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Stimulated quasiparticles in spin-split superconductors

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In superconductors spin split by an exchange field, thermal effects are coupled to spin transport. We show how an oscillating electromagnetic field in such systems creates spin imbalance, that can be detected with a spin-polarized probe. The sign and magnitude of the probe signal result from a competition between processes converting field-induced spin energy imbalance to spin imbalance, dominant at low frequencies, and microwave-driven pair breaking at high frequencies. In the presence of spin-flip scattering, we show that ac excitation also leads to multistabilities in the superconducting state.

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I. INTRODUCTION

Long-lived spin excitations are interesting for spintronics applications, and the spin transport in superconductors has recently attracted renewed attention in this context [1–3]. Spin accumulation in a superconductor can be generated by injecting current from a spin-polarized electrode, for example, a ferromagnet. A second approach for spin injection studied in a number of recent experiments [4–9] is to use a magnetic field or proximity to ferromagnetic insulators to Zeeman split the density of states of the superconductor [10,11] (cf. Fig. 1), so that injection of current from an unpolarized probe also generates observable spin accumulation. The spin splitting also changes the quasiparticle physics so that one component of the imbalance only relaxes via inelastic scattering [4,5,12–14], leading to long observed spin lifetimes and relaxation lengths.

The physics of the long-ranged quasiparticle spin accumulation in spin-split superconductors is closely connected to their thermolectric properties. Magnetic interactions in superconductors break the spin-resolved electron-hole symmetry, enabling large thermolectric responses. This is predicted to occur due to magnetic impurities [15], spin-active interfaces [16–18], and in superconductor-ferromagnet systems in the presence of exchange fields [19]. The large thermolectric effect in spin-split superconductor/ferromagnet tunnel junction has been observed very recently (see Ref. [20]).

The thermolectric mechanisms are also connected to photoelectric effects in superconductors [21,22], where absorbed radiation is converted to a dc voltage observed in a probe electrode. Based on the above discussion, a photo-spin-electric effect should be present also in spin-split superconductors—the absorbed radiation generates spin imbalance, which relaxes slowly via inelastic scattering. This is interesting to consider, e.g., in the context of measurements that use microwave signals to probe spin resonances of the quasiparticles [9].

In this work, we discuss how an electric ac field in diffusive spin-split superconductors produces spin imbalance [see Fig. 1(a)]. We find that the ac driving generates spin imbalance that is either parallel or antiparallel to the exchange field, depending on the drive frequency. Nonequilibrium states generated by ac fields in conventional superconductors have long been studied [23], and we extend the picture to include spin splitting. Although interaction with the fields conserves spin, combining it with elastic spin-flip scattering from, e.g., magnetic impurities results to a nonequilibrium steady state with nonzero spin imbalance [see Fig. 1(b)]. We discuss how the effect can be detected via ferromagnetic probes [Eq. (13)]. Similar photoelectric effects are known to occur also in the absence of spin splitting [21,22], but they require weak elastic scattering. We also predict that the spin imbalance results to an instability in the superconducting order parameter, permitting multiple nonzero values for it in a temperature range around \( T_c \), leading to hysteresis and providing a second characteristic signature of the effect.

The manuscript is organized as follows. We outline the model in Sec. II and discuss observables accessible with spin-polarized electrical probes in Sec. III. Modification of the superconducting order parameter is discussed in Sec. IV, and we conclude in Sec. V.

II. KINETIC EQUATIONS

We consider a diffusive superconductor film with a Zeeman field induced either by an external magnetic field [10] or, for example, proximity to a magnetic insulator [11,24]. In order to describe a nonequilibrium situation we apply the quasiclassical Keldysh-Green function formulation [25–29], and write the Usadel equation for the spin-split superconductor (here and below, we set \( \hbar = e = k_B = 1 \)):

\[
D \nabla \cdot \left( \tilde{g} \nabla \tilde{g} \right) + [i\epsilon \tilde{\tau}_3 - i(\mathbf{h} \cdot \mathbf{S})\tilde{\tau}_3 - \tilde{\Delta} - i\tilde{\sigma} \cdot \tilde{g}] = 0.
\]  

(1)

The function \( \tilde{g}(t, \mathbf{r}) \) is a matrix which in the Keldysh-Nambu-spin space has the form,

\[
\tilde{g} = \begin{pmatrix} \tilde{g}^R & \tilde{g}^K \\ 0 & \tilde{g}^A \end{pmatrix}.
\]

where \( \tilde{g}^{R,A,K} \) are the retarded, advanced, and Keldysh 2x2 matrices in the Nambu \( (\tau_j) \) and spin \( (\sigma_j) \) spaces and \( S = (s_1, s_2, s_3) \). The exchange field \( \mathbf{h} \) is induced by an external magnetic field [10] or, for example, proximity to a magnetic insulator [11,24]. Here, \( D \) is the diffusion constant of the superconductor, \( \Delta \) is the order parameter, and \( \sigma \) a self-energy corresponding to spin-flip and inelastic scattering (electron-phonon or electron-electron). We use a gauge where the
electric potential is $\phi = 0$, and coupling to electromagnetic fields is via a vector potential $A$ appearing in the covariant derivative $\nabla X = \nabla X - i A \delta X$.

We assume that the superconducting film is in a uniform time-dependent electric field $E(t) = A_0 \delta_0(t) \sin(\omega_0 t)$. We follow a similar approximation procedure as in Ref. [23]. In a spatially uniform situation, assuming the film is thinner than the skin depth, the vector potential enters equivalently as a space charge density distribution function $f_0$ and perturb the electron distribution (dotted). Elastic spin-flip scattering transforms quasiparticles to the opposite spin species, converting energy imbalance to imbalance.

The self-energy term in Eq. (1) also takes into account $\phi$ dependence in the leading order in the small parameter $DA_0^2 \ll \omega_0$.

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Considering time dynamics implied by Eq. (1), this expression describes the effect of the ac field in the leading order in the small parameter $DA_0^2 \ll \omega_0$.

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III. SPIN IMBALANCE

Experimentally, spin imbalance in the superconductor can be probed by electrical measurements that use spin-filtering probe junctions, for example, ferromagnets. The dc current-voltage relation will in these cases contain a component that depends on the polarization of the probe and the spin imbalance in the sample.

The current measured by a spin-filtering tunnel probe (see Fig. 2) coupled to the superconductor and biased at \( V = 0 \) is given by [32]

\[
R_{\text{det}} I_{\text{det}} = \mu + P_{\text{det}} \mu_z \tag{9}
\]

\[
\mu = \frac{1}{4} \int_{-\infty}^{\infty} d\epsilon [N_\uparrow f_{\uparrow\downarrow} + N_\downarrow f_{\downarrow\uparrow}], \tag{10}
\]

\[
\mu_z = \frac{1}{4} \int_{-\infty}^{\infty} d\epsilon [N_\uparrow (f_{\uparrow\downarrow} - f_{\downarrow\uparrow}) - N_\downarrow (f_{\downarrow\uparrow} - f_{\uparrow\downarrow})], \tag{11}
\]

where \( P_{\text{det}} \) is the detector polarization in the \( z \) direction, \( R_{\text{det}} \) the junction resistance, and \( f_{\uparrow\downarrow}, f_{\downarrow\uparrow} \) the equilibrium distribution in the detector. The current in the detector is a measure of the charge (\( \mu \)) and spin (\( \mu_z \)) imbalances.

The ac drive only excites the modes \( f_{\uparrow\downarrow}, f_{\downarrow\uparrow} \) which carry no charge imbalance, so that on this level of analysis, no photoelectric effect is present (\( \mu = 0 \)). Moreover, Eq. (2) together with a spin-independent relaxation time yields no spin imbalance (\( \mu_z = 0 \)). This follows from \( R_{L,\sigma} (E, \pm E) = R_{L,\sigma} (E, E) \) and \( \mu_z \propto \int_{-\infty}^{\infty} dE \sum_\sigma \sigma \tau_{\text{in},\sigma} \propto \int_{-\infty}^{\infty} dE \sum_\sigma \sigma \tau_{\text{in},\sigma} = 0 \), reflecting conservation of spin.

In practice, however, elastic spin-flip scattering cannot be ignored, and the associated scattering times can be short compared to the inelastic collisions, \( \tau_{\text{in}} \ll \tau_{\text{m}} \). Under such conditions, ac drive can result to nonzero spin imbalance \( \mu_z \neq 0 \) in the steady state. Away from the strict diffusive limit, magnetic impurities result to a photoelectric effect of order \( \ell_{\text{c}}^2 A_0^2 \alpha \) also in the absence of the Zeeman splitting [21,22]. Here, we concentrate only on the diffusive limit and hence the Zeeman splitting is crucial.

Writing the solution of the kinetic equations [Eqs. (2) and (9)] using the relaxation time approximation for inelastic processes, and considering the limit of weak spin-flip scattering \( \tau_{\text{in}} \ll \tau_{\text{m}}, DA_0^2 \tau_{\text{in}} \ll 1 \), we find

\[
f_{L,\sigma} - f_{L,\sigma}^{(0)} \simeq \frac{2 T_{A,\sigma} \tau_{\text{in}}}{N_\sigma} - \sigma \frac{S_1}{2 \tau_{\text{m}}} N_\sigma T_{A,\sigma}^{(0)} - N_\downarrow T_{A,\downarrow}^{(0)} - N_\uparrow T_{A,\uparrow}^{(0)}, \tag{12}
\]

where \( T_{A,\sigma} = T_{A,\sigma} (f_{L,\sigma}^{(0)}), f_{L,\sigma}^{(0)} \) and \( f_{L,\sigma}^{(0)} = \tanh \frac{E}{T} \) is the equilibrium distribution. As noted above, the first term does not contribute to spin imbalance, but the second term does. The result is illustrated in Fig. 1(b): The transitions driven by the ac field generate an imbalance of quasiparticles inside both spin bands, which the spin-flip scattering converts to a spin imbalance at energies \( |E| > |\Delta| + |h| \).

In a typical situation, however, we expect that spin-flip scattering is fast compared to inelastic relaxation (\( \tau_{\text{in}} \ll \tau_{\text{m}} \)). In this limit we have for the detector current,

\[
I_{\text{det}} = -\frac{P_{\text{det}} D A_0^2 \tau_{\text{in}}}{4 R_{\text{det}}} \int_{-\infty}^{\infty} dE \sum_\pm \left[ f_{L,\uparrow}^{(0)} (E) - f_{L,\downarrow}^{(0)} (E \pm \omega) \right] \times \frac{N_\downarrow (E) R_{L,\downarrow} (E, E \pm \omega) - N_\uparrow (E) R_{L,\uparrow} (E, E \pm \omega)}{N_\downarrow (E) + N_\uparrow (E)} \tag{13}
\]

The result is shown in Fig. 3 for representative parameters. The figure also shows results computed numerically using a phonon model (see Appendix A). The two are qualitatively similar, up to differences mainly originated from temperature and energy dependence in the relaxation rates. As the frequency increases, the amplitude of the signal also increases up to the point \( \omega \approx 2h \), where the process depicted in Fig. 1(b) saturates. At large frequencies \( \omega \gtrsim 2 \Delta \) the detector current changes sign, as microwave-driven pair breaking starts to contribute. In this regime, the resulting signal can be understood as a thermo-electric current [16,18,19] driven by a temperature difference \( T_S > T_F \) caused by the heating of the superconductor by the drive.

In addition to the nonequilibrium-generated signal, oscillating electric fields can introduce a voltage drop \( V_{\text{ac}} \) across the detector tunnel barrier. This results to an additional signal from photoassisted tunneling. Within the above approach, the total current is given by the Tien-Gordon result [33,34],

\[
I = \sum_{n=-\infty}^{\infty} j_n^2 \left( \frac{e V_{\text{ac}}}{\hbar \omega_{\text{ac}}} \right) I_{\text{det}} (V_{\text{dc}} + n \hbar \omega_{\text{ac}} / e), \tag{14}
\]
where \( I_{\text{det}}(V) \) is the current-voltage relation in the absence of \( V_{ac} \), and \( I_0 \) are Bessel functions. In the leading order in driving amplitude (\( V_{ac} \to 0 \)) and without dc bias (\( V = 0 \)),

\[
I \simeq I_{\text{det}}(0) + I_{0,T},
\]

\[
I_{0,T} = \frac{V^2}{2} \eta \omega^2 \left[ I_{eq}(\omega) + I_{eq}(-\omega) \right],
\]

where \( I_{\text{det}}(0) \) is the nonequilibrium current (9), \( I_{0,T} \) the photoassisted current, and \( I_{eq}(V) \) is the IV relation of the FIS junction when the superconductor is at equilibrium. We have

\[
I_{0,T} = \frac{P_{\text{det}} V_{ac}^2}{2 R_{\text{det}} \omega^2} \int_{-\infty}^{\infty} dE N_0(E) \left[ f_L^{(0)}(E + h) - f_L^{(0)}(E - h) \right] \frac{1}{2}.
\]

(17)

where \( N_0 = \text{Re} \frac{iE}{\sqrt{1-E^2}} \) is the BCS density of states. Consider now \( \omega, h \lesssim T \). We find

\[
I_{0,T} = - \frac{P_{\text{det}} V_{ac}^2 h}{2 R_{\text{det}} \Lambda_0^2} \eta(T/\Delta),
\]

(18)

where \( \eta \sim 1 \). The photoassisted tunneling current has the same sign \( I < 0 \) as a thermoelectric current generated by heating the ferromagnet, \( T_S < T_F \), and therefore also the same sign as the nonequilibrium effect (13) at the low frequencies. The ratio of the photoassisted tunneling to the nonequilibrium one is \( I_{\text{det}}(0)/I_{0,T} \propto (A_{01}^0 \ell_{in}/V_{ac})^2 \) where \( \ell_{in} = \sqrt{D \tau_{in}} \).

In principle, \( V_{ac} \) can be suppressed by suitable circuit design. An approach for suppressing the photoassisted tunneling used in a previous experiment [35] measuring the gap enhancement due to microwave drive was to use a large-area later tunnel junction with a high capacitance. Assuming the microwave currents in the \( S \) film and through the detector junction are of similar magnitude (cf. Ref. [36]),

one has \( A_{01} \ell_{in}/V_{ac} \propto Z_S \ell_{in}/\tau_F/\ell_{in} \), where \( Z_S \ell_{in} \) is the impedance of the \( S \) film of length \( \ell_{in} \) and \( \tau_F \) is the junction impedance. Moreover, the presence of photoassisted tunneling can in principle be recognized from the appearance of frequency replicas in the tunneling I-V relation, which should not be present in the nonequilibrium signal.

IV. GAP INSTABILITY

The nonequilibrium spin accumulation affects the magnitude of the superconducting order parameter, potentially leading to large changes and collapse of superconductivity for large driving amplitudes.

Let us consider the effect of the driving on the superconducting order parameter,

\[
\Delta = \frac{\lambda}{8} \int_{-E_0}^{E_0} dE \tau_1 + i \tau_2 \frac{1}{2} \frac{1}{\hbar} \frac{2}{E} \delta^4(E).
\]

Here, we assume singlet pairing, and \( \lambda \) is the corresponding coupling constant and \( E_0 \) the BCS cutoff. The simplest situation is obtained by neglecting spin-flip and spin-orbit scattering. In this case, we can observe that the only spin structure in the equations arises from the Zeeman term, \( h \cdot S \). Treating inelastic collisions within a relaxation time approximation, we find in leading order,

\[
\frac{N_0 \delta I_{\text{L}1}}{\tau_{in}} = - \frac{DA^2_0}{2} \sum_{\pm} R_{L,0}(E,E \pm \omega) \times \left[ f_L^{(0)}(E) - f_L^{(0)}(E \pm \omega) \right].
\]

(20)

The nonequilibrium part of the gap equation now reads

\[
\delta \Delta = \frac{\lambda}{4} \frac{DA^2_0}{2 L \tau_{in}} \int_{-\infty}^{\infty} dE \sum_{\sigma=\pm} \sum_{\pm} \frac{R_L^{(0)}(E) F_0(E)}{N_0(E)} \times \left[ f_{\pm}^{(0)}(E + \sigma h) - f_{\pm}^{(0)}(E + \sigma h \pm \omega) \right]
\]

\[
\approx \frac{\lambda}{4} \frac{DA^2_0}{2 T} \int_{-\infty}^{\infty} dE \sum_{\pm} \frac{R_L^{(0)}(E) F_0(E)}{N_0(E)} \times \left[ f_{\pm}^{(0)}(E) - f_{\pm}^{(0)}(E \pm \omega) \right]
\]

(21)

where on the second line we expand around \( T \gg h, \Delta \), and \( F_0(E) = \frac{1}{2} \text{Im} \tau_1 \delta^4(E) \). The result is the same as for zero Zeeman field. Without spin-flip scattering, the exchange field does not have a significant effect at high temperatures, and the result coincides with known results in Ref. [23]: The superconducting gap is enhanced by the driving, and the superconducting branch extends to \( T > T_c \). Numerical calculations also indicate that the exchange field does not cause significant qualitative changes at lower temperatures either (see below).

The spin imbalance generated by the spin-flip scattering, however, modifies the above conclusion, provided these processes are not slow compared to energy relaxation. As above, let us now assume \( \tau_{in} \ll \tau_{ac} \). In this case we find

\[
\delta \Delta = \frac{\lambda}{4} \frac{DA^2_0}{2 L \tau_{in}} \int_{-\infty}^{\infty} dE \sum_{\sigma=\pm} \sum_{\pm} \frac{R_L^{(0)}(E) F_0(E)}{N_0(E)} \times \left[ f_{\pm}^{(0)}(E + \sigma h) - f_{\pm}^{(0)}(E + \sigma h \pm \omega) \right]
\]

\[
\approx \frac{\lambda}{4} \frac{DA^2_0}{2 T} \int_{-\infty}^{\infty} dE \sum_{\pm} \frac{R_L^{(0)}(E) F_0(E)}{N_0(E)} \times \left[ f_{\pm}^{(0)}(E) - f_{\pm}^{(0)}(E \pm \omega) \right]
\]

(22)

where \( R_L = \frac{1}{2} \sum \sigma R_{L,\sigma} \), and \( F_{\sigma} = \text{Re} \sigma \tau_1 \delta^4(E) \) is the coherence function. The difference to Eq. (21) is in that the elastic spin-flip scattering forces the quasiparticle distributions for both spins to be the same, rather than being copies of a single distribution shifted by the exchange field.

In order to obtain analytical results, let us consider a situation in which the effect of scattering on the spectral functions is small, \( \tau_{in} \ll \tau_{ac}, \tau_{in}, \tau_{eh} \). Then, \( N_0(E) = N(E - \sigma h), F_{\sigma}(E) = F(E - \sigma h), \) where \( N(E) = \text{Re} [E/\sqrt{\Delta^2 - E^2}] \) is the BCS density of states, and \( F(E) = \text{Re} (-E) - \text{Re} [\Delta/\sqrt{\Delta^2 - E^2}] \) the BCS coherence function. Close to the critical temperature \( T \approx T_c(h) \) and neglecting the orbital effect, the gap equation can be expanded to the form [27,37,38],

\[
\ln \frac{T_c}{T} = \frac{\tau_1}{8 \pi^2 T^2} \frac{7 \zeta(3) - 186 \zeta(5)}{8 \pi^2 \tau_1^2} \frac{h^2}{5 \hbar^2} \Delta^2 - \frac{DA^2_0}{4 T} \frac{\tau_{in}}{\Delta^2} \frac{1}{\hbar} \frac{2}{E} \delta^4(E).
\]

(23)

The nonequilibrium part \( \delta \Delta \) results to an extra term [23,27], whose parameter dependence is given by the dimensionless
These limits do not include the feature at $\Delta = h$.

Function $P$ for different exchange fields $h$. The resonant dips are located at $\Delta = h$.

$$P(w,y) = \int_{-\infty}^{\infty} dx \frac{Z(x,y) - Z(x + w,y)}{\sqrt{x^2 - 1}} \frac{1}{\sqrt{(x+w)^2 - 1}} + \theta(w-2) \int_{w-1}^{w-1} dx \frac{Z(x,y) |x(w-x)|}{\sqrt{x^2 - 1}}$$

Here, $Z(x,y) = \frac{1}{2} \sum_{\alpha = \pm} F(x \Delta + (\alpha + \gamma)y \Delta)$

In the absence of spin splitting, $Z(x,y = 0) = 1/x$. The function $P$ is plotted in Fig. 4. We can also find its asymptotic behavior for $w \to 0$.

$$P(w,y) \simeq \begin{cases} \sqrt{\frac{2w}{\pi w}} + \frac{w \ln \frac{1}{\sqrt{2}}} {2w} & w \ll y < 1, \\ \sqrt{\frac{8w}{\sqrt{2}} - (\ln \frac{2}{w} - 1)w - \frac{w^{3/2}}{\sqrt{2}}} & w \ll 1 \ll y, \\ (\ln (\sqrt{2}w/8) - 1)w & y \ll w \ll 1. \end{cases}$$

For $w \gg 1$, on the other hand,

$$P(w,y) \simeq \begin{cases} \pi /w & y \ll 1 \ll w, \\ 2/w & 1 \ll w, y \gg 1. \end{cases}$$

These limits do not include the feature at $\Delta = h$.

As follows from the gap equation (24), close to $T = T_c$, the $\Delta(T)$ relation is determined by $T/T_c - 1 \propto P(\Delta/\omega, h/\omega)$. The order parameter $\Delta(T)$ is then given by the curves in Fig. 4, with the $y$ axis $\propto T - T_c$. Additional features in the $\Delta(T)$ relation may appear at $\Delta(T) = h$. In particular, the relation is multivalued around this point, indicating that we can expect discontinuous transitions as a function of temperature. This is related to changes in possible relaxation channels around $\Delta \sim h$: for $\Delta(T) > h$ the spin-averaged density of states is gapped at $E < |\Delta| - |h|$, but for $\Delta(T) < h$ the averaged DOS is gapless, as the spin splitting is large enough to separate the energy gaps of the two spin species. The gap enhancement by microwave driving, however, continues to increase in the $\Delta(T) < h$ regime as long as $\omega < 2\Delta(T)$, and is larger at some frequencies than without spin splitting.

To obtain a more accurate picture, we can solve Eqs. (1) and (19) numerically. Such results are shown in Figs. 5 and 6.

Figure 5(a) indicates the appearance of a multivalued gap, when $h \gtrsim \omega$ and $\Delta \approx h$, as found analytically. From Fig. 6 we can note that the feature occurs for a wide range of scattering times. The feature is absent if there is no exchange field, and also if there is no spin-flip scattering, so that the qualitative conclusions based on Eq. (21) also apply at lower temperatures.

V. DISCUSSION

Microwave electric field in a spin-split superconductor drives the quasiparticles lying above the superconducting gap into a nonequilibrium state. In the absence of spin-flipping processes, it, however, cannot generate charge or spin imbalance in the diffusive limit. Presence of spin-flip scattering enables generation of spin imbalance, and results to a photoelectric signal observable with ferromagnetic probes. The effect is closely related to the thermoelectric effects in magnetic superconductors. In addition, the excitation causes an instability in the superconducting order parameter when the energy gap becomes comparable to the exchange splitting of the spectrum.
To conclude, we describe production of spin imbalance and a photo-spin-electric effect in spin-split superconductors, and present calculations in relevant parameter ranges. The effects are experimentally accessible with state-of-the-art methods. A number of recent experiments in similar systems exist [5,8,9], including also microwave excitation in the GHz frequency range [9]. In addition to using ferromagnetic probes for detecting the photo spin signal, the effect can also be seen indirectly by observing the discontinuous transitions in the superconducting order parameter. These are experimentally accessible via measurements of the tunneling DOS in the superconductor, or, for example, via measurement of the supercurrent.

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APPENDIX A: ELECTRON-PHONON INTERACTION

We use a simplified model for inelastic relaxation due to electron-phonon interaction [27,39],

\[ \sigma^n_R (t) = - g_{eph} \{ \hat{D}^n_R (t) \hat{g}^R (t) + \hat{D}^n_L (t) \hat{g}^R (t) \} , \]
\[ \sigma^n_K (t) = - g_{eph} \{ [\hat{D}^n_R (t) - \hat{D}^A_R (t)] [\hat{g}^R (t) - \hat{g}^A (t)] + \hat{D}^R (t) \hat{g}^R (t) \} , \]

where the Fourier transformed functions,

\[ \hat{D}^R/A (\omega) = \pm i \omega^2 \text{sgn} \omega , \]
\[ \hat{D}^K (\omega) = [\hat{D}^R (\omega) - \hat{D}^A (\omega)] \coth \frac{\omega}{2T_{ph}} , \]

arise from weighed Fermi surface averages of the phonon Green functions. Parts that do not contribute to the collision integral have been subtracted. The collision integral assumes a standard form,

\[ \mathcal{I}_{\sigma} = g_{eph} \int_{-\infty}^{\infty} \frac{d\omega}{8\pi} |\omega| \text{tr} [\hat{g}^{RA}(E) \hat{g}^{RA}(E - \omega) s_{\sigma}] \]
\[ \times \left\{ f_{\sigma,E} f_{\sigma,E - \omega} - 1 + [f_{\sigma,E} - f_{\sigma,E - \omega}] \coth \frac{\omega}{2T_{ph}} \right\} , \]

APPENDIX B: NUMERICAL DETAILS

We solve Eq. (1) numerically via a Jacobian-free Newton-GMRES method [40]. To obtain a preconditioner, we use automatic differentiation to compute the Jacobian of the energy-local terms, excluding the self-energy parts [\hat{\delta}_{eph} + \hat{\delta}_A, \hat{g}]. The energy convolutions in the self-energies, after energy discretization, are computed via fast Fourier transforms.
[26] K. D. Usadel, Phys. Rev. Lett. 25, 507 (1970).
[27] N. B. Kopnin, Theory of Nonequilibrium Superconductivity, International Series of Monographs on Physics, No. 110 (Oxford University Press, Oxford, 2001).
[28] J. P. Morten, A. Brataas, and W. Belzig, Phys. Rev. B 70, 212508 (2004).
[29] J. P. Morten, A. Brataas, and W. Belzig, Phys. Rev. B 72, 014510 (2005).
[30] N. Poli, J. P. Morten, M. Urech, A. Brataas, D. B. Haviland, and V. Korenivski, Phys. Rev. Lett. 100, 136601 (2008).
[31] W. Belzig, C. Bruder, and G. Schön, Phys. Rev. B 54, 9443 (1996).
[32] F. S. Bergeret, A. Verso, and A. F. Volkov, Phys. Rev. B 86, 214516 (2012).
[33] P. K. Tien and J. P. Gordon, Phys. Rev. 129, 647 (1963).
[34] J. Tucker and M. Feldman, Rev. Mod. Phys. 57, 1055 (1985).
[35] R. E. Horstman and J. Wolter, Phys. Lett. A 82, 43 (1981).
[36] J. E. Mooij and T. M. Klapwijk, Phys. Rev. B 27, 3054 (1983).
[37] K. Maki and T. Tsuneto, Prog. Theor. Phys. 31, 945 (1964).
[38] K. Maki, Prog. Theor. Phys. 32, 29 (1964).
[39] G. M. Eliashberg, Sov. Phys. JETP 34, 668 (1972).
[40] D. Knoll and D. Keyes, J. Comp. Phys. 193, 357 (2003).