Domain walls in a Chern-Simons theory

by

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Abstract

We study an Abelian Maxwell-Chern-Simons model in 2 + 1 dimensions which includes a magnetic moment interaction. We show that this model possesses domain wall as well as vortex solutions.

1 The model

Theories with gauge fields coupled to matter fields in (2 + 1) space dimensions present novel effects as compared to the (3+1) dimensional case. In planar systems the Chern-Simons term (CS) can supplement (sometimes replace) the Maxwell term in the action for the gauge field $[1]$. Additionally, the covariant coupling to the scalar field can be modified by the inclusion of an extra term, which is interpreted as an scalar magnetic moment interaction $[2, 3]$. This is enforced without spoiling neither the covariance nor the gauge invariance of the theory. The presence of these two terms produce interesting new effects. We mention in particular, that Chern-Simons solitons carry electric charge as well as magnetic flux; in addition they possess fractional spin so they behave as anyon-like objects $[4]$.

We consider a scalar $QED$ model in 2 + 1 dimensions with the addition of the Chern-Simons term and an anomalous magnetic interaction, it is described by the following effective Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa}{4} \epsilon^{\mu\nu\alpha} A_\mu F_{\nu\alpha} + \frac{1}{2} |D_\mu \phi|^2 - \frac{1}{2} m^2 |\phi|^2,$$

where $\kappa$ is the topological mass and we select for the scalar field a simple $\phi^2$ potential. The covariant derivative includes both the the usual minimal coupling plus the magnetic moment interaction

$$D_\mu = \partial_\mu - ie A_\mu - i \frac{g}{4} \epsilon_{\mu\nu\alpha} F^{\nu\alpha} \equiv \partial_\mu - ie A_\mu - i \frac{g}{2} F_\mu,$$

with $g$ the magnetic moment and we have defined the dual field $F_\mu = \epsilon_{\mu\nu\alpha} F^{\nu\alpha}$. The possibility of including a magnetic moment for scalar particles is a characteristic property of the space dimensionality.

There is a limit in which the gauge field equations reduce from second- to first-order differential equations similar to those of the pure CS theory. Indeed, if the relation $\kappa = -\frac{2e}{g}$
holds the equation of motion for the gauge field is given by

$$\kappa F_\mu = J_\mu,$$  \hspace{1cm} (3)

where \( J_\mu \) is the conserved Noether current. This equation of motion implies that any object carrying magnetic flux (\( \Phi_B \)) must also carry electric charge (\( Q \)), with the two quantities related as \( Q = \kappa \Phi_B \). If we work within the pure CS limit a self-dual Maxwell-Chern-Simons gauge model can be constructed \[5\]. This is attained if the scalar potential has the particular \( \phi^2 \) form and the scalar mass is made equal to the CS mass; the energy obeys a Bogomol'nyi lower bound that is saturated by the fields that satisfy the self-duality equations. The vortex solutions of this self-dual model have been studied in detail \[6, 7\]. Additionally, it is also possible to find domain wall configurations for this model.

## 2 Domain walls

The present \( \phi^2 \) theory possesses a single minimum, yet it is possible to find one dimensional soliton solutions of the domain wall type. Consider a one dimensional structure depending only on the \( x \) variable, both at \( x \to \infty \) and \( x \to -\infty \) the scalar field should vanish. However, there can be an intermediate region where \( \phi \neq 0 \), \( i.e., \) a region of false vacuum. The maximum of \( \phi \) determines the position of the wall. The domain wall carries both magnetic flux and electric charge per unit length. Seeking a domain wall solution parallel to the \( y \)-axis, the translational invariance of the theory implies that all the fields depend only on \( x \). By an appropriate gauge transformation the scalar field is made real everywhere \( \phi = (\kappa/e)f \) and the gauge potential \( A \) is selected along the \( y \) axis. Using the pure CS equations of motion (3), the expression for the energy can be written as

$$E = \frac{1}{2} \int d^2r \left[ \left( \frac{(1-f^2)^{1/2}}{f} \frac{dA_y}{dx} \right) + \frac{\kappa f A_y}{(1-f^2)^{1/2}} \right]^2 + \frac{\kappa^2}{e^2} \left( \frac{df}{dx} \pm mf \right)^2 \pm \frac{m \kappa^2}{e^2} \left( \frac{dA_y^2}{dx} \right) \pm \kappa dA_y^2 \right].$$  \hspace{1cm} (4)

The boundary conditions for the scalar field are \( f(\infty) = f(-\infty) = 0 \). The magnetic flux per unit length (\( \gamma \)) is given by \( \gamma = A_y(\infty) - A_y(-\infty) \), so \( A_y(\infty) \neq A_y(-\infty) \) is required in order to get a non-vanishing magnetic flux. A configuration is sought which has a definite symmetry with respect to the position \( X \) of the domain wall, then \( A_y(\infty) = -A_y(-\infty) \equiv \gamma/2 \) is selected.

The static solution is obtained minimizing the energy per unit length with \( \gamma \) fixed. The boundary conditions cannot be satisfied if the same upper (or lower) signs in (4) are used for all \( x \). Rather, the upper signs in the region to the right of the domain wall (\( x > X \))
are selected, whereas for \( x < X \) we take the lower signs. With this selection the minimum energy per unit length becomes

\[
E = \frac{\kappa^2 m}{e^2} f_0^2 + \frac{\kappa}{4} \gamma^2,
\]

where \( f_0 \equiv f(X) \). This result is obtained provided that the fields satisfy the following equations:

\[
\frac{df}{dx} = \pm mf,
\]
\[
\frac{dA_y}{dx} = \pm \frac{\kappa f^2}{(1 - f^2)} A_y,
\]

where the upper (lower) sign must be taken for \( x > X \) (\( x < X \)). These equations are easily integrated to give

\[
f(x) = e^{-m|x-X|},
\]
\[
A_y(x) = \text{sgn}(x-X) \frac{\gamma}{2} \left(1 - e^{-2m|x-X|}\right)^{\kappa/2m}.
\]

This is a domain wall configuration localized at \( x = X \) with a width of order \( 1/m \). The solution to the first equation in (6) does not restrict the value of \( f_0 \). However, \( f_0 = 1 \) has to set so the gauge field be continuous everywhere. The anti-kink configuration is obtained by simply reversing the signs of the fields in (6).

The domain wall carries a magnetic flux and charge per unit length given by \( \gamma \) and \(-\kappa \gamma\) respectively. The magnetic field is given by

\[
B = \frac{\kappa \gamma}{2} e^{-m|x-X|} \left(1 - e^{-2m|x-X|}\right)^{\kappa/2m-1}.
\]

Notice that for \( \kappa < 2m \) the magnetic field is concentrated near \( x = X \) and falls off rapidly away from the wall. Instead for \( \kappa \geq 2m \) the magnetic field vanishes at \( x = X \) and the profile of \( B \) is doubled peaked with maximums at \( x = X \pm \frac{\ln \left( \kappa^2/2m\right)}{m} \).

To investigate the conditions required to have stable configurations consider the decay of the domain wall by the emission of scalar particles. Because of charge conservation a decaying wall should radiate \( \kappa \gamma/e \) quanta of scalar particles per unit length. Thus, the energy of the elementary excitations per unit length at rest will be \( \kappa \gamma m/e \). The stability condition requires this energy to be bigger than the soliton energy in (5):

\[
\frac{\kappa^2 m}{e^2} f_0^2 + \frac{\kappa}{4} \gamma^2 \leq \frac{\kappa \gamma m}{e}.
\]
This condition implies $\kappa < m$ and also yields both an upper and a lower bound to the values of the magnetic flux per unit length:

$$\gamma^- \leq \gamma \leq \gamma^+, \text{ where}$$

$$\gamma^\pm = \frac{2m}{e} \left[ 1 \pm \sqrt{1 - \frac{\kappa}{m}} \right],$$

(10)

The domain wall solutions in (7) can be utilized to study rotationally symmetric configurations of the vortex type. The vortex configurations simplify in the large vorticity limit. In this large $-n$ limit the vortex can be considered as a ring of large radius $R \approx n/\kappa$ and thickness of order $1/m$ separating two regions of vacuum. The magnetic flux is concentrated within this domain of width $\sim 1/m$ where a region of false vacuum ($\phi \neq 0$) is trapped. In this limit $R \gg 1/m \sim 1/\kappa$, and the fields near the ring is well approximated by the domain wall solution [7].

This model raises a number of interesting questions for further investigation. In particular a complete description of the multisoliton solution deserves to be clarified. It may also be interesting to investigate the properties of the Chern-Simons vortices and domain walls upon quantization.

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References

[1] S. Deser and R. Jackiw and S. Templeton, Ann. Phys. 140 (1982) 372.

[2] J. Stern, Phys. Lett. B. 265 (1991) 119.

[3] I. Kogan, Phys. Lett. B 262 (1991) 83.

[4] R. Jackiw and S. Y. Pi, Prog. Theor. Phys. Suppl. 107 (992) 1.

[5] M. Torres, Phys. Rev. D. 46 (1992) 2295.

[6] M. Torres, Phys. Rev. D. 51 (1995) 4533.

[7] A. Antillón and J. Escalona and M. Torres, Phys. Rev. D. 55 (1997) 6327.