Evolution of an open system as a continuous measurement of this system by its environment*

Michael B. Mensky
P.N. Lebedev Physics Institute, 117924 Moscow, Russia

Abstract

The restricted-path-integral (RPI) description of a continuous quantum measurement is rederived starting from the description of an open system by the Feynman-Vernon influence functional. For this end the total evolution operator of the compound system consisting of the open system and its environment is decomposed into the sum of partial evolution operators. Accordingly, the influence functional of the open system is decomposed into the integral of partial influence functionals (PIF). If the partial evolution operators or PIF are chosen in such a way that they decohere (do not interfere with each other), then the formalism of RPI effectively arises. The evolution of the open system may then be interpreted as a continuous measurement of this system by its environment. This is possible if the environment is macroscopic or mesoscopic.

1 Introduction

When a quantum system is measured, its state is unavoidably changed. This change may be described as decoherence (see [1] and references therein). Continuous measuring a system leads to its gradual decoherence. This type of evolution seems to be general, so that the evolutions of a wide class of open systems may be described as continuous fuzzy measurements of these

*Reported at 3d Sakharov Conference on Physics, Moscow, June 2002
systems by their environments. In the present paper we shall formulate this statement in more detail and present additional arguments supporting it.

Usually the evolution of a continuously measured (open, decohering) system is described non-selectively, i.e. without taking into account the concrete readout of the measurement (the concrete state of the environment). In this case the state of the system is presented by a density matrix which in Markovian approximation satisfies the time-differential equation called master equation. The most general form of this equation was derived by Lindblad.

The same physical process of continuous measurement or gradual decoherence may be presented selectively, i.e. with the readout of the measurement (or the state of the environment) taken into account. If some continuous measurement (continuous decoherence) of a system is described selectively, the state of the system stays pure and may be presented by a wave function (state vector) instead of a density matrix.

The selective description of the process may be obtained without making use of any explicit model of the environment if one apply the approach based on restricted path integrals (RPI), see and references therein. RPI approach to continuous quantum measurement is thus model-independent. All that one needs in this approach is the information obtained in the measurement (recorded in the environment of the measured system). The concrete realization of the environment and the form of recording the information are irrelevant. This is an important feature of the process in question, and this feature is quite clearly revealed by the RPI approach.

If some observables are continuously measured, and resolution of time in this measurement is considered to be absolute (the Markovian approximation), then the evolution of the measured system is presented by Schrödinger equation with a complex Hamiltonian. The transition to the non-selective description gives then Lindblad equation. In the non-Markovian approximation, when the resolution of time is assumed finite, the evolution is readily presented by RPI but cannot be reduced to a time-differential equation.

The description of continuous measurements (in other words, of slow decoherence) by RPI was first derived in the model-independent way from

---

1Of course, this is valid only in the ideal case when complete information about the measurement readout is known and taken into account. If this information is partly unknown or partly not taken into account, the state of the system is mixed, but the degree of mixture is less than in the completely non-selective description.
Feynman’s path-integral form of quantum mechanics. Later the restricted-path-integral description of some processes of this type was derived from concrete models by conventional quantum-mechanical methods [3, 5]. In particular, an explicit model of a wide class of fuzzy continuous measurements on a two-level system was considered in this way and the Gaussian RPI (equivalently, quadratic imaginary potential) was derived for the resulting evolution. This may be interpreted as a special case of the quantum version of Central Limiting Theorem [2].

In the present paper the most general derivation of the RPI description for the evolution of an open system is given.

For this end an arbitrary open system is considered and the evolution operator is constructed for the larger system consisting of the system of interest together with its environment. Then this evolution operator is decomposed into the sum of partial evolution operators. The condition is presented which provides that the partial evolutions decohere. This condition is analogous to the consistency condition of Gell-Mann and Hartle [4]. If this condition is satisfied, probabilities may be correctly ascribed to the alternative partial evolutions. In this case the partial evolutions may be considered as classical alternatives.

Besides, a stronger so-called environment-decoherence condition is introduced which means that decoherence of the alternatives follow from the properties of the environment rather than of the system of interest. The environment-decoherence condition is analyzed in terms of the partial influence functionals (PIF) which provide a decomposition of the Feynman-Vernon influence functional [8].

One more condition on PIF is also considered which is formally more strong but practically equivalent if the environment is mesoscopic or macroscopic. This condition for decoherence of PIF is shown to lead to the RPI description of the evolution of the open system. If this condition is valid, the influence of the environment on the system of interest may be interpreted as a continuous measurement of this system by its environment.

It is argued that the choice of the set of decohering partial evolutions (and therefore interpretation of the open system as a continuously measured one) is possible if the environment is macroscopic or mesoscopic.
2 Evolution of an open system

Let us consider an open system $S$ which interacts with its environment $E$ according to the following scheme:

\[
\text{System} \leftrightarrow \text{Environment}
\]

We shall consider the evolution of the compound system $S + E$ and interpret it in terms of alternative ‘partial evolutions’. Our goal will be to formulate the conditions on the choice of the set of alternatives which provide that the evolution may be interpreted as a continuous measurement of the (open) system $S$ by its environment $E$, with the given alternatives being the measurement readouts.

Note that the environment $E$ may be a measuring device constructed in a special way to perform a continuous measurement of $S$, or a ‘reservoir’ interacting with the system of interest $S$ in an uncontrollable way. However even in the latter case the process may be interpreted as a measurement.

2.1 Evolution of the system and its environment

The compound system consisting of $S$ and $E$ is assumed closed (isolated). Let the Hamiltonian of this system have the form

\[
H_{\text{tot}}(p, q, P, Q) = H(p, q) + H_{\mathcal{E}}(P, Q) + H_{\mathcal{I}}(p, q, P, Q)
\]  

(1)

where the Hamiltonians $H$, $H_{\mathcal{E}}$ and $H_{\mathcal{I}}$ describe correspondingly our system, its environment and their interaction (of course, all the variables may be many-component, the Hamiltonians may in principle depend on time).

The evolution of the compound system is given by the formula

\[
\mathcal{R} = U^{\text{tot}} \mathcal{R}_{\text{in}} (U^{\text{tot}})^\dagger
\]  

(2)

where $\mathcal{R}$ is the density matrix of the compound system at an arbitrary time moment, $\mathcal{R}_{\text{in}}$ the density matrix at the initial time and $U^{\text{tot}}$ the evolution operator of the compound system during the corresponding time interval.

The evolution operator may be expressed in the form of the Feynman
path integral

\[ U^\text{tot} = \int d[p, q] \int d[P, Q] \]

\[ \times \exp \left\{ \frac{i}{\hbar} \left[ A[p, q] + A_E[P, Q] - \int_0^t dt H_I(p, q, P, Q) \right] \right\} \]  

where the following notations are introduced:

\[ A[p, q] = \int_0^t dt [p \dot{q} - H(p, q)], \quad A_E[P, Q] = \int_0^t dt \left[ P \dot{Q} - H_E(P, Q) \right]. \]  

The evolution of the subsystem \( S \) may be presented by the reduced density matrix \( \rho \) obtained from \( \mathcal{R} \) by tracing in the degrees of freedom of the subsystem \( \mathcal{E} \):

\[ \rho = \text{Tr}_\mathcal{E} \mathcal{R} = \text{Tr}_\mathcal{E} \left[ U^\text{tot} \mathcal{R}_{\text{in}} (U^\text{tot})^\dagger \right]. \]  

2.2 Decomposition of the evolution operator

We need a decomposition of the evolution operator \( U^\text{tot} \) in the sum (integral) of partial evolution operators \( U^\text{tot}_\alpha \)

\[ U^\text{tot} = \int d\alpha U^\text{tot}_\alpha \]  

(later on the alternatives \( \alpha \) will be interpreted as readouts of a continuous measurement). The partial evolution operators may be defined as

\[ U^\text{tot}_\alpha = \int d[p, q] \int d[P, Q] W_\alpha[P, Q] \]

\[ \times \exp \left\{ \frac{i}{\hbar} \left[ A[p, q] + A_E[P, Q] - \int_0^t dt H_I(p, q, P, Q) \right] \right\} \]  

where the weight functionals \( W_\alpha[P, Q] \) form a decomposition of unity, i.e. the relation

\[ \int d\alpha W_\alpha[P, Q] = 1 \]  

\[ ^2\text{We use here an integral over paths } [p, q, P, Q] \text{ in phase space of the compound system, correspondingly the action is expressed in terms of the Hamiltonian of this system.} \]

\[ ^3\text{Note that the weight functionals may be chosen in principle in a more general form } W_\alpha[p, q, P, Q] \text{ (for example in Eqs. (11, 12)), but we need only the special case } W_\alpha[P, Q]. \]
is valid with a certain measure $d\alpha$ on the set of all possible alternatives $\alpha$.

Making use of the decomposition (6) we can present the density matrix $\mathcal{R}$ of the compound system as an integral of partial density matrices expressing the ‘partial evolutions’:

$$
\mathcal{R} = \int d\alpha d\beta \mathcal{R}_{\alpha\beta}, \quad \mathcal{R}_{\alpha\beta} = U^\text{tot}_\alpha \mathcal{R}_\text{in} (U^{\dagger}_\beta). 
$$  (9)

If the compound system’s density matrix is decomposed as in Eq. (9) then the corresponding decomposition of the reduced density matrix of the subsystem $S$ results:

$$
\rho = \int d\alpha d\beta \rho_{\alpha\beta}, \quad \rho_{\alpha\beta} = \text{Tr}_\mathcal{E} \mathcal{R}_{\alpha\beta}. 
$$  (10)

## 3 Decoherence

Let us now formulate the condition under which the alternatives denoted as $\alpha$ are characterized by probabilities instead of probability amplitudes so that there are no interference effects between them. This is necessary for $\alpha$ to be interpreted as classical alternatives (specifically, as measurement readouts of some measurement).

### 3.1 Decoherence of the system and its environment

The (total) trace of the partial density matrices gives the set of generalized decoherence functionals

$$
P_{\alpha\beta} = \text{Tr} \mathcal{R}_{\alpha\beta}, \quad \int d\alpha d\beta P_{\alpha\beta} = 1. 
$$  (11)

They are analogous to the decoherence functionals introduced by Gell-Mann and Hartle [?] but for the alternatives $\alpha$ presented by $W_\alpha$ (approximately presented by sets of paths or ‘corridors of paths’) instead of quantum histories (sequences of projectors) used in [?].

In analogy with the definition of Gell-Mann and Hartle, the alternatives $\alpha$ are said to decohere if the following generalized consistency condition is valid:

$$
P_{\alpha\beta} = \text{Tr} \mathcal{R}_{\alpha\beta} = \delta(\alpha, \beta) P_\alpha, \quad \int d\alpha P_\alpha = 1. 
$$  (12)
Here $\delta(\alpha, \beta)$ is a delta-function in respect to the measure $d\alpha$ used in (8). Eq. (12) provides that the alternatives $\{\alpha\}$ decohere, i.e. are characterized by the probabilities $P_\alpha$ (more precisely, by probability densities in respect to the measure $d\alpha$) rather than probability amplitudes. This is necessary for the set $\{\alpha\}$ to be considered as the set of classical alternatives. Usually condition (12) is satisfied only approximately.

3.2 Decoherence induced by the environment

Sometimes (typically in the situation of measurement) decoherence may be a consequence of some properties of the environment $\mathcal{E}$, but not of the system $\mathcal{S}$. Intuitively it is clear that this must take place if the environment has great number of degrees of freedom, especially if this number is macroscopic. However, instead of great number of degrees of freedom this may be provided by special properties of the environment (namely, by ‘orthogonality’ of the sets of paths corresponding to different alternatives).

The important situation when decoherence is caused only by the environment may be mathematically formulated as follows: delta-function $\delta(\alpha, \beta)$ arises not only after total tracing as in Eq. (12) but already after tracing in the environments’s degrees of freedom. This means that the relation

$$\rho_{\alpha\beta} = \text{Tr}_\mathcal{E} \mathcal{R}_{\alpha\beta} = \delta(\alpha, \beta) \rho_\alpha$$  \hspace{1cm} (13)

should be valid with a set $\rho_\alpha$ of ‘decohering partial reduced density matrices’. The total reduced density matrix will then be decomposed as

$$\rho = \int d\alpha \rho_\alpha.$$  \hspace{1cm} (14)

The condition (13) is stronger then the (generalized) consistency condition (12). It may be called the environment-decoherence condition. One may expect that this condition is approximately fulfilled if the environment contains many degrees of freedom but not only in this case. Below we shall show that a slightly stronger condition provides that the evolution of the system $\mathcal{S}$ may be presented as a result of its continuous measurement by the environment $\mathcal{E}$. The alternatives $\alpha$ may then be interpreted as measurement readouts for this continuous measurement.
4 Partial influence functionals and RPI

Now we shall introduce Feynman-Vernon influence functional, decompose it into partial influence functionals (PIF) and express the environment-decoherence condition as well as some slightly stronger condition in terms of PIF. This will allow us to go over to the restricted-path-integral (RPI) description of the evolution of the open system. The process may then be interpreted as a continuous measurement.

4.1 Partial influence functionals

With Eq. (3) taken into account, the reduced density matrix (5) may be expressed in the form of a multiple functional integral. Assuming that the initial density matrix of the compound system is factorized

\[ \mathcal{R}_{\text{in}}(q, Q|\bar{q}, \bar{Q}) = \mathcal{R}_{\text{in}}^\varepsilon(Q, \bar{Q}) \rho_{\text{in}}(q, \bar{q}) \]

and performing the functional integrations in two steps, we may put the reduced density matrix into the form

\[ \rho(q', \bar{q}') = \int dq \int d\bar{q} \int^q d[p, q] \int^{\bar{q}} d[\bar{p}, \bar{q}] 
\times \exp \left\{ \frac{i}{\hbar} (A[p, q] - A[\bar{p}, \bar{q}]) \right\} F[p, q|\bar{p}, \bar{q}] \rho_{\text{in}}(q, \bar{q}) \]

where

\[ F[p, q|\bar{p}, \bar{q}] = \int dQ' \int dQ \int d\bar{Q} \int^{Q'} d[P, Q] \int^{Q'} d[\bar{P}, \bar{Q}] \mathcal{R}_{\text{in}}^\varepsilon(Q, \bar{Q}) \]
\[ \times \exp \left\{ \frac{i}{\hbar} \left[ A\varepsilon[\bar{P}, \bar{Q}] - \int_0^t dt H_I(p, q, P, Q) \right] \right\} \]
\[ \times \exp \left\{ -\frac{i}{\hbar} \left[ A\varepsilon[\bar{P}, \bar{Q}] - \int_0^t dt H_I(\bar{p}, \bar{q}, \bar{P}, \bar{Q}) \right] \right\} \]

is Feynman-Vernon’s influence functional \[8\] in phase-space representation.

The partial reduced density matrices \[10\] are then expressed as

\[ \rho_{\alpha\beta}(q', \bar{q}') = \int dq \int d\bar{q} \int^{q'} d[p, q] \int^{\bar{q}'} d[\bar{p}, \bar{q}] 
\times \exp \left\{ \frac{i}{\hbar} (A[p, q] - A[\bar{p}, \bar{q}]) \right\} F_{\alpha\beta}[p, q|\bar{p}, \bar{q}] \rho_{\text{in}}(q, \bar{q}) \]
in terms of the partial influence functionals

\[
F_{\alpha\beta}[p, q | \bar{p}, \bar{q}] = \int dQ' \int dQ \int d\bar{Q} \int^Q_{Q'} d[P, Q] \int_{Q'}^Q d[\bar{P}, \bar{Q}]
\times W_\alpha[P, Q] R_{\text{in}}(Q, \bar{Q}) W_\beta^*[\bar{P}, \bar{Q}]
\times \exp \left\{ \frac{i}{\hbar} \left[ A_\alpha[P, Q] - \int_0^t dt H_I(p, q, P, Q) \right] \right\}
\times \exp \left\{ -\frac{i}{\hbar} \left[ A_\beta[\bar{P}, \bar{Q}] - \int_0^t dt H_I(\bar{p}, \bar{q}, \bar{P}, \bar{Q}) \right] \right\}
\] (19)

The complete influence functional \( F \) is equal to the integral of \( F_{\alpha\beta} \) in analogy with Eqs. (9, 10).

The environment-decoherence condition (13) means that the partial influence functional \( F_{\alpha\beta} \) is diagonal, i.e. contains the delta-function \( \delta(\alpha, \beta) \) as a factor. We shall argue that under rather general assumptions the stronger condition

\[
F_{\alpha\beta}[p, q | \bar{p}, \bar{q}] = \delta(\alpha, \beta) w_\alpha[p, q] w_\beta^*[\bar{p}, \bar{q}] \] (20)

takes place. This condition may be called the condition for decoherence of the partial influence functionals. It will be shown (see Sect. 4.3) to lead to the RPI description of the evolution of \( S \) so that the influence of the environment may be considered as a continuous measurement of the system \( S \).

4.2 Analysis of the condition for the measurement-

type evolution

The condition (20) turns out to be (approximately) valid if the alternatives \( \alpha \) (determined by the weight functionals \( W_\alpha \), see Eq. (7)) are chosen properly, so that the corresponding sets (corridors) of paths \([P, Q]\) are sufficiently wide, but not too wide.

More precisely, the corridors (denote them \( \alpha_\mathcal{E} \)) must be chosen in such a way that 1) if a corridor contains no classical trajectory (of the system \( \mathcal{E} \)), then the integral over this corridor is negligibly small, and 2) if a corridor contains some classical trajectory then the integral over this corridor is well

\footnote{We mean the sets of paths \([P, Q]\) in phase space of the environment \( \mathcal{E} \). The sets (corridors) of paths \([p, q]\) in the phase space of the system \( S \) which are described by the weight functionals \( w_\alpha[p, q] \) may turn out to be arbitrarily narrow.}
approximated by the exponential of the action along this trajectory (the stationary phase approximation).

If these conditions are satisfied, then $F_{\alpha\beta}$ is not negligible in the sole case when both corridors $\alpha_\mathcal{E}$, $\beta_\mathcal{E}$ contain classical trajectories and all these classical trajectories are close to each other. Therefore, $F_{\alpha\beta}$ is negligible each time when either one of $\alpha_\mathcal{E}$, $\beta_\mathcal{E}$ contains no classical trajectory or both of them contain classical trajectories but those in $\alpha_\mathcal{E}$ strongly differ from those in $\beta_\mathcal{E}$.

These features are approximately presented by the formula (20). The functional $w_\alpha[p,q]$ in this formula turns out to be negligible if the corresponding set of paths $\alpha_\mathcal{E}$ of the environment (determined by the functional $W_\alpha[P,Q]$) contains no classical trajectory.

Let us be somewhat more concrete and derive the concrete formulas for those PIF which are not negligible, i.e. those $F_{\alpha\beta}$ that both corridors $\alpha_\mathcal{E}$ and $\beta_\mathcal{E}$ (defined correspondingly by $W_\alpha[P,Q]$ and $W_\beta[P,Q]$) contain classical trajectories. The corridors were assumed to be sufficiently wide. This is why the path integrals in (19) may be approximately presented by the exponentials of the classical action calculated along these classical trajectories:

$$F_{\alpha\beta}[p,q|\bar{p},\bar{q}] \approx \int_{I_{\text{fin}}(\alpha)} dQ' \int_{I_{\text{fin}}(\beta)} dQ \int_{I_{\text{in}}(\beta)} d\bar{Q} \mathcal{R}_{\text{in}}(Q,\bar{Q}) \times \exp \left\{ \frac{i}{\hbar} \left[ S_{\text{cl}}(Q,Q',[p,q]) - S_{\text{cl}}(\bar{Q},\bar{Q}',[\bar{p},\bar{q}]) \right] \right\}. \quad (21)$$

Here $I_{\text{in}}(\alpha)$ (correspondingly $I_{\text{fin}}(\alpha)$) is the set of initial (correspondingly final) points of those classical trajectories which lie in the corridor $\alpha_\mathcal{E}$. The classical action $S_{\text{cl}}(Q,Q',[p,q])$ depends on the initial and final points $Q,Q'$ of the corresponding classical trajectory. Dependence on the path $[p,q]$ arises because, according to Eq. (19), this path determines the force acting on the system $\mathcal{E}$ so that the shape of the classical trajectory of this system depends on $[p,q]$. The same is valid for $S_{\text{cl}}(\bar{Q},\bar{Q}',[\bar{p},\bar{q}])$.

The structure of the double path integral (19) allows one to consider it as a single path integral over trajectories ‘closed in time’ (going from the initial time moment to the time moment $t$ and then backward to the initial time moment $t$).

\footnote{The classical trajectories contained in the same corridor cannot strongly differ because of the assumed properties of the corridors.}
moment). In the part of the integral, which corresponds to the inverse order of time, one has to change the signs of the momentum $P$ and velocity $\dot{Q}$.

This is why the classical trajectory coming during the time interval $[0, t]$ from the point $Q$ to the point $Q'$, should then, during the interval $[t, 0]$, return closely to the initial point. Therefore the points $Q$ and $\bar{Q}$ should be close to each other. Approximately we may accept that $\bar{Q} = Q$. Eq. (21) takes then the form

$$F_{\alpha,\beta}[p, q | \bar{p}, \bar{q}] \sim \int_{I_{\text{in}}(\alpha) \cup I_{\text{fin}}(\beta)} dQ' \int_{I_{\text{in}}(\alpha) \cup I_{\text{fin}}(\beta)} dQ \mathcal{R}^\varepsilon_{\text{in}}(Q, Q)
\times \exp \left\{ \frac{i}{\hbar} [S_{\text{cl}}(Q, Q', [p, q]) - S_{\text{cl}}(Q, Q', [p, q])] \right\}, \quad (22)$$

Eq. (22) allows one to justify the approximate expression (20) for PIF, particularly and clarify the relations between the weight functionals $W_{\alpha}[P, Q]$ (or the corresponding corridors of paths $\alpha_E$) for the environment $E$ and the functionals $w_{\alpha}[p, q]$ (and the corresponding corridors which will be denoted by $\alpha$) for the system $S$.

First of all, the integration regions $I_{\text{in}}(\alpha) \cup I_{\text{fin}}(\beta)$ and $I_{\text{fin}}(\alpha) \cup I_{\text{fin}}(\beta)$ in Eq. (22) show that the integral is non-zero only if the alternatives $\alpha$ and $\beta$ are very close to each other. This is approximately presented by the delta-function $\delta(\alpha, \beta)$.

Consider now the dependence of the classical trajectories, along which the classical actions in Eq. (22) are calculated, correspondingly on the paths $[p, q], [\bar{p}, \bar{q}]$. The expression Eq. (22) is written under the assumption that the paths $[p, q], [\bar{p}, \bar{q}]$ are such that both classical trajectories are close to the middles of the corridors $\alpha_E, \beta_E$.

Denote by $[p_{\alpha}, q_{\alpha}]$ that path of $S$ which provides the classical trajectory of the system $E$ to be in the middle of the corridor $\alpha_E$. If the alternatives $\alpha, \beta$ coincide with each other and both $[p, q], [\bar{p}, \bar{q}]$ coincide with $[p_{\alpha}, q_{\alpha}]$, then the expression (22) is real and has maximum absolute value. If $[p, q], [\bar{p}, \bar{q}]$ deflect from $[p_{\alpha}, q_{\alpha}]$, then the expression (22) is not valid. Instead, Eq. (20) arises. The factors $w_{\alpha}[p, q], w_{\beta}[\bar{p}, \bar{q}]$ become smaller and smaller in absolute value with the deflections increasing, and may become complex. The factor $w_{\alpha}[p, q]$ (correspondingly $w_{\beta}[\bar{p}, \bar{q}]$) become negligible when $[p, q]$ (correspondingly $[\bar{p}, \bar{q}]$) is such that the corridor $\alpha_E$ (correspondingly $\beta_E$) does not contain classical trajectories.

\[6\] They precisely coincide if the paths $[p, q]$ and $[\bar{p}, \bar{q}]$ coincide.
4.3 Derivation of RPI

Let the alternatives $\alpha$ be chosen properly (as it is discussed in Sect. 4.2) so that Eq. (20) is valid. In this case the alternatives $\alpha$ decohere, i.e. can be considered as classical alternatives. It may be readily shown that in this case the system $S$ evolves just as it is suggested in the restricted-path-integral (RPI) description of continuous measurements. The alternatives $\alpha$ play then the role of measurement readouts.

Indeed, if the condition (20) is valid, then, according to (18), the partial density matrices of the system $S$ take the form

$$\rho_{\alpha\beta} = \delta(\alpha, \beta) \ U_\alpha \rho_{\text{in}} \ U_\alpha^\dagger$$

(23)

where the following partial evolution operator for the open system $S$ arises:

$$U_\alpha = \int d[p, q] \ w_\alpha[p, q] \ \exp \left\{ \frac{i}{\hbar} \int_0^t [p \dot{q} - H(p, q)] \right\}.$$  

(24)

This leads to the following evolution law for the open system $S$:

$$\rho_\alpha = U_\alpha \rho_{\text{in}} \ U_\alpha^\dagger, \ \rho = \int d\alpha \rho_\alpha = \int d\alpha U_\alpha \rho_{\text{in}} \ U_\alpha^\dagger.$$  

(25)

The first formula in (23) presents a selective description of the evolution of $S$ (the evolution conditioned by the alternative $\alpha$) while the second formula gives the non-selective description of the same process (all possible alternatives are taken into account).

The selective form of the evolution law may equivalently be expressed in terms of the state vectors instead of density matrices:

$$|\psi_\alpha\rangle = U_\alpha |\psi_{\text{in}}\rangle.$$  

(26)

The formulas (23, 24, 25) are characteristic for the RPI approach to description of a continuous measurement [4]. The alternatives $\alpha$ are interpreted in this approach as alternative readouts of the continuous measurement. Therefore, we derived the RPI description of a continuous measurement starting from the standard description of an open system and choosing

---

7It should not be confused with the partial evolution operator $U_\alpha^{\text{tot}}$ of the closed compound system $S + \mathcal{E}$.  

12
the set of alternatives $\alpha$ in such a way that the environment-decoherence condition (13) and the close condition (21) on the partial influence functionals are satisfy.

This gives an one more justification of RPI description of continuously measured (open, gradually decohering) systems. Moreover, this shows that a wide class of open systems may be interpreted as systems continuously measured by their environments.

5 Conclusion

In the previous papers the restricted-path-integral (RPI) description of continuous quantum measurements was derived directly from the Feynman’s path-integral form of quantum mechanics. Besides, the RPI behavior of measured systems was derived also by conventional quantum-mechanical methods in the framework of some models of measurements.

Here we considered an open system and decomposed its evolution into the sum of partial evolutions. It was showed that the evolution of the open system may be correctly described in the framework of the RPI approach provided that the set of alternative partial evolutions decohere as a consequence of the specific features of the environment.

Mathematically the condition for the measurement-type evolution of an open system may be formulated as the condition for decoherence of the partial influence functionals (21) or with the help of the weaker (but practically equivalent for large environments) environment-decoherence condition (13).

These conditions are (approximately) valid if the alternatives $\alpha$ are presented by such sets of paths (corridors of paths) of the environment which possess classical properties. The choice of decohering alternatives is possible if the environment have many degrees of freedom (is macroscopic or mesoscopic).

The results of the above consideration give a one more justification of the restricted-path-integral approach to quantum continuous measurements. This confirms that Feynman’s theory of amplitudes and Feynman’s path integral technics are valid not only for closed systems, but also in case of open systems. This is important because Feynman’s formalism provides a physically transparent presentation of quantum mechanics and is efficient both for calculation and foruristic considerations.
Acknowledgement. This work is supported in part by the Russian Foundation for Basic Research under grant 02-01-00534.

References

[1] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, and H. D. Zeh. *Decoherence and the Appearance of a Classical World in Quantum Theory*. Springer, Berlin etc., 1996.

[2] M. B. Mensky. *Quantum Measurements and Decoherence. Models and Phenomenology*. Kluwer Academic Publishers, Dordrecht, Boston and London, 2000. Russian translation: Moscow, Fizmatlit, 2001.

[3] G. Lindblad, On the generators of quantum dynamical semigroups. *Comm. Math. Phys.* 48 (1976) 119.

[4] M. B. Mensky, *Continuous Quantum Measurements and Path Integrals*, IOP Publishing, Bristol and Philadelphia, 1993 (in English; Japanese extended translation: Yoshioka, Kyoto, 1995).

[5] M. B. Mensky, Finite resolution of time in continuous measurements: Phenomenology and the model, *Physics Letters A* 231 (1997) 1.

[6] M. B. Mensky, Quantum restrictions for continuous observation of an oscillator. *Phys. Rev. D* 20 (1979) 384; Quantum restrictions on measurability of parameters of motion for a macroscopic oscillator. Zh. Eksp. Teor. Fiz. 77 (1979) 1326 [English translation: *Sov. Phys.-JETP* 50 (1979) 667].

[7] M. Gell-Mann and J. Hartle, Classical equations for quantum systems. *Phys. Rev. D* 47 (1993) 3345.

[8] R. P. Feynman and F. L. Vernon, jr, The theory of a general quantum system interacting with a linear dissipative system. *Ann. Phys. (N.Y.)* 24 (1963) 118.