Realistic Deep Learning May Not Fit Benignly

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Abstract

Studies on benign overfitting provide insights for the success of overparameterized deep learning models. In this work, we examine the benign overfitting phenomena in real-world settings. We found that for tasks such as training a ResNet model on ImageNet dataset, the model does not fit benignly. To understand why benign overfitting fails in the ImageNet experiment, we analyze previous benign overfitting models under a more restrictive setup where the number of parameters is not significantly larger than the number of data points. Under this mild overparameterization setup, our analysis identifies a phase change: unlike in the heavy overparameterization setting, benign overfitting can now fail in the presence of label noise. Our study explains our empirical observations, and naturally leads to a simple technique known as self-training that can boost the model’s generalization performances. Furthermore, our work highlights the importance of understanding implicit bias in underfitting regimes as a future direction.

1 Introduction

Modern deep learning models achieve good generalization performances even with more parameters than data points. This surprising phenomenon is referred to as benign overfitting, and differs from the canonical learning regime where good generalization requires limiting the model complexity (Mohri et al., 2018). One widely accepted explanation for benign overfitting is that optimization algorithms benefit from implicit bias and find good solutions among the interpolating ones under the overparametrized settings. The implicit bias can vary from problem to problem. Examples include the min-norm solution in regression settings or the max-margin solution in classification settings (Gunasekar et al., 2018a; Soudry et al., 2018; Gunasekar et al., 2018b). These types of bias in optimization can further result in good generalization performances (Bartlett et al., 2020; Zou et al., 2021; Frei et al., 2022). These studies provide novel insights, yet they sometimes differ from the deep learning practice: state of the art models, despite being overparameterized, often do not interpolate the data points (e.g., He et al. (2016); Devlin et al. (2018)).

In this work, we first examine the existence of benign overfitting in realistic setups by testing whether ResNet (He et al., 2016) models can overfit data benignly under common experiment setups: image classification on CIFAR10 and ImageNet. Our results are shown in Figure 1. In particular, we trained ResNet18 on CIFAR10 for 200 epochs and the model interpolates the train data. In addition, we also trained ResNet101 on ImageNet for 160 epochs, as opposed to the common schedule that stops at 90 epochs. Surprisingly, we found that although benign overfitting happens on the CIFAR10 dataset, overfitting is not benign on the ImageNet dataset—the test loss increased as the model further fits the train set. This phenomenon cannot be explained by known analysis on benign overfitting for classification tasks, as no negative results were studied yet.

Motivated by the above observation, our work aims to understand the cause for the two different overfitting behaviors in ImageNet and CIFAR10, and to reconcile the empirical phenomenon with previous analysis on benign overfitting. Our first hint comes from the level of overparameterization. Previous results on benign overfitting in the classification setting usually requires that $p = \omega(n)$, where $p$ denotes the number of parameters and $n$ denotes the training sample size (Wang et al., 2021a; Cao et al., 2021; Chatterji et al., 2021; Frei et al., 2022). However, in practice many deep learning models...
models fall in the mild overparameterization regime, where the number of parameters is only slightly larger than the number of samples despite overparameterization. In our case, the sample size $n = 10^6$ in ImageNet, whereas the parameter size $p \approx 10^7$ in ResNets.

To close the gap, we study the overfitting behavior of classification models under the mild overparameterization setup where $p = \Theta(n)$ (this is sometimes referred to as the asymptotic regime). In particular, following [Wang et al. (2021a); Cao et al. (2021); Chatterji et al. (2021); Frei et al. (2022)], we analyze the solution of stochastic gradient descent for the Gaussian mixture models. We found that a phase change happens when we move from $p = \Omega(n \log n)$ (studied in Wang et al. (2021a)) to $p = \Theta(n)$. Unlike previous analysis, we show that benign overfitting now provably fails in presence of label noise (see Table 1 and Figure 2). This aligns with our empirical findings as ImageNet is known to suffer from mislabelling and multi-labels (Yun et al., 2021; Shankar et al., 2020).

More specifically, our analysis (see Theorem 3.1 for details) under the mild overparameterization ($p = \Theta(n)$) setup supports the following statements that align with our empirical observations in Figure 1 and 5:

- When the labels are noiseless, benign overfitting still holds under similar conditions as in previous analyses.
- When the labels are noisy, the interpolating solution can provably lead to a positive excess risk that does not diminish with the sample size.
- When the labels are noisy, early stopping can provably lead to better generalization performances compared to interpolating models.

Furthermore, our analysis naturally leads to a simple technique, known as self-training, that can improve a model’s generalization performance by removing “hard data points”. The technique was studied in (Hinton et al., 2015; Allen-Zhu et al., 2020; Huang et al., 2020). We confirm through our experiment design that the gain in generalization is coupled with the level of label noise.

More importantly, the empirical and theoretical results in our work point out that modern models may not operate under the benign interpolating regime. Hence, it highlights the importance of characterizing implicit bias when the model, though mildly overparameterized, does not interpolate the train data.

2 Related Works

Although the benign overfitting phenomenon has been systematically studied both empirically (Belkin et al., 2018b; 2019; Nakkiran et al., 2021) and theoretically (see later), our work differs in that we aim to understand why benign overfitting fails in the classification setup. In particular, among all theoretical studies, the closest ones to us are (Chatterji et al., 2021), (Wang et al. 2021b) and (Cao et al. 2021), as the analyses are done under the classification setup with Gaussian mixture models. Our work provides new result by moving from heavy overparameterization $p = \omega(n)$

Figure 2: Phase Transition in Noisy Regimes. We conduct experiments on simulated GMM data with $|\mu| = 40$ and $\sigma = 1$ using SGD to train a linear classifier and plot the excess risk. The brighter grid means that the classifier has smaller excess error, and therefore does not overfit. Figure (a) shows that training on noiseless data does not cause overfitting, and Figure (b) shows that training on noisy data causes overfitting with overparameterization. Besides, we find that overfitting consistently happens under noisy regimes, despite theoretical guarantee that the model overfits benignly when $p = \omega(n)$.

![Figure 2](image.png)

### Table 1: Comparison of Previous Work on Benign Overfitting

| Setting                    | Classification Noiseless | Classification Noisy | Regression Noisy |
|----------------------------|--------------------------|----------------------|------------------|
| Mild Overparameterization  | Work (Ours)              | Fail (Ours)          | Fail             |
| $p = \Theta(n)$            |                          |                      |                  |
| Heavy Overparameterization | Work (Cao et al., 2021)  | Work (Chatterji et al., 2021) | Work (Bartlett et al., 2020) |
| $p = \omega(n)$            |                          |                      |                  |

### More Related Works on Benign Overfitting

Researchers have made a lot of efforts to generalize the notation of benign overfitting beyond the novel work on linear regression (Bartlett et al., 2020), e.g., variants of linear regression (Tsigler et al., 2020; Muthukumar et al., 2020; Zou et al., 2021), linear classification (Liang et al., 2020a; Belkin et al., 2018a) with different distribution assumptions (instead of Gaussian mixture model), kernel-based estimators (Liang et al., 2018; 2020b; Mei et al., 2022), neural networks (Frei et al., 2022).

**Gaussian Mixture Model (GMM)** represents the data distribution where the input is drawn from a Gaussian distribution with different centers for each class. The model was widely studied in hidden Markov models, anomaly detection, and many other fields (Hastie et al., 2001; Xuan et al., 2001; Reynolds, 2009; Zong et al., 2018). A closely related work is Jin (2009) which analyzes the lower bound for excess risk of Gaussian Mixture Model under noiseless regimes. However, their analysis cannot be directly extended to either the overparameterization or noisy label regimes. Another closely related work (Mai et al., 2019) focus on the GMM setting with mild overparameterization, but they...
require a noiseless label regime and rely on a small signal to noise ratio on the input, and thus cannot be generalized to our theoretical results.

Asymptotic (Mildly Overparameterized) Regimes. This paper considers asymptotic regimes where the ratio of parameter dimension and the sample size is upper and lower bounded by absolute constants. Previous work studied this setup with different focus than benign overfitting (e.g., double descent), both in regression perspective (Hastie et al., 2019) and classification perspective (Sur et al., 2019; Mai et al., 2019; Deng et al., 2019). We study the mild overparameterization case because it generally happens in realistic machine learning tasks, where the number of parameters exceeds the number of training samples but not extremely.

Label Noise: Label noise often exists in real world datasets, e.g., in ImageNet (Yun et al., 2021; Shankar et al., 2020). However, its effect on generalization remains debatable. Recently, Damian et al. (2021) claim that in regression settings, label noise may prefer flat global minimizers and thus helps generalization. Another line of work claims that label noise hurts the effectiveness of empirical risk minimization in classification regimes, and proposes to improve the model performance by explicit regularization (Mignacco et al., 2020) or robust training (Brody and et al., 1996; Guan et al., 2011; Huang et al., 2020). Among them, Bagherinezhad et al. (2018) studies how to refine the labels in ImageNet using label propagation to improve the model performance, demonstrating the importance of the label in ImageNet. This paper mainly falls in the latter branch, which analyzes how label noise acts in the mild overparameterization regimes.

3 Overfitting under Mild Overparameterization

We observed in Figure 1 that training ResNets on CIFAR10 and ImageNet can result in different overfitting behaviors. This discrepancy was not reflected in the previous analysis of benign overfitting. In this section, we provide a theoretical analysis by studying the Gaussian mixture model under the mild overparameterization setup. Our analysis shows that mild overparameterization along with label noise can break overfitting.

3.1 Overparameterized Linear Classification

In this subsection, we study the generalization performance of linear models on the Gaussian Mixture Model (GMM). We assume that linear models are obtained by solving logistic regression with stochastic gradient descent (SGD). This simplified model may help explain phenomenons in neural networks, since previous works show that neural networks converge to linear models as the width goes to infinity (Arora et al., 2019; Allen-Zhu et al., 2019).

We next introduce GMM under two setups of overparameterized linear classification: the noiseless regime and the noisy regime.

Noiseless Regime. Let $y \sim \text{Unif}\{-1, 1\} \in \mathbb{R}$ denote the ground truth label. The corresponding feature is generated by

\[
x = y\mu + \epsilon \in \mathbb{R}^p,
\]

where $\epsilon$ denotes noise drawn from subGaussian distribution. We denote the dataset $D = \{(x_i, y_i)\}_{i \in [n]}$ where $(x_i, y_i)$ are generated by the above above mechanism.

Noisy Regime with contamination rate $\rho$. For the noisy regime, we first generate noiseless data $(x, y)$ using the noiseless regime, and then consider the data point $(x, \tilde{y})$ where $\tilde{y}$ is the contaminated version of $y$. Formally, given the contamination rate $\rho$, the contaminated label $\tilde{y} = -y$ with probability $\rho$ and $\tilde{y} = y$ with probability $1 - \rho$. And the returned dataset is $\tilde{D} = \{(x_i, \tilde{y}_i)\}$.

For simplicity, we assume that the data points in the train set are linearly separable in both noiseless and noisy regimes. This assumption holds almost surely under mild overparameterization. Besides, we make the following assumptions about the data distribution:

Assumption 1 (Assumptions on the data distribution.).

A1 The noise $\epsilon$ when generating feature $x$ is drawn from the Gaussian distribution, i.e., $\epsilon \sim \mathcal{N}(0, \sigma^2 I)$.

A2 The signal-to-noise ratio satisfies $\frac{||\mu||}{\sigma} \geq c\sqrt{\log n}$ for a given constant $c$. 

4
A3 The ratio \( p/n = r > 1 \) is a fixed constant.

The three assumptions are all crucial but can be made slightly more general (see Section 4). The first Assumption [A1] stems from the requirement that we need to derive a lower bound for excess risk under a noisy regime. The second Assumption [A2] is widely used in the analysis. For a smaller ratio, the model may be unable to learn even under the noiseless regime and return vacuous bounds. The third Assumption [A3] is the main difference from the previous analysis, where we consider a mild overparameterization instead of heavy overparameterization (i.e., \( p = \omega(n) \)).

**Training Procedure.** We consider the multi-pass SGD training with logistic loss \( \ell(w; x, y) = \log(1 + \exp(-yx^\top w)) \). During each epoch, each data is visited exactly once randomly without replacement. Formally, at the beginning of each epoch \( E \), we uniformly random sample a permutation \( P_E : \{1, ..., n\} \to \{1, ..., n\} \), then at iteration \( t \), given the learning rate \( \eta \) we have

\[
    w(t + 1) = w(t) - \eta \nabla l(w(t); x(t), y(t)),
\]

where \( x(t) = x_{P_E(t)}, y(t) = y_{P_E(t)} \), given \( t = nE + i, 1 \leq i \leq n \).

Under the above procedure, Proposition 3.1 shows that the classifier under the GMM regime with multi-pass SGD training will converge in the direction of the max-margin interpolating classifier. This paper considers zero initialization where \( w(0) = 0 \) for simplicity.

**Proposition 3.1** (Interpolator of multi-pass SGD under GMM regime, from [Nacson et al., 2019]). Under the regime of GMM with logistic loss, denote the iterates in multi-pass SGD by \( w(t) \). Then for any initialization \( w(0) \), the iterates \( w(t) \) converges to the max-margin solution almost surely, namely,

\[
    \lim_{t \to \infty} \frac{w(t)}{\|w(t)\|} = \tilde{w},
\]

where \( \tilde{w} = \arg \min_{w \in \mathbb{R}^d} \|w\|^2 \) s.t. \( w^\top x_i \geq 1, \forall i \in [n] \) denotes the max-margin solution.

For simplicity, we denote \( w_+(t) \) as the parameter at iteration \( t \) in the noiseless setting, and \( w_-(t) \) as the parameter in the noisy setting. By the proposition above, we know that both \( w_+(\infty) \) and \( w_-(\infty) \) are max-margin classifiers on the training data points.

During the evaluation process, we also focus on the 0-1 loss, where the population 0-1 loss is \( \mathcal{L}_{01}(w) = \mathbb{P}(yw^\top w < 0) \). Based on the above assumptions and discussions, we state the following Theorem 3.1 indicating the different performances between noiseless setting and noisy setting.

**Theorem 3.1.** We consider the above GMM regime with Assumption [A1-A3]. Specifically, denote the noise level by \( \rho \) and the mild overparameterization ratio by \( r = p/n \). Then there exists absolute constant \( c_1, c_2, c_3, c_4, c_5 > 0 \) such that the following statements hold with probability \( 1 - \frac{1}{n} \) at least 1 - \( c_1/n \):

1. **Under the noiseless setting,** the max-margin classifier \( w_+(\infty) \) obtained from SGD has non-vacuous 0-1 loss, namely,
   \[
   \mathcal{L}_{01}(w_+(\infty)) \lesssim n^{-c_2}.
   \]

2. **Under the noisy setting,** the max-margin classifier \( w_-(\infty) \) has vacuous 0-1 loss with constant lower bound, namely, the following inequality holds for any training sample size \( n \),
   \[
   \mathcal{L}_{01}(w_-(\infty)) \geq \min \left\{ \Phi(-2), \frac{\rho}{c_3 r} \exp(-\frac{c_3 r}{\rho}) \right\}.
   \]

3. **Under the noisy setting,** if the learning rate satisfies \( \eta < \frac{1}{c_5 n \max \|x_i\|^2} \), there exists a time \( t \) such that the trained early-stopping classifier \( w_-(t) \) has non-vacuous 0-1 loss, namely,
   \[
   \inf_t \mathcal{L}_{01}(w_-(t)) \lesssim n^{-c_4}.
   \]

\(^1\) The probability is taken over the training set and the randomness of algorithm.
Figure 3: Noisy CIFAR10 under mild overparameterization. In each experiment, the validation accuracy first increases and then dramatically decreases. This confirms point 2 and 3 in Theorem 3.1 that model would overfit under noisy label regimes.

Intuitively, Theorem 3.1 illustrates that although SGD leads to benign overfitting under noiseless regimes, it provably overfits when the labels are noisy. In particular, it incurs $\Omega(1)$ error on noiseless data and hence would incur $\Omega(1) + \rho$ error on noisy labeled data, since label noise in the test set is independent of the algorithm. Furthermore, Theorem 3.1.3 shows that the overfitting is avoidable through early stopping. Therefore, it would be insufficient to consider only the interpolators under the noiseless regimes, and further studying on the early stopping classifier is necessary.

One may doubt the fast convergence rate in Statement One and Statement Three, which seems too good to be true. The strange phenomenon happens because we split the randomness in the training set/algorithm and the randomness in the test set. Note that the first probability is in order $O(1/n)$, and therefore, the total 0-1 loss is approximate $O(1/n)$ after union bound. We refer to Cao et al. (2021); Wang et al. (2021a) for similar types of bounds.

Comparison against benign overfitting under heavy overparameterization: Previous work usually analyze the GMM model under the heavy overparameterization regime, e.g., $p = \Omega(n^2)$ (Cao et al., 2021; Chatterji et al., 2021) or $p = \Omega(n \log(n))$ (Wang et al., 2021a). In comparison, our paper focuses on the mild overparameterization regime, where $p = \Theta(n)$. We note that this leads to a phase change that the overfitting model under noisy settings now provably overfits.

3.2 Experiment in Neural Networks: Overfitting in Noisy CIFAR10

In Section 3.1 we prove that under noisy label regimes with mild overparameterization, early-stopping classifiers and interpolators perform differently. This section aims to verify if the phenomenon empirically happens in the real-world dataset. Specifically, we generate a noisy CIFAR10 dataset where each label is randomly flipped with probability $\rho$. Moreover, we show that the test error first increases and then dramatically decreases, demonstrating that the interpolator performs worse than the models in the middle of the training.

Setup. The base dataset is CIFAR10, where each sample is randomly flipped with probability $\rho \in \{0.1, 0.2, 0.3, 0.4\}$. We use the ResNet18 and use SGD to train the model with cosine learning rate decay. For each model, we train for 200 epochs, test the validation accuracy and plot the training accuracy and validation accuracy in Figure 3. More details can be found in the code.

Figure 3 illustrates a similar phenomenon to the results in linear models under mild overparameterization analysis (see Section 3.1). Precisely, the interpolator achieves suboptimal accuracy under the mild overparameterization regimes, but we can still find a better classifier through early stopping. In
Figure 4: **Three Phase Phenomenon.** The Bayesian optimal classifier is simple and smooth. In phase one, the classifier fits the data at a macroscopic level and ignores those hard-to-fit samples, which is sufficient to reach a good test error. In phase two, the classifier starts to fit those hard-to-fit samples slowly. In phase three, the classifier completely fits those samples, and the test error decreases.

Figure [3], such a phenomenon is manifested as the validation accuracy curve increases and decreases. We also notice that the degree becomes sharper as the noise level increases.

There is another interesting phenomenon during the training process in Figure [3], where the neural networks have an oscillation phase between the increasing time and the decreasing time. That is to say, the training process can be roughly split into three phases:

- **Phase One (Climbing Phase).** The training accuracy and the test accuracy both increase.
- **Phase Two (Oscillation Phase).** The training accuracy and the test accuracy both oscillate.
- **Phase Three (Overfitting Phase).** The training accuracy increases while the test accuracy decreases.

These three phases lead us to make the following conjecture on the neural network training process.

**A conjecture on the training trajectory.** [Nagarajan et al., 2019] conjecture that in overparameterized deep networks, SGD finds a fit that is simple at a macroscopic level but also has many microscopic fluctuations. Stemmed from this insight and the experiment observation, we conjecture that the process to fit at the macroscopic and microscopic levels can be separable under mild overparameterization regimes. Precisely, during the training process, SGD first fits the features at the macroscopic level, which leads to Phase One, where the training accuracy and the test accuracy both increase. Then the model oscillates in preparation for fitting the noise, leading to Phase Two, where the training accuracy and the test accuracy oscillate. Finally, the model oscillates to a proper position and starts to fit the noise, leading to poor generalization in Phase Three. We plot a sketch map in Figure [4] to illustrate such a process. Previous work mainly considers the last iterate at Phase Three, which may heavily rely on the noiseless assumption or heavy overparameterization assumption. However, such analysis can rarely hold in practice since (a) the realistic data usually contains heavy noise due to data collection and data poisoning, and (b) the heavy overparameterization regime becomes impossible as we collect increasingly more data points.

We propose the following experiments based on our conjecture.

### 3.3 Control Experiment: Avoid Overfitting in Noisy Label Regimes

Section [3.2] illustrates that noisy labels can indeed lead to overfitting in Cifar10. This section aims to test whether removing label noise can avoid overfitting. Therefore, motivated by the conjecture in the previous section, this section proposes an experiment that prevents the model from fitting noisy labels by dropping out hard-to-fit samples during training. The procedure of removing hard data points resembles the self-training techniques [Guan et al., 2011; Huang et al., 2020], where they train neural networks using the predictions from another trained network (referred to as the teacher network) and achieve better generalization than the teacher network. Therefore, our analysis and three-phase phenomenon conjecture in Section [3] provide an explanation for the self-training methods. On the other hand, the gain achieved in self-training further validates our three-phase conjecture empirically.

**Setup.** For the self-training experiment, we consider two models based on noisy CIFAR10 and ImageNet, individually and named them as self-trained.

For noisy CIFAR10, we random flip each sample with probability $\rho = 0.2$, and train it with model ResNet18. Starting from epoch 132, in each iteration, we remove the hard data points if the (noisy) train dataset label differs from the model prediction.
Figure 5: Self-training Experiment on CIFAR10 and ImageNet. In both tasks, we first load the same pretrained model for the baseline model and the self-trained model. We then train the self-trained model by iteratively removing hard data points, and train the baseline model using the label from the train set. We note that the self-trained model does not overfit, whereas the baseline model suffers from overfitting.

For the ImageNet dataset, which naturally contains label noise in our conjecture, we train it with model ResNet101. Starting from epoch 89, in each iteration, we remove data points with incorrect top-5 predictions and continue to train the model with the remaining data points.

As a comparison, we also continue to train the models without removing hard data points, named as Baseline.

Result. We plot the training accuracy and the test accuracy of the self-trained model and the baseline in Figure 5 where the training accuracy is calculated based on the whole training set. For both models, the training accuracy increases. Besides, in baseline training, the validation accuracy decreases dramatically due to overfitting the label noise, as observed in Section 3.2. However, in self-training, the trend is effectively stopped, and the validation accuracy in our Self-train experiments consistently increased compared to the early stopped model. This fact is consistent with the previous result of self-training, and researchers use a similar approach to use the model itself to distinguish the possibly mislabeled data [Zhu et al., 2003; Huang et al., 2020].

Another interesting phenomenon is that the training accuracy (calculated on the whole training set) increases in self-training experiments, which means the model is able to correct its own mistake while only training on the data that it has correctly labeled. We leave the analysis for future work.

4 Challenges in Proving Theorem 3.1

This section provides more details to the three statements in Theorem 3.1 with milder assumptions than those in Assumption 2. We also explain why existing analysis cannot be applied to our setup.

Assumption 2. The following assumptions are more general,

A4 The noise \( e \) in \( x \) is generated from a \( \sigma \)-subGaussian distribution.

A5 The signal-to-noise ration satisfies \( \frac{\| \mu \|}{\sigma} \geq c_6 \left( \frac{p}{n} \right)^{\frac{1}{2}} \).

A6 The signal-to-noise ration satisfies \( \frac{\| \mu \|}{\sigma} = \omega \left( \left( \frac{p}{n} \right)^{\frac{1}{2}} \right) \).

We compare the assumptions in Assumption 1 and Assumption 2. Assumption [A4] is a relaxation of Assumption [A1], and Assumption [A5, A6] can be obtained by Assumption [A2, A3]. Therefore, we
conclude that Assumption 2 is weaker than Assumption 1. We next introduce the generalized version of the three arguments in Theorem 3.1 including Theorem 4.1, Theorem 4.2 and Theorem 4.3.

**Theorem 4.1 (Statement One).** Under the noiseless setting, for a fixed \( \delta_1 \geq \max \{ \frac{c}{n}, \exp(-c_8 p) \} \), under Assumption \([A2, A4, A5]\), there exists constant \( c_2 > 0 \) such that the following statement holds with probability at least \( 1 - \delta_1 \),

\[
L_{01}(w_+(\infty)) \lesssim n^{-c_2}.
\]

Previous results on noiseless GMM (e.g., Cao et al. (2021); Wang et al. (2021a)) rely on the heavy overparameterization \( p = \omega(n) \) and assumption \( \left( \frac{p}{n} \right)^{\frac{3}{2}} \leq \frac{\|\mu\|_2}{\sigma} \leq \frac{p}{n} \). In contrast, our results only requires \( \frac{\|\mu\|_2}{\sigma} \geq \left( \frac{\rho}{n} \right)^{\frac{3}{2}} \) and can be deployed in the mild overparameterization regimes. Therefore, the existing results cannot directly imply Theorem 4.1.

**Theorem 4.2 (Statement Two).** Under the noisy regime with noisy level \( \rho \) and mild overparameterization ratio \( r = p/n \), for a fixed \( \delta_2 \geq \max \{ \exp(-c_8 p), c_9 \exp(-c_{10} \rho^2 n) \} \), under Assumption \([A1, A3]\), there exists constant \( c_3 \) such that the following statement holds with probability at least \( 1 - \delta_2 \),

\[
L_{01}(w_-(\infty)) \geq \min \left\{ \Phi(-2), \frac{\rho}{c_3 r} \exp\left(-\frac{c_3 r}{\rho}\right) \right\}.
\]

Therefore, \( w_-(\infty) \) has constant excess risk, given that \( r \) and \( \rho \) are both constant.

Previous results (e.g., Chatterji et al. (2021); Wang et al. (2021a)) mainly focus on deriving the non-vacuous bound for noisy GMM, which also relies on the heavy overparameterization assumption \( p = \omega(n) \). Instead, our results show that the interpolator dramatically fails and suffers from a constant lower bound under mild overparameterization regimes. Therefore, heavy overparameterization performs differently from mild overparameterization cases. We finally remark that although it is still an open problem when the phase change happens, we conjecture that realistic training procedures are more close to the mild overparameterization regime according to the experiment results.

**Theorem 4.3 (Statement Three).** Under the noisy regime, if one runs the SGD update with initialization \( w(0) = 0 \) and learning rate \( \eta < \frac{1}{c_{14} \max_{(x, y) \in D} \|x\|_2^4} \) where \((x, y) \in D\) denotes the data point. For a fixed \( \delta_3 \geq \max \{ \frac{c_{14}}{n}, c_{12} \exp(-c_{13} \rho^2 n) \} \), under Assumption \([A2, A4, A6]\), the following statement holds with probability at least \( 1 - \delta_3 \),

\[
\inf_{t} L_{01}(w_-(t)) \leq \exp \left( -c_{14} \frac{\|\mu\|_2^4}{\frac{\|\mu\|_2^2 \sigma^2}{n} + \sigma^4} \right).
\]

Therefore, the above bound is nonvacuous under the Assumption \([A6]\).

One may wonder whether we can apply the results of stability-based bound (Bousquet et al. 2002; Hardt et al. 2016) into the analysis since the training process is convex. However, the analysis might not be proper due to a bad Lipschitz constant during the training process. Therefore, the stability-based analysis may only return vacuous bound under such regimes. Besides, the previous results on convex optimization with one-pass SGD (e.g., Sekhari et al. 2021) cannot be directly applied to the analysis, since most results on one-pass SGD are expectation bounds, while we provide a high probability bound in Theorem 4.3.

### 5 Conclusions and Discussions

In this work, we aim to understand why benign overfitting happens in training ResNet on CIFAR10, but fails on ImageNet. We start by identifying a phase change in the theoretical analysis of benign overfitting. We found that when the model parameter is in the same order as the number of data points, benign overfitting would fail due to label noise. We conjecture that the noise in labels leads to the different behaviors in CIFAR10 and ImageNet. We verify the conjecture by injecting label noise into CIFAR10 and adopting self-training in ImageNet. The results support our hypothesis.

Our work also left many questions unanswered. First, our theoretical and empirical evidence shows that realistic deep learning models may not work in the interpolating scheme. Still, although there is a larger number of parameters than data points, the model generalizes well. Understanding the implicit bias in deep learning when the model underfits is still open. A closely related topic would be algorithmic stability (Bousquet et al. 2002; Hardt et al. 2016), however, the benefit of overparameterization within the stability framework stills requires future studies. Second, the GMM model provides a convenient way for analysis, but how the number of parameters in the linear setup relates to that in the neural network remains unclear.
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Appendix

This appendix provides proofs of theorems in Section A and show the experiment details in Section C.

A Detailed Proofs

A.1 Proof of Theorem 4.1

The first statement in Theorem 3.1 states that the interpolator has a non-vacuous bound in a noiseless setting, which is a direct corollary of the following Theorem 4.1. The proof of Theorem 4.1 mainly depends on bounding the projection of the classifier $w$ on $\mu$, which relies on a sketch of the classification margin.

Theorem 4.1 (Statement One). Under the noiseless setting, for a fixed $\delta_1 \geq \max\{c_2 n, \exp(-c_8 p)\}$, under Assumption [A2, A4, A5], there exists constant $c_2 > 0$ such that the following statement holds with probability at least $1 - \delta_1$,

$$L_{01}(w_+(\infty)) \lesssim n^{-c_2}.$$ 

Proof of Theorem 4.1. We denote the classifier $w_+(\infty)$ by $w$ during the proof for simplicity. Due to Proposition 3.1, the final classifier converges to its max-margin solution. Without loss of generality, let the final classifier $w$ satisfy $\|w\| = 1$. Therefore, the following equation for margin $\gamma(\cdot)$ holds since $w$ is max-margin solution:

$$\gamma(w) \geq \gamma(\mu/\|\mu\|),$$

where $\gamma(w) = \min_i y_i x_i^T w$ denotes the margin of classifier $w$ for the dataset. We next consider the margin for the classifier $\mu/\|\mu\|$. Note that the margin can be rewritten as

$$\gamma(\mu/\|\mu\|) = \min_i y_i x_i^T w \gtrsim \|\mu\|.$$ (1)

From the other hand, we rewrite the margin as

$$\min_i y_i x_i^T w \leq \mu^T w + \min_i y_i e_i^T w \leq \mu^T w + \frac{1}{n} \sum_{i \in [n]} y_i e_i^T w.$$ (2)

We note that $y_i e_i^T \mu/\|\mu\|$ is $\sigma$-subGaussian due to the definition of subGaussian random vector. Therefore, due to Claim [B.1], we have

$$\gamma(w) \gtrsim \gamma(\mu/\|\mu\|) \geq \|\mu\| - \sigma \sqrt{\log n} \gsim \|\mu\|,$$

where the last inequality is due to Assumption [A2].

We next bound the term $\mu^T w$ via the above margin. From one hand, we notice that by the definition of the margin function,

$$\gamma(w) = \min_i y_i x_i^T w \gtrsim \|\mu\|. \tag{1}$$

From the other hand, we rewrite the margin as

$$\min_i y_i x_i^T w = \mu^T w + \min_i y_i e_i^T w \leq \mu^T w + \frac{1}{n} \sum_{i \in [n]} y_i e_i^T w.$$ (2)
The right hand side can be bounded as

\[
\frac{1}{n} \sum_{i \in [n]} y_i \epsilon_i^T w \\
\leq \| \frac{1}{n} \sum_{i \in [n]} y_i \epsilon_i \| \\
\leq \sigma \sqrt{\frac{p}{n}},
\]

where the final equation is due to Claim B.2. Therefore, combining the above Equation 1, Equation 2 and Equation 3 we bound the projection of \( w \) on \( \mu \) as:

\[
\mu^T w \gtrsim \| \mu \| - \sigma \sqrt{\frac{p}{n}} \gtrsim \| \mu \|,
\]

where the last equation is due to Assumption [A5]. We rewrite Equation (4) as

\[
\mu^T w \geq c_5 \| \mu \|,
\]

then we can bound the 0-1 loss as follows for a given constant \( c_6 \):

\[
L_{01}(w_{\infty}) = P(ye^T w < 0) = P(\mu^T w + y_i \epsilon_i^T w < 0) = P(ye^T w \leq -c_6 \| \mu \|) \leq \exp\left(-c_6 \frac{\| \mu \|^2}{\sigma^2}\right).
\]

Due to Assumption [A2], \( \| \mu \| / \sigma > c \sqrt{n} \), and therefore, by setting \( c_2 = c_6 c^2 \), we have

\[
L_{01}(w) \lesssim n^{-c_2}.
\]

The proof is done. \( \square \)

### A.2 Proof of Theorem 4.2

Theorem 4.1 shows that the interpolator can be non-vacuous under noiseless regimes with mild overparameterization. However, things can be much different in noisy regimes. Statement Two proves a vacuous lower bound for interpolators in noisy settings, which can be derived by the following Theorem 4.2. The core of the proof lies in controlling the distance between the center of wrong labeled samples and the point \( \mu \), which further leads to an upper bound of \( |\mu^T w_{\infty}| \). One can then derive the corresponding 0-1 loss for classifier \( w_{\infty} \).

**Theorem 4.2** (Statement Two). Under the noisy regime with noisy level \( \rho \) and mild overparameterization ratio \( r = p/n \), for a fixed \( \delta_2 \geq \max\{\exp(-c_8 p), c_9 \exp(-c_{10} \rho^2 n)\} \), under Assumption [A1, A3], there exists constant \( c_3 \) such that the following statement holds with probability at least \( 1 - \delta_2 \),

\[
L_{01}(w_{\infty}) \geq \min \left\{ \Phi(-2), \frac{\rho}{c_3 r} \exp\left(-\frac{c_3 r}{\rho}\right) \right\}.
\]

Therefore, \( w_{\infty} \) has constant excess risk, given that \( r \) and \( \rho \) are both constant.

**Proof of Theorem 4.2** We denote \( w_{\infty} \) as \( w \) for simplicity, and assume that \( \| w \| = 1 \) without loss of generality. Let \( y \) denote the original label and \( \tilde{y} \) denote its corrupted label.

Without loss of generality, we consider those samples with \( y_i = 1 \) while \( \tilde{y}_i = -1 \), which are indexed by \( \mathcal{K} = \{i : y_i = 1, \tilde{y}_i = -1\} \). Consider the center point of \( \mathcal{K} \), which is

\[
\bar{x}_\mathcal{K} = \frac{1}{|\mathcal{K}|} \sum_{i \in \mathcal{K}} x_i = \mu \frac{1}{|\mathcal{K}|} \sum_{i \in \mathcal{K}} y_i + \frac{1}{|\mathcal{K}|} \sum_{i \in \mathcal{K}} \epsilon_i = \mu + \frac{1}{|\mathcal{K}|} \sum_{i \in \mathcal{K}} \epsilon_i.
\]
Due to the interpolation in Proposition 3.1 and $\tilde{y} = -1$, we derive that
\[
x^*_K w < 0.
\] (5)

**Case 1:** $\mu^T w < 0$. In this case, $\mathcal{L}_{01}(w)$ naturally has a lower bound of $1/2$ since it even fails in the center point $\mu$.

**Case 2:** $\mu^T w > 0$. In this case, the classifier $w$ satisfies $\mu^T w > 0$ and $\bar{x}_K^T w < 0$. Therefore, the distance between $\mu$ and $\bar{x}_K$ must be less than the distance from $\mu$ to its projection on the separating hyperplane that perpendicular to $w$ through the origin. Formally,
\[
|\mu^T w| \leq \frac{1}{|K|} \left\| \sum_{i \in K} \epsilon_i \right\|.
\] Note that $\epsilon_i$ is independent and $\sigma$-subGaussian, and therefore applying Claim B.2, we have that $\left\| \sum_{i \in K} \epsilon_i \right\| \lesssim \sigma \sqrt{|K|}$. Besides, we derive by Claim B.3 that $|K| \gtrsim \rho n$. Therefore,
\[
|\mu^T w| \leq \frac{1}{|K|} \left\| \sum_{i \in K} \epsilon_i \right\| \lesssim \sigma \sqrt{|K|} \lesssim \sigma \sqrt{\frac{p}{\rho n}}.
\] (6)

We next consider the corresponding test error of $w$, where we consider the test error on noiseless regime instead of noisy regime. Note that the two arguments are equivalent, we refer to Claim B.4 for more details. We rewrite Equation 6 as $|\mu^T w| \leq c \sigma \sqrt{\frac{p}{\rho n}}$, where we abuse the notation $c > 0$ as a fixed constant. Therefore,
\[
\mathbb{P}(y x^T w < 0) \\
= \mathbb{P}(y e^T w < -\mu^T w) \\
\geq \mathbb{P}(y e^T w/\sigma < -c \sqrt{\frac{p}{\rho n}}) \\
= \Phi(-c \sqrt{\frac{p}{\rho n}}),
\]
where $\epsilon$ is sampled from Gaussian distribution, and $\Phi$ denotes the CDF of standard Gaussian Random Variable.

**Case 1.** If $c \sqrt{\frac{p}{\rho n}} \leq 2$, then $\Phi(-c \sqrt{\frac{p}{\rho n}}) \geq \Phi(-2)$.

**Case 2.** If $c \sqrt{\frac{p}{\rho n}} > 2$, note that $\Phi(-t) \geq \left(\frac{1}{t} - \frac{1}{t^3}\right) \exp(-t^2/2) \geq 1/t^2 \exp(-t^2/2)$ if $t > 2$.

Therefore, $\Phi(-c \sqrt{\frac{p}{\rho n}}) \geq \frac{1}{c^2 \rho n} \exp(-c^2 \frac{p}{\rho n}) \geq \frac{1}{c^2 \rho n} \exp(-c^2 \frac{p}{\rho n})$.

Taking the above two cases together and denoting $r = p/n$, we have
\[
\mathcal{L}_{01}(w) \geq \min \left\{ \Phi(-2), \frac{\rho}{c^3 r} \exp(-\frac{c^4 p}{\rho}) \right\},
\] which is a constant lower bound under Assumption [A3]. The proof is done.

### A.3 Proof of Theorem 4.3

Statement Two shows that the interpolator fails in the noisy regime with mild overparameterization. How can we derive a non-vacuous bound under such regimes? The key is early-stopping. To show that, Statement Three provides a non-vacuous bound for early-stopping classifiers in noisy regimes, which is induced by the following Theorem 4.3.

**Theorem 4.3** (Statement Three). Under the noisy regime, if one runs the SGD update with initialization $w(0) = 0$ and learning rate $\eta < \frac{1}{c_5 \sqrt{n} \max_{x_i} \|x_i\|^2}$, where $(x_i, \tilde{y}_i) \in D$ denotes the data point. For
Therefore, the above bound is nonvacuous under the Assumption [A6].

To show the relationship between Theorem 4.3 and Theorem 3.1, one can directly use Assumption [A2, A3] in Theorem 4.3 to reach generalization bound in Theorem 3.1 (Statement Three). The derivation of Theorem 4.3 relies on the analysis on one-pass SGD, where we show that one-pass SGD is sufficient to reach non-vacuous bound. The proof of Theorem 4.3 again, relies on bounding the term c but in a different way. Different from the previous approaches where we can directly assume ∥w_+(∞)∥ = ∥w_-(∞)∥ = 1, the classifier w_-(t) is trained in this case and we need to first bound it. We then define a surrogate classifier and show that (a) the surrogate classifier is close to the trained classifier for a sufficiently small learning rate, and (b) the surrogate classifier can return satisfying projection on the direction μ. Therefore, we bound the term μ_t w_-(t)/∥w_-(t)∥ which leads to the results.

**Proof of Theorem 4.3** We abuse the notation w_n to represent w_-(t) which is returned by one-pass SGD. We first lower bound the term μ∥w_n∥, where μ is the optimal classification direction. To achieve the goal, we bound the term μ_t w_n and ∥w_n∥ individually.

Before diving into the proof, we first introduce a surrogate classifier  \tilde{w}_n = \frac{1}{2} \sum_t x_t \tilde{y}_t. From the definition, we have that for update step size η,

\begin{align}
  w_{t+1} &= w_t + \eta x_t \tilde{y}_t \frac{\exp(-\tilde{y}_t x_t w_t)}{1 + \exp(-\tilde{y}_t x_t w_t)}, \\
  \tilde{w}_{t+1} &= \tilde{w}_t + \frac{1}{2} \eta x_t \tilde{y}_t.
\end{align}

Therefore, we have that

\begin{align}
  w_{t+1} - \tilde{w}_{t+1} &= η \sum_t x_t \tilde{y}_t \frac{\exp(-\tilde{y}_t x_t^T w_t) - 1}{2(1 + \exp(-\tilde{y}_t x_t^T w_t))}, \\
  \mu^T w_{t+1} - \mu^T \tilde{w}_{t+1} &= η \sum_t \mu^T x_t \tilde{y}_t \frac{\exp(-\tilde{y}_t x_t^T w_t) - 1}{2(1 + \exp(-\tilde{y}_t x_t^T w_t))}.
\end{align}

**Bounding the term \mu^T w.** We fist bound the different between \mu_t^T w_t and \mu_t^T \tilde{w}_t. We note that

To bound the above different, the fist step is to bound the term max_{t \in [n]} \mu_t x_t \tilde{y}_t. Since \mu_t^T e_t/∥\mu∥ is σ^2-subGuassian, we have that with probability 1 - δ_1 with δ_1 \leq 1/n,

\begin{align}
  \max_{t \in [n]} |\mu^T x_t \tilde{y}_t| &\leq ∥\mu∥^2 + \max_t |\mu_t^T e_t| \leq ∥\mu∥^2 + ∥\mu∥ σ \sqrt{\log(n/δ_1)} \leq ∥\mu∥^2 + ∥\mu∥ σ \sqrt{\log(n)} \leq ∥\mu∥^2.
\end{align}

We then bound the term \frac{\exp(-\tilde{y}_t x_t^T w_t) - 1}{2(1 + \exp(-\tilde{y}_t x_t^T w_t))}. Note that \exp(u) - 1 \leq 2|u| when |u| < \frac{1}{2}. By the iteration in Equation 8, we have that

\begin{align}
  \max_{t \in [n]} ∥w_t∥ &\leq n η (\max_t ∥x_t∥), \\
  \max_{t \in [n]} |x_t^T w_t| &\leq n η (\max_t ∥x_t∥^2) \leq \frac{1}{2}.
\end{align}

where the first equation is due to the iteration, and the last equation is due to η \leq \frac{1}{2n \max_t ∥x_t∥} (by setting c_5 > 2). Therefore,

\begin{align}
  \left| \frac{\exp(-\tilde{y}_t x_t^T w_t) - 1}{2(1 + \exp(-\tilde{y}_t x_t^T w_t))} \right| &\leq \left| \frac{\exp(-\tilde{y}_t x_t^T w_t) - 1}{2} \right| \leq |x_t^T w_t| \leq \frac{1}{2}.
\end{align}
Combining Equation [10] and Equation [11] we have that with probability at least \(1 - \delta_1\),
\[
\max_{t \in [n]} |\mu^T w_t - \mu^T \tilde{w}_t| \lesssim \eta n \|\mu\|^2.
\] (12)

On the other hand, we show the bound for \(\mu^T \tilde{w}_n\). Note that \(\tilde{w}_{t+1} = \eta/2 \sum_t x_t \tilde{y}_t\), therefore,
\[
\frac{2}{n \eta} \mu^T \tilde{w}_n = \|\mu\|^2 \frac{1}{n} \sum_{i \in [n]} I_i(\rho) + \frac{1}{n} \sum_{i \in [n]} \mu^T \epsilon_i \tilde{y}_i,
\]
where we denote \(I_i(\rho) \in \{-1, 1\}\) as a random variable which takes value \(-1\) with probability \(\rho\) and takes value \(+1\) with probability \(1 - \rho\).

Due to Claim B.3 we have \(\frac{1}{n} \sum_{i \in [n]} I_i(\rho) \gtrsim \rho\), where we note that \(I(\rho) \in \{-1, 1\}\). Besides, since \(\mu^T \epsilon_i \tilde{y}_i\) is \(\|\mu\|\sigma\text{-subGaussian},\) we have that with probability \(1 - \frac{1}{n}\),
\[
\frac{1}{n} \sum_{i \in [n]} \mu^T \epsilon_i \tilde{y}_i \lesssim \|\mu\| \sigma \sqrt{\log(n)},
\]
where the term \(\log(n)\) comes from the probability \(1/n\). In summary, we have that
\[
\frac{2}{n \eta} \mu^T \tilde{w}_n \gtrsim \rho \|\mu\|^2 - \|\mu\| \sigma \sqrt{\log(n)} \gtrsim \|\mu\|^2,
\] (13)
where the last equation follows Assumption [A2].

Combining Equation (12) and Equation (13), we have
\[
\mu^T \tilde{w}_n \gtrsim \mu^T \tilde{w}_n - \mu^T \tilde{w}_n - \mu^T w_n \gtrsim \eta n \|\mu\|^2.
\] (14)
We additionally note that Equation 14 holds by choosing proper constant in Equation 11 (and in the choice of \(\eta\)).

Bounding the norm \(\|w_n\|\). We next bound the norm \(\|w_n\|\). Before that, we first bound the norm \(\|\tilde{w}_t\|\). Note that
\[
\max_{t \in [T]} \frac{2}{\eta} \|\tilde{w}_t\| = \max_{t \in [T]} \|\sum_{i \in [t]} x_i \tilde{y}_i\|
\]
\[
= \max_{t \in [T]} \|\mu^T \sum_{i \in [t]} I_i(\rho) + \sum_{i \in [t]} \epsilon_i \tilde{y}_i\|
\]
\[
\leq \max_{t \in [T]} \|\mu^T \sum_{i \in [t]} I_i(\rho)\| + \|\sum_{i \in [t]} \epsilon_i \tilde{y}_i\|
\]
\[
\lesssim T \|\mu\| + \sigma \sqrt{pT},
\] (15)
where the last equation is due to Claim B.1 by choosing probability \(\delta \gtrsim \exp(-p)/n\).

Therefore, we have
\[
\sup_{t \in [n]} \|\tilde{w}_t\| \lesssim \eta n \|\mu\| + \eta \sigma \sqrt{m}.
\]

Note that according to the iteration in Equation 8 we have that
\[
\|w_{t+1} - \tilde{w}_{t+1}\| \leq \eta t \max_i \|x_i\|^2 \|w_i\| \leq \frac{1}{2} \|w_t\|,
\]
where the last equation is due to \(\eta \leq \frac{1}{2n \max_i \|x_i\|^2}\).

Therefore, we bound the norm \(w_1\) as
\[
\|w_n\| \leq \|w_n - \tilde{w}_n\| + \|\tilde{w}_n\| \leq \frac{1}{2} \|w_{n-1}\| + \|\tilde{w}_n\|
\]
\[
\leq \frac{1}{4} \|w_{n-2}\| + \frac{1}{2} \|w_{n-1}\| + \|\tilde{w}_n\|
\]
\[
\leq \ldots
\]
\[
\leq 2 \eta n \|\mu\| + n \sigma \sqrt{m}
\]
\[
\lesssim \eta \|\mu\| + \eta \sigma \sqrt{m}.
\] (16)
Combining Equation [14] and Equation [16] we derive that with probability at least \(1 - c/n\),
\[
\frac{\mu^\top w_n}{\|w_n\|} \geq \frac{\eta n\|\mu\|^2}{\eta n\|\mu\| + \eta\sigma^2\sqrt{m}}.
\]

We next consider the probability on the testing point, note that given the dataset \((x_i, y_i)\) and taking probability on the testing point \((x, y)\), we have
\[
P(yx^\top w < 0) = P(yx^\top w/\|w\| \leq -\mu^\top w/\|w\|) \leq \exp\left(-c\|\mu\|^2/\|w\|^2\sigma^2\right),
\]
where \(yx^\top w = \mu^\top w + ye^\top w\) and \(ye^\top w/\|w\|\) is \(\sigma\)-subGaussian. Plugging Equation [17] into the above equation, we have that with high probability,
\[
P(yx^\top w < 0) \leq \exp\left(-c_{14}\|\mu\|^4/\|\mu\|^2\sigma^2 + \sigma^4\frac{c}{n}\right).
\]

If \(\|\mu\|/\sigma > \sqrt{p/n}\), \(\|\mu\|^4/\|\mu\|^2\sigma^2 + \sigma^4\frac{c}{n} \geq \|\mu\|/\sigma\).

If \(\|\mu\|/\sigma < \sqrt{p/n}\), \(\|\mu\|^4/\|\mu\|^2\sigma^2 + \sigma^4\frac{c}{n} \geq \frac{\|\mu\|^2}{\sigma^2}\sqrt{n/p}\), which is large when \(\|\mu\|/\sigma = \omega((p/n)^{1/4})\).

Therefore, the above bound is non-vacuous if \(\|\mu\|/\sigma = \omega((p/n)^{1/4})\).

Note that for given a constant \(c = p/n\), under Assumption [A2], we have
\[
P(yx^\top w < 0) \leq \exp\left(-c_{14}\frac{\|\mu\|^4}{\|\mu\|^2\sigma^2 + \sigma^4\frac{c}{n}}\right) \leq \exp\left(-c_{14}\frac{\|\mu\|^4}{\sigma^2}\right) \leq n^{-c_4}.
\]

The proof is done. \(\square\)

### B Technical Lemmas

We next provide some technical claims. The first claims bound the maximum of a sequence of subGaussian random variables:

**Claim B.1** (Maximum of a sequence of subGaussian random variables.). For a sequence of \(\sigma\)-subGaussian random variables \(X_1, \ldots, X_n \in \mathbb{R}\). We have that with probability at least \(1 - \delta\),
\[
\max_i |X_i| \leq 2\sigma(\sqrt{\log n} + \sqrt{\log(1/\delta)})
\]
Specifically, under the condition that \(\delta \gtrsim 1/n\), we have
\[
\max_i |X_i| \lesssim \sigma\sqrt{\log n}.
\]

**Proof of Claim B.1** By taking union bound, we have
\[
P(\max_i X_i \geq u) = P(\exists i : X_i \geq u)
\leq nP(X_1 \geq u)
\leq n\exp\left(-\frac{u^2}{2\sigma^2}\right).
\]

By setting \(u = \sqrt{2}\sigma(\sqrt{\log n} + \sqrt{\log(1/\delta)})\), we have
\[
P(\max_i X_i \geq \sqrt{2}\sigma(\sqrt{\log n} + \sqrt{\log(1/\delta)}))
\leq n\exp\left(-\frac{\sqrt{2}\sigma(\sqrt{\log n} + \sqrt{\log(1/\delta)}))}{\sigma^2}\right)
\leq n\exp\left(-\left(\sqrt{\log n} + \sqrt{\log(1/\delta)}\right)^2\right)
\leq \delta.
\]
Therefore, with probability at least $1 - \delta$, we have
\[
\max_i |X_i| \leq \sqrt{2\sigma (\sqrt{\log n} + \sqrt{\log(1/\delta)})}
\]

The next claim bounds the norm of sum of epsilon.

**Claim B.2.** Under Assumption [A4] which assumes that $\epsilon_i$ are subGaussian, we have that with probability at least $1 - \delta$,
\[
\left\| \frac{1}{n} \sum_{i \in [n]} \epsilon_i \right\| \leq 4\sigma \sqrt{\frac{p}{n} + 2\sigma \frac{\log(1/\delta)}{n}}.
\]

Specifically, under the condition that $\delta \gtrsim \exp(-4p)$, it holds that
\[
\left\| \frac{1}{n} \sum_{i \in [n]} \epsilon_i \right\| \lesssim \sigma \sqrt{\frac{p}{n}}.
\]

**Proof of Claim B.2.** Note that since $\epsilon_i$ is $\sigma^2$-subGaussian, its summation $\sum_{i \in [n]} \epsilon_i$ is $n\sigma^2$-subGaussian. Therefore, we have that
\[
\left\| \frac{1}{n} \sum_{i \in [n]} \epsilon_i \right\| \leq 4\sigma \sqrt{\frac{p}{n} + 2\sigma \frac{\log(1/\delta)}{n}}.
\]

The next claim shows that the number of noisy data is approximately in the same order with $\rho n$.

**Claim B.3 (Number of noisy data).** Denote $|K|$ as the number of samples with flipped labels. With probability at least $1 - \delta$, there exists a constant $c$ such that
\[
||K| - \rho n| \leq \sqrt{cn \log(2/\delta)}.
\]

Furthermore, when $\delta \gtrsim 2 \exp\left(-\frac{1}{2} \rho^2 n\right)$, we have that with probability at least $1 - \delta$,
\[
|K| \gtrsim \frac{1}{2} \rho n \text{ and } |K| \lesssim \frac{3}{2} \rho n.
\]

**Proof of Claim B.3.** Note that $K = \sum_i u_i$, where $u_i$ are drawn from independent Bernoulli distribution with parameter $\rho$. Therefore, by Hoeffding’s inequality, there exists constant $c$ such that with probability $1 - 2 \exp\left(-\frac{ct^2}{n}\right)$,
\[
||K| - \rho n| \leq t.
\]

By setting $t = \sqrt{\frac{n}{c} \log\left(\frac{2}{\delta}\right)}$, we have that with probability at least $1 - \delta$,
\[
||K| - \rho n| \leq \sqrt{\frac{n}{c} \log\left(\frac{2}{\delta}\right)}.
\]

We then rewrite the constant $c$ to reach the above conclusion.

The next Claim B.4 shows that analyzing $\mathcal{L}_{01}$ loss on noiseless data is sufficient under noisy regimes.

**Claim B.4.** Under the noisy regime, the 0-1 population loss on noiseless data $\mathcal{L}_{01}(w) = \mathbb{P}(y^T x^T w < 0)$ has the same order with the 0-1 population loss (excess risk) on noisy data $\tilde{\mathcal{L}}_{01}(w) - \eta = \mathbb{P}(\tilde{y}^T x^T w < 0) - \eta$. Therefore, we analyze $\mathcal{L}_{01}(w)$ in this paper as a surrogate of $\tilde{\mathcal{L}}_{01}(w)$.
Proof of Claim B.4. Note that we here consider the $\mathcal{L}_0$ on the noiseless data, and one can get $\tilde{\mathcal{L}}_0$ on the noisy data by just adding $\rho$ to $\mathcal{L}_0$. That is to say,

\[
\tilde{\mathcal{L}}_0 = \mathbb{P}(\tilde{y} \mathbf{x}^\top \mathbf{w} < 0) = \eta \mathbb{P}(-y \mathbf{x}^\top \mathbf{w} < 0) + (1 - \eta) \mathbb{P}(y \mathbf{x}^\top \mathbf{w} < 0) = \eta (1 - \mathcal{L}_0) + (1 - \eta) \mathcal{L}_0 = \eta + (1 - 2\eta) \mathcal{L}_0.
\]

And therefore $\tilde{\mathcal{L}}_0 - \eta$ has the same order with $\mathcal{L}_0$. \qed

C Experiments

C.1 Implemented Tasks

We conduct experiments on three public available datasets along with synthetic GMM data. First, we briefly introduce each dataset.

**ImageNet** is an image classification dataset containing 1.2 million pictures belonging to 1000 different classes. **CIFAR10** is an image classification dataset containing 60000 pictures belonging to 10 different classes. **Penn Tree Bank (PTB)** is a dataset containing natural sentences with 10000 different tokens. We use this dataset for language modelling task. **Synthetic GMM data** is generated following the setup in Section 3.1. We generate data for $n \in \{2^4, ..., 2^9\}$, $p \in \{2^4, ..., 2^9\}$, $\rho \in \{0, 0.4\}$, $|\mu| = 40$, $\sigma = 1$, $n \leq p$.

We then describe experiment details for each dataset.

For **ImageNet**, we train a ResNet101 from scratch. We use standard cross entropy loss. We first train the model for 90 epochs using **SGD** as optimizer, with learning rate $1e^{-2}$, momentum 0.9 and weight decay $1e^{-4}$. The learning rate will decay by a factor of 10 for every 50 iterations. Then we train the model for another 70 epochs, with initial learning rate $8e^{-4}$ and learning rate decay at epoch 10 and 60 by a factor of 1.25. For control experiments, we replace the second phase of training using algorithm 1 with initial learning rate $8e^{-5}$.

For **CIFAR10**, we randomly flip the label with probability $\{0, 0.1, 0.2, 0.3, 0.4\}$ and for each train a ResNet18 from scratch. We use standard cross entropy loss. We train each model for 200 epochs using **SGD** as optimizer, with learning rate $1e^{-1}$, momentum 0.9 and weight decay $5e^{-4}$. We use a cosine learning rate decay scheduler. For control experiment, we use the checkpoint saved at epoch 132 for dataset with label noise 0.2 during the previous training. For both baseline and self-train experiment, we reinitialize a cosine learning rate decay scheduler for 68 epochs and initial learning rate $1e^{-3}$.

For **Penn Tree Bank**, we train a standard transformer from scratch. We train the model for 4800 epochs using **ADAM** as optimizer, with learning rate $5e^{-4}$, beta $(0.9, 0.98)$ and weight decay $1e^{-2}$. We use an inverse square learning rate scheduler.

For **Synthetic GMM data**, we train a linear classifier for each dataset, initialized from 0. We use logistic loss. We train each model using **SGD** as optimizer with learning rate $1e^{-5}$ until training loss decrease below 0.05.

C.2 Overfitting on PTB with language modelling

In this subsection, we show that the training a small Transformer on PTB for language modelling also leads to overfitting, as shown in Figure 6. This is expected based on our analysis. If we view language modelling as a classification task: predicting the next word based on a prefixed sequence, then the ground truth label will naturally be noisy.

C.3 Self-Train Algorithm

In Section 3.3 we introduce a self-train algorithm, as shown in Algorithm 1. Furthermore, we specify its implementation details on ImageNet and Cifar10 as follows.
Figure 6: **Overfitting Behavior for language modelling.** We use transformer to train language modelling on Penn Tree Bank and plot the training loss as well as validation loss. We find that transformer overfits Penn Tree Bank significantly.

**Input:** Pretrained Model $M_0$, Batch Size $B$, Epoch Number $N$, Tolerance Number $K$, Loss criterion $L$

**Output:** Finetuned Model $M_t$

**Data:** Training Set $S = \{x_i, y_i, 0 \leq i \leq n\}$

**Function** Self Train:

1. Load model: $M = M_0$
2. for epoch = 0, 1, ..., $N - 1$ do
3.   for iter = 0, 1, ..., $n / B$ do
4.     losses = list()
5.     for i = 0, 1, ..., $B - 1$ do
6.       logit = $M(x_i)$
7.       if $y_i$ is in the largest $K$ number of logit then
8.         losses.append($L$($\text{logit}, y_i$))
9.     end
10.    end
11.   loss = losses.mean()
12.   loss.backward()
13.  Update model;
14. end
15. $M_t = M$

return