New Non-Trivial Vacuum Structures in Supersymmetric Field Theories

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Abstract. In this talk, we present three examples of new non-trivial vacuum structures that can occur in supersymmetric field theories, along with explicit models in which they arise. The first vacuum structure is one in which supersymmetry is broken at tree-level in a perturbative theory that also contains a supersymmetry-preserving ground state. Models realizing this structure are uniquely characterized by the fact that no flat directions appear in the classical potential, all vacua appear at finite distances in field space, and no non-perturbative physics is required for vacuum stability. The second non-trivial vacuum structure we discuss consists of large (and even infinite) towers of metastable vacua, and we show that models which give rise to such vacuum towers exhibit a rich set of instanton-induced vacuum tunneling dynamics. Finally, our third new non-trivial vacuum structure consists of an infinite number of degenerate vacua; this leads to a Bloch-wave ground state and a vacuum “band” structure. Models with such characteristics therefore experience time-dependent vacuum oscillations. Needless to say, these novel vacuum structures lead to many new potential applications for supersymmetric field theories, ranging from the cosmological-constant problem to the string landscape, supersymmetry breaking, and $Z'$ phenomenology.

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INTRODUCTION

The vacuum structure of any physical theory plays a significant and often crucial role in determining the physical properties of that theory. In this talk, we describe three hitherto-unexplored and highly non-trivial vacuum structures which can arise in supersymmetric field theories.

The first scenario we present [1] is an example of a metastable supersymmetry-breaking model in which all relevant features arise at tree-level in a completely calculable, perturbative framework. These include a supersymmetric, $R$-symmetry-preserving ground state; a metastable state in which both supersymmetry and $R$-symmetry are broken; and a vacuum energy barrier between the two of a sort that results in a long lifetime for the metastable vacuum. Neither the ground state nor the metastable vacuum involve runaways or flat directions; moreover, the salient features of the vacuum potential are perturbative and robust against quantum corrections, and the lifetimes of metastable vacua can be calculated reliably. As far as we are aware, the model we present is the first such model with these properties presented in the literature.

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The second vacuum structure we present consists of large (and even infinite) towers of metastable vacua, each with a distinct vacuum energy and particle spectrum. We present a series of models which realize this vacuum structure explicitly. As the number of vacua grows towards infinity in such models, the energy of the highest vacuum stays fixed while the energy of the ground state tends towards zero. The instanton-induced tunneling dynamics associated with such vacuum towers results in a variety of distinct decay patterns; these include not only regions of vacua experiencing direct collapses and/or tumbling cascades, but also other regions of vacua whose stability is protected by “great walls” as well as regions of vacua populating “forbidden cities” into which tunnelling cannot occur.

Finally, the third vacuum structure we discuss arises as a limiting case of the previous scenario. In this limit, all of the metastable vacua in a given vacuum tower become degenerate, and a shift symmetry emerges relating one vacuum to the next. In such a scenario, the true ground states of such theories are therefore nothing but Bloch waves across these degenerate ground states, with energy eigenvalues approximating a continuum and giving rise to a vacuum “band” structure.

Needless to say, these novel vacuum structures give rise to many new potential applications for supersymmetric field theories, ranging from the cosmological-constant problem to the string landscape, supersymmetry breaking, and $Z'$ phenomenology. In this talk, we shall merely present these three different vacuum structures and briefly sketch some possible applications; further details can be found in Refs. [1, 2, 3].

I. TREE-LEVEL METASTABLE SUPERSYMMETRY-BREAKING

The first model we discuss is a simple one which can be used as a “kernel” for developing more complete models of metastable supersymmetry-breaking. This model consists of two $U(1)$ gauge groups, $U(1)_1$ and $U(1)_2$ with couplings $g_1 = g_2 = g$, and five chiral superfields $\Phi_i$, $i = 1, ..., 5$, with charge assignments $(-1,0)$, $(1,-1)$, $(0,1)$, $(1,1)$, and $(-1,-1)$ respectively. Given these charges, the most general renormalizable superpotential is given by

$$W = \lambda \Phi_1 \Phi_2 \Phi_3 + m \Phi_4 \Phi_5.$$  (1)

We shall also generally permit non-zero Fayet-Iliopoulos terms $\xi_a$ for each $U(1)_a$ gauge group. Note that the $m = 0$ version of this model was originally discussed in Ref. [4] as part of a study of the string-theory landscape.

This model has a rich vacuum structure depending on the particular choices for the parameters $(\lambda, m, \xi_a, \xi_b, g)$. In general, the scalar potential of such a theory includes both $F$-term and $D$-term contributions and can be written in the form

$$V = \frac{1}{2} \sum_a g_a^2 D_a^2 + \sum_i |F_i|^2$$  (2)

where $D_a = \xi_a + \sum_i Q_{ai} |\phi_i|^2$ (with $\phi_i$ the scalar component of $\Phi_i$), where $F_i \equiv -\partial W^* / \partial \phi_i^*$, and where $Q_{ai}$ is the $U(1)_a$-charge of $\Phi_i$. For concreteness here, however, let us focus on the special case $(\lambda, m, \xi_a, \xi_b, g) = (1,1,5,0,1)$. We then find that the scalar potential $V$ has three relevant critical points: two stable minima and
TABLE 1. The classical vacuum structure of the model in Eq. (1) with $(\lambda, m, \xi_a, \xi_b, g) = (1, 1, 5, 0, 1)$. Here Solutions A and B correspond to stable vacua, while C corresponds to a saddle point. For each of these solutions, we have listed the corresponding field VEVs $(v_1, v_3, v_5)$, along with the value of the scalar potential, the stability and supersymmetry properties of that extremum, and the surviving (unbroken) gauge group. Note that each solution has $v_2 = v_4 = 0$. We have also indicated whether $R$-symmetry is broken or unbroken at that extremum, assuming that each of the chiral superfields in this model carries an appropriate non-zero $R$-charge discussed in Ref. [1]; likewise, that all dimensionful quantities are quoted in dimensionless units.

| Label | $(v_1, v_3, v_5)$ | $V$ | Stability | SUSY | $R$-symmetry | Gauge Group |
|-------|------------------|-----|-----------|------|--------------|-------------|
| A     | $(\sqrt{5}, 0, 0)$ | 0   | Stable    | Yes  | Yes          | $U(1)_b$    |
| B     | $(0, 2, 2)$       | $9/2$ | Metastable| No   | No           | None        |
| C     | $(\sqrt{3}/2, \sqrt{7}/2, \sqrt{5}/2)$ | $45/8$ | Unstable  | No   | No           | None        |

one saddle point between them. These occur at the specific finite field-space locations $v_i \equiv \langle \Phi_i \rangle$ shown in Table 1.

As we see from Table 1, Solution A represents a supersymmetric, $R$-symmetry-preserving ground state with vanishing vacuum energy, while Solution B represents a metastable minimum in which supersymmetry and $R$-symmetry are both broken. Solution C represents the saddle-point solution through which the classical path between the two vacua passes. This vacuum structure is shown explicitly in Fig. 1.

In order to be of phenomenological interest for model-building, the lifetime of any given metastable vacuum must be at least on the order of the present age of the universe.

FIGURE 1. *Left figure:* A surface plot of the scalar potential $V$ evaluated on the unique two-dimensional plane within the three-dimensional $(v_1, v_3, v_5)$ field space which simultaneously contains the true vacuum solution A, the metastable vacuum solution B, and the saddle-point solution C between them. Projected below the surface plot is a contour plot for $V$, showing the shortest path (blue) in field space connecting these three solutions. *Right figure:* The scalar potential $V$ evaluated along this shortest path. Field-space distances are quoted relative to the metastable vacuum B along this path in dimensionless units.
This lifetime can be determined using standard instanton methods \[5, 6\], and it can be shown that the metastable minimum in our model is stable on cosmological time scales over a large region of parameter space, including the point \((\lambda, m, \xi_a, \xi_b, g) = (1, 1, 5, 0, 1)\) discussed above. These calculations are discussed more fully in Ref. \[1\].

As we have shown, the supersymmetry and \(R\)-symmetry in our construction are broken at tree level in a perturbative theory where no flat directions appear in the classical potential and where all minima appear at finite distances in field space. Thus, our construction can be viewed as an alternative to those which have appeared in much of the prior literature on metastable supersymmetry-breaking (for original papers, see, e.g., Ref. \[7\]). Indeed, most previous models actually give rise to “vacua” containing either classical flat directions or runaway behavior. Compared with previous constructions, the key feature of our model is that the supersymmetry-breaking arises not only through \(F\)-term breaking, but also through \(D\)-term breaking. Since all of the relevant physics is perturbative, we are able to perform explicit calculations of the lifetimes and particle spectra associated with such vacua and demonstrate that these lifetimes can easily exceed the present age of the universe. Models which have these properties are extremely rare, and we are not aware of any models with these characteristics in the prior metastability literature.

II. METASTABLE VACUUM TOWERS

The second scenario \[2\] we discuss involves a class of models which give rise to towers of non-degenerate, metastable vacua. The underlying structure of these models is that of an \(N\)-site orbifold Abelian moose consisting of \(N\) different \(U(1)\) gauge groups with a common coupling \(g\) and \(N + 1\) chiral superfields \(\Phi_i\). To this structure, we then add three critical ingredients, each of which is vital for the emergence of our metastable vacuum towers. The first of these is the introduction of a single Wilson-line operator

\[
W = \lambda N+1 \prod_{i=1}^{N+1} \Phi_i
\]

which represents the most general superpotential that can be formed from the fields of the theory. The second and third ingredients both exploit the Abelian nature of our gauge groups: one is to introduce non-zero Fayet-Iliopoulos terms \(\xi_1\) and \(\xi_N\) for the “endpoint” gauge groups \(U(1)_1\) and \(U(1)_N\) respectively, while the other is to introduce kinetic mixing \[8\] among the various \(U(1)\) factors in the theory. The effect of including kinetic mixing is that the gauge-kinetic part of the Lagrangian is modified to include a mixing matrix \(X_{ab}\) and hence takes the form \[9\]

\[
\mathcal{L} \supset \frac{1}{32} \int d^2 \theta \ W_{a\alpha} X_{ab} W^\alpha_b \quad \text{where } X_{ab} \equiv \begin{pmatrix}
1 & -\chi_{12} & \cdots & -\chi_{1N} \\
-\chi_{12} & 1 & \cdots & -\chi_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-\chi_{1N} & -\chi_{2N} & \cdots & 1
\end{pmatrix}.
\]

To simplify our analysis, we will focus on the case in which mixing occurs only between nearest-neighbor sites on the moose, with a common mixing parameter \(\chi\) (i.e., \(X_{ab} = \chi_{ij}\))...
\[ \chi \delta_{a,b+1} \); likewise, we shall assume that \( 0 < \chi < 1/2 \). We shall also take \( \xi_1 = \xi_N \equiv \xi \) for simplicity. Note that a similar model, but with \( \chi = 0 \) and \( \lambda \to 0 \), was previously considered in Ref. [10,4].

Given this setup, we find that the vacuum structure of this model contains \( N - 1 \) stable vacua. These can be labelled with an index \( n = 1, \ldots, N - 1 \) in order of decreasing vacuum energy. We then find that the vacuum energy of the \( n \)-vacuum is given by

\[ V_n = \frac{1}{2} \left( \frac{1}{\chi R_n} \right) \quad \text{where} \quad R_n \equiv \left( \frac{1}{\chi} - 2 \right) n + 2 , \]

and corresponds to the solution with

\[
\nu_j^2 = \begin{cases} 
1 + 1/R_n & \text{for } j = 1 \\
1/R_n & \text{for } 2 \leq j \leq N - n \\
0 & \text{for } j = N - n + 1 \\
(R_j - N + n - 1)/R_n & \text{for } N - n + 2 \leq j \leq N \\
0 & \text{for } j = N + 1 
\end{cases} \]

(6)

where we continue to list all quantities in dimensionless units and where all dependence on the overall coupling \( g \) has been rescaled away. This vacuum tower is illustrated for the \( N = 20 \) case in Fig. 2.

As required, these vacua are separated from one another by saddle-point solutions in field space. The general expressions for the field-space coordinates and barrier heights associated with these saddle points are generally quite complicated, but they are controlled primarily by the Wilson-line coefficient \( \lambda \). As a result, the stability of any given vacuum state in the tower is therefore also primarily governed by \( \lambda \): indeed, for any \( N \), we find that the \( n \)-vacuum will be stable as long as \( \lambda > \lambda_{N,n}^* \)

\[
\lambda_{N,n}^* = \frac{\Gamma(y)}{\Gamma(n+y)} \frac{R_n^{N-2}}{\chi(1+R_n)} .
\]

(7)

Here \( y \equiv \chi/(1 - 2\chi) = n/(R_n - 2) \) and \( \Gamma(z) \) is the Euler \( \Gamma \)-function [for which \( \Gamma(z) = (z-1)! \) when \( z \in \mathbb{Z}^+ \)]. Consequently, all of the vacua in the tower will be stable as long as \( \lambda \) exceeds the maximum value of \( \lambda_{N,n}^* \). Moreover, there is nothing which prevents us from taking \( N \to \infty \) in all of our results. As a result, we see that we can achieve a vacuum structure containing literally an infinite tower of metastable vacua.

Given the existence of such vacuum towers, we find that instanton-induced tunneling can produce a variety of highly non-trivial vacuum decay patterns. These include not only regions of vacua experiencing direct “collapses” (in which a given metastable vacuum decays directly to the ground state) and/or tumbling “cascades” (in which a given metastable vacuum decays to another metastable vacuum, and so forth). There are also other regions of vacua whose stability is protected by “great walls” as well as regions of vacua populating “forbidden cities” into which tunnelling cannot occur.

One example [2] which illustrates all of these features simultaneously is shown in Fig. 3. In this example, the \( n = 3 \) vacuum decays into the \( n = 6 \) vacuum, which in turn decays (even more rapidly) into the \( n = 10 \) vacuum; this in turn decays (even more rapidly) into the \( n = 15 \) vacuum, which in turn decays directly into the ground state. In
FIGURE 2. The vacuum structure of the $N = 20$ model, plotted for $\chi = 1/5$. This model gives rise to a tower of 18 metastable vacua above the true ground state. Vacuum energy is plotted on the vertical axis, while the horizontal axis indicates the cumulative distances in field space along a trajectory which begins at the $n = 1$ vacuum and then proceeds along straight-line path segments to the $(1,2)$ saddle point, then to the $n = 2$ vacuum, then to the $(2,3)$ saddle point, and so forth.

FIGURE 3. A schematic of the vacuum cascade dynamics that arises in a model with $N = 5000$ and $\chi = 2.8 \times 10^{-4}$. Vacua in the stable region to the left of the “Great Wall” have lifetimes exceeding the age of the universe, while vacua in the cascade region decay to other (lower) metastable vacua in the vacuum tower. By contrast, vacua in the collapse region decay directly to the ground state of the vacuum tower. Finally, vacua which populate the “Forbidden City” cannot be reached from outside the Forbidden City: such vacua can be populated only as an initial condition at the birth of the vacuum configuration.
fact, in this specific example, there are four independent potential cascade trajectories, each of which unfolds with increasing speed (i.e., decreasing lifetimes):

\begin{itemize}
  \item $3 \rightarrow 6 \rightarrow 10 \rightarrow 15 \rightarrow \text{GS}$
  \item $4 \rightarrow 8 \rightarrow 13 \rightarrow \text{GS}$
  \item $5 \rightarrow 9 \rightarrow 14 \rightarrow \text{GS}$
  \item $7 \rightarrow 11 \rightarrow \text{GS}$
\end{itemize}

where ‘GS’ signifies the ground state. It is therefore only an initial condition that determines which trajectory a given system ultimately follows.

There are, of course, limits to this cascade region, both at the top and at the bottom. For example, the top two vacua have decay rates which fall below those needed for cosmological stability; these vacua, if initially populated, are therefore deemed stable on cosmological time scales. Likewise, at the $n = 11$ vacuum and beyond, we enter the collapse region in which all subsequent decays automatically proceed directly to the ground state.

Clearly, there are also a number of possible applications for a vacuum structure of this sort. Perhaps the application which most immediately springs to mind concerns a potential solution to the cosmological-constant problem. Over the past decade, several scenarios have been proposed in which a small cosmological constant emerges as a consequence of a large number of vacua [11, 12, 13, 14]. Scenarios of this sort tend to posit the existence of a “landscape” of vacua with certain gross properties, including a vacuum state whose energy is nearly vanishing. One then imagines that the universe either dynamically tumbles down to this special vacuum state, or is somehow born there. However, to the best of our knowledge, no explicit model with a vacuum structure exhibiting such properties has ever been constructed. It is our hope that the model we have presented here might provide an explicit field-theoretic realization of such a scenario.

Another potential implication of such a vacuum structure concerns statistical studies [15] of the string landscape [16]. One important issue that needs to be addressed when performing such studies concerns the proper definition of a measure across the landscape: in what manner are the different string theories to be weighted relative to each other? Clearly, the most naïve approach is to count each string model equally, interpreting each as contributing a single vacuum state to the landscape as a whole. However, moose theories of the sort we have been discussing here often appear as the actual low-energy (deconstructed) limits of flux compactifications [4], and as we have seen, such theories give rise to infinite towers of metastable vacua. Thus, if the true underlying landscape measure is based on vacua rather than models, then a theory with infinite towers of vacua is likely to dominate any statistical study of the string landscape. As such, the phenomenological properties of these sorts of models will dominate the properties of the landscape as a whole.

Further potential applications of such scenarios involve supersymmetry breaking, $Z'$ phenomenology, and cosmological evolution. Full discussions of these and other ideas can be found in Ref. [2].
III. DEGENERATE VACUA AND BLOCH WAVES

The results presented in Sect. II concerning our $N$-site moose are applicable in the range $0 < \chi < 1/2$, where $\chi$ is our kinetic-mixing parameter. Indeed, each successive vacuum in the resulting vacuum tower has a lower energy than the previous one, and consequently there exists a net direction for dynamical flow.

However, in the $\chi \to 1/2$ special case, a new behavior develops. For $\chi = 1/2$, we find that $R_n = 2$ for all $n$, and thus the expressions for the vacuum energies $V_n$ become independent of the vacuum index $n$. As a result, the resulting vacuum structure consists of $N - 1$ degenerate vacua. These turn out to be separated from each other by a set of equivalent saddle-point potential barriers of uniform height. The field-space distances between all pairings of vacua also become equal in this case. This behavior is shown in Fig. 4.

As a result, the true ground state of such a theory when $\chi = 1/2$ is no longer any of the individual $n$-vacua by itself. Instead, what emerges is an infinite set of Bloch waves across the entire set of degenerate vacua. Moreover, the vacuum energies associated with such Bloch waves fill out a continuous band. The vacuum energy of the true ground state of the theory will consequently be smaller than the vacuum energy of any individual vacuum.

It is interesting to speculate that the vacuum energy of this true ground state might actually vanish (thereby restoring supersymmetry) or alternatively merely approach zero.

![FIGURE 4.](image)

FIGURE 4. The manner in which the vacuum structure of our model depends on $\chi$. Here we have focused on the $N = 6$ model, and taken (a) $\chi = 0.2$, (b) $\chi = 0.4$, (c) $\chi = 0.5$, and (d) $\chi = 0.54$. As $\chi$ increases from zero, we see that the “slope” of our vacuum “staircase” decreases, ultimately becoming completely flat at $\chi = 0.5$. 
as $N \to \infty$ (yielding a very small cosmological constant in a manner reminiscent of the proposal in Ref. [13]). It is also interesting to note that regardless of the ground-state energy, the fact that our Bloch vacuum states are linear combinations of our individual $n$-vacua implies that a system originally populating a given $n$-vacuum will experience non-trivial time-dependent oscillations across the set of $n$-vacua as a whole. Such vacuum oscillations are analogous to multi-flavor neutrino oscillations, and can potentially have dramatic effects on the phenomenology resulting from such theories. These and other issues will be explored more fully in Ref. [3].

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