ΛΛ interaction and hypernuclei

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1 Introduction

Whether the existence of hyperonic matter is present in the central regions of neutron stars with densities are in excess of nuclear saturation density is unknown. One of the fundamental pieces to determine this is the interaction among baryons bearing strangeness. A neutron star has been considered as a gigantic nucleus of \(N \sim 10^{58}\) nucleons [1] with a radial structure closely related to the equation of state of nuclear matter and, in turn, to the nuclear interaction. Due to the large complexity of the interior of this macroscopic object with typical radii of \(\sim 12\) km and masses of \(1.5M_\odot\) is not possible to obtain an in-detail description so far. However we can consider a sort of replica and helpful insight from finite nuclei orders of magnitude much smaller. Even, from the existence of hypernuclei, nuclei where part of its content consists of hyperons. In this sense non-relativistic [2] as well as relativistic field models have been developed (and continue to present) in the literature [3] to describe the phenomenology.

In particular, because of the lack of data from scattering experiments, ΛΛ hypernuclei provide a valuable method to learn details on the baryon-baryon interaction in the strangeness \(S = -2\) sector. Furthermore, in the last years a considerable effort has been done, both by the experimental and theoretical communities, in the physics of single and double Λ hypernuclei. Many single Λ hypernuclei and some double Λ hypernuclei have been observed and their energies have been measured.

In this work we present the model of Ref. [4] to calculate the binding energy of double Λ hypernuclei, defined as

\[
B_{\Lambda\Lambda} = -[M(\Lambda\Lambda + 2Z) - M(\Lambda Z) - 2m_\Lambda].
\]  

In what follows, we will outline the main features of the scheme of reference [4] to account for the modification of the interaction between the two Λ baryons by the presence of the nuclear core.
2 Model for ΛΛ hypernuclei

We model a ΛΛ hypernucleus as an interacting pair of Λ hyperons plus a nuclear core. We adopt a variational approach to determine the intrinsic wave function and to calculate the binding energies.

Once we have removed the center of mass, we write the intrinsic Hamiltonian as

\[ H = h_{sp}(1) + h_{sp}(2) + V_{ΛΛ}(1, 2) - \frac{\nabla_1 \cdot \nabla_2}{M_A} \]

\[ h_{sp} = -\frac{\nabla_i^2}{2\mu_A} + V_Λ(|\vec{r}_i|) \]

where \( \mu_A \) and \( M_A \) are the reduced mass of the Λ hyperon-nucleus system and the mass of the nuclear core, respectively.

The \( V_Λ \) potential in the single particle Hamiltonian \( h_{sp} \) accounts for the Λ nucleus interaction and has been adjusted to reproduce the binding energy \( B_Λ = -[M(Λ^{A+1}) - M(Λ^A) - m_Λ] \) of the corresponding single Λ hypernuclei. \( V_{ΛΛ} \) represents the interaction between the two Λ hyperons in the nuclear medium. The presence of a second Λ hyperon results in a dynamical reordering of the nuclear core. This reordering effect in the nuclear core and the free space ΛΛ interaction itself contributes to \( \Delta B_{ΛΛ} = B_{ΛΛ} - 2B_Λ \), although the former effect is suppressed compared to the latter by, at least, by one power of the nuclear density. We have assumed that this nuclear core dynamical reordering effect amounts to be around 0.5 MeV for light ΛΛ-hypernuclei, as suggested by the α–cluster model calculations, and negligible for heavy ones. This uncertainty is of the order of the experimental errors [5]. One should mention as well that this reordering effect is also partially taken into account in the RPA calculation described below.

3 Free space ΛΛ interaction

We use Bonn-Jülich models to construct the free space ΛΛ interaction. We consider the exchange of Σ \( (I = 0, J^P = 0^+) \), \( \omega \) and \( \phi \) \( (I = 0, J^P = 1^-) \) mesons between the two Λ hyperons. Furthermore, monopolar form factors are used, leading to extended expressions for the potentials. Once the form factors are included, the potentials turn out to be:

\[ V_σ(r) = -m_σ \frac{g_{σΛΛ}^2}{4\pi} \left\{ \tilde{Y}(σ, r) + \frac{1}{2m_Λ^2} \left[ \left( \tilde{∇} \tilde{Y}(σ, r) \right) \cdot \tilde{∇} + \tilde{Y}(σ, r) \tilde{∇}^2 \right] \right\} \]

\[ V_α(r) = \frac{m_α}{4\pi} \left\{ \frac{g_{αΛΛ}^2}{2m_α^2} \frac{2m_αΛ_{αΛΛ}}{2m_αΛ_{αΛΛ}} e^{-Λ_{αΛΛ}r} - \right.$

$$
\left. - \frac{3g_{αΛΛ}^2}{2m_Λ^2} \left[ \left( \tilde{∇} \tilde{Y}(α, r) \right) \cdot \tilde{∇} + \tilde{Y}(α, r) \tilde{∇}^2 \right] \right\}, \quad α = \omega, φ \]
with

\[
\tilde{g}_{\alpha \Lambda \Lambda}^2 = g_{\alpha \Lambda \Lambda}^2 - \frac{1}{2} \left( \left( \frac{m_\alpha}{m_\Lambda} \right)^2 \frac{3 g_{\alpha \Lambda \Lambda}^2}{2} + \frac{m_\Lambda}{m_N} g_{\alpha \Lambda \Lambda} f_{\alpha \Lambda \Lambda} + \left( \frac{m_\Lambda f_{\alpha \Lambda \Lambda}}{m_N} \right)^2 \right)
\]

\[
\tilde{Y}(\alpha, r) = Y(m_\alpha r) - \left\{ 1 + \frac{r}{2 \Lambda_{\alpha \Lambda \Lambda}} \left( \Lambda_{\alpha \Lambda \Lambda}^2 - m_\alpha^2 \right) \right\} \frac{\Lambda_{\alpha \Lambda \Lambda}}{m_\alpha} Y(\Lambda_{\alpha \Lambda \Lambda} r)
\]

\[
Y(x) = \frac{e^{-x}}{x}.
\]

In the above expressions \( \alpha \) stands for \( \omega \) and \( \phi \). The value of the couplings constants and the cutoff masses \([6, 7]\) are summarized in Table 1.

Bonn-Jülich models use the \( SU(6) \) symmetry to relate the coupling constants of the \( \omega \) and \( \phi \) mesons to the \( \Lambda \) hyperons to those of these mesons to the nucleons. Besides, we adopt the so called "ideal mixing" and consider that the \( \phi \) meson is a \( s\bar{s} \) state, leading to a vanishing \( g_{\phi NN} \) coupling constant. Hence, the \( g_{\phi \Lambda \Lambda} \) coupling constant is determined from \( g_{\omega \Lambda \Lambda} \). Besides, as the \( \phi \) meson does not couple to nucleons, exists a larger uncertainty in the value of its cutoff. We assume for \( \Lambda_{\phi \Lambda \Lambda} \) a value similar or greater than \( \Lambda_{\omega \Lambda \Lambda} \), considering finally three values for \( \Lambda_{\phi \Lambda \Lambda} \): 1.5, 2 and 2.5 GeV.

### 4 In medium contribution

Now we describe the summation of the diagrams of Fig. 1. We will do it first in nuclear matter and then will apply that results to finite nuclei.

To evaluate the series of diagrams of Fig. 1 in nuclear matter, we consider the case of a noninteracting Fermi gas of nucleons with density \( \rho \). The series of diagrams we are interested in is just a diagrammatic representation of a Dyson type equation, which modifies the propagation in nuclear matter of the carriers (\( \sigma \), \( \omega \) and \( \phi \)) of the interaction. As explained before, the \( \phi \) meson does not couple to nucleons and thus, its propagation is not modified in the nuclear medium. The \( \sigma - \omega \) propagator in the medium, is determined by the Dyson equation

\[
D(Q) = D^0(Q) + D^0(Q) \Pi(Q) D(Q)
\]

| Vertex   | \( g_\alpha / \sqrt{4\pi} \) | \( f_\alpha / \sqrt{4\pi} \) | \( \Lambda_\alpha \) (GeV) |
|----------|------------------------------|------------------------------|-----------------------------|
| \( \omega \Lambda \Lambda \) | 2.981                        | -2.796                       | 2                           |
| \( \sigma \Lambda \Lambda \) | 2.138                        | -                            | 1                           |
| \( \phi \Lambda \Lambda \) | -2.108                       | -3.954                       | 1.5–2.5                     |

Table 1: Coupling constants and cutoffs used in Eq. (3). These values have been taken from model \( A \) of Ref. [7].
where $D^0$ is the $5 \times 5$ matrix composed of the free $\sigma$ and $\omega$ propagators,

$$D^0(Q) = \begin{bmatrix} D_{\omega}(Q) & 0 \\ 0 & D^\sigma(Q) \end{bmatrix},$$  

(6)

and $\Pi$ is the $\sigma - \omega$ self-energy in the nuclear medium,

$$\Pi(Q) = \begin{bmatrix} \Pi(\mu \nu) & \Pi(\mu) \\ \Pi(\nu) & \Pi(\nu) \end{bmatrix},$$  

(7)

where $\Pi_{\mu \nu}$ and $\Pi_s$ accounts for excitations over the Fermi sea driven by the $\omega$ and $\sigma$ mesons, respectively and $\Pi_{\mu}$ generates mixing for scalar and vector meson propagation in the medium. This term vanish in the vacuum.

We assume the following approximations in the evaluation of $\Pi(Q)$: (i) We approximate $G_{\Lambda N}$ and $G_{NN}$ by the free space diagonal $\Lambda N$ and $NN$ potentials, well described by $\sigma$ and $\omega$ exchanges in the isoscalar $^1S_0$ channels. The $\Lambda \Lambda \sigma$ and $\Lambda \Lambda \omega$ vertices were discussed above and the $NN\sigma$ and $NN\omega$ Lagrangians and coupling constants can be found in [8]. (ii) We have only considered $p - h$ excitations over the Fermi sea. This amounts to evaluate the diagrams of Fig. 2 plus the corresponding crossed terms. (iii) We have worked in a nonrelativistic Fermi sea and evaluate the $p - h$ excitations in the static limit.

With all these approximations and taking the transferred four-momentum between the two $\Lambda$ hyperons as $Q = (q^0 = 0, 0, q)$, the elements of the matrix $\Pi(0, q)$ can be written as:

$$\Pi_{ij}(0, q) = U(0, q; \rho)C_i^N(q)C_j^N(q); \quad i, j = 1, \ldots, 5$$

$$C_B \equiv (g_{\omega BB}(q), 0, 0, g_{\sigma BB}(q))$$

$$g_{\alpha BB}(q) = g_{\alpha BB} \frac{\Lambda_{\alpha BB}^2 - m_{\alpha}^2}{\Lambda_{\alpha BB}^2 + q^2}$$  

(8)

where $\alpha = \omega, \sigma$ and $B = \Lambda, N$. In the Lindhard function $U(0, q; \rho)$ a finite excitation gap has been included for particles. We have used values for the gap between 1 and 3
MeV, to account for typical excitation energies in finite nuclei. In the case of \(^4\)He, we used a value of 20 MeV for the gap. With Eq. (8), we can invert the Dyson equation, and one gets

\[
D(Q) = (I - D^0(Q)\Pi(Q))^{-1}D^0(Q).
\] (9)

With this propagator, the RPA series of diagrams of Fig. 2 (from the second diagram on) can be evaluated and the RPA contribution to the \(\Lambda\Lambda\) interaction in nuclear matter results to be

\[
\delta V_{\Lambda\Lambda}^{RPA}(q, \rho) = \sum_{ij=1}^{5} C_{i}^{\Lambda}(q) \left[ D(Q) - D^0(Q) \right]_{ij} C_{j}^{\Lambda}(q)
\]

\[
= U(0, q; \rho) \frac{(W_{\Lambda N}^{\sigma} - W_{\Lambda N}^{\omega})^2}{1 + U(W_{NN}^{\sigma} - W_{NN}^{\omega})}
\] (10)

where we have subtracted \(D^0(Q)\) to avoid double counting and we have defined \(W_{BB'}^{\alpha} = \frac{g_{\alpha BB}(q)g_{\alpha BB'}(q)}{q^2 + m_{\alpha}^2}\). We have neglected in this nonrelativistic approach the spatial and tensor \((f^{\omega\Lambda\Lambda})\) couplings of the \(\omega\) meson to the \(\Lambda\). \(\delta V_{\Lambda\Lambda}^{RPA}(r_{12}, \rho)\) depends on the distance between the two \(\Lambda\)'s, \(r_{12}\) and the constant density \(\rho\). In the case of a finite nuclei, the carrier of the interaction feels different densities when it is travelling from one hyperon to the other. To account for this fact in the case of finite nuclei, we average over the densities the carrier feels along its flight. We assume meson straight line trajectories and the local density approximation, thus we obtain

\[
\delta V_{\Lambda\Lambda}^{RPA}(1, 2) = \int_{0}^{1} d\lambda \delta V_{\Lambda\Lambda}^{RPA}(r_{12}, \rho(|\vec{r}_2 + \lambda\vec{r}_{12}|))
\] (11)

where \(\rho\) is the nucleon center density given in Table 4 of Ref. [6].
5 Variational approach

We take advantage of the Variational Theorem to find the energy of the ground state of the Hamiltonian of Eq. (2). We have used a family of $^1S_0$ ΛΛ wave functions of the form $\Phi_{\Lambda\Lambda}(\vec{r}_1, \vec{r}_2) = NF(r_{12})\phi_{\Lambda}(r_1)\phi_{\Lambda}(r_2)\chi_{S=0}$, with $\chi_{S=0}$ the spin singlet. $N$ is a normalization constant and $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$. The functions $\phi_{\Lambda}(r_i)$ are exact solutions of the single particle Hamiltonian $h_{sp}$. $F(r_{12})$ is a Jastrow correlation function of the form

$$F(r_{12}) = \left(1 + \frac{a_1}{1 + (\frac{r_{12}}{b_1})^2}\right) \prod_{i=2}^3 \left(1 - a_i e^{-b_i r_{12}^2}\right)$$

(12)

where $a_i, b_i, R, \ i = 1, 3$ are free parameters. The values of the parameters for which the expected value of the Hamiltonian reaches a minimum are summarized in Table II of Ref. [4].

6 Results and discussion

Using the Λ nuclear core potentials summarized in Ref. [6], together with the ΛΛ interaction and the variational wave functions described above, we obtain the results [4] of Table 2. We have also considered the dependence of the results on the couplings, by varying them ±10% around their SU(6) values, finding appreciable variations.

In Table 2 the experimental values for the binding energy of $^6\Lambda\Lambda$He and $^{13}\Lambda\Lambda$B[9], are more updated with respect to those included in Ref. [4]. Ref. [9] reports a value of $B_{\Lambda\Lambda}(^{10}\Lambda\Lambda$Be) = 11.90 MeV, much smaller than those reported in the same reference for $^{11}\Lambda\Lambda$Be and $^{12}\Lambda\Lambda$Be, 20.49 and 22.23 MeV respectively, for which the nuclear cores only have one and two neutrons more. In contrast, Ref. [10] reports $B_{\Lambda\Lambda}(^{10}\Lambda\Lambda$Be) = 17.7 MeV.

From the results of Table 2, we conclude that in order to explain simultaneously the experimental energy of $^6\Lambda\Lambda$He, $^{10}\Lambda\Lambda$Be and $^{13}\Lambda\Lambda$B, RPA effects should be taken into account. Besides, the RPA resummation leads to a new nuclear density or A dependence of the ΛΛ potential in the medium which notably changes $\Delta B_{\Lambda\Lambda}$ and that provides, taking into account theoretical and experimental uncertainties, a reasonable description of the currently accepted masses of these three ΛΛ hypernuclei.

These results we find are in agreement with existing calculations showing that in-medium corrections are important. In particular we find that including RPA effects, from the particle-hole excitation picture provides larger binding energies. This is in agreement with calculations [11] showing that, in turn, allows the hyperon appearance at lower densities in matter. Additional effects may happen linked to the hadronic interaction showing its crucial role in neutron stars. For example, the existence of...
hyperons has been shown to generate too soft an equation of state, so that the maximum mass of neutron stars falls below the mass measured for some compact objects. Inclusion of three-body effects seem not to help in solving this issue. The cooling pattern in a neutron stars seem to be largely affected by the presence of unpaired hyperons that would induce faster cooling via direct URCA neutrino processes. Remarkably, the role of hyperons shows again important for the transport coefficients. In particular, in bulk viscosity governing the r-mode instability of rotating neutron stars \[12\]. Gravitational wave emission is allowed when the neutron star is unstable with respect to the r-modes. Future detections of this type of radiation depend, again, on the microscopic description and strength of the YN interaction.

### Table 2: Binding energies $B_{\Lambda\Lambda}$ calculated with our model.

|       | $B_{\Lambda\Lambda}^{\text{exp}}$ | Without RPA $\Lambda_{\phi\Lambda\Lambda}$ [GeV] | With RPA $\Lambda_{\phi\Lambda\Lambda}$ [GeV] |
|-------|----------------------------------|-----------------------------------------------|---------------------------------------------|
|       |                                  | $\text{no } \phi$ 1.5 2.0 2.5                  | $\text{no } \phi$ 1.5 2.0 2.5               |
| $^6_\Lambda\Lambda$ He | 6.91 ± 0.13 [9]                  | 6.15 6.22 6.53 6.84                             | 6.34 6.41 6.82 7.33                           |
| $^{10}_\Lambda\Lambda$ Be | 17.7 ± 0.4 [10]                 | 13.1 13.2 13.7 14.2                             | 14.5 14.6 15.6 16.8                           |
| $^{10}_\Lambda\Lambda$ Be | 11.9 ± 0.13 [9]                 | 22.5 22.6 23.2 23.8                             | 24.2 24.2 25.4 27.0                           |
| $^{13}_\Lambda\Lambda$ B | 23.3 ± 0.7 [9]                  | 37.2 37.3 37.7 38.1                             | 38.3 38.2 39.1 40.1                           |
| $^{42}_\Lambda\Lambda$ Ca | −                               | 44.1 44.2 44.4 44.7                             | 44.6 44.7 45.2 46.0                           |
| $^{92}_\Lambda\Lambda$ Zr | −                               | 53.1 53.1 53.3 53.4                             | 53.4 53.4 53.7 54.1                           |

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