Spin selective Aharonov-Bohm oscillations in a lateral triple quantum dot

F. Delgado, Y.-P. Shim, M. Korkusinski, L. Gaudreau, S. A. Studenikin, A. S. Sachrajda, and P. Hawrylak

Abstract

We present a theory for spin selective Aharonov-Bohm oscillations in a lateral triple quantum dot. We show that to understand the Aharonov-Bohm (AB) effect in an interacting electron system within a triple quantum dot molecule (TQD) where the dots lie in a ring configuration requires one to not only consider electron charge but also spin. Using a Hubbard model supported by microscopic calculations we show that, by localizing a single electron spin in one of the dots, the current through the TQD molecule depends not only on the flux but also on the relative orientation of the spin of the incoming and localized electrons. AB oscillations are predicted only for the spin singlet electron complex resulting in a magnetic field tunable “spin valve”.

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The Aharonov-Bohm (AB) effect results from the accumulation of phase by a charged particle moving in a ring threaded by a magnetic flux [1, 2]. AB oscillations are detected e.g. in the magnetization of a macroscopic number of electrons in metallic rings [4] as well as in the optical emission from a charged exciton in a nanoscale semiconductor quantum ring [2]. The preparation, manipulation and detection of individual spins of localized electrons in nanoscale semiconductor systems are important elements of nanoscale spintronics [6, 7, 8], with efficient generation and detection of spin polarized carriers playing a crucial role. The electron spins can be localized in single and coupled semiconductor quantum dots (QDs) defined and controlled electrostatically [3, 11, 12, 13] with potential applications as elements of electron-spin based circuits [14, 15], coded qubits [16], entanglers [17], rectifiers and ratchets [14, 19]. The Spin blockade technique in a double dot system used for the conversion of spin to charge information has played an important role in the development of such applications [21].

The possibility of the co-existence of spin blockade with AB oscillations in a lateral TQD in a ring geometry [12, 13, 14] was discussed in Ref. 22. In this paper we describe a TQD, shown schematically in Fig. 1(a), where two dots, 1 and 3, are connected to the leads and in addition to dot number 2. A single electron spin is localized in dot 2 by lowering the confining potential. The electronic properties of these spin selective AB oscillations in transport through a TQD is equal to the chemical potential of the TQD weakly connected to two non-interacting leads. The current flows when the chemical potential of the TQD is equal to the chemical potential of the leads. This can also be understood in terms of degeneracies of many electron charge configurations (N_1, N_2, N_3), with N_i the number of electrons in dot i. The degeneracy point described here, referred to as the quadrupole point (QP), involves the one electron configurations (0, 0, 0) and two-electron configurations (1, 1, 0), (0, 2, 0) and (0, 1, 1), with one electron always confined in dot 2, as shown in Fig. 1(b).

For clarity we only present results of the Hubbard model with one orbital per dot [22, 24]. The Hamiltonian of the TQD subject to a uniform perpendicular magnetic field, \( B = B\hat{z} \), is given by

\[
H = \sum_{i,\sigma} E_i \delta_{i\sigma} d_{i\sigma}^\dagger d_{i\sigma} + \sum_{\sigma, i, j; \tau \neq \sigma} \tilde{t}_{ij} \delta_{\sigma\tau} d_{i\sigma}^\dagger d_{j\tau} + \sum_i U_i n_{i\uparrow} n_{i\downarrow} + \frac{1}{2} \sum_{i, j; i \neq j} V_{ij} \delta_{i\uparrow} \delta_{j\downarrow},
\]

where the operators \( d_{i\sigma} (d_{i\sigma}^\dagger) \) annihilate (create) an electron with spin \( \sigma = \pm 1/2 \) on orbital \( i (i = 1, 2, 3) \). \( n_{i\sigma} = d_{i\sigma}^\dagger d_{i\sigma} \) and \( \delta_i = n_{i\uparrow} + n_{i\downarrow} \) are the spin and charge density on orbital level \( i \). Each dot is represented by a single orbital with energy \( E_{i\sigma} = E_i + g^* \mu_B B \delta_i + E_0 \), where
$g^*$ is the effective Landé $g$-factor, $\mu_B$ is the Bohr magneton and $E_0$ is the common energy shift of the three dots measured from the Fermi level of the leads which is tunable by external gates. The dots are connected by magnetic field dependent hopping matrix elements $t_{ij} = t_{ij}e^{2\pi i\phi_{ij}} [26]$. For the three dots in an equilateral configuration $\phi_{12} = \phi_{23} = \phi_{31} = -\phi/3$ and $\phi_{ji} = -\phi_{ij}$, where $\phi = BA/\phi_0$ is the number of magnetic flux quanta threading the area $A$ of the triangle, $\phi_0 = \hbar c/e$ is the magnetic flux quantum, $e$ is the electron charge, $c$ is the speed of light and $h$ is the Planck’s constant. The interacting part of the Hamiltonian is parametrized by the on-site Coulomb repulsion, $U_i$, and the interdot direct repulsion term $V_{ij}$.

In order to describe transport through the TQD we first determine the QP of the isolated TQD. We start by determining the “classical QP” where we neglect the inter-dot tunneling and require the four configurations $A \equiv (1, 1, 0)$, $B \equiv (0, 2, 0)$, $C \equiv (0, 1, 1)$ and $D \equiv (0, 1, 0)$ to have equal energy. Their energies are $\epsilon_A = E_1 + E_2 + V_{12} + 2E_0$, $\epsilon_B = 2E_2 + U_2 + 2E_0$, $\epsilon_C = E_2 + E_3 + V_{23} + 2E_0$, and $\epsilon_D = E_2 + E_0$. The QP condition without tunneling requires $\epsilon_A = \epsilon_B = \epsilon_C = \epsilon_D + \mu_L$, where $\mu_L$ is the chemical potential of the leads. This implies that at the QP $E_{Q_1} = \mu_L - E_0 - V_{12}$, $E_{Q_2} = \mu_L - E_0 - U_2$ and $E_{Q_3} = \mu_L - E_0 - V_{23}$.

Let us consider now the case of finite tunneling matrix elements. The $(0, 1, 0)$ charge configuration describes the two spin states of an electron localized in dot 2, $|2\sigma\rangle \equiv d_{22}^\dagger |0\rangle$ with energy $E_2 (|0\rangle)$ (the vacuum state). The two electron classical charge configurations $(1, 1, 0)$, $(0, 2, 0)$ and $(0, 1, 1)$ correspond to the following quantum spin singlet configurations: $|S_1\rangle = \frac{1}{\sqrt{2}} (d_{11}^\dagger d_{21}^\dagger + d_{11}^\dagger d_{11}^\dagger) |0\rangle$, $|S_2\rangle = d_{21}^\dagger d_{21}^\dagger |0\rangle$ and $|S_3\rangle = \frac{1}{\sqrt{2}} (d_{21}^\dagger d_{31}^\dagger + d_{31}^\dagger d_{21}^\dagger) |0\rangle$. The Hamiltonian describing the motion of the spin singlet pair takes the form

$$\hat{H}_S = \begin{pmatrix} \epsilon_A & \sqrt{2}t_{12} e^{-2\pi i \phi/3} & \sqrt{2}t_{13} e^{2\pi i \phi/3} \\ \sqrt{2}t_{12} e^{2\pi i \phi/3} & \epsilon_B & \sqrt{2}t_{23} e^{-2\pi i \phi/3} \\ \sqrt{2}t_{13} e^{-2\pi i \phi/3} & \sqrt{2}t_{23} e^{2\pi i \phi/3} & \epsilon_C \end{pmatrix}.$$  

The result-the QP, we have $\epsilon_A = \epsilon_B = \epsilon_C$. If $t_{23} = t_{12} = \sqrt{2}t_{13}$, we can diagonalize the Hamiltonian exactly by Fourier transforming into a new basis: $|K_1\rangle = 1/\sqrt{2} (|1\rangle + |2\rangle + |3\rangle)$, $|K_2\rangle = 1/\sqrt{3} (|1\rangle + e^{2\pi i /3} |2\rangle + e^{4\pi i /3} |3\rangle)$ and $|K_3\rangle = 1/\sqrt{3} (|1\rangle + e^{-2\pi i /3} |2\rangle + e^{-4\pi i /3} |3\rangle)$ with eigenvalues $\epsilon_1 = E - 2|t| \cos (2\pi \phi /3)$, $\epsilon_2 = E - 2|t| \cos (2\pi \phi + 1/3)$ and $\epsilon_3 = E - 2|t| \cos (2\pi \phi - 1/3)$, respectively. Since one of the electrons is kept in dot 2, the energy spectrum of a pair of singlet electrons is essentially the same as that of a single electron added to a resonant TQD, with the energy levels oscillating with a period of one flux quantum $[22]$. Away from the resonance the level crossing is replaced by anti-crossing.

A pair of spin triplet electrons describes only $(1, 1, 0)$ and $(0, 1, 1)$ charge configurations. The corresponding two spin triplet configurations for $S_Z = 1$ are $|T_1\rangle = d_{11}^\dagger d_{22}^\dagger |0\rangle$ with energy $\epsilon_A (B)$ and $|T_2\rangle = d_{21}^\dagger d_{31}^\dagger |0\rangle$ with

![FIG. 1: (a) Schematic diagram of the TQD close to the considered QP. (b) Electrons with antiparallel spins can form a loop an the corresponding energy levels $E$ experiment AB like oscillations with the magnetic flux $\phi$ while electrons with parallel spin are spin-blockaded.](image-url)
By comparing the singlet and triplet eigenvalues we see charge configurations (indicated by the white circle. at \( V \times \) eigenenergies of the 2 triplet Hamiltonian are remarkable result that triplet states do not oscillate with magnetic field. Even at this qualitative level, we obtain a corrections that are obtained numerically, with \( \phi_C = A B C / \phi_0 \), indicated in Fig. 2 Above \( \phi_C \) the triplet is the ground state. Hence the presence of a trapped electron should lead to AB oscillations of the tunneling electron, interrupted and eventually terminated by the singlet-triplet transitions.

Figure 3 shows the dominant charge ground state configurations of the TQD at two different values of the magnetic flux quantum, \( \phi = 0 \) (upper panel) and \( \phi = 0.44 \) (lower panel), versus the voltages \( V_{g1} \) and \( V_{g2} \) for \( E_0 = -|t| \). Here it has been assumed that the on-site energies \( E_i \)'s change linearly with the voltages \( V_{g1} \) and \( V_{g2} \), \( E_i = \alpha_i V_{g1} + \beta_i V_{g2} + \gamma_i \), with \( \alpha_i, \beta_i \) extracted from experiment in Ref. 13. For the chosen value of \( E_0 \), the zero magnetic field stability diagram shows only a triple point, while at \( \phi = 0 \) the QP is clearly visible.

We now turn to the illustration how these spin selective AB oscillations can be observed in transport experiment. Following Ref. 27, the Hamiltonian of the TQD connected to two leads is given by \( H = H_L + H_{TQD} + H_{LD} \), where \( H_L \) is the Hamiltonian describing the two non-interacting leads, \( H_{TQD} \) corresponds to the isolated triplet dot where we assume that the on-site energies change with the applied bias \( \Delta V \) as \( E_{i\sigma} \rightarrow E_{i\sigma} - \Delta V / 2 \) and \( H_{LD} \) is the tunneling Hamiltonian between the leads and the TQD. The leads are described with a one-dimensional tight-binding model with nearest neighbor hopping \( t_L \), on-site energies \( \epsilon_L \) and \( \epsilon_R \) for the left (right) leads and coupling strength between dots and leads \( t_{LD} \). The current through the system is evaluated using a set of master equations for the occupation probabilities within the sequential tunneling approximation 27. In this approach we neglect higher order processes such as cotunneling which is important for high tunnel-coupling strengths and for temperatures below the Kondo temperature 28, 29. The occupation probabilities are then calculated using a detailed balance condition imposed by the conservation of charge. The spin components of the current in the linear regime at the lowest order in the coupling \( t_{LD} \) and at zero temperature are then given by \( I^s = \epsilon / (2\hbar) |t_{LD}|^2 \rho(\epsilon_F) \Delta V C^s \delta(\epsilon_F - (\epsilon_{2,G} - \epsilon_{1,G})) \) where \( \epsilon_{2,G} (\epsilon_{1,G}) \) is the ground state energy of the two (one) electrons and \( \rho(\epsilon_F) \) is the density of states in the

\[
\epsilon_C(B) = \epsilon_A + g^* \mu_B BS_z.
\]

The eigenenergies of the 2x2 triplet Hamiltonian are \( \epsilon_{1\sigma} = 1/2 \{ \epsilon_A(B) + \epsilon_C(B) \pm \left[ (\epsilon_A(B) - \epsilon_C(B))^2 + 4||_{13}^2 \right]^{1/2} \}. \)

By comparing the singlet and triplet eigenvalues we see that singlet is the ground state at \( B = 0 \) and the eigenvalues of the triplet do not oscillate as a function of the magnetic field. Even at this qualitative level, we obtain a remarkable result that triplet states do not oscillate with the magnetic field while singlets do.

In the case of finite tunneling each classical configuration is no longer an eigenstate of the system. Therefore, we will define QP as the point in the parameter space where the ground state energies of two and one electron dot differ by \( \mu_L \) and the three degenerate two-electron configurations are found with the same probabilities. Then, at the QP \( E_i = E^Q_i + \delta_i \), where the energies \( \delta_i \) are quantum corrections that are obtained numerically, with \( \delta_1 = \delta_3 \) for the symmetric case described here.

We shall analyze now the magnetic field dependence of the two electron energy spectrum close to the QP. The numerical CI calculations include the full Hilbert space generated from the three orbital levels. Hubbard parameters were obtained from the LCHO calculation with an interdot distance of 61.2 nm: \( t = -0.23 \text{ meV} \), \( U_i = 50|t| \) and \( V_{ij} = 10|t| \), \( g^* = -0.44 \) corresponding to GaAs will be assumed. The upper panel of Fig. 2 shows the lower part of the energy spectrum for \( E_0 = -|t| \), while the lowest panel indicates the total spin of the ground state. As was described above, the singlet (solid line) is the ground state at \( B = 0 \). The lowest energy of a singlet oscillates with period of one flux quantum while the energy of a triplet decreases monotonically with increasing magnetic field due to Zeeman energy. Notice that triplets show a small oscillation due to a coupling with higher energy configurations. The oscillating singlet energy and monotonically decreasing triplet energy leads to a number of transitions between singlet and triplet with increasing magnetic field. These transitions interrupt the AB oscillations of the singlet, and lead to their end at a critical value of the magnetic field, \( B_C = -\Delta ST / \mu B \) with \( \phi_C = A B C / \phi_0 \), indicated in Fig. 2 Above \( \phi_C \) the triplet is the ground state. Hence the presence of a trapped electron should lead to AB oscillations of the tunneling electron, interrupted and eventually terminated by the singlet-triplet transitions.
leads at the Fermi level. Here we make the assumption \( \rho_L(\varepsilon_{F,L}) \approx \rho_R(\varepsilon_{F,R}) \). \( C^\sigma = 1/3 \) for \( \sigma = \downarrow \) (singlet ground state) and \( C^\sigma = 1 \) for \( \sigma = \uparrow \) (triplet ground state).

Next we present the results for the linear conductance \( G = I/\Delta V \). The calculations were done at 50 mK \( (k_B T = 0.0145|t|) \), \( \Delta V = 2 \times 10^{-3}|t| \) and \( \mu_L = 0 \). In addition, \( |t_L| = 23.72 \text{ meV} \gg |t|, E_0, \Delta V \). Since transport through the TQD is allowed whenever the single-particle ground state and the two-particle ground state are on resonance, the AB oscillations of the energy spectra lead to repeated peaks in current. The spin components of the conductance \( G^\sigma = I^\sigma/\Delta V \) are shown in Fig. 4. At low magnetic fields, the spin down current is dominant and transport is mainly through the lowest oscillating singlet state. When the ground state of two particles becomes triplet, spin up current is dominant until the current is totally suppressed.

In summary, the presence of an extra electron localized in one dot of a ring-like TQD leads to spin selective AB oscillations as a function of magnetic field. The energy of the singlet ground state oscillates as a result of the interference between the two possible paths while the triplet state does not oscillate since one of the paths is spin blocked by the presence of a localized particle. The magnetic field orients the spin of the localized particle leading to the transport of electrons with a specific spin polarization. The AB oscillation of the singlet electron pair is reflected as peaks in the spin-down polarized current. At higher magnetic field, the Zeeman energy causes a singlet-triplet transition, which results in a change of the dominant spin component of the current.

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