Flavor violation in supersymmetric theories with gauged flavor symmetries

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Abstract

In this paper we study flavor violation in supersymmetric models with gauged flavor symmetries. There are several sources of flavor violation in these theories. The dominant flavor violation is the tree-level $D$-term contribution to scalar masses generated by flavor symmetry breaking. We present a new approach for suppressing this phenomenologically dangerous effects by separating the flavor-breaking sector from supersymmetry-breaking one. The separation can be achieved in geometrical setups or in a dynamical way. We also point out that radiative corrections from the gauginos of gauged flavor symmetries give sizable generation-dependent masses of scalars. The gaugino mass effects are generic and not suppressed even if the dominant $D$-term contribution is suppressed. We also analyze the constraints on the flavor symmetry sector from these flavor-violating corrections.

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1 Introduction

Supersymmetric extension of the standard model (SM) has been found to be very attractive, in particular, as a solution to the hierarchy problem. Superpartners of the SM fields are expected to be detected in future experiments. Even at present, supersymmetry (SUSY) breaking parameters are constrained from flavor-changing neutral current (FCNC) processes as well as CP violation [1]. That is the so-called SUSY flavor problem and requires sfermion masses between the first and second generations being degenerate, unless they are sufficiently heavy or fermion and sfermion mass matrices are aligned quite well. Such requirements for sfermion masses have been tried to be realized by considering flavor-blind SUSY breaking and/or mediation mechanisms. Actually, various types of flavor-blind mechanisms have been proposed in the literature [2]-[6].

Understanding the origin of fermion masses and mixing angles is also one of the important issues in particle physics. Three copies of SM generations have the exactly same quantum numbers except for their masses, i.e. Yukawa couplings to the Higgs field. In the SM, the exact flavor universality is violated only in the Yukawa sector. One expects that the hierarchical structure of Yukawa matrices is explained by some dynamics beyond the SM, and such additional dynamics necessarily leave some imprint of flavor violation. In supersymmetric models, a mechanism for realistic fermion masses breaks the flavor universality and generally makes the corresponding sfermion masses flavor-dependent.

One of the salient mechanisms for generating hierarchical Yukawa couplings is the Froggatt-Nielsen (FN) mechanism with additional $U(1)$ gauge symmetries [7, 8]. In the FN scenarios, flavor-dependent $U(1)$ charges are assigned to matter fields so that realistic Yukawa matrices are effectively realized in terms of higher-dimensional operators. There is a certain reason to believe that the $U(1)$ symmetries should be gauged; any global symmetry is expected to be unstable against quantum gravity and hence accidental. Therefore the $U(1)$ flavor symmetries that exactly control such operators should be gauged. Consequently, after flavor symmetry breaking, the auxiliary $D$ fields of the $U(1)$ vector multiplets give additional contribution to sfermion masses, which is proportional to $U(1)$ charges and hence flavor-dependent [9, 10]. That indeed gives significant modification of sparticle spectrum and in some cases is confronted with the SUSY flavor problem. This is called the $D$ problem in the present paper.

In this paper, we present an idea for suppressing the flavor-dependent $D$-term contribution to sfermion masses. The $D$-term contribution is generated when one integrates out heavy fields which develop vacuum expectation values (VEVs) that break the $U(1)$ gauge symmetries. As will be reviewed below, the modification of low-energy spectrum is determined by soft SUSY-breaking masses of the heavy fields. Our idea is that the $D$-term contribution is suppressed if the soft masses of the heavy fields can be made small. We will present illustrative models where this idea is realized in a dynamical or geometrical way. Unlike other approaches, the models presented here have an advantage that origins of the Yukawa hierarchy can be addressed within the same framework.

However, even if the dominant $D$-term contribution is suppressed, there still remains
a flavor-violating effect from the $U(1)$ gauge symmetries; flavor-dependent scalar masses are radiatively generated by $U(1)$ gaugino loop graphs, which involve the gaugino-fermion-sfermion vertices proportional to $U(1)$ quantum numbers. Such radiative effect is described in terms of renormalization-group equations (RGEs) for scalar masses. Unlike the above tree-level $D$-term contribution, the gaugino loop effect does not depend on SUSY-breaking scalar masses. Therefore even with reduced $D$-term contribution, e.g. by assuming the universal sparticle spectrum, flavor violation from the $U(1)$ flavor symmetries still remains and turns to be detectable signatures for flavor physics.

This paper is organized as follows. In Section 2 we describe scalar masses in supersymmetric theories with a gauged abelian flavor symmetry. In addition to the $D$-term contribution, we point out subleading but sizable flavor violation due to the abelian gaugino. In Section 3, we will discuss how to suppress the dominant $D$-term flavor violation. After a brief survey of the existing proposals in Section 3.1, we present in Section 3.2 the models with extra dimensions, while another model based on four-dimensional superconformal dynamics is presented in Section 3.3. We also examine how much degeneracy of sfermion masses is expected in these models. In Section 4, radiative corrections from the soft gaugino mass are shown to be potentially dangerous and give significant constraints on model parameters such as soft gaugino masses. Section 5 summarizes our results.

2 Sfermion masses with $U(1)$ gauge symmetry

2.1 $D$-term contribution

In this section, we discuss sfermion masses in the presence of a $U(1)$ horizontal gauge symmetry, denoted by $U(1)_X$ throughout this paper. The $D$-term effect has been considered in various contexts [11]. As simple examples, we study two types of models where a non-vanishing VEV of the auxiliary component $D$ of $U(1)_X$ vector multiplet is actually generated. However the properties presented here are generic for any model of $D$-term contribution.

As the first example, let us consider a pseudo-anomalous abelian gauge symmetry, which often appears in string models [12]. In this case, the Fayet-Iliopoulos (FI) term is generated by a non-vanishing VEV of the dilaton or moduli field, whose nonlinear shift cancels the $U(1)_X$ anomaly. We treat the coefficient of the FI term as a constant, for simplicity.* The supersymmetric scalar potential relevant to the $U(1)_X$ gauge sector is written as

$$V_{\text{SUSY}} = \frac{-1}{2g_X^2} D^2 + D \left( \xi_{\text{FI}} + \sum_{i=1}^{N} q_i |\phi_i|^2 + \sum_{j=1}^{N} \bar{q}_j |\bar{\phi}_j|^2 \right) + \cdots. \quad (2.1)$$

*If the dilaton and moduli fields are treated as dynamical fields, the FI term and Kähler metric of the FN field depend on these fields. In this case, as was shown in Ref. [10], the formula for $D$-term contribution becomes different from that in softly-broken global SUSY models with a constant FI term. The suppression mechanism presented in this paper should be reconsidered in such a case.
Here $g_X$ is the gauge coupling and $\xi_{\text{FI}}$ is the coefficient of the FI term, which we take positive without lose of generality. The $U(1)_X$ charges of scalars $\phi_i$ and $\bar{\phi}_i$ are denoted as $q_i (> 0)$ and $\bar{q}_i (< 0)$, respectively. The equation of motion for $D$ is

$$D \frac{D}{g_X^2} = \xi_{\text{FI}} + \sum_{i=1}^N q_i |\phi_i|^2 + \sum_{j=1}^{\bar{N}} \bar{q}_j |\bar{\phi}_j|^2. \quad (2.2)$$

Then in the supersymmetric limit $D = 0$, the negatively-charged fields $\bar{\phi}_i$ generally develop nonzero VEVs and the abelian gauge symmetry is broken at $M_X \sim \xi_{\text{FI}}^{1/2}$. The scalar fields with vanishing VEVs remain massless but $\bar{\phi}_i$ decouple around the $M_X$ scale.

In this work, we assume that the $U(1)_X$ breaking scale $M_X \sim \xi_{\text{FI}}^{1/2}$ is smaller than a cutoff $\Lambda$ of the theory. It is known [7, 8] that the ratio $\xi_{\text{FI}}^{1/2}/\Lambda$ can be an origin of the hierarchy of Yukawa couplings. Consider, for example, a superpotential $W$ includes the following non-renormalizable operators

$$W = y_{ij} \left( \frac{\bar{\phi}}{\Lambda} \right)^{n_{ij}} \phi_i \phi_j H, \quad (2.3)$$

where $H$ denotes the electroweak Higgs field, and $\bar{\phi}$ is a negatively-charged field that develops a nonzero VEV of order $\xi_{\text{FI}}^{1/2}$. The power $n_{ij}$ is determined by the $U(1)_X$ charge conservation to be $n_{ij} = (q_i + q_j + q_H)/|\bar{q}|$. The operators (2.3) induce the effective Yukawa couplings

$$y'_{ij} = \lambda^{n_{ij}} y_{ij}, \quad \lambda \equiv \left( \frac{\langle \bar{\phi} \rangle}{\Lambda} \right)^{|\bar{q}|}. \quad (2.4)$$

The factor $\lambda$ represents a unit of hierarchy of Yukawa couplings and is usually taken to be of the order of the Cabibbo angle. In this way, a realistic hierarchy of low-energy Yukawa couplings can be obtained by assigning different charges to matter fields $\phi$ [7, 8].

We add a remark on possible realization of the FN mechanism in weakly-coupled heterotic string models. In this case, the $U(1)$ anomaly is cancelled by a nonlinear shift of the dilaton-axion multiplet [13], and the FI term $\xi_{\text{FI}}$ is generated at loop level. Consequently $\xi_{\text{FI}}$ naturally has an appropriate size for the Yukawa hierarchy. This possibility provides us with a strong motivation for regarding an anomalous $U(1)$ as a gauged flavor symmetry. Note also that in the case of anomalous $U(1)$ symmetry, the axion-gauge mixing generates an additional contribution to the gauge boson mass, $2(\xi_{\text{FI}}/\Lambda)^2$. Since this contribution is suppressed by $\xi_{\text{FI}}/\Lambda^2$ compared with that from the scalar VEV $\langle \bar{\phi} \rangle$, we will neglect this effect in our analysis below.

The SUSY vacuum is shifted when soft SUSY-breaking masses for scalars are introduced. The scalar potential is now given by

$$V = V_{\text{SUSY}} + \sum_{i=1}^N m_i^2 |\phi_i|^2 + \sum_{j=1}^{\bar{N}} \mathring{m}_j^2 |\bar{\phi}_j|^2 + \cdots, \quad (2.5)$$

where $m_i^2$ and $\mathring{m}_j^2$ are arbitrary mass parameters. The ellipsis denotes other SUSY-breaking terms irrelevant to our analysis. In $V_{\text{SUSY}}$, the $D$ component is replaced with scalar fields.
through the equation of motion (2.2). Minimizing the potential with respect to negatively-charged fields, we find that the $D$ component obtains a VEV

$$\langle D \rangle = \frac{\bar{m}^2}{|\bar{q}|}.$$  

(2.6)

In this expression, $\bar{m}^2/|\bar{q}|$ is the minimum value of $\bar{m}^2/|\bar{q}_i|$ in the model, since the effective potential around the minimum along the $\phi_i$ direction takes a value of $O(\bar{m}^2/|\bar{q}_i|) \xi_{FI}$. Without superpotential, only the scalar fields with such minimal value of ratio, $\bar{m}^2/|\bar{q}|$, can contribute to the above equation. For example, in the case of universal scalar masses, (a combination of) scalar fields with the largest negative value of $U(1)$ charge obtain the VEV (2.6). In this way, the $D$-flat direction is lifted by SUSY-breaking masses of scalar fields with negative quantum numbers. From Eqs. (2.1) and (2.6), one finds a formula for the $D$-term contribution to the masses of light scalars:

$$m^2_{D_i} = q_i \langle D \rangle = \frac{q_i}{|\bar{q}|} \bar{m}^2.$$  

(2.7)

The most important property of the formula (2.7) is that the induced scalar masses squared are proportional to their $U(1)_X$ charges. This fact gives a significant implication to flavor physics. As explained above, realistic low-energy Yukawa couplings are generated by assigning different charges to matter fields. If all SUSY-breaking masses are of the same order of magnitude, this scalar mass difference leads to large FCNC amplitudes. For example, lepton-flavor violation from flavor symmetry $D$-terms was discussed in [14].

Besides the flavor problems, the induced scalar masses (2.7) have several interesting properties. Firstly, the contribution is independent of the $U(1)_X$ gauge coupling constant $g_X$. Therefore the formula (2.7) remains valid even if $g_X$ is small. [Note, however, that the complete global limit ($g_X \to 0$) cannot be taken since the $U(1)_X$ symmetry is broken only if the condition $g_X^2 > \bar{m}^2/(-\xi_{FI})$ is satisfied so that the scalar potential (2.5) is unstable around the origin of the moduli space.] Secondly, the $D$-term contribution does not depend on the symmetry-breaking scale $M_X$, either. The $D$-term contribution is proportional to a tiny deviation from supersymmetric conditions (flat directions), and the deviation is determined by supersymmetry breaking independently of the gauge symmetry breaking [11]. As a result of these properties, the scalar masses induced via the $D$ term appear for any value of gauge coupling and symmetry-breaking scale.\textsuperscript{1} It is also found from (2.7) that the normalization of $U(1)_X$ charges does not affect the size of $D$-term contribution.

The $D$-term contribution also appears for non-anomalous abelian gauge symmetries. Here we consider a model which contains vector-like fields $Y$ and $\bar{Y}$ with $U(1)_X$ charges $\pm q_Y$, and a gauge singlet $Z$. Taking a renormalizable and gauge-invariant superpotential

$$W = f Z (Y \bar{Y} - M^2_X),$$  

(2.8)

\textsuperscript{1}This is true provided that soft SUSY-breaking terms are present already at the scale of scalar VEVs. On the other hand, it will be intuitively clear that the $D$-term contribution is absent if soft masses arise only at low energy as in the gauge mediation of SUSY breaking [3]. This can be seen from the fact that, to correctly determine the scalar VEVs, one has to use the renormalization-group improved potential [15] in which all running parameters are evaluated at the VEV scale.
and introducing soft SUSY-breaking terms, we obtain at the minimum of the scalar potential a VEV

\[ |\langle Y \rangle|^2 - |\langle \bar{Y} \rangle|^2 \simeq \frac{1}{2q_Y g_X^2} (m_Y^2 - m_{\bar{Y}}^2), \tag{2.9} \]

where \( m_Y^2 \) and \( m_{\bar{Y}}^2 \) are the soft scalar masses of \( Y \) and \( \bar{Y} \), respectively. If one introduces an \( R \) symmetry under which the singlet \( Z \) has charge +2, the superpotential (2.8) is the most generic one. The abelian gauge symmetry is broken at \( \langle Y \rangle \simeq \langle \bar{Y} \rangle \simeq M_X \). The result (2.9) is not modified even if one stabilizes scalar fields with other types of superpotentials. When the theory contains a chiral multiplet with a charge \( q_i \), the induced mass for its scalar component is given by

\[ m_{D_i}^2 = q_i q_Y g_X^2 |\langle Y \rangle|^2 - |\langle \bar{Y} \rangle|^2 \simeq \frac{q_i}{2q_Y} (m_Y^2 - m_{\bar{Y}}^2). \tag{2.10} \]

This has the same form as in Eq. (2.7) and shares the properties discussed above with the anomalous \( U(1) \) case. Note that in this model the \( D \)-term contribution vanishes as long as soft scalar masses are universal. However unless the exact universality is a result of symmetries of models, the degenerate spectrum is expected to be split by radiative corrections governed by renormalization-group (RG) evolutions. Then non-vanishing \( D \)-terms are generated.

In this way, the \( D \)-term contribution generically appears in supersymmetric models with abelian gauge symmetries. We note that abelian gauge factors always appear when the rank of gauge group is reduced through gauge symmetry breaking, e.g. as in grand unified theories (GUTs). For example, there are several proposals for realistic Yukawa matrices where matter multiplets of different generations belong to different representations of GUT gauge group [16]. After gauge symmetry breaking, non-vanishing VEVs of abelian \( D \) components induce violation of flavor universality. One important notice in this case is that the \( U(1)_X \) gauge couplings cannot be arbitrarily small because they are unified into GUT gauge group which also contains the SM ones.

### 2.2 scalar soft masses

In this work, we focus on the effects of \( U(1)_X \) gauge symmetry on the scalars with the same quantum numbers of the SM gauge symmetry. We neglect effects from Yukawa interactions. This is justified for the light generations, for which the experimental constraints are severe. When one includes the third generation with large Yukawa couplings (e.g. for top quark), flavor violation generated by these Yukawa couplings may be large. Experimental constraints are, however, still weak for the third generation and will be a target of next generation of experiments. We leave it to future investigations.

The scalar mass under consideration generally takes the form

\[ m_i^2 = m_{D_i}^2 + m_g^2 + m_{X_i}^2 + m_{\bar{Y}}^2. \tag{2.11} \]

at the flavor symmetry breaking scale \( M_X \). The first term on the right-handed side is an initial value of the scalar mass. This part is expected to be generated by SUSY-breaking
dynamics and its structure is highly model-dependent. To focus on the $U(1)_X$ part, we take an assumption that $m_{0_i}^2$ are not dominant sources of flavor violation. In what follows, $m_{0_i}^2$ are taken to be generation-independent for simplicity, but this assumption is not necessary in our concrete models in Section 3.2.

The second term $m_g^2$ contains the effects from all flavor-blind gauge interactions such as GUT or SM ones. For example, the RG evolution due to soft gaugino masses is described as

$$m_g^2 = -\frac{8}{16\pi^2} \sum_a C_2^a(R) \int_\Lambda^{M_X} \frac{d\mu}{\mu} g_a^2 M_{\lambda_a}^2 + \cdots,$$  \hspace{1cm} (2.12)

where $g_a$ and $M_{\lambda_a}$ are the gauge coupling constant and soft gaugino mass of the $G_a$ gauge sector, respectively. The scale $\mu$ denotes the renormalization point and $C_2^a(R)$ is the quadratic Casimir of $G_a$ for the corresponding scalar in the representation $R$. In addition, there is another type of gauge contribution if the theory includes a generation-independent abelian factor like the SM hypercharge. That introduces $D$-term contribution and the $\text{Tr}(Qm^2)$ term in scalar mass RGEs. It is potentially important that low-energy sparticle spectrum is modified in the presence of such extra $U(1)$ symmetries. Generation-blind but intergeneration-dependent $D$-term contribution has been investigated in various contexts [11]. In RG evolutions of scalar masses, the net effect is to shift the low-energy value of hypercharge $D$ term which is proportional to $\text{Tr}(Q_Y m^2)$. All of these contributions just scale overall magnitude of soft scalar masses and therefore are harmless to the flavor problems.

The remaining two factors, the third and last terms in (2.11), are possible flavor-violating terms associated with the $U(1)_X$ gauge symmetry. The form of $m_{D_i}^2$ was given in the previous section,

$$m_{D_i}^2 = q_i \langle D \rangle.$$  \hspace{1cm} (2.13)

The VEV $\langle D \rangle$ is written in terms of scalar VEVs which break the flavor gauge symmetry, and the scalar VEVs are determined by their soft masses. On the other hand, the last term represents radiative corrections via RGEs from the $U(1)_X$ gauge interactions and is explicitly given by

$$m_{X_i}^2 = -\frac{8q_i^2}{16\pi^2} \int_\Lambda^{M_X} \frac{d\mu}{\mu} g_X^2 M_{\lambda_X}^2 + \frac{2q_i}{16\pi^2} \int_\Lambda^{M_X} \frac{d\mu}{\mu} g_X^2 \text{Tr}(Q_X m^2) + \cdots,$$  \hspace{1cm} (2.14)

where $M_{\lambda_X}$ is the $U(1)_X$ soft gaugino mass, $Q_X$ is the charge operator, and the trace is taken over all scalar fields charged under the $U(1)_X$ symmetry. The second term in (2.14) is generally non-vanishing for the case of non-universal soft scalar masses and also for anomalous $U(1)_X$ case even with universal scalar masses. However it should be noted that the masses squared $\tilde{m}_i^2$ of symmetry-breaking scalars $\tilde{\phi}_i$ are also affected by the same factor $\text{Tr}(Q_X m^2)$ in RG evolutions. That is, $\tilde{m}_i^2$ evaluated at the scale $M_X$ contains a term

$$\tilde{m}_i^2 = \cdots + \frac{2q_i}{16\pi^2} \int_\Lambda^{M_X} \frac{d\mu}{\mu} g_X^2 \text{Tr}(Q_X m^2).$$  \hspace{1cm} (2.15)
Interestingly, when substituted into the formula (2.7), this contribution exactly cancels the second term in (2.14). The same is true for non-anomalous $U(1)$ cases, (2.10). Therefore we can safely drop this type of effects in the following analysis and concentrate on the first term in (2.14).

We here comment on characteristic features of two flavor-violating contributions from abelian gauge dynamics, $m_{D_i}^2$ and $m_{X_i}^2$. They have rather different properties from each other. The $D$-term contribution $m_{D_i}^2$ is the dominant, tree-level source of flavor violation from the gauged flavor symmetry. The induced masses squared of light scalars are linearly proportional to their $U(1)_X$ charges, but independent of the gauge coupling strength and the symmetry-breaking scale. On the other hand, $m_{X_i}^2$ is the gaugino radiative correction and depends on the values of gauge coupling and symmetry-breaking scale. As a result, although loop-suppressed, this sub-dominant contribution can be enhanced and become comparable with $m_{D_i}^2$. Moreover, $m_{X_i}^2$ does not depend on the magnitude of soft scalar masses, unlike $m_{D_i}^2$. This fact implies that even if the apparently dominant contribution $m_{D_i}^2$ were suppressed, for example, by taking the universality assumption, $m_{X_i}^2$ is left unchanged and becomes the main source of flavor violation. Combining these two properties of $m_{X_i}^2$, we find that the $U(1)_X$ gaugino effect is an interesting and unexplored effect in supersymmetric models with flavor $U(1)_X$ symmetries. This issue will be discussed in Section 4.

3 Suppressing $D$-term contributions

The aim of this section is to discuss how to suppress the tree-level $D$-term contribution which is known to be a dominant source of flavor violation with $U(1)$ flavor symmetry. We first mention several possible approaches to the $D$ problem, some of which have been discussed in the literature. Our concrete models will be presented in Sections 3.2 and 3.3.

3.1 Approaches to the $D$ problem

A well-known solution to the SUSY flavor problems is to take $m_{D_i}^2$ very large, i.e. of the order of $O(10)$ TeV [17]. With such heavy scalars, flavor- and CP-violating processes involving these fields are suppressed by their large masses and do not lead to any severe constraints on SUSY-breaking parameters. It is also interesting that together with the FN mechanism, a larger $U(1)_X$ charge simultaneously leads to a larger scalar mass and a smaller Yukawa coupling. This is precisely the situation relevant to the first and second generations on which the experimental constraints are severer. This elegant solution, however, suffers from some unsatisfactory points, including the destabilization of the true electroweak vacuum at two-loop level [18], fine-tuning of parameters required for the CP-violation constraints [19], etc. This solution also rules out supersymmetry as a possible explanation of the experimental result for the anomalous magnetic moment of the muon.
The $D$ problem becomes less severe if one assigns the same $U(1)_X$ charge to the matter fields in the same representation under the SM gauge symmetry. It was pointed out that universal charges for three-generation left-handed leptons can explain the large flavor mixing observed at the neutrino experiments (but with a bit large value of $U_{e3}$ matrix element [20]). Within the FN mechanism, however, this could only be applied to the lepton sector. Mass hierarchy and small flavor mixing in the quark sector are realized by assigning different $U(1)_X$ quantum numbers.

Another possibility is a cancellation of $m^2_D$ with other contributions to scalar masses. An obvious choice comes from superpotential $F$-terms. It has been argued that $F$- and $D$-term effects add up to zero in specific situations [9, 10]. A cancellation may occur in $D$-term itself if there are several scalar fields developing non-vanishing VEVs. However cancellation of more than one flavor-violating effects seems unnatural unless it follows from some underlying mechanism.

Now, there is a simple and natural possibility which, to our knowledge, has not been addressed in the literature. As was seen in the previous section, the $D$-term contribution is proportional to a deviation from the $D$-flat direction, and such deviation is determined by the soft masses of the scalars that acquire flavor symmetry-breaking VEVs. Accordingly, if these soft masses can be reduced, the $D$-flat direction is not lifted and $D$-term contributions become negligible. It is this possibility that we shall pursue in the following part of this section. We will present two illustrative examples where our idea can be realized. The point is to separate the $U(1)_X$-breaking sector from the SUSY-breaking sector. We construct toy models in which this separation is achieved in a geometrical or dynamical way. In what follows, we assume, for simplicity of presentations, that only a single field $\chi$ obtains a VEV of the $U(1)_X$ flavor symmetry breaking.

### 3.2 Suppression via extra dimensions

We now describe how our idea of suppressing $D$-term contributions can be realized by considering a suitable field configuration in higher-dimensional spacetime. For definiteness, we consider a five-dimensional theory and suppose that there are two four-dimensional branes at different positions in the fifth dimension; the visible-sector matter fields, i.e. quarks and leptons, are assumed to be confined on one brane, while supersymmetry breaking occurs on another brane (called the hidden brane). As was pointed out in Ref. [21], this setup naturally explains the absence of uncontrollable gravity-mediated effects including large flavor-violating ones, provided that there is no light mode whose wavelength is longer than the distance between the two branes. This is one of the most attractive points of considering the presence of additional dimensions. However the $D$-term contribution to scalar masses can be added [22]. If flavor $U(1)_X$ gauge symmetries are introduced in this framework, suppressing their $D$ terms is required.

We apply the idea of geometrical splitting to separate the flavor symmetry-breaking sector from SUSY-breaking one. Specifically we require that, in addition to the quarks and leptons, the $\chi$ field is also confined to the visible brane. This is a setup crucial for
our scenarios. We further suppose that the $U(1)_X$ vector multiplet is also confined on the visible brane. This is just a technical assumption for simplifying discussion. If $U(1)_X$ is put in the bulk, the supersymmetric five-dimensional abelian gauge theory has a rich structure of vacua, which requires complicated analyses. We do not pursue this possibility in this paper. On the other hand, there are two choices for the SM gauge sector; on the visible brane or in the bulk.

First let us consider the SM gauge multiplets on the visible brane. In this case, all the SM fields and $\chi$ are on the visible brane. Though couplings to the hidden brane are suppressed, soft terms do not completely vanish. It has been shown that SUSY-breaking effects are transmitted at loop level to the visible sector via superconformal anomaly [4]. The anomaly-mediated spectrum is roughly given by

$$
M_{\lambda_{SM}} \sim \frac{g_{SM}^2}{16\pi^2} m_{3/2}, \quad m_{2_{SM}}^2 \sim \left(\frac{g_{SM}^2}{16\pi^2} m_{3/2}\right)^2,
$$

where $M_{\lambda_{SM}}, m_{2_{SM}}$ and $g_{SM}$ are the gaugino masses, sfermion masses, and the gauge coupling constants, respectively, all in the SM sector, and $m_{3/2}$ is the gravitino mass. Notice that the absence of tree-level contribution to $m_{2_{SM}}^2$ is guaranteed in our setup thanks to the separation of $\chi$ from SUSY breaking. Moreover the singlet $\chi$ receives only the soft scalar mass squared proportional to $g_X^4$. Therefore with a sufficiently small value of the gauge coupling $g_X$, $m_{\chi}^2$ is negligibly small which in turn implies that the $D$-term contribution is suppressed.‡ Actually enough suppression of $D$-term flavor violations requires a condition roughly estimated as

$$
g_X^4 \lesssim \epsilon g_{SM}^4. \quad (3.2)
$$

The factor $\epsilon$ denotes an order of tuning in the sfermion masses in order to satisfy the constraints from flavor physics experiments. For example, a limit from the $K^0-\bar{K}^0$ mixing phenomena implies $\epsilon \sim 10^{-(2-3)}$. This constraint is satisfied with a natural value of $g_X$ comparable to $g_{SM}$.

Another interesting pattern of spectrum is obtained by assuming that the SM gauge multiplets reside in the five-dimensional bulk and other fields are stuck on the visible brane. This situation (for the SM sector) corresponds to gaugino-mediated supersymmetry breaking [5]. The sparticle spectrum besides $D$-term contributions is given by

$$
M_{\lambda_{SM}} \sim \frac{1}{\sqrt{M_5 R}} m_{3/2}, \quad m_{2_{SM}}^2 \sim \frac{g_{SM}^2}{16\pi^2} M_{\lambda_{SM}}^2 \ln(M_S R) + \left(\frac{g_{SM}^2}{16\pi^2} m_{3/2}\right)^2,
$$

$$
M_{\lambda_X} \sim \frac{g_X^2}{16\pi^2} m_{3/2}, \quad m_{\chi}^2 \sim \left(\frac{g_X^2}{16\pi^2} m_{3/2}\right)^2, \quad (3.3)
$$

‡In the anomaly mediation scenario, there is a case where $D$-term contribution is offset by threshold corrections at leading order [23].
where $M_5$, $M_S$ and $R$ are the five-dimensional Planck mass, SUSY-breaking scale, and the compactification radius of the fifth dimension, respectively. The spectrum of the $U(1)_X$ sector, $M_\lambda$ and $m_\chi^2$, are the same as in (3.1). In the formula for the squark and slepton masses squared $m^2_{\text{SM}}$, the first term represents gaugino-mediated contribution through RGEs [corresponding to $m^2_g$ in Eq. (2.11)] and the last term is that of anomaly mediation. The both terms are generation-independent. The anomaly mediation would be dominant if $g^2_{\text{SM}} > 16\pi^2/(M_5 R) \ln(M_S R)$ were satisfied. However this inequality requires too large value of $R$ even when one assumes the five-dimensional theory is strongly coupled. In other words, it requires unrealistically small $g_{\text{SM}}$. Therefore for the SM gauge sector, the gaugino-mediated contribution is expected to be always larger than that of anomaly mediation. As a result, we find that flavor violation induced by the $D$-term contribution is suppressed if

$$g^4_X \lesssim \varepsilon \frac{16\pi^2 g_{\text{SM}}^2}{M_5 R} \ln(M_S R).$$

(3.4)

This can be easily fulfilled with an $O(1)$ gauge coupling $g_X$, provided $g_{\text{SM}}$ is properly reproduced following from the naive dimensional analysis [24].

In this way, we conclude that $D$-term contribution can indeed be suppressed in the higher-dimensional framework without conflicting with other phenomenological requirements such as FCNC constraints and naturalness. Other field configurations or SUSY-breaking mechanisms in higher dimensions could also lead to a similar conclusion.

### 3.3 Suppression via superconformal dynamics

Recently there has been proposed a new approach for realizing hierarchical Yukawa matrices by coupling the SM sector to superconformal (SC) gauge theories. See Ref. [25] for the original proposal and also Ref. [26] for another. The SC sectors are strongly coupled and assumed to have infrared fixed points [27]. In this type of scenarios, the SM matter fields couple to the SC sectors and gain large anomalous dimensions through the strong SC dynamics. As a result, their Yukawa couplings are suppressed at the scale where the SC sectors decouple from the SM sector.

The SC fixed points also have an interesting implication for SUSY breaking. The infrared convergence property of the fixed points implies that SUSY-breaking scalar masses are also suppressed towards the infrared [28].\(^5\) Below the SC decoupling scale, scalar masses receive radiative corrections due to soft gaugino masses. If this mechanism is applied to squarks and sleptons, their masses are expected to be approximately degenerate at low energy. Such possibility was mentioned in Ref. [25] and detailed studies have been done [28, 30], where it was argued that a certain amount of degeneracy in sfermion masses can be obtained. (See also Ref. [31] for a different SC approach.)

Let us discuss the $D$-term contribution in the SC framework. As stated before, the point to avoid $D$-term flavor violation is to keep the $U(1)_X$-breaking sector away from

\(^5\)This kind of behavior was first noticed in softly-broken SUSY QCD [29].
the SUSY-breaking one. This can be accomplished if SC dynamics couples to the $\chi$ field whose VEV breaks the flavor symmetry and generates flavor-dependent sfermion masses. Then strongly-coupled SC dynamics suppresses the soft mass of $\chi$ and consequently the $D$-term contribution to charged sfermion masses. In the following, we suppose for simplicity that the SC sector does not couple to the SM fields; the hierarchical structure of Yukawa couplings are generated by the conventional FN mechanism.

Now, we present a toy model by introducing a SC gauge group $G_{SC}$ in addition to the SM gauge and $U(1)_X$ flavor symmetries. Consider the following cubic superpotential term

$$W = h \chi \Phi \bar{\Phi},$$

where $\Phi$ and $\bar{\Phi}$ belong to non-trivial representations under $G_{SC}$. The SM fields are $G_{SC}$-singlets while $\chi$ couples to the SC sector through the above $W$. The operator $\Phi \Phi \bar{\Phi}$ has a nonzero $U(1)_X$ charge so that (3.5) is gauge invariant. Let us suppose that as in Section 2, the SM-singlet field $\chi$ develops a non-vanishing VEV $\langle \chi \rangle$ which breaks the flavor symmetry and generates effective Yukawa couplings through non-renormalizable operators. An interesting point of the present setup is that the required decoupling of the SC sector is achieved by the same VEV $\langle \chi \rangle$, giving a mass term for $\Phi$ and $\bar{\Phi}$. Thus both the SC and $U(1)_X$ sectors decouple from the SM one around the scale of $M_X$ for $h = O(1)$.

Strong $G_{SC}$ interactions force the trilinear coupling $h$ as well as the $G_{SC}$ gauge coupling to approach their infrared fixed-point values. Hence the field $\chi$ gains a large and positive anomalous dimension $\gamma_\chi$. For simplicity, we assume that the couplings at a cutoff scale $\Lambda$ are close to their fixed-point values. If we neglect perturbatively small couplings in the SM sector, SUSY-breaking scalar masses are suppressed (near the SC fixed point) according to the following form of evolution equations [28]

$$\mu \frac{d m_a^2}{d \mu} = \mathcal{M}_{ab} m_b^2,$$

where the index $b$ runs over all fields coupled to the SC sector including the $\chi$ field. The matrix $\mathcal{M}_{ab}$ can be calculated by use of the Grassmannian expansion method [32], given the anomalous dimensions $\gamma_a$ as the functions of coupling constants in the model. Since $\mathcal{M}_{ab}$ is determined by the first derivative of $\gamma_a$, the infrared convergence property of the fixed points guarantees that this matrix is positive definite. It follows that certain combinations of scalar masses rapidly become suppressed through the RG evolution from $\Lambda$ down to the the SC-decoupling scale $\langle \chi \rangle$. Specifically, the scalar mass squared $m_\chi^2$ of the $\chi$ field behaves like

$$m_\chi^2(\langle \chi \rangle) \approx \left( \frac{\langle \chi \rangle}{\Lambda} \right)^{\Gamma_\chi} m_\chi^2(\Lambda), \quad \Gamma_\chi \approx \gamma_\chi = O(1).$$

Here the damping factor $\Gamma_\chi$ is related to the eigenvalues of $\mathcal{M}_{ab}$ and is as large as $\gamma_\chi = O(1)$. In this way, the required separation of the flavor-breaking sector from the SUSY-breaking one can be realized by a strong dynamics of four-dimensional SC gauge theory. As a result, the $D$-term contribution can be reduced for any value of SUSY-breaking parameters at the cutoff scale.
It is important, however, to take into account two effects that make the desired suppression (3.7) insufficient. One is the violation of SC symmetry due to the \( U(1)_X \) gauge interaction (as well as the SM gauge interactions). See Ref. [30] for a detailed discussion on this point. If we include the 1-loop correction from the \( U(1)_X \) sector, the evolution equation (3.6) is modified into
\[
\mu \frac{dm_b^2}{d\mu} = M_{ab} m_b^2 - \frac{8g_b^2 g_X^2}{16\pi^2} M_{\lambda_X}^2 + \cdots. \tag{3.8}
\]
Note that a model-dependent factor proportional to \( g_X^2 \text{Tr}(Q_X m^2) \) can safely be dropped in discussing \( D \)-term induced scalar masses, as was shown in Section 2.2. The above type of correction makes the suppression of \( m_b^2 \) incomplete, but obviously such RG effect is less significant if the gauge coupling \( g_X \) and soft gaugino mass \( M_{\lambda_X} \) are not so large.

Another and more significant effect comes from the fact that the RG running distance in the SC regime is finite. This effect is more important than the previous one because the running distance is related to the hierarchy of Yukawa couplings. In the FN models (2.3), the effective Yukawa couplings \( y'_{ij} \) at the SC-decoupling scale are now suppressed, in addition to the usual factor \( (\langle \phi \rangle / \Lambda)^{n_{ij}} \), by the large wavefunction renormalization of \( \chi \) and are given by
\[
y'_{ij} \simeq \left( \frac{\langle \chi \rangle}{\Lambda} \right)^{n_{ij}} \left( \frac{\langle \chi \rangle}{\Lambda} \right)^{\gamma_X/2} y_{ij} \equiv (\tilde{\lambda})^{n_{ij}} y_{ij}. \tag{3.9}
\]
Thus the degrees of suppression for Yukawa couplings and scalar masses are correlated to each other. In particular, it is a non-trivial issue whether \( m_b^2 \) becomes small enough to suppress FCNC with a fixed Yukawa hierarchy \( \tilde{\lambda} \), i.e. with a fixed length of the SC region. Given a value of \( \tilde{\lambda} \), the \( \chi \) scalar mass (3.7) is written as
\[
m_b^2(\langle \chi \rangle) \simeq (\tilde{\lambda})^{2\gamma_X/\gamma_X} m_b^2(\Lambda). \tag{3.10}
\]
The deviation (3.10) from the exact convergence to the conformal fixed point generates the \( D \)-term contribution to sfermion masses. That implies the FCNC constraint for the first and second generation squarks
\[
(\tilde{\lambda})^{1 + \frac{2\gamma_X}{\gamma_X + \gamma_X}} \left[ \frac{m_b^2(\Lambda)}{m_{\text{ave}}^2} \right] \lesssim 10^{-2} \left( \frac{m_{\text{ave}}}{500 \text{ GeV}} \right), \tag{3.11}
\]
where the extra factor of \( \tilde{\lambda}^1 \) arises when we diagonalize the quark mass matrices. The averaged squark mass \( m_{\text{ave}} \) contains initial values of squark masses at high energy and radiative corrections from the SM gauginos. The above constraint requires that the damping factor \( \Gamma_\chi \) has an appropriately large value. Otherwise, the \( D \) problem will still be there; a high-energy value of \( m_b^2 \) has to be small compared to other SUSY-breaking parameters. Similar remarks apply also to the slepton sector.

In this way, suppressing the \( D \)-term can in principle be achieved within four-dimensional framework, but for the mechanism discussed in this subsection to work in practice, it is necessary to have a concrete model of the SC sector in which \( \Gamma_\chi \) is as large as possible whereas \( \gamma_\chi \) is a reasonably small.
4 Radiative corrections from flavor sector

In the previous section, we have discussed the dominant tree-level $D$-term contribution which must be suppressed for models to be phenomenologically viable. We have illustrated that the suppression can be achieved by use of four-dimensional SC dynamics as well as a geometrical setup in extra-dimensional scenarios. It is, however, important to notice that even if the $D$-term contribution is sufficiently reduced, radiative corrections due to the $U(1)_X$ gaugino may lead to sizable non-degeneracy of sfermion masses, because the corrections depend on their quantum numbers and then become flavor-dependent. As shown in Section 2.2, the only relevant type of radiative corrections comes from the $U(1)_X$ soft gaugino mass. In this section, we study this type of corrections, supposing that tree-level $D$-term contribution is already suppressed.

We again assume for simplicity that there is only a single field $\chi$ which breaks the gauged $U(1)_X$ flavor symmetry. As was discussed in Eqs. (2.13) and (2.14), the radiative correction to sfermion masses is given by

$$ m^2_{Xi} = \frac{-8}{16\pi^2}(q_i^2 + q_i^2 q_i) \int_{\Lambda}^{M_X} \frac{d\mu}{\mu} g_X^2 \lambda_{X}^2 M_X^2, $$

(4.1)

where $\Lambda$ is a cutoff which we take to be the Planck scale in the following. The first term on the right-handed side originates from a direct effect of the $U(1)_X$ gaugino in the RGE of sfermion masses. The second term is obtained through the $D$-term contribution with the RGE effect on $m^2_\chi$ included. Since the scalar masses (4.1) are the only sources of flavor dependence, the mass difference between the scalars with different quantum numbers $q_i$ and $q_j$ is scale invariant and found to be

$$ m^2_i - m^2_j = \frac{2}{b_X}(q_i - q_j)(q_i + q_j + q_i^2)[M^2_{X}(\Lambda) - M^2_{X}(M_X)]. $$

(4.2)

The beta-function coefficient $b_X$ for $g_X$ is determined once $U(1)_X$ charge assignment is specified. The above mass difference can be larger than those discussed in the previous section, that is, a small deviation from the complete suppression of $D$-term contributions.

Now let us study low-energy consequences of the gaugino radiative correction (4.2). The degeneracy of sfermion masses is estimated by a ratio of off-diagonal element of scalar mass matrix to the averaged mass $m_{ave}$

$$ \delta_{ij} \equiv \frac{V^*_{ki} m^2_k V_{kj}}{m^2_{ave}}, $$

(4.3)

where the matrix $V$ rotates the flavor basis so that Yukawa matrix is diagonal. In the case of flavor-independent masses ($q_i = q_j$), $\delta_{ij}$ vanishes due to the unitarity. In the hierarchical case, the degeneracy is given by

$$ \delta_{ij} \approx \lambda^{2(q_i - q_j)} \frac{m^2_i - m^2_j}{m^2_{ave}}. $$

(4.4)
Since the low-energy scalar masses take the form (2.11), $m_{\text{ave}}^2$ is given by

$$m_{\text{ave}}^2 = m_0^2 + m_G^2 + m_{\text{SM}}^2 + (m_{X_i}^2 + m_{X_j}^2)/2,$$

(4.5)

where $m_0^2$ is the initial value (for which we take a conservative assumption of the universality), and $m_G^2$ and $m_{\text{SM}}^2$ are the GUT and SUSY SM contributions,

$$m_G^2 = \frac{2C^G_g(R)}{b_G} \left[ M_{\lambda_G}^2(\Lambda) - M_{\lambda_G}^2(M_G) \right],$$

(4.6)

$$m_{\text{SM}}^2 = \sum_{a=1,2,3} \frac{2C^a_g(R)}{b_{a,\text{SM}}} \left[ M_{\lambda_{a,\text{SM}}}^2(M_G) - M_{\lambda_{a,\text{SM}}}^2(M_S) \right],$$

(4.7)

where $b$'s are the beta function coefficients of the gauge couplings, and $M_G$ is the GUT-breaking scale. In the following analysis, we will take a simplifying assumption that the SM gaugino masses unify to $M_{\lambda_G}$ at the $M_G$ scale.

As an illustrative example, we will focus on the first and second families and take their $U(1)_X$ charges as $q_1 = 3$ and $q_2 = 2$ (and $q_3 = -1$). For these first two generations, there are severe experimental constraints on scalar masses [1]. As for the squark sector, the limits on the flavor-changing scalar masses from the Kaon system are roughly given by

$$\delta Q_{12} \lesssim 10^{-2} \left( \frac{m_{\text{ave}}}{500 \text{ GeV}} \right),$$

(4.8)

$$\sqrt{\delta Q_{12}^2 + \delta D_{12}^2} \lesssim 10^{-3} \left( \frac{m_{\text{ave}}}{500 \text{ GeV}} \right).$$

(4.9)

The former bound (4.8) is milder than (4.9) by an order of magnitude, but is relevant for the case motivated by recent analyses of fermion mass matrices where the Yukawa hierarchy is ascribed to the structure of the matter fields in 10-dimensional representation of $SU(5)$ [33]. For the sleptons, the $\mu \rightarrow e\gamma$ process requires

$$\delta L_{12} \lesssim 10^{-3} \left( \frac{m_{\text{ave}}}{100 \text{ GeV}} \right)^2.$$  

(4.10)

The constraint on the right-handed slepton $\delta E_{12}$ is rather weak unless the trilinear couplings for the sleptons are very large. This is mainly because they interact only to the weak $U(1)$ hypercharge.

The above experimental upper bounds turn out to put upper bounds to $U(1)_X$ gauge effects (the gauge coupling $g_X$ and the gaugino mass $M_{\lambda_X}$). Assuming the minimal supersymmetric SM below the GUT scale, there remain only a few free parameters in the mass difference (4.4). Figure 1 shows typical upper bounds on the parameters in the $U(1)_X$ sector. The vertical axis denotes the ratio of $M_{\lambda_X}$ to the GUT gaugino soft mass estimated at the Planck scale, and the horizontal one represents $g_X^2/g_G^2(\Lambda)$. The left-lower regions below the lines are allowed by the FCNC experiments (4.8) and (4.9). In the figure, we take as an input the unified gaugino mass $M_{\lambda_{a,\text{SM}}}(M_G) = 300$ GeV, which is suitable for

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Figure 1: The upper bounds from $K^0$-$\bar{K}^0$ on the $U(1)_X$ gauge coupling and soft gaugino mass at the Planck scale. The experimentally allowed region is below each line. The dashed and solid lines represent the constraints from (4.8) and (4.9), respectively, with $b_X = 100$ (bold) and $b_X = 300$ (thin line). In this figure we take $M_{\text{asym}}(M_G) = 300$ GeV and $m_0 = 0$.

Figure 2: The upper bounds from $\mu \rightarrow e\gamma$ on the $U(1)_X$ gauge coupling and soft gaugino mass at the Planck scale. The others are the same as in Figure 1.
the naturalness criterion and experimental lower bounds of photino and slepton masses. Effects of taking other values of $M_{\lambda_{\text{SM}}}$ almost cancel in the ratios $\delta$'s and only change the overall scale of $m_{\text{ave}}$ on the right-handed sides of (4.8)–(4.10). In Fig. 2, we also plot similar experimental constraints derived from (4.10). These two figures show that there are non-trivial limits on the parameters of the $U(1)_X$ sector; the experimental results rule out large values of the gauge coupling and soft gaugino mass compared to those of the SM sector. The $U(1)_X$ soft gaugino mass has an upper bound which is universal in a relatively wide range of $g_X$, except for a small $g_X$ case. For example, weakly-coupled heterotic string theory predicts equal gaugino soft masses $M_{\lambda_G} = M_{\lambda_X}$ at high energy, and then a tiny value of $g_X$ is required.

Given fixed values of gauge couplings and gaugino masses, the FCNC constraints give a lower bound for the $U(1)_X$ gauge beta function $b_X$. Note that models with abelian flavor symmetry generally contain a number of charged fields to realize large Yukawa hierarchy (and also to stabilize GUT Higgs potential). The $U(1)_X$ gauge beta-function can therefore be relatively large. In Figs. 3 and 4, we show typical lower bounds for $b_X$ with the unification boundary conditions $g_G = g_X$ and $M_{\lambda_G} = M_{\lambda_X}$ at the Planck scale. The two types of lines

![Figure 3: The lower bounds of $b_X$ from the $K^0 - \bar{K}^0$ constraint (4.9). The horizontal axis denotes the initial universal scalar soft mass $m_0$. The bold (thin) line correspond to the case $g_G^2(\Lambda)/4\pi^2 = 1/20 \ (1/10)$. We assume $M_{\lambda_G}(M_G) = 300 \text{ GeV}$ and the boundary conditions $g_G = g_X$ and $M_{\lambda_G} = M_{\lambda_X}$ at the Planck scale.]

in the figures represent the cases $g_G^2(\Lambda)/4\pi = 1/20$ (bold) and $1/20$ (thin), respectively. The horizontal axis denotes the initial universal scalar soft mass $m_0$ which is given at the Planck scale by some SUSY-breaking dynamics and then could not be much larger than gaugino masses. For the sleptons (Fig. 4), introducing a nonzero $m_0$ considerably changes
the $b_X$ bounds than in the squark sector (Fig. 3). We find from these figures that for a fixed model (a fixed $b_X$), a rather small value of $g_{X}$ and/or a large initial scalar mass $m_0$ must be satisfied. The constraints derived in this section generically exist and are independent of mechanisms for avoiding flavor-violating tree-level $D$ terms. Therefore they could cast a serious problem on the construction of realistic flavor $U(1)_X$ models from high-energy fundamental theories.

In Section 3, we have found that the dominant $D$-term contribution can be suppressed by use of particular setups if the $U(1)_X$ gauge coupling satisfies a weak condition like (3.2). The bounds discussed here may be stronger than those obtained in Section 3. Furthermore the bounds from the radiative corrections are roughly model-independent and can be applied to any dynamics suppressing $D$-term contribution. For example, in the extra-dimensional models in Section 3.2, the $U(1)_X$ vector multiplet is taken to be stuck on the visible brane. Then a suppressed initial value of $U(1)_X$ soft gaugino mass is obtained because of the absence of local operators with which the gaugino directly couples to the SUSY-breaking brane. For the superconformal scenarios in Section 3.3, the suppression of $M_{\lambda X}$ cannot naively be achieved due to the fact that an abelian factor does not have any infrared fixed point of the gauge coupling. Embedding it to a non-abelian gauge group may cure the problem, but in that case it might be non-trivial to construct realistic Yukawa hierarchy.

Our analysis here shows that flavor-dependent radiative corrections due to the $U(1)_X$ gaugino is important from a viewpoint of experimental constraints on flavor physics. In this paper we have only discussed flavor-violating scalar masses induced by gauged abelian flavor symmetries. In addition to this, CP-violation phenomena, RGE effects on scalar
trilinear couplings, etc. would give severer bounds on gauged flavor symmetries. Those issues will be discussed elsewhere.

5 Summary

We have studied sfermion masses in supersymmetric models with gauged $U(1)$ flavor symmetries. It has been known that these models generally suffer from large flavor-violating effects due to tree-level sfermion masses proportional to individual $U(1)$ quantum numbers. We have presented a simple idea for suppressing the $D$-term contribution; a separation of flavor-symmetry breaking from supersymmetry breaking. Illustrative models have been described by extra-dimensional frameworks or strongly-coupled superconformal dynamics.

We have also pointed out that generation-dependent radiative corrections due to the $U(1)$ soft gaugino mass lead to sizable non-degeneracy in sfermion masses. The corrections are also dangerous to the FCNC constraints and put a non-trivial limit on the parameters in the flavor symmetry sector. Our analysis indicates that it is preferable to have the $U(1)$ gauge coupling and soft gaugino mass as small as possible. Required small values of parameters could be obtained, for example in extra-dimensional models, by volume suppression factors or strong RGE effects.

We add a remark that none of the flavor-violating effects is present if one could take the global limit $g \to 0$. However if one takes serious the argument that any global symmetry is not preserved in the presence of quantum gravity, one must seek for solutions to avoid the flavor-violating $U(1)$ gauge contributions. Moreover, it seems difficult to experimentally test such models with global flavor symmetry. On the other hand, the gauged flavor symmetry will provide us with detectable signatures especially through the flavor-dependent radiative corrections from $U(1)$ gaugino.

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