lambda_0$ on temperature is here taken into account using the interfering modes $LP_{01-02}$. The optical fiber sensor was theoretically investigated in the temperature range of 0–40 °C, where we suppose the temperature coefficient of the refractive indices to be constant and independent on wavelength. Then, in the investigated temperature range the equalization wavelength $\lambda_0$ for modes $LP_{01-02}$ changes from 1284.2 nm to 844.1 nm which favors this method for temperature measurements with resolution better than 0.1 °C.

Such a proposed sensor can be used for temperature measurements in all range of liquid state of toluene from −93 °C to 110 °C. In this wide temperature interval for exact analysis of sensitivity the dependence of refractive indices of toluene and fused silica on the wavelength and temperature should be included in calculations. Then including such relations allows exploitation of this method in a wide temperature range with good accuracy.

Other modifications of the proposed optical fiber sensor are possible where another liquid medium could be used in the core of the optical fiber. Such variability opens a new area of temperature sensing elements where the requested sensitivity and temperature range can be selected by an appropriate choice of liquid medium.

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