Superconducting system for adiabatic quantum computing

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Abstract. We study the Hamiltonian of a system of inductively coupled flux qubits, which has been theoretically proposed for adiabatic quantum computation to handle NP problems. We study the evolution of a basic structure consisting of three coupled rf-SQUIDs upon tuning the external flux bias, and we show that the adiabatic nature of the evolution is guaranteed by the presence of the single-SQUID gap. We further propose a scheme and the first realization of an experimental device suitable for verifying the theoretical results.

1. Introduction

Adiabatic quantum computation (AQC)[1] is a powerful method to solve NP-hard minimization problems. Whereas a conventional quantum algorithm is implemented as a sequence of discrete unitary operations, involving many energy levels of the computer and complicated superposition of them, the adiabatic algorithm works by keeping the quantum computer close to the instantaneous ground state of the Hamiltonian, which varies adiabatically in time. As a consequence, it has an inherent robustness against errors and decoherence[2], illustrating the principle that it may be possible to design suitable quantum hardware which effectively resists to noise.

From a theoretical point of view, a scalable superconducting architecture has been proposed for solving any class of NP problems[3], while from the experimental side it has been shown that an inductance transducer can be effectively used to read out the results of the adiabatic evolution algorithm[4].

We also propose to use a superconducting device for AQC consisting of coupled flux qubits (rf SQUIDs). The system presented in this paper is inspired by the fact that it is NP-hard to calculate the ground state of an antiferromagnetic Ising model on a planar graph with local coordination number \leq 3 and in a uniform magnetic field equal to the exchange coupling [5],[6]. In other words, if \( G \) is a planar graph with vertices (sites) \( V \) and edges (bonds) \( E \), such that each vertex is connected by edges to \( \leq 3 \) other vertices, then it is NP-hard to find the ground state of

\[
\mathcal{H} = \sum_{i \in V} \sigma_i^z + \sum_{(i,j) \in E} \sigma_i^z \sigma_j^z
\]
where $\sigma_z^j$ is the Pauli spin operators along the $z$-axis (set by the direction of an uniform magnetic field), $\sigma_z^j = \pm 1$ for all $j \in V$. Eq. (1) represents an Ising model from the family

$$H = \sum_{i \in V} h_i \sigma_z^i + \sum_{(i,j) \in E} \Lambda_{ij} \sigma_z^i \sigma_z^j$$

(2)

with equal antiferromagnetic coupling strength and the energy difference between the two low-lying states equal to the coupling energy: $h_i = \Lambda_{ij}$.

In this paper we show how the most general Hamiltonian, Eq. (2), can be implemented in a coupled SQUID system, and we address the problem of adiabatically reaching the interesting limit of Eq. (1) from a known ground state in the simplest non-trivial case of three coupled qubits. Moreover, a first device is shown in view of the experimental measurements.

2. Theoretical background

If we consider a system of identical SQUIDs with capacitance $C$ and critical current $I_c$, the Hamiltonian of the coupled qubits is given by

$$H = \sum_i \left[ -\frac{h}{2C} \left( \frac{2\pi}{\Phi_0} \right)^2 \frac{\partial^2}{\partial \phi_i^2} + \frac{I_c \Phi_0}{2\pi} \cos \phi_i \right] + \frac{1}{2} \left( \frac{\Phi_0}{2\pi} \right)^2 \sum_{ij} (\phi_i - \phi_x) \Lambda_{ij}^{-1} (\phi_j - \phi_x)$$

(3)

where $L_{ij}$ represents the inductance matrix of the system of coupled SQUIDs. In Eq. (3), the flux $\Phi_i$ penetrating the $i$-th superconducting ring and the external flux $\Phi_x$ applied to the SQUID have been normalized and shifted:

$$\phi_{i(x)} = \frac{2\pi}{\Phi_0} \left( \Phi_{i(x)} + \frac{1}{2} \Phi_0 \right)$$

(4)

Using $U_0 = \frac{1}{L} \left( \frac{\Phi_0}{2\pi} \right)^2$ as the energy scale, where $L$ is the self-inductance of each SQUID, and introducing the dimensionless parameters $\mu = C \frac{2\pi}{\Phi_0}$ and $\beta L = \frac{2\pi I_c \Phi_0}{\Phi_0}$ the Hamiltonian becomes

$$\frac{H}{U_0} = \sum_i \left[ -\frac{1}{2\mu} \frac{\partial^2}{\partial \phi_i^2} + U_i(\phi_i) \right] + \Lambda \sum_{(ij)} \phi_i \phi_j$$

$$U_i(\phi) = \beta L \cos \phi_i + \frac{1}{2} (\phi_i - \phi_x)^2 - \frac{z_i}{2} \Lambda (\phi_i - \phi_x) \phi_x$$

(5)

where we assume a constant coupling, $\Lambda = L L_{ij}^{-1}$, between nearest neighbors and neglect the interaction with further neighbors. The number of neighbors interacting with the $i$-th SQUID is given by $z_i$.

Even in the case of three SQUIDs, full diagonalization of this Hamiltonian is a challenging task, we have therefore approached the calculation of the evolution of the spectrum and of the ground state of the above Hamiltonian making use of the two-level approximation (2LA).

Within the 2LA, the Hamiltonian of a system of identical inductively coupled rf-SQUIDs can be represented as[7]:

$$H = H \sum_i \sigma_i^x + J \sum_{ij} \sigma_i^x \sigma_j^x + \Delta \sum_i \sigma_i^x .$$

(6)

where $H$ is the potential difference between the right and left minimum of $U_i$, $\Delta$ is the symmetric tunnel splitting of the single SQUID ($\phi_x = 0$) and we have assumed a constant coupling $J > 0$ for nearest neighbors and $J = 0$ otherwise.
The above Hamiltonian differs from our Hamiltonian of interest, Eq.(2), by the presence of the off-diagonal coupling to a transverse field $\Delta$. This term leads to level mixing of the Ising Hamiltonian, but in practice it is ineffective away from any level crossing. On the other hand it opens the possibility of removing the degeneracy at the level crossings of the unperturbed Hamiltonian, Eq.(2), thus maintaining a finite gap over the ground state.

The tunable Hamiltonian, Eq. (6), lends itself to a proof-of-principle implementation of an AQC algorithm. Such an algorithm can be realized by adiabatically changing the external overall magnetic field, interpolating between a starting Hamiltonian with a given $H = H_0$, whose ground state is known, to the final Hamiltonian with $H = J$ whose ground state encodes the desired solution to the problem[6]. During the whole evolution the ground state must always be non-degenerate, and the spectrum above the ground state needs to remain gapful all the time.

A route can be taken by starting from the fully polarized state $|\beta\rangle = |\uparrow \uparrow \ldots \uparrow\rangle$ for $H > \max\{z_i\}J$ and decreasing $H$ down to $J$. We require the point $H = J$ to be away from any level crossing of the unperturbed Hamiltonian, in order to be able to safely read out the state of the system at that field value.

We now focus on the specific case of 3 SQUIDs in a linear array. The adiabatic evolution starts from the fully polarized state $|\beta\rangle = |\uparrow \uparrow \uparrow\rangle$ toward the final state $|\alpha\rangle = |\uparrow \downarrow \uparrow\rangle$.

The expectation value of the central q-bit and the gap over the ground state for the 2LA Hamiltonian are shown in Figs.1 and 2. We observe a clear transition from state $\alpha$ to state $\beta$ when the flux of the central SQUID is inverted. Simultaneously, the minimal single-SQUID excitation gap hits a minimum corresponding to the single-SQUID symmetric tunnel splitting.

![Figure 1. 2LA results for the expectation value of the central q-bit as a function of $\phi_x$ and $\Lambda$ (three-SQUID system, $\beta_L = 1.2$, $\mu = 26.5$, $C = 50$ fF).](image1)

![Figure 2. 2LA results for the expectation value of the gap in the spectrum as a function of $\phi_x$ and $\Lambda$ (three-SQUID system, $\beta_L = 1.2$, $\mu = 26.5$, $C = 50$ fF).](image2)

3. Experimental device
In Figs. 3 and 4 we present a sketch of a possible experimental design for a three-qubit system and the practical realization. The rf SQUIDs can be inductively coupled, either by simply designing them close to each other, or via superconducting flux transformers. The natural inductive coupling is ferromagnetic, but a gradiometric superconducting flux transformer[8] interrupted by a vertical Josephson interferometer provides actually an inductive antiferromagnetic coupling, which can be switched on and off by applying an external field parallel to the chip plane[9]. Such
a control is ineffective on the dynamics of the qubits and can be disregarded in the rf SQUID Hamiltonian. Moreover each rf-SQUID is coupled via a superconducting flux transformer to a read-out system, consisting of a dc SQUID integrated on the same chip, which provides a voltage signal proportional to the qubit signal. The external magnetic flux $\Phi_x$, driving the evolution from the initial to the final state of the whole system, is orthogonal to the qubits and is generated by a current flowing in a solenoid embedding the device.

**Figure 3.** Sketch of the experimental design of a multi-qubit system, driven by the external magnetic flux $\Phi_x$. Rf SQUIDs are coupled through a switchable flux transformer guaranteeing the antiferromagnetic coupling. Each qubit signal is read by a dc SQUID sensor.

**Figure 4.** Experimental device including 3 rf SQUIDs coupled through switchable flux transformers, as shown in the inset.

In conclusion, we have studied the Hamiltonian of three coupled flux qubits in the context of adiabatic quantum computation, focusing on the evolution of the ground state upon gradually changing of the overall magnetic field. We have approached the calculation of the evolution of the spectrum using the two-level approximation. In order to have an adiabatic process we need the spectrum above the ground state to remain gapful during the whole evolution. Upon decreasing the field, we find that the minimum gap is of the order of the symmetric tunnel splitting of the single SQUID, and therefore the shortest time for the adiabatic evolution is $\tau_a \approx 1$ ns [10],[11]. Moreover we have realized a device including 3 rf SQUIDs having a switchable coupling in order to verify the theoretical results.

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