The productiveness of Bootstrap simulator in evaluating the accuracy parameters of measurement system for ball screw

Zun Liang Chuan¹, Muhamad Husnain Mohd Noh², Mohd Akramin Mohd Romlay² and Mu Wen Chuan³

¹ Centre for Mathematical Science, Universiti Malaysia Pahang, Lebuhraya Tun Razak, 26300 Gambang Kuantan, Pahang DM, Malaysia
² Faculty of Mechanical & Manufacturing Engineering, Universiti Malaysia Pahang, 26600 Pekan, Pahang DM, Malaysia
³ Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 UTM Skudai, Johor, Malaysia

Email: chuanzl@ump.edu.my

Abstract. Ball screw is an essential mechanical component of computer numerical controlled (CNC) milling machine, which the positioning accuracy of ball screw is highly associated with lead angle accuracy and axial clearance. In particular, the inaccuracy of lead angle and axial clearance of ball screw can be massively affected the inaccuracy of positioning, leading to the degraded quality of manufactured products. Therefore, a reliable and productive measurement system analysis is indeed in monitoring the accuracy parameters of the ball screw. The main objective of this study is to propose using the Bootstrap simulator in monitoring the accuracy parameters of measurement system for ball screw, with the abstraction of cost and time. The accuracy parameters of the measurement system are including stability, bias and linearity. Based on the simulation results, it can be concluded that the Bootstrap simulator is more productive in monitoring the accuracy parameters of measurement system for ball screw compared to the Monte Carlo simulator. This is due to the Bootstrap simulator can be yielded a lower uncertainty of simulation compared to the Monte Carlo simulator. Furthermore, the Bootstrap simulator is also more advantages compared to the Monte Carlo simulator as this simulator can be carried out with small sample size of measurement data.

1. Introduction

In principle, an ideal measurement system for the manufacturing process is exhibits a zero error. However, this statistical characteristic is beyond the bounds of possibility in practical. This is due to the manufacturing process in real life is frequently involved two types of errors, namely Type I and Type II errors. In specific, Type I error is an error occurred when the acceptance products misjudged as the defective products, while a Type II error is an error occurred when the defective products misjudged as the acceptance products. Since these types of errors can massively cause negative and expensive effects on manufacturers such as reputation, cost, and profit, therefore evaluating the accuracy parameters of measurement system using statistical analysis must be practically implemented in reducing the risk occurrence of these types of errors. Furthermore, conducting a statistical analysis on measurement system is one of the essences of the standard ISO/TS16949 by identifying the sources
of variation in the measurement process [1], where ISO/TS16949 is a globally well-known quality management standard for the automotive industry.

Computer numerical controlled (CNC) milling machine has been widely used in industries such as automotive manufacturing industry [2]. One of the vital mechanical components for CNC milling machine is ball screw [3]. In CNC milling machine, ball screw plays an important role as mechanical linear actuator that translates rotational motion into linear motion with minimal friction. However, the positioning inaccuracy of ball screw can be degraded the quality of manufactured products. The main causes of the occurrence of the positioning inaccuracy are due to the inaccuracy of lead angle and axial clearance of ball screw [4]. Due to the impose time that require the schedule resource to be smoothed and financial constraints, therefore a reliable simulator for a high quality measurement system is indeed when the sample size of measurement data is small. This is because the simulation can be reduced the uncertainty of simulation in monitoring the accuracy parameters of measurement system for ball screw without increase the cost and time. For instance, Pai et al. [4] proposed the use of the Monte Carlo simulation in evaluating the accuracy parameters of measurement system for ball screw, such as stability, bias and linearity.

Therefore, the main objective of this study is to extend the study of Pai et al. [4] by monitoring the accuracy parameters of measurement system for ball screw using the Bootstrap simulator. In general, the Monte Carlo simulator generated a set of random sample from a statistical distribution with predefined parameters. Alternatively, the Bootstrap simulator is a non-parametric version of the Monte Carlo simulator, which is not underlying any statistical assumption. The main differences between the Monte Carlo and Bootstrap simulators are the technique to generate the random samples. In this study, the Bootstrap approach is selected due to this approach has been successfully applied in several researches [5, 6]. In order to authenticate the productiveness of the proposed simulator, the performance comparison between the Monte Carlo and Bootstrap simulators in monitoring the stability, bias and linearity of measurement system for ball screw are evaluated based on the degree of uncertainty. Moreover, root mean square error and false positive rate is also used as the additional performance evaluation of the stability and bias in the measurement system for ball screw, respectively. The rest of this paper is organized as follows. In Section 2, the methodological framework and a brief overview of theoretical background are presented, while Section 3 discussed the simulation results based on the Monte Carlo and Bootstrap simulators. Finally, the concluding remarks are rendered in Section 4.

2. Methodological framework and theoretical framework

In this study, all the simulation analysis in monitoring the accuracy parameters of measurement system for ball screw is conducted using R statistical software. Moreover, all the measurement data applied for simulation are acquired from the publication of Pai et al. [4], where the measurement data are collected by a skillful quality control staff using a dial gauge with the code PG-02. The simulation methodological framework in monitoring the stability, bias and linearity of measurement system for ball screw involved in this study as described as below.

Step 1. Select a random seed to provide a fixed simulation environment.
Step 2. Simulate a set of measurement data based on an appropriate sample size for the stability, bias and linearity of measurement system, respectively.
Step 3. Construct the appropriate statistical control charts using the simulated measurement data in determining the stability of the measurement system.
Step 4. Compute the $(1-\alpha)100\%$ confidence intervals using the simulated measurement data of the bias and linearity of measurement system, respectively.
Step 5. Conduct the appropriate statistical hypothesis testing in determining the significance of the bias and linearity of the measurement system.
Step 6. Determine the best simulator based on the uncertainties of simulation which provides narrow widths of control limits and $(1-\alpha)100\%$ confidence intervals.
2.1 Stability

The stability analysis is used to identify whether the bias of measurement system changes over time or vice versa. This analysis is very helpful in determining the proper increment calibration and repair intervals. In practical, statistical control charts are used in evaluating the acceptability of the measurement system stability. Based on the control chart decision tree, \( \bar{X} \) and \( R \) charts are used to monitor the mean and variation of measurement data at the consecutive day shifts. This is because the measurement data collected is continuous and the subgroup sizes are between two to nine, namely three consecutive day shifts. Suppose that \( X = \left[ x_{ij} \right]_{ij} ; i,j = 1,2,\ldots,I,J \) represents a matrix of simulated measurement data collected from \( i \)th day on \( j \)th shift. Therefore, the lower \( (\text{LCL}_X) \) and upper \( (\text{UCL}_X) \) control limits for the \( \bar{X} \) control chart can be obtained based on equations (1) and (2), respectively.

\[
\text{LCL}_X = \bar{X} - A_2 \bar{R} \quad (1) \\
\text{UCL}_X = \bar{X} + A_2 \bar{R} \quad (2)
\]

Meanwhile, the lower \( (\text{LCL}_R) \) and upper \( (\text{UCL}_R) \) control limits for the \( R \) control chart are given as

\[
\text{LCL}_R = D_3 \bar{R} \quad (3) \\
\text{UCL}_R = D_4 \bar{R} \quad (4)
\]

where \( \bar{R} = \frac{1}{I} \sum_{i=1}^{I} R_i \) is the average of range, \( R_i = \max_i (x_{ij}) - \min_i (x_{ij}) \), \( \bar{X} = \frac{1}{J} \sum_{j=1}^{J} X_i \) is the average of an average for \( X \) with \( \bar{X}_i = \sum_{j=1}^{J} x_{ij} \). \( A_2 \), \( D_3 \) and \( D_4 \) are the control chart constants. If there is no any measurement data beyond the LCL or UCL, therefore the stability of measurement system can be accepted. Conversely, if there is any measurement data that beyond the LCL or UCL, identify the possible causes and recollection of measurement data is indeed. In addition, this study also applied the root mean square error (RMSE) in investigating the accurateness between the observed and simulated measurement data. The value of RMSE is zero indicates a perfect fit of the data.

2.2 Bias

Bias analysis is a study concerning the deviation of the reference value and the measurement data using a similar instrument in gauging a similar part of the ball screw. Based on the MSA guidelines [7], there are two approaches can be used to evaluate the acceptability of the bias in the measurement system. For graphical approach, the bias of measurement system is accepted when there is no any anomaly presented on the histogram. The main drawback of using a graphical approach in monitoring bias of measurement system is this approach incompetent to provide the statistical evidence. Therefore, a statistical hypothesis based on confidence interval technique is applied in this study. The \( (1-\alpha)100\% \) confidence bounds can be obtained based on equation (5).

\[
\left( \bar{E} - t_{\alpha/2}, v = I-1 \frac{s_e}{\sqrt{I}} , \bar{E} + t_{\alpha/2}, v = I-1 \frac{s_e}{\sqrt{I}} \right) \quad (5)
\]
where \( \bar{\varepsilon} = \frac{1}{n} \sum_{k=1}^{n} \varepsilon_k \) is the average of bias, and \( s_{\varepsilon}^2 = \frac{1}{n-1} \sum_{k=1}^{n} (\varepsilon_i - \bar{\varepsilon})^2 \) is the variance of bias with \( \varepsilon_k = y_k - \theta \), \( y_k \) is the simulated measurement data, \( \theta = \frac{1}{n} \sum_{k=1}^{n} y_k \) is the reference value, and \( n \) is the size of the simulation. The bias of measurement system is accepted when the zero value falls within the \((1-\alpha)100\%\) confidence bounds.

2.3 Linearity

The main principle of linearity analysis is to investigate the effect of parts size on the bias of the measurement system. In other words, this analysis is desired to determine whether the accuracy is consistent in all parts of ball screw to be measured. As the stability and bias analysis, the linearity of measurement system also can be evaluated using graphical or numerical approaches. In this study, the numerical approach based on linearity hypothesis testing is selected due to this approach can be provided statistical evidence and vice versa for the graphical approach. The test statistic of linearity of measurement system can be defined as

\[
|\bar{\beta}| = \frac{|\hat{\beta}_1|}{se(\hat{\beta}_1)}
\]

(6)

where \( \hat{\beta}_1 = \frac{\sum_{m=1}^{gp} (\tau_m \phi_m) - \frac{1}{gp} \sum_{m=1}^{gp} \tau_m \sum_{m=1}^{gp} \phi_m}{\sum_{m=1}^{gp} \tau_m^2 - \frac{1}{gp} \left(\sum_{m=1}^{gp} \tau_m\right)^2} \) is the estimated slope and its standard error,

\[
se(\hat{\beta}_1) = \sqrt{\frac{S_{\phi \phi} - \hat{\beta}_1 S_{\tau \phi}}{S_{\tau \tau} (gp - 2)}},
\]

\( g \) is the number of parts of the ball screw to be measured, and \( p \) the number replicates in measuring each part of the ball screw. Meanwhile the variance of bias, \( S_{\tau \tau} \), variance of reference value, \( S_{\phi \phi} \) and covariances between bias and reference value, \( S_{\tau \phi} \) can be expressed as

\[
S_{\tau \tau} = \frac{\sum_{m=1}^{gp} \tau_m^2 - \left(\frac{\sum_{m=1}^{gp} \tau_m}{gp}\right)^2}{gp}
\]

(7)

\[
S_{\phi \phi} = \frac{\sum_{m=1}^{gp} \phi_m^2 - \left(\frac{\sum_{m=1}^{gp} \phi_m}{gp}\right)^2}{gp}
\]

(8)

\[
S_{\tau \phi} = \frac{\sum_{m=1}^{gp} (\tau \phi)_m - \sum_{m=1}^{gp} \tau_m \sum_{m=1}^{gp} \phi_m}{gp}
\]

(9)
respectively. The linearity of measurement system is accepted if $|t| \leq t_{\alpha/2,g,n-1}$. Moreover, the $(1-\alpha)100\%$ confidence bounds for $\hat{\beta}_i$ is given as

$$
\left( \hat{\beta}_0 + \hat{\beta}_1 \tau_h \right) \pm t_{\alpha/2,g,p} \sqrt{\frac{1}{g_p} + \frac{\tau_h - \bar{\tau}}{\sum_{h=1}^{g} (\tau_h - \bar{\tau})^2}} \right]
$$

where $\hat{\beta}_0 = \bar{\phi} - \hat{\beta}_1 \bar{\tau}$ is the estimated intercept, $\tau; h = 1, 2, ..., g$ is the reference point for $h$th measured part of ball screw, and $\omega^2 = \frac{\sum_{m=1}^{g_p} \phi_m^2 - \hat{\beta}_0 \sum_{m=1}^{g_p} \phi_m - \hat{\beta}_1 \sum_{m=1}^{g_p} \tau_m \phi_m}{g_p - 2}$.

3. Analysis results

Based on the measurement collected by Pai et al. [4], the $I$ and $J$ for the stability study are 30 days and 3 consecutive day shifts. Meanwhile, the control chart constants, $A_2, D_3$ and $D_4$ are 1.023, 0 and 2.574. In addition, the $g$ and $p$ for the linear analysis are 5 and 12, which fulfil the evaluation guideline as stated in [7]. Moreover, the size of simulations, $n$ and the false positive rate, $\alpha$ applied in this study is 10000 and 5%. In order to pursue the objective of this study, the performance comparison of the Monte Carlo and Bootstrap simulators in monitoring the stability, bias and linearity of measurement system for ball screw is presented in tables 1-2 and figures 1-4.

Based on $\bar{X}$ (figure 1) and $\bar{R}$ (figure 2) control charts simulated based on the Monte Carlo and Bootstrap simulators, respectively. These figures showed that the stability of measurement system for ball screw is accepted. This is because there is no any measurement points go beyond the lower and upper control limits. In addition, this study also employed the RMSE in calibrating the accurateness between the observed and simulated measurement data. The value of RMSE depicted in table 1 shows that the simulated measurement data for both simulators are very close to the observed data as the values of RMSE are approached to zero.

Figure 4 illustrated the histograms constructed using simulated measurement data based on the Monte Carlo and Bootstrap simulators, respectively. Since there is no any anomaly presented in both histograms, therefore the bias of measurement system for ball screw can be claimed as accepting. In order to authenticate this claim’s, a statistical hypothesis testing based on confidence interval technique are applied. The 95% confidence bounds (table 2) for the simulated measurement data based on the Monte Carlo and Bootstrap simulators are $(-5.2396 \times 10^{-3}, 5.1837 \times 10^{-3})$ and $(-1.3182 \times 10^{-3}, 1.2818 \times 10^{-3})$, respectively. The value of zero falls within both 95% confidence bounds, therefore this study concluded that the bias of measurement system based on both simulators is significantly accepted. Furthermore, false positive rate also used to evaluate the performance of both simulators. Based on table 2, it found that the Bootstrap simulator is a more productiveness in simulating measurement data compared to the Monte Carlo simulator. This is because the Bootstrap simulator can be yielded a value closer to the fixed false positive rate compared to the Monte Carlo simulator, which the fixed false positive rate is 5%.

Meanwhile, figure 4 depicted the fitted regression line and its 95% confidence bounds based on both simulated measurement data in monitoring the linearity of the measurement system. Based on figure 4, it can be observed the Bootstrap simulator is performing better than the Monte
Carlo simulator. This is because 95% confidence bounds based on the Bootstrap simulator is narrower compared to the Monte Carlo simulator, leading to the lower degree of uncertainty. However, since both 95% confidence bounds are not bounded the values of zero for all five measured parts, therefore it can be concluded that the linearity of measurement system is significantly unaccepted. The similar conclusion also can be reached when a linearity statistical hypothesis testing based on $\beta_1$ is conducted. One of the possible causes that the linearity of measurement system unaccepted is a violation of an assumption of normality and heteroscedasticity in perspective of statistics. This is due to the data points is deviated to form a straight line in normal probability plot and there is a specific pattern reveals in the residual plot.

**Figure 1.** The $\bar{X}$ chart of stability analysis between the observed and simulated measurement data using the (a) Monte Carlo simulator; (b) Bootstrap simulator, respectively.
Figure 2. The $R$ chart of stability analysis between the observed and simulated measurement data using the (a) Monte Carlo simulator; (b) Bootstrap simulator, respectively.
Table 1. The performance comparison of stability analysis between the observed and simulated measurement data using the Monte Carlo and Bootstrap simulators, respectively.

| Simulation approach | Control chart | Central limit | Control limits | RMSE |
|---------------------|---------------|---------------|----------------|------|
|                     |               |               | Lower          | Upper|      |
| Observed            | \( \bar{X} \) | 14.0005       | 13.9967        | 14.0042 | n/a |
|                     | \( R \)      | 0.0037        | 0.0000         | 0.0095 | n/a |
| Monte Carlo         | \( \bar{X} \) | 14.0006       | 13.9967        | 14.0045 | 0.0016 |
|                     | \( R \)      | 0.0038        | 0.0000         | 0.0099 | 0.0023 |
| Bootstrap           | \( \bar{X} \) | 14.0005       | 13.9969        | 14.0042 | 0.0013 |
|                     | \( R \)      | 0.0035        | 0.0000         | 0.0091 | 0.0025 |

Figure 3. The histogram of bias analysis simulated using the
(a) Monte Carlo simulator; (b) Bootstrap simulator, respectively.
Table 2. The performance comparison of bias analysis between the observed and simulated measurement data using the Monte Carlo and Bootstrap simulators, respectively.

| Simulation approach | Average of bias   | 95% bounds       | False positive rate |
|---------------------|-------------------|-------------------|---------------------|
|                     | Lower bound       | Upper bound       |                     |
| Observed            | $2.1316 \times 10^{-5}$ | $-6.6303 \times 10^{-7}$ | $2.8371 \times 10^{-5}$ | n/a          |
| Monte Carlo         | $1.5760 \times 10^{-5}$ | $-5.2396 \times 10^{-5}$ | $5.1837 \times 10^{-5}$ | 5.64%        |
| Bootstrap           | $-1.4996 \times 10^{-5}$ | $-1.3182 \times 10^{-5}$ | $1.2818 \times 10^{-5}$ | 5.08%        |

Figure 4. The analysis of simulated linearity using the Monte Carlo and Bootstrap simulators, respectively.

4. Conclusion
This study presented a comparison study of the Monte Carlo and Bootstrap simulators in monitoring the accuracy parameters of measurement system for ball screw. The accuracy parameters of measurement system include stability, bias and linearity. The analysis results showed that the stability and bias of the measurement system are accepted, while the linearity of measurement system is significantly unaccepted, which evaluated based on the simulated measurement data. Based on the perspective of statistics, one of the possible causes that the linearity of measurement system is unaccepted is the violation of normality and heteroscedasticity assumptions. Furthermore, the analysis results also showed that the Bootstrap simulator is more productiveness compared to the Monte Carlo simulator in monitoring the accuracy parameters for ball screw. This is due to the width of the control limits and 95% confidence bounds for all accuracy parameters based on the Bootstrap simulator are comprehensively narrower compared to the Monte Carlo simulator. In other words, the Bootstrap simulator frequently yielded lower uncertainty of simulation compared to the Monte Carlo simulator. In summary, the Bootstrap simulator is more advantageous than the Monte Carlo simulator as the Bootstrap simulator is competent resulted low uncertainty of simulation in the condition of small measurement data without increase the cost and time.
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