Influence of Dilation on the Strength of Sand – Steel Interfaces

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Abstract. In this paper the sand-steel interface behaviour is analysed on the basis of frictional state theory. It is shown that for a small stress level the influence of dilatancy is fundamentally similar for sand-steel and sand-sand behaviour for a direct shear test. The use of the new parameter describes the influence of surface roughness on stress-dilatancy relationship. The peak strength of sand-steel is analysed. Values of the new parameter grow with normalised roughness for smooth surface and are constant for intermediate and rough surfaces. It is also shown that values of normalised friction coefficient obtained theoretically are similar with those shown in literature.

1. Introduction
The relative displacement generates shear stress and changes of volume in a narrow band of material between a bulk of granular assembly and a bulk of solid material, known as the interface.

The stress-strain-displacement response of interface plays an important role in the behaviour of deep foundations, retaining walls and reinforcement of soils, which was extensively investigated experimentally and theoretically [1-9]. The mechanical behaviour of interfaces is a function of solid material roughness; soil state and stress level [10-13].

The present paper shows that stress-strain-displacement response of sand-steel interfaces may be calculated based on frictional state theory [14]. The experiment conducted by Lings and Dietz [11-13] is the basis of the analysis.

In this paper it is postulated that the peak strength of dense sand-steel interfaces is a sum of two components: one independent of the change of volume (dilatancy) and the other directly generated by dilatancy. The dilatancy component may be simply calculated from the stress-dilatancy relationship developed by Szypcio [14] for direct shear condition.

On the basis of present analysis, the type of surface roughness may be interpreted anew.

2. Experimental data
Lings and Dietz [12] extensively tested the peak strength-dilatancy for sand-steel interfaces. Three sands of various granularity and shape of grains were used in the experiment: the coarse Leighton Buzzard sand with rounded grains termed as VLB, the medium golden colour sand with sub-rounded grains MGS, and fine silver-grey colour sand with sub-rounded grains SFS. The basic physical properties of sands are shown in Table 1.
Table 1. Physical properties of sands [12].

| Sand   | Name | $e_{min}$ | $e_{max}$ | $D_{10}$ | $D_{50}$ | $D_{60}$ | UC  | Shape         |
|--------|------|-----------|-----------|----------|----------|----------|-----|---------------|
| COARSE | VLB  | 0.506     | 0.802     | 0.64     | 0.78     | 0.81     | 1.27| Rounded       |
| MEDIUM | MGS  | 0.494     | 0.806     | 0.34     | 0.44     | 0.45     | 1.32| Sub-rounded   |
| FINE   | SFS  | 0.684     | 1.017     | 0.09     | 0.13     | 0.14     | 1.56| Sub-rounded   |

The tests were carried out using a modified conventional 100 mm square DSA. The modified DSA apparatus overcomes the tendency to overestimate friction and underestimate dilation [11]. The lower frame of shear box was replaced by a series of steel blocks with various roughness surfaces, which were in contact with sand. Four distinct steel roughness magnitudes were investigated (POL, GND, ALO, SIC). In another test three different roughness magnitudes were investigated, formed by fixing each of the three test sands to steel blocks (SFS, MGS, VLB). The formative processes and properties of surfaces and the average roughness are shown in Table 2.

Table 2. Characteristic properties of surfaces [12].

| Material | Name | Formative process                                                                 | Ra [µm] |
|----------|------|-----------------------------------------------------------------------------------|----------|
| Steel    | POL  | Abrasion of ground surface using progressively finer grades of abrasive paper, before polishing with Brasso | 0.147    |
|          | GND  | Grinding wheel, with direction of striations perpendicular to direction of shearing | 0.356    |
|          | ALO  | Shot blasting with 120 grit aluminium oxide ($D_{50} = 0.2mm$)                     | 2.49     |
|          | SIC  | Shot blasting with 16 grit silicon carbide ($D_{50} = 1.8mm$)                     | 9.40     |
| Sand     | SFS  | Pluviation of SFS onto a uniform coating of Araldite smeared across the block      | 33.7     |
|          | MGS  | Pluviation of MGS onto a uniform coating of Araldite smeared across the block      | 114      |
|          | VLB  | Pluviation of VLB onto a uniform coating of Araldite smeared across the block      | 180      |

The normal stress and rate of displacement (1.2 mm/min) were constant during the tests. The horizontal force, horizontal and vertical displacement were automatically recorded. A relative roughness is marked as

$$R_n = \frac{R_a}{D_{50}}$$ (1)

where $R_a$ is average roughness, $D_{50}$- diameter of mean grain.

The tests data for normal stress $\sigma_n \approx 25kPa$ and various densities $D_t$ are shown in Table 3. The friction angle $\delta$ is calculated from equation

$$\tan \delta = \frac{\tau}{\sigma_n}$$ (2)

And dilatancy angle $\xi$ from equation

$$\tan \xi = \frac{\delta h}{\delta s}$$ (3)

where $h$ is vertical displacement of DSB frame (positive for upwards displacement) and $s$- shear displacement.
Table 3. Data of interface tests.

| Sand | Surface | $R_n$  | $D_r$ [%] | $\sigma_n'$ [kPa] | $\delta_p$ [deg] | $\xi_p$ [deg] | $\delta_{pp}$ [deg] | $\xi_{pp}$ [deg] |
|------|---------|--------|-----------|-------------------|-----------------|--------------|-----------------|-----------------|
| VLB  | POL     | 0.000189 | 89      | 25.3             | 11.7            | 0.4          | 9.3             | -0.2            |
| VLB  | GND     | 0.000456 | 94      | 25.2             | 16.7            | 0.2          | 13.4            | 0.0             |
| VLB  | ALO     | 0.00319  | 92      | 25.3             | 26.0            | 1.9          | 23.8            | 0.3             |
| VLB  | SIC     | 0.0121   | 88      | 25.1             | 33.2            | 9.6          | 26.6            | 0.1             |
| VLB  | SFS     | 0.0432   | 83      | 25.2             | 39.9            | 16.4         | 28.5            | 0.2             |
| VLB  | MGS     | 0.1462   | 89      | 25.2             | 47.5            | 24.8         | 30.9            | -0.7            |
| VLB  | VLB     | 0.2308   | 86      | 25.2             | 48.0            | 26.6         | 30.8            | -0.1            |
| VLB  | ALO     | 0.00319  | 93      | 25.2             | 29.1            | 5.2          | 26.1            | 0.0             |
| VLB  | SIC     | 0.0121   | 90      | 25.1             | 31.4            | 8.6          | 23.5            | 0.6             |
| MGS  | POL     | 0.000334 | 94      | 25.2             | 13.7            | 0.4          | 10.8            | 0.0             |
| MGS  | GND     | 0.000809 | 93      | 25.3             | 15.3            | 0.0          | 11.8            | -0.1            |
| MGS  | ALO     | 0.00566  | 94      | 25.1             | 27.7            | 3.7          | 24.3            | 0.5             |
| MGS  | SIC     | 0.0214   | 93      | 25.2             | 39.0            | 14.1         | 27.7            | 0.3             |
| MGS  | SFS     | 0.0766   | 92      | 25.3             | 47.1            | 24.5         | 30.5            | 0.7             |
| MGS  | MGS     | 0.2591   | 93      | 25.3             | 49.0            | 26.3         | 31.7            | 0.7             |
| MGS  | GND     | 0.000809 | 78      | 25.1             | 13.3            | 0.1          | 9.3             | 0.1             |
| MGS  | GND     | 0.000809 | 69      | 25.2             | 11.5            | 0.1          | 7.1             | -0.1            |
| MGS  | GND     | 0.000809 | 23      | 25.1             | 10.8            | 0.1          | 8.0             | -0.4            |
| MGS  | ALO     | 0.00566  | 79      | 25.3             | 25.1            | 3.2          | 22.4            | -0.1            |
| MGS  | ALO     | 0.00566  | 69      | 25.3             | 25.1            | 2.3          | 21.6            | 0.1             |
| MGS  | ALO     | 0.00566  | 24      | 25.3             | 22.6            | 0.0          | 21.3            | -0.3            |
| MGS  | ALO     | 0.0214   | 75      | 25.3             | 35.8            | 12.7         | 28.4            | 0.6             |
| MGS  | SIC     | 0.0214   | 68      | 25.4             | 33.4            | 11.2         | 28.5            | 0.5             |
| MGS  | SIC     | 0.0214   | 22      | 25.1             | 29.1            | 1.5          | 28.2            | 0.0             |
| MGS  | SFS     | 0.0766   | 70      | 25.2             | 42.8            | 18.2         | 32.0            | 1.1             |
| MGS  | SFS     | 0.0766   | 26      | 25.3             | 33.7            | 4.2          | 32.5            | 1.3             |
| MGS  | MGS     | 0.2591   | 78      | 25.3             | 43.4            | 18.6         | 32.3            | 0.5             |
| MGS  | MGS     | 0.2591   | 27      | 25.2             | 33.9            | 3.6          | 31.8            | -0.5            |
| MGS  | MGS     | 0.2591   | 27      | 25.2             | 33.9            | 3.6          | 31.8            | -0.5            |
| MGS  | MGS     | 0.2591   | 27      | 25.2             | 33.9            | 3.6          | 31.8            | -0.5            |
| MGS  | MGS     | 0.2591   | 27      | 25.2             | 33.9            | 3.6          | 31.8            | -0.5            |
| MGS  | MGS     | 0.2591   | 27      | 25.2             | 33.9            | 3.6          | 31.8            | -0.5            |
| SFS  | POL     | 0.00113  | 99      | 25.3             | 10.9            | 0.4          | 9.3             | -0.1            |
| SFS  | GND     | 0.00274  | 96      | 25.3             | 15.3            | 0.1          | 10.8            | -0.1            |
| SFS  | ALO     | 0.0192   | 94      | 25.2             | 36.5            | 10.2         | 28.2            | 0.0             |
| SFS  | SIC     | 0.0723   | 92      | 25.3             | 42.8            | 15.2         | 30.6            | 0.1             |
| SFS  | SFS     | 0.2592   | 97      | 25.2             | 43.2            | 17.0         | 31.6            | 0.1             |

As in the original paper [12], $\delta_p$ and $\delta_{pp}$ are referred to as friction angles and $\xi_p$ and $\xi_{pp}$ as dilation angles at peak and post-peak (ultimate) state respectively.

3. Stress-dilatancy for soils

General stress-dilatancy for soils for direct shear [14] is

$$\frac{\tau}{\sigma'_n} = \frac{\sqrt{3} n \cos \phi' \cos \theta}{3 + n \left( \sin \theta - \sqrt{3} \sin \phi' \cos \theta \right)}$$

(4)

where

$$n = \frac{q}{p'} = \frac{M'_b}{A_b} \left( \alpha + \beta D^r \right)$$

(5)

$$M'_b = \frac{3 \sin \phi'}{\sqrt{3} \cos \theta - \sin \phi' \sin \theta}$$

(6)
\[ A_s^o = \frac{1}{\cos(\theta - \theta_0)} \left\{ 1 - \frac{2}{3} M_s^o \sin \left( \theta - \frac{2}{3} \pi \right) \right\} \] (7)

q is stress invariant, \( p' \) - mean effective stress.

Neglecting elastic strain, we can write

\[ D^p = -\sqrt{3} \frac{\delta h/\delta s}{\sqrt{1 + \frac{4}{3} \left( \frac{\delta h}{\delta s} \right)^2}} \] (8)

\[ \tan \theta_e = \frac{1}{\sqrt{3}} \frac{\delta h/\delta s}{\sqrt{1 + \left( \frac{\delta h}{\delta s} \right)^2}} \] (9)

The angle \( \phi^o \) is the critical frictional state angle [15]. For many sands (\( \phi^o=\phi_{cv} \)) but for Leighton Buzzard sand \( \phi^o=27.5^o \), smaller than \( \phi_{cv} \) [14].

For plane strain condition in direct shear test value of Lode angle it may be accepted that \( \theta=15^o \) [14, 16]. Angle \( \theta_e \) is the Lode angle for strains and \( (\theta - \theta_e) \) is a non-coaxiality angle [15].

At simple and direct shear condition for sands it is accepted that \( \alpha=0 \) and \( \beta=1.4 \) [14].

The equation (5) may be written in the form

\[ \eta = Q_s - A_s D^p \] (10)

where \( Q_s = M_s^o - \alpha A_s^o \) and \( A_s = \beta A_s^o \)

4. Friction of sand-steel interface

The stress ratio \( \eta \) is a linear function of dilatancy \( D^p \). The value \( Q_s \) represents friction at zero dilatancy state and \( A_s \) is a parameter which represents the influence of dilatancy on friction for direct shear condition.

It is postulated in this paper that for interface equation (10) has the modernized form:

\[ \eta_s = \kappa Q_s - A_s D^p \] (11)

where \( \kappa \) is a parameter which represents influence of surface roughness on stress ratio.

So it is postulated that the roughness of surface generates characteristic dilatancy in interface, but influence of dilatancy (value of parameter \( A_s \)) on stress ratio is identical to the one for conventional direct shear of sands.

5. Influence of roughness on peak strength

The influence of surface roughness on the peak strength characterised by peak angle of friction \( \delta' \) is calculated from equation (4) with \( \eta=\eta_0 \) calculated from equation (10) for peak values of dilatancy \( \theta=15^o \), \( \alpha=0.0 \), \( \beta=1.4 \). In this paper it is accepted that for VLB sand \( \phi^o=27.5^o \) [14] for MGS and SFS sands \( \phi^o \) values are obtained from “back analysis” in post peak strength (\( D^p \approx 0 \)). It is accepted that \( \phi^o=31.4^o \) for MGS and \( \phi^o=30.7^o \) for SFS sands. The values of parameter \( \kappa \) taken for calculation were obtained by the “trial and error” method. The accepted values of \( \kappa \) are those for which calculated values \( \delta' \) are equal to the experimental ones.

Analysing these dependencies it is interesting to note that for all dense sands (\( \delta h/\delta s \) ) \( \approx 0 \) for \( R_s<0.003 \) grows for 0.003 <\( R_s < 0.2 \) and reaches the constant maximum value for \( R_s > 0.2 \). Values of
parameter $\kappa$ grow with normalized roughness for $R_n < 0.003$ and are almost constant for bigger values of $R_n$.

The peak friction angle $\delta'_p$ as a function of $(\delta h/\delta s)_p$ and $\kappa$ grows with normalized roughness $R_n$ for $R_n < 0.2$.

For loose tested MGS sand the values of parameter $\kappa$ grow with normalized roughness for $R_n < 0.2$. The underlying mechanism for this behaviour is not clear. Further investigation is needed to explain this fact.

The analysis of dependence $\delta'_p$, $(\delta h/\delta s)_p$, and $\kappa$ on normalized roughness enables a new interpretation of surface roughness for sand-steel interface. Table 4 enables the definition of various roughness of surfaces: smooth, intermediate and rough.

In Figures 1, 2, 3 the dependence of $\delta'_p$, $(\delta h/\delta s)_p$, and $\kappa$ on normalized surface roughness $R_n$ are shown for VLB, MGS and SFS sands respectively.

![Figure 1. Dependence of characteristic parameters on surface roughness for VLB sand: (a) peak friction angle $\delta'_p$ – relative roughness $R_n$, (b) incremental displacement ratio at peak $(\delta h/\delta s)_p$ – relative roughness $R_n$, (c) parameter $\kappa$ – relative roughness $R_n$](image)

| Surface    | $R_n$       | $(\delta h/\delta s)_p$ | $\kappa$  | $\delta'_p$     |
|------------|-------------|--------------------------|-----------|-----------------|
| Smooth     | $R_n < 0.003$ | 0                        | Grows with $R_n$ | Grows with $R_n$ |
| Intermediate | $0.003 < R_n < 0.2$ | Grows with $R_n$ | Constant | Grows with $R_n$ |
| Rough      | $R_n > 0.2$  | Constant                 | Constant  | Constant        |
Figure 2. Dependence of characteristic parameters on surface roughness for MGS sand:
(a) peak friction angle $\delta'_p$ – relative roughness $R_n$,
(b) incremental displacement ratio at peak $(\delta h/\delta t)_p$ – relative roughness $R_n$, (c) parameter $\kappa$ – relative roughness $R_n$.

Figure 3. Dependence of characteristic parameters on surface roughness for SFS sand:
(a) peak friction angle $\delta'_p$ – relative roughness $R_n$,
(b) incremental displacement ratio at peak $(\delta h/\delta t)_p$ – relative roughness $R_n$,
(c) parameter $\kappa$ – relative roughness $R_n$. 
The dependence of $\tan \delta'_p$ on $(\delta h/\delta s)_p$ is shown in Figure 4 and has the form

$$\tan \delta'_p = 0.42 + 1.440 \frac{\delta h}{\delta s}$$

(12)

The dependence of $\delta'_p$ on $\xi_p$ is shown in Figure 5 and has the form

$$\delta'_p = 22.8^\circ + \xi_p$$

(13)

Figure 4. Dependence of $\tan \delta'_p$ on incremental displacement ratio at peak $(\delta h/\delta s)_p$

Figure 5. Dependence of $\delta'_p$ on peak dilatancy angle $\xi_p$

Lings and Dietz [12] give similar equation

$$\delta'_p = 25^\circ + \xi_p$$

(14)

6. Normalized friction coefficient

The normalized friction coefficient is defined [17] as
\[ f_n = \frac{\tan \delta_p}{\tan \phi_{ds}} \]  

(15)

where \( \phi_{ds} \) is an angle of friction for direct shear test of sand.

The angle of friction for direct shear may be obtained theoretically using equations (4) to (11) with \( \kappa = 1 \), \( \alpha = 0 \) and \( \beta = 1.4 \) and maximum values of \( (\delta h/\delta s)_{p} \) for rough surfaces. The dependence of normalized friction coefficient calculated by this manner is shown in Figure 6.

![Figure 6](image)

Figure 6. Dependence of normalized friction coefficient \( f_n \) on relative roughness \( R_n \)

For the analysed experimental data the normalised friction coefficient grows with surface roughness for \( R_n < 0.2 \) and reach almost constant value \( f_n \approx 0.9 \) for \( R_n > 0.2 \). Similar values of \( f_n \) are published in literature [17-19].

7. Conclusions
No fundamental difference exists between sand-steel and sand-sand behaviour. The frictional state theory may be used to describe the influence of dilatancy on the peak strength of dense sand-steel interfaces at small stress level.

Using the frictional state theory, a new interpretation of surface roughness is possible.

More investigation is needed to confirm this consideration, especially for loose sand and a higher level of stress.

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