Exotic baryons with charm number ±1 from Skyrme model

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We illustrate the exotic SU(3) baryon sub-multiplets in the SU(4) baryon multiplets predicted from the flavor SU(4) collective-coordinate quantization, and investigate the exotic states with charm number \( C = \pm 1 \) up to leading order of \( 1/N_c \) under chiral SU(3)_L \times SU(3)_R symmetry and the heavy quark limit from bound state approach. We find that there exist the 15-plets and 24-plets with \( C = 1 \) and the 6-plets, 15-plets and 15'-plets with \( C = -1 \) bounded in this approach, qualitatively consistent with predictions from the flavor SU(4) collective-coordinate quantization. By fitting one unique parameter of leading SU(3) flavor symmetry breaking term with the mass difference between \( \Xi_c \) and \( \Sigma_c \) and up to \( O(m_0^0 N_c^{-1}) \), we give all the average masses of baryons in 6-plet and 15-plet. Several general relations of these masses without any parameter are also introduced.

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I. INTRODUCTION

Skyrme’s soliton picture of nucleons [1] has been successfully extended to the case of three light flavors of quarks by directly generalizing collective-coordinate method from SU(2) to the SU(3) case [2]. And there is an alternative approach that hyperons are treated as bound states of solitons and \( K \) mesons by Callan and Klebanov (CK) [3]. CK method was first suggested to be applicable to charmed or bottomed baryons in [4]. And Walliser extended the collective-coordinate method in the flavor SU(3) Skyrme model to the general SU(\( N_f \)) case. However, because the mass of the heavy flavor quarks \( m_{c,b,t} \) is larger than \( \Lambda_{QCD}(\sim 200 \text{ MeV}) \) in contrast with that of the light flavors \( m_{u,d,s} \), the physical world respects the approximate light flavor SU(3) chiral symmetry more reliably. A physical system involving both light flavors\( (u, d, s) \) and heavy flavors\( (c, b) \) has the heavy quark flavor symmetry and the heavy quark spin symmetry in the heavy quark limit (\( m_Q \to \infty \)), as well as the chiral symmetry for the light quark system, i.e., the dynamics of the system is unchanged under the exchange of heavy quark flavors and under arbitrary transformations on the spin of the heavy quarks. In this limit, the spin of heavy quark \( S_Q \) is conserved as well as the total angular momentum \( J \), thus the spin of light degrees of freedom \( S_l = J - S_Q \) is also conserved. Combined with these symmetries, the CK approach has been successfully extended to describe the static properties of the heavy baryons with one charm or bottom quark [5, 6, 7, 8, 9], and also been used to chiral SU(3) case [10].

Both the flavor SU(4) collective-coordinate quantization and the bound state approaches predicted the existence of charmed baryon multiplets with exotic baryons. From the SU(\( N_f \)) symmetric collective Hamiltonian [5], we can see that the 60-plet with spin \( S=1/2 \) and the 140-plets with \( S=(3/2,1/2) \) are the lightest SU(4) baryon multiplets next to the SU(4) 20-plet with spin 1/2 and...
20'-plet with spin 3/2 \[11\]. We illustrate the weights of the SU(4) representations 60 = (0, 2, 1) and 140 = (2, 1, 1) in Figs. \[4\] and \[2\] where the minimal quark contents of the exotic states are also suggested. In the SU(4) 60-plet, there exists a light flavor SU(3) baryon multiplet 15-plet with \(C = 1\) and \(S = 1/2\) and a \(\bar{6}\)-plet with \(C = -1\) and \(S = 1/2\), while in the SU(4) 140-plets, there are 15-plets with \(C = 1\) and \(S = (3/2, 1, 2)\), 24-plets with \(C = 1\) and \(S = (3/2, 1/2)\) and 15-plets with \(C = -1\) and \(S = (3/2, 1/2)\). These SU(3) baryon multiplets all contain exotic states with \(C = \pm 1\). Bound state approaches also predicted the existence of heavy antiquark states with minimal five-quark configurations \(uudd\) and \(uuddb\) in the quark language \[12\,13\].

The main purpose of this paper is to study the exotic states with charm number \(C = \pm 1\) from bound state approach in three light flavor case under chiral SU(3)\(_L\) \(\times\) SU(3)\(_R\) symmetry and the heavy quark limit. There are two approaches to describe heavy baryons as bound states of solitons and heavy mesons. In \[4\,5\,6\], solitons are first quantized to produce light baryon states. Then, heavy baryons are constructed as bound states of these light baryons and heavy mesons with explicit spin and isospin. This is different from the approach in \[8\,9\], where heavy-meson-soliton bound states are first found and then quantized by the collective coordinate method to give baryon states with heavy quarks. In this paper, we investigate exotic states with one anti-quark (\(7\)) as well as those with one heavy quark (\(c\)) and compare the results with those in Figs. \[4\] and \[2\] predicted from the flavor SU(4) collective-coordinate quantization, and give the mass spectra of the \(\bar{6}\)-plet and 15-plet.

The paper is organized as follows. In Sec. II, we give both the Lagrangian of a physical system only involved with heavy mesons containing one flavor of heavy quark and that with their anti-particles. Then, in Sec. III, we briefly review the approach in \[6\] under chiral SU(3)\(_L\) \(\times\) SU(3)\(_R\) symmetry and the heavy quark limit, and generalize it to describe baryons with one heavy anti-quark (\(\bar{c}\)). After that, we study the exotic states with \(C = \pm 1\) by this approach, and calculate the average masses of the \(\Theta^0(udd\bar{c})\) and \(\Theta^*_{1}(uudd\bar{c}, uudd\bar{c}, uuud\bar{c})\) states, which belong to the \(\bar{6}\)-plets with \(S_l = 1\) and 15-plet with \(S_l = 1\) in Figs. \[4\] and \[2\] respectively. In Sec. V, we quantize the chiral SU(3) symmetric effective Lagrangian and give the mass spectra of the \(\bar{6}\)-plet and 15-plet baryons. And in Sec. VI, we compare the results with those in Figs. \[4\] and \[2\] and give our conclusion.

## II. THE EFFECTIVE LAGRANGIAN

We first introduce the effective Lagrangian which respects both chiral symmetry of the light flavors and heavy flavor and spin symmetry. The SU(3) symmetric action for the light flavor system is of the form

\[
I_l = \frac{L^2}{4} \int d^4 x \text{Tr} (\partial_{\mu} \Sigma \partial_{\nu} \Sigma^\dagger) + \frac{1}{2} \int d^4 x \text{Tr} (\partial_{\mu} \Sigma \Sigma^\dagger, \partial_{\nu} \Sigma \Sigma^\dagger)^2 + N \langle \Sigma \rangle \equiv \int d^4 x \mathcal{L}_l,
\]

where \(f_\pi \approx 93\) MeV is the observed pion decay constant; the second term is the so-called Skyrme quartic term, which contains the dimensionless parameter \(e\) to stabilize the solitons; \(\Gamma\) is the Wess-Zumino-Witten term; the chiral fields \(\Sigma(x)\) are elements of SU(3).

The heavy meson field for the ground state \(Q\bar{q}_a\) (0\(^-\) and 1\(^-\)) mesons can be represented by a \(4 \times 4\) matrix field \(H_a\) \[14\]

\[
H_a = \frac{1 + \gamma_5}{2} [P_{a\mu} \gamma^\mu - P_{a\gamma_5}],
\]

where \(P_a\) and \(P_{a\mu}\) represent the pseudoscalar (0\(^-\)) and the vector (1\(^-\)) meson fields respectively, which annihilate the \(s_l = 1/2\) meson multiplet and move with a four velocity \(v_\mu\). However, \(P_a\) and \(P_{a\mu}\) do not create the corresponding antiparticles by the particle projector \(\frac{1 + \gamma_5}{2}\) in the heavy quark limit, thus, a new field \(\tilde{H}_a\) is introduced to describe the antiparticles of the heavy mesons \(q_a\bar{Q}\) \[15\]

\[
\tilde{H}_a = [\bar{P}_{a\mu} \gamma^\mu - \bar{P}_{a\gamma_5}] \frac{1 - \gamma_5}{2},
\]

where \(\bar{P}_a\) and \(\bar{P}_{a\mu}\) annihilate the antiparticles of the \(s_l = 1/2\) meson multiplet and move with a four velocity \(\bar{v}_\mu\).

The fields \(H_a\) and \(\tilde{H}_a\) transform as a \((1/2, 1/2)\) representation of \(S_Q \oplus S_l\) and \(S_l \oplus S_Q\) respectively. Under SU(2)\(_Q\) \(\times\) SU(2)\(_l\), \(H_a(x)\) and \(\tilde{H}_a\) transform as

\[
H_a \to S_Q H_a S_l^\dagger,
\]

\[
\tilde{H}_a \to S_l \tilde{H}_a S_Q^\dagger,
\]

where \(S_Q \in SU(2)_Q\) and \(S_l \in SU(2)_l\). Similarly, under the isospin transformation, \(I_H \in SU(2)_l\), we have

\[
H_a \to H_a I_H,
\]

\[
\tilde{H}_a \to I_{Hab} \tilde{H}_b,
\]

For considering the couplings of heavy mesons to the
FIG. 2: The exotic pentaquark states in the SU(4) 140-plet

pseudo-Goldstone bosons, it is convenient to introduce

$$\xi = \sqrt{\Sigma},$$

which under the SU(3)$_L \times$ SU(3)$_R$ chiral symmetry transforms as

$$\xi \rightarrow L \xi U^\dagger = U \xi R^\dagger,$$

where $U$ is a function of $L$, $R$ and the chiral field. Then, under chiral symmetry, $H_a$ and $\tilde{H}_a$ transform as

$$H_a \rightarrow H_b U_{ba}^\dagger,$$

$$\tilde{H}_a \rightarrow U_{ab} \tilde{H}_b.$$  \hfill (10)

By including allowed terms with one derivation for the heavy-meson-light-meson interaction, the Lagrangians which respect both chiral symmetry and heavy quark spin and flavor symmetry are of the form \cite{13, 14}

$$\mathcal{L} = \mathcal{L}_l - i\nu_\mu \text{Tr}(D^\mu H \overline{H}) + g \text{Tr}(H \gamma^\mu \gamma_5 A_\mu \overline{H}),$$

$$\tilde{\mathcal{L}} = \tilde{\mathcal{L}}_l - i\nu_\mu \text{Tr}(\overline{H} D^\mu \tilde{H}) + g \text{Tr}(\overline{H} \gamma^\mu \gamma_5 A_\mu \tilde{H}),$$

with

$$D_\mu H = H(\overline{\partial}_\mu + V^\dagger_\mu) = H(\overline{\partial}_\mu - V_\mu),$$

$$D_\mu \tilde{H} = (\partial_\mu + V_\mu) \tilde{H},$$

$$\overline{H} = \gamma^0 H_{2\gamma_5}^\dagger, \quad \overline{H} = \gamma^0 \tilde{H}_{2\gamma_5}^\dagger,$$

where $V_\mu$ and $A_\mu$ are defined as

$$V_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger),$$

$$A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger).$$

However, because of the singularity in $\xi$, it is convenient to redefine $H_a$ as \cite{6}

$$H'_a = H_b \xi_{ba},$$

and, similarly, we redefine $\tilde{H}_b$ as

$$\tilde{H}'_a = \xi_{ab} \tilde{H}_b.$$
Then, the Lagrangians become

$$\mathcal{L} = \mathcal{L}_I - iv_\mu \text{Tr}(\partial^\mu H^\dagger \mathbf{T}^\dagger) + \frac{i}{2} g_\mu \text{Tr}(H^\dagger \Sigma^I \partial^\mu \Sigma \mathbf{T}^\dagger),$$

$$\tilde{\mathcal{L}} = \mathcal{L}_I - iv_\mu \text{Tr}(\mathbf{H}^\dagger \partial^\mu \mathbf{H}^\dagger) - \frac{i}{2} g_\mu \text{Tr}(\mathbf{H}^\dagger \Sigma^I \partial^\mu \Sigma \mathbf{H}^\dagger),$$

(21)

(22)

Then, under the transformations (11)-(17), we have the following operators from above Lagrangians

$$S^k_{iH} = \int d^3x \text{Tr}(\frac{1}{2} H^\dagger \sigma^k \mathbf{T}^\dagger),$$

$$S^k_{iH} = -\int d^3x \text{Tr}(\frac{1}{2} H^\dagger \sigma^k \tilde{\mathbf{H}}^\dagger),$$

$$I^k_{iH} = \int d^3x \text{Tr}(\frac{1}{2} H \lambda^k \mathbf{T}^\dagger),$$

$$I^k_{iH} = -\int d^3x \text{Tr}(\frac{1}{2} H \lambda^k \tilde{\mathbf{H}}^\dagger),$$

(23)

(24)

(25)

(26)

where $\sigma^i = \frac{1}{2} i \epsilon^{ijk}[\gamma^j, \gamma^k]$, $i, j, k = 1, 2, 3$, and $\lambda^k$ are the tree generators of the SU(3) group.

III. THE FORMULISM FOR HEAVY BARYONS FROM BOUND STATE APPROACH

Under the assumption of maximal symmetry, there exists a solitonic solution (with unit baryonic charge) of the equation of motion from (11)-(2)

$$\Sigma_1(\mathbf{x}) = \begin{pmatrix} \exp[i(\mathbf{r} \cdot \tau)F(r)] & 0 \\ 0 & 1 \end{pmatrix},$$

(27)

where $F(r)$ is the spherical-symmetric profile of the soliton with $F(0) = \pi$ and $F(\infty) = 0$, $\tau$ are the three Pauli matrices, and $\mathbf{r}$ is the unit vector in space. The action is invariant under the SU(3)$_L \times$ SU(3)$_R$ transformation. However, in order to describe soliton solutions with the same vacuum, it is only necessary to deal with those with the form:

$$\Sigma(x) = A(t)\Sigma_1(\mathbf{x})A(t)^{-1}, \quad A \in SU(3).$$

(28)

With this assumption, after quantizing Eq. (11) in collective coordinate $A$, we will get the SU(3) symmetric Hamiltonian (2) and from it we can see the lowest representations of light solitons (baryons) are the octet with $s_l=1/2$, the decuplet with $s_l=3/2$, the anti-decuplet with $s_l=1/2$, the 27-plet with $s_l=3/2, 1/2$, etc. The light baryon wave functions in the collective coordinates $A$ are

$$\Psi^{(\mu)}_{\nu \nu'}(A) = \sqrt{\text{dim}(\mu)} D^{(\mu)}_{\nu \nu'}(A),$$

(29)

where $(\mu)$ denotes an irreducible representation of the SU(3) group; $\nu$ and $\nu'$ denote $(Y, I, I_3)$ and $(1, J, -J_3)$ quantum numbers collectively; $Y$ is the hypercharge of the corresponding light baryons; $I$ and $I_3$ are the isospin and its third component respectively; $J$ is the third component of spin $J$; and $D^{(\mu)}_{\nu \nu'}(A)$ are representation matrices of the SU(3) group.

Besides the collective coordinate quantized chiral symmetric Hamiltonian, the interaction Hamiltonian of Eq. (21) by semiclassical approximation is

$$H'_I = \frac{gF'(0)}{2} \int d^3x \text{Tr}(H^\dagger \gamma^5 A \lambda_3 A^I \mathbf{H}^\dagger) = 2gF'(0)I^k_{iH}S^k_{iH} D^{(8)}_{kj},$$

(30)

and from Eq. (22), we have

$$\tilde{H}'_I = \frac{gF'(0)}{2} \int d^3x \text{Tr}(\mathbf{H}^\dagger \gamma^5 A \lambda_3 A^I \tilde{\mathbf{H}}^\dagger) = -2gF'(0)I^k_{iH}S^k_{iH} D^{(8)}_{kj},$$

(31)

with

$$I^k_{iH} H = -\frac{\sigma^k}{2},$$

$$S^k_{iH} H = -\frac{\sigma^k}{2},$$

(32)

(33)

and

$$I^k_{iH} \tilde{H} = \frac{\sigma^k}{2} \tilde{H},$$

$$S^k_{iH} \tilde{H} = \frac{\sigma^k}{2} \tilde{H},$$

(34)

(35)

where $D^{(8)}_{kj}$ is the adjoint representation of the SU(3) group and defined as

$$D^{(8)}_{kj}(A) = \frac{1}{2} \text{Tr}(A^\dagger \lambda_k A \lambda_j),$$

(36)

$$S^k_{iH}, I^k_{iH}, S^k_{iH} \text{ and } I^k_{iH}$$

are the quantum mechanics operators corresponding to (28)-(30). We can see that the interaction $\tilde{H}'_I$ is the negative of $H'_I$, this is because

$$H'^{\gamma^5} \gamma^5 = -H'^{\gamma^5},$$

$$\gamma^5 \gamma^5 \tilde{H}' = \sigma^5 \tilde{H}',$$

(37)

(38)

and we define

$$H_I = 2V_I I^k_{iH} S^k_{iH} D^{(8)}_{k},$$

(39)

with

$$V_I = \begin{cases} \frac{gF'(0)}{2}, & H_I = H'_I; \\
-\frac{gF'(0)}{2}, & H_I = H'_I. \end{cases}$$

(40)

It is convenient to define

$$K \equiv \mathbf{I}_H + S_{iH}.$$
with the constraint

$$Y = Y_1 + Y_k = 1 + Y_k,$$

where the isospin projector $\hat{U}_I$ is the exponential of the isospin generators, $|\Sigma_0\rangle$ are the states of a soliton with $A = 1$, $|K k\rangle$ are the representations of $K$, which describe the heavy (anti-)meson states $\tilde{T}(q)$ in this case, and $H_I'$ and $H_I^I$ act on the products of $|\Sigma_0\rangle$ and $|K k\rangle$ as

$$H_I|\Sigma_0\rangle|K k\rangle = V_I (K^2 - I^2_2 - S^2_I(H)|\Sigma_0\rangle|K k\rangle,$$

$I$ and $Y$ are the isospin and hypercharge of the $SU(3)$ state $c$ of the representation $R$, and $Y_K$ is the hypercharge of $\tilde{T}(q)$-states with $K$. For the ground states of heavy mesons and their antiparticles, $S_{lh} = 1/2$ and $I_H = (0,1/2)$, Eq. (31) gives

$$\begin{align*}
H_I & = -3V_I/2, \quad K = 0, \\
H_I & = 0, \quad K = 1/2, \\
H_I & = V_I/2, \quad K = 1.
\end{align*}$$

(45)

The $\tilde{T}$-states and $q$-states transform as the $\tilde{T}$ and 3 representations of the $SU(3)$ group respectively. Under the $SU(2) \times SU(1)$ subgroup ($I_{Y_k}$), they are decomposed as $\tilde{T} \to 1/2 - 1/2, 0 \oplus 0 - 1/2$, and $3 \to 1/2, 1/2, 1$ and accordingly we have $\tilde{T}$-states with $\{K=0,1; Y_K = -1/2\}$, $\{K = 1/2, Y_K = 1/2\}$ and $q$-states with $\{K=0,1; Y_K = 1/2\}$, $\{K = 1/2, Y_K = -1/2\}$. Thus, the allowed $SU(3)$ representations for a baryon-heavy-meson bound states with spin $s_I$, isospin $I$ and hypercharge $Y$ are those satisfying

$$(a) \quad I = s_I, \quad Y = \frac{1}{2},$$

$$(b) \quad 1 \leq I \leq s_I, \quad Y = \frac{3}{2},$$

$$(c) \quad \frac{1}{2} \leq I \leq s_I, \quad Y = \frac{3}{2},$$

and baryon-anti-heavy-meson bound states should satisfy

$$(a) \quad I = s_I, \quad Y = \frac{1}{2},$$

$$(b) \quad 1 \leq I \leq s_I, \quad Y = \frac{3}{2},$$

$$(c) \quad \frac{1}{2} \leq I \leq s_I, \quad Y = \frac{1}{2}.$$
where $\Delta M$ is the mass difference between $N$ and $\Delta$. Then, we get an exotic bound state with $H_I = -\frac{g \ell'(0)}{2} + \frac{1}{3} \Delta M$ and the average mass is

$$m_{e^*} = m_N + m_H - g \ell'(0)/2 + \frac{1}{3} \Delta M = 2802 \text{ MeV}. \quad (55)$$

The results above are the same as those in Ref. [13], which indicates the equivalence of the two approaches.

### B. The Light Flavor SU(3) Exotic Baryon Multiplets

In SU(3), exotic heavy baryon multiplets of the $SU(3)$ representations are

$$15 = (1, 2), \quad 24 = (3, 1),$$

$$15 = (2, 1), \quad 6 = (0, 2),$$

$$3 = (1, 0), \quad 15' = (4, 0), \quad (56)$$

which under the $SU(2) \times SU(1)$ subgroup decompose as

$$15 \rightarrow \frac{1}{2}^{+} \oplus 0^{+} \oplus 1^{+} \oplus \frac{3}{2}^{-} \oplus \frac{1}{2}^{-} \oplus 1^{-}, \quad (57)$$

$$24 \rightarrow \frac{3}{2}^{+} \oplus 1^{+} \oplus \frac{1}{2}^{+} \oplus \frac{3}{2}^{-} \oplus \frac{1}{2}^{-} \oplus 0^{-},$$

$$0 \rightarrow \frac{1}{2}^{-} \oplus \frac{1}{2}^{-} \oplus 1^{-}, \quad (58)$$

$$15 \rightarrow 0^{+} \oplus \frac{1}{2}^{+} \oplus 1^{+}, \quad (59)$$

$$3 \rightarrow \frac{1}{2}^{+} \oplus 0^{-}, \quad (60)$$

$$15' \rightarrow \frac{3}{2}^{+} \oplus \frac{1}{2}^{+} \oplus 1^{-} \oplus \frac{1}{2}^{-} \oplus 0^{-}. \quad (61)$$

According to [11] and [12], there are the following allowed exotic states denoted by $\{K, \gamma\}$

$$15 \rightarrow 0, \quad \{K = 0, \frac{3}{2}\}, \quad H_I = -3/2g \ell'(0);$$

$$\{K = 0, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 1/2g \ell'(0);$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 1/2g \ell'(0); \quad (63)$$

$$24 \quad s_l = 1, \quad \{K = 0, \frac{3}{2}\}, \quad H_I = -3/2g \ell'(0);$$

$$\{K = 0, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 1/2g \ell'(0);$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 1/2g \ell'(0);$$

$$6 \quad s_l = 0, \quad \{K = 0, \frac{3}{2}\}, \quad H_I = 3/2g \ell'(0);$$

$$\{K = 0, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 1/2g \ell'(0);$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 1/2g \ell'(0); \quad (64)$$

$$15 \quad s_l = 0, \quad \{K = 1, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 0; \quad (65)$$

$$15' \quad s_l = 1, \quad \{K = 1, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 0;$$

$$\{K = 1, \frac{3}{2}\}, \quad H_I = 0; \quad (66)$$

### V. COLLECTIVE COORDINATION QUANTIZATION

In Sec. IV, we see that $C = \pm 1$ bound states are degenerated in mass respectively. To obtain physical heavy baryons states with correct spin and isospin, we have to calculate up to $1/N_c$ order still in the heavy quark limits, i.e., $O(m_c^0 N_c^{-1})$. Substituting the hedgehog Ansatz (22) into (17) and (18), we get

$$V = \begin{pmatrix} \frac{i \sin^2(F(r)/2)}{r} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$A = \begin{pmatrix} 1 \frac{\sin^2(F(r))}{r} - F'(r)(\tau \cdot \hat{r}) - \frac{\sin(F(r))}{r} \\ 0 & 0 & 1 \end{pmatrix}.$$
and from (21) and (1), we get

\[ V_{cl} = g \int d^3 x \text{Tr}(H \gamma_5 \cdot A \bar{H}), \quad (66) \]

\[ \bar{V}_{cl} = g \int d^3 x \text{Tr}(H \gamma_5 \cdot A \bar{H}). \quad (67) \]

To evaluate \( V_{cl} \) and \( \bar{V}_{cl} \), we first find the classical approximation to the wave functions of heavy mesons with the general form

\[ H = \bar{H} = \left( \begin{array}{cc} \sqrt{\frac{\mp \phi_\pm(\tau \cdot \vec{r})}{k}} \chi(r) \\ 0 \end{array} \right), \quad (68) \]

where \( \phi_\pm \) are the two eigenstates of the isospin of heavy mesons, the explicit formulae of \( \chi \) are defined in (13), and the radio functions \( h(r) \) of the lowest energy eigenstate can be approximated by delta function, which is normalized \( \int r^2 |h(r)|^2 dr = 1 \). Accordingly, \( H \) is normalized as

\[ -\int d^3 x \text{Tr}(H \bar{H}) = 1, \quad (69) \]

In (28), we introduce the collective coordinate \( A(t) \), and the heavy meson fields transform simultaneously as

\[ H = H_{bf}A(t) \,, \quad \bar{H} = A(t)\bar{H}_{bf}, \quad (70) \]

where \( H_{bf} \) is the heavy meson field in the isospin body-fixed frame. Then, we obtain the Lagrangians as

\[ L = -M_{cl} - V_{cl} - i \int d^3 x \text{Tr}(\partial_0 H_{bf} \bar{H}_{bf}) + \frac{1}{2} I_\pi \sum_{a=1}^{3} \omega^a \omega^a + \frac{1}{2} I_k \sum_{b=4}^{7} \omega^b \omega^b + \sum_{c=1}^{8} I_{bf}^c \omega^c - \frac{N_c B}{2 \sqrt{3}} \omega^8, \quad (71) \]

\[ \bar{L} = -M_{cl} - \bar{V}_{cl} - i \int d^3 x \text{Tr}(H_{bf} \partial_0 \bar{H}_{bf}) + \frac{1}{2} I_\pi \sum_{a=1}^{3} \omega^a \omega^a + \frac{1}{2} I_k \sum_{b=4}^{7} \omega^b \omega^b + \sum_{c=1}^{8} \bar{I}_{bf}^c \omega^c - \frac{N_c B}{2 \sqrt{3}} \omega^8, \quad (72) \]

where \( M_{cl} \) is the classical soliton mass, \( I_\pi \) and \( I_k \) are moments of inertia, \( \omega^a \) are defined by \( A^1 \partial_0 A = i \frac{1}{2} \omega^a \lambda^a \) and \( I_{bf}^a \) and \( \bar{I}_{bf}^a \) are the rotation operator of heavy mesons on the flavor \( SU(3) \) group in the body fixed frame, defined as

\[ I_{bf}^a = -\frac{1}{4} \int d^3 x [H_{bf}(\xi_0 \tau^a \xi_0 + \xi_0 \tau^a \xi_0) \bar{H}_{bf}], \quad (73) \]

\[ \bar{I}_{bf}^a = \frac{1}{4} \int d^3 x [H_{bf}(\xi_0^\dagger \tau^a \xi_0 + \xi_0^\dagger \tau^a \xi_0^\dagger) \bar{H}_{bf}]. \quad (74) \]

By defining the canonical momenta conjugate to \( \omega^a \) as

\[ R_a = -\frac{i}{\epsilon^{a \mu}} \omega^\mu, \]

we get the Hamiltonians

\[ H = M_{cl} + V_{cl} + \frac{1}{2 I_\pi} \sum_{a=1}^{3} (R_a + I_{bf}^a)^2 \]

\[ + \frac{1}{2 I_k} \sum_{b=4}^{7} (R_b + \bar{I}_{bf}^b)^2, \quad (75) \]

\[ \bar{H} = M_{cl} + \bar{V}_{cl} + \frac{1}{2 I_\pi} \sum_{a=1}^{3} (R_a + \bar{I}_{bf}^a)^2 \]

\[ + \frac{1}{2 I_k} \sum_{b=4}^{7} (R_b + \bar{I}_{bf}^b)^2, \quad (76) \]

and \( \frac{1}{2} R_8 = \frac{2}{3}, \frac{4}{3} \) respectively. Due to the embedding of \( SU(2) \) heavy meson wave functions into \( SU(3) \) (28), \( I_{bf}^a \) and \( \bar{I}_{bf}^a \) with \( a = 4, 5, 6, 7 \) are zero and the mass formulae are as follow

\[ M = M_{cl} + V_{cl} + \frac{1}{2 I_\pi} C_2(SU(3)) + \frac{1}{2} \left( \frac{1}{I_\pi} - \frac{1}{I_k} \right) j_{sol}(j_{sol} + 1) \]

\[ - \frac{1}{2 I_k} R_8^2 + \frac{1}{2 I_\pi} (I_{bf}^2 + R \cdot I_{bf}), \quad (77) \]

where \( C_2(SU(3)) = 1/3(p^2 + q^2 + pq) + (p + q), \) \( (p, q) \) denotes an irreducible representation of the \( SU(3) \) group, \( j_{sol} \) are the angular momenta of soliton, and the last two terms reduce to \( SU(2) \) case in Ref. (3). And due to the spin-grand-spin transmutation, we have \( s_l = j_{sol} + K \).

Accordingly, the wave functions are

\[ \Psi_{\nu \nu'}^{(\mu)}(A) = \sqrt{\frac{\text{dim}(\mu)}{\text{dim}(\nu)}} D_Y^{(\mu)}(Y_{H_1}Y_{H_3} - j_{sol} - j_{sol}^*) (A), \quad (78) \]

with the constraints that \( Y_R = 2/3, 4/3 \) respectively for \( C = \pm 1 \). Then, these constraints mean \( 0 \)-plet with \( j_{sol} = 0, 15 \)-plet with \( j_{sol} = 1, 15 \)-plet with \( j_{sol} = (0, 1), \) and \( 24 \)-plet with \( j_{sol} = (1, 2) \). To get the mass splitting by strange quark mass, we have to take into consideration the following terms (14)

\[ L_{SB} = \int d^3 x Tr[\sigma_1 H (\xi m_7 \xi + \xi^\dagger m_7 \xi^\dagger) \bar{H}]
\]

\[ + \sigma_1 H \bar{H} (\Sigma^1 m_7^a + m_7 \Sigma) - \Delta (\Sigma^1 m_7^a + m_7 \Sigma)], \quad (79) \]

with

\[ m_7 = \begin{pmatrix} 1 & 0 \\ 0 & \hat{m} \end{pmatrix}, \quad (80) \]

where \( \hat{m} = 2m_s/(m_u + m_d) \) and the difference between \( m_u \) and \( m_d \) is neglected. For both \( C = \pm \), the symmetry breaking hamiltonians are the same, and due to the orthogonality of \( H_{bf} \), the first two terms in (69) give the same term as the last after evaluating \( H_{bf} \), and omitting the constant terms, we have (10)

\[ H_{SB} = \tau(1 - D_{88}^{(8)}), \quad (81) \]
where we use $\tau$ to denote all other parameters. Up to now, we can use perturbation theory to calculate the mass splitting, the results of the $6$-plet and $15$-plet baryons are shown in Table 1, where the input masses of $\Theta^0_6$ and $\Theta^*_6$ are the results in Sec. IV, and the Clebsch-Gordan coefficients are listed in Appendix. To fix the parameter $\tau$, we calculate the mass splitting between $\Sigma_c$ and $\Sigma_c$, whose experimental value is $\Delta m = m_{\Sigma_c} - m_{\Sigma_c} = 89$ MeV on the average of the degrees of freedom of spin. From the symmetry breaking hamiltonian, we can get

$$\langle \mathcal{H}_{SB} \rangle^c = 2m_{\Sigma_c} - 2m_{\Sigma_c} = 2m_{\Sigma_c} - 2m_{\Sigma_c};$$

(82)

$$m_{\Sigma_c} + m_{\Sigma_c} = m_{\Sigma_c} + m_{\Sigma_c};$$

(83)

$$m_{\Sigma_c} + m_{\Sigma_c} = m_{\Sigma_c} + m_{\Sigma_c};$$

(84)

$$m_{\Sigma_c} - m_{\Sigma_c} = 2(m_{\Sigma_c} - m_{\Sigma_c});$$

(85)

$$m_{\Sigma_c} - m_{\Sigma_c} = 2(m_{\Sigma_c} - m_{\Sigma_c});$$

(86)

$$m_{\Sigma_c} + m_{\Sigma_c} = 2m_{\Sigma_c}.$$  
(87)

Table 1. Masses of the $6$-plet and $15$-plet baryons

| Baryon States | $I(S_J^P)$ | $\langle \mathcal{H}_{SB} \rangle^c$ | $M$ (MeV) |
|---------------|------------|---------------------------------|----------|
| $\Theta^0_6$  | $0(1^+)$   | $\frac{3}{4} \tau$             | 2704     |
| $N^*_1$       | $\frac{3}{2}(1^+)$ | $\frac{3}{10} \tau$           | 2882     |
| $\Sigma^*_1$  | $1(1^+)$   | $\frac{3}{2} \tau$            | 3060     |
| $\Theta^*_6$  | $1(1^+)$   | $\frac{3}{4} \tau$            | 2802     |
| $\Delta^*_1$  | $\frac{3}{2}(1^+)$ | $\frac{3}{2} \tau$            | 3049     |
| $N^*_1$       | $\frac{3}{2}(1^+)$ | $\frac{3}{2} \tau$            | 3001     |
| $\Sigma^*_1$  | $1(1^+)$   | $\frac{3}{2} \tau$            | 3099     |
| $\Delta^*_1$  | $0(1^+)$   | $\tau$                         | 3000     |
| $\Xi^*_1$     | $\frac{3}{2}(1^+)$ | $\frac{3}{2} \tau$            | 3148     |

VI. SUMMARY AND DISCUSSION

In this paper, we first illustrate the $SU(3)$ exotic baryon multiplets as the sub-multiplets of $SU(4)$ baryon multiplets predicted from the generalized flavor $SU(4)$ collective-coordinate quantization in Figs. 1 and 2. Then, we generalize the approach in Fig. 3 to heavy baryons with anti-charm quark, and calculate the average masses of the states $\Theta^0_6$ and $\Theta^*_6$ in the light flavor $SU(2)$ case and investigate the binding energy of the heavy baryon multiplets with exotic states up to leading order of $1/N_c$ under the light flavor $SU(3)$ symmetry and the heavy quark limit. Then, by collective coordinate quantization, we give the mass spectra of the $6$-plet and $15$-plet baryons. In the light flavor $SU(2)$ case, we calculate the average masses of $\Theta^0_6$ and $\Theta^*_6$, which are estimated to be 2704 MeV and 2802 MeV respectively, and in the picture of the light flavor $SU(3)$ symmetry, the $\Theta^0_6$ and $\Theta^*_6$ states belong to the $6$-plets and $15$-plet with $S = (1/2, 3/2)$. These masses are the same as that in $[13]$, but in that approach they also predicted the existence of $\Theta^0_6$ and $\Theta^*_6$ with negative parity. Calculations in the light flavor $SU(3)$ case show that there exist $15$-plets and $24$-plets with $C = 1$ and $5$-plet and $15$-plets with $C = -1$ bounded in the bound state approach, qualitatively consistent with the results in Figs. 1 and 2. However, in this approach $SU(3)$ baryon multiplets $6$-plet with $S = 3/2$ and $15$-plets with $S = (1/2, 3/2, 3/2, 5/2)$ are also bounded. We find that these bounded states with $C = 1$ and those with $C = -1$ have equal binding energy respectively, this is because and $\Theta^*_6$ are only up to the leading order of $1/N_c$. Then, we quantize the chiral $SU(3)$ Lagrangians and calculate the splitting masses of the $6$-plet and $15$-plet baryons and give some relations between them, which may be of greater significance qualitatively than quantitatively because in the case of $SU(3)$ chiral symmetry with only light baryons, such a way to deal with symmetry breaking term does not agree with experiments very well.

In summary, both bound state approaches and the generalized $SU(N_c)$ collective-coordinate quantization approach give us exotic baryon states with $C = \pm 1$, and the search of $\Theta^0_6$ are expected further to verify the validation of the generalization of soliton picture to study exotic baryon states.

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APPENDIX: The Clebsch-Gordan Coefficients used in the paper

Because the Clebsch-Gordan coefficients used in this paper are seldom used in physics before, we list the Clebsch-Gordan Coefficients in this appendix. To be convenient, we define

$$\mu^{(\lambda)}_{(\nu)} = \sum_{\nu_1, \nu_2} \left(\frac{\mu_1 \mu_2 \mu}{\nu_1 \nu_2 \nu}\right) \mu_{1(\nu_1)} \mu_{2(\nu_2)},$$

(88)

where $\mu^{(\lambda)}_{(\nu)}$ denote the eigenstates of the representation $\mu$ contained in the direct sum of $\mu_1 \otimes \mu_2$, whose engin-states are $\mu_{1(\nu_1)}$ and $\mu_{2(\nu_2)}$ respectively, $\lambda$ is used to distinguish identical but independent representations which are all contained in $\mu_1 \otimes \mu_2$, $\nu_1$ and $\nu_2$ denote quantum number ($Y/IJ_3$) collectively, and $\left(\frac{\mu_1 \mu_2 \mu}{\nu_1 \nu_2 \nu}\right)$ are the $SU(3)$ Clebsch-Gordan coefficients. For the product $\overline{8} \otimes 8 = 3 \oplus 5 \oplus 15 \oplus 24$, we give the decomposition of $\overline{8}$ as follows.
Similarly, in the product $15 \otimes 8 = 42 \oplus 2\overline{15} \oplus 15^{(1)} \oplus 15^{(2)} \oplus 6 \oplus 3$, the representations $15^{(1)}$ and $15^{(2)}$ are given as
\[
15^{(1)}(\frac{1}{4}, 1, 0) = \frac{\sqrt{21}}{14} 15(\frac{1}{4}, 1, 0) S(0, 1, 0) + \frac{\sqrt{7}}{7} 15(\frac{1}{4}, 1, 0) S(0, 0, 0) - \frac{\sqrt{21}}{14} 15(\frac{1}{4}, 1, 0) S(0, 1, 1) - \frac{3\sqrt{7}}{28} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(1, \frac{1}{2}, \frac{1}{4})
+ \frac{3\sqrt{7}}{28} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(1, \frac{1}{2}, \frac{1}{4})
\]
\[
15^{(1)}(\frac{1}{4}, 1, 0) = \frac{\sqrt{21}}{14} 15(\frac{1}{4}, 1, 0) S(0, 0, 0) - \frac{\sqrt{21}}{14} 15(\frac{1}{4}, 1, 0) S(0, 0, 0) - \frac{3\sqrt{7}}{28} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(1, \frac{1}{2}, \frac{1}{4})
+ \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(0, 1, 1)
\]
\[
15^{(1)}(\frac{1}{4}, 1, 0) = \frac{3\sqrt{7}}{28} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(-1, \frac{1}{2}, \frac{1}{4}) - \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(0, 0, 0) - \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(0, 1, 1) + \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(1, \frac{1}{2}, \frac{1}{4})
\]
\[
15^{(1)}(\frac{1}{4}, 1, 0) = \frac{3\sqrt{7}}{28} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(0, 0, 0) - \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(0, 0, 0) - \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(0, 1, 1) + \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(1, \frac{1}{2}, \frac{1}{4})
\]
\[
15^{(1)}(\frac{1}{4}, 1, 0) = \frac{3\sqrt{7}}{28} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(-1, \frac{1}{2}, \frac{1}{4}) - \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(0, 0, 0) - \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(0, 1, 1) + \frac{\sqrt{7}}{7} 15(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) S(1, \frac{1}{2}, \frac{1}{4})
\]
\[
15^{(1)}_{\left(-\frac{2}{3}, 0, 0\right)} = \frac{\sqrt{14}}{28} 15_{\left(-\frac{2}{3}, 0, 0\right)} 8_{(0,0,0)} - \frac{\sqrt{17}}{28} 15_{\left(-\frac{2}{3}, 0, 0\right)} 8_{(0,1,0)} + \frac{\sqrt{17}}{28} 15_{\left(-\frac{2}{3}, 0, 0\right)} 8_{(1,0,0)} + \frac{\sqrt{17}}{28} 15_{\left(-\frac{2}{3}, 0, 0\right)} 8_{(1,1,0)}
\]
\[
15^{(2)}_{(\frac{1}{2},-\frac{1}{2})} = -\frac{5}{63}\sqrt{15} (\frac{1}{2},1,0) S_{(-1,\frac{1}{2},-\frac{1}{2})} - \frac{10}{63}\sqrt{14} (\frac{1}{2},\frac{1}{2},\frac{1}{2}) S_{(0,1,-1)} - \sqrt{7}\frac{14}{63} (\frac{1}{2},-\frac{1}{2},\frac{1}{2}) S_{(0,1,-1)} - \sqrt{42}\frac{12}{63} (\frac{1}{2},1,1) S_{(-1,\frac{1}{2},-\frac{1}{2})} + \frac{4\sqrt{2}}{63} (\frac{1}{2},-\frac{1}{2},\frac{1}{2}) S_{(0,1,0)} - \frac{2\sqrt{14}}{63} (\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) S_{(0,1,0)} - \frac{5\sqrt{7}}{63} (\frac{1}{2},1,-1) S_{(-1,\frac{1}{2},\frac{1}{2})} + \frac{4\sqrt{2}}{63} (\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) S_{(0,1,1)}
\]

\[
15^{(2)}_{(\frac{1}{2},-\frac{1}{2})} = -\frac{\sqrt{14}}{21} (\frac{1}{2},1,1) S_{(-1,\frac{1}{2},-\frac{1}{2})} - \frac{5\sqrt{7}}{63} (\frac{1}{2},\frac{1}{2},-\frac{1}{2}) S_{(0,1,0)} - \frac{2\sqrt{14}}{63} (\frac{1}{2},-\frac{1}{2},\frac{1}{2}) S_{(0,1,0)} - \frac{5\sqrt{7}}{63} (\frac{1}{2},1,-1) S_{(-1,\frac{1}{2},\frac{1}{2})} + \frac{4\sqrt{2}}{63} (\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) S_{(0,1,1)}
\]

\[
15^{(2)}_{(\frac{1}{2},-\frac{1}{2})} = -\frac{\sqrt{14}}{21} (\frac{1}{2},1,1) S_{(-1,\frac{1}{2},-\frac{1}{2})} - \frac{5\sqrt{7}}{63} (\frac{1}{2},\frac{1}{2},-\frac{1}{2}) S_{(0,1,0)} - \frac{2\sqrt{14}}{63} (\frac{1}{2},-\frac{1}{2},\frac{1}{2}) S_{(0,1,0)} - \frac{5\sqrt{7}}{63} (\frac{1}{2},1,-1) S_{(-1,\frac{1}{2},\frac{1}{2})} + \frac{4\sqrt{2}}{63} (\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) S_{(0,1,1)}
\]

\[
15^{(2)}_{(\frac{1}{2},-\frac{1}{2})} = -\frac{\sqrt{14}}{21} (\frac{1}{2},1,1) S_{(-1,\frac{1}{2},-\frac{1}{2})} - \frac{5\sqrt{7}}{63} (\frac{1}{2},\frac{1}{2},-\frac{1}{2}) S_{(0,1,0)} - \frac{2\sqrt{14}}{63} (\frac{1}{2},-\frac{1}{2},\frac{1}{2}) S_{(0,1,0)} - \frac{5\sqrt{7}}{63} (\frac{1}{2},1,-1) S_{(-1,\frac{1}{2},\frac{1}{2})} + \frac{4\sqrt{2}}{63} (\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) S_{(0,1,1)}
\]

\[
15^{(2)}_{(\frac{1}{2},-\frac{1}{2})} = -\frac{\sqrt{14}}{21} (\frac{1}{2},1,1) S_{(-1,\frac{1}{2},-\frac{1}{2})} - \frac{5\sqrt{7}}{63} (\frac{1}{2},\frac{1}{2},-\frac{1}{2}) S_{(0,1,0)} - \frac{2\sqrt{14}}{63} (\frac{1}{2},-\frac{1}{2},\frac{1}{2}) S_{(0,1,0)} - \frac{5\sqrt{7}}{63} (\frac{1}{2},1,-1) S_{(-1,\frac{1}{2},\frac{1}{2})} + \frac{4\sqrt{2}}{63} (\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) S_{(0,1,1)}
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