Viability of primordial black holes as short period gamma-ray bursts

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It has been proposed that the short period gamma-ray bursts, which occur at a rate of \( \sim 10 \text{yr}^{-1} \), may be evaporating primordial black holes (PBHs). Calculations of the present PBH evaporation rate have traditionally assumed that the PBH mass function varies as \( M_\odot^{3/2} \). This mass function only arises if the density perturbations from which the PBHs form have a scale invariant power spectrum. It is now known that for a scale invariant power spectrum, normalised to COBE on large scales, the PBH density is completely negligible, so that this mass function is cosmologically irrelevant. For non-scale-invariant power spectra, if all PBHs which form at given epoch have a fixed mass then the PBH mass function is sharply peaked around that mass, whilst if the PBH mass depends on the size of the density perturbation from which it forms, as is expected when critical phenomena are taken into account, then the PBH mass function will be far broader than \( M_\odot^{3/2} \).

In this paper we calculate the present day PBH evaporation rate, using constraints from the diffuse gamma-ray background, for both of these mass functions. If the PBH mass function has significant finite width, as recent numerical simulations suggest, then it is not possible to produce a present day PBH evaporation rate comparable with the observed short period gamma-ray burst rate. This could also have implications for other attempts to detect evaporating PBHs.

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\[ \frac{dM_{\text{BH}}}{dt} = -\alpha(M_{\text{BH}}) \frac{M_{\text{BH}}^2}{M_{\odot}^2}. \]  

\[ \frac{dn_{\text{BH}}}{dt} = \frac{dn_{\text{BH}}}{dM_{\text{BH}}} \frac{dM_{\text{BH}}}{dt}, \]

For the standard model of particle physics \( \alpha(M_{\text{BH}}) \geq 7.8 \times 10^2 \text{g} \text{s}^{-1} \) for \( M_{\text{BH}} \sim 5 \times 10^{14} \text{g} \). PBHs of this mass, which are evaporating today, would have \( T_{\text{BH}} > 20 \text{MeV} \) in the final stages of their evaporation. Cline and collaborators argue that \( \alpha(M_{\text{BH}}) \) could then be significantly increased by the effective number of degrees of freedom due to the quark-gluon phase transition, resulting in a burst of radiation with duration consistent with the observed duration of the short period GRBs.

To calculate the PBH evaporation rate the PBH mass function and local density enhancement are needed. Cline and collaborators take the bound on the global number density of PBHs per logarithmic mass interval, \( \mathcal{N}_g \), at \( M = M_* \), from the diffuse gamma-ray background to be \( \leq 10^5 \text{pc}^{-3} \). They combine this with a local density enhancement factor \( \eta = \rho_l / \rho_g = 5 \times 10^5 \), where ‘l’ and ‘g’ denote local and global values respectively, to obtain the local number density of PBHs per logarithmic mass interval at \( M = M_* \), \( \mathcal{N}_l = \eta \mathcal{N}_g \sim 10^{10} \text{pc}^{-3} \). The local PBH evaporation rate is then given by

\[ \frac{dn_{\text{BH}}}{dt} = \frac{\alpha(M_*)}{M_*^2} \mathcal{N}_l \sim 10 \text{yr}^{-1}. \]

I. INTRODUCTION

The physical origin of gamma-ray bursts (GRBs) is one of the outstanding problems in astrophysics. The majority of gamma-ray burst are now known to have cosmological origin, however Cline and co-authors have studied the short (duration < 200ms) GRB population, and suggest that they may be due to the evaporation of primordial black holes (PBHs) located in the galactic halo. These short events, which make up roughly 2% of the total population, have simple time histories and hard spectra, relative to the longer duration GRBs, and are consistent with a Euclidean source distribution, suggesting a local origin.

Primordial black holes (PBHs) are black holes which form in the early universe. There are a number of possible mechanisms for their formation, including the collapse of cosmic string loops and the collision of bubbles formed at phase transitions. The most natural formation mechanism is the collapse of large inflationary density perturbations. PBHs evaporate via the emission of Hawking radiation and PBHs with mass \( M_{\text{BH}} = M_* \sim 5 \times 10^{14} \text{g} \) would be evaporating today. In the standard picture of PBH evaporation all particles with rest mass less than the black hole temperature \( T_{\text{BH}} \) where

\[ T_{\text{BH}} = \frac{h c^3}{8 \pi GM_{\text{BH}}} = 1.06 \left( \frac{10^{13} \text{g}}{M_{\text{BH}}} \right) \text{GeV}, \]

and \( M_{\text{BH}} \) is the PBH mass in grams, are emitted. The rate of mass loss therefore depends on the number of particle degrees of freedom, \( \alpha(M) \):

\[ \frac{dM_{\text{BH}}}{dt} = -\alpha(M_{\text{BH}}) \frac{M_{\text{BH}}^2}{M_{\odot}^2}. \]
The constraint on the global PBH number density \( N_\gamma = dn_{\text{PBH}}/d \ln M_{\text{BH}} \) has traditionally been calculated \([14,13]\) by assuming that the PBH mass function is a power law \( dn_{\text{PBH}}/dM_{\text{BH}} \propto M_{\text{BH}}^{-5/2} \) in which case the initial PBH number density is given by \([13]\):

\[
N_\gamma = \frac{\Omega_{\text{PBH},0} \rho_c}{2M_\star},
\tag{5}
\]

where \( \Omega_{\text{PBH},0} \) is the present day fraction of the critical energy density, \( \rho_c \) in PBHs. Using MacGibbon and Carr’s 1991 evaluation of the diffuse gamma-ray bound (which assumes the \( M^{-5/2} \) mass function) \([14]\):

\[
\Omega_{\text{PBH}} \leq 7.6(\pm 2.6) \times 10^{-9} h^{-1.95 \pm 0.15},
\tag{6}
\]

where \( h \) is the Hubble parameters in units of 100 km/s/Mpc^{-1}, gives \( N_\gamma = 4.2 \times 10^3 \, \text{pc}^{-3} \).

The power law PBH mass function, \( dn_{\text{PBH}}/dM_{\text{BH}} \propto M_{\text{BH}}^{-5/2} \), was derived by Carr in the 1970s \([15]\) for scale-invariant density perturbations. In this paper we outline the arguments which show that this mass function is cosmologically irrelevant. We then calculate the constraint on the global PBH evaporation rate for the sharply peaked mass function which arises from assuming that all PBHs which form at a given epoch have the same mass, and also for the far broader mass function found by recent numerical studies \([14]\), due to near critical gravitational collapse \([17]\). Finally we review the calculation of the local density enhancement factor.

## II. PBH MASS FUNCTION

Early studies by Carr \([15]\), which assumed that all PBHs which form at a given epoch have the same mass \( M_{\text{BH}} \approx \gamma_\star^{3/2} M_\text{H} \) where \( M_\text{H} \) is the horizon mass at that epoch and \( \gamma_\star \) parameterises the equation of state: \( p = \gamma \rho \), found that an extended PBH mass function is only possible if the primordial power spectrum is scale-invariant (equal power on all scales). For Gaussian distributed fluctuations the probability distribution of the smoothed density field \( p(\delta(M_\text{H})) \) is given by

\[
p(\delta(M_\text{H})) \, d\delta(M_\text{H}) = \frac{1}{\sqrt{2\pi \sigma(M_\text{H})}} \exp \left( -\frac{\delta^2(M_\text{H})}{2\sigma^2(M_\text{H})} \right) \, d\delta(M_\text{H}),
\tag{7}
\]

where \( \sigma(M_\text{H}) \) is the mass variance evaluated at horizon crossing \([18]\). For power law power spectra, \( P(k) \propto k^n \), where \( P(k) = (|\delta_k|^2) \) and \( n \) is the spectral index,

\[
\sigma(M_\text{H}) = \sigma(M_{\text{H},0}) \left( \frac{M_{\text{eq}}}{M_0} \right)^{(1-n)/6} \left( \frac{M_{\text{H}}}{M_{\text{eq}}} \right)^{(1-n)/4},
\tag{8}
\]

where ‘0’ and ‘eq’ denote quantities evaluated at the present day and matter–radiation equality, and

\[
\sigma(M_{\text{H},0}) = 9.5 \times 10^{-5} \text{ using the COBE normalisation} \tag{19}.\text{ Using the Press–Schechter formalism Kim and Lee found the initial mass function produced by a power law power spectrum} [21]:
\]

\[
\frac{dn_{\text{PBH}}}{dM_{\text{BH}}} = \frac{n + 3}{4} \frac{2}{\pi} \frac{\gamma^{7/4}}{\sqrt{2\pi}} \frac{\rho_i}{\rho_{\text{eq}}^{1/2}} \frac{M_{\text{H}}^{5/2}}{M_{\text{BH}}^{5/2}} \frac{1}{\sigma_{\text{H}}^2} \exp \left( -\frac{\gamma^2}{2\sigma_{\text{H}}^2} \right),
\tag{9}
\]

where \( \rho_i \) and \( M_{\text{H}} \) are the energy density and horizon mass when the PBHs form, immediately after reheating finishes at the end of inflation and the universe becomes radiation dominated. The horizon provides a sharper lower cut-off in the mass function at \( M_{\text{BH}} \approx \gamma_\star^{3/2} M_\text{H} \).

For \( n \neq 1 \) the exponential term cuts the mass function off sharply so that it is peaked around \( M_{\text{BH}} \sim M_\text{H} \). Only if \( n = 1 \) does the mass function have the traditionally used \( M_{\text{BH}}^{-5/2} \) form. However for a scale invariant power spectrum normalised to COBE on large scales the PBH density is completely negligible \([22]\); the fraction of the energy density of the universe in PBHs at the time they form is given by \([13]\):

\[
\frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}} \approx \sigma(M_\text{H}) \exp \left( -\frac{1}{18 \sigma^2(M_\text{H})} \right).
\tag{10}
\]

If \( n = 1 \) then \( \sigma(M_\text{H}) = \sigma(M_{\text{H},0}) = 9.5 \times 10^{-5} \) and \( \beta \sim 10^{-6} \text{ or } 10^6 \). The \( M_{-5/2} \) PBH mass function is therefore cosmologically irrelevant. To produce an interesting density of PBHs either a power-law power-spectrum with more power on short scales \( (n > 1) \), or a spike in the density perturbations spectrum is required \([22]\). In both of these cases if all PBHs which form at a given epoch have the same mass then the mass function is very sharply peaked.

Niemeyer and Jedamzik \([16]\) have shown using numerical simulations that, as a consequence of near critical gravitational collapse \([17]\), at a fixed epoch PBHs with a range of masses form. The PBH mass is determined by the size of the fluctuation from which it formed:

\[
M_{\text{BH}} = k M_\text{H} (\delta - \delta_\epsilon) \gamma,
\tag{11}
\]

where \( \gamma \), \( k \) and \( \delta_\epsilon \) are constant for a given perturbation shape (for Mexican Hat shaped fluctuations \( \gamma = 0.36 \), \( k = 2.85 \) and \( \delta_\epsilon = 0.67 \)).

It has been found that, for both power-law power spectra and flat spectra with a spike on a particular scale, in the limit where the number of PBHs formed is small enough to satisfy the observational constraints on their abundance, it can be assumed that all the PBHs form at a single horizon mass \([23]\). It is therefore possible to calculate the PBH initial number density per unit mass interval analytically \([22, 23]\):

\[
\frac{dn_{\text{PBH}}}{dM_{\text{BH}}} = \frac{\rho_i}{\sqrt{2\pi \gamma \sigma(M_\text{H}) M_{\text{BH}} M_\text{H}}} \left( \frac{M_{\text{BH}}}{k M_\text{H}} \right)^{1/\gamma}
\]
The physical number density of PBHs dilutes to this mass function as the Niemeyer-Jedamzik mass function. The present day number densities, ignoring evaporation, are the same ($\Omega_{\text{PBH}} = 1 \times 10^{-8}$). For clarity the Carr and Niemeyer-Jedamzik mass functions are multiplied by a factor of ten.

$\Omega_{\text{PBH}} = 1 \times 10^{-8}$.

The three initial mass functions (Carr $M_{\text{BH}}^{-5/2}$, Kim-Lee and Niemeyer-Jedamzik) are plotted in Fig. 1 with the parameters of each mass function chosen such that the energy density in PBHs is the same, corresponding to a present day PBH energy density of $\Omega_{\text{PBH}} = 1 \times 10^{-8}$. In Fig. 2 we plot the present day mass functions with parameters chosen such that $N_{g} = 4 \times 10^{3} \text{pc}^{-3}$. The Kim-Lee mass function, which arises from assuming that all PBHs which form at a given epoch have the same mass, is very sharply peaked. The Niemeyer-Jedamzik is far broader, with a long tail of low mass PBHs which would have evaporated since $z \sim 700$ and would contribute to the diffuse gamma-ray background.

### III. PBH EVAPORATION RATE

The abundance of PBHs evaporating today is constrained by their contribution to the diffuse gamma-ray background \[12,13,15,23\]. All PBHs which have evaporated since $z = 700$ contribute to the diffuse gamma-ray background \[25\], therefore the resulting constraint on the number of PBHs evaporating today depends on the PBH mass function; the broader the mass function the tighter the constraint on the number of PBHs evaporating today. The diffuse gamma-ray bound has been recalculated using recent measurements by COMPTEL \[26\] and EGRET \[27\] for the Kim-Lee mass function \[21\] and also for the Niemeyer-Jedamzik mass function \[23\].

Since the Kim-Lee and Carr $M_{\text{BH}}^{-5/2}$ mass function both have a sharp lower cut-off, the diffuse gamma-ray constraint on the present day PBH evaporation rate is the same for both mass functions, with the recent COMPTEL and EGRET measurements tightening the constraints on $N_{g}$ by a factor of about 1.5 \[23\]. For the Niemeyer-Jedamzik mass function the relevant constraint on PBHs with mass $M = M_{\star}$ is $\sigma(M_{\star}) = 5.4 \times 10^{14} \text{g} < 0.056$. Inserting this in Eq. (12) leads to $N_{g} < 2.7 \times 10^{-4} \text{h}^{-2} \text{pc}^{-3}$ so that

$$\frac{dn_{\text{BH}}}{dt} = \frac{\alpha(M_{\star})}{M_{\star}^{2}} N_{g} \approx 5.2 \times 10^{-14} \text{h}^{2} \eta \text{pc}^{-3} \text{yr}^{-1}.$$ \(15\)

This calculation assumes that the cosmological constant is zero, $\Omega_{\Lambda} = 0$. If $\Omega_{\Lambda} = 0.7$, as current observation

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*Strictly speaking in the case of the Niemeyer-Jedamzik mass function the present day density will be less than this as the low mass tail of PBHs with $M_{\text{BH}} < 5 \times 10^{14} \text{g}$ will have evaporated

†The resulting constraint on the mass density in PBHs of all masses, $\Omega_{\text{PBH}}$, is far tighter for the Kim-Lee mass function however, since that mass function is far more sharply peaked.
appear to indicate \[^{21}\] then the constraint is tightened to \(\sigma(M_\star) < 0.023\) \[^{23}\] and \(N_\phi < 4.5 \times 10^{-157} h^2 \text{pc}^{-3}\) so that the present day rate of PBH evaporation is completely negligible.

We caution that the constraints on \(N_\phi\) are very sensitive to small changes in the limits on \(\sigma(M_\star)\) (or equivalently \(n\)) due to the exponential factor which arises in the expressions for \(\ln \Omega_{\text{PBH}} / \ln M_{\text{PBH}}\) (Eqs. \(^{9}\) and \(^{12}\)). This illustrates the fact that to produce an interesting (i.e. non-negligible) density of PBHs (evaporating today or otherwise) requires fine tuning of the size of the density perturbations from which they form. We have also seen that if the PBH mass function has significant finite width then the diffuse gamma-ray bound (which constrains the abundance of all PBHs which have evaporated since a redshift of \(z = 700\)) places far tighter limits on the present day rate of PBH evaporation than if the PBH mass function is sharply peaked.

### IV. PBH CONCENTRATION

The local density enhancement factor is usually calculated \[^{14}\] by assuming an isothermal halo so that the density at galactocentric radius \(R\) (outside the core radius \(R_c\)) is related to the asymptotic circular velocity \(V_\infty \approx 220 \text{kms}^{-1}\) by \[^{30}\]

\[
\rho_h(R) = \frac{V^2_\infty}{4\pi G R^2} \\
\approx 6.1 \times 10^{25} \left(\frac{V_\infty}{220 \text{kms}^{-1}}\right)^2 \left(\frac{R}{10 \text{kpc}}\right)^{-2} \text{g cm}^{-3}.
\]

Taking our galactocentric radius to be \(R_\odot = 8.5(\pm 1.1)\text{kpc}\) gives a local density enhancement factor

\[
\eta \approx 4.5(\pm 0.6) \times 10^5 h^{-2} \left(\frac{\Omega_h}{0.1}\right)^{-1},
\]

where \(\Omega_h\) is the fraction of the critical density in galactic halos. Halzen et. al. \[^{13}\] claim, without explicit calculation, a larger value for the local density enhancement:

\[
\eta \approx 1.36(\pm 0.9) \times 10^7 h^{-2} \left(\frac{\Omega_h}{0.1}\right)^{-1},
\]

which apparently \[^{21}\] relies on the assumption that PBHs are concentrated to the same extent as luminous matter.

An alternative procedure is to calculate \(\eta\) directly from estimates of the local halo density. The local halo density is poorly known, with the most recent estimates giving \(\rho_h = 9.2^{+3.8}_{-3.1} \times 10^{-25} \text{g cm}^{-3}\). Taking the currently favoured value of the present day mass density \(\Omega_m = 0.3\) \[^{20}\] gives \(\rho_m \approx 5.6 \times 10^{-30} h^{-2} \text{g cm}^{-3}\). Combining these values produces

\[
\eta = 1.6^{+1.8}_{-0.8} \times 10^5 h^{-2} \left(\frac{\Omega_m}{0.3}\right)^{-1},
\]

which is in broad agreement with the value calculated from the asymptotic circular velocity assuming that the halo is isothermal. Since evaporating PBHs could only be detected by BATSE out to distances of order a parsec \[^{3}\], it appears that, if the PBH mass function has significant width as recent numerical simulations suggest, then the local density enhancement factor is not large enough to produce a local PBH evaporation rate comparable with the observed frequency of short period gamma-ray bursts \((10 - 20 \text{yr}^{-1})\).

### V. CONCLUSIONS

Calculations of the present PBH evaporation rate have traditionally assumed that the PBH mass function varies \(\sim M_{\text{PBH}}^{-5/2}\). This mass function only arises if the density perturbations from which the PBHs form have a scale invariant power spectrum, in which case the PBH density is completely negligible \[^{21}\]. We have recalculated the present PBH evaporation rate, using the bounds which arise from the diffuse gamma-ray background \[^{21}\] for the sharply peaked mass function which arises if all PBHs which form at a given epoch have the same mass and also for the broader PBH mass function found by recent numerical simulations. We find that if the PBH mass function has significant finite width it is not possible to produce a present day PBH evaporation rate comparable with the observed short period gamma-ray burst rate. This could also have implications for other attempts to detect evaporating PBHs.

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