Possible methods for the determination of the $P$-parity of the $\Theta^+$-pentaquark in NN-collisions.

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We present two possibilities to determine the P-parity of the pentaquark $\Theta^+$, in a model independent way, via the measurement of polarization observables in $p+p \to \Theta^+ + \Sigma^+$, or $n + p \to \Theta^+ + \Lambda^0$, in the near threshold region. Besides the measurement of the spin correlation coefficient, $A_{xy} = A_{yx}$, (in collisions of transversally polarized nucleons), the coefficient $D_{xx}$ of polarization transfer from the initial proton to the final $\Sigma^+ (\Lambda^0)$ hyperon is also unambiguously related to the $\Theta^+$ parity.

In 1999, N. K. Pak and M. Rekalo proposed [1] two new methods for the determination of the P-parity of the $K$-meson, through the measurement of different polarization observables in $K$-meson production in proton proton collisions near threshold, $p + p \to K^+ + Y^0 + p$ ($Y^0 = \Lambda$ or $\Sigma$-hyperon). One method is based on the measurement of the sign of the spin correlation coefficient $A_{yy}$, for collisions of transversally polarized protons. The second one is based on the measurement of the polarization transfer coefficient from the initial proton to the produced hyperon, $D_{nn}$. Both methods apply in threshold conditions, where all final particles are in S-state. Note, in this respect, that the DISTO collaboration showed the feasibility of the second method, by measuring the $D_{nn}$ coefficient at proton momentum of 3.67 GeV/c, which showed that "$D_{nn}$ is large and negative ($\approx -0.4$) over most of the kinematic region." [2]. It was mentioned in [1] that a nonzero value of $D_{nn}$, in the threshold region, can be considered as the experimental confirmation of the pseudoscalar nature of the $K^+$ meson.

It is straightforward to adapt these methods to the determination of the P-parity of the $\Theta^+$-hyperon, which is presently object of an intensive theoretical discussion.

The simplest reactions of $NN$-collisions, which can be considered for this aim are the following:

\begin{align*}
    p + n & \to \Lambda^0 + \Theta^+, \\
    p + p & \to \Sigma^+ + \Theta^+, \\
    p + p & \to \pi^+ + \Lambda^0 + \Theta^+,
\end{align*}

(1) to (3)

in threshold conditions ($S$-wave production). It is important to note that the $\Lambda^0$ or the $\Sigma^+$ hyperons, produced in these reactions, are self analyzing particles, therefore the polarization transfer method - with measurement of the polarization transfer coefficient $D_{nn}$, seems more preferable, from the experimental point of view.

The following analysis of reactions (1-3) is based on general symmetry properties of the strong interaction, such as the P-invariance, the conservation of the total angular momentum, the Pauli principle, for the $pp$-system, and the generalized Pauli principle for the $np$ system, which holds at the level of the isotopic invariance of the strong interaction.

These symmetry properties, being applied to $S$-wave production in the processes (1-3) allow to establish the spin structure of the corresponding matrix elements, for both cases of the considered P-parity.

We consider here, for simplicity, the case of spin 1/2 $\Theta^+$ hyperon, but this formalism can be extended to any $\Theta^+$ spin.

Firstly, let us establish the general spin structure of double polarization observables for the processes (1-3) at threshold.

The dependence of the cross section, total or differential, on the polarizations $\vec{P}_1$ and $\vec{P}_2$ of the colliding nucleons can be written as:

$$\sigma(\vec{P}_1, \vec{P}_2) = \sigma_0 (1 + A_1 \vec{k} \cdot \vec{P}_1 + \vec{k} \cdot \vec{P}_2),$$

(4)

where $\sigma_0$ is the cross section for the collision of unpolarized nucleons, $\vec{k}$ is the unit vector along the three momentum of the colliding nucleons, in the reaction CM system. The real coefficients $A_1$ and $A_2$, which are different for different reactions, depend on the parity of the $\Theta^+$ hyperon. Taking the $z$-axis along $\vec{k}$, one can find the following expression...
for the spin correlation coefficients $A_{ab}$, in terms of $A_1$ and $A_2$:

$$A_{xx} = A_{yy} = A_1, \quad A_{zz} = A_1 + A_2.$$  \hspace{1cm} (5)$$

The dependence of the polarization $\vec{P}_Y$ of the produced hyperon, $\Lambda$ or $\Sigma^+$, on the polarization $\vec{P}$ of the initial nucleon (beam or target) can be written as:

$$\vec{P}_Y = p_1 \vec{P} + p_2 \hat{k} \cdot \vec{P},$$  \hspace{1cm} (6)$$

where $p_{1,2}$ are real coefficients, which depend on the $\Theta^+$ parity, so that for the non-zero polarization transfer coefficients, $D_{ab}$ one can write:

$$D_{xx} = D_{yy} = p_1, \quad D_{zz} = p_1 + p_2.$$  \hspace{1cm} (7)$$

Let us calculate these coefficients for the reactions (1-3), it terms of S-wave partial amplitudes, considering both values of the $\Theta^+$ parity.

$n + p \rightarrow \Lambda + \Theta^+$. This reaction has the lowest threshold, and seems very interesting for the measurement of the $D_{nn}$ coefficient, due to the large asymmetry and branching ratio of the decay $\Lambda \rightarrow p + \pi^-$ (\(\alpha = 0.642 \pm 0.013\) and Br=\((63.9 \pm 0.5)\%\) [3]).

The spin structure of the threshold matrix element depends on the discussed P-parity (assuming that the isotopic spin of $\Theta^+$ is zero):

$$M^{(-)}_\Lambda = f^{(-)}(\Lambda)\bar{\chi} \otimes \vec{\sigma} \cdot \vec{k}, \quad \text{if } P(\Theta^+) = -1,$$

$$M^{(\pm)}_\Lambda = f^{(\pm)}_1(\Lambda)\bar{\chi} \otimes \vec{\sigma} \cdot \vec{k} + f^{(\pm)}_2(\Lambda)(\sigma_m - \hat{k}_m \sigma \cdot \hat{k}) \otimes \sigma_m, \quad \text{if } P(\Theta^+) = \pm 1$$

where the upper index ($\pm$) for the partial amplitudes $f(\Lambda)$ corresponds to $P(\Theta^+) = \pm 1$. Henceforward we use the following abbreviation

$$A \otimes B = (\chi_2 \sigma_y A\chi_1)(\chi_4^\dagger B\sigma_y \chi_3^\dagger),$$  \hspace{1cm} (10)$$

where $\chi_1$ and $\chi_2$ ($\chi_3$ and $\chi_4$) are the two-component spinors of the initial (final) baryons.

Using Eqs. (8) and (9) one can find the following formulas for double spin polarization observables:

$$A^{(-)}_{xx}(\Lambda) = A^{(-)}_{yy}(\Lambda) = A^{(-)}_{zz}(\Lambda) = -1, \quad D^{(-)}_{ab} = 0, \quad \text{if } P(\Theta^+) = -1,$$

and

$$D^{(+)}_{\Lambda} A^{(+)}_{xx}(\Lambda) = D^{(+)}_{\Lambda} A^{(+)}_{yy}(\Lambda) = |f^{(+)}_1(\Lambda)|^2, \quad D^{(+)}_{\Lambda} A^{(+)}_{zz}(\Lambda) = 2 \left(-|f^{(+)}_1(\Lambda)|^2 + |f^{(+)}_2(\Lambda)|^2 \right)$$

$$D^{(\dagger)}_{\Lambda} D^{(\dagger)}_{xx}(\Lambda) = D^{(\dagger)}_{\Lambda} D^{(\dagger)}_{yy}(\Lambda) = 2 Re f^{(+)}_1(\Lambda) f^{(+)*}_2(\Lambda), \quad D^{(\dagger)}_{\Lambda} D^{(\dagger)}_{zz}(\Lambda) = 2 |f^{(+)}_2(\Lambda)|^2,$$

with

$$D^{(\dagger)}_{\Lambda} = |f^{(+)}_1(\Lambda)|^2 + 2 |f^{(+)}_2(\Lambda)|^2, \quad \text{if } P(\Theta^+) = +1,$$

So, comparing the two possibilities for the P-parity, one can predict, in model independent way:

$$A^{(-)}_{xx}(\Lambda) = A^{(-)}_{yy}(\Lambda) = -1, \quad D^{(-)}_{yy}(\Lambda) = 0, \quad \text{if } P(\Theta^+) = -1,$$

$$A^{(+)}_{yy}(\Lambda) \geq 0, \quad D^{(-)}_{yy}(\Lambda) \neq 0, \quad \text{if } P(\Theta^+) = +1$$

with evident sensitivity of these observables to the parity of the $\Theta^+$ hyperon.
\[ p + p \to \Sigma^+ + \Theta^{+1} \]. The spin structure of the threshold matrix element is different from \( \Lambda \) production (due to the difference in the value of the total isotopic spin for the colliding nucleons) and depends on \( P(\Theta^+) \):

\[
\mathcal{M}^{(+)}_\Sigma = f^{(+)}(\Sigma)I \otimes I, \quad \text{if } P(\Theta^+) = +1
\]  

\[
\mathcal{M}^{(-)}_\Sigma = f_1^{(-)}(\Sigma)\hat{\sigma} \hat{k} \otimes I + i f_2^{(-)}(\Sigma)(\hat{\sigma} \times \hat{k})_m \otimes \sigma_m, \quad \text{if } P(\Theta^+) = -1
\]

where \( f^{(\pm)}(\Sigma) \) are the corresponding partial amplitudes for \( P(\Theta^+) = \pm 1 \), in case of \( \Sigma \) production.

Using these matrix elements, one can find the following form for the corresponding double polarization observables:

\[
D^{(-)}_\Sigma A^{(-)}_{xx}(\Sigma) = D^{(-)}_\Sigma A^{(-)}_{yy}(\Sigma) = |f_1^{(-)}(\Sigma)|^2,
\]

\[
D^{(-)}_\Sigma A^{(-)}_{zz}(\Sigma) = -|f_1^{(-)}(\Sigma)|^2 + 2|f_2^{(-)}(\Sigma)|^2,
\]

\[
D^{(-)}_\Sigma D^{(-)}_{xx}(\Sigma) = D^{(-)}_\Sigma D^{(-)}_{yy}(\Sigma) = 2Re(f_1^{(-)}(\Sigma)f_2^{(-)*}(\Sigma)),
\]

\[
D^{(-)}_\Sigma D^{(-)}_{zz}(\Sigma) = 2|f_1^{(-)}(\Sigma)|^2,
\]

with \( D^{(-)}_\Sigma = |f_1^{(-)}(\Sigma)|^2 + 2|f_1^{(-)}(\Sigma)|^2 \), in case of negative parity of the \( \Theta^+ \), and

\[
A^{(+)}_{xx}(\Sigma) = A^{(+)}_{yy}(\Sigma) = A^{(+)}_{zz}(\Sigma) = -1, \quad D^{(+)}_{ab}(\Sigma) = 0
\]

in case of positive parity.

Therefore, the measurement of the quantities \( A_{yy}(\Sigma) \) and \( D_{yy}(\Sigma) \) allows to determine the P-parity:

\[
A^{(-)}_{yy}(\Sigma) \geq 0, \quad D^{(-)}_{yy}(\Sigma) \neq 0 \quad \text{if } P(\Theta^+) = -1
\]

\[
A^{(+)}_{yy}(\Sigma) = -1, \quad D^{(+)}_{yy}(\Sigma) = 0 \quad \text{if } P(\Theta^+) = +1.
\]

Comparing Eqs. (15) and (19), one can see that the reactions of \( \Theta^+ \) production in \( NN \)-collisions, \( p + p \to \Sigma^+ + \Theta^+ \) and \( n + p \to \Lambda^0 + \Theta^+ \), which look similar at first sight, show a very different dependence of the \( A_{yy} \) and \( D_{yy} \) observables on the P-parity of the \( \Theta^+ \) hyperon. For example, the signs of \( A_{yy}(\Sigma) \) and \( A_{yy}(\Lambda) \) are different, independently on \( P(\Theta^+) \). A large difference is also present in the \( D_{yy}(\Sigma) \) and \( D_{yy}(\Lambda) \) observables.

\[ p + p \to \pi^+ + \Lambda + \Theta^+ \]. The spin structure of the corresponding matrix elements can be written as follows:

\[
\mathcal{M}^{(-)} = f^{(-)}(\Sigma)I \otimes I, \quad \text{if } P(\Theta^+) = -1
\]

\[
\mathcal{M}^{(+)} = f_1^{(+)}(\Sigma)\hat{\sigma} \hat{k} \otimes I + i f_2^{(+)}(\Sigma)(\hat{\sigma} \times \hat{k})_m \otimes \sigma_m, \quad \text{if } P(\Theta^+) = +1
\]

i.e. similar to the reaction \( p + p \to \Sigma^+ + \Theta^+ \), but with opposite parity. So, for \( p + p \to \pi^+ + \Lambda + \Theta^+ \), the necessary polarization observables can be described by Eqs. (17) and (18), taking care to interchange the P-parities.

This analysis shows that all the reactions (1-3) are well adapted to the determination of \( P(\Theta^+) \). In all these reactions two polarization observables, namely \( A_{yy} \) and \( D_{yy} \) are sensitive to this parity. The signature of the parity of \( (\Theta^+) \) is the sign of the asymmetry \( A_{yy} \), or a a value of the transfer polarization tensor, different from zero. These statements are model independent.

Let us briefly discuss the expected values of the polarization observables, in the reactions (1-3), which depend on two amplitudes, \( f_1 \) and \( f_2 \). For this aim, we will take, as an example, a model of \( K \)-meson exchange, which has been

\[ \text{The collisions of polarized protons in this reaction have been considered in } \text{[3].} \]
applied to the Λ and Σ⁰ production in pp-collisions \[5\], and reproduces quite well the sign and the absolute value of \(D_{nn}\) in \(\bar{p} + p \rightarrow \bar{\Lambda} + K^+ + p\). Considering the contribution of both diagrams in Fig. 1, one can find:

\[
f_1^{(-)}(\Sigma) = -f_2^{(-)}(\Sigma)
\]  
(21)

This relation does not depend on many details of the reaction mechanism, such as the values of the two coupling constants, \(g_{p\Theta K}\) and \(g_{p\Sigma K}\), on the width of \(\Theta^+\) and on the form of the phenomenological form factors, which has to be taken into account in such considerations.

The relation (21) allows to predict:

\[
A_{yy}^{(-)}(\Sigma) = +1/3, \quad D_{yy}^{(-)}(\Sigma) = -2/3.
\]  
(22)

Let us note that \(|D_{yy}^{(-)}(\Sigma)|\) is different from zero and large. This will make easier to discriminate the value of the P-parity.

The same relation holds also for the amplitudes \(f_{1,2}^{(+)}(\Lambda)\) for the process \(n + p \rightarrow \Lambda + \Theta^+\) (for \(K\)-exchange), with corresponding predictions for polarization effects.

Note, however, that final state \(\Sigma\Theta\)-interaction, which is different, generally, in singlet and triplet states, can affect the relation (21). There are arguments which show that these effects can not be large \[4\]. In any case we considered here \(K\)-exchange only for illustrative purposes, for a quick estimation of polarization phenomena, without any claim that this is a realistic model for these reactions \[6\]; the main result of this paper does not depend on model considerations.

The experimental study of all three reactions (1-3) will give a non ambiguous signature of the \(\Theta^+\) parity.

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