Sub-Doppler resolution with double coherently driving fields

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Abstract

We propose a four-level model where sub-Doppler resolution as well as enhanced absorption of a weak probe field are realized by using two coherently driving fields. We show that spectral resolution can be improved by modifying the coherent fields intensity and frequencies.

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I Introduction

Multi-photon transitions are embedded in a variety of optical effects based on correlated absorption and emission of two or more photons. Step-wise and multi-photon processes can be distinguished by their frequency-correlation properties [1, 2]. Sub-Doppler resolution for an inhomogeneously broadened medium based on multi-photon processes has attracted much attention in recent decades. The physical essence for this method lies in the fact that a detuning for multi-photon transition stipulated by Doppler frequency shift can be reduced or eliminated by adopting appropriate light propagation direction, since the detuning is the sum or difference of multiple single-photon transition detunings. Sub-Doppler nonlinear optical resonance as appearance of quantum coherence and interference in the context of frequency-correlation properties of the coherent components have been proposed in [3]. A widely used method is Doppler-free two- or multi-photon absorption where the velocity dependent detuning could be removed in the case that the wave vectors of the interacting beams sum down to zero (for a review, see [4]). This type of sub-Doppler spectrum is characterized by a large detuning from the intermediate resonance, measuring the fluorescence from the upper level as well as by small cross sections for these multi-photon processes. Enhanced by intermediate resonance sub-Doppler processes based on the use of strong driving fields and on a change of frequency-correlation properties of resonant multi-photon process in strong resonant fields, have been proposed in [5] and further developed in [6] (also in [7]). Substantial enhancement in absorption, gain and fluorescence controlled with the auxiliary appropriately propagating electro-magnetic wave has been predicted. An interference nature of the spectrum modification has been stressed, which implies that along the growth of the absorption (gain) in certain spectral intervals, integral over the frequency may even decrease. A role of increase of intensity of a probe field as well as features of Doppler-free lasers were explored too.

Control of atomic response with intense coupling lasers has been a subject of many intensive studies in the context of electromagnetically induced transparency (EIT) [8], amplification without inversion (AWI) [8, 9, 11], enhancement of the refractive index without absorption [12] and so on (for review see [8, 11, 13]). Sub-Doppler resolution by using intense coherently driving field(s) at other transition(s) has been recently further explored in [14, 15]. In Ref. [14] probe weak field absorption spectrum in three-level schemes was considered. It was pointed out that the linewidth of one of the two Autler-Townes absorption peaks can be reduced by match of the coupling field intensity and frequency. In a later work [15], an experimental observation of sub-Doppler linewidth in a Doppler-broadened Λ-type Rb atomic system was reported. Related works to reduce Doppler

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broadening with atomic coherence effects could be found in Ref. [10] and multiple sub-Doppler lines have been shown to achieve with a strong coupling field and a saturation effects in a three-level system[17].

In the recent papers [13], cancellation of Doppler broadening by applying different frequency fields was proposed with one or two coherently driving lasers. As it has been outlined, sub-Doppler structures can be induced without population redistribution, but four-wave mixing output can be enhanced through Doppler-free coupling controlled with auxiliary co- or counter-propagating driving electromagnetic radiation.

In this paper, we propose a four-level scheme, where both upper and lower levels of a probe transition are coupled to other levels by strong coherent fields. In this scheme, four absorption peaks could be found in Ref. [16] and multiple sub-Doppler lines have been obtained on the basis of the developed theory are accompanied with numerical simulation addressed to real experimental schemes. Despite the decreased integral intensity, we predict enhanced laser-induced sub-Doppler absorption peaks along with transparency windows. The features are interpreted in the terms of quantum coherence and interference processes.

The paper is organized into four sections. In the next section, we present a four-level model and the density-matrix equations describing the system. The solution in the limit of a weak probe and steady-state condition is obtained and the absorption spectrum is then analyzed, first without Doppler broadening. In Sec. III, taking into consideration Doppler broadening, we demonstrate that the multiple sub-Doppler lines as well as enhanced absorption at the resonance could be obtained in this scheme. From numerical illustrations of the effect in a practical medium, the conditions for and features of Doppler-free resonance, stipulated by compensation of Doppler shifts with light shifts are discussed. In Sec. IV, we summarize the results.

II Model and absorption spectrum at homogeneously broadened probe transition

We consider a closed four-level scheme shown in Fig.1(a). In this scheme two coherent driving fields $E_c$ and $E_d$ with coupling Rabi frequencies $\Omega_c = -E_c \cdot d_{13}/\hbar$ and $\Omega_d = -E_d \cdot d_{24}/\hbar$ interact with the transitions labeled |4⟩ − |2⟩ and |3⟩ − |1⟩, respectively. The transition |4⟩ − |1⟩ is probed by the weak nonperturbating field $E_p$. Absorption index $\alpha_p$ for this field, reduced by its value $\alpha^0$ at $\omega_p = \omega_{41}$, $\Omega_c = \Omega_d = 0$ is found as $\alpha_p/\alpha^0 = Re\{\chi_p/\chi^0_p\}$, where $\chi_p$ is corresponding dressed susceptibility. The later is convenient to calculate with aid of dressed nonlinear polarization $P^NL$ and density matrix $\rho_{ij}$ as $P^NL(\omega_p) = N\chi_p E_p^2/2 = N\rho_{ij}d_{ij}$ ($d_{ij}$ is transition electric dipole moment, $N$ - number density of atoms). In the framework of semi-classical theory and using the standard density matrix formalism with the rotating wave approximation, the description equations of this scheme can be written in general form as:

$$L_{nm}\rho_{nm} = q_n - i[V, \rho]_{nm} + \sum_{m>n} \gamma_{mn}\rho_{mn}; L_{14}\rho_{14} = L_p\rho_p = -i[V, \rho]_{14} \text{ (etc.)},$$

where $L_{ij} = d/dt + \Gamma_{ij}$; $V_{14} = \Omega_p \cdot \exp\{i\Delta_p t\}$; $\Omega_p = -E_p \cdot d_{14}/2\hbar$; $\Delta_p = \omega_p - \omega_{41}$ (etc.) are frequency detunings from the corresponding resonance; $\Gamma_{ij}$ - homogeneous half-widths of transitions (in absence of collisions $\Gamma_{mn} = (\Gamma_m + \Gamma_n)/2$); $\Gamma_n = \sum_j \gamma_{nj}$ - inverse lifetimes of levels; $\gamma_{mn}$ - rate of relaxation from the level $m$ to $n$, $q_n = \sum_j w_{nj} r_{j} - \text{rate of incoherent excitation to a state } n \text{ from the underlying levels}$.

We represent density matrix elements as $\rho_{14} = r_p \cdot \exp\{i\Delta_p t\}$, $\rho_{35} = r_3$ etc. Then in a steady-state regime a set of density-matrix equation may be reduced to the set of algebraic equations for the amplitudes $r_{ij}$. Analytical solution of this system of 10 coupled equation both for closed and open four-level scheme is given in [11, 20].

In the case under consideration incoherent excitation of the upper levels as well as relaxation from the level 3 to 2 are supposed to be negligible small. Then $r_2 = r_4 = r_c = 0$ and equation system (1) reduces to

$$P_pr_{1} = i\rho_{p}r_{1} + i\rho_{12}r_{2} + i\rho_{13}r_{3} + i\rho_{14}r_{4} - i\Omega_d r_{3}2,$$

$$P_{3}r_{3} = -i\rho_{3}^2r_{3} + i\rho_{32}r_{2} + i\rho_{34}r_{4} - i\Omega_d r_{3}2,$$

$$P_{r} = i\Omega_d (r_1 - r_3),$$

where $\rho_{12} = \rho_{34} = \rho_{14} = \rho_{34} = 0$ and $\rho_{35} = \rho_{33} = \rho_{34} = 0$.

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$$P_{3}r_{3} = -i\rho_{3}^2r_{3} + i\rho_{32}r_{2} + i\rho_{34}r_{4} - i\Omega_d r_{3}2,$$

$$P_{r} = i\Omega_d (r_1 - r_3),$$

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$$P_{r} = i\Omega_d (r_1 - r_3),$$

where $\rho_{12} = \rho_{34} = \rho_{14} = \rho_{34} = 0$ and $\rho_{35} = \rho_{33} = \rho_{34} = 0$.
Here \( P_{14,13} = P_{p,d} = \Gamma_{p,d} + i \Delta_{p,d} \). \( P_{12} = \Gamma_{p,d} + i (\Delta_p - \Delta_c) \), \( P_{34} = \Gamma_{p,d} + i (\Delta_p - \Delta_d) \), \( P_{23} = \Gamma_{p,d} + i (\Delta_c - \Delta_d) \). For atom moving with speed \( v \), Doppler shift of resonances must be taken into account by substituting \( \Delta_j \) for \( \Delta_j = \) \( \Delta_j - k_j v \).

Under considered approximation solution for two level system \([1] - [3]\) is found apart of the other elements.

\[
r_d = \frac{i \Omega_d \rho_d}{\Gamma_d(1 + \omega_d) + \Delta_d^2}, \quad r_3 = \frac{\Gamma_d^2 \alpha_e / 2}{\Gamma_d^2(1 + \omega_d) + \Delta_d^2}, \quad \alpha_e = \frac{4|\Omega_d|^2}{\Gamma_d^3 \Gamma_d^r}
\]

Dressed susceptibility for the probe field is found in the notations of \([11, 20]\) as:

\[
\chi_p \equiv \frac{\Gamma_p}{\chi_{p0}^2} \Gamma_p R_p, \quad R_p = \frac{r_3}{1 + g_5 + v_5} - \frac{(r_1 - r_3)(1 + g_5 + v_5)g_1}{1 + g_5 + v_5 + g_4(1 + g_5 + v_5) + v_4(1 + g_5 + v_5)},
\]

\[
g_1 = \frac{|\Omega_d|^2}{P_{d34}^2} g_4 = \frac{|\Omega_d|^2}{P_{p34}^2} g_5 = \frac{|\Omega_d|^2}{P_{p34}^2} g_6 = \frac{|\Omega_d|^2}{P_{p34}^2} g_4 = \frac{|\Omega_d|^2}{P_{p34}^3} g_5 = \frac{|\Omega_c|^2}{P_{p34}^3} g_6 = \frac{|\Omega_c|^2}{P_{p34}^3} g_6 = \frac{|\Omega_c|^2}{P_{p34}^3} g_6.
\]

At \( \Omega_d = 0 \) all \( g_i = 0 \) and the equation (6) reduces to that describing \( \Delta \) scheme:

\[
\frac{\alpha_p}{\alpha_{p0}} = \text{Re} \left( \frac{\Gamma_p \Gamma_{12} + i(\Delta_p - \Delta_c)}{(\Gamma_p + i \Delta_p) \Gamma_{12} + i(\Delta_p - \Delta_c)} \right) = \text{Re} \left( \frac{\Gamma_p \Gamma_{12} + i(\Delta_p - \Delta_c)}{(\Gamma_p + i \Delta_p) \Gamma_{12} + i(\Delta_p - \Delta_c)} \right),
\]

\[
\delta_{1,2} = \frac{\Delta_c + i(\Gamma_{12} + \Gamma_p)}{2} \mp \sqrt{\frac{\Delta_c + i(\Gamma_{12} + \Gamma_p)}{2} + |\Omega_c|^2}.
\]

Following to \([8]\), we introduce frequency-correlation factor

\[
M_{1,2} = \frac{d \delta_{1,2}}{d \Delta_c} = \frac{1}{2} \left[ 1 \mp \frac{\Delta_c}{\sqrt{4|\Omega_c|^2 + \Delta_c^2}} \right].
\]

The denominator in \([8]\) displays two resonances. At \( \Omega_c \to 0 \) we obtain \( \delta_1 \to 0 + i \Gamma_p, \delta_2 \to \Delta_c + i \Gamma_{12} \). This indicates one resonance at \( \Delta_p = 0 \), the HWHM is \( \Gamma_p \), which corresponds to one-photon resonance with no correlation with \( \omega_c \) (\( M_1 = d \delta_1 / d \Delta_c = 0 \)). The second resonance at \( \Delta_p = \Delta_c \) is of HWHM \( \Gamma_{12} \) that corresponds to two-photon resonance, fully correlated with \( \omega_c \) (\( M_2 = d \delta_2 / d \Delta_c = 1 \)). With growth of the coupling Rabi frequency \( \Omega_c \) the resonance becomes split in two component, their frequency-correlation properties modify, and \( M_1 \approx M_2 \approx 1/2 \) at \( |\Omega_c|^2 \gg |\Delta_c|^2, \Gamma_{12}^2 \), which neither correspond to one- nor to two-photon processes. HWHM of the resonances becomes also nearly equal to each other and to \( (\Gamma_{12} + \Gamma_p)/2 \). Note that all the effects are determined by the coherence \( \rho_{12} \) induced in the transition \([1] - [2]\) by two coupled fields, and that always \( M_1 + M_2 = 1 \). More detailed discussion can be found in \([8, 10, 13]\).

In the alternative case \( \Omega_c = 0 \) all \( \nu_i = 0 \) and the equation \([8]\) reduces to that describing \( V \) scheme:

\[
\frac{\alpha_p}{\alpha_{p0}} = \text{Re} \left( \frac{\Gamma_p \Gamma_{14} + i(\Delta_p - \Delta_d)}{(\Gamma_p + i \Delta_p) \Gamma_{14} + i(\Delta_p - \Delta_d)} \right) \frac{r_3}{\Gamma_p + i \Delta_p} \frac{|\Omega_d|^2}{\Gamma_p + i \Delta_p} \Gamma_{14} + i(\Delta_p - \Delta_d)} + |\Omega_d|^2
\]

The structure of the denominator is similar to \([8]\) and determined by the coherence \( \rho_{34} \). Additional term in the nominator is stipulated by the coherence induced at the transition \([1] - [3]\) with not zero populations of the levels unlike the transition \([2] - [4]\). Alongside with the coherence \( \rho_{34} \) this term is a source of the nonlinear interference effects (NIEF) in absorption (gain) and refraction. Specific features of NIEF in coupled Doppler broadened \( \Lambda, V \) and ladder schemes were explored in \([2, 19]\). It has been outlined that frequency integrated absorption index is proportional to \( r_1 - r_4 \) only, i.e. it’s change is determined only by the population change. As it was first outlined in \([3, 10]\), indeed NIEF give rise to difference in the line shapes of pure absorption and emission spectra. As the consequence of this effect, the appearance of amplification without inversion on the base of NIEF has been predicted. Amplification without inversion was introduced and its features were explicitly analyzed and illustrated for the model of neon transitions in the early publication \([8]\).

Specific feature of the case under consideration in this paper is that both of the levels of the probe transition can be driven independently. This gives rise to multiple resonance structure, corresponding to the roots of denominator in \([8]\). If resonance splitting is much greater than their widths, we can set \( \Gamma_{ij} = 0 \), and the resonance values of \( \Delta_p \) are described by the equation:

\[
[\Delta_p - \Delta_d - \Delta_c][\Delta_p(\Delta_p - \Delta_d)(\Delta_p - \Delta_c) - (\Delta_p - \Delta_c)\Omega_d^2 - (\Delta_p - \Delta_d)\Omega_c^2] - \Delta_p(\Delta_p - \Delta_d)\Omega_d^2 + (\Delta_p - \Delta_c)\Omega_c^2 + (\Omega_d^2 - \Omega_c^2)^2 = 0.
\]
Basically, this equation possesses four roots, that indicates appearance of four nonlinear resonances, determined by splitting of each of the levels $|1\rangle$ and $|4\rangle$ into two quasi levels.

In the case $\Delta_d = -\Delta_c = \Delta$ resonance positions are given by the equation:

$$\Delta_p^2 = (\Delta^2 + 2\Omega_d^2 + 2\Omega_c^2)/2 \pm \sqrt{[(\Delta^2 + 2\Omega_d^2 + 2\Omega_c^2)/2]^2 - (\Omega_d^2 - \Omega_c^2)^2}. \quad (12)$$

At $|\Omega_d| = |\Omega_c| = |\Omega|$ two resonances merge in one not shifted resonance at $\Delta_p^{(1)} = 0$, the other two resonances are given by

$$\Delta_p^{(2,3)} = \pm \sqrt{\Delta^2 + 4\Omega^2}. \quad (13)$$

In the case $\Delta_d = \Delta_c = 0$ resonance positions are found as:

$$\Delta_p^{(1,2)} = \pm (|\Omega_d| - |\Omega_c|); \quad \Delta_p^{(3,4)} = \pm (|\Omega_d| + |\Omega_c|). \quad (14)$$

We shall illustrate major outcomes of the paper with numerical simulations addressed to the conditions of the experiment \([22]\). Transitions $|1\rangle - |3\rangle - |2\rangle - |4\rangle - |1\rangle$ in Fig. 2a are attributed to those of the sodium dimers \(Na_2\):

\[X'\Sigma_g^{+}(v'' = 0, J'' = 45) - A'\Sigma_g^{+}(6, 45)(\lambda_d = 655 \text{ nm}) - X'\Sigma_g^{+}(14, 45)(\lambda_{32} = 756 \text{ nm}) - B'\Pi_u(5, 45)(\lambda_c = 532 \text{ nm}) - X'\Sigma_g^{+}(0, 45)(\lambda_p = 480 \text{ nm}). \]

The Doppler widths of the transition at wavelength $\lambda_p = 480 \text{ nm}$ at the temperature about 450 C is $2D_p \approx 1.7 \text{ GHz}$. Then the Boltzmann's population of the level $|2\rangle$ makes about 1.5% from that of the level $|1\rangle$. The following relaxation parameters are used:

\[\Gamma_4 = \Gamma_3 = 120, \quad \Gamma_2 = \Gamma_1 = 20, \quad \gamma_{42} = 5, \quad \gamma_{41} = 10, \quad \gamma_{32} = 4, \quad \gamma_{31} = 7, \quad \Gamma_{12} = 20, \quad \Gamma_{34} = 120, \quad \Gamma_{23} = \Gamma_c = \Gamma_d = \Gamma_p = 70, \]

(all in $10^6 \text{ s}^{-1}$). For numerical simulation we have used the full set of the equations from \([13, 20]\), accounting for various relaxation transitions and not zero population of the level $|2\rangle$.

We begin our discussion with the case when the Doppler-broadening is not included. Performing numerical calculation for Eq. (6), we depict the absorption profiles in Fig. 2. From Fig. 2a and b, it is easy to find the typical EIT and absorption spectrum with $|\Omega_d| = 0$ (Fig. 2a) and $|\Omega_c| = 0$ (Fig. 2b). Under increasing Rabi frequency $|\Omega_c|$ ($|\Omega_d|$), the four-peak spectrum forms, when the driving fields become comparable to or larger than the spectral width $\Gamma_p$ of the atomic transition. The emergence of the four-peak spectrum is attributed to the fact that levels $|1\rangle$ and $|4\rangle$ have been driven into two dressed states (Fig. 3). As $|\Omega_d|$ approaches $|\Omega_c|$, the four-peak spectrum degenerates into a three-peak spectrum since two dressed-state transitions have the same resonant frequency and contribute to the same central component. As already known, a three-peak spectrum occurs in resonance fluorescence (RF) in the strong field limit of a two-level system (for a review, see \([10]\)). Even though the dressed-state diagrams are similar in the two cases, the four-peak spectrum can not appear in RF
because in the case of RF the dynamic Stark splitting for the two doublets must be the same, while in our system
the two doublet splittings are controlled separately by different driving field intensity and frequency. We would
like to stress that the multiple EIT windows can appear simultaneously with the multiple absorption peaks. In
Fig. 2c, the absorption amplitude $A$ at $\Delta_p = 0$ is plotted against the Rabi frequencies for two resonant driving
fields. An important feature is that the maximum absorption occurs under the condition that the two Rabi
frequencies are equal, which is due to the fact that the two dressed-state transitions simultaneously contribute
to the central component. This result may provide a way to measure the field intensity as well as atomic
parameters.

Fig. 3 depicts absorption spectrum for the molecules with the velocity projection on the propagation direction
of the coupled fields $v = 0$. The plots illustrate dependencies, described by the equations (12) - (14). The
appearance of the four-peak spectrum is attributed to the fact that levels $|1\rangle$ and $|4\rangle$ have been driven into two
dressed states. Direct computing shows that frequency integral absorption for the plot 3 is the same as for 5,
whereas for the other plots it is about 0.5 of that for the graph 5. This is due to strong saturation of population
difference at the transition $|1\rangle - |3\rangle$.

III Coherence induced resonances in Doppler-broadened medium of
sodium dimers

In the previous sections, Doppler broadening has not been accounted for. In order to consider this effect,
Doppler shifts must be introduced in the three interacting field resonance detunings in Eq. (6). Below we will

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Figure 4: Absorption spectrum at inhomogeneously broad-
ened probe transition $|1\rangle - |4\rangle$ (the scheme Fig. 1(a)) con-
trolled with two driving field. All fields are co-propagating, $\Delta_d = \Delta_c = 0$, $\Omega_c = 1$ GHz, $\Omega_d$ is as follows: 1 — (solid) $\Omega_d = 1$
GHz; 2 — (dash) $\Omega_d = 0.7$ GHz; 3 — (dot) $\Omega_d = 0.3$ GHz; 4 —
(solid) $\Omega_c = \Omega_d = 0$.

Figure 5: Compensation of residual Doppler broadening and
sub-Doppler structures at the probe transition $|1\rangle - |4\rangle$ (the
scheme Fig. 1(b) coherently driven with two strong fields. Main
plot — all fields are co-propagating, 1 — 3 — all parameters
and notations are the same as in Fig. 4; 4 — (dash-dot): $\Omega_d =$
0.075 GHz. Insets (a) and (b) are corresponding zoomed peaks.
Inset (c) — same as plot 1 but: 1 — only field $c$ is counter
propagating; 2 — only field $d$ is counter propagating; 3 — both
driving fields are counter-propagating in respect to the probe
field.

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...proceed with analysis of the absorption spectrum under effects of Doppler-broadening. Figure 4 displays same
dependencies, but for velocity averaged absorption index and co-propagating probe and driving fields. While
number of peaks and their positions are identical to the previous case, the specific feature of the inhomogeneous
broadening is that plot 1 displays enhanced absorption in the central component despite the fact that integral
intensity of this graph is about 2 times less than that for the plot 4 (due to the strong saturation of the
populations at the transition $|1\rangle - |3\rangle$). In absence of driving fields only small fraction of the molecules (about
$\Gamma_p/D_p \approx 10^{-3}$) can be coupled by probe field. Overlap of the two dressed transitions gives rise to increased
amount of the molecules coupled with the probe field and consequently to enhanced absorption as compared to
not perturbed absorption in the center of Doppler broadened transition.
As it was shown in [3], resonance requirements for molecules at velocity $v$ and at $max\{|\Omega_d|^2, \Delta_d|^2\} \gg k_d u$ ($u$ is thermal velocity, $k_d$ - wave number) are described by the equation

$$\Delta_p - k_p v = \delta_{1,2} - M_{1,2} k_d v.$$  \hspace{1cm} (15)

The equation indicates that cancellation of Doppler shifts is possible even if $k_p \neq k_d$, if $k_p < k_d$. This is due to the interplay of the Doppler and ac Stark shifts giving rise to variation of $M_1$ in the interval $0...1/2$, whereas $M_2$ – in the interval $1...1/2$. Possible enhancements of cross-section of optical processes through concurrent coupling of molecules from wide velocity interval by means of the above outlined approach are shown in Fig. 4. The frequencies of the probe and $E_d$ fields are interchanged (see Fig. 1b), all other relaxation parameters remain the same. Plot 1 displays enhanced sub-Doppler resonances with the FWHM comparable with the natural linewidth (see insets (a),(b)) for co-propagating wave. Dramatic change as compared to similar graphs in Fig. 3 is explicitly seen. The other plots show evolution of the structures with variation of the intensity of one of the driving field. As plot 4 indicates, the sub-Doppler structures exist only under certain ratio of the driving field intensities. Even more dramatic change occurs while driving fields (especially shorter wavelength one) are counter propagating (see inset (c)).

IV Conclusion

In conclusion, we have explored spectral properties of the molecular transitions in the case that both upper and lower levels are coupled to other atomic levels and Doppler effects play an important role. The features of the four-peak or three-peak spectra induced by the driving fields are investigated. Despite the dominated Doppler-broadening, two or more peaks may possess the sub-Doppler resolution. A crucial role of the ratio of the frequencies of the coupled transition is shown. With manipulation the detunings and/or intensities of two coupling fields, one can improve the spectral resolution. The enhanced absorption in the sub-Doppler peak is shown to be realized, whereas the peak can be induced in the center of the inhomogeneously broadened transition. The predicted effects are attributed to the quantum coherence and interference processes, while frequency-correlation properties of multi-photon processes experience substantial modification with the growth of the driving fields intensities, which leads to corresponding dramatic changes of the role of Doppler effects.

The similar sub-Doppler technique can also be implemented for other atomic coherence effects, such as EIT, LWI and for enhancing various resonant nonlinear optical phenomena in inhomogeneously broadened media. In one of our previous papers [22], the inversionless amplification has been considered with the important feature that incoherent excitation to the upper level is not necessary in the same scheme as considered in this letter. Similar scheme is often used in quantum optics. Most recently, electromagnetically induced absorption was studied in a similar four-level system [23]. We believe that, by utilizing this sub-Doppler technique, the above effects in inhomogeneously broadened media can be further enhanced and manipulated.

We want to stress that in optically thick media the processes similar to optical parametric amplification and involved in the schemes Fig. 1a,b may play a crucial role, that imposes dramatic consequences on the features of the output probe signal [24].

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