Phenomenological Study of Type II Seesaw with $\Delta(27)$ Symmetry

Itishree Sethi$^1$, and Sudhanwa Patra$^2$

$^1$Department of Physics, IIT Hyderabad, Kandi-502285, India
$^2$Department of Physics, IIT Bhilai, GEC Campus, Raipur, India

Abstract

We discuss the phenomenology of type-II seesaw by extending the Standard Model with additional Higgs doublets and scalar triplets additionally invoked with $\Delta(27)$ flavor symmetry for the explanation of non-zero neutrino masses and mixings, matter-antimatter asymmetry and lepton flavor violation. The non-zero neutrino masses can be realized via type-II seesaw mechanism by introducing scalar triplets transforming as triplets under $\Delta(27)$ while we add additional $SU(2)_L$ scalar doublets to have correct charge lepton masses. We further demonstrate with detailed numerical analysis in agreement with neutrino oscillation data like non-zero reactor mixing angle, $\delta_{CP}$, the sum of the light neutrino masses, two mass squared differences and its implication to neutrinoless double beta decay. We also discuss on the matter-antimatter asymmetry of the universe through leptogenesis with the decay of TeV scale scalar triplets and variation of CP-asymmetry with input model parameters. Finally, we comment on implication to lepton flavor violating decays like $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ processes.
I. INTRODUCTION

Over the past few decades, Standard Model (SM) of particle physics has emerged as a broadly accepted theoretical model which accounted for the interplay of three fundamental forces - strong, weak, electromagnetic forces and elementary particles including quarks and leptons. The SM is remarkably impressive in terms of the accuracy and is enormously successful in anticipating a wide range of phenomena which has been experimentally verified. However, the theory is still unanswerable to many observed phenomena such as gravity, hierarchy problem, neutrino mass, dark matter, matter-antimatter asymmetry and many more. Neutrinos are considered as massless in SM due to the absence of the right-handed neutrinos \([5, 6]\). However, the massiveness of neutrinos is evident from the neutrino oscillation experimental results which provide overwhelming confirmation that three neutrino flavors are mixed with each other and have non-zero mass. Current experimental observations give a consistent picture with a mixing structure parameterized by the \(3 \times 3\) Pontecorvo-Maki-Nakagawa-Sakata (PMNS) \([7–9]\) matrix which gives at least two non-zero massive neutrinos contradicting the theory of SM \([10, 11]\). The essence of beyond the standard model (BSM) arises, which can be addressed by extending the SM symmetry or by adding some new particles into SM.

The explanation of the neutrino mass origin is one of the intriguing problems in particle physics on the account of the smallness of the neutrino mass and its hierarchy structure. The neutrino oscillation phenomenology has revealed from the solar, atmospheric, accelerator and reactor neutrino experiments \([12]\) which determined the squared difference between the masses \(m_1\) and \(m_2\), \(m_1^2 - m_2^2\), and the squared difference between the masses \(m_1\) and \(m_3\) with mass-squared difference \(m_3^2 - m_1^2\), are in the order of \(10^{-5}\) eV\(^2\) and \(10^{-3}\) eV\(^2\) \([13]\). Yet, the absolute values of \(m_1\), \(m_2\) and \(m_3\) as well as, the doubtfulness of whether or not \(m_2\) is heavier than \(m_3\), remain unknown as the neutrino oscillation data can only provide the squared difference of the masses. However, from the cosmological point of view it is already confirmed that the mass of the neutrino should be below the eV scale \([12]\) and its value is \(m_\nu < 0.12\) eV \([14]\).

The famous seesaw mechanism \([15, 16]\) is much acclaimed to explain the tiny neutrino masses. The variants of the seesaw in the extension of the SM with some heavy fields include type-I, type-II and type-III \([17–25]\). Leptogenesis \([26]\), is the beautiful mechanism to explain matter-antimatter asymmetry observed in our universe. One of its appealing features is establishing a connection between neutrino physics at low and high energies through seesaw mechanisms. The CP-violating out of equilibrium decay of heavy messenger particle mediating seesaw mechanism leads to lep-
ton asymmetry which can then be converted to required baryon asymmetry by non-perturbative Sphaleron \[27\] process. There have been many studies in this direction and to name a few includes \[28–43\].

The non-abelian symmetries are more impactful in obtaining TBM \[44, 45\] which will give vanishing reactor angle and no CP violation. Like other non-abelian discrete symmetries \[46\], \(\Delta(27)\) symmetry is extensively used in neutrino phenomenology as it gives directly the tribimaximal(TBM) mixing pattern, which conflicts more or less with the standard neutrino mixing. However, in this case, the reactor mixing angle will be zero. Following the discovery of non-zero \(\theta_{13}\) mixing angle(2012) from several experiments Daya Bay \[47–49\], RENO \[50\], T2K \[51, 52\], Double Chooz \[53\], MINOS \[54\], the next heated debate hovering around neutrino physics is CP-violation. Theoretically, the cp phase violation can be explained together with the three mixing angles \(\theta_{12}, \theta_{23}\) and \(\theta_{13}\).

Apart from probing the neutrino mass scale and its mass hierarchy, we still need to perceive the nature of the neutrinos, i.e whether they are Dirac or Majorana type particles and measure the value of the CP violation Dirac phase, or both of the Dirac and Majorana phases, if neutrinos are Majorana type of particle. The neutrino mass hierarchy and CP-violating Dirac phase can be determined in the long base-line neutrino oscillation experiments \[55–57\]. The only well known possible experiments which can disclose the Majorana nature of massive neutrinos are finding the neutrinoless double beta \((\beta\beta)_0\) decay process \[58, 59\]. However, we cannot proceed any further in determining the neutrino masses and mixing without knowing the nature, whether it is Dirac or Majorana. There are several experiments for \((\beta\beta)_0\) decay, which take the data or are under preparation at present like GERDA, EXO, KamLand-ZEN, COURE, SNO+, MAJORANA, etc. From these above experiments, studying neutrinoless double beta \((\beta\beta)_0\) decay the effective Majorana mass has turned out to be \(|m_{ee}| \approx (0.01 - 0.05)\)eV \[60–65\].

In this work, we consider a minimal extension of SM with non-abelian discrete \(\Delta(27)\) flavor symmetry while extending SM with two Higgs doublets which corrects the charged lepton mass and three Higgs triplets for implementation of the type-II seesaw mechanism for light neutrino masses. This SM extension improves the quality of the predictability of the model by explaining different phenomenological consequences like neutrino mass, neutrinoless double beta decay(NDBD), leptogenesis, Lepton Flavor Violation (LFV) compatible with the current experimental values.

The manuscript is structured in the following way. Section-II proceeds with a brief description of the model and the particle content along with the full Lagrangian for charged lepton and neutrinos for type-II seesaw mechanism with \(\Delta(27)\) flavor symmetry. In this section, we also discuss the flavor structure of the neutrino masses along with the mixing matrix. In subsequent section-III, we
present our numerical results for non-zero neutrino masses, the behavior of the ratio between two measured neutrino mass-squared differences with input model parameters, complementary relation between measured neutrino mixing angles with input model parameters like internal mixing angle and phases and their implications to neutrinoless double beta decay. We discuss scalar triplet leptogenesis in section-IV with quantification of CP-asymmetry, while we briefly demonstrate lepton flavor violation with TeV scale scalar triplets in section-V. In section-VI, we summarise and conclude the phenomenological consequence of the type-II seesaw models with $\Delta(27)$ flavor symmetry.

II. MODEL DESCRIPTION

We briefly discuss extension of SM with additional discrete flavour symmetry $\Delta(27)$ which includes two Higgs doublets $H_2, H_3$ along with SM Higgs doublet $H_1$ and three additional scalar triplets $\Delta_1, \Delta_2, \Delta_3$. This choice of adding two Higgs doublets and three additional scalar triplets in the context of $\Delta(27)$ framework have been explored in several previous works \[66–73\]. The non-abelian discrete group $\Delta(27)$ has 27 elements divided into 11 equivalence classes. It has 9 one-dimensional irreducible representations $1_i (i = 1, \ldots, 9)$ and 2 three-dimensional representation $3$ and $\bar{3}$. The multiplication rules under $\Delta(27)$ symmetry group are described in the Appendix.

In this work, we have tried to explain neutrino phenomenology (neutrino mass, non-zero reactor mixing angle $\theta_{13}$, large cp-violation(\(\delta_{cp}\)), Jarlskog parameter(\(J_{cp}\))) by introducing three Higgs triplets. We also aim to study implications of the framework to leptogenesis, lepton flavour violation and neutrinoless double beta decay. The complete field content with their corresponding charges is provided in the table:\[1\]

| Field | \{L_{eL}, L_{\mu L}, L_{\tau L}\} | \{e_R, \mu_R, \tau_R\} | $H_1$ | $H_2$ | $H_3$ | $\{\Delta_1, \Delta_2, \Delta_3\}$ |
|-------|---------------------------------|-----------------|-----|-----|-----|----------------------------------|
| SU(2)$_L$ | 2 | 1 | 2 | 2 | 2 | 3 |
| $\Delta(27)$ | 3 | $3^*$ | 1 | 2 | 3 | 3 |

TABLE I. Complete field content with their corresponding charges of the proposed model.

A. Lagrangian and charged lepton mass matrix

The Yukawa interaction Lagrangian for charged leptons is given by

$$\mathcal{L}_{\text{Yuk}}^\ell = Y_{k}^{ij} \left[ \bar{L}_{iL} \otimes \ell_{jR} \right] \otimes H_k$$

$$= Y_1^{ij} \left[ \bar{L}_{iL} \otimes \ell_{jR} \right]_{11} \otimes [H_1]_{11} + Y_2^{ij} \left[ \bar{L}_{iL} \otimes \ell_{jR} \right]_{13} \otimes [H_2]_{12} + Y_3^{ij} \left[ \bar{L}_{iL} \otimes \ell_{jR} \right]_{12} \otimes [H_3]_{13}. \quad (1)$$
The charged lepton mass matrix after scalar field taking their respective vacuum expectation values (VEVs) is given by

\[
M_\ell = \begin{pmatrix}
Y_1 v_1 + Y_2 v_2 + Y_3 v_3 & 0 & 0 \\
0 & Y_1 v_1 + \omega^2 Y_2 v_2 + \omega Y_3 v_3 & 0 \\
0 & 0 & Y_1 v_1 + \omega Y_2 v_2 + \omega^2 Y_3 v_3
\end{pmatrix}.
\]

where \(v_1, v_2\) and \(v_3\) are the VEVs of the scalar fields and \(Y_1, Y_2\) and \(Y_3\) are the three Yukawa couplings respectively.

**B. Lagrangian and neutrino mass matrix**

The Lagrangian for neutrino mass is

\[
-\mathcal{L}_Y = f_{ij}^{\alpha} \bar{\ell}_i \ell_j \otimes \Delta_\alpha + f_{ij}^{\prime} \bar{\ell}_i \ell_j \otimes \Delta_\alpha + f_{ij}^{\prime\prime} \bar{\ell}_i \ell_j \otimes \Delta_\alpha.
\]

After the VEV gain the neutrino mass matrix will be

\[
M_\nu = \begin{pmatrix}
f_{\alpha v_{\Delta_1}} & f_{\alpha v_{\Delta_2}} & f_{\alpha v_{\Delta_3}} \\
f_{\alpha v_{\Delta_2}} & f_{\alpha v_{\Delta_3}} & f_{\alpha v_{\Delta_1}} \\
f_{\alpha v_{\Delta_3}} & f_{\alpha v_{\Delta_1}} & f_{\alpha v_{\Delta_2}}
\end{pmatrix}.
\]

where \(v_{\Delta_1}, v_{\Delta_2}, v_{\Delta_3}\) are three VEVs of \(\Delta_1, \Delta_2, \Delta_3\).

Considering, \(f_{\alpha} = f_{\alpha}^{\prime} = f_{\alpha}^{\prime\prime}\), then the mass matrix will be

\[
M_\nu = \begin{pmatrix}
f_{\alpha v_{\Delta_1}} & f_{\alpha v_{\Delta_2}} & f_{\alpha v_{\Delta_3}} \\
f_{\alpha v_{\Delta_2}} & f_{\alpha v_{\Delta_3}} & f_{\alpha v_{\Delta_1}} \\
f_{\alpha v_{\Delta_3}} & f_{\alpha v_{\Delta_1}} & f_{\alpha v_{\Delta_2}}
\end{pmatrix}.
\]

For a clear understanding of the neutrino parameters and their correlations, the light neutrino mass matrix \(M_\nu\) in Eq.(4) can be rewritten in the following form, but it has to be symmetric,

\[
M_\nu \simeq \begin{pmatrix}
\lambda a & c & b \\
c & \lambda b & a \\
b & a & \lambda c
\end{pmatrix}.
\]

where the parameters \(a, b, c\) are proportional to the three arbitrary VEVs of scalar triplets \(\Delta_1, \Delta_2, \Delta_3\).

For simplicity, we have chosen \(\lambda = 1\) for analytic as well as numerical calculations.

The light neutrino mass matrix can be completely diagonalised by two steps,
• The neutrino mass matrix ($M_\nu$) in eq.(5) can be diagonalized using the tri-bimaximal (TBM) mixing matrix [45], which implies that,

$$M_\nu^b = U_{\text{TBM}}^T M_\nu U_{\text{TBM}}. \quad (6)$$

where the form of the tribimaximal mixing matrix is

$$U_{\text{TBM}} = \begin{pmatrix} \frac{\sqrt{2}}{3} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (7)$$

In the subsequent step, we will get $M_\nu^b$ will be a block diagonalized matrix, to make it diagonal first we have to diagonal the $2 \times 2$ block diagonal matrix, which is solved in the following way

$$M_{\nu}^{b_{\text{bd}}} = \begin{pmatrix} m_{11} & m_{13} \\ m_{31} & m_{33} \end{pmatrix} = \begin{pmatrix} a - \frac{1}{2}(b + c) & \frac{\sqrt{3}}{2}(b - c) \\ \frac{\sqrt{3}}{2}(b - c) & -a + \frac{1}{2}(b + c) \end{pmatrix}. \quad (8)$$

With few steps of simple algebra calculation, the physical masses for light neutrinos are given by

$$m_1 = -\sqrt{a^2 - ab + b^2 - ac - bc + c^2} = |m_1|e^{i\phi_1},$$
$$m_3 = \sqrt{a^2 - ab + b^2 - ac - bc + c^2} = |m_1|e^{i\phi_1},$$
$$m_2 = a + b + c. \quad (9)$$

In the above, it is quite clear from eq.(9), two eigenvalues $m_1$ and $m_2$ are degenerate, which contradicts the neutrino oscillation experimental data.

• Thus, we add a perturbation term $\epsilon$ to all diagonal terms in order to generate non-degenerate neutrino masses. This small perturbation can also be generated by the inclusion of additional fields but here we express all diagonal terms as sum of leading terms plus this perturbation term $\epsilon \neq 0$. Adding this small perturbation the light neutrino mass matrix will be read as,

$$M_\nu = \begin{pmatrix} a + \epsilon & c & b \\ c & b + \epsilon & a \\ b & a & c + \epsilon \end{pmatrix}. \quad (10)$$
The process of block diagonalization using $U_{\text{TBM}}$ matrix leads to

$$M_{\nu}^{\text{bd}} = U_{\text{TBM}}^T M_{\nu} U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{pmatrix} \cdot \begin{pmatrix} a + \epsilon & c & b \\ c & b + \epsilon & a \\ b & a & c + \epsilon \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{3} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \end{pmatrix}$$

$$= \begin{pmatrix} a + \epsilon - \frac{1}{2}(b + c) & 0 & \sqrt{\frac{3}{2}}(b - c) \\ 0 & a + b + c + \epsilon & 0 \\ \sqrt{\frac{3}{2}}(b - c) & 0 & -a + \epsilon + \frac{1}{2}(b + c) \end{pmatrix}.$$  \hspace{1cm} (11)

In general, the above mass matrix can be written in the following form,

$$M_{\nu}^{\text{bd}} = \begin{pmatrix} m_{11} & 0 & m_{13} \\ 0 & m_{22} & 0 \\ m_{31} & 0 & m_{33} \end{pmatrix}. \hspace{1cm} (12)$$

Therefore, a further rotation by a unitary mixing matrix $U_{13}$ makes the above block diagonalized matrix $M_{\nu}^{\text{bd}}$ matrix to a completely diagonalized matrix $M_{\nu}^{\text{d}}$ as,

$$M_{\nu}^{\text{d}} = U_{13}^T M_{\nu}^{\text{bd}} U_{13} = U_{13}^T U_{\text{TBM}}^T M_{\nu} U_{\text{TBM}} U_{13}.$$

The unitary mixing matrix is parametrized in terms of rotation angle $\theta$ and phase $\delta$. Using $U_{13}$, the block diagonalized matrix is completely diagonalized as follows,

$$M_{\nu}^{\text{d}} = \begin{pmatrix} \cos \theta & 0 & -\sin \theta e^{i\delta} \\ 0 & 1 & 0 \\ -\sin \theta e^{-i\delta} & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} m_{11} & 0 & m_{13} \\ 0 & m_{22} & 0 \\ m_{31} & 0 & m_{33} \end{pmatrix} \begin{pmatrix} \cos \theta & 0 & \sin \theta e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin \theta e^{i\delta} & 0 & \cos \theta \end{pmatrix}.$$  \hspace{1cm} (14)

Now as $M_{\nu}$ is diagonalized and the physical mass eigenvalues for light neutrinos are expressed in terms of model parameters as,

$$m_1 = \epsilon + \sqrt{a^2 + b^2 + c^2 - (ab + bc + ca)},$$
$$m_2 = a + b + c + \epsilon,$$
$$m_3 = \epsilon - \sqrt{a^2 + b^2 + c^2 - (ab + bc + ca)}.$$  \hspace{1cm} (15)

and the value of internal angle $\theta$ is related to the input model parameters as,

$$\tan 2\theta = \frac{\sqrt{3}(b - c)}{(-2a + b + c) \cos \delta + 2\epsilon \sin \delta}, \hspace{1cm} (16)$$

For rest of our analysis we will use $\delta = 0$ and as a result of this, the mixing angle $\theta$ is then given by

$$\tan 2\theta = \frac{\sqrt{3}(\alpha_1 - \alpha_2)}{-2 + \alpha_1 + \alpha_2}. \hspace{1cm} (17)$$
III. RESULTS

Denoting $\alpha_1 = |\frac{\pi}{6}|$, $\alpha_2 = |\frac{\pi}{3}|$, $\alpha_3 = |\frac{\pi}{4}|$ and $\phi_{ba} = \phi_b - \phi_a$, $\phi_{ca} = \phi_c - \phi_a$, $\phi_{ea} = \phi_e - \phi_a$ are the phase differences between (b, a), (c, a) and ($\epsilon$, a) respectively, the light neutrino mass eigenvalues can be expressed in terms of their absolute values and corresponding phases as follows,

$$m_{\nu_1} = m_1 e^{i\phi_1} = |a| \left[ \alpha_3 e^{i\phi_{ca}} + \sqrt{1 + \alpha_1^2 e^{2i\phi_{ba}} + \alpha_2^2 e^{2i\phi_{ea}} - (\alpha_1 e^{i\phi_{ba}} + \alpha_2 e^{i\phi_{ea}} + \alpha_3 e^{i\phi_{ca}})} \right] e^{i\phi_1},$$

$$m_{\nu_2} = m_2 e^{i\phi_2} = |a| \left[ 1 + \alpha_1 e^{i\phi_{ba}} + \alpha_2 e^{i\phi_{ea}} + \alpha_3 e^{i\phi_{ca}} \right] e^{i\phi_2},$$

$$m_{\nu_3} = m_3 e^{i\phi_3} = |a| \left[ \alpha_3 e^{i\phi_{ca}} - \sqrt{1 + \alpha_1^2 e^{2i\phi_{ba}} + \alpha_2^2 e^{2i\phi_{ea}} - (\alpha_1 e^{i\phi_{ba}} + \alpha_2 e^{i\phi_{ea}} + \alpha_3 e^{i\phi_{ca}})} \right] e^{i\phi_3}.$$

Thus, the physical masses for light neutrinos are given by,

$$m_1 = |a| \left[ (\alpha_3 \cos \phi_{ca} + C)^2 + (\alpha_3 \sin \phi_{ca} + D)^2 \right]^\frac{1}{2},$$

$$m_2 = |a| \left[ (1 + \alpha_1 \cos \phi_{ba} + \alpha_2 \cos \phi_{ca} + \alpha_3 \cos \phi_{ea})^2 + (\alpha_1 \sin \phi_{ba} + \alpha_2 \sin \phi_{ca} + \alpha_3 \sin \phi_{ea})^2 \right]^\frac{1}{2},$$

$$m_3 = |a| \left[ (\alpha_3 \cos \phi_{ca} - C)^2 + (\alpha_3 \sin \phi_{ca} - D)^2 \right]^\frac{1}{2},$$

where $C$ and $D$ are defined as,

$$C = \left( \frac{A + \sqrt{A^2 + B^2}}{2} \right)^\frac{1}{2}, \quad D = \left( \frac{-A + \sqrt{A^2 + B^2}}{2} \right)^\frac{1}{2}.$$

$$A = 1 + \alpha_1^2 \cos 2 \phi_{ba} \alpha_2^2 \cos 2 \phi_{ca} - (\alpha_1 \cos \phi_{ba} + \alpha_2 \cos \phi_{ca} + \alpha_1 \alpha_2 \cos (\phi_{ba} + \phi_{ca})),$$

$$B = \alpha_1^2 \sin 2 \phi_{ba} \alpha_2^2 \sin 2 \phi_{ca} - (\alpha_1 \sin \phi_{ba} + \alpha_2 \sin \phi_{ca} + \alpha_1 \alpha_2 \sin (\phi_{ba} + \phi_{ca})). \quad (18)$$

Furthermore the phases associated with light neutrino mass eigenvalues can be written as,

$$\phi_1 = \tan^{-1} \left[ \frac{\alpha_3 \sin \phi_{ca} + D}{\alpha_3 \cos \phi_{ca} + C} \right],$$

$$\phi_3 = \tan^{-1} \left[ \frac{\alpha_3 \cos \phi_{ca} - D}{\alpha_3 \sin \phi_{ca} - C} \right],$$

$$\phi_2 = \tan^{-1} \left[ \frac{\alpha_1 \sin \phi_{ba} + \alpha_2 \sin \phi_{ca} + \alpha_3 \sin \phi_{ea}}{1 + \alpha_1 \cos \phi_{ba} + \alpha_2 \cos \phi_{ca} + \alpha_3 \cos \phi_{ea}} \right]. \quad (19)$$

We examine the correlation between model parameters compatible with 3$\sigma$ limits of the current oscillation data for which we present a random scan of these model parameters over the following ranges:

$$a \in [-0.1, 0.1] \text{ eV}, \epsilon \in [-0.01, 0.01] \text{ eV}, \quad \alpha_1 \in [0, 0.3],$$

$$\alpha_2 \in [0, 1], \quad \alpha_3 \in [0, 0.03], \quad \phi_{ba,ca,ea} \in [-\pi, \pi]. \quad (20)$$
| Parameter | Best fit ± 1σ | 2σ range | 3σ range |
|-----------|--------------|----------|----------|
| $\Delta m_{21}^2 [10^{-5} eV^2]$ | 7.56±0.19 | 7.20–7.95 | 7.05–8.14 |
| $|\Delta m_{31}^2| [10^{-3} eV^2]$ (NO) | 2.55±0.04 | 2.47–2.63 | 2.43–2.67 |
| $|\Delta m_{31}^2| [10^{-3} eV^2]$ (IO) | 2.47±0.04 | 2.39–2.55 | 2.34–2.59 |
| $\sin^2 \theta_{12}/10^{-1}$ | 3.21±0.18 | 2.89–3.59 | 2.73–3.79 |
| $\sin^2 \theta_{23}/10^{-1}$ (NO) | 4.30±0.20 | 3.98–4.78 & 5.60–6.17 | 3.84–6.35 |
| $\sin^2 \theta_{23}/10^{-1}$ (IO) | 5.98±0.17 | 4.09–4.42 & 5.61–6.27 | 3.89–4.88 & 5.22–6.41 |
| $\sin^2 \theta_{13}/10^{-2}$ (NO) | 2.155±0.090 | 1.98–2.31 | 1.89–2.39 |
| $\sin^2 \theta_{13}/10^{-2}$ (IO) | 2.155±0.076 | 1.98–2.31 | 1.90–2.39 |

TABLE II. The experimental values of Neutrino oscillation parameters for 1σ, 2σ and 3σ range [74, 75].

A. With $\phi_{ca} = 0, \phi_{ba} = 0$

Using the model parameters defined in previous discussion, the square of masses and their differences are derived to be,

$$|m_1|^2 = |a|^2(\alpha_3^2 + K^2 + 2K\alpha_3 \cos \phi_{ca}),$$

$$|m_2|^2 = |a|^2(G + \alpha_3 e^{-i\phi_a})(G + \alpha_3 e^{i\phi_a}),$$

$$|m_3|^2 = |a|^2(\alpha_3^2 + K^2 - 2K\alpha_3 \cos \phi_{ca}).$$ (21)

where

$$K = \sqrt{1 + \alpha_1^2 + \alpha_2^2 - (\alpha_1 + \alpha_2 + \alpha_2)}, G = 1 + \alpha_1 + \alpha_2.$$ (22)

In the model, the parameters are $|a|, \alpha_1, \alpha_2, \alpha_3$ and $\phi_{ca}$ (after setting $\phi_{ba} = \phi_{ca} = 0$), which can be constrained by the neutrino oscillation data through the ratio of the solar and atmospheric mass-squared differences, with the relation $r = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$, which is given in [76, 77]. Where the neutrino mass squared differences for solar and atmospheric neutrino oscillations are $\Delta m_{21}^2 = \Delta m_{23}^2 = m_2^2 - m_1^2$ and $|\Delta m_{31}^2| = |\Delta m_{31}^2| \simeq |\Delta m_{32}^2|$ respectively. Using the Eq.(21) and Eq.(22), we can acquire the mathematical expression of $r$ in terms of our model parameter. which is given by

$$r = \frac{(K^2 - G^2 + 2(K - G)\alpha_3 \cos \phi_{ca})(\alpha_3^2 + K^2 - 2K\alpha_3 \cos \phi_{ca})}{4K\alpha_3 \cos \phi_{ca}(G^2 + \alpha_3^2 + 2G\alpha_3 \cos \phi_{ca})}.$$ (23)

From the above Table-II, the best fit values of solar and atmospheric mass-squared difference are $\Delta m_{21}^2 = 7.56 \times 10^{-5}$ (for both NO and IO), $\Delta m_{31}^2 = 2.55 \times 10^{-3}$ (for NO) and $\Delta m_{31}^2 = 2.47 \times 10^{-3}$ (for IO) respectively. Using these experimental results in the Eq.(23), the value of $r$ ($r = 0.032 \pm 0.006$)
is fixed by the data, this relation implies a strong correlation between the values of the parameters $\alpha_3$ and $\cos \phi_{ea}$. Noting that the sign of $\sin \phi_{ea}$ cannot be constrained by the low energy data.

Relations between the phases associated with the masses can be written in the following way,

\[
\begin{align*}
\phi_1 &= \text{arg}(\epsilon + \sqrt{a^2 + b^2 + c^2 - (ab + bc + ca)}), \\
\phi_2 &= \text{arg}(a + b + c + \epsilon), \\
\phi_3 &= \text{arg}(\epsilon - \sqrt{a^2 + b^2 + c^2 - (ab + bc + ca)}).
\end{align*}
\]  

In this framework, the value of two Majorana phases $\alpha$ and $\beta$ can be acquired by the neutrino oscillation data. After few steps of algebraic manipulation, the two CP-violating phases $\alpha$ and $\beta$ with $\alpha_1, \alpha_2, \alpha_3$ and $\phi_{ea}$ model parameters can be related by the following expression:

\[
\begin{align*}
\tan \alpha &= \frac{-\alpha_3 \sin \phi_{ea}}{\alpha_3 \cos \phi_{ea} + G}, \\
\tan \beta &= \frac{2K \alpha_3 \sin \phi_{ea}}{\alpha_3^2 - K^2}.
\end{align*}
\]  

We talk about the dependence of the different model parameters, which are the accurate 3$\sigma$ range of the neutrino oscillation results. The correlation and restrictions on these model parameters are presented in Fig.1 to Fig.5. Note that with $\alpha_2=1$ and the value of $\alpha_1$, $\alpha_3$ varies from 0 to 0.3 and 0 to 0.03 respectively. Fig 1 represents the correlation between the phases $\phi_2$ and $\phi_1$ in (a), $\phi_2$ and $\phi_3$ in (b), lastly $\phi_3$ and $\phi_1$ in (c) respectively. The correlation of $\phi_3$ with $\phi_{ea}$ (a) and $\phi_3$ with $\phi_{ba}$ (b) is presented in Fig. 2. Similarly, the correlation of $a$ with $\phi_{ba}$ (a) and $\phi_{ea}$ (b) are shown in left and right panels of the Fig.3 respectively. We found Majorana like phases $\phi_1$ have the allowed ranges (in radian) of -0.28 to 0.314 from Fig 4(a). Fig 4(b) shows the correlation between $\Sigma m_\nu$ and $m_{eff}$, where the $\Sigma m_\nu$ should lie within a range 0.12 to 0.29, from cosmological observation of total neutrino mass. In a similar way, Fig. 4(c) represents a strong constraint on the parameter $a$ from cosmological observation of total active neutrino mass, which should lie within a range of $\pm 0.025$ to $\pm 0.035$ eV. Fig. 5 represents the correlation between the atmospheric squared neutrino mass with $r$ which obeys the present oscillation data.
FIG. 1. This plot shows the variation between $\phi_2$ and $\phi_1$ (a) $\phi_2$ and $\phi_3$ (b) $\phi_3$ and $\phi_1$ (c).

FIG. 2. This plot represents the variation between (a) $\phi_3$ and $\phi_{ea}$ (b) $\phi_3$ and $\phi_{ba}$.

FIG. 3. This plot represents the variation between (a) a and $\phi_{ba}$ (b) a and $\phi_{ea}$.
FIG. 4. This plot shows the correlation between (a) total neutrino mass with $\phi_1$ (b) total neutrino mass with effective neutrino mass (c) total neutrino mass with $a$.

FIG. 5. This plot shows the correlation between the atmospheric squared neutrino mass with $r$.

B. Correlations between neutrino mixing angles

The neutrino mixing angles within the unitary mixing matrix called as $U_{PMNS}$ [78, 79] is parametrized as follows,

$$U_{PMNS} = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta_{CP}} \\
  -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}s_{23} \\
  s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{12}c_{23}
\end{pmatrix}, \quad (26)$$

where $s_{ij}(c_{ij})$ is the sine (cosine) angle of solar, atmospheric and reactor mixing angles, whose values are known from various neutrino oscillation experiments and thus we can constrain input model parameters as these mixing angles are related to the input model parameters.

It has also been demonstrated that light neutrino masses are diagonalized by $U_{TBM}$, $U_{13}$ containing
the mixing angle $\theta$ and phases. The form of the mixing matrix is expressed in terms of $\theta$, $\delta$ and other phases in the following way \cite{80, 81},

$$
U \equiv U_{\text{PMNS}} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix} = (U_{TB} \cdot U_{13}) \cdot P
$$

(27)

where $\alpha$ and $\beta$ are the two Majorana phases.

The neutrino mixing angles like solar mixing angle $\theta_{12}$, atmospheric mixing angle $\theta_{23}$, reactor mixing angle $\theta_{13}$ and Dirac CP-phase are related to the elements of the $U_{\text{PMNS}}$ using the following set of equations

\begin{align*}
\sin^2 \theta_{13} &= |U_{e3}|^2, \\
\sin^2 \theta_{12} &= \frac{1}{1 - |U_{e3}|^2}, \\
\tan^2 \theta_{23} &= \frac{|U_{\mu3}|^2}{1 - |U_{e3}|^2}.
\end{align*}

(28)

To more explicitly, the mixing angles are related to input model parameters like mixing angle $\theta$ and phase $\delta$ as,

\begin{align*}
\sin^2 \theta_{13} &= \frac{2}{3} \sin^2 \theta, \\
\sin^2 \theta_{12} &= \frac{1}{2 + \cos 2\theta}, \\
\sin^2 \theta_{23} &= \frac{1}{2} (1 + \sqrt{3} \sin 2\theta \cos \delta).
\end{align*}

(29)

The mixing angles prediction are shown as a function of $\theta$ in Fig.6.

Another key parameter known as Jarlskog rephrasing invariant is given by

$$
J_{\text{CP}} = \text{Im} \left[ U_{e1} U_{\mu2} U_{e2}^* U_{\mu1}^* \right] = \frac{\sin \theta_{13}}{3\sqrt{2}} \sin \delta \sqrt{1 - \frac{3}{2} \sin^2 \theta_{13}},
$$

(30)

Using $\sin \theta_{13} \approx 0.16$ and $|\sin \delta| > \frac{1}{2}$, the allowed range $0.026 < |J_{\text{CP}}| < 0.036$ is thus obtained.

With few steps of simple algebra, $J_{\text{CP}}$ and $\delta_{\text{CP}}$ are expressed in terms of input model parameters
FIG. 6. Variation of solar, reactor and atmospheric mixing angles with internal mixing angle $\theta$. The green, blue and red-coloured bands show the $3\sigma$ allowed region for $\theta_{23}, \theta_{12}$ and $\theta_{13}$ respectively [82].

The mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ prediction are shown as a function of $\theta$ and CP-violating phase $\delta_{CP}$. We have plotted the correlations between these parameters in Fig. 7. The coloured bands represent the $3\sigma$ range in the mixing angles from recent global fit data [82].

**C. Comment on neutrinoless double beta decay**

The measure of lepton number violation in neutrinoless double beta decay is called effective Majorana parameter with following form,
FIG. 7. Variation of solar, reactor and atmospheric mixing angles along with Dirac CP-phase with internal mixing angle $\theta$.

Mixing angles($\theta_{12}, \theta_{23}, \theta_{13}$) In terms of $\theta$

|                |              |
|----------------|--------------|
| $\sin \theta_{13}$ | $\frac{2}{\sqrt{6}} \sin \theta$  |
| $\cos \theta_{13}$ | $\frac{2+\cos 2\theta}{3}$  |
| $\sin \theta_{12}$ | $\frac{1}{\sqrt{2+\cos 2\theta}}$  |
| $\cos \theta_{12}$ | $\frac{1}{\sqrt{1+\cos 2\theta}} \sqrt{2+\cos 2\theta}$  |
| $\sin \theta_{23}$ | $\frac{1}{6} \sqrt{1 - \frac{\sqrt{3} \sin 2\theta \cos \delta}{2 + \cos 2\theta}^2}$  |
| $\cos \theta_{23}$ | $\frac{1}{6} \sqrt{5 + \frac{\sqrt{3} \sin 2\theta \cos \delta}{2 + \cos 2\theta}^2}$  |

TABLE III. Relation between the sin and cosine of the solar, reactor and atmospheric mixing angles in terms of input model parameter.

$$m_{ee} = \left| U^2 \right| c_1m_1 + U^2 c_2 m_2 e^{i\alpha/2} + U^2 e_3 m_3 e^{i\beta/2} \right|$$

$$= \frac{2}{3} m_1 \cos^2 \theta + \frac{1}{3} m_2 e^{i\alpha/2} + \frac{2}{3} m_3 \sin^2 \theta e^{i\beta/2}.$$  \hspace{1cm} \text{(32)}

where $m_1, m_2, m_3$ are light neutrino masses, $\alpha$ & $\beta$ are Majorana phases.
FIG. 8. Contour plot for \( m_{ee} \) element of neutrino mass matrix in the plane of input model parameters.

The two Majorana phases, \( \alpha \) and \( \beta \), affect neutrino double decay (see Petr Vogel’s lectures). Their dependence in the neutrinoless double beta decay matrix element is,

\[
|m_{ee}|^2 = m^2_1 |U_{e1}|^4 + m^2_2 |U_{e2}|^4 + m^2_3 |U_{e3}|^4 + 2m_1m_2 |U_{e1}|^2 |U_{e2}|^2 \cos \alpha \\
+ 2m_1m_3 |U_{e1}|^2 |U_{e3}|^2 \cos \beta \\
+ 2m_2m_3 |U_{e2}|^2 |U_{e3}|^2 \cos(\alpha - \beta).
\] (33)

The plot in Fig.8 shows the variation of effective Majorana mass as a function of the lightest neutrino mass, and the model parameter \( \theta \). Similarly, Fig.9 shows the variation of \( m_{ee} \) element of neutrino mass matrix and its square value with input model parameters.

IV. LEPTOGENESIS WITH SCALAR TRIPLET

We briefly discuss here the phenomenology of type-II seesaw mechanism to leptogenesis for accounting matter-antimatter asymmetry of the universe via decay of scalar triplets. The interaction Lagrangian involving scalar triplets is given by,

\[
\mathcal{L}_\Delta = \left( \overline{D}_\mu \Delta^\alpha \right) \left( \overline{D}^\mu \Delta^\alpha \right)^\dagger - m^2_{\Delta \alpha} \text{Tr} \left[ \Delta^\alpha \Delta^\alpha \right] \\
+ f^{ij}_\alpha \ell^{T}_i C i \tau_2 \left( \frac{\tau \cdot \Delta^\alpha}{\sqrt{2}} \right) \ell_j + \mu_{\Delta \alpha} \bar{H} \left( \frac{\tau \cdot \Delta^\alpha}{\sqrt{2}} \right)^\dagger H + h.c.,
\] (34)
FIG. 9. Variation of ee-element of neutrino mass matrix and its square value with input model parameters.

where $\ell^T_L = (\nu_L, e_L)$ and $H^T = (H^+, \frac{v+h+iA}{\sqrt{2}})$ the leptons and scalar boson SU(2) doublets, $\bar{H} = i\tau_2 H^*, \tau^T = (\tau_1, \tau_2, \tau_3)$ (with $\tau_i$ the $2 \times 2$ Pauli matrices) and the scalar $\Delta_\alpha$ transforming under $SU(2)$ as triplets with components, $\Delta_\alpha = (\Delta^1_\alpha, \Delta^2_\alpha, \Delta^3_\alpha)$. Here $f_\alpha$ is the $3 \times 3$ Yakuwa Majorana coupling matrix in flavour space and $C$ is the charge conjugation matrix.

The covariant derivative involving scalar triplet is

$$D_\mu = \partial_\mu - ig T W_\mu - ig B_\mu, \quad (35)$$

where $T$ are the dimension three representations of the SU(2) generators. The fundamental $SU(2)_L$ scalar triplet representation have not all well defined electric charges, electric charge eigenstates are instead given by

$$\Delta_\alpha \equiv \frac{\tau . \Delta_\alpha}{\sqrt{2}} = \begin{pmatrix} \Delta^+ & \Delta^{++} \\ \Delta^0 & -\Delta^+ \end{pmatrix}_{\sqrt{2}}, \quad (36)$$

where

$$\Delta^0_\alpha = \frac{1}{\sqrt{2}} (\Delta^1_\alpha + i \Delta^2_\alpha), \Delta^+_\alpha = \Delta^3_\alpha, \Delta^{++}_\alpha \equiv \frac{1}{\sqrt{2}} (\Delta^1_\alpha - i \Delta^2_\alpha). \quad (37)$$

The interactions involving scalars induced a non-zero vacuum expectation values derived from potential minimization as

$$\langle \Delta_\alpha \rangle = v_{\Delta_\alpha} \simeq \frac{\mu_{\Delta_\alpha} v^2}{2 m_{\Delta_\alpha}^2},$$

The type-II seesaw contribution to light neutrino masses is given by

$$M^\nu = M^\nu_{\text{II}} = \sum_\alpha M^\nu_{\Delta_\alpha} = \sum_\alpha f_\alpha v_{\Delta_\alpha} = \sum_\alpha f_\alpha \frac{\mu_{\Delta_\alpha} v^2}{2 m_{\Delta_\alpha}^2}. \quad (38)$$
As discussed in previous section of neutrino masses and mixing, the light neutrino mass spectrum is derived from diagonalization method using neutrino mixing matrix $U_{PMNS} = U_{TBM} \cdot U_{13} \cdot P$ as,

$$U_{PMNS} = U_{TBM} \cdot U_{13} \cdot P$$

(39)

Defining $\mathbb{M}_\nu^\prime = \text{Diag}\{m_1, m_2, m_3\}$ one can express light neutrino mass matrix in terms of physical mass eigenvalues $m_1, m_2, m_3$ and above mentioned mixing matrix as follows,

$$\mathbb{M}_\nu^\prime = U_{PMNS} \cdot \mathbb{M}_\nu^\prime \cdot U_{PMNS}^T.$$  

(40)

FIG. 10. Tree level diagrams contributing to the asymmetry in scalar triplet decays.

A. Decay rates and CP-symmetry

The tree-level decay of scalar triplets (Fig.10) involve leptonic and scalar final states. The leptonic partial decay widths, depending on the lepton flavor composition of the final states, involve extra factors of $\frac{1}{2}$ which avoid overcounting:

$$\Gamma(\Delta_{\alpha} \rightarrow \ell_i \ell_j) = \frac{m_{\Delta_{\alpha}}}{8\pi} |f_{ij}^{\Delta_{\alpha}}|^2 [1 + |Q - 1|(1 - \delta_{ij})].$$  

(41)

where $Q$ stands for the electric charges of the different SU(2) triplet components, $\Delta_Q = (\Delta_0^0, \Delta_+^+, \Delta_-^+)$. On the other hand, scalar triplet decay modes can be written according to

$$\Gamma(\Delta_{\alpha} \rightarrow HH) = \frac{|\mu_{\Delta_{\alpha}}|^2}{8\pi m_{\Delta_{\alpha}}},$$  

(42)

The total decay rate from scalar triplets decay is given by

$$\Gamma_{\Delta_{\alpha}} = \frac{1}{8\pi} \frac{m_{\Delta_{\alpha}}^2 m_{\Delta_{\alpha}} B_{\alpha}^H + B_{\alpha}^H}{v^2} \frac{1}{\sqrt{B_{\ell}^\alpha B_{H}^\alpha}}.$$  

(43)

18
where the neutrino mass-like parameter $\tilde{m}_{\Delta_a}$ is defined as

$$\tilde{m}_{\Delta_a}^2 = |\mu_{\Delta_a}|^2 \frac{v^4}{m_{\Delta_a}^4}.$$  \hspace{1cm} (44)

with $B_\ell^\alpha$ and $B_H^\alpha$ standing for the $\Delta_a$ triplet decay branching ratios to lepton and scalar final states:

$$B_\ell^\alpha = \sum_{i=e,\mu,\tau} B_{\ell_i}^\alpha = \sum_{i,j=e,\mu,\tau} B_{\ell_{ij}}^\alpha = \sum_{i,j=e,\mu,\tau} \frac{m_{\Delta_a}}{8\pi \Gamma_{\Delta_a}} |f_{ij}^\alpha|^2,$$

$$B_H^\alpha = \frac{|\mu_{\Delta_a}|^2}{8\pi \Gamma_{\Delta_a}}.$$  \hspace{1cm} (45)

where the relation $B_\ell^\alpha + B_H^\alpha = 1$. As can be seen directly from the above equations, for fixed $\tilde{m}_{\Delta_a}$ and $m_{\Delta_a}$, $\Gamma_{\Delta_a}$ exhibits a minimum at $B_\ell^\alpha = B_H^\alpha = \frac{1}{2}$. Thus, the farther we are from $B_\ell^\alpha = B_H^\alpha = \frac{1}{2}$, the faster the scalar triplet decays.

FIG. 11. One-loop diagrams contributing to the asymmetry in scalar triplet decays.

The CP-asymmetry arising from interference between tree level decay of scalar triplets $\Delta_a$ and

FIG. 12. Variation of CP-asymmetry via decay of scalar triplets with input model parameters like $\delta$ (left panel) and $\theta$ (right panel).
one-loop self-energy corrected diagrams (which has drew in Fig. 11) can be put in following form,

$$\epsilon_{i\Delta_a} = \epsilon_{i\Delta_a}^{(L,F)} + \epsilon_{i\Delta_a}^{(F)},$$

$$\epsilon_{i\Delta_a}^{(L,F)} = \frac{1}{2\pi} \sum_{\alpha \neq \beta} \frac{m_{\Delta_a}^2}{m_{\Delta_\beta}} \text{Im} \left[ \left( Y^{\dagger}_\alpha Y_\beta \right)_{ii} \mu_{\Delta_\alpha}^{*} \mu_{\Delta_\beta} \right] g \left( \frac{m_{\Delta_\alpha}^2}{m_{\Delta_\beta}^2} \right),$$

$$\epsilon_{i\Delta_a}^{(F)} = \frac{1}{2\pi} \sum_{\alpha \neq \beta} \frac{m_{\Delta_a}^2}{m_{\Delta_\beta}} \text{Tr} \left[ Y_\alpha Y^{\dagger}_\beta \right] - \left| \mu_{\Delta_\alpha} \right|^2 g \left( \frac{m_{\Delta_\alpha}^2}{m_{\Delta_\beta}^2} \right),$$

with 

$$g(x) = \frac{x(1-x)}{(1-x)^2 + xy} \quad \text{and} \quad y = \left( \frac{\Gamma_{\Delta_a}}{m_{\Delta_a}} \right)^2.$$

As a result of this, the total CP-asymmetry can be written as,

$$\epsilon_{\Delta_a} = \sum_{i=e,\mu,\tau} \epsilon_{i\Delta_a},$$

(47)

The total flavored CP-asymmetries can be recasted as \[83\],

$$\epsilon_{i\Delta_1} = -\frac{1}{2\pi v^2} \sum_{\beta \neq \alpha} \frac{m_{\Delta_\beta}^2}{m_{\Delta_\alpha}} \sqrt{B^\alpha_B B^\alpha_H} \text{Im} \left[ \left( M^{\dagger}_{\Delta_\alpha} M^\alpha_{\Delta_\beta} \right)_{ii} \left( 1 + \frac{m_{\Delta_\beta}}{m_{\Delta_\alpha}} \frac{\text{Tr} \left[ M^\alpha_{\Delta_\alpha} M^{\dagger}_{\Delta_\beta} \right]}{m_{\Delta_\alpha} m_{\Delta_\beta}} \right) \sqrt{B^\alpha_H B^\alpha_H} \right].$$

(48)

After simplification the modified expression for CP-asymmetry due to decay of lightest scalar triplets (assuming \( m_{\Delta_1} \sim \text{TeV} \) and other two triplets around 10 TeV so that \( g(x) \to x \)) is given by

$$\epsilon_{i\Delta_1} \sim -\frac{1}{2\pi v^2} \frac{m_{\Delta_1}^2}{m_{\Delta_1}} \frac{\sqrt{B^\alpha_B B^\alpha_H}}{m_{\Delta_1}} \text{Im} \left[ \left( M^\alpha_{\Delta_1} M^{\dagger}_{\Delta_1} \right)_{ii} \right].$$

(49)

After determining the lepton asymmetry \( \epsilon_1 \), the corresponding baryon asymmetry can be obtained by

$$\gamma_B = \kappa \cdot c \cdot \frac{\epsilon_{\Delta_1}}{g_*},$$

(50)

through electroweak sphaleron processes. Here the factor \( c \) is measure of the fraction of lepton asymmetry being converted into baryon asymmetry and is approximately equal to \(-0.55\). The plot in Fig.12 shows the variation of CP-asymmetry via decay of scalar triplets with input model parameters like \( \theta \) (in the left panel) and \( \delta \) (in the right panel) respectively. Similarly, the plot in Fig.13 represents the CP-asymmetry in the plane of \( \theta \) and \( \delta \) (in the left panel) and in the plane of \( m_{\text{lightest}} \) and the parameter \( \delta \) (in the right panel).

V. LEPTON FLAVOR VIOLATION

It is quite clear that light neutrino contribution to lepton flavor violating (LFV) decays, \( \mu \to e\gamma \) with the exchange of \( W_L \) in the loop diagram is indeed suppressed ( estimated values for this is
FIG. 13. Contour plot for CP-asymmetry in the plane of δ and θ (in left-panel) and in the plane of $m_{\text{lightest}}$ and δ (in right-panel).

$\text{Br}(\mu \to e\gamma) < 10^{-50}$ whereas the current experimental bound has put a bound $\text{Br}(\mu \to e\gamma) < 10^{-12}$). Of late, many works have discussed dominant LFV contributions, however, we planned to focus on low energy lepton flavor (LFV) processes like $\mu \to e\gamma$, $\mu \to eee$ and $\mu \to e$ conversion in nuclei with exchange of TeV scale scalar triplets and their relation with input model parameters like internal mixing angle, phases and lightest neutrino mass. The relevant charged current interaction Lagrangian involving lightest scalar triplet and leptons is given by

$$\mathcal{L}_{\Delta^{\pm}} = \frac{\Delta^{+}}{\sqrt{2}} \left[ \overline{\nu}_L f \ell_L L + \overline{\nu}_L f \nu_L \right] + \text{h.c.},$$

(51)

$$\mathcal{L}_{\Delta^{\pm \pm}} = \Delta^{++} P_L f \ell_L + \Delta^{--} f^\dagger P_R \ell_L.$$  

Before numerically estimating all LFV contributions, let us define,

$$\Gamma^{(0)}_{\mu} \equiv \Gamma(\mu^+ \to e^+ \nu_e \bar{\nu}_e),$$

$$\Gamma_{\text{capt.}}^Z \equiv \Gamma(\mu^- + A(Z,N) \to \nu_\mu + A(Z - 1, N + 1)),$$

and for Branching ratios,

$$\text{Br}_{\mu \to e\gamma} \equiv \frac{\Gamma(\mu \to e\gamma)}{\Gamma^{(0)}_{\mu}},$$

$$R^A_{\mu \to e} \equiv \frac{\Gamma(\mu + A(N, Z) \to e + A(N, Z))}{\Gamma_{\text{capt.}}^Z},$$

$$\text{Br}_{\mu \to 3e} \equiv \frac{\Gamma(\mu \to 3e)}{\Gamma^{(0)}_{\mu}}.$$  

in which case, the recent experimental bound and future sensitivity in near future search experiments are presented in table IV.
| LFV Decays (with Branching Ratios) | Present Bound | Near Future Sensitivity at ongoing search experiments |
|-----------------------------------|---------------|-----------------------------------------------------|
| \( \text{Br} (\mu \rightarrow e\gamma) \) | \( 5.7 \times 10^{-13} \) | \( 6 \times 10^{-14} \) |
| \( \text{Br} (\tau \rightarrow e\gamma) \) | \( 3.3 \times 10^{-8} \) | \( 10^{-9} \) |
| \( \text{Br} (\tau \rightarrow \mu\gamma) \) | \( 4.4 \times 10^{-8} \) | \( 3 \times 10^{-9} \) |
| \( \text{Br} (\mu \rightarrow eee) \) | \( 1.0 \times 10^{-12} \) | \( 10^{-15} \) |
| \( \text{Br} (\tau \rightarrow eee) \) | \( 3.0 \times 10^{-8} \) | \( 10^{-9} \) |
| \( \text{Br} (\tau \rightarrow \mu\mu\mu) \) | \( 2.0 \times 10^{-8} \) | \( 3 \times 10^{-9} \) |

TABLE IV. Branching ratios for different LFV processes and their present experimental bound and future sensitivity values.

![Contour plot](image)

FIG. 14. Contour plot for \(|f f|_{e\mu} 10^4\) in the plane of \(\theta\) and \(\delta\) contributing to the branching ratio \(\mu \rightarrow e\gamma\).

A. \( \mu \rightarrow e\gamma \) Decay

Denoting lightest scalar triplet mass as \(m_{\Delta_1} \approx m_{\Delta}\), vacuum expectation value as \(v_{\Delta}\), the branching ratio of \(\mu \rightarrow e\gamma\) is given below

\[
\text{Br.}(\mu \rightarrow e\gamma) \approx \frac{\alpha_{em}}{192\pi} \frac{|(f^\dagger f)_{e\mu}|^2}{G_F^2} \left( \frac{1}{m_{\Delta_+}^2} + \frac{8}{m_{\Delta_{++}}^2} \right)^2, \tag{52}
\]
FIG. 15. Variation of $|ff|_{e\mu}$ vs. $\theta$ (left-panel) and $\delta$ (right-panel) contributing to the branching ratio $\mu \to e\gamma$.

Considering $m_{\Delta^+} \simeq m_{\Delta^{++}} \equiv m_{\Delta}$, the upper limit of the branching ratio of $\mu \to e\gamma$ from MEG experiment is given by the following bound on $|(f^\dagger f)_{e\mu}|$,

$$|(f^\dagger f)_{e\mu}| < 2.8 \times 10^{-4} \left( \frac{m_\Delta}{1 \text{ TeV}} \right)^2.$$  \hspace{1cm} (53)

We can use the above upper bound to obtain a lower bound on the vacuum expectation value of $\Delta^0$, $v_\Delta$. From which we can calculate

$$|(f^\dagger f)_{e\mu}| = \frac{1}{4v_\Delta^2} |U_{e2}^{} U_{2\mu}^\dagger \Delta m_{21}^2 + U_{e3}^{} U_{3\mu}^\dagger \Delta m_{31}^2|,$$  \hspace{1cm} (54)

where $U = U_{PMNS}$ is the unitary matrix and the above equation is correct. Here, the prediction for $|(f^\dagger f)_{e\mu}|$ and Br.($\mu \to e\gamma$) depends on the Dirac CP-violating phase $\delta_{CP}$ of the standard parametrization of $U_{PMNS}$ or internal phase $\delta$. The figure no.14 represents $|ff|_{e\mu}$ in the plane of $\theta$ and $\delta$ contributing the branching ratio $\mu \to e\gamma$. Similarly, Fig.15 shows the variation between $|ff|_{e\mu}$ with $\theta$ (in the left panel) and with $\delta$ (in the right panel) contributing the branching ratio $\mu \to e\gamma$.

B. The $\mu \to 3e$ Decay

The branching ratio of $\mu \to 3e$ decay within type-II seesaw mechanism with TeV scalar triplets is given below

$$\text{Br.}(\mu \to 3e) = \frac{1}{G_F^2} \left| \left( f^\dagger \right)_{ee} (f)_{\mu e} \right|^2 = \frac{1}{G_F^2 m_{\Delta^+}^4} \frac{|m_{ee}^* m_{\mu e}|^2}{16v_\Delta^4},$$  \hspace{1cm} (55)

At present, the upper limit on this branching ratio is Br.($\mu \to 3e$) < $10^{-12}$ and this bound can be translated to bound on $|(f^\dagger)_{ee}(f)_{\mu e}|$ as follows,
FIG. 16. Contour plot for branching ratio $\mu \to 3e$ in the plane of $\theta$ and $m_{\text{lightest}}$.

$$\left| (f^\dagger)_{ee} (f)_{\mu e} \right| < 1.2 \times 10^{-5} \left( \frac{m_{\Delta^+}}{1 \text{ TeV}} \right)^2. \quad (56)$$

Similarly, here $\text{Br.(} \mu \to 3e)$ depends on the factor $|m^*_{ee} m_{\mu e}|$, which involves the light neutrino masses, input model parameters and Dirac CPV phases in the PMNS matrix $U_{\text{PMNS}}$. For the values of $m_{\Delta^+}$ and $m_{\Delta^{++}}$ in the range of $\sim (1 \text{ TeV})$ GeV and of $v_\Delta \leq 1 \text{ eV}$ of interest, $m_{ee}$ practically coincides with the effective Majorana mass in neutrinoless double beta decay $<m>$.

$$|m_{ee}| = \left| \sum_{j=1}^{3} m_j U^2_{ej} \right| \simeq \langle m \rangle. \quad (57)$$

The Fig.16 shows the variation of the branching ratio $\mu \to 3e$ in the plane $\theta$ and $m_{\text{lightest}}$.

VI. CONCLUSION

In this article, we have discussed the generation of nonzero $\theta_{13}$ in a $\Delta(27)$ symmetric framework. For this, we have extended the particle content of the SM model by adding two Higgs doublets in the Model, which corrects the charged lepton mass and three Higgs triplets, that accounts for the mass to the neutrinos via type-II seesaw mechanism. The choice of these particles helped us to calculate the neutrino mass matrix as well as the neutrino Yukawa matrices dictated by the flavor symmetry imposed $\Delta(27)$ which helps in studying the mixing angles involved in the $U_{PMNS}$ matrix. This model can reproduce all the mixing angles (which are related to the model parameters $\theta$ and $\delta$) consistent with recent experimental findings for a restricted range of parameter space for $\alpha_1$ involved in the theory. This model also describes the non-zero $\delta_{CP}$ violating phase and Jarlskog parameter($J_{CP}$).
Also the effective Majorana parameter is studied $|m_{ee}|$ in terms of two Majorana phases $\alpha$ and $\beta$. We have also discussed on the matter-antimatter asymmetry of the universe through leptogenesis with the decay of TeV scale scalar triplets and variation of CP-asymmetry with input model parameters. Finally, this model explained lepton flavor violating decays like $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ processes.

VII. ACKNOWLEDGMENT

IS would like to acknowledge the Ministry of Human Resource Development (MHRD) for its financial support. IS would also like to thank her PhD supervisor Dr. Raghavendra Srikanth Hundi for his support throughout this project and Dr. Biswaranjan Behera for the initial discussion on the project.

VIII. APPENDIX

A. $\Delta(27)$ Symmetry

The group $\Delta(3n^2)$ is a non-abelian finite subgroup of SU(3) of order $3n^2$. It is isomorphic to the semidirect product of the cyclic group $Z_3$ with $(Z_n \times Z_n)$ \[84].

$$\Delta(3n^2) \sim (Z_n \times Z_n) \times Z_3.$$ 

For $n=3$,

$$\Delta(27) \sim (Z_3 \times Z_3) \times Z_3.$$ 

B. Multiplication Table

The non-abelian $\Delta(27)$ flavor symmetry includes nine one-dimensional representation and two three dimensional irreducible representations $3$ and $\overline{3}$.

If $\left( a_1, a_2, a_3 \right)$ and $\left( b_1, b_2, b_3 \right)$ are the triplets of $\Delta(27)$. Tensor products of these three triplets
are given as following

\[
\begin{pmatrix}
a_1 \\ a_2 \\ a_3
\end{pmatrix}_3 \otimes \begin{pmatrix}
b_1 \\ b_2 \\ b_3
\end{pmatrix}_3 = \overline{3} \oplus \overline{3} \oplus \overline{3} \\
= \begin{pmatrix}
a_1 b_1 \\ a_2 b_2 \\ a_3 b_3
\end{pmatrix}_3 \oplus \begin{pmatrix}
a_2 b_3 \\ a_3 b_1 \\ a_1 b_2
\end{pmatrix}_3 \oplus \begin{pmatrix}
a_3 b_2 \\ a_1 b_3 \\ a_2 b_1
\end{pmatrix}_3.
\]

and the other important multiplication rule between two triplets is given by,

\[
\begin{pmatrix}
a_1 \\ a_2 \\ a_3
\end{pmatrix}_3 \otimes \begin{pmatrix}
b_1 \\ b_2 \\ b_3
\end{pmatrix}_3 = \sum_{i=1}^{9} \oplus 1_i \text{ i=1,2,..9}.
\]
TABLE V. The singlet multiplications of the group $\Delta(27)$.

where,

\[
\begin{align*}
1_1 &= a_1 \bar{b}_1 + a_2 \bar{b}_2 + a_3 \bar{b}_3 \\
1_2 &= a_1 \bar{b}_1 + \omega a_2 \bar{b}_2 + \omega^2 a_3 \bar{b}_3 \\
1_3 &= a_1 \bar{b}_1 + \omega^2 a_2 \bar{b}_2 + \omega a_3 \bar{b}_3 \\
1_4 &= a_1 \bar{b}_2 + a_2 \bar{b}_3 + a_3 \bar{b}_1 \\
1_5 &= a_1 \bar{b}_2 + \omega a_2 \bar{b}_3 + \omega^2 a_3 \bar{b}_1 \\
1_6 &= a_1 \bar{b}_2 + \omega^2 a_2 \bar{b}_3 + \omega a_3 \bar{b}_1 \\
1_7 &= a_2 \bar{b}_1 + a_3 \bar{b}_2 + a_1 \bar{b}_3 \\
1_8 &= a_2 \bar{b}_1 + \omega^2 a_3 \bar{b}_2 + \omega a_1 \bar{b}_3 \\
1_9 &= a_2 \bar{b}_1 + \omega a_3 \bar{b}_2 + \omega^2 a_1 \bar{b}_3.
\end{align*}
\]

with

\[
\omega = e^{\frac{2\pi i}{3}}, \text{i.e.} 1 + \omega + \omega^2 = 0. \tag{58}
\]

The singlets multiplications are given in Table [IV].

C. Scalar Potential and symmetry breaking pattern

Considering all the scalar content of the model, the potential can be written as

\[
V = \mu^2_{\phi_1}(\phi_1^1 \phi_1) + \lambda_{\phi_1}(\phi_1^1 \phi_1)^2 + \mu^2_{\phi_2}(\phi_2^1 \phi_2) + \lambda_{\phi_2}(\phi_2^1 \phi_2)^2 + \mu^2_{\phi_3}(\phi_3^1 \phi_3) + \lambda_{\phi_3}(\phi_3^1 \phi_3)^2 + \mu^2_{\zeta_1}(\zeta_1^1 \zeta_1) + \\
\lambda_{\zeta_1}(\zeta_1^1 \zeta_1)^2 + \mu^2_{\zeta_2}(\zeta_2^1 \zeta_2) + \lambda_{\zeta_2}(\zeta_2^1 \zeta_2)^2 + \mu^2_{\zeta_3}(\zeta_3^1 \zeta_3) + \lambda_{\zeta_3}(\zeta_3^1 \zeta_3)^2 + \lambda_{\phi_1 \phi_2}(\phi_1^1 \phi_1)(\phi_2^1 \phi_2) + \\
\lambda_{\phi_1 \phi_3}(\phi_1^1 \phi_1)(\phi_3^1 \phi_3) + \lambda_{\phi_2 \phi_3}(\phi_2^1 \phi_2)(\phi_3^1 \phi_3) + \lambda_{\phi_1 \phi_2 \phi_3}(\phi_1^1 \phi_1)(\phi_2^1 \phi_2)(\phi_3^1 \phi_3) + \\
\lambda_{\phi_1 \phi_2 \phi_3}(\phi_1^1 \phi_1)(\phi_2^1 \phi_2)(\phi_3^1 \phi_3) + \lambda_{\phi_1 \phi_2 \phi_3}(\phi_1^1 \phi_1)(\phi_2^1 \phi_2)(\phi_3^1 \phi_3) + \\
\lambda_{\phi_1 \phi_2 \phi_3}(\phi_1^1 \phi_1)(\phi_2^1 \phi_2)(\phi_3^1 \phi_3).
\]
\[
\lambda_{\phi_1\phi_3}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{\phi_1\xi_1}(\phi_1^\dagger\phi_1)(\xi_1^\dagger\xi_1) + \lambda_{\phi_2\xi_3}(\phi_2^\dagger\phi_2)(\xi_3^\dagger\xi_3) + \lambda_{\phi_3\xi_1}(\phi_3^\dagger\phi_3)(\xi_1^\dagger\xi_1) + \lambda_{\phi_2\phi_3}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + \\
\lambda_{\phi_2\xi_1}(\phi_2^\dagger\phi_2)(\xi_1^\dagger\xi_1) + \lambda_{\phi_2\xi_3}(\phi_2^\dagger\phi_2)(\xi_3^\dagger\xi_3) + \lambda_{\phi_3\xi_1}(\phi_3^\dagger\phi_3)(\xi_1^\dagger\xi_1) + \lambda_{\phi_3\xi_3}(\phi_3^\dagger\phi_3)(\xi_3^\dagger\xi_3) + \\
\lambda_{\phi_2\xi_2}(\phi_2^\dagger\phi_2)(\xi_2^\dagger\xi_2) + \lambda_{\phi_3\xi_2}(\phi_3^\dagger\phi_3)(\xi_2^\dagger\xi_2) + \lambda_{\phi_3\xi_3}(\phi_3^\dagger\phi_3)(\xi_3^\dagger\xi_3).
\]

From symmetry breaking pattern, first \(\Delta(27)\) symmetry is broken by the flavon fields. The VEV alignment of the scalar fields are denoted as follows. 

\[
<\phi_i> = \frac{v_i}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad i = 1, 2, 3.
\]

[1] M. Herrero, “The Standard model,” NATO Sci. Ser. C 534 (1999) 1–59, [arXiv:hep-ph/9812242](http://arxiv.org/abs/hep-ph/9812242).

[2] M. K. Gaillard, P. D. Grannis, and F. J. Sciulli, “The Standard model of particle physics,” Rev. Mod. Phys. 71 (1999) S96–S111, [arXiv:hep-ph/9812285](http://arxiv.org/abs/hep-ph/9812285).

[3] S. F. Novaes, “Standard model: An Introduction,” in Particles and fields. Proceedings, 10th Jorge Andre Swieca Summer School, Sao Paulo, Brazil, February 6-12, 1999, pp. 5–102. 1999. [arXiv:hep-ph/0001283](http://arxiv.org/abs/hep-ph/0001283).

[4] G. Altarelli, “The Standard model of particle physics,” [arXiv:hep-ph/0510281](http://arxiv.org/abs/hep-ph/0510281).

[5] J. R. Ellis, “Limits of the standard model,” in PSI Zuoz Summer School on Exploring the Limits of the Standard Model Zuoz, Engadin, Switzerland, August 18-24, 2002. 2002. [arXiv:hep-ph/0211168](http://arxiv.org/abs/hep-ph/0211168).

[6] H. M. Lee, “Lectures on Physics Beyond the Standard Model,” 2019. [arXiv:1907.12409](http://arxiv.org/abs/1907.12409).

[7] B. Pontecorvo, “Inverse beta processes and nonconservation of lepton charge,” Sov. Phys. JETP 7 (1958) 172–173. [Zh. Eksp. Teor. Fiz.34,247(1957)].

[8] B. Pontecorvo, “Mesonium and anti-mesonium,” Sov. Phys. JETP 6 (1957) 429. [Zh. Eksp. Teor. Fiz.33,549(1957)].

[9] Z. Maki, M. Nakagawa, and S. Sakata, “Remarks on the unified model of elementary particles,” Prog. Theor. Phys. 28 (1962) 870–880. [34(1962)].

[10] M. Honda, Y. Kao, N. Okamura, A. Pronin, and T. Takeuchi, “Constraints on New Physics from Long Baseline Neutrino Oscillation Experiments,” [arXiv:0707.4545](http://arxiv.org/abs/0707.4545).

[11] KamLAND, S. Abe et al., “Precision Measurement of Neutrino Oscillation Parameters with KamLAND,” Phys. Rev. Lett. 100 (2008) 221803, [arXiv:0801.4589](http://arxiv.org/abs/0801.4589).

[12] Particle Data Group, K. A. Olive et al., “Review of Particle Physics,” Chin. Phys. C38 (2014) 090001.

[13] Particle Data Group, M. Tanabashi et al., “Review of Particle Physics,” Phys. Rev. D98 (2018)
Planck, N. Aghanim et al., “Planck 2018 results. VI. Cosmological parameters,” arXiv:1807.06209

P. Minkowski, “$\mu \rightarrow e\gamma$ at a Rate of One Out of $10^9$ Muon Decays?,” Phys. Lett. 67B (1977) 421–428

R. N. Mohapatra and G. Senjanovic, “Neutrino Mass and Spontaneous Parity Nonconservation,” Phys. Rev. Lett. 44 (1980) 912 [231(1979)].

M. Magg and C. Wetterich, “Neutrino Mass Problem and Gauge Hierarchy,” Phys. Lett. 94B (1980) 61–64

J. Schechter and J. W. F. Valle, “Neutrino Decay and Spontaneous Violation of Lepton Number,” Phys. Rev. D25 (1982) 774.

J. Schechter and J. W. F. Valle, “Neutrino Masses in SU(2) × U(1) Theories,” Phys. Rev. D22 (1980) 2227.

T. P. Cheng and L.-F. Li, “Neutrino Masses, Mixings and Oscillations in SU(2) × U(1) Models of Electroweak Interactions,” Phys. Rev. D22 (1980) 2860

G. Lazarides, Q. Shafi, and C. Wetterich, “Proton Lifetime and Fermion Masses in an SO(10) Model,” Nucl. Phys. B181 (1981) 287–300

R. N. Mohapatra and G. Senjanovic, “Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation,” Phys. Rev. D23 (1981) 165

R. Foot, H. Lew, X. G. He, and G. C. Joshi, “Seesaw Neutrino Masses Induced by a Triplet of Leptons,” Z. Phys. C44 (1989) 441.

E. Ma, “Pathways to naturally small neutrino masses,” Phys. Rev. Lett. 81 (1998) 1171–1174 arXiv:hep-ph/9805219

S. M. Barr, “A Different seesaw formula for neutrino masses,” Phys. Rev. Lett. 92 (2004) 101601 arXiv:hep-ph/0309152

M. Fukugita and T. Yanagida, “Baryogenesis Without Grand Unification,” Phys. Lett. B174 (1986) 45–47

V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, “On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe,” Phys. Lett. 155B (1985) 36

P. Langacker, R. D. Peccei, and T. Yanagida, “Invisible Axions and Light Neutrinos: Are They Connected?,” Mod. Phys. Lett. A1 (1986) 541

R. N. Mohapatra and X. Zhang, “Electroweak baryogenesis in left-right symmetric models,” Phys. Rev. D46 (1992) 5331–5336

M. Flanz, E. A. Paschos, and U. Sarkar, “Baryogenesis from a lepton asymmetric universe,” Phys.
[31] L. Covi, E. Roulet, and F. Vissani, “CP violating decays in leptogenesis scenarios,” Phys. Lett. B384 (1996) 169–174, arXiv:hep-ph/9605319.

[32] A. Pilaftsis, “CP violation and baryogenesis due to heavy Majorana neutrinos,” Phys. Rev. D56 (1997) 5431–5451, arXiv:hep-ph/9707235.

[33] E. Ma and U. Sarkar, “Neutrino masses and leptogenesis with heavy Higgs triplets,” Phys. Rev. Lett. 80 (1998) 5716–5719, arXiv:hep-ph/9802445.

[34] R. Barbieri, P. Creminelli, A. Strumia, and N. Tetradi, “Baryogenesis through leptogenesis,” Nucl. Phys. B575 (2000) 61–77, arXiv:hep-ph/9911315.

[35] T. Hambye, “Leptogenesis at the TeV scale,” Nucl. Phys. B633 (2002) 171–192, arXiv:hep-ph/0111089.

[36] S. Davidson and A. Ibarra, “A Lower bound on the right-handed neutrino mass from leptogenesis,” Phys. Lett. B535 (2002) 25–32, arXiv:hep-ph/0202239.

[37] G. F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, “Towards a complete theory of thermal leptogenesis in the SM and MSSM,” Nucl. Phys. B685 (2004) 89–149, arXiv:hep-ph/0310123.

[38] T. Hambye and G. Senjanovic, “Consequences of triplet seesaw for leptogenesis,” Phys. Lett. B582 (2004) 73–81, arXiv:hep-ph/0307237.

[39] P.-h. Gu and X.-j. Bi, “Thermal leptogenesis with triplet Higgs boson and mass varying neutrinos,” Phys. Rev. D70 (2004) 063511, arXiv:hep-ph/0405092.

[40] W. Buchmuller, P. Di Bari, and M. Plumacher, “Leptogenesis for pedestrians,” Annals Phys. 315 (2005) 305–351, arXiv:hep-ph/0401240.

[41] S. Davidson, E. Nardi, and Y. Nir, “Leptogenesis,” Phys. Rept. 466 (2008) 105–177, arXiv:0802.2962.

[42] F. F. Deppisch, J. Harz, and M. Hirsch, “Falsifying High-Scale Leptogenesis at the LHC,” Phys. Rev. Lett. 112 (2014) 221601, arXiv:1312.4447.

[43] A. Kusenko, K. Schmitz, and T. T. Yanagida, “Leptogenesis via Axion Oscillations after Inflation,” Phys. Rev. Lett. 115 (2015) no. 1, 011302, arXiv:1412.2043.

[44] P. F. Harrison, D. H. Perkins, and W. G. Scott, “A Redetermination of the neutrino mass squared difference in tri-maximal mixing with terrestrial matter effects,” Phys. Lett. B458 (1999) 79–92, arXiv:hep-ph/9904297.

[45] P. F. Harrison, D. H. Perkins, and W. G. Scott, “Tri-bimaximal mixing and the neutrino oscillation
data,” Phys. Lett. B530 (2002) 167, arXiv:hep-ph/0202074.

[46] E. Ma, “Non-Abelian discrete flavor symmetries,” arXiv:0705.0327

[47] Daya Bay, F. P. An et al., “Observation of electron-antineutrino disappearance at Daya Bay,” Phys. Rev. Lett. 108 (2012) 171803, arXiv:1203.1669

[48] Daya Bay, F. P. An et al., “Improved Measurement of Electron Antineutrino Disappearance at Daya Bay,” Chin. Phys. C37 (2013) 011001, arXiv:1210.6327

[49] Daya Bay, F. P. An et al., “Independent measurement of the neutrino mixing angle $\theta_{13}$ via neutron capture on hydrogen at Daya Bay,” Phys. Rev. D90 (2014) no. 7, 071101, arXiv:1406.6468.

[50] RENO, J. K. Ahn et al., “Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment,” Phys. Rev. Lett. 108 (2012) 191802, arXiv:1204.0626

[51] T2K, K. Abe et al., “Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam,” Phys. Rev. Lett. 107 (2011) 041801, arXiv:1106.2822

[52] T2K, K. Abe et al., “Observation of Electron Neutrino Appearance in a Muon Neutrino Beam,” Phys. Rev. Lett. 112 (2014) 061802, arXiv:1311.4750.

[53] Double Chooz, Y. Abe et al., “Indication of Reactor $\bar{\nu}_e$ Disappearance in the Double Chooz Experiment,” Phys. Rev. Lett. 108 (2012) 131801, arXiv:1112.6353

[54] MINOS, P. Adamson et al., “Improved search for muon-neutrino to electron-neutrino oscillations in MINOS,” Phys. Rev. Lett. 107 (2011) 181802, arXiv:1108.0015.

[55] NOvA, R. B. Patterson, “The NOvA Experiment: Status and Outlook,” arXiv:1209.0716 [Nucl. Phys. Proc. Suppl.235-236,151(2013)].

[56] G. Vladilo, “A scaling law for interstellar depletions,” Astrophys. J. 569 (2002) 295, arXiv:astro-ph/0112338

[57] M. Aoki, K. Hagiwara, Y. Hayato, T. Kobayashi, T. Nakaya, K. Nishikawa, and N. Okamura, “Prospects of very long baseline neutrino oscillation experiments with the KEK / JAERI high intensity proton accelerator,” Phys. Rev. D67 (2003) 093004, arXiv:hep-ph/0112338

[58] S. M. Bilenky and S. T. Petcov, “Massive Neutrinos and Neutrino Oscillations,” Rev. Mod. Phys. 59 (1987) 671 [Erratum: Rev. Mod. Phys.60,575(1988)].

[59] W. Rodejohann, “Neutrino-less Double Beta Decay and Particle Physics,” Int. J. Mod. Phys. E20 (2011) 1833–1930, arXiv:1106.1334.

[60] CUORE, F. Alessandria et al., “Sensitivity of CUORE to Neutrinoless Double-Beta Decay,” arXiv:1109.0494.

[61] I. Abt et al., “A New Ge$^{76}$ Double Beta Decay Experiment at LNGS: Letter of Intent,”
[62] GERDA, M. Agostini et al., “Results on Neutrinoless Double-β Decay of $^{76}$Ge from Phase I of the GERDA Experiment,” Phys. Rev. Lett. 111 (2013) no. 12, 122503, arXiv:1307.4720

[63] Majorana, C. E. Aalseth et al., “The Majorana neutrinoless double beta decay experiment,” Phys. Atom. Nucl. 67 (2004) 2002–2010, arXiv:hep-ex/0405008 [Yad. Fiz.67,2025(2004)].

[64] EXO-200, M. Auger et al., “Search for Neutrinoless Double-Beta Decay in $^{136}$Xe with EXO-200,” Phys. Rev. Lett. 109 (2012) 032505, arXiv:1205.5608.

[65] SNO+, M. C. Chen, “The SNO+ Experiment,” in Proceedings, 34th International Conference on High Energy Physics (ICHEP 2008): Philadelphia, Pennsylvania, July 30-August 5, 2008. arXiv:0810.3694.

[66] E. Ma, “Near tribimaximal neutrino mixing with Delta(27) symmetry,” Phys. Lett. B660 (2008) 505–507, arXiv:0709.0507.

[67] W. Grimus and L. Lavoura, “A Model for trimaximal lepton mixing,” JHEP 09 (2008) 106, arXiv:0809.0226.

[68] A. Damanik, “Neutrino Masses via a Seesaw with Heavy Majorana and Dirac Neutrino Mass Matrices from Discrete Subgroup $\Delta(27)$ of SU(3),” in Proceedings, 16th International Seminar on High Energy Physics (QUARKS 2010): Kolomna, Russia, June 6-12, 2010, pp. 126–131. 2010. arXiv:1011.5903 http://quarks.inr.ac.ru/2010/proceedings/www/p4_experiment/Damanik.pdf.

[69] P. M. Ferreira, W. Grimus, L. Lavoura, and P. O. Ludl, “Maximal CP Violation in Lepton Mixing from a Model with Delta(27) flavour Symmetry,” JHEP 09 (2012) 128, arXiv:1206.7072.

[70] E. Ma, “Neutrino Mass Matrix from Delta(27) Symmetry,” Mod. Phys. Lett. A21 (2006) 1917–1921, arXiv:hep-ph/0607056.

[71] I. de Medeiros Varzielas, “Neutrino tribimaximal mixing from (27),” AIP Conf. Proc. 903 (2007) no. 1, 397–400, arXiv:hep-ph/0610351.

[72] E. Ma, “Neutrino Mixing and Geometric CP Violation with Delta(27) Symmetry,” Phys. Lett. B723 (2013) 161–163, arXiv:1304.1603.

[73] A. E. Crescendo Hernández, H. N. Long, and V. V. Vien, “A 3-3-1 model with right-handed neutrinos based on the $\Delta(27)$ family symmetry,” Eur. Phys. J. C76 (2016) no. 5, 242, arXiv:1601.05062.

[74] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola, and J. W. F. Valle, “Status of neutrino oscillations 2018: 3σ hint for normal mass ordering and improved CP sensitivity,” Phys. Lett. B782 (2018) 633–640, arXiv:1708.01186.

[75] S. Gariazzo, M. Archidiacono, P. F. de Salas, O. Mena, C. A. Ternes, and M. Trtola, “Neutrino
masses and their ordering: Global Data, Priors and Models,” JCAP 1803 (2018) no. 03, 011, arXiv:1801.04946.

[76] C. Hagedorn, E. Molinaro, and S. T. Petcov, “Majorana Phases and Leptogenesis in See-Saw Models with $A(4)$ Symmetry,” JHEP 09 (2009) 115, arXiv:0908.0240.

[77] G. Altarelli and D. Meloni, “A Simplest $A_4$ Model for Tri-Bimaximal Neutrino Mixing,” J. Phys. G36 (2009) 085005, arXiv:0905.0620.

[78] I. de Medeiros Varzielas and D. Emmanuel-Costa, “Geometrical CP Violation,” Phys. Rev. D84 (2011) 117901, arXiv:1106.5477.

[79] M. Holthausen, M. Lindner, and M. A. Schmidt, “CP and Discrete Flavour Symmetries,” JHEP 04 (2013) 122, arXiv:1211.6953.

[80] M. Sruthilaya, R. Mohanta, and S. Patra, “$A_4$ realization of Linear Seesaw and Neutrino Phenomenology,” Eur. Phys. J. C78 (2018) no. 9, 719, arXiv:1709.01737.

[81] B. Karmakar and A. Sil, “An $A_4$ realization of inverse seesaw: neutrino masses, $\theta_{13}$ and lepton non-unitarity,” Phys. Rev. D96 (2017) no. 1, 015007, arXiv:1610.01909.

[82] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz, “Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity,” JHEP 01 (2017) 087, arXiv:1611.01514.

[83] D. Aristizabal Sierra, M. Dhen, and T. Hambye, “Scalar triplet flavored leptogenesis: a systematic approach,” JCAP 1408 (2014) 003, arXiv:1401.4347.

[84] C. Luhn, S. Nasri, and P. Ramond, “The Flavor group $\Delta(3n^{**2})$,” J. Math. Phys. 48 (2007) 073501, arXiv:hep-th/0701188.