In this work, a theoretical model with a numerical solution is brought forward for a bio-nanofluid with varying fluid features over a slippery sheet. The partial differential equations (PDEs) involving temperature-dependent quantities have been translated into ordinary differential equations (ODEs) by using similarity variables. Numerical verifications have been done in three different methods: finite difference method, shooting method, and bvp4c. To figure out the influence of parameters on the flows, the graphs are plotted for the velocity, temperature, concentration, and microorganism curves. The boundary layer thickness of the microorganism profile reduces with the Schmidt number and Peclet number. In addition to adding radiative heat flux, we added heat generation, rate of chemical reaction, and first-order slip. Adding these parameters brought new aspects to the underlying profiles. Moreover, the obtained data of the skin friction coefficient, the local Nusselt number, the local Sherwood number, and the local density of motile microorganisms are tabulated against various parameters for the physical parameters. From the results, it is apparent that the local Nusselt number decreases with the Brownian and thermophoretic parameters. The data obtained for physical parameters have a close agreement with the published data. Finally, the graphs for slip conditions are significantly different when the comparison is drawn with no-slip condition.

1. Introduction

Thermal analysis has attracted attention from the scientific community because of its role in our daily lives. The applications of heat transfer range from electrical devices and power plants to the heating and cooling devices, boiler, condenser, and evaporators within houses where efficiency of these devices plays a key role. The efficient devices not only reduce energy consumption but also give additional life to it.

Nanofluid is a mixture of a base fluid with 100 nm-size nanoparticles. Since the work by Choi and Eastman [1] on nanofluid, the research in this direction took a huge stride. The thermal conductivity is significantly higher than that of the traditional fluids as it was reported in Lee et al. [2]. There are many applications in the field of nanofluids including lubricants, automation, electronics, and biomedicine. For the list of references which took multiple paths considering nanofluid in their study, one is referred to in [3–7].

Bioconvection is another phenomenon which occurs due to the density difference of the fluid. Raees et al. [8] recorded the homotopy analysis method (HAM) solution for an unsteady bioconvection flow in a channel and showed that the velocity component decreases with the increase in time. Uddin et al. [9] discussed bioconvection nanofluid over a wavy surface with slip flow in application to nano-biofuel cells. Khan and Makinde [10] explored bioconvection flow due to gyrotactic microorganisms. They noticed that, with rising the values of the convective variable, the dimensionless temperature on the surface rises. Uddin et al. [11] investigated Stefan blowing with multiple slip effects in bioconvection. For finding similarity transformation, they used Lie group
analysis. The resources for further study on this topic can be found in [12–16].

One of the ways through which heat transfer occurs is thermal radiation. It has diverse technological applications in combustion, furnace design, turbines, and solar collectors. The thermal radiation with variable fluid properties is reported in [17]. The author found that the skin friction coefficient increases with viscosity parameter. RamReddy and Naveen [18] reported results for activation energy and thermal radiation. Aziz et al. [19] discussed free convection flow in nanofluids. Shahsavar et al. [20] investigated the impact of variable fluid properties in hybrid nanofluid. Naganthran et al. [21] presented multiple slip effects in the presence of microorganisms. Ali et al. [22] discussed hydromagnetic fluid flow in microorganisms. Mabood et al. [23] combined electrical and magnetic equipment[26].Ali et al. [27] discussed hybrid nanofluid with geothermal energy extractions, plasma studies, chemical conductivity, thereaderis referred to in [43–46]. For experimental investigation on nanofluids, the reader is referred to in [47].

Most theoretical studies mentioned above are focused on the idea of constant fluid properties in fluid flows. The viscosity of a fluid, however, relies heavily on temperature than on other factors, such as pressure. It comes out that the use of variable properties offers distinct effects on fluid flow motion.

This paper is ordered in the following way: the flow model is presented in Section 2. The numerical procedure for the solution is presented in Section 3. Results and discussion are given in Section 4. Conclusion of the paper is drawn in Section 5.

2. Flow Model

Consider the movement of a nano fluid containing gyrotactic microorganisms past a stretching sheet with variable physical properties. The magnetic field $B_0^\parallel$ is applied normal to the surface. Due to low magnetic Reynolds number, the induced magnetic field is assumed negligible. The stretching velocity is $U_w = ax(1 - A_1 t)^{-1}$. The governing model is [48]
and the boundary condition corresponding to the considered model is taken as
\[ \tilde{u}_1 = U_w(x, t) + N_1 \frac{\partial \tilde{u}_1}{\partial y}, \quad v = 0, \]
\[ \tilde{T}_1 = T_w(x, t) + D_1 \frac{\partial \tilde{T}_1}{\partial y}, \]
\[ C_1 = C_w, N_1 = N_w, \quad \text{at} \quad y = 0, \]
\[ \tilde{u}_1 \rightarrow 0, \tilde{T}_1 \rightarrow T_{\infty}, \]
\[ C \rightarrow C_{\infty}, N_1 \rightarrow N_{\infty}, \quad \text{as} \quad y \rightarrow \infty, \]
where all the variables are defined in the glossary.

The similarity variables are defined as
\[ \eta = \frac{a}{\sqrt{1 - A_1 t}}, \]
\[ \psi = \frac{av}{1 - A_1 t} \chi f(\eta), \]
\[ \theta(\eta) = \frac{\tilde{T}_1 - T_{\infty}}{T_w - T_{\infty}}, \]
\[ \phi(\eta) = \frac{\tilde{C}_1 - C_{\infty}}{C_w - C_{\infty}}, \]
\[ \chi(\eta) = \frac{N_1}{N_w}. \]

Inserting equation (7) into equations (1)–(6), we get
\[ \left( \frac{\mu(T_1)}{\mu_{\infty}} f'' \right)' - f'' + f f'' - \left( \frac{\eta}{2} f'' \right) - \left( M + K \rho \left( \frac{\mu(T)}{\mu_{\infty}} \right) \right) f' = 0, \]  
(8)
\[ \left( \frac{k(T_1)}{k_{\infty}} \theta' \right)' + \frac{4}{3} \frac{R \theta''}{\theta'} + \frac{Nb}{D_{B_{\infty}}} (\frac{D_B(C)}{D_{B_{\infty}}} \theta' \phi' + \frac{\eta}{2} \theta'^2 + Nt \theta'^2 + Pr_{\infty} \left( f' - \eta \rho \theta' + Ec \left( \frac{\mu(T_1)}{\mu_{\infty}} \right) f'' + ME \left( \frac{\mu(T)}{\mu_{\infty}} \right) f'' + s \theta \right) = 0, \]  
(9)
\[ \left( \frac{D_B(C)}{D_{B_{\infty}}} \phi' \right)' + \frac{Nt}{Nb} \phi' + Sc \left( f \phi' - \frac{A_2}{2} \phi'' - Kr \phi \right) = 0, \]  
(10)
\[ \left( \frac{D_m(C)}{D_{m_{\infty}}} \chi' \right)' + Sb \left( f \chi' - \frac{A_2}{2} \chi'' \right) - Pe \left( \phi' \chi' + \chi \phi'' \right) = 0, \]  
(11)
\[ \begin{aligned}
 f(0) &= 0, \\
 f''(0) &= 1 + \delta f''(0), \\
 \theta(0) &= 1 + \eta \theta'(0), \\
 \phi(0) &= 1, \\
 \chi(0) &= 1, \\
 f'(\infty) &= 0, \\
 \theta(\infty) &= 0, \\
 \phi(\infty) &= 0, \\
 \chi(\infty) &= 0.
\end{aligned} \]  
(12)

Following Amirsom et al. [48], the physical quantities consisting of viscosity, thermal conductivity, nanoparticle, and microorganism diffusivities are written as
\[ \begin{aligned}
 \mu(T_1) &= \mu_{\infty} \left( 1 + h_1 \left( T_{\infty} - \tilde{T}_1 \right) \right), \\
 k(T_1) &= k_{\infty} \left( 1 + h_1 \left( T_{\infty} - \tilde{T}_1 \right) \right), \\
 D_B(C_1) &= D_{B_{\infty}} \left( 1 + h_2 \left( \tilde{C}_1 - C_{\infty} \right) \right), \\
 D_m(C_1) &= D_{m_{\infty}} \left( 1 + h_3 \left( \tilde{C}_1 - C_{\infty} \right) \right).
\end{aligned} \]  
(13)
Equation (13) when used into equations (8)–(11), one can get

\[
(1 + h_2 - h_2\theta)f'' - h_2\theta f'' - f'' + f f'' - A\left(f' + \frac{\eta}{2}f^\prime\right) - (M + Kp (1 + h_2 - h_2\theta)) f' = 0, \\
\left(1 + h_2\theta + \frac{4}{3} Rd\right)\theta' + h_2\theta^2 + Nb (1 + h_0\phi)\theta' + Nt\theta'^2 \\
+ Pr_{co} \left(f\theta' - \frac{An}{2}\theta' + Ec (1 + h_2 - \theta h_2) f'' + MEc f'' + Kp Ec (1 + h_2 - h_2\theta) f'' + s\theta'\right) = 0, \tag{14}
\]

\[
(1 + h_0\phi)\phi'' + h_0\phi^2 + Sc \left(f \phi' - \frac{An}{2} \phi' - Kr\phi\right) + \frac{Nt}{Nb} \theta' = 0, \\
(1 + h_0\phi)\chi'' + h_0\phi\chi' + Sb \left(f \chi' - \frac{An}{2} \chi'\right) - Pe \left(\phi' \chi' + \phi\phi''\right) = 0.
\]

All these parameters are grouped into

\[
A = \frac{A_1}{a}, \\
Kp = \frac{v_{co} (1 - A_1 t)}{ak^2}, \\
M = \frac{\sigma A^3 (1 - A_1 t)}{\rho_{co} a}, \\
Pr_{co} = \frac{v_{co}}{\alpha_{co}}, \\
Rd = \frac{4r T_{co}^3}{k_k k_{co}} \\
Nb = \frac{r D_{B,co} (C_w - C_{co})}{\alpha_{co}}, \\
Nt = \frac{r D_{T} (T_w - T_{co})}{T_{co} a}, \\
Ec = \frac{u_w^2}{c_y (T_w - T_{co})}, \\
s = \frac{Q (1 - A_1 t)}{a} \\
Sc = \frac{v_{co}}{D_{B,co}}, \\
K_c = K_c (1 - A_1 t) a, \\
Sb = \frac{v_{co}}{D_{m,co}}, \\
Pe = \frac{bw_k}{D_{m,co}} \\
\delta = N \left(\frac{a}{v_{co} (1 - A_1 t)}\right)^{(1/2)}, \\
y = D \left(\frac{a}{v_{co} (1 - A_1 t)}\right)^{(1/2)}.
\]

The physical quantities of the interest in this study are

\[
C_{fx} = \frac{\mu (T) (\partial u/\partial y)_{y=0}}{\rho u_w^2}, \\
Nu_x = -k (T) x (\partial T/\partial y)_{y=0} \frac{k (T)(T_w - T_{co})}{k (T)(T_w - T_{co})}, \\
Sh_x = -D_{B,co} x (\partial C/\partial y)_{y=0} \frac{D_{B,co} (C_w - C_{co})}{D_{B,co} (C_w - C_{co})}, \\
Nn_x = -D_{m,co} x (\partial N/\partial y)_{y=0} \frac{(D_{m,co} N_w)}{(D_{m,co} N_w)}.
\]

Inserting equation (7) into equation (13) yields the following expressions:

\[
Re_x^{1/2} C_{fx} = -(1 + h_0\phi) f'' (0), \\
Re_x^{1/2} Nu_x = \left(1 + \frac{4}{3} Rd\right) \theta' (0), \\
Re_x^{1/2} Sh_x = -\phi' (0), \\
Re_x^{1/2} Nn_x = -\chi' (0),
\]

where the local Reynolds number is defined as

\[
Re_x = (U_w x / \nu).
\]

3. Numerical Process

3.1. Shooting Method. A boundary value problem ((8)–(12)) can be solved with the shooting method. The stable iterative scheme, Newton–Raphson method, has been used in locating the roots followed by obtaining the solution from the fifth-order Runge–Kutta solver. The system of first-order ODEs is
\[ f = y_1, \quad f' = y_2, \quad f'' = y_3, \]
\[ f'' = y_4' = \frac{1}{1 + h_2 - h_4 y_4} \left( h_2 y_5 y_3 + y_2' - y_1 y_3 \right) A \left( y_2 + \frac{h_2}{2} y_3 \right) + (M + \text{Kp}(1 + h_2 - h_4 y_4) y_2'). \]
\[ y_4 = \theta, \quad y_5 = \theta', \quad y_5'' = y_4' = \frac{-1}{1 + h_4 y_4 + (4/3)\text{Rd}} \]
\[ \cdot \left( h_4 y_5^2 + \text{Nb}(1 + h_6 y_6) y_3 y_7 + \text{Nt} y_5^2 + \text{Pr}_{\text{co}} \left( y_1 y_3 - \frac{h_2}{2} A y_3 + \text{Ec}(1 + h_2 - y_2 y_4) y_3^2 + s y_4 + \text{M} y_2^2 + \text{KpEc}(1 + h_2 - h_4 y_4) y_2^2 \right) \right). \]
\[ y_6 = \phi, \quad y_7 = \phi', \quad y_7'' = y_6' \]
\[ = \frac{-1}{1 + h_8 y_8} \left( \frac{N_f}{N_f} y_9' + h_6 y_7^2 + \text{Sc} y_1 y_7 + \frac{\text{Sc} A}{2} y_7 - \text{ScK} y_8 \right). \]
\[ y_8 = \chi, \quad y_9 = \chi', \quad y_9'' = y_8' = \frac{-1}{1 + h_9 y_9} \left( h_8 y_7 y_9 - \text{Sb} \left( \frac{h_2}{2} y_9 - y_9 y_8 \right) - \text{Pe}(y_9 y_9 + y_9 y_9') \right). \]

The results’ verification is achieved from the bvp4c solver. For details on bvp4c, the reader is referred to in [49].

3.2. Finite Difference Method. In this section, we present the finite difference method to solve boundary value problem (8)–(12). The spatial discretization is given by first defining \( f'' = F \) in the momentum equation:

\[
(1 + h_2 - h_4 \theta) \left( \frac{F_{t+2} - 2F_{t+1} + F_t}{(\Delta \eta)^2} \right) - h_2 \theta \left( \frac{F_{t+1} - F_t}{\Delta \eta} \right) - F_t^2 + f \left( \frac{F_{t+1} - F_t}{\Delta \eta} \right) \]
\[ - A \left( F_t + \frac{A}{2} \left( \frac{F_{t+1} - F_t}{\Delta \eta} \right) \right) - (M + \text{Kp}(1 + h_2 - h_4 \theta)) F_t = 0, \]
\[ \cdot \left( 1 + h_4 \theta \right) \left( \frac{2 \theta_{t+2} - 2 \theta_{t+1} + \theta_t}{(\Delta \eta)^2} \right) + \left( h_4 + \text{Nt} \right) \left( \frac{\theta_{t+1} - \theta_t}{\Delta \eta} \right)^2 \]
\[ + \text{Nb} \left( 1 + h_6 \phi \right) \left( \frac{\theta_{t+1} - \theta_t}{\Delta \eta} \right) \left( \phi_{t+1} - \phi_t \right) + \text{Pr}_{\text{co}} \left( F_t \left( \frac{\theta_{t+1} - \theta_t}{\Delta \eta} \right) + s \right) \left( \frac{\theta_{t+1} - \theta_t}{\Delta \eta} \right) + \text{Ec}(1 + h_2 - h_4 \theta) \]
\[ \cdot \left( \frac{F_{t+1} - F_t}{\Delta \eta} \right)^2 + \text{M} \frac{F_2^2}{\Delta \eta} + \text{KpEc}(1 + h_2 - h_4 \theta) F_t^2 + s \theta = 0, \]
\[ (1 + h_8 \phi_9) \left( \frac{\phi_{t+2} - 2 \phi_{t+1} + \phi_t}{(\Delta \eta)^2} \right) + h_6 \left( \frac{\phi_{t+1} - \phi_t}{\Delta \eta} \right)^2 \]
\[ + \text{Sc} \left( f_t \left( \frac{\phi_{t+1} - \phi_t}{\Delta \eta} \right) - \frac{A}{2} \left( \frac{\phi_{t+1} - \phi_t}{\Delta \eta} \right) - \text{Kp} \phi_t + \frac{\text{Nt} \theta_{t+2} - 2 \theta_{t+1} + \theta_t}{(\Delta \eta)^2} \right) = 0, \]
\[ \cdot \left( 1 + h_8 \phi_9 \right) \left( \chi_{t+2} - 2 \chi_{t+1} + \chi_t \right) + h_9 \left( \frac{\phi_{t+1} - \phi_t}{(\Delta \eta)} \right) \left( \chi_{t+1} - \chi_t \right) + \text{Sb} \left( f_t - \frac{A}{2} \right) \left( \chi_{t+1} - \chi_t \right) = 0, \]
\[ - \text{Pe} \left( \frac{\phi_{t+1} - \phi_t}{(\Delta \eta)} \right) \left( \chi_{t+1} - \chi_t \right) + \chi_t \left( \frac{\phi_{t+2} - 2 \phi_{t+1} + \phi_t}{(\Delta \eta)^2} \right) = 0, \]

and the boundary conditions are
\[ f_0 = 0, \]
\[ F_0 = 1 + \delta \left( \frac{F_1 - F_0}{\Delta \eta} \right), \]
\[ \theta_0 = 1 + \gamma \left( \frac{\theta_1 - \theta_0}{\Delta \eta} \right), \]
\[ \phi_0 = 1, \chi_0 = 1, F_{\infty} = 0, \theta_{\infty} = 0, \phi_{\infty} = 0, \chi_{\infty} = 0. \]

4. Results and Discussion

An excellent agreement with published results is obtained for a comparison of the skin friction coefficient \(-f''(0)\) which is shown in Tables 1–3. The data in Table 4 show computational results for the local Nusselt number, the local Sherwood number, and the local density number of motile microorganisms obtained with \(b_{v_p}c\). The local Nusselt number \(N_u\) is reduced against Brownian motion parameter \(N_b\), thermophoretic parameter \(N_t\), Eckert number \(E_c\), heat source parameter \(s\), and thermal conductivity parameter \(N_k\).

With increasing values of Prandtl number \(Pr\) and radiation parameter \(Rd\), the local Nusselt number shows an upward trend.

The physical parameter, the local Sherwood number \(Sh\), depicts an upward trend against Brownian motion parameter \(N_b\), thermophoretic parameter \(N_t\), Schmidt number \(Sc\), and chemical reaction parameter \(K_r\). However, a decreasing trend for the local Sherwood number is observed for rising values of mass diffusivity parameter \(h_0\).

Finally, the values of the local density number of motile microorganisms \(N_m\) decline with the increase of mass diffusivity parameter \(h_0\) and microorganism diffusivity parameter \(h_0\). However, there is an upsurge for increasing values of the bioconvection Schmidt number \(Sb\) and Peclet number \(Pe\).

Figures 1 and 2 illustrate the effects of magnetic parameter \(M\) and porosity parameter \(K_p\) on the velocity profile with and without hydrodynamic slip. The boundary layer thickness reduces with increasing values of \(M\) and \(K_p\). When fluid flow encounters the Lorentz forces, the velocity of the fluid decelerates which affects the boundary layer thickness. The same argument holds for \(K_p\).

Figure 3 is plotted to perceive the effect of Prandtl number \(Pr\) on the temperature profile. It is noted that an enhancement in Prandtl number \(Pr\) causes reduction in the temperature distribution. The smaller values of \(Pr\) correspond to the increase in thermal conductivities which causes reduction in a thermal boundary layer. For Prandtl number \(Pr \geq 1\), the momentum diffusivity is dominant in fluid behavior. Thus, less thermal diffusivity contributes to lowering the thermal boundary layer thickness.

Figure 4 depicts the influence of radiation parameter \(Rd\) on the temperature profile. It is seen that an increase in \(Rd\) enhances the temperature of the fluid. Larger values of radiation parameter transfer more heat to the fluid which overall increases the temperature and its profile.

Figure 5 reports the influence of Eckert number \(E_c\) on the temperature profile. The higher values of Eckert number \(E_c\) cause an increase in the thermal boundary layer thickness. The Eckert number \(E_c\) enhances kinetic energy, which increases fluid’s temperature.

Figure 6 illustrates the impact of heat source parameter \(s\) on the temperature distribution. It is observed that temperature of the fluid increases with an increment in the heat generation parameter. The higher values of \(s\) provide more heat to the fluid resulting in the rise of the temperature of the fluid.

Figure 7 examines the effect of temperature-dependent thermal conductivity parameter \(h_4\) on temperature. It is noted that the thermal boundary layer thickness increases by increasing parameter \(h_4\).

Figures 8 and 9 are drawn to perceive the effect of Brownian motion parameter \(N_b\) on the temperature and concentration profiles. It is revealed in the figure that, by increasing Brownian motion parameter \(N_b\), thermal boundary layer thickness rises, while concentration boundary layer thickness declines. The Brownian parameter appears due to the presence of nanoparticles’ concentration.

Figures 10 and 11 convey the impacts of thermophoresis parameter \(N_t\) on temperature and concentration distributions. The temperature and concentration profile rise for rising values of \(N_t\). The thermophoresis term appears due to the temperature gradient in particulate flows. Larger values of \(N_t\) transmit more temperature to the fluid along with the concentration profile.

Figure 12 portrays the influence of chemical reaction parameter \(K_r\) on the concentration profile. The rising values of \(K_r\) suppress diffusion which lowers the concentration boundary layer.

Figure 13 depicts the effects of Schmidt number \(S_b\) on the concentration distribution. The rise in \(S_b\) causes reduction in the concentration profile. The higher the Schmidt number, the lower the mass diffusivity which is the reason for reduction in the concentration boundary layer thickness.

Figure 14 presents the influence of mass diffusivity parameter \(h_6\) on the concentration profile. One can observe that rise in mass diffusivity parameter \(h_6\) results in an increase of the concentration profile.

Figure 15 describes the influence of Peclet number \(P_e\) on the density of motile microorganism profile. The incremental values of Peclet number \(P_e\) cause reduction in motile microorganisms’ boundary layer thickness. The Peclet number appears in the study of transport processes. It measures the importance of convection over diffusion. For larger values of the Peclet number, the convection is dominant and diffusion is negligible which is happening here in the motile microorganisms’ boundary layer thickness.

Figure 16 investigates the impact of bioconvection Schmidt number \(S_b\) on the density of motile microorganism profile. It is shown that rising values of bioconvection Schmidt number \(S_b\) lower the boundary layer thickness of
the motile microorganism profile. In high values of Sb, the particles are giant which means these diffuse slowly.

Figures 17 and 18 are drawn to perceive the effect of mass diffusivity parameter $h_6$ and microorganism diffusivity parameter $h_8$. Increasing the values of mass diffusivity parameter and microorganism diffusivity parameter elevates the boundary layer thickness of the motile microorganism profile.

Table 1: Comparison of skin friction coefficient $-f''(0)$ for different values of $M$ when $Pr_\infty = 1$ and $K_p = \delta = \gamma = h_2 = h_4 = h_6 = h_8 = 0$.

| $M$  | Hayat et al. [50] | Mabood and Mastroberardino [51] | Amirsom et al. [48] | Shooting method | bvp4c |
|------|------------------|-------------------------------|---------------------|----------------|-------|
| 0    | 1.0000           | 1.0000008                     | 1.0000002           | 1.0000         | 1.0000 |
| 1    | 1.41421          | 1.4142135                     | 1.4142221           | 1.4142         | 1.4142 |
| 5    | 2.44948          | 2.4494897                     | 2.4494901           | 2.4495         | 2.4495 |
| 10   | 3.31662          | 3.3166247                     | 3.3166229           | 3.3166         | 3.3166 |
| 50   | 7.14142          | 7.1414284                     | 7.1414279           | 7.1414         | 7.1414 |
| 100  | 10.04987         | 10.049875                     | 10.049868           | 10.0499        | 10.0499 |
| 500  | 22.38302         | 22.383029                     | 22.383031           | 22.3830        | 22.3830 |
| 1000 | 31.63858         | 31.638584                     | 31.638578           | 31.6386        | 31.6386 |

Table 2: Comparison of skin friction coefficient $-f''(0)$ for different values of $M$ when $Pr_\infty = 1$ and $K_p = \delta = \gamma = h_2 = h_4 = h_6 = h_8 = 0$.

| $M$  | Hayat et al. [50] | Mabood and Mastroberardino [51] | Amirsom et al. [48] | FDM |
|------|------------------|-------------------------------|---------------------|-----|
| 0    | 1.0000           | 1.0000008                     | 1.0000002           | 1.0001 |
| 1    | 1.41421          | 1.4142135                     | 1.4142221           | 1.4142 |
| 5    | 2.44948          | 2.4494897                     | 2.4494901           | 2.4495 |
| 10   | 3.31662          | 3.3166247                     | 3.3166229           | 3.3166 |
| 50   | 7.14142          | 7.1414284                     | 7.1414279           | 7.1414 |
| 100  | 10.04987         | 10.049875                     | 10.049868           | 10.0499 |
| 500  | 22.38302         | 22.383029                     | 22.383031           | 22.3830 |
| 1000 | 31.63858         | 31.638584                     | 31.638578           | 31.6386 |

Table 3: Comparison of skin friction coefficient $-f''(0)$ for different values of $\delta$ when $Pr_\infty = 1$ and $K_p = M = \gamma = h_2 = h_4 = h_6 = h_8 = 0$.

| $\delta$ | Andersson [52] | Hamad et al. [53] | Amirsom et al. [48] | Shooting method | bvp4c |
|----------|----------------|------------------|---------------------|----------------|-------|
| 0        | 1.0000         | 1.00000000       | 1.00000000          | 1.0000         | 1.0001 |
| 0.1      | 0.8721         | 0.87208247       | 0.87204247          | 0.8721         | 0.8722 |
| 0.2      | 0.7764         | 0.77637707       | 0.77593307          | 0.7764         | 0.7765 |
| 0.5      | 0.5912         | 0.59119548       | 0.59119589          | 0.5912         | 0.5913 |
| 1.0      | 0.4302         | 0.43015970       | 0.43016000          | 0.4302         | 0.4303 |
| 2.0      | 0.2840         | 0.28397959       | 0.28398932          | 0.2840         | 0.2841 |
| 5.0      | 0.1448         | 0.14484019       | 0.14464013          | 0.1448         | 0.1449 |
| 10.0     | 0.0812         | 0.08124198       | 0.08124091          | 0.0812         | 0.0813 |
| 20.0     | 0.0438         | 0.04378834       | 0.04378790          | 0.0438         | 0.0439 |
| 50.0     | 0.0186         | 0.01859623       | 0.01857868          | 0.0186         | 0.0186 |
| 100.0    | 0.0095         | 0.00954997       | 0.00954677          | 0.0095         | 0.0096 |
Table 4: Numerical values of $N_{\infty}x, S_{\infty}x, \text{and } N_{\varepsilon}x$ for several values of involved parameters $Pr_{\infty}, Rd, Nb, Nt, Ec, s, Sc, Kr, Sb, Pe, h_{2}, h_{4}, h_{6}, \text{and } h_{8}$ with $A = 0.1, M = 0.5, Kp = 0.2, \delta = 0$, and $\gamma = 1$ (bvp4c).

| $Pr_{\infty}$ | $Rd$ | $s$ | $Ec$ | $Nb$ | $Nt$ | $Sc$ | $Kr$ | $Sb$ | $Pe$ | $h_{2}$ | $h_{4}$ | $h_{6}$ | $h_{8}$ | $bvp4c$ $- (1 + (4/3) Rd)d^{3}(0)$ | $bvp4c$ $- \varphi^{(0)}(0)$ | $bvp4c$ $- \chi^{(0)}(0)$ |
|--------------|------|-----|------|------|------|------|------|------|------|-------|-------|-------|-------|-----------------------------|-----------------------------|-----------------------------|
| 4            | 0.5  | 0.1 | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.4578                      | 1.6306                      | 2.0202                      |
| 5            | 0.513 | 1.6123 | 2.0097 | 2.0020 | 1.6018 | 2.0002 | 1.6165 | 2.0072 | 1.6330 | 2.0171 | 1.5947 | 2.0020 | 1.7713 | 2.1544                      | 2.0200                      | 2.1461                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.9198                      | 2.2889                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.6452                      | 2.0171                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.6505                      | 2.0170                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.6314                      | 2.0170                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.7580                      | 2.1461                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.7580                      | 2.1461                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 2.1563                      | 2.7886                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 2.7811                      | 3.4623                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.1072                      | 1.6118                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.5947                      | 2.0020                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 2.4438                      | 2.7096                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.1491                      | 1.6338                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 2.0765                      | 2.4069                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 2.6584                      | 2.9040                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.5947                      | 1.6129                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.5947                      | 1.7956                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.5947                      | 2.2000                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 4.6914                      | 2.0200                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.6336                      | 2.0454                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.6336                      | 2.0454                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.6336                      | 2.0454                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.6336                      | 2.0454                      |
| 6.8          | 0.5  | 0.1  | 0.1  | 0.2  | 5    | 0.2  | 2    | 1    | 0.1   | 0.1   | 0.1   | 0.1   | 0.5745                      | 1.6336                      | 2.0454                      |
Figure 1: Velocity profile $f'(\eta)$ for different $M$.

Figure 2: Velocity profile $f'(\eta)$ for different $K_p$.

Figure 3: Temperature profile $\theta(\eta)$ for different $Pr_{\infty}$.

Figure 4: Temperature profile $\theta(\eta)$ for different $R_d$. 
**Figure 5:** Temperature profile $\theta(\eta)$ for different $E_c$.

**Figure 6:** Temperature profile $\theta(\eta)$ for different $s$.

**Figure 7:** Temperature profile $\theta(\eta)$ for different $h_4$.

**Figure 8:** Temperature profile $\theta(\eta)$ for different $N_b$. 

\[ \text{Pr}_\infty = 6.8, K_p = N_t = K_r = 0.2, \text{Rd} = M = 0.5, \delta = 0, \\
A = N_b = E_c = h_2 = h_4 = h_6 = h_8 = 0.1, \text{Sc} = S_b = \text{Pe} = 5 \]
\[ \phi(\eta) \]

\[ \eta \]

\[ \gamma = 0 \ (N_b = 0.1) \]
\[ \gamma = 0 \ (N_b = 0.2) \]
\[ \gamma = 0 \ (N_b = 0.5) \]
\[ \gamma = 1 \ (N_b = 0.1) \]
\[ \gamma = 1 \ (N_b = 0.2) \]
\[ \gamma = 1 \ (N_b = 0.5) \]

\[ \text{Figure 9: Concentration profile } \phi(\eta) \text{ for different } N_b. \]

\[ \text{Figure 10: Temperature profile } \phi(\eta) \text{ for different } N_t. \]

\[ \text{Figure 11: Concentration profile } \phi(\eta) \text{ for different } K_r. \]
\[ \text{Pr}_\infty = 6.8, \text{Kr} = K_p = N_t = 0.2, \text{Rd} = M = 0.5, \delta = 0, \]
\[ s = A = E_c = N_b = h_2 = h_4 = h_6 = h_8 = 0.1, S_b = \text{Pe} = 5 \]

Figure 13: Concentration profile \( \phi(\eta) \) for different Sc.

\[ \gamma = 0 \ (\text{Pe} = 1) \]
\[ \gamma = 0 \ (\text{Pe} = 2) \]
\[ \gamma = 0 \ (\text{Pe} = 3) \]
\[ \gamma = 1 \ (\text{Pe} = 1) \]
\[ \gamma = 1 \ (\text{Pe} = 2) \]
\[ \gamma = 1 \ (\text{Pe} = 3) \]

Figure 15: Microorganisms’ profile \( \chi(\eta) \) for different Pe.

\[ \gamma = 0 \ (h_b = 0.1) \]
\[ \gamma = 1 \ (h_b = 0.1) \]
\[ \gamma = 0 \ (h_b = 0.5) \]
\[ \gamma = 1 \ (h_b = 0.5) \]
\[ \gamma = 0 \ (h_b = 0.9) \]
\[ \gamma = 1 \ (h_b = 0.9) \]

Figure 16: Microorganisms’ profile \( \chi(\eta) \) for different Sb.
5. Conclusion

The focus of the paper involves unsteady MHD flow of bio-nanofluid in a permeable medium taking thermal radiation and chemical reaction into account over a stretching sheet with variable thermophysical properties. The notable findings of the problem are outlined in the following [51]:

(i) The incremental values of Brownian motion parameter $N_b$, thermophoresis parameter $N_t$, thermal radiation parameter $R_d$, Eckert number $E_c$, and heat source parameter $s$ magnify the thermal boundary layer thickness, while an increase in Prandtl number $Pr_\infty$ causes reduction in the thermal boundary layer thickness.

(ii) The concentration boundary layer thickness rises for thermophoresis parameter $N_t$ and mass diffusivity parameter $h$, whereas it declines for higher values of the Brownian motion parameter $N_b$, Schmidt number $S_c$, and chemical reaction parameter $K_r$.

(iii) The increment in bioconvection Schmidt number $S_b$ and Peclet number $P_e$ reduces the boundary layer thickness of motile microorganisms, while the motile microorganisms’ boundary layer shows inverse behavior for mass diffusivity parameter $h$ and microorganism parameter $h$.

(iv) Graphs have been drawn with and without slip conditions. Difference can be clearly seen through graphs as the boundary layer thickness of the slip condition is different when compared without the slip flow case.

**Notations**

- $a > 0$: A constant ($s^{-1}$)
- $(u, v)$: Fluid velocities along and normal to the flow ($ms^{-1}$)
- $(x, y)$: Orthogonal Cartesian coordinates (m)
- $A$: Unsteadiness parameter
- $A_1$: Dimensionless parameter
- $\beta_m$: Applied magnetic field (Nm$^{-1}$ A$^{-1}$)
- $\mu$: Dynamic viscosity (Pas)
- $\rho$: Fluid’s viscosity (kgm$^{-3}$)
- $\sigma$: Fluid’s electrical conductivity (Sm$^{-1}$) (S is siemens)
- $M$: Dimensionless magnetic variable
- $K_p$: Porosity variable
- $T$: Fluid’s temperature (K)
- $T_w$: Constant wall temperature (K)
- $T_\infty$: Free-stream temperature (K)
- $k(T)$: Variable thermal conductivity (Wm$^{-1}$K$^{-1}$)
- $\alpha$: Thermal diffusivity (m$^2$s$^{-1}$)
- $\delta$: Slip parameter
- $C_p$: Heat capacity at constant pressure (Jkg$^{-1}$K$^{-1}$)
- $q_r$: Flux due to radiation (Wm$^{-2}$)
- $Q$: Internal heat generation/absorption
- $C_f$: Skin friction coefficient
- $N_u$: Local Nusselt parameter
- MHD: Magnetohydrodynamics
- PDEs: Partial differential equations
s: Heat source/sink parameter
Rd: Thermal radiation parameter
Pr_{eff}: Free-stream Prandtl number
D_B: Brownian coefficient (m^2s^{-1})
D_T: Thermophoretic coefficient (m^2s^{-1})
D_m: Diffusivity of microorganisms (m^2s^{-1})
D_h: Diffusivity coefficient (m^2s^{-1})
τ_e: Ratio of effective heat capacitance of the nanoparticle to the base fluid
(ρc)_p: Nanoparticle heat capacity (JK^{-1}m^3)
Nt: Thermophoresis parameter
C: Concentration
C_w: Concentration at the wall
C_{inf}: Ambient fluid concentration
N: Concentration of microorganisms
N_m: Microorganisms at the wall
N_{inf}: Microorganisms far from the wall
Le: Lewis number
Sb: Bioconvection Schmidt number
b: Chemotaxis constant (m)
w_μ^*: Maximum cell swimming speed (ms^{-1})
Pe: Peclet number
Sh_μ: Local Sherwood parameter
N_{μ*}: Local density parameter of the motile microorganisms
ODEs: Ordinary differential equations
FDM: Finite difference method.

Data Availability
No data were used to support this study.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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