Parallelism of quantum computations from prequantum classical statistical field theory (PCSFT)

Andrei Khrennikov
School of Mathematics and Systems Engineering
University of Växjö, S-35195, Sweden

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Abstract

This paper is devoted to such a fundamental problem of quantum computing as quantum parallelism. It is well known that quantum parallelism is the basis of the ability of quantum computer to perform in polynomial time computations performed by classical computers for exponential time. Therefore better understanding of quantum parallelism is important both for theoretical and applied research, cf. e.g. David Deutsch [1]. We present a realistic interpretation based on recently developed prequantum classical statistical field theory (PCSFT). In the PCSFT-approach to QM quantum states (mixed as well as pure) are labels of special ensembles of classical fields. Thus e.g. a single (!) “electron in the pure state” $\psi$ can be identified with a special “electron random field,” say $\Phi_{\psi}(\phi)$. Quantum computer operates with such random fields. By one computational step for e.g. a Boolean function $f(x_1, ..., x_n)$ the initial random field $\Phi_{\psi_0}(\phi)$ is transformed into the final random field $\Phi_{\psi_f}(\phi)$ “containing all values” of $f$. This is the objective of quantum computer’s ability to operate quickly with huge amounts of information – in fact, with classical random fields.

Keywords: quantum parallelism, prequantum classical statistical field theory, random field, ensemble interpretation, Copenhagen interpretation, random field Copenhagen interpretation, gap between classical and quantum parallelism.
1 Introduction

Recent tremendous development of quantum information theory and especially quantum computing and cryptography stimulated research in foundations of quantum mechanics, see, e.g., [2] – [18], problems which have been of merely theoretical (or even philosophic) interest became extremely important for understanding of processing of quantum information. Nowadays quantum foundations have important implications for engineering and nanotechnology. In this note we discuss one exciting problem of quantum computing, namely, quantum parallelism. It is well known that quantum algorithms could solve in polynomial time some problems which need exponential time for known classical algorithms. The ability to reduce essentially computational time is one of the main motivations for development of quantum computers. This ability is closely related to one of the main problems in quantum foundations, namely interpretation of superposition of quantum states. In quantum computing the ability to operate with superpositions is called quantum parallelism.

We emphasize that any realistic interpretation of quantum parallelism which is accepted by the majority of the quantum information community has not yet been provided. Of course, one may just follow the orthodox Copenhagen interpretation. But it is not a realistic one. Some creators of quantum computing were not satisfied by the orthodox Copenhagen interpretation of quantum parallelism. An interesting attempt to provide a kind of realistic understanding of quantum parallelism was done by Deutsch[1] who used the many worlds interpretation of quantum computing. In the many worlds approach quantum parallelism can be understood in the realistic way as classical parallelism in many worlds. However, the many worlds interpretation is not (at least yet) commonly accepted. The quantum computing majority would prefer the orthodox Copenhagen interpretation.

Recently I proposed a new realistic ground for QM, see [19]-[23]. It was shown that QM can be represented as an asymptotic projection of classical statistical mechanics with infinite dimensional phase space. By representing this space as

$$\Omega = L_2(\mathbb{R}^3) \times L_2(\mathbb{R}^3)$$

we represent its points by classical vector fields $\phi(x) = (q(x), p(x))$. In our model - prequantum (classical statistical) field theory (PCSFT) -
these classical field might be considered as a kind of hidden variables. The corresponding complex representation of such vector fields is given Riemann-Silberstein vector (which is used for the complex representation of the classical electrodynamics; in particular, Maxwell equations in empty space are transformed into Schrödinger’s type equation):

$$\phi(x) = q(x) + ip(x).$$

The correspondence rules between PCSFT and QM differ from rules which are typically considered in theories with hidden variables, e.g., rules which are formalized in known “NO-GO” theorems (von Neumann, Kochen-Specker, Bell,...). A classical field $\phi(x)$ of PCSFT does not determine the values of conventional quantum observables (as it should be in a theory with hidden variables).

In the PCSFT-framework we define so called prequantum variables which are given by functionals $f(\phi)$ of classical fields. Corresponding (conventional) quantum observable $\hat{A}$ is given by the second derivative of a functional ("classical variable") $f(\phi)$. We set $A \equiv T(f) = f''(0)/2$. On the one hand, this rule for correspondence between prequantum variables and quantum observables, $f \rightarrow T(f)$, satisfies an important assumption of the von Neumann “NO-GO” theorem:

$$T(f_1 + \ldots + f_n) = \hat{A}_1 + \ldots + \hat{A}_n,$$

where operators $\hat{A}_1, \ldots, \hat{A}_n$ need not commute, [24]. On the other hand, the “spectral postulate” is violated: the ranges of values of a prequantum variable $f(\phi)$ and the corresponding quantum observable $\hat{A} \equiv T(f)$ do not coincide.

Nevertheless, average with respect to a classical random field $\Phi(\phi)$ (here $\phi \in \Omega$ plays the role of the random parameter):

$$<f>_{\Phi} \equiv Ef(\Phi(\phi))$$

can be approximated by quantum average given by the von Neumann trace-formula, [24]:

$$<A>_{\rho} \equiv Tr\rho \hat{A}.$$

Here the operator $\rho$ is obtained by normalization of the covariance operator of the random field $\Phi(\phi)$. It has all features of the von Neumann density operator, [24]. The quantum average $<A>_{\rho}$ gives the first order approximation of classical field average $<f>_{\Phi}$. The
small parameter of this asymptotic expansion of $< f >_{\Phi}$ is given by
dispersion of the random field:

$$\kappa = E\|\Phi(\phi)\|^2,$$

where $\| \cdot \|$ is the norm on the Hilbert space $\Omega$.

To distinguish PCSFT from conventional theories with hidden
variables, we shall call classical fields $\phi \in \Omega$ \textit{ontic hidden variables}, cf. [6].
“Ontic” does not mean that effects such variables could not be mea-
sured in principle. However, to find effects of “prequantum fields” one
should develop new measurement technologies corresponding to “pre-
quantum variables” given by functionals of classical fields (electron
field, proton field and so on). So, for such more advanced technologies
they will become observables.

We point out that a similar problem is present in some other mod-
els with hidden variables, but typically not so much attention is paid
to it. For example, in one of the most known models of this type –
Bohmian mechanics – momentum is not the conventional momentum
of QM. I would consider Bohmian mechanics as a model with “semi-
ontic” hidden variables. Position $q$ is the conventional hidden variable,
but momentum $p$ is the ontic one.

In our approach quantum states (mixed as well as pure) are images
of ensembles of classical fields. They are mathematically described by
measures $\mu$ on $\Omega$. Such ensembles can be considered as \textit{random fields},
see also [25]. Therefore in PCSFT quantum computations can be
represented as processing of random fields. Quantum parallelism is
classical parallelism, but for ensembles of fields.

It might be that we found the main source of tremendous ability
of quantum computer to operate with huge amounts of information.
In contrast to classical computer, quantum computer operates with
infinite dimensional objects composing infinitely large ensembles.

One may say that operations with quantum bits are performed in
the finite dimensional Hilbert spaces. However, the same arguments
that I. V. Volovich presented in [26] for quantum cryptography should
also be applied to quantum computing. Quantum computer operates
in physical space – by our model with physical fields which are infinite
dimensional objects.
1.1 Copenhagen interpretations

The main distinguishing feature of the Copenhagen interpretation is association of the wave function $\psi$ – pure quantum state – with an individual quantum system. For example, one (e.g., Heisenberg, Pauli, Dirac, Fock or Landau\(^1\)) would speak about the wave function of the electron or in other words about the electron having the concrete pure state $\psi$.

Opposite to such an individual interpretation, by the ensemble interpretation (e.g., Einstein and Ballentine, [28], [29]) the wave function $\psi$ is associated not with an individual quantum system, but with an ensemble of quantum systems prepared under the same complex of experimental physical conditions – preparation procedure, [28], [29], [30]. By the ensemble interpretation there is no difference between pure and mixed quantum states. Pure quantum states represent ensembles of systems as well as mixed states.

By the Copenhagen interpretation QM is complete. One could not introduce a mathematical model in which quantum systems are described by hidden variables determining the values of quantum observables. By the ensemble interpretation such variables can be introduced. The wave function $\psi$ is just a mathematical symbol for an ensemble of quantum systems and each system $s$ in this ensemble can be characterized by a value of the hidden variable $\lambda \equiv \lambda_s$.

Typically the Copenhagen interpretation is considered as a “NO-GO”-interpretation. It seems that its completeness does not permit a more detailed description of physical reality than the one given by QM. On the other hand, the ensemble interpretation does not claim that QM is complete and that the QM description is the final one.

*Is the Copenhagen interpretation really a “NO-GO”-interpretation?*

It is correct, but only to some extent. In fact, we can proceed in the following way.

Let us interpret quantum particles as classical random fields. For example, any electron is a classical random field\(^2\) $\Phi(x, \omega)$, where $x \in \mathbb{R}^3$ and $\omega$ is a random parameter. In our approach the wave function $\psi$ (describing by the Copenhagen interpretation a pure quantum state $\psi$).

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\(^1\)It is not clear at all whether Bohr would assign himself [27] to such an interpretation, see Plotnitsky [15]–[18] for discussion.

\(^2\)Thus in such a model we have a variety of fields corresponding to different “quantum particles”, e.g., the electron field or the proton field.
state) is considered as some characteristic of the corresponding field \( \Phi(x, \omega) \). As usual in random field theories, one can choose the random parameter \( \omega = \phi \in \Omega \). We can not introduce a conventional hidden variable associated with the electron and determining values of quantum observables. The concrete value is the result of interaction of the random field with the corresponding measuring device. An individual fluctuation \( \Phi(x, \omega_0) \) (i.e., for the fixed \( \omega_0 \)) does not determine this value. As was pointed out in introduction it is more natural to call prequantum fields ontic hidden variables.

We remark that at each preparation act a preparation procedure produces not a single fluctuation of e.g. the electron field, but an ensemble of electron fields. Such an ensemble has the label “electron” in QM.

This interpretation we call the \textit{random field Copenhagen interpretation}.

The simplest model of such type can be obtained by considering two time scales, \[31\]:

a) a quick time scale (“prequantum time scale”);

b) slow time scale – the scale of measurements (“quantum time scale”).

Denote quick and slow times by symbols \( s \) and \( t \), respectively. A random field describes fluctuations on the prequantum time scale. Such fluctuations are not visible at the quantum time scale. Measurements are averages with respect to \( s \)-time. An instant of quantum (laboratory, physical) time correspond to a huge interval of prequantum time.

In this model it is clear why prequantum observables are different from quantum ones. These are two classes of observables corresponding to two different time scales.

The main problem for justification of the random field Copenhagen interpretation is to create a random field model which would couple in a natural way classical random field averages with averages given by the QM-formalism, namely, by von Neumann’s trace formula:

\[
< \hat{A} >_\rho = \text{Tr} \rho \hat{A}.
\]

\[3\] By following Bohr \[27\] I consider QM as theory of measurements, see e.g. Plotnitsky \[15\]–\[18\] for details. Therefore it is natural for me to call the time scale of measurements the quantum time scale. I understood well that this terminology might be misleading, since many authors (especially in quantum cosmology and string theory) use the terminology the quantum time scale for the Planck time scale.
2 Prequantum classical statistical field theory

We define “classical statistical models” in the following way: a) physical states \( \omega \) are represented by points of some set \( \Omega \) (state space); b) physical variables are represented by functions \( f : \Omega \to \mathbb{R} \) belonging to some functional space \( V(\Omega) \); c) statistical states are represented by probability measures on \( \Omega \) belonging to some class \( S(\Omega) \); d) the average of a physical variable (which is represented by a function \( f \in V(\Omega) \)) with respect to a statistical state (which is represented by a probability measure \( \rho \in S(\Omega) \)) is given by

\[
< f >_\rho \equiv \int_\Omega f(\phi) d\rho(\phi).
\]

(1)

A classical statistical model is a pair \( M = (S, V) \).

The conventional quantum statistical model with the complex Hilbert state space \( \Omega_c \) is described in the following way: a) physical observables are represented by operators \( A : \Omega_c \to \Omega_c \) belonging to the class of continuous self-adjoint operators \( \mathcal{L}_s \equiv \mathcal{L}_s(\Omega_c) \); b) statistical states are represented by von Neumann density operators (the class of such operators is denoted by \( \mathcal{D} \equiv \mathcal{D}(\Omega_c) \)); d) the average of a physical observable (which is represented by the operator \( A \in \mathcal{L}_s(\Omega_c) \)) with respect to a statistical state (which is represented by the density operator \( D \in \mathcal{D}(\Omega_c) \)) is given by von Neumann’s formula:

\[
< A >_D \equiv \text{Tr} DA
\]

(2)

The quantum statistical model is the pair \( N_{\text{quant}} = (\mathcal{D}, \mathcal{L}_s) \).

We are looking for a classical statistical model \( M = (S, V) \) which will provide “dequantization” of the quantum model \( N_{\text{quant}} = (\mathcal{D}, \mathcal{L}_s) \). By dequantization we understand constructing of a classical statistical model such that averages given by this model can be approximated by quantum averages. Approximation is based on the asymptotic

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4 We recall that classical statistical mechanics on the phase space \( \Omega_{2n} = \mathbb{R}^n \times \mathbb{R}^n \) gives an example of a classical statistical model. But we shall not be interested in this example in our further considerations. We shall develop a classical statistical model with an infinite-dimensional phase-space.

5 Of course, discontinuous (unbounded) operators are important in QM. However, as was pointed by von Neumann [24], it is always possible to restrict consideration to continuous operators, since discontinuous ones can be approximated by continuous ones.
expansion of classical averages with respect to a small parameter. The
main term of this expansion coincides with the corresponding quantum
average.

We choose the phase space $\Omega = Q \times P$, where $Q = P = H$ and $H$
is the real (separable) Hilbert space. We consider $\Omega$ as the real Hilbert
space with the scalar product $(\phi_1, \phi_2) = (q_1, q_2) + (p_1, p_2)$. We denote
by $J$ the symplectic operator on $\Omega : J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Let us consider
the class $\mathcal{L}_{\text{symp}}(\Omega)$ of bounded $\mathbf{R}$-linear operators $A : \Omega \to \Omega$ which
commute with the symplectic operator:

$$AJ = JA.$$ (3)

This is a subalgebra of the algebra of bounded linear operators $\mathcal{L}(\Omega)$.
We also consider the space of $\mathcal{L}_{\text{symp}, s}(\Omega)$ consisting of self-adjoint op-
erators.

By using the operator $J$ we can introduce on the phase space $\Omega$ the
complex structure. Here $J$ is realized as $-i$. We denote $\Omega$ endowed
with this complex structure by $\Omega_c : \Omega_c \equiv Q \oplus iP$. We shall use it
later. At the moment consider $\Omega$ as a real linear space and consider
its complexification $\Omega^C = \Omega \oplus i\Omega$.

Let us consider the functional space $\mathcal{V}_{\text{symp}}(\Omega)$ consisting of func-
tions $f : \Omega \to \mathbf{R}$ such that: a) the state of vacuum is preserved:
$f(0) = 0$; b) $f$ is $J$-invariant: $f(J\Phi) = f(\Phi)$; c) $f$ can be extended
to the analytic function $f : \Omega^C \to \mathbf{C}$ having the exponential growth:
$|f(\phi)| \leq c_f e^{r_f \|\phi\|}$ for some $c_f, r_f \geq 0$ and for all $\phi \in \Omega^C$.

The following trivial mathematical result plays the fundamental role in establishing classical $\rightarrow$ quantum correspondence: Let $f$ be a
smooth $J$-invariant function. Then $f''(0) \in \mathcal{L}_{\text{symp}, s}(\Omega)$. In particular,
a quadratic form is $J$-invariant iff it is determined by an operator
belonging to $\mathcal{L}_{\text{symp}, s}(\Omega)$.

We consider the space statistical states $S_{G, \text{symp}}^\kappa(\Omega)$ consisting of
measures $\rho$ on $\Omega$ such that: a) $\rho$ has zero mean value; b) it is a Gauss-
ian measure; c) it is $J$-invariant; d) its dispersion has the magnitude $\kappa$.
Thus these are $J$-invariant Gaussian measures such that

$$\int_\Omega \Phi d\rho(\phi) = 0 \text{ and } \sigma^2(\rho) = \int_\Omega \|\Phi\|^2 d\rho(\phi) = \kappa, \kappa \to 0.$$  

Such measures describe small Gaussian fluctuations. The following
trivial mathematical result plays the fundamental role in establishing

\footnote{The vacuum state is such a classical field which amplitude is zero at any point $x$.}
classical → quantum correspondence: Let a measure $\rho$ be $J$-invariant. Then its covariation operator $B = \text{cov} \rho \in \mathcal{L}_{\text{symp},s}(\Omega)$. Here

$$(B y_1, y_2) = \int (y_1, \Phi)(y_2, \Phi) d\rho(\Phi).$$

We now consider the complex realization $\Omega_c$ of the phase space and the corresponding complex scalar product $\langle \cdot, \cdot \rangle$. We remark that the class of operators $\mathcal{L}_{\text{symp}}(\Omega)$ is mapped onto the class of $\mathbb{C}$-linear operators $\mathcal{L}(\Omega_c)$. We also remark that, for any $A \in \mathcal{L}_{\text{symp},s}(\Omega)$, real and complex quadratic forms coincide: $(A\Phi, \phi) = \langle A\Phi, \phi \rangle$. We also define for any measure its complex covariation operator $B^c = \text{cov}^c \rho$ by

$$\langle B^c y_1, y_2 \rangle = \int \langle y_1, \Phi \rangle \langle \Phi, y_2 \rangle d\rho(\Phi).$$

We remark that for a $J$-invariant measure $\rho$ its complex and real covariation operators are related as $B^c = 2B$. As a consequence, we obtain that any $J$-invariant Gaussian measure is uniquely determined by its complex covariation operator. As in the real case [1], we can prove that for any operator $A \in \mathcal{L}_{\text{symp},s}(\Omega)$:

$$\int_{\Omega} \langle A\Phi, \phi \rangle d\rho(\phi) = \text{Tr} \text{cov}^c \rho A.$$ We point out that the trace is considered with respect to the complex inner product.

We consider now the one parameter family of classical statistical models:

$$M^\kappa = (S^\kappa_{G,\text{symp}}(\Omega), \mathcal{V}_{\text{symp}}(\Omega)), \kappa \geq 0,$$

By making in the Gaussian infinite-dimensional integral the change of variables (field scaling):

$$\Phi = \sqrt{\kappa}\Psi,$$

we obtain the following result [23]:

Let $f \in \mathcal{V}_{\text{symp}}(\Omega)$ and let $\rho \in S^\kappa_{G,\text{symp}}(\Omega)$. Then the following asymptotic equality holds:

$$\langle f \rangle_{\rho} = -\frac{\kappa}{2} \text{Tr} D^c f''(0) + O(\kappa^2), \kappa \to 0,$$

where the operator $D^c = \text{cov}^c \rho/\kappa$. Here

$$O(\kappa^2) = \kappa^2 R(\kappa, f, \rho),$$

where $|R(\kappa, f, \rho)| \leq cf \int_{\Omega} e^{\|\Psi\|} d\rho_{D^c}(\Psi)$. 

9
Here $\rho_{Dc}$ is the Gaussian measure with zero mean value and the complex covariation operator $Dc$.

We see that the classical average (computed in the model $M^\kappa = (S^\kappa G, \mathcal{V}_{\text{symp}}(\Omega))$ by using the measure-theoretic approach) is coupled through (6) to the quantum average (computed in the model $N_{\text{quant}} = (D(\Omega_c), \mathcal{L}_s(\Omega_c))$ by the von Neumann trace-formula).

The equality (6) can be used as the motivation for defining the following classical $\rightarrow$ quantum map $T$ from the classical statistical model $M^\kappa = (S^\kappa G, \mathcal{V}_{\text{symp}})$ onto the quantum statistical model $N_{\text{quant}} = (D, \mathcal{L}_s)$:

$$T : S^\kappa G_{\text{symp}}(\Omega) \rightarrow D(\Omega_c), \quad D^c = T(\rho) = \text{cov}^c \rho \kappa$$

(8)

(the Gaussian measure $\rho$ is represented by the density matrix $D^c$ which is equal to the complex covariation operator of this measure normalized by $\kappa$);

$$T : \mathcal{V}_{\text{symp}}(\Omega) \rightarrow \mathcal{L}_s(\Omega_c), \quad A_{\text{quant}} = T(f) = \frac{1}{2} f''(0).$$

(9)

Our previous considerations can be presented in the following form

**Beyond QM Theorem.** The one parametric family of classical statistical models $M^\kappa = (S^\kappa G, \mathcal{V}_{\text{symp}}(\Omega))$ provides dequantization of the quantum model $N_{\text{quant}} = (D(\Omega_c), \mathcal{L}_s(\Omega_c))$ through the pair of maps (8) and (9). The classical and quantum averages are coupled by the asymptotic equality (6).

### 3 The random field Copenhagen interpretation for PCSFT

In the series of papers [19]-[23] I used the ensemble interpretation (in the spirit of Einstein, Margenau, Ballentine) to couple my model with ontic hidden variables PCSFT with QM. By this interpretation a classical statistical state $\mu$ of a prequantum theory represents an ensemble of hidden variables, say $E_\mu$. In our case the theory with (ontic) hidden variables is PCSFT and hidden variables are classical fields, $\phi(x), x \in \mathbb{R}^3$. In [19]-[23] I did not proceed carefully and I did not distinguish conventional hidden variables form the ontic ones, see the discussion in introduction.
If a quantum system, e.g., an electron, has the quantum state
\( \rho = T(\mu) = \text{cov}^\mu \mu \), then we assumed that, in fact, his state is given
by the fixed field \( \phi \in E_\mu \). By considering an ensemble of electrons
prepared in the state \( \rho \) we reproduce the ensemble of classical fields
\( E_\mu \).

Recently I found that such an ensemble interpretation is not the
only possible interpretation for coupling of PCSFT with QM. Surpris-
ingly PCSFT-QM coupling could also be interpreted in the Copen-
hamen’s way.

We recall that by the orthodox Copenhagen interpretation a pure
quantum state \( \psi, \| \psi \| = 1 \), describes an \textit{individual quantum system}.

This interpretation of QM we combine with PCSFT in the follow-
ing way. Suppose that \( \mu \equiv \mu_\psi \) has the covariance operator

\[
B = \kappa \psi \otimes \psi.
\]

In the new interpretation \( \mu_\psi \) represents a random field: "mixture of
fields \( \phi \in \Omega \) with weights \( \mu_\psi (\phi) \)."

We emphasize that there is considered mixture and not superpo-
sition. We denote this random field by \( \Phi_\psi (\phi) \); here \( \phi \in \Omega \) plays the
role of a random parameter.

We take a quantum system having the QM-state \( \psi \), e.g., an elec-
tron in the state \( \psi \). We now consider \( \psi \) as the symbol denoting the
random field \( \Phi_\psi (\phi) \). This is nothing else than \textit{random field Copen-
hamen interpretation}. By such an interpretation each quantum system
is nothing else than a mixture of classical fields\footnote{There is no problem with the superposition principle. It holds true for any classical
prequantum field.} At the moment (e.g., due to technological problems) we are not able to distinguish those
fields. By identifying a quantum system with a random field we ex-
plain the origin of quantum randomness. Opposite to von Neumann
\cite{24}, we do not consider quantum randomness as irreducible.

We remark that if one uses the \textit{random field Copenhagen inter-
pretation} then quantum randomness is not reduced to randomness for
an ensemble of quantum systems, e.g., electrons. Nevertheless, in our
approach quantum randomness is reduced to ensemble randomness,
namely, to ensembles of classical fields.
4 Quantum parallelism

The main distinguishing feature of quantum computation, see, e.g., Simon’s algorithm, is the possibility to prepare the quantum state

$$
\psi_0 = \frac{1}{2^{n/2}} \sum_x |x>,
$$

(10)

containing all possible values of the argument $x$, and then to transform the state $\psi_0$ into the quantum state

$$
\psi_f = \sum_x |x > |f(x)>,
$$

(11)

containing all values of $f$. The possibility to create the state $\psi_f$ by one step of quantum computations (by using oracal $U_f$) implies the possibility to perform on quantum computer in polynomial time calculations which are done in nonpolynomial time on classical computer.

Typically the difference between quantum parallelism and classical parallelism is emphasized. For example, in the book of A. S. Holevo [32], he pointed out that all values $f(x)$ are present in $\psi_f$ in the latent form and one should not identify this latent presence with the result of parallel computations on a classical computer. (In the latter case there are really produced all values $f(x).$)

In PCSFT the gap between quantum parallelism and classical parallelism is essentially less. By the random field Copenhagen interpretation states $\psi_0$ and $\psi_f$ are symbols denoting random fields $\Phi_{\psi_0}$ and $\Phi_{\psi_f}$. Therefore all values of the argument $x$ are really present in the ensemble of classical fields $\Phi_{\psi_0}(\phi), \phi \in \Omega$. Oracle $U_f$ really transfers these values into corresponding values of $f$ which are all contained in the random field $\Phi_{\psi_f}(\phi), \phi \in \Omega$.

We remark that by PCSFT, see [19]–[23],

$$
\Phi_\psi(U_t \Phi) = \Phi_{U_t \psi}(\phi),
$$

for any one parametric group of unitary operators $U_t$ and hence, in particular, for any unitary operator $U_f$ representing quantum computation.

The crucial difference from the classical parallelism is that we are not able to extract all these values from the final random field. A measurement destroys the structure of a random field. Therefore, to repeat this measurement, we should produce a new random field.
Conclusion. Quantum parallelism can be interpreted in the realistic way in the framework of PCSFT. By the random field Copenhagen interpretation this is parallelism of computations over ensembles of classical fields - random fields.

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