Abstract

For the two-sensor multichannel autoregressive moving average (ARMA) signals with time-delayed measurements, an equivalent state space model with time-delayed measurements is obtained. Then a measurement transformation method is presented, which can transform this state space model with measurement delays into the state space model without measurement delays. Furthermore, based on the modern time series analysis method, local Kalman predictors are obtained. Then the covariance intersection (CI) fusion Kalman predictor is presented, which avoids computing the cross-covariance compared with the fused Kalman predictor weighted by matrices. It is proved that its accuracy is higher than each local predictor, and lower than that of the fused Kalman predictor weighted by matrices. The geometric interpretations of the local and fused predictors’ accuracy relation are given based on covariance ellipses. A Monte-Carlo simulation example shows that the CI Kalman fuser has higher accuracy and good performance.

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Keywords: Time-delayed measurements; ARMA signal; covariance intersection fusion; covariance ellipse; Kalman predictor.

1. Introduction

Recently, multisensor information fusion Kalman filtering has been applied into many fields [1], such as target tracking, GPS positioning, signal processing and so on. Many results are published about the
weighted fusion estimators weighted by matrices [2,3], which can facilitate fault detection more conveniently and increase the input data rates significantly, but they are just globally suboptimal and require to calculate the cross-covariance among the local estimation errors. However, in many practical applications, the cross-covariances are unknown or it is difficult to compute the cross-covariances, or the computational burden of the cross-covariances is very large. For overcoming this limitation, the covariance intersection (CI) fusion method was presented [4-6] to solve the fusion problem with unknown cross-covariance. This method is of consistency and robustness, because it can give an upper bound of actual filtering error variance, and this upper bound is independent of unknown cross-covariances.

In the existing literature, most of them [2-5] are presented for the systems without measurement delays, only a few [7,8] are presented for the systems with time-delayed measurements, where the information fusion methods adopted are usually centralized fusion or fusion method weighted by matrices. In this paper, for the multichannel ARMA signal system with time-delayed measurement, the equivalent ARMA signal system is obtained by using the measurement transformation method. The local optimal Kalman predictors are presented based on the modern time series analysis method. Then the CI fusion Kalman predictor is obtained, which has high accuracy and good performance.

2. Problem Formulation

Consider two-sensor multichannel ARMA signal system with time-delayed measurements

\[ A(q^{-1})s(t) = C(q^{-1})w(t) \]  
\[ z_i(t) = s(t - \tau_i) + \xi_i(t), \quad i = 1, 2 \]

where \( t \) is the discrete time, \( \tau_i > 0 \) is the measurement delay of the \( i \)th sensor, \( s(t) \in \mathbb{R}^n, z_i(t) \in \mathbb{R}^n \) are the signal and measurement, \( w(t) \in \mathbb{R}^r, \xi_i(t) \in \mathbb{R}^n \) are uncorrelated white noises with zero mean and variances \( Q_s \) and \( Q_{\xi_i} \), respectively. \( A(q^{-1}) \) and \( C(q^{-1}) \) are polynomial matrices of \( q^{-1} \) with the form 

\[ A(q^{-1}) = I_m + A_1 q^{-1} + \cdots + A_n q^{-n}, \quad C(q^{-1}) = C_1 q^{-1} + \cdots + C_n q^{-n} \]

where \( q^{-1} \) is the backward shift operator, \( q^{-1}s(t) = s(t-1) \), \( I_m \) is the \( m \times m \) identity matrix, \((A(q^{-1}), C(q^{-1}))\) are left coprime, with the order \( n_s \geq n_c \).

The aim is to find the local steady-state optimal Kalman predictors \( \hat{z}_i(t \mid t+N) \) \((N>0)\), \( i = 1, 2 \), and the CI fusion Kalman predictor \( \hat{z}_{CI}(t \mid t+N) \).

3. The local steady-state optimal Kalman predictor

The multichannel ARMA signal system (1) and (2) can be transformed into the state space model

\[ x(t+1) = \Phi x(t) + \Gamma w(t) \]
\[ z_i(t) = Hx(t - \tau_i) + \xi_i(t), \quad i = 1, 2 \]
\[ s(t) = Hx(t) \]

where \( \Phi = \begin{bmatrix} -A_1 & \vdots & I_{m(n_s-1)} \\ \vdots & \ddots & \vdots \\ -A_n & 0 & \cdots & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} C_1 \\ \vdots \\ C_n \end{bmatrix}, \quad H = [I_m \ 0 \ \cdots \ \ 0], \quad C_i = 0(i > n_s). \)

Introducing a new measurement \( y_i(t) \) and a new measurement noise \( v_i(t) \)

\[ y_i(t) = z_i(t + \tau_i), v_i(t) = \xi_i(t + \tau_i) \]
So the new measurement equation is obtained as

\[ y_i(t) = Hx(t) + v_i(t) , \quad i = 1,2 \]  

(7)

where \( v_i(t) \) is white noise with zero mean and variance matrix \( Q_{vi} = Q_{ii} \), and it is independent with \( w(t) \).

Denoting the linear space spanned by the stochastic variables \((z_i(t + N), z_i(t + N - 1), \cdots)\) as \( L(z_i(t + N), z_i(t + N - 1), \cdots) \), and the linear space spanned by the stochastic variables \((y_i(t + N - \tau_i), y_i(t + N - \tau_i - 1), \cdots)\) as \( L(y_i(t + N - \tau_i), y_i(t + N - \tau_i - 1), \cdots) \), and we have the relation as \( L(z_i(t + N), z_i(t + N - 1), \cdots) = L(y_i(t + N - \tau_i), y_i(t + N - \tau_i - 1), \cdots) \).

Defining the linear minimum variance predictor \( \hat{s}_i(t | t + N - \tau_i) \) of \( s(t) \) based on \((y_i(t + N - \tau_i), y_i(t + N - \tau_i - 1), \cdots)\) and the linear minimum variance predictor \( \hat{z}_i(t | t + N) \) of \( s(t) \) based on \((z_i(t + N), z_i(t + N - 1), \cdots)\). And the relation between the predictors is given by

\[ \hat{s}_i(t | t + N) = \hat{s}_i(t | t + N - \tau_i), N < 0, \tau_i > 0 \]  

(8)

Hence the relation of the predicting errors \( \hat{s}_i(t | t + N) \) and \( \hat{s}_i(t | t + N - \tau_i) \) is

\[ \hat{s}_i(t | t + N) = \hat{s}_i(t | t + N - \tau_i), i = 1,2 \]  

. Furthermore, the steady-state predicting error variances

\[ P_{si}(N) = E[\hat{s}_i(t | t + N)\hat{s}_i(t | t + N)] \]  

and the steady-state predicting error cross-covariances

\[ P_{si}(N) = E[\hat{s}_i(t | t + N)\hat{s}_j(t | t + N)] \]  

have the relation

\[ P_{si}(N) = P_{si}(N - \tau_i, N - \tau_i), P_{si}(N) = P_{si}(N - \tau_i, N - \tau_i), i, j = 1,2, i \neq j \]  

(9)

where \( P_{si}(N - \tau_i, N - \tau_i) = E[\hat{s}_i(t | t + N - \tau_i)\hat{s}_j(t | t + N - \tau_i)], P_{si}(N - \tau_i, N - \tau_i) = E[\hat{s}_i(t | t + N - \tau_i)\hat{s}_j(t | t + N - \tau_i)] \).

Therefore, the problem of getting the local steady-state optimal Kalman predictor \( \hat{s}_i(t | t + N) \) \((N < 0)\) is converted to that of finding the local steady-state optimal Kalman predictor \( \hat{s}_i(t | t + N - \tau_i) \).

Applying the projection property \([9]\), from (5) we have the following relation:

\[ \hat{s}(t | t + k) = H\hat{x}(t | t + k), k_i = N - \tau_i \]  

(10)

Therefore, the signal estimation problem is converted into the state estimation problem. From (1), (2) and (6), we have the local ARMA innovation models as

\[ A(q^-)y_i(t) = D_i(q^-)e_i(t) , \quad i = 1,2 \]  

(11)

where \( D_i(q^-) = D_{io} + D_{oi}q^- + \cdots + D_{w_i}q^-^\infty \) is stable (i.e. all zeros of det \( D_i(x) \) lie outside the unit circle), \( D_{io} = I_n \), the innovation process \( e_i(t) \in R^n \) is white noise with zero mean and variance matrix \( Q_{ei} \), and

\[ D_{i}(q^-)e_i(t) = C(q^-)w(t) + A(q^-)v_i(t) \]  

(12)

where \( D(q^-) \) and \( Q_{ei} \) can be obtained by Gevers-Wouters iterative algorithm [10].

**Lemma 1** [2]. For the two-sensor system (3) and (7), the local steady-state Kalman predictor \( \hat{x}_i(t + 1 | t) \) of \( x(t) \) is given by

\[ \hat{x}_i(t + 1 | t) = \Psi_{pi}\hat{x}_i(t | t - 1) + K_{pi}y_i(t), i = 1,2 \]  

(13)

\[ \Psi_{pi} = \Phi - K_{pi}H, K_{pi} = \begin{bmatrix} H \\ H\Phi \end{bmatrix}^{(M_{11})} \begin{bmatrix} M_{11} \\ M_{12} \\ \vdots \\ H\Phi^{R-1} \end{bmatrix} \]  

(14)
where $\Psi_p$ is a stable matrix, $K_p$ is the predictor gain, $X^* = (X^T X)^{-1} X^T$. $M_j$ can recursively be computed as $M_j = -A_j M_{j-1} - \cdots - A_{j-n} M_{j-n} + D_j$, with $M_0 = 0$ ($j < 0$), $M_{-1} = I$. The predicting error variances $\Sigma_i$, and the predicting error cross-covariances $\Sigma_{ij}$ satisfy the Lyapunov equations

$$\Sigma_i = \Psi_p \Sigma_{pp}^T + \Gamma Q_i \Gamma^T + K_p Q_m K_p^T, \Sigma_{ij} = \Psi_p \Sigma_{pj}^T + \Gamma Q_i \Gamma^T, i, j = 1, 2, i \neq j$$

(15)

The $-k_i$ steps steady-state Kalman predictor $\hat{x}_i(t | t + k)$ is given by

$$\hat{x}_i(t | t + k) = \Phi^{-k_i-1} \hat{x}_i(t + k_i + 1 | t + k), k_i \leq -2$$

(16)

Without loss of generality, taking $\tau_i < \tau_j$, $k_i = N - \tau_i, k_j = N - \tau_j$, the local steady–state Kalman predicting error variances $P_i(k, k) = E[\hat{x}_i(t | t + k)\hat{x}_i(t | t + k)^T]$ and the local steady–state Kalman predicting error cross-covariances $P_{ij}(k, k) = E[\hat{x}_i(t | t + k)\hat{x}_j(t | t + k)^T]$ are obtained by

$$P_i(k, k) = \Phi^{-k_i-1} \Sigma_i \Phi^{k_i+1} + \sum_{r=0}^{-k_i-2} \Phi^T Q_i \Gamma^T \Phi^{r}, k_i \leq -2$$

$$P_{ij}(k, k) = \Phi^{-k_i-1} \Sigma_i \Phi^{k_i+1} \Psi_p \Gamma^T + \sum_{r=0}^{-k_i-2} \Phi^T Q_i \Gamma^T \Phi^{r}, k_j \leq k_i \leq -2$$

(17)

(18)

Then, according to (10), the local steady-state Kalman predictors of the signal $s(t)$ are obtained

$$\hat{x}_i(t | t + k) = H\hat{x}_i(t | t + k), k_i \leq -1$$

(19)

The local predicting error variance matrices $P_i(k, k) = E[\hat{x}_i(t | t + k)\hat{x}_i(t | t + k)^T]$ and the cross-covariance matrices $P_{ij}(k, k) = E[\hat{x}_i(t | t + k)\hat{x}_j(t | t + k)^T]$ are given as

$$P_i(k, k) = H P_i(k, k) H^T, P_{ij}(k, k) = H P_{ij}(k, k) H^T, k_i \leq k_j \leq -1$$

(20)

where $P(-1,-1) = \Sigma_i, P_{ij}(-1,-1) = \Sigma_{ij}$.

4. The fused steady-state optimal Kalman predictor

4.1. Kalman predictor weighted by matrices

For the two-sensor system (1) and (2), the optimal information fusion Kalman signal predictor weighted by matrices is given by [2]

$$\hat{x}_m(t | t + N) = \sum_{i=1}^2 \Omega_i \hat{x}_i(t | t + N) = \sum_{i=1}^2 \Omega_i \hat{x}_i(t | t + N - \tau_i)$$

(21)

the weighting matrix is given by $[\Omega_1, \Omega_2] = (e^T P^{-1} e)^{-1} e^T P^{-1}$, where $e^T = [I_m, I_m]$, $P = (P_m)_{2n \times 2n}$, the fusion error variance $P_m$ weighted by matrices is obtained by $P_m = (e^T P^{-1} e)^{-1}$, where defining $P_n = P_n(k, k), P_m = P_m(k, k)$ for convenience.

4.2. CI fusion Kalman predictor

For the two-sensor system (1) and (2), when $P_n, P_m$ are unknown, the CI fusion Kalman predictor is

$$\hat{x}_ci(t | t + N) = P_{ci} [\omega P_{ci}^{-1} \hat{x}_i(t | t + N) + (1 - \omega) P_{ci}^{-1} \hat{x}_2(t | t + N)]$$

(22)
the CI fusion error variance matrix $P_{CI}$ is given by $P_{CI} = [\omega P_{s1}^{-1} + (1-\omega)P_{s2}^{-1}]^{-1}$, where the weighted coefficient $\omega \in [0,1]$ and it minimizes the performance index, $J = \min_{\omega} tr P_{CI} = \min_{\omega} tr \{[\omega P_{s1}^{-1} + (1-\omega)P_{s2}^{-1}]^{-1}\}$.

The optimal weighting coefficient $\omega$ can be solved by applying the gold section method or the Fibonacci method. The actual predicting error variance

$$
E[\hat{z}(t|t+N)] = \{[\omega P_{s1}^{-1} + (1-\omega)P_{s2}^{-1}]^{-1}\}
$$

Theorem 1. For the two-sensor system (1) and (2), the accuracy relations of local and fused predictor error variance matrices are

$$
P_m \leq \bar{P}_{CI} \leq P_n, \text{tr} P_m \leq \text{tr} \bar{P}_{CI} \leq \text{tr} P_n, i=1,2
$$

Proof. It has been proved that $P_m \leq P_n$ [11] and $\bar{P}_{CI} \leq P_{CI}$ [4]. The unbiasedness of local predictor $\hat{z}_i(t|t+N)$ can yield the fused predictors $\hat{z}_i(t|t+N)$ and $\hat{z}_f(t|t+N)$ are also unbiased. Reference [12] have proved that the error variance matrix of the linear minimum variance unbiased fused predictor weighted by matrices is less than or equal to that of any other linear unbiased predictors. From (22), CI fused predictor $\hat{z}_f(t|t+N)$ can be considered as one kind of linear unbiased predictor weighted by matrices, so $P_m \leq \bar{P}_{CI} \leq P_{CI} \leq P_n$. In CI fusion algorithm, if taking $\omega = 0$, we have $J = \text{tr} P_n$, and if taking $\omega = 1$, we have $J = \text{tr} P_m$. Hence the optimal weighting coefficient $\omega \in [0,1]$ yields $\text{tr} P_{CI} \leq \text{tr} P_i, i=1,2$. So it is obvious that $\text{tr} P_m \leq \text{tr} \bar{P}_{CI} \leq \text{tr} P_n$.

5. Monte-Carlo simulation example

Consider the two-sensor target tracking system with time-delayed measurements

$$(I_A - Aq^{-1})s(t) = Cq^{-1}w(t)
$$

$$z_i(t) = s(t-\tau_i) + \xi_i(t), i=1,2
$$

where $s(t)=[s_1(t), s_2(t)]^T$, $\tau_i$ is the time-delayed measurement, $s_1(t), s_2(t)$ are the position, speed of the target at time $tT_0$, where $T_0$ is the sampled period, $y_i(t)$ is the measurement of the $i$th sensor for position, $w(t)$ and $\xi_i(t)$ are white noise with zero mean and variances $Q_w$ and $Q_{\xi_i}$, respectively. In simulation, we take that $A_1 = \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 0.5T_0^2 \\ T_0 \end{bmatrix}, T_0 = 0.5, Q_w = 2, Q_{\xi_1} = \begin{bmatrix} 3.6 & 0 \\ 0 & 0.01 \end{bmatrix}, Q_{\xi_2} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.25 \end{bmatrix}, \tau_1 = 1, \tau_2 = 2$.

In order to give a powerful geometric interpretation with respect to accuracy relations of local and fused predictors, the covariance ellipse for a covariance matrix $P$ is defined as the locus of point $x: x^TP^{-1}x = c$ where $c=1$ is assumed without loss of generality. It was proved [6] that $P_m \leq P_n$ is equivalent to that the ellipse for $P_m$ is enclosed in the ellipse for $P_n$. The simulation results are shown in Fig1. For [23], it can be proved that the ellipse of $P_{CI}$ contains the intersection of the ellipses of $P_i$ and $P_j$, and passes their four intersection points [11]. $P_m \leq \bar{P}_{CI}$ means that the ellipse of $P_m$ is enclosed in the ellipse of $\bar{P}_{CI}$. $\bar{P}_{CI} \leq P_{CI}$ means that the ellipse of $\bar{P}_{CI}$ is enclosed in that of $P_{CI}$. $P_m \leq \bar{P}_{CI} \leq P_{CI} \leq P_n (i=1,2)$ means that the ellipses of $P_m$, $\bar{P}_{CI}$ and $P_n$ are enclosed in the intersection of the ellipses of $P_{s1}$ and $P_{s2}$. In order to verify the theoretical results for accuracy relation, $N=50$ Monte-Carlo runs for $t=1,\cdots,300$. The mean square errors (MSE) at time $t$ for local and fused Kalman predictors is defined as sampled average

$$
\text{tr} P_j = \text{tr} E[(\hat{z}_j(t|t-2) - s(t))(\hat{z}_j(t|t-2) - s(t))^T]
$$
\[ \text{MSE}_j(t) = \frac{1}{N} \sum_{i=1}^{N} \left[ (\hat{s}_j^{(i)}(t | t-2) - s^{(i)}(t))^2 + (\hat{s}_j^{(i)}(t | t-1) - s^{(i)}(t))^2 \right], \quad j = 1, 2, m, CI, \]

where \( \hat{s}_j^{(i)}(t | t-2) \) and \( s^{(i)}(t) \) denote the \( r \)-th realization of \( \hat{s}_j(t | t-2) \) and \( s(t) \), respectively. According to the ergodicity, it holds that \( \text{MSE}_j(t) \to \text{trP}_j, j = 1, 2, m, CI \).

The simulation results are shown in Table 1, Fig 1 and Fig 2. From Table 1, it is obvious that the accuracy relation (23) hold. From Fig 2, it is easy to find that the \( \text{MSE}_j(t) \) values of the local and fused Kalman predictors fluctuate around the corresponding theoretical values \( \text{trP}_j \), so the ergodicity holds, and the following relations hold: \( \text{MSE}_m(t) \leq \text{MSE}_{CI}(t) \leq \text{MSE}_2(t) \leq \text{MSE}_1(t) \).

![Fig.1. The accuracy comparison of \( P_j, j = s1, s2, m, CI \) and \( \text{P}_{CI} \) based on covariance ellipses](image)

![Fig.2. Comparison of MSE curves for local and fused Kalman predictors](image)
Table 1. The accuracy comparison of local and fused Kalman predictors

|       | $\text{tr}P_m$ | $\text{tr}\tilde{P}_{CI}$ | $\text{tr}P_{CI}$ | $\text{tr}P_{r1}$ | $\text{tr}P_{r2}$ |
|-------|----------------|---------------------------|------------------|------------------|------------------|
| Values| 3.7944         | 4.2245                    | 6.0549           | 6.4782           | 7.7282           |

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