NUMERICAL SIMULATION OF COUPLED HEAT TRANSFER THROUGH A CONCRETE HOLLOW BRICK

B. JAMAL, M. BOUKENDIL, A. ABDELBAKI, Z. ZRIKEM

Cadi Ayyad University, Faculty of Sciences Semlalia, Department of Physics, LMF, B.P. 2390, Marrakesh, Morocco.

m.boukendil@uca.ac.ma

Abstract
The present study aims to investigate coupled heat transfer by natural convection and conduction through a concrete hollow brick. The governing equations for conservation of mass, momentum and energy are discretized by the finite volume approach and solved by the SIMPLE algorithm. The numerical simulations were conducted to investigate the effect of Rayleigh number \(10^3 \leq Ra \leq 10^5\) on the heat transfer and fluid flow within the structure.

Keywords: Three dimensional numerical simulation; Concrete hollow brick; Natural Convection; Conduction

1. Introduction
The concrete hollow bricks are usually used in the construction of building walls in Morocco. These bricks were designed to increase the thermal resistance of the building walls and subsequently contribute to the reduction of energy consumption requirements for cooling and heating.

During the last two decades, several experimental and numerical studies [1-3] have been presented for describing two-dimensional combined heat transfers through hollow bricks. However, three-dimensional studies dealing with coupled heat transfers through concrete hollow bricks are few. Li et al. [4] conducted a numerical study to find the optimum configuration of external hollow clay brick of differing size. The results showed that the best configuration of hollow clay brick has five holes lengthwise, four holes widthwise and one hole heightwise. Sun and Fang [5] investigated numerically the heat transfer performance of concrete hollow bricks for different configurations. The effects of enclosure configurations with the same void volume fraction and the enclosure staggered were analyzed.

The main objective of the present work is to study numerically combined heat transfer by natural convection and conduction through a concrete hollow brick. The effect of Rayleigh number on the heat transfer and fluid flow within the structure is presented and examined.

2. Mathematical formulation
The configuration under investigation is shown in Figure 1. It represents a concrete hollow brick which is commonly used in the construction of building walls in Morocco. This hollow brick is formed by three cubical cavities surrounded by solid partitions. The lateral surfaces are differentially heated while the other faces of the brick are considered adiabatic.

In formulating governing equations, the fluid in the hollow brick is assumed Newtonian and incompressible in steady laminar state. The viscous heat dissipation is neglected in the energy equation and the thermophysical properties are assumed constant, except the density in the buoyancy term for which the Boussinesq approximation is adopted.

The governing equations are written in dimensionless form as follows:

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} = - \frac{\partial p}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right) \tag{1}
\]

\[
\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} = - \frac{\partial p}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right) \tag{2}
\]

\[
\frac{\partial W}{\partial \tau} + U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} = - \frac{\partial p}{\partial Z} + Pr \left( \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right) + Ra \cdot Pr \cdot \theta_a \tag{3}
\]

\[
\frac{\partial \theta_a}{\partial \tau} + U \frac{\partial \theta_a}{\partial X} + V \frac{\partial \theta_a}{\partial Y} + W \frac{\partial \theta_a}{\partial Z} = \frac{\partial^2 \theta_a}{\partial X^2} + \frac{\partial^2 \theta_a}{\partial Y^2} + \frac{\partial^2 \theta_a}{\partial Z^2} \tag{4}
\]
where \( U, V \) and \( W \) represent the velocities, \( \theta_a \) is the fluid temperature, \( P \) is the pressure, \( Pr \) is the Prandtl number and \( Ra \) is the Rayleigh number.

The dimensionless equation of conductive heat transfer in the solid partitions is expressed as follows:

\[
\frac{\partial \theta_s}{\partial \tau} = \frac{\alpha_s}{\alpha_s} \left( \frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} + \frac{\partial^2 \theta_s}{\partial Z^2} \right)
\]

where \( \theta_s \) represents the dimensionless solid temperature and \( \alpha_s \) is the solid thermal diffusivity.

The boundary conditions of the investigated hollow brick are as follows:

\[
\begin{align*}
U=V=W=0 & \text{ on the inner sides of each cavity.} \\
\theta_s(X, 0, Z) = 1 & \text{ at } \theta_s(X, 1, Z) = 0 \\
\frac{\partial \theta_s}{\partial X} |_{X=0} = \frac{\partial \theta_s}{\partial X} |_{X=1} = 0 & \\
\frac{\partial \theta_s}{\partial Z} |_{Z=0} = \frac{\partial \theta_s}{\partial Z} |_{Z=1} = 0 \\
\end{align*}
\]

At the fluid-solid interface, the boundary conditions are given by:

\[
\theta_s(X, Y, Z) = \theta_a(X, Y, Z) \quad \text{and} \quad \frac{\partial \theta_s}{\partial n} = N_k \frac{\partial \theta_a}{\partial n}
\]

where \( \eta \) represents the dimensionless coordinate normal to the wall, \( N_k \) is the thermal conductivity ratio \( k_s/k_a \).

The average heat fluxes at the outside and inside structure surfaces are given, respectively, by the following expressions:

\[
Q_a = - \int_0^1 \int_0^1 \frac{\partial \theta_s(X, Z)}{\partial Y} \left|_{Y=0} \right. dX dZ \\
= - \int_0^1 \int_0^1 \frac{\partial \theta_s(X, Z)}{\partial Y} \left|_{Y=1} \right. dX dZ
\]

Figure 1. Schematic representation of the studied concrete hollow brick

3. **Computational methodology and validation**

The governing equations subjected to the previous mentioned boundary conditions are discretized using the finites differences method based on the control volumes approach. The power law scheme is adopted to treat the convective terms of energy conservation equations. The coupling of pressure-velocity in momentum equations is treated by the SIMPLE algorithm. The resulting algebraic equations are solved using the iterative Tri-diagonal Matrix Algorithm (TDMA). The numerical algorithm is implemented in a self-developed FORTRAN code to carry out the numerical calculations. An optimization study of the grid size and time step has been conducted to choose the suitable grid size and time step that ensure independent solutions. As a result, the grid size of 120x60x60 and time step of \( 10^{-5} \) are used to carry out the numerical calculations.

In order to check the validity of the developed computer code, the obtained results are compared with those available in the literature in the case of natural convection in a cubical cavity filled with air. Table 1 illustrates these comparisons in terms of convective Nusselt number obtained for different Rayleigh numbers \( Ra \) \( 10^3 \leq Ra \leq 10^6 \). The comparative results show a good agreement, with a maximum deviation of 1.16%.

| Ra  | Frederick and Moraga [6] | Tric et al. [7] | Present work |
|-----|--------------------------|-----------------|--------------|
| \( 10^3 \) | 1.071                    | 1.070           | 1.069        |
| \( 10^4 \) | 2.057                    | 2.054           | 2.049        |
| \( 10^5 \) | 4.353                    | 4.337           | 4.322        |
| \( 10^6 \) | 8.740                    | 8.640           | 8.638        |

4. **Results and discussion**

The isotherms obtained for different values of Rayleigh number are shown in Figure 2. The results show that for \( Ra = 10^3 \), the isotherms are almost parallel to the active walls of the structure indicating that the heat transfers are dominated by conduction. Increasing Rayleigh number intensifies the natural flow and induces a deformation of the isotherms. Indeed, the distortion of the isotherms in the central zone of the structure becomes more pronounced as \( Ra \) increases indicating an increase of the natural convection intensity.

The velocity profiles obtained at mid-height (\( Z = 0.5 \)) and at mid-length (\( X = 0.5 \)) of the concrete hollow brick are shown in Figure 3. The Figure shows that the velocity increases with Rayleigh number, especially for high values of \( Ra \) (\( Ra \geq 10^6 \)). It is also noted that for a given \( Ra \), the fluid rises and then descends along the hot and cold walls, respectively. The velocity profiles are symmetrical with respect to the center of the structure.

The impact of Rayleigh number \( Ra \) on the temperature profiles is presented at mid-height (\( Z = 0.5 \)) and at mid-length (\( X = 0.5 \)) of the structure in Figure 4. As shown, the temperature of the fluid undergoes a considerable increase when Rayleigh number varies from \( 10^3 \) to \( 10^6 \). For a given value of \( Ra \), the temperature is high at the level of the hot wall and then stabilizes along the central zone before falling near the cold wall. This decrease of the temperature near the hot surface wall results from the
important heat transfer which occurs on the first contact of the cold air with the hot inner surface. The variation of the dimensionless overall heat flux $Q_a$ as a function of Rayleigh number is illustrated in Figure 5. The results indicate that the global heat flux increases with $Ra$; at the beginning the increase is low then it becomes important when $Ra$ exceeds $10^6$. Indeed, the dimensionless heat flux increases significantly by 227% when $Ra$ varies from $10^3$ to $10^7$.

5. Conclusion
This work was devoted to the analysis of coupled heat transfer by natural convection and conduction through a concrete hollow brick. A mathematical model has been elaborated to evaluate the effects of the Rayleigh number on the heat exchange and fluid flow within the structure. The Rayleigh number alters significantly the flow structure and the thermal fields. In addition, for high values of $Ra$ ($Ra \geq 10^6$), the overall heat transfer exchanged between the active walls is strongly increased.

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Figure 2. Isotherms obtained for the concrete hollow brick: (a) $Ra=10^5$, (b) $Ra=10^6$ and (c) $Ra=10^7$.

Figure 3. Velocity profiles obtained for different values of $Ra$.

Figure 4. Temperature profiles obtained for different values of $Ra$.

Figure 5. Effect of the Rayleigh number on the dimensionless heat flux through the hollow brick.