On the question of the existence of solutions of one nonlinear boundary-value problem for the system of differential equations of the theory of shallow shells of Timoshenko type

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Abstract. The study is devoted to the solvability of one system of nonlinear second order partial differential equations with given initial conditions is considered. The research method consists in reducing the initial system of equations to one nonlinear operator equation. The solvability of equation is established with the use of the principle of contracting mappings.

1. Introduction
Let us introduce in the plane bounded domain $\Omega$ and consider a system of nonlinear differential equations in the form

$$
\begin{align*}
& w_{1a} + \mu_1 w_{1a} + \mu_2 w_{2a} = f_1, \\
& \mu_1 w_{2a} + w_{2a} + \mu_2 w_{3a} = f_2, \\
& k^2 \left( w_{3a} + w_{3a} + \psi_{1a} + \psi_{2a} \right) + k_{3} w_{1a} + k_{4} w_{2a} - k_{5} w_{3a} + \\
& + k_{3} w_{3a} / 2 + k_{4} w_{2a} / 2 + \beta_3 \left[ (T^{3a} w_{3a})_{a} + R^3 \right] = 0
\end{align*}
$$

(1)

under the following conditions at the boundary $\Gamma$:

$$
\begin{align*}
& \psi_{1a} + \mu \psi_{1a} + \mu_2 \psi_{2a} = \tilde{g}_1, \\
& \mu \psi_{2a} + \psi_{2a} + \mu_2 \psi_{3a} = \tilde{g}_2,
\end{align*}
$$

(2)

where $w_1, w_2, w_3$ are the unknown functions; $\mu, \mu_1, \mu_2$ are the coefficients of the system; $f_1, f_2$ are the right-hand sides of the equations; $\psi_{1a}, \psi_{2a}, \psi_{3a}$ are the functions of the boundary; $\tilde{g}_1, \tilde{g}_2$ are the boundary conditions; $T^{3a}$ is the operator of the third order; $R^3$ is the operator of the third order.
In (1)–(6) the following notations are used:
\[ f_j = f_j(w_3) = k_j w_{3a_1} - w_{3a_1} w_{3a_2} - \mu_2 w_{3a_2} w_{3a_3} - \mu_1 w_{3a_3} w_{3a_4} - \beta_j R_j^i, \]
\[ \tilde{g}_j = g_j + k_0 \psi_j, \quad g_j = g_j \left( w_3 \right) = k_0 w_{3a_1} - \beta_j L_j, \quad j = 1, 2, \mu_i = \left( 1 - \mu \right) / 2, \quad \mu_2 = \left( 1 + \mu \right) / 2. \] (7)
\[ \phi_i \left( w_3 \right) \left( t \right) = \beta_2 P^i \left( s \right) + \left[ -k_4 w_3 \left( t \right) + \mu w_3^2 \left( t \right) / 2 + w_3^2 \left( t \right) / 2 \right] d \xi / ds - \mu_i w_{3a_1} \left( t \right) w_{3a_4} \left( t \right) d \xi / ds, \]
\[ \phi_j \left( t \right) = \beta, N^j \left( s \right), \quad j = 1, 2, \quad t = \alpha(s) + i \alpha' \left( s \right) \in \Gamma, \quad k_3 = k_2 + \mu k_2, \quad k_4 = k_2 + \mu k_1, \]
\[ k_5 = k_1 + k_2 + 2 \mu k_1, \quad k_6 = 6 k^2 \left( 1 - \mu \right) / h^2, \quad \beta_1 = 12 \left( 1 - \mu^2 \right) / \left( h^4 E \right), \quad \beta_2 = \left( 1 - \mu^2 \right) / \left( E h \right). \]

The system (1) together with the boundary conditions (2)–(6) describes the state of equilibrium isotropic elastic homogeneous shallow shell with simply supported edges within the framework of Timoshenko shear model [1, pp. 168-170, 269]. Here \( T^j \) are stresses \( \left( \lambda, \mu = 1,3 \right); \) \( w_3 \) are the Cartesian coordinates of the points in the domain \( \Omega \); \( \psi_i \left( t = 1.2 \right) \) are rotation angles of normal cross-section of \( S_0; \) \( R^i \left( j = 1,3 \right), \) \( L^j \left( k = 1,2 \right), \) \( N^i, \quad N^2, \) \( P^2, \) \( P^3 \) are components of the external forces acting on the shell; \( \mu = \text{const} \) is the Poisson coefficient, \( E = \text{const} \) is Young’s modulus, \( k_3, k_2 = \text{const} \) are principal curvatures; \( k^2 = \text{const} \) is the shear coefficient; \( h = \text{const} \) is the shell width; \( \alpha^i, \alpha^2 \) are the Cartesian coordinates of the points in the domain \( \Omega. \)

**Problem A.** Find a solution to system (1) under boundary conditions (2)-(6).

There are a number of works devoted to the solvability of nonlinear problems in the framework of the Timoshenko displacement model [2–11]. The studies in [2–11] are based on integral representations for generalized displacements containing arbitrary holomorphic functions. Which are found in such a way that the generalized displacements satisfy the given boundary conditions. Two approaches are used for their construction. The first approach is based on the application of explicit representations of solutions of problems of Riemann–Hilbert for holomorphic functions in the unit circle. Therefore, plane domain assumed from the beginning the unit circle [2–5], [7–8], or conformal mappings on the unit circle [6], [9]. The second approach uses the theory of one-dimensional singular integral equations to determine holomorphic functions [10, 11]. The nonlinear problem for arbitrary shallow shells under other boundary conditions is studied by the conformal mapping method in this paper.

Consider boundary-value problem A in a generalized formulation. Let the following conditions hold true: (a) \( \Omega \) is a simply connected domain with the boundary \( \Gamma \in C^\rho; \) (6) external forces \( R^i \left( j = 1,3 \right), L^j \left( k = 1,2 \right), \) \( N^i, \quad N^2, \) \( P^2, \) \( P^3 \in C^\beta \left( \Gamma \right); \) in what follows \( p > 2, 0 < \beta < 1. \)

**Definition.** The vector of generalized displacements \( a = \left( w_1, w_2, w_3, \psi_1, \psi_2 \right) \in W^p \left( 2 \right) \left( \Omega \right), \) \( p > 2, \) is a generalized solution to the problem if the vector satisfies almost everywhere the equations of system (1) and it satisfies boundary conditions (2) – (6) in pointwise fashion.

2. **Solution to problem A with respect to tangential displacements and angles of rotation**

Let us consider the system of the first two equations in (1) and initially assume that \( w_3 \) is fixed. The general solution of the system is in the form \([2, 3]\]
\[ \omega_1 \left( z \right) = w_1 + i w_1 = \Phi_1 \left( z \right) + i T^i \left( \Phi_1 + T^j \right) \left( z \right), \quad z = \alpha^1 + i \alpha^2, \quad f = \left( f_1 + i f_2 \right) / 2, \] (8)
where \( \Phi_1 \left( z \right) \in C_\alpha \left( \tilde{\Omega} \right), \Phi_2 \left( z \right) \in C_\alpha \left( \tilde{\Omega} \right) \) are an arbitrary holomorphic function;
\[ T^j = - \frac{1}{\pi} \int_{\gamma} \frac{f \left( \xi \right)}{\xi - z} d \xi d \eta, \quad \xi + i \eta, \quad d \left[ g \right] = d_1 g + d_2 \overline{g}, \quad d_j = \left( \mu_j + \left( -1 \right)^j / 4 \mu_j \right), \quad j = 1, 2. \]
The holomorphic functions $\Phi_j(z), j=1,2$ find so that the tangential displacements $w_1, w_2$ (8) will satisfy boundary conditions (2), (3). Following work [9], for tangential displacements $w_1, w_2$ when the condition is satisfied
\[
\int r^2 \, ds + \oint \kappa^2 \, d\alpha \, d\alpha = 0,
\]
we get the desired view
\[
H_0 w_j(z) = H_0[f(w_j); l(w_j)](\psi(z)) + i Td[\Phi_j[l(w_j)](\psi(\zeta)) + Tf(w_j)(\zeta)](z),
\]
where
\[
\Phi_j[l(w_j)](\psi(z)) = \frac{1}{2\pi i} \int \frac{f(t)}{t - \psi(z)} \, dt,
\]
\[
\Phi_j[l(w_j)](\zeta) = 2(\mu - 1) S_{jk}(\Re Td[Tf](\phi(t)))(\zeta) + \frac{(\mu - 1)}{\pi} \int \frac{f(t)}{t - \zeta} \, dt,
\]
and
\[
l(w_j)(\tau) = \frac{\varphi_j(w_j)(\tau)}{\mu - 1} + \Re[t'Sd[Tf]^{\tau}(\tau) - \mu \alpha] / ds \Re Tf(\tau) = \ln[f(w_j); \varphi_j(\psi(z), \tau) \in \Gamma;
\]
via the $Sd[\Phi_j^{\tau}(\tau)$ means the limit of the function $Sd[\Phi_j](z)$ as $z \to \tau \in \Gamma$ from the interior of the domain $\Omega$; $z = \phi(\zeta)$ is conformal mapping of the unit disk $K: |z| \leq 1$ to the area $\Omega$; $\zeta = \psi(z)$ reverse function to $z = \phi(\zeta)$; $t' = dt / d\sigma, d\sigma$ is part of an arc of a circle $\partial K$; $c_0$ is an arbitrary real constant.

We turn to functions $\psi_1, \psi_2$ in the last two equations (1). These functions should satisfy boundary conditions (4), (5).

Let us note that the structure of left-hand sides in the last two equations (1) coincides with the structure of left-hand sides in boundary conditions (4) and (5). Relations for tangential displacements differ only in the right-hand sides. Therefore at fixed right-hand sides for rotation angles we obtain similar view (8)
\[
\psi = \psi_2 + i \psi_1 = H_0[\tilde{\psi}(\nu); \tilde{I}(\nu)] + \psi_\nu(z), \quad z \in \Omega
\]
where accepted designation
\[
\nu = \nu_1 + i \nu_1, \quad \tilde{\psi}(\nu) = (\tilde{\psi}_1(\nu) + i \tilde{\psi}_2(\nu))/2, \quad \tilde{I}(\nu) = \tilde{I}_1(\nu) + i \tilde{I}_2(\nu),
\]
\[
\nu_j = w_{s\nu_j} + \nu_j, \quad \tilde{\psi}_j(\nu) = k_0 \nu_j - \beta_{1j} L_{1j}, \quad \tilde{I}_j(\nu) = \tilde{I}_1(\nu) + (\mu - 1) + h_j \tilde{\psi}_j(\nu),
\]
\[
h_j \tilde{\psi}_j(\nu)(t) = \Re[i t'Sd[T \tilde{\psi}_j(\nu)]^+(t) - (1 + j)(\mu - 1) \mu \alpha^{\nu_j} / ds \Re T \tilde{\psi}_j(\nu)(t)] \in \Gamma,
\]
\[
\psi_\nu(z) = \psi_\nu_2(z) + i \psi_\nu_1(z) = c_1 \tilde{z}/(1 - \mu) + c_2 + i c_3, \quad j = 1, 2.
\]
c_j (j = 1, 3) are an arbitrary real constants; $H_0(\tilde{\psi}(\nu); \tilde{I}(\nu))$ is defined by formula in (10), in which
\[
\Phi_j[l(w_j)](\psi(z)) = \frac{1}{2\pi i} \int \ln \left(1 - \frac{\psi(z)}{t}\right) h_j[l(w_j)](t) \, dt,
\]
\[
h_j[l(w_j)](t) = \varphi_j(t)/[l(w_j)(\phi(t)) + t \varphi_j(t) Sd[\Phi_j[l(w_j)]^{\nu_j}((\phi(t))] + \mu_j \varphi_j(t) \Phi_j[l(w_j)](t) / 2, t \in \partial K;
\]
by $\ln(1 - \psi(z)/t)$ means a branch that vanishes when $\psi(z) = 0$;
\[
\Phi_1[I(w_3)](z) = \Re[I(\tilde{w}_3)](z), \quad z \in K,
\]
\[
B[I(w_3)](z) = \frac{i}{\pi \mu_0 \phi'(z)} \left\{ \frac{1}{\pi} \int_{\xi} \frac{\phi'(\zeta)}{\phi'(\xi)} \frac{A(w_3)}{(1 - \xi z)^2} d\xi + A(w_3)(z) \right\},
\]
\[
A(w_3)(z) = A[I(\tilde{w}_3)](z) = \int_{\tilde{z}} \frac{\tilde{I}(w_3)(\phi(t))}{(t - z)}|\phi'(t)| dt, \quad \tilde{I}(w_3) = \tilde{I}(w_3) + i\tilde{I}_2(w_3),
\]

\(\Re g\) is a linear bounded operator in \(C_a(\bar{K})\), the existence of which follows from Fredholm's third theorem [12].

As this takes place, the conditions of solvability, must be executed

\[
\beta_j \left( \int_{\Gamma} N^j(s)ds + \int_{\Omega} \alpha^j d\alpha d^2 = 0, \quad j = 1, 2, \right.
\]
\[
\beta_j \left( \int_{\Gamma} (\alpha^j N^j - \alpha^2 N^j)ds + \int_{\Omega} (\alpha^j L^j - \alpha^2 L^j) d\alpha d^2 = 0, \right.
\]

where \(N^j, L^j\) are components of external load, \(\nu_j \in \mathcal{W}_p(\Omega)\) \((j = 1, 2)\) are functions introduced in (12), which are temporarily considered fixed.

Thus, under the conditions (9), (16), the problem \(A\) at fixed \(w_3, \nu_j (j = 1, 2)\) solvable relative to the tangential displacement and rotation angles; its solutions are given by formulas (10), (12).

Let us present the relations (12) to a more convenient form for further investigation. For \(\tilde{g}(\nu), \tilde{I}(\nu)\) from (12) we obtain

\[
\tilde{g}(\nu) = \tilde{g}^0(\nu), \quad \tilde{I}(\nu) = \tilde{I}^0(\nu), \quad \tilde{g}^k = (\tilde{g}_{1k} + i\tilde{g}_{2k})/2, \quad \tilde{I}^k = \tilde{I}_{1k} + i\tilde{I}_{2k} (k = 0, 1),
\]

\[
\tilde{g}_{j0} = -\beta_j L^j, \quad \tilde{g}_{j1} = k_0 \nu_j, \quad \tilde{I}_{j0} = \phi_j I(1 - \mu) + h_j \tilde{g}^0, \quad \tilde{I}_{j1} = h_j \tilde{I}^1(\nu), \quad \phi_j = \beta_j N^j(s), \quad j = 1, 2.
\]

Note that \(\tilde{g}_j(\nu), \tilde{I}_j(\nu)\) are homogeneous operators with respect to \(\nu\) of order \(j\).

Now, substituting the expression (17) in (12), we come to the desired representations for the angles of rotation and their derivatives

\[
\psi = \psi(\nu) = \psi^0 + \psi^1(\nu) + \psi^2, \quad \psi_{j\alpha} = \psi_{j\alpha}(\nu) = \psi_{j\alpha^0} + \psi_{j\alpha^1} + \psi_{j\alpha^2},
\]

\[
\psi^{n}(\nu) = \psi_{2n}(\nu) + i\psi_{1n}(\nu) = H_{1} [\tilde{g}^{n}(\nu); \tilde{I}^{n}(\nu)], \quad \psi_{j\alpha^k}(\nu) = H_{j} [\tilde{g}^{n}(\nu); \tilde{I}^{n}(\nu)] \quad j, k = 1, 2, n = 0, 1,
\]

where \(\psi^{n}(\nu), \psi_{j\alpha^k}(\nu)\) are homogeneous operators with respect to \(\nu\) of order \(n\).

Before moving on to the third equation in (1), the deflection \(w_3\) and its derivatives are expressed in terms of \(\nu_j (j = 1, 2)\). From the formula (13), taking into account (18), we obtain

\[
w_{3a^j} = w_{3a^j}(\nu) = \psi_{3a} + w_{3a^j}(\nu) + w_{3a^2}, \quad (19)
\]

\[
w_{3a^j} = -\psi_{j\alpha}, \quad w_{3a^j}(\nu) = \psi_{j^2} - \psi_{j^1}(\nu), \quad w_{3a^2} = -\psi_{j^1}, \quad j = 1, 2.
\]

From the ratio (19) we derive the representation

\[
w_{3} = w_{3}(\nu) = w_{30} + w_{31}(\nu) + w_{32}, \quad w_{3} = -c_2 \alpha^2 + c_1 \alpha^2 + c_1, \quad (20)
\]

\[
w_{30} = \int_{(0,0)} \psi_{10} d\alpha^1 + \psi_{20} d\alpha^2, \quad w_{31}(\nu) = \int_{(0,0)} [\nu_1 - \psi_{11}(\nu)] d\alpha^1 + [\nu_2 - \psi_{21}(\nu)] d\alpha^2.
\]

Note that the curvilinear integrals in (20) do not depend on the integration path and therefore the constant \(c_1 = 0\). This is taken into account when displaying the expression \(w_{3}^*\). Then the formula \(\psi_{.}(z)\) in (13) will take a simpler form
\[ \psi_r(z) = \psi_{2r}(z) + i\psi_{1r}(z) = c_2 + ic_3, \quad (21) \]

The representations (19), (20) substitute first in (7), then the resulting expression substitute in (10). Then, for tangential displacements and their derivatives, we obtain partitions into linear and nonlinear operators

\[ \omega_0 = \omega_{0i}(\nu) + \omega_{0j}(\nu) + \omega_{0k}(\nu), \quad w_{i,j} = w_{j,i}(\nu) + w_{i,j}(\nu) + w_{j,i}(\nu), \quad j = 1, 2, \quad (22) \]

where

\[ \omega_{0i}(\nu) = w_{i,j}(\nu) + i\omega_{i,j}(\nu) = H_0[j^i(\nu); l^j(\nu)], \quad w_{i,j}(\nu) = H_{jk}[f^i(\nu); l^j(\nu)], \quad \omega_{i,k} = w_{i,k} + i\omega_{i,k} = H_{ik}[f^i; l^k], \quad j, k, n = 1, 2; \quad (23) \]

operators \( H_0[f; g] \) are defined in (10), \( f^i(\nu), l^j(\nu) \) and \( f^j(\nu), l^j(\nu) \) are homogeneous terms of the first order and are nonlinear terms are relatively \( \nu, f^*, l^* \) are known functions that depend on \( w_j^*(j = 1, 3), \psi_{+k}(k = 1, 2) \).

Note that the functions \( w_j^*(j = 1, 3), \psi_{+k}(k = 1, 2) \) defined by the equations in (23), (20), (21), are rigid displacements of the shell as an absolutely rigid body. That is, they zero out the strain components \( \gamma_{ij}(i, j = 1, 3, k = 0, 1) \). Then for \( w_j^*, \psi_{+k} \) we obtain explicit representations in the form

\[ w_j^* = c_2 + c_2k(\alpha^2 - k_1(\alpha^2)) / 2 - k_2c_2\alpha^2 + (c_1k_2 - c_2^2 / 2)\alpha^2 + (c_1 - c_2c_2 / 2)\alpha^2 - c_2(k_1 - k_2) / 4 + c_6, \quad (24) \]

3. Reduction of system (1) to a single equation and its study

We pass to the third equation in (1). Replacing the generalized displacements with expressions from (18) – (22), taking into account the boundary condition (6) and following [7], we lead the third equation to the equivalent equation with respect to \( \nu = \nu_3 + i\nu_1 \)

\[ \nu - G_\nu = 0, \quad (25) \]

where \( G_\nu \) is a nonlinear bounded operator in \( W^1_p(\Omega) \), \( 2 < p < 2/(1 - \beta) \); moreover, for arbitrary \( \nu^j(j = 1, 2) \in W^1_p(\Omega) \) that lie in the ball \( \| \nu^j(\Omega) \| < r \), the estimate holds

\[ \| G_\nu \nu^j - G_\nu \nu^j \|_{W^1_p(\Omega)} \leq q_r \| \nu^j - \nu^j \|_{W^1_p(\Omega)}. \]

Thus there is a solvability condition of the form

\[ \int (k_1 \alpha^2 T^1(a) + k_2 \alpha^2 P^2 + P^3) ds + \int (k_3 \alpha^2 R^1 + k_4 \alpha^2 R^2 + R^3) d\alpha^2 d\alpha^2 = 0, \]

\[ T^1(a) = T^1 d\alpha^2 / ds + T^1 d\alpha^1 / ds, \]

which is performed by selecting a constant \( c_2 \).

Suppose that the radius \( r \) of the ball and the external forces acting on the shell satisfy the conditions

\[ q_r < 1, \quad \| G_\nu(0) \|_{W^1_p(\Omega)} < (1 - q_r) r. \quad (26) \]

Under these conditions, for equation (25) one can use the contraction mapping principle [13], accordingly to which equation (25) in the ball \( \| \nu \|_{W^1_p(\Omega)} < r \) has a unique solution \( \nu \in W^1_p(\Omega) \), \( 2 < p < 2/(1 - \beta) \). The solution can be represented in the form \( \nu = \Re G_\nu(0) \), where \( \Re \) is the operator resolvent \( G_\nu(\nu) - G_\nu(0) \).

Knowing \( \nu = \Re G_\nu(0) \) by formulas (18), (20), (22) we find generalized displacements \( w_1, w_2, w_3, \quad \psi_1, \psi_2 \in W^1_p(\Omega), \quad 2 < p < 2/(1 - \beta) \). As a result, we obtain a generalized solution \( a = (w_1, w_2, w_3, \psi_1, \psi_2) \)
to the problem A, which can be represented in the form $a = a_0 + a^*$, where $a^* = (0, w^*_2, -c_3, \alpha_3', c_4, c_5', c_6')$ ($w^*_2$ is defined in (24)); $a_0$ is vector with components $w_{j1}(\nu) + w_{j2}(\nu)$ $(j = 1,2)$, $w_{30} + w_{31}(\nu), \psi_{j0} + \psi_{j1}(\nu)$ $(j = 1,2)$, which are found by formulas (23), (20), (18).

Now the solution $\nu = \nu_2 + i\nu_1 = \Re G_\nu(0)$ of equation (25) we substitute to the conditions in (16). Then, it is easy to make sure that the third condition in (16) is fulfilled identically, the first two conditions can be transformed to form

$$
\int (N^i + [k_2(\alpha^2) - k_1(\alpha')^2]T^i(a)/2 - k_2\alpha'^2P - \alpha^2P^3)ds +
+ \int (L^i + [k_2(\alpha^2) - k_1(\alpha')^2]T^i(a)/2 - k_2\alpha'^2R - \alpha^2R^3)ds + \int T^i(a)\mathcal{Z}ds + \int R^i\mathcal{Z}d\alpha' d\alpha^2 = 0,
$$

$$
\int (N^i - [k_2(\alpha^2) - k_1(\alpha')^2]T^i(a)/2 - k_2\alpha'^2T^i(a) - \alpha^2T^i(a)P^3)ds +
+ \int (L^i - [k_2(\alpha^2) - k_1(\alpha')^2]T^i(a)/2 - k_2\alpha'^2R - \alpha^2R^3)ds + \int P^i\mathcal{Z}ds + \int R^i\mathcal{Z}d\alpha' d\alpha^2 = 0,
$$

where $\mathcal{Z} = w_{j1}(\nu) = w_{30} + w_{31}(\nu) + w_{j2}^*$ is known function; herein $\nu = \Re G_\nu(0)$ and $w_{30}, w_{31}(\nu), w_{j2}^*$ are defined by equality (20). Note that $\mathcal{Z}$ depends only on external forces and rigid displacements $w_{j2}^*$.

Note that the conditions (9), (27) are not only sufficient, but also necessary conditions for the solvability of the problem A. Indeed, if $a = (w_1, w_2, \psi_1, \psi_2)$ is a generalized solution to the problem A, then integrating over the domain $\Omega$ of equation (1) except for the first. To do this, by involving the integration formula in parts and taking into account the boundary conditions (2) – (6), we come to the conditions (9), (27).

So, we have proved

Theorem. Let conditions (a), (6) in Section 1 be fulfilled and inequality (26) holds. Then conditions (9), (27) are necessary and sufficient for the solvability of the geometrically nonlinear equilibrium problem for shallow elastic shells of the Timoshenko type under the boundary conditions (2) – (6). Then the problem has generalized solution $a = (w_1, w_2, \psi_1, \psi_2) \in W^2_2(\Omega), 2 < p < 2/(1 - \beta)$, of the form $a = a_0 + a^*$ with accuracy to rigid displacements $a^*$ of the shell as an absolutely rigid body.

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