Research Article

Solution Procedure for Inventory Models with Linear and Fixed Backorder Costs

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Several researchers applied the algebraic method to solve the optimal solution of EOQ and EPQ models with linear and fixed backorder costs. However, there are questionable derivations in their solution procedures such that researchers provided further examinations. We study two papers that are considered the same inventory models under different notation and slightly different objective functions to provide detailed discussions for their questionable results and our improvements.

1. Introduction

After Sphicas [1] applied the algebraic method to find the optimal solution of inventory systems with two backorder costs, linear and fixed, several papers continued trying to solve these types of inventory models using other algebraic approaches or calculus because the partition of Sphicas [1] is not well understood. Cárdenas-Barrón [2] and Chung and Cárdenas-Barrón [3] claimed Sphicas [1] derived the partition from previously published papers using calculus such that he could divide his solution structure into two parts and then solve the optimal problem. Several researchers tried to develop a new solution method to replace the unexpected partition approach proposed by Sphicas [1]. In this study, we focus on Omar et al. [4] to show the new solution method proposed by Omar et al. The study [4] contains severely questionable findings such that the simplification proposed by Omar et al. [4] cannot be treated as a substitute for Sphicas [1]. Moreover, we will discuss a related paper of Chung and Cárdenas-Barrón [3] which had cited Omar et al. [4].

Eight papers referred to Omar et al. [4] in their references: Cárdenas-Barrón [2], Chung and Cárdenas-Barrón [3], Wee et al. [5, 6], Cárdenas-Barrón et al. [7], Khalilpourazari et al. [8], Afshar-Nadjafi et al. [9], and Jonrinaldi et al. [10]. The second group contains Cárdenas-Barrón [2] and Chung and Cárdenas-Barrón [3].

For the six papers in the first group, Wee et al. [5] developed new inventory models with partial backorders. Wee et al. [6] considered EPQ models with nonsynchronized screening and reworking. Cárdenas-Barrón et al. [7] constructed EPQ models with delivery and rework. Khalilpourazari et al. [8] studied the multiproduct EPQ model with partial backorders. Afshar-Nadjafi et al. [9] examined inventory models with backorders and inspection. Jonrinaldi et al. [10] constructed EOQ/EPQ models with multiple items and variable demand. These six articles only mentioned Omar et al. [4] in their introduction and then focused on their own inventory models such that they were not really related to Omar et al. [4].

For the second group, we briefly review these two papers in Section 1, and then in Section 3, we offer detailed examinations of them. Cárdenas-Barrón [2] used the analytic geometry of a parabola and applied it to complete the square method to obtain the minimum solution for inventory models with linear and fixed backlog costs. Cárdenas-Barrón [2] mentioned that the algebraic development of Omar et al. [4] is complex. We remind the readers that Omar et al. [4] only used one algebraic method of completing a square that will be further discussed in Section 4. We will demonstrate the algebraic development of Omar et al. [4] is incomplete but that is not complex. Lin [11] already provided a detailed
discussion for Cárdenas-Barrón [2] to point out several questionable results in Cárdenas-Barrón [2]. Chung and Cárdenas-Barrón [3] used calculus to find the first partial derivative system to obtain the optimal solution. Chung and Cárdenas-Barrón [3] mentioned five related papers that did not provide a complete solution. One of them is Omar et al. [4]. However, Chung and Cárdenas-Barrón [3] did not detail the shortcomings of Omar et al. [4]. Similarly, a detailed discussion of Chung and Cárdenas-Barrón [3] will be presented in Section 3.

Based on the above discussions, we point out that those eight papers were too eager to derive new inventory models such that they did not undertake detailed examinations of Omar et al. [4]. The purpose of our paper is to present.

2. Notation and Assumptions

To be compatible with Omar et al. [4], we use the same notation as theirs. Moreover, we need to compare Omar et al. [4] with Sphicas [1] and Cárdenas-Barrón [2], so different expressions for the same meaning are also listed.

2.1. Notation

c = the purchasing cost per unit, used by Cárdenas-Barrón [2].
C_k = the backorder cost per unit, per unit of time (linear backorder cost) that is denoted as p (the lowercase letter) in Sphicas [1], and π in Cárdenas-Barrón [2].
C_p = the ordering cost per order that is denoted as K in Sphicas [1] and A in Cárdenas-Barrón [2].
C_p = the holding cost per unit, per unit of time that is denoted as h in Sphicas [1] and Cárdenas-Barrón [2].
D = the constant demand, per unit of time that is also used by Sphicas [1] which is denoted as d in Cárdenas-Barrón [2].
K' = PK/(P - D) = K/r, an auxiliary expression to compare our EOQ and EPQ models with the model by Sphicas [1].
Q = the ordering quantity.
R = the constant production rate per unit of time for the EPQ model that is denoted as P (the capital letter) in Sphicas [1] and p (the lowercase letter) in Cárdenas-Barrón [2].
S = the backlog quantity, and in Sphicas [1], he also used S which is denoted as B in Cárdenas-Barrón [2] such that the beginning inventory level is Q - S.

a = 1 – (D/R) for the EPQ model proposed by Omar et al. [4], which is denoted as r = 1 – (d/p) by Cárdenas-Barrón [2] and r = 1 – (D/P) by Sphicas [1].
π = the backlog cost per unit (fixed backorder cost).
TC = the average cost for the EOQ model of Omar et al. [4], denoted as TC(Q,S) by Sphicas [1] and Cárdenas-Barrón [2] and as TC(1,Q,S) by Chung and Cárdenas-Barrón [3].

2.2. Assumptions

(1) The purchasing cost per unit time is cd in Cárdenas-Barrón [2] and is a constant term. The EOQ and EPQ models of Sphicas [1] and Omar et al. [4] did not contain this constant term. Since the purchasing cost is a constant term, using or not using this term will not alter the optimal solution for the ordering quantity, Q and backorder quantity, B of Cárdenas-Barrón [2], or S of Sphicas [1] and Omar et al. [4].

(2) There is only one kind of product for the EOQ or EPQ models.

(3) The production rate is greater than the demand rate, which is R > D for EPQ models which are denoted as P > D for Sphicas [1] and P > d for Cárdenas-Barrón [2].

(4) Shortages are allowed and all shortages are fully backordered, which is expressed as B in Cárdenas-Barrón [2] and denoted as S in Sphicas [1] and Omar et al. [4].

(5) There are two kinds of backorder costs: linear and fixed backorder costs. The linear backorder cost, C_b, is related to the shortage quantity and backorder waiting duration time, which is denoted as p in Sphicas [1] and π in Cárdenas-Barrón [2]. The fixed backorder cost, π, is related to the shortage quantity that is not related to the backorder waiting duration time.

3. A Review of Chung and Cárdenas-Barrón[3]

Cárdenas-Barrón [2] is an important related paper for Omar et al. [4]. However, Lin [11] already provided a detailed discussion for Cárdenas-Barrón [2] to show its questionable results and to offer its revisions. Interested readers, please refer to Lin [11] for Cárdenas-Barrón [2] for a complete discussion. In the following, we only provide a brief discussion for Cárdenas-Barrón [2].

We recall the objective function of Cárdenas-Barrón [2]:

$$TC(Q,B) = \frac{Ad}{Q} + \frac{h(Q - B)^2}{2Q} + \frac{\pi B^2}{2Q} + \frac{\pi Bd}{Q} + cd. \quad (1)$$

We point out that Cárdenas-Barrón [2] added an extra term, cd, which is the average purchasing cost per unit of time. It is a constant term that will not alter the optimal solution for the proposed inventory models.

Cárdenas-Barrón [2] applied algebraic methods to find
\[ B^* = \frac{hQ^* - \pi d}{h + \pi}, \quad (2) \]
\[ Q^* = \sqrt{\frac{2Ad(h + \pi) - (\pi d)^2}{h^2}}. \quad (3) \]

However, Cárdenas-Barrón [2] did not check whether or not \( B^* \geq 0 \) or the term inside the square root, \( 2Ad(h + \pi) > (\pi d)^2 \). For the EPQ model, Cárdenas-Barrón [2] repeated his approach for the EOQ model such that the same questionable derivations are committed again. We point out that Cárdenas-Barrón [2] did not know, when \( 2Adh \leq d^2\pi^2 \), the optimal solution will occur on the boundary with \( Q^* = \sqrt{2Ad/h} \). Hence, the solution of Cárdenas-Barrón [2] is deficient.

We recall the objective function of Chung and Cárdenas-Barrón [3]:
\[ TC_1(Q, B) = \frac{Ad}{Q} + \frac{h(Q - B)^2}{2Q} + \frac{\pi B^2}{2Q} + \frac{\pi B d}{Q}. \quad (4) \]

Chung and Cárdenas-Barrón [3] derived \( \partial TC_1(Q, B)/\partial Q \) and \( \partial TC_1(Q, B)/\partial B \), and then setting \( \partial TC_1(Q, B)/\partial Q = 0 \) and \( \partial TC_1(Q, B)/\partial B = 0 \), they found
\[ B^* = \frac{hQ - d\pi}{h + \pi}, \quad (5) \]
\[ Q^* = \sqrt{\frac{2Ad(h + \pi) - d^2\pi^2}{h^2}}. \quad (6) \]
which are identical to the results of Cárdenas-Barrón [2] using the algebraic method of equations (2) and (3).

Chung and Cárdenas-Barrón [3] substituted equation (6) into equation (5), to obtain a restriction of \( 2Adh \geq d^2\pi^2 \), to guarantee \( B^* \geq 0 \).

We point out that Chung and Cárdenas-Barrón [3] should derive the restriction to assure \( B^* > 0 \), since Chung and Cárdenas-Barrón [3] used calculus to compute partial derivatives which are valid for interior points in the domain. If \( B^* = 0 \), then the optimal solutions are on the boundary \( B = 0 \), such that the objective function is changed from equation (4) to
\[ TC(Q, B = 0) = \frac{Ad}{Q} + \frac{hQ}{2}. \quad (7) \]

The solution of \( dTC(Q, B = 0)/dQ = 0 \) is \( Q^* = \sqrt{2Ad/h} \) which is identical to \( Q^* = \sqrt{\frac{2Adh}{\pi}} \), under the condition \( 2Adh > d^2\pi^2 \), by solving \( B^* = 0 \) in equation (5).

However, when \( 2Adh = d^2\pi^2 \), based on equations (5) and (6) with \( B^* = 0 \), there are two expressions of \( Q^* \) as \( Q^* = \frac{d\pi h}{2} \) and \( Q^* \) of equation (6), and Chung and Cárdenas-Barrón [3] were not aware that these two expressions are the same as the restriction of \( 2Adh = d^2\pi^2 \).

When \( 2Adh < d^2\pi^2 \), we cite the derivations of Chung and Cárdenas-Barrón [3], “Case (B): \( 2Adh < d^2\pi^2 \) Case (B) implies that there are three cases to occur:

(i) \( 2DK(h + \pi) - D^2\pi^2 > 0 \).
In this case, equation (19) (that is, equation (6) of this paper) is well defined. Substitute (19) into (18) (that is plugging equation (6) into equation (5), we get \( B^* < 0 \). Hence, \( Q^*, B^* \) does not exist.

(ii) \( 2DK(h + \pi) - D^2\pi^2 = 0 \).
In this case, \( Q^* = 0 \) and \( B^* = -D\pi/(h + \pi) \). Evidently, \( Q^*, B^* \) does not exist.

(iii) \( 2DK(h + \pi) - D^2\pi^2 < 0 \).
In this case, \( Q^* \) is not well defined. Equation (18) (that is, equation (5) of this paper) implies that \( B^* \) does not exist. Obviously, \( Q^*, B^* \) does not exist.

Therefore, under Case (B), situations (i)–(iii) conclude \( Q > 0 \) and \( B > 0 \), then \( (Q, S) \) is never the optimal solution of \( TC(B, Q) \) on \( Q > 0 \) and \( B > 0 \). So, if the optimal solution of \( TC(B, Q) \) on \( Q > 0 \) and \( B > 0 \) exists, then \( B^* = 0 \).”

Based on the above quotation, we mention that Chung and Cárdenas-Barrón [3] did not know that the analytic approach is only applicable to interior points. We provide the following two minimum problems to illustrate the questionable solution procedure proposed by Chung and Cárdenas-Barrón [3]:

Example 1: \( f(x) = -x^2 + 8x - 9 \), for \( 3 \leq x \leq 6 \)
Example 2: \( g(x) = -x^2 + 8x - 6 \), for \( 1 \leq x \leq 5 \).

For Examples 1 and 2, if researchers follow the solution procedure proposed by Chung and Cárdenas-Barrón [3], based on the same condition \(-2x + 8 = 0\), then researchers try to find \( x^* = 6 \) for Example 1 and \( x^* = 1 \) for Example 2 which is impossible. Hence, the application of their results from interior points to boundaries is misleading.

For two boundaries \( a = 0 \), no shortage case, and \( b = Q \), no inventory case, researchers should consider different objective functions. Chung and Cárdenas-Barrón [3] neglected the local minimum along the boundary, \( B = Q \). Therefore, the analytic solution procedure of Chung and Cárdenas-Barrón [3] is deficient for two boundaries: \( a = 0 \), no shortage case, and \( B = Q \), no inventory case.

4. A Review of the Algebraic Method of Omar et al. [4] and Our Revisions

Omar et al. [4] claimed to find the minimum of
\[ \left( \frac{a}{x} \right)^2 + bx, \quad (8) \]
which occurs when \( x = \sqrt{ab} \), where \( a, b \) are constants.

We must point out that Omar et al. [4] overlooked the sign of \( a \) and \( b \) such that their solution method is questionable, and this will be explained in the following.

If \( a > 0 \) and \( b > 0 \), we assume \( f(x) = (ax/b) + bx \) for \( x > 0 \), then
\[ f(x) = \left( \sqrt{\frac{a}{x}} - \sqrt{bx} \right)^2 + 2\sqrt{ab}, \quad (9) \]
such that the minimum value is $2\sqrt{ab}$ and the minimum point is $x^* = \sqrt{ab}$. This is the case handled by Omar et al. [4].

For the other six cases, Case (a) $a < 0$; Case (b) $a = 0$ and $b < 0$; Case (c) $a = 0$ and $b = 0$; Case (d) $a = 0$ and $b > 0$; Case (e) $a > 0$ and $b < 0$; and Case (f) $a > 0$ and $b = 0$, except for Case (c), we will prove that the minimum value does not exist for a positive solution $x$, with $x > 0$.

For Case (a) with $a < 0$, we know $\lim_{x \to 0} f(x) = -\infty$ such that the minimum problem has no solution.

For Case (b) with $a = 0$ and $b < 0$, we obtain $\lim_{x \to 0} f(x) = -\infty$, and then we still cannot find a positive $x$ which attains the inferior value, $-\infty$. 

For Case (c) with $a = 0$ and $b = 0$, we find $f(x) \equiv 0$ which is the constant function with zero values.

For Case (d) with $a = 0$ and $b > 0$, we derive $\lim_{x \to 0} f(x) = 0$ such that we cannot find a positive $x$ which attains the inferior value, zero.

For Case (e) with $a > 0$ and $b < 0$, we derive that $\lim_{x \to -\infty} f(x) = -\infty$ such that the minimum problem has no solution.

For Case (f) with $a > 0$ and $b = 0$, we evaluate that $\lim_{x \to -\infty} f(x) = 0$, and then we still cannot find a positive $x$ which attains the inferior value, zero.

From the above discussion, we summarize our results in the next theorem.

**Theorem 1.** For the minimum problem of $f(x) = (a/x) + bx$ for $x > 0$, if $a > 0$ and $b > 0$, then the minimum value is $2\sqrt{ab}$ and the minimum point is $x^* = \sqrt{ab}$. For the other five cases related to signs of $a$ and $b$, except for the constant zero function, we verify that the minimum value does not exist.

From our Theorem 1, we provide a revision for Omar et al. [4].

Omar et al. [4] mentioned for a general cost function, 

$$C(x, y) = \frac{a}{x} + bx + \frac{1}{2} \left[ cy^2 + (v + wx)y \right].$$

With $u = b + c$, $v = d$, and $w = -2b$, they rewrote equation (10) as

$$C(x, y) = \frac{a}{x} + bx + \frac{1}{x} \left[ uy^2 + (v + wx)y \right].$$

They completed the square for the variable $y$, treating $x$ and $(v + wx)$ as a constant, to derive 

$$C(x, y) = \frac{a}{x} + bx + \frac{u}{2a} \left( y^2 + \frac{v + wx}{2a} \right)^2 - \frac{u}{x} \left( y + \frac{v + wx}{2a} \right)^2 = \left( \frac{4au - v^2}{4u} \right) \frac{1}{x} + \frac{4bu - w^2}{4u} + \frac{u}{x} \left( y + \frac{v + wx}{2a} \right)^2 - \frac{uw}{2a}.$$ 

(12)

Based on equation (12), Omar et al. [4] claimed 

$$x^* = \sqrt{\frac{4au - v^2}{4bu - w^2}}.$$ 

(13)

and 

$$y^* = \frac{v + wx^*}{2u}.$$ 

(14)

Omar et al. [4] applied their findings of equations (13) and (14) to solve the EOQ and EPQ models with linear and fixed backorder costs.

We must point out that Omar et al. [4] completely overlooked the restriction for their optimal solutions $x^*$ and $y^*$ that should satisfy the following conditions:

$$0 \leq y^* \leq x^*$$ 

(15)

and

$$0 < x^*.$$ 

(16)

Without checking the restrictions of equations (15) and (16), Omar et al. [4] implied a simple but questionable solution procedure.

We use the EOQ model with linear and fixed backorder costs to demonstrate our point of view. The objective function is

$$TC = \frac{C_p D}{Q} + \frac{C_o (Q - S)^2}{2Q} + \frac{C_k S^2}{2Q} + \frac{\pi DS}{Q}.$$ 

(17)

With $a = C_o D$, $b = C_p/2$, $c = C_k/2$, $d = \pi D$, $u = b + c = C_p + C_k$, $v = d$, and $w = -2b = -2C_p$, Omar et al. [4] derived 

$$Q^* = x^* \sqrt{\frac{4au - v^2}{4bu - w^2}} = \frac{4a(b + c) - d^2}{4(b + c) - 4b^2}.$$ 

(18)

$$S^* = y^* \sqrt{\frac{v + wx^*}{2u}} = \frac{2b x^* - d}{2(b + c)} = \frac{C_p x^* - \pi D}{C_p + C_k}.$$ 

(19)

Omar et al. [4] mentioned that “where the above results are similar with those derived by Sphicas [1].”

Under his notation, Sphicas [1] found, if $\sqrt{2KDh} > \pi D$, the optimal solution was derived as equations (18) and (19). On the other hand, when $\sqrt{2KDh} \leq \pi D$, the optimal solution becomes $Q^* = \sqrt{2KDh}$ and $S^* = 0$. We convert the findings of Sphicas [1] under the expressions of Omar et al. [4].

Hence, Sphicas [1] found that if

$$\sqrt{2C_o DC_p} > \pi D,$$ 

(20)

then the optimal solution was derived as equations (18) and (19). On the other hand, when $\sqrt{2C_o DC_p} \leq \pi D$, the optimal solution becomes $Q^* = \sqrt{2C_o DC_p}$ and $S^* = 0$. 


When \( 2C_oDC_p \leq \pi^2D^2 \), Omar et al. [4] did not obtain the optimal solution of \( Q' = \sqrt{2C_oD/C_p} \) and \( S^* = 0 \). Therefore, the assertion of Omar et al. [4] to claim that “where the above results are similar with those derived by Sphicas [1]” is a false statement. Consequently, we point out that the solution procedure proposed by Omar et al. [4] is deficient.

Next, we provide our improvement for Omar et al. [4].

We recall that Omar et al. [4] did not check the requirements of equations (15) and (16) for their proposed method. It means that Omar et al. [4] did not examine whether or not the term in the square root of equation (18) was positive:

\[
2C_oD(C_p + C_k) - \pi^2D^2 > 0, \tag{21}
\]

and the nonnegativity of \( S^* \) in equation (19),

\[
C_p \sqrt{\frac{2C_oD(C_p + C_k) - \pi^2D^2}{C_pC_k}} - \pi D \geq 0. \tag{22}
\]

If Omar et al. [4] checked two inequalities of equations (21) and (22), owing to the nonnegativity of equation (22), implying the positivity of equation (21), we only need to check the inequality of equation (20)

\[
2C_oD(C_p + C_k) - \pi^2D^2 \geq \frac{C_k\pi^2D^2}{C_p}. \tag{23}
\]

We can further simplify equation (23) as

\[
2C_oDC_p \geq \pi^2D^2. \tag{24}
\]

If we compare the findings of equations (21) and (24), we can know that the inequality of equation (24) can imply the inequality of equation (21). Hence, we can merge inequalities of equations (21) and (24) into a single relation as equation (24).

Remark 1. The result of Sphicas [1] in equation (20) and our derivation of equation (24) are not identical. When \( 2C_oDC_p = \pi^2D^2 \), we can plug \( 2C_oDC_p = \pi^2D^2 \) into equation (18) to derive that \( Q' = \sqrt{2C_oD/C_p} \), such that we further simplify equation (19) as \( S^* = 0 \) which is the same result proposed by Sphicas [1]. Therefore, the difference between equations (20) and (24) will not cause any problem.

5. Direction for Future Research

We point out the seemingly simple solution procedures of Omar et al. [4], Cárdenas-Barrón [2], and Chung and Cárdenas-Barrón [3]; all contained severely questionable results. Sphicas [1] already solved inventory models with linear and fixed backorder costs using the algebraic method to obtain the right partition of the solution structure as (i) \( \sqrt{2KDh} > \pi D \) (in other words, \( 2C_oDC_p > \pi D^2 \)) and (ii) \( \sqrt{2KDh} \leq \pi D \) (in other words, \( 2C_oDC_p \leq \pi D^2 \)). However, the motivation for his partition is not well explained such that Omar et al. [4] and Cárdenas-Barrón [2] tried to use the algebraic approach and Chung and Cárdenas-Barrón [3] attempted to use calculus to derive an easy method to find an optimal solution.

There are two related papers, Sphicas [12] and Luo [13], that are worthy to mention. Sphicas [12] claimed that he extended Sphicas [1]. However, Luo [13] claimed that Sphicas [12] did not provide a meaningful explanation for Sphicas [1], but also contained several other severe questionable results.

In the future, researchers should pay attention to the original work of Sphicas [1] for his solution procedure to the EOQ and EPQ inventory models with the algebraic method to find a reasonable explanation for his partition.

6. Conclusion

Sphicas [1] used the algebraic method to solve the optimal solution of the EOQ and EPQ models with linear and fixed backorder costs. However, there was no proper motivation for his partition such that Omar et al. [4], Cárdenas-Barrón [2], and Chung and Cárdenas-Barrón [3] tried to provide an easy solution approach to replace the complicated one proposed by Sphicas [1]. Recently, Lin [11] pointed out that Cárdenas-Barrón [2] contained severe questionable results. Following this trend, we show the seemingly simple derivations of Omar et al. [4] and Chung and Cárdenas-Barrón [3] also contained questionable findings. Using a simple but incomplete solution approach to replace a complicated but sound solution method is not an acceptable simplification in academia. We suggest researchers return to the original derivation of Sphicas [1] to find proper motivation for his partition.

Data Availability

The inventory model discussed in this paper is cited from Omar et al. [4] that was published in Computing and Industrial Engineering in 2010.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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