Elliptic flow from event-by-event hydrodynamics

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Abstract. We present an event-by-event hydrodynamical framework which takes into account the initial density fluctuations arising from a Monte Carlo Glauber model. The elliptic flow is calculated with the event plane method and a one-to-one comparison with the measured event plane $v_2$ is made. Both the centrality- and $p_T$-dependence of the $v_2$ are remarkably well reproduced. We also find that the participant plane is a quite good approximation for the event plane.

1. Introduction

Hydrodynamical models using smooth initial conditions cannot reproduce the centrality dependence of the elliptic flow coefficients $v_2$, measured using the event plane method (see e.g. Fig. 7.5 in [1]). To study this more carefully we present here an event-by-event hydrodynamical framework introduced in Ref. [2] and make a one-to-one comparison with the experimental event plane results for the elliptic flow. Also the $v_2$ with respect to the participant plane is considered.

2. Event-by-event hydrodynamics framework

The initial state is obtained from a Monte Carlo Glauber model. First, nucleons are randomly distributed into the nuclei using a standard Woods-Saxon potential. Then the impact parameter $b$ is sampled from a distribution $dN/db \sim b$. A nucleon $i$ and a nucleon $j$ from different nuclei collide if their transverse locations are close enough,

$$\left( x_i - x_j \right)^2 + \left( y_i - y_j \right)^2 \leq \frac{\sigma_{NN}}{\pi},$$

(1)

where $\sigma_{NN}$ is the inelastic nucleon-nucleon cross section. For $\sqrt{s_{NN}} = 200$ GeV we use $\sigma_{NN} = 42$ mb. The impact parameter defines the reaction plane (RP) and the participant plane (PP) is obtained from the participant configuration by maximizing the eccentricity. Centrality classes are defined using the number of participants $N_{\text{part}}$ and slicing their distribution into $N_{\text{part}}$ intervals as shown in Ref. [2].
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Figure 1. The transverse momentum spectra of positively charged pions in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The event-by-event hydrodynamical calculations with two different fluctuation size parameters and the smooth initial state calculations are shown at two centralities. The data are from the PHENIX Collaboration [7]. From [2].

The initial energy density $\epsilon$ in the transverse plane is obtained by distributing energy around the participants using a 2-dimensional Gaussian smearing,

$$\epsilon(x, y) = \frac{K}{2\pi\sigma^2} \sum_{i=1}^{N_{\text{part}}} \exp \left(-\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma^2}\right),$$

(2)

where $K$ is an overall normalization factor and $\sigma$ is a free parameter, which controls the width of the Gaussian. The overall normalization as well as the initial time $\tau_0 = 0.17$ fm are fixed from the EKRT model [3].

For each event, we solve the ideal hydrodynamical equations $\partial_\mu T^{\mu\nu} = 0$ assuming longitudinal boost-invariance and zero net-baryon density. We further need to specify an equation of state (EoS) $P = P(\epsilon)$ to close the set of equations. Our choice is the EoS from Ref. [4]. The freeze-out is assumed to happen at a constant temperature $T_F = 160$ MeV, and the thermal spectra are calculated using the Cooper-Frye method. Hadrons are sampled from the calculated spectra and they are given to PYTHIA 6.4 [5], which does all the strong and electromagnetic resonance decays.

In order to make a one-to-one comparison with the experiments, the elliptic flow is calculated with the event plane (EP) method. The event flow vector for the second harmonic is

$$Q_2 = \sum_i (p_{T_i} \cos(2\phi_i), p_{T_i} \sin(2\phi_i)),$$

(3)

where we sum over all particles, and the event plane angle $\psi_2$ is obtained from this as

$$\psi_2 = \frac{\arctan(Q_{2,y}/Q_{2,x})}{2}.$$

(4)
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Since we have only a finite number number of particles in each event, the event plane fluctuates around the true event plane, which would be the one determined from infinitely many particles. The correction $R$ from these fluctuations is calculated with the 2-subevent method \cite{6}. The final event plane elliptic flow is thus

$$v_2\{\text{EP}\} = \langle \cos(2(\phi_i - \psi_2)) \rangle / R. \quad (5)$$

\[ \begin{array}{c}
(a) \vphantom{2} \quad (b) \vphantom{2} \\
\begin{array}{c}
\text{v2 smooth} \\
\text{v2[RP]} \\
\text{v2[EP]} \\
\text{v2[PP]} \\
\end{array} \quad \begin{array}{c}
\text{PHENIX} \\
\text{STAR} \\
\end{array}
\end{array} \]

\[ \begin{array}{c}
(c) \vphantom{2} \quad (d) \\
\begin{array}{c}
\text{0-5\% charged} \\
\text{5-10\% charged} \\
\end{array} \quad \begin{array}{c}
\text{10-20\% charged} \\
\text{20-30\% charged} \\
\end{array}
\end{array} \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{The elliptic flow of charged hadrons in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The event-by-event calculations with respect to different reference planes and the calculations with smooth initial states are from \cite{2}. The data are from the PHENIX and STAR Collaborations \cite{8,9,10}.}
\end{figure}

3. Results

For each centrality class we have made 500 hydro runs and to increase the event statistics we have sampled each thermal hadron spectra 20 times. The smooth initial states are obtained by taking an average over 20 000 initial energy density profiles. The obtained transverse momentum spectra of pions are plotted in Fig. 1. The density fluctuations increase the number of high-$p_T$ particles since the initial pressure gradients are larger. However, if the fluctuation size parameter $\sigma$ is sufficiently large, the difference to the smooth result is small.

In Fig. 2 we have plotted the elliptic flow with respect to the different reference planes. The EP results agree very well with the measured data at all centralities shown.
in the figure. If we use PP instead of EP, the elliptic flow is a little bit smaller, but PP seems to be a quite good approximation for the EP. The $v_2\{\text{RP}\}$ is generally smaller than $v_2\{\text{EP}\}$, but it is very close to the smooth result. Thus the fluctuations alone do not generate more elliptic flow, but the reference plane definition is very important.

In Fig. 3 is shown the correlation of the EP with the PP, and with the RP. Note that the trivial fluctuations of $\psi_2$ around the true EP are included in the figure. The correlation between the EP and the PP is, as expected, stronger than that between the EP and the RP.

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