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A summary of the SOTANCP3 workshop

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Abstract. This text summarizes the talks presented at the 3rd International Workshop on State of the Art in Nuclear Cluster Physics (SOTANCP3) held in Yokohama (Japan) from 26 to 30 May 2014. Some personal opinions are also expressed on two much debated topics: the $^{12}$C spectrum in the continuum and conflicting interpretations of cluster wave functions.

1. Introduction
Summarizing the talks presented at a conference is of course an impossible task, for several reasons. Physical reasons are the limited time allowed to a summary and the existence of parallel sessions which would require some kind of quantum nonlocality for the person in charge of this summary. Psychological reasons are bias introduced by personal tastes and, above all, a limited field of expertise (specially the limited experimental knowledge of a theorist).

Anyway, I tried to find a reasonable first approximation. I thus start with some statistics that reveal some properties of the field. Then comes an attempt to answer the question “What is a cluster?” based on the notions of cluster encountered in the various talks. I also give my opinion on two topics which led to hot debates and which are of special interest for me. A brief summary follows.

Because of space limitations, talks are referred to just by the name of the presenter.

2. Some statistics
The SOTANCP3 workshop has attracted more than 130 physicists to Yokohama. About 40% of them came from abroad.

In 16 plenary sessions, 56 invited talks were presented. Among them, 60% covered theoretical aspects and 40% covered experimental aspects. As the organizers rightly decided to have a maximum of oral presentations, parallel (or clusterized?) sessions were organized. The three parallel sessions allowed 35 oral contributions, 80% concerning theory and 20% experiment. Among a dozen posters, about 90% concerned theory.

These statistics lead to questioning the notion of cluster. Is it more a theoretical concept than an experimental fact? The answers from the participants are summarized in the next sections.

3. What is a cluster?, or What is the nuclear cluster physics?
3.1. A definition
At the fifth cluster conference held in Kyoto in 1988, Ikeda proposed a definition of a cluster [1]. A cluster is a “spatially localized substructure composed of strongly correlated nucleons”. It is
characterized by “internally strong and externally weak interactions”. Moreover, “α clustering is the most prominent”. A well-developed cluster structure appears in general near thresholds. This last point is exemplified by the Ikeda diagram [1].

In fact, the present workshop shows that several concepts of “cluster” consistent with this definition coexist: (i) microscopic clusters (where all nucleons are described and the Pauli antisymmetrization is performed), (ii) clusters as structureless component particles, and (iii) clusters as emitted nuclei. So maybe should one rephrase the question “What is a cluster?” as “What is the nuclear cluster physics?”.

3.2. Several cluster concepts
The cluster concept is different when treated by theory or by experiment.

In theoretical nuclear cluster physics, a cluster is a pre-assumed substructure in bound states but also in the description of collisions. These pre-assumed clusters describe nuclear properties while simplifying the theoretical treatment and interpretation. As already mentioned different types of models may involve microscopic clusters (§4.1), clusters treated as particles (§4.2) or emitted clusters arising from some fission-like process (§4.3).

A significant part of theoretical nuclear cluster physics concerns the development of efficient calculation techniques (§4.4), which may also be useful in other fields of quantum physics.

An important, rather recent, topic is the explanation of the cluster existence from first principles. There is an increasing number of works aiming at showing the presence of clusters in models where no clusters are assumed to exist at the start (§4.5).

In experimental nuclear cluster physics, the aim is a search for evidence and properties of cluster structures in nuclei. The main topic is α clustering (§5.1) but other clustering aspects appear in the study of nuclear molecules and halo nuclei (§5.2). Cluster radioactivity and fission are examples of the emitted-cluster aspect (§5.2). The cluster concept is also useful in the study of hypernuclei or nuclear properties useful for astrophysics (§5.3).

4. Theoretical nuclear cluster physics
4.1. Microscopic clusters
Microscopic clusters are a 77-year old idea. They were introduced by Wheeler [2] who presented a molecular picture where “the constituent neutrons and protons are divided into various groups (such as alpha-particles)”. These groups are our clusters. Wheeler also introduced an antisymmetrized wave function based on α clusters which is now known as the resonating-group wave function.

The microscopic α clusters are in general made of two protons and two neutrons in a (0s)⁴ translation-invariant harmonic-oscillator state. Horiuchi has insisted on the notion of duality: several apparently different interpretations may coexist (cluster model, shell model, …). A number of talks have been devoted to the Tohsaki-Horiuchi-Schuck-Röpke (THSR) basis function and its generalizations. Funaki, Horiuchi, and Schuck have presented condensate interpretations of cluster states and suggested that the THSR wave functions imply a non-localized clustering. Suhara has shown that the ¹²C and ¹⁶O wave functions can have a large overlap with 3α and 4α linear chain states. Zhou has generalized this basis for the description of the positive- and negative-parity bands of ²⁰Ne.

An important topic, the resonating-group method (RGM), was not covered in plenary talks but its results served as a reference for several tests of the THSR functions. In parallel sessions, Tchuvil’sky discussed the α(³He,γ)⁷Be radiative capture with the algebraic RGM and Dohet-Eraly used the RGM to study the α(α,αγ)α bremsstrahlung reaction.

Another traditional model, the two-centre microscopic (Brink) model in a generalized form was used by Ito to study coupled configurations of Be isotopes and the corresponding monopole strengths.
The tensor force is the enemy of the simple \((0s)^4\) α cluster which is too simple to take its effect into account. Its introduction requires a generalization. An important progress is reached with generalized microscopic clusters allowing the efficient use of realistic forces. Generalized cluster internal functions are obtained from variational calculations. The cluster structure describes bound/scattering states in the spirit of the RGM, with possible additional configurations at short distances. Navrátíl combined the no-core shell model and resonating-group method (NCSM-RGM) to study with realistic forces two-body collisions and the three-body continuum of \(^6\)He. Neff described the \(^{12}\)C bound states and the \(\alpha+^8\)Be scattering by a coupled-channel cluster model.

4.2. Clusters as structureless component particles
Many cluster-model studies treat clusters as particles (i.e. non-microscopic clusters). This requires using effective forces between the clusters.

Again the most convenient cluster is the \(\alpha\) particle. This model was used to describe the \(3\alpha \rightarrow ^{12}\)C+\(\gamma\) reaction. How can this 3-body radiative-capture problem with the additional complication of Coulomb forces be treated at very low temperatures? There is a controversy which started with a CDCC calculation of Ogata et al [3, 4] (see §6.1). To clarify this 3\(\alpha\) problem, Ishikawa solved the Faddeev equation while Yabana performed a direct calculation of the capture rate with the imaginary-time method without reference to cross sections. In both cases, they find a 3\(\alpha\) rate in qualitative agreement with the rate in the 1999 NACRE compilation [5], i.e. without the suggested enhancement at very low temperatures.

Photon emissions can reveal the cluster structure. Garrido related the molecular structure to transitions in the continuum of \(^8\)Be and \(^{12}\)C.

Heavier clusters such as \(^{12}\)C and \(^{16}\)O can also be useful. Uegaki studied alignment effects of oblate nuclei in \(^{12}\)C+\(^{12}\)C resonances. W. Horiuchi reached convergence in a 5-body \(^{12}\)C+p+p+n+n model of \(^{16}\)O. A particular case is the core cluster appearing in 3-body models of halo nuclei. Kikuchi studied the breakup of core+n+n nuclei (\(^6\)He, \(^{11}\)Li) with complex scaling. Yarmukhamedov discussed the asymptotic normalization factors for core+n+n systems with an application to \(^6\)He.

Hypernuclei can also be studied with a similar model where a \(\Lambda\) particle is added. Hiyama presented a \(t+n+n+\Lambda\) model disfavoring the existence of \(^6\)\(^\Lambda\)H and an \(\alpha+n+n+\Lambda\) model of \(^7\)\(^\Lambda\)He.

4.3. Clusters as emitted particles
The name “cluster” also applies to final particles, in other words to observable nuclei emitted in some process.

The simplest example are elastic scattering and breakup. Capel discussed the breakup of halo nuclei with CDCC and eikonal methods. Matsumoto used CDCC for the 3-body breakup of \(^6\)He on \(^{12}\)C and the one- and two-neutron stripping in the \(^6\)He+\(^{208}\)Pb collision.

Cluster formation can be studied in different models. With an extended version of antisymmetrized molecular dynamics (AMD), Ono showed the importance of cluster effects in fragmentation and their relation to the symmetry energy in the equation of state (EOS). With a mean-field approach, Robledo related the octupole degree of freedom with cluster emission and discussed nuclei with exotic shapes.

The fine structure in \(\alpha\) decay was studied by Ren with a multichannel cluster model for deformed nuclei. Von Oertzen analyzed potential energy surfaces for collinear ternary fission.

4.4. Techniques related to cluster structures
A much used technique is the complex-scaling method (CSM). Lazauskas applied it to the elastic scattering and breakup of 3-body systems as well as to 4-nucleon scattering. Doté combined a coupled-channel CSM with Feshbach projection to the K\(^-\)pp system. Using the CSM with
a complex Gaussian basis, Kamimura described a successful search for the elusive $0^+_1$ state predicted in $^{12}$C. Katō studied the bound states and resonances in $A = 5-8$ nuclei with the CSM using Green’s function. The CSM is an efficient tool which was also used in several other talks.

How to extract information from collisions was discussed by Khoa. He obtained transition strengths in $^{12}$C from the inelastic $\alpha + ^{12}$C scattering.

Cluster aspects of the EOS and of phases of matter were also treated. Khan introduced a localization parameter for the transition between quantum-liquid and crystal states. Typel discussed a relativistic density functional with light and heavy clusters and its relation with the EOS.

4.5. Cluster existence from first principles
The appearance of non pre-assumed clusters was obtained for some time with AMD [6]. With this method, Kimura displayed the variety of cluster structures encountered in $^{24}$Mg. Isaka showed that there is no parity inversion in the spectrum of the $^{12}_\Lambda$Be hypernucleus, unlike in the corresponding $^{11}$Be (halo) nucleus.

The nuclear lattice effective-field theory was applied by Epelbaum in a lattice study of $^4$He, $^8$Be, $^{12}$C, $^{16}$O.

Surprisingly, the shell model can also reveal cluster structures. Myo showed the role of the tensor interaction with the tensor-optimized shell model applied to He, Li, Be isotopes. Launey presented a symmetry-guided reduction of the no-core shell-model space and its application to $^8$Be, $^{12}$C, . . . Yoshida employed Monte Carlo shell model wave functions to derive intrinsic densities of Be isotopes, displaying clustering.

5. Experimental nuclear cluster physics
5.1. Alpha clustering
In bound states, clustering is not an observable but rather an interpretation. One can thus only rely on indirect experimental evidence. In collisions, it is customary to consider as “clusters” the incoming-channel nuclei or some reaction products. The indirect experimental constraints for the cluster interpretation can come in increasing order of severity from: (i) searches for bands or for missing states in assumed bands, (ii) determinations of $\alpha$ widths and transition probabilities, (iii) measurements of multiplicities and correlation functions of the emitted particles.

Many experiments concentrate on $\alpha$ cluster structures. A well-known but efficient tool is resonant $\alpha$ scattering. Rogachev used it to study $^{10}$Be and $^{18}$O and asymptotic normalization coefficients (ANC) in oxygen isotopes. Yamaguchi presented results on resonant $\alpha$ scattering on $^7$Li/$^7$Be and $^{10}$Be. Lombardo studied the $\alpha$ resonant scattering on $^9$Be.

Inelastic $\alpha$ scattering also provides useful information. Itoh studied near-threshold resonances from inelastic $\alpha$ scattering on $^{12}$C and $^{16}$O. Kokalova and Marin-Lambardi discussed a new excited state of $^{12}$C. Kawabata measured monopole strengths of $^{12}$C, $^{16}$O, . . . , $^{40}$Ca.

Another approach consists in studying $\alpha$ decay products in some reaction. Gai discussed the Hoyle $2^+$ excitation and proposed a $D_{3h}$ symmetry of the $^{12}$C spectrum. Kirsebom detected three $\alpha$ particles produced in $p + ^{11}$B reactions. Akimune measured high $\alpha$ multiplicities in the $\alpha + ^{36}$Ar inelastic scattering.

Other probes can also be used to test clustering. Ong presented a $(p,d)$ study of tensor-force effects. Rodrigues studied $\alpha$ clustering with $(^6$Li,d) transfer reactions.

5.2. Nuclear molecules
Nuclear molecules are weakly bound systems involving $\alpha$ or heavier clusters. The clusters can be nuclei in the entrance channel. Courtin studied $^{12}$C+$^{12}$C and $^{12}$C+$^{16}$O molecular resonances by radiative capture. Jenkins excited superdeformed states with the $^{12}$C($^{20}$Ne,$\alpha$)$^{28}$Si reaction.
Luong showed in subbarrier reactions with $^6$Li, $^7$Li, $^9$Be that nucleon transfer is important and reduces fusion. Heavy clusters also appear in the output channel. Kohley discussed the two-neutron decays of unbound nuclei with a first measurement of the lifetime of the $^{26}$O nucleus. Kondo discussed the spectroscopy of the unbound $^{29}$O and $^{26}$O nuclei. Andreyev presented results on the $\beta$-delayed multimodal (i.e. symmetric and asymmetric) fission of some nuclei.

5.3. Hypernuclei and astrophysics
Experiments on hypernuclei were also discussed. Feliciello presented evidence for the $^6_\Lambda$H hyperhydrogen (evidence not supported by a J-PARC experiment and by theory). Nakazawa explained the search for double $\Lambda$ hypernuclei with automated emulsion scanning. Clusters are also useful for nuclear astrophysics. In addition to the many talks on the $^{12}$C spectrum already quoted, Tumino presented applications of the Trojan-horse method, which involves a spectator cluster.

6. Comments on two hot topics
6.1. The $^{12}$C spectrum
Why is there such an excitement for the spectrum of this “well-known”, abundant, stable nucleus? One third of the plenary talks discussed its unbound spectrum! My opinion is that it is the simplest case where resonances display a surprising variety of shapes while remaining accessible to both accurate experiment and theory. The (relatively) small number of nucleons allows a variety of precise model or ab initio studies. The fact that the nucleus is stable and the low density of the continuum allow detailed experiments that should be much more difficult for heavier or unstable nuclei.

The structure and interpretation of this continuum remain nevertheless quite difficult for experiment (overlaps of resonances, large widths) and for theory (unusual structures for which models must be adapted). Of particular interest are the Hoyle resonances and the triple-$\alpha$ rate. There has been a long experimental hunt for the $2_2$+ “Hoyle-band” resonance. This resonance was predicted long ago by various authors [7, 8, 9] and this prediction was so strong that, despite the lack of experimental confirmation, its effect was included in the NACRE compilation [5]. Another strange state is the $0_3^+$ resonance predicted by Kurokawa and Katô [10] and confirmed by Kamimura in this workshop [11]. The study of further $0^+$ states, other $J^+$ resonances and the general structure of the continuum spectrum of $^{12}$C are of great interest.

Let me come back to the problems of the triple-$\alpha$ rate for astrophysics. The role of the Hoyle state is well known. At high temperatures, other resonances in addition to $2_2^+$ might affect the rate. At low temperatures, a hot debate about a possible strong rate enhancement with respect to the values in the NACRE compilation was initiated by the CDCC calculation of Ref. [3]. I think that this controversy is to a large extent solved by the calculations of Ishikawa [12] and Yabana [13] presented at this conference. In addition, Yabana proposed a plausible explanation for the qualitative validity of the simple description in NACRE.

The problem is however not completely settled. It would be interesting to understand the origin of the discrepancies in other approaches. An unpleasant aspect of comparing references is that the various calculations of the $3\alpha$ rate usually differ by the $\alpha\alpha$ effective potential. A comparison of models with a benchmark potential (usable in the different techniques) is crucial for closing the debate. In the longer term, an ab initio calculation will be of great interest.

6.2. Localized clustering vs nonlocalized clustering, or THSR vs Brink
The THSR basis function provides an amazingly good description of several states of some nuclei with a single (though rather complicated) basis function. It has recently been generalized to
other angular momenta and to negative parity. Its validity is established by an almost perfect overlap with accurate RGM wave functions!

So, should we forget the RGM-GCM description with Brink basis functions? The generator-coordinate method (GCM) makes use of linear combinations of angular-momentum and parity projected Slater determinants. More basis states are necessary in this approach. Nevertheless it is very flexible and applicable to most light nuclei. Its results are not improved by the THSR basis which only provides a more elegant presentation. Moreover, its extension to the continuum and to reactions is natural.

I think that while the elegant mathematical simplicity of THSR is a striking discovery, the THSR basis is not competitive (yet?) with the flexibility of Brink expansions in the RGM-GCM. There are various apparently conflicting visions of the cluster structure. The THSR wave function has led to the suggestion of an interpretation where the clusters are non localized, in contradiction with the definition in §3.1. In order to clarify this question, let me summarize in Fig. 1 the structure of a cluster wave function. To fix ideas, this figure presents a microscopic $3\alpha$ cluster description of $^{12}\text{C}$. In this physical cluster-model wave function, one encounters from right to left three translation-invariant internal wave functions $\phi_\alpha$ of the $\alpha$ particles, a relative wave function depending on two Jacobi coordinates $x$ and $y$ expanded in some basis of $N$ functions $f_n$, the antisymmetrization projector $\mathcal{A}$ and a projector $P_L$ on angular momentum and parity. The relative wave function $\sum_\alpha c_n f_n(x, y)$ can be of Brink or THSR or other types. It can thus receive various mathematical interpretations but these are not physically relevant. In the THSR case, the number $N$ of basis states can be very small in some cases. After antisymmetrization, one obtains an intrinsic wave function which can receive a cluster-model interpretation but is not yet physical. Only when good quantum numbers are recovered after projection does one obtain physical, though approximate, wave functions in the laboratory frame.

In the laboratory frame, there is no clear indication of clustering in the properties of $0^+$ states obtained with the cluster model. Here one can talk of non-localized clustering. In the intrinsic frame, both the Brink and THSR expansions lead to a similar structure since they have an almost perfect overlap. Intrinsic states may then display a localized clustering in both approaches.

This is exemplified in Figs. 15 and 16 of the $\textit{ab initio}$ $^8\text{Be}$ calculation of Ref. [14]. The $0^+$ ground-state density is spherical, displaying no clustering, but the intrinsic density clearly displays two $\alpha$ clusters. More strikingly, the $4^+$ density is elongated because of the centrifugal effect and thus different, but the corresponding intrinsic density is very similar to the $0^+$ one. More recent $\textit{ab initio}$ calculations show similar patterns. Epelbaum obtained cluster structures

![Figure 1. Schematic structure of a microscopic wave function: the $3\alpha$ example.](image-url)
on an intrinsic lattice. Yoshida also obtained clusters by transforming from the laboratory-frame shell model to an intrinsic frame.

A number of interpretations concern the relative wave function \( \sum_n c_n f_n(x, y) \) and are mathematical interpretations: \( \alpha \) condensate, particles in a container, localized clusters in the GCM, nonlocalized clusters in THSR functions, gas-like states, \ldots The cluster-model interpretation is based on intrinsic densities in the intrinsic frame: linear chain, shell-model-like, more or less localized clusters. This approach is the first step in many models: RGM-GCM, THSR, AMD, FMD, lattice EFT, \ldots Physical interpretations require a projection on good quantum numbers in the laboratory frame where observable properties can be compared with experiment. Such wave functions are directly obtained in \textit{ab initio} calculations: Green-function Monte Carlo, shell models, no-core shell model, \ldots In this frame, clusters may seem to be absent, or be said nonlocalized or in a gas state.

An example of the difference between the intrinsic and mathematical interpretations can be seen in the THSR 3\( \alpha \) description of Funaki. Before antisymmetrization, the clusters do not seem to be localized in the density \( \left| \sum_n c_n f_n(x, y) \right|^2 \). After antisymmetrization but before angular-momentum projection, the clusters are localized in the intrinsic THSR density. Another aspect has been illustrated by Zhou: the THSR and RGM intrinsic densities of \( \alpha + ^{16}\text{O} \) look very similar and the clustering into \( \alpha \) and \( ^{16}\text{O} \) can also be said localized.

7. Summary of the summary

Clusters are a useful simplifying approximation in (hyper)nuclear physics. There is a variety of cluster concepts and applications, which is a richness. The theoretical interpretation of a microscopic cluster wave function may be misleading if it is done before antisymmetrization. As the cluster structure is hidden in the laboratory frame, it must be analyzed in the intrinsic frame. Cluster ideas are also very useful in scattering, reactions, decays and many other aspects of nuclear physics, up to now mostly for light nuclei. Significant progresses have recently been made in \textit{ab initio} explanations of the occurrence of a cluster substructure.

The cluster structure is not an observable. Experimental evidence can thus only be indirect. Evidence for a cluster structure requires a variety of experimental information. A new generation of experiments appeared which will allow making more stringent tests of the models.

Nuclear cluster physics has many facets and a lot of exciting work remains to be done. Many thanks to Yamada-san and his team for having perfectly organized this interesting workshop.

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