Role of specimen size upon the measured toughness of cellular solids

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Abstract. It is well known that the mechanical properties of cellular solids depend critically upon the specimen size and that a ‘sufficiently’ large test specimen is needed to obtain representative bulk values. Notwithstanding, the fracture toughness of cellular solids is still measured experimentally based on standards, such as the ASTM E399 and E813, developed for solid materials that do not possess an intermediate, ‘cell-level’ length scale. Experimental data in the literature appears to show that the toughness of stochastic 3D foams is, also, size-dependent. This paper presents the results of a detailed finite element (FE) study that will quantify, and identify the physical origin of, the size-dependent effect. Three-point bending of a single-edge notched (or SEN(B)) specimen, with a 2D Voronoi micro-architecture, is modelled numerically to obtain estimates of fracture toughness which are compared to those obtained with a ‘boundary-layer’ analysis.

1. Introduction

An important consideration in the mechanical testing of cellular materials is the effect of specimen size upon their measured properties. It is well known that the elastic modulus and yield strength of stochastic 3D foams increase with specimen size until their bulk values are reached; by contrast, their shear modulus and shear strength decreases. The former is due to a reduction, and the latter an increase, in the area fraction of weak boundary layer that develops next to the stress-free edges with increasing specimen size. Under bending, the weak boundary layers that develop at the top and bottom surface have an enhanced contribution to the mechanical response compared to the bulk material, leading to greater sensitivity to specimen size than, say, in compression [1–3].

Size effects were also reportedly observed in toughness measurements of stochastic 3D foams [4–6]; however, data scatter did not allow conclusive inference to be made. In this paper, the results of a detailed FE study are presented to quantify, and identify the physical origin of, the size dependent effect. The single edge notch specimen, with a 2D Voronoi micro-architecture, is modelled numerically using FE to obtain estimates of its fracture toughness which are compared to those obtained with a ‘boundary-layer’ analysis described in [7, 8].
2. Methodology and model set-up

Voronoi tessellations are employed to generate random planar lattices. A global cell regularity parameter, viz. \( \Lambda = d/d_0 \in (0, 1] \), controls the minimum distance \( d \) between the nuclei of any two adjacent cells and \( d_0 \) is the corresponding distance in a regular hexagonal lattice, see Christodoulou and Tan [7] for details. \( \Lambda = 1 \) and \( \Lambda \to 0^+ \) corresponds to a regular hexagonal and a completely random \( \Gamma \)-lattice, respectively. It was shown in [7] that the lattice ligaments deform by bending regardless of \( \Lambda \); all specimens modelled have identical cell regularity of \( \Lambda = 0.5 \).

Since the effects of relative density \( \rho \) upon toughness are already well known, see [8], only results for \( \rho = 0.1 \) are shown here.

![Figure 1. (a) SEN(B) specimen geometry and (b) Schematic of model set-up in the 'boundary-layer' analysis](image)

Twelve Voronoi tessellations with \( \Lambda = 0.5 \) were generated. Each tessellation is trimmed to size giving seven SEN(B) specimens of different width, viz. \( W/d_0 = 10, 15, 20, 25, 30, 50 \) and 100 where \( d_0 = \sqrt{3}l \) for a regular hexagonal lattice with cell wall length \( l \). The crack-tip in each specimen, generated from a given tessellation, is always centred at the same location. This eliminates variability of the calculated toughness caused by different crack-tip cell geometry [7]. A schematic of the SEN(B) specimen is shown in Figure 1a. The crack length \( \alpha (= 0.5W) \) and span \( S (= 4W) \) of the specimen were chosen according to [9]. The minimum diameter of the loading and support pins satisfy \( D_{\text{Load}}/W > 0.25 \) and \( 0.5 < D_{\text{Support}}/W < 1 \), respectively [9]. Fracture toughness of the SEN(B) specimen is calculated using [9]

\[
K_{IC} = 3 \frac{P_Q}{\sqrt{W}} \frac{S}{W} \left[ \frac{1}{W} \left( \sqrt{\frac{\alpha}{W}} \right) \left( 1 - \frac{\alpha}{W} \right) \left( 2.15 - 3.93 \frac{\alpha}{W} + 2.7 \left( \frac{\alpha}{W} \right)^2 \right) \right]^{3/2}
\]

where \( P_Q \) is the peak load during testing at the onset of fracture.

Additionally, the toughness of each tessellation is also calculated using a ‘boundary-layer’ analysis, see details given in [7, 8], where the displacement and rotational fields corresponding to the asymptotic \( K \)-field are applied to the nodes along the boundary of the mesh shown in Figure 1b. The size of the mesh used has been chosen to avoid size effects.

Both the SEN(B) and ‘boundary-layer’ specimens were implemented in ABAQUS/standard® where each ligament is modelled using four Timoshenko beam (B21) elements. Surface-solid body interaction was used to model contact between the pins and specimen, with the former as solid non-deformable bodies. Load is applied by prescribing a finite displacement to the rigid loading pin in the negative \( x_2 \) direction and constraining all its other degrees of freedom. This approach is preferred to a prescribed displacement BC which would have resulted in a...
more compliant response. An elastic-brittle failure criterion is adopted where ligament failure is assumed to occur if the predicted tensile stress reaches the fracture strength of the ligament $\sigma_f$.

3. Results

3.1. Effect of pin size

The toughness is found to be sensitive to the diameter of the pins and the manner in which they come into contact with the specimen. To illustrate these effects, a regular hexagonal lattice is used. Given a SEN(B) specimen with even number of cells along its width ($W$), the loading pin comes into contact with a closed cell whilst the support pins come into contact with open cells. The opposite occurs with odd number of cells along its width.

![Figure 2.](image)

Figure 2. Variation of non-dimensional fracture toughness with (a) loading and (b) support pin diameters for specimen with $W/d_0 = 10, 11, 50$ and 51 cells. Black and red lines indicates a specimen with even and odd number of cells along its width, respectively. The vertical dotted line indicates the minimum pin diameter recommended by the ASTM E399 [9].

Figure 2a plots the variation of the non-dimensional fracture toughness $K_{IC}/(\sigma_f \bar{p}^2 \sqrt{l})$ against loading pin diameter $D_{Load}/W$ for a fixed $D_{Support}/W = 0.5$. Results from both small ($W/d_0 = 10, 11$) and large ($W/d_0 = 50, 51$) sized specimens with odd and even number of cells along its width are shown. The diameter of the loading pin has a negligible influence upon the toughness of the specimen. The recommended value of $D_{Load}/W = 0.25$ by the ASTM E399 [9] gives converged results for all values of $W/d_0$.

By contrast, the diameter of the support pins has a considerable influence upon the specimen toughness; this is clearly evident in Figure 2b. For the smaller-sized specimens ($W/d_0 = 10, 11$), there is a significant difference between their predicted toughness for the entire range of $D_{Support}/W$. Even at $D_{Support} = W$, the results do not appear to have converged in a satisfactory manner. The larger specimens ($W/d_0 = 50, 51$), on the other hand, give converged results between $0.5 < D_{Support}/W < 1$. The numerical data suggest that smaller-sized specimens are likely to be the more sensitive to local cell topological variations along the specimen boundary where the pins come into contact. As it wouldn’t be possible to control the local morphological variations of the cells in stochastic lattices where the support pins come into contact, one could expect a significant increase in the variations of the measured toughness if smaller-sized specimens were used.
3.2. Specimen size

SEN(B) specimens of various size $W/d_0 = 10, 15, 20, 25, 30, 50$ and $100$ were generated from each of the twenty Voronoi tessellations created. Figure 3 plots the non-dimensional toughness for three, out of the twenty, tessellations. The predicted toughness by ‘boundary-layer’ analysis is also shown for comparison. The diameter of the pins used were that recommended by the ASTM E399 [9].

In general, an increase in toughness is observed with increasing specimen width $W$ for a given $d_0$. Each Voronoi tessellation gives slightly different dependence upon specimen size. For instance, the increase in toughness with $W/d_0$ is more dramatic for the tessellation shown in Figure 3b than in Figure 3a. Figure 3c, on the other hand, appears to show converged results although this was only observed in two, of the twenty, tessellations studied. More importantly, the ‘boundary-layer’ analysis consistently predicts lower toughness compared to that estimated by the largest SEN(B) specimen, for all tessellations. Note that the toughness predicted by the ‘boundary-layer’ analysis is representative of the lattice material since it is unaffected by size effects.

Figure 4 plots the average toughness of twenty tessellations obtained from the SEN(B) specimens and ‘boundary-layer’ analyses. The error-bars and shaded region correspond to ± one standard deviation. An average reduction in toughness, by up to $\approx 24\%$, is noted as specimen size decreases. The results show that a minimum specimen size of approximately 50 cells is needed to obtain representative lattice toughness if it is to be unaffected by size effects. The lattice toughness predicted by the ‘boundary-layer’ analysis is considerably lower, by approximately $10\%$, compared to one obtained using a SEN(B) specimen.

4. Discussions

To better appreciate the physical origin of the size effects as seen in Figure 4, continuous strain maps are needed to explain the cell-level deformation in the specimens. Strain maps are generated by triangulating each cell in the lattice, using a technique known as Delaunay triangulation, and assuming that each of the triangulated regions have a constant strain, i.e. a constant strain triangle; the details are described in [7]. Figure 5 plots the normalised equivalent strain, $\bar{\varepsilon}_{eq} = (\sigma_f \sqrt{3/2 \varepsilon_{ij} \varepsilon_{ij}})/(\bar{\rho}E_s)$, for three SEN(B) specimens of different size, and the those obtained by a corresponding ‘boundary-layer’ analysis, at the onset of first ligament fracture. The strain maps shown are the average of twenty tessellations.
Figure 4. Variation of non-dimensional toughness with specimen size for Voronoi lattices (average of twenty tessellations are given) and regular hexagonal lattices. Results from the ‘boundary-layer’ analysis are shown for comparison and the shaded region corresponds to ± one standard deviation.

Figure 5. Strain maps showing the normalised equivalent strain $\bar{\varepsilon}_{eq}$ for three SEN(B) specimens of different size $W/d_0 = 10, 20, 50$. The corresponding strain map from the ‘boundary-layer’ analysis is also shown. The strain maps shown are the average of twenty Voronoi lattices.

For the smaller SEN(B) specimen ($10d_0$ and $20d_0$), the strong interaction between the crack tip fracture process zone and the specimen boundary (upper), in the vicinity where the loading pin makes contact with the specimen, is clearly evident. A region of elevated equivalent strain appears in the un-cracked region ahead of the crack-tip; interestingly, the highest equivalent strain occurs near the specimen boundary rather than the crack tip. This is contrary to using a pre-cracked specimen where the purpose is to generate a region of $K$-dominance around the crack tip. As $W$ increases, the interaction between the fracture process zone and the specimen
boundary becomes somewhat weaker; at $W = 50d_0$, the highest equivalent strain now occurs in the vicinity of the crack-tip and is comparable to that observed in a ‘boundary-layer’ analysis. The strain maps appear to show that the physical origin of size effect in lattice materials are due to interactions between the crack-tip fracture process zone with the stress field generated by local indentation due to the loading pin. This disrupts the $K$-field in the vicinity of the crack-tip and renders LEFM inapplicable.

Since fracture toughness is a material property, it is not expected to be sensitive to specimen geometry and the measurement technique employed. However, the difference in predicted toughness by the two methods is significant, approximately 10%, between specimens thought to be unaffected by size-effects, see Fig. 4. Figure 5 shows that, unlike the SEN(B) specimen of width $W = 50d_0$, hardly any deformation is observed ahead of the crack tip in the ‘boundary-layer’ analysis. This suggests that there exist an additional component of the deformation field in the SEN(B) specimen which might not be present in a ‘boundary-layer’ analysis. A likely explanation is the presence of positive $T$-stress ahead of the crack-tip in the SEN(B) specimen. The biaxiality ratio of the SEN(B) specimen modelled here is estimated to be $B = 0.2$ and would typically result in a 200% increase in fracture toughness for hexagonal and random lattices [7, 10]. This is part of an on-going investigation to be reported elsewhere.

5. Conclusions

The predicted fracture toughness of random Voronoi 2D lattices is compared for single-edge notch specimen and one obtained by a ‘boundary-layer’ analysis. The SEN(B) specimen was found to be sensitive to size effects and at least 50 cells along the specimen width are needed to avoid size effects. It was found that size effects in lattice materials are caused by interactions between the crack-tip fracture process zone with the stress field generated by local indentation due to the loading pin, which disrupts the $K$-field in the vicinity of the crack-tip. Additionally, a significant difference between the predicted toughness of the SEN(B) specimen and the ‘boundary-layer’ analysis was found to exist between specimens unaffected by size effect and this is attributed to the presence of higher order terms in the stress-field around the crack-tip.

References

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