Development of the numerical model for evaluating the temperature field and thermal stresses in structural elements of aircrafts

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Abstract. An approximate method for estimating the thermal stresses of the aircraft key components of the simple geometric shape (the edges of the hull and wings, the nose fairing) has been developed. The mathematical model of such estimates is based on the solution of the quasi-static thermoelasticity problem. The solution is evaluated in the area with curvilinear boundaries, and the shape of these boundaries changes under the influence of thermal and mechanical loads. Thus the computational domain is transformed to an area where the regular Cartesian (structured) grid can be introduced. The initial validation and verification of the developed numerical methodology was carried out. Numerical modeling of temperature fields and thermal stresses in the simplest components of aircraft structures (cylinder blunted over the sphere and the shell) is performed.

1. Introduction

One of the most difficult problems of aerothermodynamics is the problem of modelling the conjugate heat transfer on the surface of a high-speed aircraft, as well as the problem of calculation the resulting thermal stresses in its hull. In this case, when the aircraft moves, the boundary layer can be regarded as a narrow spatial region adjacent to the surface of the streamlined body in which there is the intense heat release due to energy dissipation processes. Dissipation processes are accompanied by a strong change in the thermophysical (density, pressure, temperature, viscosity, thermal conductivity, etc.), the dynamic properties of the gas, heat fluxes directed to the surface of the aircraft, construction material and performance characteristics. However, carrying out real physical experiments in this area of the aircraft motion is of high cost, due to a multitude of technological and technical difficulties. Therefore, the mathematical modelling of thermal stresses and thermophysical processes in the aircraft, as well as near its surface, is highly important for optimizing the performance characteristics and design of the aircraft.

2. The statement of the problem

The aim of this work is to develop an approximate method for estimating thermal stresses in the aircraft (or in their key components: body and wing edges, nose cone) of a simple spatial shape, for
example: in the form of a sphere conjugated to a cylinder. The mathematical model of these estimates, intended for the calculation of thermal stresses in individual elements of the design of aircraft, is based on the solution of the quasi-static thermoelasticity problem [1-5]. It includes the equations of mechanical equilibrium of a linearly elastic medium with allowance for temperature stresses (however, this mathematical model does not allow a complete description of thermomechanical processes, since it lacks the basic physical mechanisms that take into account the plastic deformation of structural materials) and the heat equation of a special type. As a rule, the physical area (in which the solution is sought) has curvilinear boundaries, and their shape changes in the process of modelling under the influence of thermal and mechanical loads. To solve a problem with such spatial boundaries, the computational domain is transformed to an area in which a regular Cartesian (structured) grid can be introduced.

When evaluating thermal stresses in the aircraft the following simplifying assumptions are introduced:

- it is assumed that the structural material is an isotropic continuous medium;
- geometry and heat stress state of the structural element of the aircraft is described in two-dimensional (2D) setting;
- external mechanical (forces, pressure fields) and thermal effects (convective and radiation heat fluxes) are initially set;
- it is considered that the speed of acoustic waves in the structural elements of aircraft is much greater than the speed of propagation of thermal waves in them.

### 3. The mathematical formulation of the problem

The system of two-dimensional equations of quasi-static thermoelasticity allows one to determine the temperature and displacement fields, the components of stress and strain tensors in the structural elements of an aircraft [2-8]:

\[
\mu \text{div}\left(\text{grad}\left(\vec{U}\right)\right) + \left(\lambda + \mu\right)\text{grad}\left(\text{div}\vec{U}\right) - \left(3\lambda + 2\mu\right)\alpha_T\text{grad}\left(\theta\right) = 0, \tag{1}
\]

\[
\rho c_v \frac{\partial \theta}{\partial t} + \alpha_T \left(3\lambda + 2\mu\right) T_0 \text{div}\left(\frac{\partial \vec{U}}{\partial t}\right) = \text{div}\left(\lambda_0 \text{grad}\theta\right), \tag{2}
\]

where \(t, x_j\) are the time and Descartes coordinates, \(\vec{U}\) is vector describing the displacement of the body point, \(T\) is the body temperature, \(T_0\) is the initial body temperature, \(\theta = T - T_0\) is the excess temperature, \(\rho\) is the density, \(c_v\) is the heat capacity at zero strain, \(\mu, \lambda\) are Lame parameters, \(\alpha_T\) is the coefficient of thermal expansion.

The boundary conditions for the temperature are formulated by specifying the heat flux on the surface of the element of the aircraft structure:

\[
-\lambda_0 \frac{\partial T}{\partial x_i} \bigg|_\Gamma = q_i, \tag{3}
\]

where \(T_0\), \(q\) are given constants, \(\Gamma\) is the aircraft surface.

Mechanical boundary conditions in the form of stresses are given in a manner analogous to [3].

The initial conditions (for the time \(t = 0\)) are determined in the form of the initially assigned spatial distribution of the temperature and displacements.

The boundary conditions necessary for solving the quasi-static thermoelasticity problem by a finite-difference method are conveniently realized when the boundaries of the computational domain coincide with the coordinate lines in a certain generalized coordinate system. In this case, the calculation area goes into the parametric area (for example: in a rectangle).
We introduce a coordinate transformation of the form:

\[
 r = r(\xi, \eta), \quad z = z(\xi, \eta).
\]  

(4)

If the coordinates of grid nodes known in physical space are known in the calculated domain, in the general case, metric coefficients can be found by numerical differentiation according to formulas:

\[
 J = \frac{\partial (r, z)}{\partial (\xi, \eta)} = \frac{\partial r}{\partial \xi} \frac{\partial z}{\partial \eta} - \frac{\partial r}{\partial \eta} \frac{\partial z}{\partial \xi},
\]

(5)

\[
 \xi = J^{-1} \frac{\partial z}{\partial \eta}, \quad \eta = -J^{-1} \frac{\partial z}{\partial \xi}, \quad \xi_\alpha = J^{-1} \frac{\partial r}{\partial \xi}, \quad \eta_\alpha = J^{-1} \frac{\partial r}{\partial \xi},
\]

(6)

gде \( J = \frac{\partial (r, z)}{\partial (\xi, \eta)} \) is the Jacobian of transition from a cylindrical coordinate system \( r, z \) to a curvilinear coordinate system \( \xi, \eta \).

To find functions, the system of equations obtained in [9] can be used. These equations guarantee that the functions found are smooth functions of the coordinates. However, in some cases, the calculated grid can be considered unsuccessful for one reason or another. In this situation (together with differential methods) it is expedient to use analytic algebraic transformations [10].

The system of thermoelasticity equations (1)-(2) in a vector semi-divergent form in an arbitrary curvilinear coordinate system has the following form:

\[
 A\vec{U} = B \frac{\partial \vec{F}}{\partial \xi} + B \frac{\partial \vec{G}}{\partial \eta} + C_\xi \frac{\partial \vec{I}}{\partial \xi} + C_\eta \frac{\partial \vec{I}}{\partial \eta} + D_\xi \frac{\partial \vec{W}}{\partial \xi} + D_\eta \frac{\partial \vec{W}}{\partial \eta} + \vec{S} = 0 .
\]

(7)

The vectors entering into the given system of equations are written in the following form:

\[
 \vec{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \quad \vec{G} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \quad \vec{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \quad \vec{W} = \begin{bmatrix} \vec{\rho} \\ \vec{\theta} \end{bmatrix},
\]

(8)

\[
 B = \begin{bmatrix} \frac{\mu}{J} & 0 \\ 0 & \frac{\mu}{J} \end{bmatrix}, \quad C_\xi = \begin{bmatrix} (\lambda + \mu) \xi_t & 0 \\ 0 & (\lambda + \mu) \xi_z \end{bmatrix}, \quad C_\eta = \begin{bmatrix} (\lambda + \mu) \eta_t & 0 \\ 0 & (\lambda + \mu) \eta_z \end{bmatrix},
\]

(9)

\[
 D_\xi = \begin{bmatrix} \alpha_t(3\lambda + 2\mu) \xi_t & 0 \\ 0 & \alpha_t(3\lambda + 2\mu) \xi_z \end{bmatrix}, \quad D_\eta = \begin{bmatrix} \alpha_t(3\lambda + 2\mu) \eta_t & 0 \\ 0 & \alpha_t(3\lambda + 2\mu) \eta_z \end{bmatrix},
\]

(10)

\[
 I = \text{div} \vec{U} = \frac{1}{J} \frac{\partial J(U_t)}{\partial \xi} + \frac{1}{J} \frac{\partial J(U_\eta)}{\partial \eta},
\]

(11)

\[
 F_t = J(\xi^2_t + \xi^2_z)u_t, \quad F_\eta = J(\xi^2_t + \xi^2_z)u_\eta, \quad G_t = J(\eta^2_t + \eta^2_\xi)u_t, \quad G_\eta = J(\eta^2_t + \eta^2_\xi)u_\eta,
\]

(12)

\[
 \vec{S} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \quad \left\| S_1 \right\| + \left\| S_2 \right\|,
\]

(13)

\[
 S_1 = \frac{1}{J} \frac{\partial \{ J(\xi, \eta, \xi, \eta) u_t \}}{\partial \xi} + \frac{1}{J} \frac{\partial \{ J(\eta, \xi, \eta, \xi) u_t \}}{\partial \eta} + \alpha \frac{1}{r} \frac{\partial \{ \xi u_t + \eta u_\xi \}}{\partial \eta} + \alpha \frac{1}{r} \frac{\partial \{ \xi u_\eta + \eta u_\xi \}}{\partial \eta},
\]

(14)
\[ S_2 = \frac{1}{J} \left[ \frac{\partial}{\partial \xi} \left( J \left( \xi \eta_t + \xi \eta_r \right) v_\eta \right) \right] + \frac{1}{J} \left[ \frac{\partial}{\partial \eta} \left( J \left( \eta \xi_t + \eta \xi_r \right) v_\xi \right) \right] + \frac{1}{r} \left[ \xi \eta_t \frac{\partial v}{\partial \xi} + \eta \xi_t \frac{\partial v}{\partial \eta} \right], \] (15)

\[ S_1 = (\lambda + \mu) \left[ \xi \left( \frac{\partial (\alpha u)}{\partial \xi} \right)_r + \eta \left( \frac{\partial (\alpha u)}{\partial \eta} \right)_r \right], \quad S_\zeta = (\lambda + \mu) \left[ \xi \left( \frac{\partial (\alpha u)}{\partial \xi} \right)_r + \eta \left( \frac{\partial (\alpha u)}{\partial \eta} \right)_r \right], \] (16)

where \( u(r,z,t) = U_r \) and \( v(r,z,t) = U_z \) are projections of the displacement vector \( \vec{U}(r,z,t) \) on the \( r \) and \( z \) axis, \( U_\xi = \xi_u + \xi_v \), \( U_\eta = \eta_u + \eta_v \) are contravariant components of a vector \( \vec{U} \), which describes the displacement of a point of a body in a curvilinear coordinate system \( \xi, \eta \), \( \alpha = 0 \) corresponds to the case of plane deformation, \( \alpha = 1 \) corresponds to the case of axisymmetric deformation.

Now we transform, using the transition to the curvilinear coordinate system, the equation associated with the transfer of internal energy by the heat conduction process:

\[ \rho c_v \frac{\partial \theta}{\partial t} = \frac{1}{J} \frac{\partial}{\partial \xi} \left( J \lambda (\xi \eta_t + \xi \eta_r) \theta_\eta \right) + \frac{1}{J} \frac{\partial}{\partial \eta} \left( J \lambda (\eta \xi_t + \eta \xi_r) \theta_\xi \right) + \frac{\lambda}{J} \left( \xi \frac{\partial \theta}{\partial \xi} + \eta \frac{\partial \theta}{\partial \eta} \right) + f, \] (17)

where \( f = S_\lambda + D_\lambda \),

\[ S_\lambda = \frac{1}{J} \frac{\partial}{\partial \xi} \left( J \lambda (\xi \eta_t + \xi \eta_r) \theta_\eta \right) + \frac{1}{J} \frac{\partial}{\partial \eta} \left( J \lambda (\eta \xi_t + \eta \xi_r) \theta_\xi \right) + \frac{\lambda}{J} \left( \xi \frac{\partial \theta}{\partial \xi} + \eta \frac{\partial \theta}{\partial \eta} \right), \] (18)

\[ D_\lambda = -\alpha_1 (3\lambda + 2\mu) T_0 \left[ \frac{1}{J} \frac{\partial}{\partial \xi} (\xi U_r) + \frac{1}{J} \frac{\partial}{\partial \eta} (\eta U_r) \right] - \alpha_2 (3\lambda + 2\mu) T_0. \] (19)

In the numerical realization of equations (7), (17), a rectangular grid is introduced in the parametric domain \( \alpha_\theta = \{ \xi_j, \eta_k : j = 0, 1, \ldots, J, k = 0, K \} \), \( \alpha_\theta \in \Omega_\theta \). Here \( h_\xi = \xi_j - \xi_{j-1} \), \( \xi_{j+1/2} = \xi_j + 0.5h_\xi \), \( \xi_{j+1/2} = \xi_{j+1/2} - 0.5h_\xi \), \( h_\eta = \eta_k - \eta_{k-1} \), \( \eta_{k+1/2} = \eta_k + 0.5h_\eta \), \( \eta_{k-1/2} = \eta_k - 0.5h_\eta \).

The numerical solution of the system of equations (7), (17) is carried out in two stages. In the first stage, the heat conduction equation of a special form (17) is implicitly solved. Then the second stage is the solution of the equations of thermoelasticity (7).

When solving the “thermal” part of the two-dimensional equations of quasi-static thermoelasticity, which describe the transfer of internal energy by the thermal conductivity process, the following two-step difference scheme is used [11]:

\[ \frac{\theta_{j+1/2,k}^{n+1/2} - \theta_{j,k}^{n+1/2}}{\Delta \theta} = a_{j+1/2,k} \left( \theta_{j+1/2,k}^{n+1/2} - \theta_{j,k}^{n+1/2} \right) - a_{j,k+1/2} \left( \theta_{j,k+1/2}^{n+1/2} - \theta_{j+1,k+1/2}^{n+1/2} \right) + F_{\xi}^n + O \left( h_\xi^2, h_\eta^2 \right), \] (20)

\[ \theta = \theta(\xi, \eta), \quad F_{\xi} = \frac{\partial}{\partial \eta} \left( J \lambda (\eta^2 + \eta^2) \theta_\xi \right) + Jf, \] (21)

\[ \frac{\theta_{j+1/2,k}^{n+1/2} - \theta_{j,k}^{n+1/2}}{\Delta \theta} = a_{j+1/2,k} \left( \theta_{j+1/2,k}^{n+1/2} - \theta_{j,k}^{n+1/2} \right) - a_{j,k+1/2} \left( \theta_{j,k+1/2}^{n+1/2} - \theta_{j+1,k+1/2}^{n+1/2} \right) + F_{\eta}^{n+1/2} + O \left( h_\xi^2, h_\eta^2 \right), \] (22)

\[ \theta = \theta(\xi, \eta), \quad F_{\eta} = \frac{\partial}{\partial \xi} \left( J \lambda \left( \xi^2 + \xi^2 \right) \theta_\eta \right) + Jf, \] (23)
where \( n \) is the superscript referring to the moment of "time" \( t = n\Delta t \), \( \Delta t \) is the "time" step, 
\[
c = \rho c_v = \frac{\partial}{\partial t} J; \quad a_x = J \lambda_q \left( \varepsilon_x^2 + \varepsilon_y^2 \right); \quad a_\eta = J \lambda_\eta \left( \eta_x^2 + \eta_y^2 \right).
\]
This difference scheme (along spatial directions) is easily solved by scalar sweep.

To solve the equations of thermoelasticity, we use the method of establishing [12]. In this case, we find the step in "time" using an iterative method of variational type. To do this, we define the vector of residuals:

\[
R^e_{x,j,k} = \left( AU_{j,k} \right)_1^n - b^e_{x,j,k}, \quad R^f_{2,j,k} = \left( AU_{j,k} \right)_2^n - b^f_{2,j,k},
\]

\[
\tilde{b}^e_{j,m} = \left\{ b^e_{x,j,k}, b^f_{2,j,k} \right\} = \{0,0\}, \quad j = 1, J - 1, k = 1, K - 1.
\]

We introduce the scalar product as follows [11]:

\[
(a, b) = \sum_{j=1}^{J-1} \sum_{k=1}^{K-1} a_{x,j,k} h_1 h_2 + \sum_{j=1}^{J-1} \sum_{k=1}^{K-1} a_{2,j,k} h_2^2 h_3,
\]

\[
a_{j,k} = (a_x, a_\eta)_{j,k}, \quad \tilde{b}_{j,k} = (h_1, h_2)_{j,k}.
\]

We will minimize the value of the discrepancy, using a modified version of the variational-type iteration method – the method of minimal residuals [13]. In this case, iterations should be carried out according to the formulas:

\[
U^{n+1}_{j,k} = U^n_{j,k} + c\Delta t R^o_{j,k}, \quad c \approx 0.9, \quad \Delta t = \frac{\left( \bar{R}, A\bar{R} \right)}{\left( A\bar{R}, A\bar{R} \right)}.
\]

In the numerical realization of the method of minimal residuals for approximating the derivatives of vectors and variables, let us define the following finite-difference representation of the derivatives on the grid introduced above [14]:

\[
\frac{\partial}{\partial \xi} \left[ a \frac{\partial g}{\partial \xi} \right]_{j,k} = \frac{a_{j+1/2,k} \left( g_{j+1,k} - g_{j,k} \right) - a_{j-1/2,k} \left( g_{j,k} - g_{j-1,k} \right)}{h_2^2} + O\left(h_2^3\right), \quad a_{j=1/2,k} = \frac{a_{j,k} + a_{j+1,k}}{2},
\]

\[
\frac{\partial}{\partial \xi} \left[ a \frac{\partial g}{\partial \eta} \right]_{j,k} = \frac{a_{j,k+1} \left( g_{j+1,k+1} - g_{j+1,k} \right) - a_{j-1,k} \left( g_{j-1,k+1} - g_{j-1,k} \right)}{4h_2 h_3} + O\left(h_2^3 h_3^2\right).
\]

Note that for the correct operation of the described algorithm it is necessary:

- after every calculation step (because deformation of the boundaries of the computational domain occurred), reorganize the computational grid (for example: using the method of [9]);
- then interpolate (taking into account the conservation laws and the required degree of smoothness of the solution, see [15]) and values known on the computational grid at the time point on the grid obtained after its reconstruction.

4. Results of numerical simulation

To test the efficiency of the formulated numerical methodology, a group of test problems was solved. In all series of calculations, the system of equations of the linear elasticity theory in Euler coordinates and the heat equation were used.

The first test (validation) task is to find the equilibrium thermoformable state of the steel beam, which is fixed on both sides by fixed hinges (Figure 1) and is in a uniform temperature field \( T(x, y) = T_0 + \Delta T \) (where \( T_0 \) is the initial value of temperature, \( \Delta T \) is the value of uniform temperature
heating). Only the problem of mechanics in a given uniformly distributed temperature field $T(x, y) = T_0 + \Delta T$ is solved. The exact solution of this problem is as follows:

$$U_y(x) = \frac{h}{2} \left\{ \cos(\lambda x) \left( \cos \left( \frac{\lambda l}{2} \right) \right)^{1/2} \right\}^{-1}, \quad \lambda = \left( \frac{P}{D} \right)^{1/3}, \quad D = \frac{Eh^3}{12},$$

where $h$ is the thickness of the beam, $l$ is the length of the beam, $E$ is the modulus of elasticity of the first kind, $P$ is the reaction force of the support, which is found from the balance equation of the beam.

**Figure 1.** The scheme of the test problem: the bending of a uniformly heated fixed beam under the influence of temperature stresses.

In Fig. 2 shows the results (at: $h = 2.5$ mm, $l = 100$ mm, $E = 2 \cdot 10^{11}$ Pa, $\alpha_T = 1.2 \cdot 10^{-5}$ C$^{-1}$, $\Delta T = 36$ °C) of the numerical calculation of the first test problem for determining the deflection of a uniformly heated fixed beam under the influence of temperature stresses (the maximum value of the relative error was 2%).

**Figure 2.** Numerical calculation results of the test problem for determining the deflection of a uniformly heated fixed beam under the influence of temperature stresses.

It is known from systematic calculations [18, 19] that the convective heat flux $q_w$ near the front critical point can be determined using the formulas ($R$ is the radius of blunting):

$$q_w = 1.93 \times 10^{-4} V_{w1.08} (H_0 - H_v) \left( \frac{\rho_c}{R} \right)^{1/2}$$

in the case of the laminar flow,
\[ q_w = 4.69 \times 10^{-4} V_\infty^{1.25} (H_0 - H_w) \left( 1 + \frac{T_w}{T_0} \right)^{\frac{2}{3}} \frac{\rho_\infty^{0.8}}{R^{0.2}} \] in the case of the turbulent flow. (33).

From these formulas it follows that the heat transfer coefficient \( \alpha \) at the front critical point can be estimated using the relation \( \alpha \sim R^{\frac{1}{2}} \). In the case of turbulent flow this estimation is \( \alpha \sim R^{-0.2} \).

Therefore, at high velocities \( M > 6 \) and, respectively, large deceleration temperatures, as the radius of blunting decreases at the critical point, the values of the convective and radiation fluxes sharply increase (if the inverse heat flux \( q_w \) of the body surface temperature \( T_w \) is not taken into account). These formulas also demonstrate a noticeable effect on the convective heat flux \( q_w \) of the surface temperature \( T_w \) of the aircraft element.

The calculations performed in this work (Figures 3 ... 10) allow us to estimate the effect on the convective heat flux \( q_w \) of the blunt radius, the geometric shape, the structural material and the surface temperature distribution of the \( T_w \) element of the aircraft.

**Figure 3.** The temperature distribution \( T \) [K] in a wedge-shaped shell (Inconel 617 alloy), blunt in the cylinder. The radius of blunting of the cylinder is \( R = 3.18 \) mm, the Mach number in the oncoming stream is \( M = 5.25 \).

**Figure 4.** The distribution of displacements \( U_x \) [mm] in the direction of the \( X \) axis in a wedge-shaped shell (Inconel 617 alloy), blunt in the cylinder. The radius of blunting of the cylinder is \( R = 3.18 \) mm, the Mach number in the oncoming stream is \( M = 5.25 \). Solid lines indicate the outlines of the original undeformed body.
For the validation and verification of the developed numerical technique, the data of the papers [20, 21] were taken as the second test problem. The geometry (Fig. 3 ... 5) of the test problem is a 2D wedge-shaped shell (the shell material is Inconel 617 alloy, the wedge length is 7.62 mm, the shell thickness is constant and equal to 11 mm, the opening angle of the wedge 60°) blunt in the cylinder (the blunting radius is \( R = 3.18 \text{ mm} \)).

The convective heat flux arriving at the surface of the wedge-shaped shell, as well as the aerodynamic loads distributed along its surface, are taken from [20, 21]. The parameters of the flow of air flowing onto the element of the aircraft (the edges of the wings, nose cone) are determined by the Mach number \( M = 5.25 \). It is assumed that the stabilization of internal energy in an aircraft element, which is rigidly embedded, is carried out by thermal radiation.

![Figure 5](image)

**Figure 5.** The distribution of displacements \( U_y \text{ [mm]} \) in the \( Y \)-axis direction in a wedge-shaped shell (Inconel 617 alloy) blunt in the cylinder. The radius of blunting of the cylinder is \( R = 3.18 \text{ mm} \), the Mach number in the oncoming stream is \( M = 5.25 \). Solid lines indicate the outlines of the original undeformed body.

The results of calculating the equilibrium thermodetectable state corresponding to the second test problem (Fig. 3 ... 5) show:

1. The maximum value of the relative error, both in the temperature distribution and in the values of the displacements, is lower than 3%.
2. There is a noticeable change in the temperature distribution \( T_w \) along the surface of the aircraft element \( T_w = 1100 - 3100 \text{ K} \);
3. The displacement \( U_x \) of the front part of the aircraft element in the direction of the \( X \)-axis are equal to 2 mm towards the elongation of the body;
4. The displacement \( U_y \) of the front part of the aircraft element in the \( Y \)-axis direction: 0.25 mm - in the lower part of the body, 0.5 mm - in the upper part of the body;
5. The radius of blunting \( R \) varies and reaches \( R = 2.10 \text{ mm} \) (resulting in an increase in the convective heat flux by 23%), which is 34% less than the original radius \( R = 3.18 \text{ mm} \).

The next problem consisted in finding the distribution of temperature and displacements in the structural element of the aircraft - in a cylinder blunted over the sphere. In solving this problem, the following geometry of the aircraft element (spherical blunting of radius 1.27 cm, cylinder length 10 cm) was given, as well as the convective heat flux (Fig. 6) incident on the surface of the element under consideration.
Figure 6. The distribution of the convective heat flow density along the surface of a cylinder blunted over a sphere [15]. The radius of blunting is $R = 1.27$ cm.

As gas-dynamic design data in the oncoming external flow, the data of [22] were used: pressure $P = 0.23 \cdot 10^3$ Pa; density $\rho = 0.178 \cdot 10^{-5}$ g cm$^{-3}$; speed $V = 4.167 \cdot 10^5$ cm s$^{-1}$; temperature $T = 450$ K. The surface temperature of the element was assumed to be constant and equal to $T_w = 300$ K. The density distribution of the convective heat flux along the flowing non-catalytic surface was taken from [20] and is shown in Fig. 6. In Fig. 7. The distribution of the temperature of the gas along the surface of a cylinder blunted along a sphere is given. We note that these aerothermophysical values correspond to the hypersonic regime of gas flow ($M = 9.8$) [23-25].

Figure 7. The temperature distribution $T$ [K] along the surface of the cylinder blunted over the sphere. The radius of blunting is $R = 1.27$ cm, the velocity of the oncoming stream is $V_\infty = 4.17$ km/s (the Mach number is $M = 9.8$, the height is $h = 22$ km).
In solving this problem, just as before, it is assumed that the loss of internal energy by an airborne element is realized only through thermal radiation. The element of the aircraft itself (Figures 8 ... 10) is made of tungsten and rigidly embedded.

In Fig. 8 ... 10 shows the results of calculations performed with the help of the above technique for the element of the aircraft under consideration.

**Figure 8.** The temperature distribution $T$ [K] in a cylinder blunted over the sphere. The radius of blunting is $R = 1.27$ cm, the velocity of the oncoming stream is $V_\infty = 4.17$ km/s (the Mach number is $M = 9.8$, the height is $h = 22$ km).

**Figure 9.** The distribution of displacements $Ur$ [mm] in the direction of the $R$ axis in a blunt cylinder. The radius of blunting is $R = 1.27$ cm, the velocity of the oncoming stream is $V_\infty = 4.17$ km/s (the Mach number is $M = 9.8$, the height is $h = 22$ km).
Figure 10. The distribution of displacements $Uz$ [mm] in the direction of the $Z$ axis in a blunted cylinder. The radius of bluntness of the sphere is $R = 1.27$ cm, the velocity of the oncoming stream is $V_\infty = 4.17$ km/s (the Mach number is $M = 9.8$, the height is $h = 22$ km).

From the graphical dependencies shown in Fig. 8 ... 10 it follows:

1. the temperature distribution of the surface $T_w = 1500 - 2200$ K, which is streamlined by the external flux of the aircraft element, differs from the constant and significantly exceeds the temperature ($T_w = 300$ K) of the element surface, which was accepted as the boundary condition in the calculation of the aerothermodynamics of the aircraft;

2. The radius of blunting $R$ changes by 4.6% (from $R = 12.7$ mm to $R = 12.12$ mm). At the same time, the convective heat flux at the front critical point due to the change in the radius of curvature $R$ increases by 2.4%. However, if we take into account the substantial growth of the surface temperature $T_w = 2200$ K, then the change will be at the level of 1%

3. The displacement of the points of the solid element in question is small and, in the main, concentrated near the embankment;

4. The convective heat flux (Figure 6), taken from [22], must be corrected by 70% along the generator of the cylinder, taking into account the solutions of the equations of quasi-static thermoelasticity (due to a significant increase in temperature in the body and on the surface of the element).

Conclusions

A 2D computational and theoretical methodology is developed. It is designed to find the solution of the system of thermoelasticity equations with boundary conditions of a general form. The initial validation and verification of the developed methodology was carried out. Numerical modeling of temperature fields and thermal stresses in the simplest elements of aircraft components (a wedge-shaped shell blunted along a cylinder, a cylinder blunted over a sphere) is performed.

The calculations show that it is necessary to carry out numerical simulation of the physical processes taking place on the surface (and in the immediate vicinity) of high-speed aircrafts on the basis of complex, mutually consistent, coupled mathematical models of aerothermodynamics, heat transfer and thermal resistance. The edges of the hull, wings and the nose cone of the aircraft are advisable to be performed in the form of a solid part. In this case their deformations will be minimal.

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