Negative vacuum energy densities and the causal diamond measure

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Arguably a major success of the landscape picture is the prediction of a small, non-zero vacuum energy density. The details of this prediction depends in part on how the diverging spacetime volume of the multiverse is regulated, a question that remains unresolved. One proposal, the causal diamond measure, has demonstrated many phenomenological successes, including predicting a distribution of positive vacuum energy densities in good agreement with observation. In the string landscape, however, the vacuum energy density is expected to take positive and negative values. We find the causal diamond measure gives a poor fit to observation in such a landscape — in particular, 99.6% of observers in galaxies seemingly just like ours measure a vacuum energy density smaller than we do, most of them measuring it to be negative.

I. INTRODUCTION

If our universe is one vacuum phase among many in an eternally inflating multiverse, our expectations for cosmological observables depend on the answers to three major questions: (1) what is the distribution of the relevant physical parameters among generic vacuum states in the landscape, (2) how do conditional constraints associated with other characteristics of our universe, most notably anthropic constraints, affect this distribution, and (3) how do we add up / compare the distinct observations of the diverging number of observers. The landscape prediction of the cosmological constant \( \Lambda \) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], made possible by our confidence in addressing questions (1) and (2) in this scenario, is arguably a major success of the multiverse picture. Question (3) — the so-called measure problem — remains unanswered, yet one may calculate expectations for \( \Lambda \) given any specific proposal.

One such proposal is the causal diamond measure [11, 12], which regulates the diverging spacetime volume of the multiverse by restricting attention to only the spacetime volume of the causal horizon about a given worldline. This measure has been shown to avoid the “youngness paradox” [13, 14], “runaway inflation” [15, 16, 17], and under certain conditions “Boltzmann brain domination” [18, 19, 20, 21, 22, 23, 24] — three phenomenological pathologies in which the overwhelming majority of observers see a world in stark contrast with what we observe.\(^1\) The causal diamond measure has been combined with an entropy-counting anthropic selection factor to successfully explain the observed value of \( \Lambda \), at least when attention is limited to positive \( \Lambda \) [8], along with the observed values of other cosmological parameters [32]. (See also Ref. [36]; it has also found phenomenological success using more explicit anthropic selection criteria [37].)

On the other hand, it takes only one poor prediction to raise suspicion on a given theory. We calculate the full distribution (positive and negative values) of \( \Lambda \), using the causal diamond measure and assuming a flat “prior” distribution among the very small values of \( |\Lambda| \) in the landscape. The result is in poor agreement with observation, with about 99.6% of observers seeing a value smaller than what we measure. This includes the effects of anthropic selection; in fact compared to other calculations in the literature we use very restrictive selection criteria — counting only galaxies with mass and virialization density very similar to those of the Milky Way — to avoid counting hypothetical observers where possibly none in fact exist. Choosing very restrictive anthropic criteria can be viewed as simply further conditioning the predicted distribution, and therefore should only lead to increased accuracy [38].

One might be willing to accept our measurement of \( \Lambda \) as a rare statistical fluke, especially in light of any compelling theoretical considerations and successful predictions of other cosmological observables, or if one believes that even our restrictive anthropic criteria significantly overcount the number of observers measuring values of \( \Lambda \) smaller than what we do. Alternatively, given that we do not possess a firmly-grounded theoretical derivation of this or any other regulator of eternal inflation, one might see this result as providing guidance as to what directions to pursue with respect to formalizing other promising measures (see for example footnote \(^{11}\)) or modifying the causal diamond measure.

The remainder of this paper is organized as follows. In Section \(^{11}\) we provide background to the problem, describing first our assumptions about the landscape, next the relevant aspects of the causal diamond measure, and finally our specific anthropic criteria. We perform the calculation in Section \(^{11}\) where we also comment on some of the uncertainties of the analysis. A final discussion is given in Section \(^{11}\).
II. BACKGROUND

A. Landscape Assumptions

String theory apparently possesses an enormous number of metastable vacua, each with potentially different low-energy particle physics and/or vacuum energy densities \[39, 40, 41, 42\]. We restrict attention to vacua indistinguishable from our own except for the vacuum energy density, and also, as we explain below, to measurements of observers in galaxies much like the Milky Way. In hindsight \[1, 2\], this limits the magnitude of the vacuum energy density to a range of values that is microscopic compared to the range of possibilities in the landscape. For these reasons we assume the distribution of vacuum energy densities in relevant states of the landscape is essentially continuous (e.g., finer than the resolution of foreseeable observation) and flat,

\[
I(\Lambda) = \text{constant}. \tag{1}
\]

Each of the various vacua of the landscape are physically realized during eternal inflation, during which spacetime regions in one de Sitter (dS) vacuum may tunnel to other dS or anti-de Sitter (AdS) vacua, forming new pocket universes (note that AdS vacua may not tunnel back to dS vacua). This tunneling may be seen as proceeding through potential barriers between local minima (vacua) in the landscape. We assume the various tunneling transition rates into a vacuum with very small \(|\Lambda|\) are uncorrelated with the precise value \(\Lambda\); this should be the case whenever the vacua with \(\Lambda\) in any small interval \(d\Lambda\) are surrounded by a diverse set of landscape potential barriers — another property expected of an enormous landscape. This conclusion has been demonstrated in several more simple landscape scenarios, see for example Refs. \[43, 44, 45, 46, 47\].

B. Causal Diamond Measure

The causal diamond measure \[11, 12\] regulates the diverging spacetime volume of eternal inflation as follows. One focuses on a single worldline, beginning in a given dS vacuum,\(^2\) and considers the ensemble all possible future “histories” of that worldline. All except a set of measure zero of the worldlines in this ensemble have finite duration, eventually terminating on an AdS singularity. The multitude of possible histories in the above ensemble are weighted against each other according to their normalizable quantum-mechanical branching ratios. For a given worldline in the ensemble, the causal diamond is constructed by finding the intersection of the future lightcone of the point at the beginning of the worldline, and the past lightcone of the point at which the worldline terminates. The fraction of observers measuring a given value of \(\Lambda\) is then calculated by cataloging the full set of observers in the set of causal diamonds generated by the ensemble of worldline histories.

To proceed one can employ an important simplifying approximation. First note that vacua suitable to observers, which we call “anthropic” vacua, should be very rare among the full set of vacua in the landscape. Second, transitions from an anthropic dS vacuum to other dS vacua are suppressed by a factor \(e^{-3/\Lambda G}\) relative to transitions to AdS vacua, and meanwhile AdS vacua cannot transition back to dS vacua at all. These imply that among the worldline “histories” in the above ensemble, observers should overwhelmingly appear in just two situations: either in dS vacua (with very small vacuum energies) that subsequently decay to AdS vacua (with relatively large-magnitude vacuum energy densities), in which case the worldline quickly terminates after decay to AdS, or in AdS vacua with small-magnitude vacuum energy densities, in which case the worldline terminates at the AdS singularity. Therefore, to good approximation we can restrict attention to histories in the above ensemble in which only one anthropic vacuum is encountered, as either the second-last or the last vacuum before the worldline is terminated.

The causal diamond measure can then be conveniently divided into two parts. In the first part one assigns a “prior” probability \(I\) to each type of pocket, according to the relative likelihood of the above worldline encountering such a pocket. An example of such a calculation is made explicit in Ref. \[11\]; however for us the details are unimportant because, as described above, the prior distribution of \(\Lambda\) is flat, \(I(\Lambda) = \text{constant}\). Each anthropic pocket then receives an additional weight \(A(\Lambda)\), corresponding to the typical number of observers measuring \(\Lambda\) in the causal diamonds of the ensemble that include that anthropic pocket. Since by hypothesis the worldline terminates soon after the decay of dS (or at the singularity of AdS), the typical intersection of a causal diamond with an anthropic pocket is simply the past lightcone of a random point on the hypersurface of decay (or the final singularity) of the anthropic pocket.\(^3\) Henceforth we refer to this past lightcone as the causal diamond.

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\(^2\) In general, the predictions of the causal diamond measure depend on the choice of initial dS vacuum — or how an ensemble of such vacua are weighted against each other. Although one might find it desirable that eternal inflation erase such dependence on initial conditions, there is at present no theoretical reason to demand that this be the case.

\(^3\) To see this, note that the causal diamond stops growing, and begins to shrink, when half of the conformal time has elapsed between the start of the worldline and its end. Meanwhile, the time rate of change of conformal time is one over the scale factor. Since the scale factor grows exponentially during inflation, the pace of conformal time is exponentially suppressed at late times, the effect of which is the causal diamond almost always begins to shrink before a worldline reaches the reheating hypersurface of an anthropic vacuum. It might help to consider the following...
Note that, as defined and implemented above, the “causal diamond” measure makes the same anthropic predictions as another form of “causal patch” measure, which is defined exactly as above except with the role of the causal diamond played by the past lightcone of the future endpoint of a given worldline. (This equivalence is trivial, since as described above the intersection of the causal diamond with the anthropic pocket is taken to be equal to the past lightcone in the anthropic pocket.) Thus, the terms “causal diamond” and “causal patch” have been used interchangeably in the literature.

Putting the above results together, we write

\[ dP(\Lambda) \propto A(\Lambda) I(\Lambda) d\Lambda, \]  

(2)

where again \( I(\Lambda) = \text{constant} \). The typical number of observers in the causal diamond can be written

\[ A(\Lambda) \propto \int_0^{\tau_f} \rho_{\text{obs}}(\Lambda, \tau) V_\circ(\Lambda, \tau) d\tau, \]  

(3)

where \( \rho_{\text{obs}} \) is the average number of observers per unit four-volume, \( V_\circ \) denotes the physical three-volume in the causal diamond as a function of proper time \( \tau \), and \( \tau_f \) is the time of vacuum decay in an anthropic dS vacuum or the time of the final singularity in an anthropic AdS vacuum. Defining conformal time \( \eta = \int d\tau / a(\tau) \) (where \( a \) is the scale factor) and choosing the integration constant so that \( \eta \) is negative and approaches zero as \( \tau \rightarrow \tau_f \), we can write

\[ V_\circ(\Lambda, \tau) \propto -a^3(\Lambda, \tau) \eta^3(\Lambda, \tau). \]  

(4)

C. Anthropic Selection

We seek a distribution of \( \Lambda \) from which the value we measure can be considered as randomly drawn. Although one might speculate that this applies to the (properly regulated) set of measurements made by all observers, it is less presumptuous to take our measurement as typical of those made by observers very much like us. The only danger in specifying a more narrow notion of observer is the possibility that such observers exist for only a slim range of \( \Lambda \), leaving little opportunity to falsify the prediction. This will not be the case with our analysis.

We do not possess the technical skill to track the density of observers just like us. One step to simply the problem is to restrict attention to a slice of the landscape on which all parameters except \( \Lambda \) are fixed to the values we measure. Thus we ask the conditional question: given what we know about the parametrization of our universe, what value of \( \Lambda \) should we expect to measure?

A second, more significant step to simplify the problem is to develop an astrophysical proxy for an observer. It is of course crucial that the set of proxies faithfully represents the set of observers. We take as our proxy “Milky-Way like” (MW) galaxies, by which we mean galaxies constrained as much as is reasonably possible to resemble the Milky Way (details are given below). Unless our existence in a MW galaxy is itself atypical of observers like us — a circumstance made unlikely by the fact that a diverse range of galaxies exist even in our universe — this approach should only increase the accuracy of our prediction relative more general approaches. Regardless, restricting attention to MW galaxies can simply be viewed as performing a more conditioned prediction.

To identify MW galaxies, we adopt the following simplified picture of structure formation [18]. Consider first a comoving sphere enclosing a mass \( M \), with density contrast \( \sigma > 0 \). The evolution of \( \sigma \) can be analyzed using linear perturbation theory, or for instance by studying the evolution of a spherical top-hat overdensity. The collapse density threshold \( \delta_c \) is defined as the amplitude of \( \sigma \), according to the linear analysis, when the spherical top-hat analysis says it has collapsed. The collapse density threshold is not a constant when \( \Lambda \neq 0 \), but the variation is relatively small and to good approximation we can simply set \( \delta_c = 1.69 \) [19 2].

In a more sophisticated analysis, the overdensity does not collapse to a point, but virializes as a halo with density \( 18\pi^2 \) times the average cosmic matter density at that time it would have collapsed [3]. In fact, structure formation is hierarchical: smaller comoving regions collapse and virialize first, and larger halos form as these accrete matter, merge, and \text_quote_left\textquoteright\textquoteright\virialize.\textquote_right In our universe the rate of such mergers decreases with time, furthermore the baryons cool and collapse into galaxies containing stars. We approximate the full process of structure formation by identifying a critical time \( \tau_* \), before which the baryons in a halo are continuously revirializing, and after which the evolution of the baryons depends only on the mass \( M \) and the average density at \( \tau_* \).

We define a MW galaxy as one that has the same mass, virialization density, and age as the Milky Way, where by age we mean the time lapse \( \Delta\tau \) between \( \tau \) and when observers arise. The time \( \tau_* \) is different in different universes; it is found by solving \( \rho_m(\tau_*) = \overline{\rho}_m \), where \( \rho_m \) is the cosmic matter density and \( \overline{\rho}_m \) is that when the Milky Way last virialized. On the other hand \( \Delta\tau \) should be the same across the set of universes that we consider; it is simply the difference between the present cosmic time and that when the Milky Way last virialized.

The distribution of \( \sigma \) over comoving spheres enclos-
ing mass $M$ is Gaussian with a standard deviation $\sigma_{\text{rms}}(M, \tau)$ that depends on $M$ and grows with time. The probability that any such region collapses in a small interval $d\tau$ about $\tau_*$ is therefore

$$
\frac{dP}{d\rho} \propto \frac{\delta_{\text{rms}}}{\sigma_{\text{rms}}^2} \exp\left(\frac{1}{2} \frac{\delta_\tau^2}{\sigma_{\text{rms}}^2}\right) \left|_{M, \tau_*}\right. \ d\tau ,
$$

(5)

where the dot denotes differentiation with respect to $\tau$ (we have used that $\sigma/\sigma_{\text{rms}}$ is constant in time according to the linear analysis). Thus we write

$$
\rho_{\text{obs}}(\tau) d\tau \propto \delta(\tau - \tau_*) \rho_m(\tau) dP_{\text{coll}} ,
$$

(6)

where the delta function arises because we have so constrained the set of observers, that they all arise at a single time in any given universe.

There is another condition that should be considered. Above we assumed that after some time $\tau_*$, the evolution of a galaxy is approximately independent of the surrounding cosmic environment. However this approximation must fail at least for late times and negative $\Lambda$, as ultimately such spacetimes collapse to a singularity. To account for this, we impose the additional constraint,

$$
\tau_* + \Delta \tau \leq \tau_f / 2 ,
$$

(7)

which ensures that we only count galaxies before any AdS vacua begin to collapse.

Note it is possible that Eq. (7) does not go far enough. In universes with (positive and negative) values of $\Lambda$ smaller than ours, the rate of halo mergers and galaxy collisions (as a function of matter density) will be greater than in our universe. If these events are disruptive to the development of observers, for instance by repeatedly resetting the process of star formation or by disrupting stable stellar systems with stellar fly-bys during galaxy collisions after star formation, then our calculation of $\rho_{\text{obs}}$ overestimates the number of observers measuring $\Lambda$ smaller than we do. A full consideration of this issue appears to be rather formidable, and is not attempted in this work. Instead we make some basic observations about our results at the end of Section III.

### III. DISTRIBUTION OF $\Lambda$

Combining Eqs. (23), the distribution of vacuum energy densities $\rho_\Lambda$ can be written

$$
\frac{dP}{d\rho_\Lambda} \propto - \left[ a^3 \eta^3 \rho_m^2 \right]_{\tau_* + \Delta \tau} \left[ \sigma_{\text{rms}} \exp\left(\frac{1}{2} \frac{\delta_\tau^2}{\sigma_{\text{rms}}^2}\right) \right]_{\tau_*} ,
$$

(8)

where again $a$ is the scale factor, $\eta$ is the conformal time, $\rho_m$ is the matter density, $\sigma_{\text{rms}}$ is the root-mean-square (rms) density contrast evaluated on a comoving scale enclosing mass $M$, and $\delta_c = 1.69$ is the collapse density threshold. The first term in brackets is proportional to the total matter in the causal diamond when observers arise, a time $\Delta \tau$ after halo virialization at $\tau_*$, whereas the second term in brackets is proportional to the probability that a volume enclosing mass $M$ will have virialized at time $\tau_*$. One should also bear in mind we impose the constraint Eq. (7). We now describe all of these pieces.

As explained in Section III, we restrict attention to pocket universes that are in every way like ours except for their vacuum energy densities. In fact, since observers like us do not arise before matter domination, we can ignore the early radiation-dominated eras in these universes. This allows for an analytic solution to the Einstein field equations, thus greatly simplifying the analysis. The cosmic matter density is then

$$
\rho_m(\tau) = |\rho_\Lambda| \sin^{-2}\left(\frac{3}{2} \frac{\tau}{\tau_\Lambda}\right) \quad \rho_\Lambda \leq 0
$$

(9)

$$
\rho_m(\tau) = \rho_\Lambda \sinh^{-2}\left(\frac{3}{2} \frac{\tau}{\tau_\Lambda}\right) \quad \rho_\Lambda \geq 0 ,
$$

(10)

where $\rho_\Lambda$ is the vacuum energy density and

$$
\tau_\Lambda = \sqrt{3/8\pi G |\rho_\Lambda|} .
$$

(11)

As we restrict attention to pockets indistinguishable from ours except for the value of $\rho_\Lambda$, we normalize the scale factor so that at early times, it is independent of $\rho_\Lambda$. Thus we write

$$
a = (\rho_\Lambda^2/\rho_m)^{1/3} .
$$

(12)

The conformal time is defined $\eta = \int d\tau/a(\tau)$. We set the constant of integration so that $\eta$ is negative and approaches zero as $\tau \to \tau_f$. As before, $\tau_f$ is the time of vacuum decay in dS vacua, in which case we can safely take $\tau_f \to \infty$, and $\tau_f$ is the time of the future singularity in AdS space, $\tau_f = (2\pi/3) \tau_\Lambda$. This gives

$$
\eta(\tau) = - \left(\frac{3\tau_\Lambda}{8\pi G}\right)^{1/3} \left\{ \Delta \eta + \frac{2}{3} \cos\left(\frac{3}{2} \frac{\tau}{\tau_\Lambda}\right) \right. \left. 2F_1 \left[\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \cos^2\left(\frac{3}{2} \frac{\tau}{\tau_\Lambda}\right)\right] \right\} \quad \rho_\Lambda < 0
$$

(13)

$$
\eta(\tau) = - \left(\frac{3\tau_\Lambda}{8\pi G}\right)^{1/3} \cosh^{-2/3}\left(\frac{3}{2} \frac{\tau}{\tau_\Lambda}\right) \left. 2F_1 \left[\frac{5}{6}, \frac{1}{3}; \frac{4}{3}; \cosh^{-2}\left(\frac{3}{2} \frac{\tau}{\tau_\Lambda}\right)\right] \right\} \quad \rho_\Lambda \geq 0 ,
$$

(14)
where $\Delta \eta \approx 2.429$ is simply an integration constant and $\gamma F_1$ is the hypergeometric function.\footnote{As an anthropic AdS vacuum collapses, its radiation density will grow and eventually dominate the total energy density. The appendix of Ref.\cite{8} shows that the conformal time is changed negligibly by ignoring this radiation and instead tracking the matter density all the way to $a \to 0$.}

The evolution of the rms density contrast $\sigma_{\text{rms}}$ cannot be written in closed form, even in our simplified model. However, fitting functions accurate to the percent level are available in the literature. We use \cite{7}

$$
\sigma_{\text{rms}} = \frac{3}{5} \sigma_{\text{eq}} \left[ \frac{(\rho_a / \rho_{\text{eq}})^{1/3} \left( \frac{3}{2} \left( \frac{\tau}{\tau_a} \right) \right)^{2/3}}{1 + 1.68 \left( \frac{\tau}{\tau_f} \right)^{2.18}} \right]^{1/2} \left[ 1 - 4 \left( \frac{\tau}{\tau_f} \right)^2 \right],
$$

for the case $\rho_a < 0$, and \cite{8}

$$
\sigma_{\text{rms}} = \frac{3}{5} \sigma_{\text{eq}} \left( \frac{\rho_a}{\rho_{\text{eq}}} \right)^{2/3} \left( \frac{G_{\infty}^2 \rho_{\text{eq}}}{\rho_a} \right)^{\alpha - 1/3 \alpha},
$$

for the case $\rho_a \geq 0$, where $\alpha = 159/200$ and $G_{\infty} = 1.437$. Here $\rho_{\text{eq}}$ corresponds to the matter density at matter-radiation equality (as measured in our universe). Of course our model does not contain radiation, but $\rho_{\text{eq}}$ is still a convenient reference to ensure our model matches onto the observed evolution of $\sigma_{\text{rms}}$. Note that $\sigma_{\text{rms}}$ near to and before matter-radiation equality differs significantly from that in Eqs. (15) and (16), due to the presence of a non-growing mode, but this discrepancy is inconsequential since in both cases $\sigma_{\text{rms}}$ is negligibly small at these times.

The time of last virialization $\tau_*$ is described in Section II C — it corresponds to the time at which MW galaxies must virialize in order to have the same virialization density as the Milky Way, in our simple model. If we take the Milky Way virialization time to be $\tau_0$, then because the virialization density is proportional to the cosmic matter density at the time of virialization,

$$
\tau_* = \frac{3}{2} \rho_a \arcsin \left( \frac{\rho_a}{\rho_a} \sinh \left( \frac{3}{2} \frac{\tau_0}{\tau_a} \right) \right) \rho_a < 0 \quad (17)
$$

$$
\tau_* = \frac{2}{3} \rho_a \arcsinh \left( \frac{\rho_a}{\rho_a} \sinh \left( \frac{3}{2} \frac{\tau_0}{\tau_a} \right) \right) \rho_a \geq 0, \quad (18)
$$

where everywhere we use bars to denote quantities evaluated in our universe. Note also that the time lapse $\Delta \tau$, included in our analysis to allow for the requisite evolution (including planet formation and biological evolution) to bring the virialized halo to the present state of the Milky Way, is simply

$$
\Delta \tau = \tau_0 - \tau_*,
$$

where $\tau_0$ is the present age of our universe.

All that is left to determine Eq. (8) is to fill in the various cosmological and astrophysical parameters. We use WMAP-5 mean-value cosmological parameters \cite{50}, along with CMBFAST \cite{51} to determine the density contrast on a comoving scale enclosing $M = 10^{12}$ solar masses,\footnote{For convenient reference we note the relevant cosmological parameter values are $\Omega_a = 0.742$, $\Omega_m = 0.258$, $\Omega_k = 0.044$, $n_s = 0.96$, $h = 0.719$, and $\Delta^2(k = 0.02 \text{ Mpc}^{-1}) = 2.21 \times 10^{-9}$. These give $\sigma_{\text{rms}}(M = 10^{12} \text{ M}_\odot, \tau = \tau_0) = 2.03$ in our universe.} corresponding roughly to the mass scale of the Milky Way. In fact, depending on the choice of $\Delta \tau$ one should choose a somewhat smaller value of $M$, to account for accretion and minor mergers during the interval $\Delta \tau$. However our choice of $M$ is already a bit of an underestimate, and the primordial density contrast has a rather weak dependence on $M$. The remaining astrophysical parameter is $\Delta \tau$. Interestingly, we find the results to be very insensitive to reasonable choices of $\Delta \tau$. For the moment we choose $\Delta \tau = 5 \times 10^9$ years and later comment on the effect of changing this.

We first compare to previous results. Fig. 1 displays the distribution of positive values of $\Lambda$ for the parameter choices described above. Although this distribution has not appeared before in the literature, Ref. \cite{8} calculated the distribution of positive values of $\Lambda$ using an entropy-counting approximation to implement anthropic selection, and restricting attention to the “inner” causal diamond, i.e. the intersection of what we have referred to as the causal diamond with the future lightcone of a point on the surface of reheating. Comparing to Fig. 1 or Fig. 8 in Ref. \cite{8}, we see that our approach shifts the distribution to smaller values of $\Lambda$; however the curve still gives an acceptable fit to observation.\footnote{Although it is not evident from Fig. 1 the distribution of $\Lambda$ diverges at $\Lambda = 0$. Yet the divergence is integrable over continuous distribution of $\Lambda$, so this is only a problem if observers can arise in vacuo with $\Lambda$ exactly equal to zero. As such vacua are expected to be supersymmetric, this is not the case for observers like us, and seemingly not the case for observers in general.} Note that Fig. 1 is somewhat deceptive due to the long tail of the
distribution toward small $\Lambda$. In fact only about 6% of observers measuring positive $\Lambda$ measure it to be larger than the value we measure.

The difference between our result and that of Ref. [8] has two distinct sources: (1) Ref. [8] restricts attention to the full causal diamond, whereas we use the full causal diamond (or, equivalently, the past lightcone, see footnote 3 and surrounding discussion), and (2) Ref. [8] estimates the number of observers by integrating the entropy production, whereas we count the number of galaxies with mass, virialization density, and age equal to those of the Milky Way. The choice of inner causal diamond was made in Ref. [8] to avoid counting the entropy produced at reheating, which would otherwise dominate the calculation [4]. Note however that this choice technically constitutes a different measure, the definition of which appears ad hoc. For instance, the motivation to restrict to a causal patch is based on an analogy to black-hole complementarity [11, 12]; however there is clearly no problem with receiving information from beyond the inner causal diamond, as this occurs when we observe the cosmic microwave background. Meanwhile, to the extent that counting entropy production differs from counting MW galaxies, we consider the former to include “observers” who are rather unlike ourselves (or to omit observers who are much like ourselves), in which case the result is biased by observations from which it is more presumptuous for us to consider our measurement as randomly drawn. To disentangle these effects for the interested reader we note that, if one restricts to the inner causal diamond but otherwise adheres to our approach (equating observers with MW galaxies, etc.), one finds that about 11% of observers measuring positive $\Lambda$ measure it to be larger than the value we measure.

The situation is much worse when we include negative values of $\Lambda$. The full distribution of $\Lambda$ is displayed in Fig. 2. The range of the plot is chosen so as to chop the distribution when the constraint of Eq. (7) is violated. Due in large part to the relatively large number of observers who measure negative $\Lambda$, the fraction of observers measuring $\Lambda$ to be smaller than the value we measure is 99.6% — making our measurement appear as a roughly three standard deviation statistical fluke. (If one restricts to the inner causal diamond, this fraction is 98.9%.) The reason so many observers measure negative $\Lambda$ is easy to understand: due to differences in global geometry, the comoving three-volume of the causal diamond on any constant-time hypersurface is much larger in AdS space than in dS space, for a given value of $|\Lambda|$.

Because much of the problem is rooted in the relative volume-weighting of AdS and dS space, we do not expect the details of our attempt to identify Milky Way-like galaxies to significantly change the result. To illustrate this, consider two very different choices of $\Delta \tau$: $\Delta \tau = 0$ and $\Delta \tau = 10^{10}$ years (for the moment we keep $M$ fixed). Respectively, in these cases we find 98.7% and 99.7% of observers measure a value of $\Lambda$ that is smaller than what we measure. Actually, in the case of larger $\Delta \tau$, we should note that over such a time interval the mass of a typical galaxy grows significantly, so one might want to also decrease $M$. Yet this is hardly helpful, due to the weak dependence of $\sigma_{\text{rms}}$ on $M$. If for instance we choose $\Delta \tau = 10^{10}$ years and $M = 10^{11}$ solar masses, we still find 99.7% of observers measure a value of $\Lambda$ less than what we measure.

Although our results are robust to choosing different time lapses $\Delta \tau$, we cannot exclude the possibility that we make a different type of error: counting MW galaxies as if they have observers when in fact they do not. As described at the end of Section II C this might be the case if halo mergers and galaxy collisions — which should occur at higher rates in universes with smaller values of $\Lambda$ — are detrimental to the development of observers. A full analysis of this issue appears rather formidable, so we instead make some basic observations.

First of all, if an increased rate of mergers keeps our basic model of structure formation intact, but merely shifts the last virialization time $\tau_*$ away from the values given by Eqs. (17-18), then it is unlikely to significantly affect our results, for much the same reason that adjusting $\Delta \tau$ does not significantly affect the results.

Second, the merger rate itself should differ from one universe to another in a way closely related to the differing evolution of $\sigma_{\text{rms}}$. Yet, because our anthropic constraints already limit us to a rather narrow range of $\Lambda$, the evolution of $\sigma_{\text{rms}}$ is not dramatically different across the range of anthropic conditions we consider. This is illustrated in Fig. 3. The top panel plots the time evolution of $\sigma_{\text{rms}}$: it is a decreasing function of time, except at late times in vacua with negative $\Lambda$, and at fixed times it is an increasing function of decreasing $\Lambda$. The two vertical bars indicate the time of last virialization, in our universe, for the two choices $\Delta \tau = 5 \times 10^9$ years and $\Delta \tau = 10^{10}$ years.

The bottom panel of Fig. 3 more clearly indicates the dependence on $\Lambda$ at two critical times in our model: the
time of last virialization $\tau_*$ and the time of observation $\tau_0$. Note that the curve for $\tau_*(\Delta\tau = 10^{10} \text{ yrs})$ is nearly constant — this means selecting MW galaxies according to the corresponding fixed virialization density is not much different than selecting MW galaxies according to when $\dot{\sigma}_{\text{rms}}$ falls below a certain critical level. When the curve for $\tau_0$ is below the curve for $\tau_*$, this indicates that $\dot{\sigma}_{\text{rms}}$ never rises above that critical level (before the arrival of observers). These curves might be taken to indicate a stronger bound on small $\Lambda$ than our anthropic criteria assume; yet even if we set a lower bound as tight as $\Lambda \geq -\overline{\Lambda}/4$, then still 99.4% of observers measure a value of $\Lambda$ smaller than we do (99.6% for $\Delta\tau = 10^{10} \text{ yrs}$).

Finally, we note that because halo mergers and galaxy collisions are probabilistic events, modest increases in the average rates can be overcome by fortuitous circumstances. In the context of the above discussion, $\dot{\sigma}_{\text{rms}}$ may be seen to give a measure of the average merger rate, but the actual merger rate will vary from region to region depending on the local properties of the density contrast. Demanding fortuitous circumstances will introduce a statistical suppression factor when counting observers in vacua with smaller values of $\Lambda$, but if this factor does not change the order of magnitude of the density of observers, then the results would still be rather discouraging for the causal diamond measure.

**IV. DISCUSSION**

We have calculated a distribution of positive and negative values of $\Lambda$, using the causal patch measure, restricting attention to universes otherwise like ours, and assuming a flat distribution of $\Lambda$ when $|\Lambda|$ is very small. We found the value we measure to be roughly a three standard deviation outlier, with 99.6% of observers measuring values smaller than we do.

This result appears to be rather robust. Our calculation used restrictive anthropic criteria, attempting to avoid mistakenly counting hypothetical observers where none in fact would exist. Our restrictive anthropic criteria might have missed observers, but these observers would live in galaxies that look rather different than ours. Ignoring such observers is equivalent to simply further conditioning the distribution that we calculate. We explored adjusting the parameters by which we attempt to identify Milky Way-like galaxies (these parameters are, to a large degree, uncertain), but found the results were insensitive to these changes. Yet it is possible that even our restrictive anthropic criteria are not restrictive enough — for example if we underestimate the effects of the increased numbers of mergers for smaller values of $\Lambda$. Although we have not ruled out this possibility, we provided some basic observations to suggest that this should not drastically change the results.

Still, depending on one’s attitude, this sort of discrepancy on a single data point might not be seen as a significant problem. One source of interest comes from the perspective that the causal diamond measure, though motivated by considerations of holography and black-hole complementarity, is not firmly grounded in fundamental theory (of course, neither are any of the other proposed regulators of eternal inflation). If one presumes the “correct” measure gives a better fit to observation, then our result might be seen as highlighting certain paths toward the correct measure.

For instance, it has recently been shown that the causal diamond measure, given suitable initial conditions and restricting attention to dS vacua, is equivalent to a seemingly very different approach based on an analogy to AdS/CFT complementarity [29]. Yet the approach of Ref. [29] appears to open up new possibilities for treating AdS and Minkowski vacua in the multiverse. Our result highlights the phenomenological importance that proposals to resolve these ambiguities do not fall prey to ‘over-weighting’ the volume of AdS pockets.

Alternatively, one might find motivation to pursue other spacetime measures. Let us here note that in a Bayesian analysis that compares one measure to another, one would not compare the goodness-of-fits, but instead the probability that each measure assigns to $\Lambda$ being in a small interval $d\Lambda$ about the value we measure. The anthropic criteria used in this paper are easily translated to a measure that weights fixed comoving thermalized volumes equally (such a measure suffers from Boltzmann brain domination, but nevertheless may serve as a refer-
ence). In such a measure the probability assigned to the value of $\Lambda$ we observe is eight times that of the causal diamond measure. Comparison to the “no collapse” scale-factor cutoff measure \[27\] is somewhat more involved. We have checked and the scale-factor cutoff assigns about 32 times the probability of the causal diamond measure to the value of $\Lambda$ we measure. (Both of these measures were shown to provide a good fit to the observed value of $\Lambda$ in for instance Ref. \[8\] ) Of course how one uses these differences depends on one’s theoretical priors.

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