Scalar Particle Spectrum in broken fermion family symmetries

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Abstract

We study the spectrum and possible observability of Higgs-like scalar particles associated with spontaneous breaking at high energies of fermion family symmetries such as $SU(3)_f$ proposed sometime ago by King and Ross. We treat the energy scale at which the symmetry is broken as a variable not necessarily as high as the GUT scale used by King and Ross. We compare and contrast a non-supersymmetric treatment of the fermion family symmetry model introduced here, and the supersymmetric treatment used by King and Ross.

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1 Introduction

In a number of studies, King, Ross and collaborators [1-6] argued that if one attributes the unsolved standard model problem of fermion family replication, mass hierarchy, and flavor mixing, to the existence of a continuous family symmetry such as $SU(3)_f$ among the fermions, one can reproduce the observed fermion mass patterns and flavor mixing. The family symmetry is postulated to be exact at some high energy scale well above the electroweak scale, but becomes completely broken spontaneously, before one reaches the electroweak scale. The spontaneous family symmetry breaking is taken to occur in stages, in a manner that allows the creation of three distinct fermion families observed at electroweak scale.

For this sequential breaking of the family symmetry, King and Ross introduced a number of new scalar fields that trigger the symmetry breaking. This family symmetry breaking is expected to yield as signature, a number of Higgs-like particles which may be observable depending on the scale at which the symmetry is broken. The formulation of King and Ross puts the scale of the family symmetry breaking at GUT scale or above, at which scale any Higgs-like particles produced are too heavy and decouple from low energy dynamics or observability. If we relax the need to tie the $SU(3)_f$ breaking scale to GUT scale or above, we may find a variable $SU(3)_f$ breaking scale that puts its physical scalar particle spectrum within experimental reach.

The new scalar particles to be observed, have peculiar properties. They carry family labels (family charge), being triplets or anti-triplets of $SU(3)_f$ like the quarks and leptons. The new scalars are however singlets of the standard model $SU(2)_L \otimes U(1)_Y$, meaning the new scalar particles have zero electric charge and zero weak hypercharge. Their masses are completely unknown and our interest in this paper, is to ascertain the physical spectrum of these peculiar family symmetry scalar particles, and what Higgs-like masses they may have. The hope is that if these masses are not too high, the ongoing LHC experiments at CERN may see these scalar bosons or set limits on them.

Before we plunge into these concerns however, we review in section 2, the salient features of the King-Ross formulation of the fermion family symmetry model, built on GUTs and supersymmetry, to be contrasted later with a non-Guts and non-supersymmetric path we followed.
2 The King-Ross Family Symmetry Model of Fermion masses

A starting point is that King and Ross formulated the fermion mass problem into an effective field theory. Such a formulation requires that we regard the standard model as a good theory capable of correctly describing electroweak physics but only within a certain limited energy scale (the cut-off scale $\Lambda_c$ of the standard model). The physics of fermion mass replication and flavor mixing is specifically believed to lie beyond scale $\Lambda_c$. Rather at some high energy scale $M > \Lambda_c$, it is believed that some more exact or fundamental theory of electroweak physics exists, that provides a direct or natural explanation for the fermion mass patterns and mixing observed at standard model level. What this scale $M$ is, and the dynamics there become matters of conjecture and model building.

Whatever other details of the dynamics of scale $M$, it is believed that an fermion family symmetry such as $SU(3)_f$ exists there. The symmetry is exact and fermion family triplets like $(u, c, t)$ or $(d, s, b)$ or $(e, \mu, \tau)$ are all massless there, and correspond to triply degenerate states $U^i, D^i, E^i$, with $i = 1, 2, 3$ running through each family. The family symmetry is however subsequently broken, at or below scale $M$.

King and Ross chose scale $M$ to be as high or higher than the GUT scale of $10^{16}$ GeV which means that besides a gauged fermion family symmetry $SU(3)_f$ at scale $M$, there are other gauge symmetries there particularly GUT $SO(10)$. The fields present at such high energy scale are taken to be a variety of heavy particles (bosons and fermions) along side the light fermions and Higgs boson $H$ of the standard model.

For the dynamics at scale $M$, King and Ross adopt the Froggatt-Nielsen model [7] where one has a variety of tree graphs mediated by heavy messenger particles (bosons and fermions) coupled directly to standard model fermions as external legs. When these heavy messenger particles are later integrated out, one is led to the low energy effective Lagrangian or effective Yukawa couplings of the standard model, and thence to fermion mass matrices.

The model of King and Ross is chosen to be supersymmetric, and the supersymmetry played two central roles in the results obtained by King and Ross. First the Supersymmetry tied in a scalar potential needed to break the family symmetry, this potential being some $F$-term of a scalar superpotential.
given generally by [8]:

\[ W(\varphi_i) = \sum_i a_i \varphi_i + \frac{1}{2} \sum_{ij} m_{ij} \varphi_i \varphi_j + \frac{1}{6} \sum_{ijk} g_{ijk} \varphi_i \varphi_j \varphi_k + \frac{1}{24} \sum_{ijkl} h_{ijkl} \varphi_i \varphi_j \varphi_k \varphi_l + \ldots \to \infty \]  

(1)

where \( \varphi_i \) is the scalar field component of the \( i \)th left chiral superfield in the system, and \( i, j, k, l \ldots \) run over all left chiral superfields in the system. Because one is dealing with effective theory situation where operators of higher dimensions are permitted, the \( W(\varphi_i) \) can run beyond \( n \leq 3 \) in powers of \( \varphi_i \) normally required for renormalizable theories.

Every term of \( W(\varphi_i) \) is required to be \( SU(3)_f \) invariant. The first term \( W_1(\varphi_i) = a_i \varphi_i \) is present only if the system contains singlet scalar fields of \( SU(3)_f \). The King-Ross model specifically admitted a number of such (heavy) singlet scalar fields at scale \( M \), denoted generically by \( X \), and assigned the role of generating F-terms and vacuum alignment of other scalar fields (triplets and anti-triplets \( \phi_3, \bar{\phi}_3, \phi_{23}, \bar{\phi}_{23} \) ) used to break \( SU(3)_f \) in a desired pattern. Sample terms of \( W(\varphi_i) \) used to align the scalar fields and achieve spontaneous symmetry breaking in specific F-flat directions include the following:

\[ W_3(\varphi_i) = X(\bar{\phi}_3^j \phi_{3i} - \mu^2) \]

(2)

which generates the F-term \( F_X = \bar{\phi}_3^j \phi_{3i} - \mu^2 \) and the alignment of \( \phi_3 \) and \( \bar{\phi}_3 \) from the minimization \( |F_X| = \bar{\phi}_3^j \phi_{3i} - \mu^2 = 0 \). Also

\[ W_{34}(\varphi_i) = X_1(\bar{\phi}_3^j \phi_{2i}) + X_2(\bar{\phi}_3^j \phi_{3i} \bar{\phi}_{2j}^j - \mu^4) + X_3(\bar{\phi}_{23}^j \phi_{23i}) + \ldots \]

(3)

whose separate terms align \( \phi_2 \) relative to \( \bar{\phi}_3 \); \( \phi_{23} \) relative to \( \phi_2 \), and \( \phi_{23} \) relative to \( \bar{\phi}_{23} \), etc. The end result is the following specific alignment used by King-Ross to break the fermion family symmetry \( SU(3)_f \to SU(2)_f \to 0 \):

\[
\begin{align*}
\phi_3 &= \begin{pmatrix} 0 \\ 0 \\ a_3 \end{pmatrix}; \quad \bar{\phi}_3 = (0, 0, a_3); \\
\phi_{23} &= \begin{pmatrix} 0 \\ b \\ b \end{pmatrix}; \quad \bar{\phi}_{23} = (0, b, -b)
\end{align*}
\]

(4)

It is with these particular alignments, and the GUT- \( SU(3)_f \) scale \( M \) dynamics that King and Ross obtained their result that fermion \( SU(3)_f \) family symmetry could be the source of the observed hierarchy of fermion masses and flavor mixing. Notably however, because the \( SU(3)_f \) symmetry is broken at so high scale \( M \geq \) GUT, no lower energy scale observable signature survives in the King-Ross model, neither the very existence of the family symmetry \( SU(3)_f \), nor its spontaneous breaking. Only the indirect inference
exists through the equally high scale Froggatt-Nielsen heavy messenger particles, that the observed fermion mass pattern may be due to the existence and spontaneous breaking of a fermion family symmetry. It is against this background that we consider a modified approach to the problem of $SU(3)_f$ fermion family symmetry, and what signatures we may find for it.

3 A re-formulation of the fermion $SU(3)_f$ family symmetry model

We retain the same effective theory framework of King and Ross with a scale $M$ that we treat as a variable, not necessarily as high as GUT scale. We focus on $SU(3)_f$ as our main gauge symmetry group at scale $M$. We assume that this $SU(3)_f$ is broken sequentially as before by the same scalar fields listed in equation (4) and aligned similarly, but not necessarily arising from any $F$-term or any assumed supersymmetry. In fact we seek to construct directly [9] two separate scalar potentials $V(\phi_3, \bar{\phi}_3)$ and $V(\phi_{23}, \bar{\phi}_{23})$ that we take to be responsible for the two stage breaking : $SU(3)_f \rightarrow SU(2)_f \rightarrow 0$. They create Higgs-like physical particles we can monitor or tag through varying scale $M$. With our minimal assumptions, we can still construct an effective Yukawa Lagrangian weighted by our scale $M$, in a manner similar to the King-Ross model. Specifically, we write down the following effective Yukawa couplings in our model

$$L_Y = \frac{1}{M^2} \bar{\phi}^3 \psi_i \phi^c_i H + \frac{1}{M^2} \bar{\phi}^3 \psi_i \phi^c_i H + ...$$

These terms can generate fermion masses exactly as before.

Leaving aside the issue of fermion masses, we focus on the scalar potentials that break the $SU(3)_f$ symmetry, and the scalar mass spectrum they may produce at variable scale $M$.

4 The potential $V(\phi_3, \bar{\phi}_3)$ and $SU(3)_f \rightarrow SU(2)_f$ Breaking

Up to fourth order we write $V(\phi_3, \bar{\phi}_3)$ as follows:

$$V(\phi_3, \bar{\phi}_3) = \mu_1^2 \phi_3^i \phi_3^i + \mu_2^2 \phi_3^i \phi_3^i + \mu_3^2 \phi_3^i \phi_3^i + \mu_4^2 \phi_3^i \phi_3^i$$

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two conjugate fields. This leads to our putting general as follows:

\[ V^2 \text{real, while we drop } V \text{ obtaining:} \]

\[ \frac{1}{2} \left( \phi_3 \phi_3^* + \phi_3^* \phi_3 \right) + \lambda_5 \frac{1}{2} \left( \phi_3 \phi_3^* + \phi_3^* \phi_3 \right) + \frac{\lambda_6}{2} \left( \phi_3 \phi_3^* + \phi_3^* \phi_3 \right) + \frac{\lambda_7}{2} \left( \phi_3 \phi_3^* + \phi_3^* \phi_3 \right) \quad (6) \]

Requiring the potential to be hermitian means the unknown parameters \( \mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) are all real, while parameters \( \mu_{12}^2, \lambda_5, \lambda_6, \lambda_7 \) are all complex. We can additionally require \( V(\phi, \bar{\phi}) \) to be symmetric with respect to its two conjugate fields. This leads to our putting \( \mu_1^2 = \mu_2^2; \lambda_1 = \lambda_2; \) and \( \lambda_5, \mu_{12}^2 \)
real, while we drop \( \lambda_6 \) and \( \lambda_7 \) terms as not tenable. We then rewrite equation (6) obtaining:

\[ V(\phi_3, \bar{\phi}_3) = \frac{\mu_2^2}{2} (\phi_3 \phi_3^* + \phi_3^* \phi_3) + \frac{\mu_{12}^2}{2} (\phi_3 \phi_3^* + \phi_3^* \phi_3) \]

\[ + \lambda \left[ (\phi_3 \phi_3^* + (\phi_3^* \phi_3)^2) + \lambda_3 (\phi_3 \phi_3^* + (\phi_3^* \phi_3)^2) \right] \]

\[ + \lambda_4 (\phi_3 \phi_3^* + (\phi_3^* \phi_3)^2) + \frac{\lambda_5}{2} \left[ (\phi_3 \phi_3^* + (\phi_3^* \phi_3)^2) \right] \quad (7) \]

where \( \mu^2/2 = \mu_1^2 = \mu_2^2; \) and \( \lambda/2 = \lambda_1 = \lambda_2. \)

The scalar fields \( \phi_3 \) and \( \bar{\phi}_3 \) are both complex and can be parameterized in general as follows:

\[ \phi_3 = \begin{pmatrix} \eta_1 + i\eta_2 \\ \eta_3 + i\eta_4 \\ \eta_5 + i\eta_6 \end{pmatrix}; \quad \langle \phi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ a_3 \end{pmatrix} \quad (8) \]

\[ \bar{\phi}_3 = (\eta_1 - i\eta_2, \eta_3 - i\eta_4, \eta_5 - i\eta_6); \quad \langle \bar{\phi}_3 \rangle = (0, 0, a_3) \quad (9) \]

Similarly:

\[ \phi_{23} = \begin{pmatrix} \eta_7 + i\eta_8 \\ \eta_9 + i\eta_{10} \\ \eta_{11} + i\eta_{12} \end{pmatrix}; \quad \langle \phi_{23} \rangle = \begin{pmatrix} 0 \\ b \\ b \end{pmatrix} \quad (10) \]

\[ \bar{\phi}_{23} = (\eta_7 - i\eta_8, \eta_9 - i\eta_{10}, \eta_{11} - i\eta_{12}); \quad \langle \bar{\phi}_{23} \rangle = (0, b, -b) \quad (11) \]

We obtain from equations (8) and (9) the following final form for \( V(\phi_3, \bar{\phi}_3) \):

\[ V(\eta_1, \eta_2...\eta_6) = \mu^2(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_5^2 + \eta_6^2) + \mu_{12}^2(\eta_1^2 - \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 - \eta_6^2) \]

\[ + 2\lambda(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_5^2 + \eta_6^2)^2 \]
where we find $\lambda_3$ not a different coupling from $\lambda$ and so have combined the two terms.

We are ready to analyze the potential (12) to find the physical spectrum and masses of the scalar particles produced in this first stage breaking of $SU(3)_f$. We use equation (4) or equations (8) and (9), as what defines the vacuum of the potential. We take the first and second derivatives of equation (12) evaluated at the vacuum, and obtain the following constraint equations on the $\mu_i, \lambda_i$ parameters.

\[
\frac{\partial V}{\partial \eta_i}\big|_{\text{vac}} = 0; \ i = 1, 2, 3, 4, 6.
\]  
\[
\frac{\partial V}{\partial \eta_5}\big|_{\text{vac}} = \mu^2 + \mu_{12}^2 + 4\lambda a_3^2 + 2\lambda_4 a_3^2 + 2\lambda_5 a_3^2 = 0
\]  
\[
\frac{1}{2} \frac{\partial^2 V}{\partial \eta_5^2}\big|_{\text{vac}} = M_{11}^2 = \mu^2 + \mu_{12}^2 + 4\lambda a_3^2 + 2\lambda_4 a_3^2 + 2\lambda_5 a_3^2
\]  
\[
\frac{1}{2} \frac{\partial^2 V}{\partial \eta_3^2}\big|_{\text{vac}} = M_{33}^2 = \mu^2 + \mu_{12}^2 + 4\lambda a_3^2 + 2\lambda_4 a_3^2 + 2\lambda_5 a_3^2
\]  
\[
\frac{1}{2} \frac{\partial^2 V}{\partial \eta_5^2}\big|_{\text{vac}} = M_{55}^2 = \mu^2 + \mu_{12}^2 + 12\lambda a_3^2 + 6\lambda_4 a_3^2 + 6\lambda_5 a_3^2
\]  
\[
\frac{1}{2} \frac{\partial^2 V}{\partial \eta_3^2}\big|_{\text{vac}} = M_{22}^2 = \mu^2 - \mu_{12}^2 + 4\lambda a_3^2 - 2\lambda_4 a_3^2 - 2\lambda_5 a_3^2
\]  
\[
\frac{1}{2} \frac{\partial^2 V}{\partial \eta_3^2}\big|_{\text{vac}} = M_{44}^2 = \mu^2 - \mu_{12}^2 + 4\lambda a_3^2 - 2\lambda_4 a_3^2 - 2\lambda_5 a_3^2
\]  
\[
\frac{1}{2} \frac{\partial^2 V}{\partial \eta_6^2}\big|_{\text{vac}} = M_{66}^2 = \mu^2 - \mu_{12}^2 + 4\lambda a_3^2 + 2\lambda_4 a_3^2 - 6\lambda_5 a_3^2
\]  
\[
\frac{1}{2} \frac{\partial^2 V}{\partial \eta_i \partial \eta_j}\big|_{\text{vac}} = M_{ij}^2 = 0; \ i \neq j
\]

We deduce from equations (14) - (16) that $M_{11}^2 = M_{33}^2 = 0$, and that combining this with equation (17) we obtain that

\[
M_{55}^2 = 4a_3^2(2\lambda + \lambda_4 + \lambda_5) \geq 0
\]

Equations (15) - (17) are seen to define one group of three particles (a family triplet), while equations (18) - (20) define another group of particles (a family
anti-triplet). The symmetry broken by $V(\phi_3, \bar{\phi}_3)$ is $SU(3)_f \rightarrow SU(2)_f$ and we expect five massless Nambu Goldstone bosons two of which are already identified by $M^2_{11} = M^2_{33} = 0$. Similarly treating the second group of particles, we set $M^2_{22} = M^2_{44} = 0$ which leads to two more Nambu Goldstone bosons and a residual mass:

$$M^2_{66} = 4a^2_3(\lambda_4 - \lambda_5) \geq 0$$  \hspace{1cm} (23)

Looking at equation (12) we see that $\lambda_4$ and $\lambda_5$ essentially define the same coupling constant and we can set $\lambda_4 = \lambda_5$. Then $M^2_{66} = 0$ also, yielding the required fifth Nambu Goldstone boson, and a final third generation heavy neutral physical particle $H_5$ of mass:

$$M^2_{55} = 8a^2_3(\lambda + \lambda_4)$$  \hspace{1cm} (24)

related to our field $\eta_5$ by: $H_5 = \eta_5 - a_3$ since $\langle H_5 \rangle = \langle \eta_5 \rangle - a_3 = 0$. We can on the basis of equation (24), attempt to estimate the mass of this Higgs-like particle $H_5$ produced in this first $SU(3)_f$ symmetry breaking. We argue as follows.

First we argue that if $M$ is the scale at which the $SU(3)_f \rightarrow SU(2)_f$ breaking occurred, we can take the vev parameter $a_3$ of the symmetry breaking as of the same order of magnitude as scale $M$. That is, we put $a_3 \approx M$ in equation (24).

Next the parameters $\lambda$ and $\lambda_4$ in equation (24), are both quartic scalar field couplings at the high energy scale $M$ where the potential $V(\phi_3, \bar{\phi}_3)$ operates. We can assume that $\lambda \approx \lambda_4 = \lambda_M$ where $\lambda_M$ is a running coupling constant at scale $M$ for a general $\lambda \phi^4$ system.

Equation (24) becomes rewritten as:

$$M^2_{55} = 8M^2 \lambda_M$$  \hspace{1cm} (25)

Next we use the renormalization group to relate the high scale coupling $\lambda_M$ to a the standard model $\lambda_{SM}$ of Higgs model at electroweak scale. To one loop order the RG relationship is [10]:

$$\lambda_M = \frac{\lambda_{SM}}{1 - \frac{3\lambda_{SM}}{16\pi^2} \ln \frac{M}{\mu}}$$  \hspace{1cm} (26)

where $\mu$ is the electroweak scale taken here to be the $M_Z = 91.2$ GeV $\approx 10^2$ GeV. Taking $\lambda_{SM}$ to be very small and perturbative based on indications from
the standard model, we find even for large values of scale $M$ that we can write the relationship equation (26) in the approximate form:

$$\lambda_M = \lambda_{SM}[1 + a\lambda_{SM}] \quad (27)$$

where $a = (3/(16\pi^2))\ln(M/\mu)$.

Plugging equation (27) into (25) we obtain:

$$M_{55}^2 = 8M^2\lambda_{SM}[1 + a\lambda_{SM}] \quad (28)$$

Next, we write down the standard model equation that relates $\lambda_{SM}$ to standard model Higgs mass. It is:

$$M_{Higgs}^2 = 2\lambda_{SM}v^2 \quad (29)$$

where $v = 246$ GeV.

Eliminating $\lambda_{SM}$ between equations (28) and (29) we finally obtain an equation relating the standard model Higgs mass and the new Higgs-like particle $M_{55}$ produced in the breaking of $SU(3)_f \rightarrow SU(2)_f$. The relation is:

$$\frac{M_{55}^2}{M^2} = 8\frac{M_{H}^2}{v^2} + 4a \left( \frac{M_{H}^2}{v^2} \right)^2 \quad (30)$$

This is our main finding, namely that if $SU(3)_f$ fermion family symmetry exists at some high energy scale $M$ where it is spontaneously broken by some heavy scalar fields, the ratio of the Higgs-like particle produced to the symmetry breaking scale $M$, is related in a distinct way to a similar ratio between the standard model Higgs mass and standard electroweak breaking scale (vev) $v$. Thus if a Higgs boson should turn up in the ongoing LHC experiments with a measured mass $M_H$, one can use equation (30) to estimate what $SU(3)_f$ breaking Higgs-like mass to expect at a higher energy scale $M$.

5 The second stage $V(\phi_{23}, \bar{\phi}_{23})$ Breaking

We consider next the complementary potential $V(\phi_{23}, \bar{\phi}_{23})$ and the spontaneous $SU(2)_f \rightarrow$ nothing symmetry breaking it induces. The potential is given by:

$$V(\phi_{23}, \bar{\phi}_{23}) = \frac{\mu^2}{2}(\phi_{23}^\dagger\phi_{23} + \bar{\phi}_{23}^\dagger\bar{\phi}_{23}) + \frac{\mu^{'2}}{2}(\phi_{23}\phi_{23} + \bar{\phi}_{23}\bar{\phi}_{23})$$
\[ V(\eta_7, \eta_8, \eta_9, \eta_{10}, \eta_{11}, \eta_{12}) = \mu'^2 \left[ \eta_7^2 + \eta_8^2 + \eta_9^2 + \eta_{10}^2 + \frac{1}{2}(\eta_{11}^2 + \eta_{12}^2) \right] + \mu'^2 (\eta_7^2 + \eta_8^2 + \eta_9^2 + \eta_{10}^2 + \eta_{11}^2 - \eta_{12}^2) \]

or using equations (10) and (11) we rewrite the potential as:

\[ \begin{align*}
V(\eta_7, \eta_8, \eta_9, \eta_{10}, \eta_{11}, \eta_{12}) &= \mu'^2 \left[ \eta_7^2 + \eta_8^2 + \eta_9^2 + \eta_{10}^2 + \frac{1}{2}(\eta_{11}^2 + \eta_{12}^2) \right]
+ \mu'^2 (\eta_7^2 - \eta_8^2 + \eta_9^2 - \eta_{10}^2 + \eta_{11} \eta_{12} - \eta_{12}^2) \\
&\quad + \lambda'_1 \left( \eta_7^2 + \eta_8^2 + \eta_9^2 + \eta_{10}^2 + \eta_{11}^2 + \eta_{12}^2 \right) \\
&\quad + \lambda'_2 \left( \eta_7^2 + \eta_8^2 + \eta_9^2 - \eta_{10}^2 + \eta_{11}^2 + \eta_{12}^2 \right) \\
&\quad + \lambda'_3 \left( \eta_7^2 - \eta_8^2 + \eta_9^2 - \eta_{10}^2 + \eta_{11} \eta_{12} - \eta_{12}^2 \right) \\
&\quad + \lambda'_4 \left( 2 \eta_7 \eta_8 + 2 \eta_9 \eta_{10} + \eta_{11} \eta_{12} + \eta_{11} \eta_{12} \right) \\
&\quad + \lambda'_5 \left( \eta_7^2 - \eta_8^2 + \eta_9^2 - \eta_{10}^2 + \eta_{11} \eta_{12} - \eta_{12}^2 \right) \\
&\quad - \lambda'_6 \left( 2 \eta_7 \eta_8 + 2 \eta_9 \eta_{10} + \eta_{11} \eta_{12} + \eta_{11} \eta_{12} \right)^2
\end{align*} \] (31)

It is this potential we analyze for the additional set of scalar particles created by the \( V(\phi_{23}, \bar{\phi}_{23}) \). We find as follows:

\[ \frac{\partial V}{\partial \eta_i} \bigg|_{\text{vac}} = 0; \ i = 7, 8, 10, 12. \] (33)

\[ \frac{\partial V}{\partial \eta_9} \bigg|_{\text{vac}} = \mu'^2 + \mu'^2_1 + 4\lambda'^2 (\lambda' + \lambda'_3) = 0 \] (34)

\[ \frac{\partial V}{\partial \eta_{11}} \bigg|_{\text{vac}} = \mu'^2 - \mu'^2_1 + 4\lambda'^2 (\lambda' + \lambda'_3) = 0 \] (35)

\[ \frac{\partial^2 V}{2 \partial \eta_7^2} \bigg|_{\text{vac}} = M_{77}^2 = \mu'^2 + \mu'^2_1 + 4\lambda'^2 (\lambda' + \lambda'_3) \] (36)

\[ \frac{\partial^2 V}{2 \partial \eta_8^2} \bigg|_{\text{vac}} = M_{88}^2 = \mu'^2 - \mu'^2_1 + 4\lambda'^2 (\lambda' + \lambda'_3) \] (37)

\[ \frac{\partial^2 V}{2 \partial \eta_9^2} \bigg|_{\text{vac}} = M_{99}^2 = \mu'^2 + \mu'^2_1 + 8\lambda'^2 (\lambda' + \lambda'_3) + 4\lambda'^2 (\lambda'_4 + \lambda'_5) \] (38)

\[ \frac{\partial^2 V}{2 \partial \eta_{10}^2} \bigg|_{\text{vac}} = M_{10,10}^2 = \mu'^2 - \mu'^2_1 + 4\lambda'^2 (\lambda' + \lambda'_3 + \lambda'_4 - \lambda'_5) \] (39)
\[
\frac{1}{2} \partial^2 V \bigg|_{\text{vac}} = M_{11,11}^2 = \frac{1}{2} \mu'^2 + b^2 (4 \lambda' + 2 \lambda'_3 + \lambda'_4 + \lambda'_5) \quad (40)
\]

\[
\frac{1}{2} \partial^2 V \bigg|_{\text{vac}} = M_{12,12}^2 = \mu'^2 - \mu''^2 + 4b^2 (\lambda' + \lambda'_3) \quad (41)
\]

The only mixing found in the system is between fields \( \eta_9 \) and \( \eta_{11} \) where

\[
\frac{1}{2} \partial^2 V \bigg|_{\text{vac}} = M_{9,11}^2 = M_{11,9}^2 = 2b^2 (\lambda' + \lambda'_3 - \lambda'_4 - \lambda'_5) \quad (42)
\]

Before discussing this equation, we draw some conclusions from equations (34) - (41). Based on these equations, the fields are seen again to divide into two groups: Fields \( \eta_7, \eta_9, \eta_{11} \) form a triplet while fields \( \eta_8, \eta_{10}, \eta_{12} \) represent anti-triplet effect. Based on equation (34) we deduce \( M_{77}^2 = 0 \); while equation (38) becomes:

\[
M_{99}^2 = 4b^2 (\lambda' + \lambda'_3 + \lambda'_4 + \lambda'_5) \geq 0 \quad (43)
\]

Also based on equation (35) we see \( M_{88}^2 = M_{12,12}^2 = 0 \) while equation (39) simplifies to :

\[
M_{10,10}^2 = 4b^2 (\lambda'_4 - \lambda'_5) \quad (44)
\]

We have thus three massless particles \( \eta_7, \eta_8, \eta_{12} \) we expect as Nambu Goldstone bosons for \( SU(2) \rightarrow \text{nothing} \). In addition if we make the assumption that \( \lambda'_4 \approx \lambda'_5 \) (both being part of a general scalar running quartic coupling stated below), we get a fourth massless particle \( \eta_{10} \). There remain two fields to account for: \( \eta_9 \) and \( \eta_{11} \). They appear to mix as indicated by equation (42). If \( H_9 \) and \( H_{11} \) are two physical particles associated with \( \eta_9 \) and \( \eta_{11} \), then upon diagonalization we find these particle masses to be

\[
M_{H_9}^2 = -\frac{1}{2} B + \frac{1}{2} \sqrt{B^2 - 4C} \quad (45)
\]

\[
M_{H_{11}}^2 = -\frac{1}{2} B - \frac{1}{2} \sqrt{B^2 - 4C} \quad (46)
\]

where:

\[
B = -(4b^2 X_1 + \frac{1}{2} \mu'^2 + b^2 X_3)
\]

\[
C = (4b^4 X_1 X_3 + 2b^2 \mu^2 X_1 - 4b^4 X_2^2)
\]

\[
X_1 = \lambda'_1 + \lambda'_3 + \lambda'_4 + \lambda'_5
\]

\[
X_2 = \lambda'_1 + \lambda'_3 - \lambda'_4 - \lambda'_5
\]

\[
X_3 = 4 \lambda'_1 + 2 \lambda'_3 + \lambda'_4 + \lambda'_5 \quad (47)
\]
We can now try to estimate the scale of these masses by arguing variously as before. First we argue that all the couplings \( \lambda_1', \lambda_3', \lambda_4', \lambda_5' \) are quartic scalar field couplings at the same high energy scale \( M' \) where \( SU(2)_f \to \) nothing occurs. As such we can represent each of them by the same running coupling \( \lambda_{M'} \) at scale \( M' \) of a \( \lambda \phi^4 \) theory. Then we write:

\[
\begin{align*}
X_1 &= 4\lambda_{M'} \\
X_2 &= 0 \\
X_3 &= 8\lambda_{M'} \\
\end{align*}
\] (48)

Notably too, the interference term \( M_{11,9}^2 \) in equation (42) drops out, leaving us to deal directly with equations (40) and (43). From equation (43) we get

\[
M_{99}^2 = 16b^2\lambda_{M'}
\] (49)

while from equation (40) we get:

\[
M_{11,11}^2 = \frac{1}{2}\mu'^2 + 8b^2\lambda_{M'}
\] (50)

Then similar to equation (27) from the renormalization group, we can take:

\[
\lambda_{M'} = \lambda_{SM}[1 + a'\lambda_{SM}]
\] (51)

where \( a' = (3/(16\pi^2))\ln(M'/\mu) \) This transforms equation (49) and (50) to:

\[
\begin{align*}
M_{99}^2 &= 16b^2\lambda_{SM}[1 + a'\lambda_{SM}] \\
M_{11,11}^2 &= \frac{1}{2}\mu'^2 + 8b^2\lambda_{SM}[1 + a'\lambda_{SM}]
\end{align*}
\] (52) (53)

Finally we bring in standard model Higgs mass as a reference scale related to \( \lambda_{SM} \) and \( v = 246 \) GeV by equation (29), \( M_{Higgs}^2 = 2v^2\lambda_{SM} \). Also we can take the vacuum expectation value \( b \) of the \( SU(2)_f \to \) nothing breaking at scale \( M' \) as comparable in value to \( M' \). That is we put \( b = M' \). Then equations (52) and (53) take the final form

\[
\frac{M_{99}^2}{M'^2} = 8\frac{M_H^2}{v^2} + 4a'\left(\frac{M_H^2}{v^2}\right)^2
\] (54)

\[
2M_{11,11}^2 - M_{99}^2 = c
\] (55)

where \( c \) is a constant. We see equation (54) is the same formula as equation (30), but holds at scale \( M' \) where the secondary spontaneous breaking \( SU(2)_f \to \) nothing, takes place.
6 Summary and Conclusions

We summarize our results and draw conclusions. We set out to examine the physical spectrum and masses of new scalar particles that should exist if continuous family symmetries such as SU$_3$$_f$ symmetry proposed by King et. al.[1-6] exist and become completely broken down spontaneously above the electroweak scale. Our analysis shows that besides the expected numbers of massless Nambu Goldstone bosons, some massive scalar bosons $H_5, H_9, H_{11}$, much heavier than the Higgs boson should also exist. We obtained the main result stated in equations (30) and (54), that the ratio of the mass of these heavy Higgs bosons to the scale $M$ at which their SU$_3$$_f$ symmetry is broken, can be determined by the standard model Higgs boson mass and the electroweak scale $v$. We consider that this formula can be a guide in searching for heavy Higgs bosons as evidence of SU$_3$$_f$ type fermion family symmetry.

These particles are postulated to carry no electroweak quantum numbers and no color charges. They carry only family or generation quantum number through which they couple to fermions that also carry generation numbers. If the coupling of these heavy scalar bosons is by the gauge principle as we assumed, then new gauge bosons other than the electroweak and gluonic gauge bosons must exist to mediate the family (generation) force at high energies.

By way of other competing new Higgs bosons, we mention other models that propose new scalar particles in the electroweak system. We have in particular, the two Higgs doublet model 2HDM [11,12], and the minimal supersymmetric model MSSM [11,12], the latter being however only a special case of 2HDM. Each of these two models independently predicts a total of five new Higgs-like scalar particles compared to the one Higgs particle of the standard model. The five particles are usually denoted by: $H^\pm, A^0, H^0, h^0$ of which $h^0$ is believed to be the lightest. Their masses are however unknown except for various bounds and limits placed from experimental searches or unseen decay rates [13-16]. Many workers in these models think however, that none of the five particles is likely to have mass exceeding 700 GeV. This is likely to be much lighter than our own particles $H_5, H_9, H_{11}$, unless the scale $M$ at which our SU$_3$$_f$ family symmetry is actually broken turns out not to be exceedingly high.

References:

1. S. F. King and G. G. Ross, Phys. Letters B520 (2001) 243.
2. S. F. King and G. G. Ross, Phys. Letters B574 (2003) 239.

3. Ivo de Medeiros Varzielas, University of Oxford Ph.D. thesis, hep-ph/0801.2775 (2008)

4. Ivo de Medeiros Varzielas and G. G. Ross, Nucl. Phys. B733 (2006) 31

5. G. G. Ross and L. Velasco-Sevilla, Nucl. Phys. B653 (2003) 3

6. S. F. King and M. Malinsky, JHEP 0611 : 071, (2006)

7. C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B147 (1979) 277

8. A. Signer, ABC of SUSY, hep-ph/0905.4630 (2009)

9. L. F. Li, Phys. Rev. D9 (1974) 1723

10. T. P. Cheng and L. F. Li, Gauge theory of elementary particle physics, Clarendon Press Oxford, 1984.

11. J. L. Diaz-Cruz and A. Mendez, Nucl. Phys. B380 (1992) 39

12. J. F. Gunion and H. Haber, Nucl. Phys. B272 (1986) 1

13. S. Heinemeyer, Higgs Physics at the LHC: Some theory aspects, arXiv:hep-ph/08072514 (2008).

14. Shinya Kanemura et. al. Distinctive Higgs signals of a type II 2HDM at the LHC : arXiv:hep-ph/09010204 (2009).

15. J. Ellis, Outlook for charged Higgs Physics, arXiv:hep-ph/09011120 (2009)

16. B. Dudley and C. Kolda, Constraining the charged Higgs mass in the MSSM: arXiv:hep-ph/09013337 (2009).