Confinement, chiral symmetry breaking and the mass generation of hadrons*

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A key question to QCD is what mechanism generates the hadron mass in the light quark sector, where both confinement and chiral symmetry breaking are in the game. Are confinement and chiral symmetry breaking in the vacuum uniquely interconnected? Can hadrons survive chiral symmetry restoration? If yes, what happens with their mass and what symmetries beyond the chiral symmetry are there? We review our recent insights. In particular, in a dynamical lattice simulation we artificially restore chiral symmetry by removing the low-lying Dirac modes of the valence quark propagators, which is a well defined procedure and keep gluodynamics intact. Hadrons survive this artificial chiral restoration and their mass is surprisingly large. All hadrons fall into chiral multiplets and some of them are degenerate, i.e. the spectrum reveals some higher symmetry, that includes the chiral symmetry as a subgroup. The $U(1)_A$ symmetry does not get restored after removal of the chiral modes from the valence quarks.

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1. Introduction

QCD is already 40 years old but we do not know yet the answer to a key question about mass generation of hadrons. In the light quark sector, with the quark masses of a few MeV, practically the whole hadron mass consists of the energy of the quantized gluonic field. This straightforward answer from the trace anomaly of QCD is correct but not satisfactory. We are interested in the mechanism of the mass generation. Are confinement and dynamical chiral symmetry breaking interconnected and how do they contribute to the hadron mass? It is also important to shed the light on this issue if we want to understand the phase diagram of QCD.

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It was believed that the chiral symmetry breaking in the vacuum is the crucial phenomenon responsible for the hadron mass generation: The hadron mass in the light quark sector is determined mainly by the quark condensate of the vacuum. This is certainly true for the pion, which as the (pseudo) Goldstone boson originates from the dynamical chiral symmetry breaking. Is it true, however, that the nucleon, the rho-meson and other hadron masses also come mostly from the quark condensate of the vacuum? Given this view, it was expected that upon chiral restoration masses of these and other hadrons should drop off \cite{1} and eventually beyond the chiral restoration phase transition (crossover) the hadrons should disappear. In other words, without the chiral symmetry breaking in the vacuum there cannot be any confined hadrons.

The ’t Hooft anomaly matching conditions \cite{2} formally state, that at zero temperature and density in the confining mode the chiral symmetry should be indeed spontaneously broken, though they do not suggest any insight why it should be so. This generic statement does not imply, however, that the hadron mass should be made mostly of the quark condensate of the vacuum. The latter view had essentially phenomenological \cite{3,4} and model grounds starting from the bag model in the past up to contemporary Schwinger-Dyson approaches to hadrons. The ’t Hooft anomaly matching conditions do not constrain, however, the interrelation between the confinement and chiral symmetry breaking at nonzero temperatures and densities.

Another argument, according to Casher \cite{5}, was that the quark cannot be confined without the chiral symmetry breaking in the vacuum. If so, hadrons cannot exist in the world with unbroken chiral symmetry. However, it was shown that the Casher argument is not general and can be easily bypassed \cite{6}. In particular, at least within the manifestly confining model hadrons with rather large mass can still exist in a dense medium at low temperatures where the chiral symmetry is restored \cite{7}.

Another interesting issue is whether the highly excited hadrons reveal or not effective restoration of the chiral symmetry \cite{8}. If yes (it should be confirmed or disproved experimentally), then their mass should not be influenced by the quark condensate of the vacuum. There are also some experimental hints, that hadrons in this regime reveal some higher symmetry, that includes $SU(2)_L \times SU(2)_R$ as a subgroup.

We want to shed some light on all these questions. For this purpose we can use lattice QCD as a tool to explore the interrelation between confinement and chiral symmetry breaking and to check whether or not hadrons can still exist in a world without breaking of chiral symmetry in a vacuum. If yes, what happens with their mass and what symmetries do they have in this regime?

The idea, that the low-lying modes of the Dirac operator, that are re-
sponsible for the chiral symmetry breaking are crucial for hadron masses such as nucleon or $\rho$-meson, etc., has its roots in the instanton liquid model of the QCD vacuum [9]. Subsequently, the effect of the low-lying modes of the Dirac operator on different hadron correlators was studied on a small lattice within the quenched approximation [10]. We pose just the opposite question: Will hadrons survive if we remove the low-lying modes keeping the gluodynamics intact? Such a procedure is a well-defined one [11] and for the $\pi$, $\rho$, $a_0$ and $a_1$ mesons was implemented in [12]. Here we study both baryons and mesons as well their symmetries after such artificial chiral restoration [13].

2. The setup

The quark condensate of the vacuum is related to a density of the lowest quasi-zero eigenmodes of the Dirac operator [14]:

$$\langle 0 | \bar{q} q | 0 \rangle = -\pi \rho(0).$$  \hspace{1cm} (1)

Here first the infinite volume limit is assumed at a finite quark mass and then the chiral limit should be taken.

From the lattice calculations in a given finite volume we cannot say a priori which and how many lowest eigenmodes of the Dirac operator are responsible for the quark condensate of the vacuum. We remove an increasing number of the lowest Dirac modes from the valence quark propagators,

$$S_{\text{red}(k)} = S - S_{\text{lm}(k)} \equiv S - \sum_{i \leq k} \mu_i^{-1} |v_i \rangle \langle v_i| \gamma_5,$$ \hspace{1cm} (2)

and study the effects of the (remaining) chiral symmetry breaking on the masses of hadrons. Here $S$ is the untruncated quark propagator, the $\mu_i$ are the (real) eigenvalues of the Hermitian Dirac operator $D_5 = \gamma_5 D_4$, $|v_i \rangle$ are the corresponding eigenvectors and $k$ represents the number of the removed lowest-lying modes.

We perform our calculations on the unquenched two-flavor configurations with chirally improved fermions [15] on the lattice size of 2.4 fm at the pion mass $m_\pi = 322$ MeV.

3. Existence of hadrons after unbreaking of chiral symmetry

An interesting observation is that for all hadrons under our study, except for a pion, the quality of the exponential decay of the correlators essentially improves by increasing the number of removed eigenmodes. The exponential decay of the correlator with the given quantum numbers indicates that there is a state with the same quantum numbers. Assume that after removal of
a sufficient amount of the low-lying modes the exponential decay signals from all hadrons would disappear. This would indicate that hadrons also disappear, i.e. there is not confinement without the chiral modes of the Dirac operator. We observe, however, a very clean signal from all hadrons, except for a pion. The hadrons survive this artificial unbreaking of the chiral symmetry. Even more, the nucleon and rho-meson masses do not decrease upon chiral restoration, see Figs. 1 and 2!

4. Meson degeneracies and splittings and what they tell us

If hadrons survive the restoration of the $SU(2)_{L} \times SU(2)_{R}$ chiral symmetry in the vacuum, they must fall into parity-chiral multiplets \[8\]. These multiplets for the $J = 1$ mesons are as follows:

\begin{align*}
(0, 0) & : \quad \omega(0, 1^{--}) \quad f_{1}(0, 1^{++}) \\
\left(\frac{1}{2}, \frac{1}{2}\right)_{a} & : \quad h_{1}(0, 1^{+-}) \quad \rho(1, 1^{--}) \\
\left(\frac{1}{2}, \frac{1}{2}\right)_{b} & : \quad \omega(0, 1^{--}) \quad b_{1}(1, 1^{+-}) \\
(0, 1) + (1, 0) & : \quad a_{1}(1, 1^{++}) \quad \rho(1, 1^{--})
\end{align*}

If the $U(1)_{A}$ symmetry is unbroken the states from two distinct multiplets $\left(\frac{1}{2}, \frac{1}{2}\right)_{a}$ and $\left(\frac{1}{2}, \frac{1}{2}\right)_{b}$ that have the same isospin but opposite spatial parity are connected to each other by the $U(1)_{A}$ transformation. In our real world $U(1)_{A}$ is broken both via the axial anomaly and via the quark condensate of the vacuum. In the world with restored $SU(2)_{L} \times SU(2)_{R} \times U(1)_{A}$ symmetry a $\rho$ meson, that is the chiral partner to the $h_{1}$ meson, must be degenerate with the $b_{1}$ state.

On Fig. 1 we show the mass evolution of the isovector mesons with $J = 1$ upon the truncation of the low-lying modes. Both the number of the removed modes $k$ as well as the maximal energy $\sigma$ of these modes are shown.

At the truncation energy $\sigma \sim 40$ MeV the $\rho - a_{1}$ splitting vanishes, a direct indication of the chiral $SU(2)_{L} \times SU(2)_{R}$ restoration in the physical states. The large $b_{1} - \rho$ and $b_{1} - \rho'$ splittings persist, however. This means that the $U(1)_{A}$ breaking does not disappear. While that $U(1)_{A}$ breaking component that is due to the chiral condensate should vanish with the condensate, the $U(1)_{A}$ breaking via the axial anomaly still persists. Indeed, the quark determinant that contains the $U(1)_{A}$ breaking in the vacuum, is not affected by our truncation of the valence quarks. However, this result does show that there is not direct interconnection of the lowest lying modes of valence quarks and the mechanism of the $U(1)_{A}$ splittings in QCD.

The $\rho$ and $\rho'$ mesons become degenerate, too. These $\rho$ and $\rho'$ are different states because they appear in different eigenvalues of the correlation
matrix (i.e. their eigenvectors are orthogonal) and because they are well
split before the removal of the low modes. This degeneracy indicates some
higher symmetry that includes chiral $SU(2)_L \times SU(2)_R$ as a subgroup.

### 5. Baryon chiral multiplets

If baryons survive chiral restoration they have to fall into baryonic
parity-chiral multiplets:

$$(\frac{1}{2}, 0) + (0, \frac{1}{2}), \ (\frac{3}{2}, 0) + (0, \frac{3}{2}), \ (\frac{1}{2}, 1) + (1, \frac{1}{2}).$$  \hspace{1cm} (3)

The first representation consists of nucleons of positive and negative
parity of the same spin. Another representation contains both positive
and negative parity $\Delta$'s with the same $J$. Finally, the third representation combines one nucleon and one Delta parity doublet with the same spin.

We observe at least two degenerate nucleon parity doublets. This indicates a higher symmetry for the $J = I = \frac{1}{2}$ states.

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